STATUS OF COSMOLOGY

Joel R. Primack
University of California, Santa Cruz, CA 95064 U.S.A.

Abstract. The cosmological parameters that I will emphasize are the Hubble parameter $H_0 \equiv 100h$ km s$^{-1}$ Mpc$^{-1}$, the age of the universe $t_0$, the average matter density $\Omega_m$, the baryonic matter density $\Omega_b$, the neutrino density $\Omega_\nu$, and the cosmological constant $\Omega_\Lambda$. The evidence currently favors $t_0 \approx 13$ Gyr, $h \approx 0.65$, $\Omega_m \approx 0.3$, $\Omega_\Lambda \approx 0.7$.

1. Introduction

In this brief summary I will concentrate on the values of the cosmological parameters. The other key questions in cosmology today concern the nature of the dark matter and dark energy, the origin and nature of the primordial inhomogeneities, and the formation and evolution of galaxies. I have been telling my theoretical cosmology students for several years that these latter topics are their main subjects for research, since determining the values of the cosmological parameters is now mainly in the hands of the observers.

In discussing cosmological parameters, it will be useful to distinguish between two sets of assumptions: (a) general relativity plus the assumption that the universe is homogeneous and isotropic on large scales (Friedmann-Robertson-Walker framework), or (b) the $\Lambda$CDM family of models. The $\Lambda$CDM models assume that the present matter density $\Omega_m$ plus the cosmological constant (or its equivalent in “dark energy”) in units of critical density $\Omega_\Lambda = \Lambda/(3H_0^2)$ sum to unity ($\Omega_m + \Omega_\Lambda = 1$) to produce the flat universe predicted by simple cosmic inflation models. The $\Lambda$CDM family of models was introduced by Blumenthal et al. (1984), who worked out the linear power spectra $P(k)$ and a semi-analytic treatment of structure formation compared to the then-available data. We did this for the $\Omega_m = 1$, $\Lambda = 0$ “standard” cold dark matter (CDM) model, and also for the $\Omega_m = 0.2$, $\Omega_\Lambda = 0.8$ $\Lambda$CDM model. In addition to $\Omega_m + \Omega_\Lambda = 1$, these $\Lambda$CDM models assumed that the primordial fluctuations were Gaussian with a Zel’dovich spectrum ($P_p(k) = Ak^n$, with $n = 1$), and that the dark matter is mostly of the cold variety.

The table below summarizes the current observational information about the cosmological parameters. The quantities in brackets have been deduced using at least some of the $\Lambda$CDM assumptions. The rest of this paper discusses these issues in more detail. But it should already be apparent that there is impressive agreement between the values of the parameters determined by various methods.
Table 1. Cosmological Parameters [results assuming ΛCDM in brackets]

| Parameter          | Value                              |
|--------------------|------------------------------------|
| Hubble parameter   | $H_0 = 100 \, h \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}$, $h = 0.65 \pm 0.08$ |
| Age of universe    | $t_0 = 9-16 \, \text{Gyr}$ (from globular clusters) $[9-17 \, \text{Gyr}]$ |
| Baryon density     | $\Omega_b h^2 = 0.019 \pm 0.001$ (from D/H) $> 0.015$ from Lyα forest opacity |
| Matter density     | $\Omega_m = 0.4 \pm 0.2$ (from cluster baryons) $= 0.34 \pm 0.1$ from Lyα forest $P(k)$ $= 0.4 \pm 0.2$ from cluster evolution $> 0.3$ (2.4σ, from flows) $\approx \frac{1}{4} \Omega - \frac{1}{2}$ from SN Ia |
| Total density      | $\Omega_m + \Omega_\Lambda \approx 1 \pm 0.3$ (from CMB peak location) $< 0.73$ (2σ) from radio QSO lensing |
| Dark energy density| $\Omega_\Lambda = 0.8 \pm 0.3$ (from previous two lines) $< 0.001$ (from Superkamiokande) $\lesssim [0.1]$ |
| Neutrino density   | $\Omega_\nu \approx 0.001$ (from Superkamiokande) $\lesssim [0.1]$ |

2. Age of the Universe $t_0$

The strongest lower limits for $t_0$ come from studies of the stellar populations of globular clusters (GCs). In the mid-1990s the best estimates of the ages of the oldest GCs from main sequence turnoff magnitudes were $t_{GC} \approx 15 - 16 \, \text{Gyr}$ (Bolte & Hogan 1995; VandenBerg, Bolte, & Stetson 1996; Chaboyer et al. 1996). A frequently quoted lower limit on the age of GCs was 12 Gyr (Chaboyer et al. 1996), which was then an even more conservative lower limit on $t_0 = t_{GC} + \Delta t_{GC}$, where $\Delta t_{GC} \gtrsim 0.5 \, \text{Gyr}$ is the time from the Big Bang until GC formation. The main uncertainty in the GC age estimates came from the uncertain distance to the GCs: a 0.25 magnitude error in the distance modulus translates to a 22% error in the derived cluster age (Chaboyer 1995).

In spring of 1997, analyses of data from the Hipparcos astrometric satellite indicated that the distances to GCs assumed in obtaining the ages just discussed were systematically underestimated (Reid 1997, Gratton et al. 1997). It follows that their stars at the main sequence turnoff are brighter and therefore younger. Stellar evolution calculation improvements also lowered the GC age estimates. In light of the new Hipparcos data, Chaboyer et al. (1998) have done a new Monte Carlo analysis of the effects of varying various uncertain parameters, and obtained $t_{GC} = 11.5 \pm 1.3 \, \text{Gyr}$ (1σ), with a 95% C.L. lower limit of 9.5 Gyr. The latest detailed analysis (Carretta et al. 1999) gives $t_{GC} = 11.8 \pm 2.6 \, \text{Gyr}$ from main sequence fitting using parallaxes of local subdwarfs, the method used in the 1997 analyses quoted above. These authors get somewhat smaller GC distances when all the available data is used, with a resulting $t_{GC} = 13.2 \pm 2.9 \, \text{Gyr}$ (95% C.L.).

Stellar age estimates are of course based on standard stellar evolution calculations. But the solar neutrino problem reminds us that we are not really sure that we understand how even our nearest star operates; and the sun plays an important role in calibrating stellar evolution, since it is the only star whose age...
we know independently (from radioactive dating of early solar system material). An important check on stellar ages can come from observations of white dwarfs in globular and open clusters (Richer et al. 1998).

What if the GC age estimates are wrong for some unknown reason? The only other non-cosmological estimates of the age of the universe come from nuclear cosmochronometry — radioactive decay and chemical evolution of the Galaxy — and white dwarf cooling. Cosmochronometry age estimates are sensitive to a number of uncertain issues such as the formation history of the disk and its stars, and possible actinide destruction in stars (Malaney, Mathews, & Dearborn 1989; Mathews & Schramm 1993). However, an independent cosmochronometry age estimate of $15.6 \pm 4.6$ Gyr has been obtained based on data from two low-metallicity stars, using the measured radioactive depletion of thorium (whose half-life is 14.2 Gyr) compared to stable heavy r-process elements (Cowan et al. 1997, 1999). This method could become very important if it were possible to obtain accurate measurements of r-process element abundances for a number of very low metallicity stars giving consistent age estimates, and especially if the large errors could be reduced.

Independent age estimates come from the cooling of white dwarfs in the neighborhood of the sun. The key observation is that there is a lower limit to the luminosity, and therefore also the temperature, of nearby white dwarfs; although dimmer ones could have been seen, none have been found (cf. however Harris et al. 1999). The only plausible explanation is that the white dwarfs have not had sufficient time to cool to lower temperatures, which initially led to an estimate of $9.3 \pm 2$ Gyr for the age of the Galactic disk (Winget et al. 1987). Since there was evidence, based on the pre-Hipparcos GC distances, that the stellar disk of our Galaxy is about 2 Gyr younger than the oldest GCs (e.g., Stetson, VandenBerg, & Bolte 1996, Rosenberg et al. 1999), this in turn gave an estimate of the age of the universe of $t_0 \approx 11 \pm 2$ Gyr. Other analyses (cf. Wood 1992, Hernanz et al. 1994) conclude that sensitivity to disk star formation history, and to effects on the white dwarf cooling rates due to C/O separation at crystallization and possible presence of trace elements such as $^{22}$Ne, allow a rather wide range of ages for the disk of about $10 \pm 4$ Gyr. One determination of the white dwarf luminosity function, using white dwarfs in proper motion binaries, leads to a somewhat lower minimum luminosity and therefore a somewhat higher estimate of the age of the disk of $\sim 10.5^{+2.5}_{-1.5}$ Gyr (Oswalt et al. 1996; cf. Chabrier 1997). More recent observations (Leggett, Ruiz and Bergeron 1998) and analyses (Benvenuto & Althaus 1999) lead to an estimated age of the galactic disk of $8 \pm 1.5$ Gyr.

We conclude that $t_0 \approx 13$ Gyr, with $\sim 11$ Gyr a lower limit. Note that $t_0 > 13$ Gyr implies that $h \leq 0.50$ for matter density $\Omega_m = 1$, and that $h \leq 0.73$ even for $\Omega_m$ as small as 0.3 in flat cosmologies (i.e., with $\Omega_m + \Omega_\Lambda = 1$). If $t_0$ is as low as $\sim 11$ Gyr, that would allow $h$ as high as 0.6 for $\Omega_m = 1$.

3. Hubble Parameter $H_0$

The Hubble parameter $H_0 \equiv 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$ remains uncertain, although no longer by the traditional factor of two. The range of $h$ determinations has been shrinking with time (Kennicutt, Freedman, & Mould 1995). De Vau-
couleurs long contended that $h \approx 1$. Sandage has long contended that $h \approx 0.5$, although a recent reanalysis of the Type Ia supernovae (SNe Ia) data coauthored by Sandage and Tammann concludes that the latest data are consistent with $h = 0.6 \pm 0.04$ (Saha et al. 1999).

The Hubble parameter has been measured in two basic ways: (1) Measuring the distance to some nearby galaxies, typically by measuring the periods and luminosities of Cepheid variables in them; and then using these “calibrator galaxies” to set the zero point in any of the several methods of measuring the relative distances to galaxies. (2) Using fundamental physics to measure the distance to some distant object(s) directly, thereby avoiding at least some of the uncertainties of the cosmic distance ladder (Rowan-Robinson 1985). The difficulty with method (1) was that there was only a handful of calibrator galaxies close enough for Cepheids to be resolved in them. However, the HST Key Project on the Extragalactic Distance Scale has significantly increased the set of calibrator galaxies. The difficulty with method (2) is that in every case studied so far, some aspect of the observed system or the underlying physics remains somewhat uncertain. It is nevertheless remarkable that the results of several different methods of type (2) are rather similar, and indeed not very far from those of method (1). This gives reason to hope for convergence.

3.1. Relative Distance Methods

One piece of good news is that the several methods of measuring the relative distances to galaxies now mostly seem to be consistent with each other. These methods use either “standard candles” or empirical relations between two measurable properties of a galaxy, one distance-independent and the other distance-dependent. The favorite standard candle is SNe Ia, and observers are now in good agreement. Taking account of an empirical relationship between the SNe Ia light curve shape and maximum luminosity leads to $h = 0.65 \pm 0.06$ (Riess, Press, & Kirshner 1996), $h = 0.64^{+0.08}_{-0.06}$ (Jha et al. 1999), or $h = 0.63 \pm 0.03$ (Hamuy et al. 1996, Phillips et al. 1999), and the slightly lower value mentioned above from the latest analysis coauthored by Sandage and Tammann agrees within the errors. The HST Key Project result using SNe Ia is $h = 0.65 \pm 0.02 \pm 0.05$, where the first error quoted is statistical and the second is systematic (Gibson et al. 1999), and their luminosity-metallicity relationship (Kennicutt et al. 1998) has been used (this lowers $h$ by 4%). Some of the other relative distance methods are based on old stellar populations: the tip of the red giant branch (TRGB), the planetary nebula luminosity function (PNLF), the globular cluster luminosity function (GCLF), and the surface brightness fluctuation method (SBF). The HST Key Project result using these old star standard candles is $h = 0.66 \pm 0.04 \pm 0.06$. The old favorite empirical relation used as a relative distance indicator is the Tully-Fisher relation between the rotation velocity and luminosity of spiral galaxies. The “final” value of the Hubble constant from the HST Key Project taking all of these into account is $h = 0.71 \pm 0.06$ (Ferrarese et al. 1999, and this conference, for a nice summary).

3.2. Fundamental Physics Approaches

The fundamental physics approaches involve either Type Ia or Type II supernovae, the Sunyaev-Zel’dovich (S-Z) effect, or gravitational lensing of quasars.
All are promising, but in each case the relevant physics remains somewhat uncertain.

The $^{56}$Ni radioactivity method for determining $H_0$ using Type Ia SNe avoids the uncertainties of the distance ladder by calculating the absolute luminosity of Type Ia supernovae from first principles using plausible but as yet unproved physical models for $^{56}$Ni production. The first result obtained was that $h = 0.61 \pm 0.10$ (Arnet, Branch, & Wheeler 1985; Branch 1992); however, another study (Leibundgut & Pinto 1992; cf. Vaughn et al. 1995) found that uncertainties in extinction (i.e., light absorption) toward each supernova increases the range of allowed $h$. Demanding that the $^{56}$Ni radioactivity method agree with an expanding photosphere approach leads to $h = 0.60^{+0.14}_{-0.11}$ (Nugent et al. 1995). The expanding photosphere method compares the expansion rate of the SN envelope measured by redshift with its size increase inferred from its temperature and magnitude. This approach was first applied to Type II SNe; the 1992 result $h = 0.6 \pm 0.1$ (Schmidt, Kirschner, & Eastman 1992) was subsequently revised upward by the same authors to $h = 0.73 \pm 0.06 \pm 0.07$ (1994). However, there are various complications with the physics of the expanding envelope (Ruiz-Lapuente et al. 1995; Eastman, Schmidt, & Kirshner 1996).

The S-Z effect is the Compton scattering of microwave background photons from the hot electrons in a foreground galaxy cluster. This can be used to measure $H_0$ since properties of the cluster gas measured via the S-Z effect and from X-ray observations have different dependences on $H_0$. The result from the first cluster for which sufficiently detailed data was available, A665 (at $z = 0.182$), was $h = (0.4 - 0.5) \pm 0.12$ (Birkinshaw, Hughes, & Arnoud 1991); combining this with data on A2218 ($z = 0.171$) raised this somewhat to $h = 0.55 \pm 0.17$ (Birkinshaw & Hughes 1994). The history and more recent data have been reviewed by Birkinshaw (1999), who concludes that the available data give a Hubble parameter $h \approx 0.6$ with a scatter of about 0.2. But since the available measurements are not independent, it does not follow that $h = 0.6 \pm 0.1$; for example, there is a selection effect that biases low the $h$ determined this way.

Several quasars have been observed to have multiple images separated by $\theta \sim$ a few arc seconds; this phenomenon is interpreted as arising from gravitational lensing of the source quasar by a galaxy along the line of sight (first suggested byRefsdal 1964; reviewed in Williams & Schechter 1997). In the first such system discovered, QSO 0957+561 ($z = 1.41$), the time delay $\Delta t$ between arrival at the earth of variations in the quasar’s luminosity in the two images has been measured to be, e.g., $409 \pm 23$ days (Pelt et al. 1994), although other authors found a value of $540 \pm 12$ days (Press, Rybicki, & Hewitt 1992). The shorter $\Delta t$ has now been confirmed (Kundic et al. 1997a, cf. Serra-Ricart et al. 1999 and references therein). Since $\Delta t \approx \theta^2 H_0^{-1}$, this observation allows an estimate of the Hubble parameter. The latest results for $h$ from 0957+561, using all available data, are $h = 0.64 \pm 0.13$ (95% C.L.) (Kundic et al. 1997a), and $h = 0.62 \pm 0.07$ (Falco et al. 1997), where the error does not include systematic errors in the assumed form of the lensing mass distribution.

The first quadruple-image quasar system discovered was PG1115+080. Using a recent series of observations (Schechter et al. 1997), the time delay between images B and C has been determined to be about $24 \pm 3$ days. A simple model for the lensing galaxy and the nearby galaxies then leads to $h = 0.42 \pm 0.06$. 
(Schechter et al. 1997), although higher values for $h$ are obtained by more sophisticated analyses: $h = 0.60 \pm 0.17$ (Keeton & Kochanek 1996), $h = 0.52 \pm 0.14$ (Kundic et al. 1997b). The results depend on how the lensing galaxy and those in the compact group of which it is a part are modelled.

Another quadruple-lens system, B1606+656, leads to $h = 0.59 \pm 0.08 \pm 0.15$, where the first error is the 95% C.L. statistical error, and the second is the estimated systematic uncertainty (Fassnacht et al. 1999). Time delays have also recently been determined for the Einstein ring system B0218+357, giving $h = 0.69^{+0.13}_{-0.19}$ (95% C.L.) (Biggs et al. 1999).

Mainly because of the systematic uncertainties in modelling the mass distribution in the lensing systems, the uncertainty in the $h$ determination by gravitational lens time delays remains rather large. But it is reassuring that this completely independent method gives results consistent with the other determinations.

3.3. Conclusions on $H_0$

To summarize, relative distance methods favor a value $h \approx 0.6 - 0.7$. Meanwhile the fundamental physics methods typically lead to $h \approx 0.4 - 0.7$. Among fundamental physics approaches, there has been important recent progress in measuring $h$ via the Sunyaev-Zel’dovich effect and time delays between different images of gravitationally lensed quasars, although the uncertainties remain larger than via relative distance methods. For the rest of this review, we will adopt a value of $h = 0.65 \pm 0.08$. This corresponds to $t_0 = 6.52h^{-1}\text{Gyr} = 10 \pm 2$ Gyr for $\Omega_m = 1$ — probably too low compared to the ages of the oldest globular clusters. But for $\Omega_m = 0.2$ and $\Omega_\Lambda = 0$, or alternatively for $\Omega_m = 0.4$ and $\Omega_\Lambda = 0.6$, $t_0 = 13 \pm 2$ Gyr, in agreement with the globular cluster estimate of $t_0$. This is one of the several arguments for low $\Omega_m$, a non-zero cosmological constant, or both.

4. Hot Dark Matter Density $\Omega_\nu$

The recent atmospheric neutrino data from Super-Kamiokande (Fukuda et al. 1998) provide strong evidence of neutrino oscillations and therefore of non-zero neutrino mass. These data imply a lower limit on the hot dark matter (i.e., light neutrino) contribution to the cosmological density $\Omega_\nu \gtrsim 0.001$. $\Omega_\nu$ is actually that low, and therefore cosmologically uninteresting, if $m(\nu_\tau) \gg m(\nu_\mu)$, as is suggested by the hierarchical pattern of the quark and charged lepton masses. But if the $\nu_\tau$ and $\nu_\mu$ are nearly degenerate in mass, as suggested by their strong mixing, then $\Omega_\nu$ could be substantially larger. Although the Cold + Hot Dark Matter (CHDM) cosmological model with $h \approx 0.5$, $\Omega_m = 1$, and $\Omega_\nu = 0.2$ predicts power spectra of cosmic density and CMB anisotropies that are in excellent agreement with the data (Primack 1996, Gawiser & Silk 1998), as we have just seen the large value measured for the Hubble parameter makes such $\Omega_m = 1$ models dubious. It remains to be seen whether including a significant amount of hot dark matter in low-$\Omega_m$ models improves their agreement with data. Primack & Gross (1998) found that the possible improvement of the low-$\Omega_m$ flat (ΛCDM) cosmological models with the addition of light neutrinos appears to be rather limited, and the maximum amount of hot dark matter
decreases with decreasing $\Omega_m$ (Primack et al. 1995). For $\Omega_m \lesssim 0.4$, Croft, Hu, and Davé (1999) find that $\Omega_\nu \lesssim 0.08$. Fukugita et al. (1999) find more restrictive upper limits with the constraint that the primordial power spectrum index $n \leq 1$, but this may not be well motivated.

5. Cosmological Constant $\Lambda$

The strongest evidence for a positive $\Lambda$ comes from high-redshift SNe Ia, and independently from a combination of observations indicating that $\Omega_m \sim 0.3$ together with CMB data indicating that the universe is nearly flat. We will discuss these observations in the next section. Here we will start by looking at other constraints on $\Lambda$.

The cosmological effects of a cosmological constant are not difficult to understand (Felton & Isaacman 1986; Lahav et al. 1991; Carroll, Press, & Turner 1992). In the early universe, the density of energy and matter is far more important than the $\Lambda$ term on the r.h.s. of the Friedmann equation. But the average matter density decreases as the universe expands, and at a rather low redshift ($z \sim 0.2$ for $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$) the $\Lambda$ term finally becomes dominant. Around this redshift, the $\Lambda$ term almost balances the attraction of the matter, and the scale factor $a \equiv (1 + z)^{-1}$ increases very slowly, although it ultimately starts increasing exponentially as the universe starts inflating under the influence of the increasingly dominant $\Lambda$ term. The existence of a period during which expansion slows while the clock runs explains why $t_0$ can be greater than for $\Lambda = 0$, but this also shows that there is an increased likelihood of finding galaxies in the redshift interval when the expansion slowed, and a correspondingly increased opportunity for lensing by these galaxies of quasars (which mostly lie at higher redshift $z > 2$).

The observed frequency of such lensed quasars is about what would be expected in a standard $\Omega = 1$, $\Lambda = 0$ cosmology, so this data sets fairly stringent upper limits: $\Omega_\Lambda \leq 0.70$ at 90% C.L. (Maoz & Rix 1993, Kochanek 1993), with more recent data giving even tighter constraints: $\Omega_\Lambda < 0.66$ at 95% confidence if $\Omega_m + \Omega_\Lambda = 1$ (Kochanek 1996b). This limit could perhaps be weakened if there were (a) significant extinction by dust in the E/S0 galaxies responsible for the lensing or (b) rapid evolution of these galaxies, but there is much evidence that these galaxies have little dust and have evolved only passively for $z < 1$ (Steidel, Dickinson, & Persson 1994; Lilly et al. 1995; Schade et al. 1996). An alternative analysis by Im, Griffiths, & Ratnatunga 1997 of some of the same optical lensing data considered by Kochanek 1996 leads them to deduce a value $\Omega_\Lambda = 0.64^{+0.15}_{-0.26}$, which is barely consistent with Kochanek’s upper limit. Malhotra, Rhodes, & Turner (1997) presents evidence for extinction of quasars by foreground galaxies and claims that this weakens the lensing bound to $\Omega_\Lambda < 0.9$, but this is not justified quantitatively. Maller, Flores, & Primack (1997) shows that edge-on disk galaxies can lens quasars very effectively, and discusses a case in which optical extinction is significant. But the radio observations discussed by Falco, Kochanek, & Munoz (1998), which give a $2\sigma$ limit $\Omega_\Lambda < 0.73$, are not affected by extinction. Recently Chiba and Yoshii (1999) have suggested that a reanalysis of lensing using new models of the evolution of elliptical galaxies gives $\Omega_\Lambda = 0.7^{+0.1}_{-0.2}$.
but Kochanek et al. (1999, see especially Fig. 4) show that the available evidence disfavors the models of Chiba and Yoshii.

A model-dependent constraint appeared to come from simulations of ΛCDM (Klypin, Primack, & Holtzman 1996) and OpenCDM (Jenkins et al. 1998) COBE-normalized models with $h = 0.7$, $\Omega_m = 0.3$, and either $\Omega_\Lambda = 0.7$ or, for the open case, $\Omega_\Lambda = 0$. These models have too much power on small scales to be consistent with observations, unless there is strong scale-dependent antibiasing of galaxies with respect to dark matter. However, recent high-resolution simulations (Klypin et al. 1999) find that merging and destruction of galaxies in dense environments lead to exactly the sort of scale-dependent antibiasing needed for agreement with observations for the ΛCDM model. Similar results have been found using simulations plus semi-analytic methods (Benson et al. 1999, but cf. Kauffmann et al. 1999).

Another constraint on Λ from simulations is a claim that the number of long arcs in clusters is in accord with observations for an open CDM model with $\Omega_m = 0.3$ but an order of magnitude too low in a ΛCDM model with the same $\Omega_m$ (Bartelmann et al. 1998). This apparently occurs because clusters with dense cores form too late in such models. This is potentially a powerful constraint, and needs to be checked and understood. It is now known that including cluster galaxies does not alter these results (Meneghetti et al. 1999; Flores, Maller, & Primack 1999).

### 6. Measuring $\Omega_m$

The present author, like many theorists, has long regarded the Einstein-de Sitter ($\Omega_m = 1$, $\Lambda = 0$) cosmology as the most attractive one. For one thing, there are only three possible constant values for $\Omega$ — 0, 1, and $\infty$ — of which the only one that can describe our universe is $\Omega_m = 1$. Also, cosmic inflation is the only known solution for several otherwise intractable problems, and all simple inflationary models predict that the universe is flat, i.e. that $\Omega_m + \Omega_\Lambda = 1$. Since there is no known physical reason for a non-zero cosmological constant, it was often said that inflation favors $\Omega = 1$. Of course, theoretical prejudice is not a reliable guide. In recent years, many cosmologists have favored $\Omega_m \sim 0.3$, both because of the $H_0 - \tau_0$ constraints and because cluster and other relatively small-scale measurements have given low values for $\Omega_m$. (For a summary of arguments favoring low $\Omega_m \approx 0.2$ and $\Lambda = 0$, see Coles & Ellis 1997. A review that notes that larger scale measurements favor higher $\Omega_m$ is Dekel, Burstein, & White 1997.) But the most exciting new evidence has come from cosmological-scale measurements.

**Type Ia Supernovae.** At present, the most promising techniques for measuring $\Omega_m$ and $\Omega_\Lambda$ on cosmological scales use the small-angle anisotropies in the CMB radiation and high-redshift Type Ia supernovae (SNe Ia). We will discuss the latter first. SNe Ia are the brightest supernovae, and the spread in their intrinsic brightness appears to be relatively small. The Supernova Cosmology Project (Perlmutter et al. 1997a) demonstrated the feasibility of finding significant numbers of such supernovae. The first seven high redshift SNe Ia that they analyzed gave for a flat universe $\Omega_m = 1 - \Omega_\Lambda = 0.94^{+0.34}_{-0.28}$, or equivalently $\Omega_\Lambda = 0.06^{+0.34}_{-0.28}$ (at the 95% confidence level) (Perlmutter et al.
But adding one \( z = 0.83 \) SN Ia for which they had good HST data lowered the implied \( \Omega_m \) to 0.6 ± 0.2 in the flat case (Perlmutter et al. 1997a). Analysis of their larger dataset of 42 high-redshift SNe Ia gives for the flat case \( \Omega_m = 0.28^{+0.09+0.05}_{-0.08-0.04} \) where the first errors are statistical and the second are identified systematics (Perlmutter et al. 1999). The High-Z Supernova team has also searched successfully for high-redshift supernovae to measure \( \Omega_m \) (Garnavich et al. 1997, Riess et al. 1998), and their three HST SNe Ia, two at \( z \approx 0.5 \) and one at 0.97, imply \( \Omega_m = 0.4 \pm 0.3 \) in the flat case. The main concerns about the interpretation of this data are evolution of the SNe Ia (Drell, Loredo, & Wasserman 1999) and dimming by dust. A recent specific supernova evolution concern that was discussed at this workshop is that the rest frame rise-times of distant supernovae may be longer than nearby ones (Riess et al. 1999). But a direct comparison between nearby supernova and the SCP distant sample shows that they are rather consistent with each other (Aldering, Nugent, & Knop 1999). Ordinary dust causes reddening, but hypothetical grey dust would cause much less reddening and could in principle provide an alternative explanation for the fact that high-redshift supernovae are observed to be dimmer than expected in a critical-density cosmology. It is hard to see why the largest dust grains, which would be greyer, should preferentially be ejected by galaxies (Simonsen & Hannestad 1999). Such dust, if it exists, would also absorb starlight and re-radiate it at long wavelengths, where there are other constraints that could, with additional observations, rule out this scenario (Aguirre & Haiman 1999). But another way of addressing this question is to collect data on supernovae with redshift \( z > 1 \), where the dust scenario predicts considerably more dimming than the \( \Lambda \) cosmology. The one \( z > 1 \) supernova currently available, SCP’s “Albinoni” (SN1998eq) at \( z = 1.2 \), will help, and both the SCP and the High-Z group are attempting to get a few more very high redshift supernovae.

**CMB anisotropies.** The location of the first Doppler (or acoustic, or Sakharov) peak at angular wavenumber \( l \approx 250 \) indicated by the presently available data (Scott, this volume) is evidence in favor of a flat universe \( \Omega_m + \Omega_\Lambda \approx 1 \). New data from the MAXIMA and BOOMERANG balloon flights apparently confirms this, and the locations of the second and possibly third peak appear to be consistent with the predictions (Hu, Spergel, & White 1997) of simple cosmic inflation theories. Further data should be available in 2001 from the NASA Microwave Anisotropy Probe satellite.

**Large-scale Measurements.** The comparison of the IRAS redshift surveys with POTENT and related analyses typically give values for the parameter \( \beta_I \equiv \Omega_m b_I^6 \) (where \( b_I \) is the biasing parameter for IRAS galaxies), corresponding to \( 0.3 \leq \Omega_m \leq 3 \) (for an assumed \( b_I = 1.15 \)). It is not clear whether it will be possible to reduce the spread in these values significantly in the near future — probably both additional data and a better understanding of systematic and statistical effects will be required. A particularly simple way to deduce a lower limit on \( \Omega_m \) from the POTENT peculiar velocity data was proposed by Dekel & Rees (1994), based on the fact that high-velocity outflows from voids are not expected in low-\( \Omega \) models. Data on just one nearby void indicates that \( \Omega_m \geq 0.3 \) at the 97% C.L. Stronger constraints are available if we assume that the probability distribution function (PDF) of the primordial fluctuations was Gaussian. Evolution from a Gaussian initial PDF to the non-Gaussian mass distribution observed today requires considerable gravitational nonlinearity, i.e. large \( \Omega_m \).
The PDF deduced by POTENT from observed velocities (i.e., the PDF of the mass, if the POTENT reconstruction is reliable) is far from Gaussian today, with a long positive-fluctuation tail. It agrees with a Gaussian initial PDF if and only if $\Omega_m \sim 1$; $\Omega_m < 1$ is rejected at the $2\sigma$ level, and $\Omega_m \leq 0.3$ is ruled out at $\geq 4\sigma$ (Nusser & Dekel 1993; cf. Bernardeau et al. 1995). It would be interesting to repeat this analysis with newer data.

**Measurements on Scales of a Few Mpc.** A study by the Canadian Network for Observational Cosmology (CNOC) of 16 clusters at $z \sim 0.3$, mostly chosen from the Einstein Medium Sensitivity Survey (Henry et al. 1992), was designed to allow a self-contained measurement of $\Omega_m$ from a field $M/L$ which in turn was deduced from their measured cluster $M/L$. The result was $\Omega_m = 0.19 \pm 0.06$ (Carlberg et al. 1997). These data were mainly compared to standard CDM models, and they appear to exclude $\Omega_m = 1$ in such models.

**Estimates on Galaxy Halo Scales.** Work by Zaritsky et al. (1993) has confirmed that spiral galaxies have massive halos. They collected data on satellites of isolated spiral galaxies, and concluded that the fact that the relative velocities do not fall off out to a separation of at least 200 kpc shows that massive halos are the norm. The typical rotation velocity of $\sim 200 - 250$ km s$^{-1}$ implies a mass within 200 kpc of $\sim 2 \times 10^{12} M_\odot$. A careful analysis taking into account selection effects and satellite orbit uncertainties concluded that the indicated value of $\Omega_m$ exceeds 0.13 at 90% confidence (Zaritsky & White 1994), with preferred values exceeding 0.3. Newer data suggesting that relative velocities do not fall off out to a separation of $\sim 400$ kpc (Zaritsky et al. 1997) presumably would raise these $\Omega_m$ estimates.

**Cluster Baryons vs. Big Bang Nucleosynthesis.** White et al. (1993) emphasized that X-ray observations of the abundance of baryons in clusters can be used to determine $\Omega_m$ if clusters are a fair sample of both baryons and dark matter, as they are expected to be based on simulations (Evrard, Metzler, & Navarro 1996). The fair sample hypothesis implies that

$$\Omega_m = \frac{\Omega_b}{f_b} = 0.3 \left( \frac{\Omega_b}{0.04} \right) \left( \frac{0.13}{f_b} \right).$$  \hspace{1cm} (1)

We can use this to determine $\Omega_m$ using the baryon abundance $\Omega_b h^2 = 0.019 \pm 0.001$ from the measurement of the deuterium abundance in high-redshift Lyman limit systems, of which a third has recently been discovered (Kirkman et al. 1999). Using X-ray data from an X-ray flux limited sample of clusters to estimate the baryon fraction $f_b = 0.075 h^{-3/2}$ (Mohr, Mathiesen, & Evrard 1999) gives $\Omega_m = 0.25 h^{-1/2} = 0.3 \pm 0.1$ using $h = 0.65 \pm 0.08$. Estimating the baryon fraction using Sunyaev-Zel’dovich measurements of a sample of 18 clusters gives $f_b = 0.77 h^{-1}$ (Carlstrom et al. 1999), and implies $\Omega_m = 0.25 h^{-1} = 0.38 \pm 0.1$.

**Cluster Evolution.** The dependence of the number of clusters on redshift can be a useful constraint on theories (e.g., Eke et al. 1996). But the cluster data at various redshifts are difficult to compare properly since they are rather inhomogeneous. Using just X-ray temperature data, Eke et al. (1998) conclude that $\Omega_m \approx 0.45 \pm 0.2$, with $\Omega_m = 1$ strongly disfavored.

**Power Spectrum.** In the context of the $\Lambda$CDM class of models, two additional constraints are available. The spectrum shape parameter $\Gamma \approx \Omega_m h^2 \approx 0.25 \pm 0.05$, implying $\Omega_m \approx 0.4 \pm 0.1$. A new measurement $\Omega_m = 0.34 \pm 0.1$
comes from the amplitude of the power spectrum of fluctuations at redshift \( z \sim 3 \), measured from the Lyman \( \alpha \) forest (Weinberg et al. 1999). This result is strongly inconsistent with high-\( \Omega_m \) models because they would predict that the fluctuations grow much more to \( z = 0 \), and thus would be lower at \( z = 3 \) than they are observed to be.

7. Conclusion

One of the most striking things about the present era in cosmology is the remarkable agreement between the values of the cosmological parameters obtained by different methods — except possibly for the quasar lensing data which favors a higher \( \Omega_m \) and lower \( \Omega_A \), and the arc lensing data which favors lower values of both parameters. If the results from the new CMB measurements agree with those from the other methods discussed above, the cosmological parameters will have been determined to perhaps 10%, and cosmologists can turn their attention to the other subjects that I mentioned at the beginning: origin of the initial fluctuations, the nature of the dark matter and dark energy, and the formation of galaxies and large-scale structure. Cosmologists can also speculate on the reasons why the cosmological parameters have the values that they do, but this appears to be the sort of question whose answer may require a deeper understanding of fundamental physics — perhaps from a superstring theory of everything.

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