Deflection Calculation of Steel-Concrete Composite Beams Considering Effects of Shear Lag and Slip

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Abstract. Based on the proposed longitudinal deformation equation considering the slip between steel and concrete and the shear lag, the energy variation function is deduced, then the formula to calculate the deflection of steel-concrete composite beams considering the effects of shear lag and slip under the action of uniformly distributed load is derived. A group of experimental steel-concrete composite beams with different space of stud is made and the experimental data are compared with theoretical values showing that the formula holds high calculation accuracy.

1. Introduction
Steel-concrete composite material is widely used in large-span structures and high-rise buildings for their good working performance and high economy. A steel-concrete composite beam is composed of concrete slab, steel beam and shear connectors. The shear lag of flange slab, shear deformation of shear connectors and relative slip between steel and concrete affect the deflection of the composite beams [2-6]. Up-to-present, two group approaches to calculate the deflection as analytical methods [2-9] and numerical methods [10-12] are broadly used, including conversion section method [2], modified conversion section method [3], interpolation method based on conversion section method [4], reduction stiffness method [5-7], energy principle method and so on.

In order to simplify the analytical calculation of composite beams, some studies consider inter laminar slip but omit the effect of shear lag [8, 9], while some studies omit inter laminar slip but consider the effect of shear lag [10]. The experimental and theoretical studies show that the interlayer slip obviously reduces the overall stiffness of composite beams resulting in increasing the deflection of the beams [6, 8-9]. Therefore, the comprehensive effects of shear lag and slip should be considered in the calculation of deflection of steel-concrete composite beams.

Studies show that the spacing of studs, the shear strength of single studs and the cross-section characteristics affect the slip [5]. However, the existing analytical formulae fail to consider the influence of the above parameters comprehensively. In this paper, the energy variation function of steel-concrete composite box girder is established based on the longitudinal displacement equation considering the effect of interlayer slip and shear lag, then the formula for calculating the deflection of simply supported steel-concrete composite box girder under the action of uniformly distributed load is derived by using energy variation method.
2. Derivation of Basic Differential Equations

2.1. Basic Considerations

Fig. 1 is the elevation coordinate schematic diagram of the composite beam, and Fig. 2 is the cross-section schematic diagram of the composite beam. Set the maximum width among the width of webs of the concrete slabs, the width of the cantilever, and the width of the upper and lower flanges of the steel girder as “b”, then the other widths are expressed as $\xi_i b$ $(i=1,2,3,4)$. Unless otherwise stated, the subscript “c” stands for concrete and “s” stands for steel.

**Figure 1.** Schematic diagram of elevation coordinate  
**Figure 2.** Schematic diagram of section

By introducing three generalized displacements, the vertical deflection, the longitudinal displacement and the generalized slip between layers of the beam, the longitudinal displacement mode of any point at any section of the composite beam can be written as follows:

\[
U_i(x, y) = -Z_i \left[ \frac{dw}{dx} + \left[ 1 - \frac{y}{\xi_i b} \right]^3 u(x) \right] - g_c \cdot S(x) \\
U_i(x, y) = +Z_{m(b)} \left[ \frac{dw}{dx} + \left[ 1 - \frac{y}{\xi_i b} \right]^3 u(x) \right] + g_s \cdot S(x)
\]

Where \(u(x)\) is the maximum difference of shear angle, \(Z_i\) and \(Z_{m(b)}\) respectively are the distances from the middle surface of the concrete and the upper and lower flanges of the profiled steel to the cross-sectional centroid axis of composite beam, \(S(x)\) is the relative slip.

For a simply supported steel-concrete composite beams under the action of uniformly distributed load, the relative slip between steel and concrete can be deduced as:

\[
S(x) = \beta q \left[ \frac{a_l}{\alpha (1 + e^{-a_l})} \right] + x
\]

Where \(\alpha = \sqrt{\frac{K_n (1 + I_s/A_s + I_c/A_c) + (0.5h)^2}{E_s d (1 + I_s/E_s)}}\), \(\beta = \frac{h d}{0.58 n_s A_s \sqrt{E_s A_s}}\); \(I_s\) and \(I_c\) are inertia moments of steel and concrete slabs, respectively; \(A_s\) and \(A_c\) are cross-sectional areas of steel and concrete slabs, respectively; \(n_s\) is the number of studs in the same cross-section, \(q\) is the uniform load; \(l\) is the calculation span; \(d\) is longitudinal spacing of studs.

When establishing the energy equation, it is assumed that the steel section and the concrete section satisfy the plane hypothesis at the section of the webs, the beam body works elastically.

2.2. Establishment of Function of Potential Energy

According to the principle of virtual work, the expression of total potential energy of composite beams under the action of vertical uniformly distributed load is
The load potential energy of composite beams is

\[ \Pi = \bar{W}_b - \bar{W} \quad (4) \]

Where \( \bar{W}_b \) is the deformation potential energy of steel webs, concrete slab, upper and lower flanges of section steel, and \( \bar{W} \) is the load potential energy of composite beams.

In consideration of \( \varepsilon_a = \frac{\partial U_a(x,y)}{\partial x} \), \( \gamma_a = \frac{\partial U_a(x,y)}{\partial y} \), \( \varepsilon_a = \frac{\partial U_a(x,y)}{\partial x} \), \( \gamma_a = \frac{\partial U_a(x,y)}{\partial y} \), the total potential energy is obtained as:

\[
\bar{W}_b = 2 \times 2 \times \frac{1}{2} \int_0^l \frac{1}{2} \int_0^h \left( \sum_{i=1}^2 \left( E_i \varepsilon_i^2 + G_i \gamma_i^2 \right) \right)_{uu} dxdy + 2 \times 2 \times \frac{1}{2} \int_0^l \frac{1}{2} \int_0^h \left( E_i \varepsilon_i^2 + G_i \gamma_i^2 \right)_{ll} dxdy
\]

\[ +4 \times 2 \times \frac{1}{2} \int_0^l \frac{1}{2} \int_0^h \left( E_i \varepsilon_i^2 + G_i \gamma_i^2 \right)_{ll} dxdy + 2 \times \frac{1}{2} \int_0^l E_i I_u \left( w^* \right) dx \quad (5) \]

\[ \bar{W} = -2 \int_0^l M(x)w^*(x)dx \quad (6) \]

Where \( I_u \) is inertia moment of steel web and \( G \) is shear modulus.

Substitute Equations 5 and 6 into Equation 4, the total potential energy is obtained.

3. Deflection and Stress

When the beam is under the action of uniformly distributed load \( q \), according to the principle of minimum potential energy, \( \delta \Pi = \delta (\bar{W}_b - \bar{W}) = 0 \), the following formulae are derived:

\[
\begin{aligned}
\alpha^* - k^2 u &= \frac{7n}{6} \left[ \frac{Q(x)}{E_i I_a} + \frac{\beta q h}{Z_i h} I_a \left( \frac{1}{I_a} - \frac{1}{I_l} \right) g^*(x) \right] \\
\beta^* &= -\frac{1}{E_i I_a} \left[ M(x) + \frac{3}{4} E_i I_u u^* + \frac{\beta q h}{Z_i h} E_i I_g \left( x \right) \right] - \frac{3}{4} \frac{9G I_2}{5E b^2 I_4} u - \frac{3\beta q h I_1}{4Z_i h_3} g^* = 0
\end{aligned} \quad (7) \]

Where \( n = (1 - 0.875 I_1/I_4)^{-1} \), \( k = 14n I_1 G_i/(5E b^2 I_4) \), \( I_1 = I_{out} + I_{u2} + n_1 I_3 + n_1 I_4 \), \( I_2 = I_{out} + \frac{1}{z_2} I_{u2} \), \( I_3 = I_{out} + I_{u2} + n_3 I_5 + n_3 I_4 \), \( I_4 = n_3 I_5 + I_1 \), \( I_5 = 2t_h b Z_c^2 \), \( I_6 = 2t_h b Z_c^2 \), \( I_7 = 2t_h b Z_c^2 \), \( I_8 = 4t_h b Z_c^2 \),

\[ g(x) = 1 - e^{-\frac{a}{e}x}\left( e^{-\frac{a}{e}x} - e^{-\frac{a}{e}x} \right) \]

Substituting \( M(x) = \frac{1}{8} ql^2 - \frac{1}{2} qx^2 \) and \( Q(x) = qx \) into Equation 7, and considering boundary conditions \( u(x = l/2) = 0 \) and \( u'(x = l/2) = 0 \), \( u(x) \) is obtained, hence the deflection of beam can be derived from Equation 8 as:

\[
\begin{aligned}
w(x) &= \frac{5}{384E_i I_a} ql^4 + \frac{\beta q h I_3}{Z_i h I_a} \left( \frac{l^2}{8} - \frac{1}{\alpha^2} \right) + \frac{3l^2}{4 I_a} \left( \frac{C_{0}}{k^2} - \frac{7nq}{6k^2} \frac{l^2}{8E_i I_a} + \frac{\beta h I_3 I_I - I_{I4}}{I_{I4}} \right) I_a \left( I_{I4} - I_{I4} \right) \right] \\
&\quad \frac{2q x^2 - 3q l^2 x^2}{48E_i I_a} + \frac{\beta q h I_3}{Z_i h I_a} \left\{ \frac{e^{-\frac{a}{e}x} + e^{-\frac{a}{e}x}}{\alpha^2 (1 + e^{-\alpha x})} - \frac{1}{2} x^2 \right\}
\end{aligned} \]

3
Concrete stress can be calculated by \( \sigma_x = E_x \frac{\partial U(x,y)}{\partial x} \) and steel stress can be calculated by \( \sigma_s = E_s \frac{\partial U(x,y)}{\partial x} \).

4. Examples and Analysis

4.1. Test Beams

In order to verify the accuracy of the Equation 9, three simply supported steel-concrete composite beams with bolts spacing of 120 mm, 180 mm and 300 mm were fabricated, in which the shear connectivity degrees are 1.33, 0.88 and 0.5 (calculated according to reference [1]). The dimension of the test beams are shown in Fig. 3 and Fig.4. The uniformly distributed load is \( 8.9 \text{kN/m} \); the concrete slab is made of C40 concrete, \( E_c = 34.5 \text{GPa} \); the steel beam is made of Q235, \( E_s = 206 \text{GPa} \). The shear connector is made of cylindrical head welding nails with the diameter of 12.8 mm and the length of 45 mm.

In this test, the deflections at mid-span are measured.

\[
+ \frac{3l}{4l_i} \left\{ \frac{7ng}{6k} \left[ \frac{x^2}{2E_i I_s} + \frac{\beta h}{Z, h} \left( \frac{I_s - I_s}{I_s} \right) \right] - \left( \frac{1}{E_i I_s} \frac{\alpha^2 \beta h}{Z, h} \frac{I_s - I_s}{I_s} \right) \frac{14nqh}{kI} \frac{l}{2} \right\} 
\]

(9)

The following conclusions can be drawn from Table 1:

(1) When the bolt spacing is 120 mm, 180 mm and 300 mm respectively, the relative differences between the calculated values by Formula 9 and the experimental data are 4.61%, 4.55% and 6.88%, respectively. The calculated values are close to the experimental value, showing that the calculation accuracy of Formula 9 is satisfactory.

(2) As the spacing of bolts increases, the deflection at mid-span increases.

In order to understand further the effect of degree of shear connection between concrete slab and steel structure to deflection of steel-concrete composite beams, the theoretical deflections at mid-span of beam varying with the spacing of bolts are calculated by Equation 9, which are drawn in Fig. 5. For the purpose of verify the computation accuracy of Equation 9, ANSYS is used to calculate the deflection in case of two extreme conditions, as no slip (steel and concrete is completely connected) and full free

| Bolt spacing/mm | Deflections/mm |
|-----------------|---------------|
|                 | Experimental data | Values by Formula 9 | Relative difference |
| d=120           | 4.12           | 4.31                | 4.61%               |
| d=180           | 4.62           | 4.83                | 4.55%               |
| d=300           | 5.23           | 5.59                | 6.88%               |
slipping between concrete slab and steel structure. The finite element theoretical values are also drawn in Fig. 5

![Graph showing relationship between mid-span deflection and longitudinal space of stud](image)

**Figure 5.** The relationship between mid-span deflection and longitudinal space of stud

From Fig.5 we can make following conclusions:

1. The mid-span deflection increases with the increase of the bolt spacing, while the rate of the increase reduces gradually.
2. In two extreme conditions, there is no slip and fully free slipping between concrete slab and steel structure, the calculated values by Formula 9 are close to the finite element theoretical values (ANSYS). When the steel and the concrete is completely connected (no slipping), the calculated value by Formula 9 is 2.78 mm and the result by ANSYS is 2.608 mm, its relative difference is 6.59%; when the shear connection between steel and concrete is complete free, the calculated value by Formula 9 is 8.62 mm and the result by ANSYS is 8.716 mm, its relative difference between them is -1.10%, showing that Formula 9 gives reliable deflection of steel-concrete composite beams.

5. Conclusion

Based on the proposed longitudinal displacement model considering the effect of interlayer slip, the analytical formula for calculating the deflection of simply supported steel-concrete composite beams under the action of uniformly distributed load considering the effect of shear lag and slip is established by using the energy variation method. The calculated values by the established formulae are agreeable with the experimental data showing that the formulae hold high calculation accuracy. The presented formulae can easily consider the influence of the number, spacing and the shear capacity of the bolts, and the characteristics of the cross-sections.

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