Wilson lines and transverse-momentum dependent parton distribution functions: A renormalization-group analysis

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Abstract

The renormalization-group properties of gauge-invariant transverse-momentum dependent (TMD) parton distribution functions (PDF) in QCD are addressed. We perform an analysis of their leading-order anomalous dimensions, which are local quantities, making use of the renormalization properties of contour-dependent composite operators in QCD. We argue that attaching individual gauge links with transverse segments to quark fields in the light-cone gauge, the associated gauge contours are joined at light-cone infinity through a cusp-like junction point. We find that the renormalization effect on the junction point creates an anomalous dimension which has to be compensated in order to recover the results in a covariant gauge. To this end, we include in the definition of the TMD PDF an additional soft counter term (gauge link) along that cusped contour. We show that the eikonal factors entering this counter term are peculiar to the Mandelstam field formalism and are absent when one uses a direct gauge contour.

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I. INTRODUCTION

Theoretical interest in the use of Wilson lines (also termed gauge links or eikonal phases) has been greatly stimulated recently by both theoretical investigations and experiments on single spin asymmetries (SSA) [1, 2, 3, 4, 5, 6]—see also [7] for a quite recent review and further references. This interest appears in the context of a gauge-invariant formulation of parton distribution functions (PDF) in terms of hadronic matrix elements. Because these matrix elements involve quark–antiquark field operators at different spacetime points, one has to introduce a path-ordered gauge link of the form

\[ [y,x|C] = \mathcal{P} \exp \left[ -ig \int_{x|C}^{y} dz_{\mu} A_{\mu}^{a}(z)t_{a} \right] \] (1)

that restores gauge invariance, albeit introducing an implicit, i.e., functional dependence on the contour \( C \) adopted. Here, \( C \) is, in general, an arbitrary path in Minkowski space and \( \mathcal{P} \) denotes the path-ordering instruction that orders the Lie-algebra valued gluon fields with the earliest contour point \( x \) furthest to the right, whereas the final point \( y \) is put furthest to the left. [Throughout this work, \( t_{a} \) stands for the generator \( T^{(F)}_{a} \) of the fundamental representation (labelled \( F \)) of color SU(\( N_{c} \)). Note that there is an implied sum over the color index \( a \).]

The concept of gauge links is so pervasive throughout Yang-Mills theories because it ensures local gauge invariance independent of the particular dynamical theory. The concept of using contour-dependent operators in QCD is an old one and mostly studied in connection with the renormalization of singularities caused by contour obstructions, like end- or cross-points, and cusps—see, for instance, in \([8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31]\) and further references cited therein. The renewed interest in Wilson lines in SSA and day-to-day applications is due to the potential breakdown of universality in transverse-momentum dependent (TMD) parton distribution functions (or PDF for short) \([32, 33, 34, 35, 36, 37]\) and the ensuing discussion about a process dependence of the gauge link caused by the color flow of the particular hard-scattering process. As a consequence, it is argued that there is a change of the overall sign of the single transverse spin asymmetry from the deeply inelastic scattering (DIS) to the Drell-Yan (DY) case and, hence, a breakdown of universality \([32]\). The content and formulation of this question is coterminous with the properties of Wilson lines within the purview of specific gauge-fixing prescriptions and the imposition of boundary conditions at light-cone infinity.

This brings in questions about the role of final (initial) state interactions (FSI) between the struck quark and the target spectators that may yield important effects like shadowing and SSA \([38, 39, 40]\). The core issue here is the choice of the gauge adopted for the Wilson line that has to be inserted to ensure gauge invariance. For singular gauges, like the light-cone gauge \( A^{+} = 0 \), the gauge link vanishes by choice without any restriction put on the gauge potential \( A_{\perp} \) at infinity. Then, to avoid light-cone singularities at \( k^{+} = 0 \), specific boundary conditions on the gluon propagator have to be imposed in order to exhaust the remaining gauge freedom and recover the results obtained in non-singular gauges, say, in the Feynman gauge. It was shown in \([41]\), and further worked out and detailed in \([33]\), that the effects of FSI in the light-cone gauge can be properly taken into account by including a gauge link (an “eikonalized” quark line) which involves a path in the transverse direction. In covariant, i.e., non-singular, gauges this additional term does not contribute and, therefore, the modified definition, proposed by Belitsky, Ji, and Yuan, for the TMD PDF reduces to the correct gauge-invariant one. These important findings notwithstanding, yet a consistent
gauge-invariant picture for TMD PDFs in the whole phase space is still incomplete, because the behavior of the gauge contour at infinite transverse distance is largely arbitrary [41]. The crucial point is—as we have recently shown [42]—that splitting the gauge link and allowing the separated contours to stretch out to infinity in the transverse configuration space, induces an additional contribution that cannot be dispensed with by imposing suitable boundary conditions [33]. Instead, one has to compensate this new contribution by incorporating into the definition of the TMD PDF an eikonal factor which provides a soft counter term in the sense of Collins and Hautmann [43, 44, 45].

In the present work, we shall investigate these issues from the point of view of the renormalization group and address parton distribution functions—integrated and unintegrated—aiming for a more suitable definition of TMD PDFs. Our considerations will employ contour-dependent operators and we will calculate the leading gluon radiative corrections in the light-cone gauge. The renormalization of such operators is supremely simple to deal with when stated in terms of anomalous dimensions because these quantities are local and do not depend on the length of the gauge contour. Therefore, they provide a powerful tool to access the renormalization properties of Wilson lines and take into account those contributions originating from geometrical obstructions, notably, endpoints, or sharp bends in the contour, as first pointed out by Polyakov [8]. In fact, we will show in more detailed form than in our brief presentation in [42] (see also [46]) that, adopting the light-cone gauge, the leading gluon radiative corrections associated with the transverse gauge link give rise to an anomalous dimension that exhibits a \( \ln p^+ \) behavior—characteristic of a contour with a cusp. We will fathom out the physics underlying this finding—in particular, the renormalization effect on the junction point of two individual gauge contours joined non-smoothly through a cusp. Moreover, we will present a new definition for the TMD PDF that (i) reduces to the correct integrated case and (ii) coincides with the result obtained in the Feynman gauge that is untainted by contour obstructions (the reason being, we reiterate, that in this case \( A_\perp \) vanishes at infinity).

The remainder of the paper is organized as follows. In the next section, we give a summary of the kinematics used and sketch the spacetime picture of DIS. Section III discusses integrated PDFs and their gauge-invariance and renormalization-group properties. In Sec. IV we reinvigorate our statements on the transverse gauge link, presented briefly in [42], by a formal derivation, making use of a “classical” current along the lines of thought described by Jackiw, Kabat, and Ortiz in [47]. The calculation of the leading one-loop anomalous dimension of the TMD PDF in the light-cone gauge is outlined in Sec. V. Here we also present a generalized factorization rule for gauge links which takes into account the possibility that the contours may be joined non-smoothly via a cusp-like junction point. The same section contains the evaluation of the soft counter term to supplement the definition of the TMD PDF and its interpretation as an “intrinsic Coulomb phase”, in analogy to the phase found by Jakob and Stefanis [48] for QED within a manifestly gauge-invariant formulation in terms of Mandelstam fields [49, 50]. In Sec. VI we turn our attention to the real-gluon contributions and the evolution equations, providing a tangible proof that the integrated PDF obtained from our modified TMD PDF definition coincides with the correct one with no artefact of the cusped contour left over. Section VII addresses the application of our approach to the Drell-Yan case. Finally, a summary and further discussion of our findings together with our conclusions is given in Sec. VIII.
II. KINEMATICS AND SPACETIME PICTURE OF (SI)DIS

In what follows, we employ null-plane coordinates with
\[ P^\mu = (P^+, P^-, P_\perp) \text{, } P^\pm = (P^0 \pm P^3)/\sqrt{2} \text{, } P^2 = 2P^+P^- - P_\perp^2, \]
where \( P^+ > 0 \) is large and the other components \( P^- \), \( P_\perp > 0 \) are small. Moreover, we will visualize the spacetime structure of the chief hadronic reactions, like DIS, Semi-Inclusive Deep-Inelastic lepton-nucleon Scattering (SIDIS), etc., as a series of snapshots on a plane of equal \( x^+ \) as depicted in Fig. 1 [For a pedagogical exposition of this graphical method, see, e.g., Refs. [51, 52], and the review in Ref. [53].]

Let us now fix the kinematics relevant for the cases to be considered in our work. We introduce two light-cone vectors
\[ n^\mu = \Omega(1,1,0_\perp), \text{ } n^\mu = \frac{1}{2\Omega}(1,-1,0_\perp) \]
with the following properties of their plus/minus light-cone components
\[ n^{+} = \sqrt{2}\Omega, \text{ } n^{-} = 0, \text{ } n^{+} = 0, \text{ } n^{-} = \frac{1}{\sqrt{2}\Omega}, \text{ } n^+n = 1, \text{ } (n^+)^2 = n^2 = 0, \]
where \( \Omega \) is an arbitrary parameter having the dimension of mass. Then, the momentum of the initial hadron reads
\[ P^\mu = n^\mu + \frac{M^2}{2}n^\mu, \text{ } P^2 = M^2. \]
The momentum of the struck quark before being “measured” \([51]\) by the photon is
\[ k_{\text{in}}^\mu = xP^\mu + k_\perp^\mu \rightarrow k_{\text{in}}^+ = xP^+ = \sqrt{2}x\Omega \]
with
\[ k_{\text{in}}^- = \frac{xM^2}{2\sqrt{2}\Omega}, \text{ } k_\perp^\mu = (0^+, 0^-, k_\perp), \text{ } k_{\text{in},\perp} = k_\perp, \]
whereas the off-shell photon has the momentum
\[ q^\mu = -x'n^\mu + \frac{Q^2}{2x'}n^\mu \rightarrow q^+ = -\sqrt{2}x'\Omega, \text{ } q^- = \frac{Q^2}{2\sqrt{2}x'\Omega}. \]
After the interaction with the highly virtual photon the struck quark acquires the momentum
\[ k_{\text{out}}^\mu = (k_{\text{in}}^\mu + q^\mu) = (x - x')n^\mu + \frac{xx'M^2 + Q^2}{2x'}n^\mu. \]
Choosing a specific Lorentz frame upon setting \( \Omega = Q/(2x') \), corresponds to an almost lightlike hadron that moves along the plus direction. In the Bjorken limit \( Q^2 \rightarrow \infty \), the variables \( x' \) and \( x \) coincide up to \( O(M^2/Q^2) \) terms, so that one gets for the photon
\[ q^+ = -\frac{Q}{\sqrt{2}}, \text{ } q^- = \frac{Q}{\sqrt{2}}, \]
meaning that the photon moves along the negative \( x^3 \)-direction, whereas the struck quark (after being probed by the photon) moves along the minus direction to infinity:
\[ k^+ = 0, \text{ } k^- = \frac{Q^2}{2\sqrt{2}x\Omega} = \frac{Q}{\sqrt{2}} = q^- . \]
\[
\begin{align*}
\delta x_+^{-} & \sim 1/P^+ \to \text{small}, \quad \delta x_+^{+} \sim 1/P^- \to \text{large}, \\
\delta x_+^{k^+} & \sim 1/k^+ \to \text{large}, \quad \delta x_+^{k^-} \sim 1/k^- \to \text{small}.
\end{align*}
\]

In the next section, the spacetime picture, described above, will be used to motivate the introduction of the extra transverse gauge link.

**III. INTEGRATED PDFs: GAUGE-INvariance AND REnormalization-GROUP PROPERTIES**

A well-known example for an integrated PDF is provided by the single parton distribution of a quark of fractional longitudinal momentum \(x\) and flavor \(i\) in a fast moving hadron which contains the nonperturbative physics in DIS:

\[
\begin{align*}
\bar{f}_i/H(x) &= \frac{1}{2} \int \frac{d\xi^-}{2\pi} e^{-ik^+\xi^-} \langle H(P) | \bar{\psi}_i(\xi^-, 0_\perp) \gamma^+ \psi_i(0^-, 0_\perp) | H(P) \rangle, 
\end{align*}
\]

where the quark momentum is defined by \(k^+ = xP^+\) with \(x = Q^2/(2P \cdot q)\).

To give this expression a physical meaning, i.e., elevate it to an observable, one has to ensure that it is gauge invariant.\(^2\) To achieve this goal, one usually introduces ad hoc a gauge-link operator

\[
[y^-, x^-] = \mathcal{P} \exp \left[ -ig \int_{x^-}^{y^-} dz^- A^+_a(0, z^-_\perp, 0_\perp) t_a \right]
\]

\(^1\) Strictly speaking, this is an unrenormalized quantity. The renormalization properties of the operators will be considered shortly in detail.

\(^2\) Otherwise, one would have to define a “standard” gauge to be used in all calculations in order to make them comparable because what is a quark in one gauge is a quark plus gluons in another.
which is path ordered along a lightlike line from the point \(x^-,\) where the quark was removed from the hadronic state by the annihilation operator \(\psi_i(0^-, 0_\perp)\), to the point \(y^-,\) where it was recreated by \(\bar{\psi}_i(\xi^-, 0_\perp)\). The gauge-link operator represents the struck quark as an eikonal parton \([54]\) with fixed color charge \(g\), after its interaction with the highly virtual photon \((0 < Q^2 = -q^2 \gg 1 \text{ GeV}^2)\) in the DIS process, as it moves with (almost) the speed of light along a lightlike line in the \(x^-\) direction. The spacetime visualization of this process is illustrated in Fig. 1(b).

It should be clear that, ultimately, expressions (14) and (15) have to be quantized, using, for instance, functional-derivative techniques. This means that in employing Eq. (15) in Eq. (14), the gluon potential in the gauge link has to be Wick contracted with corresponding terms in the interaction Lagrangian, accompanying the Heisenberg fermion (quark) field operators. The consequence of this operation is that all terms in the expectation value in (14), proportional to the gauge parameter and originating from the gluon propagator in a covariant gauge, will ultimately cancel. This has been explicitly proved for the quark propagator in leading-order perturbation theory long ago in [14] (see also [12, 13] and [55] for alternative formulations). Indeed, it has been shown in these works that after renormalization in a \(\overline{\text{MS}}\) scheme, the anomalous dimension associated with the ordinary quark self-energy—which is gauge dependent—gets additional contributions, stemming from the two endpoints of the gauge link (coined in [14] the “connector”),\(^3\) that exactly cancel its gauge-parameter term, so that the anomalous dimension of the composite gauge-independent quark propagator (termed “hybrid” in [14]) is indeed free of the gauge parameter. [Below, the terminology of [14] is adopted.] This translates into the following sum rule for the anomalous dimensions

\[
\gamma_{\text{hybrid}} = \gamma_{2q} + \gamma_{\text{connector}},
\]

where

\[
\gamma_{2q} = \frac{\alpha_s C_F a}{4\pi} + \mathcal{O}(\alpha_s^2),
\]

\[
\gamma_{\text{connector}} = -\frac{\alpha_s C_F (3 + a)}{4\pi} + \mathcal{O}(\alpha_s^2),
\]

and

\[
\gamma_{\text{hybrid}} = -\frac{3\alpha_s C_F}{4\pi} + \mathcal{O}(\alpha_s^2),
\]

with \(\alpha_s = g^2/4\pi\), \(C_F = (N_c^2 - 1)(2N_c) = 4/3\), and \(a\) being the gauge parameter. Note that we use the same conventions for the definition of the anomalous dimension as in [14], i.e., we write

\[
\gamma_{\text{hybrid}} = \frac{\mu}{2 Z_{\text{hybrid}}} \frac{1}{d\mu} \frac{dZ_{\text{hybrid}}}{d\mu}
\]

with analogous expressions for the other anomalous dimensions.

The above sum rule (16) is nothing but a “logarithmic”, i.e., additive, version of the Ward-Takahashi/Slavnov-Taylor, identities in terms of ratios of the various renormalization constants of the QCD Lagrangian in the \(\overline{\text{MS}}\) scheme \([9, 12, 13, 14]\):

\[
\frac{Z_{\text{connector}} Z_{1q}}{Z_{\text{hybrid}}} = \frac{Z_3}{Z_1} = \frac{Z_3}{Z_1} = 1 + \frac{N_c \alpha_s}{8\pi} (3 + a) \frac{1}{\epsilon},
\]

\(^3\) These contributions are generated by singularities at the endpoints of the line integrals that are dimensionally regularized in \(D = 4 - \epsilon\) dimensions and give \(1/\epsilon\) poles.
where, $Z_{\text{hybrid}} = Z_{\text{connector}} Z_{2q}$, with $Z_{2q}$, $Z_{3}$, $\tilde{Z}_{3}$ being, respectively, the renormalization constants of the quark, the gluon, and the ghost field, whereas $Z_{1q}$, $Z_{1}$, $\tilde{Z}_{1}$ are the renormalization constants pertaining to the quark-gluon-ghost vertex, the three-gluon vertex, and the ghost-gluon-ghost vertex, respectively.

Some important comments are here in order: (i) This leading-order result can be formally proved for smooth contours in every order of QCD perturbation theory \[10, 11, 56\]. (ii) It was shown in \[9, 12, 13, 56\] that the connector can be renormalized by multiplying it with an appropriate renormalization constant and by replacing in the exponent the strong coupling by its renormalized version. Indeed, the renormalization constants $Z_{2q}$ and $Z_{1q}$ do not depend on the path chosen in any order of $g$ \[10, 12, 13, 56\] and the crucial Slavnov-Taylor identities are satisfied. This has been explicitly shown in \[9, 13\] up to the order $g^4$ and to all orders in $g$ in \[10\]. (iii) The straight line is actually enough for the renormalization of the connector for any smooth contour \[23, 57, 58\]. The reason is that what matters are only the singularities induced by the endpoints of the contour, which are multiplicatively renormalizable using exclusively dimensional regularization \[14\], whereas the specific path itself (for instance, its length) is irrelevant—provided no local obstructions like cusps and self-intersections are involved. Such obstructions would induce additional anomalous dimensions because of discontinuities in the contour slope, as discussed in detail in \[8, 12, 16\], that have to be taken into account in the anomalous-dimensions sum rule.\[5\] We will show below the key role of a cusped gauge contour in the eikonalized TMD PDF with a transverse gauge link.

The “eikonalized” quark PDF reads \[54\] (see also \[53, 60\])

\[
f_{i/a}(x) = \frac{1}{2} \int \frac{d\xi}{2\pi} e^{-ik\cdot \xi} \left< P | \bar{\psi}_i(\xi, 0, 0) \gamma^+ [\xi^-, 0^-] \psi_i(0, 0, 0) | P \right>,
\]

and is a manifestly gauge invariant quantity, but has an anomalous dimension that comprises contributions stemming from the two endpoints of the lightlike contour $C_{\perp, 0^-}$ (recall the remarks on the anomalous dimensions given above). Alternatively, one may be tempted to split the connector $[\xi^-, 0^-]$ into two gauge links that connect the points $0^-$ and $\xi^-$ through a point at infinity, the aim being to associate each of them with a quark field operator. Inserting a complete set of intermediate states, one can then recast Eq. \[22\] in the following form

\[
f_{i/a}^{\text{split}}(x) = \frac{1}{2} \sum_n \int \frac{d\xi}{2\pi} e^{-ik\cdot \xi} \left< P | \bar{\psi}_i(\xi, 0, 0) [\xi^-, \infty^-] | n \right> \gamma^+ \left< n | [\infty^-, 0^-] \psi_i(0, 0, 0) | P \right>,
\]

\[
= \frac{1}{2} \sum_n \int \frac{d\xi}{2\pi} e^{-ik\cdot \xi} \left< P | \bar{\Psi}_i(\xi, 0, 0, C_1) | n \right> \gamma^+ \left< n | \Psi_i(0, 0, 0, C_2) | P \right>,
\]

where we have introduced the path-dependent Mandelstam fields \[49, 50\]

\[
\Psi(x|C_2) = \mathcal{P} \exp \left[ -ig \int_{\infty[C_2]}^{x} d\xi_{\mu} A_{\mu}^a(\xi) t_a \right] \psi(x),
\]

\[4\] It is worth noting that in the covariant gauge $a = -3$, the gauge-contour divergences cancel among themselves and the Slavnov-Taylor identity \[24\] becomes trivially satisfied \[12, 14\].

\[5\] Just recently, Pobylitsa \[59\] has studied inequalities of a particular class of anomalous dimensions depending on cusp angles.
FIG. 2: Space-time picture of the DIS process with a gauge connector (double line) (a) and a split-gauge link (two double lines) at light-cone infinity (b). The infinitely distant parts of the contours, where the behavior of the fields is not specified, are represented by the symbol for “ground” (earth) in electricity, introduced in [48]. Else, the same designations as in Fig. [1] are used.

for the fermion and

\[
\bar{\Psi}(x|C_1) = \psi^\dagger(x) \mathcal{P} \left[ ig \int_{x^-}^{x^+} d\xi^\mu A_\mu^a(\xi) t_a \right] \gamma^0. \tag{25}
\]

for the antifermion. These field operators represent the struck quark as an eikonal line [53] along a light-like ray in the minus light-cone direction while interacting with the gluon field of the hadron, thus mimicking the motion of a struck quark in a real experiment [51].

The above two definitions (14) and (23) are equivalent, because the anomalous dimension of the direct gauge link \([\xi^-, 0^-]\) is preserved when splitting the contour into two distinct contours through infinity. This means, in particular, that the junction point—which is shifted to infinity—is not creating any anomalous-dimension artefact. By virtue of the smooth connection of the contours \(C_1\) and \(C_2\), direct calculation shows that the renormalization of the junction point \(z\) preserves the algebraic identity

\[
[x_2, z \mid C_1] [z, x_1 \mid C_2] = [x_2, x_1 \mid C = C_1 \cup C_2] \tag{26}
\]

in any order of the coupling [13]. This is not trivial because in a local gauge-invariant theory the factorized gauge link does depend, in general, on the junction point. However, for purely lightlike smooth contours \(C_1\) and \(C_2\), the above gauge-link factorization property is satisfied and the transport of color information by two different routes does not depend on the junction point (the latter being “hidden” at infinity in our case), with complete cancellation of the contributions at infinity. This is a well-established property which has been demonstrated by many authors in the early literature on studies of path-ordered exponentials. The analogous situation for non-smooth contours, which are not purely lightlike, and the generalization of Eq. (26) will be discussed in the next section.

IV. DERIVATION OF THE TRANSVERSE GAUGE LINK

To substantiate the use of the transverse gauge link and prepare the ground for the gauge-invariant formulation of unintegrated PDFs, which bear a full transverse-momentum
dependence, let us introduce a Coulomb source in terms of a "classical" current and write

\[ j_\mu(y) = g \int dy'_\mu \delta^{(4)}(y - y'), \quad y'_\mu = v_\mu \tau, \tag{27} \]

which corresponds to a charged point-like particle moving with the four-velocity \( v_\mu \) along the straight line \( v_\mu t \). The gauge field related to such a current has the form

\[ A^\mu(\xi) = \int d^4y \, D^{\mu\nu}(\xi - y)j_\nu(y), \tag{28} \]

where \( D^{\mu\nu} \) is the gluon Green’s function in an arbitrary covariant gauge.

Here and in below, we will use two dimensionless light-cone vectors \( n^\pm \)

\[ n^\pm = \frac{1}{\sqrt{2}}(1, 0, \pm 1), \quad (n^+)^2 = (n^-)^2 = 0, \quad (n^+ \cdot n^-) = 1, \tag{29} \]

and the metric tensor

\[ g^{\mu\nu} = g_T^{\mu\nu} + (n^+)^\mu(n^-)^\nu + (n^+)^\nu(n^-)^\mu \quad \text{with} \quad g_T^{\mu\nu} = -\delta^{\mu\nu}. \tag{30} \]

Appealing to the spacetime structure of this process, illustrated in Fig. 2(a), we recast the current in the form

\[ j_\mu(y) = g \left[ n^+_\mu \int_{-\infty}^{0} d\tau \, \delta^{(4)}(y - n^+\tau) + n^-_\mu \int_{0}^{\infty} d\tau \, \delta^{(4)}(y - n^+\tau) \right] \]

\[ = g \, \delta^{(2)}(y) \left[ n^+_\mu \delta(y^-) \int \frac{dq^-}{2\pi} \frac{e^{-iq^-y^+}}{q^+ + iq^+} - n^-_\mu \delta(y^+) \int \frac{dq^+}{2\pi} \frac{e^{-iq^+y^-}}{q^- - iq^-} \right], \tag{31} \]

which makes it clear that the first term in this expression corresponds to a gauge field created by a source moving from minus infinity to the origin in the plus light-cone direction, before being struck by the photon, whereas the second term corresponds to a gauge field being created by a source moving, after the collision, from the origin to plus infinity along a light-cone ray in the minus light-cone direction. Note that the underlying kinematics are those of Sec. III.

To continue, we approximate the gluon Green’s function \( D^{\mu\nu} \) by the free gluon propagator in the light-cone gauge \( A^+ = 0 \) and obtain

\[ D^{\mu\nu}(z) = \int \frac{d^4k}{(2\pi)^4} e^{-ikz} \tilde{D}^{\mu\nu}(k) \]

\[ = -\int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikz}}{k^2 + i0} \left( g^{\mu\nu} - \frac{k^\mu(n^-)^\nu + k^\nu(n^-)^\mu}{[k^+]} \right), \tag{32} \]

where \( 1/[k^+] \) denotes the regularization prescription to handle the light-cone (pole) singularity at \( k^+ \). Taking into account that the \( n^- \)-part of the current \( j_\mu \) does not contribute in the \( A^+ = (A \cdot n^-) = 0 \) gauge, one is able to obtain the transverse components \( \mu = i = 1, 2 \) of the gauge field; viz.,

\[ A_\perp^i(\xi) = -g \, n^+_\nu \int \frac{d^4k}{(2\pi)^4} e^{-ik\xi} \tilde{D}^{\mu\nu}(k) \int dy^+dy^-d^2y_\perp e^{ik\cdot y} \delta(y^-)\delta^{(2)}(y_\perp) \]

\[ = -\frac{1}{2} g \int \frac{dk^+}{2\pi} \frac{e^{-ik^+\xi^-}}{[k^+]} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{k_\perp^i}{k_\perp^2} e^{ik_\perp \cdot \xi_\perp}. \tag{33} \]
The second, purely transverse, integral over $d^2k_\perp$ in Eq. (33) gets factorized and finally yields
\begin{equation}
\int \frac{d^2k_\perp}{(2\pi)^2} \frac{k_\perp^4}{k_\perp^2} e^{ik_\perp \cdot \xi_\perp} = -\frac{i}{2\pi} \nabla^i \ln |\xi_\perp| ,
\end{equation}
while the first integral over $dk^+$ can be performed on account of the preferred regularization prescription for the pole at $k^+ = 0$. Note that in Eq. (34), $\lambda$ is an auxiliary infrared (IR) regulator which ultimately drops out from all physical quantities. The connection between the regularization procedure in the momentum-space representation with the behavior of the gauge field at light-cone infinity can be anticipated from the following expression
\begin{equation}
\int dk^+ \frac{e^{-ik^+\xi^-}}{2\pi} = i\kappa \int_0^\infty dl \delta(\kappa l + \xi^-) ,
\end{equation}
where $\kappa = \pm 1$. Then, one readily finds
\begin{equation}
A_\perp(\infty^-; \kappa = -1) = \frac{g}{4\pi} C_\infty \nabla \ln |\xi_\perp| ,
\end{equation}
emphasizing that this expression is dependent on the boundary conditions via the parameter $C_\infty$. Obviously, the longitudinal components $A^\pm$ vanish.

Note that the magnitude of $A_\perp$ in our expression (36) turns out to be two times smaller than that obtained in Ref. [47] (see also [61] and references cited therein). The reason for this difference lies in the fact that in our case the source travels not along the whole plus lightlike axis, but only along half of it, changing its direction at the origin, as a result of its collision with the hard photon, and in agreement with the physical picture of the process.

Let us continue by supplying within the same approach the gauge field in a general covariant gauge, characterized by the gauge parameter $a$, and using the gluon propagator
\begin{equation}
\tilde{D}^{\mu\nu}(k) = -\frac{1}{k^2 + i0} \begin{pmatrix}
g^{\mu\nu} - (1 - a) \frac{k^\mu k^\nu}{k^2 + i0}
\end{pmatrix} .
\end{equation}
Starting from Eq. (28), one gets after some simple algebraic manipulations
\begin{equation}
A'_\perp = 0 , \quad A'^- = 0 , \quad A'^+ (\xi) = -\frac{g}{4\pi} \delta(\xi^-) \ln l|\xi_\perp| ,
\end{equation}
where fields in a covariant gauge are marked by a prime accent in order to distinguish them from those in the light-cone gauge [cf. Eq. (9) in Ref. [47]]. Notice that the $a-$dependent terms, which are proportional to $\sim k^-$ under the $d^4k^-$ integral, do not contribute by virtue of the delta-function $\delta(k^-)$. Next we give the (singular) gauge transformation which connects these two representations in the light-cone gauge and in covariant gauges:
\begin{equation}
A^LC_\mu = A'_\mu + \partial_\mu \phi , \quad \phi(\xi) = -\int^{\xi^-}_{-\infty} d\xi'^- A'^+(\xi'^-) .
\end{equation}

Now we are ready to discuss the origin of the transverse contribution in Mandelstam’s gauge-invariant formalism in the light-cone gauge. First, note that the analogous expression to the Mandelstam field (24) in a covariant gauge reads
\begin{equation}
\Psi_{\text{cov}}(\xi; n^+) = \mathcal{P} \exp \left[ -ig \int_{\xi^-}^{\infty^-} dz^- A^+_{\text{cov}}(z^-, \xi_\perp) \right] \psi_{\text{cov}}(\xi^-, \xi_\perp) ,
\end{equation}
where the gauge field $A^+_{\text{cov}}$ differs from the special case, $A^+$, given by Eq. \((38)\). Second, performing a regular gauge transformation

$$U(x^-) = \exp \left(-ig \int^{x^-} dz^- A^+\right)$$  \((41)\)

in the light-cone gauge on both sides of \((40)\), one can eliminate the Wilson-line integral in the phase. However, the regular transformation $U(x^-)$ does not exhaust the gauge freedom in the light-cone gauge completely and is, therefore, insufficient to trivialize the interaction of the struck quark with the gluon field of the spectators. More explicitly, a residual singular transformation $U_{\text{sing}}(\infty^-; \xi_\perp)$ is still allowed and one realizes that the singular gauge transformation \((39)\) reflects exactly this remaining gauge freedom. Carrying out this additional gauge transformation, one generates an additional phase that is now associated with the quark field itself; viz.,

$$\psi(\xi^-, \xi_\perp)_{\text{LC}} = U_{\text{sing}}(\infty^-; \xi_\perp) \psi_{\text{LC}}(\xi^-, \xi_\perp)$$

\((42)\)

which is now completely gauge-fixed, and hence, represents a quark with a fixed color charge. [This is indicated by a wide hat over the label LC which abbreviates ‘light cone’.] Finally, by taking into account expression \((36)\), one finds that the quark wave function in the light-cone gauge acquires a phase that may formally be written as

$$\psi(\xi^-, \xi_\perp)_{\text{LC}} = \left[1 + ig \int_{\xi^-}^{\infty^-} d\xi_\perp A^+_{\text{source}}(\xi^-; \xi_\perp) + O(g^2)\right] \psi_{\text{LC}}(\xi^-, \xi_\perp)$$

\((43)\)

The above arguments make it clear that a complete gauge fixing can be achieved in \((42)\) by inserting the additional singular gauge transformation $U_{\text{sing}}$ which contains the cross-talk effects of the struck parton with the light-cone source. As a result, taking the product of two (local) quark field operators in the fixed light-cone gauge differs from what one finds in a covariant gauge. This difference is encapsulated in two phase factors so that one gets

$$[\bar{\psi}(\xi^-, \xi_\perp) \gamma^+ \psi(0^-, 0_\perp)]_{\text{LC}} = \bar{\psi}_{\text{LC}}(\xi^-, \xi_\perp) \mathcal{P} \exp \left[+ig \int_{\xi^-}^{\infty^-} d\xi_\perp A_{\text{source}}^{\text{LC}}(\xi^-, \xi_\perp) + O(g^2)\right] \gamma^+$$

\((44)\)

\[\times \mathcal{P} \exp \left[-ig \int_{0_\perp}^{\infty_\perp} d\xi_\perp A_{\text{source}}^{\text{LC}}(\xi^-, \xi_\perp) \right] \psi_{\text{LC}}(0^-, 0_\perp)\]

In the next section, we will use the results obtained above in order to demonstrate the role of the transverse link in the restoration of the prescription-independence of the anomalous dimension of the TMD PDF. Specifically, the explicit expression for the transverse gauge field at infinity, Eq. \((33)\), will be used in the diagrammatic calculations of the gluon radiative corrections pertaining to the anomalous dimension of the TMD PDF.

V. CALCULATION OF THE ONE-LOOP ANOMALOUS DIMENSION OF THE TMD PDF IN THE LIGHT-CONE GAUGE

We have stressed before the importance of the renormalization effect on the junction point of the decomposed transverse contours at infinity. In this section we will explain exactly what
this means in mathematical detail. We will prove that the factorization of the gauge link into factors, each associated with a distinct contour starting (ending) at light-cone infinity, has to be modified to include an additional phase factor which accounts for the cusp anomalous dimension induced by the junction point of these decomposed contours.

A. Definitions

Our starting point is the operator definition of the (unpolarized) TMD distribution of a quark with momentum $k_\mu = (k^+, k^-, k_\perp)$ in a quark with momentum $p_\mu = (p^+, p^-, 0_\perp)$:

$$f_{q\bar{q}}(x, k_\perp) = \frac{1}{2} \int \frac{d\xi^- d^2 \xi_\perp}{2\pi (2\pi)^2} e^{-ik^+ \xi^-} e^{ik_\perp \cdot \xi_\perp} \langle q(p) | \bar{\psi}(\xi^-, \xi_\perp) | \xi^- = 0, \xi_\perp \rangle \bigg|_{\xi^+ = 0},$$

where the lightlike and transverse gauge links are defined, respectively, by

$$[\infty^-, z_\perp; z^-, z_\perp] \equiv \mathcal{P} \exp \left[ ig \int_0^\infty d\tau n^- A^\mu_a t^a (z + n^- \tau) \right],$$

$$[\infty^-, \infty_\perp; \infty^-, \xi_\perp] \equiv \mathcal{P} \exp \left[ ig \int_0^\infty d\tau l \cdot A^\mu_a t^a (\xi_\perp + l\tau) \right].$$

Let us emphasize that in the definition of the transverse gauge link the contour is defined in terms of the two-dimensional vector $l$ which is absolutely arbitrary. We will show explicitly that this arbitrariness does not affect the local properties of the gauge link—in particular the anomalous dimension.

Employing the light-cone axial gauge

$$A^+ = (A \cdot n^-) = 0, \quad (n^-)^2 = 0,$$

the gluon propagator has an additional pole, related to the plus light-cone component of the gluon momentum, and reads

$$D_{\mu\nu}^{LC}(q) = \frac{-i}{q^2 - \lambda^2 + i0} \left( g_{\mu\nu} - \frac{q_\mu n_\nu^+ + q_\nu n_\mu^+}{[q^+]} \right).$$

To give this expression a mathematical meaning, we apply the following pole prescription

$$\left. \frac{1}{[q^+]} \right|_{\text{Ret/Adv}} = \frac{1}{q^+ \pm i\eta}, \quad \left. \frac{1}{[q^+]} \right|_{\text{PV}} = \frac{1}{2} \left[ \frac{1}{q^+ + i\eta} + \frac{1}{q^+ - i\eta} \right],$$

where $\eta$ has the dimension of mass, to be kept small but finite, and where we used the abbreviations Ret for retarded, Adv for advanced, and PV for the principal value.\(^6\) In what follows, we regularize collinear poles by means of the quark virtuality $p^2 < 0$, whereas IR

---

\(^6\) We remark that another possible prescription—the so-called Mandelstam-Leibbrandt pole prescription \(^{62, 63, 64}\)—is outside the scope of the present investigation, though we will make some related comments in connection with the anomalous dimension in Eq. (79).
FIG. 3: One-loop gluon contributions (curly lines) to the UV-divergences of the TMD PDF in a general covariant gauge. Double lines denote gauge links. Diagrams (b) and (c) are absent in the light-cone gauge. The omitted Hermitian conjugate diagrams are symbolically abbreviated by \( (h.c.) \).

singularities are regularized by an auxiliary gluon mass \( \lambda \) which is put at the end back to zero. The described regularization procedure works well in the one-loop order, while at a higher loop order one may need to apply more sophisticated methods.

In the tree approximation, where the gauge links are equal to unity and the quark-gluon interactions vanish, one trivially gets

\[
\nonumber
f^{(0)}_{q/A}(x, \xi_{\perp}) = \frac{1}{2} \int \frac{d\xi^2 - d^2 \xi_{\perp}}{2\pi(2\pi)^2} e^{-ik^+\xi^- + ik_{\perp} \cdot \xi_{\perp}} \langle p | \bar{\psi}(\xi^-) \gamma^+ \psi(0^-) | 0_{\perp} \rangle | p \rangle \\
= \delta(p^+ - xp^+) \delta(2)(k_{\perp}) \frac{1}{2} \bar{u}(p) \gamma^+ u(p),
\]

where \( u(p) \) denotes a quark spinor and summation over spin indices is tacitly assumed. Moreover, we use the short-hand notation

\[
\bar{u}(p) \gamma^+ u(p) \equiv \phi_0(p),
\]

which implies

\[
\nonumber
f^{(0)}_{q/A}(x, \xi_{\perp}) = \delta(1 - x) \delta(2)(k_{\perp}) \phi_0(p).
\]

B. One-loop calculation

Our objective here is to discuss the leading-order (LO) \( g^2 \) quark-gluon interactions, stemming, one on one hand, from the standard QCD vertices, and, on the other hand, from the interactions of the quarks with the gauge links. Because in the TMD case the distance between the quark fields is spacelike, i.e., \( \xi_{\mu} \xi^\mu = -\xi_{\perp}^2 \neq 0 \), the UV-divergent contributions arise only due to virtual gluon corrections. Therefore, in LO (alias, at the one-gluon exchange level), diagrams (a), (b), and (c) contribute in a covariant gauge (see Fig. 3), while in the light-cone gauge, only diagrams (a) and (d) give non-vanishing contributions—with the latter diagram being associated with the transverse gauge link.

The Hermitian-conjugate (\( h.c. \) for short) contributions (omitted in Fig. 3) are generated by the corresponding “mirror” diagrams. In what follows, we consider first the “left” set of diagrams (in order to show explicitly how the transverse gauge link comes into play) and then we take the sum “left + right” which gives the total contribution to the TMD PDF.
These diagrams yield contributions proportional to delta-functions:

\[ f_{\gamma q}^{1-\text{loop}}(x, k_\perp; \mu, \eta) = \delta(1 - x) \delta^{(2)}(k_\perp) \phi_\gamma(p) \Sigma(p, \alpha_s, \mu, \eta), \]

where \( \Sigma^{1-\text{loop}} \) results from the diagrams shown in Fig. 3. The quark self-energy diagram \((a)\) gives (in dimensional regularization with \( \omega = 4 - 2\epsilon \))

\[ \Sigma^{(a)}(p, \alpha_s; \mu, \eta, \epsilon) = -g^2 C_F \mu^{2\epsilon} \int \frac{d^\omega q}{(2\pi)^\omega} \frac{\gamma_\mu(\hat{p} - \hat{q})\gamma_\nu}{(p - q)^2(q^2 - \lambda^2 + i0)} \frac{d^{\mu\nu}(q) i\hat{p}}{p^2} \]  

(53)

with

\[ d^{\mu\nu}_\text{LC}(q) = g^{\mu\nu} - \frac{q^\mu(n^-)^\nu}{[q^+]^2} - \frac{q^\nu(n^-)^\mu}{[q^+]^2}, \]  

(54)

where the dependence on the auxiliary mass scale \( \eta \) is “hidden” in the pole prescription \([q^+]\) and \( \hat{a} \equiv (\gamma \cdot a) \).

The \( g^{\mu\nu}\)-proportional term gives a “Feynman”-like contribution, namely,

\[ \Sigma^{(a)}_{\text{Feynman}}(p, \alpha_s, \mu, \epsilon) = -g^2 C_F \mu^{2\epsilon} \int \frac{d^\omega q}{(2\pi)^\omega} \frac{\gamma_\mu(\hat{p} - \hat{q})\gamma_\mu}{(p - q)^2(q^2 - \lambda^2)} \frac{i\hat{p}}{p^2} \]  

(55)

and generates no extra light-cone singularities. After carrying out the momentum integral, one gets

\[ \Sigma^{(a)}_{\text{Feynman}}(p, \alpha_s, \mu, \epsilon) = -\frac{\alpha_s}{4\pi} C_F \Gamma(\epsilon) \left( -4\pi \frac{\mu^2}{p^2} \right)^\epsilon / (1 - \epsilon) \int_0^1 dx \left[ x(1 - x) \left( 1 - \frac{\lambda^2}{xp^2} \right) \right]^{-\epsilon}. \]  

(56)

Performing the remaining integral, one finally finds

\[ \Sigma^{(a)}_{\text{Feynman}}(p, \alpha_s, \mu, \epsilon) = -\frac{\alpha_s}{4\pi} C_F \Gamma(\epsilon)(1 - \epsilon) \left( -4\pi \frac{\mu^2}{p^2} \right)^\epsilon \times \left[ 1 + \epsilon \left( 2 + \frac{\lambda^2}{p^2} \ln \frac{\lambda^2 - p^2}{\lambda^2} - \ln \frac{p^2 - \lambda^2}{p^2} \right) + O(\epsilon^2) \right]. \]  

(57)

Note that the ”mirror” diagram gives precisely the same contribution, doubling this result.

C. Evaluation of the pole-prescription-dependent contributions

The calculation of the \([q^+]\)-dependent part

\[ \Sigma^{(a)}_{\text{pole}}(p, \alpha_s, \mu, \eta, \epsilon) = g^2 C_F \mu^{2\epsilon} \int \frac{d^\omega q}{(2\pi)^\omega} \frac{1}{(p - q)^2(q^2 - \lambda^2)} \left( \frac{(\hat{p} - \hat{q})\gamma^+}{[q^+]} + \frac{\gamma^+(\hat{p} - \hat{q})\hat{q}}{[q^+]} \right) \frac{i\hat{p}}{p^2} \]  

(58)

is more demanding, owing to the presence of light-cone singularities, and we will consider its evaluation in detail. After some simple transformations of the numerator, we find

\[ \Sigma^{(a)}_{\text{pole}} = g^2 C_F \mu^{2\epsilon} \int \frac{d^\omega q}{(2\pi)^\omega} \left[ (\hat{p}\gamma_\mu\gamma^+ + \gamma^+ \gamma_\mu\hat{p}) \frac{(p - q)^\mu}{(p - q)^2 - 2\gamma^+} \right] \frac{1}{(q^2 - \lambda^2)[q^+] \frac{i\hat{p}}{p^2}}. \]  

(59)

One observes that the integral \( \int \frac{d^\omega q}{(q^2 - \lambda^2)[q^+] \frac{i\hat{p}}{p^2}} \) vanishes (which is true for the considered pole prescriptions but not for the Mandelstam-Leibbrandt one), while the rest can be recast in the form

\[ \Sigma^{(a)}_{\text{pole}} = g^2 C_F \mu^{2\epsilon} (\hat{p}\gamma_\mu\gamma^+ + \gamma^+ \gamma_\mu\hat{p}) \left[ p^\mu \sigma_1(p) + n^\mu \sigma_2(p) \right] \frac{i\hat{p}}{p^2}, \]  

(60)
where
\[ \sigma_1(p) = \frac{i}{(4\pi)^{s/2}} \frac{\Gamma(\epsilon)}{-p^2} \int_0^1 dx \frac{1-x}{[xp^+]} \left[ x(1-x) \left( 1 - \frac{\lambda^2}{x^2} \right) \right]^{-\epsilon} \]  \hspace{1cm} (61)

bearing in mind that by virtue of \( \gamma^+ \gamma^+ = (n^-)^2 = 0 \) the term \( \sigma_2(p) \) does not contribute. Thus, one has
\[
\left[ \Sigma^{(a)}_{\text{Feynman}} + \Sigma^{(a)}_{\text{pole}} \right] (p, \alpha_s, \mu, \eta, \epsilon) = \frac{\alpha_s}{4\pi} C_F \left( -4\pi \frac{\mu^2}{p^2} \right)^{\epsilon} \Gamma(\epsilon) \left\{ (1 - \epsilon) \right. \\
\times \left[ 1 + \epsilon \left( 2 + \frac{\lambda^2}{p^2} \ln \frac{\lambda^2 - p^2}{\lambda^2} - \ln \frac{p^2 - \lambda^2}{p^2} \right) \right. \\
- \frac{2\gamma^+ \hat{p}}{p^+} \int_0^1 dx \frac{(1-x)}{[x]} \left\{ 1 - \epsilon \ln \left[ x(1-x) \left( 1 - \frac{\lambda^2}{x p^2} \right) \right] \right\} \\
+ O(\epsilon^2) \right\}. \hspace{1cm} (62)
\]

In order to evaluate the integral \( \int dx (1-x)/[x] \), one has to use a specific pole prescription for \([x] \). Let us consider three possible prescriptions: Advanced, Retarded and Principal Value:
\[
\frac{1}{[x]}_{\text{Ret}} = \frac{1}{x + i\eta} \hspace{1cm} \frac{1}{[x]}_{\text{Adv}} = \frac{1}{x - i\eta} \hspace{1cm} \frac{1}{[x]}_{\text{PV}} = \frac{1}{2} \left( \frac{1}{x + i\eta} + \frac{1}{x - i\eta} \right) \hspace{1cm} (63)
\]

using temporarily for convenience the short-hand notation \( \bar{\eta} = \eta/p^+ \). In the limit of small \( \bar{\eta} \), we keep only logarithmic terms and omit any powers of \( \bar{\eta} \). The UV-divergent part (in the \( \overline{\text{MS}} \)-scheme) then reads
\[
\Sigma^{(a)}_{\text{UV}} = -\frac{\alpha_s}{4\pi} C_F \frac{1}{\epsilon} \left[ 1 - \ln 4\pi + \gamma_E - \frac{2\gamma^+ \hat{p}}{p^+} \left( 1 + \ln \frac{\eta}{p^+} - \frac{i\pi}{2} - i\pi C_\infty \right) \right], \hspace{1cm} (64)
\]
where the numerical constant \( C_\infty \) depends on the pole prescription according to (cf. \( [33] \))
\[
C_\infty = \begin{cases} 
0 \hspace{1cm} \text{Advanced} \\
-1 \hspace{1cm} \text{Retarded} \\
-\frac{1}{2} \hspace{1cm} \text{Principal Value} 
\end{cases} \hspace{1cm} (65)
\]

One the other hand, the finite part of the pole-prescription dependent gluon radiative corrections is
\[
\Sigma^{(a)}_{\text{finite}} (p, \alpha_s, \mu, \eta, \epsilon) = -\frac{\alpha_s}{4\pi} C_F \left( 1 + \ln \frac{\mu^2}{p^2} + \frac{\lambda^2}{p^2} \ln \frac{\lambda^2 - p^2}{\lambda^2} - \ln \frac{p^2 - \lambda^2}{p^2} \right) \\
- \frac{2\gamma^+ \hat{p}}{p^+} \left\{ \left( 1 + \ln \frac{\eta}{p^+} - \frac{i\pi}{2} - i\pi C_\infty \right) \ln \frac{\mu^2}{p^2} \\
+ \int_0^1 dx \frac{(1-x)}{[x]} \ln \left[ x(1-x) \left( 1 - \frac{\lambda^2}{x p^2} \right) \right] \right\} \right\} \hspace{1cm} (66)
\]

Evaluating this UV-finite integral \( [66] \) by setting \( \lambda^2 = 0 \) (which is justified given that this integral is IR finite and the magnitude of the IR regulator is irrelevant), we obtain
\[
\int_0^1 dx \frac{(1-x)}{x \mp i\eta} \ln [x(1-x)] = 2 + (1 \mp i\eta) \left[ \text{Li}_2 \left( \frac{1}{i\eta} \right) - \text{Li}_2 \left( \frac{1}{1-i\eta} \right) \right]. \hspace{1cm} (67)
\]
As a result, the complete prescription-dependent finite part for $\lambda^2 = 0$ becomes

$$
\Sigma^{(a)}_{\text{finite}}(p, \alpha_s, \mu, \eta, \epsilon) = -\frac{\alpha_s}{4\pi} C_F \left( 1 + \ln \frac{\mu^2}{p^2} + \frac{2\gamma^+ \hat{\rho}}{p^+} \left\{ \left( 1 + \ln \frac{\eta}{p^+} - \frac{i\pi}{2} - i\pi C_\infty \right) \ln \frac{\mu^2}{p^2} + 2 + (1 \mp i\eta) \left[ \text{Li}_2 \left( \frac{1}{\pm i\eta} \right) - \text{Li}_2 \left( \frac{1}{1 \mp i\eta} \right) \right] \right\} \right). \tag{68}
$$

A key remark here is that any dependence on the pole prescription in Eqs. (64) and (66) (or equivalently (68)). Before we proceed, note that, as it is obvious from Eq. (64), the UV-divergent part depends not only on the pole prescription but also on the logarithmic $p^+$-term. The effects related to this latter dependence will be considered subsequently.

In Sec. IV, we have worked out the transverse components of the gluon field in the light-cone gauge and found Eq. (33). This expression can be further evaluated to read

$$
A_\perp(\infty^-, 0^+; l_\perp \tau) = \int \frac{dq^+}{2\pi} \exp \left[ i\epsilon q^+ \infty \right] \int \frac{d^2q_\perp}{(2\pi)^2} e^{iq_\perp \cdot l_\perp} A_\perp(q), \tag{70}
$$

finally assuming the form

$$
\int_0^\infty d\tau l_\perp \cdot A_\perp(\infty^-, 0^+; l_\perp \tau) = \int \frac{dq^+}{2\pi} \exp \left[ -i\epsilon q^+ \infty \right] \int \frac{d^2q_\perp}{(2\pi)^2} l_\perp \cdot A_\perp(q) \frac{i}{(q_\perp \cdot l_\perp + i0)}. \tag{71}
$$

Consider now the free gluon propagator resulting from the correlation between longitudinal and transverse gluons; viz.,

$$
\langle A^\mu(q) A^i_\perp(q') \rangle = -\frac{q^i n^\mu}{(q^2 - \lambda^2) q^+} (-i)(2\pi)^4 \delta^{(4)}(q + q'), \tag{72}
$$

and use the relation

$$
\frac{e^{-i\epsilon q^+ \infty}}{[q^+]} = 2\pi i C_{\infty}\delta(q^+) \tag{73}
$$

to find the contribution of the diagram in Fig. 3(d):

$$
\Sigma^{(d)}_{\perp}(p, \mu, g; \epsilon) = g^2 C_F \mu^{2\epsilon} 2\pi i C_{\infty} \int \frac{d^2q}{(2\pi)^2} \delta(q^+) \frac{\gamma^+(\hat{p} \cdot \hat{q})}{(p - q)^2 (q^2 - \lambda^2)^2} \times \frac{\gamma^+(\hat{p} - \hat{q})}{p^+} \int_0^1 \frac{1 - \delta(x)}{x(1 - x)} \left( 1 - \frac{\lambda^2}{xp^2} \right)^{-\epsilon} \tag{74}
$$
FIG. 4: Graphical representation of a generic TMD PDF (shaded oval) in coordinate space. The double lines denote the lightlike and transverse gauge links, connecting the quark field points $(0^-, 0_{\perp})$ and $(\xi^-, \xi_{\perp})$, by a composite contour through light-cone infinity. The latter is marked by the typical symbol for the ground in an electrical circuit. The contour obstruction at infinite transverse and lightlike distance $(\infty^-, \infty_{\perp})$ is symbolized by a cross, whereas the broken line indicates that this obstruction is “hidden”.

One sees explicitly that by virtue of the relation

$$\frac{1}{[x]} = \lim_{\eta \rightarrow 0} \frac{1}{x \pm i\eta} = \text{PV} \frac{1}{x} \mp i\pi\delta(x),$$

(75)

the transverse-gauge link contribution (74) exactly cancels the dependence on the pole prescription in both the UV-divergent part $\Sigma^{(a)}_{\text{UV}}$ and in the finite part $\Sigma^{(a)}_{\text{finite}}$. To show this explicitly, we collect all pole-prescription dependent terms of diagram (a) in Fig. 3 and add to them the contribution from the transverse gauge link, i.e., Eq. (74). Then, we have

$$\lim_{\eta \rightarrow 0} \int_0^1 dx (1 - x) \left[ \frac{1 + C_{\infty}}{x - i\eta} - \frac{C_{\infty}}{x + i\eta} - i2\pi C_{\infty} \delta(x) \right] \ln (x(1 - x))$$

$$= \lim_{\eta \rightarrow 0} \int_0^1 dx \frac{1 - x}{x - i\eta} \ln (x(1 - x))$$

(76)

which establishes the independence of the result on the parameter $C_{\infty}$—the latter encoding the adopted pole-prescription. As a result, the complete UV-divergent part of the TMD PDF $f_{q/q}(x, k_{\perp})$ is

$$\Sigma^{(a+d)}_{\text{UV}}(\rho, \mu, \alpha_s; \epsilon) = -\frac{\alpha_s}{\pi} C_F \frac{1}{\epsilon} \left[ \frac{1}{4} - \frac{\gamma^+ \tilde{p}}{2p^+} \left( 1 + \ln \frac{\eta}{p^+} - \frac{i\pi}{2} - i\pi C_{\infty} + i\pi C_{\infty} \right) \right]$$

$$= -\frac{\alpha_s}{\pi} C_F \frac{1}{\epsilon} \left[ 1 - \frac{\gamma^+ \tilde{p}}{2p^+} \left( 1 + \ln \frac{\eta}{p^+} - \frac{i\pi}{2} \right) \right].$$

(77)

Next, taking into account that

$$\frac{\gamma^+ \tilde{p}}{2p^+} = \gamma^+$$

and recalling that the mirror (which we termed before “right”) counterparts of the evaluated diagrams yield the complex-conjugated contributions, one can conclude that the imaginary
terms above mutually cancel, so that the UV-divergent part of diagrams (a) and (d) contains
only contributions due to the $p^+-$dependent term, notably,
\[
\Sigma_{\text{UV}}^{(a+d)}(\alpha_s, \epsilon) = 2 \frac{\alpha_s}{\pi} C_F \left[ \frac{1}{\epsilon} \left( \frac{3}{4} + \ln \frac{\eta}{p^+} \right) - \gamma_E + \ln 4\pi \right],
\]  
(78)
plus those terms originating from the standard $\overline{\text{MS}}$ renormalization. Hence, there is an extra anomalous dimension associated with the $p^+$-dependent term which at the one-loop level, considered here, is given by
\[
\gamma_{1-\text{loop}}^{\text{LC}} = \frac{\alpha_s}{\pi} C_F \left( \frac{3}{4} + \ln \frac{\eta}{p^+} \right) = \gamma_{\text{smooth}} - \delta\gamma. 
\]  
(79)
The difference $\delta\gamma$ between $\gamma_{\text{smooth}}$ and $\gamma_{\text{LC}}$ is exactly that term induced by the additional divergence which ultimately has to be compensated by a suitable redefinition of the TMD PDF, if we want to reproduce the same anomalous dimension as in a covariant gauge.

Here, some comments are in order. It was shown (see, e.g., [65]) that the Mandelstam-Leibbrand (ML) prescription \[62, 63, 64\]
\[
\frac{1}{|q^+|_{\text{ML}}} = \frac{1}{q^+ + i0} \frac{1}{q^-} = \frac{q^-}{q^+q^- + i0} 
\]  
(80)
yields a $p^+$-independent anomalous dimension of the quark fields, i.e.,
\[
\gamma_{\text{ML}}^{\text{LC}} = \frac{3\alpha_s}{4\pi} C_F + O(\alpha_s^2). 
\]  
(81)
Moreover, the ML-prescription, in contrast to the PV/Adv/Ret ones (cf. Eq. (65)), entails additional poles in the complex $q^0$-plane which allow for a Wick rotation and, therefore, it does not break the standard power counting rules. However, on the one hand, it is not clear how the ML-prescription can be related to any boundary conditions on the gauge field at light-cone infinity, thus making the popular initial/final state interactions interpretation questionable. On the other hand, the ML-regularization appears to be not sufficient for the calculation of the transverse gauge field at light-cone infinity (in the form of an expression analogous to, say, Eq. (49)). The latter issue is potentially crucial for reproducing the results obtained in covariant gauges, while within the PV/Adv/Ret methods, the similarity between the light-cone and covariant gauges can be explicitly established. These issues will be further investigated and quantified in a separate work.

To continue, recall that in a covariant gauge the gluon field vanishes at infinity and, hence, the only anomalous dimensions ensuing from the gauge link stem from its endpoints that are joined by a smooth direct contour.\footnote{It is worth reiterating that all smooth gauge contours yield the same anomalous dimensions, $\gamma_{\text{smooth}}$, as the straight line between the endpoints, because only the latter are relevant.} Actually, $p^+ = (p \cdot n^-) \sim \cosh \chi$ defines an angle $\chi$ between the direction of the quark momentum $p_\mu$ and the lightlike vector $n^-$. Then, in the large $\chi$ limit, one has $\ln p^+ \to \chi$, $\chi \to \infty$. Thus, we come to recognize that the “defect” of the anomalous dimension, $\delta\gamma$, can be identified with the well-known cusp anomalous dimension \[17\]
\[
\gamma_{\text{cusp}}(\alpha_s, \chi) = \frac{\alpha_s}{\pi} C_F \left( \chi \coth \chi - 1 \right),
\]
\[
\frac{d}{d\ln p^+} \delta\gamma = \lim_{\chi \to \infty} \frac{d}{d\chi} \gamma_{\text{cusp}}(\alpha_s, \chi) = \frac{\alpha_s}{\pi} C_F. 
\]  
(82)
FIG. 5: Renormalization effect on the junction point due to gluon corrections (illustrated by a shaded oval with gluon lines attached to it) for (a) two smoothly joined gauge contours $C_1$ and $C_2$ at point 3 and (b) the same for two contours joined by a cusp (indicated by the symbol $\otimes$) at infinite transverse distance (marked by the earth symbol) off the light cone. All contours shown are assumed to be arbitrary non-lightlike paths in Minkowski space.

This is an important observation that deserves to be discussed in some detail.

As we mentioned earlier, splitting the gauge contour for non purely lightlike contours through the light-cone infinity, is not equivalent to the situation with a direct contour between the two field points. To understand the deeper reason for this difference, we have to study again the algebraic identity (26) for decomposing (factorizing) gauge contours (links). The crucial question here is whether the defect of the anomalous dimension, we calculated, is compatible with this identity when the junction point is assumed to be at infinite distance in the transverse configuration space. To answer this question, consult Fig. 5. Panel (a) of this figure shows the renormalization effect (illustrated by a shaded oval with gluon lines attached to it) on the junction point in the algebraic identity (26). The contour $C_1 \cup C_2$ is smooth and non-self-intersecting owing to the assumption that $C_1$ and $C_2$ are smoothly connected at 3. Then, both contours the direct one, $C$, and the decomposed one, $C_1 \cup C_2$, between the endpoints 1 and 2, cannot be distinguished from each other by switching on gluon quantum corrections. In particular, no anomalous dimension emerges from the junction point 3 and thus (symbolically)

$$\gamma_C = \gamma_{C_1 \cup C_2}.$$  \hspace{1cm} (83)

Now we may ask what changes are induced, if we allow the junction point 3 to be shifted to infinity in the transverse direction off the light cone. The graphics at right of Fig. 5 helps the eye catch the key features of the situation involving two non-lightlike contours $C_1$ and $C_2$. It turns out that the naive assumption that

$$\gamma_C = \gamma_{C_1^\infty \cup C_2^\infty}$$ \hspace{1cm} (84)

is incorrect for contours containing transverse segments. Instead, we found that in this case

$$\gamma_C = \gamma_{C_1^\infty \cup C_2^\infty} + \gamma_{\text{cusp}}.$$ \hspace{1cm} (85)

Consequently, the validity of the algebraic identity (26) is not conserved and we have to replace it by the generalized gauge-link factorization rule

$$[2, 1|C] = [2, \infty|C_2^\infty]^{\dagger}[\infty, 1|C_1^\infty]e^{i\Phi_{\text{cusp}}},$$ \hspace{1cm} (86)

which is valid for arbitrary paths in Minkowski space. In this expression, $\Phi_{\text{cusp}}$ takes care of the effect induced by the cusp-like junction point. We will consider an explicit example
of such a phase in the next subsection, where we show how to compensate it by an eikonal factor in the definition of the TMD PDF. One may associate this phase with final (or initial) state interactions, as proposed by Ji and Yuan in [41], and also by Belitsky, Ji, and Yuan in [33]. However, these authors (and also others) did not recognize that the junction point in the split contour (taking a detour to light-cone infinity) is no more a simple point, but becomes a cusp obstruction $\sim \ln p^+$ that entails an anomalous dimension as the result of a non-trivial renormalization effect owing to gluon radiative corrections.

These arguments make it clear that the naive decomposition of gauge contours that stretch out to light-cone infinity along the transverse direction is erroneous, simply because the basic algebraic identity (26), which is tacitly assumed, is inapplicable to such contours and has to be replaced by Eq. (86). It almost goes without saying that the modified factorization rule, expressed through Eqs. (85) and (86), is valid when one is composing non-smoothly any gauge contours with a cusp obstruction at the junction point.\footnote{A similar factorization rule holds for contours joined through a self-crossing point.}

What marks out a cusped contour from all the others, however, is that it gives rise to an anomalous dimension proportional to $\ln p^+$, i.e., to a jump in the four-velocity. This is a salient ingredient in describing correctly a DIS process in spacetime, because if the two quarks (the struck one and a spectator) are separated also in the transverse coordinate space, the gluons emitted mismatch in rapidity and, hence, the contour liaising them has to have a sharp bend and cannot be the direct one.

**E. Compensating the defect of the anomalous dimension by a soft counter term**

The defect of the anomalous dimension owing to the gauge-contour cusp at light-cone infinity represents a distortion of the gauge-invariant formulation of the TMD PDF in the light-cone gauge. To restore its consistency, we have to dispense with the anomalous-dimension artefact of the cusp. This can be achieved by supplying the original definition of $f_{q/q}(x, k_\perp)$ by a soft counter term in the sense of Collins and Hautmann [43, 44, 45, 66]:

$$ R \equiv \Phi(p^+, n^-|0)\Phi^\dagger(p^+, n^-|\xi) , $$

where the eikonal factors are given by

$$ \Phi(p^+, n^-|0) = \left\langle 0 \left| \mathcal{P} \exp \left[ ig \int_{C_{\text{cusp}}} d\zeta^\mu t^a A^a_{\mu}(\zeta) \right] 0 \right\rangle , $$

$$ \Phi^\dagger(p^+, n^-|\xi) = \left\langle 0 \left| \mathcal{P} \exp \left[ -ig \int_{C_{\text{cusp}}} d\zeta^\mu t^a A^a_{\mu}(\xi + \zeta) \right] 0 \right\rangle , $$

and evaluate $R$ along the non-smooth (non-lightlike) integration contour $C_{\text{cusp}}$, defined by

$$ C_{\text{cusp}} : \zeta_\mu = \left\{ [p^+_{\mu} s, -\infty < s < 0] \cup [n^-_{\mu} s', 0 < s' < \infty] \cup [l_\perp \tau, 0 < \tau < \infty] \right\} , $$

with $n^-_\mu$ being the minus light-cone vector, as illustrated in Fig. 6.

Contour (90) is obviously cusped: at the origin, the four-velocity $p^+_{\mu}$, which is parallel to the plus light-cone ray, is replaced—non-smoothly—by the four-velocity $n^-_{\mu}$, which is parallel...
to the minus light-cone ray. This jump in the four-velocity becomes visible in the standard leading-order term

$$g^2 \int_0^\infty ds \int_0^\infty ds' \frac{(v_1 \cdot v_2)}{(v_1 s - v_2 s')^2} = g^2 \int_0^\infty ds \int_0^\infty ds' \frac{(v_1 \cdot v_2)}{v_1^2 s + v_2^2 s' - 2(v_1 \cdot v_2)ss'},$$  \quad (91)

in which $v_1 = p^+$, $v_2 = n^-$, and the change of the four-velocity at the origin produces an angle-dependence via $(v_1 \cdot v_2) = p^+$. This means that exactly at this point the contour has a cusp that is characterized by the angle $\chi \sim \ln p^+ = \ln(p \cdot n^-)$, and, therefore, the corresponding eikonal factor (89) gives rise to a cusp anomalous dimension. Obviously, this is exactly what we need in order to compensate the extra term in the anomalous dimension found in the preceding subsection.

Next, we show that the one-loop gluon virtual corrections, contributing to the UV divergences of $R$ and displayed in Fig. 7, yield an anomalous dimension that neutralizes the cusp artefact $\delta \gamma$. Note that in the light-cone gauge $A^+ = (n^- \cdot A) = 0$ only the first lightlike ray $-\infty < s < 0$ and also the transverse segment contribute, since the other eikonal line along the minus lightlike ray depends on the longitudinal component of the gauge field and vanishes due to the gauge condition.

Calculate first the diagram (a) in Fig. 7. In leading order, the first nontrivial term in Eq. (89) reads

$$\Phi_a^{(1-\text{loop})}(u, \eta) = ig^2 \mu^2 C_F u_\mu u_\nu \int_0^\infty d\sigma \int_0^\sigma d\tau \int \frac{d^2 q}{(2\pi)^\omega} \frac{e^{-iq \cdot u \cdot (\sigma - \tau)}}{q^2 - \lambda^2} \left( g^{\mu\nu} - \frac{q^\mu n^- \nu + q^\nu n^- \mu}{[q^+]} \right)$$

$$= ig^2 \mu^2 C_F \int \frac{d^2 q}{(2\pi)^\omega} \frac{1}{q^2 - \lambda^2} \left[ -\frac{u^2}{(q \cdot u - i0)^2} + \frac{2u^+}{(q \cdot u - i0)[q^+]} \right].$$  \quad (92)

The first term in the square bracket vanishes since $u_\mu$ is chosen to point along the $p^+$-direction, i.e., $u_\mu = (p^+, 0^-, 0_\perp)$, $u^2 = 0$, and by recalling that in dimensional regularization $u^2/(u^2 - \epsilon) = 0$. Notice that in a covariant gauge this diagram would be tantamount to the self-energy contribution of the struck quark. However, in the light-cone gauge, we are employing, a second term in the parenthesis in the first line of Eq. (92) appears which stems from the gluon propagator in that gauge. This term, being not lightlike, also entails a contribution to the cusp-dependent part, as we will now show. Indeed, one has

$$\Phi_a^{(1-\text{loop})}(u, \eta) = ig^2 \mu^2 C_F 2p^+ \int \frac{d^2 q}{(2\pi)^\omega} \frac{1}{(q^2 - \lambda^2)(q \cdot u - i0)[q^+]}.$$

(93)
an expression which would correspond to a vertex-like contribution of the pure gauge link in a covariant gauge—as one may appreciate from Eq. (91).

The pole-prescription dependent integral can be evaluated in analogy to our previous calculations in the preceding subsection, so that

\[ \Phi^{(1-\text{loop})}_a(u, \eta) = -g^2 C_F 2 \left( \frac{4\pi \mu^2}{\lambda^2} \right) \frac{\epsilon \Gamma(\epsilon)}{(4\pi)^2} \int_0^1 dx \frac{1}{x^2} \left[ -\bar{x} x \right] \],

where the bracketed term in the denominator is to be evaluated with the aid of Eq. (75).

The last step in obtaining an explicit expression for \( \Phi^{(1-\text{loop})}_a(u, \eta) \) is to carry out the line integral over \( x \) (which enters because of the appearance of the gauge link in the lightlike direction). To do so, we have to take care of the additional (logarithmic) singularity \( \sim \ln \tau \), which cannot be regularized by the parameter \( \eta \), where \( \tau \) is an extra regulator. The origin of this singularity is related to the vector \( u_\mu \) which defines the lightlike direction. There are, of course, several possibilities how to regularize this type of integral. For instance, one can get a regular expression in \( \tau \)—after the integration over \( d\eta \)—as discussed in Refs. (19, 21) having recourse to the fact that the derivative \( \partial \Phi^{(1-\text{loop})}_a(u, \eta) / \partial \eta \) is \( \tau \)-independent. However, for technical convenience, we apply here a different regularization technique by making use of an auxiliary regulator \( \tau \) and absorb the light-cone singularity \( \ln \tau \) inside a redefined parameter \( \tilde{\eta} = 2\tau \eta \), the latter being contained inside the pole-prescription contribution—cf. Eq. (75). This is possible, given that \( \tau \) does not depend on the scale parameter \( \eta \) and hence does not contribute to the evolution of the considered quantity (see Sec. VI).

Performing all these operations and taking into account that \( u^+ = p^+ \), one gets for the UV part of diagram (a) in Fig. 7

\[ \Phi^{(1-\text{loop})}_a(u, \eta) = -\frac{\alpha_s}{\pi} C_F \frac{1}{\epsilon} \left( \ln \frac{\eta}{p^+} - i \frac{\pi}{2} - i \pi C_\infty \right) \].

Evidently, this expression yields a cusp-dependent contribution, \( \sim \ln p^+ \), as we already mentioned, which would be completely absent in a covariant gauge, thus, underlying their mutual difference. Besides, there is a dependence on the choice of the pole prescription (via the numerical parameter \( C_\infty \)). In order to cancel this latter dependence, one needs to take into account the contribution of diagram (d) in Fig. 7 viz.,

\[ \Phi^{(d)} = -g^2 C_F \mu^2 p^+ \int \frac{d^2 q}{(2\pi)^2} \int d^2 q^+ e^{-iq^+\infty} \int d^2 q'_+ \frac{1}{(2\pi)^2 (q^2 - \lambda^2)[q^+]} \times (-i)(2\pi)^4 \delta^{(4)}(q + q') \frac{1}{q \cdot u + i0} \frac{1}{q'_+ \cdot l_\perp + i0} \].

FIG. 7: Virtual gluon contributions to the UV-divergences of the soft counter term, given by Eq. (87). The designations are as in Fig. 3.

\[ \Phi^{(1-\text{loop})}_a(u, \eta) = -g^2 C_F 2 \left( \frac{4\pi \mu^2}{\lambda^2} \right) \frac{\epsilon \Gamma(\epsilon)}{(4\pi)^2} \int_0^1 dx \frac{1}{x^2} \left[ -\bar{x} x \right] \].

\[ (94) \]
Using Eqs. (70)–(73), we find
\[
\Phi^{(d)} = \frac{\alpha_s C_F}{\pi} \frac{1}{\epsilon} \left( -4\pi \frac{\mu^2}{l^2} \right) ^\epsilon \tag{97}
\]
and extracting the UV-pole and adding it to Eq. (95) we finally arrive at
\[
\Phi^{(a+d)}_{UV}(\eta) = -\frac{\alpha_s}{\pi} C_F \frac{1}{\epsilon} \left( \ln \frac{\eta}{p^+} - \frac{i\pi}{2} - i\pi C_\infty + i\pi C_\infty \right) = -\frac{\alpha_s}{\pi} C_F \frac{1}{\epsilon} \left( \ln \frac{\eta}{p^+} - \frac{i\pi}{2} \right) . \tag{98}
\]
This result exhibits the independence on the pole prescription of the soft factor, in close analogy to Eq. (79). Taking into account the corresponding “mirror” diagram (which doubles the real part and cancels the imaginary one), we obtain the total UV-divergent part of the soft factor in the one-loop order:
\[
\Phi^{(1\text{-loop})}_{UV}(\eta) = -\frac{\alpha_s}{\pi} C_F \frac{1}{\epsilon} \ln \frac{\eta}{p^+} , \tag{99}
\]
making it apparent that there is no dependence on the pole prescription, as now all \(C_\infty\)-dependent terms are absent.

To conclude, we have shown at the one-loop level that the soft counter term (soft eikonal factor) has the following two important properties:
(i) it gives rise to the same cusp anomalous dimension as \(f_{q/q}(x, k_\perp)\), but with an opposite sign, and
(ii) it bears no dependence on the choice of the pole prescription to go around the light-cone singularity in the light-cone gauge (with corresponding terms cancelling among themselves).

Therefore, it is reasonable to redefine the conventional TMD PDF by including into its definition the soft counter term ab initio. This provides
\[
f_{q/q}^{\text{mod}}(x, k_\perp; \mu, \eta) = \frac{1}{2} \int \frac{d\xi d^2\xi_\perp}{2\pi(2\pi)^2} e^{-ik^+\xi^- + i\xi_\perp \cdot \xi_\perp} \langle q(p) | \bar{\psi}(\xi^-, \xi_\perp) \nu(\xi^- - y) D_{\mu\nu}(x - y) \rangle , \tag{100}
\]
which represents one of the main results of our investigation here and in [42].

Let us finish this section by giving a physical interpretation to the soft counter term, using Mandelstam’s formalism [49, 50]. To this end, we utilize the exponentiation theorem for non-Abelian path-ordered exponentials [17] and recast the exponential operator (89) in the form
\[
\Phi(u, n^-) = \exp \left[ \sum_{n=1}^{\infty} \alpha_n \Phi_n(u, n^-) \right] , \tag{101}
\]
where the functions \(\Phi_n\) have, in general, a complicated structure that is, however, irrelevant for our purposes here. The leading term in this series, \(\Phi_1\), is just a non-Abelian generalization of the Abelian expression
\[
\Phi_1(u, n^-) = -4\pi C_F \int_{C_{\text{cusp}}} dx_\mu dy_\nu \theta(x - y) D^{\mu\nu}(x - y) . \tag{102}
\]
Then, by virtue of the current
\[
f_{\nu}^b(z) = t^b v_\nu \int_{C_{\text{cusp}}} d\tau \delta^{(4)}(z - v\tau) , \tag{103}
\]
evaluated along the contour $C_{\text{cusp}}$ (cf. Eq. (90)) and where the velocity $v_\nu$ equals either $u_\nu$, $n^-$, or $l_\perp$ (depending on the segment of the contour along which the integration is performed), one can rewrite (102) as follows

$$\Phi_1(u, n^-) = -t^a 4\pi \int_{C_{\text{cusp}}} dx_\mu \int d^4z \delta^{ab} D^{\mu\nu}(x - z) j^b_\nu(z).$$

(104)

This expression looks formally very similar to the “intrinsic” Coulomb phase found by Jakob and Stefanis (JS) [48] in QED for Mandelstam charged fields involving a gauge contour which is a timelike straight line. The name “intrinsic” derives from the fact that this phase is different from zero even in the absence of external charge distributions. Its origin was ascribed by JS to the long-range interaction of the charged particle with its oppositely charged counterpart that was removed “behind the moon” after their primordial separation.\(^9\)

This phase is acquired during the parallel transport of the charged field along a timelike straight line from infinity to the point of interaction with the photon field and is absent in the local approach, i.e., for local charged fields joined by a connector. It is different from zero only for Mandelstam fields with their own gauge contour attached to them and keeps track of its full history since its primordial creation. Keep in mind that the connector is introduced ad hoc in order to restore gauge invariance and is not part of the QCD Lagrangian. In contrast, when one associates a distinct contour with each quark field, one, actually, implies that these Mandelstam field variables should also enter the QCD Lagrangian (see [48] for more details). However, a consistent formulation of such a theory for QCD is still lacking and not without complications of its own.

The analogy to our case is the following. First, formally adopting a direct contour for the gauge-invariant formulation of the TMD PDF in the light-cone gauge (Figure 8(a) shows an example of the contributing diagrams), the connector gauge link does not contribute any anomalous dimension—except at the endpoints; this anomalous dimension being, however, irrelevant for the issue at stake. Hence, there is no intrinsic Coulomb phase in that case. Second, splitting the contour and associating each branch to a quark field, transforms it into a Mandelstam field and, as a result, adding together all gluon radiative corrections at the one-loop order, a $p^+$-dependent term survives that gives rise to an additional anomalous dimension. We have shown that this extra anomalous dimension can be viewed as originating from a contour with a discontinuity in the four-velocity $\dot{x} (\sigma)$ at light-cone infinity—a cusp obstruction.

Classically, it is irrelevant how the two distinct contours $C_1$ and $C_2$ in Fig. 5 are joined, i.e., smoothly or by a sharp bend. But switching on gluon quantum corrections, the renormalization effect on the junction point reveals that the contours are not smoothly connected, but go instead through a cusp. Here, we have a second analogy to the QED case discussed above. Similarly to the “particle behind the moon”, this cusp-like junction point is “hidden” and manifests itself only through the path-dependent phase (104). Note in the same context that integrating over the transverse momentum (see next section), the $p^+$-dependent terms, resulting from virtual gluon corrections, cancel against their counterparts from real-gluon corrections, so that this cusp-induced phase disappears [see for illustration Fig. 8(b)].

In our previous paper [42], we concentrated on the anomalous dimension of the TMD PDF, and, therefore, only the UV-divergent parts were studied. In the present work, however,\(^9\) the existence of a balancing charge “behind the moon” was postulated before by several authors—see [48] for related references—in an attempt to restore the Lorentz covariance of the charged sector of QED.
we take also into account the UV-finite parts and, consequently, the dependence on the transverse momentum appears explicitly, as we discussed above.

VI. REAL-GLUON CONTRIBUTIONS AND EVOLUTION EQUATIONS

In this section, we concentrate our efforts on two subjects: (i) First we discuss in some detail the evolution behavior of the TMD PDF and establish the connection between our approach and that of Collins and Soper [67]. (ii) Second, we prove that the integrated PDF, obtained from our modified definition, coincides with the standard one with no any artefact of the cusped contour used in the TMD PDF left over.

(i) Evolution behavior. The modified TMD PDF (100) depends on two arbitrary mass-scale parameters: the UV scale \( \mu \) and the extra regulator \( \eta \). The \( \mu \)-dependence is described by the standard renormalization-group evolution equation (see below) and is controlled by the UV-anomalous dimension, which arises as the sum of the anomalous dimensions of all the ingredients of the TMD PDF (100):

\[
\gamma_{f_{q/q}} = \gamma_{2q} + \sum_{i=1}^{4} \gamma_{\text{gauge link}} + \gamma_R
= \frac{3}{4} \frac{\alpha_s}{\pi} C_F + O(\alpha_s^2). \tag{105}
\]

The anomalous dimensions associated with the quark fields and the soft counter term \( R \) are marked by self-explaining labels. We have used for convenience a short-hand notation to denote the anomalous dimension of each gauge link on the right-hand side of Eq. (100) by a number in the order the gauge link appears from the left to the right.

Before we proceed, a couple of important remarks are here in order. One realizes that the anomalous dimension of \( f_{q/q} \) coincides with the anomalous dimension of the conventional quark propagator in the light-cone gauge, but with the opposite sign due to the different Dirac structure. Up to the sign, this result also coincides with the anomalous dimension of the gauge-invariant quark propagator in a covariant gauge [14]. The anomalous dimension of \( R, \gamma_R \), cancels precisely those contributions in the sum above which contain the \( p^+ \)-dependent terms.
FIG. 9: The leading-order real gluon contributions to the TMD PDF are shown. The diagrams (b) and (d) with a transverse gauge link do not contribute to the TMD PDF in the light-cone gauge. The dashed line marks the cut.

On the other hand, the dependence on $\eta$ is more complicated and is described by an integral kernel to be determined below. In order to derive the corresponding evolution equation, one needs to calculate the real-gluon contributions, depicted in Fig. 9. Here, we present this calculation in the small-\(\eta\) limit (that corresponds to the large-rapidity $\zeta \to \infty$ limit within the Collins-Soper approach [67]). Let us emphasize that in the case of the integrated PDFs, where the dependence on the regularization parameter $\eta$ appears at the intermediate steps of the calculations (in the light-cone gauge), it cancels out in the final expression. This will be demonstrated below. In contrast, in the unintegrated PDFs, this dependence remains and, thus, it should be treated by means of a corresponding evolution equation. Note that the diagrams in Fig. 9 are UV-finite and do not contribute to the anomalous dimensions. However, they do depend on the regularization parameter $\eta$. The diagram (a) yields

$$\Sigma^{(a)}_{\text{real}} = -g^2 C_F \int \frac{d^4 q}{(2\pi)^4} \frac{\gamma_\mu(\hat{p} - \hat{q})\gamma^+(\hat{p} - \hat{q})\gamma_\nu}{(p - q)^4} \text{Disc} [D^{\mu\nu}(q)] \times \delta(p^+ - k^+ - q^+) \delta^{(2)}(q_\perp - k_\perp) ,$$

where the absorptive part of the gluon propagator reads

$$\text{Disc} [D^{\mu\nu}(q)] = 2\pi \theta(q^+) \delta(q^2 - \lambda^2) \left( -g^{\mu\nu} + \frac{q^\mu n^- + q^\nu n^-}{[q^+]_{\text{PV}}} \right) .$$

In the last equation we have adopted the PV-prescription, because the real gluon contributions are prescription-independent and the diagrams with transverse gauge links do not contribute. The $\eta$-divergences can be isolated by means of the standard rules given in [68]:

$$\frac{1 - x}{(1 - x)^2 + (\eta/p^+)^2} = -\delta(1 - x) \ln \frac{\eta}{p^+} + \frac{1}{(1 - x)_+} .$$

After some standard calculations, one gets the “Feynman” ($\eta$-independent) part

$$\Sigma^{(a)\text{real}}_{\text{Feynman}} = \frac{\alpha_s}{2\pi^2} C_F \frac{|1 - x|}{p^+} \frac{k_\perp^2 + x\lambda^2 + x(x - 3)p^2}{[k_\perp^2 + x\lambda^2 - x(1 - x)p^2]^2} .$$

The $\eta$-dependence appears through the pole-contributions, i.e.,

$$\Sigma^{(a)\text{real}}_{\text{pole}} = \frac{\alpha_s}{\pi^2} C_F \left\{ \frac{x}{(1 - x)_+} - \delta(1 - x) \ln \frac{\eta}{p^+} \right\} \frac{1}{k_\perp^2 + x\lambda^2 - x(1 - x)p^2} .$$
On the other hand, the diagram (c) in Fig. 9 yields

$$
\Sigma_{\text{real}}^{(c)} = ig^2 C_F u_\mu u_\nu \delta(p^+ - xp^+) \int_0^\infty d\sigma \int_0^\infty d\tau \int \frac{d^4 q}{(2\pi)^4} e^{-iqu(\sigma-\tau)}\delta^{(2)}(q_\perp - k_\perp) \\
\times 2\pi\theta(q^+)(q^2 - \lambda^2) \left(g^{\mu\nu} - \frac{q^{\mu}n^{\nu} + q^{\nu}n^{\mu}}{|q^+|_{\text{PV}}}\right).
$$

(111)

In the small-$\eta$ limit, a straightforward calculation gives

$$
\Sigma_{\text{real}}^{(c)} = \frac{\alpha_s}{2\pi^2} C_F \delta(1-x) \frac{1}{k_\perp^2 + \lambda^2} \left(1 - \ln \frac{\eta}{p^+}\right).
$$

(112)

Finally, the (logarithmic) dependence of the modified TMD PDF on $\eta$ is determined in terms of the equation

$$
\eta \frac{d}{d\eta} f_{q/q}^{\text{mod}}(x, k_\perp; \mu, \eta) = \left[K(\mu) + G(\mu, \eta)\right] \otimes f_{q/q}^{\text{mod}}(x, k_\perp; \mu, \eta).
$$

(113)

We can recast this equation, which governs evolution with respect to $\eta$, in a form which formally resembles the standard Collins-Soper evolution equation\[67, 69\] with respect to $\mu$, namely,

$$
\eta \frac{d}{d\eta} f_{q/q}^{\text{mod}}(x, k_\perp; \mu, \eta) = \left[K(\mu) + G(\mu, \eta)\right] \otimes f_{q/q}^{\text{mod}}(x, k_\perp; \mu, \eta).
$$

(114)

The renormalization-group behavior of the functions $K(\mu)$ and $G(\mu, \eta)$\[70\] is determined by the universal cusp anomalous dimension

$$
\frac{1}{2} \mu \frac{d}{d\mu} \ln K(\mu) = -\frac{1}{2} \mu \frac{d}{d\mu} \ln G(\mu, \eta) = \gamma_{\text{cusp}} = \frac{\alpha_s}{\pi} C_F + O(\alpha_s^2).
$$

(115)

Extracting explicit expressions for $K(\mu)$ and $G(\mu, \eta)$ from our Eq. (113), we can readily show that they each satisfy Eq. (115) with respect to the cusp anomalous dimension. We emphasize that the parameter $\eta$ in our approach plays a role akin to the rapidity parameter $\zeta$ in the additional evolution equation of Collins and Soper, with Eq. (113) being the analogue of the Collins-Soper equation.

Therefore, the dependence on the dimensional regularization scale $\mu$ of the re-defined TMD PDF (100) is given by the following renormalization-group equation

$$
\frac{1}{2} \mu \frac{d}{d\mu} \ln f_{q/q}^{\text{mod}}(x, k_\perp; \mu, \eta) = \frac{3\alpha_s}{4\pi} C_F + O(\alpha_s^2).
$$

(116)

It is important to appreciate that only the modified TMD PDF, given by Eq. (100), obeys such a simple UV-evolution; without the soft counter term, non-trivial extra contributions would arise in the corresponding anomalous dimension on the right-hand side of Eq. (105).

Taking logarithmic derivatives of $f_{q/q}^{\text{mod}}(x, k_\perp; \mu, \eta)$ with respect to both scales $\mu$ and $\eta$, we get

$$
\mu \frac{d}{d\mu} \left[\eta \frac{d}{d\eta} f_{q/q}^{\text{mod}}(x, k_\perp; \mu, \eta)\right] = 0,
$$

(117)
which establishes the formal analogy between our approach and the Collins-Soper one. This equation ensures the absence of extra UV-singularities related to artefacts owing to the light-cone gauge and is equivalent to our initial requirement of the cancellation of undesirable divergences.

(ii) Integrated modified PDF. Consider now the integration of Eq. \((100)\) over \(k_\perp\). We collect all \(\eta\)-dependent terms from the virtual and the real-gluon contributions—contributing UV divergences—and perform the \(k_\perp\) integration. We find that the result is \(\eta\)-independent, so that the DGLAP evolution of this quantity is guaranteed. Below, we demonstrate this cancellation explicitly. The integration of the UV-divergent term \((78)\) trivially gives

\[
2\frac{\alpha_s}{\pi} C_F \frac{1}{\epsilon} \ln \eta \delta(1-x) \int d^2k_\perp \delta^{(2)}(k_\perp) = 2\frac{\alpha_s}{\pi} C_F \frac{1}{\epsilon} \ln \frac{\eta}{p^+} \delta(1-x) .
\]  

On the other hand, the integration (in the dimensional regularization) of the \(\eta\)-dependent real-gluon contribution \((110)\) yields

\[
-\frac{\alpha_s}{\pi^2} C_F \ln \frac{\eta}{p^+} \delta(1-x) \mu^2 \int d^2-2\kappa \frac{1}{k_\perp^2 + \Lambda^2} = -2\frac{\alpha_s}{\pi} C_F \Gamma(\epsilon) \ln \frac{\eta}{p^+} \times \delta(1-x) \left(\frac{4\pi \mu^2}{\Lambda^2}\right)^\epsilon ,
\]  

where \(\Lambda^2 = x\lambda^2 - x(1-x)p^2\). After extracting the UV-divergent term

\[
-2\frac{\alpha_s}{\pi} C_F \frac{1}{\epsilon} \ln \frac{\eta}{p^+} \delta(1-x) ,
\]  

we observe that it exactly cancels the right-hand-side of Eq. \((118)\). The same cancellation occurs between the \(\eta\)-dependent terms in the virtual and the real-gluon contributions of the soft factor. Integration over the transverse momentum (using dimensional regularization) in the modified TMD PDF \((100)\) yields—at least formally—the integrated PDF

\[
\int d^{n-2}k_\perp f^{\text{mod}}_{i/a}(x,k_\perp;\mu,\eta) = f_{i/a}(x,\mu)
\]

which bears no \(\eta\)-dependence as well.

From the above considerations it becomes apparent that the renormalization-group properties of this distribution are described by the DGLAP equation

\[
\mu \frac{d}{d\mu} f_{i/a}(x,\mu) = \sum_j \int_x^1 \frac{d\zeta}{\zeta} P_{ij} \left(\frac{x}{\zeta}\right) f_{j/a}(\zeta,\mu) ,
\]

where the integral kernel reads (in leading order)

\[
P_{ij}(x) = \frac{\alpha_s}{\pi} C_F \left[ \frac{3}{2} \delta(1-x) + \frac{1 + x^2}{(1-x)_+} \right] + O(\alpha_s^2)
\]

and the \((\cdot)_+\)-regularization is defined in the standard manner by

\[
\int_0^1 dz \frac{f(z)}{(1-z)_+} \equiv \int_0^1 dz \frac{f(z) - f(0)}{1-z} .
\]

Thus, one may conclude that the extra cusp-dependent terms are not present in the integrated case and, consequently, the UV properties of the standard PDFs (governed by the DGLAP equation) are not affected by the additional parameter \(\eta\), as expected.
VII. DRELL-YAN AND UNIVERSALITY

The study presented in the preceding sections was performed for the semi-inclusive DIS (SIDIS). In this section we discuss the DY case within our approach and comment on universality.

We now ask ourselves to what extent our analysis can be applied to other reactions, like the DY lepton-pair production. It was shown by Collins [32] that the direction of the integration contours in the lightlike gauge links entering the definition of the TMD PDF should be reversed relative to the SIDIS, see Fig. 10:

\[
\left[ \xi^-, \xi_\perp; \infty^-, \infty_\perp \right]_{\text{SIDIS}} \rightarrow \left[ \infty^-, \xi_\perp; \xi^-, 0_\perp \right]_{\text{DY}}. \tag{125}
\]

Later, Belitsky-Ji-Yuan [33] argued that the transverse gauge links, they had introduced to exhaust the gauge invariance in the light-cone gauge, should be used in the DY case with the reverse sign, keeping in mind that the lightlike gauge links do not contribute. Hence, one has for DY the following combination of gauge links

\[
\left[ \infty^-, 0_\perp; \infty^-, \infty_\perp \right]_{\text{SIDIS}} \rightarrow \left[ \infty^-, 0_\perp; \infty^-, 0_\perp \right]_{\text{DY}}. \tag{126}
\]

In our approach, the latter replacement should be supplied with a change of sign of the additional regulator \( \eta \) (cf. the pole prescription in Eq. (49)), i.e.,

\[
\eta_{\text{SIDIS}} \rightarrow -\eta_{\text{DY}}. \tag{127}
\]

This change reflects, in fact, the different behavior of the gauge fields at the plus and minus light-cone infinity subject to the proviso of different pole-prescriptions. In the non-polarized
case (we exclusively discuss in the present investigation), this affects only the purely imaginary terms of the UV-divergent parts—consult Eq. (64). Thus, in the DY case, one has

\[ \Sigma^{(a)}_{\text{UV}} \bigg|_{\text{DY}} = -\frac{\alpha_s}{4\pi} C_F \frac{1}{\epsilon} \left[ 1 - \ln 4\pi + \gamma_E - \frac{2\gamma^\pm \hat{p}}{p^+} \left( 1 + \ln \left( \frac{\eta}{p^+} + \frac{i\pi}{2} + i\pi C_{\text{DY}}^\infty \right) \right) \right], \quad (128) \]

where the numerical factor \( C_{\text{DY}}^\infty \) differs from the SIDIS case and is defined as

\[ C_{\text{DY}}^\infty = \begin{cases} -1, \text{ Advanced} \\ 0, \text{ Retarded} \\ \frac{1}{2}, \text{ Principal Value} \end{cases} \quad (129) \]

These imaginary terms occur at the intermediate steps of the calculations, but do not contribute to the final expressions for the unpolarized TMD PDFs. Exactly the same arguments hold for the additional soft factor (92). This means that the (real-valued) UV anomalous dimension of the unpolarized TMD PDFs is universal as regards the SIDIS and the DY processes:

\[ \gamma_{f_{q/q}}^{\text{SIDIS}} = \gamma_{f_{q/q}}^{\text{DY}}. \quad (130) \]

This, however, may not be true for the spin-dependent TMD PDFs, since in that case the imaginary parts play a crucial role and, thus, a sign change (expressed in (128)) might indeed affect the renormalization-group properties and the corresponding evolution equations. This is an interesting task which will be pursued separately elsewhere.

VIII. SUMMARY AND CONCLUSIONS

In this paper we have applied renormalization-group techniques to TMD PDFs, defined in a gauge-invariant way. We have shown by explicit calculation in the light-cone gauge of the one-loop gluon radiative corrections to the quantity \( f_{q/q}(x, k^\perp) \) that a contribution appears, which is proportional to \( \ln p^+ \). This contribution gives rise to an anomalous dimension that is formally equal to the universal cusp anomalous dimension and helps unravel a cusp obstruction in the composed gauge contour at light-cone infinity. The origin of this anomalous dimension can be traced to the renormalization effect on the junction point of the split contours, each associated with a gauge link and attached individually to a quark field—transforming it into a Mandelstam path-dependent field [49, 50]. Guided by this finding, we derived a generalized factorization rule for cusp-connected gauge links which contains an additional phase factor and worked it out. For gauge links joined along lightlike contours, this factor reduces to unity, while for more convoluted contours which run off to light-cone infinity in the transverse configuration space, this eikonal factor contributes an anomalous dimension due to the cusp. In this context, we emphasize that we verified that integrating over the transverse momenta in \( f_{q/q}(x, k^\perp) \) no artefact owing to the contour cusp remains, thus invigorating the validity of the standard integrated PDF.

In order to eliminate the cusp anomalous dimension and recover the well-known results in a covariant gauge, where the gauge field vanishes at infinity, we proposed a new definition for the TMD PDF, which includes a soft counter term in the sense of Collins and Hautmann [43, 44], in order to eliminate the contribution from the cusp anomalous dimension. This counter term enters in addition to the transverse gauge inks, previously introduced by Belitsky, Ji, and Yuan [33], and comprises two eikonal factors caused by a particle-like current flowing along a cusped contour meandering from \((0^-,-\infty^+,0^\perp)\) to \((\xi^-,\infty^+,(\xi^\perp)\) with a sharp bend.
in the transverse direction. We have argued that each of these eikonal factors resembles in crucial aspects the “intrinsic Coulomb phase” found before by Jakob and Stefanis \[48\] in a formulation of QED in terms of Mandelstam fields. In the present case, the cusp-like junction point of the two individual gauge contours plays a similar role as the so-called “particle behind the moon”, postulated in QED in connection with the Lorentz-covariance restoration of its charged sector. Both quantities share the feature of being “hidden” at infinity and reveal themselves only in terms of (path-dependent) phases, being independent of external charge distributions (QED case) and unrelated to boundary conditions to avert light-cone singularities (TMD PDF case in QCD). The origin of the phenomenon is in both cases the same and peculiar to the inclusion of the individual path-dependent exponential into the field operator supposed to describe the quark as a Mandelstam field. No such effect appears in cases where the dynamics of the process allow one to use a direct contour between the two field points. In that case, one has to deal only with the connector which has well-known renormalization properties \[14, 67\].

The “intrinsic Coulomb phase” in QED tells us that each charged particle, though primordially separated from its balancing counterpart, is still in harness with it. In the TMD PDF case, this phase accumulates effects due to the interaction of the struck quark with its target spectators, as pointed out by Ji and Yuan in \[41\] and reinforced by Belitsky, Ji, and Yuan in \[33\]. However, the existence of a cusp at light-cone infinity went unnoticed, because in previous works the UV divergences of the TMD PDF were not considered. In \[33\] UV divergences were addressed, but only within the Collins-Soper approach, which is formulated off-the-light-cone and, hence, the \(\ln p^+\) term does not appear there at all.

The appearance of the cusp anomalous dimension in the present context is, in actual fact, not really surprising. We know from the so-called modified factorization of exclusive reactions that retaining transverse degrees of freedom amounts to the inclusion of Sudakov factors for each quark in the hard-scattering subprocess \[71\]—see for a review \[72\]. The connection of the Sudakov factors to the cusp anomalous dimension within the modified factorization scheme was worked out in detail in \[73\] up to the level of the next-to-leading-order logarithmic accuracy.

Our results may have a wide range of applications. Chief among them:

- More precise data analyses of various experimental data on hard-scattering cross sections.
- Development of more accurate Monte-Carlo event generators (to estimate exclusive components of inclusive cross sections) \[66, 74\].
- Better description of polarized TMD PDFs and the phenomenology related to SSA and spin physics \[2, 3, 32, 75, 76, 77\].

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