Active identification of object parameters with non-scalar inputs-outputs

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Abstract. In this work, within the framework of the active identification problem, the recurrent least squares method is used to determine the object parameters with non-scalar signals at the input and output of the system. The proposed technique is shown using the example of a third order object. Calculations are executed in Simulink environment. Parameter estimates are received for a discrete description of a third-order object as a result of modeling in Simulink environment. Convergence graphs of parameter estimates to the base values of the in question object are given. It should be noted the rapid convergence of parameter estimates. A meander type signal was used to test the proposed parameter estimation algorithm.

1. Introduction
It should be noted that when calculating control systems of various purposes, there are errors in indicating values of object parameters. Therefore the task of finding an optimal input signal for the object parameters estimating remains relevant in practical applications. Different approaches to the development of the system identification algorithms are reflected in the works [1-7]. The task of the object parameters determination arises in many practical applications. One approach to the solving such problems is the ability to send test signals to the system. In works [8, 9] the Fisher matrix is built to obtain the estimates of the object parameters. The authors of the work [10] consider the least squares method for linear time-varying data model. In work [11] the iterative procedure modeling of the parameters estimation was executed for the case where two unknown parameters are present in the describing of a dynamic object.

2. Problem definition
A static object with a matrix input and a noisy matrix output is considered as follows:

\[ Y = X^T A + V, \]

where \( X \) – the matrix input signal of the \( n \times n \) dimension; \( Y \) – the matrix output signal of the \( n \times n \) dimension; \( V \) – the matrix Gaussian noise with zero mean; \( A = [a_{ij}]_{n \times n} \) - the matrix of unknown parameters. A recursive least squares method is described in work [3] for the case where \( X \) is the vector input and \( Y \) is the scalar output. You want to define unknown parameters of the \( A = [a_{ij}]_{n \times n} \) matrix. This article proposes a procedure for the case where a non-scalar (matrix) input signal and a non-scalar (matrix) output signal are used to describe an object.
The parameters estimates are obtained by the recursive least squares method [3]:

\[ A_n = (X_n^T X_n)^{-1} X_n^T Y_n, \]  

where \( X_n = (X_1, X_2, \ldots, X_N)^T \), \( Y_n = (Y_1, Y_2, \ldots, Y_N)^T \) and \( N \) is measurements.

The following formula is used for the \( P_n \) matrix of the \( n \times n \) dimension:

\[ P_n = (X_n^T X_n)^{-1}. \]  

The formula (3) can be written for \( N + 1 \) measurement as follows:

\[ P_{n+1} = (X_n^T X_n + X_{n+1}^T X_{n+1})^{-1}. \]

An iterative algorithm for the unknown parameters determination is given below:

\[ A_{n+1} = A_n + K_{n+1}(Y_{n+1} - X_{n+1}^T A_n), \]

\[ K_{n+1} = P_n X_{n+1}(I + X_{n+1}^T P_n X_{n+1})^{-1}, \]

\[ P_{n+1} = (I - P_n X_{n+1} X_{n+1}^T (I + X_{n+1}^T P_n X_{n+1})^{-1}) P_n, \]

where \( K_{n+1} \) is gain matrix of the \( n \times n \) dimension, \( P_{n+1} \) matrix is calculated from \( N + 1 \) measurements.

3. Experimental

Due to the fact that the formulas are rather cumbersome, we will further consider the case \( n = 3 \). The static object of the third order is considered in the following form:

\[
\begin{bmatrix}
  y_{11} & y_{12} & y_{13} \\
  y_{21} & y_{22} & y_{23} \\
  y_{31} & y_{32} & y_{33}
\end{bmatrix} = \begin{bmatrix}
  x_{11} & x_{12} & x_{13} \\
  x_{21} & x_{22} & x_{23} \\
  x_{31} & x_{32} & x_{33}
\end{bmatrix}^T \begin{bmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{bmatrix} + \begin{bmatrix}
  v_{11} & v_{12} & v_{13} \\
  v_{21} & v_{22} & v_{23} \\
  v_{31} & v_{32} & v_{33}
\end{bmatrix}
\]  

Simulation was performed at the following base values: \( a_{11} = 1, \ a_{12} = 2.5, \ a_{13} = 0.5, \ a_{21} = 0.5, \ a_{22} = 1, \ a_{23} = 0.5, \ a_{31} = 1, \ a_{32} = 0.9, \ a_{33} = 0.5 \). Matrix with random values formed by random number generator from the [0,1] interval is selected as initial value of \( P \) matrix: \( P_0 = [0.9, 0.2, 0.1; 0.3, 1.0, 0.2; 0.3, 0.5, 1.0] \). The object and input signals are simulated in the Simulink environment. Each step of the recurrent estimation based on the least squares method is designed as an independent computing block [11]. Input and output parameters are defined for each block. Blocks for the calculation of parameters estimates are shown in Figure 1.

![Figure 1. The calculation of parameters estimates.](image-url)
signals for the object (7) are given in Figure 2 (input signal \((x_{12}, x_{22}, x_{32})^T\) with periods \(T = 14\), \(T = 10\), \(T = 9\)).

![Figure 2. Test signals.](image)

The output signals are given in Figure 3 (output signal \((y_{12}, y_{22}, y_{32})^T\)). The following values of estimated parameters were obtained for case \(N = 50\) measurements: \(a_{11} = 1.02\), \(a_{12} = 2.576\), \(a_{13} = 0.55\), \(a_{21} = 0.546\), \(a_{22} = 1.023\), \(a_{23} = 0.521\), \(a_{31} = 1.017\), \(a_{32} = 1.065\), \(a_{33} = 0.546\).

![Figure 3. Output signals.](image)

The results of the parameters estimation at \(N = 50\) measurements are demonstrated in Figure 4 (for case \((a_{12}, a_{22}, a_{32})^T\)). The following values of the estimated parameters were obtained for case
After 50-60 steps, it can be observed in graphs that parameter estimates converge to base values.

Figure 4. The estimated parameters for case $N = 50$ measurements.

The behavior of the gain matrix coefficients included in the recursive least squares algorithm is shown in Figure 5 (coefficients $(K_{12}, K_{22}, K_{32})^T$). The following values of the gain matrix coefficients were obtained for case $N = 50$ measurements: $K_{11} = 0.0059$, $K_{12} = -0.0059$, $K_{13} = -0.0073$, $K_{21} = 0.0049$, $K_{22} = -0.0049$, $K_{23} = -0.099$, $K_{31} = 0.0039$, $K_{32} = -0.0039$, $K_{33} = 0.0112$.

Figure 5. The behavior of the gain matrix coefficients.
The following values of $P$ matrix coefficients were obtained for case $N = 100$ measurements:

\[ P_{11} = 0.0033, \quad P_{12} = 0.0003, \quad P_{13} = -0.0003, \quad P_{21} = 0.0002, \quad P_{22} = 0.0039, \quad P_{23} = -0.0014, \]

\[ P_{31} = -0.0004, \quad P_{32} = -0.0014, \quad P_{33} = 0.0038. \]

4. Conclusion

The use of the recurrent least squares method to the static object parameters determination with a matrix input and a matrix output is shown in this paper. Estimates schedules obtained during the operation of the algorithm are plotted on the example of a third-order object. The fast convergence of parameter estimates is noted. A meander type input signal is used to derive parameter estimates. The impact of periods and amplitudes of test inputs on the accuracy of the calculated estimates remains an open question. The case where the dimensions of the $X$ input, $Y$ output and $A$ object matrices do not coincide is also of interest for further investigation. The proposed approach can be used, for example, in solving the active identification problem of an inverted pendulum control system, which is a good model for describing missile take-off control. To specify the object parameters included in the automatic control system, low-order regulators can be used, in which the regulator parameters tuning procedure is realized.

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