Self-attraction effect and correction on three absolute gravimeters

E Biolcati¹, S Svitlov² and A Germak¹

¹ Istituto Nazionale di Ricerca Metrologica INRIM, Turin, Italy
² Institute of Optics, Information and Photonics, University of Erlangen-Nuremberg, Erlangen, Germany

E-mail: e.biolcati@inrim.it

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Abstract
The perturbations of the gravitational field due to the mass distribution of an absolute gravimeter have been studied. The so-called self-attraction effect (SAE) is crucial for the measurement accuracy, especially for International Comparisons, and for the uncertainty budget evaluation. Three instruments have been analysed: MPG-2, FG5-238 and IMGC-02. The SAE has been calculated using a numerical method based on finite element method simulation. The modelled effect has been treated as an additional vertical gravity gradient. The self-attraction correction (SAC) to be applied to the computed $g$ value is of the order of $1 \times 10^{-8} \text{ m s}^{-2}$.

(Some figures may appear in colour only in the online journal)

1. Introduction

Modern transportable absolute gravimeters (AGs) measure the local value of the free-fall acceleration $g$ using the reconstructed trajectory of a falling object in vacuum. The masses of the parts constituting each AG apparatus (such as laser interferometer, vibration isolation system and vacuum chamber) are sources of an additional gravitational field, which can systematically perturb the motion of the object. The so-called self-attraction effect (SAE), as demonstrated in [1, 2], is greater than 1 µGal (1 µGal = $10^{-8} \text{ m s}^{-2}$), which is not negligible in the uncertainty budgets of modern AGs.

The knowledge of the SAE and the specific self-attraction correction (SAC) is crucial for each measurement carried out by the AGs, because it can improve the measurement accuracy. It becomes very important when the measurements are used to calculate the Key Comparison Reference Value (not physically known) during dedicated International Comparisons.

To calculate the SAE, a detailed study on three different gravimeters MPG-2 [3], FG5-238 [4] and IMGC-02 [5] has been performed. A finite element method (FEM) simulation to calculate the contributions of each part of the gravimeters, as proposed in [2, 6], has been used. To simplify calculation of the SAC, the SAE has been treated as an additional constant gravity gradient.

The paper is structured as follows. In section 2 a brief description of the three instruments is presented. Section 3 describes the adopted method used to evaluate the SAE and the results obtained for the AGs. The SAC values are calculated separately for the three AGs in section 4. The uncertainty of the SAC is discussed in section 5, whilst the main conclusions are drawn in section 6.

2. Absolute gravimeters

Three different transportable AGs have been studied:
- the MPG-2, designed in Germany by the Max Planck Institute for the Science of Light (MPL), prototype instrument;
- the FG5-238, a commercial instrument produced by the USA company Micro-g LaCoste Inc. (the results coming from this AG can be considered for the other instruments of the same type, i.e. FG5-2xx with vertical legs of the supporting tripod);
- the IMGC-02, developed in Italy by the Istituto Nazionale di Ricerca Metrologica (INRIM), prototype instrument.

For each of them, the measurement of the $g$ value is obtained using the reconstructed trajectory of a corner-cube prism (or a hollow retroreflector) which moves vertically in vacuum. The IMG-02 takes into account both the rise and fall motions of the flying object, whilst the other two instruments measure the acceleration during the free-fall motion only.
Automated systems are employed to centre, launch and receive the object event by event with nominal rates of about 0.02 Hz to 0.1 Hz during data-taking sessions of several hours. An interferometric system is implemented to obtain time and distance coordinates of the trajectory using a visible laser beam. The interferometer measures the distance between the free-falling corner-cube test mass and a second retroreflector mounted on the quasi-inertial mass of a vibration isolation system (which is used to isolate it from ground vibrations). A detailed description of the three AGs can be found in [3–5].

The distribution of the heaviest and nearest parts of each instrument and the path of the flying object have been considered to characterize the perturbing gravitational field \( \Gamma(Z) \). Each gravimeter can be essentially divided into three main parts:

- **read-out electronic case:** it can be easily moved farther than 1 m from the flying object trajectory, so the SAE from this source is negligible\(^3\);
- **measuring system:** supporting tripod, seismometer or super-spring system with its own support, several detectors and the interferometer;
- **launch system:** the vacuum chamber with the dropping mechanism and its basis with all the accessories, in this case the effect must be analysed in detail because those objects are very close to the trajectory of the flying object.

The geometry of the last two parts has been drawn using the COMSOL\textsuperscript{®} 3D module and it is shown for the three AGs in figure 1. Screws, cables and small holes are not simulated because their influence is negligible for the SAE.

In the free-fall AGs, a co-falling system is implemented. It consists of a support which moves ahead of the test mass to catch it at the end of the free fall [3, 4]. The distance between such system and the test mass varies during the trajectory from zero to a few millimetres. For this reason a non-constant SAE is present. It cannot be included in the time-independent FEM simulation, hence an approximation of the average effect has been evaluated with the appropriate uncertainty.

### 3. Self-attraction effect

To evaluate the gravitational field perturbation of each single part of an AG, an accurate knowledge of the geometry and the mass distribution of the source is needed. Due to the complexity of the single parts (edges, different materials, irregular shape), a FEM simulation has been preferred instead of a mathematical modelling.

The COMSOL\textsuperscript{®} software [7] has been adapted to this purpose. A package dedicated to the gravitational effects is not implemented in the original software. The electrostatic module is then used, exploiting the analogy between gravitational and electrical interactions, as already used for this or a similar purpose in [2, 6].

The component along the direction orthogonal to the floor on which a gravimeter is located (called the \( Z \) coordinate) of the gravitational field (\( \Gamma_Z \)) has been considered to evaluate possible effects on the measurement of the \( g \) value. The other two components along the \( X \)- and \( Y \)-axes can influence the trajectory of the flying object by introducing rotation or shift components. This effect has not been treated in this study because the perturbations are expected to be negligible with respect to the one along the \( Z \) coordinate.

The origin of the reference frame has been set on the ground, with the \( Z \)-axis directed upwards and crossing the centre of the flying object. Modern AGs measure the \( g \) value at height values between about 0.4 m and 1.3 m. The \( \Gamma_Z \) value is

\[^3\] As an example, a mass of 200 kg, at least twice as big as typical AG electronics, placed at 1 m from the trajectory axis produces an effect along the vertical direction less than 0.1 µGal.
A range larger than the distance covered by the free-falling object has been chosen in order to show the perturbation of $\Gamma Z$ due to all the simulated parts.

The FEM simulation consists of three main steps: drawing of geometry, mesh implementation and solution of equations.

In the first step, the parts of the AG are drawn and their mass values implemented. Two different approaches can be used for each part:

- if its shape is regular (such as a cylinder or parallelepiped) and the mass distribution is almost homogeneous, the mass density and volume values are introduced in the software;
- in the case of complex shapes, e.g. different parts with screws and holes, or inhomogeneous mass distribution, such as a seismometer, it is approximated to a regular solid with equivalent volume and mass.

A detailed study has been performed on a single object in order to prove that the two approaches are equivalent with respect to the gravitational field uncertainty required for this report. The result of this test has been omitted and a similar procedure can be found in [2].

For this study seismometers, frames, launch chambers and vacuum pumps have been simulated using the second approach. The other components have been simulated with the proper geometry and density parameters.

The parts so defined are then embedded in empty volume where the field can be propagated. The shape is spherical with the radius value about one order of magnitude larger than the sizes of the AGs, in order to have edges far from the centre of the instrument, so as to minimize the boundary effects [2].

In the second step, the mesh geometry and size are implemented. Such parameters must be tuned to have the maximum resolution of the gravitational field in the range of interest.

To validate the FEM simulation parameters, i.e. the mesh size and the boundary sphere, a study of two simple objects was performed first. A steel sphere centred at $(0, 0, 0)$ with radius $r_1 = 0.12$ m has been located together with an aluminium sphere centred at $(0, 0, 0.6)$ m and $r_2 = 0.1$ m (figure 2).

Materials, positions and sizes of the spheres have been chosen to cover the actual ranges of the SAE of several microgals. The radius of the boundary sphere is $r_0 = 3$ m. In figure 2(left) the gravitational field (only the component along the $Z$ coordinate at $X = 0$ and $Y = 0$) obtained from the FEM simulation is plotted. In the same figure the distribution of the residuals between values coming from theoretical model and simulation is plotted. A root mean square value of the residuals is $0.08$ µGal with the mean about zero, figure 2(right). Some $89.9\%$ of the residuals are below $0.1$ µGal (absolute value). Such a result will be considered as a measure of the discrepancy between the theoretical model and FEM simulation.

In the last step of the method, the equations of the problem are numerically solved. In this case the only parameters that can be set refer to the CPU consumption for the algorithm, with modifications of the results below $0.01\%$ [2].

The main contribution to the uncertainty of the simulation comes from the approximations made in the first step. In order to estimate it, several simulations of different complex parts (e.g. seismometer, launch chamber) have been performed varying the ratio between the sub-part masses. Using such an approach, an uncertainty of $0.1$ µGal has been estimated.

The method has been applied to the three AGs with equal FEM parameters. A mesh called finer (COMSOL\textsuperscript{®} notation) has been implemented. The mesh has been adapted to the edges and the contact points of the objects to have about $300$ k finite elements for each AG. The gravitational field has been calculated in a boundary sphere. The component $Z$ of the gravitational field along the line centred with the trajectory of the flying object has been extracted for the global simulation. The contributions due to the measuring and the launch systems have been separately estimated.

In figure 3 the total effect and contributions due to the individual instrument components are shown for the three AGs. For the MPG-2, the largest values are located at $Z \simeq 0.8$ m and $Z \simeq 1.2$ m, due to the presence of the launch

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\textsuperscript{4} In other words, an object of known mass is constituted of sub-parts of unknown mass. A simulation with a ratio between the sub-parts set to $1/3, 1/3, 1/3$ is performed. Then it is repeated with different ratio values such as $1/8, 3/8, 1/2$, etc.
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The SAE computed above is approximately linear in the measurement range of every AG (figure 4, when the straight line is fitted to the SAE, the coefficient of determination exceeds 0.9 for every instrument). Such a linear gravity variation corresponds to the constant self-attraction gravity gradient \( \gamma_{SAE} \) with some constant offset \( g_{0SAE} \) at a chosen origin, where the free-fall acceleration from the Earth’s gravity field equals \( g_0 \). The superposition property of gravitational fields from several sources allows us to treat the SAE in an AG similarly and in addition to the conventional vertical gradient \( \gamma \) of the Earth’s gravity field. Then the perturbed height-varied free-fall acceleration is approximately given by

\[
g(z) = (g_0 + \gamma z) + (g_{0SAE} + \gamma_{SAE} z). \tag{1}
\]

The \( z \)-axis in (1) is directed towards the centre of Earth (opposite to the \( Z \)-axis in the previous figures), and the signs of \( g_0 \) and \( \gamma \) are chosen to be positive downwards (a freely falling object accelerates downwards, the magnitude of the acceleration increases towards the Earth’s surface). For the purpose of the SAC calculation, the \( z \)-origin is adjusted to the first measurement position (start of the total measurement distance interval \( H \), figure 4).

Typically, AGs employ the linear free-fall motion model for the unperturbed free-fall [3,4,8]. The measurement result \( g_m \) is obtained as the mean-weighted value of the perturbed free-fall acceleration (the weighting functions of different gravimeters are given in [9]). It is equal to an instantaneous value of the linearly varied free-fall acceleration at the specific level below the level \( z = 0 \) (start of \( H \)), called the effective measurement height \( h_{eff} \). Such a parameter is independent of the constant gravity gradient and its uncertainty. It can be found analytically for different gravimeters, as shown in [9–13].

Then at \( z = h_{eff} \) we have from (1)

\[
g_m = g(h_{eff}) = (g_0 + \gamma h_{eff}) + (g_{0SAE} + \gamma_{SAE} h_{eff}). \tag{2}
\]

The second term in (2) is the additive systematic error, which appears in the measurement result due to the SAE. When taken with the opposite sign, it is the self-attraction correction:

\[
\Delta g_{SAC} = -(g_{0SAE} + \gamma_{SAE} h_{eff}). \tag{3}
\]

After the measurement result \( g_m \) is corrected by (3), we assume that the instrument is removed from the observation site. Then this corrected result can be transferred to the desired height level, using the conventional vertical gradient \( \gamma \).
Figure 4 illustrates the SAC calculation. The total measurement interval $H$ is offset from the start of drop by some distance interval $h_{\text{eff}}$, which is different for different gravimeters (in the case of the IMGC-02, the start of drop corresponds to the apex of the trajectory parabola).

The MPG-2 is a multiple-level free-fall AG with the linear free-fall motion model [3]. A SAC of $-1.59 \mu$Gal is obtained by the linear interpolation of the SAE to the level $z = h_{\text{eff}}$ (figure 4(left)). A slight deviation of the SAE from the straight line is considered as the systematic effect in the uncertainty calculation (see below).

The FG5-238 is a multiple-level free-fall AG with the linear free-fall motion model, which includes the gravity gradient as an external known quantity [4]. Such a transformation of the linear model automatically shifts the mean-weighted measurement result (2) from the level $z = h_{\text{eff}}$ to the start of measurement interval $H$ with a correction of $\Delta g_{\gamma} = -\gamma h_{\text{eff}}$. Since $h_{\text{eff}}$ does not depend on the constant gravity gradient or on its distortion by the SAE, the same correction (3) is valid for both results reported at $z = h_{\text{eff}}$ or $z = 0$. (Including the known, but uncertain gravity gradient $\gamma$ into the fitted model causes an additional associated measurement uncertainty of $u_{\gamma} h_{\text{eff}}$, which is irrelevant to the SAC and is not considered in this paper.) The computed SAC is $-1.13 \mu$Gal (figure 4 (centre)).

These two SAC values have to be increased by 0.1 $\mu$Gal due to the co-falling system SAE, i.e. it becomes $-1.69 \mu$Gal for the MPG-2 and $-1.23 \mu$Gal for the FG5-238.

The IMGC-02 is a multiple-level rise-and-fall AG with the non-linear model, which contains the gravity gradient $\gamma$ as an unknown parameter [14]:

$$z(t) = z_0 + g_a \left[ \frac{(t - t_a)^2}{2} - \frac{(t - t_a)^3}{6} + \frac{g_b}{24}(t - t_a)^4 \right],$$

where $g_a$ is the free-fall acceleration, $t_a$ is the time at the apex of the trajectory parabola and $\gamma$ the friction coefficient of the residual air in the launch chamber.

Since this model is non-linear in the parameters $g_a$ and $\gamma$, the superposition principle for the parameter estimates is not valid, and a posteriori correction for a time (or height) varying perturbation is not applicable in a strict sense. Instead, a straightforward approach to modify the registered time–distance coordinates before the non-linear least-squares adjustment might be used by taking into account the a priori computed SAE. However, the obtained results might not be applicable to other measurement conditions or changed parameters of an instrument. Then such a modification is necessary at every new measurement session.

In the IMGC-02 a measurement result is usually reported at the best measurement height $z_b$ below the apex of the parabola [14]:

$$g_b = g_a + \gamma z_b,$$

where $g_a$ and $\gamma$ are directly estimated from the trajectory data using the model (4), and $z_b$ is computed from the elements of the variance–covariance matrix and also depends on a quantity of the upper portion of the trajectory removed from the least-squares fit. With typical parameters of the IMGC-02 the best measurement height is close to the effective measurement height $h_{\text{eff}} = H/6$ derived for the rise-and-fall AG with a linear free-fall motion model and without removing the upper portion of the trajectory [9]:

$$z_b \simeq \frac{H}{5.8},$$

with $H$ being the total distance interval travelled by a falling object twice.

To evaluate the impact of the SAE on the measurement result (figure 4(right)), we assume that at the level $z_b$ the estimate $g_b$ (5) is not correlated with the estimate $\gamma$ [14]. This indicates analogy between $z_b$ and $h_{\text{eff}}$. Then the bias of $g_b$ can be evaluated similar to that, which contributes to the measurement result at the level $z = h_{\text{eff}}$ (3). Consequently, the SAC is given by

$$\Delta g_{\text{SAC}} = -[g_{b,\text{SAE}} + \gamma(z_b - h_0)].$$

The SAC is then computed as $+0.61 \mu$Gal (figure 4(right)).
5. Uncertainty of the SAC

If the errors of the computed SAE were independent random variables, the uncertainty of the SAC, as given by (1), could be estimated using the conventional uncertainty propagation procedures. Since in our case the errors are not entirely independent (this was noticed from separate investigations of the residuals in figure 2) and in addition the SAE deviates from the straight line (figure 4), we avoid the analytical uncertainty propagation and consider the uncertainty components of the SAC in the following way.

Incomplete modelling. The incomplete knowledge of density and geometry of the parts of gravimeters induces some systematic error of the computed SAC. According to simulations shown in section 3, we assign 0.1 µGal for every instrument.

FEM simulation. The discrepancy between the FEM simulation and the mathematical model depends on the magnitude and curvature of the SAE in a complicated way (not shown in this report). As the overestimate, we assign the standard deviation of the single value of the SAE (0.1 µGal) to the uncertainty of the SAC, obtained by the linear interpolation (3), for all the three instruments.

SAE non-linearity. We consider the maximal deviations of the SAE from the straight lines (figure 4) and assign it (with the overestimate) to the uncertainties of the computed SAC. This gives 0.13 µGal, 0.15 µGal and 0.06 µGal for the MPG-2, FG5-238 and IMGC-02, respectively.

Co-falling system. We have evaluated the correction due to such peculiarity of the free-fall systems as −0.1 µGal with the associated uncertainty of ±0.1 µGal.

Combining together, we find the SAC uncertainty of 0.2 µGal, 0.2 µGal and 0.1 µGal for the MPG-2, FG5-238 and IMGC-02, respectively. Table 1 summarizes the computed SAC values and uncertainties for the three AGs together with proper parameters such as initial velocity, measurement range, effective height and initial offset.

6. Conclusions

Three AGs have been studied in order to calculate the SAE due to the masses of their single parts, using a FEM simulation method. The correction to be applied to the final measure of g has then been calculated for each AG. It should be pointed out that, in addition to magnitude and curvature of the observed SAE, the SAC numerically depends on the method of a free-fall, adopted motion model, number and method of data location and portion of the reconstructed free-fall motion trajectory used in the least-squares adjustment (all of this is accumulated in the specific weighting function [9] of an AG).

The MPG-2 presents the largest SAE. A SAC of (−1.7 ± 0.2) µGal can be applied to the reported g value. For the FG5-238 a similar SAE has been found. In this case a SAC of (−1.2 ± 0.2) µGal can be applied to the measurement result reported at the start of drop or at the effective measurement height. This correction is consistent with the results reported in [1]. For the IMGC-02 all the ranges of the SAE appear to be negligible with respect to the declared combined standard uncertainty of 4.3 µGal. However, an a priori approach to implement a SAC of (+0.6 ± 0.1) µGal has been described.

Such results can be related to the measurements performed during all the previous and future comparisons, especially the International Comparisons of Absolute Gravimeters (ICAGs). The most recent published results of an ICAG (i.e. 2005 [15]) have been chosen as a reference in the following.

In the case of the MPG-2 (not present at the ICAG 2005) the computed SAC exceeds the preliminary declared uncertainty of 0.5 µGal due to this effect. Therefore, the results reported at previous comparisons by such AG can be updated with the obtained SAC, removing the contribution of 0.5 µGal from the uncertainty budget.

Regarding the IMGC-02, the application of the SAC of 0.6 µGal should not significantly change the g values with respect to the combined standard uncertainty of about 4.3 µGal, which already included a contribution of 0.3 µGal for SAE [15].

The results found for the FG5-238 can be considered for the other AGs of the same type (i.e. FG5-2.x.x) which represent the majority of the instruments participating in the comparisons. Usually the g measurements performed using FG5-x.x.x did not present a SAC and only a few participants considered the SAC in the uncertainty budget evaluation. At the ICAG 2005 [15], a contribution of 0.1 µGal has been estimated for the instruments and it is has been used to calculate the so-called conventional uncertainty for all the FG5-x.x.x [16]. Hence, a revision of the values of the previous comparison is proposed, also because the abundance...
of FG5 instruments strongly influences the Key Comparison Reference Value.

The SAE, computed using the FEM simulations, was treated as an additive perturbation of the Earth’s gravity field. The SAC was obtained as the mean-weighted value of the SAE over the measurement range. In the case of the linear SAE, the SAC corresponds to the SAE value, linearly interpolated to the effective measurement height (with the opposite sign). If the SAE significantly deviated from linearity, the SAC could be computed using the weighting function [9] of a particular instrument. The presented approach to evaluate the SAE and SAC is applied to three different AGs, but it has general validity. For this reason, the whole procedure can be applied to other AGs, knowing their peculiar features as geometry, mass values and weighting functions.

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