A Genetic Algorithm to Solve Capacity Assignment Problem in a Flow Network

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Abstract: Computer networks and power transmission networks are treated as capacitated flow networks. A capacitated flow network may partially fail due to maintenance. Therefore, the capacity of each edge should be optimally assigned to face critical situations—i.e., to keep the network functioning normally in the case of failure at one or more edges. The robust design problem (RDP) in a capacitated flow network is to search for the minimum capacity assignment of each edge such that the network still survived even under the edge’s failure. The RDP is known as NP-hard. Thus, capacity assignment problem subject to system reliability and total capacity constraints is studied in this paper. The problem is formulated mathematically, and a genetic algorithm is proposed to determine the optimal solution. The optimal solution found by the proposed algorithm is characterized by maximum reliability and minimum total capacity. Some numerical examples are presented to illustrate the efficiency of the proposed approach.

Keywords: Flow network, capacity assignment, network reliability, genetic algorithms.

1 Introduction

The capacity assignment problem (CAP) is generally defined as the search for the optimal capacities that minimize the delay in the network and maximize its reliability [Zantuti (2008)]. Capacity assignment plays an important role in various industries, including communication networks [Chang and Gavish (1993); Ahlert, Corsten and Gössinger (2009)] and shipping industries [Wang and Yun (2013); Hung and Chen (2013)].

The maximum capacity of each component (arc or node) is essential in evaluations of the system reliability of a stochastic-flow network (SFN). Many papers have introduced algorithms or methods based on minimal paths or minimal cuts to evaluate the reliability of SFNs [Lin, Jane and Yuan (1995); Lin (2001, 2002); Lin and Yeh (2015)]. These studies are based on knowing the capacity of each component in advance. The maximum capacity of each component has been discussed by Chen et al. [Chen and Lin (2008)].

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Received: 03 April 2020; Accepted: 06 May 2020.
The analysis of a network structure has an effect on the capacity assignment strategy, as this analysis helps determine the critical nodes (the reliability value of an SFN equals zero when the critical node has zero capacity). Consequently, the best solution to the CAP must be found in order to decrease the number of critical nodes.

Chen et al. [Chen and Lin (2008)] studied the CAP for SFNs with node failure—where the node has several states or capacities and may fail. They also studied the reliability of an SFN in the case of an existing critical node. Later, Chen et al. [Chen and Lin (2010)] defined the CAP as a robust design problem for an SFN and proposed an algorithm to solve the problem. Chen [Chen (2012)] discussed the robust design problem for an SFN in the case of each edge having several capacities and the potential to fail, proposing an algorithm to determine the minimum capacity assignment of each edge so that the network can still function. Chen [Chen (2012)] also stated that the problem is an NP-hard problem and proposed an exact algorithm to solve it.

The capacity assignment in a stochastic-flow network is known to be NP-hard [Ball (1986); Chen and Lin (2010)], a relatively fast optimization algorithm would be beneficial. A genetic algorithm (GA) is a heuristic search method used in optimization problems. To solve reliability optimization problems, many based GA approaches were proposed such as [Younes and Hassan (2011)]. Also, GA is used to solve multiple objective optimization problems, [Taboada, Espiritu and Coit (2008); Coello and Christiansen (2000); Deb, Pratap, Agarwal et al. (2000); Azaron, Perkgoz, Katagiri et al. (2009); Hassan and Abdou (2018)]. Therefore, this paper presents a GA to solve the capacity assignment problem. The objective is to minimize the total capacities while meeting the network reliability requirement.

The rest of this paper is organized as follows. Section 2 sets forth the preliminaries. Section 3 presents the problem formulation. Then, Section 4 explains the proposed genetic algorithm (GA). Section 5 provides the steps of the entire algorithm. Section 6 includes several examples demonstrating the usability of the proposed approach. Finally, Section 7 draws conclusions and outlines possibilities for future work.

2 Preliminaries

2.1 Network reliability $R_d$

Given the demand $d$ and the set of minimal paths ($mp$), the system reliability $R_d$ is defined as Lin [Lin (2001)]:

$$R_d = Pr\{X|V(X) \geq d\}$$

where $X$ is a lower boundary point for $d$ and $V(X)$ is the maximum flow under $X$, $X=(x_1, x_2, ..., x_m)$. $X$ can be deduced from $F=(f_1, f_2, ..., f_m)$ through the following equation:

$$x_i = \sum_{j=1}^{m} f_j |a_i \in mp_j| \text{ for each } i = 1, 2, ..., n$$

2.2 Critical edge

The arc $a_i$ is said to be critical if and only if the network reliability $R_{d,i}$ is zero, where $R_{d,i}$ is the reliability of the given SFN when zero capacity is assigned to $a_i$ (failed).
2.3 Assumptions

The following assumptions should be satisfied for the given SFN:

(i) Each node is perfect and has an infinite capacity.
(ii) The capacity of each arc has an integer-valued random variable.
(iii) The arcs are perfect and unlimited in capacity.
(iv) Flow of the given SFN satisfies the flow-conservation law [Ford and Fulkerson (1962)].
(v) The arcs are statistically independent.

3 Problem formulation

Let $M = (M^1, M^2, ..., M^n)$ as assigned capacities to the set of edges $(a_1, a_2, ..., a_n)$. The mathematical formulation of the problem is:

Minimize $C = \sum_i M^i$ \hspace{1cm} (3)

Subject to: $R_d \geq R_0$ \hspace{1cm} (4)

where $R_d$ is reliability corresponding to the assigned capacities under demand $d$ and $R_0$ is the network reliability requirement. $M^i$ ranges from 1 to $d$, except for critical edges [Lin (2008)], $M^i = d$.

4 The genetic algorithm (GA)

The following subsections describe the different components of the presented GA.

4.1 Representation, crossover and mutation

The chromosome $M$ is represented by a string of length $n$, where $n$ is the number of edges, as follows:

$$(M^1, M^2, ..., M^n)$$

Figure 1: Chromosomal representation

The one-cut point crossover is used to generate two new offspring, and a simple mutation process is used to mutate the offspring. Figs. 2 and 3 show the crossover process and the mutation process, respectively.

Figure 2: Crossover process

Figure 3: Mutation process
4.2 Fitness function
The following penalty function is used as a fitness function [Coit and Smith (1996); Altiparmak, Dengiz and Smith (1997); Gen and Cheng (2000)]:

\[
\mathbb{C}(M) = \mathbb{C}(M) + \delta[M_{\text{max}}(R_d(M) - R_{\text{min}})]^2
\]  

(3)

where \( M_{\text{max}} \) is the maximum capacity of \( M \), and

\[
\delta = \begin{cases} 
0, & \text{if } R_d(M) \geq R_{\text{min}} \\
1, & \text{otherwise}
\end{cases}
\]  

(4)

The fitness function is

\[
\text{Fit}(M) = \mathbb{C}_{\text{max}}(M) - \mathbb{C}_{\text{obj}}(M)
\]  

(5)

where \( \mathbb{C}_{\text{max}}(M) \) is the maximum value of Eq. (9) for the current population.

4.3 Selection process
The algorithm uses the roulette wheel selection mechanism to select new parents [Hassan (2020)], and the selection of a chromosome is based on its fitness value.

5 Entire algorithm
The following steps describe the GA used to solve the capacity assignment problem:

\begin{tabular}{l}
Start \\
Input the information of the network: number of edges, their reliabilities, the demand \( d \), and paths. \\
Set the GA parameters. \\
Generate initial population. \\
Evaluate the initial population and calculate \( \text{Fit}(M) \) for each chromosome. \\
\hspace{1cm} \textbf{while} \ g \leq g_{\text{max}}, \textbf{do} \\
\hspace{1.5cm} \textbf{while} \ p \leq p_{\text{size}}, \textbf{do} \\
\hspace{2.5cm} Select two parents by using roulette wheel selection. \\
\hspace{2.5cm} Generate new child by applying crossover and mutation according to \( \lambda \) and \( \mu \), respectively. \\
\hspace{2.5cm} \hspace{1cm} p = p + 1; \\
\hspace{2.5cm} \textbf{End do.} \\
\hspace{1.5cm} Evaluate the current population. \\
\hspace{1cm} g := g + 1; \\
\hspace{1cm} \textbf{End do.} \\
End
\end{tabular}

6 Studied cases
We used MATLAB 2013a to implement the presented algorithm. It is applied to three network examples. The GA parameters were \( g_{\text{max}}=100, p_{\text{size}}=20, \lambda=0.90, \) and \( \mu=0.10 \).

The network in Fig. 4 includes four nodes and six edges [Chen (2012)]. The available reliabilities of the edges are 0.91, 0.93, 0.94, 0.93, 0.87, and 0.91. The best solutions
found for R and C are 0.9997 and 22, respectively. Two other network examples were also studied, as shown in Figs. 5 and 6 [Chen (2012)]. The best solutions found are listed in Tab. 1.

**Table 1:** The best solution found for each network

| Studied Network | $R_{\text{min}}$ | $M$                     | $C(M)$ | $R_d(M)$ |
|-----------------|-----------------|------------------------|--------|----------|
| Four-node       | 0.9             | 5 5 1 1 4 5            | 21     | 0.9987   |
|                 | 0.999           | 5 5 1 1 5 5            | 22     | 0.9997   |
| Eight-node      | 0.9             | 4 4 1 1 1 1 1 1 1 1 1 1 2 2 | 22     | 0.9591   |
|                 | 0.999           | 4 4 1 1 1 1 1 1 1 1 1 1 1 2 2 | 22.0028| 0.9637   |
| Thirteen-node   | 0.999           | 4 4 4 4 4 3 3 4 4 4 4 4 1 1 1 2 1 1 4 | 102.0028| 0.9560*  |

*The $R_d(M)$ values for eight-node and thirteen-node networks are less than those found by Chen [Chen (2012)], but the values were verified as correct for the corresponding $M$.

**Figure 4:** Four-node network with 6 edges

7 Conclusions

The capacity assignment problem was discussed in this paper and was formulated mathematically. A genetic algorithm was proposed to solve the problem. The presented algorithm successfully determined the optimal capacities with minimum total capacities and maximum reliability. The critical edges were treated in a special manner by being assigned capacity values equal to the demand. In comparison with Chen [Chen 2012], this paper finds the best capacity distribution to all studied cases. The proposed solution approach may be applied to various problem related to the capacity assignment problem.
Figure 5: Eight-node network with 14 edges

Figure 6: Thirteen-node network with 33 edges
Acknowledgement: The authors thank the anonymous referees for their careful readings and provisions of helpful suggestions to improve the presentation.

Funding Statement: The author(s) received no specific funding for this study.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

References
Ahlert, K. H.; Corsten, H.; Gössinger, R. (2009): Capacity management in order-driven production networks- a flexibility-oriented approach to determine the size of a network capacity pool. International Journal of Production Economics, vol. 118, no. 2, pp. 430-441.
Azaron, A.; Perkgoz, C.; Katagiri, H.; Kato, K.; Sakawa, M. (2009): Multi-objective reliability optimization for dissimilar-unit cold-stand by systems using a genetic algorithm. Computers and Operations Research, vol. 36, no. 5, pp. 1562-1571.
Chang, S. G.; Gavish, B. (1993): Telecommunications network topological design and capacity expansion: formulations and algorithms. Telecommunication Systems, vol. 1, no. 1, pp. 99-131.
Chen, S. G. (2012): An optimal capacity assignment for the robust design problem in capacitated flow networks. Applied Mathematical Modeling, vol. 36, pp. 5272-5282.
Chen, S. G.; Lin, Y. K. (2008): Capacity assignment for a stochastic-flow network based on structural importance. Asian International Workshop Advances Reliability Modeling, Taichung, Taiwan.
Chen, S. G.; Lin, Y. K. (2010): An approximate algorithm for the robust design in a stochastic-flow network. Communications in Statistics-Theory and Methods, vol. 39, no. 13, pp. 2440-2454.
Coello, C. A.; Christiansen, A. D. (2000): Multi-objective optimization of trusses using genetic algorithms. Computers and Structures, vol. 75, no. 6, pp. 647-660.
Deb, K.; Pratap, A.; Agarwal, S.; Meyarivan, T. (2002): A fast and elitist multi-objective genetic algorithm: NSGA-II. IEEE Transactions on Evolutionary Computation, vol. 6, no. 2, pp. 182-198.
Ford, L. R.; Fulkerson, D. R. (1962): Flows in networks, NJ: Princeton University Press.
Gen, M.; Cheng, R. (2000): Genetic algorithms and engineering optimization. Wiley Series in Engineering, Design, and Automation.
Hassan, M. R. (2020): Maximizing reliability of the capacity vector for multi-source multi-sink stochastic-flow networks subject to an assignment budget, Journal of Industrial and Management Optimization, doi: 10.3934/jimo.2020020.
Hassan, M. R.; Abdou, H. (2018): Multi-objective components assignment problem subject to lead-time constraint. Indian Journal of Science and Technology, vol. 11, no. 21, pp. 1-9.
Hung, Y. F.; Chen, C. H. (2013): An effective dynamic decision policy for the revenue management of an airline flight. *International Journal of Production Economics*, vol. 144, no. 2, pp. 440-450.

Lin, J. S.; Jane, C. C.; Yuan, J. (1995): On reliability evaluation of a capacitated-flow network in terms of minimal path sets. *Networks*, vol. 25, pp. 131-138.

Lin, Y. K. (2001): A simple algorithm for reliability evaluation of a stochastic-flow network with node Failure. *Computer Operation Research*, vol. 28, pp. 1277-1285.

Lin, Y. K. (2002): Two-commodity reliability evaluation for a stochastic-flow network with node failure. *Computer Operation Research*, vol. 29, pp. 1927-1939.

Lin, Y. K.; Yeh, C. T. (2011): Maximizing network reliability for stochastic transportation networks under a budget constraint by using a genetic algorithm. *International Journal of Innovative Computational, Information and Control*, vol. 7, pp. 7033-7050.

Lin, Y. K.; Yeh, C. T. (2015): System reliability maximization for a computer network by finding the optimal two-class allocation subject to budget. *Applied Soft Computing*, vol. 36, pp. 578-588.

Taboada, H. A.; Espiritu, J. F.; Coit, D. W. (2008): MOMS-GA: a multi-objective multi-state genetic algorithm for system reliability optimization design problems. *IEEE Transactions on Reliability*, vol. 57, no. 1, pp. 182-191.

Wang, W. F.; Yun, W. Y. (2013): Scheduling for inland container truck and train transportation. *International Journal of Production Economics*, vol. 143, no. 2, pp. 349-356.

Younes, A.; Hassan, M. R. (2011): A genetic algorithm for reliability evaluation of a stochastic-flow network with node failure. *International Journal of Computer Science and Security*, vol. 4, no. 6, pp. 528-536.

Zantutti, A. F. (2008): Algorithms for capacities and flow assignment problem in computer networks, *19th International Conference on Systems Engineering*, pp. 315-318.