Abstract

It is shown that the mass spectrum of supermembrane theory is continuous. This fact is due to the trivial circumstance that membrane can emit ”needles” of zero area with no cost in energy. In supersymmetric case, this classical degeneracy is not lifted by quantum corrections. This unpleasant property may be cured, perhaps, for the supermembrane with a modified action involving higher derivative terms.
1 Introduction

Recently the impressive progress in studying string theories which impelled many high energy theorists to believe that the Theory of Everything based on string philosophy would be built soon has slowed down. The task proved to be more complicated than it seemed to be several years ago and a decisive breakthrough has not occurred yet. In these circumstances, the attention of theorists was distracted to studying alternative possibilities to construct the theory of fundamental interactions. One of these alternative candidates is the supermembrane theory - the main subject of the present meeting. The main conclusion of my talk is that, unfortunately, the standard supermembrane with the action being a supersymmetry extension of the Nambu action in 2+1 dimensions meets serious difficulties. It involves the continuous mass spectrum and hence, unlike string theory, the effective field theory of lowest mass states cannot be built in supermembrane case. This unpleasant fact has a very transparent physical interpretation. In the light cone gauge, the supermembrane mass operator is an integral of some density over membrane surface (see Refs. [1,2] and the lectures by I.Hoppe, C.Pope and others at this Conference)

\[ M^2_{\text{supermembrane}} = \int d^2\sigma \left[ P'^2_I + \frac{1}{2} \{X_I, X_J\}^2 + 2i\bar{\theta}\Gamma_I\{\theta, X_I\} \right], \]

where \( I, J = 1, \ldots, 9; \{X_I, X_J\} = \epsilon^{rs}\partial_r X_I \partial_s X_J \) and \( P'_I \) involves only nonzero modes contribution. But the membrane (unlike string) can be deformed without increasing its area: a ”needle” of zero area can be attached at any point of membrane surface (see Fig.1).

\[ H_{\text{SYM QM}} = E_i E_i^A + \frac{1}{2} f_{ABE} f_{CDE} A_i^A A_j^B A_k^C + 2 i f_{ABC} \lambda^A \Gamma_i \lambda^B A_i^C, \]

Figure 1:

The classical energy of these degenerate configurations is the same. Thus, we are faced with the valley in configuration space, the motion along which is infinite, and the spectrum is continuous. The statement is that, in supersymmetric case, this classical degeneracy is not lifted by quantum corrections. We use the wonderful fact discovered in Ref. [2] that the supermembrane hamiltonian (1) may be thought of as \( N \to \infty \) limit of the hamiltonian of the Super-Yang-Mills quantum mechanics with the \( SU(N) \) group,
where \( f_{ABC} \) are the \( SU(N) \) structure constants, \( A_I^A \) and \( \lambda_\alpha \) are the boson and fermion components of supergauge field, \( \Gamma_I \) are the 10-dimensional \( \Gamma \)-matrices. The Hamiltonian (2) is obtained by reducing the 10-dimensional SYM theory to \((0+1)\) space (i.e. by considering only the fields not depending on the space variables) and choosing the gauge \( A_0^A = 0 \). The correspondence with the supermembrane Hamiltonian is the following: \( A_I^A \) correspond to the spherical harmonics of the coordinates

\[
X_I(\sigma) = \sum_{LM} (X_I^{LM} \equiv A_I^A) Y^{LM}(\theta, \phi),
\]

(3)

\( P_I^{LM} \equiv E_I^A, \theta_\alpha^{LM} \equiv \lambda_\alpha^A \). There is a subtlety that, at finite \( N \), the structure constants of \( SU(N) \) do not coincide with the structure constants of the area-preserving diffeomorphism group which, in fact, enter eq. (1), but only tend to them in the \( N \to \infty \) limit. But for the physical implications, the property

\[
\lim_{N \to \infty} f_{ABC}^{SU(N)} = g_{ABC}^{SDiff^2}
\]

(4)

is quite sufficient. One may think of the Hamiltonian (2) as of a convenient way to perform ultraviolet regularization of the 2-dimensional field theory (1). It is very close in spirit to familiar lattice regularization of YM theory where one cares to respect the gauge symmetry of the field theory. As \( N \to \infty \) and we involve in the analysis the spherical harmonics with higher and higher \( L \)'s, the physical properties of the Hamiltonians of eq. (2) and of eq. (1) are equivalent. The Hamiltonian (2) involves valleys, i.e. the regions in configuration space where the classical bosonic potential turns to zero. A sufficient (but for \( N > 2 \) not necessary - see the discussion in Sect.3) condition of the valley is

\[
A_I^{A(val)} = c_I \eta^A.
\]

(5)

In that case, the commutator \([A_I, A_J]\) together with the potential \( \propto \text{Sp} \{[A_I, A_J]^2\} \) turn to zero. The existence of these valleys and also the fact that, in supersymmetric case, they are not lifted by quantum corrections was first noted by E. Witten in the celebrated paper where the concept of the Witten index had been introduced [3].

Witten was interested in supersymmetric gauge field theories and studied the question if the supersymmetry is broken spontaneously. To this end, he considered a theory placed in a small spatial box with \( L \ll \Lambda \) so that \( g^2(L) \ll 1 \) and perturbative expansion makes sense, and imposed periodic boundary conditions on the fields

\[
\Phi(x_1, x_2, x_3) = \Phi(x_1 + L, x_2, x_3)
\]

(6)

etc. (Witten considered also \( \text{t'} \text{Hooft} \) twisted boundary conditions but, for our purposes, they are not relevant). In this case, only the gauge transformations respecting the boundary conditions (6) are allowed and, in contrast to the field theory in infinite volume, the field configuration \( A_I^A = \text{const} \) is generally not gauge equivalent to the configuration \( A_I^A = 0 \). Thus, for the gauge theory in finite volume, the valley of constant configurations \( A_I^A \) satisfying the condition \([A_I, A_J] = 0\) is present. As long as \( L \) is nonzero, the valley has finite length \( \propto 2n/gL \) (the configurations \( A_I^2(x) \) and \( A_J^2(x) + 2\pi n/gL, \) where
$n_I$ is an integer vector, are gauge equivalent [3]. But at small $L$, the valley length is large and in the supersymmetric case, where, as Witten argued, the valleys are preserved after accounting for the quantum corrections, the low energy spectrum of the system is determined by the motion along the valleys: the characteristic excitation energies due to the valley motion are $g^2(L)/L$, which is much less than the characteristic energies of ”fast variables” excitations $\propto 1/L$. To resolve the problem at hand, one should go one step further and set $L = 0$. In this case (where field theory is reduced to quantum mechanics), the valleys have infinite length and the spectrum of the corresponding hamiltonian is continuous. In the rest of the talk, I’ll try to argue why, in the supersymmetric case, the valleys are not lifted. I start with the analysis of the simple case of quantum mechanics of $d = 4$ $SU(2)$ SYM theory, which is presented in the following section. In Sect.3, I consider its extension to higher dimensions and higher groups. In Sect.4, I’ll discuss briefly chiral gauge theories where the effective valley hamiltonian is not so trivial. Conclusive remarks are given in the last section.

2 Quantum mechanics of $SU(2)$ $d = 4$ super-Yang-Mills

The hamiltonian and supercharges of the model have the form [4, 5]

$$H = \frac{1}{2} P_i A_i + \frac{g^4}{4} \epsilon^{ABE} \epsilon^{CDE} A_i^A A_j^B A_k^C + ig \epsilon^{ABC} \bar{\lambda}^A \sigma_i \lambda^B A_i^C,$$  \hspace{1cm} (7)

where $i, j = A, B, C; \alpha = 1, 2$ and $H_i^A = \frac{g}{2} \epsilon^{ABC} \epsilon^{ijk} A_j^B A_k^C$. We introduced the coupling constant $g$ which can be easily scaled away by the transformation $A = g^{-1/3} \hat{A}$, $\lambda = g^{-1/3} \hat{\lambda}$. The supercharges (8) satisfy the following supersymmetry algebra

$$\{Q_\alpha, \bar{Q}_\beta\} = \delta^\beta_\alpha H - (\sigma_i)_\alpha^\beta A_i G^A,$$  \hspace{1cm} (9)

where

$$G^A = \epsilon^{ABC} (P_j^B A_j^C + i \bar{\lambda}_{\alpha}^B \sigma_{\alpha} A_i^C)$$  \hspace{1cm} (10)

is the Gauss constraint operator. The second term in the r.h.s. of eq. (9) is due to the known fact that a superposition of two supertransformations involves a gauge transformation besides translation. In the Hilbert space of physical gauge-invariant states, $G^A |\Psi_{\text{phys}}\rangle = 0$ and the standard $N = 2$ SQM algebra is realized. The hamiltonian (7) involves the valleys defined by the condition (5) (in the $SU(2)$ case, it is both sufficient and necessary). The $A_i^{A_{\text{val}}}$ of Eq. (5) can be freely rotated in colour space. This is
an unphysical gauge degree of freedom, and one is allowed to choose the gauge where \( \eta^A = \delta^A_3 \). In the Born-Oppenheimer spirit, we may classify the physical bosonic variables in two groups: the "slow" variables \( c_i \) which describe the motion along the valley and the "fast" variables \( A^a_i \), where \( a = 1, 2 \) and \( A^a_i c_i = 0 \). Only three of four variables \( A^a_i \) are physical, while the fourth is the gauge degree of freedom corresponding to a gauge rotation around the third colour axis. Our task is to construct the effective Hamiltonian depending only on the slow variables \( c_i \) and describing the low-energy spectrum of the system. To this end, we assume \( |c| \gg |A^a| \) and classify the various terms in the full Hamiltonian (7) by the powers of the formal parameter \( x^{\text{fast}}/x^{\text{slow}} \). We get

\[
H = H^{(0)} + H^{(2)} + H^{(2)},
\]

(11)

where

\[
H^{(0)} = -\frac{1}{2} \frac{\partial^2}{(\partial A_m^a)^2} + \frac{g^2 \epsilon^2}{2} (A_m^a A_m^a) + igc \epsilon^{a b} \chi^{a a} (\sigma_3)^{b b}
\]

(12)

(we have chosen the z direction along \( c \) so that \( c_i = c \delta_{i3} \) and \( a, m = 1, 2 \)) and \( H^{(1)}, H^{(2)} \) involve cubic and quartic in \( A_m^a \) terms, correspondingly.

The main observation is that the Hamiltonian (12) describes the supersymmetric oscillator. Its ground state can be found explicitly:

\[
\Psi^{(0)}_C (A_m^a) \propto \exp \left\{ -\frac{g c}{2} A_m^a A_m^a \right\} \left\{ \lambda^{b a} \lambda_b^b + i \epsilon^{b c} \lambda^{b a} (\sigma_3)^{b c} \lambda^c_c \right\}.
\]

(13)

It satisfies automatically the constraint \( G^3 |\Psi^{(0)}_C \rangle = 0 \) and has zero energy. In other words, in supersymmetric case, zero-point quantum fluctuations of the fast variables in the direction across the valley yield zero contribution in the effective Hamiltonian. Note that, in the nonsupersymmetric case, it is not true. For the pure YM quantum mechanics, \( H^{(0)} \) does not involve the third term and the bosonic zero-point energy \( \sim gc \) has no fermionic counterpart to cancel with. Thus, in the pure bosonic case, the classical valleys are lifted by quantum corrections, the motion along the valley is prohibited and the spectrum is discrete. A clear illustration what is going on in the pure bosonic case is given in Fig. 2. As \( c \) grows, the walls of the valley get steeper and steeper and the zero-point energy of transverse fluctuations gets higher and higher.

![Figure 2](image_url)

Surely, the effective valley Hamiltonian can be obtained in the rigorous and regular way. The straightforward way to do it is to use second order perturbation theory in the...
parameter $x_{\text{fast}}/x_{\text{slow}}$ and to write

$$H^{\text{eff}} = \langle 0 | H^{(2)} | 0 \rangle - \sum_n \frac{\langle 0 | H^{(1)} | n \rangle \langle n | H^{(1)} | 0 \rangle}{E_n}, \quad (14)$$

where $|0\rangle$ and $|n\rangle$ are the ground and excited states of the hamiltonian $H^{(0)}$. More simple and more convenient is to find out first the effective supercharge

$$Q^{\text{eff}}_{\alpha} = \langle 0 | Q_{\alpha} | 0 \rangle \quad (15)$$

with $Q_{\alpha}$ of eq. (8), and then to built up $H^{\text{eff}}$ as the anticommutator of $Q^{\text{eff}}_{\alpha}$ and $\bar{Q}^{\text{eff}}_{\alpha}$. Both methods give the same answer

$$Q^{\text{eff}}_{\alpha} = -i \frac{(\sigma_k)^\alpha_{\beta} \lambda_{\beta}}{\sqrt{2}} \frac{\partial}{\partial c_k},$$

$$H^{\text{eff}} = -\frac{1}{2} \frac{\partial^2}{(\partial c_i)^2}, \quad (16)$$

i.e. the motion along the valley is unbounded, indeed.

Let us discuss now the region of applicability of these results. The characteristic values of $A^a_i$ in the wave function (13) are $(A^a_i)_{\text{char}} \sim 1/\sqrt{gc}$. The condition $(A^a_i)_{\text{char}} \ll c$ necessary for the classification (11) to make sense is fulfilled provided $gc^3 \gg 1$. The parameter $1/gc^3$ is the true Born-Oppenheimer expansion parameter in the effective hamiltonian. If $gc^3 \gg 1$, the corrections to the lowest order effective hamiltonian (14) are expected to be small. We calculated explicitly these corrections in the case of supersymmetric QED, which is technically more simple [6]. Supersymmetric QED involves the Abelian gauge field $A_i$, its supersymmetric counterpart $\lambda_{\alpha}$, and two chiral multiplets $(\phi_{\alpha}, \xi_{\alpha})$ and $(\chi_{\alpha}, \eta_{\alpha})$ with opposite charges. The slow valley variables in this case are just $A_i$ and the fast variables are $\phi$ and $\chi$. The result of calculation of the effective hamiltonian for the massless SQED quantum mechanics is

$$H^{\text{eff}} = -\frac{1}{2} f (\partial^2/\partial A^2_k) f + i \epsilon_{kpl} \bar{\lambda} \sigma_i \lambda f (\partial f/\partial A_p) \frac{\partial}{\partial A_k} + \frac{1}{6} f (\partial^2 f/\partial A^2_k) \bar{\lambda} \sigma_i \lambda \bar{\lambda} \sigma_i \lambda, \quad (17)$$

where

$$f(A) = 1 - \frac{1}{4eA^3} \quad (18)$$

(the corresponding effective supercharges may be found in Ref. [6]). We see that, as long as $eA^3 \gg 1$, the second term in the r.h.s. of eq.(18) is small and $H^{\text{eff}}$ is reduced to the free hamiltonian in eq.(16). But in the region where $A$ is small, the corrections grow large and the Born-Oppenheimer approximation is not applicable. However, for the conclusion that the spectrum of $H^{\text{eff}}$ and hence the low energy spectrum of the total hamiltonian is continuous this is not important as this conclusion depends only on the fact that, at large values of $A$, the motion is free.
3 Higher groups and higher dimensions

For supermembrane, relevant is the quantum mechanics of 10-dimensional $SU(N)$ Super-Yang-Mills.

Consider first the SYM theory with the $SU(2)$ gauge group in 10 dimensions. One can be convinced that this case can be treated quite analogously to the 4-dimensional case. We have now 9 valley variables $A^3_I = c_I$ and 16 fast variables $A^a_M$ ($a = 1, 2$ and $M = 1, \ldots, 8$), one of which corresponds to the gauge degree of freedom connected with rotations around the third colour axis. Directing $c_I$ along the 9-th axis: $c_I = c\delta_{I9}$ and assuming $gc^3 \gg 1$, we can expand the total hamiltonian over the formal parameter $x^{\text{fast}}/x^{\text{slow}}$. Then the analog of $H^{(0)}$ is

$$H^{(0)}_{10\dim} = -\frac{1}{2} \frac{\partial^2}{(\partial A^a_M)^2} + \frac{g^2c^2}{2} (A^a_M A^a_M) + igc\epsilon^{ab} \lambda^a (\Gamma_9)^3 \lambda^b,$$

(19)

$\alpha, \beta = 1, \ldots, 8$. It is still a supersymmetric oscillator with the ground state wave function

$$\Psi_C^{(0)}(A^a_M) \propto \exp \left\{ -\frac{gc}{2} A^a_M A^a_M \right\} \{ \lambda^a \lambda^a + i\epsilon^{bc} \lambda^b (\Gamma_9)^3 \lambda^c \}^4,$$

(20)

which has zero energy. All the arguments of the previous section are repeated unchanged with the result that the lowest order effective hamiltonian describes the free motion in the $c_I$ - space and, in the region $gc^3 \gg 1$, the corrections are small.

Let us discuss now a slightly more complicated case of higher $SU(N)$ groups. Consider first the $SU(3)$ case. The valley condition $[\hat{A}_I, \hat{A}_J] = 0$ is satisfied provided all $\hat{A}_I$ can be simultaneously diagonalized by a gauge transformation:

$$\hat{A}_I^{(\text{val})} = \begin{pmatrix} a_I & 0 & 0 \\ 0 & b_I - a_I & 0 \\ 0 & 0 & -b_I \end{pmatrix}.$$  

(21)

This representation is more convenient than the equivalent form $\hat{A}_I = A^3_{I3} + A^8_{I8}$. Thus, in the $SU(3)$ case, the valley variables are $a_I$ and $b_I$ - the weights of the Cartan subalgebra. The valley configuration (21) is not presented, generally, in the form (5) - the vectors $a_I$ and $b_I$ are not necessarily parallel. As earlier, we substitute now $\hat{A}_I = \hat{A}_I^{\text{val}} + \hat{A}_I^{\text{fast}}$ in the hamiltonian (2) and pick up the quadratic in $\hat{A}_I^{\text{fast}}$ terms. Then the potential part of $H^{(0)}$ is

$$V^{(0)} = \frac{g^2(2a - b)^2}{2} A^{a=1,2}_{I=a=1,2} \left[ \delta_{IJ} - \frac{(2a - b)_I (2a - b)_J}{(2a - b)^2} \right] + \frac{g^2(a + b)^2}{2} A^{a=4,5}_{I=a=4,5} \left[ \delta_{IJ} - \frac{(a + b)_I (a + b)_J}{(a + b)^2} \right] + \frac{g^2(2b - a)^2}{2} A^{a=6,7}_{I=a=6,7} \left[ \delta_{IJ} - \frac{(2b - a)_I (2b - a)_J}{(2b - a)^2} \right].$$

(22)

Thus, there are 48 fast variables divided naturally in three groups: $A^{1,2}_I$ satisfying the condition $(2a - b) A^{1,2}_I = O$, $A^{4,5}_I$ satisfying the condition $(a + b) A^{4,5}_I = O$ and $A^{6,7}_I$...
satisfying the condition \((2b - a)A^{6,7} = 0\). Two of these 48 variables are gauge degrees of freedom corresponding to the action of the generators \(G^3\) and \(G^8\). Besides, there are 6 more gauge degrees of freedom: \((A^{1,2}_I)_{\text{gauge}} = X^{1,2}_b (2a - b)_I\); \((A^{4,5}_I)_{\text{gauge}} = X^{4,5}_g (a + b)_I\) and \((A^{6,7}_I)_{\text{gauge}} = X^{6,7}_g (2b - a)_I\) corresponding to the rotations generated by \(G^{1,2,4,5,6,7}\) that act nontrivially on the valley configuration (21).

Each of the three terms in the r.h.s. of eq. (22) combined with the corresponding terms in the kinetic part of \(H^{(0)}\) and the part involving fermions has the form of eq.(19). Thus, in the \(SU(3)\) case, \(H^{(0)}\) is represented as the sum of three hamiltonians, each of them having the same form as in the \(SU(2)\) case, i.e. representing supersymmetric oscillator with zero energy ground state of eq.(20). The total ground state wave function representing the product of three eq.(20)-like factors satisfies automatically the conditions \(G^3|\Psi_0\rangle = G^8|\Psi_0\rangle = 0\). The effective supercharges and hamiltonian can be found in the same way as earlier. They have the form

\[
Q^{\text{eff}}_\alpha = -\frac{i}{\sqrt{2}} (\Gamma_I)^\alpha \left[ \lambda^{(a)}_\beta \frac{\partial}{\partial a_I} + \lambda^{(b)}_\beta \frac{\partial}{\partial b_I} \right],
\]

\[
H^{\text{eff}} = -\frac{2}{3} \left[ \frac{\partial^2}{(\partial a_I)^2} + \frac{\partial^2}{(\partial b_I)^2} + \frac{\partial^2}{\partial a_I \partial b_I} \right].
\]

The hamiltonian in (23) describes the free motion in the space of weights \(a_I, b_I\) (there is a subtlety that the wave function is required to be invariant under Weyl transformations permuting weights but, for our purposes, it is not important). The corrections to the lowest order \(H^{\text{eff}}\) of eq. (23) are small provided \(g |2a - b|^3, g |a + b|^3, g |2b - a|^3 \gg 1\), and the spectrum is continuous.

The higher \(N\) case is treated with an equal ease. Thus, for \(SU(4)\), we have 3x9 = 27 valley variables \(a_I, b_I, c_I\) and 96 fast variables divided in 6 groups, which include three gauge degrees of freedom corresponding to \(G^3, G^8\) and \(G^{15}\) rotations; for \(SU(5)\) there are 4x9 = 36 valley variables and 160 fast variables divided in 10 groups, etc. The answer is the same: \(H^{\text{eff}}\) describes the free motion in the space of weights and the spectrum is continuous. The similar analysis with the same result has been done in Ref. [7], but in the particular case where the valley configuration is representable in the form (5), i.e. when all the weights vectors \(a, b, \ldots\) are parallel. In this respect, their treatment is not quite complete, though it is quite sufficient to justify the continuity of the spectrum.

4 Digression: chiral gauge theories

In some cases, the lowest order effective hamiltonians are not so trivial as those in eqs.(16,23) and are rather funny. Nontrivial contributions arise for gauge theories with chiral matter content - the only case not analyzed in Witten’s paper [3]. In our works [8,9], the effective hamiltonians for chiral supersymmetric electrodynamics (to be anomaly free, the charges of different chiral multiplets should satisfy the condition \(\sum_f (Q_f)^3 = 0\)), the chiral supersymmetric \(SU(3)\) theory involving a right sextet and 7 left triplets, and the chiral \(SU(5)\) theory with a left quintet and a right decuplet has been built. We have found effective hamiltonians both for field theories in a finite volume (in the spirit of
Witten’s approach) and in the zero-volume (quantum mechanical) limit. We present here the supercharges and hamiltonian for quantum mechanics of chiral SQED with one chiral multiplet found in Ref. [8] (in the quantum mechanical limit, the problem of anomaly does not arise). They depend on the slow variables $A_i, \lambda$ and have the form

$$Q^\alpha_{\text{eff}} = -\frac{i}{\sqrt{2}} \lambda \gamma \left( \frac{\sigma_j}{\bar{\sigma}_i} A_j - i A_j \right) + \frac{\delta_{\alpha}^i}{2A},$$

$$H^\text{eff} = -\frac{1}{2} \left( \frac{\partial}{\partial A_j} - i A_j \right)^2 + \frac{1}{8A^2} + \frac{A_j}{8A^3} \bar{\lambda} \sigma_j \lambda,$$

where $\mathcal{A}$ is a vector function of the fields $\mathcal{A}$ coinciding with the Dirac monopole vector potential. The hamiltonian in (24) describes the motion of scalar (in the sectors $F = 0, 2$) and spinor (in the sector $F = 1$) particle in the field of a magnetic monopole placed at the point $\mathcal{A} = 0$ and also in the scalar potential $1/8A^2$. The spectrum of the hamiltonian in (24) is still continuous as, at large $A$, the potentials (both the scalar and the vector) and the monopole magnetic field $h_j = -A_j/2A^3$ vanish. (The continuity of spectrum depends only on two premises: \(i\) the existence of unbounded valleys and \(ii\) supersymmetry implying the zero ground state energy of the fast variables hamiltonian $H^0$. ) The Born-Oppenheimer parameter is $1/eA^3$ as earlier and, at small $A$, the corrections are large and the Born-Oppenheimer approximation is not applicable.

There is a gauge freedom in the phase choice of eigenfunctions of the hamiltonian in (24). This phenomenon is closely connected to the singularity of $H^\text{eff}$ at $\mathcal{A} = 0$ where the energy gap due to the fast variables excitations vanishes and is known in the literature as Berry’s phase [10].

5 Discussions and conclusions

There are two more questions to be discussed. The first one is the status of the Witten index approach to this problem. Our statement is that the Witten index is not a suitable tool for studying the supermembrane spectrum. The reason is that, for systems with the continuous spectrum, the concept of index, i.e. the number of unpaired zero-energy states, is poorly defined — the gap between the ground state and the excited states is absent here. Attempts to calculate Witten index in the systems not involving this gap may lead to rather puzzling results such as the fractional values for the index [11]. Our early calculation of the Witten index in the $SU(2)$ SYM quantum mechanics performed in Ref. [5] has an unclear status by the same reason.

Note in passing that the calculation of the index for supersymmetric nonchiral gauge theories performed in Ref. [3] with the method based on the periodic boundary conditions is also not well grounded. The reason for that is not the continuous spectrum (in the finite volume, the motion is finite), but the fact that there are regions in slow variables configuration space where the Born-Oppenheimer approximation is not applicable. This remark resolves the apparent contradiction between Witten’s result $I_W = \text{rank of the group} + 1$ and the calculation of index based on the instanton calculus for higher orthogonal and exceptional groups (see the detailed discussion in Refs. [6, 12]).
Finally, I would like to speculate on possible ways to cure the continuous spectrum illness. We have seen earlier that the basic reason for this is the instability of supermembrane which tends to smear out emitting “needles” of zero area. Seemingly, this instability could be suppressed if the action would include terms preventing the membrane from bending. Such an action has been built in case of strings in ref. [13], has been extended on the supersymmetric case in ref. [14] and is known as “rigid string” action. It involves the square of extrinsic curvature of the world sheet and hence high curvature configurations of rigid string are suppressed. So one can suggest the program: to built up the analog of Polyakov action for supermembranes and look at the mass spectrum of its mass operator. Presumably, it is discrete.

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