Distribution Network Sag Assessment under Stochastic Faults  
Based on Network Transformation and Fitting Integral

Hong-xin Li*, Hua-ying Zhang, Jing-wen Ai and Yan Li
Electric Power Research Institute Shenzhen Power Supply Co., Ltd., Shenzhen 518000, China
*Corresponding author

Keywords: Network transformation, Fitting integral, Random faults, Voltage sag assessment.

Abstract. In this paper, according to the structure and load characteristics of the distribution network, a network transformation algorithm is proposed. The simplified analysis circuit with the root node of the distribution network as the observation port is obtained by a series-parallel transformation method, and then the network state is solved. The fitting method is used to obtain the relationship between the voltage of each node and the fault location, and weighted integral is used to evaluate the expected value of voltage sag probability of all nodes and the whole distribution network system. Taking the typical systems of IEEE 33-bus as the simulation object, the results show that the error between the integral fitting algorithm and the Monte Carlo stochastic algorithm is within the allowable range, and the former greatly improves the calculation speed.

Introduction

The short-circuit fault is the most common fault in the power system[1]. When a short-circuit occurs, the current at the short-circuit point will increase sharply and the voltage will drop sharply[2]. Other nodes in the network will also be affected to a certain extent, making the voltage level of the whole network to a certain extent, which may have a negative impact on the power system and users[3]. How to better evaluate the voltage sag caused by a short-circuit fault is worth exploring.

For voltage sag assessment, different scholars have put forward many different evaluation methods according to different indicators[4-6]. For example, in reference [7], the concept of entropy weight is quoted, and the weights are distributed according to load characteristics. The degree of voltage sag is evaluated by the Monte Carlo algorithm. In reference [8], the virtual node method is combined with the Monte Carlo method to set the voltage. Threshold, voltage sag region can be obtained by the forward and backward solution. Reference [9] synthesizes the temporary assessment indicators of domestic and foreign studies and puts forward a relatively complete evaluation system. The evaluation and estimation methods mentioned above are often combined with Monte Carlo random probability distribution. It is necessary to repeat a large number of random numbers to ensure better accuracy while ignoring the problem of time-consuming calculation, which is more disadvantageous to the increasingly large and complex distribution network.

Starting with the network transformation of the fault network, this paper analyses the simplest circuit of a network. According to its structural characteristics, the functional relationship between sag voltage and fault location is approximated, and then the sag expectation value of the whole distribution system is evaluated by integrating the function.

Network Transformation

Load and Line Model

The distribution network consists of transmission lines and user load. Compared with the transmission network, the load information and network information of the distribution network are
not strictly symmetrical in three phases, so the three-phase form is often used to describe the line and load situation to reflect the physical characteristics of each phase.

For user load, there is no direct coupling between phases, so it can be expressed in the form of matrix $Z_{\text{load}}^{a,b,c}$ as follows:

$$Z_{\text{load}}^{a,b,c} = \text{diag}(Z_{\text{load,a}}, Z_{\text{load,b}}, Z_{\text{load,c}})$$  \hspace{1cm} (1)

The user load can be calculated directly from the actual operation data in the distribution network.

$$Z_{\text{load,a}} = \frac{|V|^2}{P_a - jQ_a}$$  \hspace{1cm} (2)

For transmission lines, there is a certain coupling relationship between lines, that is, mutual inductance between different phase lines cannot be ignored. The impedance model of the three-phase transmission line can be expressed by matrix $Z_{\text{line}}^{a,b,c}$:

$$Z_{\text{line}}^{a,b,c} = \begin{bmatrix}
Z_{a-a} & Z_{a-b} & Z_{a-c} \\
Z_{b-a} & Z_{b-b} & Z_{b-c} \\
Z_{c-a} & Z_{c-b} & Z_{c-c}
\end{bmatrix}$$  \hspace{1cm} (3)

The diagonal elements of $Z_{\text{line}}^{a,b,c}$ in the matrix represent the self-impedance parameters of their respective phases, while the non-diagonal elements are the mutual impedance parameters.

**Equivalent Model of Short Circuit Fault**

When a short-circuit fault occurs in the power system, the most obvious change is that the voltage level of the network decreases as a whole, because the existence of short-circuit points reduces the equivalent impedance of each node of the network to the ground. The physical characteristics at the short-circuit point can be simulated by a three-phase grounded branch, as shown in Figure 1.

![Figure 1. Simulated grounding model for short-circuit fault.](image)

The equivalent models of different forms of short-circuit fault can be represented by the above figure, but the corresponding parameters are different.

When the three-phase short-circuit fault occurs, there is $Z_a = Z_b = Z_c = Z_g = 0$; When the single-phase grounding short-circuit fault (e.g. A phase) occurs, there is $Z_a = Z_g = 0, Z_b = Z_c = \infty$; When the two-phase short-circuit fault (e.g. B and C phase) occurs, there is $Z_b = Z_c = 0, Z_a = Z_g = \infty$; When the two-phase grounding short-circuit fault (e.g. B and C phase) occurs, there is $Z_b = Z_c = Z_g = 0, Z_a = \infty$.

When the short-circuit fault occurs at the node of the distribution network, it is equivalent to the node accessing the fault branch A shown in Figure 1. When the short-circuit fault occurs on the line, the original line is cut into two lines and connected to the fault branch A at the cut-off point. The number of nodes increases by 1 accordingly.

**Series-Parallel Transformation**

Series-parallel transformation is one of the commonly used methods in circuit analysis. Its characteristic is that it can be simplified and has no effect on the electrical operation of its external
structure when it is replaced by another new circuit structure. The principle of three-phase series-to-parallel conversion is briefly described below.

Assuming that there are two branches \( m \) and \( n \), the corresponding impedance matrices are \( Z_{m}^{a,b,c} \) and \( Z_{n}^{a,b,c} \), respectively. When two branches are connected in series, the equivalent impedance matrix is:

\[
Z_{eq}^{a,b,c} = Z_{m}^{a,b,c} + Z_{n}^{a,b,c}
\]  

(4)

While there are connect in parallel, the equivalent impedance matrix is:

\[
Z_{eq}^{a,b,c} = \left( (Z_{m}^{a,b,c})^{-1} + (Z_{n}^{a,b,c})^{-1} \right)^{-1}
\]  

(5)

In the distribution network, it is often a radial hybrid network, which generally does not form a ring network. As long as the above series-parallel transformation formula is used, the equivalent circuit can be gradually obtained from the end of the load to the root node of the distribution network, and the information of the network state can be accurately obtained.

**Fitting and Integral**

**Short-Circuit Voltage Fitting**

For short-circuit grounding fault in distribution network, the simplified equivalent of fault phase can be shown in Figure 2 below.

Assuming that a short-circuit fault occurs in branch \( Z_{L} \), \( K \) is the node to be calculated. \( Z_{N} \) and \( Z_{G} \) represent the general series-parallel relationship with branch \( Z_{L} \). When the fault occurs in branch \( Z_{L} \), it is equivalent to dividing the line into two sections at the fault point and using \( \lambda \) to indicate the exact location of the fault point and the fault line. According to the equivalent topology, we can get:

\[
U_{K} (\lambda) = U_{M} \cdot \frac{\lambda Z_{L} \cdot Z_{g} + Z_{N}}{\lambda Z_{L} + Z_{g} + Z_{N} + Z_{M}} = \frac{U_{M} \cdot (Z_{g} + Z_{N}) \cdot \lambda Z_{L} + U_{M} \cdot Z_{N} \cdot Z_{g}}{(Z_{g} + Z_{N} + Z_{M}) \cdot \lambda Z_{L} + Z_{N} \cdot Z_{g} + Z_{M} \cdot Z_{g}}
\]  

(6)

According to the characteristics of observation Eq.(6), the fitting function of fault phase voltage underground fault can be set as follows:

\[
U_{K, eq} (\lambda) = \frac{a\lambda + b}{\lambda + c}
\]  

(7)

The network transformation of phase-to-phase faults is more complex than that of grounding faults, but the node \( K \) voltage \( U_{K} \) increases monotonously with the increase of the relative ratio of fault location \( \lambda \). Therefore, the fitting of \( U_{K} \) by using the commonly used primary function can also meet the fitting error requirements.
Voltage Sag Integral Evaluation

In order to evaluate the effect of short-circuit fault on voltage sag, the expected value of voltage sag can be obtained by integral form. Combining with the fitting formula $U_K$ proposed above, the voltage sag of a calculated node $K$ is integrated with the $[0,1]$ interval.

$$
E_0(U_K) = \int_0^1 U_{K_{eq}}(\lambda) \, d\lambda
$$

(9)

In actual power system operation, the sensitivity of each user to power quality requirements, the economic losses caused by voltage sags in history and the impact on social stability are different. Therefore, the differential analysis of each user should be given the corresponding weight. At the same time, the types of faults and the probability of faults should be fully taken into account, so that the voltage sag assessment of the whole distribution network can have higher reference value. Assuming the failure probability of $b$ branches and four types of short-circuit fault probability, the fault location and fault type are selected randomly to simulate and calculate the voltage of $n$ nodes. The expected voltage sag of the whole distribution network can be expressed by the following formula.

$$
E_0 = \sum_{j=1}^{b} \sum_{k=1}^{n} t(j)l(k)w(i)E_0(U_i)
$$

(10)

From historical fault information, various types of fault proportions $t(j)$ can be determined. The failure probability of a branch $l(k)$ can be set according to the ratio of the length of the line $l$ to the length of the whole network bus $\sum_{l=1}^{b} l$. $w(i)$ represents the sensitivity of user $i$ to voltage sag and can determine the weight distribution according to the actual impact.

Case Simulation

In order to verify the validity of the fitting integration method based on network transformation, the typical systems of IEEE 33-bus is selected in the simulation of this paper, which is shown in Figure 3.

From historical fault information, various types of fault proportions $t(j)$ can be determined. The failure probability of a branch $l(k)$ can be set according to the ratio of the length of the line $l$ to the length of the whole network bus $\sum_{l=1}^{b} l$. $w(i)$ represents the sensitivity of user $i$ to voltage sag and can determine the weight distribution according to the actual impact.

Table 1. Comparisons of fitting method and network transformation method.

| Fault location | Fitting method (p.u.) | Network Transformation (p.u.) | maximum error |
|----------------|-----------------------|------------------------------|---------------|
| \( \lambda \)  |          |                             |               |
| 10%            | 0.957    | 0.459                       | 0.963         |
| 30%            | 0.959    | 0.463                       | 0.964         |
| 50%            | 0.959    | 0.484                       | 0.965         |
| 70%            | 0.960    | 0.506                       | 0.966         |
| 90%            | 0.961    | 0.517                       | 0.968         |
The results of voltage function fitting algorithm are very close to those of network transformation algorithm. The maximum absolute error is only 1.38%, which is less than 5% of the reasonable error. It can better reflect the relationship between sag voltage and fault location.

And then, Fitting integral method and Monte Carlo method were used to simulate 1500 random faults to complete the temporary assessment of The typical systems of IEEE 33-bus. The results are shown in Table 2.

| Expectations of Node 8 (p.u.) | Expectations of Node 14 (p.u.) | Expectations of Node 22 (p.u.) | Expectations of Full Distribution Network (p.u.) | computing time (s) |
|------------------------------|-------------------------------|--------------------------------|-----------------------------------------------|--------------------|
| Fitting integral method      | 0.872                         | 0.691                          | 0.907                                         | 0.864              |
| Monte Carlo method           | 0.891                         | 0.672                          | 0.921                                         | 0.879              |
| error                        | 1.02%                         | 2.83%                          | 1.52%                                         | 1.71%              |

Table 2 shows that the evaluation results of Fitting integral method and Monte Carlo simulation method are very similar. At the same time, the two methods have the same expected evaluation order for randomly selected nodes 8, 14 and 22.

**Conclusion**

Through the simulation example of the typical systems of IEEE 33-bus, the following conclusions are obtained:

(1) When a short-circuit grounding fault occurs in the distribution network, the fitting function of each node voltage obtained by network transformation has higher accuracy. Especially when the phase of grounding fault is fitted, the maximum error of simulation results is only 1.38%. It can be seen that using this form to fit the voltage sag of the distribution network when the short circuit occurs has reference significance.

(2) Fitting integral method is used to evaluate the voltage sag of the distribution network. Compared with the traditional Monte Carlo simulation, the ranking of node evaluation of the two methods is identical. For the whole distribution network system, the error between the two methods is less than 2%, and the former has higher accuracy. At the same time, the former calculation speed is faster than the traditional Monte Carlo simulation, and the speed advantage is more obvious in the larger scale of distribution network sag assessment.

**Acknowledgement**

This research was financially supported by the Electric Power Research Institute Shenzhen Power Supply Co., Ltd. Science and Technology Project.

**References**

[1] An-ping Hu, Yi-bin Tao, Jia-yuan Chen, Jun Zhuang. Review of Voltage Sag Mitigation Measures and Equipment, J. Power Electronics. 53(07) (2019) 1-5+10.

[2] Yun-wei Li, Jin-wei He. Distribution system harmonic compensation methods [J]. IEEE Industrial Electronics Magazine, 12 (2014) 18-31.

[3] Lin Zhou, Hong-chun Wu, Jing Meng, et al. Study of the voltage sag analysis methods[J].High Voltage Engineering, 5 (2008) 1010-1016.
[4] Xuan Li, Ke Xu, Li Xiao, Analysis of Voltage Sag in Regional Power Grid and Research on Its Evaluation Method, J. Sichuan Electric Power Technology. 41(06) (2018) 41-46.

[5] Xian-yong Xiao, Xiu-mei Tan, Ying Wang, Zhong Xu, Voltage Sag Economic Losses Assessment Based on Electrical Characteristic-physical Attribute-perceived Loss, J. Proceedings of the CSEE. 38(S1) (2018), 105-110.

[6] Xi Ye, Kai-pei Liu, Zhi-wei Li, Voltage sag frequency assessment considering action characteristics of line protection in uncertain conditions, J. Electric Power Automation Equipment. 38(03) (2018) 169-176.

[7] Jiang Zeng, Dong-yang Cai, A Monte Carlo Assessment Method of Voltage Sags Based on Combination Weight, J. Power System Technology. 40(05) (2016)1469-1475.

[8] Kun Li, Xiao-Bo Liu, Ming-Hao Zhang, Juan He, Forward and backward analysis of voltage sag based on Monte Carlo method and virtual node method, J. Electric Power Science and Engineering. 34(08) (2018)3-49.

[9] J. van der Geer, J.A.J. Hanraads, R.A. Lupton, Study on Assessment System of Voltage Sags, J. Power System Technology. 34(06) (2010)128-133.