Strength evaluation of butt joint by stress intensity factor of small edge crack near interface edge

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Abstract. Failure of the bonded dissimilar materials generally initiates near the interface, or just from the interface edge due to the stress singularity at the interface edge. In this study, the stress intensity factor of an edge crack close to the interface between the dissimilar materials is analyzed. The small edge crack is strongly dominated by the singular stress field near the interface edge. The analysis of stress intensity factor of small edge crack near the interface in bi-material and butt joint plates is carried out by changing the length and the location of the crack and the region dominated by the interface edge is examined. It is found that the dimensionless stress intensity factor of small crack, normalized by the singular stress at the crack tip point in the bonded plate without the crack, is equal to 1.12, independent of the material combination and adhesive layer thickness, when the relative crack length with respect to the crack location is less than 0.01. The adhesive strength of the bonded plate with various adhesive layer thicknesses can be expressed as the constant critical stress intensity factor of the small edge crack.

1. Introduction
The development of high-strength adhesives has encouraged the use of bonded joint for various engineering applications. In adhesively bonded joint components under mechanical or thermal loading condition, the singular stresses near the interface free-edge occur and these high stresses may result in the initiation of failure from the interface edge. Many studies on adhesive strength have been carried out analytically and experimentally using the intensity of the singular stress field at the interface corner of bonded heterogeneous material. It is well known that the singular stress distribution in the vicinity of the interface edge can be expressed by two parameters, that is, the index and the intensity of the singular stress. In late years, Kubo-Ohji [1] and Chen-Nisitani [2] have reinvestigated the singularities of stress field in the vicinity of the interface corner by the method of eigenvalue expansion. Munz-Yang [3] have showed the extrapolation technique for single term singularity corner in order to determine the intensity of the singular stress distribution at the edge of interface, and Xu et al [4] have proposed the numerical method to calculate the indexes of multiple singular terms and the corresponding stress intensity coefficients. Noda et al [5,6] have reported the efficient technique to evaluate the intensity of the singular stress field (ISSF) at the interface corner by using the ratio of the stress value near the interface edge obtained by the finite element method (FEM). Furthermore, Noda et al [7-10] have presented that the fictitious interface crack model in bonded materials is useful for evaluating the adhesive strength of the bonded structures.
On the other hand, the problem of cracks induced near the free-edge of interface has also been researched. Fett et al [11,12] have examined about the weight functions of the sub-interface cracks. Xu et al [13] have presented the approximate analytical solution of the stress intensity factor (SIF) for edge crack close to the interface by using the superposition method.

In this study, a problem of small edge crack close to the interface in a bi-material plate and butt joint plate will be considered. The stress intensity factors (SIFs) for small edge crack will be analysed by changing the crack length and the distance from the interface, and the reasonable expression of solution for the edge crack will be examined. In addition, the fracture criterion of the bonded plate will be discussed by using the small edge crack model.

2. Crack tip stress method
In this study, a small edge crack problem near the interface in the bonded plates is analyzed. As shown in figure 1, ‘a’ expresses the crack length and ‘c’ stands for the distance from the interface to the crack. When the crack is close to the free-edge of interface, the edge crack is strongly affected by the corner stress singularity. The stress intensity factor of the edge crack is calculated by the finite element analysis and the crack tip stress method [14,15]. In this analysis, the versatile FEM code MSC.Marc/Mentat is employed and the four-node-quadrilateral element is used. The pattern of finite element mesh around the crack tip is systematically and finely created.

The crack tip stress method has been originally proposed by Nisitani [14,15] and a stress intensity factor for two dimensional crack problem can be obtained accurately by using the FEM analysis. According to the principle of the method, the ratio between the stress intensity factors of two different problems is nearly equal to the ratio between the stress values at the crack tip nodes in y-direction obtained by FEM. When the same mesh patterns around the crack tip are used for two different crack problems, the singular stress fields can be represented by the crack tip stress \( \sigma_{y0,FEM} \) [14,15]. Based on the above fact, the equation (1) is obtained. In this relation, \( K_I \) is the mode I stress intensity factor, \( \sigma_{y0,FEM} \) means the stress value in y-direction at the crack-tip-node calculated by FEM and the asterisk * indicates the value of the reference problem.

\[
\frac{K_I}{\sigma_{y0,FEM}} = \frac{K_I^*}{\sigma_{y0,FEM}^*}
\]  

(1)

If we know the \( K_I^* \) for the reference problem in advance, the value of \( K_I \) for the unknown problem can be evaluated from the equation (1) because the stress values \( \sigma_{y0,FEM} \) and \( \sigma_{y0,FEM}^* \) for the reference and the unknown problems can be readily computed by FEM. As the reference problem, the closed-form solution of a crack in an infinite homogeneous plane is selected in this calculation.
3. Numerical results and discussion

3.1. Stress intensity factors of small edge crack close to interface in bi-material plate

The stress intensity factors of small edge crack close to the interface illustrated in figure 1 are analyzed. In figure 1, the plate length $L=400$ mm, the plate width $W=100$ mm and the distance from the interface to the crack $c=2$ mm. As the combination of the materials used for the analysis, the medium carbon steel $S45C$ ($E_1 = 206$ GPa, $\nu_1 = 0.3$ for material ‘1’) and the epoxy resin ($E_2=4.93$ GPa, $\nu_2=0.33$ for material ‘2’ are assumed. The singularity index of interface corner is $1 - \lambda = 0.2823$.

In this study, the normalized stress intensity factors are defined by the singular stress near the free-edge of interface as the next expression [16]:

$$F_1 = K_1 / \sigma_{y,eq} \sqrt{\pi a}, \quad F_{II} = K_{II} / \sigma_{y,eq} \sqrt{\pi a}$$ \hspace{1cm} (2)

As shown in figure 1, $\sigma_{y,eq}$ stands for the y-directional stress value at the crack tip position in the same bi-material plate without the crack. The stress $\sigma_{y,eq}$ close to the edge of interface in material ‘1’ has been given as following equations [13]:

$$\sigma_{y,eq} = \frac{\rho (\sin \theta)^{1-\lambda}}{4} \left[ ((\lambda + 2 - \cos \lambda \pi) d + b \sin \lambda \pi \right] \cos (\lambda - 1) \theta$$

$$\left. - [ (\lambda + 2 + \cos \lambda \pi) b + d \sin \lambda \pi \right] \sin (\lambda - 1) \theta$$

$$\left. + (\lambda - 1)[d \cos (\lambda - 3) \theta - b \sin (\lambda - 3) \theta - b \sin (\lambda - 3) \theta] \right]$$ \hspace{1cm} (3)

Here,

$$\rho = \frac{\lambda c^{\lambda-1} K_\sigma}{\lambda^2 - \sin^2 (\lambda \pi / 2)}, \quad \theta = \tan^{-1} (c/a),$$

$$b = 2F - \frac{1}{\lambda} [ \sin \lambda \pi - F (1 - \cos \lambda \pi) ], \quad d = 2 - \frac{1}{\lambda} (1 - \cos \lambda \pi + F \sin \lambda \pi),$$

$$F = \frac{\lambda D + \sin (\lambda \pi / 2) \cos (\lambda \pi / 2) - D \sin^2 (\lambda \pi / 2) + \sin (\lambda \pi / 2) \cos (\lambda \pi / 2)}{\lambda + \sin^2 (\lambda \pi / 2) + D \sin (\lambda \pi / 2) \cos (\lambda \pi / 2)},$$

$$D = \frac{2 \beta [\lambda^2 + (2\lambda + 1) \sin^2 (\lambda \pi / 2)] - 2 \alpha [\lambda^2 - \lambda \cos (\lambda \pi)]}{\sin (\lambda \pi) [1 - 2\alpha (\beta - \alpha)]}.$$ \hspace{1cm} (4)

In the equations (3) and (4), $K_\sigma$ is the fracture mechanics parameter expressing the intensity of the singular stress field in the vicinity of the interface corner, and $\alpha$, $\beta$ are Dundurs’ composite parameters describing the elastic mismatch.

Figure 1 illustrates the numerical results of the normalized stress intensity factor $F_1$ for various crack lengths and crack locations. As shown in this figure, it is found that the normalized stress intensity factors are converged 1.12 when the relative crack length with respect to the crack distance $a/c<0.01$, independent of the crack distance from interface. In this state, the mode II factors $F_{II}$ are nearly zero. Therefore, the stress intensity factor of small edge crack close to the interface can be represented by

$$K_1 = 1.12 \sigma_{y,eq} \sqrt{\pi a}, \quad K_{II} = 0.$$ \hspace{1cm} (5)

Furthermore, in the case of $a/c<0.01$, it is shown that the equations (3), (4) and (5) express the linear relationship between the stress intensity factor $K_1$ of the small edge crack and the intensity $K_\sigma$ of corner singular stress.

3.2. Stress intensity factor of bonded plate with adhesive layer
Next, the edge crack problem in butt joint plate with adhesive layer is considered. As illustrated in figure 2, the S45C (material 1) rectangular plates are adhered the epoxy resin (material 2). The stress intensity factors of the edge crack for various crack lengths and locations are calculated by using the crack tip stress method. Figure 2 shows the relation between the normalized factor $F_1$ and the relative crack length with respect to the bond line thickness $a/h$ when the bond line thickness is $h=1$ mm. In this case, the stress value $\sigma_{y,eq}$ is calculated by FEM, not the equation (3). As shown in figure 2, the normalized stress intensity factors for the different crack locations are also converged with 1.12 when $a/h<0.01$. Therefore, even in the case of the bonded plate with the adhesive layer, the stress intensity factor of the small edge crack within the corner singularity area can be evaluated by $K_1 = 1.12\sigma_{y,eq}\sqrt{\pi a}$.

![Figure 2. Relation between $F_1$ and $a/h$ for butt joint (S45C-Epoxy) when $h=1$ mm.](image)

Table 1 shows the comparison of $K_\sigma$ between the present results obtained from equations (3), (5) and the results analyzed by Noda et al [5] when the crack length is fixed and the crack position is systematically changed in the range of $a/c<0.01$ and $a/h<0.01$. Table 2 also presents the present $K_\sigma$ calculated from equations (3), (5) when the crack position is fixed and the crack length is systematically changed. From tables 1 and 2, even when the crack position is changed within the range of $a/c<0.01$ and $a/h<0.01$, the results of both are in good agreement.

| $c$ (mm) | $K_\sigma$ Present results | Noda et al [5] |
|---------|----------------------------|----------------|
| 0.09    | 70.7                       | 68.9           |
| 0.07    | 70.56                      | 68.9           |
| 0.05    | 70.34                      | 68.9           |
| 0.03    | 70.18                      | 68.9           |

| $a$ (mm) | $K_\sigma$ Present results | Noda et al [5] |
|---------|----------------------------|----------------|
| 0.0009  | 70.76                      | 68.9           |
| 0.0008  | 70.73                      | 68.9           |
| 0.0007  | 70.70                      | 68.9           |
| 0.0006  | 70.66                      | 68.9           |

Table 3 indicates the stress intensity factors $K_1$ calculated from equation (5) and by the crack tip stress method (CTSM) when $c = 0.09$ mm and the crack length is changed from $a = 0.0006$ to 0.0009 mm. As
shown in table 3, the difference between both results is small within 0.1% error and equation (5) is useful for evaluating the stress intensity factor of small edge crack close to the interface when \( a/c < 0.01 \) and \( a/h < 0.01 \).

**Table 3.** Comparison of stress intensity factors. \((c=0.09 \text{ mm}, h=0.1 \text{ mm})\).

| \( a \) (mm) | \( K_I \) | Error (%) |
|-------------|-----------|-----------|
| Equation (5) | CTSM      |           |
| 0.0009      | 6.015     | 6.019     | 0.067     |
| 0.0008      | 5.671     | 5.670     | 0.018     |
| 0.0007      | 5.305     | 5.306     | 0.005     |
| 0.0006      | 4.912     | 4.912     | 0.003     |

4. **Application to adhesion strength evaluation**

Next, we examine the strength evaluation of butt joint specimens by using the stress intensity factor of edge crack near the interface. The tensile test for the bonded plates with various adhesive thicknesses has been carried out by Suzuki [17]. In this study, the critical stress intensity factor \( K_{IC} \) is calculated by using the experimental fracture stress \( \sigma_c \) obtained by Suzuki [17]. The material properties for two patterns of bonded specimens used by Suzuki are shown in table 4.

**Table 4.** Material combinations [17].

|          | E (GPa) | \( \nu \) |
|----------|---------|-----------|
| S35C Epikote828 | 210     | 0.3       |
|           | 3.14    | 0.37      |
| S35C Epikote871 | 210     | 0.3       |
|           | 2.16    | 0.38      |

![Figure 3](image-url)

**Figure 3.** Relation between \( K_{IC} \) and \( h \) when \( a = 10^{-5} \text{ mm} \) and \( c = 10^{-3} \text{ mm} \) \((a/h \leq 0.1)\).

Figure 3 shows the relationship between the critical stress intensity factor \( K_{IC} \) and the adhesive layer thickness \( h \) when the crack location \( c = 10^{-3} \text{ mm} \) and the crack length \( a = 10^{-5} \text{ mm} \). The left figure is S35C-Epikote828 and the right is S35C-Epikote871. As indicated in figure 3, when the bond line thickness \( h \) is larger than the edge crack distance \( c \) from the interface, that is, \( c/h < 1 \), the values of \( K_{IC} \) are almost constant. On the other hand, when \( c/h > 1 \), the values of \( K_{IC} \) deviate because the dominated area of the interface corner singularity is related to the adhesive layer thickness.

If the distance \( c \) from the interface is larger than the adhesive thickness \( h \), the edge crack is out of
the singularity area, the stress intensity factor $K_I$ does not correspond to the intensity of corner singular stress field $K_\sigma$.

Figure 4 also indicates the relationship between the critical stress intensity factor $K_{IC}$ and the adhesive layer thickness $h$ when the location and length of the edge crack are changed, $c = 10^{-4}$ mm and $a = 10^{-6}$ mm. In this case, since the adhesive layer thickness $h$ is always greater than the distance $c$ to the interface in the calculation range, the values of $K_{IC}$ are approximately constant including experimental error, as indicated by the blue dashed line in figure 4. These results are equivalent to the results evaluated by the intensity of the corner singular stress field $K_\sigma$ [9,10].

![Figure 4](image)

**Figure 4.** Relation between $K_{IC}$ and $h$ when $a = 10^{-6}$ mm and $c = 10^{-4}$ mm ($a/h \leq 0.01$).

5. **Conclusions**

In this study, the evaluation method of the adhesive strength for butt joint specimens were considered by means of the edge crack model. The stress intensity factors of the small edge crack close to the interface were analyzed by the crack tip stress method and the relation between the stress intensity factor (SIF) of edge crack and the intensity of corner singular stress field (ISSF) were examined. Then, the numerical results of edge crack model were applied to estimation of the fracture strength of butt joint specimens. The conclusions can be summarized as follows:

- The normalized stress intensity factors on the basis of the singular stress $\sigma_{y,eq}$ were converged with the constant value 1.12 when the ratio between the length and the location of the crack $a/c < 0.01$. The stress value $\sigma_{y,eq}$ at the crack tip position in the bonded dissimilar plate without the crack was calculated by the equation (3) or the finite element method.
- The stress intensity factor of the small edge crack close to the free-edge of interface can be shown by $K_I = 1.12\sigma_{y,eq}\sqrt{\pi a}$. Since the singular stress $\sigma_{y,eq}$ is proportional to the intensity $K_\sigma$ of the singular stress field at the interface corner, $K_I$ and $K_\sigma$ have a one-to-one relationship.
- The critical stress intensity factors $K_{IC}$ became almost constant irrespective of the bond line thickness $h$ when $a/h < 0.01$ and $c/h < 1$. Therefore, the stress intensity factor of the small edge crack close to the interface can be an alternative parameter of the intensity of the interface edge singular field, and useful for evaluating the adhesive strength of bonded specimen.

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