Fractal and Smooth Complexities in Electroencephalographic Processing

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Abstract

The importance of the electroencephalogram (EEG) rests upon the fact that it provides useful information of the normal and pathological brain functions. However, the relations among abnormal EEG, brain functions and disorders are not well known yet. We have proposed numerical quantifiers of the EEG signal, coming from the methodology of fractal mathematics and the theory of approximation. In the first part we describe an alternative to the computation of nonlinear dimensions for this kind of signals. The approach used here is based on a fractal interpolation of the data. In the second part, we describe a method for the computation of smooth complexities based on the interpolation of EEG signals by means of polynomial splines. This kind of functions is used to find quadrature formulas for the spectral moments. Both procedures are applied to treat the electroencephalographic discrimination of a group of children suffering from an Attention Deficit with Hyperactivity Disorder (ADHD).

Keywords: Electroencephalogram; Fractal dimensions; Hjorth parameters; Attention Deficit Hyperactivity Disorder

Introduction

Hans Berger discovered in 1924 the oscillations of the electric potential of the human brain, and their trace is called electroencephalogram (EEG) since then. The importance of the EEG rests upon the fact that it provides useful information of the normal and pathological brain function. The EEG waves contain a huge quantity of information which deserves to be discovered.

All the signals with their different frequencies, amplitudes and shapes may have a value of physiological or pathological type, but the relations among abnormal EEG, brain functions and disorders are not well known yet. In many cases the discrimination of normal and pathological EEG waves is rather difficult, and the digital quantifiers may help a possible diagnosis.

The analysis of computerized EEG is an important tool nowadays in the brain sciences and psychopharmacology. The neurophysiological literature provides relevant information about the advances and applications of the field.

Our team has proposed numerical quantifiers of the EEG signal, coming from the methodology of fractal mathematics and the theory of approximation. The objective of our study consists in the determination of useful parameters characterizing the signal and its different rhythms.

Several one-single channel indices are described, along with numerical procedures for their computation. In every case, an experiment related to a disorder of attention is described. The recordings are analyzed by means of the use of the proposed quantifiers.

Fractal Complexity

A common way of facing the computation of fractal dimensions of experimental signals is the use of a phase-space model. From a single sampled signal, a whole trajectory in a higher-dimensional space is reconstructed, considering as coordinates the delays of the recording. However, this method generates a large number of algorithmic problems some of which we briefly summarize here [1].

The need to reach a convergence value for the dimension with respect to the number of delay variables is not fulfilled in general, as communicated in many papers about the subject [3,4].

We have proposed an alternative to the computation of fractal dimensions by means of these procedures. The computation of an unknown function fitting a sampled signal can be approached by means of fractal interpolation [5,6]. A specific characteristic is the fact that the graph of these interpolants (as geometric object of the real plane) possesses a fractal dimension. This parameter constitutes a numerical index of the signal that can be used as a measure of the complexity of a variable Figure 1.

We have implemented several method of data approximation by means of fractal interpolation functions, and proved the validity and convergence of the procedures if the sampling frequency is high enough [7,8].

From the point of view of the application, we look for electroencephalographic differences between normality patterns and a syndrome of lack of attention in children.

Fractal interpolation functions

In this Subsection we describe shortly the mathematical foundations of the fractal interpolation functions.

Let K be a complete metric space respect the distance \(d(x,y), \forall x,y \in K\). Let \(H\) be the set of all compact not empty subsets of \(K\). Let \(\forall n=1, 2, ..., N, w_n; K \to K\), be a set of continuous maps. Then, the set \([K, w_n]\)

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Received November 06, 2014; Accepted December 31, 2014; Published January 10, 2015

Citation: Navascués MA, Sebastián MV, Valdizán JR (2015) Fractal and Smooth Complexities in Electroencephalographic Processing. J Appl Comput Math 4: 196. doi:10.4172/2168-9679.1000198

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$n=1,2,...,N\}$ is an Iterated Function System (IFS). Define the mapping $W: H \rightarrow H$ by

$W(A) = \bigcup W_n(A) \quad \forall A \in H$

Any set $G \in H$ such that $W(G)=G$ is an attractor of the IFS.

Let $t_0 \leq t_1 \leq \ldots \leq t_N$ be real numbers, and $I=[t_0, t_N]$ the closed interval that contains them. Let $I$ be a subset of the data points $\{(t_n, x_n) \in I \times R: n=0,1,2,...,N\}$ be given. Let $I_n=[t_{n-1}, t_n]$ and let $I,I_n \in \{1,2,...,N\}$, be contractions, homeomorphisms such that:

$F_n(t_0,x_0)=x_n$ (1)

$\left|L_n(c_1)-L_n(c_2)\right| \leq I \quad \forall c_1,c_2 \in I$ (2)

or some $0 \leq I < 1$.

Let $I=[t_0, t_N]$ for some $-\infty < c < d < \infty$ and $N$ continuous mappings, $F_n: I \rightarrow R$ be given satisfying:

$F_n(t_0,x_0)=x_n$ (3)

for $n=1,2,...,N$, and

$\left|F_n(t,x)-F_n(t,y)\right| \leq \alpha(t)|x-y|$ (4)

with $t \in I$ and $x, y \in R$.

Now define functions $\forall n=1,2,...,N$.

$w_n(t,x) = \left(L_n(t),F_n(t,x)\right)$

Theorem [5,6]: The Iterated Function System (IFS) $\{F, w_n: n=1,2,...,N\}$ defined above admits a unique attractor $G$. $G$ is the graph of a continuous function $F: I \rightarrow R$ which obeys $f(t_0)=x_0$ for $n=0,1,2,...,N$.

The previous function is called a Fractal Interpolation Function (FIF) corresponding to $\{(L_n(t),F_n(t,x))\}_{n=1,...,N}$.

Let $G$ be the set of continuous functions $f:[t_0, t_0] \rightarrow [c, d]$ such that

$f(t_j)=x_j \quad j=1,2,...,N$.

$G$ is a complete metric space respect to the uniform norm. Define a mapping $T: G \rightarrow G$ by:

$T\{f_n(t_0)\}(t) = F_n\{L_n\}(t_0,t)\}

\forall t \in [t_0, t_0]$, $n=1,2,...,N$.

$T$ is a contraction mapping on the metric space $(G, \left\|\cdot,\right\|_1)$

$\left\|Tg-Tf\right\|_1 \leq |r_n| \left\|f-g\right\|_1$ (5)

where $|a|_{\max n=1,2,...,N}$.

Since $\left\|T\right\|_1 < 1$, $T$ possesses a unique fixed point on $G$, that is to say, there is $f \in G$ such that $Tf(t) = f(t) \quad \forall t \in [t_0, t_0]$. This function is the FIF corresponding to $w_n$ and it is the unique $f \in G$ satisfying the functional equation [5,6]:

$f(t)=F_n\{L_n\}(t_0,t)\}

n=1,2,...,N$.

The most widely studied fractal interpolation functions so far are defined by the IFS

$\{L_n(t)=a_n t + b_n\}

\forall a_n = \frac{t_n-t_{n-1}}{t_N-t_0}$ and $b_n = \frac{t_{n-1}-t_n t_N-t_0}$ (8)

$a_n$ is called a vertical scaling factor of the transformation $w_n$ and $\alpha=(\alpha_1, \alpha_2, ..., \alpha_N)$ is the scale vector of the IFS. If $q_n(t)$ is a line, the FIF is termed affine Figure 2. In this case, by (3) $q_n(t)=a_n t + b_n$ with:

$q_n(t) = \frac{x_n-x_{n+1}}{t_N-t_0}$ and $b_n = \frac{t_{n-1}-t_n t_N-t_0}$ (9)

$q_n(t) = \frac{x_N-x_n}{t_N-t_0}$ and $b_n = \frac{t_{n-1}-t_n t_N-t_0}$ (10)

Computation of scaling factors

Let $\{(t_j, x_j)\}_{n=0,...,N}$ be a subset of the data, that here we consider equidistant, $t_j=t_0+n h$. That values are used as interpolation nodes, and we consider some intermediate points of the signal $t \in I \rightarrow [c, d]$, $j=1,2,...,m-1$ as target points to define the fit. If $t$ are also equidistant:

$t_j = \frac{m-j}{h} t_0 + \frac{j}{m} t_N$ (11)

The value of the FIF at the point $t$ is given by the equation (6). Replacing the value of the function $f$ at $L_n^{-1}(t)$ by the value of the polygon $f_j$ (with interpolation nodes $t_j$):

Figure 2: 3D representation of the complexity of an individual executing a visual test. The closest side corresponds to the occipital area.
\[ f(t') \cong \alpha_n f_0 \circ L_n (t') + q_n \circ L_n (t') \quad (12) \]

By (9), (10) and (11),

\[ q_n = L_n (t') = \left( \frac{m-j}{m} x_{n+1} + j x_n - \alpha_n \frac{(m-j) x_j + j x_n}{m} \right) \]

Therefore, following (12) and (11),

\[ f(t') \cong \alpha_n \left( f_0 \left( \frac{(m-j) t_{n+1} + j t_n}{m} \right) - \frac{(m-j) x_j + j x_n}{m} \right) + \left( \frac{m-j}{m} x_{n+1} + j x_n - \alpha_n \frac{(m-j) x_j + j x_n}{m} \right) \]

\[ x' = f(t') \cong \alpha_n a(j) + v_j (j) \]

Now we compute \( \alpha_n \) by means of least squares approximation:

\[ \min E(\alpha_n) = \sum_{j=r}^{n} (\alpha_n a(j) + v_j (j) - x_j')^2 \]

In this way, the following value of \( \alpha_n \) is obtained:

\[ \alpha_n = - \sum_{j=r}^{n} v_j (j) a(j) \sum_{j=r}^{n} u(j)^2 \]

where

\[ u(j) = f_0 \left( \frac{(m-j) t_{n+1} + j t_n}{m} \right) - \frac{(m-j) x_j + j x_n}{m} \]

\[ v_j (j) = x' - \frac{(m-j) x_{n+1} + j x_n}{m} \]

If \( h = t_n - t_n \) tends to zero, \( v_j (j) \) goes to zero. As a consequence, \( \alpha \rightarrow 0 \) if \( h \rightarrow 0 \). This fact allows us to obtain \( h \) low enough to get \( \alpha \| \leq 1 \).

Bounds of interpolation error and study of convergence are treated in the reference [9].

**Fractal dimension**

The first step is the reconstruction of the signal by means of fractal interpolation functions, computing the parameters of the IFS associated to the data according to the fit proposed in the previous paragraph. The computation of the fractal dimension is then performed by the use of explicit formulas.

Following some theorems concerning Iterated Function Systems [6,10], the fractal dimension \( D \) of the graph of an affine FIF verifies the equation:

\[ \sum_{n=1}^{N} |a_n a_{n+1}|^{-1} = 1 \quad (13) \]

where \( a_n \) is the scaling vertical factor of the IFS and \( a_{n+1} \) the coefficients defined in (8). If the nodes are equidistant, \( a_n = 1/N \) and

\[ D = 1 + \frac{\log(\sum_{n=1}^{N} |a_n|)}{\log N} \quad (14) \]

This formula for the dimension is valid in the case \( 1 < \Sigma |a_n| \). Otherwise, the fractal dimension is one [6]. This parameter lies between 1 and 2.

**Experiment**

**Patients:** The procedures described were applied to the study of the EEG recordings of two samples of children: a healthy control group and a set diagnosed with an Attention Deficit with Hyperactivity Disorder (ADHD). The clinical manifestations of the ADHD are characterised by a lack of attention, impulsive cognitive and behaviour styles and by an excessive motor activity. Its incidence is estimated between 3 and 5 % of the school population and one or two children with deficient attention per classroom during the first school years may be observed. By a mere visual inspection of the EEG, no difference was observed in the patient group.

Former ADHD EEG studies show the existence of an excessive slow activity, mainly localized in the frontal zone. The differences found in this area aim at a predominant role of the frontal lobe in the study of the attention. In fact, some authors report similarities between the ADHD children and the patients suffering some kind of injury in this region. Both pathologies become apparent by problems of attention and control of impulses, leading to unsuitable behaviour.

The children of control group were selected randomly by the teachers and belong to the same school groups than the children with ADHD. 19 children diagnosed with ADHD were chosen, with an average age of 9.3 and a standard deviation of 1.5. The sample was compared with a control group of 13 children with similar age (9.2) and standard deviation (1.3).

For every subject, the following signals were recorded: (i) an EEG of rest with closed eyes, (ii) an EEG during the execution of a test consisting in the recognition of a face different from the others, in series of three.

Six locations of the cortical surface were analyzed, following the 10-20 International System of Jasper: F3, F4, O1, O2, F7, F8. This method was developed to ensure standardized reproducibility so that a subject’s studies could be compared over time and subjects could be compared to each other. The “10” and “20” refer to the fact that the actual distances between adjacent electrodes are either 10% or 20% of the total front–back or right–left distance of the skull. Each site has a letter to identify the lobe and a number to identify the hemisphere location. In our case the letters F and O stand for frontal and occipital lobes, respectively. Even numbers refer to electrode positions on the right hemisphere, whereas odd numbers refer to those on the left hemisphere. The recording of the signal was performed by an electroencephalograph Grass, connected to the program Rhythm, version 5. The equipment included filters of 0.18 Hz for low frequencies and 35 Hz for high frequencies. The sensibility was 7 microvolts per millimeter. The sampling frequency was 128 Hz. A segment of 30 seconds was analyzed inside the second minute.

The fractal dimension of the EEG was obtained by the method proposed, with one intermediate point between every pair of nodes used for the fractal interpolation (\( N \) even, \( m=2, j=1 \)).

To compare the EEG of rest with the EEG recorded while the execution of the exercise described, the test of the sign hierarchized of Wilcoxon was used. To compare both groups during the execution of the same task the test of Mann-Whitney was performed.

**Results and discussion:** Table 1 shows the average values of the fractal dimension for each group, EEG and channel. This parameter
undergoes a general increasing on the whole cortical surface by the execution of the visual test in the ADHD group, but the difference is only significant in some locations.

In the comparison of the data obtained in the computation of the fractal dimension of the EEG during the test of faces recognition with respect to the rest EEG, in the group of children with ADHD the differences were found in O2 to the significance level of 0.01, whereas in the control group no difference was found. These variations show the activation of the occipital zone (primary visual area) in the achievement of tasks of visual attention, and an increase in frontal area, responsible for cognitive processes. The need of the children with ADHD to activate more cortical networks to perform the same test may be evident.

The results described aim at a lower dimensionality in the resting EEG in general. This fact agrees with the studies of several authors [3] for different pathologies and brain processes.

Some differences between the group of children diagnosed with ADHD and the healthy control at rest were also found. In F8 at level 0.01, whereas in the control group no difference was found. This fact agrees with the studies of several authors [3] for different pathologies and brain processes.

Table 1: Average values of the fractal dimension for each group, EEG and control.

|                | Control | Deficient Attention |
|----------------|---------|---------------------|
|                | Rest    | Test               |
| F3             | 1.71192 | 1.70421            |
| F4             | 1.70788 | 1.76038            |
| O1             | 1.72900 | 1.75976            |
| O2             | 1.74617 | 1.72834            |
| F7             | 1.74366 | 1.72131            |
| F8             | 1.74458 | 1.73599            |

In the present study, quadrature formulae for the computation of smooth complexities of any order. Several groups have employed these parameters for the EEG processing. To mention some examples, it is worth emphasizing the work of Elbert et al. [11], which use this procedure to evaluate the EEG of a sample of schizophrenic patients. Mouzé-Amady and Horwat [12] compare the results obtained by means of normalized slope descriptors with a conventional FFT analysis on EMG signals recorded from a group of adult subjects while they were executing a manual task of repetitive type. Biring et al. [8] perform a quantitative electroencephalographic analysis of children with learning problems. Ziller et al. [14] carry out a comparative study between Hjorth and nonlinear parameters by means of factor analysis.

In the present study, quadrature formulae for the computation of smooth complexities of arbitrary order are proposed. The expressions for the descriptors are obtained by means of an interpolation of the signal by polynomial splines.

To illustrate the procedure, an application to the electroencephalographic study of the Attention Deficit with Hyperactivity Disorder (ADHD) is described.

Slope descriptors

Let \( x(t) \) and \( \hat{x}(w) \) be, respectively, the representation of an EEG signal in the time and frequency (Fourier Transform) domains. The spectral moment of order \( q \) is defined as

\[
m_q = \frac{1}{T} \int x(t)^q \, dt = \|x\|^q
\]

Where \( S(w) = \hat{x}(w) \hat{x}(w)^* \), and \( \hat{x}(w)^* \) is the complex conjugate of \( \hat{x}(w) \). These moments can be expressed as:

\[
m_q = \frac{1}{T} \int x(t)^q \, dt = \|x\|^q
\]

The slope descriptors are defined in terms of the moments as:

\[
A = m_2
\]

\[
M = \frac{m_4}{m_2}
\]

\[
C = \frac{m_4}{m_2} \left( \frac{m_4}{m_2} \right)^{1/2}
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\]
The coefficients \(d_i\) satisfy the conditions:

\[
\sum_{i=1}^{N} d_i \sigma(t_i) = 0, \quad K = 0, \ldots, q - 1
\]

The above equalities along with the interpolation conditions \(\sigma(t_i) = x_i\) for \(i = 1, 2, \ldots, N\) allow us to find the coefficients \(a_i, d_i\) and obtain the polynomial spline of interpolation. This function represents the respective relations between the different types of parameters, along with the election of the most suitable quantifiers (or their combinations) in order to describe specific mental processes and pathologies.

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