Limited-Feedback-Based Channel-Aware Power Allocation for Linear Distributed Estimation

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Abstract—This paper investigates the problem of distributed best linear unbiased estimation (BLUE) of a random parameter at the fusion center (FC) of a wireless sensor network (WSN). In particular, the application of limited-feedback strategies for the optimal power allocation in distributed estimation is studied. In order to find the BLUE estimator of the unknown parameter, the FC combines spatially distributed, linearly processed, noisy observations of local sensors received through orthogonal channels corrupted by fading and additive Gaussian noise. Most optimal power-allocation schemes proposed in the literature require the feedback of the exact instantaneous channel state information from the FC to local sensors. This paper proposes a limited-feedback strategy in which the FC designs an optimal codebook containing the optimal power-allocation vectors, in an iterative offline process, based on the generalized Lloyd algorithm with modified distortion functions. Upon observing a realization of the channel vector, the FC finds the closest codeword to its corresponding optimal power-allocation vector and broadcasts the index of the codeword. Each sensor will then transmit its analog observations using its optimal quantized amplification gain. This approach eliminates the requirement for infinite-rate digital feedback links and is scalable, especially in large WSNs.

Index Terms—Limited feedback, best linear unbiased estimator (BLUE), generalized Lloyd algorithm, power allocation, distributed estimation, fusion center, wireless sensor networks.

I. INTRODUCTION

Wireless sensor networks (WSNs) are typically formed by spatially distributed sensors with limited communications and processing capabilities that cooperate with each other to achieve a common goal. One of the most important applications of such networks is distributed estimation, which is a key enabling technology for a wider range of applications such as event classification and object tracking. In a WSN performing distributed estimation, sensors make noisy observations that are correlated with an unknown parameter to be estimated, process their local observations, and send their processed data to a fusion center (FC), which then combines all of the locally processed samples to perform the ultimate global estimation.

Recently, the problem of distributed estimation in WSNs has extensively been studied in the literature [1–5]. The type of local processing that is performed on each sensor’s noisy observation before it is transmitted to the FC differentiates these works and can be either a local quantization [1, 2] or an amplify-and-forward strategy [3–5]. We consider the second approach in this paper due to its simplicity and practical feasibility. One of the main issues to be addressed in the case of analog amplify-and-forward local processing is finding the optimal local amplification gains. The values of these gains set the instantaneous transmit power of sensors; therefore, we refer to their determination as the power allocation to sensors.

Cui et al. [5] have proposed an optimal power-allocation scheme to minimize the variance of the best linear unbiased estimator (BLUE) for a random scalar parameter at the FC of a WSN, given a total transmission-power constraint in the network. In their proposed approach, the optimal local amplification gains depend on the instantaneous fading coefficients of the channels between the sensors and FC. Therefore, in order for the FC to achieve the minimum estimation variance of the BLUE estimator, it must feed the exact channel fading gains back to sensors through infinite-rate, error-free links. This requirement is not practical in most WSN applications, especially when the number of sensors is large. In this paper, we investigate the application of a limited-feedback strategy for the optimal power-allocation scheme proposed in [5]. We use the generalized Lloyd algorithm with modified distortion functions to design an optimal codebook, which is then used to quantize the space of the optimal power-allocation vectors used by the sensors to set their amplification gains.

Note that the approach proposed in this paper is different from other works that have applied limited feedback to distributed estimation. In particular, the phrase “limited feedback” has a different meaning in this paper compared to other works in the field of distributed estimation. For example, Banavar et al. [4] have investigated the effects of feedback error and the impact of the availability of different amounts of full, partial, or no channel state information at local sensors on the estimation variance of the BLUE estimator of a scalar random parameter at the FC. The problem of distributed estimation using amplify-and-forward local processing over unknown parallel fading channels is studied in [6]. A two-phase approach is proposed based on pilot-based channel estimation followed by source parameter estimation at the FC, where the estimated channel between each sensor and the FC is used at the sensor for local power optimization.

The rest of this paper is organized as follows: In Section II, the system model of the WSN under study is introduced.

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Suppose that the local noisy observation at each sensor is a random variable with zero mean and variance $\sigma^2_i$. Note that no further assumption is made on the distribution of this random variable. Consider a wireless sensor network (WSN) composed of $K$ sensors, each of which contains a local signal source, a local amplifier, and a sensor node. The local amplifier is used to amplify the local signal before it is transmitted to the fusion center (FC). The received signal at sensor $i$ is given by:

$$ y_i = h_i z_i + w_i, \quad i = 1, 2, \ldots, K, $$

where $h_i$ is the multiplicative fading coefficient of the channel between sensor $i$ and the FC, $z_i$ is the signal transmitted from sensor $i$ to the FC, and $w_i$ is the spatially independent and identically distributed additive Gaussian noise with zero mean and variance $\sigma^2_i$.

The received signal at the FC is a sum of all the received signals from the sensors:

$$ y = \sum_{i=1}^{K} h_i z_i + \sum_{i=1}^{K} w_i. $$

The performance of this scheme is analyzed in terms of the channel signal-to-noise ratio (CSNR) at sensor $i$ as $\gamma_i = \frac{|h_i|^2}{\sigma^2_i}$, $i = 1, 2, \ldots, K$.

### Problem Statement

Suppose that a power-allocation scheme is available and a realization of the fading gains, the FC combines the set of received signals from sensors to find the best linear unbiased estimator (BLUE) for the unknown parameter $\theta$ as [7, Chapter 6]

$$ \hat{\theta} = \left( \sum_{i=1}^{K} \frac{\beta_i \gamma_i a_i^2 \sigma_o^2}{1 + \gamma_i a_i^2 \sigma_o^2} \right)^{-1} \sum_{i=1}^{K} \frac{\beta_i \gamma_i a_i^2 \sigma_o^2}{1 + \gamma_i a_i^2 \sigma_o^2} z_i, $$

where the corresponding estimator variance can be found as

$$ \text{Var}(\hat{\theta}|a, g) = \left( \sum_{i=1}^{K} \frac{\beta_i \gamma_i a_i^2 \sigma_o^2}{1 + \gamma_i a_i^2 \sigma_o^2} \right)^{-1}. $$

In which $a = [a_1, a_2, \ldots, a_K]^T$ and $g = [g_1, g_2, \ldots, g_K]^T$ are column vectors containing the set of local amplification gains $a_i$ and fading coefficients of the channels $g_i$, respectively.

Cui et al. [5] have derived the optimal local amplification gains or equivalently, the optimal power-allocation scheme to minimize the BLUE-estimator variance, defined in (6), given a total transmission-power constraint in the network as:

$$ a_i = \begin{cases} \sqrt{\frac{1}{\gamma_i a_i^2} (\sqrt{\delta_i (K_1 - 1)} - 1)}, & i \leq K_1 \\ 0, & i > K_1 \end{cases}, $$

where $\delta_i = \frac{\beta_i \gamma_i}{1 + \beta_i}, i = 1, 2, \ldots, K$, the sensors are sorted so that $\delta_1 \geq \delta_2 \geq \cdots \geq \delta_K$. The function $\rho(\cdot)$ is defined for any integer argument $n$ as

$$ \rho(n) = \frac{P_{\text{Total}} + \sum_{i=1}^{n} \frac{\beta_i}{\delta_i}}{\sum_{i=1}^{n} \frac{\beta_i}{\delta_i}}, $$

where $K_1$ is the unique largest integer for which $\sqrt{\delta_{K_1}} \rho(K_1) > 1$ and $\sqrt{\delta_{K_1+1}} \rho(K_1 + 1) \leq 1$, and $P_{\text{Total}}$ is the constraint on the total power consumed in the entire network so that $\sum_{i=1}^{K} P_i \leq P_{\text{Total}}$. The above power-allocation strategy assigns a zero amplification gain or equivalently, zero transmit power to sensors that satisfy the power constraint.
power to the sensors for which $\delta_i \leq [\rho(K_1)]^{-2}$, because either the sensor’s observation SNR or its channel SNR is too low. The assigned instantaneous transmit power to other sensors is non-zero and based on the value of $\delta_i$, for each sensor. Note that based on the above power-allocation scheme, there is a unique one-to-one mapping between $g$ and $a$ that could be denoted as $a = f(g)$.

It can be observed that the optimal power-allocation scheme proposed in [5] is based on the assumption that the complete forward channel state information (CSI) is available at local sensors. In other words, Equation (7) shows that the optimal value of the local amplification gain at sensor $i$ is a function of its channel SNR $\gamma_i$, which in itself is a function of the instantaneous fading coefficient of the channel between sensor $i$ and the FC. Therefore, in order for the FC to achieve the estimator variance given by (6), it must feed the exact instantaneous amplification gain $a_i$ back to each sensor.¹ This requirement is not practical in most applications, especially in large-scale WSNs, since the feedback information is typically transmitted through finite-rate digital feedback links.

In this paper, we propose a limited-feedback strategy to alleviate the above-mentioned requirement for infinite-rate digital feedback links from the FC to the local sensors. In the proposed limited-feedback strategy, the FC and local sensors must agree on a codebook of the local amplification gains or equivalently, a codebook of possible power-allocation schemes. Before the sensors transmit their amplified observations, the FC reliably estimates the channels between them and itself (i.e., $g_i$), and finds the optimal power allocation to the sensors for the given channel realization, using the approach proposed in [5]. Note that the FC has access to the perfect backward CSI; i.e., the instantaneous fading gain of the channel between each sensor and itself. Therefore, it can find the exact power-allocation strategy of the entire network based on (7), given any channel realization. The FC will then identify the closest codeword to the optimal power-allocation vector, and broadcasts the index of that codeword to all sensors over an error-free digital feedback channel.² The optimal codebook can be designed offline by quantizing the space of the optimal local amplification gains and to design the optimal codebook $C$ in an iterative process.

In order to implement the generalized Lloyd algorithm, a distortion metric must be defined for the codebook and for each codeword. Let $D_B(C)$ denote the average distortion for codebook $C$ defined as

$$D_B(C) \triangleq \mathbb{E}_g \left[ \min_{\ell \in \{1, 2, \ldots, 2^L\}} D_W(a_\ell | g) \right],$$

where $\mathbb{E}_g [\cdot]$ denotes the expectation operation with respect to the fading coefficients of the channel and $D_W(a_\ell | g)$ represents the conditional codeword distortion resulting from assigning a suboptimal quantized power-allocation vector $a_\ell$ (instead of the optimal one, denoted by $a^{OPT}$ and found using (7)), given a realization of the vector of channel fading coefficients $g$. We define this conditional codeword distortion as

$$D_W(a_\ell | g) \triangleq \operatorname{Var}(\hat{\theta} | a_\ell, g) - \operatorname{Var}(\hat{\theta} | a^{OPT}, g),$$

where the estimation variance $\operatorname{Var}(\hat{\theta} | g)$ is found using (6).

Let $G \subseteq \mathbb{R}^{K+}$ and $A \subseteq \mathbb{R}^{K+}$ be the $K$-dimensional vector spaces of fading coefficients of the channel and the optimal local amplification gains, respectively, whose entries are chosen from the set of real-valued non-negative numbers. Note that each vector of channel fading coefficients $g \in G$ is uniquely mapped into an optimal power-allocation vector $a^{OPT} \in A$ by applying (7) to find each element of $a^{OPT}$. We denote this mapping by $f : G \to A$. Given the distortion functions for the codebook $C$ and for each one of its codewords defined in Equations (9) and (10), respectively, the two main conditions of the generalized Lloyd algorithm could be reformulated for our vector-quantization problem as follows [8, Chapter 11].

**Nearest-Neighbor Condition:** This condition finds the optimal quantization regions (or Voronoi cells) for the vector space to be quantized, given a fixed codebook. Based on this condition, each point $a \in A$ in the vector space of the optimal local amplification gains is assigned to partition $\ell$ represented by codeword $a_\ell \in C$ if and only if its distance to codeword $a_\ell$, with respect to the conditional codeword distortion function vectors into $2^L$ disjoint regions. Note that $L$ is the total number of feedback bits broadcast by the FC, and not the number of bits fed back to each sensor. A codeword is chosen in each quantization region. The length of each codeword is $K$, and its $i$th entry is a real number representing a quantized version of the optimal local amplification gain for sensor $i$. The proposed quantization scheme could then be considered as a mapping from the space of channel state information to a discrete set of $2^L$ length-$K$ real-valued power-allocation vectors.

Let $C = [a_1, a_2, \ldots, a_M]^T$ be a $2^L \times K$ codebook matrix of the optimal local amplification gains, where $[C_{\ell,i}]$ denotes its element in row $\ell$ and column $i$ as the optimal gain of sensor $i$ in codeword $\ell$. Note that each $a_\ell, \ell = 1, 2, \ldots, 2^L$ is associated with a realization of the fading coefficients of the channels between local sensors and the FC, denoted by $g_\ell$. We apply the generalized Lloyd algorithm with modified distortion functions to solve the problem of vector quantization in the space of the optimal local amplification gains and to design the optimal codebook $C$ in an iterative process.

### IV. CODEBOOK DESIGN USING LLOYD ALGORITHM

Let $L$ be the number of feedback bits that the FC uses to quantize the space of the optimal local power-allocation

¹Note that instead of feeding $a_i$ back to each sensor, the FC could send back the fading coefficient of the channel between each sensor and the FC. However, the knowledge of $g_i$ alone is not enough for sensor $i$ to compute the optimal value of its local amplification gain $a_i$. The sensor must also know whether it needs to transmit (i.e., $i \leq K_1$) or stay silent (i.e., $i > K_1$). There are two ways that the extra data can be fed back to the sensors: This information could be encoded in an extra one-bit command instructing the sensor to transmit or stay silent, or the sensor could listen for the entire vector of $g_i$ sent by the FC over a broadcast channel. Sending back each value of $a_i$ avoids the problems of either having to send each sensor an additional bit, or requiring the sensor to listen to the entire vector of $g$.

²Since the rate of the feedback link is very low, an error-free channel can be realized by using capacity-approaching channel codes.
defined in (10), is less than its distance to any other codeword in the codebook. In this paper, given a codebook $C$, the space $G$ of channel fading coefficients is divided into $2^L$ disjoint quantization regions with the $\ell$th region defined as

$$G_\ell = \{g \in G : D_W(\mathbf{a}_k|g) \leq D_W(\mathbf{a}_k|g), \forall k \neq \ell\}. \quad (11)$$

Since for every $g \in G$, there is a unique $a \in A$ that can be found using (7), we could define the Voronoi cell $A_\ell$ as the image of $G_\ell$ under mapping $A_\ell = f(G_\ell)$. In other words, to find the optimal partition for each $a \in A$, its corresponding vector of channel fading coefficients $g \in G$ is considered. The distortion of using any codeword $a_\ell \in C$ instead of the optimal power-allocation vector for that channel realization is found using (10), and $a$ is assigned to the region with the lowest conditional codeword distortion, given $g$.

**Centroid Condition:** This condition finds the optimal codebook, given a specific partition of the vector space of the optimized power-allocation vectors $\{A_1, A_2, \ldots, A_{2^L}\}$. Based on this condition, the optimal codeword associated with each Voronoi cell $A_\ell \subseteq A$ is the centroid of that cell with respect to the conditional codeword-distortion function introduced in (10), and is defined as

$$a_\ell = \arg \min_{a \in A_\ell} \mathbb{E}_{g \in G_\ell} [D_W(\mathbf{a}|g)], \quad (12)$$

where the expectation operation is performed over the set of realizations of the channel fading coefficients, whose associated optimal power-allocation vectors are members of partition $A_\ell$. This set is denoted by $G_\ell$.

The optimal codebook is designed offline by the FC using the above two conditions. It can be shown that the average codebook distortion is monotonically non-increasing through the iterative usage of the Centroid Condition and the Nearest-Neighbor Condition [8, Chapter 11]. Details of the codebook design process are summarized in Algorithm I. The optimal codebook is stored in the FC and all sensors.

Upon observing a realization of the channel fading vector $g$, the FC finds its associated optimal power-allocation vector $a^{\text{OPT}}$. It then identifies the closest codeword in the optimal codebook $C$ to $a^{\text{OPT}}$ with respect to the conditional codeword distortion defined in (10). Finally, the FC broadcasts the index of that codeword to all sensors as

$$\ell = \arg \min_{k \in \{1,2,\ldots,2^L\}} D_W(\mathbf{a}_k|g). \quad (13)$$

Upon reception of the index $\ell$, each sensor $i$ knows its local amplification gain as $|C|_{\ell,i}$, where $\ell$ and $i$ are the row and column indexes of the codebook $C$, respectively.

### V. Numerical Results

In this section, numerical results are provided to verify the effectiveness of the proposed limited-feedback strategy in achieving a BLUE-estimator variance close to that of a WSN with full feedback. In our simulations, the local observation gains are randomly chosen from a Gaussian distribution with unit mean and variance 0.09, i.e., $h_i \sim \mathcal{N}(1,0.09)$. In all simulations, the average power of $h_i$ across all sensors is set to be 1.2. The observation and channel noise variances are set to $\sigma_0^2 = 10$ dBm and $\sigma_2^2 = -90$ dBm, respectively. The following fading model is considered for the channels between local sensors and the FC:

$$g_i = \eta_0 \left( \frac{d_i}{d_0} \right)^{-\alpha} f_i, \quad i = 1, 2, \ldots, K, \quad (14)$$

where $\eta_0 = -30$ dB is the nominal path gain at the reference distance $d_0 = 1$ meter, $d_i$ is the distance between sensor $i$ and the FC, and uniformly distributed between 50 and 150 meters, $\alpha = 2$ is the path-loss exponent, and $f_i$ is the independent and identically distributed (i.i.d.) Rayleigh-fading random variable with unit variance. The size of the training set in the optimal codebook-design process described in Algorithm I is set to $M = 5,000$. The codebook-distortion threshold for stopping the iterative algorithm is assumed to be $\epsilon = 10^{-6}$. The results are averaged over 50,000 Monte-Carlo simulations.

Figure 2 illustrates the effect of $L$ as the number of feedback bits from the FC to local sensors on the performance of the BLUE estimator. It should be emphasized that $L$ is the total number of feedback bits broadcast by the FC, and not the number of bits fed back to each sensor. This figure depicts the average BLUE-estimator variance versus the total transmit power $P_{\text{total}}$ for different values of the number of feedback bits $L$, when there are $K = 5$ or $K = 10$ sensors in the network, shown in Figs. 2a and 2b, respectively. The results for the case of full-feedback from the FC to local sensors proposed in [5] are shown with solid lines as a benchmark. As it can be seen in this figure, the performance of the BLUE estimator

| ALGORITHM I: The process of optimal codebook design based on the generalized Lloyd algorithm with modified distortion functions. |
| --- |
| **Require:** $K$, $L$, and channel-fading model. |
| **Require:** $M$. $\triangleright$ $M$ is the number of training vectors in space $A$. |
| **Require:** $\epsilon$. $\triangleright$ $\epsilon$ is the distortion threshold to stop the iterations. |
| **1. Initialization** |
| $2. G_0 \leftarrow$ A set of $M$ length-$K$ vectors of channel-fading realizations based on the given fading model of the channels between local sensors and the FC. $\triangleright$ $M \geq 2L$ and $G_0 \subseteq G$. |
| $3. A_s \leftarrow$ The set of optimal local power-allocation vectors associated with the channel fading vectors in $G_s$, found by applying Eq. (7). $\triangleright$ $A_s$ is the set of training vectors, and $A_s \subseteq A$. |
| **4. Randomly select** $2^L$ optimal power-allocation vectors from the set $A_s$ as the initial set of codewords. Denote the codewords by $a_0^A$. |
| **5.** $C_0 \leftarrow [a_0^1 a_0^2 \ldots a_0^{2^L}]^T \triangleright C_0$ is the initial codebook. |
| **6.** $j \leftarrow 0$ and NewCost $\leftarrow D_B(C_0^j)$. $\triangleright$ The average distortion of codebook is found using Eq. (9). |
| **7. EndInitialization** |
| **8. repeat** |
| **9.** $j \leftarrow j + 1$ and OldCost $\leftarrow$ NewCost |
| **10.** Given codebook $C_{j-1}$, optimally partition the set $A_s$ into $2^L$ disjoint subsets based on the Nearest-Neighbor Condition using Eq. (11). Denote the resulted optimal partitions by $A_{j-1}$. |
| **11. for all** $A_l^{j-1}$, $l = 1, 2, \ldots, 2^L$ $\triangleright$ Optimal codeword associated with partition $A_l^{j-1}$ based on the Centroid Condition using Eq. (12). Denote this new optimal codeword as $a_j^l$. |
| **12.** Find the optimal codeword associated with partition $A_{j-1}^{l}$ using Eq. (14). $\triangleright$ Denote this new optimal codeword as $a_j^l$. |
| **13.** end for |
| **14.** $C_j \leftarrow [a_1^j a_2^j \ldots a_{2^L}^j]^T \triangleright C_j$ is the new codebook. |
| **15.** NewCost $\leftarrow D_B(C_j^j)$. |
| **16.** until OldCost $-$ NewCost $\leq \epsilon$ |
| **17.** return $C_{j_{\text{OPT}}} \leftarrow C_j^j$. |
VI. CONCLUSIONS

In this paper, a limited-feedback strategy was proposed to be applied in an adaptive power-allocation scheme for distributed BLUE estimation of a random scalar parameter at the FC of a WSN. The proposed approach eliminates the requirement for infinite-rate feedback of the instantaneous forward CSI from the FC to local sensors in order for them to find their optimal local amplification gains. The generalized Lloyd algorithm with modified distortion functions was used to quantize the vector space of the optimal local amplification gains and to design an optimal codebook for this space. Upon observing the CSI, the FC broadcasts the index of the closest codeword to the corresponding optimal power-allocation vector, rather than feeding back the exact instantaneous CSI. Numerical results showed that even with a small number of feedback bits, the average estimation variance of the BLUE estimator with adaptive power allocation based on the proposed limited-feedback strategy is close to that with perfect CSI feedback.

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