Even an Initially Top-Hat Jet can Fit the Afterglow of GW170817 / GRB170817A

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(Received; Revised; Accepted)
Submitted to ApJL

ABSTRACT
The afterglow of GRB 170817A/GW 170817 was very unusual, slowly rising as $F_\nu \propto t_0^{0.8} \nu^{-0.6}$, peaking at $\sim 150$ days, and sharply decaying as $\sim t_0^{-2.2}$. VLBI observations revealed an unresolved radio afterglow image whose flux centroid apparently moved superluminally with $v_{\text{app}} \approx 4c$ between 75 and 230 days, clearly indicating that the afterglow was dominated by a relativistic jet’s compact core. Different jet angular structures successfully explained the afterglow lightcurves: Gaussian and steep power-law profiles with narrow core angles $\theta_c \lesssim 5^\circ$ and significantly larger viewing angles $\theta_\text{obs}/\theta_c \sim 3 - 5$. However, a top-hat jet (conical with sharp edges at $\theta = \theta_0$) was unanimously ruled out since it appeared to produce an early flux rise much steeper ($\propto t_\text{obs}^a$ with $a \gtrsim 3$) than observed. Here we clearly show that 2D relativistic hydrodynamic simulations of an initially top-hat jet can fit GW 170817/GRB 170817A’s afterglow lightcurves and flux centroid motion, for $\theta_\text{obs}/\theta_0 \approx 3$ and $\Gamma_0 = \Gamma(t_0) \gtrsim 10^{2.5}$ where $t_0$ is the simulation’s start time, and find a minimal jet energy of $E_\text{min} \approx 2 \times 10^{48}$ erg. On the dynamical time a bow-shock like structure develops with slower material on the jet’s sides, whose emission is not strongly beamed away from observers at $\theta_\text{obs} \gtrsim 2\theta_0$, causing a shallow flux rise. The steep initial flux rise is an artifact caused by the simulation’s finite $t_0$, missing its flux contributions from $t < t_0$ and sometimes “compensated” using an analytic top-hat jet. While an initial top-hat jet is not very physical, it can still reproduce the afterglow data, which require that most of the jet’s energy initially resides in a narrow core and sharply drops outside of it.

Keywords: gamma-ray burst; general — ISM: jets and outflows — hydrodynamics — methods: numerical — relativistic processes — gravitational waves

1. INTRODUCTION
The first gravitational wave (GW) detection of a binary neutron star (NS) merger, GW 170817 (Abbott et al. 2017a), was accompanied by the first electromagnetic counterpart to any GW detection – the weak, short duration gamma-ray burst, GRB 170817A (Abbott et al. 2017b), that originated in the nearby ($D \approx 40$ Mpc) elliptical galaxy NGC 4993 (Coulter et al. 2017). An impressive observational campaign detected the quasi-thermal kilonova emission in the NIR-optical-UV energy bands over the next few weeks (see, e.g., Abbott et al. 2017c, and references therein). The non-thermal afterglow emission was detected after 8.9 days in X-rays (Troja et al. 2017) and after 16.4 days in the radio (Hollin et al. 2017).

GW 170817/GWB 170817A’s long-lived X-ray to radio afterglow emission was highly unusual. In contrast to the flux decay seen in almost all GRB afterglows, it showed an exceptionally long-lasting flux rise, as $F_\nu(t_\text{obs}) \propto t_\text{obs}^{0.8} \nu^{-0.8}$, up to the peak at $t_\text{obs, pk} \sim 150$ days post merger (e.g. Margutti et al. 2018; Mooley et al. 2018a), followed by a sharp decay as $F_\nu \propto t_\text{obs}^a$ where $a \approx -2.2$ (Mooley et al. 2018b; van Eerten et al. 2018). The broadband (X-rays, radio, and late-time optical) afterglow emission is consistent with arising from a single power-law segment (PLS) of the afterglow synchrotron spectrum, $\nu_{\text{ms}} \leq \nu \leq \nu_{\text{c}}$.1

Almost all successful off-axis jet models for this afterglow have an angular profile that is either a (quasi-) Gaussian or a narrow core with sharp power-law wings (Lamb & Kobayashi 2018; Lazzati et al. 2018; Troja et al. 2017; D’Avanzo et al. 2018; Gill & Granot 2018; Gill et al. 1998).

1 Here $\nu_{\text{ms}}$ is the synchrotron frequency of minimal energy electrons and $\nu_{\text{c}}$ of electrons that cool on the dynamical time (Sari et al. 1998).
Margutti et al. 2018; Resmi et al. 2018; Troja et al. 2018). Moreover, several works have argued that a top-hat jet can be ruled out (e.g., Margutti et al. 2018; Moley et al. 2018a) since it would produce a very sharp initial flux rise ($F_{\nu} \propto t_{\text{obs}}^{-\alpha}$ with $\alpha \gtrsim 3$) compared to the observed one. Such a sharp initial flux rise was obtained both numerically from 2D hydrodynamic simulations (e.g., van Eerten & MacFadyen 2011; Granot et al. 2018a), and analytically assuming an idealized top-hat jet (e.g., Granot et al. 2002; Eicher & Granot 2006; Nakar & Piran 2018).

Here we show that while an idealized top-hat jet would indeed produce sharply rising early lightcurves for off-axis observers, a more realistic description of the dynamics (using numerical simulations) for an initially top-hat jet leads to a much shallower flux rise that can explain the GRB 170817A afterglow observations (lightcurves, flux centroid motion, and upper limits on the image size). The main difference arises since within the simulation’s first dynamical time an initial top-hat jet develops a bow-shock like angular structure, which produces afterglow emission resembling that from a core-dominated structured jet, with a much shallower flux rise. Numerical simulations have a finite lab-frame start time, $t = t_0 > 0$, thus missing contributions to $F_{\nu}$ from $t < t_0$. This is often compensated for by adding emission at $t < t_0$ from a conical wedge from the Blandford & McKee (1976, hereafter BM76) spherical self-similar solution (e.g., van Eerten et al. 2012; De Colle et al. 2012a,b; Bietenholz et al. 2014; Granot et al. 2018a,b). This still results in an unphysically sharp flux rise at early observed times, $t_{\text{obs}} \lesssim 2t_{\text{obs},0}$, corresponding to lab-frame times $t \lesssim 2t_0$.

The effects of $t_0$ including $t_{\text{obs},0}(\theta_{\text{obs}}, t_0)$ are analytically explained in § 2. The effect of starting the simulations with a larger Lorentz factor (LF) $\Gamma_0 = \Gamma(t_0)$ and correspondingly smaller $t_0$ is shown in § 3 through 2D relativistic hydrodynamical simulations. We fit simulated lightcurves to the afterglow data of GW 170817/GRB 170817A and also calculate and compare the flux centroid location and the image size and shape with radio measurements. Our conclusion are discussed in § 4.

2. THE EFFECT OF SIMULATION START TIME

We perform 2D relativistic hydrodynamical simulations with initial conditions of a conical wedge of half-opening angle $\theta_0$ taken out of the BM76 solution. This initially narrow and relativistic jet expands into a cold circumburst medium (CBM) with a power-law restmass density profile with radius $R$ from the central source, $\rho(R) = AR^{-k}$, where for uniform (wind-like) density environment $k = 0$ ($k = 2$). The BM76 spherical self-similar phase occurs after the original outflow is significantly decelerated and most of the energy is in the shocked CBM behind the forward (afterglow) shock. The material just behind the shock moves with velocity $\beta c$, with $c$ being the speed of the light, and bulk LF $\Gamma = (1 - \beta^2)^{-1/2} = \Gamma_{\text{shock}}/\sqrt{2}$. The BM76 phase reasonably holds for a top-hat jet while $\Gamma > 1/\theta_0$ (assuming $\Gamma_0\theta_0 \gg 1$, as typically inferred for GRBs) before significant lateral spreading can occur.

The radial width behind the forward shock containing most of the blastwave’s energy is $\Delta \sim 0.1 R/\Gamma^2$. During the BM76 self-similar phase, $\Gamma^4 R^{3-k} = \Gamma_0^4 R_0^{3-k} = (17-4k)E_{k,\text{iso}}/\alpha_{\gamma}^2$ is a constant, $R_0 = R(t_0) \approx c t_0$ being the initial shock radius. Thus the initial radial width $\Delta_0 = \Delta(t_0) \sim 0.1 R_0/\Gamma_0^2 \propto R_0^{-3-k} \propto \Gamma_0^{-2(3-k)/(3-k)} \propto (\Gamma_0^{-8/3}$ for $k = 0)$ becomes much narrower and harder to resolve for larger $\Gamma_0$ or correspondingly smaller $t_0 \approx R_0/c \propto \Gamma_0^{-2/(3-k)} \propto \Gamma_0^{-2/3}$ for $k = 0$). This practically limits $\Gamma_0$ from above and $t_0$ from below.

An on-axis observer ($\theta_{\text{obs}} < \theta_0$) receives the first photons from the simulation after a radial time delay of

$$t_{\text{obs},r} = \frac{t_0 - R_0}{c} \approx \frac{R_0}{4(4-k)c\Gamma_0^2} \approx \frac{t_0}{4(4-k)c\Gamma_0^2}$$

(1)

$z$ being the source’s cosmological redshift. For an off-axis observer ($\Delta \theta \equiv \theta_{\text{obs}} - \theta_0 > 0$), there is an additional angular time delay,

$$\frac{t_{\text{obs},\theta}}{1+z} \approx \frac{R_0}{c} \frac{1 - \cos(\Delta \theta)}{2} \approx \frac{\Delta \theta^2}{2} \frac{t_0}{c}$$

$$\approx \frac{\Delta \theta^2}{2} \left[ \frac{(17-4k)E_{k,\text{iso}}}{16\pi\alpha_{\gamma}^2\Gamma_0^2} \right]^{1/4}$$

(2)

(e.g., Granot et al. 2017), which dominates the total time delay $t_{\text{obs},0} = t_{\text{obs},r} + t_{\text{obs},\theta} \approx t_{\text{obs},\theta}$ for $\Delta \theta > 1/\Gamma_0$.

For such off-axis viewing angles one can conveniently express $\Gamma_0 \propto t_{\text{obs},0}^{-3-k}/2$, which for $k = 0$, $E_{k,\text{iso}} \approx (2/\theta_0^2)E$ and $z \ll 1$ gives

$$\Gamma_0 \approx \sqrt{\frac{17E\theta_0^{-2}(\Delta \theta)^2}{64\pi m_p c^5 t_{\text{obs},0}^{-3/2}}}$$

$$= \frac{149E_{500}^{1/2} \theta_0^{-1}}{3.6 \theta_0^{-1}} \frac{t_{\text{obs},0}}{10 d}^{-3/2}$$

(3)

where for the numerical value we normalize by our best-fit model parameters derived in § 3, for which $t_{\text{obs},0} = 38.1, 23.0, 18.3$ days for $\Gamma_0 = 20, 40, 60$.

The compactness argument implies that GRB jets typically have $\Gamma_0 \gtrsim 100$ for the emission region to be optically thin to $\gamma\gamma$-annihilation (e.g., Lithwick & Sari 2001). Such large $\Gamma_0$ are very difficult to simulate, and current numerical works usually set $\Gamma_0 \sim 20 - 25$ (see, however, van Eerten & MacFadyen 2013).

Simulations initialized at $t_0$ do not contribute any flux at $t_{\text{obs}} < t_{\text{obs},0}$ (see Fig. 1). Over the first dynamical time ($t_0 < t \lesssim 2t_0$), as the simulated jet relaxes from its artificially sharp top-hat initial condition, the flux sharply rises at times $t_{\text{obs},0} \lesssim t_{\text{obs}} \lesssim 2t_{\text{obs},0}$, after which
the flux evolves smoothly with time. During this relaxation phase, the top-hat jet is slowed down due to its interaction with the CBM and develops a bow-shock like structure (e.g. Granot et al. 2001; van Eerten & Ramirez-Ruiz 2012). Its structure at this point resembles a ‘structured jet’ with a highly energetic core, whose velocity is almost radial, surrounded by less energetic slower-moving material whose velocity points more sideways. Therefore, an initially top-hat jet viewed off-axis is within an angle $\sim\max(\Gamma^{-1}, \Delta \theta)$ of the point in the jet closest to the observer (where $\hat{\theta} \approx \Delta \theta$), occupying a solid angle $\Omega_{\omega} \sim \min(\max(\Gamma^{-2}, \Delta \theta)^2, \theta_0^2)$. During the early flux-rising phase while the radiation is beamed away from the observer ($\Gamma > 1/\Delta \theta$), $\Omega_{\omega} = \text{const}$ and one can use the scalings of $L'_{\nu}$ for a spherical flow, $L'_{\nu} \propto R^3 \nu^b \propto R^3 \delta_{BP}$, where the PLS-dependent power-law indices $a$ and $b$ are explicitly calculated in Granot (2005). Therefore, $F_{\nu} \propto \delta_{BP}^{-b} R^c$ where (e.g. Salmonson 2003; Granot 2005) $\delta_{BP} \approx 2/\Gamma \Delta \theta^2 \propto R^{3(b-1)/2} \Rightarrow F_{\nu} \propto R^{(2a+b)(3-b)/(2a+b)}$. For GRB 170817A, PLS G is relevant and $a = [15 - 9p - 2(k(3 - p))/4, b = (1 - p)/2$. From Eq. (2), $t_{\text{obs}} \propto R$ which implies $F_{\nu} \propto t_{\text{obs}}^2$ for a uniform CBM ($k = 0$).

3. DIFFERENT $\Gamma_0$ FITS TO THE AFTERGLOW DATA OF GW170817/GRB 170817A

Here we show results of 2D hydrodynamic simulations using the special-relativistic hydrodynamics code Mezcal, post-processed by a complimentary radiation code (see De Colle et al. 2012a,b, for details). The simulations are initialized with a conical wedge of half-opening angle $\theta_0 = 0.1, 0.2$ rad and initial LF $\Gamma_0 = 20, 40, 60$ expanding into a uniform CBM ($k = 0$) of rest-mass density $\rho_0 = n m_p$ and number density $n$, $m_p$ being the proton mass. The outflow has an isotropic-equivalent kinetic energy $E_{k,\text{iso}} = 10^{55}$ erg, corresponding to a true jet energy of $E = (1 - \cos \theta_0) E_{k,\text{iso}} \approx 5 \times 10^{50}$ erg for $\theta_0 = 0.1$ and $E \approx 2 \times 10^{51}$ erg for $\theta_0 = 0.2$.

We consider synchrotron radiation from relativistic electrons that are accelerated at the afterglow shock to a power-law energy distribution, $dN_e/d\gamma_e \propto \gamma_e^{-p}$ for $\gamma_e > \gamma_m$ with $p = 2.16$, which are a fraction $\xi_e$ of all post-shock electrons, and hold a fraction $\epsilon_e = 0.1$ of the post-shock internal energy density, where a fraction $\epsilon_B = 0.1$ goes to the magnetic field. The radiation is calculated numerically for a fixed set of model parameters ($E, n, \epsilon_e, \epsilon_B, p, \theta_0$) and for a grid of $\theta_{\text{obs}}$ values. We then use the scaling relations described in Granot (2012) for arbitrary values of ($E, n$), as well as the scaling with

$$F_{\nu}(t_{\text{obs}}) = \frac{(1 + z)}{4\pi d_L^2(z)} \int dt \delta \propto \delta_L^2 L'_{\nu} \propto \delta_L^2 L'_{\nu}, \quad (4)$$

where $d_L(z)$ is the luminosity distance, the $\delta$-function, $\delta_t = \delta (t - t_{\text{obs}}/1 + z - R\hat{\mu}/c)$, accounts for the photon arrival times (Granot et al. 1999), $R\hat{\mu} = \hat{n} \cdot \hat{R}$ where $\hat{n}$ is the direction to the observer and $\hat{R}$ is the radius vector (measured from the central source) of each fluid element having velocity $\vec{v} = \hat{\beta}_c$ and Doppler factor $\delta_D = [(\Gamma(1 - \hat{n} \cdot \hat{\beta})^{-1}/\Gamma)]$. For radial velocities (e.g. a spherical shell), $\hat{n} \cdot \hat{\beta} = \beta \mu$ and $\delta_D \approx 2 \Gamma/[1 + (\Gamma \hat{\theta})^2]$ for $\Gamma \gg 1$. In Eq. (4), $F_{\nu} \propto \delta_L^2 L'_{\nu}$ holds where $L'_{\nu}$ and $\delta_D$ are those of the source that dominates the observed emission, which for a top-hat jet matches the analytical flux scaling for an off-axis relativistic top-hat jet (the slightly shallower slope towards $t_{\text{obs}} \propto R$ arises because of its proximility to $t_{\text{obs}, pk}$).
the shock microphysical parameters in each PLS (Table 2 of Granot & Sari 2002). See Granot et al. (2017) for further details.

Our afterglow lightcurve fitting is guided by the measured peak at $t_{\text{obs, pk}} \sim 150$ days (Dobie et al. 2018) and the data points near the peak. Figure 2 shows the fit to the afterglow data for different initial $\Gamma_0$ (left-panel) and viewing angles $\theta_{\text{obs}}$ (middle-panel). We do not attempt to fit the early time data at $t_{\text{obs}} \lesssim 40$ days, before the simulated lightcurves contain the dominant and dynamically relaxed contribution from the hydrodynamic simulation. Nevertheless, we obtain a reasonable fit to the afterglow data for different values of $\Gamma_0$, where our lightcurves for larger $\Gamma_0$ extend to earlier times and can adequately explain the data at $t_{\text{obs}} \gtrsim 40$ days.

The best constrained parameters are (also see Granot et al. 2018b): (i) $p \approx 2.16$, and (ii) $\theta_{\text{obs}}/\theta_0 \approx 3.1 \pm 0.1$, since it significantly affects the shape of the lightcurve before and around the peak time. In the right-panel of Figure 2, we compare the model lightcurves for $\theta_0 = 0.1, 0.2$ and show that in both cases $\theta_{\text{obs}}/\theta_0 = 3.1$, while obtaining the same values for the shock microphysical parameters. The main difference between the two fits is in the true jet energy $E$ and CBM density $n$.

We also compare the lightcurve obtained from the publicly available afterglow modeling code BOXFITv2 (van Eerten et al. 2012), which has been widely used to fit afterglow observations of GRB 170817A. Lightcurves obtained from our numerical simulations are in excellent agreement with that obtained from BOXFITv2. We also show the extension of the lightcurve to $t_{\text{obs}} < t_{\text{obs, 0}} \sim 40$ days, where we reproduce the analytic flux scaling derived in §2, since BOXFITv2 also supplements the lightcurve at early times ($t < t_0 \leftrightarrow t_{\text{obs}} < t_{\text{obs, 0}}$) with the flux from a conical wedge out of the BM76 self-similar solution (also used for the initial conditions). Although BOXFITv2 allows the user to not include this extension in the final lightcurve, many works indeed do include it, even when fitting to observations. Either way, the flux at $t_{\text{obs}} \lesssim 2t_{\text{obs, 0}}$ is strongly affected by the arbitrary simulation start time $t_0$. Initializing the simulation at a smaller $t_0$ corresponding to a larger $\Gamma_0$ would shift this feature to earlier times and recover the much shallower flux rise in the lightcurve.

Since the model parameters outnumber the effective constraints on them, the model parameter space is degenerate, and a unique solution cannot be obtained. Moreover, there is an additional degeneracy (Eichler & Waxman 2005) where the afterglow flux is invariant under the change $E \rightarrow E/\xi$ and $\nu \rightarrow \nu/\xi$, while obtaining the same values for the shock microphysical parameters. The main difference between the two fits is in the true jet energy $E$ and CBM density $n$. For the lightcurve fits we assume $\xi = 1$, and use the dependence on the shock microphysical parameters in PLS G from Granot & Sari (2002), now including the degeneracy due to $\xi$ (e.g. van Eerten & MacFadyen 2012), $F_{v,G} \propto \nu^{p-1/4} \xi^{2-p} B^{(p+1)/2}$. We also use the global scaling relations (Granot 2012), which are conveniently parameterized through length and time, $\propto \ell^p/\ell = t_{\text{obs}}/t_{\text{obs}}$, and through mass and energy, $E = m^p/m = E^p/E$, where the rescaled parameters are denoted with a prime, $F = F_{v,G} (\ell', \nu', B', \xi')/F_{v,G} (\ell, \nu, B, \xi)$.

$$
F = \xi^{(p+1)/4} \alpha^{-3(p+1)/4} \left( \frac{\ell'}{\ell} \right)^{p-1} \left( \frac{\nu'}{\nu} \right) \left( \frac{B'}{B} \right)^{(p+1)/2} \left( \frac{\xi'}{\xi} \right) \xi^2. \quad (5)
$$

Next, we constrain $E$ from below by using these scaling relations and our (partly degenerate) best-fit parameters: $E < 10^{50.3}$ erg, $n < 10^{-3.6}$ cm$^{-3}$, $\epsilon_e = 10^{-1.8}$, $\epsilon_B = 10^{-3}$, $\theta_{\text{obs}}/\theta_0 = 3.1$ (fixing $\xi = 1$, $p = 2.16$, $\theta_0 = 0.1$). Matching the peak time of the simulated lightcurve to $t_{\text{obs, pk}} \approx 150$ days requires no significant time rescaling, and yields $\alpha = t_{\text{obs}}/t_{\text{obs, pk}} \approx 1$. Matching the peak flux to that observed requires equating Eq. (5) to unity. Altogether, replacing the unprimed quanti-
ties by the best-fit values, and then making the rescaled quantities unprimed, and solving for $\zeta$, yields

$$
\zeta = \frac{E}{10^{50.3} \text{erg}} \approx \left( \frac{\epsilon_e}{10^{-1.8}} \right)^{\frac{4(1-p)}{p+5}} \left( \frac{\epsilon_B}{10^{-3}} \right)^{\frac{-(p+1)}{p+5}} \frac{\xi_e^{4(p+5)}}{\xi_e^{(p+5)}}.
$$

The jet energy increases with $\epsilon_e$, $\epsilon_B$ and does so weakly with $\xi_e$, implying a minimal energy (using $p = 2.16$) of about

$$
E_{\text{min}} \approx 1.84 \times 10^{48} \epsilon_e^{-0.089} \epsilon_B^{-0.648} \xi_e^{-0.44} \text{erg}.
$$

Next we compare the afterglow image size and flux centroid motion on the plane of the sky as obtained from our simulations to the GW 170817/GRB 170817A radio observations. VLBI observations between 75 and 230 days revealed an unresolved source whose flux centroid showed apparent superluminal motion with $v_{\text{app}}/c = (\beta_{\text{app}}) = 4.1 \pm 0.5$ (Mooley et al. 2018b). The flux centroid’s location on the plane of the sky is defined as

$$
\bar{r}_{\text{fc}} = (\bar{x}_{\text{fc}}, \bar{y}_{\text{fc}}) = \frac{\int dF_r \bar{r}}{\int dF_r} = \frac{\int d\bar{x} d\bar{y} I_{\nu} \bar{r}}{\int d\bar{x} d\bar{y} I_{\nu}}.
$$

(e.g., Granot et al. 2018a), where $dF_r = I_{\nu} d\Omega = I_{\nu} dA \Delta S_{\perp}$, with $I_{\nu}$ being the specific intensity, $dA$ the angular distance, and $\Delta S_{\perp} = d\bar{x} d\bar{y}$ a transverse area element on the plane of the sky. The jet symmetry axis is in the $\bar{x}$-$\bar{z}$ plane, where the $\bar{z}$-axis points to the observer. Because of the flow’s axisymmetry, the image has the reflection symmetry $I_{\nu}(\bar{x}, \bar{y}) = I_{\nu}(\bar{x}, -\bar{y})$. Therefore, $\bar{r}_{\text{fc}} = (\bar{x}_{\text{fc}}, 0)$ and the flux centroid moves along the $\bar{x}$-axis. Since $I_{\nu} = d^2_{\nu} dF_r/dS_{\perp} \propto F_r/S_{\perp}$ where $S_{\perp} \propto \ell^2$, it scales in PLS G as $I = I_{\nu,G}(t_{\text{obs}}, \bar{x}', \bar{y}')/I_{\nu,G}(t_{\text{obs}}, \bar{x}, \bar{y})$,

$$
I = \zeta \left( \frac{\nu_{\text{obs}}}{\nu_{\text{G}}} \right)^{\frac{\nu_{\text{obs}}}{\nu_{\text{G}}} + 2} \left( \frac{\epsilon_e}{\epsilon_B} \right)^{\frac{1}{p+1}} \left( \frac{\epsilon_B}{\xi_e} \right)^{2-p}.
$$

Figure 3. The observed mean radio flux centroid velocity between 75 and 230 days, $\langle \beta_{\text{app}} \rangle = 4.1 \pm 0.5$ (horizontal lines; Mooley et al. 2018b), is compared to that from our best-fit simulation (thick red line) as a function of $\alpha$. It corresponds to $\alpha = 0.715^{+0.260}_{-0.154}$ (vertical lines) or a 1σ confidence interval $0.561 < \alpha < 0.975$.

Figure 4. Top: The evolution of the afterglow image flux-centroid location ($\bar{x}_{\text{fc}}$; deep purple), and best-fit parameters to an elliptical Gaussian: semi-minor axis $\sigma_x$ (blue), semi-major axis $\sigma_y$ (red), and center $\bar{x}_{\text{cl}}$ (magenta). Solid lines are for our fiducial model, and dotted lines of the same color are for our best-fit length-time rescaling parameter $\alpha = 0.715$. Our model calculations are compared to observational upper limits (Mooley et al. 2018b; Ghirlanda et al. 2019) on the semi-major (red) and semi-minor (blue) axes (the ones with an ellipse symbol at 230 days assume a 4:1 axis ratio), as well as when assuming a circular Gaussian image (circle symbol, in black as it applies to both axes). The vertical dotted black lines indicate the two epochs (75 and 230 days) between which $\langle \beta_{\text{app}} \rangle = 4.1 \pm 0.5$ was measured (Mooley et al. 2018b).

Bottom: The evolution of the flux-centroid location ($\bar{x}_{\text{fc}}$; left $y$-axis) for our fiducial model (deep purple) and its rescaled version to best fit the measured $\langle \beta_{\text{app}} \rangle$ (shaded region of matching color for the 1σ confidence region), as well as of the flux centroid’s apparent velocity (right $y$-axis). For the latter we show both the mean apparent velocity from $t = 0$, $\langle \beta_{\text{app}} \rangle = \bar{x}_{\text{fc}}/c t_{\text{obs}}$ (dark green), and for the instantaneous $\beta_{\text{app}} = |\bar{x}_{\text{fc}}|/c t_{\text{obs}}$ (blue).
The image size, flux centroid location, and observed time all scale as \( \alpha = \hat{x}^* / \hat{x} = \hat{y}^* / \hat{y} = \hat{x}_c^* / \hat{x}_c = t^*_\text{obs} / t^*_\text{obs} \), independent of the r.h.s of Eq. (9). The flux centroid’s apparent velocity \( \beta_{\text{app}} \) remains unchanged, but shifts to the rescaled observer time (see, e.g. Sec. 4 of Granot et al. 2018a, for more details).

Fig. 3 shows how our best-fit simulated \( \langle \beta_{\text{app}} \rangle \) varies with \( \alpha \). The measured \( \langle \beta_{\text{app}} \rangle = 4.1 \pm 0.5 \) corresponds to \( \alpha = 0.715^{+0.154}_{-0.141} \), and is consistent (at the 1.0σ level) with our fiducial model that fits the afterglow lightcurve (\( \alpha = 1 \)), which thus passes an important consistency check.

To calculate the afterglow image size and shape, we fit the surface brightness to an elliptical Gaussian, \( I \propto \exp[-(\hat{x} - \hat{x}_c)^2/2\sigma_x^2 - (\hat{y} - \hat{y}_c)^2/2\sigma_y^2] \) centered at \( (\hat{x}_c, \hat{y}_c, 0) \), where \( \sigma_x, \sigma_y \) are the standard deviations of the minor and semi-major axes along the \( \hat{x} \)-axis and \( \hat{y} \)-axis, respectively (Granot et al. 2018a). The top-panel of Fig. 4 shows the evolution of the afterglow flux-centroid location, and the afterglow image size and shape for \( \alpha = 1 \) and for the \( \langle \beta_{\text{app}} \rangle \) best-fit \( \alpha = 0.715 \). Our image size is consistent with the upper limits from radio VLBI observations (Mooley et al. 2018b; Ghirlanda et al. 2019).

The bottom-panel of Fig. 4 shows the flux centroid’s location, \( \hat{x}_c(t_{\text{obs}}) \), as well as its instantaneous \( \langle \beta_{\text{app}} \rangle = |x^c / d(t_{\text{obs}})\rangle \) and mean \( \langle \langle \beta_{\text{app}} \rangle \rangle = |x^c / c(t_{\text{obs}})\rangle \) apparent velocities, for our fiducial model (\( \alpha = 1 \)), and over the 1σ confidence interval of \( \alpha \) derived in Fig. 3. We find that \( \beta_{\text{app}} \) (\( t_{\text{obs}} / c \)) \( \approx \langle \beta_{\text{app}} \rangle \).

The measured \( \langle \beta_{\text{app}} \rangle \) favors a slightly larger \( \theta_0 \) compared to our \( \theta_0 = 0.1 \). The lightcurve peak occurs when \( 1/\Delta \theta = \Gamma(t_{\text{obs}} / c) \approx \beta_{\text{app}}(t_{\text{obs}} / c) \approx \beta_{\text{app}} \), implying \( \theta_0 \approx \langle \beta_{\text{app}} \rangle (\theta_{\text{obs}} / \theta_0 - 1) \)\(^{-1} \approx 0.116^{-0.016}_{+0.016} \) using the measured \( \langle \beta_{\text{app}} \rangle = 4.1 \pm 0.5 \), and our inferred \( \theta_{\text{obs}} / \theta_0 = 3.1 \pm 0.1 \). The latter implies \( \Gamma(t_{\text{obs}} / c) \propto \theta_0^{-1} \), which in turn for the measured \( t_{\text{obs}}(\theta_0) \), \( \approx 150 \) days, and either pre- or post-jet break simple analytic dynamics, implies \( E / n \propto \theta_0^{-6} \). This agrees with the best-fit values for our \( \theta_0 = 0.1, 0.2 \) to within 34%, \( 0.2/0.1)^{6}(10^{50.22}/10^{-2})/(10^{50.3}/10^{-3.6}) \approx 1.337 \). Even for \( \theta_0 = 0.2 \), a derivation of \( E_{\text{min}} \) following the one done above for \( \theta_0 = 0.1 \) gives a result very similar to Eq. (7), implying that it is quite robust. Even for \( \theta_0 = 0.2 \), we find \( E_{\text{min}} \) very similar to that found for \( \theta_0 = 0.1 \) in Eq. (7), implying that it is quite robust.

4. DISCUSSION AND CONCLUSIONS

In this work we demonstrate using afterglow lightcurves and image size, shape and flux centroid motion, all derived from 2D hydrodynamical numerical simulations, that even an initially top-hat jet can fit the afterglow observations of GW 170817/GW 170817A. We show that simulations of initially top-hat jets that are limited to only modest \( \Gamma_0 \sim 20 - 25 \) can only be used to fit the late time observations near the peak of the lightcurve at \( t_{\text{obs}} \approx 150 \) days. To fit the early time observation at \( t_{\text{obs}} < 60 \) days, \( \Gamma_0 > 25 \) is required.

The numerical simulations are initialized using a conical wedge with the self-similar BM76 dynamics; a similar setup is used in the BOXFITv2 code. Having only modest \( \Gamma_0 \) the simulation is initialized at a finite lab-frame time \( t_0 \) when \( \Gamma(t_0) = \Gamma_0 \). Therefore, no flux contributions are obtained from the simulated region at \( t < t_0 \). Artificially supplementing the lightcurve at those times with flux arising from the initial condition (a top-hat jet) over a wide time-range produces an early sharply rising flux for an off-axis (\( \theta_0 > \theta_0 \)) observer. However, within a dynamical time \( (t_0 - t < 2t_0 \Rightarrow t_{\text{obs},0} < t_{\text{obs}} \leq 2t_{\text{obs},0}) \), as the outflow relaxes from the initial condition it develops a bow-shock-like angular structure that resembles a structured jet having an energetic relativistic core surrounded by mildly (and sub-) relativistic low-energy material. Outside the relativistic core, whose emission is highly beamed, the sub-relativistic material makes the dominant contribution to the flux for an off-axis observer due to its much wider beaming cone. As the jet’s core decelerates, its beaming cone widens and the observer sees a gradual rise in flux until the entire core becomes visible, at which point the flux peaks and starts to decline thereafter, gradually joining the on-axis lightcurve.

We demonstrate here that by using increasingly larger \( \Gamma_0 = 20, 40, 60 \) the initial observed time can be shifted to correspondingly earlier times, \( t_{\text{obs},0} \approx 38.1, 23.0, 18.3 \) days, thereby replacing the sharply rising flux with that rising much more gradually. In the case of GRB 170817A, the shallow flux rise seen from \( t_{\text{obs},0} \approx 10 \) days can be simulated if \( \Gamma_0 > 10^{2.5} \), requiring higher resolutions and longer computation times.

Our results clearly show that a highly ideal initial top-hat jet can adequately reproduce the afterglow data for GRB 170817A. This requires most of the jet’s energy to initially reside within a narrow core (\( \theta_0 \approx 0.1 \)) and sharply drop outside of it, and an off-axis observer with \( \theta_{\text{obs}} / \theta_0 \approx 3 \). We use flux scaling relations to obtain the lightcurve and image properties for any set of model parameters, and by comparing the model flux to observations at \( t_{\text{obs},pk} \) for our best-fit model, we find a minimal jet energy of \( E_{\text{min}} \approx 2 \times 10^{48} \text{ erg} \) when the microphysical parameters attain their physically maximal values. Apart from obtaining a good fit to the lightcurve, we verify the consistency of our best fit model by comparing the mean flux centroid motion, \( \langle \beta_{\text{app}} \rangle = 4.1 \pm 0.5 \), and image size obtained from VLBI observations to that obtained from our simulations. These are important constraints that any afterglow jet structure model must satisfy.

R.G. and J. G. are supported by the Israeli Science Foundation under grant No. 719/14. FDC acknowledges support from the UNAM-PAPIIT grant IN117917. We acknowledge the support from the Miztli-UNAM
supercomputer (project LANCAD-UNAM-DGTIC-281) in which the simulations were performed.

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