ABSTRACT Renewable energy is being increasingly integrated into distribution systems worldwide in response to technological, economic, and environmental challenges. The assessment of hosting capacity allows us to determine the maximum installation capacity of distributed energy resources (DERs) in a distribution system within its operational limits to obtain more benefits. In this study, a new multistage algorithm is developed based on an analytical approach and optimal power flow (OPF) for the assessment of DERs’ hosting capacity (DERHC) with single and multiple multi-type DERs. In the first stage, the optimal locations of DERs are determined analytically, and the second stage involves the calculation of optimal DERs sizes for the assessment of the maximum locational and total DERHC. This method provides mathematical and global optimum certainty considering the constraints to maintain the reliability and protection of the system. Moreover, the proposed method is tested using a standard IEEE 33-bus distribution system, and different scenarios are created based on the number and type of DERs to achieve the best-case results of DERHC. The obtained results are compared with those of the conventional OPF iterative method that are encouraging and validate the accuracy and robustness of the proposed methodology.

INDEX TERMS Analytical optimal power flow method, hosting capacity assessment, multi-type distributed generation, optimal power flow.

I. INTRODUCTION Owing to significant increase in the demand for electric power over the last few decades, distributed energy resources (DERs) have received attention as potential solutions to the energy crisis. Therefore, electrical distribution systems are progressively featuring DERs to satisfy electrical power requirements. Nevertheless, excessive integration of DERs into a distribution system negatively affects the performance of the system and may lead to various problems, such as overvoltage or undervoltage, thermal overload, power quality deterioration, excessive line losses, and protection failure [1]. Therefore, the maximum capacity of the distribution system needs to be evaluated to host DERs within the limits of operational constraints, called the hosting capacity (HC). The assessment of HC takes considerable amount of computational time because it requires solving multiple power flow tasks. However, DER integration requires fast analysis [2], which provides robustness to the grid system and enables the active control of DERs.

Distributed energy resources’ hosting capacity (DERHC) is a well-known concept that has been extensively studied in recent years [2], [3], [4], [5], [6]; however, some problems still need to be addressed. Several methods have been proposed for the evaluation of DERHC using various programming and simulation based iterative [7], stochastic [8], [9], streamlined [10], and heuristic methods. In the iterative methods, the DER sizes are increased stepwise for all
combinations of total numbers of buses until they violate the operational constraints. It is a large time-consuming method because it requires large number of iterations. A detailed iterative method is presented in [11] for DERHC assessment. Stochastic approaches, such as the Monte Carlo method, simulate the uncertainties in the DER installation location, ratings, and load variations [12]. These approaches are faster than the iterative methods, and their accuracy depends on the computational resources and performance of the computer system. The streamlined method was developed by the Electric Power Research Institute (EPRI) to evaluate DERHC on a system wide basis. It is simple but has accuracy problems in complex systems [13]. Heuristic methods are based on artificial intelligence and probabilistic models, such as particle swarm optimization [14], the firefly algorithm [15], the genetic algorithm [16], robust optimization [17], the slime mould algorithm [18], multiestimation optimization [19] and the Bass diffusion model [20]. These techniques are widely used for the evaluation of DERHC, and their main feature is computational robustness [21]. However, these methods cannot locate global optimum solutions because they are based on random variables without mathematical certainty and approximation errors.

Analytical methods have great importance in field of distribution system planning and operation because they are fast and easy to implement. It often follows different ways to simplify the optimization problems. Previously, this method has been widely used for DER allocation in distribution systems with various objective functions [22]. It is extended to efficient analytical (EA) method and validated by comparison with other methods for the power loss minimization in [23]. The comprehensive analytical expressions are also developed to quantify the energy losses, voltage deviation, line congestion margin and voltage stability index of distribution system with PV integration in [24]. However, no such method is present for the assessment of DERHC of distribution system with multi-type DERs.

The literature review for the assessment of DERHC indicates that the methodology to evaluate the maximum DERHC of distribution systems in less computational time, with mathematical certainty of the global optimum solution and accuracy, has not been adequately investigated. In addition, existing methods cannot provide the optimal solution for the DERHC of a distribution system with a combination of different DER types. The optimal power flow (OPF) iterative method handles this problem easily and yields the best results; however, it requires a lot of simulation time. Table 1 summarizes the review of research articles on the distribution system planning and operation.

The aforementioned literature review and research gap gives the motivation for developing a fast systematic procedure for the assessment of the DERHC of distribution system with multi-type DERs having high probability of finding a global optimum solution. In this study, the objective function for the assessment of DERHC is formulated and solved using the proposed multistage analytical OPF algorithm. The first stage is based on the novel concept of DERHC to obtain the optimal locations for the placement of multi-type single and multiple DERs using an analytical approach. The second stage of the algorithm evaluates the maximum DERs capacity injection into the distribution system using the OPF to determine the locational and total maximum DERHC. The numerical and simulation results were obtained using IEEE 33-bus distribution test systems.

The main contribution of this study can be summarized as follows:

- **Novel direction toward concept of DERHC assessment:** In this article, a comprehensive analysis of the assessment of the DERHC of a distribution system is presented based on a novel direction toward assessment of DERHC.

- **Analytical OPF method:** A multistage analytical OPF method is proposed for the assessment of the DERHC of distribution system with single and multiple multi-type DERs. This method provides accuracy and robustness with the high probability of finding a global optimum solution, and its mathematical certainty is verified through the results obtained on the IEEE 33-bus test system. In addition, the robustness of the proposed method is also verified for an increased number of buses, i.e., the IEEE 69-bus distribution system.

- **Assessment of multi-type DERHC:** The DERHC of a distribution system with single and multiple multi-type DERs are evaluated using the proposed method. The simulation results obtained for the IEEE 33-bus distribution system are compared with those of the iterative OPF method that are encouraging. Furthermore, the DERHC of the distribution system is also determined by combining all types of DERs.

The remainder of this paper is organized as follows. A new direction for the DERHC concept is presented in Section II. Section III focuses on the proposed DERHC methodology, including the mathematical formulation and classification of DERs. Section IV presents a detailed description of the test system, the simulation and numerical results, and a discussion of the obtained results. Section V presents the conclusion of this study.

### II. CONCEPT OF DERHC

The study of DERHC has been recently started to research the impacts of increasing the penetration of DERs into the distribution network of power systems. The literature review defines the DERHC as, “the maximum integration of DERs into the distribution system, subjected to the operational constraints, at which the power system operates satisfactorily” [25], [26], [27]. A locational DERHC is usually referred to as the maximum capacity of the DER that can be integrated at each location (bus) of a distribution system without disturbing the operation of the power system.

The concept of the DERHC of a distribution system is illustrated in Fig. 1. The distribution grids, substations, and transmission system are modeled based on the specified limits of operational indices for protection and better performance.
The limits of the operational indices must be established based on reliable sources, standards, and reasonable assumptions. The increase in the integration of the capacity of DERs in a distribution system causes the operational limitations to be disrupted. The point of maximum penetration of DERs at which the distribution system still performs according to the standards and constraints within the surface of the inbound region is called the DERHC [28]. As shown in this figure the capacity of DER integration is represented by two curves. The first curve shows the DERHC at lower DER capacity point and this point is increased to the maximum DERHC for accommodating additional DERs capacity through the optimal allocation of DERs.

### A. OPF ITERATIVE METHOD

The distribution systems are radial with a high R/X ratio and unbalanced loads. Therefore, conventional power flow approaches, such as the Newton–Raphson method and fast decoupled power flow analysis, are not preferred when dealing with distribution system challenges. However, it is essential to use the distribution version of the power flow method [29].

The iterative based OPF is a conventional method used for the assessment of DERHC by the Pacific Gas and Electric (PG&E) distribution system operator [30]. This method involves creating two matrices of all possible combinations of the number of buses with respect to the number of DERs and DER capacities ranging from minimum to maximum. It follows a systematic procedure by iteratively selecting combinations. Power flow calculations are performed, and the limits of the operational constraints are checked at every iteration. The DER capacities that are within the boundaries of the operational constraints are saved in an array, and the same procedure is performed until all possible combinations of both matrices are investigated. Thus, the combination with the maximum sum of DER capacities is selected as the DERHC of distribution system. The flow diagram of this method is presented in Fig. 2.

### B. PROPOSED CONCEPT OF DERHC ASSESSMENT

The objective function of DERHC from its definition is non-linear and complex; therefore, the state-of-the-art methods for the assessment of DERHC have limitations. The proposed methodology guarantees a global optimum solution with mathematical certainty. In this paper, a new concept of the DERHC assessment is presented which lead to formulation of the objective function and propose an algorithm. DERHC is defined as the maximum power injection of DERs into the distribution system within the limits of operational constraints. It gives an intuition of direct proportionality of DERHC and calculated power injection without...
### III. PROPOSED DERHC ASSESSMENT

This section presents the mathematical formulation of the objective function and the proposed methodology for the assessment of DERHC with multi-type single and multiple DERs in a distribution system.

#### A. MATHEMATICAL FORMULATION

The objective function for the assessment of DERHC is formulated based on the definition, “the summation of total DERHC capacity injection into a distribution system subject to operational constraints,” as follows:

\[
\text{Objective function} = \text{Maximum} \sum_{i=1}^{n} S_{i}^{\text{DERs}} \tag{1}
\]

subjected to

\[
V_{\text{min}} \leq V_{i} \leq V_{\text{max}} \tag{2}
\]

\[
I_{\text{min}} \leq I_{i} \leq I_{\text{max}} \tag{3}
\]

where \( S_{i}^{\text{DERs}} \) is the total apparent power from the DERs at bus number \( i \) of a distribution system with \( n \) number of buses.

Equations (2) and (3) represent the maximum and minimum limits of the voltage and current constraints, respectively. The minimum and maximum limits of voltage are selected as 0.95 and 1.05 p.u., respectively, and the branch current limits are obtained from [31].

1) POWER INJECTION FOR SINGLE DER

The equations for the active and reactive power injections of a distribution system with \( n \) number of buses is as follows:

\[
P_{i}^{\text{inj}} = \sum_{k=1}^{n} |V_{i}| |V_{k}| (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}) \tag{4}
\]

\[
Q_{i}^{\text{inj}} = \sum_{k=1}^{n} |V_{i}| |V_{k}| (G_{ik} \cos \theta_{ik} - B_{ik} \sin \theta_{ik}) \tag{5}
\]

where \( P_{i}^{\text{inj}} \) and \( Q_{i}^{\text{inj}} \) are the active and reactive power injections, respectively, at bus \( i \) of the distribution system. \( k \) is the number of buses starting from the first to the last bus; and \( V_{i} \) and \( V_{k} \) are the voltage magnitudes at buses \( i \) and \( k \), respectively. \( G_{ik} + jB_{ik} \) is the line admittance.

2) POWER INJECTION FOR MULTIPLE DERS

To determine the power injection for multiple DERs, a matrix of all possible combinations, \( N_{c} \), is constructed using (6) with the total number of eligible buses \( N_{B} \) and the number of integrated DERs \( N_{\text{DERs}} \) [23].

\[
N_{c} = \frac{N_{B}!}{N_{\text{DERs}}! \times (N_{B} - N_{\text{DERs}})!} \tag{6}
\]

The active power injection for multiple DERs into bus \( i \) \( P_{i,\text{multi}}^{\text{inj}} \) is then calculated by summing the power injection of \( N_{\text{DERs}} \) number of DERs on all combinations using (7), and the combination of buses with the maximum power injection is obtained using (8).

\[
P_{i,\text{multi}}^{\text{inj}} = \sum_{N_{\text{DERs}}=1}^{n} P_{i,N_{\text{DERs}}}^{\text{inj}} \tag{7}
\]
The reactive power injection for multiple DERs $Q_{i,\text{multi}}^{\text{inj}}$ is determined by rewriting (7) and (8) as follows:

$$P_{i,\text{multi}}^{\text{inj},\text{max}} = \max \{P_{i,\text{multi}}^{\text{inj}}\} \quad (8)$$

$Q_{i,\text{multi}}^{\text{inj}} = \sum_{N_{\text{DER}}=1}^{n} Q_{i,N_{\text{DER}}}^{\text{inj}} \quad (9)$

$$Q_{i,\text{multi}}^{\text{inj},\text{max}} = \max \{Q_{i,\text{multi}}^{\text{inj}}\} \quad (10)$$

### B. DISTRIBUTED ENERGY RESOURCES

In this study, the various DER technologies are generally categorized into three types based on the characteristics of their active and reactive power generation capabilities [23]. This categorization of the types of DERs and the bounds of the decision variables specified for active and reactive DER powers are described below.

**Type 1:** DERs with fixed power factor and reactive power generation ($Q_{\text{DER}}^\text{Fixed}$). These DERs were treated as a negative load during the simulation, with a unity power factor. Thus, these DERs support only active power in the distribution system. The characteristic equations of the active and reactive powers from the DERs are as follows:

$$P_{\text{DER}}^\text{min} \leq P_{i}^{\text{DER}} \leq P_{\text{DER}}^\text{max} \quad (11)$$

$$Q_{i} = Q_{i}^{\text{Fixed}} \quad (12)$$

The most common example of a type 1 DER is a photovoltaic array with a static power converter.

**Type 2:** DERs with fixed power factor and active power generation ($P_{\text{DER}}^\text{Fixed}$). These DERs were also treated as a negative load during the simulation, but with zero power factor. Thus, these DERs support only reactive power in the distribution system. The characteristic equations for this type of DERs are given as

$$Q_{i}^{\text{DER}} \leq Q_{i} \leq Q_{i}^{\text{max}} \quad (13)$$

$$P_{i}^{\text{DER}} = P_{i}^{\text{Fixed}} \quad (14)$$

An example of type 2 DER is a wind turbine with a squirrel cage induction machine.

**Type 3:** DERs capable of injecting both active and reactive powers into a distribution system with an unspecified power factor. Their power factors are viewed as decision variables and thus require special treatment to obtain accurate simulation results. The characteristic equations of active and reactive power generation are given in (15) and (16), respectively.

$$P_{\text{DER}}^\text{min} \leq P_{i}^{\text{DER}} \leq P_{\text{DER}}^\text{max} \quad (15)$$

$$Q_{i}^{\text{DER}} \leq Q_{i} \leq Q_{i}^{\text{max}} \quad (16)$$

An example of type 3 DER is a gas turbine with a synchronous machine.

### C. PROPOSED MULTISTAGE ALGORITHM

The proposed multistage algorithm addresses the objective function for the assessment of the maximum DERHC for the integration of single and multiple multi-type DERs into the distribution system subjected to operational constraints. The methodology is described in two stages.

1) **ANALYTICAL STAGE: TO FIND THE OPTIMAL LOCATIONS**

   **Step 1:** Read the line and load data of the distribution system, the number of DERs to be integrated, and their type.

   **Step 2:** Run single-time power flow analysis to obtain the bus voltages.

   **Step 3:** Construct the bus admittance matrix.

   **Step 4:** Calculate the active and reactive power injections using (4)–(10) for different numbers of integrated DERs.

   **Step 5:** Extract the number of locations where the power injection is maximum according to the number of DERs to be integrated.

   **Step 6:** Print them as an optimal location.

The assessment of the maximum DERHC of the distribution system is dependent on the placement of the DERs; therefore, the optimal locations for the integration of DERs need to be determined. The first stage provides an accurate and robust analytical optimal location for the placement of DERs with mathematical certainty. The flow diagram of this stage is shown in Fig. 3.

2) **OPF STAGE: ASSESSMENT OF MAXIMUM DERHC**

   **Step 1:** Import the optimal location from the first stage of algorithm.

   **Step 2:** Create a matrix of all possible combinations of DER capacities, starting from the minimum to the maximum capacity of a DER with a step increase in capacity using (6).

   **Step 3:** Start iterations from 1 to the length of the matrix of DER capacities.
Step 4) Perform optimal power flow analysis for each iteration and create an array of those iterations whose DER capacities are within the operational constraints.

Step 5) Select the combination of DER capacities whose sum is the maximum.

Step 6) Print the optimal locations, maximum locational and total DERHC of the distribution system with simulation time.

The second stage of the OPF was used to calculate the maximum DERHC of the optimal locations obtained from the first stage. The novel attribute of this solution is that it guarantees a global optimum solution that does not adhere to the local optimal point because this stage is based on iterations where each iteration is simulated independently and the maximum DERHC is obtained. The steps are illustrated in Fig. 4.

![Second stage of the proposed algorithm to calculate maximum DERHC](image)

This multistage analytical OPF method is used to calculate the maximum DERHC of the distribution system, and the results validate the accuracy, robustness, and global optimum solution with mathematical certainty.

IV. RESULTS AND DISCUSSIONS

This section presents a verification of the proposed algorithm. Intensive simulation tests were conducted for the integration of multi-type single and multiple DERs into the distribution system for the assessment of DERHC compared with the OPF iterative method. The locational and total DERHC were determined for different scenarios created for single and multiple DERs and combinations of multi-type DERs. The proposed algorithm and tests were programmed in MATLAB with a 3.20 GHz PC with 16.00 GB of RAM.

The IEEE 33-bus radial distribution system is used as a test system to investigate the performance of the proposed algorithm and to determine the best-case scenario of the maximum locational and total DERHC of the distribution system. The topology of the standard IEEE 33-bus distribution system is illustrated in Fig. 5. The test system is a 12.66 KV system with one feeder substation as bus 1 (slack bus) and 32 PQ buses that are considered candidate buses for DER integration. The total active and reactive powers provided by the load buses are 3715 KW and 2300 KVAR, respectively.

![Single-line diagram of the IEEE 33-bus radial distribution system](image)

The locational DERHC was evaluated for the IEEE 33-bus system using the OPF iterative method. It is plotted against the calculated power injection without considering the operational constraints using the first step of the proposed algorithm to validate the accuracy of the proposed algorithm. The results are presented in Fig. 6. In the figure, the DERHC of each bus and the calculated power injection are represented by bars. These bars on each bus have the same nature of increasing or decreasing the KVA capacity with respect to other buses but with different capacities. This indicates that if any bus has a high DERHC, the calculated power injection value is also high, and vice versa. This validates the accuracy of finding the optimal location for the placement of DERs for the assessment of DERHC using the first stage of the proposed algorithm.

The assessment of the maximum DERHC with multi-type single and multiple DERs is presented in the following subsections.

A. DER TYPE 1

The proposed algorithm was first applied to single and multiple type 1 DERs that generate only active power. The simulation results of the maximum locational and total DERHC of the 33-bus distribution system with the integration of one, two, and three type 1 DERs are obtained through the proposed
analytical OPF method. These results are compared with those of the OPF iterative method in terms of the DERHC and simulation time, as shown in Fig. 7. This figure represents the locational DERHC, total DERHC, and simulation time for the stem, bar, and diamond shapes, respectively. The total DERHC of the distribution system are 8300 KVA, 8200 KVA, and 8300 KVA for the integration of one, two, and three type 1 DERs, respectively, which are the closest to the OPF iterative method. The simulation time required by the proposed algorithm was very short. The numerical data of the obtained DERHC are listed in Table 2.

B. DER TYPE 2
Tests were also performed for single and multiple type 2 DERs. The simulation results of the DERHC of the 33-bus distribution system with the integration of one, two, and three type 2 DERs are presented in Fig. 8. This figure shows the locational and total DERHC, and the simulation time obtained by the proposed algorithm compared with those of the iterative OPF method. The total DERHC of the distribution system are obtained as 5600 KVA, 5700 KVA, and 6000 KVA, respectively, with the integration of one, two, and three type 2 DERs.

C. DER TYPE 3
This section describes the results of the DERHC of a distribution system with one, two, and three type 3 DERs for the 33-bus distribution system. Fig. 9 shows a comparison of the obtained results of the DERHC and simulation time using the proposed method and the OPF iterative method. The total DERHC of the distribution system are 9615 KVA, 9167 KVA, and 9503 KVA with the integration of one, two, and three DERs of type 3 at the optimal locations, respectively. The simulation results of the proposed algorithm are almost the same in terms of DERHC and show a significant improvement in the simulation time compared with those of the iterative OPF method.

The simulation results of the total DERHC of the distribution system with the integration of multi-type single and multiple DERs are presented in Table 2. The locational DERHC on the optimal number of buses are also mentioned. With the integration of the multi-type single and multiple DERs, the obtained results show that the total DERHC is maximum with the integration of three DERs all of type 3, as compared with the DERHC with three DERs of type 1 and type 2. The locational DERHC is also maximum with the integration of single and multiple DERs of type 3 at the optimal locations of the distribution system. Furthermore, it can be observed that three is the optimal number of integrated DERs in the 33-bus distribution system. A detailed comparison of the DERHC and simulation time of the proposed analytical OPF algorithm and the OPF iterative method is presented in Table 2. The simulation time of the proposed methodology shows a significant improvement, and the DERHC results are almost similar to those of the OPF iterative method. The computational time of the OPF iterative method is much longer, but in
this study, the step increase in the capacity of multiple DERs were taken high to get the results faster but still the simulation time is much better for proposed algorithm. These results and comparisons verify the robustness, accuracy, and high probability of finding the global optimum solution of the proposed algorithm with mathematical certainty for the DERHC of distribution systems with single and multiple multi-type DER technologies.

D. EFFECTIVENESS ON HIGHER BUS SYSTEM

The simulation results of the DERHC of the distribution system with multi-type DERs validate the accuracy of the proposed method with a significant improvement in the simulation time. In this section, the analysis of the computational burden with an increase in the number of buses is presented. Accordingly, the DERHC of the distribution system with three type 3 DERs was obtained for the IEEE 69-bus system. The simulation times obtained using the proposed method and the OPF iterative method are shown in Fig. 10. The proposed method shows a tremendous improvement in simulation time compared with that of the OPF iterative method with a higher bus system.

E. COMBINATION OF DIFFERENT DER TYPES

This section presents a solution methodology for the DERHC of a distribution system with a combination of three types of DERs for different scenarios using the proposed algorithm. The scenarios are created with three DERs, as it is the optimal number of DER integration for the 33-bus system. All three types of DERs were combined differently, resulting in six scenarios.

| Type of DER | No. of DERs | Method | Optimal Bus No. | Locational DERHC (KVA) | Total DERHC (KVA) | Simulation Time (sec) |
|------------|-------------|--------|----------------|------------------------|------------------|----------------------|
| 1          | 1           | OPF    | 2              | 8300                   | 8300             | 2.91                 |
|            |             | A-OPF  | 2              | 8300                   | 8300             | 0.29                 |
| 2          | 2           | OPF    | 2,3            | 4600,3600             | 8200             | 116.2                |
|            |             | A-OPF  | 2,26           | 5000,3200             | 8200             | 4.4                  |
| 3          | 1           | OPF    | 2,10,26        | 4600,3100,600         | 8300             | 9852.39              |
|            |             | A-OPF  | 2,26,11        | 4800,1100,2400        | 8300             | 161.54               |
| 2          | 1           | OPF    | 2              | 5600                   | 5600             | 2.64                 |
|            |             | A-OPF  | 2              | 5600                   | 5600             | 0.26                 |
| 2          | 2           | OPF    | 5,15           | 4600,1100             | 5700             | 116.87               |
|            |             | A-OPF  | 2,11           | 3100,2700             | 5800             | 4.68                 |
| 3          | 1           | OPF    | 6,7,8          | 4600,1100,100         | 5800             | 9717.17              |
|            |             | A-OPF  | 2,11,26        | 1000,2200,2800        | 6000             | 184.48               |
| 3          | 1           | OPF    | 2              | 9615                   | 9615             | 3.04                 |
|            |             | A-OPF  | 2              | 9615                   | 9615             | 0.39                 |
| 2          | 2           | OPF    | 2,3            | 5143,4024             | 9167             | 124.50               |
|            |             | A-OPF  | 2,26           | 5590,3801             | 9391             | 5.20                 |
| 3          | 1           | OPF    | 5,10,22        | 5422,1509,2627        | 9559             | 9519.93              |
|            |             | A-OPF  | 2,26,11        | 4360,3801,1342        | 9503             | 208.83               |

A-OPF = Analytical OPF

FIGURE 10. Simulation time (sec) for the IEEE 69-bus distribution system.

Scenario 1. DER type 1, 2, and 3
Scenario 2. DER type 1, 3, and 2
Scenario 3. DER type 2, 1, and 3
Scenario 4. DER type 2, 3, and 1
Scenario 5. DER type 3, 1, and 2
Scenario 6. DER type 3, 2, and 1

The optimal locations for these scenarios were obtained using the first stage of the proposed algorithm as buses 2, 11, and 26. Different types of DERs were sequentially placed at the optimal locations, and the second stage of the proposed algorithm was used to calculate the DERHC. Table 3 provides a comparison of the DERHC for all the scenarios created by the combination of different DER types using the proposed algorithm. The results show that Scenario 2 has a maximum DERHC of 11953 KVA. In this scenario, DERs of type 1, 3, and 2 of capacity 7150 KVA, 1453 KVA, and 3350 KVA were placed on bus number 2, 11, and 26, respectively. These
results are also presented in the bar graph in Fig. 11. Here, the three types of DERs are shown by numbering the bars, and the colors represent their placement on the corresponding bus locations, also mentioned on the x-axis. The y-axis represents the locational DERHC of the DERs, and the total DERHC is mentioned at the top of each scenario by summing up the locational DERHC of each scenario.

The main target is to select the optimal scenario in terms of the DERHC with a combination of different types of DERs. An analysis of the simulation results yielded the following observations:

- The total DERHC is significantly improved for all scenarios compared with the DERHC with the integration of three DERs of the same type.
- The maximum total DERHC was observed in Scenario 2; therefore, this is the optimal scenario. This scenario provides active power from DER type 1 near the substation on bus 2 and reactive power support from type 3 and 2 on bus number 11 and 26, respectively.

This study provides the best-case scenario for the system under study, and it can be extended to any DER combination scenario.

V. CONCLUSION

This paper presented a comprehensive explanation and analysis of the assessment of the locational and total DERHC of a distribution system with multiple types of single and multiple DERs. The algorithm was proposed and developed in two stages based on an analytical approach and an OPF for the assessment of DERHC. This method provided the global optimum results of the DERHC of a distribution system with multi-type single and multiple DERs with promising mathematical certainty. The operational constraints were also considered to maintain the reliability and protection of the distribution system. Furthermore, the simulation time for this solution decreased, which improved the robustness of the grid system. The investigation was supported by a cohesive and critical analysis of the simulation results of the DERHC obtained for a 33-bus radial distribution system that verified the accuracy of the proposed algorithm. The effectiveness of proposed method in terms of computational robustness is also verified on higher bus system i.e., IEEE 69-bus system. Based on the simulation results, the following conclusions can be drawn:

- The DERHC is a location-dependent concept, and a higher capacity of DERs can be integrated by determining the optimal locations without any enhancement techniques.
- The DERHC of the distribution system with the same type of DERs is maximum at 9503 KVA for three DERs of type 3 in comparison with the integration of three type 1 and type 2 DERs.
- For a combination of different DER types, Scenario 2 had the maximum DERHC of 11953 KVA compared with the other scenarios.

This study makes a significant contribution to the research on DERHC analysis assessment. In the future, it would be more interesting to study the enhancement of hosting capacity by using energy storage devices, reconfiguration or reinforcement, and smart inverter techniques considering the DER uncertainties and load variability.

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