Estimates of the Nucleon Tensor Charge

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I. INTRODUCTION

The spin composition of the nucleon has been intensely studied. Its study has produced important and surprising insights, beginning with the revelation that the longitudinal spin is carried more by gluons than quarks. Considerable effort has gone into understanding, predicting

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and measuring the corresponding transversity distribution of the nucleon constituents. The leading twist transversity structure function, $h_1(x)$, is as fundamental to understanding the nature of the non-perturbative QCD regime of hadronic physics as is the longitudinal function $g_1(x)$, yet the transversity distribution can not be measured directly in deep inelastic lepton scattering since it is chiral odd.

Like the isoscalar and isovector axial vector charges defined from the forward nucleon matrix element of the axial vector current, the nucleon tensor charge is defined from the corresponding forward matrix element of the tensor current,

$$\langle P, S_T \left| \bar{\psi} \sigma^\mu \gamma_5 \lambda^a \psi \right| P, S_T \rangle = 2\delta q^a \left( \mu_0^2 (P^\mu S^\nu_T - P^\nu S^\mu_T) \right),$$ \hspace{1cm} (1)

$S_T^Q \sim (|+| > \pm |− >)$ for the moving nucleon is the transversity, a variable introduced originally by Moravcsik and Goldstein [1] to reveal an underlying simplicity in nucleon–nucleon spin dependent scattering amplitudes. Unlike the axial vector isovector and isosinglet charges, no sum rule has been written that enables a clear relation between the tensor charge and a low energy measurable quantity. So, aside from model calculations, there are no definitive theoretical predictions of the tensor charge. Among the various approaches, from the QCD sum rule [2–4], which estimates the flavor contributions to the tensor charge by analyzing the bilocal tensor current on the light cone, to lattice calculations [5] and light cone quark models [6], there appears to be a range of expectations and a disagreement concerning the sign of the down quark contribution.

Given the numerous experiments at, RHIC-Brookhaven (the PHENIX and STAR collaborations [7,8]), CERN (COMPASS experiment [9]), and HERA-DESY (the HERMES experiment [10]) and proposals [11,12] for extracting quark transversity distributions and, in turn, the flavor contributions to the nucleon’s chiral odd tensor charge, there is reason to expect that in the not too distant future one will be able to reliably compare the data to the theoretical estimates of these quantities.

The various charges, which are first moments of the quark helicity and transversity distribution functions $\Delta q^a(x)$ and $\delta q^a(x)$ respectively, in principle can be measured in hard scattering processes. In their systematic study of the chiral odd distributions, Jaffe and Ji [13] related the
first moment of the transversity distribution to the flavor contributions to the nucleon tensor charge:

\[ \int_0^1 (\delta q^a(x) - \delta q^a(x)) \, dx = \delta q^a. \]  (2)

Because there must be a helicity flip of the struck quark in order to probe the transverse spin polarization of the nucleon, the transversity distribution (and thus the tensor charge) decouple at leading twist in deep inelastic scattering. No such suppression appears in Drell-Yan scattering where Ralston and Soper [14] first encountered the transversity distribution entering the corresponding transverse spin (both the beam’s and target’s spin being transversely polarized to the incident beam direction) asymmetries. Consequently, the charge is difficult to measure and its non-conservation [15] makes it difficult to predict. While bounds placed on the leading twist quark distributions through positivity constraints suggest that they satisfy the inequality of Soffer [16]:

\[ |2\delta q^a| \leq q^a + \Delta q^a, \]  (3)

(where \( q^a \) denotes the unpolarized quark distribution), model calculations yield a range of theoretical predictions [17].

Here we offer another estimate. Our motivation stems in part from the result that the tensor charge does not mix with gluons under QCD evolution and therefore behaves as a nonsinglet matrix element. This, in conjunction with the fact that the tensor current is charge conjugation odd (it does not mix quark-antiquark excitations of the vacuum, since the latter is charge conjugation even, nor does it mix with gluonic operators under evolution, since gluonic operators are even), suggests that the tensor charge is more amenable to a valence quark model analysis. With this in mind we estimate the tensor charge by using axial vector dominance and an approximate phenomenological mass symmetry among the light axial vector mesons.

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1 The quark lines are not correlated in the hard quark target amplitude.

2 This is to be contrasted with the axial charges.
\((a_1(1260), b_1(1235) \text{ and } h_1(1170))\). The \(b_1\) and \(h_1\) could couple to the quark tensor current in the nucleon at low energies, and via the symmetry, their coupling to the leptons is related to the \(a_1\) production in \(\tau\) decay.

Next we present our determination of the tensor charge under conditions of axial vector dominance in the context of our phenomenological symmetry represented by \(SU(6)_W \otimes O(3)\) spin-flavor symmetry. We use this symmetry to relate the parameters of the previous section to measurable quantities. Then we present our results for the isoscalar and isovector and, in turn, up and down quark contributions to the the tensor charge of the nucleon. Finally we compare our results with QCD sum rule and other models calculations.

II. TENSOR CHARGE

We now apply axial vector dominance to estimate the tensor charge. That is, we choose to determine the value of the tensor charge at a scale where the matrix element of the tensor current is dominated by the lowest lying axial vector mesons \([18]\). Under these conditions, the matrix element of the tensor current, Eq. (1) becomes

\[
\sum_{\mathcal{M}} \frac{\langle 0 \varphi \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi | \mathcal{M} \rangle \langle \mathcal{M}, P, S_T | P, S_T \rangle}{M_{\mathcal{M}}^2 - k^2}.
\]  

(4)

The summation is over those mesons with quantum numbers, \(J^{PC} = 1^{+-}\) that couple to the nucleon via the tensor current; namely the charge conjugation odd axial vector mesons – the isoscalar \(h_1(1170)\) and the isovector \(b_1(1235)\). To analyze this expression in the limit \(k^2 \rightarrow 0\) we require the vertex functions for the nucleon coupling to the \(h_1\) and \(b_1\) meson and the corresponding matrix elements of the meson decay amplitudes which are related to the meson to vacuum matrix element via the quark tensor current. The former yield the nucleon coupling constants \(g_{NNN}\) defined from the matrix element

\[
\langle MP|P\rangle = \frac{ig_{NNN}}{2M_N} \pi(P, S_T) \sigma^{\mu\nu} \gamma_5 u(P, S_T) \epsilon_{\mu} P_{\nu},
\]  

(5)

where \(P_{\mu}\) is the nucleon momentum, and the latter yield the meson decay constant, \(f_{\mathcal{M}}\)

\[
\langle 0 | \varphi \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi | \mathcal{M} \rangle = if_{\mathcal{M}}^a (\epsilon_{\mu} k_{\nu} - \epsilon_{\nu} k_{\mu}),
\]  

(6)
where the $k_\mu$ and $\varepsilon_\nu$ are the meson momentum and polarization respectively. For transverse polarized Dirac particles, $S^\mu = (0, S_T)$ we project out the tensor charge using the constraint on the vector meson, $k \cdot \varepsilon_M = 0$

$$\delta q^a(\mu^2) = \frac{f_M g_{MNN} (S_T \cdot k) A}{P \cdot \varepsilon M^2_M},$$

(7)

where

$$A = \frac{P \cdot \varepsilon (S_T \cdot k)}{2 M_N}$$

(8)

is the nucleon-meson vertex function. In order to evaluate the tensor charge at the scale dictated by the axial vector meson dominance model we must determine the isoscalar and isovector meson coupling constants. We take a hint from the valence interpretation of the tensor charge and exploit the phenomenological mass symmetry among the lowest lying axial vector mesons that couple to the tensor charge; we adopt the spin-flavor symmetry characterized by an $SU(6)_W \otimes O(3)$ [19,20] multiplet structure. Thus, the $1^{+-} h_1$ and $b_1$ mesons fall into a $(35 \otimes L = 1)$ multiplet that contains $J^{PC} = 1^{++}, 0^{++}, 1^{++}, 2^{++}$ states. This analysis enables us to relate the $a_1$ meson decay constant measured in $\tau^- \rightarrow a_1^- + \nu_\tau$ decay [22,21]

$$f_{a_1} = (0.19 \pm 0.03) \text{GeV}^2,$$

(9)

and the $a_1NN$ coupling constant

$$g_{a_1NN} = 9.3 \pm 1.0,$$

(10)

to the meson decay constants, $f_{b_1}$, $f_{h_1}$ and coupling constants, $g_{b_1NN}$ and $g_{h_1NN}$. We find

$$f_{b_1} = \frac{\sqrt{2}}{m_{b_1}} f_{a_1}, \quad g_{b_1NN} = \frac{5}{3\sqrt{2}} g_{a_1NN},$$

(11)

where the $5/3$ appears from the $SU(6)$ factor $(1+F/D)$ and the $\sqrt{2}$ arises from the $L=1$ relation between the $1^{++}$ and $1^{+-}$ states. [3] These, in turn, enable us to determine the isovector and isoscalar contributions

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[3] We note that our value of $f_{b_1} \approx 0.21 \pm 0.03$ agrees well with an independent sum rule determination of $0.18 \pm 0.03$ [23,4].
\[ \delta q' = f_{b_1} g_{b_1 NN} \frac{\langle k_{\perp}^2 \rangle}{\sqrt{2} M_N M_{b_1}^2}, \quad \text{and} \quad \delta q'' = f_{h_1} g_{h_1 NN} \frac{\langle k_{\perp}^2 \rangle}{\sqrt{2} M_N M_{h_1}^2} \]

(12)

respectively (where, \( \delta q' = (\delta u - \delta d) \), and \( \delta q'' = (\delta u + \delta d) \)). Transverse momentum appears in these expressions because the tensor couplings involve helicity flips that carry kinematic factors of momentum transfer. The intrinsic \( k_\perp \) of the quarks in the nucleon is non-zero, as determined from Drell-Yan and heavy vector boson production processes. Using a Gaussian momentum distribution, and letting \( \langle k_{\perp}^2 \rangle \) range from \((0.58 \text{ to } 1.0 \text{GeV}^2)\) \[24\] results in the up and down quark transversity ranging from

\[ \delta u(\mu_0^2) = (0.53 \text{ to } 0.92) \pm 0.20 \quad \delta d(\mu_0^2) = -(0.33 \text{ to } 0.58) \pm 0.20. \]

(13)

This range of values for the \( u \)-quark tensor charge lies slightly lower than most other estimates while the \( d \)-quark charge is negative and of a comparable magnitude. Note that many predictions have the ratio \( \delta d/\delta u \) near \(-1/4 \) or \( (1 - \sqrt{3})/(1 + \sqrt{3}) \), the value resulting from an \( SU(3) \) degeneracy between the \( \pi^0 \) and the \( \eta(8) \) octet elements in their coupling to the \( u \)-quark and the \( d \)-quark, i.e. the isoscalar coupling to \( u \) and \( d \)-quarks is \( 1\sqrt{3} \) times the isovector. In our calculation the isoscalar and isovector axial vector couplings to the nucleon also enter as factors in the expressions for the charges, with the \( D/F \) ratio being \( 3/2 \) in exact \( SU(6) \). Loosening the \( SU(6) \) constraint and incorporating mixing of the \( h_1(1170) \) with the \( h_1(1380) \) will alter the \( u \) to \( d \) ratio. These variations are being explored. Several other model calculations are summarized in the Table \[4\].
TABLE I. Sampling of estimates of the flavor contribution to the nucleon tensor charge calculated in: lattice (Lat) \[5\]; QCD sum rules (SR) \[2\]; the bag model (BAG) \[25,2\]; the constituent quark model with Goldstone boson effects (CQ) \[26\]; a quark soliton model calculation (QS) \[27\]; and the NJL chiral soliton model (NJL) \[28,29\] with the associated momentum ranging from 0.40 – 1.0 GeV^2 (errors range from 10-40%).

| $\delta q^a$ | Lat | SR  | BAG | CQ  | QS  | NJL |
|-------------|-----|-----|-----|-----|-----|-----|
| $\delta q_1^a(Q^2)$ | 0.84 | 1.33 | 1.09 | 1.19 | 1.07 | 0.82 |
| $\delta q_2^a(Q^2)$ | -0.23 | 0.04 | -0.27 | 0.12 | -0.38 | -0.07 |

The calculation has been carried out at the scale $\mu_0 \approx 1$ GeV, which is set by the nucleon mass as well as being the mean mass of the axial vector meson multiplet. The appropriate evolution to higher scales (wherein the Drell-Yan processes are studied) is determined by the anomalous dimensions of the tensor charge \[15\] via

$$\delta q(\mu^2) = \left(\frac{\alpha(\mu^2)}{\alpha(\mu_0^2)}\right)^{4_{3.3-2n_f}} \delta q(\mu_0^2).$$

(14)

This is straightforward but slowly varying.

In conclusion, our axial vector dominance model with $SU(6)_W \otimes O(3)$ coupling relations provide simple formulae for the tensor charges. This simplicity belies the considerable subtlety of the (non-perturbative) hadronic physics that is summarized in those formulae. We obtain the same order of magnitude as most other calculation schemes. This result supports the view that the underlying hadronic physics, while quite difficult to formulate from first principles, is essentially a $1^{+-}$ meson exchange process. Forthcoming experiments will begin to test this notion.
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[1] G.R. Goldstein and M.J. Moravcsik, Ann. Phys. (NY) 98, 128 (1976); Ann. Phys. (NY) 142, 219 (1982); Ann. Phys. (NY) 195, 213 (1989).

[2] H. He and X. Ji, Phys. Rev. D52, 2960 (1995); Phys. Rev. D56, 6897 (1996).

[3] X. Jin and J. Tang, Phys. Rev. D56, 5618 (1997).

[4] V.M. Belyaev and A. Oganesian, Phys. Lett. B395, 307 (1997).

[5] S. Aoki, et al., Phys. Rev. D 56, 433 (1997).

[6] B-Q Ma, I. Schmidt and J. Soffer, Phys. Lett. B441, 461 (1998); I. Schmidt and J. Soffer, Phys. Lett. B407, 331 (1997).

[7] H. Enyo (PHENIX collab.), RIKEN Rev. 28, 3 (2000).

[8] L.C. Bland (STAR collab.), RIKEN Rev. 28, 8 (2000).

[9] G. Baum et al. (COMPASS collab.), COMPASS: A Proposal for a Common Muon and Proton Apparatus for Structure and Spectroscopy, CERN preprint CERN-SPSLC-96-14 (1996).

[10] V. A. Korotkov and W.-D. Nowak, Future Measurements of Transversity, e-print [hep-ph/0102013].

V. A. Korotkov and W.-D. Nowak and K.A. Oganessyan, Transversity Distributions and Polarized Fragmentation Function from Semi-Inclusive Pion Electroproduction, e-print [hep-ph/0002268].

[11] M. Anselmino, et al. (TESLA-N Study Group collab.), Electron Scattering with Polarized Targets at TESLA, e-print [hep-ph/0011299].
[12] K.A. B. Aune et al., ELEFE at CERN: Conceptual Design Report, CERN preprint CERN-99-00 (1999).

[13] R. L. Jaffe and X. Ji, Phys. Rev. Lett. 67, 552 (1991); Nucl. Phys. B375, 527 (1992).

[14] J. Ralston and D. E. Soper, Nucl. Phys. B152, 109 (1979).

[15] X. Artu and M. Mekhfi, Z. Phys. C45 (1990) 669.

[16] J. Soffer, Phys. Rev. Lett. 74, 1292 (1995); G. R. Goldstein, R. L. Jaffe and X. Ji, Phys. Rev. D52, 5006 (1995).

[17] V. Barone, A. Drago and P. G. Ratcliffe, Transverse Polarization of Quarks in Hadrons, e-print hep-ph/0104283.

[18] S. Weinberg Phys. Rev. Lett. 18, 507 (1967).

[19] B. Sakita, Phys. Rev. 136, B1756 (1964); F. Gürsey and L.A. Radicati, Phys. Rev. Lett. 13, 173 (1964).

[20] F.E. Close, An introduction to quarks and partons (Academic Press, New York 1979).

[21] W.-S. Tsai, Phys. Rev. D4, 2821 (1971).

[22] M. Birkel and H. Fritzsch, Phys. Rev. D53, 6195 (1996).

[23] P. Ball and V.M. Braun, Phys. Rev. D54, 2182 (1996).

[24] R.K. Ellis, W.J. Stirling and B.R. Webber, QCD and Collider Physics (Cambridge University Press, Cambridge, U.K. 1996), p.305.

[25] R. L. Jaffe and X. Ji, Nucl. Phys. B375, 527 (1992).

[26] K. Suzuki and T. Shigetani, Nucl. Phys. A626, 886 (1997).

[27] H.-C. Kim, M. Polyakov and K. Goeke, Phys. Rev. D53, 4718 (1994); (See also, M. Wakamatsu and T. Kubota, Phys. Rev. D60, 034020 (1999) ).

[28] R. Alkofer, H. Reinhardt and H. Weigel, Phys. Rep. 265, 139 (1996).
[29] L. Gamberg, H. Weigel and H. Reinhardt, Phys. Rev. D58, 054014 (1998); L. Gamberg, “Structure Functions and Chiral Odd Quark Distributions in the NJL Soliton Model of the Nucleon” in Proceedings of Future Transversity Measurements, RIKEN BNL Research Center Workshop, September 18-20 (2000).