Experimental observation of an entire family of four-photon entangled states

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A single linear optical set-up is used to observe an entire family of four-photon entangled states. This approach breaks with the inflexibility of present linear-optical set-ups usually designed for the observation of a particular multi-partite entangled state only. The family includes several prominent entangled states that are known to be highly relevant for quantum information applications.

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Multi-partite entanglement is the vital resource for numerous quantum information applications like quantum computation, quantum communication and quantum metrology. So far, the biggest variety of multi-partite entangled states was studied using photonic qubits (e.g. [1, 2, 3, 4, 5, 6]). As there is no efficient way of creating entanglement between photons by direct interaction, entangled photonic states are generally observed by a combination of a source of entangled photons and their further processing via linear optical elements and conditional detection. Based on this approach, experiments were designed for the observation of a single, e.g. [1, 2, 3, 4, 5], or two [6] multi-partite entangled state(s).

Here, we break with this inflexibility by designing a single linear optics set-up for the observation of an entire family of four-photon entangled states. The states of the family are conveniently chosen by one experimental parameter. Thereby, states that differ strongly in their entanglement properties are accessible in the same experiment [6]. We demonstrate the functionality of the scheme by the observation and analysis of a selection of distinguished entangled states.

The family that can be observed experimentally is given by the superposition of the tensor product of two Bell states and a four-qubit GHZ state:

\[ |\Psi(\gamma)\rangle = \alpha(|\psi^+\rangle \otimes |\psi^+\rangle) + \sqrt{1 - \alpha^2} |\text{GHZ}\rangle, \]

where \( |\psi^+\rangle = 1/\sqrt{2}(|HV\rangle + |VH\rangle) \) and \( |\text{GHZ}\rangle = 1/\sqrt{2}(|HHVV\rangle + |VVHH\rangle) \) [8,9]. We use the notation for polarization encoded qubits, where, e.g., \( |HHVV\rangle = |H\rangle_a \otimes |H\rangle_b \otimes |V\rangle_c \otimes |V\rangle_d \) and \( |H\rangle \) and \( |V\rangle \) denote linear horizontal and vertical polarization and the subscript denotes the spatial mode of each photon. Here, the real amplitude \( \alpha(\gamma) \) with \( |\alpha(\gamma)| \leq 1 \) is determined by a single, experimentally tunable parameter \( \gamma \), which is set by the orientation of a half-wave plate (HWP). Thus, we are able to change continuously from the product of two Bell states over a number of interesting genuinely four-partite entangled states to the four-qubit GHZ state.

According to the four-qubit SLOCC (stochastic local operations and classical communication) classification in [10], the family \( |\Psi(\gamma)\rangle \) is a subset of the generic family \( G_{abcd} \) of four-qubit entangled states. Note, \( |\Psi(\gamma)\rangle \) represents a different class of SLOCC equivalent states for each value of \( |\alpha(\gamma)| \).

The experimental set-up that allows a flexible observation of the family \( |\Psi(\gamma)\rangle \) is depicted in Fig. 1. Four photons originate from the second order emission of a spontaneous parametric down conversion (SPDC) process [11] in a 2 mm thick \( \beta \)-Barium borate (BBO) crystal arranged in non-collinear type II configuration. The crystal is pumped by UV pulses with a central wavelength of 390 nm and an average power of 600 mW obtained from a frequency-doubled Ti:Sapphire oscillator (pulse length 130 fs). The four photons are emitted into two spatial modes \( a \) and \( b \) [12]:

\[ 1/(2\sqrt{3})[(a_H^d b_V^d)^2 + (a_V^d b_H^d)^2 + 2a_H^d a_V^d b_H^d b_V^d] \langle \text{vac} |, \]

where \( m_j^\dagger \) is the creation operator of a photon having polarization \( j \) in mode \( m \) and \( \langle \text{vac} | \) is the vacuum state. A
HWP and a 1 mm thick BBO crystal compensate walk-off effects. The spatial modes $a$ and $b$ are defined by coupling the photons into single mode (SM) fibres. Spectral selection is achieved by 3 nm FWHM interference filters (IF) centered around 780 nm. A HWP in mode $a$ transforms the polarization of the photons. The orientation of the optical axis, $\gamma$, of this HWP is the tuning parameter of the family. Subsequently, the modes $a$ and $b$ are overlapped at a polarizing beam splitter (PBS) with its output modes denoted by $c$ and $d$. A HWP oriented at $\pi/4$ behind the PBS transforms the polarization of the photons in mode $c$ from $H(V)$ into $V(H)$. Subsequently, the modes $c$ and $d$ are split into the output modes $e, f$ and $g, h$, respectively, via polarization-independent beam splitters (BS). Birefringence of the beam splitters is compensated by a pair of perpendicularly oriented birefringent Yttrium-Vanadate (YVO$_4$) crystals. Finally, the polarization state of each photon is analyzed with a HWP, a quarter-wave plate (QWP) and a PBS. The photons are detected by fiber-coupled single photon detectors and registered by a multichannel coincidence unit.

Under the condition of detecting one photon of the second order SPDC emission in each spatial output mode, the family of states $|\Psi(\gamma)\rangle$ is observed, where the amplitude $\alpha(\gamma)$ depends on the HWP angle $\gamma$ via $\alpha(\gamma) = (2\cos 4\gamma)/\sqrt{48\beta(\gamma)}$ with $\gamma \in [0, \pi/4]$. This occurs with a probability $\beta(\gamma) = (5 - 4\cos 4\gamma + 3\cos 8\gamma)/48$ (Fig. 2). Only for few states of the family a dedicated set-up is known \cite{2, 3, 4, 5}. For these particular cases the respective state is observed with equal or higher probability. Here, however, we profit from the flexibility to choose various entangled states using the same set-up.

Let us illustrate the described state observation scheme by examining the action of the HWP together with the PBS. We note that only the case where two photons are found in each spatial mode $c$ and $d$ behind the PBS, respectively, can lead to a detection event in each of the four output modes $e, f, g, h$. First, we consider a HWP oriented at $\gamma = 0$. This setting leaves the polarization of each photon unchanged. Each of the first two terms of Eq. (2) results in four photons in the same spatial mode behind the PBS and, thus, does not contribute to the state in each of the four output modes $e, f, g, h$. First, we consider a HWP oriented at $\gamma = 0$. This setting leaves the polarization of each photon unchanged. Each of the first two terms of Eq. (2) results in four photons in the same spatial mode behind the PBS and, thus, does not contribute to the state $|\Psi(\gamma)\rangle$.

This family contains useful states, which, moreover, differ strongly in their entanglement properties. For example, the well known GHZ state $|GHZ\rangle = |\Psi(\pi/2)\rangle$, i.e. $\alpha = 0$ \cite{2} belongs to the graph states \cite{13} and finds numerous applications in quantum information, e.g. \cite{14}. The entanglement of the symmetric Dicke states \cite{13} is known to be very robust against photon loss. Out of these states we observe with $\alpha = \sqrt{2/3}$ the state $|D_4^{(2)}\rangle = |\Psi(\pi/12)\rangle$ \cite{3}. Remarkably, this state allows to obtain, via a single projective measurement, states out of each of the two inequivalent classes of genuine tri-partite entanglement \cite{3, 10}. The states $|\Psi_4^-\rangle = |\Psi(\pi/4)\rangle (\alpha = -\sqrt{1/3})$ \cite{4} and $|\psi^+\rangle \otimes |\psi^-\rangle$ \cite{17} that are equivalent under local unitary (LU) operations to $|\Psi_4^+\rangle = |\Psi(\approx 0.098\pi)\rangle (\alpha = \sqrt{1/3})$ \cite{3} and $|\psi^+\rangle \otimes |\psi^+\rangle = |\Psi(0)\rangle (\alpha = 1)$, respectively are invariant under any action of the same LU transformation on each qubit and, therefore, they form a basis for decoherence-free communication \cite{18}.

To characterize the family of states we consider the correlations of $|\Psi(\gamma)\rangle$. Of all 256 correlations $T_{ijkl}$ in the standard basis, the family $|\Psi(\gamma)\rangle$ exhibits at most 40 that are non-zero. The modulus of these correla-
tions, $|T_{ijkl}|$, shows five distinct dependencies on $\gamma$, which are shown in Fig. 2. Interestingly, one finds the aforementioned states at the crossing points of some correlations. Consequently, we can identify other distinguished states at the remaining four crossing points. These are found at $\gamma \approx 0.0767\pi$ [$\alpha = (1/6(3 + \sqrt{3}))^{1/2}$], $\gamma \approx 0.091\pi$ [$\alpha = (1/6(3 + \sqrt{3}))^{1/2}$], and for $\gamma \approx 0.174\pi$ [$\alpha = -(1/6(3 - \sqrt{3}))^{1/2}$]. We label them for brevity by $|S^a\rangle$, $|S^b\rangle$, $|S^c+\rangle$ and $|S^c-\rangle$, respectively.

We select these nine states for an experimental characterization. As the setup is stable and delivers the states $\Psi(0)$, we apply generic entanglement witnesses in the computational basis as a polarization filter and, thus, leads to an additional reduction of the fidelity. Considering these effects, the question arises whether the fidelity of particular states is higher when these states were observed with dedicated linear optics set-ups. For example, the states $|D_4(2)\rangle$ and $|\Psi_4^-\rangle$ were recently observed with fidelities of $F_{D_4(2)} = 0.844 \pm 0.008$ [23] and $F_{\Psi_4^-} = 0.901 \pm 0.01$ [4], respectively. Here we achieved $0.809 \pm 0.014$ and $0.932 \pm 0.008$, respectively, comparable with the dedicated implementations.

Finally, for proving genuine four-partite entanglement of the observed states we apply generic entanglement witnesses $W_{\Psi(\gamma)}$ [4, 22]. Their expectation value depends directly on the fidelity: $\text{Tr}(W_{\Psi(\gamma)} \rho_{\text{exp}}) = c(\gamma) - F_{\Psi(\gamma)}$, where $c(\gamma)$ is the maximal overlap of $|\Psi(\gamma)\rangle$ with all bi-separable states. A fidelity larger than $c(\gamma)$ (solid curve in Fig. 4) detects genuine four-qubit entanglement of $\rho_{\text{exp}}$. We find that all experimental fidelities, of course except $F_{\Psi(0)}$, are larger than $c(\gamma)$, thus, proving four-qubit entanglement. For the bi-separable entangled state $|\psi(0)\rangle$ we apply the witness given in [24] on each pair and find $-0.466 \pm 0.006$ and $-0.461 \pm 0.006$, respectively, detecting the entanglement of each pair.

To summarize, we are able to observe an entire family of highly entangled four-photon states with high fidelity by using the same linear optics set-up. For this purpose,
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a single SPDC source and one overlap on a PBS were 
the presented set-up, or replace the PBS with a BS. Both 
ables the observation of different families of states \cite{23}. 
Even if the weak photon-photon coupling does not allow 
enables the observation of different families of states \cite{25}.

FIG. 4: Experimentally determined fidelities of nine distin-
guished states from the family \( |\Psi(\gamma)\rangle = a(\gamma) |\psi^+\rangle \otimes |\psi^+\rangle + \sqrt{1-a(\gamma)^2} |GHZ\rangle \). The minimal fidelity for proving gen-
true four-qubit entanglement is depicted as solid curve.

a single SPDC source and one overlap on a PBS were 
sufficient. This is a clear improvement compared to previ-
ous dedicated linear optics realizations, where basically 
only one state could be observed. The general principle of 
commonly manipulating multi-photon states followed 
by interferometric overlaps at linear optical components, 
of course, can be easily extended: For example, one can 
use the six photon emission from the SPDC source and the 
presented set-up, or replace the PBS with a BS. Both 
enables the observation of different families of states \cite{23}. 
Even if the weak photon-photon coupling does not allow 
the design of simple quantum logic gates, the utilization 
of higher order emissions from an SPDC source together 
with multi-photon interference will enable further flexi-
ble experiments, each with numerous different and highly 
relevant multi-partite entangled states.

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\( T_{ijkl} = \langle \psi |(\sigma^i \otimes \sigma^j \otimes \sigma^k \otimes \sigma^l) |\psi\rangle \), where 
\( i, j, k, l \in \{0, 1, 2, 3\} \), \( \sigma^0 = \mathbb{I} \) and \( \sigma^1, \sigma^2, \sigma^3 \) are the usual 
Pauli operators.
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\( \rho_{\text{exp}} = \sum_{ijkl} S_{ijkl}(\sigma^i \otimes \sigma^j \otimes \sigma^k \otimes \sigma^l) /16 \), where \( S_{ijkl} \) 
are the correlations of the experimentally observed state, we 
find \( F_{\Psi(\gamma)} = \sum_{ijkl} S_{ijkl} T_{ijkl} /16 \). Only terms with non-
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is overlapped on the PBS. Subsequently, the photons are 
distributed onto six spatial modes. Then, the HWP(\( \gamma \)) 
in front of the PBS tunes the observable six photon states: 
\( \beta(\gamma) |GHZ_0\rangle + \sqrt{1-\beta(\gamma)^2} \sqrt{2} (|W_3\rangle \otimes |W_3\rangle - |W_3\rangle \otimes |\bar{W}_3\rangle) \), where 
\( |GHZ_0\rangle = \frac{1}{\sqrt{2}} (|HHHVVV\rangle - |VVVVHHH\rangle) \), 
\( |W_3\rangle = \frac{1}{\sqrt{3}} (|HHH\rangle + |HHV\rangle + |VHV\rangle) \), 
\( |\bar{W}_3\rangle = \frac{1}{\sqrt{3}} (|VHH\rangle + |VHV\rangle + |HVV\rangle) \).

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