We show results for the $B$ meson decay constant calculated both for $B$ mesons at rest and those with non-zero momentum and using both the temporal and spatial components of the axial vector current. It is an important check of lattice systematic errors that all these determinations of $f_B$ should agree. We also describe how well different smearings for the $B$ meson work at non-zero momentum - the optimal smearing has a narrow smearing on the $b$ quark.

1. Introduction

Matrix elements involving moving $B$ and $D$ mesons are important for studies of $B \to D$ and $B \to \pi$ decay. It is necessary to understand the systematic errors in lattice QCD that come from the presence of non-zero momenta and the optimal way in which to handle such mesons on the lattice. An easy place to study these effects is in the determination of the $B$ meson decay constant, $f_B$.

In the absence of discretisation errors it should be true that

$$<0| A_\mu |B > = f_B p_\mu .$$

(1)

We use both $A_0$ and $A_k$ with all current corrections and renormalisation through $\alpha_s/M_b$.

Results shown are for an ensemble of 278 $12^3 \times 24$ lattices at $\beta = 5.7$. Clover light quarks are used with $\kappa = \kappa_s$ and NRQCD heavy quarks are used with masses in lattice units of 2 and 8 ($m_{Q_a} = 4.0$). All momenta are considered up to $(pa)^2$ of 16 in units of $(2\pi/L)^2$, i.e. roughly 4 GeV$^2$ at this lattice spacing.

2. Smearing

We generated heavy and light quark propagators with 3 different smearings: a delta function, a narrow Gaussian (width 1) and a broad Gaussian (width 3). We combined these smearings together to make 6 different smearings for the heavy-light meson and analysed the resulting smeared-smeared meson correlators using the constrained curve fitting methods described in [3]. The results are shown in Figure 1, which plots the amplitude of the ground state $B$ meson as a function of squared momentum for some of these smearings. It is clear that the optimal smearing for a moving heavy-light meson is one in which the smearing on the heavy quark is a narrow one and the smearing on the light quark is a broad one. This is not surprising if one considers that the heavy quark is carrying almost all of the meson momentum. The overlap as a function of momentum is maximized if the heavy quark has a delta function smearing, but in that case the statistical noise is large because of the well-known problem that an unsmeared heavy quark has a poor signal/noise ratio [4]. At zero momentum this has led to the received wisdom that the heavy quark should have a broad smearing - Figure 1
Figure 1. The amplitude for the ground state in various heavy-light smeared-smeared correlators as a function of $\vec{p}^2$ in units of $(2\pi/L)^2$. The heavy mass was $m_a = 8$.

shows that this is not correct at non-zero momentum.

Figure 2 amplifies this point by showing the rapid plateau and consequent small error on the kinetic energy possible with a good finite-momentum smearing, in contrast to that obtained with a smearing that might have been considered a good one at zero momentum.

3. $f_B$ from $A_0$

We used smearing ‘5’ from the above analysis and the constrained curve fitting methods to determine $f_B$ for moving $B$ mesons and the temporal axial current. Matrix elements for $A_0$ are obtained from a simultaneous fit to all $A_0^i$ (i=0,1,2) and $A_i^k$ (i=0,1,2,3,4) current correlators at a given momentum. The construction of the continuum $A_0$ from $A_0^{latt}$ is described in [2].

Figure 3 shows results for the ratio:

$$R(p) = \frac{\langle 0|A_0|B(p)\rangle/\sqrt{E}}{\langle 0|A_0|B(0)\rangle/\sqrt{M}}$$  \hspace{2cm} (2)
for $ma=8$ using $\alpha_s(2/a)$ in the renormalisation. The expected curve $\sqrt{E/M}$ is also shown. No disagreement is seen until the highest momentum. At $ma=2$, larger discrepancies appear [5].

4. $f_B$ from $A_k$

The matrix elements for the spatial axial current behave rather differently to those for the temporal axial current since they must vanish when there is no component of $\vec{p}$ along the direction of the current. Even the leading order current, $A^0_k = \vec{\gamma}_5 \gamma_k Q$ then has a matrix element $O(p_k/M)$ with respect to $A^0$. This is of the same order as the ‘$1/M$’ suppressed current contributions from $A^1_k = 7\gamma_5 \gamma_k (\vec{\gamma} \cdot \vec{D})/2M)Q$ and $A^3_k = \vec{\gamma}_5 (\vec{D}/2M)Q$. $A^1_k$ contributes at tree level and $A^3_k$ at one-loop to the final result for $A_k$. To understand the size of different current contributions it is important to use a power-counting in both the external velocity and in $\Lambda_{QCD}/M$ [5].

Figure 4 shows results for the ratio:

$$R_k(p) = \frac{\langle 0|A_k|B(p)\rangle}{\langle 0|A_0|B(p)\rangle}$$

as a function of $p_k$ for a range of values of $\vec{p}^2$, for $ma=8$. The line shown is $p_k/E$ (variation with $\vec{p}^2$ of this line is not significant). Good agreement with the line is found for $p_k a = 1.2 \times 2\pi/L$ at all $\vec{p}$ except for $p_k a = 3, 4$ which show signs of deviation, presumably from missing current corrections that are higher order in the external velocity.

Figure 4 shows agreement between $f_B$ from $A_0$ and $A_k$ which resolves a long-standing problem. Ref. [1] found a disagreement of $O(10\%)$ when comparing the tree-level matrix elements for clover fermions. This ignores the contribution of $A^3_k$ and does not allow for the clover equivalent of the different renormalisation of $A^0_{0,k}$ and $A^0_{0,k}$. Both these effects are included here.

5. Conclusions

We have demonstrated the usefulness of optimal (i.e. narrow) smearing for heavy quarks in moving heavy-light mesons, and of constrained curve fitting in extracting results out to much higher momenta than have previously been attempted. Good agreement is found for $f_B$ at zero and non-zero momentum. For the temporal axial current at large $ma$ this holds out to the highest momenta studied. To understand the behaviour of the currents at non-zero momentum it is important to use a power-counting appropriate to moving heavy-light mesons.

Acknowledgements SC, GC, CD and JH are members of the UKQCD collaboration. We are grateful to the following bodies for support of this work: PPARC, DoE, NSF and the EU under HPRN-2000-00145 Hadrons/Lattice QCD.

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