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RADIATIVE QCD CORRECTIONS: A PERSONAL OUTLOOK

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ABSTRACT

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1. Introduction

Some time ago, with other members of the INR group, when we were completing our joined work on the analytical evaluation of the high-order QCD corrections to the physical quantity, using the already developed methods and the symbolic manipulation program SCHOONSCHIP, we were frequently asking ourselves questions about the possible outcome of our long-term project, which resulted later on in the completing of the calculations of the next-to-leading-order (NLO) QCD corrections to the several deep-inelastic scattering (DIS) sum rules and of the next-to-next-to-leading-order (NNLO) QCD corrections to \( R(s) = \sigma_{tot}(e^+ e^- \rightarrow \text{hadrons})/\sigma_{tot}(e^+ e^- \rightarrow \mu^+ \mu^-) \). Today the main outcomes of these studies are more clear to the whole scientific community.

First of all it is important to stress that any cumbersome calculation of the effects of radiative QCD corrections represents the classical example of a “theoretical experiment”. Therefore, as it happens sometimes with a real phenomenological experiment, the experience gained after its completion might result in arriving to results that were not expected at the beginning of the project. Of course, the most obvious aim of a precise QCD calculation is related to the phenomenology. Indeed, the work on the extraction of the values of various QCD parameters from the existing experimental data clearly necessitates a detailed consideration of the perturbative QCD effects. These theoretical contributions can also be further considered as the QCD background to the effects of electroweak (EW) physics or to the possible effects of new physics, say supersymmetry. As the other argument, which favours a detailed consideration of the perturbative QCD effects, today we can quote other theoretical studies, pushed ahead by the NNLO QCD calculations of \( R(s) \), namely the rediscovery on the more quantitative level of the interesting world of renormalons in QCD. These works demonstrated that in order to understand the role of certain non-perturbative effects it is also necessary to consider in more detail the asymptotic structure of the perturbative QCD predictions in the deep Euclidean region.

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Other examples of the importance of the studies of the perturbative QCD effects to various physical quantities were given in a number of talks at this very productive Symposium. Here we will summarize the main results of the work of its QCD part and will add some new information about several subjects, important from our point of view, which slipped away from the scientific part of the Symposium.

2. Determination of the QCD Parameters

It is known that the standard Lagrangian of the QCD has the following form

\[ L = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_{f=1}^{6} \bar{q}_f (i \hat{D} - m_f) q_f, \]  

(1)

where \( \hat{D} = \hat{\partial} - ig \hat{A}^a A^a / 2 \). The parameters \( m_i = m_i(\mu) \) are the current quark masses that depend on the normalization point. The traditional methods of the determination of the masses of light quarks \( m_u, m_d \) and \( m_s \) are the chiral perturbation theory and the QCD sum rules approach. Recently the new modern approach based on the application of lattice calculations was also used to determine the value of \( m_s \). The masses of heavy quarks, namely \( m_c \) and \( m_b \), can be determined using the QCD sum rules and the methods provided by the potential models. The theoretical determination of the top-quark mass is rather delicate subject. One of the approaches is based on the study of the fixed points in the solution of the renormalization-group (RG) equations for the Yukawa couplings in the general renormalized theory. And of course one can use experimental data of Fermilab or LEP machines to get phenomenologically motivated information about the pole mass of the top-quark. This subject is one of the hottest at present. However, we will not concentrate on it in this talk. Its more detailed discussion was presented at this Symposium by Altarelli. Here we will make several comments about the extractions of the masses of lighter quarks.

Let us first mention that the chiral perturbation theory, which gives the possibility to determine values of the ratios of light-quark masses that are independent of the normalization point (for a recent review see), does not allow us to fix the scales at which the corresponding absolute values of the current quark masses \( m_u = 4 \) MeV, \( m_d = 7 \) MeV and \( m_s = 140-150 \) MeV are defined. It is possible to solve this problem after application of the finite-energy QCD sum rules technique (FESR) to the two-point function

\[ \Pi(q^2) = i \int e^{iqx} \langle T J(x) J(0) \rangle_0 d^4 x \]  

(2)

of the (pseudo)scalar quark currents \( J = m_j \bar{\sigma}_j (\gamma_5) q_j \). In the application of this technique, the values of the light-quark masses explicitly depend on the local duality interval, namely \( m_j = m_j(s_0) \). The transformation to another normalization point can be made using the solution of the RG equation:

\[ - \frac{\partial \ln m(\mu)}{\partial \ln(\mu^2)} = \gamma_m(\alpha_s) = \sum_{i\geq 0} \gamma_i \left( \frac{\alpha_s}{\pi} \right)^{i+1}, \]  

(3)
where the perturbative expression for the mass anomalous dimension function is known at the three-loop level\(^{14}\).

Usually the FESR values of the light-quark masses\(^{15}\),\(^{16}\),\(^{17}\) are normalized at 1 GeV. The commonly used values are \(m_u(1\text{ GeV})=6\text{ MeV}, m_d(1\text{ GeV})=10\text{ MeV}, m_s(1\text{ GeV})=195\pm 33\text{ MeV}\)\(^{14}\), which are in agreement with other determinations\(^{15}\),\(^{16}\). However, it was shown\(^{18}\), using the Borel sum rules method\(^{19}\), that at the scale of over 1 GeV it is important to take into account the sizeable instanton-type contributions to the two-point function of the (pseudo)scalar quark currents. The more concrete calculations\(^{20}\) gave even the explicit form of these instanton contributions to the theoretical part of the FESR

\[
\int_0^{s_0} \rho^{th}(s) ds = \frac{3}{4\pi^2} m(s_0)^2 s_0 \left( 1 + R_{ins}(s_0) + R_{pt}(s_0) \right),
\]

where \(R_{pt}(s_0)\) is the perturbative contribution, which is known at the three-loop level\(^{21}\), and the expression for \(R_{ins}(s_0)\) reads\(^{20}\):

\[
R_{ins}(s_0) = \frac{11}{4\pi} \frac{\hat{m}_s}{\sqrt{s_0}} \left( \log \frac{\sqrt{s_0}}{\Lambda} \right)^{8/9} \left( \frac{5.17\Lambda}{\sqrt{s_0}} \right)^9.
\]

The result of Eq. (5) is comparable with the one-loop expression at \(s_0 = 13\Lambda^2 \approx 2\) GeV\(^2\). Therefore it is necessary to understand in more detail the place of these additional theoretical contributions in the FESR determinations of light-quark masses. It is possible that these effects are automatically taken into account in the recent lattice determinations of \(m_s\)\(^{10}\), since this non-perturbative method was aimed at the calculation of the \(m_s\) value at the high energy scale. The result obtained in Ref. \(^{10}\) reads \(m_s(2\text{ GeV})=127\pm 18\text{ MeV}\). Evolving it to the traditional normalization point using the solution of the RG equation of Eq. (3) at the three-loop level we get \(m_s(1\text{ GeV})\approx 180\pm 30\text{ MeV}\), which is in very good agreement with the recent determination of the \(s\)-quark mass from the Borel sum rules for the divergence of the vector current, namely \(m_s(1\text{ GeV})=189\pm 32\text{ MeV}\)\(^{22}\), the latter being in its turn in agreement with FESR results\(^{15}\),\(^{16}\),\(^{17}\). This fact gives us the idea that apart from the instanton-type contributions to the correlator of the (pseudo)scalar quark currents, other uncertainties in the determinations of the \(s\)-quark mass are well under control.

A few words should be added to the discussions of the heavy-quark properties, presented at the Symposium in the theoretical\(^{23}\),\(^{24}\),\(^{25}\) and experimental\(^{26}\),\(^{27}\) talks. First of all, it is important to mention that the pole masses of the heavy quarks are related to the running quark masses by the following numerical expression\(^{28}\)

\[
m_q(m_q^p) = m_q^p \left[ 1 - \frac{4\alpha_s(m_q^p)}{3\pi} - \left( 14.33 - 1.04 \sum_{i=u}^{q-1} \left( 1 - \frac{4m_i^p}{3m_q^p} \right) \left( \frac{\alpha_s(m_q^p)}{\pi} \right)^2 \right) \right],
\]

where \(m_q^p\) are the pole masses and the coefficient of the \(m_q^p/m_q^p\) term is the approximation, which should be used only up to the value of the ratio up to 0.3. Traditionally Eq. (6) was used to determine the values of the running quark masses from the values of \(m_q^p\). However, it was discovered recently\(^{29}\) that the resummation of the renormalon-type contributions, non-considered previously, to the relations between heavy-quark
running and pole masses result in the appearance of an additional contribution, which has the following approximate form:

\[ m_q(m^p_q) \approx m^p_q - \frac{2}{3\beta_0} \Lambda_{QCD} \tag{7} \]

where \( \beta_0 \) is the first coefficient of the QCD \( \beta \)-function, defined as \( \beta(\alpha_s) = \mu^2 \partial a / \partial \mu^2 \) \((a = \alpha_s / \pi)\), and \( \Lambda_{QCD} \) is the QCD scale parameter in some non-fixed scheme. Therefore, the pole mass is not defined to an accuracy better than \( \Lambda_{QCD} \) within perturbation theory \[29\]. The indirect indication of this problem comes from detailed studies \[30\] of the effects of the QCD corrections to \( \Gamma(H^0 \rightarrow b\bar{b}) = \Gamma_{Hb\bar{b}} \) in the case when the value of \( m^p_b \) is considered as the input parameter and the RG-improved version of the relation of Eq. (6) is used to sum up the RG-controllable \( \log(M_H/m^p_b) \)-terms and to present the final expression for \( \Gamma_{Hb\bar{b}} \) in terms of \( m_b(M_H) \).

Indeed, it was observed \[30\] that for reasonably large values of the Higgs boson mass \( M_H \) the corresponding NNLO corrections to the ratio \( R_{Hb\bar{b}} = \Gamma_{Hb\bar{b}} / \Gamma^{(0)}_{Hb\bar{b}} \) (where \( \Gamma^{(0)}_{Hb\bar{b}} = 3\sqrt{2}/(8\pi)G_F M_H(m^p_b)^2 \)) are larger than the NLO ones. This observation was further considered as an indication of the asymptotic explosion of the corresponding NNLO approximation, which is related to the one of Eq. (6). The latter feature might indicate the importance of the careful treating of the renormalon contributions to Eq. (6) responsible for the asymptotic structure of the corresponding perturbative relation. Now we can convince ourselves that in order to avoid the problem of the study of the asymptotic behaviour of the perturbative series related to Eq. (6) it might be better to use, in the phenomenological considerations, the value of \( m_b(\mu \approx M_H/2 \approx m^p_b) \), evolve it using the RG equation to any high-energy scale, and then to study the scheme dependence of the corresponding results. This point of view is in agreement with the one of Marciano \[31\] and with the studies of the dependence of the results of calculations of the QCD corrections to the \( \rho \)-parameter on the definition of the top-quark mass \[32\]. An analogous observation came previously from the results of calculations of the \( O(m^2_b/M_Z^2) \) corrections to \( \Gamma(Z^0 \rightarrow \text{hadrons}) \) \[33\]. Note that a definite attempt to estimate the renormalon-type contributions in Eq. (6) was recently made \[34\]. This analysis was based on the following values of the \( c \)-quark and \( b \)-quark running masses in the \( \overline{MS} \) scheme: \( m_c(m^p_c) = 1.23^{+0.02}_{-0.03} \pm 0.03 \) GeV, \( m_b(m^p_b) = 4.23^{+0.03}_{-0.02} \pm 0.02 \) GeV. The numerical values of the renormalon contributions \( \Delta m_q \) to Eq. (7) were estimated to be \( \Delta m_c \approx 30 \pm 20 \) MeV and \( \Delta m_b \approx 70 \) MeV \[34\]. However, we think that this phenomenological analysis is only the first step in the direction of future, more definite studies of the values of these newly discovered effects.

Let us now turn to more classical problems. The precise determination of the value of the coupling constant \( \alpha_s(\mu) = g(\mu^2)/(4\pi) \) and of the QCD scale parameter \( \Lambda_{\overline{MS}} \) in the \( \overline{MS} \) scheme is considered at present as one of the basic phenomenological tests of QCD. The detailed update of the determinations of \( \alpha_s \) values from different processes was presented at the Symposium by Bethke \[36\]. The results presented are included in Table 1, taken from a later review \[37\].
Table 1. World summary of measurements of $\alpha_s$. Abbreviations: DIS = deep-inelastic scattering; GLS-SR = Gross-Llewellyn-Smith sum rules; Bj-SR = Bjorken sum rules; LGT = lattice gauge theory; resum. = resummed NLO. Reference: S. Bethke, Proc. of the QCD’94, Montpellier, France, July, 1994; Aachen preprint PITHA-94-30. The comments on the first result and its uncertainties are presented in Section 4.

Of course, the application of the $\overline{MS}$ scheme represents the example of the phenomenological convention between theoreticians and experimentalists. Indeed, it does not allow us to avoid the theoretical ambiguities due to the existence of the scale-scheme dependence problem, reviewed at the Symposium by Brodsky. However, it is known that in QED the following relation takes place: $\alpha_{\overline{MS}}(m_e) = \alpha_{OS}[1 + O(\alpha_{OS}^2)]$ where $\alpha_{OS}$ is the QED fine structure constant. Therefore, the NLO on-shell scheme QED phenomenology is identical to the $\overline{MS}$ scheme one. Moreover, we think that the $\overline{MS}$ scheme has also other attractive features and can be really considered as the conventional referential scheme, which should be used in the studies of the phenomenological QCD predictions. The results of Table 1 give the following modern (but current) world average value of $\alpha_s$ at the $M_Z$ scale: $\alpha_s(M_Z) = 0.117 \pm 0.006$,
which corresponds to the following values of the parameters $\Lambda_{\overline{MS}}$:

$$\Lambda_{\overline{MS}}^{(5)} = 195^{+80}_{-60} \text{ MeV}, \Lambda_{\overline{MS}}^{(4)} = 280^{+95}_{-80} \text{ MeV}, \Lambda_{\overline{MS}}^{(3)} = 380^{+130}_{\pm 90} \text{ MeV}. \quad (8)$$

The cited world average for $\alpha_s$ is in agreement with $\alpha_s(M_Z) = 0.118 \pm 0.006$ given in another recent review work.\[23\]

As was also stressed by Bethke\[36\], in order to provide a more reliable estimate of the theoretical uncertainties in the extractions of $\alpha_s$ from jet cross sections it is urgently necessary to calculate still unknown order-$O(\alpha_s^3)$ NLO QCD corrections to the characteristics of 4-jet production in $e^+e^-$-annihilation. The theoretical background lying under this problem was discussed at the Symposium by Lampe\[39\]. The problem of the calculation of the NLO QCD corrections is also important for the analysis of the jet production in $p\bar{p}$ collisions. Indeed, in the talk by Lynn\[40\], a plot of the data of the CDF collaboration, taken from Ref.\[41\], was presented.

One can see that the CDF data\[41\] lie (1.5-2.4)$\sigma$ below the range of the QCD predictions. Clearly this fact stimulates further experimental and theoretical studies. The status of the theoretical program of the jet cross section calculations in the $p\bar{p}$ collisions was discussed by Giele\[42\]. It was emphasized that the interest in the corresponding NLO calculations is stimulated by the fact that the high-multiplicity jet cross sections are only known at the leading-order (LO) level. One of the most important messages given by Giele\[42\] is that the continuous development of the theoretical technology\[43\] can result in the appearance of the NLO calculations to $p\bar{p} \rightarrow 3$ jets and $p\bar{p} \rightarrow W, Z + 2$ jets cross sections before the end of this year.

The interesting phenomenological result, related to the measurement of the 3-jet cross sections by the CLEO group from Cornell, was presented at the Symposium by Sanghera\[27\]. These measurements allowed one to determine the value of $\alpha_s$ in the energy region with four active flavours and to obtain the following result $\alpha_s(10.53 \text{ GeV}) = 0.164 \pm 0.004(\text{exp}) \pm 0.015$ (theory). The RG evolution of this result through the threshold of the production of the $b$-quark gave the following value: $\alpha_s(M_Z) = 0.113 \pm 0.002 \pm 0.007$. This result is in reasonable agreement with the world average value of $\alpha_s$. Note, however, that the measurement of $\alpha_s$ from the jets rates directly at a $Z^0$-pole give the larger central value of $\alpha_s$ (see Table 1).

In general the determination of $\alpha_s$ from the LEP measurements creates a definite problem in the comparison with certain other results, say with the one presented by Sanghera\[27\]. Indeed, the central values of the results obtained from LEP data are usually higher than the results extracted from DIS, which are in agreement with the result of the CLEO analysis. For example, the most detailed recent extraction of $\alpha_s$ from the hadronic width of the $Z^0$ using three independently written computer codes BHM, TOPAZ0 and ZFITTER gave the following value\[3]: $\alpha_s(M_Z) = 0.120 \pm 0.007$ (exp) $\pm \delta\alpha_s(\text{theor})$, where

$$\delta\alpha_s(\text{theor}) = \pm 0.002(\text{EW}) \pm 0.002(QCD)^{+0.004}_{-0.003}(m_t^p, M_H). \quad (9)$$

This result, discussed in the talks of Bethke\[36\], Fleischer\[24\] and Hollik\[25\] (who described the current status of the comparisons of the analyses of the EW×QCD effects at the $Z^0$-pole by means of the different computer codes within the program of the LEP 1 Theoretical Working Group\[23\]) is in agreement with the one obtained with the help of the LEPTOP program\[23\], namely $\alpha_s(M_Z) = 0.125 \pm 0.005$ (exp) $\pm 0.002$ (theory). Clearly, the results of Refs.\[36, 24\] are larger than the extractions of $\alpha_s$ from the
DIS data (see Table 1 and Ref. 49). Moreover, the corresponding value of \( \Lambda_{\overline{MS}}^{(3)} \), which follows from the analysis of the LEP data, lies above the bound \( \Lambda_{\overline{MS}}^{(3)} \leq 400 \text{ MeV} \), extracted from the analysis of the low-energy \( e^+e^- \) data by means of the QCD sum rules approach. This deviation creates a problem in the understanding of the relation of the results of the QCD sum rules analysis, which are known to be quite successful in describing the low-energy hadronic phenomenology, to the above-mentioned LEP values of \( \alpha_s \). We think that before making any definite conclusions it is necessary to reconsider the problem of the extraction of the parameter \( \Lambda_{\overline{MS}}^{(3)} \) from the low-energy \( e^+e^- \) data, taking into account the sizeable \( O(\alpha_s^3) \) perturbative contributions to the Borel sum rules, which are known from the results of Ref. 51.

3. New Theoretical Results

The work on the precise analysis of the LEP experimental data stimulates the calculations and estimates of new theoretical effects. It is clear that large corrections in the Standard Model can come in particular from two sources, namely from the top-quark mass-dependent effects and from the effects of the higher-order perturbative QCD corrections. Two new results of the calculation of the top-quark mass-dependent QCD contributions were actively discussed in the talks of Hollik 46, Kniehl 25, and Fleischer 24. The first important result is the analytical calculation of the three-loop QCD corrections to the Veltman \( \rho \)-parameter, which is defined as the ratio of the neutral to charged currents amplitudes:

\[
\rho = \frac{G_{NC}(0)}{G_{CC}(0)} = 1/(1 - \Delta \rho),
\]

where

\[
\Delta \rho \approx 3x_t(1 + \rho^{_{\text{EW}}})\left(1 + \delta^{_{\text{QCD}}}\right)
\]

\[
x_t = \sqrt{2}G_\mu(m_t^p)^2/(16\pi^2)
\]

and \( \rho^{_{\text{EW}}} \) is the two-loop EW contribution, calculated for different values of \( M_H \) in several works 53. The QCD contribution to Eq. (10), normalized at the pole top-quark mass, has the following numerical form 52:

\[
\delta^{_{\text{QCD}}} = -2.86a(m_t^p) - (21.27 - 1.79n_f)a(m_t^p)^2,
\]

where \( a(\mu) = \alpha_s(\mu)/\pi \). As was mentioned by Kniehl 25 and Fleischer 24, it is interesting to study the numerical effects of this result using different prescriptions to fixing the renormalization-scheme ambiguities, namely the effective charges approach (ECH) 54, the principle of minimal sensitivity (PMS) 55, the BLM procedure 56, and the different definitions of the top-quark mass. Even more important is the detailed consideration of the scheme dependence of the EW contributions to the \( \rho \)-parameter 57.

Other recent works 58 concentrated on the calculation of the \( O(\alpha^3) \) contributions to \( \Gamma_{Z^0} = \Gamma(Z^0 \rightarrow \text{hadrons}) \) from the singlet diagrams with the loop formed by the propagator of the virtual top-quark. Including the previously calculated similar term of order \( O(\alpha^2) \) 59, and the results of Refs. 4 6, one gets the following \( O(\alpha^3) \) QCD approximation for \( \Gamma_{Z^0} \):

\[
\Gamma_{Z^0} = \Gamma_{\text{QPM}}^{_{_{\text{QM}}}} \left[ \sum_{i=u}^b (g_i^V)^2(1 + a_5 + 1.409a_5^2 - 12.767a_5^3) + \left( \sum_{i=u}^b g_i V \right)^2 (-0.413a_5) \right]
\]

\[
+ \sum_{i=u}^b (g_i^A)^2(1 + a_5 + 1.409a_5^2 - 12.767a_5^3)
\]
where \( \Gamma^{QPM} = G_F M_Z^2 / (8\pi \sqrt{2}) \), \( g_i^V = 2I_i - 4Q_i s_W^2 \), \( g_i^A = 2I_i \), and where \( a_{(5)} = a(M_Z) \) corresponds to \( f = 5 \) numbers of flavours and \( \overline{m}_t = m_t(m_t^*). \) For simplicity we neglected the known \( O(m_t^2/M_Z^2) \) corrections \( \text{[3]} \) and the \( O(\alpha_s) \) corrections \( \text{[4]} \), which should be taken into account in the precise analysis of the LEP data.

In the present situation, when the LEP experimental data on \( \Gamma_Z \) is continuously increasing, and in view of the existence of the certain deviation of the central values of \( \alpha_s(M_Z) \) (and thus \( \Lambda_{\overline{MS}} \)) from the ones, extracted from the DIS and low-energy hadronic phenomenology, one can ask whether taking into account higher-order perturbative QCD terms can affect the outcomes of the fits of the LEP data. In order to study this question the effects of the higher-order QCD corrections were estimated \( \text{[6]–[8]} \) using the procedure, proposed in Ref. \( \text{[3]} \) of the re-expansion of the optimized expression for the physical quantities into the initial \( \overline{MS} \) scheme. The results of applications of this procedure to the Euclidean \( D \)-function

\[
D(Q^2) = Q^2 \int_0^\infty \frac{R(s)}{(s + Q^2)^2} ds = 3\Sigma Q_f^2 \left[ 1 + a + \sum_{i \geq 1} d_i a_i^{i+1} \right] + (\Sigma Q_f)^2 [\tilde{d}_2 a^3 + O(a^4)],
\]

for different values of the number of flavours \( f \), are presented in Table 2 taken from Ref. \( \text{[8]} \).

| \( f \) | \( d_2^{est} \) | \( d_3^{est} \) | \( d_4^{est} \) | \( d_4^{est} - c_3 d_4 \) |
|---|---|---|---|---|
| 1 | 14.11 | 7.54 | 75 | 474 |
| 2 | 10.16 | 6.57 | 50 | 261 |
| 3 | 6.37 | 5.61 | 27.5 | 111 |
| 4 | 2.76 | 4.68 | 8 | 23 |
| 5 | -0.69 | 3.77 | -8 | -15 |
| 6 | -3.96 | 2.88 | -21 | -1.8 |

Table 2. The results of estimates of the \( O(a^3) \), \( O(a^4) \) and \( O(a^5) \) corrections in the series for the \( D \)-functions.

The estimated value of the NNLO term \( d_2^{est} \) in reasonable agreement with the result \( d_2^{ex} \) of the exact calculations \( \text{[3]–[4]} \).

In order to obtain the related estimates of the coefficients of \( R(s) = 3\Sigma Q_f^2 [1 + a_s + \sum_{i \geq 1} r_i a_i^{i+1}] + (\Sigma Q_f)^2 [\tilde{r}_2 a^3 + ...] \) it is necessary to take into account the effects of the Minkowskian region. The corresponding coefficients \( r_i \) have the following form: \( r_1 = d_1, r_2 = d_2 - \pi^2 \beta_0^4/3, \tilde{r}_2 = \tilde{d}_2, r_3 = d_3 - \pi^2 \beta_0^4 (d_1 + 5/6 c_1) \) and \( r_4 = d_4 - \pi^2 \beta_0^4 (2d_2 + \frac{2}{9} c_1 d_1 + \frac{4}{9} c_2 + c_2) + \frac{2}{3} \beta_0^4, \) where \( c_i \) are defined through the coefficients of the QCD \( \beta \)-function as \( \beta(a) = -\beta_0 a^2 (1 + \sum_{i \geq 1} c_i a^i) \). Using these expressions, we arrive at the results of Table 3, taken from the revised version of Ref. \( \text{[4]} \).
The results of estimates of the $O(a^4)$, $O(a^4)$ and $O(a^5)$ corrections in the series for $R(s)$. It should be stressed that for $\alpha_s(M_Z) \approx 0.12$ the corresponding $O(a^4)$ contributions to $\Gamma_{Z^0}$, as estimated in Refs. [23,24], namely

$$
\delta \Gamma_{Z^0} = \Gamma_{QP} \left[ \sum_u (g_i^u)^2 + (g_i^d)^2 \right] (-97a_{(5)})
$$

are of the same order of magnitude as the $O(m_t^2/M_Z^2)$ corrections, involved in the current analysis of the LEP data. However, since they are negative, they can only increase slightly the central value of $\alpha_s(M_Z)$ and will thus not remove the deviations of the values of $\alpha_s(M_Z)$ extracted from different processes. The positive $O(a^5)$ corrections to $\Gamma_{Z^0}$, as estimated in Ref. [24] (see also the last column of Table 2), are small and can be safely neglected.

Let us now turn to the analysis of the perturbative predictions for the ratio $R_\tau = \Gamma(\tau \rightarrow \nu_\tau + \text{hadrons})/\Gamma(\tau \rightarrow \nu_\tau \mu_\tau e)$, which were not discussed at this Symposium. It is known that this ratio is related to $R(s)$ by the following physical FESR [25] :

$$
R_\tau = 2 \int_0^{M^2_\tau} \frac{ds}{M^2_\tau} (1 - s/M^2_\tau)^2 (1 + 2s/M^2_\tau) \tilde{R}(s) \simeq 3 [1 + a_\tau + \sum_{i \geq 1} r_i^a a^{i+1}_\tau ]
$$

where $a_\tau = \alpha_s(M^2_\tau)/\pi$ and $\tilde{R}(s)$ is $R(s)$ with $f = 3, (\Sigma Q_f)^2 = 0, 3\Sigma Q^2_f$ substituted for $3\Sigma |V_{f\mu}|^2$ and $|V_{ud}|^2 + |V_{us}|^2 \approx 1$.

It was shown [25] that it is convenient to express the coefficients of the series (15) through those of the series (13) for the $D$-function in the following form : $r_1^\tau = d_1^\overline{MS} (f = 3) + g_1$, $r_2^\tau = d_2^\overline{MS} (f = 3) + g_2$, $r_3^\tau = d_3^\overline{MS} (f = 3) + g_3$, where the numerical expressions for the coefficients $g_i$ read $g_1 = 3.563$, $g_2 = 19.99$, $g_3 = 78.00$. One of the pleasant features of these coefficients is that they are absorbing all effects of the analytical continuation. Following the lines of Ref. [25] we derive the corresponding expression for the coefficient $r_4^\tau$: $r_4^\tau = d_4^\overline{MS} (f = 3) + g_4$ where

$$
g_4 = -[4d_3 + 3d_2c_1 + 2d_1c_2 + c_3] \beta_0 I_1 + [6d_2 + 7c_1d_1 + \frac{3}{2}c_1^2 + 3c_2] \beta_0^2 I_2
$$

$$
- [4d_1 + \frac{13}{3}c_1] \beta_0^3 I_3 + \beta_0^4 I_4 = 3.6c_3(f = 3) + 14.25d_3(f = 3) - 466.8
$$

\begin{table}
| $f$ | $r_2^{ex}$ | $r_2^{est}$ | $r_3^{est}$ | $r_4^{est} - c_3d_1$ |
|-----|-------------|-------------|-------------|---------------------|
| 1   | -7.84      | -14.41      | -166        | -1748               |
| 2   | -9.04      | -12.63      | -147        | -1156               |
| 3   | -10.27     | -11.03      | -128        | -669                |
| 4   | -11.52     | -9.58       | -112        | -263                |
| 5   | -12.76     | -8.29       | -97         | 64                  |
| 6   | -14.01     | -7.17       | -83         | 334                 |
\end{table}
and \( I_4 = \frac{41041}{864} - 265\pi^2/36 + \pi^4/5 \approx -5.668 \), while the expressions for \( I_1, I_2 \) and \( I_3 \) are known from the considerations of Ref. [64]. Using now the estimates of Table 2 we get the estimates of the corresponding higher-order coefficients of \( R_{\tau}^6 \), \( R_{\tau}^7 \):

\[
(\tau^3)_{est} \approx 105.5a_4^4, \quad (\tau^7)_{est} \approx 94a_7^5.
\]

It will be interesting to include these estimates in the analysis of the experimental data.

Some work in this direction was recently done in Ref. [65], where the experimental data of the ALEPH group for \( R_{\tau} \) and their moments was used to estimate from the fit the value of the coefficient \( d_3(f = 3) \) of the \( D \)-function. The result of this fit is presented in Fig. 2 taken from Ref. [65].
In the process of the fits of Ref. 65, it was assumed that the non-perturbative contributions to $R_\tau$ can be neglected. For $\mu = 1$ GeV the result of the fit is $d_3^{est} = 29 \pm 4 \pm 2^{65}$, where the second uncertainty is due to the theoretical uncertainties of the procedure used. The obtained result is in very good agreement with the results of theoretical considerations (see also Table 2). In fact this agreement is not very surprising, since the estimate of Ref. 65 was obtained with the help of the “optimization” of the experimental part of the relation $R_{\tau}^{\text{theory}} = R_{\tau}^{\text{exp}}$, while the theoretical estimates were obtained from the “optimized” theoretical l.h.s. of this relation. However, it is pleasant to see that this procedure is respected by the analysis of the experimental data, presented in Fig. 2. This fact can be considered as the additional argument in favour of applicability of the methods of the estimates of the higher-order perturbative corrections proposed in Ref. 55 and further used in the theoretical considerations of Ref. 61.

4. Polarized Deep-Inelastic Scattering

The detailed analysis of DIS is traditionally considered as one of the basic problems of the physics of strong interactions. Special attention is nowadays paid to the experimental and theoretical consideration of the polarized DIS processes and especially of the polarized DIS sum rules (SRs), namely of the Bjorken polarized SR

$$\Gamma_1^p - \Gamma_1^n = \int_0^1 [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] dx$$

(17)

and of the Ellis-Jaffe SR

$$\Gamma_{1}^{p(n)} = \int_0^1 g_{1}^{p(n)}(x, Q^2) dx.$$  

(18)

The current status of the investigations of this very hot topic was discussed at the Symposium in the beautiful experimental-oriented talk of Stuart 66 and theoretically-oriented talk of Forte 67. Clearly, experiment is going ahead of the theory in this field. Indeed, two new measurements of the proton polarized structure function $g_1^p(x, Q^2)$ in different energy regions became recently available. The result of the measurement at the energy scale $Q^2 = 10$ GeV$^2$, as presented at Fig. 3, taken from Ref. 68, was obtained by the SMC group at CERN. The preliminary data of the E143 group, who measured $g_1^p$ at $Q^2 = 3$ GeV$^2$, depicted in Fig. 4, was presented by Stuart 66.

After extrapolation of the results of the measurements of the SMC and E143 groups in the region of small and large $x$ and integration over the whole region $0 \leq x \leq 1$ the following new experimental results were obtained 68, 69:

$$\Gamma_1^p(Q^2 = 10 \text{ GeV}^2) = 0.136 \pm 0.011 \pm 0.011$$

$$\Gamma_1^n(Q^2 = 3 \text{ GeV}^2) = 0.129 \pm 0.004 \pm 0.010.$$  

(19)

The latter result is of particular interest since, after combining it with the previous neutron data of the E142 collaboration at an average $Q^2$ of 2 GeV$^2$, namely $\Gamma_1^n = -0.022 \pm 0.011$ 70, it is possible to “measure” the value of the polarized Bjorken SR in the low-energy region. The re-analysis of these data, made in Ref. 71, resulted in the definite modification of the original E142 result 70. In fact the authors of Ref. 71 obtained the following number:

$$\Gamma_1^n(Q^2 = 2 \text{ GeV}^2) = -0.028 \pm 0.006(\text{stat}) \pm 0.009(\text{syst}).$$  

(20)
Combining Eq. (21) with the first, preliminary, E143 result for the proton, namely \( \Gamma_1^n(Q^2 = 3 \text{ GeV}^2) = 0.133 \pm 0.004 \pm 0.012 \), the authors of Ref. [7] extracted the value of the Bjorken polarized SR at \( Q^2 = 2.5 \text{ GeV}^2 \):

\[
\Gamma_1^n - \Gamma_1^n(Q^2 = 2.5 \text{ GeV}^2) = 0.161 \pm 0.007 \pm 0.015. \tag{21}
\]

This result was further used to compare the results discussed above, which were experimentally motivated, with the theoretical QCD predictions, which we are now going to discuss.

A number of works are devoted to the calculation of the theoretical contributions into the polarized Bjorken SR. Summarizing the available information, let us present the theoretical expression for this fundamental SR, normalized to \( f = 3 \) numbers of flavours:

\[
\Gamma_1^n(Q^2) - \Gamma_1^n(Q^2) = \frac{1}{6} \left[ g_A \right] \left[ 1 - a - 3.583a^2 - 20.215a^3 - 130a^4 - O(a^5) \right] - O\left( \frac{1}{Q^2} \right). \tag{22}
\]

The exact expressions for the \( O(a^2) \) and \( O(a^3) \) corrections are known from the calculations of Refs. [1], [2] and Ref. [3] respectively. The \( O(a^4) \) coefficient was estimated in Refs. [3,4], while a less substantiated estimate of the \( O(a^5) \) term can be read from the results of the studies of Ref. [5]. The estimates of the \( O(a^4) \) terms are in qualitative agreement with the results of applications of the Padé resummation technique [6].

The expression for the Ellis-Jaffe SR consists of two parts, namely from the non-singlet and the singlet contributions. The \( O(a^2) \) correction to the singlet contribution was calculated in Ref. [7] (see also Ref. [8]), while the estimates of the \( O(a^3) \) corrections were given recently [9]. Combining these results with the ones for the non-singlet contribution (which coincide with the Bjorken polarized SR) one can arrive at the following expression for the Ellis-Jaffe SR in the \( \overline{\text{MS}} \) scheme:

\[
\Gamma_1^{(n)}(Q^2) = \left[ 1 - a - 3.583a^2 - 20.215a^3 - 130a^4 \right] \times \left( \pm \frac{1}{12}a_3 + \frac{1}{36}a_8 \right)
\]

\[
+ \left[ 1 - a - 1.096a^2 - 3.7a^3 \right] \frac{1}{9} \Delta \Sigma(Q^2) + O\left( \frac{1}{Q^2} \right) \tag{23}
\]

where \( a_3 = \Delta u - \Delta d, a_8 = \Delta u + \Delta d - 2\Delta s, \Delta \Sigma = \Delta u + \Delta d + \Delta s \) and \( \Delta u, \Delta d \) and \( \Delta s \) can be considered as the measure of polarization of the quarks in a nucleon.

The results of Eqs. (22), (23) were recently used [10] to determine the values of \( \alpha_s, \Delta s \) and \( \Delta \Sigma \) from the available experimental data on polarized SRs. At the first stage of the analysis, the authors of Ref. [11] neglected the higher-twist (HT) \( O(1/Q^2) \) contributions to Eqs. (22), and (23), used the experimentally motivated result of Eq. (21) that they derived for the polarized Bjorken SR and extracted the value of \( \alpha_s(Q^2 = 2.5 \text{ GeV}^2) \) in different orders of perturbation theory. The outcomes of this analysis are depicted at Fig. 5, taken from Ref. [12]. The final result, obtained by taking account of the estimates of the \( O(a^4) \) corrections given in Refs. [3,13], is \( \alpha_s(2.5 \text{ GeV}^2) = 0.375^{+0.062}_{-0.085} \) [13]. This result corresponds to the following value of \( \alpha_s \) at the \( M_Z \)-pole: \( \alpha_s(M_Z) = 0.122^{+0.005}_{-0.009} \) [14]. Using this result the authors managed further on to adjust all available experimental data on the polarized Ellis-Jaffe SR with the assumption that \( \Delta s \neq 0 \). The outcomes of this analysis, presented in Fig. 6 taken from Ref. [15], give \( \Delta s = -0.10 \pm 0.03 \).
And finally, using the facts that the perturbative corrections to the Ellis-Jaffe SR are negative and that the absolute values of the perturbative contributions to the non-singlet part are significantly larger than the ones to the singlet part (see Eq. (15), the authors of Ref. 72 extracted the average value of \( \Delta \Sigma(Q^2 = 10 \text{ GeV}^2) \) from the fits of all the data available up to now (see Fig. 7 taken from Ref. 72). Notice that Fig. 7 demonstrates that it is possible to satisfy all available data, including the neutron data of the E142 collaboration, by the condition \( \Delta \Sigma = 0.31 \pm 0.07 \) only after taking into account higher-order perturbative corrections.

This conclusion gives one more example of the importance of the consideration of the higher-order perturbative QCD effects.

In spite of the fact that the authors of Ref. 72 used in their analysis very preliminary E143 data, which were higher than the final result of Ref. 69 due to the non-careful treatment of the high \( x \)-extrapolation (for a detailed discussion of this important subject see the talk by Forte 67), their results have rather solid status. Indeed, they are in agreement with the results \( \Delta s(\infty) = -0.097 \pm 0.018, \Delta \Sigma(\infty) = 0.33 \pm 0.04 \) and \( \alpha_s(2 \text{ GeV}^2) = 0.49 \pm 0.06 \), which corresponds to \( \alpha_s(M_Z) = 0.125 \pm 0.06 \). These results were obtained from the careful fits of all available data on polarized DIS in the approximation when HT contributions are neglected 24. The interesting physics, lying beyond the outcomes of the fits of Refs. 72, 79 was discussed at the Symposium by Forte 67 and more recently in the work of Ref. 79.

Let us now summarize the current status of the studies of the HT corrections to DIS SRs. The concrete form of the matrix elements contributing to the HT corrections are known from the considerations of Ref. 80. The numerical values of these matrix elements were calculated using different approaches. It is known that the results of the three-point function QCD sum rules calculations 81 are larger than the ones obtained with the help of the bag model calculations 82. The original considerations were recently reanalysed with the help of a similar approach. The estimates obtained in Ref. 83 turned out to be larger than the original results of 81 but they have surprisingly small error bars. If one adopts the 50% error bars to the results of Ref. 83 which are typical of the physical outcomes of the three-point function QCD sum rules calculations, one can convince oneself that within these error bars the results of Ref. 81 remain true. Moreover, these results were recently confirmed by the outcomes of the application of the same method to the three-point function with different interpolating currents. In our opinion, these facts provide a solid background to the applications of the results 81 in the phenomenological studies of the experimental data for the polarized Bjorken SR. In fact this was already done in Ref. 72, where it was demonstrated that the incorporation of the HT corrections into the theoretical expression for the polarized Bjorken SR gives a smaller value \( \alpha_s(M_Z) = 0.118^{+0.007}{-0.014} \). This result is in agreement with the extraction of \( \alpha_s \) from the Gross-Llewellyn Smith SR, taking into account HT corrections, namely \( \alpha_s(M_Z) = 0.115 \pm 0.006(\text{exp}) \pm 0.003(\text{theory}) \) (see the second result presented in Table 1). The second result 82 is also in agreement with the value of \( \alpha_s \) obtained from other DIS processes (see Table 1), while the results obtained in Refs. 72, 79 without HT corrections are in better agreement with the LEP measurements. The estimated effects of the HT contributions 81, were also included in the studies of Ref. 84 aimed at a careful analysis of the SMC and E142 data, using the detailed parametrization of \( g_1(x) \) at different values of \( x \) and fixed values of \( Q^2 \). In the process of another interesting work 87 the authors extracted the values of the HT contributions directly from the experimental data. The results obtained are in qualitative agreement with the outcomes of the
QCD sum rules analysis. It is also worth while to mention that the authors of Ref. 88 advocate the conclusion that the HT contributions to the Ellis-Jaffe SR might be even more important than the ones to the polarized Bjorken SR. All these examples demonstrate that the problem of better understanding of the effects of the HT terms is more than a pure theoretical one. The detailed information about the values of the HT corrections will be even more important for the study of the theoretical predictions for moments of the $g_2(x)$ structure function of the polarized DIS. Notice that the experimental measurements of this structure function are already pushed ahead. However, in view of the absence of theoretical information about the behaviour of $g_2(x)$ in the region of small $x$, these measurements did not yet allow us to check the validity of the Burkhardt-Cottingham sum rule

$$\int_0^1 g_2(x, Q^2) dx = 0$$

(24)

which contrary to the claim of Ref. 22 does not receive perturbative QCD corrections.

To conclude the discussions of this section we would like to emphasize that the experimental and theoretical studies of the polarized DIS functions are really important. These offer new understanding of the details of the nucleon structure. Clearly the studies of these problems should be continued, and we believe that the time and money necessary for the continuation of the future experimental investigations in this field will not be wasted.

5. Fragmentation Functions and Unpolarized Deep- Inelastic Scattering

There is a close analogy between the considerations of the theoretical behaviour of the fragmentation functions (FFs) $D(z, \mu^2)$ of the process $e^+e^- \rightarrow hX$, discussed at the Symposium in the talk of Mele 92 and those of the SFs $F_i(x, Q^2)$ of the DIS processes. Indeed, as in the case of the SFs, the high-energy behaviour of the FFs is governed by the Altarelli-Parisi equation, which in the non-singlet case has the following form:

$$\frac{dD_{NS}(x, \mu)}{d \ln(\mu^2)} = \frac{\alpha_s(\mu^2)}{\pi} \int_x^1 \frac{dy}{y} D_{NS}(y, \mu)V_{NS}(x/y, \mu),$$

(25)

where the Mellin moments from the splitting function $V_{NS}(x)$ determine the corresponding anomalous dimension function:

$$\int_0^1 x^{n-1}V_{NS}(x)dx = \gamma_D^{(n)}(\alpha_s) = \sum_{i \geq 0} \gamma_i^{(n)} \left( \frac{\alpha_s}{\pi} \right)^{i+1}.$$

(26)

In the talk of Mele 22, which was based on the detailed discussion of Ref. 93, it was stressed that the calculations of the NLO perturbative QCD approximation of the FFs $D_i(x, \mu)$ are not yet completed. The important problem still remaining is the NLO calculation of the longitudinal FF, which can provide a new method of measuring $\alpha_s$ 92, 93.

Contrary to the case of FFs, the behaviour of the DIS SFs and in particular of the non-singlet ones is known at the NNLO. At this order of perturbation theory the expression for the SF $F_2(x, Q^2)$ is known from the results of Ref. 24, while the
NNLO corrections to the SF $xF_3(x, Q^2)$ of the deep-inelastic neutrino scattering were calculated in Ref. [73]. It should be stressed that it is possible to check the results of the complicated calculations of Ref. [94] by comparing them with the ones obtained in Ref. [95] for the Mellin moments:

$$M_{NS}^{(n)}(Q^2) = \int_0^1 x^{n-1} F_i^{NS}(x, Q^2) dx.$$  \hspace{1cm} (27)

These moments obey the RG equation with the anomalous-dimension term. Its solution has the following form:

$$\frac{M_{NS}^{(n)}(Q^2)}{M_{NS}^{(n)}(Q_0^2)} = \exp\left[ \int_{\alpha_s(Q_0^2)}^{\alpha_s(Q^2)} \frac{\gamma^{(n)}(x)}{\beta(x)} dx \right] \frac{C_{NS}^{(n)}(\alpha_s(Q^2))}{C_{NS}^{(n)}(\alpha_s(Q_0^2))}.$$ \hspace{1cm} (28)

In the case of $n = 2, 4, 6, 8, 10$, the NNLO corrections to the coefficient functions of the non-singlet moments of $F_2(x, Q^2)$ were calculated analytically [95]. It is pleasant to stress that these results coincide with the ones obtained from the results of Ref. [94] after taking the corresponding Mellin moments. The possibility to compare the cumbersome results of Refs. [95, 94] obtained by means of different calculational methods demonstrates the attractive feature of the exact analytical calculations.

The results of Refs. [95, 94] were taken into account in the non-singlet fits of the BCDMS data in Ref. [96]. The results of the fits are presented in Table 4 taken from Ref. [96].
| Deuterium        | $\Lambda_{\text{MS}}$ (MeV) | $\kappa$ (MeV) | $\chi^2(F_2)/\text{dof}$ | $\chi^2(R)/\text{dof}$ |
|------------------|------------------------------|---------------|--------------------------|--------------------------|
| $F_2$            | LO                           | 182 ± 32      | 63.5/65                  |                          |
|                  | NLO                          | 182 ± 30      | 62.7/65                  |                          |
|                  | SI(N)                        | 159 ± 25      | 62.4/65                  |                          |
|                  | NNLO                         | 168 ± 27      | 62.5/65                  |                          |
|                  | SI(NN)                       | 164 ± 26      | 62.4/65                  |                          |
| $F_2$ and $R$    | LO                           | 223 ± 35      | 65.1/65                  | 91.5/43                  |
|                  | NLO                          | 235 ± 34      | 65.5/65                  | 90.9/43                  |
|                  | SI(N)                        | 238 ± 28      | 70.8/65                  | 66.3/43                  |
|                  | NNLO                         | 218 ± 27      | 65.5/65                  | 79.9/43                  |
| (Including twist-4) | LO                        | 181 ± 31      | 207 ± 13                 | 63.5/65                  |
|                  | NLO                          | 180 ± 29      | 198 ± 14                 | 62.7/65                  |
|                  | SI(N)                        | 157 ± 25      | 184 ± 17                 | 62.4/65                  |
|                  | NNLO                         | 159 ± 21      | 200 ± 12                 | 62.8/65                  |
| Proton           | $F_2$                        | LO            | 171 ± 27                 | 51.2/60                  |
|                  | NLO                          | 175 ± 26      | 47.6/60                  |                          |
|                  | SI(N)                        | 159 ± 23      | 46.0/60                  |                          |
|                  | NNLO                         | 168 ± 25      | 46.4/60                  |                          |
|                  | SI(NN)                       | 169 ± 25      | 45.8/60                  |                          |
| $F_2$ and $R$    | LO                           | 200 ± 30      | 52.8/60                  | 92.3/43                  |
|                  | NLO                          | 222 ± 29      | 50.0/60                  | 82.8/43                  |
|                  | SI(N)                        | 231 ± 25      | 54.5/60                  | 59.2/43                  |
|                  | NNLO                         | 209 ± 23      | 48.8/60                  | 79.7/43                  |
| (Including twist-4) | LO                        | 168 ± 27      | 205 ± 14                 | 52.0/60                  |
|                  | NLO                          | 175 ± 26      | 195 ± 14                 | 48.2/60                  |
|                  | SI(N)                        | 158 ± 23      | 179 ± 17                 | 46.6/60                  |
|                  | NNLO                         | 158 ± 22      | 189 ± 14                 | 46.5/60                  |

Table 4. The results of the NS fits of the BCDMS data for $F_2$ and the SLAC data for $R = F_L/2xF_1$. The SI(N) and SI(NN) lines indicate the outcomes of the scheme-invariant analysis at the NLO and NNLO respectively. The fitted values of $\Lambda_{\text{MS}}$ are normalized to $f = 4$ numbers of flavours.
The analysis of Ref. 96 was made with the help of the method of the reconstruction of the SF from their moments using the expansion of the SF over the Jacobi polynomials 97, 98:

\[
F^{N_{\text{max}}}_i = x^\alpha (1 - x)^\beta \sum_{n=0}^{N_{\text{max}}} \Theta_n^\alpha \beta (x) \sum_{j=0}^{n} c_j^{(n)} (\alpha, \beta) M^{(j+2)} (Q^2) \tag{29}
\]

where \(c_j^{(n)} (\alpha, \beta)\) are the coefficients that are expressed through \(\Gamma\)-functions and the parameters \(\alpha, \beta\) can be chosen such as to achieve the fastest convergence of the series in the r.h.s. of Eq. (29). It should be stressed that in order to perform the fit of the data at the NNLO self-consistently it is necessary to take into account the NNLO corrections to the corresponding anomalous dimension functions. For the non-singlet moments \(M^{(n)}_{\text{NS}}\) of the SF \(F_2\), they were recently analytically calculated in the case of \(n = 2, 4, 6, 8\) 99. For \(n = 3, 5, 7\) the NNLO coefficients of the \(\gamma_{\text{NS}}\)-function were estimated 96 using the smooth interpolation of the results of Ref. 99 to the case of odd moments.

Let us make several comments on the physical results presented in Table 4. One can see that the inclusion directly in the \(\overline{\text{MS}}\) scheme of the NNLO corrections in the analysis of the experimental data slightly decreases by over 20 MeV the NLO values of the parameter \(\Lambda^{(4)}_{\overline{\text{MS}}}\). Therefore, taking into account these effects cannot remove the slight difference existing at present between the central values of \(\alpha_s\) extracted from the previous analysis of DIS data (see Table 1) and from the LEP measurements. The most important phenomenological outcome of the corresponding NNLO analysis is the observation of the minimization of the difference between the results obtained within the framework of the \(\overline{\text{MS}}\) scheme and the scheme-invariant approach. This observation is in agreement with the results of the study of the higher-order QCD corrections to \(R(s)\) and to the Gross-Llewellyn Smith SR within different approaches of fixing the scheme-dependence ambiguities 100, 49.

The progress discussed above in the detailed consideration of different effects of perturbative QCD for the SFs and their moments in the kinematical region \(Q^2 \rightarrow \infty, x\) fixed, should not shadow down the continuous interest in the description of the dynamics of the behaviour of the SFs in the limit \(x \rightarrow 0, Q^2\) fixed. The behaviour of the singlet SFs in this limit is governed by the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation. The partonic picture of the evolution of the SF \(F_2(x, Q^2)\) both for increasing \(Q^2\) and for decreasing \(x\) was discussed at the Symposium by Kirschner 101. It should be stressed that contrary to the case of polarized DIS, theory is going ahead of the experiment in the investigations of the physics typical to the low \(x\) limit. Indeed, the results of the recent experimental measurements at HERA at values of \(x\) down to \(10^{-4}\), presented in the detailed talk of Obrock 102, demonstrate the fast rise of \(F_2(x, Q^2)\) at low \(x\)-values. This effect can be associated with the prediction, which follows from the BFKL considerations, namely

\[
F_2(x, Q^2) \approx \left( \frac{1}{x} \right)^{\omega_0} \tag{30}
\]

where \(\omega_0 = (g^2/2\pi^2)N2 \ln 2\) and \(N\) is the number of colours. Another preliminary experimental result of HERA, namely the counting of the number of the events typical of the forward jets in the data of the H1 collaboration, is presented in Table 5, taken from the report of Obrock 102.
Table 5. Comparison of the number of events seen in the preliminary data of H1 with theoretical expectations.

One can conclude that this analysis is very promising to understand whether HERA results are sensitive to the BFKL evolution.

Apart from the physics at low $x$ the experimental program of HERA is also includes the study of “classical” problems. Indeed, as was emphasized by Obrock it is possible to measure the value of $\alpha_s$ at HERA using the process of the gluon jet emission in the 2+1 outcoming jets. The preliminary results of these measurements are compared with the LEP results at Fig. 8 taken from the talk of Obrock.

The preliminary result with all estimated uncertainties reads $\alpha_s(M_z) = 0.121 \pm 0.003 \pm 0.003 \pm 0.006 \pm 0.005 \pm 0.006$. It is obvious that more statistics are necessary in order to get a more precise result.

The further accumulation of experimental data is also important for the more detailed studies of the physics of low $x$. The theoretical investigations of the effects typical of this region are also in the process of fast development. The interesting possibility that the physics of low $x$ can be identical to the physics of two-dimensional exactly solvable models was discussed in detail by Kirschner. In one of the recent works it was even shown that the low- $x$ limit can be described by the generalization of the Heisenberg XXX model, namely by a periodic lattice spin-0 model.

In conclusion of this section, let us return from the world of low $x$ to the world of large $Q^2$, namely from HERA to LEP. In the talk of Fürstenau the new result of the extraction of $\alpha_s(M_Z)$ from the LEP data for FFs was presented. The obtained preliminary value $\alpha_s(M_Z) = 0.118 \pm 0.005$ is in agreement with the world average value of $\alpha_s$ presented by Bethke. As was already mentioned, the theoretical formalism of the description of the behaviour of the FFs is very similar to the one commonly used in the analysis of the SFs. This fact gives us the idea that we can expect soon the appearance of new, more detailed, results of the determination of $\alpha_s$ from the LEP experimental data for FFs based on the generalization of the methods of the DIS analysis and their application to the physics of FFs.

6. The Connections Between Physical Quantities

In all previous discussions we considered the $\overline{MS}$ scheme as the reference method for fixing the scheme-dependence ambiguities of the perturbative QCD predictions. However, there are also other approaches to treat these ambiguities in the higher orders of perturbation theory. The appearance of several complete NNLO QCD results makes it possible to apply these different prescriptions in practice and to study the related phenomenological and theoretical outcomes of these analyses. The definite steps in the direction of detailed studies of the results of applications of the ECH prescription and of the PMS method have already been done.

Some other aspects of these studies were discussed at the Symposium in the talk by Brodsky, which was based on some recent work. In this work the ECH method (or the so-called scheme-invariant perturbation theory) was used to express the NNLO perturbative QCD predictions for one observable quantity through another one. The considered physical quantities were $R(s)$, $R_\tau$ and the DIS SRs. At the second step of their considerations, in order to get rid of the dependence on the
number of flavours $f$ in the resulting “commensurate scale relations”, the authors of Ref. [107] applied the BLM prescription [108] and the variant of its NNLO generalization [109]. Several comments on the typical features of the methods used in the process of the derivation of these relations, which provide the additional possibilities for testing perturbative QCD predictions, are now in order.

First, the application of the ECH prescription does not eliminate the ambiguities typical of the perturbative expansions. Indeed, instead of solving the guess about the “basic phenomenological scheme” for the definition of $\alpha_s$, one should now write down the agreement about the basic physical quantity, which should be used as the reference in all corresponding perturbative expansions. The authors of Ref. [107] are proposing for this role the effective quark potential. However, in view of the absence of information about the NNLO perturbative corrections to this quantity, it is impossible at present to check the advantages of this choice self-consistently.

Secondly, one should be careful in applying the BLM prescription in the high orders of perturbation theory. For example, the comparison of the results of the application of this procedure to the analysis of the effects of the perturbative QCD corrections to the $\rho$ parameter with the results of the exact calculations demonstrates that the BLM approach is missing the numerical contribution most important in this case, which comes from the three-loop diagram with the triangle fermion loop insertions. Quite fortunately, in the case of $R_\tau$ and of the polarized Bjorken sum rule the light-by-light-type graphs are absent, while the contribution of these graphs to $R(s)$ and to the Gross-Llewellyn Smith SR is small. However, even in this case the applications of the main relation derived in Ref. [107], namely

$$\hat{\alpha}_{g_1}(Q) = \hat{\alpha}_R(Q^*) - \hat{\alpha}_R^2(Q^{**}) + \hat{\alpha}_R^3(Q^{***}) + \ldots,$$  

where $\hat{\alpha} = (3C_F/4\pi)\alpha$ and $\alpha_{g_1}$ and $\alpha_R$ are the effective charges of the polarized Bjorken SR and $R(s)$, the BLM method is shadowing the effects of new physics, which obviously manifest themselves in the $\overline{MS}$ scheme.

Indeed, the simplicity of the relation of Eq. (31) is connected with the Crewther quark-parton relation [111] between three fundamental quantities, namely the anomalous constant $S$, associated with the amplitude of the $\pi^0 \to \gamma\gamma$ decay, the polarized Bjorken SR BjpSR and the Adler $D$-function in the annihilation channel. The derivation of this relation relied on the conformal and the chiral invariance. However, it was shown [112] that the naive Crewther relation, at the NLO and NNLO of perturbation theory, receives additional radiative corrections, which are proportional to the $\beta$-function, or more definitely to the factor $\beta(a)/a$:

$$\Delta_S = BjpSR \times D - 1 = \frac{\beta(a)}{a} \left[ S_1 C_F a + \left( S_2 T_F f + S_A C_A + S_F C_F \right) C_F a^2 \right] + O(a^4) \quad (32)$$

where $C_F = 4/3$, $C_A = 3$, $T_F = 1/2$ in the case of QCD and $S_1 = -21/2 + 12\zeta(3)$, $S_2 = 326/3 - (304/3)\zeta(3)$, $S_A = -629/2 + (884/3)\zeta(3)$ and $S_F = 397/6 + 136\zeta(3) - 240\zeta(5)$ are the analytical numbers independent of the structure of the gauge group. It is now possible to understand that Eq. (31) can be obtained from Eq. (32) in the conformal invariant limit, namely after the nullification of the factor $\Delta_S$. However, the applications of the BLM approach presented in Ref. [107] do not allow us to understand the origin of the appearance of this factor in the fixed schemes. Work on the study of this fact is now in progress [113]. The puzzle, discovered in Ref. [114] of the cancellations of the $C_F^2$ factors in the $N$-th order approximation for $\Delta_S$ is already understood [115]. This cancellation is the consequence of the Adler-Bardeen theorem. The rigorous
theoretical explanation of the remaining “wonders” from the “seven” ones observed in Ref. 112 is on the agenda.

We hope that the possible outcomes of these studies will have both theoretical and phenomenological consequences, which might be important for the more detailed understanding of the experimental numbers obtained for the polarized Bjorken SR and for the Gross-Llewellyn Smith SR, extracted from the data of the CCFR collaboration 115 in Refs. 116, 117.

7. Conclusion

Summarizing the discussions of the current status of QCD, presented at this Symposium, it is necessary to stress that in general this theory is in good condition. However, a lot of important problems are still waiting for a solution. Among them are

1. the calculations of the NLO corrections to the 4-jets characteristics;

2. more rigorous and precise determinations of the values of light and heavy quark masses;

3. the necessity to understand the origin of the certain discrepancies between the values of \( \alpha_s \) extracted from different processes.

4. Clearly, the desire to understand this problem is supporting new experimental measurements and new more precise calculations, say the above-mentioned calculations of the higher-order perturbative QCD corrections to the cross sections of the \( e^+e^- \rightarrow 4 \) jets and \( pp \rightarrow 3 \) jets processes.

5. There are a number of calculational problems, which are related to the recent development of the new rigorous theory of the study of the heavy quarkonium annihilation rates 118, which was discussed at the Symposium in the talk by Braaten 119. This theory allows to calculate the related characteristics from the first principles, the only input being the heavy-quark mass and the QCD coupling constant. Concrete applications of this formalism are on the agenda.

6. The next important problem is related to the necessity of more detailed studies of the effects of the HT corrections in the polarized DIS. The future works can provide a more rigorous control of this type of the theoretical uncertainties in the description of the corresponding experimental data. This problem is of particular interest in view of the existence of programs of future measurements of the polarized DIS SFs at SLAC and CERN.

7. In a future analysis of the new data on both polarized and non-polarized DIS the problem of the parametrization of the behaviour of the SFs at low \( x \) should be considered very carefully. Note that this limit is not totally described by the perturbation theory. The possibilities provided by the non-perturbative methods, and in particular by the mathematical methods typical of the exactly solvable models, should be developed to the new level.

8. There are still a number of problems that should be understood on the new level within the framework of the perturbation theory. For example different points of view on different methods of treating the scheme-dependence problem still do not allow us to write down the convention about the best way of controlling the theoretical uncertainties in the analysis of the effects of the higher-order perturbative QCD corrections.
9. Even more important can be the breakthrough in the understanding of the theoretical structure of the perturbative series in QCD and gauge models.

10. It is important to find some new methods of testing the predictions of the perturbative QCD. We believe that the future, more detailed studies of the Crewther relation and its different generalizations might give us the opportunity of comparing theoretical and experimental results for the DIS SRs with the ones for the annihilation processes on the new level.

To conclude, let us hope that the era of perturbative QCD is not finished and that we can expect the appearance of new interesting and important results.

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