Supermassive black hole mass regulated by host galaxy morphology

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ABSTRACT
We investigated the relationship between supermassive black hole (SMBH) mass and host starburst luminosity in Seyfert galaxies and Palomar–Green quasi-stellar objects, focusing on the galaxy host morphology. Host starburst luminosity was derived from the 11.3 μm polycyclic aromatic hydrocarbon luminosity. We found that the SMBH masses of elliptical-dominated host galaxies are more massive than those of disc-dominated host galaxies statistically. We also found that the SMBH masses of disc-dominated host galaxies seem to be suppressed even under increasing starburst luminosity. These findings imply that final SMBH mass is strongly regulated by host galaxy morphology. This can be understood by considering the radiation drag model as the SMBH growth mechanism, taking into account the radiation efficiency of the host galaxy.

Key words: galaxies: active – galaxies: nuclei – galaxies: starburst.

1 INTRODUCTION
Recent observations have found that the mass of supermassive black holes (SMBH) in Galactic Centres correlates strongly with bulge mass in both active and inactive galaxies (Kormendy & Gebhardt 2001; Merritt & Ferrarese 2001; Tremaine et al. 2002; Marconi & Hunt 2003; Barth, Greene & Ho 2005). Also, it has been revealed that SMBH mass does not correlate with galaxy disc mass; in fact, the SMBH–disc mass ratio is much smaller than the SMBH–bulge mass ratio (Salucci et al. 2000; Kormendy & Gebhardt 2001). These observational findings suggest that SMBH formation is strongly connected to the bulge component, not the disc component. However, the physical mechanism of SMBH formation that can lead to these observational results has remained unclear.

Investigations of starbursts around active galactic nuclei (AGN) may prove pertinent to this question. Observations of polycyclic aromatic hydrocarbon (PAH) emission have gradually revealed evidence of starburst phenomena around AGN and a connection between AGN activity and starbursts (Imanishi & Gebhardt 2001; Imanishi & Wada 2004; Schweitzer et al. 2006; Maiolino et al. 2007; Netzer et al. 2007; Shi et al. 2007; Lutz et al. 2008; Watabe, Kawakatu & Imanishi 2008). This means that the mass accretion process on to the SMBH, i.e. the SMBH growth mechanisms, could be closely connected with starburst phenomena.

Umemura (2001) and Kawakatu & Umemura (2002) suggested that the physical mechanism of the link between the SMBH and bulge formation may be the radiation drag effect (the Poynting–Robertson effect) from bulge starbursts. Especially, Kawakatu & Umemura (2004) showed that the radiation efficiency differs for starbursts in the bulge compared with the disc. The bulge is round, providing high radiation efficiency. Disc starbursts are less efficient than bulge starbursts in the same starburst-luminosity range due to photon escape from the disc surface and edge-on opacity consideration, so the final SMBH mass of a host galaxy with a disc starburst cannot be large. Thus, to understand SMBH growth mechanisms, it may be necessary to consider where the starbursts occur in their host galaxies. In order to understand the radiation effects from the host starburst and confirm the radiation drag model, we must investigate the relationship between SMBH mass and host starburst activity, focusing on host galaxy morphology.

To date, the morphology of Seyfert galaxies (low-luminosity AGN) has been well studied and is almost all spiral (e.g. McLeod & Rieke 1995; Hunt & Malkan 1999). Moreover, the starbursts in the galaxy disc have been investigated (Watabe et al. 2008). For quasi-stellar objects (QSO) (high-luminosity AGN), although it has been difficult to determine their morphology due to their luminous nuclei, recent high-resolution imaging observations in the optical and near-infrared, and those exploiting adaptive optics (AO), have gradually revealed their morphology in detail (Taylor et al. 1996; Bahcall et al. 1997; McLeod & McLeod 2001; Márquez et al. 2001; Percival et al. 2001; Hamilton, Casertano & Turnshek 2002; Dunlop et al. 2003; Guyon, Sanders & Stockton 2006; Veilleux et al. 2006). They found not only the elliptical (which is equivalent to the bulge, classified as the spheroid) component but also the prominent disc component in the host of lower luminosity QSOs and radio-quiet QSOs (Dunlop et al. 2003). Also, we now know that QSO host
galaxies are gas-rich (which is not normal for elliptical galaxies) (Evans et al. 2001, 2006; Scoville et al. 2003) and that starbursts occur in the host galaxies (Haas et al. 2003; Barthel 2006; Schweitzer et al. 2006; Maiolino et al. 2007; Netzer et al. 2007; Shi et al. 2007; Lutz et al. 2008).

In this paper, to clarify the SMBH growth mechanism that satisfies the observational results, we investigated the SMBH mass–host starburst connection, taking into account the host galaxy morphology for Seyfert galaxies and QSOs. Throughout this paper, we adopted $H_0 = 80 \text{ km s}^{-1} \text{Mpc}^{-1}$, $\Omega_M = 0.3$ and $\Omega_A = 0.7$.

2 DATA AND ANALYSIS

2.1 Sample data

To investigate the effect of starbursts and host galaxy morphology on the final SMBH mass, we selected AGN based on estimated SMBH mass, PAH emission and morphology. The samples in this paper are Palomar–Green (PG) QSOs, selected at B band to have blue $U - B$ colour (Schmidt & Green 1983). Since the $B$- and $U$-band emissions of PG QSOs are dominated by the AGN, there are no biases for the presence of starbursts.

Seyfert galaxies in the CfA (Huchra & Burg 1992) and 12 $\mu$m (Rush, Malkan & Spinoglio 1993) samples, selected on the basis of their host galaxy magnitudes and IRAS 12 $\mu$m fluxes, respectively. These samples do not also include some biases for the presence of starbursts.

2.2 Black hole mass

To estimate SMBH mass, $M_{\text{BH}}$, of PG QSOs and Seyfert 1 galaxies, we used a method based on the reasonable assumption that the motion of ionized gas clouds around the SMBH is dominated by the gravitational force and that the clouds within the broad-line region (BLR) are virialized (e.g. Peterson & Wandel 1999). The velocity dispersion $v$ can be estimated from the full width at half-maximum (FWHM) of H$\beta$ broad-line emission $v = f v_{\text{FWHM}}$ by assuming the isotropic spherical virial coefficient, $f = \sqrt{3}/2$ (Netzer 1991). We selected H$\beta$ because H$\beta$ lines radiate by simple photoionization mechanisms and their line profiles reflect gravitational potential of the H$\beta$ emission region. Adopting an empirical relationship (Kaspi et al. 2000) between the size of the BLR and the rest-frame optical continuum luminosity, $\lambda L_{\lambda}(5100 \, \text{Å})$, and using reverberation mapping, we obtain the following formula:

$$M_{\text{BH}} = 4.9 \times 10^6 \left( \frac{\lambda L_{\lambda}(5100 \, \text{Å})}{10^{44} \text{ erg s}^{-1}} \right)^{0.70} \left( \frac{v_{\text{FWHM}}}{10^3 \text{ km s}^{-1}} \right)^2 M_\odot. \quad (1)$$

For the error of SMBH mass, McGill et al. (2008) compared 12 formulae taken from the literature, showing that SMBH mass estimates can differ on average 0.13 ± 0.05 or 0.38 ± 0.05 dex in the case of the same or different virial coefficient, respectively. For Seyfert 2 galaxies, we used SMBH mass data (Bian & Gu 2007) whose mean error was within a factor of 1.6. These were estimated from the SMBH mass–stellar velocity dispersion relation (Tremaine et al. 2002).

2.3 Host galaxy starburst luminosity

Since the PAH molecules are excited by far-ultraviolet photons in the photodissociation region around the H II region, and strong PAH emission is often observed from even a weak starburst (Imanishi 2002), we can use PAH emission as an indicator of starburst activity. We used the PAH emission estimated by Watabe et al. (2008) for Seyfert galaxies (6.2, 7.7 and 11.3 $\mu$m) and Shi et al. (2007) for PG QSOs (7.7 and 11.3 $\mu$m). We select the 11.3 $\mu$m PAH emission. The 7.7 $\mu$m PAH emission is sometimes affected by the broad and strong 9.7 $\mu$m silicate absorption, especially in Seyfert 2 galaxies. Thus, it could be difficult to distinguish between 7.7 $\mu$m PAH emission and 9.7 $\mu$m silicate absorption. PAH emission was obtained with the Spitzer Space Telescope Infrared Spectrograph (IRS; Houck et al. 2004) (Werner et al. 2004). For Seyfert galaxies, since the PAH emission was observed with the slit-scan mode (PID 3269, Principal Investigator: J. Gallimore), the entire host galaxy regions of the Seyfert galaxies were covered. For PG QSOs, PAH emission was obtained by the slit width of short-low (SL) (SL1: 3.7 arcsec, SL2: 3.6 arcsec) and long-low (LL) (LL1: 10.7 arcsec, LL2: 10.5 arcsec) modules (for PID and Principal Investigator, see Shi et al. 2007). The SL slit width is roughly comparable to the effective radius of PG QSO host galaxies (Guyon et al. 2006) and several hundred-parsec to kiloparsec-scale starbursts have been considered as the origin of the far-infrared radiation in PG QSOs (Haas et al. 2003; Barthel 2006; Netzer et al. 2007). Thus, we consider that these slit observations cover almost all starburst activities in PG QSO host galaxies.

2.4 Host galaxy morphology

We used host galaxy morphology classifications for PG QSOs based on the literature, evaluated by two-dimensional $y^2$ fitting of the obtained image (references listed in Table 1). We carefully checked the morphology classifications and defined a host galaxy as elliptical- or disc-dominated only in cases for which the literature listed said galaxy as only elliptical or disc, respectively. We also defined a bulge + disc host galaxy in cases where a two-dimensional fit favoured a two-component (bulge + disc) model, or in cases where the morphology decision given in the literature varied (see Table 1). We also used a homogeneous morphology classification criterion to check our results, by separately examining objects classified only by Guyon et al. (2006). This sample consisted of a number of PG QSOs (20 objects) investigated by near-infrared AO imaging with the Gemini-N and Subaru Telescope. For the Seyfert galaxies, since their hosts are almost all spiral galaxies (McLeod & Rieke 1995; Hunt & Malkan 1999), we assumed that their morphologies were disc-dominated.

3 RESULTS: SMBH MASS AND HOST STARBURST GEOMETRY RELATIONSHIP

We plotted the 11.3 $\mu$m PAH luminosity and SMBH mass specifying the host galaxy morphology in Fig. 1. The left- and the right-hand panels of this figure show that the morphology classification is used in Table 1 and only Guyon et al. (2006), respectively.

We applied detailed statistical tests about the difference of these distributions (Takeuchi et al., in preparation). To summarize, the SMBH mass distributions of the elliptical- and disc-dominated host galaxies are significantly different. However, the difference of the PAH luminosity is not very clear. (We showed this statistical analysis in the Appendix.) Averaged SMBH mass and its dispersions are $(\log M_{\text{BH}}/M_\odot) = 8.42 (\pm 0.44)$ and 7.48 (±0.36) for the elliptical- and disc-dominated host galaxies, respectively. Also, bulge + disc host galaxies are distributed in both the elliptical- and the disc-dominated host galaxy regions. In particular, for the disc-dominated host galaxy, although the PAH luminosity increases.
by about three orders of magnitude, SMBH mass increases by only about one order. These results indicate that the final SMBH mass is strongly connected with host galaxy morphology; the SMBH of a disc-dominated host galaxy is suppressed, while a more massive SMBH can form in an elliptical-dominated host galaxy. In order to remove the differences in observations and morphology classification methods, we checked our findings using data from Guyon et al. (2006) only in the right-hand panel of Fig. 1. Although the sample size decreases, the tendency of our results does not change; \( \log \frac{M_{BH}}{M_{\odot}} = 8.40 \pm 0.41 \) and \( 7.42 \pm 0.34 \) for the elliptical- and disc-dominated host galaxies, respectively.

4 DISCUSSION: AGN FUELING MECHANISM

To interpret our findings, we must consider the SMBH growth mechanism including both the host starburst effects and the host galaxy morphology. Although galaxy mergers (e.g. Hernquist 1989) and stellar bars (e.g. Noguchi 1988) have also been considered as SMBH growth mechanism candidates, the relationship between final SMBH mass and these mechanisms is still unknown. Therefore, it is difficult to explain the difference in SMBH mass for the same starburst-luminosity range using these mechanisms. Thus, in order to relate the host starburst and host galaxy morphology with SMBH
formation, we focused on the radiation-hydrodynamic effect from the host starburst. The radiation drag is a relativistic effect known as the Poynting–Robertson effect. It is a possible mechanism for extracting angular momentum from the gas and driving SMBH mass accretion (Umemura, Fukue & Mineshige 1997; Umemura 2001; Kawakatu & Umemura 2002). Final SMBH mass is connected with the absorption efficiency of the amount of radiation energy from the starburst. This radiation drag efficiency is strongly affected by host geometry (Umemura, Fukue & Mineshige 1997, 1998; Ohsuga et al. 1999). Kawakatu & Umemura (2004) explored the possibility that SMBH mass of a disc-dominated host galaxy could be one to two orders of magnitude smaller than that of an elliptical-dominated host galaxy due to the effects of geometrical dilution and opacity. If an elliptical-dominated host galaxy begins with starbursts in a highly inhomogeneous and optically thick interstellar medium (e.g. Sanders et al. 1988; Gordon, Calzetti & Witt 1997), radiation drag could effectively work to extract the angular momentum (Kawakatu & Umemura 2002). In contrast, a large number of photons escape from the disc surface of a disc-dominated host galaxy. Also, radiation from a disc-starburst is shielded due to edge-on optical depth. Therefore, from a radiation-hydrodynamic point of view, final SMBH mass could be strongly connected to starburst location. Also, there seems to be no possibility that the radiation efficiency is equivalent for disc- and elliptical-dominated host galaxies, and the SMBH mass of a disc-dominated host galaxy is currently small but will grow up in the future due to SMBH growth delay. It is because SMBH masses of the inactive spiral galaxies are comparable to those of the disc-dominated host galaxies with AGN and follow the same SMBH–bulge mass relation (Pastorini et al. 2007). Therefore, in the case that all inactive spiral galaxies have passed the AGN phase (Marconi et al. 2004), the final SMBH masses of the disc-dominated host galaxies with AGN could not reach those of the elliptical-dominated host galaxies. Thus, our result flows naturally from a radiation drag model that includes the radiation efficiency due to host galaxy morphology.

5 SUMMARY

We investigated the SMBH mass–host starburst connection, taking into account the host galaxy morphology for Seyfert galaxies and PG QSOs. We checked the statistical test about the difference of these distributions. As a result, we found that host galaxy morphology may strongly regulate final SMBH mass, as the SMBH masses of the elliptical-dominated host galaxies were more massive than those of the disc-dominated host galaxies. Also, the SMBHs of disc-dominated host galaxies showed suppressed mass even in the case of increased starburst luminosity. These findings indicate that the SMBH growth mechanism is strongly connected to radiation efficiency dependent on the geometry of the host starbursts.

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\[ x^2 = \frac{4D^2n_{ell}n_{disc}}{n_{ell} + n_{disc}} \]  

(A1)

obey the \( \chi^2 \)-distribution with a degree of freedom (d.o.f.) of 2, under the null hypothesis that these are drawn from the same DF. The distances \( D \) from the sample of \( a \) and \( b \) are 0.818 and 0.882. Then, the probabilities having the \( \chi^2 \) values (equation A1) corresponding to these values are \( 5 \times 10^{-4} \) and 0.0001, respectively. Thus, in both cases, the null hypothesis is very clearly rejected.

Next, we examine the horizontal marginal distribution of the samples. Along with the abcissa in Fig. 1, we have a significant number of upper limits in both elliptical- and disc-galaxy samples. In this case, usual two sample tests are no longer valid. We first estimate DFs of the PAH luminosity, \( F(L_{\text{PAH}}) \) of both samples by the Kaplan–Meier (KM) estimator (Kaplan & Meier 1958; Kalbfleisch & Prentice 2002). The KM estimator is designed to obtain the so-called survival function (or survivor function), \( S(t) \), of a given sample including right censored data (i.e. data with lower limits). A survival function is related to the DF by the following relation: \( S(t) = 1 - F(t) \). To construct this estimator, we first mathematically formulate the current problem according to Feigelson & Nelson (1985). Since most of the astronomical observational data are selected with a certain detection limit, usually a sample includes upper limits, instead of lower limits. In statistical terminology, lower limit data are called ‘left-censored’. Let \( T_{1}^L, T_{2}^L, \ldots, T_{n}^L \) denote measurements, where the superscript \( L \) means ‘left’.

If \( T_{1}^L < A_{i} \), where \( A_{i} \) is its upper limit, then we cannot obtain the exact value of \( T_{1} \), but the upper limit \( A_{i} \). The information available is a combination of \( \{X_{1}^L, \delta_{1}^L\} = \{T_{1} , 1\} \) and \( \{A_{i} , 0\} \) for \( T_{1}^L \geq A_{i} \) and \( T_{1}^L < A_{i} \), respectively. Suppose that \( T_{1}^L \) is a random sample drawn from a distribution \( F(t) = P(T_i \leq t) \), where \( P(E) \) denotes the probability of an event \( E \). If \( \{A_{i}\}_{i=1,2,...,n} \) are mutually independent, identically distributed (referred to as IID) and independent of the true measurements \( \{T_{i}^L\}_{i=1,2,...,n} \), this statistical model is called random censorship. We can easily transform left-censored data to right-censored ones by setting a constant \( M; T_i = M - T_i^L, X_i = M - X_i^L, C_i = M - A_i, \delta_i = \delta_i^L \). Then, \( \{X_i, \delta_i\}_{i=1,2,...,n} \) represent right-censored data. The KM estimator has the following form:

\[ \hat{S}(t) = \begin{cases} \prod_{j|x_{j}^L < t} \left(1 - \frac{d_{j}}{n} \right)^{ \delta_{j}^L} & \text{if } t > x_{j}^L, \\ 1 & \text{if } t \leq x_{j}^L. \end{cases} \]  

(A2)

where \( x_{j}^L \) denote distinct, ordered observed values in which ties are identified, and \( n_j = \# \{k; x_k \geq x_{j}^L\}, d_j = \# \{k; x_k = x_{j}^L\}. \) A detailed and comprehensive derivation is found in, for example, Feigelson & Nelson (1985) or Kalbfleisch & Prentice (2002).

Once we get the KM estimate of the survival function, \( \hat{S}(t) \), we can convert it into the estimate of the DF, \( \hat{F}(t) \), as

\[ \hat{F}(t) = P(T \leq t) = \begin{cases} P(M - T \leq t) & \text{if } T \leq M - t, \\ P(T \leq M - t) & \text{if } T > M - t. \end{cases} \]  

(A3)

Then,

\[ \hat{F}(t) = \hat{S}(M - t). \]  

(A4)

In this analysis, we set \( M = 0 \), i.e. we made a flip of a sign \( t \rightarrow -t \).

In order to see if these two DFs are different, we should perform a statistical test. Though there are a few methods to do so, we adopt the Mantel–Haenszel (MH) two-sample test (also often referred to as the logrank test, but the latter denotes some variations). The derivation is shown in, for example, Kalbfleisch & Prentice (2002). The MH test makes use of the values as follows: first, we sort all the galaxy samples along with the (minus logarithmic) luminosity, \( -\log L_{\text{PAH},j} \). Then, we define mean numbers of galaxies at each luminosity \( -\log L_{\text{PAH},j} \) for both samples, \( m_{k,j} \equiv d_{j} n_{k,j} \) where \( k \) denotes the label of the two samples, ‘ell’ and ‘disc’, and \( n_{j} = n_{\ell,j} + n_{\text{disc},j} \). We also define variance

\[ v_{k,j} = v(t_{k,j}) = m_{k,j} \left(1 - \frac{n_{k,j}}{n_{j}} \right) \left( \frac{n_{j} - d_{j}}{n_{j} - 1} \right) \]  

(A5)

\( k \) and \( \ell \) again denote ‘ell’ and ‘disc’), and covariance;\(^{1} \) \( v_{k,\ell,j} = -v_{k,j} \). Consider a deviation vector \( \{\sum_{j} \Delta_{\ell,j}, \sum_{j} \Delta_{\text{disc},j}\} = \langle \sum_{j} \delta_{\ell,j} - m_{\ell,j}, \sum_{j} \delta_{\text{disc},j} - m_{\text{disc},j} \rangle \). Since the d.o.f. is 2 - \( l = 1 \) in the current problem, \( \sum_{j} \Delta_{\ell,j} = -\sum_{j} \Delta_{\text{disc},j} \). Then, simply we can use a statistic \( \sum^{2}_{j} \Delta^{2}_{k,j}/v_{k,j} \). If the two samples are drawn from the same DF (this is the null hypothesis to be tested), this statistic should obey the \( \chi^2 \)-distribution with d.o.f. = 1. From the sample of \( a \) and \( b \), we have \( \chi^2 = 0.694 \) and 0.951, respectively. Probabilities of having these values under the \( \chi^2 \)-distribution are 0.405 and 0.329. Then, if we set the confidence limit of 0.05, we cannot reject the null hypothesis, i.e. the two DFs are not significantly different.

\(^{1}\) However, in this case, we have only two samples. They are equivalent but the signs are different. We use the variance-covariance matrix when we perform \( n \)-sample test \( (n > 2) \).

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