Relativistic models of the neutron-star matter equation of state

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Motivated by a recent astrophysical measurement of the pressure of cold matter above nuclear-matter saturation density [1], we compute the equation of state of neutron star matter using accurately calibrated relativistic models. The uniform stellar core is assumed to consist of nucleons and leptons in beta equilibrium; no exotic degrees of freedom are included. We found the predictions of these models to be in fairly good agreement with the measured equation of state. Yet the Mass-vs-Radius relations predicted by these same models display radii that are consistently larger than the observations.

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The quest for the Holy Grail of Nuclear Physics—the equation of state (EOS) of hadronic matter—remains an area of intense activity that cuts across a variety of disciplines. Indeed, the limits of nuclear existence, the dynamics of heavy-ion collisions, the structure of neutron stars, and the collapse of massive stellar cores all depend sensitively on the equation of state. With the advent and commissioning of sophisticated new radioactive beam facilities, powerful heavy-ion colliders, telescopes operating at a variety of wavelengths, and more sensitive gravitational wave detectors, one will be able to probe the nuclear dynamics over a wide range of nucleon asymmetries, temperatures, and densities. However, in the present contribution we focus on the dynamics of cold matter under extreme conditions of density (both small and large) and for this case neutron stars remain the tool of choice [2–4]. Being both very compact and extremely dense, neutron stars are unique laboratories for probing the equation of state of neutron-rich matter under conditions unattainable by terrestrial experiments.

Intimately connected to the equation of state of cold, neutron-rich matter is the mass-vs-radius (M-R) relationship of neutron stars. Indeed, an EOS is the sole ingredient that must be supplied to solve the equations of stellar structure (i.e., the Tolman-Oppenheimer-Volkoff equations). Conversely, knowledge of the M-R relation is sufficient to uniquely determine the equation of state of neutron star matter [5]. As argued by Lindblom almost 20 years ago, the availability of such information—even from a single neutron star—will provide interesting information about the equation of state [5]. Viewed in this light, the recent report of combined mass-radius measurement for three neutron stars and the subsequent determination of the equation of state is significant [1]. In particular, the conclusion that the EOS so determined is softer than those containing only nucleonic degrees of freedom is both interesting and provocative.

In this contribution we compute the equation of state of neutron star matter and the resulting M-R relation using accurately-calibrated relativistic mean-field models. These models have been calibrated to the properties of infinite nuclear matter at saturation density [6], to the ground-state properties of finite nuclei [7–8], or to both [9]. Unlike the former two, the latter parametrization predicts a significantly soft symmetry energy, a feature that appears consistent with the behavior of dilute neutron matter (see Ref. [10] and references therein). A detailed explanation of the role of the model parameters on the equation of state is given below. We note, however, that none of the models considered in this work include exotic degrees of freedom, such as hyperons, meson (condensates), or quarks. In this regard, our results are mixed when compared with the conclusions of Ref. [1]. On the one hand, the stellar radii predicted by the relativistic models are larger than observed, seemingly confirming that such equations of state are too stiff. On the other hand, the agreement between the predicted and observed EOS suggests the opposite.

The structure of neutron stars is sensitive to the equation of state of cold, fully catalyzed, neutron-rich matter over an enormous range of densities [2–4]. For the low-density outer crust we employ the equation of state of Baym, Pethick, and Sutherland [11]. At densities of about a third to a half of nuclear-matter saturation density, uniformity in the system is restored and for this (liquid-core) region we use an EOS derived from a representative set of accurately calibrated relativistic mean-field models [9–10]. It has been speculated that the region between the outer crust and the liquid core consists of complex and exotic structures, collectively known as nuclear pasta [12–13]. Whereas significant progress has been made in simulating this exotic region [15], a detailed equation of state is still missing. Hence, we resort to a fairly accurate polytropic EOS to interpolate between the solid crust and the uniform liquid interior [15]. To compute the transition density from the liquid core to the solid crust we employ a relativistic random-phase-approximation (RPA) analysis to
TABLE I: Parameter sets for the four models used in the text to generate the equation of state. The parameter κ and the meson masses $m_n$, $m_\nu$, and $m_\rho$ are all given in MeV. The nucleon mass has been fixed at $M=939$ MeV in all the models.

| Model | $m_n$  | $m_\nu$ | $m_\rho$ | $g_\phi^2$ | $g_\rho^2$ | $g_\rho^4$ | $\kappa$ | $\lambda$ | $\zeta$ | $\Lambda_v$ |
|-------|--------|---------|---------|------------|-----------|-----------|---------|---------|---------|------------|
| NL3   | 508.194| 782.501 | 763.000 | 104.3871   | 165.5854  | 79.6000   | 3.8599  | -0.0159 | 0.00    | 0.00       |
| MS    | 485.000| 782.500 | 763.000 | 111.0426   | 216.8998  | 70.5941   | 0.5082  | +0.02772| 0.06    | 0.00       |
| FSU   | 491.500| 782.500 | 763.000 | 112.1996   | 204.5469  | 138.4701  | 1.4203  | +0.02376| 0.06    | 0.03       |
| XS    | 491.500| 782.500 | 763.000 | 131.0059   | 258.1044  | 213.9596  | 0.0079  | +0.04339| 0.09    | 0.04       |

search for the critical density at which the uniform system becomes unstable to small amplitude density oscillations [19].

Accounting for most of the stellar radius and practically all of its mass, the liquid core is structurally the most important component of the star. Matter in the liquid core is assumed to be composed of neutrons, protons, electrons, and muons in chemical equilibrium. We reiterate that no exotic degrees of freedom are included in the model. Both electrons and muons are treated as non-interacting relativistic Fermi gases. For the hadronic component, the equation of state is generated using non-interacting relativistic Fermi gases. For the hadronic component, the equation of state is generated using accurately-calibrated relativistic models. Details on the calibration procedure may be found in Refs. [20–23]. The model includes a nucleon field ($\psi$) interacting via standard Yukawa couplings to two isoscalar mesons (a scalar $\phi$ and a vector $V^\mu$) and one vector-isovector meson ($b^\mu$) [20, 21]. Such an interacting Lagrangian density may be written as follows [6, 20, 21]:

$$\mathcal{L}_{\text{int}} = \bar{\psi} \left[ g_\phi \phi - \left( g_\phi V_\mu + \frac{g_\rho}{2} \tau \cdot b_\mu \right) \gamma^\mu \right] \psi - U(\phi, V^\mu, b^\mu).$$

(1)

In addition to the Yukawa couplings ($g_\phi$, $g_\rho$, and $g_\rho$), the model is supplemented by non-linear meson interactions given by

$$U(\phi, V^\mu, b^\mu) = \frac{\kappa}{3!} (g_\phi \phi)^3 + \frac{\lambda}{4!} (g_\phi \phi)^4 - \frac{\zeta}{4!} (g_\rho^2 V_\mu V^\mu)^2 - \Lambda_v \left( g_\rho^2 b_\mu \cdot b^\mu \right) \left( g_\rho^2 V_\mu V^\mu \right).$$

(2)

The inclusion of scalar cubic ($\kappa$) and quartic ($\lambda$) self-interactions dates back to the late seventies [24] and is instrumental for softening the incompressibility coefficient of symmetric nuclear matter, as required to explain the excitation of the nuclear breathing mode [25].

Of particular interest and of critical importance to the present study are the vector self-interaction ($\zeta$) and the isoscalar-isovector mixing term $\Lambda_v$ [9, 22]. That both of these parameters are zero in the enormously successful NL3 model (see Table I) suggests that existing laboratory data are fairly insensitive to the physics encoded in these two parameters. Indeed, Müller and Serot found possible to build models with different values of $\zeta$ that reproduce the same observed properties at normal nuclear densities, yet produced maximum neutron star masses that differ by almost one solar mass [6]. This result indicates that observational data on neutron stars—rather than laboratory experiments—may provide the only meaningful constraint on the high-density component of the equation of state. Further, it indicates that the empirical parameter $\zeta$ provides an efficient tool to control the high-density component of the equation of state.

The isoscalar-isovector coupling constant $\Lambda_v$ was introduced in Ref. [22] to modify the poorly known density dependence of the symmetry energy. The symmetry energy represents the energy cost involved in changing protons into neutrons (and vice-versa). To a good approximation, it is given by the difference in energy between pure neutron matter and symmetric nuclear matter. With only one isovector parameter ($g_\rho$) to adjust, relativistic mean-field models have traditionally predicted a stiff symmetry energy. The addition of $\Lambda_v$ provides a simple—yet efficient and reliable—method of softening the symmetry energy without compromising the success of the model in reproducing well determined ground-state observables [9]. Indeed, whereas models with different values of $\Lambda_v$ reproduce the same exact properties of symmetric nuclear matter, they yield vastly different predictions for both the neutron radii of heavy nuclei and for the radius of neutron stars [22, 20]. Given that the neutron star radius is believed to be primarily controlled by the symmetry pressure at intermediate densities [4], the upcoming Parity Radius Experiment (PREx) at the Jefferson Laboratory (with an imminent start date of March, 2010) will provide a unique laboratory constraint on a fundamental neutron star property [27, 25].

In summary, the two empirical parameters $\zeta$ and $\Lambda_v$ provide a highly economical and efficient control of the softness of the high-density component of equation of state and of the symmetry pressure at intermediate densities, respectively—with the former primarily controlling the maximum neutron star mass and the latter the stellar radius. Parameter sets for all the models employed in this work are listed in Table I.
In Fig. 1 we compare observational results for three neutron star masses and radii against the model predictions. These neutron stars are in the binaries 4U 1608-52 [29], EXO 1745-248 [30], and 4U 1820-30 [31]. The very stiff behavior of the NL3 equation of state is immediately evident. With both empirical parameters $\zeta$ and $\Lambda_v$ set equal to zero, it is not surprising that the NL3 model predicts neutron star masses as large as $2.8 M_\odot$ with very large radii. As compared to the observational data, the NL3 model suggests a radius for a 1.7 solar-mass neutron star that is about 6 km too large. Moreover, the NL3 equation of state is so stiff that gravity in a 2.8 $M_\odot$ neutron star can compress matter to only about four times normal nuclear density (see Table II). All these, even when the model provides an excellent description of many laboratory observables.

As first suggested by Müller and Serot [6], adding a vector self-interaction (with $\zeta = 0.06$) dramatically reduces the repulsion at high densities and ultimately the limiting neutron star mass. As compared to the NL3 parameter set, the maximum neutron star mass predicted by Müller and Serot (MS) is reduced by almost one solar mass (see Fig. 1 and Table II). Consistent with this softening is a significant increase in the compactness of the star. For example, for a neutron star mass of 1.8 $M_\odot$, NL3 predicts a stellar radius that is more than 3 kilometers larger than MS. Note, however, that the density dependence of the symmetry energy predicted by NL3 and MS is practically identical (see inset in Fig. 2). In particular, this is reflected in the identical prediction of 0.28 fm for the neutron-skin thickness of $^{208}\text{Pb}$. This suggests that tuning the density dependence of the symmetry energy—via the addition of the isoscalar-isovector mixing term $\Lambda_v$—may yield a further reduction in neutron star radii [20], as suggested by observation.

Incorporating information on nuclear collective modes in the calibration procedure of the FSUGold model favors a non-zero value for $\Lambda_v$ [9]. Further, it now seems that the resulting softening of the symmetry energy is consistent with the EOS of dilute neutron matter predicted by various microscopic approaches (see Refs. [10, 32, 34] and references therein). That the addition of $\Lambda_v$ produces the intended effect can be appreciated in Fig. 1 and Table II. That is, although one has adopted the same value of $\zeta$ for both MS and FSUGold, their predictions for the radius of a “canonical” 1.4 solar-mass neutron star differ by more than one kilometer. Related to this fact is the significantly smaller neutron-skin thickness of $^{208}\text{Pb}$ predicted by FSUGold (0.21 fm vs 0.28 fm). However, it appears that the combined softening of the EOS at high densities (through $\zeta$) and of the symmetry pressure (through $\Lambda_v$) is insufficient to explain the observational data; the minimum stellar radius predicted by the FSUGold model is about 11 km, significantly larger than suggested by observation.

In an effort to describe the observational data, we have constructed an “Extra Soft” (XS) relativistic mean-field model constrained by the properties of symmetric nuclear matter at saturation density (i.e., equilibrium density, binding energy per nucleon, and incompressibility coefficient). In regards to these properties, the model is indistinguishable from FSUGold. The only additional constraint imposed on the model is that its limiting mass be no smaller than 1.6 solar masses. We feel that lowering this limiting value any further may start conflicting with the observational data. Although no exhaustive parameter search was conducted, we trust that the resulting extra-soft equation of state (as given in Table I) is representative of the softness that may be achieved with present-day relativistic mean-field models. With such a soft model, neutron star radii get significantly reduced indeed (see Fig. 1). For example, the neutron radius of a 1.4 $M_\odot$ neutron star is reduced by almost one kilometer relative to the FSUGold prediction (see Table II) and by more than 1.5 km at its limiting mass of 1.6 $M_\odot$. Still, the minimum neutron star radius of $R = 10.41$ km predicted by the model remains outside the reported 1$\sigma$

| Model | $\rho$ | $P$ | $M$ | $R$ | $R_{1.4}$ |
|-------|------|----|-----|----|--------|
| NL3   | 0.667| 440.58 | 2.78 | 13.39 | 15.05 |
| MS    | 1.040| 311.92 | 1.81 | 11.64 | 13.78 |
| FSU   | 1.153| 345.78 | 1.72 | 10.97 | 12.66 |
| XS    | 1.252| 345.37 | 1.60 | 10.41 | 11.73 |
Do we then conclude that the results presented in Fig. [1] are indicative of relativistic equations of state that are too stiff? Do the observational results unambiguously called for a softer equation of state, as would be produced by exotic states of matter, such as meson condensates and/or quark matter? To answer this question we compare in Fig. [2] the various equations of state used to generate Fig. [1] against the values extracted from the observational data [1]. The inset in the figure displays the symmetry pressure for the models under consideration. The observed softening of the symmetry pressure between models is entirely due $\Delta^*$, Note, however, that unlike the neutron-skin thickness of neutron-rich nuclei, the radius of a neutron star is not uniquely constrained by the symmetry pressure at low to intermediate densities [26]. Thus models with similar symmetry pressures may—and do—predict significantly different stellar radii. Contrary to the expectations generated by Fig. [1] most of the equations of state are not too stiff. Indeed, with the exception of NL3, the remaining equations of state appear, if anything, slightly too soft at the highest density. Based on these results—and these results alone—nucleonic equations of state do not seem to be in conflict with the observational data.

In summary, the Mass-vs-Radius relation of neutron stars was computed using equations of state derived from relativistic mean-field models. Although the models are calibrated in the vicinity of nuclear-matter saturation density, it is possible to tune their high-density behavior in a highly efficient and economical manner. In this contribution we have used two parameters to control the maximum neutron star mass and the stellar radius. As we compared our predictions to the observational data a conflict emerged. Whereas one could generate equations of state that are in agreement with observation, the predicted stellar radii are too large. This result is particularly intriguing given that “inversion” methods exist for extracting the equation of state of stellar matter directly from masses and radii of neutron stars [3]. Thus, one would expect that if the $M$-$R$ predictions do not match observation, neither would the equations of state. Clearly, to reconcile these facts much work remains to be done in both the observational and theoretical fronts. For now we must conclude—although the existence of exotic stars is very appealing—that the downfall of the purely nucleonic equations of state may be premature.

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FIG. 2: (Color online) Equation of state (Pressure vs baryon density) of neutron star matter predicted by the four relativistic mean-field models discussed in the text. The three data point are from the observational extraction as described in Ref. [1]. The symbols (stars) indicate the central density and pressure for the maximum-mass neutron star. The inset shows the symmetry pressure, given as the pressure of pure neutron matter minus that of symmetric nuclear matter.

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