Modification of the Unitarity Relation for $\sin 2\beta - V_{ub}$ in Supersymmetric Models

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Abstract

Recently, a more than $2\sigma$ discrepancy has been observed between the well measured inclusive value of $V_{ub}$ and the predicted value of $V_{ub}$ from the unitarity triangle fit using the world average value of $\sin 2\beta$. We attempt to resolve this tension in the context of grand unified SO(10) and SU(5) models where the neutrino mixing matrix is responsible for flavor changing neutral current at the weak scale and the models with non-proportional $A$-terms (can be realized simply in the context of intersecting D-brane models) and investigate the interplay between the constraints arising from $B_s,d$-$\bar{B}_{s,d}$ mixings, $\epsilon_K$, $\text{Br}(\tau \to \mu\gamma)$, $\text{Br}(\mu \to e\gamma)$ and a fit of this new discrepancy. We also show that the ongoing measurement of the phase of $B_s$ mixing will be able to identify the grand unified model. The measurement of $\text{Br}(\tau \to e\gamma)$ will also be able to test these scenarios, especially the models with non-proportional $A$-terms.
1 Introduction

Recent measurement of $B_s - \bar{B}_s$ oscillation [1] not only can examine the Kobayashi-Maskawa theory [2], but also can probe the existence of new physics such as supersymmetric (SUSY) models. Accurate measurements of the mass differences for $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$, time dependent CP asymmetry of $B_d \rightarrow J/\psi K$ decay [3], and Cabibbo mixing can determine the unitarity triangle. Since one angle and two sides are determined from these measurements, one can predict the remaining angles and the side. The experimental measurements of the remaining two angles still have large errors, while the measurement of the remaining side (naively $|V_{ub}/V_{cb}|$) has become accurate recently, especially from inclusive $B$ decay data. One can thus compare the experimental measurement of $|V_{ub}|$ with the value from the global fit of the CKM parameters [4, 5, 6] and it turns out that there exists a more than $2\sigma$ discrepancy between the experimental measurement and the unitarity prediction of $|V_{ub}/V_{cb}|$ [4, 5]. This discrepancy may be an indication of new physics.

The SUSY models are the most attractive candidates for new physics. The gauge hierarchy problem can be solved and a natural aspect of the theory can be developed from the weak scale to the ultra high energy scale. In fact, the gauge coupling constants in the standard model (SM) can unify at a high scale using the renormalization group equations (RGEs) involving the particle contents of the minimal SUSY standard model (MSSM), which indicates the existence of grand unified theories (GUTs). While the experimental result for gauge unification is successfully explained in the gauge sector, the flavor sector remains a problem in SUSY theories. The SUSY extension of the SM needs SUSY breaking terms, and there can be a lot of parameters in the flavor sector in general. The SUSY breaking scalar masses in general can induce large flavor changing neutral currents (FCNCs), which contradict experimental results. In order to avoid the SUSY FCNCs, the masses of SUSY scalar particles (squarks and sleptons) need to be degenerate especially for the first two generations as long as the SUSY particles are lighter than 2-3 TeV [7].

The flavor degeneracy of SUSY breaking scalar masses is often assumed at a cutoff scale such as the GUT scale or the Planck scale. The degeneracy can be realized when the Kähler metric is universal in flavor indices. However, even if the degeneracy is realized at a scale, the RGE evolution still can induce flavor violation for squarks and sleptons at low energy. The flavor violation induced by RGEs is suppressed by loop factors, and thus, can satisfy the current FCNC experimental bounds. The ongoing and future experiments may reveal the existence of small flavor violation, and therefore the footprint of GUT or Planck physics may be obtained since the pattern of flavor violation depends on the unification of matters, contents of heavy particles, and the nature of SUSY breaking.
In the scenarios where a small amount of flavor violation is generated at low energy through RGEs, the seeds of flavor violation are implanted in the Yukawa couplings and SUSY breaking scalar trilinear couplings ($A$-terms). The up- and down-type Yukawa matrices are not diagonalized simultaneously, and thus, flavor mixings are generated in charged current interaction. The FCNCs via RGEs can originate from the mixing matrices characterized by the CKM (Cabibbo-Kobayashi-Maskawa) and the MNSP (Maki-Nakagawa-Sakata-Pontecorvo) matrices for quarks and leptons, respectively. In the MSSM, the induced FCNCs in the quark sector are not large since the CKM mixings are small. On the other hand, sizable FCNC effects can be generated in the lepton sector since two neutrino mixings in MNSP matrix are large due to the observation of solar and atmospheric neutrino oscillations. Thus, a testable amount of flavor violating lepton decay can be observed [8]. In grand unified models, as a consequence of the quark-lepton unification, the large neutrino mixings can also generate flavor violation in the quark sector [9].

There could be another source of flavor violation. In the minimal supergravity mediated SUSY breaking, the $A$-term couplings are proportional to the Yukawa couplings, and therefore the effects are already discussed above. In general, the $A$-term couplings can possess non-proportional terms which can be new sources of flavor violation. This can happen even if the Kähler metric is flavor universal. Actually, the $A$-term includes the following three parts: 1) universal term which originates from local SUSY breaking, 2) non-universal term which comes from non-trivial Kähler connection (namely, non-canonical Kähler metric), 3) non-proportional term which is generated when the Yukawa coupling is a function of moduli fields which acquire non-zero $F$-components. The $A$-term coupling in first and second parts can be proportional to the Yukawa coupling when the Kähler metric is flavor universal to make the SUSY breaking scalar mass degenerate. In the third part, the $A$-term couplings are proportional to the derivative of the Yukawa couplings in moduli fields, and the derivative of the Yukawa matrix is not necessarily proportional to the Yukawa coupling itself. Thus, it is possible that $A$-terms contain the seeds of flavor violation which does not depend on the CKM and MNSP mixings, even if the SUSY breaking masses are degenerate at the string scale.

In this paper, we investigate the prediction of flavor violation when the unitarity relation of $V_{ub}$ and CP asymmetry of $B_d \to J/\psi K$ is modified by the SUSY FCNC contribution. We assume that the SUSY breaking scalar masses are degenerate at the unification scale due to the nature of Kähler metric of the matter sector. The sources of flavor violation induced via RGEs can be the Yukawa couplings and the $A$-terms. We will consider the following two typical cases when the FCNCs are induced in SUSY breaking scalar mass matrices via RGEs. In the first case, the FCNCs originate from large neutrino mixings, and in the second case, the scalar trilinear couplings are not proportional to Yukawa couplings.
In the first case, we consider the SU(5) and SO(10) grand unified models. In a SU(5) model, the right-handed down-type quarks and the left-handed lepton doublet are unified in one multiplet. Thus, a sizable flavor violation can be generated in the SUSY breaking mass matrix for the right-handed down-type squarks. In a SO(10) model, all the matter fields are unified in one multiplet, and thus, both left- and right-handed squark mass matrices can have sizable flavor mixing. We will see that SO(10) models can have larger FCNC effects in \( B - \bar{B} \) mixings compared to SU(5) models.

In the second case, we will investigate the intersecting D-brane model [10, 11] where the non-proportional \( A \)-terms have been realized explicitly. In this scenario, the SUSY breaking scalar masses are degenerate [12]. In a simple intersecting D-brane model, the Yukawa matrices are rank 1 matrices plus small corrections [13] [14], and consequently, the non-proportional part of the \( A \)-term has a simple structure [15].

In both cases, the important constraints come from flavor violating lepton decays, \( \mu \to e\gamma \) and \( \tau \to \mu\gamma \), mass difference of \( B_s - \bar{B}_s \), and CP violation in \( K - \bar{K} \) mixing. We will see that the experimental constraints restrict the parameters in neutrino mixings and the SUSY particle spectrum. Once the discrepancy between the unitarity prediction and experimental measurements is explained by the SUSY contribution, the phase of \( B_s - \bar{B}_s \) mixings [13] [17] and flavor violating lepton decays in these models get constrained. We will investigate the predictions of the models. It is interesting to note that the phase of \( B_s - \bar{B}_s \) mixing is being measured by the time dependent CP asymmetry of \( B_s \to J/\psi\phi \) decay and the CP asymmetry of semileptonic \( B_s \) decay, and we already have a value for this phase from DØ [18].

This paper is organized as follows. In section 2, we will go through the unitarity relation of \( V_{ub} \) and CP asymmetry of \( B_d \to J/\psi K \) to understand the \( 2\sigma \) discrepancy between the unitarity prediction and experimental measurement of \( |V_{ub}| \). In section 3, we will study the origin of flavor violation from the neutrino mixings in GUT models, and investigate the constraint and the consequences of the modification of unitarity relation by SUSY contributions. In section 4, we will investigate the non-proportional \( A \)-term in the context of intersecting D-brane models, and see the prediction of the models when it explains the \( 2\sigma \). Section 5 is devoted to conclusions and discussions.

2 Unitarity prediction of \( |V_{ub}| \)

The recent accurate measurement of \( B_s - \bar{B}_s \) mass difference, as well as \( B_{d,s} - \bar{B}_{d,s} \), can determine one side of the unitarity triangle. In addition, one angle obtained from the CP asymmetry of \( B_d \to J/\psi K \) and one side of the unitarity triangle determined by the Cabibbo mixing are accurate quantities. Therefore, the unitarity triangle can be determined accurately. The
remaining side and the angles are predicted if there is no effect from new physics.

The unitarity of the CKM matrix gives rise to the following equation. The unitarity condition,

\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \]  

(1)
can be rewritten to obtain the following relation,

\[ |V_{ud}V_{ub}^*|^2 = |V_{cd}V_{cb}^*|^2 + |V_{td}V_{tb}^*|^2 - 2|V_{cd}V_{cb}^*V_{td}V_{tb}^*|\cos\beta, \]  

(2)

where \( \beta = \phi_1 \equiv \arg (V_{td}V_{tb}^*/V_{cd}V_{cb}^*) \). Using approximate relations, \( |V_{cb}| \approx |V_{ts}| \) and \( V_{tb} \approx 1 \), we obtain

\[ |V_{ub}| \approx \left| \frac{V_{cb}}{V_{ud}} \right| \sqrt{|V_{cd}|^2 + \left| \frac{V_{td}}{V_{ts}} \right|^2 - 2 \left| \frac{V_{cd}V_{tb}^*}{V_{ts}} \right|\cos\beta}. \]  

(3)
The recent measurement of \( B_s - \bar{B}_s \) oscillation indicates \( |V_{td}/V_{ts}| = 0.206^{+0.008}_{-0.006} \). Using world average of \( \sin2\beta \) obtained from \( B_d \to J/\psi K \) [19, 20],

\[ \sin2\beta = 0.674 \pm 0.026, \]  

(4)

\( |V_{cd}| = 0.2258 \), and \( |V_{cb}| = (41.6 \pm 0.6) \times 10^{-3} \) [4], we obtain

\[ |V_{ub}| = (3.49 \pm 0.17) \times 10^{-3} \text{ (unitarity)}. \]  

(5)
The \( |V_{ub}| \) obtained above can be compared to the combined data analysis \( |V_{ub}| = (3.5 \pm 0.18) \times 10^{-3} \) obtained by UTfit and CKMfitter [5, 6, 21, 22]. We note that the \( |V_{ub}| \) prediction from the unitarity triangle is insensitive to \( |V_{td}/V_{ts}| \) error since \( \alpha = \phi_2 \equiv \arg (V_{ud}V_{td}^*/V_{td}V_{tb}^*) \) is close to 90°. The prediction for \( \alpha \) from the unitarity triangle using two sides \( |V_{cd}|, |V_{td}/V_{ts}| \) and an angle \( \beta \) is \( \alpha = (93.2 \pm 5.7)^\circ \). The \( \sin 2\beta - V_{ub} \) relation is plotted in Figure 1.

The experimental measurement of \( |V_{ub}| \) [1, 20] is

\[ |V_{ub}| = (4.49 \pm 0.19 \pm 0.27) \times 10^{-3} \text{ (inclusive)}, \]  

(6)
\[ |V_{ub}| = (3.84^{+0.67}_{-0.49}) \times 10^{-3} \text{ (exclusive)}. \]  

(7)
The tension between the experimental data and the unitarity prediction of \( |V_{ub}| \) is significant especially for the inclusive \( B \) decay data which has less theoretical error compared to the exclusive case. The unitarity condition does not agree with the inclusive data within 99% confidence level. The data from the exclusive decay \( B \to \pi \ell \bar{\nu} \) includes large uncertainty from lattice calculation. The statistical error of the inclusive data is reduced in recent experiments at Babar and Belle [20, 23], and the inclusive data has become more reliable.

The discrepancy between the unitarity condition and the experimental measurement may indicate new physics. We consider a scenario in which the unitarity condition, Eq. (1), is really
true, but the SUSY contribution modifies the sin 2\(\beta\) measurement by \(B_d \rightarrow J/\psi K\). We will study different types of boundary conditions (SU(5), SO(10) GUTs or non-proportional \(A\)-terms at the unification scale) that can explain the discrepancy without contradicting other experimental results.

3 Origin of FCNC in GUT Models

In SUSY theories, the SUSY breaking terms can be the sources of flavor violations. In general, it is easy to include sources of flavor violation by hand and the discrepancy between the unitarity condition and the experimental measurement can be fitted since the SUSY breaking masses with flavor indices are parameters in the model. However, if these parameters are completely general, too much FCNCs are induced. Therefore, as a minimal assumption of the SUSY breaking, universality is often considered, which means that all the SUSY breaking scalar masses are universal to be \(m_0^2\), and the scalar trilinear couplings are proportional to Yukawa couplings (the coefficient is universal to be \(A_0\)) at unification scale. In this case, the angles in the unitarity triangle are not changed. Since the measurements of the \(V_{ub}\) are obtained from tree-level processes, the SUSY particles do not contribute to its determination. Therefore, we need to include flavor violating sources at the unification scale to explain the discrepancy.

In GUT models, the flavor violating sources in SUSY breaking parameters can be induced from neutrino Dirac Yukawa couplings, \(Y_\nu \bar{\nu}_5 N^c H_5\) [8, 9]. Since the left-handed lepton doublet, \(L\), and the right-handed down-type quarks, \(D^c\), are unified in \(5\) multiplet, the SUSY breaking
mass matrix for $D^c$ is corrected by colored-Higgs and right-handed neutrino loops. As a result, the flavor violation in the quark sector can be generated from the neutrino coupling. The contribution is naively proportional to $Y_\nu Y_\nu^T$. Our purpose in this section is to search for a solution of the $V_{ub}\sin 2\beta$ discrepancy using the neutrino mixings in the context of a GUT scenario.

We will work in a basis where the right-handed neutrino Majorana mass matrix, $M_R$, and charged-lepton Yukawa matrix, $Y_\nu$, are diagonal,

$$M_R = \text{diag}(M_1, M_2, M_3).$$ (8)

The neutrino Dirac Yukawa coupling matrix is written as

$$Y_\nu = V_L^c Y_\nu^{\text{diag}} V_R^{c\dagger},$$

where $V_{L,R}^c$ are diagonalizing unitary matrices. We note that $V_L^c$ corresponds to the (conjugate of) MNSP neutrino mixing matrix, $U_{\text{MNSP}}$, in type I seesaw, $m^\text{light}_\nu = Y_\nu M_R^{-1} Y_\nu^T \langle H_u^0 \rangle^2$ [24], up to a diagonal phase matrix when $V_R^c = 1$ (identity matrix), which we will assume for simplicity. Through RGE, the off-diagonal elements of the SUSY breaking mass matrix for the left-handed lepton doublet gets the following correction

$$\delta M^2_{L,ij} \simeq -\frac{1}{8\pi^2} (3m_0^2 + A_0^2) \sum_k (Y_\nu)_{ik} (Y_\nu^*)_{jk} \ln \frac{M_s}{M_k},$$ (10)

where $M_s$ is a cutoff scale and the SUSY breaking parameters are universal. Neglecting the threshold of the GUT and the Majorana mass scales, we can write down the boundary conditions as

$$M^2_5 = M^2_5 - M^2_L = m_0^2 \left(1 - \kappa V_L^c \begin{pmatrix} k_1 \\ k_2 \\ 1 \end{pmatrix} V_L^{c\dagger}\right),$$ (11)

where $\kappa \simeq (Y_\nu^{\text{diag}})^2 \frac{\delta m^2_{\text{sol}} + \delta m^2_{\text{atm}}}{8\pi^2 \ln M_s/M_{\text{GUT}}}$, and $k_2 \simeq \sqrt{\frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}} M_2/M_3}$. We parameterize the unitary matrix $V_L^c$ as

$$V_L^c = \begin{pmatrix} e^{i(\alpha_1-\delta)} \\ e^{i\alpha_2} \\ 1 \end{pmatrix} \begin{pmatrix} e^{c_\delta} & c_{12} e^{c_{13}} \sin \theta_{13} e^{-i\delta} & s_{12} c_{13} - e^{c_\delta} \sin \theta_{13} e^{-i\delta} \\ -s_{12} c_{23} e^{c_{13}} \sin \theta_{13} e^{-i\delta} & c_{12} c_{23} - s_{12} s_{23} \sin \theta_{13} e^{-i\delta} & c_{23} e^{c_{13}} \sin \theta_{13} e^{-i\delta} \\ s_{12} s_{23} \sin \theta_{13} e^{-i\delta} & s_{12} c_{23} \sin \theta_{13} e^{-i\delta} & s_{23} e^{c_{13}} \sin \theta_{13} e^{-i\delta} \end{pmatrix},$$

where $s_{ij}$ and $c_{ij}$ are sin and cos of mixing angles $\theta_{ij}$. In the limit $k_{1,2} \to 0$, $\alpha_1$ and $\alpha_2$ are the phase of the 13 and the 23 element of $M^2_5$. Since we are assuming that $V^c_L = 1$, $\theta_{12}$ and $\theta_{23}$ correspond to solar and atmospheric neutrino mixings, respectively, which are large. On the other hand, $s_{13}^c$ is bounded by CHOOZ experiments, $s_{13}^c \lesssim 0.2$ [25]. The SUSY breaking mass
for 10 multiplet \((Q, U^c, E^c)\) is also corrected by the (colored-)Higgsino loop, but it arises from CKM mixings and the effect is small. So, we assume that the boundary condition at the GUT scale for 10 multiplet is

\[
M_{10}^2 = M_Q^2 = M_{U^c}^2 = M_{E^c}^2 = m_0^2 \mathbf{1},
\]

(13)
neglecting the small contribution arising from the CKM mixings. The boundary conditions, Eqs.(11,13), are typical assumptions in the case of SU(5) GUT. The Yukawa coupling matrices for up- and down-type quarks and charged-leptons are given as

\[
Y_u = V_{qeL} V_{CKM}^T Y_u^{\text{diag}} P_u V_{uR},
\]

(14)
\[
Y_d = V_{qeL} Y_d^{\text{diag}} P_d V_{qeR}^T,
\]

(15)
\[
Y_e = Y_e^{\text{diag}} P_e,
\]

(16)
where \(Y_{u,d,e}^{\text{diag}}\) are real (positive) diagonal matrices and \(P_{u,d,e}\) are diagonal phase matrices. In a minimal SU(5) GUT, in which only \(H_5\) and \(\bar{H}_5\) couple to matter fields, we have \(V_{uR} = V_{CKM} V_{qeL}^T, V_{qeL} = V_{qeR} = \mathbf{1}\), and \(Y_d^{\text{diag}} = Y_e^{\text{diag}}\). We may consider non-minimal SU(5), but we assume \(V_{uR}, V_{qeL}, V_{qeR} \simeq \mathbf{1}\). Otherwise, the simple relations between the flavor violations in the quark and lepton sectors which we see below will be lost.

All matter fields are unified in the spinor representation 16 in SO(10) models. Since the right-handed neutrino is also unified to other matter fields, the neutrino Dirac Yukawa coupling does not have large mixings \((i.e. \, V_L^e \simeq \mathbf{1})\) in a simple fit of the Yukawa couplings. In this case, the proper neutrino masses with large mixings can be generated by the type II seesaw mechanism \([26]\), in which the interaction term \(\frac{1}{2} f L L \Delta L\) (where \(\Delta L\) is a SU(2)\(_L\) triplet) induces the light neutrino masses, \(m_{\nu}^{\text{light}} = f \langle \Delta L^0 \rangle\). Due to the unification under SO(10), the left-handed Majorana coupling, \(f\), is unified to the other matter fields, and the off-diagonal terms in the sparticle masses are induced by loop effect which are proportional to \(f f^\dagger\). Neglecting the GUT scale threshold, we can write the boundary condition in SO(10) as

\[
M_{16}^2 = m_0^2 \left( \mathbf{1} - \kappa_{16} U \begin{pmatrix} k_1 & k_2 \\ 0 & 1 \end{pmatrix} U^\dagger \right),
\]

(17)
where \(\kappa_{16} \simeq 15/4 (f_3^{\text{diag}})^2 (3 + A_0^2 / m_0^2) / 8 \pi^2 \ln M_*/M_{\text{GUT}}, \) and \(k_2 \simeq \Delta m^2_{\text{sol}} / \Delta m^2_{\text{atm}}\) in this case. Note that the parameters \(\kappa_{16}, k_{1,2}\) are of course different from those given in Eq.(11) using the set-up for type I seesaw, but we use the same notation to make the description simple. The unitary matrix \(U\) is the (conjugate of) MNSP neutrino mixing matrix up to a diagonal phase matrix, which is parameterized in the same way as Eq.(12). The Yukawa couplings are also given as Eq.(14,15,16). If we do not employ 120 Higgs fields, the Yukawa matrices are
symmetric, and thus, \( V_{uR} = V_{CKM} V_{qeL}^T, V_{qR} = V_{qeL} \). The unitary matrix \( V_{qeL} \) is expected to be close to \( 1 \) if there is no huge fine-tuning in the fermion mass fits.

Now let us consider the necessary conditions to solve the discrepancy between the unitarity condition and experimental measurements using the boundary conditions, Eqs. (11,13) in SU(5) and Eq. (17) in SO(10) respectively. Due to the smallness of \( k_{1,2} \) and \( s_{e_{13}} \), the 12 and 13 elements of \( M_{5}^2 \) and \( M_{16}^2 \) are expected to be smaller than the 23 element. In the limit of \( k_{1,2}, s_{e_{13}} \to 0 \), only the 23 element is non-zero and then \( \sin 2\beta \) can not be modified. In this limit, the \( B_s-\bar{B_s} \) mass difference \( \Delta m_s \) is modified, and therefore \( |V_{td}/V_{ts}| \) prediction is changed, and one of the sides of the unitarity triangle is modified. However, if \( |V_{td}/V_{ts}| \) is modified to obtain larger \( |V_{ub}| \) from the unitarity relation, CP violation in \( K^- \bar{K} \) mixing, \( \epsilon_K \), will not fit well with the experimental range. Thus, we need to modify \( \sin 2\beta \) itself employing the finite contributions of the 13 element in SUSY breaking mass matrix. If at least one of \( k_2 \) and \( s_{e_{13}} \) is non-zero, the 13 element is generated. Then the 12 element of SUSY breaking mass matrix is also generated in general. However, if the 12 element is generated, due to the quark-lepton unification, \( \mu \to e\gamma \) process is enhanced through the chargino diagram. In order to satisfy the experimental bound for the branching ratio of \( \mu \to e\gamma \) process, \( \text{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11} \) [27], the 12 element needs to be small to suppress the chargino contribution. It can happen by a cancellation when \( k_1 \ll k_2, s_{e_{13}} \sim k_2 \sin 2\theta_{12}/2 \) and \( \delta \simeq \pi \).

In order to obtain a sizable effect in \( \sin 2\beta \), \( \kappa \) needs to be large enough. However, if \( \kappa \) is too large, \( \tau \to \mu\gamma \) process may exceed the experimental bound, \( \text{Br}(\tau \to \mu\gamma) < 4.5 \times 10^{-8} \) [28] for large \( \theta_{23} \) mixing. A large value of \( m_0 \sim 1 \) TeV can avoid the excess of \( \tau \to \mu\gamma \) due to the following reason. The off-diagonal elements of SUSY breaking mass matrices are proportional to \( m_0^2 \), whereas the diagonal element of squark matrix at the weak scale is insensitive to \( m_0 \) when \( m_0 \lesssim 500 \) GeV for gluino mass \( \sim 1 \) TeV, since RGE contribution from the gluino is large. Consequently, the flavor violations from SUSY contributions in the quark sector become maximal for \( m_0 \sim 1 \) TeV. On the other hand, the diagonal elements of slepton mass matrix are sensitive to \( m_0 \), and thus, lepton flavor violating processes are suppressed for large \( m_0 \).

Now let us study different results that can be produced by type I seesaw for SU(5) boundary condition, Eq. (11,13), and type II seesaw for SO(10) boundary conditions, Eq. (17). If the boundary condition is flavor universal, which means \( \kappa = 0 \), the chargino contribution is the dominant SUSY contribution to the \( B_{d,s}-\bar{B}_{d,s} \) mixing amplitude, \( M_{12}(B_{d,s}) \). Using a general parameter space for the soft SUSY breaking terms, the gluino box diagram dominates the SUSY contribution. The gluino contribution (divided by the SM contribution) can be written naively in the following mass insertion form

\[
\frac{M_{12}^g}{M_{12}^{SM}} \simeq a (\delta_{LL}^d)^2_{3i} + (\delta_{RR}^d)^2_{3i} - b \delta_{LL}^d (\delta_{RR}^d)_{3i}
\]

\[\text{(18)}\]
where $i = 1$ for $B_d$ mixing, $i = 2$ for $B_s$ mixing, $a$ and $b$ depend on squark and gluino masses, and $\delta_{LL, RR}^d = (M_2^d)_{LL, RR}/\bar{m}^2$ ($\bar{m}$ is an averaged squark mass). The matrix $M_2^d$ is a down-type squark mass matrix ($\tilde{Q}, \tilde{D}^c) M_2^d (\tilde{Q}^\dagger, \tilde{D}^c)^T$ at weak scale in the basis where down-type quark mass matrix is real (positive) diagonal. When the squark and the gluino masses are less than 1 TeV, $a \sim O(1)$ and $b \sim O(100)$. We also have contributions from $\delta_{LR}^d$, but we neglect them since they are suppressed by $(m_b/m_{\text{SUSY}})^2$.

Using the SU(5) boundary condition, Eq. (11,13), we find that $\delta_{RR}^d$ can be large due to large neutrino mixings, but $\delta_{LL}^d$ cannot be very large since it is induced from CKM related mixing. On the other hand, using the SO(10) boundary conditions we find that both $\delta_{LL}^d$ and $\delta_{RR}^d$ can be large. As a result, the SO(10) boundary condition can generate larger SUSY contribution in $B\bar{B}$ mixings rather than the SU(5) boundary condition for the same parameters for $\kappa$, $k_{1,2}$ and mixing angles $\theta_{ij}$ since $b \gg a$ in Eq. (18).

The $B\bar{B}$ mixing amplitude is given as

$$M_{12}(B)^{\text{full}} = M_{12}(B)^{\text{SM}} + M_{12}(B)^{\text{SUSY}}.$$  (19)

The angle $2\beta$ for unitarity triangle is the argument of $M_{12}(B_d)^{\text{SM}}$ in the particle data group phase notation of CKM matrix. The observed angle $2\beta_{\text{eff}}$ by $B_d \rightarrow J/\psi K$ decay is the argument of $M_{12}(B_d)^{\text{full}}$. The SUSY contribution of $B_d\bar{B}_d$ mixing is naively proportional to $(s_{13}^c)^2$. As was noted, it is necessary that $s_{13}^c \sim k_2 \sin \theta_{12}/2$ to suppress $\mu \rightarrow e\gamma$ process. In the case of type I seesaw in SU(5) model, $k_2$ is a free parameter, and thus, $s_{13}^c$ is free. On the other hand, using type II seesaw in the SO(10) model, $k_2$ is almost determined by the ratio of mass squared differences for solar and atmospheric neutrino oscillations. As a result, the suppression of $\mu \rightarrow e\gamma$ process indicates $s_{13}^c \sim 0.02$. Due to the smallness of $s_{13}^c$, the modification of sin $2\beta$ will be small for type II seesaw SO(10) boundary condition. For type I seesaw SU(5) boundary condition, on the other hand, the modification can be larger than the SO(10) case since $s_{13}^c$ can be sizable $s_{13}^c \sim 0.1$, though the modification is smaller if we use the mixing angle $s_{13}^c \sim 0.02$.

The absolute value of $M_{12}(B_{d(s)})^{\text{SUSY}}$ is almost determined by $\kappa$, $s_{13}^c$ ($s_{23}^c$), and squark, gluino masses, whereas, the argument of $M_{12}(B)^{\text{SUSY}}$ can be anything since $\alpha_{1,2}$ in Eq. (12) and phases $P_{d,u}$ in Yukawa matrices in Eq. (14,15) are all free. The minimal value of sin $2\beta_{\text{eff}}$ for a given size of SUSY contribution can be obtained by choosing the phases. The branching ratios of lepton flavor violating processes do not depend on the phases since the branching ratios are proportional to the squared absolute values of the amplitudes of SUSY contributions.

We plot the relation between the minimal value of sin $2\beta_{\text{eff}}$ and the branching ratio of $\tau \rightarrow \mu\gamma$ decay in Figure 2. In the plot, we use $m_0 = 1.2$ TeV, $m_{1/2} = 300$ GeV for gaugino mass at GUT scale, and $\tan \beta_H = 10$ for a ratio of vacuum expectation values of up- and down-type Higgs fields. In the SU(5) case, one can tune $k_2$ for a given $s_{13}^c$ to suppress $\text{Br}(\mu \rightarrow e\gamma)$ to
Figure 2: Relation of the minimal value of \( \sin 2\beta_{\text{eff}} \) and branching ratio of \( \tau \to \mu \gamma \) decay for \( |V_{ub}| = 0.0041 \) (left), 0.0045 (right). The dotted line stands for the experimental bounds.

be less than \( 10^{-11} \). With type II seesaw in the SO(10) model, \( k_2 \) is fixed by observation of neutrino oscillation and the suppression of \( \text{Br}(\mu \to e\gamma) \) implies \( s_{13}^e \sim 0.02 \). Since there are also neutralino contributions for both left- and right-handed muon decay, one can not cancel \( \text{Br}(\mu \to e\gamma) \) completely. In the range where \( \sin 2\beta_{\text{eff}} \) is in agreement with experiment, we obtain \( \text{Br}(\mu \to e\gamma) \gtrsim 10^{-12} - 10^{-13} \). In the plot, we show the cases \( s_{13}^e = 0.08 \), and 0.15 for SU(5), and \( s_{13}^e = 0.022 \) for SO(10). As was expected, one can read off that the modification of \( \sin 2\beta_{\text{eff}} \) can become large for larger \( s_{13}^e \) for the same value of \( \text{Br}(\tau \to \mu \gamma) \).

So far we have obtained the parameter region which satisfies the observed \( \sin 2\beta_{\text{eff}} \) and the inclusive data of \( |V_{ub}| \), as well as the bounds on the leptonic flavor violating decay in a grand unified scenario. We will next study the consequence of the fit in the phase of \( B_s - \bar{B}_s \) mixing. Although the measurements of mass differences, \( \Delta M_{d,s} \equiv 2|M_{12}(B_{d,s})_{\text{full}}| \), are in good agreement with SM prediction, there can still be room for SUSY contribution due to the following two reasons. One reason is that there exists large uncertainties \( \sim 20\% \) in the lattice calculation for the decay constants \( f_{B_{d,s}} \) and the bag parameters \( B_{B_{d,s}} \). The SM prediction of the ratio \( \Delta M_s/\Delta M_d \) can be more accurate since the ratio for \( f_B \) and \( B_B \) have less uncertainty. The other reason is that we still have an ambiguity for the phase of \( B_s-\bar{B}_s \) mixing. For instance, one can satisfy \( |M_{12}(B_s)_{\text{full}}| = |M_{12}(B_s)_{\text{SM}}| \) by choosing a phase of \( M_{12}(B_s)_{\text{SUSY}} \) as long as \( |M_{12}(B_s)_{\text{SUSY}}| < 2|M_{12}(B_s)_{\text{SM}}| \). As we have noted, the phase of \( M_{12}(B_s)_{\text{SUSY}} \) is completely free for the boundary conditions in GUT models. The SM prediction of the phase of \( B_s-\bar{B}_s \) oscillation (\( 2\beta_s = -\text{arg} M_{12}(B_s)_{\text{SM}} \)) is about 2° (\( \sin 2\beta_s \simeq 0.04 \)). If the SUSY contribution modifies \( \sin 2\beta_{\text{eff}} \) and resolves the tension of the \( \sin 2\beta_{\text{eff}}-V_{ub} \) unitarity relation, the phase of \( B_s-\bar{B}_s \) (\( 2\beta_{\text{eff}} = -\text{arg} M_{12}(B_s)_{\text{full}} \)) must be different from the SM prediction.
Figure 3: Random dot plot for $\sin 2\beta_{\text{eff}}$ and $\phi_{B_s}$ for $|V_{ub}| = 0.0041$ using the SO(10) boundary condition. The shaded area shows the $1\sigma$ range of $\sin 2\beta$ for the world average [20].

It is convenient to define the deviation from SM prediction by two real parameters $C_{B_s}$ and $\phi_{B_s}$ as

$$C_{B_s} e^{2i\phi_{B_s}} \equiv \frac{M_{12}(B_s)^{\text{full}}}{M_{12}(B_s)^{\text{SM}}}.$$  (20)

If there is no SUSY contribution, $C_{B_s} = 1$ and $\phi_{B_s} = 0$. In this notation, $\beta_{s}^{\text{eff}} = \beta_s - \phi_{B_s}$. We are interested in the value of $\phi_{B_s}$ in the range where $\sin 2\beta_{\text{eff}}$ is in agreement with the experimental data for inclusive $V_{ub}$. In the case of SU(5) boundary condition, the value is $\phi_{B_s} \sim \pm 3^\circ$, namely $\sin 2\beta_{\text{eff}} \sim 0.14, -0.07$. On the other hand, in the case of SO(10) boundary condition, $\phi_{B_s}$ can be sizable. We show $\phi_{B_s}$ and $\sin 2\beta_{\text{eff}}$ in SO(10) type II seesaw by random dot plot in Figure 3. We choose the same parameters as in the plot for Figure 2. Constraint arising from the Br($\tau \to \mu \gamma$) < $4.5 \times 10^{-8}$ rules out larger values of $|\phi_{B_s}| > 35^\circ$. We also filter out the points by the constraint from $K$-$\bar{K}$ mixing, and $\Delta M_s/\Delta M_d$. Especially, points at the left-bottom corner in the plot are ruled out by $K$-$\bar{K}$ mixing data. The reason for the allowed left-top corner is that the SUSY contribution becomes $\epsilon_{K}^{\text{SUSY}} \sim -2\epsilon_{K}^{\text{SM}}$. From the figure, we find the preferred value to be $\phi_{B_s} \sim +(20 - 30)^\circ$, which corresponds to $\sin 2\beta_{\text{eff}} \sim -(0.6 - 0.9)$. The recent measurement by DØ collaboration [18] shows a large negative central value for $\sin 2\beta_{\text{eff}} = -0.71$ with a sizable error and if this result holds then, definitely, the SO(10) model will be preferred.

We summarize what we have accomplished in this section. We have considered the parameter space where we can resolve the discrepancy between the unitarity prediction and the experimental measurement for the boundary conditions for SU(5) with type I seesaw and for SO(10) with type II seesaw. To obtain the solution we need large $m_0 \sim 1$ TeV to suppress $\tau \to \mu \gamma$ decay. In order to suppress $\mu \to e\gamma$, we need to adjust the 13 mixings in the Dirac
Yukawa couplings for type I seesaw and the Majorana couplings for type II seesaw. Consequently, the neutrino oscillation parameter is constrained as follows. The CP phase in neutrino oscillation $\delta_{\text{MNSP}}$ needs to be close to $\pi$. In type II seesaw case, the $13$ neutrino mixing should be around $0.02$, while in type I seesaw case, the $13$ mixing is free. The phase of $B_s - \bar{B}_s$ is not the same as that can be obtained in the SM. The deviation is small for the SU(5) case, and large for the SO(10) case.

The crucial assumptions we made to obtain above results are that $V_{eR} \approx 1$ in the Dirac Yukawa coupling, Eq.(9), in type I seesaw for SU(5), and the triplet part dominates the light neutrino mass rather than the type I seesaw part for type II seesaw for SO(10). If we do not assume them, $V_{eL}$ and $U$ in the boundary conditions do not take part in the construction of the neutrino mixing matrix. Even in this case, we may need the relation, $s_{13}^e \sim k_2 \sin 2\theta_{12}/2$, and $\delta \simeq \pi$ to suppress $\mu \to e\gamma$ decay, but the mixing angles and the phase are not necessarily related to the observed neutrino mixings. Moreover, since $s_{23}^e$ is not necessarily large, $\tau \to \mu\gamma$ can be suppressed by choosing a small $s_{23}^e$ even for smaller $m_0 \lesssim 500$ GeV. If this is the case, depending on the $s_{23}^e$, the deviation of the $B_s-\bar{B}_s$ phase $\phi_{B_s}$ will not be large compared to the above. The possible feature of this case is that $\text{Br}(\tau \to e\gamma)$ may be comparable to $\text{Br}(\tau \to \mu\gamma)$ since $s_{13}^e$ needs to be around $0.1$ to modify $\sin 2\beta_{\text{eff}}$ while we get $\text{Br}(\tau \to e\gamma) \ll \text{Br}(\tau \to \mu\gamma)$ in a usual scenario.

4 Origin of FCNC Using Non-proportional $A$-term

In the previous section, we have considered the scenario where the neutrino mixings are the origin of flavor violation at low energy, assuming the universality for SUSY breaking scalar masses and trilinear scalar couplings at unification scale. The SUSY breaking scalar mass and trilinear scalar couplings are given as

$$m_{a\bar{a}}^2 = m_{3/2}^2 + V_0 - \sum_{M,N} F^M F^N \partial_M \partial_N \ln K_{a\bar{a}},$$  \hspace{1cm} (21)$$

$$A_{abc} = \sum_M F^M \left[(K_M - \partial_M \ln(K_{a\bar{a}} K_{b\bar{b}} K_{c\bar{c}})) Y_{abc} + \partial_M Y_{abc}\right],$$  \hspace{1cm} (22)$$

where $K$ is the Kähler potential and $F^M$ is the $F$-term of moduli field $M$. The flavor invisibleness of Kähler potential is necessary for the universality of SUSY breaking parameters. However, the invisibleness is not a sufficient condition to make the $A$-term flavor universal. Actually, when the Yukawa coupling depends on moduli and the $F$-term of the moduli is non-zero, the $A$-terms are no longer proportional to Yukawa coupling, even if the Kähler potential is flavor invisible. In the intersecting D-brane models [11], the Kähler potential can be flavor
invisible, but Yukawa couplings depend on $U$-moduli (shape moduli), and they can generate flavor violation in the SUSY breaking scalar mass matrices at low energy [15]. In this section, we will consider non-proportional $A$-terms to resolve the $\sin 2\beta - V_{ub}$ tension.

In the intersecting D-brane models, the SUSY breaking scalar masses are flavor universal when the generations are simply replicated at the intersection of the branes. The Yukawa couplings are induced by the three-point open string scattering, and they are determined by the triangle area formed by branes. As we have noted, the couplings depend on $U$-moduli.

The Yukawa coupling matrices are given in a factorized form [13],

$$Y_{ij}^0 = x_i^L(U_1)x_j^R(U_2),$$

(23)

if the left- and right-handed matter fields are replicated on different tori. The couplings are the function of Jacobi theta function $\vartheta \left[ \begin{array}{c} \delta \\ \phi \end{array} \right](t)$. The coupling $x_i$ is given as

$$x_1 : x_2 : x_3 = \vartheta \left[ \begin{array}{c} \varepsilon + \frac{1}{3} \\ 0 \end{array} \right](t) : \vartheta \left[ \begin{array}{c} \varepsilon - \frac{1}{3} \\ 0 \end{array} \right](t) : \vartheta \left[ \begin{array}{c} \varepsilon \\ 0 \end{array} \right](t),$$

(24)

where $\varepsilon$ stands for a brane shift parameter, and the Wilson line phase $\phi$ is chosen to be zero for simplicity. The variable $t$ is proportional to $U$-moduli. Since the Yukawa matrices are rank 1, only third generation becomes massive after the electroweak phase transition. We, therefore, need to introduce corrections to Yukawa couplings, which can arise due to higher dimensional operators. If the correction $\delta Y$ is given as $\delta Y = \text{diag}(0, 0, \epsilon)$ in the basis where the light neutrino Majorana mass is diagonal, $U_{e3}$ (the 13 mixing in neutrino oscillation) is exactly zero. In the quark sector, the correction $\delta Y = \text{diag}(0, 0, \epsilon)$ leads to the relation $m_s/m_b \sim V_{cb}$.

The first generation fermions are still massless with the above correction to the rank 1 Yukawa matrix and we need more corrections. Thus, $U_{e3}$ is small and relates to the size of the Cabibbo angle in the quark-lepton unification picture. Furthermore, $\tan \theta_{\text{sol}} \lesssim 1$ for the solar mixing angle, and $\sin^2 2\theta_{\text{atm}} \sim 1$ for the atmospheric neutrino mixing can be satisfied naturally in the perturbative region of the Yukawa coupling. The details can be found in the Refs. [14, 15].

If the $F$-terms of $U$-moduli, $F_U$, are zero and the Kähler potential is flavor invisible, the $A$-term is proportional to the Yukawa couplings. We consider the case where the Kähler potential is flavor invisible whereas $F^U \neq 0$. The rank 1 matrix $Y^0$ is given as in Eq.(23). For simplicity, we consider that the matrix is symmetric: $x_i^{L,R} = x_i(U)$. The diagonalization unitary matrix of $Y^0$ is

$$U^0 = \begin{pmatrix} \cos \theta_s & -\sin \theta_s & 0 \\ \cos \theta_a \sin \theta_s & \cos \theta_s \cos \theta_a & -\sin \theta_a \\ \sin \theta_s \sin \theta_a & \sin \theta_a \cos \theta_s & \cos \theta_a \end{pmatrix},$$

(25)

where $\tan \theta_s = x_1/x_2$ and $\tan \theta_a = \sqrt{x_1^2 + x_2^2}/x_3$. We work in a basis where the light neutrino mass is diagonal assuming that the correction for the charged-lepton Yukawa matrix is $\delta Y_e \simeq$
diag(0, 0, \epsilon). Then, MNSP neutrino mixing matrix is given as

\[ U_{\text{MNSP}} = V_e^* U^0, \]  

(26)

where \( V_e \) is a diagonalization matrix of \( U^0(\delta Y_e)(U^0)^T \). If the quark-lepton unification is realized, we obtain \( V_e \approx V_{\text{CKM}}^\dagger \). The solar and atmospheric neutrino mixing is close to \( \theta_s, a \) respectively, but solar mixing is corrected by \( (V_e)_{12} \). The 13 neutrino mixing is given as

\[ U_{13} \approx \left( \frac{(V_e)_{12}}{\sqrt{2}} \right), \] 

The non-proportional part of \( A \)-term is proportional to \( \partial_{\mu} Y \). The derivative of Yukawa coupling \( Y^0 \) is calculated as

\[ U^0(\partial_{\mu} Y^0)(U^0)^T = \begin{pmatrix} 0 & 0 & y \sin \theta_a \partial_{\mu} \theta_s \\ 0 & 0 & y \partial_{\mu} \theta_a \\ y \sin \theta_a \partial_{\mu} \theta_s & y \partial_{\mu} \theta_a & \partial_{\mu} y \end{pmatrix}, \] 

(27)

where \( y = \sum x_i^2 \). The \( ij \) \( (i, j = 1, 2) \) elements of \( \partial Y^0 \) are zero, and thus, the 12 element of non-proportional the \( A \)-term is suppressed to be of the order of \( \sim \lambda^2 \partial Y_{13} + \lambda^3 \partial Y_{23} \) (where \( \lambda \sim 0.2 \)) in the basis where the charged-lepton Yukawa matrix is diagonal. This is a general property of the almost rank 1 Yukawa matrix, independent of any set-up.

We note that the non-proportional pieces of the \( A \)-term can be considered in the non-minimal scenario of hidden sector model with flavor symmetries. However, in such a scenario, the non-proportional \( A \)-term is hierarchical and thus, large 13 off-diagonal elements are not expected. On the other hand, in our scenario of \( U \)-moduli origin of the \( A \)-terms, we can realize large 13 elements of the \( A \)-term.

In general, if the \( A \)-term has non-proportional pieces, the off-diagonal elements of SUSY breaking scalar mass matrices are generated via RGEs, e.g. \( dM^2_{D_{13}}/\ln Q = 1/(4\pi^2)A_{13}^a A_{a}^* + \cdots \). Since our purpose is to modify \( \sin 2\beta \), we need the 13 \((31)\) element of the \( A \)-term for left (right) handed squark mass matrices in the basis where Yukawa matrices are diagonal. The RGE induced 13 element of SUSY breaking mass matrices can be larger when \( A_{33} \) (=\( A_b \)) is large. Therefore, a large coefficient of \( A \)-term, \( i.e. \) \( A_0 \), is preferable. The negative \( A_0 \) can generate large contribution to \( \sin 2\beta \) for gluino mass \( M_3 > 0 \) since the RGE is given as \( dA_b/d\ln Q = +8/(3\pi)\alpha_3 M_3 y_b + \cdots \), where \( y_b \) is a bottom quark Yukawa coupling.

When both \( A_{13} \) and \( A_{23} \) elements are introduced at boundary condition, the 12 elements of SUSY breaking mass matrices are generated at low energy even if \( A_{12} \) is zero in the boundary condition. The 12 elements of SUSY breaking mass matrices are unwanted for \( K-\bar{K} \) mixing and \( \mu \to e\gamma \). Thus, for our purpose to modify \( \sin 2\beta \), we need to suppress \( A_{23} \) \((A_{32})\) rather than \( A_{13} \) \((A_{31})\). Such a situation can be satisfied when \( \partial_{\mu} \theta_a = 0 \). There is a solution of \( \partial_{\mu} \theta_a = 0 \) when \( \frac{1}{6} < \epsilon < \frac{1}{4} \) where \( \epsilon \) is a brane shift parameter in Eq.(24). Since \( \partial_{\mu} \theta_a = 0 \), the atmospheric
mixing as a function of $U$ is maximized for a given $\varepsilon$. The predicted atmospheric mixing is $\frac{8}{9} < \sin^2 2\theta_a < 1$ and $\theta_a > 45^\circ$.

For the numerical studies, we assume the following boundary condition for the non-proportional $A$-term:

$$A_{u,p} = A_{d,p} = A_{e,p} = \begin{pmatrix} 0 & 0 & \delta \\ 0 & 0 & 0 \\ \delta & 0 & 0 \end{pmatrix} A_0,$$

in the basis where Yukawa matrices are diagonal. The $A$-terms for up- and down-type quarks and charged-leptons are given as $A_{u,d,e} = A_0 Y_{u,d,e} + A_{u,d,e}^{np}$. In general, there is no reason that $A^{np}$’s are unified, but we assume the unification in the context of $G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$ model construction of intersecting D-brane: The $A^{np}$ may originate from $G_{422}$ invariant term, while the corrections to the Yukawa coupling breaks $G_{422}$. The $A^{np}$ is assumed to be symmetric just for simplicity.

We plot $\sin 2\beta_{\text{eff}}$ and $\text{Br}(\tau \to e\gamma)$ in Figure 4, varying $\delta$ in Eq. (28) from 0 to $y_b$. In the plot, we use $m_0 = 200$ GeV, $m_{1/2} = 300$ GeV, $\tan \beta_H = 10$ and $|V_{ub}| = 4.1 \times 10^{-3}$. When $\text{sgn}(y_d) = -\text{sgn}(y_b)$ for Yukawa couplings of down and bottom quarks, the SUSY contribution of $\sin 2\beta_{\text{eff}}$ becomes negative and satisfy the observation from CP asymmetry of $B_d \to J/\psi K$. The positive values of $A_0$ do not modify $\sin 2\beta$ much as we have explained. The $\text{Br}(\tau \to e\gamma)$ do not depend on the signature of $A_0$ because RGE running of $A_\tau$ is not large.

Since we have assumed that $A_{12}$ and $A_{23}$ are zero, $\epsilon_K$ and $B_s\bar{B}_s$ mixing ($\Delta M_s$ and $\phi_{B_s}$) do not get modified from the SM values. We do not get any enhancement in the branching ratios of $\mu \to e\gamma$ and $\tau \to \mu\gamma$, neither. When we introduce $A_{23}$ elements, all these quantities can be

Figure 4: The $\sin 2\beta_{\text{eff}}$ and $\text{Br}(\tau \to e\gamma)$ relation employing non-proportional $A$-term. The dotted lines stand for the experimental bounds.
modified. The strongest constraint comes from $\epsilon_K$ when $\sin 2\beta_{\text{eff}}$ is in the experimental range. Interestingly, in this scenario we have solutions for smaller values of $m_0$. Thus, the solution can be consistent with the muon $g-2$ from $e^+e^-$ data [29]. The main feature of this solution is that $\text{Br}(\tau \to e\gamma)$ is larger than $\text{Br}(\tau \to \mu\gamma)$.

5 Conclusion

The measurements of $B_{d,s}-\bar{B}_{d,s}$ mass differences, $\Delta M_{d,s}$, and the CP asymmetry of $B_d \to J/\psi K$, $\sin2\beta$, determines the unitarity triangle, and generates predictions for other angles, CP violation in $K-\bar{K}$ mixing, as well as $|V_{ub}/V_{cb}|$. The predictions are consistent with the measurements within errors except for $|V_{ub}/V_{cb}|$. The recent development of $|V_{ub}|$ measurement from the inclusive $B$ decay indicates a more than $2\sigma$ discrepancy compared to the value obtained from the unitarity relation. This may imply the presence of new physics.

Since the inclusive $B$ decay to measure $|V_{ub}|$ is a tree level process, it does not include new physics. On the other hand, $B-\bar{B}$ mixing and $\sin 2\beta$ arise from box diagrams, and thus, they may include new physics contributions, since the new particles can propagate in the diagram and can shift the experimental measurements of $\Delta M_{s,d}$ and $\sin 2\beta$.

We have studied whether the SUSY contributions can modify the unitarity relation to make the experimental measurements to fit well, and investigated the predictions for other measurements, such as the CP asymmetry in $B_s$ decays and branching ratios of flavor violating lepton decay modes. As a consequence, the modification of the unitarity relation of $|V_{ub}|$ and $\sin 2\beta$ shifts the tension to the phase of $B_s-\bar{B}_s$ mixing or $\text{Br}(\tau \to e\gamma)/\text{Br}(\tau \to \mu\gamma)$.

We assumed that the SUSY breaking scalar masses are universal at a cutoff scale. The origin of flavor violations can be in the Yukawa interaction and scalar trilinear couplings. We considered two different set-ups to modify the unitarity relation. In the first set-up, the flavor violation originates from the large neutrino mixings in SU(5) and SO(10) grand unified models. In the second set-up, A-term couplings include pieces which are not proportional to Yukawa couplings in the context of intersecting D-brane models.

In GUT models, responsible for quark-lepton unification, the neutrino Dirac couplings and the Majorana couplings can induce the off-diagonal elements of squark mass matrices via the GUT scale particle loops. In order to modify the unitarity relation appropriately, we need large contribution in 13 elements in SUSY breaking scalar mass matrices. However, a large 13 neutrino mixing is bounded by experiments, since it leads to large effects in flavor violating lepton decay such as $\tau \to \mu\gamma$ and $\mu \to e\gamma$. When we suppress the branching ratio of the lepton flavor violating decays, neutrino mixing parameters and the SUSY spectrum is restricted. As a result, the CP violating phase in the neutrino oscillation needs to be close to $\pi$ when the
large solar and atmospheric mixings are responsible for the structures of neutrino Dirac Yukawa coupling in type I seesaw and for the Majorana coupling in type II seesaw. In type II seesaw case, the 13 neutrino mixing is restricted to be around 0.02. The squarks and the sleptons need to be heavy for the solution, and the universal scalar mass $m_0$ is about 1 TeV. As a consequence of resolving the $V_{ub}$-$\sin2\beta$ tension by the SUSY contribution in the GUT models, the phase of $B_s$-$\bar{B}_s$ mixing becomes different from the SM prediction, $\beta_{s}^{SM} \simeq 1^\circ$. The prediction of the phase in the SU(5) model is not very different from the SM prediction, $\beta_{s}^{SU(5)} \sim \pm 3^\circ$, while it becomes large in the SO(10) boundary condition to be $\beta_{s}^{SO(10)} \sim \pm (20 - 30)^\circ$. In the SO(10) case, the contribution of $\epsilon_K$ is also large. In order to fit the $\epsilon_K$ data, the negative value of $\beta_{s}^{\text{eff}}$ is preferred. The DØ collaboration has reported the direct measurement of the phase, and the phase $\beta_{s}^{\text{eff}}$ can be a large negative value, $\beta_{s}^{\text{eff}} = (-23 \pm 16)^\circ$ \cite{18}. The combined analysis of the CP asymmetry in the $B_s \to J/\psi\phi$ decay as well as asymmetry in semileptonic $B_s$ decay improves $\beta_{s}^{\text{eff}} = (-16 \pm 12)^\circ$. The measurements are expected to become more accurate soon and the prediction of the models can be tested.

The $A$-term can include a non-minimal term which is not proportional to the Yukawa coupling if the Yukawa coupling is the function of moduli and the moduli acquires a non-zero $F$-component which breaks SUSY. In the intersecting D-brane models, it is natural that the SUSY breaking scalar mass matrices are flavor universal while the $A$-term can have non-proportional terms. In this model, when the atmospheric neutrino mixing angle is maximized as a function of $U$-moduli, only the 13 and the 31 elements of the non-proportional $A$-term can be large. As a result, the phase of $B_d$-$\bar{B}_d$ mixing can be modified to make the unitarity relation consistent with the $|V_{ub}|$ data. The feature of this model is that Br$(\tau \to e\gamma)$ is larger than Br$(\tau \to \mu\gamma)$. The current bounds of the branching ratios are Br$(\tau \to \mu\gamma) < 4.5 \times 10^{-8}$ and Br$(\tau \to e\gamma) < 1.2 \times 10^{-7}$ \cite{28}.

Finally, we comment on the $\sin2\beta^{\text{eff}}$ from $b \to s$ penguin modes such as $B_d \to \phi K$ decay in our solutions to resolve the $V_{ub}$-$\sin2\beta$ tension. In GUT models, since the squarks are heavy $\sim 1$ TeV in the solutions, the contribution of the penguin diagrams are small. Thus, $\sin2\beta^{\text{eff}}_{B_d \to J/\psi K} \sim \sin2\beta^{\text{eff}}_{B_d \to \phi K}$. In our solutions using the non-proportional $A$-terms, the $b \to s$ penguin effects can become larger depending on the model parameters since the squarks are light. However, when 23 and 32 elements of the non-proportional $A$-term are zero as we have assumed in Eq.\cite{28}, the $b \to s$ penguin effects are not significant.

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