Aeroacoustic Fields of Supersonic Twin Jets at the Ideally Expanded Condition

Yuta OZAWA,† Taku NONOMURA, Yuji SAITO, and Keisuke ASAI

Department of Aerospace Engineering, Tohoku University, Sendai, Miyagi 980–8579, Japan

The aeroacoustic fields of twin jets and the equivalent single jet were experimentally investigated using particle image velocimetry (PIV), schlieren visualization, and acoustic measurement. The present study focuses on the aeroacoustic fields of the twin jets, and the effect of the interaction between each jet was investigated using various nozzle spacing. The PIV results indicated that strong interaction causes elliptical jet growth on a cross-stream plane and a decrease in the Reynolds stress of the inner shear layer on a plane containing both jet axes. The dynamic mode decomposition of the double-pulsed schlieren images extracted the interaction of each jet, which relates to the Mach wave generation. The noise of the twin jets was basically quieter than the noise of an equivalent single jet because of a shielding effect and a reduction in the Reynolds stress resulting in a decrease in the overall sound pressure level.

Key Words: Twin Jets, Supersonic Jet, Aeroacoustics

Nomenclature

- $A$: cross-sectional area
- $D$: nozzle exit diameter
- $f$: frequency
- $L_{sh}$: shock-cell length
- $L_{vf}$: wave length of velocity fluctuation
- $M$: Mach number
- $r$: cylindrical coordinate
- $r_0$: jet half-width
- $Re$: Reynolds number based on the jet velocity and the nozzle diameter
- $s$: nozzle spacing
- $St$: Strouhal number
- $U, u$: axial velocity
- $u_c$: convective velocity
- $x, y, z$: Cartesian coordinates
- $\delta$: shear layer thickness
- $\gamma$: specific heat ratio
- $\phi$: azimuthal angle
- $\theta$: streamwise angle

Subscripts

- $d$: design condition
- $e$: equivalent single jet condition
- $j$: fully expanded condition
- $\infty$: ambient condition

1. Introduction

A supersonic jet emitted from a rocket engine produces strong acoustic waves and the strong vibration can cause an artificial satellite to malfunction. Therefore, predicting and reducing supersonic jet noise are significantly important for aerospace engineering, and the supersonic jet noise of a single jet has been investigated by many researchers.1–5 Recent rockets often have a propulsion system that consists of multiple small rocket engines in contrast to conventional rockets that used a single powerful engine. This change in the design of propulsion systems has led to the mass production of rocket engines and a reduction in the manufacturing cost. Moreover, multiple rocket engine systems improve the reliability of space missions because a rocket can continue the mission using other normal engines if one engine stops. Therefore, rocket designs using multiple engines may become the mainstream for future rocket propulsion systems. Although rockets are recently equipped with multiple rocket engines, the aeroacoustic fields of multiple supersonic jets have rarely been investigated.

Several previous studies have investigated the aeroacoustic fields of multiple supersonic jets. The acoustic field of linear-arrayed multiple supersonic jets has been investigated by Raman and Taghavi6 and Miles.7 They showed that noise is reduced due to the rapid mixing of the jets when the screech instability of each jet is synchronized. Umeda and Ishii8,9 investigated the oscillation modes of screeching multiple jets and identified the different oscillation modes that develop as the nozzle pressure ratio (NPR) increases. Recently, Coltrin et al.10,11 experimentally investigated the aeroacoustic fields of $8 \times 8$ arrays of supersonic jets. Their results show that the interaction of shock waves strongly correlates with the sound pressure level. Accordingly, these studies only focus on the imperfectly expanded jets that contain the shock waves. However, the fundamental understanding of jet interaction should be investigated using a simplified model such as twin jets under the ideally expanded condition.

The aeroacoustic fields of twin jets have been investigated by many researchers under both subsonic and supersonic conditions,12 and there are two distinct effects of the twin...
jets interaction: strong coupling of the screech instability modes and jet shielding. The effects of twin jet couple modes were first identified by Seiner, Manning, and Ponton.\textsuperscript{15,16} Tam and Seiner\textsuperscript{17} explained the mechanism of synchronized resonant oscillations using the vortex sheet model, but were unable to estimate the peak frequency quantitatively. Therefore, many efforts are being dedicated to this field to comprehend the mechanism of the resonance and to suppress the screech intensity.\textsuperscript{15–18} Their results indicated that closely spaced twin jets can couple regardless of the screech radiation while the particular coupling mode dominates the aeroacoustic fields of the twin jets when screech appears. This particular coupling mode strongly depends on the operating conditions, nozzle spacing, and nozzle geometry.\textsuperscript{19,20} Several recent studies have also investigated the aeroacoustic fields of twin jets under imperfectly expanded conditions,\textsuperscript{21–23} and they provided further details regarding the twin jets noise and coupling modes.

The shielding effect is a reduction in the sound pressure level due to the total reflection of the noise generated from one jet at the shear layer of the other jet. The noise reduction using twin jets was first observed in a plane through two nozzles by Greatrex and Brown,\textsuperscript{24} Bhat\textsuperscript{25} and Kantola\textsuperscript{26} showed that the sound pressure level of twin jets in a plane through the jet center axis is lower than that of a single jet. Consequently, the shielding effect of twin jets consists of several phenomena: reflection, refraction, diffraction, and scattering.\textsuperscript{27–29} Bozak and Henderson\textsuperscript{30} revisited the shielding effect and showed a noise reduction of 3 dB at the maximum. Bozak\textsuperscript{31} also developed an empirical model of twin jets noise based on far-field acoustic data. Although the mechanism of the noise reduction was not taken into account, the model can predict the sound pressure within an error of approximately 0.5 dB. Pineau and Bogey\textsuperscript{32} computationally investigated the shielding effect of strongly heated supersonic twin jets, and observed a maximum noise reduction of 2 dB.

The discussions of the previous studies indicate that the aeroacoustic fields of twin supersonic jets have been investigated applying limited conditions for the objectives of understanding the coupling mode or shielding effect. Many of the studies conducted in the 1980s and the aeroacoustic fields of twin jets have been discussed without considering the dynamics of the fundamental jet interaction. Although recent studies of twin jets provide quantitative data, they only discuss imperfectly expanded twin jets because the acoustic vibration is significant when the jets are operated under imperfect conditions. However, the Mach wave is the dominant noise for the supersonic jet and causes significant vibration during rocket launch. Therefore, a fundamental understanding of jet interaction under the ideally expanded conditions, where it is free from shock-associated noise, should be obtained. To investigate the interaction between each jet, and to obtain fundamental knowledge of the jet interaction, a simplified model of twin jets under the ideally expanded conditions can be used. This will contribute to the construction of an accurate acoustic prediction model for multiple supersonic jets. The present study utilizes twin jets with different nozzle spacing for the experiment. The aeroacoustic fields of the twin jets were investigated using particle image velocimetry (PIV), schlieren visualization, and far-field acoustic measurements.

2. Configuration of the Twin Supersonic Jets

Figure 1 shows the cross-sectional geometries of the convergent-divergent (CD) nozzle, of which the downstream part from the throat was designed using the method of characteristics of compressible flow. The design Mach number $M_d$, which is determined by the ratio of the diameter between the nozzle throat and the exit, was set to 2.0. An ideally expanded supersonic jet of $M = 2.0$ was reproduced at a NPR of 7.8, corresponding to a total pressure of 790 kPa. The jet facility can manually control the total pressure within an accuracy of ±5 kPa. The mass flow rate of a single jet and twin jets is supposed to be constant in the present study because the thrust power of the rocket remains constant. Therefore, the cross-sectional area of each nozzle for the twin jets is half that of the equivalent single jet. The cross-sectional area, diameter at the nozzle exit, and Reynolds number based on the diameter at the nozzle exit are summarized in Table 1. Although the low-Reynolds number jet exhibits the laminar-to-turbulent transition resulting in amplification of the acoustic waves,\textsuperscript{33,34} the Reynolds number effect can be negligible because the jets in the present study have already been turbulent.

The equivalent single nozzle was made of resin manufactured using stereolithography. The nozzles of the twin jets were made of stainless steel manufactured using an electrical-discharge machine, and they are reusable when reproducing twin jets with different nozzle spacing. The outer diameter of the twin jets nozzles was set to 9 mm, which is the minimum diameter that can be fabricated.

Figure 2 shows the definition of the coordinate for the twin jets. The nozzle spacing of the twin jets configuration

| Exit area [mm²] | $A$ | 78.5 | 39.3 |
|----------------|-----|------|------|
| Exit diameter [mm] | $D$ | 10.0 | 7.07 |
| Throat diameter [mm] | $D_t$ | 7.69 | 5.45 |
| Reynolds number | $Re$ | $7.0 \times 10^5$ | $1.0 \times 10^6$ |

Fig. 1. Cross-sectional geometry of the axisymmetric nozzle.
is represented by \( s \), and the present study investigates the aeroacoustic fields of twin jets with a range of \( s/D = 1.55, 2, 3, 4, \) and 5. The origin of the coordinate was the center point of the two nozzles in the present study. The \( x \) axis was set to be the streamwise direction and the \( y \) axis was set to be normal to the \( x \) axis and corresponds to the plane through the two nozzles. The \( z \) axis was normal to the \( x \) and \( y \) axes. The streamwise angle \( \theta \) was defined as the angle between the \( x \) axis and the \( y \) axis, and the downstream side was set to be 0 deg. The azimuthal angle \( \phi \) was defined as the angle between the \( y \) axis and the \( z \) axis, and 0 deg was set to be the plane through the two nozzles.

### 3. Experimental Setup

#### 3.1. PIV measurement

The equivalent single jet and the twin jets were realized by injecting high-pressure air into a jet-generating system installed in the anechoic room at Tohoku University. The schematic images of the jet-generating system and the anechoic room are shown in Fig. 3(a). The jet-generating system consists of a pump, a high-pressure air tank, a plenum chamber, and the nozzle for a supersonic jet. The study by Ozawa et al.\(^{33}\) describes more details of the jet-generating system and the performance of the anechoic room.

The conventional planar PIV was performed in the present study. The two seeding devices were employed for PIV measurement and they filled both the supersonic jet and the ambient air with seeding particles. An aqueous 50% glycerin solution was used for the tracer particles. The particle size was approximately several micrometers. A double-pulsed laser system (LDY-300PIV, Litron Lasers) and a high-speed camera (Phantom V611, Vision Research) were utilized for high-speed imaging. The frame-straddling technique with a short time interval of 1.2 \( \mu \)s was applied to the PIV system because the local velocity at the potential core of the jet is approximately 510 m/s. A Nikon lens (Nikkor 80-200 mm f/2.8) was used for the optical system and an image frame of 150 \( \times \) 70 mm was visualized using an image sensor having a size of 1,280 \( \times \) 600 pixels. The schematic image of the field-of-view (FOV) is shown in Fig. 3(b), (c). The present study visualizes the twin jets using two types of FOV and investigates the shear layer development of a three-dimensional space. A total of 20,000 image pairs were acquired with a sampling rate of 1,000 Hz for each case.

The acquired particle images were analyzed using two algorithms. One is the conventional spatial correlation method, which can estimate instantaneous velocity fields. This was calculated using commercial software for PIV (Dynamic Studio 6.7, Dantec Dynamics). The multi-grid approach called Adaptive-PIV was also used and the maximum and minimum sizes of the interrogation areas were set to be 32 \( \times \) 32 and 8 \( \times \) 8 pixels, respectively. After the vector calculation, a moving average filter of 3 \( \times \) 3 was applied for error vector correction. The standard deviation and Reynolds stress were calculated using the instantaneous velocity fields.

The other algorithm is the single-pixel ensemble correlation method, which can calculate the temporal averaged velocity field with using high spatial resolution instead of the instantaneous velocity fields.\(^{35}\) This method can effectively calculate the temporal averaged velocity fields of a supersonic jet with high spatial resolution.\(^{33,36}\) The papers of Westerweel et al. and Ozawa et al.\(^{33,35,36}\) provide more details on single-pixel ensemble correlation.

#### 3.2. Schlieren visualization

Double-pulsed schlieren visualization, which can visualize the density gradient of the flow field, was performed. A standard Z-type schlieren system was used for the visualization. We used a pulsed light-emitting-diode (LED) light system (LE CG P3A 01-6V6W-1, OSRAM) as the point light source.\(^{36-38}\) Parabolic mirrors having a diameter of 200 mm and a convex lens with a focal length of 300 mm were used for the optical system. A knife-edge was placed at the focal position of the parabolic mirror and the density gradient in the streamwise direction was emphasized. The same high-speed camera as that used for PIV measurement was used for schlieren imaging. The FOV was approximately 200 \( \times \) 160 mm using an image sensor having a size of 1,280 \( \times \) 800 pixels. The exposure time was approximately
400 ns, and the frame-straddling technique was applied to schlieren images with a short time interval of 2 μs. The acquired double-pulsed schlieren images can follow the displacement of the visualized turbulent structures so that the exact dynamic mode decomposition (DMD) analysis, which provides information about the dynamics of the flow fields,30 can be applied.

3.3. Acoustic measurement

Far-field acoustic measurements of the equivalent single jet and twin jets were performed in the anechoic room using microphones. The nozzles and plenum chamber were covered by glass wool panels to prevent the reflection of acoustic waves. Figure 4 depicts the observer positions for far-field acoustic measurements. The distribution of the far-field sound pressure level was acquired in the range of 30 ≤ θ ≤ 100 and 0 ≤ φ ≤ 90. Note that the effect of azimuthal angle φ does not appear in the case of the equivalent single jet. The spatial resolution of the acoustic measurements is 10 deg for both angles, which is sufficiently high for the discussion on the directivity of acoustic waves. The acoustic signal was recorded for 3 s at each position using eight 1-inch condenser microphones (TYPE4158N, ACO). The signals of microphones were amplified using two power supplies (TYPE5006/4, ACO) and recorded using a computer through two DAQ analyzers (USB-6363, National Instruments) and two shielded connector blocks (BNC-2120, National Instruments). The microphones were calibrated to 1,000 Hz for 94 dB, which corresponds to the fluctuation amplitude of 1 Pa, using an acoustic calibrator (TYPE2127, ACO). The sampling frequency was set to 192 kHz, which is sufficiently high compared to the frequency of the supersonic jet noise, and the aliasing effect was minimized. Frequency analysis using fast Fourier transformation (FFT) was applied for the measured data. The FFT analysis was conducted using a data length of 1,024 points with 50% overlapping. The acoustic spectra were calculated as the 1,000 times averaging of the FFT results. The Strouhal number St is utilized as the nondimensionalized frequency as follows:

\[ St = f D_e / U_j. \] (1)

Here, \( f \) and \( U_j \) are the frequency and theoretical axial velocity of the supersonic jet calculated using the isentropic flow relation (\( U_j = 510 \text{ m/s} \)). Note that the Strouhal number was calculated based on the diameter of the equivalent single jet regardless of the twin jets or the single jet. The overall sound pressure level (OASPL) is calculated using the frequency range of 0.02 ≤ \( f \) ≤ 0.50 for the present study.

4. Results and Discussion

4.1. Temporal averaged velocity fields

Figure 5 shows the distribution of the temporal averaged velocity fields of twin jets on the xy plane calculated using single-pixel ensemble correlation. The velocity field of the single jet is also shown in this figure. The spatial resolution of the velocity fields is approximately 130 μm. The temporal averaged velocity fields clearly show the shear layer and potential core of each jet. The position where the interaction of the velocity fields occurs moves downstream as the nozzle spacing \( s/D \) increases. The streamwise distributions of the axial velocity on the symmetry line (\( y = 0 \)) and the jet centerline (\( y = 0.5s \)) of one jet in the twin jets are shown in Fig. 6. The distributions of the axial velocity on the jet centerline are not affected by the nozzle spacing \( s \), and they agree well with that of the single jet. The velocity fluctuations observed in the potential core seem to be due to the shock-cell structures. The reason why the shock-cell struc-

![Fig. 4. Location of the microphone for far-field acoustic measurement.](image)

![Fig. 5. Temporal averaged velocity fields of twin jets on xy plane.](image)

![Fig. 6. Streamwise distribution of axial velocity on the center line and the symmetry line.](image)
ture appears even under the ideally expanded condition seems to be due to the change in the actual nozzle diameter because the boundary layer inside the nozzle makes the diameter of the nozzle exit smaller. The distributions of the axial velocity on the symmetry line ($y = 0.5s$) are considered to represent the interaction of the velocity fields between each jet. The streamwise position where the axial velocity on the symmetry line increases indicates the position that the interaction begins. This moves downstream by increasing the nozzle spacing, as expected. In addition, the velocity gradient on the symmetry line becomes steep with decreasing the nozzle spacing. Therefore, the small nozzle spacing seems to lead to a stronger interaction in the velocity fields.

The shear layer thickness $\delta$ and the jet half-width $r_{0.5}$ on the $xy$ and $xz$ planes were calculated, and the spatial development of twin jets was quantitatively investigated. The half-width $r_{0.5}$ is the radial position where the axial velocity becomes half of that on the jet centerline. These were calculated using the definition proposed by Troutt and McLaughlin, as shown in Eqs. (2) and (3):

$$\tilde{u}/U(\eta) = \begin{cases} \exp[-2.773(\eta + 0.5)^2] & \text{for } \eta > -0.5 \\ 1 & \text{for } \eta \leq -0.5 \end{cases} \quad (2)$$

$$\eta = \frac{r - r_{0.5}}{\delta}. \quad (3)$$

Here, $\tilde{u}$, $U$, and $r$ are the temporally averaged velocity, jet centerline velocity at the given streamwise position, and radial position, respectively. The shear layer thickness was calculated using curve fitting to the measured velocity profile. The half-Gaussian fitting of the velocity profile is validated in Appendix A. The shear layer of the twin jets on the $xy$ plane can be divided into two types of shear layers because one side of the twin jets is affected by another jet. Figure 3(b) shows the definition of the shear layer in the twin-jets configuration on the $xy$ plane. The outer shear layer is supposed to be on the opposite side of the symmetry line and the inner shear layer is supposed to be on the symmetry line side in the present study.

First, the outer shear layer of the $xy$ plane and the shear layer of the $xz$ plane were calculated as shown in Fig. 7, and the uniformity of the spatial development on the $xz$ plane of twin jets is investigated. The shear-layer-growth rate and the jet half-width do not have a difference between the $xy$ and $xz$ planes, except for the case of $s/D = 1.55$. This indicates that each jet of the twin jets uniformly develops in the $xz$ plane for the case of $s/D \geq 2$. Therefore, the effect of the interaction on the spatial development of the twin jets is relatively small in those cases. The shear-layer-growth rate on both planes in the case of $s/D \geq 2$ asymptotically approaches that of a single jet as nozzle spacing increases. In addition, the shear-layer-growth rate and jet half-width for both planes agree well with those of a single jet in the case of $s/D = 5$. Therefore, the interaction of each jet appears to be sufficiently small in the case of $s/D = 5$. On the other hand, the jet half-width of $s/D = 1.55$ shows a large difference between the $xy$ and $xz$ planes, while the shear-layer-growth rate on both planes is not different. The jet half-width on the $xy$ plane is smaller than that on the $xz$ plane. This small half-width indicates that the growth direction of the shear layer is not only towards the outside, but also towards the potential core side. Therefore, these results imply that the twin jets with $s/D = 1.55$ elliptically spreads downstream because the narrow nozzle spacing leads to strong jet interaction.

Figure 8 shows the outer and inner shear layer thicknesses on the $xy$ plane. Note that the inner shear layers are only shown in the region where the inner shear layer fits well with
the half-Gaussian profile of Eqs. (2) and (3). Therefore, they are only plotted up to the streamwise position where the two jets merge completely. The shear-layer-growth rate of the inner shear layer asymptotically approaches that of the outer shear layer as the nozzle spacing increases in the case of $s/D \geq 3$. This trend implies that the inner-shear-layer-growth rate may be affected by the acoustic waves or the interaction of velocity fields in the $xy$ plane. The physical mechanism of the acoustic excitation of the instability waves in the shear layer was investigated by Tam.\textsuperscript{41,42} When considering the two-dimensional free shear layer and incident acoustic waves, there are two necessary conditions to interact with each other. One is the matching of the instability wave and acoustic wave frequencies, the other is the matching of their wave numbers.\textsuperscript{42} Here, the instability waves and acoustic waves generated from one jet of the twin jets are the same as those of the other jet because each jet is operated under the same conditions. The incident acoustic waves to one jet have the same frequencies and wave numbers as those of the instability waves in the shear layer. Therefore, the interaction between the instability waves and incident acoustic waves excites the instability waves and makes the growth rate large. This effect is considered to decrease as the nozzle spacing increases because the amplitude of the incident acoustic waves decreases due to distance attenuation. Consequently, the twin jets with a nozzle spacing of $s/D = 5$ do not have any interaction in the temporal averaged velocity fields.

### 4.2. Turbulent statistics of the velocity fields

Figures 9 and 10 show the lateral distributions of the standard deviation and the Reynolds stress in the $xy$ plane at each streamwise position. These turbulent statistics of the velocity fields are calculated using the conventional spatial correlation method. Note that the spatial resolution of the conventional spatial correlation method was 1/8 of that of the single-pixel ensemble correlation method. In the present study, we assume that the velocity calculation may have an error in the region where the shear layer thickness $\delta$ is less than four times the minimum spatial resolution of the conventional spatial correlation method. Four times of the minimum spatial resolution was approximately 4 mm, and the streamwise position where the shear layer thickness is larger than 4 mm is $x/D \approx 6$. Therefore, the velocity profiles of $x/D \leq 6$ are not shown in these figures. The ordinates of these figures are set to $(y - 0.5s)/D$, where the centerline of one of the twin jets is zero. Therefore, the positive and negative values of $(y - 0.5s)/D$ represent the outer and inner sides of the twin jets, respectively.

The standard deviation and the Reynolds stress show symmetric distributions with respect to the centerline of one jet in the cases of $s/D = 4$ and 5. This is reasonable, as the interaction of each jet does not appear in the temporal averaged velocity fields. On the other hand, the effect of the nozzle spacing appears in the case of $s/D \leq 3$. These symmetry distributions start to break as the nozzle spacing decreases and the peak values of the standard deviation and Reynolds stress decrease in the inner shear layer. As the nozzle spacing decreases, the position of the peak in the inner shear layer moves towards the centerline of the jet. While the effect of nozzle spacing on the outer shear layer is relatively smaller than that on the inner shear layer, the peak height of the standard deviation and the Reynolds stress in the outer shear layer increase as the nozzle spacing decreases. These results of the narrow nozzle spacing indicate that the turbulent mixing is suppressed due to relaxing the inner shear layer velocity gradient, whereas the turbulent mixing is enhanced in the outside shear layer.

### 4.3. Density gradient fields

Figure 11 shows the instantaneous schlieren images and temporal averaged schlieren images of $s/D = 1.55$ and 5. The latter discussion of the schlieren images focuses on the cases of $s/D = 1.55$ and 5 because the flow field monotonically changes as the nozzle spacing increases. The instantaneous schlieren images clearly visualize the density gradient of the flow and acoustic fields of the twin jets. The Mach waves, which are emitted towards the downstream side, are observed on both the outer and inner sides. Since strong
acoustic waves might excite turbulence in the inner shear layer, the large growth rate of the inner shear layer shown in Fig. 8 appears to be due to the Mach wave emission. The averaged schlieren images in Fig. 11(b) show the shock-cell structures inside the potential core of each jet. Although the shock-cell structure does not clearly appear in the instantaneous images, periodic shock waves and expansion waves are observed in the averaged images. This indicates that the strength of the shock waves is not significantly large. The presence of the shock-cell structure was also observed in the axial velocity fluctuation at the jet centerline, as shown in Fig. 6. The wavelength of the velocity fluctuation and the shock-cell length are calculated from the PIV data and schlieren images in the case of s/D = 1.55. The wavelength of the velocity fluctuation Lvf is defined as the distance between the streamwise positions where the velocity gradient reaches the local minimum. The shock-cell length Lsh is defined as the distance between the streamwise position where the pixel intensity of the schlieren image on the center axis of one jet reaches the local maximum. Figure 12 shows a comparison of Lsh and Lvf at each streamwise position. The wavelength of the velocity fluctuation and the shock-cell length basically agree well with each other regardless of the streamwise position. Therefore, the present results of PIV and schlieren visualization are consistent.

The exact-DMD analysis was applied to the double-pulsed schlieren image pairs, and the coherent structures that generate the acoustic waves were estimated at each frequency. The 2,500 pairs of schlieren images were subjected to DMD analysis and a detailed explanation of this analysis technique is illustrated in Appendix B.

Figures 13 and 14 summarize the results of the DMD analysis. These figures consist of the power spectrum, eigenvalues, and the spatial distribution of DMD modes. The power spectra of these figures were computed from the DMD-mode amplitudes of Eq. (12). The plotted eigenvalues also show the unit circle and amplification of each mode. The eigenvalues of all of the modes are less than unity regardless of the nozzle spacing and all of the DMD modes obtained are decaying modes.

Figures 13 and 14 show that two types of peaks are observed in the power spectra of DMD modes for both cases. One is a type of high-frequency peak (0.25 \( \lesssim S_{DMD} \lesssim 0.35 \)) with the largest amplitude. Here, the frequency of the DMD modes f is nondimensionalized by D and \( U_\infty \) as the Strouhal number of \( S_{DMD} = fD/U_\infty \). The reason why the DMD modes calculated from the schlieren images have large amplitudes on the high-frequency side seems to be due to the characteristics of the schlieren image. The schlieren image visualizes the density gradient fields, not the original density fields. The derivative emphasizes the variations with a larger wave number, which corresponds to the high-frequency fluctuation. Therefore, DMD analysis of the schlieren image tends to extract the DMD modes on the high-frequency side with large amplitude.

The DMD modes on the high-frequency side are labeled H1 and H2 as depicted in these figures. Especially, the DMD amplitude of H1 and H2 of s/D = 1.55 is significantly high compared to that of s/D = 5. The other is DMD modes labeled as L1 and L2, which relate to the Mach wave generation. The criteria for selecting the L1 and L2 modes are frequency and the spatial distribution of the DMD modes. Previous studies of the supersonic jet noise indicate that the Mach wave has a broadband peak in the frequency domain.
with a peak Strouhal number of 0.1 ~ 0.2.2,3 Therefore, the DMD modes at $S_{f,DMD}$ = 0.1 ~ 0.2 with the spatial distribution of the Mach wave pattern were selected. The spatial distributions of DMD modes consist of three figures: the real part, the amplitude, and the phase, respectively. The real part and phase are considered to represent an instantaneous acoustic propagation pattern, and the direction in which the phase increases corresponds to the propagation pattern of the acoustic waves. The amplitude is considered to represent the sound pressure level.

Discussion of the spatial distributions of DMD modes first focuses on the case of $s/D = 1.55$, as shown in Fig. 13. The H1 and H2 modes show a distinct pattern of the aeroacoustic fields in the case of $s/D = 1.55$. The asymmetric patterns with respect to the symmetry line ($y = 0$) are observed in the H1 and H2 modes. These modes appear to be a helical mode of the supersonic jet because the helical mode is the most unstable in the supersonic jet of $M = 2.0$. Note that the schlieren images integrate the density gradient fields along with the optical path so that the obtained results cannot directly indicate the helical structure. The interaction of the helical instabilities asymmetrically occurs with respect to the symmetry line ($y = 0$), and its amplitude reaches near the end of the potential core.

On the other hand, the L1 and L2 modes show the symmetric patterns with respect to the symmetry line, and the area with the large amplitude mainly spreads on the downstream side ($x/D \geq 12$). This indicates that the large-scale coherent structures are formed on the downstream side. In addition, the L2 mode shows strong interaction between each jet on the downstream side with a large amplitude. This implies that the large-scale coherent structures that generate lower-frequency noise are formed due to the pairing of the large eddies in each jet. Therefore, the Mach waves of $s/D = 1.55$ are generated from not only the upstream outer shear layers, but also the downstream coherent structures. Here, the Mach waves from the upstream outer shear layer are considered to be dominant because the natural growth of the instability wave contributes to modulation of the instability wave that is important for generating the Mach wave. On the other hand, the large-scale coherent structure only forms on the downstream side and convects as its speed decreases. Therefore, the Mach waves generated from the coherent structures are considered to hardly affect noise amplification and the frequency is low.

The H1 and H2 modes of $s/D = 5$ show an asymmetric pattern with respect to the symmetry line as in the case of $s/D = 1.55$. While the amplitude of the asymmetric pattern is biased for each jet, the phase distribution clearly indicates the interaction of twin jets. Therefore, interaction at a high frequency is asymmetric regardless of the nozzle spacing, and the interaction appears to become weak as the nozzle spacing increases. The L1 and L2 modes of $s/D = 5$ show symmetric patterns with respect to each centerline of the jets, and the area with large amplitude mainly spreads on the downstream side, as in the case of $s/D = 1.55$. However, the phase distribution does not show a distinct pattern of interaction between each jet. This indicates that the interaction of each jet is weak and the Mach wave generation is isolated. Therefore, the low-frequency aeroacoustic interaction occurs only in the case of narrow nozzle spacing.

4.4. Far-fields acoustics

The acoustic power of two incoherent jets with the same diameter is approximately 3 dB ($\approx 10 \log_{10} 2$ dB) higher than that of a single jet with the same diameter because the acoustic power becomes double. Therefore, “the noise level of a single jet with the same diameter $+3$ dB” can be a reference of the acoustic power emitted from the twin jets. However, in the present study, both twin-jets and equivalent jet noises at $r = 100D_e = 100\sqrt{2}D$ leads to a distance attenuation for the acoustic power of approximately $-3$ dB ($\approx 20 \log_{10} 1/\sqrt{2}$ dB) of the jet from the nozzle with a small diameter, which is used for the twin-jet configuration when assuming the point noise source. Since the Reynolds number effect on the aeroacoustic fields can be negligible in the present study, the noise intensity of the equivalent single jet ($D_e = 10$ mm) measured at $r = 100D_e$ is equivalent to that of the downscaled single jet ($D = 7.07$ mm) measured at $r = 100D$. Therefore, the difference in the noise intensity between the twin jets and the equivalent single jet ($\Delta$OASPL) can simply be calculated as the direct subtraction of the twin jets’ noise from the equivalent single jet noise.

The difference in the total acoustic power was calculated using the spherical surface integration of $\Delta$OASPL distribution, as shown in Fig. 15. Table 2 summarizes the noise difference per unit surface in each case. The twin jets’ noise is...
basically quieter than the equivalent single jet noise, and the noise intensity decreases as the nozzle spacing decreases. This implies that the strong interaction of each jet leads to noise reduction rather than noise amplification when considering the total acoustic power.

Figure 15 shows the distribution of $\Delta$OASPL in each nozzle spacing. Significant noise reduction is observed at $0^\circ \leq \phi \leq 40^\circ$ and $30^\circ \leq \theta \leq 50^\circ$. This noise reduction is considered to be due to the shielding effect since noise reduction is only observed in the shadow zone where one jet is covered by the other. The reason why the shielding effect is remarkable at the downstream appears to be mainly due to the total reflection of the acoustic waves when the incident angle of the acoustic waves to the shear layer decreases towards the downstream side. The other phenomena that can cause a shielding effect are the scattering and the refraction of the acoustic waves. However, scattering of the acoustic waves should be negligible because it is only remarkable when the wavelength of the acoustic waves is smaller than the turbulent scale, and this is not the case in the present relatively low-Mach-number jet condition. The effect of refraction on noise reduction is also minor because the refraction slightly shifts the effective noise source position for the observer. Moreover, the region where the shielding effect is remarkable decreases as the nozzle spacing increases. The shadow zone becomes smaller as the nozzle spacing increases if the source jet is supposed to be a point source. Therefore, this is due to changing the size of the shadow zone.

On the other hand, Fig. 15 shows that the noise amplification at $50^\circ \leq \phi \leq 90^\circ$ is significant in the cases of $s/D \geq 3$, and noise amplification is suppressed by decreasing the nozzle spacing. The noise sources at $\phi = 90^\circ$ are considered to be the inner shear layer on the $xz$ plane and the outer shear layer on the $xy$ plane of each jet. Although the PIV results of the $xz$ plane are not shown in this article, the distribution of the Reynolds stress on the $xz$ plane does not significantly change regardless of the nozzle spacing. On the other hand, the Reynolds stress in the inner shear layer of the $xy$ plane decreases as the nozzle spacing decreases, as shown in Fig. 10. Therefore, reducing the Reynolds stress in the inner shear layer is considered to suppress the noise amplification at $\phi = 90^\circ$.

Figures 16 and 17 show the OASPL distributions of the equivalent single jet and the twin jets at $\phi = 0^\circ$ and $90^\circ$. Note that the OASPL of the equivalent single jet does not have any azimuthal distribution because of the axisymmetric jet. Regardless of the azimuthal angle $\phi$, the OASPL shows a large peak at $\theta \approx 30^\circ$ in all of the cases because of the directivity of the Mach waves. The peak of the Mach wave at this operating condition of the jets has been observed at $\theta = 30^\circ \sim 40^\circ$ in previous studies, so a peak at $\theta \leq 30^\circ$ will not appear in the present study. The effect of the nozzle spacing on the OASPL does not appear at $\phi = 0^\circ$ because one jet behind the other cannot be seen from the observer. Therefore, when the observer position is $\phi = 0^\circ$, one jet in the twin-jet schema seems to be downscaled when compared to the equivalent single jet, resulting in a decrease in OASPL. On the other hand, Fig. 17 shows the effect of the nozzle spacing on the OASPL distributions at $\phi = 90^\circ$. The peak height of the Mach wave decreases as the nozzle spacing decreases. This reduction in the Mach wave emission also seems to be due to the decrease in the Reynolds stress at the inner shear layer.

Figures 18 and 19 show the power spectral density (PSD) of the sound pressure level at $(\theta, \phi) = (30^\circ, 0^\circ)$ and $(30^\circ, 90^\circ)$. Note that the Strouhal number of all of the cases is calculated using the diameter of the equivalent single jet ($D_e = 10 \text{ mm}$). In addition, the frequencies of the DMD modes are also shown in the figures. The PSDs of the twin jets appear to basically shift towards the high-frequency side because the diameter of each nozzle is downscaled. The peak frequency of the Mach wave was determined considering the largest PSD level within the frequency range of $St = 0.1 \sim 0.2$; that is the peak frequency of the Mach waves observed in previous studies. The peak frequencies of the Mach waves at $(\theta, \phi) = (30^\circ, 0^\circ)$ are found to be $St \approx 0.1$, 

| $s/D$ $[-]$ | $\Delta$OASPL $[\text{dB/m}^2]$ |
|------------|-------------------|
| 1.55       | -1.49             |
| 2          | -1.83             |
| 3          | -1.65             |
| 4          | -1.59             |
| 5          | -1.41             |
whereas those of the twin jets at $(\theta, \phi) = (30^\circ, 90^\circ)$ are observed at $St \approx 0.2$. Focusing on the Mach wave peak at $(\theta, \phi) = (30^\circ, 90^\circ)$, the peak amplitude at $St \approx 0.2$ increases as the nozzle spacing increases, as shown in Fig. 19. This effect of the nozzle spacing agrees well with that observed in the OASPL distribution at $\phi = 90^\circ$ shown in Fig. 17. In addition, the peak amplitude of $s/D = 5$ reaches the same level as that of the equivalent single jet. These results indicate that the Reynolds stress in the inner shear layer appears to contribute to the noise amplification at $\phi = 90^\circ$ because the observer position is exposed to the $xy$ plane, including the outer and inner shear layers. On the other hand, in the case of $s/D = 1.55$, the peak amplitude of the Mach waves is low, whereas the PSD levels on the low-frequency side ($St \leq 0.09$) increase when compared with other twin jets. This may have a relation with the L1 and L2 modes that represent the pairing of the large eddies since the large-scale coherent structures are considered to be increased for the sound pressure level of the low-frequency component.

Although the spectra of the turbulent mixing noise should monotonically drop after the peak frequency of the Mach waves ($0.1 \leq St \leq 0.2$), there is another bump in PSD at $St \geq 0.3$. This seems to be the shock-associated noise since the peak frequency is relatively higher than that of the Mach waves and the PIV results indicate the presence of shock cells. The peak frequency of the screech tone $f_s$, which is one of the shock-associated noise, is estimated using Eq. (4) proposed by Powell,

$$n f_s = \frac{L_{th}(1 + u_e/c_{\infty})}{u_c}. \quad (4)$$

Here, $n$, $L_{th}$, $u_c$, and $c_{\infty}$ are the mode number, shock-cell length, convective velocity, and ambient sound speed, respectively. The averaged shock-cell length shown in Fig. 12 is 19.4 mm, and the convective velocity was empirically determined as $u_c = 0.7U_1$. The estimated screech frequency is 18 kHz ($St = 0.35$) when $n = 2$, and corresponds to the frequency of the H1 and H2 modes in the case of $s/D = 1.55$. Therefore, generation of the shock-associated noise in the twin jets is considered to be observed in the H1 and H2 modes of the DMD analysis because these modes appear in the same frequency range with a distinct pattern. Therefore, the twin jets with a narrow nozzle spacing may enhance the generation of the shock-associated noise due to the strong interaction of each jet, even under the ideally expanded condition.

5. Conclusions

The aeroacoustic fields of the twin jets under the ideally expanded condition were experimentally investigated applying various nozzle spacings for each jet. The twin jets and an equivalent single jet of $M = 2.0$ were experimentally investigated using PIV, schlieren visualization, and acoustic measurement. The present study focuses on the aeroacoustic fields of the twin jets, and the effect of the interaction between each jet is discussed.

The noise of the twin jets was basically quieter than the equivalent single jet noise and the noise intensity of the twin jets decreases as the nozzle spacing decreases. The shielding effect causes a decrease in OASPL at $\phi = 0^\circ$, and this is remarkable on the downstream side. This is because total reflection occurs on the downstream side, where the incident angle of the acoustic waves decreases. On the other hand, the peak amplitude at $St = 0.2$ increases as nozzle spacing increases. This is because the Reynolds stress of the inner shear layer on the $xy$ plane appears to contribute to noise amplification at $\phi = 90^\circ$.

Exact DMD analysis of double-pulsed schlieren images extracted a characteristic fluctuation in the aeroacoustic fields. The L1 and L2 modes basically represent the Mach wave emission, and the L2 mode of $s/D = 1.55$ showed a pairing of large eddies, which generates acoustic waves at low frequencies. On the other hand, even when the twin jets were operated at the ideally expanded condition, the strong interaction between each jet ($s/D = 1.55$) excites the helical instability in the shear layer and causes amplification of the shock-associated noise $St \approx 0.3 \sim 0.5$. This is observed in the H1 and H2 modes of DMD analysis as an asymmetric pattern with respect to the symmetry line. Therefore, twin jets with narrow nozzle spacing may enhance the generation of the shock-associated noise.
Acknowledgments

This work was supported by JSPS KAKENHI (17H03473, 19KK0361 and 20H00278).

References

1) Seiner, J.: Advances in High Speed Jet Aeroacoustics, 9th Aeroacoustics Conference, 1984, p. 2275.
2) Tam, C. K.: Supersonic Jet Noise, Ann. Rev. Fluid Mech., 27 (1995), pp. 17–43.
3) Raman, G.: Supersonic Jet Scream: Half-century from Powell to the Present, J. Sound Vib., 225 (1999), pp. 543–571.
4) Morris, P. J.: Jet Noise Prediction: Past, Present and Future, Can. Acoust., 35 (2007), pp. 16–22.
5) Bailly, C. and Fuji, K.: High-speed Jet Noise, Mech. Eng. Rev., 3 (2016), 15-00496.
6) Raman, G. and Taghavi, R.: Resonant Interaction of a Linear Array of Supersonic Rectangular Jets: an Experimental Study, J. Fluid Mech., 309 (1996), pp. 93–111.
7) Miles, I.: Collective Interaction in a Linear Array of Supersonic Rectangular Jets – A Linear Spatial Instability Study, 37th Aerospace Sciences Meeting and Exhibit, 1999, p. 82.
8) Umeda, Y. and Ishii, R.: Oscillation Modes of Supersonic Multijets, J. Acoust. Soc. Am., 101 (1997), pp. 3336–3360.
9) Umeda, Y. and Ishii, R.: Oscillation Modes of Supersonic Multijets Exhausting from Very Adjacent Multiple Nozzles, J. Acoust. Soc. Am., 110 (2001), pp. 1873–1877.
10) Coltrin, I. S., Blotter, J. D., Maynes, R. D., and Gee, K. L.: Shock-cell Structures and Corresponding Sound Pressure Levels Emitted from Closely Spaced Supersonic Jet Arrays, Appl. Acoust., 74 (2013), pp. 1519–1526.
11) Coltrin, I. S., Maynes, R. D., Blotter, J. D., and Gee, K. L.: Influence of Nozzle Spacing and Diameter on Acoustic Radiation from Supersonic Jets in Closely Spaced Arrays, Appl. Acoust., 81 (2014), pp. 19–25.
12) Raman, G., Panckar, P., and Chelliah, K.: Aeroacoustics of Twin Supersonic Jets: a Review, Int. J. Aeroacoustics, 11 (2012), pp. 957–984.
13) Seiner, J. M., Manning, J. C., and Ponton, M. K.: Dynamic Pressure Loads Associated with Twin Supersonic Plume Resonance, AIAA J., 26 (1988), pp. 954–960.
14) Tam, C. K. and Seiner, J.: Analysis of Twin Supersonic Plume Resonance, 11th Aeroacoustics Conference, 1987, p. 2695.
15) Norum, T. and Shearin, J.: Dynamic Loads on Twin Jet Exhaust Nozzles due to Shock Noise, J. Aircr., 23 (1986), pp. 728–729.
16) Seiner, J., Manning, J., and Ponton, M.: Model and Full Scale Study of Twin Supersonic Plume Resonance, 25th AIAA Aerospace Sciences Meeting, 1987, p. 244.
17) Shaw, L.: Twin-jet Scream Suppression, J. Aircr., 27 (1990), pp. 708–715.
18) Walker, S.: Twin Jet Scream Suppression Concepts Tested for 4.7 Percent Axisymmetric and Two-dimensional Nozzle Configurations, 26th Joint Propulsion Conference, 1990, p. 2150.
19) Wlezien, R.: Nozzle Geometry Effects on Supersonic Jet Interaction, AIAA J., 27 (1989), pp. 1361–1367.
20) Zilz, D. and Wlezien, R.: The Sensitivity of Near-field Acoustics to the Orientation of Twin Two-dimensional Supersonic Nozzles, 26th Joint Propulsion Conference, 1990, p. 2149.
21) Sabareesh, V. B., Srinivasan, K., and Sundararajan, T.: Acoustic Characteristics of Equal and Unequal Twin Circular Slot Jets, J. Sound Vib., 342 (2015), pp. 90–112.
22) Gao, J., Xu, X., and Li, X.: Numerical Simulation of Supersonic Twin 2018 jet Noise with High-order Finite Difference Scheme, AIAA J., 56, 1 (2018), pp. 290–300.
23) Bell, G., Soria, J., Honnery, D., and Edgington-Mitchell, D.: An Experimental Investigation of Coupled Underexpanded Supersonic Twin-jets, Exp. Fluids, 59 (2018), p. 139.
24) Greatore, F. and Brown, D.: Progress in Jet Engine Noise Reduction, The First International Council of the Aeronautical Sciences, 1958, pp. 364–392.
25) Bhat, W.: Experimental Investigation of Noise Reduction from Two Parallel-flow Jets, AIAA J., 16 (1978), pp. 1160–1167.
26) Cantola, R.: Acoustic Properties of Heated Twin Jets, J. Sound Vib., 79 (1981), pp. 79–106.
27) Gerhold, C. H.: Analytical Model of Jet Shielding, AIAA J., 21 (1983), pp. 694–698.
28) Yu, J. and Fratello, D.: Measurement of Acoustic Shielding by a Turbulent Jet, J. Sound Vib., 98 (1985), pp. 183–212.
29) Simonich, J. C., Amiet, R. K., and Schlinker, R. H.: Jet Shielding of Jet Noise, NASA-CR-3966, 1986.
30) Bozak, R. and Henderson, B.: Aeroacoustic Experiments with Twin Jets, 17th AIAA/CEAS Aeroacoustics Conference (32nd AIAA Aeroacoustics Conference), 2011, p. 2790.
31) Bozak, R.: Twin Jet Effects on Noise of Round and Rectangular Jets: Experiment and Model, 26th AIAA/CEAS Aeroacoustics Conference, 2014, p. 2890.
32) Pineau, P. and Boggey, C.: Acoustic Shielding and Interaction Effects for Strongly Heated Supersonic Twin Jets, AIP Adv., 11 (2021), 075114.
33) Ozawa, Y., Nonomura, T., Oyama, A., and Asai, K.: Effect of the Reynolds Number on the Aeroacoustic Fields of a Translational Supersonic Jet, Phys. Fluids, 32 (2020), 046108.
34) Nonomura, T., Ozawa, Y., Abe, Y., and Fuji, K.: Computational Study on Aeroacoustic Fields of a Translational Supersonic Jet, J. Acoust. Soc. Am., 149 (2021), pp. 4484–4502.
35) Westerweel, J., Geelhoff, P., and Lindken, R.: Single-pixel Resolution Ensemble Correlation for Micro-PIV Applications, Exp. Fluids, 37 (2004), pp. 375–384.
36) Ozawa, Y., Buki, T., Nonomura, T., Suzuki, K., Komuro, A., Ando, A., and Asai, K.: Single-pixel Resolution Velocity/Convection Velocity Field of a Supersonic Jet Measured by Particle/Schlieren Image Velocimetry, Exp. Fluids, 61 (2020), 129.
37) Komuro, A., Takashima, K., Konno, K., Tanaka, N., Nonomura, T., Kaneko, T., Ando, A., and Asai, K.: Schlieren Visualization of Flow-field Modification over an Airfoil by Near-surface Gas-density Perturbations Generated by a Nanosecond-pulse-driven Plasma Actuator, J. Phys. D: Appl. Phys., 50 (2017), 215202.
38) Komuro, A., Takashima, K., Suzuki, K., Kanno, S., Nonomura, T., Kaneko, T., Ando, A., and Asai, K.: Influence of Discharge Energy on the Lift and Drag Forces Induced by a Nanosecond-pulse-driven Plasma Actuator, Plasma Sources Sci. Technol., 28 (2019), 065006.
39) Tsu, J. H., Rowley, C. W., Lichtenburg, D. M., Brunton, S. L., and Kutz, J. N.: On Dynamic Mode Decomposition: Theory and Applications, J. Comput. Dyn., 1 (2014), pp. 391–421.
40) Troutt, T. and McLaughlin, D.: Experiments on the Flow and Acoustic Properties of a Moderate-Reynolds-number Supersonic Jet, J. Fluid Mech., 116 (1982), pp. 123–156.
41) Tam, C. K.: Excitation of Instability Waves in a Two-dimensional Shear Layer by Sound, J. Fluid Mech., 89 (1978), pp. 357–371.
42) Tam, C.: Excitation of Instability Waves by Sound—A Physical Interpretation, J. Sound Vib., 105 (1986), pp. 169–172.
43) Tanna, H.: An Experimental Study of Jet Noise Part I: Turbulent Mixing Noise, J. Sound Vib., 50 (1977), pp. 405–428.
44) Nonomura, T., Nakano, H., Ozawa, Y., Terakado, D., Yamamoto, M., Fuji, K., and Oyama, A.: Large Eddy Simulation of Acoustic Waves Generated from a Hot Supersonic Jet, Shock Waves, 29 (2019), pp. 1133–1154.
45) Powell, A.: On the Mechanism of Choked Jet Noise, Proc. Phys. Soc. Sect. B, 66 (1953), p. 1039.

Appendix

A. Half-Gaussian fitting of the velocity profile

The half-Gaussian fitting of the velocity profile was validated for both inner and outer shear layers. Figure 20 shows the half-Gaussian fitting of the outer shear layers in the cases of $s/D = 1.55$ and 5. The result indicates that the velocity profile of the outer shear layer agrees well with the half-
Fig. 20. Half-Gaussian fitting of the velocity profile at the outer shear layer.

Fig. 21. Half-Gaussian fitting of the velocity profile at the inner shear layer.

Fig. 22. Streamwise distribution of the fitting error at the inner shear layer.

Gaussian profile regardless of the streamwise position or the nozzle spacing. Figure 21 shows the fitting of the inner shear layers for all of the cases. The velocity profile of the inner shear layer basically agrees well with the half-Gaussian profile before each jet merges. However, the velocity profile on the low-velocity side does not match the reference curve on the downstream side where the jets interact with each other. This is because the streamwise velocity at the symmetry line gradually increases downstream. Therefore, the fitting error of the inner shear layer is calculated and the applicable limit of the fitting is evaluated. The fitting error at the given streamwise position is defined as in Eq. (5); that is, the root-mean-square of the difference between the observed and modeled data.

\[ e = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \frac{\bar{u}_i - v_i}{U} \right)^2} . \]  

Here, \( N \), \( \bar{u}_i \), and \( v_i \) are the total number of measurement points in the radial velocity profile at the given streamwise position, temporally averaged velocity observed by PIV, and modeled velocity at the given point, respectively. The modeled velocity was calculated using Eqs. (2) and (3). Figure 22 shows the fitting error of the inner shear layers. The error rapidly increases when each jet merges, although the error linearly increases downstream. Therefore, the present study assumes that the half-Gaussian fitting can be applied to the inner shear layer until the error rapidly increases. Consequently, Fig. 8 only shows the inner shear layer in the region where the inner shear layer fits well with the half-Gaussian profile.

B. Exact-DMD analysis

Exact-DMD analysis was applied to the double-pulsed schlieren image pairs, and the coherent structures that generate the acoustic waves were estimated at each frequency. This analysis method considers the dataset of \( \{ (x_1, y_1), \ldots, (x_m, y_m) \} \), where \( (x_m, y_m) \) is the data vector of the \( m \)th image pair obtained in a short time interval. The 2,500 pairs of the schlieren images are then subjected to DMD analysis. The exact DMD is then defined for the following new data matrices,

\[ X = \begin{bmatrix} x_1 & x_2 & \cdots & x_m \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 & y_2 & \cdots & y_m \end{bmatrix}. \]  

The datasets are assumed to be generated using linear dynamics for unknown matrix \( \hat{A} \) as follows:

\[ y_k = \hat{A}x_k. \]  

Matrix \( \hat{A} \) in Eq. (7) is constant for all \( k \) and can be obtained using pseudoinverse operation as follows:

\[ A \triangleq Y X^\dagger, \]  

where \( X^\dagger \) is the pseudoinverse of \( X \). Matrix \( \hat{A} \) for the exact DMD is approximated as follows:

\[ \hat{A} \triangleq U_r^* Y V_r \Sigma_r^{-1}, \]  

where \( U_r^* \), \( V_r \), and \( \Sigma_r^{-1} \) are the matrices obtained using the truncated singular value decomposition of \( X \) with remaining leading \( r \) singular values and vectors. The original data were...
truncated to the low-order descriptions of 30 modes in the present study. The number of modes for truncation is relatively small because a large number of modes leads to an estimation error due to the noisy experimental data. The eigenvalues $\lambda$ and eigenvectors $\mathbf{w}$ of matrix $\tilde{\mathbf{A}}$ are then computed where $\tilde{\mathbf{A}} \mathbf{w} = \lambda \mathbf{w}$. The DMD mode $\Phi$ corresponding to the eigenvalue $\lambda$ is given by

$$
\Phi \triangleq \frac{1}{\lambda} \mathbf{Y} \mathbf{V} \Sigma^{-1} \mathbf{w}.
$$

Here, the amplitudes of the DMD modes can be defined for each snapshot $\mathbf{x}_i$ as follows:

$$
\mathbf{b}^{(i)} = \Phi^i \mathbf{x}_i,
$$

where $\mathbf{b}^{(i)}$ is a vector of the DMD amplitude in the $i$th snapshot. This vector consists of the amplitudes of the $k$th DMD mode $b_k^{(i)} (1 \leq k \leq r)$. In the present study, it is assumed that the estimated DMD mode always has the same amplitude within the dataset. Therefore, the amplitude of the $k$th DMD mode can be calculated as the average of the amplitude for all of the snapshots.

$$
b_k = \frac{1}{m} \sum_{i=1}^{m} |b_k^{(i)}|.
$$