The string tension from smeared Wilson loops at large N

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Abstract

We present the results of a high statistics analysis of smeared Wilson loops in 4 dimensional SU(N) Yang-Mills theory for various values of N. The data is used to analyze the behaviour of smeared Creutz ratios, extracting from them the value of the string tension and other asymptotic parameters. A scaling analysis allows us to extrapolate to the continuum limit for N=3, 5, 6 and 8. The results are consistent with a $1/N^2$ approach towards the large N limit. The same analysis is done for the TEK model (one-point lattice) for N=841 and a non-minimal symmetric twist with flux of $k = 9$. The results match perfectly with the extrapolated large N values, confirming the validity of the reduction idea for this range of parameters.
There is considerable interest in gauge theories at large N for their simplicity, proximity to phenomenologically interesting field theories and their presumed connection to string theory. Lattice gauge theory has proved to be a fundamental tool in deriving the non-perturbative properties of Yang-Mills theories at small N. In approaching large N, the standard pathway is to study the theory at increasing values of N and to extrapolate the results to infinite N. This is, no doubt, a costly procedure, with the additional risks involved in any extrapolation procedure. Nevertheless, results point towards a somewhat fast approach to the large N limit in many of its observables [1]-[2]. An alternative would be to use the simplifications involved in the large N theory to find a way to simulate it directly. An idea going in this direction is that of reduction or volume independence [3]-[4]-[5]-[6]. This allows the possibility of trading the space-time degrees of freedom with those of the group. The essential ingredient for the idea to work is invariance under $Z^4(N)$ symmetry, which is broken in the original proposal [3]-[4]. In the twisted Eguchi-Kawai model (TEK) [5], introduced by the present authors, an invariance subgroup is preserved at sufficiently weak coupling, enabling reduction to work. Recently, it was reported in Ref. [7]-[8]-[9] that symmetry-breaking takes place at intermediate couplings and $N > 100$. To circumvent this problem we proposed a slight variation of the model [10]. It exploits the freedom associated with an integer parameter entering the formulation, and representing the chromomagnetic flux through each two-dimensional plane. Traditionally this parameter was kept fixed when taking the large N limit, while we advocated the need to scale it with $\sqrt{N}$ in order to avoid symmetry-breaking phase transitions. In practice, the modification involves no additional technical or computational cost. Our initial tests [10] were free from the problems reported earlier. To further test the validity of this idea demanded performing state of the art computations of the large N observables and comparing them with those obtained for the TEK model. Furthermore, even if reduction operates at the level of the lattice model, our ultimate goal is the continuum theory, so a scaling analysis is necessary. These were our original motivations for embarking in the present work.

Although other observables are possible, we have focused upon the string tension. This can be obtained as the slope of the linear quark-antiquark potential. Lately, the best determinations of the potential and of the string tension have been obtained by compactifying one dimension, and studying the connected correlation function of Polyakov lines [2]. In the large N limit this is subleading with respect to the disconnected term, and it is unclear how to make the connection. Thus, we stick to the traditional way in which the string tension is obtained from the expectation value of Wilson loops $W(T, R)$. Here one meets a technical but severe difficulty, since large Wilson loops are very
noisy quantities. Furthermore, the Wilson loops themselves are affected by ultraviolet divergences so that we will rather focus on the traditional Creutz ratios:

$$\chi(T, R) = -\log \frac{W(T + 0.5, R + 0.5)W(T - 0.5, R - 0.5)}{W(T + 0.5, R - 0.5)W(T - 0.5, R + 0.5)}$$ (1)

which are defined for half-integer $R$ and $T$. In the limit $R << T$ these quantities are lattice approximants to the force $F(R)$ among quarks separated by a distance $R$. Although, Creutz ratios get rid of the constant and perimeter divergences in Wilson loops, they do so through a cancellation, which makes them even more numerically challenging. To reduce the errors we resort to the well-known Ape-smearing procedure[11] for the ordinary theory. The corresponding smearing for the TEK model is given by

$$U_{\mu}^{\text{smeared}} = \text{Proj}_N \left[ U_{\mu} + c \sum_{\nu \neq \mu} \left( z_{\mu \nu} U_{\mu} U_{\mu}^\dagger U_{\nu}^\dagger + z_{\mu \nu} U_{\nu} U_{\mu}^\dagger U_{\mu}^\dagger \right) \right]$$ (2)

with $z_{\mu \nu}$ the twist tensor. $\text{Proj}_N$ stands for the operator that projects onto SU(N) matrices. This process can be iterated several times and produces a considerable noise reduction in the data. One could extract the string tension from the force $F(R)$ obtained through Creutz ratios for $R << T$ smeared in the three directions transverse to $T$. This is, however, very impractical in our case. It is much more effective to employ four-dimensional smearing and values $R \approx T$. The problem that arises in this approach is that not only the error, but also the value of the Wilson loops and Creutz ratios vary with the number of smearing steps. This could be an important source of systematic uncertainties, which might prevent a precision determination of the string tension from this source. To circumvent this problem, our strategy has been to use the smeared Creutz ratio values to extrapolate back and obtain un-smeared values. The extrapolated Creutz ratios do not depend on the number of smearing steps, and the errors are considerably smaller than the original un-smeared ratios. Having explained the main observables that we will be using, let us summarize in the next paragraph the goals and methodology used in this work.

Our main goal is the determination of the string tension for large N Yang-Mills theory by means of the study of smeared Creutz ratios on the lattice. The large N value will be obtained by extrapolation of data taken at $N=3,5,6,8$ and by direct use of Twisted Eguchi-Kawai model at $N = 841$ and symmetric twist with flux $k = 9$. Indirectly, since the same procedures will be used to study the reduced and ordinary model, our results will serve to validate the reduction achieved by the TEK model in this physical range of parameters. Since the goal is the continuum result, we have simulated the model with Wilson action at several values of its coupling $\beta \equiv 2N^2 \beta = 2N^2/\lambda_L$,
FIG. 1. $n_s$ dependence of the Creutz ratio $\chi(4.5, 4.5)$ for $N=841$ and $\lambda_L = 2.5974$

where $\lambda_L$ is ‘t Hooft coupling on the lattice. The list of parameters and main lattice results are summarized in Table I.

The analysis of data and presentation of results follows the following steps:

1. **Measurement of Wilson loops and Creutz ratios.**

   For $N=3, 5, 6, 8$ lattice gauge theory, simulations are made on a $32^4$ lattice with 260 configurations used for each $\lambda_L$. The number of configurations used in the TEK model for each $\lambda_L$ is 5400, except at $\lambda_L=2.77778$ where it is 2300. In both the ordinary theory and the TEK model, all configurations are separated by 100 sweeps, one sweep being defined by one-heat-bath update followed by five overrelaxation updates.

   We determined the Creutz ratios from Wilson loops smeared up to 20 times with $c = 0.1$ in the range $R, T \in [3.5, 8.5]$. Errors were estimated by jack-knife. Smaller values of $R, T$ were also obtained, but dismissed for the analysis for being more sensitive to lattice artifacts. Larger loops can also be obtained but are too noisy and/or the number of smearing steps falls too short for them.

2. **Extrapolation to the un-smeared Creutz ratio with error.**

   The extrapolation procedure depends on the values of $R$ and $T$. For small values the smeared Creutz ratio are very well fitted to a dependence $a(1 - \exp[-b/(n_s + \delta)])$ where $n_s$ is the number of smearing steps. This dependence is suggested by perturbation theory. In Fig. [1] we show, as an
| $N$ | $\lambda_L$ | $u_P$ | $\kappa$ | $\gamma$ |
|-----|-------------|-------|----------|----------|
| 3   | 3.05085     | 0.58184 | 0.06737(75) | 0.2110(22) |
| 3   | 3.00000     | 0.59370 | 0.04696(75) | 0.2020(92)  |
| 3   | 2.95082     | 0.60414 | 0.03296(32) | 0.2460(48)  |
| 3   | 2.90323     | 0.61361 | 0.02471(33) | 0.2309(64)  |
| 3   | 2.85714     | 0.62242 | 0.01828(27) | 0.2411(58)  |
| 3   | 2.81250     | 0.63064 | 0.01374(11) | 0.2453(7)   |
| 3   | 2.76923     | 0.63836 | 0.01055(12) | 0.2399(14)  |
| 5   | 2.84625     | 0.57441 | 0.04028(28) | 0.2516(40)  |
| 5   | 2.78676     | 0.58892 | 0.02668(24) | 0.2501(53)  |
| 5   | 2.76564     | 0.59378 | 0.02244(20) | 0.2662(15)  |
| 5   | 2.72242     | 0.60338 | 0.01654(21) | 0.2700(47)  |
| 5   | 2.65125     | 0.61836 | 0.01014(10) | 0.2650(15)  |
| 6   | 2.82408     | 0.56997 | 0.04126(35) | 0.2580(75)  |
| 6   | 2.76923     | 0.58390 | 0.02711(15) | 0.2706(12)  |
| 6   | 2.74872     | 0.58883 | 0.02351(14) | 0.2700(13)  |
| 6   | 2.70677     | 0.59850 | 0.01729(12) | 0.2747(29)  |
| 6   | 2.63736     | 0.61363 | 0.01090(13) | 0.2657(34)  |
| 8   | 2.80179     | 0.56548 | 0.04279(26) | 0.2540(19)  |
| 8   | 2.74973     | 0.57930 | 0.02840(17) | 0.2672(13)  |
| 8   | 2.72869     | 0.58456 | 0.02407(14) | 0.2724(13)  |
| 8   | 2.68902     | 0.59405 | 0.01811(10) | 0.2728(11)  |
| 8   | 2.62134     | 0.60931 | 0.01112(12) | 0.2726(33)  |
| 841 | 2.77778     | 0.55801 | 0.04234(103) | 0.3019(170) |
| 841 | 2.73973     | 0.56902 | 0.03181(60) | 0.2764(118) |
| 841 | 2.70270     | 0.57895 | 0.02474(56) | 0.2623(134) |
| 841 | 2.66667     | 0.58805 | 0.01852(45) | 0.2692(94)  |
| 841 | 2.63158     | 0.59651 | 0.01418(41) | 0.2722(88)  |
| 841 | 2.59740     | 0.60442 | 0.01101(24) | 0.2677(49)  |

**TABLE I.** We list the values of $N$ and lattice couplings $\lambda_L$ studied, together with the plaquette expectation value $u_P$ and best fit parameters for Eq. 3. The $N=841$ case corresponds to the TEK model.
example, the $n_s$ dependence of the $R = T = 4.5$ Creutz ratio for the TEK model and $\lambda_L = 2.5974$, together with the corresponding best fit curve. For larger values of $R \approx T$ the first smearing steps represent a certain transient behaviour, which is then followed by a plateau, before decaying consistently with the previous formula. The extrapolated value is set to the plateau value. Details apart, it is important to emphasize that once the protocol to determine the un-smereared Creutz ratios was defined, it was applied by a program in exactly the same way for all values of $N$ and $\lambda_L$, for both the reduced and ordinary model. Whenever bad fits or ambiguous behaviour was present, the errors were set to reflect the different options.

3. **Analysis of square $R = T$ Creutz ratios**

The square Creutz ratios for large values of $R = T$ are expected to behave as

$$\chi(R, R) = \kappa + \frac{2\gamma}{R^2} + \ldots$$

(3)

A non-zero lattice string tension $\kappa$ is the consequence of Confinement. The linear term in $1/R^2$ is the predicted behaviour both from perturbation theory and from a string description of the quark-antiquark flux-tube. The dots contain corrections from different sources both of continuum and lattice origin.

The data of both models and values of $N$ show a very clear linear behaviour in $1/R^2$, even at the smallest values of $R$. A good description of the data can be obtained with a three parameter fit based on Eq. 3 plus an additional term of the form $\eta/R^4$. The reduced chi square $\sqrt{\chi^2/ndf}$ was typically of order 1 and never exceeded 2. The best fit parameters are listed in Table. I.

4. **Scaling analysis**

Since our goal is continuum physics we should extrapolate our results to the continuum limit. Scaling implies that, close enough to the continuum limit, results obtained at different values $\lambda_L$ should coincide once the lattice spacing $a(\lambda_L)$ is chosen appropriately. In particular, the length of both sides of a rectangular Wilson loop are given in physical units by $t = Ta(\lambda_L)$ and $r = Ra(\lambda_L)$. Using this fact and the definition of Creutz ratios one concludes:

$$\chi(T, R) = a^2(\lambda_L)\tilde{F}(t, r) + \frac{a^4(\lambda_L)}{24} \left( \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^2} \right) \tilde{F}(t, r) + \ldots$$

(4)

where the dots contain higher powers of $a(\lambda_L)$. The continuum function $\tilde{F}(t, r)$ is given by

$$\tilde{F}(t, r) = -\frac{\partial^2 \log W(t, r)}{\partial t \partial r}$$

(5)

where $W(t, r)$ is the value of the continuum $t \times r$ Wilson loop. Notice that, although the Wilson loop itself has perimeter and corner divergences, these disappear when taking the second derivative
with respect to \( t \) and \( r \). Thus, \( \tilde{F}(t, r) \) is a well-defined continuum function having the dimensions of energy square.

In perturbation theory one gets

\[
\tilde{F}(t, r) = \gamma_p(z) \left( \frac{1}{r^2} + \frac{1}{t^2} \right) \tag{6}
\]

where \( \gamma_p \) is a given function of the aspect ratio \( z = r/t \). For the full non-perturbative theory, one can study the behaviour of the function as \( t \) and \( r \) go to infinity. One expects

\[
\tilde{F}(t, r) = \sigma + \gamma(z) \left( \frac{1}{r^2} + \frac{1}{t^2} \right) + \ldots \tag{7}
\]

where \( \sigma \) is the string tension, and the dots represent subleading terms starting with \( 1/((\min(t, r))^4 \). The expansion is also exactly the same as predicted by an effective string theory description of the Wilson loop expectation value.

This analysis justifies the parametrization used previously for square Creutz ratios with \( \gamma = \gamma(1) \) and \( \kappa(\lambda_L) = \sigma a^2(\lambda_L) \). In order to compute the continuum string tension we need to determine \( a(\lambda_L) \). For very small values of \( \lambda_L \) perturbation theory dictates its form:

\[
a(\lambda_L) = \frac{1}{\Lambda_L} \exp\left\{ -\frac{1}{2\beta_0\lambda_L} \right\} \left( \beta_0\lambda_L \right)^{-\beta_1/\left(2\beta_0^2\right)} \equiv \frac{1}{\Lambda_L} f(\lambda_L) \tag{8}
\]

However, it is well-known that scaling seems to work much beyond the region where Eq. 8 provides a good approximation. There are several proposals in the literature, which have been discussed and tested in many papers, which argue that Eq. 8 can be extended to the whole scaling region using improved couplings \( \lambda_I(\lambda_L) \) in the previous formula, instead of \( \lambda_L \) itself. All these proposals can be considered perturbative renormalization prescriptions, and the ratio of the corresponding scales is obtainable by a perturbative calculation, i.e. the ratio of lambda parameters. A particular proposal that has shown good results in previous studies was done by Parisi [12] and used in the analysis of Ref. [13]. When expressed in \( \Lambda_{\overline{\text{MS}}} \) units it is given by \( a_E = \frac{\Lambda_{\overline{\text{MS}}}}{\Lambda_E} f(\lambda_E) \), where \( \lambda_E = (1 - u_P)8N^2/(N^2 - 1) \). A somewhat different proposal resulted from the analysis of Allton et al. [2]. It is based on a different definition of the effective coupling \( \lambda_A = \lambda_L/u_P \) (and a somewhat modified expression for \( f(\lambda_A) \)).

Scaling then implies that the continuum string tension can be determined in \( \Lambda_{\overline{\text{MS}}} \) units as follows:

\[
\frac{\sigma}{\Lambda_{\overline{\text{MS}}}^2} = \lim_{a_E \to 0} \frac{\kappa}{a_E^2} \tag{9}
\]
Our data are consistent with the limit appearing in the right-hand side of the previous equation being approached linearly in $a_E^2$. The extrapolated values for the ratio $\Lambda_{\text{MS}}/\sqrt{\sigma}$ are displayed in Fig. 2 as a function of $1/N^2$. Again a linear fit to the data with parameters $0.515(3) + 0.34(1)/N^2$ is quite satisfactory. The same procedure to obtain the string tension in the continuum limit was followed for the TEK model and $N=841$. The result, also displayed in the figure, is $0.513(6)$. The agreement with the large $N$ extrapolated value of $\Lambda_{\text{MS}}/\sqrt{\sigma}$ is very remarkable, and serves as a non-trivial test that reduction is operative for the TEK model in this range.

Another remarkable feature of our result is that the $N$ dependence matches perfectly with that obtained in Ref. [2], which used different observables, techniques and range of ‘t Hooft couplings. The actual value of the large $N$ ratio given in that reference was $0.503(2)$, which seems inconsistent with our result on statistical grounds. However, the estimated systematic errors quoted in Ref. [2] are as large as 0.04. We should also mention that a recent analysis, largely complementary to ours, has obtained an estimate of the large $N$ string tension which is consistent with our result [14].

In order to give a robust prediction for the large $N$ string tension, we should also estimate our systematic errors. The most important source of these errors arises from an overall scale. If we repeat the procedure replacing the expression of $a_E$ by the formula given by Allton et al. [2] our estimate of the large $N$ ratio $\Lambda_{\text{MS}}/\sqrt{\sigma}$ becomes $0.525(2)$. This is a 2 percent change in the predicted value, which is 5 times bigger than the statistical error.

To give a more precise prediction we should use a non-perturbative renormalization prescription to fix $a(\lambda_L)$. It is possible to give a prescription based on Wilson loops and which follows the same...
philosophy as the one used to define the Sommer scale \[15\]. Let us consider the dimensionless function \( G(r) \equiv r^2 \tilde{F}(r, r) \). A scale \( \bar{r} \) can be defined as the one satisfying \( G(\bar{r}) = \bar{G} \). If scaling holds, the choice of \( \bar{G} \) is irrelevant (provided the equation has a solution), since it amounts to a change of units. For our analysis we took \( \bar{G} = 1.65 \), by analogy with Sommer scale. However, we checked that taking other choices (\( \bar{G} = 2 \) and \( \bar{G} = 2.5 \)) give consistent results up to a change of units. We recall that the idea of considering Creutz ratios with different aspect ratios \( z = R/T \) to define the scale appears in Ref. \[16\].

One possible way to determine the scale is by solving for \( \bar{R}(\lambda L) \) in the equation

\[
\bar{G} = \bar{R}^2(\lambda L) \chi(\bar{R}(\lambda L), \bar{R}(\lambda L); \lambda L)
\]

This gives us \( a(\lambda L) = \bar{r}/\bar{R}(\lambda L) \). Although, our data points are defined only for half integer \( R \), it is easy to interpolate and obtain any real \( \bar{R} \). Interpolation is a much more robust procedure than extrapolation, and one can use different interpolating functions to estimate errors.

The main problem of the previous procedure is that, as explained previously, the Creutz ratios have intrinsic scaling violations given by the second term in Eq. \(4\). Hence, a much better procedure is to make a simultaneous fit to all the square Creutz ratio data \( \chi(R, R) \) for a particular value of \( N \) (and all values of \( \lambda L \)). Combining Eq. \(4\) and the expansion formula Eq. \(7\), one is led to the following functional form

\[
\frac{\bar{r}^2}{a^2} \chi(R, R; \lambda L) = \sigma \bar{r}^2 + 2 \bar{r}^2 \chi(r, r) + 4 \bar{r}^4 (c + d \frac{a^2}{\bar{r}^2})
\]

Indeed, this formula describes remarkably well all our data, with chi squares per degree of freedom of order 1. Notice that the coefficient \( d \) accounts for the scaling violations arising from the definition of the Creutz ratios. The remaining terms parametrize the continuum function \( \tilde{F}(r, r) \) for large values of \( r \). This depends on 3 parameters \( \sigma \bar{r}^2 \), \( \gamma \) and \( c \). However, only two are independent since, by definition, \( \bar{r}^2 \tilde{F}(\bar{r}, \bar{r}) = 1.65 \). This fixes \( 4c = 1.65 - \sigma \bar{r}^2 - 2\gamma \). If we substract the term proportional to \( d \) from the data, all data points should lie in a universal curve given by the function \( \tilde{F}(r, r) = \frac{\bar{r}^2}{n^2} G(r) \). In Fig. 3 we display the corresponding curve for SU(8) together with the best fit function extracted from Eq. \(11\). Notice how the values obtained from different couplings fall into a universal curve. Errors are displayed but hard to see at the scale of the graph. Similar curves are obtained for other values of \( N \) and for the TEK model.

The value of the parameters extracted from the fit are given in Table II. Notice that they are very similar for all theories. This makes the large \( N \) extrapolation very stable. A safe estimate of
the large N value of $\sigma \bar{r}^2$ is 1.105(10), where the error now includes both statistical and systematic uncertainties. It is clear that a good part of the N dependence and systematic error found before resides in the ratio of scales $\bar{r} \Lambda_{\text{MS}}$.

In addition to the determination of the string tension, which sets the long-distance behaviour of Creutz ratios, there is considerable interest in the parameters that determine the approach to this long-distance limit. In particular, our results show that the large N slope parameter $\gamma$ has a value of $0.272(5)$. The slope takes a non-zero value in perturbation theory equal to $\gamma_p(1) = \frac{(\pi+2\lambda)}{16\pi^2}(1-1/N^2)$. Using this formula to define an effective coupling, our data implies $\lambda_{\text{eff}} \approx 8.4$. At long distances, however, a new perspective arises which describes this term as arising from the fluctuation of the chromo-electric flux-tube stretching among the quark and the anti-quark. In the limit in which their separation is large compared to the thickness of this flux tube, an effective string theory description of the dynamics arises. The picture predicts [17] that the coefficient of the $1/r^2$ contribution to the force $F(r)$ is $\gamma(0) = \frac{\pi}{12}$. This prediction has been verified by lattice data.

In our present case, it would be possible to study the function $\tilde{F}(r, t)$ for $r \neq t$ using information of non-square smeared Creutz ratios. In particular the function $\gamma(z)$ provides interesting information about the properties of the effective string theory. We can use our data to determine the function $\gamma(z)$ for the large N theory. The value for $z = 1$ coincides with the parameter $\gamma$ appearing in Table. II. Since by definition $\gamma(z) = \gamma(1/z)$, we can parametrize this function in the vicinity of $z = 1$ as $\gamma(z) = \gamma(1)(1 + \tau \frac{(z-1)^2}{2z})$. Our data for $z > 0.5$ allow a determination of $\tau$. For all values of $N$ and $\lambda$ we get $\tau = 0.31(6)$.

As mentioned previously, the string picture predicts $\gamma(0) = \pi/12$. However, the leading string fluctuation prediction for $\gamma(1)$ is $\approx 0.16$. Our numerical result for $\gamma(1)$ is far from this value and rather close to $\pi/12$. The same happens for the $\tau$ coefficient, which is predicted to be close to 2.

| $N$ | $\sigma \bar{r}^2$ | $\gamma$ | $c$  | $d$  |
|-----|-------------------|----------|------|------|
| 3   | 1.180( 4)         | 0.239( 2)| -0.0019| 0.27( 1) |
| 5   | 1.133( 7)         | 0.263( 3)| -0.0023| 0.29( 2) |
| 6   | 1.120( 4)         | 0.270( 2)| -0.0026| 0.30( 1) |
| 8   | 1.117( 4)         | 0.272( 2)| -0.0027| 0.31( 1) |
| 841 | 1.130(17)         | 0.267( 8)| -0.0036| 0.44( 5) |

TABLE II. The best fit parameters corresponding to Eq. [11]
FIG. 3. The continuum function $\tilde{F}(r, r) = \frac{r^2}{r^2} G(r)$ is plotted for SU(8) from our data. The solid lines correspond to the quadratic function Eq. 11 and to the linear part of this function.

Remarkably, lowest order perturbation theory also has a prediction for $	au = 2/(\pi + 2) \approx 0.39$, which is consistent with our data. The whole issue of string fluctuations for Wilson loops with different aspect ratios is being investigated at present [18].

In summary, we have presented a very precise measurement of the string tension for SU(N) Yang-Mills theory in the large N limit. It is remarkable that the N dependence is consistent with that obtained from correlation of Polyakov lines covering a different range of scales and distances $r \sqrt{\sigma}$ [2]. The large N result is also consistent with that obtained from the TEK single-site model, as predicted by the reduction idea. A.G-A thanks the GGI Institute, the organizers and participants of the 2011 workshop on Large N gauge theories for the opportunity to discuss about this topic. Financial support from Spanish grants FPA2009-08785, FPA2009-09017, CSD2007-00042, HEPHACOS S2009/ESP-1473, PITN-GA-2009-238353 (ITN STRONGnet) and CPAN CSD2007-00042 is acknowledged. M. O is supported in part by Grants-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, Science and Technology (No 23540310).

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