ABSTRACT An equivalent fractional order orbital angular momentum (OAM) mode based on concentric uniform circular array (CUCA) is proposed in this paper. It is known that the azimuthal resolution of electromagnetic vortex imaging is closely related to the number of OAM modes. Although the multiple-in-multiple-out (MIMO) scheme gives better azimuthal resolution than the multiple-in-single-out (MISO) scheme, it suffers aliasing problem, which means the scattering points with their azimuthal difference larger than 180 degree cannot be separated correctly. One possible way to solve this is adding fractional modes in MIMO scheme, but unfortunately, fractional modes are unstable and cannot transmit to far field. In this paper, an equivalent fractional order OAM mode is introduced to deal with the problem to achieve higher azimuthal resolution by MIMO scheme. The transmitted and received modes are set to be two adjacent integer modes to realize an equivalent fractional mode. Since the radiation patterns generated by uniform circular array (UCA) for different modes are not aligned, concentric uniform circular array (CUCA) is applied and optimized to overcome this shortage. Simulation results demonstrate the validity of the proposed method.

INDEX TERMS Orbital angular momentum, radar imaging, fractional mode, concentric uniform circular antenna.

I. INTRODUCTION

Recently, the electromagnetic waves carrying orbital angular momentum have been applied in 2D radar staring imaging taking advantage of the dual relationship between the azimuthal angle and the OAM mode [1]–[6]. The targets can be reconstructed without relative motion with the antenna. In the past, one feasible approach to achieve staring imaging is assisted by the generation of stochastic radiation fields [7]–[9]. However, as the radiation pattern is not focus, it may not be practical for far field imaging. The electromagnetic vortex wave, with a “doughnut” beam, is more useful since it is focused at elevation direction.

The EM vortex wave based radar imaging has been realized using the uniform circular array (UCA). Although various methods exist to generate EM vortex wave [1]–[6], [10]–[17], the UCA is more practical for its flexibility and the ability to generate numerous OAM modes with one antenna array.

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Due to the orthogonality between different OAM modes, the EM vortex wave has been used in the communication system [18]–[20]. However, there are also some limitations in practical application. It is more suitable for near-field communication, or the receiving aperture needs to be very large at long distance [21]. Other researchers have studied the scattering properties from targets by different OAM modes [22]–[25]. More researches have been done in radar imaging domain. The EM vortex wave based radar imaging has been realized using the uniform circular array (UCA).

In [2], two schemes of the OAM based radar imaging were introduced including MISO scheme and MIMO scheme, and Liu pointed out that the azimuthal resolution is correlated to the range of OAM mode. Both of the two schemes can be used to achieve 2D imaging, but the MIMO scheme gives better azimuth resolution. However, it suffers aliasing problem, the scattering points with their azimuthal angle difference larger than 180 degree cannot be separated correctly. To take full advantage of the higher azimuth resolution of the MIMO scheme, fractional modes should be added to increase sample
point in OAM domain. But unfortunately, fractional OAM modes are unstable and can not transmit to far field [26].

In this paper, an equivalent fractional order orbital angular momentum mode based on concentric uniform circular array (CUCA) [4], [5] is proposed to overcome the aliasing problem of the MIMO scheme to realize high resolution. The main lobes of different OAM modes are optimized to a same elevation angle with low side lobes. Then the transmitted and received OAM modes are set to be two adjacent integers to realize an equivalent fractional mode. The remainder of the paper is organized as follows. The aliasing problem at azimuth angle of the target can be obtained through FFT by making the azimuthal resolution for MIMO scheme is better than the MISO mode in FFT is two times larger than the MISO scheme. However, as the equivalent mode interval become lager, it leads to aliasing problem. The scattering points with their azimuthal difference larger than 180 degree can not be separated correctly.

The aliasing problem can be explained in another way by the correlation function of the echo from two adjacent scattering points with same elevation angle and a different azimuth angle $\Delta \phi$. The correlation functions for MISO and MIMO schemes are

\[ g_{\text{MISO}}(\Delta \phi) = \int_{-\infty}^{\infty} A e^{i2\alpha \varphi} J_0^2(ka \sin \theta) A e^{-i2\alpha \varphi(\varphi + \Delta \phi)} J_0^2(ka \sin \theta) d\alpha \]

\[ \approx \sum_{\alpha=-\infty}^{\infty} A e^{i2\alpha \varphi} J_0^2(ka \sin \theta) A e^{-i2\alpha \varphi(\varphi + \Delta \phi)} J_0^2(ka \sin \theta) \]

\[ g_{\text{MIMO}}(\Delta \phi) = \int_{-\infty}^{\infty} A e^{i2\alpha \varphi} J_0^2(ka \sin \theta) A e^{-i2\alpha \varphi(\varphi + \Delta \phi)} J_0^2(ka \sin \theta) d\alpha \]

\[ \approx \sum_{\alpha=-\infty}^{\infty} A e^{i2\alpha \varphi} J_0^2(ka \sin \theta) A e^{-i2\alpha \varphi(\varphi + \Delta \phi)} J_0^2(ka \sin \theta) \]

in which $A = \sigma e^{i2kr}/r^2$, and for simplicity, the scattering coefficient $\sigma=1$.

The normalized correlation function is equal to 1 when the two scattering points coincide ($\Delta \phi = 0$). And then the magnitude of the correlation function decrease when $\Delta \phi$ is gradually increased so that the two points can just be distinguished when $g(\Delta \phi)$ is small enough.

As shown in Fig.2, the normalized correlation function of MIMO scheme has a periodic of 180 degree while the MISO scheme is 360 degree which means the difference azimuthal angle of the scattering points larger than 180 degree can not be reconstructed correctly by the MIMO scheme. The scattering point with a azimuth angle of ($\varphi + 180$) may be wrongly reconstructed at $\varphi$. Although the MISO mode does not suffer the aliasing problem, its azimuthal resolution is not as good as the MIMO scheme. This also can be seen from Fig.2 that the main lobe of the MISO mode is much wider than the MIMO scheme.

**FIGURE 1. Diagram of the EM vortex wave based radar imaging system.**
The aliasing problem can be solved by adding fractional modes in the MIMO scheme. When fractional order OAM modes are used, for instance $\alpha = -l$, $-l + 0.5$, $-l + 1$, $l-0.5$, $l$, then $2\alpha = -2l$, $-2l + 1$, $-2l + 2$, $2l - 2$, $2l - 1$, $2l$. Thus the equivalent sampling interval in OAM mode domain can be kept at 1. But unfortunately, fractional modes are unstable and can not transmit to far field [26]. In this paper, equivalent fractional order OAM mode is introduced in the following.

### B. EQUIVALENT FRACTIONAL OAM MODE

In this section, an equivalent fractional order OAM mode is realized through a simple approach, the transmitted and received modes are set to be two different integers $\alpha$ and $\beta$ respectively, then the echo can be represented as

$$S(\alpha) = \sum_{m=1}^{M} \bar{\sigma}_m e^{i(\alpha + \beta) \psi_m} J_{\alpha}(ka \sin \theta_m)J_{\beta}(ka \sin \theta_m)$$  

(8)

where $\alpha$, $\beta$ are the transmitted and received OAM modes. As we can see, there is $e^{i(\alpha + \beta) \psi_m}$ in (8). Then the echo can be considered as the MIMO scheme with an equivalent mode of $(\alpha + \beta)/2$. If $\beta = \alpha + 1$, the equivalent mode is $\alpha + 0.5$.

However, as the main lobes of different OAM modes generated by a same UCA are not matched [4] as shown in Fig.3. The magnitude of the echo (8) maybe too small which is bad for radar imaging.

In order to collimate the beams of different OAM modes to a same elevation direction, the concentric uniform circular array (CUCA), as shown in Fig.4 was proposed in [4].

And the excitation amplitudes for each circle can be optimized to suppress the side lobe of each OAM mode [5] to improve the imaging quality.

In this paper, covariance matrix adaptation evolution strategy (CMA-ES) [27], [28] is used to collimate and optimize the beams of different OAM modes at 6GHz. CMA-ES is an adaptive algorithm which can learn from the relativity of the complex parameters. Compared with the common used genetic algorithm, this adaptive algorithm has faster optimization speed and better robustness, and is more suitable for solving the problem with large number of optimization variables.

The radius of each circle of the CUCA are listed in table 1. The designed elevation angle of the main lobe is point to $\theta = 11$degree. And the unit radiation pattern is assumed as

$$f(\theta, \phi) = \frac{1 + \cos(\pi \sin \theta)}{\cos \theta}$$  

(9)

It should be noted that Eq. (9) can be replaced by other kind of element pattern. The optimized feed amplitudes for each circle are listed in table 1 and the optimized normalized radiation patterns of different OAM mode are shown in Fig.5. As we can see, the main lobes of different OAM modes are almost directed to a same elevation angle of $\theta = 11$degree, and the side-lobe level is suppressed lower than $-20$dB.

According to the MIMO scheme, the transmitted signal by CUCA of $\alpha$ mode after optimization is received by each element which can be represented as [5]

$$S_r(\alpha) = \sum_{m=1}^{M} e^{-i2kr_m} \sum_{h=1}^{H} I_h J_\alpha(k_h \sin \theta_m)$$  

(10)
in which $I_h$ is the feed amplitude of the $h$th circle. When the received mode is $\beta$, which means the received signal of the $n$th array element is multiplied by the phase of $e^{i\beta\phi_n}$, the final echo can be represented as

$$S_r(\alpha, \beta) = AB \sum_{m=1}^{M} \bar{\sigma}_m e^{-i2kr_m} e^{i(\alpha+\beta)\phi_m}$$

(11)

in which

$$A = \sum_{h=1}^{H} I_h J_a(ka_h \sin \theta_m)$$

(12)

$$B = \sum_{h=1}^{H} \bar{I}_h J_\beta(ka_h \sin \theta_m)$$

(13)

In this paper, the received mode is set as $\beta = \alpha + 1$. The transmitted OAM modes are $-10 \sim 10$. The equivalent fractional modes are $-9.5 \sim 9.5$. Then 2D imaging can be achieved through fast Fourier transform.

### III. SIMULATION IMAGING RESULTS

In this section, numerical experiments are conducted to verify the proposed method. Firstly, 1-D imaging is realized at single frequency, assume that two scattering points are located at a same distance and a same elevation angle but different azimuth angle. For case1, the difference of the azimuth angle is less than 180 degree, while for case2 the difference is larger than 180 degree as shown in table 2. The full range of the OAM mode is $\Delta \alpha = -10$, $-9.5$, $-9$, $..., 9, 9.5, 10$, and the frequency is fixed at 6GHz. The optimized parameters of the CUCA are given in table 1.

The imaging results are shown in Fig.6, for case 1, as we can see, the azimuth angle of the two points can be reconstructed correctly if their difference is less than 180 degree. While for case2, the scattering points cannot be reconstructed correctly if only integer modes are applied. The equivalent fractional OAM mode is feasible to handle this problem.

Secondly, the resolution of the two schemes are compared by another 1-D imaging experiment. The frequency is fixed at 6GHz, and for MISO scheme, the OAM mode applied in the experiment is $-10 \sim 10$ but only the integer part, while for MIMO scheme the OAM mode is the same as the first experiment. The two scattering points are with a same distance and same elevation angle but different azimuth angle similar to the first experiment, and the two scattering points used here are much closer in the azimuth direction as shown in table 3.

The imaging results are shown in Fig.7. For case 1, as we can see, both of the two schemes can reconstruct the azimuth angles of the two points correctly, demonstrating that the MISO scheme doesn’t suffer the aliasing problem. However, for case 2, the two scattering points cannot be separated through the MISO scheme since the resolution of the MIMO scheme is better than the MISO scheme.

### TABLE 2. Locations of the two scattering points in the first experiment.

| scattering points | $P_1(r_1, \theta_1, \phi_1)$ | $P_2(r_1, \theta_1, \phi_1)$ |
|-------------------|-----------------------------|-----------------------------|
| case1             | (1000m, 11°, 54°)           | (1000m, 11°, 90°)           |
| case2             | (1000m, 11°, 54°)           | (1000m, 11°, 270°)          |

### TABLE 3. Locations of the two scattering points in the second experiment.

| scattering points | $P_1(r_1, \theta_1, \phi_1)$ | $P_2(r_1, \theta_1, \phi_1)$ |
|-------------------|-----------------------------|-----------------------------|
| case1             | (1000m, 11°, 50°)           | (1000m, 11°, 245°)          |
| case2             | (1000m, 11°, 50°)           | (1000m, 11°, 65°)           |
OAM mode. While by the traditional MIMO scheme with only integer modes, only one “plane” appears in the imaging result.

IV. CONCLUSION
The OAM based radar imaging method is a new computational imaging scheme, targets can be reconstructed by applying multiple OAM modes and the computation of the echo without relative motion. In this paper, an equivalent fractional OAM mode is proposed for the first time to solve the aliasing problem in MIMO scheme to realize high resolution in EM vortex radar imaging. The transmitted and received OAM modes are set as two adjacent integers to achieve an equivalent fractional mode. Considering different modes of the EM vortex wave generated by one single UCA radiate to different elevation angle, the optimized CUCA is used to collimate the radiation patterns of different modes to a same angle with low side-lobes. Then 1D and 2D imaging based on MIMO mode is realized through FFT, simulation results validate the effectiveness of the proposed method.

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