FINANCING STRATEGIES FOR A CAPITAL-CONSTRAINED SUPPLIER UNDER YIELD UNCERTAINTY

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Abstract. We consider a supply chain consisting of a supplier and a distributor, in which the supplier has a capital constraint and faces productivity yield uncertainty. To solve the capital constraint problem, we propose an advance payment with risk compensation (APRC) mechanism, under which the distributor finances the supplier with an advance payment, and the supplier provides a price discount to compensate the distributor for the supplier’s bankruptcy risk. The optimal solutions are derived under the APRC mechanism and the results indicate that under the APRC, the whole supply chain performs as well as if there is no capital constraint, in terms of profits and optimal strategies. Therefore, the APRC is an efficient solution for the supplier’s capital constraint issue. In addition, when the deficit is big, the APRC provides an alternative financing arrangement and it can bring higher profits for both parties. Another very interesting finding is that, when the capital deficit is small, the supplier can do better with the bank loan financing, despite that a higher interest rate needs to be paid in this case.

1. Introduction. In a supply chain, due to capital constraints, the supplier may suffer a capital deficit during the production period, and the distributor may suffer a capital deficit during the sales season. The supplier or the distributor with capital deficit can get financed using bank loans. Another very common solution to the supplier’s capital deficit problem is the advance payment (or trade credit) from the downstream companies (the distributors). In practice, when the supplier faces capital deficit, the downstream firm (the distributor) is usually willing to offer an advance payment to the supplier, in order to secure the supply. When a distributor pays an advance payment, the supplier usually offers some price discount. For example, in order to guarantee the operation of suppliers, PSA Peugeot Citroen offers

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advance payments to suppliers to mitigate the suppliers’ capital deficit problems. Another example is that, in China, to stabilize the supply of coal, the downstream firms such as electricity companies, almost always offer some advance payments to the coal supply companies, and the coal companies typically offer price discounts on the pre-ordered quantity.

In addition to capital deficit problems, a supplier may have productivity yield uncertainty (Gurnani and Gerchak [12], Federgruen and Yang [11], Chen and Xiao [5], Li and Li [23]). For example, there are yield uncertainties in supply chains of the agricultural and forestry products due to the impacts of weather and other natural conditions. In addition, production systems with uncertain yield can be found in many other industries, such as the coal industry, the steel industry, the chemical industry (e.g., for the production of special chemicals or tailor made chemicals) and the electronics industry (e.g., for the production of special processors or silicon chips) (Keren [18], Hu et al. [13], Peng et al. [32], Caro et al. [4], Peng et al. [30]).

When a supplier faces an uncertain yield, the bank (or the creditor) who lends the money to the supplier may face a risk of the supplier’s bankruptcy when the yield is not as strong as expected. Because of the bankruptcy risk, it can be very difficult for the supplier to get loans from banks or other financial institutions. Therefore, it is very meaningful to consider other alternative financing strategies for suppliers with uncertain yield.

In this paper, we consider a supply chain in which the supplier has a capital constraint and faces a yield uncertainty. We first derive the optimal operation strategies for the supplier and the distributor with the bank loan financing. Then, we propose a mechanism of advance payment with risk compensation (APRC) under which the distributor provides an advance payment to the supplier, and the supplier offers a price discount to the distributor as a risk compensation. We derive the optimal strategies for the supplier and the distributor under the APRC. Further analysis indicates that in most situations (if the supplier’s capital deficit is not too small), the APRC mechanism will provide a perfect solution to the capital deficit issue and it can realize a Pareto improvement for the supply chain system over the bank loan financing. When the capital deficit is small, the supplier may do better with the bank loan financing, despite of the higher interest rate. This is a very interesting result, and we will give some explanations to it.

Suppliers’ capital constraint problem has considered by many researchers. Lai et al. [21] consider the effectiveness of the supply chain under the circumstances of the supplier’s capital constraint in reservation, delegate, and mixed forms. Mateut and Zanchettin [27] and Thangam [35] consider the optimal price discount and batch-ordering policy of perishable goods in supply chains with advance payments. Capital deficit, to some extent, may occur to both the supplier and the distributor in a supply chain at the same time. Raghavan and Mishra [33] point out that a bank is also willing to grant loans to the distributor as it does for the supplier. Kouvelis and Zhao [19] analyze the decisions involved in optimally structuring the trade credit contract from the supplier’s perspective, while the supplier needs to get a bank loan. They conclude that a risk-neutral supplier should always finance the distributor with a trade credit at rates less than or equal to the risk-free bank rate. Further, Kouvelis and Zhao [20] investigate contract design and coordination of a supply chain with one supplier and one retailer, both of which are capital constrained and in need of short-term financing for their operations. Peng et al. [31] propose a mutual-aid mechanism for a supply chain with capital constraints,
under which the whole supply chain will get a Pareto improvement comparing to the bank loan financing. Although there have been some literature on the financing strategies for the supplier with capital constraints, to our best knowledge, nobody has ever considered the financing problems for the supplier with a productivity yield uncertainty. In addition, we introduce the risk compensation for the advanced payment in our model, which is also new.

While we focus on the supplier’s deficit issue in this paper, some researchers also consider the distributor’s capital deficit issues. Buzacott and Zhang [2], Dada and Hu [10] and Yan and Sun [36] consider the situation where the distributor applies for loans from financial institutions. In those papers, the authors combine the asset-based financing with inventory decisions, and establish leader-follower games of the bank and the distributor based on the classical Newsvendor Model. Some others focus on comprehensive decisions in forms of trade credits from varies aspects, including trade credit and comprehensive decision of inventory (Huang et al. [14], Moussawi-Haidar and Jaber [28], Chen and Teng [6]); trade credit and comprehensive decision of optimal ordering quantity (Lou and Wang [24], Ouyang et al. [29], Annadurai and Uthayakumar [1]); optimal production quantity decision with trade credit (Lou and Wang [24], Teng et al. [34]); trade credit considered as endogenous variable and motivation tool to study supply chain coordination (Chen and Wang [8], Chern et al. [9], Lee and Rhee [22]). There is also literature comparing the bank loan financing with the trade credit financing. According to Cai et al. [3], complementary exists in these two financing methods when the distributor’s initial capital is at an extremely low level. Jing et al. [16] and Jing and Seidmann [17] show that when the production cost is relatively low, the trade credit financing can reduce the double marginal effect more effectively than the bank loan financing and otherwise the bank loan financing performs better. Chen [7] finds that in a wholesale price contract, the trade credit better integrates the channel than the bank credit by centralizing the manufacturer’s financing at the distribution. Yan et al. [37] design a partial credit guarantee contract for supply chain financing systems, incorporating the bank credit financing and the manufacturer’s trade credit guarantee. In this paper, we also compare the bank loan financing with the trade credit (advance payment), but we focus on the capital constrained supplier, instead of the distributor.

The contributions of this paper are listed as follows. First, to the authors’ best knowledge, this is the first paper to consider financing problems for the supplier with capital constraint and yield uncertainty. Secondly, we propose a APRC mechanism which can solve the supplier’s capital constraint problem very efficiently. We find that the profits for both the supplier and the distributor under the APRC are the same with those when the supply chain has no capital constraint. Thirdly, by comparing the optimal solutions under the APRC with those under the bank loan financing, we find that when the supplier’s capital deficit is not too small, the APRC will realize a Pareto improvement to the supply chain over the bank loan financing. The effect of the APRC is more significant when the supplier has a big deficit. Finally, when the capital deficit is small, the bank loan financing can bring a higher profit to the supplier, which is an interesting result.

Some extensions can be considered as topics for future research. For example, one may consider the supply chain with yield uncertainty in which both the supplier and the distributor have capital deficits. Further, one may consider the effects of the interest rates on the optimal strategies, profits and the associated bankruptcy
risks. Moreover, one can consider supply chains in which the supplier and/or the distributor have some monopolistic or oligopolistic power. In those cases, the supplier and/or the distributor will have the power to negotiate the prices, so the wholesale/retail prices should be treated endogenously as decision variables. We hope that this paper can pave ways to more research works on supply chain finance with yield uncertainties.

The rest of the paper is organized as follows. In Section 2, we set up the model and give some preliminary results. The bank loan financing is discussed in Section 3. Then in Section 4, we propose the APRC mechanism and derive the optimal strategies for the supplier and the distributor. In Section 5, we compare the solutions under the bank loan financing with the solutions under the APRC mechanism. Further discussions and some illustrating numerical results are also given in Section 5. We conclude the paper in Section 6.

2. Model formulation.

We consider a supply chain consisting of a supplier and a distributor. Assume that the supplier has a capital constraint and faces some capital deficit during the production period. Further, we assume that there is a yield uncertainty with the supplier’s productivity and the distributor faces a market demand uncertainty. In this paper, we focus on the supplier’s financing strategies, so we assume that there is no capital constraint for the distributor. In addition, we only consider the optimal strategies for the supplier and the distributor in the production period.

For convenience, we use subscript $s$ for the supplier, and subscript $d$ for the distributor. We assume that for one unit of the product, the retail price is $p$, the wholesale price is $w$ and the production cost is $c$. We also assume that the product scrap value at the end of sales period is zero, and both the supplier and the distributor are risk neutral.

Suppliers with random yields are usually small and medium enterprises in vulnerable and weak competitive positions. Most of the suppliers with random yields have little say in price decision and they have to sell the products at the market price or a given price. For example, in supply chains of the agricultural products, due to the market competitions, the farmers usually take the given market wholesale price (or negotiates a little bit from a given wholesale price) and the retailer sells the agriculture products at the market price. Neither of them can freely choose the wholesale prices or the market prices. For this reason, many researchers who study supply chain management with random yields consider the wholesale price and the retail price as fixed values (Gurnani and Gerchak [12], Keren [18], Federgruen and Yang [11], Hu et al. [13], Peng et al. [32], Caro et al. [4], Chen and Xiao [5], Li and Li [23], Luo and Chen [26]). So, we consider the wholesale price $w$ and the retail market price $p$ both to be exogenous and fixed values in this paper. For supply chains in which suppliers and/or distributors have some monopolistic or oligopolistic market powers, the prices should be considered as endogenous and they may be treated as decision variables. Those topics are out of the scope of this paper and they can be topics for future research.

Let $X$ be the random variable stands for the supplier’s productivity yield. That means, if the supplier’s planned production quantity is $q$, then the actual production quantity is $qX$ at the end of the production period. $X > 1$ means that the production quantity is higher than the objective and $X \leq 1$ means the production quantity is less than the objective. This productivity yield uncertainty is very common in the real world. For example, a farmer may expect to get 10 tons of beans
from his farmland. However, at the end, due to the weather and other factors, the production quantity of the beans may higher or lower than 10 tons. Apparently, there is a productivity yield uncertainty. Productivity uncertainty is also very common in coal mining industry, which has some effects on the coal-electricity supply chain. We assume that $X$ is a positive continuous type random variable and its probability density function is $f(x)$, its distribution function is $F(x)$, and its survival function is $F(x)$. We know that $F(x)$ is monotonically non-decreasing, and $F(x)$ is monotonically non-increasing.

In addition, we assume that the market demand, $Y$, for the product is also uncertain. Assume that $Y$ is a positive, continuous random variable with probability density function $g(y)$, distribution function $G(y)$, and survival function $G(y)$. We know that $G(y)$ is monotonically non-decreasing, and $G(y)$ is monotonically non-increasing.

We further assume that the yield uncertainty $X$ and the demand $Y$ both follow increasing generalized failure rate (IGFR) distributions, that is, $h_X(x) = x f(x) / F(x)$ and $h_Y(y) = y g(y) / G(y)$, are monotonically increasing (or non-decreasing). It is easy to verify that the widely used increasing failure rate (IFR) distributions are all IGFR distributions, but it is not true vice versa.

The distributor places an order of $q_d$ before the production period. $q_d$ is the decision variable of the distributor, who needs to choose an optimal $q_d$ to maximize the expected profit, taking the demand uncertainty and the productivity yield uncertainty into consideration.

After the distributor places an order $q_d$, the supplier chooses a production quantity $q_s$ at the beginning of the production period. Because of the yield uncertainty, the actual production quantity is $q_s X$ at the end. $q_s$ is the decision variable of the supplier and the supplier needs to choose the optimal value of $q_s$ to maximize the expected profit.

We assume that the initial capital of the supplier is $\zeta$ and $\zeta < cq_1^*$, where $q_1^*$ is the optimal product quantity under the bank loan financing. So, the supplier faces a capital deficit (Buzacott and Zhang [2]). In this paper, we focus on the effect of the supplier’s capital deficit and assume that the initial capital of the distributor is enough.

Usually, the supplier tends to finance the capital deficit by bank loans. We will derive the optimal strategies under the bank loan financing in Section 3. Then we propose an advance payment with risk compensation (APRC) mechanism, in which the distributor will provide an advance payment to finance the supplier’s capital deficit. The supplier will offer a price discount (as a risk compensation), with a price discount factor of $r_d$ to the distributor. We will derive the optimal strategies for the supplier and the distributor under the APRC mechanism. Details will be given in Section 4.

Before we move to the next section, we present two lemmas that will be frequently used later.

**Lemma 2.1.** Let $Z_1 \geq 0, Z_2 \geq 0$ be two independent random variables with survival functions $F_1, F_2$ respectively. Assume that $\lambda > 0, u > 0$ are two constants. Then we have

$$E[\min\{u, \lambda Z_1\}] = \lambda \int_0^u F_1(x)dx = \int_0^u F_1\left(\frac{x}{\lambda}\right)dx,$$  \hspace{1cm} (1)
\[ \mathbb{E}[\min\{Z_1, u, \lambda Z_2\}] = \int_0^u F_1(x) F_2 \left( \frac{x}{\lambda} \right) dx. \]  
(2)

(Proof of Lemma 2.1 is in Appendix A.1.)

**Lemma 2.2.** Define \([a - b]^+ = \begin{cases} a - b, & \text{if } a - b \geq 0; \\ 0, & \text{if } a - b < 0. \end{cases}\) Then we have \([a - b]^+ = a - \min(a, b)\) for any \(a, b\).

The proof is straightforward, and we omit it here.

### 3. Bank loan financing.

In this section, we consider the situation that the supplier’s capital deficit can only be financed by bank loans. We use subscript 1 for this situation. When the supplier gets financed through a bank, we now have a supply chain financing system that consists of a supplier, a distributor and a bank.

Typically, after the distributor chooses the order quantity \(q_{d1}\), the supplier will choose the optimal production quantity \(q_{s1}\) based on \(q_{d1}\). Then, once \(q_{d1}, q_{s1}\) are known, the bank can evaluate the bankruptcy risk of the supplier and determines the nominal rate \(r_b\) for the loan to the supplier. This forms a sequential game problem and we can use the standard backward induction to solve the problem. The first step is to figure out how the bank decides the nominal rate for the loan to the supplier, based on any given \(q_{d1}\) and \(q_{s1}\). The second step is to figure out the supplier’s optimal strategy to determine \(q_{s1}\) for any given \(q_{d1}\), assuming that the supplier knows the strategy of the bank choosing the nominal rate. The final step is to derive the optimal strategy of the distributor on \(q_{d1}\), assuming that the distributor knows the optimal decisions of the supplier and the bank.

First, we figure out the interest rate decision for the bank. In general, a bank charges the same interest rate for risk-free loans. We assume that the loan interest rate is \(r\). However, if the borrower has some default risk (such as bankruptcy risk), the bank usually charges a higher nominal rate for risk compensation. In our model, due to the yield uncertainty, there is a bankruptcy risk with the supplier. Here we follow Kouvelis and Zhao [19, 20], and assume that the capital market is perfect that all bank loans are competitively priced, which implies that the bank’s nominal interest rate \(r_b\) is chosen so that the bank is indifferent between issuing the loan to the supplier and issuing the loan to the firms without bankruptcy risk (i.e., earning the risk-free rate \(r\)). Therefore, the bank will charge a nominal rate of \(r_b\), such that the expected return rate is \(r\) (Kouvelis and Zhao [19, 20]; Peng et al. [31]). So, the nominal rate \(r_b\) is determined by

\[ (1 + r)(cq_{s1} - \zeta) = \mathbb{E}[w \min\{q_{d1}, q_{s1}, X\}, (1 + r_b)(cq_{s1} - \zeta)], \]

where \(w \min\{q_{d1}, q_{s1}, X\}\) is the sales income and \((1 + r_b)(cq_{s1} - \zeta)\) is the principal and interest of the bank loan.

#### 3.1. The supplier’s optimal strategy.

Now we consider the optimization problem faced by the supplier. Due to the capital constraint, the supplier needs to apply for a bank loan with the amount of \(cq_{s1} - \zeta\). The expected profit of the supplier is

\[ \pi_{s1}(q_{s1}) = \mathbb{E}[w \min\{q_{d1}, q_{s1}, X\} - (1 + r_b)(cq_{s1} - \zeta)]^+ - \zeta, \]

where \(w \min\{q_{d1}, q_{s1}, X\} - (1 + r_b)(cq_{s1} - \zeta)]^+\) is the supplier’s terminal cash flows. If the yield uncertainty results in a bad revenue such that the supplier cannot repay
the principal and interest of the bank loan, the supplier will go bankrupt, and the residual equity becomes zero. Define
\[
\theta_1 = \frac{(1 + r_b)(cq_{s1} - \zeta)}{wq_{s1}},
\]
(5)
it is easy to check that \( \theta_1 \) is the critical value of \( X \), and \( X < \theta_1 \) will cause the supplier to go bankrupt.

According to the standard backward induction, the bank’s decision on the loan interest rate (see (3)) should influence the supplier’s decision. The decision model for the supplier is:
\[
\max_{q_{s1}} \pi_{s1}(q_{s1}), \quad \text{subject to } (3).
\]
(6)
By virtue of (4), (3), Lemma 2.1 and Lemma 2.2, we can get
\[
\pi_{s1}(q_{s1}) = \mathbb{E}[w \min\{q_{d1}, q_{s1}, X\}] - \mathbb{E}[\min\{w \min\{q_{d1}, q_{s1}, X\}, (1 + r_b)(cq_{s1} - \zeta)\} - \zeta

= w \int_{q_{d1}}^{q_{s1}} F\left(\frac{x}{q_{s1}}\right) dx - (1 + r_b)(cq_{s1} - \zeta) - \zeta

= wq_{s1} \int_{q_{d1}}^{q_{s1}} F(x) dx - (1 + r_b)(cq_{s1} - \zeta) - \zeta.
\]
(7)
We have the following result:

**Proposition 3.1.** Under the bank loan financing, for any given order quantity \( q_{d1} \), the supplier’s optimal production quantity \( q_{s1}^* \) is given by
\[
q_{s1}^* = k_1 q_{d1},
\]
(8)
where \( k_1 \) satisfies
\[
\int_0^{q_{d1}} x f(x) dx = \frac{(1 + r)c}{w}.
\]
(9)
The supplier’s optimal expected profit \( \pi_{s1}^* \) is given by
\[
\pi_{s1}^*(q_{s1}^*) = wq_{d1} F\left(\frac{1}{k_1}\right) + r\zeta.
\]
(10)
(Proof of Proposition 3.1 is in Appendix A.2.)

**Remark 1.** As we can see from (8) and (9), the bank’s decision on the expected interest rate \( r \) does have an effect on the supplier’s optimal strategy. Moreover, since the bank is risk-neutral, the nominal interest rate \( r_b \) is chosen based on (3) and eventually, it is the expected interest rate \( r \) instead of the nominal rate \( r_b \) that has an effect on the supplier’s optimal production quantity \( q_{s1} \). Further, as we will see in the next subsection, the supplier’s optimal strategy (see (8)) will affect the distributor’s optimal ordering strategy (see (14)). So, the bank’s decision of the interest rate actually influence both the supplier’s and the distributor’s decisions directly or indirectly.
3.2. The distributor’s optimal strategy. Now we consider the optimization problem faced by the distributor. The expected profit function of the distributor is
\[ \pi_{d1}(q_{d1}) = E[p \min\{Y, q_{d1}, q_{s1}, X\} - w \min\{q_{d1}, q_{s1}, X\}] \]
\[ = \int_0^{q_{d1}} \int_0^{\infty} y f(x)g(y)dydx + \int_{q_{d1}}^{\infty} \int_{q_{s1}}^{\infty} q_{s1}q_{s1}x f(x)g(y)dydx \]
\[ + \int_{q_{d1}}^{\infty} \int_{q_{s1}}^{\infty} q_{d1}q_{s1}x f(x)g(y)dydx - w \int_0^{q_{d1}} F\left(\frac{x}{q_{s1}}\right) dx, \tag{11} \]
where \( p \min\{Y, q_{d1}, q_{s1}, X\} \) is the sales revenue and \( w \min\{q_{d1}, q_{s1}, X\} \) is the wholesale cost. By virtue of Lemma 2.1, we can get
\[ \pi_{d1}(q_{d1}) = p \int_0^{q_{d1}} G(x)F\left(\frac{x}{q_{s1}}\right) dx - w \int_0^{q_{d1}} F\left(\frac{x}{q_{s1}}\right) dx \]
\[ = \int_0^{q_{d1}} [pG(x) - w]F\left(\frac{x}{q_{s1}}\right) dx. \tag{12} \]

We assume that the distributor makes the decision (choosing the optimal \( q_{d1} \)) based on the assumption that, for any given ordering quantity \( q_{d1} \), the supplier will choose the optimal planned production quantity \( q^*_s \) given by (8). The bank’s decision on loan interest rate does not influence the distributor’s decision directly. It influences the ordering decision through the decision of the optimal production quantity. So the decision model for the distributor is:
\[ \max_{q_{d1}} \pi_{d1}(q_{d1}), \quad \text{subject to} \quad q^*_s = k_1 q_{d1}, \tag{13} \]
where \( k_1 \) is given by (9). We have the following result:

Proposition 3.2. Under the bank loan financing, the distributor’s optimal ordering quantity \( q^*_{d1} \) satisfies
\[ \frac{1}{k_1} [pG(q_{d1}) - w] + k_1 \int_0^{\frac{1}{k_1}} [pG(k_1 q_{d1}^*) x - w]xf(x)dx = 0, \tag{14} \]
where \( k_1 \) is given by (9).

(Proof of Proposition 3.2 is in Appendix A.3.)

From (12), (13) and (14), we can see that the optimal strategy and the maximal profit of the distributor do not depend on the supplier’s initial capital \( \zeta \). Actually, they depend on \( k_1 \), which is determined by (9) and does depend on \( r \).

4. Advance payment with risk compensation. There are two issues with the bank loan financing for the supplier. First, the bank charges interest on the loan to the supplier, and it causes some extra financing cost to the supplier. Secondly, due to the yield uncertainty, the supplier’s default (bankruptcy) risk may be too high for the bank to issue a loan. Even if the risk can be compensated with the higher nominal interest rate, sometimes, the high bankruptcy risk may disqualify the supplier to get a loan from the bank due to bank regulations. Therefore, if the bank does not know too much about the supplier, or the bankruptcy risk is too high, the bank may not be able to issue the loan to the supplier.

Here, we propose an advance payment with risk compensation (APRC) mechanism, an alternative financing solution to the supplier that can solve the two issues
mentioned above. The APRC works as follows. Before the supplier’s initial production period, the distributor provides an advance payment to the supplier so as to finance the supplier’s capital deficit. The supplier offers price discount to the distributor as a risk compensation.

Since the distributor orders the product from the supplier, as a stake holder, the distributor may be willing to finance the supplier’s capital deficit. The risk is compensated by a discounted wholesale price. Unlike the bank loan financing, there is no regulations prohibiting the distributor to provide an advance payment to the supplier, even if the supplier’s bankrupt risk is high. Further, as well-established longterm partners, the supplier and the distributor usually know each other very well, and the distributor may be willing to offer an advance payment as a financing solution to the supplier. Finally, unlike the bank loan financing, the APRC is a financing solution within the supply chain, and it does not involves any third party. Therefore, the APRC financing is usually easier to implement and it brings no extra financing cost to the supplier.

Before we get the optimal strategies under the APRC, we first present the results when there is no capital deficit for comparison purpose. Later we will show that the results under the APRC are the same with those without any capital deficits. So the proposed APRC can solve the supplier’s capital deficit issue efficiently.

4.1. Solutions without capital deficits. Assume that the supplier does not have any capital deficit and we use the subscript 0 for variables in this case. With adequate capital, the supplier’s profit is simply

\[ \pi_s(q_s) = \mathbb{E}[w \min\{q_d, q_s X\}] - cq_s. \] (15)

By virtue of Lemma 2.1, we can get

\[ \pi_s(q_s) = wq_s \int_0^{q_d} F(x)dx - cq_s. \] (16)

Comparing (16) to (7), we can see that if we take \( r = 0 \) in (7), it will be in the same form as (16), except that \( q_s, q_d \) are replaced by \( q_s, q_d \), respectively. Therefore, similar to Proposition 3.1, we can get:

**Proposition 4.1.** With adequate capital, for any given order quantity \( q_d \), the supplier’s optimal production quantity \( q^*_s \) is given by

\[ q^*_s = k_0 q_d, \] (17)

where \( k_0 \) satisfies

\[ \int_0^{1/k_0} xf(x)dx = \frac{c}{w}. \] (18)

The supplier’s optimal expected profit \( \pi^*_s \) is given by

\[ \pi^*_s(q^*_s) = wq_d F \left( \frac{1}{k_0} \right). \] (19)

Further, as we discussed in Subsection 3.2, the supplier’s capital does not have any effect on the decision of the distributor, and the supplier’s financing cost \( r \) only influences the distributor’s decision through the constant \( k_1 \) determined by (9). Therefore, similar to (12) and Proposition 3.2, we have

\[ \pi_d(q_d) = \int_0^{q_d} [pG(x) - w]F \left( \frac{x}{q_s} \right)dx, \] (20)
and the following result:

**Proposition 4.2.** With adequate capital, the distributor’s optimal ordering quantity \( q^*_d \) is given by

\[
F \left( \frac{1}{k_0} \right) [pG(q^*_d) - w] + k_0 \int_0^{\theta} [pG(k_0 q^*_d x) - w] x f(x) dx = 0,
\]

where \( k_0 \) is given by (18).

4.2. The supplier’s optimal strategy under the APRC. The optimization problem is a two-player game under the APRC, in which the distributor is the leader and the supplier is the follower. We use subscript 2 for variables under this situation. We start with the optimal strategy for the supplier first, then we derive the optimal strategy for the distributor.

Under the APRC, the supplier requests the distributor to prepay part of the order with the amount of \( cq^*_s - \zeta \). As a compensation to the bankruptcy risk, the supplier offers the distributor a discounted wholesale price, and we call it the pre-order price. Here we assume that the pre-order price is \( w + r_d \), where \( r_d \) is the discount factor (Jiang and Hao [15]). So the pre-order quantity of the products is \( (1 + r_d)(c q^*_s - \zeta) \). The supplier’s terminal cash flow is

\[
w \left[ \min\{q^*_d, q^*_s X\} - \frac{(1 + r_d)(c q^*_s - \zeta)}{w} \right]^+ = [w \min\{q^*_d, q^*_s X\} - (1 + r_d)(c q^*_s - \zeta)]^+.
\]

Define

\[
\theta_2 = \frac{(1 + r_d)(c q^*_s - \zeta)}{w q^*_s}.
\]

Then it is easy to verify that when \( X < \theta_2 \), the supplier would not be able to supply enough products to cover the pre-order quantity of the products. So \( \theta_2 \) is a critical value of the supplier’s productivity yield under the APRC.

From (22), we can see that the discount factor \( r_d \) can be treated as the interest rate of the advance payment to compensate the bankruptcy risk of the supplier.

When the supplier has no yield uncertainty risk, the distributor always offers advance payment without interest (Zhang et al. [38]). Similar to the assumptions of Kouvelis and Zhao [19, 20], we assume that the risk compensation discount factor \( r_d \) is chosen so that the distributor is indifferent between making advance payments to the supplier and to the firms which has no bankruptcy risk. Therefore, \( r_d \) satisfies

\[
c q^*_s - \zeta = E[\min\{w \min\{q^*_d, q^*_s X\}, (1 + r_d)(c q^*_s - \zeta)\}].
\]

Using (22), we can get that the supplier’s expected profit under the APRC is

\[
\pi_{s_2}(q^*_s) = E[\min\{w \min\{q^*_d, q^*_s X\} - (1 + r_d)(c q^*_s - \zeta)\}^+] - \zeta.
\]

By virtue of Lemma 2.2, (24) and Lemma 2.1, we can rewrite (25) as

\[
\pi_{s_2}(q^*_s) = E[w \min\{q^*_d, q^*_s X\} - \min\{w \min\{q^*_d, q^*_s X\}, (1 + r_d)(c q^*_s - \zeta)\}] - \zeta
\]

\[
= E[w \min\{q^*_d, q^*_s X\}] - c q^*_s = w q^*_s \int_0^{\theta_2} F(x) dx - c q^*_s.
\]

We have the following result:
Proposition 4.3. Under the APRC, for any given ordering quantity \( q_{d_2} \), the supplier’s optimal production quantity \( q_{s_2}^* \) is given by:

\[
q_{s_2}^* = k_2 q_{d_2},
\]

(27)

where \( k_2 \) satisfies

\[
\int_0^{\frac{c}{w}} xf(x) dx = \frac{c}{w}.
\]

(28)

In addition, the supplier’s maximal expected profit \( \pi_{s_2}^* \) is given by

\[
\pi_{s_2}^* (q_{s_2}^*) = w q_{d_2} F \left( \frac{1}{k_2} \right).
\]

(29)

The proof is very similar to the proof of Proposition 3.1, so we omit it.

From Proposition 4.1 and Proposition 4.3, we can get that the optimal strategy and the profit function of the supplier under the APRC are the same with those when the supplier has enough capital. So the APRC mechanism can relieve the supplier’s financial cost significantly.

It seems like the supplier would always participate the APRC mechanism instead of using a bank loan. However, although this is true for most situations, there are some exceptions. Our research results indicate that under certain conditions, the supplier can achieve a higher profit by using the bank loan financing instead of the APRC. This result seems counterintuitive, but there is a reason for that. More details will be given in Section 5.

4.3. The distributor’s optimal strategy under the APRC. Now let us consider the optimal strategy of the distributor under the APRC. In this case, the distributor pays an advance payment of \( cq_{s_2} - \zeta \) and receives a price discount with the price discount factor, \( r_d \). By virtue of (22), we can get the profit function of the distributor:

\[
\pi_{d_2} (q_{d_2}) = E \left\{ p \min\{Y, q_{d_2}, q_{s_2}X\} - [w \min\{q_{d_2}, q_{s_2}X\} - (1 + r_d)(cq_{s_2} - \zeta)]^+ \right\} - (cq_{s_2} - \zeta).
\]

(30)

On the right hand side of the above equation, the first term is the sales revenue of the products, the second term is the wholesales expenses of the products beyond the pre-order products and the third item is the advance payment amount. Plugging (24) into (30) and using (2), we can get that

\[
\pi_{d_2} (q_{d_2}) = E[p \min\{Y, q_{d_2}, q_{s_2}X\} - w \min\{q_{d_2}, q_{s_2}X\}] + E[w \min\{q_{d_2}, q_{s_2}X\}] - \left( 1 + r_d \right) \left( cq_{s_2} - \zeta \right) - (cq_{s_2} - \zeta)\]

\[
= E[p \min\{Y, q_{d_2}, q_{s_2}X\} - w \min\{q_{d_2}, q_{s_2}X\}] - \left( 1 + r_d \right) \left( cq_{s_2} - \zeta \right)
\]

\[
= \int_0^{q_{d_2}} \int_0^{\infty} y f(x)g(y) dx dy + \int_0^{q_{d_2}} \frac{q_{s_2}}{q_{s_2}} \int_0^{\infty} x f(x) g(y) dy dx
\]

\[
+ \int_0^{\infty} \int_0^{q_{d_2}} q_{d_2} f(x) g(y) dy dx - w \int_0^{q_{d_2}} F \left( \frac{x}{q_{s_2}} \right) dx,
\]

\[
= \int_0^{q_{d_2}} \left[ pq_G(x) - w \right] \mathcal{F} \left( \frac{x}{q_{s_2}} \right) dx.
\]

(31)

From (31) and (20), we can see that the distributor’s profit function under the APRC has the same form with that under the case of no capital deficit.
The distributor’s goal is to maximize the expected profit \( \pi_{d2} \) by choosing the optimal ordering quantity \( q_{d2}^* \) based on the assumption that, for any given ordering quantity \( q_{d2} \), the supplier will choose the optimal planned production quantity \( q_{s2}^* \) given by (27). So the decision model for the distributor is:

\[
\max_{q_{d2}} \pi_{d2}(q_{d2}), \quad \text{subject to} \quad q_{s2}^* = k_2 q_{d2},
\]

where \( k_2 \) is given by (28). We have the following result for the distributor’s optimal ordering quantity:

**Proposition 4.4.** Under the APRC, the distributor’s optimal ordering quantity \( q_{d2}^* \) satisfies

\[
\mathbb{P} \left( \frac{1}{k_2} \left[ p \mathcal{G}(q_{d2}^*) - w \right] + k_2 \int_0^{q_{d2}^*/k_2} \left[ p \mathcal{G}(k_2 q_{d2}^* x) - w \right] x f(x) dx = 0. \]

(33)

The proof is very similar to the proof of Proposition 3.2, so we omit it.

From (27) and (33), we can see that, under the APRC (equivalent to \( r = 0 \) case), the optimal decision of the production and ordering quantity, \( q_{s2}^* \) and \( q_{d2}^* \) do not depend on the supplier’s initial capital \( \zeta \).

**Remark 2.** We can see that the profits and the strategies of the supply chain under the APRC are the same with those when the supplier has enough capital. The results show that the proposed APRC program can solve the capital deficit problem. Therefore, the APRC is a very good supply chain financing mechanism for the supplier’s capital deficit issue. In addition, there is an essential difference between the bank loan financing and the APRC for the supplier. The bank loan financing is an external financing model and the APRC is an internal financing mechanism of the supply chain so it is relatively easy to implement.

Some researchers show that the trade credit (Cai et al.[3], Jing et al. [16], Jing and Seidmann [17], Kouvelis and Zhao [19]) and advance payment (Thangam [35], Zhang et al. [38]) may be better than bank credit under certain conditions. But the strategies and profits of the capital constrained supply chain cannot achieve the level when the supplier has enough capital. However, our result shows that the profits and the strategies of the supply chain under the mechanism of advance payment with risk compensation (APRC) are the same with those when the supplier has enough capital.

5. Discussions and numerical results. In this section, we compare the results under the bank loan financing and the results under the APRC mechanism. Here we denote the expected interest rate as \( \tau \), while \( \tau = r \) for the bank loan financing and \( \tau = 0 \) for the APRC mechanism. For qualitative analysis and comparison purposes, in this section we use variables \( \pi_s, \pi_d, q_s, q_d, k, \tau \) such that \( \pi_s = \pi_{s1}, \pi_d = \pi_{d1}, q_s = q_{s1}, q_d = q_{d1}, k = k_1, \tau = \tau_1 \) under the bank loan financing, and \( \pi_s = \pi_{s2}, \pi_d = \pi_{d2}, q_s = q_{s2}, q_d = q_{d2}, k = k_2, \tau = 0 \) under the APRC.

Meanwhile, for illustration purpose, we present a numerical example here. We consider a supply chain with a supplier and a distributor, in which the product demand \( Y \) is uniformly distributed over \([0, 1000]\) and the productivity yield random variable \( X \) is uniformly distributed over \([0, 1.5]\). The parameters of the supply chain are as follows: \( p = 1000, w = 650, c = 300, r = 0.06 \). The numerical results are presented in Figure 1 to Figure 6 and those results will be discussed along with our qualitative analysis results.
We first present a lemma which will be used later.

**Lemma 5.1.** The distributor’s optimal ordering quantity $q_d^*$ satisfies:

$$pG(q_d^*) - w < 0.$$  \hspace{1cm} (34)

(Proof of Lemma 5.1 is in Appendix A.4.)

5.1. **Ordering and production quantities.** From (9) and (28), we can see that $k_2 > k_1$. Since the supplier’s optimal strategy is always $q_s_i = k_i q_d_i$, $i = 1, 2$, $k_2 > k_1$ means that, with the same order quantity, the supplier is going to produce more products under the APRC than that under the bank loan financing. In other words, the supplier would have a higher production enthusiasm under the APRC than that under the bank loan financing. Actually, we have the following results:

**Proposition 5.2.** Assume that the demand $Y$ follows an IGFR distribution (see Section 2). Then the optimal production quantity is higher under the APRC, i.e. $q_{s_2}^* > q_{s_1}^*$. If we further assume that the yield uncertainty $X$ follows an IGFR distribution, then the optimal ordering quantity is lower under the APRC, i.e. $q_{d_2}^* < q_{d_1}^*$.

(Proof of Proposition 5.2 is in Appendix A.5.)

The numerical results showed in Figure 1 are consistent with Proposition 5.2. Moreover, from Figure 1, we can get that, under the bank loan financing, when the expected interest rate $r$ increases, $q_{s_1}^*$ decreases and $q_{d_1}^*$ increases, and vice versa. Therefore, as the financing cost $r$ increases, the supplier tends to produce less. To encourage the supplier to produce more, the distributor tends to order more. In addition, we can see that the ratio of $k = \frac{q_s}{q_d}$ is a decreasing function of $r$, which further confirms that as the financing cost $(r)$ increases, the supplier would choose to produce less. Finally, from Figure 1, we can also get that when $r = 0$, the values are the same with those under the APRC, which indicates that the APRC is a solution for the supplier’s capital deficit issue.
In addition, we consider the sensitivity of the optimal decisions under the APRC with respect to the whole sale price \( w \) and the production cost \( c \). The numerical
results are given in Figure 2 and Figure 3. From (28), we can see that $k_2$ increases with $w$ and decreases with $c$. In other words, with the same order quantity, the supplier is going to produce more products with higher wholesale price or lower production cost. Figure 2 indicates that, under the APRC, the distributor’s ordering quantity $q_{d2}$ and the supplier’s planned production quantity $q_{s2}$ both decrease as the wholesale price $w$ increases. Higher wholesale price means a higher cost for the distributor, so it is not surprising that the distributor will order less under a higher wholesale price. In addition, although $k_2$ increases with $w$, the production quantity of the supplier may decrease with the wholesale price due to the lower ordering quantity of the distributor.

Figure 3 implies that the production quantity of the supplier decreases with the production cost. However, when supplier’s production cost increases, the distributor may order more production to encourage the supplier produce more.

5.2. Price discount factor. Now we consider the price discount factor $r_d$ under the APRC. We have the following result.

**Proposition 5.3.** The price discount factor $r_d$ decreases with the amount of the supplier’s initial capital $\zeta$, i.e. $\frac{dr_d}{d\zeta} < 0$.

(Proof of Proposition 5.3 is in Appendix A.6.)

The intuition behind Proposition 5.3 is that, the less the supplier’s initial capital is, the higher bankruptcy risk the distributor faces, so the supplier has to provide more price discount to compensate the bankruptcy risk. Moreover, from (42) in Appendix A.6, we can see that the price discount factor $r_d$ also depends on the yield uncertainty ($\tilde{F}(\theta_2)$).

From Figure 4, we can see that, in general, the financing cost under APRC mechanism ($r_d$) is always lower than that under the bank loan financing ($r_b$). In other words, the APRC mechanism can decrease the financing cost for the supplier.
5.3. Profit of the distributor. Comparing (31) with (12), we can see that the distributor’s expected profit under the APRC has the same form with that when the supplier does not request an advance payment.

We assume that both the supplier and the distributor are risk-neutral. So as long as their expected profits under the APRC mechanism are no less than their profits under the bank loan financing, they will participate in the APRC mechanism. More detailed discussions on the supplier’s profit will be given in subsection 5.4. Here we consider the distributor first. We have the following result:

**Proposition 5.4.** Under the APRC, the optimal profit of the distributor is higher than that under the bank loan financing. That is, \( \pi^*_d(q^*_s, q^*_d) > \pi^*_d(q^*_s, q^*_d) \).

(Proof of Proposition 5.4 is in Appendix A.7.)

From the proof of Proposition 5.4, we can see that the distributor’s profit is an increasing function of \( k \). From (9) and (28), we can get that \( k \) is an decreasing function of \( r \). The intuition is that the lower the interest rate, the more products the supplier is willing to produce, and this will benefit the distributor.

Proposition 5.4 indicates that a risk-neutral distributor should participate in the APRC mechanism and benefits from the APRC mechanism. In addition, the APRC mechanism can stabilize the product supply, which also benefits the distributor. Moreover, by participating in the APRC, the supplier can finance the capital deficit with a lower expected interest rate (\( \bar{r} = 0 \)) compared to the bank loan financing (\( \bar{r} = r > 0 \)). It seems like the supplier will always want to participate in the APRC. However, an interesting finding is that this is not always true. Actually, when the initial capital deficit is relatively small, the supplier can achieve a higher profit with the bank loan financing than that with the APRC financing. Next we will discuss it in details.

5.4. The supplier’s financing choice: Bank or APRC?. As we have discussed, by participating in the APRC, the supplier can finance the capital deficit with a zero expected interest rate. However, the saving on interest may not always improve the supplier’s expected profit.

Now we compare the supplier’s maximal profits under the two situations. From (19) and (29), we can see that the expected profit of the supplier under the APRC has the same form with that when the supplier has no capital deficit. Therefore, \( \pi^*_s \) is also the supplier’s maximal profit when there is no capital deficit.

From (10) and (29), we can see that under the bank loan financing, the supplier’s optimal profit is a linear increasing function of the initial capital \( \zeta \), while the supplier’s optimal profit does not depend on \( \zeta \) under the APRC. Therefore, when \( \zeta \) is relatively large, the supplier’s profit under the APRC may be less than that under the bank loan financing. In this case, the supplier may not participate in the APRC mechanism. Actually, we have the following result:

**Proposition 5.5.** Under the APRC, the supplier’s maximal profit \( \pi^*_s \) satisfies

\[
\begin{align*}
\pi^*_s & \geq \pi^*_s, & \text{if } \zeta \leq \zeta_0, \\
\pi^*_s & < \pi^*_s, & \text{if } \zeta > \zeta_0,
\end{align*}
\]

where \( \zeta_0 \) is given by

\[
\zeta_0 \equiv \frac{w}{r} \left[ q^*_d \mathcal{F} \left( \frac{1}{k_2} \right) - q^*_d \mathcal{F} \left( \frac{1}{k_1} \right) \right],
\]

(36)

\( k_1, k_2, q^*_d \) and \( q^*_d \) are given by (9), (28), (14) and (33), respectively.
The proof is very straightforward, so we omit it here.

From Proposition 5.5, we get a very interesting result that, when the supplier’s initial capital is relatively large ($\zeta > \zeta_0$), or equivalently, when the capital deficit is relatively small, the supplier’s profit under the bank loan financing will be higher than that under the APRC despite of the zero financing cost. It is inconsistent with the results for supply chains without yield uncertainty (Thangam [35], Zhang et al. [38]). Intuitively, because of financing cost, the profit of a supplier with capital deficit should be less than that when the supplier has sufficient capital. Therefore, the result of Proposition 5.5 seems to be counterintuitive. However, there is a reason behind that. When the supplier borrows money from the bank, the supplier pays interest, and the optimal strategy is to choose $k_1$ instead of $k_2$. In this case, the distributor will place an order $q_d$ based on the assumption that $q_s^* = k_1 q_d$ instead of $q_s^* = k_2 q_d$. Based on the results in Proposition 5.2, we know that $q_s^*$ will be bigger if the supplier chooses $k_1$ (bank loan financing) instead of $k_2$ (APRC or no capital deficit). Therefore, by using the bank loan financing, the supplier will get a bigger order from the distributor. When the benefit of the bigger order surpasses the extra interest cost, the supplier will achieve a higher profit by choosing the bank loan financing. This is only true when the deficit is relatively small so that the supplier’s interest cost does not exceed the benefit of the bigger order quantity.

For the numerical example we consider, we can get that $\zeta_0 = 99235$, which is very close to the total production cost $cq_s^* = 111900$. From Figure 5, we can see that, when $\zeta > \zeta_0 = 99235$, $\pi_{s_2}^* < \pi_{s_1}^*$. Therefore, it is better for the supplier to participate in the APRC, when the capital deficit is not very small (or equivalently $\zeta < \zeta_0$). In this case, by participating the APRC, both the supplier and the distributor can achieve higher profits. Therefore, the APRC can realize a Pareto
improvement of the whole supply chain. On the other hand, when the capital deficit is small (when $\zeta > \zeta_0$), it is better for the supplier to use the bank loan financing, and the APRC would not work in this case.

5.5. Bankruptcy risk of the supplier. Because of the yield uncertainty, the supplier faces a bankruptcy risk under the bank loan financing or the APRC financing. We use the bankruptcy probability to measure the bankruptcy risk of the supplier. That is

$$u = P(X < \theta) = F(\theta),$$

(37)

where $\theta$ is the critical value of $X$. For the bank loan financing case, $\theta = \theta_1$, and for the APRC case, $\theta = \theta_2$. Because $F(x)$ is monotonically non-decreasing, the bankruptcy risk of the supplier, $u$ is non-decreasing with respect to $\theta$. We have the following results regarding the relation between the bankruptcy probability and the initial capital.

**Proposition 5.6.** The bankruptcy risk of the supplier, $u$ decreases with the initial capital of the supplier $\zeta$, i.e. $\frac{du}{d\zeta} \leq 0$.

(Proof of Proposition 5.6 is in Appendix A.8).

The intuition behind Proposition 5.6 is that, the less the supplier’s initial capital is, the higher bankruptcy risk the supplier faces. The numerical results showed in Figure 6 are consistent with Proposition 5.6. In addition, from Figure 6, we can also see that, the bankruptcy risk of the supplier is higher under the bank loan case when the initial capital of the supplier is not relatively high ($\zeta \leq 90000$), and it is higher under the APRC case when the capital deficit is small ($\zeta > 90000$). As we can see here, the supplier’s bankruptcy risk not only depends on the initial capital, but also depends on the financing cost (i.e., the nominal interest rate, $r_b$ or $r_d$). Actually from (5) and (23), we can see that the bankruptcy risk of the supplier increases with respect to the nominal interest rate, the planned production quantity and the initial capital deficit of the supplier. Therefore, when the initial
capital deficit is small, since the financial cost with the bank loan financing is higher than that of the APRC, the bankruptcy risk under the bank loan financing is higher than that under the APRC. As the initial capital \( \zeta \) increases, the capital deficit is decreasing, so the bankruptcy risks decreases under both cases, but they decreases with different speeds. So as \( \zeta \) becomes large enough (\( \zeta > 90000 \)), the supplier’s bankruptcy risk becomes higher under the APRC case.

Although the nominal interest rate or the price discount factor depend on the supplier’s bankruptcy risk, the expected interest rate under the bank loan financing and the expected financing cost under the APRC are fixed. Therefore, the bankruptcy risk does not influence the optimal decisions of the supplier and the distributor. In addition, the less the supplier’s initial capital, the higher bankruptcy risk, so the higher the nominal interest rate of the bank loan and the higher the price discount factor under the APRC (see Figure 4).

6. Conclusions. In this paper, we consider a supply chain in which the supplier has a capital constraint and faces a yield uncertainty. First we derive the optimal strategies under the bank loan financing. Then, we propose an APRC mechanism, under which the distributor provides an advance payment to the supplier, and the supplier offers a price discount to the distributor as a risk compensation.

We derive the optimal strategies under the APRC mechanism, and it turns out that all the strategies and profits are exactly the same with those without any capital deficits. Therefore, the proposed APRC mechanism is a perfect solution for the supplier’s capital deficit issue.

Further, by comparing the profits under the APRC with those under the bank loan financing, we find that the APRC mechanism can bring a Pareto improvement to the supply chain when the supplier’s capital is not relatively big (\( \zeta < \zeta_0 \)), or equivalently, when the capital deficit is not too small. However, when the supplier’s capital deficit is small (or equivalently, \( \zeta > \zeta_0 \)), it is optimal for the supplier to borrow money from the bank. By doing that, the supplier can achieve a higher profit. This is a very interesting result. It seems counterintuitive, but further analysis can justify the result.

Some extensions such as a supply chain with capital constraints for both the supplier and the distributor, optimal choice of the nominal interest rates can be subjects for future research. In addition, supply chain financing problems with endogenous wholesale and retail prices can be considered, too.

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APPENDIX.

A.1. Proof of Lemma 2.1. Let \( f_1(x) \) be the probability density function of \( Z_1 \). Then, using integration by parts, we can get

\[
\int_0^u x f_1(x) dx = -uF_1(u) + \int_0^u F_1(x) dx.
\]

So we have

\[
E[\min(u, Z_1)] = \int_0^u x f_1(x) dx + uF_1(u) = \int_0^u F_1(x) dx.
\]
Then we can get
\[
\mathbf{E}[\min(u, \lambda Z_1)] = \lambda \mathbf{E} \left[ \min \left( \frac{u}{\lambda}, Z_1 \right) \right] = \lambda \int_0^{\frac{u}{\lambda}} F_1(x) dx = \int_0^u F_1 \left( \frac{x}{\lambda} \right) dx.
\]
Define $Z_0 = \min(Z_1, \lambda Z_2)$. Then $Z_0 \geq 0$. Let $\bar{F}_0$ be the survival function of $Z_0$. Then we have
\[
\bar{F}_0(x) = Pr(Z_0 > x) = Pr(Z_1 > x, \lambda Z_2 > x) = Pr(Z_1 > x) Pr(\lambda Z_2 > x) = \bar{F}_1(x) \bar{F}_2 \left( \frac{x}{\lambda} \right).
\]
Then using (38) and the above equation, we can get
\[
\mathbf{E}[\min(Z_1, u, \lambda Z_2)] = \mathbf{E}[\min(\min(Z_1, \lambda Z_2), u)] = \mathbf{E}[\min(Z_0, u)] = \int_0^u \bar{F}_0(x) dx = \int_0^u \bar{F}_1(x) \bar{F}_2 \left( \frac{x}{\lambda} \right) dx.
\]
This completes the proof.

A.2. Proof of Proposition 3.1. From the definition of $\pi_{s_1}$, it is easy to see that
\[
\frac{d\pi_{s_1}}{dq_{s_1}} = w \int_0^{q_{s_1}} x f(x) dx - (1+r)c, \quad \frac{d^2\pi_{s_1}}{dq_{s_1}^2} = -\frac{wq_{s_1}^2 f(w_{s_1})}{q_{s_1}^2} < 0.
\]
Thus, $\pi_{s_1}(q_{s_1})$ is a concave function of $q_{s_1}$. Set $\frac{d\pi_{s_1}}{dq_{s_1}} = 0$, and we can get that
\[
\int_0^{q_{s_1}} x f(x) dx = \frac{(1+r)c}{w}.
\]
Denote $T(u) \equiv \int_0^u x f(x) dx$, then $T(u)$ is a monotone function of $u$, which is obvious from $\frac{dT(u)}{du} = uf(u) > 0$. So (9) has a unique solution. Further, by virtue of (8), (9) and (7), we can get (10).

A.3. Proof of Proposition 3.2. By virtue of the constraint $q_{s_1}^* = k_1 q_d$, and (12), we can get that
\[
\frac{d\pi_d}{dq_d} = \mathcal{F} \left( \frac{1}{k_1} \right) \left[ p\mathcal{G}(q_d) - w \right] + k_1 \int_0^{\frac{1}{k_1}} \left[ p\mathcal{G}(k_1 q_d x) - w \right] x f(x) dx,
\]
\[
\frac{d^2\pi_d}{dq_d^2} = -\mathcal{F} \left( \frac{1}{k_1} \right) g(q_d) + k_1^2 \int_0^{\frac{1}{k_1}} g(k_1 q_d x) x^2 f(x) dx < 0.
\]
Thus, under the bank loan financing, the expected profit of the distributor is a concave function of $q_d$. Set $\frac{d\pi_d}{dq_d} = 0$, and we can get (14).

A.4. Proof of Lemma 5.1. From (14), (21) and (33), we can see that $q_d = q_{d_1}, q_{d_0}, q_{d_2}$ satisfies
\[
\mathcal{F} \left( \frac{1}{k} \right) \left[ p\mathcal{G}(q_d^*) - w \right] + k \int_0^{\frac{1}{k}} \left[ p\mathcal{G}(k q_d^* x) - w \right] x f(x) dx = 0.
\]
Now assume that $p\mathcal{G}(q_d^*) - w \geq 0$. Using the fact that $\mathcal{G}$ is non-increasing, we can get that $\int_0^{\frac{1}{k}} \left[ p\mathcal{G}(k q_d^* x) - w \right] x f(x) dx > \left[ p\mathcal{G}(q_d^*) - w \right] \int_0^{\frac{1}{k}} x f(x) dx \geq 0$. It is a contradiction with (39). So $p\mathcal{G}(q_d^*) - w < 0$ holds for $q_d^* = q_{d_1}, q_{d_0}, q_{d_2}$.
A.5. Proof of Proposition 5.2. Let $H(x) = \tilde{G}(x) - xg(x)$. It is easy to check that when the distribution of $Y$ is IGFR, $H(x)$ is a decreasing function of $x$. In addition, we can rewrite (39) as the following

$$\Phi(k, q^*_d) = 0, \quad \text{where} \quad \Phi(k, q_d) \equiv \int_0^k [pH(kq_d) - w]F(x)dx.$$ \hspace{1cm} (40)

Using $q^*_s = kq^*_d$, we can also get

$$\Phi(k, q^*_s) = \int_0^k [pH(q^*_s) - w]F(x)dx \equiv \hat{\Phi}(k, q^*_s).$$

So we have $\hat{\Phi}(k_1, q^*_s) = \hat{\Phi}(k_2, q^*_s) = 0$. By Lemma 5.1, we can get that $pH \left( \frac{q^*_s}{k_2} \right) - w < 0$. Noting that $k_2 > k_1$ and $H(x)$ is a decreasing function of $x$, we can get that $pH(q^*_s) - w < 0$ for $\frac{k_2}{k_1} \leq x \leq \frac{k_1}{k_1}$. Therefore,

$$\hat{\Phi}(k_1, q^*_s) = \int_0^{\frac{k_1}{k_1}} [pH(q^*_s) - w]F(x)dx \leq \hat{\Phi}(k_2, q^*_s) + \int_{\frac{k_1}{k_1}}^{\frac{k_1}{k_2}} [pH(q^*_s) - w]F(x)dx = \int_{\frac{k_1}{k_2}}^{\frac{k_1}{k_1}} [pH(q^*_s) - w]F(x)dx < 0 = \hat{\Phi}(k_1, q^*_s).$$ \hspace{1cm} (41)

Noting that $H$ is decreasing, we must have $\frac{\partial \hat{\Phi}(k, q^*_d)}{\partial q^*_d} < 0$. Then we can get $q^*_s > q^*_d$ by (41).

On the other hand, we can rewrite (40) as $\Phi(k, q^*_d) = \frac{1}{k} \int_0^1 [pH(q^*_d y) - w]F \left( \frac{y}{k} \right)dy = 0$. $H(x)$ is a decreasing function of $x$, so there exists a $y_0 \in [0, 1]$, such that when $y \in [y_0, 1]$, $pH(q^*_d y) - w > 0$, and when $y \in [y_0, 1]$, $pH(q^*_d y) - w \leq 0$. If $X$ follows an IGFR distribution, $\frac{x f(x)}{F(x)}$ is an increasing function with respect to $y$. So we can get

$$\frac{\partial \Phi(k, q^*_d)}{\partial k} = \frac{1}{k^2} \int_0^1 [pH(q^*_d y) - w]F \left( \frac{y}{k} \right) \frac{x f(x)}{F(x)}dy,$$

and

$$\leq \frac{w x f \left( \frac{y}{k} \right)}{F \left( \frac{y}{k} \right)} \frac{1}{k^2} \int_0^1 [pH(q^*_d y) - w]F \left( \frac{y}{k} \right)dy = 0.$$ \hspace{1cm} (42)

So, $\Phi(k_2, q^*_d) < \Phi(k_1, q^*_d) = \Phi(k_2, q^*_d)$. (40) is also the first order condition to determine the optimal order quantity $q_d$ and $\Phi(k, q_d) = \frac{dx_d(q_d)}{dq_d}$, so we must have that $\frac{\partial \Phi}{\partial q_d} < 0$. Then, from $\Phi(k_2, q^*_d) < \Phi(k_2, q^*_d)$, we can get that $q^*_d < q^*_d$. \hspace{1cm} \Box

A.6. Proof of Proposition 5.3. From (24) and (23), we can get that

$$(1 + r_d)(cq_{s_2} - \zeta)\bar{F}(\theta_2) + wq_{s_2} \int_0^{q_{s_2}} xf(x)dx = cq_{s_2} - \zeta.$$ \hspace{1cm} (42)

From the above equation, we can see that $(1 + r_d)\bar{F}(\theta_2) < 1$. In addition, from (42), we can get:

$$\frac{dr_d}{\zeta} = - \frac{1 - (1 + r_d)\bar{F}(\theta_2)}{(cq_{s_2} - \zeta)\bar{F}(\theta_2)} < 0.$$ \hspace{1cm} \Box
A.7. Proof of Proposition 5.4. From (12), (31) and (39), we can get that
\[
\frac{d\pi_d^*}{dk} = \frac{\partial \pi_d^*}{\partial k} + \frac{\partial \pi_d^*}{\partial q_d^*} \frac{d q_d^*}{dk} = -d^*_d \frac{1}{k} [p \bar{G}(q_d^*) - w].
\]
By virtue of (34), we know that \( \frac{d\pi_d^*}{dk} > 0 \). Because \( k_2 > k_1 \), the distributor will achieve higher profit under the APRC, that is \( \pi_d^*(k_2, q_d^*) > \pi_d^*(k_1, q_d^*) \). \( \square \)

A.8. Proof of Proposition 5.6. From (3), (5), (22) and (23), we can get that
\[
\frac{d\theta_i}{d\zeta} = \frac{\partial \theta_i}{\partial \zeta} + \frac{\partial \theta_i}{\partial r_i} \frac{dr_i}{d\zeta} = -\frac{1 + r_i}{w q_s} - \frac{c q_s - \zeta}{w q_s} \frac{1}{(c q_s - \zeta) F(\theta_i)} < 0.
\]
Noting that \( u \) is non-decreasing with respect to \( \theta_i \), we can get that \( \frac{du}{d\zeta} \leq 0 \). \( \square \)

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