Universal conductance dips and fractional excitations in a two-subband quantum wire

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We theoretically investigate a quantum wire based on a quasi-one-dimensional Kondo lattice formed by localized spins and itinerant electrons, where the lowest two subbands of the quantum wire are populated. We uncover a backscattering mechanism involving helically ordered spins and Coulomb interaction between the electrons. The combination of these ingredients results in scattering resonances and partial gaps which give rise to non-standard plateaus and conductance dips at certain electron densities. The positions and values of these dips are independent of material parameters, serving as direct transport signatures of this mechanism. While our theory describes a generic Kondo lattice, an experimentally relevant realization is provided by quantum wires made out of III-V semiconductors hosting nuclear spins such as InAs. Observation of the universal conductance dips would not only confirm the presence of a nuclear spin helix but also identify a strongly correlated fermion system hosting fractional excitations, resembling the fractional quantum Hall states even without external magnetic fields.

I. INTRODUCTION

Quasi-one-dimensional conductors, such as semiconductor nanowires or quantum point contacts, are typical elements of nanocircuits. On the one hand, they embody the ultimate quantum limit upon shrinking a conductor. On the other hand, they provide a testbed for fundamental physics of low-dimensional interacting fermions. From both aspects, conductance is the quantity of prime interest and of most direct experimental access. The observation of conductance quantization, at integer multiples of the conductance quantum $G_0 = e^2/h$, where $e$ is the elementary charge and $h$ the Planck constant, was a landmark achievement in experimental realization of conductors in the quantum limit. It initiated extensive research activities on the quantum conductance, both in experiment and theory, which continue unabated. While the ballistic conductance is expected to be robust against interactions, deviations from the universal values are routinely observed [6–9], including mysterious conductance features that are unexpected from standard single-particle quantum mechanics, such as dips and new plateaus at fractional conductance values, strongly suggesting the importance of many-body interaction effects.

Among these observations, the systems with a single transverse subband received most attention, targeting features such as uniform reduction of the conductance plateau [6–8, 13, 20, 21], emergence of a small plateau around $0.7 \times 2e^2/h$ (also known as the 0.7 anomaly) [8, 12, 22, 24], re-entrant behavior [15, 25], and signature of discrete single-particle quantum mechanics, such as dips and new plateaus at fractional conductance values, strongly suggesting the importance of many-body interaction effects.

The paper is organized as follows. In Sec. II we describe our setup and review the properties of the nuclear spin helix. In Sec. III we explain how the helix induces an energy gap opening in the electron subsystem. The direct transport signatures of our theory, universal conductance dips, and the conditions for their appearance are presented in Sec. IV. The helix-assisted higher-order scatterings for the even- and odd-denominator fillings are discussed in Sec. V and Sec. VI, respectively. Finally, we discuss the experimental realization and verification of our theory in Sec. VII. The details of the calculation are given in Appendix A.
II. SETUP

We consider a quantum wire with anisotropic transverse confinements (see Fig. [1]). The geometry is chosen to separate the subbands corresponding to the stronger confinement direction so that the chemical potential, tuned by a voltage gate, intersects with the lowest two transverse subbands, each being spin degenerate. In typical semiconductor devices, nuclear spins are present and coupled to conduction electrons through the hyperfine interaction. The conduction electrons mediate indirect RKKY coupling between nuclear spins. With sufficiently strong electron-electron interactions, which enhance the RKKY coupling, a nuclear spin helix is stabilized in a finite-length wire at dilution fridge temperature, e.g., \(O(10 \text{ mK}) - O(100 \text{ mK})\) for GaAs and InAs wires [21][26][27][30][31]. This parameter regime is where we focus for the rest of this article.

As the helix is crucial for our mechanism, we briefly review its properties, established in Refs. [21][26][27][30][35]. When only the lowest transverse subband is populated (one-subband regime), a nuclear spin helix is stabilized by the RKKY coupling. The ordered spins induce a spatially rotating (Overhauser) magnetic field, which acts back on electrons and causes spin-flip backscattering between right- and left-moving electrons. Since the RKKY coupling arises from resonant scattering of electrons at the Fermi energy, the spatial period \(\pi/k\) of the helix and the Overhauser field is determined by the Fermi wavevector \(k_{Fj}\). As a consequence, the helix-induced backscattering opens a partial gap at the Fermi points. In transport measurements, this gap manifests itself as uniform reduction of the conductance plateau when the wire is cooled down below the helix ordering temperature, as observed in Ref. [13].

When the chemical potential is adjusted to populate the second subband (two-subband regime), the \(\pi/k_{Fj}\) helix remains stabilized by the lower-spin subbands. A second, additional helix with a spatial pitch of \(\pi/k_{F2}\) could be induced by the upper-subband electrons [32]. However, since the two helices have in general different ordering temperatures (with typical difference \(\sim 10\%\)), it is possible, by adjusting the temperature, to reach the regime in which only the \(\pi/k_{F1}\) helix is present. As outlined here, previous works focused on either the one-subband regime or the double-helix phase in the two-subband regime with incommensurable configuration between \(k_{F1}\) and \(k_{F2}\) (that is, the ratio \(k_{F1}/k_{F2}\) is not an integer). In contrast, we turn our attention to the temperature range where there is a single helix, as illustrated in Fig. [1]. Here we investigate scattering processes due to the single helix with two commensurate subbands, where the chemical potential is adjusted to make \(k_{F1}/k_{F2}\) an integer. The combination of the helix and electron-electron interactions leads to helix-assisted scattering and opens a gap in the upper subband spectrum.

III. HELIX-INDUCED GAP OPENING

We model the system by expanding the electron operator around the Fermi points \(k_{Fj}\),

\[
\psi_{j\sigma}(x) = e^{ik_{Fj}x} R_{j\sigma}(x) + e^{-ik_{Fj}x} L_{j\sigma}(x),
\]

with the slowly varying right(left)-moving fields \(R_{j\sigma}\) \((L_{j\sigma})\), the subband index \(j \in \{1, 2\}\), and the spin index \(\sigma \in \{\uparrow, \downarrow\}\) (we assign the up/down-spin orientation to be in the \(\pm x\) direction). We will suppress the coordinate \(x\) along the wire in the argument unless it may cause confusion. In analogy to the fractional quantum Hall states, we define the ratio \(\nu \equiv k_{F2}/k_{F1}\) as the “filling factor”. The commensurability condition is fulfilled when its inverse, \(1/\nu\), is an integer. In terms of the chemical potential measured from the bottom of the lowest subband, the condition reads

\[
\mu_\nu = \frac{E_g}{1 - \nu^2},
\]

with the subband spacing \(E_g\), see Fig. [1].

The Hamiltonian consists of three parts: \(H = H_0 + H_{\text{int}} + H_B\). The first term \(H_0\) is the kinetic energy. The second term \(H_{\text{int}}\), describing electron-electron interactions, can be separated into the forward-scattering \(H_f\) and backscattering \(H_b\) parts. The term \(H_0 + H_f\) describes a two-subband Tomonaga-Luttinger liquid parametrized by the charge and spin interaction parameters \(K_{jc} < 1\) and \(K_{js} \approx 1\) (for subband \(j\)). Finally, the ordered nuclear spins induce the Overhauser field,

\[
B(x) = B \left[ e_y \cos(2k_{F1}x) + e_z \sin(2k_{F1}x) \right],
\]
In Eq. (4), there is a single resonant (that is, non-oscillatory) term in the integrand,

$$H_B = \sum_{\mu=x,y,z} \int dx \, B^\mu \left( \frac{1}{2} \sum_{j,\sigma,\sigma'} \bar{\psi}_{j}\sigma \sigma' \psi_{j}\sigma' \right),$$

with the Pauli matrix $\sigma^\mu$ defined as

$$\sigma^x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma^y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^z = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \quad (5)$$

In Eq. (4), there is a single resonant (that is, non-oscillatory) term in the integrand,

$$O_B = \frac{g^{(1)}}{2} R_{1\downarrow}^\dagger L_{1\uparrow} \text{ H.c.}, \quad (6)$$

with the coupling strength $g^{(1)} \propto B$, which is of order $O(100 \, \mu eV)$ for typical semiconductors. The operator $O_B$ describes spin-flip backscattering with momentum transfer $2k_F$, which gaps out the $R_{1\downarrow}$ and $L_{1\uparrow}$ modes in the lower subband \cite{26,27}, leading to a partial gap, as shown in Fig. 1.

The other modes ($R_{1\uparrow}$ and $L_{1\downarrow}$) remain gapless, resulting in a helical spin texture also in the electron subsystem. These gapless modes mediate the RKKY interaction and maintain the helix, ensuring the self-consistency of our theory. In this limit, operators that do not commute with $O_B$ cannot be ordered.

The remaining, oscillating integrand in Eq. (4) does not lead to a gap opening by itself due to momentum mismatch. However, as the subbands are commensurate, the combination of this oscillating term and the $H_B$ term of the electron-electron interaction allows for higher-order scattering processes that preserve both the momentum and the spin. If such higher-order backscattering commutes with the operator $O_B$, it can open a gap in the upper subband of the energy spectrum. As a result, the primary experimental consequence of the helix-assisted backscattering is reduction of the conductance at certain fillings, which we present next.

IV. UNIVERSAL CONDUCTANCE DIPS

The conductance as a function of the chemical potential in the one- and two-subband regime is presented in Fig. 1. The conductance values at the dips, as well as the criterion for interaction strength, are given by

$$G_\nu = \begin{cases} 1, & \text{for even } 1/\nu \text{ and } K_{2c} < 2\nu^2, \\ (3\nu^2 + 1)/(\nu^2 + 1), & \text{for odd } 1/\nu \text{ and } K_{2c} < 3\nu^2. \end{cases} \quad (7)$$

At even-denominator fillings (that is, $\nu = 1/(2n)$ with a positive integer $n$), the upper subband is fully gapped, so that the conductance comes from an open channel through the half-gapped lower subband, with the value $e^2/h$. In contrast, at odd-denominator fillings ($\nu = 1/(2n+1)$), both subbands are partially gapped. The upper subband contributes a fractional conductance depending on the filling factor $\nu$ and an open channel in the lower subband gives $e^2/h$. In consequence, upon decreasing $\nu$, the dip value decreases towards unity. In either case, the appearance of the dips rely on strong electron-electron interactions, parametrized by $K_{2c}$. In principle, the smaller the filling factor is, the higher-order backscattering and thus the stronger interaction (smaller $K_{2c}$) is required for the occurrence of...
FIG. 3. Scattering process in the upper subband for ν = 1/2 (that is, k_{F1} = 2k_{F2}); for clarity, the two spin branches are separated and plotted in different colors. The lower subband, identical to the one plotted in Fig. 1 is not shown. (a) Scattering process O_{σ}^{(2)}. The H_{B} term brings an electron from L_{2\sigma}(-k_{F2}) to the intermediate state R_{2\sigma}′(3k_{F2}) (green dashed arrow). Electron-electron interactions allow R_{2\sigma}′ to forward scatter to R_{2\sigma}(k_{F2}) (brown dashed arrow). The momentum difference is compensated by a backscattering process L_{2\sigma} ′ → R_{2\sigma} (brown solid arrow). (b) Diagram for O_{σ}^{(2)} scattering process in panel (a). The red (blue) arrows indicate up-(down-)spin fermion fields. The wavy line and green circle mark Coulomb interaction and helix-induced spin-flip backscattering, respectively.

FIG. 4. The same as in Fig. 3 but for an odd filling ν = 1/3 (k_{F1} = 3k_{F2}). (a) Scattering process O_{σ}^{(3)}. The H_{B} term brings an electron from L_{2\sigma}(-k_{F2}) to R_{2\sigma}′′(5k_{F2}) (green dashed arrow). With the help of electron-electron interactions, the electron scatters from R_{2\sigma}′′, through R_{2\sigma}(3k_{F2}) to R_{2\sigma}(k_{F2}) (brown dashed arrows), accompanied by the L_{2\sigma} ′→ R_{2\sigma} and L_{2\sigma} → R_{2\sigma}′ backscattering processes at the Fermi points (brown solid arrows). (b) Diagram for O_{σ}^{(3)} scattering in panel (a). The labels are the same as those given in Fig. 3.

V. EVEN-DENOMINATOR FILLING

When the chemical potential is at the even commensurability ν = 1/(2n), it allows for the (2n)th-order helix-assisted scattering in the upper subband. In the renormalization-group (RG) framework, we keep the most relevant scattering, which consists of two terms, denoted as

\[ O_{σ}^{(2n)} = \frac{g_{2n}}{2} (R_{2a}^{\dagger} L_{2\sigma})(R_{2a} L_{2\sigma})^{n} (R_{2\sigma}^{\dagger} L_{2\sigma})^{n-1} + \text{H.c.}, \]

with \( g_{2n} \propto B(U_{2k_{F2}})^{2n-1} \), the Fourier component of the Coulomb potential U, and \( σ = -\bar{σ} \in \{↑ = +, ↓ = -\} \).

Figure 3 illustrates the scattering process for \( n = 1 \) and \( σ = + \). The Overhauser field in H_{B} brings an electron from L_{2\sigma}; at the Fermi point -k_{F2} to the intermediate state R_{2\sigma}′ at 3k_{F2}, which subsequently forward scatters to R_{2\sigma}′ at k_{F2} by electron-electron interactions. The momentum difference is compensated by a backscattering process, which brings an electron from L_{2\sigma} at the Fermi point -k_{F2} to R_{2\sigma} at k_{F2}. One can generalize the diagram to describe scattering processes for \( σ = -\), as well as for general \( n \).

With the bosonization technique (see Appendix A), we express the operator as \( O_{\bar{σ}}^{(2n)} = g_{2n} \cos \phi \) in terms of the boson fields \( \phi \). Importantly, we show that \( O_{\bar{σ}}^{(2n)} \) commute with each other and with \( O_{\bar{σ}} \), so that the three operators can be ordered simultaneously. In wires with sufficiently strong interactions, meaning sufficiently small \( R_{2\sigma} \), the two terms in Eq. (8) open a full gap in the upper subband, leading to dips in the conductance; see Eq. (7).

VI. ODD-DENOMINATOR FILLING

Now we turn to the odd case \( ν = 1/(2n+1) \) and investigate the (2n+1)th-order helix-assisted scattering. In contrast to the even case, here the most RG relevant process consists of a single term,

\[ O^{(2n+1)} = \frac{g_{2n+1}}{2} (R_{2\sigma}^{\dagger} L_{2\sigma})(R_{2\sigma}^\dagger L_{2\sigma})^{n} (R_{2\sigma}^\dagger L_{2\sigma})^{n} + \text{H.c.}, \]

with \( g_{2n+1} \propto B(U_{2k_{F2}})^{2n} \); Figure 4 illustrates the \( O^{(3)} \) process, which is similar to Fig. 3 but involves higher-order scattering processes.

Again, we bosonize the operator and write it as \( O^{(2n+1)} = g_{2n+1} \cos(2\sqrt{2n+1}\Phi) \); see Appendix A. In the presence of strong interactions, the \( O^{(2n+1)} \) term leads to a partial gap in the upper subband. As a result, the system contains a fractional (helical) Tomonaga-Luttinger liquid in the upper (lower) subband. The conductance, with contributions from both subbands, is summarized in Eq. (7). In the bosonic language, when \( O^{(2n+1)} \) is ordered, the \( \Phi \) field is pinned at multiples of \( \pi/√{2n+1} \), while the excitations correspond to kinks in \( \Phi \), where the field changes its value between neighboring
minima. We find that such a kink carries a charge of \( q_k = \nu e \) and zero spin. This fractional charge can be examined through shot noise, similar to the proposed setup in Ref. [30]. Here, \( q_k \) reveals the hierarchy of the Laughlin states [29], similar to an array of one-subband quantum wires [28, 37, 40] and Rashba wires [41, 43] in magnetic fields.

VII. DISCUSSION

In this work, we demonstrate a mechanism for universal conductance dips and fractional excitations in the absence of external magnetic fields, which relies on ingredients naturally present in semiconductor wires. In general terms, our model consists of a Kondo lattice in which the couplings between itinerant charge carriers and localized spins are weak so that the RKKY coupling dominates over the Kondo screening [44]. With the tunable dimensionality of our setup, we demonstrate that Coulomb interaction between the charge carriers can trigger the formation of strongly correlated fermions even in this weak-coupling limit. While we mainly discuss nuclear spins here, as motivated by experiments on III-V semiconductors, our theory covers a wide variety of materials and setups. For instance, magnetic dopants (e.g., Mn) can substitute nuclear spins in creating the required internal helical magnetic field. Alternatively, the quasi-one-dimensional Kondo lattice can be fabricated out of heavy-fermion compounds, where itinerant electrons and localized spins interact through exchange couplings [45]. In these scenarios, both the transition temperature and the helical field strength are much larger due to larger exchange couplings [46]. Yet differently, the helical field can be generated by the external magnetic field in Rashba wires [47] or by depositing nanomagnets [48, 49].

For the purpose of observation, stronger transverse confinement is preferable, as it leads to larger subband spacing, allowing to resolve dips at different commensurate fillings, especially near the plateau edge. Further, it leads to stronger electron-electron interactions, a crucial factor for higher-order scattering. Since the helix is susceptible to magnetic fields [33] and elevated temperatures [21, 42], field- and temperature-dependent conductance might be used to seek additional signatures, such as disappearance of the dips and restoration of the standard plateaux. The observation of universal conductance dips would indicate the helix-assisted backscattering and therefore the presence of the helix itself, supplementary to uniformly dropped plateaux [13]. Since the predicted conductance dips can be observed in straightforward transport measurements, our prediction can be verified through systematic investigations on the temperature-dependent conductance upon varying carrier density.

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Appendix A: Details of the calculation

Here we present the details of our calculation. To analyze the helix-assisted backscattering processes, we employ the bosonization technique [28, 32, 37, 39, 50, 52]. We express the slowly varying fields [see Eq. (1)] as

\[
R_{j\sigma}(x) = \frac{1}{\sqrt{2\pi a}} e^{i\phi_{Rj\sigma}(x)}, \quad L_{j\sigma}(x) = \frac{1}{\sqrt{2\pi a}} e^{i\phi_{Lj\sigma}(x)},
\]

with the chiral bosonic fields \( \phi_{Rj\sigma} \) and the short-distance cutoff \( a \). In the above, we omit the Klein factors, and use the indices \( \ell \in \{R, L\}, j \in \{1, 2\}, \) and \( \sigma \in \{\uparrow, \downarrow\} \) to label the chirality, subband, and spin, respectively.

For the odd-denominator filling \( \nu = 1/(2n+1) \), we impose the following commutation relation

\[
[\phi_{lj\sigma}(x), \phi_{lj'\sigma'}(x')] = i\pi\delta_{lj'\ell'}\delta_{jj'}\delta_{\sigma\sigma'}\text{sign}(x-x').
\]

Next, we express the operator \( O^{2n+1} \) as a cosine [28, 37, 39, 51, 52].

\[
O^{2n+1} = g^{2n+1} \cos\left(2\sqrt{2n+1}\Phi\right),
\]

with

\[
\Phi = \frac{1}{2\sqrt{2n+1}} \left[ -n\phi_{R2L} - (n+1)\phi_{R2L} + (n+1)\phi_{L2L} \right. + \left. n\phi_{L2L} \right].
\]

The linear-response conductance from the fractional Tomonaga-Luttinger liquid, as well as the charges of the excitations, can be computed straightforwardly as in Refs. [41, 50]. The results are given in Eq. (7).

For even-denominator fillings \( \nu = 1/(2n) \), we use the commutator in Eq. (A2) for \( j = 1 \), and the following generalized commutation relation for \( j = 2 \),

\[
[\phi_{lj\sigma}(x), \phi_{lj'\sigma'}(x') ] = i\pi M_{lj\sigma lj'\sigma'}\text{sign}(x-x'),
\]

with \( M_{lj\sigma lj'\sigma'} \) being an integer depending on the chirality and the spin. In addition, the chiral boson fields for \( j = 1 \) and \( j = 2 \) commute with each other. Defining the new index,

\[
p, p' \in \{1 \equiv R \uparrow, 2 \equiv R \downarrow, 3 \equiv L \uparrow, 4 \equiv L \downarrow\},
\]

we can write \( M_{pp'} \) in the following matrix form,

\[
\begin{pmatrix}
1 & 2n - 1 & 0 & 0 \\
(2n - 1) & 1 & 0 & 1 \\
-1 & 0 & -1 & 2n - 1 \\
0 & 1 & 2n - 1 & -1 \\
\end{pmatrix}
\]

Denoting \( \Psi \equiv (\Phi_+, \Phi_-, \Theta_+, \Theta_-)^t \), \( \psi \equiv (\phi_{R2L}, \phi_{R2L}, \phi_{L2L}, \phi_{L2L})^t \), the transpose operator \( t \), we...
define the transformation \( \Psi = T \psi \), with the matrix \( T \) given by

\[
T = \begin{pmatrix}
-n & -n & n+1 & n-1 \\
-n & n+1 & -n & -n \\
-f_n & -(2n-1)f_n & 2n(n-1)f_n & 0 \\
0 & -2n(n-1)f_n & (2n^2-1)f_n & f_n
\end{pmatrix},
\]

(A8)

with \( f_n \equiv 1/[4n(4n^2 - 4n - 1)] \). With Eqs. (A5) and (A8), the new fields \( \Phi_{\pm} \) are conjugate to \( \partial_x \Theta_{\pm} \) and the operators \( O_{\pm}^{(2n)} \) can be expressed in terms of \( \Phi_{\pm} \), as given in the main text.

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