Parity Violation and Standard Model Tests

Krishna S Kumar
Department of Physics, University of Massachusetts, Amherst, MA 01003, USA
E-mail: kkumar@physics.umass.edu

Abstract. We discuss the status and prospects of an experimental program of parity-violating asymmetry measurements in the scattering of longitudinally polarized electrons off unpolarized electrons and nucleons. In particular, we focus on those measurements where judicious choices of target and kinematics makes the theoretical predictions dependent purely on fundamental electroweak couplings. If such asymmetries are measured with sufficient precision, they are sensitive to new physics at the TeV scale. The physics implications of recent results, experiments under construction and plans for the future are discussed.

1. Introduction
Over the past three decades, experiments have been searching for clues to address the many shortcomings of the SU(2)$_L \times$U(1)$_Y$ gauge theory of electroweak interactions. Experimental measurements are trying to access the TeV scale in two broad thrusts: high energy colliders and complementary low energy precision electroweak measurements. While low energy measurements are being carried out both at accelerator as well as non-accelerator facilities, the accelerator-based measurements focus on electroweak observables that can be calculated to high accuracy, and that can achieve sufficient precision so that indirect effects of new dynamics at the TeV scale might become manifest. In the following, we describe an important subset of such measurements which use the technique of parity-violating electrons scattering.

Soon after the discovery of parity violation in beta decay 50 years ago, Zel’dovich speculated that there might be an analogous parity violating neutral current interaction [1]. He noted that if such an interaction existed, then parity violation would be manifested in lepton-nucleon scattering due to the interference between the weak and electromagnetic amplitudes. He predicted that if one scatters longitudinally polarized electrons off unpolarized protons and flipped the sign of the beam polarization, the fractional difference in the cross-section would be:

\[ A_{PV} \equiv \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \simeq \frac{|A_Z|}{|A_\gamma|} \simeq \frac{G_F Q^2}{4\pi \alpha} \simeq 10^{-4} Q^2 \]  (1)

For typical fixed target experiments, $A_{PV}$ ranges from roughly $10^{-4}$ to as small as $10^{-7}$. In the mid-seventies, parity violation in deep inelastic electron nucleon scattering was first observed at SLAC [2], from which the electron-quark weak neutral current coupling could be extracted. The measurement was an important validation of the Standard Model, and the extracted value of the electroweak mixing angle $\sin^2 \theta_W$ matched the corresponding value obtained from neutral current neutrino scattering experiments.
Over the past 20 years, the experimental techniques employed to measure these tiny left-right asymmetries have been steadily refined such that statistical errors approaching 0.01 parts per million (ppm) and systematic errors of a few parts per billion (ppb) are possible [3]. Depending on the choice of target and kinematic variables, this has facilitated measurements in several important physics topics, such as many-body nuclear physics, nucleon structure and searches for physics beyond the standard model at the TeV scale.

In this review, we focus on the last topic. Electroweak measurements continue to probe for the indirect effects of new physics at the TeV scale by making more and more precise measurements of electroweak parameters [4]. For example, weak neutral current interactions at $Q^2 \ll M_Z^2$ can probe for heavy Z' bosons or leptoquarks whose effects might be highly suppressed in measurements on the Z pole. Since Z pole measurements are imaginary, there are no interference terms with new, real amplitudes. At low $Q^2$ on the other hand, interference effects might be measurable if sufficient accuracy is achieved [5]. The goal of low energy neutral current measurements is to reach a sensitivity on contact interactions at the 10 TeV level, similar sensitivity to that of the highest energy colliders.

2. Ongoing Program of Low Energy Measurements

Over the past decade, three measurement have reached such sensitivity and set important new constraints on new contact interactions at the TeV scale. The weak charge measurement in atomic Cesium [6], the NuTeV neutrino deep-inelastic scattering measurement [7] and the measurement of $A_{PV}$ in electron-electron (Møller) scattering [8] at SLAC. We start with a discussion of E158, which has the best low energy measurement of $\sin^2 \theta_W$ to date.

2.1. The Purely Leptonic Process of Electron-Electron (Møller) Scattering

The Feynman diagrams for Møller scattering involve both direct and exchange diagrams that interfere with each other. $A_{PV}$ is given by [9]

$$A_{PV} = mE \frac{G_F}{\sqrt{2} \pi \alpha} \frac{4 \sin^2 \theta}{(3 + \cos^2 \theta)^2} Q_{W}^e = mE \frac{G_F}{\sqrt{2} \pi \alpha} \frac{2y(1-y)}{1 + y^4 + (1-y)^4} Q_{W}^e$$

(2)

where $\alpha$ is the fine structure constant, $E$ is the incident beam energy, $m$ is the electron mass, $\theta$ is the scattering angle in the center of mass frame, $y \equiv 1 - E'/E$, $E'$ is the energy of one of the scattered electrons, and $Q_{W}^e$ (proportional to the product of the electron’s vector and axial-vector couplings to the Z0) is the electron’s weak charge.

The electroweak theory prediction at tree level in terms of the weak mixing angle is $Q_{W}^e = 1 - 4 \sin^2 \theta_W$; this is modified at the 1-loop level [10, 11] and becomes dependent on the energy scale at which the measurement is carried out, i.e. $\sin^2 \theta_W$ “runs”. It increases by ~3% compared to its value at the scale of the Z0 boson mass, $M_Z$.

2.1.1. E158 at SLAC

The SLAC E158 experiment carried out the first measurement of parity violation in Møller scattering [8]. The experiment was constructed between 2000 and 2002 and final data collection completed in late 2003. The grand average result for the parity-violating asymmetry in Møller scattering at $Q^2 = 0.03$ GeV$^2$ was found to be $A_{PV} = -131 \pm 14\text{(stat)} \pm 10\text{(syst)} \ (\text{ppb})$. This can be compared to the theoretical prediction within the electroweak framework to put limits on new contact interactions at the TeV scale.

A fairly general and model-independent way to quantify the effects of new high energy dynamics in low energy processes is to express the resulting new amplitudes in terms of 4-fermion contact interactions among leptons and quarks. Specializing here to vector and axial-vector interactions between electrons and/or positrons, such an interaction Lagrangian takes the
form [12]:

\[ \mathcal{L}_{e_1e_2} = \sum_{i,j=L,R} \frac{g_{ij}^2}{2\Lambda^2} \epsilon_i \gamma_\mu \epsilon_j \gamma^\mu e_i, \]

where \( e_{L/R} = \frac{1}{2} (1 \mp \gamma_5) \psi_e \) are chirality projections of the electron spinor, \( \Lambda \) is the mass scale of the contact interaction, \( g_{ij} = \tilde{g}_{ij}^* \) are coupling constants, and \( g_{RL} = g_{LR} \).

From the measured result, and assuming that the only contributing chiral structure comes from the \( \Lambda_{LL} \) term, the 95% C.L. limit is 7 TeV or 16 TeV depending on the sign of the contact interaction term.

2.1.2. MOLLER at JLab There is strong motivation to make further improvements in the accuracy of low energy leptonic and semi-leptonic weak neutral current coupling constants at low energy. Improved measurements would keep pace with the improved sensitivity for discovery at the multi-TeV scale by experiments at the Large Hadron Collider. With the upgrade of Jefferson Laboratory to 12 GeV, a new project called MOLLER (Measurement of Lepton-Lepton Electroweak Reaction) is being designed to improve on the SLAC E158 measurement of the weak charge of the electron \( Q_e^W \) by a factor of five [13].

Using \( \alpha_{\text{EM}}, G_F, M_Z, m_t \) as input, precision measurements of electroweak parameters such as \( m_W \) and \( \sin^2 \theta_W \) can be used to test the electroweak theory at the level of electroweak radiative corrections. Consistency (or lack thereof) of various precision measurements can then be used to constrain \( m_H \) and search for indications of physics beyond the Standard Model. While there is no significant conflict between such indirect limits of \( m_H \) with direct collider searches, the \( m_H \) constraints from the two most precise single measurements of \( \sin^2 \theta_W \) are very different [14]. The proposed MOLLER measurement would provide a third independent measurement of \( \sin^2 \theta_W \) with comparable precision.

In addition, since MOLLER would be carried out a very low energy scale in contrast to the two collider measurements, there is greatly enhanced sensitivity to as yet undiscovered superweak interactions at the TeV scale. For the 2.3% total uncertainty envisioned, the sensitivity can be expressed as:

\[ \frac{\Lambda}{\sqrt{|g_{RR}^2 - g_{LL}^2|}} = \frac{1}{\sqrt{2G_F|\Delta Q_W^W|}} \simeq \frac{246.22 \text{ GeV}}{\sqrt{0.023Q_W^W}} = 7.5 \text{ TeV}. \]

For example, models of lepton compositeness are characterized by strong coupling dynamics. Taking \( \sqrt{|g_{RR}^2 - g_{LL}^2|} = 2\pi \) shows that mass scales as large as \( \Lambda = 47 \text{ TeV} \) can be probed, far beyond the center of mass energies of any current or planned high energy accelerator. This allows electron substructure to be studied down to the level of \( 4 \times 10^{-21} \text{ m} \). The strongest constraints on the coefficients in Eqn. (3) come from LEP 2, approaching 5 TeV for specific chiral combinations. However, the parity-conserving cross-sections and forward-backward asymmetries studied at LEP 2 are blind to the parity-violating combination \( g_{RR}^2 - g_{LL}^2 \) probed by MOLLER.

Using Eqn. (4), it is also straightforward to examine its reach in specific models [4]. The sensitivity to specific R-Parity-violating SUSY interactions would be greatly improved [15]. In addition, the MOLLER measurement would provide unique new constraints on combinations of left- and right-handed leptonic couplings to new neutral gauge bosons between 1 and 2 TeV [16], complimenting LHC measurements [17]. Finally, the sensitivity to lepton-number violating interactions mediated by doubly charged scalars would be greatly extended.

The MOLLER collaboration, a group of ~ 100 authors, submitted a proposal to the Jefferson Laboratory Program Advisory Committee in December 2008. The project was awarded stage-I approval with strong endorsement. The collaboration is currently carrying out an R&D plan to develop the technical design of the apparatus. The MOLLER collaboration is about to submit a proposal to the US Department of Energy, Division of Nuclear Physics to obtain primary...
project funding, and also hope to obtain supplementary funding from the US National Science Foundation and international funding agencies.

The goal is to launch MOLLER with significant R&D funding by late 2011, so that construction of the apparatus can begin in 2013. The MOLLER experiment could then be constructed by 2015, soon after full luminosity beams become available.

2.2. Semi-leptonic Processes

The interactions of the Z-boson and heavier particles can be approximated by four-fermion contact interactions. The parity-violating part of the electron-hadron interaction can then be given in terms of phenomenological couplings $C_{ij}$

$$L^{PV} = \frac{G_F}{\sqrt{2}} \left[ \bar{\tau}_i \gamma^\mu \gamma_5 (C_{1u} \bar{\tau}_i \gamma_\mu u + C_{1d} \bar{\tau}_i \gamma_\mu d) + \bar{\tau}_i \gamma^\mu (C_{2u} \bar{\tau}_i \gamma_\mu u + C_{2d} \bar{\tau}_i \gamma_\mu d) \right]$$

with additional terms as required for the heavy quarks. Here $C_{1j}$ ($C_{2j}$) gives the vector (axial-vector) coupling to the $j^{th}$ quark. Within the framework of the Standard Model, these four couplings are purely a function of $\sin^2 \theta_W$. The atomic Cesium weak charge measurement [6] measured one combination of $C_{1u}$ and $C_{1d}$ precisely.

2.2.1. Elastic Scattering

At sufficiently forward angles and low $Q^2$, the hadronic structure uncertainty in the WNC elastic electron-proton amplitude becomes small enough such that one can measure the underlying coherent $2u + d$ e-q amplitude combination to high precision, thus precisely constraining $2C_{1u} + C_{1d}$. This combination is proportional to $1 - 4\sin^2 \theta_W$, so that a 4% measurement of $A_{PV}$ would achieve a precision of $\delta(\sin^2 \theta_W) = 0.0007$.

Such a measurement is being carried out by the Qweak collaboration in Hall C at Jefferson Lab [18]. Qweak was recently successfully commissioned and is in the process of collecting the first physics data. It is hoped the full statistics will be accumulated by 2012, before Jefferson Laboratory shuts down to upgrade its beam energy to 12 GeV. If successful, the combination of APV in $^{133}\text{Cs}$ and Qweak will precisely constrain both $C_{1u}$ and $C_{1d}$, providing new, unique sensitivity to TeV scale dynamics.

2.2.2. Deep Inelastic Scattering (DIS) $A_{PV}$ in DIS can be written as

$$A_{PV} = \frac{Q^2}{2\sqrt{2} \pi \alpha} \left[ a(x) + \frac{1 - (1 - y)^2}{1 + (1 - y)^2} b(x) \right],$$

$$a(x) = \sum_i f_i(x) C_{1i} q_i / \sum_i f_i(x) q_i^2,$$

$$b(x) = \sum_i f_i(x) C_{2i} q_i / \sum_i f_i(x) q_i^2.$$

Here, $x$ is the fraction of the nucleon momentum carried by the struck quark, $f_i(x)$ are parton distribution functions and $q_i$ are the electromagnetic charges. The $a(x)$ term, which are functions of $C_{1i}$, is the dominant term. For an isoscalar target such as deuterium, the dependence on structure largely cancels out in the $A_{PV}$ ratio of the weak and electromagnetic amplitudes:

$$a(x) = \frac{6}{5} \left[ (C_{1u} - \frac{1}{2} C_{1d}) + \text{corrections} \right];$$

$$b(x) = \frac{6}{5} \left[ (C_{2u} - \frac{1}{2} C_{2d}) \frac{q(x) - \bar{q}(x)}{q(x) + \bar{q}(x)} + \text{corrections} \right],$$

where $q(x) = u(x) + d(x)$. 

\[4\]
In trying to make precision measurements of $A_{PV}$ in PVDIS, several important issues should be taken under consideration. The $a(x)$ term is a factor of 5 to 10 larger than the $b(x)$ term because the latter involves the vector coupling of the electron, which is small in the electroweak theory. In order to be relevant in constraining new physics at the TeV scale, $A_{PV}$ must be measured to 1% relative accuracy or better. To avoid uncertainties from sea-quark distributions and higher-twist effects, it is important to have $Q^2 \geq 2$ GeV$^2$ and $x \geq 0.35$. The DIS cross-section is steeply falling as $x$ increases. The above factors have precluded a followup precision measurement to E122 for the past two decades.

The upgrade of Jefferson Laboratory (Jlab) to 11 GeV incident energy will allow precision measurements for the first time. While the first measurements were carried out at 6 GeV [19] and a 11 GeV experiment has been proposed for Hall C [20], a dedicated spectrometer could for the first time allow the accumulation of high statistics at high $x \sim 0.7$. PV DIS provides access to novel aspects of nucleon structure, such as charge symmetry violation and investigation of higher-twist effects [21, 22]. Most importantly, PV DIS allows the isolation of the linear combination $2C_{2u} + C_{2d}$, which is difficult to measure using elastic scattering.

To comprehensively address these physics topics experimentally, a series of $A_{PV}$ measurements with better than 1% accuracy are required for the $x$ range from 0.3 to 0.7, with a lever arm of a factor of 2 in $Q^2$ while keeping $W^2_{\text{min}} > 4$ and $Q^2_{\text{min}} > 1$. However, to achieve sufficient statistics at the highest possible $Q^2$, a spectrometer with at least 50% acceptance in the azimuth is required. It turns out that the central tracking magnetic field of one of the collider experiments BaBar, CDF or CLEO would be ideal for this purpose. A proposal [23] exploiting this idea was recently approved by the JLab PAC for this comprehensive program. An engineering design of the detectors and support structures inside the solenoid are under way. It is hoped that this project will be funded in a similar fashion to the MOLLER project, roughly one to two years afterward.

3. EW Physics at a Lepton-Ion Collider

At a polarized electron light ion collider (EIC), where longitudinally polarized 4 to 20 GeV electrons would collide with 60 to 250 GeV polarized light ions, many of the hadronic physics issues are alleviated. In addition, because data can be obtained over a wide range of $Q^2$ and $x$ with high precision, it would be easy to separately measure $a(x)$ and $b(x)$, with very little impact of higher-twist effects and axial-current uncertainties. Indeed, it might be possible to directly measure higher-twist effects at fixed $x$ given the luminosity and kinematic reach of the EIC. In particular, it has emerged [22] that the higher-twist coefficient for the parity-violating asymmetry off $^2\text{H}$ is dynamically interesting and isolates a quark-quark correlator that might ultimately be calculable on the lattice.

3.1. Target Spin-Flip PV Asymmetries

While the EIC would make modest improvements on fixed target electroweak measurements planned for Jefferson Laboratory with a 12 GeV beam, it is with target-flip parity-violating asymmetries that the EIC offers an entirely new opportunity. In order to describe the physics, it is instructive to review the most general hadronic tensor that is compatible with Lorentz and CP invariance. It is well-known that, if P conservation is additionally imposed, there are four structure functions, $F_1, F_2$ in the unpolarized case, and $g_1, g_2$ with the inclusion of lepton and quark spin degrees of freedom. If P-violation is allowed, then there are additionally four new structure functions: $F_3, g_3, g_4$ and $g_5$. This phenomenology has been explored by many authors [24, 25, 26, 21].

The EIC will have sufficient luminosity so that the weak-electromagnetic interference amplitude can be extracted from the parity-violating asymmetry in the collisions of unpolarized electrons off longitudinally polarized light ions. At sufficiently high $Q^2$ and $W^2$, one can
extract two independent new structure functions $g_{1}^{\gamma Z_{1}}$ and $g_{5}^{\gamma Z_{5}}$, which will provide unique new
information about polarized parton densities in the nucleon, augmenting existing measurements
in electromagnetic double-spin asymmetries.

Studies of $Q^{2}$ evolution of these new structure functions could potentially provide new insights
into the QCD structure of the nucleon. Cumulatively, it is envisioned that the EIC will collect
data with three polarized light ion species: $^1$H, $^2$H and $^3$He. It will be possible to measure both
single- and double-spin asymmetries with longitudinal polarization in the neutral current process,
as well as analogous asymmetries in the charged current process, thus accessing electroweak
amplitudes with $\gamma$ exchange, $W$ exchange and $\gamma - Z$ interference. Putting all these structure
functions together would provide stringent new tests of the QCD structure of the nucleon and
help alleviate the limiting systematic errors from poor knowledge of the electron and light-ion
longitudinal polarizations.

4. Outlook
Parity-violating electron scattering is a mature field and addresses fundamental questions in
a variety of different topics. The E158 experiment at SLAC has produced the most precise
measurement of the weak mixing angle at low energy. Future measurements at Jefferson
Laboratory will lead to important new insights on the structure of the nucleon and yield new
and more precise measurements of the weak mixing angle. In the far future, it is possible to
probe the nucleon in novel ways at a polarized electron-polarized light ion collider.

Acknowledgments
It is a pleasure to thank the organizers for a stimulating meeting. The contributions of all the
experimental collaborations discussed in this review are acknowledged. This work is funded in
part by US Department of Energy Grant No. DE-FG02-88R40415-A018.

References
[1] Zel’dovich, Ya B, J. Exptl. Theoret. Phys. (U.S.S.R.), 36, 1959, pp. 964-966.
[2] Prescott, C Y, et.al., Phys. Lett., B84, 1979, 524.
[3] Kumar, K S and Souder, P A, Prog. Part. Nucl. Phys., 45, 2000, pp. S333-S395.
[4] Ramsey-Musolf, M J, Phys. Rev., C60, 1999, 015501.
[5] Kumar, K S, et.al., Mod. Phys. Lett., A10, 1995, 2979.
[6] Wood, C S, et.al., Science, 275, 1997, 1759.
[7] Zeller, G P, et.al., Phys. Rev. Lett., 88, 2002, 091802.
[8] Anthony, P L, et.al, Phys. Rev. Lett., 95, 2005, 081601.
[9] Derman, E and Marciano W J, Annals Phys. 121, 147, (1979).
[10] Czarnecki, A and Marciano W J, Phys. Rev. D 53, 1066, (1996) [arXiv:hep-ph/9507420].
[11] Erler, J and Ramsey-Musolf M J, Phys. Rev. D72, 073003 (2005) [arXiv:hep-ph/0409169].
[12] Eichten, E, Lane K D and Peskin M E, Phys. Rev. Lett. 50, 811 (1983).
[13] Jefferson Lab Proposal PR-09-005, K S Kumar, Contact Person
[14] Marciano, W J, AIP Conf. Proc. 870, 236 (2006).
[15] Ramsey-Musolf, M J and Su, S, Phys. Rept. 456, 1 (2008) [arXiv:hep-ph/0612057].
[16] Erler, J and Langacker, P, Phys. Lett. B456, 68 (1999) [arXiv:hep-ph/9903476].
[17] Li, Y, Petriello, F and Quackenbush, S, Phys. Rev. D 80, 055018 (2009) [arXiv:0906.4132 [hep-ph]].
[18] Qweak Experiment at Jefferson Laboratory, R Carlini, Contact Person
[19] Jefferson Lab Experiment E05-007, X Zheng, Contact Person
[20] Jefferson Lab Experiment E05-007, X Zheng, Contact Person
[21] Jefferson Lab Proposal PR12-07-102, P.A. Souder, Contact Person
[22] Anselmino, M, Efremov, A and Leader, E, Phys. Rept. 261, 1 (1995) [Erratum-ibid. 281, 399 (1997)] 
[arXiv:hep-ph/9503369].
[23] Ji, X D, Nucl. Phys. B 402, 217 (1993).
[24] Stratmann, M, Weber, A and Vogelsang, W, Phys. Rev. D 53, 138 (1996) [arXiv:hep-ph/9509236].