Instantaneous Measurement of Non-local Variables

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Seventy years ago Landau and Peierls \cite{1} claimed that the instantaneous measurability of nonlocal variables (i.e., variables which related to more than one small region of space) contradicts relativistic causality. Twenty years ago, Aharonov and Albert \cite{2} showed that some nonlocal variables (e.g., the Bell operator, see below) can be measured instantaneously and that this does not contradict causality. They also showed explicitly how the possibility of performing instantaneous von Neumann measurements of some other nonlocal variables does contradict causality. The question: “What are the observables of relativistic quantum theory?” remains topical even today \cite{3}.

A variable can obtain the status of an observable if it can be measured. However, the standard (von Neumann) definition of quantum measurement is too restrictive for defining a physical observable: the von Neumann definition requires that eigenstates of the measured variable are not changed due to the measurement process. The existence of a verification measurement which yields the eigenvalue of a variable with certainty, if prior to the measurement the quantum system was in the corresponding eigenstate, is enough for giving the status of an observable for such a variable, even if the measurement does not leave the system in this eigenstate as the von Neumann measurement does. (If, initially, the system is in a superposition or mixture of the eigenstates of the observable, then the verification measurement yields one of the corresponding eigenvalues according to the quantum probability law.)

The meaning of “instantaneous measurement” is that in a particular Lorentz frame, at time \( t \), observers perform local actions for a duration of time which can be as short as we wish. At the end of the measurement interactions, the information about the outcome of the measurement is classically recorded in the results of local (irreversible) measurements. In order to infer which eigenvalue of the nonlocal variable the system had originally, or to generate correctly distributed probabilistic outcome, these classical results are later combined at a point within the future light cones of all the observers.

Note the difference with the case of exchange measurements \cite{4} which can also be performed for all nonlocal variables. In the exchange measurement, local operations of swapping lead to swapping between the quantum state of the composite system and the quantum state of the local separated parts of the measuring device. In order to find out which eigenvalue the system had originally, it is required coherent maintaining of all these parts until they enter the forward light cone of all of the original subsystems one wishes to measure where final local joint measurement is performed. After instantaneous swapping, the outcome of the measurement is not written in the form of classical information and, in fact, the outcome of the quantum measurement does not exist yet: at this stage the exchange measurement can be reversed and the system can be brought back to its original (in general unknown) state.

In this Letter, I will show that apart from variables related to the spread-out fermionic wave function, all nonlocal variables have the status of observables in the framework of relativistic quantum mechanics, i.e., all variables related to two or more separate sites are measurable instantaneously using verification measurements. This includes variables with entangled eigenstates and nonlocal variables with product eigenstates \cite{5}. Verification measurements have been considered before. It has been shown \cite{6} that verification measurements of some nonlocal variables erase local information and, therefore, cannot be ideal von Neumann measurements. Recently, Groisman and Reznik \cite{7} showed that there are instantaneous verification measurements for all spin variables of a system of two separated spin-\( \frac{1}{2} \) particles. Consider, for example a nonlocal variable of two spin-\( \frac{1}{2} \) particles located in separate locations \( A \) and \( B \), whose eigenstates are the following product states:

\[
\begin{align*}
|\Psi_1\rangle &= |\uparrow_z\rangle_A |\uparrow_z\rangle_B, \\
|\Psi_2\rangle &= |\uparrow_z\rangle_A |\downarrow_z\rangle_B, \\
|\Psi_3\rangle &= |\downarrow_z\rangle_A |\uparrow_z\rangle_B, \\
|\Psi_4\rangle &= |\downarrow_z\rangle_A |\downarrow_z\rangle_B.
\end{align*}
\]

An instantaneous ideal von Neumann measurement of
this variable does contradict causality. Assume that at
time \( t \) such an ideal measurement is performed. Then we
can send superluminal signal from \( A \) to \( B \) in the follow-
ing way. We prepare in advance the system in the state
\( |\Psi_1\rangle \) and agree that Bob at site \( B \) measures the spin \( z \)
component of his particle shortly after time \( t \). Now, in
order to send a superluminal signal, Alice at site \( A \) can at
a very short time before time \( t \) flip her spin. If she does
so, then after the nonlocal measurement at time \( t \), the
system will end up either in state \( |\Psi_1\rangle \) or in state \( |\Psi_4\rangle \).
In both cases Bob has a nonvanishing probability to find
his spin “down” in \( \hat{z} \) direction, while this probability is
zero if Alice decides not to flip her spin.

The method for the verification measurement I present
here uses teleportation technique \( \otimes \). The first step is the
teleportation of the state of the spin from \( B \) (Bob’s site)
to \( A \) (Alice’s site). Bob and Alice do not perform the full
teleportation (which invariably requires a finite period of
time), but only the Bell measurement at Bob’s site which
might last, in principle, as short a time as we wish. (I
will continue to use the term “teleportation” just for this
first step of the original proposal \( \otimes \).)

In the teleportation procedure for a spin-\( \frac{1}{2} \) particle we
start with a prearranged EPR(Bohm) pair of spin-\( \frac{1}{2} \)
particles one of which is located at Bob’s site and another at
Alice’s site, \( |\Psi_{-}\rangle_{AB} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_{A}|\downarrow\rangle_{B} - |\downarrow\rangle_{A}|\uparrow\rangle_{B}) \).
The procedure is based on the identity
\[
|\Psi_{-}\rangle_{1,2,3} = \frac{1}{2} (|\Psi_{-}\rangle_{1,2,3} + |\Psi_{+}\rangle_{1,2,3} +
|\Phi_{-}\rangle_{1,2,3} + |\Phi_{+}\rangle_{1,2,3})
\]
(2)
where \( |\Psi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle \pm |\downarrow\rangle|\uparrow\rangle \) are eigenstates of the Bell operator and \( |\Phi_{\pm}\rangle \) signifies the state \( |\Psi\rangle \) rotated by \( \pi \) around \( \hat{z} \) axis, etc. Thus,
the Bell operator measurement performed on the two par-
ticles in Bob’s site “collapses” (or effectively collapses) to
one of the branches of the superposition, the RHS of \( \otimes \),
and, therefore, teleports the state \( |\Psi\rangle \) of Bob’s particle
to Alice except for a possible rotation by \( \pi \) (known to
Bob) around one of the axes.

The second step is taken by Alice. She can perform it
at time \( t \) without waiting for Bob. She measures the spin
of her particle in the \( z \) direction. If the result is “up”, she
measures the spin of the particle teleported from Bob in
the \( z \) direction, and if her spin is “down”, she measures
the spin of the Bob’s particle in the \( x \) direction.

This completes the measurement except for combining
local results together for finding out the result of the non-
local measurement. Indeed, the eigenstates of the spin
in the \( z \) direction and in the \( x \) direction are teleported
without leaving their lines. Thus, Bob’s knowledge about
possible flip together with Alice’s results distinguish un-
ambiguously between the four eigenstates \( \otimes \).

The method I presented above can be modified for mea-
surement of other nonlocal variables of two spin-\( \frac{1}{2} \)
particles. However, I will turn now to another, univer-
sal, method which is applicable to any nonlocal variable
\( O(q_1,q_2,...) \), where \( q_A \) belongs to region \( A \), etc. I will
not try to optimize the method or consider any realistic
proposal: my task is to show that, given unlimited re-
sources of entanglement and arbitrary local interactions,
any nonlocal variable is measurable.

I will start with the case of a general variable of a com-
posite systems with two parts. First, (for simplicity),
Alice and Bob perform unitary operations which swap
the states of their systems with the states of sets of K
spin-\( \frac{1}{2} \) particles. In this way Alice and Bob will need the
teleportation procedure for spin-\( \frac{1}{2} \) particles only. Tele-
portation of the states of all \( K \) individual spins leads to
teleportation the state of the set, be it entangled or not.

The general protocol is illustrated in Fig. 1. The
resources include numerous teleportation channels ar-
ranged in a particular way: two channels for the first
round of back and forth teleportations, then \( 4^K - 1 \)
clusters, each includes two channels for the second round of
back and forth teleportations and \( 4^{2K} - 1 \) sub-clusters.
Each sub-cluster, in turn, includes two channels for the
third round of teleportation and \( 4^{2K} - 1 \) similar sub-
clusters, etc. The protocol consists of the following steps:

- Bob teleports his system (\( K \) spin-\( \frac{1}{2} \) particles) to Alice
  and records the outcome of the Bell measurements \( n \).

As before, “teleports” means that Bob performs the
Bell measurements but does not send the outcome to
Alice. The number of possible outcomes is \( N = 4^K \). We
signify them by \( n = 1,2,...,N \), with \( n = 1 \) corresponding
to singlets in all Bell measurements, i.e. to teleportation
without distortion.

- Alice performs a unitary operation \( U \) on the com-
  posite system of her and the teleported spins which, un-
der the assumption of non-distorted teleportation, trans-
forms the eigenstates of the nonlocal variable (which now
actually are fully located in Alice’s site) to product states
in which each spin is either “up” or “down” along the \( z \)
direction.

- Alice teleports the complete composite system (\( 2K \)
  spin-\( \frac{1}{2} \) particles) to Bob.

Note that if the system is in one of the product states
in the spin \( z \) basis, then it will remain in this basis.

- If \( n = 1 \) Bob measures the teleported system in the
  spin \( z \) basis.

In this case (the probability for which is \( \frac{1}{4^K} \)), Bob gets
the composite system in one of the spin \( z \) product states
and his measurements in the spin \( z \) basis complete the
measurement of the nonlocal variable.

If \( n \neq 1 \) Bob teleports the system back to Alice in
the teleportation channel of cluster \( n \). He records the
outcome of the Bell measurements \( m_1 \) which can have
values from 1 to \( M = 4^{2K} \).

Since in this case Alice’s operations do not bring the
eigenstates of the nonlocal variable to the spin \( z \) basis,
Bob teleports the system back to Alice “telling” her the
outcome of his previous Bell measurements via the channel he uses for the teleportation.

- Alice performs unitary operations on each system in $N-1$ teleportation channels of the second round which, under the assumption of no distortion in these teleportations, transforms the eigenstates of the nonlocal variable to product spin $z$ eigenstates.

Alice's operations include corrections required due to her and Bob's teleportations and her unitary transformation of the first round.

- Alice teleports all $N-1$ systems back to Bob.
- If $m_1 = 1$ Bob measures the system teleported from Alice in cluster $n$ in the spin $z$ basis.

Again, in that case, the spin measurements complete the measurement of the nonlocal variable, since their results together with the outcomes of Alice's and Bob's Bell measurements specify uniquely the eigenvalue of the nonlocal variable.
If $m_1 \neq 1$ Bob teleports the system back to Alice in the teleportation channel of sub-cluster $m_1$ of cluster $n$. He records the outcome of the Bell measurements $m_2$.

- Alice performs unitary operations on each system in $(N-1)(M-1)$ teleportation channels of the third round. The operation on each system is such that if Bob, indeed, teleported the system in this channel, and if his last teleportation happened to be without distortion, then the eigenstates of the nonlocal variable are transformed into product spin $z$ states.

Alice’s operations include corrections required due to her and Bob’s teleportations and her unitary transformations of the first and second rounds.

- Alice teleports all $(N-1)(M-1)$ systems back to Bob.

- If $m_2 = 1$ Bob measures the system teleported from Alice in sub-cluster $m_1$ of cluster $n$ in the spin $z$ basis. If $m_2 \neq 1$ Bob teleports the system back to Alice in the teleportation channel of sub-sub-cluster $m_2$ of sub-cluster $m_1$ of cluster $n$. He records the outcome of the Bell measurements $m_3$.

Alice and Bob continue this procedure. The nonlocal measurement is completed when, for the first time, Bob performs a teleportation without distortion. Since, conceptually, there is no limitation for the number of teleportation rounds, and each round (starting form the second) has the same probability for success, the measurement of the nonlocal variable can be performed with probability arbitrarily close to 1. Given the desired probability of the successful nonlocal measurement, Alice and Bob decide about the number of rounds of teleportations. The number of entangled pairs required for each round grows exponentially with the number of rounds. While Bob uses only one teleportation channel in each round and stops after his first teleportation without distortion, Alice has to perform all teleportations in all channels.

The generalization to a system with more than two parts is more or less straightforward. Let us sketch it for three-part system. First, Bob and Carol teleport their parts to Alice. Alice performs a unitary transformation which, under the assumption of undisturbed teleportations of both Bob and Carol, transforms the eigenstates of the nonlocal variable to product states in the spin $z$ basis. Then she teleports the complete system to Bob. Bob teleports it to Carol in a particular channel $n_B$ depending on the results of the Bell measurement of his first teleportation. Carol teleports all the systems from the teleportation channels from Bob back to Alice. In particular, the system from channel $i_B$ she teleports in the channel $(n_B, n_C)$ depending on her Bell measurement result $n_C$. The system corresponding to $(n_B, n_C) = (1, 1)$ is not teleported, but measured by Carol in the spin $z$ basis. Alice knows the transformation performed on the system which arrives in her channels $(n_B, n_C)$ except for corrections due to the last teleportations of Bob and Carol. She assumes that there were no distortion in those, and teleports all the systems back to Bob after the unitary operation which transforms the eigenstates of the variable to product states in the spin $z$ basis. Alice, Bob and Carol continue the procedure until the desired probability of successful measurement is achieved.

The required resources, such as the number of teleportation channels and required number of operations are very large, but this does not concern us here. We have shown that there are no relativistic constraints preventing instantaneous measurement of any variable of a quantum system with spatially separated parts, answering the above long standing question. This question is relevant for quantum cryptography and quantum computation performed with distributed systems. The practical advantage of the method presented in this Letter is that it relies on prior entanglement and does not require coherent transportation of quantum systems.

Can this result be generalized to a quantum system which itself is in a superposition of being in different places? The key to this question is the generality of the assumption of the possibility to perform any local operation. If a quantum state of a particle which is in a nonlocal superposition can be locally transformed to (an entangled) state of local quantum systems, then any variable of the particle is measurable through the measurement of the corresponding composite system. However, while for bosons it is clear that there are such local operations (transformation of photon state to entangled state of atoms has been achieved in the laboratory), for fermion states the situation is different. If the transformation of a superposition of a fermion state to entangled state of atoms is possible, then these local separated in space variables should fulfill anti-commutation relations. This is the reason to expect super-selection rules which prevent such transformations.

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