Kalman Filtering Algorithm for Integrated Navigation System in Unmanned Aerial Vehicle

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Abstract. Through the complementarity of the satellite navigation positioning system (GNSS) and the inertial navigation system (INS), the combination of GNSS/INS can make up for the shortcomings of a single system that is difficult to improve, thereby greatly improving the accuracy of the integrated navigation system. In this paper, the transformation relationship between the carrier coordinate system and the navigation coordinate system is deduced, and the INS position and velocity measurement methods are derived. Based on the theory of GNSS/INS loose combined system and Kalman combined filter, the system equation and measurement equation of combined filter Kalman are studied in depth. The principle of discrete Kalman filter is researched, and five basic equations of Kalman filter are given. Then the speed and position model of Kalman filter of GNSS/INS loose combined navigation is derived in detail, and according to the speed and position model perform semi-physical simulation experiments. The experimental results show that the inertial sensor can provide navigation information during the GNSS receiving signal gap, and the position and speed information of the GNSS can also correct the navigation information of the inertial sensor.

1. Introduction
There are many ways to combine GNSS and INS systems. The difference lies in the "depth" of the interaction and the shared information between the systems. The more common combination methods are decoupled combination, loose combination, tight combination and super-compact combination. The system works independently and provides two different results [1]. GNSS is generally considered more accurate and can be used as a result of the system. In addition, the results of GNSS are usually used to correct (or reset) the results of INS, but do not estimate the cause of sensor drift. In the absence of GNSS data, the system results are provided entirely by the inertial sensor, but the inertial sensor will drift severely for a long time. For this limitation, a decoupling combination strategy is usually not used.

In the loose combination method, the GNSS results are fused with the results obtained by the inertial sensor to obtain the final output of the combined system, while in the tight combination method, the combination is "deeper" because the GNSS raw measurement is directly filtered with the INS information in a suitable filter. Combined in the device [2,3]. In the super tight combination, GNSS and INS devices no longer work as independent systems. GNSS measurements are used to estimate INS errors and INS measurements to assist the GNSS receiver tracking loop; this combination is obviously at a deep level and requires access to the receiver's firmware, so it is usually implemented by the receiver manufacturer or software receiver [4,5].

Since the inertial sensors and GNSS receivers used in this article are both low-cost devices, this article will use a loose combination approach suitable for low-cost integrated navigation. The loose combination is relatively simple in these methods and avoids understanding the limitations of the
coupled inertial sensor. In this way, the inertial navigation settlement process and the GNSS information reception and filtering process are independent. Information fusion, and calculate the position, velocity and attitude information after fusion.

Figure 1. Loose combined navigation system.

Generally speaking, the loose combination is performed in a closed-loop manner [6], and the fused INS error is sent from the Kalman filter to the IMU module to correct the state of the INS, and the error estimated by the sensor can also compensate the original IMU measurement value. The other method is an open-loop method, where the INS works independently of the Kalman filter, and the fused INS error is not sent back to the IMU; in this way, the inertial sensor error remains the same, and the inertial navigation state error increases rapidly. Therefore, a larger error is introduced in the combined system. This method can only be used for high-cost inertial sensors, which is characterized by small errors; for low-cost or MEMS-based inertial systems, a closed-loop method is generally used.

2. Selection of State Variables and Establishment of System Equations

The main purpose of the combined Kalman filter is to estimate the error of the INS system, and the GNSS information is used as an external aid[7,8,9]. The basic state vector to be estimated includes nine navigation parameter errors[10,11,12], three attitude errors $\phi_x, \phi_y, \phi_z$; three speed errors $\delta V_E, \delta V_N, \delta V_U$; three position errors $\delta L, \delta \lambda, \delta h$, and includes six device error parameters, gyroscope drift error $\varepsilon_x, \varepsilon_y, \varepsilon_z$; accelerometer bias $V_x, V_y, V_z$.

2.1. Carrier Attitude Angle Error Equation

$$\dot{\phi}_E = (\omega_x \sin L + \frac{V_E}{R+h} \tan L) \phi_N - (\omega_x \cos L + \frac{V_E}{R+h} \omega_y ) \phi_N - \frac{\delta V_N}{R+h} + \varepsilon_E \tag{1}$$

$$\dot{\phi}_N = -(\omega_x \sin L + \frac{V_E}{R+h} \tan L) \phi_E - \frac{V_N}{R+h} \phi_N - \frac{\delta V_E}{R+h} - \omega_y \sin L \delta L + \varepsilon_N \tag{2}$$

$$\dot{\phi}_U = (\omega_x \cos L + \frac{V_E}{R+h} \omega_y ) \phi_E + \frac{V_N}{R+h} \phi_N + \frac{\delta V_E}{R+h} \tan L + (\omega_x \cos L + \frac{V_E}{R+h} \sec^2 L) \delta L + \varepsilon_U \tag{3}$$

Where, $V_E, V_N, V_U$ representing east, north, and sky speeds; $L$ representing the latitude of the earth; $\omega_x = 7.29 \times 10^4 rad/ s$ is the earth rotation angular velocity; $R = 6378137$ is the earth radius; $\varepsilon_E, \varepsilon_N, \varepsilon_U$ are the Gyro drifts in the east, north and sky for the carrier.
2.2. Carrier Velocity Error Equation

\[ \delta V_E = -f_E \phi_N + f_N \phi_U + \frac{V_N}{R+h} \tan L - \frac{V_U}{R+h} \delta V_E + (2 \omega_\alpha \sin L + \frac{V_N}{R+h} \tan L) \delta V_N - (2 \omega_\alpha \cos L + \frac{V_U}{R+h}) \delta V_U + (2 \omega_\alpha \cos L_N + 2 \omega_\alpha \sin L_N + \frac{V_N}{R+h} \sec^2 L) \delta L + \nabla_E \]

(4)

\[ \delta V_N = f_E \phi_E - f_N \phi_U - (2 \omega_\alpha \sin L + \frac{V_E}{R+h} \tan L) \delta V_E - \frac{V_N}{R+h} \delta V_N - \frac{V_U}{R+h} \delta V_U - (2 \omega_\alpha \cos L + \frac{V_E}{R+h} \sec^2 L) \delta V_E \delta L + \nabla_N \]

(5)

\[ \delta V_U = -f_E \phi_E + f_N \phi_U + 2 \omega_\alpha \cos L + \frac{V_E}{R+h} \delta V_E + \frac{2 V_N}{R+h} \delta V_N - 2 \omega_\alpha \sin L \delta V_E \delta L + \nabla_U \]

(6)

among them \( f_E, f_N, f_U \) respecting specific forces measured by the accelerometer in the east, north, and sky directions, and \( \Delta_E, \Delta_N, \Delta_U \) are the east, north, and sky accelerations.

2.3. Carrier Position Error Equation

\[ \delta L = \frac{1}{R+h} \delta V_N \]

(7)

\[ \delta \dot{h} = \delta V_U \]

(8)

\[ \delta \dot{x} = \frac{\sin L}{(R+h) \cos L} \delta V_E + \frac{V_E \tan L}{(R+h) \cos L} \delta L \]

(9)

2.4. Gyro Random Drift Error

\[ \varepsilon = \varepsilon_b + \varepsilon_r + \omega_g \]

(10)

Where \( \varepsilon_b \) is the random constant error, \( \varepsilon_r \) is the first-order Markov process, \( \omega_g \) is white noise.

2.5. Accelerometer Error

\[ \nabla = \nabla_r + \omega_h \]

(11)

Where, \( \nabla_r \) represents a first-order Markov drift.

Combining equations (1)–(11), the state equation of the Kalman filter of the loose combination system can be obtained as:

\[ X_k = \Phi_{k-1:k-1} X_{k-1} + \Gamma_{k-1:k-1} W_{k-1} \]

(12)

among them:

\[ X = [\delta L, \delta \dot{h}, \delta \dot{x}, \delta V_E, \delta V_N, \delta V_U, \phi_E, \phi_N, \phi_U, \epsilon_r, \epsilon_s, \epsilon_t, \nabla_x, \nabla_y, \nabla_z]^T \]

System noise matrix:

\[ W = [\eta_{ax}, \eta_{ay}, \eta_{az}, \eta_{ax}, \eta_{ay}, \eta_{az}, \eta_{bx}, \eta_{by}, \eta_{bz}, \eta_{bx}, \eta_{by}, \eta_{bz}, \eta_{ax}, \eta_{ay}, \eta_{az}]^T \]

Where \([\eta_{ax}, \eta_{ay}, \eta_{az}]\) is the accelerometer noise, \([\eta_{bx}, \eta_{by}, \eta_{bz}]\) is the gyroscope noise, \([\eta_{bx}, \eta_{by}, \eta_{bz}]\) are Gauss-Markov driving noise of accelerometer and gyroscopes respectively.

Therefore, the noise driving matrix \( \Gamma \) and the state transition matrix \( \Phi \) can be deduced:
\[
\Gamma = \begin{bmatrix}
-C^n_b & O_{3\times3} & O_{3\times3} & O_{3\times3} \\
O_{3\times3} & C^n_b & O_{3\times3} & O_{3\times3} \\
O_{3\times3} & O_{3\times3} & O_{3\times3} & O_{3\times3} \\
O_{3\times3} & O_{3\times3} & I_{3\times3} & O_{3\times3}
\end{bmatrix}_{15\times12}
\]

(13)

\[
\Phi = \begin{bmatrix}
\Phi_N & \Phi_S \\
O & \Phi_M
\end{bmatrix}_{15\times15}
\]

(14)

\(\Phi_N\) is the system matrix of 9 basic navigation parameters. Its non-zero elements can be determined by equations (15), (16).

\[
\Phi_S = \begin{bmatrix}
-C^n_b & O_{3\times3} \\
O_{3\times3} & C^n_b \\
O_{3\times3} & O_{3\times3}
\end{bmatrix}
\]

(15)

\[
\Phi_M = \text{diag}\left\{\frac{1}{T_{rx}}, \frac{1}{T_{ry}}, \frac{1}{T_{rz}}, \frac{1}{T_{ax}}, \frac{1}{T_{ay}}, \frac{1}{T_{az}}\right\}
\]

(16)

3. Establishment of Kalman Measurement Equation

The measurement equation of the loose combination system is:

\[
Z_k = H_kX_k + V_k
\]

(17)

Among them, the measurement state amount \(Z_k\) is composed of the difference between the position and speed measured by GNSS and the position and speed solved by inertial navigation. Its expression is:

\[
Z = \begin{bmatrix}
\mathbf{r}_{\text{GNSS}} - \mathbf{r}_{\text{INS}} \\
V_{\text{GNSS}} - V_{\text{INS}}
\end{bmatrix}
\]

(18)

For the measurement state, the position and velocity results measured by GNSS are used directly, so the measurement matrix can be obtained:

\[
H_k = \begin{bmatrix}
I_{3\times3} & O_{3\times3} & O_{3\times6} \\
O_{3\times3} & I_{3\times3} & O_{3\times6}
\end{bmatrix}_{6\times15}
\]

(19)

Measurement noise variance matrix \(V_k\) determined by the variance of the position and velocity measured by the GNSS receiver.

4. Experiment

According to the theoretical derivation, select the observation position information, that is the difference between the longitude, latitude, and altitude calculated by the inertial navigation solution and the longitude, latitude, and altitude output by the GNSS receiver. Perform a semi-physical simulation experiment based on actual physical data. The actual data conditions are: the navigation coordinate system is the northeast sky coordinate system, the carrier coordinate system is the front right upper coordinate system; the initial speed is 0; the initial heading angle is -90°, and the pitch and roll angles are 0.
Figure 2. Position-error contrast under normal condition.

Figure 2 shows the position curve of the GNSS/INS loose combination, where the blue line is the position curve of the GNSS, the red line is the curve after the GNSS/INS combination. The three pictures from top to bottom are latitude and longitude curve, and the height curve. As can be seen from the figure, in the gap between GNSS data, INS provides navigation information for the carrier, and the error of INS accumulates with time. When GNSS data appears, GNSS will modify the data of INS to eliminate the time accumulation error. In the attitude angle curve, the change of the heading angle corresponds to the heading in the position curve. Because of driving on a relatively flat road, the changes of the pitch angle and roll angle are not obvious. In the speed curve, the change in the eastward speed and the change in the northward speed correspond to the heading in the position curve. Similarly, the value of the skyward speed is relatively small due to driving on a relatively flat road surface with little change.

5. Conclusion

According to the theory of GNSS/INS loose combination system, a Kalman combination filter is established. The system equations and measurement equations of the Kalman combination filter are studied in depth to complete the semi-physical simulation experiment of actual physical data. Based on the GNSS receiver chip and IMU inertial device, the hardware platform of the integrated navigation system and the experiment are completed. The Kalman filter speed and position model of the GNSS/INS loose integrated navigation and semi-physical simulation experiments based on the speed and position models are shown. The integrated navigation results show that the combined results are significantly better than the respective results of GNNS and INS.

6. References

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