Factorization in leptonic radiative $B \to \gamma e \bar{\nu}$ decays

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Abstract

We discuss factorization in exclusive radiative leptonic $B \to \gamma e \bar{\nu}$ decays using the soft-collinear effective theory. The form factors describing these decays can be expanded in a power series in $\Lambda_{QCD}/E_\gamma$ with $E_\gamma$ the photon energy. We write down the most general operators in the effective theory which contribute to the form factors at leading order in $\Lambda_{QCD}/E_\gamma$, proving their factorization into hard, jet and soft contributions, to all orders in $\alpha_s$. 

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I. INTRODUCTION

Recently, there has been substantial progress in the notoriously difficult theoretical treatment of exclusive hadronic decays of $B$-mesons. One on hand, it was shown that in the heavy quark limit and in low orders of QCD 'factorization' holds in certain processes; on the other hand, the soft-collinear effective theory (SCET) formulated recently in Refs. promises a new systematic way to tackle all orders in the strong coupling. Such applications were discussed in.

Beside purely hadronic two-body decays such as $B \rightarrow K\pi$, also the simpler exclusive radiative decays are of great interest. For instance, the rare radiative weak decays $b \rightarrow (s,d)\gamma$ are an important source of information about the Standard Model and may provide a window to possible 'New Physics'. For this reason their study has received a great deal of attention, both experimentally and theoretically.

While the theory of inclusive $b \rightarrow (s,d)\gamma$ decays is relatively well under control, the corresponding exclusive decays are less understood, although several model calculations of the matrix elements exist (see, for instance, Refs. and some progress towards the factorization of these matrix elements has been achieved recently in the framework of the QCD factorization for exclusive processes.

The simplest $B$ decay involving a hard photon is the radiative leptonic decay $B \rightarrow \gamma e\bar{\nu}$. This decay is not only a testing ground for the various new methods of tackling exclusive decays, but is also interesting in its own right because it is a clear probe of some of the $B$ meson properties and weak couplings. The motivation for the present study is connected with the fact that many ingredients required for the hadronic decay $B \rightarrow V\gamma$ already appear in this case but in a simplified setting.

The goal of the present paper is therefore to present a complete all-order (in $\alpha_s$) proof of the factorization ansatz,

$$A\left(\bar{B} \rightarrow \gamma e\bar{\nu}\right) = C\left(E_\gamma/m_b\right) \cdot J \otimes \Psi_B$$

using the soft-collinear effective theory formulated recently in Refs. Here, $C$ stands for the Wilson coefficients of the SCET operators and encode the hard gluons effects, $J$ are the so-called jet functions and take into account contributions from collinear loop momenta, $\Psi_B$ is the $B$-meson wave function and includes ultra-soft gluons effects; $\otimes$ denotes a convolution along the light-cone component of the spectator momentum ($k_+ = n\cdot k$). This decay was first considered in QCD factorization in, and more recently in Ref. where a factorization formula of type is proved at one-loop order.

The effective theory greatly simplifies factorization proofs of perturbative QCD, by reducing a complicated diagrammatic analysis to simple field transformations on operators in the effective Lagrangian. Using SCET methods, we first identify the operators contributing to this decay and then prove the factorization of the hadronic matrix elements for this process in a form similar to to all orders in $\alpha_s$.

The contents are organized as follows. The effective theory is briefly summarized in Sec. II, where we recapitulate the basic notions needed for the rest of the paper. In Sec. III, we consider the leptonic radiative $B$ decay $\bar{B} \rightarrow \gamma e\nu$. We show, in particular, that the form factors describing this decay can be written as a product of hard, collinear and soft factors. The symmetries of the effective theory give symmetry relations among $B \rightarrow \gamma$ form factors, to all orders in $\alpha_s$. We find that the five form factors describing these decays reduce at leading order in $\Lambda/E_\gamma$ to just one independent function, proving a result conjectured in from a one-loop computation. Necessary technical details are presented in the Appendix.
II. BASICS OF THE SOFT-COLLINEAR EFFECTIVE THEORY (SCET)

The effective theory developed in [3, 4, 5, 6] describes particles (quarks and gluons) relevant for decays of a heavy meson into fast light hadrons. The relevant scales in this problem are $Q, Q\lambda$ and $Q\lambda^2$, where $Q$ is the large energy of the final fast hadron $\simeq m_b$ and $\lambda^2 \simeq p_T^2/Q^2$ with $p_{\perp}$ the typical transverse momentum of the particles. The goal is to obtain an expansion in powers of $\lambda$; to this extent, the $\lambda$-scaling of momenta and fields is important.

Referring to the papers [3, 4, 5, 6] for a detailed explanation of the theory, we present here the few basic points that will be required in the following. We take the hadrons moving with large momentum and small invariant mass in a light-cone direction, $n$. The other light-cone vector, $\bar{n}$, is chosen such that $n^2 = \bar{n}^2 = 0$ and $n\cdot\bar{n} = 2$. Momenta are decomposed along the light cone as $p = (n\cdot p, \bar{n}\cdot p, p_{\perp}) = (p^+, p^-, p_{\perp})$ with

$$p^\mu = \frac{n\cdot p}{2} \bar{n}^\mu + \frac{\bar{n}\cdot p}{2} n^\mu + p_{\perp}^\mu,$$  \hspace{1cm} (2)

In the effective theory one introduces distinct fields for each relevant momentum region. These include the following: collinear modes for quarks ($\xi_{n,p}$) and gluons ($A_{n,q}^\mu$) with momenta scaling as $Q(\lambda^2, 1, \lambda)$; ultra-soft modes ($A_{us}$ and $q_{us}$) with momenta scaling as $Q(\lambda^2, \lambda^2, \lambda^2)$ and soft modes whose momenta scale as $Q(\lambda, \lambda, \lambda)$. The fields also have a well definite scaling in powers of $\lambda$: $A_{n,q} \sim (\lambda^2, 1, \lambda)$, $A_{us} \sim \lambda^2$, $\xi_{n,p} \sim \lambda$ and $q_{us} \sim \lambda^3$.

The collinear fields are related to the full theory fields by [32]

$$\phi(x) = \sum_{\tilde{p}} e^{-i\tilde{p}x} \phi_{n,p}(x)$$  \hspace{1cm} (3)

where $\tilde{p} = \frac{1}{2}(\bar{n} p) n + p_{\perp}$ is the large part of the momentum, treated as a label on the collinear field. Although in the following we will for convenience often write $p$ instead of $\tilde{p}$ in the exponents, one should always keep in mind that algebraic manipulations of exponents (and fields) should only involve the appropriate components.

It is convenient to introduce a ‘label’ operator $P$ [4] which acts on the collinear fields and picks up their large momentum $P^\mu \xi_{n,p} = \left(\frac{n\cdot p}{2} n^\mu + p_{\perp}^\mu\right) \xi_{n,p}$. We will frequently use a special notation which associates a well-defined momentum label index to an arbitrary product of collinear fields. This is defined according to

$$[\phi^{(1)}_n \cdots \phi^{(j)}_n]_P \equiv \delta_{\tilde{p}p,\tilde{p}P} [\phi^{(1)}_n \cdots \phi^{(j)}_n]$$  \hspace{1cm} (4)

where $\delta_{\tilde{p}p,\tilde{p}P}$ acts only inside the square brackets. In this and in the following expressions, we usually omit the momentum labels of the collinear fields $A_{n,q}$ and $\xi_{n,p}$. Moreover, appropriate summation over the (omitted) labels of the fields is usually implied.

The longitudinal component of collinear gluons moving in the $n_\mu$ direction, $\bar{n} A_{n,q}$, appears only in the combination $W_n = \exp(-\frac{g}{\bar{n} p} \bar{n} \cdot A_n)$ which is essentially a Wilson line along the $\bar{n}$ light cone direction. The most general gauge invariant collinear operators can be built using the $\xi_n, A_n, W_n$ and $P_\mu$ building blocks.

Particular products of collinear fields appear often; for convenience of writing we will sometimes use

$$\chi_n \equiv W_n^{\dagger} \xi_n, \hspace{1cm} \Phi_n \equiv W_n^{\dagger} \left(P_{\perp} + g A_{n,\perp}\right) \frac{\bar{n}}{2} \xi_n.$$  \hspace{1cm} (5)
The precise definition of the expansion parameter $\lambda$ is specific to each problem. For the $B \to \gamma e\bar{\nu}$ decay discussed in Sec. III, $\lambda$ is defined by the typical virtuality of the light quark struck by the emitted photon, $p^2 \sim Q^2 \lambda^2 \sim E_\gamma \Lambda_{\text{QCD}}$ with $E_\gamma$ the photon energy in the rest frame of the $B$ meson, which gives $\lambda^2 \sim \Lambda_{\text{QCD}}/E_\gamma$. With this definition, the typical momentum of the spectator in the $B$ meson is of order $\Lambda_{\text{QCD}} \sim Q \lambda^2$, such that its momentum is ultra-soft. On the other hand, for the $\bar{B} \to D\pi$ decay, the typical virtuality of the collinear partons in the pion is of order $\Lambda_{\text{QCD}}^2$, leading to $\lambda = \Lambda_{\text{QCD}}/m_b$; in this case, the momenta of the $B$ constituents are soft.

The interactions among the effective theory fields are described by an effective Lagrangian. It consists basically of the most general combination of fields and/or suitable products of fields (such as in Eq. (5)), compatible with gauge invariance and reparameterization invariance [4, 25]. This Lagrangian is organized in increasing powers of $\lambda$, such that the currents of the full theory are matched onto operators in the effective theory, and can be obtained by matching to the full theory, order by order in $\lambda$.

The couplings of the effective theory fields are described by the soft-collinear effective Lagrangian. At leading order $O(\lambda^0)$, this is given by [4]

$$
\mathcal{L}^{(0)} = \xi_n \left\{ n \cdot D + g n \cdot A_n + (\mathcal{P}_\perp + g A_{n\perp}) \frac{1}{\bar{n} \cdot \mathcal{P} + g \bar{n} \cdot A_n} (\mathcal{P}_\perp + g A_{n\perp}) \right\} \frac{\gamma^\mu}{2} \xi_n. \tag{6}
$$

Here, the covariant derivative $D_\mu = \partial_\mu - ig A^a_\mu T^a$ contains ultra-soft gauge fields $A_\mu$ only.

The form of operators in the effective theory is severely restricted by collinear gauge invariance. These constraints have been discussed at length in [3, 5] for the case of operators contributing to semileptonic and non-leptonic weak decays. For processes containing one hard photon we will find it useful to consider invariance under the combined $SU(3)$ and $U(1)_{\text{em}}$ collinear gauge transformations. Although we work only to first order in the electromagnetic coupling, this provides a convenient way of automatically incorporating electromagnetic couplings in a gauge invariant way.

To this extent, we consider the couplings of a charged collinear quark moving along the $n$ direction $\xi_n$ to collinear gluons $A_n$ and photons $A_n$ moving along the same direction. A simple extension of the argument presented in [5] shows that the latter must appear at lowest order in the combination

$$
W[\bar{n} \cdot A_n, \bar{n} \cdot A_n] = \exp \left[ \frac{1}{\bar{n} \cdot \mathcal{P}} (-g \bar{n} \cdot A_n - e Q \bar{n} \cdot A_n) \right], \tag{7}
$$

where $Q$ is the electric charge matrix defined in flavor space. Under a general e.m. $n$-collinear gauge transformation

$$
U = \sum_P e^{-iPx} U_P(x), \tag{8}
$$

with $U_P$ a matrix in flavor space, the charged $n$ collinear fields transform according to

$$
\xi^{(q)}_{n,p} \rightarrow U^{(q)}_{p-p} \xi^{(q)}_{n,p}, \quad W \rightarrow U_PW. \tag{9}
$$

Finally, the heavy quarks are treated according to the heavy quark effective theory. We write the generic heavy quark field with four-velocity $v$ as $Q(x) = \exp (-imv \cdot x) Q_v(x)$ and decompose it into a large and small component $Q_v = h_v + H_v$, with

$$
h_v = \frac{1 + \gamma^\mu}{2} Q_v. \tag{10}
$$
III. FACTORIZATION IN $B \to \gamma e \nu$ DECAYS

The simplest $B$ decay involving a hard photon is the radiative leptonic decay $\bar{B} \to \gamma e \bar{\nu}$. Our main result will be to show that the $\bar{B} \to \gamma e \bar{\nu}$ amplitude factorize according to

$$A(\bar{B} \to \gamma e \bar{\nu}) = C(E_\gamma/m_b) \int \frac{dn \cdot k}{4\pi} J(n \cdot k) \Psi_B(n \cdot k)$$

where $C(E_\gamma/m_b)$ is an effective theory Wilson coefficient, $J$ is a collinear function and $\Psi_B$ is the usual light-cone $B$ meson wave function. Using the effective theory formulation we will prove also symmetry relations between form factors parametrizing $B \to \gamma$ transitions.

The $B \to \gamma e \bar{\nu}$ amplitude is given in QCD by

$$A(B^- (v) \to \gamma(q, \varepsilon) e \bar{\nu}) =$$

$$ie Q_q \frac{A_G F}{\sqrt{2}} V_{ub} \int d^4 x e^{iqx} \langle 0 | T([\bar{q} \gamma_\mu P_L b](0), [\bar{q} \gamma^\nu q](x)) | \bar{B}(v) \rangle [\bar{u}(p_e) \gamma_\mu P_L v(p_\nu)]$$

with $P_L = \frac{1}{2} (1 - \gamma_5)$.

We fix the kinematics by taking the photon momentum as $q_\mu = E_\gamma n_\mu$ with the light-cone vector given by $n = (1, 0, 0, 1)$. The direction orthogonal to $n$ along the light-cone is parameterized by $\bar{n} = (1, 0, 0, -1)$. The electromagnetic current $\bar{q} \gamma^\nu q$ takes the ultra-soft spectator anti-quark in the $B$ meson carrying momentum $k \simeq \Lambda_{QCD}$ into a collinear quark with a large momentum component along the $n$ light-cone direction $q - k$.

The weak current $\bar{q} \Gamma b$, coupling a heavy quark to an energetic light quark, is matched in SCET according to [4]

$$\bar{q} \Gamma b = \sum_i C_i(\bar{n} \cdot p, \mu) \bar{\chi}_{n, \mu} \Gamma_i h_v + O(\lambda)$$

with $C_i(\omega, \mu)$ Wilson coefficients. The explicit choice of the Dirac structure in these operators differs from that used in [4], but they are simply related by algebraic manipulations. To leading order in $\lambda$, the vector and axial currents are matched onto the effective theory operators (with $\omega = \bar{n} \cdot p$)

$$\begin{align*}
\bar{q} \gamma_\mu b &= C_1^{(v)}(\omega, \mu) \bar{\chi}_{n, \omega} \gamma_\mu h_v + C_2^{(v)}(\omega, \mu) \bar{\chi}_{n, \omega} v_\mu h_v + C_3^{(v)}(\omega, \mu) \bar{\chi}_{n, \omega} n_\mu h_v \\
\bar{q} \gamma_5 b &= C_1^{(a)}(\omega, \mu) \bar{\chi}_{n, \omega} \gamma_5 h_v + C_2^{(a)}(\omega, \mu) \bar{\chi}_{n, \omega} v_\mu \gamma_5 h_v + C_3^{(a)}(\omega, \mu) \bar{\chi}_{n, \omega} n_\mu \gamma_5 h_v.
\end{align*}$$

Explicit results for the Wilson coefficients $C_i^{(v,a)}(\omega, \mu = m_b)$ at one-loop order can be found in Eq. (33) of Ref. [4]. Only two of these Wilson coefficients are needed in the following

$$C_1^{(v)}(\omega, m_b) = C_1^{(a)}(\omega, m_b) = 1 - \frac{\alpha_s C_F}{4\pi} \left\{ 2 \log^2 \left( \frac{\omega}{m_b} \right) + 2 \text{Li}_2(1 - \frac{\omega}{m_b}) \\
+ \log \left( \frac{\omega}{m_b} \right) \frac{3\omega - 2m_b}{m_b - \omega} + \frac{\pi^2}{12} + 6 \right\}$$

For completeness we give also the matching of the tensor current $\bar{q} \sigma_{\mu \nu} b$, which contributes to the amplitude for the rare decay $B_s \to e^+ e^- \gamma$ [31]. This is given at leading order in $\lambda$ in terms of four Wilson coefficients $C_i^{(t)}(\omega, \mu)$ defined as

$$\begin{align*}
\bar{q} i \sigma_{\mu \nu} b &= C_1^{(t)}(\omega, \mu) \bar{\chi}_{n, \omega} i \sigma_{\mu \nu} h_v + C_2^{(t)}(\omega, \mu) \bar{\chi}_{n, \omega} (n_\mu \gamma_\nu - n_\nu \gamma_\mu) h_v \\
+ C_3^{(t)}(\omega, \mu) \bar{\chi}_{n, \omega} (v_\mu \gamma_\nu - v_\nu \gamma_\mu) h_v + C_4^{(t)}(\omega, \mu) \bar{\chi}_{n, \omega} (n_\mu v_\nu - n_\nu v_\mu) h_v
\end{align*}$$
The corresponding matching conditions at one-loop order are \[4]\n
\[ C_1^{(t)}(\omega, m_b) = 1 - \frac{\alpha_s C_F}{4\pi} \left\{ 2 \log^2 \left( \frac{\omega}{m_b} \right) + 2 \text{Li}_2(1 - \frac{\omega}{m_b}) + 2 \log \left( \frac{\omega}{m_b} \right) \frac{2\omega - m_b}{m_b - \omega} + \frac{\pi^2}{12} + 6 \right\} \]

\[ C_2^{(t)}(\omega, m_b) = \frac{\alpha_s C_F}{4\pi} \frac{2\omega}{m_b - \omega} \log \left( \frac{\omega}{m_b} \right) \]

\[ C_3^{(t)}(\omega, m_b) = C_4^{(t)}(\omega, m_b) = 0. \]

The matching of the light quark photon vertex $\bar{q}\gamma_\perp q$ onto SCET operators is slightly more complicated. In addition to a local term as in the case of the heavy-light current, this matching contains also a non-local term:

\[ (\bar{q}\gamma_\perp q)(x) = \varepsilon^\mu J_\mu = \sum_p e^{-i p x} (\bar{q}\gamma_\perp \chi_{n,p})(x) + i \int d^4 y T(J_g(x), L_{q\xi}^{(1)}(y)). \] (19)

The local term describes contributions in which the energetic photon itself turns the ultrasoft quark $q$ directly into a collinear quark $\xi_n$. The nonlocal term, on the other hand, describes the process in which the $O(\lambda)$ subleading Lagrangian $L_{q\xi}^{(1)}$ merely turns the ultrasoft quark into a collinear quark; its coupling to the photon is then described by a new current $J_g$. The right hand side must be evaluated in the presence of the interaction \[4\].

The expression for the subleading Lagrangian $L_{q\xi}^{(1)}$ has been obtained in Ref. \[4\] and reads

\[ L_{q\xi}^{(1)}(y) = [\bar{q} W_n^\dagger(p_\perp^\perp) + g A_{n\perp}] \xi_n(y) + h.c. \] (20)

The explicit form of $J_g$ can be obtained by coupling the photon as a collinear field and expanding the Lagrangian \[4\] to linear order in $e$ (with $\Phi_{n,p}$ as defined in \[4\])

\[ J_g(x) = \sum_{p_1, p_2} e^{i(q - p_1 - p_2) x} \left\{ [\bar{\chi}_{n,p_2} \gamma_\perp \frac{1}{n \cdot P} \Phi_{n,p_1}](x) + [\bar{\Phi}_{n,p_2} \frac{1}{n \cdot P} \gamma_\perp \xi_{n,p_1}](x) \right\}. \] (21)

Note that, despite the $O(\lambda)$ suppression of $L_{q\xi}^{(1)}$, both terms in \[19\] contribute to the same order in $\lambda$. The first term scales like $\lambda^4$ by simple power counting of the fields \[20\]. In the second term, $J_g(x)$ scales like $\lambda^3$ and $L_{q\xi}^{(1)}$ like $\lambda^5$. Taking into account a factor $\lambda^{-4}$ from the $y$-integration yields a total scaling of $\lambda^4$ for the second term as well. We have not added any new Wilson coefficients in \[19\] since the form of the operators is restricted to all orders by the arguments in \[4\]. This is also checked by direct computation in the Appendix, where we show explicitly that, at one-loop order, Eq. \[19\] reproduces correctly the IR behavior of the full QCD.

Inserting Eqs. \[19\] and \[13\] into the time-ordered product that appears in Eq. \[12\] one obtains the hard-collinear factorization of the matrix element

\[ A(B(v) \to \gamma(q, \varepsilon) e\bar{\nu}) = ie Q_q \frac{4G_F}{\sqrt{2}} V_{ub} \sum_{i=1}^{6} \sum_{np} C_i (\bar{u}(p_e) \gamma_\mu P_L v(p_\nu)) \] (22)
where the decay amplitudes in the effective theory are given by time-ordered products of ultra-soft and collinear fields

\[ A^{i(1)}_{\text{eff}} = \int \! d^4 x e^{i(q-p_2) x} \langle 0 | T(\{ [\tilde{\chi}_{\alpha n p_2} \Gamma_\gamma h_\nu] (0), [\bar{q}_\perp^{* \nu} \chi_{n p_2}] (x) \}) | \bar{B}(v) \rangle \]  

\[ + i \int \! d^4 x d^4 y \langle 0 | T(\{ [\tilde{\chi}_{\alpha n p_1} \Gamma_\gamma h_\nu] (0) ; J_g (x), L_{q\perp}^{(1)} (y) \}) | \bar{B}(v) \rangle . \]  

In the next step the ultra-soft fields are decoupled from the collinear fields \( \xi_{n p} \) and \( A_n \). This is achieved via the field redefinitions [6]

\[ \xi_{n p} (x) \rightarrow Y_n (x) \xi_{n p}^{(0)} (x), \quad A_n (x) \rightarrow Y_n (x) A_n^{(0)} (x) Y_n^+ (x) \]  

where \( Y_n \) is a Wilson line along the light cone direction \( n \) containing the ultra-soft gluon field

\[ Y_n (x) = \text{P} \exp \left( i g \int_{-\infty}^{x/2} d\lambda n \cdot A_{\text{us}} (\lambda n) \right) . \]  

The integration over \( x \) allows to express the label momentum \( \bar{n} \cdot p_2 \) in terms of the photon momentum as \( \bar{n} \cdot p_2 = \bar{n} \cdot q = 2 E_\gamma \). The collinear factor is the propagator of the \( \chi_n \) fields, which, in momentum space, reads [6]

\[ \langle 0 | T(\chi_{\alpha \gamma n p_2}^{(0)} (x) , \chi_{\gamma n p_1}^{(0)} (0)) | 0 \rangle = i \delta_{\gamma \gamma_{n p_1} n p_2} \int \frac{d^4 k}{(2\pi)^4} e^{-i k x} J(\bar{n} \cdot p_1; n \cdot k) \left( \frac{g}{2} \right)_{\alpha \beta} . \]  

This function was introduced in Ref. [6] in connection with the factorization of the photon energy spectrum in the endpoint of \( B \rightarrow X_s \gamma \) decay. The discontinuity of \( J(\bar{n} \cdot p; n \cdot k) \) across its cut gives the jet function appearing in the factorization formula for \( B \rightarrow X_s \gamma \). Here we need the function \( J(\bar{n} \cdot p; n \cdot k) \) itself in a region away from the cut, where it can be computed in perturbation theory provided that the virtuality \( p^2 = \bar{n} \cdot p \cdot n \cdot k \) is sufficiently large, i.e. \( p^2 \gg \Lambda_{QCD}^2 \). The explicit result at one loop order is [3] [4] (with \( L = \log(\bar{n} \cdot p \cdot n \cdot k / \mu^2) \))

\[ J(\bar{n} \cdot p; n \cdot k) = \frac{1}{n \cdot k} \left[ 1 + \frac{\alpha_s C_F}{4\pi} \left( 2L^2 - 3L + 7 - \frac{\pi^2}{3} \right) \right] . \]  

Note that the collinear function \( J(\bar{n} \cdot p; n \cdot k) \) depends on \( k \) only through \( n \cdot k \) to all orders in perturbation theory since the soft–collinear Lagrangian contains only the \( n \cdot \partial \) derivative [6].
FIG. 1: One-loop effective theory graphs contributing to the T-product (23). Only the collinear graphs are shown; the topology of the uSOFT graphs is identical to that of the QCD graphs. The graphs (a)-(d) are produced by the first term in (23), and (e)-(g) by the second term. The lower vertex in (a)-(d) denotes the $\bar{\xi}_\varepsilon / \perp W q$ operator; in (e)-(g) the photon attaches to the circled cross vertex, denoting the $J_g$ operator, and the lower vertex is the $L^{(1)}_\xi q$ subleading Lagrangian.

this implies that the coordinate $x$ in Eq. (26) is constrained to be on the light cone (i.e. $x_\perp = n \cdot x = 0$). We obtain, finally, the factorized form of the first term in Eq. (23) as a one-dimensional integral

$$A^{(1)}_{\text{eff}} = i \delta_{\bar{n} q, \bar{n} p_1} \delta_{\bar{n} p_1, \bar{n} p_2} \int d\bar{n} \cdot x \int \frac{dn \cdot k}{4\pi} e^{i \bar{n} \cdot kx} J(\bar{n} \cdot p; n \cdot k) \text{Tr} \left[ \frac{g}{2} \Gamma_i \Psi_B(\bar{n} \cdot x) \bar{\xi}_\varepsilon / \perp \right].$$

(29)

The ultra-soft matrix element on a $B$ meson defines its light-cone wave function

$$\Psi_B(\bar{n} \cdot x) \equiv \langle 0 | T(q^{\beta} \frac{\bar{n} \cdot x}{2} n, Y_n(\frac{\bar{n} \cdot x}{2} n, 0), h^a_v(0)) | B(v) \rangle$$

$$= \int \frac{dn \cdot k}{4\pi} e^{-\frac{i}{2} n \cdot kx} \Psi_B^{a\beta}(n \cdot k)$$

(30)

with $Y_n(x, 0) = Y^*_n(0)Y_n(x)$.

In the heavy quark limit, the most general parameterization for $\Psi_B$ involves two functions $\psi_\pm(n \cdot k)$, which are usually defined as $[1, 2, 14, 15]$

$$\Psi_B(n \cdot k) = \frac{1 + \frac{g}{2} \left( \frac{\bar{\eta} \eta}{4} \psi_+(n \cdot k) + \frac{\bar{\eta} \eta}{4} \psi_-(n \cdot k) \right) \gamma_5}{f_B m_B}.$$ 

(31)

With this definition, the light cone wave functions $\psi_\pm(n \cdot k)$ are normalized as

$$\int \frac{dn \cdot k}{2\pi} \psi_\pm(n \cdot k) = f_B m_B.$$

(32)
Expressed in terms of the momentum space wave functions, the first term in the effective theory amplitude (23) is given in terms of one of the two functions $\psi_\perp(n\cdot k)$ and is written as

$$A_{\text{eff}}^{(1)}(\bar{B} \to \gamma e\bar{\nu}) = i\delta_{\eta q,\eta q_1}\delta_{\eta p_1,\eta p_2}\int \frac{d^n k}{4\pi} J(2E_\gamma, n\cdot k)\psi_\perp(n\cdot k)\text{Tr}\left[\frac{g^\gamma_i}{2} \gamma_\perp \frac{1}{2} \gamma_5 f_{\perp}^*\right].$$  \hspace{1cm} (33)

The factorization of the second term in Eq. (23) can be proved in a similar way. After the field redefinition [24] the relevant collinear matrix element is (the collinear field $\Phi_{n,p}$ is defined in Eq. (5))

$$i \int d^4 x e^{i(q-p_1+p_2)x} \langle 0|T(\Phi_{n,0}^\alpha(y), \mathcal{J}_g^{(0)}(\vec{n}, p_2, \vec{n}, p_1, x), \bar{\chi}_{n,p}^{(0)}(0))|0\rangle = i \int d^4 x \langle 0|T(\Phi_{n,0}^\alpha(y), \mathcal{J}_g^{(0)}(\vec{n}, p_1 - \vec{n}, q, \vec{n}, p_1, x), \bar{\chi}_{n,p}^{(0)}(0))|0\rangle$$

$$\equiv i\delta_{\eta q,\eta q_1}\delta_{\eta p_1,\eta p_2}\int \frac{d^4 k}{(2\pi)^4} e^{-iky} J_\ell(\vec{n}, p; n\cdot k) \left[\frac{g^\gamma_i}{4} f_{\perp}^*\right].$$

This defines a new collinear function $J_\ell(\vec{n}, p; n\cdot k)$, similar to $J(\vec{n}, p; n\cdot k)$ introduced in (27). The operator $\mathcal{J}_g^{(0)}$ is related to $\mathcal{J}_g$ introduced in [21] by the field redefinition [24] and is given explicitly by

$$\mathcal{J}_g^{(0)}(\vec{n}, p_2, \vec{n}, p_1, y) = \bar{\chi}_{n,p}^{(0)}(0)\frac{1}{\vec{n}, p_1} \Phi_{n,p_2}^\alpha + \bar{\Phi}_{n,p_2}^\alpha \frac{1}{\vec{n}, q} \chi_{n,p_1}.$$  \hspace{1cm} (35)

In contrast to the function $J$ which starts at tree level, $J_\ell$ starts at $O(\alpha_s)$ since the sub-leading operator $E_q^{(1)}$ has only Feynman rules with at least one collinear gluon. The three graphs contributing to $J_\ell$ at lowest order are shown in Figs. [1(e)-(g)]. Their computation is described in the Appendix, and the result reads (in Feynman gauge)

$$J_\ell(\vec{n}, p; n\cdot k) = \frac{1}{n\cdot k} \frac{\alpha_s C_F}{4\pi} \left(-L^2 + 3L - \frac{\pi^2}{6}\right).$$  \hspace{1cm} (36)

Note that, using the same argument as in the $J(\vec{n}, p; n\cdot k)$ case, the collinear function $J_\ell(\vec{n}, p; n\cdot k)$ also depends on $k$ only through $n\cdot k$ at all orders in perturbation theory.

Inserting Eq. (34) into the factorized matrix element (23), multiplying with the ultra-soft matrix element on the $B$ meson, and performing the integration over $x$ gives $A_{\text{eff}}^{(2)}$ similar to $A_{\text{eff}}^{(1)}$ but with the replacement $J(2E_\gamma, n\cdot k) \to J_\ell(2E_\gamma, n\cdot k)$. This determines $A_{\text{eff}}^{(2)} = A_{\text{eff}}^{(1)} + A_{\text{eff}}^{(2)}$ to be given by an expression similar to (33) with the replacement $J \to J + J_\ell$.

The final result for the $B \to \gamma e\bar{\nu}$ decay amplitude is obtained by inserting the factorized soft-collinear matrix elements as in Eq. (33) into the expansion (22) and performing the sum over all Dirac structures for the current in the effective theory. One defines separate form factors $f_{V,A}(E_\gamma)$ for the vector and axial currents in QCD by (with the convention $\varepsilon^{0123} = 1$)

$$\frac{1}{e} \langle \gamma(q, \varepsilon)|\bar{q}\gamma_\mu b|B(v)\rangle = i\varepsilon_{\mu\alpha\beta\delta}\varepsilon^*_{\gamma\delta} v_\beta q_\delta f_V(E_\gamma)$$

$$+ (v\cdot\varepsilon^*) v_\mu \frac{1}{v\cdot q} f_B m_B.$$  \hspace{1cm} (37)
The second term in the matrix element of the axial current is required by gauge invariance \cite{27} and cancels the photon emission amplitude from the charged lepton line. With these definitions, one finds the following explicit results for the $B \rightarrow \gamma$ form factors at leading order in $1/E_\gamma$

$$f_V(E_\gamma) = C_1^{(v)}(2E_\gamma, \mu) \frac{Q_q}{E_\gamma} I(\mu),$$  
(39)

$$f_A(E_\gamma) = C_1^{(a)}(2E_\gamma, \mu) \frac{Q_q}{E_\gamma} I(\mu),$$  
(40)

where the usoft-collinear part of the matrix element is contained in the integral

$$I(\mu) \equiv \int \frac{dn \cdot k}{4\pi} (J(2E_\gamma; n \cdot k) + J_\ell(2E_\gamma, n \cdot k)) \psi_+(n \cdot k).$$  
(41)

The results \cite{39}-\cite{41} can be summarized at one-loop by writing

$$f_V(E_\gamma) = f_A(E_\gamma) = \frac{Q_q}{E_\gamma} \int \frac{dn \cdot k}{4\pi} \frac{1}{n \cdot k} \left\{ 1 + \frac{\alpha_s C_F}{4\pi} T^{(1)}(n \cdot k, \mu) \right\} \psi_+(n \cdot k, \mu)$$  
(42)

where the one-loop correction to the hard scattering kernel $T^{(1)}(n \cdot k, \mu)$ is given by

$$T^{(1)}(n \cdot k, \mu) = -\frac{1}{2} \log^2 \left( \frac{\mu^2}{m_b^2} \right) + \log \left( \frac{\mu^2}{m_b^2} \right) \left( 2 \log \frac{2E_\gamma}{m_b} - \frac{5}{2} \right) - \frac{3E_\gamma - m_b}{m_b - 2E_\gamma} \log \frac{2E_\gamma}{m_b}$$

$$+ \log^2 \left( \frac{2E_\gamma (n \cdot k)}{\mu^2} \right) - 2 \log^2 \left( \frac{2E_\gamma}{m_b} \right) - 2 \text{Li}_2 \left( 1 - \frac{2E_\gamma}{m_b} \right) - \frac{\pi^2}{4} - 7.$$  
(43)

This expression coincides with the one-loop results presented in Ref. \cite{30}.

Similar results are obtained for the form factors of the tensor current, which can be defined as

$$\frac{1}{e} \langle \gamma(q, \varepsilon)|\bar{q}\sigma_{\mu\nu}|B(p)\rangle = g_+(E_\gamma \gamma \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{\alpha}(p + q)^\beta + g_-(E_\gamma \gamma \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{\alpha}(p - q)^\beta$$

$$- 2h(E_\gamma \gamma \varepsilon^\alpha \cdot p) \varepsilon_{\mu\nu\alpha\beta} p^\beta q^\beta.$$  
(44)

The explicit leading order results for these form factors are

$$g_+(E_\gamma) = \frac{1}{2} \left\{ C_1^{(t)}(2E_\gamma, \mu) + C_2^{(t)}(2E_\gamma, \mu) + \left( 1 - \frac{E_\gamma}{m_B} \right) C_3^{(t)}(2E_\gamma, \mu) \right\} \frac{Q_q}{E_\gamma} I(\mu)$$  
(45)

$$g_-(E_\gamma) = -\frac{1}{2} \left\{ C_1^{(t)}(2E_\gamma, \mu) + C_2^{(t)}(2E_\gamma, \mu) + \left( 1 + \frac{E_\gamma}{m_B} \right) C_3^{(t)}(2E_\gamma, \mu) \right\} \frac{Q_q}{E_\gamma} I(\mu)$$  
(46)

$$h(E_\gamma) = \frac{1}{2m_B^2} C_3^{(t)}(2E_\gamma, \mu) \frac{Q_q}{E_\gamma} I(\mu).$$  
(47)

Writing the form factors in this form makes explicit their factorization into a hard ($C_i$), collinear ($J$) and ultra-soft ($\psi_+$) components. This result gives a field-theoretical interpretation of the contributions coming from different momentum regions. Corrections to factorization can be expected from subleading operators in the matching of the currents and
Lagrangian insertions, for which the field redefinition (24) fails to decouple ultrasoft from collinear fields [6].

In the numerical evaluation of the form factors, a convenient choice for the scale $\mu$ is $\mu^2 \sim 2E_\gamma \Lambda$, for which the logs appearing in the collinear functions $J, J_\ell$ are minimized. With this choice, these logs are shifted instead into the Wilson coefficients $C_i(2E_\gamma)$, where they can be resummed using renormalization group (RG) methods. It has been shown in [3, 4] that the Wilson coefficients $C_i(\bar{n} \cdot p, \mu)$ satisfy RG equations of the form

$$\mu \frac{d}{d\mu} C_i(\bar{n} \cdot p, \mu) = \gamma(\bar{n} \cdot p/\mu) C_i(\bar{n} \cdot p, \mu)$$

with a universal anomalous dimension $\gamma$. The form of the resummed result has been discussed in detail in [4] to which we refer for explicit results. A similar resummation can be performed using the methods of [28, 29], which are equivalent to the effective theory approach. At leading order, the resummation of the Sudakov double logs $\log(2E_\gamma/\mu)$ has been given in [23] using the approach of [28, 29], and a next-to-leading analysis was done recently in [30] using SCET methods.

The factorized form (39)-(40) exhibits a symmetry relation between radiative leptonic form factors of different currents. At $O(\lambda^0)$, the only difference between $f_V(E_\gamma)$ and $f_A(E_\gamma)$ comes from the Wilson coefficients of the heavy light currents. However, it has been shown in [4] that, for massless collinear quarks, the Wilson coefficients $C_i^{(v)}$ and $C_i^{(a)}$ are equal to all orders in $\alpha_s$ as a consequence of the chiral symmetry of the effective theory. This proves the symmetry relation $f_V(E_\gamma) = f_A(E_\gamma)$ to all orders in $\alpha_s$, confirming a result conjectured in [23] from an explicit one-loop computation. Analogous relations can be given for the form factors of the tensor current following from (45)-(47).

Finally, we comment on the relation of our results to a factorization formula for $B \to \gamma e\nu$ conjectured in [23] from a one-loop computation. The main difference is the simpler one-dimensional form of the convolution formula for the form factors (39), (41) (and also in [30]), compared to the three-dimensional factorization formula proposed in [23]. This is due to a different definition for the B meson wave function adopted in [23], which shifts part of the usoft matrix element ($k_\perp$-dependent) into the hard scattering amplitude $T$.

Another difference which must be taken into account when comparing with the results of [23] is the hierarchy $m_b \gg 2E_\gamma \gg \Lambda$ adopted in that paper. This corresponds to expanding the hard scattering kernel $T$ in $2E_\gamma/m_b$ and keeping only the leading term. On the other hand, the more general approach used here allows $m_b, 2E_\gamma$ to be of comparable magnitude, subject only to the condition $m_b, 2E_\gamma \gg \Lambda$.

IV. CONCLUSIONS

We have studied in this paper the consequences of the presence of two considerably different scales, $2E_\gamma \sim m_b$ and $\Lambda_{QCD}$. Although this is a general feature of radiative $B$ decays involving an energetic photon, the basic physical features of such a problem can be seen on the simpler case of the leptonic radiative decay $B \to \gamma e\nu$. The presence of a small parameter $\lambda$, where $\lambda^2 = \Lambda_{QCD}/m_b$, suggests using the soft-collinear effective theory which is tailored to systematically treat the expansion in $\lambda$.

We discussed in this paper the most general form of the SCET operators contributing to the leptonic radiative decay $\bar{B} \to \gamma e\bar{\nu}$ at leading order in $\lambda$. Using this result, we showed
that the form factors describing this decay factorize into hard, collinear and soft factors
to all orders in perturbation theory. Further work is needed to complete the proof of the
factorization relation for a generic radiative B decay. One can hope that the application
of the soft-collinear effective theory along the lines of the present paper can be fruitful in
unravelling the general form of such a result.

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APPENDIX A: ONE-LOOP MATCHING OF THE ULTRA-SOFT–COLLINEAR
CURRENT

We consider here in some detail the coupling of an energetic photon $\gamma(q, \varepsilon)$ to an ultra-
soft quark $q$, turning it into a collinear quark $\xi_n$. Proceeding in analogy to the heavy-light
current $[3, 4]$ one would be led to match this coupling onto a local operator in the effective
theory. At order $O(\lambda^0)$ the most general operator compatible with collinear gauge invarian-
c is $\bar{\xi}_n W^\varepsilon_q / \xi_n$. In Sec. III it was shown that the complete matching can include also a nonlocal
term and it reads

$$\bar{q}\xi^\varepsilon_q \to \bar{\xi}_n W^\varepsilon_q + iT \{ J_g, L^{(1)}_\xi \}. \tag{A1}$$

The operators appearing in the nonlocal term are defined as

$$J_g = \bar{\xi}_{n,p} \left[ \frac{1}{\bar{n} \cdot \mathcal{P} + g \bar{n} \cdot A_n} (\mathcal{P}_\perp + gA_\perp) + \frac{1}{\bar{n} \cdot \mathcal{P} - \bar{n} \cdot q + g\bar{n} \cdot A_n} \frac{\bar{q}\xi_{n,p}}{2} \right]$$

$$L^{(1)}_\xi = \bar{\xi}_n (\mathcal{P}_\perp + gA_\perp) W q. \tag{A2}$$

The SCET diagrams involving collinear gluons that describe the one-loop matching of the
ultra-soft-collinear current are given in Fig. 2; we do not draw the diagram with one ultra-
soft gluon since it has the same topology as in full QCD. The graph in Fig. 2(a) is the
contribution analogous to the one present in the heavy-light correction; it stems from the
first term in Eq. (A1) and the two vertices denote, respectively, an insertion of the $\bar{\xi}_n W^\varepsilon_q$
operator (the blob) and of the leading order Lagrangian $L^{(0)}$. The two remaining graphs,
Figs. 2(b)-(c), appear only for the light-light case and contain one insertion of the sub-leading
Lagrangian $L^{(1)}_\xi$ and one insertion of the operator $J_g$ (the diagram in Fig. 2(b) contains also
an insertion of the leading order lagrangian). Using Eqs. (A1) and (A2) it is immediate to
extract the Feynman rules for the various vertices that appear in Fig. 2.

In the explicit computation of the diagrams we assume that the photon moves along
the $n$ direction with light cone momentum components $q = (n \cdot q, \bar{n} \cdot q, q_\perp) = (0, \bar{n} \cdot q, 0)$.
The momenta of the ingoing ultrasoft quark and of the outgoing collinear quark are $p$ and
$p' = p - q$, respectively. The ultrasoft quark is taken on-shell with $p'^2 = m^2$.  

12
The contribution of the local term in (A1), Fig. 2(a), is identical to the one-loop correction to the heavy-light vertex \(3\), and its computation gives (we use everywhere in the following the Feynman gauge)

\[
\Gamma_c^{(a)} = -2ig^2C_F\mu^2e \int \frac{d^d k}{(2\pi)^d} \frac{n \cdot (k - q)}{(p - q + k)^2} \bar{\xi}_n(p')\xi^*_\perp u(p) = -2\alpha_s C_F \frac{\Gamma(-\epsilon)\Gamma(2 - \epsilon)}{\Gamma(2 - 2\epsilon)} \left( \frac{n \cdot p n \cdot q}{\mu^2} \right)^{-\epsilon} \bar{\xi}_n(p')\xi^*_\perp u(p) \]

where \(d = 4 - 2\epsilon\), \(L = \log(\frac{n \cdot p n \cdot q}{\mu_E^2}) = \log(-p'^2/\mu_E^2)\) and \(\mu_E^2 = \mu^2 e^{\gamma_E - \log 4\pi}\). The result contains a double logarithm \(3\). In an analogous way we compute the contributions from the nonlocal term in (A1), Figs. 2(b)-(c), and obtain

\[
\Gamma_c^{(b+c)} = \alpha_s C_F 4\pi \left( \frac{2}{\epsilon^2} - \frac{3 - 2L}{\epsilon} - L^2 - 3L + \frac{\pi^2}{6} - 8 \right) \bar{\xi}_n(p')\xi^*_\perp u(p) 
\]

Finally, the sum of the three diagrams reads

\[
\Gamma_c = \alpha_s C_F 4\pi \left( -\frac{1}{\epsilon} + \log \left( \frac{-p'^2}{\mu_E^2} \right) - 4 \right) \bar{\xi}_n(p')\xi^*_\perp u(p) 
\]

where we restored the logarithm. Note that in Eq. (A5) the double logarithms cancel.

To complete the computation of the vertex correction in SCET we need to include also the usoft diagram that comes at one-loop order only from one insertion of the local term in (A1), and is equal to

\[
\Gamma_s = -2ig^2C_F\mu^2e \int \frac{d^d k}{(2\pi)^d} \frac{1}{[(k + p)^2 - m^2 + i\varepsilon][(k^2 + i\varepsilon)]} \bar{\xi}(p')\xi^*_\perp u(p) 
\]

\[
\Gamma_s = \alpha_s C_F 4\pi \frac{\Gamma(-\epsilon)\Gamma(2 - \epsilon)}{\Gamma(2 - 2\epsilon)} \left( \frac{m^2}{\mu^2} \right)^{-\epsilon} \bar{\xi}(p')\xi^*_\perp u(p) 
\]

\[
\Gamma_s = \alpha_s C_F 4\pi \left( \frac{2}{\epsilon} - 2\log \left( \frac{m^2}{\mu_E^2} \right) + 4 \right) \bar{\xi}(p')\xi^*_\perp u(p) 
\]

\[\text{FIG. 2: Collinear graphs for photon emission from a light quark in the effective theory. The wiggly line denotes the photon. The graph (a) corresponds to the local operator in the matching, and the remaining two (b), (c) come from the nonlocal term. The blob in (a) denotes the } \xi\xi^*_\perp Wq \text{ operator; in (b), (c) the shaded blob denotes the } L^{(1)}_{\xi q} \text{ subleading Lagrangian and the circled cross is the insertion of } J_g.\]
where \( m \) is the mass of the ultrasoft quark that we use as IR regulator.

The total SCET contributions to the usoft-collinear vertex is

\[
\Gamma_{\text{SCET}} = \frac{\alpha_s C_F}{4\pi} \left[ \frac{1}{\epsilon} + \log \left( \frac{-p'^2}{\mu_E^2} \right) - 2 \log \left( \frac{m^2}{\mu_E^2} \right) \right] \bar{\xi}(p')\frac{\not\epsilon}{u(p)} \]  

(A7)

Finally, computing the vertex correction in full QCD, we obtain

\[
\Gamma_{\text{QCD}} = -ig_s^2 C_F \mu^2 \epsilon \int \frac{d^dk}{(2\pi)^d} \frac{\gamma^\mu(p' + k + m)\not\epsilon}{((p + k)^2 - m^2)^2} \frac{\gamma^\mu(p' + k + m)}{((p' + k)^2 - m^2)} = \Gamma_{\text{SCET}} + O(m^2(n\cdot q)^2, p'^2(n\cdot q)^2) . \]  

(A8)

This exercise shows that the IR behavior of QCD is exactly reproduced in the SCET at the one-loop order; moreover, it also shows to what extent the effective theory computation reproduces the full QCD result.

Before concluding this appendix let us describe briefly the computation of the jet functions \( J \) and \( J_l \) introduced in Eqs. (28) and (36). The diagrams that produce the latter are exactly those drawn in Figs. 2(b)-(c) in which the ultra-soft field on the left is dropped and a collinear propagator is inserted on the right (as can be readily seen expanding explicitly the T-product in Eq. (34)). In fact, after subtracting the divergences (remember that \( J_l \) is defined as the vacuum-to-vacuum matrix element of a T-product of effective theory fields) \( \Gamma^{(b+c)}_C \) exactly reproduces Eq. (36).

The leading order contribution to \( J \) is simply the propagator of the collinear field while the \( O(\alpha_s) \) correction is given by the sum of three graphs. The first diagram is given by the diagram in Fig. 2(a) in which the ultra-soft field on the left is dropped and a collinear propagator is inserted on the right. The second diagram is obtained from the first one by moving the collinear propagator to the left side. These two diagrams yield identical results and their contribution to \( J \) is \( 1/(n\cdot k)\alpha_s C_F/(4\pi)(2L^2 - 4L + 8 - \pi^2/3) \). The third diagram is simply the one loop correction to the quark propagator (analogous to QCD). Its contribution was computed in Eq. (36) of Ref. [4] and yields \( 1/(n\cdot k)\alpha_s C_F/(4\pi)(L - 1) \). The sum of the three graphs reproduces Eq. (28).

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