1 Introduction

1.1 Hints

A number of results testifies for non-zero neutrino masses and mixing:

- Solar neutrino spectroscopy.
- Results on atmospheric neutrinos.
- Large scale structure of the Universe. (Its formation may imply some amount of the hot dark matter (HDM)).
- LSND results.
- Hydrogen ionization in the Universe.
- Peculiar velocities of pulsars.
- Excess of events in tritium spectrum.

First four items are reviewed by Y. Suzuki [1], the fifth item was discussed in [2], and the last two will be presented in sect. 2.

1.2 Upper bounds

Majority of results gives just upper bounds on neutrino masses and mixing. The most strong bounds relevant for the discussion come from:

- reactor oscillation experiments BUGEY [3], Krasnoyarsk [4] ($\bar{\nu}_e - \nu_x$ oscillations);
- meson factory oscillation experiments (KARMEN [5], LSND [6]);
- accelerator experiments E531 ($\nu_\mu \to \nu_\tau$) [7] and E776 ($\nu_\mu - \nu_\tau$) [8];
- direct kinematic searches of neutrino mass: tritium experiment in Troitzk on $\bar{\nu}_e$ [9] (see also Mauz experiment [10]), PSI experiment on the mass of $\nu_\tau$ [11], LEP ALEPH result on $\nu_\tau$ [12] (see also OPAL result [13], new possibility to measure the mass of $\nu_\tau$ has been suggested in [14]);
- searches for the neutrinoless double beta decay in Heidelberg-Moscow experiment [15] (see also IGEX [16]);
- supernova 1987A data [17,18,19], dynamics of supernovas [20];
- nucleosynthesis in supernovas [21,22];
- cosmology [23];
- structure formation in the Universe [24,25,26].

These results give important restrictions on possible pattern of neutrino masses and mixing.

1.3 Lower bounds on neutrino mass?

Neutrinos are the only fermions for which the Standard model predicts masses. It predicts that neutrino masses are zero. This follows from the content of the model, namely, from the fact that in the model there is

- no right handed neutrino components,
- no Higgs triplets which can give the Majorana mass for the left handed neutrinos.

The absence of the $\nu_R$ gives an explanation of strong upper bounds on the neutrino masses. However the absence of $\nu_R$ looks rather anesthetic.

The Standard Model is not the end of the story and we know that at least there is the gravity. The gravity can questioned both above items:
1. One point is related to a consistency of the theory. It is argued that invariant (Pauli-Villars) regularization in the case of local Lorentz invariance requires an existence of 16 spinors, i.e. an additional spinor with properties of $\nu_R$. Once $\nu_R$ exists there is no reasons not to introduce the Dirac mass term for neutrinos.

2. It is believed that gravity breaks global quantum numbers. In the SM the lepton number invariance requires an existence of 16 spinors, or condensates) can be even much smaller than the one estimated from (3). In particular, $M < M_P$ can be a combination of other mass parameters $M'$, $m_{3/2}$ which are much smaller than $M$ itself: e.g. $M = (M')^2/m_{3/2}$.

A phenomenological lower bound on $m_{\nu}$ has been suggested recently. The exchange of massless neutrinos leads to the long range neutrino forces. In particular, two body potential due to the exchange of the $\bar{\nu}\nu$ - pair gives:

$$ V_0^{(2)} = \frac{aG_F^2}{4\pi^3r^5} , $$

where $a$ is known coefficient. Many body (four, six ... $k$ ...) potentials contain additional factors $(G_F/r^2)^k$ which are extremely small for $r = R_{ns}$ (radius of neutron star). However in compact stellar objects like neutron stars and white dwarfs, the contributions of these many body interactions to energy of the star are greatly enhanced due to combinatorial factor.

The contribution of $k$-body interactions, $W^k$, to the total energy is proportional to number of combinations of $k$-neutrons from total number of neutrons in a star. The combinatorial factor leads to the series parameter $W_+^{k+2}/W^k \sim (G_{F}nR_{ns})^2 \sim (10^{13})^2$, where $n$ is the number density of neutrons. So that the six body contribution to the energy dominates over the four body contribution etc. It turns out that the energy due to the eight body interactions overcomes the mass of a star. According to the only way to resolve this paradox is to suggest that all neutrinos have nonzero masses: $m_{\nu} > 0.4 \text{ eV}$. Neutrino mass cuts off the forces at $r > 1/m_{\nu}$. In section 2.3 we will argue that there is another resolution of the paradox and the mass of neutrino can be zero.

2 Refraction and neutrino masses

There are several new results on neutrino refraction and propagation in media which have important implications to the neutrino mass problem.

2.1 Effective potentials

In transparent medium neutrinos undergo essentially elastic forward scattering. The effect of the scattering is described by

$$ H = G_F\sqrt{2} \bar{\nu}\gamma^\mu(1 - \gamma_5)\nu (\bar{\psi}_e\gamma_\mu(g_\nu + gA\gamma_5)e|\psi_e) , $$

where $\psi_e$ is the wave function of medium. (We took into account the interactions with electrons only). For ultrarelativistic neutrinos the expression can be reduced to

$$ H = \sqrt{2} G_F V \nu^\dagger \nu , $$
where $V$ is the effective potential. Let us summarize the results on the potentials for some cases:

1. Unpolarized medium at the rest: Only $\gamma^0$ component of the vector current contribute to $V$ and its matrix element gives the density of electrons, $n_e$. As the result we get:

$$V = \sqrt{2}G_F n_e g_V.$$  \hspace{1cm} (7)

2. Polarized medium at the rest. The axial vector current, $\vec{\gamma}\gamma_5$, also gives the contribution which is proportional to the vector of spin $\vec{\gamma}$:

$$V = \sqrt{2}G_F n_e \left[ g_V + g_A 2(\vec{k} \cdot \langle \vec{s} \rangle) \right],$$  \hspace{1cm} (8)

where $\vec{k} \equiv \vec{p}/p$, and $\vec{p}$ is the momentum of neutrino, $\langle \vec{s} \rangle$ is the averaged spin of electrons in medium. The second term can be rewritten as $\sqrt{2}G_F g_A (n_+ - n_-).$ Here $n_+, n_-$ are the concentrations of the electrons with polarization along and against the neutrino momentum.

3. In the case of moving medium also spatial components of the vector current give non-zero contribution: $\langle \psi_e | \vec{\gamma} \psi_n \rangle \propto \vec{v} \cdot \vec{k}$ and

$$V = \sqrt{2}G_F n_e g_V (1 - v \cdot \cos \theta),$$ \hspace{1cm} (9)

where $\theta$ is the angle between the momentum of the electrons and neutrino. In the case of isotropic distribution the correction disappears. In this case non zero effect of the motion appears via the correction to the propagator of the vector boson: $G_F \rightarrow G_F (1 + q^2/m_W^2)$, where $q^2$ is the four momentum of the intermediate boson squared. \hspace{1cm} (10)

In thermal bath $q^2 \sim T^2$ and one gets:

$$V \sim \sqrt{2}G_F n_e A \frac{T^2}{m_W^2},$$ \hspace{1cm} (11)

where $A$ is the constant which depends on the composition of plasma. In all the cases, apart from the thermal correction, $V$ has opposite signs for neutrinos and antineutrinos.

2.2 Neutrino sea and the long range neutrino forces

At low energies a medium is transparent for neutrinos and main effect is the refraction. Refraction index equals:

$$(n_r - 1) = \frac{V}{\rho} \propto \frac{G_F n}{\rho}.$$ \hspace{1cm} (12)

At usual conditions: $E \sim 1$ MeV, $\rho = 1$ g/cm$^3$, the deviation of the refraction index from 1 is extremely small: $(n_r - 1) \sim 10^{-20} - 10^{-19}$. However at very low energies this deviation can be of the order one, leading to complete inner reflection of neutrinos in stars. For neutron star with $\rho \sim 10^{14}$ g/cm$^3$ the complete reflection takes place for neutrinos with energies $E < 50$ eV. In other terms, a star can be considered as a potential well with the depth $V$. The potential has different signs for neutrinos and antineutrinos. Therefore, neutrinos are trapped, whereas antineutrinos are expelled from the star. In such a way strongly degenerate sea is formed with chemical potential $\mu$:

$$\mu \sim V \sim G_F n.$$ \hspace{1cm} (13)

In neutron stars the density of neutrinos from the sea is $n_\nu \sim 10^{17}$ cm$^{-3}$ and the total energy in the sea is very small in comparison with mass of a star. In spite of this, an existence of the sea can play an important role. The degenerate sea in stars leads to Pauli blocking of the long range forces. Instead of (14) we get for two body potential:

$$V^{(2)}_\mu = \frac{G_F^2}{4\pi^3 \rho^5} (\cos 2\mu r + \mu r \sin 2\mu r).$$ \hspace{1cm} (15)

Note that $1/\mu \sim 10^{-5}$ cm $\ll R_{ns}$. Rapidly oscillating factors in (15) lead to effective cut off of the forces at $r > 1/\mu$. Similar oscillating factors appear for many body interactions. As the result the many body forces do not dominate in self energy of star. This can resolve the energy paradox suggested in \cite{13} even for massless neutrinos.

Another objection to Fischbach result is related to resummation of series over the $k$-body interactions. The interaction with medium modifies the dispersion relation for neutrinos:

$$q_0 = |\vec{q}| \pm V,$$ \hspace{1cm} (16)

and correspondingly, the propagator of neutrino:

$$S(q) = \frac{i}{q_0} = \frac{i}{(q_0 - V)^2 - \vec{q}^2}.$$ \hspace{1cm} (17)

This dressed propagator is the sum of free propagator and the results of elastic forward scattering on one neutron, two neutrons .... $k \rightarrow \infty$ neutrons in medium. If the neutrino forms closed loop, then this process is equivalent to summation of 0, 2, 4, .... $k$ body interactions due to neutrino exchange.
Therefore the energy density due to the neutrino exchange can be written as
\[
w = \int \frac{d^4 q}{(2\pi)^4} (-i) Tr \left[ q^0 \gamma_0 \frac{1 - \gamma_5}{2} \right]. \tag{16}
\]

The energy density due to the interactions, \(\Delta w\), is the difference of \(w\) given in (16) and \(w_0\) - the energy density for vacuum propagator: \(\Delta w = w - w_0\). Total energy of star is the integral of \(\Delta w\) over the volume of star. In approximation of uniform medium, \(V = \text{const}\), one can redefine the integration variable in (16):
\[
q'_0 = q_0 - V. \tag{17}
\]

After redefinition \(w\) is reduced to \(w_0\), so that \(\Delta w = 0\). Thus the energy of a star in this approximation is zero. However, this proof corresponds to infinite and uniform medium. Real star has finite size and the distribution of neutrons is non-uniform. In this case the redefinition of variables (17) is impossible and non-zero self energy of the star appears.

### 2.3 Oscillations in Magnetized Medium

Let us consider neutrino propagation in the thermal bath with magnetic field. Effect of the medium can be calculated as the correction to self-energy. Two diagrams appear: The loop diagram with \(W\)-boson: \(\nu \rightarrow We \rightarrow \nu\), where for the electron we should use the effective propagator in thermal bath. (ii) the tadpole diagram with \(Z\) and electron in the loop. The electrons couple to the electromagnetic field

\[
V = \sqrt{2} G_F n_e g_\nu + \frac{e g_A G_F}{\sqrt{2}} \left( \frac{3}{\pi^2} \right) (\hat{k} \cdot \hat{B}). \tag{18}
\]

The correction originates from the axial vector current. It influences dynamics of the neutrino conversion. In particular, the correction modifies the resonance condition:
\[
V + \frac{\Delta m^2}{2E} \cos 2\theta = 0 \tag{19}
\]

shifting position of the resonance in comparison with the case of zero magnetic field. It also influences the adiabaticity condition.

There are however wrong statements that the magnetic term can compensate or even be bigger that the first (vector current) term. It would induce new resonances and open the possibility to have the flavor resonances both for neutrinos and antineutrinos in the same medium. Actually magnetic (axial) term can not be bigger than the vector one. This can be seen immediately from another approach to the problem.

Indeed, the effect of the magnetic field is reduced to polarization of electrons, so that one can use the result \(\delta\) for the effective potential and calculate the average polarization of the electrons. For flavor oscillations the matter effects is determined by charge current scattering on electrons for which \(g_A = g_\nu = 1\) and therefore
\[
V = \sqrt{2} G_F n_e (1 + 2(s) \cos \alpha). \tag{20}
\]

Here \(\alpha\) is the angle between the neutrino momentum and the polarization of electrons and \(\langle \vec{s} \rangle = \langle \hat{s}(B) \rangle\). Obviously, second term can not be bigger than 1, so that one can get at most the compensation of the effective potential: \(V = 0\) in the case of the complete polarization of electrons in the direction against the neutrino momentum. Complete polarization can be achieved in the case of very big magnetic field and zero temperature.

The polarization equals \((n_+ - n_-)/n_e\), where \(n_+, n_-\): are the concentrations of the electrons with polarization +1 and -1. The energy spectrum of electrons in the magnetic field is quantized:
\[
\varepsilon(p_z, n, \lambda) = \sqrt{p_z^2 + m_e^2 + |e| B(2n + 1 - \lambda)}, \tag{21}
\]

where \(\lambda = -2s_z\). It consists of main Landau level, \(n = 0, \lambda = 1\), and pairs of the degenerate levels with opposite polarizations. Therefore the polarization effect is determined by concentration of electrons in Landau level,
\[
2(s) = \frac{n_+ - n_-}{n_e} = \frac{n_0}{n_e}. \tag{22}
\]

For strongly degenerate gas:
\[
n_0 = \frac{e B p_F}{2\pi^2}, \tag{23}
\]

where the Fermi momentum, \(p_F\), is determined by the normalization
\[
n_e = \frac{e B p_F}{2\pi^2} + \sum_{n=1}^{n_{\text{max}}} \frac{2 e B \sqrt{p^2 - 2 e B n}}{2\pi^2}. \tag{24}
\]
The first term corresponds to the main Landau level \( n = 0, \lambda = 1 \); and the second one is the result of summation over all other levels. The complete polarization corresponds to \( 2eB \geq p_F^2 \), when \( n_{\text{max}} \leq 1 \), and the sum vanishes. In this case all electrons are in the main Landau level: \( n_e = eBp_F/2\pi^2 \), from this one gets \( p_F = 2\pi^2n_e/eB \), and consequently, \( n_0 = n_e \). In the limit of small field: \( p_F \approx (3\pi^2n_e)^{1/3} \) and

\[
n_0 = \frac{eB}{2\pi^4} 3n_e . \tag{25}
\]

This leads to the result (18).

For oscillation to sterile neutrinos, however, the effective \( g_A \) can be bigger than \( g_V \) and the level crossing phenomena induced by magnetization are possible(4).

2.4 Neutrino mass and the peculiar velocities of pulsars

Important application of results described in sect. 2.3. has been found by Kusenko and Segre (4). There is the long standing problem of explanation of the high peculiar velocities of pulsars (\( v \sim 500 \) km/s). Non-symmetric collapse, effects in binary systems etc., give typically smaller velocities.

It looks quite reasonable to relate these velocities with neutrino burst. The momenta of pulsars are \( 10^{-3} - 10^{-2} \) of the integral momentum carried by neutrinos. Therefore, \( 10^{-3} - 10^{-2} \) asymmetry (anisotropy) in neutrino emission is enough for explanation of the peculiar velocities (4).

The anisotropy of neutrino properties can be related to the magnetic field. It was suggested that very strong magnetic field (\( 10^{15} - 10^{16} \) Gauss) can influence the weak processes immediately: the probability of emission of neutrino along the field and against the field are different.

According to mechanism suggested in magnetic field influences the resonance flavor conversion leading to angular asymmetry of the conversion with respect to the magnetic field. The latter results in asymmetry of the neutrino properties.

It is assumed that the resonance layer for the conversion \( \nu_e - \nu_e \) lies between the \( \nu_e - \)neutrinosphere and \( \nu_e - \)neutrinosphere (the latter is deeper than the former due to weaker interactions of \( \nu_e \)). Thus the \( \nu_e \) which appear in the resonance layer will propagate freely and \( \nu_e \) are immediately absorbed. The resonance layer becomes the “neutrinosphere” for \( \nu_e \). (In fact, in presence of the magnetic field the neutrinosphere becomes “neutrinoellipsoid” and this is crucial for the mechanism).

It is assumed that inside the protoneutron star there is a strong magnetic field of the dipole type. Then in one semishpere the field is directed outside the star, so that for neutrinos leaving the star \( (\vec{k} \cdot \vec{B}) > 0 \), whereas in another semishpere the field points towards the center of star and \( (\vec{k} \cdot \vec{B}) < 0 \). Since the electronic gas in the star is strongly degenerate we can use the expression (18) for the effective potential. According to (18) the magnetic field modifies the resonance condition differently in these two semishperes. In semishpere with \( (\vec{k} \cdot \vec{B}) > 0 \), the resonance condition is satisfied at larger densities and larger temperatures; \( \nu_e \) emitted from this semishpere will have bigger energies. On the contrary, in the neutrinosphere with \( (\vec{k} \cdot \vec{B}) > 0 \) the resonance is at lower densities and lower temperatures and neutrinos have smaller energies. Thus presence of the magnetic field leads to difference in energies of \( \nu_e \) emitted in different directions and therefore neutrino burst knocks the star. The observed velocities imply the polarization effect \( 10^{-3} - 10^{-2} \), or according to (18)

\[
eB\left(\frac{3n_e}{\pi^4}\right)^{1/2} \sim 10^{-3} - 10^{-2} .
\]

Below the \( \nu_e - \)neutrinosphere: \( n_e > 10^{11} \) cm\(^{-3} \) which gives \( B \sim 10^{14} \) Gauss. From the condition that the resonance should be below the \( \nu_e - \)neutrinosphere one gets

\[
\Delta m^2 > 10^4 \text{ eV}^2 , \text{ or } m_3 > 100 \text{ eV} . \tag{26}
\]

The mixing angle can be rather small: from the adiabatic condition it follows \( \sin^2 2\theta > 10^{-8} \).

Thus explanation of the peculiar velocities of pulsars based on the resonance flavor conversion implies the mass of the heaviest (\( \sim \nu_e \)) neutrino bigger than 100 eV. To avoid the cosmological bound on mass, the neutrino must decay (e.g. with Majoron emission). The attempts to diminish \( m_3 \) by means of very large magnetic field (so that the polarization effect compensates the density) lead to very strong asymmetry \( \sim 1 \). Another problem is that due to relatively high temperatures very strong polarization and consequently, the compensation are impossible.
In connection with Kusenko-Segre proposal it is interesting to mark recent results on measurements of the beta spectrum in tritium decay [1]. There are two features in the spectrum: (i) Excess of events near the end point, \( Q \), of the spectrum \( Q - E_e \lesssim 10 \text{ eV} \), (peak in the differential spectrum) which leads to the negative value of the \( m^2 \) in usual fit. (ii) Excess of events at lower energies of the electrons: \( Q - E_e \gtrsim 200 \text{ eV} \). The excess in this region was also observed by Mainz group. One possible explanation of this anomaly is an existence of neutrino with mass \( m \sim 200 \text{ eV} \) whose admixture in the electron neutrino state is characterized by probability \( P \sim 1 - 2 \% \). This is precisely in the range implied by pulsar velocities.

As far as the first anomaly is concerned (the negative \( m^2 \)) one possible explanation is the tachionic nature of neutrinos [10]. It should be stressed, however, that position of the peak depends on condition of the experiment: In the run of experiment in 1994 the peak was at \( Q - E_e \approx 7 \text{ eV} \) whereas in the run 1996 the peak is at \( Q - E_e \sim 11 \text{ eV} \). There were some changes of the experiment in run 1996, in particular, the strength of the magnetic field was higher. The shift of the peak indicates that it may have the instrumental origin, rather than the origin in neutrino properties.

### 2.5 Lepton asymmetry in the Early Universe

According to (10) in the Early Universe the difference of the potentials for different neutrino species can be written as

\[
\Delta V = \sqrt{2} G_F n_\gamma \left( \Delta L + A \frac{T^2}{m_W^2} \right),
\]

where \( n_\gamma \) is the photon density, \( \Delta L = (n_L - n_L)/n_\gamma \) is the leptonic asymmetry and \( n_L, n_\bar{L} \) are the concentrations of the active neutrinos and antineutrinos.

Matter effects can be important for oscillations into sterile neutrinos. Matter influences differently the neutrino and antineutrino channels, so that transitions \( \nu_\tau \rightarrow \nu_\tau \), and \( \bar{\nu}_\tau \rightarrow \bar{\nu}_\tau \) can create the \( \nu_\tau - \bar{\nu}_\tau \) asymmetry in the Universe.

Since \( V \) depends on concentration of neutrinos themselves, and consequently, on conversion probability, the task becomes non-linear. Due to this, depending on values of parameters, a small original asymmetry (one can expect \( \Delta L_0 \sim \Delta \rho_{\nu_\tau} \sim 10^{-6} \)) can be further suppressed or blow up [8].

The leptonic asymmetry influences the primordial nucleosynthesis. It was realized recently, that it can suppress production of sterile neutrinos, so that the concentration of these neutrinos is much smaller than the equilibrium concentration even in the case of large mixing angle and large mass squared difference.

Scenario suggested in [8] is the following. Suppose \( \nu_\tau \) mixes with \( \nu_\nu \) and parameters of the system are: \( \Delta m^2 \sim 5 \text{ eV}^2 \) and \( \theta_{\mu\nu} \sim 10^{-4} \). On the contrary, \( \nu_\mu - \nu_\nu \) has large mixing \( \theta_{\mu\nu} \sim 1 \) and \( \Delta m^2 \sim 10^{-2} \text{ eV}^2 \), so that \( \nu_\mu - \nu_\nu \) oscillations can solve the atmospheric neutrino problem. It turns out that in spite of this large mixing the concentration of sterile neutrinos is small.

Let us consider the evolution of system with decrease of temperature. There are two important scales determined by the equality of the \( T \)-term in \( \Delta V \) and level splitting due to mass difference:

\[
\sqrt{2} G_F n_\gamma A \frac{T^2}{m_W^2} = \frac{\Delta m^2}{2T}.
\]

For \( \Delta m^2 \) corresponding to \( \nu_\tau - \nu_\nu \) and \( \nu_\mu - \nu_\nu \) channels we get from (28) \( T_\tau \approx 14 \text{ MeV} \) and \( T_\mu \approx 2 \text{ MeV} \). (i) For \( T > T_\tau \) the oscillation transitions are suppressed by \( T \)-term in the potential. (ii) For \( T \approx T_\mu \) the \( T \)-term drops enough and oscillations become possible. Due to non-linearity of the equations the amplitude of oscillations blows up and the asymmetry reaches (practically during the same epoch \( t \sim 10^{-2} \text{ sec} \) \( \Delta L \sim 10^{-5} \)). With further diminishing of temperature the asymmetry may slowly increase up to \( 10^{-2} \) or even higher. (The mixing is chosen to be small enough, so that the concentration of sterile neutrinos, \( n_\nu \gtrsim n_\nu \Delta L \), is still smaller than the equilibrium one). (iii) In the epoch \( T \lesssim T_\mu \), when transition \( \nu_\mu - \nu_\nu \) could be important, the effective (matter) mixing \( \nu_\mu - \nu_\nu \) is suppressed by leptonic asymmetry (\( \Delta L \)-term of the potential) produced previously in \( \nu_\tau \) oscillations. We discuss the application of this result in sect. 4.7.

### 3 Pattern of neutrino masses and mixing

#### 3.1 Neutrino anomalies

Existing neutrino anomalies imply strongly different scales of \( \Delta m^2 \). For the solar neutrinos, the atmospheric neutrinos and LSND we have corre-
spondingly:
\[
\Delta m_{\odot}^2 \sim (0.3 - 1.2) \cdot 10^{-5} \text{ eV}^2 ,
\]
\[
\Delta m_{\text{atm}}^2 \sim (0.3 - 3) \cdot 10^{-2} \text{ eV}^2 ,
\]
\[
\Delta m_{\text{LSND}}^2 \sim (0.2 - 2) \text{ eV}^2 .
\]

That is
\[
\Delta m_{\text{LSND}}^2 \gg \Delta m_{\text{atm}}^2 \gg \Delta m_{\odot}^2 .
\]

The mass scale which gives desired HDM component of the Universe, \( m_{HDM} \):
\[
m_{HDM}^2 \sim (1 - 50) \text{ eV}^2
\]

can cover the LSND range.

In the case of three neutrinos there is an obvious relation:
\[
\Delta m_{21}^2 + \Delta m_{32}^2 = \Delta m_{31}^2 ,
\]
and inequality (32) can not be satisfied. That is with three neutrinos it is impossible to reconcile all the anomalies. Furthermore, additional bigger scale is needed for explanation of the pulsar velocities [20].

Three different possibilities are discussed in this connection. One can

- suggest (stretching the data) that
  \[
  \Delta m_{\text{LSND}}^2 = \Delta m_{\text{atm}}^2 ;
  \]
  Also the possibility \( \Delta m_{\odot}^2 = \Delta m_{\text{atm}}^2 \) was discussed [2]
- “sacrifice” at least one anomaly, e.g. the LSND result, or atmospheric neutrinos;
- introduce additional neutrino states.

In what follows we will consider examples which realize these three possibilities.

There is another important mass scale: the upper bound on the effective Majorana mass of the electron neutrino which determines the rate of the neutrinoless double beta decay:
\[
m_{ee} = \sum_{i=1,2,3} U_{ei}^2 m_i .
\]

Here \( U_{ei} \) are the elements of the lepton mixing matrix. Taking into account uncertainties in the nuclear matrix element one gets from the data \( m_{ee} \lesssim 0.5 \text{ - } 1.5 \text{ eV} \). Forthcoming experiments (NEMO-III [3]) will be able to strengthen the bound by factor 3. Note that typically \( m_{HDM} > m_{ee} \).

Figure 1: Qualitative pattern of the neutrino masses and mixing. Boxes correspond to different mass eigenstates. The sizes of different regions in the boxes determine fluctuations (|\( |U_{ij}|^2 \) of given eigenstates. Weakly hatched regions correspond to the electron flavor (admixture of \( \nu_e \)), strongly hatched regions depict the muon flavor and black regions present the tau flavor. Arrows connect the eigenstates involved in oscillation/conversion which solve \( \nu_e \) - solar, ATM - atmospheric, LSND - problems. Scenario shown here reproduces simultaneously \( \nu_\odot \), ATM, LSND and HDM.

### 3.2 Everything with three neutrinos?

It is assumed that LSND and atmospheric neutrino scales coincide [3].
\[
\Delta m_{23}^2 = \Delta m_{\text{LSND}}^2 = \Delta m_{\text{atm}}^2 \sim 0.2 - 0.3 \text{ eV}^2 .
\]

\( \nu_1 \) and \( \nu_2 \) are strongly mixed in \( \nu_\mu \) and \( \nu_\tau \). The neutrino \( \nu_1 \) has dominant \( e \) - flavor and weakly mixes with \( \nu_2 \). The mass splitting between these two states \( \Delta m_{12}^2 \approx \Delta m_{2}^2 \) (see fig.1). Basic features of this scenario are the following:

(i) \( \nu_\mu \rightarrow \nu_\tau \) oscillations explain the atmospheric neutrino deficit. However, since \( \Delta m_{23}^2 \) is rather big, no appreciable angular dependence is expected for multi GeV events in Kamiokande and SuperKamiokande.

(ii) The solar neutrino problem is solved by \( \nu_e \rightarrow \nu_\mu \) resonance conversion.

(iii) The probability of the LSND/KARMEN oscillations is determined by
\[
P \propto 4 |U_{3e}|^2 |U_{3\mu}|^2 .
\]

(iv) The scenario can supply three component HDM, if the absolute values of the masses are in eV range. In this case the spectrum is degenerate: \( m_1 \approx m_2 \approx m_3 \approx 1 \text{ eV} \).

(v) If neutrinos \( \nu_i \) are the Majorana particles, then \( m_{ee} \approx m_1 \approx 1 \text{ eV} \) is at the level of present upper experimental bound.

This scheme is a variant of the previously considered schemes with three degenerate neutrinos.
and an order of magnitude smaller mass splitting: 
\( \Delta m^2_{23} \sim 10^{-2} \text{eV}^2 \) (see sect. 4.4).

One can modify the scenario assuming mass hierarchy, so that \( \Delta m^2_{23} \approx m_3^2 \). In this case \( m_3 \sim 0.5 \text{eV} \), \( m_2 \approx 3 \cdot 10^{-3} \text{eV} \) and \( m_1 \ll m_2 \). The contribution to HDM is small and signal in \( \beta\beta \rightarrow \text{decay} \) searches is negligible.

3.3 Sacrifice solar neutrinos

The scheme consists of two heavy degenerate neutrinos \( \nu_2 \), \( \nu_3 \) strongly mixed in \( \nu_\mu \), \( \nu_\tau \) and one light weakly mixed state \( \nu_1 \) (fig. 2):

\[
\begin{align*}
\nu_1 & \ll m_2 \approx m_3 \approx 1 \text{eV}, \\
\Delta m^2_{23} & \approx 10^{-2} \text{eV}^2
\end{align*}
\]  

\( \nu_1 \approx \nu_e \). This scenario is realized e.g. in the Zee model. Basic features of the scenario are the following:

(i) Atmospheric neutrino problem is solved by \( \nu_\mu - \nu_e \) oscillations.

(ii) \( \nu_1 \) and \( \nu_2 \) form two components HDM.

(iii) The probability of oscillations in LSND/KARMEN experiments is determined by \( e \) and \( \mu \) flavors of the lightest state:

\[
P \propto 4|U_{e1}|^2|U_{\mu 1}|^2.
\] (40)

Mixing elements \( U_{e1} \) and \( U_{\mu 1} \) are restricted by BUGEY and BNL E776 experiments.

(iv) No observable signal of \( \nu_\mu - \nu_e \) oscillations is expected in CHORUS/NOMAD experiments \( \square \square \), however these experiments may discover \( \nu_\tau - \nu_e \) oscillations.

(v) \( \beta\beta \rightarrow \text{decay} \) is strongly suppressed.

Modification of the scenario is suggested with the same mass spectrum but inverse flavor hierarchy \( \square \square \) (fig. 3). Electron flavor is essentially in heavy states and its admixture in \( \nu_1 \) is small. Tau and \( \mu \) flavors have comparable admixtures in all three states. Some features of the scenario are:

(i) All three flavors and both mass squared differences contribute to the oscillations of atmospheric neutrinos. The generic 3\( \nu \)-case is realized. \( \nu_\mu - \nu_e \) has unsuppressed mode of oscillations with \( \Delta m^2_{23} \), and \( \nu_\mu - \nu_\tau \) has both \( \Delta m^2_{12} \) and \( \Delta m^2_{23} \) modes. CHOOZ experiment will put the bound on this possibility.

(ii) Effective Majorana mass of the electron neutrino is \( m_{ee} \approx m_0(U_{e2}^2 + \eta\gamma P U_{e3}^2) \), where \( \eta\gamma P \) is the relative CP-parity of two massive neutrinos. The bound from \( \beta\beta \rightarrow \text{decay} \) can be satisfied by some amount of cancelation.

(iii) If \( m^2_0 > 4 \text{ eV}^2 \), CHORUS/NOMAD may observe signal of \( \nu_\mu - \nu_e \) oscillations.

(iv) Due to inverse flavor/mass hierarchy the scenario predicts strong resonance conversion of antineutrinos in supernova: \( \nu_\mu \rightarrow \nu_e \rightarrow \nu_e \). The conversion results in permutation of \( \nu_\tau, \bar{\nu}_e \) energy spectra which is disfavored by SN87A data \( \square \).

In these schemes the solar neutrino data can be explained by virtue of introduction of the additional (sterile) neutrino states.

3.4 Sacrifice LSND. Degenerate neutrinos

Solar, atmospheric and HDM problem can be solved simultaneously, if neutrinos have strongly degenerate mass spectrum \( m_1 \approx m_2 \approx m_3 \sim 1 - 2 \text{ eV} \), with \( \Delta m^2_{12} = \Delta m^2_{23} = 6 \cdot 10^{-6} \text{eV}^2 \) and \( \Delta m^2_{31} = \Delta m^2_{\text{atm}} = 10^{-2} \text{eV}^2 \) (fig. 4). The corresponding mass matrix may have the form:

\[
m = m_0 I + \delta m,
\] (41)

where \( I \) is the unit matrix, \( \delta m \ll m_0 \sim 1 - 2 \text{ eV} \). Moreover \( \square \square \) can be realized in unique see-
saw mechanism with non zero direct Majorana masses of the left components. Main contribution, $m_0$, originates from interaction with Higgs triplets which respects some horizontal symmetry like $SU(2), S_4$ or permutation symmetry. It looks quite interesting that the desired mass splitting $\delta m$ can be generated by the standard see-saw contribution with $M_R \sim 10^{13}$ GeV. The effective Majorana mass $m_{ee} \approx m_0$ at the level of upper bound from the $\beta \beta_{0y} - decay$.

Figure 4: The same as in fig.1. Scenario with strongly degenerate neutrino spectrum.

The mass $m_{ee}$ can be suppressed if the electron flavor has large admixture in $\nu_1$ and $\nu_2$, so that the solar neutrino problem is solved by the large mixing MSW solution. Now the effective Majorana mass equals $m_{ee} \approx m_0(1 - \sin^2 2\theta)$, and for $\sin^2 2\theta = 0.7$ one gets suppression factor $0.3$. However simple formula (11) does not work.

No observable signals are expected in CHORUS/NOMAD and LSND/KARMEN.

Another possibility (fig.5) is to sacrifice the HDM assuming (if needed) that some other particles (e.g. sterile neutrinos, axino etc..) are responsible for structure formation in the Universe. In this case $m_3 \sim 0.1$ eV and $\nu_\mu - \nu_\tau$ oscillations explain the atmospheric neutrino deficit.

Figure 5: The same as in fig.1. Scenario for solar and atmospheric neutrinos.

Strong $\nu_\mu - \nu_\tau$ mixing, could be related to relatively small mass splitting between $m_2$ and $m_3$, which implies the enhancement of the mixing in the neutrino Dirac mass matrix. It could be related to the see-saw enhancement mechanism, or with strong mixing in charge lepton sector $L$. 

3.5 Without the atmospheric neutrino problem

The schemes are suggested which can accommodate solar neutrinos, HDM, and the LSND result. According to (fig. 6) all neutrinos are in the eV range, first two neutrinos are strongly degenerate: $\Delta m_{12}^2 \approx \Delta m_{23}^2$, whereas $\Delta m_{13}^2 = \Delta m_{\text{LSND}}^2$. Mixing is small: the electron flavor dominates in $\nu_1$, the tau flavor - in $\nu_2$, and the muon flavor in the heaviest state $\nu_3$. Remarks:

(i) Solar neutrino problem is solved by the $\nu_e - \nu_\tau$ small mixing MSW solution.
(ii) All three neutrinos give comparable contributions to the HDM.
(iii) $\nu_\mu - \nu_e$ oscillations can be in the range of sensitivity of the LSND/KARMEN.
(iv) The Majorana mass $m_{ee}$ is at the level of upper experimental bound.

Another version is characterized by $m_1 \ll m_2 \approx m_3 \sim 1$ eV with $\Delta m_{23}^2 = \Delta m_{\odot}^2$. Heavy components $\nu_1$ and $\nu_3$ are strongly mixed in $\nu_\tau$ and $\nu_e$ and the lightest state has mainly the muon flavor (inverse hierarchy). (In contrast with scheme of sect. 3.3 , now the splitting between heavy states explain the solar neutrino problem.) Comments:

(i) $\nu_e - \nu_\tau$ conversion gives large mixing MSW solution to the solar neutrino problem.
(ii) The mass $m_{ee}$ can be at the experimental
bound, although the cancelation is possible.
(iii) One expects strong $\bar{\nu}_\mu \rightarrow \nu_e$ conversion in the supernova, which is disfavored by SN87A data.
(iv) scenario supplies two component HDM and explanation of the LSND result.

3.6 “Standard” scenario

The scenario is characterized by strong mass hierarchy $m_1 \ll m_2 \ll m_3$ and weak mixing (fig.7). Basic features are:

(i) $m_3 = m_{HDM}$, so that $\nu_3$ forms the HDM.
(ii) Second mass, $m_2$, is in the range $m_2 = (2-3) \times 10^{-3}$ eV, and the $\nu_e \rightarrow \nu_\mu$ resonance conversion solves the solar neutrino problem.
(iii) There is no solution of the atmospheric neutrino problem.
(iv) The depth of $\bar{\nu}_\mu - \nu_e$ oscillations with $\Delta m^2 \approx m_3^2$ equals $4 |U_{3\mu}|^2 |U_{3e}|^2$. Existing experimental bounds on these matrix elements give the upper bound on this depth: $< 10^{-3}$ which is too small to explain the LSND result.
(v) Parameters of $\nu_\mu - \nu_e$ oscillations can be in the range of sensitivity of the CHORUS/NOMAD.

There is a number of attractive features of this scenario: It naturally follows from the see-saw mechanism with Dirac mass matrix $m_\nu^D \sim m_{up}$ and the intermediate mass scale for the Majorana mass matrix of the RH neutrinos: $M_1 \sim 10^{13}$ GeV. More precisely, for the eigenstates of this matrix one gets

\[
M_2 \sim (2 - 4) \cdot 10^{10} \text{ GeV} \\
M_3 \sim (4 - 8) \cdot 10^{12} \text{ GeV}.
\]

These values of masses are in agreement with “linear” hierarchy: $M_2/M_3 \approx m_e/m_t$.

The decays of the RH neutrinos with mass $10^{10} - 10^{12}$ GeV can produce the lepton asymmetry of the Universe which can be transformed by sphalerons into the baryon asymmetry.

The mixing angle desired for solution of the $\nu_\odot$ problem is consistent with expression

\[
\theta_{e\mu} = \sqrt{\frac{m_e}{m_\mu}} e^{i\phi} \theta_\nu,
\]

where $m_e$ and $m_\mu$ are the masses of the electron and muon, $\phi$ is a phase and $\theta_\nu$ is the angle which comes from diagonalization of the neutrino mass matrix. The relation between the angles and the masses (43) is similar to relation in quark sector. Such a possibility can be naturally realized in terms of the see-saw mechanism.

3.7 More neutrino states?

Another way to accommodate all the anomalies is to introduce new neutrino state which mixes with active neutrinos (see e.g. [30,32]). As follows from LEP bound on the number of neutrino species this state should be sterile (singlet of SM symmetry group). Mixing of sterile and active neutrinos leads to oscillations and the oscillations result in production of sterile neutrinos in the Early Universe. Presence of the sterile component in the epoch $t \sim 1$ sec could influenced the Primordial Nucleosynthesis. Several comments are in order.

1. At present a situation with bound on the effective number of the neutrino species, $N_\nu$, is controversial. Depending on the abundance of primordial deuterium one uses in the analysis the bound ranges from $N_\nu < 2.5$ to 3.9. Certain model of evolution of the deuterium is used. A conservative analysis which does not rely on any model leads to $N_\nu < 4.5$. If $N_\nu > 4$ is admitted then obviously there is no bound on oscillation parameters of the sterile neutrinos.

2. Even if $N_\nu < 4$, strong mixing of the sterile and active neutrinos is not excluded. The bound can be avoided in presence of large enough ($\Delta L \gtrsim 10^{-5}$) lepton asymmetry in the Universe, as it was discussed in sect. 2.5.

There are bounds on oscillation parameters of sterile neutrinos from SN87A observations. Thus at present it seems possible to introduce sterile neutrinos for explanations of different neutrino anomalies.
3.8 Rescue the standard scenario

The atmospheric neutrino deficit is the problem for the standard scenario. To solve it one can assume that an additional light singlet fermion exists with the mass \( m \sim 0.1 \text{ eV} \), which mixes mainly with muon neutrino, so that \( \nu_\mu - \nu_s \) oscillations explain the data \(^{12}\). In this case one arrives at the scheme (fig. 8) \(^{12}\). Production of \( \nu_s \) singlets in the Early Universe can be suppressed (if needed) by generation of the lepton asymmetry \(^{39}\) in the \( \nu_\tau - \nu_s \) and \( \nu_\tau - \nu_e \) oscillations \(^{40}\). The presence of large admixture of the sterile component in \( \nu_\tau \) influences resonance conversion of solar \( \nu_e \), and also can modify the \( \nu_\mu - \nu_\tau \) oscillations \(^{41}\).

3.9 The safest possibility?

Even without lepton asymmetry strong nucleosynthesis bound is satisfied, if \( \nu_s \) has the parameters of the solar neutrino problem. In this scenario \(^{13}\) (fig. 9) \( m_1 < m_S \ll m_2 \approx m_3 \) . Remarks
(i) Sterile neutrino has the mass \( m_S \sim (2-3) \times 10^{-3} \text{ eV} \) and mixes with \( \nu_e \), so that the resonance conversion \( \nu_e \rightarrow \nu_s \) solves the solar neutrino problem; (ii) Masses of \( \nu_\mu \) and \( \nu_\tau \) are in the range 2 - 3 eV, they supply the hot component of the DM; (iii) \( \nu_\mu \) and \( \nu_\tau \) form the pseudo Dirac neutrino with large (maximal) mixing and the oscillations \( \nu_\mu - \nu_\tau \) explain the atmospheric neutrino problem; (iv) The \( \nu_\mu - \nu_e \) mixing can be strong enough to explain the LSND result. (v) No effect is expected for \( \nu_\mu - \nu_\tau \) oscillations in CHORUS/NOMAD as well as in future searches of the \( \beta \beta_{0 \nu} \rightarrow \text{ decay} \).

4 On the models of neutrino mass

4.1 Predicting neutrino mass

Majority of attempts to predict neutrino masses are reduced to establishing relations between quarks and leptons. Then known parameters in quark sector are used as an input to make some conclusions on mass and mixing in lepton sector. The see-saw mechanism allows one to realize the quark - lepton symmetry most completely. To make the predictions one should fix the Dirac mass matrix of neutrinos, \( m^D \nu \), as well as the Majorana mass matrix of the right handed components, \( M_R \). Usually the direct Majorana masses of the left handed components are neglected. To find \( m^D \nu \), one can use GUT relation, e.g. \( m^D = m^{\nu \nu} \) at GUT scale. For \( M_R \) different ansatze \(^{72}\) were suggested. Also minimality of the Higgs sector can be postulated \(^{73}\) which allows one to get some relation between structure of \( M_R \) and quark mass matrices. The pattern of masses and mixing of the light neutrinos strongly depends on structure of \( M_R \), so that even for fixed \( m^D \nu \), practically any scenario can be realized.

Relations between quarks and leptons can be based also on certain horizontal symmetries.

Recent attempts to predict neutrino masses are based on
- GUT models with \( SO\text{\textsubscript{10}} \) symmetry,
- Models with anomalous \( U(1) \) symmetry,
- SUSY Models with R-parity violation,
• Models with radiative neutrino mass generation.

Also one can introduce some ansatze for the quark and lepton matrices.

4.2 An ansatz for large lepton mixing

It is postulated\footnote{Previously dihedral group $\Delta(75)$ was suggested in\footnote{\cite{hall}}.} that fermion mass matrices have the following structure in certain basis (the scale is not specified)

\[
M_i = c_i M_{dem} + \Delta M_i^{diag}
\]

where

\[
M_{dem} = M_0 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}
\]

is the democratic matrix, and

\[
\Delta M^{diag} = \begin{pmatrix} \delta & 0 & 0 \\ 0 & \rho & 0 \\ 0 & 0 & \epsilon \end{pmatrix}
\]

with $\delta, \rho, \epsilon \ll M_0$. It is assumed that parameters $c_i$ are proportional to electric charges of fermions:

\[
c_i \propto Q_i .
\]

For quarks and charged leptons first term in (44) dominates leading to big mass in one generation and big mixing angles which diagonalize matrices. As the result of two similar rotations for the upper and for the down quarks, the mixing in quark sector is small. The situation in lepton sector is different. For neutrinos: $c_i = 0$, and therefore the neutrino mass matrix is diagonal:

\[
M_{\nu} = \Delta M^{diag} .
\]

The lepton mixing follows from diagonalization of the charge lepton mass matrix and since $M_l \approx M^{diag}$, the mixing in leptonic sector is automatically large. In a sense large mixing in lepton sector is related to smallness of the neutrino mass.

All three neutrinos are strongly mixed. If $\delta \approx \rho \approx \epsilon \approx 1$ eV and splitting is very small: $(\delta^2 - \rho^2) \approx 10^{-10}$ eV$^2$ and $(\rho^2 - \epsilon^2) \sim 10^{-2}$ eV$^2$, one can explain the solar neutrino data via just-so oscillations, the atmospheric neutrino deficit, and presence of the HDM.

However there is no understanding why $\Delta M^{diag}_{\nu} \ll \Delta M^{diag}_{u,d,l}$.

4.3 $SO(10)$ model

The (supersymmetric) model\footnote{\cite{so10}} is based on $G = SO(10) \times \Delta(48) \times U(1)$ symmetry, where dihedral group $\Delta(48)$ (subgroup of $SU(3)$) was used as the family symmetry\footnote{\cite{rparity}}. Fermions are in $(16,3)$ representation of $SO(10) \times \Delta(48)$. The fermion masses are generated by Froggatt-Nielsen mechanism and for this a number of new chiral superfield is introduced. The symmetry is broken to the SM symmetry in three steps: $G \rightarrow SO(10) \rightarrow SU(5) \rightarrow SM$ at mass scales $M_P, v_{10},$ and $v_5$ correspondingly. All fermion are predicted in terms of 4 continuous parameters: $v_5/M_P, v_{10}/M_P,$ the ratio of MSSM VEV: $\tan \beta$, and universal Yukawa coupling $\lambda$. An additional $U(1)$ symmetry is also used to get desired structure of the mass matrices. The neutrino Dirac mass matrix has the following hierarchical structure of the elements: $m_{33} \gg m_{12} \gg m_{23} \approx m_{22}$, for the Majorana mass matrix one gets: $M_{22} \gg M_{12} \gg M_{13}$, (the matrices are symmetric and all other elements are zero). This leads via the see-saw mechanism to the pattern of the light masses with $m_2 \approx m_3 \approx m_1$. Also additional sterile neutrino is introduced to explain the solar neutrino problem, thus the model reproduces the pattern discussed in sect. 3.9.

It should be stressed however that the pattern is the result of ad hoc introduction of the large number of new supermultiplets and special $U(1)$-charge prescription. In fact, these $U(1)$ charges should be considered as new free parameters, so that high predictivity becomes not so impressive.

4.4 Neutrino-neutralino mixing

This is low scale realization of the see-saw mechanism. The neutrino mass equals $m_{\nu} \sim m^2_{\nu N}/m_N$, where $m_{\nu N}$ is the mixing mass term, and $m_N$ is the typical neutralino mass. Mixing of neutrinos and neutralinos implies violation of the R-parity. It was realized recently that Hall - Suzuki model\footnote{\cite{hall}} endowed by the universality of some SUSY breaking mass terms leads naturally to $m_{\nu N} \ll m_N$, and therefore to smallness of the neutrino mass\footnote{\cite{so10}}.

In terms of the MSSM multiplets, the superpotential of the model at GUT scale is

\[
W = \mu_l L H_2 + h_0 Q_3 L_0 D^c_3
\]
(i = 0, 1, 2, 3), where $H_2$ is the Higgs doublet, and $L_0 \equiv H_1$ is defined as the only component which has Yukawa couplings at GUT scale; (we took into account the Yukawa couplings of the third generation only). $Q_3$ is the quark doublet, $D^c_\mu$ is the superfield with the RH quark component.

The model implies that the R-parity is broken by dimension three (and less) operators only.

Basic feature of the superpotential is that only one component of the quartet, $L_0$ has the Yukawa couplings. This can be related to $R$-symmetry $\mathbb{Z}_3$.

The fields $L_0$ and $L_a$ ($a = 1, 2, 3$) may have different $R$-charges: e.g. $R(L_0, H_2) = 2$, whereas $R(Q, U^c, D^c, L_a) = 0$. In this case the $R$-parity breaking Yukawa couplings are suppressed. Moreover, the $\mu$-terms can be generated by nonrenormalizable interactions with new fields $z_i$. The $R$-symmetry is broken spontaneously by the VEV of these field $z_i$: $(z_i) \ll M_P$, and the $\mu$ parameters of the superpotential may have the hierarchy determined by $(z_i)^n/M^n_{P}$, where $n$ is fixed by the $R$-charge of $z_i$.

It is assumed (here we will follow discussion in [4]) that soft SUSY breaking terms for $L_i$ are universal at, e.g., GUT scale:

$$V = B \cdot \mu_i L_i H_2 + m^2_{0}\tilde{L}_i |^2 + \ldots$$

Due to the universality one can diagonalize the $\mu$ term in the superpotential, and simultaneously in the potential (50), by rotation $L_i \rightarrow L'_i$:

$$\mu_i \tilde{L}_i H_2 \rightarrow \mu L'_0 H_2.$$  (51)

(This rotation generates simultaneously the $R$-parity violating Yukawa couplings). There is no terms like $\tilde{L}'_a H_2$ ($a = 1, 2, 3$) at GUT scale. These terms however appear at the electroweak scale due to the renormalization group effect. Indeed, Yukawa coupling (19) distinguishes different components of $L_i$ and this leads to different renormalization of terms with $\tilde{L}_0$ and $\tilde{L}_a$ in (54). The universality turns out to be broken, and the rotation (51) will not diagonalize the potential. We get after rotation (51) the mixing term

$$\frac{\mu_i}{\mu} \times [\delta m^2 L'_0 + \delta B \cdot \mu H_2] \tilde{L}'_i + \text{h.c.}$$

where $\delta m^2$ and $\delta B$ describe the renormalization group effect. After electroweak symmetry breaking the mixing terms (52) (linear in $L'$), together with soft symmetry breaking masses, induce a VEV of “sneutrino” (neutral component of the doublet in $L'_i$) of the order:

$$\langle \tilde{\nu} \rangle \approx v \frac{\mu_i}{\mu} \left( \frac{\delta m^2}{m^2_0} \cos \beta + \frac{\delta B \cdot \mu}{m^2_0} \sin \beta \right).$$

Here $v$ is the electroweak scale. The VEV of sneutrino leads via the gauge coupling to the neutrino-gaugino mixing: $m_{\nu N} \sim g\langle \tilde{\nu} \rangle$. In turn the seew-saw mechanism results in the mass

$$m_\nu \approx \frac{g^2 + \frac{\mu}{\mu}}{2} \frac{\langle \tilde{\nu} \rangle^2}{M^2_Z}$$

$$\approx \frac{g^2}{16\pi^2} \frac{m_Z^2}{M^2_Z} \left( \frac{\mu_i}{\mu} \right)^2 \left[ \frac{h^2}{16\pi^2} \log \frac{M^2_{\nu}}{m^2_{\nu}} \right]^2.$$  (54)

This contribution to neutrino mass is typically larger than the one produced by the loop-diagram stipulated by $R$-parity violating Yukawa couplings. For $\mu_i \sim \mu$ and large $\tan \beta$ ($h_B \sim 1$) we find $m_\nu \sim O(10 \text{ MeV})$. This neutrino can be identified with $\nu_\tau$.

There are several possibilities to get much smaller mass. For small $\tan \beta (\sim 1)$ the Yukawa coupling is small and the $m_\nu$ is of the order $10 \text{ eV}$. Also the mass can be suppressed if there is the hierarchy of $\mu$: $\mu_i/\mu \ll 1$. For $\mu_i/\mu \sim M_{\text{GUT}}/M_{\text{Pl}}$ : $m_\nu \sim 10 \text{ eV}$ even for large $\tan \beta$.

Another possibility for suppression of $m_\nu$ is a cancelation between the two terms in (53). If there is no cancellaion, the neutrino mass turns out to be related to the $R$-parity violating Yukawa coupling generated by rotation (51): $\lambda' \approx C m_\nu m_Z^2 (\mu_i/\mu)^2$, where $C$ is known constant [5].

Thus certain relation between the probabilities of $R$-parity violating processes (due to $\lambda'$) and neutrino mass gives signature of this mechanism.

In the case of three generations only one neutrino acquires the mass due to neutrino-neutrino mixing. Loop corrections induced by $R$-parity violating couplings make all neutrinos massive. In certain region of parameters one can explain solar neutrino problem and supply HDM (i.e. reproduce the standard scenario). Also simultaneous solution of the solar and atmospheric neutrino problems is possible [6].

4.5 Models with anomalous $U(1)_A$ symmetry

Masses of neutrinos are generated by the seew-saw mechanism. Structure of the neutrino mass ma-
trines is determined by $U(1)_A$ charges of neutrinos and quarks. Relation between the neutrino and the quark mass matrices is established via the charges (rather than immediately, as in the simplest GUT theories). It is assumed that charges of neutrinos coincide with charges of (electrically charged) leptons:

$$Q(\nu L) = Q(\nu'_L) = Q(l) = Q_l.$$  \hspace{1cm} (55)

Masses are generated *a la* Froggatt-Nielsen mechanism, and elements of the mass matrix appear as

$$m_{ij} \sim m_0 \left( \frac{\theta}{M} \right)^{|Q_i + Q_j + Q_{\Sigma}|},$$  \hspace{1cm} (56)

where $\theta$ is the VEV of singlet field with unit $U(1)_A$ charge and $M$ is the mass scale of new heavy scalar fields. There are different mass parameters for the upper, $M_2$, and down, $M_1$, fermions. The equality of charges $\Sigma$ leads to the following relation between the neutrino Dirac mass matrix and the mass matrix of the charged leptons:

$$m^D_{ij} \sim \tan \beta \cdot m_0 (M_1 \rightarrow M_2).$$  \hspace{1cm} (57)

(In the leptonic matrix one should substitute $M_1 \rightarrow M_2$.)

The Majorana mass matrix of the RH neutrino components is generated by coupling with new singlet field $\Sigma$:

$$M_R \sim \langle \Sigma \rangle \left( \frac{\theta}{M_3} \right)^{|Q_i + Q_j + Q_{\Sigma}|}. $$  \hspace{1cm} (58)

Depending on the charge of the $\Sigma$, $Q_{\Sigma}$, (which is, in fact, unknown) one can get different structures of $M_R$ and eventually of the mass matrix of light neutrinos. The $\Sigma$ can appear as the composite operator:

$$\Sigma = s \cdot s \cdot \frac{\theta}{M_P}. $$  \hspace{1cm} (59)

Here $M_P$ is the Planck mass. For $\langle s \rangle \sim 10^{16}$ GeV one gets $\langle \Sigma \rangle \sim 10^{13}$ GeV.

Depending on charge prescription (especially for $\Sigma$) one can accommodate the solar neutrino data and HDM (sect. 3.6, fig. 7), or the solar and atmospheric neutrino problems (sect. 3.4, fig.5).

### 4.6 Zee model revisited

Zee model includes the charged scalar field $h$, being a singlet of the $SU(2)$, and two doublets of the Higgs bosons. In virtue of the gauge symmetry the singlet $h$ has antisymmetric (in flavor) couplings to lepton doublets $L_{iL} \equiv (\nu_l, l^-)$, $l = e, \mu, \tau$

$$\mathcal{L}_{\text{Zee}} = f_{\ell\ell'} L^T_{iL} i\tau_2 L_{iL} h + \text{h.c.},$$  \hspace{1cm} (60)

$f_{\ell\ell'} = -f_{\ell'\ell}$. Neutrino mass is generated in one loop. Neutrino mass terms are proportional to masses of the charge leptons squared. As the consequence of the antisymmetry of the couplings and hierarchy of charge leptons masses, the Zee model gives very distinctive pattern of neutrino masses and mixing $\Sigma$. For not too strong hierarchy of couplings $f_{\ell\ell'}$ the two heavy neutrinos, $\nu_2, \nu_3$, are degenerate and mix in $\nu_\mu$ and $\nu_\tau$ almost maximally. The first neutrino $\nu_1$ practically coincides with $\nu_e$ and has much smaller mass: $m_1 \ll m_2 \approx m_3$. This pattern coincides with the one (fig. 2) needed for a solution of the atmospheric neutrino problem by $\nu_\mu \leftrightarrow \nu_\tau$ oscillations and for existence of the two component hot dark matter in the Universe $\Sigma$. Furthermore, the oscillations $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ can be in the range of sensitivity of KARMEN/LSND experiments $\Sigma$. Thus the model reproduces the pattern discussed in sect. 3.3, 3.7. The analysis shows that scenario implies large values and inverse flavor hierarchy of couplings of the Zee boson with fermions: $f_{\tau\tau} \ll f_{\mu\tau} \leq f_{\mu\mu} \sim 0.1$.

Main signatures of scenario are: strongly suppressed signal of $\nu_\mu \leftrightarrow \nu_\tau$ oscillations in CHORUS/NOMAD, so that positive result from these experiments will rule out the scenario; possibility of observation of $\nu_e \rightarrow \nu_\tau$ oscillations by CHORUS/NOMAD; corrections to the muon decay and neutrino-electron scattering at the level of experimental errors; branching ratio $B(\mu \rightarrow e\gamma)$ bigger than $10^{-13}$. The solar neutrino problem can be solved by introduction of additional very light singlet fermion without appreciable modification of the active neutrino pattern.

### 4.7 Sterile neutrinos: window to the hidden world?

Common wisdom is that existence of light sterile neutrinos is unnatural. Indeed, introducing sterile neutrino, $\nu_s$, one encounters several questions: What is the origin of this neutrino? How it mixes with usual neutrinos? What protects the mass of $\nu_s$ and makes it small? Therefore discovery of sterile neutrino will mean something very non trivial. In fact, forthcoming solar neutrino experiments, as well as the atmospheric neutrino exper-
iments and long base line experiments will be able to establish, if solar or/and atmospheric neutrinos are converted into sterile neutrinos.

What could be behind this discovery? There are several studies of this question recently.

1. Immediate candidate for $\nu_s$ is the RH neutrino component. However in this case the see-saw mechanism does not operate.

2. Sterile neutrino could be the component of multiplet of some extended gauge symmetry - like $SO(10)$-singlet from 27-plet of $E_6$. The mass of the $\nu_s$ is generated by separate see-saw mechanism and its value is protected by $U(1)$ symmetry which is embedded in $E_6$ and broken at low scale.

3. In [2] it was suggested that $\nu_s$ is the mirror neutrino from the mirror standard model. The mass of $\nu_s$ is generated by the see-saw mechanism in the mirror world which, however, has the electroweak symmetry breaking scale $\langle H_M \rangle$ about two orders of magnitude bigger than in usual world. (Here $H_M$ is the mirror Higgs doublet.) Generalizing (1) we get $m_s = \langle H_M \rangle^2 / M_P$. Mixing of usual neutrinos with the mirror one proceeds via the gravitational interactions

$$\frac{1}{M_P} L H L_M H_M + h.c.,$$

where $L_M$ is the mirror lepton doublet. Therefore the mixing angle is determined essentially by the ratio of VEV: $\langle H \rangle / \langle H_M \rangle$.

4. The origin and properties of $\nu_s$ can be related to SUSY. A number of singlet superfields was introduced for different purposes: to generate $\mu$ term, to realize PQ-symmetry breaking, to break spontaneously the lepton number, etc. String theory typically supplies a number of singlets. Fermionic components of these superfield could be identified with desired sterile neutrino.

It was shown in [2] that masses and mixing of $\nu_s$ can be protected by $R$-symmetry.

5. Another possibility is that $\nu_s$ is the would be Nambu-Goldstone fermion, the superpartner of the Nambu-Goldstone boson which appears as the result of spontaneous violation of some $U(1)$ global symmetry like Pececi-Quinn symmetry or lepton number symmetry etc. (i.e. $\nu_s$ is the axino, or majorino ....). General problem is that SUSY breaking generates typically the mass of $\nu_s$ of the order the gravitino mass and further suppression is needed. One can use here the ideas of non-scale supergravity, or possibly, gauge mediated SUSY breaking.

6. Sterile neutrino as modulino? Suppose that there is a singlet $S = \nu_s$ which is massless in the supersymmetric limit and couples with observable sector via the gravitational interactions. The mass and effective interactions are induced when supersymmetry is broken. For some reasons (e.g. related to cancellation of the cosmological constant) $S$ may not acquire the mass in the order $m_{3/2}$. Then natural scale of mass of $S$ is

$$m_S \sim \frac{m_{3/2}^2}{M_P}. \quad (62)$$

The mixing of $S$ with active neutrinos involves electroweak symmetry breaking. The simplest appropriate effective operator is $(m_{3/2}/M_P) LSH$. It generates the mixing mass parameter

$$\tilde{m} \sim \frac{m_{3/2} \langle H \rangle}{M_P}. \quad (63)$$

For small electron neutrino mass $m_{\nu_e} \ll m_S$ the $\nu_e - \nu_S$ mixing angle $\theta_{es}$ is of the order

$$\theta_{es} \sim \frac{\langle H \rangle}{m_{3/2}}. \quad (64)$$

For $M_P \sim 2 \times 10^{18}$ GeV and $m_{3/2} \sim 10^3$ GeV one gets $m_S$ and $\tilde{m}$

$$m_S \sim 10^{-3} \text{eV}, \quad \tilde{m} \sim 2 \cdot 10^{-4} \text{eV} \quad (65)$$

precisely in the range desired for a solution of the solar neutrino problem via resonance conversion $\nu_e \rightarrow S$ in the sun. Moreover, varying the parameters (constants of the order 1) and taking into account the renormalization group effect it is easy to achieve both small and large mixing solutions to the problem.

Fermion $S$ can also mix with the other neutrino species. If the coupling of $S$ with fermion generations is universal; i.e. $\tilde{m}$ are the same (or of the same order) for all generations, then $S$-mixing with $\nu_\mu$ and $\nu_\tau$ are naturally suppressed as the mixing angles behave as $\theta_i \sim \tilde{m}/m_i$. For instance, taking $m_2 \sim 10^{-1}$ eV and $m_3 \sim 1$ eV we get $\sin^2 \theta_{S\mu} \sim 10^{-5}$ and $\sin^2 \theta_{S\tau} \sim 10^{-7}$. Thus the lightest neutrino has naturally the biggest mixing with $\nu_s$.

The desired properties of $S$ could be realized for some fields in hidden sector, and probably for fermionic components of some moduli field[s].
5 Conclusion

1. New effects of the neutrino refraction in media have been considered recently which may have important impact on pattern of neutrino masses and mixing.

   Neutrino conversion in polarized and magnetized media opens new possibility in explanation of peculiar velocities of pulsars. This implies $m_\nu \gtrsim 100$ eV.

   Large leptonic asymmetry in the Early Universe due to oscillation into sterile neutrinos may have serious impact on primordial nucleosynthesis and the nucleosynthesis bounds on neutrino parameters.

   Modification of long range forces stipulated by the neutrino exchange in dense medium allows one to resolve the energy paradox in compact stellar objects (neutrons stars, white dwarfs etc..)

2. Several possible patterns (scenarios) of neutrino masses and mixing were elaborated on the basis of present neutrino data (hints and bounds). This allows one to check a consistency of different positive results and gives a guideline for further studies.

   The data indicate that structure of the mass spectrum and lepton mixing may differ strongly from those in quark sector. In particular, spectrum may show complete degeneracy, pseudo Dirac structure, or even inverse hierarchy. The mixing can be large or even maximal. New sterile states may exist which mix with active neutrinos.

   Different scenarios have rather distinctive predictions and forthcoming experiments (SK, SNO, CHOOZ CHORUS/NOMAD, NEMO ....) will be able to discriminate among them.

3. Neutrinos may have several different sources of mass: usual see-saw contribution, radiative effects, mixing with neutralinos (in models with $R$-parity violation). Structure of the mass matrices can be related to supersymmetry and $R$-symmetry. The neutrino mass and mixing can have a connection to quark-lepton symmetry, GUT, to new mass scales and new symmetries.

   However it will be difficult to identify mechanism of neutrino mass generation just from neutrino data (even if in future we will know neutrino parameters with good precision). As an illustration: two different models discussed in sect.4 radiative Zee model and GUT $SO(10)$ with horizontal symmetry lead to precisely the same pattern in lepton sector. To identify the mechanism one will need an information about other elements of models: e.g. the discovery of proton decay, processes with $R$-parity violation, Zee singlet etc. , will clarify many points.

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Questions

D. R. O. Morrison, CERN

Dr. Smirnov has mentioned a paper that I did not have time to submit as I was working on molecular genetics. However I did send him a paper written earlier this year which raises worries about 3 things: Errors, the Sun’s luminosity and motion inside the Sun.

1. ERRORS: We seem to agree there is a problem with the very different errors of different SSM’s.
2. LUMINOSITY: What we measure is what we see on the surface of the Sun over the last few years. But what we need is an average over the last few million years as the time for thermal information to travel from the core to the surface is between one and ten million years (Douglas
Gough’s estimate). The latest satellite measurements show the luminosity follows the sunspot cycle. If we were living near the year 1700, the luminosity would have been quite different as the earth’s temperature was much lower - in London people had fairs with bonfires on the ice on the Thames - and there were no sunspots between 1650 and 1710. Similarly there were few sunspots about 1400 when there was another cold spell whereas near 1200, there was a hot period with extra sunspots. In other words, the surface of the Sun changes in ways not included in the SSM which does not consider sunspots nor variation of the apparent luminosity. Going back further, for many million years, the sea level was much higher indicating that the luminosity was much greater. For example when the dinosaurs were extinguished, the sea level was consistently about 200 metres higher than now and half of the present land surface was under water. We do not have a good measurement of the luminosity over a suitably long time period and hence the error on the luminosity should be greatly increased.

3. INTERNAL MOTION: There are three pieces of evidence. Initially the Sun was a T Tauri star - very bright and rotating quickly. Standard Solar Models cannot slow this rotation to zero, so one expects a differential rotation even to the core of the Sun. This is supported by helioseismological measurements which show that the rotation at the poles and at the equator is different down to 0.2 of the Sun’s radius. Helium-3 has an unusual distribution being sharply peaked at a radius of 0.3. Calculations by W. Haxton have shown that a motion of only 700 metres per year, is enough to cause this Helium-3 to move and to be burnt thus changing the temperature of the Sun’s core appreciably. Lithium-7 has a measured abundance which is less than one hundredth of that predicted by the SSM. Also looking at other stars, the Boesgaard dip is not explained by the SSM. Sylvie Vauclair et al. have explained this by meridional motion inside the Sun. However they cut the motion at a radius higher than 0.3 and hence do not allow any Helium-3 movement, and so find little change in the neutrino flux. Without this cut which seems in contradiction to the helioseismological results which show effects down to at least 0.2 radius, the neutrino flux would have been changed.

You said so many things that I forget what they were.

D. R. O. Morrison:
Errors, Luminosity, Internal motion.

A. Yu. S.:
1. At present the solar neutrino problem can be formulated practically without reference to a specific standard solar model. The problem can be formulated as discrepancy between different experimental results, and in this connection more relevant question is how reliable are the experimental results.

2. Possible variations of the Solar luminosity are certainly much smaller than those which could correspond to depletion of the Gallium result by factor 2.

3. W. Haxton has suggested unusual mixing of elements. It involves fast filamental flow of matter from the layers with maximal concentration of $^3$He downward, and slow restoring flow upward. This leads to enhancement of the pp-I branch and therefore suppression of the $^7$Be neutrino flux. However, (i) the suppression achieved by mixing is not enough for good description of data. (ii) In fact, no consistent solar model has been elaborated which incorporates the mixing, and it is unclear what is a feedback of the mixing on other observable characteristics of the Sun. (iii) It has been shown by Bahcall and collaborators, that mixing suggested by Haxton strongly contradicts the helioseismology data.

Results obtained by S. Vauclair et al. show that solution of the Lithium-7 problem has no serious impact on solar neutrino fluxes if one takes into account the helioseismological data. The rotation-induced mixing of elements below the connective zone is introduced; the agreement with the helioseismological data imply that mixing should terminate below $R < 0.4R_\odot$. This is enough to solve the lithium problem, but this is not enough to change appreciably the neutrino fluxes which are generated in deeper layers.

In paper presented at Rencontres de Blois Vauclair et al., add some mixing in the central regions ($R \sim (0.1 - 0.2)R_\odot$) to have better description of the helioseismology data. It turns out that the mixing should be weak and its effect on neutrino fluxes is of the order 20% only.