Depletion of advection in turbulent scalar mixing

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Abstract. In turbulent scalar mixing the mean square advection is strongly suppressed with respect to its Gaussian estimate. This effect is particularly important in the small scales and related to the scales in which diffusion plays a role. The link with the generation of passive scalar fronts is discussed and it is argued that scalar fronts are the consequence of the underlying suppression of nonlinearity observed in a wide class of flows for which the dynamics are governed by quadratic nonlinearities or pseudo-nonlinearities.

1. Depletion of nonlinearity

Turbulence displays a tendency to self-organize its small scales to a state in which nonlinearity is reduced. This effect was investigated by Kraichnan & Panda (1988), and shown to reduce significantly the nonlinearity compared to the case in which the velocity field is Gaussian. Investigating other types of model equations, so-called Betchov models, they inferred that this effect might be more general.

In a follow-up paper, Chen et al. (1989), the Direct Interaction Approximation (Kraichnan (1959)) was used to investigate this effect in more detail and comparisons with Direct Numerical Simulations (DNS) showed that statistical descriptions such as DIA are able to capture this effect. The advantage of statistical approaches is that smooth statistics at high Reynolds number can be obtained at low computational cost.

In the present communication we will extend this work by showing that the advection term in the equation for the mixing of a passive scalar is also strongly reduced in the small scales and we argue that the mechanism responsible for this is analogous to the effect discovered by Kraichnan and Panda. Furthermore, this observation elucidates the origin of strong gradients in the fine scales of turbulence.

2. Fronts in passive scalar turbulence

A typical feature observed in independent realizations of a scalar mixed by a turbulent, or random flow is the persistence of small scale gradients, or fronts. These fronts are even generated when the advecting velocity field is structureless (Holzer & Siggia (1994); Shraiman & Siggia (2000)). This suggested that the problem of small scale intermittency could be studied by considering the scalar advection-diffusion equation in the hope to learn about the more complicated case of Navier-Stokes turbulence (see e.g. reference Celani et al. (2001)). In Holzer & Siggia (1994) and Shraiman & Siggia (2000) a phenomenological explanation of the generation
of fronts was proposed, based on the convergence of fluid parcels. In the present communication we do not propose another mechanism of front-generation but we show that the mechanism proposed in Holzer & Siggia (1994) and Shraiman & Siggia (2000) can be seen as a consequence of a more general phenomenon called depletion of nonlinearity of which the discovery can be attributed to Kraichnan and Panda.

3. Numerical and theoretical results

In the present communication we present results from both high-resolution DNS and two-point closure theory to investigate the depletion of advection in isotropic turbulence as a function of lengthscale. We therefore define the spectrum of the advection term as,

$$\int w_\theta(k)dk = |\mathbf{u} \cdot \nabla \theta|^2. \quad (1)$$

We will compare this spectrum to its Gaussian estimate $$w_{\theta}^G(k)$$, in which we will consider a scalar field $$\varphi_\theta(x,t)$$, with the same scalar variance spectrum as the true field, in other words, $$\varphi_\theta(k,t)\varphi_\theta^*(k,t) = (2\pi k^2)^{-1}E_\varphi(k,t)$$, with $$E_\varphi(k,t)$$ the scalar variance spectrum. In the following we will in particular investigate the ratio $$w_\theta(k)/w_{\theta}^G(k)$$. This ratio can be interpreted as a measure of non-Gaussianity of the strength of advection as a function of lengthscale. If this quantity is equal to one, the scalar advection is as strong as it would be if the scalar field was Gaussian.

In direct numerical simulations $$w_\theta(k)$$ can be determined directly from the scalar field. The Gaussian estimate $$w_{\theta}^G(k)$$ is then obtained by replacing the vector-field $$\mathbf{\theta}(x,t)$$ by $$\varphi_\theta(x,t)$$. This field can be readily obtained by randomizing the phases of the Fourier-coefficients $$\hat{\mathbf{\theta}}(k,t)$$. This will yield a field with Gaussian statistics and leaves unchanged the wavenumber spectrum since the values of the amplitudes of the Fourier-coefficients are not touched. This procedure was carried out using data of high resolution (1024^3 gridpoint) pseudospectral direct numerical simulations (DNS) of an isotropic scalar advected by isotropic turbulence. Details on the data and the simulation can be found in Watanabe & Gotoh (2004).

A closure expression for $$w_\theta(k)$$ was obtained following the procedure described in Chen et al. (1989). The final expression, that will be published elsewhere, is a function of the energy spectrum and the scalar spectrum. These two quantities are computed using the lagrangian markovianized field approximation described in Bos & Bertoglio (2006). In Figure 1 we compare the results of DNS and closure. In the top figure we show the spectrum of the scalar variance in compensated form. It is well-known that closures of the Lagrangian DIA (Kraichnan (1965)) family are underestimating the inertial range level of the scalar spectrum. This is here also the case, but we will not focus further on this issue here. In the bottom figure we show the comparison of $$w_\theta(k)/w_{\theta}^G(k)$$. Even though the quantitative agreement is only approximate, both curves show an important suppression of advection in the small scales. It is possible that this agreement will improve even more if the DNS results for $$E(k)$$ and $$E_\varphi(k)$$ are used in the closure expression for $$w_\theta(k)/w_{\theta}^G(k)$$, but that will be left for future work. It is at this point enough to say that the effect we investigate is clearly observed both in DNS and closure. In Figure 2 we show the behavior of this quantity for different Prandtl numbers. It is observed that the range in which depletion takes place is coupled to the diffusive range of lengthscales of the scalar field.
Figure 1. Top: compensated scalar spectra in isotropic turbulence at a Taylor-scale Reynolds number of 427 and $Sc = 1$. Bottom: comparison of the spectrum of the mean square advection term of the scalar equation in isotropic turbulence to its Gaussian value. DNS and theoretical results.
Figure 2. Comparison of the spectrum of the mean square advection term of the scalar equation in isotropic turbulence to its Gaussian value at a Taylor-scale Reynolds number of 1000 and $Sc = 0.01, 0.1, 1$. Inset: scalar spectra in isotropic turbulence.

4. The creation of strong gradients in the small scales of an advected passive scalar field

The physical relevance of the observed mechanism is that it links the origin of fronts (a phenomenon we can also call frontogenesis) observed in scalar mixing (e.g. Watanabe & Gotoh (2004)) and the depression of nonlinearity observed in turbulent flows. Indeed, scalar Beltramization corresponds to the case in which the velocity is perpendicular to the scalar gradients. It is thereby a mechanism which will not mix strong gradients by turbulent diffusion. The large scales will however continue to stretch the small scale scalar patches so that gradients will be enhanced. We infer from this that the observed fronts in passive scalar turbulence might be a consequence of the more general mechanism discovered by Kraichnan & Panda (1988) in the case of hydrodynamic turbulence.

5. Structures and two-point closures

It might seem surprising that if the depression of advection is linked to the marked fronts observed in the fine scales, it could be captured by statistical closures. Indeed it is often
mistakenly assumed that these approaches can not predict anything on structure related issues since all phase-information is averaged out. The apparent paradox stems from the fact that structures are a dynamical consequence of the underlying equations and the statistical theories are derived from these equations. It is therefore not completely surprising that, if the assumptions used in deriving the closures are physically sound, the statistics observed from closures can be related to the structures observed in experiments and simulations.

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References

Bos, W.J.T. & Bertoglio, J.-P. 2006 A single-time two-point closure based on fluid particle displacements. *Phys. Fluids* **18**, 031706.

Celani, A., Lanotte, A., Mazzino, A. & Vergassola, M. 2001 Fronts in passive scalar turbulence. *Phys. Fluids* **13**, 1768.

Chen, H., Herring, J.R., Kerr, R.M. & Kraichnan, R.H. 1989 Non-gaussian statistics in isotropic turbulence. *Phys. Fluids A* **1**, 1844.

Holzer, M. & Siggia, E. 1994 Turbulent mixing of a passive scalar. *Phys. Fluids A* **6**, 1820.

Kraichnan, R.H. 1959 The structure of isotropic turbulence at very high Reynolds numbers. *J. Fluid Mech.* **5**, 497–543.

Kraichnan, R.H. 1965 Lagrangian-history closure approximation for turbulence. *Phys. Fluids* **8**, 575.

Kraichnan, R.H. & Panda, R. 1988 Depression of nonlinearity in decaying isotropic turbulence. *Phys. Fluids* **31**, 2395.

Shraiman, B. & Siggia, E. 2000 Scalar turbulence. *Nature* **405**, 639.

Watanabe, T. & Gotoh, T. 2004 Statistics of a passive scalar in homogeneous turbulence. *New J. Phys.* **6**, 40.