Renormalization group approach to Fermi Liquid Theory

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We show that the renormalization group (RG) approach to interacting fermions at one-loop order recovers Fermi liquid theory results when the forward scattering zero sound (ZS) and exchange (ZS') channels are both taken into account. The Landau parameters are related to the fixed point value of the "unphysical" limit of the forward scattering vertex. We specify the conditions under which the results obtained at one-loop order hold at all order in a loop expansion. We also emphasize the similarities between our RG approach and the diagrammatic derivation of Fermi liquid theory.

Much of our understanding of interacting fermions is based on the Fermi liquid theory (FLT). Although the latter was first formulated as a phenomenological theory, its microscopic foundation was rapidly established using field theoretical methods. The discovery of new materials showing non Fermi liquid behavior, like high-$T_c$ superconductors, has motivated a lot of theoretical work in order to clarify the validity and the limitations of the FLT.

Several authors have recently applied renormalization group (RG) methods to interacting fermions (see [5,7] and references therein). While these methods are well known in the context of one and quasi-one dimensional interacting fermions systems where they have been very successful, their application to isotropic systems of dimension $d$ greater than one is more recent. In his study of interacting fermions in $d = 2, 3$, Shankar used both RG methods and standard perturbative calculation. While RG arguments were used to identify the relevant couplings, the low-energy degrees of freedom were explicitly integrated out in the Landau interaction channel by means of standard diagrammatic calculations. Extending Shankar’s approach to finite temperature, Chitov and Sénéchal have recently shown how this interaction channel can be treated by RG without use of any additional perturbative calculation. Moreover, the finite temperature formalism clearly establishes the difference between the "physical" and "unphysical" limits of the forward scattering vertex and therefore differentiates the Landau function (i.e. the Landau parameters $F^*_i$ and $F^*_j$) from the (physical) forward scattering amplitude. It is clear that both approaches amount to summing the series of bubble diagrams in the forward scattering zero sound channel. Since it is well known that such an RPA-type calculation reproduces the results of FLT, the agreement between FLT and RG approach is a posteriori not surprising. Although the selection of Feynman diagrams appearing in the RG procedure was justified on the basis of an expansion in the small parameter $\Lambda_0/K_F$ ($K_F$ is the Fermi wave-vector and $\Lambda_0$ a low-energy cut-off), it is nevertheless rather unexpected that the RG approach reduces to an RPA calculation while the diagrammatic microscopic derivation of FLT is obviously more than a simple RPA calculation.

The aim of this paper is to reconsider the RG approach to interacting fermions along the lines developed in Refs. [5,7] in order to clarify its connection with FLT. First, we derive the RG equation for the "physical" ("unphysical") limit of the forward scattering vertex $\Gamma^Q$ ($\Gamma^{Q'}$) at one-loop order. In order to respect the Fermi statistics, the forward scattering zero sound (ZS) and exchange (ZS') channels are both taken into account. As a result, we find that both the flows of $\Gamma^Q$ and $\Gamma^{Q'}$ are non zero. We show that the antisymmetry of $\Gamma^Q$ under exchange of the two incoming or outgoing particles is conserved under RG, while the antisymmetry of $\Gamma^{Q'}$ is lost. We then solve (approximately) the RG equations to obtain a relation between the fixed point (FP) values $\Gamma^{Q*}$ and $\Gamma^{Q*}$. The standard relation between $\Gamma^{Q*}$ and the Landau parameters $F^*_i$, $F^*_j$ (which is one of the key results of the microscopic diagrammatic derivation of FLT) is recovered if one identifies these latter with $\Gamma^{Q*}$. This result differs from previous RG approaches where the Landau parameters were identified with the bare interaction of the low-energy effective action (which is the starting point of the RG analysis). We show that the relation between $\Gamma^{Q*}$ and the Landau parameters obtained at one-loop order holds at all order if one assumes that the only singular contribution to the RG flow is due to the one-loop ZS graph.

We consider a two-dimensional system of interacting spin one-half fermions with a circular Fermi surface (the results obtained in this paper can be straightforwardly extended to the three-dimensional case). Following Refs. [5], we write the partition function as a functional integral over Grassmann variables, $Z = \int \mathcal{D}\psi^* \mathcal{D}\psi e^{-S}$, where $S$ is a low-energy effective action (we set $\hbar = k_B = 1$):

$$S = -\sum_{K,\sigma} \psi^*_\sigma(\vec{K})(i\omega - \epsilon(K) + \mu)\psi_{\sigma}(\vec{K}) + \frac{1}{4\beta'} \sum_{K_1, \ldots, K_4} \sum_{\sigma_1, \sigma_2, \sigma_3, \sigma_4} U_{\sigma_1, \sigma_2, \sigma_3, \sigma_4}(K_1, K_2, K_3, K_4)
\times \psi^*_{\sigma_4}(\vec{K}_4)\psi^*_{\sigma_3}(\vec{K}_3)\psi_{\sigma_2}(\vec{K}_2)\psi_{\sigma_1}(\vec{K}_1)
\times \delta_{K_1+K_2-K_3+K_4} \delta_{\omega_1+\omega_2+\omega_3+\omega_4 + \cdots},$$ (1)
where the dots denote terms which are irrelevant at tree-level. Here $\mathbf{K}$ is a two-dimensional vector with $|K - K_F| < \Lambda_0 \ll K_F$. $\mu$ is the chemical potential, $K_F$ the Fermi wave-vector and the cut-off $\Lambda_0$ fixes the energy scale of the effective action. $\hat{K} = (\mathbf{K}, \omega)$ and $\omega$ is a fermionic Matsubara frequency. $\beta = 1/T$ is the inverse temperature and $\nu$ the size of the system. $\sigma = \uparrow, \downarrow$ refers to the electron spins. The antisymmetric coupling function $\Gamma_{\uparrow\sigma_1\sigma_2\sigma_3\sigma_4}(\mathbf{K}_1, \mathbf{K}_2, \mathbf{K}_3, \mathbf{K}_4)$ is assumed to be a non-singular function of its arguments. Ignoring irrelevant terms, we write the single particle energy as $\epsilon(\mathbf{K}) = \mu + v_F k$ where $K = K_F + k$ and $v_F$ is the Fermi velocity. The summation over the wave vectors is defined by

$$ \frac{1}{\nu} \sum_{\mathbf{K}} \equiv \int \frac{d^2 \mathbf{K}}{(2\pi)^2} = K_F \int_{-\Lambda_0}^{\Lambda_0} dk \int_{-\pi}^{\pi} d\theta \frac{2\pi}{2\pi}.$$

keeping only the relevant term in the integration measure at tree-level. Shankar’s analysis of the coupling functions of the quartic interaction shows that only two such functions survive under the RG flow for $\Lambda_0 \ll K_F$: the forward scattering coupling function and the BCS coupling function. In the following, we neglect this latter by assuming it is irrelevant at one-loop order so that no BCS instability occurs. The forward scattering coupling function is denoted by $\Gamma_{\sigma_1}(K_1, K_2, K_3 - \hat{Q}, K_4 + \hat{Q})$ where $\hat{Q} = (\mathbf{Q}, \Omega)$ with $Q \ll K_F$ and $\Omega$ is a bosonic Matsubara frequency (we use the notation $\Gamma_{\sigma_1} = \Gamma_{\sigma_1\sigma_2\sigma_3\sigma_4}$). Since the dependence on $k_{1,2}$ and $\omega_{1,2}$ is irrelevant, we introduce the coupling function $\Gamma_{\sigma_1}(\theta_1, \theta_2, \hat{Q}) = \Gamma_{\sigma_1}(\mathbf{K}_F, \mathbf{K}_F, \mathbf{K}_F - \hat{Q}, \mathbf{K}_F + \hat{Q})$ where $\mathbf{K}_F = K_F \mathbf{K}/K = K_F (\cos \theta, \sin \theta)$ is a wave-vector on the Fermi surface. The forward scattering coupling function can be decomposed into a singlet and triplet amplitude $\Gamma_t$ and a spin singlet amplitude $\Gamma_s$.

$$ \Gamma_{\sigma_1}(\theta_1, \theta_2, \hat{Q}) = \Gamma_t(\theta_1, \theta_2, \hat{Q}) (\delta_{\sigma_1\sigma_2} \delta_{\sigma_3\sigma_4} + \delta_{\sigma_1\sigma_3} \delta_{\sigma_2\sigma_4}) \rightleftharpoons \frac{1}{2} (\delta_{\sigma_1\sigma_2} \delta_{\sigma_3\sigma_4} - \delta_{\sigma_1\sigma_3} \delta_{\sigma_2\sigma_4}).$$

We now introduce the “physical” ($\Gamma^Q$) and “unphysical” ($\Gamma^\Omega$) limits of the forward scattering vertex:

$$\Gamma^Q_{\sigma_1}(\theta_1 - \theta_2) = \lim_{Q \to 0} \Gamma_{\sigma_1}(\theta_1, \theta_2, \hat{Q}) \Big|_{\Omega = 0}, \rightleftharpoons \frac{1}{2} (\delta_{\sigma_1\sigma_2} \delta_{\sigma_3\sigma_4} - \delta_{\sigma_1\sigma_3} \delta_{\sigma_2\sigma_4}).$$

$\Gamma^Q_{\sigma_1}(\theta_1 - \theta_2) = \lim_{Q \to 0} \Gamma_{\sigma_1}(\theta_1, \theta_2, \hat{Q}) \Big|_{\Omega = 0}$.

$\Gamma^\Omega_{\sigma_1}(\theta_1 - \theta_2)$ can be decomposed into singlet and triplet amplitudes according to (3). The only remnant of the antisymmetry of $\Gamma_{\sigma_1}$ is then the condition $\Gamma^Q_{\sigma_1}(\theta) = 0$ for the bare vertices.

We now derive the RG equation (using the Kadanoff-Wilson approach) for the coupling $\Gamma^\Omega_{\sigma_1}$ when the cut-off $\Lambda_0$ is reduced according to $\Lambda(t) = \Lambda_0 e^{-t}$. Three diagrams have to be considered at one-loop order, corresponding to the ZS, ZS' and BCS channels. As pointed out in Ref. [3], the ZS graph alone does not respect the Fermi statistics. Indeed, if one exchanges the two incoming or outgoing lines, the ZS graph transforms into the ZS' graph and vice versa (Fig. 1). It is therefore necessary to consider the ZS and ZS' graphs on the same footing. We ignore momentarily the symmetry-preserving contribution of the BCS diagram which will be discussed later.

The contribution of the ZS graph is

$$\frac{d\Gamma^Q_{\sigma_1}(\theta_1 - \theta_2)}{dt} = -\frac{N(0)\beta_R}{\cosh^2(\beta R)} \int d\theta \sum_{\sigma,\sigma'} \Gamma^Q_{\sigma_1\sigma\sigma'}(\theta_1 - \theta) \Gamma^Q_{\sigma\sigma\sigma}(\theta - \theta_2),$$

where $\beta_R = v_F \beta(1/2)$ is a dimensionless inverse temperature and $N(0) = K_F / 2\pi v_F$ the density of states per spin. Since $\lim_{\beta \to \infty} (\beta/4) \cosh^2(\beta x/2) = \delta(x)$, the ZS graph gives a singular contribution to the RG flow of $\Gamma^Q$ when $T \to 0$. Consider now the contribution of the ZS' graph. For a given $\hat{Q}$, this graph involves the quantity (Fig. 1)

$$ T \sum_{\omega} G(\mathbf{K}) G(\mathbf{K} + \mathbf{K}_F - \hat{Q}) = 
\frac{1}{2} \tanh \left[ \frac{\beta}{2} \epsilon(\mathbf{K}) \right] - \tanh \left[ \frac{\beta}{2} \epsilon(\mathbf{K}) \right] - \frac{1}{2} \Omega + \epsilon(\mathbf{K}) - \epsilon(\mathbf{K} + \mathbf{K}_F - \hat{Q}).$$

Here $G(\mathbf{K}) = (i \omega - v_F k)^{-1}$ is the one-particle Green’s function and $\mathbf{K}_F = \mathbf{K}_F - \mathbf{K}_F$. In general, the limit $\hat{Q} \to 0$ can be taken without any problem (and is independent of the order in which the limits $Q, \Omega \to 0$ are taken) and gives a smooth contribution to the flow of $\Gamma^Q$ and $\Gamma^\Omega$. As pointed out by Mermin [3], problems arise when $\mathbf{K}_F$ is small since the limits $\hat{Q} \to 0$ and $\mathbf{K}_F \to 0$ do not commute. For small $\mathbf{K}_F \to \mathbf{K}_F$, i.e. for $|\theta_1 - \theta_2| \ll T / E_F$, $\mathbf{K}$ becomes

$$\mathbf{K} = \mathbf{K}_F / K$$

where $\mathbf{K} = \mathbf{K} / K$ is a unit vector. This quantity (apart from the thermal factor $\beta \cosh^2(\beta v_F k/2)$) has been analyzed in detail by Mermin who showed that the antisymmetry of the vertex is strongly related to the order in which the different limits are taken. Following Ref. [3], we first take the limit $\hat{Q} \to 0$ (which is well defined for $\mathbf{K}_F \neq \mathbf{K}_F$) and then $\theta_1 \to \theta_2$. This ensures that $\Gamma^Q_{\sigma_1}(\theta)$ is a continuous function at $\theta = 0$. We then have

$$\lim_{\theta_{1,2} \to 0} \left[ T \sum_{\omega} G(\mathbf{K}) G(\mathbf{K} + \mathbf{K}_F) \right] = -\beta / 4 \cosh^2(\beta v_F k/2).$$

(8)
Eq. (8) shows that when $T \to 0$ the ZS' graph gives a singular contribution to the RG flow of $\Gamma^Q,\Omega(\theta)$ for $|\theta| \ll T/E_F$. Taking into account the spin dependence of the coupling, we obtain that the contributions of the ZS and ZS' graphs to the RG flow of $\Gamma^Q_i(\theta = 0)$ cancel each over. Consequently, $\Gamma^Q_i(\theta = 0) = 0$ for any value of the flow parameter $t$. The antisymmetry of $\Gamma^Q$ is therefore conserved under RG. Since the contribution of the ZS graph to the RG flow of $\Gamma^Q_\sigma(\theta = 0)$ vanishes, while the contribution of the ZS' graph does not, the antisymmetry of $\Gamma^\Omega$ is not conserved under RG. This results agrees with standard diagrammatic calculations.

Taking into account both the contributions of the ZS and ZS' graphs, the RG equations of $\Gamma^Q,\Omega$ can be written:

$$
\frac{d\Gamma^Q_\sigma}{dt} = \frac{d\Gamma^Q_\sigma}{dt} \bigg|_{ZS} + \frac{d\Gamma^Q_\sigma}{dt} \bigg|_{ZS'} ,
$$

$$
\frac{d\Gamma^\Omega_\sigma}{dt} = \frac{d\Gamma^\Omega_\sigma}{dt} \bigg|_{ZS} + \frac{d\Gamma^\Omega_\sigma}{dt} \bigg|_{ZS'} .
$$

(9)

The two preceding equations can be combined (using also Eq. (8)) to obtain

$$
\frac{d\Gamma^Q_\sigma}{dt}(\theta_1 - \theta_2) = \frac{d\Gamma^Q_\sigma}{dt}(\theta_1 - \theta_2) - \frac{N(0)\beta_R}{\cosh^2(\beta_R)} \sum_{\sigma,\sigma'} \Gamma^Q_{\sigma_1',\sigma_2,\sigma_3}(\theta_1 - \theta) \Gamma^Q_{\sigma_2',\sigma_3'}(\theta - \theta_2) .
$$

(10)

In order to solve this RG equation, we Fourier transform $\Gamma^{Q,\Omega}_\sigma(\theta)$ and introduce the spin symmetric ($A^{Q,\Omega}$) and antisymmetric ($B^{Q,\Omega}$) parts:

$$
\Gamma^{Q,\Omega}_\sigma = \int \frac{d\theta}{2\pi} e^{-i\theta \omega} \Gamma^{Q,\Omega}_\sigma(\theta) ,
$$

$$
2N(0)\Gamma^{Q,\Omega}_\sigma = A^{Q,\Omega}_\sigma \delta_{\sigma,\sigma_1} \delta_{\sigma_2,\sigma_3} + B^{Q,\Omega}_\sigma \tau_{\sigma_1,\sigma_2,\sigma_3} + \tau_{\sigma_2,\sigma_3} ,
$$

(11)

where $\tau$ denote the Pauli matrices. Eq. (11) holds when the spin dependent part of the particle interaction is due purely to exchange. Eq. (11) then takes the simple form

$$
\frac{dA^Q_i}{dt} = \frac{dA^Q_i}{dt} - \frac{\beta_R}{\cosh^2(\beta_R)} A^{Q2}_i ,
$$

(12)

and the same equation relating $B^Q_i$ and $B^\Omega_i$. Integrating these equations between 0 and $t$, we obtain (writing explicitly the $t$ dependence)

$$
A^Q_i(t) = A^Q_i(0) - \int_0^t dt' \frac{\beta_R}{\cosh^2(\beta_R)} A^{Q2}_i(t')^2
$$

(13)

and a similar equation for $B^Q_i(t)$. The RG equations in their symmetry preserving form (44,45) relate two FP-s $\Gamma^Q_\sigma^\ast$ and $\Gamma^\Omega_\sigma^\ast$ in a fashion more general than the standard RPA-like form (see Eq. (15) below) with all harmonics decoupled. Deferring study of such a fixed point, which is beyond the scope of the present paper, we concentrate now on the approximation leading to the standard FLT results. Because of the thermal factor $\beta_R/\cosh^2(\beta_R)$, the second term of the rhs of (13) becomes different from zero only when $\Lambda(t) \leq T/E_F$. On the other hand, we have shown above that $\Gamma^\Omega_\sigma^\ast(\theta)$ is a smooth function of $\Lambda(t)$ except for $|\theta| \ll T/E_F$. The Fourier transform $\Gamma^\Omega_\sigma^\ast(\theta)$ is also a smooth function of $\Lambda(t)$. At low temperature, we can therefore make the approximation $A^{Q}_\sigma|_{\Lambda(t) \leq T/E_F} \approx A^{Q*}_\sigma$, where $A^{Q*}_\sigma = A^{Q}_\sigma|_{\Lambda(t)=0}$ is the FP value of $A^{Q}_\sigma$. This allows us to rewrite (44) for $\Lambda(t) \leq T/E_F$ as

$$
A^Q_i(t) = A^{Q*}_i - \int_0^t dt' \frac{\beta_R}{\cosh^2(\beta_R)} A^{Q2}_i(t')^2 .
$$

(14)

Eq. (14) is solved by introducing the parameter $\tau = \tanh \beta_R$. The FP values of $A^Q_i$ and $B^Q_i$ are given in the zero-temperature limit by

$$
A^{Q*}_i = \frac{A^{Q*}_i}{1 + A^Q_i}; \quad B^{Q*}_i = \frac{B^{Q*}_i}{1 + B^Q_i} .
$$

(15)

Eq. (15) shows that the standard results of the microscopic FLT are recovered if one identifies the Landau parameters with the FP values of $A^Q_i$ and $B^Q_i$:

$$
F_i^L = A^{Q*}_i; \quad F_i^L = B^{Q*}_i .
$$

(16)

Alternatively, (15) can be written as $f_{\sigma,\sigma'}(\theta) = \Gamma^{Q*}_{\sigma,\sigma'}(\theta)$ where $f_{\sigma,\sigma'}(\theta)$ is Landau’s quasi-particle interaction function. Since the singular contribution (8) of the ZS’ graph to $\Gamma^Q_\sigma$ was neglected when approximating (13) by (14), $\Gamma^Q_\sigma(\theta)^\ast$ obtained from (14) is correct only for $|\theta| \geq T/E_F$.

The determination of $\Gamma^Q_\sigma(\theta)^\ast$ for $|\theta| \leq T/E_F$ would require the consideration of the singular contribution of the ZS’ graph. It should be noted that the diagrammatic derivation of FLT also neglects the zero-angle singularity in the ZS’ channel and therefore does not respect the antisymmetry of the “physical” limit of the forward scattering vertex. The condition $\Gamma^Q_i(\theta = 0) = 0$ is usually enforced, giving the “amplitude sum rule” of FLT. For physical quantities (like collective modes or response functions) which probe all values of the angle $\theta$, it is nevertheless justified to neglect the singularity of the ZS’ channel.

The relation (44) between $\Gamma^Q$ and the Landau parameters has been obtained at one-loop order. It appears therefore as an approximate relation whose validity is restricted to the weak coupling limit. However, it turns out that (44) holds at all orders in a loop expansion if we assume that the only singular contribution to the RG flow comes from the one-loop ZS graph (again we neglect the singular contribution of the ZS’ graph). Note that the same kind of assumption is at the basis of the diagrammatic derivation of FLT. In this case, the RG flows of $\Gamma^Q$ and $\Gamma^\Omega$ are determined by
The contribution of the one-loop ZS graph (first term of the rhs of (17)) has been separated from the non-singular contributions. Note that we have included in this latter the one-loop BCS graph which had been neglected up to now. Eqs. (17,18) can be combined to obtain
\[
\frac{d\Gamma_{\sigma_i}}{dt} = \frac{d\Gamma_{\sigma_i}^{Q}}{dt} \bigg|_{ZS} + \frac{d\Gamma_{\sigma_i}^{\Omega}}{dt} \bigg|_{ZS'}, \, \text{BCS, 2 loops...}
\]
where the second term of the rhs of (19) is given by (3). Since, according to our assumption, \(\Gamma_{\sigma_i}^Q\) is a non-singular function of \(\Lambda(t)\), Eq. (19) is similar to Eq. (10) and can be solved in the same way, yielding again the result (13). Thus, higher order contributions change the FP value \(\Gamma_{\sigma_i}^{Q*}\) obtained at one-loop order, but not the relation (13) between \(\Gamma_{\sigma_i}^{Q*}\) and \(\Gamma_{\sigma_i}^{Q}\).

It should be pointed out that the Landau parameters are not determined by the bare coupling function \(U_{\sigma_i}(K_1, \ldots, K_4)\) of the effective action (1), but are related to the FP value \(\Gamma_{\sigma_i}^{Q*}\). Usually, the FP values of physical quantities are related to some bare effective values, so that their calculation by means of the RG (within the framework of a loop expansion) is approximative and valid only in the weak coupling regime. The FLT, which assumes that the only singular contribution to the RG flow is due to the one-loop ZS graph, does not rely on any kind of weak coupling condition.

Eq. (13) (together with the analog Eq. for \(B_{1}^{Q}\)) is nothing else but the Bethe-Salpeter equation in the ZS channel for the vertex \(\Gamma_{\sigma_i}^{Q}\), with \(\Gamma_{\sigma_i}^{Q*}\) the irreducible two-particle vertex. This shows that the integration of the RG equations generates the same Feynman diagrams as those considered by Landau (3). From this point of view, there is therefore a strict equivalence between the present RG approach and the diagrammatic microscopic derivation of FLT.

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\[\begin{align*}
\frac{d\Gamma_{\sigma_i}^{Q}}{dt} & = \frac{d\Gamma_{\sigma_i}^{\Omega}}{dt} \bigg|_{ZS} + \frac{d\Gamma_{\sigma_i}^{Q}}{dt} \bigg|_{ZS'}, \, \text{BCS, 2 loops...} \\
\frac{d\Gamma_{\sigma_i}^Q}{dt} & = \frac{d\Gamma_{\sigma_i}^{\Omega}}{dt} \bigg|_{ZS} + \frac{d\Gamma_{\sigma_i}^{Q}}{dt} \bigg|_{ZS'} \, \text{BCS, 2 loops...}
\end{align*}\]

The quantities \(\Gamma^A\) and \(\Gamma^B\) introduced in Ref. (8) correspond to \(-N(0)\Gamma^Q/2\) and \(N(0)\Gamma^Q/2\) respectively.

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13. Note that \(K\) and \(K + K_{2}^{F} - Q\) can both be within an infinitesimal shell near the cut-off \(\Lambda(t)\) only if \(Q, K_{2}^{F} \to 0.\) When this latter condition is not realized, the ZS' diagram is generated from the 6-point vertex function. The importance of \(m\)-point (\(m \geq 6\)) vertex functions, which was pointed out by Shankar in a preliminary version of Ref. (3), will be discussed in detail elsewhere.
14. G. Chitov, in preparation.