Cosmology from a gauge induced gravity

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Abstract. The main goal of the present work is to analyze the cosmological scenario of the induced gravity theory developed in previous works. Such a theory consists on a Yang-Mills theory in a four-dimensional Euclidian spacetime with $SO(m, n)$ such that $m + n = 5$ and $m \in \{0, 1, 2\}$ as its gauge group. This theory undergoes a dynamical gauge symmetry breaking via an Inönü-Wigner contraction in its infrared sector. As a consequence, the $SO(m, n)$ algebra is deformed into a Lorentz algebra with the emergency of the local Lorentz symmetries and the gauge fields being identified with a vierbein and a spin connection. As a result, gravity is described as an effective Einstein-Cartan-like theory with ultraviolet correction terms and a propagating torsion field. We show that the cosmological model associated with this effective theory has three different regimes. In particular, the high curvature regime presents a de Sitter phase which tends towards a $\Lambda$CDM model. We argue that $SO(m, n)$ induced gravities are promising effective theories to describe the early phase of the universe.
1 Introduction

The Standard Model (SM) has shown to be a very efficient theoretical framework of
the three fundamental interactions with the LHC continuously providing remarkable
experimental confirmations. The SM is constructed over a consistent quantum gauge
theoretical basis known as Yang-Mills theories. On the other hand, the fourth fundamental
interaction lacks a consistent quantum description, i.e., quantum gravity remains an open
issue with many theoretical proposals. Promising candidates for quantum gravity are
(to name a few): Loop Quantum Gravity [1, 2], Higher Derivative Quantum Gravity
[3, 4, 5], Causal Sets [6], Causal Dynamical Triangulations [7, 8], String Theory [9, 10, 11],
Asymptotic Safety [12, 13, 14, 15], and Emergent Gravities [3, 16, 17]. As widely known,
all of these theories have their share of goals and problems which, for the sake of objectivity,
we will not enumerate here. Nevertheless, string theories and emergent gravities share a
common aspect that deserves attention: they do not deal with the direct quantization
of the gravitational field. These theories have gravity as an emergent phenomenon. In
other words, gravity, as a geometrodynamical phenomenon, is only a classical limit of a
completely different quantum theory.

In the wide range of emergent gravities, there are several articles about gauge theories
which employ a physical mechanisms to “free” the geometrical degrees of freedom, see for
instance [18, 19, 20, 21, 22, 23, 24, 25, 26]. As a general aspect, the gravities which are
generated in this way are described in first order variables [27, 28, 29]. These particular
scenarios are very interesting in virtue of its resemblance with the SM, putting gravity on
equal footing with the other three fundamental interactions. In this sense, geometrodynam-
ics would be only a classical manifestation of another gauge theory, just like spontaneous
symmetry breaking and hadronization are low energy effects of the different sectors of the
SM.

The theory developed in [25] is based on the Yang-Mills action for the group
$SO(m, n)$. From this theory, a generalized gravity action emerges due to a non-perturbative dynamical
symmetry breaking. Differently from the other approaches, the physical mechanism
responsible for this breaking is not the Higgs mechanism, but the soft BRST breaking
associated to the so-called Gribov problem. The running of the Gribov and coupling
parameters work together for an Inönü-Wigner contraction [30] $SO(m, n) \mapsto ISO(m! − 1, p)$
where $p = 5 − m!$. Since the group $ISO(m! − 1, p)$ does not represent a symmetry
of the original action, the theory suffers a symmetry breaking for the common stability
group, namely $SO(m! − 1, p)$. This symmetry breaking is responsible for “freeing” the
gravitational variables (see section 2 for more details). The resulting theory describes a
geometrodynamical action for the vierbein and spin connection composed by the Einstein-
Hilbert term, the cosmological constant, a quadratic torsion term, and a quadratic curvature
term.

In the present paper, we focus on the study of the cosmological scenario provided by
the emergent gravity mentioned above. We are particular interested in the cosmological
solutions that may describe the early universe. In fact, the current paradigm to describe
the early universe is inflation [31, 32, 33, 34]. In this particular scenario, the standard
cosmological model, namely, the $\Lambda$CDM model is preceded by an exponentially accelerated
phase which should be responsible for solving the problems associated with the Friedmann-
Lemaître-Robertson-Walker (FLRW) metric such as the flatness, isotropy, horizon, and
monopole excess. In addition, one of the most attractive features of inflation is the

\[1\] The quantities $m$ and $n$ obey the bound with $m + n = 5$ with $m \in \{0, 1, 2\}$. 
prediction of primordial quantum fluctuations that seed the large scale structure with the correct scale-invariant spectrum. Apart from all its success, the inflationary scenario has some weakening points related to the existence of an initial singularity [35] and the open issue if inflation can indeed solve the homogeneity problem [36, 37, 38]. The proper manner to tackle these issues is to work with a complete quantum gravity theory.

Recently, there has been an increased interest in bouncing models in the literature. These models are valuable alternatives to inflation [39, 40, 41]. Bouncing universes are non-singular models which can be generated within higher derivative and quantum gravity theories [42, 43, 44, 45, 46, 47, 48, 49]. All these results suggest that the early universe is a promising arena to investigate theories associated to very high energy physics.

Our paper has the following structure: In section 2 we briefly point out the principal building blocks of our theory of gravity. In section 3 we construct a cosmological model from the induced gravity that we built. In section 4 we display our conclusions and further discussions.

2 Induced gravity from a Yang-Mills theory

In a previous paper [50], some of us have used a Yang-Mills theory in order to describe gravity at the quantum level. The main idea is to follow Quantum Chromodynamics (QCD) techniques such that, at low energies, a geometrodynamical theory of gravity is generated. In this section, we will briefly review the main features of these gauge theories.

We start with a pure gauge theory based on the $SO(5,m,n)$ group with $m + n = 5$ and $m \in \{0,1,2\}$. When $m = 0$, we have the orthogonal group. For $m = 1$ and $m = 2$, we have, respectively, de Sitter and anti-de Sitter groups. The spacetime is an Euclidian four-dimensional differential manifold, $\mathbb{R}^4$. The algebra of the group is given by

$$\left[ J^{AB}, J^{CD} \right] = -\frac{1}{2} \left[ \left( \eta^{AC} J^{BD} + \eta^{BD} J^{AC} \right) - \left( \eta^{AD} J^{BC} + \eta^{BC} J^{AD} \right) \right],$$

where $J^{AB} = -J^{BA}$ are the 10 anti-hermitian generators of the gauge group. Capital Latin indexes are chosen to run as $\{5,0,1,2,3\}$. As a Lie group, $SO(m,n)$ can be seen as a five-dimensional flat space, $\mathbb{R}^{m,n}_S$, with invariant Killing metric given by $\eta^{AB} \equiv \text{diag}(\epsilon, \varepsilon, 1, 1, 1)$ where $\epsilon = (-1)^{(2-m)!}$ and $\varepsilon = (-1)^{m! + 1}$. It is worth mentioning that the two spaces $\mathbb{R}^4$ and $\mathbb{R}^{m,n}_S$ are not dynamically related to each other, i.e., the gauge group has no relation with spacetime whatsoever.

The $SO(m,n)$ group can be decomposed as a direct product, $SO(m,n) \equiv SO(m!-1,p) \times S(4)$ where $S(4) \equiv SO(m,n)/SO(m!-1,p)$ is a symmetric coset space and $p = 5 - m!$. This can be accomplished by projecting the group space into its fifth coordinate direction $A = 5$. For convenience, let us label $J^{ba} = J^a$, where small Latin indexes vary as $\{0,1,2,3\}$. Thus, the algebra (1) can be written as

$$\left[ J^{ab}, J^{cd} \right] = -\frac{1}{2} \left[ \left( \eta^{ac} J^{bd} + \eta^{bd} J^{ac} \right) - \left( \eta^{ad} J^{bc} + \eta^{bc} J^{ad} \right) \right],$$

with $\eta^{ab} \equiv \text{diag}(\varepsilon, 1, 1, 1)$.
Now, we construct the Yang-Mills action as
\[ S_{YM} = \frac{1}{2} \int F_A^B * F_A^B, \quad (3) \]
where \( F_A^B \) is the field strength 2-form, \( F = dY + \kappa YY \), \( d \) is the exterior derivative, \( \kappa \) is the coupling parameter and \( Y \) is the gauge connection 1-form. The gauge field is the fundamental field and it lies on the adjoint representation of the gauge group. The “\(*\)” operator denotes the Hodge dual operator in Euclidean spacetime. The action (3) is invariant under \( SO(m,n) \) gauge transformations, \( Y \mapsto -\rightarrow U^{-1} (\frac{1}{\kappa} d + Y) U \), with \( U \in SO(m,n) \). The infinitesimal version of the gauge transformation is \( Y \mapsto Y + \nabla \alpha \) where \( \nabla = d + \kappa Y \) is the full covariant derivative and \( \alpha \) is the infinitesimal gauge parameter.

Following the above prescription, the gauge field is decomposed as follows
\[ Y = Y_A^B J_B^A = A^a_b J_b^a + \theta^a J_a. \quad (4) \]
Thus, the decomposed field strength reads
\[ F = F_A^B J_A^B = \left( \Omega^a_b - \frac{\epsilon \kappa}{4} \theta^a \theta_b \right) J^b_a + K^a J_a, \quad (5) \]
where we have defined \( \Omega^a_b \equiv dA^a_c + \kappa A^a_c A^c_b \) and \( K^a \equiv d\theta^a + \kappa A^a_b \theta^b \). It is straightforward to rewrite the Yang-Mills action (3) as
\[ S_{YM} = \frac{1}{2} \int \left[ \Omega^a_b * \Omega^b_a + \frac{1}{2} K^a * K_a - \frac{\epsilon \kappa}{2} \Omega^a_b * (\theta^a \theta^b) + \frac{\kappa^2}{16} \theta^a \theta^b * (\theta^a \theta^b) \right]. \quad (6) \]
From the physical point of view, the actions (3) and (6) are indistinguishable - the action (6) is only a different way to write down the Yang-Mills action for the group \( SO(m,n) \).

Before we go further, let us point out some interesting aspects of Yang-Mills theories and how we can make a consistent analogy with a possible quantum gravity theory. Yang-Mills theory presents two important properties, namely, perturbative renormalizability and asymptotic freedom [51]. Renormalizability is ensured by the BRST symmetry [52]. Asymptotic freedom [53, 54] means that the coupling parameter increases as the energy decreases. Hence, perturbation theory can only be employed at high energies. At low energies, the theory is settled in a highly non-perturbative regime. It is exactly at this regime that enters the Gribov problem [55, 56]. As it is well-known, the Faddeev-Popov gauge fixing is not sufficient to eliminate all spurious degrees of freedom. In this sense, a residual gauge symmetry survives and it is quite relevant at low energies. The procedure to eliminate the Gribov copies is not completely understood but we do know that one needs a mass parameter and a soft BRST symmetry breaking related to the Gribov mass parameter, see [58, 59, 60, 61, 62, 63, 64]. The BRST symmetry breaking, the Gribov parameter and the asymptotic freedom are the crucial effects that lead the action (6) to a gravity one. For details, see [25, 26].

It is immediate to check that the \( \theta \) field has the same degrees of freedom of a vierbein field, \( e \). However, as a piece of the gauge field \( Y \), the \( \theta \) field has canonical dimension 1 while the vierbein field is dimensionless. Nevertheless, in despite of its transformation rule, the Gribov mass parameter can be used to adjust this dimension discrepancy. In fact, we

\[ \text{Recently, it was shown that, although standard BRST symmetry is broken, a non-pertubative generalization of the standard BRST symmetry can be defined [57].} \]
can employ the following rescalings

\[ A \rightarrow \frac{1}{\kappa} A, \quad \theta \rightarrow \kappa \theta, \]

which modify the de Sitter algebra (2) to

\[ [J^{ab}, J^{cd}] = -\frac{1}{2}\left(\eta^{ac} J^{bd} + \eta^{bd} J^{ac}\right) - \left(\eta^{ac} J^{bc} + \eta^{bc} J^{ac}\right), \] (8a)

\[ [J^{a}, J^{b}] = -\frac{\gamma^2}{2\kappa^2} J^{ab}, \] (8b)

\[ [J^{ab}, J^{c}] = \frac{1}{2}\left(\eta^{ac} J^{b} - \eta^{bc} J^{a}\right). \] (8c)

In addition, the action (6) can be recasted as

\[ S = \frac{1}{2\kappa^2} \int \left( \Omega_{a}^{b} \ast \Omega_{a}^{b} + \frac{\gamma^2}{2} K_{a} \ast K_{a} - \frac{\gamma^2}{2} \Omega_{a}^{b} \ast (\theta_{a}\theta^{b}) + \frac{\gamma^4}{16} \theta_{a}\theta_{b} \ast (\theta_{a}\theta^{b}) \right), \] (9)

where \( \Omega_{a}^{b} \equiv dA_{a}^{b} + A_{c}^{a} A_{b}^{c} \), \( K_{a} \equiv D\theta^{a} \) and \( D = d + A \) is the covariant derivative with respect to the \( SO(m! - 1, p) \) sector.

The connection of the action (9) with a gravity theory is attained from the analysis of running of the ratio \( \gamma^2/\kappa^2 \). It has been shown in [26] that this ratio vanishes at an energy scale near Planck energy. Hence, from (8), it is clear that an Inönü-Wigner contraction takes place, i.e. \( SO(m,n) \rightarrow ISO(m! - 1, p) \). However, since the action (9) is not invariant under \( ISO(m! - 1, p) \) gauge transformations, this contraction induces a symmetry breaking in the theory that goes to the common stability group \( SO(m! - 1, p) \).

Given the symmetry breaking, the theory is now highly non-perturbative and it has also reached its quantum boundary to become a classical theory. This motivate us to find the possible physical observables. In QCD these are hadrons and glueballs. In a classical theory of gravity, these are geometrical entities. Hence, one may identify the effective metricity and affinity of spacetime with the gauge invariant quantities \( g_{\mu\nu} = \eta_{ab} \langle \theta_{a}^{\mu}\theta_{b}^{\nu} \rangle \) and \( \Gamma_{\mu\nu}^{a} = \langle \partial_{\mu}\theta_{a}^{\nu} + A_{\mu}^{a}\theta_{a}^{\nu} \rangle \), respectively. This idea allows a map between the gauge field pieces and the first order gravitational variables

\[ \delta_{a}^{b} \delta_{b}^{a} A_{a} = \omega_{a}^{b}, \]

\[ \delta_{a}^{b} \theta^{a} = e^{a}, \] (10)

where indexes \( \{a, b, c, \ldots\} \) are related to the tangent space of the deformed spacetime, \( \omega_{a}^{b} \) is the spin connection 1-form and \( e^{a} \) the vierbein 1-form. Additionally, it is convenient to identify

\[ \gamma^2 = \frac{\kappa^2}{4\pi G} = \frac{4\Lambda}{3}, \] (11)

where \( G \) and \( \Lambda \) are, respectively, Newton’s constant and the gravitational cosmological constant. With these redefinitions, the action (9) finally becomes a gravity action

\[ S_{\text{Grav}} = \frac{1}{16\pi G} \int \left( \frac{3}{2\Lambda} R_{b}^{a} \ast R_{a}^{b} + T_{a} \ast T_{a} - \frac{\epsilon}{2} \varepsilon_{abcd} R_{ab}^{c} e^{d} + \frac{\Lambda}{12} \varepsilon_{abcd} e^{a} e^{b} e^{c} e^{d} \right), \] (12)

where \( R_{b}^{a} = d\omega_{a}^{b} + \omega_{c}^{b} \omega_{a}^{c} \) is the curvature 2-form and \( T_{a} = de^{a} + \omega_{a}^{b} e^{b} \) is the torsion
2-form. In the above equation, the “⋆” operator represents the Hodge dual operator in the deformed spacetime $\mathbb{M}^4$.

Needless to say, the last two terms in (12) are recognized as the Einstein-Hilbert and the cosmological constant terms in the first order formalism. The other terms account for generalized terms of our effective gravity action, namely, a quadratic curvature and a quadratic torsion term.

It is worth emphasizing that, in this theory, Newton’s constant and the cosmological constant are coupled via (11). One-loop semi-perturbative estimations [26] predict an exact agreement for Newton’s constant but an extremely large value for $\Lambda$. This prediction seems to fail when compared with the observed cosmological constant (which we call $\tilde{\Lambda}$) [65, 66]. However, the Quantum Field Theory (QFT) prediction for the SM vacuum also predicts a discrepant value for the cosmological constant (here denoted by $\Lambda_{\text{QFT}}$) [67, 68, 69, 70]. In order to adjust the theoretical predictions with the observational value, following [71, 72, 25], we assume that the value of the observed cosmological constant is due to a cancellation of the bare cosmological constant with the contributions coming from the expectation vacuum of the matter quantum fields, hence, we write the net renormalized cosmological constant as $\tilde{\Lambda} = \Lambda + \Lambda_{\text{QFT}}$.

The vacuum field equations of this gravity theory can be found by applying the variational principle to action (12). Its extremization with respect to the vierbein field yields

$$\frac{3}{2\Lambda} R^b c \star (R_{bc} e_a) + T^b \star (T_b e_a) + D \star T_a - \varepsilon_{abcd} \left( \varepsilon R_{bc} e_d - \frac{\tilde{\Lambda}}{3} e_b e_c e_d \right) = 0, \quad (13)$$

while the extremization with respect to the spin connection field gives

$$3 \Phi \star \left( (\Lambda^{-1} R_{ab}) + \varepsilon_b \star T_a - \varepsilon_a \star T_b - \varepsilon_{abcd} T^c e_d \right) = 0. \quad (14)$$

We notice that the quadratic curvature term is inversely proportional to the bare cosmological constant. For a weak curvature regime, this correction term can be neglected and for a torsionless situation we arrive at the standard Einstein-Hilbert theory with a cosmological constant. In fact, by looking at (14) at a weak curvature regime, the solution $T \approx 0$ is immediate (at least for vanishing spin-densities). It is interesting to note that the above system of equation is different from the one coming from the Einstein-Cartan-Sciama-Kibble theory. Indeed, the field equation (14) is not an algebraic equation for the spin connection and shows that, for a strong curvature regime, torsion generally behaves as a propagating field.

### 3 Cosmology

The present Standard Cosmological Model (SCM) is a complex set of models that, combined together, gives us a coherent description of the evolution of the universe. Despite the delicate open issues, such as the nature of dark energy and dark matter, the SCM is a consistent framework that accounts for all present observations. The core of the SCM is the assumption that the universe is homogeneous and isotropic over scales larger than 200 Mpc. The current CMB data seems to validate the isotropy of the universe around us but, since astronomical observations give access only to the past null-light cone, the homogeneity remains only as a profitable assumption.

The effective theory of gravity briefly described in the last section provides us a rich
cosmological scenario. As will be shown, there are three distinctive regimes which, if properly connected, could outline the evolution of the universe from energies close to the Planck energy up to the MeV scale where the SCM nucleosynthesis took place.

At the present analysis, we will use the term cosmological model for the evolution of a homogeneous and isotropic metric according to the field equations (13) and (14). The matter content will be assumed to have negligible spin contribution and be represented as non-interacting fluids. In addition, in order to have a Riemannian spacetime, we will disregard torsion effects, i.e. the torsion 2-form will be assumed everywhere zero, $T^a(x) = 0$.

The energy scale of our cosmological model has a wide range of validity. Thus, we need to account for the QFT contribution to the cosmological constant. The net effect of this contribution is a change only in the cosmological constant term $\Lambda \rightarrow \tilde{\Lambda} = \Lambda + \Lambda_{\text{QFT}}$ leaving intact the factor $1/\Lambda$ in the curvature square term (see last section and [25, 26]). Thus, our cosmological model will be based on the action

$$S = \int (\epsilon L_{RR} + \epsilon L_{CC} + L_{EH} + L_m) ,$$  \hspace{0.5cm} (15)

where

$$L_{RR} = \frac{3}{32\pi G} R^a_b \star R^b_a ,$$  \hspace{0.5cm} (16)

$$L_{CC} = \frac{\tilde{\Lambda}}{192\pi G} \epsilon_{abcd} e^a e^b e^c e^d ,$$  \hspace{0.5cm} (17)

$$L_{EH} = \frac{-1}{32\pi G} \epsilon_{abcd} R^{ab} e^c e^d .$$  \hspace{0.5cm} (18)

and $\epsilon = \pm 1$. As mentioned before, $L_{EH}$ and $L_{CC}$ denote, respectively, the Einstein-Hilbert and the cosmological constant Lagrangians and both of them are already present in General Relativity (GR). The Lagrangian $L_m$ sets the matter content that, as already said, will be considered as a combination of non-interacting fluids with negligible net spin. The extra lagrangian $L_{RR}$ deviates our dynamics from GR’s by introducing a curvature square correction - even though it is suppressed by a $\Lambda$ denominator.

It must be mentioned that the identification (11) induces an energy-dependent running on $\Lambda$ at a semi-classical level - this will be discussed in subsection 3.1. In addition, we clearly have distinct contributions from each of the terms in the action (15). Therefore, we can distinguish three main sectors of our model, namely, the deep ultraviolet (deep UV), the ultraviolet (UV), and the infrared (IR) sectors. Before detailing each of these three regimes, let us re-write the dynamical equations governing this scenario using spacetime indexes.

Consider a generic fluid with four-velocity field given by $v^\mu$. Its energy-momentum tensor can always be decomposed as

$$T^{\mu\nu} = \rho v^\mu v^\nu + ph^{\mu\nu} + v^\mu q^\nu + v^\nu q^\mu + \pi^{\mu\nu} ,$$  \hspace{0.5cm} (19)

where the thermodynamic quantities $\rho$, $p$, $q^\mu$ and $\pi^{\mu\nu}$ are, respectively, the energy density, the pressure, the heat flux vector and the anisotropic pressure tensor. The $h^{\mu\nu}$ tensor is the projector defined as $h^{\mu\nu} \equiv v^\mu v_\nu + \delta^\mu_\nu$. Let us define, for later convenience, the Einstein tensor $G^\mu_\nu \equiv R^\mu_\nu - \frac{1}{2} R \delta^\mu_\nu$ and a tensor $\Upsilon^\mu_\nu$ describing the UV correction as

$$\Upsilon^\mu_\nu \equiv R^{\alpha\beta\lambda\sigma} R_{\alpha\beta\lambda\sigma} \delta^\mu_\nu - 2 R^{\alpha\beta\lambda\mu} R_{\alpha\beta\lambda\nu} .$$  \hspace{0.5cm} (20)
In the presence of matter and for vanishing torsion, the field equations (13) and (14) can be written in a coordinate basis as

$$\frac{3\epsilon}{8\Lambda} \nabla_\mu \nabla_\nu + C_\mu^{\nu} + \epsilon \tilde{\Lambda} \delta_\mu^{\nu} = \chi \nabla_\mu \nabla_\nu,$$

$$\nabla_\alpha \left( \Lambda^{-1} \nabla_\rho \nabla_\sigma \right) = 0,$$

where we introduced the gravitational coupling constant $\chi \equiv 8\pi G$. Equation (22) can still be further simplified by using the Bianchi identities. In an arbitrary Riemannian geometry the Bianchi identities guarantee that $2\nabla_a \nabla^a = \nabla_\beta \nabla^\beta$. Thus, the second equation above can be recasted as

$$\frac{1}{\Lambda} \partial_\mu \nabla = \frac{1}{\Lambda} \nabla_\mu \nabla_\alpha \nabla_\alpha \ln \Lambda.$$

To consider a homogeneous and isotropic metric means that there is a special foliation where each spatial section is maximally symmetric. Therefore, the metric must be of a Friedmann-Lemaître-Robertson-Walker (FLRW) type and, hence, the interval can be written as

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right),$$

where $a(t)$ is the scale factor, $d\Omega^2$ is the solid angle and the constant $k \in \{-1, 0, 1\}$ defines the curvature of the spatial sections.

In a geometric theory of gravity, isometries must come accompanied by symmetries of the energy-momentum tensor. In the particular case of a FLRW metric, the matter field acting as a source field must have an energy-momentum of a perfect fluid, i.e. equation (19) simplifies to

$$\nabla_\mu = (\rho + p)v^\mu v_\nu + p \delta_\mu^{\nu}.$$

In the preferred coordinate system in which the interval takes the form (24), the field equations reduce to a time evolution of the scale factor combined with homogeneous and isotropic thermodynamic quantities. It is useful to define two variables encoding time derivatives of the scale factor as

$$l \equiv \frac{\dot{a}}{a}, \quad h \equiv \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2}.$$

In terms of these variables, in a FLRW spacetime, the non-zero components of the relevant geometrical objects are

$$\mathcal{R}^i_j = (l+2h)\delta^i_j, \quad \Upsilon^i_j = 4(2l^2+h^2)\delta^i_j, \quad \mathcal{R}^0_0 = 3l, \quad \Upsilon^0_0 = 12h^2, \quad \mathcal{R} = 6(l+h).$$

Defining two energy densities associated with the two cosmological constants as $\rho_\Lambda \equiv \chi^{-1}\Lambda$ and $\rho_{\tilde{\Lambda}} \equiv \chi^{-1}\tilde{\Lambda}$, the field equations read

$$\frac{3\epsilon}{2\chi \rho_\Lambda} h^2 - h + \frac{\chi}{3}(\rho + \epsilon \rho_{\tilde{\Lambda}}) = 0,$$

$$\frac{3\epsilon}{2\chi \rho_\Lambda} l^2 - l - \frac{\chi}{6}(\rho + 3p - 2\epsilon \rho_{\tilde{\Lambda}}) = 0,$$

$$\partial_l (l + h) - l \partial_l \ln \Lambda = 0.$$

This is the system of equations governing the evolution of this cosmological scenario.
distinct behavior in the different sectors comes from the importance of each of the terms in the above system of equations. In the following, we will analyze each of the three sectors going from very high energies to low energy scales.

3.1 Deep UV sector: running in $\rho_\Lambda$

In section 2, we briefly described the emergency of a geometrodynamical theory of gravity from a pure Yang-Mills theory. The energy scale of this phase transition turned out to be in the same order of magnitude of Planck energy. Therefore, the period of this transition is compatible with the end of what is known in cosmology as the Planck Era. Hence, we expect that this classical description of gravity to be valid up to very high energies.

The gravitational constant and the cosmological constant are related to the $\kappa^2$ parameter through equation (11). As long as the coupling parameter $\kappa^2$ and $\gamma^2$ can display running effects, either $\Lambda$ or $G$ should also vary at the thin transition region around Planck scale. Without loss of generality, we will assume a constant $G$ and a running $\Lambda$. However, we do expect this running to saturate already at extremely high energies. Indeed, the semi-perturbative analysis in [26, 73] shows that this is exactly the case: the runnings rapidly become negligible below Planck energy. Nevertheless, we will consider the possibility of such a small window, really close to the Planck frontier, in which gravity behaves classically but $\Lambda$, for instance, still has an inherited running. We will refer to this short period as the deep ultraviolet (deep UV) sector of this theory of gravity.

The term “ultraviolet” is used in this context to explicit the fact that we are referring to a really high energy regime. In a geometric theory of gravity, this can be translated as saying that spacetime has a large curvature. However, both statements only make sense if the curvature (or energy) is being compared with some other equivalent quantity. This quantity would be the characteristic energy scale of the model, given by $\Lambda$ or, equivalently, by $\rho_\Lambda$. For instance, we can compare the linear terms $h$ and $l$ with $\rho_\Lambda$. If $h$ and $l \gg \rho_\Lambda$, then the quadratic terms in (28) and (29) are comparable with the linear terms themselves and the full equations must be taken into account. In other words, the “UV qualifier” in this gravity context means that the UV correction terms cannot be neglected.

In order to solve the system of equations (28), (29) and (30) we need to specify the time dependence of $\Lambda$. The behavior $\Lambda(t)$ is closely tied to the behavior of $\gamma^2$ in the non-perturbative sector of the original Yang-Mills theory - where the identification (11) took place. Unfortunately, we lack a method to evaluate this dependence since perturbative techniques are no longer valid. Thus, we have no firm ground to go beyond our remarks and we will leave this sector for future investigations.

3.2 UV sector: high curvature and constant parameters regime

Fairly below Planck energy the gravitational parameters $\Lambda$ and $G$ have already reached their saturation values - any running effects can be safely neglected. In spite of this, we can still assume a high curvature situation. This scenario will be called the UV sector of this gravity theory and its cosmological dynamics is given by (28), (29) and

$$\partial_t (l + h) = 0 .$$

(31)

In what follows we will outline separately the evolution of a vacuum and a matter filled spacetime.
3.2.1 Vacuum case

In a vacuum universe, equations (28) and (29) resume to

\[
\frac{3\epsilon}{2\Lambda} h^2 - h + \frac{\epsilon \Lambda}{3} = 0, \tag{32}
\]

\[
\frac{3\epsilon}{2\Lambda} l^2 - l + \frac{\epsilon \Lambda}{3} = 0. \tag{33}
\]

These are algebraic relations for \( h \) and \( l \), respectively. In fact, both equations have the same structure showing that \( h \) and \( l \) share the same spectrum. It is straightforward to identify the roots as

\[
\Lambda_{\text{dS} \pm} \equiv \frac{\epsilon \Lambda}{3} \left( 1 \pm \sqrt{1 - \frac{2}{1 - 2\frac{\epsilon \Lambda}{\Lambda}}(\Lambda_{\text{dS} \pm})} \right). \tag{34}
\]

Since both \( h \) and \( l \) are constants, equation (31) is automatically satisfied. The Ricci scalar reads \( R = 6(l + h) \) which for \( l = h \) gives \( R = 12\Lambda_{\text{dS} \pm} \) and for \( l \neq h \) gives \( R = 4\epsilon \Lambda \). The evolution of the scale factor can automatically be integrated from the equation \( h = \Lambda_{\text{dS} \pm} \).

The solutions are

\[
a(t) = \frac{1}{\Lambda_{\text{dS} \pm}} \begin{cases} 
\cosh \left( \sqrt{\Lambda_{\text{dS} \pm} t} \right) & ; \ k = -1, \\
\exp \left( \sqrt{\Lambda_{\text{dS} \pm} t} \right) & ; \ k = 0, \\
\sinh \left( \sqrt{\Lambda_{\text{dS} \pm} t} \right) & ; \ k = 1.
\end{cases} \tag{35}
\]

If \( \epsilon = 1 \) one can immediately recognize these as being three different foliations of a de Sitter universe with an effective cosmological constant given by \( \Lambda_{\text{dS} \pm} \). However, if \( \epsilon = -1 \), then \( \Lambda_{\text{dS} \pm} \) is negative and the hyperbolic functions are, in fact, regular trigonometric functions. This latter case means that (35) represents different foliations of an anti-de Sitter universe.

The deceleration parameter

\[
q \equiv -\frac{\ddot{a}}{a^2} = -\frac{1}{1 - k/(a^2\Lambda_{\text{dS} \pm})}, \tag{36}
\]

is always negative, manifesting the accelerated expanding phase. The effective cosmological constant \( \Lambda_{\text{dS} \pm} \) depends on both \( \Lambda \) and \( \Lambda_{\text{dS}} \). In the limit \( \Lambda_{\text{dS}}/\Lambda \ll 1 \), we can expand it up to first order to obtain

\[
\Lambda_{\text{dS} \pm} \approx \frac{\epsilon \Lambda}{3} \left[ 1 \pm \left( 1 - \frac{\Lambda_{\text{dS} \pm}}{\Lambda_{\text{dS}}} \right) \right]. \tag{37}
\]

The root \( \Lambda_{\text{dS} +} \) is then approximately given by \( \epsilon 2\Lambda / 3 \) while the root \( \Lambda_{\text{dS} -} \) is approximately given by \( \epsilon \Lambda / 3 \). Given the enormous value of \( \Lambda \), if \( \epsilon = 1 \) then the first root represents an universe with a violent de Sitter phase. Therefore, it may be associated with an inflationary expansion. On the other hand, the second root, also if \( \epsilon = 1 \), would correspond to a smoothly accelerating phase, similar to the late time expansion in the \( \Lambda CDM \) model.

3.2.2 Matter case

A generic perfect fluid has an energy-momentum tensor given by eq. (25). To complete its specification, one has to provide an equation of state which, in cosmology, is generally a functional dependence of the pressure in terms of the energy density, i.e. \( p = p(\rho) \). Fluids
that obey this equation of state are known as barotropic fluids. In GR the fluid equation of state is of major importance to completely specify the dynamical system. We are about to see that this is not the case for the UV sector of our model.

The dynamics of the matter filled spacetime is given by the system (28), (29), and (31). In principle, we have a system of three equations and three variables, namely, $a(t)$, $\rho(t)$ and $p(t)$ that will determine the evolution of spacetime and matter. Taking into account the thermodynamic equation of state would only make this system overdetermined. Indeed, equations (28), (29) and (31) are solvable without the need of an equation of state. A possible way to reconcile this situation with a thermodynamic description of matter is to interpret the UV sector as a regime where the gravitational field does not distinguish the nature of the matter fields. In other words, in the UV sector any perfect fluid gravitates in the same manner.

To solve the above system of equation, we first focus on (31). This equation states that the Ricci scalar, $R = 6(l + h)$, has to be a constant which we write as $R_0$. Thus, we can substitute $l = R_0/6 - h$ in (29) to obtain a new equation for the variable $h$, which can be combined with (28) to provide

$$h = \frac{\chi^2 \rho - R_0}{4\chi \rho - R_0} (\rho + p) + \frac{R_0}{12}. \quad (38)$$

Note, however, that from the definition of $h$ itself - see (26) - we have $\dot{h} = 2\dot{a}a^{-1}(l - h)$ and hence

$$\dot{h} = \frac{1}{3a} (R_0 - 12h) \Rightarrow h = \frac{\xi_0}{a^4} + \frac{R_0}{12}, \quad (39)$$

where $\xi_0$ is a constant of integration. The above equation can be further integrated to obtain the time evolution of the scale factor. In order to do this, we recast (39) by writing $a^2(t) = x(t)$ such that

$$x^2 - \frac{R_0}{3} x^2 + 4kx = 4\xi_0. \quad (40)$$

The solutions of this differential equation are branched in basically three situations. When $R_0 \neq 0$ we have

$$x(t) = x_0 e^{\pm\alpha t} + \frac{9k^2 - 3R_0 \xi_0}{R_0^2 x_0} e^{\mp\alpha t} + \frac{6k}{R_0}, \quad (41)$$

where $\alpha = \sqrt{R_0/3}$ and $x_0 > 0$ is a constant of integration. Of course, the characteristic of these solutions depends on the interplay among the constants $k$, $R_0$ and $\xi_0$. In particular, it can describe a bounce if the second term has the same sign as the first one, namely, if $R_0 \xi_0 < 3k^2$.

For a null Ricci scalar $R_0 = 0$ but nonzero spatial section, the scale factor evolves accordingly to

$$a(t) = \sqrt{\xi_0 k} \left( t \pm \sqrt{|\xi_0|} \right)^2, \quad (42)$$

where we chose the origin of time to correspond to the (classical) singularity. The ± sign within the square term does not change qualitatively the evolution. For $k = 1$ we have a Big Bang\Big Crunch solution with an initial and a final singularity. The scale factor has maximal range of $\Delta t = 2\sqrt{\xi_0}$ where $a_{\text{max}} = \sqrt{\xi_0}$. For $k = -1$ we have two disjoint branches: either an expanding universe with initial singularity or a collapsing universe with a future singularity. For instance, for the plus sign the initial singularity is located at
$t = 0$, while the final singularity in the collapsing phase is at $t = -2\sqrt{\xi_0}$.

Finally, for a flat spatial section, we have that the constant $\xi_0$ must be positive-definite and the solution is given by

$$a(t) = \begin{cases} \sqrt{-2\sqrt{\xi_0}t} & \text{if } t < 0, \\ \sqrt{+2\sqrt{\xi_0}t} & \text{if } t > 0. \end{cases} \quad (43)$$

where the constant of integration was chosen to locate the (classical) singularity at $t = 0$. Once more we have a collapsing phase for $t < 0$ that reaches the singularity and an expanding phase originating at an initial singularity at $t = 0$. For a qualitative view of all these solutions, see fig. 1.

Figure 1: Three possible qualitatively distinct behavior of the scale factor. The solid line describes a non-singular bounce dynamics. The dashed line depict a typical Big Bang/Big Crunch model starting from and ending on a singularity. The dot-dashed lines represent two other possible singular models either starting from a singularity and expanding forever or an universe that contracts from infinity and ends on a singularity.

The dynamics of the energy density and pressure can be found using the other two equations. Eq. (39) can be combined with equations (28) and (38) resulting in the evolution of the two thermodynamic quantities

$$\rho = \frac{3\xi_0(4\chi\rho_\Lambda - R_0)}{4\chi^2\rho_\Lambda}a^{-4} - \frac{9\xi_0^2}{2\chi^2\rho_\Lambda}a^{-8} - \left(\rho_\Lambda - \frac{R_0}{4\chi} + \frac{R_0^2}{32\chi^2\rho_\Lambda}\right), \quad (44)$$

$$p = -\rho + \left(\frac{4\chi - R_0}{\chi^2\rho_\Lambda}\right)\xi_0a^{-4}. \quad (45)$$

The particular case $\xi_0 = 0$ freezes the value of the energy density and the pressure becomes $p = -\rho$. This case reflects a simple redefinition of $\rho_\Lambda$ and hence we will assume $\xi_0 \neq 0$. The solutions (44) and (45), plotted in fig. 2, show the universal behavior of the pressure and energy density, independently of the equation of state of the fluid. Of course, we can use the solutions for the scale factor to obtain $\rho(t)$ and $p(t)$. For instance, see fig. 3. On the other hand, if we had assumed a barotropic equation of state of the fluid, say $p = w\rho$ with constant $w$, we would have found that equations (38) and (39) necessarily implies
$w = -1$. Thus, this model in its UV sector does not accept a barotropic equation of state describing matter. In fact, the only consistent barotropic equation of state in the UV sector is for the observational cosmological constant, $p_{\Lambda} = -\rho_{\Lambda}$.

In a FLRW universe, the conservation of the energy-momentum tensor of a barotropic fluid provides the energy density in terms of the scale factor. In particular, if $p = w\rho$ then $\rho \propto a^{-3(1+w)}$. Hence, it is clear that the energy density and pressure given by (44) and (45) must be associated with a non-conservation of the matter energy-momentum tensor. Indeed, taking the time derivative of equation (28) and using equations (29) and (39) we arrive at

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) + \frac{\dot{a}^2}{8(\dot{a}/a)(\rho + \rho_{\Lambda} - \rho_{\Lambda}/2)} = 0,$$

(46)

confirming the non-conservation of the matter energy-momentum tensor. The first two terms are the usual components while the last non-linear term comes from the geometrical UV corrections.

### 3.3 IR sector: connecting to the $\Lambda$CDM model

The IR sector is characterized by a low energy regime for gravity, i.e. low spacetime curvature. In fact, if $h$ and $l \ll \rho_{\Lambda}$, then the quadratic terms in (28) and (29) become negligible. Also, as discussed in section 2, equation (31) can be disregarded completely. Effectively, one can reach the IR regime by taking the limit $\Lambda \rightarrow +\infty$. In this case, the dynamics is given by

$$h = \frac{\chi}{3}(\rho + \epsilon\rho_{\Lambda}),$$

(47)

$$l = -\frac{\chi}{6}(\rho + 3p - 2\epsilon\rho_{\Lambda}),$$

(48)

which are exactly the Einstein field equations with a cosmological term. Note, however, that the sign of the cosmological term is determined by the parameter $\epsilon$. In particular, only if $\epsilon = 1$ their solutions will contemplate the late time expansion of the $\Lambda$CDM model.
The values of the parameters are $\xi_0 = 1.8$, $\chi \rho_\Lambda = 2.0$, $\chi \rho_{\tilde{\Lambda}} = 0.32$ and $R_0 = 1.66$.

The phase transitions originating the present gauge induced theory of gravity is expected to happen at energy scales of the order of $10^{16}$ TeV. Therefore, the IR regime should be valid much before the primordial nucleosynthesis that occurred at MeV. Clearly, the IR cosmological sector mimics the $\Lambda$CDM model with a cosmological constant given by $\tilde{\Lambda}$.

4 Conclusions

The present standard cosmological model is heavily based upon the FLRW metric. In this work we addressed the FLRW-like models in the effective gravity scenario discussed in section 2. The induced gravity theory developed can be seen as a non-Riemannian generalization of GR with UV completion terms. As a first approach, we assumed a Riemannian spacetime which can be accomplished by annulling all torsion’s degrees of freedom. Notwithstanding, the UV correction terms have shown to be sufficient to bring new features to the dynamics of the model.

In section 3 we showed that our model displays three distinct regimes. These dynamical regimes can be identified by comparing the curvature invariants with the bare cosmological constant $\Lambda$. In the IR sector, which is defined as a low curvature regime with negligible UV contributions, the theory mimics GR with a renormalized cosmological constant $\tilde{\Lambda}$. For a homogeneous and isotropic universe we naturally recover the $\Lambda$CDM model, hence smoothly connecting with the standard cosmological model. In order to secure all present early universe observation we need to connect our model with the $\Lambda$CDM at least before nucleosynthesis. Indeed, as long as the bare cosmological constant suppressing the UV corrections is really high, $\Lambda^{1/2} \sim 10^{16}$ TeV, we expect that the IR regime is reached much before the primordial nucleosynthesis which happened at MeV. Nevertheless, we should emphasize that this feature is only true for the $SO(5)$ induced gravity analyzed in the
present work. Different $SO(m, n)$ induced gravity models might be in contradiction with the standard cosmological model.

The UV sector is characterized by a high curvature regime which is of the same order of magnitude of $\Lambda$. In this regime, the energy scale is comparable with customary inflationary phase. We analyzed separately a vacuum universe and a perfect fluid permeated spacetime. In the former, equations (32) and (33) were solved and we obtained three solutions for the scale factor, depending on $\epsilon$ and the curvature of the spatial section as listed in (35). In order to obtain an expanding de Sitter phase one must consider only the $\epsilon = 1$ solutions. This is the same condition that was required to associate the IR sector with the $\Lambda$CDM model. Hence, for the $SO(5)$ induced gravity, the cosmological model can have a de Sitter primordial phase consistently connected with a $\Lambda$CDM universe.

The induced gravity theory was developed in a first order formalism which increases the number of dynamical equations. For GR, the extra field equation associated with the spin connection’s degrees of freedom is not properly a dynamical equation. In fact, it establishes the affine nature of the spacetime by requiring the connection to be identified with Christoffel symbols. In our case, we do have an extra dynamical equation that turns the set of cosmological equations into a determined system. As a consequence, one cannot introduce an equation of state for the perfect fluid. As we have argued in section 3.2.2, the proper way to interpret this result is to consider that, in the UV regime, gravity do not distinguish different fluids. All perfect fluids gravitate in the same manner. Among the possible solutions, there are singular solutions with single past or future singularities, Big Bang\textbackslash Big Crunch and also nonsingular solution with a single bounce that can be symmetric or antisymmetric.

The deep UV sector is also a high curvature regime. However, the main difference with respect to the UV sector is a possible running on $\Lambda$. This energy dependence comes from the running of the coupling parameter $\kappa^2$. The dynamic systems governing this period is enclosed in equations (28)-(30). The energy scale of the deep UV sector is in the order of Planck energy and, to adequately describe this regime one needs to specify the behavior of $\gamma^2$ in the non-perturbative sector of the original Yang-Mills theory. Unfortunately, at present moment, we lack a method to evaluate this dependence since perturbative techniques are no longer valid.

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