Are the B-anomalies evidence for heavy neutrinos?

Xiao-Gang He\textsuperscript{1,2,3,}\textsuperscript{*} and German Valencia\textsuperscript{4,}\textsuperscript{†}

\textsuperscript{1}Physics Division, National Center for Theoretical Sciences, Hsinchu, Taiwan 30013.
\textsuperscript{2}T-D Lee Institute, Department of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai.
\textsuperscript{3}Department of Physics, National Taiwan University, Taipei.
\textsuperscript{4}School of Physics and Astronomy, Monash University, 3800 Melbourne Australia.

(Dated: June 30, 2017)

Abstract

The existing anomalies appearing in decays of the form $b \rightarrow s \ell^+ \ell^-$ constitute a possible hint for new physics. We point out that modifications to the SM results due to heavy neutrinos could account for the observed deviations while satisfying existing constraints from lepton flavor violating processes. The required mixing angle, however, is an order of magnitude larger than suggested by recent global fits to lepton flavor conserving processes. We frame our discussion in terms of a Type-I seesaw model, but it can be made more general.

\textsuperscript{*} Electronic address: hexg@phys.ntu.edu.tw
\textsuperscript{†} Electronic address: German.Valencia@monash.edu
I. INTRODUCTION

Experimental data have suggested anomalies in the flavour-changing neutral current (FCNC) process \( b \to s \mu^+ \mu^- \) for some time now [1–5]. At the same time it appears that the related mode with electrons instead of muons \( b \to se^+ e^- \), is consistent with the standard model (SM) expectations [6]. A particularly interesting discrepancy between experiment and the SM occurs in the ratios \( R_K, R_K^* = B(B \to K(K^*)\mu^+ \mu^-)/B(B \to K(K^*)e^+ e^-) \) [7–9], where lepton universality appears to be violated.

As expected, the anomalies in the \( b \to s \ell^+ \ell^- \) measurements have received considerable attention in the literature and multiple models have been put forward as possible new physics explanations [10]. There are also several model independent analyses of these experimental results in the form of global fits to the Wilson coefficients of the relevant low energy effective Hamiltonian [11].

One of the scenarios preferred by these global fits affects primarily the \( C_9 \) and \( C_{10} \) Wilson coefficients. These coefficients are defined by the operators,

\[
\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* (C_9 \mathcal{O}_9 + C_{10} \mathcal{O}_{10}) ,
\]

\[
\mathcal{O}_9 = \frac{e^2}{16\pi^2} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \ell) , \quad \mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \gamma_5 \ell) ,
\]

where \( P_L = (1 - \gamma_5)/2 \) and, in the absence of flavour universality, \( C_{9\mu,10\mu} \neq C_{9e,10e} \).

The SM predicts that \( C_{9,10} \) are approximately the same for all leptons with \( C_{9\text{SM},10\text{SM}} \approx 4.1 \), and \( C_{10\text{SM}} \approx -4.1 \). The model discussed in this paper will introduce corrections to these coefficients with the pattern \( C_9^{\text{NP}}(M_W) = -C_{10}^{\text{NP}}(M_W) \) and therefore our benchmark will be the best fit with \( C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}} \) in the 1\(\sigma\) range \([-0.73, -0.48]\) found by Ref. [11].

II. THE MODEL

In Type I Seesaw models [12] there are three light and \( N \) heavy neutrinos and the general mass term for the neutrinos can be written as

\[
\mathcal{L}_M = -\bar{L}_\nu Y_\nu \phi \nu_R - \frac{1}{2} \bar{\nu}_R^c M_R \nu_R + h.c.
\]

where \( \phi \) is the usual Higgs doublet with a vacuum expectation value \( \langle \phi \rangle = v/\sqrt{2} \).

The neutrino mass matrix then takes the following form in the \((\nu_L^c, \nu_R)\) basis

\[
M_\nu = \begin{pmatrix}
0 & \frac{v}{\sqrt{2}} Y_\nu \\
\frac{v}{\sqrt{2}} Y_\nu^T & M_R
\end{pmatrix},
\]

which is a symmetric matrix that can be diagonalized by the transformation

\[
\hat{U}^T M_\nu \hat{U} = \hat{M}_\nu.
\]
$\bar{U}$ is a unitary matrix and the diagonal neutrino mass matrix is then $\hat{M} = \text{diag}(m_1, m_2, m_3, M_4, \cdots, M_{3+N})$ with $m_i$ and $M_i$ the light and heavy mass eigenvalues respectively. We will denote the heavy neutral mass eigenstates by $\bar{N}$. $\bar{U}$ is a $(3+N) \times (3+N)$ matrix which can be written as two block $3 \times (3+N)$ matrices

$$\bar{U} = \begin{pmatrix} \bar{U}_L^T \\
\bar{U}_R \end{pmatrix}, \quad (5)$$

In order to accommodate the known neutrino oscillation data, which shows that there are at least two massive light neutrinos, $N$ should be equal to or larger than two.

The charged current interaction between the $W$-boson and quarks and leptons in the weak interaction basis is given by

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} W^\mu \bar{\ell} \gamma_\mu \nu^m - \frac{g}{\sqrt{2}} W^\mu \bar{\nu} \gamma_\mu \ell^m + h.c. \quad (6)$$

where $L = (e, \mu, \tau)^T$, $\nu = (\nu_e, \nu_\mu, \nu_\tau)^T$, $U = (u, c, t)^T$, and $D = (d, s, b)^T$. If there are no right-handed $W$-boson interactions, the heavy right-handed neutrinos are not connected to the charged leptons by the $W$ boson. However, the left-handed neutrinos will have heavy neutrino components in the mass eigenstate basis and the charged current becomes

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} W^\mu \bar{\nu}^{\ell m} \gamma_\mu P_L U_L^{\ell m} \nu^m - \frac{g}{\sqrt{2}} W^\mu \bar{\nu}^{\ell m} \gamma_\mu P_L V D^m + h.c.$$ \quad (7)

Here we have introduced the matrix $U_L^{\ell m} = \sum_{i=1}^3 S_L^{\ell i} \bar{U}_{ij}^s$ with $\ell = e, \mu, \tau$. $S_L$ is the matrix that diagonalizes the left-handed charged lepton mass matrix: $\ell^m_L = S_L \ell_L$ and $V = V_{KM}$ is the standard Kobayashi-Maskawa (KM) matrix.

In what follows we will drop the superscript “m” from the fermion fields and always refer to mass eigenstates. We will write the $3 \times (3+N)$, $U_L = (S_L)^T \bar{U}_L$, matrix in the following form

$$U_L = \begin{pmatrix}
U_{e_1}^L & U_{e_2}^L & U_{e_3}^L & U_{e_4}^L & \cdots & U_{e_{3+N}}^L \\
U_{\mu_1}^L & U_{\mu_2}^L & U_{\mu_3}^L & U_{\mu_4}^L & \cdots & U_{\mu_{3+N}}^L \\
U_{\tau_1}^L & U_{\tau_2}^L & U_{\tau_3}^L & U_{\tau_4}^L & \cdots & U_{\tau_{3+N}}^L
\end{pmatrix}, \quad (8)$$

and note that it satisfies the unitarity condition

$$\sum_{j=1}^{3+N} U_{ij}^L U_{ij}^L = \delta_{\ell \ell'}. \quad (9)$$
FIG. 1: Box diagram responsible for the process $b \rightarrow d_j \ell\ell'$.

III. LOW ENERGY EFFECTIVE LAGRANGIAN

The model is particularly simple, as the only new contribution to $B$ decay arises from the box diagram depicted in Figure 1 (plus associated diagrams involving would-be Golstone bosons).

These diagrams have been calculated before for the case of lepton flavor violating (LFV) $B$ decays and the result is known [13–16]. In our case we must be careful not to discard the terms that vanish due to the GIM mechanism on the neutrino side for LFV processes, but do not vanish for lepton flavor conserving processes. We find the contribution to the effective Lagrangian for $b \rightarrow d_j \ell\ell'$ at the $M_W$ scale to be,

$$L = -\frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi s_W^2} \sum_{i=u,c,t} \sum_{\alpha=1}^{N+3} V^*_{ij} V_{ik} U_{\alpha j} U_{\alpha k} (4B(\lambda_i) + E(\lambda_i, \lambda_a)) \bar{\ell}_\mu P_L \ell' \gamma^\mu P_L b$$

where $d_j$ refers to a $d$ or an $s$ quark, $\lambda_i = m_i^2/m_W^2$, and the Inami-Lim functions [17] $B(\lambda_i)$ and $E(\lambda_i, \lambda_a)$ are given by,

$$B(\lambda_i) = \frac{1}{4} \left( \frac{\lambda_i}{1 - \lambda_i} + \frac{\lambda_i \log(\lambda_i)}{(1 - \lambda_i)^2} \right)$$

$$E(\lambda_i, \lambda_a) = \lambda_i \lambda_a \left\{ -\frac{3}{4} \frac{1}{(1 - \lambda_i)(1 - \lambda_a)} + \left[ \left( \frac{1}{4} \frac{3}{2 \lambda_i - 1} - \frac{3}{4} \frac{1}{(\lambda_i - 1)^2} \right) \frac{\log \lambda_i}{\lambda_i - \lambda_a} + \left( \frac{1}{4} - \frac{3}{2 \lambda_a - 1} - \frac{3}{4} \frac{1}{(\lambda_a - 1)^2} \right) \frac{\log \lambda_a}{\lambda_a - \lambda_i} \right] \right\}.$$  (11)

$B(\lambda_i)$ is just the usual function that reproduces the SM box diagram contribution to $b \rightarrow d_j \ell\ell'$ [18]. The new term is given by $E(\lambda_i, \lambda_a)$ and it subtracts from the SM as illustrated in Figure 2.

Neglecting for simplicity the contribution from the charm-quark intermediate state, our result in Eq. 10 implies that

$$C_{9}^{NP}(M_W) = -C_{10}^{NP}(M_W) = \frac{1}{4s_W^2} \sum_{N} U_{\mu N}^L U_{\mu N}^L E(\lambda_\mu, \lambda_N).$$  (12)

\footnote{Note that $E(\lambda_i, \lambda_a) = -E_L(\lambda_i, \lambda_N)/4$ in the notation of Ref. [15].}
FIG. 2: Inami-Lim function $E(\lambda_t, \lambda_N)$ for physical top-quark mass as a function of heavy neutrino mass in TeV, normalized to the SM box function $B(\lambda_t)$.

As shown in Ref. [15], the main constraints on the mixing angles $U_{\ell N}$ arise from radiative lepton decay and the corresponding low energy effective Lagrangian for this type of process is given by

$$
\mathcal{L} = \frac{4 G_F}{\sqrt{2}} \frac{e}{16\pi^2} \sum_N U_{\ell N}^L U_{\ell N}^L F(\lambda_N) \ m_\ell e^e \sigma_{\mu\nu} P_R \ell' \ell'
$$

(13)

where $F^{\mu\nu}$ is the electromagnetic field strength tensor and the Inami-Lim function in this case is

$$
F(\lambda_N) = \left[ \frac{3\lambda_N^3 \log \lambda_N}{4(1-\lambda_N)^4} + \frac{2\lambda_N^3 + 5\lambda_N^2 - \lambda_N}{8(1-\lambda_N)^3} \right].
$$

(14)

IV. RESULTS AND CONCLUSION

Using Eq. (13) and Eq. (10), we can update the constraints on $U_{\ell N}^L$ arising from radiative lepton decay and LFV $B$ decay with the experimental limits given in Table I.

TABLE I: Summary of current experimental bounds for $\ell \to \ell'\gamma$ and $B \to \ell^\pm\ell'^\mp$ taken from the Particle Data Book [19].

| Process | limit | Process | limit |
|---------|-------|---------|-------|
| $B(\mu \to e\gamma)$ | $4.2 \times 10^{-13}$ | $B \to e^\pm \mu^\mp$ | $2.8 \times 10^{-9}$ |
| $B(\tau \to e\gamma)$ | $3.3 \times 10^{-8}$ | $B \to e^\pm \tau^\mp$ | $2.8 \times 10^{-5}$ |
| $B(\tau \to \mu\gamma)$ | $4.4 \times 10^{-8}$ | $B \to \mu^\pm \tau^\mp$ | $2.2 \times 10^{-5}$ |
| $K_L \to e^\pm \mu^\mp$ | $4.7 \times 10^{-12}$ | $B_s \to e^\pm \mu^\mp$ | $1.1 \times 10^{-8}$ |

For simplicity we now consider the contribution of only one heavy neutrino and we find that its mass is not significantly constrained. Taking, for example,
$M_N = 2$ TeV and assuming that all the elements $U_{\ell N}^L$ are real, we illustrate two scenarios in Figure 3. Although not necessary, it is possible to choose $U_{e N}^L = 0$, which will automatically remove any constraints from LFV processes involving electrons, and we show this case in the right panel. The constraints in this case arise exclusively from $\tau \to \mu \gamma$.

![Figure 3: Constraints on $U_{\mu N}^L$ for $M_N = 2$ TeV in two illustrative scenarios.](image)

The parameter space that both, reproduces the best fit scenario of Ref. [11] at $1\sigma$, $C_9^{NP}(m_b) \approx -C_{10}^{NP}(m_b) \sim [-0.73, -0.48]$, and satisfies the LFV constraints is shown in Figure 3. To accommodate the B-anomalies with heavy neutrino masses

\footnote{Notice that this is only approximate for $C_9^{NP}(M_W) = -C_{10}^{NP}(M_W)$ as the QCD running changes $C_9$ but not $C_{10}$.}
in the TeV range, would thus require $U_{\mu N}^L \sim 0.3$. This would produce a contribution to the muon $g - 2$ of $a_{\mu} = -5.7 \times 10^{-10}$ which is at the level of the error in the measurement and below the current anomaly. At the same time, the large mixing angle needed is at odds with recent global fits [20, 21].

Experimental anomalies in the tree-level dominated semileptonic decay of $b$ to $\tau$-leptons have also been reported [22–24]. These ones, $R(D) = B(B \rightarrow D\tau^-\bar{\nu}_\tau)/B(B \rightarrow D\ell^-\bar{\nu}_\ell)$ and $R(D^*)$ (where a $D^*$ replaces the $D$) cannot be explained by the mechanism described in this paper. Interestingly, however, there exist possible explanations involving additional light neutrinos for this case [25, 26].

Acknowledgments

X-G He was supported in part by MOE Academic Excellent Program (Grant No. 102R891505) and MOST of ROC (Grant No. MOST104-2112-M-002-015-MY3), and in part by NSFC (Grant Nos. 11175115 and 11575111) and Shanghai Science and Technology Commission (Grant No. 11DZ2260700) of PRC.

[1] LHCb Collaboration, PRL 111 (2013) 191801, arXiv:1308.1707 [hep-ex].
[2] LHCb Collaboration, JHEP 1406 (2014) 133, arXiv:1403.8044 [hep-ex].
[3] LHCb Collaboration, arXiv:1512.04412 [hep-ex].
[4] LHCb Collaboration, JHEP 1307 (2013) 084, arXiv:1305.2168 [hep-ex].
[5] LHCb Collaboration, arXiv:1506.08777 [hep-ex].
[6] LHCb Collaboration, JHEP **1504** (2015) 064, [arXiv:1501.03038](https://arxiv.org/abs/1501.03038) [hep-ex].
[7] LHCb Collaboration, Phys. Rev. Lett. **113** (2014) 151601, [arXiv:1406.6482](https://arxiv.org/abs/1406.6482) [hep-ex].
[8] A. Abdesselam *et al.* [Belle Collaboration], [arXiv:1604.04042](https://arxiv.org/abs/1604.04042) [hep-ex].
[9] S. Wehle *et al.* [Belle Collaboration], Phys. Rev. Lett. **118** (2017) no.11, 111801, [arXiv:1612.05014](https://arxiv.org/abs/1612.05014) [hep-ex].
[10] A partial list can be found in [11] for example.
[11] See for example B. Capdevila, A. Crivellin, S. Descotes-Genon, J. Matias and J. Virto, [arXiv:1704.05340](https://arxiv.org/abs/1704.05340) [hep-ph], and references therein.
[12] P. Minkowski, Phys. Lett. B **67**, 421 (1977); T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979), p. 95; Prog. Theor. Phys. **64**, 1103 (1980); M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979), p. 315; P. Ramond, [arXiv:hep-ph/9809459](https://arxiv.org/abs/hep-ph/9809459); S.L. Glashow, in *Proceedings of the 1979 Cargese Summer Institute on Quarks and Leptons*, edited by M. Levy *et al.* (Plenum Press, New York, 1980), p. 687; R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980); J. Schechter and J.W.F. Valle, Phys. Rev. D **22**, 2227 (1980); Phys. Rev. D **25**, 774 (1982).
[13] P. Langacker, S. Uma Sankar and K. Schilcher, Phys. Rev. D **38**, 2841 (1988);
[14] Z. Gagyi-Palffy, A. Pilaftsis and K. Schilcher, Phys. Lett. B **343**, 275 (1995) [arXiv:hep-ph/9410201];
[15] X. G. He, G. Valencia and Y. Wang, Phys. Rev. D **70**, 113011 (2004) doi:10.1103/PhysRevD.70.113011 [hep-ph/0409346].
[16] T. Fujihara, S. K. Kang, C. S. Kim, D. Kimura and T. Morozumi, Phys. Rev. D **73**, 074011 (2006) doi:10.1103/PhysRevD.73.074011 [hep-ph/0512010].
[17] T. Inami and C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981) [Erratum-ibid. **65**, 1772 (1981)].
[18] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996) [arXiv:hep-ph/9512380].
[19] C. Patrignani *et al.* [Particle Data Group], Chin. Phys. C **40**, no. 10, 100001 (2016). doi:10.1088/1674-1137/40/10/100001
[20] S. Antusch, C. Biggio, E. Fernandez-Martinez, M. B. Gavela and J. Lopez-Pavon, JHEP **0610**, 084 (2006) doi:10.1088/1126-6708/2006/10/084 [hep-ph/0607020].
[21] E. Fernandez-Martinez, J. Hernandez-Garcia and J. Lopez-Pavon, JHEP **1608**, 033 (2016) doi:10.1007/JHEP08(2016)033 [arXiv:1605.08774] [hep-ph].
[22] A. Bozek *et al.* [Belle Collaboration], Phys. Rev. D **82**, 072005 (2010) [arXiv:1005.2302] [hep-ex].
[23] J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. Lett. **109**, 101802 (2012) doi:10.1103/PhysRevLett.109.101802 [arXiv:1205.5442] [hep-ex].
[24] LHCb Collaboration, Phys. Rev. Lett. **115**, no. 11, 111803 (2015) Addendum: [Phys. Rev. Lett. **115**, no. 15, 159901 (2015)] doi:10.1103/PhysRevLett.115.159901, 10.1103/PhysRevLett.115.111803 [arXiv:1506.08614] [hep-ex].
[25] X. G. He and G. Valencia, Phys. Rev. D **87**, no. 1, 014014 (2013) doi:10.1103/PhysRevD.87.014014 [arXiv:1211.0348] [hep-ph].
[26] G. Cvetic, F. Halzen, C. S. Kim and S. Oh, [arXiv:1702.04333](https://arxiv.org/abs/1702.04333) [hep-ph].