Study of turbulent natural convection in a tall differentially heated cavity filled with either non-participating, participating grey and participating semigrey media

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Abstract. Turbulent natural convection in a tall differentially heated cavity of aspect ratio 5:1, filled with air under a Rayleigh number based on the height of $4.5 \times 10^{10}$, is studied numerically. Three different situations have been analysed. In the first one, the cavity is filled with a transparent medium. In the second one, the cavity is filled with a semigrey participating mixture of air and water vapour. In the last one the cavity contains a grey participating gas. The turbulent flow is described by means of Large Eddy Simulation (LES) using symmetry-preserving discretizations. Simulations are compared with experimental data available in the literature and with Direct Numerical Simulations (DNS). Surface and gas radiation have been simulated using the Discrete Ordinates Method (DOM). The influence of radiation on fluid flow behaviour has been analysed.

1. Introduction

Experimental measurements of a tall differentially heated cavity of aspect ratio 5:1 were conducted by Cheeswright et al. \cite{1}. Measured temperatures at floor and ceiling differ from adiabatic conditions. The authors attributed this fact to the heat losses through the adiabatic horizontal walls and through the vertical front and back walls.

Several numerical studies of the present case, using either low Reynolds number two-equation eddy-viscosity turbulent models \cite{2, 3, 4, 5} or LES \cite{6}, have been presented in the literature. In general, they had difficulties in reproducing experimental results, even using thermal resistance coefficients to reproduce the heat losses \cite{2}. Xin et al. \cite{7} studied experimentally and numerically turbulent natural convection inside a parallelepiped cavity (with square vertical cross section). They found that passive walls (not only horizontal, but also front and rear walls) were far from adiabatic and the coupling between convection and surface radiation must be considered. If gases contained inside the cavity also participate in radiation, both surface and gas radiation affect the fluid flow. In this case, the governing equations of the fluid motion are coupled to the integro-differential equation of radiative transport (RTE) through a new term, the divergence of radiative heat transfer. This term appears in the energy equation relating the net gain of energy due to radiation inside the fluid. The RTE represents an energy balance associated with
radiation, and depends not only on spatial position, but also on direction of propagation of radiation.

In [7] they studied the surface radiation effect on the fluid flow of a turbulent natural convection inside a square cavity. They did a balancing between convection and radiation at the front an rear walls and the balancing among surface radiation, convection in air and conduction in polyurethane foam at the top and bottom walls.

2. Mathematical Formulation

In the present work the turbulent flow is described by means of LES using symmetry-preserving discretizations [8]. The spatial filtered Navier-Stokes equations can be written as,

$$ \mathbf{M} \frac{\partial \mathbf{u}_c}{\partial t} + \mathbf{C}(\mathbf{u}_c) \mathbf{u}_c + \nu \mathbf{D} \mathbf{u}_c + \nabla \cdot \mathbf{f}_c = \mathbf{C}(\mathbf{u}_c) \mathbf{u}_c - \mathbf{C}(\mathbf{u}_c) \mathbf{u}_c \approx -\mathbf{M}_c \mathbf{T}_c $$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{G}$ are the divergence, convective, diffusive and gradient operators, respectively, $\Omega$ is a diagonal matrix with the sizes of control volumes, $\mathbf{u}_c$ is the filtered velocity, $\mathbf{f}_c$ is the body force term (in this paper the Boussinesq approximation for the density has been adopted), $\mathbf{M}_c$ represents the divergence operator of a tensor, and $\mathbf{T}_c$ is the SGS stress tensor, which is defined as,

$$ \mathbf{T}_c = -2\nu_{sgs} \mathbf{S}_c \approx (\mathbf{T}_c : I) I/3 $$

where $\mathbf{S}_c = \frac{1}{2} [\mathbf{G}(\mathbf{u}_c) + \mathbf{G}^*(\mathbf{u}_c)]$. To close the formulation, a suitable expression for the subgrid-scale viscosity, $\nu_{sgs}$, must be introduced. In the present work the variational multiscale method - WALE (VMS) SGS model is used, since it has shown good results in previous works [9] The main issues related to this model is given below.

The filtered temperature transport equation is

$$ \Omega_c \frac{\partial T_c}{\partial t} + \mathbf{C}(\mathbf{u}_c) T_c + \frac{\nu}{P_r} \mathbf{D} T_c - \nabla \cdot q_{rad} = \mathbf{C}(\mathbf{u}_c) T_c - \mathbf{C}(u_c) T_c \approx -\mathbf{M}_c T'_c $$

In the $T'_c$ term the diffusion term is approximated by $\nu_{sgs}/Pr_l$, where $Pr_l$ is the turbulent Prandtl (0.4 in this paper).

The $\nabla \cdot q_{rad}$ is obtained from the RTE. In the present study RTE is solved for an emitting-absorbing non-scattering grey medium:

$$ \frac{dI}{ds} = -\kappa I + \kappa I_b $$

The filtered RTE can be written as

$$ \frac{dT}{ds} = -\kappa T + \kappa T_b = -\kappa T - \left(\kappa T - \kappa T_b\right) + \kappa T_b \approx -\kappa T + \kappa T_b $$

The simplest way to close the filtered RTE is used, i.e. the terms in parenthesis are ignored and $T_b \approx I_b(T)$. The filtered radiative source term is approximated using the same criteria as in the filtered RTE,

$$ \nabla \cdot q_{rad} = 4\pi \kappa I_b - \int_{4\pi} \kappa I d\omega \approx 4\pi \pi I_b - \int_{4\pi} \pi I d\omega $$

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2.1. VMS, WALE model with a variational multiscale framework [10]
In VMS three classes of scales are considered: large, small and unresolved scales. The first two classes are solved with LES, whereas the unresolved scales are modeled. In this model, the subgrid small-scale term $T'_c$, is modeled instead of $T_c$,

$$T'_c = -2\nu_s S'_{ij} + \frac{1}{3} T_c \delta_{ij} \tag{8}$$

$$\nu_{sgs} = (C_{wms} \Delta)^2 \frac{(V'_i : V'_j) \delta_{ij}}{(S'_i : S'_j)^2 + (V'_i : V'_j)^2}$$

$$S'_{ij} = \frac{1}{2} [G(\overline{u'_c}) + G^*(\overline{w'_c})]$$

$$V'_{ij} = \frac{1}{2} [G(\overline{w'_c})^2 + G^*(\overline{u'_c})^2] - \frac{1}{3} (G(\overline{u'_c})^2 I)$$

where $C_{wms} = 0.325$ and is the equivalent of the WALE coefficient for the small-small VMS approach.

2.2. The Planck-mean absorption coefficient
The semigrey model is computed by means of the Planck mean absorption coefficient (see [11] for more details)

$$\kappa_P = \int_0^{\infty} \kappa_{\eta} I_{\eta \eta} d\eta = \frac{1}{I_k} \sum_i I_{\eta_i} \int_0^{\infty} \kappa_{\eta \eta} d\eta \tag{9}$$

$\kappa_{\eta \eta}$ values for water vapour over the range from 500K to 2500K have been obtained from the HITEMP database [12]. The absorption coefficient, $\kappa$, is then obtained as a function of temperature and the water vapour pressure inside the cavity. The latter is evaluated based on the relative humidity (HR).

3. Numerical method
Numerical results are carried out by using the CFD code Termofluids which is an intrinsic 3D parallel CFD object-oriented code applied to unstructured-collocated meshes [13]. Fully conservative second-order schemes for spatial discretization and third order explicit time integration are used. The pressure-velocity linkage is solved by means of an explicit finite volume fractional step procedure.

The generated domain is periodic in depth. Therefore the system of equations is reduced to a set of 2D systems by means of a Fourier diagonalization method. These systems are solved using a Direct Schur complement-based domain decomposition method in conjunction with a Fast Fourier Transform [14].

In the code, radiation can be simulated using either the Discrete Ordinates Method (DOM) [15, 16] or the Finite Volume Method (FVM) [17, 16]. In both methods the space is discretized by means of finite volumes and the difference lies on the angular discretization procedure. In DOM, the directional variation of the radiative intensity is represented by a discrete number of ordinates, and integrals over solid angles are approximated by numerical quadratures which are usually designed to preserve symmetry and satisfy several moments. The FVM ensures the conservation of radiative energy by means of integrating the RTE not only in finite volumes (spatial domain) but also in control angles (angular domain). In the present work, some results using DOM with a $S_6$ angular discretization and the step scheme, which is the analogous of the upwind scheme in CFD, are presented. The periodic boundary condition is performed by means of setting the entering intensity at one side of the periodic boundary, $I_w(x_i, y_i, z_i)$, the value of the outgoing intensity at the same location of the other side of the periodic boundary $I_w(x_i, y_i, z_o)$. 

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The discretized algebraic radiative transfer equations are solved using a parallel sweep solver [18]. Parallel computation is performed in a Gigabit Ethernet networked Beowulf cluster of PCs.

3.1. Test cases
The studied case corresponds to a cavity of aspect ratio 5:1 (width $L_x = 0.5$ and height $L_y = 2.5$) filled with air (Prandtl number evaluated at mean temperature of the isothermal walls is 0.7) and a Rayleigh number based on the height of $4.5 \cdot 10^{10}$. Three different cases have been considered:

**Case A:** In the first one the medium is considered transparent to the radiation and surface radiation is neglected.

**Case B:** In the second one the medium is participating in radiation, with a Planck number (defined as $Pr = \sigma T_o^4 / (\lambda DT_o / H)$) of 1253.8 and a temperature ratio ($\delta = DT_o / T_o$) of 0.1399. The four walls are considered black. The cavity is filled with a semi-grey ($\tau$ depends on the temperature, see section 2.2 for more details) moist air. Two different relative humidities have been studied: HR=6%, which corresponds to an optical thickness evaluated at the average temperature ($\tau(T_o) = \tau(T_o) H$) of 1.0, and HR=61%, which corresponds to $\tau(T_o) = 10$.

**Case C:** The third case is equal to the second one but considering a grey medium ($\tau$ is constant): $\tau = 1$ and $\tau = 10$ are analysed.

Barlaghi and Davidson [6] stated that a spanwise width of 0.372 was large enough to capture the required turbulent scales. The same value, a spanwise depth of $L_z = 0.2$, has also been chosen in this work.

4. Results
Different meshes have been tested [9] and it was concluded that the accuracy of the solution is mainly dependant on the quality of the mesh, not on the used turbulence model. The key aspect in the appropriate discretization of the present case is to highly discretize the direction of the buoyancy flow. This was attributed to the fact that LES models are deactivated in the laminar region. Then, it is necessary to use a high resolution mesh in this region in order to correctly solve it. Discretization in horizontal and periodic directions seem not being so decisive.

In the present work only results with a structured mesh of 55 x 340 x 8 CV’s ($1.50 \cdot 10^5$ CV’s) that was specially designed for the computation of the case where radiation is taken into account are presented. This mesh is concentrated near the horizontal walls in order to capture the steep gradients and avoid instabilities due to the radiation heat fluxes. In the 20% of the height located near the horizontal walls 80 CV’s are concentrated using an hyperbolic function. In the remaining 60% of the height, which is located in the central region, 180 CV’s are distributed uniformly.

The mesh is uniform in the periodic ($z$) direction and distributed using a hyperbolic function in the horizontal ($x$) direction. The distribution in the horizontal direction has been designed taking into account that the thickness of the thermal boundary layer predicted by Grö tzbach [19] and Patterson and Imberger [20] is $\delta_\theta = 5.15 \cdot 10^{-3}$ and $\delta_\theta = 5.43 \cdot 10^{-3}$, respectively. Three control volumes have been placed inside the smallest thermal boundary layer value.

DNS results were obtained with a mesh 318 x 862 x 128 CV’s ($3.51 \cdot 10^7$ CV’s). Numerical details of the DNS simulations are out of the scope of the present work and can be found in [21].

4.1. No Radiation (transparent medium)
Natural turbulent convection inside cavities include regions of laminar, transitional and turbulent flow. From a numerical point of view the prediction of the transitional point is a difficult task.

From an experimental point of view it is also very difficult to establish insulated boundaries. Cheesewright et al. [1] reported heat losses in their experiment that implied a destabilizing effect (negative temperature gradient at the wall) through the ceiling causing the boundary layer to be turbulent. This means that the vertical boundary layer is turbulent along all the cold wall. As heat losses at the bottom wall are smaller, flow stabilizes along that wall and becomes
laminar. Therefore, flow is laminar at the beginning of the hot wall until it becomes turbulent at approximately 20% of the height of the cavity. Traditionally numerical simulations have used perfect adiabatic boundary conditions and the aforementioned experimental transition point around 20% of the height as the reference value. However, since the flow is extremely dependent on the boundary conditions, the experimental value seems not very reliable. This is confirmed by the DNS simulation with perfect adiabatic boundary condition (see figure 1a), which predicts the transition at 70% of the height ($y = 1.7$). DNS results show a laminar flow zone up to $\approx 65\%$ of the height, a transition flow zone between $\approx 65\%$ and $\approx 76\%$ and after that a fully developed turbulent flow zone.

Figure 1. Case A. (a) Nusselt number along hot wall. (b) Dimensionless average horizontal velocity at mid-width of the cavity ($x/L_y = 0.1$), ($V_{ref} = \sqrt{g \beta L_y \Delta T}$). (c) Dimensionless time averaged isotherms with LES. (d) Dimensionless instantaneous turbulent viscosity.

In figure 1a can be observed that LES results exactly reproduce the laminar and the turbulent flow zones in the boundary layer and main discrepancies take place around the transition zone. The average Nusselt number obtained with LES differs 2.5% with respect to the DNS.

LES simulation reproduces DNS results, even the horizontal velocity profile at mid-width (see figure 1b), which is difficult to predict due to the fact that velocities are very small, are pretty similar to DNS results (see [9] for more results).

4.2. Radiation (participating grey medium)
Numerical simulations of coupled convection and radiation (both surface and gas) have been obtained starting with the initial map of the stationary results without radiation.

Figure 2 depicts the temperature isotherms map for the cases with a low and high percentage of humidity. If it is compared with figure 1c, where neither surface nor gas radiation are considered, it can be observed that in the radiative situation, isotherms become closer the top of the hot wall and nearer the bottom of the cold wall. This phenomenon is coupled with a downward shift of the isotherm for the average temperature (($T - T_c)/\Delta T = 0.5$) and a decrease in vertical gradient of the temperature. The latter effect can be also clearly observed in figure 4. These effects are enhanced as the humidity of the air is increased (the medium becomes more participant).

Although numerical results with radiation are closer to the experimental data than the results without radiation, the differences suggest that experimental temperature measurements in the top and bottom walls should be used instead of assuming adiabaticity hypothesis in order to reproduce experimental profiles.
An outstanding change is that the case becomes more turbulent (see figure 3) specially near the floor and the ceiling. This phenomenon is accompanied with higher horizontal velocities (see figure 4) implying an increase of the intensity flow. Maximum velocity near the top and bottom walls are not equal in the radiation case. In general, it can be stated that radiation breaks the symmetry of the flow.

Dimensionless time averaged temperature and vertical velocities at different heights are depicted in figures 5, 6 and 7. Radiation thickens the boundary layers formed along the isothermal surfaces. This effect becomes more noticeable as the medium is more participant.

Figures 4, 5, 6 and 7 show that there are almost no discrepancies between considering a semigrey model or considering a grey medium with an optical thickness evaluated at the average temperature.

5. Conclusions

Turbulent natural convection in a tall differentially heated cavity of aspect ratio 5:1 filled with air, with a Rayleigh number based on the height of $4.5 \cdot 10^{10}$, has been studied numerically...
with LES models. It has been found that the mesh quality highly affects the quality of the LES solution in the turbulent boundary layer due to the fact that the mesh is used as a passive filter in the physical space LES implementations. Comparison between DNS and experimental data confirm that experiment differ by far from adiabatic boundary conditions. It has also been analysed the effect of radiation in participating medium on the fluid flow. Radiation breaks the symmetry of the case, increases the intensity of the flow and reduces the dimensionless stratification. It has been stated that there are no noticeable differences between considering a semigrey model and a grey medium with an optical thickness evaluated at the average temperature.

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Figure 7. Dimensionless time averaged temperature and velocity profiles at $y/L_y = 0.05$. (Left): temperature. (Right): vertical velocity.

References

[1] Cheeswright R, King K and Ziai S 1986 Proceedings of Significant Questions in Buoyancy Affected Enclosure or Cavity Flows pp 75–81
[2] Ince N, Betts P and Launder B 1986 Proc. of Eurotherm Seminar 22 - Turb. nat. convect. in enclosures. A computational and experimental benchmark study ed Henkes R and Hoogendoorn C pp 76–87
[3] Henkes R and Hoogendoorn C 1986 Proc. of Eurotherm Seminar 22 - Turb. nat. convect. in enclosures. A computational and experimental benchmark study ed Henkes R and Hoogendoorn C pp 64–75
[4] Capdevila R, Perez-Segarra C D, Colomer G and Oliva A 2006 Proceedings of the V International Symposium on Turbulence, Heat and Mass Transfer pp 535–538
[5] Albets-Chico X, Oliva A and Perez-Segarra C D 2008 Journal of Heat Transfer 130 1–11
[6] Barhaghi D and Davidson L 2007 Physics of Fluids 19 125106
[7] Xin A, Salat J, Joubert P, Sergent A Le Quéré P and Penot F 2006 Proceedings of the 13th International Heat Transfer Conference (Sydney, Australia)
[8] Rodriguez I, Borrell R, Lehmkuhl O, Perez-Segarra C D and Oliva A 2011 Journal of Fluid Mechanics 679 263
[9] Capdevila R, Lehmkuhl O, Trias F X, Colomer G and Perez-Segarra C D 2011 Journal of Physics: Conference Series 318 042048
[10] Hughes T J R, Mazzei L and Hanzen K E 2000 Computing and Visualization in Science 3 47–59
[11] Zhang H and Modest M 2002 Journal of Quantitative Spectroscopy and Radiative Transfer 73 349–360
[12] Rothman L e a 1998 Journal of Quantitative Spectroscopy and Radiative Transfer 60 665–710
[13] Lehmkuhl O, Perez-Segarra C D, Borrell R, Soria M and Oliva A 2007 Proceedings of the Parallel CFD 2007 Conference pp 1–8
[14] Borrell R, Lehmkuhl O, Trias F X and Oliva A 2011 Journal of Computational Physics 230
[15] Fiveland W 1984 Journal of Heat Transfer - Transactions of ASME 106 699–706
[16] Capdevila R, Perez-Segarra C D and Oliva A 2010 Journal of Quantitative Spectroscopy and Radiative Transfer 111 264–273
[17] Raitby G and Chui E 1990 Journal of Heat Transfer 112 415–423
[18] Colomer G, Borrell R, Lehmkuhl O and Oliva A 2010 Proceedings of International Heat Transfer Conference 14, IHTC14 (Washington, DC, USA)
[19] Grötzbach G 1983 Journal of Computational Physics 49 241–264
[20] Patterson J and Imberger J 1980 Journal of Fluid Mechanics 100 65–86
[21] Trias F X, Gorobets A, Soria M and Oliva A 2010 International Journal of Heat and Mass Transfer 665–673