NUCLEON TO PION TRANSITION DISTRIBUTION AMPLITUDES IN A LIGHT-CONE QUARK MODEL

M. Pincetti‡, B. Pasquini, S. Boffi
Dip. di Fisica Nucleare e Teorica, Università degli Studi di Pavia and INFN, Sezione di Pavia, Pavia, 27100, Italy
‡E-mail: manuel.pincetti@pv.infn.it

We present a general representation for the nucleon distribution amplitudes and for the nucleon to pion transition distribution amplitudes in terms of light-cone wave functions. We apply our formalism to a light-cone constituent quark model giving some numerical results for both the classes of observables.

Keywords: Distribution Amplitudes; Transition Distribution Amplitudes; Meson Cloud

1. Introduction

Until few years ago all matrix elements taken into account in the description of exclusive reactions at high momentum transfer were those between initial and final hadron states, giving rise to the so-called Generalized Parton Distributions (GPDs)¹, or those involving an hadronic and a vacuum state, describing the well-known Distribution Amplitudes (DAs)². The generalization of such distributions to the case where the initial and final states correspond to different particles has recently been proposed. This new mathematical objects, initially called skewed distribution amplitudes³, are now established as Transition Distribution Amplitudes⁴ (TDAs), since they describe a transition between two different hadronic states.

In this article, we perform a Fock-state decomposition of the nucleon state with the aim to understand the structure of the nucleon in terms of its Light-Cone Wave Function (LCWF) representation. First, we derive the general LCWF representation for the nucleon DAs taking into account the lower Fock-state component made up of three valence quarks only. Then, for the first time in the literature we apply the LCWF formalism to the nucleon to pion TDAs for which we are forced to consider at least the state composed of five partons, i.e. three valence quarks and a quark-antiquark pair describing the mesonic cloud surrounding the nucleon. The nucleon to pion TDAs are believed to be the most direct observables to test the pion-cloud contribution to the nucleon structure. Their experimental study is ongoing (JLab) or is planned (GSI, CERN).

The outline of the paper is as follows. In section 2 we give a detailed...
description of the LCWF representation for the DAs in the valence sector
and we give some numerical results for the DAs and their moments within
a light-cone quark model, in comparison with other model calculations and
lattice predictions. Then in section 3 in the framework of the convolution
model of Ref. 5 we present the representation of the nucleon to pion TDAs
in terms of LCWFs giving also some numerical predictions for three of
them.

2. DAs in the LCWFs formalism

2.1. Definitions and properties

In general the notion of DAs refers to hadron to vacuum matrix elements of
nonlocal operators built up of quark and gluon fields at light-like separation.
In this paper we deal with the three-quark matrix element

\[ \langle 0 | \epsilon^{ijk} u^i_n(z_1n)[z_1;0]\epsilon^{ij'}_{\alpha}(z_2n)[z_2;0]\epsilon^{jk'}_{\gamma}(z_3n)[z_3;0]|P(p_1, \lambda) \rangle, \]

where \( |P(p_1, \lambda) \rangle \) denotes the proton state with momentum \( p_1 (p_1^2 = M^2) \)
and helicity \( \lambda \). The Latin letters \( i, j \) and \( k \) refer to colour, while the Greek
letters \( \alpha, \beta \) and \( \gamma \) stand for Dirac indices. We define the light-cone vectors
\( p \) and \( n \) \((p^2 = n^2 = 0)\) such that \( 2p \cdot n = 1 \). The most general decomposition
of the matrix element in Eq. (1) involves 24 invariant functions\(^6\), but to
leading twist accuracy only three of them survive, i.e.

\[ 4\mathcal{F} \left( \langle 0 | \epsilon^{ijk} u^i_n(z_1n)u^j_j(z_2n)d^k_k(z_3n)|P(p_1, \lambda) \rangle \right) = f_N \left[ V^p(\gamma^5 C)_{\alpha\beta}(N^+)\gamma \right. \]

\[ + A^p(\gamma^5 C)_{\alpha\beta}(N^+)\gamma + T^p(\sigma_{\mu\nu} C)_{\alpha\beta}(\gamma^\mu\gamma^5 N^+)\gamma \right] \]

where \( \sigma^{\mu\nu} = 1/2[\gamma^\mu, \gamma^\nu] \), \( \sigma^{\mu\nu}_{\rho\sigma} \) is a shorthand notation for \( p_{\rho\sigma}\gamma^{\mu\nu} \), \( C \) is the
charge conjugation matrix, \( N^+ \) is the “good” or “large” component of the
nucleon spinor \( N \), \( \mathcal{F} \) represents a Fourier transform\(^a\), and \( f_N \) is the value
of the nucleon wave function at the origin. The three functions \( V^p, A^p \) and
\( T^p \) depend on the variables \( x_i \) \( (0 < x_i < 1, \sum_i x_i = 1) \) which correspond to the longitudinal momentum fractions carried by the quarks inside the
nucleon. Because of the symmetry properties of the operator in Eq. (2) it
is easy to see that the three invariant functions can be expressed in term

\(^a\)The symbol \( \mathcal{F} \) is a shorthand notation for the Fourier transform

\[ (p \cdot n)^3 \int \frac{d^3z}{(2\pi)^3} \exp[ix_kz_kp \cdot n]. \]
of a single function Φ. Indeed, labelling with 1, 2 or 3 the arguments of the
DAs the following relations hold:

\[ V(1, 2, 3) = V(2, 1, 3), \quad A(1, 2, 3) = -A(2, 1, 3), \quad T(1, 2, 3) = T(2, 1, 3). \quad (3) \]

and

\[ 2T(1, 2, 3) = \Phi(1, 3, 2) + \Phi(2, 3, 1), \quad \Phi(1, 2, 3) = V(1, 2, 3) - A(1, 2, 3). \quad (4) \]

Defining with the symbol \( M^{\uparrow/\downarrow}_{\alpha\beta\gamma} \) the matrix element in the left-hand side of Eq. (2), we can express the three DAs as

\[ V^p = \frac{1}{N} \frac{1}{\sqrt{2}} (p_1^+)^{-\frac{3}{2}} \left( M^\uparrow_{12,1} + M^\downarrow_{21,1} \right), \quad (5) \]

\[ A^p = \frac{1}{N} \frac{1}{\sqrt{2}} (p_1^+)^{-\frac{3}{2}} \left( M^\downarrow_{21,1} - M^\uparrow_{12,1} \right), \quad (6) \]

\[ T^p = -\frac{1}{N} \frac{1}{\sqrt{2}} (p_1^+)^{-\frac{3}{2}} M^\uparrow_{11,2}. \quad (7) \]

Thus to calculate the leading twist DAs we must give an expression for the \( M^{\uparrow/\downarrow}_{\alpha\beta\gamma} \) matrix elements.

### 2.2. LCWFs representation

The representation in terms of LCWFs has been applied to a large range of observables like form factors\(^7\), the transversity distribution\(^8\), GPDs\(^9\) and transverse momentum dependent distributions\(^10\) as a useful formalism to disentangle the contribution of the different partonic configurations in the nucleon. Here we sketch the derivation of the LCWF representation for the DAs. Starting from the left-hand side of Eq. (2) and substituting the general Fourier expansion in momentum space of the free quark field one finds,

\[
\langle 0 | 4(p_1^+)^3 \epsilon^{ijk} \int \frac{dk_1^+ d^2 k_{1\perp}}{16\pi^3 k_1^+} \frac{dk_2^+ d^2 k_{2\perp}}{16\pi^3 k_2^+} \frac{dk_3^+ d^2 k_{3\perp}}{16\pi^3 k_3^+} \Theta(k_1^+) \Theta(k_2^+) \Theta(k_3^+)
\times \sum_{\lambda_1, \lambda_2, \lambda_3} b_1^\dagger(\tilde{k}_1, \lambda_1)b_2^\dagger(\tilde{k}_2, \lambda_2)b_3^\dagger(\tilde{k}_3, \lambda_3) u_{+\alpha}(k_1^+, \lambda_1) u_{+\beta}(k_2^+, \lambda_2) u_{+\gamma}(k_3^+, \lambda_3)
\times \delta(x_1 p_1^+ - k_1^+) \delta(x_2 p_1^+ - k_2^+) \delta(x_3 p_1^+ - k_3^+) |P(p_1, s_1)\rangle,
\]

\(^7\)In principle one has to face with 128 matrix elements \( (4 \times 4 \times 4 \times 2) \) because there are three Dirac indices and two helicity states, but for symmetry reasons only three of them are independent.
where \( \tilde{k}_i \equiv (k_i^+, k_{i\perp}) \) is a shorthand notation for the plus and transverse parton momentum components, \( \lambda_i \) represents the parton helicity and \( b \) is the annihilator of the “good” component of the quark fields. Truncating the Fock expansion to the minimal configuration corresponding to three valence quarks, the proton state in Eq. (8) can be written as

\[
|P(p_1, s_1)\rangle = \sum_{\lambda_i, \tau_i, c_i} \int \frac{3!}{\sqrt{2\pi^3}} \frac{dy_i}{\sqrt{y_i}} \int \frac{\prod_{i=1}^{3} d^2k_{i\perp}}{[2(2\pi)^3]^2} \delta \left( 1 - \sum_{i=1}^{3} y_i \right) \delta^{(2)} \left( \sum_{i=1}^{3} k_{i\perp} \right) \\
\times \tilde{\Psi}_\lambda^{[f]}(\{y_i, k_{i\perp}; \lambda_i, \tau_i, c_i\}) \prod_{i=1}^{3} |y_ip_{i\perp}^+, k_{i\perp} + y_ip_{i\perp}^+, k_{i\perp}; \lambda_i, \tau_i, c_i; q\rangle,
\]

(9)

where \( \lambda_i, \tau_i \) and \( c_i \) are spin, isospin and colour variables of the quarks, respectively. Then, after some algebra one obtains\(^c\)

\[
M_{\mu\beta, \gamma}^{1/1} = -\frac{24}{\sqrt{x_1x_2x_3}} \sum_{\lambda_1, \lambda_2, \lambda_3} u_{+\alpha}(x_1p_1^+, \lambda_1)u_{+\beta}(x_2p_2^+, \lambda_2)u_{+\gamma}(x_3p_3^+, \lambda_3) \\
\times \int \frac{\prod_{i=1}^{3} d^2k_{i\perp}}{[2(2\pi)^3]^2} \delta^{(2)} \left( \sum_{i=1}^{3} k_{i\perp} \right) \\
\times \tilde{\Psi}_\lambda^{[f]}(\{x_1, k_{1\perp}; \lambda_1, 1/2\}; x_2, k_{2\perp}; \lambda_2, 1/2\{x_3, k_{3\perp}; \lambda_3, -1/2\}).
\]

(10)

The previous equation is a formal general representation for the matrix element involved in the DAs definition. To get the final representation for the matrix element what is missing is an expression for the LCWFs. In this work we follow the same procedure adopted in Refs. 11, 12. We refer to these works for the details and here we give the final results for the DA \( \Phi \)

\[
\Phi = \frac{8\sqrt{2}}{F_N} \left[ \frac{1}{M_0} \frac{\omega_1\omega_2\omega_3}{x_1x_2x_3} \right]^{\frac{1}{2}} \int \frac{\prod_{i=1}^{3} d^2k_{i\perp}}{16\pi^3} \delta^{(2)} \left( \sum_{i=1}^{3} k_{i\perp} \right) \psi(k_1, k_2, k_3) \\
\times \left\{ \left[ a_1 \kappa_L^R \kappa_L^R \right] + \frac{1}{2} \left[ a_1 a_2 a_3 \right] - \frac{1}{2} \left[ \kappa_L^R \kappa_R^L a_3 \right] \right\} \\
\times \sqrt{\left[ a_1^2 + k_{1\perp}^2 \right] \left[ a_2^2 + k_{2\perp}^2 \right] \left[ a_3^2 + k_{3\perp}^2 \right]},
\]

(11)

where \( a_i = m + x_i M_0 \) and \( \kappa_i^{L,R} = \kappa_i^L \mp \kappa_i^R \), with \( m \) being the quark mass and \( M_0 \) the eigenvalue solution of the free Hamiltonian. For the description

\(^c\)The colour dependence in the LCWF \( \tilde{\Psi}_\lambda^{[f]} \) has already been taken into account in the numerical prefactor.
of the momentum dependence part $\psi(k_1,k_2,k_3)$ of the LCWF we adopt the parameterization of Ref. 21.

2.3. Numerical Results

In the literature there are many different papers trying to give a good description of the nucleon DAs 13, 14, 15, 16, 17, 18, 19, 20. In particular, in Fig. 1 the DA $\Phi$ from the data fit of Ref. 20 is compared with our model calculation. Our result is in good agreement with the data fit of Bolz and Kroll. In Table 1 we report our predictions for some moments of the DA $\Phi$ in comparison with the results from Refs. 13, 14, 18, 19, 20, 22. The moments are defined as

$$\Phi^{(l,m,n)}(x_1,x_2,x_3) = \frac{\int [dx] x_1^l x_2^m x_3^n \Phi(x_1,x_2,x_3)}{\int [dx] \Phi(x_1,x_2,x_3)},$$

where $[dx] = \delta(1 - \sum_{i=1}^3 x_i) \prod_{i=1}^3 dx_i$. Our results are pretty close to the BK data parameterization and to the very recent lattice predictions.

3. Nucleon to pion TDAs

In this section we apply the same formalism used for the DAs to the study of the TDAs in the baryonic sector and in particular for the description of the nucleon to pion transition. Such quantities can be analyzed in reactions such as backward pion electroproduction, e.g., $ep \rightarrow e'p'\pi^0$, and nucleon-antinucleon annihilation, e.g., $NN \rightarrow \gamma^*\pi$. As in Ref. 4 we rewrite the general matrix element describing a transition from a nucleon to a pion in terms of eight TDAs: two vectorial $V_{1,2}^{\pi^0}(x_i,\xi,\Delta^2)$, two axial $A_{1,2}^{\pi^0}(x_i,\xi,\Delta^2)$ and four tensorial $T_{1,\ldots,4}^{\pi^0}(x_i,\xi,\Delta^2)$, i.e.
encoded into two LCWFs, one for the 3

The calculations of the LCWF representation follows exactly the same line

DAs, defining the left-hand side of Eq. (13) with the symbol

The TDAs depend on the light-cone momentum fractions

| (l,m,n) | COZ | KS | DF | SB | BK | Our | LAT |
|--------|-----|----|----|----|----|-----|-----|
| 0 0 0  | 1   | 1  | 1  | 1  | 1  | 1   | 1   |
| 1 0 0  | 0.54| 0.46| 0.582| 0.572| 0.381| 0.346| 0.394|
| 0 1 0  | 0.18| 0.18| 0.213| 0.184| 0.309| 0.331| 0.302|
| 0 0 1  | 0.20| 0.22| 0.207| 0.244| 0.309| 0.323| 0.304|
| 2 0 0  | 0.32| 0.27| 0.367| 0.338| 0.179| 0.152| 0.18  |
| 0 2 0  | 0.065| 0.08| 0.085| 0.066| 0.125| 0.142| 0.132|
| 0 0 2  | 0.09| 0.10| 0.83 | 0.170| 0.125| 0.137| 0.138|

Here, \( p_1 = (1 + \xi)p + \frac{M^2}{1+\xi^2}n, \) \( p_\pi = (1 - \xi)p + \frac{m_\pi^2 + \Delta T^2}{1-\xi^2}n + \Delta_T, \) and \( \Delta = p_\pi - p_\pi, \) where we have defined \( \xi = -\Delta^+/2P^+ \) with \( P = \frac{1}{2}(p_1 + p_\pi). \)

The TDAs depend on the light-cone momentum fractions \( x_i, \) and on the skewedness parameter \( \xi \) that describes the change of longitudinal momentum from the initial and the final state, and on \( \Delta^2. \) As for the DAs, defining the left-hand side of Eq. (13) with the symbol \( D_{\alpha\beta\gamma}^{\Delta} \) it is possible to express the TDAs in terms of eight of these matrix elements. The calculations of the LCWF representation follows exactly the same line we used for the DAs. However to match the final one-pion state the nucleon state must now be composed at least by five partons with their dynamics encoded into two LCWFs, one for the \( 3q \) structure and one for \( gg \) pair, convoluted with a splitting function, \( \phi_{N,(N\pi)}^{\lambda} \), that weighs the different possible configurations. For the details about the description of the nucleon as a bound state of a baryon surrounded by its meson cloud we refer to Ref. 5, while we postpone all the details regarding the calculation to a forthcoming paper23. We report here the general LCWF representation for the matrix

\[
\mathcal{F}\left( \pi^0(p_\pi)[\bar{u}^i(z_1n)u^j(z_2n)d^k(z_3n)|P(p_1, s_1)] \right) = 
\int_N \frac{d^{-}}{f_{q}} \left[ V_{1}(p)\frac{\pi^0(q)+A_{1}(p)\gamma^{5}c_{\alpha\beta}(\gamma^{5}c_{5}+\gamma^{5}c_{5})_{\alpha\beta}}{m_{\pi}} \right] 
+ T_{1}(p)\frac{\gamma^{5}c_{\alpha\beta}(\gamma^{5}c_{5}+\gamma^{5}c_{5})_{\alpha\beta}}{m_{\pi}} + M^{-1}T_{2}(p)\frac{\gamma^{5}c_{\alpha\beta}(\gamma^{5}c_{5}+\gamma^{5}c_{5})_{\alpha\beta}}{m_{\pi}} 
+ M^{-1}T_{3}(p)\frac{\gamma^{5}c_{\alpha\beta}(\gamma^{5}c_{5}+\gamma^{5}c_{5})_{\alpha\beta}}{m_{\pi}}.
\]
element $D_{\alpha\beta,\gamma}^\lambda$

$$\begin{align*}
&= -\frac{24}{\sqrt{x_1x_2x_3}} \left(\frac{1}{2\xi}\right)^{\frac{3}{2}} \sum_{\lambda_1,\lambda_2,\lambda_3} u_{\alpha}(x_1P^+,\lambda_1)u_{+\beta}(x_2P^+,\lambda_2)u_{+\gamma}(x_3P^+,\lambda_3) \\
&\times \sum_{\lambda'} \int \frac{dyd^2k_{\perp}}{4\pi} \phi_{\lambda'(N,N')}(y,k_{\perp}/\sqrt{y(1-y)}) \delta \left(1-y - \frac{p_+}{p_1}\right) \\
&\times \left(\prod_{i} d^2k_{i\parallel} \delta^{(2)}(p_{\perp} + k_{\perp}) \delta^{(2)}(\sum_{i=1}^{3} k_{i\parallel}) \right) \\
&\times \tilde{\Psi}_{\lambda'} \left(\left\{ \frac{x_1}{2\xi}, k_{1\parallel}; \lambda_1, 1/2 \right\}, \left\{ \frac{x_2}{2\xi}, k_{2\parallel}; \lambda_2, 1/2 \right\}, \left\{ \frac{x_3}{2\xi}, k_{3\parallel}; \lambda_3, -1/2 \right\} \right), \quad (14)
\end{align*}$$

where, apart from some kinematical factors, it is easy to recognize the same structure found for the DAs. In Eq. (14) we note that the dependence from the light-cone momentum fraction in the LCWF is rescaled by a factor $2\xi$ with respect to the DAs case, as found in Ref. 4 in the soft pion limit $^{24,25}$ ($\Delta_T = 0, \xi \to 1$) and for the pion-nucleon GDAs in Ref. 26. In Fig. 2 the results for $V_{1}^{0\pi}$, $A_{1}^{0\pi}$ and $T_{1}^{0\pi}$ are shown.

![Figure 2. Results for the TDAs $V_{1}^{0\pi}$, $A_{1}^{0\pi}$ and $T_{1}^{0\pi}$ at $\Delta^2 = -1\text{GeV}^2$ and $\xi = 0.9$.](image)
For a detailed numerical analysis of all the TDAs we refer to Ref. 23.

Acknowledgments
M.P. acknowledges useful discussions with F. Conti, A. Courtoy and O. Teryaev.

Bibliography
1. For recent reviews on the subject see: M. Diehl, Phys. Rep. 388, 41 (2003); Xiangdong Ji, Ann. Rev. Nucl. Part. Sci. 54, 413 (2004); A.V. Belitsky and A.V. Radyushkin, Phys. Rept. 418, 1 (2005); S. Boffi and B. Pasquini, Riv. Nuovo Cim. 30, 387 (2007), arXiv:0711.2625 [hep-ph].
2. V.L. Chernyak and A.R. Zhitnitsky, Phys. Rep. 112, 173 (1984); G.P. Lepage and S.J. Brodsky, Phys. Rev. D 22, 2157 (1980).
3. L.L. Frankfurt et al., Phys. Rev. D 60, 014010 (1999); hep-ph/9808449.
4. B. Pire and L. Szymanowski, Phys. Rev. D 71, 111501 (2005); Phys. Lett B 622, 83 (2005); J.P. Lansberg, B. Pire and L. Szymanowski, Phys. Rev. D 75, 074004 (2007); arXiv:0709.2567 [hep-ph]; Phys. Rev. D 76, 111502 (2007).
5. B. Pasquini and S. Boffi, Phys. Rev. D 73, 094001 (2006).
6. V. Braun et al., Nucl. Phys. B 589, 381 (2000); ibidem 607, 433 (2001).
7. S.D. Drell and T. Yan, Phys. Rev. Lett. 24, 181 (1970); G.B. West, Phys. Rev. Lett. 24, 1206 (1970).
8. S. Boffi, B. Pasquini and M. Pincetti, Phys. Rev. D 76, 034020 (2007).
9. M. Diehl et al., Nucl. Phys. B 596, 33 (2001); ibidem, 99 (2001).
10. X. Ji, J.-P. Ma, F. Yuan, Nucl. Phys. B 652, 383 (2003); B. Pasquini, S. Cazzaniga and S. Boffi, arXiv:0806.2298 [hep-ph].
11. S. Boffi, B. Pasquini and M. Traini, Nucl. Phys. B 649, 243 (2003).
12. S. Boffi, B. Pasquini and M. Traini, Nucl. Phys. B 680, 147 (2004); ibidem A 755, 545 (2005); Phys. Rev. D 71, 034022 (2005); S. Boffi, B. Pasquini and M. Pincetti, Phys. Rev. D 72, 094029 (2005).
13. V.L. Chernyak, A.A. Ogioblin, and I.R. Zhitnitsky, Yad. Fiz. 48, 841 (1988).
14. I.D. King and C.T. Sachrajda, Nucl. Phys B 279, 785 (1987).
15. M.F. Gari and N.G. Stefanis, Phys. Rev. D 35, 1074 (1987).
16. Z. Dziembowski, Phys. Rev. D 37, 2030 (1988).
17. A. Schäfer, Phys. Lett B 217, 545 (1989).
18. Z. Dziembowski and J. Franklin, Phys. Rev. D 42, 905 (1990).
19. N.G. Stefanis and M. Bergmann, Phys. Rev. D 47, R3685 (1993).
20. J. Bolz and P. Kroll, Z. Phys A 356, 327 (1996).
21. F. Schlumpf, J. Phys. G 20, 237 (1994).
22. QCDSF/UKQCD collaborations, Göckeler et al., arXiv:0804.1877 [hep-lat].
23. B. Pasquini, M. Pincetti and S. Boffi, forthcoming.
24. S. Adler and R. Dashen, Currents Algebra (W.A. Benjamin, New York, 1968).
25. P.V. Polyvyatsa, M.V. Polyakov and M. Strikman, Phys. Rev. Lett. 87, 022001 (2001); M.V. Polyakov and S. Stratman, hep-ph/0609045.
26. V.M. Braun et al., Phys. Rev. D 75, 014021 (2007).