COMPETITION IN A DUAL-CHANNEL SUPPLY CHAIN
CONSIDERING DUOPOLISTIC RETAILERS WITH DIFFERENT BEHAVIOURS

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Abstract. We study competition in a dual-channel supply chain in which a single supplier sells a single product through its own direct channel and through two different duopolistic retailers. The two retailers have three competitive behaviour patterns: Cournot, Collusion and Stackelberg. Three models are respectively constructed for these patterns, and the optimal decisions for the three patterns are obtained. These optimal solutions are compared, and the effects of certain parameters on the optimal solutions are examined for the three patterns by considering two scenarios: a special case and a general case. In the special case, the equilibrium supply chain structures are analysed, and the optimal quantity and profit are compared for the three different competitive behaviours. Furthermore, both parametric and numerical analyses are presented, and some managerial insights are obtained. We find that in the special case, the Stackelberg game allows the supplier to earn the highest profit, the retailer playing the Collusion game makes the supplier earn the lowest profit, and the Stackelberg leader can gain a first-mover advantage as to the follower. In the general case, the supplier can achieve a higher profit by raising the maximum retail price or holding down the self-price sensitivity factor.

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1. **Introduction.** In recent years, the rapid development of e-commerce has encouraged many companies to sell products directly to consumers online [25]. This direct channel not only reduces the link and shortens the time of transaction but also improves the economic benefits of enterprises [11]. Therefore, dual-channel and multiple-channel structures, which combine the retail channel and the direct channel in the supply chain, are becoming increasingly popular. Finding examples of the dual-channel structure is not difficult. Several leading manufacturers in the United States, such as Apple, IBM and Dell, sell their products to consumers through independent retailers such as Circuit City and Best Buy and through their e-commerce websites [12]. In China, companies such as Suning and Gome have also established e-commerce websites.

In dual-channel supply chain, the retail price at supplier or retailers will be mainly affected by two aspects. On one hand, the quantities that released to the market will affect it. In our life, customers have alternatives to select the channel that satisfies their shopping requirements, therefore the supplier should decide the quantities to be sold through the retailer and directly on line. On the other hand, duopolistic retailers are very common in the market. III Media Research shows that Tmall and JD.com account for a 83.8% share of the B2C online retail market in China in the first half of 2018. These duopolistic retailers are interdependent. In order to take the most advantageous action, each duopolistic retailer will anticipate the impact of this action on the other retailers and the possible reactions of other retailers. The behaviour (cooperative behaviour or non-cooperative behaviour) between the two duopolistic retailers will also affect the price at a given firm. Therefore, it is significant to discuss the optimal decision considering retailers with different behaviours.

In practice, various forms of competition, such as Cournot, Collusion and Stackelberg, exist between the two retailers in the downstream supply chain. The three different behaviours represent different decision orders between the duopolistic retailers. When the duopolistic retailers are similar or identical to each other and make decisions simultaneously, such as Suning and Gome, AMD and Intel, they follow Cournot behaviour. Cournot behaviour of duopolistic retailers is a common phenomenon in the market [19]. When the duopolistic retailers have different decision order, one retailer takes decision based on the decision of the other retailer, their competition follow the Stackelberg behaviour. As mentioned above, in the network retail market of China, Tmall and JD.com are two competitive retailers. Tmall holds China Double 11 Shopping Festival in 2009 while 618 Shopping Festival was started by JD.com in 2010. In this Stackelberg behaviour, Tmall acts as leader and JD.com acts as follower. When the duopolistic retailers take decision jointly, they will follow Collusion behaviour. Collusion occurs when rival firms agree to work together. Since overt collusion is usually illegal, most collusion behaviors among the retailers are confidential, but it is prevalent in many retail markets. For example, Kroger, the second-largest retailer in the U.S., has partners with Alibaba Group to sell nuts, supplements and other products in China in 2018.

For the three competitive behaviours mentioned above, one question is that how the competitive behaviour effects on the optimal decisions for the member of the supply chain. In this paper, we will consider a two-echelon and dual-channel supply chain with duopolistic retailers in the downstream that have three competitive behaviours: Cournot, Collusion and Stackelberg. The optimal quantity decision
problem of the supplier and the two retailers is studied. Under the optimal decision process, the supplier’s and retailers’ quantity decisions are divided into a two-stage game. In the first stage, the supplier announces the total quantity to be sold through retailers and the retail quantity to be sold in the direct channel. In the second stage, the two retailers choose their order quantities. Based on the above discussion, there are three main objectives of this paper. First, it analyzes the optimal decisions of the supplier and the two retailers for the three competitive mode. Second, it compares the relation of the optimal quantities, price and profit for the three behaviours respectively. Third, it examines the effect of the market environment on the optimal decisions.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the notations and assumptions of our models. Section 4 presents dual-channel supply chain models for the retailers’ three competitive behaviours. Section 5 compares the optimal solutions and analyses the effects of parameters on the optimal solutions by considering two scenarios. Conclusions are discussed in Section 6. All the proofs are provided in the appendices.

2. Literature review. For a dual channel supply chain with a supplier and two competitive retailers, this paper focus on the comparison of the optimal solution for the three different behaviours of the two retailers. Thus, literatures related to competition between two retailers, competition between the supplier’s direct and indirect channel, channel structure, are reviewed and summarized in this section.

2.1. Competition between two retailers. Competition between two retailers is widely studied in many literature. Many studies have focused on the supply chain coordination. Ingene and Parry (1995) [10] explored channel coordination in a two-echelon single channel supply chain with one manufacture that sells through competing retailers and that treats these retailers equal. Yao et al. (2008) [27] investigated a revenue-sharing contract for coordinating a single channel supply chain comprising one manufacturer and two competing retailers under stochastic demand. The competing retailers are to determine the quantities to be ordered from the manufacturer simultaneously. David and Adida (2015) [6] studied competition and coordination in a dual-channel supply chain with one supplier and N retailers. The supplier operates a direct channel and indirect channels. The retailers competed in quantity form an oligopoly over the end market. Sadigh et al. (2016) [15] studied the coordination of pricing decisions in a three-echelon single channel supply chain including multiple retailers, one manufacturer, and multiple suppliers. The retailers determine their strategies simultaneously. Modak et al. (2016) [12] explored channel coordination issues of single supply chain with a manufacturer-distributor-retailers, in which the duopolistic retailers played Bertrand, Collusion and Stackelberg behaviours.

Some researchers studied competitive decision strategy. Yang and Zhou (2006) [26] considered the pricing decisions in singe channel supply chain with a manufacturer and two competitive retailers. They studied the effects of the retailers’ Bertrand, collusion and Stackelberg behaviors on the supply chains’ performance. Adida and DeMiguel (2011) [1] studied competition in a supply chain where multiple manufacturers compete in quantities to supply a set of products to multiple risk-averse retailers who compete in quantities to satisfy the uncertain consumer demand. Huang et al. (2016) [9] considered a pricing competition problem in a two-echelon single channel supply chain with one manufacturer and two duopoly
The two retailers played Bertrand and collusion behaviors, they explored the effects of the retailers' behaviors on the supply chains' performance. Wu et al. (2019) [22] considered single supply chain with a manufacturer and two asymmetric retailers, they studied inventory (order quantity) competition in the presence of demand uncertainty and an exogenously given retail price. Wang et al. (2019) [21] studied the closed-loop supply chain network consists of manufacturers, retailers and consumer markets engaging in a Cournot-Nash game. They presented an equilibrium model and discussed properties of the variational inequality model.

The above studies about competitive between retailers either considered coordination and competitive problems in single channel supply chain, or only considered one or two competitive behaviours between retailers. In contrast to these works, we focus on the quantities competition in a dual-channel considering three different behaviours between the two retailers.

2.2. Competition between the suppliers direct and indirect channel. The advent of the direct channel added a new concept to the supply chain and competition. Some of the previous studies focused on the competition between direct and retail channel. Balasubramanian (1998) [2] early attempted to analyze the context of direct versus retail competition from a strategic viewpoint. He modeled the role of information in multiple-channel markets and shown that the level of information disseminated by the direct marketer have strategic implications. Vinhas and Anderson (2005) [16] modeled the degree to which direct and indirect channels compete destructively against each other when contacting the same customers in a subset of firms that use both channel types in the same market. Brynjolfsson et al. (2009) [3] studied the level of competition between Internet retailers and traditional stores varies across products that have different levels of search costs in traditional stores. Lai et al. (2018) [11] found that when moving from the asymmetric information case to the full information case, the indirect channel will experience a decreased demand and increased price, the direct channel demand will increase, price will not change. Sadeghi et al. (2019) [14] studied competition problems between chains with different channel structures. They showed that the direct channel is most powerful channel that always attracts the highest customer attention to itself through its high proposing discount.

Many suppliers face a channel distribution decision of whether to add a new retailing channel in addition to their existing channels. Some authors studied whether should manufacturers add a direct channel or not. Chiang et al. (2003) [5] analyzed the incentives for a manufacturer to create its own direct channel to compete with its retailer, conceptualizes the impact of customer acceptance of a direct channel, the degree to which customers accept a direct channel as a substitute for shopping at a traditional store. Dumrongsi et al. (2008) [7] found that the manufacturer is likely to be better off using dual channels than using the single channel when the retailer's marginal cost is high, and the wholesale price, customer valuation, and demand variability are low. Xiao et al. (2014) [23] found that adding the direct channel raises the unit wholesale price and retail price in the indirect channel due to customization in the direct channel.

The above studies mainly focused on the competition between direct and indirect channel. They investigated which channel is most economic or whether is worth to open a direct channel. They ignore different competitive behaviours among multiple retailers on the profit or demand of manufacturers or retailers. In contrast to these works, we do not study the competition issues between direct and indirect
channel, we mainly compare the optimal solutions for three different behaviours of the retailers.

2.3. Channel structure. The advent of e-commerce has prompted many enterprises to develop an online channel for direct sales with the traditional retail channel being maintained to establish an online/offline dual-channel distribution system. The channel structure problem has been widely studied in the literature. For the single-channel supply chain problem, the researchers mainly focus on the issues of supply chain coordination and price competitive decision strategy. Moon and Feng (2017) [13] studied channel coordination with a single-period supply chain model with one supplier and independent retailers. Chen and Xiao (2017) [4] built a model by considering competing retailers and proposed a Grove wholesale price contract for coordination. Wang et al. (2012) [18] explored the optimal decision models under decentralized and integrated systems with a non-linear demand function.

For the multiple-channel supply chain problem, channel selection, Pricing decisions and coordination mechanisms, have received increasing attention from researchers. Wang et al. (2016) [20] explored the channel selection and pricing strategy in a supply chain. They found that the gap between the online and offline channels’ operating costs was critical to the retailer’s choice of its channel selection strategy. Han et al.(2016) [8] studied the reverse channel selection issue from both profitability and robustness perspectives. They found the direct channel outperforms indirect channel under a favorable operational environment, while the indirect channel performs better in an adverse environment. Xu et al. (2014) [24] investigated the impact of establishing a dual-channel supply chain coordinating contract when the supply chain agents are risk averse. Wang and Song et al. (2019) [17] considered the pricing problem of complementary products in a fuzzy dual-channel supply chain environment.

It is sometimes beneficial for the supplier to operate a direct channel, so in contrast to these works that investigate single-channel supply chain problem, we focus on dual-channel supply chain about quantities competition. In contrast to the studies on dual-channel problems, we focus on comparison of the optimal solution for three different behaviours and analyze the effect of the market environment on the optimal decisions.

Our work differs from previous papers by considering duopolistic retailers competing via three different behaviours simultaneously in a dual-channel supply chain. Our model is closely related to that proposed by Yang and Zhou (2006) [26], who considered a two-echelon supply chain. However, There are three differences in contrast to Yang and Zhou (2006) [26]. First, their model discusses retailers’ competitive behaviours in only a single channel, we add a direct retail channel and consider duopolistic retailers’ different behaviours in a dual-channel structure. The quantity and price in direct channel and indirect channel influence each other, so the models are more complex. Second, their model considers price competition, while our model considers quantities competition. Third, we give the conditions when supplier acts only as a wholesale supplier or a monopoly retailer, so we discuss the equilibrium supply chain structure in a special case and propose the condition when the dual-channel will occur. We make the following specific contributions in this study: (1) The dual-channel supply chain equilibrium quantities are obtained when the retailers are asymmetric in demand characteristics. (2) The equilibrium supply chain structure is given when the retailers are symmetric in demand characteristics, and the relations about price, quantities and profit are given. (3) We
present some analytical results and numerical analysis of the effect of parameters on optimal solutions. Some of the findings for symmetric retailer are also hold in general case.

3. Model notations and assumptions. The notations used to develop the proposed models are as follows.

| Symbol | Description |
|--------|-------------|
| **Indices** | | |
| \(i\) | Index of retailers \((i = 1,2)\) |
| \(j\) | Index of three patterns of the two retailers \((j = \text{co, cn, st})\) |
| **Parameters** | | |
| \(a_0\) | The maximum retail price at the supplier |
| \(a_i\) | The maximum retail price at the retailer \(i\) |
| \(\theta_0\) | The price sensitivity of demand at the supplier with respect to its own quantity |
| \(\theta_i\) | The price sensitivity of demand at each retailer with respect to its own quantity |
| \(\beta\) | The cross-sensitivity of demand |
| **Decision Variables** | | |
| \(q_0\) | The quantity sold in the direct channel |
| \(q_i\) | The quantities ordered by retailer \(i\) or the deterministic demand faced by retailer \(i\) |
| **Dependent Variables** | | |
| \(w\) | The wholesale price per unit charged to the retailers by the supplier |
| \(c\) | The cost per unit of the supplier |
| \(q\) | The total quantity selected by the supplier to be sold through retailers |
| \(p_0\) | The retail price at the supplier’s direct channel |
| \(p_i\) | The retail price at retailer \(i\) |
| \(\Pi_0\) | The profit of the supplier |
| \(\Pi_i\) | The profit of retailer \(i\) |

Our model considers a single supplier that supplies a single product to two duopolistic retailers and operates a direct retail channel. The supply chain structure is shown in Fig. 1.

**Assumption 1.** The supplier and the two retailers engage in a Stackelberg game, where the supplier is a Stackelberg leader and chooses his supply quantity to be sold through the dual channels to maximize his profit. The retailers then react by choosing their wholesale market order quantity to maximize their profit from retail sales, while the wholesale price \(w\) is set to clear the wholesale market.

**Assumption 2.** The competitive behaviour of the two retailers has three patterns: (1) Cournot, i.e., the retailers simultaneously react by choosing their retail quantities; (2) Collusion, i.e., the retailers jointly make a decision to maximize their total profit; and (3) Stackelberg, i.e., one of the two retailers makes the order quantity decision by considering the response function of the other.

**Assumption 3.** Similar to David and Adida (2015) [6], it is assumed that the retail demand satisfies the following linear inverse demand function:

\[
p = a - Bq.
\]
Figure 1. Supply chain structure

where \( p = (p_0, p_1, p_2) \), \( q = (q_0, q_1, q_2) \) and \( a = (a_0, a_1, a_2) \). The matrix \( B \in \mathbb{R}^{3 \times 3} \) is the price sensitivity matrix given by

\[
B = \begin{pmatrix}
\theta_0, & \beta, & \beta \\
\beta, & \theta_1, & \beta \\
\beta, & \beta, & \theta_2
\end{pmatrix}
\]

Assumption 4. The parameters \( \theta_k > 0 \) \((k = 0, 1, 2)\) and \( \beta \) are independent. It is assumed that increasing the order quantity of a firm’s product will decrease the price and that increasing a competitor’s quantity will also decrease the firm’s price; therefore, \( \theta_k > 0 \) and \( \beta > 0 \). The change in a competitor’s quantity affects a firm’s price less so than a change in the firm’s own quantity; thus, \( \beta < \theta_k \).

Assumption 5. The wholesale prices have to be higher than the supplier’s cost of the products, that is, \( w > c \), because the supplier wants to obtain a positive profit. Similarly, the retailers’ prices must be higher than the wholesale prices because of the requirement for positive retailer profit; therefore, \( p_i > w \).

4. The model. According to the above assumptions, the profit functions of the supplier and retailers are given as follows:

\[
\Pi_0 = p_0q_0 + (w-c)q = (a_0 - \theta_0q_0 - \beta q_1 - \beta q_2)q_0 + (w-c)q, \tag{1}
\]

\[
\Pi_1 = (p_1 - w)q_1 = (a_1 - \beta q_0 - \theta_1 q_1 - \beta q_2 - w)q_1, \tag{2}
\]

\[
\Pi_2 = (p_2 - w)q_2 = (a_2 - \beta q_0 - \beta q_1 - \theta_2 q_2 - w)q_2. \tag{3}
\]

Remark 1. We assume that the supplier’s optimization problem is constrained by the wholesale market clearing condition: \( q = q_1(w) + q_2(w) \), where \( q_i(w) \) denotes the quantity selected by retailer \( i \) when the wholesale price is \( w \). Therefore, according to the market clearing condition, the wholesale price \( w \) and the total quantity \( q \) have a one-to-one correspondence. Then, the supplier’s profit \( \Pi_0 \) can be expressed as

\[
\Pi_0 = (a_0 - \theta_0q_0 - \beta q)q_0 + (w(q) - c)q. \tag{4}
\]

In the following sub-sections, we will discuss how the members of the two-echelon supply chain make decisions under three patterns.
4.1. Pattern 1: Retailers play a Cournot game. In this pattern, both retailers independently respond with their order quantities, and their optimal order quantities can be solved by the simultaneous-equation models constructed by their respective response functions. We will use backward induction to determine the equilibrium solution of the two-stage game. First, the retailers simultaneously determine the optimal order quantities $q_1^{co}$ and $q_2^{co}$ for any given $q_0$ and $w$. Then, the supplier determines the optimal retail quantities $q_0^{co}$ and $q^{co}$ based on the two quantities $q_1^{co}$ and $q_2^{co}$.

For any given $q_0$ and $w$, the optimal order quantities of retailers 1 and 2 are given by (solving $\partial \Pi_1/\partial q_1 = 0$ and $\partial \Pi_2/\partial q_2 = 0$)

$$q_1^{co} = \frac{[2\theta_2(a_1 - \beta q_0 - w) - \beta(a_2 - \beta q_0 - w)]/(4\theta_1\theta_2 - \beta^2)}{\beta},$$  
$$q_2^{co} = \frac{[2\theta_1(a_2 - \beta q_0 - w) - \beta(a_1 - \beta q_0 - w)]/(4\theta_1\theta_2 - \beta^2)}{\beta}.$$  

According to $q = q_1^{co} + q_2^{co}$, the wholesale price $w$ is

$$w = \frac{A - qE}{D} - \beta q_0.$$  

where $A = 2\theta_2 a_1 - \beta a_2 + 2\theta_1 a_2 - \beta a_1$, $D = 2(\theta_1 + \theta_2 - \beta)$ and $E = 4\theta_1\theta_2 - \beta^2$.

By Eq. (4), the supplier’s profit function with respect to $q_0$ and $q$ is

$$\Pi_0^{co} = (a_0 - \theta_0 q_0 - \beta q_0 q_0 + (A - qE)/D - \beta q_0 - c)q.$$  

Proposition 4.1. The supplier’s profit function $\Pi_0^{co}$ is a concave function with respect to $q_0$ and $q$.

Proof. see Appendix.  

From Proposition 4.1, solving $\partial \Pi_0^{co}/\partial q_0 = 0$ and $\partial \Pi_0^{co}/\partial q = 0$ will yield the optimal quantities $q_0^{co}$ and $q^{co}$ set by the supplier.

$$q_0^{co} = \frac{[\beta(A - cD) - a_0 E]/(2\beta^2 D - 2\theta_0 E)}{\beta},$$  
$$q^{co} = \frac{[- \theta_0 (A - cD) + \beta Da_0] / (2\beta^2 D - 2\theta_0 E)}{\beta}.$$  

By substituting Eqs. (8) and (9) into Eq. (7), we can obtain the optimal wholesale price of the supplier as follows:

$$w^{co} = \frac{A + cD}{2D}.$$  

Combining Eqs. (8) – (9) with Eqs. (1) – (3) and Eqs. (5) – (6) will easily lead to the optimal quantities and the corresponding profits of the retailers and supplier.

4.2. Pattern 2: Retailers play a Collusion game. In this pattern, assume that the duopolistic retailers recognize their interdependence. To maximize the total profit in the downstream retail market, the retailers agree to act in a union. Hence, the total profit of the downstream retail market is

$$\Pi_r = \Pi_1 + \Pi_2 = (a_1 - \beta q_0 - \theta_1 q_1 - \beta q_2 - w)q_1 + (a_2 - \beta q_0 - \beta q_1 - \theta_2 q_2 - w)q_2.$$  

Proposition 4.2. $\Pi_r$ is a concave function with respect to $q_1$ and $q_2$.

Proof. see Appendix.
Proposition 4.3. Π as follows:

\[ q_1^{cn} = \left[ \theta_2(a_1 - \beta q_0 - w) - \beta(a_2 - \beta q_0 - w) \right] / \left[ 2(\theta_1 \theta_2 - \beta^2) \right], \quad (11) \]
\[ q_2^{cn} = \left[ \theta_1(a_2 - \beta q_0 - w) - \beta(a_1 - \beta q_0 - w) \right] / \left[ 2(\theta_1 \theta_2 - \beta^2) \right]. \quad (12) \]

According to \( q = q_1^{cn} + q_2^{cn} \), we obtain

\[ w = \frac{F}{G} - \beta q_0 - \frac{H}{G} q, \quad (13) \]

where \( F = a_1(\theta_2 - \beta) + a_2(\theta_1 - \beta) \), \( G = \theta_1 + \theta_2 - 2\beta \), and \( H = 2(\theta_1 \theta_2 - \beta^2) \).

By Eq. (4), the supplier’s profit is

\[ \Pi_0^{cn} = (a_0 - \theta_0 q_0 - \beta q)q_0 + \left( \frac{F}{G} - \beta q_0 - \frac{H}{G} q - c \right)q. \]

**Proposition 4.3.** \( \Pi_0^{cn} \) is a concave function with respect to \( q_0 \) and \( q \).

**Proof.** see Appendix.

The optimal quantities \( q_0^{cn} \), sold in the direct channel, and \( q^{cn} \), ordered by retailers, are (by solving \( \partial \Pi_0^{cn} / \partial q_0 = 0 \) and \( \partial \Pi_0^{cn} / \partial q = 0 \))

\[ q_0^{cn} = \left( a_0 H - \beta F + G\beta c \right) / \left[ 2(\theta_0 \beta G) \right], \quad (14) \]
\[ q^{cn} = \left( \theta_0 F - \theta_0 Gc - a_0 \beta G \right) / \left[ 2(\theta_0 \beta G) \right]. \quad (15) \]

By substituting Eqs. (14) – (15) into Eq. (13), we can obtain the optimal wholesale price of the supplier as follows:

\[ w^{cn} = \frac{F + Gc}{2G}. \quad (16) \]

Combining Eqs. (14) – (16) with Eqs. (1) – (3) and Eqs. (11) – (12) will allow us to easily obtain the optimal quantities and the corresponding profits of the retailers and supplier.

4.3. **Pattern 3: Retailers play a Stackelberg game.** In this pattern, the Stackelberg leader (i.e., retailer 1) makes the optimal order quantity decision by considering the response function of the Stackelberg follower (i.e., retailer 2). Then, the Stackelberg follower decides his optimal order quantity based on the Stackelberg leader’s decision.

For any given \((q_0, q_1, q_2, w)\), retailer 2 observes his reaction function \( q_2 = (a_2 - \beta q_0 - \beta q_1 - w) / (2\theta_2) \) (solving \( \partial \Pi_2^{rt} / \partial q_2 = 0 \)) and adjusts his order quantity to maximize his profit according to the given order quantity decision of retailer 1. Substituting the reaction function of retailer 2 into Eq. (1), the profit function of retailer 1 is given as

\[ \Pi_1^{rt} = \left[ 2\theta_2(a_1 - \beta q_0 - \theta_1 q_1 - w) - \beta(a_2 - \beta q_0 - \beta q_1 - w) \right] q_1 / (2\theta_2). \]

Clearly, retailer 1’s profit is a function of \( q_1 \) alone and a concave function of \( q_1 \) because \( \beta < \theta_k \). Thus, for any \( q_0 \) and \( w \) set by the supplier, retailer 1 obtains his optimal order quantity by setting \( \partial \Pi_1^{rt} / \partial q_1 = 0 \).

\[ q_1^{rt} = \frac{2\theta_2(a_1 - \beta q_0 - w) - \beta(a_2 - \beta q_0 - w)}{2(2\theta_1 \theta_2 - \beta^2)}. \quad (17) \]
Retailer 2’s optimal order quantity can be determined by substituting retailer 1’s optimal quantity into his reaction function. Then, the relevant expression is

\[ q_2^* = \left[ (4\theta_1 \theta_2 - \beta^2)(a_2 - \beta q_0 - w) - 2\theta_2 \beta (a_1 - \beta q_0 - w) \right] / \left[ 4\theta_2 (2\theta_1 \theta_2 - \beta^2) \right]. \] (18)

According to \( q = q_1^* + q_2^* \), we obtain

\[ w = -\beta q_0 - \frac{4\theta_2 M}{N} q + \frac{K}{N}. \] (19)

where \( M = 2\theta_1 \theta_2 - \beta^2 \), \( N = 4\theta_2^2 - 4\theta_2 \beta + 4\theta_1 \theta_2 - \beta^2 \) and \( K = a_1 (4\theta_2^2 - 2\theta_2 \beta) + a_2 (4\theta_1 \theta_2 - \beta^2 - 2\theta_2 \beta) \).

The supplier follows the retailers’ reactions \( q_1^* \) and \( q_2^* \) for any \( q_0 \) and \( q \) values that he sets. The profit function of the supplier is

\[ \Pi_0^s = (a_0 - \theta_0 q_0 - \beta q) q_0 + (-\beta q_0 - \frac{4\theta_2 M}{N} q + \frac{K}{N} - c) q. \]

**Proposition 4.4.** \( \Pi_0^s \) is a concave function with respect to \( q_0 \) and \( q \).

**Proof.** see Appendix.

The optimal quantities \( q_0^* \) sold in the direct channel and \( q^* \) ordered by retailers are (by solving \( \partial \Pi_0^s / \partial q_0 = 0 \) and \( \partial \Pi_0^s / \partial q = 0 \))

\[ q_0^* = \frac{(Nc - K) \beta + 4a_0 \theta_2 M}{8\theta_0 \theta_2 M - 2\beta^2 N}, \] (20)

\[ q^* = \frac{a_0 \beta N + (Nc - K) \theta_0}{2(\beta^2 N - 4\theta_0 \theta_2 M)}. \] (21)

By substituting Eqs. (10) and (21) into Eq. (19), we can obtain the optimal wholesale price of the supplier as follows:

\[ w^* = \frac{Nc + K}{2N}. \] (22)

Combining Eqs. (20) – (22) with Eqs. (1) – (3) and Eqs. (17) – (18) allows us to easily obtain the optimal quantities and the corresponding profits of the retailers and supplier.

5. Comparison of the optimal solutions and parametric sensitivity analysis. In this section, we will compare the optimal solutions and examine the effects of several parameters on the optimal solutions for the three patterns. Two scenarios are considered. First, a special case, where \( a_1 = a_2 \) and \( \theta_1 = \theta_2 \), is discussed. Second, a general case, where \( a_1 \neq a_2 \) or \( \theta_1 \neq \theta_2 \), is discussed.

5.1. **Scenario 1:** \( a_1 = a_2 \) and \( \theta_1 = \theta_2 \). In this section, we consider the two retailers to be symmetric in terms of demand parameters, that is, \( a_1 = a_2 = a \) and \( \theta_1 = \theta_2 = \theta \); according to the results obtained in Section 4, the optimal solutions consisting of optimal quantities, profits and prices can be solved.

However, the quantities sold in the direct and indirect channels may be less than 0. Specifically, \( q_0^* \leq 0 \); then, we take \( q_0^* = 0 \), where the supplier acts as a wholesale supplier. If \( q_1^* \leq 0 \), then we take \( q_1^* = 0 \), where the supplier acts as a monopoly retailer. Let \( \delta = \frac{a_0}{a_0 - c} \) denote the relative strength of the supplier’s direct channel compared to the indirect channel. The equilibrium supply chain structure, optimal profits, quantities, and prices for the three patterns are given in Table 1. Several insights are obtained from Table 1.
The equilibrium supply chain structure about the three different competitive behaviors

| Structure       | Wholesale supplier | Dual-channel | Monopoly retailer |
|-----------------|--------------------|--------------|------------------|
| Cournot         | \( \delta \leq \frac{2a}{\theta + \beta} \) | \( \frac{2a}{\theta + \beta} < \delta < \frac{a_0}{\theta} \) | \( \delta \geq \frac{a_0}{\theta} \) |
|                  | \( q_0^{\infty} \) | \( q_1^{\infty} \) | \( q_2^{\infty} \) | \( w^{\infty} \) | \( p_0^{\infty} \) | \( p_1^{\infty} \) | \( p_2^{\infty} \) | \( \Pi_0^{\infty} \) | \( \Pi_1^{\infty} \) | \( \Pi_2^{\infty} \) |
| Collusion        | \( \delta \leq \frac{a}{\theta + \beta} \) | \( \frac{a}{\theta + \beta} < \delta < \frac{a_0}{\theta} \) | \( \delta \geq \frac{a_0}{\theta} \) |
|                  | \( q_0^n \) | \( q_1^n \) | \( q_2^n \) | \( w^n \) | \( p_0^n \) | \( p_2^n \) | \( \Pi_0^n \) | \( \Pi_1^n \) | \( \Pi_2^n \) |
| Stackelberg      | \( \delta \leq \frac{4a}{\theta + \beta} \) | \( \frac{4a}{\theta + \beta} < \delta < \frac{a_0}{\theta} \) | \( \delta \geq \frac{a_0}{\theta} \) |
|                  | \( q_0^t \) | \( q_1^t \) | \( q_2^t \) | \( w^t \) | \( p_0^t \) | \( p_1^t \) | \( p_2^t \) | \( \Pi_0^t \) | \( \Pi_1^t \) | \( \Pi_2^t \) |

where \( T = 4\delta(2\theta^2 - \beta^2) \), \( U = 8\theta^2 - 4\theta\beta - \beta^2 \), \( V = a_0\beta - \theta_0(a - c) \)
Insight 5.1. If retailers play a Cournot game and \( \delta \leq \frac{2\beta}{\theta + \beta} \), then \( q_0^j \leq 0 \). The supplier’s direct channel is weak compared with the indirect channel; therefore, the supplier acts only as a wholesale supplier to its retailers. The dual-channel case occurs if \( \frac{2\beta}{\theta + \beta} < \delta < \frac{\theta}{\beta} \). The Cournot results in Table 1 is consistent with the results in David and Adida [6]. When the parameters in David and Adida [6] satisfies some conditions. Similarly, when retailers play the Collusion game, the supplier acts as a wholesale supplier if \( \delta \leq \frac{\beta}{\theta + \beta} \), and the dual-channel case occurs if \( \frac{\beta}{\theta + \beta} < \delta < \frac{\theta}{\beta} \). When retailers play a Stackelberg game, the supplier acts as a wholesale supplier if \( \delta \leq \frac{\beta(8\theta^2 - 4\theta \beta - \beta^2)}{4\theta(2\theta - \beta^2)} \), and the dual-channel case occurs if \( \frac{\beta(8\theta^2 - 4\theta \beta - \beta^2)}{4\theta(2\theta - \beta^2)} < \delta < \frac{\theta}{\beta} \). Therefore, regardless of the type of pattern that the duopolistic retailers play, the dual-channel case occurs if \( \frac{\beta(8\theta^2 - 4\theta \beta - \beta^2)}{4\theta(2\theta - \beta^2)} < \delta < \frac{\theta}{\beta} \).

In addition, regardless of the type of pattern that the retailers play, if \( \delta \geq \frac{\theta}{\beta} \), then \( q_1^j \leq 0 \); that is, the quantity sold in the indirect channel is not greater than zero.

The supplier’s direct channel is strong enough to make retailers exit the market, and the supplier acts as a monopoly retailer.

Insight 5.2. When the supplier acts only as a wholesale supplier or when the dual-channel case occurs, the retailers’ different competitive behaviours do not affect the optimal price \( w^j \). \( w^j \) depends only on the parameters \( a \) and \( c \), and \( w^co = w^cn = w^st \). When the supplier acts only as a monopoly retailer or when the dual-channel case occurs, the retailers’ different competitive behaviours do not affect the optimal price \( p^0_0 \). \( p^0_0 \) depends only on the parameter \( a_0 \), and \( p^co_0 = p^cn_0 = p^st_0 \). When the supplier acts as a monopoly retailer, the retailers’ different competitive behaviours do not affect the optimal quantity \( q_0^j \) and the optimal profit \( \Pi^j_0 \). \( q^0_1 \) and \( \Pi_0^j \) depend only on the parameters \( a_0 \) and \( \theta_0 \), and they have the following relations: \( q^co_0 = q^cn_0 = q^st_0 \) and \( \Pi^co_0 = \Pi^cn_0 = \Pi^st_0 \).

Insight 5.3. Regardless of the type of equilibrium supply chain structure, when retailers play the Cournot game and the Collusion game, the optimal profits, quantities, and prices of the two retailers are equal; that is, \( \Pi^1_1 = \Pi^2_1 = \Pi^co_1 = \Pi^cn_1 = \Pi^st_1 \), \( q^co_1 = q^cn_1 = q^st_1 \), \( p^co_1 = p^cn_1 = p^st_1 \), and \( \Pi^co_2 = \Pi^cn_2 = \Pi^st_2 \).

By comparing the optimal prices, quantities and profits of the supplier and the retailers under the three patterns, we have the following propositions from Table 1.

**Proposition 5.1.** When the supplier acts as a wholesale supplier:

1. For the supplier, the optimal profits have the following relation:
   \[ \Pi^st_0 > \Pi^co_0 > \Pi^cn_0. \]

2. For the retailers, the optimal quantities, prices and profits have the following relations:
   \[ q^st_1 > q^co_1 = q^co_2 > q^st_2 > q^cn_2 > q^cn_1, \]
   \[ p^cn_1 = p^cn_2 > p^co_1 = p^co_2 > p^st_2 > p^st_1, \]
   \[ \Pi^cn_1 = \Pi^cn_2 > \Pi^st_1 > \Pi^st_2 > \Pi^co_1 = \Pi^co_2 > \Pi^co_2. \]

**Proof.** see Appendix.

**Proposition 5.2.** When the dual-channel case occurs:
(1) For the supplier, the optimal quantities and profits have the following relations:
\[ q_0^{cn} > q_0^{co} > q_0^{st}, \quad \Pi_0^{st} > \Pi_0^{co} > \Pi_0^{cn}. \]

(2) For the retailers, the optimal quantities, prices and profits have the following relations:
\[ q_1^{st} = q_1^{co} > q_2^{st} > q_1^{cn} = q_2^{cn}, \]
\[ p_1^{st} = p_2^{cn} > p_1^{co} = p_2^{co} > p_2^{st} > p_1^{st}, \]
\[ \Pi_1^{st} > \Pi_2^{st}, \quad \Pi_1^{co} = \Pi_2^{co} > \Pi_2^{st}, \]
\[ \text{When } \theta_0 > 4\beta, \quad \Pi_1^{co} = \Pi_2^{cn} > \Pi_1^{st} = \Pi_2^{st}. \]

Proof. see Appendix. \( \square \)

Remark 5.1. From Propositions 5.1 and 5.2, among the duopolistic retailers’ three patterns, the Stackelberg game allows the supplier to earn the highest profit. Collusion makes the duopolistic retailers charge the highest price and obtain the minimum quantity, while the Stackelberg leader (i.e., retailer 1) charges the lowest price and obtains the largest quantity. When duopolistic retailers play the Stackelberg game, the more powerful leader obtains more profit than the follower, which is consistent with our intuition.

Proposition 5.3. Regardless of the type of pattern that duopolistic retailers play, when the supplier acts only as a wholesale supplier, the supplier’s profit is greater than the total profit of the two duopolistic retailers, that is, \( \Pi_0^i > (\Pi_1^i + \Pi_2^i) \).

Proof. see Appendix. \( \square \)

5.2. Scenario 2: \( a_1 \neq a_2 \) or \( \theta_1 \neq \theta_2 \). In this section, when the dual-channel case occurs, let \( a_1 \neq a_2 \) or \( \theta_1 \neq \theta_2 \). We will examine the effects of \( a_k \) \( (k = 0, 1, 2) \), \( \theta_k \) and \( \beta \) on the optimal solutions, present some analytical results and conduct a numerical analysis. Three cases are considered. First, \( a_0, \theta_1 \) and \( \theta_2 \) change, while the other variations do not change. Second, \( \theta_0, \theta_1 \) and \( \theta_2 \) change, while the other parameters are static. Finally, we consider that only one parameter \( \beta \) changes.

5.2.1. Effect of \( a_k \) on the optimal quantity and profit. For the case in which \( a_k \), \( a_1 \) and \( a_2 \) change and the other variations do not change, we will discuss the effect of \( a_k \) on the optimal quantity \( q_k \) for the three different competitive behaviours. When the retailers play the Cournot game, taking the partial derivatives of Eqs. (5), (6) and (8) with respect to \( a_0 \),

\[
\frac{\partial q_k^{co}}{\partial a_0} = \frac{4\theta_1\theta_2 - \beta^2}{4\beta^2(\theta_1 + \theta_2 - \beta) - 2\theta_0(4\theta_1\theta_2 - \beta^2)} = \frac{-E}{2(\beta^2 D - \theta_0 E)},
\]
\[
\frac{\partial q_k^{co}}{\partial a_0} = \frac{4\beta^2(\theta_1 + \theta_2 - \beta) - 2\theta_0(4\theta_1\theta_2 - \beta^2)}{\beta(2\theta_2 - \beta)} = \frac{2(\beta^2 D - \theta_0 E)}{\beta(2\beta_2 - \beta)},
\]
\[
\frac{\partial q_k^{co}}{\partial a_0} = \frac{4\beta^2(\theta_1 + \theta_2 - \beta) - 2\theta_0(4\theta_1\theta_2 - \beta^2)}{\beta(2\theta_2 - \beta)} = \frac{2(\beta^2 D - \theta_0 E)}{\beta(2\beta_2 - \beta)}.
\]

Because \( 0 < \beta < \theta_k, 2(\theta_0 E - \beta^2 D) = (\theta_0 + \beta)(\theta_1 - \beta)(\theta_2 - \beta) + 3\theta_0\theta_1\theta_2 - 2\theta_0\beta^2 - \theta_1\theta_2\beta + (\theta_2 + \theta_1\beta)(\theta_0 - \beta) + \beta^3 > 0 \). Hence, \( \frac{\partial q_0^{co}}{\partial a_0} > 0, \frac{\partial q_1^{co}}{\partial a_0} < 0, \) and \( \frac{\partial q_2^{co}}{\partial a_0} < 0, \) which means that \( q_0^{co} \) increases in \( a_0 \) but \( q_k^{co} \) decreases in \( a_0 \).

Similarly, to examine the effects of \( a_1 \) and \( a_2 \) on \( q_k^{co} \), we take the partial derivatives of Eqs. (5)–(8) with respect to \( a_1 \) and \( a_2 \), respectively. In addition, we analyse
the sensitivity of the optimal quantity $q_k^j$ to $a_k$ for the other patterns. The results are shown in Table 2.

### Table 2. Partial derivatives of optimal quantity with respect to $a_k$ in three patterns

| Model      | $a_0$ | $a_1$ | $a_2$ |
|------------|------|------|------|
| Cournot    | $a_0$ | $a_1$ | $a_2$ |
| $q^c_0$    | $-\frac{E}{2P}$ | $\frac{\beta(2\theta_2-\beta)}{2P}$ | $\frac{\beta(2\theta_1-\beta)}{2P}$ |
| $q^c_1$    | $-\frac{\beta(\beta-2\theta_2)}{2P}$ | $\frac{P(2\theta_2+D+E-\beta^2(2\theta_2-\beta)^2)}{2P}$ | $-P(\beta D+E-\beta^2(2\theta_2-\beta)(2\theta_2-\beta)D)$ |
| $q^c_2$    | $-\frac{\beta(\beta-2\theta_1)}{2P}$ | $-P(\beta D+E-\beta^2(2\theta_1-\beta)(2\theta_2-\beta)D)$ | $P(2\theta_1+D+E-\beta^2(2\theta_1-\beta)^2)D$ |
| Collusion  | $a_0$ | $a_1$ | $a_2$ |
| $q^{cn}_0$ | $\frac{g_1\theta_2-\beta^2}{Q}$ | $\frac{-\beta(\theta_1-\theta)}{2Q}$ | $-\frac{\beta(\theta_1-\theta)}{2Q}$ |
| $q^{cn}_1$ | $\frac{\beta(\beta-\theta_2)}{2Q}$ | $\frac{Q(2\theta_2+G+H)+2G\beta^2(\theta_2-\beta)^2}{4Q}$ | $-Q(2\beta G+H)+2G\beta^2(\theta_1-\beta)(\theta_2-\beta)$ |
| $q^{cn}_2$ | $\frac{\beta(\beta-\theta_1)}{2Q}$ | $-Q(2\beta G+H)+2G\beta^2(\theta_1-\beta)(\theta_2-\beta)$ | $Q(2\beta G+H)+2G\beta^2(\theta_2-\beta)^2$ |
| Stackelberg | $a_0$ | $a_1$ | $a_2$ |
| $q^{ct}_0$ | $\frac{2M\theta_2}{R}$ | $-\frac{\theta_2^2(2\theta_2-\beta)}{R}$ | $-\frac{\theta_2^2(2\theta_2-\beta)}{R}$ |
| $q^{ct}_1$ | $\frac{\theta_2^2(2\theta_2-\beta)}{R}$ | $\frac{R(\theta_2 N+2\theta_2 M)+\theta_2^2(2\theta_2-\beta)^2 N}{4RMN}$ | $R(4M\theta_2+\beta N)+\beta^2(2\theta_2-\beta)SN$ |
| $q^{ct}_2$ | $\frac{\beta(2\theta_2-\beta-E)}{4RMN}$ | $-R(4M\theta_2+\beta N)+\beta^2(2\theta_2-\beta)SN$ | $RNE+S^2(\beta^2 N-2M\theta_2)\theta_2$ |

where $P = \beta^2 D - \theta_0 E$, $Q = \theta_0 H - \beta^2 G$, $R = 4M\theta_2 - \beta^2 N$, $S = E - 2\theta_2\beta$.

From the results in Table 2, we can obtain several propositions.

**Proposition 5.4.** Regardless of the type of pattern that duopolistic retailers play:
1. The optimal quantity $q^c_0$ sold in the direct channel increases in $a_0$ but decreases in $a_1$ or $a_2$.
2. The optimal quantity $q^c_1$ ordered by retailer 1 increases in $a_1$ but decreases in $a_0$ or $a_2$.
3. The optimal quantity $q^c_2$ ordered by retailer 2 increases in $a_2$ but decreases in $a_0$ or $a_1$.

**Proof.** see Appendix.

**Proposition 5.5.** Regardless of the type of pattern that the retailers play, the profits of retailers 1 and 2, $\Pi_1^1$ and $\Pi_2^1$, respectively, decrease in $a_0$, and $\Pi_1^1$ increases in $a_1$. $\Pi_2^1$ increase in $a_2$.

**Proof.** see Appendix.

From the results in Section 4, the optimal retail price $p_0$ and the wholesale price $w$ have several notable results, which are discussed in Insight 5.4.

**Insight 5.4.** The optimal retail price in the supplier’s direct channel under different competitive behaviours of the two retailers is a constant, that is, $p^{co}_0 = p^{cn}_0 = p^{ct}_0 = \frac{a_0}{2}$, which is consistent with Insight 5.2. However, the wholesale price under different competitive patterns is equal if and only if $a_1 = a_2$.

Because of the complexity of the models, a numerical study is performed to demonstrate the proposed models and to obtain several insights. Suppose that $a_0 = 16$, $a_1 = 25$, $a_2 = 30$, $\theta_0 = 1$, $\theta_1 = 1$, $\theta_2 = 2$, $\beta = 0.5$, and $c = 2$. The optimal solutions for the three different competitive behaviours are presented in Table 3.
Table 3. The optimal solutions for three different competitive behaviors.

| Optimal | \( q_0 \) | \( q_1 \) | \( q_2 \) | \( w \) | \( p_0 \) | \( p_1 \) | \( p_2 \) | \( \Pi_0 \) | \( \Pi_1 \) | \( \Pi_2 \) |
|---------|---------|---------|---------|------|------|------|------|-------|------|-------|
| Cournot | 4.83   | 3.44   | 2.90   | 14.25 | 8.00 | 17.69 | 20.06 | 116.36 | 11.85 | 16.86 |
| Collusion | 5.20   | 2.81   | 2.60   | 14.13 | 8.00 | 18.24 | 20.74 | 108.01 | 11.57 | 17.23 |
| Stackelberg | 4.76   | 3.58   | 2.90   | 14.23 | 8.00 | 17.59 | 20.03 | 117.35 | 12.04 | 16.82 |

Based on the results in Table 3, the effects of \( a_k \) on \( \Pi_0 \) and \( \Pi_1 - \Pi_2 \) are discussed. \( a_k \) is set between 15 and 30. From Fig. 2(A) – (C), we can obtain the following two insights.

Insight 5.5. Regardless of the type of pattern that the retailers play, the profit of the supplier is monotonically increasing in \( a_k \). From a practical perspective, the supplier may obtain more profit in all three patterns by increasing the maximum retail price of the supplier or retailer.

Insight 5.6. With increasing \( a_k \), the profit relations \( \Pi_0^c > \Pi_0^t \) and \( \Pi_0^s > \Pi_0^c \) hold, that is, the Collusion game makes the supplier always obtain the smallest profit, which implies that the Collusion game is unacceptable from the supplier’s perspective. In each figure, the profit value in the Stackelberg game is very close to that in the Cournot game. When \( a_0 \) or \( a_2 \) increases, the profit relation \( \Pi_0^t > \Pi_0^c \) always holds, but the difference of \( \Pi_0^t \) and \( \Pi_0^c \) decreases if \( a_0 \) increases. From Fig. 2(B), the profit relation is \( \Pi_0^c > \Pi_0^t \), but when \( a_1 \) increases, the reverse case will occur.

The effect of \( a_k \) on the profit difference of two retailers is shown in Fig. 2(D) – (F). We can obtain Insight 5.7.

Insight 5.7. With increasing \( a_0 \), the profit value difference \( (\Pi_1 - \Pi_2) \) is not monotonically increasing; it first decreases and then increases, and it is always negative. Increasing \( a_1 \) makes the negative value become a positive value. The value of \( (\Pi_1 - \Pi_2) \) is monotonically decreasing in \( a_2 \), and the relation \( \Pi_1 > \Pi_2 \) becomes \( \Pi_1 < \Pi_2 \) with increasing \( a_2 \). Therefore, if a retailer wants to obtain more profit than his competitors, he can decrease his maximum retail price.

5.2.2. Effect of \( \theta_k \) on the optimal quantity and profit. First, the effects of \( \theta_0 \) on \( q_0 \), \( q_1 \), and \( q_2 \) are discussed. We can obtain Proposition 5.6.

Proposition 5.6. Regardless of the type of pattern that duopolistic retailers play, the optimal quantity \( q_0^* \) sold in the direct channel decreases in \( \theta_0 \) but \( q_i^* \) \((i = 1, 2)\) increases in \( \theta_0 \).

Proof. see Appendix.

Based on the results in Table 3, \( \theta_k \) is set between 1 and 5, and the effects of \( \theta_k \) on the optimal quantities and profit for the three different competitive behaviours are shown in Fig. 3. Several insights are obtained from Fig. 3.

Insight 5.8. Regardless of the type of behaviour that the retailers follow, if the self-price sensitivity factor \( \theta_k \) increases, the supplier’s profit will decrease. In other words, the supplier can achieve higher profit by holding down the self-price sensitivity factor \( \theta_0 \). Furthermore, increasing \( \theta_0 \) reduces the quantity sold in the direct channel, but increasing \( \theta_1 \) or \( \theta_2 \) increases the quantity sold in the direct channel.
Hence, $\theta_1$ and $\theta_2$ have inverse effects on the quantities and profit of the supplier. Therefore, when the market demand increases, the supplier can achieve higher order quantities in the direct channel by holding down the self-price sensitivity factor $\theta_0$. This can also increase the profit of the supplier under this condition.

**Insight 5.9.** As the self-price sensitivity factor $\theta_k$ increases, a greater quantity is sold in the direct channel when the retailers play the Collusion game; the relation $q^{cn}_0 > q^{co}_0 > q^{ct}_0$ is established, but the relation of the supplier’s profit is $\Pi^{ct}_0 > \Pi^{co}_0 > \Pi^{cn}_0$, and the supplier’s profit in the Cournot game or the Stackelberg game is much greater than that in the Collusion game. Therefore, the Collusion game makes the supplier obtain the largest quantity but charge the lowest price, while the Stackelberg game makes the supplier obtain the minimum quantity but charge the highest price, which is consistent with our Proposition 5.2.
Figs. 4 and 5 show the effects of $\theta_k$ on the quantities and profit of retailers in terms of the three different competitive behaviours. From Fig. 4, Insight 5.10 and Insight 5.11 can be obtained.

**Insight 5.10.** Regardless of the type of pattern that the retailers play, increasing $\theta_0$ or $\theta_2$ increases the profit of retailer 1, but increasing $\theta_1$ reduces the profit of retailer 1. Therefore, retailer 1 can achieve a higher profit by holding down the self-price sensitivity factor $\theta_1$ or holding up $\theta_0$ and $\theta_2$. Furthermore, increasing $\theta_0$ or $\theta_2$ increases the quantities ordered by retailer 1, but increasing $\theta_1$ decreases the quantities ordered by retailer 1. Therefore, when the market demand increases, retailer 1 can achieve higher order quantities by holding down the self-price sensitivity factor $\theta_1$ or holding up $\theta_0$ and $\theta_2$. 
Insight 5.11. The relation $q_1^{st} > q_1^{co} > q_1^{cn}$ is established; in other words, the Stackelberg game results in greater order quantities than the other two patterns for all values of $\theta_k$. When $\theta_1$ increases, the profit relation is $\Pi_1^{st} > \Pi_1^{co} > \Pi_1^{cn}$; thus, the Stackelberg game results in higher profit for retailer 1 than the other two patterns. In other words, the Stackelberg game is acceptable from the perspective of retailer 1. When $\theta_0$ or $\theta_2$ increases, the profit relation changes to $\Pi_1^{cn} > \Pi_1^{st} > \Pi_1^{co}$. Therefore, the Stackelberg game makes retailer 1 obtain the largest quantity, but he can obtain the largest profit only before certain fixed point, while the Collusion game makes retailer 1 obtain the largest profit after this fixed point.

According to Fig. 5, Insight 5.12 and Insight 5.13 can be obtained.

Insight 5.12. Regardless of the type of pattern that the retailers play, increasing $\theta_0$ increases the profit of retailer 2, while increasing $\theta_2$ reduces the profit of retailer 2,
and increasing $\theta_1$ first increases and then decreases the profit of retailer 2. Retailer 2 can obtain a higher profit by holding up the self-price sensitivity factor $\theta_0$, holding down $\theta_2$, or keeping $\theta_1$ at its best value, which can stimulate market demand and cause an increase in the order quantities of retailer 2. Therefore, when the market demand increases, retailer 2 can achieve higher order quantities by holding down the self-price sensitivity factor $\theta_2$; it can also increase the profit of retailer 2 under this condition.

**Insight 5.13.** When $\theta_0$ or $\theta_2$ increases, the quantity relation $q_2^c > q_2^t > q_2^n$ is established, but when $\theta_1$ increases, the quantity relation will change to $q_2^t > q_2^c > q_2^n$. Note that when $\theta_k$ increases, the quantity difference between $q_2^c$ and $q_2^t$ decreases, which implies that the market order quantity of the Cournot game for retailer 2 is fairly close to that of the Stackelberg game. When $\theta_0$ increases, the
profit relation $\Pi_{2}^{cn} > \Pi_{2}^{co} > \Pi_{2}^{st}$ is established, but when $\theta_{i}$ increases, the profit relation will change to $\Pi_{2}^{cn} > \Pi_{2}^{st} > \Pi_{2}^{co}$. Therefore, when the self-price sensitivity factor $\theta_{i}$ is greater than some fixed point, the retailer prefers to play the Cournot game.

To compare the profits of the two retailers for the three different competitive behaviours, Figs. 6(A) – (C) show the difference between the profits of the two retailers ($\Pi_{1} - \Pi_{2}$).

**Insight 5.14.** Regardless of the type of pattern that the retailers play, with increasing $\theta_{k}$, retailer 1 initially obtains less profit than retailer 2. However, with increasing $\theta_{0}$, the profit difference between retailer 1 and retailer 2 decreases. With increasing $\theta_{1}$, the profit difference between retailer 1 and retailer 2 increases. When $\theta_{2}$ increases, the profit difference between retailer 1 and retailer 2 increases sharply, and the relation ($\Pi_{1}^{c} < \Pi_{2}^{c}$) changes to ($\Pi_{1}^{c} > \Pi_{2}^{c}$). Therefore, retailers 1 and 2 can obtain higher profits in competition by decreasing their price sensitivity.

**5.2.3. Effect of $\beta$ on the optimal quantity and profit.** When $\beta$ changes from 0 to 1, the effect of $\beta$ on the optimal quantity and profit for the three competitive behaviours is as shown in Fig. 7, from which several insights can be obtained.

**Insight 5.15.** Regardless of the type of pattern that the retailers play, if the cross-sensitivity factor $\beta$ increases, the quantity $q_{0}$ and the profit $\Pi_{0}$ will decrease. When the cross-sensitivity factor $\beta$ increases, the quantities $q_{1}$ and $q_{2}$ ordered by retailer 1 and retailer 2, respectively, will decrease when retailers play the Cournot game and
Figure 7. Effect of $\beta$ on $q_k$ or $\Pi_k$ in three patterns

the Collusion game, and $q_1$ and $q_2$ first decrease and then increase when retailers play the Stackelberg game. When $\beta$ increases, the variation trends of $\Pi_i$ are similar to $q_i$; that is, if $\beta$ increases, $\Pi_i$ will decrease when retailers play the Cournot game or the Collusion game, while $\Pi_i$ first decreases and then increases when retailers play the Stackelberg game. Therefore, to obtain a higher profit, the supplier and retailers prefer to sell more goods in direct and indirect channels, respectively, which is consistent with our intuition.

Insight 5.16. As shown in Fig. (7), the quantity $q_0$ is at its minimum when the retailers play the Stackelberg game, and $q_0$ is at its maximum when they play the Collusion game. However, $\Pi_0$ is at its maximum when the retailers play the Stackelberg game, and $q_0$ is at its minimum when they play the Collusion game. Therefore, the relations $q_{0c} > q_{00} > q_{0s}$ and $\Pi_{0s} > \Pi_{00} > \Pi_{0c}$ hold, which is
consistent with our Proposition 5.2. Based on the range of changes in the cross-sensitivity factor $\beta$, the quantity relation $q_{1t}^{st} > q_{1t}^{co} > q_{1t}^{cn}$ holds, and when $\beta$ changes from 0.3 to 1, the profit relation $\Pi_{1t}^{st} > \Pi_{1t}^{co} > \Pi_{1t}^{cn}$ is established. $q_2$ is minimized and $\Pi_2$ is maximized when the retailers play the Collusion game. Furthermore, Fig. 6(D) shows that the profit of retailer 1 is always lower than that of retailer 2. Therefore, based on the range of changes in the cross-sensitivity factor $\beta$, regardless of the type of pattern that the retailers play, the profit $\Pi_0$ of the supplier is the maximum and the profit $\Pi_1$ of retailer 1 is the minimum for the three patterns with the same $\beta$. Additionally, when the duopolistic retailers play the Stackelberg game, the more powerful leader obtains less profit than the follower. This finding is in contrast to intuitive expectations. However, as $\beta$ increases, the leader will obtain more profit than the follower, which is consistent with our Proposition 5.2.

6. Conclusions. This paper considered a two-echelon and dual-channel supply chain with a single supplier and two retailers. The retailers have three competitive behaviours: Cournot, Collusion and Stackelberg. Three models are constructed, and optimal solutions for the three different types of competitive behaviours are proposed. Two scenarios, a special case and a general case, are presented to compare the optimal solutions and to analyse the effects of certain parameters on the optimal solutions for the three different competitive behaviours.

In the special case, our investigations revealed several insights: (1) When the supplier acts as a wholesale supplier or when the dual-channel case occurs, among the duopolistic retailers’ three patterns, the Stackelberg game allows the supplier to earn the highest profit, and the Collusion game makes the supplier earn the lowest profit. (2) When the supplier acts as a wholesale supplier, the Collusion game allows the retailers to achieve the highest profit. When the dual-channel case occurs, the Cournot game allows the retailers to obtain more profit than the Stackelberg follower (i.e., retailer 2). When the price sensitivity of demand at the supplier with respect to its own quantity is much larger than the cross-sensitivity of demand, the Collusion game allows the retailers to earn more profit than does the Cournot game. However, regardless of the supply chain structure employed, the Stackelberg leader (i.e., retailer 1) obtains more profit than the follower, that is, the leader can gain a first-mover advantage, which is consistent with our intuition. (3) When the supplier acts as a wholesale supplier, the supplier’s profit is greater than the total profit of the two retailers. In the general case, managerial insights from our study can be summarized as follows: (1) The supplier can achieve higher profit by raising the maximum retail price at the supplier or retailer or holding down the self-price sensitivity factor. (2) Regardless of which parameter changes, retailers prefer to sell more goods to obtain a higher profit. (3) When the cross-sensitivity of demand or the maximum retail price at the supplier or retailer changes, in the Collusion game, the order quantities of the retailer are at their minimum.

This paper proposed Equilibrium quantities for the three behaviours (Cournot, Collusion and Stackelberg) between two retailers when the retailers are asymmetric in demand characteristics, and analyzed the effect of these behaviours on optimal solutions. The insights are helpful for retailers to make reasonable decision on competitive strategies when they face with competitors in market. This study has the following two limitations that should be addressed in future research. First, the demand function is a linear function related to the price and is quite different from the actual market demand. One direction for further research would be to
add demand functions as uncertain variables, which could be considered as having probability functions following fuzzy set theory. Second, in reality, many other factors, such as product quality, service level, logistics response time, and advertising promotion efforts, affect supply chain decisions. Integrating multiple factors into the model for comprehensive decision making and analysis is another direction for future research.

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Appendix.

Proof of the Proposition 4.1.

Proof. Taking the second-order partial derivatives of $\Pi_0^co$ with respect to $q_0$ and $q$, we obtain

$$\frac{\partial^2 \Pi_0^{co}}{\partial q_0^2} = -2q_0, \quad \frac{\partial^2 \Pi_0^{co}}{\partial q^2} = -2\frac{E}{D} \quad \text{and} \quad \frac{\partial^2 \Pi_0^{co}}{\partial q_0 \partial q} = \frac{\partial^2 \Pi_0^{co}}{\partial q \partial q_0} = -2\beta.$$

Let $\Delta_1^{co}$ and $\Delta_2^{co}$ denote the first and second-order principal minors of Hessian matrix of $\Pi_0^co$, then

$$\Delta_1^{co} = -2q_0 < 0,$$

$$\Delta_2^{co} = 4\frac{\theta_0 E}{D} - 4\beta^2 = 4\frac{\theta_0 E - D\beta^2}{D} \bigg(4\theta_0 E - 2\beta^2 + 2\theta_2 \beta_1 + 2\beta^3 \bigg) / D$$

$$= 4\left[\left(3\theta_0^2 \theta_1 \theta_2 - 2\theta_0 \beta^2 - 2\theta_0 \theta_1 \beta + 2\beta^3 \bigg) / D\right. \right.$$

$$\left. + \theta_0 \theta_1 \theta_2 - 2\theta_0 \beta^2 + 2\theta_1 \beta^2 + 3\theta_0 \theta_1 \beta \bigg) / D\right]\bigg) / D$$

$$> 4\left[\left(3\theta_0^2 \theta_1 \theta_2 - 2\theta_0 \beta^2 - 2\theta_1 \beta^2 + 2\theta_2 \beta + 3\theta_0 \theta_1 \beta \bigg) / D\right. \right.$$

$$\left. + \theta_0 \theta_1 \theta_2 - 2\theta_0 \beta^2 + 2\theta_1 \beta^2 - 2\theta_0 \theta_1 \beta + 3\theta_0 \theta_1 \beta \bigg) / D\right]\bigg) / D$$

$$= 4\left[\left(3\theta_0^2 \theta_1 \theta_2 - 2\theta_0 \beta^2 - 2\theta_0 \theta_1 \beta + 2\beta^3 + (\theta_0 - \beta)(\theta_1 - \beta)(\theta_2 + \beta) \bigg) / D\right. \right.$$

$$\left. + \theta_0 \theta_1 \theta_2 - 2\theta_0 \beta^2 + 2\theta_0 \theta_1 \beta - 3\theta_0 \theta_1 \beta \bigg) / D\right]\bigg) / D$$

$$> 0.$$

The first and second inequality hold because $0 < \beta < \theta_i (i = 0, 1, 2)$, hence the Hessian matrix of $\Pi_0^co$ is negative definite, which implies that $\Pi_0^co$ is a concave function with respect to $q_0$ and $q$. \qed

Proof of the Proposition 4.2.

Proof. Taking the second-order partial derivatives of $\Pi_r$ with respect to $q_1$ and $q_2$, we obtain

$$\frac{\partial^2 \Pi_r}{\partial q_1^2} = -2q_1, \quad \frac{\partial^2 \Pi_r}{\partial q_2^2} = -2q_2 \quad \text{and} \quad \frac{\partial^2 \Pi_r}{\partial q_1 \partial q_2} = \frac{\partial^2 \Pi_r}{\partial q_2 \partial q_1} = -2\beta.$$

Let $\Delta_1$ and $\Delta_2$ denote the first and second-order principal minors of Hessian matrix of $\Pi_r$, then $\Delta_1 = -2q_1 < 0$, $\Delta_2 = 4\theta_1 \theta_2 - 4\beta^2 > 0$.

Hence the Hessian matrix of the total profit of the retailers is negative definite, which implies that $\Pi_r$ is a concave function with respect to $q_1$ and $q_2$. \qed
Proof of the Proposition 4.3.

Proof. Similar to the proof of Proposition 4.1, the second-order partial derivatives of $\Pi_0^{cn}$ with respect to $q_0$ and $q$ are

$$\frac{\partial^2 \Pi_0^{cn}}{\partial q_0^2} = -2\theta_0, \quad \frac{\partial^2 \Pi_0^{cn}}{\partial q^2} = -\frac{2H}{G}, \quad \text{and} \quad \frac{\partial^2 \Pi_0^{cn}}{\partial q_0 \partial q} = \frac{\partial^2 \Pi_0^{cn}}{\partial q \partial q_0} = -2\beta,$$

Let $\Delta_1^{cn}$ and $\Delta_2^{cn}$ denote the first- and second-order principal minors of the Hessian matrix of $\Pi_0^{cn}$, then

$$\Delta_1^{cn} = -2\theta_0 < 0,$$

$$\Delta_2^{cn} = 4 \frac{\theta_0 H}{G} - 4\beta^2 = \frac{4}{G} \left[ \theta_0 (2\theta_1 \theta_2 - 2\beta^2) - (\theta_1 + \theta_2 - 2\beta) \beta^2 \right]$$

$$= \frac{4}{G} \left[ \left( \theta_0 \theta_1 \theta_2 - 2\theta_2 \beta^2 - \theta_1 \theta_1 \beta + \beta^3 \right) \left( \theta_0 \theta_1 \theta_2 - 2\theta_1 \beta^2 + \theta_0 \theta_2 \beta - \theta_1 \beta^2 + \beta^3 \right) \right]$$

$$\geq \frac{4}{G} \left[ \left( \theta_0 \theta_1 \theta_2 - 2\theta_2 \beta^2 - \theta_2 \theta_1 \beta + \beta^3 \right) \left( \theta_0 \theta_1 \theta_2 - 2\theta_1 \beta^2 + \theta_0 \theta_2 \beta - \theta_1 \beta^2 + \beta^3 \right) \right]$$

$$= \frac{4}{G} \left[ (\theta_2 - \beta)(\theta_0 \theta_1 - \beta^2) + (\theta_0 - \beta)(\theta_1 - \beta)(\theta_2 + \beta) \right] > 0.$$

The first and second inequality hold because $0 < \beta < \theta_i (i = 0, 1, 2)$, hence the Hessian matrix of $\Pi_0^{cn}$ is negative definite, which implies that $\Pi_0^{cn}$ is a concave function with respect to $q_0$ and $q$.

Proof of the Proposition 4.4.

Proof. Similar to the proof of Proposition 4.1, the second-order partial derivatives of $\Pi_0^{ct}$ with respect to $q_0$ and $q$ are

$$\frac{\partial^2 \Pi_0^{ct}}{\partial q_0^2} = -2\theta_0, \quad \frac{\partial^2 \Pi_0^{ct}}{\partial q^2} = -\frac{8\theta_2 M}{N}, \quad \text{and} \quad \frac{\partial^2 \Pi_0^{ct}}{\partial q_0 \partial q} = \frac{\partial^2 \Pi_0^{ct}}{\partial q \partial q_0} = -2\beta,$$

Let $\Delta_1^{ct}$ and $\Delta_2^{ct}$ denote the first- and second-order principal minors of Hessian matrix of $\Pi_0^{ct}$; then,

$$\Delta_1^{ct} = -2\theta_0 < 0,$$

$$\Delta_2^{ct} = \frac{16\theta_0 \theta_2 M}{N} - 4\beta^2$$

$$= 16 \left( 2\theta_0 \theta_1 \theta_2 - \theta_0 \theta_2 \beta^2 - \theta_2 \beta^2 + \theta_2 \beta^2 - \theta_1 \beta^2 + \frac{1}{4} \beta^4 \right) / N$$

$$> 16 \beta \left( 2\theta_0 \theta_1 \theta_2 - \theta_0 \theta_2 \beta - \theta_2 \beta^2 - \theta_1 \beta^2 + \frac{1}{4} \beta^3 \right) / N$$

$$= 16 \beta \left( (\theta_0 - \beta)(\theta_1 - \beta)(\theta_2 + \beta) + (\theta_2 - \beta)(\theta_0 \theta_1 - \theta_0 \beta) + \theta_1 \beta^2 - \frac{3}{4} \beta^3 \right) / N.$$

The inequality holds because $0 < \beta < \theta_2$. Since $0 < \beta < \theta_i (i = 0, 1, 2)$, then $\Delta_2^{ct} > 0$, hence the Hessian matrix of $\Pi_0^{ct}$ is negative definite, which implies that $\Pi_0^{ct}$ is a concave function with respect to $q_0$ and $q$.\qed
Proof of the Proposition 5.1.

Proof. From Table 1, when the supplier acts as a wholesale supplier,

(1) For the supplier, since $0 < \beta < \theta$, hence $\beta - 2\theta < 0$ and $8\theta^2 - \beta^2 - 4\theta\beta < 0$, then

$$
\frac{1}{\Pi^i_0} - \frac{1}{\Pi^co_0} = \frac{2\beta^2(\beta - 2\theta)}{(a - c)^2(8\theta^2 - \beta^2 - 4\theta\beta)} < 0,
$$

$$
\frac{1}{\Pi^co_0} - \frac{1}{\Pi^cn_0} = \frac{-2\beta}{(a - c)^2} < 0.
$$

Combining with the above two equations, we obtain $1 > \frac{\Pi^i_0}{\Pi^co_0} > \frac{\Pi^co_0}{\Pi^cn_0}$, therefore $\Pi^i_0 > \Pi^co_0 > \Pi^cn_0$.

(2) For the retailers, since $0 < \beta < \theta$, hence $2\theta^2 - \beta^2 < 0$ and $2\theta^2 - \theta\beta - \beta^2 < 0$, then

$$
q^{st}_1 - q^{co}_1 = \frac{(a - c)(a - c)^2}{4(2\theta^2 - \beta^2)(2\theta + \beta)} > 0,
$$

$$
q^{co}_1 - q^{st}_2 = \frac{(a - c)(a - c)^3}{8\theta(2\theta^2 - \beta^2)(2\theta + \beta)} > 0,
$$

$$
q^{st}_2 - q^{st}_1 = \frac{\beta(a - c)(2\theta^2 - \theta\beta - \beta^2)}{8\theta(2\theta^2 - \beta^2)(\theta + \beta)} > 0.
$$

Combining with the above three equations, we obtain $q^{st}_1 > q^{co}_1 = q^{co}_2 > q^{st}_2 > q^{st}_1 = q^{st}_2$.

$$
p^{st}_1 - p^{co}_1 = \frac{\beta(a - c)}{4(2\theta + \beta)} > 0,
$$

$$
p^{co}_1 - p^{st}_2 = \frac{(a - c)(a - c)^3}{8\theta(2\theta + \beta)(2\theta^2 - \beta^2)} > 0,
$$

$$
p^{st}_2 - p^{st}_1 = \frac{\beta^2(a - c)(\theta - \beta)}{8\theta(2\theta^2 - \beta^2)} > 0.
$$

Combining with the above three equations, we obtain $p^{st}_2 = p^{st}_2 > p^{co}_2 = p^{co}_2 > p^{st}_2 > p^{st}_1$.

Since $0 < \beta < \theta$, hence $16\theta^3 - 8\theta\beta^2 - \beta^3$, then

$$
\Pi^{st}_i - \Pi^{st}_1 = \frac{(\theta - \beta)(a - c)^2}{32\theta(2\theta^2 - \beta^2)(\theta + \beta)} > 0,
$$

$$
\Pi^{st}_1 - \Pi^{co}_1 = \frac{\beta^4(a - c)^2}{32\theta(2\theta^2 - \beta^2)(2\theta + \beta)^2} > 0,
$$

$$
\Pi^{co}_1 - \Pi^{st}_2 = \frac{\beta^3(a - c)^2(16\theta^3 - 8\theta\beta^2 - \beta^3)}{64\theta(2\theta^2 - \beta^2)^2(2\theta + \beta)^2} > 0.
$$

Combining with the above three equations, we obtain $\Pi^{st}_i = \Pi^{st}_2 > \Pi^{st}_1 > \Pi^{co}_1 = \Pi^{co}_2 > \Pi^{st}_2$.

Proof of the Proposition 5.2.

Proof. When the dual-channel case occurs, from Table 1,
(1) For the supplier,
\[ q^c_0 - q^c_1 = \frac{\beta^2[\theta_0(a - c) - a_0\beta]}{2[\beta^2 - \theta_0(2\theta + \beta)][\theta_0(\theta + \beta) - \beta^2]}, \]
\[ q^t_0 - q^c_0 = \frac{\beta^3(2\theta - \beta)[a_0\beta - \theta_0(a - c)]}{2[\beta^2(8\theta^2 - 4\theta\beta - \beta^2) - 4\theta_0\theta(2\theta^2 - \beta^2)]}, \]
Since \( q^c_0 > 0 \) and \( 2\beta^2 - \theta_0(2\theta + \beta) < 0 \), then \( a_0\beta - \theta_0(a - c) < 0 \), furthermore, since \( q^t_0 > 0 \), then \( \beta^2(8\theta^2 - 4\theta\beta - \beta^2) - 4\theta_0\theta(2\theta^2 - \beta^2) < 0 \), hence \( q^c_0 > q^c_1 > q^t_0 \).

Similarly,
\[ \Pi^c_0 - \Pi^c_1 = \frac{-\beta[a_0\beta - \theta_0(a - c)]}{[\beta^2 - \theta_0(2\theta + \beta)][\theta_0(\theta + \beta) - \beta^2]} > 0, \]
\[ \Pi^t_0 - \Pi^c_0 = \frac{\beta^2(2\theta - \beta)[a_0\beta - \theta_0(a - c)]^2}{4[\beta^2(8\theta^2 - 4\theta\beta - \beta^2) - 4\theta_0\theta(2\theta^2 - \beta^2)][\beta^2 - \theta_0(2\theta + \beta)]} > 0. \]

Combining with the above two equations, we obtain \( \Pi^t_1 > \Pi^c_0 > \Pi^c_1 \).

(2) For the retailers,
\[ \frac{1}{q^t_i} - \frac{1}{q^c_i} = \frac{\beta^2(2\theta_0\theta - \beta^2)}{\theta(\theta + \beta)}[a_0\beta - \theta_0(a - c)], \]
\[ \frac{1}{q^c_i} - \frac{1}{q^c_t} = \frac{2\beta^3(\theta_0 - \theta)}{(4\theta^2 - \beta^2 - 2\theta\beta)[a_0\beta - \theta_0(a - c)]}, \]
\[ \frac{1}{q^t_2} - \frac{1}{q^c_2} = \frac{4\theta_0\beta(2\theta^2 - \beta^2 - \theta\beta) + 2\beta^4}{(4\theta^2 - \beta^2 - 2\theta\beta)[a_0\beta - \theta_0(a - c)]}. \]
Since \( q^c_0 > 0 \) and \( 2\beta^2 - \theta_0(2\theta + \beta) < 0 \), then \( a_0\beta - \theta_0(a - c) < 0 \), hence
\[ \frac{1}{q^t_i} - \frac{1}{q^c_i} < 0, \quad \frac{1}{q^c_i} - \frac{1}{q^c_t} < 0, \quad \frac{1}{q^t_2} - \frac{1}{q^c_2} < 0. \]

Hence \( \frac{1}{q^t_i} < \frac{1}{q^c_i} < \frac{1}{q^c_t} < \frac{1}{q^c_2} \), therefore \( q_i^t > q_i^c > q_2^c > q_1^c = q_2^i > q_2^t > q_1^t = q_2^c \).

Similarly, Let \( \bar{p}_i^t = p_i^t - \frac{\theta + c}{2} \) (i = 1, 2, j = co, cn, st), we can get the following results.
\[ \frac{1}{\bar{p}_i^c} - \frac{1}{\bar{p}_i^c} = \frac{2\beta(2\beta^2 - \theta_0\theta - \theta_0\beta)}{\theta(\theta + \beta)[a_0\beta - \theta_0(a - c)]}, \]
\[ \frac{1}{\bar{p}_2^c} - \frac{1}{\bar{p}_1^c} = \frac{2\beta^3(\beta - \theta)}{(4\theta^2 - \beta^2 - 2\theta\beta)[a_0\beta - \theta_0(a - c)]}, \]
\[ p_i^t - p_2^t = \frac{\beta^2(2\theta - \beta)[a_0\beta - \theta_0(a - c)]}{4[\beta^2(8\theta^2 - 4\theta\beta - \beta^2) - 4\theta_0\theta(2\theta^2 - \beta^2)]}. \]
Since \( 0 < \beta < \theta \), we also got \( a_0\beta - \theta_0(a - c) < 0 \) and \( \beta^2(8\theta^2 - 4\theta\beta - \beta^2) - 4\theta_0\theta(2\theta^2 - \beta^2) < 0 \), hence
\[ \frac{1}{\bar{p}_i^c} < \frac{1}{\bar{p}_2^c} < \frac{1}{\bar{p}_2^t} \] and \( p_1^t < p_2^t \), therefore \( p_1^t = p_2^c > p_2^c > p_2^t > p_1^t \).

Similarly,
\[ \Pi^t_1 - \Pi^t_2 = \frac{(4\theta^2 - 3\theta\beta^2)[a_0\beta - \theta_0(a - c)]^2}{4[\beta^2(8\theta^2 - 4\theta\beta - \beta^2) - 4\theta_0\theta(2\theta^2 - \beta^2)]^2}, \]
\[
\frac{1}{\sqrt{\Pi_2^c}} - \frac{1}{\sqrt{\Pi_1^c}} = \frac{2\beta^3(\beta - \theta_0)}{\sqrt{\theta(a_0\beta - \theta_0(a - c))(4\theta^2 - 2\theta\beta - \beta^2)}}.
\]

Hence \(\Pi_1^t - \Pi_2^t > 0\) and \(\frac{1}{\sqrt{\Pi_2^c}} - \frac{1}{\sqrt{\Pi_1^c}} > 0\), therefore \(\Pi_1^t > \Pi_2^t\) and \(\Pi_1^c = \Pi_2^c > \Pi_2^t\).

\[
\frac{1}{\Pi_1^c} - \frac{1}{\Pi_2^c} = 4\beta^2 \left[\theta_0(\theta + \beta)(\theta_0 - 4\beta) + 4\beta^3\right] - \frac{\theta(\theta + \beta)(a_0\beta - \theta_0(a - c))^2}{\theta^2(\theta - \beta)^2}.
\]

When \(\theta_0 > 4\beta\), \(\frac{1}{\Pi_1^c} - \frac{1}{\Pi_2^c} > 0\), hence \(\Pi_1^c = \Pi_2^c > \Pi_1^c = \Pi_2^c\).

**Proof of the Proposition 5.3.**

*Proof.* From Table 1, since \(0 < \beta < \theta\), we have

\[
\frac{1}{\Pi_0^c} - \frac{1}{\Pi_1^c + \Pi_2^c} = \frac{-2(2\theta^2 + \beta^2 + 3\theta\beta)}{\theta(a - c)^2} < 0,
\]

\[
\Pi_0^c - (\Pi_1^c + \Pi_2^c) = \frac{(a - c)^2}{\theta(\theta + \beta)} > 0,
\]

\[
\Pi_0^c - (\Pi_1^t + \Pi_2^t) = \frac{(a - c)^2[32\theta^2(\theta^2 - \beta^2) + \beta^3(5\beta + 4\theta)]}{64\theta(2\theta^2 - \beta^2)^2} > 0.
\]

Combining the above three equations, we obtain

\[
\Pi_0^c > (\Pi_1^c + \Pi_2^c),
\]

\[
\Pi_0^c > (\Pi_1^c + \Pi_2^c),
\]

\[
\Pi_0^c > (\Pi_1^t + \Pi_2^t).
\]

Therefore, the proof of Proposition 5.3 is completed. \(\square\)

**Proof of the Proposition 5.4.**

*Proof.* In the Cournot model, since \(0 < \beta < \theta_i (i = 1, 2)\), so \(E > 0\). From the proof process of Proposition 4.1, we got \(\beta^2D - \theta_0E < 0\), hence

\[
\frac{\partial q_0^c}{\partial a_0} > 0, \frac{\partial q_1^c}{\partial a_0} < 0, \frac{\partial q_2^c}{\partial a_0} < 0.
\]

Since \(0 < \beta < \theta_i (i = 1, 2)\) and \(\beta^2D - \theta_0E < 0\), then \((\beta^2D - \theta_0E)(2\theta_2D + E) - \beta^2(2\theta_2 - \beta)^2D < 0\), and

\[
- (\beta^2D - \theta_0E)(\beta D + E) - \beta^2(2\theta_1 - \beta)(2\theta_2 - \beta)D
\]

\[
= - (\beta^2D - \theta_0E)(\beta D + E) - \beta^2(E - \beta D)D
\]

\[
= - 2\beta^2DE + \theta_0E\beta + \theta_0E^2
\]

\[
= \beta DE(\theta - \beta) + E(\theta_0E - \beta^2D)
\]

\[
> 0.
\]

Hence

\[
\frac{\partial q_0^c}{\partial a_1} < 0, \frac{\partial q_0^c}{\partial a_2} < 0, \frac{\partial q_1^c}{\partial a_1} > 0, \frac{\partial q_1^c}{\partial a_2} < 0, \frac{\partial q_2^c}{\partial a_1} < 0, \frac{\partial q_2^c}{\partial a_2} > 0.
\]
In the Collusion model, since $0 < \beta < \theta_i (i = 1, 2)$ and $\theta_0 H - \beta^2 G > 0$ from the proof process of Proposition 4.3, hence

$$\frac{\partial q_i}{\partial a_0} > 0, \frac{\partial q_i}{\partial a_1} < 0, \frac{\partial q_i}{\partial a_2} < 0.$$  

Since $0 < \beta < \theta_i (i = 1, 2)$ and $\theta_0 H - \beta^2 G > 0$, then $(\theta_0 H - \beta^2 G)(2\theta_2 G + H) + 2\beta G(\theta_2 - \beta)^2 > 0$, and

$$(\theta_0 H - \beta^2 G)(2\beta G + H) + 2\beta G(\theta_2 - \beta)$$

$$= -[(\theta_0 H - \beta^2 G)(2\beta G + H) + G\beta^2(\theta_2 - \beta)] < 0.$$

Hence

$$\frac{\partial q_i}{\partial a_0} < 0, \frac{\partial q_i}{\partial a_1} > 0, \frac{\partial q_i}{\partial a_2} < 0.$$  

In the Stackelberg model, since $0 < \beta < \theta_i (i = 1, 2)$ and $4\theta_0 \theta_2 M - \beta^N > 0$ from the proof process of Proposition 4.4, hence

$$\frac{\partial q_i}{\partial a_0} > 0, \frac{\partial q_i}{\partial a_1} < 0, \frac{\partial q_i}{\partial a_2} < 0.$$  

Since $0 < \beta < \theta_i (i = 1, 2)$ and $4\theta_0 \theta_2 M - \beta^N > 0$, then $(4M\theta_0 \theta_2 - \beta^2 N)(\theta_2 N + 2\theta_2 M) + \theta_2^2(2\theta_2 - \beta)^2 N > 0$, and

$$- (4\theta_0 \theta_2 M - \beta^2 N)(4\theta_2 M + \beta N) + \beta^2(2\theta_2 - \beta)(E - 2\theta_2 \beta)N$$

$$= -4\theta_0 \theta_2 M(4\theta_2 M + \beta N) + \beta^2 N(4\theta_2 M + 8\theta_1 \theta_2^2 - 4\theta_2^2 - 4\theta_1 \theta_2 \beta + \beta^3)$$

$$= -4\theta_0 \theta_2 M(4\theta_2 M + \beta N) + \beta^2 N(4\theta_2 M + 8\theta_1 \theta_2^2 - 4\theta_2^2)$$

$$= -4\theta_0 \theta_2 M(4\theta_2 M + \beta N) + \beta_2^2 MN$$

$$= 4\theta_2 M(-4\theta_0 \theta_2 M + \beta^2 N) + 4\theta_2 \beta MN(\beta - \theta_0)$$

< 0.

Hence

$$\frac{\partial q_i}{\partial a_0} < 0, \frac{\partial q_i}{\partial a_1} < 0, \frac{\partial q_i}{\partial a_2} < 0.$$  

Therefore, the proof of Proposition 5.4 is completed.

**Proof of the Proposition 5.5.**

**Proof.** We first take the partial derivative of $p_0, p_1, p_2$, with respect to $a_0$. Since $0 < \beta < \theta_i (i = 1, 2)$, $\beta^2 D - \theta_0 E < 0$, $\theta_0 H - \beta^2 G > 0$ and $4\theta_0 \theta_2 M - \beta^2 N > 0$, then

$$\frac{\partial p_0}{\partial a_0} = \frac{\beta(2\theta_1 \theta_2 - \theta_1 \beta)}{2(\beta^2 D - \theta_0 E)} < 0, \quad \frac{\partial p_1}{\partial a_0} = \frac{\beta(2\theta_1 \theta_2 - \theta_2 \beta)}{2(\beta^2 D - \theta_0 E)} < 0,$$

$$\frac{\partial p_2}{\partial a_0} = \frac{\beta(\beta^2 - 2\theta_1 \theta_2)}{2(\theta_0 H - \beta^2 G)} < 0, \quad \frac{\partial p_3}{\partial a_0} = \frac{\beta(\beta^2 - 2\theta_0 \theta_2)}{2(\theta_0 H - \beta^2 G)} < 0,$$
\[
\begin{align*}
\frac{\partial p_i^t}{\partial a_i} & = \frac{\beta(2\theta_i\theta_2\beta - 4\theta_1\theta_2^2 + 2\theta_2\beta^2 - \beta^3)}{2(4\theta_0\theta_2 M - \beta^2 N)} < 0, \\
\frac{\partial p_i^t}{\partial a_0} & = \frac{\beta(2\theta_i^2\beta - 4\theta_1\theta_2^2 + \theta_2\beta^2)}{2(4\theta_0\theta_2 M - \beta^2 N)} < 0,
\end{align*}
\]

From Proposition 5.4, \( \frac{\partial q_i^j}{\partial a_0} < 0 \) (i = 1, 2, j = co, cn, st), hence

\[
\frac{\partial \Pi_i^j}{\partial a_0} = \frac{\partial p_i^j}{\partial a_0} q_i^j + (p_i^j - w) \frac{\partial q_i^j}{\partial a_0} < 0, \quad i = 1, 2, j = co, cn, st.
\]

Therefore, \( \Pi_i^1 \) and \( \Pi_i^2 \), respectively, decrease in \( a_0 \). Similarly,

\[
\begin{align*}
\frac{\partial p_i^t}{\partial a_1} &= \frac{(D - \theta_1)E + 2\theta_1\theta_2 D + \beta E}{2DE} - \frac{\beta^2 \theta_1 (2\theta_2 - \beta)^2}{2E(\beta^2 D - \theta_0 E)}, \quad \frac{\partial w^c_i}{\partial a_1} = \frac{2\theta_2 - \beta}{2D}, \\
\frac{\partial p_i^t}{\partial a_0} &= \frac{3G - \theta_1 + \beta}{4G} + \frac{\beta^2 (\theta_2 - \beta)}{4(\theta_0 H - \beta^2 G)}, \quad \frac{\partial w^c_i}{\partial a_0} = \frac{\theta_2 - \beta}{2G}, \\
\frac{\partial p_i^t}{\partial a_1} &= \frac{12\theta_2^2 - 8\theta_2 \beta + 8\theta_1 \theta_2 - 3\beta^2}{4N} + \frac{\beta^2 (2\theta_2 - \beta)^2}{4(4\theta_0\theta_2 M - \beta^2 N)} + \frac{\beta^2 (2\theta_2 - \beta)^2}{4(4\theta_0\theta_2 M - \beta^2 N)} > 0.
\end{align*}
\]

Since \( 0 < \beta < \theta_1 (i = 1, 2) \) and \( \beta^2 D - \theta_0 E < 0, \theta_0 H - \beta^2 G > 0, 4\theta_0\theta_2 M - \beta^2 N > 0 \), then

\[
\begin{align*}
\frac{\partial p_i^t}{\partial a_1} - \frac{\partial w^c_i}{\partial a_1} &= \frac{(D - 2\theta_1)E + \theta_1 (2\theta_2 - \beta)^2 + 2\beta E}{2DE} + \frac{\beta^2 \theta_1 (2\theta_2 - \beta)^2}{2E(\beta^2 D - \theta_0 E)} > 0, \\
\frac{\partial p_i^t}{\partial a_1} - \frac{\partial w^c_i}{\partial a_1} &= \frac{2\theta_1 - \beta}{2D}, \quad \frac{\partial w^c_i}{\partial a_1} = \frac{\theta_2 - \beta}{2G}, \\
\frac{\partial p_i^t}{\partial a_1} - \frac{\partial w^c_i}{\partial a_1} &= \frac{4\theta_2^2 - 4\theta_2 \beta + 8\theta_1 \theta_2 - 3\beta^2}{4N} + \frac{\beta^2 (2\theta_2 - \beta)^2}{4(4\theta_0\theta_2 M - \beta^2 N)} > 0.
\end{align*}
\]

From Proposition 5.4, \( \frac{\partial q_i^j}{\partial a_1} > 0 \) (i = 1, 2, j = co, cn, st), hence

\[
\frac{\partial \Pi_i^j}{\partial a_1} = (\frac{\partial p_i^j}{\partial a_1} - w_j) q_i^j + (p_i^j - w) \frac{\partial q_i^j}{\partial a_1} > 0, \quad i = 1, 2, j = co, cn, st.
\]

Therefore, \( \Pi_i^1 \) increases in \( a_1 \).

\[
\begin{align*}
\frac{\partial p_i^t}{\partial a_2} &= \frac{(D - \theta_2)E + 2\theta_1\theta_2 D + \beta E}{2DE} + \frac{\beta^2 \theta_2 (2\theta_1 - \beta)^2}{2E(\beta^2 D - \theta_0 E)}, \quad \frac{\partial w^c_i}{\partial a_2} = \frac{2\theta_1 - \beta}{2D}, \\
\frac{\partial p_i^t}{\partial a_2} &= \frac{3G - \theta_2 + \beta}{4G} + \frac{\beta^2 (\theta_1 - \beta)}{4(\theta_0 H - \beta^2 G)}, \quad \frac{\partial w^c_i}{\partial a_2} = \frac{\theta_1 - \beta}{2G}, \\
\frac{\partial p_i^t}{\partial a_2} &= \frac{N + 2\theta_2^2 - \theta_0 (\theta_2^2 - \beta^2 G)}{2N} - \frac{\theta_0 (\theta_2^2 - \beta^2 G)}{2(4\theta_0\theta_2 M - \beta^2 N) N}, \quad \frac{\partial w^c_i}{\partial a_2} = \frac{E - 2\theta_2 \beta}{2N}.
\end{align*}
\]

Since \( 0 < \beta < \theta_1 (i = 1, 2) \) and \( \beta^2 D - \theta_0 E < 0, \theta_0 H - \beta^2 G > 0, 4\theta_0\theta_2 M - \beta^2 N > 0 \), then

\[
\begin{align*}
\frac{\partial p_i^t}{\partial a_2} - \frac{\partial w^c_i}{\partial a_2} &= \frac{(D - 2\theta_1)E + \theta_2 (2\theta_1 - \beta)^2 + 2\beta E}{2DE} + \frac{\beta^2 \theta_1 (2\theta_2 - \beta)^2}{2E(\beta^2 D - \theta_0 E)} > 0,
\end{align*}
\]
respectively, we can obtain

\[\frac{\partial p_{2}^{cn}}{\partial a_{2}} - \frac{\partial q_{2}^{st}}{\partial a_{2}} = \frac{2(\theta_{2} - \beta) + (\theta_{1} - \beta)}{4G} + \frac{\beta^{2}(\theta_{2} - \beta)}{4(\theta_{0} H - \beta^{2}G)} > 0,\]

\[\frac{\partial q_{2}^{st}}{\partial a_{2}} - \frac{\partial w^{st}}{\partial a_{2}} = \frac{2\theta^{2}}{N} - \frac{\theta_{0}\theta_{2}(E - 2\theta_{2}\beta)(-\theta_{0}\theta_{2}^{2} + \beta^{2} + 2\theta_{2}\beta)}{2(4\theta_{0}\theta_{2} M - \beta^{2}N)N} > 0.\]

From Proposition 5.4, \(\frac{\partial q_{i}^{j}}{\partial a_{2}} > 0 (i = 1, 2, j = co, cn, st)\), hence

\[\frac{\partial \Pi_{2}^{j}}{\partial a_{2}} = (\frac{\partial p_{1}^{j}}{\partial a_{2}} - w^{j} \frac{\partial q_{1}^{j}}{\partial a_{2}}) > 0, i = 1, 2, j = co, cn, st.\]

Therefore, \(\Pi_{2}^{j}\) increase in \(a_{2}\). \(\square\)

**Proof of the Proposition 5.6.**

*Proof.* Take the partial derivative of Eqs. (8), (14) and (20) with respect to \(\theta_{0}\). Since \(\beta^{2}D - \theta_{0} E < 0, \theta_{0} H - \beta^{2}G > 0 \theta_{0}\theta_{2} M - \beta^{2}N > 0\) and \(q_{0} > 0\), so \(\beta(A - cD) - \theta_{0} E < 0, \theta_{0} H - \beta F + G\beta c > 0\) and \((Nc - K)\beta + 4\theta_{0}\theta_{2} M > 0\),

\[\frac{\partial q_{0}}{\partial \theta_{0}} = \frac{2E[\beta(A - cD) - \theta_{0} E]}{(2\beta^{2}D - 2\theta_{0} E)^{2}} < 0,\]

\[\frac{\partial q_{0}}{\partial \theta_{0}} = -\frac{H(\theta_{0} H - \beta F + G\beta c)}{2(H\theta_{0} - \beta^{2}G)^{2}} < 0,\]

\[\frac{\partial q_{0}}{\partial \theta_{0}} = -\frac{2\theta_{2} M[(Nc - K)\beta + 4\theta_{0}\theta_{2} M]}{(4\theta_{0}\theta_{2} M - \beta^{2}N)^{2}} < 0.\]

Therefore, the optimal quantity \(q_{i}^{j}\) sold in the direct channel decreases in \(\theta_{0}\).

Similarly, taking the partial derivative of \(q_{i}^{co}, q_{i}^{cn}\) and \(q_{i}^{st}\) with respect to \(\theta_{0}\) respectively, we can obtain

\[\frac{\partial q_{i}^{co}}{\partial \theta_{0}} > 0, \quad \frac{\partial q_{i}^{cn}}{\partial \theta_{0}} > 0, \quad \frac{\partial q_{i}^{st}}{\partial \theta_{0}} > 0.\]

\[\frac{\partial q_{i}^{co}}{\partial \theta_{0}} > 0, \quad \frac{\partial q_{i}^{cn}}{\partial \theta_{0}} > 0, \quad \frac{\partial q_{i}^{st}}{\partial \theta_{0}} > 0.\]

Therefore, \(q_{i}^{j}\) (\(i = 1, 2\)) increases in \(\theta_{0}\). \(\square\)

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