Deep Learning of Near Field Beam Focusing in Terahertz Wideband Massive MIMO Systems

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Abstract—Employing large antenna arrays and utilizing large bandwidth have the potential of bringing very high data rates to future wireless communication systems. However, this brings the system into the near-field regime and also makes the conventional transceiver architectures suffer from the wideband effects. To address these problems, in this letter, we propose a low-complexity frequency-aware beamforming solution that is designed for hybrid time-delay and phase-shifter based RF architectures. To reduce the complexity, the joint design problem of the time delays and phase shifts is decomposed into two subproblems, where a signal model inspired online learning framework is proposed to learn the shifts of the quantized analog phase shifters, and a low-complexity geometry-assisted method is leveraged to configure the delay settings of the time-delay units. Simulation results highlight the efficacy of the proposed solution in achieving robust performance across a wide frequency range for large antenna array systems.

Index Terms—Near field communication, deep learning, massive MIMO, Terahertz communications.

I. INTRODUCTION

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ploying large antenna arrays and utilizing the large bandwidth available at millimeter wave and terahertz (THz) bands are two key trends in current and future wireless communication systems. However, these new characteristics also bring new design challenges. Two particular challenges are: (i) With a large antenna array aperture, the system is more likely to operate in the near-field region. This makes the conventional far-field planar wave propagation assumption invalid and challenges the beamforming design problems. (ii) With the wide bandwidth, the classical phase shifter based analog-only beamforming experiences beam squinting/misfocusing, leading to degradation in the beamforming gain and increase in the introduced network interference. These problems motivate adopting new transceiver architectures and developing novel beamforming solutions for near-field wideband communication systems, which is the focus of this letter.

Prior Work: A common approach to improve the performance of a wideband system is by equipping the antenna arrays with the time delay (TD) units which can introduce a constant group delay to the different frequency components of the transmit/receive signals. For instance, in [1], [2], [3], the authors demonstrate the potential of such TD units based arrays in achieving robust beamforming gain performance and in facilitating other operations (such as beam training and alignment) in the wideband systems. However, the TD-based arrays come with a higher manufacture cost and they also consume more power than the conventional phase-shifter (PS) based antenna arrays. With the motivation to address this increased cost and power consumption, [4], [5] studied the near-field wideband beam focusing problem using PS-only architectures. Although the average performance within the considered bandwidth is optimized, the peak gains are still majorly sacrificed. To strike a balance between performance and hardware cost/power consumption, the hybrid TD-PS based architecture is introduced, where the wideband beamforming problem is studied under both far-field [6] and near-field [7] channel models. However, the achieved performance in the existing work relies on accurate channel knowledge, which incurs high training and acquisition overhead given the large number of antennas.

Contribution: In this letter, we develop a low-complexity beamforming approach for TD-PS architecture that is capable of learning and optimizing the near-field wideband beams without requiring any channel knowledge. Specifically, an online learning framework that is inspired by the signal model is developed to learn the phase shifts of the quantized analog PSs and achieve fast convergence despite the large numbers of antennas. Further, a low-complexity geometry-assisted method is devised to configure the delay settings of the TD units. The proposed framework relies only on the power measurements of the received signal at different subcarriers, and does not require any explicit channel or location information. Simulation results demonstrate the capability of the proposed solution in learning optimized near-field beams that achieve robust performance in practical wideband large-antenna array systems.

II. SYSTEM AND CHANNEL MODELS

A. System Model

We consider a system where a THz massive MIMO base station (BS) with M antennas is communicating with a single-antenna user equipment (UE). The system adopts OFDM transmission/reception with K subcarriers and operates at a center frequency $f_c$ over a bandwidth $B$. In the uplink, if the UE transmits a symbol $s_k \in \mathbb{C}$ at the $k$-th subcarrier, then the received signal at the BS after combining can be expressed as

$$y_k = w^H h_k s_k + w^H n_k, \quad \forall k = 1, \ldots, K,$$

(1)
where we assume that the transmitted symbol $s_k$ satisfies the average power constraint $\mathbb{E}\{ |s_k|^2 \} = \frac{P_t}{K}$ with $P_t$ denoting the total transmit power. $\mathbf{w}$ is the combining vector, $\mathbf{h}_k \in \mathbb{C}^{M \times 1}$ is the uplink channel vector between the UE and the BS at the $k$-th subcarrier, and $\mathbf{n}_k \sim \mathcal{CN}(0, \sigma_k^2 \mathbf{I})$ is the receive noise vector at the BS with $\sigma_k^2$ being the noise power.

The combining vector $\mathbf{w}$ in (1) takes different forms in different transceiver architectures. In this letter, we consider a practical system where the BS has only one RF chain and employs analog-only beamforming using a network of $r$-bit quantized PSs. Further, to mitigate the severe beam split effect in the wideband systems, we assume that the system is equipped with time-delay units. These TD units are RF circuit blocks that introduce a constant group delay to the different frequency components of the received signal [3]. For example, if the $n$-th TD unit is configured to introduce a delay of $\tau_n$ seconds, then the phase shift incurred for the signal at the $k$-th subcarrier is $-2\pi f_k \tau_n$. As shown in Fig. 1, we assume that every $P$ PSs are connected to one TD unit, i.e., forming a partially connected architecture. Without loss of generality, we assume that the system adopts a total number of $N$ TD units such that $M = NP$. With this architecture, and unlike the conventional PS-only beamforming, the beamforming becomes frequency-dependent. Specifically, the effective combining vector at the frequency $f_k$ is given by

$$
\mathbf{w}_k = \frac{1}{\sqrt{M}} e^{j(-2\pi f_k \tau \otimes 1_P + \theta)},
$$

(2)

where $\otimes$ is Kronecker product, $\mathbf{r} \in \mathbb{R}^{N \times 1}$ is the delay vector, $\theta \in \mathbb{R}^{M \times 1}$ is the phase vector (of the phase shifters), and $1_P$ denotes the $P \times 1$ all one vector. In the rest of this letter, we will use the approximation

$$
\mathbf{w}(f) \approx \mathbf{w}_k, \forall f \in \left[ f_k - \frac{B}{2K}, f_k + \frac{B}{2K} \right],
$$

(3)

i.e., we will assume that the effective beamforming is constant across each subcarrier.

**B. Near Field Wideband Channel Model**

Without loss of generality, we assume a 2-D geometry, where both the BS antenna array and the UE’s antenna are at the $x$-$y$ plane. The BS is assumed to employ a linear array that is aligned with the $y$ axis and that has a half aperture of $\frac{D}{2}$. The center of the BS array is assumed to be at the origin. Based on that, the positions of the $M$ BS antenna elements can be written as $\mathbf{p}_m = \left[ 0, \frac{D}{2} \alpha_m \right]^T, \forall m = 1, 2, \ldots, M$, where $\alpha_m$ takes a value in $[-1, 1]$, i.e.,

$$
-1 \leq \alpha_M < \alpha_{M-1} < \cdots < \alpha_2 < \alpha_1 \leq 1.
$$

(4)

It is worth noting that we do not assume that the BS adopts a uniform linear array (ULA), meaning that $\alpha_m, \forall m$ can be any values in $[-1, 1]$ and only need to satisfy the sequential constraint, i.e., $\alpha_M < \cdots < \alpha_2 < \alpha_1$. Finally, we denote the UE’s position as $\mathbf{q} = [q_x, q_y]^T \in \mathbb{R}^2$. Hence, the distance between the UE’s antenna and the $m$-th BS antenna element is given by $d_m = \| \mathbf{p}_m - \mathbf{q} \|_2$.

Given the large aperture of the array adopted by the BS, we consider a practical scenario where the UE is within the near field region of the BS. Therefore, and based on the above geometry, the channel coefficient between the UE and the $m$-th BS antenna at the $k$-th subcarrier is given by

$$
\mathbf{h}_k[m] = \frac{\rho_k \lambda_k}{4\pi d_m} e^{-\frac{j\pi}{\lambda_k} d_m}, \forall k = 1, \ldots, K, m = 1, \ldots, M,
$$

(5)

where $\lambda_k$ is the wavelength of the $k$-th subcarrier and $\rho_k$ subsumes all the other factors such as antenna pattern of the BS array, pathloss, and channel gain at the $k$-th subcarrier.

**III. PROBLEM FORMULATION**

Given the system and channel models described in Section II, we cast the problem as a joint design of phase shifts of the quantized phase shifters and the delay configurations of the TD units. The overall objective is to maximize the average beamforming gain, or equivalently, the signal-to-noise ratio (SNR), of the system across all the subcarriers. Formally, this objective can be written as

$$
\max_{\tau, \theta} \frac{1}{K} \sum_{k=1}^{K} |\mathbf{w}_k^H \mathbf{h}_k|,
$$

(6)

s. t. $\tau_n \in [0, \tau_{\text{max}}], \forall n = 1, 2, \ldots, N$, $\theta_m \in \Psi, \forall m = 1, 2, \ldots, M$,

(7)

(8)

where $\mathbf{w}_k$ is given by (2), $\tau_{\text{max}} \in \mathbb{R}^+$ denotes the maximum delay that the TD unit supports, and $\Psi$ is a finite set with $2^P$ possible phase values drawn uniformly from $(-\pi, \pi)$. In addition to the constraints (7) and (8), we also assume that the channels, $\mathbf{h}_k, \forall k$, are unknown, as acquiring the channel state information (CSI) is typically challenging in systems with fully analog transceiver architectures [8].

**IV. PROPOSED SOLUTION**

In this section, we present our proposed near field wideband beam focusing solution. The proposed approach decomposes the original joint design problem (6) into two subproblems: (i) The quantized analog phase shifter design subproblem, which focuses on maximizing the beamforming gain at the center frequency, while setting all the TD units to have zero delays; and (ii) the TD unit design subproblem, which focuses on maximizing the average beamforming gain achieved across all the subcarriers by properly configuring the TD units. Despite this decomposition, we will show in Section V that the performance achieved by the proposed solution approaches
that of the prior work [7] which (different than our solution) requires full channel knowledge and uniform array geometry. The motivation of this decomposition is partially originating from the observation that unless the number of elements in each sub-array is very large, then the performance degradation caused by the wideband effect for this sub-array is negligible.

A. Decomposition of the Original Problem

The combining vector in (2) can be expressed as $w_k = w_{TD,k} \odot w_{PS}$, where $w_{TD,k} = e^{-j2\pi f_c \tau_k}1_P$ is the TD-based frequency-dependent combining vector and $w_{PS} = \frac{1}{\sqrt{M}}e^{j\theta}$ is the phase-shifter based frequency-flat combining vector. The symbol $\odot$ denotes the element-wise product. Correspondingly, the design of the quantized analog phase shifters can be formulated as the following optimization problem

$$\theta^* = \arg \max_\theta \left| w_{PS}^H h_f \right|,$$  
\hspace{1cm} (9)

\hspace{1cm} s.t. \hspace{0.2cm} \left| w_{PS} \right|_m = \frac{1}{\sqrt{M}} e^{j \theta_m},

$$\theta_m \in \Psi, \forall m = 1, 2, \ldots, M,$$  
\hspace{1cm} (10)

where $h_f$ denotes the channel at the center frequency. After obtaining $\theta^*$, the design problem of the TD units can be expressed as follows

$$\tau^* = \arg \max_\tau \frac{1}{K} \sum_{k=1}^K \left| \left( w_{TD,k} \odot \tilde{w}_{PS} \right)^H h_k \right|,$$  
\hspace{1cm} (12)

\hspace{1cm} s.t. \hspace{0.2cm} $w_{TD,k} = e^{-j2\pi f_c \tau_k}1_P$,

$$\tau_n \in [0, \tau_{\text{max}}], \forall n = 1, 2, \ldots, N,$$  
\hspace{1cm} (13)

$$\left| \theta \right|_m = \arg \min_\theta \left| \theta - (\theta^*)_m + 2\pi f_c \tau_n \right|,$$  
\hspace{1cm} (14)

\hspace{1cm} $\forall n, \forall m \in \{ (n-1)P + 1, \ldots, nP \}$,

where $\tilde{w}_{PS} = \frac{1}{\sqrt{M}} e^{j\theta^*}$. The constraint (15) is to ensure that the gain at the center frequency, i.e., the objective function of (9), will not be influenced after the introduction of the TD units. Next, in Sections IV-B and IV-C, we present the proposed solutions for solving (9) and (12).

B. The Design of the Analog Phase Shifters

As mentioned before, the design of the quantized analog PSs focuses on maximizing the beamforming/combining gain at the center frequency $f_c$. We adopt the similar reinforcement learning (RL) formulation as in the previous work [9]. Specifically, we define the state $s_t$ as a vector that consists of the phases of all the phase shifters at the $t$-th iteration, that is, $s_t = [\theta_1, \theta_2, \ldots, \theta_M]^T$. We define the action $a_t$ as the element-wise changes to all the phases in $s_t$. Since the phases can only take values in $\Psi$, a change of a phase represents the action that a phase shifter selects a value from $\Psi$. Therefore, the action is directly specified as the next state, i.e., $a_t = s_{t+1}$, which can be viewed as a deterministic transition in the Markov Decision Process (MDP). We define the reward as the achieved beamforming gain difference between the action (i.e., new beam, denoted as $w_{PS,t}$) and the state (i.e., the previous beam, denoted as $w_{PS,t-1}$). Formally, $r_t = \left| w_{PS,t}^H h_f \right|^2 - \left| w_{PS,t-1}^H h_f \right|^2$.

Moreover, to improve the sample efficiency of the proposed algorithm in learning over large antenna arrays, we develop a signal model inspired online RL-based search algorithm. Specifically, we improve the existing actor-critic architecture based beam learning algorithm [9] by designing a special critic network that better utilizes the underlying signal model. Formally, the critic network takes the following form

$$f(s_t, a_t) = w_{a_t}^H QH w_{a_t} - w_{s_t}^H QH w_{s_t},$$  
\hspace{1cm} (16)

where $Q \in \mathbb{C}^{M \times v}$ is the model parameters with $v \in \mathbb{N}_+$ being a hyperparameter, and $w_{a_t} = \frac{1}{\sqrt{M}} e^{ja_t}$ and $w_{s_t} = \frac{1}{\sqrt{M}} e^{j\theta_t}$. The complete beam learning framework is illustrated in Fig. 2.

C. The Design of the Time Delay Units

In this subsection, we introduce the proposed low complexity geometry-assisted TD unit design algorithm. Before we delve into the details, it is worth mentioning that the motivation of leveraging geometry to design the delay configuration of the TD units can be found in the following observations. First, the TD units are mainly adopted to compensate for the different delays experienced at the different parts of the large array. This implies that the design can be determined based on the distance differences between the elements on the antenna array and the UE. Second, the absolute distance difference between any two antenna elements on the array and the UE is upper bounded by the array aperture, i.e., $D$. Third, depending on the relative positions of the BS and UE, the delay profile on the BS’s array (i.e., the experienced delay value as a function of antenna element) presents different monotonicity properties. These observations provide useful insights and guidance for the considered TD units design as they help constrain the search space of the delay configurations. Moreover, as indicated by the first observation, the introduction of the TD units is to compensate for the differences in the propagation delays experienced at the different receive antenna elements, which are directly related to the different propagation distances. Therefore, understanding and finding such distance differences could facilitate the design of the TD units. Based on this, we introduce the concept of the distance difference function (DDF), which directly determines the design of the delay units. To be more specific, we set the set $P_{\text{ref}} = [0, D]^T$ as the reference point of the BS array. Further, we define $\Delta = 1 - \alpha$ as the relative coefficient with respect to the reference point,\(^{1}\)

\(^{1}\)As described in Section II-B, the position of an antenna is determined via its coefficient $-1 \leq \alpha \leq 1$. 

Fig. 2. The adopted actor-critic learning architecture where the actor network is implemented using the fully-connected neural network and the critic network is designed to reflect the underlying signal model.
which implies that $\Delta \in [0, 2]$. Therefore, the reference distance, defined as the distance between $p_{\text{ref}}$ and the considered UE position $q$, is given by $d_{\text{ref}} = \|p_{\text{ref}} - q\|_2$. Based on the reference distance, we define the DDF $f_d$ for any point on the array $p_\Delta = [0, (1 - \Delta) D]^T$ and the UE as

$$f_d(\Delta, q) = d(\Delta) - d_{\text{ref}},$$

where $d(\Delta) = \|p_\Delta - q\|_2$. As can be seen, the DDF depends only on $\Delta$ and $q$. Besides, $d_{\text{ref}}$ corresponds to $\Delta = 0$, and $f_d$ is monotonically increasing when $q_y > 0$. Moreover, an important observation is that the monotonicity property of $f_d$ depends on the UE position $q$ relative to the BS array. Without loss of generality, we assume $q_x > 0$. Then, the UE position space can be divided into three regimes based on the value of $q_y$: (i) $q_y > \frac{D}{2}$; (ii) $q_y < -\frac{D}{2}$; and (iii) $|q_y| < \frac{D}{2}$. For these regimes, $f_d$ is monotonically increasing when $q_y > \frac{D}{2}$ and is monotonically decreasing when $q_y < -\frac{D}{2}$. When $|q_y| < \frac{D}{2}$, $f_d$ first decreases and then increases. It is worth noting that the delay configuration of the TD units is closely related to $f_d$ which essentially characterizes the additional propagation distance at different parts of the array. Specifically, the ideal delay configuration of the $m$-th antenna is given by $\tilde{f}_d(\Delta_m)$, where $\Delta_m = 1 - a_m$. However, as can be seen from (17), in order to obtain the exact expression of $f_d$, the UE position $q$ should be known. Since in practice systems, however, the UE position is either unavailable or noisy, it is important to develop search algorithms that are able to find an approximation of $f_d$, and hence design the delay configurations, without requiring the UE position. Next, we describe the proposed low complexity delay search method. Each search cycle in this method consists of the following three steps:

1) Select a Linear Approximation: Inspired by the behavior of the DDF $f_d$, we propose a low complexity linear approximation based search algorithm to design the delay configurations of the TD units. Specifically, we use the following piecewise linear function to approximate $f_d$, that is

$$\tilde{f}_d(\Delta; a_x, a_y, b) = \begin{cases} \frac{a_x}{2} \Delta, & 0 \leq \Delta \leq a_x, \\ \frac{a_y}{2} (\Delta - a_x) + a_y, & a_x < \Delta \leq 2, \end{cases}$$

where the approximation function $\tilde{f}_d$ is parameterized by $a_x, a_y, b$, with $(a_x, a_y)$ modelling the coordinate of the only breakpoint of (18) and $b$ modelling the intersection of the second part of (18) with $\Delta = 2$. Therefore, the approximation problem essentially becomes a joint search problem of $a_x, a_y$ and $b$. Since $a_x, a_y$ and $b$ should satisfy the conditions determined by the properties of $f_d$, $0 \leq a_x \leq 2$, $-\frac{D}{2} a_y \leq a_y \leq \frac{D}{2} a_y$, $-D \leq b \leq D$, we discretize the original continuous search space into a finite size discrete grid. Specifically, we denote $A_x \subseteq [0, 2]$ as the finite search set that contains the search points of $a_x$. $A_y [a_y] \subseteq [-\frac{D}{2} a_y, \frac{D}{2} a_y]$ as the finite search set of $a_y$ given $a_x$, and $B \subseteq [-D, D]$ as the finite search set of $b$. In each search cycle, one combination of the possible values of $a_x, a_y, b$ and $d$ are selected, which gives one candidate approximation of $f_d$.

2) Compute the Delay Configurations: After obtaining $\tilde{f}_d$, the delay search is then conducted based on sampling of the corresponding delay difference function, defined as $\hat{f}_d = \tilde{f}_d$. If we set one sampling point at the center of each sub-array to have the set $\{\Delta_1, \Delta_2, \ldots, \Delta_N\}$, with $0 \leq \Delta_1 < \Delta_2 < \cdots < \Delta_N \leq 2$, then, the delay configuration vector will be

$$\tau = [\hat{f}_d(\Delta_1), \hat{f}_d(\Delta_2), \ldots, \hat{f}_d(\Delta_N)]^T.$$  

3) Evaluate the Configured Delays: In this step, the analog phase shifters are first reconfigured to compensate for the additional phase shifts introduced by the TD units, according to (15). Then, the system measures the receive beamforming gain at the different subcarriers to determine the performance of the current delay configurations. This completes one cycle of the proposed delay search algorithm.

D. Complexity Analysis

For the analog PS design problem, the proposed signal model-based learning architecture reduces the computational complexity when comparing with the signal model-unaware architecture [9]. Specifically, the complexity reduction of the proposed solution mainly comes from the following aspects. First, the signal model-based critic network design reduces its complexity from $O(M^2 N_{n_{\text{iter}}})$, where we assume a fully-connected neural network-based critic network architecture as in [9], to $O(M N_{n_{\text{iter}}})$, with $N_{n_{\text{iter}}}$ denoting the mini-batch size and $N_{\text{iter}}$ denoting the total number of iterations. Second, from training perspective, the convergence of the proposed solution is much faster than the signal model-unaware architectures, meaning that $N_{\text{iter}}$ is normally also small. To be more specific, from the simulation result, it indicates that, for a 256-antenna BS with 3-bit quantized PSs, the training of the critic network, depending on the number of real power measurements, is able to converge within 1000 ~ 5000 iterations. And with a well-trained critic network, the actor network is typically able to converge within only 20 ~ 50 iterations. Finally, due to the decoupling of the original joint TD-PS design problem (6), the search space becomes much smaller, which generally leads to less search complexity.

V. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed near field wideband beamforming design algorithm.

A. Simulation Setup

In this simulation, we consider the scenario where a BS receiver adopts a linear array with 256 elements. While the positions of the antenna elements within the array are randomly sampled, giving rise to an unknown and non-uniform array geometry, we assume that the aperture of the array is given by $(M - 1) \lambda_c$, with $\lambda_c$ being the wavelength of the center frequency and $M = 256$. Furthermore, we assume that the system is operating at a center frequency of 100 GHz (hence $\lambda_c = 0.3$ centimeter) and has a bandwidth of 10 GHz. Moreover, each antenna of the array is followed by 3-bit analog PSs. The signals from the $P$ adjacent PSs are first combined and then processed by the same TD unit, as illustrated in Fig. 1. The value of $P$ depends on the number of TD units in the specific simulations.
B. Numerical Results

In Fig. 3(a), we visualize the beam focusing performance of the proposed solution at different frequencies for a user at $q = [2, -2]^T$, where we show the lowest, center and highest frequencies of the considered bandwidth. The average beamforming gain performance at these frequencies is plotted on the considered user grid. Specifically, the figure to the left shows the performance of the PS-only architecture and the figure to the right shows the performance of the TD-PS hybrid architecture with the proposed beam focusing algorithm. (b) shows the beamforming gain performance with different numbers of TD units.

![Beam focusing at different frequencies](image)

![Wideband performance](image)

Fig. 3. The evaluation of the proposed solution. (a) shows the beam focusing performance at different frequencies (lowest, center and highest frequencies of the considered bandwidth) to a user at $q = [2, -2]^T$ that is marked by the red square, where the figure to the left shows the performance of PS-only architecture and the figure to the right shows the performance of the TD-PS hybrid architecture with the proposed beam focusing algorithm. (b) shows the beamforming gain performance with different numbers of TD units.

VI. Conclusion

In this letter, we developed a low-complexity wideband beamforming solution for hybrid TD-PS based RF architectures operating in the near-field regime. To perform beamforming, the proposed solution does not require any explicit channel knowledge and relies only on the power measurements of the received signal at different subcarriers. The sample efficiency of the proposed solution is enhanced by designing a signal model inspired actor-critic architecture to learn the phase shifts, and by leveraging a geometry-assisted linear approximation model to configure the TD units. Important future extensions of this letter include near-field wideband beam codebook learning, interference aware near-field beam pattern learning, and beam/codebook learning under user mobility.

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