Study on nonlinear dynamics of the marine rotor-bearing system under yawing motion

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Abstract—After considering the yawing motion, the dynamic model of the marine rotor-bearing system is developed on the short bearing theory. Theoretical analysis shows that the yawing motion causes gyroscopic moment and strong nonlinear time-varying transport inertia moment. The dynamic characteristics of the rotor-bearing system with and without yawing motion are compared. The result shows that the yawing motion significantly affects the dynamic characteristics of the rotor-bearing system, which makes the rotor system be dominated by the quasi-period. The yawing motion makes the rotor amplitude increase significantly at low speed. Finally, under the influence of yawing motion, the dynamic characteristic of the rotor shows chaos at high speed.

1. INTRODUCTION

When a marine is sailing, due to the influence of complex navigation conditions, it will produce yawing motion, pitch and other motions as shown in Fig. 1. All of these forms of motion will have a serious impact on marine rotor-bearing system. Yawing is a typically transport motion which is worth studying.

Figure 1. Schematic diagram of transport motion of ship.

At present, the studies on the influence of transport motions on the dynamic characteristics of the rotor bearing system focus on maneuvering flight. For example, Hou et al. [1-2] carried out the dynamic simulation of a nonlinear rotor system in Hebert maneuvering flight and hovering flight, and Lin et al. [3] investigated the nonlinear dynamics of a cracked rotor system in an aircraft maneuvering with constant velocity or acceleration, especially focused on the influence of the aircraft climbing angle on the cracked rotor system response. Stringer et al. [4] presented a methodology for conducting modal reduction on a geared rotor dynamic system under the influences of general damping and gyroscopic effects. Avramov et al. [5-7] studied the nonlinear dynamic characteristics of rotor system subjected to variable base motions.
In addition, several studies have been published on the effect of the dynamic response of ship under transport motion. Zhang et al. [8] paid attention to the pressure distribution and nonlinear oil-film force of journal bearing with different forms of ship movements. Dake et al. [9] used the implicit Newmark time-step integration scheme to solve the linear/nonlinear equations of motion of the rotating rotor in bending with respect to the moving rigid support. Liu et al. [10] established a mathematic model of marine rotor-bearing, in which the air bag and floating raft are embedded.

In the research mentioned above, there are few studies on the influence of low frequency and large amplitude implicated motion on rotor bearing system. In this paper, the nonlinear dynamic model of rotor bearing system under the yawing motion is derived. Moreover, a variety of nonlinear dynamic stability analysis methods are introduced to study the dynamic characteristics of the rotor with or without yaw motion.

2. NONLINEAR DYNAMIC MODEL

Considering the rotor model as shown in Fig. 2, where \( O \) is the geometric center of the disk, \( O_1 \) and \( O_r \) are the geometric center of two shaft journals. Fig. 3 depicts the marine rotor bearing system under the influence of the transport motion. For the convenience of discussion, the following assumptions are introduced:

- The bearing basement and the disc are regarded as rigid bodies with mass.
- The oil film force model is derived from the oil film force theory of short bearing.

\[
\begin{align*}
\text{Figure 2. Profile of a rotor-bearing system.} \\
\text{Figure 3. Schematic diagram of a marine rotor-bearing system.}
\end{align*}
\]

Let \( m, e, J_1, J_2 \) are the mass of disc, eccentricity, the moment of inertia about the \( y \) and \( z \) axes, respectively. \( O_0X_0Y_0Z_0 \) is a fixed reference system, \( O_1X_1Y_1Z_1 \) is the one attached to the ship, and \( OXYZ \) is established in the rotor-bearing system. The motion of the rotor is modeled by four degrees of freedom of the disc: the vertical displacement \( x \), the horizontal displacement \( y \) and the corresponded rotation angles \( \theta_x \) and \( \theta_y \). And the angular displacement of the marine under yawing motion is \( \theta_0 \).

2.1. Equations of motion

Based on analytical mechanics, the kinetic energy and potential energy of the rotor are obtained as follows:
\[ T = T_i + T_e \]
\[ = \frac{1}{2} m \left( \dot{x}^2 + \dot{y}^2 - 2 ye \cos \omega t + 2 xe \cos \omega t + e^2 \omega^2 \right) \]
\[ + \frac{1}{2} m \left( \dot{\theta}_o^2 y^2 + 2 \dot{\theta}_o \dot{y} e \cos \omega t + \dot{\theta}_o^2 e^2 \cos^2 \omega t \right) \]
\[ + \frac{1}{2} \left( J_1 + J_2 \right) \left( \dot{\theta}_1 + \dot{\theta}_2 \right)^2 \sin^2 \theta + \frac{1}{2} J_1 \left[ \left( \dot{\theta}_1 + \dot{\theta}_2 \right)^2 + \dot{\theta}_1^2 \right] \]
\[ + \frac{1}{2} J_2 \left[ \dot{\theta}_2^2 + 2 \omega \left( \dot{\theta}_1 + \dot{\theta}_2 \right) \sin \theta \right] \]
\[ U = -mg(x + e \sin \omega t) \tag{2} \]

Corresponding with \( x, y, \theta_1, \theta_2 \), the additional forces can be written as:
\[ \begin{bmatrix} Q_x \\ Q_y \\ Q_{\theta x} \\ Q_{\theta y} \end{bmatrix} = \begin{bmatrix} F_{x1} + F_{x2} \\ F_{y1} + F_{y2} \\ F_{xy} l - F_{y1} l \\ F_{y1} l - F_{xy} l \end{bmatrix} \tag{3} \]

The equations of motion can be obtained as follows by using Lagrange's equation of the form (4).
\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} = Q_j \quad (j = x, y, \theta_x, \theta_y) \tag{4} \]

\[ \begin{bmatrix} m \ddot{x} = mg + me \omega^2 \sin \omega t + F_{x1} + F_{x2} \\ m \ddot{y} = my \dot{\theta}_o^2 + m e \dot{\theta}_o \omega \cos \omega t + ma \dot{\theta}_o \cos \omega t + F_{y1} + F_{y2} \\ J_1 \ddot{\theta}_1 = -J_1 \omega \dot{\theta}_1 - J_1 \dot{\theta}_1 + F_{xy} l - F_{y1} l - 2 \theta_1 \dot{\theta}_1 \left( J_1 + J_2 \right) \left( \dot{\theta}_1 + \dot{\theta}_2 \right) \\ J_2 \ddot{\theta}_2 = J_2 \omega \dot{\theta}_2 + J_2 \omega \dot{\theta}_2 + F_{xy} l - F_{y1} l + \theta_1 \left( J_1 + J_2 \right) \left( \dot{\theta}_1 + \dot{\theta}_2 \right)^2 \end{bmatrix} \tag{5} \]

where \( \omega \) is the speed of rotor; \( F_{x1}, F_{x2}, F_{y1}, F_{y2} \) are the nonlinear oil film force components in \( x \) and \( y \) directions, respectively; \( m \dot{\theta}_o^2, me \dot{\theta}_o \omega \cos \omega t, -2(\theta_1 + J_2) \dot{\theta}_1 \dot{\theta}_2, J_1 \dot{\theta}_1, J_2 \dot{\theta}_2, (J_1 + J_2) \theta_1 (\dot{\theta}_1^2 + 2 \dot{\theta}_1 \dot{\theta}_2) \) are induced by the yawing motion of the ship.

2.2. Oil film force

Fig. 4 shows a depiction of a hydrodynamic bearing. In this study, the hydrodynamic short journal bearing theory is introduced, and the Reynolds equation can be written as [11]:
\[ \frac{\partial}{\partial z} \left( \frac{h^3}{\mu} \frac{\partial \phi}{\partial z} \right) = 6 \left( \Omega - 2 \dot{\theta} \right) \frac{\partial h}{\partial \phi} + 12 \dot{e} \cos \phi, \quad h = c(1 + e \cos \theta) \tag{6} \]

where \( \phi \) and \( h \) are circumferential azimuth and bearing oil film thickness, respectively, \( e \) is the eccentricity ratio (\( e = e/c \), \( e = \sqrt{x^2 + y^2} \) is the radial displacement), and \( \theta \) is angle of the journal equilibrium position.
After considering the semi-Sommerfeld condition the nonlinear oil film expressions of radial and tangential direction are obtained:

\[
\begin{align*}
F_r &= 2\mu BR \left( \frac{R}{c} \right)^2 \left( \frac{B}{2R} \right)^2 \left[ \Omega - 2 \frac{d\theta}{dt} G_1 + 2 \frac{d\epsilon}{dt} G_2 \right] \\
F_t &= 2\mu BR \left( \frac{R}{c} \right)^2 \left( \frac{B}{2R} \right)^2 \left[ \Omega - 2 \frac{d\theta}{dt} G_1 + 2 \frac{d\epsilon}{dt} G_2 \right]
\end{align*}
\]

where \( R, c, \mu, B \) are the bearing radius, clearance, lubricant film viscosity, and bearing length, respectively.

### 2.3. Dimensionless equations

By using the dimensionless parameter expressions shown in Table I and letting \( \dot{x} = dx/d\tau, \dot{y} = dy/d\tau \), \( X' = dX/d\tau, Y' = dY/d\tau \), the dimensionless equations of (5) and (7) can be obtained as follows:

\[
\begin{align*}
f_r &= \frac{\sigma \lambda^2 \omega \left[ (1-2\varphi^2) G_1 + 2\epsilon G_2 \right]}{3} \\
f_t &= \frac{\sigma \lambda^2 \omega \left[ (1-2\varphi^2) G_1 + 2\epsilon G_2 \right]}{3}
\end{align*}
\]

| Parameter Description | Expression |
|------------------------|------------|
| \( \tau \)             | \( \tau = \omega t \) |
| \( \alpha \)           | \( \alpha = \epsilon/c \) |
| \( \Omega_0 \)         | \( \Omega_0 = \omega \sqrt{c/g} \) |
| \( f_{ui} \)           | \( f_{ui} = F_{ui}/mg, i = 1,2 \) |
| \( \lambda \)          | \( \lambda = B/2R \) |
| \( M \)                | \( M = mg l / J, O^2 \) |
| \( X \)                | \( X = x/c \) |
| \( Y \)                | \( Y = y/c \) |
| \( \sigma \)           | \( \sigma = \sigma / m \sqrt{gc} \) |
The nondimensional oil film force in the $x$ and $y$ directions are expressed as:

$$
\begin{align*}
    f_{x,i} &= -f_i \cos \varphi - f_i \sin \varphi \\
    f_{y,i} &= -f_i \sin \varphi - f_i \cos \varphi
\end{align*}
$$

(10)

$$
\begin{align*}
X^* &= \frac{1}{\Omega_0^2} + \alpha \cos \tau + \frac{f_{x,1}}{\Omega_0^2} + \frac{f_{x,2}}{\Omega_0^2} \\
Y^* &= Y_1^2 + \alpha \cos \tau + \frac{f_{y,1}}{\Omega_0^2} + \frac{f_{y,2}}{\Omega_0^2} \\
\theta_{x}^* &= -\theta_0^* + J \theta^*_x - 2(1 + J) \theta_0^* (\theta_0^* + \theta_0^*) + Mf_{y,2} - Mf_{y,1} \\
\theta_{y}^* &= J \theta^*_y + J \theta^*_y + (1 + J) \theta_0^* (\theta_0^* + \theta_0^*) + Mf_{x,1} - Mf_{x,2}
\end{align*}
$$

(11)

where $\tau$ is the nondimensional time, $\Omega_0$ is the dimensionless speed, $\alpha$ is the Nondimensional eccentricity, $f_{x,i}, f_{y,i}(i=1,2)$ are dimensionless oil film forces in the $x$ and $y$ directions, respectively, $J$ is the nondimensional moment of inertia, $M$ is the dimensionless moment due to gravity. In view of the existing theoretical study, it is reasonable to assume that ship yawing is $\theta_0 = A_0 \sin(\omega_0 t)$ [8].

3. NUMERICAL RESULTS AND DISCUSSIONS

To get the effects of the yawing motion on the dynamic characteristics of the rotor system, the Runge-Kutta method is employed in the following computations.

![Bifurcation diagram](image)

Figure 5. The bifurcation diagram of displacement $Y$ and its largest Lyapunov exponents. (a) The steady-state response of the rotor-bearing system and its largest Lyapunov exponents without yawing. (b) The steady-state response of the rotor-bearing system and its largest Lyapunov exponents under yawing.

The displacement bifurcation diagram and the largest Lyapunov exponent curve of the rotor system without yawing are shown in Fig. 5(a), in which $\sigma = 3, \alpha = 0.05, \lambda = 0.2$, the dimensionless speed $\Omega_0$...
changes from 0.4 to 3.6. At low speeds, when the dimensionless speed $\Omega_0$ changes from 0.4 to 2.24, the rotor system appears period 1 motion with small amplitude under the effect of eccentricity. With the increase of the dimensionless speed $\Omega_0$, the steady-state response of rotor increases suddenly. While $\Omega_0$ changes from 2.25 to 2.6, the dynamic characteristics of the system alter from period 1 to period 2. The characteristic of period 2 doesn’t persist long and then it goes back to period 1 when $\Omega_0$ is equal to 2.6. While the speed $\Omega_0$ changes from 2.86 to 3.6, the system has a transition from period 1 to quasiperiodic bifurcation; the displacement of steady-state response begins to gradually increase at this time. It can be seen that the system exhibits abundant dynamic characteristics under the action of unbalanced mass and nonlinear oil film forces. Fig. 5(b) illustrates the bifurcation diagram of the displacement of the rotor and the curve of the largest Lyapunov exponent under yawing motion. It is obtained by the numerical integration method with the rotation speed $\Omega_0$ ranging from 0.4 to 3.6, in which parameters are $\sigma = 3$, $\alpha = 0.05$, $\lambda = 0.2$, $A_0 = \pi/12$, $\omega_0 = \pi/4$. At low speeds, such as $\Omega_0$ changes from 0.4 to 2.24, the displacement of the rotor system in the y direction is not that different but its amplitude is significantly higher compared with the case without yawing. While $\Omega_0$ varies from 2.25 to 2.6, the displacement of rotor changes dramatically. With the speed $\Omega_0$ increasing further, the displacement of the system steady-state response starts to increase when $\Omega_0$ changes from 2.61 to 3.53. When $\Omega_0$ is greater than 3.53, the largest Lyapunov exponent of the rotor system becomes greater than 0, and the system behaves as chaos at this stage. Before that, the system always behaved as a pseudo-period. It is obvious that under the influence of yawing motion, the bifurcation diagram shows richer dynamic behaviors than without yawing.

Fig. 6 indicates the steady-state response, frequency spectrum, rotor trajectory, shaft journal trajectory and Poincaré map of the rotor system, in which $\Omega_0 = 0.7$, $A_0 = \pi/12$, $\omega_0 = \pi/4$. Fig. 6(a) and Fig. 6(c) illustrate that the trajectories of $O_l$ and $O_r$ are similar, but there are large differences from the trajectory of $O$, which means that the rotor will deflect under the influence of yawing motion. It can be seen from the displacement response that the system has small amplitude in the $Y$ direction and a small change range, but the displacement changes greatly with $\tau$, which indicates that the yawing motion will have a greater impact on the displacement of the rotor system when $\Omega_0$ is small. It can be seen from Fig. 6(e) that the frequency $f_0$ caused by the yaw occupies the main component. In addition, the frequency $f$ caused by the unbalanced force and its frequency multiplication components $2f$, $3f$ etc. also appear. Although the combined frequency of $(f + f_0)/2$ does exist, its amplitude is smaller than that of $f$ and $f_0$. At the current speed, the Poincaré map shows a closed curve. The largest Lyapunov exponent is -0.0097. It shows that the system is in quasiperiodic motion.

Fig. 7 illustrates the steady-state response of the rotor in which the rotation speed $\Omega_0 = 3.55$, $A_0 = \pi/12$, $\omega_0 = \pi/4$. It can be seen from the orbits of $O_l$, $O$, and $O_r$ that the trajectory of the rotor is confined to a disorderly and irregular oscillation in an ellipsoidal domain, and the curvature changes greatly in some positions. Although the existence of combined frequencies and power frequency can be identified in the frequency response diagram, the spectrum is already in a continuous state due to the appearance of a large number of harmonic components. It can be seen from Fig. 7(d) that the amplitude in the $Y$ direction changes drastically and irregularly. In Fig. 7(f), Poincaré map illustrates a set of points distributed in an elliptical region. The largest Lyapunov exponent of the system is 0.00216, which means that the system is chaotic.
4. CONCLUSION

The nonlinear dynamics of the marine rotor-bearing system under yawing motion are discussed in this article, and the conclusions are obtained as follows:

The yawing motion has a significant impact on the dynamic characteristics of the rotor-bearing system. For example, the dynamic response of the rotor without yaw is: period1→period2→period1→quasi-period. And the dynamic response of the rotor under the influence of yawing motion is: quasi-period→chaos.

Yawing motion can advance the speed of the rotor into complex quasi-periodic stage. The amplitude of the rotor system increases significantly under the influence of yawing motion, especially at low speeds.

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