Influence of modification of an asymmetric outline on teeth deformation in the spur gears

JAN BUREK
MICHAŁ CHLOST *

* Dr hab. inż. Jan Burek, prof. PRz, jburek@prz.edu.pl, https://orcid.org/0000-0003-2664-5248 – Katedra Technik Wytwarzania i Automatyzacji, Wydział Budowy Maszyn i Lotnictwa, Politechnika Rzeszowska, Rzeszów, Polska

Mgr inż. Michał Chlost, m.chlost@prz.edu.pl, https://orcid.org/0000-0001-9420-4239 – Katedra Technik Wytwarzania i Automatyzacji, Wydział Budowy Maszyn i Lotnictwa, Politechnika Rzeszowska, Rzeszów, Polska

The aim of the article was to determine the value of deformation of straight teeth with an asymmetric outline with variation of the pressure angle and minimization of tooth undercutting. The geometry of the gear was developed as a parametric model in the NX 10 system. Simulation tests of tooth deflection were performed using the Pre/Pos module available in the NX10 program.

KEYWORDS: gears, asymmetric tooth, FEM, 5-axis milling

To reduce the deformation of the elastic teeth of gears under load, their outline is often modified. One of the modification methods is to introduce an asymmetric tooth outline. Previous FEM studies focused mainly on the analysis of deformation and strength of asymmetric teeth while modifying the transition curve at the base of the tooth [1]. A commonly used method of gear tooth modification is to change the pressure angle $\alpha_1$ and $\alpha_2$ on the tool, which is the Maag gear (fig. 1). Parameters of this modification are presented in tab. I [1, 2].

![Fig. 1. Geometry of Maag gear](https://example.com/fig1.png)

TABLE I. Sizes defining a gear tool

| Designation | Pattern | Description |
|-------------|---------|-------------|
| $\alpha$    | –       | pressure angle |
| $h$         | $h = h_1 + h_2$ | tooth height |
| $h_a$       | $h_a = (y + x) \cdot m$ | addendum |
| $h_f$       | $h_f = (y - x) \cdot m + c$ | dedendum |
| $m$         | –       | module |
| $p$         | $p = \pi \cdot m$ | circular pitch |
| $s$         | $s = p/2$ | tooth thickness |
| $l_w$       | $l_w = c \cdot m$ | top clearance |

where: $c$ – vertex clearance factor, $y$ – tooth height factor, $x$ – tooth correction factor
This method, however, has a major disadvantage, namely: as a result of the rolling away of the rack gear at small pressure angles, i.e. $\alpha_2<20^\circ$, an undercut of the tooth is created, weakening its base (fig. 2). Whether an undercut occurs also depends on the limit number of teeth, which for a gear tool is determined by the relationship [2, 3]:

$$z_g = \frac{y}{\sin^2 \alpha_i}$$

where: $y$ - tooth height factor, $\alpha_i$ - pressure angle ($i = 1, 2$).

Therefore, new methods of asymmetric tooth outline modification are being sought. One of them is the machining of teeth on five-axis machines, which allows for obtaining the asymmetric modification of the tooth profile as a result of a change in the tilt angle of the cutter axis $\beta$ (fig. 3). Machining is carried out by tangentially positioning the cutter to the side surface of the tooth. The main feed movement takes place along the tooth line, and the positioned tool axis is implemented by changing the angle $\beta$. This angle changes depending on the position of the tool on the tooth contour line. The range of angle $\beta$ variation is limited by the diameter of the vertices $d_a$ and the diameter of the feet $d_f$ (tab. II). As a result of the $\beta$ angle control, the required tooth side curvature and the specified angle of the action line can be obtained.

The change of angle $\beta$ depending on the diameter $d_i$ on which this angle is determined, is described by the equation:

$$\beta = \arccos \left( \frac{\cos \alpha \cdot d_i}{d_i} \right) + \tan \left( \arccos \left( \frac{\cos \alpha \cdot d_i}{d_i} \right) \right) \cdot \frac{180}{\pi - \arccos \left( \frac{\cos \alpha \cdot d_i}{d_i} \right)}$$

This relationship makes it possible to determine the $\beta$ value for each gear diameter $d_i$ between the limiting values $d_a$ and $d_f$.

**TABLE II. Characteristic diameters in gear**

| Designation | Pattern | Description |
|-------------|---------|-------------|
| $d$         | $d = m \cdot z$ | pitch diameter |
| $d_a$       | $d_a = d + 2 \cdot h_a$ | tip diameter |
| $d_f$       | $d_f = d - 2 \cdot h_f$ | base diameter |
| $d_b$       | $d_b = \cos \alpha \cdot d$ | root diameter |

The course of $\beta$ angle change depending on the diameter, on which the tool works, and the change of the $\alpha_i$ pressure angle are shown in fig. 3. It should be noted that the relationship (2) applies in the case of the $\alpha_2$ pressure angle for the opposite side of the tooth.
Taking into account the cited dependencies, a parametric CAD model of the gear was developed with the module \( m_n = 5 \) and the number of teeth \( z = 18 \), which allows for obtaining asymmetry of processed teeth with different pressure angles - both for the active side and the passive side of the tooth [4].

**FEM analysis**

Tooth deformation tests for geometry with limited undercutting were performed in the NX Pre/Pos module. As variable parameters in the first case, the pressure angle on the active flank of the tooth \( \alpha_1 \) was adopted with a constant angle on the passive flank \( \alpha_2 \). In the second case, the reverse was considered.

FEM analysis was performed on a single tooth, for which a 2 mm Tetrahedral 3D mesh model was adopted, with local density on the force interaction surface and on the transition curve, at the level of 0.5 mm (fig. 4).
The test consisted in applying a concentrated force $P = 1000$ N to the active surface of the tooth, acting along the action line, and determining the value of the deflection $f$ the tooth at its tip. Steel with Young’s modulus $E = 206$ GPa, Poisson’s ratio $\nu = 0.3$ and Kirchhoff modulus $G = 80$ GPa [1, 5, 6], was adopted as the gear material.

In the case of an asymmetric tooth profile, there is a non-uniformity of force acting on the action line. This is due to the number of pairs of teeth working together at any given time. In this case, the greatest forces occur in the middle area of the tooth outline. Force $P$ was applied in the upper part of the area, in which only one pair of teeth cooperate, so that it is possible to determine the maximum deformation of the tooth [5]. Eleven tests were carried out for the adopted parameters - for each case (fig. 5).

In the first case, angle $\alpha_1$ was increased by 2° in the range from 20° to 40°, with a constant value of angle $\alpha_2$ of 20°. In the second case, the $\alpha_2$ pressure angle on the tooth’s passive surface was changed in the range from 10° to 20° by 1°, while the angle $\alpha_1$ was 40°.

The limit values - $\alpha_1 = 40°, \alpha_2 = 20°$ - were selected in such a way as to prevent shearing the tooth tip. Changes in tooth deformation at subsequent values of angle $\alpha_1$ are shown in fig. 6.
By introducing different pressure angles $\alpha_1$ and $\alpha_2$, the cross-sectional area of the tooth at its base was changed. In the first case, for $\alpha_1 = 40^\circ$, there is the largest cross-sectional area at the base of the tooth and the smallest deformation $f$.

**Analysis of simulation results of tooth deformation change**

Based on the simulation results, charts were developed showing the value of tooth deformation $f$ with a variable angle on the active flank $\alpha_1$ (fig. 7) and on the passive flank of tooth $\alpha_2$ (fig. 8).

![Fig. 7. Diagram of the dependence of deformation $f$ on the pressure angle $\alpha_1$](chart1)

![Fig. 8. Diagram of the dependence of deformation $f$ on the pressure angle $\alpha_2$](chart2)

The graphs show that as the value of angle $\alpha_1$ increased, the value of the tooth's $f$ deformation at its tip decreased twice. When the angle $\alpha_2$ changes in the range of 10° to 20°, deformation shows a downward trend, but with a much milder character. The smallest deformation value was observed for the pair of angles $\alpha_1$ and $\alpha_2$ equal to 40° and 10°, respectively, and the highest for the pair of 20° and 20°.

With constant material and geometrical parameters, the change for the pressure angle $\alpha_1$, defined for the active flank of the tooth, was described by the formula (3) as an exponential function, and for the angle $\alpha_2$, describing the passive flank of the tooth, it shows a polynomial form expressed by equation (4):

$$f = 0.0043 \cdot e^{-0.033\alpha_1} \tag{3}$$

$$f = -10^{-6} \cdot \alpha_2^2 + 10 - 5 \cdot \alpha_2 + 0.0014 \tag{4}$$

**Conclusions**

Possibility of modifying the tooth profile in the kinematics of a five-axis machine tool by tangentially guiding the tool to the machined surface and using the tilting angle of the tool axis $\beta$ was confirmed, which allows for obtaining the desired values of the $\alpha$ angle on both flank of the tooth.
Modification of the gear tooth profile has a significant impact on reducing tooth deformation and, consequently, on increasing the strength of the entire transmission. This involves increasing the ranges of transmitted power, as well as slimming the entire structure.

The analysis showed a much smaller impact of angle $\alpha_2$ on tooth tip deformation, however, the change in the value of this angle is dictated by geometric relationships and is achieved by obtaining the largest angle $\alpha_1$ with the elimination of tooth tip shearing. The case studied, referring to a straight-tooth wheel, indicates that this type of modification should be used on heavily loaded wheels, also with helical teeth, as well as on bevel gears.

REFERENCES

[1] Masuyama T., Miyazaki N. "Evaluation of load capacity of gears with an asymmetric tooth profile". *International Journal of Mechanical and Materials Engineering*. 11, 11 (2016): 1–9.

[2] Ochęduszko K. „Kołazębate – konstrukcja”. Tom I'. Warszawa: Wydawnictwa Naukowo-Techniczne, 1976.

[3] Ochęduszko K. „Koła zębatye – wykonanie i montaż”. Tom II’. Warszawa: Wydawnictwa Naukowo-Techniczne, 1976.

[4] Burek J., Płodzień M., Gdula M., Buk J. „Kształtowanie zarysu koła zębatego w programowaniu dialogowym i parametrycznym”. *Mechanik*. 2 (2015): CD. DOI: http://dx.doi.org/10.17814/mechanik.2015.2.29

[5] Masuyama, T., Mimura Y., Inoue K. "Bending strength simulation of asymmetric involute tooth gears". *Journal of Advanced Mechanical Design, Systems, and Manufacturing*. 9, 5 (2015).

[6] Documentation of System NX 10.