Sampling and Distribution Parameter Analysis for Estimating Three-dimensional Fracture Orientation Distributions from a Circular Sampling Window

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Abstract. Precise evaluation of three-dimensional (3D) fracture orientation distributions is vital to a reliable discrete fracture network (DFN) model for rock mass. This study analyzed the accuracy and dispersion of the evaluated orientation distributions of fracture sets with different fracture geometric parameters. The geometric probability method by two-dimensional (2D) trace data is introduced and compared with the three methods of the Terzaghi family by one-dimensional (1D) orientation data. The Monte Carlo method generated many groups of 3D fracture networks to analyze the influence of uniform orientation distribution, average fracture diameter, and volume density on the evaluated orientation distributions. The results indicate that the piecewise distribution increases the computational complexity of the geometric probability method. This method can be used to effectively evaluate the orientation distributions of fracture sets with an average diameter greater than 0.25 m and volume density greater than 0.2 m⁻³.

1. Introduction
Discrete fracture network (DFN) model is a commonly used description method to generate an artificial fracture network that simulates the real three-dimensional (3D) rock mass geometric characteristics [1]. Precise evaluation of 3D fracture orientation distributions is vital to model a reliable DFN model for rock mass [2]. In contrast, the orientation distributions cannot be evaluated directly from the in situ orientation measurements [3]. Terzaghi reported that the significant sampling bias caused by the intersection angle between a fracture and the sampling tool exists in the observation data and subsequently proposed a frequency-based method to evaluate orientation distributions using 1D measurements along a scanline [4]. Many researchers studied advanced ways of reducing the sampling bias in the estimation results [5–10]. The Terzaghi method mainly considers the intersection angle between the fractures and the scanline, and the fractures are assumed to be a group of quasi-parallel infinite planes [4]. Mauldon and Mauldon further considered the size of fractures and boreholes as a significant modification of the Terzaghi method [5]. Fouché and Diebolt proposed an improved Terzaghi method, by employing the intersection angle and sample size as the parameters to
correct the sampling bias [6]. Recently, Zhang et al. proposed a geometric probabilistic method to accurately evaluate the fracture orientation distributions from a circular sampling window [10]. In this method, the Poisson Disk model described the fractures as a number of thin disks uniformly distributed in the rock mass [11, 12], and the relationships between 2D trace data and 3D fracture orientation distributions were derived. However, the fracture diameter and volume density, as critical geometric parameters of fractures, are the prerequisites for this method, and their effects on estimation results were not discussed. The application of the piecewise probability density function, for example the uniform distribution, to the proposed method was not verified.

Hence, in this study, the geometric probabilistic method by 2D trace data was introduced and compared with the three methods of the Terzaghi family by 1D orientation data. An analytical geometric method is proposed to detect the intersection between fractures and a vertical borehole. 3D DFN models and three types of sampling tools were simulated by the Monte Carlo method. The estimation accuracy and dispersion of the geometric probabilistic approach for different uniform orientation distributions are analyzed based on the Monte Carlo simulations. The average fracture diameter and volume density suitable for the geometric probabilistic method are determined with the Terzaghi family of methods.

2. Methods of estimating fracture orientation distributions by 1D or 2D measurements

2.1. A geometric probabilistic estimation method based on 2D trace measurements
Zhang et al. derived the geometric probabilistic relationships between 2D trace statistics and 3D fracture orientation distributions to evaluate the distribution parameters with low sampling bias [10]. In this method, a circular sampling window is set up to obtain the trace data. It is assumed that the fracture diameter distribution and the volume density of the fracture center are known. As illustrated in Figure 1, the fracture orientation is defined as dip direction, $\alpha$ within $(0, 2\pi)$ and dip angle, $\beta$ within $(0, \pi/2)$, in the upper hemispheric coordinate system. According to the relative position of the observed trace endpoints and the sampling window boundary, the traces can be divided into three categories: (a) transecting traces with both endpoints outside the window; (b) dissecting traces with only one endpoint in the window; and (c) contained traces with both endpoints inside the window.

![Figure 1](image_url) **Figure 1.** Schematic diagram of fracture orientation and three types of traces. (a) Fracture dip direction $\alpha$ and dip angle $\beta$ in the upper hemispheric coordinate system. The excavation face is perpendicular to the x-axis; (b) The three types of traces intersecting the sampling window.
The estimation process of this method involves three types of error functions: the moment estimation error functions \( r_1 \) to \( r_5 \) [Eq. (1) and Eq. (2)], the number estimation error functions \( r_6 \) to \( r_{10} \) [Eqs. (3-5)], and the normalization error functions \( r_6 \) and \( r_{10} \) [Eq. (6) and Eq. (7)] as:

\[
r_i = \left( -1 \right) \frac{1}{N_{TT} + N_{TD} + N_{TC}} \sum_{j=1}^{N_{TT} + N_{TD} + N_{TC}} \left( \theta_j \right)^i \int_{a-\pi/2}^{a+\pi/2} \int_{\beta-\pi/2}^{\beta+\pi/2} \frac{\left[ \arctan(\cos \alpha \tan \beta) \right] g_\alpha(\alpha)g_\beta(\beta)d\alpha d\beta}{2 \sin^2 \alpha \sin^2 \beta + g_\alpha(\alpha)g_\beta(\beta)d\alpha d\beta - 1} + \frac{1}{N_{TT} + N_{TD} + N_{TC}} \sum_{j=1}^{N_{TT} + N_{TD} + N_{TC}} \left( \theta_j - \bar{\theta} \right)^2 \int_{a-\pi/2}^{a+\pi/2} \int_{\beta-\pi/2}^{\beta+\pi/2} \frac{\left[ \arctan(\cos \alpha \tan \beta) - E[\arctan(\cos \alpha \tan \beta)] \right]^2 g_\alpha(\alpha)g_\beta(\beta)d\alpha d\beta}{2 \sin^2 \alpha \sin^2 \beta + g_\alpha(\alpha)g_\beta(\beta)d\alpha d\beta - 1} - 1
\]

\[
r_6 = \frac{\rho_v}{N_{TT}} \int_{\beta=0}^{\beta=\pi/2} \int_{y_\max}^{y_\min} \frac{\pi}{2} D^2 - \left[ \min[h(y), D] \right]^2 - \frac{D^2}{2} \sin h(y) D \frac{\min[h(y), D]}{D} \frac{g_\beta(\beta)}{D} dy dy D
\]

\[
r_7 = \frac{\rho_v}{N_{TD}} \int_{D=0}^{D=\pi/2} \int_{y_\max}^{y_\min} \frac{\pi}{2} D^2 - \left[ \min[h(y), D] \right]^2 + \frac{D^2}{2} \sin h(y) D \frac{\min[h(y), D]}{D} \frac{g_\beta(\beta)}{D} dy dy D
\]

\[
r_8 = \frac{\rho_v}{N_{TC}} \int_{D=0}^{D=\pi/2} \int_{y_\max}^{y_\min} \frac{\pi}{2} D^2 - \left[ \min[h(y), D] \right]^2 - \frac{D^2}{2} \sin h(y) D \frac{\min[h(y), D]}{D} \frac{g_\beta(\beta)}{D} dy dy D
\]

\[
r_9 = \int_{a=0}^{a=\pi/2} g_\alpha(\alpha) d\alpha - 1
\]

\[
r_{10} = \int_{\beta=0}^{\beta=\pi/2} g_\beta(\beta) d\beta - 1
\]

where \( N_{TT} \), \( N_{TD} \), and \( N_{TC} \) denote the numbers of transecting traces, dissecting traces, and contained traces in the sampling window, respectively, \( \theta \) denotes the inclined angle of a trace relative to the \( y \)-axis, \( \bar{\theta} \) denotes the mean value of all the trace samples, \( h(y) \) denotes the vertical chord length of the sampling window at coordinate \( y \), \( y_{\max} \) and \( y_{\min} \) denote the upper and lower limits of the sampling window boundary on the \( y \)-axis, \( \rho_v \) denotes the volume density of fracture center, \( g_\alpha(\alpha) \), \( g_\beta(\beta) \), and \( g_\alpha(\beta) \) denote the distributions of fracture dip direction, fracture dip angle, and fracture diameter, respectively.

The parameters of the fracture orientation distribution are calculated by minimizing the quadratic sum of three types of error functions.

\[
(\mu_\alpha, \mu_\beta, \sigma_\alpha, \sigma_\beta) = \arg \min \left[ \sum_{i=1}^{10} (r_i)^2 \right]
\]

where \( \mu_\alpha \) and \( \mu_\beta \) denote the mean values of the distributions of dip direction and dip angle, respectively; and \( \sigma_\alpha \) and \( \sigma_\beta \) denote the standard deviations of the two distributions.

2.2. Three frequency-based methods of the Terzaghi family using 1D orientation measurements

The Terzaghi method and the two key improvements [4–6] were introduced to compare the employed method. The Terzaghi family methods consider and reduce the sampling bias for the intersection angle between the fracture sets and the sampling tool, depicting that the sample size of the fractures
observed from the sampling tool decreases as the intersection angle. Figure 2 demonstrates the steps employed in the Terzaghi family of methods: (a) counting orientation poles within each grid cell, (b) weighting the frequencies, and (c) fitting the distributions by the weighted relative frequencies. The orientation samples were obtained along a scanline in the Terzaghi and Fouché methods, while only the observations intersecting the borehole surface were required in the Mauldon method. The frequencies of orientation observations collected along them are multiplied by a weighting factor to reduce the sampling bias, expressed as:

\[ P_{wi} = P_{oi} \cdot w_i \]  

where \( P_{wi} \) denotes the weighted frequency over the \( i^{\text{th}} \) grid cell; \( P_{oi} \) denotes the observed frequency over the \( i^{\text{th}} \) cell; and \( w_i \) denotes the weighting factor of \( P_{oi} \) and its specific expression in each method for a vertical scanline or borehole is summarized in Table 1.

![Figure 2. Workflow for the Terzaghi family of methods to evaluate the fracture orientation distributions by observation data along a scanline or borehole.](image)

### Table 1. Weighting factors of the three methods of the Terzaghi family for vertical linear sampling tools.

| Method name     | Weighting factor \( w_i \)                                                                 |
|-----------------|-------------------------------------------------------------------------------------------|
| Terzaghi method | \( w_i = 1 / \cos \beta_i \)                                                                  |
| Fouché method   | \( w_i = \left[ 1 + \left( n - 1 \right) / \cos \beta_i \right] / n \)                  |
| Mauldon method  | \[
\begin{align*}
W_{L_y} L_y / (\sqrt{2} \pi r_c \sqrt{1 + \cos^2 \beta_i}), \quad & \text{if} \quad \overline{d}_l < 2r_c \cos \beta_i \\
4W_{L_y} L_y / [J_0 - 4(J_1 + J_2 - J_3 - J_4)], \quad & \text{if} \quad 2r_c \cos \beta_i \leq \overline{d}_l < 2r_c \\
4W_{L_y} L_y / J_0, \quad & \text{if} \quad \overline{d}_l \geq 2r_c
\end{align*}
\] |

where
\[
\begin{align*}
J_0 &= \pi \bar{d}^2 \cos \bar{\beta} + 2\sqrt{\pi r_1 \bar{d}} \sqrt{1 + \cos^2 \bar{\beta}} + \pi r_1^2 \\
J_1 &= r_1^2 \arccos\left(\frac{\bar{d}_i^2 - 2r_2^2(1 + \cos^2 \bar{\beta})}{2r_2^2 \sin^2 \bar{\beta}}\right) \\
J_2 &= 0.5 \bar{d}_i^2 \chi_i \cos \bar{\beta}, \quad \text{where } \chi_i = 0.5 \arccos\left(\frac{\bar{d}_i^2(1 + \cos^2 \bar{\beta}) - 8r_2^2 \cos^2 \bar{\beta}}{(\bar{d}_i^2 \sin^2 \bar{\beta})}\right) \\
J_3 &= \int_0^\pi \sqrt{\sin^2 t + \cos^2 \bar{\beta} \cos^2 t} \, dt \\
J_4 &= \int_0^\pi \left(\sin^2 t + \cos^2 \bar{\beta} \cos^2 t\right)^{-3/2} \, dt
\end{align*}
\]

\*The average dip angle of fractures over the \(i\)th grid cell.
\*\(n\) is the sample size of fractures over all the cells, and \(|x|\) is the largest integer not greater than \(x\).
\*The average diameter of the fractures over the \(i\)th grid cell.

The evaluated mean value, \(\hat{\mu}\) and standard deviation, \(\hat{\sigma}\) of an orientation distribution, are calculated by weighted frequencies as:

\[
\hat{\mu} = \frac{\sum_{i=1}^{n_{\text{cell}}} (P_{wi} \cdot Y_i)}{\sum_{i=1}^{n_{\text{cell}}} P_{wi}} \quad (10)
\]

\[
\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n_{\text{cell}}} (P_{wi} \cdot Y_i^2)}{\sum_{i=1}^{n_{\text{cell}}} P_{wi}} - \hat{\mu}^2} \quad (11)
\]

where \(Y_i\) represents the dip direction or dip angle of the centroid of the \(i\)th grid cell, and \(n_{\text{cell}}\) represents the total number of grid cells in a projection net.

For detecting the intersection between a fracture and a vertical borehole, the number of solution sets \((\eta, \xi)\) to the following equation set [Eq. (12)] derived by the analytic geometry can be used to determine whether a fracture with centroid coordinate \((c_x, c_y, c_z)\), diameter \(d\), and orientation \((\alpha, \beta)\) intersects with the borehole surface with axis coordinate \((x_c, y_c)\).

\[
\begin{align*}
(\eta - \eta_o)^2 + (\xi - \xi_o)^2 &= d^2 / 4 \\
(\eta \cdot c_n / r_c)^2 + (\xi / r_c)^2 &= 1
\end{align*}
\]

where

\[
\begin{align*}
\eta_o &= (c_n \sqrt{a_n^2 + b_n^2})^{-1} \cdot \left(\frac{(c_x - x_c)^2 [(a_n^2 + c_n^2)] + (c_y - y_c)^2 [(b_n^2 + c_n^2)]}{(a_n^2 + b_n^2) - a_n^2 c_n^2} + 2a_n b_n (c_x - x_c) (c_x - c_n) (a_n^2 + b_n^2 + c_n^2)\right) \\
\xi_o &= (c_n \sqrt{a_n^2 + b_n^2})^{-1} \cdot \left(b_n (c_y - y_c) - a_n (c_y - y_c)\right) \\
a_n &= \sin \beta \sin \alpha, \quad b_n = \sin \beta \cos \alpha, \quad \text{and } c_n = \cos \beta
\end{align*}
\]

Let \(d_o\) denote the critical length of the fracture diameter, which is expressed as:

\[
d_o = 2\sqrt{(c_x - x_c)^2 (a_n^2 + c_n^2) + (c_y - y_c)^2 (b_n^2 + c_n^2) + 2a_n b_n (c_x - x_c) (c_y - y_c) / c_n} \quad (13)
\]

The magnitude of \(\eta_o, \xi_o, d_o\), and intersection detection between the fracture and the vertical borehole are discussed in the three circumstances.

(a) When \(d < d_o\) and \((\eta_o c_n / r_c)^2 + (\xi_o / r_c)^2 \geq 1\), the fracture intersects the borehole surface if Eq. (12) has at least two sets of real solutions.

(b) When \(d \geq d_o\) and \((\eta_o c_n / r_c)^2 + (\xi_o / r_c)^2 \geq 1\), the fracture intersects the borehole surface.

(c) When \((\eta_o c_n / r_c)^2 + (\xi_o / r_c)^2 < 1\), the fracture intersects the borehole surface if Eq. (12) has at least one set of real solutions.
For a vertical scanline, the fracture intersects the scanline with coordinate \((x, y'_c)\) when the fracture diameter satisfies \(d > d_c\).

3. Parameter analysis with Monte Carlo simulations

The Monte Carlo method generated a 3D fracture network model with different fracture geometric parameters to simulate the fractured rock mass based on the Poisson Disk model. The effects of uniform orientation distribution, average fracture diameter, and volume density on the estimation results of geometric fracture parameters are investigated. Figure 3 shows that the generation region (24 m \(\times\) 16 m \(\times\) 16 m) is larger than the simulation region (18 m \(\times\) 10 m \(\times\) 10 m). The window, scanline, and borehole were set up in the rock mass space to obtain the fracture data, where the sampling window has the same diameter as the borehole. The fracture diameter conformed to a lognormal distribution with a standard deviation of 0.707 m. Each simulation having the same geometric fracture parameters contains five distinct stochastic fracture networks to calculate the statistical indexes such as mean relative error (MRE) and relative standard deviation (RSD).

\[
\text{MRE} = \frac{\bar{X} - \mu}{\mu} \times 100\% \quad (15)
\]

\[
\text{RSD} = \sqrt{\frac{\sum_{i=1}^{k} (X_i - \bar{X})^2}{(5\mu^2)}} \quad (16)
\]

where \(X_i\) denotes the \(i\)th estimator; \(\bar{X}\) denotes the average of the estimators; and \(\mu\) denotes the true value.

Figure 3. Size and layout of the sampling tools and 3D fracture network model. (a) The sampling window, scanline, and borehole. \(r_c\) is the radius of the borehole. \(L_s, W_s\), and \(H_s\) are the edge lengths of the simulation region. \(L_g, W_g\), and \(H_g\) are the edge lengths of the generation region; (b) The 3D fracture network model.

3.1. Uniform orientation distributions

The fracture dip direction and dip angle are uniformly distributed with the same standard deviation of 5.73°. The mean values of \(\alpha\) and \(\beta\) are from 30° to 75° at 15° interval and 45° to 315° at 90° interval, respectively. The mean values of the fracture diameter distribution of the 16 groups of simulations and the volume density are 4 m, and 0.5 m\(^{-3}\), respectively. The MRE and RSD of the evaluated mean values \(\mu_\alpha\) and \(\mu_\beta\) were computed using the geometric probabilistic method, with the fitting surface shown in Figure 4.

The MREs of the evaluated \(\mu_\alpha\) and \(\mu_\beta\) are generally lower than 5% and 15%, except for one simulation, and the statistical error of dip angle is about three times that of dip direction. This may be
because the length of the definition domain of dip angle is smaller than that of dip direction, but their standard deviations are identical. The RSDs of the evaluated $\mu_\alpha$ and $\mu_\beta$ are all within 0.4, indicating the dispersion degree of the estimators of dip direction and dip angle are similar. Compared with the estimation results reported by Zhang et al. [10], the maximum MRE of the evaluated $\mu_\beta$ of the uniformly distributed dip angle is considerably greater than those of the normally and lognormally distributed dip angles with the same standard deviation. This is partly because the computational complexity of finding the optimal global solution of the total error functions increased by the piecewise orientation distributions. Due to the computational error, the piecewise distribution, such as uniform distribution, could be evaluated at a low accuracy via this method.

![Figure 4](image)

**Figure 4.** Variation of the statistical indexes of the mean values of the uniformly distributed fracture dip direction and dip angle evaluated by the Zhang method with the true mean values of them. (a) MRE of $\mu_\alpha$; (b) RSD of $\mu_\alpha$; (c) MRE of $\mu_\beta$; (d) RSD of $\mu_\beta$.

3.2. Fracture diameter

Five groups of stochastic fracture networks were generated using the fracture diameter's mean values, $\mu_D$ of 0.25 m, 0.5 m, 1 m, 2 m, and 4 m. The fracture dip direction and dip angle are normally distributed with the mean values of $\alpha$ and $\beta$ of 45° and 30°, but the same standard deviation of 5.73°. The volume density is 0.5 m$^{-3}$. The geometric probabilistic method and the three methods of the Terzaghi family were used to evaluate the fracture orientation distributions.

In Figure 5 (a) and (c), the maximum MREs of the evaluated $\mu_\alpha$ and $\mu_\beta$ by the four methods are all within 4% when $\mu_D$ is greater than 0.25 m, while the MREs of $\mu_\alpha$ and $\mu_\beta$ versus $\mu_D$ at 0.25 m increased to about 8% and 10%, respectively. In Figure 5 (b) and (d), the RSD by the Zhang method is slightly higher than those obtained by the Terzaghi family of methods in most of the simulations. The
error of the Zhang method is a little larger than that of the other three methods in a few simulations but within the acceptable range, where the most accurate results were computed using the Mauldon method. The Terzaghi and Fouché methods gave similar estimation accuracies. The error of the Zhang method is primarily due to the computation. The Mauldon method considered the fracture and borehole sizes compared with the other two methods of the Terzaghi family. According to the estimation results of these simulations, the four methods are more applicable to the fracture set with a diameter greater than 0.25 m.

![Figure 5](image.png)

**Figure 5.** Variation of the statistical indexes of the mean values of the distributions of fracture dip direction and dip angle evaluated by the four methods with the true mean value of the fracture diameter distribution. (a) MRE of $\mu_\alpha$; (b) RSD of $\mu_\alpha$; (c) MRE of $\mu_\beta$; (d) RSD of $\mu_\beta$.

3.3. Volume density of fracture center

Five groups of stochastic fracture networks are generated with the volume density of the fracture center $\rho_v$ of 0.05 m$^{-3}$, 0.15 m$^{-3}$, 0.25 m$^{-3}$, 0.35 m$^{-3}$, and 0.45 m$^{-3}$. The fracture orientation distributions are the same as those of the simulation in section 3.2, and the mean value of fracture diameter is 4 m.

Figure 6 shows that all the evaluated results yielded very low MREs (less than 2.5%) for dip direction and 5% for dip angle, respectively, when the $\rho_v$ is greater than 0.2 m$^{-3}$. At low volume densities ($\rho_v < 0.2$ m$^{-3}$), the MREs by the Zhang method were higher than those by the other three
methods, which are within 15% for dip direction and 10% for dip angle, respectively. The dispersion degree of the Zhang method is slightly higher than that of the other three methods. For fracture sets with very low densities, the Zhang method is more sensitive to the sample size. The four methods can be used to accurately evaluate the fracture orientation distributions when the $\rho_V$ is greater than 0.2 $m^{-3}$.

![Figure 6](image)

Figure 6. Variation of the statistical indexes of the mean values of the distributions of fracture dip direction and dip angle evaluated by the four methods with the true volume density of fracture center. (a) MRE of $\mu_\alpha$; (b) RSD of $\mu_\alpha$; (c) MRE of $\mu_\beta$; (d) RSD of $\mu_\beta$.

4. Conclusion
In this study, the accuracy and dispersion of the evaluated orientation distributions of fracture sets with different fracture geometric parameters are analyzed. The geometric probabilistic method by 2D trace data is introduced and compared with the three methods of Terzaghi family by 1D orientation data, and an analytical geometric method is proposed to detect the intersection between fractures and a vertical borehole. 3D disc fractures and three types of sampling tools are simulated by Monte Carlo method to analyse the influence of uniform orientation distribution, average fracture diameter, and volume density on the evaluated orientation distributions using the four estimation methods. The results are as following.
(1) The piecewise distribution such as uniform distribution increases the computational complexity of the Zhang method, and the evaluated orientation distributions may have low accuracy due to the calculation error.

(2) The estimation error of the Zhang method is higher than that of the other three methods in the simulations with small-sized fracture sets owing to the small sample size but still within the acceptable range. The calculation error mainly controls the accuracy of the Zhang method. The results of Mauldon method are more consistent with the true orientation distributions compared with the other two methods of Terzaghi family. The four methods are more applied to the fracture set with diameter greater than 0.25 m.

(3) The Zhang method is more sensitive to sample size for fracture sets with very low volume density. The four methods can be used to effectively evaluate the fracture orientation distributions when the volume density is greater than 0.2 m$^{-3}$.

The Zhang method is applied to fresh tectonic joint, and requires good site construction quality and less disturbance to the original rock, such as high-precision smooth blasting and mechanical excavation. In addition, the input data of the Zhang method are only trace angles and quantities, and it is more precise and efficient to extract these trace samples by Lidar or digital photography technique than to directly obtain orientation samples by a borehole. Obtaining the 2D trace data from an excavation face usually requires only 15 minutes by a Lidar and 5 minutes by a digital camera, at almost no labor cost, whereas it may be difficult to construct a large number of boreholes mainly because of the huge time and economic waste.

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References
[1] Gudmundsson A, De Guidi G and Scudero S 2013 Length-displacement scaling and fault growth Tectonophysics 608 1298-1309
[2] Fernandes AJ, Maldaner CH, Negri F, Rouleau A and Wahnfried ID 2016 Aspects of a conceptual groundwater flow model of the Serra Geral basalt aquifer (Sao Paulo, Brazil) from physical and structural geology data Hydrogeol J. 24 1199-1212
[3] Kulatilake P 1986 Bivariate Normal-distribution fitting on discontinuity orientation clusters Math. Geol. 18 181-195
[4] Terzaghi RD 1965 Sources of error in joint surveys Géotechnique 15 287-304
[5] Mauldon M and Mauldon JG 1997 Fracture sampling on a cylinder: From scanlines to boreholes and tunnels Rock Mech. Rock Eng. 30 129-144
[6] Fouche O and Diebolt J. 2004 Describing the geometry of 3D fracture systems by correcting for linear sampling bias Math. Geol. 36 33-63
[7] Huang L, Tang H, Wang L and Huang CH 2019 Minimum scanline-to-fracture angle and sample size required to produce a highly accurate estimate of the 3-D fracture orientation distribution Rock Mech. Rock Eng. 52 803-825
[8] Huang L, Jiang CH and Tang H 2020 Assessing error in the 3D discontinuity-orientation distribution estimated by the Fouche method Comput. Geotech. CG119(2020)103293
[9] Huang L, Su X and Tang H 2020 Optimal selection of estimator for obtaining an accurate three-dimensional rock fracture orientation distribution Eng. Geol. EG270(2020)105575
[10] Zhang Q, Wang X, He L and Tian L 2021 Estimation of fracture orientation distributions from a sampling window based on geometric probabilistic method Rock Mech. Rock Eng. 54 3051-
75

[11] Zhang LY and Einstein HH 2000 Estimating the intensity of rock discontinuities Int. J. Rock Mech. Min. Sci. 37 819-837

[12] Jimenez-Rodriguez R and Sitar N 2006 Inference of discontinuity trace length distributions using statistical graphical models Int. J. Rock Mech. Min. Sci. 43 877-893