Cross section fluctuations and chaoticity 
in heavy–ion dynamics

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CROSS SECTION FLUCTUATIONS AND CHAOTICITY IN HEAVY–ION DYNAMICS

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ABSTRACT

Cross section fluctuations in nuclear scattering are briefly reviewed in order to show the main important features. Then chaotic scattering is introduced by means of a very simple model. It is shown that chaoticity produces the same kind of irregular fluctuations observed in light heavy–ion collisions. The transition from order to chaos allows a new general framework for a deeper understanding of reaction mechanisms.

1. INTRODUCTION

In this presentation we will discuss recent investigations about chaos in nuclear scattering. Though chaos is not a new concept in nuclear physics, only recently one is completely realizing its important consequences [1]. The problem, especially the quantal aspect of it, is rather elusive and delicate, however it shows very general features which are of extreme interest for several fields [2-4]. Since most of the material here presented has recently been published in several papers [5-9] , we will not go into the technicalities, which can be found in the quoted references, trying on the other hand to explain in a simple and schematic language the reason of chaoticity onset, the meaning of it and the experimental implications according to the present understanding. This framework is still too young to draw general conclusions, however it opens new fascinating horizons, connecting nuclear physics with the novel interdisciplinary research of nonlinear dynamics and the evolution of complex systems.

The paper is organized as follows. The main experimental results in the scattering of light nuclei are briefly summarized in section 2. The concept of deterministic chaos is discussed in section 3. Here the classical and the quantal dynamics of a simple nuclear scattering problem exhibiting chaoticity are illustrated. Section 4 deals with a general discussion on the implications of the transition from order to chaos. A summary of the main important results is done in section 5.
2. EXPERIMENTAL BACKGROUND

Cross section fluctuations have been observed since the 60s, when nucleon-nucleus reactions started to be intensively studied [10]. Predicted by Ericson [11,12], fluctuations in compound nucleus cross sections were detected at excitation energies above the neutron evaporation barrier. That is in the energy region of strongly overlapping resonances, where the level spacing $D$ is very small in comparison with the level width $\Gamma$, $\Gamma/D \gg 1$. Fluctuations are generated by the random action of the very many intermediate levels which connect the entrance and the exit channels. According to Ericson's theory, autocorrelation functions of experimental data have a Lorentzian shape whose width $\Gamma$, the coherence length, gives the energy range within which the intermediate levels are excited coherently. Therefore $\Gamma$ represents the average level width of the intermediate compound nucleus and gives information on the average lifetime of the compound nucleus $\tau = \hbar/\Gamma$ and on the level density. Fluctuations have a statistical nature, but are experimentally reproducible. This view is nicely confirmed by a vast literature [13]. However, experiments with heavier projectiles - performed almost at the same time - revealed excitation function fluctuations with different features. The first system to be studied was $^{12}C + ^{12}C$ [14]. In this case fluctuations started around the Coulomb barrier presenting structures with widths of different sizes. In general these structures, which were present in several reaction channels, became broader as the incident energy increased. The coherence lengths extracted from these experiments were larger than those previously found in nucleon-nucleus scattering - 100-300 KeV against 10-50 KeV- and correlation analyses showed a nonstatistical origin. Similar characteristics were observed for $^{12}C + ^{16}O$ [15], and $^{16}O + ^{16}O$ [16] among several other systems [17]. Due to the peripheral kind of these reactions and the unusual strongly attractive nucleus-nucleus potential at large distances, it was postulated that these oscillating structures should have a molecular-like nature substantially different from that of the average compound nucleus. Actually, some evidence of this molecular origin has been found for $^{12}C + ^{12}C$ and $^{12}C + ^{16}O$. At variance, in other cases, for example $^{16}O + ^{16}O$, the situation remains more ambiguous [17].

Going to heavier systems, a more complex behaviour was detected. In correspondence of excitation function fluctuations, anomalous large and highly oscillating angular distributions were observed. Typical examples of this behaviour are the systems $^{16}O + ^{28}Si$ [18] and $^{12}C + ^{28}Si$, $^{32}S$ [19], where these features were first measured. Again a dinuclear molecule was thought to be the origin, but the mechanism soon appeared much more complicated: systems leading to the same dinuclear composite showed different structures; it was not always possible to understand the angular distributions in terms of only one single wave, on the contrary several angular momenta around the grazing were involved [20]. The phenomenon has been intensively studied and, as in the case of Ericson’s fluctuations, a vast literature can be found on the subject. Fundamental review papers, both on the many experiments performed and the theoretical models proposed to explain heavy–ion resonances, are those of Erb and Bromley [17] and Braun-Munzinger and Barrette [21]. They say clearly that fluctuating phenomena in light systems seem to have a common nature: there are only quantitative, but not qualitative differences from system to system. However, notwithstanding the great effort spent during these years, there is not yet a quantitative theoretical understanding of this behaviour: all the advanced models fail - partly or completely - in reproducing the large set of existing data. The only model-independent consideration which comes out naturally from the experimental analysis is the unexpected presence of a very
weak surface absorption. In other words, a relatively small number of channels is involved.

Though the interest in these mysterious phenomena diminished in the 80s, some groups have continued the experimental research. Thus fluctuations were recently found in the elastic and inelastic cross sections of heavier nuclei like $^{28}\text{Si} + ^{28}\text{Si}$ [22], $^{24}\text{Mg} + ^{24}\text{Mg}$ [23] and $^{24}\text{Mg} + ^{28}\text{Si}$ [24]. At the same time excitation function fluctuations were observed also in deep inelastic collisions of several systems like $^{19}\text{F} + ^{89}\text{Y}$ [25], $^{28}\text{Si} + ^{64}\text{Ni}$, $^{28}\text{Si} + ^{48}\text{Ti}$ [26]. Again, differences from Ericson’s theory were found, mainly because correlations between several channels and a clear angular dependence were evidenced. In ref. [27] cross section fluctuations were measured for several windows of energy loss, establishing this way a connection between oscillating phenomena in elastic and damped reactions. Further aspects on the current experimental studies of excitation function fluctuations in light nuclei can be found in the contributions by G. Pappalardo and D. Vinciguerra to this conference.

This is very briefly the puzzling state of the art of the experimental fluctuations in light nuclei collisions, which is still waiting for a sound theoretical comprehension. In what follows the concept of chaotic scattering will be introduced and one will see that this new perspective is able to justify the experimental phenomenology presented above.

3. CHAOTIC SCATTERING

The intensive studies on nonlinear dynamical systems have demonstrated that simple deterministic laws can exhibit a complex and unpredictable evolution for infinitesimal variations of the initial conditions. This irregular behaviour, which was first discovered by Poincaré at the end of the last century [28], but has been thoroughly investigated only since the 70s, goes under the name of deterministic chaos [29]. In general, considering classical hamiltonian systems, one should expect chaoticity when the system is non-integrable, i.e. the number of degrees of freedom is greater than that of the constants of motion. In other words chaos shows up if a symmetry breaking occurs and a strong coupling between the equations of motion exists. Let us consider for example the following hamiltonian describing a generic two-body problem whose interaction has a central and a non-central term

$$H(r, p, \theta, I) = \frac{p^2}{2m} + \frac{I^2}{2I} + V(r) + \alpha U(r, \theta). \quad (1)$$

Here $p$ is the momentum, $m$ the reduced mass, $I$ the angular momentum, $\Omega$ the moment of inertia, $V(r)$ the central potential and $\alpha$ the strength of the non-central potential $U(r, \theta)$. If $\alpha = 0$ the system is integrable: we have 2 degrees of freedom, $r$ and $\theta$, and 2 conserved quantities, the energy $E$ and the angular momentum $I$. It is possible to solve exactly our problem by decoupling the classical hamiltonian equations. But if $\alpha \neq 0$ then the rotational symmetry is destroyed, the only conserved quantity is the energy $E$ and it is not possible to find an exact solution. The system can show an irregular behaviour whose degree of chaoticity depends on the strength $\alpha$. The transition from a ordered dynamics to a completely chaotic one is gradually regulated by the coupling strength. In general the two extreme cases, order ($\alpha = 0$) and hard chaos ($|\alpha| \gg 0$), are rather rare. On the other hand, very often one has to deal with a mixed behaviour, called soft chaos, where $\alpha$ is not so sufficiently small that perturbation theory can be applied [2,30].

In quantum mechanics, due to the limitations imposed by the uncertainty principle, such an irregular behaviour should not be present. This is one of the most debated
Figure 1. Final scattering angle $\phi_f$ as a function of the initial orientation angle $\theta_i$ for three different incident energies.

Figure 2. Deflection function for a fixed orientation angle $\theta_i$ (a). In (b) and (c) two blow-ups are shown in order to show the fractal structure. See text.

topics in the current literature [2-4]. According to the present understanding of the phenomenon, the transition from order to chaos occurs also in quantum mechanics, but it exhibits smoother features substantially different from the fractal structure of classical chaos. In general one calls quantum chaos the quantal analogue of those systems which are classically chaotic.

Coming back to the classical case and to eq. (1), we can study a bounded dynamics or a scattering problem. Strictly speaking, the scattering problem is integrable at large distances where the potential goes to zero. However, due to the fact that one probes an interaction region which is chaotic, the final observables (final scattering angle, final angular momentum, etc.) present wild oscillations at all scales and a mixture of regular and irregular islands as a function of the initial conditions. This phenomenon is called chaotic scattering [2,31-34].

In ref.[5,6] it has been proved that, when considering a deformed nucleus impinging on a spherical one, chaotic scattering occurs if the nuclei are not very heavy and one is not very far from the grazing condition. The hamiltonian in this case can be written as

$$\mathcal{H} = \frac{p^2}{2m} + \frac{I^2}{2m} + \frac{(L - I)^2}{2mr^2} + V(r, \theta),$$

where $L$ and $I$ are respectively the total angular momentum and the spin of the rotor. The potential $V(r, \theta)$ is the sum of the Coulomb and the nuclear interaction. In particular, as a typical example, the system $^{28}Si + ^{24}Mg$ was considered (see refs. [5,6] for further details). Solving numerically the equations of motion, one gets for the final observables the fluctuations shown in figs. 1 and 2. In fig.1, for the $^{28}Si + ^{24}Mg$ reaction, the final scattering angle $\phi_f$ is displayed as a function of the initial orientation $\theta_i$. Three different incident energies are considered. Each plot contains 1800 points and each point is the result of the numerical integration.
for a particular initial orientation. This irregular behaviour occurs at all scales as fig.2 shows for the case of the deflection function (see also figs. 3,4 of ref.[6]). Here the final scattering angle $\phi_f$ is reported as a function of the total angular momentum $L$, for a fixed incident energy and initial orientation angle $\theta_i$. The value of $L$ corresponds to the initial orbital angular momentum $\ell_i$ being $I_i=0$. In the same figure two blow-ups of smaller regions are shown in order to display the fractal structure, characteristic signature of chaoticity.

These impressing chaotic features are not due to the peculiarities of our model and have been recently found also in a different nuclear scattering problem considering vibrations as the only degrees of freedom [35]. In general, such fractal fluctuations are generated by trapped trajectories which remain in the internal pocket for a very long time. Since the interaction zone is chaotic, it is not possible to predict this trapping time (or delay time) which depends in a very sensitive way on the initial conditions. This is illustrated in the schematic pictorial view of fig.2. Here the oscillations of the potential, due to the coupling of the relative motion to the rotor spin, and the chaotic evolution of one trajectory are shown together with a generic excitation and deexcitation of the internal degrees of freedom. In this scheme the absorption has been neglected, but from the experimental overview of the previous section, one knows that this is not a bad approximation for light heavy–ion systems. The role of the absorbtion will be discussed later in connection with the quantal dynamics.

Concluding the review of the classical dynamics, one can claim that irregular scattering is a general phenomenon which coexists with a regular one when - and this occurs very often - the problem is non-integrable.

One should not forget that nuclei are quantum objects and so the question is now if this new perspectives can influence the quantum description. This is especially important when the semiclassical approximation can be applied. In the following the quantal analogue of the above discussed classical model will be reviewed. Several recent studies on dynamical systems have demonstrated that the classical transition from order to chaos has a quantal analogue [2-4, 31,32,36]. We will see that this is the case also in our model.

The quantal approach which corresponds to the above discussed classical scattering is the 2-dimensional coupled-channels model studied in refs.[6-9]. These investigations have demonstrated that the quantal analogue of chaotic scattering is the appearance of sharp irregular fluctuations in the transition probabilities as a function of energy. The irregular structures are caused by the presence of an asymmetric pocket and a strong coupling term between the different channels [9]. It has been checked that these quantal irregularities are present in the same energy and angular region where classical chaos shows up. More precisely fluctuations start at the potential barrier and their widths become broader and broader until they disappear completely by increasing energy and/or angular momentum. An example of these oscillations, for the same system investigated in the classical case, is displayed in fig.4. In the figure only the elastic channel is displayed. The total angular momentum is $L=15\ h$, while the initial spin of the rotor is $I=0$. The energy step used was 20 KeV, no structures below this energy step were found [6-9]. Similar features have been obtained taking into account vibrations in a nuclear reaction [37]. Actually, various chaotic scattering problems [31,32,36] show an identical behaviour.

A more quantitative analysis of the fluctuations illustrated in fig.4 was done by investigating the autocorrelation functions, both in the classical and in the quantal case [8,9]. The coherence lengths extracted in the two cases were comparable within a factor of two and of the order of 100-250 KeV. Therefore as
Figure 3. Pictorial view of chaotic scattering. Due to the strong coupling, the interaction region shows large oscillations for infinitesimal variations of the initial conditions. Therefore the incident nucleus is trapped and excited, until it finally succeeds in escaping again. Of course, classically the internal degrees of freedom are not discrete.

Figure 4. Quantal elastic transition probability as a function of incident energy. The calculations were performed by means of a 2-d coupled-channels approach, which is the quantal analogue of the classical chaotic scattering problem. The energy step used in the calculations is 20 KeV. See text.

in ref.[31,32], a firm correspondence exists between the classical and the quantal dynamics, notwithstanding the semiclassical theory is not valid a priori in the problem into consideration. Unfortunately, the small number of resonances has not permitted the study of their distribution as in [31] where a GOE law was evidenced.

The irregularities in the transition probabilities are reflected drastically in the excitation functions and the angular distributions as a function of energy. This result has been checked in refs.[8,9], where it was shown how transition probabilities summed over different incident waves continued to display irregular fluctuations. More realistic 3-dimensional calculations were performed [9] by means of the code FRESCO [38]. They confirmed the presence of irregular fluctuations above the Coulomb barrier, both in the excitation functions and in the angular distributions. Fig.5 displays these cross section fluctuations as a function of energy for the channels considered, i.e. only the states $0^+, 2^+, 4^+$ in $^{24}$Mg. Fig. 6 shows the correspondent autocorrelation functions (full curves) with the Lorentzian fits (dashed curves) and the extracted coherence lengths. A preliminary analysis of the cross sections reported in fig.5 has revealed some correlation among the con-
Figure 5. Excitation functions calculated for the system $^{28}\text{Si} + ^{24}\text{Mg}$ by means of the code FRESCO [38]. The c.m. angle is $178^\circ$. See text.

Figure 6. Autocorrelation functions of the fluctuations displayed in fig.5 (full curves). The dashed curves are Lorentzian fits whose widths are reported in the figure.

Figure 7. Elastic angular distributions, with $86^\circ < \theta_{cm} < 178^\circ$, as a function of incident energy calculated by means of the code FRESCO [38], see text.
sidered channels. On the other hand, as fig.6 shows, there are clear deviations from the Lorentzian shape predicted by Ericson’s statistical theory [9]. These are important points which will be discussed in the next section. In fig.7 the angular distributions are shown as a function of the incident energy. This plot illustrates nicely how, analogously to the experimental data, theoretical calculations exhibit an irregular behaviour which starts at the Coulomb barrier and is more evident at backward angles, where the absolute value of cross section is smaller [9]. The cross sections of figs.5 and 7 were calculated using a small imaginary part of the potential, according to ref.[39] and to the experimental evidence [17,21]. In ref. [7] it was shown that the strength of the absorption is a crucial point, since it can smooth and even wash out completely the fluctuations.

The 3-d calculations give therefore a satisfactory justification of the experimental anomalies discussed in section 2. At the same time, on the other hand, they connect the experimental data to chaotic scattering, being the code FRESCO a more refined version of the 2-d quantal model whose classical analogue is chaotic.

Concluding this section, it is important to note that preliminary calculations for the systems $^{12}C + ^{24}Mg$ and $^{16}O + ^{28}Si$ have revealed the same features [40].

4. DISCUSSION

The phenomenology which comes out of chaotic scattering calculations is therefore identical to that observed experimentally for light nuclear systems. These theoretical investigations indicate strongly that the experimental fluctuations have a chaotic origin. At the same time they demonstrate the smoother but impressing sensitiveness of quantum chaos to small changes of the input parameters. In this sense chaos maintain a clear identification also in quantum mechanics. In refs. [27,31] the chaotic origin of the scattering between light nuclei was already claimed. In this presentation and in refs.[5-9] this hypothesis has been investigated in detail and a sound support has been given to it. It is true, however, that a systematic investigation and a more complete quantitative analysis is needed. It should be stressed that a quantitative study has a meaning only within a statistical analysis. In general, in fact, it has no sense trying to reproduce the single fluctuation produced by the interplay of several resonances. We are in a regime where minimal - even if not infinitesimal - changes of the input parameters have a strong influence in the final results. Our knowledge of nuclear physics is not so good to trust the parameters used to that accuracy and several parametrizations are able to fit the same structures [39].

If it is true that this theoretical analysis is only at the beginning, more refined experiments should be also performed. The nowadays available $4\pi$ detectors and more sophisticated experimental techniques could give a crucial contribution in order to draw definitive conclusions. In particular irregular fluctuations should be detected also in the gamma multiplicities as preliminary chaotic scattering calculations indicate [40].

Therefore chaotic scattering outlines a new framework in which old puzzles find their natural justifications and a deeper insight of nuclear reaction mechanisms can be gained. In particular two important considerations should be underlined. The first is that a few degrees of freedom are able to produce a very complicated dynamics. This is a very general warning and should be always borne in mind, especially when the semiclassical approximation is used. The second is that both order and chaos are two extreme regimes. We know very well how to describe these two cases. In the former the optical model works pretty well, while in the
ORDER elastic scattering optical model

\[ \downarrow \]

MIXED REGIME non-equilibrated TDHF
d and rotating coupled-channels dinuclear system transport theories

\[ \downarrow \]

CHAOS excited compound Random Matrix Theory nucleus

\[ \downarrow \]

Ericson’s fluctuations

Table 1. Schematic table illustrating the different stages of the transition from order to chaos (1\textsuperscript{st} column) in correspondence of the physical picture (2\textsuperscript{nd} column) and the standard models use in nuclear reaction theory (3\textsuperscript{rd} column).

| Direct reactions | Complex reactions | Compound reactions |
|------------------|-------------------|--------------------|
| \[ \uparrow \] | \[ \uparrow \] | \[ \uparrow \] |
| \[ \tau \sim 10^{-22} \text{ s.} \] | \[ \tau \sim 10^{-21} \text{ s.} \] | \[ \tau \sim 10^{-20} \text{ s.} \] |
| \[ \Gamma \sim 2 - 4 \text{ MeV} \] | \[ \Gamma \sim 200 - 400 \text{ KeV} \] | \[ \Gamma \sim 10 - 50 \text{ KeV} \] |

Figure 8. Pictorial view of the three kinds of reactions which contribute to the cross sections in the mixed regime (soft chaos). The characteristic coherence widths and reaction times are also displayed.

latter statistical theories, like Random Matrix Theory or the Ericson one, can be successfully used [1,31,41]. In reality the most general situation is very often in between [2]. This mixed regime - soft chaos - is, however, the most difficult case to treat: fluctuations are not completely statistical and dynamical correlations exist. The problems discussed in section 2 and 3 seem to belong just to this regime: both the experimental data and the dynamical model presented show deviations from Ericson’s theory. That is why several models in the past, like for example
time-dependent Hartree-Fock, coupled-channels approaches and transport theo-
ries, have had only a partial success: in general only average quantities have
been correctly described. The transition from order to chaos allows to reinterpre-
t standard models offering, at the same time, a deeper dynamical insight. Table 1
summarizes this schematic discussion, illustrating the different physical situations
(\textit{2}nd column) in correspondence of the chaoticity regime (\textit{1}st column) and of some
well-known theoretical approaches (\textit{3}rd column).

Coming back to Ericson’s fluctuations and Random Matrix Theory, in our
view, they apply only in the complete chaotic regime. In this sense the excited
compound nucleus reactions are nice examples of chaotic behaviour [41,42]. How-
ever, from the above discussion, one should expect deviations in the case of soft
chaos. Actually, since the first papers [12], Ericson himself warned that his formu-
lation did not take into account possible coherence effects, which could generate
interference between \textit{direct} and \textit{compound} reactions. In general, is not always pos-
sible to decompose the cross section into a direct term and a compound one. This
is what happens in the mixed regime, where a third kind of reactions, we can call
them \textit{complex reactions}, should be considered. In the classical picture these are
those who create the islands of regularity inside the chaotic regions. Such colli-
sions not only can enlarge the coherence length, having different reaction times,
but they can often produce small oscillations in the autocorrelation function at
energies greater than the coherence length [25,26,40,43]. The three different kind
of reactions are illustrated in a pictorial view in fig.8, where also the characteristic
coherence lengths and reactions times are reported.

This new scenario may seem too generic and chaotic scattering a superficial
rephrasing of old theories or an easy explanation for unsolved problems. In reality,
though a lot has still to be done in order to be more precise and characterize the
different stages with an increasing degree of chaoticity (or complexity), the chaotic
framework is the new emerging interdisciplinary paradigm of natural sciences. It
is a novel powerful language whose limits are still unknown, which can refresh
nuclear physics and be successful where the old way of thinking failed [44,45].

5. SUMMARY

It has been shown that chaotic scattering represents a real possibility in col-
lisions between light nuclei and that it can explain the irregular fluctuations ob-
served experimentally. This is an important result both for nuclear physics and for
more fundamental questions like the existence and the features of quantum chaos.
These investigations allow to reinterpret standard approaches - although for the
moment only in a generic way - in the new framework of the transition from order
to chaos. The study of heavy–ion scattering is particularly interesting due to its
privileged position between the classical and the quantum world.

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