Darwin and Phenomenology Beyond the Standard Model *

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Abstract

After preamble about Darwin, my talk described the conformality approach to extending the standard model of particle phenomenology using an assumption of no conformal anomaly at high energy. Topics included quiver gauge theory, the conformality approach to phenomenology, strong-electroweak unification at 4 TeV, cancellation of quadratic divergences, cancellation of U(1) anomalies, and a dark matter candidate.

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Preamble

I would first like to thank the organizers, Carlos Montufar and Bruce Hoeneisen for the invitation. Since this is the only evening talk of the meeting and conferees are still coming in from dinner, I have a few minutes to talk about the local hero, Charles Darwin.

Of the participants at this conference, I was born nearest to Shrewsbury which is Darwin’s birthplace. I was born and raised in Worcestershire from which Shrewsbury is just across the county line in Shropshire. When I was growing up the answer to the question “Who was the greatest scientist ever born in England?” was unambiguously “Newton”. In the fifty years since then that has changed and Darwin is now placed on a pedestal equally as high as Newton’s. Darwin’s fame is helped by results of polls in America showing half of the people do not believe in evolution.

Indeed, when I was growing up, Shrewsbury had no museum or even a marker on Darwin and most people probably did not know of him. Now in Shrewsbury there is a large statue of a seated elderly Darwin, even larger than the one of a standing young Darwin here on Frigatebird Hill. There is also a Darwin museum in Shrewsbury and it is now an important visit in the well-organized English tourist industry.

One lecturer on the first day made an analogy between Darwin’s visit to the Galapagos in 1835, towards the end of his five year voyage on the Beagle, and the commissioning of the LHC next year. Let me pursue his analogy further. Firstly, after Darwin returned to England it took him 4 or 5 years to figure out the theory of evolution: there was no sudden epiphany here in the Galapagos. It may take a similar time to deduce the correct theory from LHC data. Secondly, although the Turtles, Iguanas and Boobies are some of the more obvious inhabitants of the Galapagos the best examples for Darwin’s theory were the beaks of the small finches which could not fly in between the islands. Perhaps one should look for the analogs of the beaks of the finches which would be small details in a less conspicuous subset of the LHC data.

It is startling that these two greatest scientists, Newton and Darwin, were both born in England with one per cent of the world’s population.
I have boasted enough about England. Now I can begin my talk.

In case there is any misunderstanding, this is not a public lecture as someone asked me this morning. What the organizers requested was that speakers “bring original ideas on viable extensions of the standard model”. What I will talk about is certainly original; whether it is viable or not may have to await the LHC or at least the discussion section but I will do my best.

Introduction

In this talk I will describe an approach to extending the standard model of particle phenomenology to make predictions for the Large Hadron Collider which is based on conformality, defined as the absence of conformal anomaly at high energy.

The contents of the talk are as follows:

1. Quiver gauge theories.
2. Conformality phenomenology.
3. 4 TeV Unification.
4. Quadratic divergences.
5. Anomaly Cancellation and Conformal U(1)s
6. Dark Matter Candidate.
7. Summary.

Before I start here is a concise history of string theory:

• Began 1968 with Veneziano model.
• 1968-1974. Dual resonance models for strong interactions. Replaced by QCD around 1973. DRM book 1974. Hiatus 1974-1984
• 1984. Cancellation of hexagon anomaly.
• 1985. $E(8) \times E(8)$ heterotic strong compactified on Calabi-Yau manifold gives temporary optimism of TOE.
• 1985-1997. Discovery of branes, dualities, M theory.

• 1997. Maldacena AdS/CFT correspondence relating 10 dimensional superstring to 4 dimensional gauge field theory.

• 1997-present. Insights into gauge field theory including possible new states beyond standard model. String not only as quantum gravity but as powerful tool in nongravitational physics.

**QUESTION:** What is meant by *duality* in theoretical physics?

**ANSWER:** Quite different looking descriptions of the same underlying theory.

The difference can be quite striking. For example, the AdS/CFT correspondence describes duality between an $\mathcal{N} = 4, d = 4$ SU(N) GFT and a $d = 10$ superstring. Nevertheless, a few non-trivial checks have confirmed this duality.

In its simplest version, one takes a Type IIB superstring (closed, chiral) in $d = 10$ and one compactifies on:

$$ (AdS)_5 \times S^5 $$

We recall old results on the perturbative finiteness of $\mathcal{N} = 4$ SUSY Yang-Mills theory:

• Was proved by Mandelstam, Nucl. Phys. B213, 149 (1983); P. Howe and K. Stelle, ICL preprint (1983), Phys.Lett. **B137**, 135 (1984). L. Brink, talk at Johns Hopkins Workshop on Current Problems in High Energy Particle Theory, Bad Honnef, Germany (1983).
• The Malcacena correspondence is primarily aimed at the $N \to \infty$ limit with the 't Hooft parameter of $N$ times the squared gauge coupling held fixed.

• Conformal behavior assumed valid here also for finite $N$.

• After exploiting initially the duality of gauge theory with string theory, we shall thereafter focus exclusively on the gauge theory description.
SECTION 1. QUIVER GAUGE THEORIES

Breaking supersymmetries:

To approach the real world, one needs less or no supersymmetry in the (conformal?) gauge theory.

By factoring out a discrete (abelian) group and composing an orbifold:

\[ S^5/\Gamma \]

one may break \( \mathcal{N} = 4 \) supersymmetry to \( \mathcal{N} = 2, \ 1, \ \text{or} \ 0 \). Of special interest is the \( \mathcal{N} = 0 \) case.

We may take \( \Gamma = Z_p \) which identifies \( p \) points in \( C_3 \).

The rule for breaking the \( \mathcal{N} = 4 \) supersymmetry is:

\[
\Gamma \subset SU(2) \quad \Rightarrow \quad \mathcal{N} = 2
\]

\[
\Gamma \subset SU(3) \quad \Rightarrow \quad \mathcal{N} = 1
\]

\[
\Gamma \not\subset SU(3) \quad \Rightarrow \quad \mathcal{N} = 0
\]

In fact to specify the embedding of \( \Gamma = Z_p \) we need to identify three integers \((a_1, a_2, a_3)\):

\[
C_3 : (X_1, X_2, X_3) \xrightarrow{Z_p} (\alpha^{a_1} X_1, \alpha^{a_2} X_2, \alpha^{a_3} X_3)
\]

with

\[
\alpha = exp \left( \frac{2\pi i}{p} \right)
\]
Matter representations:

- The $Z_p$ discrete group identifies $p$ points in $C_3$.

- The $N$ converging D3-branes meet on all $p$ copies, giving a gauge group: $U(N) \times U(N) \times \ldots \times U(N)$.

- The matter (spin-1/2 and spin-0) which survives is invariant under a product of a gauge transformation and a $Z_p$ transformation.

One can draw $p$ points and arrows for $a_1, a_2, a_3$.

\[ e.g. \quad Z_5 \quad (1, 3, 0) \]

Quiver diagram (Douglas-Moore).

Scalar representation is: $\sum_{k=1}^{3} \sum_{i=1}^{p} (N_1, \tilde{N}_{i \pm a_k})$
For fermions, one must construct the 4 of R-parity $SU(4)$:

From the $a_k = (a_1, a_2, a_3)$ one constructs the 4-spinor $A_\mu = (A_1, A_2, A_3, A_4)$:

\[
\begin{align*}
A_1 &= \frac{1}{2}(a_1 + a_2 + a_3) \\
A_2 &= \frac{1}{2}(a_1 - a_2 - a_3) \\
A_3 &= \frac{1}{2}(-a_1 + a_2 - a_3) \\
A_4 &= \frac{1}{2}(-a_1 - a_2 + a_3)
\end{align*}
\]

These transform as $exp\left(\frac{2\pi i}{\rho}A_\mu\right)$ and the invariants may again be derived (by a different diagram):
\[ e.g. A_\mu = 2; \quad p = 5. \]

These lines are oriented. Specifying the four \( A_\mu \) is equivalent to the three \( a_k \) and group theoretically more fundamental.

One finds for the fermion representation

\[
\sum_{\mu=1}^{4} \sum_{i=1}^{p} (N_i, \bar{N}_{i+A_\mu})
\]

Two interesting reference for hierarchy and naturalness are:

K.G. Wilson, Phys. Rev. D3, 1818 (1971) - already discussed in the late 1960s

K.G. Wilson [hep-lat/0412043] – what a difference 3 decades make.

We also note that superconformal symmetry exemplified by \( \mathcal{N} = 4 \) SU(N) Yang-Mills in 1983 and related to string theory in 1997 for infinite N can be lessened to

supersymmetry which answered, though as we shall see not uniquely, Wilson’s objection, rescinded in 2004, about quadratic divergences in 1974 and generated > 10^4 papers.

or conformality according to \( \mathcal{N} = 4 \rightarrow \mathcal{N} = 0 \) by orbifolding \( \rightarrow U(N)^n \) with, so far, < 10^2 papers.

Within a few years the Large Hadron Collider can experimentally discriminate between the two possibilities.
SECTION 2. CONFORMALITY PHENOMENOLOGY

- Hierarchy between GUT scale and weak scale is 14 orders of magnitude. Why do these two very different scales exist?

- How is this hierarchy of scales stabilized under quantum corrections?

- Supersymmetry answers the second question but not the first.

The idea is to approach hierarchy problem by Conformality at TeV Scale.

- Will show idea is possible.

- Explicit examples containing standard model states.

- Conformality more rigid constraint than supersymmetry.

- Predicts additional states at TeV scale for conformality.

- Gauge coupling unification.

- Naturalness: cancellation of quadratic divergences.

- Anomaly cancellation: conformality of U(1) couplings.

- Dark Matter candidate.
Conformality as hierarchy solution.

- Quark and lepton masses, QCD and weak scales small compared to TeV scale.

- May be put to zero suggesting:

- Add degrees of freedom to yield GFT with conformal invariance.

- ’t Hooft naturalness since zero mass limit increases symmetry to conformal symmetry.

The theory is assumed to be given by the action:

\[
S = S_0 + \int d^4x \alpha_i O_i
\]

where \( S_0 \) is the action for the conformal theory and the \( O_i \) are operators with dimension below four which break conformal invariance softly.

The mass parameters \( \alpha_i \) have mass dimension \( 4 - \Delta_i \) where \( \Delta_i \) is the dimension of \( O_i \) at the conformal point.

Let \( M \) be the scale set by the parameters \( \alpha_i \) and hence the scale at which conformal invariance is broken. Then for \( E >> M \) the couplings will not run while they start running for \( E < M \). To solve the hierarchy problem we assume \( M \) is near the TeV scale.
Experimental evidence for conformality:

Consider embedding the standard model gauge group according to:

\[ SU(3) \times SU(2) \times U(1) \subset \bigotimes_i U(N_d_i) \]

Each gauge group of the SM can lie entirely in a \( SU(N_d_i) \) or in a diagonal subgroup of a number thereof.

Only bifundamentals (including adjoints) are possible. This implies no (8, 2), etc. A conformality restriction which is new and satisfied in Nature!

No \( U(1) \) factor can be conformal and so hypercharge is quantized through its incorporation in a non-abelian gauge group. This is the “conformality” equivalent to the GUT charge quantization condition in \( e.g. \ SU(5) \)!

Beyond these general consistencies, there are predictions of new particles necessary to render the theory conformal.
SECTION 3. 4 TeV UNIFICATION

- Above 4 TeV scale couplings will not run.
- Couplings of 3-2-1 related, not equal, at conformality scale.
- Embeddings in different numbers of the equal-coupling $U(N)$ groups lead to the 4 TeV scale unification without logarithmic running over large desert.

When we arrive at $p = 7$ there are viable models. Actually three different quiver diagrams can give:

1) 3 chiral families.
2) Adequate scalars to spontaneously break $U(3)^7 \rightarrow SU(3) \times SU(2) \times U(1)$
   and
3) $\sin^2 \theta_W = 3/13 = 0.231$

The embeddings of $\Gamma = Z_7$ in $SU(4)$ are:

7A. $(\alpha, \alpha, \alpha, \alpha^4)$
7B. $(\alpha, \alpha, \alpha^2, \alpha^3)^* \quad$ C-H-H-W-H-W
7C. $(\alpha, \alpha^2, \alpha^2, \alpha^2)$
7D. $(\alpha, \alpha^3, \alpha^5, \alpha^5)^* \quad$ C-H-W-H-H-W
7E. $(\alpha, \alpha^4, \alpha^4, \alpha^5)^* \quad$ C-H-W-W-H-H-H
7F. $(\alpha^2, \alpha^4, \alpha^4, \alpha^4)$

* have properties 1), 2) and 3).
7B  $4 = (1, 1, 2, 3)$  $6 = (2, 3, 3, -3, -3, -2)$

7D  $4 = (1, 3, 5, 5)$  $6 = (1, 1, 3, -3, -1, -1)$

7E  $4 = (1, 4, 4, 5)$  $6 = (1, 2, 2, -2, -2, -1)$
The simplest abelian orbifold conformal extension of the standard model has \( U(3)^7 \rightarrow SU(3)^3 \) trinification \( \rightarrow (321)_{SM} \).

In this case we have \( \alpha_2 \) and \( \alpha_1 \) related correctly for low energy. But \( \alpha_3(M) \approx 0.07 \) suggesting a conformal scale \( M \geq 10 \) TeV - too high for the L.H.C.

A more unified model which introduces the 4 TeV scale is:

Taking as orbifold \( S^5/Z_{12} \) with embedding of \( Z_{12} \) in the SU(4) R-parity specified by \( 4 \equiv \alpha^{A_1}, \alpha^{A_2}, \alpha^{A_3}, \alpha^{A_4} \) and \( A_\mu = (1, 2, 3, 6) \).

This accommodates the scalars necessary to spontaneously break to the SM.

As a bonus, the dodecagonal quiver predicts three chiral families (see next page).
\[ A_\mu = (1, 2, 3, 6) \]

\[ SU(3)_C \times SU(3)_H \times SU(3)_H \]

\[ 5(3, \bar{3}, 1) + 2(\bar{3}, 3, 1) \]
without thresholds: all states at $M_U$

hep-ph/0302074

PHF+Ryan Rohm + Tomo Takahashi
thresholds: CH fermions at 2 TeV

hep-ph/0302074

PHF+Ryan Rohm + Tomo Takahashi
thresholds: CW fermions at 2 TeV

hep-ph/0302074

PHF+Ryan Rohm + Tomo Takahashi
SECTION 4. QUADRATIC DIVERGENCES.

Classification of abelian quiver gauge theories

We consider the compactification of the type-IIB superstring on the orbifold $\text{AdS}_5 \times S^5/\Gamma$ where $\Gamma$ is an abelian group $\Gamma = \mathbb{Z}_p$ of order $p$ with elements $\exp(2\pi i A/p)$, $0 \leq A \leq (p - 1)$.

The resultant quiver gauge theory has $\mathcal{N}$ residual supersymmetries with $\mathcal{N} = 2, 1, 0$ depending on the details of the embedding of $\Gamma$ in the $SU(4)$ group which is the isotropy of the $S^5$. This embedding is specified by the four integers $A_m, 1 \leq m \leq 4$ with

$$\Sigma_m A_m = 0 \text{(mod } p\text{)}$$

which characterize the transformation of the components of the defining representation of $SU(4)$. We are here interested in the non-supersymmetric case $\mathcal{N} = 0$ which occurs if and only if all four $A_m$ are non-vanishing.

It can be shown by group theory arguments that:

*In an $\mathcal{N} = 0$ quiver gauge theory, chiral fermions are present if and only if all scalars are in bifundamental representations.*

The following Table lists all abelian quiver theories with $\mathcal{N} = 0$ from $\mathbb{Z}_2$ to $\mathbb{Z}_7$:
| No | p | $A_m$ | $a_i$ | scal bfs | scal adj | chir frms | SM |
|----|---|------|------|----------|----------|----------|----|
| 1  | 2 | (1111) | (000) | 0        | 6        | No       | No |
| 2  | 3 | (1122) | (001) | 2        | 4        | No       | No |
| 3  | 4 | (2222) | (000) | 0        | 6        | No       | No |
| 4  | 4 | (1133) | (002) | 2        | 4        | No       | No |
| 5  | 4 | (1223) | (011) | 4        | 2        | No       | No |
| 6  | 4 | (1111) | (222) | 6        | 0        | Yes      | No |
| 7  | 5 | (1144) | (002) | 2        | 4        | No       | No |
| 8  | 5 | (2233) | (001) | 2        | 4        | No       | No |
| 9  | 5 | (1234) | (012) | 4        | 2        | No       | No |
| 10 | 5 | (1112) | (222) | 6        | 0        | Yes      | No |
| 11 | 5 | (2224) | (111) | 6        | 0        | Yes      | No |
| 12 | 6 | (3333) | (000) | 0        | 6        | No       | No |
| 13 | 6 | (2244) | (002) | 2        | 4        | No       | No |
| 14 | 6 | (1155) | (002) | 2        | 4        | No       | No |
| 15 | 6 | (1245) | (013) | 4        | 2        | No       | No |
| 16 | 6 | (2334) | (011) | 4        | 2        | No       | No |
| 17 | 6 | (1113) | (222) | 6        | 0        | Yes      | No |
| 18 | 6 | (2235) | (112) | 6        | 0        | Yes      | No |
| 19 | 6 | (1122) | (233) | 6        | 0        | Yes      | No |
| 20 | 7 | (1166) | (002) | 2        | 4        | No       | No |
| 21 | 7 | (3344) | (001) | 2        | 4        | No       | No |
| 22 | 7 | (1256) | (013) | 4        | 2        | No       | No |
| 23 | 7 | (1346) | (023) | 4        | 2        | No       | No |
| 24 | 7 | (1355) | (113) | 6        | 0        | No       | No |
| 25 | 7 | (1114) | (222) | 6        | 0        | Yes      | No |
| 26 | 7 | (1222) | (333) | 6        | 0        | Yes      | No |
| 27 | 7 | (2444) | (111) | 6        | 0        | Yes      | No |
| 28 | 7 | (1123) | (223) | 6        | 0        | Yes      | Yes |
| 29 | 7 | (1355) | (113) | 6        | 0        | Yes      | Yes |
| 30 | 7 | (1445) | (113) | 6        | 0        | Yes      | Yes |
Cancellation of quadratic divergences

The lagrangian for the nonsupersymmetric $Z_p$ theory can be written in a convenient notation which accommodates simultaneously both adjoint and bifundamental scalars as

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu;r,r}^{ab} F_{\mu\nu;r,r}^{ba} + i \lambda_{r+A_4}^{ab} \gamma^{\mu} D_{\mu} \lambda_{r+A_4}^{ba} + 2 D_{\mu} \Phi_{r+a_i,r}^{ab} D_{\mu} \Phi_{r,r+a_i}^{ba} + i \bar{\Psi}_{r+A_m,r}^{ab} \gamma^{\mu} D_{\mu} \Psi_{r,r+A_m}^{ba}$$

$$-2ig \left[ \bar{\Psi}_{r+r+A_1}^{ab} P_L \Phi_{r+A_1,r+A_4}^{bc} \Phi_{r+r+A_4}^{ca} - \bar{\Psi}_{r,r+A_1}^{ab} P_L \Phi_{r+A_1,r-A_4}^{bc} \lambda_{r-A_4}^{ca} \right]$$

$$-\sqrt{2}ig \epsilon_{ijk} \left[ \bar{\Psi}_{r+r+A_1}^{ab} P_L \Phi_{r+r+A_1,r+A_4}^{bc} \Phi_{r-A_4}^{ca} - \bar{\Psi}_{r,r+A_1}^{ab} P_L \Phi_{r+r+A_1,r-A_4}^{bc} \lambda_{r-A_4}^{ca} \right]$$

$$-g^2 \left( \Phi_{r+r+a_i}^{ab} \Phi_{r+r-a_i}^{bc} - \Phi_{r-r-a_i}^{ab} \Phi_{r-r+a_i}^{bc} \right) \left( \Phi_{r+r,a_j}^{cd} \Phi_{r+r,a_j}^{da} - \Phi_{r-r,a_j}^{cd} \Phi_{r-r,a_j}^{da} \right)$$

$$+ 4g^2 \left( \Phi_{r+r+a_i}^{ab} \Phi_{r+r+a_i}^{bc} \Phi_{r+r,a_j}^{da} - \Phi_{r+r,a_i}^{ab} \Phi_{r+r,a_i}^{bc} \Phi_{r+r,a_j}^{da} \right)$$

where $\mu, \nu = 0, 1, 2, 3$ are lorentz indices; $a, b, c, d = 1$ to $N$ are $U(N)^p$ group labels; $r = 1$ to $p$ labels the node of the quiver diagram; $a_i$ ($i = \{1, 2, 3\}$) label the first three of the $6$ of $SU(4)$; $A_m$ ($m = \{1, 2, 3, 4\}$) = $A_{i_1}$ label the $4$ of $SU(4)$. By definition $A_4$ denotes an arbitrarily-chosen fermion ($\lambda$) associated with the gauge boson, similarly to the notation in the $\mathcal{N} = 1$ supersymmetric case.

Let us first consider the quadratic divergence question in the mother $\mathcal{N} = 4$ theory. The $\mathcal{N} = 4$ lagrangian is like Eq. (2) but since there is only one node all those subscripts become unnecessary so the form is simply

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^{ab} F_{\mu\nu}^{ba} + i \lambda_{r+A_4}^{ab} \gamma^{\mu} D_{\mu} \lambda_{r+A_4}^{ba} + 2 D_{\mu} \Phi_{r+a_i}^{ab} D_{\mu} \Phi_{r,r+a_i}^{ba} + i \bar{\Psi}_{r+A_m}^{ab} \gamma^{\mu} D_{\mu} \Psi_{r,r+A_m}^{ba}$$

$$-2ig \left[ \bar{\Psi}_{r+r+A_1}^{ab} P_L \Phi_{r+A_1,r+A_4}^{bc} \Phi_{r+A_4}^{ca} - \bar{\Psi}_{r,r+A_1}^{ab} P_L \Phi_{r+A_1,r-A_4}^{bc} \lambda_{r-A_4}^{ca} \right]$$

$$-\sqrt{2}ig \epsilon_{ijk} \left[ \bar{\Psi}_{r+r+A_1}^{ab} P_L \Phi_{r+r+A_1,r+A_4}^{bc} \Phi_{r-A_4}^{ca} - \bar{\Psi}_{r,r+A_1}^{ab} P_L \Phi_{r+r+A_1,r-A_4}^{bc} \lambda_{r-A_4}^{ca} \right]$$

$$-g^2 \left( \Phi_{r+r+a_i}^{ab} \Phi_{r+r-a_i}^{bc} - \Phi_{r-r-a_i}^{ab} \Phi_{r-r+a_i}^{bc} \right) \left( \Phi_{r+r,a_j}^{cd} \Phi_{r+r,a_j}^{da} - \Phi_{r-r,a_j}^{cd} \Phi_{r-r,a_j}^{da} \right)$$

$$+ 4g^2 \left( \Phi_{r+r+a_i}^{ab} \Phi_{r+r+a_i}^{bc} \Phi_{r+r,a_j}^{da} - \Phi_{r+r,a_i}^{ab} \Phi_{r+r,a_i}^{bc} \Phi_{r+r,a_j}^{da} \right)$$

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All $\mathcal{N} = 4$ scalars are in adjoints and the scalar propagator has one-loop quadratic divergences coming potentially from three scalar self-energy diagrams: (a) the gauge loop (one quartic vertex); (b) the fermion loop (two trilinear vertices); and (c) the scalar loop (one quartic vertex).

For $\mathcal{N} = 4$ the respective contributions of (a, b, c) are computable from Eq. (2) as proportional to $g^2 N(1, -4, 3)$ which cancel exactly.

The $\mathcal{N} = 0$ results for the scalar self-energies (a, b, c) are computable from the lagrangian of Eq. (2). Fortunately, the calculation was already done in the literature. The result is amazing! The quadratic divergences cancel if and only if $x = 3$, exactly the same “if and only if” as to have chiral fermions. It is pleasing that one can independently confirm the results directly from the interactions in Eq. (2). To give just one explicit example, in the contributions to diagram (c) from the last term in Eq. (2), the $1/N$ corrections arise from a contraction of $\Phi$ with $\Phi^\dagger$ when all the four color superscripts are distinct and there is consequently no sum over color in the loop. For this case, examination of the node subscripts then confirms proportionality to the kronecker delta, $\delta_{0,a_i}$. If and only if all $a_i \neq 0$, all the other terms in Eq. (2) do not lead to $1/N$ corrections to the $\mathcal{N} = 4$. 
The lagrangian for the nonsupersymmetric $Z_n$ theory can be written in a convenient notation which accommodates simultaneously both adjoint and bifundamental scalars as mentioned before.

As also mentioned above we shall restrict attention to models where all scalars are in bifundamentals which requires all $a_i$ to be non zero. Recall that $a_1 = A_2 + A_3$, $a_2 = A_3 + A_1$; $a_3 = A_1 + A_2$.

The lagrangian is classically $U(N)^p$ gauge invariant. There are, however, triangle anomalies of the $U(1)_pU(1)^2_q$ and $U(1)_pSU(N)^2_q$ types. Making gauge transformations under the $U(1)_r$ $(r = 1, 2, \ldots, n)$ with gauge parameters $\Lambda_r$ leads to a variation

$$\delta \mathcal{L} = -\frac{g^2}{4\pi^2} \Sigma_{p=1}^{p=n} A_{pq} F^{(p)}_{\mu\nu} \tilde{F}^{(p)\mu\nu} \Lambda_q$$

which defines an $n \times n$ matrix $A_{pq}$ which is given by

$$A_{pq} = \text{Tr}(Q_p Q^2_q)$$

where the trace is over all chiral fermion links and $Q_r$ is the charge of the bifundamental under $U(1)_r$. We shall adopt the sign convention that $N$ has $Q = +1$ and $N^*$ has $Q = -1$.

It is straightforward to write $A_{pq}$ in terms of Kronecker deltas because the content of chiral fermions is

$$\Sigma_{m=1}^{m=4} \Sigma_{r=1}^{r=n} (N_r, N^*_r + A_m)$$

This implies that the antisymmetric matrix $A_{pq}$ is explicitly

$$A_{pq} = -A_{qp} = \Sigma_{m=1}^{m=4} (\delta_{p,q-A_m} - \delta_{p,q+A_m})$$
Now we are ready to construct $L^{(1)}_{\text{comp}}$, the compensatory term. Under the $U(1)_r$ gauge transformations with gauge parameters $\Lambda_r$ we require that

$$\delta L^{(1)}_{\text{comp}} = -\delta L$$

$$= + \frac{g^2}{4\pi^2} \sum_{p=1}^{p=n} A_{pq} F_{\mu\nu}^{(p)} \tilde{F}^{(p)\mu\nu} \Lambda_q$$  \hspace{1cm} (6)

To accomplish this property, we construct a compensatory term in the form

$$L^{(1)}_{\text{comp}} = \frac{g^2}{4\pi} \sum_{p=1}^{p=n} \sum_{k} B_{pk} \text{ImTr} \ln \left( \frac{\Phi_k}{\nu} \right) F_{\mu\nu}^{(p)} \tilde{F}^{(p)\mu\nu}$$  \hspace{1cm} (7)

where $\Sigma_k$ runs over scalar links. To see that $L^{(1)}_{\text{comp}}$ of Eq.(7) has SU($N$)$^n$ invariance rewrite $\text{Tr ln} \equiv \text{exp det}$ and note that the SU($N$) matrices have unit determinant.

We note en passant that one cannot take the $\nu \rightarrow 0$ limit in Eq.(7); the chiral anomaly enforces a breaking of conformal invariance.

We define a matrix $C_{kq}$ by

$$\delta \left( \sum_{p=1}^{p=n} \sum_{k} \text{ImTr} \ln \left( \frac{\Phi_k}{\nu} \right) \right) = \sum_{q=1}^{q=n} C_{kq} \Lambda_q$$  \hspace{1cm} (8)

whereupon Eq.(6) will be satisfied if the matrix $B_{pk}$ satisfies $A = BC$. The inversion $B = AC^{-1}$ is non trivial because $C$ is singular but $C_{kq}$ can be written in terms of Kronecker deltas by noting that the content of complex scalar fields in the model implies that the matrix $C_{kq}$ must be of the form

$$C_{kq} = 3\delta_{kq} - \sum_i \delta_{k,a_i,q}$$  \hspace{1cm} (9)
**Evolution of U(1) gauge couplings.**

In the absence of the compensatory term, the two independent $U(N)^n$ gauge couplings $g_N$ for SU(N) and $g_1$ for U(1) are taken to be equal $g_N(\mu_0) = g_1(\mu_0)$ at a chosen scale, e.g. $\mu_0=4$ TeV to enable cancellation of quadratic divergences. Note that the $n$ SU(N) couplings $g_N^{(p)}$ are equal by the overall $Z_n$ symmetry, as are the $n$ U(1) couplings $g_1^{(p)}$, $1 \leq p \leq n$.

As one evolves to higher scales $\mu > \mu_0$, the renormalization group beta function $\beta_N$ for SU(N) vanishes $\beta_N = 0$ at least at one-loop level so the $g_N(\mu)$ can behave independent of the scale as expected by conformality. On the other hand, the beta function $\beta_1$ for U(1) is positive definite in the unadorned theory, given at one loop by

$$b_1 = \frac{11N}{48\pi^2} \quad (10)$$

where $N$ is the number of colors.

The corresponding coupling satisfies

$$\frac{1}{\alpha_1(\mu)} = \frac{1}{\alpha_1(M)} + 8\pi b_1 \ln \left( \frac{M}{\mu} \right) \quad (11)$$

so the Landau pole, putting $\alpha(\mu) = 0.1$ and $N = 3$, occurs at

$$\frac{M}{\mu} = \exp \left[ \frac{20\pi}{11} \right] \approx 302 \quad (12)$$

so for $\mu = 4$ TeV, $M \sim 1200$ TeV. The coupling becomes “strong” $\alpha(\mu) = 1$ at

$$\frac{M}{\mu} = \exp \left[ \frac{18\pi}{11} \right] \approx 171 \quad (13)$$

or $M \sim 680$ TeV.

We may therefore ask whether the new term $\mathcal{L}_{\text{comp}}$ in the lagrangian, necessary for anomaly cancellation, can solve this problem for conformality?
Since the scale $v$ breaks conformal invariance, the matter fields acquire mass, so the one-loop diagram \(^2\) has a logarithmic divergence proportional to

$$\int \frac{d^4p}{v^2} \left[ \frac{1}{(p^2 - m_k^2)} - \frac{1}{(p^2 - m_{k'}^2)} \right] \sim - \frac{\Delta m_{kk'}^2}{v^2} \ln \left( \frac{\Lambda}{v} \right)$$

(14)

the sign of which depends on $\delta m_{kk'}^2 = (m_k^2 - m_{k'}^2)$.

To achieve conformality of $U(1)$, a constraint must be imposed on the mass spectrum of matter bifundamentals, \textit{viz}

$$\Delta m_{kk'}^2 \propto v^2 \left( \frac{11N}{48\pi^2} \right)$$

(15)

with a proportionality constant of order one which depends on the choice of model, the $n$ of $Z_n$ and the values chosen for $A_m$, $m = 1, 2, 3$. This signals how conformal invariance must be broken at the TeV scale in order that it can be restored at high energy; it is interesting that such a constraint arises in connection with an anomaly cancellation mechanism which necessarily breaks conformal symmetry.

\(^2\)The usual one-loop $\beta$–function is of order $h^2$ regarded as an expansion in Planck’s constant: four propagators each $\sim h$ and two vertices each $\sim h^{-1}$ (c.f. Y. Nambu, Phys. Lett. B26, 626 (1968)). The diagram considered is also $\sim h^2$ since it has three propagators, one quantum vertex $\sim h$ and an additional $h^{-2}$ associated with $\Delta m_{kk'}^2$.  

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SECTION 6. DARK MATTER CANDIDATE

Definition of a $Z_2$ symmetry

In the nonsupersymmetric quiver gauge theories, the gauge group, for abelian orbifold $AdS_5 \times S^5/Z_n$ is $U(N)^n$. In phenomenological application $N = 3$ and $n$ reduces eventually after symmetry breaking to $n = 3$ as in trinification. The chiral fermions are then in the representation of $SU(3)^3$:

$$ (3, 3^*, 1) + (3^*, 1, 3) + (1, 3, 3^*) $$

This is as in the 27 of $E_6$ where the particles break down in to the following representations of the $SU(3) \times SU(2) \times U(1)$ standard model group:

$$ Q, \ u^c, \ d^c, \ L \ e^c \ N^c $$

transforming as

$$ (3, 2), \ (3^*, 1), \ (3^*, 1), \ (1, 2), \ (1, 1), \ (1, 1) $$

in a 16 of the $SO(10)$ subgroup.

In addition there are the states

$$ h, \ h^*, \ E, \ E^* $$

transforming as

$$ (3, 1), \ (3^*, 1), \ (2, 1), \ (2, 1) $$

in a 10 of $SO(10)$ and finally

$$ S $$

transforming as the singlet

$$ (1, 1) $$

It is natural to define a $Z_2$ symmetry $R$ which commutes with the $SO(10)$ subgroup of $E_6 \rightarrow O(10) \times U(1)$ such that $R = +1$ for the first 16 of states. Then it is mandated that $R = -1$ for the 10 and 1 of $SO(10)$ because
the following Yukawa couplings must be present to generate mass for the fermions:

\[
16_f 16_f 10_s, \quad 16_f 10_f 16_s, \quad 10_f 10_f 1_s, \quad 10_f 1_f 1_s \tag{23}
\]

which require \( R = +1 \) for \( 10_s, 1_s \) and \( R = -1 \) for \( 16_s \).
Contribution of the Lightest Conformality Particle (LCP) to the cosmological energy density:

The LCP act as cold dark matter WIMPs, and the calculation of the resultant energy density follows a well-known path. Here we follow the procedure in a recent technical book by Mukhanov.

The LCP decouple at temperature $T_*$, considerably less than their mass $M_{LCP}$; we define $x_* = M_{LCP}/T_*$. Let the annihilation cross-section of the LCP at decoupling be $\sigma_*$. Then the dark matter density $\Omega_m$, relative to the critical density,

$$\Omega_m h_{75}^2 = \frac{g^*_{1/2}}{g_*} x^{3/2}_* \left( \frac{3 \times 10^{-38} \text{cm}^2}{\sigma_*} \right)$$

where $h_{75}$ is the Hubble constant in units of $75 \text{km/s/Mpc}$. $g_* = (g_b + \frac{7}{8} g_f)$ is the effective number of degrees of freedom (dof) at freeze-out for all particles which later convert their energy into photons; and $\tilde{g}_*$ is the number of dof which are relativistic at $T_*$.  

Discussion of LCP Dark Matter:

The LCP is a viable candidate for a cold dark matter particle which can be produced at the LHC. The distinction from other candidates will require establishment of the $U(3)^3$ gauge bosons, extending the 3-2-1 standard model and the discovery that the LCP is in a bifundamental representation thereof.

To confirm that the LCP is the dark matter particle would, however, require direct detection of dark matter.
Status of conformality

It has been established that conformality can provide (i) naturalness without one-loop quadratic divergence for the scalar mass and anomaly cancelation; (ii) precise unification of the coupling constants; and (iii) a viable dark matter candidate. It remains for experiment to check that quiver gauge theories with gauge group $U(3)^3$ or $U(3)^n$ with $n \geq 4$ are actually employed by Nature.

SECTION 7. SUMMARY

- Phenomenology of conformality has striking resonances with the standard model.
- 4 TeV Unification predicts three families and new particles around 4 TeV accessible to experiment (LHC).
- The scalar propagator in these theories has no quadratic divergence iff there are chiral fermions.
- Anomaly cancellation in effective lagrangian connected to consistency of U(1) factors.
- Dark matter candidate (LCP) will be produced at LHC, then directly detected.

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