The fracture of yield stress fluid jet in air and in viscous fluids

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Abstract. One of the largest family of complex materials are represented by fluids with yield stress. In this category are included creams, pastes, greases, gels, in general materials which start to flow at a certain value of the imposed shear stress. A common characteristic of yield stress materials is the association of material instability with the onset of the fluid behavior. The present paper is concerned with the experimental investigations and numerical modeling of the dynamics of a yield stress fluid jet in air or immersed in viscous/viscoelastic liquids. A cream jet at constant flow rate is generated through a capillary with a syringe pump and the visualizations of its motion in a vessel filled with an immiscible fluid is performed with normal and high-speed cameras. One goal of the study is to test the capability of the VoF code to simulate the jetting phenomena, the rheology of the sample being modeled with the Carreau equation and the Herschel-Bulkley relation, respectively. The visualizations show the specific yield stress instabilities from jetting to coiling, buckling and fracture.

1. Introduction

The yield stress fluids are materials with a complex elasto-viscoplastic rheological behavior which start to flow at a critical level of the internal shear stress, called the yield stress \( \sigma_0 > 0 \). In this category are included pastes, creams, solid dispersion, gels, greases, in general nano-micro solid structures or solid particles in high concentration embedded in viscous liquids, [1], [2].

Below this threshold value of the shear stress, the material behaves more as a soft solid, keeping its own configuration but characterized by high deformations for \( \sigma < \sigma_0 \). The yield shear stress is associated with the yield shear rate of deformation \( \gamma_0 \), respectively with the yield deformation \( \gamma_0 \). From the rheological point of view, the yield stress materials are elastoplastic solids at small deformations \( (\gamma < \gamma_0) \), but they are considered as fluids with very high constant viscosity \( \eta_0 \) (zero shear viscosity) for \( \gamma > \gamma_0 \) and \( \dot{\gamma} < \dot{\gamma}_0 \).

Beyond the critical values of shear stress and shear rate, respectively \( (\sigma > \sigma_0, \gamma > \gamma_0 \) and \( \dot{\gamma} > \dot{\gamma}_0 \) \) the yield stress fluids present a strong shear thinning behavior, with a relatively small viscosity as shear rates tend to infinite, i.e. \( \eta_\infty \ll \eta_0 \), [2], [3], [4], [5].

The paper is concerned with the experimental and numerical simulation of the yield stress jet flows in air or immersed in Newtonian and viscoelastic liquids. The visualizations display all three flow regimes: (i) jetting, (ii) coiling, and (iii) buckling, [6], [7]. The main aim of the study is to test the capability of the VoF method, [8], in the commercial ANSYS Fluent code, to simulate the jetting flow regime, from the sample’s exit in the outer fluid to its fracture and rupture.
2 Experimental and numerical simulations

2.1. Experimental

2.1.1 Samples rheometry. In the absence of elasticity, the Cauchy extra-stress tensor \( \mathbf{T} \) has the expression of a generalized Newtonian fluid, linear in the strain rate kinematic tensor \( \mathbf{D} \), and a viscosity dependent function of the strain (shear) rate (i.e. \( \dot{\gamma} \) the second invariant of \( \mathbf{D} \)),

\[
\mathbf{T} = 2\eta(\dot{\gamma})\mathbf{D}.
\] (1)

The most used formulas for the viscosity function suitable to represent the rheology of yield stress fluids are given by the Carreau model:

\[
\frac{\eta(\dot{\gamma}) - \eta_\infty}{\eta_0 - \eta_\infty} = \left[ 1 + (\lambda \dot{\gamma})^2 \right]^{\frac{n-1}{2}}
\] (2)

with the thinning exponent (flow index) \( n = 0 \), and Herschel-Bulkley (HB) models (which have the origin in the Bingham relation, \( \sigma = \sigma_0 + k \dot{\gamma}^n \), at \( \sigma > \sigma_0 \)), e.g.

\[
\eta(\dot{\gamma}) = \frac{\sigma_0}{\dot{\gamma}} + k \left( \frac{\dot{\gamma}}{\gamma_0} \right)^{n-1}
\] (3)

for \( \dot{\gamma} > \gamma_0 \), and

\[
\eta(\dot{\gamma}) \equiv \eta_0 = \frac{\sigma_0}{\gamma_0} + k
\] (4)

at \( \dot{\gamma} < \gamma_0 \), [2, 5, 9].

In (2) \( \eta_\infty \) is the infinite viscosity and \( \lambda \) the time constant of the model. In (3) \( k \) [Pas] is the consistency with the flow index \( n < 1 \). In both models the condition \( \gamma > \gamma_0 \) is considered to be fulfilled.

The tested yield stress sample is a commercial cosmetic cream (mixture between glycerin, vegetal oils and polyisobutene) having the density \( \rho_c = 1050 \text{ kg/m}^3 \). The viscosity measurements have been performed in a cone-plate configuration (50 mm diameter, Anton Paar rheometer MC 301) in controlled strain mode, at different time/point inputs, 10 points/decade in the range \( \dot{\gamma} \in (10^{-4}, 10^2) \text{ s}^{-1} \).

The experimental results and the fitting with the two models are shown in figure 1.

![Figure 1](image1.png)

**Figure 1.** Flow curve (shear stress vs. shear rate) and the corresponding viscosity function for the cream sample: measurements and fitting with Carreau relation (2): \( \eta_0 = 15000 \text{ Pa.s, } \eta_\infty = 10 \text{ Pa.s, } \lambda = 400 \text{ s, } n = 0 \) and Herschel - Bulkley (HB) model (3-4): \( \sigma_0 = 70 \text{ Pa, } \gamma_0 = 0.01 \text{ s}^{-1}, k = 25 \text{ Pa.s, } n = 0.9 \).
The measured curves are quasi-steady, especially at very low shear rates where a long time is needed to reach the steady state. The experiments from figure 1 depict the typical pattern for fluids with yield stress in simple shear: (i) hysteresis between up- and down-flow curves (down-curve always illustrates a well-defined yield stress and an infinite zero shear viscosity), (ii) well defined plateau in shear stress, (iii) dependence of the input conditions at low shear rates, (iv) strong shear thinning, [1], [2], [3].

2.1.2 Experimental set-up. The designed setup for visualization of fluid jets is presented in figure 2. Using a Harvard syringe pump, the sample is ejected at a constant flow rate from a capillary into atmosphere or into a vessel filled with Newtonian oil. The capillary has internal diameter: \( d_{\text{int}} = 2.25 \, \text{mm} \), with \( d_{\text{ext}} = 2.80 \, \text{mm} \).

The exit mean velocity is \( v_0 = 1 \, \text{mm/s} \), which corresponds to an input syringe pump flow rate of \( 0.24 \, \text{ml/min} \). Assuming fluid adherence at the wall, the wall shear rate in the capillary is approximated by: \( \dot{\gamma}_w = 8v_0/d_{\text{int}} \approx 3.5 \, \text{s}^{-1} \), [10].

From figure 1, the corresponding viscosity value is 20 \( Pa \cdot s \) and therefore the flow is characterized by a very low Reynolds number (Stokes approximation):

\[
Re = \frac{\rho \cdot d_{\text{int}} \cdot v_0}{\eta \cdot \dot{\gamma}_w} < 10^{-5}. \tag{5}
\]

The flow regime corresponds to the plateau region in the flow curve from figure 1. Since the shear rate is maximum at the wall, there is almost a constant shear stress in the flow domain (the yield stress \( \sigma_0 \approx 75 \, Pa \)), which determines a plug flow (constant velocity) in the center of the capillary, where the shear rate tends to zero and the viscosity is maximum.

![Figure 2](image)

**Figure 2.** Experimental setup: the cream jet is generated in the vessel at constant flow rate imposed by a syringe pump. The jet is visualized with digital cameras.

The visualizations are performed with two cameras: (1) Nikon 1J4 at 60 fps with 10 Mpixels resolution, (2) Phantom VEO-E 340L high speed camera, with the maximum resolution of 2560 \( \times \) 1600 pixels at 800 fps. The movies are shot with the frequency in the range of 800 \( \div \) 4000 fps, according to the characteristic time of the phenomenon and the picture desired resolution.

The cream jet is ejected in a close glass vessel having the diameter of 40 \( mm \) and height of 150 \( mm \) respectively, the outer fluid being air or a Newtonian oil with density \( \rho_e = 920 \, \text{kg/m}^3 \) and shear viscosity \( \eta_e = 55 \, \text{mPas} \), at a working temperature of 23° C.
2.2 Flows visualizations

The visualizations of the cream jet at the exit from the capillary in air or in oil confirm the existence of the plug flow. In the absence of buoyance (Archimedes) force and friction, i.e. cream jet in air, the jet’s fracture occurs at a distance of 3-4 diameters from the exit, when the weight force induces internal normal stresses beyond the coherence stress within the sample (generated by yield, elasticity and surface tension), figure 3.

![Image](image1.png)

**Figure 3.** Evolution of the cream jet in air (the time gap between the pictures is 1 ms). The evolution of the rupture’s location in the jet is marked.

The visualizations from figure 3 confirm the jetting regime, followed by the jet thinning and fracture, [6]. The jetting regime is better observed in the case of the cream jet flow in oil, figure 4.a. The buoyancy and friction forces keep the internal normal stress within the sample below the fracture limit. Therefore, the continuous straight jet reaches the bottom of the vessel, which immediately generates the onset of the coiling regime, figure 4.b. In the absence of air bubbles or material discontinuities, the coiling regime continues and evolves to the buckling regime, figure 4.c [6], [7].

![Image](image2.png)
Figure 4. a) Jetting followed by coiling (b) at the contact with the bottom of the vessel; (c) coiling followed by buckling after the contact with the wall (cream immersed in Newtonian oil).

The three flow regimes of the jet dynamics are visualized in the case of the cream jet ejected in a viscoelastic liquid, figure 5. The phenomenon shown in figure 5 is generated by the interaction between the yield stress of the cream and the high viscous/elastic stresses induced at the jet’s surface by the outer complex fluid.

Figure 5. Jetting, coiling and buckling flow regimes of the dynamics of the cream jet immersed in viscoelastic fluid.

2.3 Numerics

The numerical simulations of the cream jet flow immersed in air or viscous fluids is based on coupling of the Cauchy/Navier Stokes equations of motion with the continuity equation. Non-stationary local solutions of the equations of motion (Newtonian and generalized Newtonian models under incompressible and isothermal conditions) for the whole flow domain are obtained with the commercial ANSYS Fluent code, the dynamics of the interface being computed using the VoF model, [8].

The time step is 0.1 ms with 400 maximum iterations per time step and $10^{-8}$ precision. Under these conditions, the Courant number was maintained below one. The geometry is axial-symmetric and contains 300,000 quadrilateral cells. The working PC has 16 parallel processors at 3 GHz and 128 GB RAM memory. The computation time to obtain the rupture of the cream jet at $v_j = 0 \text{mm/s}$ is at least 15 days.

The boundary conditions are: (i) adherence at solid walls (including the capillary), (ii) free surface at the upper limit of the domain, (iii) constant velocity at the entrance in the capillary, (iv) contact angle...
90° between the tested fluid and capillary wall. The interface between the immiscible fluids (cream and air/oil) is traced using the VoF implicit scheme [8]; the line with volume fraction VF = 0.5 is considered to represent the interface between the two fluids.

The errors in tracing the interface profile are given by the width band between VF = 0.1 and VF = 0.9, which, for thinning part of the jet in the vicinity of the fracture is almost 50% from the nominal measured dimension. This is the limit of the calculus; of course, it is established by the mesh quality (the number of nodes/cells, higher mesh density in the region of the jet) and directly related with the computation time step and the available resources.

The numerical simulations presented in figure 6 are consistent with the experiments from figure 3. The numerics offer a fair representation of the cream jet dynamics; the evolution of jet’s boundary in air and its fracture being correctly reproduced.

![Figure 6. Numerical simulation (HB model) of the jet fracture (cream in air, figure 3); a) viscosity distribution (red maximum - \( \eta_0 \), blue minimum - \( \eta_{\infty} \), or \( \eta_e \), respectively), b) jet geometry (blue – cream, red – air).](image)

The analysis of the numerical solutions gives relevant information about the viscosity/shear rate distributions: (i) viscosity is constant in the middle of capillary, which proves the existence of the plug flow in that region (one main characteristic of the yield stress fluids [1], [2], [11], [12]); (ii) before jet’s fracture, the viscosity in the very vicinity of the rupture region tends to \( \eta_{\infty} \) and immediately post-rupture viscosity is going to \( \eta_0 \) in both separated cream jets. The detached jet and the jet after the thinning zone depict a solid like motion.

The simulations of the jet’s flow in mineral oil reveal the plug flow with high constant viscosity from the exit of the cream in the oil, figure 7.a. The pathlines within the jet are straight lines, but the pathlines in the oil are curved, due to the imposed boundary conditions at the wall of the vessel and the formation of symmetric vortices. In figure 7.b. are shown the distribution of the vorticity lines in the oil at the jet’s boundary. This is an indication of the mechanism generation of the vortices in the viscous fluid by the plug flow of the yield stress jet.
Figure 7. Numerical simulation (HB model) of the cream jet in Newtonian oil (see figure 4.a); a) Pathlines coloured with the level of viscosity (red – max., blue – min.); b) Corresponding distribution of the iso-vorticity magnitude in the vicinity of the immersed jet.

Comparisons between the models (2) and (3-4) at two times are displayed in figure 8.

Figure 8. Viscosity distributions in the cream jet immersed in oil; comparison between the Carreau and HB models; $\eta_e = 55 \text{ mPa}s$ (blue) for the values of $\eta_0$ (red) see figure 1, (2) and (4), respectively.
The results from figure 8 are expected. The simulations with Carreau relation show a continuous variation of viscosity within the jet and a pronounced contraction of the jet at the exit from the capillary, where the shear thinning characteristic of the viscosity is remarkable (rheological behavior observed also in the case of viscoelastic liquids, [13]).

The viscosity distribution in the HB jet model is sharper, which is an indication of a pronounced plug flow at the exit of the cream in the oil. The jet’s contraction is almost absent in the case of HB jet, which determines a slower advance of the jet in comparison to the Carreau model.

3. Final remarks and conclusions
The study investigated the correlation between the experiments and 2D numerical simulations of the jetting flow regime of the yield stress jet in air or immersed in a Newtonian viscous liquid. The qualitative analyses of the results confirm the capability of the VoF – ANSYS Fluent code to correctly reproduce the flow, from the jet exit in the outer fluid to its fracture and rupture limits. Of course, the numerical results are only approximative in the very vicinity of the rupture, due to the limit of the mesh density in that region.

The performed visualizations also show more complex flow regimes of the yield stress jet, as coiling and buckling. These phenomena are in present under study in our research group, especially the experimental investigations of the yield stress jet impact on free surfaces of viscoelastic fluids.

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