Fuel/Air Mixing in Reacting and Non-reacting Flows within a Dual-mode Combustor*

Takuya ARAKAWA,† Kiyoshi NOJIMA,† Kan KOBAYASHI, and Sadatake TOMIOKA*‡

1) Department of Aerospace Engineering, Tohoku University, Sendai, Miyagi 980–8579, Japan
2) Kisukasa Space Center, Japan Aerospace Exploration Agency, Kakuda, Miyagi 981–1525, Japan

An experimental investigation on fuel and air mixing characteristics within a dual-mode combustor is presented. To determine the dominant parameters of mixing characteristics of fuel with airflow, fuel (H\textsubscript{2} or C\textsubscript{2}H\textsubscript{4}) or inert gas (He, N\textsubscript{2} or Ar) were perpendicularly injected from plural circular orifices into high- or room-temperature M2.5 airflowes decelerated through pseudo-shockwave systems. Under similar dynamic pressure ratios, fuel mass flux contours downstream of the injector were similar between the H\textsubscript{2} and C\textsubscript{2}H\textsubscript{4} reacting cases. However, fuel mass flux contours were different between the C\textsubscript{2}H\textsubscript{4} reacting and Ar non-reacting cases due to the heat-release effect during local mixing. On the other hand, the heat-release effect on mixing efficiency was found to be minor in bulk-scale mixing evaluation. It was found that C\textsubscript{2}H\textsubscript{4} mixing efficiency with an equivalence ratio of 1.6 could be predicted within an error of a few percent from Ar mixing results under similar dynamic pressure ratio and similar peak pressure.

Key Words: Dual-mode Combustor, Ramjet-mode Operation, Mixing Parameters

Nomenclature

- \(A\): cross-sectional area
- \(C\): ratio of oxygen mass fraction between cold and hot airflows
- \(d\): injector orifice diameter
- \(G\): mass flux (\(G_{ave}\) as averaged)
- \(J\): dynamic pressure ratio
- \(K\): recovery ratio
- \(m\): mass flow rate
- \(M\): Mach number
- \(M_w\): convective Mach number
- \(p, P\): static pressure, total pressure
- \(t, T\): temperature, total temperature
- \(t_r\): normalized reaction delay time
- \(V\): velocity
- \(x\): streamwise location from injector
- \(y\): spanwise location from symmetry plane on injector-side wall
- \(z\): height from injector-side wall
- \(\alpha\): constant coefficient to convert injectant into fuel, Eq. (5)
- \(\phi, \phi'\): total, local equivalence ratio
- \(\gamma\): specific heat ratio
- \(\eta_c, \eta_f\): bulk, local combustion efficiency
- \(\lambda\): mass flux ratio
- \(\rho\): density
- \(\delta/x\): combustible shear growth rate

Subscripts

- \(a\): airflow
- \(c\): cold (unheated) airflow
- \(i\): injectant
- \(h\): hot (heated) airflow

1. Introduction

A supersonic combustion ramjet (scramjet) engine has been studied for high-speed flight in the atmosphere because the engine performance of the scramjet was higher than that of a rocket in the Mach 5–20 range using H\textsubscript{2} as fuel, and the Mach 5–10 range using hydrocarbon as fuel. At flights with lower Mach numbers, the combustor can be operated under subsonic combustion (ramjet-mode) using a throttling effect due to thermal choking in the scramjet engine flow-pass, thereby enhancing airflow compression similar to a conventional ramjet. The scramjet combustor capable of operating in both ramjet-mode and scramjet-mode is termed as a 'dual-mode combustor.'

A one-dimensional analysis showed that thermal choking in the combustor’s diverging section enables further fuel injection and further thrust production. Moreover, the choking location in the diverging section had an impact on the specific impulse during ramjet-mode operation. Therefore, the choking location should be controlled according to the flight Mach number. The choking location depends on heat release distribution, being related to (fuel and air) mixing and reaction. Thus, mixing or reacting should be controlled to attain choking at the targeted location. For reaction control, a possible idea is to set many flame holders such as cavities aligned in a streamwise direction. However, this idea is not practical because the combustor could suffer severe weight penalty and cooling requirements. Mixing control,
on the other hand, sounds practical because injection schemes with different characteristics\(^3\) and/or multi-staged injections\(^4\) are available.

It is known that a pseudo-shockwave system (PSW) occurs upstream of the fuel injector location during ramjet-mode operation.\(^5\) Then, the PSW dramatically changes fuel and air mixing characteristics compared to those of scramjet-mode operation.\(^6\) Thus, mixing characteristics under ramjet-mode operation should be investigated with the presence of the shock train. However, observation of mixing in reacting flows is hard to perform. The first problem is on the setup. Reacting tests need facilities to heat up the airflow for simulating enthalpy at the combustor entrance in a real flight. As a result, these facilities make experiments very costly. The second problem is measurement. Probes for gas sampling need to be cooled and the size limited in order to avoid aerodynamic choking within the combustor for intrusive measurement. Optical measurements still have the problem of uncertainty and optical access is difficult as well.

On the other hand, observation of mixing in non-reacting flows with room-temperature air and inert gas can mitigate problems. Furthermore, the PSW location and strength can be easily controlled by adjusting a mechanical valve downstream of the test section to attain the targeted pressure-rise. However, fuel and air conditions are different between reacting and non-reacting tests, even when the PSW strength is set to be equivalent.

The major challenge is to find a dominant parameter to match mixing characteristics between reacting and non-reacting flows when using the PSW. Research on fuel mixing has been conducted, thus some parameters have been proposed as candidates for the dominant parameter of the mixing process. Zakkay et al. reported that the potential core decay of coaxial jets was affected by the mass flux ratio, while mass concentration decay of the coaxial jets was not affected by the molecular weight of the injected gas.\(^7\) Similarly, the molecular weight effect on the injectant mass concentration decay was found to be small in the perpendicular injection case.\(^8\) Diskin et al. proposed empirical equations of H\(_2\) mixing efficiency distribution with the equivalence ratio as the dominant parameter.\(^9\),\(^10\) There are reports showing that fuel penetration would be related to the dynamic pressure ratio from H\(_2\) and air mixing and combustion tests.\(^11\),\(^12\) The dynamic pressure ratio was also used as the dominant parameter of the injectant trajectory.\(^13\) In research on turbulent shear growth rate, velocity ratio and density ratio were found to be effective parameters.\(^14\),\(^15\) However, few studies comparing reacting tests with non-reacting tests have been conducted, thus dominant parameters for matching mixing flow fields between reacting and non-reacting flows has not been proposed yet. These parameters, in turn, would lead to further general expression of the empirical mixing model compared to these mentioned above.

In the present study, fuel was injected into high-temperature airflows (i.e., reacting tests) and inert gas was injected into room-temperature airflows (i.e., non-reacting tests) to compare mixing characteristics with the PSW. In this study, H\(_2\) or C\(_2\)H\(_4\) were used as fuel in the reacting case, while He, N\(_2\) or Ar were used as the simulant fuel for safety reasons in the non-reacting case. To simulate pressure-rise by heat release in the non-reacting case, back-pressure was attained by adjusting the mechanical valve opening. Wall pressure distributions were measured in the streamwise direction to confirm that the PSW strength was close in every test condition. To evaluate mixing characteristics, fuel mass flux contours were deduced based on Pitot pressure measurement and gas sampling downstream of the injector. Then, mixing efficiency and combustion efficiency were evaluated with referencing NASA’s definition,\(^16\) to convert inert gas mixing efficiency in non-reacting flows into fuel mixing efficiency in reacting flows. However, mixing efficiency could not be evaluated in the inert gas injection case because the equivalence ratio is required to calculate mixing efficiency in the definition. Therefore, this study also discussed deducing inert gas mixing efficiency by assuming the inert gas equivalence ratio, to determine the feasibility of comparing the mixing efficiency between reacting and non-reacting flows.

2. Experimental Apparatus and Measurements

2.1. Experimental apparatus

Figure 1 shows a schematic diagram of the experimental apparatus used in reacting and non-reacting tests. All tests were conducted using a blowdown-type wind tunnel facility at the Japan Aerospace Exploration Agency (JAXA), Kakuda Space Center. This facility had a vitiation air heater by burning H\(_2\) with set-up O\(_2\). The heater could provide high enthalpy mixed with pure inert gas during hypersonic flight. The burnt gas was ‘vitiated’ because it included water vapor generated from H\(_2\) combustion. A Mach 2.5 facility nozzle with an exit of 94.3 × 51.0 mm was directly connected to the heater. To prevent pressure-rise propagation into the facility nozzle due to combustion or rising back-pressure, an isolator section 470–730 mm in length (i.e., varied for each condition) was added at the facility nozzle exit. In the non-reacting case, however, pressure-rise propagation reached the facility nozzle even with the isolator installed. Instead, a duct with four wedges placed on its corners was added to stabilize the PSW origin.

A 272-mm-long combustor section had an injector with four circular orifices with diameters (\(d\)) of 3.0 mm. The spanwise interval of orifices was 6.7\(d\), while the outer orifice center to side wall spacing was 5.7\(d\). Each injectant jet was choked at the orifice to keep the same flow rate if the back-pressure increased. In this paper, a right-hand system was used with the origin of the streamwise direction (\(x\)) at the injection location, that of the spanwise direction (\(y\)) at the combustor symmetry plane, and that of the height direction (\(z\)) at the injector wall. In the non-reacting case, back-pressure was attained by adjusting the mechanical valve opening. A settling chamber was added upstream of the mechanical valve to relieve flow asymmetry.

Sampling probes were settled 25\(d\) downstream of the injector location, where the injectant plume remained clearly...
even with the PSW.\textsuperscript{16} Two types of sampling probes, shown in Figs. 2(a) and 2(b), were used. One was a ‘water-cooled’ with a 10 mm interval for the reacting case, as shown in Fig. 2(a). This probe had a fine tip of 0.3 mm in diameter. Then, following expansion, water-cooled heat exchange ensured quenching of the reactions in the probe.\textsuperscript{17} For the non-reacting case as shown in Fig. 2(b), the other was non-cooled with a 4.0 mm interval (i.e., 1.0–3.0 mm near the wall) and a probe tip diameter of 0.6 mm.

### 2.2. Measurement and data reduction

The pressure in the air and injectant plenum chambers was monitored by pressure transducers. The flow rates of H\textsubscript{2} and O\textsubscript{2} using the vitiation air heater were measured using sharped-edged orifice flowmeters. The total pressure of the vitiated airflow was measured, while the total temperature of the vitiated airflow was evaluated using one-dimensional calculation. The airflow conditions were presented in Table 1. The repeatability of total pressure and temperature was within 3.9\% and 5.6\%, respectively. Wall pressure distributions were measured on the injector-sidewall and on the opposite-sidewall in the streamwise direction using electrical scanning pressure sensors (PSI System 8400). Sensor ranges were 0–340 kPa and 0–670 kPa, while an uncertainty was ±0.1\% of the full scale. Pitot pressure was also measured using these sensors.

Gas sampling was carried out only within the half-cross-section after checking the symmetry at y = 0. Sampled gas was analyzed by gas chromatography using thermal conductivity detector capable of detecting various gases (i.e., H\textsubscript{2}, O\textsubscript{2}, N\textsubscript{2}, CO, CH\textsubscript{4}, CO\textsubscript{2}, C\textsubscript{2}H\textsubscript{4}, C\textsubscript{2}H\textsubscript{6} and C\textsubscript{3}H\textsubscript{8} could be detected in this study) with an uncertainty of ±0.3 vol.\% for each component. The local Mach number was calculated from local composition, Pitot pressure and nearest wall pressure (assumed as static pressure).

In the reacting cases, the local equivalence ratio (\(\phi\)) and the local combustion efficiency (\(\eta_c\)) were evaluated at every sampling point. In this study, bulk combustion efficiency (\(\eta_{bc}\)) was defined as the ratio of fuel reacting with the amount of available fuel for reaction (i.e., denoted as ‘injectant-based’ evaluation). Injectant-based evaluation was used to deter-

| Test conditions. | \(P_{\text{a},\text{t}}\) | \(T_{\text{a},\text{t}}\) | Injectant | \(C_d\) | \(\lambda\) | \(\phi\) | \(M_{\text{a},\text{t}}\) | h/s |
|-----------------|----------------|----------------|----------------|--------|--------|--------|----------------|--------|
| H\textsubscript{2} | 0.55  | 1845 | 0.90 | 2.5 | 2.8 | 0.48 | 1.04 | 0.010 |
| C\textsubscript{2}H\textsubscript{4} | 0.54  | 2217 | 0.91 | 2.5 | 12.1 | 0.93 | 2.30 | 0.025 |
| He | 1.0 | 282 | 0.85 | 5.1 | 3.0 | 1.07 | 0.018 |
| C\textsubscript{2}H\textsubscript{4} | 0.54  | 2204 | 0.89 | 4.6 | 22.3 | 1.69 | 2.29 | 0.024 |
| He | 0.98  | 287 | 0.83 | 5.1 | 9.5 | 1.93 | 0.019 |
| Ar | 0.39  | 290 | 0.84 | 12.8 | 24.0 | 1.96 | 0.018 |
mine combustion efficiency ($\eta_c$) as follows,

$$
\eta_c = \left(1 - \frac{\sum (Y_i \rho V \Delta A)}{m_1 K_1} \right) \frac{1}{\phi}
$$

(1)

where $\phi$ denotes the equivalence ratio. The injectant recovery ratio ($K_i$) was defined as the ratio of the sampled injectant atoms and supplied injectant atoms. The error for $\eta_c$ was estimated by counting undetected injectant as either reacted (plus error) or unreacted (minus error).

Mixing efficiency ($\eta_m$) was defined as the amount of fuel that would react if complete reaction occurred without further mixing, divided by the amount of fuel that would react if the mixture was uniform. The mixing efficiency was formed as follows,

$$
\eta_m = \frac{\sum (Y_i \rho V \Delta A / \phi')}{m_1 K_1 / \phi'}
$$

(2)

As with the combustion efficiency, the error for $\eta_m$ was estimated by counting undetected injectant as either mixed (plus error) or unmixed (minus error). In Eq. (2), $\phi'$ should be defined for the non-reacting case with inert gas injection, so the present attempt to define the inert gas equivalence ratio will be discussed later.

Especially with C2H4 injection, other hydrocarbons such as CH4, C2H6 and CO and H2 were generated due to low O2 combustion at high-temperatures. As the result, C2H4 combustion efficiency was defined as the ratio of the amount of O2 consumed to the amount of available O2 for reaction (i.e., denoted as ‘O2-based’ evaluation), while mixing efficiency was defined as the amount of O2 would react if complete reaction occurred without further mixing, divided by O2 that would react if a mixture was uniform. The combustion efficiency and the mixing efficiency using O2-based evaluation were formed as follows;

$$
\eta_c = \left(1 - \frac{\sum (Y_o \rho V \Delta A)}{m_o K_o} \right) \frac{1}{\phi'}
$$

(3)

$$
\phi' = \begin{cases} 
1 & (\phi' \leq 1) \\
\phi' & (\phi' > 1)
\end{cases}
$$

(4)

Table 2. Combustion efficiency and mixing efficiency for the H2 injection case.

| Injectant-based | Error | O2-based | Error |
|-----------------|-------|----------|-------|
| $\eta_c$ | 0.64 | +0.04 | -0.07 |
| $\eta_m$ | 0.89 | +0.01 | -0.10 |

The results are shown in Table 2. The combustion efficiency and the mixing efficiency using O2-based evaluation were 0.03–0.04 points lower than those using the injectant fuel-based evaluation; the differences being within the error allowed for combustion and mixing efficiencies using the injectant-based evaluation. In the following, injectant-based evaluation was used to calculate the combustion and mixing efficiencies for the H2 and inert gas injection cases, while the O2-based evaluation was used only for the C2H4 injection case.

### 2.3. Test conditions

The test conditions are summarized in Table 1. The total temperature of the injectant gases was room-temperature in all cases. Dynamic pressure ratio ($J$) and mass flux ratio ($\lambda$) were deduced from the injectant condition at the injector orifice and the airflow condition at the facility nozzle exit as mere representative values. As will be shown later, one-dimensional assessed dynamic pressure of airflow that passed through the PSW was similar in all cases. Therefore, matching dynamic pressure based on the undisturbed airflow condition was similar to matching based on the airflow dynamic pressure after compression using PSW. Convective Mach number ($M_c$) and compressible shear growth rate ($\delta / \chi$) were also presented as references. Note that these parameters were for the shear layer experiment, as reported by Papamoschou and Roshko, while the injection scheme in the present study was perpendicular. Thus, the dynamic pressure ratio and the mass flux ratio were the major concerns in the present study. The dynamic pressure ratio was important for fuel penetration and trajectory as well as orifice diameter, while the mass flux ratio was also an important parameter for matching injectant potential core decay. In this study, test conditions matching both parameters at the same time between reacting and non-reacting tests were not available. Therefore, reacting and non-reacting tests were conducted under either similar dynamic pressure ratio or similar mass flux ratio.

At first, H2 mixing characteristics were targeted using a similar dynamic pressure ratio to that of the He injection case ($J = 5.1$). Unfortunately, pressure-rise by thermal choking penetrated the facility nozzle. Alternatively, H2 reacting tests were conducted using a similar mass flux ratio to that of the He injection case. Then, C2H4 reacting tests were conducted using a similar dynamic pressure ratio to that of the H2 reacting case. Moreover, the mixing flow field in the C2H4 reacting tests was simulated using Ar injection tests at a similar dynamic pressure ratio or similar mass flux ratio.
3. Results and Discussions

3.1. Comparison of mixing characteristics between reacting and non-reacting flow

3.1.1. Comparison using H₂, C₂H₄ and He injection

Figure 3 shows wall pressure distributions for the H₂, C₂H₄, and He injection cases. Wall pressure distributions were normalized with airflow total pressure to mitigate run-to-run deviations in test conditions. In the H₂ and C₂H₄ injection cases, pressure-rise penetration was observed upstream of the injector location. Pressure-rise peak was located at the injector location. These features indicated that the PSW was generated by back-pressure due to thermal choking. The airflow decelerated to subsonic speed upstream of the fuel injector.\(^{18}\)

For He injection, back-pressure was attained by adjusting the mechanical valve and settled downstream of the sampling point. The peak value using He injection corresponded reasonably well with that of H₂ injection, while the PSW penetration length (i.e., length between pressure-rise origin and injection location in the streamwise direction) using He injection was shorter than that of H₂ injection. The reason was momentum thickness difference in the boundary layer, revealing that the PSW should be anchored by wedges in the isolator section at \(x/d \sim -150\) when using He injection. However, the effect of PSW penetration length on fuel and air mixing was small in the present case. The injection location was in the mixing region of the PSW because the Mach number of the airflow at injection location (i.e., calculated using quasi-one-dimensional analysis\(^{4}\)) was about 0.6 for both H₂ and He injection cases. In the mixing region of the PSW, a shock-train was not formed,\(^{5}\) so the injectant would not be influenced by the shock-train. Although wedges changed the momentum thickness (i.e., PSW origin), they did not change that the injection location was in the mixing region. Therefore, the effect of PSW penetration length on mixing would be small in the present case with or without corner wedges.

The effect of PSW penetration length on the dynamic pressure ratio and mass flux ratio was evaluated using quasi-one-dimensional analysis.\(^{4}\) In this analysis, the total mass flow rate was preserved and heat loss to the wall was considered in solving the energy equation. Skin friction and heat transfer were calculated based on mean flow states over the cross-section using van Driest’s method\(^{19}\) and Reynolds’ analogy. However, momentum equation was violated because the measured wall pressure distributions were input for calculation. Table 3 shows a summarization of the dynamic pressure ratio and mass flux ratio deduced from wall pressure at the injection location calculated using quasi-one-dimensional analysis for three injection cases. The dynamic pressure ratio at the injection location using H₂ injection was similar to that using C₂H₄ injection, while the mass flux ratio at the injection location using H₂ injection was almost similar to that using He injection. These results indicate that the condition with similar dynamic pressure ratio or similar mass flux ratio can be preserved to the injection location under similar pressure-rise.

Figures 4(a), 4(b), and 4(c) show deduced injectant mass flux contours for H₂, C₂H₄ and He injection cases. Each contour only shows the half-cross-section at \(x/d = 25\) and dots denote the sampling points. The arrows on the \(y\)-axis show the spanwise location of injection orifices. Mass flux contours were normalized with the averaged value of injectant mass flux \((G_{ave})\) to enable fair comparison of contours for different injectants. The average value of injectant mass flux was defined as the total injectant mass flow rate detected divided by the cross-sectional area. The case of H₂ injection is shown in Fig. 4(a), with the injectant plume spreading in the spanwise direction. The plume near the sidewall \((y/d = 13.3)\) diffused in the height direction more than the plume near the symmetric plane \((y/d = 0)\). A high mass flux region over 2.0 was not observed near the sidewall, while one was observed at \(z/d = 5.2\) near the symmetric plane. These trends agreed with the C₂H₄ mass flux contour in Fig. 4(b), although the high mass flux region near the symmetric plane was closer to the injection sidewall than that with H₂ injection. In contrast, the He mass flux contour in Fig. 4(c) was different from that of the H₂ injection. A high mass flux region over 2.0 existed more extensively. For the plume near the sidewall at \(y/d = 13.3\), a steep gradient of mass flux was observed. Vertical diffusion was suppressed below \(z/d = 12\), showing that the high concentration region of the plume remained when compared to that of the H₂ injection.

3.1.2. Comparison using C₂H₄ and Ar injection

Figure 5 shows wall pressure distributions for the cases using C₂H₄ injection, Ar injection using a similar dynamic pressure ratio (i.e., denoted as ‘Ar-case1’), and Ar injection using a similar mass flux ratio (i.e., denoted as ‘Ar-case2’).
As with Fig. 3, the PSW was observed upstream of the injector in the case of C2H4 injection. In Ar-case2, the throttling effect resulting from perpendicular injection induced a rise of pressure as well as mechanical throttling. The peak pressure value was almost similar in all cases. Table 4 summarizes the dynamic pressure ratio and mass flux ratio from the injectant and air flow states at the injection location. These ratios were calculated using quasi-one-dimensional analysis as mentioned above. Conditions with similar dynamic pressure ratio and similar mass flux ratio between C2H4 and Ar were preserved.

Figures 6(a), 6(b), and 6(c) show the normalized injectant mass flux contours for C2H4 injection, Ar-case1 and Ar-case2. In spite of similar dynamic pressure ratios for the C2H4 injection case and Ar-case1, the mass flux contours were different from each other; a region of high mass flux over 2.0 was not observed in the C2H4 injection case, as shown in Fig. 6(a), while a region of high mass flux existed at z/d ~ 6 for Ar-case1, as shown in Fig. 6(b). Conversely, for Ar-case2, as shown in Fig. 6(c), the Ar high mass flux region near the symmetric plane penetrated higher than that of the C2H4 injection case. Argon plume diffusion in the height direction at y/d = 10 and 13.3 was slower than that of C2H4, with wide spreading in the height direction, as shown in Fig. 6(a).

A comparison of the mass flux contours shown in Fig. 4 shows that the injectant mass flux contours in the cross-section were similar between H2 injection and C2H4 injection using a similar dynamic pressure ratio. This trend was also observed when comparing the injectant mass flux contours between He injection and Ar injection (i.e., comparison between non-reacting and non-reacting flows) using a similar dynamic pressure ratio in the previous study. However, as shown in Fig. 6, matching dynamic pressure ratios between C2H4 injection and Ar injection are insufficient to simulate injectant mass flux contours in the cross-section.

These results indicate that the heat release effect on local mixing is not negligible, even with similar dynamic pressure ratios between reacting and non-reacting flows. Mahle et al. investigated the heat release effect on compressible mixing layers using direct numerical simulation (DNS). They also reported that turbulent kinetic energy and Reynolds shear stress were reduced by heat release due to a decrease in mean density around the flame sheet at convective Mach numbers between 0.15 and 1.1. Pouransari et al. indicated heat release has a damping effect on velocity fluctuation using DNS for turbulent reacting wall-jet flows. They also reported that fine-scale structures of turbulence were damped, and the vortices became larger due to heat release; the
peak location of kinetic energy shifting to the wall on the injection-side.\(^{21}\)

In the present study, pressure-rise as a result of heat release was simulated by creating back-pressure during inert gas mixing tests. However, density and temperature changes due to heat release could not be simulated in the non-reacting flows. As a result, changes in vortex center position and vortex strength were induced by changing density and temperature. Changing vortex center position and vortex strength were affected by changing injectant plume. It is unknown to what extent the vortices affected the mixing characteristics in the present measurements. Furthermore, how heat release affected the evaluation of mixing efficiency (i.e., the evaluation of bulk-averaged mixing in the cross-section) is still unknown.

A major concern in this study was the possibility of predicting heat release distribution during ramjet-mode operation by converting the results of (inert gas and air) mixing tests into reacting cases (i.e., prediction of the combustion efficiency in reacting flows by converting mixing efficiency in non-reacting flows). For this purpose, there were some technical challenges. One challenge was correlation between the combustion efficiency and the mixing efficiency in reacting flows, allowing these efficiencies to be compared. Another challenge was converting the inert gas mixing efficiency in non-reacting tests into the fuel mixing efficiency in reacting tests, where an inert gas equivalence ratio is required for deducing the inert gas mixing efficiency. Here, it is assumed that the inert gas equivalence ratio is equal to the H\(_2\) or C\(_2\)H\(_4\) equivalence ratios: 1) under similar dynamic pressure ratios, or 2) under similar mass flux ratios. Then, which assumption is more appropriate for comparing reacting and non-reacting mixing characteristics, is discussed. After that, the above-mentioned heat release effects on mixing efficiency were evaluated by comparing actual and converted (from Ar mixing results) C\(_2\)H\(_4\) mixing efficiencies.

### 3.2. Evaluation of combustion efficiency and mixing efficiency in reacting flows

In this section, the relationship between bulk combustion efficiency (\(\eta_{B} \)) and mixing efficiency (\(\eta_{M} \)) in reacting flows is examined. Figure 7 shows the combustion efficiency and mixing efficiency for H\(_2\) and C\(_2\)H\(_4\) injection cases against a dynamic pressure ratio of \(x/d = 25\). Note that the O\(_2\)-based evaluation is used for C\(_2\)H\(_4\) injection. Open circles show the mixing efficiency and closed circles show the combustion efficiency using H\(_2\) injection. Open and closed triangles show mixing and combustion efficiencies using C\(_2\)H\(_4\) injection, respectively.

In Fig. 7, mixing efficiency was not equal to combustion efficiency. Fine-scale mixing at the molecular level is required to cause combustion. Thus, deduced combustion efficiency is equal to ‘fine-scale mixing efficiency’ for H\(_2\) injection, as fast chemistry is expected. In contrast, a mixing efficiency was deduced based on time- and spatially-averaged gas samples in the present study. Thus, deduced mixing efficiency was different from ‘fine-scale’ mixing efficiency.
In general, it takes more time for fine-scale mixing, so 'time- and spatially-averaged' 'bulk' mixing efficiency should be higher than 'fine-scale' mixing efficiency (combustion efficiency). Thus, one should note that over-estimation of mixing efficiency could occur. For H$_2$ injection, mixing efficiency decreased in proportion to the dynamic pressure ratio and the ratio of combustion efficiency to mixing efficiency was almost constant regardless of the dynamic pressure ratio at $J$ of 1.7 and 2.5. This result shows that bulk combustion efficiency can be predicted from mixing efficiency. Mitani et al. reported so-called ‘intensive combustion’ (i.e., large rise in pressure in a scramjet engine at M6 flight conditions) was experienced under mixing-controlled conditions. Under mixing-controlled conditions, local combustion efficiency was minimum at a local equivalence ratio of unity. On the other hand, they also reported that so-called ‘weak combustion’ (i.e., without a significant rise in pressure) was experienced under reaction-controlled conditions, where local combustion efficiency was scattered against a local equivalence ratio. Figure 8(a) shows the correlation between the local equivalence ratio and the local combustion efficiency using H$_2$ injection, in which open and closed circles are data at $J$ of 1.7 and 2.5, respectively. Although some data with poor combustion efficiency was observed at $J$ of 2.5, the mixing-controlled feature could be seen with the H$_2$ injection case.

With C$_2$H$_4$ injection in Fig. 7, the bulk combustion efficiency decreased as dynamic pressure ratio increased, while the mixing efficiency increased when the equivalence ratio became more than unity. Figure 8(b) shows the correlation between the local equivalence ratio and the local combustion efficiency for C$_2$H$_4$ injection at $J$ of 2.5 and 4.6, respectively. For a $J$ of 2.5, the local combustion efficiency was at minimum near the local equivalence ratio of unity, showing that the mixing-controlled conditions feature is valid. For a $J$ of 4.6, although some plots of local combustion efficiency were similar to those with a $J$ of 2.5, most plots were about 20–40% lower than those with a $J$ of 2.5. Note that local combustion efficiency was also at minimum near the local equivalence ratio of unity with a $J$ of 4.6. These results indicate that C$_2$H$_4$ combustion at the sampling location (i.e., 25d downstream from the injection location) with a $J$ of 4.6 was affected by not only mixing, but also reaction delay. In other words, a reduced mixture temperature at a high equivalence ratio would result in longer reaction delay using C$_2$H$_4$ injection.

Table 5 summarizes the reaction delay time and the ratio of bulk combustion efficiency to mixing efficiency ($\eta_b/\eta_m$) for H$_2$ and C$_2$H$_4$ injections. The reaction delay time, which is defined as the time when OH concentration reaches a peak, was calculated using the CHEMKIN-PRO software code. Zero-dimensional chemical reaction calculation was used under constant pressure and adiabatic conditions. In this study, Knowledge-basing Utilities for Complex Reaction Systems (KUCRS) was used as the chemical reaction mechanism. Flow properties at the injection location were estimated using quasi-one-dimensional analysis. The fuel and air mixture gas (i.e., assumed complete mixing) mole fractions, pressure, and temperature obtained were used as input parameters to calculate the reaction delay time. The reaction delay time was normalized ($t_r$) using the time-of-arrival at the sampling location (25d/V).

The C$_2$H$_4$ reaction delay time with a $J$ of 2.5 was almost similar to that when using H$_2$. The reaction delay time with a $J$ of 4.6 was longer than that when using H$_2$. The value of $\eta_b/\eta_m$ increased as reaction delay time decreased, showing that reaction delay affects the relationship between combustion efficiency and mixing efficiency. In the present study, quantitative evaluation of the reaction delay effect was difficult because the reaction delay time could not be separated from the time required for fine-scale mixing.

### 3.3. Conversion of inert gas mixing efficiency into fuel mixing efficiency

As shown in Fig. 4, injectant mass flux contours in the cross-section were similar between H$_2$ and C$_2$H$_4$ injection.

| Injectant | $J$ | $p_s$, kPa | $T$, K | $V$, m/s | $t_r$ | $\eta_b/\eta_m$ (exp.) |
|-----------|----|-----------|------|-------|------|-------------------|
| H$_2$     | 1.7| 129       | 1383 | 676   | 0.13 | 0.738             |
|           | 2.5| 168       | 1370 | 545   | 0.08 | 0.703             |
| C$_2$H$_4$| 2.5| 168       | 1766 | 530   | 0.14 | 0.613             |
|           | 4.6| 183       | 1629 | 477   | 0.33 | 0.453             |
cases under similar dynamic pressure ratios. However, H₂ and C₂H₄ mixing efficiencies did not match with the J of 2.5 in Fig. 7. With similar mass flux ratios, the equivalence ratio (i.e., consequently the mixing efficiency) differs according to the injectant. Conversely, there is a possibility to convert injectant mixing efficiency to the injectant. Conversely, there is a possibility to convert inert gas mixing efficiency into H₂ or C₂H₄ mixing efficiency by converting the equivalence ratio. In this study, an attempt was made to convert inert gas mixing efficiency into H₂ or C₂H₄ mixing efficiency by assuming an inert gas equivalence ratio, which was accomplished by comparing the mixing efficiency between reacting and non-reacting flows.

At first, the inert gas equivalence ratio ($\phi_i$) was defined as follows,

$$\phi_i = \alpha \frac{m_i}{m_{O_2}} / (\frac{m_{O_2}}{m_{W_2}})$$  \hspace{1cm} (5)

where $\alpha$ is a constant coefficient for converting injectant into H₂ or C₂H₄. Next, it is assumed that the inert gas equivalence ratio is equal to the fuel equivalence ratio if the dynamic pressure ratio in non-reacting flows is equal to that in the reacting flows (i.e., denoted as ‘J-based assumption’). For example, when converting He into H₂ using the J-based assumption, one derives

$$\frac{m_{He} V_{He}}{m_{cold} V_{cold}} = \frac{m_{H_2} V_{H_2}}{m_{hot} V_{hot}}$$  \hspace{1cm} (6)

where the injector orifice diameter and the airflow path cross-sectional area are equivalent for He and H₂ injections. By equating the He equivalence ratio ($\phi_{He}$) to H₂ equivalence ratio ($\phi_{H_2}$), $\alpha$ is obtained as follows,

$$\alpha = \frac{1}{2} \frac{m_{He}}{m_{H_2}} / \frac{m_{O_2}}{m_{W_2}} \frac{m_{O_2}}{m_{H_2}}$$

$$\frac{V_{He}}{V_{hot}} \frac{V_{hot}}{V_{cold}}$$  \hspace{1cm} (7)

where C denotes the O₂ mass fraction ratio between cold and hot airflows; this ratio being almost unity in the present study. The velocities of He and H₂ were derived at the injector locations, while the velocities of the cold and hot airflows were derived from calculation at the injector locations using quasi-one-dimensional analysis. Constant $\alpha$ for different inert gas and fuel types could be obtained in the same way. Inert gas local equivalence ratio ($\phi_i$) was also required to deduce inert gas mixing efficiency. The local equivalence ratio was evaluated in the same manner as the inert gas equivalence ratio.

Another assumption was that the inert gas equivalence ratio was equal to the fuel equivalence ratio when the mass flux ratio in the non-reacting flows was equal to that in the reacting flows (i.e., denoted as ‘J-based assumption’). To convert He into H₂ using the J-based assumption, it is derived as follows.

$$\frac{m_{He}}{m_{cold}} = \frac{m_{H_2}}{m_{hot}}$$  \hspace{1cm} (8)

Then, $\alpha$ is obtained as follows.

Finally, inert gas mixing efficiency is deduced using Eq. (2) and regarded as ‘converted’ fuel mixing efficiency.

Figure 9(a) shows converted H₂ mixing efficiency compared to the converted total equivalence ratio using the J-based assumption. Note that the PSW was observed in all cases. The dashed line was obtained from Diskin and Northam, where H₂ was injected from one perpendicular orifice. The H₂ mixing efficiency at $\phi < 0.48$ was higher than that in Diskin and Northam, due to overestimating the present mixing efficiency by ignoring ‘fine-scale’ mixing. In addition, there was a difference in combustor geometry in terms of the cross-sectional aspect ratio (i.e., ratio of duct height to injector orifice spacing) of the fuel diffusing area per injector. The cross-sectional aspect ratio was about 0.5 in Diskin and Northam, while it was 1.9 at the outer injector and 2.6 at the inner injector in this study. Tomioka et al. indicated that the cross-sectional aspect ratio has a sizable effect on mixing characteristics through interaction of the injectant plume with the wall and neighboring jets. Although it remains uncertain what the effect of the cross-sectional aspect ratio on mixing efficiency is, mixing efficiency distribution should be qualitatively consistent.

In Fig. 9(a), the discrepancy of mixing efficiency between H₂ and C₂H₄ (converted into H₂) was 4.7% at $\phi \sim 0.48$. 

\[ \frac{m_{He} V_{He}}{m_{cold} V_{cold}} = \frac{m_{H_2} V_{H_2}}{m_{hot} V_{hot}} \]

\[ \frac{V_{He}}{V_{hot}} \frac{V_{hot}}{V_{cold}} \]
Moreover, the rate of converted mixing efficiency decreased compared to the equivalence ratio was similar to that of the H2 mixing efficiency. This trend is in good agreement with that reported by Diskin and Northam,9) at $\phi < 1$. Although test data using H2 injection at $\phi > 0.48$ could not be obtained due to the failure of the facility nozzle to start in the present study, the rate that mixing efficiency increased (i.e., converted from the inert gas mixing) compared to the equivalence ratio is also in good agreement with Diskin and Northam,9) in $1 < \phi < 2.5$.

Figure 9(b) shows plots of H2 mixing efficiency compared to converted equivalence ratio using the $\lambda$-based assumption. The converted mixing efficiency using He injection had a good correlation to the H2 mixing efficiency at $\phi \sim 0.48$ within 5%. However, when $\phi > 1$, mixing efficiencies (i.e., converted from inert gas and C2H4 data) were scattered. When $\phi < 1$ (i.e., low $\lambda$), vertical penetration of the injectant decreased more than that when $\phi > 1$ because the dynamic pressure ratio also decreased. The mass flux ratio is related to the shear growth rate. In this case, fuel was located near the injection sidewall, so that the shear mixing area between fuel and airflow would be small. Therefore, shear mixing would not be so affected by mixing efficiency. When $\phi > 1$ (i.e., high $\lambda$), on the other hand, vertical penetration increased to the mass flux ratio, and then the shear mixing area also increased. As the result, shear mixing affected the mixing efficiency, and evaluation using the $\lambda$-based assumption would become inappropriate. The results in Fig. 9(a) and 9(b) suggest that the $J$-based assumption is more appropriate for converting inert gas mixing efficiency into H2 mixing efficiency.

To evaluate the heat release effect on bulk mixing efficiency, the following three cases were compared: C2H4 mixing efficiency as measured, C2H4 mixing efficiency converted from Ar data using the $J$-based assumption, and converted from H2 data using $J$-based assumption. Figure 10 shows C2H4 mixing efficiency compared to the equivalence ratio using the $J$-based assumption. The mixing efficiency converted from H2 data was 4.7% lower than the mixing efficiency measured at $\phi \sim 0.9$. On the other hand, the mixing efficiency converted from Ar data was 1.0% lower than measured mixing efficiency at $\phi \sim 1.8$; the deviation being within the mixing efficiency error shown in Fig. 10. Thus, converted mixing efficiencies were in good agreement with those of measured mixing efficiency within an error of a few percent in 0.9 < $\phi$ < 1.8. These results suggest that the heat release effect on bulk mixing efficiency is small. Thus, C2H4 bulk mixing efficiency can be predicted within the error of a few percent using Ar mixing results, a similar dynamic pressure ratio and similar peak pressure value.

4. Conclusions

Dominant parameters to match mixing characteristics between reacting and non-reacting flows in dual-mode combustors were experimentally investigated. Fuels (i.e., H2 or C2H4) were injected into high-temperature airflows, while inert gas (i.e., He, N2 or Ar) was injected as a simulant fuel into room-temperature airflows. All mixing tests were under presence of the pseudo-shockwave system, with mechanical throttling for the non-reacting case. Both dynamic pressure ratio and mass flux ratio were major candidates for the mixing parameter, accordingly, fuel or an inert gas was injected at similar dynamic pressure ratios or similar mass flux ratios. The normalized injectant mass flux contours were similar between H2 injection and C2H4 injection under similar dynamic pressure ratios. On the other hand, they did not match when comparing C2H4 injection and Ar injection under similar dynamic pressure ratios. Heat release effects on local mixing cannot be ignored under similar dynamic pressure ratios between reacting and non-reacting flows.

In an attempt to predict the mixing efficiency in reacting flows using that in non-reacting flows, the latter was converted to the former. In this attempt, it was assumed that the injectant equivalence ratio is equal to that of fuel under similar dynamic pressure ratios ($J$-based assumption) or similar mass flux ratios ($\lambda$-based assumption).

Comparing the H2 mixing efficiency measured using the one converted from inert gas mixing results showed that the $J$-based assumption is more appropriate for conversion. Comparing C2H4 mixing efficiency measured from the one converted using the $J$-based assumption showed that heat release effect on bulk-scale mixing efficiency is small. In the present study, the mixing efficiency for the C2H4 reacting case having an equivalence ratio of 1.6 could be predicted within an error of a few percent using the Ar mixing results under similar dynamic pressure ratios and under presence of the pseudo-shockwave system.

References

1) Fry, R. S.: A Century of Ramjet Propulsion Technology Evolution, J. Propul. Power, 20 (2004), pp. 27–58.
2) Tomioka, S., Tani, K., Masumoto, R., and Ueda, S.: Dual-mode Operation of a Rocket-Ramjet Combined Cycle Engine, Trans. JSASS Aerospace Technology Japan, 8 (2010), pp. Pa.13–Pa.18.
3) Schetz, J. A., Thomas, R. H., and Billig, F. S.: Mixing of Transverse Jets and Wall Jets in Supersonic Flow, Separated Flows and Jets, Kozlov, V. V. and Dovgal, A. V. (eds.), Springer-Verlag, Berlin, 1991, pp. 807–835.
4) Tomioka, S., Murakami, A., Kudo, K., and Mitani, T.: Combustion Tests of a Staged Supersonic Combustor with a Strut, *J. Propul. Power*, 17 (2001), pp. 293–300.

5) Matsuo, K., Miyazato, Y., and Kim, H. D.: Shock Train a Pseudo-Shock Phenomena in Internal Gas Flow, *Progr. Aerospace Sci.*, 35 (1999), pp. 33–100.

6) Kouchi, T., Tomioka, S., Hirano, K., Matsuo, A., and Masuya, G.: Supersonic Combustion Using Multiple Stinger Shaped Injectors, *Trans. Jpn. Soc. Aeronaut. Space Sci.*, 60 (2017), pp. 56–59.

7) Zakay, V., Krause, E., and Woo, S. D. L.: Turbulent Transport Properties for Axisymmetric Heterogeneous Mixing, *AIAA J.*, 2 (1964), pp. 1939–1947.

8) Torrence, M. G.: Effect of Injectant Molecular Weight on Mixing of a Normal Jet in a Mach 4 Airstream, NASA TN D-6061, 1971.

9) Diskin, G. S. and Northam, G. B.: Effects of Scale on Supersonic Combustor Performance, *AIAA Paper* 1993, pp. 499–514.

10) Heiser, W. H. and Pratt, D. T.: Combustion System Processes and Components, *Hypersonic Airbreathing Propulsion*, AIAA Education Series, AIAA, Washington, DC, 1994, pp. 302–307.

11) Gruber, M. R., Nejad, A. S., Chen, T. H., and Dutton, J. C.: Mixing and Penetration Studies of Sonic Jets in a Mach 2 Freestream, *J. Propul. Power*, 11 (1995), pp. 315–323.

12) Cohen, L. S., Coulter, L. J., and Egan, W. J., Jr.: Penetration and Mixing of Multiple Gas Jets Subjected to a Cross Flow, *AIAA J.*, 9 (1971), pp. 718–724.

13) Watanabe, J., Kouchi, T., Takita, K., and Masuya, G.: Characteristics of Hydrogen Jets in Supersonic Crossflow: Large-Eddy Simulation Study, *J. Propul. Power*, 29 (2013), pp. 661–674.

14) Papamoschou, D. and Roshko, A.: The Compressible Turbulent Shear Layer: an Experimental Study, *J. Fluid Mech.*, 197 (1988), pp. 453–477.

15) Dimotakis, P. E.: Turbulent Free Shear Layer Mixing and Combustion, *High-Speed Flight Propulsion Systems*, Murthy, S. N. B. and Curran, E. T. (eds.), Progress in Astronautics and Aeronautics, AIAA, Washington, DC, 1991, pp. 265–340.

16) Arakawa, T. and Tomioka, S.: Research on Fuel Mixing in a Non-Reacting Situation Simulating a Dual-Mode Combustor, *Trans. JSASS Aerospace Technology Japan*, 17 (2019), pp. 82–99.

17) Mitani, T., Takahashi, M., Tomioka, S., Hiriwa, T., and Tani, K.: Analyses and Application of Gas Sampling to Scramjet Engine Testing, *J. Propul. Power*, 15 (1999), pp. 572–577.

18) Billig, F. S.: Research on Supersonic Combustion, *J. Propul. Power*, 9 (1993), pp. 499–514.

19) van Driest, E. R.: Turbulent Boundary Layer in Compressible Fluids, *J. Aeronaut. Sci.*, 18 (1951), pp. 145–160.

20) Mahle, I., Foysi, H., Sarkar, S., and Friedrish, R.: On the Turbulence Structure in Inert and Reacting Compressible Mixing Layers, *J. Fluid Mech.*, 593 (2007), pp. 171–180.

21) Pouransari, Z., Vervisch, L., and Johansson, A.: Heat Release Effects on Mixing Scales of Non-Premixed Turbulent Wall-Jets: A Direct Numerical Simulation Study, *Int. J. Heat Fluid Flow*, 40 (2013), pp. 65–80.

22) Mitani, T., Chinzei, N., and Kanda, T.: Reaction and Mixing Controlled Combustion in Scramjet Engines, *J. Propul. Power*, 17 (2001), pp. 308–314.

23) Mével, R. and Shepherd, J. E.: Ignition Delay-time behind Reflected Shock Waves of Small Hydrocarbon-Nitrous Oxide-(oxygen) Mixture, *Shock Waves*, 25 (2015), pp. 217–229.

24) ANSYS CHEMKIN-Pro 19.2, ANSYS, 2019.

25) Miyoshi, A.: KUCRS Software Library, Available from: [http://akrmys.com/KUCRS](http://akrmys.com/KUCRS), 2011 (accessed November 10, 2018).

26) Tomioka, S., Izumikawa, M., Sakuranaka, N., Shibata, H., Yamasaki, H., Kobayashi, T., and Matsuo, A.: Mixing Control by Wall Flush-Mount Injectors in Dual-Mode Combustor, AIAA Paper 2010-6959, 2010.