Intraband electron focusing in bilayer graphene

Csaba G Péterfalvi\textsuperscript{1,3}, László Oroszlány\textsuperscript{2}, Colin J Lambert\textsuperscript{1} and József Cserti\textsuperscript{2}

\textsuperscript{1} Department of Physics, Lancaster University, LA1 4YB, UK
\textsuperscript{2} Department of Physics of Complex Systems, Eötvös University, H-1117 Budapest, Pázmány Péter sétány 1/A, Hungary
E-mail: c.peterfalvi@lancaster.ac.uk

New Journal of Physics 14 (2012) 063028 (13pp)
Received 19 March 2012
Published 21 June 2012
Online at http://www.njp.org/
doi:10.1088/1367-2630/14/6/063028

Abstract. We propose an implementation of a valley selective electronic Veselago lens in bilayer graphene. We demonstrate that in the presence of an appropriately oriented potential step, low-energy electrons radiating from a point source can be re-focused coherently within the same band. The phenomenon is due to the trigonal warping of the band structure that leads to a negative refraction index. We show that the interference pattern can be controlled by an external mechanical strain.

Contents

1. Introduction 1
2. Electron focusing in novel materials 2
3. Focusing in bilayer graphene 5
4. Effects of broken symmetry 10
5. Conclusion 11
Acknowledgments 11
References 12

1. Introduction

Since its first experimental isolation [1, 2] graphene has had a profound impact on the solid state community. The past year has seen reinvigorated experimental and theoretical investigations of bilayer graphene, as sufficiently clean samples have been fabricated to probe the low-energy

\textsuperscript{3} Author to whom any correspondence should be addressed.
spectrum [3]. A key conclusion of these investigations is that mechanical distortion as well as electron–electron (e–e) interaction can lead to profound changes in the topology of the band structure and to symmetry breaking in the electronic system [4–6]. In recent years, new ideas on electron optics have utilized the exotic electronic structure of graphene and other novel materials such as topological insulators. Electron lenses utilizing negative refraction [7–10] and valley-polarized electron beam splitters [11–14] among others have been proposed to manipulate electron beams in mesoscopic devices. Apart from the manipulation of the valley degree of freedom, graphene devices have already been proposed for spin-polarized electron beam engineering using magnets [15], and for spatial separation of electrons and holes using normal–superconducting–normal junctions [16].

Unlike the two-dimensional electron gas (2DEGS) in more conventional semiconductors [17,18], which require a magnetic field to provide the focusing of electrons and the imaging of the interference pattern of injected electrons is hindered by the fact that the 2DEG is often buried inside a heterostructure [19], graphene-based devices can be self-focusing and their interference patterns are exposed on the surface. In this paper, we propose an electron-optical device whose valley-dependent interference pattern can be controlled by strain. By analogy with conventional 2DEGS, where the control of focusing by an external magnetic field leads to sensitive magnetic field sensors even at room temperature [20], our graphene-based device forms the basis for a sensitive detector of strain.

2. Electron focusing in novel materials

The focusing of electrons in graphene by a planar $pn$ junction was first proposed by Cheianov et al [7]. In their proposal, particles are focused due to the fact that in the valence band, the group velocity points in the opposite direction to the wavevector. Hence, electrons emitted from a point source on one side of a $pn$ junction converge on the other side at a focal point, thereby realizing a Veselago lens [21,22]. Circular $pn$ junctions have also been investigated recently in single-layer [8] and bilayer [23] graphene, where apart from the focal point, a hierarchy of caustics was discovered and described using the semiclassical approach of catastrophe optics [24]. Later it was pointed out that warping of the band structure [25] can also induce a focusing effect even when particles on the two sides of the junction are in the same band [9]. This was demonstrated with the hexagonally warped surface states of the three-dimensional (3D) topological insulator Bi$_2$Te$_3$ [26]. In this work, we propose a flat interface setup (shown in figure 1) that can focus electrons in bilayer graphene due to the trigonal warping [25] of the band structure. In this section, we review some aspects of electron optics with regard to focusing particles by a flat interface.

For a particle incident on the barrier with wavevector $k_i$ and group velocity $v_i = (\partial_k E(k)/\hbar)|_{k=k_i}$, the refractive index $n$ at the barrier is defined by the states that conserve the momentum parallel to the interface and propagate away from the interface. Let $k_{i/r}$ and $v_{i/r}$ denote the wavevector and group velocity of the transmitted/reflected particles, respectively. If the velocities $v_{i/r}$ make an angle $\phi_{i/r}$ with the normal of the barrier, then the refractive index is

$$n = \frac{\sin(\phi_i)}{\sin(\phi_t)}.$$  

(1)

In general, $n$ depends on the direction of $k_i$. 

New Journal of Physics 14 (2012) 063028 (http://www.njp.org/)
Figure 1. Single-band focusing in bilayer graphene with a potential step. A particle emitted from a source disperses or focuses depending on which valley it came from. Measurement of the maxima of the transmitted electron density with an STM tip can be used to indirectly gain information about the low-energy topology of the band structure.

The intraband focusing effect of a planar interface is closely related to the concave geometry of the $E(k)$-energy contours of the dispersion relation. It can be shown that electrons radiating from a point source into a small region in $k$-space will be focused on the other side of the junction if and only if the curvatures of the energy contours on the two sides at the corresponding $k$-wavevectors differ in sign. If this holds around the $k$-vectors aligned with the optical axis, that is, for rays that are nearly perpendicular to the interface, then the different signs of the curvatures result in a negative refractive index, which is however not a necessary condition for focusing if we move away from the optical axis. In particular, if the $E(k)$-contours are locally symmetric to the direction of the optical axis, then a simple, direct relation can be formulated between the refractive index for these rays and the curvatures on the two sides [9]:

$$n = \frac{c(k, E_i)}{c(k, E_f)},$$

(2)

where $E_i$ is the chemical potential on the incoming side of the junction and $E_f$ is the chemical potential on the other side of the junction. It also follows that around the optical axis, the negative sign of $n$ is needed for the focusing phenomenon. In figure 2, we show the trajectory of a single particle hitting a planar potential step in a bilayer graphene sample. (The barrier coincides with the $y'$-axis.) A classical trajectory can be parameterized by $\phi_i$ which determines all the remaining angles.

If a point source is situated at $(-x_s, 0)$, then on the other side of the interface, a generic trajectory has the form

$$y = x_s \tan(\phi_i(\psi_i)) + x \tan(\phi_i(\psi_i)).$$

(3)
Figure 2. Real space trajectory and \(k\)-space directions of a single particle incident on a potential step in the valley \(K'\) in a bilayer graphene junction at \(w = 0\) and \(\alpha = 0\), i.e. where the \((x, y)\) and \((x', y')\) systems coincide. Angles \(\varphi\) denote the direction of the wavevectors, whereas angles \(\phi\) stand for the direction of the group velocities. Note that in this anisotropic system the group velocity and the wavevector of a given state are, in general, not parallel. A particle emitted from the point source \(S\) at \(-2.07\) \(\mu m\) on the \(x\)-axis hits the boundary at the \(y\)-axis with an angle of incidence \(\phi_i\) and, along with electrons on neighbouring trajectories, gets diverted towards a cusp-type caustic, the maximum of which we call the focus \(F\), for the sake of simplicity. The electrons’ energy is 80 meV on the left and 10 meV on the right side of the junction. Blue arrows represent the incident particle, green arrows the transmitted, and red arrows the reflected ones. The wavevectors are denoted by dashed arrows and the classical trajectory of the particle by solid lines. Solid lines depict the electrons’ dispersion curves on both sides of the junction. Note that the curvature has different signs on the two sides for particles close to the optical axis.

Some of the refracted rays touch each other at certain points. The sets of such points are called caustics, which are envelopes of a family of rays at which the density of rays is singular. In the classical theory of geometrical optics, this singularity is unphysical; however, the results of the semiclassical catastrophe optics \([27]\) agree well with the quantum mechanical treatment, which predicts a finite particle density with local maxima \([24]\). The focal point of particles corresponds to the global maximum of a cusp-type caustic, which evolves into two fold-type caustics. The coordinates of these curves can be obtained by differential geometry:

\[
\partial_{\psi_i} y = 0.
\] (4)

In order to describe the system quantum mechanically, one has to solve the Schrödinger equation on both sides of the junction and match the solutions at the boundary. The boundary conditions can be derived from the Schrödinger equation by applying it for an infinitesimally narrow stripe containing the junction. In the case of the model Hamiltonian used in this work, these conditions require the wavefunction to be smooth and the same for its first derivative with respect to the direction perpendicular to the junction. This can be satisfied with two planewave-like solutions to the homogeneous Schrödinger equation on either side of the junction. For a given \(k_{x', y'}\), which must be conserved during the scattering process, there always exist one or two propagating waves and one or zero decaying wave, giving always exactly two solutions on either side needed to satisfy the boundary conditions. By solving this system of equations, we obtain the coefficients for the reflected and refracted rays matched to the incident ray.
Since all the calculations can be extended in a straightforward manner for several rays, for the sake of simplicity, wherever applicable, we will assume that we only have one refracted wave propagating to the right direction. To calculate the density of particles, the contributions of all the refracted rays with the corresponding transmission amplitudes were summed up in the far-field approximation. By the summation of the rays, a homogeneous distribution of the source electrons in \( \varphi_{\text{r}} \) is assumed.

3. Focusing in bilayer graphene

The central idea of our work is to use bilayer graphene with a potential step for focusing electrons, where particles remain in the same band. As we will demonstrate, this idea retains all the neat features of earlier setups [7, 9], including focusing and high transmission, and even incorporates the idea of valley-dependent electron beam manipulation [11, 12, 26].

We start with the Hamiltonian of the system under consideration

\[
H = \begin{pmatrix} V(r) & -\frac{1}{2m}p_+^2 + \xi v_3 p_+ + w^* \\ -\frac{1}{2m}p_+^2 + \xi v_3 p_+ + w & V(r) \end{pmatrix},
\]

where \( p_\pm = p_x \pm p_y \) are the momentum operators and \( \xi = 1 \) for valley \( K \) and \( \xi = -1 \) for \( K' \). The material parameters of bilayer graphene \( m \approx 0.035m_e \) and \( v_3 \approx 10^5 \text{ m s}^{-1} \) are defined among others in [28]. For very low energies (\( \lesssim 1 \text{ meV} \)), the complex parameter \( w \) describes symmetry breaking due to the e–e interaction [4], but it also describes the external mechanical strain [5] for higher energies as well. The relation between \( w \) and the mechanical distortion of the lattice can be formulated as

\[
w = \eta (\delta - \delta') \exp(-2i\theta),
\]

where \( \eta \) is a real constant describing the mechanical properties of the lattice, \( \delta > \delta' \) are the principal values of the strain tensor \( \frac{1}{2}(\partial_i d_j + \partial_j d_i) \), and \( \theta \) is the angle between the principal axes and coordinate axes \((x, y)\). In the strain tensor, \( i \) and \( j \) are the \((x, y)\) coordinate-indexes and \((d_i, d_j)\) denote displacements from the equilibrium positions. In expression (6), we neglect the interlayer shear shift. The potential \( V(r) \) describes a potential step that is sufficiently smooth on the atomic scale so that it does not introduce inter-valley scattering, and we can also neglect a term proportional to the gradient of the potential that arises in the Schrieffer–Wolff transformation in the derivation of (5) [25, 29]. We assume \( V(r) \) to have the form

\[
V(r) = V_1 + V_2 \Theta(x - \tan(\alpha)y),
\]

i.e. the potential on the two sides of the step is constant \( V_1 \) and \( V_2 \) and the barrier makes an angle \( \alpha \) with the crystallographic \( \Gamma K \)-direction (\( k_x \)-axis). It is convenient to introduce a coordinate system \((x', y')\) where \( y' \) is defined by the planar potential step, and the source is sitting on the axis \( x' \), which means that \( x \) and \( x' \) and also \( y \) and \( y' \) make an angle denoted by \( \alpha \). We do not take into account a gap term in the Hamiltonian that accounts for the electrostatic asymmetry of the two graphene layers, since this would not alter the main message of this work. We also neglect second nearest neighbour hopping in the individual graphene planes that has a negligible effect on the results presented here. In the absence of the potential \( V(r) \) the spectrum of (5) has been studied in [5, 25]. For low energies, depending on the value of \( w \), the spectrum consists of four or two massless Dirac cones, whereas at higher energies, the band structure merges to form only a single pocket per valley. A typical energy scale of this model is the Lifshitz energy.
Figure 3. Curvature $c$ as a function of energy along the direction indicated by the dashed line in the inset. The inset shows the energy contours around the $K'$ point for $w = 0$ meV. In the main figure, the solid line denotes curvature for $w = -6$ meV, the dashed line is for $w = 0$ meV and the dotted line is for $w = 6$ meV. Black dots denote the particles’ energy on the left (80 meV) and right (10 meV) sides of the junction as in figure 2. The electrons’ energy is chosen to find optimal conditions for focusing, that is, a negative and slowly varying refractive index $n$ in a possibly wide range of the angle of incidence $\phi$.

$E_L = m v_f^2 / 2 \approx 1$ meV, which defines the Lifshitz transition of the band structure’s topology from multiple pockets to a single pocket in the absence of strain ($w = 0$). Since the Lifshitz energy is quite small, it was until recently difficult to gain experimental data from energies below it [3].

As shown in the previous section, focusing is controlled by the curvature of the band structure. If on the two sides of the potential step there exist regions in the $k$-space on the given energies with overlapping projections to the dimension parallel to the junction, and for these corresponding $k$-regions the curvature of the dispersion relation has different sign, then the system can focus electrons. In figure 3, we present the curvature $c$ of the $E(k)$ dispersion curve of the considered model as a function of energy along the $\Gamma K$-direction for different values of $w$. It is clear that the curvature in the absence of $w$ changes sign roughly at $E \approx 42.1$ meV. This energy scale is still sufficiently small compared to the direct interlayer hopping [25, 28] $\gamma_1 \approx 0.4$ eV; that is, it falls in the energy range where the two-band model (5) is still valid. Since a changing $w$ can produce a large change in the energy of zero curvature, one can study the focused electrons in bilayer graphene samples with a planar potential step, and yield information about the value of $w$ even in samples that are only moderately gated ($\sim 50$ meV).

Our setup also has the capability of separating electrons in different valleys, without the need for an unrealistic potential barrier height of about 1 eV, as in previous graphene-based electron optical setups [11, 14]. This is demonstrated in figure 4, where we follow the classical trajectories of several particles emitted from a point source, in real and $k$-space for the case of undistorted samples, with the interface aligned on the $y$-axis, i.e. $w = 0$ meV and $\alpha = 0$. It is
Figure 4. Real space trajectories and $k$-space directions for both valleys with barrier parameters the same as in figure 2. In panels (b) and (c), blue arrows show the direction of the group velocity of incident particles with wavevectors corresponding to the arrows’ position on the $E(k)$ energy contour (grey solid curves). Green arrows are for the transmitted, and red ones for the reflected electrons in both valleys. The arrows in (a) and (d) show the real space trajectories of particles emitted from the source (blue) and partially transmitted through (green) or reflected by the junction (red). Note that particles in valley $K'$ focus, while particles in the valley $K$ diverge. We only show trajectories that have a finite probability of transmission at 80 and 10 meV.

clear from the figure that electrons from the two valleys behave differently. Particles in valley $K$ disperse upon passing the barrier, whereas those in valley $K'$ gather in a focus point on the other side. If the barrier is not aligned with the crystallographic orientation of the lattice, then in general there will be two or three foci, each gathering particles from different valleys as shown in figure 5.
Figure 5. Total particle densities and theoretical curves of the caustics (blue lines) for different values of $\alpha$. Note that a given focus point gathers particles from only a single valley. The position of the point source is at $-2.07 \, \mu m$ on the $x'$-axis. The electrons’ energy is 80 meV on the left and 10 meV on the right side of the junction. The equation of the sheared cusp caustics can be derived from equations (3) and (4) [9, 30].

One key ingredient that is not captured by the ray tracing analysis is the transmission probability of particles reaching the junction. This can be calculated by evaluating first the expectation values of the current operator along the $x'$-axis. Then the transmission probability $T$ will be the ratio of the current carried by the refracted wave in the $x'$ direction to that of the incoming wave also projected to $x'$. The reflection probability $R$ can be calculated on a similar basis from the reflected wave. Note that the evanescent waves do not carry current.
Figure 6. Quantum mechanical transmission probability as a function of the angle of incidence in valley $K'$ for different values of $w$ as in figure 3. The barrier parameters are the same as in figure 2.

Figure 7. Calculated density of transmitted particles for different values of $w$. The particles’ energy is 80 meV on the left side and 10 meV on the right side of the junction. The source is at $-2.07 \mu m$ on the $x'$-axis. The sub-figures correspond to (a) $w = 0$ meV, (b) $w = 6i$ meV, (c) $w = -6i$ meV, (d) $w = -4$ meV and (e) $w = 2 - 2i$ meV.

In figure 6, we show the transmission probability as a function of the angle of incidence for different values of $w$ in the case of $\alpha = 0$. Remarkably, the transmission probability is high and roughly independent of the angle of incidence $\phi_i$ for particles reaching the junction at a wide range of angles of incidence around the $x'$-axis. This is a beneficial result of scattering within the same band. For particles that change band during refraction at the junction, the transmission probability is reduced, is very sensitive to $\phi_i$ and vanishes exactly for perpendicular incidence [23], unless gaps are induced by external gating [10].

For finite $\alpha$, when the junction is not aligned with the $y$-axis, the focus point will not remain on the $x$-axis but will continuously move with $\alpha$. This is depicted in figure 5, where the total particle density is plotted with the contributions from both valleys and for different values of
Figure 8. A semiclassical estimation of the position of the tip of the cusp caustic formed by electrons from the valley $K'$ as a function of the parameter $w$ for $\alpha = 0$. The electrons’ energy is 80 meV on the left side and 10 meV on the right side of the junction. The position of the source is at $-2.07 \mu m$ on the $x$-axis. Inside the green curve the low-energy spectrum has four Dirac cones, between the red and green curves there are two Dirac cones and a local minimum, while outside the red curve the spectrum has two Dirac points but no local minimum. The nonzero value of $w$ is attributed to a distortion of the lattice that is about or less than 1% on this map [5]. Here we note that the e–e interaction renormalizes $w$, thus rendering it slightly energy dependent and changing its relation to the applied strain. This, however, does not affect our results qualitatively.

4. Effects of broken symmetry

As shown in [4], the e–e interaction in bilayer graphene leads to a spontaneous breaking of the threefold rotational symmetry, giving rise to a finite value of $w$ in the effective low-energy Hamiltonian (5). However in this description of the e–e interaction, the Fermi energy as well as the energy of the excitations around it need to be of the same order of magnitude as the Lifshitz energy ($\sim 1$ meV), which is not the case discussed in this paper. Also, $w$ originating from the interaction can have very different values on the two sides of the junction, which are unknown. On the other hand, later, it was argued in [5] that mechanical distortion leads to the same low-energy effective theory, with a constant $w$ on a wide energy range. A key consequence of a finite $w$ is that depending on its value, the low-energy spectrum of the system can undergo a topological phase transition. For small magnitudes of $w$, the spectrum consists of four Dirac
cones, while at larger $w$, there are only two Dirac cones in the spectrum. The small value of the Lifshitz energy renders it difficult to infer directly the number of Dirac points. Therefore it is essential to seek indirect methods for determining the value of $w$ and hence the number of Dirac points.

As presented in figure 3, the curvature of the energy contours depends considerably on the value of $w$. This suggests that the position of the focus point will be similarly sensitive. Indeed, this is clear from figure 7, where we show particle densities from valley $K'$ for different values of $w$. The most important information one can extract from these data is the position of the maxima that can be used to directly infer the value of the complex parameter $w$. In figure 8, we show the semiclassical estimation for the position of the tip of the cusp caustic as a function of the real and the imaginary part of $w$ for $\alpha = 0$. From expression (6), it is apparent that the imaginary phase of $w$ will be $-2\theta$, reflecting a direct relation to the geometrical deformation of the lattice. It is also worth mentioning that if $\theta$ is a multiple of $\pi/2$, then $w$ is real, but if mod($\theta, \pi/2$) = $\pi/4$, then $w$ is purely imaginary, and otherwise it is a general complex number. Interestingly, if $\delta = \delta'$, then $w$ vanishes. This corresponds to a ‘hydrostatic’ rescaling of the lattice and only affects the values of $m$ and $v_3$. Based on the results of [5] and [31], it can be showed that a distortion of 1% changes $m$ and $v_3$ by just a few per cent and thus the effect on the diffraction pattern is negligible. On the other hand, we have shown that the position of the foci sensitively depends on the value of $w$. Thus by experimentally investigating the focusing effect, one can measure the external strain present in the sample.

In clean systems, the most important factor that determines the lifetime of excitations is electron–phonon scattering. According to recent calculations [32, 33], the mean free path for excitations in the considered energy range around 50 meV is of the order of microns, which is accessible by current experimental techniques [34].

5. Conclusion

In this paper, we investigated the anisotropic electron optics of bilayer graphene. We demonstrated that a moderate potential step can be used to focus electrons within the same band in a valley-selective manner with high transmission probability. We also investigated the effects of broken symmetry in the electronic structure due to mechanical distortion. The presented results clearly show that the proposed device can be used to determine symmetry breaking and extract indirect information about the low-energy topology of the band structure of bilayer graphene samples. These results are expected to form key ingredients in future device design, where one can envisage using bilayer graphene as building blocks for more complex electron-optical devices.

Acknowledgments

The authors are grateful to E McCann, M Berry, Cs Tőke, V I Fal’ko, A Cortijo, D A Gradinar and M Mucha-Kruczyński for fruitful discussions. This work was supported by the Hungarian research funds OTKA K81492, K76010, K75529, NK72916, NNF78842, TAMOP-4.2.1/B-09/1/KMR-2010-0002, TAMOP-4.2.1./B-09/1/KMR-2010-0003 and the EU grant NanoCTM.
References

[1] Novoselov K S, Geim A K, Morozov S V, Jiang D, Katsnelson M I, Grigorieva I V, Dubonos S V and Firsov A A 2005 Two-dimensional gas of massless Dirac fermions in graphene Nature 438 197
[2] Novoselov K S, McCann E, Morozov S V, Fal’ko V I, Katsnelson M I, Zeitler U, Jiang D, Schedin F and Geim A K 2006 Unconventional Quantum Hall effect and Berry’s phase of $2\pi$ in bilayer graphene Nature Phys. 2 177
[3] Mayorov A S et al 2011 Interaction-driven spectrum reconstruction in bilayer graphene Science 333 860
[4] Lemonik Y, Aleiner I L, Tőke Cs and Fal’ko V I 2010 Spontaneous symmetry breaking and Lifshitz transition in bilayer graphene Phys. Rev. B 82 201408
[5] Mucha-Kruczyński M, Aleiner I L and Fal’ko V I 2011 Strained bilayer graphene: band structure topology and Landau level spectrum Phys. Rev. B 84 041404
[6] Gradinar D A, Schomerus H and Fal’ko V I 2012 Conductance anomaly near the Lifshitz transition in strained bilayer graphene Phys. Rev. B 85 165429
[7] Cheianov V V, Fal’ko V and Altshuler B L 2007 The focusing of electron flow and a Veselago lens in graphene $p$–$n$ junctions Science 315 1252
[8] Cserti J, Pályi A and Péterfalvi C 2007 Caustics due to a negative refractive index in circular graphene $p$–$n$ junctions Phys. Rev. Lett. 99 246801
[9] Hassler F, Akhmerov A R and Beenakker C W J 2010 Flat-lens focusing of electrons on the surface of a topological insulator Phys. Rev. B 82 125423
[10] Park S and Sim H-S 2011 $\pi$ Berry phase and Veselago lens in a bilayer graphene $n$–$p$ junction Phys. Rev. B 84 235432
[11] Garcia-Pomar J L, Cortijo A and Nieto-Vesperinas M 2008 Fully valley-polarized electron beams in graphene Phys. Rev. Lett. 100 236801
[12] Pereira J M Jr, Peeters F M, Costa Filho R N and Farias G A 2009 Valley polarization due to trigonal warping on tunneling electrons in graphene J. Phys.: Condens. Matter 21 045301
[13] Abergel D S L and Chakraborty T 2009 Generation of valley polarized current in bilayer graphene Appl. Phys. Lett. 95 062107
[14] Wang Z and Liu F 2010 Manipulation of electron beam propagation by hetero-dimensional graphene junctions ACS Nano 4 2459
[15] Michetti P, Recher P and Iannaccone G 2010 Electric field control of spin rotation in bilayer graphene Nano Lett. 10 4463–9
[16] Gómez S, Burset P, Herrera W J and Levy Yeyati A 2012 Selective focusing of electrons and holes in a graphene-based superconducting lens Phys. Rev. B 85 115411
[17] van Houten H, van Wees B J, Mooij J E, Beenakker C W J, Williamson J G and Foxon C T 1988 Coherent electron focussing in a two-dimensional electron gas Europhys. Lett. 5 721
[18] Topinka M A, Westervelt R M and Heller E J 2003 Imaging electron flow Phys. Today 56 47
[19] Gilbertson A M, Kormányos A, Buckle P D, Fearn M, Ashley T, Lambert C J, Solin S A and Cohen L F 2011 Room temperature ballistic transport in InSb quantum well nanodevices Appl. Phys. Lett. 99 242101
[20] Aidala K E, Parrott R E, Kramer T, Heller E J, Westervelt R M, Hanson M P and Gossard A C 2007 Imaging magnetic focusing of coherent electron waves Nature Phys. 3 464
[21] Veselago V G 1968 The electrodynamics of substances with simultaneously negative values of $\epsilon$ and $\mu$ Sov. Phys.—Usp. 10 509
[22] Pendry J B 2000 Negative refraction makes a perfect lens Phys. Rev. Lett. 85 3966
[23] Péterfalvi Cs, Pályi A and Cserti J 2009 Electron flow in circular $n$–$p$ junctions of bilayer graphene Phys. Rev. B 80 075416
[24] Péterfalvi C, Pályi A, Rusznyák Á, Koltaï J and Cserti J 2010 Catastrophe optics of caustics in single and bilayer graphene: fine structure of caustics Phys. Status Solidi b 247 2949
[25] McCann E and Fal’ko V I 2006 Landau-level degeneracy and Quantum Hall effect in a graphite bilayer Phys. Rev. Lett. 96 086805

New Journal of Physics 14 (2012) 063028 (http://www.njp.org/)
[26] Liang Fu 2009 Hexagonal warping effects in the surface states of the topological insulator Bi$_2$Te$_3$ Phys. Rev. Lett. 103 266801
[27] Berry M V and Upstill C 1980 Catastrophe optics: morphologies of caustics and their diffraction patterns Prog. Opt. 18 257
[28] Koshino M 2009 Electronic transport in bilayer graphene New J. Phys. 11 095010
[29] Poole C J 2010 On the applicability of the two-band model to describe transport across $n$--$p$ junctions in bilayer graphene Solid State Commun. 150 632
[30] Nye J F and Hannay J H 1984 The orientations and distortions of caustics in geometrical optics Opt. Acta: Int. J. Opt. 31 115
[31] Mucha-Kruczyński M, Aleiner I L and Fal’ko V I 2011 Landau levels in deformed bilayer graphene at low magnetic fields Solid State Commun. 151 1088
[32] Borysenko K M, Mullen J T, Barry E A, Paul S, Semenov Y G, Zavada J M, Buongiorno Nardelli M and Kim K W 2010 First-principles analysis of electron–phonon interactions in graphene Phys. Rev. B 81 121412
[33] Borysenko K M, Mullen J T, Li X, Semenov Y G, Zavada J M, Buongiorno Nardelli M and Kim K W 2011 Electron–phonon interactions in bilayer graphene Phys. Rev. B 83 161402
[34] Zhang Y, Brar V W, Wang F, Girit C, Yayon Y, Panlasigui M, Zettl A and Crommie M F 2008 Giant phonon-induced conductance in scanning tunnelling spectroscopy of gate-tunable graphene Nature Phys. 4 627