Phenomenological Aspects of Isospin Violation in the Nuclear Force

by

U. van Kolck
Department of Physics
University of Washington
Seattle, WA 98195

and

J.L. Friar and T. Goldman
Theoretical Division
Los Alamos National Laboratory
Los Alamos, NM 87545

Abstract

Phenomenological Lagrangians and dimensional power counting are used to assess isospin violation in the nucleon-nucleon force. The $\pi NN$ coupling constants (including the Goldberger-Treiman discrepancy), charge-symmetry breaking, and meson-mixing models are examined. A one-loop analysis of the isospin-violating $\pi NN$ coupling constants is performed using chiral perturbation theory. Meson-mixing models and the $^3\text{He} - ^3\text{H}$ mass difference are also discussed in the context of naturalness.
Isospin violation in the strong interaction remains one of the least understood aspects of the nuclear force [1]. After the long-range and well-understood electromagnetic interaction is removed from the nucleon-nucleon (NN) interaction, small but significant differences (\(\sim 1\%\)) exist between the \(nn\), \(pp\), and (\(T = 1\)) \(np\) interactions. We briefly discuss these differences below in the context of phenomenological Lagrangians that exhibit (broken) chiral symmetry as well as isospin violation [2]. A detailed discussion will be presented elsewhere [3]. Our topics are: (1) isospin violation in the \(\pi NN\) coupling constants; (2) the sizes of \(\rho - \omega\) and \(\pi - \eta\) mixing forces [4, 5], as well as forces from \((h_1, b_1)\) and \((a_1, f_1)\) meson mixing; (3) naturalness and the \(^3\text{He} - ^3\text{H}\) mass difference.

Isospin violation in the NN force is usually described [1] in terms of classes of possible isospin operators for nucleons 1 and 2. These can be taken to be: (I) \(1 \cdot t(1)\); (II) \(t_z(1)t_z(2) - t(1) \cdot t(2)\); (III) \(t_z(1) + t_z(2)\); (IV) \([t(1) \times t(2)]_z\) and \((t_z(1) - t_z(2))\), and are referred to as: (I) charge-independent; (II) charge-dependent; (III) and (IV) charge asymmetric or charge-symmetry breaking (CSB). Classes (II) and (IV) forces vanish for \(pp\) and \(nn\) systems, while class (III) vanishes for the \(np\) system. Class (IV) forces involve \(T = 0\) and \(T = 1\) mixing in the \(np\) system. The best evidence for class (II) forces is obtained from NN phase-shift analyses (PSA). The recent Nijmegen PSA [6, 7] finds a fairly strong difference between \(np\) (\(T = 1\)) and \(pp\) \(1^S_0\) phase shifts and determines them separately. The numbers that best characterize this charge dependence are the \(1^S_0\) scattering lengths after removal of all long-range electromagnetic effects: \(a_{np} (-23.75 \text{ fm})\) and \(a_{pp} (-17.3 \text{ fm})\). This PSA also finds that the \(3^P_J\) waves are charge dependent.

The various \(\pi NN\) coupling constants \(f^2\) can be written in the generic form

\[
f^2 = \left( \frac{1}{4\pi} \right) \left( \frac{g_A m_{\pi^+}}{2f_\pi} \right)^2,
\]

where \(g_A\) is the axial vector coupling constant (1.2573(28)) [5], \(f_\pi\) is the pion decay constant (92.4(3) MeV) [5], the charged pion mass is conventional and is entered to make \(f^2\) dimensionless, while \(d - 1\) is the Goldberger-Treiman (GT) discrepancy [4]. The latter is positive and a measure of chiral-symmetry breaking and is defined in terms of the pseudoscalar form of the \(\pi NN\) coupling constant, \(G\), and the nucleon mass, \(M\), by \(G/M = g_A d/f_\pi\). Because the Nijmegen PSA explicitly includes the tail of the NN force (OPEP), this procedure can be used to determine all of the \(\pi NN\) coupling constants, \(\pi^0 pp\), \(\pi^0 nn\), and \(\pi^\pm np\), as well as the masses of the exchanged pions. Proton-proton scattering determines both the \(\pi^0\) mass and \(f_p^2 = f_{pp\pi^0}\), while neutron-proton scattering determines \(f_{np\pi^0} = f_{nn\pi^0} f_{pp\pi^0}\) and \(f_c^2 = f_{np\pi^-} f_{pn\pi^+}\) and the charged-pion mass. This PSA finds [6] an exchanged \(\pi^0\) mass of 135.6(13) MeV and a \(\pi^\pm\) mass of 139.4(10) MeV and the \(pp\) [6] and \(np\) [7] results (separated by a bar) listed in Table 1.
Table 1: Pion-nucleon coupling constants determined by the Nijmegen\[6, 7\] PSA.

| $f^2_{\pi^0 pp}$ | $f_{\pi^0 pp}f_{\pi^0 nn}$ | $f_{\pi^0 np}$ |
|-----------------|-----------------|----------------|
| 0.0745(6)       | 0.0745(9)       | 0.0748(3)      |

We will return to these results shortly. We further note that setting $d$ to 1 in eqn. (1) produces $f^2(d = 1) = 0.0718(5)$, which demonstrates that the measured values are close to the chiral limit.

A separate experiment \[10\] on $\pi^- + d \rightarrow n + n + \gamma$ determined the $nn$ $^1S_0$ scattering length. After removal of small electromagnetic effects \[1\] this is found to be $a_{nn} = -18.8(4)$ fm. The difference of $-1.5(5)$ fm between $a_{nn}$ and $a_{pp}$ is the best experimental information \[1\] that we have on CSB in the $NN$ force. Other evidence comes from the $^3\text{He} - ^3\text{H}$ binding energy difference of 764 keV. Approximately 693 keV is attributable to the Coulomb interaction between protons (648 keV) and to small nucleon magnetic moment and velocity-dependent parts of the Breit interaction and to the nucleons’ mass difference in the kinetic energy (45 keV) \[1\]. The remaining $\sim 70(25)$ keV is consistent with any short-range CSB interaction that produces the $-1.5(5)$ fm difference between $nn$ and $pp$ scattering lengths.

A systematic development was recently made \[2\] of isospin-violating interactions in the context of the general effective chiral Lagrangian, which we briefly summarize now. These have three distinct origins: (1) the mass difference of the $u$- and $d$-quarks in QCD specified by $\epsilon \equiv \frac{m_d - m_u}{m_d + m_u} \sim 0.3$ \[12\]; (2) the (frozen-out) effect of high-frequency photons exchanged between quarks in a nuclear system; (3) the effect of soft photons that are exchanged or modify vertices. Because only a subset of the last type of process has been implemented (or even calculated) we will not further consider these corrections. They are expected to be of order $(\alpha/\pi)$ times the OPEP coupling constants and therefore probably not larger than the errors on those quantities \[13\].

This formalism is designed to separate the effects of the short-range QCD dynamics from those of long range, which are mostly constrained by chiral symmetry. The former are usually calculated using simple models, while in our approach they appear in the Lagrangian as parameters whose values are not determined by symmetry. We make only the minimal assumption of naturalness \[14\], namely, that unless suppressed by some symmetry these parameters are given by naive dimensional analysis \[15\]. The (magnitudes of the) dimensionless factors that result from such an analysis should be numbers that are $O(1)$ (we will see later that in our case all are in the range 0.5-2.0). The long-range effects are obtained from the pion cloud, including the contributions of loops.
We wish to perform a one-loop calculation of isospin violation in the one-pion-exchange potential (OPEP) due to the first two mechanisms listed above, and later discuss shorter-range effects. We can organize the calculation using power counting \cite{10} to characterize the individual vertices: \(\Delta = d + n_F/2 - 2\), where \(d\) is the number of derivatives or pion masses at each vertex and \(n_F\) is the number of fermion fields, while \(\Delta\) determines the number of powers of a typical small momentum (i.e., \(Q^\Delta\)) characteristic of an amplitude. Chiral symmetry dictates that \(\Delta\) must be nonnegative for strong interactions. Adding a loop to an amplitude constructed from these building blocks increases the exponent of \(Q\) by 2. (Electromagnetic interactions can produce vertices with smaller values of \(\Delta\), but are suppressed by powers of \(\alpha(\sim 1/137)\).)

There are three relevant isospin-conserving interactions of lowest order (\(\Delta = 0\)):

\[
L_{IC}^{(0)} = -\frac{g_A}{f_\pi} N \overrightarrow{\sigma} \cdot \nabla (t \cdot \pi) N (1 - \frac{\pi^2}{4f_\pi^2}) - \frac{1}{2f_\pi^2} N t \times \pi \cdot \pi N + \frac{m_\pi^2 \pi^4 - 2\pi^2 (\partial_\mu \pi)^2}{8f_\pi^2}. \quad (2a)
\]

There are three appropriate isospin-violating interactions arising from quark-mass differences (\(L_{qm}^{(\Delta)}\))

\[
L_{qm} = -\delta M \overrightarrow{N} (t_3 - \frac{\pi_3 t \cdot \pi}{2f_\pi^2}) N + \frac{\beta_1}{2f_\pi} \overrightarrow{N} \sigma \cdot \nabla \pi_3 N + \frac{\delta M}{2M^2} \overrightarrow{N} p^2 t_3 N, \quad (2b)
\]

corresponding to \(\Delta = 1, 2,\) and 3, respectively. The first and third terms reflect differences in the nucleon rest and kinetic masses (\(\delta M \sim \epsilon m_\pi^2 / \Lambda\)), while \(\beta_1 \sim \epsilon m_\pi^2 / \Lambda^2\) is an isospin-violating \(\pi NN\) coupling constant. The constants in eqn. (2b) are chiral-symmetry breaking and hence proportional to \(\epsilon m_\pi^2\), while \(\Lambda\) is the QCD large-mass scale (\(\sim 1\) GeV). The powers of \(m_\pi\) contribute to the power counting.

There are also isospin-violating interactions of the EM type (\(L_{EM}^{(\Delta)}\))

\[
L_{EM} = -\frac{\tau m_\pi^2}{2} (\pi^2 - \pi_3^2) (1 - \frac{\pi^2}{2f_\pi^2}) - \delta M \overrightarrow{N} t_3 N + \frac{\beta_3}{2f_\pi} \overrightarrow{N} \sigma \cdot \nabla \pi_3 N
\]
\[
+ \frac{\beta_{10}}{2f_\pi} \overrightarrow{N} t_3 \sigma \cdot \nabla \pi_3 N + \left(\frac{\beta_4 + \beta_5}{2f_\pi}\right) \overrightarrow{N} \sigma \cdot \nabla (\pi \times t)_3 N, \quad (2c)
\]

where \(\tau m_\pi^2 \sim \alpha \Lambda^2 / \pi\), \(\delta M \sim \delta m_\pi^2 / \Lambda\), and \(\beta_i \sim \delta m_\pi^2 / \Lambda^2\). The first two terms are of orders \(-2\) and \(-1\), respectively, while the remaining three are of order 0. The last term in eqn. (2c) will not contribute to OPEP because of symmetry, while \(\delta M\) can be added to \(\delta M\) to form the complete nucleon mass and \(\beta_3 + \beta_1\) is the complete CSB \(\pi NN\) vertex, which we will continue to denote by \(\beta_1\).

The large pion-mass difference is primarily of electromagnetic origin, and this allows the relative sizes of the interactions due to quark-mass differences (\(L_{qm}^{(\Delta)}\)) and
hard electromagnetic processes \( (L_{EM}^{(\Delta)}) \) to be related \([2, 3]\). One expects on this basis that \( L_{EM}^{(-2)} \sim L_{qm}^{(1)} \), and the first, second, and remaining terms in eqn. (2c) can be taken to be comparable to terms in eqn. (2b) of orders 1, 2, and 3. In order to be consistent to this order we also need one-loop corrections (equivalent to \( \Delta = 3 \)) obtained from a single \( L_{qm}^{(1)} \) or \( L_{EM}^{(-2)} \) and various \( L_{IC}^{(0)} \) terms from our Lagrangian

\[
\Delta L = L_{IC} + L_{qm} + L_{EM} + \cdots .
\] (2d)

Only nonanalytic terms are kept, since the others can be absorbed into the parameters of the Lagrangian (see, for example, ref.\([17]\)).

Figure 1: The pion-mass-difference effect in a pion vertex correction is shown in (a), while the same effect on the nucleon self energy is illustrated in (b) and (c). Pions are indicated by dashed lines, nucleons by solid lines, and the charged-neutral pion-mass difference is indicated by a cross. The circled cross indicates an off-shell pion.

We find that 6 classes of diagrams have nonvanishing contributions to this order. Four classes cancel in pairs: isospin-violating nucleon self-energy graphs containing nucleon-mass differences cancel the corresponding vertex corrections, while the isospin-conserving pion loops containing a pion-mass difference in vertex bubbles (from the first term in \( L_{IC} \)) cancel the corresponding pion self-energy bubbles (from the last term in \( L_{IC} \)). The graphs in Figure (1) remain. The nucleon self-energy graphs are isospin conserving. The vertex correction in Fig. (1a) gives the sole isospin-violating nonanalytic term

\[
\Delta \bar{\beta}_{10} = \frac{2 \delta m_{\pi}^2 g_A^3 \log(\mu/m_{\pi})}{(4\pi f_{\pi})^2},
\] (3)

where \( \mu \) is the renormalization scale. Choosing \( \mu \sim m_{\rho} \) gives \( \Delta \bar{\beta}_{10} = 6 \cdot 10^{-3} \). Such terms are special because of their analytic structure and because they are typically larger. Our results below for \( \bar{\beta}_{10} \) will include \( \Delta \bar{\beta}_{10} \).
The one-pion-range force generated by the interactions in eqn. (2) has the form in momentum space

\[ V_\pi(q) = \left[ \frac{g_A \sigma(1) \cdot q \sigma(2) \cdot q}{q^2 + m_\pi^2} \right] \left( g_A d^2 t(1) \cdot t(2) - \frac{\beta_1}{2} (t_z(1) + t_z(2)) - \bar{\beta}_{10} t_z(1)t_z(2) \right), \]

and generates charge-asymmetric and charge-dependent πNN coupling constants in the usual (charge-independent) OPEP. Because the latter is conventionally defined in terms of \( f^2 \) (as in eqn. (1)), it involves \( d \) and is proportional to \( \frac{g_A^2 d^2}{f^2} t_1 \cdot t_2 \). In the experimental data we identify an isospin-conserving Goldberger-Treiman discrepancy, \( d - 1 \), and attribute any isospin violation to \( \beta_1 \) and \( \bar{\beta}_{10} \). That is, we use eqn. (1) to define and extract from the data the quantities \( d_{pp}, d_{nn}, d_c, \) and \( d_0(\approx 1) \), from which \( d, \beta_1, \) and \( \bar{\beta}_{10} \) can be easily extracted:

\[ \beta_1 = \frac{g_A}{2} (d_{nn} - d_{pp}), \]

\[ d = d_c, \]

\[ \bar{\beta}_{10} = g_A (2d_c - d_{pp} - d_{nn}). \]

If \( pp \) data are not used we cannot determine \( \beta_1 \) and we must replace eqn. (5c) by

\[ \bar{\beta}_{10} = 2g_A (d_c - d_0). \]

The results are shown in Table 2.

| Type      | \( d - 1 \)     | \( \beta_1 \)          | \( \bar{\beta}_{10} \) |
|-----------|-----------------|-------------------------|-------------------------|
| \( np \)  | 2.1(5)%         | 0(9) \cdot 10^{-3}     | 5(18) \cdot 10^{-3}     |
| \( pp \)  | 2.1(5)%         | 5(18) \cdot 10^{-3}    |                         |

As noted earlier, the GT discrepancy is smaller than many previous values, but is consistent with the dimensional estimate: \( d - 1 \approx m_\pi^2 / \Lambda^2 \approx 2\% \). The isospin-violating parameters \( \beta_1 \) and \( \bar{\beta}_{10} \) are consistent with zero. The parameter \( \beta_1 \) should be of order \( (\epsilon m_\pi^2 / \Lambda^2) \) or \( \approx 6 \cdot 10^{-3} \), which is slightly smaller than the uncertainty in \( \beta_1 \). A direct determination of \( \beta_1 \) would probably require reducing the uncertainty by a factor of two, and this would be quite difficult. The nonanalytic contribution to \( \bar{\beta}_{10} \) produced in the one-loop calculation is of comparable size. Unfortunately the error bars are twice as large in this case.

Note that the rest masses of the nucleons can play no role outside of loops, while the different nuclear kinetic masses in eqn. (2b) are properly treated in the Nijmegen PSA.
In addition to the one-pion-range potentials there are isospin-violating short-range potentials. The leading-order terms have $\Delta = 2$ and are given by \[2, 3\]

$$L^{(2)}_{\text{qm}} = \gamma_s(Nt_3N)(\overline{NN}) + \gamma_{\sigma}(Nt_3\sigma N) \cdot (\overline{N}\sigma N),$$  

(6)

where $\gamma_i \sim \epsilon m^2_\pi/f^2_\pi \Lambda^2$. According to the definitions presented earlier, these interactions will generate nuclear forces of class (III). As expected from power counting, the pion-mass difference from eqn. (2c) generates a relatively large and well-known contribution to class (II) through differences in the range of OPEP that it produces.

Class (IV) forces, on the other hand, appear only in higher orders; as a consequence of the structure of the chiral Lagrangian, they are therefore the smallest \[2, 3\].

We now briefly discuss mesonic models. In the well-studied mesonic sector it has been found that the parameters of the chiral Lagrangian are reliably estimated if they are assumed to be saturated by resonance exchange \[18\]. Analogous models based on isospin mixing have been used extensively to estimate CSB in the $NN$ force (see for example refs.\[4, 5\]). The isospin mixing of $\rho$ and $\omega$ mesons produces a class (III) force \[4\], which for small $q^2(\ll m^2_\omega \sim m^2_\rho)$ takes the momentum-space form

$$V_{\rho\omega}(q) = -\frac{g_\rho g_\omega \langle \rho^0 | H | \omega \rangle}{2 m^4_\rho} [t_z(1) + t_z(2)].$$  

(7a)

Comparison with eqn. (6) shows that this mechanism provides a contribution to $\gamma_s$:

$$\gamma^\rho_{\omega} = \frac{g_\rho g_\omega \langle \rho^0 | H | \omega \rangle}{2 m^4_\rho} \equiv c_{\rho\omega}(\epsilon m^2_\pi f^2_\pi) \Lambda^2.$$  

(7b)

Using standard values for coupling constants and matrix elements measured in $e^+ + e^- \rightarrow \pi^+ + \pi^-$ in storage rings for $s \sim m^2_\rho$ ($g_\rho \sim 5.5$, $g_\omega \sim 15.2$, and $\langle \rho^0 | H | \omega \rangle \sim -4500$ MeV$^2$ \[1, 4\]) we can calculate $\gamma^\rho_{\omega} \sim -0.54$ GeV$^{-2}$. Using $\Lambda \sim m_\rho$, we can solve eqn. (7b) for the dimensionless parameter $c_{\rho\omega}$ and find $c_{\rho\omega} \approx -0.5$, a value that is not unnatural. The on-shell mixing-matrix element ($\equiv -0.8(\epsilon m^2_\rho)$), the $\rho NN$ coupling constant \[13\] ($g_\rho/m_\rho \approx 0.66/f_\pi$) and the $\omega NN$ coupling constant ($g_\omega/m_\omega \approx 1.8/f_\pi$) are also natural. Therefore we can conclude that the assumption of saturation of $\gamma_s$ by on-shell $\rho - \omega$ mixing is consistent with naturalness. This has an important consequence. The fact that the on-shell $\rho - \omega$ mixing model does provide the necessary CSB in the $NN$ system implies that the latter can more generally be explained by a chiral Lagrangian with natural parameters. Then it is obvious that any mechanism or sum of mechanisms that produces a $\gamma_s$ of natural size (and correct sign) can potentially account for the observed CSB, both in $NN$ scattering and in the $^3\text{He} - ^3\text{H}$ mass difference. For example, a big off-shell suppression of $\rho - \omega$ mixing (as suggested, e.g., in ref.\[21\]) would mean that we have to relax the saturation assumption so
that other processes (including those not involving resonances) restore naturalness. This demonstrates the strength of the model-independent chiral perturbation theory approach advocated here.

Other meson-mixing forces can also be calculated. The π − η mixing force \[4, 5\] has the form (and again neglecting \(q^2\) with respect to \(m_\eta^2\)):

\[
V_{\pi\eta}(q) = \frac{g_\eta}{2f_\pi^2m_\eta^2} \frac{\sigma(1) \cdot q \sigma(2) \cdot q}{(q^2 + m_\pi^2)} (t_z(1) + t_z(2)),
\]

where we have defined \(g_\eta/M \equiv g_\eta/f_\pi\) and have used the GT relation. Comparing to eqn. (4) leads to a model-dependent prediction for \(\beta_1\):

\[
\beta_1^{\eta} = \frac{\bar{\eta}}{m_\eta^2} \langle \pi^0 | H | \eta \rangle \equiv c_\eta (\frac{\epsilon m_\pi^2}{\Lambda^2}),
\]

Using \[1, 4, 5\] \(\bar{\eta} \sim 0.25\), \(m_\eta = 550\) MeV, and \(\langle \pi^0 | H | \eta \rangle \sim -4200\) MeV\(^2\) (\(\approx -0.7(\epsilon m_\pi^2)\)), we find \(\beta_1^{\eta} \sim -3.5 \cdot 10^{-3}\). This is less than the upper limit for \(\beta_1\) that we found earlier. Using \(\Lambda \sim 1\) GeV, we can solve eqn. (8b) for \(c_\eta\) and find \(c_\eta \approx -0.6\), so that saturation of \(\beta_1\) by on-shell mixing is also consistent with naturalness. This force does not include the effect of \(\pi - \eta'\) mixing, which may be nonnegligible. Studies of \(\pi - \eta\) mixing suggest that it is not greatly suppressed in a nucleus \[21\].

What simple mechanism could, likewise, produce a natural \(\gamma_\sigma\) term in eqn. (6)? If we restrict ourselves to single-meson exchanges in the static limit without derivative coupling (which are simplest and perhaps dominant), that meson must couple to one of the Dirac operators (1, \(\gamma_5\), \(\gamma^\mu\), \(\gamma_5\gamma^\mu\), \(\sigma_{\mu\nu}\)). The matrix \(\gamma_5\) has no static limit, while \(\gamma^\mu\) and 1 have a spin-independent static limit. A tensor meson has a symmetric spin tensor, which cannot couple to the antisymmetric \(\sigma_{\mu\nu}\). This leaves \(\gamma_5\gamma^\mu\), whose static limit is \(\sigma\). Hence, axial-vector \((1^+)\) meson exchange can generate \(\gamma_\sigma\). Candidates \[8\] with appropriate isospins are \(h_1(1170) - b_1(1235)\) and \(a_1(1260) - f_1(1285)\).

We conclude that the isospin-violating mechanisms that have been used to explain the \(^3\)He - \(^3\)H mass difference are of natural size, although the effect of axial-vector-meson mixing has never been calculated (or considered heretofore). In particular, as we have shown, there is no evidence for large isospin violation in the πNN coupling constants, one of which subsumes the effect of \(\pi - \eta\) mixing. If the \(\rho - \omega\) mixing force is much smaller than the dimensional estimate, one expects some other mechanism to restore the naturalness of \(\gamma_\sigma\); otherwise, we would be facing a less than ideal situation where a number of small effects would have to conspire coherently to make up the difference. However, the more important conclusion transcends models. We find that the observed magnitude of CSB by class (III) nuclear forces results from the
symmetries and naturalness of QCD. This is the power of chiral perturbation theory: questions of specific mechanisms need neither be posed nor answered.

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