Multi-objective optimization and experimental study of the milling parameters of cycloid gear

JianMin XU1,3, Lizhi GU2,3, ShanMing LUO1

1 School of Mechanical and Automotive Engineering, Xiamen University of Technology, Xiamen, Fujian, 361024, China
2 School of Mechanical and Electric Engineering, Quanzhou University of Information Engineering, Quanzhou, Fujian, 362000, China
3 Fujian University Key Laboratory of Virtual Manufacturing Technology, Quanzhou, Fujian, 362000, China
E-mail:xujianmin1020@163.com

Abstract. To improve production efficiency and reduce machining cost of end milling cycloid gear, a multi-objective milling parameter optimization method which is based on the dynamic characteristics of the machining system is proposed. Using optimized milling parameters for finishing milling cycloid gear, the maximum tool tip deformation is reduced from 0.258μm to 0.092μm by 64.3%, the material removal rate is reduced from 1.56 cm³/mm to 1 cm³/mm by 35.9%, the tool life and the machining accuracy are significantly improved. Effectiveness of the milling parameter optimization model for cycloid gear is verified by milling experiments.

1. Introduction
Cycloid gear is one of the fundamental parts of a cycloid gear reducer. The extreme hardness of the gear material, 20CrMnTi, makes its machinability poor. In the process of high-speed milling of cycloid gear, processing deformation is too difficult to be controlled and milling vibration is also generated. The traditional CAD/CAM system generally requires the craft personnel to manually select milling process parameters to create a CNC machining program, the quality of the CNC machining process is difficult to control. Therefore, the optimization of milling parameters is of major importance in improving milling efficiency and machining quality.

The process parameters for high-speed CNC milling were optimized. The relationship between milling parameters, surface roughness, and material removal rate was established by multi-objective genetic algorithm [1]. Xiong et. al. [2] proposed a processing parameter optimization method based on the Grid Direct Optimization Algorithm and Numerical Simulation. Zhang et al. [3] proposed a method to optimize the parameters of high-speed milling based on the dynamic characteristics of the spindle system, and used an intelligent colony algorithm to optimize the milling parameters. The optimization of milling parameters of an aluminum alloy was achieved by using an adaptive particle swarm optimization algorithm [4]. The cuckoo search algorithm for the selection of optimal machining parameters was researched in milling operations [5]. Pawar et al [6] explored a teaching-learning-based optimization algorithm to optimize parameters of machining processes. Özel et al [7] studied cutting errors in tooth profiles of cycloidal gears manufactured using a CNC milling machine.

At present, most of the milling parameters optimization do not take into account the stability of the machining system. In addition, it is difficult to make prediction using the empirical method and difficult
to avoid the occurrence of milling chatter. Therefore, this aim is achieved by development of a mathematical model for optimizing cycloid gear milling parameters by the optimization algorithm and analysis of the quantity of cycloid gear by experiment contrasts.

2. Mathematical model for optimization of milling parameters

2.1. Design variables

The cutting amount is mainly a function of spindle speed $n ( \text{r/min})$, cutting speed $V_c (\text{m/min})$, feed per tooth $f_t (\text{mm/z})$, feed per revolution $f_r (\text{mm/r})$, feed rate $V_f (\text{mm/min})$, axial cut depth $a_p (\text{mm})$ and radial cut depth $a_e (\text{mm})$. In addition, there is the following relationship between the milling cutter diameter $D$, the cutting speed $V_c$ and the spindle speed $n$, as shown in Eq. (1)

$$V_c = \frac{n \pi D}{1000} \quad (1)$$

Further, the feed per tooth $f_t (\text{mm/z})$, the feed per revolution $f_r (\text{mm/r})$, and the feed rate $V_f (\text{mm/min})$ are related as follows:

$$f_r = n f_t = n N f_t$$

Where $N$ is the number of cutter teeth. Therefore, for the milling process, there are only four independent design variables: the spindle speed $n (\text{r/min})$, feed per tooth $f_t (\text{mm/z})$, axial cut depth $a_p (\text{mm})$, and radial cut depth $a_e (\text{mm})$. Thus, the design parameter vector can be written as

$$X = [n, f_t, a_p, a_e]^T \quad (3)$$

2.2. Objective function

(1) Material removal rate $MRR$. For the milling process, the material removal rate $MRR$ can be expressed as a function of axial cut depth $a_p (\text{mm})$, radial cut depth $a_e (\text{mm})$, spindle speed $n (\text{r/min})$, feed per tooth $f_t (\text{mm/z})$, and number of cutter teeth $N$, as shown in Eq. (4)

$$MRR = n N a_p a_e f_t \quad (4)$$

(2) Processing costs $C_u$. The cost of producing a single piece can be expressed in the following equation:

$$C_u = C_{\text{mat}} + C_m + C_f + C_c + C_t$$

Where $C_{\text{mat}}$ is the material cost of a single piece, $C_m$ is the manufacturing cost of a single piece, $C_f$ is the idle cost of machine tools, $C_c$ is the cost of changing a tool, and $C_t$ is the cost of a single tool. When the cutting process is divided into roughing and finishing, manufacturing costs of a single-piece can be expressed as follows:

$$C_m = (C_{\text{mat}} + C_m)[\frac{L_{\text{fin}}}{n_s N f_t} + \sum_{i=1}^{N_{\text{fin}}} \frac{L_{f_i}}{n_s N f_{t_i}}]$$

Where $C_{\text{mat}}$ is the cost of using the machine tool per unit time, $C_b$ is the labor cost per unit time, $n_s$ is the spindle speed in precision machining, $n_{s r}$ is the spindle speed in rough machining, $f_{t o}$ is the feed per tooth in precision machining, $f_{t r}$ is the feed per tooth in rough machining, $L_{\text{fin}}$ is the tool path length of the finishing layer, and $L_{f i}$ is the length of the tool path for rough machining.

For the numerical control milling, the multi-objective linear weighting method is used to optimize the milling parameters in this paper. The obtained multi-objective optimization model is as follows [8]:

$$F(x) = \omega_1 \frac{MRR}{MRR_0} + \omega_2 \frac{C(x)}{C_{\text{to}}}, \omega_1 + \omega_2 = 1$$

Where and represent the weight of the processing efficiency and processing cost, respectively, and $MRR_0$ and $C_{\text{to}}$ represent the material removal rate and processing costs, respectively.

2.3. Constraints
(1) Cutting force constraint. The milling force cannot exceed the permissible milling force of the spindle, which is as follows:

\[ g_1(X) = F_c - \eta F_{c_{\text{max}}} \leq 0 \]  

Where \( F_{c_{\text{max}}} \) is the maximum milling force for the milling machine, the unit is N, and \( \eta \) is the safety factor. (\( \eta = 0.8 \) in this paper)

(2) The constraints of machining stability. Because the relationship between the stability of the machining system and the cutting parameters \( f(x) \) is difficult to be expressed in a formula, it can only be expressed by a flutter stability leaf diagram obtained by the simulation. The cutting parameters used in the stable cutting can only be selected in the stable cutting area. The stable cutting area is denoted as \( f(x_T) \), which is as follows:

\[ g_2(X) = f(x_T) - f(x) \leq 0 \]  

(3) Milling power constraints. The cutting power constraint can be expressed as [8]:

\[ g_3(X) = P_t - P_c \eta_s \leq 0 \]  

Where \( P_t \) is the instantaneous cutting power, \( P_c \) is the rated power of the spindle motor, and \( \eta_s \) is the transmission efficiency of the milling machine.

(4) Milling parameter constraints. Milling speed and feed rate cannot exceed the scope of the milling machine, and the milling depth generally cannot exceed 10% of the diameter of the cutter. The constraint equation can be defined by Eq. (10). Where \( n_{\text{min}}, n_{\text{max}}, f_{\text{min}} \) and \( f_{\text{max}} \) are the minimum and maximum values of the milling machine speed and feed rate, respectively.

\[ n_{\text{min}} \leq n \leq n_{\text{max}} \]
\[ f_{\text{min}} \leq f_t \leq f_{\text{max}} \]
\[ 0 < a < 0.1D \]  

(5) Spindle torque constraints. Tool torque should be less than the spindle allowable torque. Supposing that \( F_t \) represents the cutting force, \( T_t \) represents the torque the milling machine can provide, it can be defined by

\[ g_4(X) = \frac{DF_t}{2000} - T_t \leq 0 \]  

(6) Processing quality constraints of the finishing surface. The roughness constraint for the side surfaces \( R_a \) and bottom surfaces \( R'_a \) during milling are given by Eq. (13). Where \( f_t \) is the feed per tooth, \( r_e \) is the radius of the tip arc, and \( R_{a_{\text{max}}} \) is the maximum allowable surface roughness.

\[ R_a = 318.5 \frac{f_t^3}{4D} \leq R_{a_{\text{max}}} \]
\[ R'_a = \frac{f_t^3}{8r_e} \leq R_{a_{\text{max}}} \]  

(7) Deformation constraints. Machining deformation constraints can be expressed as:

\[ \delta \leq \delta_{\text{max}} \]  

Where \( \delta \) is the instantaneous deformation and \( \delta_{\text{max}} \) is the maximum allowable deformation. Instantaneous deformation can be obtained by simulation. The tool deformation is not considered in this paper, and the work piece deformation is used as the instantaneous deformation amount during the simulation.

(8) Tool durability constraints. Durability constraints of the milling tool can be expressed as:

\[ T_i = m_1 \sqrt{\frac{C_i K_i D^n}{f_t^m a_p^m a_v^m N^n}} \leq T_{i_{\text{life}}} \]  

Where \( C_i \) and \( K_i \) are constant coefficients, \( m_1, m_2, m_3, m_4 \) and \( m_5 \) are the exponential constants associated with \( f_t, a_p, a_v, N, D \), respectively, and \( T_{i_{\text{life}}} \) is the allowable durability of the milling cutter. Hence, the multi-objective function is defined by
3. Optimization of cycloid gear milling parameters

3.1. The process and algorithm of milling optimization

Because the mathematical optimization model established in this paper is non-linear and multi-objective, the genetic algorithm has a very effective optimization effect on this kind of issue. The material removal rate MRR and machining cost are taken as optimization objective functions on the premise of satisfying the constraints of cutting force, cutting power, cutting torque, tool life and tip deformation. The MATLAB Genetic Algorithm Toolbox (GADS) was used with the following settings, as shown in Table 1. The cutter radius is 3 mm and there are 2 teeth. tool path is end milling. The cutting force constraint is set to 500 N in 3 directions, X, Y, Z, the roughness constraint is set to 0.8μm, the power constraint is 5kw, the torque constraint is 50 Nm.

Table 1. Genetic algorithm parameter settings

| population size | maximum run | crossover probability | mutation probability | ω1 | ω2 |
|-----------------|-------------|-----------------------|----------------------|----|----|
| 200             | 100         | 0.5                   | 0.05                 | 0.5| 0.5|

3.2. Comparison of the milling effect before and after optimization

(1) Processing efficiency comparison. According to Table 2, It is denoted that: after the simulated optimization, the material removal rate was reduced to 1 cm³/min. The material removal rate is decreased by 35.9%. This is because that the optimal parameter combination is obtained by weighing the overall effect of various factors in the process of multi-objective optimization. After optimizing the end milling parameters, the tangential axial depth ratio is greatly reduced. Although the removal rate of materials per unit time is reduced, the tool life and the overall machining quality of cycloid gear are greatly improved.

Table 2. Comparison of finish milling parameters before and after optimization.

| Optimization | Spindle speed (r/min) | Feed rate (mm/min) | Radial cutting depth (mm) | Axial cutting depth (mm) | Material removal rate (cm³/min) | Processing effect |
|--------------|-----------------------|--------------------|---------------------------|--------------------------|-------------------------------|------------------|
| Before       | 5000                  | 1000               | 1.2                       | 1.3                      | 1.56                          | common           |
| After        | 20000                 | 1000               | 2                         | 0.5                      | 1                             | good             |

\[ X = [X_1, X_2, X_3, X_4]^T = [n, f_r, a_p, a_v]^T \]

\[ \min F(X) = \frac{MRR_0}{MRR(X)} + \omega_2 \frac{C(X)}{C_0}, \quad \omega_1 + \omega_2 = 1 \]
Comparison of precision milling processes before and after optimization

Figure 1. Comparison of precision milling processes before and after optimization

(2) Comparison of the main milling process. Figure 1(a) is the comparison of cutting force simulation results in X direction before and after optimization. Figure 1(b) is the comparison of cutting force simulation results in Y direction before and after optimization. Figure 1(c) is the comparison of cutting force simulation results in Z direction before and after optimization. Figure 1(d) is the comparison of milling power before and after optimization. Figure 1(e) is the comparison of milling torque before and after optimization. There is a conclusion that when the optimized parameters are used to process cycloid gear, the force of the cutter in three directions can be greatly improved.

Moreover, milling forces in the three directions X, Y, and Z are greatly reduced in the end milling process with the end mill, as shown in Table 3. It is worth noting that the maximum tool nose deformation is reduced from 0.258μm to 0.092μm by 64%. The main reason is that the optimized cutting parameters are used to end milling cycloid gear, the force of the cutter on all sides is greatly improved. The machining accuracy and surface quality is enhanced, and tool life increases from 1018 min to 2872 min. Tool life and workpiece machining quality can be effectively improved.

Table 3. Comparison of the maximum milling process volume before and after optimization

| Milling process volume                        | Before optimization | After optimization | Contrast before and after optimization (after / before) |
|-----------------------------------------------|---------------------|--------------------|--------------------------------------------------------|
| Maximum milling force in X direction /N       | 45                  | 17                 | 0.38                                                   |
| Maximum milling force in Y direction /N       | 51                  | 16                 | 0.31                                                   |
| Maximum milling force in Z direction /N       | 20                  | 8                  | 0.4                                                    |
| Maximum milling force in XY plane /N          | 70                  | 25                 | 0.36                                                   |
| Maximum cutting power /kw                     | 0.096               | 0.065              | 0.68                                                   |
| Maximum torque /Nm                            | 0.183               | 0.062              | 0.34                                                   |
| Maximal cutter deformation /μm                | 0.258               | 0.092              | 0.36                                                   |
| Tool life /min                                | 1018                | 2872               | 2.82                                                   |

4. Experimental verification of milling parameters optimization of the cycloid gear

4.1. Milling test conditions and High-speed end milling of a cycloid gear

The Heidenhain iTNC530 machine control system is used, the spindle speed is 20-24000 rpm; the maximum feed speed is 30 m/min. The five-axis machining center can fully meet the processing requirements of cycloid gears in this paper. In the process, the spindle should deflect at a certain angle, and the top-down linear end milling method and the projection curve finishing method were used to realize the processing procedures of high precision and high efficiency, as shown in Figure 2, Figure 3 and Figure 4.
4.2. Contrast of surface profile of actual cycloidal gear profile before and after optimization

Figure 5 and Figure 6 show the tooth profile of the cycloid gear machined with the finishing milling parameters before and after optimization.

As can be seen from Figure 5.a, when the cycloid gear was processed with the before optimized parameters, there were obvious fishscale-like chatter tool marks on the tooth surface of the cycloid gear root, such as area 1. When the cutter leaves the workpiece, there are plenty of burrs remaining on the edge of the tooth surface, such as area 2 and 3. As can be seen from Figure 5.b, the 3D morphology of cycloid gear processed by optimizing the processing parameters with the above optimization method at the root of the tooth is greatly improved, and the burr on the edge of the tooth surface is basically removed.

According to Figure 5.a and Figure 6.a, there is noted that when the machining parameters before optimization are used to end mill cycloid gear, the chatter tool mark at the root of the tooth(area 1) is wider than that at the top of the tooth(area B), and the burr of tooth surface edge at the root of tooth(area 3) is larger than that at the top of tooth(area A). The main reasons that the curvature of cycloid gear changes greatly at the tooth root, and the cutter point is denser when the end milling cutter is at the tooth root of cycloid gear. The chipping space between the cutter and the workpiece is small, and the chipping cannot be discharged in time, which causes the cutter to be squeezed by the chipping while cutting the workpiece. Thus, the force of the cutter in the three directions of X, Y, Z is uneven, which obviously aggravates the vibration of the cutter. By contrast, optimized milling parameters were used to end milling the cycloid gear, and the chatter tool mark at the top of the tooth was basically eliminated and the surface quality was improved, as shown in Figure 6.b.
5. Conclusions

Aiming at the finishing process of cycloid gear tooth profile, a multi-objective optimization scheme of milling parameters based on the dynamic characteristics of the machining system is proposed. And the mathematical model of multi-objective optimization of milling parameters is then established.

The optimization of cutting parameters of finishing cycloidal gear is carried out, the maximum deformation is decreases by 64%, the maximum tool tip deformation decreased by 64%, the material removal rate increased by 36%. A comparison of the surface morphology of the gear tooth profile before and after parameter optimization confirms that the optimized milling parameters can greatly improve the surface topography of the machined surface, which verifies the effectiveness of the optimization method.

Acknowledgements

This work was supported by the National Natural Science Foundation of China under No. 51975499, the Educational Research Fund for Young and Middle-aged Teachers in Fujian Province under No. JT180445. The financial and technique supports are gratefully acknowledged.

References

[1] Thangarasu V S, Devaraj G and Sivasubramanian R 2013 High speed CNC machining of AISI 304 stainless steel; Optimization of process parameters by MOGA, International Journal of Engineering. Science and Technology, 4(3): 66-77.

[2] Xiong Y, Wu J, Deng C, Wang Y H. Machining process parameters optimization method for heavy-duty CNC machine tools. Computer Integrated Manufacturing Systems, 2012, 18(4): 729-737.

[3] Zhang Z, Li A P, Bao J and Liu X 2015 Parameters optimization of high speed milling based on the dynamic behaviour of spindle system. Journal of Tongji University, 43(1): 113-120.

[4] Li Y and Liu Q 2015 Service-oriented Research on Multi-pass Milling Parameters Optimization for Green and High Efficiency. Journal of Mechanical Engineering, 51(11): 89-98.

[5] Yildiz and Ali R 2013 Cuckoo search algorithm for the selection of optimal machining parameters in milling operations. International Journal of Advanced Manufacturing Technology, 64(1): 55-61.

[6] Pawar P J and Rao R V 2013 Parameter optimization of machining processes using teaching-learning-based optimization algorithm. The International Journal of Advanced Manufacturing Technology, 67(5-8): 995-1006.

[7] Özel C and Ortaç Y 2016 A study on the cutting errors of the tooth profiles of the cycloidal gears manufactured in CNC milling machine. International Journal of Materials and Product Technology, 53(1): 42-60.

[8] Liu Q, Li Z Q. 2011 Simulation and optimization of CNC milling process. (Beijing: Aviation Industry Press).