Investigation of stressed state of shell structure

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Abstract. The main fundamental difference of wave gears from all other meshing gears is the presence of a flexspline which is implemented in the form of a shell structure, and which, by means of a wave generator located on the input shaft, modifies the rotational motion of the wave generator into permanent elastic wave deformations of the flexspline, which are then transformed in the rotation of the output shaft. The design features of harmonic gear drives provide them with a number of significant advantages, such as smaller dimensions and weight, in comparison with conventional ones, including planetary gear train of the same torque. They have the highest kinematic accuracy, including damping behavior and low level of noise and vibration. A unique feature of harmonic gear drives is the ability to transfer rotation into a sealed space with almost zero fluid leakage. Harmonic gear drives are capable of carrying out large gear ratios in one stage: for steel flexspline, from a minimum of about 60 to a maximum of 300. At the same time, their efficiency is quite high - in the reduction mode of 80 ... 90 \%, as in planetary gears with the same gear ratio. When operating in the torque multiplication mode, the efficiency noticeably declines. The disadvantage of harmonic gear drives is a relatively low frequency of the wave generator rotation, approximately within the range of 1500 ... 3500 min\textsuperscript{-1} with the radii of small flexspline from 125 to 25 mm, respectively. Therefore, the power of harmonic gear drives, despite the high transmitted torques, is limited to relatively small values - from 0.1 to 48 kW. Their service life is limited to approximately 10\textsuperscript{4} hours; a year and a half with continuous operation throughout the day. Specific weight and cost of harmonic drives is 1.5 -2 times lower than those of planetary type, which represents technical and economic advantages and commercial interest for road and construction machinery.

1. Introduction

The disks generate elastic wave deformations of a flexible gear rim, the rotation component of which is taken by the shell to the output shaft. The stressed-deformed state of a flexspline deformed by the disks of the wave generator was obtained by joint arrangement of the ring and the shell. The external load on the gear rim is converted by a delta function. The solutions obtained made it possible to create large harmonic gear drives that meet the technical requirements of construction and transport engineering [1-3]. Deformations of the flexspline develop multi-flow gearing of high load capacity [4-5]. The harmonic gear drives with a cam wave generator [6], the action of which on a flexspline differs from the action of a disk wave generator are serially produced [7, 8]. Deformations of a flexspline by a disk wave generator are fundamentally different from the deformations caused by the action of a cam wave generator. The scale factor makes it difficult to transfer scientific developments from small samples with a cam generator to large gears with a disk generator. The designing of a harmonic gear drive, taking into account the special aspects of the stress-deformed state of a flexspline, is a topical
issue of an up-to-date mechanical drive, the solution of which is aimed at increasing the single-unit capacities and productivity of machines, reducing overall dimensions and weight. A flexspline is considered in the form of a cylindrical shell of length \( l \), with a flexible gear rim of width \( b_a \) at one end, and splines of width \( b_o \) at the opposite end [9]. The ratio of the shell thickness \( h \) to the diameter of the circle dividing the shell wall in half \( 2a \) makes 0.012. Satisfactory technical and economic results have been obtained for relatively small serially produced harmonic gear drives with torques up to 5 kN\( \cdot \)m [10]. The aim of the paper is to increase the load capacity of wave gear trains.

2. Materials and methods

The stress state of the shell is considered as a perturbation field due to the edge effect at the end face of the shell, imposed on the tangential stress field with a uniform distribution of tangential forces along the end face of the shell. The perturbation field due to the edge effect is found using the theory of V.Z. Vlasov, under the following conditions:

1. Bending \( M_\theta \) and torque moments \( M \) in the sections perpendicular to the generatrix, and the tangential force in these sections are not taken into consideration.
2. There is no deformation strain of the middle surface.
3. Poisson’s ratio is taken to be zero \( (\nu = 0) \).

The assumptions taken are equivalent to the condition: \( \frac{\partial^2 f}{\partial x^2} << \frac{\partial^2 f}{\partial y^2} \), where \( f \) – power and geometric factors; \( S_x, S_y \) – elements of the length of coordinate lines in the axial and circumferential directions. The shell is a part of flexspline which is free from external load and which transfers the torque. Perturbation field, caused by non-uniform deformation of the shell end face, determines its strength. Irregular deformation in the circumferential direction is a determinant, it significantly exceeds the longitudinal deformation. Geometric and force factors in the direction of the generating line of the shell change much more slowly than in the circumferential direction. Let us set: \( \xi^2 = \frac{h^2}{12a^2} \);

\[-\frac{h}{2} \leq \xi \leq +\frac{h}{2} \].

We establish tangential stresses \( \tau \) out of outer load \( M \)

\[
\tau = \frac{M(a + \xi)}{I_p} = \frac{M(1 + \frac{\xi}{a})}{2\pi a^2 h(1 + 3\xi^2)}
\]

Let’s point out the shell element by means of two planes, passing through the shell axis, and of two orthogonal planes of this axis. Let us transfer power factors operating in the sections to the middle surface element and apply them along the coordinate lines of local coordinates \( x, y, z \). The transfer of the stress tensor components to the coordinate lines of the middle surface \((x = \text{const}; \theta = \text{const})\) has determined power factors \( N_x, N_\theta, M_\theta, S, Q_\theta, \) referred to the unit of length of one of the coordinate lines (Fig. 1). We use the dimensionless coordinates: \( x = \frac{X}{a} \) and \( \theta \), where \( X \) – is linear dimension of axis \( x \). Then \( \frac{\partial}{\partial x} = \frac{1}{a} \frac{\partial}{\partial x} \).

Let’s set up equilibrium equation for the element of the cylindrical shell relative to the local coordinate system \((x, y, z)\)

\[
\frac{\partial N_x}{\partial x} + \frac{\partial S}{\partial \theta} = 0, \quad \frac{\partial S}{\partial x} + \frac{\partial N_\theta}{\partial \theta} + Q_\theta = 0, \quad \frac{\partial Q_\theta}{\partial \theta} - N_\theta = 0, \quad -aQ_\theta + \frac{\partial M_\theta}{\partial \theta} = 0.
\]

Power factors referred to the middle surface \((N_\theta, N_\theta, Q_\theta, M_\theta)\) are associated with its strain by physical equations.
Figure 1. Power factors applied to the shell element.

\[ N_x = Eh \cdot \varepsilon_x, \quad M_\theta = D \chi_\theta, \quad \text{(3)} \]

where \( \varepsilon_x \) is relative strain in the direction of axis \( x \); \( D = \frac{Eh^3}{12} \) – is cylindrical stiffness of the shell with the proviso that \( \nu = 0 \). Physical equations are consequence of Hooke’s law and hypothesis of linear normals, will be obtained by summing up power factors and their moments in the direction of axis \( z \).

Components of point displacement vector of the middle surface in the direction of coordinates \( x, y, z \) will be identified as \( u, v, w \).

Relative strain in the direction of axis \( x \), \( \varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u}{\partial \alpha} \). Total relative strain in the direction of axis \( y \), \( \varepsilon_\theta = \frac{1}{a} \left( \frac{\partial v}{\partial \theta} + w \right) \). Total change of curvature of coordinate line \( x = \text{const} \) in the direction of the angle \( \theta \), t. e. \( \chi_\theta = \frac{1}{a^2} \left( -\frac{\partial v}{\partial \theta} + \frac{\partial^2 w}{\partial \theta^2} \right) \). Physical equations are obtained with due consideration of the assumption \( \nu = 0 \), let’s substitute the values \( \varepsilon_x \) and \( \chi_\theta \) into formulae (3)

\[ N_X = \frac{Eh}{a} \frac{\partial u}{\partial x}, \quad M_\theta = D \chi_\theta = -\frac{D}{a^2} \left( -\frac{\partial v}{\partial \theta} + \frac{\partial^2 w}{\partial \theta^2} \right). \quad \text{(4)} \]

Let’s express components of the displacement vector through the function of stresses \( \Phi \)

\[ u = \frac{1}{a} \frac{\partial \Phi}{\partial x}, \quad v = -\frac{1}{a} \frac{\partial \Phi}{\partial \theta}, \quad w = \frac{1}{a} \frac{\partial^2 \Phi}{\partial \theta^2}. \quad \text{(5)} \]

Let’s substitute the values (5) into formulae (4)

\[ N_X = \frac{Eh}{a^2} \frac{\partial^2 \Phi}{\partial x^2}, \quad M_\theta = -\frac{D}{a^4} \left( \frac{\partial^3 \Phi}{\partial \theta^3} + \frac{\partial^5 \Phi}{\partial \theta^5} \right). \quad \text{(6)} \]

From the forth equation of equilibrium (2) we obtain shearing force \( Q_\theta \)

\[ Q_\theta = \frac{1}{a} \frac{\partial M_\theta}{\partial \theta} = -\frac{D}{a^4} \left( \frac{\partial^3 \Phi}{\partial \theta^3} + \frac{\partial^5 \Phi}{\partial \theta^5} \right). \quad \text{(7)} \]

From the third equation of equilibrium (2) we obtain force \( N_\theta \)
From the first equation of equilibrium (2) we obtain derivative
\[
\frac{\partial N_x}{\partial \theta} = -\frac{\partial N}{\partial x} = -\frac{Eh}{a^2} \frac{\partial^3 \Phi}{\partial \theta^3}
\]

From the second equation (2) we deduce the resolving equation for function \( \Phi(x, \theta) \). We differentiate the mentioned equation on parameter \( \theta \)
\[
\frac{\partial}{\partial \theta} \left( \frac{\partial S}{\partial \theta} \right) + \frac{\partial^2 N_x}{\partial \theta^2} + \frac{\partial Q}{\partial \theta} = 0.
\]

In equation (10) we replace \( Q, N_x, \frac{\partial S}{\partial \theta} \) using the expressions (7–9)
\[
\frac{\partial^4 \Phi}{\partial x^4} + 2 \left( \frac{\partial^8 \Phi}{\partial \theta^8} + 2 \frac{\partial^6 \Phi}{\partial \theta^6} + \frac{\partial^4 \Phi}{\partial \theta^4} \right) = 0
\]

\[
\frac{\partial}{\partial x^4} \left( d_k (x) \cos k \theta \right) = d_k^{IV} (x) \cos k \theta; \quad \frac{\partial}{\partial \theta^4} \left( d_k (x) \cos k \theta \right) = k^4 d_k (x) \cos k \theta;
\]
\[
\frac{\partial}{\partial \theta^6} \left( d_k (x) \cos k \theta \right) = k^6 d_k (x) \cos k \theta; \quad \frac{\partial}{\partial \theta^8} \left( d_k (x) \cos k \theta \right) = k^8 d_k (x) \cos k \theta.
\]

3. Results and discussion
The solution for function \( \phi(x, \theta) \) получил will be obtained as an expansion in series of cosines
\[
\phi(x, \theta) = \sum_{k=1}^{\infty} d_k (x) \cos k \theta.
\]

After substituting the values (12) and (13) into equation (11), we obtain
\[
\sum_{k=1}^{\infty} \left[ k^4 (k^2 - 1) \xi^2 d_k (x) + d_k^{IV} (x) \right] \cos k \theta = 0
\]

Since \( \cos k \theta \) at \( k=1; 2… \) linearly independent orthogonal in the range \([0; \pi]\) of function (14), the expressions in brackets should go to zero
\[
d_k^{IV} (x) + \xi^2 k^4 (k^2 - 1)^2 d_k (x) = 0, \quad (k = 1; 2; 3…).
\]

Characteristic equation for number \( k \) of terms of series for function \( \Phi \)
\[
\lambda^4 + \xi^2 k^4 (k^2 - 1)^2 = 0
\]

From formula (16) in view of Euler’s formulae we obtain:
\[
\lambda^2 = -\xi^2 k^4 (k^2 - 1)^2; \quad \lambda^2 = \pm \xi k^2 (k^2 - 1) i; \quad \lambda^2_{1,2} = k^2 (k^2 - 1) e^{\mp \frac{\pi}{2} i}.
\]

Roots of characteristic equation (16)
\[
\lambda_{1,2,3,4} = \pm k \sqrt{\frac{k^2 - 1}{2}} \cdot (1 \pm i).
\]

According to the roots (17) of the equation (16), elementary linearly independent solutions of differential equation (15) will be the functions:
The solution for an urgent scientific and technical problem of improving a mechanical drive, which is based on the progressive principles of reduction of rotation by means of its transformation into permanent wave transmission of the crane turning mechanism. Construction and road machines, updated procedure for calculating and designing large harmonic drives as applied to construction and road machinery. The results obtained have been implemented into the production of machinery and equipment for heavy and transport engineering.

4. Summary

The solution for an urgent scientific and technical problem of a mechanical drive, which is based on the progressive principles of reduction of rotation by means of its transformation into permanent bending deformation of a flex spline, is presented. Increase of the load capacity and efficiency of large harmonic gear drives on the basis of improving theoretical and experimental research methods, optimizing design, improving technical and operational indicators, reducing material consumption, increasing competitiveness to the level of the best world-class products, represents a solution to the urgent scientific and technical problem of improving a mechanical drive.

5. References

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Acknowledgements
This work was realized in the framework of the Program of flagship university development on the base of the Belgorod State Technological University named after V G Shukhov. The work was realized using equipment of High Technology Center at BSTU named after V G Shukhov.