Self-focusing Hybrid Skyrmions in Spatially Varying Canted Ferromagnetic Systems

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Magnetic Skyrmions are quasiparticle configurations in a magnetic film that can act as information carrying bits for ultrasmall, all electronic nonvolatile memory. The skyrmions can be nucleated and driven by spin orbit torque from a current driven in a heavy metal underlayer. Along its gyrotropic path, a Magnus force can cause a skyrmion to be annihilated at the boundaries. By combining interfacial and bulk Dzyaloshinskii-Moriya interactions (DMI), for instance by using a B20 material on top of a heavy metal (HM) layer with high spin-orbit coupling, it is possible to engineer a hybrid skyrmion that will travel parallel to the racetrack with zero Magnus force. We show that by using a spatially varying interfacial DMI, a hybrid skyrmion will automatically self-focus onto such a track as its domain angle evolves along the path. Furthermore, using a gate driven voltage controlled magnetic anisotropy (VCMA), we can control the trajectory of the hybrid skyrmion and its eventual convergence path and lane selection in a racetrack geometry.

Magnetic skyrmions are localized spin textures, potentially much smaller than domain walls. Individual skyrmions can be driven like particles in clean magnetic films, making them interesting candidates for high density information storage such as racetrack memory. Skyrmions are typically nucleated as metastable states on the magnet’s energy landscape through a competition between exchange, anisotropy, DMI and stray field energies. In particular, the DMI energy comes from breaking bulk or interfacial inversion symmetry. The former, bulk DMI (bDMI), exists in chiral magnets such as B20 materials (MnSi, FeGe) and results in Bloch skyrmions with a ninety degree domain angle \( \psi \) (Fig. 1 inset), while the latter interfacial DMI (iDMI) prefers Néel skyrmions with zero domain angle. These two types of skyrmions are topologically equivalent, but their dynamical behavior are different. In particular, they tend to move orthogonal to each other due to a topologically generated Magnus force under the action of a spin orbit torque. Eliminating that Magnus force typically requires ferrimagnets at their angular momentum compensation point, or tracks with raised edges - operating thus at specific temperatures or along pre-set tracks. In a structure with both types of symmetry breaking however, we get hybrid skyrmions with a velocity aligned between Néel and Bloch. For a specific domain angle the Magnus force can cancel and allow the skyrmion to move linearly along the current path - but that requires a precise confluence of parameters setting the ratio of bulk and inversion asymmetry contributions.

In this paper, we show that hybrid skyrmions can be made to naturally self-focus along a racetrack with spatially varying DMI, for instance when it sits on a HM with varying thickness or composition (e.g. Pt\(_x\)W\(_{1-x}\)). Using analytical results and numerical simulations describing skyrmion movement, we show that the domain angle for a traveling skyrmion will keep changing with varying DMI until it reaches the cancellation point for the Magnus force. Furthermore, we can control the converging lane of the skyrmion through a voltage controlled magnetic anisotropy (VCMA) at its interface with a top oxide layer. We can thus gate control the skyrmion trajectory to resonate into specific lanes, while allowing it to diffuse along between resonances.

![Figure 1. Schematic view of a skyrmion in a ferro/ferrimagnetic material (orange). In a FM/HM hetero-junction, an applied current in a HM underlayer (blue) generates a spin Hall effect that separates opposite spins, resulting in spin injection and torque applied in the FM layer. For a given current, Néel and Bloch skyrmions with orthogonal domain wall angle \( \psi \) will move perpendicular to each other, so that a suitably engineered hybrid can move along the current direction.](image-url)
symmetry of the HM/FM stack. In our example, the polarization direction is $\hat{P} = \pm \hat{j} \times \hat{n}$, where $\hat{j}$ is the current direction and $\hat{n}$ is interface normal vector, assumed $\parallel \hat{z}$.

In a FM/HM structure (Fig. 1), the driving force from current induced SOT on a Néel skyrmion in the FM layer will be in the $\hat{z}$ direction, and for a Bloch skyrmion in the $-\hat{y}$ direction. The Magnus force for a Néel skyrmion will be in the $\hat{y}$ direction and Bloch skyrmion in the $\hat{x}$ direction. The net force from the SOT and Magnus effect generate perpendicular motion of Néel and Bloch skyrmions, with a hybrid skyrmion moving in between. We see this from the solution of the Thiele equation for the skyrmion velocity $v$ for a given domain angle $\psi$ (for details see supplementary)\[13\] [14]

$$F + G \times v - D_\alpha(\alpha v) + 4\pi B \theta_{SH} R(\psi) j_{hm} = 0$$ (1)

where $F$ includes unaccounted forces like gradients of effective fields or skyrmion-skyrmion interactions, $G = (0, 0, -4\pi N_{sk})$ is the gyrotropic vector, $N_{sk} = \pm 1$ is the skyrmion winding number, $D$ is the dissipation tensor (assumed isotropic with diagonals $D_{xx}$), $\alpha$ is Gilbert damping, $B = \pi n N_{sk} / 2\epsilon M_r \tau_{FM}$ is the SOT pre-factor, $\epsilon$ is the electron charge, $\tau_{FM}$ reduced Planck’s constant, $\gamma$ is the gyromagnetic ratio, $t_{FM}$ is the FM thickness, $M_r$ saturation magnetization, $I = \Delta I_d(\rho, \Delta)$ is the integral of the DMI energy, $\rho = R_{skm}/\Delta$ is the size of the skyrmion core $R_{skm}$ relative to the domain wall width $\Delta$. $R(\psi) = \begin{pmatrix} \cos \psi & \sin \psi \\ -\sin \psi & \cos \psi \end{pmatrix}$ is the 2D rotation matrix involving the domain angle $\psi$, $\theta_{SH}$ is spin hall angle, and $j_{hm}$ is the current density in the HM layer, assumed to be along the $x$ direction. Solving for $v$:

$$v = M^{-1} \left( F + 4\pi B \theta_{SH} R(\psi) j_{hm} \right), \quad M = \alpha D + G$$ (2)

where $G = 4\pi \begin{pmatrix} 0 & N_{sk} \\ -N_{sk} & 0 \end{pmatrix}$ is the gyrotropic tensor. From this equation, we can extract the skyrmion hall angle $\phi_{skm} = \tan^{-1}(v_y/v_x)$, and the critical domain angle $\psi_c$ where the Magnus force vanishes ($\phi_{skm} = 0$). At zero force $F = 0$, we get

$$\phi_{skm} = \tan^{-1} \left( \frac{G \cos \psi - \alpha D_{xx} \sin \psi}{G \sin \psi + \alpha D_{xx} \cos \psi} \right)$$ (3)

$$\psi_c = \tan^{-1} \left( \frac{G}{\alpha D_{xx}} \right) \text{ when } \phi_{skm} = 0$$ (4)

with $G = 4\pi N_{sk}$. The domain angle $\psi$, set by the ratio of bulk to interfacial DMI, determines if a skyrmion is Néel (Figs. 2, 3) ($\psi = 0, \pi$), Bloch ($\psi = \frac{\pi}{2}, \frac{3\pi}{2}$) or hybrid ($\psi = \pi, \frac{\pi}{2}, \frac{3\pi}{2}$). For a given $R_{skm}$ and $\Delta$, the velocities of Bloch and Néel skyrmions are perpendicular to each other ($v_{\text{block}} \cdot v_{\text{Néel}} = 0$), so that a proper hybrid skyrmion at domain angle $\psi_c$ moves along the current direction. However, reaching this domain wall angle requires a precise tuning of parameters.

Spatially varying iDMI for self-focusing skyrmions. Let us now consider a hybrid skyrmion moving in a FM layer with both bulk DMI (e.g. a B20 material like FeGe or MnSi) and a linearly varying interfacial DMI from HM spin-orbit coupling [15]

$$D_{int} = D_0 + \lambda y$$ (5)

The DMI energy density $\epsilon_{DMI} = D_{int} \cos \psi - D_{bulk} \sin \psi$. Solving $\partial \epsilon_{DMI} / \partial \psi = 0$, we can then get the evolution of $\psi$ as a function of $y$ from Eq.[5]

$$\psi(y) = \tan^{-1} \left( \frac{-D_{bulk}}{D_0 + \lambda y} \right)$$ (6)

meaning the skyrmions pick up increasingly more Néel characteristic during transit. The corresponding force from the linearly varying interfacial DMI [16]:

$$F_{DMI} = -\frac{\partial \epsilon_{DMI}}{\partial r} = 2\pi \lambda \frac{\gamma}{M_s} \Delta I_d \cos \psi \hat{y}$$ (7)

Using a 2$\pi$ domain wall model for the azimuthal angle $\theta(r)$ (distinct from domain wall angle $\psi$), $\theta(r) = 2 \tan^{-1}[\sin \rho / \sin(\rho / \Delta)]$, the DMI integral $I_d$ can be approximated as $I_d \approx 2\pi \rho \Delta$. Putting in $F_{DMI}$ in the Thiele equation, we get a correction to the critical angle: $\tan \psi_c = (1 + C) G / \alpha D_{xx}$ with $C = 4\lambda et_{FM} \alpha D_{xx} / \pi n N_{sk} G \theta_{SH} j_{hm}$. As Fig.2 shows, the convergent lane depends on the anisotropy parameter $K_u$, which in turn affects the skyrmion size $\rho$ and thereby the dissipation tensor $D_{xx}$ responsible for the critical domain
angle $\psi_c$. The dissipation integral $\mathcal{D}_{xx}$ is defined as [17]:

$$\mathcal{D}_{xx}(\rho) = \int dx dy \left( \partial_x \mathbf{m} \right)^2$$

$$= 2\pi \sinh^2 \rho \int_0^\infty \frac{[\cosh 2r + 1]r + [\cosh 2r - 1]/r}{(\sinh^2 r + \sin^2 \rho)^2} dr$$

in a 2D domain wall model. The integral above does not have an analytical solution to our knowledge. However, a simple fit that works quite well for $\rho \sim 0.5 - 4$ is $\mathcal{D}_{xx} \approx 5.96 \sqrt{4.285 + \rho^2}$, so that:

$$\psi_c \approx \tan^{-1}\left( \frac{G}{5.96 \alpha \sqrt{4.285 + \rho^2}} (1 + C) \right)$$

(9)

The anisotropy dependence of the skyrmion size $\rho$ can be seen from an evaluation of the energy integrals within a 2D model [18-19], and can be approximated as (with zero external magnetic field)

$$\rho(y) = \left( \frac{D(y)}{D_c} \right)^2 \frac{C_1}{\sqrt{1 - C_2(D(y)/D_c)^4}}$$

(10)

where $D_c = 4\sqrt{A_{ex}K_a}/\pi$, $C_1 \approx 12.7$, $C_2 \approx 1.06$ and $D(y) = D_{int}(y) \cos \psi - D_{bulk} \sin \psi$ is the total DMI. Since $D_{int}$ is changing with $y$ (Eq. [5]), $\rho$ is a function of $y$ for a given anisotropy (Fig. [2] inset). Solving $\psi_c = \tan^{-1}\left( \frac{-D_{bulk}}{D_0 + 2y} \right)$ for $y$, we get the convergence lane (Fig. [2]).

We perform micromagnetic (MuMax3 [20]) simulations to verify our assumptions and see skyrmion behavior in a linearly varying iDMI with different anisotropies (Fig. [3]). The parameters used in simulation are taken from [9-10 [21] 22] for FeGe, $A_{ex} = 7.5e-12$ J/m, $D_{bulk} = 1$ mJ/m$^2$, $\alpha = 0.2$, $K_a = 185$, 200 (assuming interfacial effects for ultra thin FeGe), $\theta_{SH} = 0.15$ and $J_{hm} = 10^{12}$ A/m$^2$. The FM layer thickness $t_{FM}$ is taken to be 2 nm. We apply a spin current of polarization $\hat{y}$ in the $\hat{z}$ direction, arising for instance from SOT in the HM layer from a charge current in the $\hat{x}$ direction. The Magnus force will cause the skyrmion to have a velocity component in the $-\hat{y}$ direction. As the skyrmion comes down the FM, the interfacial DMI decreases, which leads to an increasing domain wall angle $\psi$ (Eq. [6]), until it reaches the critical angle $\psi_c$ where it no longer has a velocity component along $\hat{y}$ and will have a rectilinear motion, self-focusing into a lane in the process.

Fig. [2] shows the tracks for a skyrmion injected at the low iDMI end, as obtained from a numerical simulation of the Landau-Lifschitz-Gilbert equation (which goes beyond the rigid skyrmion Thiele approximation), for various bulk anisotropy values $K_a$. The horizontal dashed lines show the convergent path coordinates $y$ obtained from analytical approximations [23]. The decrease in skyrmion size $\rho$ along the travel path $y$ means that smaller skyrmions (e.g. with higher anisotropy) travel further down before self-focusing.

Figure 3. Possible stacking options a) Using $Pt_x(Ir,Au,W)_{1-x}$ by varying $x$ value from left to right we can achieve a spatially varying interfacial DMI. b) By changing Pt thickness, or equivalently a varying thickness MgO underlayer between Pt and FM. The DMI from Pt increases for thicknesses of 1-3 nm and saturates thereafter, and can be used to get a spatially varying interfacial DMI. The top MgO-electrode stack sits on a select part of the race-track for applying a Voltage Controlled Magnetic Anisotropy (VCMA, Fig. [4]).

To get a linearly varying iDMI we suggest two possible approaches (Fig. [3]) - one is to use a varying thickness of HM layer, or more realistically of an MgO film between the HM layer and FM layer, to tune the iDMI. According to the experiments done in [23-25], by putting a MgO layer between HM and FM, the iDMI can be increased substantially and this effect increases with thickness of MgO until it saturates for a thickness of around 2 nm. A second approach is to use a non uniform composition of HM, e.g. $Pt_x(Ir,Au,W)_{1-x}$ [26-27]. It has been seen by using a composition of $Pt_x(Ir,Au,W)_{1-x}$ that the iDMI increases with increasing $x$ from 0 to 1. The main fabrication challenge is growing high quality B20 thin films on heavy metals and underlying the MgO.

VCMA for gate dependent lane selection. From Eq. [9] if we increase $\rho$, $\psi_c$ would decrease. One way to increase $\rho$ is by reducing the anisotropy $K_a$ and thus $D_c$ (Eq. [10]). We can gate control the skyrmion size through Voltage Controlled Magnetic Anisotropy (VCMA), in effect, voltage gating the Stark shift of the bond between the magnet’s $d_z^2$ orbital and the oxygen $p_z$ orbital of an oxide layer like MgO grown on the side of the sample opposite the HM layer (Fig. [3]). The equation describing the perpendicularly magnetic anisotropy variation with gate voltage in a VCMA set-up is

$$K_a(V) = K_a(0) - \xi V / t_{ox} t_{FM}$$

(11)

with $t_{ox}$ the oxide thickness and $\xi$ the VCMA coefficient. This means with a gate voltage, we can change the anisotropy and $D_c$, the size $\rho$ of the skyrmion, the dissipation tensor $\mathcal{D}_{xx}$ and ultimately the critical angle $\psi_c$ - making the skyrmion converge to a different lane. Assuming a thickness of 1 nm for oxide and FM layers, we would need a $\xi = 50$ fJ/V/m to get a $10^5$ J/m$^3$ change in anisotropy per 1V. $\xi$ of 50-100 fJ/Vm has been achieved for CoFeB of 1 nm thickness [23-29], whereas by doping FM/oxide interface $\xi$ larger than 100 fJ/Vm has been reported [30-31], suggesting that our device parameters are quite realizable for lane control. To get a higher separation of lanes, we can choose a smaller $\lambda$
term in Eq 5. This can be done by making the doping $x$ variation slower in Fig. 3a, or using Pt with a slower thickness variation, Fig. 3b.

Fig. 4 shows how skyrmions can be made to self-focus into a set of prefabricated racetracks using a VCMA gate [32]. Anisotropy in the fabricated lanes is $K_u = 190 \text{kJ/m}^3$, separated by regions with higher magnetic anisotropy $K_u = 220 \text{kJ/m}^3$. While the skyrmion domain angle evolves during transport along $y$, the VCMA gate changes the local anisotropy and alters the bending of the skyrmion track, attempting to set them into alignment with the racetracks. When the alignment is not perfect, the skyrmions instead enter the region between the lanes. The lane locations and relative anisotropies are designed so the skyrmions entering the regions in between the tracks see a Magnus force in the $-\hat{y}$ direction, as well as an attractive force towards the nearest racetrack. The net force will determine which track the non-resonant skyrmions finally converge to. Once they enter a lane, the repulsive force from the interstitials cancels out the Magnus force and the skyrmions stay in lane. As a result, we get a stepfunction-like quantized behaviour (Fig 4b) for the final $y$ coordinate of the convergent skyrmion lanes as a function of the VCMA engineered anisotropy, implying a robust well controlled scheme for directing skyrmions into the lanes. We emphasize that this ability to control lanes dynamically is unique to skyrmions with flexible domain angles, and is not achievable with domain walls.

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