Channel Estimation and Hybrid Precoding for Distributed Phased Arrays Based MIMO Wireless Communications

Yu Zhang, Yiming Huo, Member, IEEE, Dongming Wang, Member, IEEE, Xiaodai Dong, Senior Member, IEEE, and Xiaohu You, Fellow, IEEE

Abstract

Distributed phased arrays based multiple-input multiple-output (DPA-MIMO) is a newly debuted architecture that enables both spatial multiplexing and beamforming while facilitating highly reconfigurable hardware implementation in millimeter-wave (mmWave) frequency bands. With a DPA-MIMO system, we focus on channel state information (CSI) acquisition and hybrid precoding. As benefited from a coordinated and open-loop pilot beam pattern design, all the subarrays can simultaneously perform channel sounding with less training overhead compared to the time-sharing operation of each subarray. Furthermore, two sparse channel recovery algorithms, known as joint orthogonal matching pursuit (JOMP) and joint sparse Bayesian learning with $\ell_2$ reweighting (JSBL-$\ell_2$), are proposed to exploit the hidden structured sparsity in the beam-domain channel vector. Finally, successive interference cancellation (SIC) based hybrid precoding through subarray grouping is illustrated for the DPA-MIMO system, which decomposes the joint subarray RF beamformer design into an interactive per-subarray-group handle. Simulation results show that the proposed two channel estimators fully take advantage of the partial coupling characteristic of DPA-MIMO channels to perform channel recovery, and the proposed hybrid precoding algorithm is suitable for such array-of-subarrays architecture with satisfactory performance and low complexity.

Index Terms

Distributed phased arrays based multiple-input multiple-output (DPA-MIMO), millimeter-wave (mmWave), array-of-subarrays, channel estimation, orthogonal matching pursuit (OMP), sparse Bayesian learning (SBL), successive interference cancellation (SIC), hybrid precoding.

Y. Zhang, D. Wang and X. You are with the National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China, e-mail: \{yuzhang, wangdm, xhyu\}@seu.edu.cn.

Y. Huo and X. Dong are with the Department of Electrical and Computer Engineering, University of Victoria, Victoria, BC V8P 5C2, Canada, e-mail: (ymhuo@uvic.ca, xdong@ece.uvic.ca).
I. INTRODUCTION

Driven by the tremendous growth in demand for wireless data, many new technologies have been proposed for the fifth generation wireless communications (5G) to enable orders of magnitude increases in the network capacity [1]–[4]. In the physical layer (PHY), the exploration of new spectrum in the so-called 5G upper bands, for example, from 6 GHz up to 100 GHz, including the millimeter-wave (mmWave) frequencies, has made multi-gigabit-per-second wireless communications more promising and feasible. In fact, mmWave communication has been widely used in long-distance point-to-point communication via satellite and terrestrial applications [5]. However, applying mmWave communications to commercial cellular networks is very challenging mainly due to, first, much higher propagation losses compared to those at lower microwave frequencies; second, strict constraints on hardware designs and implementations which include but are not limited to, antenna performance and dimension [6]–[9], circuits and systems integration challenges [10]–[14], power consumption and power supply, form factor (particularly critical for a mobile handset device), etc., according to [15]. Fortunately, a large number of antenna elements working at mmWave bands can be accommodated into a limited hardware area due to shorter wavelength. This fact facilitates a compact design of large antenna arrays to provide beamforming gain for combating large pathloss and establishing stable links with reasonable signal-to-noise ratio (SNR) values.

Compared to multiple-input multiple-output (MIMO) systems designed for 4G long-term evolution (LTE) standards, it is impractical to assign one dedicated ratio frequency (RF) chain that includes the digital-to-analog (D/A) / analog-to-digital (A/D) converter, signal mixer and power amplifier to each antenna element in mmWave MIMO systems, in light of the state-of-the-art hardware implementation techniques and power consumptions [3], [14], [16]–[19]. The hybrid analog-digital solution, which was first investigated in [20] for the diversity and spatial multiplexing performance improvement, divides the signal processing in MIMO systems into low-dimensional digital precoders/combiners, and high-dimentional analog precoders/combiners that are implemented with low-cost phase shifters [21]. This hybrid transceiver topology is further categorized into the fully-connected and partially-connected structures in terms of how RF chains are mapped to antennas. In a fully-connected structure, each RF chain enables full array gain through individual connection to all antennas [22]–[25]; while for the latter partially-connected structure also known as the subarray based structure, each RF chain is only connected to partial
antennas, which reduces complexity at the penalty of degrading beamforming gain [26].

In order to achieve an overall design trade-off among the complexity, cost and performance for large-scale antenna arrays, a novel hybrid adaptive receive subarray architecture was proposed in [27]. Such architecture consists of multiple analog subarrays each of which has its own digital processing chain. Thus, it can provide flexible beamforming designs with spatial multiplexing capability for long-range high data-rate mmWave communications. Specifically, this architecture is composed of two different configurations, namely, hybrid arrays of interleaved and side-by-side subarrays. The former one has narrower beamwidth and is therefore more suitable for multibeam transmission in space-division multiple access (SDMA) applications, while the latter one can better support the system with a relatively larger angle of arrival (AoA) without causing serious grating lobes. A unified wideband AoA estimation and beamforming algorithm was further developed for these two hybrid configurations with the mutual coupling effect considered for each subarray [28]. A two-level approach which includes multiple subarrays at both the transmitter (TX) and the receiver (RX) investigates the subarray spacing impact on the spatial multiplexing performance for outdoor Urban mmWave communications [29]. In this approach, the subarray spacing is defined as the distance between the centers of two adjacent subarrays, and it increases with the number of antenna elements deployed within each subarray. In contrast to compact subarray architectures [26], [27], [29]–[32], where the edge spacing of adjacent subarrays is usually half wavelength of the carrier frequency, a novel distributed phased arrays based MIMO (DPA-MIMO) architecture which can be easily applied to both base station (BS) and user equipment (UE) designs, has recently been proposed for practical system and hardware designs [15]. This highly reconfigurable architecture facilitates the multi-beam multi-stream based 5G system and hardware designs under the realistic constraints and resources limitation which include but not limited to antenna arrays, RF front ends, baseband processing, thermal dissipation performance and form factor. Therefore, the DPA-MIMO architecture enables appealing advanced features for both academic research and industrial applications [7], [33]–[36]. Subsequently, further investigation should be conducted on the baseband processing techniques such as high-efficiency channel estimation and low-complexity hybrid precoding.

A. Related Works

In mmWave wireless systems, channel state information (CSI) acquisition is unconditionally required prior to beamforming to enable sufficient link margin. Most of the previous research
works in this area focus on the fully-connected structure. For instance, a multi-stage channel sounding approach for this structure was developed by exploiting the sparse scattering nature of the mmWave channel [37]. From the compressed sensing (CS) perspective, this feedback based divide-and-conquer searching process which uses a hierarchical multi-resolution codebook leads to an adaptive equivalent measurement matrix with fewer measurements. Compared to the closed-loop beam training methods in [37]–[39], a CS-based open-loop channel estimator can decrease feedback overhead by using deterministic pilot beam patterns which are designed through minimizing the total coherence of the equivalent measurement matrix [40]. Following the design criteria of multi-resolution codebooks in [37], subarray based coordinated beam training with time-delay phase shifters is proposed for sub-Terahertz (sub-THz) communication systems with an array-of-subarrays architecture [41]. Due to the high complexity of jointly optimizing the RF beam directions across multiple subarrays, two complementary approaches are developed to obtain a small set of dominant candidate directions in [32].

The demand of energy-efficient subarray architectures has motivated many researchers to investigate high-efficiency hybrid precoding schemes. For example, assisted by the mechanism of successive interference cancellation (SIC) in multi-user detection, the hybrid precoder design can be simplified by decomposing the total achievable rate optimization problem, with nonconvex constant amplitude constraints of phase shifters, into a series of subrate optimization problems each of which handles one subarray [26]. Due to the block diagonal structure of RF precoders in partially-connected hybrid MIMO systems, the RF and digital precoders can be optimized based on the principle of alternating minimization [42]. During each iteration, a semidefinite relaxation (SDR) problem is formed to obtain the optimal digital precoder while the optimal RF precoder has a closed-form expression. In order to incorporate the merits of both the fully-connected and partially-connected structures, a novel multi-subarray structure, where each subarray consists of multiple RF chains and each RF chain connects to all the antennas corresponding to the subarray, is proposed to provide high spectral efficiency (SE) and energy efficiency (EE) [31].

B. Main Contributions

Most of the aforementioned results for subarray based mmWave systems are restricted to the case where all the subarrays at the TX (RX) have the same angle of departures (AoDs) (AoAs). However, this may not be valid for DPA-MIMO systems at high frequencies where the inter-subarray channel coupling should be taken into account. In this paper, we consider cooperative
multi-subarray based channel estimation and hybrid precoding for DPA-MIMO systems. The main contributions are summarized below.

- We exploit joint channel sparsity among distributed subarrays. The inter-subarray coupling channel model motivates us to take advantage of a multi-subarray coordinated channel sounding strategy which undoubtedly decreases the training overhead. Based on this strategy, we formulate the DPA-MIMO channel estimation problem as a structured single measurement vector (SMV) recovery problem in CS [43]. Furthermore, by minimizing the total coherence of the equivalent measurement matrix [44], we design non-feedback pilot beam patterns which have successful applications in fully-connected hybrid MIMO systems [40].

- We propose two customized algorithms to find the optimal sparse channel vector with equi-length structured blocks each of which has both the individual and common supports. The first one is an orthogonal matching pursuit (OMP) based greedy algorithm, which is divided into two intuitive parts, namely the common support identification and the individual support identification following it. The second one is called the joint sparse Bayesian learning (JSBL)-\(\ell_2\) algorithm which adapts the SBL framework [45] to the structured DPA-MIMO channel estimation problem by capitalizing on a dual-space transform.

- We propose a novel low-complexity SIC-based hybrid precoding scheme through subarray grouping for the array-of-subarrays architecture. For the design of the RF beamformers, the idea of SIC is used to decompose the original SE maximization problem into several subproblems each of which is only related to one group of subarrays, thereby facilitating efficient handling of the subarrays group by group.

C. Paper Organization and Notations

The rest of the paper is organized as follows. Section II introduces a joint sparse DPA-MIMO channel model and a scheme of cooperative multi-subarray beam training. Section III formulates the DPA-MIMO channel estimation problem and proposes the subarray based pilot beam pattern design. Section IV presents two channel recovery algorithms based on the structured channel sparsity. Section V specifies the SIC-based hybrid precoding design through subarray grouping, with simulation results analyzed in Section VI. Finally, Section VII concludes this paper.

Notations: bold uppercase \(A\) (bold lowercase \(a\)) denotes a matrix (a vector). We denote \([A]_{i,j}\) and \([A]_{:,j}\) as its \((i,j)\)th element and \(j\)th column, respectively. \(\text{vec} (\cdot)\) stacks the columns of a matrix into a tall vector, and \(\text{Tr} [\cdot]\) stands for the matrix trace operation. \(I_N\) and \(0_{M,N}\) denote the
Fig. 1. An architecture of the DPA-MIMO system

Fig. 2. Illustration of the joint sparse DPA-MIMO channel due to common and local scattering.

$N \times N$ dimensional identity matrix and the $M \times N$ dimensional all-zero matrix, respectively. $(\cdot)^H$, $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^{-1}$ and $(\cdot)^\dagger$ stand for the conjugate transpose, transpose, conjugate, inverse and pseudo-inverse, respectively. $\text{diag}[\mathbf{a}]$, $\text{diag}[\mathbf{A}]$ and $\text{blkdiag} [\mathbf{A}_1 , \cdots , \mathbf{A}_N]$ represent a diagonal matrix with $\mathbf{a}$ along its main diagonal, a vector constructed by the main diagonal of the matrix $\mathbf{A}$, a block diagonal matrix whose diagonal entries are given by $[\mathbf{A}_1 , \cdots , \mathbf{A}_N]$, respectively. $\otimes$ denotes the Kronecker product of two matrices. $\ell_0$, $\ell_1$ and $\ell_2$ norm of vectors are denoted by $\| \cdot \|_0$, $\| \cdot \|_1$ and $\| \cdot \|_2$, respectively. $\| \mathbf{A} \|_F$ denotes the Frobenius norm and the mixed $\ell_{1,2}$ norm is defined as $\| \mathbf{A} \|_{1,2} \triangleq \sum_i \sum_j |[\mathbf{A}]_{i,j}|^2$. $\mathcal{CN}(\mu, \mathbf{R})$ denotes the complex Gaussian distribution with mean $\mu$ and covariance matrix $\mathbf{R}$. $\mathbb{E}[\cdot]$ is the expectation operator.

II. SYSTEM MODEL

In this section, we present the joint sparse DPA-MIMO channel model and the cooperative multi-subarray beam training design in the DPA-MIMO system.

A. Joint Channel Sparsity Model

Consider a $M_t \times M_r$ DPA-MIMO system shown in Fig. 1 where a TX with $M_t$ subarrays communicates $N_s$ data streams to a RX with $M_r$ subarrays. We denote by $N_{t\text{tot}}$ ($N_{r\text{tot}}$) the total number of antennas at the TX (RX) end. Note that $N_{t\text{tot}} = M_t \times N_{t\text{sub}}$ and $N_{r\text{tot}} = M_r \times N_{r\text{sub}}$. Furthermore, each subarray is an uniform linear array (ULA) and all the subarrays are arranged
in the same axis at both TX and RX ends. \( d_e \) is antenna element spacing inside each subarray and \( d_a \) defines the inter-subarray spacing.

Compared to the rich scattering channel model often used for microwave frequencies [2], [46], mmWave channels are better characterized by a limited number of scattering clusters [5]. Thus, the \( L_{m,n} \)-path narrowband channel matrix between the \( n \)th TX subarray and the \( m \)th RX subarray is formulated as

\[
H_{m,n} = \sqrt{\frac{N_{\text{sub}} N_{\text{sub}}}{L_{m,n}}} \left( \alpha_{m,n}^{(0)} a_r(\vartheta_{m,n}^{(0)}) a_t^H(\psi_{m,n}^{(0)}) + \sum_{i=1}^{L_{m,n}-1} \alpha_{m,n}^{(i)} a_r(\vartheta_{m,n}^{(i)}) a_t^H(\psi_{m,n}^{(i)}) \right),
\]

where \( \alpha_{m,n}^{(0)} \) is the complex gain of the line-of-sight (LoS) component with \( \vartheta_{m,n}^{(0)} \) and \( \psi_{m,n}^{(0)} \) representing its spatial directions composed of an AoA and an AoD, respectively. \( \alpha_{m,n}^{(i)} \) is the complex gain of the \( i \)th non-line-of-sight (NLoS) component with \( \vartheta_{m,n}^{(i)} \) and \( \psi_{m,n}^{(i)} \) denoting its spatial directions composed of an AoA and an AoD, respectively. The path amplitudes are assumed to be Rayleigh distributed, i.e., \( \alpha_{m,n}^{(0)} \sim CN(0, \sigma_{\text{LoS}}^2) \) and \( \alpha_{m,n}^{(i)} \sim CN(0, \sigma_{\text{NLoS}}^2) \), where \( \sigma_{\text{LoS}}^2 \) and \( \sigma_{\text{NLoS}}^2 \) are the variances of the LoS and NLoS path gain, respectively. For an ULA with \( N \) antennas, the array response vector is \( a(\psi) = \sqrt{\frac{1}{N}} [e^{-j2\pi u_1}, \ldots, e^{-j2\pi u_N}] \), where \( \mathcal{I}(N) = \{ l - \frac{N-1}{2}, l = 0, \ldots, N - 1 \} \) is a symmetric set of indices centered around zero. The spatial direction is defined as \( \psi = \frac{d_e}{\lambda_c} \sin \theta \), where \( \theta \) is the physical direction and \( \lambda_c \) is the carrier wavelength. We use \( a_t(\cdot) \) and \( a_r(\cdot) \) to denote the array response vectors for the TX and RX subarrays, respectively, and define the entire channel matrix between all TX and RX subarrays using \( H \).

The highly directional propagation and high dimensionality of mmWave MIMO channels make beam-domain representation a natural choice [2]. The physical spatial domain and the beam domain are related through a spatial discrete Fourier transform (DFT) matrix [47], which contains the array steering vectors of uniformly spaced orthogonal spatial directions covering the entire space as \( U = [a(\bar{\psi}_1), \ldots, a(\bar{\psi}_N)] \), where \( \bar{\psi}_i = \frac{2}{N} \left( i - \frac{N+1}{2} \right) \) for \( i = 1, \ldots, N \). We use \( U_t \) (\( U_r \)) to denote the spatial DFT matrix for each TX (RX) subarray. Thus, the beam-domain channel matrix between the \( n \)th TX subarray and the \( m \)th RX subarray can be represented as \( H_{m,n} = U_r G_{m,n} U_t^H \). Subsequently, we can express the relationship between the entire spatial channel and the entire beam-domain channel for the DPA-MIMO system as \( H = A_r G A_t^H \),

\(^1\)In order to avoid causing too serious grating lobes and significant channel capacity degradation in the DPA-MIMO system, the antenna spacing and inter-subarray spacing usually satisfies \( d_e = \frac{\lambda_c}{2} \) and \( d_a \geq \frac{3}{2} \lambda_c \) respectively [15], [46].
where $A_t = I_{M_t} \otimes U_t$ and $A_r = I_{M_r} \otimes U_r$ constitute the beam-domain transform matrix for the TX and the RX, respectively, and $G$ is the entire beam-domain channel matrix.

The earlier sub-6 GHz experimental result has shown that the angular power spectrum and power variation across a large-scale array differ in the measured channels at 2.6 GHz and it makes some antennas contribute more than others [48]. When moving to higher frequencies, shorter wavelength enables accommodating a larger number of antenna elements into a small area, thereby making antenna elements within each subarray highly correlated. However, with the spacing between adjacent subarrays much larger than the wavelength, independent scatterers appear over different subarrays, which is illustrated for indoor THz Communication [49]. On the other hand, as constrained by hardware dimension and power consumption, a reasonable distance between adjacent subarrays should be set, which leads to partial coupling characterized sparse channels at both TX and RX ends [15]. Similar to the joint channel sparsity structure in the multi-user massive MIMO system [50], the channel coupling phenomena among subarrays at both sides, can be observed from both the distributed and individual joint sparse structure in the beam domain. Based on the above discussion, we have the following assumption on the beam-domain channel matrices in the DPA-MIMO system.

**Definition 1 (Joint Sparse DPA-MIMO Channel):** The channel matrices $\{G_{m,n} : \forall m, n\}$ have the following properties:

- **Individual joint sparsity due to local scattering at each subarray:** Denote $\text{supp} \{A\}$ as the index set of non-zero entries of the matrix $A$. Then, $\{G_{m,n} : \forall m, n\}$ are simultaneously sparse, i.e., the index sets $\Omega_{m,n}$ ($0 < |\Omega_{m,n}| \ll N_{\text{sub}}^t N_{\text{sub}}^r$, $\forall m, n$) which satisfies
  \[
  \Omega_{m,n} \triangleq \text{supp} \{G_{m,n}\}. 
  \]  

- **Distributed joint sparsity due to the LoS path and common scattering at each subarray:**
  Different $\{G_{m,n} : \forall m, n\}$ share a common support, i.e., the index set $\Omega_c$ which satisfies
  \[
  \Omega_c \triangleq \bigcap_{m=1}^{M_t} \bigcap_{n=1}^{M_r} \Omega_{m,n}. 
  \]

From Definition[1] it is observed that the DPA-MIMO channel sparsity support is parameterized by $\{\Omega_c, \{\Omega_{m,n} : \forall m, n\}\}$, where $\Omega_{m,n}$ and $\Omega_c$ determine the individual sparsity support and the

$^2$ $d_a$ has relationship with joint sparsity and it is worth pointing out that this relationship needs to be validated through extensive measurement and study of mmWave channels.
shared common sparsity support, respectively. When \( \Omega_{m,n} = \Omega_c \) for \( \forall m, n \), it is simplified to the most common case where all subarrays at the TX (RX) have the same AoDs (AoAs) \([26],[29],[32]\). Moreover, when \( \Omega_{m,n} = \emptyset \) for \( \forall m, n \), this transforms to another case where independent scatterers are present for each subarray \([49]\). The illustration of the proposed DPA-MIMO channel is given in Fig. 2. Assume a \( 2 \times 2 \) DPA-MIMO system with \( N_{\text{sub}}^t = N_{\text{sub}}^r = 4 \), and there are two common paths among all subarrays and one individual path within each transceiver subarray pair. By applying the vectorized operation to each beam-domain channel matrix \( G_{m,n} \) and putting the obtained channel vectors together, we can formulate a new structured sparse matrix which is row-sparse plus element-sparse as shown in the right part of this figure. This observed feature will be taken advantage of to develop channel estimators in Section IV.

B. Cooperative Multi-Subarray Beam Training

For the noncooperative training process, each TX subarray sends pilot beams in a time division manner to avoid inter-subarray interference. Furthermore, without any prior channel information, each TX subarray usually needs to send \( N_{\text{sub}}^t N_{\text{sub}}^r \) training beams defined in DFT based RF codebooks, which leads to a total probing overhead as \( N_{\text{tot}}^t N_{\text{tot}}^r \) \([32],[38]\). In order to decrease the probing overhead, we adopt a cooperative multi-subarray beam training scheme. For CSI acquisition, the TX uses \( N_{\text{beam}}^t \) (\( N_{\text{beam}}^t \leq N_{\text{tot}}^t \)) pilot beam patterns denoted as \( \{f_p \in \mathbb{C}^{N_{\text{tot}}^t \times 1} : \|f_p\|_2^2 = 1, p = 1, \cdots, N_{\text{beam}}^t \} \), and the RX adopts \( N_{\text{beam}}^r \) (\( N_{\text{beam}}^r \leq N_{\text{tot}}^r \)) pilot beam patterns denoted as \( \{w_q \in \mathbb{C}^{N_{\text{tot}}^r \times 1} : \|w_q\|_2^2 = 1, q = 1, \cdots, N_{\text{beam}}^r \} \).

During the training period, the TX successively sends its training beam patterns \( \{f_p : \forall p\} \) which are received by the RX through its beam patterns \( \{w_q : \forall q\} \). Since the RX has \( M_r \) subarrays enabling \( M_r \) pilot beam patterns simultaneously (assume that \( N_{\text{beam}}^r \) is multiples of \( M_r \)), the \( \bar{q} \)th received vector for the \( p \)th TX beam pattern is given by

\[
y_{\bar{q},p} = W_{\bar{q}}^H H_{\bar{q}}^{\text{TX}} x_p + W_{\bar{q}}^H z_{\bar{q},p},
\]

where \( y_{\bar{q},p} \in \mathbb{C}^{M_r \times 1} \) for \( \bar{q} \in \left\{ 1, \cdots, \frac{N_{\text{beam}}^r}{M_r} \right\} \), \( x_p \) is the transmitted pilot symbol, \( W_{\bar{q}} = \begin{bmatrix} w_{(\bar{q}-1)M_r+1} & \cdots & w_{\bar{q}M_r} \end{bmatrix} \in \mathbb{C}^{N_{\text{tot}}^r \times M_r} \) and \( z_{\bar{q},p} \in \mathbb{C}^{N_{\text{tot}}^r \times 1} \) is a noise vector with \( \mathcal{CN}(0, \sigma_z^2 \mathbf{I}_{N_{\text{tot}}^r}) \).

\(^3\)The hierarchical codebook based scheme \([37],[39],[41]\) only requires \( SL^2 \left[ SL/M_r \right] \log_S \left( N_{\text{tot}}^r / L \right) \) TX training beams, where \( L_{m,n} = L \) for \( \forall m, n \), \( N_{\text{tot}}^t = N_{\text{tot}}^r = N_{\text{tot}} \) and \( S \) is a design parameter that is usually set to be 2. However, this low overhead scheme is not suitable for the partial coupling DPA-MIMO channel characterized in Definition 1.
Collecting \( y_{\bar{q},p} \) for \( \bar{q} \in \{1, \ldots, N_{\text{beam}}\} \), we have \( y_p \in \mathbb{C}^{N_{\text{beam}} \times 1} \) given by

\[
y_p = W^H H f_p x_p + \text{blkdiag} \left[ W_1^H, \cdots, W_{N_{\text{beam}}/M_r}^H \right] z_p,
\]

where \( W = \left[ W_1, \cdots, W_{N_{\text{beam}}/M_r} \right] \in \mathbb{C}^{N_{\text{beam}} \times N_{\text{beam}}} \) and \( z_p = \left[ z_{1,p}^T, \cdots, z_{N_{\text{beam}}/M_r,p}^T \right]^T \in \mathbb{C}^{N_{\text{beam}} \times 1} \).

To represent the received signals for all TX beam patterns, we collect \( y_p \) for \( p \in \{1, \ldots, N_t\} \) yielding

\[
Y = W^H H F X_p + Z,
\]

where \( Y = \left[ y_1, \cdots, y_{N_t} \right] \in \mathbb{C}^{N_{\text{beam}} \times N_{\text{beam}}} \) and \( F = \left[ f_1, \cdots, f_{N_{\text{beam}}} \right] \in \mathbb{C}^{N_{\text{total}} \times N_{\text{beam}}} \) is the complete TX processing matrix and the noise matrix \( Z \in \mathbb{C}^{N_{\text{beam}} \times N_{\text{beam}}} \) is given by

\[
Z = \text{diag} \left[ W_1^H, \cdots, W_{N_{\text{beam}}/M_r}^H \right] \cdot \left[ z_1, \cdots, z_{N_{\text{beam}}} \right].
\]

The pilot matrix \( X_p \in \mathbb{C}^{N_{\text{beam}} \times N_{\text{beam}}} \) is diagonal with \( \{x_p\}_{p=1}^{N_{\text{beam}}} \) on its principal diagonal. In general, we choose \( X_p = \sqrt{P_p} I_{N_{\text{beam}}} \) where \( P_p \) is the pilot power per transmission.

In our array-of-subarrays architecture, the TX and RX processing matrices are decomposed as \( F = F_{\text{RF}} F_{\text{BB}} \) and \( W = W_{\text{RF}} W_{\text{BB}} \). Thus, (6) can be written as

\[
Y = \sqrt{P_p} W_{\text{BB}}^H W_{\text{RF}}^H H F_{\text{RF}} F_{\text{BB}} + Z,
\]

where \( F_{\text{RF}} \in \mathbb{C}^{N_{\text{RF}} \times N_{\text{RF}}} \) and \( W_{\text{RF}} \in \mathbb{C}^{N_{\text{RF}} \times N_{\text{RF}}} \) denote RF beamforming matrices at the TX and the RX, respectively, while \( F_{\text{BB}} \in \mathbb{C}^{N_{\text{BB}} \times N_{\text{BB}}} \) and \( W_{\text{BB}} \in \mathbb{C}^{N_{\text{BB}} \times N_{\text{BB}}} \) denote the TX and RX baseband processing matrices, respectively. Since there are \( M_t \) (\( M_r \)) subarrays at the TX (RX), \( F_{\text{RF}} \) and \( W_{\text{RF}} \) can be partitioned into \( N_{\text{RF}}^{\text{block}} = N_{\text{RF}}/M_t = N_{\text{sub}} \) and \( N_{\text{RF}}^{\text{block}} = N_{\text{RF}}/M_r = N_{\text{sub}} \) sub-RF beams \( [40] \), i.e., \( F_{\text{RF}} = \left[ F_{\text{RF},1}, \cdots, F_{\text{RF},N_{\text{RF}}^{\text{block}}} \right] \) and \( W_{\text{RF}} = \left[ W_{\text{RF},1}, \cdots, W_{\text{RF},N_{\text{RF}}^{\text{block}}} \right] \) where \( F_{\text{RF},\bar{p}} \in \mathbb{C}^{N_{\text{RF}} \times M_t}, W_{\text{RF},\bar{p}} \in \mathbb{C}^{N_{\text{RF}} \times M_t}, \bar{p} \in \{1, \cdots, N_{\text{RF}}^{\text{block}}\} \) and \( \bar{q} \in \{1, \cdots, N_{\text{RF}}^{\text{block}}\} \). Similarly, \( F_{\text{BB}} \) and \( W_{\text{BB}} \) are block diagonal matrices given by \( F_{\text{BB}} = \text{diag} \left[ F_{\text{BB},1}, \cdots, F_{\text{BB},N_{\text{BB}}^{\text{block}}} \right] \) and \( W_{\text{BB}} = \text{diag} \left[ W_{\text{BB},1}, \cdots, W_{\text{BB},N_{\text{BB}}^{\text{block}}} \right] \) where \( F_{\text{BB},\bar{p}} \in \mathbb{C}^{N_{\text{BB}} \times N_{\text{BB}}^{\text{block}}}, W_{\text{BB},\bar{p}} \in \mathbb{C}^{N_{\text{BB}} \times N_{\text{BB}}^{\text{block}}} \). Furthermore, each column of the RF precoder matrix is zero except for a continuous block of non-zero entries (consisting of the beamforming weights used on the corresponding subarray), i.e., \( F_{\text{RF},\bar{p}} = \text{blkdiag} \left[ f_{\text{RF},\bar{p},1}, \cdots, f_{\text{RF},\bar{p},M_t} \right] \) and \( W_{\text{RF},\bar{p}} = \text{blkdiag} \left[ w_{\text{RF},\bar{p},1}, \cdots, w_{\text{RF},\bar{p},M_r} \right] \), where \( f_{\text{RF},\bar{p},n} \in \mathbb{C}^{N_{\text{sub}} \times 1} \) and \( w_{\text{RF},\bar{p},n} \in \mathbb{C}^{N_{\text{RF}} \times 1} \) denote the RF precoder and combiner for the \( n \)-th TX subarray and the \( m \)-th RX subarray, respectively.
III. FORMULATION OF DPA-MIMO CHANNEL ESTIMATION PROBLEM AND PILOT BEAM PATTERN DESIGN

In this section, we first exploit the joint sparse nature of the DPA-MIMO channel, and formulate the mmWave channel estimation problem as a structured sparse vector recovery problem. Then, we propose a deterministic beam training scheme.

A. Sparse Formulation of DPA-MIMO Channel Estimation Problem

To exploit the sparse nature of the mmWave channel, it is necessary to vectorize the received signal matrix \( \mathbf{Y} \) in (8). By denoting \( \text{vec} \left( \mathbf{Y} \right) \) by \( \mathbf{y} \in \mathbb{C}^{N_t^\text{beam} N_r^\text{beam} \times 1} \), we have

\[
\mathbf{y} = \sqrt{P_r} \left( (\mathbf{F}_{\text{BB}}^T \mathbf{F}_r^T) \otimes (\mathbf{W}_{\text{BB}}^H \mathbf{W}_{r}^H) \right) \text{vec} \left( \mathbf{H} \right) + \mathbf{z} = \mathbf{Q} \cdot \text{vec} \left( \mathbf{G} \right) + \mathbf{z}, \tag{9}
\]

where (a) follows from the equivalent noise vector \( \mathbf{z} \in \mathbb{C}^{N_t^\text{beam} N_r^\text{beam} \times 1} \) as

\[
\mathbf{z} = \left( \mathbf{I}_{N_t^\text{beam}} \otimes \text{blkdiag} \left[ \mathbf{W}_1^H, \ldots, \mathbf{W}_{N_t^\text{beam}/M_r}^H \right] \right) \left[ \mathbf{z}_1^T, \ldots, \mathbf{z}_{N_t^\text{beam}}^T \right]^T \tag{10}
\]

and the properties of Kronecker product, \( \text{vec} \left( \mathbf{ABC} \right) = (\mathbf{C}^T \otimes \mathbf{A}) \text{vec} \left( \mathbf{B} \right) \) and \( (\mathbf{A} \otimes \mathbf{B})^T = \mathbf{A}^T \otimes \mathbf{B}^T \) [51]. (b) follows from \( \text{vec} \left( \mathbf{H} \right) = (\mathbf{A}_r^T \otimes \mathbf{A}_r) \text{vec} \left( \mathbf{G} \right) \) and \( (\mathbf{A} \otimes \mathbf{B}) (\mathbf{C} \otimes \mathbf{D}) = (\mathbf{AC}) \otimes (\mathbf{BD}) \). The matrix \( \mathbf{Q} \in \mathbb{C}^{N_t^\text{beam} N_r^\text{beam} \times N_t^\text{tot} N_r^\text{tot}} \) can be written as

\[
\mathbf{Q} = (\mathbf{F}_{\text{BB}}^T \mathbf{F}_r^T \mathbf{A}_r^T) \otimes (\mathbf{W}_{\text{BB}}^H \mathbf{W}_r^H \mathbf{A}_r). \tag{11}
\]

The formulation of the vectorized received signal in (9) represents a sparse formulation of the channel estimation problem as \( \text{vec} \left( \mathbf{G} \right) \) has only \( N_0 = |\Omega_c| + \sum_{m=1}^{M_r} \sum_{n=1}^{M_t} (|\Omega_{m,n}| - |\Omega_c|) \) non-zero elements and \( N_0 \ll N_t^\text{tot} N_r^\text{tot} \). This implies that the number of required measurements \( N_t^\text{beam} N_r^\text{beam} \) to detect the non-zero elements can be much less than \( N_t^\text{tot} N_r^\text{tot} \). Given this formulation in (9), CS tools can be leveraged to design estimation algorithms to determine the non-zero elements in the beam-domain channel matrix. Our goal is to exploit the hidden joint sparsity in the beam-domain channel to reduce the required training and improve the performance of channel estimation. For making better use of the structured sparsity, we exchange the order of elements in \( \text{vec} \left( \mathbf{G} \right) \) to get a new vector with \( M_t M_r \) equi-length blocks as

\[
\mathbf{x} = \left[ \mathbf{x}_1^T, \ldots, \mathbf{x}_{M_t M_r}^T \right]^T
\]

\[
= \begin{bmatrix}
\text{vec}^T \left( \mathbf{G}_{1,1} \right), & \ldots, & \text{vec}^T \left( \mathbf{G}_{M_t,1} \right), & \ldots, & \text{vec}^T \left( \mathbf{G}_{1,M_r} \right), & \ldots, & \text{vec}^T \left( \mathbf{G}_{M_t (M_r - 1) + 1}, M_r \right), & \ldots, & \text{vec}^T \left( \mathbf{G}_{M_t M_r, M_r} \right)
\end{bmatrix}^T, \tag{12}
\]
where the block size is $N_t^{\text{sub}} N_r^{\text{sub}}$. This manipulation is also shown in Fig. 2. In this case, the corresponding equivalent measurement matrix $\Phi$ is obtained by exchanging the column order of $Q$, such that $\Phi x = Q \cdot \text{vec}(G)$. As a result, the problem of DPA-MIMO channel recovery at the RX can be formulated as Problem $\mathcal{P}_1$:

$$
\min_x \| y - \Phi x \|_2^2
$$

s.t. $x$ complies with the joint sparsity model as in Definition 1

(13)

However, Problem $\mathcal{P}_1$ is very challenging due to the individual and distributed joint sparsity requirement in the constraint which is quite different from the conventional CS-recovery problem with a simple sparsity ($\ell_0$-norm) constraint. We will propose customized algorithms for solving this structured sparsity recovery problem in the next section. In addition, the equivalent measurement matrix has to be carefully designed to guarantee the recovery of the non-zero elements of the vector with high probability by using a small number of measurements.

B. Open-Loop Pilot Beam Pattern Design

Instead of randomized sensing matrices frequently used for CS-based channel estimation [52], a deterministic measurement matrix designed by minimizing its total coherence can improve the recovery performance [44]. This strategy has been recently applied to pilot beam pattern design for the fully-connected structure [40]. Due to the excellent performance improvement and zero feedback overhead of total coherence based pilot beam patterns, we apply this strategy to channel sounding in the DPA-MIMO system.

The total coherence of any matrix is defined by $\mu^\text{tot} (\Phi) = \sum_{k=1}^{N_t^\text{tot}} \sum_{l \neq k} \left( [\Phi]_{:,k} [\Phi]_{:,l} \right)^2$, where $\Phi$ is the equivalent measurement matrix in (13). Then, we have

$$
\mu^\text{tot} (\Phi) \overset{(a)}{=} \mu^\text{tot} (Q) \overset{(b)}{\leq} \mu^\text{tot} (F_{BB}^T F_{RF}^T A_t^*) \cdot \mu^\text{tot} (W_{BB}^H W_{RF}^H A_r),
$$

(14)

where $(a)$ follows from the definition of the total coherence and $(b)$ follows from [40, Lemma 7]. Therefore, we can decompose the design problem of minimizing $\mu^\text{tot} (\Phi)$ into two separate designs, namely the design of $F_{BB}$ and $F_{RF}$ via minimizing $\mu^\text{tot} (F_{BB}^T F_{RF}^T A_t^*)$ and the design of $W_{BB}$ and $W_{RF}$ via minimizing $\mu^\text{tot} (W_{BB}^H W_{RF}^H A_r)$. Next, we focus on the $F_{BB}$ and $F_{RF}$ by solving the following problem

$$
\min_{F_{RF}, F_{BB}} \mu^\text{tot} (F_{BB}^T F_{RF}^T A_t^*)
$$

s.t. $\| f_p \|_2 = 1, p = 1, \cdots, N_t^\text{beam}$.

(15)
Via limiting the RF precoder to the unitary matrix and using [40, Theorem 2], the optimal baseband precoder of the $\bar{p}$th block is given by

$$F^*_{BB,\bar{p}} = \bar{U}_t \begin{bmatrix} I_{N_t^{\text{beam}}} & 0_{N_t^{\text{beam}} 	imes M_t - N_t^{\text{beam}}} \\ \end{bmatrix}^{T} \bar{V}_t^H,$$

(16)

where $\bar{U}_t \in \mathbb{C}^{M_t \times M_t}$ and $\bar{V}_t \in \mathbb{C}^{N_r^{\text{block}} \times N_r^{\text{beam}}}$. In order to make RF pilot beams cover a full range of AoDs, we choose the unitary DFT matrix as the solution

$$\left( f^{[i]}_{RF,1}, \cdots, f^{[i]}_{RF,N_t^{\text{block}}} \right) = \text{circshift} \left( F_{N_t^{\text{sub}}, i - 1} \right),$$

(17)

where $F_N$ denotes the $N$-dimensional unitary DFT matrix and the symbol $\text{circshift} (A, i)$ represents moving the columns of a matrix $A$ to the right for $(i)$ columns in a circular manner. In this way, every TX subarray simultaneously probes different spatial directions using RF beams.

Similar operation can be applied to the RX, leading to the optimal baseband combiners and the optimal RF combiners as

$$W^*_{BB,\bar{q}} = \bar{U}_r \begin{bmatrix} I_{N_r^{\text{beam}}} & 0_{N_r^{\text{beam}} 	imes M_r - N_r^{\text{beam}}} \\ \end{bmatrix}^{T} \bar{V}_r^H,$$

(18)

$$\left[ \left( w^{[i]}_{RF,1} \right)^*, \cdots, \left( w^{[i]}_{RF,N_r^{\text{block}}} \right)^* \right] = \text{circshift} \left( F_{N_r^{\text{sub}}, i - 1} \right),$$

(19)

where $\bar{U}_r \in \mathbb{C}^{M_r \times M_r}$ and $\bar{V}_r \in \mathbb{C}^{N_t^{\text{block}} \times N_t^{\text{beam}}}$. Moreover, the noise vector after the designed RF and baseband processing remains i.i.d. Gaussian with $CN \left( 0, \sigma^2_z I_{N_t^{\text{beam}}N_r^{\text{beam}}} \right)$ without prewhitening, which provides much convenience for the further design of CSI recovery algorithms in the next section.

### IV. DPA-MIMO Channel Estimation Algorithms

To solve Problem $P_1$ in this section, we present two customized algorithms, i.e., an OMP based greedy algorithm with low complexity and a SBL inspired algorithm with excellent accuracy. For notational simplicity, we define $N \triangleq N_t^{\text{beam}}N_r^{\text{beam}}, B \triangleq N_t^{\text{sub}}N_r^{\text{sub}}$ and $K \triangleq M_tM_r$, respectively.

#### A. Proposed JOMP Algorithm

In the existing literature, classical CS-based algorithms, e.g., basis pursuit (BP) [53], OMP [40], [52], [54] and SBL [55], are often used to recover the sparse signal vector without structured sparsity for their easy implementation and fairly good recovery performance. In later
Algorithm 1 JOMP Algorithm

Input: $\Phi$, $y$, $T_1$, $T_2$, $\delta_1$, $\delta_2$

- **Part 1** (*Common Support Identification*): Initialize $\Omega^c_e = \emptyset$, $\Omega^a = \{1, \cdots, BK\}$, $r_1 = y$, and $\Omega^K_b = \{b, B + b, \cdots, (K - 1) B + b\}$ for $1 \leq b \leq B$.

  while $t_1 \leq T_1$ or $\|r_1\|_2^2 > \delta_1$ do
  1. (Support Estimate): $b^* = \arg \max_{1 \leq b \leq B} \left\| \left( \Phi_{\Omega^K_b} \right)^H r_1 \right\|_2^2$.
  2. (Support Update): $\Omega^K_b = \Omega^K_b \cup \{b^*\}$.
  3. (Residual Update): $r_1 = y - \Phi_{\Omega^K_b} \left( \Phi_{\Omega^K_b} \right)^\dagger y$.
  4. (Iteration Update): $t_1 = t_1 + 1$.
  end while

- **Part 2** (*Individual Support Identification*): Set $\Omega^r_i = \Omega^c_e$, $r_2 = r_1$ and $\Omega^r = \Omega^a \setminus \Omega^c_e$.

  while $t_2 \leq T_2$ or $\|r_2\|_2^2 > \delta_2$ do
  5. (Support Estimate): $j^* = \arg \max_{j \in \Omega^r} \left| \left[ \Phi_{\Omega^r_i} \right]^H r_2 \right|^2$.
  6. (Support Update): $\Omega^r_i = \Omega^r_i \cup \{j^*\}$.
  7. (Residual Update): $r_2 = y - \Phi_{\Omega^r_i} \left( \Phi_{\Omega^r_i} \right)^\dagger y$.
  8. (Iteration Update): $t_2 = t_2 + 1$.
  end while

Output: $\hat{x}_{\Omega^r_i} = \Phi_{\Omega^r_i} \left( \Phi_{\Omega^r_i} \right)^\dagger y$ and $\hat{x}_{\Omega^a \setminus \Omega^c_e} = 0$.

Signal processing applications, the realistic structured sparse features lurking behind the signal coefficients have been gradually exploited and received considerable attention [43], [56]. For instance, a simultaneous OMP (SOMP) algorithm efficiently provides good solutions to the multiple measurement vector (MMV) problems [57]. From an empirical Bayesian perspective, a multiple response extension of the standard SBL framework (M-SBL) algorithm is also proposed to solve the simultaneous sparse recovery problems. The most related work in [50] provides a novel distributed compressive CSI estimation scheme, which aims at performing CSI recovery at the BS for multiple users by exploiting the hidden joint sparsity lying behind their beam-domain channel matrices. However, none of the previous works cover the case of the beam-domain channel vector in (12) and hence, the known associated algorithms are not suitable to solve our SMV problem with the special structure defined in Definition [1]. In this subsection, we develop an innovative joint OMP algorithm (JOMP) tailored to the block vector in (12) to solve Problem
More specifically, this algorithm is designed by adapting OMP [54] to Problem $\mathcal{P}_1$.

The details of the proposed JOMP algorithm are described in Algorithm [1]. For the input parameters, $\delta_1$ and $\delta_2$ are the predetermined thresholds; $T_1$ ($T_1 \leq |\Omega_c|$) and $T_2$ ($T_2 \geq \sum_{m=1}^{M_c} \sum_{n=1}^{M_c} (|\Omega_{m,n}| - |\Omega_c|)$) are the maximal number of iterations to guarantee the convergence. This algorithm is divided into two parts, where the first part aims at common support identification, and the second part continues the individual support identification. Note that the estimation target $x$ in [12] has non-zero elements at the same positions of each block. Therefore, motivated by the simultaneous sparse approximation algorithm proposed for MMV problems in [57], we wish to find a group of equi-spaced atoms in the equivalent measurement matrix $\Phi$ by maximizing the sum of their absolute correlations with the residual $r_1$. This procedure is done in step 1 and the absolute sum has the equivalent expression $\sum_{k=1}^{K} \left\| \Phi_{[:b+(k-1)b]} r_1 \right\|_2^2 = \left\| \left( \Phi_{\Omega_c^e} \right)^H r_1 \right\|_2^2$. After the common support $\Omega_c^e$ is detected, the standard OMP method in [54] is used to identify the individual support $\Omega_c^e$ as realized from step 5 to 8. Finally, depending on the estimated support index, the least square (LS) method is used to recover the channel vector.

B. Proposed JSBL-$\ell_2$ Algorithm

1) Introduction to SBL: Consider the classical sparse recovery model without the structured sparsity $y = \Phi x + z$, where $z$ is a noise vector with $CN(0, \lambda I_N)$ and $\lambda$ is the known noise variance, i.e., $\lambda = \sigma_z^2$. Thus we have the Gaussian likelihood model $p(y|x) = CN(\Phi x, \lambda I_N)$.

Assume the Gaussian prior $p(x) = N(0, \Gamma)$, where $\Gamma = \text{diag}[\gamma]$ with a vector of hyperparameters $\gamma$ governing the prior variances of the elements in $x$. For a fixed $\gamma$, using the Bayesian rules we can obtain the Gaussian posterior density of $x$ as $p(x|y) = CN(\mu_x, \Sigma_x)$, where

$$\mu_x = \Gamma \Phi^H \left( \lambda I_N + \Phi \Gamma \Phi^H \right)^{-1} y,$$

$$\Sigma_x = \Gamma - \Gamma \Phi^H \left( \lambda I_N + \Phi \Gamma \Phi^H \right)^{-1} \Phi \Gamma. \tag{20b}$$

The next key task is to estimate the latent variables $\gamma$. By treating $x$ as hidden variables and integrating them out [55], we obtain the maximum a posterior (MAP) estimate on $\gamma$ as

$$\gamma^{(II)} = \arg \max_{\gamma \geq 0} \int p(y|x) p(x; \gamma) dx = \arg \min_{\gamma \geq 0} y^H \Sigma_y^{-1} y + \ln |\Sigma_y|,$$ \tag{21}

where the covariance matrix of $y$ denotes $\Sigma_y = \lambda I_N + \Phi \Gamma \Phi^H$. Once $\gamma^{(II)}$ is obtained, a commonly accepted point estimate for $x$ naturally emerges as

$$x^{(II)} = \mathbb{E}[x|y; \gamma^{(II)}] = \Gamma^{(II)} \Phi^H \left( \lambda I + \Phi \Gamma^{(II)} \Phi^H \right)^{-1} y. \tag{22}$$
This procedure is referred to Type II estimation, also called empirical Bayesian. From (22), it can be observed that a sparse $\gamma_{(II)}$ leads to a corresponding sparse estimate $x_{(II)}$. Note that the logarithm term $\ln |\Sigma_y|$ in (21) is a concave function with respect to $\gamma$ according to [55, Lemma 1], thereby favoring a sparse $\gamma$, which further results in a sparse $x$ through (22).

2) SBL-Inspired Cost Function: By reshaping the vector $x$, we define a new matrix $X \triangleq [x_1, \ldots, x_K] \in \mathbb{C}^{B \times K}$ which is both row-sparse and element-sparse, as shown in Fig. 2. In order to promote such a structure, $X$ can be viewed as the summation of an element-sparse matrix $S \triangleq [s_1, \ldots, s_K] \in \mathbb{C}^{B \times K}$ and a row-sparse matrix $C \triangleq [c_1, \ldots, c_K] \in \mathbb{C}^{B \times K}$ [58, 59]. Furthermore, with the use of convex approximation, Problem $P_1$ can be transformed into an unconstrained optimization problem

$$
\min_{c,s} \|y - \Phi x\|_2^2 + \beta_1 \sum_{k=1}^K \|s_k\|_1 + \beta_2 \|C\|_{1,2},
$$

where $\beta_1 \geq 0$ and $\beta_2 \geq 0$ are weights regarding element-sparsity and row-sparsity respectively.

In order to promote sparsity of the solution, we transform the cost function of (23) in $x$-space to the SBL-like cost function in $\gamma$-space by using a dual-space view [60], where the following variational representations are used [59]:

$$
\|x_k\|_1 = \min_{\gamma_{bk} \geq 0} \frac{1}{\gamma_{bk}} \sum_{b=1}^B x_{bk}^2 + \gamma_{bk},
$$

$$
\|X\|_{1,2} = \min_{\gamma_{b} \geq 0} \frac{1}{\gamma_{b}} \sum_{b=1}^B \sum_{k=1}^K x_{bk}^2 + \gamma_{bk}^c,
$$

where $x_{bk} = [X]_{b,k}$, $\gamma_{bk}^c$ and $\gamma_{bk}^e$ are scalars, $\gamma^c = [\gamma_1^c, \ldots, \gamma_B^c]^T$ is a vector common to all columns of $X$, and $\gamma^e = [\gamma_1^e, \ldots, \gamma_{B,1}^e, \ldots, \gamma_{1,K}^e, \ldots, \gamma_{B,K}^e]^T$ is a vector with each element corresponding to that of $vec(X)$. Furthermore, by using the identity derived in Appendix A

$$
y^H (\Sigma^{se})^{-1} y = \min_{c,s} \frac{1}{\lambda} \|y - \Phi (c + s)\|_2^2 + s^H (\Gamma^s)^{-1} s + c^H (I_K \otimes \Gamma^e)^{-1} c,
$$

where $s = [s_1^T, \ldots, s_K^T]^T \in \mathbb{C}^{BK \times 1}$, $c = [c_1^T, \ldots, c_K^T]^T \in \mathbb{C}^{BK \times 1}$, $\Gamma^e = \text{diag}[\gamma^e]$ and $\Gamma^s = \text{diag}[\gamma^s]$, we can further express the convex cost function of (23) in $\gamma$-space as

$$
L_{(I)}(\gamma^e, \gamma^s) = y^H \Sigma^{se} y + \beta_1^2 \text{Tr} (\Gamma^s) + \text{Tr} (I_K \otimes \Gamma^e).
$$

where $\Sigma^{se} = \lambda I_N + \Phi (\Gamma^s + I_K \otimes \Gamma^e) \Phi^H$. Comparing the data-related term $y^H \Sigma^{se} y$ in (25) and that of the SBL cost function in (21), we can observe that the common component $\gamma^e$ and the individual component $\gamma^s$ interact with each other in a manner like $\Gamma = \Gamma^s + I_K \otimes \Gamma^e$. 
Following the innovative decoupling idea in [59], we can replace the convex penalties in the existing models with the SBL counterparts to obtain some of the corresponding benefits, even without any formal probabilistic model for this derivation. Therefore, we put forth a new cost function in $\gamma$-space from (26) as

$$L_{(II)}(\gamma^c, \gamma^s) = y^H (\Sigma^{sc})^{-1} y + \beta \ln |\Sigma^s| + \ln |\Sigma^c|,$$

where $\Sigma^s = \frac{1}{2} I_N + \Phi \Gamma^s \Phi^H$ and $\Sigma^c = \frac{1}{2} I_N + \Phi (I_K \otimes \Gamma^c) \Phi^H$. Since the log-determinant function is concave and nondecreasing, the term $\ln |\Sigma^c|$ and the term $\ln |\Sigma^s|$ promote a sparse common component $\gamma^c$ and a sparse individual component $\gamma^s$, respectively. Moreover, the weight $\beta$ which is regarded as the tradeoff between row sparsity and element sparsity in the defined matrix $X$, should be tuned with training data or given with prior information.

3) $\ell_2$ Reweighting Scheme: Following an extension of the duality space analysis for the basic SBL framework [60], we can transform the cost function of (27) from $\gamma$-space to $x$-space. First, via using the identity (25) and standard determinant identities, we can upper-bound (27) by

$$L(\gamma^c, \gamma^s, c, s) = \frac{1}{\lambda} \|y - \Phi(c + s)\|^2 + \beta \ln |\Gamma^s| + K \ln |\Gamma^c| + BK(\beta + 1)\ln \left(\frac{\lambda}{2}\right)$$

$$+ \beta h_s(z^s) + h_c(z^c) + \sum_{b=1}^{B} \sum_{k=1}^{K} \frac{|s_{bk}|^2}{\gamma_{bk}} + \sum_{b=1}^{B} \sum_{k=1}^{K} \frac{|c_{bk}|^2}{\gamma_{bk}^c},$$

where we define two concave functions, namely $h_s(\gamma^s) \triangleq \ln |(\Gamma^s)^{-1} + \frac{2}{\lambda} \Phi^H \Phi|$ and $h_c(\gamma^c) \triangleq \ln |(I_K \otimes \Gamma^c)^{-1} + \frac{2}{\lambda} \Phi^H \Phi|$. Meanwhile, this upper-bound is tight if the followings are satisfied:

$$s^* = \Gamma^s \Phi^H (\Sigma^{sc})^{-1} y,$$

$$c^* = (I_K \otimes \Gamma^c) \Phi^H (\Sigma^{sc})^{-1} y.$$  (29a)

Due to the duality of concave conjugate functions, we have the following upper bounds:

$$h_s(\gamma^s) = \min_{z^s \geq 0} \sum_{b=1}^{B} \sum_{k=1}^{K} \frac{z_{bk}^s}{\gamma_{bk}^s} - h^*_s(z^s),$$  (30a)

$$h_c(\gamma^c) = \min_{z^c \geq 0} \sum_{b=1}^{B} \frac{z_{bk}^c}{\gamma_{bk}^c} - h^*_c(z^c).$$  (30b)

By using (28), (30a) and (30b), we can then perform coordinate descent optimization over the following approximation with irrelevant terms are dropped:

$$\min_{c, s, \gamma^s, \gamma^c, z^s, z^c} \|y - \Phi(c + s)\|^2 + \lambda \left[ \sum_{b=1}^{B} \sum_{k=1}^{K} \left( \frac{|s_{bk}|^2 + \beta z_{bk}^s}{\gamma_{bk}^s} + \beta \ln \gamma_{bk}^s \right) - \beta h_s(z^s) \right]$$

$$+ \sum_{b=1}^{B} \left( \frac{\sum_{k=1}^{K} |c_{bk}|^2 + z_{bk}^c}{\gamma_{bk}^c} + K \ln \gamma_{bk}^c \right) - h_c(z^c).$$  (31)
To further simplify the expression, we now calculate the optimal values of $z^c$ and $z^s$. According to the relations (30a) and (30b), we can directly obtain the optimal values as follows:

$$(z^s)^* = \nabla \left\{ (\gamma^s_{bk})^{-1} \ln \left( (\Gamma^s)^{-1} + \frac{2}{\lambda} \Phi H \Phi \right) = \text{diag} \left[ \Gamma^s - \Gamma^s \Phi H (\Sigma^s)^{-1} \Phi \Gamma^s \right] \right\},$$

$$(z^c)^* = \nabla \left\{ (\gamma^c_{bk})^{-1} \ln \left( (I_K \otimes \Gamma^c)^{-1} + \frac{2}{\lambda} \Phi H \right) \right\} = \Xi \cdot \text{diag} \left[ I_K \otimes \Gamma^c - (I_K \otimes \Gamma^c) \Phi H (\Sigma^c)^{-1} \Phi (I_K \otimes \Gamma^c) \right],$$

where the Moore-Penrose pseudo-inverse [51] is used for computation reduction and $\Xi = [I_B, \cdots, I_B] \in \mathbb{C}^{B \times BK}$. Finally, by fixing others, the optimal hyperparameters are given by

$$(\gamma^s_{bk})^* = \frac{|s_{bk}|^2 / \beta + z^s_{bk}}{K}, \quad (33a)$$

$$(\gamma^c_{bk})^* = \left( \sum_{k=1}^{K} |c_{bk}|^2 + z^c_{bk} \right) / K. \quad (33b)$$

Therefore, by alternately minimizing and repeatedly updating the upper-bound function (31), we obtain the reweighted algorithm described in Algorithm 2.

---

**Algorithm 2 JSBL-\ell_2 Algorithm**

**Input:** $\Phi$, $y$, $\lambda$, $\beta$, $T_{\text{max}}$, $\epsilon$

**while** $t \leq T_{\text{max}}$ or $\|s + c - s_{\text{old}} - c_{\text{old}}\|^2 > \epsilon$ **do**

1. $s_{\text{old}} = s$ and $c_{\text{old}} = c$.
2. Update $s^*$ and $c^*$ using (29a) and (29b).
3. Update $(z^s)^*$ and $(z^c)^*$ using (32a) and (32c).
4. Update $(\gamma^s_{bk})^*$ and $(\gamma^c_{bk})^*$ using (33a) and (33b).
5. $t = t + 1$.

**end while**

**Output:** $\hat{x} = (\Gamma^s + I_K \otimes \Gamma^c) \Phi H (\Sigma^c)^{-1} y$.

---

**V. SIC-BASED HYBRID PRECODING THROUGH SUBARRAY GROUPING**

As a result of CSI acquisition in Section IV, we now consider the channel is known at both TX and RX ends. The task of this section is to design the hybrid precoding and combining matrices for the DPA-MIMO system. The processed received signal after combining is given by

$$y_d = W_{BB}^H W_{RF}^H F_{RF} F_{BB} s_d + W_{BB}^H W_{RF}^H z_d,$$  \quad (34)
where \( s_d \in \mathbb{C}^{N_s \times 1} \) is the signal vector such that \( \mathbb{E}[s_ds_d^H] = (P_d/N_s)I_{N_s} \), \( P_d \) is the average transmit power, \( z_d \) is a Gaussian noise vector with \( \mathcal{CN}(0, \sigma_z^2 I_{N_{tot}}) \), \( \mathbf{F}_{RF} = \text{blkdiag} \left[ \mathbf{f}^{[1]}_{RF}, \ldots, \mathbf{f}^{[M_f]}_{RF} \right] \) and \( \mathbf{W}_{RF} = \text{blkdiag} \left[ \mathbf{w}^{[1]}_{RF}, \ldots, \mathbf{w}^{[M_r]}_{RF} \right] \). With Gaussian signaling employed at the TX, the instantaneous achievable SE is

\[
R = \log_2 \left| \mathbf{I}_{N_{tot}} + P_d (N_s \sigma_z^2)^{-1} \mathbf{W}_t \left( \mathbf{W}_t^H \mathbf{W}_t \right)^{-1} \mathbf{W}_t^H \mathbf{H} \mathbf{F}_{RF} \mathbf{F}_{BB}^H \right|,
\]

(35)

where \( \mathbf{F}_t = \mathbf{F}_{RF} \mathbf{F}_{BB} \) and \( \mathbf{W}_t = \mathbf{W}_{RF} \mathbf{W}_{BB} \). Thus we can find the optimal hybrid precoders at the TX and the optimal hybrid combiners at the RX by solving Problem \( \mathcal{P}_2 \):

\[
\max_{\mathbf{F}_{RF}, \mathbf{F}_{BB}, \mathbf{W}_{RF}, \mathbf{W}_{BB}} R \quad \text{(36a)}
\]

\[
\text{s.t.} \quad \| \mathbf{F}_{RF} \mathbf{F}_{BB} \|^2_F = N_s \quad \text{(36b)}
\]

\[
\left[ \mathbf{F}_{RF} \right]_{i,j} = \left( N_{\text{sub}}^t \right)^{-1/2}, \quad \forall (i,j) \in \mathcal{F}_t \quad \text{(36c)}
\]

\[
\left[ \mathbf{W}_{RF} \right]_{i,j} = \left( N_{\text{sub}}^r \right)^{-1/2}, \quad \forall (i,j) \in \mathcal{W}_r, \quad \text{(36d)}
\]

where \( \mathcal{F}_t (\mathcal{F}_r) \) is the set of non-zero elements of the RF precoder (combiner).

In addition to the non-convex constraints on the elements of the RF beamformers, Problem \( \mathcal{P}_2 \) also involves joint optimization of the hybrid precoders and combiners, which makes it impractical to obtain the exact optimal solutions [61]. To simplify the transceiver design, we decouple the joint TX-RX optimization problem into two separate subproblems each of which deals with the TX design and the RX design, respectively [62]. Specifically, we first design the hybrid precoders by assuming the optimal RX, and then design the hybrid combiners with the already obtained TX. In lieu of maximizing SE, we design \( \mathbf{F}_{RF} \) and \( \mathbf{F}_{BB} \) to maximize the mutual information achieved by Gaussian signaling. Then, the hybrid precoder design problem can be written as

\[
\max_{\mathbf{F}_{RF}, \mathbf{F}_{BB}} \log_2 \left| \mathbf{I}_{N_{tot}} + P_d (N_s \sigma_z^2)^{-1} \mathbf{H} \mathbf{F}_{RF} \mathbf{F}_{BB} \mathbf{F}_{RF}^H \mathbf{F}_{BB}^H \right| \quad \text{(37a)}
\]

\[
\text{s.t.} \quad \text{(36b) and (36c)} \quad \text{(37b)}
\]

In fact, this problem can be solved by using alternating optimization methods [63]. However, at each iteration, an SDR problem should be solved by conventional optimization tools, such as CVX [42]. In the following part, we present a novel low-complexity hybrid precoding method for the array-of-subarrays architecture.
A. Hybrid Precoder Design

1) Digital Precoder Design: Since \( N_s \leq M_t \), we first group the subarrays at the TX according to the number of data streams \( N_s \). We simply assign successive equal number of subarrays to each data stream if \( M_t \) is multiples of \( N_s \), otherwise, redundant subarrays are all assigned to any data stream.\(^4\) We use the vector \( \mathbf{d}_t = [d_1, \cdots, d_{N_s}] \) to indicate the number of subarrays assigned to each data stream. Thus, we have the precoders with the new structures as

\[
\bar{\mathbf{F}}_{RF} = \text{blkdiag} \left[ \bar{\mathbf{f}}_{RF}^{[1]}, \cdots, \bar{\mathbf{f}}_{RF}^{[N_s]} \right] \in \mathbb{C}^{N_{\text{tot}} \times N_s}, \quad \bar{\mathbf{f}}_{RF}^{[i]} \in \mathbb{C}^{d_i N_{\text{sub}} \times 1} \quad \text{and} \quad \bar{\mathbf{F}}_{BB} \in \mathbb{C}^{N_s \times N_s}.
\]

With the defined variables \( \bar{\mathbf{F}}_{RF} \) and \( \bar{\mathbf{F}}_{BB} \) substituted into (37), we follow the common decoupling procedure \([23], [30]\) that given a fixed RF precoder \( \bar{\mathbf{F}}_{RF} \) and an equivalent channel matrix \( \bar{\mathbf{H}}_{eq} = \bar{\mathbf{H}}_{RF} \), the optimal digital precoder has a closed-form water-filling solution as

\[
\bar{\mathbf{F}}_{BB}^* = D_t^{-1/2} U_e \Lambda_e^{1/2},
\]

where \( D_t = \bar{\mathbf{H}}_{RF}^H \bar{\mathbf{F}}_{RF} = \text{diag} \left[ \mathbf{d}_t \right] \), \( U_e \) is the set of right singular vectors corresponding to the \( N_s \) largest singular values of \( \bar{\mathbf{H}}_{eq} D_t^{-1/2} \), and \( \Lambda_e \) is a diagonal matrix with the allocated powers to each data stream on its main diagonal. We further adopt an equal power allocation scheme for all data streams, i.e., \( \Lambda_e \approx \mathbf{I}_{N_s} \), which shows a little loss in performance for moderate and high SNR regimes. Accordingly, the digital precoder can be approximated as \( \bar{\mathbf{F}}_{BB} \approx D_t^{-1/2} U_e \).

2) RF Precoder Design: In this part, we begin to design the RF precoder by assuming the approximately optimal digital precoder \( \bar{\mathbf{F}}_{BB} \approx D_t^{-1/2} U_e \). Under this assumption, the RF precoder can be obtained by solving the following problem

\[
\max_{\bar{\mathbf{F}}} \quad \log_2 \left| \mathbf{I}_{N_{\text{tot}}} + P_d \left( N_s \sigma_z^2 \right)^{-1} \bar{\mathbf{H}} \bar{\mathbf{F}}^H \bar{\mathbf{F}} \bar{\mathbf{H}}^H \right| \\
\text{s.t.} \quad \left| \bar{\mathbf{F}} \right|_{i,j} = \left( d_j N_{\text{sub}} \right)^{-1/2}, \quad \forall (i, j) \in \bar{\mathcal{F}}_t.
\]

where \( \bar{\mathcal{F}} = \bar{\mathbf{F}}_{RF} D_t^{-1/2} \in \mathbb{C}^{N_{\text{tot}} \times N_s} \) and \( \bar{\mathcal{F}}_t \) is the set of non-zero elements of \( \bar{\mathbf{F}}_{RF} \).

According the block structure of \( \bar{\mathbf{F}} \), it is observed that the optimization problem (39) with nonconvex constraints can be decomposed into a series of simple subproblems, each of which only considers one specific group of subarrays \([26], [31]\). In particular, we can divide the matrix

\(^4\)The proposed homogeneous grouping strategy is not a special case of the hybridly connected structure based partition strategy in \([31]\). The optimal grouping strategy for the DPA-MIMO system is scheduled for future research.
where \( \bar{f} \) as \( \bar{f} = \begin{bmatrix} \bar{F}_{N_s-1} \bar{f}_{N_s} \end{bmatrix} \), where \( \bar{f}_{N_s} \) is the \( N_s \)th column and \( \bar{F}_{N_s-1} \) is a matrix containing the first \( N_s - 1 \) columns of \( \bar{F} \), respectively. Thus, the cost function of (39) can be written as

\[
\begin{align*}
\text{(39a)} & \log_2 \left| I_{N_t'} + P_d \left( N_s \sigma_z^2 \right)^{-1} \left( \bar{H} \bar{F}_{N_s-1} \bar{F}_{N_s-1}^H + \bar{H} \bar{f}_{N_s} \bar{f}_{N_s}^H \right) \right| \\
& (a) \log_2 \left| R_{N_s-1} \right| + \log_2 \left| I_{N_t'} + P_d \left( N_s \sigma_z^2 \right)^{-1} \bar{f}_{N_s}^H \bar{f}_{N_s} \right| \\
& (b) \sum_{n=1}^{N_s} \log_2 \left| 1 + P_d \left( N_s \sigma_z^2 \right)^{-1} \bar{f}_n^H \bar{r}_n \right|
\end{align*}
\]

where we define \( R_{n-1} = I_{N_t'} + P_d \left( N_s \sigma_z^2 \right)^{-1} \bar{H} \bar{f}_{n-1} \bar{f}_{n-1}^H \) and \( \bar{r}_0 = I_{N_t'} \). (a) follows from \( |I + AB| = |I + BA| \) and \( |AB| = |A||B| \) [31], and (b) is the result of \( N_s - 1 \) iterations.

As seen from (40), the total achievable rate can be a summation of the subrates of all data streams. Motivated by the idea of SIC for multiuser detection, we can first optimize the achievable subrate of the first data stream and then update \( R_1 \). After some iterations, the optimal RF precoder design subproblem for the \( n \)th group of subarrays can be equivalently represented by

\[
\hat{f}_n^* = \arg \max_{\hat{f}_n} \log_2 \left| 1 + P_d \left( N_s \sigma_z^2 \right)^{-1} \hat{f}_n^H \hat{r}_n \right|
\]

where \( \hat{r}_n = H^H R_{n-1}^H \hat{f}_n \). Due to the special structure of \( \hat{f}_n \), (41) can be further simplified to

\[
\hat{f}_n^* = \arg \max_{\hat{f}_n} \log_2 \left| 1 + P_d \left( N_s \sigma_z^2 \right)^{-1} \hat{f}_n^H \hat{r}_n \right|
\]

where \( \hat{r}_n \) is a \( d_n N_{t} \times d_n N_{s} \) Hermitian matrix formed as a submatrix of matrix \( T_{n-1} \) by taking the \( \left( N_s \sum_{i=1}^{n-1} d_i + 1 \right) \)th row and column to the \( \left( N_s \sum_{i=1}^{n} d_i \right) \)th row and column of \( T_{n-1} \). (Note that \( T_{n-1} \) can be iteratively obtained without matrix inverse [31], however, this simple procedure is omitted due to space limitation.) Define the eigen value decomposition (EVD) of \( \hat{T}_{n-1} \) as \( \hat{T}_{n-1} = \hat{V} \hat{\Sigma} \hat{V}^H \), where \( \hat{V} \) is a unitary matrix and \( \hat{\Sigma} \) is a diagonal matrix with the eigen values arranged in a decreasing order. From (42), the optimal unconstrained RF precoder is the first column \( \hat{v}_1 \) of \( \hat{V} \) [62], and the problem of (42) is equivalent to minimizing the Euclidean distance [26]

\[
\left\| \hat{r}_n - \hat{v}_1 \right\|_2^2 = 1 + \frac{1}{d_n} - \frac{2}{\sqrt{d_n}} \Re \left( \hat{v}_1^H \hat{a}_n \right)
\]

where \( \hat{r}_n = \frac{1}{\sqrt{d_n}} \hat{a}_n \) and every element of \( \hat{a}_n \) has a constant amplitude that equals to \( \frac{1}{\sqrt{N_{s}}} \). By maximizing \( \Re \left( \hat{v}_1^H \hat{a}_n \right) \), we can obtain the optimal \( \hat{f}_n \) of (41) as

\[
\hat{f}_n^* = \begin{bmatrix} e^{j\angle(\hat{V}_1)} & 0_{1,N_s} \sum_{i=1}^{n-1} d_i, 0_{1,N_s} \sum_{i=n+1}^{N_s} d_i \end{bmatrix}^T
\]

where the symbol \( \angle (\cdot) \) extracts the corresponding phases of the elements.
B. Hybrid Combiner Design

We now consider the hybrid combiner design with the obtained precoders. We group the subarrays at the RX in the same way as the TX. Furthermore, without causing loss of optimality, we decouple the design of $\hat{\mathbf{W}}_{\text{RF}}$ and $\hat{\mathbf{W}}_{\text{BB}}$ by first optimizing the RF combiner with assumed ideal digital combiner and then finding the optimal digital combiner for the obtained RF combiner [23]. As a result, the RF combiner design problem can be formulated as

$$
\max_{\mathbf{W}_{\text{RF}}} \quad \log_2 \left| \mathbf{I}_{N_s} + P_d \left( N_s \sigma_z^2 \right)^{-1} \left( \mathbf{W}_{\text{RF}}^H \mathbf{W}_{\text{RF}} \right)^{-1} \mathbf{W}_{\text{RF}}^H \mathbf{H}_{\text{eq}} \mathbf{H}_{\text{eq}}^H \mathbf{W}_{\text{RF}} \right| 
$$

(45a)

subject to

$$
\left[ \mathbf{W}_{\text{RF}} \right]_{i,j} = \left( N_s \right)^{-1/2}, \quad \forall (i, j) \in \tilde{W}_{r}.
$$

(45b)

Since $\mathbf{W}_{\text{RF}}^H \mathbf{W}_{\text{RF}} = \mathbf{D}_r = \text{diag} \left[ d_r \right]$ where $d_r = [\tilde{d}_1, \cdots, \tilde{d}_{N_s}]$, we can define $\hat{\mathbf{W}} = \hat{\mathbf{W}}_{\text{RF}} \mathbf{D}_r^{-1/2}$, which results in the similar problem as (39):

$$
\max_{\mathbf{W}} \quad \log_2 \left| \mathbf{I}_{N_s} + P_d \left( N_s \sigma_z^2 \right)^{-1} \hat{\mathbf{W}}^H \mathbf{H}_{\text{eq}} \mathbf{H}_{\text{eq}}^H \hat{\mathbf{W}} \right| 
$$

(46a)

subject to

$$
\left[ \hat{\mathbf{W}} \right]_{i,j} = \hat{d}_j \left( N_s \right)^{-1/2}, \quad \forall (i, j) \in \tilde{W}_{r}.
$$

(46b)

Moreover, assuming all other beamformers are fixed, the optimal digital combiner based on the MMSE criterion is formulated as

$$
\hat{\mathbf{W}}_{\text{BB}}^* = \mathbb{E} \left[ \mathbf{s}_d \mathbf{y}_d^H \right] \mathbb{E} \left[ \mathbf{y}_d \mathbf{y}_d^H \right]^{-1} = \frac{P_d}{N_s} \mathbf{J}^{-1} \mathbf{W}_{\text{RF}}^H \mathbf{H} \mathbf{f}_t,
$$

where $\mathbf{J} = \frac{P_d}{N_s} \mathbf{W}_{\text{RF}}^H \mathbf{F}_t \mathbf{F}_t^H \mathbf{W}_{\text{RF}} + \sigma_z^2 \mathbf{D}_r \in \mathbb{C}^{N_s \times N_s}$ and $\mathbf{f}_t = \mathbf{F}_t^* \mathbf{f}_{\text{BB}}^*$.

VI. SIMULATION RESULTS

The evaluated performance is examined through simulation with the following parameters. The TX and RX subarrays are ULAs with $M_t = M_r = 4$. The channel coefficients are generated through (1) with the variances of the channel paths as $\sigma_{\text{LoS}}^2 = 1$ and $\sigma_{\text{NLoS}}^2 = 10^{-0.5}$ [64]. We denote the channel common sparsity as $L_c = |\Omega_c|$ and assume the equal channel individual sparsity among different subarrays as $L = |\Omega_{m,n}| = 5$. In Algorithm [1] the threshold parameters are set to be $\delta_1 = N \sigma_z^2$ and $\delta_2 = 0.1 N \sigma_z^2$; the maximal iteration numbers are chosen as $T_1 = \max \{ L_c - 1 \}$ and $T_2 = (L + 1 - T_1) K$. In Algorithm [2] we set the weight $\beta = 3.3$, the maximal iteration number $T_{\text{max}} = 10^2$ and the error tolerance $\epsilon = 10^{-4}$. In the following, two types of SNRs are considered: one is the pilot-to-noise ratio (PNR) defined as $10 \log_{10} (P_p/\sigma_z^2)$,
and the other is the data-to-noise ratio (DNR) defined as $10 \log_{10} \left( \frac{P_d}{\sigma_z^2} \right)$. The performance metric for channel estimation is the normalized mean square error (NMSE) defined as

$$\text{NMSE} \triangleq 10 \log_{10} \left( \mathbb{E} \left[ \| \mathbf{H} - \hat{\mathbf{H}} \|_F^2 / \| \mathbf{H} \|_F^2 \right] \right) = 10 \log_{10} \left( \mathbb{E} \left[ \| \mathbf{G} - \hat{\mathbf{G}} \|_F^2 / \| \mathbf{G} \|_F^2 \right] \right).$$ \hspace{1cm} (48)

Besides, the hybrid precoding schemes based on the channel estimates is evaluated through the aggregate SE defined in (35).

This section consists of two parts. In the first one, we compare the NMSE of the proposed JOMP and JSBL-$\ell_2$ estimators with the conventional OMP and SBL estimators by employing the designed training beam patterns. In the second part, we investigate the performance of the proposed hybrid precoding scheme termed as Group-SIC, the SDR-AltMin scheme \(^5\) and the optimal fully-digital precoding scheme. We further compare the SE realized through the proposed hybrid beamformers based on the channel estimates obtained in the previous part.

### A. Performance Evaluation of Channel Estimation

In Fig. 3, we compare the NMSE of the estimated channels versus the PNR when $N_{\text{sub}}^t = N_{\text{sub}}^r = 10$, $N_{t\text{beam}} = N_{r\text{beam}} = 20$ (partial-training case) and $L_c = 3$. As expected, the JSBL-$\ell_2$ estimator substantially outperforms the other three estimators for all the range of PNR. It is further observed that when PNR $\geq$ 10 dB, the greedy JOMP estimator achieves the same channel estimation performance as the SBL estimator. Additionally, the greedy JOMP estimator keeps a fixed 5 dB lower gap than the OMP estimator that is unable to exploit the structured sparsity of the DPA-MIMO channel.

\(^5\) Since the SDR-AltMin algorithm is based on the matrix decomposition via alternating optimization, for fair comparison in the simulation, we adopt the optimal water-filling transmit matrix of $\mathbf{H}$ as the objective transmit matrix. Furthermore, we take the MMSE receiver based on the designed hybrid precoders as the objective combiner \(^6\).

---

**Fig. 3.** NMSE versus PNR. **Fig. 4.** NMSE versus common sparsity. **Fig. 5.** NMSE versus number of training beams.
In Fig. 4, we investigate the NMSE of the estimated channels versus the common sparsity $L_c$ when $N_{\text{sub}} = N_{\text{sub}}^r = 10$, $N_{\text{beam}}^t = N_{\text{beam}}^r = 20$, and $	ext{PNR} = 10$ dB. For both the proposed JOMP and JSBL-$\ell_2$ estimators, better channel estimation performance is obtained with an increasing number of the common support $L_c$, while the OMP and SBL estimators keep the constant NMSE. This is because the two customized estimators take advantage of the joint sparse characteristic of the DPA-MIMO channel to enhance the quality of estimated channels. Moreover, for $L_c > 3$, the JOMP estimator surpasses the SBL estimator in the accuracy of channel estimates.

In Fig. 5, we show the NMSE of the estimated channels versus the number of training beams $N_{\text{beam}}^t$ when $N_{\text{beam}}^t = N_{\text{beam}}^r = N_{\text{beam}}$, $N_{\text{sub}} = N_{\text{sub}}^r = 8$, $	ext{PNR} = 10$ dB and $L_c = 3$. With an increasing $N_{\text{beam}}$, the NMSE of all estimators decrease monotonically. More specifically, in the full-training case ($N_{\text{beam}} = N_{t}^t = 32$), the OMP estimator approaches the same NMSE as the JOMP estimator since the channel support recovery probabilities of these greedy schemes gradually approximate 100% with a higher number of measurements. Additionally, it is evident that the JSBL-$\ell_2$ estimator has the highest reconstruction accuracy among all estimators.

In Fig. 6, we depict the NMSE of the estimated channels versus the number of subarray antennas $N_{\text{sub}}$ when $N_{\text{sub}} = N_{\text{sub}}^r = N_{\text{sub}}^t$, $N_{\text{beam}}^t = N_{\text{beam}}^r = 20$, $	ext{PNR} = 10$ dB and $L_c = 3$. As shown in this figure, the channel estimation performance of all estimators increases with a bigger $N_{\text{sub}}$ due to higher resolution of AoAs (AoDs) in the beam domain. Additionally, the JOMP estimator can obtain better channel estimate than the SBL estimator when $N_{\text{sub}} = 12$. The rational behind this phenomenon is that a larger number of subarray antennas contributes to a greater correlation calculated in step 1 of Algorithm which provides better recovery of the distributed joint sparsity in the DPA-MIMO channel.
Fig. 9. SE versus DNR.  
Fig. 10. SE versus common sparsity Fig. 11. SE versus common sparsity where \( N_s = 1 \).  
where \( N_s = 3 \).

B. Performance Evaluation of Hybrid Precoding

First, we compare the SE versus the DNR for various number of subarrays at the TX (RX) with perfect CSI when \( N_{\text{sub}} = N_{\text{rsub}} = 10 \), \( L_c = 3 \) and \( M_t = M_r = M \). As illustrated in Fig. 7 when single data stream is transmitted, both the Group-SIC and SDR-AltMin schemes achieve almost the same performance as the fully-digital precoding scheme with different number of subarrays. While two data streams are transmitted, it is observed from Fig. 8 that there are distinct gaps between the fully-digital precoding scheme and the other two hybrid precoding schemes, which results from the beamforming gain loss of the subarray based structure compared to the fully-digital structure [42]. We further find that the proposed Group-SIC scheme can successfully compete with the SDR-AltMin scheme while with far less complexity. The efficiency advantage of the proposed Group-SIC scheme over the SDR-AltMin scheme is verified through average central processing unit (CPU) processing time. By using the same configuration as the Matlab experiment demonstrated in Fig. 7 and Fig. 8 with \( M = 4 \), the average CPU processing time of the Group-SIC and SDR-AltMin schemes are 0.29 sec and 5.53 sec, respectively. Therefore, in the following, we only employ the proposed Group-SIC scheme to design the hybrid beamformers based on the estimated channels obtained in Subsection VI-A.

In Fig. 9 we investigate the SE versus the DNR when \( N_{\text{sub}} = N_{\text{rsub}} = 10 \), \( L_c = 3 \) and \( N_s = 3 \). Among the practical estimators, the proposed JSBL-\( \ell_2 \) estimator provides the best performance which is followed closely by the proposed JOMP estimator. Furthermore, it is interesting that although with slightly worse quality of estimated channels in some cases depicted in Subsection VI-A, the proposed JOMP estimator can achieve higher SE than the SBL estimator. This can be explained by the fact that the transmitted symbols should be preferentially sent through the common channel paths among all subarrays to obtain significant array gain in the DPA-MIMO
system, and JOMP has better estimation performance of the common channel paths than SBL.

Fig. 10 shows the SE versus the common sparsity $L_c$ when $N_{t\text{ub}}^s = N_{r\text{ub}}^s = 10$ and DNR = 10 dB. When single data stream is transmitted, the SE of the proposed two channel estimators is slightly above that of the other two channel estimators for all the range of $L_c$. Moreover, the performance of all estimators varies within a very small range over $L_c$. However, the situation is totally different with three data streams transmitted, i.e., the SE of every channel estimator becomes higher with an increasing common sparsity $L_c$, which is illustrated in Fig. 11.

In Fig. 12, we depict the SE versus the number of training beams when $N_{t\text{ub}}^s = N_{r\text{ub}}^s = 10$, $L_c = 3$, $N_s = 3$ and DNR = 10 dB. It is evident that the SE of all estimators increases with a larger number of training beams, and in the full-training case equal SE which approximates that of perfect CSI is obtained. In Fig. 13, we demonstrate the SE versus the number of subarray antennas when $L_c = 3$, $N_s = 3$ and DNR = 10 dB. It is clear that a larger number of subarray antennas which provides more array gain leads to higher SE for all estimators, and JSBL-$\ell_2$ outperforms other algorithms.

VII. Conclusion

In this paper, we have focused on modeling and analysis of the narrowband DPA-MIMO based transceiver system, and designed efficient mmWave channel estimation and hybrid precoding schemes for such distributed array-of-subarrays architecture. Based on the reasonable analysis in Section II, the DPA-MIMO channel has high probability to manifest a hidden structured sparsity in the beam-domain channel vector due to the partially shared scatterers among the distributed subarrays at mmWave frequencies at the TX (RX). In light of this characteristic, we formulate a structured SMV problem that estimates the AoDs, AoAs and the corresponding gain of significant paths. In order to guarantee the good recovery performance and decrease the training
feedback overhead, the open-loop training beam patterns are designed through minimizing the total coherence of the equivalent measurement matrix. The simulation and comparison results demonstrate that the proposed channel estimators can better exploit the structured channel properties defined in Definition 1 than the existing CS-based estimators such as the OMP and SBL estimators, and the proposed hybrid precoding method enjoys the low-complexity while achieving the nearly same performance as the well-known alternating optimization based method termed as SDR-AltMin. Interesting topics for future research in DPA-MIMO systems include many diversified situations, such as broadband channel modeling, multi-user channel acquisition and hybrid precoding schemes, and two-dimensional (2D) antenna subarray deployments.

**APPENDIX A**

**DERIVATION OF (25)**

Let $s = x - c$, $A = (\Gamma^*)^{-1}$ and $B = (I_K \otimes \Gamma^c)^{-1}$, thus we can transform the objective function in (25) to the following

$$
\min_{x, c} \frac{1}{\lambda} \|y - \Phi x\|^2_2 + (x - c)^H A (x - c) + c^H B c.
$$

(49)

For the fixed $x$, we have an unconstrained quadratic function only with respect to $c$ and get its optimal solution as $c^* = (A + B)^{-1} A x$. After submitting the optimal $c^*$ into (49), we have

$$
\min_{x} \frac{1}{\lambda} \|y - \Phi x\|^2_2 + x^H (A - A (A + B)^{-1} A) x
$$

$$
\overset{(a)}{=} \min_{x} \frac{1}{\lambda} \|y - \Phi x\|^2_2 + x^H (A^{-1} + B^{-1})^{-1} x
$$

$$
\overset{(b)}{=} y^H (\lambda I_N + \Phi (A^{-1} + B^{-1}) \Phi^H)^{-1} y,
$$

(50)

where (a) follows from the Woodbury identity [51] and (b) follows from the identity [60]

$$
y^H (\lambda I_N + \Phi \Gamma \Phi^H)^{-1} y = \min_{x} \frac{1}{\lambda} \|y - \Phi x\|^2_2 + x^H \Gamma^{-1} x.
$$

(51)

**REFERENCES**

[1] S. Malkowsky et al., “The world’s first real-time testbed for massive MIMO: Design, implementation, and validation,” *IEEE Access*, vol. 5, pp. 9073–9088, Jun. 2017.

[2] D. Wang, Y. Zhang, H. Wei, X. You, X. Gao, and J. Wang, “An overview of transmission theory and techniques of large-scale antenna systems for 5G wireless communications,” *Sci. China Inf. Sci.*, vol. 59, no. 8, pp. 1–18, Aug. 2016.

[3] L. Li, D. Wang, X. Niu, Y. Chai, L. Chen, L. He, X. Wu, F. Zheng, T. Cui, and X. You, “mmWave communications for 5G: Implementation challenges and advances,” *Sci. China Inf. Sci.*, vol. 61, no. 2, pp. 1–19, Feb. 2018.
[4] G. Yue et al., “Demonstration of 60 GHz millimeter-wave short-range wireless communication system at 3.5 Gbps over 5 m range,” Sci. China Inf. Sci., vol. 60, no. 8, pp. 1–7, Aug. 2017.

[5] J. G. Andrews, T. Bai, M. N. Kulkarni, A. Alkhateeb, A. K. Gupta, and R. W. Heath, “Modeling and analyzing millimeter wave cellular systems,” IEEE Trans. Commun., vol. 65, no. 1, pp. 403–430, Jan. 2017.

[6] W. Zhai et al., “Dual-band millimeter-wave interleaved antenna array exploiting low-cost PCB technology for high speed 5G communication,” in Proc. IEEE MTT-S Int. Microw. Symp. (IMS), San Francisco, CA, USA, May 2016, pp. 1–3.

[7] B. Yu, K. Yang, C. Sim, and G. Yang, “A novel 28 GHz beam steering array for 5G mobile device with metallic casing application,” IEEE Trans. Antennas Propag., vol. 66, no. 1, pp. 462–466, Jan. 2018.

[8] T. Deckmyn et al., “A novel 60 GHz wideband coupled half-mode/quarter-mode substrate integrated waveguide antenna,” IEEE Trans. Antennas Propag., vol. 65, no. 12, pp. 6915–6926, Dec. 2017.

[9] Y. Huo et al., “A wideband artificial magnetic conductor Yagi antenna for 60-GHz standard 0.13-µm CMOS applications,” in Proc. IEEE Int. Solid-State and Integrated Circuit Technology (ICSICT), Guilin, China, Oct. 2014, pp. 1–3.

[10] Y. Huo, X. Dong, L. Li, M. Xie, and W. Xu, “26/40 GHz CMOS VCOs design of radio front-end for 5G mobile devices,” in Proc. IEEE International Conference on Ubiquitous Wireless Broadband (ICUWB), Nanjing, China, Oct. 2016, pp. 1–4.

[11] Y. Huo, X. Dong, W. Xu, and M. Yuen, “Cellular and WiFi co-design for 5G user equipment,” in Proc. IEEE in IEEE 5G World Forum (5GWF), Santa Clara, California, USA, Jul. 2018, pp. 256–261. [Online]. Available: https://arxiv.org/abs/1803.06943

[12] B. Floyd, “High-performance millimeter-wave beamformers with built-in self-test,” in Proc. IEEE Integr. Circuits Conf. (CICC), San Diego, CA, USA, Apr. 2018, pp. 1–68.

[13] Y. Huo, X. Dong, L. Li, M. Xie, and W. Xu, “Design and analysis of two K-band CMOS VCOs for next generation wireless systems,” in Proc. Asia-Pacific Microwave Conference (APMC), Nanjing, China. Dec. 2015, pp. 1–3.

[14] B. Sadhu et al., “A 28-GHz 32-element TRX phased-array IC with concurrent dual-polarized operation and orthogonal phase and gain control for 5G communications,” IEEE J. Solid-State Circuits, vol. 52, no. 12, pp. 3373–3391, Dec. 2017.

[15] Y. Huo, X. Dong, and W. Xu, “5G cellular user equipment: From theory to practical hardware design,” IEEE Access, vol. 5, pp. 13 992–14 010, Aug. 2017.

[16] T. Sowlati et al., “A 60 GHz 144-element phased-array transceiver with 51dBm maximum EIRP and ±60° beam steering for backhaul application,” in Proc. IEEE Int. Solid-State Circuits Conf. (ISSCC), San Francisco, CA, Feb. 2018, pp. 66–68.

[17] H. Kim et al., “A 28-GHz CMOS direct conversion transceiver with packaged 2 × 4 antenna array for 5G cellular system,” IEEE J. Solid-State Circuits, vol. 53, no. 5, pp. 1245–1259, May 2018.

[18] M.-Y. Huang, T. Chi, F. Wang, T.-W. Li, and H. Wang, “A 23-to-30GHz hybrid beamforming MIMO receiver array with closed-loop multistage front-end beamformers for full-FoV dynamic and autonomous unknown signal tracking and blocker rejection,” in Proc. IEEE Int. Solid-State Circuits Conf. (ISSCC), San Francisco, CA, USA, Feb. 2018, pp. 68–70.

[19] S. Hu, F. Wang, and H. Wang, “A 28GHz/37GHz/39GHz multiband linear Doherty power amplifier for 5G massive MIMO applications,” in Proc. IEEE Int. Solid-State Circuits Conf. (ISSCC), San Francisco, CA, USA, Feb. 2017, pp. 32–33.

[20] X. Zhang, A. F. Molisch, and S.-Y. Kung, “Variable-phase-shift-based RF-baseband codesign for MIMO antenna selection,” IEEE Trans. Signal Process., vol. 53, no. 11, pp. 4091–4103, Nov. 2005.

[21] R. W. Heath, N. Gonzalez-Prelcic, S. Rangan, W. Roh, and A. M. Sayeed, “An overview of signal processing techniques for millimeter wave MIMO systems,” IEEE J. Sel. Topics Signal Process., vol. 10, no. 3, pp. 436–453, Apr. 2016.

[22] L. Liang, W. Xu, and X. Dong, “Low-complexity hybrid precoding in massive multiuser MIMO systems,” IEEE Wireless Commun. Lett., vol. 3, no. 6, pp. 653–656, Dec. 2014.

[23] F. Sohrabi and W. Yu, “Hybrid digital and analog beamforming design for large-scale antenna arrays,” IEEE J. Sel. Topics Signal Process., vol. 10, no. 3, pp. 501–513, Apr. 2016.
[24] W. Ni and X. Dong, “Hybrid block diagonalization for massive multiuser MIMO systems,” IEEE Trans. Commun., vol. 64, no. 1, pp. 201–211, Jan. 2016.

[25] L. Pan, L. Liang, W. Xu, and X. Dong, “Framework of channel estimation for hybrid analog-and-digital processing enabled massive MIMO communications,” IEEE Trans. Commun., vol. 66, no. 9, pp. 3902–3915, Sep. 2018.

[26] X. Gao, L. Dai, S. Han, L. Chih-Lin, and R. W. Heath, “Energy-efficient hybrid analog and digital precoding for mmWave MIMO systems with large antenna arrays,” IEEE J. Sel. Areas Commun., vol. 34, no. 4, pp. 998–1009, Apr. 2016.

[27] X. Huang, Y. J. Guo, and J. D. Bunton, “A hybrid adaptive antenna array,” IEEE Trans. Wireless Commun., vol. 9, no. 5, pp. 1770–1779, May 2010.

[28] X. Huang and Y. J. Guo, “Frequency-domain AoA estimation and beamforming with wideband hybrid arrays,” IEEE Trans. Wireless Commun., vol. 10, no. 8, pp. 2543–2553, Aug. 2011.

[29] J. Lota et al., “5G uniform linear arrays with beamforming and spatial multiplexing at 28, 37, 64, and 71 GHz for outdoor urban communication: A two-level approach,” IEEE Trans. Veh. Technol., vol. 66, no. 11, pp. 9972–9985, Nov. 2017.

[30] S. Park, A. Alkhateeb, and R. W. Heath, “Dynamic subarrays for hybrid precoding in wideband mmWave MIMO systems,” IEEE Trans. Wireless Commun., vol. 16, no. 5, pp. 2907–2920, May 2017.

[31] D. Zhang, Y. Wang, X. Li, and W. Xiang, “Hybridiy connected structure for hybrid beamforming in mmWave massive MIMO systems,” IEEE Trans. Commun., vol. 66, no. 2, pp. 662–674, Feb. 2018.

[32] J. Singh and S. Ramakrishna, “On the feasibility of codebook-based beamforming in millimeter wave systems with multiple antenna arrays,” IEEE Trans. Wireless Commun., vol. 14, no. 5, pp. 2670–2683, May 2015.

[33] F. Sandoval, G. Poti, and F. Gagnon, “Hybrid peak-to-average power ratio reduction techniques: Review and performance comparison,” IEEE Access, vol. 5, pp. 27145–27161, Nov. 2017.

[34] Y. Huo and X. Dong, “Millimeter-wave for unmanned aerial vehicles networks: Enabling multi-beam multi-stream communications,” Oct. 2018. [Online]. Available: https://arxiv.org/abs/1810.06923

[35] Y. Huo, X. Dong, T. Lu, W. Xu, and M. Yuen, “Distributed and multi-layer UAV network for the next-generation wireless communication,” May 2018. [Online]. Available: https://arxiv.org/abs/1805.01534

[36] Pasternack, “5G update: Standards emerge, accelerating 5G deployment,” Microwave Journey, vol. 61, no. 5, May 2018.

[37] A. Alkhateeb, O. El Ayach, G. Leus, and R. W. Heath, “Channel estimation and hybrid precoding for millimeter wave cellular systems,” IEEE J. Sel. Topics Signal Process., vol. 8, no. 5, pp. 831–846, Oct. 2014.

[38] S. Hur, T. Kim, D. J. Love, J. V. Krogmeier, T. A. Thomas, and A. Ghosh, “Millimeter wave beamforming for wireless backhaul and access in small cell networks,” IEEE Trans. Commun., vol. 61, no. 10, pp. 4391–4403, Oct. 2013.

[39] Z. Xiao, P. Xia, and X.-G. Xia, “Codebook design for millimeter-wave channel estimation with hybrid precoding structure,” IEEE Trans. Wireless Commun., vol. 16, no. 1, pp. 141–153, Jan. 2017.

[40] J. Lee, G.-T. Gil, and Y. H. Lee, “Channel estimation via orthogonal matching pursuit for hybrid MIMO systems in millimeter wave communications,” IEEE Trans. Commun., vol. 64, no. 6, pp. 2370–2386, Jun. 2016.

[41] C. Lin, G. Y. Li, and L. Wang, “Subarray-based coordinated beamforming training for mmWave and sub-THz communications,” IEEE J. Sel. Areas Commun., vol. 35, no. 9, pp. 2115–2126, Sep. 2017.

[42] X. Yu, J.-C. Shen, J. Zhang, and K. B. Letaief, “Alternating minimization algorithms for hybrid precoding in millimeter wave MIMO systems,” IEEE J. Sel. Topics Signal Process., vol. 10, no. 3, pp. 485–500, Apr. 2016.

[43] R. G. Baraniuk, V. Cevher, M. F. Duarte, and C. Hegde, “Model-based compressive sensing,” IEEE Trans. Inf. Theory, vol. 56, no. 4, pp. 1982–2001, Apr. 2010.

[44] L. Zelnik-Manor, K. Rosenblum, and Y. C. Eldar, “Sensing matrix optimization for block-sparse decoding,” IEEE Trans. Signal Process., vol. 59, no. 9, pp. 4300–4312, Sep. 2011.
[45] W. Chen, D. Wipf, Y. Wang, Y. Liu, and I. J. Wassell, “Simultaneous Bayesian sparse approximation with structured sparse models,” IEEE Trans. Signal Process., vol. 64, no. 23, pp. 6145–6159, Dec. 2016.

[46] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, “Scaling up MIMO: Opportunities and challenges with very large arrays,” IEEE Signal Process. Mag., vol. 30, no. 1, pp. 40–60, Jan. 2013.

[47] A. M. Sayeed, “Deconstructing multiantenna fading channels,” IEEE Trans. Signal Process., vol. 50, no. 10, pp. 2563–2579, Oct. 2002.

[48] X. Gao, O. Edfors, F. Tufvesson, and E. G. Larsson, “Massive MIMO in real propagation environments: Do all antennas contribute equally?” IEEE Trans. Commun., vol. 63, no. 11, pp. 3917–3928, Nov. 2015.

[49] C. Lin and G. Y. Li, “Adaptive beamforming with resource allocation for distance-aware multi-user indoor terahertz communications,” IEEE Trans. Commun., vol. 63, no. 8, pp. 2985–2995, Aug. 2015.

[50] X. Rao and V. K. Lau, “Distributed compressive CSIT estimation and feedback for FDD multi-user massive MIMO systems,” IEEE Trans. Signal Process., vol. 62, no. 12, pp. 3261–3271, Jun. 2014.

[51] K. B. Petersen and M. S. Pedersen, The Matrix Cookbook, 2012.

[52] W. U. Bajwa, J. Haupt, A. M. Sayeed, and R. Nowak, “Compressed channel sensing: A new approach to estimating sparse multipath channels,” Proceedings of the IEEE, vol. 98, no. 6, pp. 1058–1076, Jun. 2010.

[53] E. J. Candes and T. Tao, “Decoding by linear programming,” IEEE Trans. Inf. Theory, vol. 51, no. 12, pp. 4203–4215, Dec. 2005.

[54] J. A. Tropp and A. C. Gilbert, “Signal recovery from random measurements via orthogonal matching pursuit,” IEEE Trans. Inf. Theory, vol. 53, no. 12, pp. 4655–4666, Dec. 2007.

[55] D. P. Wipf and B. D. Rao, “Sparse bayesian learning for basis selection,” IEEE Trans. Signal Process., vol. 52, no. 8, pp. 2153–2164, Aug. 2004.

[56] M. F. Duarte and Y. C. Eldar, “Structured compressed sensing: From theory to applications,” IEEE Trans. Signal Process., vol. 59, no. 9, pp. 4053–4085, Sep. 2011.

[57] J. A. Tropp, A. C. Gilbert, and M. J. Strauss, “Algorithms for simultaneous sparse approximation. Part I: Greedy pursuit,” Signal Processing, vol. 86, no. 3, pp. 572–588, Aug. 2006.

[58] A. Jalali, P. Ravikumar, and S. Sanghavi, “A dirty model for multiple sparse regression,” IEEE Trans. Inf. Theory, vol. 59, no. 12, pp. 7947–7968, Dec. 2013.

[59] W. Chen, “Simultaneous sparse Bayesian learning with partially shared supports,” IEEE Signal Process. Lett., vol. 24, no. 11, pp. 1641–1645, Nov. 2017.

[60] D. P. Wipf, B. D. Rao, and S. Nagarajan, “Latent variable Bayesian models for promoting sparsity,” IEEE Trans. Inf. Theory, vol. 57, no. 9, pp. 6236–6255, Sep. 2011.

[61] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, “Joint Tx-Rx beamforming design for multicarrier MIMO channels: A unified framework for convex optimization,” IEEE Trans. Signal Process., vol. 51, no. 9, pp. 2381–2401, Sep. 2003.

[62] O. El Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath, “Spatially sparse precoding in millimeter wave MIMO systems,” IEEE Trans. Wireless Commun., vol. 13, no. 3, pp. 1499–1513, Mar. 2014.

[63] W. Ni, X. Dong, and W.-S. Lu, “Near-optimal hybrid processing for massive MIMO systems via matrix decomposition,” IEEE Trans. Signal Process., vol. 65, no. 15, pp. 3922–3933, Aug. 2017.

[64] X. Gao, L. Dai, S. Han, I. Chih-Lin, and X. Wang, “Reliable beamspace channel estimation for millimeter-wave massive MIMO systems with lens antenna array,” IEEE Trans. Wireless Commun., vol. 16, no. 9, pp. 6010–6021, Sep. 2017.