Suppressed fluctuations in non-stretched-twist-fold turbulent helical dynamos

by

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Abstract

Suppression of fluctuations of normally perturbed magnetic fields in dynamo waves and slow dynamos along curved (folded), torsioned (twisted) and non-stretched, diffusive filaments are obtained. This form of fluctuations suppression has been recently obtained by Vainshtein et al [PRE 56, (1997)] in nonlinear ABC and stretch-twist-fold (STF) dynamos by using a magnetic Reynolds number of the order of $Rm \approx 10^4$. Here when torsion does not vanish an expression between magnetic Reynolds number and length scale $L$ as with constant torsion $\tau_0$ itself is obtained, such as $Rm \approx \frac{\tau_0 L}{\eta}$ is obtained. At coronal loops $Rm \approx 10^{12}$ and torsion of the twisted structured loop from astronomical data by Lopez-Fuentes et al [Astron. and Astrophys. (2003)] of $\tau \approx 9.0 \times 10^{-10}cm^{-1}$ is used to compute a very slow magnetic diffusion of $\eta \approx 10^{-8}$. The slow dynamo obtained here is in agreement with Vishik argument that fast dynamo cannot be obtained in non-stretched dynamo flows. When torsion vanishes helical turbulence is quenched and but $\alpha$-dynamos cannot be maintained since exponential stretching depends on torsion. This is actually Zeldovich antidynamo theorem for torsion-free or planar filaments which has been discussed by the other also recently in another context [Astr Nach. (2008)]. The suppression of magnetic field fluctuations is actually a result of the coupling of the magnetic diffusion and Frenet torsion of helical turbulence. PACS numbers: 02.40.Hw:differential geometries. 91.25.Cw-dynamo theories.
I Introduction

Earlier M. Vishik [1] has argued that only slow dynamos can be obtained from non-stretching dynamo flows and no fast dynamos so well-known to be obtained from the Vainshtein and Zeldovich [2] work on stretch-twist and fold (STF) [3] magnetic dynamo generation mechanism on ropes and magnetic filaments. Ropes are essentially thin twisted tubes filled with plasma flow and or magnetic fields which are compressed by the stretching of the tube giving rise to an amplification of the magnetic field. Diffusion processes which have been investigated in the context of Riemannian geometry by S. Molchanov [4] have also been given strongly importance in obtaining slow dynamos. Such slow dynamos which have previously obtained by Soward [5] by simply aligning magnetic and flow velocity to obtain helicity distinct from zero. In this report one finds two solutions to the self-induction equations, first an analytical solution which is a torsioned turbulent filament with very small torsion and diffusion. When torsion vanishes filaments are planar and by Zeldovich anti-dynamo theorem [6] cannot support dynamo action, actually it is shown that it generates a static magnetic initial field and a steady perturbation which may be a marginal dynamo at maximum. Thus as the same way fast dynamo are generated by stretch, folding and twisting of the loops or filaments it seems that non-stretched, fold and twisted filaments leads to slow dynamos filaments. The paper is organized as follows: In section 2 a brief review on dynamics of holonomic Frenet frame is presented. In section 3 the self-induction equation is solved in this frame for slow dynamos and magnetic torsion is obtained from Rm and solutions are shown in the torsion-free case to be consistent with Zeldovich anti-dynamo theorem. In this section one also shows that chaos is restricted [7] in the $\alpha$ – dynamos. In section 4 conclusions are presented.
II Non-stretched dynamo filamentary flows in Frenet frame

This section deals with a very brief review of the Serret-Frenet holonomic frame [8] equations that are specially useful in the investigation of STF Riemannian flux tubes in magnetohydrodynamics (MHD) with magnetic diffusion. Frenet frame has been used by solar physicists and astrophysicists [8, 9] to investigate twisted solar flux tubes in the solar photosphere. Here the Frenet frame is attached along the magnetic flux tube axis which possesses Frenet torsion and curvature [7], which completely determine topologically the filaments, one needs some dynamical relations from vector analysis and differential geometry of curves such as the Frenet frame \((t, n, b)\) equations

\[
\begin{align*}
    t' &= \kappa n \\
    n' &= -\kappa t + \tau b \\
    b' &= -\tau n
\end{align*}
\]

The holonomic dynamical relations from vector analysis and differential geometry of curves by \((t, n, b)\) equations in terms of time

\[
\begin{align*}
    \dot{t} &= [\kappa' b - \kappa \tau n] \\
    \dot{n} &= \kappa \tau t \\
    \dot{b} &= -\kappa' t
\end{align*}
\]

along with the flow derivative

\[
\dot{t} = \partial_t t + (\vec{v} \cdot \nabla) t
\]

From these equations and the generic flow

\[
\dot{X} = v_t t + v_n n + v_b b
\]

one obtains

\[
\frac{\partial l}{\partial t} = (-\kappa v_n + v_s') l
\]

where \(l\) is given by

\[
l := (X' \cdot X')^{\frac{1}{2}}
\]
which shows that if $v_s$ is constant, which fulfills the solenoidal incompressible flow

$$\nabla \cdot \mathbf{v} = 0$$  \hspace{1cm} (II.11)

and $v_n$ vanishes, one should have a non-stretched twisted flux tube. This is exactly the choice $\mathbf{v} = v_0 t$, where $v_0 = constant$ is the steady flow one uses here. The solution

$$\mathbf{B} = B_s(s, t) t$$  \hspace{1cm} (II.12)

shall be considered here. This definition of magnetic filaments is shows from the solenoidal character of the magnetic field

$$\nabla \cdot \mathbf{B} = 0$$  \hspace{1cm} (II.13)

where $B_s$ is the toroidal component of the magnetic field. In the next section one shall solve the diffusion equation in the steady case in the non-holonomic Frenet frame as

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$  \hspace{1cm} (II.14)

where $\eta$ is the magnetic diffusion. Since in astrophysical scales, $\eta \nabla^2 \approx \eta L^{-2} \approx \eta \times 10^{-20} cm^{-2}$ for a solar loop scale length of $10^{10} cm$ [6] one notes that the diffusion effects are not highly appreciated in astrophysical dynamos, though they are not neglected here. Let us now consider the magnetic field definition in terms of the magnetic vector potential $\mathbf{A}$ as

$$\mathbf{B} = \nabla \times \mathbf{A}$$  \hspace{1cm} (II.15)

the gradient operator is

$$\nabla = t \partial_s$$  \hspace{1cm} (II.16)
III Turbulent helical dynamos and dynamo waves in filaments

In this section we review the magnetic perturbation which leads to the dynamo waves in filaments [10]. Let us start by assuming that the filaments are perturbed accordingly to the laws

\[ B = B_0 + B_1 \tag{III.17} \]

where \( B_0 = B_0 t \) is the initial field and \( B_1 \) is the normally perturbed magnetic field w.r.t to the filament axis itself. This field in Frenet frame is given by

\[ B_1 = b_1 n + b_2 b \tag{III.18} \]

By performing the substitution into the turbulent structure of the magnetic induction equation in the steady state

\[ \eta \nabla^2 B_1 + (B_0, \nabla)v = 0 \tag{III.19} \]

where one has considered that \( V = V_0 + v \), and \( V_0 \) vanishes. The turbulent averages \( <B_1> \) and \( <v> \) both vanish and term

\[ v \times B_1 - <v \times B_1> = 0 \tag{III.20} \]

By taking Parker's \( \alpha \)-effect hypothesis

\[ <v \times B_1> = \alpha B_0 \tag{III.21} \]

the remaining equation is

\[ \partial_t B_0 = \eta \nabla^2 B_0 + \text{rot}(\alpha B_0) \tag{III.22} \]

which is the \( \alpha \)-dynamo equation. Here the term \( \text{rot}(V_0 \times B_0) \) vanishes since \( V_0 \). The first equation one shall solve is \( \text{div}B = 0 \) which yields

\[ \partial_\tau B_0 = \tau_0 b_1 \tag{III.23} \]

where torsion \( \tau_0 \) and curvature \( \kappa_0 \) of helical filament coincides and are constants.
Taking the dynamo ansatz $B_0 = b_0 e^{\omega t + ik_s s}$, where the constant factor $k_s$ is the dynamo wave number, and $\omega$ plays the role of dynamo factor, we may solve the equation (III.22). By examining this factor below in the limit of $Rm \to \infty$, one shall be able to check if the dynamo is fast or slow. Equation (III.22) now yields

$$\partial^2 s B_0 = -k_s^2 B_0$$

(III.24)

and

$$B_0 \tau_0 (v_0 - \tau_0) n + B_0 \omega = B_0 \alpha \tau_0 \times n - ik^2_s \alpha - \eta k_s^2 B_0$$

(III.25)

Using the equations above for the dynamics of Frenet frame and transvecting this equation with scalar product of $B_0$ yields

$$\omega = -\eta k_s^2$$

(III.26)

which is similar to Parker’s dynamo wave frequency, with only basic difference that now it is a real number. The phase velocity is

$$v_{ph} := \frac{\omega}{k_s} = -\eta k_s$$

(III.27)

This is already an indication that the dynamo is slow since it violates the condition for fast dynamo cause this condition is given by the fact that $\omega > 0$ in the limit of $Rm \to \infty$, which is certainly not the case and $\omega \leq 0$ since the diffusivity $\eta$ is always positive. Since $Rm = \frac{v_0 L}{\eta}$ actually $\omega$ vanishes and the dynamo is slow or marginal in this case. Nevertheless since diffusivity here is finite and $Rm$ is also finite a non-trivial solution may be obtained here. Note that by transvecting the equation (III.22) with the vector product of $B_0$ one obtains

$$[B_0 \alpha \tau_0 \times n] \times B_0 = 0$$

(III.28)

This expression is fundamental since implies that $\tau_0 \alpha = 0$, which means that either $\alpha$ effect vanishes or torsion vanishes. We rather choose the first option since we shall show that torsion suppresses dynamo effect fluctuations. Transvecting equation (III.22) again with $n$ yields that

$$B_0 \tau_0 (v_0 - \tau_0) = 0$$

(III.29)

where we have used simple vector expressions $B_0 \cdot n = 0$ and $B_0 \times B_0 = 0$. 
Expression (III.29) then yields $\tau_0 = v_0$. The remaining equation for $B_1$ yields

$$\eta [b''_1 + 2\tau_0 (b'_1 - b'_2) - b_2 \tau_0^2] + v_0 \tau_0 B_0 = 0 \quad (\text{III.30})$$

$$b''_2 - b_2 \tau_0^2 = 0 \quad (\text{III.31})$$

$$2b'_1 - b_2 \tau_0 = 0 \quad (\text{III.32})$$

Summing up the last two equations yields

$$b''_1 = 2b'_1 \tau_0 \quad (\text{III.33})$$

Equation (III.24) yields

$$b'_1 = \frac{i k_s \tau_0}{\tau_0} B_0 \quad (\text{III.34})$$

Integration of this expression yields $b_1(s, t)$ as

$$b_1 = \frac{1}{\tau_0} B_0 \quad (\text{III.35})$$

Substitution of (III.35) and (III.35) into equation (III.30) yields

$$i [k^4_s + 2\tau_0 k_s] + 2[k^3_s + 5\tau_0^2 k_s] = 0 \quad (\text{III.36})$$

where one has assumed that the wave length $k_s$ can be a complex number. In the limit of long wave-length, the dynamo wave number modulus is $|k_s| \ll 1$, and we approximate $O(k^4_s) = O(k^3_s) \approx 0$, this result yields the equation

$$i 2\tau_0 k^2_s + 5\tau_0^2 k_s = 0 \quad (\text{III.37})$$

which yields

$$k_s = \frac{i 5}{2} \tau_0 \quad (\text{III.38})$$

which shows that the exponential stretching of the magnetic field $B_1$ is strongly suppressed since

$$b_1 = \frac{1}{\tau_0} e^{-\frac{5}{2}\tau_0 (s - \eta \tau_0 t)} \quad (\text{III.39})$$

note that since torsion of the solar loop is rather small, the second term in the exponent of this expression takes a long time to increase since $\eta$ is also rather weak. Actually the formula (III.39) shows that the fluctuation $b_1$ is suppressed by a coupling between torsion and magnetic
diffusion constant $\eta$, as can be seen from the second term of the exponent in the RHS of that equation. At the $t = 0$ these fluctuations are rather suppressed as one goes along the loop. At the end of the loop where $s = L$, $b_1$ is very weak and suffers a strong damping. Analogous result holds for $b_2$ and therefore for $\mathbf{B}_1$. Despite of the fact that the initial amplitudes of the fluctuations are great since $b_1 \approx \tau_0^{-1}$. Since $v_0 = \tau_0$ the Reynolds number $R_m$ is

$$R_m = \frac{\tau_0 L}{\eta}$$

which for a Reynolds $R_m$ number in coronal solar loop regions of $R_m$ and the above torsion yields a diffusion constant of $\eta \approx 10^{-8}$, which is a very small diffusion as happens in the highly conductive coronal solar region. This shows that the model presented here is compatible with the fact that the dynamo begins his action in the convective zone and undergoes magnetic buoyancy to raise to coronal regions of the Sun. Concerning finally the Zeldovich anti-dynamo theorem one notes that in all above solutions, the vanishing of torsion implies that turbulence is planar and the magnetic field cannot be amplified and any dynamo action is suppressed.

IV Conclusions

Vishik’s idea that the non-stretched dynamos cannot be fast is tested once more here, by showing that the fluctuations modes of the solar dynamos are strongly suppressed when the long wavelength dynamo modes, or small dynamo wave numbers are effective. Stretching are therefore fundamental in the effective dynamo convective region of the Sun [6]. The twist or Frenet torsion is also small, and slow dynamo are shown to be present in this astrophysical loops. Thus $\alpha$-dynamo model is suppressed by helical turbulent diffusion in dynamo waves. Simplifications in the model as the constant toroidal flow along the plasma flow velocity along the tube axis and the small constraint of twist given by Lopez-Fuentes et al [13] are given. STF Zeldovich-Vainshtein [14] fast dynamo generation method, is not the only Riemannian method that can be applied as in Arnold’s cat map but other conformal fast kinematic dynamo models as the conformal Riemannian one [15] has been recently obtained. Small scale dynamos in Riemannian spaces can therefore be very useful for our understanding of more large scale astrophysical dynamos. Other applications of plasma filaments such as stretch-twist and fold fractal dynamo mechanism which are approximated Riemannian metrics have been recently
put forward by Vainshtein et al [14]. Here one has relaxed the stretch used in STF fast dynamo method and instead use the NTF kinematic slow dynamos in Frenet frame, showing that small torsion is fundamental for the suppression of fluctuations. Finally one has shown that the fluctuations suppression, results from an interaction between the magnetic diffusion constant and the Frenet torsion.

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