Output Power Maximization of DFIG Wind Turbine using Linear MPC Technique

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Abstract. Wind energy conversion systems have been attracting wide attention as a renewable energy source. To extract maximum energy from the wind turbine, an efficient controller plays an important role. The target of this paper is to develop a Linear Model Predictive Control (MPC) to maximize power production according to wind speed. Firstly, the DFIG wind turbine model was linearized at a specific operating point by using the Jacobian method. The MPC then was developed based on the linearized model where wind speed equal to 8 m/s is chosen as its operating area. The controller was tested to deal with different wind speed. A presence of a certain range of wind speed errors was included to evaluate the controller efficiency. Numerical simulation was done by using MATLAB software. The proposed controller has shown great performances when within its operating area but downgraded when moving away from its operating area. Imprecise wind speed measurement has shown a significant impact on the controller efficiency.

1. Introduction

Wind energy is getting a lot of attention from the industry nowadays as of the source of energy conversion. It is one of the most common and widely used forms of renewable energy. Due to the nature of the wind that always fluctuates, an effective control strategy is highly desired especially by the wind industry to improve the performance of wind turbine. The controller typically used to regulate the rotor speed to trace its optimal value to maximize power production from the wind turbine, as a result, reduce the power generation cost. There is a huge quantity of study covering control of a wind turbine such as operational control, control of power electronics in wind energy, and control of supervisory level. In this work, the focus will be only limited to operational control.

Nowadays, a doubly-fed induction generator (DFIG) is mostly used for wind turbines because of its characteristic that it is able to operate at variable speed, making it simpler to work at different speeds of the wind. A DFIG can gracefully control at a consistent voltage and steady frequency while its rotor revolution speed changes [1]. DFIG wind turbines use a wound rotor induction generator, where the
rotor winding is driven by sequential frequency factor, voltage source and converters [2]. The stator is normally wired directly to the grid, and the rotor is interfaced by a variable frequency converter. Due to the dependability to the wind speed, variable speed wind turbine’s operation can be classified into four regions, as shown in figure 1. In the first region (Region I), the wind speed is below the cut-in speed, \( v_{\text{min}} \) which means that the wind speed is too low to move the wind turbine. For the second region (Region II) which is also also known as partial load region, the wind is below the rated speed, \( v_{\text{rated}} \). In this region is where the controller plays a very important role as the wind turbine need to extract as much energy as possible from the wind. For full load region known as Region III, since there is a limit for the wind turbine and the equipment itself, the wind turbine is only allowed to operate according to the rated power, \( P_{\text{rated}} \). If the wind speed is exceeding the rated speed \( v_{\text{max}} \), which is situated in Region IV, the wind turbine will stop operating to prevent and mechanical damage to the wind turbine itself.

However, in this paper, the proposed controller will only cover in region II.

The controller’s key purpose is to enable the wind turbine to generate a pre-specified desired amount of power from the wind according to the wind speed. In addition, it is also intended to ensure that the closed-loop system is stable over the entire operating period. From the literature, Model Predictive Controller (MPC) is proven one of the best controllers for wind turbine regulation [3-5]. Criteria provided by the MPC make it proficient to extract the optimum power from wind turbine especially in dealing with multivariable constrained control problems. A tractable predictive control of the nonlinear wind turbine is formulated as the concept of the linear MPC is extended. Based on the MPC developed and explored for DFIG [6], the controller was assumed to be able to use wind speed over a particular future time horizon (within seconds to minutes), offering the maximum comparison quality over a specific future horizon. Potential wind speed projection can be achieved through the use of statistical models or focused on nonlinear systems, i.e. neural networks or fuzzy models. It is worth to mention that MPC needs optimization online, which would be a considerable computing burden. Consequently, MPC was primarily used in relatively slow dynamic systems. However, recently, several MPC approaches have been suggested to minimize digital mathematical complexity [6], [7] enabling us to extend the MPC to fast-dynamic systems such as mechanical and electrical structures.

This paper will discuss the performances of Linear MPC in maximizing the output power of the DFIG wind turbine. The effectiveness of the proposed control scheme is evaluated through MATLAB simulation in several situations.

![Figure 1. Power curve of a variable speed wind turbine](image)

**2. DFIG Wind Turbine Model**

In this work, 2 MW DFIG wind turbine is considered. All the important specifications used in the simulation are listed in Table 1. For controller design purpose, the DFIG wind turbine model is initially converted into an equivalent state-space representation such as follow:

\[
\dot{x} = Ax + Bu \\
y = Cx + Du
\]  

where the state, input and output vectors are defined as follows:
\[ x = [i_{dr} \ i_{ds} \ i_{qr} \ i_{qs} \ \omega_r] \]
\[ u = [v_{dr} \ v_{qr}]^T \]
\[ u = [\omega_r] \]  

(2)

To derive the state-space, Jacobian method is used to linearize the following nonlinear system at the specific operating point, which consists of three main parts i.e the aerodynamic part, the electrical part and mechanical part:

2.1. Aerodynamic part
The output power can be estimated from [8] [9]
\[ P_m = 0.5 \rho \pi R^2 v_w^3 C_p(\lambda, \theta) \] 

(3)

where \( \rho \) denotes the air density, \( R \) is the rotor radius, \( v_w \) is the wind speed in m/s and \( C_p(\lambda, \theta) \) is the wind turbine power coefficient as a function of \( \lambda \) and \( \theta \), which are respectively the tip-speed ratio and the blade pitch angle. The \( C_p \) characteristic function can be expressed as

\[ C_p = 0.5 \left( \frac{0.98}{\lambda_i} - 0.4 \theta - 5 \right) e^{16.5 \theta} \]

\[ \frac{1}{\lambda_i} = \frac{1}{\lambda + 0.08 \theta} - \frac{0.035}{\theta^3 + 1} \]

(4)

where \( \lambda \) is given by
\[ \lambda = \frac{\omega_r R}{v_w} \]

(5)

\( \omega_r \) in equation (5) defines the turbine rotor speed. The generator speed, \( \omega_g \) is associated with \( \omega_r \) through the gear ratio, \( \eta_g \), hence can be simply estimated from \( \omega_g = \eta_g \omega_r \).

2.2. Electrical part
The electrical dynamics that drive the DFIG are

\[ \epsilon \frac{di_{dr}}{dt} = (L_m V_{ds} - L_s V_{dr}) + (L_m \omega_s \psi_{qs} - L_s (\omega_s - \omega_m) \psi_{qr}) + (L_s R_r i_{dr} - L_m R_s i_{ds}) \]

\[ \epsilon \frac{di_{ds}}{dt} = (L_m V_{dr} - L_r V_{ds}) - (L_r \omega_s \psi_{qs} - L_m (\omega_s - \omega_m) \psi_{qr}) + (L_r R_s i_{ds} - L_m R_r i_{dr}) \]

\[ \epsilon \frac{di_{qr}}{dt} = (L_m V_{qs} - L_s V_{qr}) - (L_m \omega_s \psi_{ds} - L_s (\omega_s - \omega_m) \psi_{dr}) + (L_s R_r i_{dr} - L_m R_q i_{qs}) \]

\[ \epsilon \frac{di_{qs}}{dt} = (L_m V_{qr} - L_r V_{qs}) + (L_r \omega_s \psi_{ds} - L_m L_s (\omega_s - \omega_m) \psi_{dr}) + (L_r R_s i_{qs} - L_m R_r i_{qr}) \]

(6)

where \( \epsilon = (L_m^2 - L_r L_s)/2\pi(50) \), \( \psi_{ds} = L_s i_{ds} + L_m i_{dr} \), \( \psi_{qs} = L_s i_{qs} + L_m i_{qr} \), \( \psi_{dr} = L_r i_{dr} + L_m i_{ds} \) and \( \psi_{qr} = L_r i_{qr} + L_m i_{qs} \).

As a result, the electrical torque is generated from

\[ T_e = \psi_{qr} i_{dr} - \psi_{dr} i_{qr} = \psi_{ds} i_{qs} - \psi_{qs} i_{ds} \] 

(7)
2.3. Mechanical part

Meanwhile, the wind turbine rotor dynamics can be represented by

$$J_m \frac{d \omega_g}{dt} = 0.5(T_m - T_e - F \omega_g)$$

(8)

where $T_m = P_m/\omega_m$ is the mechanical torque and $F$ is the viscous damping.

3. Model Predictive Controller Design

The principle of the MPC is to choose the command move by addressing an optimal control issue digitally again and again. The aim is to minimize a criterion of performance known as cost function, $J$ over a future horizon and may be subject to constraints on controlled inputs and outputs. Future outputs are predicted by using an internal model where the prediction uses both previous and current input and output values to plan optimal controlling actions for the future.

The proposed MPC is designed based on the linearized wind turbine model as per equation (1) to (2). For Region II wind turbine operating area, the primary objective is to maximize the output power by maximizing the aerodynamic efficiency. From equation (4), $C_p$ is possibly maximized if an optimum $\lambda$ is achieved while $\theta = 0$ (typically assumed for below-rated region). In this case, the optimum $\lambda$ is known approximately to 6.8 for that particular wind turbine. Thus, the main task of the controller is to provide optimal rotor speed tracking by using the wind speed overview in which, the optimum rotor speed, recognized as $\omega_d$ can be derived from equation (5) when the optimum $\lambda$ is substituted. The following cost function is used in the proposed MPC as for output reference tracking purpose;

$$J(Z_k) = \sum_{j=1}^{n_y} \sum_{i=1}^{p} \left( \frac{w_{i,j}^y}{s_{j}^y} \left[ \omega_{d,j}(k+i|k) - \omega_{r,j}(k+i|k) \right] \right)^2$$

(9)

where each notation is defined as;
- $k =$ current control interval
- $p =$ prediction horizon
- $n_y =$ number of plant output variables
- $\omega_{d,j}(k+i|k)$ = predicted value of plant output (rotor speed)
- $\omega_{r,j}(k+i|k)$ = reference value of plant output (desired rotor speed)
- $s_{j}^y =$ scale factor for plant output

| Table 1: Important specifications of 2 MW wind turbine |
| --- | --- | --- |
| Symbol | Parameter | Value |
| $v_{\text{min}}$ | Cut-in wind power | 4 m/s |
| $v_{\text{rated}}$ | Rated wind speed | 12.35 m/s |
| $v_{\text{max}}$ | Cut-out wind speed | 20 m/s |
| $R$ | Rotor radius | 37.5 m |
| $J_m$ | Rotor inertia | 4.5207 s |
| $F$ | Viscous damping | 0.01 |
| $\eta_g$ | Gear ratio | 84.15 |
| $R_s$ | Stator resistance | 0.0049 p.u. |
| $R_r$ | Rotor resistance | 0.0055 p.u. |
| $L_s$ | Stator inductance | 4.0454 p.u. |
| $L_r$ | Rotor inductance | 4.0525 p.u. |
| $L_m$ | Mutual inductance | 3.9530 p.u. |
$w_{ij}^y$ = tuning weight for plant output
$Z_k$ = QP decision, given by

$$Z_k = [u(k|k)^T \ u(k+1|k)^T \ \cdots \ u(k+p-1|k)^T \ e_k]$$

At each sampling instant, the MPC will solve the following optimal control problem:

$$\min_{u_k} J(Z_k)$$

subject to

$$u_{\text{min}} < u_k < u_{\text{max}}$$

In most cases, the controlled variables have been set with minimum and maximum constraints to satisfy the hardware limitations.

![Overall closed-loop system](image)

**Figure 2.** Overall closed-loop system

### 4. Case study and Simulation Result

The overall closed-loop system that includes the proposed Linear MPC is displayed in figure 2. The system was developed and tested using Matlab software. As aforementioned, the proposed controller was particularly designed for the operating point at $v_w = 8 \text{ m/s}$. Two case studies are conducted to evaluate the controller performance i.e. when dealing with various wind speed in between 6 to 11 m/s and also when the wind speed reading contains a range of error.

#### 4.1. Controller efficiency analysis at different wind speed

The data that has been collected is the controlled variables $v_{dr}$ and $v_{qr}$ and the rotor speed $\omega_r$. Subsequently, the generator speed $\omega_g$, aerodynamic power, $P_m$ and power coefficient, $C_p$ can be calculated from equation (3) to (4). All simulation results are recorded and filled in Table 2.

| Wind speed $v_w$ (m/s) | Desired rotor speed $\omega_d$ (p.u) | Controlled variable | Output $v_{qr}$ (p.u) | $\omega_r$ (p.u) | $\omega_g$ (p.u) | $P_m$ (p.u) | $C_p$ (p.u.) |
|------------------------|-----------------------------------|---------------------|------------------------|----------------|----------------|-------------|-------------|
| 6                      | 0.5                               | 0.25                | 0.01344                | 0.5866         | 49.3623        | 0.1364      | 0.4666      |
| 7                      | 0.6                               | 0.2254              | 0.01344                | 0.6004         | 50.5230        | 0.2183      | 0.4703      |
| 8                      | 0.7                               | 0.08933             | 0.007174               | 0.7            | 58.905         | 0.3256      | 0.47        |
| 9                      | 0.8                               | -0.02653            | -0.002131              | 0.8            | 67.32          | 0.4633      | 0.4697      |
| 10                     | 0.9                               | -0.1424             | -0.01143               | 0.9            | 75.735         | 0.6551      | 0.4694      |
| 11                     | 1.0                               | -0.25               | -0.01344               | 0.9676         | 81.4235        | 0.8462      | 0.4699      |
Figure 3. Controlled $v_{dr}$ at different wind speed, $v_w$

As depicted in figure 3, $v_{dr}$ reaches its highest value when the wind speed is at 6 m/s and reaches its lowest value when the wind speed is at 11 m/s. The maximum and minimum for $v_{dr}$ is 0.25 p.u and -0.25 p.u respectively. The values do not exceed its maximum and minimum value because of the constraints that have been set by the MPC.

Figure 4. Controlled $v_{qr}$ at different wind speed, $v_w$

The value of $v_{qr}$ is a lot smaller than $v_{dr}$. From figure 4, it can be seen that the value of $v_{qr}$ also has the same characteristic as $v_{dr}$ where it reaches its highest value when the wind speed is at 6 m/s and reaches its lowest value when the wind speed is at 11 m/s. The maximum and minimum value of $v_{qr}$ is 0.01344 p.u. and -0.01344 p.u. respectively satisfying the constraints set by the controller.
As shown in Figure 5, the rotor speed starts to operate from the initial point (0.7771) then gradually follows the desired rotor speed that has been set. However, for the desired rotor speed of 0.5 p.u and 1.0 p.u, the rotor speed failed to achieve and only manage to achieve the rotor speed of 0.5866 p.u and 0.9676 p.u respectively. This is due to the desired rotor speed is far from the operating point where the controller is specifically designed for.

The output power is highly dependent on wind speed. The plotted graph in Figure 6 has demonstrated that higher power will be produced by higher wind speed. The lowest power managed to be produced by the system is 0.1364 p.u. while the highest power is 0.8462 p.u. The maximum and minimum power measured may vary according to the operating point of the controller. As referred to the graph, $C_p$ shows a significant drop as the system moves away from the operating point.

4.2. Controller efficiency analysis due to imprecise wind speed measurement
When the measured wind speed experiencing an error, subsequently the desired rotor speed will also produce an error. This will lead the controller to drive the rotor speed to achieve the false value. This situation hence will result in the power coefficient, $C_p$, to drop as can be seen from Figure 7. The added wind speed error can be computed from $v_f - v_w$. It is noticed that, from the operating point ($v_w = 8$
the controller’s efficiency drops up to 4.19% as $v_f$ increase to 11 m/s but not significantly affected as $v_f$ decrease to 6 m/s.

Obviously shown the rotor speed and motor speed increase as the wind speed increase. The output power, $P_m$ was produced according to the rotor speed as per intended by the MPC. The value of $v_{dr}$ and $v_{qr}$ also does not exceed the value of the constraints that have been set by the MPC. The power coefficient, $C_p$ does produce some error during random windspeed. This is because the controller just plays its role to produce the desired output without knowing any wind speed error, thus resulting in the efficiency to drop.

5. Conclusion

DFIG wind turbine has been modelled and represented in block diagrams in Matlab Simulink. Linear MPC has been specifically designed for the wind turbine to produce optimum power at partial load region. The controller efficiency has been studied and analyzed by applying a random wind speed. It is found that the proposed controller managed to produce the desired output while maintaining the constraints set by the MPC but must be around its operating point. The controller’s efficiency started to downgrade once moving out from its operating area. The controller also has demonstrated poor performances if wind speed measurements contain some errors. One of the reasons is the controller has followed the imprecise set point as a result of the imprecisely measured wind speed. From the analysis,

| Measured wind speed | Desired rotor speed | Controlled variable | Output |
|---------------------|---------------------|---------------------|--------|
| $v_f$ (m/s)         | $\omega_d$ (p.u)   | $v_{dr}$ (p.u)      | $v_{qr}$ (p.u) | $\omega_r$ (p.u) | $\omega_y$ (p.u) | $P_m$ (p.u) | $C_p$ (p.u) |
| 6                   | 0.5                 | 0.25                | 0.01344       | 0.5866            | 49.3624         | 0.3257      | 0.4701      |
| 7                   | 0.6                 | 0.2254              | 0.01344       | 0.6004            | 50.5236         | 0.3259      | 0.4704      |
| 8                   | 0.7                 | 0.08933             | 0.007174      | 0.7               | 58.905          | 0.3256      | 0.47        |
| 9                   | 0.8                 | -0.02653            | -0.002131     | 0.8               | 67.32           | 0.3225      | 0.4655      |
| 10                  | 0.9                 | -0.1424             | -0.01143      | 0.9               | 75.735          | 0.3169      | 0.4575      |
| 11                  | 1.0                 | -0.25               | -0.01344      | 0.9676            | 81.4235         | 0.3119      | 0.4503      |

- Table 3. Simulation results at different imprecise measured wind speed, $v_f$ while the real wind speed, $v_w$ is at 8 m/s

- Figure 7. Power coefficient, $C_p$ versus maximum power coefficient, $C_{p_{max}}$ at different imprecise measured wind speed, $v_f$ while the real wind speed, $v_w$ is at 8 m/s
it is verified that a set of Linear MPC needs to be designed to cover the whole operating region of the nonlinear wind turbine. Also, accurate readings of wind speed are very crucial in the wind turbine control system to ensure the output power is maximized.

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