Determination of the parameters of diffusion and unsteady filament currents in a cylindrical chalcogenide glassy semiconductor

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Abstract. For the Ge-Sb-Te system, the heat equation describing the current filament in the cylindrical semiconductor plate is approximately solved. The scale of the lateral current flowing perpendicular to the filament is estimated. It is shown that for the infinitely long times, the current in the filament is proportional to the square of the maximum temperature at the centre of the filament and inversely proportional to the applied electric field. It is established that the lateral current is small as compared to the current flowing in the filament, so the occurrence of lateral filaments growing from the main filament is unlikely.

1. Introduction
Chalcogenide glassy semiconductors (CGSs) were discovered in the late 1950s [1], and they possess many interesting properties and characteristics that have been studied since. For example, a phenomenon of reversible breakdown (switching effect) occurs in these semiconductors when a low-resistance state changes for a high-resistance state and backwards. In this case, a section with an increased current density (current filament) may form in the material [2]. Also, these semiconductors can remain in a state of high or low conductivity after turning off the field (memory effect). Due to this property, CGSs can be used in phase-change memory (PCM) devices, where the memory effect is enabled by a large resistance contrast between amorphous and crystalline states in the material. The amorphous phase typically offers high electrical resistivity, while the crystalline phase may exhibit resistivity that can be three or four orders of magnitude lower. It is believed that CGS-based PCM devices may soon replace the current generation of Flash Nand memory media [3-5]. However, high currents in the filament can destroy the semiconductor, and knowing as many characteristics of the filament as possible is very useful for designing PCM CGS-based elements.

Reversible breakdown was discovered by S. Ovshinsky in 1963 [6]. If the breakdown is caused solely by the heating, this is a thermal breakdown; in other cases, an electro-thermal breakdown occurs. In the general case, the conductivity may depend on temperature, electric field, activation energy and effective electron mass, and can be affected by tunnel effects and the presence of impurities.

The purpose of this work was to prove the existence of a stable current filament that is not subject to decay over time, even under the influence of some important factors, including diffusion. We obtained a simple analytical description of the current filament taking into account the dependence of its parameters on time and coordinates. We found that diffusion currents do not make a significant contribution to the destruction of the filament due to their small value.
2. Analytical solution

Let us consider the current filament that is formed in a cylindrical semiconductor plate with heat removal according to Newton’s law. Let us also take conductivity and activation energy values typical of GST (Ge-Sb-Te) system, which is most promising for memory applications [7]. A constant voltage is applied to the ends of the plate. Conductivity can be taken in the form that summarizes the temperature and electric field dependences proposed in references [8] and [9], respectively:

\[ \sigma = \sigma_0 e^{(\frac{-\Delta E}{kT} + \frac{F}{F_0})}, \]

where \( \Delta E \) is the energy of activation of conductivity, \( T \) is the temperature, \( k \) is the Boltzmann constant, and \( F \) is the electric field. This expression for the conductivity is in fact the simplest one among those that take into account the electro-thermal breakdown.

In the 1960s, a stationary heat equation was solved [10], which describes the current distribution:

\[ t'\prime(x) + \frac{t(x)}{x} - (t - t_0) + \beta e^{(\frac{-1}{\tau})} = 0. \]

Here \( t = kT/\Delta E \) is the dimensionless temperature, \( x = r/r_0 \) is the dimensionless coordinate,

\[ \beta = \frac{F^2}{F_0^2} e^{(\frac{F}{F_0})}, F_0^2 = \frac{2\Delta E}{\sigma_0 L k}. \]

\( F_0 = 10^6 \) V/m, \( L \) is the film thickness and \( \lambda \) is the heat sink coefficient. In the present work, equation (2) was transformed into a time-dependent one:

\[ 2 \frac{t'(x)}{x} + at + b = \frac{\partial t}{\partial u}. \]

Boundary conditions:

\[ \frac{\partial t}{\partial x}(0, u) = 0; \quad t(x, \infty) = t_o + (t_m - t_0) e^{(\frac{-ax^2}{4})} \]

\[ b = \beta e^{-1}_m (1 - \frac{1}{t_m}) + t_0; \quad a = \frac{\beta}{t_m} e^{-1}_m - 1; \quad u = \frac{ct}{r_0^2} \]

where \( r_0 = 1.4 \times 10^{-6} \) m, \( c = 2 \times 10^{-6} \) m²/s is the thermal diffusivity, \( \ell \) is time.

The second condition is the approximate solution of equation (2). The second derivative is transformed into \( t'(x)/x \), taking into account L’Hôpital’s rule and the extremum of the first derivative at the zero coordinate, while the exponent is expanded into a Taylor series to the first term. Homogeneous and heterogeneous solutions have a physical meaning:

\[ t(x, u) = t_0 + (t_m - t_0) \theta(u) \theta(x). \]

which is a homogeneous solution (and \( \theta \) is the Heaviside step function), and

\[ t(x, u) = -\frac{b}{a} + (t_m + \frac{b}{a}) e^{(\frac{-ax^2}{4})} - (t_m + \frac{b}{a}) e^{(\frac{-a(x-\frac{b}{a})^2}{4})}, \]

which is a heterogeneous solution.

\[ t_m^2 - t_m + t_0 = 0, \quad t_0 = -\frac{b}{a}. \]

The solution of equation (6) expanded into a Taylor series coincides with the stationary solution [10] of equation (2) near zero. The dimensionless temperature dependence on the distance from the centre of the filament is presented in figure 1. The heterogeneous solution describes the current filament. At the initial point of time, the solution of this equation is homogeneous only near zero and at infinity, while with \( a \gg 1 \) the equation is homogeneous with high accuracy. The maximum of the current temperature is not stable and it is drifting to the zero coordinate for several nanoseconds. Let us note, that in reference [2] the filament is reported to move to the borders of the semiconductor.
The roots of equation (6) are:

$$t_{m1,2} = \frac{1 \pm \sqrt{1 - 4t_0}}{2}.$$  \hspace{1cm} (7)

The coordinate is taken as a filament radius:

$$\frac{\partial^2 x}{\partial x^2} = 0.$$ \hspace{1cm} (8)

This is the inflection point at which the gradient of the modulus of the diffusion flow of the particles reaches its maximum due to the temperature difference. Thus,

$$x_{cond}(u = \infty) = \frac{r_0}{\sqrt{a}},$$ \hspace{1cm} (9)

$$I = \langle \sigma \rangle \pi x_{cond}^2 F \propto \frac{r_0^2}{F}.$$ \hspace{1cm} (10)

At infinite time, this dependence for the current completely coincides with the dependence on the field and the maximum temperature obtained by Gelmont and Tsendin [10]. The current filament exists with positive differential conductivity, which becomes much higher than it used to be. According to equation (9), with electric field increasing, the cross-section of the filament decreases, so the current density increases. Thus, switching occurs in the sample. The density of the current flowing in the filament \(j_f = 10^8 \text{A/m}^2\).

The side current consists of the drift current \(j_n\) and the diffusion current \(j_{diff}\) and is calculated by the formula:

$$j_{side} = j_n + j_{diff} = \sigma_n \left( \frac{d\Phi}{dx} + \frac{kT \delta n}{en \delta x} \right).$$ \hspace{1cm} (11)

The drift current:

$$j_n = \sigma_n \frac{kT \delta n}{e} = \sigma_n \frac{\Delta \delta n \delta t}{e}.$$ \hspace{1cm} (12)

It appears that \(j_n \approx 0.1 \text{A/m}^2\) if \(t \approx 0.1\). The dependence of the value of the drift current on the distance from the centre of the filament is presented in figure 3. We consider that the concentration of

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**Figure 1** Dependence of the dimensionless temperature on the distance from the centre of the filament, calculated for \(F = 4 \times 10^6 \text{V/m}, F_0 = 6 \times 10^6 \text{V/m}, \alpha = 100\).

**Figure 2.** Dependence of filament radius on voltage calculated for \(t_0 \approx 0.05, \Delta E = 0.5 \text{eV}, t_m = 0.94, L = 10^{-6} \text{m}\).
charge carriers is constant and the diffusion current $j_{diff}$ is zero. So, the scale of the radial current is many orders of magnitude smaller than the value of the current in the filament.

3. Conclusion

In this work, the simplest analytical model of current crowding was developed taking into account the dependence on the coordinate and time. This model was used to estimate the scale of currents perpendicular to the current of the filament and arising from the uneven heating of the semiconductor. It is shown that the filament current is able to hold for a long time without breaking. The formula describing the filament has a physical meaning and is consistent with the data obtained earlier. However, it is valid only at relatively small distances, which are approximately equal to the radius of the filament. It is shown that the side current does not affect the filament parameters, since its maximum value is several orders of magnitude lower than the amount of current in the filament, which equals hundreds of microamperes. Thus, the occurrence of the side channels of the current filament, which threaten with material damage, is extremely unlikely.

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