Modeling mechanical, magnetic and thermal processes in high school lab activities: an experiment with a rotating disc

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Abstract. In this paper we present the case study of a rotating disc stopped by a dissipative interaction arising from the relative motion between some magnets, fixed on top of the disc, and a copper plate kept at rest at a very short distance above them. We measure the time dependence of the angular velocity of the disc and the temperature change in the copper plate, during both the braking phase and the subsequent cooling process. The experiment is analysed and modelled step by step:

- from the mechanical point of view, the dry friction (always present) and the magnetic braking interaction are separately modelled: indeed, comparing the model results with the collected data, from the measured angular velocity vs. time it is possible to determine the value of all the phenomenological coefficients;
- from the thermal point of view, the model stresses the quantitative connection between the dissipative process (due to the eddy currents generated by the magnetic interaction), the thermal conduction with the surroundings, the temperature changes of the copper plate and that indicated by a thermometer placed inside it;
- from the energy point of view, testing this way the coherence of the whole model.

From a didactical perspective, each sub-model emerged step by step developing independently the various parts of the model can be considered as a quantitative conceptual map, and represents therefore a powerful tool for interaction with the thought schemes used by the students.

1. Introduction

Teaching physics in high school is often limited to situations and examples in which the interconnections between the several aspects of a real process are in fact decoupled or artificially suppressed, determining the recurrent and persistent difficulties, experienced and documented by both teachers and students in constructing a coherent and consistent teaching sequences, in particular in an interdisciplinary perspective. This is due partly to the comprehensible concern to limit the inquiry to situations in which only one aspect at a time is presented and highlighted; and partly because the more complex situations often require advanced mathematical tools for a quantitative description. This case study can help also (advanced) high school students to bridge this gap: today indeed, dynamical modelling, exploiting numerical methods well supported by easy-to-use software, provides an important tool to enable access to a large number of interesting situations even at a high school level.

This activity that represents one of the possible destinations of a teaching approach, whose general framework is discussed in a separate communication presented at this seminar (D’Anna 2016) and founded on the following points: 1) a rigorous and systematic distinction between being constant and being conserved; 2) the systematic link to the production of entropy for all processes in which energy dissipation occurs; 3) the introduction of process diagrams as graphic tools to allow students to follow
both the energy exchanges *between* a system and its surroundings, and the energy transfer (release and/or upload) *inside* a system during a process; 4) the introduction of *fields* as real physical systems that can store or transport energy.

This approach is significantly fostered by several experimental situations which can be presented to the students at the different levels both as demonstrations experiments and lab activities. Regarding the introduction of the energy concept, a series of experimental situations dealing with analogies were presented in an earlier GIREP conference (D’Anna 2010), while a detailed analysis of the energy flows and energy balance of a magnetically damped mechanical oscillator, to clarify the different roles played by the spring forces and the dissipative force(s) in the energy exchanges, was presented in (Corridoni *et al.* 2014).

Here the case study is introduced by two preparatory experimental situations that can be presented to the students as lab activities at an early stage of the teaching sequence. These experiments were also chosen in order to show how this approach to energy naturally enhances the interconnections between different aspects of natural phenomena and also allows students to be introduced to dynamical modelling activities.

2. Preparatory lab activities focused on friction and dissipation

This sequence of lab activities concerning friction processes, starting from simple mechanical observations, is aimed at testing the conservation of energy, by measuring the thermal aspects at the same time.

*Sliding box.* The first experiment can be carried out in classroom and/or executed directly by students in lab activities. A box of total mass $M$, containing some chalk sticks, is set in motion manually along the table, towards a motion sensor which starts measuring when the box approaches it more than a predetermined distance (Fig. 1a). The velocity of the box linearly decreases in time (Fig. 1b), so that students can conclude that the friction braking force is constant in time, determining also its value.

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Fig. 1. The sliding box experiment: (a) experimental set up; (b) box’s velocity vs time
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Thus they can analyse quantitatively the experimental results from the point of view of the energy exchanges, representing graphically the value of the energy flow which goes out of the system vs. time, and determining (graphically and algebraically) the amount of energy dissipated. Energy conservation can then be verified by comparing this value with the initial kinetic energy.

*Magnetically braked cart.* As a challenge, a slightly modified version of the previous experience can be presented to the students, with at present a time depending friction: some neodymium magnets are fixed under a cart, thrown manually along an aluminium track (Fig. 2a). The experiment is repeated under the same conditions as in the previous example, the students basically receiving the same questions. However, for students, it is a more challenging situation, because the time dependence of the velocity makes it impossible for them to reach the result algebraically (Fig. 2b).
Students can be asked to comment on this new situation in words, to represent the exchange of energy between the cart and the track graphically, as well as to predict the value of the energy dissipated in the whole process. If the experiment is carried out with students of advanced courses\textsuperscript{11}, it is possible to use this example to introduce them to the dynamics modelling techniques, to let them grasp precisely the different role of physical relationships\textsuperscript{12}. For example, they learn to distinguish between general relations that structure the whole physical description of natural phenomena (i.e. principles, general laws…), and the particular relationships characterizing individual systems (constitutive laws).

This version of the braked cart experiment can also be useful to address the effects of energy dissipation: the answer generally provided by the students – as obvious as it is problematic – is that “somewhere heat is produced”, so that, in theory, they expect both the cart as well as the track to be heated. An experimental verification of this prediction (with the equipment described above) is practically impossible, since the dissipation happens all along the track. In order to measure the thermal effects it is necessary to localize the dissipation process, which is exactly the aim of the next experiment.

3. Localizing the dissipative process: the rotating disc experiment

A plastic disc is made to rotate by hand and rapidly braked by the dissipative interaction between some magnets, fixed on the disc, and a little copper plate (Fig. 3a), held just above the magnets and thermally insulated from the surroundings as efficiently as possible. Fig. 3b shows the detail of the thermometer inserted in the plate to monitor the thermal effects of this dissipative process. A short video of the experiment can be downloaded (QR code, Fig. 3c).

The size of the disc, the number and the strength of the magnets and the dimensions of the copper plate are chosen to allow measuring the heating of the copper plate with school equipment. Since there are many side effects occurring simultaneously, the complete dynamical model is obtained assembling the models relative to two preparatory experiments, whose parameters are determined independently comparing the measured data with models predictions\textsuperscript{13}. From the didactical point of view, each sub-model can be considered as a quantitative conceptual map: it represents a powerful and concrete tool to interact with the thought schemes used by the students.

\textsuperscript{11} In our educational system there are optional courses that students can choose during the last two years of high school. Among these there is also \textit{Fisica e applicazioni della matematica} (Physics and mathematics applications).

\textsuperscript{12} For an introduction to dynamical modeling, see for example (Fuchs 2002).

\textsuperscript{13} In particular the moment of inertia of the various parts of the rotating system (axle, disc with magnets) are experimentally determined measuring the angular velocity increase / decrease under the action of a constant torque.
In a first experiment, the disc is put in rotation after the copper plate has been removed: it is observed that the rotation is slowed down and that the disc gradually stops. By measuring the angular velocity decrease due to the friction torque, it is possible to fix this “mechanical” parameter. Fig. 4a shows the scheme used to model the situation: the core is essentially the balance equation for the angular momentum. Fig. 4b shows both the model’s prediction and the data measured for the angular velocity: the essentially linear decrease allows for the modelling of this interaction as dry friction.

Fig. 3. A magnetically braked rotating disc: (a) experimental setup; (b) thermometer inserted in the copper plate; (c) QR code to download a video of the experiment

Fig. 4. A rotating disk braked by dry friction only: (a) model; (b) angular velocity vs time: simulated (1 – blue) and measured (2 – red) values
In a second preparatory experiment, the temperature of the copper plate is measured in time after the disc has been magnetically braked, and consequently the copper plate heated. By measuring the temperature of the copper plate during the cooling, it is possible to fix all the thermal parameters characterizing its interaction with the surroundings (to obtain repeatable results, it is important to operate in a room which is absolutely free of air currents). Fig. 5a illustrates the corresponding model. The thermal process is modelled assuming that the copper plate is involved in heat exchanges – at different rates – with both the insulating support as well as with the thermistor acting as a thermometer. The thermal interaction with the surroundings is supposed to be only of a conductive type. The thermal capacities of the thermistor and the support as well as the different conduction coefficients are fixed by matching the model predictions with the measured data. The result is shown in Fig. 5b.

![Diagram of the experimental setup](image1)

**Fig. 5.** A rotating disk braked by dry and magnetic friction: (a) model; (b) temperature of the copper plate after the braking vs time: simulated (1 – blue) and measured (2 – red) values

At this point all the system parameters are set, and the experience on which we are focused can be performed: data collection is started and the disc, giving a vigorous initial spin by hand, is put into rotary motion. This time the disc requires a much shorter time interval to stop: this is due to the action of the magnetic brake. The trend registered for the angular velocity (Fig. 6a) suggests that a braking force of a viscous type must be added in the mechanical sector of the model, i.e. that the balance equation for the angular momentum must be completed with an exchange term proportional to the angular velocity (Fig. 6b).

Also in this case, the proportionality constant (which depends mainly on the strength of the magnets and the distance between them and the copper plate – typically between 1 and 2 mm) is fixed through the comparison between the experimental data and the measured values, as shown in Fig. 6a: the mechanical part is now modelled in a satisfactory way.

This suggests asking students to consider the experiment also from the energy point of view: in fact, all aspects can be calculated without adding any new element, besides the constitutive relations that define the various energy terms. Fig. 6b shows the model extended to consider these aspects, while Fig. 6c shows the model predictions. The energy exchange associated with every process is calculated separately from its own constitutive law. It can be noted that the “mechanical” dissipation is less than 10% of the “magnetic” one, and also that the total energy – calculated as the sum of the rotational energy and the dissipated energy – turns out to be constant: this tells us that the model is coherent with the energy conservation principle.
Fig. 6. A rotating disk braked by dry and magnetic friction: (a) angular velocity vs. time: simulated (1 – blue) and measured (2 – red) values; (b) model including energy aspects; (c) simulated values of the energy vs. time: rotation energy (1 – blue), energy dissipated by dry (2 – red) or magnetic friction (3 – green). The sum of the three contributes represents the total energy of the system (4 – black).

At this point the two parts of the model can be easily coupled, recognizing that only the magnetic interaction contributes to the temperature rise in the copper plate. Furthermore, we assume that the dissipative processes in the neodymium magnets can be neglected, due to their high magnetic stiffness: in order to test these ideas, in the model it is sufficient to interpret the power dissipated in the magnetic interaction as an additional exchange (inflow) in the balance equation of the (internal) energy of the copper plate (Fig. 7a). Fig. 7b shows the result for the change in temperature (comparison between measured data and model predictions).

Fig. 7. A rotating disk braked by dry and magnetic friction: (a) the model: only the magnetic interaction contributes to the temperature rise in the copper plate; (b) temperature of the copper plate during the braking vs time: simulated (1 – blue) and measured (2 – orange) values.
4. Conclusions
We presented and discussed an experiment with a rotating disc braked magnetically. As far as the mechanical aspects are concerned, our work confirms the principal results of previous studies of several aspects involved in rotational motion affected by friction (Amrani 2006, Eadkhong et al. 2012, Mungan 2013, Kladivova et al. 2016 and references therein), but it also allows us to extend the study to the thermal aspects, localizing the dissipative processes. Our model is, in fact, constructed step by step from the mechanical and thermal constitutive laws of interacting parts, fixing in separate experimental situations all the model parameters without the necessity to introduce any hypothesis concerning energy. This way, the resulting model can be seen as a real experimental test of the properties of energy itself, in particular energy conservation: the agreement of the model predictions with the whole set of measured data (relative to both the mechanical and thermal behaviour) shows, therefore, the robustness and the coherence of the whole model, not only as a mathematical structure, but above all as the result of the general approach used to describe energy exchanges.

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