Predicting the Bubble of Non-Fungible Tokens (NFTs): An Empirical Investigation

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Abstract—Our study empirically predicts the bubble of Non-Fungible Tokens (NFTs): transferrable and unique digital assets on public blockchains. This subject is important because, despite their strong market growth in 2021, NFTs have not been studied in terms of bubble prediction. To achieve the purpose, we applied Logarithmic Periodic Power Law (LPPL) model to the time-series price data of major NFT projects, retrieved from nonfungible.com. Results implied that, as of December 20, 2021, (i) NFTs in general are in a small bubble (predicting price decline), (ii) Decentraland project is in a medium bubble (predicting price decline), and (iii) Ethereum Name Service and ArtBlocks projects are in a small negative bubble (predicting price increase). Future works are to refine the prediction by considering heterogeneity of NFTs, comparing other methods, and using more enriched data.

Index Terms—non-fungible token, blockchain, bubble prediction, logarithmic periodic power law model

I. INTRODUCTION

Non-Fungible Tokens (NFTs) are “transferrable and unique digital assets on public blockchains” [1], developed as an extension of cryptocurrencies (e.g., Bitcoin; BTC [6]) that enabled decentralized consensus-building on transaction records. Unlike fungible (homogeneous) cryptocurrencies, NFTs can be unique digital assets (e.g., arts, games, collectibles) once they are associated with unique metadata or image [5], [10]. NFTs are in practice often minted on a project basis, as a group of similar images. For example, CryptoPunks—one of the earliest NFT projects—minted 10,000 NFTs, each with a human face drawn in 24 × 24 pixels (Fig. 1). According to coinmarketcap.com, there are currently at least 1056 such projects on Ethereum (ETH) [2], [3].

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One of the noteworthy features of NFTs, at the time of writing this paper, is their strong market growth. Fig. 2 is monthly volume of NFTs traded in the world largest NFT marketplace OpenSea, which shows that the market has a growth trend since 2021 and especially has an extraordinary growth in August. We can also see the same trend in luxury NFTs traded outside of OpenSea. Table I is the ranking of the most expensive NFTs traded on seven marketplaces (SuperRare, Nifty Gateway, Foundation, hic et nunc, MakersPlace, KnownOrigin, and Async Art), which shows that the prices of top five NFTs were all recorded in 2021. These trends of strong market growth naturally lead people to question whether the current NFT market is a bubble [11], [12] or what the future price movements will be [13], [14].

To answer these questions, our study aims to predict the bubble of NFTs from empirical data. Specifically, we applied Logarithmic Periodic Power Law (LPPL) model [15]–[17]—a standard method for bubble prediction—to the time-series price data of four major NFT projects, retrieved from nonfungible.com (see Section III for details). Results of fitting implied that, as of December 20, 2021, (i) NFTs in general are in a small bubble (predicting price decline), (ii) Decentraland project is in a medium bubble (predicting price decline), and (iii) Ethereum Name Service and ArtBlocks projects are...
| rank | title                        | artist  | price (USD)      | date         |
|------|------------------------------|---------|------------------|--------------|
| 1    | EVERYDAYS: THE FIRST 5000 DAYS | beeple  | $69,346,250.00   | 11 Mar 2021  |
| 2    | HUMAN ONE                    | beeple  | $28,985,000.00   | 10 Nov 2021  |
| 3    | Stay Free (Edward Snowden, 2021) | snowden | $9,516,829.60    | 16 Apr 2021  |
| 4    | CROSSROAD                    | beeple  | $6,600,000.00    | 25 Feb 2021  |
| 5    | OCEAN FRONT (beeple)         | beeple  | $6,000,000.00    | 20 Mar 2021  |

Source: MOST EXPENSIVE ARTWORKS (https://cryptoart.io/), accessed December 20, 2021, created by the author.

II. Related Works

A. Empirical Study on NFT Markets

Thanks to the transparency of transaction records, empirical studies on NFT markets, using richer data (both in quality and quantity), are now emerging.

Nadini, et al. [4], [18] is the first empirical study on NFTs, which comprehensively analyzed their massive transaction data (6.1 million NFT trades between June 23, 2017 and April 27, 2021) with a variety of methods including network analysis for community detection, neural network for image classification, and linear regression for price estimation. Ante [1] specifically focused on the spillover effect among NFT projects, which found a Granger causality between the number of active wallets in established projects and that in emerging projects. Moreover, Ante [19] and Dowling [20], [21] have studied the spillover between NFTs and cryptocurrencies, where the former found Granger causalities from BTC price to NFT sales, from ETH price to the number of active NFT wallets, and from BTC price to ETH price, while the latter pointed out that there is little causality between NFTs and cryptocurrencies in terms of price volatility.

For this research topic, our study has an academic significance in that it covers the bubble prediction while focusing on NFT markets.

B. Bubble Prediction on Cryptocurrencies

Bubble prediction on cryptocurrencies, especially BTC, has been a popular topic as we can directly apply preceding techniques used in the traditional financial markets.

To our best knowledge, Macdonell [22] is the first study on this topic, which used the LPPL model to the weekly moving-average prices of BTC (from July 2010 to August 2013), thereby predicting the bubble crash in December 2013. Subsequent studies have extended this approach in several ways, such as using the LPPL model to other cryptocurrencies [23], [24], adding new terms into the model [25], [26], and modifying the model itself to accommodate highly-volatile BTC price data [27]. The LPPL model is the standard, but not the only, method for the bubble prediction. Preceding studies have also adopted other methods including augmented Dickey-Fuller test [25], [28], [29], sentiment analysis [30]–[32], and machine learning [33]–[35].

For this research topic, our study has an academic significance in that it covers NFT markets while focusing on the bubble prediction.

III. Data

The time-series data used in this analysis are the weekly moving-average prices of NFTs, retrieved from nonfungible.com, displayed on a daily basis in US dollars. We extracted the moving-average prices not only from all available NFTs (from 2017-06-23 to 2021-12-20), but also from each of the

Fig. 2: Monthly Volume of NFTs Traded in OpenSea
four major projects with different time-scales and categories: Decentraland (from 2018-03-19 to 2021-12-20), CryptoPunks (from 2018-05-17 to 2021-12-20), Ethereum Name Service (from 2019-05-04 to 2021-12-20), ArtBlocks (from 2020-11-27 to 2021-12-20). Fig. 3 plots the retrieved data with log-scale prices on the vertical axis. Our analysis will use the LPPL model for each of these 1 + 4 time-series price data.

Note that these data suggest two simplifications. First, we use weekly moving-average price data rather than daily moving-average. This is in order to apply the LPPL model to the highly-volatile NFT market. Second, we use price data associated with projects (or all available NFTs) rather than with each NFT. This is in order to apply the LPPL model to the NFT market where each one has inherently unique prices. In other words, we assume that all NFTs are homogeneous in each project. Relaxing these two simplifications (which undermine the original characteristics of the NFT market) is one of our future studies.

IV. METHODS

A. About LPPL Model

The LPPL model [15]–[17] is to predict bubbles using only time-series price data. Specifically, it approximates \( \ln[p(t)] \) —the log-price of data at a given period \( t \) as follows:

\[
\ln[p(t)] \approx A + B(t_c - t)^m + C(t_c - t)^m \cdot \cos[\omega \ln(t_c - t) - \phi],
\]

where the right-hand side contains three linear parameters \( A, B, C \) and four nonlinear parameters \( t_c, m, \omega, \phi \).

To reduce the computational complexity, our calibration first uses Filimonov and Sornette’s method that eliminates a nonlinear parameter \( \phi \) by expanding Equation 1 as follows:

\[
\ln[p(t)] \approx A + (t_c - t)^m \cdot \left[ B + C_1 \cos[\omega \ln(t_c - t)] + C_2 \sin[\omega \ln(t_c - t)] \right],
\]

where \( C_1 = C \cos \phi \) and \( C_2 = C \sin \phi \).

For the remaining nonlinear parameters \( t_c, m, \omega \), we set the following conditions, which are derived from the empirical evidence of previous bubbles and are commonly adopted as the stylized features of the LPPL model [39]–[41]:

1. The critical time is also called singularity, which is why the LPPL model is also called the LPPLS model.
2. Note that the power law acceleration works toward increasing prices if \( B < 0 \) and toward decreasing prices if \( B > 0 \). We will denote the former case as a positive bubble and the latter case as a negative bubble [37].
3. Since \( |C| < 1 \), condition \( C_1^2 + C_2^2 < 1 \) holds.
max \left\{ t_2 - 60, \frac{t_2 - 0.5}{t_2 - t_1} \right\} < t_c < \min \left\{ t_2 + 252, \frac{t_2 + 0.5}{t_2 - t_1} \right\},
(3)
\begin{align}
0 & < m < 1, \\
2 & < \omega < 15.
\end{align}
(4)

Calibration for a given \([t_1, t_2]\) ends if the estimated parameters satisfy all of the above stylized features\(^{13}\).

We then make bubble predictions at each period of the data, by letting this calibration iterate for the shrinking time window \([t_1, t_2]\). Here, \([t_1, t_2]\) is in daily units according to the daily-basis data (Section III); \(t_2\) denotes a fictitious today corresponding to \(t\); \(t_1\) denotes an earlier day, respectively. For a given \(t_2\), the iterative calibration sets the initial range of the time window as 120 days and the shrinking interval of \(t_1\) as 5 days. That is, we need to estimate parameters 24 times for each \(t_2\) (e.g., \([1, 120]\), \([5, 120]\), \(\ldots\), \([115, 120]\) for \(t_2 = 120\); \([2, 121]\), \([6, 121]\), \(\ldots\), \([116, 121]\) for \(t_2 = 121\))\(^4\). It should be emphasized that this process, as the prediction, uses only historical data as input: the outcome of \(t_2\) depends only on data from \(t_2\) to the last 120 days.

C. Bubble Indicator for Visualization

The LPPL model visualizes its own predictions as the bubble indicator (or confidence indicator) \(^{41}\). For a given \(t_2\) with 24 outcomes, the bubble indicator first counts the number of \(B < 0\) and \(B > 0\) cases, where the former implies the price increases faster than exponential (i.e., positive bubble) and the latter implies the price decreases faster than exponential (i.e., negative bubble). We will denote these numbers as \([B < 0]_{\text{count}}\) and \([B > 0]_{\text{count}}\) for convenience.

Moreover, the 24 outcomes, now classified into \(B < 0\) and \(B > 0\) groups, are filtered to obtain higher confidence (thereby preventing the type 1 error). It specifically sets the following filter conditions for nonlinear parameters \(t_c, m, \omega\):

\[
\frac{\omega}{2\pi} \cdot \ln \frac{t_c - t_1}{t_c - t_2} > 2.5, \quad (6)
\]

\[
\frac{m|B|}{\omega|C|} > 0.5, \quad (7)
\]

where we will denote the number of outcomes that satisfied the above filter conditions in \(B < 0\) and \(B > 0\) groups as \([B < 0]^{\text{count}}\) and \([B > 0]^{\text{count}}\), respectively.

Finally, we can compute bubble indicators (of a given \(t_2\)) as follows:

\[
\text{bubbleindicator}(\text{pos}) = \frac{[B < 0]^{\text{count}}}{[B < 0]^{\text{count}}}, \quad (8)
\]

\[
\text{bubbleindicator}(\text{neg}) = \frac{[B > 0]^{\text{count}}}{[B > 0]^{\text{count}}}, \quad (9)
\]

where \(\text{bubbleindicator}(\text{pos})\) indicates how much of a positive bubble the price at \(t_2\) is in the \([0, 1]\) range, which quantifies the possibility of a price decline in the near future; on the other hand, \(\text{bubbleindicator}(\text{neg})\) indicates how much of a negative bubble the price at \(t_2\) is in the \([0, 1]\) range, which quantifies the possibility of a price increase in the near future\(^{15}\).

The LPPL model derives these positive and negative bubble indicators for all \(t_2\), thereby visualizing its own predictions.

V. Results

Fig. 4 summarizes the prediction results, where Fig. 4a-4e correspond to the aforementioned 1 x 4 time-series price data, with the \(\text{bubbleindicator}(\text{pos})\) depicted in red and the \(\text{bubbleindicator}(\text{neg})\) in green.

Overall, the LPPL model seems to capture the trend of both positive and negative bubbles in NFT markets. The results for all available NFTs (Fig. 4a) are generally successful in predicting the direction of price changes, although prices have risen further after \(\text{bubbleindicator}(\text{pos})\) reached its highest in late August 2020. This is true for individual projects as well; the \(\text{bubbleindicator}(\text{neg})\) successfully predicts the shift to an upward price trend after July 2021 that is common across the Decentraland (Fig. 4b), Cryptopunks (Fig. 4c), and Ethereum Name Service (Fig. 4d), although the \(\text{bubbleindicator}(\text{pos})\) failed to predict the continuous price increase of the Cryptopunks (Fig. 4c) from around October 2020 to March 2021.

Now that we have confirmed the accuracy of the LPPL model, let us focus on the latest indicators. Near December 20, 2021, the bubble indicators are signaling in four cases, except for CryptoPunks (Fig. 4c): \(\text{bubbleindicator}(\text{pos}) \approx 0.2\) in all available NFTs (Fig. 4a), \(\text{bubbleindicator}(\text{pos}) \approx 0.4\) in Decentraland (Fig. 4b), and \(\text{bubbleindicator}(\text{neg}) \approx 0.1\) in Ethereum Name Service (Fig. 4d) and ArtBlocks (Fig. 4e). These results imply that, as of December 20, 2021, (i) NFTs in general are in a small bubble (predicting price decline), (ii) Decentraland project is in a medium bubble (predicting price decline), and (iii) Ethereum Name Service and ArtBlocks projects are in a small negative bubble (predicting price increase), respectively.

VI. Conclusion

This paper empirically predicted the bubble of NFTs, by applying the LPPL model to the time-series price data of four major NFT projects, retrieved from nonfungible.com. Results implied that, as of December 20, 2021, (i) NFTs in general are

\(^{13}\)Accordingly, this calibration is stochastic rather than deterministic (i.e., calibration results are not unique and change with each run).

\(^{14}\)Thus, this analysis has a lag for approximately four months: the results for all available NFTs from 2017-06-23 to 2021-12-20 range from 2017-10-20 to 2021-12-20.

\(^{15}\)Here, we regard the bubble indicator as zero if the denominator, \([B < 0]^{\text{count}}\) or \([B > 0]^{\text{count}}\), is zero.
in a small bubble (predicting price decline), (ii) Decentraland project is in a medium bubble (predicting price decline), and (iii) Ethereum Name Service and ArtBlocks projects are in a small negative bubble (predicting price increase). To our best knowledge, this is the first empirical investigation on the NFT bubble.

On the other hand, this study, as a first step of NFT-bubble prediction, needs future works to improve its quality. Below are three potential directions of future works:

\section*{A. Considering Heterogeneity of NFTs}

We assumed that, to apply the LPPL model, all NFTs are homogeneous in each project (Section III). However, NFTs are inherently unique and heterogeneous, and this is exactly what differentiates NFTs from cryptocurrencies (and flat currencies). It is therefore worth addressing to develop some new method for bubble prediction which can take into account of the heterogeneity of NFTs.

\section*{B. Comparing Other Methods}

The LPPL model is only one method for bubble prediction, and there are other methods such as augmented Dickey-Fuller test, sentiment analysis, and machine learning (Section II). Using these methods and comparing their results would be another direction of future works. We need to find the best mix of preceding methods for the accuracy of NFT-bubble prediction. It will probably use a variety of data (other than time-series price) as input.

\section*{C. Using More Enriched Data}

Our prediction deals with only four major NFT projects, even thought it also covers the weekly moving-average prices of all available NFTs (Section III). More enriched data from other projects would refine our analysis. In addition, while we used weekly moving-average prices (to address the high volatility), it would also be important to develop new methods that can leverage daily or hourly time-series price data as inputs.

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\section*{References}

[1] L. Ante, “Non-fungible token (NFT) markets on the Ethereum blockchain: Temporal development, cointegration and interrelations,” Available at SSRN 3904683, 2021.

[2] V. Buterin et al., "A next-generation smart contract and decentralized application platform," white paper, vol. 3, no. 37, 2014.

[3] G. Wood et al., "Ethereum: A secure decentralised generalised transaction ledger," Ethereum project yellow paper, vol. 151, no. 2014, pp. 1–32, 2014.

[4] M. Nadini, L. Alessandretti, F. Di Giacinto, M. Martino, L. M. Aiello, and A. Baronchelli, "Mapping the nft revolution: market trends, trade networks and visual features," arXiv preprint arXiv:2106.00647, 2021.

[5] Q. Wang, R. Li, Q. Wang, and S. Chen, "Non-fungible token (NFT): Overview, evaluation, opportunities and challenges," arXiv preprint arXiv:2105.07447, 2021.

[6] S. Nakamoto, "Bitcoin: A peer-to-peer electronic cash system," Decentralized Business Review, p. 21260, 2008.

[7] S. Meiklejohn and C. Orlandi, "Privacy-enhancing overlays in bitcoin," in International Conference on Financial Cryptography and Data Security. Springer, 2015, pp. 127–141.

[8] M. Conti, E. S. Kumar, C. Lal, and S. Rui, "A survey on security and privacy issues of bitcoin," IEEE Communications Surveys & Tutorials, vol. 20, no. 4, pp. 3416–3452, 2018.

[9] N. Amarasinghe, X. Boyen, and M. Mcaque, "A survey of anonymity of cryptocurrencies," in Proceedings of the Australasian Computer Science Week Multiconference, 2019, pp. 1–10.

[10] M. Franceschet, G. Colavizza, T. Smith, B. Finucane, M. L. Ostachowksi, S. Scala, J. Perkins, J. Morgan, and S. Hernandez, “Crypto art: A decentralized view,” Leonardo, vol. 54, no. 4, pp. 402–405, 2021.

[11] E. Howcroft, "Nft sales surge as speculators pile in. sceptics see bubble,” REUTERS, 2021. [Online]. Available: https://www.reuters.com/technology/nft-sales-surge-speculators-pile-sceptics-see-bubble-2021-08-25/

[12] L. Castano, “Nfts to drive our parallel universe – if a bubble doesn’t pop first,” Nasdaq, 2021. [Online]. Available: https://www.nasdaq.com/articles/nfts-to-drive-our-parallel-universe-if-a-bubble-doesnt-pop-first-2021-09-20

[13] Zoup, “Sorry... no nft market crash so far!” nonfungible.com analysis, 2021. [Online]. Available: https://nonfungible.com/news/analysis/sorry-no-nft-market-crash

[14] W. Canny, "Jefferies sees the nft market reaching more than $800 in value by 2025," CoinDesk, 2022. [Online]. Available: https://www.coindesk.com/business/2022/01/20/jefferies-sees-the-nft-market-reaching-more-than-80-billion-in-value-by-2025/

[15] A. Johansen, D. Sornette, and O. Ledoit, "Predicting financial crashes using discrete scale invariance," arXiv preprint cond-mat/9903321, 1999.

[16] A. Johansen, O. Ledoit, and D. Sornette, "Crashes as critical points," International Journal of Theoretical and Applied Finance, vol. 3, no. 02, pp. 219–255, 2000.

[17] D. Sornette, Why stock markets crash. Princeton University Press, 2009.

[18] M. Nadini, L. Alessandretti, F. Di Giacinto, M. Martino, L. M. Aiello, and A. Baronchelli, “Mapping the nft revolution: market trends, trade networks, and visual features,” Scientific Reports, vol. 11, no. 1, p. 20902, 2021. [Online]. Available: https://doi.org/10.1038/s41598-021-00553-8

[19] L. Ante, “The non-fungible token (nft) market and its relationship with bitcoin and ethereum,” Available at SSRN 3861106, 2021.

[20] M. Dowling, "Fertile land: Pricing non-fungible tokens," Finance Research Letters, p. 102096, 2021.

[21] ----, “Is non-fungible token pricing driven by cryptocurrencies?” Finance Research Letters, p. 102097, 2021.

[22] A. MacDonell, "Popping the bitcoin bubble: An application of log-periodic power law modelling to digital currency," University of Notre Dame working paper, pp. 1–33, 2014.

[23] R. Rokcos, "Evaluating asset bubbles within cryptocurrencies using the lppl model," Ph.D. dissertation, Duke University Durham, 2018.

[24] M. Bianchetti, C. Ricci, and M. Scaringi, “Are cryptocurrencies real financial bubbles? evidence from quantitative analyses,” Evidence from Quantitative Analyses (February 24, 2018). A version of this paper was published in Risk, vol. 26, 2018.

[25] S. Wheatley, D. Sornette, T. Huber, M. Reppen, and R. N. Gantner, “Are bitcoin bubbles predictable? combining a generalized metcalfe’s law and the log-periodic power law singularity model,” Royal Society open science, vol. 6, no. 6, p. 180538, 2019.

[26] J. Xiong, Q. Liu, and L. Zhao, “A new method to verify bitcoin bubbles: Based on the production cost,” The North American Journal of Economics and Finance, vol. 51, p. 191095, 2020.

[27] M. Shu and W. Zhu, "Real-time prediction of bitcoin bubble crashes," Physica A: Statistical Mechanics and its Applications, vol. 548, p. 124477, 2020.

[28] A. Cheung, E. Roca, and J.-J. Su, “Crypto-currency bubbles: an application of the phillips–shi–yu (2013) methodology on mt. gox bitcoin prices,” Applied Economics, vol. 47, no. 23, pp. 2348–2358, 2015.

[29] S. Corbet, B. Lucey, and L. Yarovaya, "Datestamping the bitcoin and ethereum bubbles,” Finance Research Letters, vol. 26, pp. 81–88, 2018.

[30] J. Bukovina, M. MartíckeÁ et al., “Sentiment and bitcoin volatility,” University of Brno, 2016.
[31] V. Karalevicius, N. Degrande, and J. De Weerdt, "Using sentiment analysis to predict interday bitcoin price movements," *The Journal of Risk Finance*, 2018.

[32] C. Y.-H. Chen and C. M. Hafner, "Sentiment-induced bubbles in the cryptocurrency market," *Journal of Risk and Financial Management*, vol. 12, no. 2, p. 53, 2019.

[33] D. C. Mallqui and R. A. Fernandes, "Predicting the direction, maximum, minimum and closing prices of daily bitcoin exchange rate using machine learning techniques," *Applied Soft Computing*, vol. 75, pp. 596–606, 2019.

[34] Z. Chen, C. Li, and W. Sun, "Bitcoin price prediction using machine learning: An approach to sample dimension engineering," *Journal of Computational and Applied Mathematics*, vol. 365, p. 112395, 2020.

[35] A. M. Khedr, I. Arif, M. El-Bannany, S. M. Alhashmi, and M. Sreedharan, "Cryptocurrency price prediction using traditional statistical and machine-learning techniques: A survey," *Intelligent Systems in Accounting, Finance and Management*, vol. 28, no. 1, pp. 3–34, 2021.

[36] N. Kyriazis, S. Papadamou, and S. Corbet, "A systematic review of the bubble dynamics of cryptocurrency prices," *Research in International Business and Finance*, vol. 54, p. 101254, 2020.

[37] W. Yan, R. Woodard, and D. Sornette, "Diagnosis and prediction of tipping points in financial markets: Crashes and rebounds," *Physics Procedia*, vol. 3, no. 5, pp. 1641–1657, 2010.

[38] V. Filimonov and D. Sornette, "A stable and robust calibration scheme of the log-periodic power law model," *Physica A: Statistical Mechanics and its Applications*, vol. 392, no. 17, pp. 3698–3707, 2013.

[39] D. Sornette and A. Johansen, "Significance of log-periodic precursors to financial crashes," *Quantitative Finance*, vol. 1, no. 4, p. 452, 2001.

[40] A. Johansen, D. Sornette et al., "Shocks, crashes and bubbles in financial markets," *Brussels Economic Review*, vol. 53, no. 2, pp. 201–253, 2010.

[41] D. Sornette, G. Demos, Q. Zhang, P. Cauwels, V. Filimonov, and Q. Zhang, "Real-time prediction and post-mortem analysis of the shanghai 2015 stock market bubble and crash," *Swiss Finance Institute Research Paper*, no. 15-31, 2015.
Fig. 4: Prediction Results