First-passage dynamics of linear stochastic interface models: numerical simulations and entropic repulsion effect

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Abstract. A fluctuating interfacial profile in one dimension is studied via Langevin simulations of the Edwards–Wilkinson equation with non-conserved noise and the Mullins–Herring equation with conserved noise. The profile is subject to either periodic or Dirichlet (no-flux) boundary conditions. We determine the noise-driven time-evolution of the profile between an initially flat configuration and the instant at which the profile reaches a given height $M$ for the first time. The shape of the averaged profile agrees well with the prediction of weak-noise theory (WNT), which describes the most-likely trajectory to a fixed first-passage time. Furthermore, in agreement with WNT, on average the profile approaches the height $M$ algebraically in time, with an exponent that is essentially independent of the boundary conditions. However, the actual value of the dynamic exponent turns out to be significantly smaller than predicted by WNT. This ‘renormalization’ of the exponent is explained in terms of the entropic repulsion exerted by the impenetrable boundary on the fluctuations of the profile around its most-likely path. The entropic repulsion mechanism is analyzed in detail for a single (fractional) Brownian walker, which describes the anomalous diffusion of a tagged monomer of the interface as it approaches the absorbing boundary. The present study sheds light on the accuracy and the limitations of the weak-noise approximation for the description of the full first-passage dynamics.
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**Keywords:** fluctuation phenomena, macroscopic fluctuation theory, extreme value, large deviations in non-equilibrium systems

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1. Introduction

In the present study, first-passage events arising in the Edwards–Wilkinson and the Mullins–Herring equation for various boundary conditions are investigated based on Langevin simulations. The obtained results for the spatio-temporal evolution of the profile are confronted to WNT—which has been discussed in a preceding paper [1]—and to reduced models of (fractional) Brownian walkers. In order to make the present study self-contained, the relevant models are briefly reviewed in the following.

We consider a one-dimensional interfacial profile \( h(x,t) \), defined on a domain of size \( L (0 \leq x \leq L) \) governed by either the Edwards–Wilkinson (EW) equation [2]

\[
\partial_t h = \eta \partial_x^2 h + \zeta, \tag{1.1}
\]

or the stochastic Mullins–Herring (MH) equation [3–5]

\[
\partial_t h = -\eta \partial_x^4 h + \partial_x \zeta. \tag{1.2}
\]

The noise \( \zeta \) is a Gaussian random variable of zero mean and correlation

\[
\langle \zeta(x,t)\zeta(x',t') \rangle = 2D \delta(x-x') \delta(t-t'). \tag{1.3}
\]

The ratio between the friction coefficient \( \eta \) and the noise strength \( D \) is related to the temperature via a fluctuation-dissipation relation (see below). The initial configuration is generally taken to be flat,

\[
h(x,t=0) = 0, \tag{1.4}
\]

and the profile is assumed to fulfill either periodic boundary conditions (p)

\[
h^{(p)}(x,t) = h^{(p)}(x + L, t), \tag{1.5}
\]

or Dirichlet boundary conditions (D)

\[
h^{(D)}(0,t) = 0 = h^{(D)}(L, t). \tag{1.6}
\]

When using the latter in conjunction with the MH equation, we additionally impose a no-flux condition at the boundaries,

\[
\partial_x^2 h^{(D')}(0,t) = 0 = \partial_x^2 h^{(D')}(L,t), \tag{1.7}
\]

which is indicated by a primed superscript (D’). For the MH equation with periodic or Dirichlet no-flux boundary conditions, the area under \( h \),

\[
A([h], t) \equiv \int_0^L dx \, h(x,t), \tag{1.8}
\]

henceforth called the ‘mass’, is conserved in time:

\[
A([h], t) = 0. \tag{1.9}
\]

In contrast, due to the presence of \( \zeta \) instead of \( \partial_x \zeta \) in equation (1.1), the mass is generally not conserved for the EW equation. In particular, for periodic boundary conditions, \( A([h], t) \) behaves diffusively at large times [5, 6], while for Dirichlet boundary conditions, \( \langle A([h], t) \rangle = 0 \) holds only as a time-average. In order to enforce equation (1.9) also
for EW dynamics with periodic boundary conditions, we consider in this case instead of $h^{(p)}$ the profile

$$
\tilde{h}^{(p)}(x, t) \equiv h^{(p)}(x, t) - A(h^{(p)}, t)/L, \quad \text{(EW)}
$$

(1.10)

which fulfills $A(\tilde{h}^{(p)}, t) = 0$. In the simulations discussed here, the prescription in equation (1.10) is applied at each time step. In order to simplify notation, the tilde will be dropped henceforth. We emphasize that equation (1.10) is rather artificial from a physical point of view and is imposed here mainly in order to compare the different models under the common condition $\langle A([h], t) \rangle = 0$.

We focus on the stochastic evolution of $\hat{h}(x, t)$ until the (random) first-passage time $T$, at which the profile has reached a given maximum height $M > 0$ for the first time:

$$
\max_x h(x, T) = M.
$$

(1.11)

The resulting (random) coordinate $x$ will be denoted in the following by $x_M$. Equation (1.11) implies an absorbing boundary condition for the profile at the height $M$ [7, 8]. The absorbing boundary condition acts over the whole domain $[0, L]$ and represents an impenetrable repulsive barrier to the profile (see also [9, 10]). For a highly correlated system, such as an profile in the presence of a mass constraint (equation (1.9)), analytical solutions of the first-passage problem are technically difficult and are available only in certain limits (see, e.g. [11–17]). The first-passage dynamics of the profile is thus addressed here via numerical simulation of equations (1.1) and (1.2), as well as by relying on reduced descriptions of the effective (fractional) Brownian dynamics of a ‘tagged monomer’, i.e. of $h(x_M, t)$. Note that, in the absence of an absorbing boundary, the stochastic process governed by equations (1.1) and (1.2) is fully Gaussian and underlies the well-studied phenomenon of interfacial roughening (see, e.g. [5, 18] as well as appendix F).

A tractable approximation to the first-passage problem discussed here is provided by weak-noise theory (WNT), also known as macroscopic fluctuation theory [17, 19, 20]. WNT of equations (1.1) and (1.2) has been discussed in detail in [1]. WNT represents a leading-order saddle-point approximation to the first-passage problem and describes the most-likely (‘optimal’) trajectory between two states. Specifically, within WNT, equation (1.11) is replaced by a height constraint, $h(x, T) = M$, and the first-passage time $T$ is taken as a free, but constant, parameter. Accordingly, WNT neither takes into account fluctuation-induced interactions with the absorbing boundary nor the fact that the first-passage time $T$ follows a certain probability distribution. However, it is shown here that, despite these limitations, WNT accurately captures the scaling functions of the averaged profile shape. A significant difference nevertheless arises in the value of the dynamic exponent characterizing the time-dependence of the first-passage profile. Based on insights from models of (fractional) Brownian walkers, this difference is argued to be a genuine consequence of the fluctuations around the most-likely path near an impenetrable boundary.

The first-passage problem of the MH equation discussed here and in [1] is, inter alia, physically relevant for noise-driven rupture of liquid films on substrates. So far, typically films have been considered which are either linearly unstable with respect to small fluctuations of the interface or where the rupture proceeds via hole nucleation in the presence of disjoining pressure [21–35]. Here and in [1], we focus on linearized
models in one dimension and assume absence of any deterministic force beside surface tension. In particular, we neglect the influence of disjoining pressure, which is experimentally justified for colloidal fluids \[36, 37\]. Accordingly, in this case film rupture is solely driven by noise. This situation is analogous to the noise-driven breakup of a liquid nanojet, which has been analyzed within WNT in \[38\] and studied experimentally and by simulations in \[39–41\]. Physical realizations of one-dimensional interfaces occur, e.g. in lipid bilayer membranes below their demixing transition \[42, 43\]. The extension of the present study to two-dimensional interfaces as well as the incorporation of an interface potential are reserved for future work.

2. Model and simulations

2.1. General aspects

In the following, a number of relevant properties of the considered models are summarized. It is useful to note that, dimensionally \[\eta \sim [L]^z/[T]\], \([D] \sim [M]^2[L]^{-1}/[T]\], \([D/\eta] = [M]^2/[L]\), where \([M]\), \([L]\), and \([T]\) represent the fundamental dimensions of height, length, and time, respectively. In order to facilitate the analysis of the first-passage dynamics, we recall the phenomenology of interfacial roughening (see appendix F for details). To this end, we consider equations (1.1) and (1.2) in the absence of an absorbing boundary condition. In this situation, one can analytically determine the trajectory \(h(x,t)\) as well as the roughness \(\langle |\delta h(x,t)|^2 \rangle\), where \(\delta h(x,t) \equiv h(x,t) - h(x,0)\) is the relative height fluctuation. We consider either a flat initial condition, \(h(x,0) = 0\), or a thermal one. In the latter case, the roughness is calculated as an average over an ensemble of equilibrium profiles \(h(x,0)\). Since, for Dirichlet boundary conditions, the variance depends on position, we evaluate in the following \(\langle h(x,t) \rangle\) at a fixed location \(x_M\) far from the boundaries (the precise value of \(x_M\), however, is irrelevant for the general scaling behavior). The roughness resulting from equations (1.1) and (1.2) is characterized by three regimes \[5, 6, 18, 44–54\]:

\[
\langle |\delta h(x_M,t)|^2 \rangle \sim \begin{cases} t, & t \lesssim \tau, \\ t^{1/z}, & \tau_x \lesssim t \lesssim \tau, \\ \text{const}, & t \gtrsim \tau, \end{cases}
\]

where

\[
z \equiv \begin{cases} 2, & \text{EW equation,} \\ 4, & \text{MH equation,} \end{cases}
\]

is the dynamic index and \(\tau\) denotes the roughening time. The latter coincides with the relaxation time of the (eigen-)mode with the largest ‘wavelength’ that can be accommodated in the system:

\[
\tau = \left( \frac{L}{\omega_1} \right)^z, \quad \omega_1 \equiv \begin{cases} 2\pi, & \text{periodic,} \\ \pi, & \text{standard Dirichlet,} \\ 4.73 \ldots & \text{Dirichlet no flux boundary conditions.} \end{cases}
\]
Dirichlet no-flux boundary conditions are considered only for the MH equation \((z = 4)\), in which case the value \(\omega_1 \simeq 4.73\) represents the smallest positive solution of the associated eigenvalue equation, \(\cos \omega \cosh \omega = 1\) (see appendix E). Within WNT, \(\tau\) is in fact the characteristic time scale for the development of the first-passage profile in an equilibrium system (see [1]). This property is confirmed by the present simulations. Furthermore, \(\tau_\times\) in equation (2.1) represents a cross-over time related to the presence of a microscopic cutoff. While \(\tau_\times = 0\) in the continuum limit, for a one-dimensional lattice one has (see appendix G)

\[
\tau_\times = \tau \left( \frac{\omega_1}{\omega_{k_\times}} \right)^z ,
\]

with \(\omega_{k_\times}^{(o)} = 2L/\Delta x\) and \(\omega_{k_\times}^{(D)} = L/\Delta x\) for periodic and (standard) Dirichlet boundary conditions, respectively, where \(\Delta x\) is the lattice spacing. For Dirichlet no-flux boundary conditions, a numerical analysis yields \(k_\times \gtrsim 0.5L/\Delta x\), with the actual value depending on the particular problem under study (see appendix G.2; the corresponding value of \(\omega_{k_\times}^{(D)}\) follows from the eigenvalue equation in equation (E.9)).

According to equation (2.1), a tagged monomer of the profile exhibits standard Brownian diffusion at early times, followed by a subdiffusive regime characterized by a Hurst exponent \([55, 56]\)

\[
H = \frac{1}{2z} .
\]

For a sufficiently large system, the latter regime dominates the roughening behavior. A tagged monomer thus diffuses the distance \(M\) approximately within the time (see equation (F.28))

\[
\tau_D = \frac{M^{1/H}}{2[(2/\pi)\Theta \Gamma(1-z^{-1})]^z1/2} ,
\]

where \(\Theta\) is the temperature (see equation (2.8) below). The numerical prefactors in equation (2.6) arise from a detailed analysis (see appendix F) along with the two-time correlation function of the relative height fluctuations \(\delta h(x,t)\) (see equation (F.26a)),

\[
\langle \delta h(x,t)\delta h(x,s) \rangle_{\text{flat}} \simeq (2/\pi)\eta^{1/2} \Gamma(1-z^{-1}) \Theta \left[(t+s)^{1/2} - |t-s|^{1/2}\right] , \tag{2.7a}
\]

\[
\langle \delta h(x,t)\delta h(x,s) \rangle_{\text{th}} \simeq (2/\pi)\eta^{1/2} \Gamma(1-z^{-1}) \Theta \left[t^{1/2} + s^{1/2} - |t-s|^{1/2}\right] , \tag{2.7b}
\]

corresponding to flat and thermal initial conditions, respectively. The Gaussian stochastic process described by equation (2.7b) is a fractional Brownian motion (fBM) \([57–60]\).

For times \(t \geq \mathcal{O}(\tau)\), all memory of the initial condition has been lost and the interface has reached its equilibrium roughness. In this regime, the profile \(h(x,t)\) follows a time-independent joint Gaussian distribution with a temperature (see appendix B)

\[
\Theta = \frac{D}{2\eta} .
\]

\(^1\) Note that, occasionally, different definitions of fractional Brownian motion are used in the literature, see, e.g. [98, 99].
This equation represents a fluctuation-dissipation relation for equations (1.1) and (1.2). For periodic boundary conditions, the one-point variance $\langle h(x,t)^2 \rangle$ is independent of position $x$ and is in equilibrium given by (see equation (B.4))

$$\langle |h^{(p)}|^2 \rangle = \frac{1}{6} \Theta L.$$  \hfill (2.9)

For Dirichlet boundary conditions, the equilibrium variance at the mid-point $x = L/2$ is given by (see equations (B.7) and (B.9))

$$\langle |h^{(D)}(L/2,t)|^2 \rangle = \frac{1}{2} \Theta L \quad \text{(2.10a)}$$

and

$$\langle |h^{(D)}(L/2,t)|^2 \rangle = \frac{7}{8} \Theta L, \quad \text{(2.10b)}$$

in the cases without and with an additional mass constraint (equation (1.9)), respectively.

The first-passage dynamics is generally distinct in the transient and the equilibrium regime, which, within WNT, correspond to $T/\tau \ll 1$ and $T/\tau \gg 1$, respectively. However, for the actual stochastic equations (1.1) and (1.2), the first-passage time $T$ is a random quantity and $T/\tau$ is therefore not an appropriate parameter. We thus define instead the \textit{reduced height}

$$\mathcal{M} \equiv \frac{M}{\sqrt{\Theta L}} \sim \frac{M}{(h^2)^{1/2}}, \quad \text{(2.11)}$$

which is essentially the ratio between the maximum height $M$ and the equilibrium variance of the profile (see equations (2.9) and (2.10)). For $\mathcal{M} \ll 1$, the profile is likely to reach the height $M$ within its roughening phase, whereas for $\mathcal{M} \gg 1$, the profile is fully equilibrated before the first-passage event occurs. The definition in equation (2.11) is consistent with the fact that the transient regime corresponds to diffusion times $\tau_D \ll \tau$ (see equations (2.3) and (2.6)). We henceforth take $\tau_D$ and $\tau$ as the fundamental time scales for the first-passage dynamics in the transient and equilibrium regimes, respectively.

\subsection*{2.2. Implementation}

The stochastic equations in (1.1) and (1.2) are discretized on a one-dimensional lattice comprising $N = L/\Delta x$ nodes with spacing $\Delta x$ and are solved using a standard forward Euler scheme with time step $\Delta t$ (see, e.g. [11, 61]):

$$h(i, t + \Delta t) = h(i, t) - \eta \Delta t (-\nabla^2)^{3/2} h(i, t) + \sqrt{2D \Delta t} \nabla^{2-1} \zeta(i, t), \quad \text{(2.12)}$$

with $i = 0, \ldots, N - 1$. The noise variables $\zeta(i)$ are uncorrelated Gaussian variables of zero mean and unit variance, $\langle \zeta(i, t) \zeta(j, t') \rangle = \delta_{ij} \delta_{t,t'}$. The discretized forms of the derivative operators $\nabla^2$ and $\nabla^4$ as well as further technical details on the numerical simulations are provided in appendix G. In the simulations, a profile is generally initialized in a flat configuration (equation (1.4)). If an equilibrated system is required at

\footnote{In fact, the mean first-passage time $\langle T \rangle$ is a complicated function of the system parameters and is not exactly known for the models considered here.}
the first-passage event, the height $M$ is chosen sufficiently large such that $T \gg \tau$ (see also section 3). Figure 1 exemplifies a typical time evolution of a profile governed by equation (1.1) close to the first-passage event.

The main object of the present study is the averaged profile $\langle h(x, \delta t) \rangle$, which is obtained in the following way: let $\{h^{(s)}(x,t)\}, s = 1, \ldots, S$ be an ensemble of profiles obtained from a total number of $S$ simulations. Let $T^{(s)}$ be the corresponding first-passage time, such that $h^{(s)}(x_M^{(s)}, T^{(s)}) \geq M$ for the first time for any $x_M^{(s)}$. The averaged profile is defined as

$$\langle h(x, \delta t) \rangle \equiv \frac{1}{N(T \geq \delta t)} \sum_{s=1}^{N(T \geq \delta t)} h^{(s)}(x - X^{(s)}, T^{(s)} - \delta t), \quad (2.13)$$

where $N(T \geq \delta t) \leq S$ denotes the number of profiles for which $T^{(s)} \geq \delta t$. Note that the averaged profile is a function of the time variable $\delta t$, which is defined such that the first-passage event corresponds to $\delta t = 0$, i.e. $\langle h(x_M, 0) \rangle = M$. Depending on the model and the regime considered, we set either $X^{(s)} \equiv 0$ or $X^{(s)} \equiv x_M^{(s)} - L/2$, where the latter choice induces a shift of the location of the maximum $x_M^{(s)}$ to the center $L/2$.

The finite value of the time step in equation (2.12) gives rise to two potentials errors: first, a profile can ‘overshoot’ the boundary, i.e., instead of equation (1.11) one finds $h^{(s)}(x_M, T) = M + \delta M^{(s)}$ with $\delta M^{(s)} > 0$. This effect is taken into account by subtracting the individual $\delta M^{(s)}$ on the r.h.s. of equation (2.13). While the overshoot leads to slight changes of the observed scaling of the peak $\langle h(x, \delta t) \rangle$ for small $\delta t$, it turns out to not significantly affect the intermediate asymptotics. Second, there is a certain finite probability that between two discrete time steps the profile has crossed the boundary [62, 63]. It has been checked by decreasing the time step in a few cases that the results here are essentially insensitive to this effect.

3. First-passage time

Before addressing the profile dynamics, we briefly turn to the first-passage time $T$, i.e. the time at which the profile, starting from the initial configuration in equation (1.4), reaches the given height $M$ for the first time. We remark that related first-passage problems of linear interface and polymer models have been studied previously in, e.g. [11, 15, 64–66]. Closed analytical expressions are, however, available only within certain approximations [12, 13, 16].

The first-passage distribution $P_1(T)$ is discussed separately in appendix A. For the models considered here, we find that $P_1(T)$ decays either exponentially or algebraically for large $T$, with an exponent smaller than $-2$. Consequently, the mean first-passage time

$$\langle T \rangle = \int_0^\infty dT T P_1(T) \quad (3.1)$$

As will be justified in the corresponding sections, we set $X^{(s)} = x_M^{(s)} - L/2$ generally in the transient regime and for periodic boundary conditions also in the equilibrium regime. For Dirichlet boundary conditions, we set $X^{(s)} = 0$ in the equilibrium regime.
is finite. In order to obtain an estimate for $\langle T \rangle$ in the transient regime, we recall that a tagged monomer traverses the distance between $h = 0$ and $M$ within a time of order of $t^\alpha$, with $\alpha = 1/(2z)$. Specifically, based on equation (2.6) one expects

$$\langle T \rangle \sim \frac{M^{1/\alpha}}{\Theta^z}. \quad (3.2)$$

However, instead of the naive value $\alpha = 1/(2z)$, we use in equation (3.2) the effective values $\alpha_{\text{EW}} \simeq 0.27 - 0.3$ and $\alpha_{\text{MH}} \simeq 0.16 - 0.18$ in the case of EW and MH dynamics, respectively, which coincide with the value of the exponent characterizing the averaged path (see below). As demonstrated in figure 2(a), the scaling behavior of the mean first-passage time in the transient regime is well captured by the scaling relation (3.2)$^4$.

In the equilibrium regime, equation (3.2) does not provide a satisfactory description of the mean first-passage time. Instead we recall that the steady-state probability distribution of the profile is Gaussian with a single-site variance given in equations (2.9) and (2.10). We can thus consider a tagged monomer $h(x_M,t)$ as a fractional Brownian walker ($H = 1/(2z) < 1/2$, see equation (2.7b)) in an effective harmonic potential $U(h) \simeq h^2/\Theta L$. To leading order, the monomer dynamics can be approximated by a Markovian Brownian process ($H = 1/2$), such that the present first-passage problem reduces to the well-known Kramers escape problem [7, 67]. Accordingly, the mean-first-passage time of a tagged monomer in the equilibrium regime is expected to behave as

$^4$ We remark that, formally, using a value of $\alpha \neq 1/(2z)$ requires a factor with dimension $|M|^{2z-1/\alpha}$ to be present on the r.h.s. of equation (3.2).
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\[ \langle T \rangle \simeq c_1 L^z \eta^{-1} \exp \left( c_2 \frac{M^2}{\Theta L} \right), \]  

(3.3)

where \( c_1 \) and \( c_2 \) are fit parameters (independent of \( M, L \) and \( \Theta \))\(^5\). Essentially the same form as in equation (3.3) has been obtained in [68] for a fBM in a parabolic potential as well as in [16] in the case of a Rouse polymer chain. As demonstrated in figure 2(b), the simulation data pertaining to each model falls onto distinct master curves described by equation (3.3).

\[ \langle T \rangle \simeq c_1 L^z \eta^{-1} \exp \left( c_2 \frac{M^2}{\Theta L} \right), \]  

Figure 2. Mean first-passage time \( \langle T \rangle \) for a profile with a flat initial configuration (equation (1.4)) in (a) the transient and (b) the equilibrium regime, corresponding to \( M \ll 1 \) and \( M \gg 1 \), respectively (see equation (2.11)). In (a), effective values \( \alpha_{\text{EW}} = 0.29 \) and \( \alpha_{\text{MH}} = 0.17 \) are used for the exponent \( \alpha \) in the scaling relation (3.2). Time is expressed in units of the simulation time step \( \Delta t \). In (b), the axes are scaled according to equation (3.3). The bulk dynamic equation and the boundary conditions are indicated by the labels near the data (purple solid symbols connected by solid lines: EW equation with periodic bc.s; black open symbols connected by dashed lines: EW equation with Dirichlet bc.s; blue solid symbols connected by dashed lines: MH equation with periodic bc.s; orange open symbols connected by solid lines: MH equation with Dirichlet no-flux bc.s).

4. Edwards–Wilkinson equation

We now turn to the first-passage dynamics of a profile governed by equation (1.1) with periodic and Dirichlet boundary conditions.

4.1. Summary of WNT

Before discussing the simulation results, we summarize a few relevant predictions of WNT of the EW equation (see [1] for details). The following expression for \( h(x, \delta t) \)

\[ \text{Prefactor of the exponential in equation (3.3) can be motivated based on dimensional considerations: noting that } \eta \sim L^z/T \text{ and } [D] \sim M^2 L^{-1}/T, \text{ a dimensionally consistent ansatz for the prefactor is given by } L^a M^b \eta^{-1} \exp \left( \frac{M^2}{\Theta L} \right), \text{ with } a = z - b/2. \text{ It turns out that a satisfactory scaling collapse of the data is possible with the simplest choice, } b = 0, \text{ which implies equation (3.3).} \]
are to be understood as the leading-order contribution to the averaged profile \( h(x, \delta t) \). Note that, differently from [1], we use \( \delta t = T - t \) as the time variable. Within WNT, the first-passage time \( T \) is a fixed parameter and the transient and the equilibrium regime are distinguished by the value of \( T/\tau \). In the transient regime \( (T \ll \tau) \), a scaling profile at time \( \delta t = 0 \) results from WNT as

\[
h(x, \delta t = 0) \bigg|_{T\ll\tau} = M H \left( \frac{x - L/2}{(2T)^{1/z}} \right), \quad z = 2,
\]

with the scaling function

\[
H(\xi) = \exp \left( -\frac{\xi^2}{4} \right) + \frac{1}{2} \sqrt{\pi} |\xi| \left[ \text{erf} \left( \frac{|\xi|}{2} \right) - 1 \right].
\]

For \( 0 < \delta t \ll T \), one obtains the dynamic scaling profile

\[
h(x, \delta t) \bigg|_{T\ll\tau} = M - M \left( \frac{\delta t}{2T} \right)^{1/z} \tilde{H} \left( \frac{x - L/2}{\delta t^{1/z}} \right), \quad z = 2,
\]

with the scaling function

\[
\tilde{H}(\xi) = \exp \left( -\frac{\xi^2}{4} \right) + \frac{1}{2} \sqrt{\pi} \xi \text{erf} \left( \frac{\xi}{2} \right).
\]

When applying equations (4.1) and (4.3) to simulation results, we consider the quantity \( T \) as a fit parameter. In the equilibrium regime \( (T \gg \tau) \) for \( \delta t = 0 \), one finds the following asymptotic first-passage profiles for periodic and Dirichlet boundary conditions, respectively:

\[
h^{(p)}(x, \delta t = 0) \bigg|_{T\to\infty} = M - 6 \left| \frac{x}{L} - \frac{1}{2} \right| + 6 \left( \frac{x}{L} - \frac{1}{2} \right)^2,
\]

\[
h^{(D)}(x, \delta t = 0) \bigg|_{T\to\infty} = M - \left| 1 - \frac{2x}{L} \right|.
\]

These profiles attain their maximum at \( x_M = L/2 \). They follow readily from the constrained minimization of the corresponding equilibrium free energy. For times \( 0 < \delta t \ll T \), one finds a dynamic scaling form,

\[
h(x, \delta t) \bigg|_{T\gg\tau} \simeq M - M(\delta t)^{1/z} \Gamma(1 - 1/z) \tilde{H} \left( \frac{x - L/2}{\delta t^{1/z}} \right), \quad z = 2,
\]

with the same scaling function as in equation (4.4). Note that, unless otherwise indicated, the above scaling forms apply to all boundary conditions considered here. Exact analytical expressions for the profile \( h(x, t) \) obtained within WNT can be found in [1] and are not repeated here.

We emphasize that the above expressions pertain to a continuum system. As shown in [1], the presence of a microscopic cutoff (e.g. a lattice constant) modifies the dynamics for times \( \delta t \ll \tau_x \), where \( \tau_x \) is the crossover time in equation (2.4). Upon taking this...
effect into account, the time-evolution of the profile \( h(x, \delta t) \) at \( x = x_M \) is given within WNT by

\[
1 - h(x_M, \delta t)/M \propto \begin{cases} \delta t, & \delta t \lesssim \tau_x, \\ \delta t^{1/z}, & \delta t \gtrsim \tau_x. \end{cases}
\] (4.7)

This result is independent of the boundary conditions and applies to both the transient and equilibrium regime (see equation (4.3) and (4.6)).

### 4.2. Periodic boundary conditions

We now turn to the discussion of the first-passage properties of a profile governed by the EW equation (1.1) with periodic boundary conditions. We recall that, in this case, the constraint of zero mass (equation (1.9)) is imposed via equation (1.10) at each time step in the simulation. (Within WNT, this constraint is reflected by the absence of the zero mode in the series solution for the profile, see [1]). Figure 3 illustrates the spatial shape of the averaged profile at the first-passage event, \( \langle h(x, \delta t = 0) \rangle \), for various reduced heights \( M^6 \). The asymptotic scaling profiles predicted by WNT in the transient and the equilibrium regime (equations (4.1) and (4.5a) solid lines) agree well with the numerical results in the limits \( M \ll 1 \) and \( M \gg 1 \). According to equation (4.1), the analytical profile in the transient regime still depends on \( T \), which is considered here as a fit parameter and effectively controls the width of the profile. Furthermore, since equation (4.1) is obtained by neglecting the mass constraint (equation (1.9)), it applies only to an inner region of the profile. In contrast, the full solution of WNT provides an accurate description for \( M \lesssim 1 \) also in the outer regions, as is illustrated below. Part of the remaining discrepancies between the analytically and numerically obtained profiles in figure 3 can be attributed to the fact that WNT neglects fluctuations around the saddle point solution. Such fluctuations can give rise to an effective repulsion from the boundary. We will return to this aspect in section 6.

In figures 4 and 5, the spatio-temporal evolution of the averaged profile approaching the first-passage event \( \langle h(x = x_M, 0) \rangle = M \) is illustrated in the transient and equilibrium regimes, respectively. As observed in figures 4(a) and 5(a)\(^7\), both in the transient and the equilibrium regime, the peak of the profile, \( \langle h(x_M, \delta t) \rangle \) (with \( x_M = L/2 \)), approaches the maximum height \( M \) algebraically,

\[
M - \langle h(x_M, \delta t) \rangle \propto \delta t^\alpha.
\] (4.8)

For times \( \delta t \) larger than a cross-over time \( \tau_x \) (see below), one obtains an exponent

\[
\alpha \simeq 0.28 - 0.3,
\] (4.9)

while \( \alpha = \alpha_0 \simeq 0.5 \) for \( \delta t \lesssim \tau_x \). The extent of the intermediate asymptotic regime described by equation (4.9) grows upon increasing the system size \( L \), as illustrated in figure 5(a). Notably, the above values of the exponent \( \alpha \) differ significantly from the values \( \alpha_{\text{WNT}} = 1/z = 1/2 \) and \( \alpha_{0,\text{WNT}} = 1 \) predicted by WNT in equation (4.7). An

\[^6\]Simulations in the equilibrium regime are found to be computationally feasible only for reduced maximum heights of \( M \sim O(1) \), since the probability equation (B.1) to observe significantly larger height fluctuations becomes exponentially small, see appendix B.

\[^7\]The data underlying the time-evolution of the peak shown here and in the other figures occasionally stem from two separate simulations, which have been performed with identical parameters but different time resolutions.
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Figure 3. Averaged profile $\langle h(x, \delta t = 0) \rangle$ (broken lines) at the first-passage event, as obtained from simulations of the EW equation (1.1) with periodic boundary conditions and for various reduced heights $M$ (equation (2.11)). A constraint of zero mass (equation (1.9)) is imposed via equation (1.10). The solid lines represent the asymptotic scaling profiles predicted by WNT in the transient regime ($M \to 0$, equation (4.3), sharply peaked curve) and in the equilibrium regime ($M \to \infty$, equation (4.5a), broadly peaked curve). In the former case, the parameter $T$ results from a fit as $T \simeq 1.5 \times 10^{-9} \tau^{(p)}$.

The explanation of these findings, which are analogously obtained also for the other models considered in this study, is provided in section 6. As seen in figure 5, in the equilibrium regime, the first-passage evolution of the profile happens essentially within a timescale of the order of $\tau^{(p)}$ (see equation (2.3)), as predicted by WNT. In the transient regime, the characteristic time scale is taken here to be the effective diffusion time $\tau_{\text{eff}}$. The latter is defined by equation (2.6), using for the dynamic exponent $z$ the effective value $1/(2\alpha) \simeq 1.7$ with $\alpha$ given in equation (4.9). Using instead the value $z = 2$ predicted by WNT leads to a significant underestimation of the first-passage time scale. The non-vanishing cross-over time $\tau_x$ arises due to the finite lattice spacing $\Delta x$ in the simulations. In agreement with the numerical data, equation (2.4) predicts $\tau_x / \tau^{(p)} \sim 10^{-5}$ ($L = 1000\Delta x$) and $\tau_x / \tau^{(p)} \sim 2 \times 10^{-4}$ ($L = 200\Delta x$) for the two system sizes considered in figure 5(a).

In figures 4(b) and 5(b), the shape of the averaged profile is illustrated for various times $\delta t$ (solid lines). In figure 5(b) the dashed lines represent the time-dependent profiles obtained within WNT (equation I-(2.19)). Since the actual time-dependence of $\langle h(x, t) \rangle$ differs from the prediction of WNT due to a different value of the dynamic exponent $\alpha$, analytical profiles do in general not match the numerical solutions well for $\delta t > 0$. These discrepancies are found to be more severe in the transient regime figure 4(b), where we show only the scaling profile given in equation (4.1) (dashed line).

In figures 4(c) and 5(c), the dynamic scaling behavior asymptotically predicted by WNT (see equations (4.3) and (4.6)) is tested. To this end, the profile height $\langle h \rangle$ and the coordinate $x$ are rescaled accordingly and the scaling function $cH$ in equation (4.4) is fitted via the parameter $c$. In order to account for the renormalization of the dynamic exponent $\alpha$, we use for $1/z$ in equations (4.3) and (4.6) an effective value which is close to the value for $\alpha$ reported in equation (4.9). As shown in figures 4(c) and 5(c), this

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8 While within WNT, the exponent $z$ is identified with $1/\alpha$ (see, e.g. equation (4.7)), beyond WNT, it turns out that $z$ has to be identified with a value close to $1/(2\alpha)$. This fact is also used in the definition of $\tau_{\text{eff}}$. See section 6 for further discussion.

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results in a satisfactory matching (in an inner region) of the numerical profiles with the scaling function $\tilde{H}$ in equation (4.4) (dashed line). The outer parts of the profiles deviate from the scaling function due to the influence of the boundary conditions.

### 4.3. Dirichlet boundary conditions

We now turn to the rare event dynamics of a profile governed by the EW equation with standard Dirichlet boundary conditions (equation (1.6)). We recall that, in this case, the mass constraint in equation (1.9) is not fulfilled by the individual realizations of the profile. The probability distribution $P_1(x_M)$ of the location $x_M$ of the first-passage event...
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Figure 5. Averaged profile \( \langle h(x, \delta t) \rangle \) obtained for the EW equation (1.1) with periodic boundary conditions in the equilibrium regime \((\mathcal{M} \gg 1)\). The first passage of the height \( M \) occurs at the time \( \delta t = 0 \). Utilizing translational invariance, the individual profiles obtained from simulation are shifted such that the height \( M \) is reached at location \( x_M = L/2 \). Time is normalized to the relaxation time scale \( \tau \) (equation (2.3)). (a) Time-evolution of the peak of the profile, \( \langle h(x_M, \delta t) \rangle \), for two different system sizes (in units of the lattice spacing \( \Delta x \)). In the intermediate asymptotic regime one has \( M - \langle h(x_M, \delta t) \rangle \propto \delta t^\alpha \) with \( \alpha \approx 0.28 \). (b) Spatio-temporal evolution of the averaged profile. The solid curves represent the profiles obtained from numerical simulations with \( L = 400\Delta x, \mathcal{M} \approx 1.6 \), while the dashed curves indicate the prediction of WNT (see equation (2.19) in [1]). (c) Test of the dynamic scaling behavior of \( \langle h(x, \delta t) \rangle \) as predicted by WNT according to equation (4.6), using a value of \( 1/z \approx 0.3 \). The dashed curve represents the scaling function \( \tilde{c}\mathcal{H} \) in equation (4.4), with a prefactor \( c \approx 1.6 \) determined from a fit.

(see equation (1.11)) is shown in figure 6 for various reduced heights \( \mathcal{M} \). For \( \mathcal{M} \ll 1 \), \( \mathcal{P}_1 \) is essentially flat, in agreement with the prediction of WNT in the transient regime (see [1]). For \( \mathcal{M} \gg 1 \), instead, the first-passage event is most likely to occur at the center of the system. In this regime, \( \mathcal{P}_1 \) can be well fitted by the analytical expression reported in equation I-(2.16), using a value of \( \eta M^2/DL \approx 1.8 \) and \( T/\tau^{(D)} \gg 1 \) (the precise value of the latter parameter is immaterial since \( \mathcal{P}_1 \) becomes independent of it provided it is sufficiently large). In the crossover region between the transient and the equilibrium regime, \( \mathcal{P}_1 \) depends within WNT on both \( T/\tau^{(D)} \) and \( \eta M^2/DL \) and, therefore, a fit is less meaningful. Differently from WNT, \( \mathcal{P}_1 \) develops two maxima near the boundaries for \( \mathcal{M} \sim \mathcal{O}(1) \).
In figures 7 and 8, the spatio-temporal evolution of the averaged profile in the transient and equilibrium regimes, respectively, is illustrated. Since the distribution \( P_1(x_M) \) of the first-passage location is flat in the transient regime, the averaged profiles shown in figure 7 are obtained by shifting each realization such that the first-passage event occurs at \( x_M = L/2 \) (see equation (2.13)). Since the profile is strongly localized in the transient regime, such a shift does not significantly affect its averaged shape. As shown in figure 7(b), a fit via the parameter \( T \) of the asymptotic profile of WNT reported in equation (4.1) yields satisfactory agreement with the data. In the equilibrium regime, the averaged profile is computed according to equation (2.13) without a shift (\( X(s) = 0 \)). In this case, the finite width of \( P_1(x_M) \) (see figure 6) is reflected by the rather strong deviation of \( \langle h(x, \delta t) \rangle \) from the prediction of WNT (equation (4.5b), dashed lines in figure 8(b)) as well as by the fact that \( \langle h(x_M, \delta t = 0) \rangle < M \). These deviations diminish upon increasing \( M \).

As shown in figures 7 and 8, both in the transient and equilibrium regime, the peak \( \langle h(x_M, \delta t) \rangle \) follows the same algebraic time-evolution as in equation (4.8) and is characterized by two distinct dynamic exponents. Similarly to periodic boundary conditions (see equation (4.9)), we obtain \( \alpha \simeq 0.29 \) and \( \alpha = \alpha_0 \simeq 0.5 \) for the values of the dynamic exponent at late and early times \( \delta t \), respectively, which are different from the prediction of WNT in equation (4.7). Despite this discrepancy, the time-dependent averaged profiles of WNT qualitatively match the simulation results in the equilibrium regime (see figure 8(b)). Deviations are more significant in the transient regime (not shown), although the qualitative behavior agrees with WNT.

In figures 7(c) and 8(c), the dynamic scaling behavior predicted in equations (4.3) and (4.6), respectively, is tested. Using an effective value of \( 1/z = \alpha \simeq 0.29 \) for the dynamic exponent, a satisfactory fit of the numerical profiles with the scaling function

\[ \langle h(x_M, \delta t) \rangle \sim \left( \frac{\delta t}{\tau(z)} \right)^{\alpha/z} \]

is obtained, as shown in figures 7(c) and 8(c).
in equation (4.4) is obtained. The agreement between WNT and simulations generally improves as $\delta t \to 0$.

5. Mullins–Herring equation

We proceed with the discussion of the first-passage dynamics for the MH equation (1.2). For the considered boundary conditions, the mass (equation (1.8)) is conserved in time
and, in fact, $A(h, t) = 0$ owing to the initial condition in equation (1.4). Due to the larger value $z = 4$ of the dynamic index (see equation (2.3)), simulations are more time-demanding than for the EW equation. Moreover, it turns out that the cross-over regions between the different asymptotic regimes are broader, making it more difficult to identify clear power-laws.

5.1. Summary of WNT

Before proceeding to the simulation results, we summarize the essential predictions of WNT (see [1], as well as [17] in the case of periodic boundary conditions). As before, we use $\delta t = T - t$ as the time variable and the following expressions for $h$ are to be understood as the leading-order contributions to the averaged profile $\langle h \rangle$. Asymptotically for

\[ 1 - \langle h(X, \delta t) \rangle / M \propto \delta t^\alpha \]

with $\alpha \approx 0.29$. (b) Spatio-temporal evolution of the averaged profile. The solid curves represent the profiles obtained from numerical simulations, while the dashed curves indicate the prediction of WNT (see equation I-(2.21)). (c) Test of the dynamic scaling behavior of $\langle h(x, t) \rangle$ as predicted by WNT in equation (4.6), using a value of $1/z \approx 0.29$. The dashed curve represents the scaling function $cH$ in equation (4.4), with a prefactor $c \approx 1.5$ determined from a fit. In order to properly exhibit the scaling behavior, in panels (a) and (c) the individual profiles are shifted before averaging such that $h(s)(L/2, T(s)) = M$ (see equation (2.13)).

Figure 8. Averaged profile $\langle h(x, \delta t) \rangle$ for the EW equation (1.1) with Dirichlet boundary conditions in the equilibrium regime ($\mathcal{M} \gg 1$). The first passage of the height $M$ occurs at the time $\delta t = 0$. Time is normalized to the relaxation time $\tau(D)$ (equation (2.3)). (a) Time-evolution of the peak of the profile, $\langle h(x_M, \delta t) \rangle$, which exhibits an intermediate asymptotic regime $M - \langle h \rangle \propto \delta t^\alpha$ with $\alpha \approx 0.29$. (b) Spatio-temporal evolution of the averaged profile. The solid curves represent the profiles obtained from numerical simulations, while the dashed curves indicate the prediction of WNT (see equation I-(2.21)). (c) Test of the dynamic scaling behavior of $\langle h(x, t) \rangle$ as predicted by WNT in equation (4.6), using a value of $1/z \approx 0.29$. The dashed curve represents the scaling function $cH$ in equation (4.4), with a prefactor $c \approx 1.5$ determined from a fit. In order to properly exhibit the scaling behavior, in panels (a) and (c) the individual profiles are shifted before averaging such that $h(s)(L/2, T(s)) = M$ (see equation (2.13)).
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$T \to 0$ in the transient regime, one obtains the following static scaling profile at the first-passage event:

$$h(x, \delta t = 0)|_{T \ll \tau} = M \mathcal{H} \left( \frac{x - L/2}{(2T)^{1/z}} \right), \quad z = 4,$$

(5.1)

with the scaling function

$$\mathcal{H}(\xi) = _1F_3 \left( \begin{array}{c} -1/4, 1/2, 3/4 \\ 1/4, 3/4, 3/4 \end{array} \right) + \xi^2 \frac{\Gamma(1/4)}{8\Gamma(1/2)} _1F_3 \left( \begin{array}{c} 1/4, 5/4, 3/4 \\ 1/4, 3/4, 3/4 \end{array} \right) - \frac{\pi}{2\Gamma(1/4)} |\xi|,$$

(5.2)

which applies to periodic as well as Dirichlet no-flux boundary conditions. $_1F_3$ is a hypergeometric function [69]. A dynamic scaling profile for times $\delta t > 0$ with $\delta t \ll T$ is given, to leading order in $\delta t/T$, by

$$h(x, \delta t)|_{\delta t \ll T} = M - M \left( \frac{\delta t}{2T} \right)^{1/z} \mathcal{H} \left( \frac{x - L/2}{(\delta t)^{1/z}} \right), \quad z = 4,$$

(5.3)

with the scaling function

$$\mathcal{H}(\xi) = _1F_3 \left( \begin{array}{c} -1/4, 1/2, 3/4 \\ 1/4, 3/4, 3/4 \end{array} \right) + \xi^2 \frac{\Gamma(1/4)}{8\Gamma(1/2)} _1F_3 \left( \begin{array}{c} 1/4, 5/4, 3/4 \\ 1/4, 3/4, 3/4 \end{array} \right).$$

(5.4)

In the equilibrium regime, the static profile $h^{(0)}(x, \delta t = 0)|_{T \to \infty}$ minimizing the corresponding free energy for periodic boundary conditions (see [1]) coincides with the one in equation (4.5a). For Dirichlet no-flux boundary conditions, instead, one finds

$$h^{(D')}(x, \delta t = 0)|_{T \to \infty} = h^{(0)}(x + L/2 - x_M, \delta t = 0)|_{T \to \infty}$$

(5.5)

with

$$x_M^{(D')}|_{T \to \infty} = \frac{L}{2} \left( 1 \pm \frac{1}{\sqrt{3}} \right).$$

(5.6)

For definiteness, we choose henceforth the smaller value for $x_M^{(D')}$, such that equation (5.5) can be explicitly written as

$$h^{(D')}(x, \delta t = 0)|_{T \to \infty}/M = \begin{cases} 6\frac{\pi}{L} \left( \frac{x}{L} + \frac{1}{\sqrt{3}} \right), & x \leq x_M^{(D')}, \\ 6 \left( \frac{x}{L} - 1 \right) \left( \frac{x}{L} - 1 + \frac{1}{\sqrt{3}} \right), & x > x_M^{(D')} \end{cases}.$$  

(5.7)

In the equilibrium regime for times $\delta t > 0$ with $\delta t \ll T$, a dynamic scaling profile for periodic and Dirichlet boundary conditions is given by

$$h(x, \delta t)|_{\delta t \ll T} \simeq M - M (\delta t)^{1/z} \Gamma(1 - 1/z) \mathcal{H} \left( \frac{x - x_M}{\delta t^{1/z}} \right), \quad z = 4,$$

(5.8)

with the same scaling function as in equation (5.4). Note that the above expressions pertain to a continuum system. In the presence of an upper bound to the eigenmode spectrum, the time evolution of the peak $h(x_M, \delta t)$ of the profile exhibits two regimes:

$$1 - h(x_M, \delta t)/M \propto \begin{cases} \delta t, & \delta t \leq \tau_\times, \\ \delta t^{1/z}, & \delta t \gtrsim \tau_\times. \end{cases}$$

(5.9)

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\[ \tau \times \] is the crossover time (see equation (2.4)). As was the case for the EW equation (see equation (4.7)), equation (5.9) is independent of the boundary conditions and applies to both the transient and the equilibrium regime. Explicit expressions for the first-passage profiles obtained within WNT for all times are reported in [1].

5.2. Periodic boundary conditions

Here, we discuss simulation results obtained for the MH equation with periodic boundary conditions. Figures 9 and 10 illustrate the time evolution of the averaged profile \( \langle h(x, \delta t) \rangle \) towards the first-passage event in the transient and equilibrium regimes, respectively.

\[ \text{Figure 9.} \quad \text{Averaged profile} \quad \langle h(x, \delta t) \rangle \quad \text{for the MH equation (1.2) with periodic boundary conditions in the transient regime} \quad (M \ll 1). \text{The first passage of the height} \quad M \quad \text{occurs at the time} \quad \delta t = 0. \text{Utilizing translational invariance, the individual profiles obtained from simulation are shifted such that the height} \quad M \quad \text{is reached at location} \quad x_M = L/2. \text{Time is normalized to the diffusion time scale} \quad \tau_D \quad \text{(equation (2.6)) using for the exponent} \quad z \quad \text{an effective value of} \quad 1/(2 \alpha) \approx 2.9 \quad (\alpha \approx 0.17) \quad \text{instead of} \quad 4, \text{as implied by panel (a).} \]

\[ \text{(a)} \]

\[ \text{(b)} \]

\[ \text{(c)} \]

where \( \tau_x \) is the crossover time (see equation (2.4)). As was the case for the EW equation (see equation (4.7)), equation (5.9) is independent of the boundary conditions and applies to both the transient and the equilibrium regime. Explicit expressions for the first-passage profiles obtained within WNT for all times are reported in [1].
As shown in panels (a), in both regimes, the peak $\langle h(x_M = L/2, \delta t) \rangle$ approaches the height $M$ via a power-law, $M - \langle h(x_M = L/2, \delta t) \rangle \propto \delta t^\alpha$, with $\alpha \simeq 0.16 - 0.17$ at intermediate times ($\delta t \gtrsim \tau_x$) and $\alpha = \alpha_0 \simeq 0.5$ at early times ($\delta t \lesssim \tau_x$). Analogously to the finding for EW dynamics (see section 4), these values of the dynamical exponent are significantly smaller than the prediction $\alpha = 1/4$ and $\alpha_0 = 1$ obtained from WNT (equation 5.9)). This finding is rationalized in section 6 below. In order to account for this quantitative change in the dynamics, in figure 9(a) we rescale time by an effective diffusion time scale $\tau_D^{\text{eff}}$, which results from equation (2.6) by replacing $z$ by the value $1/(2\alpha) \simeq 2.9 - 3.1$ (see also section 4.2). For the systems considered in figures 9(a) and 10(a), the crossover time defined in equation (2.4) follows as $\tau_x/\tau^{(p)} \simeq 2 \times 10^{-7}$ and $6 \times 10^{-8}$, respectively, which is in good agreement with the simulation data.

Figures 9(b) and 10(b) illustrate the spatio-temporal evolution of the averaged profile. The deviations from the prediction of WNT (dashed curves) can be mainly

Figure 10. Averaged profile $\langle h(x, \delta t) \rangle$ for the MH equation (1.2) with periodic boundary conditions in the equilibrium regime ($M \gg 1$). The first passage of the height $M$ occurs at the time $\delta t = 0$. Utilizing translational invariance, the individual profiles obtained from simulation are shifted such that the height $M$ is reached at location $x_M = L/2$. Time is normalized to the relaxation time $\tau^{(p)}$ (equation (2.3)). (a) Time-evolution of the peak of the profile, $\langle h(x_M, \delta t) \rangle$, which exhibits an intermediate asymptotic regime $M - \langle h(x_M, \delta t) \rangle \propto \delta t^\alpha$ with $\alpha \simeq 0.16$. (b) Spatio-temporal evolution of the averaged profile. The solid curves represent the numerical simulations, while the dashed curves indicate the prediction of WNT (see equation I-(3.17) as well as [17]). (c) Test of the dynamic scaling behavior of $\langle h(x, t) \rangle$ as predicted by WNT according to equation (5.8), using a value of $1/z \simeq 0.17$. The dashed curve represents the scaling function $c H$ in equation (5.4), with a prefactor $c \simeq 1.7$ determined from a fit.
attributed to the fact that simulations operate in the finite-noise regime. As shown in figure 10(b), in the equilibrium regime, the time-dependent profile shapes obtained from simulations are qualitatively similar to WNT, although the difference in the value of the dynamic exponent $\alpha$ leads to a faster time evolution in the latter case.

Figures 9(c) and 10(c) demonstrate that, in an inner region, the profiles follow the scaling behavior implied by equations (5.3) and (5.8). The agreement improves upon decreasing $\delta t$. Scaling collapse is obtained here by using in equations (5.3) and (5.8) for $1/z$ an effective value of $0.17 - 0.19$, consistent with the value of the exponent ($\alpha$) that governs the time-evolution of the peak of the profile (see figures 9(a) and 10(a)).

### 5.3. Dirichlet no-flux boundary conditions

In contrast to standard Dirichlet boundary conditions, which entail a fixed chemical potential at the boundaries (see [1]) and thus a non-conserved mass, the no-flux condition (equation (1.7)) ensures mass conservation for the MH equation. In fact, due to the initial condition in equation (1.4), the mass $\mathcal{A}([h], t)$ (equation (1.8)) vanishes at all times. Figure 11 shows the probability distribution $\mathcal{P}_1^{(D)}(x_M)$ of the first-passage location $x_M$. We find that the essential predictions of WNT (see figure I-7) are recovered by the simulations. Asymptotically in the transient regime ($\mathcal{M} \to 0$), $\mathcal{P}_1$ is generally constant as a function of $x_M$ for $0 < x_M < L$. At the boundaries, $\mathcal{P}_1^{(D)}$ vanishes as a consequence of Dirichlet boundary conditions. Upon increasing $\mathcal{M}$ towards values of $\mathcal{O}(1)$, a peak develops in the central region of $\mathcal{P}_1^{(D)}$. Upon increasing $\mathcal{M}$ further, this peak diminishes, while two symmetric peaks develop near the location $x_M^{(D)}$ (equation (5.6)) predicted by WNT. One expects $\mathcal{P}_1^{(D)}(x_M) \to \delta \left(x_M \pm x_M^{(D)}\right)$ as $\mathcal{M} \to \infty$, which represents a particular realization of the weak-noise limit.

In figures 12 and 13, the profile dynamics obtained from simulations in the transient and equilibrium regimes, respectively, is illustrated. In panels (a), the averaged time evolution of the peak, $\langle h(x_M, \delta t) \rangle$, is shown as a function of the time $\delta t$ until the
first-passage event. In order to account for the spread in the distribution of $x_M$ in these two panels $\langle h(x_M, \delta t) \rangle$ is computed according to equation (2.13) by shifting the individual profiles $h^{(s)}$ to the common first-passage location $L/2$, such that $h^{(s)}(L/2, T^{(s)}) = M$. The peak is found to evolve algebraically, $M - \langle h(x_M, \delta t) \rangle \propto \delta t^\alpha$ with $\alpha = \alpha_0 \simeq 0.5$ for times $\delta t \lesssim \tau_x$ and $\alpha \simeq 0.16 - 0.17$ for $\tau_x \lesssim \delta t \lesssim \tau(D')$. These values for $\alpha$ practically coincide with the ones for periodic boundary conditions (see section 5.2) and are further discussed in section 6. In order to estimate the crossover time $\tau_x$ (see equation (2.4)), we assume that the largest mode which can be accommodated by the system is given by $k_x \simeq 0.5L/\Delta x$ (see appendix G.2 for further discussion). This renders the estimates $\tau_x \simeq 2.4 \times 10^{-7} \tau_D^{\text{eff}}$ and $\tau_x \simeq 3.2 \times 10^{-9} \tau(D')$ in the transient and equilibrium regimes.
respectively, which are seen to agree with the simulation data within an order of magnitude.

The time-dependent averaged profile $\langle h(x, \delta t) \rangle$ in the transient regime is illustrated in figure 12(b). The average (see equation (2.13)) is computed here again by translating each profile $h^{(s)}$ to the common first-passage location $L/2$. This transformation does not significantly affect the profile shape because the profiles are strongly localized and the distribution $P_1^{(D)}(x_M)$ is approximately flat in the transient regime (see figure 11). In the equilibrium regime, in contrast, $P_1^{(D)}(x_M)$ is symmetric around $L/2$ and the first-passage event is most likely to occur at either of the two locations given in equation (5.6). In this case, the averaged profile $\langle h(x, \delta t) \rangle$ shown in figure 13(b) is obtained by mirroring at $x = L/2$ all profiles $h^{(s)}$ which belong to a simulation with $x_M^{(s)} > L/2$. The spatio-temporal evolution of the profile displayed in the plots qualitatively agrees with the predictions of WNT (see [1]). As a consequence of the finite width of $P_1^{(D)^*}$
around each of its two peaks, the maximum of $\langle h(x, \delta t) \rangle$ in figure 13(b) is smaller than $M$, despite the fact that each stochastic realization fulfills $h(x_M^0, T^0) = M$.

Close to the first-passage event, WNT predicts a universal dynamic scaling behavior of the profile, as expressed in equations (5.3) and (5.8). As shown in figures 12(c) and 13(c), this property is recovered in the simulations: upon accounting for the renormalized dynamic exponent $1/z \to \alpha \simeq 0.17$, the profiles superimpose onto the scaling function $cH$ (equation (5.4)) within an inner region, where $c$ is a fit parameter.

6. Discussion

As demonstrated in the preceding sections, a crucial difference between the results of the Langevin simulations and the predictions of WNT arises in the time-dependence of the averaged profile. Both in simulations and within WNT, the peak of the profile $\langle h(x_M, \delta t) \rangle$ approaches the first-passage height $M$ algebraically,

$$M - \langle h(x_M, \delta t) \rangle \propto \begin{cases} \delta t^\alpha_0, & \delta t \lesssim \tau_x, \\ \delta t^\alpha, & \tau_x \lesssim \delta t \lesssim \tau. \end{cases}$$

(6.1)

However, simulations yield the values

$$\alpha_0 \simeq 0.5, \quad \alpha \simeq \begin{cases} 0.27 - 0.3, & EW, \\ 0.16 - 0.17, & MH, \end{cases}$$

(6.2)

for the dynamic exponents, while WNT predicts (see [1])

$$\alpha_{0,WNT} = 1, \quad \alpha_{WNT} = 1/z = \begin{cases} 1/2, & EW, \\ 1/4, & MH. \end{cases}$$

(6.3)

We emphasize that these results are independent of the boundary conditions and apply both in the transient and in the equilibrium regime. The crossover time $\tau_x$ (equation (2.4)) and the roughening time $\tau$ (equation (2.3)) correspond to the relaxation time of the shortest and largest fluctuation wavelengths, respectively, that can be accommodated by the system. Since $\tau \propto L^z$, the intermediate asymptotic regime characterized by the exponent $\alpha$ dominates for sufficiently large systems. As detailed in the preceding sections, we furthermore recall that the time-evolution of the peak $\langle h(x_M, \delta t) \rangle$ is determined based on a slightly different averaging procedure than the one used for the full profile (see also equation (2.13)).

In order to gain a basic understanding of the discrepancy between equations (6.2) and (6.3), we first consider a (Markovian) Brownian walker $h(t)$, initially at $h(t = 0) = 0$, in the presence of an absorbing boundary at a fixed height $h = M$ (see appendices C and D for details). Within WNT, the averaged path of the walker between the points $(t = 0, h = 0)$ and $(T, M)$, with $T$ fixed, is the one minimizing the associated action (see appendix D). This results in a linear time-dependence of the walker approaching the absorbing boundary (see equation (D.11)),

$$M - \langle h(\delta t) \rangle_{WNT} \propto \delta t. \quad \text{(standard Brownian motion).}$$

(6.4)
As before, the average is defined here such that the first-passage event occurs at \( \delta t = 0 \). For a Markovian Brownian walker, the averaged path to an impenetrable boundary can however also be calculated exactly, i.e. including all corrections beyond WNT (see appendix C.1). For a fixed endpoint \((T, M)\), this yields

\[
M - \langle h(\delta t) \rangle \propto \delta t^{1/2} \quad \text{(standard Brownian motion, fixed } T) \tag{6.5}
\]

as \( \delta t \to 0 \). The difference between the dynamic exponents in equations (6.5) and (6.4) arises from the ‘entropic repulsion’ (see e.g. \([8, 70]\)) exerted by the absorbing boundary onto fluctuations of the walker \emph{around} the most-likely path described by WNT. Averaging also over the first-passage time distribution results in (see appendix C.2)

\[
M - \langle h(\delta t) \rangle \propto \delta t^{1/2} \quad \text{(standard Brownian motion, first-passage path)} \tag{6.6}
\]

and accordingly does not alter the trajectory asymptotically close to the boundary compared to equation (6.5). Far from the boundary, however, significant changes in the walker path are induced by this additional average (see figures C1 and C3).

The preceding results can be extended to \emph{fractional Brownian motion}, i.e. to a Gaussian random process \( h(t) \) characterized by the correlation function in equation (C.19). On its most-likely path, the walker approaches the endpoint \((t = T, h = M)\) algebraically (see equation (D.10)):

\[
M - \langle h(\delta t) \rangle_{\text{WNT}} \propto \delta t^{2H}, \quad \text{(fractional Brownian motion)} \tag{6.7}
\]

where \( H \) is the Hurst exponent of the process \((H = 1/2 \text{ for standard Brownian motion})\). Beyond the weak-noise approximation, numerical simulations (see appendix C.2.2) show that the actual first-passage path of a fractional Brownian walker behaves as

\[
M - \langle h(\delta t) \rangle \propto \delta t^{H}. \quad \text{(fractional Brownian motion)} \tag{6.8}
\]

Note that, as in equation (6.6), the average is performed here also over the first-passage time distribution. Equations (6.7) and (6.8) are straightforward generalizations of the Markovian expressions in equations (6.4) and (6.6). We conclude that taking into account fluctuation-induced interactions with the absorbing boundary effectively leads to a reduction of the dynamic exponent characterizing the averaged path of a Brownian walker from the value \(2H\) predicted by WNT to the value \(H\).

We now apply these insights to a fluctuating profile \( h(x, t) \). To this end, we recall that a tagged monomer \( h(x_M, t) \) follows a Gaussian stochastic process characterized by the Hurst exponents

\[
H_0 = 1/2 \quad \text{and} \quad H = \frac{1}{2z}, \quad \text{(profile)} \tag{6.9}
\]

which, \emph{inter alia}, determine the variance as (see equation (2.7))

\[
\langle [\delta h(x_M, t)]^2 \rangle^{1/2} \sim \begin{cases} t^{H_0}, & t \lesssim \tau_x \\ t^H, & \tau_x \lesssim t \lesssim \tau. \end{cases} \tag{6.10}
\]

\[9\] Note that, in contrast to the Markovian case, for fBM the influence of the entropic repulsion effect and the random character of the first-passage time could not be separated here. This requires a new simulation method and is left for future work.

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For times \( t \gtrsim \tau \), a tagged monomer experiences the ‘self-generated’ effective potential of the mass-conserving profile, as reflected by the Gaussian equilibrium variance (see equations (2.10) and (2.9)).

We first turn to equilibrium initial conditions, for which the stochastic process described by equations (6.9) and (6.10) is actually a fractional Brownian motion (see equation (2.7b)). In this case, equation (6.7) predicts, based on equation (6.9), the values \( \alpha_{0,\text{WNT}} = 2H_0 = 1 \) and \( \alpha_{\text{WNT}} = 2H = 1/z \) for the dynamic exponents in equation (6.1), in agreement with the explicit WNT results in equation (6.3). (Note that the weak-noise approximation here is insensitive to the presence of an impenetrable boundary.) Beyond WNT, equation (6.8) accordingly predicts

\[
\alpha_0 = H_0 = \frac{1}{2}, \quad \alpha = H = \frac{1}{2z} = \begin{cases} 1/4, & \text{EW}, \\ 1/8, & \text{MH} \end{cases} \quad (\text{prediction})
\]

for the dynamic exponents of a profile near a first-passage event. These values are indeed close to the simulation results in equation (6.2), especially in the case of the short-time exponent \( \alpha_0 \). Possible reasons for the discrepancy of the late-time exponent \( \alpha \) are discussed below.

For non-equilibrium initial conditions, corresponding to the transient first-passage regime (\( M \ll 1 \)), the stochastic process underlying equation (6.10) is not a fractional Brownian motion (see equation (2.7a)). However, the above reasoning concerning the averaged profile essentially relies only on the Hurst characterization of the dynamics of a tagged monomer. In particular, this process has the same subdiffusive scaling behavior in the equilibrium and the transient regime, suggesting equation (6.11) to apply also in the latter. Indeed, the values for \( \alpha \) obtained from the simulations in the two regimes are practically identical.

The prediction in equation (6.11) is based on the equivalence of a fractional Brownian walker and a tagged monomer of an unconstrained interface. However, for the first-passage dynamics considered here, the absorbing boundary condition at the height \( M \) (equation (1.11)) is essential. This boundary condition constrains the profile as a whole and, owing to the long-range correlations of the profile, it can in principle lead to deviations in the behavior of a tagged monomer from the behavior expected for a single fractional Brownian walker. To which extent this effect is responsible for the discrepancy between the values for \( \alpha \) reported in equation (6.2) and the predictions in equation (6.11) demands further studies.

Here it is possible to clarify at least the impact of the spatially extended nature of the absorbing boundary condition. To this end, we perform simulations in which an absorbing boundary acts only on a monomer at a single location \( x_M \). Figure 14(a) shows \( \langle h(x_M, \delta t) \rangle \) as a function of time obtained in this case for the EW equation with periodic boundary conditions (solid curve). One observes that \( \langle h(x_M, \delta t) \rangle \) still follows the algebraic behavior in equation (6.1), with a value of \( \alpha \) that is essentially identical to the one obtained for an absorbing boundary acting on all monomers (dash-dotted curve; see also figure 5(a)). As figure 14(b) shows, also the averaged profile at the first-passage event is not significantly affected by the spatially extended character of the absorbing boundary condition. This insensitivity can be attributed to the rather sharply peaked shape of the first-passage profile, which is already predicted by WNT (see also figure 5(b)).
Overall, the results in figure 14 suggest that the spatial extension of the absorbing boundary has a negligible influence on the behavior of the averaged profile.

We finally remark that, in principle, also insufficiently large values of the system size $L$ or of the reduced height $\mathcal{M}$ can contribute to the deviations between the observed dynamic exponent and the prediction of the fBM model. In fact, the crossover to the short-time diffusive regime in equation (6.10) happens earlier for smaller systems, which can result in an artificially large effective value of $\alpha$ (see, e.g., figure 5(a)). A similar effect can also be observed in the case of roughening (see, in particular, figure G3(c)). However, for the largest values of $L$ used here, we have not observed a significant $L$-dependence of the effective dynamic exponent. This indicates that the residual finite-size corrections to the values in equation (6.2) are rather small (see, e.g., figure 5(a)).

Note furthermore that, within the applicability of its underlying approximations, WNT is expected to become exact in the two limits $\mathcal{M} \ll 1$ and $\mathcal{M} \gg 1$ [1]. Indeed, the spatial profile shapes are accurately captured by WNT in these limits. However, since WNT disregards by construction some fundamental aspects of the first-passage process (see the above discussion), we expect no convergence of the values of $\alpha$ to the predictions of WNT.

7. Summary

In the present study, the first-passage dynamics of an interfacial profile governed by the EW or MH equations (equations (1.1) and (1.2)) have been analyzed based on numerical solutions. We have considered here periodic as well as Dirichlet boundary conditions. In the case of the MH equation, the latter are imposed in conjunction with a no-flux condition in order to ensure conservation of the mass (equation (1.8)). For the EW equation with periodic boundary conditions, mass conservation is explicitly imposed during the time evolution via the rule in equation (1.10). The first-passage
event is defined as the instant at which the profile reaches a given height $M > 0$ for the first time. Accordingly, an absorbing boundary condition acts at the height $M$ (equation (1.11)).

The obtained results are compared here to weak-noise theory (WNT) as well as to effective Brownian walker models describing the anomalous diffusion of a tagged ‘monomer’ of the profile. WNT can be considered as a saddle-point approximation to the first-passage problem and thus neglects the entropic repulsion effect of the impenetrable boundary and the random character of the first-passage time. The present study elucidates the accuracy of WNT for the description of the noise-activated dynamics of a spatially extended, finite and highly correlated stochastic system.

We find that the shape of the averaged profile $\langle h(x, \delta t) \rangle$ is in general well described by WNT. In particular, the dynamic scaling behavior predicted by WNT is qualitatively recovered in the simulations. In the transient regime (corresponding to small reduced heights, $M \ll 1$ (see equation (2.11))), the averaged profile is sharply peaked and independent from the boundary conditions. In the equilibrium regime (corresponding to $M \gg 1$), the profile is insensitive to the boundary conditions only in an inner region, where a dynamic scaling behavior applies. The associated scaling function and scaling exponents are universal. Consistent with WNT, the roughening time $\tau$ (see equation (2.3)) sets the characteristic time scale for the creation of the first-passage fluctuation.

A significant difference between WNT and the fully stochastic model (equations (1.1) and (1.2)) concerns the dynamic exponent $\alpha$, which characterizes the approach of the profile towards first-passage event at the height $M$ via $M - \langle h(x, \delta t) \rangle \propto \delta t^{\alpha}$. Here, instead of the value $\alpha = 1/z$ predicted by WNT (see equation (6.3)), a value close to $1/(2z)$ is found in the simulations (see equations (6.2) and (6.11)), with $z = 2$ for the EW and $z = 4$ for the MH equation. This ‘renormalization’ of the dynamic exponent can be understood based on the equivalence between a tagged monomer in equilibrium and a fractional Brownian walker with Hurst index $H = 1/(2z)$. For the walker, it is shown here analytically and via dedicated numerical simulations, that the dynamic exponent $n$ describing the averaged trajectory near an absorbing boundary at height $M$, $M - \langle h(\delta t) \rangle \propto \delta t^{n}$, changes from $n = 2H$ within WNT to $n = H$ when fluctuation-induced (entropic) interactions between the walker and the boundary are taken into account. Accordingly, the renormalization of the profile exponent $\alpha$ can be attributed to the fluctuations of the profile around its most-likely path as it approaches the first-passage event (see discussion in section 6). We remark that our numerical solutions yield a value for $\alpha$ slightly larger (see equation (6.2)) than the prediction $\alpha = 1/(2z)$ (equation (6.11)), which might be related to the fact the mapping between a tagged monomer and a Brownian walker is formally obtained in the absence of an absorbing boundary. This aspect deserves further studies.

The inadequacy of WNT to capture the exact time-dependence of the first-passage dynamics becomes particularly clear for standard Brownian motion, in which case the problem can be solved exactly (see appendix C.2). A Brownian path with fixed endpoints is sensitive to the presence of the absorbing boundary only close to it (see equation (C.4)). In the weak-noise limit, the effect of the absorbing boundary diminishes, such that the averaged path reduces to the classical one (see equation (C.6)). Upon averaging over the first-passage distribution, the influence of the boundary effectively ‘spreads’ over the whole path (see equations (C.17) and (C.18)). However, in the absence of noise,
the first-passage distribution trivially vanishes, as does the first-passage path (see equation (C.16)). For future studies it would be interesting to improve WNT by taking into account the distribution of first-passage times and to include the fluctuations around the most-likely path in the presence of an impenetrable boundary. This would allow one to rigorously assess the various approximations involved in WNT.

As a by-product of our simulations, we have obtained the mean first-passage time \( \langle T \rangle \). In the equilibrium regime, \( \langle T \rangle \) is found to grow exponentially with the square of the reduced height \( M^2 \) (equation (2.11)). This reflects the self-generated harmonic potential in which a tagged monomer of an equilibrated profile moves. In the transient regime, instead, we find an algebraic dependence of \( \langle T \rangle \) on the actual height \( M \), which reflects the sub-diffusive motion of a tagged monomer. It turns out that mass conservation (equation (1.9)) as well as the extended nature of the absorbing boundary (equation (1.11)) can significantly affect the first-passage distribution (see appendix A).

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**Appendix A. First-passage time distribution**

The distribution \( P_1(T) \) of the first-passage time to the height \( M \) obtained in the transient regime is illustrated in figure A1(a). Note that \( T \) is normalized here by the mean first-passage time \( \langle T \rangle \), which is discussed separately in section 3. We find that \( P_1 \) generally exhibits a well-defined maximum for \( T \approx \langle T \rangle \). In the case of MH dynamics, which conserves mass (see equation (1.9)), \( P_1 \) decays exponentially. This is also found in the case of EW dynamics with periodic boundary conditions, in which case mass conservation is explicitly enforced via equation (1.10). In contrast, if equation (1.10) is not imposed (curve in figure A1(a) labeled by 'unc.'), the first-passage distribution decays algebraically for large \( T \), \( P_1 \sim T^{-n} \), with \( n \approx 3.5^{10} \). A similar algebraic decay is also observed in the case of EW dynamics with Dirichlet boundary conditions, where mass is conserved only as a time average.

The behavior of \( P_1 \) is also sensitive to the spatially extended character of the absorbing boundary condition (see equation (1.11)). This is illustrated in figure A1(b), which shows \( P_1 \) obtained in the transient regime for an absorbing boundary acting only on the monomer at \( x_M \). Compared to figure A1(a), \( P_1 \) decays here slower for large \( T \), although still approximately exponentially. Lifting, in the case of EW dynamics, additionally the mass constraint results in an algebraic decay, \( P_1 \sim T^{-n} \) with \( n \approx 2.2 \). This value of \( n \) is smaller than the one obtained in the case of a spatially extended absorbing boundary (see figure A1(a)). It is, however, close to the prediction \( n \approx 2.5 \) given in [11], where the transient persistence probability of an interface has been investigated.

In the equilibrium regime (see figure A1(b)), both for the EW and MH equation as well as for all considered boundary conditions, we empirically find that the first-passage distribution is a simple exponential function of \( T/\langle T \rangle \):

\[ P_1(T) \propto T^{-\frac{3}{2}}. \]

\[ \text{Note that the considered profiles do not yet exhibit center-of-mass diffusion (see equation (F.29)), in which case a behavior } P_1 \propto T^{-\frac{3}{2}} \text{ is expected [8, 64, 66].} \]
The exponential behavior is in fact characteristic for a fractional Brownian walker in a parabolic potential \(^7\) and found to persist also if the absorbing boundary condition acts only on a single monomer (data not shown). Removing the mass constraint in the equilibrium regime results in a simple diffusive motion of the center-of-mass of the profile, which then dominates the first-passage distribution.

\[
P_1(T) \simeq \langle T \rangle \exp(-T/\langle T \rangle).
\]  

The exponential behavior is in fact characteristic for a fractional Brownian walker in a parabolic potential \(^68\) and found to persist also if the absorbing boundary condition acts only on a single monomer (data not shown). Removing the mass constraint in the equilibrium regime results in a simple diffusive motion of the center-of-mass of the profile, which then dominates the first-passage distribution.

Appendix B. Equilibrium distribution of height fluctuations

B.1. Periodic boundary conditions

The friction and noise parameters \(\eta\) and \(D\) in equations (1.1) and (1.2) can be determined by requiring that the ensuing steady-state probability distribution of the profile \(h(x)\) is characterized by a certain temperature \(\Theta\). For periodic boundary conditions, equations (1.1) and (1.2) yield in the steady-state a Gaussian joint-probability distribution of the form \(^71, 72\)
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\[ P_{\text{eq}}^{(p)}[h] \sim \exp \left[ -\frac{1}{4\Theta} \int_0^L \frac{dh}{dx} \right] \delta[h(0) - h(L)] \delta \left[ \int_0^L dh(x) \right], \quad (B.1) \]

with the temperature (see equation (2.8))

\[ \Theta \equiv \frac{D}{2\eta} \quad (B.2) \]

in units of \( k_B \). In equation (B.1), the \( \delta \)-functions enforce the periodic boundary conditions and the zero-mass constraint equation (1.9). The stationary single-site height distribution resulting from equation (B.1) is given by [71–73]

\[ P_{\text{eq}}^{(p)}(h) = \sqrt{\frac{3}{\pi \Theta L}} \exp \left( -\frac{3}{\Theta L} h^2 \right), \quad (B.3) \]

implying the variance (see also equation (F.14))

\[ \langle h^2 \rangle = \frac{\Theta L}{6}. \quad (B.4) \]

According to equation (B.1), a profile \( h(x) \) in equilibrium can be considered as a Brownian motion process for which \( x \) plays the role of time. Since the motion is required to start and end here at the same point, \( h(0) = h(L) \), the process is in fact a Brownian bridge, with the additional constraint of having zero area under it [74, 75]. Equation (B.2) is taken as a definition of the temperature throughout the present study, despite the fact that, for non-periodic boundary conditions, the resulting steady-state variance is different from equation (B.4).

**B.2. Dirichlet boundary conditions**

The steady-state distribution for Dirichlet boundary conditions is given by the same expression as in equation (B.1), except that \( \delta[h(0) - h(L)] \) is replaced by \( \delta[h(0)] \delta[h(L)] \) and that the mass constraint is present only for Dirichlet no-flux boundary conditions (see equations (1.6) and (1.7)). Correlation functions can be readily determined with the aid of the closely-related propagator for a Brownian particle with fixed endpoints [76, 77]:

\[ G(h, x|h_0, x_0) = \int_{h(0)=h_0}^{h(x)=h} \mathcal{D}h(\xi) \exp \left[ -\frac{1}{4\Theta} \int_{x_0}^x d\xi \left( \frac{dh(\xi)}{d\xi} \right)^2 \right] = \frac{1}{\sqrt{4\pi \Theta(x-x_0)}} \exp \left[ -\frac{(h-h_0)^2}{4\Theta(x-x_0)} \right]. \quad (B.5) \]

If, in addition to the endpoints also the area under the profile is constrained, corresponding to Dirichlet no-flux boundary conditions, the propagator is instead given by [71, 78]

\[ G(h, x, A|h_0, x_0, A_0) = \int_{h(0)=h_0}^{h(x)=h} \mathcal{D}h(\xi) \delta \left( \int_{x_0}^x d\xi h(\xi) - A \right) \exp \left[ -\frac{1}{4\Theta} \int_{x_0}^x d\xi \left( \frac{dh(\xi)}{d\xi} \right)^2 \right] \]

\[ = \sqrt{\frac{3}{2\pi \Theta(x-x_0)^2}} \exp \left[ -\frac{1}{\Theta} \frac{3}{(x-x_0)^3} \left\{ A - A_0 - (x-x_0)h \right\} \left\{ A - A_0 - (x-x_0)h_0 \right\} + \frac{1}{x-x_0}(h-h_0)^2 \right]. \quad (B.6) \]

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Given by standard Dirichlet boundary conditions, the equilibrium variance of a fluctuating profile is given by

\[
\langle h^2(x) \rangle = \frac{\int_{-\infty}^{\infty} dh' G(h', x|0, 0) h'^2 G(0, L|h', x)}{G(0, L|0, 0)} = 2\Theta L \frac{x}{L} \left(1 - \frac{x}{L}\right), \tag{B.7}
\]

while for the averaged path, \(\langle h(x) \rangle = 0\). Equation (B.7) also represents the variance of a Brownian bridge (see, e.g. [75, 79]). For a Dirichlet profile whose area is constrained to vanish, the averaged path results instead as

\[
\langle h(x) \rangle_A = \frac{\int_{-\infty}^{\infty} dA' \int_{-\infty}^{\infty} dh' G(h', x, A'|0, 0, 0) h' G(0, L, 0|h', x, A')}{G(0, L, 0|0, 0, 0)} = \frac{A x}{L L} \left(1 - \frac{x}{L}\right), \tag{B.8}
\]

while its variance is given by (we consider here only \(A = 0\), such that \(\langle h(x) \rangle_{A=0} = 0\))

\[
\langle h^2(x) \rangle_{A=0} = \frac{\int_{-\infty}^{\infty} dA' \int_{-\infty}^{\infty} dh' G(h', x, A'|0, 0, 0) h'^2 G(0, L, 0|h', x, A')}{G(0, L, 0|0, 0, 0)} = 2\Theta L \frac{x}{L} \left(1 + 3\frac{x}{L} \left(\frac{x}{L} - 1\right)\right). \tag{B.9}
\]

The above results rely on the Markovian nature of the respective stochastic process. In particular, the normalization in equations (B.8) and (B.9) follows from the Markovian nature of the joint stochastic process \((h, A)\), i.e. \(G(h, x, A|h_0, x_0, A_0) = \int_{-\infty}^{\infty} dA' \int_{-\infty}^{\infty} dh' G(h, x, A|h', x, A')G(h', x', A'|h_0, x_0, A_0)\) for any \(x_0 < x' < x\).

**Appendix C. Averaged path for a single Brownian walker**

**C.1. Averaged path with constrained endpoints**

We place an absorbing boundary at height \(h = 0\) and consider a (Markovian) Brownian walker that departs from \((h, t) = (\epsilon, 0)\) to some distant position \((M, T)\). The infinitesimal quantity \(\epsilon\) is required as a regularization and the limit \(\epsilon \to 0\) will be performed at the end of the calculation [8]. Owing to the Markovian property of the process, the averaged trajectory of the walker can be expressed as (see also [80–82])

\[
\langle h(t) \rangle_{(0,T)\to(M,T)} = \lim_{\epsilon \to 0} \frac{\int_0^{\infty} dh G_+(M, T|h, t) h G_+(h, t|\epsilon, 0)}{\int_0^{\infty} dh G_+(M, T|h, t) G_+(h, t|\epsilon, 0)} = \lim_{\epsilon \to 0} \frac{\int_0^{\infty} dh G_+(M, T|h, t) h G_+(h, t|\epsilon, 0)}{G_+(M, T|\epsilon, 0)}. \tag{C.1}
\]

The propagator \(G_+(h, t|h_0, t_0)\) represents the conditional probability for the walker to move from \((h_0, t_0)\) to \((h, t)\) without \(h\) becoming negative and is given by the well-known expression

[33https://doi.org/10.1088/1742-5468/aaa792]
\[ G_+(h, t| h_0, t_0) = \frac{1}{\sqrt{4\pi \Theta(t - t_0)}} \left[ \exp\left( -\frac{(h - h_0)^2}{4\Theta(t - t_0)} \right) - \exp\left( -\frac{(h + h_0)^2}{4\Theta(t - t_0)} \right) \right], \]  

which follows, e.g., by applying the image method to the propagator in equation (B.5) (replacing \( x \to t \)) [8]. Equation (C.1) can be evaluated analytically, yielding

\[
\langle h(t) \rangle_{M,T} = \frac{2}{M\sqrt{\pi}} \sqrt{\Theta} \left( 1 - \frac{t}{T} \right) \exp\left( \frac{M^2 t}{4\Theta T(t - T)} \right) + \left( \frac{Mt}{T} + 2d \right) \text{erf}\left( \sqrt{\frac{M}{2d\Theta T(t - T)}} \right) - \left( \frac{M}{T} + 2d \right) \text{erf}\left( \sqrt{\frac{M}{2d\Theta T(t - T)}} \right),
\]

where, in the last equation, the dimensionless scaling variables \( U \equiv M/\sqrt{\Theta T} \), \( V \equiv M/\sqrt{\Theta T} \) have been introduced. The behavior of the averaged path is illustrated in figure C1(a) as a function of \( \Theta t/M^2 = 1/U^2 \). For small times \( t \), one asymptotically has

\[
\langle h(t \to 0) \rangle_{M,T} \simeq 4\sqrt{\frac{\Theta}{\pi}} t + \mathcal{O}(t^{3/2}).
\]

At late times \( t \approx T \), the behavior of the averaged path depends on the value of \( T \) and \( M \). The associated control parameter can be determined by noting that, for \( U \sim \mathcal{O}(V) \) (with \( U > V \)), the first term in the curly brackets in equation (C.3) is small, while the error function in equation (C.3) is approximately equal to one. Accordingly, values \( \langle h \rangle_{M,T}/M \gg 1 \) are possible if \( V^2 \lesssim 1 \), i.e., the averaged path develops a ‘bow’ as seen in figure C1(a) if...
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\[ \frac{M^2}{\Theta T} \lesssim 1. \]  \hfill (C.5)

If, on the other hand, \( M^2/\Theta T \gtrsim 1 \), the averaged path behaves linearly for \( t \simeq T \):

\[ \langle h(t \to T) \rangle_{M,T} \simeq M \frac{t}{T}. \]  \hfill (C.6)

As shown in appendix D, this expression, being independent of the noise \( \Theta \), is simply the most-likely path of the walker (see equation (D.11)). The cross-over time \( \tau_c \) between the two regimes can be defined as the time where the two asymptotic laws in equations (C.4) and (C.6) are equal, yielding

\[ \tau_c \simeq \frac{16 \Theta T^2}{M^2 \pi}. \]  \hfill (C.7)

The two asymptotic laws can only be distinguished as long as \( \tau_c < T \), which gives an estimate consistent with equation (C.5). Inserting equations (C.7) into (C.6) yields the length scale

\[ h_c \simeq \frac{16 \Theta T}{\pi M}, \]  \hfill (C.8)

which characterizes the range of influence of the absorbing boundary. As a reflection of the scale-free nature of the Brownian process, this length depends on coordinates \( T \) and \( M \) arbitrarily far away from the boundary. The averaged path given in equation (C.3) is illustrated in figure C1(b) in the limit \( M^2/\Theta T \gg 1 \).

In passing, we remark that the averaged trajectory \( \langle h(t) \rangle \) of a free Brownian walker (i.e. in the absence of an absorbing boundary) between two points is a straight line,

\[ \langle h(t) \rangle \propto t. \]  \hfill (C.9)

This result follows from equation (C.1) by replacing therein \( G_+ \) by the standard diffusion propagator \( G \) given in equation (B.5). For free Brownian motion, the averaged path (equation (C.9)) coincides with the ‘classical’ (most-likely) path which follows from the minimization of the corresponding action (see equation (D.11) below).

C.2. First-passage path

Consider a Brownian (but not necessarily Markovian) walker starting at \( (h, t) = (0, 0) \) in the presence of an absorbing boundary at \( h = M > 0 \). Let \( F_1(M, T) \) be the corresponding probability distribution of the first-passage time \( T \) to the height \( M \) (see below) and \( \langle h(t) \rangle_{M,T} \) be the averaged path between the (fixed) points \((0, 0)\) and \((M, T)\). We then define the averaged ‘first-passage path’ of the walker, i.e. its averaged path near the first-passage event (see also [82]), by

\[ \langle h(t) \rangle = M - \frac{\int_1^\infty dT F_1(M, T) \langle h(T - t) \rangle_{M,T}}{\int_1^\infty dT F_1(M, T)}. \]  \hfill (C.10)

The associated transformation of the sample paths is illustrated in figure C2. Exact analytical expressions for \( F_1 \) and \( \langle h(t) \rangle_{M,T} \) are available only for Markovian Brownian
walkers (see equations (C.3) and (C.12)). In the non-Markovian case, we shall therefore resort to numerical calculations.

**C.2.1. Markovian case.** In the Markovian case, the mapping implied by equation (C.10) can be implemented by placing an absorbing boundary at \( h = 0 \) and considering a walker which starts at \( (\epsilon, 0) \) with infinitesimal \( \epsilon > 0 \) and ends at \( (M, T) \) at a random time \( T \) governed by \( F_1(M, T) \). Accordingly, using \( \langle h(t) \rangle_{M,T} \) as defined in equation (C.1), the first-passage path in equation (C.10) reduces to

\[
\langle h(t) \rangle = \frac{\int_0^\infty dT F_1(M, T) \langle h(t) \rangle_{M,T}}{\int_0^\infty dT F_1(M, T)}.
\] (C.11)

For a Markovian Brownian walker, the first-passage time \( T \) from \( h = 0 \) to a height \( h = M \) is governed by the probability distribution [8]

\[
F_1(M, T) = \frac{\Theta |M|}{\sqrt{4\pi \Theta T^3}} \exp \left( -\frac{M^2}{4\Theta T} \right) = \frac{M^2}{\Theta} f_1(\Theta T/M^2),
\] (C.12)

where in the last equation the scaling function

\[
f_1(\tau) = \frac{\exp (-1/4\tau)}{\sqrt{4\pi \tau^3}}
\] (C.13)

has been introduced. For \( G_+ \) as defined in equation (C.2), one has \( \int_0^\infty dt G_+(\epsilon, t|M, 0) = \epsilon/\Theta \), implying

\[
F_1(M, T) = \lim_{\epsilon \to 0} \frac{\Theta}{\epsilon} G_+(\epsilon, T|M, 0).
\] (C.14)

Furthermore, noting \( \int_0^\infty dT F_1(M, T) = 1 \), the quantity

\[
\int_t^\infty dT F_1(M, T) = 1 - \int_0^t dT F_1(M, T) = \text{erf} \left( \frac{M}{2\sqrt{\Theta t}} \right)
\] (C.15)

Figure C2. Transformation of the sample paths of random walkers as considered in appendix C.2. (a) In the actual setup, a random walker \( h(t) \) starts at the origin, \( h(t = 0) = 0 \), and moves until it hits a boundary at height \( M > 0 \) (shaded bar) for the first time. (b) According to equation (C.10), the paths are transformed such that the first-passage event occurs at the space-time origin. In the Markovian case, the first-passage path can be directly obtained by placing an absorbing boundary at \( h = 0 \) and considering paths that start at \( h(t = 0) = 0 \) and are conditioned to end at \( h(T) = M \), with a random time \( T \) governed by \( F_1(M, T) \).
represents the survival probability. Using equation (C.15) as well as equations (C.1) and (C.14), the first-passage path defined in equation (C.11) can be calculated analytically:

\[
\langle h(t) \rangle = \frac{1}{\int_{T}^{} \text{d}T \, F_1(M,T)} \lim_{\epsilon \to 0} \frac{\Theta}{\epsilon} \int_{0}^{\infty} \text{d}h \int_{t}^{\infty} \text{d}T \, G^+_1(h,t|\epsilon,0) \, h \, G^+_1(M,T|h,t) = M \left\{ 1 + \frac{4}{\xi \sqrt{\pi}} \left[ 1 - \exp \left( -\frac{\xi^2}{4} \right) \right] \right\} \frac{\text{erf} \left( \frac{\xi}{2} \right)}{\text{erf} \left( \frac{\xi}{\sqrt{\Theta t}} \right)} - 1 \right\}, \quad \xi \equiv \frac{M}{\sqrt{\Theta t}}, \quad \text{(C.16)}
\]

where the integral over \( T \) has been performed before the one over \( h \). Note that the term in the curly brackets is solely a function of the scaling variable \( \xi \). For small \( t \), i.e. near the absorbing boundary, equation (C.16) reduces to

\[
\langle h(t \to 0) \rangle \simeq 4 \sqrt{\frac{\Theta t}{\pi}}. \quad \text{(C.17)}
\]

The essential reason for recovering in equation (C.17) the asymptotic behavior of the path with fixed endpoints \( \langle h(t) \rangle_{M,T} \) (see equation (C.4)) is that, very close to the absorbing boundary, \( \langle h(t) \rangle_{M,T} \) is independent of the final time \( T \) and thus can be moved out of the integral in equation (C.11) in this limit. For \( \Theta t/M^2 \gg 1 \), i.e. far from the absorbing boundary, the first-passage path behaves as

\[
\langle h(t) \rangle \simeq \sqrt{\pi \Theta t}. \quad \text{(C.18)}
\]

The non-monotonic behavior of the path \( \langle h(t) \rangle_{M,T} \) (equation (C.3)) for \( t \) near \( T \) (see figure C1(a)) is reflected by a gentle ‘bump’ of the first-passage path for \( \Theta t/M^2 \simeq 1 \) (see figure C3(b)). Overall, the asymptotic trajectory of a Brownian walker to its first passage point however remains at all times close to a power-law, \( \langle h(t) \rangle \sim t^{1/2} \).

Note that, in the weak-noise limit (\( \Theta \to 0 \)), the first-passage path (equation (C.16)) vanishes. This is in contrast to the path with fixed endpoints (equation (C.3)), for which the ‘classical’ contribution, being independent of \( \Theta \), prevails as \( \Theta \to 0 \) (see equation (C.6) as well as equation (D.11) below). The time-dependence of the first-passage evolution is thus an intrinsic finite-noise property. According to equation (C.18), this applies even far from the absorbing boundary.
C.2.2. Non-Markovian case. As a specific realization of a non-Markovian random walk relevant for interfacial roughening, we consider fractional Brownian motion (fBM). FBM is a Gaussian process $h(t)$ with correlation function [58–60]

$$
\langle h(t)h(s) \rangle = \Theta \left( t^{2H} + s^{2H} - |t - s|^{2H} \right),
$$

characterized by the Hurst index $H$. The correlation function of the relative height fluctuations $\delta h(x, t) = h(x, t) - h(x, 0)$ of an equilibrated one-dimensional interface governed by equation (1.1) or (1.2) takes the same form as in equation (C.19) (see equation (2.7b) and, e.g., [11]). Standard Markovian Brownian motion results for $H = 1/2$, in which case the stochastic increments $h(t + dt) - h(t)$ are uncorrelated. For $H < 1/2$ ($H > 1/2$), instead, the increments are anti-correlated (positively correlated). In the non-Markovian case, it is known that the distribution of the first-passage time $T$ to a single boundary asymptotically behaves as [11, 83, 84]

$$
F_1(T \to \infty) \sim T^{-2+H}.
$$

Recently, an expression for the propagator of fBM with absorption has been derived perturbatively [85–87]. However, since closed analytical results are neither available for $F_1$ nor $\langle h(t) \rangle_{M,T}$, we resort in the following to numerical simulations in order to determine the first-passage path defined in equation (C.10).

We seek the averaged path of a fractional Brownian walker starting at $h(t = 0) = 0$ and being absorbed at a boundary at height $M > 0$ (see figure C2(a)). To this end, an ensemble of trajectories $\{h_{t=0,1,\ldots,N}(k)\}$, each of around $N \simeq 10^7$ steps, are created and the step $T(k)$, where $h_{T(k)}(k) \geq M$ for the first time, is determined for each trajectory $h(k)$. Owing to the long-time tail of $F_1$ (see equation (C.20)), the mean first passage time $\langle T \rangle$ to a single absorbing boundary is infinite. This is essentially a consequence of the fact the the walker can perform arbitrary large excursions in the negative half space ($h_t < 0$) before hitting the boundary at $M$ [8]. By checking different values of $N$ and $M$, we find that, in the present case, these excursions have negligible influence on the behavior of the averaged path near the boundary. The averaged first-passage path $\langle h_i \rangle$, $i = 0, \ldots, N - 1$ (with $i = 0$ now corresponding to the first-passage event) is then obtained as

$$
\langle h_i \rangle = N_{T \geq i}^{-1} \sum_k^{T \geq i} \left( M - h_{T(k)-i}^{(k)} \right),
$$

where the sum is defined to run over all $N_{T \geq i}$ paths that end at times $T \geq i$. Furthermore, the individual trajectories are shifted such that their respective first-passage times coincide (see figure C2).

The equivalence of equations (C.21) and (C.10) is readily proven: the averaged path of walkers between the fixed endpoints $(h = 0, t = 0)$ and $(M, T)$ is given by the restricted average $\langle h_i \rangle_{M,T} = \sum_k^{T} h_i^{(k)}/N_T$, where $N_T$ is the total number of such paths and the sum runs over precisely these paths. The discrete first-passage time distribution

---

11 Usually, some ‘overshoot’, $h_T > M$, is observed, in which case the whole trajectory is shifted, i.e. $h_t \to h_t - (h_T - M)$, in order to ensure that $h_T = M$ is exactly fulfilled. The averaged path turns out to be insensitive to the overshoot correction.

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can be expressed as $F_1(M, T) = \frac{N_T}{\sum_{T \geq i} N_T} = \frac{N_T}{N}$, where $N$ is the total number of paths considered in the sample. Using $N_{T \geq i} = \sum_{T \geq i} N_T$, the discrete analogue of equation (C.10) for the averaged path can accordingly be written as

$$M - \langle h \rangle = \frac{1}{\sum_{T \geq i} F_1(M, T)} \sum_{T \geq i} (h_{T-i})_{M,T} F_1(M, T) = \frac{1}{\sum_{T \geq i} F_1(M, T)} \sum_{T \geq i} \sum_k (T) \frac{h_{T-i}^{(k)}}{N} = \frac{1}{N_{T \geq i}} \sum_k (T) h_{T-i}^{(k)},$$

which coincides with equation (C.21).

Simulation results obtained for the averaged first-passage path defined in equation (C.21) are shown in figure C4. For convenience, we revert to the notation of continuous time $\delta t$. FBM is simulated based on the ‘circulant method’ [88, 89] (see [90] for a practical implementation). From the plot one infers that the walker approaches the first-passage height algebraically, $$\langle h(\delta t) \rangle \sim \delta t^H,$$

with an exponent essentially coinciding with the Hurst index $H$ of the underlying fBM process. This behavior is consistent with equation (C.17) in the Markovian case ($H = 1/2$). Since $M^2/\Theta \sim \mathcal{O}(10^6)$, the slight change of the logarithmic slope of the path for $H = 1/2$ observed in figure C3(b)) is only partly visible in figure C4. This applies also to the data for $H \neq 1/2$, if one assumes (as suggested by dimensional analysis of equation (C.19)) that the crossover time generalizes to $M^{1/H}/\Theta^{1/(2H)}$ for general fBM. Note that $\langle h(\delta t) \rangle$ can be larger than $M$ for large $\delta t$ because the walker can make excursions to the lower half-space (see figure C2(a)). Slight deviations from a pure algebraic behavior are noticeable in figure C4 for small times, which are found to be independent of the variance of the noise increments used in the numerical simulation as well as of the overshoot correction.

It is illuminating to consider here also the most-likely path of a fBM between two locations $h = 0$ and $M$. The most-likely path minimizes the dynamic action of the
associated probability functional and thus represents the weak-noise approximation of the averaged path. As shown in appendix D, within WNT, one finds
\[ h_{\text{MLP}}(\delta t) \sim \delta t^{2H} \] (C.24)
near the endpoint. The different exponents in equations (C.23) and (C.24) can be attributed to the repulsive effect exerted by the absorbing boundary on the fluctuations around the most-likely path (see section 6 for further discussion).

**Appendix D. Most-likely path of a Gaussian random process**

We determine here the most-likely path of a Gaussian random process \( h(t), 0 \leq t \leq T, \) subject to the constraints
\[ h(0) = M_0 \quad \text{and} \quad h(T) = M_T, \] (D.1)
where \( M_0 \) and \( M_T \) are constants. The following discussion is in fact a straightforward application of the constrained minimization of a quadratic functional (see, e.g. \[91\]). The Gaussian process is taken to have zero mean \( \langle h(t) \rangle = 0 \) and correlation function
\[ G(t, t') \equiv \langle h(t)h(t') \rangle. \] (D.2)
Accordingly, the joint probability distribution is given by
\[ P[h] \sim \exp(-S[h]), \] (D.3)
with the ‘action’
\[ S[h] \equiv \frac{1}{2} \int_0^T dt \int_0^T dt' h(t)G^{-1}(t, t')h(t'). \] (D.4)
The inverse \( G^{-1} \) of the correlation function is defined (in an operator sense) by
\[ \int_0^T dt G(s, t)G^{-1}(t, s') = \delta(s - s'). \] (D.5)
For a Markovian process (i.e. standard Brownian motion), one has \( G^{-1}(t, t') = -\frac{1}{2} \partial_t^2 \delta(t - t') \), while the correlation function \( G(t, t') = 2\min(t, t') \) [92]. For fractional Brownian motion, the explicit form of \( S \) is known only perturbatively [85]. In passing, we remark that the continuous time description used here should formally be understood as the limit of a multivariate Gaussian process of random variables \( \{h_i \} \) defined at discrete times \( i = 0, \Delta t, \ldots, (T/\Delta t) - 1 \), analogously to the definition of a path integral [92]. Imposing the constraints in equation (D.1) gives rise to the augmented action
\[ \tilde{S}([h], \lambda_1, \lambda_2) \equiv S[h] - \lambda_1 [h(0) - M_0] - \lambda_2 [h(T) - M_T] \] (D.6)
with the Lagrange multipliers \( \lambda_{1,2} \). Minimization of \( \tilde{S} \) with respect to \( h(\tau) \) yields
\[ 0 = \frac{\delta \tilde{S}}{\delta h(\tau)} = 2 \int_0^T dt h(t)G^{-1}(t, \tau) - \lambda_1 \delta(\tau) - \lambda_2 \delta(T - \tau), \] (D.7)
where we used the symmetry property $G(t, t') = G(t', t)$. Multiplying equation (D.7) with the inverse correlation function $G(s, \tau)$ and integrating over $\tau$, using equation (D.5), one obtains

$$h(s) = G(s, 0) \sum_k (Q^{-1})_{1k} M_k + G(s, T) \sum_k (Q^{-1})_{2k} M_k,$$

(D.8)

where

$$Q \equiv \begin{pmatrix} G(0, 0) & G(0, T) \\ G(T, 0) & G(T, T) \end{pmatrix}$$

(D.9)

is the ‘covariance matrix of the constraints’ and $M \equiv (M_0, M_T)$. These results naturally generalize to more than two constraints. Notably, the time dependence of the minimum action path in equation (D.8) is essentially determined by the correlation function.

We now specialize the above results to fBM, i.e. a Gaussian process described by the correlation function in equation (C.19). Since this correlation function is trivially zero if one of its arguments vanishes (rendering a singular covariance matrix in equation (D.9)), the evaluation of equation (D.8) is performed with a value $\epsilon > 0$ instead of 0 for the initial time. After sending eventually $\epsilon \to 0$ and setting $M_0 = 0, M_T = M$ (see equation (D.1)), equation (D.8) reduces to (see also [86])

$$h(t) = \frac{1}{2} MT^{-2H} (t^{2H} - (t - T)^{2H} + T^{2H})$$

(D.10)

where $0 < t < T$. For $t \to 0$ or $t \to T$, one has $h(t) \sim t^{2H}$ and $M - h(t) \sim (T - t)^{2H}$, respectively, showing that a fractional Brownian walker approaches the endpoints of a constrained path via a power-law with exponent $2H$. This is illustrated in figure D1. In the Markovian case, equation (D.10) reduces to a straight line,
\[ h(t) = M \frac{t}{T}, \quad (H = 1/2). \]  

**Appendix E. Review on eigenfunctions**

Here, a number of relevant properties of the eigenfunctions of the (bi-)harmonic operator $\partial^2_z$ on the interval $[0,L]$ are collected (see [1] for more details). We introduce a complete set of (‘proper’) eigenfunctions $\sigma_k$, $k \in \mathbb{Z}$, fulfilling

\[ \partial^2_z \sigma_k(x) = \gamma_k \sigma_k(x), \quad z \in \{2, 4\} \]

with eigenvalues $\gamma_k$. The eigenfunctions are subject to one of the following boundary conditions:

**periodic** : \[ \sigma_k^{(p)}(x, t) = \sigma_k^{(p)}(x + L, t), \]  

**Dirichlet zero-\(\mu\)** : \[ \sigma_k^{(D)}(0, t) = 0 = \sigma_k^{(D)}(L, t), \quad \partial^2_z \sigma_k^{(D)}(0, t) = 0 = \partial^2_z \sigma_k^{(D)}(L, t), \]  

**Dirichlet no-flux** : \[ \sigma_k^{(D')}(0, t) = 0 = \sigma_k^{(D')}(L, t), \quad \partial^2_z \sigma_k^{(D')}(0, t) = 0 = \partial^2_z \sigma_k^{(D')}(L, t). \]

The symbol $\mu$ refers to the chemical potential, which vanishes at the boundary for standard Dirichlet boundary conditions (see [1]). For this reason, the latter are also called Dirichlet zero-\(\mu\) boundary conditions here. Associated with $\sigma_k$ are a set of *adjoint* eigenfunctions $\varphi_k$, which fulfill

\[ \partial^2_z \varphi_k(x) = \gamma_k \varphi_k(x) \]

as well as one of the following adjoint boundary conditions:

**periodic** : \[ \varphi_k^{(p)}(x, t) = \varphi_k^{(p)}(x + L, t), \]  

**Dirichlet zero-\(\mu\)** : \[ \varphi_k^{(D)}(0, t) = 0 = \varphi_k^{(D)}(L, t), \quad \partial^2_z \varphi_k^{(D)}(0, t) = 0 = \partial^2_z \varphi_k^{(D)}(L, t), \]  

**Neumann zero-\(\mu\)** : \[ \partial_z \varphi_k^{(D')}(0, t) = 0 = \partial_z \varphi_k^{(D')}(L, t), \quad \partial^2_z \varphi_k^{(D')}(0, t) = 0 = \partial^2_z \varphi_k^{(D')}(L, t). \]

Note that proper and adjoint eigenfunctions in general have an identical set of eigenvalues $\gamma_k$. For periodic and Dirichlet zero-\(\mu\) boundary conditions, the operator $\partial^2_z$ is self-adjoint on $[0,L]$, implying that

\[ \varphi_k^{(p, D)} = \sigma_k^{(p, D)}. \]

In contrast, for Dirichlet no-flux boundary conditions on $\sigma_k$ (equation (E.2c)), the operator $\partial^2_z$ is not self-adjoint and the associated adjoint eigenfunctions $\varphi_k^{(D')}$ are required to satisfy the distinct boundary conditions in equation (E.4c). The eigenfunctions $\sigma_m$ and $\varphi_n$ are mutually orthogonal:

\[ \int_0^L dx \sigma_m(x) \varphi_n(x) = \kappa_n \delta_{mn} \]
First-passage dynamics of linear stochastic interface models: numerical simulations and entropic repulsion effect

**Table E1.** Eigenfunctions and related properties of $\partial^2_x$ on the interval $[0,L]$ for various boundary conditions. The eigenfunctions $\sigma_k$ coincide with the corresponding adjoint eigenfunctions $\varphi_k$ if $\partial^2_x$ is self-adjoint. Furthermore, $b = 0$ for EW dynamics, $b = 1$ for MH dynamics, and the dynamic index $z = 2b + 2$.

| $\partial^2_x$ Self-adjoint | Periodic (equation (E.2a)) | Dirichlet zero $\mu$ (equation (E.2b)) | Dirichlet no-flux (equation (E.2c)) $(b = 1)^a$ |
|-----------------------------|--------------------------|-------------------------------------|-----------------------------------|
| $\sigma_k$                  | $\frac{1}{\sqrt{L}} \exp \left( \frac{2\pi i k}{L} x \right)$ | $\frac{1}{\sqrt{L}} \sin \left( \frac{k \pi}{2L} x \right)$ | No $\sigma_k^{(D')}$ $\varphi_k^{(D')}$ $\gamma_k^{(D')}$ |
| $\varphi_k$                 | $\sigma_k$                | $\sigma_k$                          | $\sigma_k^{(D')}$ $\varphi_k^{(D')}$ |
| $k$                         | $0, \pm 1, \pm 2, \ldots^b$ | $1, 2, 3, \ldots$                   | $1, 2, 3, \ldots$ |
| $\gamma_k$ (equations (E.1) and (E.3)) | $(-1)^{k+1} \left( \frac{2\pi}{L} x \right)$ | $(-1)^{k+1} \left( \frac{2\pi}{L} x \right)$ | $(-1)^{k} \left( \frac{2\pi}{L} x \right)$ |
| $\kappa_k$ (equation (E.6))  | $1$                      | $1$                                 | $\frac{L}{2} \left( 1 - \frac{(-1)^{k+1}}{\cosh(L\gamma_k^{D'})} \right)$ |
| $\epsilon_k$ (equation (E.7)) | $[-|\gamma_k|^{1/2}]^k \kappa_k$, $\epsilon_0 = 0$ | $[-|\gamma_k|^{1/2}]^k \kappa_k$ | $-\gamma_k^{1/2} \kappa_k$ |

$^a$ Dirichlet no-flux boundary conditions are considered only for MH dynamics $(b = 1)$. Since $\sigma_k^{(D')}$ and $\varphi_k^{(D')}$ are not normalized here, the system size $L$ appears in the corresponding expression for $\kappa_k$.

$^b$ Due to the mass constraint (equation (1.9)), the zero mode $(k = 0)$ is absent from the actual solution for periodic boundary conditions.

with a real number $\kappa_n$. The star denotes complex conjugation, which is necessary in order to deal with complex-valued eigenfunctions, such as those for periodic boundary conditions. One furthermore has

$$
\int_0^L dx \varphi^*_m(x) \varphi''_n(x) = \epsilon_n \delta_{mn} \tag{E.7}
$$

with a real number $\epsilon_n$. The eigenvalues of $\partial^2_x$ (see equation (E.1)) for Dirichlet no-flux boundary conditions are given by

$$
\gamma_k^{(D')} = \left( \frac{\omega_k^{(D')}}{L} \right)^4 \tag{E.8}
$$

where $\omega_k^{(D')}$ denotes a solution to the transcendental equation

$$
\cos(\omega_k^{(D')}) \cosh(\omega_k^{(D')}) = 1. \tag{E.9}
$$

Numerically one obtains

$$
\omega_k^{(D')} = 4.7300, 7.8532, 10.9956, \ldots \quad (k = 1, 2, 3, \ldots). \tag{E.10}
$$

Since $\sigma_k^{(D')}(x) = 0$, $\sigma_k^{(D')}(x) = \sigma_k^{(D')}(x)$, and $\omega_k^{(D')} = \omega_k^{(D')}$, we restrict the eigenspectrum to $k \geq 1$. For $k \geq 4$, an accurate approximation is provided by

$$
\omega_k^{(D')} \simeq \pi \left( k + \frac{1}{2} \right), \tag{E.11}
$$

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First-passage dynamics of linear stochastic interface models: numerical simulations and entropic repulsion effect which is asymptotically exact. Explicit expressions and relevant properties of \( \sigma_k, \varphi_k \) are summarized in table E1. (Expresssions for the eigenfunctions \( \sigma_k^{(D)} \) and \( \varphi_k^{(D)} \) are reported [1].)

Appendix F. Roughening

In the absence of an impenetrable wall, the EW and the MH equation can be solved analytically. In the context of roughening, so far mainly bulk systems or systems with periodic boundary conditions have been considered [5, 6, 18, 44–50, 52, 53, 94, 95]. Here, we provide a general series solution in terms of the corresponding eigenfunctions, which can be readily specialized to various boundary conditions. We begin by casting equations (1.1) and (1.2) into the common form

\[
\partial_t h = (-1)^b \eta \partial_z^2 h + \hat{\zeta} \tag{F.1}
\]

with \( b = 0, 1 \) for the EW and MH equation, respectively, and \( z = 2b + 2 \). The noise \( \hat{\zeta} \equiv \partial^2_b \zeta \) is correlated as (see equation (1.3))

\[
\langle \hat{\zeta}(x, t)\hat{\zeta}(x', t') \rangle = (-1)^b 2D \partial^2_z \delta(x - x')\delta(t - t'). \tag{F.2}
\]

To proceed, the field \( h \) and the noise \( \hat{\zeta} \) are expanded in terms of the eigenfunctions \( \sigma_k(x) \) defined in equation (E.1):

\[
h(x, t) = \sum_k h_k(t)\sigma_k(x), \tag{F.3a}
\]

\[
\hat{\zeta}(x, t) = \sum_k \hat{\zeta}_k(t)\sigma_k(x). \tag{F.3b}
\]

The expansion coefficients follow from the orthogonality relation in equation (E.6) as

\[
h_k(t) = \int_0^L \, dx \, h(x, t)\varphi^*_k(x)/\kappa_k, \tag{F.4a}
\]

\[
\hat{\zeta}_k(t) = \int_0^L \, dx \, \hat{\zeta}(x, t)\varphi^*_k(x)/\kappa_k, \tag{F.4b}
\]

where \( \varphi_k(x) \) are the adjoint eigenfunctions (equation (E.3)) and \( \kappa_k \) is reported in table E1. Accordingly, upon using equations (E.6) and (E.7), the correlation of the noise modes follows as

\[
\langle \hat{\zeta}_m(t)\hat{\zeta}_n^*(t') \rangle = \left\langle \int_0^L \, dx \int_0^L \, dx' \, \varphi_m^*(x)\varphi_n(x')\partial^2_z \zeta(x, t)\partial^2_z \zeta(x', t') \frac{1}{\kappa_m\kappa_n} \right\rangle
\]

\[
= (-1)^b 2D \delta(t - t')\frac{\tilde{\epsilon}_m}{\kappa^2_m} \delta_{mn}. \tag{F.5}
\]
where $\xi_k \equiv \kappa_k$ for $b = 0$ and $\xi_k \equiv \epsilon_k$ for $b = 1$. The partial integrations required in the case $b = 1$ have generated the factor $(-1)^b$ in the last line of equation (F.5); the same result is obtained upon using equation (F.2). All boundary terms vanish for the boundary conditions considered here. The mass-conserving property of the noise for MH dynamics ($b = 1$) is reflected in equation (F.5) by the fact that $\xi_0/\kappa_0^2 = 0$ in this case (see table E1). For EW dynamics, instead, $\xi_0/\kappa_0^2 = 1$, such that the noise in principle contributes to the zero mode. Upon inserting the expansions given in equation (F.3) into equation (F.1) and using equation (E.1), one obtains

$$\partial_t h_m(t) = -\Lambda_m h_m(t) + \zeta_m(t), \quad \Lambda_m \equiv -(-1)^b \eta \gamma_m,$$

with $\Lambda_m \geq 0$ and the eigenvalues $\gamma_m$ (see table E1). For an arbitrary initial profile $h_m(0)$, the solution of equation (F.6) is given by

$$h_m(t) = e^{-\Lambda_m t} \left( h_m(0) + \int_0^t dt' e^{\Lambda_m t'} \zeta_m(t') \right).$$

For the EW equation with periodic boundary conditions, the zero mode $h_{m=0}$ (for which $\Lambda_0 = 0$) is absent from the spectrum due to the mass constraint (equation (1.9)) enforced by equation (1.10). The dynamics of $h_0$ obtained in the case of an unconstrained profile is discussed separately below (see equation (F.29)). In the long-time, equilibrium limit, the equal-time correlation function follows as

$$\langle h_m(t) h_n^*(t) \rangle \big|_{t \to \infty} = \frac{D(-1)^b \tilde{\epsilon}_m}{\Lambda_m \kappa_m^2} \delta_{mn} \equiv V_m \delta_{mn}.$$  

Note that $V_m = D |\tilde{\epsilon}_m|/|\Lambda_m|\kappa_m^2 > 0$, as is readily shown using table E1. Equation (F.8) does not apply to a zero mode, in which case equation (F.7) directly yields $\langle h_0(t) h_n^*(t) \rangle = 0$ for all $n$ (see also equation (F.5)).

Assuming uncorrelated initial conditions, $\langle h_m(0) h_n^*(0) \rangle \propto \delta_{mn}$, the two-time correlation function of a relative height fluctuation $\delta h_m(t) \equiv h_m(t) - h_m(0)$ follows from equation (F.7) as

$$\langle \delta h_m(t) \delta h_n^*(s) \rangle = \left\{ \langle |h_m(0)|^2 \rangle \left[ 1 - e^{-\Lambda_m t} + 1 - e^{-\Lambda_m s} - (1 - e^{-\Lambda_m (t+s)}) \right] \ight.$$

$$+ V_m \left[ 1 - e^{-\Lambda_m (t+s)} - (1 - e^{-\Lambda_m (t-s)}) \right] \delta_{mn} \right\} \delta_{mn}. \quad (F.9)$$

If the profile is initially flat, $h_m(0) = 0$, only the second term in equation (F.9) remains:

$$\langle \delta h_m(t) \delta h_n^*(s) \rangle_{\text{flat}} = V_m \left[ 1 - e^{-\Lambda_m (t+s)} - (1 - e^{-\Lambda_m (t-s)}) \right] \delta_{mn}. \quad (F.10)$$

For thermal initial conditions, where according to equation (F.8) $\langle |h_m(0)|^2 \rangle = V_m$, equation (F.9) instead becomes

$$\langle \delta h_m(t) \delta h_n^*(s) \rangle_{\text{th}} = V_m \left[ 1 - e^{-\Lambda_m t} + 1 - e^{-\Lambda_m s} - (1 - e^{-\Lambda_m (t-s)}) \right] \delta_{mn}. \quad (F.11)$$

12 For Dirichlet no-flux boundary conditions, this is readily proven by considering the limit $\omega \to 0$. 

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The real-space correlation function follows as

$$\langle \delta h(x, t) \delta h(y, s) \rangle = \sum_m \langle \delta h_m(t) \delta h^*_m(s) \rangle \sigma_m(x) \sigma^*_m(y),$$  \hspace{1cm} (F.12)

where we used the fact that $h_{-k} = h^*_k$ for periodic boundary conditions, which is a consequence of $h(x, t)$ being real.

For $t = s$ and $x = y$, the real-space correlation function reduces, both for flat and thermal initial conditions, in the long-time limit to

$$\langle |\delta h(x)|^2 \rangle_{eq} \equiv \langle |\delta h(x, t)|^2 \rangle_{t \to \infty} = \sum_m V_m |\sigma_m(x)|^2.$$  \hspace{1cm} (F.13)

For periodic boundary conditions (see table E1), one has $|\sigma_m(x)|^2 = 1/L$, and equation (F.13) becomes (see also equation (B.4))

$$\langle |\delta h^{(0)}(x)|^2 \rangle_{eq} = \frac{DL}{(2\pi)^2 \eta} \sum_{m, m \neq 0} \frac{1}{m^2} = \frac{DL}{12\eta} = \frac{\Theta L}{6},$$  \hspace{1cm} (F.14)

where we used $\sum_{m=1}^{\infty} m^{-2} = \pi^2/6$ and introduced the temperature $\Theta = D/(2\eta)$ according to equation (2.8). For Dirichlet zero-\(\mu\) boundary conditions, instead, equation (F.13) becomes (see also equation (B.7))

$$\langle |\delta h^{(D)}(x)|^2 \rangle_{eq} = \frac{2DL}{\pi^2\eta} \sum_{k=1}^{\infty} \frac{1}{k^2} \sin^2 \left( \frac{k \pi x}{L} \right) = 2\Theta L \frac{x}{L^2} \left( 1 - \frac{x}{L} \right),$$  \hspace{1cm} (F.15)

where we used $\sin^2(y) = [1 - \cos(2y)]/2$ and well-known Fourier series representations of trigonometric functions [96]. In the case of Dirichlet no-flux boundary conditions, instead of directly calculating the infinite sum in equation (F.13), we invoke a mapping to Brownian motion, which according to equation (B.9) yields

$$\langle |\delta h^{(D)}(x)|^2 \rangle_{eq} = 2\Theta L \frac{x}{L} \left( 1 + 3 \frac{x}{L} \left( \frac{x}{L} - 1 \right) \right).$$  \hspace{1cm} (F.16)

This expression is found to numerically coincide with equation (F.13).

For $x = y$, but arbitrary times, equation (F.12) becomes

$$\langle \delta h(x, t) \delta h(x, s)^* \rangle_{flat} = C(t + s, x) - C(|t - s|, x),$$  \hspace{1cm} (F.17a)

$$\langle \delta h(x, t) \delta h(x, s)^* \rangle_{th} = C(t, x) + C(s, x) - C(|t - s|, x),$$  \hspace{1cm} (F.17b)

with

$$C(t, x) = \sum_k V_k \left( 1 - e^{-\Lambda_k t} \right) |\sigma_k(x)|^2.$$  \hspace{1cm} (F.18)

The roughness of an interface is defined as one of the following equal time correlation functions:

$$\langle |\delta h(x, t)|^2 \rangle_{flat} = C(2t, x),$$  \hspace{1cm} (F.19a)

$$\langle |\delta h(x, t)|^2 \rangle_{th} = 2C(t, x).$$  \hspace{1cm} (F.19b)

The finiteness and the discreteness of the system imply the existence of a smallest and a largest mode index, $k_{\text{min}}$ and $k_{\text{max}}$. In order to obtain a closed expression for the roughness, we use

$$\sum_{k=0}^{k_{\text{max}}} V_k \left( 1 - e^{-\Lambda_k t} \right) |\sigma_k(x)|^2 = \sum_{k=0}^{k_{\text{max}}} V_k \left( 1 - \frac{1}{\Lambda_k t} \right) |\sigma_k(x)|^2.$$  \hspace{1cm} (F.20)

The expression for the roughness becomes

$$\langle |\delta h(x, t)|^2 \rangle_{eq} = \sum_{k=0}^{k_{\text{max}}} V_k \left( 1 - \frac{1}{\Lambda_k t} \right) |\sigma_k(x)|^2 + \beta \langle |\delta h(x, t)|^2 \rangle_{flat} + \gamma \langle |\delta h(x, t)|^2 \rangle_{th},$$  \hspace{1cm} (F.21)

where $\beta$ and $\gamma$ are constants.

References:

[96] J. Stat. Mech. (2018) 033212
correlation function $C(t, x)$, we replace the sum in equation (F.18) by an integral. The error arising from this approximation is small if the summands in equation (F.18) vary significantly only over a few $k$. This, in turn, applies if the system size is large and $t \ll 1/\Lambda_{k_{\text{min}}}$, since then the variation occurs for large $k$, where $\Lambda_k \sim k^2$. For periodic boundary conditions one has $k_{\text{min}}^{(p)} = 1$ and $k_{\text{max}}^{(p)} = [L/(\Delta x - 1)]/2$ (see table E1 as well as appendix G.1), such that equation (F.18) becomes

$$ C^{(p)}(t) \sim \frac{D}{\eta \pi} \int_{k_{\text{min}}^{(p)}}^{k_{\text{max}}^{(p)}} dp \frac{1 - \exp(-\eta p^2 t)}{p^2} \equiv C(t; p_{\text{min, max}}^{(p)}), \quad (F.20) $$

where we introduced the wave number $p^{(p)} \equiv 2\pi k^{(p)}/L$ associated with $k$. Note that equation (F.20) is independent of $x$ owing to translational invariance. For standard Dirichlet boundary conditions, instead, one has $k_{\text{min}}^{(D)} = 1$ and $k_{\text{max}}^{(D)} = L/\Delta x - 1$ (see appendix G.1). In order to evaluate equation (F.18), we focus on the point $x = L/2$ and note that $\sqrt{L/2} \sigma_t(L/2) = 1, 0, -1, 0, 1, \ldots$ for $k = 1, 2, 3, \ldots$, such that one obtains

$$ C^{(D)}(t, L/2) = \frac{2D}{\eta L} \sum_{n=0}^{k_{\text{max}}^{(D)}} \frac{1 - \exp(-\eta [(2n + 1)\pi/L]t)}{[2n + 1]\pi/L} \approx \frac{D}{\eta \pi} \int_{p_{\text{min}}^{(D)}}^{p_{\text{max}}^{(D)}} dp \frac{1 - \exp(-\eta p^2 t)}{p^2} \equiv C(t; p_{\text{min, max}}^{(D)}), \quad (F.21) $$

where $n \equiv 2k + 1$, $n_{\text{max}} \equiv [(L/\Delta x - 1)/2]$, and $p^{(D)} \equiv \pi k^{(D)}/L$. For Dirichlet no-flux boundary conditions and a sufficiently large integer $k'$, one may approximate, for $k \geq k'$, $|\Lambda_k|/\eta \approx [(k + 1/2)^2/\pi]$, $[\sigma_k^{(D)}(L/2)]^2 \approx 2/3$ for even $k$, while $\sigma_k^{(D)}(L/2) = 0$ for odd $k$ (see table E1 and [1]). Leaving at this point the largest mode $k_{\text{max}}^{(D)}$ unspecified\textsuperscript{13}, we accordingly obtain $(n \equiv k/2)$

$$ C^{(D')}(t, L/2) \approx \frac{2D}{\eta L} \sum_{n=0}^{k_{\text{max}}^{(D')}/2} \frac{1 - \exp(-\eta [2\pi n/L + \pi/2L]^2 t)}{(2\pi n/L + \pi/2L)^2} \approx C(t; p_{\text{min, max}}^{(D)}) \quad (F.22) $$

with $p_{\text{max}}^{(D')} = (k_{\text{max}}^{(D')} + 1/2)\pi/L$. The freedom in the choice for the lower bound $k'$ leads to a negligible error in $C^{(D')}$ at large times. We thus re-instate for the smallest wave number the exact value $p_{\text{min}}^{(D')} = \omega_1^{(D')}/L$, with $\omega_1^{(D')}$ defined in equation (E.10). In conclusion, the expression for $C(t; p_{\text{min, max}}^{(D)})$ in equation (F.20), which approximates the one-point correlation function in equation (F.18) at $x = L/2$, depends on the boundary conditions only via the integration boundaries $p_{\text{min, max}}$. The integral in equation (F.20) can be calculated in closed form, leading to

$$ C(t) = \frac{D}{\eta \pi} (\eta t)^{1/2} z^{-1} \int_{\eta t p_{\text{min}}^{(p)}}^{\eta t p_{\text{max}}^{(p)}} dx \frac{1}{x^{1/2} - 1} (1 - e^{-x}) = \frac{D}{\eta \pi} (\eta t)^{1/2} z^{-1} [\Gamma (-z^{-1}, x) - z x^{-1/2}]_{x = \eta t p_{\text{min}}^{(p)}}^{x = \eta t p_{\text{max}}^{(p)}}, \quad (F.23) $$

with $\Gamma(n, x)$ being the upper incomplete Gamma-function. To proceed, we introduce the crossover time $\tau_x = 1/(\eta z^{1/2})$, as well as the roughening time

\textsuperscript{13} The numerical analysis in appendix G.2 suggests $k_{\text{max}}^{(D')} \approx L/(2\Delta x)$.
First-passage dynamics of linear stochastic interface models: numerical simulations and entropic repulsion effect

\[ \tau_R = \begin{cases} \frac{(L/\pi)^z}{\pi} \frac{1}{\eta}, & \text{periodic}, \\ \frac{(L/\pi)^z}{\pi} \frac{1}{\eta}, & \text{standard Dirichlet}, \\ \left(\frac{L}{z\eta(0)}\right)^z \frac{1}{\eta}, & \text{Dirichlet no-flux boundary conditions}, \end{cases} \quad (F.24) \]

for which one has \( \tau_R \simeq 1/(\eta \pi^z) \) (see also equations (2.3) and (2.4)). From equation (F.6) one infers that \( \tau_R \) and \( \tau_\alpha \) are the relaxation times of the mode with largest and smallest wavelength that can be accommodated by the system. The correlation function \( C(t) \) exhibits three distinct asymptotic regimes:

\[ C(t) \simeq \frac{2\Theta}{\pi} \begin{cases} \frac{p_{\max}^z - p_{\min}^z}{z - 1}, & t \lesssim \tau_\alpha, \\ \Gamma(1 - z^{-1})(\eta t)^{1/z}, & \tau_\alpha \lesssim t \lesssim \tau_R, \\ \frac{1}{p_{\min}} - \frac{1}{p_{\max}}, & t \gtrsim \tau_R, \end{cases} \quad (F.25) \]

i.e., an initial diffusive growth followed by a subdiffusive law characterized by the dynamic index \( z \). For times of the order of \( \tau_R \), the height variance saturates to its equilibrium value\(^1\). In the subdiffusive regime, the two-time correlation functions in equation (F.17), evaluated for \( x = L/2 \), take the form \([11, 52]^{15}\)

\[ \langle \delta h(x, t) \delta h(x, s)^{*} \rangle_{\text{flat}} \simeq (2\Theta/\pi)\eta^{1/z}\Gamma(1 - z^{-1}) \left[ (t + s)^{1/z} - |t - s|^{1/z} \right], \quad (F.26a) \]

\[ \langle \delta h(x, t) \delta h(x, s)^{*} \rangle_{\text{th}} \simeq (2\Theta/\pi)\eta^{1/z}\Gamma(1 - z^{-1}) \left[ t^{1/z} + s^{1/z} - |t - s|^{1/z} \right]. \quad (F.26b) \]

Note that the prefactors in the above expression will be different if \( x \neq L/2 \). Recalling that the height fluctuations are Gaussian, equation (F.26b) shows that, in the equilibrium regime, a tagged monomer of a one-dimensional profile performs a fractional Brownian motion (see equation (C.19)) with Hurst index \([11]\)

\[ H = \frac{1}{2z}. \quad (F.27) \]

In contrast, in the transient roughening regime, the correlation function in equation (F.26a) describes a non-Markovian Gaussian process with non-stationary increments. In the subdiffusive regime, a tagged monomer \( h(x, t) \) traverses a distance \( \Delta \) within a characteristic diffusion time \( \tau_D \) determined by \( \Delta \simeq \langle \delta h(x, \tau_D)^2 \rangle^{1/2} \). Equation (F.26), which applies to \( x \simeq L/2 \), yields accordingly

\[ \tau_D \simeq \frac{\Delta^{2z}}{[(2/\pi)\Theta \Gamma(1 - z^{-1})]^2 c \eta^z}, \quad (F.28) \]

with \( c = 2 \) for flat initial conditions and \( c = 2^z \) for thermal ones. Within a time \( \tau_R \), a tagged monomer has covered a region of a typical extent set by the equilibrium variance, \( \langle h^2 \rangle^{1/2} \) (see equations (F.14) to (F.16)).

For EW dynamics with periodic boundary conditions, the zero-mode \( (k = 0) \) is absent from equation (F.7) as a consequence of enforcing the mass constraint (see

\(^{14}\) Due to the involved approximations, the equilibrium value in equation (F.25) differs from the exact results in equation (F.14) to (F.16).

\(^{15}\) The equilibrium variance results from (F.26) as \( \lim_{t \to \infty}\langle \delta h(x, t) \delta h(x, t) \rangle = 2\Theta L/\pi^2 \) and differs from the discrete result in equation (2.9) due to the continuum assumption.
equations (1.9) and (1.10)). Without this constraint, the two-time correlation function of $h_0$ (see equation (F.8)) results as

\[
\langle h_0(t) h_0^*(s) \rangle = 2D \min(t, s),
\]

representing standard Brownian diffusion of the center-of-mass of the profile.

Appendix G. Numerical implementation of the Langevin simulations

G.1. Discretization

As reported in equation (2.12), we use in our simulations a standard Euler discretization in time for equations (1.1) and (1.2):

\[
h(x_i, t + \Delta t) = h(x_i, t) - \eta \Delta t (-\nabla^2)^{1/2} h(x_i, t) + \sqrt{2D \Delta t} \nabla^1 \zeta(x_i, t),
\]

where $i = 0, 1, 2, \ldots, N - 1$, $N = L/\Delta x$ denotes the number of lattice nodes and $\Delta x$ is the lattice spacing. For notational simplicity, here we have dropped the tilde on $\zeta$. The required spatial derivatives are discretized based on a standard central difference scheme [97]:

\[
\nabla \zeta_i(x_i) \equiv \frac{1}{2} [\zeta(x_{i+1}) - \zeta(x_{i-1})],
\]

\[
\nabla^2 h(x_i) \equiv h(x_{i-1}) - 2h(x_i) + h(x_{i+1}),
\]

\[
\nabla^4 h(x_i) \equiv h(x_{i-2}) - 4h(x_{i-1}) + 6h(x_i) - 4h(x_{i+1}) + h(x_{i+2}).
\]

Here and in the following, the time argument is suppressed and length is expressed in units of $\Delta x$. We remark that the discretizations for the bi-Laplacian in equation (G.2c) and the Laplacian in equation (G.2b) are related via $\nabla^4 = \nabla^2(\nabla^2)$. We consider periodic boundary conditions,

\[
h(x_{N+i}) = h(x_i),
\]

and Dirichlet boundary conditions

\[
h(x_0) = 0 = h(x_{N-1}).
\]

For Dirichlet boundary conditions, equation (G.1) is evaluated only at the nodes $1 \leq i \leq N - 2$ (see figure G1). Since the discretized Laplacian in equation (G.2b) requires only the values of $h$ at the nearest neighbors, the boundary conditions defined in equation (G.3) and (G.4) fully determine the discretized EW dynamics. In contrast, in the case of the MH equation, the discretized bi-Laplacian in equation (G.2c) involves also next-nearest neighbors and is thus a priori undefined at the boundary nodes $i \in \{1, N - 2\}$. For Dirichlet no-flux boundary conditions, an expression of the discretized bi-Laplacian at the boundary can be determined by requiring conservation

Figure G1. On a one-dimensional lattice of $N$ nodes, Dirichlet boundary conditions are imposed at the nodes 0 and $N - 1$ (highlighted gray), i.e. $h(x_0) = 0 = h(x_{N-1})$. 

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of the total mass $A = \sum_{i=1}^{N-2} h(x_i)$ at each time step. Within the domain $2 \leq i \leq N - 3$, the deterministic contribution to the change of the mass, i.e. the contribution stemming from the second term on the r.h.s. of equation (G.1), is obtained as

$$\sum_{i=2}^{N-3} \nabla^4 h(x_i) = -3h(x_1) + 3h(x_2) - h(x_3) - h(x_{N-4}) + 3h(x_{N-3}) - 3h(x_{N-2}),$$  \hspace{1cm} (G.5)

where equations (G.2c) and (G.4) have been used and any prefactors are omitted. Accordingly, the simplest choice for $\nabla^4$ at the nodes $i \in \{1, N-2\}$ ensuring vanishing of the total deterministic mass change is given by:

$$\nabla^4 h(x_1) = 3h(x_1) - 3h(x_2) + h(x_3),$$  \hspace{1cm} (G.6a)

$$\nabla^4 h(x_{N-2}) = h(x_{N-4}) - 3h(x_{N-3}) + 3h(x_{N-2}).$$  \hspace{1cm} (G.6b)

Concerning the stochastic contribution to the mass change, the last term on the r.h.s. of equation (G.1) yields

$$\sum_{i=1}^{N-2} \nabla \zeta(x_i) = \frac{1}{2} [-\zeta(x_0) - \zeta(x_1) + \zeta(x_{N-2}) + \zeta(x_{N-1})].$$  \hspace{1cm} (G.7)

In order for this expression to vanish, a choice for $\zeta(x_i)$ at the boundary nodes $i = 0, N - 1$ must be made. Given equation (G.4), it appears natural to set

$$\zeta(x_0) = 0 = \zeta(x_{N-1}),$$  \hspace{1cm} (G.8a)

which, by equation (G.7), implies

$$\zeta(x_1) = 0 = \zeta(x_{N-2}).$$  \hspace{1cm} (G.8b)

The above choice is not unique, but minimizes artificial correlations. Alternatively, one may set $\zeta(x_0) = -\zeta(x_1)$ and $\zeta(x_{N-1}) = -\zeta(x_{N-2})$. This choice has been checked in a number of cases to yield similar results to the prescription in equation (G.8). For periodic boundary conditions, finally, it is straightforward to prove that mass is exactly conserved by equation (G.1).

It can be readily shown that equation (G.6) in fact implies a vanishing (discretized) flux at the boundaries, i.e. $\nabla \mu(x_i) = 0$ for $i = 1, N - 2$, where

$$\mu(x_i) \equiv -\nabla^2 h(x_i) = -[h(x_{i-1}) - 2h(x_i) + h(x_{i+1})]$$  \hspace{1cm} (G.9)

is the chemical potential. To this end, we introduce the forward difference

$$\nabla^F h(x_i) \equiv h(x_{i+1}) - h(x_i),$$  \hspace{1cm} (G.10)

in terms of which the Laplacian of $\mu$ can be written as

$$\nabla^2 \mu(x_i) = -\nabla^4 h(x_i) = [\mu(x_{i-1}) - \mu(x_i)] + [\mu(x_{i+1}) - \mu(x_i)]$$

$$= -\nabla^F \mu(x_{i-1}) + \nabla^F \mu(x_i), \hspace{1cm} i = 1, \ldots, N - 1.$$  \hspace{1cm} (G.11)

Upon imposing no-flux boundary conditions in the discretized form$^{16}$

$^{16}$The symmetry of equation (G.12) is more clearly revealed by writing the second relation as $\nabla^F \mu(x_{N-1}) = 0$, where $\nabla^F \mu(x_i) = \mu(x_i) - \mu(x_{i-1}) = \nabla^F \mu(x_{i-1})$ is the backward difference.
\[ \nabla^F \mu(x_0) = 0 = \nabla^F \mu(x_{N-2}), \]  
\hfill (G.12)\]

one recovers the expressions in equation (G.6):

\[ \nabla^4 h(x_1) = \mu(x_1) - \mu(x_2), \]  
\hfill (G.13a)\]

\[ \nabla^4 h(x_{N-2}) = \mu(x_{N-2}) - \mu(x_{N-3}). \]  
\hfill (G.13b)\]

We finally recall some useful properties related to the eigenmode decomposition of the profile for various boundary conditions. In the case of periodic boundary conditions, \( h(x) \) can be expressed in terms of its Fourier modes as

\[ h^{(p)}(x) = \sum_{q=0}^{N-1} \exp \left( \frac{2\pi i q}{L} x \right) h^{(p)}_q. \]  
\hfill (G.14)\]

Correspondingly, taking into account the discrete nature of \( h(x) \), the Fourier coefficients \( h_q \) are given by

\[ h_q^{(p)} = \frac{1}{N} \sum_{k=0}^{N-1} \exp \left( -\frac{2\pi i q}{L} \Delta x \right) h^{(p)}(k\Delta x), \]  
\hfill (G.15)\]

where we reinstated the lattice spacing \( \Delta x \). Note that

\[ \sum_{k=0}^{N-1} \exp \left( \frac{2\pi i}{L} qk \right) = N\delta_{q,N\mathbb{Z}}, \]  
\hfill (G.16)\]

where \( \delta_{q,N\mathbb{Z}} \equiv 1 \) if \( q \) is an integer multiple of \( N \), and zero otherwise. Since equation (G.15) implies \( h_{N-q}^{*} = h_q^{*} \), a real-valued \( h(x) \) is completely determined by its (complex) Fourier coefficients \( h_q \) within the first Brillouin zone, \( q = 0, 1, \ldots, \left[ (N-1)/2 \right] \), where \( \left[ x \right] \) denotes the largest integer smaller than or equal to \( x \).

In the case of standard Dirichlet boundary conditions, \( h(x) \) can be analogously decomposed as

\[ h^{(D)}(x) = \sum_{q=0}^{N-1} \sin \left( \frac{\pi q}{L} x \right) h_q^{(D)}, \]  
\hfill (G.17)\]

with the inverse relation

\[ h_q^{(D)} = \frac{2}{N} \sum_{k=0}^{N-1} \sin \left( \frac{\pi q}{L} k\Delta x \right) h^{(D)}(k\Delta x). \]  
\hfill (G.18)\]

The orthogonality property is given by

\[ \sum_{k=0}^{N-1} \sin \left( \frac{\pi p}{N} k \right) \sin \left( \frac{\pi q}{N} k \right) = \begin{cases} \frac{N}{2} \delta_{p,q}, & q,p \neq 0, \\ 0, & p = q = 0, \end{cases} \]  
\hfill (G.19)\]

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assuming $0 \leq q, p \leq N - 1$. Indeed, since $\sin(\pi(2N-q)k/N) = -\sin(\pi qk/N)$, the specification of the expansion coefficients $h_q$ for $0 \leq q \leq N - 1$ completely determines $h^{(D)}(x)$ on a lattice of $N = L/\Delta x$ points and, accordingly, one has $h^{(D)}_{2N-q} = -h^{(D)}_q$.

G.2. Benchmarks

We now assess the accuracy of the discretizations in equation (G.1) with a few benchmarks. Figure G2 illustrates the deterministic relaxation of a profile governed by the noiseless MH equation with Dirichlet no-flux boundary conditions. As initial configuration (at time $\delta t = 0$) we take here the first-passage profile obtained from the solution of WNT in equation I-(C63) for $\delta t = 0$ and $x_M = L/2$. Symbols represent simulation results, while solid curves represent the time-dependent first-passage profile $h(x, \delta t)$ predicted by WNT equation I-(C63) for the same value of $x_M$. Time is expressed in terms of the relaxation time $\tau^{(D)}_x$ (equation (2.3)).

Figure G2. Deterministic relaxation of a profile with Dirichlet no-flux boundary conditions for the noiseless MH equation (see equation (G.1)). The profile is initialized with the static first-passage profile obtained within WNT in the equilibrium regime, given by equation I-(C63) for $\delta t = 0$ and $x_M = L/2$. Symbols represent simulation results, while solid curves represent the time-dependent first-passage profile $h(x, \delta t)$ predicted by WNT equation I-(C63) for the same value of $x_M$. Time is expressed in terms of the relaxation time $\tau^{(D)}_x$ (equation (2.3)).

The mode $q = 0$ is usually not considered as part of the spectrum for Dirichlet boundary conditions.
algebraic growth with exponent \( 1/z \) for \( \tau_x \lesssim t \lesssim \tau \), where \( \tau_x \) and \( \tau \) are the crossover and the relaxation time, respectively (see equation (2.3) to (2.4)). In the case of MH dynamics, the slight deviation of the simulation results from the expected value \( 1/4 \) of the power-law exponent at intermediate times is found to gradually diminish upon increasing the system size \( L \).

In order to determine the crossover time \( \tau_x \), recall that for standard Dirichlet boundary conditions, one has \( L/\Delta x - 1 \approx L/\Delta x \) distinct wavemodes \( (k = 1, \ldots, L/\Delta x - 1) \) on a lattice of size \( L \) (see equation (G.19) and the related discussion). As observed in figure G3(a), the resulting crossover time \( \tau_x \) is correctly captured by the solution in equation (F.25) (dashed curve). For Dirichlet no-flux boundary conditions, there is no symmetry between the eigenmodes \( \sigma_k^{(D)}(x) \) (see table E1) for small and large \( k \); hence the largest possible eigenmode is not easily obtained. In order to gain further insight.
First-passage dynamics of linear stochastic interface models: numerical simulations and entropic repulsion effect into this issue, we determine the stability of an eigenmode via numerical simulation. To this end, the noiseless relaxation of a profile \( h(x,t) \), initialized as \( h(x,0) = \sigma_{k}^{(D')} (x) \), is simulated under MH dynamics. For a system size of, e.g. \( L = 100\Delta x \), we find that eigenmodes with \( k \lesssim L/(2\Delta x) \) typically keep their shape during the time evolution, i.e. \( h(x,t)/h(x_{\text{ref}},t) \simeq \sigma_{k}^{(D')} (x) \), where \( x_{\text{ref}} \) is a suitable reference position. For \( k \gtrsim L/(2\Delta x) \), instead, the profile is strongly disturbed during the evolution, indicating that the corresponding eigenmode is unstable under the discretization used here. The instability is amplified upon increasing \( k \). We find that using for the evaluation of equation (F.19a) a value of \( k \simeq 0.6L/\Delta x \) for the largest mode number accurately captures the crossover-time observed in figure G3(c).

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