Glue: Adaptively Merging Single Table Cardinality to Estimate Join Query Size

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ABSTRACT
Cardinality estimation (CardEst), a central component of the query optimizer, plays a significant role in generating high-quality query plans in DBMS. The CardEst problem has been extensively studied in the last several decades, using both traditional and ML-enhanced methods. Whereas, the hardest problem in CardEst, i.e., how to estimate the join query size on multiple tables, has not been extensively solved. Current methods either reply on independence assumptions or apply techniques with heavy burden, whose performance is still far from satisfactory. Even worse, existing CardEst methods are often designed to optimize one goal, i.e., inference speed or estimation accuracy, which can not adapt to different occasions.

In this paper, we propose a very general framework, called \textsc{Glue}, to tackle with these challenges. Its key idea is to elegantly decouple the correlations across different tables and losslessly merge single table CardEst results to estimate the join query size. \textsc{Glue} supports obtaining the single table-wise CardEst results using any existing CardEst method and can process any complex join schema. Therefore, it easily adapts to different scenarios having different performance requirements, i.e., OLTP with fast estimation time or OLAP with high estimation accuracy. Meanwhile, we show that \textsc{Glue} can be seamlessly integrated into the plan search process and is able to support counting distinct number of values. All these properties exhibit the potential advances of deploying \textsc{Glue} in real-world DBMS.

1 INTRODUCTION
Query optimizer (QO) plays a significant important role in modern DBMSs. It is an integral component to generate high-quality execution plans for the input SQL queries. Cardinality estimation (CardEst), which aims at estimating the result size of all sub-plans queries, is a central part in QO. It lays the foundation for cost estimation and guides the QO for join order selection. Thus, CardEst has a critical impact on the quality of the generated query plans.

Background: Due to its importance, CardEst has been extensively studied in the literature. The core task of CardEst is to build a compact sketch capturing the synopses of data and/or query information. Current open-source and commercial DBMSs mainly use two traditional CardEst methods, namely histogram \cite{1, 3, 9, 23, 27, 33} in PostgreSQL\cite{4} and SQL Server\cite{21} and sampling \cite{13, 16, 18, 19, 41} in MySQL\cite{26} and MariaDB\cite{28}. Recently, with the prosperity of machine learning (ML), there is a booming of ML-enhanced CardEst methods in the last several years \cite{5, 12, 14, 17, 20, 31, 35, 36, 39, 40, 42}. These methods are either query-driven \cite{5, 17}, which maps featurized queries to their cardinality, or data-driven, which directly model the joint distribution of all attributes \cite{7, 14, 20, 32, 36, 39, 40, 42}. They devote lots of efforts in improving the performance of CardEst algorithms in terms of different criteria, namely end-to-end query time \cite{10}, estimation accuracy, inference latency, updating speed and model size \cite{20, 42}.

Challenge and Motivation: Although the estimation accuracy, as well as other performance criteria, has been shown to be significantly improved for CardEst methods. Some challenges still exist for CardEst algorithms. We summarize them as follows.

First, join query size estimation, the hardcore problem of CardEst, is not well studied yet. Due to the booming size of the join table and the existence of cross-table correlations between attributes, the performance of existing CardEst methods degrades with the number of join tables \cite{10}. Current CardEst methods mainly apply two kinds of approaches to process join queries. The first one \cite{35, 39} directly builds a large model on the samples of the full outer join table, which causes heavy overhead and poor scalability. The second one \cite{14, 36, 42} makes strong independence assumptions among tables and builds an ensemble of small models on partial tables, which needs to be tuned by experience and would more or less harm the estimation accuracy.

Second, current CardEst methods are not adaptive to different applications. Real-world DBMS would face different datasets and query workloads, which emphasize different performance criteria of CardEst methods. For example, OLTP queries require fast estimation time while OLAP queries need high estimation accuracy \cite{10}. These criteria often conflicts with each other so it is difficult for a CardEst method to achieve both at the same time. Moreover, even for a specific metric, different CardEst methods are suitable for different data due to their independence assumptions. For example, SPN-based method \cite{14} is only accurate on attributes with low correlations. There exist no versatile CardEst method that can perform well on any data with any query workload.

As a result, a more sophisticated and universal CardEst paradigm is still missing, especially for the complex multi-table join queries.

Contributions: In this paper, we propose \textsc{Glue}, a novel CardEst framework to tackle with these challenges. Unlike with existing CardEst methods, \textsc{Glue} does not consider how to model data in each single table. Instead, it builds an upper structure that decouples the correlations between join tables using local independence, and then merge single table CardEst results to predict join query
size. The upper structure is conceptually independent of the underlying models. In comparison with existing CardEst methods, GLUE has the following advantages:

1) it is very general to support any join schema, i.e. star, chain, cycle and mixture, and join types, i.e., one-to-many and many-to-many. The upper structure is very lightweight and easy to update. Meanwhile, GLUE’s local independence assumption is data adaptive, so its estimation error is much lower.

2) it is very flexible to support any CardEst method on single table as plug-ins, or even different CardEst methods for different tables in one database. This allows the QO to steer to different optimization goals, i.e. plan quality or throughput, and adapt to different types of data, i.e. loosely or strongly correlated, by selecting proper CardEst methods. To the best of our knowledge, this establish a new paradigm for CardEst.

Besides, we show that the computation process in GLUE ensembles the dynamic programming based join order selection in QO, so it can be seamlessly integrated to speed up the plan search process. More over, we show that GLUE could also support counting distinct values. All these properties indicate that GLUE is a highly promising candidate for deploying in real-world DBMS.

**Organization:** In the following content, Section 2 introduces some preliminary knowledge, Section 3 describes the main idea of GLUE framework, Section 3 presents how to construct the structure in GLUE, Section 4 exhibits the distinct counting method in GLUE and Section 5 concludes this paper.

## 2 PRELIMINARIES

In this section, we formalize the CardEst problem and brief review representative CardEst algorithms.

### 2.1 Problem Definition

Let $T$ be a table with $k$ attributes $A = \{A_1, A_2, \ldots, A_k\}$. $T$ could either be a single relational table or a joined table. Without ambiguity, if $T$ is a joined table, we also use $T$ to represent the set of all single tables joining it. In this paper, we assume that each attribute $A_i$ for each $1 \leq i \leq k$ to be either categorical (whose values can be mapped to integers) or continuous, whose domain (all unique values) is denoted as $\text{Dom}(A_i)$. We also denote $\text{Dom}(T) = \text{Dom}(A_1) \times \text{Dom}(A_2) \times \cdots \times \text{Dom}(A_k)$ to be the domain of table $T$.

Thereafter, any selection query $Q$ on $T$ can be represented in a canonical form: $Q = \{A_1 \in R_1 \land A_2 \in R_2 \land \cdots \land A_n \in R_n\}$, where $R_i \subseteq \text{Dom}(A_i)$ is the constraint region specified by $Q$ over attribute $A_i$ (i.e. filter predicates). Without loss of generality, we have $R_i = \text{Dom}(A_i)$ if $Q$ has no constraint on $A_i$. In this paper, if a region $R \subseteq \text{Dom}(T)$ could be decomposed into the form $R = R_1 \times R_2 \times \cdots \times R_k$ where $R_i \subseteq \text{Dom}(A_i)$ for all $i$, we call $R$ a regular region. Intuitively, a regular region is formed by a number of hyper-rectangles in the domain space. Obviously, the query space of any selection query $Q$ is a regular region. In the following, we use $Q$ to denote the selection query and its region interchangeably. In this paper, we focus on evaluating selection queries on numerical or categorical attributes. We do not consider “LIKE” (or pattern matching) queries on string attributes as they follow different technical routines.

Let $\text{Card}(T, Q)$ denote the cardinality, i.e., the exact number of records in $T$ satisfying all constraints in $Q$. The CardEst problem requires estimating $\text{Card}(T, Q)$ as accurately as possible without executing $Q$ on $T$.

The CardEst problem is often interpreted and solved in a statistical perspective. Specifically, we could regard each attribute $A_i$ as a random variable defined over its domain space $\text{Dom}(A_i)$. Then, the set of attributes $A$ defines a joint probability distribution function (PDF) $\text{Pr}_T(A) = \text{Pr}_T(A_1, A_2, \ldots, A_k)$ over table $T$. Each record $t \in T$ represents an independent tuple sampled from $\text{Pr}_T(A)$.

At this time, $\text{Pr}_T(Q) = \text{Pr}_T(A_1 \in R_1, A_2 \in R_2, \ldots, A_k \in R_k)$ represents the probability that a randomly picked record $t \in T$ satisfying the query $Q$. When the number of tuples is large enough in $T$, we naturally have $\text{Card}(T, Q) = \text{Pr}_T(Q) \cdot |T|$. When $T$ is a join table, e.g. $T \bowtie S$, we have $\text{Card}(T \bowtie S, Q) = \text{Pr}_T\bowtie_S(Q) \cdot |T \bowtie S|$. Since $|T|$ or $|T \bowtie S|$ is often known or can be estimated easily, the CardEst problem is equivalent to model the joint PDF $\text{Pr}_T(A)$ and estimate the probability $\text{Pr}_T(Q)$.

### 2.2 Related Work

There exist many CardEst methods in the literature, which can be classified into three classes as follows:

**Traditional CardEst methods**, such as histogram [27] and sampling [13, 16, 18], are widely applied in DBMS and generally based on simplified assumptions and expert-designed heuristics. Many variants of histograms [1, 3, 6, 8, 9, 15, 23, 25, 29, 30, 33, 34] are proposed later to enhance their performance. Sampling-based variants include query-driven kernel-based methods [13, 16], index based methods [18] and random walk based methods [19, 41]. Some other work, such as the sketch based method [2], explores a new direction for CardEst.

**ML-based query-driven CardEst methods** try to learn a model to map each featurized query $Q$ to its cardinality $\text{Card}(T, Q)$ directly. Some ML-enhanced methods improve the performance of CardEst methods by using more complex models such as DNNs [17] or gradient boosted trees [5].

**ML-based data-driven CardEst methods** are independent of the queries. They try to model the joint PDF $\text{Pr}_T(A)$ directly so they have better generalization ability. A variety of ML-based models have been used in existing work to represent $\text{Pr}_T(A)$, the most representative of which includes deep auto-regression model [11, 39, 40] and probabilistic graphical models (PGMs) such as Bayesian networks (BN) [7, 32, 36], SPN [14], and FSPN [42]. In addition, some methods proposed recently such as [35] try to integrate both query and data information for CardEst.

## 3 GLUE FRAMEWORK

In this section, we describe the details of our GLUE framework. We first formally define local independence, the foundation tool for our GLUE framework in Section 3.1, then present the main idea of GLUE in Section 3.2. Finally, Section 3.3 shows how GLUE is applied to solve the CardEst problem.
\[
Pr_T(Q) = \sum_{q \in Q} Pr_T(q_y) \cdot Pr_T(q_X | q_y) = \sum_i \left( \sum_{q \in Q \cap (\text{Dom}(X) \times R_i)} Pr_T(q_Y) \cdot Pr_T(q_X | q_Y) \right) \cdot \left( \sum_{q \in Q \cap (\text{Dom}(X) \times R_i)} Pr_T(q_Y) \cdot Pr_T(q_X | q_Y) \right)
\]

\[
= \sum_i \left( \sum_{q \in Q \cap (\text{Dom}(X) \times R_i)} Pr_T(q_Y | R_i) \cdot Pr_T(q_X | q_Y) \right) \left( \sum_{q \in Q \cap (\text{Dom}(X) \times R_i)} Pr_T(q_Y | R_i) \cdot Pr_T(q_X | q_Y) \right)
\]

(1)

### 3.1 Local Independence

We first introduce the concept of local independence, which serves as a fundamental tool in our GLUE framework. For any table \( T \) having attributes (random variables) \( A, \) let \( X \) and \( Y \) be a division of \( A. \) For any query \( Q \) on \( T, \) we have

\[
Pr_T(Q) = \sum_{q \in Q} Pr_T(q_y) \cdot Pr_T(q_X | q_y).
\]

where \( q_X \) and \( q_Y \) represent the values of the point \( q \) on attributes \( X \) and \( Y, \) respectively. The hardness in Eq. (2) is that the term \( Pr_T(q_X | q_Y) \) is not independent of \( q_Y, \) i.e., the conditional PDF \( Pr_T(X | y) \) differs between different values \( y \) of \( Y, \) so the probability over \( X \) and \( Y \) cannot be computed and then multiplied independently. We derive a method to decouple the correlations between \( X \) and \( Y. \)

To compactly model the conditional PDF \( Pr_T(X | y), \) we partition the domain space \( \text{Dom}(T) = \text{Dom}(X) \times \text{Dom}(Y) \) into multiple regions in terms of \( Y \) as \( \text{Dom}(X) \times R_1 \), \( \text{Dom}(X) \times R_2 , \ldots, \text{Dom}(X) \times R_k. \) Each \( R_i \subseteq \text{Dom}(Y) \) is a regular subspace s.t. for any \( y, y' \in R_i, \)
\[ Pr_T(X | y) = Pr_T(X | y') \] roughly holds. At this time, we only need to maintain the PDF \( Pr_T[R_i](X) \) for each \( R_i, \) where \( T[R_i] \) denote the set of tuples in \( T \) existing in the space \( R_i. \) We have \( Pr_T(X | y) = Pr_T(R_i[X] \) for any \( y \in R_i. \) We call this the contextual condition removal, where each sub-domain \( R_i \) refers to the context.

For each sub-space \( \text{Dom}(X) \times R_i, \) we have \( Pr_T(Y) = Pr_T[R_i](Y) \) when the value of \( Y \) is restricted in \( R_i. \) Let \( Q_X \) and \( Q_Y \) denote the space of \( Q \) restricted to the domain space of \( X \) and \( Y, \) respectively. Then, we derive Eq. (1), which splits the probability of \( X \) and \( Y \) to be independent terms in each region \( Q \cap (\text{Dom}(X) \times R_i). \) Using this local independence, we could fast compute the probability of query \( Q \) in each region and then sum them together.

Next, we introduce the intuitive idea of how to divide the domain space to derive local independence. In the extreme case, the local independence for each distinct value in \( \text{Dom}(T). \) In general, we could break the correlations between variables \( X \) and \( Y \) to derive the local independence. As computing the correlations of two set of variables are expensive, we could done in a pairwise manner using a heuristic rule. Specifically, we could compute the pairwise correlation value \( s(X_i, Y_j), \) such as RDC score [22], for each pair of attributes \( X_i \in X \) and \( Y_j \in Y \) and select the \( Y_j \) maximizing \( s(X_i, Y_j), \) i.e., \( Y_j = \arg \max_j s(X_i, Y_j). \) Then we could divide the domain according to \( Y_j \) by splitting its domain into several parts. After partition, all tuples in the same part tend to have more similar values in terms of \( Y_j, \) so \( Y_j \) is not easily affected by \( X \) and more likely to be locally independent of \( X. \) In an extreme case, all records in the same part may have the same value on \( Y_j \) and obviously \( Y_j \) is independent of \( X \) in this part. This method could be iteratively done over each part until the maximum pairwise correlation value \( s(X_i, Y_j) \) is lower than a threshold.

### 3.2 Overview of GLUE

In this subsection, we generally introduce how GLUE do CardEst on join tables in a top-down manner. Let \( W = T \Rightarrow S \) be a join relation where \( T = (A_1, A_2, \ldots, A_k) \) and \( S = (B_1, B_2, \ldots, B_n) \) represent a single or join relation table. For any selection query \( Q \) on \( T \Rightarrow S, \) let \( Q_T \) and \( Q_S \) denote the sub-query on \( T \) and \( S, \) respectively. By Section 2, we have \( \text{Card}(T \Rightarrow S, Q) = Pr_{T \Rightarrow S}(Q) \cdot |T \Rightarrow S|. \) GLUE could obtain \( Pr_{T \Rightarrow S}(Q) \) by merging the information from \( T \) and \( S \) without building the entire model over \( T \Rightarrow S. \) We elaborate the details as the following three main steps.

**Step 1: Table decoupling.** Note that, some attributes \( A_i \) of \( T \) may be correlated with some \( B_j \) of \( S \) over \( W = T \Rightarrow S, \) which is also known as the cross-table correlations. Therefore, we often have \( Pr_W(Q) \neq Pr_{T \Rightarrow S}(Q_T) \cdot Pr_{T \Rightarrow S}(Q_S) \) on the whole domain space of \( \text{Dom}(T) \times \text{Dom}(S). \) At this time, we utilize the local independence tool, namely cross-table local independence, to decompose \( Pr_W(Q). \)

Specifically, let \( L_1, L_2, \ldots, L_i \) be the partition of \( \text{Dom}(T) \times \text{Dom}(S) \) and each \( L_i \) is in a regular form \( L_i = \text{Dom}(T) \times L^1_i \times L^2_i \times \cdots \times L^j_i \) with \( L^j_i \subseteq \text{Dom}(B_j) \) for each \( j. \) The local independence holds in each \( L_i. \) That is, we have

\[
Pr_{T \Rightarrow S}(Q \cap L_i) = Pr_W(Q \cap L_i) = Pr_{W[L_i]}(Q_T) \cdot Pr_{W[L_i]}(Q_S \cap L_i)
\]

for each \( L_i \) by Eq. (1). Then, using Eq. (1), we derive

\[
Pr_{T \Rightarrow S}(Q) = \sum_i Pr_{W[L_i]}(Q_T) \cdot Pr_{W[L_i]}(Q_S \cap L_i).
\]

(3)

At this time, on each \( L_i, \) we could separately compute the probability of \( Q_T \) and \( Q_S \cap L_i. \) As \( Q_T \) and \( Q_S \cap L_i \) contains no predicates over another table, we decouple the connections across the tables \( T \) and \( S. \) We next show how to obtain the probability purely using the models on \( T \) and \( S, \) respectively.

**Step 2: Local PDF correction.** The probability \( Pr_{W[L_i]}(Q_S \cap L_i) \) contains only predicates on attributes in \( S, \) but it is defined over the joint PDF of \( W = T \Rightarrow S \) but not over \( S. \) We apply the idea proposed in [14, 42] to correct the PDF from \( S \) to \( T \Rightarrow S \) using fanout (or scattering) columns. However, unlike with them, we do not need to explicitly maintain these fanout columns but just need to store several numbers.

Let \( F_{S \Rightarrow T} \) denote the fanout column from \( T \) to \( S. \) For any tuple \( t \) in \( S, \) \( F_{S \Rightarrow T}(t) \) records how many tuples in \( T \) could join with \( t. \) Notably, we set \( F_{S \Rightarrow T}(t) = 1 \) if \( t \) can not join with any tuple in \( T. \) At this time, \( t \) still occurs once (with null attributes) in \( T \Rightarrow S. \) Then, by [14, 42], we have

\[
Pr_{T \Rightarrow S}(Q_S \cap L_i) = \frac{|S|}{|T \Rightarrow S|} \sum_f Pr_S(Q_S \cap L_i) \cdot F_{S \Rightarrow T}(t) = f \cdot f.
\]

(4)
The above equation is difficult to compute directly. However, we could simplify it again using the local independence property. We call such the join-key frequency independence.

This time we divide the domain space \( \text{Dom}(S) \) to regular sub-space \( S_1, S_2, \ldots, S_k \) s.t. all attributes of \( S \) are locally independent of \( F_{S \rightarrow T} \) in each \( S_j \). That is, for any space \( Q' \in \text{Dom}(S) \), we have

\[
\Pr_S((Q' \cap S_j) \land F_{S \rightarrow T} = f) = \Pr_S([S_j]')(Q' \cap S_j) \cdot \Pr_S([S_j])(F_{S \rightarrow T} = f).
\]

Then, let \( Q'_j = Q \cap S_j \). By Eq. (1), we derive

\[
\Pr_{T \rightarrow S}([Q'_j]) = \frac{|S|}{|T \bowtie S|} \cdot \sum_j \Pr_S([[Q'_j] \cap S_j] \land F_{S \rightarrow T} = f) \cdot f
\]

\[
= \frac{|S|}{|T \bowtie S|} \cdot \sum_j \Pr_S([S_j]')(Q' \cap S_j) \cdot \Pr_S([S_j])(F_{S \rightarrow T} = f) \cdot f
\]

\[
= \frac{|S|}{|T \bowtie S|} \cdot \sum_j \Pr_S([S_j]')(Q'_j \cap S_j) \cdot \Pr_S([S_j])(F_{S \rightarrow T} = f) \cdot f
\]

\[
= \frac{|S|}{|T \bowtie S|} \cdot \sum_j \Pr_S([S_j]')(Q'_j \cap S_j) \cdot \mathbb{E}_S([S_j])(F_{S \rightarrow T}).
\]

(5)

where \( \mathbb{E}_S([S_j])(F_{S \rightarrow T}) \) denotes the expected value of \( F_{S \rightarrow T} \) restricted to all tuples of \( S \) in the sub-domain \( S_j \). We could pre-obtain and store all these expected values. The details are discussed later. All probabilities \( \Pr_S([S_j]')(Q'_j \cap S_j) \) could be recursively computed on table \( S \). After that, we easily obtain the probability \( \Pr_{T \rightarrow S}(Q \cap S_j) \).

In similar, for the probability \( \Pr_{T \rightarrow S}(Q') \), we could use the column \( F_{T \rightarrow S} \) to correct the PDF from table \( T \) to \( T \bowtie S \). We assume that the domain space \( \text{Dom}(T) \) are divided into regular sub-space \( T_1, T_2, \ldots, T_m \) s.t. all attributes of \( T \) are locally independent of \( F_{T \rightarrow S} \) in each \( T_k \). That is, for any space \( Q' \in \text{Dom}(T) \), we have

\[
\Pr_T((Q' \cap T_k) \land F_{T \rightarrow S} = f) = \Pr_T([T_k]')(Q' \cap T_k) \cdot \Pr_T([T_k])(F_{T \rightarrow S} = f).
\]

We also have

\[
\Pr_{T \rightarrow S}(Q') = \frac{|T|}{|T \bowtie S|} \cdot \sum_k \Pr_T((Q' \cap T_k) \land F_{T \rightarrow S} = f) \cdot f
\]

\[
= \frac{|T|}{|T \bowtie S|} \cdot \sum_k \Pr_T([T_k]')(Q' \cap T_k) \cdot \Pr_T([T_k])(F_{T \rightarrow S} = f) \cdot f
\]

\[
= \frac{|T|}{|T \bowtie S|} \cdot \sum_k \Pr_T([T_k]')(Q' \cap T_k) \cdot \Pr_T([T_k])(F_{T \rightarrow S} = f) \cdot f
\]

\[
= \frac{|T|}{|T \bowtie S|} \cdot \sum_k \Pr_T([T_k]')(Q' \cap T_k) \cdot \mathbb{E}_T([T_k])(F_{T \rightarrow S}).
\]

(6)

where \( \mathbb{E}_T([T_k])(F_{T \rightarrow S}) \) denotes the expected value of \( F_{T \rightarrow S} \) restricted to all tuples of \( T \) in the sub-domain \( T_k \). All probabilities \( \Pr_T(Q' \cap T_k) \) could be recursively obtained on table \( T \).

Using the join-key frequency local independence, we could obtain the probability \( \Pr_{T \rightarrow S}(Q \cap S \bowtie T) \) over table \( S \) for each \( i \) and \( \Pr_{T \rightarrow S}(Q') \) from table \( T \) individually. Putting Eq. (5) and Eq. (6) into Eq. (3), we easily obtain the probability of \( Q \) on table \( T \bowtie S \).

**Step 3: Recursive processing.** By Step 2, we need to further obtain each probability \( \Pr_S(Q'_i \cap S_j) \) from \( S \) and \( \Pr_T(Q_T \cap T_k) \) from \( T \). If \( S \) (or \( T \)) is a single table, the probability could be directly computed using the underlying model of \( \Pr_S(B_1, B_2, \ldots, B_n) \). In GLUE, we have no limitation on how the model is built. It could either be a simple histogram or a complex SPN model. The probability could even be set to some magic number in some applications. This reserves great flexibility to the underlying DBMS to choose different CardEst algorithms to meet different requirements, i.e., much faster for OLTP or more accurate for OLAP. Even more, GLUE also allows the QO to use different CardEst methods on different tables in a database, so users could specify the most suitable CardEst method according to the statistical characteristics of each table.

If \( S \) (or \( T \)) is not a single table, we could recursively decompose its joined tables for probability computation. Let \( \Pr_S(Q'_i) = \Pr_S(Q \cap S \bowtie T) \) denote each query on a sub-range of \( Q \) and \( S = U \bowtie V \). We further divide \( \text{Dom}(S) = \text{Dom}(U) \times \text{Dom}(V) \) into regular sub-domains \( L'_1, L'_2, \ldots, L'_p \) in terms of \( V \) s.t. attributes \( U \) and \( V \) are independent in \( L'_j \), i.e., the cross-table local independence holds. Meanwhile, we divide the domain space \( \text{Dom}(V) \) to regular sub-space \( V_1, V_2, \ldots, V_c \) s.t. all attributes of \( V \) are locally independent of \( F_{T \rightarrow U} \) in each \( V_j \), i.e., the join-key frequency local independence holds. After that, we need to compute \( \Pr_S(Q'_i \cap L'_j \cap V_i) \) for all \( 1 \leq i \leq p \) and \( 1 \leq j \leq c \). The decomposition processing of table \( T \) is similar. We iterate until both \( U \) and \( V \) are single tables. Notice that, we could organize all tables as a tree structure based on their decomposition manner, where each leaf node is a single table and each inner node is a join table. We defer the details on how to construct this decomposition tree in next section.

### 3.3 GLUE for CardEst

We show how GLUE could be utilized for CardEst. We first introduce the basic algorithm to process a single query, and then present how GLUE used in the plan search process of QO.

**Basic CardEst algorithm in GLUE.** We present the complete algorithm GLUE-CARDEST for CardEst on a single query using GLUE in Figure 1. We assume that the join tree has already been constructed offline with the root node \( N \) and push the query \( Q \) onto \( N \). Basically, if \( N \) is a leaf node on single table, we fetch and return the probability of \( Q \) from the underlying model over \( N \). Otherwise, we fetch the cross-table local independence division conditions and split \( Q \) to \( Q_T \) and multiple \( Q_S \cap L_j \) on the left and right child, respectively. If \( Q_T \) (or \( Q_S \)) is empty, this indicates we have no predicates constraints over attributes in table \( T \). We directly return 1 to be the probability of \( Q_T \) on node \( N \). Else, for both \( Q_T \) and \( Q_S \), we divide it into many sub-ranges \( Q'_i \) and \( Q'_j \) by intersecting with each \( T_k \) and \( S_j \), respectively. The probability of \( Q'_i \) and \( Q'_j \) are obtained by recursively calling GLUE-CARDEST on the sub-tree rooted at node \( T \) and \( S \), respectively. After collecting them, we compute the probability of \( Q_T \) and \( Q_S \) using Eq. (6) and Eq. (5), respectively. They are merged together by Eq. (3) to obtain the cardinality of \( Q \) on \( T \bowtie S \).

**Time complexity analysis.** Let \( b \) be the height of the decomposition tree and \( \ell \) be the maximum number of sub-space split in each node. Assume that the probability could be obtained in \( O(1) \) time on each leaf node and \( O(1) \) on each inner node. The time cost of
The cost is affordable in nowadays DBMS due to the following reasons: 1) $O(t)$ is often very low on single tables using simple models such as histogram or ML models such as SPN [14] or FSPN [42]; 2) $h$ is less than the number of joined tables in the DBMS, which is often a small constant number. We could also tune the tree construction method to decrease $h$; and 3) the computation of different sub-space is easy to do in parallel.

**Glue in Plan Search Process.** Next, we show how Glue could be seamlessly used in the plan search process. Unlike with existing CardEst methods, such as [14, 39, 40, 42], which compute the cardinality for each sub-plan query from scratch, Glue done in a more elegant manner. We utilize the overlaps among different sub-plan queries to avoid redundant computation.

Given the join decomposition tree, for each node $N$, we associate it with a set $CD(N)$. Each element in $CD(N)$ is a pair $(D, p)$ where $D$ is a sub-domain space and $q$ is its probability. In the first traversal, we compute the probability of query $Q$ itself. The domain space is recursively divided into multiple sub-space in each node, we store each sub-space and its related probability on each node $N$ to obtain $CD(N)$. Next time, for any sub-plan query $Q'$ of $Q$, we do not need to decompose and compute the range of $Q'$ on the overlapped parts w.r.t. $Q$. For non-overlapped sub-trees, which indicate $Q'$ does not touch the tables within this sub-tree, the sub-tree root node just returns 1 as the probability on this branch.

**4 GLUE CONSTRUCTION**

In this section, we discuss how to construct the Glue structure. We first introduce the details on space division to derive the local independence (in Section 4.1) and then outline the method for building join decomposition tree (in Section 4.2).
join or a many-to-many FK-FK join; and 3) an equal join or even unequal join.

A decomposition tree $T$ is valid if and only if: 1) the root node is full join of all tables in $\mathcal{V}$; 2) each inner node $N$ splitting its join table $S \bowtie T$ to left child $S$ and right child $T$ s.t. tables in $S$ and $T$ form connected components in $G$ with internal join edges; and 3) each leaf node corresponds to a single table in $\mathcal{V}$. Obviously, each decomposition tree corresponds to a plan tree for a query touching all tables in $\mathcal{V}$. Therefore, it could be generated using a similar method for plan generation.

In general, we could apply a dynamic programming method to construct the tree $T$. Each time on a node $N$, we split its tables to $S$ joining with $T$ such that this decomposition minimizes Cost($S$) + Cost($T$) + Cost($S$, $T$). Here Cost($S$) (or Cost($T$)) defines the recursively defined cost in terms of CardEst task over underlying node $S$ (or $T$). Cost($S$, $T$) describes the cost for combining $S$ and $T$ together. Unlike with the plan generation with a specified cost model, the case for our CardEst is a bit more complex. The cost model needs to consider the following aspects:

1) the sampling cost. As stated in Section 3.1, we need to apply samples on $T$, $S$ and $T \bowtie S$ to obtain the division conditions and expected value. Obviously, the sampling could be done easily when $T$ or $S$ contains less number of tables. For example, if we restrict $S$ to be a single table each time, in similar to the left-deep plan restriction, we could easily obtain the samples on $T \bowtie S$ by sampling tuples in $S$ w.r.t. samples in $T$ using Olken’s sampling algorithm. If $T$ or $S$ contains multiple tables, we may need to sample for $T \bowtie S$ individually and can not reuse the existing samples. We could measure this by $\min(||S||, ||T||)$, where $||T||$ denote the number of tables in $T$.

2) the error cost. The correlations between different pairs of $T$ and $S$ are different, so as the decomposition error. Intuitively, the smaller the correlation scores between $T$ and $S$, the easier of the local independence exists. We could measure this by $s(T, S) = \max_i(s(A_i, B_j))$ where $s(A_i, B_j)$ is the correlation score between attributes $A_i$ of $T$ and $B_j$ of $S$.

3) the inference cost. By the time complexity analysis in Section 3.2, the inference cost is determined by the tree height $h$ and sub-space division number $l$. Obviously, $h$ is lower when $T$ and $S$ have balanced number of tables, and $l$ is lower when the correlations between $T$ and $S$ have higher correlations. Therefore, we could measure this by $s(T, S)^{\max(||S||, ||T||)}$.

4) the modeling cost. This refers to the base cost of building CardEst methods on each single table $T$. We denote it as a function $g(T)$, which is related to the structure learning time complexity of the underlying CardEst algorithm. For histogram, it is linear w.r.t. the number of attributes in $T$. For SPN/FSPN and BN, it is polynomial and exponential w.r.t. the number of attributes in $T$, respectively.

Putting them together, we obtain the following cost model function

$$
\begin{align*}
\text{Cost}(S \bowtie T) &= \alpha \min(||S||, ||T||) + \beta s(T, S) \\
&+ \gamma s(T, S)^{\max(||S||, ||T||)} + \text{Cost}(S) + \text{Cost}(T),
\end{align*}
$$

where $\alpha$, $\beta$, and $\gamma$ are all hyper-parameters tuning the weights of each part. Some aspects, such as the sampling cost and inference cost, are conflict with each other. We could emphasize different parts in different scenarios, e.g., fast inference for OLTP or low error for OLAP. Note that, we measure each aspect using the most straightforward metric. GLUE is open for any complex cost model and it is also an interesting future research work.

Using Eq. (7), we could apply the dynamic programming method to construct the decomposition tree. The procedures are similar to plan generation. We omit it for simplicity. We could also apply some heuristic rules, such as greedy search, to find near-optimal result. Moreover, we do not restrict to build only one join decomposition tree for a database. We could build each for each frequently occurred join schema in the query workload.

**Model update.** Conceptually, the join decomposition tree is independent of the underlying models on single tables. Thus, they could be updated individually. When data changes on some tables, the corresponding CardEst models are updated accordingly. For the join decomposition tree, it is more robust for data changes. We could periodically examine whether the local independence still holds in each sub-space. If not, we re-split the sub-space accordingly.

## 5 GLUE FOR DISTINCT COUNT

In this section, we discuss how to adapt GLUE to count distinct number of values, which is frequently occurred in SQL queries with distinct predicate. We first show how to adapt existing CardEst model to support distinct count on single table, and then how the framework in GLUE could support join queries.

Following Section 2, from a statistical perspective, the distinct count of query $Q$ on table $T$ could be defined as $\text{Dis}(T, Q) = |\{q \in Q | \text{Pr}_T(q) > 0\}|$. That is, each point $q$ that can occur in the space of $Q$ is counted exactly once. Recall that, $T$ could be either single relational table or join table. In traditional methods, histogram and multiple sampling-based CardEst methods could be used to count distinct number of values. For ML-based methods, we find that the SPN model [24] and FSPN model [38] could also support distinct count with small adaptations. We elaborate the details as follows.

**Distinct count on single table.** For the SPN model, it models the joint PDF $P_{T'}(A_1, A_2, \ldots, A_k)$ using sum and product operations. Each sum node decomposes the joint PDF into weight sum of smaller models and each product node find local independence among different groups of attributes. Each leaf node in SPN maintains a histogram over a singleton attribute on some data. The distinct count could be done in similar to its probability inference as follows:

1) on leaf node modeling $P_{T'}(A_i)$, we easily obtain $\text{Dis}(T', Q_i)$ by scanning the histogram of $A_i$ in $Q_i$’s range and send it to its father node.

2) on sum node modeling $P_{T'}(A') = \sum_i w_i P_{T}(A_i)$, we restrict that each sum node divides the domain space into non-overlapping regular sub-space and each $T_i$ contains all tuples in a sub-space. As a result, we easily scan $\text{Dis}(T, Q') = \sum_i \text{Dis}(T_i, Q_i')$ from all children.

3) on product node modeling $P_{T'}(A') = \prod_j P_{T_j}(A'_j)$, since $A'_j$ are all mutually independent, we know $P_{T_j}(A'_j) > 0$ if and only if $P_{T_T}(A'_j) > 0$ for all $j$. That is, each distinct value of $A'$ must
be counted in each $A_j$. Therefore, we easily have $\text{Dis}(T', Q'_{A}) = \prod_j \text{Dis}(T_j', Q'_{A_j})$.

The distinct count on SPN is the same as probability inference, which is linear w.r.t. its node size. For FSPN, the method is similar as long as we count and add the distinct number in each sub-space specified by its factorize and split nodes.

**Distinct count in GLUE.** Following the local independence space decomposition in Section 3.2, the distinct counting in GLUE is also straightforward. First, Eq. (3) split the range of $Q$ to $Q_T$ and each $Q_S \cap L_j$. Due to the cross-table local independence, we obtain

$$\text{Dis}(T \bowtie S, Q_T) = \sum_i \text{Dis}(T \bowtie S, Q_T) \cdot \text{Dis}(T \bowtie S, Q_S \cap L_i).$$

For $\text{Dis}(T \bowtie S, Q_T)$, we could obtain it from the joint PDF over $T$.

By Eq. (6), we have

$$\text{Dis}(T \bowtie S, Q_T) = \sum_k \left\{ \begin{array}{ll}
\text{Dis}(T \bowtie S, Q_T \cap T_k), & \text{if } \mathbb{E}_{T \mid T_k}[F_T - S] > 0;
0, & \text{if } \mathbb{E}_{T \mid T_k}[F_T - S] = 0.
\end{array} \right.$$  

This is because in the sub-domain of each $T_k$, all attributes of $T$ are independent of $F_T - S$ due to join-key frequency local independence. $\mathbb{E}_{T \mid T_k}[F_T - S] = 0$ indicates $\Pr(T \bowtie S) = f$ for all $f$, so no values in this sub-domain would be scattered from $T$ to $T \bowtie S$. Otherwise, $\mathbb{E}_{T \mid T_k}[F_T - S] > 0$ indicates $\Pr(T \bowtie S) > f$ for some $f$, so a value in $Q_T$ would occur in $T \bowtie S$ as long as it occurs in $T$. For each $\text{Dis}(T \bowtie S, Q_T \cap T_k)$, it could be recursively obtained from the join table $T$. If $T$ is a single table, we return the distinct count value using the algorithm on single table.

Similarly, for each $\text{Dis}(T \bowtie S, Q_S \cap L_i)$, by Eq. (5), we have

$$\text{Dis}(T \bowtie S, Q_S \cap L_i) = \sum_j \left\{ \begin{array}{ll}
\text{Dis}(S, Q_S \cap L_i \cap S_j), & \text{if } \mathbb{E}_{S \mid S_j}[F_S - T] > 0;
0, & \text{if } \mathbb{E}_{S \mid S_j}[F_S - T] = 0.
\end{array} \right.$$  

where each $\text{Dis}(S, Q_S \cap L_i \cap S_j)$ could be recursively obtained from table $S$.

As a result, we could use the same framework of GLUE-CARD$^E$ for CardEst to count distinct value with some small modifications: 1) replacing Eq. (3), Eq. (5) and Eq. (6) to Eq. (8), Eq. (10) and Eq. (9), respectively; and 2) in the base case, returning the distinct count value over a single table.

**6 CONCLUSIONS**

We propose GLUE, a general CardEst framework that is able to merge single table CardEst results to predict join query size. GLUE is flexible to support any underlying CardEst method on single table and could steer to optimize different criteria. It is more flexible and adaptive to different datasets and query workloads, thus more suitable for deployment in real-world DBMS.

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