A Logic Programming Framework for Possibilistic Argumentation with Vague Knowledge

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Abstract

Defeasible argumentation frameworks have evolved to become a sound setting to formalize commonsense, qualitative reasoning from incomplete and potentially inconsistent knowledge. Defeasible Logic Programming (DeLP) is a defeasible argumentation formalism based on an extension of logic programming. Although DeLP has been successfully integrated in a number of different real-world applications, DeLP cannot deal with explicit uncertainty, nor with vague knowledge, as defeasibility is directly encoded in the object language. This paper introduces P-DeLP, a new logic programming language that extends original DeLP capabilities for qualitative reasoning by incorporating the treatment of possibilistic uncertainty and fuzzy knowledge. Such features will be formalized on the basis of PGL, a possibilistic logic based on Gödel fuzzy logic.

1 Introduction and motivations

In the last years defeasible argumentation frameworks have proven to be a successful approach to formalizing qualitative, commonsense reasoning from incomplete and potentially inconsistent knowledge [Chesñevar et al., 2000, Prakken and Vreeswijk, 2002]. As a consequence, argument-based frameworks have integrated in a number of real-world applications, such as automated text analysis [Hunter, 2001], intelligent web search [Chesñevar and Maguitman, 2004b], knowledge engineering [Carbogim et al., 2000] and clustering [Gómez and Chesñevar, 2004], among many others). Defeasible Logic Programming (or DeLP) [García and Simari, 2004] is one of such formalisms, combining results from defeasible argumentation theory and logic programming. Although DeLP has proven to be a suitable framework for building real-world applications that deal with incomplete and contradictory information in dynamic domains, it cannot deal with explicit uncertainty, nor with vague knowledge, as defeasible information is encoded in the object language using “defeasible rules”.

This paper introduces P-DeLP, a new logic programming language that extends original DeLP capabilities for qualitative reasoning by incorporating the treatment of possibilistic uncertainty and fuzzy knowledge. Such features will be formalized on the basis of PGL [Alsinet and Godo, 2000, Alsinet and Godo, 2001], a possibilistic logic based on Gödel fuzzy logic. In PGL formulas are built over fuzzy propositional variables and the certainty degree of formulas is expressed with a necessity measure. In a logic programming setting, the proof method for PGL is based on a complete calculus for determining the maximum degree of possibilistic entailment of a fuzzy goal.

In the context of complex logic-programming frameworks (like the one provided by extended logic programming), PGL lacks of an adequate mechanism to handle contradictory information, as conflicting derivations can be found. In P-DeLP such conflicts will be solved using an argument-based inference engine. Formulas will be supported by arguments, which will have an attached necessity measure associated with the supported conclusion. The ultimate answer to queries will be given in terms of warranted arguments, computed through a dialectical analysis.

The rest of the paper is structured as follows. First, in Section 2 we formalize the syntax, semantics and the proof method of P-DeLP. In Section 3 we introduce the central notion of argument and a procedural mechanism for obtaining arguments. In Section 4 we formalize the notions of attack among arguments and the process of warrant in P-DeLP. Finally, in Section 5 we discuss related work and present the most impor-
tant conclusions that have been obtained.

2 The P-DeLP programming language

As already pointed out our objective is to extend the DeLP programming language to deal with both vague knowledge and possibilistic uncertainty, we will refer to this extension as Possibilistic DeLP or P-DeLP. To this end, the base language of DeLP will be extended with fuzzy propositional variables and arguments will have an attached necessity measure associated with the supported conclusion. The ultimate answer to queries will be given in terms of warranted arguments, computed through a dialectical analysis.

The P-DeLP language $\mathcal{L}$ is defined from a set of fuzzy atoms (fuzzy propositional variables) \{p, q, \ldots\} together with the connectives \{\sim, \land, \leftarrow\}. The symbol $\sim$ stands for negation. A literal $L \in \mathcal{L}$ is a ground (fuzzy) atom $q$ or a negated ground (fuzzy) atom $\sim q$, where $q$ is a (fuzzy) propositional variable. A rule in $\mathcal{L}$ is a formula of the form $Q \leftarrow L_1 \land \ldots \land L_n$, where $Q, L_1, \ldots, L_n$ are literals in $\mathcal{L}$. When $n = 0$, the formula $Q \leftarrow$ is called a fact and simply written as $Q$. In the following, capital and lower case letters will denote literals and atoms in $\mathcal{L}$, respectively.

On the one hand, fuzzy propositions provide us with a suitable representation model in situations where there is vague or imprecise information about the real world. For instance, the fuzzy statement “the engine speed is low” can be nicely represented by the fuzzy proposition $\text{engine\_speed}(\text{low})$, where $\text{low}$ is a fuzzy set defined over the domain $\text{revs\_per\_minute}$, say an interval $[0, 6000]$. In the case $\text{low}$ actually denotes a crisp interval of number of revolutions, the above proposition is to be interpreted as “$\exists x \in \text{low}$ such that the engine speed is $x$”. In the case $\text{low}$ denotes a fuzzy interval with a membership function $\mu_{\text{low}} : [0, 6000] \rightarrow [0, 1]$, the above proposition is interpreted in possibilistic terms as “for each $\alpha \in [0, 1]$, $\exists x \in [\mu_{\text{low}}]_\alpha$ such that the engine speed is $x$, is certain with a necessity of at least $1 - \alpha$”, where $[\mu_{\text{low}}]_\alpha$ denotes the $\alpha$-cut of $\mu_{\text{low}}$, the set of values defined as $[\mu_{\text{low}}]_\alpha = \{ u \in [0, 6000] \mid \mu_{\text{low}}(u) \geq \alpha \}$. So, fuzzy propositions can be seen as (flexible) restrictions on an existential quantifier [Dubois et al., 1998].

On the other hand, in this framework, negation is used to contradict statements represented by fuzzy propositions. For instance, in the case $\text{low}$ denotes a crisp interval of revolutions, $\sim \text{speed}(\text{low})$ is interpreted as “$\exists x \in \text{low}$ such that the engine speed is $x$”, or equivalently “$\forall x \in \text{low}, x$ does not correspond with the engine speed”.

A rigorous (and powerful) approach should be to define a first-order language with typed regular predicates and sorted fuzzy constants (cf. [Alsinet et al., 1999]) which would indeed represent, for instance, the fuzzy statement “the engine speed is low” as $\text{engine\_speed}(\text{low})$ where $\text{engine\_speed}$ is a unary predicate of type ($\text{revs\_per\_minute}$) and $\text{low}$ is a fuzzy constant of sort $\text{revs\_per\_minute}$. In doing so, one is able to deal with partial matching between similar (fuzzy) constants, a very interesting feature. However, in this paper we are not considering yet the possibility of incorporating fuzzy unification between fuzzy constants in the language. Therefore, we will restrict ourselves to a simpler propositional language, where for instance the fuzzy statement $\text{engine\_speed}(\text{low})$ will be simply represented as a fuzzy propositional variable $\text{low\_speed}$.

Definition 1 (P-DeLP formulas) The set $\text{Wffs}(\mathcal{L})$ of wffs in $\mathcal{L}$ are facts and rules built over the literals of $\mathcal{L}$. A certainty-weighted wff in $\mathcal{L}$ or weighted clause is a pair of the form $(\varphi, \alpha)$, where $\varphi$ is a wff in $\mathcal{L}$ and $\alpha \in [0, 1]$ expresses a lower bound for the certainty of $\varphi$ in terms of a necessity measure.

The P-DeLP language is based on Possibilistic Gödel Logic or PGL [Alsinet and Godo, 2000]. There are three main reasons for choosing PGL as the underlying logic to model both uncertainty and fuzziness. First, we have proved that many-valued Gödel logic is fully compatible with an already proposed and suitable extension of necessity measures for fuzzy events, in the sense that Gödel logic allows us to define a well-behaved and featured possibilistic semantics on top of it. Second, like in classical propositional logic programming systems, PGL enables us to define an efficient proof method by derivation based on a complete calculus for determining the maximum degree of possibilistic belief with which a fuzzy propositional variable can be entailed from a set of formulas. Finally, PGL can be extended with a partial matching mechanism between fuzzy propositional variables based on a necessity-like measure which preserves completeness for a particular class of formulas [Alsinet and Godo, 2001]. In our opinion, this is a key feature that justifies by itself the interest of such a logic programming system for defeasible argumentation under vague knowledge and possibilistic uncertainty.

The semantics of PGL [Alsinet and Godo, 2000] is given by interpretations $I$ of the fuzzy propositional variables into the real unit interval $[0, 1]$ which are extended to wffs in $\mathcal{L}$ by means of the following rules:

$$I(L_1 \land \cdots \land L_n) = \min(I(L_1), \ldots, I(L_n))$$

$$I(Q \leftarrow \varphi) = \begin{cases} 1, & \text{if } I(\varphi) \leq I(Q) \\ I(Q), & \text{otherwise} \end{cases}$$
\[ I(\sim q) = \begin{cases} 1, & \text{if } I(q) = 0 \\ 0, & \text{otherwise} \end{cases} \]

Certainty weights are employed to model statements of the form “\( \varphi \) is \( \alpha \)-certain”, where \( \varphi \) represents vague knowledge about the real world. Within the possibilistic model of uncertainty, belief states are modelled by normalized possibility distributions \( \pi : \mathcal{I} \rightarrow [0, 1] \) on a set of interpretations \( \mathcal{I} \). In our framework, the truth evaluation of a wff in \( \mathcal{L} \) \( \varphi \) in each interpretation \( I \) is a value \( I(\varphi) \in [0, 1] \). Therefore, each formula does not induce a crisp set of interpretations, but a fuzzy set of interpretations \( \{ \varphi \} \), defining \( \mu_{\varphi}(I) = I(\varphi) \), for each interpretation \( I \). Hence, to measure the uncertainty induced on a formula by a possibility distribution on the set of interpretations \( \mathcal{I} \) we have to consider some extension of the notion of necessity measure for fuzzy sets, in particular for fuzzy sets of interpretations. In [Dubois and Prade, 1991] the authors propose to define

\[ N([\varphi] \mid \pi) = \inf_{I \in \mathcal{I}} \pi(I) \Rightarrow \mu_{\varphi}(I), \]

where \( \mu_{\varphi}(I) = I(\varphi) \in [0, 1] \) and \( \Rightarrow \) is the reciprocal of G\( \ddot{o} \)del's many-valued implication, which is defined as \( x \Rightarrow y = 1 \) if \( x \leq y \) and \( x \Rightarrow y = 1 - x \), otherwise.

Now let us go into formal definitions.

**Definition 2 (Possibilistic model)** Let \( \mathcal{I} \) be the set of many-valued, interpretations over the language \( \mathcal{L} \). A possibilistic model is a normalized possibility distribution \( \pi : \mathcal{I} \rightarrow [0, 1] \) on the set of interpretations \( \mathcal{I} \).

A possibility distribution \( \pi \) is normalized when there is at least one \( I \in \mathcal{I} \) such that \( \pi(I) = 1 \). In other words, belief states modelled by normalized distributions are consistent states, in the sense that at least one interpretation (or state or possible world) has to be fully plausible.

**Definition 3 (Possibilistic entailment)** A possibilistic model \( \pi : \mathcal{I} \rightarrow [0, 1] \) satisfies a clause \((\varphi, \alpha)\), written \( \pi \models (\varphi, \alpha) \), iff \( N([\varphi] \mid \pi) \geq \alpha \). Now let \( \Gamma \) be a set of clauses in \( \mathcal{L} \). We say that \( \Gamma \) entails \((\varphi, \alpha)\), written \( \Gamma \models (\varphi, \alpha) \), iff every possibilistic model satisfying all the clauses in \( \Gamma \) also satisfies \((\varphi, \alpha)\).

**Proposition 4** Let \( \Gamma \) be a set of clauses in \( \mathcal{L} \). If \( \Gamma \) is satisfiable, then \( \Gamma \models (q, \alpha), (\sim q, \beta) \) iff either \( \alpha = 0 \) or \( \beta = 0 \).

In [Alsinaet and Godo, 2000] we formalized a Hilbert-style axiomatization of PGL. Axioms of PGL are axioms of G\( \ddot{o} \)del fuzzy logic weighted by 1 plus the triviality axiom \((\varphi, 0)\), and inference rules of PGL are a generalized modus ponens rule for necessity measures and a weight weakening rule. The proof method in PGL is defined for any certainty-weighted G\( \ddot{o} \)del formula by deduction relative to the set of axioms and inference rules. In \( \mathcal{L} \) wffs are either certainty-weighted facts or rules (with positive and negative literals) and the proof method should be oriented to goals (positive and negative literals). Then, for P-DeLP we will consider a simple and efficient calculus which will not need the whole axiomatization of PGL. But before we need to introduce some extra definitions and results.

**Definition 5 (Maximum degree of possibilistic entailment)** The maximum degree of possibilistic entailment of a goal \( Q \) from a set of clauses \( \Gamma \), denoted by \( \|Q\|_{\Gamma} \), is the greatest lower bound \( \alpha \in [0, 1] \) on the belief on \( Q \) such that \( \Gamma \models (Q, \alpha) \). Thus, \( \|Q\|_{\Gamma} = \sup \{ \alpha \in [0, 1] \mid \Gamma \models (Q, \alpha) \} \).

Following [Alsinaet and Godo, 2000], one can prove that the maximum degree of possibilistic entailment of a goal \( Q \) from a set of clauses \( \Gamma \) is the least necessity evaluation of \( Q \) given by the models of \( \Gamma \).

To provide P-DeLP with a complete calculus for determining the maximum degree of possibilistic entailment we only need the triviality axiom of PGL and a particular instance of the generalized modus ponens rule of PGL:

**Axiom:** \((\varphi, 0)\)

**Generalized modus ponens (GMP):**

\[
\begin{align*}
(L_0 &\leftarrow L_1 \land \cdots \land L_k, \gamma) \\
(L_1, \beta_1), \ldots, (L_k, \beta_k) \\
(L_0, \min(\gamma, \beta_1, \ldots, \beta_k))
\end{align*}
\]

The GMP rule can be proven to be sound with respect to the many-valued and the possibilistic semantics of the underlying logic.

**Definition 6 (Degree of deduction)** A goal \( Q \) is deduced with a degree of deduction \( \alpha \) from a set of clauses \( \Gamma \), denoted \( \Gamma \vdash (Q, \alpha) \), iff there exists a finite sequence of clauses \( C_1, \ldots, C_m \) such that \( C_m = (Q, \alpha) \) and, for each \( i \in \{1, \ldots, m\} \), it holds that \( C_i \in \Gamma \), \( C_i \) is an instance of the axiom or \( C_i \) is obtained by applying the above inference rule to previous clauses in the sequence.

Due to the negation connective of P-DeLP, the GMP rule allows us to define a complete calculus for determining the maximum degree of possibilistic entailment of a goal from a set of clauses if we restrict ourselves to sets of clauses satisfying the following forward reasoning constraint: The possibilistic entailment degree of a goal \( Q \) from a set of clauses \( \Gamma \) must be univocally determined by those clauses of \( \Gamma \) having \( Q \) in their
head or leading to one of these clauses by resolving them with other clauses by applying the GMP rule. The objective of the forward reasoning constraint is to ensure that, for any goal \( Q \), \( \|Q\|_\Gamma \) can be determined only from the subset \( \Gamma_Q \) of clauses \((\varphi, \alpha)\) in \( \Gamma \) for which either \( Q \) is in the head of \( \varphi \) or \( Q \) depends\(^2\) on the head of \( \varphi \). Roughly speaking, with this requirement one wants to avoid having formulas of the form \((t \leftarrow p, 1)\) and \((\sim t, 1)\) together in \( \Gamma \) since, due to the semantics of the negation connective, we would have that the clause \((\sim p, 1)\) should be derivable from \((t \leftarrow p, 1)\) and \((\sim t, 1)\), and thus, we should enable a kind of modus tollens inference mechanism. The forward reasoning constraint is ensured when for all literal \( L \) appearing in the body of a rule, \( \Gamma \) contains explicit information about \( L \), i.e. either \((L, \alpha) \in \Gamma\) or \((L \leftarrow L_1 \wedge \cdots \wedge L_n, \alpha) \in \Gamma \) with \( \alpha > 0 \). A similar constraint was defined in [Alsinet and Godo, 2000], called context constraint, for preserving completeness when extending PGL with a fuzzy unification mechanism between fuzzy constants. At this point we are ready to define the syntactic counterpart of maximum degree of possibilistic entailment.

**Definition 7 (Maximum degree of deduction)**

The maximum degree of deduction of a goal \( Q \) from a set of clauses \( \Gamma \), denoted \( \|Q\|_\Gamma \), is the greatest \( \alpha \in [0, 1] \) such that \( \Gamma \vdash (Q, \alpha) \).

As the only inference rule of our proof method is the GMP rule within a logic programming framework in which \( \Gamma \) is always a finite set of clauses, there exists a finite number of proofs of a goal \( Q \) from \( \Gamma \), and thus, the above definition turns into \( \|Q\|_\Gamma = \max\{\alpha \in [0, 1] \mid \Gamma \vdash (Q, \alpha)\} \).

Finally, following [Alsinet and Godo, 2000, Alsinet and Godo, 2001], completeness for P-DeLP reads as follows: Let \( \Gamma \) be a set of clauses satisfying the forward reasoning constraint and let \( Q \) be a goal. Then, \( \|Q\|_\Gamma = |Q|_\Gamma \). From now on, we will consider clauses in \( L \) satisfying the forward reasoning constraint.

3 Argumentation in P-DeLP

In the last section we formalized the many-valued and the possibilistic semantics of the underlying logic of P-DeLP. In this section we formalize the procedural mechanism for building arguments in P-DeLP.

We distinguish between certain and uncertain clauses. A clause \((\varphi, \alpha)\) will be referred as certain if \( \alpha = 1 \) and uncertain, otherwise. Moreover, a set of clauses \( \Gamma \) will be deemed as contradictory, denoted \( \Gamma \vdash \perp \), if \( \Gamma \vdash (q, \alpha) \) and \( \Gamma \vdash (\sim q, \beta) \), with \( \alpha > 0 \) and \( \beta > 0 \), for some atom \( q \) in \( L \). Notice that if \( \Gamma \) is a contradictory set of clauses for some atom \( q \) in \( L \), \( \Gamma \) is not satisfiable and there exist \( \Gamma_1 \subseteq \Gamma \) and \( \Gamma_2 \subseteq \Gamma \) such that \( \Gamma_1 \) and \( \Gamma_2 \) are satisfiable and \(|q|_{\Gamma_1} > 0 \) and \(|\sim q|_{\Gamma_2} > 0 \).

**Example 8** Consider the set \( \Gamma = \{ (p \leftarrow q, 0.5), (\sim p \leftarrow q \wedge r, 0.3), (q, 0.2), (r, 1) \} \). Then \( \Gamma \) is contradictory, whereas \( \Gamma \setminus \{(r, 1)\} \) is not.

A P-DeLP program is a set of clauses in \( L \) in which we distinguish certain from uncertain information. As additional requirement, certain knowledge is required to be non-contradictory. Formally:

**Definition 9 (P-DeLP program)** A P-DeLP program \( P \) (or just program \( P \)) is a pair \((\Pi, \Delta)\), where \( \Pi \) is a non-contradictory finite set of certain clauses, and \( \Delta \) is a finite set of uncertain clauses.

**Example 10** Consider an intelligent agent controlling an engine with three switches \( sw_1 \), \( sw_2 \) and \( sw_3 \). These switches regulate different features of the engine, such as pumping system, speed, etc. This agent may have the following certain and uncertain knowledge about how this engine works, e.g.: – If the pump is clogged, then the engine gets no fuel.

- When \( sw_1 \) is on, normally fuel is pumped properly.
- When fuel is pumped properly, fuel seems to work ok.
- When \( sw_2 \) is on, usually oil is pumped.
- When oil is pumped, usually it works ok.
- When there is oil and fuel, usually the engine works ok.
- When there is heat, then the engine is usually not ok.
- When there is heat, normally there are oil problems.
- When fuel is pumped and speed is low, then there are reasons to believe that the pump is clogged.

- When \( sw_2 \) is on, usually speed is low.
- When \( sw_2 \) and \( sw_3 \) are on, usually speed is not low.
- When \( sw_3 \) is on, usually fuel is ok.

Suppose also that the agent knows some particular facts: \( sw_1 \), \( sw_2 \) and \( sw_3 \) are on, and there is heat. The knowledge of such an agent can be modelled by the program \( P_{\text{engine}} \) shown in Fig. 1. Note that uncertainty is assessed in terms of different necessity measures.

Next we will introduce the notion of argument in P-DeLP. Informally, a argument \( A \) is a tentative proof (as it relies to some extent on uncertain, possibilistic information) supporting a given conclusion \( Q \) with a necessity measure \( \alpha \). Formally:

**Definition 11 (Argument. Subargument)** Given a P-DeLP program \( P = (\Pi, \Delta) \), a set \( A \subseteq \Delta \) of uncertain clauses is an argument for a goal \( Q \) with necessity measure \( \alpha > 0 \) (denoted \( \langle A, Q, \alpha \rangle \)) iff:
Let \( \Pi \cup A \) incluce an uncertain fact \((Q, \alpha)\) on the uncertain infor-
mation used to derive a given goal \( Q \) with necessity degree \( \alpha \).
Appropriate preconditions ensure that the proof obtained always follows Cond. 2 in Def. 11. Given a program \( \mathcal{P} \), rule INTF allows to construct arguments from facts. An empty argument can be obtained for any certain fact in \( \mathcal{P} \). An argument concluding an uncertain fact \((Q, \alpha)\) in \( \mathcal{P} \) can be derived whenever assuming \((Q, \alpha)\) is not contradictory wrt the set \( \Pi \) in \( \mathcal{P} \). Rule MPA accounts for modus ponens with a uncertain rule \((L_0 \leftarrow L_1 \land L_2 \land \ldots \land L_k, \gamma)\), with \( \gamma < 1 \). Note that there must exist arguments \((A_1, L_1, \alpha_1), \ldots, (A_k, L_k, \alpha_k)\) for every literal in the antecedent of the rule. MPA is applicable whenever no contradiction wrt \( \Pi \) results when assuming the rule \((L_0 \leftarrow L_1 \land L_2 \land \ldots \land L_k, \gamma)\) and the sets \( A_1, \ldots, A_k \) corresponding to the arguments associated with the antecedent of the rule. Finally, the rule EAR stands for extending a given argument \((A, P, \alpha)\) on the basis of certain knowledge. As any argument is non contradictory wrt \( \Pi \), making new inferences from \( \Pi \cup \{P, \alpha\} \) cannot lead to contradictions, and hence \((A, P, \alpha)\) is also valid whenever \( \Pi \cup \{P, \alpha\} \vdash (Q, \alpha) \). Note that in this case the necessity measure of \( Q \) is the same as the one computed for \( P \), as no uncertainty is involved.

Example 12 Consider the program \( \mathcal{P}_{\text{eng}} \) in Example 10. An argument \((B, fuel_\text{ok}, 0.3)\) can be derived from \( \mathcal{P}_{\text{eng}} \) as follows:

1. \((\emptyset, sw1, 1)\) from (2) via INTF.
2. \((B', pump_\text{fuel}, 0.6)\) from (6) and i) via MPA.
3. \((\emptyset, fuel_\text{ok}, 0.3)\) from (7) and ii) via MPA.

where \( B' = \{(\text{pump}_\text{fuel} \leftarrow \text{sw1}, 0.6)\} \) and \( B = \{(\text{pump}_\text{fuel} \leftarrow \text{sw1}, 0.6);(\text{pump}_\text{fuel} \leftarrow \text{pump}_\text{fuel}, 0.3)\} \). Similarly, an argument \((C, oil_\text{ok}, 0.8)\) can be derived from \( \mathcal{P}_{\text{eng}} \) using the rules (3), (8) and (9) via INTC, MPA, and MPA, resp., with \( C = \{(\text{pump}_\text{fuel} \leftarrow \text{sw2}, 0.8);(\text{oil}_\text{ok} \leftarrow \text{pump}_\text{fuel}, 0.8)\} \). Finally, an argument \((A_1, engine_\text{ok}, 0.3)\) can be derived from \( \mathcal{P}_{\text{eng}} \) as follows:

1. \((B, fuel_\text{ok}, 0.3)\) as shown above.
2. \((C, oil_\text{ok}, 0.8)\) as shown above.
3. \((A_1, engine_\text{ok}, 0.3)\) from i), ii), 10) via MPA.

where \(A_1 = \{(\text{engine}_\text{ok} \leftarrow fuel_\text{ok} \land oil_\text{ok}, 0.3)\} \cup C \). Note that \((C, oil_\text{ok}, 0.8)\) and \((B, fuel_\text{ok}, 0.3)\) are subarguments of \((A_1, engine_\text{ok}, 0.3)\).
4 Computing Warrant in P-DeLP

4.1 Counterargumentation and defeat

Given a program $\mathcal{P}$, it can be the case that there exist conflicting arguments for complementary literals $\langle A_1, q, \alpha_1 \rangle$ and $\langle A_2, \neg q, \alpha_2 \rangle$. Such conflict among arguments will be formalized by the notions of counterargument and defeat presented next.

**Definition 13 (Counterargument)** Let $\mathcal{P}$ be a program, and let $\langle A_1, Q_1, \alpha_1 \rangle$ and $\langle A_2, Q_2, \alpha_2 \rangle$ be two arguments wrt $\mathcal{P}$. We will say that $\langle A_1, Q_1, \alpha_1 \rangle$ counterargues $\langle A_2, Q_2, \alpha_2 \rangle$ iff there exists a subargument (called disagreement subargument) $\langle S, Q, \beta \rangle$ of $\langle A_2, Q_2, \alpha_2 \rangle$ such that $\Pi \cup \{(Q_1, \alpha_1), (Q, \beta)\}$ is contradictory.

**Example 14** Consider $\langle A_1, \text{engine\_ok}, 0.3 \rangle$ given in Example 12 wrt the program $\mathcal{P}_{\text{eng}}$. A counterargument for $\langle A_1, \text{engine\_ok}, 0.3 \rangle$ can be found, namely $\langle A_2, \neg \text{fuel\_ok}, 0.6 \rangle$, obtained from (2), (3), (6), (14), (13) and (1) by applying INTF, INTF, MPA, MPA, MPA, and EAR, resp., with $A_2 = \{\langle \text{pump\_fuel} \leftarrow \text{sw}1, 1\rangle, \langle \text{low\_speed} \leftarrow \text{sw}2, 1\rangle, \langle \text{pump\_clog} \leftarrow \text{pump\_fuel} \land \text{low\_speed}, 1\rangle\}$. Argument $\langle A_2, \neg \text{fuel\_ok}, 0.6 \rangle$ is a counterargument for $\langle A_1, \text{engine\_ok}, 0.3 \rangle$ as there exists a subargument $\langle B, \text{fuel\_ok}, 0.3 \rangle$ in $\langle A_1, \text{engine\_ok}, 0.3 \rangle$ (see Example 12) such that $\Pi \cup \{(\text{fuel\_ok}, 0.3), (\neg \text{fuel\_ok}, 0.6)\}$ is contradictory.

Defeat among arguments involves a preference criterion on conflicting arguments, defined on the basis of necessity measures associated with arguments.

**Definition 15 (Preference criterion $\succeq$)** Let $\mathcal{P}$ be a program, and and let $\langle A_1, Q_1, \alpha_1 \rangle$ and $\langle A_2, Q_2, \alpha_2 \rangle$ be conflicting arguments in $\mathcal{P}$. We will say that $\langle A_1, Q_1, \alpha_1 \rangle$ is preferred over $\langle A_2, Q_2, \alpha_2 \rangle$ (denoted $\langle A_1, Q_1, \alpha_1 \rangle \succeq \langle A_2, Q_2, \alpha_2 \rangle$) iff $\alpha_1 \geq \alpha_2$.

If $\alpha_1 > \alpha_2$, then we will say that $\langle A_1, Q_1, \alpha_1 \rangle$ is strictly preferred over $\langle A_2, Q_2, \alpha_2 \rangle$. Otherwise, if $\alpha_1 = \alpha_2$ we will say that both arguments are equiprefered, denoted $\langle A_2, Q_2, \alpha_2 \rangle \simeq \langle A_1, Q_1, \alpha_1 \rangle$.

**Definition 16 (Defeat)** Let $\mathcal{P}$ be a program, and let $\langle A_1, Q_1, \alpha_1 \rangle$ and $\langle A_2, Q_2, \alpha_2 \rangle$ be two arguments in $\mathcal{P}$. We will say that $\langle A_1, Q_1, \alpha_1 \rangle$ defeats $\langle A_2, Q_2, \alpha_2 \rangle$ (or equivalently $\langle A_1, Q_1, \alpha_1 \rangle$ is a defeater for $\langle A_2, Q_2, \alpha_2 \rangle$) iff

1) $\langle A_1, Q_1, \alpha_1 \rangle$ counterargues $\langle A_2, Q_2, \alpha_2 \rangle$ with disagreement subargument $\langle A, Q, \alpha \rangle$.

2) Either it holds that $\alpha_1 > \alpha$, in which case $\langle A_1, Q_1, \alpha_1 \rangle$ will be called a proper defeater for $\langle A_2, Q_2, \alpha_2 \rangle$, or $\alpha_1 = \alpha$, in which case $\langle A_1, Q_1, \alpha_1 \rangle$ will be called a blocking defeater for $\langle A_2, Q_2, \alpha_2 \rangle$.

**Example 17** Consider $\langle A_1, \text{engine\_ok}, 0.3 \rangle$ and $\langle A_2, \neg \text{fuel\_ok}, 0.6 \rangle$ in Example 14. Then $\langle A_2, \neg \text{fuel\_ok}, 0.6 \rangle$ is a proper defeater for $\langle A_1, \text{engine\_ok}, 0.3 \rangle$, as $\langle A_2, \neg \text{fuel\_ok}, 0.6 \rangle$ counterargues $\langle A_1, \text{engine\_ok}, 0.3 \rangle$ with disagreement subargument $\langle B, \text{fuel\_ok}, 0.3 \rangle$, and $0.6 > 0.3$.

4.2 Computing Warrant via Dialectical Trees

Given an argument $\langle A, Q, \alpha \rangle$, the definitions of counterargument and defeat allows to detect whether other possible arguments $\langle B_1, Q_1, \alpha_1 \rangle$, ..., $\langle B_k, Q_k, \alpha_k \rangle$ are defeaters for $\langle A, Q, \alpha \rangle$. Should the argument $\langle A, Q, \alpha \rangle$ be defeated, then it would be no longer supporting its conclusion $Q$. However, since defeaters are arguments, they may on their turn be defeated. That prompts for a complete recursive dialectical analysis to determine which arguments are ultimately defeated. Ultimately undefeated arguments will be marked as $U$-nodes, and the defeated ones as $D$-nodes. To characterize this process we will introduce some auxiliary notions.

An argumentation line starting in an argument $\langle A_1, Q_1, \alpha_1 \rangle$ (denoted $\lambda_{\langle A_1, Q_1, \alpha_1 \rangle}$) is a sequence $[\lambda_{\langle A_0, Q_0, \alpha_0 \rangle}, \langle A_1, Q_1, \alpha_1 \rangle, \ldots, \langle A_n, Q_n, \alpha_n \rangle]$ that can be thought of as an exchange of arguments between two parties, a proponent (evenly-indexed arguments) and an opponent (oddly-indexed arguments). Each $\langle A_i, Q_i, \alpha_i \rangle$ is a defeater for the previous argument $\langle A_{i-1}, Q_{i-1}, \alpha_{i-1} \rangle$ in the sequence, $i > 0$. In order to avoid fallacious reasoning, argumentation theory imposes additional constraints on such an argument exchange to be considered rationally acceptable wrt a P-DeLP program $\mathcal{P}$, namely:

1) **Non-contradiction**: given an argumentation line $\lambda$, the set of arguments of the proponent (resp. opponent) should be non-contradictory wrt $\mathcal{P}$.

2) **No circular argumentation**: no argument $\langle A_j, Q_j, \alpha_j \rangle$ in $\lambda$ is a sub-argument of an argument $\langle A_i, Q_i, \alpha_i \rangle$ in $\lambda$, $i < j$.

3) **Progressive argumentation**: every blocking defeater $\langle A_i, Q_i, \alpha_i \rangle$ in $\lambda$ is defeated by a proper defeater $\langle A_{i+1}, Q_{i+1}, \alpha_{i+1} \rangle$ in $\lambda$.

The first condition disallows the use of contradictory information on either side (proponent or opponent). The second condition eliminates the “circular reasoning” fallacy. The last condition enforces the use of a stronger argument to defeat an argument which acts as a blocking defeater. An argumentation line satisfying the above restrictions is called acceptable, and can be proven to be finite.
Example 18 Consider \((A_1, \text{engine-ok}, 0.3)\) and the associated defeater \((A_2, \sim \text{fuel-ok}, 0.6)\) in Example 17. Note that \((A_2, \sim \text{fuel-ok}, 0.6)\) has the associated subargument \((A_2', \text{low-speed}, 0.8)\), with \(A_2' = \{(\text{low-speed} \leftarrow \text{sw2}, 0.8)\}\). From the program \(P_{\text{eng}}\) (Fig. 1) a blocking defeater for \((A_2, \sim \text{fuel-ok}, 0.6)\) can be derived, namely \((A_3, \sim \text{low-speed}, 0.8)\), obtained from (3), (4) and (15) via INTF, INTF and MPA, resp. Note that this third defeater can be thought of as an answer (15) via INTF, INTF and MPA, resp. Note that the situation can be expressed in the following argumentation line: \([(A_1, \text{engine-ok}, 0.3), (A_2, \sim \text{fuel-ok}, 0.6), (A_3, \sim \text{low-speed}, 0.8)]\). Note that the program’s last defeater in the above sequence could be on its turn defeated by a blocking defeater \((A_2', \sim \text{fuel-ok}, 0.6)\), resulting in \([(A_1, \text{engine-ok}, 0.3), (A_2, \sim \text{fuel-ok}, 0.6), (A_3, \sim \text{low-speed}, 0.8), (A_2', \sim \text{fuel-ok}, 0.8) \ldots]\). However, such line is not acceptable, as it violates the condition of non-circular argumentation.

Given a program \(P\) and an argument \((A_0, Q_0, \alpha_0)\), the set of all acceptable argumentation lines starting in \((A_0, Q_0, \alpha_0)\) accounts for a whole dialectical analysis for \((A_0, Q_0, \alpha_0)\) (i.e. all possible dialogues rooted in \((A_0, Q_0, \alpha_0)\), formalized as a dialectical tree.  

Definition 19 (Dialectical Tree) Let \(P\) be a DeLP program, and let \((A_0, Q_0, \alpha_0)\) be an argument w.r.t. \(P\). A dialectical tree for \((A_0, Q_0, \alpha_0)\), denoted \(T_{(A_0, Q_0, \alpha_0)}\), is a tree structure defined as follows:  
1) The root node of \(T_{(A_0, Q_0, \alpha_0)}\) is \((A_0, Q_0, \alpha_0)\).  
2) \((B', h', \beta')\) is an immediate child of \((B, h, \beta)\) iff there exists an acceptable argumentation line \(\lambda'\langle A_1, Q_1, \alpha_1\rangle = \langle A_0, Q_0, \alpha_0 \rangle, \langle A_1, Q_1, \alpha_1 \rangle, \ldots, \langle A_n, Q_n, \alpha_n \rangle, \ldots\) such that there are two elements \(\langle A_{i+1}, Q_{i+1}, \alpha_{i+1}\rangle = \langle B', h', \beta'\rangle\) and \(\langle A_i, Q_i, \alpha_i\rangle = \langle B, h, \beta\rangle\), for some \(i = 0 \ldots n - 1\).  

Example 20 Consider \((A_1, \text{engine-ok}, 0.3)\) from Example 12, and the argumentation line shown in Example 18. Note that the argument \((A_2, \sim \text{fuel-ok}, 0.6)\) has a second (blocking) defeater \((A_4, \text{fuel-ok}, 0.6)\), computed from (4), (16) via INTF and MPA, resp. The argument \((A_1, \text{engine-ok}, 0.3)\) has also a second defeater \((A_5, \sim \text{engine-ok}, 0.95)\), computed from (5), (11) via INTF and MPA, resp. There are no more arguments to consider. There are three acceptable argumentation lines rooted in \((A_1, \text{engine-ok}, 0.3)\), namely:

- \([(A_1, \text{engine-ok}, 0.3), (A_2, \sim \text{fuel-ok}, 0.6), (A_3, \sim \text{low-speed}, 0.8)]\)
- \([(A_1, \text{engine-ok}, 0.3), (A_2, \sim \text{fuel-ok}, 0.6), (A_4, \text{fuel-ok}, 0.9)]\)
- \([(A_1, \text{engine-ok}, 0.3), (A_5, \sim \text{engine-ok}, 0.95)]\)

Fig. 2 shows the corresponding dialectical tree \(T_{(A_1, \text{engine-ok}, 0.3)}\) rooted in \((A_1, \text{engine-ok}, 0.3)\). Nodes in a dialectical tree \(T_{(A_0, Q_0, \alpha_0)}\) can be marked as undefeated and defeated nodes (U-nodes and D-nodes, resp.). A dialectical tree will be marked as an AND-or tree: all leaves in \(T_{(A_0, Q_0, \alpha_0)}\) will be marked U-nodes (as they have no defeaters), and every inner node is to be marked as D-node iff it has at least one U-node as a child, and as U-node otherwise. An argument \((A_0, Q_0, \alpha_0)\) is ultimately accepted as valid (or warranted) wrt a DeLP program \(P\) iff the root of \(T_{(A_0, Q_0, \alpha_0)}\) is labelled as U-node.

Example 21 Consider the dialectical tree \(T_{(A_1, \text{engine-ok}, 0.3)}\) from Example 20. The marking procedure results in the nodes of \(T_{(A_1, \text{engine-ok}, 0.3)}\) marked as U-nodes and D-nodes as shown in Fig. 2.

Definition 22 (Warrant) Given a program \(P\), and a goal \(Q\), we will say that \(Q\) is warranted wrt \(P\) with a necessity \(\alpha\) iff there exists a warranted argument \((A, Q, \alpha)\).

For a given program \(P\), a P-DeLP interpreter will find an answer for a goal \(Q\) by determining whether \(Q\) is supported by some warranted argument \((A, Q, \alpha)\). Different doxastic attitudes are distinguished when providing an answer for the goal \(Q\) according to the associated status of warrant.  
1) Answer Yes (with a necessity \(\alpha\)) whenever \(Q\) is supported by a warranted argument \((A, Q, \alpha)\);  
2) Answer No (with a necessity \(\alpha\)) whenever for \(\sim Q^6\) is supported by a warranted argument \((A, \sim Q, \alpha)\);  
3) Answer Undecided whenever (1) and (2) do not hold. It can be shown that the cases (1) and (2) cannot hold simultaneously [García and Simari, 2004]: if there exists an warranted argument for an atom \(q\) based on a program \(P\), then there is no warranted argument for \(\sim q\) based on \(P\).
Figure 2: Dialectical tree for \( \langle A_1, \text{engine ok}, 0.3 \rangle \)

therefore be No, with \( \alpha = 0.95 \).

Consider now the same program \( P_{eng} \), and the goal \( \text{fuel ok} \). The only argument supporting \( \text{fuel ok} \) is \( \langle A_4, \text{fuel ok}, 0.6 \rangle \), which is defeated by a blocking defeater \( \langle A_2, \sim \text{fuel ok}, 0.6 \rangle \). The analysis for \( \langle A_2, \sim \text{fuel ok}, 0.6 \rangle \) is analogous, as this argument is defeated by \( \langle A_4, \text{fuel ok}, 0.6 \rangle \). Thus both arguments ‘block’ each other, neither of them being warranted.

The resulting answer is \text{UNDECIDED}.

5 Conclusions and related work

In this paper we have presented P-DeLP, a new logic programming language based on Defeasible Logic Programming which incorporates the treatment of possibilistic uncertainty and representation of fuzzy knowledge. The proposed approach improves the PGL logic programming framework, allowing to reason about conflicting goals on the basis of an argument-based procedure for computing warrant built on top of the PGL inference mechanism. In this approach, arguments are sets of uncertainty weighted formulas that support a goal, and support weights are used to resolve conflicts among contradictory goals. It must be remarked that DeLP has been successfully integrated in a number of real-world applications (e.g. clustering [Gómez and Chesn´evar, 2004]), intelligent web search [Chesn´evar and Maguitman, 2004b] and recommender systems [Chesn´evar and Maguitman, 2004a]). Several features leading to efficient implementations of DeLP have been also recently studied, particularly those related to comparing conflicting arguments by specificity [Stolzenburg et al., 2003] and defining transformation properties for DeLP programs [Chesn´evar et al., 2003]. Extensions for DeLP in the context of multiagent systems have been also been proposed [Capobianco et al., 2004].

In this context, P-DeLP keeps all the original features of DeLP while incorporating more expressivity and representation capabilities by means of possibilistic uncertainty and fuzzy knowledge. One particularly interesting feature of P-DeLP is the possibility of defining aggregated preference criteria by combining the necessity measures associated with arguments with other syntax-based criteria (e.g. specificity [Simari and Loui, 1992, Stolzenburg et al., 2003]).

In the last years the development of combined approaches based on qualitative reasoning and uncertainty has deserved much research work [Parsons, 2001]. Preference-based approaches to argumentation have been developed, many of them oriented towards formalizing conflicting desires in multiagent systems [Amgoud, 2003, Amgoud and Cayrol, 2002]. In contrast with these preference-based approaches, the P-DeLP framework involves necessity measures explicitly attached to fuzzy formulas and the proof procedure of the underlying possibilistic fuzzy logic is used for computing the necessity measure for arguments. Besides, it must be stressed that a salient feature of P-DeLP is that it is based on two logical frameworks that have already been implemented (namely PGL [Alsinet and Godo, 2001] and DeLP [García and Simari, 2004]).

There has been generic approaches connecting defeasible reasoning and possibilistic logic (e.g.[Benferhat et al., 2002]), and recently a number of hybrid approaches connecting argumentation and uncertainty have been developed. Probabilistic Argumentation Systems [Haenni et al., 2000, Haenni and Lehmann, 2003] use probabilities to compute degrees of support and plausibility of goals, related to Dempster-Shafer belief and plausibility functions. However this is not a dialectics-based framework as opposed to the one presented in this paper. In [Schweimeier and Schroeder, 2001] a fuzzy argumentation system based on extended logic programming is proposed. In contrast with our framework, this approach relies only on fuzzy values applied to literals and there is no explicit treatment of possibilistic uncertainty.

Part of our current research work will be developed into three directions: first, we will extend the existing implementation of DeLP to incorporate the new features of P-DeLP. Second, we will apply the resulting implementation of P-DeLP to improve existing real-world applications of DeLP and to develop new ones. Finally, we will analyze extending P-DeLP to first order. It must be remarked that the Generalized Modus Ponens rule used in P-DeLP is syntactically the same as the one used in possibilistic logic [Dubois et al., 1994]. As a consequence, to implement the machinery of P-DeLP the underlying possibilistic fuzzy logic PGL can be replaced by the possibilistic logic. The advantage of this approach is that the current logic programming system can be extended to first order, incorporating fuzzy unification between fuzzy constants [Alsinet and Godo, 2001].
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