Probabilistic Thinking of Senior High School Students with Low Mathematical Abilities in Solving Probability Tasks

R Raya1*, St. Suwarsono2, and A Lukito3

1 Universitas Cendrawasih, Jl. Camp Wolker, Waena, Jayapura 99358, Indonesia
2 Universitas Sanata Dharma, Jln. Stm Mrican, Caturtunggal, DIY 55002, Indonesia
3 Universitas Negeri Surabaya, Jl. Lidah Wetan, Surabaya 60231, Indonesia

*e-mail: ritaraya68@gmail.com

Abstract. Probabilistic thinking of each individual in solving probability tasks is different. Different mathematics ability also causes different process of student thinking probabilistic. This study aimed at describing the profile of probabilistic thinking of high school students with low mathematics abilities in solving probability tasks. This qualitative research was a case study. The subject in this study was a female student in 11th grade of science that had learned probability and had low mathematics ability selected based on mathematics ability test. The subject was given Problem Solving Tasks then she was interviewed based on her written answers. Data credibility used time triangulation. The results showed that in solving probability tasks, subject used personal experiences found in her own family and extended family. The subject used intuitive strategy that tended to use heuristic thinking patterns by performing attribute substitution process. In this case, subject gave a non-statistical response. The results of this study can complement the existing theories and become input for teachers to use approaches and strategies in learning that can accommodate the probabilistic ways of thinking of high school students in solving probability tasks.

Keywords: Probabilistic Thinking, Mathematical Abilities, Solving Probability

1. Introduction

Probabilities are many things found in mathematics, in other sciences and are very closely related to everyday life and also in various professions, as expressed by [1] that “Probability opened new possibilities for dealing with uncertain and random situations that occur in almost every field of our life. Today, probability has become an integral component in every area.”

Mathematical curriculum in various countries includes probability and statistics as part of the school mathematical curriculum. The current national curriculums in Indonesia namely the Curriculum 2013 (K-13) also include statistics and probability topics as a part of mathematical lesson. Statistics and probability subject matters contain some daily problems faced by students in the form of problems that contain probabilistic situations. In solving the problems, someone needs to do reasoning or think to make a decision. The degree of one's belief about the possibility of an occurrence of an event is largely determined by the presence or absence of knowledge about the incident.

When someone is thinking of a problem that has various possibilities or contains elements of uncertainty, then he is thinking of probabilistic. This is in accordance with the opinion of Falk and
Konold [2] that “probabilistic thinking is an inherently new way of processing information as the world view shifts from deterministic view of reality.” Deterministic is a certainly thing.

[3] in their research used the term probabilistic thinking to describe children's thinking in responding to probabilistic situations. This shows that the response is one of the aspects studied in probabilistic thinking.

In connection with students’ responses in probabilistic thinking, [4], explored the ideas of students aged 14-16 years in facing probabilistic situations, especially in the opportunity of fair events. Based on the results of the analysis, Sharma identified four categories of students’ probabilistic thinking based on students’ responses that are non-response, non-statistical responses, partial-statistical responses and statistical responses.

In addition, [5] in their qualitative analysis said that there are four categories in observing students' probabilistic thinking, namely: (a) types of strategies, (b) representation, (c) use of probabilistic language, (d) the nature of cognitive obstacle. It means that strategy and representation are observable aspects related to students' probabilistic thinking. Next, Jan and Amit stated that thinking strategies include: beliefs, subjective estimations, chance estimations, and probabilistic calculations. Accordingly, the findings of [6] in her research relating to probabilistic situation indicate that many students use strategies based on cultural experiences (beliefs, everyday and school experiences) and intuitive strategies.

Jan and Amit (2009) said that representation is also an observable aspect with regard to students' probabilistic thinking. This is in line with the opinion of [1] stating that “Work in probability, as with in any other mathematical topic, is conducted via different representations and models, such as , table, venn diagram, area model, tree diagram, pipe diagram, formulas, etc”. [7] stated that,”A basic foundation for successful reasoning in solving probability problems is the recognition of an adequate representation of the sample space for an experiment.” It shows that representation is one aspect that is needed in solving probability problems because representation helps students understand the concept of probability.

In addition to research that aims to identify aspects of students’ responses, strategies, and representation in solving probability problems, there are many studies that focus on cognitive processes at micro level. For example, [8] used qualitative method to explore students' ways of thinking when addressing probability problems. [8] claimed that students tried to predict probability experimental results rather than use probability expectations to determine the most likely outcomes. He called this way of thinking as an outcome approach and gave the impression that maybe students used representativeness heuristic to try to predict results.

Since the 1970s [9] began to conduct a research that focused on students’ misconception associated with the framework of heuristic and bias in uncertain situations. Heuristics and bias are research fields closely related to judgment theory research. Heuristic and bias research try to identify and explain the right and wrong answers to probability problems based on different mental processes.

The revision of the heuristic framework and bias can be focused on two types of heuristic, namely: representativeness and availability. Both of these heuristics are believed to use the same heuristic thinking pattern, called attribute substitution. The attribute substitution process explains how errors in thinking occur when a target attribute in a problem is accidentally replaced by another attribute that is easier to come to mind.

[10] in his study used [11] attribute substitution model for mathematics prospective teachers. The mathematics prospective teachers were given two answer key models from the 10 multiple choice questions quiz. They were asked to determine and give the reasons for which of the two key answers were most likely to occur. The result of his research showed that there were certain participants that, when they were given questions, they answered using different questions.

A person's ability to deal with probabilistic thinking aspects in solving probability problems is different. [1] stated that different people will give different responses in the same situation. This is influenced by the differences in knowledge that each person has about the situation and the person's ability to understand the information provided by the problem. Other factors that cause student differences in probabilistic thinking include: cultural factors [6]; language, belief, and experience [12] and students' mathematical ability [7].
For this reason the researcher feels that she needs to know how the thinking profile of low-ability mathematics students of senior high school in Jayapura in solving probability problems. This case concerns with the aspects of students’ responses strategies and representation in solving probability problems.

2. Research Method

This study aimed to know the responses, strategies, and representation thinking of students with low mathematical ability in solving probability problems. The researcher asked the subjects to solve certain probability problems using task-based clinical interview method. Thus, this study was categorized as explorative research with a qualitative approach.

The research subject was student of class XI Science who had studied the material about probability and who had low mathematical ability. The instrument used for data collection was the researcher as the main instrument; the supporting instruments were in the form of Mathematical Ability Test (TKM) used to determine the category of research subjects, the Problem Solving Task (TPM) questions given to selected subjects and interview guidance instrument. Before Mathematical Ability Test (TKM) and Problem Solving Task (TPM) questions were used, validity test was first carried out by mathematicians and mathematics education experts on these instruments.

After the subjects solved the problem in writing, the researcher asked the subjects to explain how to obtain the solution and important arguments relating to the problem solving through an interview. Before the data was analyzed, checking data validation was first carried out. Checking the data validation was done through credibility testing. In this research, credibility testing was done through extending observation, increasing perseverance, discussion with peers and triangulation (time triangulation), namely comparing data from the results of interviews I and II.

3. Result and Discussion

The problem solving task given to the research subjects was taken from the research of [9], as follows:

In the survey of families with six children, the following births of children are found: LPLLLL, PLPLLP, and LLLPPP. (L state boy and P state girl)

i) If the children birth order of LPLLLL is compared to PLPLLP, which of the following sequences of births is more likely to occur?
   a. LPLLLL
   b. PLPLLP
   c. They are equally likely
   Give a reason for your answer.

ii) If the children birth order of PLPLLP is compared to LLLPPP, which of the following sequences of births is more likely to occur?
   a. PLPLLP
   b. LLLPPP
   c. They are equally likely
   Give a reason for your answer.

Three orders of children birth in a family with six children were given to the subject, where theoretically the three children birth orders had the same probability of occurring. Subject was asked to determine which order of children birth was more likely to occur and then provide the reasons for the choice through written responses.

The written answer given by the subject is as follows:
Snippets of the interview results on the subject in solving the problem in part i) are as follows:

P: After reading this problem, to answer the first part of the question, what was your first thought?
S: In the order of the birth of these 6 children, look for the most men.

P: Why?
S: Because in my big family, women are limited and men are more and even more than the limit (while smiling)

P: Ooo, is that so?
S: Yes. Like the children of mother's older sister, she has 2 daughters and 7 sons.

P: Are the other families like that?
S: My other mother's older sister has 3 children, but all of them are boys.

P: Is there more?
S: My other mother's older sister has 5 children, 4 boys and 1 girl. Most of my uncles and aunts have more sons.

P: What about your own family?
S: For my own family, there are 3 boys and 2 girls.

The interview result for part ii) is as follows:

P: Why do you choose answer b?
S: Because the first order is a boy.

P: Why do you choose the order with boy as the first one?
S: In our big family, most boys were born first. Like my family, the order of 5 children are boy, boy, girl, girl.

P: How is the birth order of your aunt who has 9 children?
S: I don't know exactly the order but the girls are all in the middle. The first and second children are boys.

P: Is there something else?
S: The older brother of my mother has 5 children in the order of LLPPL. There are many more like that, boys are ranked first.
In the next interview subject also said that the problem of birth order was not determined by the formula, but it was a sure gift from God.

Based on the results of written answers and interview, it was revealed that the profile of students' probabilistic thinking in solving problems related to the probability of an event is as follows:

**Response.** In solving this probability problem, subject answered the first part of the question based on the family of her uncles, aunts who mostly had more sons than daughters. Likewise with her family who had more sons than daughters. For the second part of the question, subject answered based on the family of her uncles, aunts who mostly had sons in the first and second births. Likewise with her family, in which the first, second and third children were boys. It means that in solving this task, subject referred to everyday experiences or make connections with other learning areas, namely the family area. Subject also referred to beliefs. It was reflected in subject’s statement which said that the problem of birth order was not determined by the formula but it was a sure gift from God. Thus, it can be said that subject gave a non-statistical response [6].

**Strategy.** When subject understood what was known and what was asked about, she immediately remembered the number of sons and the number of daughters from the family of her uncles, aunts and her own family. Subject immediately remembered with extended family and her own family that has more sons than daughters, so subject said that LPLLLL birth order was more likely to occur than PLPLL birth order (answer a). Subject also immediately remembered the birth order of her uncles’ and aunts’ children who generally had sons in the first and second births. Likewise with her own family, sons were in the first and second births, so subject said that LLLPPP birth order was more likely to occur than the PLPLL birth order (answer b). In this probabilistic problem, the target attribute was to compare the relative probability of the children birth order. However, the target attribute did not immediately appear in the mind, but the search generated the value of other attributes that were conceptually and associatively related [11]. Thus, it can be said that the strategy used by subject in solving the task was to use intuition which tended to use heuristic thinking patterns by doing the attribute substitution process. In this case, subject used representative as a heuristic attribute to assess the probability of an event (birth order).

**Representation.** To solving this probabilistic task, subject used a representation in the form of a list of sample space members based on the similarity of birth order in the extended family and based on her own thinking. Subject also used a verbal representation to express the probability of an event occurring. It was illustrated by the statement of the subject stating that the birth order on a task that was similar to the birth order in the extended family was certain.

4. **Conclusion**

The probabilistic thinking profile of female subject who has low mathematical ability in solving probabilistic problems is as follows:

The female subject with low mathematical ability gives non-statistical responses in solving the probabilistic problem because for the first question, subject answers based on her extended family that has more sons than daughters; and for the second question, subject answers based on her extended families, most of whom has sons in the first and second births. It means that the subject's response refers to everyday experiences, making connections with other learning areas, namely the family area. In addition, the subject's response also refers to beliefs.

Female subject with low mathematical ability uses intuitive strategy that tends to use heuristic thinking patterns by doing attribute substitution process. In this probabilistic problem, the target attribute is to compare the relative probability of children births order, while the heuristic attribute for the first question is the ratio of sons to daughters in extended families. Likewise for the second question, the target attribute is replaced by another trait related to the birth order of the children, namely the birth of sons in the first and second order in the extended family as heuristic attribute.

Female subject with low mathematical ability used a representation in the form of a list of sample space members and a verbal representation to express the probability of an event occurring.

This study does not see in terms of gender differences and differences in mathematical abilities. It is recommended for other researchers to examine more deeply the profile of probabilistic thinking of high school students in terms of gender differences, differences in mathematical abilities, and others.
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