Optimum design of cryogenic pump for circulation cooling of high temperature superconducting cables

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Abstract. Pressure drops and temperature rises caused by the cooling with liquid nitrogen are evaluated for two typical examples of HTS cables. One is the three-in-one-type AC cable located inside a corrugated pipe, and the other is the single-core-type DC cable located inside a straight pipe. The theoretical expressions of pressure drop and temperature rise indicate the contributions from three components such as the friction between the liquid nitrogen and the wetted surfaces, the ratio of the heat input to the mass flow rate of liquid nitrogen and the vertical change of position along the cooling pipe. The numerically estimated results are validated by comparing them with the test results carried out by the other groups. Furthermore, the maximum feeding distance of HTS cable can be determined by adjusting the flow rate of pump if the maximum permissible pressure drop and temperature rise are fixed in advance.

1. Introduction

High temperature superconducting (HTS) wires in the form of a tape can be cooled using liquid nitrogen with the boiling temperature of 77 K at the atmospheric pressure. A lot of research-and-development (R&D) projects for power transmission cables using the HTS wires have been in progress all over the world [1,2] because the HTS cables are expected to be small energy dissipation and low installation cost under the ground. A liquid nitrogen pump with high efficiency, enough discharge pressure and allowable maintenance interval would be required to realize a long-length HTS cable in near future, but such a pump has not been developed yet. Therefore, our group aims to develop a liquid nitrogen circulation pump composed of cryogenic magnetic bearings and HTS motor [3]. The specifications of the target pump are determined as the discharge pressure of 1 MPa using a 2-stage impeller, the flow rate of 100 L/min and the rotating speed of 5000 rpm on the basis of the specific speed in turbomachinery.

In order to design the pump, it is necessary to evaluate the pressure drop and temperature rise when the subcooled liquid nitrogen is circulated inside a cryogenic pipe with HTS cable. The pressure drop has been estimated only from the contribution of the friction between the liquid nitrogen and the...
wetted surfaces of pipe and cable, whereas the temperature rise has been calculated only from the heat input to the liquid nitrogen [4]. However, the simplified theoretical expressions for the pressure drop and temperature rise derived for liquid hydrogen cooling of an MgB$_2$ cable have included both the contributions of the friction and the heat input [5]. Therefore, it is necessary to confirm the influences of the additions on the estimated results of pressure drop and temperature rise for liquid nitrogen cooling.

In this study, we evaluate the pressure drop and temperature rise for two types of model cables when the subcooled liquid nitrogen flows through the cooling pipes. The influence of the difference in height along the cooling pipe on the pressure drop and temperature rise is also estimated. Furthermore, the maximum feeding distance of HTS cable is determined by adjusting the flow rate of liquid nitrogen if the maximum permissible pressure drop and temperature rise are fixed in advance.

2. Theoretical expressions for pressure drop and temperature rise

The pressure drop and temperature rise of the cryogenic fluid flowing through the pipe containing the HTS cable as shown in figure 1 can be expressed directly from the stationary one-dimensional balances of mass, momentum and energy as [5]

\[
\frac{1 - Ma^2}{\rho} \frac{dp}{ds} = - \left\{ 1 + \frac{(\gamma - 1)Ma^2}{\alpha T} \right\} F - \frac{(\gamma - 1)Ma^2 W}{\dot{m}} - g \frac{dz}{ds},
\]

\[
\frac{1 - Ma^2}{\alpha T} \frac{dT}{ds} = \frac{1 - \gamma Ma^2}{\alpha T} F + \frac{1 - \gamma Ma^2 W}{\dot{m}} - g \frac{dz}{ds},
\]

where $p$ is the pressure, $T$ the temperature, $Ma (= v/c)$ the Mach number, $v$ the flow velocity, $c$ the speed of sound, $\rho$ the density, $\gamma (= C_p/C_v)$ the specific heat ratio, $C_p$ the specific heat at constant pressure, $C_v$ the specific heat at constant volume, $\alpha$ the thermal expansivity at constant pressure, $W$ the heat input per unit length, $\dot{m}$ the mass flow rate, and $g$ the gravitational acceleration. The flow path $s$ along the pipe is given by $s = \sqrt{x^2 + z^2}$ if only the flow in the $xz$ plane perpendicular to the $y$-axis is considered. Assuming that the symbol $\theta$ represents the angle between the horizontal direction parallel to the $x$-axis and the flow direction along the path $s$, the relationship $dz/ds = \sin \theta$ can be obtained [6]. Furthermore, the parameter $F$ representing the frictional force per mass is given by [5,7]

\[
F = \frac{f \cdot v|v|}{D_n},
\]

where $f$ is the Darcy friction coefficient, and $D_n (= 4A_{ff}/(P_p + P_c))$ the hydraulic diameter with the free-flow cross-sectional area $A_{ff}$ inside the cooling pipe, and the wetted perimeters, $P_p$ and $P_c$, for pipe inner surface and cable outer surface [8].

Equations (1) and (2) can be simplified as [5]

![Figure 1. Tilted location of cooling pipe for HTS cable.](image)


\[
\frac{1}{\rho} \frac{dp}{ds} = -F - \frac{\alpha v^2 W}{C_p} \frac{m}{\dot{m}} - g \frac{dz}{ds},
\]

(4)

\[
C_p \frac{dT}{ds} = (1 - \alpha T) F + \frac{W}{\dot{m}} - \alpha T g \frac{dz}{ds},
\]

(5)

respectively. This is because three parameters appearing in equations (1) and (2) are extremely negligible compared with unity as \(2.3 \leq 10^5 Ma^2 \leq 4.5\), \(7.7 \leq 10^7 (\gamma - 1) Ma^2 / \alpha T \leq 10.4\) and \(4.4 \leq 10^7 \gamma Ma^2 \leq 9.5\) for the subcooled liquid nitrogen with the temperature of 65-80 K by estimating them using the commercial software GASPAK [9] if the typical value for the flow velocity of \(v = 0.6 \text{ m/s}\) is used. On the other hand, another parameter, \(0.27 \leq \alpha T \leq 0.48\), is significant. The first, second and third terms in right-hand sides of equations (4) and (5) represent the effects of the friction, heat input and difference in height, respectively.

The equivalent friction coefficient \(f\) for the cooling pipe with HTS cable can be expressed by [10]

\[
f = \frac{f_p P_p + f_c P_c}{P_p + P_c},
\]

(6)

where \(f_p\) and \(f_c\) are the friction coefficients of the pipe inner surface and cable outer surface, respectively. In this study, the following expressions are used as the friction coefficient \(f_s\) for the straight pipe inner surface or the HTS cable outer surface, and the friction coefficient \(f_r\) for the corrugated pipe inner surface [11]:

\[
f_s = \begin{cases} 0.3164 Re^{0.25}, & \text{Re} \leq 1.17 \times 10^5, \\ 0.0032 + 0.221 Re^{-0.237}, & \text{Re} > 1.17 \times 10^5, \end{cases}
\]

(7)

\[
f_r = \left(1.74 - 2 \log_{10} \left(\frac{2 \varepsilon}{D_i} \right)^2\right), \quad \text{Re} > 900 \left(\frac{\varepsilon}{D_i}\right),
\]

(8)

where \(Re (= \rho v D_i / \mu)\) is the Reynolds number, \(\mu\) the viscosity, and \(\varepsilon\) the corrugation height of the corrugated pipe with the inner diameter \(D_i\). In the model A where the corrugated pipe is used as explained in the next section, the Reynolds number \(Re\) is greater than 2970, whereas \(900(\varepsilon/D_i) = 44.6\). Therefore, we can use equation (8) for the evaluation of friction coefficient. In the case of the steady flow, the product \(\rho v\) is constant from the law of conservation of mass, so that the mass flow rate \(\dot{m} (= \rho v A_H)\) also becomes constant.

3. Evaluation of properties of liquid nitrogen circulations in model cables

In order to evaluate the pressure drop and temperature rise in the circulation cooling with subcooled liquid nitrogen, it is necessary to predetermine the structures and sizes of HTS cables and cooling pipes. The HTS cable is generally operated under the condition of AC or DC. In addition, the HTS cable for AC use has a three-in-one-type, single-core-type, or tri-axial-type structure typically. Furthermore, the use of a straight pipe as an alternative to a conventional corrugated pipe is proposed for the cooling pipe [2]. Therefore, in order to obtain a few typical sets of HTS cables and cooling pipes which cover the realistic structures as many as possible, we refer to the so-called Yokohama project [1] and Ishikari project [2] in this study. The Yokohama project is the R&D of three-in-one-type AC cable implemented under the collaboration among the Tokyo Electric Power Company, Sumitomo Electric Industries and Mayekawa Manufacturing Company. The Ishikari Project is the R&D of single-core DC cable implemented mainly by Chubu University. Figure 2 and table 1 show the structures and specifications of model cables, respectively. The model A is the three-in-one-type AC cable located inside the corrugated pipe, and the model B is the single-core-type DC cable located inside the straight pipe.

In order to validate the evaluation of pressure drop and temperature rise using equations (4) and (5), the experimental data obtained previously by the other groups are compared with the corresponding
calculated results at first. In the Yokohama project [1], the pressure drop of the corrugated pipe with the three-in-one-type HTS cable has been evaluated as about 15 kPa/250 m for the flow rate of 40 L/min. Assuming the cable length of 250 m, the inlet temperature of 69 K, the inlet pressure of 0.25 MPa and the total heat input of 5 W/m, the total pressure drop can be estimated as 12.0 kPa, which is close to the above-mentioned test result. In the Ishikari project [2], on the other hand, the pressure drop for the straight pipe observed at the flow rate of 30 L/min was about 10 kPa for the cable section and the temperature rises were 0.4 K, 0.15 K and 19 mK for sections 3, 2 and 1, respectively, where the liquid nitrogen flowed through the cable pipe in the order of sections 3, 2 and 1. When the inlet temperature of 75 K, the inlet pressure of 0.25 MPa, the heat inleak of 0.818 W/m for section 3 with the length of 379 m, 0.95 W/m for section 2 with 137 m and 0.034 W/m for section 1 with 475 m are used in the calculation, the temperature rises of 369 mK, 155 mK and 21 mK are obtained for sections 3, 2 and 1, respectively, and the pressure drop is estimated as 9.7 kPa for the entire cable pipe, which are very close to the test results.

Figure 3 shows the cable length dependency of pressure drop and temperature rise in two model cables shown in figure 2. Since it is assumed that the cables are located horizontally, the effect of difference in height represented by the third terms in right-hand sides of equations (4) and (5) is not considered here. In the model A, the distance at which the pressure drop becomes equal to 1 MPa is 3.45 km together with the temperature rise of 6.47 K. In this case, the contribution of the second term in right-hand side of equation (4) is only 6.7 ppm, so that the effect of heat input can be neglected. On the other hand, the contribution of the first term in right-hand side of equation (5) is 6.0%, so that the
effect of friction cannot be ignored. In the case of the model B, the distance at which the pressure drop becomes equal to 1 MPa is 10.97 km together with the temperature rise of 7.20 K. In this case, the magnitude relationships between the terms in right-hand sides of equations (4) and (5) are similar to those for the model A. That is, the contributions of the second and first terms in right-hand sides of equations (4) and (5) are 9.9 ppm and 5.4%, respectively.

Figure 4 shows the flow rate dependency of pressure drop and temperature rise in the model cables. The pressure drops increase with increasing the flow rates, whereas the temperature rises have opposite dependency. This means that there are optimum conditions for flow rates, which are discussed in the next section. The pressure drops are proportional to the 1.90 and 1.76 powers of the flow rates in the models A and B, respectively.

Figure 5 shows the dependence of the difference in elevation on pressure drop and temperature rise in two model cables. When the inclination of the cable increases in the positive direction, where the
liquid nitrogen flows diagonally upward, the pressure drop also increases. This is due to the own weight of liquid nitrogen. The influence of difference in height is quite significant for the pressure drop, whereas it is very small for the temperature rise.

4. Optimum design of cryogenic pump

The maximum feeding distances of HTS cables can be determined by adjusting the flow rates of pump if the maximum permissible pressure drop and temperature rise are fixed in advance, as can be found from figure 4. Therefore, we construct an optimization program code to find the flow rate at which the cable length becomes longest. In the models A and B, an optimization program code is constructed to obtain the inlet flow rate $\dot{x}_2$ at which the cable length $x_1$ becomes longest within the pressure drop of 1 MPa and the temperature rise of 10 K. At that time, we use the nonlinear optimization library NLopt [12,13]. In this case, the objective function $F$ and the inequality constraint equations are defined as

$$\begin{align*}
\text{minimize} & \quad F(x_1, x_2) = -x_1, \\
\text{subject to} & \quad p_0 - p_d \leq 1 \text{ MPa}, \\
& \quad T_d - T_0 \leq 10 \text{ K},
\end{align*}$$

where $p_0$ and $T_0$ are the pressure and temperature at the cable inlet, respectively. In the case of cable pipe without return pipe, where the liquid nitrogen flow is unidirectional, $p_d$ and $T_d$ are considered as the pressure and temperature of the cable termination, respectively. In the case of cable pipe with return pipe, where the flow is bidirectional, $p_d$ and $T_d$ are considered as the pressure at the return cable end and the halfway cable termination temperature, respectively. The cable length dependency of pressure drops and temperature rises in the models A and B with optimum flow rates of liquid nitrogen is plotted in figures 6 and 7, respectively, and the optimization results are summarized in table 2. In the case of cable pipe with return pipe, there are pressure drops in the return pipes, so the optimum flow rates are less than those for no return pipe. Even if the return pipes are added, the cable lengths do not become too short because the temperatures at the return pipes do not affect the temperature limit of 10 K.

5. Conclusions

The pressure drop and temperature rise were numerically evaluated when the subcooled liquid nitrogen was circulated through two types of model cables. The pressure drop and temperature rise increased and decreased with increasing the pump flow rate, respectively. When the inclination of cable increased in the positive direction, where the liquid nitrogen flowed diagonally upward, the pressure drop also increased. This was due to the own weight of liquid nitrogen. The influence of difference in height was quite significant for the pressure drop, whereas it was very small for the
Figure 6. Profiles of pressure drop and temperature rise along model A (a) without and (b) with return pipe.

Figure 7. Profiles of pressure drop and temperature rise along model B (a) without and (b) with return pipe.

Table 2. Optimum flow rate at which the cable length becomes longest.

| HTS cable | Return pipe | Optimum inlet flow rate | Cable length | Plot            |
|-----------|-------------|--------------------------|--------------|-----------------|
| Model A   | None        | 85.2 L/min               | 4.68 km      | Figure 6(a)     |
|           | Included    | 74.7 L/min               | 4.15 km      | Figure 6(b)     |
| Model B   | None        | 88.3 L/min               | 13.74 km     | Figure 7(a)     |
|           | Included    | 76.9 L/min               | 12.10 km     | Figure 7(b)     |

temperature rise. Furthermore, we constructed the optimization program code to find the flow rate at which the cable length became longest.
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