Symmetry and geometry in
generalized Higgs effective field theory

– Finiteness of oblique corrections v.s. perturbative unitarity –

Ryo Nagai,1,2,* Masaharu Tanabashi,3,4,† Koji Tsumura,5,‡ and Yoshiki Uchida3,§

1 Institute for Cosmic Ray Research (ICRR),
The University of Tokyo, Kashiwa, Chiba 277-8582, Japan
2 Department of Physics, Tohoku University, Sendai, Miyagi 980-8578, Japan
3 Department of Physics, Nagoya University, Nagoya 464-8602, Japan
4 Kobayashi-Maskawa Institute for the Origin of Particles and the Universe,
Nagoya University, Nagoya 464-8602, Japan
5 Department of Physics, Kyoto University, Kyoto 606-8502, Japan

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Abstract

We formulate a generalization of Higgs effective field theory (HEFT) including arbitrary number of extra neutral and charged Higgs bosons (generalized HEFT, GHEFT) to describe non-minimal electroweak symmetry breaking models. Using the geometrical form of the GHEFT Lagrangian, which can be regarded as a nonlinear sigma model on a scalar manifold, it is shown that the scalar boson scattering amplitudes are described in terms of the Riemann curvature tensor (geometry) of the scalar manifold and the covariant derivatives of the potential. The one-loop divergences in the oblique correction parameters $S$ and $U$ can also be written in terms of the Killing vectors (symmetry) and the Riemann curvature tensor (geometry). It is found that perturbative unitarity of the scattering amplitudes involving the Higgs bosons and the longitudinal gauge bosons demands the flatness of the scalar manifold. The relationship between the finiteness of the electroweak oblique corrections and perturbative unitarity of the scattering amplitudes is also clarified in this language: we verify that once the tree-level unitarity is ensured, then the one-loop finiteness of the oblique correction parameters $S$ and $U$ is automatically guaranteed.

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*E-mail: r nagai@icrr.u-tokyo.ac.jp
†E-mail: tanabash@eken.phys.nagoya-u.ac.jp
‡E-mail: ko2@gauge.scphys.kyoto-u.ac.jp
§E-mail: uchida@eken.phys.nagoya-u.ac.jp
I. INTRODUCTION

What is the origin of the electroweak symmetry breaking (EWSB)? In the standard model (SM) of particle physics, the EWSB is caused by a vacuum expectation value of a complex scalar field (SM Higgs field), which linearly transforms under the $SU(2)_W \times U(1)_Y$ electroweak gauge symmetry. The Higgs sector of the SM is constructed to be minimal, as it includes only a scalar boson (SM Higgs boson) and three would-be Nambu-Goldstone bosons eaten by massive gauge bosons after the EWSB. There are no cousin particles of Higgs in the SM. The scalar particle discovered by the ATLAS and CMS experiments in 2012 with the mass of 125 GeV [1, 2] can now be successfully interpreted as the SM(-like) Higgs boson.

The Higgs sector in the SM, however, does not ensure the stability of the EWSB scale against quantum corrections. In other words, the SM itself cannot explain why the EWSB scale is an order of 100 GeV, much smaller than its cutoff scale such as Planck (or Grand Unification) scale. The SM Higgs sector is therefore inherently incomplete. It should be extended. Many extensions/generalizations of the SM Higgs sector, such as Two Higgs Doublet Model [3–24], Composite Higgs Models [25–34], Georgi-Machacek Model [35–38], etc., have been proposed. The 125GeV Higgs boson accompanies extra Higgs particles in these scenarios.

The Effective Field Theory (EFT) approach is widely used to study these beyond-SM (BSM) physics in a model independent manner. The physics below 1TeV can be described by the Standard Model Effective Field Theory (SMEFT) [39–66], which parametrizes the BSM contributions using the coefficients of SM field higher dimensional operators. The SMEFT is successful if the BSM particles are much heavier than 1TeV and they decouple from the low energy physics. The SMEFT cannot be applied, however, if the heavy BSM particles do not decouple from the low energy physics. The Higgs Effective Field Theory (HEFT) [67–83] should be applied instead. These existing EFTs cannot be applied if there exist BSM particles lighter than 1TeV. We should include these BSM particles explicitly in the EFT approach.

In this paper, we propose a generalization of HEFT (GHEFT) for this purpose. As in the HEFT, GHEFT is based on the electroweak chiral perturbation theory (EWChPT) [84–89]. In GHEFT, the BSM particles, as well as the 125GeV Higgs boson, are introduced as matter
particles in the Callan-Coleman-Wess-Zumino (CCWZ) construction [90–92] of EWChPT.

Note that the longitudinal gauge boson scattering amplitudes exceed perturbative unitarity limits at high energy in the EWChPT. The GHEFT couplings should satisfy special conditions, known as the unitarity sum rules [93–95], to keep the amplitudes perturbative in the high energy scatterings, if the model is considered to be ultraviolet (UV) complete. We also note that the EWChPT is not renormalizable. The UV completed GHEFT couplings should satisfy the finiteness conditions in order to cancel these UV divergences.

The GHEFT can also be described in a geometrical language using the scalar manifold metric, as discussed in Refs. [96, 97] in the HEFT context. We point out that both the scalar scattering amplitudes and the one-loop UV divergences in the electroweak oblique correction parameters $S$ and $U$ [98] are described by using the Riemann curvature tensor (geometry) and the Killing vectors (symmetry) of the scalar manifold. Therefore, both the unitarity sum rules and the oblique correction finiteness conditions are described in terms of the geometry and the symmetry. We find that the perturbative unitarity is ensured by the flatness of the scalar manifold (vanishing Riemann curvature). We also find that the divergences in the oblique correction parameters ($S$ and $U$ parameters) are canceled if a subset of the perturbative unitarity conditions and the $SU(2)_W \times U(1)_Y$ gauge symmetry are satisfied. These findings generalize our previous observation [99] which relates the perturbative unitarity to the one-loop finiteness of the oblique correction parameters$^1$.

This paper is organized as follows: in §. II we introduce the GHEFT Lagrangian at its lowest order ($\mathcal{O}(p^2)$). We investigate the scalar boson scattering amplitudes in §. III. §. IV and §. V are for one-loop computations with and without the gauge boson contributions. The relationship between the perturbative unitarity and the one-loop finiteness of the oblique correction parameters is clarified in §. VI. We conclude in §. VII.

II. GENERALIZED HEFT LAGRANGIAN OF $SU(2)_W \times U(1)_Y \to U(1)_{\text{em}}$

The electroweak chiral perturbation theory (EWChPT) [84–89] provides a systematic framework to describe the low energy phenomenologies of the electroweak symmetry breaking physics. It utilizes the electroweak chiral Lagrangian method for parametrizing the

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$^1$ Possible relations between the unitarity and the renormalizability have also been investigated in gravity models. See Refs. [100–104].
non-decoupling corrections, which appear ubiquitously in models with strongly interacting electroweak symmetry breaking sector. Although the original version of the EWChPT was constructed to be a Higgsless theory [94, 95, 105–114], after the discovery of the 125GeV Higgs particle, the EWChPT is extended to the Higgs Effective Field Theory (HEFT), incorporating the 125 GeV Higgs particle $h$ as a neutral spin-0 matter particle in the electroweak chiral Lagrangian [67–83]. Introducing functions $F(h)$ and $V(h)$, which parametrize the phenomenological properties of the 125GeV Higgs, the HEFT provides a systematic description for a neutral spin-0 particle in the electroweak symmetry breaking sector, including the one-loop radiative corrections [72–83]. It can parametrize the low energy properties of the 125GeV Higgs particle in the strongly interacting model context, as well as weakly interacting model context.

We need to generalize the HEFT further (generalized HEFT, GHEFT), if we want to introduce extra Higgs particles other than the discovered 125GeV Higgs particle. It is not trivial to introduce non-singlet extra particles in the EWChPT, however, since the electroweak gauge symmetry $SU(2)_W \times U(1)_Y$ is realized nonlinearly in the EWChPT. The interaction Lagrangian needs to be arranged carefully to make the theory invariant under the electroweak gauge symmetry $SU(2)_W \times U(1)_Y$.

These extra non-singlet Higgs particles can be regarded as matter particles in the EWChPT Lagrangian context. The Callan-Coleman-Wess-Zumino (CCWZ) formulation [90–92] provides an ideal framework for the concrete construction of the matter particle interaction Lagrangian in a manner consistent with the nonlinear sigma model symmetry structure.

In this section, we apply the CCWZ formulation for the construction of the GHEFT Lagrangian.

A. Electroweak chiral Lagrangian

For simplicity, in this subsection, we consider the EWChPT Lagrangian in the gaugeless limit, i.e., $g_W = g_Y = 0$. The couplings with the electroweak gauge fields will be re-introduced in §. II C. The electroweak symmetry $G = [SU(2)_W \times U(1)_Y]$ is broken spontaneously to the $H = U(1)_{\text{em}}$ symmetry in the SM Higgs sector. The most general scalar sector Lagrangian consistent with the symmetry breaking structure $G/H =$
\[ [SU(2)_W \times U(1)_Y]/U(1)_{em} \text{ can be constructed as the CCWZ nonlinear sigma model Lagrangian on the coset space } G/H. \text{ The coset manifold } G/H = [SU(2)_W \times U(1)_Y]/U(1)_{em} \text{ is coordinated by the Nambu-Goldstone (NG) boson fields } \pi^a \text{ (} a = 1, 2, 3 \text{)} \text{ as} \]

\[ \xi_W(x) = \exp \left( i \sum_{a=1,2} \pi^a(x) \frac{\tau^a}{2} \right), \quad (1) \]

\[ \xi_Y(x) = \exp \left( i \pi^3(x) \frac{\tau^3}{2} \right), \quad (2) \]

\[ \text{with } \tau^a \text{ (} a = 1, 2, 3 \text{)} \text{ being Pauli spin matrices. Under the } G = [SU(2)_W \times U(1)_Y] \text{ transformation,} \]

\[ g_W \in SU(2)_W, \quad g_Y \in U(1)_Y, \quad (3) \]

\[ \text{these NG boson fields transform as} \]

\[ \xi_W(x) \rightarrow \xi'_W(x) = g_W \xi_W(x) h^\dagger (\pi, g_W, g_Y), \quad (4) \]

\[ \xi_Y(x) \rightarrow \xi'_Y(x) = h(\pi, g_W, g_Y) \xi_Y(x) g_Y^\dagger. \quad (5) \]

\[ \text{Here } h(\pi, g_W, g_Y) \text{ is an element of the unbroken group } H, \text{ which is determined to pull-back the coset space coordinates to their original forms (1) and (2). Note that the } H \text{ transformation } h(\pi, g_W, g_Y) \text{ depends not only on the } SU(2)_W \text{ and } U(1)_Y \text{ elements } g_W \text{ and } g_Y, \text{ but also on the NG boson fields } \pi(x). \text{ The NG boson fields } \pi^a \text{ (} a = 1, 2, 3 \text{)} \text{ therefore transform nonlinearly under the } G \text{ symmetry.} \]

\[ \text{It is useful to introduce objects called Maurer-Cartan (MC) one-forms } \alpha^a_{\perp \mu} \text{ (} a = 1, 2, 3 \text{)} \text{ defined as} \]

\[ \alpha^a_{\perp \mu} = \text{tr} \left[ -\frac{1}{i} \xi_W^\dagger (\partial_\mu \xi_W) \tau^a \right], \quad (a = 1, 2) \quad (6) \]

\[ \text{and} \]

\[ \alpha^3_{\perp \mu} = \text{tr} \left[ -\frac{1}{i} \xi_W^\dagger (\partial_\mu \xi_W) \tau^3 \right] + \text{tr} \left[ -\frac{1}{i} (\partial_\mu \xi_Y) \xi_Y^\dagger \tau^3 \right]. \quad (7) \]

\[ \text{Although the NG boson fields } \pi \text{ transform nonlinearly, these MC one-forms transform homogeneously, } i.e., \]

\[ \sum_{a=1,2} \alpha^a_{\perp \mu} \frac{\tau^a}{2} \rightarrow h(\pi, g_W, g_Y) \left( \sum_{a=1,2} \alpha^a_{\perp \mu} \frac{\tau^a}{2} \right) h^\dagger (\pi, g_W, g_Y), \quad (8) \]

\[ \alpha^3_{\perp \mu} \frac{\tau^3}{2} \rightarrow h(\pi, g_W, g_Y) \left( \alpha^3_{\perp \mu} \frac{\tau^3}{2} \right) h^\dagger (\pi, g_W, g_Y), \quad (9) \]
under the $G$ symmetry. We see that the MC one-forms transform as
\[ \alpha_{\perp \mu}^a \to [\rho_\alpha(\mathfrak{h})]^a \mathfrak{h}_{\perp \mu}^b \alpha_{\perp \mu}^b, \] (10)
with $\rho_\alpha(\mathfrak{h})$ being a $3 \times 3$ matrix
\[ \rho_\alpha(\mathfrak{h}) = \exp \left( i \theta_\mathfrak{h}(\pi, \mathfrak{g}_W, \mathfrak{g}_Y) Q_\alpha \right), \quad \mathfrak{h} = \exp \left( i \theta_\mathfrak{h}(\pi, \mathfrak{g}_W, \mathfrak{g}_Y) \tau^3 \right). \] (11)
In the expression (10) and hereafter, summation $\sum_{b=1,2,3}$ is implied whenever an index $b$ is repeated in a product. Here the NG boson charge matrix $Q_\alpha$ is defined by
\[ Q_\alpha = \begin{pmatrix} -\sigma_2 & 0 \\ 0 & 0 \end{pmatrix}, \] (12)
with $\sigma_2$ being the Pauli spin matrix
\[ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \] (13)

It is now straightforward to construct the lowest order ($O(p^2)$) $G$ invariant Lagrangian of the NG bosons:
\[ \mathcal{L}_\pi = \frac{1}{2} G_{ab}^{(0)} \alpha_{\perp \mu}^a \alpha_{\perp \mu}^b, \] (14)
with
\[ G_{ab}^{(0)} = \frac{1}{4} \begin{pmatrix} v^2 & 0 \\ 0 & v^2 \end{pmatrix}, \] (15)
The Lagrangian can be rewritten as
\[ \mathcal{L}_\pi = \frac{v^2}{4} \text{tr} \left[ (\partial_\mu U^\dagger) (\partial^\mu U) \right] - \frac{v_Z^2}{8} v^2 \text{tr} \left[ U^\dagger (\partial_\mu U) \tau^3 \right] \text{tr} \left[ U^\dagger (\partial^\mu U) \tau^3 \right], \] (16)
with
\[ U := \xi_W \xi_Y. \] (17)
It should be emphasized here that $v$ and $v_Z$ (decay constants of $\pi^1,2$ and $\pi^3$) are independently adjustable parameters in the EWChPT on the $G/H = [SU(2)_W \times U(1)_Y]/U(1)_{em}$ coset space. Phenomenologically preferred relation
\[ \rho := \frac{v^2}{v_Z^2} \simeq 1 \] (18)
is realized only by a parameter tuning $v \simeq v_Z$ in this setup.
B. Matter particles coupled with the electroweak chiral Lagrangian

Thanks to the homogeneous transformation properties of the MC one-forms (10), matter particles can be introduced easily in the CCWZ formulation of the EWChPT Lagrangian (16).

We consider a set of real scalar matter fields $\phi^I$, which transforms homogeneously as

$$\phi^I \to [\rho_\phi(h)]^I_J \phi^J,$$  \hspace{1cm} (19)

under the unbroken group $H$. Here $\rho_\phi(h)$ stands for a representation matrix

$$\rho_\phi(h) = \exp \left( i \theta_\phi Q_\phi \right), \quad h = \exp \left( i \theta h \tau_3^2 \right),$$  \hspace{1cm} (20)

with $Q_\phi$ being a hermitian matrix. Note here that the $h$ transformation depends on the NG boson fields $\pi(x)$. It therefore is a local transformation depending on the space-time point $x$. If the set of scalar matter particles consists of $n_N$ species of neutral particles and $n_C$ species of charged particles, the matrix $Q_\phi$ can be expressed as a $(2n_C + n_N) \times (2n_C + n_N)$ matrix

$$Q_\phi = \begin{pmatrix} -q_1 \sigma_2 & & & \\ & \ddots & & \\ & & -q_{n_C} \sigma_2 & \\ & & & 0 \end{pmatrix} .$$  \hspace{1cm} (21)

Here $q_i$ ($i = 1, 2, \cdots n_C$) are the charges of the scalar matter particles. Since $h$ is a local transformation, $\partial_\mu \phi^I$ transforms non-homogeneously under $h$. In order to write a kinetic term for the matter field $\phi^I$, we therefore introduce a covariant derivative of the matter field $\phi^I$:

$$(D_\mu \phi)^I = \partial_\mu \phi^I + i \nu_\mu^3 [Q_\phi]^I_J \phi^J, \quad (I, J = 1, 2, \cdots, 2n_C + n_N).$$  \hspace{1cm} (22)

We take the connection $\nu_\mu^3$ as

$$\nu_\mu^3 = -\text{tr} \left[ \frac{1}{i} (\partial_\mu \xi Y) \xi Y^\dagger \tau_3 \right] + c \alpha_{3.\mu},$$  \hspace{1cm} (23)

with $c$ being an arbitrary constant. Hereafter we take $c = 0$ for simplicity. The covariant derivative (22) transforms homogeneously

$$(D_\mu \phi)^I \to [\rho_\phi(h)]^I_J (D_\mu \phi)^J,$$  \hspace{1cm} (24)
as we designed so in Eq. (22). It is now straightforward to write down an $O(p^2)$ EWChPT Lagrangian including additional scalar bosons with arbitrary charges:

$$\mathcal{L} = \frac{1}{2} G_{ab} \alpha^{a}_{\perp \mu} \alpha^{b}_{\perp} + G_{aI} \alpha^{a}_{\perp \mu} (D^\mu \phi)^I + \frac{1}{2} G_{IJ} (D_{\mu} \phi)^I (D^\mu \phi)^J - V. \quad (25)$$

Here $G_{ab}, G_{aI}, G_{IJ}$ and $V$ are functions of the scalar fields $\phi^I$. Also, $G_{ab}, G_{aI}$ and $G_{IJ}$ transform homogeneously as multiplets of corresponding representations. They satisfy

$$G_{ab} \mid_{\phi=0} = G_{ab}^{(0)}, \quad G_{aI} \mid_{\phi=0} = 0, \quad G_{IJ} \mid_{\phi=0} = \delta_{IJ}, \quad (26)$$

and

$$\frac{\partial}{\partial \phi^I} V \mid_{\phi=0} = 0, \quad \frac{\partial}{\partial \phi^I} \frac{\partial}{\partial \phi^J} V \mid_{\phi=0} = M_I^2 \delta_{IJ}, \quad (27)$$

with $M_I$ being the $\phi^I$ boson mass. The second and the third conditions in Eq. (26) can be achieved by redefining the scalar field $\phi^I$ in the Lagrangian. The first condition in Eq. (26) ensures that the extended Lagrangian (25) reproduces the lowest order EWChPT Lagrangian (14) in the absence of Higgs particles $\phi^I$. The stability around the vacuum $\phi = 0$ is guaranteed by the conditions (27).

C. Electroweak gauge fields

It is easy to reintroduce the electroweak gauge fields $W^a_\mu \ (a = 1, 2, 3)$ and $B_\mu$ in our EWChPT Lagrangian (25). When the gauge coupling is switched on, we just need to replace the derivatives $\partial_\mu \xi_W$ and $\partial_\mu \xi_Y$ by the covariant derivatives:

$$D_\mu \xi_W = \partial_\mu \xi_W - ig_W W^a_\mu \frac{\tau^a}{2} \xi_W, \quad (28)$$

$$D_\mu \xi_Y = \partial_\mu \xi_Y + ig_Y \xi_Y B_\mu \frac{\tau^3}{2}, \quad (29)$$

with $g_W$ and $g_Y$ being the $SU(2)_W$ and $U(1)_Y$ gauge coupling strengths, respectively.

The lowest order ($O(p^2)$) GHEFT Lagrangian is therefore

$$\mathcal{L} = \frac{1}{2} G_{ab} \hat{\alpha}^{a}_{\perp \mu} \hat{\alpha}^{b}_{\perp} + G_{aI} \hat{\alpha}^{a}_{\perp \mu} (D^\mu \phi)^I + \frac{1}{2} G_{IJ} (D_{\mu} \phi)^I (D^\mu \phi)^J - V$$

$$- \frac{1}{4} W^a_{\mu \nu} W^{a \mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu}, \quad (30)$$

with

$$\hat{\alpha}^{a}_{\perp \mu} = \text{tr} \left[ \frac{1}{i} \xi_W^\dagger (D_\mu \xi_W)^\tau^a \right] - g_W \text{tr} \left[ \xi_W^\dagger W^a_\mu \frac{\tau^b}{2} \xi_W \tau^a \right], \quad (a = 1, 2) \quad (31)$$
and
\[ \hat{\alpha}^2_{3\mu} = \text{tr} \left[ \frac{1}{i} \xi^i (\partial_\mu \xi^i) \tau^3 \right] + \text{tr} \left[ \frac{1}{i} (\partial_\mu \xi^i) \xi^i \tau^3 \right] - g_W \text{tr} \left[ \xi^i W^b_\mu \frac{\tau^b}{2} \xi W \tau^3 \right] + g_Y B_\mu. \] (32)

We define the covariant derivative of the matter fields \((D_\mu \phi)^I\)
\[ (D_\mu \phi)^I = \partial_\mu \phi^I + i \hat{\gamma}^3_\mu [Q_\phi]^I_J \phi^J, \] (33)
with
\[ \hat{\gamma}^3_\mu = - \text{tr} \left[ \frac{1}{i} (\partial_\mu \xi^i) \xi^i \tau^3 \right] - g_Y B_\mu. \] (34)

It should be noted that the GHEFT Lagrangian (30) reproduces HEFT Lagrangian [67–83] for \(n_N = 1\) and \(n_C = 0\). Here \(\phi^I\) stands for the 125 GeV Higgs boson field. In the HEFT, \(G_{al}\) and \(G_{IJ}\) are taken as
\[ G_{ah} = 0, \quad G_{hh} = 1. \] (35)

\(G_{ab}\) is tuned to be
\[ G_{ab} = \frac{v^2}{4} F(h) \delta_{ab}. \] (36)

D. Geometrical form of the \(\mathcal{O}(p^2)\) GHEFT Lagrangian

The lowest order \((\mathcal{O}(p^2))\) GHEFT Lagrangian (30) can also be expressed in a geometrical form:
\[ \mathcal{L} = \frac{1}{2} g_{ij}(\phi) D_\mu \phi^i D^\mu \phi^j - V(\phi) - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \] (37)
where \(\phi^i\) stands a scalar field multiplet containing both Higgs bosons \(\phi^I\) and the NG bosons \(\pi^a\) as its component, i.e.,
\[ \{ \phi^i \} = \{ \pi^a, \phi^I \}. \] (38)
The geometrical form of the GHEFT Lagrangian (37) can be understood as a gauged non-linear sigma model on a scalar manifold. The scalar manifold (internal space) is coordinated by the scalar multiplet \(\phi^i\). Both the metric \(g_{ij}(\phi)\) and the potential \(V(\phi)\) are functions of
\( \phi^i \). They should be invariant under the \( SU(2)_W \times U(1)_Y \) transformation:

\[
0 = w^k_{a} g_{ij,k} + (w^k_{a}),_{j} g_{ik} , \quad (39)
\]

\[
0 = y^k g_{ij,k} + (y^k),_{j} g_{ik} , \quad (40)
\]

\[
0 = w^k V_{,k} , \quad (41)
\]

\[
0 = y^k V_{,k} , \quad (42)
\]

with

\[
g_{ij,k} := \frac{\partial}{\partial \phi^k} g_{ij} , \quad V_{,k} := \frac{\partial}{\partial \phi^k} V , \quad (w^k_{a}),_{j} := \frac{\partial}{\partial \phi^j} w^k_{a} , \quad (y^k),_{j} := \frac{\partial}{\partial \phi^j} y^k . \quad (43)
\]

The \( SU(2)_W \) and \( U(1)_Y \) Killing vectors are denoted by \( w^k_{a} \) \((a = 1, 2, 3)\) and \( y^k \), respectively, in Eqs. \((39)-(42)\). The GHEFT Lagrangian \((30)\) provides the most general and systematic method to construct the geometrical form of the Lagrangian \((37)\) having these symmetry properties \((39)-(42)\). The translation dictionary from the GHEFT Lagrangian \((30)\) to the geometrical form \((37)\) will be published elsewhere.

The \( SU(2)_W \times U(1)_Y \) gauge interactions are introduced in the scalar sector through the covariant derivative

\[
D_{\mu} \phi^i = \partial_{\mu} \phi^i + g_{W} W_{\mu}^a w^i_{a} (\phi) + g_{Y} B_{\mu} y^i (\phi) . \quad (44)
\]

It should be noted that the gauge fields interact with the scalar sector through the \( SU(2)_W \times U(1)_Y \) Killing vectors \( w^k_{a} \) and \( y^k \).

The scalar potential \( V(\phi) \) should be minimized at the vacuum,

\[
\langle \phi^i \rangle = \bar{\phi}^i . \quad (45)
\]

Note that, since the electroweak symmetry is spontaneously broken at the vacuum, the vacuum \( \phi^i = \bar{\phi}^i \) cannot be a fixed point of the \( SU(2)_W \times U(1)_Y \) transformation, \( \text{i.e.,} \)

\[
w^i_{a} (\bar{\phi}) \neq 0 , \quad y^i (\bar{\phi}) \neq 0 . \quad (46)
\]

It should be a fixed point of the \( U(1)_{\text{em}} \) transformation,

\[
w^i_{3} (\bar{\phi}) + y^i (\bar{\phi}) = 0 , \quad (47)
\]

however. The electroweak gauge bosons (\( W \) and \( Z \)) acquire their masses

\[
M^2_{W} \propto g^2_{W} g_{ij} (\bar{\phi}) w^i_{1} (\bar{\phi}) w^j_{1} (\bar{\phi}) , \quad M^2_{Z} \propto (g^2_{W} + g^2_{Y}) g_{ij} (\bar{\phi}) w^i_{3} (\bar{\phi}) w^j_{3} (\bar{\phi}) . \quad (48)
\]
The Killing vectors at the vacuum (46) therefore play the role of the Higgs vacuum expectation value in the SM. It should be emphasized that the vanishing scalar vacuum expectation value $\bar{\phi}^i = 0$ does not imply the electroweak symmetry recovery in the GHEFT Lagrangian. Actually, in the GHEFT coordinate (38), even though the vacuum expectation values of the scalar fields are all vanishing $\bar{\phi}^i = 0$, the electroweak symmetry is still spontaneously broken by the non-vanishing Killing vectors at the vacuum (46).

The dynamical excitation fields $\varphi^i$ are obtained after the expansion around the vacuum,

$$\phi^i = \bar{\phi}^i + \varphi^i.$$ (49)

The scalar manifold metric $g_{ij}$ is expanded as

$$g_{ij} = \bar{g}_{ij} + \bar{g}_{ij,k} \varphi^k + \frac{1}{2} \bar{g}_{ij,kl} \varphi^k \varphi^l + \cdots,$$ (50)

with

$$\bar{g}_{ij} := g_{ij}(\bar{\phi}), \quad \bar{g}_{ij,k} := \frac{\partial}{\partial \phi^k} g_{ij}(\phi) \bigg|_{\phi = \bar{\phi}}, \quad \bar{g}_{ij,kl} := \frac{\partial}{\partial \phi^l} \frac{\partial}{\partial \phi^k} g_{ij}(\phi) \bigg|_{\phi = \bar{\phi}}, \quad \cdots.$$ (51)

In a similar manner, the potential term is expanded as

$$V(\phi) = \bar{V} + \bar{V}_{,i} \varphi^i + \frac{1}{2} \bar{V}_{,ij} \varphi^i \varphi^j + \frac{1}{3!} \bar{V}_{,ijk} \varphi^i \varphi^j \varphi^k + \frac{1}{4!} \bar{V}_{,ijkl} \varphi^i \varphi^j \varphi^k \varphi^l + \cdots,$$ (52)

with

$$\bar{V} := V \bigg|_{\phi = \bar{\phi}}, \quad \bar{V}_{,i} := \frac{\partial}{\partial \phi^i} V \bigg|_{\phi = \bar{\phi}}, \quad \bar{V}_{,ij} := \frac{\partial}{\partial \phi^i} \frac{\partial}{\partial \phi^j} V \bigg|_{\phi = \bar{\phi}}, \quad \cdots.$$ (53)

Since the potential $V$ is minimized at the vacuum, the potential should satisfy

$$\bar{V}_{,i} = 0.$$ (54)

The scalar manifold is coordinated by the scalar field multiplet $\phi^i$. Hereafter, we normalize/diagonalize the coordinate $\phi^i$ as

$$\bar{g}_{ij} = \delta_{ij},$$ (55)

and

$$\bar{V}_{,ij} = \delta_{ij} m_i^2,$$ (56)

so that the excitation fields $\varphi^i$ are canonically normalized and diagonalized.
III. SCALAR SCATTERING AMPLITUDES AND PERTURBATIVE UNITARITY

We next consider implications of the perturbative unitarity in the GHEFT framework. It is well known that, in the effective field theory framework, the longitudinally polarized electroweak (EW) gauge boson scattering amplitudes grow in the high energy and tend to cause violations of the perturbative unitarity [115, 116]. The effective field theory coupling constants need to be arranged to keep the perturbative unitarity in the high energy gauge boson scattering amplitudes.

For such a purpose, we use the equivalence theorem between the longitudinally polarized gauge boson scattering amplitudes and the corresponding would-be NG boson amplitudes [117–121]. The equivalence theorem allows us to estimate the longitudinally polarized gauge boson high energy scattering amplitudes by using the NG boson amplitudes in the gaugeless limit, i.e., $g_W = g_Y = 0$ with uncertainty of $O(M_W^2/E^2)$. The computation of the amplitudes is simplified greatly in the gaugeless limit.

Note that the energy growing behavior in the longitudinal polarized gauge bosons amplitudes is exactly canceled in the SM [117, 122–124]. The energy growing behavior coming from the EW gauge boson exchange and contact interaction diagrams is exactly canceled by the Higgs exchange diagram in the SM. The Higgs boson plays an essential role to keep the perturbative unitarity in the SM.

On the other hand, it is highly non-trivial whether the cancellation of the energy growing terms does work or not in the GHEFT. In fact, in order to ensure the cancellation, the coupling strengths between the Higgs boson(s) and the EW gauge bosons should satisfy special conditions known as the “unitarity sum rules” [93–95]. The unitarity sum rules provide a guiding principle to investigate the extended Higgs scenarios in a model-independent manner. Model-independent studies on extended EWSB scenarios have been done based on the unitarity argument [70, 93, 99, 125, 126].

We estimate the amplitudes of EW gauge boson scattering by the NG boson scattering with the help of the equivalence theorem. In subsequent subsections, we explicitly calculate the on-shell amplitudes among the scalar fields $\varphi^i$ in the gaugeless limit, and express the unitarity sum rules in terms of the scalar manifold’s geometry.
A. Scalar scattering amplitudes

We consider here an \( N \)-point on-shell scalar scattering amplitude at the tree-level,

\[
i\mathcal{M}(123 \cdots N) := i\mathcal{M}(\varphi^{i_1}(p_1), \varphi^{i_2}(p_2), \varphi^{i_3}(p_3), \cdots, \varphi^{i_N}(p_N)),
\]

with \( p_n \) and \( i_n (n = 1, 2, \cdots, N) \) being outgoing momenta and the particle species, respectively.

We define

\[
s_{n_1} := p_{n_1}^2, \quad s_{n_1n_2} := (p_{n_1} + p_{n_2})^2, \quad s_{n_1n_2n_3} := (p_{n_1} + p_{n_2} + p_{n_3})^2, \quad \cdots.
\]

External momenta \( p_n \) are taken on-shell,

\[
s_n = m_{i_n}^2.
\]

We note

\[
s_{n_1n_2n_3} = s_{n_1n_2} + s_{n_2n_3} + s_{n_1n_3} - m_{i_{n_1}}^2 - m_{i_{n_2}}^2 - m_{i_{n_3}}^2, \quad \cdots.
\]

The \( N \)-point amplitude (57) can thus be written as a function of the scalar particle masses \( m_{i_n}^2 \) and the generalized Mandelstam variables \( s_{n_1n_2} \).

As we will show explicitly below, the three- and four-point on-shell scattering amplitudes are described in terms of the geometry of the scalar manifold,

\[
i\mathcal{M}(123) = -i\tilde{V}_{(i_1i_2i_3)},
\]

\[
i\mathcal{M}(1234) = i\mathcal{M}(123) + i\mathcal{M}(125) [D(s_{12})]_{i_5i_6} i\mathcal{M}(346)
\]

\[
+ i\mathcal{M}(135) [D(s_{13})]_{i_5i_6} i\mathcal{M}(246) + i\mathcal{M}(145) [D(s_{14})]_{i_5i_6} i\mathcal{M}(236),
\]

with

\[
i\mathcal{M}(1234) = -i\tilde{V}_{(i_1i_2i_3i_4)} - \frac{i}{3} (\tilde{R}_{i_1i_3i_4i_2} + \tilde{R}_{i_1i_4i_3i_2}) s_{12}
\]

\[
- \frac{i}{3} (\tilde{R}_{i_1i_2i_4i_3} + \tilde{R}_{i_1i_4i_2i_3}) s_{13} - \frac{i}{3} (\tilde{R}_{i_1i_2i_3i_4} + \tilde{R}_{i_1i_3i_2i_4}) s_{14},
\]

and

\[
[D(s)]_{ij} := \frac{i}{s - m_i^2} g_{ij}.
\]

Here \( \tilde{V}_{(i_1i_2i_3)}, \tilde{V}_{(i_1i_2i_3i_4)} \) and \( \tilde{R}_{i_1i_2i_3i_4} \) stand for the totally symmetrized covariant derivatives of the potential and the Riemann curvature tensor of the scalar manifold at the vacuum.
Let us start with the three-point scalar scattering amplitude $\mathcal{M}(123)$. The interaction vertices relevant for this amplitude are

$$\mathcal{L}_3 = \frac{1}{2} \bar{g}_{jk} \varphi^k (\partial^\mu \varphi^j) (\partial^\nu \varphi^i) - \frac{1}{24} \bar{V}_{ijk} \varphi^i \varphi^j \varphi^k,$$  \quad (65)

at the tree-level. It is straightforward to evaluate the on-shell three-point amplitude

$$i\mathcal{M}(123) = \frac{i}{2} (\bar{g}_{1i2,i3} + \bar{g}_{i21,i3}) (-p_1 \cdot p_2) + \frac{i}{2} (\bar{g}_{i3i,i1} + \bar{g}_{i1i,i3}) (-p_2 \cdot p_3)$$

$$+ \frac{i}{2} (\bar{g}_{i3i,i2} + \bar{g}_{i1i,i3}) (-p_3 \cdot p_1) - i \bar{V}_{1i2i3}$$

$$= \frac{i}{2} \bar{g}_{1i2,i3} (m_{i1}^2 + m_{i2}^2 - s_{12}) + \frac{i}{2} \bar{g}_{i3i,i1} (m_{i2}^2 + m_{i3}^2 - s_{23})$$

$$+ \frac{i}{2} \bar{g}_{i3i,i2} (m_{i3}^2 + m_{i1}^2 - s_{31}) - i \bar{V}_{1i2i3},$$  \quad (66)

from the vertices in (65). The conservation of the total momentum

$$p_1 + p_2 + p_3 = 0,$$

implies

$$s_{12} = (p_1 + p_2)^2 = p_3^2 = m_{i3}^2,$$

and similarly

$$s_{23} = m_{i1}^2, \quad s_{31} = m_{i2}^2.$$  \quad (67)

The on-shell three-point amplitude (66) can therefore be expressed as

$$i\mathcal{M}(123) = \frac{i}{2} \bar{g}_{1i2,i3} (m_{i1}^2 + m_{i2}^2 - m_{i3}^2) + \frac{i}{2} \bar{g}_{i3i,i1} (m_{i2}^2 + m_{i3}^2 - m_{i1}^2)$$

$$+ \frac{i}{2} \bar{g}_{i3i,i2} (m_{i3}^2 + m_{i1}^2 - m_{i2}^2) - i \bar{V}_{1i2i3}$$

$$= \frac{i}{2} m_{i1}^2 (\bar{g}_{1i2,i3} + \bar{g}_{i1i,i2} - \bar{g}_{i2i,i3}) + \frac{i}{2} m_{i2}^2 (\bar{g}_{i3i,i1} + \bar{g}_{i1i,i3} - \bar{g}_{i3i,i2})$$

$$+ \frac{i}{2} m_{i3}^2 (\bar{g}_{i3i,i2} + \bar{g}_{i3i,i1} - \bar{g}_{i1i,i3}) - i \bar{V}_{1i2i3}.$$  \quad (67)

Note that the $m_{i1}^2, m_{i2}^2$ and $m_{i3}^2$ are related with the second derivative of the potential $V_{ij}$ by (56). The first derivative of the metric tensor in the interaction vertex (65) is related with the the Affine connection $\Gamma^l_{jk}$

$$g_{il} \Gamma^l_{jk} := \frac{1}{2} [g_{ij,k} + g_{ki,j} - g_{jk,i}].$$  \quad (68)

The amplitude (67) can then be rewritten as

$$i\mathcal{M}(123) = i \bar{V}_{1i2} \bar{\Gamma}_{i2i3}^l + i \bar{V}_{i3i} \bar{\Gamma}_{i3i1}^l + i \bar{V}_{i3i} \bar{\Gamma}_{i3i2}^l - i \bar{V}_{1i2i3},$$  \quad (69)
with $\Gamma^l_{jk}$ being the Affine connection at the vacuum

$$\bar{\Gamma}^l_{jk} := \Gamma^l_{jk} \bigg|_{\phi = \bar{\phi}} .$$

(70)

Our final task is to rewrite the amplitude (69) in terms of the covariant derivatives of the potential $V$. It is straightforward to show

$$V_{ijk} = V_{ijk} - (\Gamma^l_{ij})_k V_{,l} - \Gamma^l_{ij} V_{,lk} - \Gamma^l_{ik} V_{,lj} - \Gamma^l_{jk} V_{,li} + \Gamma^l_{ik} \Gamma^m_{lj} V_{,m} + \Gamma^l_{jk} \Gamma^m_{li} V_{,m} .$$

(71)

Since the first derivative of the potential vanishes at the vacuum, we obtain

$$\bar{V}_{ijk} = \bar{V}_{ijk} - \bar{\Gamma}^l_{ij} \bar{V}_{,lk} - \bar{\Gamma}^l_{ik} \bar{V}_{,lj} - \bar{\Gamma}^l_{jk} \bar{V}_{,li} .$$

(72)

Moreover, as we see in (72), $\bar{V}_{ijk}$ is symmetric under the $i \leftrightarrow j$, $i \leftrightarrow k$ and $j \leftrightarrow k$ exchanges. We therefore obtain

$$\bar{V}_{i(jk)} := \frac{1}{3!} \left[ \bar{V}_{ijk} + \bar{V}_{jki} + \bar{V}_{kij} + \bar{V}_{iki} + \bar{V}_{kji} + \bar{V}_{jik} \right] = \bar{V}_{ijk} .$$

(73)

It is now easy to obtain a geometrical formula for the three-point amplitude

$$iM(123) = -i\bar{V}_{i(12i3)} .$$

(74)

We next consider the four-point amplitude

$$iM(1234) = iM_0(1234) + iM(12[5])[D(s_{12})]_{i_5i_6} iM(34[6]) + iM(13[5])[D(s_{13})]_{i_5i_6} iM(24[6]) + iM(14[5])[D(s_{14})]_{i_5i_6} iM(23[6]) ,$$

(75)

where the first line comes from the four-point contact interaction vertices, while the second, the third and the fourth lines are from the $i_5$-particle exchange diagrams in the $s_{12}$, $s_{13}$ and $s_{14}$ channels, respectively. The three-point amplitude $M(ij[k])$ is for on-shell $i$ and $j$, allowing off-shell $k$-particle.

We first study $M(12[5])$,

$$M(12[5]) = -\bar{V}_{i(125)} + \bar{g}_{i_5i_6} \bar{\Gamma}^{i_5}_{i_1i_2} (s_{12} - m_{i_5}^2) ,$$

(76)
which can be related with the on-shell three-point amplitude $\mathcal{M}(125)$ as

$$M(12|5) = \mathcal{M}(125) + g_{\bar{1}23} \bar{\Gamma}_{\bar{1}23}^i (s_{12} - m_{12}^2).$$ \hspace{1cm} (77)

It is easy to rewrite the amplitude (75) as

$$i\mathcal{M}(1234) = i\mathcal{M}(1234)$$

$$+ i\mathcal{M}(125)[D(s_{12})]_{i_5} i_6 i_7 i_8 i\mathcal{M}(346)$$

$$+ i\mathcal{M}(135)[D(s_{13})]_{i_5} i_6 i\mathcal{M}(246)$$

$$+ i\mathcal{M}(145)[D(s_{14})]_{i_5} i_6 i\mathcal{M}(236),$$ \hspace{1cm} (78)

with $\mathcal{M}(1234)$ being

$$\mathcal{M}(1234) = \mathcal{M}_0(1234) - g_{\bar{1}23} \bar{\Gamma}_{\bar{1}23}^i \tilde{\Gamma}_{\bar{1}23}^i (s_{12} - m_{12}^2)$$

$$- g_{\bar{1}23} \bar{\Gamma}_{\bar{1}23}^i \tilde{\Gamma}_{\bar{1}23}^i (s_{13} - m_{13}^2) - g_{\bar{1}23} \bar{\Gamma}_{\bar{1}23}^i \tilde{\Gamma}_{\bar{1}23}^i (s_{14} - m_{14}^2)$$

$$+ \bar{V}_{i_1 i_2 i_3} \Gamma_{i_1 i_2 i_3} + \bar{V}_{i_1 i_2 i_3} \Gamma_{i_1 i_2 i_3} + \bar{V}_{i_1 i_2 i_3} \Gamma_{i_1 i_2 i_3}$$

$$+ \bar{V}_{i_1 i_2 i_3} \Gamma_{i_1 i_2 i_3} + \bar{V}_{i_1 i_2 i_3} \Gamma_{i_1 i_2 i_3} + \bar{V}_{i_1 i_2 i_3} \Gamma_{i_1 i_2 i_3} \hspace{1cm} (79)$$

The evaluation of the four-point contact interaction contribution is a bit tedious but straightforward. We obtain

$$i\mathcal{M}_0(1234) = -i\bar{V}_{i_1 i_2 i_3 i_4}$$

$$+ \frac{i}{2} \left[ - (g_{i_1 i_2, i_3 i_4} + g_{i_1 i_2, i_3 i_4}) s_{12} - (g_{i_1 i_4, i_2 i_3} + g_{i_1 i_4, i_2 i_3}) s_{13} \right.$$

$$- (g_{i_1 i_4, i_2 i_3} + g_{i_1 i_4, i_2 i_3}) s_{14}$$

$$+ (g_{i_1 i_2, i_3 i_4} + g_{i_1 i_2, i_3 i_4}) m_{i_1}^2 + (g_{i_1 i_2, i_3 i_4} + g_{i_1 i_2, i_3 i_4} + g_{i_1 i_2, i_3 i_4}) m_{i_2}^2$$

$$+ (g_{i_1 i_2, i_3 i_4} + g_{i_1 i_2, i_3 i_4} + g_{i_1 i_2, i_3 i_4}) m_{i_3}^2 + (g_{i_1 i_2, i_3 i_4} + g_{i_1 i_2, i_3 i_4} + g_{i_1 i_2, i_3 i_4}) m_{i_4}^2 \right].$$ \hspace{1cm} (80)

Combining these results, we obtain a geometrical formula for the on-shell four-point amplitude

$$i\mathcal{M}(1234) = -i\bar{V}_{i_1 i_2 i_3 i_4} - \frac{i}{3} \left( R_{i_1 i_2 i_3 i_4} + R_{i_1 i_2 i_3 i_4} \right) s_{12}$$

$$- \frac{i}{3} \left( R_{i_1 i_2 i_3 i_4} + R_{i_1 i_2 i_3 i_4} \right) s_{13} - \frac{i}{3} \left( R_{i_1 i_2 i_3 i_4} + R_{i_1 i_2 i_3 i_4} \right) s_{14} \hspace{1cm} (81)$$
We used the on-shell condition

\[ s_{12} + s_{13} + s_{14} = m_{i_1}^2 + m_{i_2}^2 + m_{i_3}^2 + m_{i_4}^2 , \]  

(82)
in the computation above. Here \( \bar{R}_{ijkl} \) and \( \bar{V}_{i(ijkl)} \) denote the Riemann curvature tensor and the totally symmetrized covariant derivatives of the potential at the vacuum:

\[ \bar{R}_{ijkl} := R_{ijkl} \bigg|_{\phi = \bar{\phi}} , \quad \bar{V}_{i(ijkl)} := V_{i(ijkl)} \bigg|_{\phi = \bar{\phi}} . \]  

(83)

We here give formulas to compute \( \bar{R}_{ijkl} \) and \( \bar{V}_{i(ijkl)} \) from the metric tensor \( g_{ij} \) and the potential \( V \):

\[ \bar{R}_{ijkl} = \frac{1}{2} (\bar{g}_{d,jk} + \bar{g}_{j,k,l} - \bar{g}_{i,k,l} - \bar{g}_{j,i,l}) + \bar{g}_{mn} \left( \Gamma^m_{i,j} \Gamma^n_{j,k} - \Gamma^m_{i,k} \Gamma^n_{j,l} \right) , \]  

(84)

and

\[ \bar{V}_{i(ijkl)} = \bar{V}_{i,jkl} - \bar{V}_{ij,m} \Gamma^m_{kl} - \bar{V}_{ik,m} \Gamma^m_{lj} - \bar{V}_{il,m} \Gamma^m_{jk} - \bar{V}_{i,j,m} \Gamma^m_{kl} - \bar{V}_{i,j,m} \Gamma^m_{lj} + \bar{V}_{i,j,m} \Gamma^m_{jk} \]  

\[ + \bar{A}_{ijkl} + \bar{A}_{jikl} + \bar{A}_{kijl} + \bar{A}_{lijk} , \]  

(85)

with

\[ \bar{A}_{ijkl} := \frac{1}{6} \bar{V}_{i,m} \bar{g}^{mn} \left[ \bar{g}_{j,k,nl} + \bar{g}_{k,l,nj} + \bar{g}_{j,l,nk} - 2(\bar{g}_{n,j,kl} + \bar{g}_{n,k,jl} + \bar{g}_{n,l,jk}) \right] \]  

\[ + \bar{V}_{i,m} \left[ \Gamma^m_{j,n} \Gamma^n_{k,l} + \Gamma^m_{k,n} \Gamma^n_{j,l} + \Gamma^m_{l,n} \Gamma^n_{j,k} \right] \]  

\[ + \frac{1}{3} \bar{V}_{i,m} \bar{g}^{mp} \left[ \Gamma^q_{p,j} \Gamma^n_{k,l} + \Gamma^q_{p,k} \Gamma^n_{j,l} + \Gamma^q_{p,l} \Gamma^n_{j,k} \right] \bar{g}_{qn} . \]  

(86)

\[ \text{B. Perturbative unitarity} \]

As we have shown in Eq. (81), the scalar four-point amplitude \( M(1234) \) contains the energy-squared terms proportional to \( s_{12}, s_{13} \) and \( s_{14} \). This implies that the perturbative unitarity of the scattering amplitude is generally violated at certain high energy scale in the GHEFT (37). In order to keep the perturbative unitarity in the high energy limit, the GHEFT Lagrangian should satisfy special conditions known as the unitarity sum rules [93–95]. We here give a geometrical interpretation for the unitarity sum rules.

Applying the on-shell condition

\[ s_{12} + s_{13} + s_{14} = m_{i_1}^2 + m_{i_2}^2 + m_{i_3}^2 + m_{i_4}^2 , \]  

(87)

18
in the four-point amplitude (81) we obtain

\[
iM(1234) = -\frac{i}{3}(\bar{R}_{i1i3i4i2} + \bar{R}_{i1i4i2i3} - \bar{R}_{i1i2i3i4} - \bar{R}_{i1i3i2i4})s_{12} \\
- \frac{i}{3}(\bar{R}_{i1i2i4i3} + \bar{R}_{i1i4i2i3} - \bar{R}_{i1i2i3i4} - \bar{R}_{i1i3i2i4})s_{13} + \mathcal{O}(E^0).
\]

(88)

Therefore, the unitarity sum rules can be summarized in the geometrical language as

\[
\bar{R}_{i1i3i4i2} + \bar{R}_{i1i4i2i3} - \bar{R}_{i1i2i3i4} - \bar{R}_{i1i3i2i4} = 0,
\]

(89)

\[
R_{i1i2i4i3} + R_{i1i4i2i3} - R_{i1i2i3i4} - R_{i1i3i2i4} = 0.
\]

(90)

Note that the Riemann curvature tensor \( R_{ijkl} \) is antisymmetric under the \( k \leftrightarrow l \) exchange:

\[
R_{ijkl} \equiv -R_{ijlk}.
\]

(91)

The unitarity sum rules (89) can thus be rewritten as

\[
2\bar{R}_{i1i3i4i2} - \bar{R}_{i1i4i2i3} - \bar{R}_{i1i2i3i4} = 0.
\]

(92)

The Bianchi identity

\[
R_{ijkl} + R_{iklj} + R_{iljk} \equiv 0
\]

(93)

can be expressed as

\[
-\bar{R}_{i1i4i2i3} - \bar{R}_{i1i2i3i4} \equiv \bar{R}_{i1i3i4i2},
\]

(94)

which enables us to simplify the unitarity sum rules (92) further. We obtain the sum rules (89) can be expressed in a simple form:

\[
3\bar{R}_{i1i3i4i2} = 0.
\]

(95)

The both of the unitarity sum rules (89) and (90) can be expressed in a compact form:

\[
\bar{R}_{ijkl} = 0.
\]

(96)

Note that the unitarity sum rules (96) imply the flatness of the scalar manifold only at the vacuum. The unitarity conditions (96) is lifted to

\[
R_{ijkl} = 0,
\]

(97)

\textit{i.e.,} the complete flatness of the entire scalar manifold at least in the vicinity of the vacuum, by imposing the perturbative unitarity in the arbitrary \( N \)-point amplitudes. See appendix. \( A \) for details.
The perturbative unitarity is violated at the certain high energy scale in an extended Higgs scenario with a curved scalar manifold. For instance, if we consider the HEFT with $\mathcal{F}(h) = (1 + (\kappa_V h)/v)^2$ and take $\kappa_V < 1$, the corresponding scalar manifold has non-zero curvature proportional to $1 - \kappa_V^2$ [96, 97]. The model causes the violation of the perturbative unitarity at $\Lambda \sim 4\pi v/(1 - \kappa_V^2)^{1/2}$. In that case, we need to introduce new particles lighter than $\Lambda$ and/or to consider non-perturbative effects for ensuring the unitarity in the model.

IV. ONE-LOOP DIVERGENCES IN THE GAUGELESS LIMIT

As we have shown in the previous section, the tree-level perturbative unitarity requires the GHEFT scalar manifold should be flat at the vacuum. What does this imply at the loop level, then? Refs. [96, 97] investigated the structure of the one-loop divergences in the nonlinear sigma model Lagrangian (37). They found the logarithmic divergences in the scalar one-loop integral are described in the gaugeless limit by

$$\Delta L_{\text{div-loop}} = \frac{1}{(4\pi)^2\epsilon} \left[ \frac{1}{12} \text{tr}(Y_{\mu\nu}Y^{\mu\nu}) + \frac{1}{2} \text{tr}(X^2) \right].$$

(98)

Here $\epsilon$ is defined as

$$\epsilon := 4 - D,$$

(99)

with $D$ being the spacetime dimension. $Y_{\mu\nu}$ and $X$ are defined as

$$[Y_{\mu\nu}]^i_j = R^{i}_{jkl}(D_{\mu}\phi)^k(D_{\nu}\phi)^l + W^a_{\mu\nu}(w^i_a)_{;i} + B_{\mu\nu}(y^i)_{;i},$$

(100)

$$[X]^i_k = R^{i}_{jkl}(D_{\mu}\phi)^j(D_{\nu}\phi)^l + g^{ij}V_{;ijkl},$$

(101)

with

$$(w^i_a)_{;i} = \frac{\partial}{\partial \phi_j} w^i_a + \Gamma^{i}_{ij} w^j_a, \quad (y^i)_{;i} = \frac{\partial}{\partial \phi_j} y^i + \Gamma^{i}_{ij} y^j,$$

(102)

and $R^{i}_{jkl} = g^{im} R_{mijkl}$.

Remember that the perturbative unitarity implies the flatness at the vacuum,

$$\bar{R}_{ijkl} = 0.$$  

(103)

It is easy to see that the unitarity condition (103) is enough to guarantee the absence of the divergences in the $(\partial_\mu \phi)^4$ type operators, which affect the scalar boson high energy four-point scattering amplitudes. The flatness of the scalar manifold at the vacuum (103) also
automatically guarantees the absence of the divergences in the anomalous triple gauge boson operators.

The divergence structure in the operators proportional to

\[ W_{\mu\nu}^a W^{b\mu\nu}, \quad W_{\mu\nu}^a B^{\mu\nu}, \quad B_{\mu\nu} B^{\mu\nu}, \]  

is not manifest, however. Note that the oblique correction parameters \( S \) and \( U \) [98] are related with the gauge-kinetic-type operators listed in (104). There is no obvious reason to ensure the absence of the one-loop divergences in the \( S \) and \( U \) parameters even in the perturbatively unitary models.

Moreover, the one-loop divergence formula (98) does not include quantum corrections arising from the gauge-boson loop diagrams, which should be evaluated to deduce the conclusion on the divergence structure for the oblique correction parameters.

V. OBLIQUE CORRECTIONS AND FINITENESS CONDITIONS

A. Vacuum polarization functions at one-loop

The electroweak oblique correction parameters \( S \) and \( U \) are defined as

\[ S := 16\pi (\Pi'_{33}(0) - \Pi'_{3Q}(0)), \]
\[ U := 16\pi (\Pi'_{11}(0) - \Pi'_{33}(0)), \]

with \( \Pi'_A(0) \) being

\[ \Pi'_A(0) := \left. \frac{d}{dp^2} \Pi_A(p^2) \right|_{p^2=0}. \]

Here \( \Pi_A(p^2) \) stands for the non-SM contribution to the gauge boson vacuum polarization function in the \( A \)-channel. \( \Pi_{11}(p^2) \) and \( \Pi_{33}(p^2) \) are charged and neutral weak \( SU(2)_W \) current correlators at momentum \( p \), respectively. \( \Pi_{3Q}(p^2) \) is the correlator between the neutral weak \( SU(2)_W \) current and the electromagnetic current. Note that, in the GHEFT, a number of scalar particles other than the 125GeV Higgs contribute to \( \Pi_A(p^2) \) at loop.

The oblique correction parameter \( T \) is related with Veltman’s \( \rho \) parameter [127],

\[ \alpha T := \rho - 1, \quad \rho = \frac{\frac{v^2}{4} + \Pi_{11}(0)}{\frac{v^2}{4} + \Pi_{33}(0)}, \]

\[ \frac{v^2}{4} + \Pi_{33}(0), \]
with $v_0$ and $v_{Z0}$ being the “bare” parameters corresponding to the charged and neutral would-be NG boson decay constants $v$ and $v_Z$. The GHEFT Lagrangian loses its predictability on the $T$-parameter, if we allow to introduce independent counter terms for $v$ and $v_Z$.

On the other hand, if we assume the counter terms for $v$ and $v_Z$ are related with each other,

$$v_0^2 = v^2 (1 + \delta_v), \quad v_{Z0}^2 = v_Z^2 (1 + \delta_v),$$

(109)

the $\rho$ is calculated as

$$\rho = \frac{v^2}{v_Z^2} \frac{1 + \delta_v + \frac{4}{v^2} \Pi_{11}(0)}{1 + \delta_v + \frac{4}{v_Z^2} \Pi_{33}(0)},$$

(110)

and we regain a counter-term independent predictability on the $\rho$ parameter

$$\rho = \frac{v^2}{v_Z} \left(1 + \alpha \tilde{T}\right),$$

(111)

with

$$\alpha \tilde{T} := 4 \left(\frac{1}{v^2} \Pi_{11}(0) - \frac{1}{v_Z^2} \Pi_{33}(0)\right).$$

(112)

In what follows, we calculate $\Pi_{11}$, $\Pi_{33}$, and $\Pi_{3Q}$ at one-loop level in the GHEFT and derive the required conditions for ensuring the UV finiteness of Eqs. (105), (106) and (112). We apply a background field method [128–133] to calculate the vacuum polarization functions to keep the gauge invariance. See Appendix B for the details of the calculation. Although there exist UV divergences in $\Pi_{11}(0)$ and $\Pi_{33}(0)$ associated with tadpole diagrams as shown in Figure 1, we assume these UV divergences are canceled by the corresponding tadpole counter terms.
1. Scalar loop

Let us start with the scalar loop corrections to the vacuum polarization functions. The relevant Feynman diagrams are shown in Figure 2, which are evaluated to be

\[
\Pi_{3Q}^{\phi\phi}(p^2) = \frac{1}{(4\pi)^2} \left[ -2 \sum_{i,j} (\bar{w}_3^i)_{i,j} \left( (\bar{w}_3^j)_i + (\bar{y}^j)_i \right) B_{22}(p^2, m^2_i, m^2_j) \right. \\
+ \sum_{i,j} (\bar{w}_3^i)_{i,j} \left( (\bar{w}_3^j)_i + (\bar{y}^j)_i \right) A(m^2_i) \right],
\]

and

\[
\Pi_{bc}^{\phi\phi}(p^2) = \frac{1}{(4\pi)^2} \left[ -2 \sum_{i,j} (\bar{w}_c^i)_{i,j} (\bar{w}_c^j)_i B_{22}(p^2, m^2_i, m^2_j) \right. \\
+ \sum_{i,j} \left[ (\bar{w}_c^i)_{i,j} (\bar{w}_c^j)_i + \bar{y}^i_j (\bar{w}_c^k) (\bar{w}_c^l) R_{kilj} \right] A(m^2_i) \right],
\]

for \( b, c = 1, 2, 3 \). Here \((\bar{w}_a)^{i}_{j}\) and \((\bar{y})^{i}_{j}\) denote the covariant derivatives of the Killing vectors at the vacuum,

\[
(\bar{w}_a)^{i}_{j} := \left. (w_a)^{i}_{j} \right|_{\phi=\bar{\phi}}, \quad (\bar{y})^{i}_{j} := \left. (y)^{i}_{j} \right|_{\phi=\bar{\phi}}.
\]

\( A \) and \( B_{22} \) are loop functions defined as

\[
\frac{i}{(4\pi)^2} A(m^2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2}, \quad (116)
\]

\[
\frac{i}{(4\pi)^2} B_{22}(p^2; m_1^2, m_2^2) = \int \frac{d^4k}{(2\pi)^4} \frac{k_{\mu}k_{\nu}}{(k^2 - m_1^2) \{(k + p)^2 - m_2^2\}} \left|_{g_{\mu\nu}} \right. .
\]

2. Scalar-Gauge loop

We next calculate the Feynman diagrams shown in Figure 3. In the ’t Hooft-Feynman gauge, we obtain

\[
\Pi_{3Q}^{\phi\phi}(p^2) = 0,
\]

(118)
FIG. 3: Feynman diagrams for $\Pi^{\rho V}_{11}$, $\Pi^{\rho V}_{33}$, and $\Pi^{\rho V}_{3Q}$. The internal lines correspond to $\varphi$ and gauge fields.

and

$$
\Pi^{\rho V}_{bc}(p^2) = -\frac{4}{(4\pi)^2} \left[ \sum_{a=1,2} \sum_{i,j} \bar{g}_{ij} (G_{Wa})^i_b (G_{Wa})^j_c B_0(p^2, M^2_W, m^2_i) + \sum_{i,j} \bar{g}_{ij} (G_Z)^i_b (G_Z)^j_c B_0(p^2, M^2_Z, m^2_i) + \sum_{i,j} \bar{g}_{ij} (G_A)^i_b (G_A)^j_c B_0(p^2, 0, m^2_i) \right],
$$

(119)

for $b, c = 1, 2, 3$. Here $(G_{Wa})^i_b$, $(G_Z)^i_b$, $(G_A)^i_b$ are defined as

$$
(G_{Wa})^i_b := g_W (\bar{w}_a^i)_j \bar{w}_b^j, \quad (a = 1, 2) \tag{120}
$$

$$
(G_Z)^i_b := \frac{1}{\sqrt{g_W^2 + g_Y^2}} \left[ g_W^2 (\bar{w}_3^i)_j - g_Y^2 (\bar{y}^i)_j \right] \bar{w}_b^j, \tag{121}
$$

$$
(G_A)^i_b := \frac{g_W g_Y}{\sqrt{g_W^2 + g_Y^2}} \left[ (\bar{w}_3^i)_j + (\bar{y}^i)_j \right] \bar{w}_b^j \tag{122}
$$

and

$$
M^2_W = \frac{g_W^2}{4} v^2, \quad M^2_Z = \frac{g_W^2 + g_Y^2}{4} v^2. \tag{123}
$$

$B_0$ is defined as

$$
\frac{i}{(4\pi)^2} B_0(p^2; m^2_1, m^2_2) = \int \frac{d^4 k}{(2\pi)^4 (k^2 - m^2_1)} \frac{1}{((k + p)^2 - m^2_2)}. \tag{124}
$$

3. Gauge and Faddeev-Popov (FP) ghost loop

Finally, we calculate the contributions which are independent of the scalar interactions. The relevant Feynman diagrams are depicted in Figure 4. In the 't Hooft-Feynman gauge,
FIG. 4: Feynman diagrams for $\Pi_{11}^{\text{Gauge,cc}}$, $\Pi_{33}^{\text{Gauge,cc}}$, and $\Pi_{3Q}^{\text{Gauge,cc}}$. The wavy and dotted lines correspond to gauge fields and Faddeev-Popov ghost fields, respectively.

we find the gauge bosons contributions are given by

$$
\Pi_{11}^{\text{Gauge}}(p^2) = \Pi_{22}^{\text{Gauge}}(p^2)
$$

$$
= \frac{4}{(4\pi)^2} \left[ -A(M_W^2) - c_W^2 A(M_Z^2) - s_W^2 A(0)
+ 2p^2 \left(c_W B_0(p^2; M_Z^2, M_W^2) + s_W^2 B_0(p^2; 0, M_W^2)\right)
+ 4 \left(c_W^2 B_{22}(p^2; M_Z^2, M_W^2) + s_W^2 B_{22}(p^2; 0, M_W^2)\right) \right],
$$

(125)

$$
\Pi_{33}^{\text{Gauge}}(p^2) = \frac{8}{(4\pi)^2} \left[ p^2 B_0(p^2; M_W^2, M_W^2) + 2 B_{22}(p^2; M_W^2, M_W^2) - A(M_W^2) \right],
$$

(126)

$$
\Pi_{3Q}^{\text{Gauge}}(p^2) = \frac{8}{(4\pi)^2} \left[ p^2 B_0(p^2; M_W^2, M_W^2) + 2 B_{22}(p^2; M_W^2, M_W^2) - A(M_W^2) \right],
$$

(127)

and Faddeev-Popov (FP) ghost contributions are calculated as

$$
\Pi_{11}^{\text{cc}}(p^2) = \Pi_{22}^{\text{cc}}(p^2)
$$

$$
= \frac{2}{(4\pi)^2} \left[ A(M_W^2) + c_W^2 A(M_Z^2) + s_W^2 A(0)
- 4 \left(c_W^2 B_{22}(p^2; M_Z^2, M_W^2) + s_W^2 B_{22}(p^2; 0, M_W^2)\right) \right],
$$

(128)

$$
\Pi_{33}^{\text{cc}}(p^2) = -\frac{4}{(4\pi)^2} \left[ 2 B_{22}(p^2; M_W^2, M_W^2) - A(M_W^2) \right],
$$

(129)

$$
\Pi_{3Q}^{\text{cc}}(p^2) = -\frac{4}{(4\pi)^2} \left[ 2 B_{22}(p^2; M_W^2, M_W^2) - A(M_W^2) \right].
$$

(130)
Here $s_W$ and $c_W$ are
\[ s_W = \frac{g_Y}{g_Z}, \quad c_W = \frac{g_W}{g_Z}, \quad g_Z = \sqrt{g_W^2 + g_Y^2}. \] (131)

**B. Finiteness of the oblique corrections**

We are now ready to derive the UV finiteness conditions for the oblique correction parameters at the one-loop level, i.e., the finiteness of Eqs. (105), (106) and (112).

For the estimation of the UV cutoff dependence, we regularize $A$, $B_0$, and $B_{22}$ functions as
\[ A(m^2) = -\Lambda^2 + m^2 \ln \frac{\Lambda^2}{\mu^2} - (4\pi)^2 A_r(m), \] (132)
\[ B_0(p^2, m_1^2, m_2^2) = \ln \frac{\Lambda^2}{\mu^2} + (4\pi)^2 B_r(m_1, m_2, p^2), \] (133)
\[ B_{22}(p^2, m_1^2, m_2^2) = -\frac{1}{2} \Lambda^2 + \frac{1}{4} \left( m_1^2 + m_2^2 - \frac{p^2}{3} \right) \ln \frac{\Lambda^2}{\mu^2} + \frac{1}{4} (4\pi)^2 B_{0r}(m_1, m_2, p^2), \] (134)

where $\Lambda$ and $\mu$ denote UV cutoff and renormalization scale, respectively. $A_r$, $B_r$, and $B_{0r}$ are $\Lambda$-independent ($\mu$-dependent) functions. The explicit expressions of the $\Lambda$-independent functions are given in Ref. [99].

1. **$S$ and $U$ parameter**

Let us focus on the UV divergences in Eqs. (105) and (106). Combining the results derived in subsection V A and Eqs. (132)-(134), we find that the UV divergent parts of $S$ and $U$ are given as
\[ S_{\text{div}} = -\frac{1}{12\pi} \langle \bar{w}_3^i \rangle_{;i} \langle \bar{y}^j \rangle_{;i} \ln \frac{\Lambda^2}{\mu^2}, \] (135)
\[ U_{\text{div}} = \frac{1}{12\pi} \left( \langle \bar{w}_1^i \rangle_{;i} \langle \bar{w}_1^j \rangle_{;j} - \langle \bar{w}_3^i \rangle_{;i} \langle \bar{w}_3^j \rangle_{;j} \right) \ln \frac{\Lambda^2}{\mu^2}. \] (136)

The gauge boson loops do not contribute to the one-loop divergences in $S$ and $U$ parameters. These results are thus identical with the results computed in the gaugeless limit [96, 97].
2. $\Pi_{11}(0)$ and $\Pi_{33}(0)$

The UV divergences in Eq. (112) other than the tadpole contributions can also be extracted using Eqs. (132)-(134). We obtain

$$
\left( \frac{1}{v^2} \Pi_{11}(0) - \frac{1}{v_Z^2} \Pi_{33}(0) \right)_{\text{div}} = \left( \frac{1}{v^2} \Pi_{11}(0) - \frac{1}{v_Z^2} \Pi_{33}(0) \right)_{\Lambda^2} + \left( \frac{1}{v^2} \Pi_{11}(0) - \frac{1}{v_Z^2} \Pi_{33}(0) \right)_{\ln \Lambda^2},
$$

(137)

where

$$
\left( \frac{1}{v^2} \Pi_{11}(0) - \frac{1}{v_Z^2} \Pi_{33}(0) \right)_{\Lambda^2} = -\frac{1}{(4\pi)^2} \left[ \frac{1}{v^2} (\bar{w}_1^i)(\bar{w}_1^i) - \frac{1}{v_Z^2} (\bar{w}_3^i)(\bar{w}_3^i) \right] \bar{R}_{ikjl} \bar{g}^{kl} \Lambda^2,
$$

(138)

and

$$
\left( \frac{1}{v^2} \Pi_{11}(0) - \frac{1}{v_Z^2} \Pi_{33}(0) \right)_{\ln \Lambda^2} = \frac{1}{(4\pi)^2} \left[ \frac{1}{v^2} (\bar{w}_1^i)(\bar{w}_1^i) - \frac{1}{v_Z^2} (\bar{w}_3^i)(\bar{w}_3^i) \right] \times
$$

$$
\times \left\{ -4g_{W}(\bar{w}_a^k,\bar{w}_a^i)\bar{g}_{kl} - 4g_{Y}(\bar{y}^i,\bar{y}^i)\bar{g}_{kl} + \bar{R}_{ikjl} (\bar{M}^2)^{kl} \right\} \ln \frac{\Lambda^2}{\mu^2},
$$

(139)

with $(\bar{M}^2)^{kl}$ being the scalar boson mass matrix in the 't Hooft-Feynman gauge:

$$
(\bar{M}^2)^{ij} := \bar{g}^{ik}\bar{g}^{jl}\bar{V}_{ikl} + g_{W}^{2}(\bar{w}_a^i)\bar{g}_{kl} + g_{Y}^{2}(\bar{y}^i)\bar{g}_{kl}, \quad \bar{V}_{ij} := V_{ij}\bigg|_{\phi = \bar{\phi}}.
$$

VI. PERTURBATIVE UNITARITY VS. FINITENESS CONDITIONS

We are now ready to discuss the implications of the perturbative unitarity to the one-loop finiteness of the oblique correction parameters. We first concentrate ourselves on the $S$-parameter, the UV-divergence of which is given by Eq. (135). As we stressed in §. IV, since there are no obvious connections between the Riemann curvature tensor (geometry) $R_{ijkl}$ and the $SU(2)_W \times U(1)_Y$ Killing vectors (symmetry) $w_a^i$ and $y^i$, the relation between the perturbative unitarity $\bar{R}_{ijkl} = 0$ and the one-loop finiteness of the $S$-parameter is not evidently understood in Eq. (135).

We note, however, that the scalar manifold should be invariant under the $SU(2)_W \times U(1)_Y$ transformations, and thus the Killing vectors should satisfy the Killing equations,

$$
0 = (w_a^k)_{ij,k} + (w_a^k)_{i} g_{kj} + (w_a^k)_{j} g_{ik}, \quad 0 = (y^k)_{ij,k} + (y^k)_{i} g_{kj} + (y^k)_{j} g_{ik}.
$$

(141)
There do exist connections between the geometry \( R_{ijkl} \) and the symmetry \( w_a^i \) and \( y^i \) embedded in the Killing equations Eqs. (141). Moreover, the Killing vectors \( w_a^i \) and \( y^i \) should obey the \( SU(2)_W \times U(1)_Y \) Lie algebra,

\[
[w_a, w_b] = \varepsilon_{abc} w_c, \quad [w_a, y] = 0, \tag{142}
\]

with

\[
w_a := w_a^i \frac{\partial}{\partial \phi^i}, \quad y := y^i \frac{\partial}{\partial \phi^i}. \tag{143}
\]

The connections can be studied most easily if we take the Riemann Normal Coordinate (RNC) around the vacuum \( \bar{\phi} \), in which the metric tensor \( g_{ij}(\phi) \) can be expressed in a Taylor-expanded form around \( \bar{\phi} \) as,

\[
g_{ij}(\phi) = \delta_{ij} - \frac{1}{3} \bar{R}_{ijkl} \phi^k \phi^l + \cdots, \tag{144}
\]

with

\[
\delta_{ij} = \bar{g}_{ij} = g_{ij}(\phi) \bigg|_{\phi = \bar{\phi}}, \quad \bar{R}_{ijkl} = R_{ijkl} \bigg|_{\phi = \bar{\phi}}. \tag{145}
\]

Solving the Killing equations (141) in terms of the Taylor expansion around the vacuum,

\[
w_a^i = \bar{w}_a^i + (\bar{w}_a^i)_{,j} \phi^j + \frac{1}{2!} (\bar{w}_a^i)_{,jk} \phi^j \phi^k + \cdots, \tag{146}
\]

\[
y^i = \bar{y}^i + (\bar{y}^i)_{,j} \phi^j + \frac{1}{2!} (\bar{y}^i)_{,jk} \phi^j \phi^k + \cdots, \tag{147}
\]

we find the Taylor expansion coefficients satisfy

\[
0 = \bar{g}_{ik}(\bar{w}_a^k)_{,j} + \bar{g}_{jk}(\bar{w}_a^k)_{,i}, \tag{148}
\]

\[
0 = \bar{g}_{ik}(\bar{y}^k)_{,j} + \bar{g}_{jk}(\bar{y}^k)_{,i}, \tag{149}
\]

\[
(\bar{w}_a^i)_{,jk} = \frac{1}{3} \left( \bar{R}_{ijkl} + \bar{R}_{ikjl} \right) \bar{w}_a^l, \tag{150}
\]

\[
(\bar{y}^i)_{,jk} = \frac{1}{3} \left( \bar{R}_{ijkl} + \bar{R}_{ikjl} \right) \bar{y}^l, \tag{151}
\]

There certainly exist connections between the geometry \( R_{ijkl} \) and the symmetry \( w_a^i \) and \( y^i \) in Eqs. (150) and (151). However, Eqs. (150) and (151) are not enough to clarify the relation between the perturbative unitarity and the \( S \)-parameter coefficient in (135). Note that the \( S \)-parameter coefficient is written in terms of the first derivative of the Killing vectors \( (\bar{w}_a^i)_{,j} \) and \( (\bar{y}^i)_{,j} \). We need physical principles to relate \( (\bar{w}_a^i)_{,j} \) and \( (\bar{y}^i)_{,j} \) with the second derivatives.
(\vec{w}^i)_{jk}, \text{ and } (\vec{y}^i)_{jk}. \text{ Actually, the } SU(2)_W \times U(1)_Y \text{ Lie algebra (symmetry) (142) plays the role. Plugging Eqs. (150) and (151) into Eq. (142), we obtain}

\begin{equation}
(T_a)_j^i = \frac{1}{2} \varepsilon_{abc} ([T_b, T_c])_j^i + \frac{1}{2} \varepsilon_{abc}(\vec{w}^k_b) (\vec{w}^l_c) \bar{R}^i_{jkl}, \tag{152}
\end{equation}

\begin{equation}
0 = ([T_a, T_Y])_j^i + (\vec{w}^k_a) (\vec{y}^l) \bar{R}^i_{jkl}, \tag{153}
\end{equation}

with \( T_a \) and \( T_Y \) being matrices denoting the first derivatives of the \( SU(2)_W \times U(1)_Y \) Killing vectors at the vacuum,

\begin{equation}
(T_a)_j^i := (\vec{w}^i_a)_j, \quad (T_Y)_j^i := (\vec{y}^i)_j. \tag{154}
\end{equation}

It is now easy to show

\begin{equation}
\text{tr}(T_3T_Y) = \frac{1}{2} \varepsilon_{3bc}\text{tr}([T_a, T_Y]) + \frac{1}{2} \varepsilon_{3bc}(\vec{w}^k_b) (\vec{w}^l_c) \bar{R}^i_{jkl}(T_Y)_i^j
\end{equation}

\begin{equation}
= \frac{1}{2} \varepsilon_{3bc}\text{tr}([T_a, T_Y]T_b) + \frac{1}{2} \varepsilon_{3bc}(\vec{w}^k_b) (\vec{w}^l_c) \bar{R}^i_{jkl}(T_Y)_i^j
\end{equation}

\begin{equation}
= -\frac{1}{2} \varepsilon_{3bc}(\vec{w}^k_c) (\vec{y}^j) \bar{R}^i_{jkl}(T_b)_i^j + \frac{1}{2} \varepsilon_{3bc}(\vec{w}^k_b) (\vec{w}^l_c) \bar{R}^i_{jkl}(T_Y)_i^j
\end{equation}

\begin{equation}
= \frac{1}{2} \varepsilon_{3bc}(\vec{w}^k_b) (\vec{w}^l_c) \bar{R}^i_{jkl}(T_b)_i^j + \frac{1}{2} \varepsilon_{3bc}(\vec{w}^k_b) (\vec{w}^l_c) \bar{R}^i_{jkl}(T_Y)_i^j. \tag{155}
\end{equation}

In the last line of Eq. (155), we used the fact that \( U(1)_{\text{em}} \) is unbroken at the vacuum Eq. (47), \( i.e., \)

\begin{equation}
0 = \vec{w}^i_3 + \vec{y}^i. \tag{156}
\end{equation}

Eq. (155) can be rewritten in a covariant form

\begin{equation}
(\vec{w}^i_3)_{,j} (\vec{y}^j)_i = \frac{1}{2} (\varepsilon_{3bc} (\vec{w}^k_b) (\vec{w}^l_c) \bar{R}^i_{jkl} (\vec{w}^l_b)_j (\vec{w}^k_c) R^i_{jkl} (\vec{y}^j)_i). \tag{157}
\end{equation}

In a similar manner, we obtain the divergent coefficient in the \( U \)-parameter (136),

\begin{equation}
(\vec{w}^i_1)_{,j} (\vec{w}^j_1)_i - (\vec{w}^i_3)_{,j} (\vec{w}^j_3)_i = \frac{1}{2} (\varepsilon_{1bc} (\vec{w}^k_b) (\vec{w}^l_c) \bar{R}^i_{jkl} (\vec{w}^l_1)_j (\vec{w}^k_c) R^i_{jkl} (\vec{y}^j)_i). \tag{158}
\end{equation}

Combining Eqs. (135), (136), (157), and (158), we find

\begin{equation}
S_{\text{div}} = -\frac{1}{12\pi} (\varepsilon_{3bc} (\vec{w}^k_b) (\vec{w}^l_c) \bar{R}^i_{jkl} (\vec{w}^l_3)_j (\vec{w}^k_c) R^i_{jkl} (\vec{y}^j)_i) \ln \frac{\Lambda^2}{\mu^2}, \tag{159}
\end{equation}

\begin{equation}
U_{\text{div}} = \frac{1}{12\pi} (\varepsilon_{1bc} (\vec{w}^k_b) (\vec{w}^l_c) \bar{R}^i_{jkl} (\vec{w}^l_1)_j (\vec{w}^k_c) R^i_{jkl} (\vec{y}^j)_i) \ln \frac{\Lambda^2}{\mu^2}. \tag{160}
\end{equation}
The relation between the symmetry and the geometry hidden in the expressions (135) and (136) is now unveiled in the expressions (159) and (160). The one-loop divergences of both $S$ and $U$ are proportional to the Riemann curvature tensor $\bar{R}_{ijkl}$ at the vacuum. Once the four-point tree-level unitarity is ensured, i.e., $\bar{R}_{ijkl} = 0$, then the one-loop finiteness of $S$ and $U$ is automatically guaranteed in Eqs. (159) and (160).

The physical implications of the $S$ and $U$ parameter formulas (159) and (160) can be studied more closely. Note that both of them vanish when

$$\left(\bar{w}_a^k\right) \left(\bar{w}_c^l\right) \bar{R}_{ijkl} = 0,$$

(161)
even if there might exist non-vanishing $\bar{R}_{ijkl}$. What does the condition (161) imply, then? Combining the equivalence theorem and the results presented in § III, we see that the condition (161) ensures the tree-level unitarity of the high energy $p$-wave scattering amplitude in the

$$V^b_L V^c_L \rightarrow \varphi^i \varphi^j$$

(162)
channel. Here $V^a_L$ stands for the longitudinally polarized massive gauge bosons, $V^{1,2}_L = W^{1,2}_L$ and $V^3_L = Z_L$. The one-loop finiteness of the $S$ and $U$ parameters does not require a completely flat scalar manifold. Once the $p$-wave tree-level unitarity in the channel (162) is somehow ensured, it is potentially possible to construct strongly interacting EWSB models without violating the one-loop finiteness of the $S$ and $U$ parameters.

Moreover, as we see in Appendix C, the covariant derivative of the Killing vector $(w^i_L)_c$ is related with the light-fermion scattering amplitudes

$$f \bar{f} \rightarrow \varphi^i \varphi^j.$$  

(163)
Here $f$ (and $\bar{f}$) stands for light quarks or leptons (light anti-quarks or anti-leptons). Since the coefficients in front of the logarithmic divergences in Eqs. (159) and (160) can be expressed in a form

$$\left(w^k_a\right) \left(w^l_c\right) \bar{R}^i_{jkl} \left(\bar{w}^j_a\right)_i,$$

(164)
the precise measurements of the $S$ and $U$ parameters can be used to constrain the high energy scattering amplitudes in

$$V^b_L V^c_L \rightarrow \varphi^i \varphi^j, \quad f \bar{f} \rightarrow \varphi^i \varphi^j$$

(165)
channels, which can be tested in future collider experiments.
Finally we make a comment on the UV finiteness condition of (137). We find that the UV finiteness of (137) is not ensured solely by the flatness of the scalar manifold. For an example, even if we assume that the scalar manifold is completely flat and $v = v_Z$ at the tree-level, an extra condition

$$
\left[ (\bar{w}_i^1)(\bar{w}_i^3) - (\bar{w}_i^3)(\bar{w}_i^3) \right] \left[ g_{W}^2 (\bar{w}_a^k)_{ij} (\bar{w}_a^i)_{kl} \tilde{g}_{kl} + g_{Y}^2 (\bar{y}_a^k)_{ij} (\bar{y}_a^i)_{kl} \tilde{g}_{kl} \right] = 0 \quad (166)
$$

is required to ensure the finiteness of the one-loop $T$ parameter correction. The Georgi-Machacek model [35–38] is one of examples where the condition (166) is not satisfied. We need to introduce independent counter terms for $v$ and $v_Z$ in these models.

VII. SUMMARY

We have formulated a generalized Higgs effective field theory (GHEFT), which includes extra Higgs particles other than the 125 GeV Higgs boson as a low energy effective field theory describing the electroweak symmetry breaking. The scalar scattering amplitudes are expressed by the geometry (Riemann curvature) and the symmetry (Killing vectors) of the scalar manifold in the GHEFT. The one-loop radiative corrections to electroweak oblique corrections are also expressed in terms of geometry and symmetry of the scalar manifold. By using the results, we have clarified the relationship between the perturbative unitarity and the UV finiteness of oblique corrections in the GHEFT.

Especially, we have shown that once the tree-level unitarity is ensured, then the $S$ and $U$ parameters’ one-loop finiteness is automatically guaranteed. The tree-level perturbative unitarity in the scalar amplitudes requires the complete flatness of the scalar manifold at the vacuum. On the other hand, the one-loop finiteness of electroweak oblique correction does not require the complete flatness. The findings enable us to verify that tree-level unitarity condition is stronger than the one-loop UV finiteness condition in extended Higgs scenarios. Interestingly, the findings also indicate possibilities of extended Higgs scenarios where perturbative unitarity is broken at certain energy scale but keeps the consistency with the electroweak precision measurements. The construction of a concrete model is deferred to our future work.

We also found connections between the coefficients of $S$ and $U$ parameter divergences and the particle scattering amplitudes which can be measured in future collider experiments.
We emphasize that future precision measurements of the discovered Higgs couplings, cross section, and oblique parameters are quite important for investigating the geometry and symmetry of the scalar manifold in the generalized Higgs sector. Combining collider/precision experimental data with our effective theoretical approach, we should be able to obtain new prospects of the physics beyond the SM.

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Appendix A: \(N\)-point amplitude

Let us consider the Taylor expansion of the scalar manifold metric tensor \(g_{ij}(\phi)\) around the vacuum point \(\bar{\phi}^i\),

\[
g_{ij}(\phi) = \bar{g}_{ij} + \bar{G}_{ijk} \varphi^k + \frac{1}{2} \bar{G}_{ijkl} \varphi^k \varphi^l + \frac{1}{3!} \bar{G}_{ijklm} \varphi^k \varphi^l \varphi^m + \frac{1}{4!} \bar{G}_{ijklmn} \varphi^k \varphi^l \varphi^m \varphi^n + \cdots, \tag{A1}
\]

with \(\phi^i = \bar{\phi}^i + \varphi^i\). The Taylor coefficients can be expressed in terms of the covariant derivatives of the Riemann curvature tensor in RNC. They are \([134, 135]\)

\[
\bar{g}_{ij} = \delta_{ij}, \tag{A2}
\]
\[
\bar{G}_{ijk} = 0, \tag{A3}
\]
\[
\bar{G}_{ijkl} = \frac{2}{3} \bar{R}_{iklj}, \tag{A4}
\]
\[
\bar{G}_{ijklm} = \bar{R}_{iklj;m}, \tag{A5}
\]
\[
\bar{G}_{ijklmn} = \frac{6}{5} \bar{R}_{iklj;mn} + \frac{16}{15} \bar{R}_{iklo} \bar{R}^{lo}_{mnj}, \tag{A6}
\]

and

\[
\bar{R}_{ijkl} := R_{ijkl}\bigg|_{\phi = \bar{\phi}}, \quad \bar{R}_{ijkl;m} := R_{ijkl;m}\bigg|_{\phi = \bar{\phi}}, \quad \bar{R}_{ijklmn} := R_{ijklmn}\bigg|_{\phi = \bar{\phi}}, \quad \cdots. \tag{A7}
\]
The one-particle-irreducible on-shell $N$-point amplitude $M(12 \cdots N)$ can thus be expressed as
\[ iM(12 \cdots N) = -\frac{i}{2} \sum_{m<n} s_{mn} \bar{G}_{(i_m i_n)(i_1 i_2 \cdots i_{m-1} \cdots i_n \cdots i_N)}, \tag{A8} \]
in the gaugeless flat-potential ($V = 0$) scalar model. Scalar particles are all massless in this model. The indices inside parentheses are understood to be totally symmetrized. The check symbols on top of \( \check{i}_m \) and \( \check{i}_n \) in the sequence $i_1 i_2 \cdots \check{i}_m \cdots \check{i}_n \cdots i_N$ denote the absence of the corresponding indices, i.e.,
\[ i_1 i_2 \cdots \check{i}_m \cdots \check{i}_n \cdots i_N = i_1 i_2 \cdots i_{m-1} i_{m+1} \cdots i_{n-1} i_{n+1} \cdots i_N. \tag{A9} \]

We show, in this appendix, that the perturbative unitarity up to the $N$-point amplitudes requires
\[ \bar{R}_{i_1 i_2 i_3 i_4} = 0, \quad \bar{R}_{i_1 i_2 i_3 i_4; i_5} = 0, \quad \bar{R}_{i_1 i_2 i_3 i_4; i_5 i_6} = 0, \quad \cdots \quad \bar{R}_{i_1 i_2 i_3 i_4; i_5 \cdots i_N} = 0. \tag{A10} \]
The scalar manifold needs to be completely flat at least in the vicinity of the vacuum. It should be stressed here, even though we already have a compact expression for the $N$-point amplitude (A8), it is nontrivial to obtain the unitarity condition (A10), since the generalized Mandelstam variables $s_{mn}$ need to satisfy the momentum conservation conditions
\[ \sum_{n=1}^{N} s_{mn} = 0, \tag{A11} \]
and the conditions coming from the four-dimensional space-time (Gram determinant conditions) [137]. We need to make full use of the Riemann tensor symmetry in order to deduce our conclusions (A10).

$N = 4$

Let us start with the analysis on the four-point scattering amplitude. We compute the amplitude in the limit
\[ s := s_{12} = s_{34} = -s_{13} = -s_{24} \neq 0, \quad s_{14} = s_{23} = 0. \tag{A12} \]
Clearly the momentum conservation conditions (A11) are satisfied in (A12). The Gram determinant conditions do not give extra conditions in $N = 4$.

\[ \text{Eq. (A8) can be regarded as a geometrical manifestation of Weinberg’s soft-theorem in on-shell amplitudes. See Ref. [136], for an example, for a recent review on the computational techniques of various on-shell amplitudes including nonlinear sigma models.} \]
In the limit above, the four-point on-shell amplitude behaves as

$$M(1234) \propto s A(1234),$$

(A13)

with

$$A(1234) := \{(12)(34)\} + \{(34)(12)\} - \{(13)(24)\} - \{(24)(13)\}. $$

(A14)

Here we introduce an abbreviation for the Riemann curvature tensor

$$\{12|34\} := \bar{R}_{i_1 i_3 i_4 i_2}. $$

(A15)

The indices inside parentheses are, again, understood to be totally symmetrized.

Considering the amplitude (A13) for large $s$, we see the perturbative unitarity requires

$$A(1234) = 0.$$  (A16)

Using the Riemann curvature tensor symmetry

$$\{12|34\} = -\{32|14\} = -\{14|32\} = \{34|12\} = \{21|43\},$$  (A17)

and the first Bianchi identity

$$\{12|34\} + \{13|42\} + \{14|23\} = 0,$$  (A18)

the coefficient $A(1234)$ can be computed as

$$A(1234) = 2\{(12)(34)\} - 2\{(13)(24)\}$$

$$= \{12|34\} + \{12|43\} - \{13|24\} - \{13|42\}$$

$$= \{12|34\} - \{13|42\} + \{14|23\} - \{13|42\}$$

$$= -3\{13|42\}.$$  (A19)

It is now easy to see that the perturbative unitarity requires the vanishing Riemann curvature tensor at the vacuum,

$$\bar{R}_{i_1 i_3 i_4 i_2} = 0.$$  (A20)

Taking the external lines $i_1, \ldots, i_4$ arbitrary, the result (A20) requires $\bar{R}_{ijkl} = 0$, which is enough to guarantee the perturbative unitarity in the arbitrary four-point amplitudes given in the form of Eq. (A8). The considerations in the limit (A12) thus provide necessary and sufficient conditions for the perturbative unitarity in the four-point amplitudes.
\( \mathcal{N} = 5 \)

We next consider the five-point scattering amplitude. Again, we consider the amplitude in the limit

\[
s := s_{12} = s_{34} = -s_{13} = -s_{24} \neq 0, \quad s_{14} = s_{23} = s_{15} = s_{25} = s_{35} = s_{45} = 0. \quad (A21)
\]

Note that the fifth particle is considered to be very soft.

We introduce an abbreviation for the covariant derivative of the Riemann curvature tensor,

\[
\{12|34; 5\} := \bar{R}_{i1i3i4i2;i5}. \quad (A22)
\]

The five-point amplitude in the limit behaves as

\[
M(12345) \propto sA(12345), \quad (A23)
\]

with

\[
A(12345) := \{(12)|(34; 5)\} + \{(34)|(12; 5)\} - \{(13)|(24; 5)\} - \{(24)|(13; 5)\}. \quad (A24)
\]

Using

\[
\{(12)|(34; 5)\} = \frac{1}{3}\{(12)|(34); 5\} + \frac{1}{3}\{(12)|(35); 4\} + \frac{1}{3}\{(12)|(45); 3\}, \quad (A25)
\]

we obtain

\[
A(12345) = \frac{1}{3}\left\{\{(12)|(34); 5\} + \{(34)|(12; 5)\} - \{(13)|(24; 5)\} - \{(24)|(13; 5)\}\right\}
+ \frac{1}{3}\left\{\{(43)|(25); 1\} - \{(42)|(35); 1\}\right\}
+ \frac{1}{3}\left\{\{(34)|(15); 2\} - \{(31)|(45); 2\}\right\}
+ \frac{1}{3}\left\{\{(21)|(45); 3\} - \{(24)|(15); 3\}\right\}
+ \frac{1}{3}\left\{\{(12)|(35); 4\} - \{(13)|(25); 4\}\right\}. \quad (A26)
\]

The first line in (A26) can be computed easily using the result on the four-point amplitude. The second and the third lines can also be computed in a manner similar to Eq. (A19). We find

\[
A(12345) = -\{13|42; 5\} - \frac{1}{2}\{42|53; 1\} - \frac{1}{2}\{31|54; 2\} - \frac{1}{2}\{24|51; 3\} - \frac{1}{2}\{13|52; 4\}. \quad (A27)
\]

Eq. (A27) can be simplified further with the help of the second Bianchi identity

\[
\{12|34; 5\} + \{14|35; 2\} + \{15|32; 4\} = 0. \quad (A28)
\]
We obtain

\[ A(12345) = -\{13|42; 5\} - \frac{1}{2}\left\{24|35; 1\} + \{25|31; 4\}\right\} - \frac{1}{2}\left\{13|45; 2\} + \{15|42; 3\}\right\} \]
\[ = -\{13|42; 5\} + \frac{1}{2}\{21|34; 5\} + \frac{1}{2}\{12|43; 5\} \]
\[ = -2\{13|42; 5\}. \]  

(A29)

The perturbative unitarity in the five-point amplitude thus requires

\[ \bar{R}_{i_1 i_3 i_4 i_2 i_5} = 0. \]  

(A30)

It is easy to see that Eq. (A30) gives necessary and sufficient conditions for the perturbative unitarity in the five-point amplitudes.

\[ N = 6 \]

It is now straightforward to derive the perturbative unitarity conditions for the six-point amplitude \( M(123456) \). It will be turned out considerations in the limit

\[ s := s_{12} = s_{34} = -s_{13} = -s_{24} \neq 0, \]  

(A31)

are enough. Generalized Mandelstam variables other than \( s_{12}, s_{34}, s_{13}, s_{24} \) are taken to be zero. Note that the fifth-particle and the sixth-particle are both considered to be very soft in this limit. Note also this choice of the Mandelstam variables is consistent with the momentum conservation constraints and the Gram determinant constraints.

We already know the Riemann curvature tensor \( \bar{R}_{ijkl} \) vanishes at the vacuum thanks to the perturbative unitarity of the four-point amplitude. We therefore concentrate ourselves to the \( \bar{R}_{ijkl;mn} \) term in (A6). The six-point amplitude coming from the \( \bar{R}_{ijkl;mn} \) term in (A6) behaves as

\[ M(123456) \propto s A(123456), \]  

(A32)

with

\[ A(123456) := \{(12)|(34;56)\} + \{(34)|(12;56)\} - \{(13)|(24;56)\} - \{(24)|(13;56)\}. \]  

(A33)

Here we introduce an abbreviation

\[ \{12|34;56\} := \bar{R}_{i_1 i_3 i_4 i_2 i_5 i_6}. \]  

(A34)
Using
\[
\{(12)|(34; 56)\} = \frac{1}{6}\{(12)|(34); (56)\} + \frac{1}{6}\{(12)|(56); (34)\} + \frac{1}{6}\{(12)|(35); (46)\} \\
+ \frac{1}{6}\{(12)|(46); (35)\} + \frac{1}{6}\{(12)|(36); (45)\} + \frac{1}{6}\{(12)|(45); (36)\}
\] (A35)
we obtain
\[
A(123456) := A_1 + A_2 + A_3 + A_4,
\] (A36)
with
\[
A_1 = \frac{1}{6}\left\{\{(12)|(34); 56\} + \{(34)|(12); 56\} - \{(13)|(24); 56\} - \{(24)|(13); 56\}\right\},
\] (A37)
\[
A_2 = \frac{1}{6}\left\{\{(12)|(35); 46\} + \{(12)|(45); 36\} + \{(34)|(15); 26\} + \{(34)|(25); 16\} \\
- \{(13)|(25); 46\} - \{(13)|(45); 26\} - \{(24)|(15); 36\} - \{(24)|(35); 16\}\right\},
\] (A38)
\[
A_3 = \frac{1}{6}\left\{\{(12)|(36); 45\} + \{(12)|(46); 35\} + \{(34)|(16); 25\} + \{(34)|(26); 15\} \\
- \{(13)|(26); 45\} - \{(13)|(46); 25\} - \{(24)|(16); 35\} - \{(24)|(36); 15\}\right\},
\] (A39)
\[
A_4 = \frac{1}{6}\left\{\{(12)|(56); 34\} + \{(34)|(56); 12\} - \{(13)|(56); 24\} - \{(24)|(56); 13\}\right\}.
\] (A40)
Here we used the fact that the covariant derivatives are commutable, justified by the vanishing curvature tensor $\tilde{R}_{ijkl} = 0$ at the vacuum. The $A_1$ term can be computed easily by using the result of $A(1234)$. The $A_2$ and $A_3$ terms can be computed in a manner similar to the computations of $A(12345)$. We obtain
\[
A_1 = A_2 = A_3 = -\frac{1}{2}\{13|42; 56\}.
\] (A41)

37
The $A_4$ term can be computed as

$$A_4 = \frac{1}{12} \left\{ \{12|56;34\} + \{12|65;34\} + \{34|56;12\} + \{34|65;12\} - \{13|56;24\} - \{13|65;24\} - \{24|56;13\} - \{24|65;13\} \right\},$$

$$= \frac{1}{12} \left\{ \{12|56;34\} + \{12|65;34\} + \{43|65;12\} + \{43|56;12\} + \{16|53;24\} + \{15|63;24\} + \{45|62;13\} + \{46|52;13\} \right\},$$

$$= \frac{1}{12} \left\{ (\{12|56;34\} + \{16|53;24\}) + (\{12|65;34\} + \{15|63;24\}) + (\{43|65;12\} + \{45|62;13\}) + (\{43|56;12\} + \{46|52;13\}) \right\}.$$

Applying the second Bianchi identity, it can be simplified further

$$A_4 = -\frac{1}{12} \left\{ \{13|52;64\} + \{13|62;54\} + \{42|63;15\} + \{42|53;16\} \right\},$$

$$= -\frac{1}{12} \left\{ \{31|25;46\} + \{26|31;45\} + \{24|36;15\} + \{35|24;16\} \right\},$$

$$= -\frac{1}{12} \left\{ (\{31|25;46\} + \{35|24;16\}) + (\{26|31;45\} + \{24|36;15\}) \right\},$$

$$= \frac{1}{12} \left\{ \{34|21;56\} + \{21|34;65\} \right\},$$

$$= -\frac{1}{6} \{13|42;56\}. \quad (A42)$$

The second Bianchi identity is used in the first- and fourth-lines in the above calculation. Combining these results, we find the six-point amplitude can be expressed in a simple form,

$$A(123456) = -\frac{5}{3} \{13|42;56\}. \quad (A43)$$

The perturbative unitarity condition in the six-point amplitude $A(123456) = 0$ can now be written in terms of the covariant derivative of the Riemann curvature

$$\bar{R}_{i_1i_4i_2i_3i_5i_6} = 0. \quad (A44)$$

It is straightforward to generalize the calculation presented above to the perturbative unitarity conditions in the $N$-point amplitude,

$$R_{i_1i_4i_2i_3i_5i_6\cdots i_N} = 0. \quad (A45)$$
Since the Taylor expansion coefficients of $R_{ijkl}(\phi)$ are required to vanish at any order, the $N$-point perturbative unitarity requires the Riemann curvature to be

$$R_{ijkl}(\phi) = 0,$$  \hspace{1cm} (A46)

at least in the vicinity of the vacuum. There may exist non-perturbative essential singularity type corrections to (A46), though.

**Appendix B: Background field method**

In this appendix, we briefly summarize the interaction terms used in the calculation of the vacuum polarization functions in the background field method at the one-loop level. The background field method is reviewed in Refs. [128–133].

We start with the lowest order ($O(p^2)$) gauged nonlinear sigma model Lagrangian (37). Let us first decompose $\phi^i$, $W^a_\mu$ and $B_\mu$ into the background fields and the fluctuation fields as

$$\phi^i := \bar{\phi}^i + \xi^i - \frac{1}{2} \bar{\Gamma}^i_{jk} \xi^j \xi^k + \cdots,$$  \hspace{1cm} (B1)

$$W^a_\mu := \bar{W}^a_\mu + W^a_\mu,$$  \hspace{1cm} (B2)

$$B_\mu := \bar{B}_\mu + B_\mu,$$  \hspace{1cm} (B3)

where $\bar{\phi}^i$, $\bar{W}^a_\mu$, and $\bar{B}_\mu$ are the background fields. The dynamical fluctuation fields are denoted by $\xi^i$, $W^a_\mu$, and $B_\mu$. $\bar{\Gamma}^i_{jk}$ represents the Christoffel symbols for the metric $g_{ij}$ at $\phi = \bar{\phi}$. The metric tensor and the Killing vector fields are expanded as

$$g_{ij} = \bar{g}_{ij} + \frac{1}{3} \bar{R}_{iklj} \xi^k \xi^l + \cdots,$$  \hspace{1cm} (B4)

$$w^i_a = \bar{w}^i_a + (\bar{w}^i_a)_j \xi^j + \frac{1}{3} \bar{R}^i_{klj} \bar{w}^j_a \xi^k \xi^l + \cdots,$$  \hspace{1cm} (B5)

$$y^i = \bar{y}^i + (\bar{y}^i)_j \xi^j + \frac{1}{3} \bar{R}^i_{klj} \bar{y}^j \xi^k \xi^l + \cdots,$$  \hspace{1cm} (B6)

where $\bar{g}_{ij}$, and $\bar{R}_{ikjl}$ denote the metric, and the Riemann curvature tensor evaluated at $\phi = \bar{\phi}$. $\bar{w}^i_a$ and $\bar{y}^i$ are the $SU(2)_W$ and $U(1)_Y$ Killing vectors, while $(\bar{w}^i_a)_j$ and $(\bar{y}^i)_j$ are the covariant derivatives of the Killing vectors evaluated at $\phi = \bar{\phi}$.

The Lagrangian (37) is expanded as

$$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots,$$  \hspace{1cm} (B7)
where $L^{(n)}$ is of order $n$ in the fluctuation fields.

The quadratic terms $L^{(2)}$ is given as

$$
L^{(2)} = -\frac{1}{2} \mathcal{W}_\mu^a \left( -\tilde{D}^2 \delta_{ab} \eta^{\mu\nu} + \tilde{D}^\nu \tilde{D}^\mu \delta_{ab} - g_W^2 \tilde{g}_{ij} \tilde{w}_a^i \tilde{w}_b^j \eta^{\mu\nu} - g_W \tilde{W}^{\mu\nu} \varepsilon^{abc} \right) \mathcal{W}_\nu^b \\
- \frac{1}{2} \mathcal{B}_\mu \left( -\partial^2 \eta^{\mu\nu} + \partial^\nu \partial^\mu - g_Y^2 \tilde{g}_{ij} \tilde{y}_i^j \eta^{\mu\nu} \right) \mathcal{B}_\nu \\
+ g_W g_Y \mathcal{W}_\mu^a \left( \tilde{g}_{ij} \tilde{w}_a^i \tilde{w}_b^j \eta^{\mu\nu} \right) \mathcal{B}_\nu \\
+ \frac{1}{2} \xi^{ij} \left( -\tilde{D}^2 \tilde{g}_{ij} - \tilde{D}_\mu \tilde{\phi}_k \tilde{D}^\mu \tilde{\phi}_l \tilde{R}_{kl} \right) \xi^{ij} \\
+ 2 g_W \mathcal{W}_\mu^a \left( \tilde{g}_{jk}(\tilde{w}_a^k)_i \tilde{D}^\mu \tilde{\phi}^j \right) \xi^i \\
+ 2 g_Y \mathcal{B}_\mu \left( \tilde{g}_{jk}(\tilde{y}_a^k)_i \tilde{D}^\mu \tilde{\phi}^j \right) \xi^i \\
- g_W \tilde{g}_{ji} \tilde{w}_a^j \left( \tilde{D}^\mu \mathcal{W}_\mu^a \right) \xi^i - g_Y \tilde{g}_{ji} \tilde{y}_j^i \left( \partial^\mu \mathcal{B}_\mu \right) \xi^i, 
$$

(B8)

with $\eta^{\mu\nu}$ being the space-time metric. Here we define

$$
\tilde{D}_\mu \mathcal{W}_\mu^a := \partial_\mu \mathcal{W}_\mu^a - g_W \varepsilon^{abc} \tilde{W}_\mu^b \mathcal{W}_\mu^c, \quad \text{(B9)}
$$

$$
\tilde{D}_\mu \tilde{\phi}^j := \partial_\mu \tilde{\phi}^j + g_W \tilde{W}_\mu^a \tilde{w}_a^i \tilde{w}_b^j + g_Y \tilde{B}_\mu \tilde{y}_i^j, \quad \text{(B10)}
$$

$$
\tilde{D}_\mu \xi^i := \partial_\mu \xi^i + \tilde{\Gamma}_{kji}^i \left( \partial_\mu \tilde{\phi}^j \right) \xi^k + g_W \tilde{W}_\mu^a \left( \tilde{w}_a^i \right)_j \xi^j + g_Y \tilde{B}_\mu \left( \tilde{y}_a^i \right)_j \xi^j, \quad \text{(B11)}
$$

$$
\tilde{V}_{ij} := V_{ij} \bigg|_{\phi = \tilde{\phi}}, \quad \text{(B12)}
$$

In order to compute radiative corrections, we introduce the gauge fixing action,

$$
L_{GF} := -\frac{1}{2\alpha_W} G_W^a G_W^a - \frac{1}{2\alpha_Y} G_Y G_Y, \quad \text{(B13)}
$$

where

$$
G_W^a := \tilde{D}^\mu \mathcal{W}_\mu^a - g_W \alpha_W \tilde{g}_{ij} \tilde{w}_a^i \xi^j, \quad \text{(B14)}
$$

$$
G_Y := \partial^\mu \mathcal{B}_\mu - g_Y \alpha_Y \tilde{g}_{ij} \tilde{y}_i^j \xi^j. \quad \text{(B15)}
$$

with $\alpha_W$ and $\alpha_Y$ being gauge fixing parameters for $SU(2)_W$ and $U(1)_Y$ symmetry, respectively. In the one-loop calculation performed in § V A, we take $\alpha_W = \alpha_Y = 1$ (’t Hooft Feynman gauge).
We then obtain

\[
\mathcal{L}^{(2)} + \mathcal{L}_{GF} = -\frac{1}{2} \mathcal{W}_\mu^{\alpha} \left( -\delta^2 \delta_{ab} \eta^{\mu
u} + \left( 1 - \frac{1}{\alpha_W} \right) \bar{D}^\mu \bar{D}_\nu \delta_{ab} - g_W^2 \bar{g}_{ij} \bar{w}_{i\alpha} \bar{w}_{j\beta} \eta^{\mu
u} - 2g_W \bar{W}^c \eta^{abc} \right) \mathcal{W}^b_\nu \\
- \frac{1}{2} B_\mu \left( -\partial^2 \eta^{\mu
u} + \left( 1 - \frac{1}{\alpha_Y} \right) \partial^\mu \partial_\nu - g_Y^2 \bar{g}_{ij} \bar{y}^i \bar{y}^j \eta^{\mu
u} \right) B_\nu \\
+ g_W g_Y \mathcal{W}_\mu^{\alpha} \left( \bar{g}_{ij} \bar{w}_{i\alpha} \eta^{\mu
u} \right) B_\nu \\
+ \frac{1}{2} \xi^i \left( -\delta^2 \bar{g}_{ij} - \bar{D}_\mu \bar{\phi}^k \bar{D}^\mu \bar{R}_{kij} - \alpha_W g_W^2 \bar{g}_{ij} \bar{w}_{i\alpha} \bar{w}_{j\beta} - \alpha_Y g_Y^2 \bar{g}_{ij} \bar{y}^i \bar{y}^j \right) \xi^j \\
+ 2g_W \mathcal{W}_\mu^\alpha \left( \bar{g}_{jk} (\bar{w}_{i\alpha}^b \bar{D}^\mu \bar{\phi}_j^a) \right) \xi^i + 2g_Y B_\mu \left( \bar{g}_{jk} (\bar{y}^k) \bar{D}^\mu \bar{\phi}_j^a \right) \xi^i.
\] 

(B16)

We also need to introduce the Faddeev-Popov (FP) action

\[
\mathcal{L}_{FP} := ig_W c_W^{\alpha} \frac{\delta G_W^{\alpha}}{\delta \theta_W^a} c_W^b + ig_Y \bar{c}_Y \frac{\delta G_Y}{\delta \bar{\theta}_Y} c_Y + ig_Y c_W^\alpha \frac{\delta G_W^{\alpha}}{\delta \theta_W^a} c_Y + ig_W \bar{c}_Y \frac{\delta G_Y}{\delta \theta_W^a} c_W^b,
\]

(B17)

associated with the gauge fixing term where

\[
\frac{\delta G_W^{\alpha}}{\delta \theta_W^a} := -\frac{1}{g_W} \left[ (\partial^\mu \delta_{ab} - g_W \varepsilon^{abc} \bar{W}_\mu^{bc}) (\partial_\mu \delta_{bc} - g_W \varepsilon^{dec} \bar{W}_\mu^{ce}) + g_W^2 \alpha_W \bar{g}_{ij} \bar{w}_{i\alpha} \bar{w}_{j\beta} \right] + \mathcal{O}(\xi)
\]

(B18)

\[
\frac{\delta G_W^{\alpha}}{\delta \bar{\theta}_Y} := -g_W \alpha_W \bar{g}_{ij} \bar{w}_{i\alpha} \bar{y}^j + \mathcal{O}(\xi),
\]

(B19)

\[
\frac{\delta G_Y}{\delta \theta_W^a} := -g_Y \alpha_Y \bar{g}_{ij} \bar{y}^i \bar{w}^j_\beta + \mathcal{O}(\xi),
\]

(B20)

\[
\frac{\delta G_Y}{\delta \bar{\theta}_Y} := -\frac{1}{g_Y} (\partial^2 + g_Y^2 \alpha_Y \bar{g}_{ij} \bar{y}^i \bar{y}^j) + \mathcal{O}(\xi).
\]

(B21)

The \( \mathcal{L}_{FP} \) is expanded as

\[
\mathcal{L}_{FP} = i \left( (\bar{D}^\mu c_W^{\alpha}) \bar{D}_\mu c_W^b - g_W^2 \alpha_W \bar{g}_{ij} \bar{w}_{i\alpha} \bar{w}_{j\beta} c_W^a c_W^b \right) \\
+ i \left( (\partial^\mu c_Y) \partial_\mu c_Y - g_Y^2 \alpha_Y \bar{g}_{ij} \bar{y}^i \bar{y}^j \bar{c}_Y c_Y \right) \\
- ig_W g_Y \alpha_W \bar{c}_W^\alpha \bar{g}_{ij} \bar{w}_{i\alpha} \bar{y}^j c_Y \\
- ig_W g_Y \alpha_Y \bar{c}_Y \bar{g}_{ij} \bar{y}^i \bar{w}^j_\alpha c_W + \cdots,
\]

(B22)

where

\[
\bar{D}_\mu c_W^{\alpha} := \partial_\mu c_W^{\alpha} - g_W \varepsilon^{abc} \bar{W}_\mu^{bc} c_W^c,
\]

(B23)

\[
\bar{D}_\mu c_W^a := \partial_\mu c_W^a - g_W \varepsilon^{abc} \bar{W}_\mu^{bc} c_W^c.
\]

(B24)

In Eq. (B22), we only show the quadratic terms of the fluctuation fields.
The one-loop vacuum polarizations among the electroweak gauge boson can be evaluated by using the quadratic Lagrangian, \( L^{(2)} + L_{\text{GF}} + L_{\text{FP}} \). In § V A, we calculate the one-loop diagrams where the internal lines are the fluctuation fields or FP ghosts.

**Appendix C: \( f \bar{f} \to \varphi^i \varphi^j \) amplitude**

The four-point scalar boson scattering amplitudes are described by the Riemann curvature tensor \( \bar{R}_{ijkl} \) and the covariant derivatives of the potential \( \bar{V}_{ij}, \bar{V}_{ijk}, \bar{V}_{ijkl} \) at the vacuum in the nonlinear sigma model, as we have shown in § III. These tensors can, therefore, be measured through the measurements of the scalar boson scattering cross sections.

When we consider a gauged nonlinear sigma model, the derivative \( \partial_{\mu} \phi^i \) is replaced by the covariant one \( (D_{\mu} \phi)^i \)

\[
(D_{\mu} \phi)^i := \partial_{\mu} \phi^i + g_V V_{\mu} v^i(\phi),
\]

with \( V_{\mu} \) and \( v^i(\phi) \) being a gauge field and its corresponding Killing vector. The gauge coupling strength is denoted by \( g_V \) in (C1). If the Killing vector \( v^i(\phi) \) does not vanish at the vacuum

\[
\bar{v}^i := v^i(\phi) \bigg|_{\phi = \bar{\phi}} \neq 0,
\]

it implies that the gauge symmetry is spontaneously broken, and the gauge boson \( V_{\mu} \) acquires its mass

\[
M^2_V = g_V^2 \bar{g}_{ij} (\bar{v}^i) (\bar{v}^j),
\]

with

\[
\bar{g}_{ij} := g_{ij}(\phi) \bigg|_{\phi = \bar{\phi}}.
\]

The magnitude of the Killing vector at the vacuum, \( \bar{g}_{ij} (\bar{v}^i) (\bar{v}^j) \), can therefore be determined by the gauge boson mass measurement.

How can we measure the first covariant derivative of the Killing vector

\[
(\bar{v}^i)_j := (v^i)_j \bigg|_{\phi = \bar{\phi}}
\]

from experimental observables in the gauged nonlinear sigma model, then? We address the issue in this appendix and show that the process \( f \bar{f} \to V_{\mu} \to \varphi^i \varphi^j \) can be used to determine \( (\bar{v}^i)_j \). Here we introduce a spin-1/2 fermion multiplet \( f \). It couples with the gauge field \( V_{\mu} \).
through its covariant derivative
\[ D_\mu f := \partial_\mu f + g_V V_\mu T^{(f)}_V f, \quad (C6) \]
with \( T^{(f)}_V \) being the charge matrix of the fermion multiplet \( f \). Note that, in order to keep the Lagrangian gauge invariant, the fermion current
\[ J^\mu_V := \bar{f} \gamma^\mu T^{(f)}_V f \quad (C7) \]
must be conserved
\[ 0 = \partial_\mu J^\mu_V. \quad (C8) \]

In order to calculate the \( f \bar{f} \to V_\mu \to \varphi^i \varphi^j \) amplitude, we consider the gauge interaction Lagrangian
\[ \mathcal{L}_{V\phi} = g_V V_\mu g_{ij}(\phi)(\partial^\mu \varphi^i)(\varphi^j), \quad (C9) \]
which can be derived from the nonlinear sigma model kinetic term,
\[ \frac{1}{2} g_{ij}(\phi)(D_\mu \phi)^i (D^\mu \phi)^j \in \mathcal{L}. \quad (C10) \]
Expanding the scalar manifold metric \( g_{ij}(\phi) \) and the Killing vector \( v^j(\phi) \) by the dynamical excitation field \( \varphi^i \), we obtain
\[ g_{ij}(\phi) = \bar{g}_{ij} + \varphi^k \bar{g}_{ij,k} + \cdots, \quad (C11) \]
\[ v^j(\phi) = \bar{v}^j + \varphi^k (\bar{v}^j)_k + \cdots, \quad (C12) \]
with
\[ \bar{g}_{ij,k} := \frac{\partial}{\partial \phi^k} g_{ij} \bigg|_{\phi = \bar{\phi}}, \quad (\bar{v}^j)_k := \frac{\partial}{\partial \phi^k} v^j \bigg|_{\phi = \bar{\phi}}, \quad (C13) \]
and
\[ \varphi^i = \bar{\varphi}^i + \varphi^i. \quad (C14) \]
The interaction Lagrangian (C9) can be expanded as
\[ \mathcal{L}_{V\phi} = g_V V_\mu \bar{g}_{ij}(\partial^\mu \varphi^i)(\bar{v}^j) + g_V V_\mu (\partial^\mu \varphi^i) \varphi^k (\bar{g}_{ij}(\bar{v}^j)_k + \bar{g}_{ij,k}(\bar{v}^j)) + \cdots. \quad (C15) \]
Note that on-shell amplitudes are not affected by total derivative terms in the Lagrangian. The interaction Lagrangian (C15) can thus be replaced by
\[ \mathcal{L}'_{V\phi} = \mathcal{L}_{V\phi} - \frac{1}{2} \partial^\mu \left( g_V V_\mu \varphi^i \varphi^k (\bar{g}_{ij}(\bar{v}^j)_k + \bar{g}_{ij,k}(\bar{v}^j)) \right) \]
\[ = g_V V_\mu \bar{g}_{ij}(\partial^\mu \varphi^i)(\bar{v}^j) - g_V (\partial^\mu V_\mu) \varphi^i \varphi^k (\bar{g}_{ij}(\bar{v}^j)_k + \bar{g}_{ij,k}(\bar{v}^j)) \]
\[ + \frac{1}{2} g_V V_\mu (\partial^\mu \varphi^i) \varphi^k (\bar{g}_{ij}(\bar{v}^j)_k + \bar{g}_{ij,k}(\bar{v}^j) - \bar{g}_{kj}(\bar{v}^j)_i - \bar{g}_{kj,i}(\bar{v}^j)) + \cdots. \quad (C16) \]
On the other hand, it is straightforward to show

\[
g_{ij}(v^j)_k - g_{kj}(v^j)_i = g_{ij}(v^j)_k + g_{ij} \Gamma^j_{kl} v^l - g_{kj}(v^j)_i + g_{kj} \Gamma^j_{kl} v^l
\]

\[
= g_{ij}(v^j)_k + \frac{1}{2} \left[ g_{id,k} + g_{ki,l} - g_{kt,i} \right] v^l
- g_{kj}(v^j)_i - \frac{1}{2} \left[ g_{kl,i} + g_{ik,l} - g_{il,k} \right] v^l
= g_{ij}(v^j)_k + g_{id,k}(v^l) - g_{kj}(v^j)_i - g_{kt,i}(v^l).
\] (C17)

Here the Affine connection \( \Gamma^j_{kl} \) is defined by

\[
\Gamma^j_{kl} = \frac{1}{2} g^{jm} \left( g_{ml,k} + g_{km,l} - g_{kl,m} \right).
\] (C18)

It is now easy to see

\[
\mathcal{L}_{V\phi} = g_V V^\mu \bar{\phi}^i (\partial^\mu \phi^j) - g_V (\partial^\mu V^\mu) \phi^i \phi^k \left( \bar{g}_{ij} (\bar{v}^j)_k + \bar{g}_{ij,k} (\bar{v}^j) \right)
+ \frac{1}{2} g_V V^\mu (\partial^\mu \phi^j) \phi^k \left( \bar{g}_{ij} (\bar{v}^j)_k - \bar{g}_{kj} (\bar{v}^j)_i \right) + \cdots.
\] (C19)

Thanks to the fermion current conservation, the term proportional to \( \partial^\mu V^\mu \) does not contribute to the \( f \bar{f} \rightarrow V^\mu \rightarrow \phi^i \phi^j \) amplitude. It is now easy to show

\[
\mathcal{M}(f \bar{f} \rightarrow V^\mu \rightarrow \phi^i \phi^j) \propto \left( \bar{g}_{ij}(\bar{v}^j)_k - \bar{g}_{kj}(\bar{v}^j)_i \right) \frac{g_V^2 \delta^{ab}}{s - M_V^2}.
\] (C20)

The first covariant derivative of the Killing vector, \( \bar{g}_{ik}(\bar{v}^k)_{ij} \), thus plays the role of the \( V^\mu-\phi^i-\phi^j \) interaction vertex in the \( f \bar{f} \rightarrow V^\mu \rightarrow \phi^i \phi^j \) amplitude.

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