A note on holographic superconductors with Weyl Corrections

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We study analytical properties of the holographic superconductors with Weyl corrections. We describe the phenomena in the probe limit neglecting backreaction of the space-time. We observe that for the conformal dimension $\Delta_+ = 3$, the minimum value of the critical temperature $T_{cMin}$ at which condensation sets, can be obtained directly from the equations of motion as $T_{cMin} \approx 0.170845 \sqrt{\rho}$, which is in very good agreement with the numerical value $T_{cMin} = 0.170 \sqrt{\rho}$ [Phys.Lett.B697:153-158,2011]. This value of $T_{cMin}$ corresponds to the value of the Weyl’s coupling $\gamma = -0.06$ in table (1) of [Phys.Lett.B697:153-158,2011]. We calculate the $T_{cMin} \approx 0.21408 \sqrt{\rho}$ for another Weyl’s coupling $\gamma = 0.02$ and the the conformal dimension $\Delta_- = 1$. Further, we show that the critical exponent is $\beta = \frac{1}{2}$. We observe that there is a linear relation between the charge density $\rho$ and the chemical potential difference $\mu - \mu_c$ qualitatively matches the numerical curves.

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I. INTRODUCTION

The anti de Sitter/conformal field theory (AdS/CFT) correspondence [1] provides a powerful theoretical method to investigate the strongly coupled field theories. It may have useful applications in condensed matter physics, especially for studying scale-invariant strongly-coupled systems, for example, low temperature systems near quantum criticality (see for example [2, 3] and references therein). Recently, it has been proposed that the

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AdS/CFT correspondence also can be used to describe superconductor phase transition \[4, 5\]. Since the high \(T_c\) superconductors are shown to be in the strong coupling regime, the BCS theory fails and one expects that the holographic method could give some insights into the pairing mechanism in the high \(T_c\) superconductors. Various holographic superconductors have been studied in Einstein theory \[6, 7\] or extended versions as Gauss-Bonnet (GB) \[8–11\] and even in Horava-Lifshitz theory \[12, 13\]. AdS/CFT can also describe superfluid states in which the condensing operator is a vector and hence rotational symmetry is broken, that is, p-wave superfluid states \[14–16\]. Here the CFT has a global \(SU(2)\) symmetry and hence three conserved currents \(J_a^\mu\), where \(a = 1, 2, 3\) label the generators of \(SU(2)\). All these works are based on a numerical analysis of the equations on motion (EOM) near the horizon and the asymptotic limit by a suitable shooting method. But as we know the analytical methods are better and easy for invoking in different problems. The first pioneering work on analytic methods in this topic was the Hertzog’s work \[17\]. He showed that at least in probe limit, by solving equations analytically, one can obtain the critical exponent and the expectation values of the dual operators. The philosophy hidden behind his calculations is perturbation theory. Near the critical point the value of the scalar field \(\psi\) is small and consequently we can treat the expectation values of the dual boundary operators \(\epsilon \equiv < O_{\Delta x} >\) as a perturbation parameter. This method has been used recently by Kanno for investigating the GB superconductors even away from the probe limit \[18\]. Applying the analytical methods has been extended to new trends (see for example \[19\] and the references in it). In \[19\] the authors have shown that one can obtain the critical exponent and the critical temperature by applying a variational method on the EOM. Their method and terminology is simple and very sound. Instead of involving in numerical problems, we can obtain the critical temperature \(T_c\) and the exponent of the criticality very easily by computing a simple variational approach. They studied different modes of super criticality s-wave, p-wave and even d-wave. Thus as we know, there is two major method for analytical study of superconductors: The small parameter perturbation theory as used by \[17\] and the variational method \[19\]. We must attend that, the variational method, which has been used in the present work, gives only the minimum value of the critical temperature \(T_c^{Min}\) for a model with a typical parameter. For example if we focus on Weyl corrections to holographic superconductors, as it has been shown in \[20\], for a large range of the coupling value \(-\frac{1}{16} < \gamma < \frac{1}{24}\), there is a universal relation for the critical
temperature $T_c \simeq \sqrt[3]{\rho}$. The proportionality constant depends on the Weyl coupling $\gamma$ and can be computed using the numerical methods. There is a low limit of superconductivity with critical temperature $T_c^{\text{Min}} = 0.170 \sqrt[3]{\rho}$ corresponds to the value $\gamma = -0.06$. Specially recently there are many interests on GB and Weyl corrected superconductors which in them, one is working with a corrected BH. The holographic superconductors with Weyl corrections has been studied recently [20]. They studied the problem numerically. Our program in this paper is studying the Weyl corrections to the superconductors analytically. Our plan is organized as follows. In section 2, we construct the basic model of the 3+1 holographic superconductor with Weyl corrections. In section 3 we present the analytical results for the condensation and minimum value of the critical temperature for different scaling and the critical exponent $\beta$ via variational bound. Conclusions and discussions follow in section 4.

II. WEYL CORRECTED S-WAVE SUPERCONDUCTORS

The s-wave holographic superconductors can be constructed from a U(1) scalar gauge field coupled to a massive charged scalar field (complex field). The simplest form of the action in five dimensions (3+1 holographic picture) with Weyl corrections, in units in which the AdS radius $L=1$, charge $e=1$, reads [21]

$$S = \int dt d^4x \sqrt{-g} \left\{ \frac{1}{16\pi G_5} (R + 12) - \frac{1}{4} (F_{\mu\nu} F^{\mu\nu} - 4 \gamma C^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}) - |D_\mu \psi|^2 - m^2 |\psi|^2 \right\} \tag{1}$$

Here $G_5$ is the gravitational constant, the $(12)$ term gives the negative cosmological constant, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and as the usual $D_\mu = \nabla_\mu - i A_\mu$. The gauge field $A_\mu$ lives in bulk and produces a conserved current $J^\mu$. This current corresponds to a global $U(1)$ symmetry.

About the action (1) we can say that since the background geometry will be an Einstein metric, we will argue that there is a unique tensorial structure correcting the Maxwell term at leading order in derivatives, arising from a coupling to the Weyl tensor and leading to the dimension-six operator in (1) parametrized by the constant $\gamma$. Other curvature couplings simply provide constant shifts when considering linearized gauge field fluctuations about the background. There is another reason for considering the Weyl correction, which is related to the quantum corrections: In any background in which additional charged matter fields
are integrated out below their mass threshold, the Weyl coupling $C_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$ is generated at 1-loop, with a coefficient $\gamma = \frac{\alpha}{m^2}$ first computed (for four dimension) by Drummond and Hathrell [22].

The Weyl’s coupling $\gamma$ is limited such that its value is in the interval $-\frac{1}{16} < \gamma < \frac{1}{24}$. In probe limit, we neglect the back reactions and in this case, the gravity sector is effectively decoupled from the matter field’s sector. In this probe limit, the exact solution for Einstein-Yang Mills equations is black brain given by

$$ds^2 = r^2 (-f dt^2 + dx_i dx_i) + \frac{dr^2}{r^2 f} \quad (2)$$

Here

$$f = 1 - \left(\frac{h}{r}\right)^4 \quad (3)$$

and the a horizon locates at $r = h$. This solution is asymptotically anti-de Sitter. The temperature of the dual conformal field theory (CFT) is nothing just the Hawking temperature and reads $T = \frac{h}{\pi}$. When the temperature of the black brane falls below a critical value $T_c$, it happens a phase transition between the normal phase and a new phase, in which the scalar field $\psi$ condenses. If the model has such a solution, we state that our field theory has a superfluid phase.

We choose a gauge as $\psi = \psi(r), A_t = \varphi(r)$. It is more convenient to work in terms of the dimensionless parameter $\xi = \frac{h}{r}$, in which the horizon is $\xi = 1$ and the boundary at infinity locates at $\xi = 0$. The resulting Yang-Mills equations

$$D_\mu D^\mu \psi - m^2 \psi = 0$$

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} F^{\mu\nu}) = i [\psi^* D^\nu \psi - \psi D^{\mu*} \varphi^*]$$

for metric (2) are given by

$$\psi'' - \frac{\xi^4 + 3}{\xi(1 - \xi^4)} \psi' + \left(\frac{\varphi^2}{h^2(1 - \xi^4)^3} - \frac{m^2}{\xi^2(1 - \xi^4)}\right) \psi = 0 \quad (4)$$

$$\varphi'' - \frac{1}{\xi} (1 + 72\gamma \xi^3) \varphi' - \frac{2\psi^2}{\xi^2(1 - \xi^4)} \varphi = 0 \quad (5)$$

where prime now denotes derivative with respect to $\xi$. Also we fix the mass of the scalar field to $m^2 = -3$ which is obviously above the Breitenlohner-Freedman bound [23]. The adequate and sufficient boundary conditions for these equations can be written on horizon
ξ = 1, the bulk’s boundary ξ = 0. On the horizon we’ve ϕ(1) = 0, ψ'(1) = \frac{2}{3}\psi(1) and on the boundary of bulk, the following asymptotic forms of the solutions must be existed

\[ \varphi \approx \mu - \frac{\rho}{h} \xi^2 \]  
\[ \psi \approx \frac{<O_{\Delta \pm}>}{\sqrt{2h\Delta_{\pm}}} \xi^{\Delta_{\pm}} = \psi^{(1)}\xi^{\Delta_{+}} + \psi^{(3)}\xi^{\Delta_{-}} \]  

μ and ρ are dual to the chemical potential and charge density of the boundary CFT, ψ(1) and ψ(3) are dual to the source and expectation value of the boundary operator O respectively and \(<O_{\Delta \pm}>) are the condensation with dimension Δ_{\pm} where the dimension Δ_{\pm} is given by

\[ \Delta_{\pm} = \{3,1\} \]  

The conformal scaling dimension Δ ≥ 1 is related to the mass and the de Sitter radius by

\[ m^2 L^2 = \Delta(\Delta - 4). \]  

III. ANALYTICAL RESULTS FOR THE CONDENSATION AND CRITICAL TEMPERATURE

We know that there is a second order continuous phase transition at the critical temperature, the solution of the EOMs (4,5) at \( T = T_c \) is

\[ \varphi = \lambda h_c(1 - \xi^2) \]  

where \( h_c \) is the radius of the horizon at \( T = T_c \). As \( T \to T_c \), the scalar filed’s EOM tends to the following form

\[ -\psi'' + \frac{\xi^4 + 3}{\xi(1 - \xi^4)}\psi' - \frac{3}{\xi^2(1 - \xi^4)}\psi = \frac{\lambda^2}{(1 + \xi^2)^2}\psi, \]  

here \( \lambda = \frac{\rho}{h_c} \). By solving the equation of (10), we can obtain the value of \( T_c \). To match the behavior at the boundary, we can define

\[ \psi(\xi) = \frac{<O_{\Delta_{\pm}>}}{\sqrt{2h\Delta_{\pm}}} \xi^{\Delta_{\pm}}\Omega(\xi) \]  

where, according to eq.(7), \( \Omega \) is normalized as \( \Omega(0) = 1 \). We deduce

\[ -\Omega'' + \frac{\Omega'}{\xi} \left( \frac{\xi^4 + 3}{1 - \xi^4} - 2\Delta_{\pm} \right) + \frac{\Delta_{\pm}^2\xi^4 - (\Delta_{\pm} - 1)(\Delta_{\pm} - 3)}{\xi^2(1 - \xi^4)}\Omega = \frac{\lambda^2}{(1 + \xi^2)^2}\Omega \]  

when \( z \to 0, \frac{\Omega'}{\xi} \) should be finite, so this equation is to be solved subject to the boundary condition \( \Omega'(0) = 0 \).
A. Variational approach

Now we use from the variation method to solve the Sturm-Liouville problem (12). The Sturm-Liouville eigenvalue problem is to solve the equation

\[ \frac{d}{d\xi}[k(\xi)\frac{d\Omega}{d\xi}] - q(\xi)\Omega(\xi) + \lambda^2 \rho(\xi)\Omega(\xi) = 0 \]  

with boundary condition

\[ k(\xi)\Omega(\xi)\Omega'(\xi)|^1_0 = 0 \]  

The Sturm-Liouville problem can be result to be a functional minimize problem

\[ F[\Omega(\xi)] = \frac{\int_0^1 d\xi(k(\xi)\Omega'(\xi)^2 + q(\xi)\Omega(\xi)^2)}{\int_0^1 d\xi \rho(\xi)\Omega(\xi)^2} \]  

Then the eigenvalue \( \lambda_n \) can also be obtained by variation Eq. (14). This eigenvalue is the minimum value of a sequence of the eigenvalues \( \{\lambda_n\}_0^\infty \) i.e. we obtain \( \lambda_0 < \lambda_n \). It is a familiar result from the functional theory. For Eq.(12) we immediately obtain

\[ k(\xi) = \xi^{2\Delta_+ - 3}(1 - \xi^4) \]  

\[ q(\xi) = -\xi^{2\Delta_+ - 5}(\Delta_+^2 \xi^4 - (\Delta_+ - 1)(\Delta_+ - 3)) \]  

\[ \rho(\xi) = \frac{\xi^{2\Delta_+ - 3}(1 - \xi^2)}{1 + \xi^2} \]  

The boundary condition (14) is very serious for our computational settings. We will discuss two different cases \( \Delta_+ = \{3, 1\} \) separately. In each case, we will chose different kind of the trial function \( \Omega(\xi) \).

1. Case \( \Delta_+ = 3 \)

If we fix \( \Delta_+ = 3 \), then the boundary condition (14) reads as

\[ \xi^3(1 - \xi^4)\Omega(\xi)\Omega'(\xi)|^1_0 = 0 \]  

In right boundary point \( \xi = 1 \) it is satisfied clearly. For left point \( \xi = 0 \) we must be careful. Indeed to have \( \lim_{\xi \to 0} \xi^3(1 - \xi^4)\Omega(\xi)\Omega'(\xi) = 0 \), the eigenfunction \( \Omega(\xi) \) must be non singular. Additionally, we impose the auxiliary conditions on it’s value at left point of the interval \( [0, 1] \). In order to use the variation method, we have to specifies the trial eigenfunction
FIG. 1: Variation of the functions $k(\xi), q(\xi), \rho(\xi)$ for the case $\Delta_+ = 3$

$\Omega(\xi)$. We impose the auxiliary boundary conditions $\Omega(0) = 1$ and $\Omega'(0) = 0$. The third order trial eigenfunction is then

$$\Omega(\xi) = 1 - \alpha \xi^2 + \beta \xi^3$$

(20)

We obtain

$$|\lambda_{\alpha,\beta}|^2 = \frac{-0.633333\alpha^2 + 1.01299\alpha\beta + 2.25\alpha - 0.375\beta^2 - 2\beta - 1.5}{0.0151862\alpha^2 + \alpha(-0.0241323\beta - 0.052961) + 0.00981385\beta^2 + 0.0393597\beta + 0.0568528}$$

(21)

which attains its minimum at $\alpha = -3.92548, \beta = 4.76575$. We obtain

$$|\lambda_{-3.92548,4.76575}|^2 \approx 41.9572$$

(22)

which can be compared with the numerical value $20$. The figure (1) shows the the functions $k(\xi), q(\xi), \rho(\xi)$ for the case $\Delta_+ = 3$.

The Minimum critical temperature $T_c^{Min}$ is

$$T_c^{Min} = \frac{h_c}{\pi} = \frac{1}{\pi} \sqrt[3]{\frac{\rho}{\xi}}$$

(23)

so for $\Delta_+ = 3$, $T_c^{Min} \approx 0.170845 \sqrt[3]{\rho}$, which is in very good agreement with the numerical value $T_c^{Min} = 0.170 \sqrt[3]{\rho}$ for $\gamma = -0.06$ of table (1) in [20]. In fact, this analytical calculation can be done even better if we include higher order of $\xi$. However, for qualitative analyze, the third order trial eigenfunction is good enough and we use it.
2. Case $\Delta_- = 1$

Now we fix $\Delta_- = 1$, then the boundary condition (14) reads as

$$\frac{(1 - \xi^4)}{\xi} \Omega(\xi)\Omega'(\xi)|^1_0 = 0$$

(24)

The second order trial eigenfunction satisfying the $\Omega(0) = 0$ is then

$$\Omega(\xi) = a\xi + b\xi^2 + c\xi^3$$

(25)

Now we obtain

$$\lim_{\xi \to 0} \frac{1 - \xi^4}{\xi}(a\xi + b\xi^2 + c\xi^3)(a + 2b\xi + 3c\xi^2) = 0$$

(26)

It results $a = 0$. Now we examine $\Omega(\xi) = b\xi^2 + c\xi^3$ in functional (15) which attains its minimum at $b = 0.465, c = -0.323$. We obtain

$$|\lambda_{0.465,-0.323}|^2 \approx 10.806$$

(27)

So for $\Delta_- = 1$, $T_{c}^{Min} \approx 0.21408\sqrt{\rho}$. This value for critical temperature corresponds to the value of the $\gamma = 0.02$ from numerical solving given by [20]. The figure (2) shows the variation of $k(\xi), q(\xi), \rho(\xi)$ with respect to $\xi$.

B. Critical exponent $\beta$

Now we begin to solve the equation for $\varphi$ to obtain the behavior of the order parameter at $T_{c}$. Away from (but close to) the critical temperature, the field eq.(5) for $\varphi$ becomes

$$(1 - 24\gamma\xi^4)\varphi'' - \left(\frac{1}{\xi} + 72\gamma\xi^3\right)\varphi' \approx \left[\frac{\langle O_{\Delta\pm}\rangle^2}{\hbar^2\Delta_{\pm}}\right]^{2\Delta_{\pm}-2} \frac{\xi^2}{1 - \xi^4}\Omega(\xi)^2\varphi$$

(28)

where the parameter $\varepsilon^2 = \frac{\langle O_{\Delta\pm}\rangle^2}{\hbar^2\Delta_{\pm}}$ is small. Because near the critical chemical $\mu_{c}$, the condensation of the operator is very small, we can expand $\varphi$ in $\varepsilon$ as [24]

$$\varphi(\xi) \sim \mu_{c} + \varepsilon\chi(\xi)$$

(29)
FIG. 2: Variation of the functions $k(\xi)$, $q(\xi)$, $\rho(\xi)$ for the case $\Delta_+ = 1$

where $\chi(\xi)$ is the general correction function to be $\chi(0) = 1$. The equation of motion of $\chi(\xi)$ is

$$
\chi''(\xi) - \frac{1}{\xi} + 72\gamma\xi^3 \chi'(\xi) = \varepsilon \mu c \frac{\xi^{2\Delta_+ - 2}}{(1 - \xi^4)(1 - 24\gamma\xi^4)} \Omega(\xi)^2
$$

(30)

Multiplying

$$
\eta(\xi) = \frac{\sqrt{-1 + 24\gamma\xi^4} e^{-3\sqrt{6}\gamma \arctanh(2\sqrt{6}\gamma\xi^2)}}{\xi}
$$

to both sides of the above equation the equation of $\chi(\xi)$ is reduced to

$$
\frac{d}{d\xi} [\eta(\xi) \frac{d\chi}{d\xi}] = -\varepsilon \mu c \frac{\xi^{2\Delta_+ - 3} e^{-3\sqrt{6}\gamma \arctanh(2\sqrt{6}\gamma\xi^2)} \Omega(\xi)^2}{(-1 + 24\gamma\xi^4)^{3/4}(1 - \xi^4)}
$$

(31)

Making integration of both sides, we get

$$
\eta(\xi) \frac{d\chi}{d\xi} \bigg|_0^1 = -\varepsilon \mu c \int_0^1 \frac{\xi^3 e^{-3\sqrt{6}\gamma \arctanh(2\sqrt{6}\gamma\xi^2)} (1 - \alpha \xi^2)^2}{(-1 + 24\gamma\xi^4)^{3/4}(1 - \xi^4)} d\xi
$$

(32)

where we have used the trial function $\Omega(\xi) = 1 - \alpha \xi^2$, we fixed $\Delta_+ = 3$. Near $\xi = 0$, $\varphi(\xi)$ can be expanded as

$$
\varphi \approx \mu - \frac{\rho}{h} \xi^2 \approx \mu c + \varepsilon (\chi(0) + \chi'(0)\xi + \frac{1}{2} \chi''(0)\xi^2 + \ldots)
$$

(33)
Comparing the coefficients of $\xi^0$ term in both sides of the above formula, we get

$$\mu - \mu_c \approx \varepsilon \chi(0)$$  \hspace{1cm} (34)

Besides, from the $\xi^1$ term in (31), we obtain that $\chi'(0) = 0$. Therefore, from the equation (29) and the boundary conditions of $\chi(z)$, we can solve $\chi(\xi)$ to be

$$\chi(\xi) = -\varepsilon \mu_c \{c' + \frac{c\xi^2}{2} + 9c\gamma \xi^4 + \left\{\frac{-1}{24} + c\gamma(1 + 108\gamma)\right\}\xi^6 + O(\xi^7)\}$$  \hspace{1cm} (35)

Here, $c, c'$ both are the integration constants. Thus we obtain $\chi(0) \approx -\varepsilon \mu_c c'$. Further we have

$$\mu - \mu_c \approx -\varepsilon \mu_c c'$$  \hspace{1cm} (36)

$$< O_{\Delta \pm} > \approx \frac{h^3}{\sqrt{-c'\mu_c}} \sqrt{\mu - \mu_c}$$  \hspace{1cm} (37)

This critical exponent $\frac{1}{2}$ for the condensation value and $\mu - \mu_c$ qualitatively match the numerical curves in Figure.1 of Ref. [20]. Further we can show that there is a linear relation between the charge density $\rho$ and the chemical potential difference $\mu - \mu_c$ qualitatively matches the numerical curves in [20]. Moreover, this linear relation between $\rho$ and $\mu - \mu_c$ can also be frequently seen in the numerical analysis.

**IV. CONCLUSIONS**

In this paper, we have studied the analytical properties of the s-wave holographic superconductor phase transitions with the Weyl corrections and obtained the analytical solutions of this model for the scalar operators of conformal dimension $\Delta = \{3, 1\}$. Actually, we have analytically obtained the minimum bound for critical temperature $T_c^\text{Min}$ in s-wave model. We found that the critical temperature $T_c \approx 0.170845 \sqrt{\rho}$ for $\Delta = 3$ and $T_c^\text{Min} \approx 0.21408 \sqrt{\rho}$ for conformal dimension $\Delta = 1$ which both are perfectly in agreement with the previous numerical values [20]. We found that the critical exponent of condensation operator is always $\beta = \frac{1}{2}$ in this model. Also, we obtained the linear relations between the charge density $\rho$ and the chemical potential difference $\mu - \mu_c$, which is also qualitatively consistent with the previous numerical results.
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