Degenerate and critical domain walls and accelerating universes driven by bulk particles

A. de Souza Dutra\textsuperscript{a,b,*}, A.C. Amaro de Faria Jr.\textsuperscript{b} and M. Hott\textsuperscript{b†}

\textsuperscript{a}Abdus Salam ICTP, Strada Costiera 11, Trieste, I-34100 Italy.
\textsuperscript{b}UNESP-Campus de Guaratinguetá-DFQ\textsuperscript{‡}
Departamento de Física e Química
12516-410 Guaratinguetá SP Brasil

March 31, 2009

Abstract

We consider a scenario where our universe is taken as a three-dimensional domain wall embedded in a five-dimensional Minkowski space-time, as originally proposed by Brito and collaborators [1]. We explore the existence of a richer class of soliton solutions and their consequences for the acceleration of the universe driven by collisions of bulk particle excitations with the walls. In particular it is shown that some of these solutions should play a fundamental role throughout the expansion process.

PACS numbers: 11.27.+d, 11.10.Lm, 95.35.+d, 98.80.Cq

\textsuperscript{*}E-mail: dutra@feg.unesp.br
\textsuperscript{†}e-mail: hott@feg.unesp.br
\textsuperscript{‡}Permanent Institution
1 Introduction

Very important systems described by quantum field theories are intrinsically nonlinear; the Standard Model and the Quantum Chromodynamics are classical examples. By exploring deeply such systems or effective nonlinear models, even at the classical level, one has shown the increasing importance of the soliton solutions (classical solutions with finite and localized energy) and their broad applications [2]-[5]. Soliton solutions in quantum field theory describe, for instance, monopoles, magnetic vortices, instantons in quantum chromodynamics, cosmic strings and magnetic domain walls. Finding exact classical solutions, particularly solitons, is one of the problems on nonlinear models with interacting fields. When one has in hands a systematic method, as the one offered by Rajaraman [6], this task becomes easier, even though one has to explore the consequences of the classical solutions, as well as their possible realizations in the nature. As pointed out by R. Rajaraman and E. Weinberg [7], in such nonlinear models more than one time-independent classical solution can exist and each one of them corresponds to a different family of quantum states, which come into play when one performs a perturbation around those classical solutions.

The method in [6], usually called trial orbits method, is a very powerful one presented for finding exact soliton solutions for nonlinear second-order differential equations of models with two interacting relativistic scalar fields in 1+1 dimensions, and it is model independent. The trial orbits method have been applied to the special cases whose soliton solutions of the nonlinear second-order differential equations are equivalent to the soliton solutions of first-order nonlinear coupled differential equations, the so-called Bolgomol’nyi-Prasad-Sommerfeld (BPS) topological soliton solutions [8]. Some years ago one of us presented a method for finding additional soliton solutions for those special cases whose soliton solutions are the BPS ones [9]. Latter, that approach was extended by considering more general models [10]. Furthermore, that method shows how to get the general equation of the orbits, explains how the different solutions connect the different vacua of the model under analysis and, as a novelty, presents a class of soliton solutions with a kink-like profile for both fields with its minimum energy (BPS energy) smaller than that of the usual solution which exhibits a kink-like configuration for one of the fields and a lump-like configuration for the other one. Moreover, the stability of the quantum states corresponding to these new soliton solutions can be shown on the same basis presented in [11].

The BPS soliton solutions have found applications in a great variety of natural systems whose dynamics can be approximately described by nonlinear quantum field models for interacting scalar fields in 1+1 dimensions [12]. Those kind of models have been generalized by including into the Lagrangian density some minimal terms that break Lorentz and CPT symmetries [13]. By analyzing the consequences of additional soliton solutions in a given nonlinear model, some of us have shown [14], by using the method developed in [9], that those nonlinear Lorentz breaking models in 1+1 dimensions exhibit additional soliton solutions whose BPS energies are smaller than those found in [13], and that more general Lorentz breaking models in 1+1 dimensions, which admits soliton solutions, can be built.

In this work, we explore more deeply the classical solutions found in [9], particularly a class of degenerate soliton solutions [15] [16], for the nonlinear model of two interacting scalar...
fields in 1+1 dimensions [12]. In special, we analyze the consequences that those additional soliton solutions bring for the scenario of accelerating universes. This scenario was recently conceived by Brito and collaborators [1] and it is within the context of the extra-dimensions [16]-[24]. Our analysis is done by following a similar approach to that of the reference [1]. It is shown that, for a critical value of the degeneracy parameter, the reflection coefficient of the bulk particles over the wall is maximized. That implies into a degree of expansion of the universe which is compatible with the present observed acceleration.

In the second section we present the model we are going to work with and discuss the influence of each set of soliton solutions, in the context of accelerating universes driven by the quantum states (bulk particles). In the third section we present final comments about the impact of those new solutions over the expanding process.

2 Domain walls and accelerating universes

A very important and intriguing modern physical problem is that of finding a way to explain the observed accelerated expansion of the universe. On the other hand, the recent cosmological data indicates that a relevant part of the energy of the universe would be a kind of dark energy [25]. In fact, that dark energy is supposed to be the responsible for that acceleration. As a consequence, many authors look for a deep understanding of these subjects. A very interesting possibility is the one associated to the so-called brane worlds [24]. In a recent work, Brito and collaborators [1] conceived a scenario where our universe is taken as a three-dimensional domain wall embedded in a five-dimensional Minkowsky space-time, where the elastic collisions of bulk particles with the walls would be the ultimate reason for the universe acceleration.

The model under analysis here is precisely the same one considered in the reference [1], which is the scalar sector of a five-dimensional supergravity theory obtained by means of dimensional compactification of a higher dimensional supergravity, where the Lagrangian density [1] looks like

$$\mathcal{L} = \left| \det g_{\mu
u} \right| \frac{1}{2} \left\{ -\frac{1}{4} M^3 R_5 + G_{AB} \left[ \partial_\mu \phi^A \partial^\mu \phi^B - \frac{1}{4} \frac{\partial W(\phi)}{\partial \phi^A} \frac{\partial W(\phi)}{\partial \phi^B} \right] + \frac{1}{3} M^3 W(\phi)^2 \right\}, \quad (1)$$

where $1/M$ is the five-dimensional Planck length, $R_5$ is the Ricci scalar, and $G_{AB}$ is the metric on the scalar target space. Now, considering the limiting situation where $M >> 1 << M_{Pl}$, so that the five-dimensional gravity decouples from the scalars, one is left with a 3-brane which is a classical 3d domain wall embedded in a 5d Minkowski space [1]. Thus, in this limit, and choosing the case with two scalar fields in the target space, we are working with a nonlinear model with interacting scalar fields in 1+1 dimensions whose time-independent equations of motion (Eqs. (6) below), are those one-dimensional static equations for the scalar fields taken into account in the reference [1]. We follow the same route as in that reference, but here we consider more general set of static solutions and compare our results with those obtained in [1] where the bulk particles, which are the quantum excitations around
the classical solutions, collide with a 3d domain wall described by a kink. The essential idea is to show that the situation is richer than analyzed in [1], and that from a complete solution as the one we discuss here, important consequences for the expanding scenario show up.

The model was introduced before to study BPS soliton solutions [? and, recently, it was explored in the context of brane worlds [16]. It consists of two interacting real scalar fields in 1+1 dimensions and it is described by the Lagrangian density

\[ \mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi)^2 + \frac{1}{2} (\partial_{\mu} \chi)^2 - V(\phi, \chi), \]  

(2)

where the potential is given by

\[ V(\phi, \chi) = \frac{1}{2} \lambda^2 (\phi^2 - a^2)^2 + (2\mu^2 + \lambda \mu) \phi^2 \chi^2 - \lambda \mu a^2 \chi^2 + \frac{1}{2} \mu^2 \chi^4. \]  

(3)

The distinctive property of this model is that its potential can be written in terms of a so-called superpotential as

\[ V(\phi, \chi) = \frac{1}{2} \left( \frac{\partial W(\phi, \chi)}{\partial \phi} \right)^2 + \frac{1}{2} \left( \frac{\partial W(\phi, \chi)}{\partial \chi} \right)^2, \]  

(4)

where the superpotential is

\[ W(\phi, \chi) = \phi \left[ \lambda \left( \frac{\phi^2}{3} - a^2 \right) + \mu \chi^2 \right]. \]  

(5)

Hence, finding the classical solutions with minimum energy for the time-independent equations of motion

\[ \frac{d^2 \phi}{dx^2} = \frac{\partial V}{\partial \phi} \quad \text{and} \quad \frac{d^2 \chi}{dx^2} = \frac{\partial V}{\partial \chi}, \]  

(6)

is equivalent to find the classical solutions with minimum energy for the time-independent first-order differential equations

\[ \phi' = W_\phi(\phi, \chi), \quad \chi' = W_\chi(\phi, \chi). \]  

(7)

In the above equations the prime means the derivative with respect to the space coordinate, and \( W_\phi (W_\chi) \) stands for the partial derivative of \( W(\phi, \chi) \) with respect to the \( \phi (\chi) \) field. The minimum energy (BPS energy) [8] for nonlinear systems described by the Lagrangian density [2] with potentials written as [1] is found to be given by

\[ E_{BPS} = \left| W(\phi_j, \chi_j) - W(\phi_i, \chi_i) \right|, \]  

(8)

where \( \phi_i \) and \( \chi_i \) mean the \( i - \text{th} \) vacuum states of the model.

Applying the method developed in [9], we note that it is possible to write the relation \( d\phi/W_\phi = dx = d\chi/W_\chi \), where the differential element \( dx \) is a kind of invariant. Thus, one is lead to
\[
\frac{d\phi}{d\chi} = \frac{W_\phi}{W_\chi}. \tag{9}
\]

This is in general a nonlinear differential equation relating the scalar fields of the model. If one is able to solve it completely for a given model, the function \( \phi(\chi) \) can be used to eliminate one of the fields, rendering the equations (7) uncoupled and equivalent to a single one. Finally, the resulting uncoupled first-order nonlinear equation can be solved in general, even if numerically. By substituting the derivatives of the superpotential (5) with respect to the fields in (9), it can be rewritten as a linear differential equation,

\[
\frac{d\rho}{d\chi} - \frac{\lambda}{\mu} \chi = \chi, \tag{10}
\]

by the redefinition of the field, \( \rho = \phi^2 - a^2 \). Now, the general solutions are easily obtained as

\[
\rho(\chi) = \phi^2 - a^2 = c_0 \chi^{\lambda/\mu} - \frac{\mu}{\lambda - 2\mu} \chi^2, \quad \text{for } \lambda \neq 2\mu, \tag{11}
\]

where \( c_0 \) is an arbitrary integration constant and, for the sake of simplicity, we will restrict our study to the case where \( \lambda \neq 2\mu \). Then, we substitute the above solutions in the differential equation (7) and obtain the following first-order differential equation for the field \( \chi(x) \), obtaining

\[
\frac{d\chi}{dx} = \pm 2\mu \chi \sqrt{a^2 + c_0 \chi^{\lambda/\mu} - \frac{\mu}{\lambda - 2\mu} \chi^2}, \quad \lambda \neq 2\mu. \tag{12}
\]

Despite the fact that in general explicit solutions for the above equation can not be obtained, one can verify numerically that the solutions belong to the same classes, and some of those classes of solutions can be written in closed explicit forms. In those last cases we are able to obtain the several types of soliton solutions, whose consequences we discuss below.

The set of solutions used by Brito et al [1], called here type-A kink [2] to distinguish it from other types of kink solutions we present for this model, can be obtained by means of the method described in the previous section just by taking \( c_0 = 0 \) in the expression (12). In this case that equation can be solved analytically for any value of \( \lambda \) and \( \mu \), in the range \( \lambda > 2\mu \). We have the following solutions for \( \chi(x) \) and \( \phi(x) \):

\[
\bar{\chi}_A(x) = a \sqrt{\frac{\lambda - 2\mu}{\mu}} \text{sech}(2\mu ax), \quad \bar{\phi}_A(x) = \pm a \tanh(2\mu ax). \tag{13}
\]

In this case the BPS energy is given by \( E_{BPS}^A = \frac{4}{3} \lambda a^3 \).

Other soliton solutions can be found when one considers the integration constant \( c_0 \neq 0 \) [15]. It was found in [9, 10, 14, 16] that in three particular cases the equation (12) can be solved analytically. For \( c_0 < -2a \) and \( \lambda = \mu \) it was found that the solutions for the \( \chi(x) \) field are lump-like solutions, which vanishes when \( x \to \pm \infty \). On its turn, the field \( \phi(x) \) exhibits a kink-like profile. They also connect the vacua of the model and also have BPS energy \( E_{BPS}^A = \frac{4}{3} \lambda a^3 \). These classical solutions can be written as

\[
\bar{\chi}_A^{(1)}(x) = \frac{2a}{\sqrt{c_0^2 - 4 \cosh(2\mu ax)} - c_0} \quad \text{and} \quad \bar{\phi}_A^{(1)}(x) = a \frac{\sqrt{c_0^2 - 4 \sinh(2\mu ax)}}{\sqrt{c_0^2 - 4 \cosh(2\mu ax)} - c_0}, \tag{14}
\]
An interesting aspect of these solutions is that, for some values of \( c_0 < -2a \), \( \bar{\phi}^{(1)}_A(x) \) exhibits a double kink profile. We can speak of a formation of a double wall structure. In Figure 1 we plot some typical profiles of the soliton solutions in the case where \( \lambda = \mu \), both when \( c_0 \) is close to its critical value (\( c_0 = -2 \) in this case) and far from it. Both fields are there represented. One can verify that the distance from one wall to the other one increases as \( c_0 \) approaches its critical value. For the critical value of \( c_0 \) the double wall structure merges into a single one.

Similar behavior is also noted in the classical solutions for \( \lambda = 4\mu \) and \( c_0 < 1/16 \). In this case the fields profile look like

\[
\bar{\chi}^{(2)}_A(x) = -\frac{2a}{\sqrt{1 - 16c_0}} \cosh(4\mu ax) + 1
\]

and

\[
\bar{\phi}^{(2)}_A(x) = \sqrt{1 - 16c_0}a \frac{\sinh(4\mu ax)}{\sqrt{1 - 16c_0}} \cosh(4\mu ax) + 1.
\]

In the above solution one can also see the double kink profile for some values of \( c_0 \), and the increasing of the distance from one wall to the other as \( c_0 \) approaches its critical value (\( c_0 = 1/16 \) in this case). Once again, at the critical value of \( c_0 \), the double wall structure coalesces into a single wall. In Figure 2 it is shown the behavior of the energy density for \( \lambda = 4\mu \). From that, it is quite evident the appearance of the double walls when one approaches \( c_0 = 1/16a^2 \).

Finally, very interesting analytical soliton solutions were shown to exist when one takes \( \lambda = \mu \) and the critical parameter \( c_0 = -2a \) and for \( \lambda = 4\mu \) and the critical parameter \( c_0 = 1/16a^2 \), in the equation (12). The novelty in these cases is the fact that both, the \( \chi(x) \) field and the \( \phi(x) \) field present a kink-like profile and the BPS energy is half of that of type-A kinks, that is \( E^{B}_{BPS} = \frac{3}{8}\lambda a^3 \). We call this set of solutions as type-B kinks. For \( \lambda = \mu \) and \( c_0 = -2a \) the classical solution for the fields can be shown to be given by

\[
\bar{\chi}^{(1)}_B(x) = \frac{a}{2}(1 \pm \tanh(\mu ax)) \quad \text{and} \quad \bar{\phi}^{(1)}_B(x) = \frac{a}{2}(\tanh(\mu ax) \mp 1).
\]

For \( c_0 = 1/16a^2 \) and \( \lambda = 4\mu \), the following set of type-B kinks is obtained

\[
\bar{\chi}^{(2)}_B(x) = -\sqrt{2}a \frac{\cosh(\mu ax) \pm \sinh(\mu ax)}{\sqrt{\cosh(2\mu ax)}} \quad \text{and} \quad \bar{\phi}^{(2)}_B(x) = \frac{a}{2}(1 \mp \tanh(2\mu ax)).
\]

The type-B solutions also connect the vacua of the model, but this time one could interpret these solutions as two kinds of torsion in a chain, represented through an orthogonal set of coordinates \( \phi \) and \( \chi \). In the plane \( (\phi, \chi) \), the type A kinks correspond to a complete torsion going from \((-a, 0)\) to \((a, 0)\) whereas the type B corresponds to a half torsion, where the system goes from \((-a, 0)\) to \((0, a)\), in the case where \( (\lambda = \mu) \) for instance.

Now, by proceeding as in [1], we perform a linear perturbation of the \( \chi(r, t) \) field around the classical solutions, that is

\[
\chi(r, t) = \bar{\chi}(r) + \zeta(r, t) \quad \text{and} \quad \phi(r, t) = \bar{\phi}(r),
\]
where $\bar{\chi}(r)$ and $\bar{\phi}(r)$ are the classical solutions (background fields) and $\zeta(r,t)$ is the quantum field. By expanding the action up to quadratic terms in the quantum fields we obtain second-order differential equations for the quantum fields

$$\partial_\mu \partial^\mu \zeta + \bar{V}_{\chi\chi}(r) \zeta = 0,$$

that is, the quantum field obeys a Klein-Gordon equation with an effective potential $\bar{V}_{\chi\chi}(r)$ which is obtained by taking the second derivative of the potential given in (3) with respect to $\chi(r)$ and evaluated at the classical solutions $\bar{\chi}(r)$ and $\bar{\phi}(r)$

$$\bar{V}_{\chi\chi}(x) = 2(2\mu^2 + \lambda \mu)\bar{\phi}^2 + 6\mu^2\bar{\chi}^2 - 2\lambda \mu a^2. \tag{21}$$

If we consider our model as a five-dimensional one and that the fluctuations can be expanded in terms of plane-waves in the space-time coordinates, that is

$$\zeta(r,t) = \zeta(r) \exp[-i(\omega t - k_xx - k_yy - k_zz)], \tag{22}$$

with $\zeta(r)$ a function of the fifth coordinate, we obtain the following Schrödinger-like equation

$$\left(-\frac{d^2}{dr^2} + \bar{V}_{\chi\chi}(r)\right) \zeta(r) = \varepsilon \zeta(r), \tag{23}$$

where $\varepsilon = \omega^2 + k_x^2 + k_y^2 + k_z^2$ and we are considering that the static solutions described in the previous section are now functions of the coordinate $r$.

Now, we compare the effect of all the above soliton solutions over the acceleration scenario proposed in [1]. For this, we begin by considering the static solutions given by the equations (14). By substituting them in the effective potential expressed in (21), we find that the Schrödinger-like equation for the quantum excitations is

$$\left(-\frac{d^2}{dr^2} + V_{\text{eff}}^{(A-1)}(r)\right) \zeta(r) = \varepsilon \zeta(r), \tag{24}$$

where

$$V_{\text{eff}}^{(A-1)}(r) = 6\mu^2 a^2 \frac{(c_0^2 - 4) \sinh^2(2\mu a r) + 4}{\left(\sqrt{c_0^2 - 4} \cosh(2\mu a r) - c_0\right)^2} - 2\mu^2 a^2, \tag{25}$$

and, when $c_0 = -2$, the effective potential, for $\mu = \lambda$, becomes

$$V_{\text{eff}}^{(B-1)}(r) = \mu^2 a^2 (1 + 3 \tanh^2(\mu a r)). \tag{26}$$

Note that, in the case analyzed in [1], the effective potential is

$$V_{\text{eff}}^{(A)}(r) = 4\mu a^2 - 4\mu a^2 \left(4 - \frac{\lambda}{\mu}\right) \text{sech}^2(2\mu a r). \tag{27}$$

In Figure 3 it is shown the behavior of the effective potential coming from the domain wall solutions in four situations: the case studied by Brito et al. and for the case of degenerate
solitons when $c_0$ is far from its critical value, near it and at the critical point ($c_0 = -2$). From that Figure one can perceive that, apart from the appearance of the two wells potential, for $c_0$ close to the critical value, the case where the reflection coefficient is bigger is probably the case studied in [1].

The more remarkable result comes to life for $\lambda = 4\mu$. By substituting the equations (15) and (16) in (21), one can verify that the quantum excitations of the $\chi(r, t)$ field satisfies the effective Schrödinger equation (23) with an effective potential given by

$$V_{\text{eff}}^{(A-2)}(r) = 12\mu^2 a^2 \frac{(1 - 16c_0) \sinh^2(4\mu a r) + 2}{\sqrt{1 - 16c_0 \cosh(4\mu a r) + 1}} - 8\mu^2 a^2,$$

and, when $c_0 = 1/16$, the effective potential becomes

$$V_{\text{eff}}^{(B-2)}(r) = \mu^2 a^2 \text{sech}^2(2\mu a r) [2 + 5 \cosh(4\mu a r) + 3 \sinh(2\mu a r)].$$

It can be observed from equation (27) that, for $\lambda = 4\mu$, one has a constant effective potential $V_{\text{eff}}^{(A)}(r) = 4\mu a^2$, so there is no reflection of the bulk particles. This is an artifact of the particular solution considered in [1]. In contrast with this, the solutions presented here lead to effective potentials such that the reflection coefficient increases as $c_0$ approaches its critical value. Those effective potentials are presented in Figure 4. One can note that, for $c_0$ far from and close to its critical value, there is a non-zero probability for the bulk particles to be transmitted, whereas, for the critical case, the effective potential acquires a ramp-like profile, so granting that the bulk particles with energy below the height of the ramp are certainly reflected by that “infinitely large barrier”.

For a given amount of energy available to build up structures in the original universe, we argue that it should be distributed, for instance, like

$$\mathcal{E}_T = \sum_{i=1}^{M} N^{(i)} \mathcal{E}_{BPS}^{(i)},$$

where $N^{(i)}$ stands for the number of defect structures of the $i$-th specimen and

$$\mathcal{E}_{BPS}^{(i)} = E_{BPS}^{(i)} + \frac{k_r^2}{2m},$$

with $E_{BPS}^{(i)}$ being the BPS energy of a given soliton configuration, and $k_r$ the momentum of the excitation with mass $m$ hitting the wall.

In order to estimate the force acting on the wall by the incoming excitations, we consider quasi-elastic collisions to guarantee the energy conservation in the process. In other words, the kinetic energy acquired by the wall comes from the hitting particles. In that case, the momentum transferred to the wall is given by

$$\Delta P \simeq N_{\text{reflected}}(2k_r) = N_{TI} R(2k_r),$$

where $R$ is the reflection coefficient and $N_{TI}$ is the total number of incoming particles. From this, we can conclude that the acceleration of the wall is
a_{wall} = \frac{1}{M_{wall}} \frac{\Delta P}{\Delta t} = \alpha R, \quad (33)

with

\[ \alpha \approx \frac{2}{M_{wall}} \frac{k_r}{N_{TI}} \Delta t, \]

and \( \frac{N_{TI}}{\Delta t} \) is the flux of incident particles.

From now on we follow the brane inflation scenario presented in [1], and similarly in [21]. In that scenario one supposes that we live in a three-dimensional domain wall which is considered as the surface of a hypersphere, whose radius increases due to the hitting bulk particles. The increasing of the radius can be modeled as \( r(t) = a(t) \tilde{r} \), where \( a(t) \) is the known scale factor and \( \tilde{r} \) is the constant radius of the bubble in the comoving system. Consequently, in this cosmological model, we have the metric of the 4d universe given by

\[ ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2). \quad (34) \]

From this point of view, the acceleration of the wall is proportional to the second-derivative in time of the scale factor, that is \( a_{wall} = \ddot{a}(t)\tilde{r} \), and we have the following relation

\[ \ddot{a}(t) = \kappa R. \quad (35) \]

where \( \kappa = \alpha/\tilde{r} \).

It can be seen from Figure 4 that the effective potential barrier width increases when the degeneracy parameter \( c_0 \) approaches its critical value \( c_0^* \). As a consequence, the reflection coefficient becomes asymptotically constant. Conversely, for \( c_0 \) far from the critical value the reflection coefficient tends to zero, since the width and height of the barrier diminish in this limiting situation. Thus, when \( c_0 \) is close to the critical value, one has

\[ a(t) \sim \frac{1}{2} k t^2, \quad (36) \]

which is compatible with the present observed behavior of the universe. Furthermore, in the opposite limit, when \( c_0 \) is far from the critical value, the reflection coefficient approaches zero. In this situation one can suppose that this happens exponentially, so the scale factor is given by

\[ a(t) \sim \frac{\beta}{2} e^{\gamma t}, \quad (37) \]

as requested by an inflationary era.

At this point it is important to remark that \( c_0^* \) is a function of the parameters \( \mu \) and \( \lambda \). This can be noted from the fact that its value changes when the relation among \( \lambda \) and \( \mu \) changes (see the cases analyzed above). This property allows us to suppose that \( c_0^* \) would have a time dependence, as well as the parameters of the potential. That dependence might have its origin in the universe expansion itself. The temperature of the universe decreases along with its expansion and, consequently, the parameters of the potential would exhibit a temperature dependence. Therefore, \( c_0^* \) approaches the value of \( c_0 \) the universe was created with.
3 Final remarks

In this work we have analyzed the impact of a general set of soliton solutions over the reflection coefficient of the bulk particle collisions with a 3d domain wall, as originally proposed in [1]. We have shown that when the potential parameters are such that $\lambda = 4\mu$, the effective potential interacting with the bulk particles have its reflection coefficient arbitrarily larger, depending on the value of the degeneracy parameter $c_0$. A remarkable situation is that when $c_0$ reaches its critical value. In that case, the effective potential becomes a kind of step potential, so that the bulk particles having energy lesser than that of the step potential will always be reflected, so producing a quadratic driven acceleration. The degenerate structure, however, in principle would be created with an arbitrary $c_0$, far from the original critical value of the universe, so leading to an exponentially growing driven acceleration. Therefore, while the universe is cooling due to its expansion, the critical value $c_0^*(\lambda(T), \mu(T))$ approaches $c_0$ and, as a consequence, the reflection coefficient increases and the acceleration also becomes quadratic. Thus, supposing that both types of topological structures were created in the origin of the universe, one can see that the degenerate structure would dominate the beginning of the process. This happens due to the fact that, when the reflection coefficient is close to one, the scale factor $a(t) \sim t^2$ and for an exponentially growing reflection coefficient, it grows exponentially too. Therefore, in the beginning the degenerate structure dominates the expansion and, as the time goes by, both structures predict a quadratic behavior, which is compatible with the present experimental bounds.

Acknowledgements: The authors ASD and MH thanks to CNPq and ACAF to CAPES for the partial financial support. We also thanks to Professor D. Bazeia for introducing us to the matter of solitons and BPS solutions. This work has been partially done during a visit (ASD) within the Associate Scheme of the Abdus Salam ICTP.

References

[1] F. A. Brito, F. F. Cruz and J. F. N. Oliveira, Phys. Rev. D 71, 083516 (2005).
[2] R. Rajaraman, Solitons and Instantons (North-Holand, Amsterdam, 1982)
[3] A. Vilenkin and E. P. S. Shellard, Cosmic Strings and Other Topological Defects (Cambridge University, Cambridge, England, 1994).
[4] T. Vachaspati, Kinks and Domain Walls: An Introduction to Classical and Quantum Solitons (Cambridge University Press, Cambrifge, England, 2006).
[5] D. Walgraef, Spatio-Temporal Pattern Formation (Springer-Verlag, Berlin, 1997).
[6] R. Rajaraman, Phys. Rev. Lett. 42, 200 (1979).
[7] R. Rajaraman an E. J. Weinberg, Phys. Rev. D 11, 2950 (1975).
[8] M. K. Prasad and C. M. Sommerfeld, Phys. Rev. Lett. 35, 760 (1975); E. B. Bologomol’nyi, Sov. J. Nucl. Phys. 24, 449 (1976).

[9] A. de Souza Dutra, Phys. Lett. B 626, 249 (2005).

[10] A. de Souza Dutra and A. C. Amaro de Faria Jr., Phys. Lett. B 642 (2006) 274.

[11] D. Bazeia, J. R. S. Nascimento, R. F. Ribeiro and D. Toledo, J. Phys. A 30, 8157 (1997)

[12] D. Bazeia, Braz. J. Phys. 32 (2002) 869. D. Bazeia, M. J. dos Santos and R. F. Ribeiro, Phys. Lett. A 208 (1995) 84. D. Bazeia, W. Freire, L. Losano and R. F. Ribeiro, Mod. Phys. Lett. A 17 (2002) 1945.

[13] M. N. Barreto, D. Bazeia and R. Menezes, Phys. Rev. D 73, 065015 (2006).

[14] A. de Souza Dutra, M. Hott and F.A. Barone, Phys. Rev. D 74, 085030 (2006).

[15] M. A. Shiffman and M. B. Voloshin, Phys. Rev. D 57 (1998) 2590.

[16] A. de Souza Dutra, A. C. Amaro de Faria Jr. and M. Hott, Physical Review D 78 (2008) 043526.

[17] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429 (1998) 263.

[18] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 4690.

[19] G. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B 485 (2000) 208.

[20] G. Dvali and G. Gabadadze, Phys. Rev. D 63 (2001) 065007.

[21] G. Dvali and S. -H.H. Tye, Phys. Lett. B 450 (1999) 72.

[22] P. S. Apostopoulou and N. Tetradis, Phys. Rev. D 71 (2005) 043506.

[23] P. S. Apostopoulou and N. Tetradis, Phys. Lett. B 633 (2006) 409.

[24] C. Bogdanos and K. Tamvakis, Phys. Lett. B 646 (2007) 39.

[25] A. G. Riess, et al., Astron. J. 116 (1998) 1009. S. Perlmutter, et al., Astron. J. 517 (1999) 565. A. G. Riess, et al., Astron. J. 607 (2004) 665.
Figure 1: Typical profiles of the soliton solutions in the case where $\lambda = \mu$ ($a = 1$) as a function of $x$, both when $c_0$ is close to its critical value ($c_0 = -2$ in this case) (solid lines) and far from it (dashed lines). The $\chi$ field is represented on the thick lines and $\phi$ on the thin ones.

Figure 2: Energy densities for the case where $\lambda = 4\mu$, where $c_0$ is near its critical value (solid line) ($c_0 = 1/16$ in this situation), and when it is far from this value (dashed line). Here it is evident the appearance of the double wall structure.
Figure 3: Comparison of the domain wall effective potentials for the case of Brito et al. (thick solid line) and degenerate solitons when $c_0$ is far from its critical value (thin solid line), near it (thick dashed line) and at the critical point ($c_0 = -2$) (thin dashed line). All of them are depicted for the case where $\lambda = \mu$.

Figure 4: Comparison of the domain wall effective potentials for the case where $\lambda = 4\mu$. The thick solid line represents the critical case ($c_0 = 1/16$). The other curves correspond to $c_0$ far from its critical value (thick dashed line) and near it (thin solid line).