Order and anarchy hand in hand in 5D SO(10)

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Abstract. A mechanism to generate flavour hierarchy via 5D wave-function localization is revisited in the context of SO(10) grand unified theory. In an extra-dimension compactified on an orbifold, fermions (living in the same 16 representation of SO(10)) result having exponential zero-modes profiles, localized around one of the brane. The breaking of SO(10) down to SU(5) $\times U(1)_X$ provides the key parameter that distinguishes the profiles of the different SU(5) components inside the same 16 representation. Utilizing a suitable set of scalar fields, a predictive model for fermion masses and mixing is constructed and shown to be viable with the current data through a detailed numerical analysis. The scalar field content of the model is also suitable to solve the doublet-triplet splitting problem through the missing partner mechanism. All the Yukawa couplings in the model are anarchical and of order unity, while the hierarchies among different fermions result only from zero-mode profiles. The naturalness of Anarchical Yukawa couplings is studied, showing a preference for a normal ordered neutrino spectrum; predictions for various observables in the lepton sector are also derived.

1. Introduction

It is well known that the Standard Model (SM), despite being so far the most complete and successful theory of particles interactions, still doesn’t account for some aspects of the observed phenomenology or doesn’t provide explanation to how particular structures of interaction originate. Among these, the flavour sector of SM remains one of the most obscure aspects: besides the necessity of introducing 3 generations of fermions, constituting the matter fields, lots of parameters are needed to describe their mass terms and flavour mixing interactions. These parameters, given in the so-called Yukawa couplings, are simply fixed by experiments, from which a peculiar pattern and a spanning of several orders of magnitude emerge, without any theoretical explanation: a huge hierarchy among the generations characterizes the masses of the quarks, especially the up-type ones ($\sim$ 1 MeV-170 GeV), while a milder hierarchy rules the down-type quarks ($\sim$ 5 MeV-5 GeV) and the charged leptons ($\sim$ 0.5 MeV-2 GeV); besides this, neutrinos are known to have a very small mass ($\sim$ eV) that is not taken into account in the SM; then, strong hierarchy appears in the flavour mixing of the quarks, while, on the other...
side, the flavour mixing of the leptons results mostly anarchical. This setup is summarized by
the following relations, expressed in terms of the parameter $\lambda \approx 0.27$, identified as the Cabibbo
mixing angle:

$$
m_u : m_c : m_t \approx \lambda^8 : \lambda^4 : 1;
$$

$$
m_d : m_s : m_b \approx \lambda^5 : \lambda^3 : 1;
$$

$$
m_e : m_\mu : m_\tau \approx \lambda^6 : \lambda^2 : 1;
$$

$$
\frac{m_{\nu_2}^2 - m_{\nu_1}^2}{m_{\nu_3}^2 - m_{\nu_2}^2} \approx \lambda^2;
$$

$$
V_{CKM} \approx \begin{pmatrix}
1 & \lambda & \lambda^3 \\
\lambda & 1 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix};
$$

$$
U_{PMNS} \approx \begin{pmatrix}
0.8 & 0.5 & 0.2 \\
0.5 & 0.6 & 0.6 \\
0.3 & 0.6 & 0.7
\end{pmatrix}.
$$

All these unexplained features in the flavour sector characterize the so called “flavour problem”
and, together with other open issues, necessarily require to extend the SM within new scenarios.
This work [1] analyzes the theoretical advantages and the phenomenological viability of realizing
this extension to a supersymmetric SO(10) grand unified theory (GUT) in extra dimensions, by
updating a primordial model originally proposed by Kitano and Li [2].

The first motivation of this scenario is the possibility of perfectly accommodating the particle
content of the SM and its gauge group into an SO(10) GUT, whose supersymmetric (SUSY)
version allows for gauge coupling unification without need of intermediate scales. All fermions of
each generation result unified in the 16 spinorial representation of SO(10), nicely explaining the
origin of the fermions quantum numbers under the SM gauge group. The description of fermion
masses and mixing angles, anyway, remains as complicated as in the SM, requiring still lots of
parameters spanning a vast range of magnitude, but some advantages arise from the structure
of SO(10) GUT:

(A) an extra component present in the 16 would be a right-handed (RH) neutrino, naturally
allowing a description of the light neutrino masses in terms of the seesaw mechanism;

(B) the possible embedding of intermediate symmetry breaking down to SU(5) would allow for
unified representation of charged leptons and down-type quarks, a picture compatible with
the similar hierarchy manifested between their masses and with the known unification of
tau lepton and bottom quark masses approximately realized at high energy in the minimal
supersymmetric standard model (MSSM).

The Yukawa interactions in SO(10) couple two 16 fermions with a Higgs field in the possible
representations: 10, 120 or 126 (any combination of them). Viable SO(10) models requires to
combine at least two of these representations (10-126 or 120-126), in order to fit all the fermion
masses and mixing angles (see e.g. [3, 4]). The use of 126 allows also a Majorana mass term for
RH neutrinos, implementing the light neutrino masses through the seesaw mechanism.
Overall, there is no qualitative difference with respect to the SM, since the number of free
parameters in the flavour sector remains very large (so that no predictions are available) and
the best fit parameters span several orders of magnitude, as much as in the SM.
It would be desirable to have a natural theory of flavour, in which all the couplings are of
the same order magnitude.
Extra dimensions are introduced in this context exactly with this purpose, by accounting for
the hierarchies of the charged fermion masses and of the quark mixing angles in terms of an
irreducible set of order-one parameters that don’t require to be fine-tuned. The constraints given
by SUSY, moreover, let some advantages in the theoretical structure, permitting to reduce the
number of free parameters with respect to the non-supersymmetric case.
The idea is to build up the observed hierarchical structure of the Yukawas starting from
fundamental Yukawas with anarchical entries of order one, combined with a mechanism that can
“reorder” these entries introducing the wanted hierarchy: this mechanism exploits the possibility
to associate a hierarchical structure in the generation space to the matter fields, in such a way
to “sandwich” the fundamental anarchical Yukawa between two diagonal hierarchical matrices.
To give roughly the idea, the fundamental Yukawa would be:
\[
Y \approx \begin{pmatrix}
\varepsilon^2 & 0 & 0 \\
0 & \varepsilon & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\
\mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\
\mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1)
\end{pmatrix}
\begin{pmatrix}
\varepsilon^2 & 0 & 0 \\
0 & \tilde{\varepsilon} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]  
(5)
where \(\varepsilon\) and \(\tilde{\varepsilon}\) are just example parameters to express the hierarchy associated to the fields by
some mechanism, and the \(\mathcal{O}(1)\) matrix is the natural, fundamental Yukawa.
This kind of scenario\(^1\) is emerging introducing a compactified extra dimension, provided the
matter fields are propagating in the bulk with a hierarchical profile: putting the Higgs field (and
so the Yukawa coupling) on a brane, the coupling results hierarchically modulated by the value
assumed by the fermions profiles on that brane, playing the role of the \(\varepsilon\) parameters above.
Exponential profiles along the extra dimension naturally emerge by introducing a bulk-mass pa-
rameter (a 5-dimensional mass term) and hierarchical Yukawa couplings are generated by small
\((< \mathcal{O}(1))\) difference of these parameters.

2. Flavour hierarchy from extra dimension: an abelian model
Before introducing the specific features of our model in SO(10), let’s first consider the basic setup
of a 5-dimensional (5D) U(1) gauge theory with N=1 SUSY. The extra dimension is considered
compactified on an orbifold \(S^1/Z_2\) of radius \(R, y \in [0, \pi R]\). As shown in \([5]\), 5D N=1 SUSY
can conveniently be written in terms of 4-dimensional N=2 SUSY formalism. Gauge fields are
described by a 5D vector supermultiplet, containing an \(N = 1\) chiral multiplet \(\Phi\) and a vector
multiplet \(V\); matter fields are described by a 5D hypermultiplet, containing a pair of \(N = 1\)
chiral multiplets, \(H\) and \(H^c\). The U(1) gauge invariant action of interacting hypermultiplet and

\(^1\) A similar scenario is realized, for example, in Froggatt-Nielsen models \([5]\) by adding a \(U(1)_{FN}\) flavour group,
introducing a coupling with flavon field and proper assignment of charges to the fields. Hierarchy is created by
different powers of the flavon VEV breaking the symmetry.
vector multiplet is written as: $[6, 7, 8]$

$$S_5 = \int dy \, d^4 x \left[ \int d^4 \theta \left( \partial_\mu V - \frac{1}{\sqrt{2}} (\Phi + \bar{\Phi}) \right)^2 + \frac{1}{4} \int (d^2 \theta \, W^a W_\alpha + \text{h.c.}) \right]$$

$$+ \int d^4 \theta \left( \bar{H}_i e^{2g_5 Q_i V} H_i + \bar{H}^c_i e^{-2g_5 Q_i V} H^c_i \right)$$

$$+ \left( \int d^2 \theta \, (\hat{m}_i + \partial_y - \sqrt{2}g_5 Q \Phi) H_i + \text{h.c.} \right), \quad (6)$$

where $W^a$ is a field strength, $g_5$ is the 5D gauge coupling constant, $\hat{m}$ is the bulk mass and $Q$ is the U(1) charge of the chiral multiplet $H_i$; $i$ is the index in the generation space. The first line describes the 5D kinetic term for the gauge fields; the second line describes the 4D part of the kinetic term for matter fields and the gauge interactions; the last line complete the previous one with the derivative with respect to the extra dimension, the gauge interaction with the field $\Phi$, related to the fifth component of the 5D gauge multiplet, and a 5D mass term for each generation.

$Z_2$ parity is assigned by boundary conditions of the fields, as: $(H; H^c) = (+; -)$ and $(V, \Phi) = (+; -)$, in such a way to break explicitly $N=2$ down to $N=1$ SUSY on the branes. To have invariance, $\hat{m}$ must be odd under $Z_2$ and the simplest choice is $\hat{m}(y) = m \text{ sgn}(y)$, $m$ being a real constant.

Eq. (6) describes the action in the bulk and, besides this, there can be contributions strictly localized on the branes, which should only respect $N = 1$ SUSY. Here we discuss the theory in the ideal limit of exact $N = 1$ SUSY.

By performing the Kaluza-Klein (KK) expansion of 5D bulk fields (for the chiral multiplet, $H(x, y) = \sum_n H_n(x) f_n(y)$) and by reduction to the 4D theory, we can obtain the profiles $f_n(y)$ of the various modes using the equations of motion (e.o.m.) and imposing the $Z_2$-boundary conditions. For the chiral superfields, e.o.m. come only from the last line of the action (6), leading to the 0-mode profiles:

$$f^i_0(y) = \sqrt{\frac{2m_i}{1 - e^{-2m_i \pi R}}} e^{-m_i y} \text{ and } f^c_0(y) = 0. \quad (7)$$

Only the $Z_2$-even fields have a 0-mode, that results exponentially modulated by the bulk mass parameter $m_i$, localized mostly on the $y = 0$ brane or the $y = \pi R$ brane, according to the sign of $m_i$. This feature is an essential ingredient: localizing the Higgs sector on one of the branes (e.g. $y = 0$) with $O(1)$ Yukawa couplings, the effective 4D Yukawa results being suppressed or enhanced depending on the profile value at that brane. The bulk masses, for a field of a given representation of the gauge group (in this case, for assigned U(1) charge) permit to distinguish the profiles with respect to the 3 generations. In this way the hierarchical pattern observed in fermion masses and mixing angles can be explained without appealing to small ad hoc parameters [9] [10] [11].

At this point it is worth to note that the VEV of the scalar field contained in the chiral superfield $\Phi$ of the gauge sector, can correct the bulk mass term by:

$$m_i \rightarrow m_i - \sqrt{2} g_5 Q \langle \phi \rangle \quad (8)$$
in a way proportional on the U(1) charge \( Q \). The only interaction with \( \Phi \) allowed by \( N = 1 \) 5D SUSY is of gauge type, so that this field couples to \( H \) and \( H^c \) through \( g_5 \), universally with respect to the generations of matter fields. The correction of Eq.\((8)\) is then universal as well.

3. The SO(10) model

The previous model can be implemented in the SO(10) framework, following the original work of \([2]\), that we improved in \([1]\) by making the theory more predictive and testing its phenomenological viability. In this context the compactification radius of the extra dimension is taken above the GUT scale (\( \frac{1}{R} > M_{\text{GUT}} \approx 10^{16} \text{ GeV} \)).

In this framework \( H \) and \( H^c \) are replaced by three copies of \( 16 \) and \( 16^c \), accommodating the three generations of SM fermions and RH neutrinos, transforming as \( 16 \) and \( 16^c \) under SO(10) respectively. The vector supermultiplet, comprising \( 45_V \) and \( 45_\Phi \), transforms in the adjoint of SO(10).

The relevant part of the action for getting the 0-modes of matter fields is given, as in the previous model, by the superpotential in the bulk, that is written in the SO(10) theory as:

\[
W_{\text{bulk}} = 16^c_i \left[ \hat{m}_i + \partial_y - \sqrt{2}g_5 \ 45_\Phi \right] 16_i
\]

This superpotential leads to the same 0-mode profile in Eq.\((7)\), identical for all the components of the \( 16_i \) multiplet of a given generation. To exploit completely the mechanism of addressing the flavour hierarchy through extra dimension, it is not sufficient to distinguish the profiles only with respect to the generations, but it would be desirable, indeed, to split the profiles with respect to the different components of the \( 16 \), implementing a breaking of the SO(10) symmetry. With this purpose, we exploit the mechanism described for the abelian model in Eq.\((8)\), using the fact that the \( 45_\Phi \) field contains a SU(5) singlet which, taking a VEV, is responsible of breaking SO(10) down to SU(5) \( \times \) U(1)\( _X \). Such a VEV generates different contributions to the bulk masses of the SU(5) components of each \( 16 \) bulk multiplet, proportional to their U(1)\( _X \) charges, causing the splitting of the profiles. Under SU(5) \( \times \) U(1)\( _X \) the \( 16 \) decomposes as

\[
16 = 10_{-1} + \bar{5}_3 + 1_{-5}
\]

with the U(1)\( _X \) charges \( Q^X \) given in subscript. Each SU(5) multiplet gets an effective bulk mass \( m^r_i \) (\( r = 10, 5, 1 \)) given by

\[
m^r_i = m_i - \sqrt{2}g_5 Q^X \sqrt{2} v_3 / 2
\]

Schematically, from the initial unified picture of the \( 16 \), we end up with the splitting of bulk masses and relative profiles, for each generation, as:

\[
m^1_i \xrightarrow{45_\Phi} m^{10}_i, m^5_i, m^1_i
\]

\[
f^{16}_i \xrightarrow{45_\Phi} f^{10}_i, f^5_i, f^1_i
\]

The effective Yukawas for the charged fermions are then, resembling the scheme in Eq.\((5)\):

\[
\mathcal{Y}_u = F_{10} Y_u F_{10}, \ \mathcal{Y}_d = F_{10} Y_d F_{\bar{5}}, \ \mathcal{Y}_e = F_{\bar{5}} Y_e F_{10}
\]
where the entries of diagonal matrices $F_r$ are the zero-mode profiles evaluated at the $y = 0$ brane:

$$F_r = \text{diag}(f_{1r}^r, f_{2r}^r, f_{3r}^r), \quad f_{ir}^r = \sqrt{\frac{2m_i^r}{1 - e^{-2m_i^r/\pi R}}}.$$  \hspace{1cm} (13)

The mass matrix of light neutrinos is obtained through the type I seesaw mechanism and is proportional to

$$m_{\nu} \propto F_5 Y_{\nu}^{-1} Y^T_{\nu} F_5.$$  \hspace{1cm} (14)

where $Y_{\nu}$ is the Yukawa of the Dirac mass term for neutrinos and $Y_R$ the one of Majorana mass term for RH neutrinos.

It’s quite remarkable that in this mechanism, designed by the gauge group structure and by the SUSY constraints, only one parameter, $\nu_{\Phi}$, is responsible of the profiles splitting into the three SU(5) submultiplets, so that, overall, only 4 parameters $(m_i, \nu_{\Phi})$ describe 9 different profiles. Implementing a compactified extra dimension at the level of the SM gauge group would require, instead, to introduce 15 different bulk masses (one for each representation and generation) without even describing RH neutrinos. Thus, the present SO(10) model would be much more predictive, but we have to verify its viability by fitting the fermion masses and mixing angles: this fit is not so trivial, since the profiles are still unified in the SU(5) multiplets and the splitting is not so flexible, but dictated by the fixed U(1)$_X$ charges.

Let’s now discuss the particular Higgs sector of our model to see how this scenario is concretely implemented.

The following expression describes the superpotential at the branes as chosen in our framework and characterizing the main difference with respect to the original model:

$$W_{\text{branes}} = \frac{\delta(y)}{\Lambda} \left[ Y_{16}^{ij} 16,16,10_H + Y_{120}^{ij} 16,16,120_H + Y_{126}^{ij} 16,16,126_H + \ldots \right]$$

$$+ \delta(y) w_0(45_H, 10_H, 126_H, \overline{126}_H, 120_H)$$

$$+ \delta(y - \pi R) w_\pi(126_H, \overline{126}_H).$$  \hspace{1cm} (15)

The first line describes the Yukawa interactions localized at the $y = 0$ brane. $\Lambda > M_{\text{GUT}}$ is a cut-off scale and here all the Yukawas arise from operators of the same dimensionality. The $\overline{126}_H$, which contains an SU(5) singlet, has the following roles: 1) it provides a Majorana mass term to the RH neutrinos; 2) its VEV is responsible for breaking the residual U(1)$_X$ symmetry; 3) together with a $126_H$ on the same brane and another pair of $\overline{126}_H$ and $126'_H$ located on the other brane, it is responsible for restoring the N=1 SUSY on the branes which otherwise is violated by the non-vanishing D-terms arising from the VEV $(45_{\Phi})$, because of its non trivial profile (for more details, see [1]); 4) it is a heavy field, with a mass $\sim O(M_{\text{GUT}})$, an important feature for solving the doublet-triplet splitting problem, as we are going to discuss.

The field $45_H$ does not appear in the Yukawas, but it is responsible of breaking SU(5) down to the SM gauge group; it is an heavy field with mass $\sim O(M_{\text{GUT}})$, also entering in the mechanism

\begin{itemize}
  \item[2] 5D N=1 SUSY, equivalent to 4D N=2 SUSY in the bulk, forbids to couple to $16$ and $16^c$ another chiral multiplet in the $45$ representation, whose coupling would not be universal, but a $3 \times 3$ matrix in the flavour space, introducing much more parameters.
  \item[3] In the original model [2], instead, higher dimension operators are necessary to correct the Yukawas, with $\Lambda$ required to be $O(M_{\text{GUT}})$, spoiling the consistency of the effective theory approach underlying the whole model.
\end{itemize}
solving the doublet-triplet splitting problem. 

The $10_H$ and $120_H$ contains pairs of weak doublets that play the role of the MSSM Higgses, being responsible of the EW symmetry breaking and giving mass to the charged SM fermions; the Yukawa couplings of charged fermions are then linear combinations of $Y_{10}$ and $Y_{120}$. The combination of these two fields permits to distinguish the Yukawa couplings of down-type quarks from those of the charged leptons, that otherwise would result equal because of the unified representation under SU(5): this choice permits to maintain viable the splitting of the profiles down to the SU(5) multiplets, without need to go further and distinguishing with respect to the SM representations (for which a similar mechanism of splitting through spontaneous symmetry breaking cannot be implemented).

The above Higgs content on the brane, with the distinction made between heavy and light fields, can address the doublet-triplet splitting problem through the missing partner mechanism. As can be seen from the decomposition under the SM gauge group, the set of light fields ($10_H$ and $120_H$) contains three pairs of weak doublets and three pairs of color triplets, while the set of heavy fields ($126_H$, $126_H$, $45_H$) contains three pairs of triplets but only two pairs of doublets.

All the doublets and triplets of the different sectors get mixed with each other when $45_H$ takes a VEV (from operators coupling these fields in the superpotential $w_0$, see [1] and [13]): the light triplets get mixed with the same number of heavy ones and all of them get a mass at the GUT scale, while in the case of doublets, all pairs get a mass at GUT scale but one linear combination of doublets remains massless because of the “missing partner” in the heavy sector: it is this combination to be identified as the MSSM Higgs doublets.

Explicitely, the Yukawa couplings of Dirac type fermions, entering Eq. (12), are given by:

$$
Y_u = 2\sqrt{2} \alpha_1 Y_{10} + (-2\sqrt{2} \alpha_2 + \frac{2}{3} i \sqrt{6} \alpha_3) Y_{120},
$$

$$
Y_d = 2\sqrt{2} \bar{\alpha}_1 Y_{10} + (-2\sqrt{2} \bar{\alpha}_2 + \frac{2}{3} i \sqrt{6} \bar{\alpha}_3) Y_{120},
$$

$$
Y_e = 2\sqrt{2} \alpha_1 Y_{10} + (-2\sqrt{2} \alpha_2 - 2i \sqrt{6} \alpha_3) Y_{120},
$$

$$
Y_e = 2\sqrt{2} \bar{\alpha}_1 Y_{10} + (-2\sqrt{2} \bar{\alpha}_2 - 2i \sqrt{6} \bar{\alpha}_3) Y_{120},
$$

where $\alpha_i$ and $\bar{\alpha}_i$ parameters are coefficients of the linear combinations of doublets describing the $H_u$ and $H_d$ Higgs fields of MSSM (up and down type), satisfying $\sum_{i=1}^3 |\alpha_i|^2 = \sum_{i=1}^3 |\bar{\alpha}_i|^2 = 1$.

The Yukawa for RH neutrino, from the Majorana mass term, is simply $Y_R = Y_{126}$.

All these couplings, considering also the symmetry properties of some representations, introduce 35 real parameters (for more details, see [1]), to be added to the 4 real parameters describing the profiles. With this set of parameters we must fit 17 observables (fermion masses and mixing angles). Despite the large number of parameters, the fit is not trivial at all, since all the 35 parameters from the Yukawa sector are required to be anarchical and $O(1)$ for naturalness: the

4 This happens because $120_H$ contains the Higgs doublets pair in different representations of SU(5), in particular the 45 and $\mathbf{35}$ of SU(5), that is known to couple to down-type quarks and charged leptons with different Clebsch-Gordan coefficients.

5 The unequal content of doublets and triplets in the heavy sector arises from the $50$, $\mathbf{50}$ of SU(5) residing in $126_H$, $\overline{126}_H$ which contain only triplets.
parameters with the leading role remain only the 4 describing the profiles \( (m_i, \nu_\Phi) \), responsible of creating the hierarchies.

4. Viability test, naturalness test and predictions

With the above setup, we have performed the numerical fit for testing the viability of the model. As input data we used the experimental values of the fermion masses and mixing parameters extrapolated at the GUT scale in the MSSM. All the parameters of the Yukawas are restricted within the narrow range \( 0.5 \leq |Y_{ij}| \leq 1.5 \). In a first approach, we have optimized all the free parameters, including the Yukawa entries, by the technique of \( \chi^2 \) minimization (for technical details we refer the reader to [1]). A good fit results possible only for large value of tan \( \beta \), in agreement with the unification of bottom quark and tau lepton masses, resulting from the SU(5) representations. Then, fixing \( \tan \beta = 50 \), the fit was done for normal ordering (NO) and inverted ordering (IO) of the neutrino masses, obtaining in both cases an acceptable solution, with \( \chi^2 \approx 0 \) for NO and \( \chi^2 \approx 5.8 \) for IO. Anyway these solutions could still be fine-tuned in the Yukawa entries so, in order to test the naturalness of the \( \mathcal{O}(1) \) Yukawa couplings, we proceed by taking a flat random distribution for all the Yukawa parameters in the range established above, and optimizing by \( \chi^2 \) minimization only the remaining parameters, \( i.e. \) the Higgs mixing parameters and the set of bulk masses and VEV parametrizing the profiles. In fig. 1 we plotted the distribution of the minimal \( \chi^2/\nu \), where \( \nu \) is the number of independent degrees of freedom, obtained in the case of NO and IO. From this plot it results very clearly how the NO is favoured, having a distribution peaked on lower \( \chi^2 \) values. In terms of goodness of fit characterized by the \( p \)-value, we find \( p > 0.05 \) for 2.2\% of cases in the NO and only for \( 10^{-3}\% \) in the IO. From this picture we conclude that the proposed model has an acceptable naturalness only for the NO case.

For the selected cases of NO with \( p > 0.05 \), which fit well the experimental data and respect naturalness, we find the distribution for some physical observables that can be predicted after the fit:

- the lightest neutrino mass, shown in fig. 2, results restricted to be \( \lesssim 5 \) meV, corresponding to a hierarchical neutrino mass spectrum. Unfortunately this prediction is far from the sensitivity of the current experiments.

![Figure 1](image-url)
The distribution for the lightest neutrino mass (NO, $\tan \beta = 50$, selected cases with $p$-value $> 0.05$).

- the effective mass $m_{\beta\beta}$, shown in fig. 3, is predicted in the range 0.1-5 meV. Also this prediction is far from the sensitivity of the current experiments.

The distribution for the effective mass $m_{\beta\beta}$ (NO, $\tan \beta = 50$, selected cases with $p$-value $> 0.05$)

- the Dirac CP violating phase, shown in fig. 4, has an almost uniform distribution, allowing the entire range in CP phase.
- the RH neutrino masses, shown in fig. 5, result in an extremely hierarchical spectrum, $M_{N_1} \ll M_{N_2} < 10^9$ GeV. This kind of spectrum turns out to be incompatible with the models of standard thermal leptogenesis, both in a flavour independent or dependent scheme, requiring $M_{N_1} \gg 10^9$ or $10^{12}$ GeV $\gtrsim M_{N_2} \gtrsim 10^9$ GeV and $M_{N_1} \ll 10^9$ GeV.

5. Conclusion

In conclusion, we have demonstrated that a model realized on 5D SO(10) SUSY GUT can nicely address the flavour problem taking fundamental anarchical $O(1)$ Yukawas and creating the observed hierarchies by introduction of only four parameters: three bulk masses describing the profiles along the extra dimension for the three generations of $16$ (unified representation of all the...
Figure 4. The distribution for Dirac CP violating phase (NO, tan $\beta = 50$, selected cases with $p$-value $> 0.05$)

Figure 5. The distribution for the RH neutrino masses (NO, tan $\beta = 50$, selected cases with $p$-value $> 0.05$)

SM fermions), plus a VEV responsible of splitting the profiles, within a mechanism of symmetry breaking, with respect to the SU(5) submultiplets. The scalar sector of the theory is chosen also to address the doublet-triplet splitting problem through the missing partner mechanism. It’s significative that in this 5D context the MSSM Higgs fields can be chosen in the 10 and the 120 representations of SO(10), a combination that in 4D doesn’t allow to fit the data well.

The viability of the model has been checked by fitting the the fermion masses and mixing angles and a test of naturalness has be performed, showing a clear preference for the case of NO in neutrino masses.

The model has some predictions for unobserved quantities, that result unfortunately not measurable in the current or next-future experiments. This makes the model not possible to be tested directly, but still falsifiable.

Let’s finally remark that this model has been analyzed in the ideal limit of exact N=1 SUSY, neglecting soft SUSY breaking contributions with a characteristic scale in the range $1 \div 10$ TeV. It would be desirable, in a further project, to make the model more realistic by proposing a viable mechanism of N=1 SUSY breaking (possibly exploiting the extra dimension) and specifying the
sparticle spectrum.

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