Quarkonia and QGP studies

D. Blaschke\textsuperscript{a,b}, C. Peña\textsuperscript{a}

\textsuperscript{a}Institute for Theoretical Physics, University of Wroclaw, 50-204 Wroclaw, Poland
\textsuperscript{b}Bogoliubov Laboratory for Theoretical Physics, JINR, 161980 Dubna, Russia

Abstract

We summarize results of recent studies of heavy quarkonia correlators and spectral functions at finite temperatures from lattice QCD and systematic T-matrix studies using QCD motivated finite-temperature potentials. We argue that heavy quarkonia dissociation shall occur in the temperature range $1.2 \leq T_d/T_c \leq 1.5$ by the interplay of both screening and absorption in the strongly correlated plasma medium. We discuss these effects on the quantum mechanical evolution of quarkonia states within a time-dependent harmonic oscillator model with complex oscillator strength and compare the results with data for $R_{AA}/R_{\text{CNM}}$ from RHIC and SPS experiments. We speculate whether the suppression pattern of the rather precise NA60 data from In-In collisions may be related to the recently discovered $X(3872)$ state. Theoretical support for this hypothesis comes from the cluster expansion of the plasma Hamiltonian for heavy quarkonia in a strongly correlated medium.

Keywords: Heavy quarkonia, Quark-gluon plasma, Mott effect

1. Introduction

In Physics we know fields of research which are opened by ingenious experimental discoveries and sometimes have to wait even decades for their proper theoretical explanation. This was so in the case of superconductivity, where after Kamerlingh-Onnes’ observation of the vanishing resistance in Mercury at liquid He temperatures \cite{1} more than four decades of theory development were necessary before a satisfactory explanation could be formulated by Bardeen, Cooper and Schrieffer \cite{2}. Sometimes it is vice-versa, like in the case of Bose-Einstein condensation \cite{3}. The verification of the theory formulated by Bose and Einstein in 1924 succeeded only 70 years later \cite{4} since extremely subtle experimental techniques had to be developed to cool a sufficient number of atoms in a trap to nano-Kelvin temperatures.

The effect of $J/\psi$ suppression (more general, heavy quarkonium suppression) was suggested by theory \cite{5} to be a signal of quark-gluon plasma formation before it was actually seen in first heavy-ion collisions at CERN SPS and thus at first glance seems to belong to the latter class of discoveries. However, after 24 years of intense experimental research and theory developments, one has the impression that heavy quarkonia became a field where theory and experiment have to work hand in hand in order to make progress, like at this workshop. A key problem to be solved is that the usage of heavy quarkonia states as a probe for the diagnostics of the quark-gluon plasma (QGP) state of matter in ultra-relativistic heavy-ion collisions requires the knowledge of a baseline, e.g., from their production and evolution characteristics in situations when a QGP is absent. For a recent review see, e.g., \cite{6} and references therein. Current issues in the experiment-theory dialogue are summarized, e.g., in \cite{7}. Aspects of quarkonium production at LHC are discussed, e.g., in \cite{8,9}. For these proceedings we first summarize basic theory issues on the use of quarkonia as probes of the QGP before outlining the aspect of the modification of quarkonium formation by a strongly correlated QGP within a quantum mechanical model.

The spectrum of low-lying heavy quarkonia states is perfectly described by solutions of the Schrödinger equation for confining potentials of the Cornell-type \cite{10}. While at zero temperature the Cornell potential is nicely reproduced by lattice QCD simulations for the change in the singlet free energy of the system when static color charges are inserted, it has been argued that in a thermal system the internal energy should be used as the potential instead \cite{11}. The proper form of potential to be
used for studying quarkonia states in the medium may be a superposition of both forms depending on the interplay of time scales for interaction and thermal relaxation in the heat bath \[^{[12]}\]. The status of this discussion is unsettled. In the literature (cf. \[^{[13]}\]) calculations with both potential models derived from fits to lattice data are found whereby the use of the free energy as a heavy-quark potential implies a weaker binding and thus lower temperatures for the Mott transition of heavy quarkonia states, see also \[^{[14]}\].

![Figure 1: The "classical" picture: modification of binding energies of heavy quarkonia states in a hot plasma by static screening leads to their dissociation at the corresponding Mott-temperatures.](image)

In Fig. 1 we illustrate the “classical” picture of bound state dissociation in a hot plasma by screening of the interaction for the example of a screened Cornell potential applied to charmonium and bottomonium states \[^{[15]}\]: at state dependent (Mott) temperatures, the binding energy vanishes and bound state merges the continuum of scattering states. Two comments of significance for experiments are in order: (i) just above the Mott temperatures, remnants of the charmonia states survive as resonances in the continuum to be identified by peaks in the spectral functions and strong correlations in the corresponding scattering phase shifts \[^{[16]}\]; (ii) before reaching the Mott temperatures, the binding energies are already sufficiently lowered so that collisions with particles from the medium may have sufficient thermal energy to overpass the threshold for impact dissociation of quarkonia \[^{[17, 18, 19]}\], see the lower panels of Fig. 1 for corresponding estimates. This effect is particularly dramatic for \( \Upsilon \) which may undergo thermal dissociation already at \( T \approx 250 \text{ MeV} \) while its Mott temperature is \( \approx 470 \text{ MeV} \).

Both effects tend to wash out the pattern of sequential suppression for heavy quarkonia states expected from the “classical” picture of the Mott effect \[^{[20]}\]. The description of quarkonia states in a hot QGP medium in the vicinity of the critical temperature should therefore treat bound and scattering states on an equal footing. This is appropriately achieved within a thermodynamical T-matrix approach, which has been developed to address the spectral properties of quarkonia \[^{[21]}\] as well as open flavor meson states \[^{[22, 23]}\]. Such an approach allows a simultaneous description of quarkonium suppression and heavy flavor diffusion, presently under scrutiny for RHIC experiments, see \[^{[13]}\] and references therein.

The lowering of the quarkonia binding energy by screening has important consequences also for the process of quarkonium formation itself, once it occurs in a QGP environment. We will illustrate this aspect within a simple model.

### 2. Time-dependent harmonic oscillator model

For our discussion of the quantum mechanical evolution of quarkonia in an evolving QCD plasma state, we will employ here a generalization of the harmonic oscillator model \[^{[24]}\] to time-dependent one with complex oscillator strengths (THO model). Aspects of an optical potential for the propagation of charmonia through a medium have been already discussed, e.g., for cold nuclear matter in \[^{[25, 26]}\] and for a quark-gluon plasma in \[^{[27, 28]}\]. The merit of such a model is its simplicity and transparency as well as tractability. We consider the time-dependent Hamiltonian for heavy quarkonia in the form

\[
H(\tau) = 2m_Q + \frac{p^2}{2\mu} + \frac{\mu}{2} \omega^2(\tau) r^2(\tau),
\]

where \( \mu = m_Q/2 \) is the reduced mass and \( m_Q \) the heavy quark mass. The complex oscillator strength \( \omega^2(\tau) = \omega_0^2(\tau) + i\omega_I^2(\tau) \) has an implicit time dependence due to the temperature evolution \( T(\tau) \) of the system surrounding the evolving heavy quarkonium state. The temperature dependence of \( \omega_{0j}(T) \) resembles screening or strengthening of the quarkonium interaction, while a nonvanishing \( \omega_I(T) \) signals for quarkonium absorption or dissociation processes. The quadratic dependence of the imaginary part of the (optical) oscillator
potential is motivated by the phenomenon of color transparency, see also [29] and references therein. The conditions \(\omega_f(0) = 0\) and \(\omega_f(\rho_f) = \omega_f\) apply for the vacuum, where the spectrum of the low-lying charmonium states is approximated by a suitably chosen oscillator strength \(\omega = \omega_f = \text{const.}\)

The general classical trajectories for the Hamiltonian \(\{1]\) are linear combinations of the two solutions

\[ r(t) = \rho(t) \exp(\pm i\phi(t)) \quad , \quad \phi(t) = \int_0^t \frac{dr'}{\rho^3(r')} . \]

(2)

The amplitude \(\rho(t)\) fulfills the Ermakov equation

\[ \ddot{\rho}(t) + \omega^2(t) \rho(t) - \frac{1}{\rho^3(t)} = 0 , \]

(3)

for which exact solutions exist [30], allowing to evaluate the time evolution operator using path integral methods \([31]\]

\[ U(t_f; t_i) = \left[ \frac{\mu \rho_f \rho_i}{2\pi i \sin(\phi_f - \phi_i)} \right]^{1/3} e^{iS_{cl}} , \]

(4)

where the classical action functional \(S_{cl}[\rho(t)]\) enters.

The survival probability (suppression factor) for \(J/\psi\) as defined in [24] can be generalized to the THO case and applied to the QGP diagnostics in collisions of heavy ions with mass number \(A\) when identified with the experimentally determined quantity

\[ \frac{R_{AA}}{R_{\text{NNM}}} = \left| \cos(\phi_f) + \left( \frac{\rho_f}{\rho_i} + i \frac{\omega_f}{\rho_i} \right) \sin(\phi_f) \right|^3 \]

(5)

where \(R_{\text{NNM}}^{\text{CNM}}\) accounts for the cold nuclear matter (CNM) effects from charmonium absorption and modification of charm production by shadowing/antishadowing of gluon distribution functions in the CNM of the colliding nuclei. Both effects have been discussed in contributions to this workshop and are in principle accessible by analysis of \(pA\) collision experiments, see \([8, 32]\) and references therein. We restrict our discussion here to ground state charmonium at rest in the QGP medium \((\rho_T = y = 0)\) so that the discussion of Lorentz boost effects on the formation process can be omitted and also a detailed discussion of feed-down from higher charmonia states will be given elsewhere [33].

In the following we show that the anomalous \(J/\psi\) suppression in both, SPS and RHIC experiments can be simultaneously described with the natural assumption that above the critical temperature relevant screening parametrized with a temperature dependent, complex oscillator strength. The time evolution of temperature itself will be given by longitudinal (Bjorken scaling) hydrodynamic evolution of a fireball volume \(V(t)\) under entropy conservation

\[ S(t) = \text{const} = s(T(i))V(t) \quad ; \quad V(t) = A_T z(t) \]

(6)

with initial conditions determined by a Glauber model for nucleus-nucleus collisions. The temperature dependence of the entropy density \(s(T)\) is taken from recent lattice QCD simulations [34] which are well parametrized by the simple ansatz

\[ s(T) = 9.0 T^3 \left[ 1 + \tanh \left( \frac{T - T_0}{0.54 T} \right) \right] , \]

(7)

with \(T_0 = 0.189\) GeV, see Fig. [2]. The resulting temperature evolution is shown in Fig. [3] for initial values of temperature which correspond to initial entropy densities given by the Bjorken formula [35] \(s_0 = 3.86 \frac{\rho_{AA}}{\rho_{NN}} s_{NN}\), where the transverse overlap area \(A_T\) is taken from [36]. For the initial time \(t_0\) of the thermodynamical fireball evolution is assumed to depend on the center of mass energy of the collision and we take here 0.6 (1.0) \(\text{fm/c}\...
for RHIC (SPS) experiments. This completes the definition of the the THO model for applications to QGP studies in heavy-ion collisions which we discuss in the next section.

3. Anomalous suppression and the In-In case

A key indicator for QGP formation in heavy-ion collisions is anomalous suppression, the deviation of experimental data for the $J/\psi$ production ratio \( \frac{dN}{dy} / R_{AA} (CM) \) from unity. This effect, first observed at CERN SPS for Pb-Pb collisions at \( \sqrt{s} = 17 \text{ GeV} \), has been qualitatively confirmed by RHIC experiments with Au-Au interactions at \( \sqrt{s} = 200 \text{ GeV} \) whereby surprising new findings were revealed: (i) the suppression is stronger at forward and backward rapidities rather than at midrapidity where the particle densities are highest, (ii) the onset of anomalous suppression and its dependence on centrality scales with the charged particle density at midrapidity rather than with energy density. While (i) is caused mainly by antishadowing and to some extent by geometry \([37]\), the second finding is still not understood. A third puzzling issue raised by Carlos Lourenço in discussions at this meeting is (iii) the dip in the centrality dependence of the $J/\psi$ suppression ratio which is seen in the rather precise data of the NA60 collaboration for In-In collisions and so far widely ignored by theorists.

We want to report rather fresh results within the THO model which might get substantially improved in near future but the main idea of which is in the spirit of this workshop, namely to provide a more appropriate theoretical basis for future experiments on heavy quarkonium production, in particular in the LHC era. We are convinced that any deeper insights into the quantum mechanical evolution of the $\bar{c}c$ state in the hot QGP medium can become very important in this context. Here we will apply the THO model to extract the real (imaginary) part of the oscillator strength and thus the amount of screening (absorption) as a function of the temperature of the QGP medium from the experimental data.

In Fig. 4 we present a fit which ignores the In-In dip and is governed by the screening of the confining interaction which monotonously drops to zero in the temperature range \( T / T_c \sim 1.2 - 1.6 \), in gross accordance with the analysis of the charmonium spectrum from the temperature dependent potential on the basis of the heavy-quark free energy from lattice QCD simulations with a Mott temperature below \( 1.4 T_c \). This fit is rather insensitive to strong variations of the absorptive part of the potential. For recent systematic, quantum field theoretic

\[
\frac{dN}{d\eta} / R_{AA} (CM) \Big|_{T=1.4T_c}
\]

This parametrization is more susceptible to changes in the absorptive part in the complex oscillator strength: if \( \omega_\phi(T) \gtrsim \omega_\phi / 2 \) a too strong $J/\psi$ suppression would result.

Let us present a speculation about the possible origin for a resonant-like strengthening of the heavy-quark potential in the vicinity of the chiral/deconfinement transition. The Lippmann-Schwinger equation for $\rho$-$J/\psi$ scattering in the $D - D^*$ channel can be depicted in the following way.

\[
\rho / J = \frac{1}{2 \pi \alpha_s} \hat{G}(q^2) \hat{V}(q^2) \rho / J
\]
and its solution may produce a new state or resonance, such as the $X(3872)$ recently discovered at Belle and BaBar. For a recent discussion, see Ref. [42], and references therein. The scattering kernel,

$$\rho \begin{bmatrix} T_x \\ T \end{bmatrix} = \rho \begin{bmatrix} U_{\text{flip}} \\ T \end{bmatrix} + \rho \begin{bmatrix} U_{\text{flip}} \\ T \end{bmatrix} \begin{bmatrix} T_x \\ T \end{bmatrix}$$

and follows from a cluster expansion of the plasma medium, for details see Ref. [43]. In the present context, we may identify the T-matrices with resonant mesonic states: $T = \rho$, $T' = D$ and $T'' = D^*$. The resulting contribution to the plasma Hamiltonian for quarkonia states is proportional to the partial density of $\rho$ mesons in the medium which is peaked just above the QCD transition temperature, so that a contribution with the shape of $\omega_{\rho}(T)$ in the lower panel of Fig. 5 could be expected. A more detailed calculation could be based in Refs. [44, 45] and is in progress [33]. Quark exchange processes for quarkonium suppression have first been introduced within nonrelativistic potential models [45] and successfully been applied to describe CERN SPS data [46]. Their reformulation within relativistic quark models has confirmed results for the the behaviour and magnitude of the $J/\psi$ dissociation cross section by $\pi$ [47] and $\rho$ meson impact [48]. These models can be used to derive formfactors for the otherwise very powerful chiral Lagrangian approaches to charmonium dissociation, see [49] and references therein. In the context of the $X(3872)$ conjecture the finding of Ref. [50] is important that the spectral broadening of D-mesons due to their Mott effect at the chiral transition entails a qualitative increase in the dissociation rate by meson impact. Here the $\rho$ meson plays the dominant role [51], which motivates the conjecture that the ladder-type iteration of the quark exchange interaction kernel $U_{\text{flip}}$ may provide sufficient strength to produce the $X(3872)$ state in that channel.

4. Conclusion

Recent studies of heavy quarkonia correlators and spectral functions at finite temperatures in lattice QCD and systematic T-matrix approaches using QCD motivated finite-temperature potentials support that heavy quarkonia dissociation shall occur in the temperature range $1.2 \leq T_d/T_c \leq 1.5$ whereby the interplay of both screening and absorption processes is important. We have discussed these effects on the quantum mechanical evolution of quarkonia states within a time-dependent harmonic oscillator model with complex oscillator strength and compared the results with data for $R_{AA}/R_{AA}(CNM)$ from RHIC and SPS experiments. Besides the traditional interpretation, with a threshold for
the onset of anomalous suppression by screening and dissociation kinetics at $dN_{ch}/d\eta \sim 300$, we suggest an alternative arising from the attempt to model the dip the suppression pattern of the rather precise NA60 data from In-In collisions at $dN_{ch}/d\eta \sim 150 - 250$. We suggest that this dip indicates the true threshold for the onset of anomalous suppression due to the coupling of charmonium to the $D_s^+ D_0$ channel with the recently discovered X(3872) state. Although details need to be worked out, the theoretical basis for supporting this hypothesis has apparently been developed in plasma physics with the concept of a plasma Hamiltonian for nonrelativistic bound states like heavy quarkonia when they are immersed in a medium dominated by strong correlations like bound states, to be systematically addressed within cluster expansion techniques. This illustrates that in the studies of the interrelation of quarkonia with the QGP there are still many interesting and challenging aspects to be clarified within the further development and close collaboration of theory and experiment.

Acknowledgments

We would like to thank to R. Arnaldi, E. Scomparin and M. Leitch for providing us with data shown in Figs. 4 and 5. This work was supported in part by the Polish Ministry for Science and Higher Education under grants NN 202 0953 33 and NN 202 2318 37.

References

[1] H. Kamertingh-Onnes, Kon. Ned. Akad. W. Proc. 13, 1274 (1911).
[2] J. Bardeen, L. N. Cooper and J. R. Schrieffer, Phys. Rev. 106, 162 (1957).
[3] S. N. Bose, Z. Phys. 26, 178 (1924); A. Einstein, Sitzungsber. Preuss. Akad. Wiss. 1, 3 (1925).
[4] M. H. Anderson et al., Science 269, 198 (1995); K. B. Davis et al., Phys. Rev. Lett. 75, 3969 (1995).
[5] T. Matsui and H. Satz, Phys. Lett. B 178, 416 (1986).
[6] R. Rapp, D. Blaschke and P. Crochet, Prog. Part. Nucl. Phys. 65, 209 (2010) [arXiv:0807.2470 [hep-ph]].
[7] N. Brambilla et al., [arXiv:1010.5822 [hep-ph]].
[8] M. Bedjidian et al., [arXiv:hep-ph/0311048].
[9] J. P. Lansberg et al., AIP Conf. Proc. 1038, 15 (2008).
[10] W. Buchmuller and S. H. H. Tye, Phys. Rev. D 24, 132 (1981).
[11] H. Satz, J. Phys. G 36, 064011 (2009).
[12] E. V. Shuryak and I. Zahed, Phys. Rev. D 70, 054507 (2004).
[13] F. Riek and R. Rapp, Phys. Rev. C 82, 035201 (2010).
[14] H. T. Ding, A. Francis, O. Kaczmarek, F. Karsch, H. Satz and W. Soeldner, [arXiv:1011.0695] [hep-lat].
[15] J. Jankowski, D. Blaschke and H. Grigorian, Acta Phys. Polon. Supp. 3, 747 (2010).
[16] D. Blaschke, O. Kaczmarek, E. Laermann and V. Yudichev, Eur. Phys. J. C 43, 81 (2005).
[17] G. Röpke, D. Blaschke and H. Schulz, Phys. Lett. B 202, 479 (1988).
[18] G. Röpke, D. Blaschke and H. Schulz, Phys. Rev. D 38, 3589 (1988).
[19] D. Blaschke, Y. Kalinovsky and V. Yudichev, Lect. Notes Phys. 647, 366 (2004).
[20] S. Digal, P. Petreczky and H. Satz, Phys. Rev. D 64, 094015 (2001).
[21] D. Cabrera and R. Rapp, Phys. Rev. D 76, 114506 (2007).
[22] H. van Hees, M. Mannarelli, V. Greco and R. Rapp, Phys. Rev. Lett. 100, 192301 (2008).
[23] H. van Hees, M. Mannarelli, V. Greco and R. Rapp, Eur. Phys. J. C 61, 799 (2009).
[24] T. Matsui, Ann. Phys. 196, 182 (1989).
[25] B. Z. Kopeliovich and J. Kaufelnsen, Lect. Notes Phys. 647, 305 (2004).
[26] D. Koudela and C. Volpe, Phys. Rev. C 69, 054904 (2004).
[27] J. Cugnon and P. B. Gossiaux, Z. Phys. C 58, 77 (1993).
[28] J. Cugnon and P. B. Gossiaux, Z. Phys. C 58, 95 (1993).
[29] D. Blaschke and J. Huitner, Phys. Lett. B 281, 364 (1992).
[30] A. D. Polyamov and V. F. Zaitsev, *Handbook of exact solutions for ordinary differential equations*, Chapman & Hall/CRC, Boca Raton (2003), p. 461; I. Gijas and A. Bhattacharjee, Phys. Rev. Lett. 68, 2413 (1992); P. G. L. Leach and K. Andriopoulos, Appl. Ann. Discrete Math. 2, 146 (2008).
[31] H. Kleinert, *Path integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets*, World Scientific, Singapore (2006), pp. 114-115.
[32] E. G. Ferreiro, F. Fleuret, J. P. Lansberg and A. Rakotozafindrala, Phys. Rev. C 81, 064911 (2010).
[33] C. Peña and D. Blaschke, in preparation (2011).
[34] S. Borsanyi et al., [arXiv:1007.2580 [hep-lat]].
[35] K. Yagi, T. Hatsuda and Y. Miyake, *Quark-Gluon Plasma*, Cambridge University Press (2005).
[36] S. S. Adler et al., [PHENIX Collaboration], Phys. Rev. C 71, 034908 (2005) [Erratum-ibid. C 71, 049901 (2005)].
[37] D. Prorok, L. Turko and D. Blaschke, Phys. Lett. B 690, 352 (2010).
[38] A. Beraudo, J. P. Blaizot and C. Ratti, Nucl. Phys. A 806 (2008) 312.
[39] M. Laine, O. Philipsen and M. Tassler, JHEP 0709 (2007) 066.
[40] R. Arnaldi et al. [NA60 Collaboration], Phys. Rev. Lett. 99, 132302 (2007).
[41] R. Arnaldi [NA60 Collaboration]. Presentation at the ECT* Workshop on “Quarkonium Production in Heavy-Ion Collisions”, Trento (Italy), May 25-29, 2009.
[42] T. J. Burns, F. Piccinini, A. D. Polosa and C. Sabelli, Phys. Rev. D 82, 074003 (2010).
[43] D. Blaschke, [arXiv:0912.4479] [hep-ph].
[44] D. Blaschke and G. Röpke, Phys. Lett. B 299, 332 (1993).
[45] K. Martins, D. Blaschke and E. Quack, Phys. Rev. C 51, 2723 (1995).
[46] C. Y. Wong, E. S. Swanson and T. Barnes, Phys. Rev. C 62, 045201 (2000).
[47] M. A. Ivanov, J. G. Körner and P. Santorelli, Phys. Rev. D 70, 014005 (2004).
[48] A. Bourque and C. Gale, Phys. Rev. C 80, 054904 (2009).
[49] D. Blaschke, H. Grigorian and Yu. L. Kalinovsky, [arXiv:0808.1705] [hep-ph].
[50] G. R. G. Burau, D. B. Blaschke and Yu. L. Kalinovsky, Phys. Lett. B506, 297 (2001).
[51] D. Blaschke, G. Burau, Yu. Kalinovsky and T. Barnes, Eur. Phys. J. A 18, 547 (2003).