QCD-Instantons at HERA – An Introduction*

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Abstract

We review our ongoing theoretical and phenomenological study of the discovery potential for instanton-induced DIS events at HERA. Constraints from recent lattice simulations will be exploited and translated into a “fiducial” kinematical region for our predictions of the instanton-induced DIS cross-section.

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1 Instantons

Non-abelian gauge theories like QCD are known to exhibit a rich vacuum structure. The latter includes topologically non-trivial fluctuations of the gauge fields, carrying an integer topological charge $Q$. The simplest building blocks of topological structure are instantons ($Q = +1$) and anti-instantons ($Q = -1$) which are well-known explicit solutions of the euclidean field equations in four dimensions \[1\].

Instantons ($I$) are widely believed to play an important rôle in various long-distance aspects \[2\] of QCD:

First of all, they may provide a solution of the famous $U_A(1)$ problem \[3\] ($m_{\eta'} \gg m_\eta$), with the corresponding pseudoscalar mass splitting related to the topological susceptibility in the pure gauge theory by the well-known Witten-Veneziano formula \[4\]. Moreover, a number of authors have attributed a strong connection of instantons with chiral symmetry breaking \[5,2\] as well as the hadron and glueball spectrum.

However, there are also very important short-distance implications \[6,7,8,9,10\] of QCD instantons to which the present report is devoted:

Instantons are known to induce certain processes which violate chirality in accord with the general axial-anomaly relation \[3\] and which are forbidden in conventional perturbation theory. Of particular interest in this context is the deep inelastic scattering (DIS) regime. Here, hard instanton-induced processes may both be calculated \[8,9,10\] within instanton-perturbation theory and possibly be detected experimentally \[1\].\[1,12,13\]. As a key feature it has recently been shown \[8\], that in deep-inelastic scattering (DIS) the generic hard scale $Q$ cuts off instantons with large size $\rho \gg Q^{-1}$, over which one has no control theoretically.

Our finalized results \[9,10\] for inclusive instanton-induced DIS cross-sections are summarized in sections 2 and 4. Their weak residual renormalization-scale dependence is quite remarkable.

As a second main point of this review (section 3), constraints from recent lattice simulations will be exploited \[9,14\] and translated into a “fiducial” kinematical region for our predictions of the instanton-induced DIS cross-section based on instanton-perturbation theory. In section 5 we discuss the expected event signature and search strategies based on our Monte Carlo generator \[1\] QCDINS 1.60. Finally (section 6), we briefly address an interesting class of “fireball” events, observed in photoproduction, in the context of instantons and put forward a promising proposal \[14\] on extending our theoretical predictions beyond the regime of strict instanton perturbation theory.

2 DIS cross-sections in instanton-perturbation theory

In $I$-perturbation theory one expands the relevant Green’s functions about the known, classical instanton solution $A_{\mu} = A_{\mu}^{(I)} + \ldots$ instead of the usual (trivial) field configuration $A_{\mu}^{(0)} = 0$ and obtains a corresponding set of modified Feynman rules. Like in conventional pQCD, the gauge coupling $\alpha_s$ has to be small.

The leading instanton-induced process in the DIS regime of $e^\pm P$ scattering for large photon virtuality $Q^2$ is illustrated in figure 1. The dashed box emphasizes the so-called instanton-subprocess with its own Bjorken variables,

$$Q'^2 = -q'^2 \geq 0; \quad x' = \frac{Q'^2}{2p \cdot q'} \leq 1.$$

(1)
It induces a total chirality violation \( \Delta \text{chirality} = 2n_f \), in accord with the corresponding axial anomaly \cite{3}. In the Bjorken limit of \( I \)-perturbation theory, the dominant \( I \)-induced contribution to the inclusive HERA cross-section may be shown to take the form \cite{9,10}

\[
\frac{d\sigma_{\text{HERA}}}{dx'dQ'^2} \simeq \frac{d\mathcal{L}_{qg}^{(I)}}{dx'dQ'^2} \cdot \sigma_{qg}^{(I)}(Q', x').
\]

(2)

The differential luminosity, \( d\mathcal{L}_{qg}^{(I)} \), accounting for the number of \( qg \) collisions per \( eP \) collision, has a convolution-like structure. It involves integrations over the gluon density, the \( \gamma \)-flux \( P_\gamma \) and the known \( q \)-flux \( P_q^{(I)} \) in the \( I \)-background (c. f. figure 1). The crucial instanton-dynamics resides in the \( I \)-subprocess total cross-section \( \sigma_{qg}^{(I)}(Q', x') \), on which we focus our attention next \cite{9,10}.

Being an observable, \( \sigma_{qg}^{(I)}(Q', x') \) involves integrations over all \( I(\overline{I}) \)-“collective coordinates”, including the \( I (\overline{I}) \)-sizes \( \rho (\overline{\rho}) \) and the \( I(\overline{I}) \)-distance 4-vector \( R_\mu \),

\[
\sigma_{qg}^{(I)} = \int_0^\infty d\rho \int_0^\infty d\overline{\rho} \int d^4R \{ \ldots \} e^{-Q'(\rho + \overline{\rho})} e^{i(p + q') \cdot R} e^{-\frac{\omega}{\alpha_s} \frac{R^2}{\rho \overline{\rho}}} \Omega \left( \frac{R^2}{\rho \overline{\rho}} \right).
\]

(3)

The \( \rho (\overline{\rho}) \)-integrals in (3) involve as generic weight the \( I(\overline{I}) \)-density \( D(\rho (\overline{\rho})) \) \cite{13,14,10}:

\[
D(\rho) = \frac{d}{\rho^5} \left( \frac{2\pi}{\alpha_s(\mu_r)} \right)^{2N_c} \exp \left( -\frac{2\pi}{\alpha_s(\mu_r)} \right) (\rho \mu_r)^{\beta_0 + \frac{\alpha_s(\mu_r)}{\beta_1} (\beta_1 - 4N_c \beta_0)}
\]

(4)

\[
d = \frac{2e^{5/6}}{\pi^2 (N_c - 1)! (N_c - 2)!} e^{-1.51137N_c + 0.29175n_f} \quad (\overline{\text{MS}} \text{ scheme});
\]

\[
\beta_0 = \frac{11}{3} N_c - \frac{2}{3} n_f; \quad \beta_1 = \frac{34}{3} N_c^2 - \left( \frac{13}{3} N_c - \frac{1}{N_c} \right) n_f,
\]

with renormalization scale \( \mu_r \) and \( N_c = 3 \).

\footnote{Both an instanton and an anti-instanton enter here, since cross sections result from taking the \textit{modulus squared} of an amplitude in the single \( I \) background.}

Figure 1: The leading instanton-induced process in the DIS regime of \( e^\pm P \) scattering \((n_f = 3)\).
The function \( \Omega(R^2/(\rho \bar{\rho}), \ldots) \) in equation (3), appearing in the exponent with a large numerical coefficient \( 4\pi/\alpha_s \), incorporates the effects of final-state gluons. Within strict I-perturbation theory, it is given in form of a perturbative expansion [17], while in the so-called \( I \bar{T} \)-valley approximation [18,19] \( \Omega \) is associated with an analytically known closed expression [19,20] for the interaction between \( I \) and \( \bar{I} \), \( \Omega \approx \alpha_s/(4\pi)S[A^I_{\mu}] - 1 \). With both methods agreeing for larger values of \( R^2/(\rho \bar{\rho}) \), we have actually used the valley method in our quantitative evaluation.

Due to the nonvanishing virtuality \( Q' \) in DIS, the “form factor” \( \exp[-Q'(\rho + \bar{\rho})] \) in (3), being associated with the off-shell quark (zero mode) \( q \), suppresses large-size instantons [8,9,10]. Hence, the integrals in (3) are finite. In fact, they are dominated by a unique saddle-point [9,10],

\[
\rho^* = \rho^* \sim 1/Q' \quad R^2 \sim 1/(p + q)^2 \quad \Rightarrow \quad \frac{R^*}{\rho^*} \sim \sqrt{\frac{x'}{1-x'}}
\]

(5)

from which it becomes apparent that the virtuality \( Q' \) controls the effective \( I \)-size, while \( x' \) determines the effective \( I \bar{T} \)-distance (in units of the size \( \rho \)).

In figure 2, the resulting \( I \)-subprocess cross-sections (3) is displayed [1] over a large range of \( \mu_r/Q' \) for fixed \( x' = 0.5 \) and \( Q'/\Lambda = 30, 50, 70 \). Apparently, we have achieved great progress in stability and hence predictivity by using the improved expression (4) of the \( I \)-density \( D(\rho) \), which is renormalization-group (RG) invariant at the 2-loop level, i.e. \( D^{-1} dD/d\ln(\mu_r) = O(\alpha^2_s) \). The residual dependence on the renormalization scale \( \mu_r \) is remarkably flat and turns out to be strongly reduced as compared to the 1-loop case! Throughout, we choose as the “best scale”, \( \mu_r = 0.15 Q' \), for which \( \partial \sigma^{(1)}_{qg}/\partial \mu_r \simeq 0 \) (c.f. figure 2). This choice agrees well with the intuitive expectation \( \mu_r \sim 1/(\rho) \sim Q'/\beta_0 = O(0.1)Q' \).

Figure 2: Illustration of the weak residual renormalization-scale (\( \mu_r \)) dependence of the resulting \( I \)-subprocess cross-section \( \sigma^{(1)}_{qg}(Q', x') \).
3 “Fiducial” region from lattice simulations

There has been much recent activity in the lattice community to “measure” topological fluctuations in lattice simulations [21] of QCD. Being independent of perturbation theory, such simulations provide “snapshots” of the QCD vacuum including all possible non-perturbative features like instantons (figure 3). Let us discuss next, how these lattice results may be exploited to provide crucial support for the theoretical basis of our calculations in DIS:

To this end, we first perform a quantitative confrontation [9,14] of the predictions from $I$-perturbation theory with a recent high-quality lattice simulation [23] of QCD (without fermions, $n_f = 0$). The striking agreement which we shall find over a range of $I$-collective coordinates is a very interesting result by itself.

Next, we recall (c.f. (3) and (5)) that the collective coordinate integrals in our DIS cross-section $\sigma_{qg}(Q',x')$ are dominated by a unique, calculable saddle-point $(\rho^*, R/\rho^*)$, in one-to-one correspondence to the conjugate momentum variables $(Q', x')$. This fact then allows us to translate the extracted range of validity of $I$-perturbation theory and the dilute $I$-gas approximation, $(\rho \leq \rho_{\text{max}}, R/\rho \geq (R/\rho)_{\text{min}})$, directly into a “fiducial” kinematical region $(Q' \geq Q'_{\text{min}}, x' \geq x'_{\text{min}})$ in momentum space!

In lattice simulations 4d-Euclidean space-time is made discrete; specifically, the recent “data” from the UKQCD collaboration [23], which we shall use here, involve a lattice spacing $a = 0.055 - 0.1$ fm and a volume $V = l_{\text{space}}^3 \cdot l_{\text{time}} = [16^3 \cdot 48 - 32^3 \cdot 64] a^4$. In principle, such a lattice allows to study the properties of an ensemble of $I$’s and $\bar{T}$’s with sizes $a < \rho < V^{1/4}$. However, in order to make instanton effects visible, a certain “cooling” procedure has to be applied first. It is designed to filter out (dominating) fluctuations of short wavelength $O(a)$ (c.f. figure 3 (a)), while affecting the topological fluctuations of much longer wavelength $\rho \gg a$ comparatively little. After “cooling”, $I$’s and $\bar{T}$’s can clearly be seen (and studied) as bumps in the lagrange density and the topological charge density (figure 3 (b), (c)). For a more detailed discussion of lattice-specific caveats, like possible lattice artefacts and the dependence of results on “cooling” etc., see Refs. [21,23].

Of course, one has to extrapolate the lattice observables to the continuum $(a \Rightarrow 0)$, before a meaningful comparison with $I$-perturbation theory can be made. This is complicated by a strong dependence of the various distributions on the number $n_{\text{cools}}$ of cooling sweeps for fixed $\beta = 6/g_{\text{lat}}^2$.

![Lagrange Density](image1.png)

**Lagrange Density**

![Topological Charge Density](image2.png)

**Topological Charge Density**

Figure 3: Instanton content of a typical slice of a gluon configuration at fixed x,y as a function of z and t [22]. (a) Lagrange density before “cooling”, with fluctuations of short wavelength $O(a)$ dominating. After “cooling” by 25 steps, 3 $I$’s and 2 $\bar{T}$’s may be clearly identified in the lagrange density (b) and the topological charge density (c).
In ref. [23], however, equivalent pairs \((\beta, n_{\text{cools}})\) were found, for which shape and normalization of the distributions essentially remain invariant. For instance, the continuum extrapolation of the data for the \((I + \bar{T})\)-density \(D_{I+\bar{T}}\) at \((\beta, n_{\text{cools}}) = (6.0, 23), (6.2, 46), (6.4, 80)\), may thus be performed quite reliably [14], by simply rescaling the arguments \(\rho \Rightarrow \rho(0)/\rho(a) \cdot \rho\). Here, \(\rho(0)\) denotes the continuum limit of the weakly varying average \(\rho\) values, \(\rho(a)\), of \(D_{I+\bar{T}}(\rho, a)\). A linear extrapolation in \((a/r_0)^2\) was employed. For consistency and minimization of uncertainties, one should use only a single dimensionful quantity to relate lattice units and physical units. Throughout our analysis, all dimensions are therefore expressed by the so-called Sommer scale [24, 25] \(r_0\), with \(2r_0 \simeq 1\) fm, which we prefer over the string tension [23]. The resulting “continuum data” for \(D_{I+\bar{T}}(\rho)\) are displayed in figure 4. They scale nicely. We are now ready to perform a quantitative comparison with the predictions of \(I\)-perturbation theory [14]. For reasons of space, let us concentrate here on the \((I + \bar{T})\)-density \(D_{I+\bar{T}}(\rho)\). The prediction (4) of \(I\)-perturbation theory is a power law for small \(\rho\), i.e. approximately \(D \sim \rho^6\) for \(n_f = 0\). Due to its 2-loop RG-invariance the normalization of \(D_{I+\bar{T}}(\rho)\) is practically independent of the renormalization scale \(\mu\) over a wide range. It is strongly and exclusively dependent on \(r_0 \Lambda_{\text{MS}} n_f = 0\), for which we take the most recent, accurate result by the ALPHA-collaboration [25], \(2r_0 \Lambda_{\text{MS}} n_f = 0 = (238 \pm 19)\) MeV fm. In figure 4(b) we display both this parameter-free prediction from (4) of \(I\)-perturbation theory and the continuum limit of the UKQCD data in a log-log plot, to clearly exhibit the expected power law in \(\rho\). The agreement in shape and normalization for \(\rho < 0.3\) \((2r_0) \simeq 0.3\) fm is striking, indeed, notably in view of the often criticized “cooling” procedure and the strong sensitivity to \(\Lambda_{\text{MS}} n_f = 0\).

By a similar analysis [14], we were able to infer from the “equivalent” UKQCD lattice data a range of validity \(R/\rho > 1\) of the valley expression for the \(I\bar{T}\)-interaction \(\Omega(R^2/(\rho \bar{\rho}), \ldots)\) in (3). Finally,
we have confirmed the approximate validity of the dilute-gas picture for sufficiently small instantons with $\rho \lesssim (0.3 - 0.5) \text{ fm}$. The latter results are based on the “packing fraction” being $< 1$ and a test of the dilute-gas identity: $\langle Q^2 \rangle = N_{\text{tot}}$. Here $Q$ is the topological charge and $N_{\text{tot}}$ the total number of charges. These results strongly support the reliability of our calculations in DIS.

By means of the discussed saddle-point correspondence, these lattice constraints may be converted into a “fiducial” region for our cross-section predictions in DIS,

$$
\begin{align*}
\rho^* & \leq \rho_{\text{max}}^* \simeq 0.3 \text{ fm}; \\
\frac{R^*}{\rho^*} & \geq \left( \frac{R^*}{\rho^*} \right)_{\text{min}} \simeq 1
\end{align*}
\Rightarrow \left\{ \begin{array}{l}
Q' \geq Q'_\text{min} \simeq 8 \text{ GeV}; \\
x' \geq x'_\text{min} \simeq 0.35.
\end{array} \right.
$$

(6)

4 HERA cross-section

Figure 5 displays our finalized $I$-induced cross-section at HERA, as function of the cuts $x'_\text{min}$ and $Q'_\text{min}$. For the minimal cuts extracted from the UKQCD lattice simulation, we obtain

![Figure 5: $I$-induced cross-section at HERA as function of the cuts in $(x', Q')$.](image)

a surprisingly large cross-section,

$$
\sigma^{(I)}_{\text{HERA}}(x' \geq 0.35, Q' \geq 8 \text{ GeV}) \simeq 126 \text{ pb}; \ x_{\text{Bj}} \geq 10^{-3}; \ 0.9 \geq y_{\text{Bj}} \geq 0.1.
$$

(7)

Hence, with the total luminosity accumulated by experiments at HERA, $\mathcal{L} = \mathcal{O}(80) \text{ pb}^{-1}$, one already expects $\mathcal{O}(10^4)$ $I$-induced events on tape from this kinematical region. Note also that the cross-section quoted in Eq. (7) corresponds to a fraction of $I$-induced to normal DIS (nDIS) events of

$$
f^{(I)} = \frac{\sigma^{(I)}_{\text{HERA}}}{\sigma^{(\text{nDIS})}_{\text{HERA}}} = \mathcal{O}(1) \%; \ \text{for} \ x_{\text{Bj}} \geq 10^{-3}; \ 0.9 \geq y_{\text{Bj}} \geq 0.1.
$$

(8)

Note that the full $(I + \bar{I})$-ensemble without the size restriction is known not to be a dilute gas.
This is remarkably close to the published upper limits on the fraction of $I$-induced events [20], which are also on the one percent level.

There are still a number of significant uncertainties in our result for the cross-section. For fixed $Q'$ and $x'$ cuts, one of the dominant uncertainties arises from the experimental uncertainty in the QCD scale $\Lambda$. In the 2-loop expression for $\alpha_s$ with $n_f = 3$ (massless) flavours we used the value $\Lambda^{(3)}_{\text{MS}} = 282 \text{MeV}$, corresponding to the central value of the DIS average for $n_f = 4$, $\Lambda^{(4)}_{\text{MS}} = 234 \text{MeV}$ [27]. If we change $\Lambda^{(3)}_{\text{MS}}$ within the allowed range, $\approx \pm 65 \text{MeV}$, the cross-section (7) varies between 26 pb and 426 pb. Minor uncertainties are associated with the residual renormalization-scale dependence (c.f. figure 2) and the choice of the factorization scale. Upon varying the latter by an order of magnitude, the changes are in the $O(20)$ % range only.

By far the dominant uncertainty in $\sigma^{(I)}_{\text{HERA}}$ arises, however, from the uncertainty in placing the $(x', Q')$ cuts (c.f. figure 3). Hence, the constraints (8) from lattice simulations are extremely valuable for making concrete and reliable predictions of the $I$-induced rate at HERA.

5 Signatures and searches

An indispensable tool for investigating the structure of the $I$-induced final state and for developing optimized search strategies is our Monte-Carlo generator for $I$-induced DIS-events, QCDINS 1.60. Besides the matrix element for the $I$-induced hard subprocess, it provides leading-log parton showers and hadronization via its interface to HERWIG 5.9.

The characteristic features of the $I$-induced final state are illustrated in figure 6 (a) displaying the lego plot of a typical event from QCDINS 1.60 (c.f. also figure 1):

![Figure 6](image-url)

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![Figure 6](image-url)

(b) An interesting real “fireball” event in photoproduction from ZEUS [30] with very large total $E_T$ and multiplicity.

Besides a single (not very hard) current-quark jet, one expects an accompanying densely populated “hadronic band”. For $x_{Bj}\text{min} \approx 10^{-3}$, say, it is centered around $\eta \approx 2$ and has a width of $\Delta \eta \approx \pm 1$. 8
The band directly reflects the isotropic production of an I-induced “fireball” of O(10) partons in the I-rest system. Both the total transverse energy \( \langle E_T \rangle \approx 15 \text{ GeV} \) and the charged particle multiplicity \( \langle n_c \rangle \approx 13 \) in the band are far higher than in normal DIS events. Finally, each I-induced event has to contain strangeness (and possibly also charm) such that the number of \( K^0 \)'s amounts to \( \approx 2.2/\text{event} \).

Despite the high expected rate (1) of I-induced events at HERA, no single observable is known (yet) with sufficient nDIS rejection. Hence, a dedicated multi-observable analysis seems to be required. Neural network filters are being tried and exhibit a very good analyzing power if applied to \( \gtrsim O(5) \) observables [28]. Strategies to produce “instanton-enriched” data samples and to reconstruct \( (Q'^2, x') \) are under study and look quite promising [28]. Clearly, in all cases, a good understanding of the perturbative QCD background in the tails of the considered distributions is required.

### 6 Going beyond instanton-perturbation theory

A class of striking “fireball” events in photoproduction, with large total \( E_T \) and large multiplicity, has been reported [29] at this meeting (see e.g. figure 6 (b)). While the quantitative analysis is still in an early stage, these events seem to exhibit all characteristics of I-induced events (c.f. figure 6 (a)). However, it appears that – unlike ordinary QCD perturbation theory – the hard photoproduction limit, \( Q^2 \rightarrow 0, (E_T)_{\text{jet}} \) large, is not within the reach of strict I-perturbation theory. The reason is that in the \( Q^2 \rightarrow 0 \) limit, we encounter a contribution to the photoproduction cross-section, which tends to diverge, if integrated over the I-size \( \rho \). This IR divergence at large \( \rho \) is independent of the \( E_T \) of the (current quark) jet and directly associated with the “bad” large-\( \rho \) behaviour of the perturbative expression (4) for the I-density, \( D(\rho) \sim \rho^{6-2/3n_f} \). In contrast, the actual form of \( D(\rho) \) (c.f. figure 4) is strongly peaked around \( \rho \approx 0.5 \text{ fm} \), and appears to vanish exponentially fast for larger \( \rho \). The above “fireball” events and more generally, the ongoing I-searches at HERA, provide plenty of motivation for trying to extend our calculational framework beyond strict I-perturbation theory. We are thus led to make the following promising proposal in this direction:

One may try and replace the most strongly varying entries in the perturbative calculations, the I-density \( D(\rho) \) and the IT-interaction \( \Omega(R^2/(\rho \bar{\rho}), \ldots) \), in (3), by their actual form as extracted from the recent non-perturbative lattice results [14].

The I-rates in photoproduction and the I-contributions to further interesting observables may then be calculated, and the \( (Q'^2, x') \) cuts in I-searches be considerably relaxed! Due to the strong peaking of \( D(\rho) \), only the region around \( \rho \approx 0.5 \text{ fm} \) enters and the dilute-gas approximation may well continue to hold up to the peak (c.f. section 3).

### References

[1] A. Belavin, A. Polyakov, A. Schwarz and Yu. Tyupkin, *Phys. Lett.* B 59 (1975) 85.

[2] T. Schäfer and E.V. Shuryak, *Rev. Mod. Phys.* 70 (1998) 323.

[3] G. ‘t Hooft, *Phys. Rev. Lett.* 37 (1976) 8; *Phys. Rev.* D 14 (1976) 3432; *Phys. Rev.* D 18 (1978) 2199 (Erratum); *Phys. Rep.* 142 (1986) 357.
[4] E. Witten, Nucl. Phys. B 156 (1979) 269; G. Veneziano, Nucl. Phys. B 159 (1979) 213.

[5] D. Diakonov, Lectures at the 1995 Enrico Fermi School, hep-ph/9602375.

[6] I. Balitsky and V. Braun, Phys. Lett. B 314 (1993) 237.

[7] A. Ringwald and F. Schrempp, hep-ph/9411217, in: Quarks ‘94, Proc. 8th Int. Seminar, Vladimir, Russia, 1994, eds. D. Giguere et al., pp. 170-193.

[8] S. Moch, A. Ringwald and F. Schrempp, Nucl. Phys. B 507 (1997) 134.

[9] A. Ringwald and F. Schrempp, hep-ph/9806528, Phys. Lett. B 438 (1998) 217.

[10] S. Moch, A. Ringwald and F. Schrempp, to be published.

[11] M. Gibbs, A. Ringwald and F. Schrempp, hep-ph/9506392, in: Proc. Workshop on Deep Inelastic Scattering and QCD, Paris, France, 1995, eds. J.-F. Laporte and Y. Sirois, pp. 341-344.

[12] A. Ringwald and F. Schrempp, hep-ph/9610213, in: Quarks ‘96, Proc. 9th Int. Seminar, Yaroslavl, Russia, 1996, eds. V. Matveev et al., Vol. I, pp. 29-54.

[13] A. Ringwald and F. Schrempp, hep-ph/9706395, in: Proc. 5th Int. Workshop on Deep Inelastic Scattering and QCD (DIS 97), Chicago, 1997, eds. J. Repond and D. Krakauer, pp. 781-786.

[14] A. Ringwald and F. Schrempp, DESY 98-201.

[15] C. Bernard, Phys. Rev. D 19 (1979) 3013; A. Hasenfratz and P. Hasenfratz, Nucl. Phys. B 193 (1981) 210; M. Lüscher, Nucl. Phys. B 205 (1982) 483.

[16] T. Morris, D. Ross and C. Sachrajda, Nucl. Phys. B 255 (1985) 115.

[17] P. Arnold and M. Mattis, Phys. Rev. D 44 (1991) 3650; A. Mueller, Nucl. Phys. B 364 (1991) 109; D. Diakonov and V. Petrov, in: Proc. 26th LNPI Winter School, (Leningrad, 1991), pp. 8-64.

[18] A. Yung, Nucl. Phys. B 297 (1988) 47.

[19] V.V. Khoze and A. Ringwald, Phys. Lett. B 259 (1991) 106.

[20] J. Verbaarschot, Nucl. Phys. B 362 (1991) 33.

[21] see e.g.: P. van Baal, Nucl. Phys. (Proc. Suppl.) 63 (1998) 126; J. Negele, hep-lat/9810053, Review at Lattice ‘98; and references cited therein.

[22] M.-C. Chu, J.M. Grandy, S. Huang and J.W. Negele, Phys. Rev. D 49 (1994) 6039.

[23] D.A. Smith and M.J. Teper, (UKQCD Collab.), hep-lat/9801008, Phys. Rev. D 58 (1998) 014505 and M. Teper, private communication.
[24] R. Sommer, hep-lat/9310022, *Nucl. Phys.* B 411 (1994) 839.

[25] S. Capitani, M. Lüscher, R. Sommer and H. Wittig, (ALPHA Collab.), hep-lat/9810063, DESY 98-154.

[26] S. Aid *et al.*, H1 Collaboration, *Nucl. Phys.* B 480 (1996) 3;
S. Aid *et al.*, H1 Collaboration, *Z. Phys.* C 72 (1996) 573;
T. Carli and M. Kuhlen, *Nucl. Phys.* B 511 (1998) 85.

[27] Review of Particle Physics, Particle Data Group, *Phys. Rev. D* 54 (1996) 1.

[28] T. Carli, private communication;
J. Gerigk, Diplome Thesis, DESY, Nov 1998, to be published.

[29] Y. Yamazaki, talk in WG2/WG3 at this conference;
C. Nath, PhD thesis, Oxford 1998.

[30] ZEUS Collaboration, event 38585, run 13990, 1995,
http://www-zeus.desy.de/~krakauer/disp/95_list.html.