Implications of the recent new measurements of \( B \to K_1\gamma \) by Belle are examined. It is shown that the new branching ratio \( B(B \to K_1(1270)\gamma) \) requires very large form factor compared to the theoretically predicted one. This is an opposite case to \( B \to K^*\gamma \) where theory expected larger branching ratio. Possible origins of the discrepancy are discussed.

Radiative \( B \) decays to kaons provide a rich laboratory to test the standard model (SM) and probe new physics. \( B \to K^*\gamma \) is a well established process among them. Higher resonant kaons such as \( K_1^*(1430) \) are also measured by CLEO [1] and the \( B \) factories [2,3]. Recently, Belle has announced the first measurement of \( K_1(1270) \) [4]:

\[
B(B^+ \to K_1^+(1270)\gamma) = (4.28 \pm 0.94 \pm 0.43) \times 10^{-5}.
\]

There is also an upper bound on \( K_1(1400) \) [4]:

\[
B(B^+ \to K_1^+(1400)\gamma) < 1.44 \times 10^{-5} \text{ (at 90\% C.L.)}.
\]

There are many reasons to focus on the higher kaon resonances. Firstly, they share lots of things with \( B \to K^*\gamma \). At the quark level, both of them are governed by \( b \to s\gamma \); all of the accumulated achievements of \( b \to s\gamma \) can be used in radiative \( B \) decays to kaon resonances. For example, the same operators in the operator product expansion, the same corresponding Wilson coefficients are available. In addition, when the hadronic descriptions are required, the resemblance between \( K^* \) and \( K_1 \) makes the analysis much easier. Especially, the light-cone distribution amplitudes (DA) are same except the overall factor of \( \gamma_5 \) which gives rise to few differences in many calculations [5].

Secondly, \( B \to K_{res} \to K\pi\gamma \) can provide a direct measurement of the photon polarization [6]. In particular, it was shown that \( B \to K_1(1400)\gamma \) can produce large polarization asymmetry of \( \approx 33\% \) in the SM. In the presence of anomalous right-handed couplings, the polarization can be severely reduced in the parameter space allowed by current experimental bounds of \( B \to X_s\gamma \) [7]. It was also argued that the \( B \) factories can now make a lot of \( BB \) pairs enough to check the anomalous couplings through the measurement of the photon polarization.

Thirdly, theorists are now facing challenges from the discrepancy between their predictions and experiments. In fact, there have been noticeable theoretical advances in \( B \to K^*\gamma \) over the last decade. QCD corrections at next-to-leading order (NLO) of \( \alpha_s \) was already considered in [8–10]. Furthermore, relevant Wilson coefficients have been improved [11,12] up to three-loop calculations. Recent developments of the QCD factorization (QCDF) [13] helped one calculate the hard spectator contributions systematically in a factorized form through the convolution at the heavy quark limit [14–16]. \( B \to K^*\gamma \) is also analyzed in the effective theories at NLO, such as large energy effective theory [17] and the soft-collinear effective theory (SCET) [18].

But the nonperturbative analyses should be taken into account to complete the phenomenological explanation. QCD sum rule or the light-cone sum rule (LCSR) is among the most reliable. It was pointed out in [17], however, that the LCSR results for the relevant form factor of \( B \to K^*\gamma \) lead to a very large branching ratio compared to the measured one. Unfortunately, there is no way to explain the gap up to now.

The situation is more complicated in \( B \to K_1\gamma \). Based on the QCD framework combined with the LCSR results, Ref. [5] predicted \( B(B^0 \to K_1^0(1270)\gamma) = (0.828 \pm 0.335) \times 10^{-5} \) and \( B(B^0 \to K_1^0(1400)\gamma) = (0.393 \pm 0.151) \times 10^{-5} \) at the NLO of \( \alpha_s \). New measurements (Eqs. (1) and (2)) certainly cast many questions about the theoretical predictions. Present work will be devoted to this issue.

The effective Hamiltonian for \( b \to s\gamma \) is

\[
H_{\text{eff}}(b \to s\gamma) = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{8} C_i(\mu) O_i(\mu),
\]

where

\[
O_2 = (s_i c_i) v_A(\bar{c}_j b_j) v_A, \quad O_7 = \frac{e m_b}{8\pi^2} s_i \sigma^{\mu\nu} (1 + \gamma_5) b_i F_{\mu\nu}, \quad O_8 = \frac{g_s m_b}{8\pi^2} s_i \sigma^{\mu\nu} (1 + \gamma_5) T^a_{\mu\nu} b_i G_{\mu\nu}^a,
\]

are the relevant operators for present analysis. Here \( i, j \) are color indices, and we neglect the CKM element \( V_{ub} V_{ts}^* \) as well as the s-quark mass. At next-to-leading order of \( \alpha_s \), the decay amplitude \( \mathcal{A} \) is given by

\[
\mathcal{A}(B \to K_1\gamma) = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^*(C_7^\gamma(O_7) + C_2^\gamma(O_2) + C_8^\gamma(O_8)),
\]
where \( \langle O_i \rangle \equiv \langle K_1 \gamma | O_i | B \rangle \). The leading contribution of \( \langle O_7 \rangle \) is given by

\[
\langle O_7 \rangle = \langle K_1 (p', \epsilon) \gamma (q, \epsilon) | O_7 | B(p) \rangle = \frac{e m_b}{4 \pi^2} F_+(0) \left[ (\epsilon^* \cdot q (p + p') \cdot \epsilon^* - \epsilon^* \cdot (p^2 - p'^2) + i \epsilon_{\mu \nu \alpha \beta} \epsilon^{\mu \nu} q^\alpha (p + p')^\beta \right],
\]

with \( \epsilon^\mu \) being the photon polarization vector. The form factor \( F_+ \) is defined by

\[
\langle K_1 (p', \epsilon) | i \sigma^\mu q^\nu b' | B(p) \rangle = F_+^A(q^2) \left[ (\epsilon^* \cdot q (p + p')_\mu - \epsilon^*_\mu (p^2 - p'^2) \right] \]

\[
+ F_+^A(q^2) \left[ (\epsilon^* \cdot q) q_\mu - \epsilon^*_\mu q^2 \right] \]

\[
+ \frac{F_0^A(q^2) \epsilon^* \cdot q}{m_B^2} [ (p^2 - p'^2) q_\mu - (p + p')_\mu q^2 ],
\]

where \( m \) and \( \epsilon^\mu \) are the mass and polarization vector of \( K_1 \), respectively, and \( q = p - p' \) is the photon momentum.

**FIG. 1.** NLO corrections to \( O_7 \). These diagrams are absorbed into the weak form factor \( F_+^A \).

All the subleading contributions to \( \langle O_7 \rangle \) shown in Fig. 1 are absorbed into the form factor \( F_+^A \), while the corresponding Wilson coefficient \( C_{7}^{\text{eff}} \) contains its NLO parts,

\[
C_{7}^{\text{eff}}(\mu) = C_{7}^{\text{eff}(0)}(\mu) + \frac{\alpha_s(\mu)}{4 \pi} C_{7}^{\text{eff}(1)}(\mu).
\]

On the other hand, the leading order \( C_{2}^{(0)} \) and \( C_{8}^{\text{eff}(0)} \) are sufficient for \( C_2 \) and \( C_8 \) since \( O_2 \) and \( O_8 \) contributions begin at NLO. The NLO contributions of \( O_{2,8} \) can be written as

\[
\langle O_i \rangle = \langle O_i \rangle_{VC} + \langle O_i \rangle_{HS} \quad (i = 2, 8),
\]

where \( \langle O_i \rangle_{VC(HS)} \) are vertex corrections (hard spectator interactions) depicted in Figs. 2 (3).

**FIG. 2.** Vertex corrections to the operators (a) \( O_2 \) and (b) \( O_8 \). Crosses denote the possible attachment of the emitted photon.

**FIG. 3.** Hard spectator interactions to (a) \( O_2 \) and (b) \( O_8 \). First diagrams are leading contributions at the heavy quark limit.

The branching ratio of \( B \to K_1 \gamma \) is simply given by

\[
B(B \to K_1 \gamma) = \frac{\alpha_s(\mu_b)}{4 \pi} C_{7}^{\text{eff}(0)}(\mu_b) \left( 1 - \frac{m_A^2}{m_B^2} \right)^{3/2} |F_+(0)|^2 |V_{tb} V_{ts}^*|^2 \times |C_{7}^{\text{eff}}(\mu_b) + A_{VC} + A_{HS}|^2.
\]

At the heavy quark limit,

\[
A_{VC} = \frac{\alpha_s(\mu_b)}{4 \pi} \left[ C_{7}^{\text{eff}}(\mu_b) \left[ - \frac{32}{9} \ln \frac{m_b}{\mu_b} + \frac{4}{27} (33 - 2 \pi^2) + 6 i \pi \right] + C_2(\mu_b) \left[ \frac{16}{3} \ln \frac{m_b}{\mu_b} + r_2 \right] \right],
\]

\[
A_{HS} = \frac{4 \pi \alpha_s(\mu_H) C_F}{N_c} f_B f_A \int \frac{d^4 q}{2 \pi^2} \left[ C_{7}^{\text{eff}}(\mu_H) \right] \frac{1}{12} \langle \mu^{-1} \rangle_\perp
\]
\[-C_2(\mu_H) \frac{1}{12} \left\langle \frac{\Delta t_5(z_0^c)}{u} \right\rangle \pm \right\}. \tag{11}\]

See [5] for details.

Keeping the hadronic parameters specifically, we have

\[B(B^0 \to K^0 \gamma) = 0.003 \times \left(1 - \frac{m_b^2}{m_B^2}\right)^3 \left|F_+^{K_1(1270)}(0) - 0.385 - i0.014\right| \]
\[+ \left|F_+^{K_1(1400)}(0) - 0.024 - i0.022\right|^2, \tag{12}\]

at the reference scales

\[(\mu_b, \mu_H) = (m_b(m_b), \sqrt{\Lambda_H m_b(m_b)}) = (4.2 \text{ GeV}, 1.45 \text{ GeV}). \tag{13}\]

It is now quite straightforward to extract the value of \(F_+^{K_1(1270)}(0)\) from the new measurements (1) and (2). We have

\[F_+^{K_1(1270)}(0) = 0.32 \pm 0.03, \]
\[F_+^{K_1(1400)}(0) < 0.19, \tag{14}\]

where \(f_{K_1(1270)} = 0.122 \text{ GeV}, f_{K_1(1400)} = 0.091 \text{ GeV}\) are used [19]. These must be compared with the LCSR results [19]

\[F_+^{K_1(1270)}(0) |_{\text{LCSR}} = 0.14 \pm 0.03, \]
\[F_+^{K_1(1400)}(0) |_{\text{LCSR}} = 0.098 \pm 0.02. \tag{15}\]

Here we have another big difference between theory and experiment other than \(K^*\). But the details of the differences are quite opposite. In short,

\[F_+^{K_1^+} \text{theory} > F_+^{K_1^+} \text{exp}, \]
\[F_+^{K_1} \text{theory} < F_+^{K_1} \text{exp}. \tag{16}\]

There are some candidates to explain the discrepancy. Higher twist effects in the light-cone DA are the first one. Usually they are process dependent, and are encoded in the coefficients of the Gegenbauer expansion. It is also known that they are asymptotically zero at \(\mu \to \infty\) where \(\mu\) is the renormalization scale. Ref. [17] estimated that the non-asymptotic correction of \(K^*\) at higher twist through the Gegenbauer moments to the operator \(O_8\) is \(\sim -20\%\). This is a bad news for \(K_1(1270)\) if a similar tendency occurs for the axial Kaons since the present analysis is based on the asymptotic form of the light-cone DA.

The second candidate is the non-zero mass effect. When calculating the hard spectator interactions in (11), it is assumed that the axial kaon is nearly massless and energetic. Although the assumption is acceptable for \(m_{K_1} \ll m_B\), the mass hierarchy of \(m_{K^*} < 1 \text{ GeV} < m_{K_1}\) might impose some doubts about the common framework for both \(K^*\) and \(K_1\). Note that the chiral symmetry is broken around 1 GeV.

But including non-zero mass corrections is very non-trivial. Since the relevant large scale in \(B \to K_1 \gamma\) is \(m_B\), possible mass corrections will appear in the form of \(m_{K_1}/m_B\). It means that to fully appreciate the mass effects, one has to consider the \(1/m_B\) (or \(1/m_0\)) corrections throughout the analysis, which is not well established so far. Since the discrepancy of (16) is quite large and \(m_{K_1}/m_B \approx 0.24\), one should expect large corrections like chiral enhancement in non-leptonic decays at \(1/m_B\).

Thirdly, the framework of QCD might not adequate for the axial kaons. The main idea of QCDF can be summarized by [16]

\[\langle V(\gamma) | O_i | B \rangle = \left[ F_{B \to V}(0) T^I_i \right. \]
\[+ \left. \int_0^1 d\xi dv \, \Phi_V(v) T^{II}_i(\xi, v) \Phi_B(\xi) \right] \epsilon, \tag{17}\]

where \(T^{I,II}_i\) are the hard scattering kernels. The kernel \(T^{II}_i\) is concerned with the hard spectator interactions. The factorization of (17) holds when the hard kernels are perturbatively calculable. All the nonperturbative physics is encapsulated in the DAs. A great discrepancy of (16) suggests that this may not be the case for \(K_1\). Some of the simple model calculations based on the heavy quark effective theory (HQET) predict rather large branching ratios; see the Table V in [5] or refer to [20]. Since the higher resonant kaons are heavy \(\gtrsim 1 \text{ GeV}\), it is quite natural and attractive to consider them as heavy mesons. In the heavy quark scheme, hard spectator interaction is inconceivable since almost all the momentum of initial heavy quark is transferred to the final one.

We can also question the reliability of the QCD sum rule or LCSR results. It is a common knowledge that the stability of an observable against the Borel parameter in the QCD sum rule gets poorer as higher resonances are involved. Still, the problem of how to describe the higher kaon resonances remains. It is also noticeable that the lattice calculation is very close to the QCD sum rule result for \(K^*\) [17, 21]. Much more reliable nonperturbative analyses are required in the near future.

Next, possible mixing in \(K_1(1270)\) and \(K_1(1400)\) cannot explain the large mismatches of (16). Quark model states \(^3P_1\) and \(^1P_1\) can mix to form physical states \(K_1(1270)\) and \(K_1(1400)\). The form factors are now written as [22]

\[F_+^{K_1(1270)}(0) = Y_A(0) \sin \theta + Y_B(0) \cos \theta, \]
\[F_+^{K_1(1400)}(0) = Y_A(0) \cos \theta - Y_B(0) \sin \theta, \tag{18}\]

where \(Y_{A,B}\) are the form factors corresponding to the angular momentum eigen states. The enhancement from maximal mixing is only a factor of \(\sqrt{2}\), assuming \(Y_A(0) \approx Y_B(0)\). A substantial growth in \(Y_A(0)\) is inevitable to explain the experimental data. On the other hand, the usefulness of mixing lies in the fact that it can naturally
explain a strong suppression of $B \to K_1(1400)\gamma$. But it is too early to say something about this point with the new upper bound of (2); the LCSR result (15) is still within the boundary. Therefore, a new observation of $B \to K_1(1400)\gamma$ is much anticipated.

Finally, it is quite unlikely that the annihilation topology would give considerable contributions, as pointed out in [15,16].

In conclusion, we surveyed the implications of the first observation of $B \to K_1\gamma$. The values of the relevant form factors are extracted from the experimental data at NLO of $\alpha_s$. We found that a very large discrepancy between theory and experiment is reproduced after $B \to K^*\gamma$. Eliminating the gap will be a great challenge in theory. Further observation by other $B$ factory as well as $K_1(1400)$ will provide much interest in coming days.

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