Topological Dephasing in the $\nu = 2/3$ Fractional Quantum Hall Regime

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We study dephasing in electron transport through a large quantum dot (a Fabry-Perot interferometer) in the fractional quantum Hall regime with filling factor $2/3$. In the regime of sequential tunneling, dephasing occurs due to electron fractionalization into counterpropagating charge and neutral edge modes on the dot. In particular, when the charge mode moves much faster than the neutral mode, and at temperatures higher than the level spacing of the dot, electron fractionalization combined with fractional statistics of the charge mode leads to the dephasing selectively suppressing $h/e$ Aharonov-Bohm oscillations but not $h/(2e)$ oscillations, resulting in oscillation-period halving.

Electron interaction is a dominant source of dephasing at low temperature [1]. It leads to electron fractionalization [2, 3] in quantum wires: An electron, injected into a wire, splits into constituents (spin-charge separation, charge fractionalization), showing reduction of interference visibility or dephasing [4]. Typically, the visibility exponentially decays with electron travel distance. When the wire is finite, the constituents recombine after bouncing at wire ends, resulting in coherence revival [5]. Fractionalization was detected [6] in a non-chiral wire, and studied in the integer quantum Hall (QH) edge [7, 12].

A fractional QH system of filling fraction $\nu$ has edge channels that support fractional charges obeying fractional braiding statistics [13]. At $\nu = 2/3$, the edge states are decomposed into a $v_{\text{edge}} = 2/3$ charge mode and a counterpropagating neutral mode [14, 15]. They originate from renormalization of two counterpropagating charge modes [16, 17], $v_{1, \text{edge}} = 1$ and $v_{2, \text{edge}} = -1/3$, and stabilize at low temperature under strong disorder. Neutral modes were detected [18] and studied [19, 20].

The present study of the $\nu = 2/3$ QH regime emphasizes another dephasing mechanism unnoticed so far: Fractionalized constituents (charge and neutral components) of an electron, satisfying fractional statistics, may braid with thermally excited anyons. Thermal average of the resulting braiding phase leads to dephasing, which we call topological dephasing, which occurs only in the interfering processes characterized by particular values of topological winding numbers.

Our analysis addresses the Aharonov-Bohm (AB) oscillation of differential conductance $G$ through a quantum dot (QD) at $\nu = 2/3$. We focus on linear response of electron sequential tunneling into the QD. $G$ is decomposed into the harmonics of the AB flux $\Phi$ in the QD,

$$G = \frac{e^2}{h} \sum_{\delta p = 0, 1, 2, \ldots} G_{\delta p} \cos(2\pi \delta p \Phi / \Phi_0).$$

Semiclassically, $\delta p$ represents the relative winding number of a fractionalized charge component, around the circumference $L$ of the QD, between two interfering paths:

An electron, after tunneling into the QD, fractionalizes into charge and neutral components; see Fig. 1. The charge (neutral) component has propagation velocity $v_c (n)$, spatial width $L_{T,c(n)} \equiv h v_c(n)/(2\pi k_B T \delta p_c(n))$ at temperature $T$, and level spacing $E_c(n) = 2\pi h v_c(n)/L$, and its scaling dimension in the electron tunneling operator is $\delta_c = 3/4$ ($\delta_n = 1/4$). $G_{\delta p}$ is determined by the overlaps of the components of the same kind between two interfering paths of relative charge winding $\delta p$.

We find two mechanisms suppressing $G_{\delta p \neq 0}$, plasmonic dephasing and topological dephasing: the former (latter) involves plasmon (zero-mode) parts of the components. In the plasmonic dephasing, $G_{\delta p}$ is contributed from the two interfering paths whose charge components overlap maximally. But, their neutral components overlap only partially, reducing $G_{\delta p}$; similar dephasing occurs in other fractionalizations [4, 5]. The topological dephasing additionally occurs, but depending on $\delta p$, in contrast to the other known mechanisms. When $v_c > v_n$ [21, 22], the first harmonics $G_{\delta p = 1}$ is suppressed at $k_B T > E_c(n)/(4\pi^2 \delta_n)$ (namely, $L > L_{T,c(n)}$). It is because the charge component gains thermally fluctuating fractional braiding phase of

![FIG. 1: (Color online) A large quantum dot (Fabry-Perot interferometer) in the fractional QH regime of $\nu = 2/3$, coupled to lead edge states of $\nu = 2/3$ (black solid lines) through quantum point contacts (QPCs) at $x_L, x_R$. Electron (rather than fractional quasiparticle) tunneling occurs through the QPCs (dotted lines). Following the tunneling, each electron (and the hole left behind in the lead edge) fractionalizes into a charge component propagating at velocity $v_c$ (solid blue arrow) and a neutral component counterpropagating at velocity $v_n$ (dashed red). The magnetic flux in the dot area is $\Phi$.](image.png)
logical dephasing. We address the occupation number fractionalization and fractional statistics cause the topological is dominated by finite range of bare parameters, monomers in the QD edge or in the bulk. By contrast, the second harmonic $G_{\delta p=2}$ is not affected by the topological dephasing (as braiding phase $\pi N_c \delta p$ and $(\pm 1)^{\delta p} = 1$ are trivial) and dominates $G$, resulting in $h/(2e) AB$ oscillations.

Setup.—The $\nu = 2/3$ QD is coupled to two lead edges via quantum point contacts (QPCs) [33]; see Fig. 1. The Hamiltonian is $H = H_0 + H_L + H_R + H_T$. $H_D$ describes the edge of the QD, while $H_{L(R)}$ the $\nu = 2/3$ left (right) lead edge. Each edge consists of the bosonic mode $\phi_1 (\nu_1^* = 1)$ and the counterpropagating $\phi_2 (\nu_2^* = -1/3)$. $\phi_{i=1,2}$ supports charge $e\nu_i^*$ and satisfies $[\phi_i(x), \phi_i'(x')] = i\pi \nu_i^* \delta (x - x') \delta_{ij}$ at positions $x, x'$. Introducing the charge mode $\phi_c \equiv \sqrt{3/2}(\phi_1 + \phi_2)$ (supporting charge $2e/3$) and the neutral mode $\phi_n \equiv (\phi_1 + 3\phi_2)/\sqrt{2}$, one writes [14] [15]

$$H_D = \frac{\hbar}{4\pi} \int_0^L dx [v_c(\partial_x \phi_c)^2 + v_n(\partial_x \phi_n)^2 + v_\delta \phi_c \partial_x \phi_n] + \int_0^L dx [\xi(x) \exp(i\sqrt{2} \phi_n) + H.c.].$$

Disorder-induced tunneling amplitude $\xi(x)$ between $\phi_1$ and $\phi_2$ is modeled by a Gaussian random variable with mean zero and variance $\xi^2(x) = W \delta (x - x')$. For a finite range of bare parameters, $\phi_c$ and $\phi_n$ decouple [14], rendering $v$ irrelevant. $H_{L,R}$ is similarly written.

The QPCs are almost closed, so electron tunneling is facilitated. Renormalization group analysis [14] [15] indicates four equally most relevant electron tunneling operators between the electron field operators, $\Psi_\pm (x_\alpha) = e^{i\sqrt{3/2} \phi_c (x_\alpha)} e^{\pm \partial_\alpha \phi_\alpha (x_\alpha)} / \sqrt{2} \pi a$ at $x_\alpha = L, R$ on the QD, and $\Psi_{\alpha, \pm}(0)$ on lead edge $\alpha$; $a$ is an ultra-violet cutoff and $\Psi_{\alpha, \pm}$ has the same form as $\Psi_\pm$. So the tunneling Hamiltonian is $H_T = \sum_{\alpha=L,R} \sum_{\nu=\pm} \int \Psi_\nu^\dagger (0) \Psi_\nu (x_\alpha) + H.c.$, where $\nu_{\alpha,ij}$ is the tunneling strength.

Topological dephasing.—We show that at $\nu = 2/3$, fractionalization and fractional statistics cause the topological dephasing. We address the occupation number operator $N_{c(n)}$ of charge (neutral) mode at the QD edge,

$$\frac{1}{3} N_c = N_1 - \frac{1}{3} N_2, \quad N_n = N_2 - N_1,$$

(3)
defined through the zero-mode parts of $\phi_{1,2}$ [34]. The number operator $N_{1(2)}$ of $\phi_{1(2)}$ is an integer since $e$ and $-e/3$ are the elementary charges of $\phi_{1,2}$; $N_c$ is an integer measuring charge excitations in the units of $e/3$ ($N_c = 1$ for a quasiparticle of charge $e/3$; $N_c = 3$ for an electron).

A quasiparticle of charge $e/3$ at position $x$ on the QD edge is written as $e^{i\phi_c(x) / \sqrt{3}} e^{i\partial_\alpha \phi_\alpha (x)} / \sqrt{2}$ [22]. Consider clockwise exchange of two such quasiparticles. Since $[\phi_c(x), \phi_c(x')] = i\pi \delta (x - x')$, the exchange of the two charge components results in statistical phase $\pi/6$,

$$e^{i\phi_c(x)} e^{i\phi_c(x')} = e^{i\pi \delta (x' - x)} e^{i\phi_c(x')} e^{i\phi_c(x)}.$$

(4)

So, after the charge component of the electron operator $\Psi_\pm$ winds once clockwise around $N_c$ charge-mode excitations on the edge, a phase $3 \times N_c \times 2 \times \pi/6 = \pi N_c$ is gained [33]. Here, 3 means the number of charge components forming $\Psi_\pm (x)$, and 2 refers to braiding (double exchanges). Similarly, the exchange of the neutral components of the two quasiparticles leads to exchange phase $-\pi/2$. So the neutral component of $\Psi_\pm (x)$ gains $\pm 1 \times N_n \times 2 \times (-\pi/2) = \mp \pi N_n$, after winding once around $N_n$ neutral-mode excitations; the number of the neutral components of $\Psi_\pm$ is $\pm 1$.

This has implications on the dynamics of an electron which enters into the QD and then fractionalizes. When $v_c \gg v_n$, there is a process where the charge component of the electron winds once around the QD, while the neutral component moves very little. In terms of the winding numbers of the charge and neutral components, $p$ and $q$, this process is denoted by $(p, q) = (1, 0)$. This process interferes with that of no winding $(p', q') = (0, 0)$, contributing to the $h/e$ harmonics $G_{\delta p=1}$; see Fig. 2. The relative winding numbers between the two interfering paths are $(\delta p = p - p', \delta q = q - q')$, and the net braiding phase gained from that winding around $N_c$ charge and $N_n$ neutral excitations on the edge is $\pi (N_c \delta p + N_n \delta q) = \pi N_c$. Since $N_c$ is an integer, thermal fluctuations of quasiparticle (or electron) excitations on the edge give rise to fluctuations of the braiding phase factor $e^{i\pi N_c} = \pm 1$ [+] (for even (odd) $N_c$), suppressing the $h/e$ harmonics. This topological braiding-induced dephasing also occurs due to thermal quasiparticle or electron fluctuations in the bulk [34].

By contrast, the main contribution to the $h/(2e)$ harmonics $G_{\delta p=2}$ comes from $(\delta p, \delta q) = (2, 0)$. In this case, the braiding phase factor $e^{i\pi (N_c \delta p + N_n \delta q)} = 1$, regardless of $N_c$ being even or odd. Hence, $G_{\delta p=2}$ is immune to the topological dephasing. In general, such dephasing occurs only with odd $\delta p + \delta q$, since the fluctuating $N_c \pm N_n$ is always even; see Eq. (3).

When $v_c \simeq v_n$, the topological dephasing does not occur, since $\delta p = -\delta q$ and $N_c \pm N_n$ is even.

Sequential tunneling.—We compute $G$ in Eq. (1) to the order of sequential tunneling,

$$G \simeq \frac{e^2}{\hbar} \sum_{j=\pm, \alpha} \int_{-\infty}^0 dt F(t) \text{Im} G_j(x_\alpha, t),$$

(5)

where $\gamma = \gamma_c \gamma_R / (\gamma_c + \gamma_R)$, $\gamma_\alpha \propto |t_{\alpha ij}|^2$ is the (renormalized) electron tunneling rate between the QD and lead edge $\alpha = L, R$, $G_j$ is the Green function describing the time ($t$) evolution of the fractionalized components of an injected electron described by $\Psi_j (x_\alpha, t = 0)$, and $e\gamma$ is a constant [33]. The injection leaves a hole behind on the lead edge. $F(t) = (\pi k_B T \hbar) \sin^2 (\delta \phi + \delta \beta)(\pi k_B T \hbar)$ accounts for the fractionalization of the hole.

$G_{\delta p}$ comes from the interference between two processes of relative charge winding number $\delta p$. At $k_B T > \cdots$
electron tunnels from lead edge $\mathcal{L}$ into the QD and fractionalizes at $x_{c}$ at time $t_{i} = 0$, while at $t_{j} = -L/v_{c}$ in the other [Fig. 2(b)]. The charge components of the two processes interfere at $x_{\varepsilon}$ at $t = 0$, contributing to $G_{1}$, after respective windings $p = 0$ and 1. At that time, the distance between the neutral components is $L - \Delta L$, leading to partial overlap, hence, to the third factor $\exp(-(L - \Delta L)/LT_{n})$ of $G_{1}$. The tunneling leaves a hole behind on $\mathcal{L}$, which also fractionalizes into charge and neutral components (not shown in Fig. 2). The partial overlap at $t = 0$ between the two charge components from the holes created at $t_{i} = 0$, and that between the two neutral components, lead to the first two exponential factors of $G_{1}$ in Eq. (6), respectively.

In the other limit of $v_{c} \gg v_{n}$, both plasmonic and topological dephasings are crucial. There are two interfering processes for $G_{1}$ shown in Figs. 2(a) and 2(c), and for $G_{2}$ in Figs. 2(a) and 2(d). When $k_{B}T \gg E_{c}$, we obtain

\begin{equation}
G_{1} \propto \tilde{\gamma}L(k_{B}T)^{3} \exp\left(-\frac{L}{LT_{c}} - \frac{\Delta L}{LT_{n}} - \frac{L - \Delta L}{LT_{n}} + \frac{(\Delta L)^{2}}{LLT_{n}}\right)
\times \exp\left[-\frac{(L - \Delta L)^{2}}{LLT_{n}}\right]
G_{2} \propto \tilde{\gamma}L(k_{B}T)^{3} \exp\left[-2\left(\frac{L}{LT_{c}} + \frac{\Delta L}{LT_{n}} + \frac{\Delta L}{LT_{n}}\right)\right].
\end{equation}

The first three exponential factors of $G_{1}$ and $G_{2}$ result from plasmonic dephasing, as those of Eq. (6). The third factor has a form different from that of Eq. (6) of $v_{c} \gtrsim v_{n}$, as the interfering neutral components in the QD are now $\Delta L$ apart in space. The factor 2 in the arguments of $G_{2}$ arises from the double winding, $\tilde{\delta} = 2$. Another exponential factor $\exp[(\Delta L)^{2}/(LLT_{n})]$ of $G_{1}$ comes from the plasmonic part of the neutral component; it is cancelled out with zero-mode contributions in other cases [5], e.g. Eq. (6) and $G_{2}$ in Eq. (7).

The last suppression factor of $G_{\tilde{\delta}p=1}$, $\exp[-(L - \Delta L)^{2}/(LLT_{n})]$, represents the topological dephasing, arising from the zero-mode parts of $G_{j}(x_{\alpha}, x_{\alpha}; t)$. The process in Fig. 2(c) (where the center of the neutral component hardly moves, while the charge component winds once around $L$) interferes with that of Fig. 2(a), contributing to $(\tilde{\delta}p, \tilde{\delta}q) = (1, 0)$. As discussed around Eq. (4), this interference with $\tilde{\delta}p + \tilde{\delta}q = 1$ is suppressed by the thermally fluctuating braiding phase factor of $e^{i\pi(\tilde{\delta}pN_{c} + \tilde{\delta}qN_{n})} = \pm 1$. The suppression factor is interestingly determined by the spatial tail (or finite $LT_{n}$) of the zero-mode part of the neutral component. The tail indicates that the neutral component can quantum mechanically wind once more than the semiclassical number $q$ of the center; the quantum mechanical winding is well defined by the Poisson formula [34]. Hence, from the processes in Figs. 2(a) and 2(c), interference with the total relative winding number of $\tilde{\delta}p + (\tilde{\delta}q + 1)$ can occur. As $\tilde{\delta}p + \tilde{\delta}q + 1$ is even, this interference avoids the topological dephasing, dominantly contributing to $G_{\tilde{\delta}p=1}$, but it is
reduced by the separation $L - \Delta L$ of the neutral components of $\delta \varphi + 1$ relative windings. This explains the factor $\exp[-(L - \Delta L)^2/(LLT_n)]$; the exponent is quadratic in $L - \Delta L$, since it originates from the zero-mode part \[5\].

Because of the topological dephasing, $G_2$ is much larger than $G_1$ when $v_c \gg v_n$; $\exp(-(L - \Delta L)^2/(LLT_n))$ is much smaller than the other factors. As a result, $G$ shows $h/(2e)$ AB oscillations. In Fig. 3 we numerically compute $G$ for both $v_c \gg v_n$ and $v_c \gg v_n$ without employing the semiclassical approximation. The result for $v_c \gg v_n$ demonstrates the topological dephasing and consequent period halving even at $k_B T < E_c$.

**Strong disorder regime.**—In the strong disorder regime of a QD edge with $v = 0$ [see Eq. (2)], we find that Eqs. (6) and (7) are hold without any modification. The reasoning is as follows. We start with the diagonalized form of $H_D$ \[14\], \[\begin{align*}
H_D &= \int dx \left[ \frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} \psi \right)^2 + v_n \psi^\dagger \partial_x \psi \right],
\end{align*}\]
where $\psi(x) \equiv \psi(x + \delta_0)/\sqrt{2}, \psi(x - \delta_0)/\sqrt{2} \right] T = U(x) \psi(x)$, the unitary matrix $U(x)$ represents random disorder scattering, $\psi \equiv \psi(x + \delta_0)/\sqrt{2}, \psi(x - \delta_0)/\sqrt{2} \right] T$ is a two-component fermionic field, and $\chi$ is an auxiliary bosonic field. The equal-position correlator $\langle \Psi_+^\dagger(x \rho, t) \Psi_+ x \rho, 0 \rangle$ is readily computed: The contribution of the charge sector is not affected by disorder, while the gauge choice $U(x \rho) = 1$ renders the evaluation of the neutral sector contribution unchanged as well \[14\] \[14\].

**Discussion and conclusion.**—We have studied electron dephasing at $\nu = 2/3$. Electron fractionalization into charge and neutral components leads to plasmonic dephasing. When $v_c \gg v_n$ (which is likely \[31\] \[32\]) and at $k_B T > E_c/(\pi^2 \delta_0)$, a new type of dephasing additionally arises. This dephasing is topological, resulting from the fractionalization and the fractional braiding statistics of the components, and occurs depending on the topological sectors characterized by the winding numbers $\theta, \delta_0$ of the components; its dependence on the even-odd parity of $\delta_0, \delta_0$ is mathematical reminiscent of the parity (integer versus half-integer spin) dependent role of the topological $\theta$ term in antiferromagnetic spin chains \[31\]. It leads to period halving of the AB oscillations.

In the presence of edge disorder, this physics is robust against the introduction of weak intermode interaction, $v \ll v_c$ (see Eq. (2)), which is achieved under a smooth QD confinement potential \[27\] \[12\]. At low temperature, $v$ renormalizes towards 0 \[14\]; at finite temperature, it causes the decay of the plasmonic part of the neutral component, accompanied by the diffusive spreading of the plasmonic part of the charge component \[14\] \[15\].

These slightly modify the first three dephasing factors in Eqs. (9) and (7), but do not affect the topological dephasing, hence the emergence of the $h/2e$ oscillations.

We note that the QH edges at $\nu = 2/3$ may undergo more complex edge reconstruction at about $T > 50$ mK \[27\] \[33\]. At lower temperature our analysis is applicable, while at higher temperature different topological dephasing may occur. Assuming $v_n \sim 10^4$ m/s and $v_c \sim 10^5$ m/s, the $h/(2e)$ oscillation will appear at $T > 20$ mK, when $L > (\hbar v_n/(2\pi k_B T \delta_0)) \sim 2 \mu$m.

Detection of the period halving supports the topological dephasing, thus, the fractional statistics of the charge component at $\nu = 2/3$. The plasmonic dephasing and the topological dephasing will occur, with modifications, in other anyon interferometers or at other $\nu$'s.

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SUPPLEMENTAL MATERIAL

In this material, we derive (a) Eq. (2), (b) Eq. (3), and (c) Eq. (5). (d) We expand, using the Poisson summation formula, the zero-mode contribution to $G$ in harmonics of winding numbers, explaining topological dephasing. (e) We get Eqs. (6) and (7). We discuss (f) quasiparticle fluctuations in the bulk and (g) why $v_c$ can be much larger than $v_n$ and $v$. 

a. Quantum-Dot Hamiltonian in Eq. (2)

We derive the Hamiltonian $H_D$ (cf. Eq. (2) in the main text). $H_D$ is written in terms of the mode $\phi_1$ of filling factor $\nu_1 = 1$ and the counterpropagating mode $\phi_2$ of $\nu_2 = -1/3$, as

$$H_D = \frac{\hbar}{4\pi} \int_0^L dx [v_1(\partial_x \phi_1)^2 + 3v_2(\partial_x \phi_2)^2 + 2v_{12}\partial_x \phi_1\partial_x \phi_2] + \int_0^L dx [\xi(x) \exp(i\phi_1 + 3i\phi_2) + H.c.]$$

(S1)

$v_{12}$ is the velocity of $\phi_{12}$ (renormalized by the intra-mode interactions) and $v_{12}$ describes the inter-mode interaction. $\phi_{1,2}$ satisfies $[\phi_i(x), \phi_i'(x')] = i\pi\nu_i\text{sgn}(x-x')\delta_{ii'}$. Each field $\phi_i(x)$ is decomposed through $\phi_i(x) = \phi_i^{pl}(x) + \phi_i^{pl}(x)$ into a plasmonic mode $\phi_i^{pl}(x)$, satisfying the periodic boundary condition of $\phi_i^{pl}(x + L) = \phi_i^{pl}(x)$, and a zero mode $\phi_i^{pl}(x)$,

$$\phi_i^{pl}(x) = \frac{2\pi\nu_i x}{L} (N_i + \frac{1}{2} - \frac{\Phi}{\Phi_0}) - \lambda_i.$$  

(S2)

The number operator $N_i$ counts the excess number of quasiparticles of charge $\nu_i e$. Its canonical conjugate $\lambda_i$ satisfies $[\lambda_i, N_i'] = i\delta_{ii'}$, and $e^{\pm i\lambda_i}$ changes $N_i$ by $\pm 1$, acting as a Klein factor. This ensures $[\phi_i^{pl}(x), \phi_i^{pl}(x')] = 2i\delta_{ii'}\pi\nu_i(x-x')/L.$
Combined with the commutation rule of the plasmonic part \([\hat{\phi}^{\dagger}_{\nu}(x), \hat{\phi}_{\nu}^{\dagger}(x')] = i\pi\nu_{L}\text{sgn}(x-x') - 2(x-x')/L\delta_{xx'}\), this leads to \([\hat{\phi}_{\nu}(x), \hat{\phi}_{\nu}(x')] = i\pi\nu_{L}\text{sgn}(x-x')\delta_{xx'}\). The term 1/2 in the bracket of Eq. (S2) is introduced, to impose the boundary condition of electron operators, \(\exp(i\hat{\phi}_{\nu}(x + L)/\nu_{L}) = \exp(i\hat{\phi}_{\nu}(x)/\nu_{L})\exp(-2\pi i\Phi/\Phi_{0})\). The magnetic flux \(\Phi\) enclosed by the QD edge states causes the shift of \(N_{v} \to N_{v} - \Phi/\Phi_{0}\) in Eq. (S2); as \(\Phi\) increases (decreases) by \(\Phi_{0}\), the edge state with filling factor \(\nu_{L}\) is energetically stabilized by removing (adding) its own quasiparticle of charge \(\nu_{L}\).

Combining \(\phi_{\nu}\)’s, one introduces the charge mode \(\phi_{c} = \sqrt{3/2}(\phi_{1} + \phi_{2})\) and the neutral mode \(\phi_{n} = \sqrt{1/2}(\phi_{1} + 3\phi_{2})\), satisfying \([\hat{\phi}_{c/n}(x), \hat{\phi}_{c/n}(x')] = \pm i\pi\text{sgn}(x-x')\) and \([\phi_{c}(x), \phi_{n}(x')] = 0\). Putting this into Eq. (S1), we derive Eq. (2).

b. Derivation of \(N_{c}\) and \(N_{n}\) in Eq. (3)

We derive Eq. (3) in the absence of disorder \((W = 0)\). The charge (neutral) mode is decomposed into the zero-mode part \(\phi^{0}_{c}(x)\) \((\phi^{0}_{n}(x))\) and the plasmonic part \(\phi^{\ast}_{c}(x)\) \((\phi^{\ast}_{n}(x))\). The latter describes edge plasmonic excitations, while the former anyon number excitations. The zero-mode parts are determined from Eq. (S2) as

\[
\phi^{0}_{c}(x) = \sqrt{\frac{2\pi}{\nu_{L}}} \left( N_{c} + 1 - 2\frac{\Phi}{\Phi_{0}} \right) - \sqrt{6} \lambda_{c}, \quad \phi^{0}_{n}(x) = -\sqrt{\frac{2\pi}{\nu_{L}}} N_{n} x - \sqrt{2} \lambda_{n}.
\]

We impose \([\lambda_{c/n}, N_{c/n}] = i\), \([\lambda_{c/n}, N_{c/n}^{\dagger}] = 0\), and \([\phi^{\dagger}_{c/n}(x), \phi^{\dagger}_{c/n}(x')] = \pm i\pi\text{sgn}(x-x') - 2\pi i(x-x')/L\). These ensure \([\phi^{0}_{c/n}(x), \phi^{0}_{c/n}(x')] = \pm 2\pi i(x-x')/L\) and \([\phi_{c/n}(x), \phi_{c/n}(x')] = \pm i\pi\text{sgn}(x-x')\). As \(\Phi\) shifts by \(\Phi_{0}/2\), the QD is energetically stabilized by removing a quasiparticle with charge \(e/3\). Comparing Eqs. (S3) and (S2), one gets Eq. (3).

We note that in the absence of disorders \((W = 0)\) and interaction \((v = 0)\) between the two modes, the Hamiltonian \(H^{0}_{D}\) of the zero-mode parts is obtained, by putting Eq. (S3) into Eq. (2) in the main text, as

\[
H^{0}_{D} = \frac{\pi \hbar v_{c}}{6L} \left( N_{c} + 1 - 2\frac{\Phi}{\Phi_{0}} \right)^{2} + \frac{\pi \hbar v_{n}}{2L} N_{n}^{2} - \frac{E_{c}}{12} \left( N_{c} + 1 - 2\frac{\Phi}{\Phi_{0}} \right)^{2} + \frac{E_{n}}{4} N_{n}^{2}, \quad E_{c/n} \equiv \frac{2\pi \hbar v_{c/n}}{L}.
\]

For later use, we derive \(H^{0}_{D}\) in the regime of strong disorder and no interaction \((v = 0)\). As in the main text, the QD Hamiltonian \(H^{0}_{D}\) is written \([4]\), using an extra free bosonic field \(\chi\), as \(H^{0}_{D} = \frac{\hbar^{2} v_{c}^{2}}{4\pi} \int_{0}^{L} dx \langle \partial_{x} \phi_{c} \rangle^{2} + i\hbar v_{n} \int_{0}^{L} dx \hat{\psi}^\dagger \hat{\partial}_{x} \hat{\psi}(x)\rangle\) via the unitary pseudo-spin rotation \(U(x) = T_{c} \exp[-i \int_{0}^{x} dx' \left( \xi(x') \sigma_{+}^{c} + \xi^{*}(x') \sigma_{-}^{c} \right) / v_{n}]\) by disorder scattering, \(\sigma_{\pm} = \sigma_{x} \pm i \sigma_{y}\), \(\sigma_{x}\) and \(\sigma_{y}\) are the Pauli matrices, and \(T_{c}\) represents the \(x\)-ordering operator. Re-bosonizing \(\hat{\psi}\) as \(\exp[i(\hat{x} + \hat{\phi}_{n})/\sqrt{2}]\) and defining the zero-mode parts as \(\hat{\phi}_{n}^{0} = -2\pi \hat{N}_{c}x/(\sqrt{2}L) - \sqrt{2} \lambda_{n}\) and \(\chi^{0} = -2\pi \hat{N}_{x}/(\sqrt{2}L) - \sqrt{2} \lambda_{X}\), \(H^{0}_{D}\) is written as

\[
H^{0}_{D} = \frac{\pi \hbar v_{c}}{6L} \left( N_{c} + 1 - 2\frac{\Phi}{\Phi_{0}} \right)^{2} + \frac{\pi \hbar v_{n}}{2L} \hat{N}_{n}^{2} + \frac{\pi \hbar v_{n}}{2L} \hat{N}_{n}^{2}. \tag{S5}
\]

\(\hat{N}_{n} (\hat{N}_{X})\) is the number operator for the quasiparticles involved in \(\hat{\phi}_{n} (\hat{\chi})\), and \(\lambda_{c/n}(\hat{\phi}_{n}(\hat{\chi})\rangle\) satisfies \([\lambda_{c/n}(\hat{\phi}_{n}(\hat{\chi})\rangle = i\).

c. Derivation of differential conductance \(G\) in Eq. (5)

We derive \(G\) in Eq. (5). The electron current \(I_{R}\) along the right lead edge is given by \(I_{R} = e \frac{d(n_{R})}{dt} = \frac{i}{\hbar} \langle [H_{T}, n_{R}]\rangle\),

\[
I_{R} = -\frac{ie}{\hbar} \sum_{i,j=\pm} \left\langle T_{t} \exp \left( \frac{i}{\hbar} \int_{-\infty}^{0} dt H_{T}(t) \right) \left[ t_{Rij} \hat{\Psi}^{\dagger}_{R_{i},t}(0,t') = 0 \right] \hat{\Psi}_{j}(x_{R},t'=0) - H.c.] T_{t} \exp \left( -\frac{i}{\hbar} \int_{-\infty}^{0} dt H_{T}(t) \right) \right\rangle.
\]

\(n_{R}\) counts electron number in the right lead edge, \(T_{t}\) is the time ordering, the operators with (without) caret are in the interaction (Heisenberg) picture, and \((\cdot)\) is the thermal average. We set \(t = 0\) at which \(I_{R}\) is measured. To the second order in \(t_{Rij}\), \(I_{R}\) is calculated as \(I_{R}^{(2)} = \langle e/\hbar^{2} \sum_{i,j=\pm} \int_{-\infty}^{0} dt \text{Re} \left[ \exp \left( -\frac{i}{\hbar} (\mu_{R} - \mu_{D})t \right) \left( G_{D_{ij}}^{K_{R,i}}(-t)(G_{R_{i},i}^{R_{i},i} - G_{A_{i},i}^{R_{i},i})(t) - (G_{D_{ij}}^{R_{i},i} - G_{A_{i},i}^{R_{i},i})(-t)G_{D_{ji}}^{K_{R,i}}(t) \right) \right] \rangle\),

\[
I_{R}^{(2)} = \frac{e}{\hbar^{2}} \sum_{i,j=\pm} \left| t_{Rij} \right|^{2} \int_{-\infty}^{0} dt \text{Re} \left[ \exp \left( -\frac{i}{\hbar} (\mu_{R} - \mu_{D})t \right) \left( G_{D_{ij}}^{K_{R,i}}(-t)(G_{R_{i},i}^{R_{i},i} - G_{A_{i},i}^{R_{i},i})(t) - (G_{D_{ij}}^{R_{i},i} - G_{A_{i},i}^{R_{i},i})(-t)G_{D_{ji}}^{K_{R,i}}(t) \right) \right] \rangle.
\]
\[ G^R_{\alpha i}(t) \] and \[ G^A_{\alpha i}(t) \] are the retarded, advanced, and Keldysh Green's functions of lead edge \( \alpha = \mathcal{L}, \mathcal{R} \) (QD), satisfying \((G^R_{\alpha i} - G^A_{\alpha i})(t) = -i\langle \{ \hat{\Psi}_{\alpha i}^+, 0(t), \hat{\Psi}_{\alpha i}^+, 0(0) \} \rangle\), \[ G^K_{\alpha i}(t) = -i\langle \{ \hat{\Psi}_{\alpha i}^+, 0(t), \hat{\Psi}_{\alpha i}^+, 0(0) \} \rangle, \] and \[ (G^R_{\alpha i} - G^A_{\alpha i})(t) = -i\langle \{ \hat{\Psi}_{\alpha i}^+ (x, t), \hat{\Psi}_{\alpha i}^+ (x, 0) \} \rangle, \] \( \mu_0 \) (\( \mu_D \)) is the chemical potential for lead edge \( \alpha \) (the QD). \( \mu_D \) is assumed to be uniform over the entire region of the QD, which is valid in the linear response regime. The expression \( I^{(2)}_\mathcal{L} \) of electron current in the left lead edge is similar to that of \( I^{(2)}_\mathcal{R} \). To the second order in the tunneling strengths, the current \( I \) through the QD is written as \( I = -I^{(2)}_\mathcal{L} = I^{(2)}_\mathcal{R} \).

Applying the current conservation of \( (I^{(2)}_\mathcal{R}) + (I^{(2)}_\mathcal{L}) = 0 \), we write the symmetrized form of \( I \) as \( \frac{\gamma_\mathcal{L}\gamma_\mathcal{R}}{\gamma_\mathcal{L} + \gamma_\mathcal{R}} \), as

\[
I = \frac{e}{4\hbar} q^{3/4}v_n^{1/4} \left[ \frac{\gamma_\mathcal{L}\gamma_\mathcal{R}}{\gamma_\mathcal{L} + \gamma_\mathcal{R}} \right] \sum_{i,j} \int_{-\infty}^{0} dt \text{Re} \left[ \left( e^{-\frac{\lambda}{\hbar}(\mu_R - \mu_D)t} - e^{-\frac{\lambda}{\hbar}(\mu_L - \mu_D)t} \right) \right. \\
\times \left. \left( G_{\mathcal{D},j}(t) (G_{\mathcal{R},i} - G_{\mathcal{A},i})(t) - (G_{\mathcal{R},j} - G_{\mathcal{A},j})(t) G_{\mathcal{D},i}(t) \right) \right].
\]

where \( \gamma_\mathcal{L}/\mathcal{R} = \sum_{i,j} |t_{\mathcal{L}/\mathcal{R},ij}|^2/(\hbar \varepsilon_n^{3/4} v_n^{1/4}) \). We used the simplification of \( H_{\mathcal{L}} = H_{\mathcal{R}} \) (symmetric lead edges), namely \( G^R_{\alpha i} = G^R_{\alpha,i}; G^A_{\alpha i} = G^A_{\alpha,i}; \) and \( G^K_{\alpha i} = G^K_{\alpha,i} \). We also used the fact that the Green's functions are independent of the index \( \mathcal{J} \) of the electron field operators in \( H_T \). The differential conductance \( G = \frac{dI}{dV}|_{V \rightarrow 0} \) is written as

\[
G = -\frac{e^2}{8\hbar^2} \left[ \int \gamma_\mathcal{L}/\mathcal{R} \sum_{i,j} \sum_{n=1}^{\infty} dt \text{Im} \left[ i \left( G_{\mathcal{K},j}(t) (G_{\mathcal{K},i} - G_{\mathcal{A},i})(t) - (G_{\mathcal{K},j} - G_{\mathcal{A},j})(t) G_{\mathcal{K},i}(t) \right) \right].
\]

where \( eV \equiv \mu_L - \mu_R \). The Green's functions for the lead edges are computed as

\[
(G^R_{\alpha i} - G^A_{\alpha i})(t) = -\frac{i}{\pi a} \text{Re} \left[ \left( \frac{\sinh \left( \frac{\pi a t \bar{B}_n}{\hbar c} \right)}{\sinh \left( \frac{\pi a t}{\hbar c} \right)} \right)^{\frac{1}{2}} \left( \frac{\sinh \left( \frac{\pi a t \bar{B}_n}{\hbar c} \right)}{\sinh \left( \frac{\pi a t}{\hbar c} \right)} \right)^{\frac{1}{2}} \right].
\]

Since \( (G^R_{\alpha i} - G^A_{\alpha i})(t) \gg G_{\mathcal{K},i}(t) \) at \( |t| > a/v_n \), and since the processes of \( |t| < a/v_n \) do not contribute to the interference in \( G \), we can ignore \( G_{\mathcal{K},i}(t) \) in the expression of \( G \). Then, one derives Eq. (5) in the main text,

\[
G = -\frac{e^2}{4\hbar^2} \left[ \sum_{i,j} \sum_{n=1}^{\infty} dt F(t) \text{Im} G_{j}(x_a, x_a, t) = \frac{e^2 c_g k_B T}{\hbar^2} \sum_{i,j} \sum_{n=1}^{\infty} dt F(t) \text{Im} G_{j}(x_a, x_a, t).
\]

Here, \( c_g = a^2/(4\hbar^2 v_n^{3/4} v_n^{1/4}) \), \( F(t) = (\pi k_B T/\hbar) \sinh^{-2}(\pi k_B T/\hbar) \) is obtained from \( t(G^R_{\alpha i} - G^A_{\alpha i})(t) \) by \( \pi a t(k_B T/\hbar) v_n^{-3/2} v_n^{-1/2} \sinh^{-2}(\pi k_B T/\hbar) \), and \( G_{\mathcal{K},i}(x_a, x_a, t) \equiv \left( \hat{\Psi}_i^+ (x_a, t), \hat{\Psi}_i (x_a, 0) \right) \) is a Green’s function of the QD; its starting position \( x_a \) is the same with the ending one in the sequential tunneling regime. The weight factor \( F(t) \) decays rapidly as \( e^{-2\pi k_B T/\hbar} r / t \) for \( t > \hbar / k_B T \), describing the plasmonic dephasing (by partial overlap due to different positions of the neutral components between the interfering paths) occurring at lead edge \( \alpha \).

Below, we further compute \( G \) in the case of \( v = 0 \), where \( G_{\mathcal{L}} \) is decomposed into the charge and neutral components,

\[
G = \frac{e^2}{\hbar^2} \left[ \frac{\pi a k_B T}{\hbar^2 v_n^{3/4} v_n^{1/4}} \right] \sum_{i,j} \sum_{n=1}^{\infty} dt F(t) \text{Im} \left[ G_{c}(t) G_{n}(t) e^{-\pi i v_n t / 2 L} e^{-\pi i v_n t / 2 L} \right] \text{Re} G_{0}(t)
\]

where the plasmonic parts \( G_{c}(t) \) and \( G_{n}(t) \) of the charge and neutral modes and the zero-mode part \( G_{0}(t) \) are

\[
G_{c}(t) = \langle e^{i \sqrt{2} \phi_0} e^{-i \sqrt{2} \phi_0} e^{-i \pi v_c t / L} \left( \theta_1 \left( \frac{-i \pi v_c t}{L}, e^{-\gamma} \right) \right)^{\frac{1}{2}} \theta_1 \left( \frac{i \pi v_c t}{L}, e^{-\gamma} \right),
\]

\[
G_{n}(t) = \langle e^{i \sqrt{2} \phi_0} e^{-i \sqrt{2} \phi_0} e^{-i \pi v_c t / L} \left( \theta_1 \left( \frac{-i \pi v_c t}{L}, e^{-\gamma} \right) \right)^{\frac{1}{2}} \theta_1 \left( \frac{i \pi v_c t}{L}, e^{-\gamma} \right),
\]

\[
G_{0}(t) = \langle e^{i \pi v_c t / 2 \Phi_0 / L} e^{-i \pi N v_n t / L} \rangle.
\]

The elliptic-theta function of the first kind is \( \theta_1(0, z) = 2q^{1/4} \sum_{n=1}^{\infty} (1 - 2q^{2n} \cos(2z)) \left( 1 - q^{n} \right) \), and \( \gamma \equiv \pi \hbar v_c / k_B T L \). In the derivation of Eq. (5), we used the relations of \( G_{\mathcal{L}/\mathcal{R}}(x_a, x_a, t) = G_{\mathcal{L}/\mathcal{R}}(x_a, x_a, t) \), \( G_{c}(t) = G_{c}(t) \), and \( F(t) = -F(-t) \). The zero-mode part \( G_{0}(t) \) will be calculated in the next section.
d. Topological dephasing

We first sketch the topological dephasing in the case of \( v_c \gg v_n \) and no disorder, and derive it, by expanding the zero-mode contribution to \( \mathcal{G} \) in harmonics of the winding numbers. The discussion is valid even with strong disorder.

As in the main text, we consider the interference between the processes in Figs. 2(a) and 2(c). In the semiclassical regime of \( L \gg L_{T,c/n} \), counting the winding numbers \( p \) and \( q \) of the center of the spatial distributions of the charge and neutral components, we find that this interference contributes mainly to \( (\delta p, \delta q) = (1, 0) \), and gains the net phase of \( \theta = \pi (N_c - 2\Phi/\Phi_0) \delta p \) from those windings that braid with \( N_c \) charge excitations and \( N_n \) neutral excitations. At lower temperature of \( L \approx L_{T,c/n} \), the tails (namely the spatial width \( L_{T,c/n} \)) of the spatial distributions of the components are non-negligible, and imply that the two processes can also contribute to quantum mechanical net windings \((\delta p_{q_m}, \delta q_{q_m})\) that can differ from \((\delta p, \delta q)\). To see the contribution to different windings, we expand the average of \( \langle e^{i\theta} \rangle_{kBT} \) over the thermal fluctuations of \( N_c \) and \( N_n \) in the harmonics of \((\delta p_{q_m}, \delta q_{q_m})\),

\[
\langle e^{i\theta} \rangle_{kBT} = \sum_{\delta p_{q_m}, \delta q_{q_m} \in \mathbb{Z}} f(\delta p_{q_m}, \delta q_{q_m}, kBT) \exp(2\pi i \delta p_{q_m} \Phi/\Phi_0) \exp(2\pi i \delta q_{q_m} \varphi_n),
\]

where we introduce a fictitious "neutral flux" \( \varphi_n \) in the mathematical analogy of \( \Phi/\Phi_0 \) in order to have the expansion; \( \theta \) is now generalized to \( \theta = \pi (N_c - 2\Phi/\Phi_0) \delta p + \pi (N_n - 2\varphi_n) \delta q \), and we put \( \varphi_n \rightarrow 0 \) at the end. The thermal fluctuations of \( N_c \) and \( N_n \) are governed by the QD-energy \( H_D = E_J(N_c - 2\Phi/\Phi_0 + 1)^2/12 + E_n(N_n - 2\varphi_n)^2/4 \) (cf. Eq. (S4)). Notice \( H_D^0(\Phi/\Phi_0, \varphi_n) |_{N_c, N_n} = H_D^0(\Phi/\Phi_0 + 1, \varphi_n + 1) |_{N_c + 2, N_n + 2} \) and \( \theta(\Phi/\Phi_0, \varphi_n) |_{N_c, N_n} = \theta(\Phi/\Phi_0 + 1) |_{N_c + 2, N_n + 2} \), meaning that \( \theta \) and \( H_D^0 \) are restored by changing \( N_c \) and \( N_n \) by 2, when each flux shifts by one as \( \Phi/\Phi_0 \rightarrow \Phi/\Phi_0 + 1 \) and \( \varphi_n \rightarrow \varphi_n + 1 \).

We decompose the amplitude \( f(\delta p_{q_m}, \delta q_{q_m}, kBT) = f_c(\delta p_{q_m}, \delta q_{q_m}, kBT) + f_o(\delta p_{q_m}, \delta q_{q_m}, kBT) \) into the average \( f_c \) over even \( N_c \) and \( N_n \) and that \( f_o \) over odd \( N_c \) or odd \( N_n \); \( N_c \) and \( N_n \) should have the same parity, according to Eq. (3) in the main text. We find a useful relation of \( f_o(\delta p_{q_m}, \delta q_{q_m}, kBT) = (-1)^{\delta p_{q_m} + \delta q_{q_m}} f_c(\delta p_{q_m}, \delta q_{q_m}, kBT) \), obtained from the fact that the thermal average of \( \langle e^{i\theta} \rangle_{kBT} \) over odd \( N_c \) and odd \( N_n \) at \( \Phi/\Phi_0, \varphi_n \) is identical to that over even \( N_c \) and even \( N_n \) at \( (\Phi/\Phi_0 + 1/2, \varphi_n + 1/2) \), according to \( H_D^0 \). This relation leads to \( f(\delta p_{q_m}, \delta q_{q_m}, kBT) = 0 \) for odd \( \delta p_{q_m} + \delta q_{q_m} \) describing the topological dephasing. For the interference between those in Figs. 2(a) and 2(c), the contribution from the semiclassical winding of \((\delta p, \delta q) = (\delta p_{q_m}, \delta q_{q_m}) = (1, 0)\) vanishes (independent of temperature \( T! \)), while \( G_{\delta p=1} \) is contributed dominantly from the quantum mechanical winding of \((\delta p_{q_m}, \delta q_{q_m}) = (1, -1) \) of the tail.

We confirm the above discussion mathematically. The flux dependence of \( \mathcal{G} \) in Eq. (1) comes from the zero-mode part of the electron field operator \( \Psi_{\pm} \), hence, from the Green’s function \( G^0 \) in Eq. (S8). Using Eq. (S4),

\[
G^0(t) = \left[ \sum_{n_c,n_n=-\infty}^{\infty} \exp \left( -\frac{E_J}{4} \left( 2n_c + 1 - \frac{2\Phi}{\Phi_0} + 1 \right)^2 + \frac{E_n}{4} (2n_n + 1)^2 \right) \right] \left[ \sum_{n_c,n_n=-\infty}^{\infty} \exp \left( -\frac{E_J}{4} \left( 2n_c + 1 - \frac{2\Phi}{\Phi_0} + 1 \right)^2 + \frac{E_n}{4} (2n_n + 1)^2 \right) \right] 
\]

\[
\times \exp \left( -\frac{E_J}{4} \left( 2n_c + 1 - \frac{2\Phi}{\Phi_0} + 1 \right)^2 + \frac{E_n}{4} (2n_n + 1)^2 \right) + \exp \left( -\frac{E_J}{4} \left( 2n_c + 1 - \frac{2\Phi}{\Phi_0} + 1 \right)^2 + \frac{E_n}{4} (2n_n + 1)^2 \right) \right]
\]

\[
\times \exp \left( -\frac{E_J}{4} \left( 2n_c + 1 - \frac{2\Phi}{\Phi_0} + 1 \right)^2 + \frac{E_n}{4} (2n_n + 1)^2 \right)
\]

The first term of Eq. (S9) comes from odd integers \( N_c = 2n_c + 1 \) and \( N_n = 2n_n + 1 \), and the second from even integers \( N_c = 2n_c \) and \( N_n = 2n_n \). Utilizing the Poisson summation formula of \( \sum_{n=-\infty}^{\infty} \exp(-a(n + \delta)^2) \exp\left(-\frac{2\pi i (n + \delta^2)}{a}\right) \) with real constants, \( a, b, \) and \( \delta \), we obtain

\[
G^0(t) = \sum_{\delta p', \delta q' = -\infty}^{\infty} \left( -1 \right)^{\delta p' + \delta q'} \exp(2\pi i \delta p' \Phi/\Phi_0) \exp \left[ -\frac{3\pi^2 k_B T}{E_n} \left( \frac{eL}{v} - \delta p' \right)^2 \right] \exp \left[ -\frac{3\pi^2 k_B T}{E_n} \left( \frac{eL}{v} + \delta q' \right)^2 \right] \exp \left[ -\frac{3\pi^2 k_B T}{E_n} \left( \frac{eL}{v} + \delta q' \right)^2 \right] \exp \left[ -\frac{3\pi^2 k_B T}{E_n} \left( \frac{eL}{v} + \delta q' \right)^2 \right]
\]

\[
G^0(t) = \sum_{\delta p', \delta q' = -\infty}^{\infty} \left( -1 \right)^{\delta p' + \delta q'} \exp(2\pi i \delta p' \Phi/\Phi_0) \exp \left[ -\frac{3\pi^2 k_B T}{E_n} \left( \frac{eL}{v} - \delta p' \right)^2 \right] \exp \left[ -\frac{3\pi^2 k_B T}{E_n} \left( \frac{eL}{v} + \delta q' \right)^2 \right] \exp \left[ -\frac{3\pi^2 k_B T}{E_n} \left( \frac{eL}{v} + \delta q' \right)^2 \right]
\]

\[
\sum_{\delta p', \delta q' \in 2\mathbb{Z}} \left( -1 \right)^{\delta p' + \delta q'} \exp(2\pi i \delta p' \Phi/\Phi_0) \exp \left[ -\frac{3\pi^2 k_B T}{E_n} \left( \frac{eL}{v} - \delta p' \right)^2 \right] \exp \left[ -\frac{3\pi^2 k_B T}{E_n} \left( \frac{eL}{v} + \delta q' \right)^2 \right] \exp \left[ -\frac{3\pi^2 k_B T}{E_n} \left( \frac{eL}{v} + \delta q' \right)^2 \right] \exp \left[ -\frac{3\pi^2 k_B T}{E_n} \left( \frac{eL}{v} + \delta q' \right)^2 \right]
\]

(S10)
Here $2\mathbb{Z}$ is the set of even integers. The denominator is approximated as 1 at high temperature $k_B T \gg \hbar v_c/L$. Then $G_\delta(t)$ is written by the harmonics of winding numbers $\delta p' \to \delta p_{\text{qm}}$ and $\delta q' \to \delta q_{\text{qm}}$.

$$G_\delta(t) \simeq \sum_{\delta p_{\text{qm}} + \delta q_{\text{qm}} \in 2\mathbb{Z}} (-1)^{\delta p_{\text{qm}}} \exp(2\pi i \delta p_{\text{qm}} \Phi_0) \exp \left[ -\frac{3\pi^2 k_B T}{E_c} \left( \frac{v_c t}{L} - \delta p_{\text{qm}} \right)^2 \right] \exp \left[ -\frac{\pi^2 k_B T}{E_n} \left( \frac{v_n t}{L} + \delta q_{\text{qm}} \right)^2 \right].$$ (S11)

Notice that the windings of odd $\delta p_{\text{qm}} + \delta q_{\text{qm}}$ do not contribute to $G_\delta(t)$, as expected before.

We also show that Eq. (S10) is valid even with strong disorder. In this regime, the Green’s function $\langle \hat{V}^\dagger(x_L,t) \hat{V}^{}(x_L,0) \rangle = \left\langle e^{-i\sqrt{\frac{2}{L}} \phi_0(x_L,t)} e^{i\sqrt{\frac{2}{L}} \phi_0(x_L,0)} e^{-i\sqrt{\frac{2}{L}} \phi_0(x_L,0)} \right\rangle$ of the QD is calculated as

$$\langle \hat{V}^\dagger(x_L,t) \hat{V}^{}(x_L,0) \rangle = G_c(t) G_n(t) \left\langle e^{-i\sqrt{\frac{2}{L}} \phi_0(x_L,t)} e^{i\sqrt{\frac{2}{L}} \phi_0(x_L,0)} e^{-i\sqrt{\frac{2}{L}} \phi_0(x_L,0)} \right\rangle = G_c(t) G_n(t) e^{-3\pi i v_c t / 2L} \left\langle e^{i\pi v_c t \left( N_n - 2k_F / \Phi_0 + 1 \right) / L} e^{-i\pi v_n t N_n / L} \right\rangle.$$ (S12)

Here, we used $\hat{V}(x_L) = \Psi(x_L)$, hence $\exp(i\phi_0(x_L)/\sqrt{2}) = \exp(i\phi_0(x_L)/\sqrt{2})$, which can be chosen with the global gauge transformation making $U(x_L) = 1$. $G_c(t)$ and $G_n(t)$ are the plasmonic-part Green’s functions and have the same expression as Eq. (S8). The occupation number operator $\hat{N}_n$ of $\phi_0$ is also an integer and has the same set of values as $N_n$. Therefore, the expression of $G^0$ in the strong disorder regime is identical to Eq. (S10).

e. Derivation of Eqs. (6) and (7): Semiclassical approximation

We compute the analytic expression of $G_{\delta p}$ for the two cases $v_c \gtrsim v_n$ (Eq. (6)) and $v_c \gg v_n$ (Eq. (7)), utilizing a semiclassical approximation such that the time $t$ in the integrand of Eq. (S7) is replaced by $\delta p L/v_c$ except for the time argument $t$ in $G_c(t)$. The approximation is based on the fact that the dominant contribution to the integrand of Eq. (S7) comes from a peak structure of $G_c(t)$ near $t = \delta p L/v_c$; the other peaks from $G_n(t)$ near $t = \delta q L/v_n$ is much more monotonous and less important because the scaling dimension ($\delta = 1/4$) of the neutral component in the electron tunneling operator is smaller than that ($\delta = 3/4$) of the charge component. This approximation is applicable when (i) the spatial distance between two interfering neutral components in the QD at $t = \delta q L/v_n$ is much larger than the width $L_{T,n} \propto \hbar v_n/k_B T$ of the neutral components (then $G_n(t)$ is sufficiently monotonous), and (ii) $L \gg \hbar v_c/k_B T$ (then $F(t)$, coming from the lead edge, is sufficiently monotonous). We focus on the contribution from near $t = \pm \pm L/v_c$ and near $\pm 2L/v_c$, since that from larger times $t = \delta p L/v_c$, of $|\delta p| > 2$ is much more smaller due to more dephasing.

We first compute $G_{\delta p = 1}$ in the $v_c \gtrsim v_n$ case with the semiclassical conditions of $k_B T \gg \hbar v_c/L$ and $k_B T \gg \hbar v_n/(L - \Delta L)$, where $\Delta L \equiv L v_n/v_c$. In Eq. (S7), the main contribution occurs at $t = \pm L/v_c$. Near $t = L/v_c$, we use the following approximations: (1) For the portion of $(\delta p_{\text{qm}}, \delta q_{\text{qm}}) = (1, -1)$ in the zero-mode part, $\text{Re}[G^0] \simeq -\cos(2\pi \Phi_0) \exp(-(L - \Delta L)^2/(L L_{T,n}))$; cf. Eq. (S8). (2) For the lead edge part, $F \simeq 4\pi k_B T L \exp(-2\pi k_B T L / \hbar v_c) / \hbar v_c = (8L/3L_{T,c}) \exp(-L/L_{T,c})$. (3) For the QD plasmon part, $G_c(t) G_n(t) e^{-3\pi i v_c t / 2L} \left\langle e^{i\pi v_c t \left( N_n - 2k_F / \Phi_0 + 1 \right) / L} e^{-i\pi v_n t N_n / L} \right\rangle$.

We have used $\theta_t(u, \exp(-\gamma)) \simeq 2(-1)^n \sqrt{\pi / \gamma} \exp[-(u - n \pi x^2) / \gamma] \exp(-\pi^2 / 4 \gamma) \sinh(\pi(u - n \pi) / \gamma)$ for $\gamma \ll 1$.

We compute Eq. (S7), merging together (1)-(3), to get Eq. (6) in the main text,

$$G_{\delta p = 1} \simeq -g_0 \gamma L (k_B T)^3 \exp(-L/L_{T,c}) \exp(-L - \Delta L/L_{T,n}) \exp(-L + L_{T,n}).$$ (S14)

Here $g_0 = 16e^2 \sqrt{2\pi \pi (a / \hbar v_c)^{13/4} (a / \hbar v_n)^{3/4}} (\Gamma(3/4))^2 / (ah)$ is a constant, $\Gamma$ is the Gamma function, and we have used the integral formula of

$$\int_{-\infty}^{\infty} dt \left( \frac{\sinh(\frac{\pi v_n t}{L})}{\sinh(\frac{\pi k_B T}{L} (1 - v_c t))} \right)^2 = \frac{2a}{v_c} \frac{2ak_B T}{\hbar v_c} \frac{1}{\Gamma(\frac{3}{4})^2}.$$ (S15)

Notice that the factor $\exp(-(L - \Delta L)^2/(L L_{T,n}))$ of $\text{Re}[G^0]$ (zero-mode part) exactly cancels out $\exp((L - \Delta L)^2/(L L_{T,n}))$ from $G_n(t)$ (the plasmonic part). This fact was found in a Luttinger liquid with finite size [2].
We next move to $G_{\delta p=1}$ in the $v_c \gg v_n$ case. The main contribution to $G_{\delta p=1}$ also occurs near $t = \pm L/v_c$. We observe the followings. (1) Because the portion of $(\delta p_{qm}, \delta q_{qm}) = (1, 0)$ in $\text{Re}[G^0]$ fully vanishes, the dominant contribution comes from the portion of $(\delta p_{qm}, \delta q_{qm}) = (1, -1)$, which leads to $\text{Re}[G^0] \approx \cos(2\pi\Phi_0) \exp(- (L - \Delta L)^2/(LL_{T,n}))$. (2) $F \approx 4\pi k_B T L \exp(-2\pi k_B T L/h v_c)/(h v_c) = (8/3L_{T,c}) \exp(-L/L_{T,c}) \exp(-\Delta L/L_{T,n})$. (3) For the QD plasmon part, the same expression is obtained as Eq. (S13), except that $L - \Delta L$ is replaced by $\Delta L$. Merging (1)-(3) and using Eq. (S15), we obtain Eq. (S16) in the main text,

$$
G_{\delta p=1} \approx -g_0 \gamma L (k_B T)^3 \exp\left(-\frac{L}{L_{T,c}}\right) \exp\left(-\frac{\Delta L}{L_{T,n}}\right) \exp\left(-\frac{\Delta L}{L_{T,n}}\right) \exp\left[-\frac{(L - \Delta L)^2 - (\Delta L)^2}{LL_{T,n}}\right]
$$

$$
= -g_0 \gamma L (k_B T)^3 \exp\left(-\frac{L}{L_{T,c}}\right) \exp\left(-\frac{L}{L_{T,c}}\right).
$$

(S16)

In the same way, we obtain Eq. (S17),

$$
G_{\delta p=2} \approx 2g_0 \gamma L (k_B T)^3 \exp\left(-\frac{2L}{L_{T,c}}\right) \exp\left(-\frac{2\Delta L}{L_{T,n}}\right) \exp\left(-\frac{2\Delta L}{L_{T,n}}\right) = 2g_0 \gamma L (k_B T)^3 \exp\left(-\frac{10L}{3L_{T,c}}\right).
$$

(S17)

f. The quasiparticle fluctuation in the bulk

We consider quasiparticle fluctuations in the bulk, differentiating the QD bulk from the edge. In the limit of bulk charging energy smaller than temperature, we show that the zero-mode part $G^0$ has the same expression as Eq. (S10) in the presence of quasiparticle fluctuations in the bulk, hence, that the period halving ($\hbar/2\pi$ oscillation) takes place.

We argue that the zero-mode part of the QD states is characterized by four numbers $(N_c, N_n, N_{qp}, N_{qp,n})$, when we additionally consider the bulk degrees of freedom. $N_c$ and $N_n$ are necessary to describe excess electrons in the QD. For an additional excess electron, the number $N_c$ of excess electrons increases by 1. This electron is decomposed into 3 additional charge components $(N_c \rightarrow N_c + 3$; the charge of the charge component is $e/3)$ and $\pm 1$ additional neutral component $(N_n \rightarrow N_n \pm 1)$, as discussed in the main text. $N_c$ counts the number of the neutral components by excess electrons in the QD, as in the main text. On the other hand, $N_{qp}$ and $N_{qp,n}$ are introduced to describe quasiparticle excitations in the QD bulk. When an additional quasiparticle excites in the bulk, the number $N_{qp}$ of quasiparticle increases by 1. This quasiparticle is decomposed into 1 charge component and $\pm 1$ neutral component $(N_{qp,n} \rightarrow N_{qp,n} \pm 1)$. $N_{qp,n}$ counts the number of the neutral components by bulk quasiparticle excitations.

Taking into account quasiparticle fluctuations in the bulk and electron fluctuations inside the QD, the zero-mode part of the QD Hamiltonian $H_{D}^{0}$ is expressed as

$$
H_{D}^{0}(N_c, N_{qp}, N_n, N_{qp,n}) = \frac{E_c}{12} \left( 3N_c + 1 - \frac{2\Phi}{\Phi_0} - N_{qp} \right)^2 + \frac{E_n}{4} (N_n - N_{qp,n})^2 + E_{bc} \left( \frac{2\Phi}{\Phi_0} + N_{qp} \right)^2,
$$

(S18)

where $E_{bc}$ is the bulk charging energy. The first (second) term describes the interaction between the charge (neutral) components on the edge while the third term describes the interaction between the charge components in the bulk. The flux dependence in the bulk-charging energy term describes charge accumulation in the bulk as the magnetic flux increases. We ignore the interactions between the neutral components in the bulk because they are of dipole type hence weaker than the interaction terms of the total charge. We also assume that it is in the Aharonov-Bohm regime neglecting electrostatic coupling between quasiparticles in the bulk and on the edge.

We consider the case that the relaxation time from the QD edge to the bulk is much longer than the QD dwell time of an electron injected from a lead edge, hence, that the electron enters only into the QD edge. Then, the zero-mode part $\exp(i\sqrt{3}/2\phi_0^{(x)} \pm i\sqrt{1/2}\phi_0^{(n)})$ of the electron operators $\Psi_\pm(x)$ is evolved by Eq. (S18) as

$$
\exp\left(i\sqrt{\frac{3}{2}}\phi_0^{(x)}(x,t) \pm i\sqrt{\frac{1}{2}}\phi_0^{(n)}(x,t)\right) = \exp\left(iH_{D}^{0}(N_c, N_{qp}, N_n, N_{qp,n})\right) \exp\left(i\sqrt{\frac{3}{2}}\phi_0^{(x)}(x) \pm i\sqrt{\frac{1}{2}}\phi_0^{(n)}(x)\right) e^{-itH_{D}^{0}(N_c, N_{qp}, N_n, N_{qp,n})}
$$

$$
= \exp\left(i\sqrt{\frac{3}{2}}\phi_0^{(x)}(x) \pm i\sqrt{\frac{1}{2}}\phi_0^{(n)}(x)\right) e^{-it\left(H_{D}^{0}(N_c, N_{qp}, N_n, N_{qp,n}) - H_{D}^{0}(N_c - 1, N_{qp}, N_n - 1, N_{qp,n})\right)}
$$

$$
= \exp\left(i\sqrt{\frac{3}{2}}\phi_0^{(x)}(x) \pm i\sqrt{\frac{1}{2}}\phi_0^{(n)}(x) + \frac{i\pi v_n t}{L} (3N_c + 1 - \frac{2\Phi}{\Phi_0} - N_{qp}) \pm \frac{i\pi v_n t}{L} (N_n - N_{qp,n})\right).
$$

(S19)
And, the zero-mode part \(G^0(t)\) of the Green’s function \(\langle \hat{\Psi}_+^\dagger(x_L,t)\hat{\Psi}_+(x_L,0) \rangle\) is written as

\[
G^0 \propto \left\{ \exp \left[ \frac{i \pi v_c t}{L} \left( 3N_e - \frac{2\Phi}{\Phi_0} + 1 - N_{qp} \right) \right] \exp(\pm \frac{i \pi v_n t}{L}(N_n - N_{qp,n})) \right\}
\]

\[
= \sum_{\delta p_{qm}, \delta p', \delta q_{qm} = -\infty}^{\infty} e^{-i\pi \delta p'/3}(1 + (-1)^{\delta p_{qm} + \delta q_{qm}})(1 + (-1)^{\delta p' + \delta q_{qm}}) \exp(2\pi i \delta p_{qm} \Phi/\Phi_0)
\]

\[
\times \exp \left[ -\frac{3\pi^2 k_B T}{E_c} \left( \frac{v_c t}{L} + \frac{\delta p'}{3} \right)^2 \right] \exp \left[ -\frac{\pi^2 k_B T}{E_n} (\delta q_{qm} \pm \frac{v_n t}{L})^2 \right] \exp \left[ -\frac{\pi^2 k_B T}{4E_{bc}} (\delta p_{qm} - \frac{\delta p'}{3})^2 \right]
\]

\[
\left/ \left\{ \sum_{\delta p_{qm}, \delta p', \delta q_{qm} = -\infty}^{\infty} e^{-i\pi \delta p'/3}(1 + (-1)^{\delta p_{qm} + \delta q_{qm}})(1 + (-1)^{\delta p' + \delta q_{qm}}) \exp(2\pi i \delta p_{qm} \Phi/\Phi_0)
\right. \right\}
\]

\[
\times \exp \left[ -\frac{3\pi^2 k_B T}{E_c} \left( \frac{\delta p'}{3} \right)^2 \right] \exp \left[ -\frac{\pi^2 k_B T \delta q_{qm}^2}{E_n} \right] \exp \left[ -\frac{\pi^2 k_B T}{4E_{bc}} (\delta p_{qm} - \frac{\delta p'}{3})^2 \right].
\]  

We applied the Poisson summation formula as in Eq. (S10). At temperature much higher than the bulk-charging energy, the portion of \(\delta p' = 3\delta q_{qm}\) survives, resulting in the same expression as Eq. (S10).

### g. Confining potential and Coulomb interaction

When the edge potential is smooth enough, we below show that \(v_c\) is much larger than \(v_n\) and \(v\); see Eq. (2) in the main text. In the absence of Coulomb interaction, the edge Hamiltonian at \(\nu = 2/3\) has the form of \(H_{con} = \frac{1}{4\pi} \int dx[v_{1,con}(\partial_x \phi_1)^2 + 3v_{2,con}(\partial_x \phi_2)^2]\) in terms of the velocities of the original edge modes, \(v_{1,con}\) and \(v_{2,con}\), which are solely determined by the edge confining potential. When the Coulomb interaction is tuned on, it leads to an interaction Hamiltonian \(H_{int} = \frac{\mu_0}{4\pi} \int dx(\partial_x(\phi_1 + \phi_2))^2\), which counts the interaction by the total charge density \(\partial_x(\phi_1 + \phi_2)/(2\pi)\); here, we assume that \(v_{int}\) is independent of momentum, (which is valid when the Coulomb interaction is short ranged due to the screening by gates). Then, the total Hamiltonian is \(H = H_{con} + H_{int} = \frac{1}{4\pi} \int dx(v_c(\partial_x \phi_1)^2 + v_n(\partial_x \phi_n)^2 + v\partial_x \phi_c \partial_x \phi_n)\), where \(v_c = 3v_{1,con}^2/2 + v_{2,con}^2/2 + 2v_{int}/3, v_n = v_{1,con}/2 + 3v_{2,con}/2,\) and \(v = -\sqrt{3}(v_{1,con} + v_{2,con})\). This shows that \(v\) and \(v_n\) are much smaller than \(v_c\), if the edge potential is smooth enough such that \(v_{int} \gg v_{1,con}, v_{2,con}\). This will be responsible for renormalizing \(v\) toward 0 at low temperatures, where the plasmonic part of the neutral component becomes decaying and the plasmonic part of the charge component propagates diffusively [4].

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