Estimating the uncorrelated dark energy evolution in the Planck era

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The equation of state (EOS), $w(z)$, is the most important parameter of dark energy. We reconstruct the evolution of this EOS in a model-independent way using the latest cosmic microwave background (CMB) data from Planck and other observations, such as type Ia supernovae (SNe Ia), the baryonic acoustic oscillation measurements (SDSS, 6dF, BOSS, and WiggleZ), and the Hubble parameter value $H(z)$. The results show that the EOS is consistent with the cosmological constant at the 2σ confidence level, not preferring a dynamical dark energy. The uncorrelated EOS of dark energy constraints from Planck CMB data are much tighter than those from the WMAP 9-year CMB data.

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I. INTRODUCTION

The acceleration of our Universe was first discovered by observing type Ia supernovae (SNe Ia) \cite{1, 2}. This unexpected discovery has been confirmed by the observations on cosmic microwave background, large scale structure, weak gravitational lensing, and Hubble parameter. Within the framework of general relativity, the accelerating expansion is attributed to mysterious dark energy, which has an equation of state (EOS), \( w = p/\rho \), where \( P \) and \( \rho \) are the pressure and energy density respectively. In addition to the cosmological constant, other many dark energy models have been suggested (for reviews see \cite{3, 4}).

EOS \( w \) is the most important parameter that describes the properties of dark energy. Whether and how it evolves with time is crucial for revealing the physics of dark energy. The evolution of EOS can be reconstructed from data using either parametric or nonparametric methods \cite{5}. Simple parametric forms of \( w(z) \) have been proposed for studying the evolution of dark energy, such as \( w(z) = w_0 + w_1 z \) \cite{6, 7} and \( w(z) = w_0 + w_1 (1 + z) \) \cite{8, 9}. Reconstructing the EOS of dark energy in a parametric method has been widely explored \cite{9}. However, a simplified dark energy parameterization is equivalent to a strong and not necessarily theoretically justified prior on the nature of dark energy \cite{10}. In order to avoid this shortcoming, the nonparametric approach was proposed \cite{11-15}. The procedure is to bin \( w \) in \( z \), and fit the \( w \) in each bin to data. This method assumes that \( w(z) \) is constant in each bin and has been widely used in the literature. The procedure is as follows: first performing a Fisher forecast to determine the eigenmodes of the covariance matrix, and then fitting the constrained modes to observational data. The Fisher matrix methodology has been used in predicting the ability of future experiments to constrain particular parameters of interest \cite{16}. The Joint Dark Energy Mission team proposed that this method can constrain binned 36 EOS parameters with future data \cite{17}. Another method has also been proposed, which puts prior directly on the space of \( w(z) \) functions \cite{18}. But the functions may be arbitrary. The Gaussian process is another method of dealing with stochastic variables \cite{14, 19}.

Using the nonparametric methods, two distinctive evolution of \( w(z) \) have been derived, one is consistent with the cosmological constant \cite{10, 12, 14, 20}, and the other supports a dynamical dark energy model \cite{21, 22}. Zhao et al. found that a dynamical dark energy model which evolves from \( w < -1 \) at \( z \sim 0.25 \) to \( z > -1 \) at higher redshift is favored using the WMAP 7-year data and other cosmological observations \cite{22}. So the evolutional behavior of \( w(z) \) is controversial. More recently, the Planck team has released the first cosmological results based on the measurement of the CMB temperature and lensing-potential power spectra, which prefer a low Hubble constant and high matter density. In this paper, we estimate the evolution of \( w(z) \) using the latest cosmological observations including the Planck data.

II. METHOD AND RESULTS

We use the nonparametric technique to constrain the uncorrelated EOS of dark energy \( w(z) \) \cite{12}. This is a modification version of the method proposed by \cite{11}. Theoretically, the luminosity distance \( d_L(z) \) is given by

\[
d_L(z) = (1 + z) \frac{c}{H_0} \times \begin{cases}
1 + \sinh \left( \sqrt{\Omega_m} \int_0^z \frac{dz}{\sqrt{1 + \Omega_k}} \right) & \text{if } \Omega_k > 0 \\
\sqrt{\Omega_m} \int_0^z \frac{dz}{\sqrt{1 + \Omega_k}} & \text{if } \Omega_k = 0 \\
1 - \sinh \left( \sqrt{\Omega_m} \int_0^z \frac{dz}{\sqrt{1 + \Omega_k}} \right) & \text{if } \Omega_k < 0
\end{cases}
\]

where \( E(z) = [\Omega_m(1 + z)^2 + \Omega_r f(z) + \Omega_k(1 + z)^2]^{1/2} \), \( \Omega_m + \Omega_r + \Omega_k = 1 \), and

\[
f(z) = \exp \left[ 3 \int_0^z \frac{1 + w(z)}{1 + \tilde{z}} \frac{dz}{\tilde{z}} \right].
\]

The function \( f(z) \) is related to the parameterization of dark energy. If the EOS is considered to be piecewise constant in redshift, then \( f(z) \) can be described as \cite{20}

\[
f(z_{n-1} < z \leq z_n) = (1 + z)^{w(z_{n-1})} \prod_{i=0}^{n-1} (1 + z_i)^{w(z_i - w(z_{i+1}))},
\]

where \( z_0 < z_1 < \cdots < z_n \) are the nodes of the parameterization in redshift.
where \( w_i \) is the EOS parameter in the \( i^{th} \) redshift bin defined by an upper boundary at \( z_i \), and the zeroth bin is defined as \( z_0 = 0 \). We choose 10 bins in our analysis, e.g. \( z_i = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.4 \). Previous studies only used less than five redshift bins. We fix \( w = -1 \) at \( z > 1.4 \). This assumption does not affect the derived results \([20, 22]\). The number of bins and the range are chosen to be large enough so that the derived results are stable to these choices.

The luminosity distance depends on the integration of the energy properties, and the constraints derived from the distance shifts \( d_L(z) \).

\[ \frac{d_L}{d_L} \approx \frac{c}{H_0} d_L(z) \]

where \( H_0 \) is the Hubble constant and \( d_L \) is the luminosity distance. We use the distance priors \([24]\) from Planck’s first data release. We use the Union 2.1 SNe Ia sample \([23]\).

For the baryonic acoustic oscillations (BAO) data, we use the SDSS DR7 BAO measurements at effective redshifts \( z = 0.2 \) and \( z = 0.35 \) \([28]\), the WiggleZ measurements at \( z = 0.44, 0.60, 0.73 \) \([29]\), the BOSS DR9 measurement at \( z = 0.57 \) \([30]\), and the 6dF Galaxy Survey measurement at \( z = 0.1 \) \([31]\). The distance ratio vector of BAO is

\[ w_{\text{BAO}} = \begin{pmatrix} d_{0.1} \\ d_{0.05}^{-1} \\ d_{0.5} \end{pmatrix} = \begin{pmatrix} 0.336 \\ 8.88 \\ 13.67 \end{pmatrix} \]

where \( d_{0.1} \) is the distance ratio at \( z = 0.1 \) and \( d_{0.05}^{-1} \) is the inverse of the distance ratio at \( z = 0.05 \). The corresponding inverse covariance matrix is

\[ C_{\text{BAO}}^{-1} = \begin{pmatrix} 4444.4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 34.602 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20.661157 & 0 & 0 & 0 \\ 0 & 0 & 0 & 24532.1 & -25137.7 & 12099.1 \\ 0 & 0 & 0 & -25137.7 & 134598.4 & -64783.9 \\ 0 & 0 & 0 & 12099.1 & -64783.9 & 128837.6 \end{pmatrix} \]

We also use the 28 independent measurements of the Hubble...
parameter between redshifts $0.07 \leq z \leq 2.3$ compiled in [32]. There are 12 cosmological parameters in our analysis, such as $\Omega_m, H_0, w_1, ..., w_{10}$. We take $\Omega_m$ as a free parameter in our calculation. According to the observations of Planck, a flat cosmology is assumed. We also adopt $\chi^2$ statistic to estimate parameters. The likelihood function $L$ is then proportional to $\exp\left(-\chi^2/2\right)$, which produces the posterior probability when multiplied by the prior probability. According to the posterior probability derived in this way, Markov chains are generated through the Monte-Carlo algorithm to study the statistical properties of the parameters. We marginalize over the Hubble constant $73.8 \pm 2.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from Cepheid variables [33].

Figure 1 shows the estimation of the uncorrelated dark energy EOS parameters at different redshift bins. The dark energy EOS is consistent with the cosmological constant at the $2\sigma$ confidence level. The EOS shows a marginal oscillation feature (from $w < -1$ to $w > -1$) around redshift $z \sim 0.5$, but from $w > -1$ to $w < -1$ at $z \sim 0.8$. The reconstructed EOS is consistent with the cosmological constant in all redshift bins at the $1\sigma$ confidence level except for these three bins. In order to determine the need for complex form of $w(z)$, we can calculate the improvement to the best fit,

$$\chi^2_{\text{eff}} = -\Delta(2 \ln L) = 2 \ln L(w = -1) - 2 \ln L(w = w_i),$$

(10)

where the likelihood function $L$ is proportional to $\exp\left(-\chi^2/2\right)$. $L(w = -1)$ and $L(w = w_i)$ represent the likelihood functions calculated in best-fit $\Lambda$CDM and the best-fit $w = w(z)$ model, respectively. In our calculation, an improvement of $\chi^2_{\text{eff}} = 4.0$ is found with 12 additional free parameters. So the cosmological constant can well fit the observational data. The additional complexity in the dark energy model is not required.

We also show the probability distributions of uncorrelated dark energy EOS $w_i$ in Figure 2. For the first four bins, the errors are very small, and the EOS is tightly constrained. The likelihood distributions are close to Gaussian. At high redshift, the probability distributions of $w_i$ span a wide range. We obtain the $1\sigma$ error bar around the best-fit value of the $w_i$ parameter. More standard candle data is required to reduce the error. Long gamma-ray bursts (GRBs) are promising candidates, because of their high luminosities and quasi-standard candle luminosity correlations. Figure 3 shows the uncorrelated dark energy EOS parameters at different redshift bins using SNe Ia+BAO+Planck+$H(z)$+GRB data. The EOS of dark energy is almost consistent with the cosmological constant at the $1\sigma$ confidence level. We use the GRB data from [35]. The GRB data covers the redshift range between $z = 0.17$ and $z = 8.2$. The redshift distribution of GRB data is shown in Figure 4. We can see that almost half of GRBs are at redshift $z > 1.4$, which is the largest SNe Ia redshift in Union 2.1 sample. So long GRBs are a promising complementary probe of dark energy at high redshifts [36]. Comparing Figure 1 and Figure 3, we can see that the errors of EOS are reduced significantly by including GRBs. In order to show the importance of observation data in the Planck era, we also present the constraint on uncorrelated dark energy EOS parameters from SNe Ia+BAO+WMAP9+$H(z)$ data in Figure 5. The distance priors from WMAP9 are used [37], including the acoustic scale ($l_a$), the shift parameter ($R$), and the decoupling redshift ($z_d$). In the calculation, we use the Hubble constant $73.8 \pm 2.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from Cepheid variables [33]. From this figure, the EOS parameter of one bin ($z \sim 0.75$) deviates the cosmological constant at the $2\sigma$ confidence level. The errors of EOS are larger than those of Figure 1. The main reason is that the distance priors from Planck are significantly tighter than those from WMAP 9-year data [24].
III. DISCUSSION

Previous investigations show that the EOS of dark energy may be consistent with the cosmological constant or prefers a dynamical dark energy. In this paper, we apply the latest CMB from Planck, SNe Ia, BAO from SDSS, 6dF, BOSS, WiggleZ, and the Hubble parameter value $H(z)$ to constrain the uncorrelated EOS of dark energy using the nonparametric technique. We model the EOS as piecewise constant values in 10 bins. Previous studies only use less than 5 bins. We find that the uncorrelated EOS is consistent with the cosmological constant at the 2σ confidence level. But if we use the CMB data from WMAP9, the results deviate from the cosmological constant at one redshift bin. At present, we find that the cosmological constant is consistent with observations, and no preference of a dynamical dark energy. We also find that the use of complex form of $w(z)$ does not provide a statistically significant improvement over the use of the cosmological constant.

With an increasing data of supernova sample, and improvement in other cosmological observations (such as CMB, BAO, $H(z)$), the binned EOS values may be constrained much more better. Future data, such as from LSST [38] and Euclid mission [39], will shed light on the evolution of dark energy EOS.

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[1] A. Riess, et al., Astron. J. 116, 1009 (1998).
[2] S. Perlmutter, et. al., Astrophys. J. 517, 565 (1999).
[3] D. H. Weinberg, et al., Physics Reports, in press, arXiv:1201.2434.
[4] M. Li, et al., Commun. Theor. Phys. 56, 525 (2011).
[5] V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D. 15, 2105 (2006).
[6] A. R. Cooray and D. Huterer, Astrophys. J. 513, L95 (1999).
[7] M. Chevallier and D. Polarski, Int. J. Mod. Phys. D, 10, 213 (2001).
[8] E. V. Linder, Phys. Rev. Lett., 90, 091301 (2003).
[9] D. Huterer and M. S. Turner, Phys. Rev. D. 60, 081301 (1999); T. Chiba and T. Nakamura, Phys. Rev. D. 62, 121301 (2000); T. D. Saini, S. Raychaudhury, V. Sahni, and A. A. Starobinsky, Phys. Rev. Lett. 85, 1162 (2000); U. Alam, V. Sahni, T. D. Saini, and A. A. Starobinsky, Mon. Not. R. Astron. Soc. 344, 1057 (2003); F. Y. Wang, Z. G. Dai and Z. H. Zhu, Astrophys. J. 667, 1 (2007); R. Lazkoz, V. Salzano and I. Sendra, Eur. Phys. J. C 72, 2130 (2012).
[10] A. Riess, et al., Astrophys. J. 659, 98 (2007).
[11] D. Huterer and G. Starkman, Phys. Rev. Lett. 90, 031301 (2003).
[12] D. Huterer and A. Cooray, Phys. Rev. D. 71, 023506 (2005).
[13] A. Shafieloo, U. Alam, V. Sahni, and A. A. Starobinsky, Mon. Not. R. Astron. Soc. 366, 1081 (2006); V. Sahni and A. Starobinsky, Int. J. Mod. Phys.D15, 2105(2006); A. Shafieloo, Mon. Not. R. Astron. Soc. 380, 1573 (2007); C. Clarkson and C. Zunke, Phys. Rev. Lett. 104, 211301 (2010); A. Shafie, J. Cosmol. Astropart. Phys. 08, 002 (2012).
[14] T. Holsclaw, U. Alam, B. Sanso, H. Lee, K. Heitmann, S. Habib, and D. Higdon, Phys. Rev. Lett. 105, 241302 (2010).
[15] A. M. Seikel, C. Clarkson, and M. Smith, J. Cosmol. Astropart. Phys. 06, 036 (2012).
[16] M. Tegmark, A. Taylor, and A. Heavens, Astrophys. J. 440, 22 (1997).
[17] A. Albrecht et al., arXiv:0901.0721.
[18] R. G. Crittenden, L. Pogosian, and G. B. Zhao, J. Cosmol. Astropart. Phys. 12, 025 (2009).
[19] A. Shafie, A. G. Kim and E. V. Linder, Phys. Rev. D. 85, 123530 (2012).
[20] S. Sullivan, A. Cooray and D. E. Holz, J. Cosmol. Astropart. Phys. 09, 004 (2007).
[21] S. Qi, T. Lu and F. Y. Wang, Mon. Not. R. Astron. Soc. 398, L78 (2009).
[22] G. B. Zhao, R. G. Crittenden, L. Pogosian and X. M. Zhang, Phys. Rev. Lett. 109, 171301 (2012).
[23] N. Suzuki et al., Astrophys. J. 746, 85 (2012).
[24] Y. Wang and S. Wang, Phys. Rev. D, 88, 043522 (2013).
[25] Planck Collaboration, P. A. R. Ade, et. al., arXiv: 1303.5062.
[26] C. L. Bennett, et al., Astrophys. J. Suppl., 208, 20 (2013).
[27] Y. Wang and P. Mukherjee, P. Phys. Rev. D, 76, 103533 (2007); E. Komatsu, et al., Astrophys. J. Suppl. 192, 18 (2011).
[28] W. J. Percival, et al., Mon. Not. R. Astron. Soc. 401, 2148 (2010).
[29] C. Blake, et al., Mon. Not. R. Astron. Soc. 418, 1707 (2011).
[30] L. Anderson, et al., Mon. Not. Roy. Astron. Soc. 428, 1036 (2013).
[31] F. Beutler, et al. Mon. Not. R. Astron. Soc. 416, 3017 (2011).
[32] O. Farooq and B. Ratra, Astrophys. J. 766, L7 (2013).
[33] A. G. Riess, et al. Astrophys. J. 730, 119 (2011).
[34] Planck Collaboration. P. A. R. Ade, et al., arXiv: 1303.5076
[35] F. Y. Wang, S. Qi, and Z. G. Dai, Mon. Not. R. Astron. Soc. 415, 3423 (2011).
[36] Z. G. Dai, E. W. Liang, and D. Xu, Astrophys. J. 612, L101 (2004); F. Y. Wang, Z. G. Dai, and Z. H. Zhu, Astrophys. J. 667, 1 (2007).
[37] G. Hinshaw, et al. Astrophys. J. Suppl., 208, 19 (2013).
[38] http://www.lsst.org
[39] http://sci.esa.int/euclid