HFB theory for nuclei near the drip-lines: continuum coupling

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We have developed a new HFB code that specifically addresses nuclear structure physics near the driplines. The HFB equations are solved on a two-dimensional lattice for axially symmetric even-even nuclei using B-Spline techniques. The quasiparticle energy spectrum is obtained by direct diagonalization of the HFB lattice Hamiltonian with LAPACK. The energy spectrum extends high into the continuum, up to several thousand MeV. Calculations with Skyrme forces and (density-dependent) delta pairing interactions are now underway.

I. INTRODUCTION

Near the neutron or proton drip lines, large pairing correlations are expected which can no longer be described by a small residual interaction. Furthermore, the outermost nucleons are weakly bound (which implies a large spatial extent), and they are strongly coupled to the particle continuum. These features represent major challenges for the mean field theories. We solve the self-consistent mean field plus pairing (Hartree-Fock-Bogoliubov) equations for axially symmetric even-even nuclei on a two-dimensional lattice. We utilize the Skyrme M* interaction, and pairing is described either by a pure delta force or by a density-dependent delta interaction (DDDI).

II. NUMERICAL METHOD AND PRELIMINARY RESULTS

High numerical accuracy is achieved by representing the operators and wavefunctions in terms of Basis-Splines; a combination of the Galerkin and collocation method is utilized. Our work represents a natural extension of the 1-D calculations for spherical nuclei by Dobaczewski et al.

We have successfully tested our numerical algorithm by solving the HFB equations for a constant pairing Hamiltonian in which case the problem becomes equivalent to HF + BCS. As an example, we show in Fig. 1 the quasiparticle energy spectrum of the normal and pairing density for \(^{22}\)Ne. The spectrum of the lattice HFB Hamiltonian is obtained by direct diagonalization with LAPACK. In this way, we obtain the whole quasiparticle energy spectrum at once, up to about \(E_n = 2000\) MeV. In calculating observables, we cut off this spectrum at an equivalent s.p. energy of about 60 MeV. A correct representation of high-energy continuum states is crucial for physics near the driplines.

We wish to emphasize that this numerical test is non-trivial because the HFB code solves a quasiparticle energy spectrum (upper and lower continuum, no lower bound) while the HF + BCS codes solve for the energy spectrum of real particles (one upper continuum only, lower bound).

![FIG. 1. Test of HFB code for constant pairing Hamiltonian: all observables are identical to HF + BCS, as expected.](image-url)
In the following, we discuss preliminary results obtained with the full HFB code. All calculations are for $^{22}$Ne with the Skyrme M* interaction in the p-h channel and a pure delta pairing interaction (strength $V_0 = -173 MeV fm^3$, taken from ref. [4]). We utilized B-Splines of order $M = 7$, a lattice size of 8 fm in radial direction, and a lattice spacing of about 1 fm.

![Graph showing quasiparticle energy spectrum of densities](image1)

**FIG. 2.** HFB code with SkM* and pairing delta interaction: quasiparticle energy spectrum of densities

Fig. 2 shows that both the normal density and the pairing density develop more structure in the quasiparticle energy spectrum, as compared to the BCS pairing case. This is in agreement with the results from 1-D calculations.

![Graph showing convergence of total nuclear binding energy](image2)

**FIG. 3.** Convergence of total nuclear binding energy as function of number of HFB iterations in LAPACK

Fig. 3 demonstrates the convergence of the total nuclear binding energy as a function of the number of iterations. The three different curves correspond to lattice spacings of $\Delta x = 1.33, 1.14, 1.00$ fm. Fig. 4 shows contour plots of the pairing density for protons and neutrons. The square of the pairing density describes the probability of finding a correlated nucleon pair with opposite spin directions at position $r$.

We are now in the process of adjusting the DI and DDDI pairing force parameters to the tin isotope chain and plan to do a detailed comparison for spherical nuclei with Dobaczewski’s 1-D HFB code [2]. After these tests are finished, we will be in a position to address the new physics near the drip lines such as neutron halos and neutron skins, proton radioactivity etc. We are going to calculate binding energies, neutron and proton separation energies, pairing gaps, normal and pairing densities, rms radii, and electric or magnetic moments.

We have adopted the Skyrme force parameterization suggested in ref. [3]. Currently, all of our HFB calculations...

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[2] Reference for the 1-D HFB code

[3] Reference for the Skyrme force parameterization

[4] Reference for the pairing interaction strength

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are carried out with the SkM* interaction. In the near future, we plan to investigate some of the more recent parameterizations, in particular SLy4 and to explore the sensitivity of observables to these forces. We will also add constraints ($Q_{20}$, $Q_{30}$, $\omega_j^x$) to the HFB code in order to calculate potential energy surfaces for nuclei far from stability. Our long-range plan is to extend the current ground state theory to excited states, i.e. quasiparticle RPA on the lattice [5].

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[1] V.E. Oberacker and A.S. Umar, in “Perspectives in Nuclear Physics”, (World Scientific Publ. Co., 1999), p. 255-266; nucl-th/9905010.
[2] J. Dobaczewski, W. Nazarewicz, T.R. Werner, J.F. Berger, C.R. Chinn, and J. Dechargé, Phys. Rev. C53, 2809 (1996)
[3] P.-G. Reinhard, D.J. Dean, W. Nazarewicz, J. Dobaczewski, J.A. Maruhn, and M.R. Strayer, Phys. Rev. C60, 014316 (1999)
[4] J. Dobaczewski, W. Nazarewicz, and T.R. Werner, Phys. Scr. T56, 15 (1995)
[5] W. Nazarewicz, ORNL, private communication