ERRATUM TO “A QUOTIENT CRITERION FOR SYzyGIES IN EQUIVARIANT COHOMOLOGY”

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Abstract. We correct a mistake in Proposition 3.3 of the paper. All other results remain unchanged.

The proof of Proposition 3.3(ii) in [1] is incorrect. To address this, we modify the statement and proof of this part. We also take the opportunity to point out two minor inaccuracies in [1]: In line 2 after equation (1.3), rank $Q$ denotes the common dimension of the orbits in $X$ lying over the interior of $Q$. In line 2 before Section 2.2, we call $K$ an isotropy subtorus occurring in $X$ if it is the identity component of the isotropy group $T_x$ of some $x \in X$.

The corrected proposition reads as follows:

Proposition 3.3.

(i) For any subtorus $K \subset T$,

$$\text{depth}_R H^*_T(X^K) \geq \text{depth}_R H^*_T(X).$$

(ii) $H^*_T(X)$ is a $j$-th syzygy if and only if

$$\text{depth}_{R_L} H^*_L(X^K) \geq \min(j, \dim L)$$

for any subtorus $K \subset T$ with quotient $L = T/K$. If $X$ is a $T$-manifold, then it suffices to look at the isotropy subtori $K$ occurring in $X$.

(iii) If $H^*_T(X)$ is a $j$-th syzygy over $R$, then so is $H^*_L(X^K)$ over $R_L$ for any subtorus $K \subset T$ with quotient $L = T/K$.

Since Proposition 3.3(ii) is only applied to $T$-manifolds in [1], this change does not affect the rest of the paper.

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Proof. Part (i) and the equivalence in part (ii) are established by the same arguments as in [1].

Consider now a $T$-manifold $X$ satisfying the depth condition for all isotropy subtori occurring in it, and let $K \subset T$ be a subtorus that does not occur. Each connected component $Y$ of $X^K$ is a $T$-manifold. Let $T/\bar{K}$ be the principal orbit type of $Y$. Then the identity component $K'$ of $\bar{K}$ occurs in $X$ and properly contains $K$, and $Y$ is a connected component of $X^{K'}$. Write $\dim K' = \dim K + s$ and $L' = T/K'$. By assumption and Lemma 3.1 (ii), we have

\[
\text{depth}_{R_L} H^*_L(Y) \geq \text{depth}_{R_L} H^*_L(X^{K'}) = \text{depth}_{R_{L'} H^*_{L'}}(X^{K'}) + s \\
\geq \min(j, \dim L') + s \geq \min(j, \dim L).
\]

Because this holds for any connected component $Y$ of $X^K$, the depth condition is satisfied for $K$, too.

The proof of part (iii) remains unchanged, except that one takes all subtori of $L = T/K$ into account and not just the ones appearing in $X$. \hfill \square

The mistake made in [1] is that for an arbitrary $T$-space $X$ and a subtorus $K \subset T$ the fixed point set $X^K$ is in general not the disjoint union of fixed point sets of subtori $K'$ occurring in $X$.

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References

[1] M. Franz, A quotient criterion for syzygies in equivariant cohomology, Transform. Groups 22 (2017), no. 4, 933–965.

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