Baryonia with open and hidden strange

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The relativistic six-quark equations are found in the framework of the dispersion relation technique. The strange baryonia are constructed without the mixing of the quarks and antiquarks. The relativistic six-quark amplitudes of the strange baryonia with the open and hidden strange are calculated. The poles of these amplitudes determine the masses of strange baryonia. 17 masses of baryonia are predicted.

PACS numbers: 11.55.Fv, 11.80.Jy, 12.39.Ki, 12.39.Mk.
Keywords: Strange baryonia, dispersion relation technique.

I. INTRODUCTION.

Theoretical studies of baryon-antibaryon bound states were started by Fermi and Yang in the study of pions as nucleon-antinucleon pairs [1]. A Nambu and Jona-Lasinio model [2, 3] was constructed in which the possibility of obtaining a pion with zero mass as a fermion-antifermion bound state with a doubled mass of a fermion was considered.

BES Collaboration observed a significant threshold enhancement of \( p\bar{p} \) mass spectrum in the radiative decay \( J/\psi \to \gamma p\bar{p} \) [4]. Recently BES Collaboration reported the results on \( X(1835) \) in the \( J/\psi \to \gamma \eta' \pi^+\pi^- \) channel [5]. Under the strong assumption that the \( p\bar{p} \) threshold enhancement and \( X(1835) \) are the same resonance, Zhu and Gao suggested \( X(1835) \) could be a \( J^{PC} = 0^{--} p\bar{p} \) baryonium [6]. Enhancement in the baryon-antibaryon channel near the threshold are expected on the basis of duality arguments [7–9] and by comparison with the systematic of resonance formation in meson-meson and meson-baryon channels [10].

A historical survey of bound states or resonances coupled to the nucleon-antinucleon channel is given in [11]. Gluonic states can couple to baryon-antibaryon channels of appropriate spin and parity.

Theoretical work speculated many possibilities for the enhancement such as the t-channel pion exchange, some kind of threshold kinematical effects, as new resonance below threshold or \( p\bar{p} \) bound state [12–19].

In Refs. [20–24] a method has been developed which is convenient for analysing relativistic three-hadron systems. The physics of the three-hadron system can be described by means of a pair interaction between the particles. There are three isobar channels, each of which consists of a two-particle isobar and the third particle. The presence of the isobar representation together with the condition of unitarity in the pair energies and of analyticity leads to a system of integral equations in a single variable. Their solution makes it possible to describe the interaction of the produced particles in three-hadron systems.

In Refs. [25–27] relativistic generalization of the three-body Faddeev equations was obtained in the form of dispersion relations in the pair energy of two interacting quarks. The mass spectrum of \( S \)-wave baryons including \( u, d, s \) quarks was calculated by a method based on isolating the leading singularities in the amplitude. We searched for the approximate solution of integral three-quark equations by taking into account two-particle and triangle singularities, all the weaker ones being neglected. If we considered such approximation, which corresponds to taking into account two-body and triangle singularities, and defined all the smooth functions of the subenergy variables (as compared with the singular part of the amplitude) in the middle point of the physical region of Dalitz-plot, then the problem was reduced to the one of solving a system of simple algebraic equations.

In Ref. [28] the relativistic six-quark equations are found in the framework of coupled channel formalism. The dynamical mixing between the subamplitudes of hexaquark are considered. The six-quark amplitudes of dibaryons

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are calculated. The poles of these amplitudes determine the masses of dibaryons. We calculated the contribution of six-quark subamplitudes to the hexaquark amplitudes.

In the previous paper [29] the relativistic six-quark equations including $u$, $d$ quarks and antiquarks are found. The nonstrange baryonia $BB$ are constructed without the mixing of the quarks and antiquarks. The relativistic six-quark amplitudes of the baryonia are calculated. The poles of these amplitudes determine the masses of baryonia.

In the our paper [30] the charmed baryonia are constructed without the mixing of the quarks and antiquarks. The relativistic six-quark amplitudes of the heavy baryonia with the open and hidden charm are calculated. The poles of these amplitudes determine the masses of charmed baryonia.

In the present paper the strange baryonia are constructed without the mixing of the quarks and antiquarks. The relativistic six-quark amplitudes of the strange baryonia with the open and hidden strange are calculated. The poles of these amplitudes determine the masses of strange baryonia.

In Sec. II the six-quark amplitudes of baryonia are constructed. The dynamical mixing between the subamplitudes of baryonia are considered. The relativistic six-quark equations are obtained in the form of the dispersion relations over the two-body subenergy. The approximate solutions of these equations using the method based on the extraction of leading singularities of the amplitude are obtained. Sec. III is devoted to the calculation results for the strange baryonia mass spectrum (Tables II, III). In conclusion, the status of the considered model is discussed.

II. SIX-QUARK AMPLITUDES OF THE STRANGE BARYONIA.

The relativistic generalization of the three-body Faddeev equations was obtained in the form of dispersion relations in the pair energy of two interacting quarks. The pair quarks amplitudes $qq \rightarrow qq$ are calculated in the framework of the dispersion $N/D$ method with the input four-fermion interaction with quantum numbers of the gluon [30].

The construction of the approximate solution is based on extraction of the leading singularities are close to the region $s_{ik} \approx 4m^2$. Such a classification of singularities makes it possible to search for an approximate solution of equations, taking into account a definite number of leading singularities and neglecting the weaker ones [28].

The relativistic six-quark equations in the framework of the dispersion relation technique are derived. We use only planar diagrams; the other diagrams due to the rules of $1/N_c$ expansion are neglected. The current generates a six-quark system. The correct equations for the amplitude are obtained by taking into account all possible subamplitudes. Then one should represent a six-particle amplitude as a sum of 15 subamplitudes:

$$A = \sum_{i<j}^6 A_{ij}.$$  \hspace{1cm} (1)

This defines the division of the diagrams into groups according to the certain pair interaction of particles. The total amplitude can be represented graphically as a sum of diagrams. We take into account the pairwise interaction of all quarks and antiquarks in the baryonia.

We use the results of our relativistic quark model and write down the pair quark amplitudes in the form:

$$a_n(s_{ik}) = \frac{G^2_n(s_{ik})}{1 - B_n(s_{ik})},$$  \hspace{1cm} (2)

$$B_n(s_{ik}) = \frac{1}{\pi} \int \frac{ds'_{ik}}{(m_i + m_k)^2} \rho_n(s'_{ik}) G^2_n(s'_{ik}).$$  \hspace{1cm} (3)

Here $G_n(s_{ik})$ are the diquark vertex functions (Table III). The vertex functions are determined by the contribution of the crossing channels. The vertex functions satisfy the Fierz relations.

These vertex functions are generated from $g_V$. $B_n(s_{ik})$ and $\rho_n(s_{ik})$ are the Chew-Mandelstam functions with cutoff $\Lambda$ and the phase spaces:

$$\rho_n(s_{ik}, J^{PC}) = \left( \alpha(n, J^{PC}) \frac{s_{ik}}{(m_i + m_k)^2} + \beta(n, J^{PC}) + \delta(n, J^{PC}) \frac{(m_i - m_k)^2}{s_{ik}} \right) \times \sqrt{(s_{ik} - (m_i + m_k)^2)(s_{ik} - (m_i - m_k)^2)}.$$  \hspace{1cm} (4)
The coefficients $\alpha(n, J^{PC})$, $\beta(n, J^{PC})$ and $\delta(n, J^{PC})$ are given in Table III.

Here $n = 1$ corresponds to $q\bar{q}$-pairs with $J^P = 0^-$, $n = 2$ corresponds to the $q\bar{q}$-pairs with $J^P = 1^-$, $n = 3$ determines $J^P = 0^+$, $n = 4$ determines $J^P = 1^+$ $q\bar{q}$-pairs.

In the present paper we consider the two types of the six-quark baryon-antibaryon $BB$ states: the baryon-antibaryons with one strange quark $qqQ\bar{q}\bar{q}$, where $q = u, d$, $Q = s$, and the baryon-antibaryons with two strange quarks $qqQ\bar{q}\bar{q}Q$, here $q = u, d, s$, $Q = s$.

The values of quark masses ($m_u, d = 410$ MeV, $m_s = 557$ MeV) are taken from the previous papers. We use the parameters of model similar to those in the previous papers: the gluon coupling constant $g = 0.314$ was used in the study of light and charmed baryonia, the cutoff $\Lambda = 11.0$ is usual for the all light six-quarks states. The cutoffs $\Lambda_{qs} = 6.54$ for baryonia with open strange and $\Lambda_{qs,ss} = 9.17$ for baryonia with hidden strange, $q = u, d$ are new parameters.

The results of our calculations are given in the Tables III.

As the example, we consider the baryonium $\Sigma_3 \Delta (uus \bar{d}d\bar{d})$ with the spin-parity $J^P = 1^-$. The system of equations for this baryonium is as follows:

\[
\alpha_1^{uu} = \lambda + 2\alpha_1^{0u} I_1(1u0\bar{u}u) + 6\alpha_1^{1u} I_1(1u1\bar{u}d) + 6\alpha_1^{0d} I_1(1u0\bar{u}d),
\]

\[
\alpha_1^{0u} = \lambda + \alpha_1^{1u} I_1(0u1\bar{u}u) + \alpha_1^{0s} I_1(0u0\bar{u}u) + 3\alpha_1^{1u} I_1(0u1\bar{u}d) + 3\alpha_1^{0d} I_1(0u0\bar{u}d) + 3\alpha_1^{1d} I_1(0u1\bar{u}d) + 3\alpha_1^{0d} I_1(0u0\bar{u}d),
\]

\[
\alpha_1^{1d} = \lambda + 2\alpha_1^{1d} I_1(1\bar{d}d1\bar{d}) + 4\alpha_1^{1d} I_1(1\bar{d}d1\bar{u}d) + 4\alpha_1^{0d} I_1(1\bar{d}d0\bar{u}d) + 2\alpha_1^{1d} I_1(1\bar{d}d1\bar{u}d) + 2\alpha_1^{0d} I_1(1\bar{d}d0\bar{u}d),
\]

\[
\alpha_1^{1u} = \lambda + 2\alpha_1^{1u} I_1(1u1\bar{u}u) + \alpha_1^{0u} I_1(1u0\bar{u}u) + 2\alpha_1^{1u} I_1(1u1\bar{d}d) + 3\alpha_1^{1u} I_1(1u1\bar{d}d) + 3\alpha_1^{0d} I_1(1u0\bar{u}d) + 2\alpha_1^{0d} I_1(1u0\bar{u}d) + 2\alpha_1^{0d} I_1(1u0\bar{u}d)
\]

\[
\alpha_1^{0u} = \lambda + \alpha_1^{1u} I_1(0u1\bar{u}u) + \alpha_1^{0u} I_1(0u0\bar{u}u) + 2\alpha_1^{1u} I_1(0u1\bar{d}d) + 3\alpha_1^{1u} I_1(0u1\bar{d}d) + 3\alpha_1^{0d} I_1(0u0\bar{u}d) + 2\alpha_1^{0d} I_1(0u0\bar{u}d) + 2\alpha_1^{0d} I_1(0u0\bar{u}d)
\]

\[
\alpha_1^{1d} = \lambda + 2\alpha_1^{1d} I_1(1\bar{d}d1\bar{d}) + 4\alpha_1^{1d} I_1(1\bar{d}d1\bar{u}d) + 4\alpha_1^{0d} I_1(1\bar{d}d0\bar{u}d) + 2\alpha_1^{1d} I_1(1\bar{d}d1\bar{u}d) + 2\alpha_1^{0d} I_1(1\bar{d}d0\bar{u}d),
\]

\[
\alpha_1^{1u} = \lambda + 2\alpha_1^{1u} I_1(1u1\bar{u}u) + \alpha_1^{0u} I_1(1u0\bar{u}u) + 2\alpha_1^{1u} I_1(1u1\bar{d}d) + 3\alpha_1^{1u} I_1(1u1\bar{d}d) + 3\alpha_1^{0d} I_1(1u0\bar{u}d) + 2\alpha_1^{0d} I_1(1u0\bar{u}d) + 2\alpha_1^{0d} I_1(1u0\bar{u}d)
\]

\[
\alpha_1^{0u} = \lambda + \alpha_1^{1u} I_1(0u1\bar{u}u) + \alpha_1^{0u} I_1(0u0\bar{u}u) + 2\alpha_1^{1u} I_1(0u1\bar{d}d) + 3\alpha_1^{1u} I_1(0u1\bar{d}d) + 3\alpha_1^{0d} I_1(0u0\bar{u}d) + 2\alpha_1^{0d} I_1(0u0\bar{u}d) + 2\alpha_1^{0d} I_1(0u0\bar{u}d)
\]

\[
\alpha_1^{1d} = \lambda + 2\alpha_1^{1d} I_1(1\bar{d}d1\bar{d}) + 4\alpha_1^{1d} I_1(1\bar{d}d1\bar{u}d) + 4\alpha_1^{0d} I_1(1\bar{d}d0\bar{u}d) + 2\alpha_1^{1d} I_1(1\bar{d}d1\bar{u}d) + 2\alpha_1^{0d} I_1(1\bar{d}d0\bar{u}d),
\]

\[
\alpha_1^{1u} = \lambda + 2\alpha_1^{1u} I_1(1u1\bar{u}u) + \alpha_1^{0u} I_1(1u0\bar{u}u) + 2\alpha_1^{1u} I_1(1u1\bar{d}d) + 3\alpha_1^{1u} I_1(1u1\bar{d}d) + 3\alpha_1^{0d} I_1(1u0\bar{u}d) + 2\alpha_1^{0d} I_1(1u0\bar{u}d) + 2\alpha_1^{0d} I_1(1u0\bar{u}d)
\]

\[
\alpha_1^{0u} = \lambda + \alpha_1^{1u} I_1(0u1\bar{u}u) + \alpha_1^{0u} I_1(0u0\bar{u}u) + 2\alpha_1^{1u} I_1(0u1\bar{d}d) + 3\alpha_1^{1u} I_1(0u1\bar{d}d) + 3\alpha_1^{0d} I_1(0u0\bar{u}d) + 2\alpha_1^{0d} I_1(0u0\bar{u}d) + 2\alpha_1^{0d} I_1(0u0\bar{u}d)
\]
\[ \alpha_{2}^{0\pi+1dd} = \lambda + \alpha_{1}^{1uu} I_{4}(0us1dd1uu) + \alpha_{1}^{0\pi} I_{4}(0us1dd0us) + 2 \alpha_{1}^{1dd} I_{4}(1dd0us1dd) + 2 \alpha_{1}^{1ud} I_{3}(0us1dd1ud) + 2 \alpha_{1}^{0\pi} I_{3}(0us1dd0ud) + 2 \alpha_{1}^{1sd} I_{3}(0us1dd1sd) + 2 \alpha_{1}^{0\pi} I_{3}(0us1dd0\pi) + 2 \alpha_{2}^{0\pi+1dd} I_{6}(0us1dd0us1dd). \] (13)

For simplicity, the amplitudes \( \alpha_{3} \) are neglected.

In Fig. 1 the graphical representation of equation (13) is given.

### III. Calculation Results.

In Fig. 1 the first and the second coefficients are equal to 2, that is, the permutation of particles 1 and 2; the third and the fourth coefficients are equal to 4, that is, the number 2 = 4 (the permutation of particles 1 and 2) \( \times \) 2 (the permutation of particles 3 and 4); the fifth coefficient is equal to 4, that is, the number 4 = 2 (the permutation of particles 1 and 2) \( \times \) 2 (the permutation of particles 3 and 4).

The similar approach allows us to take into account the coefficients in all the equations.

The poles of the reduced amplitudes \( \alpha_{1} \) correspond to the bound states and determine the masses of the strange baryonia.

We consider baryonia with the content \( qqQ\bar{q}q \) and the spin-parities \( J^{P} = 0^{-}, 1^{-}, 2^{-} \). The isospin projections are equal to \( \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \) (Table II).

The degeneration of baryonium masses with the different spin-parities \( J^{P} = 0^{-}, 1^{-}, 2^{-} \) was obtained. We cannot also calculate the bound states of baryonia with \( J^{P} = 3^{-} \).

The baryonium state \( \Sigma_{1}\Delta (uus\bar{d}d) \) for the spin-parities \( J^{P} = 0^{-}, 1^{-}, 2^{-} \) is calculated with the nine subamplitudes: seven \( \alpha_{1} \) (similar to \( \alpha_{1}^{uu+1dd} \)) and two \( \alpha_{1}^{1uu+1dd}, \alpha_{2}^{0\pi+1dd} \).

The baryonium state \( \Sigma_{1}\Delta (uus\bar{d}d) \) consists of 16 subamplitudes with the spin-parities \( J^{P} = 0^{-}, 1^{-}, 2^{-} \): 12 \( \alpha_{1} \) and 4 \( \alpha_{2} \): \( \alpha_{1}^{1uu+1dd}, \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd} \). For the case of the spin-parity \( J^{P} = 2^{-} \) the subamplitude \( \alpha_{2}^{0\pi+1dd} \) is absent. The states with spin-parities \( J^{P} = 0^{-}, 1^{-}, 2^{-} (uds\bar{u}u\bar{u}) \) are constructed with 13 subamplitudes: 10 \( \alpha_{1} \) and 3 \( \alpha_{2} \): \( \alpha_{2}^{1uu+1dd}, \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd} \). For the case \( J^{P} = 2^{-} \) the subamplitudes \( \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd} \) are absent.

We predict the mass of lowest open strange baryonium with the isospin projection \( I_{3} = \frac{1}{2} \) and the spin-parity \( J^{P} = 1^{-} (M = 2085 \text{MeV}) \).

We also predict the masses of strange baryonium with the isospin projection \( I_{3} = \frac{1}{2}, \frac{3}{2} \) and the spin-parity \( J^{P} = 0^{-} M = 2100 \text{MeV} \).

The baryonium state \( \Sigma \Sigma_{s} (uusd) \) consists of 16 subamplitudes with the spin-parities \( J^{P} = 0^{-}, 1^{-}, 2^{-} \): 12 \( \alpha_{1} \) and 4 \( \alpha_{2} \): \( \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd} \). The baryonium mass \( M = 2100 \text{MeV} \).

The baryonium state \( \Sigma \Sigma_{s} (uusd) \) is described with 16 subamplitudes: 12 \( \alpha_{1} \) and 4 \( \alpha_{2} \): \( \alpha_{2}^{1uu+1dd}, \alpha_{2}^{1uu+1dd}, \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd} \). The isospin projection is equal to \( I_{3} = 0 \) and the spin-parities \( J^{P} = 0^{-}, 1^{-}, 2^{-} \). We predict the mass of degenerated strange baryonia (Table II) with the mass \( M = 2200 \text{MeV} \). These states also have a small width with respect to their masses, \( \Gamma = 32 \text{MeV} \).

The model in question describes the baryonia \( \Sigma \Sigma_{s} (uusd) \) with 16 subamplitudes (equations), 24 \( \alpha_{1} \) and 6 \( \alpha_{2} \): \( \alpha_{1}^{0\pi+1dd}, \alpha_{1}^{0\pi+1dd}, \alpha_{1}^{0\pi+1dd}, \alpha_{1}^{0\pi+1dd} \). In the case \( J^{P} = 2^{-} \) we considered 15 subamplitudes: 12 \( \alpha_{1} \) and 3 \( \alpha_{2} \): \( \alpha_{2}^{1uu+1dd}, \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd} \). (Baryonium mass \( M = 2180 \text{MeV} \)).

For the spin-parity \( J^{P} = 2^{-} \) we used the 20 subamplitudes: 17 \( \alpha_{1} \) and 3 \( \alpha_{2} \): \( \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd}, \alpha_{2}^{0\pi+1dd} \). (Baryonium mass \( M = 2205 \text{MeV} \)).
We used the functions \( I_1, I_2, I_3, I_4, I_5, I_6 \):

\[
I_1(ij) = \frac{B_1(s_{13}^{(1)})}{B_1(s_{0}^{(1)})} \frac{(m_1 + m_2)^2 A_i}{(m_1 + m_2)^2} \int \frac{ds_{12}^2 G_1^2(s_{0}^{(1)})}{\pi} \frac{\rho_i(s_{12}^2)}{s_{12}^2 - s_{0}^{(1)}} \int dz_1(1) \frac{1}{2} \left[ 1 - B_1(s_{13}^{(1)}) \right],
\]

(14)

\[
I_2(ikl) = \frac{B_1(s_{13}^{(1)}) B_k(s_{24}^{(1)})}{B_1(s_{0}^{(1)}) B_j(s_{0}^{(1)})} \frac{(m_1 + m_2)^2 A_i}{(m_1 + m_2)^2} \int \frac{ds_{12}^2 G_1^2(s_{0}^{(1)})}{\pi} \frac{\rho_i(s_{12}^2)}{s_{12}^2 - s_{0}^{(1)}} \int dz_1(2) \frac{1}{2} \left[ 1 - B_1(s_{13}^{(1)}) \right] \left[ 1 - B_k(s_{24}^{(1)}) \right],
\]

(15)

\[
I_3(ikl) = \frac{B_k(s_{24}^{(1)})}{B_1(s_{0}^{(1)}) B_j(s_{0}^{(1)})} \frac{(m_1 + m_2)^2 A_i}{(m_1 + m_2)^2} \int \frac{ds_{12}^2 G_1^2(s_{0}^{(1)})}{\pi} \frac{\rho_i(s_{12}^2)}{s_{12}^2 - s_{0}^{(1)}} \int dz_1(3) \frac{1}{2} \left[ 1 - B_k(s_{24}^{(1)}) \right],
\]

(16)

\[
I_4(ijk) = I_1(ik),
\]

(17)

\[
I_5(ijkl) = I_2(ikl),
\]

(18)

\[
I_6(ijkl) = I_1(ik) \cdot I_1(jl).
\]

(19)

Here \( i, j, k, l, m \) correspond to the diquarks with the spin-parity \( J^P = 0^+, 1^+ \).

**IV. CONCLUSIONS.**

We calculated the masses of strange baryonia (Tables II and III). We predicted 17 masses and the degeneration of some states.

We obtain that these states have the widths depended of heavy quarks. We have obtained that some strange and charmed states possess small width.

We predict the mass of lowest open strange baryonium with the isospin projection \( I_3 = \frac{1}{2} \) and the spin-parity \( J^P = 1^- \ (M = 2085 \text{ MeV}) \).

We also predict the masses of strange baryonium with the isospin projection \( I_3 = \frac{1}{2}, \frac{3}{2} \) and the spin-parity \( J^P = 0^- \ M = 2100 \text{ MeV} \Gamma = 33 \text{ MeV}, J^P = 1^- \ M = 2100 \text{ MeV} \Gamma = 33 \text{ MeV}, \) and \( I_3 = \frac{1}{2} \ J^P = 0^- \ M = 2110 \text{ MeV} \Gamma = 23 \text{ MeV}, J^P = 1^- \ M = 2110 \text{ MeV} \Gamma = 23 \text{ MeV} \). These states have a small width with respect to their masses.

**Acknowledgments**

The authors would like to thank T. Barnes for useful discussions.
Fig. 1. The graphical equations of the reduced amplitude $\alpha_2^{uu\bar{d}d}$ with the projection of isospin $I_3 = \frac{1}{2}, \frac{3}{2}$ and the spin-parity $J^P = 1^- \sum_s \Delta(uus\bar{d}\bar{d})$. 
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TABLE I: $qqQq\bar{q}$, $q = u, d, Q = s$. Parameters of model: cutoff $\Lambda = 11.0$, $\Lambda_{qs} = 6.54$, gluon coupling constant $g = 0.314$. Quark masses $m_q = 410 \text{MeV}$, $m_c = 557 \text{MeV}$.

| Quark content | $I_3$ | $J$ | Baryonium | Mass (MeV) | Binding energy (MeV) |
|---------------|-------|-----|-----------|------------|----------------------|
| $uus \bar{u}u, dds \bar{d}d$ | $\frac{1}{2}$ | $\frac{3}{2}$ | $\Sigma^*_s \Delta, \Delta \Sigma^*_s$ | 2092 | 525 |
| $uus \bar{u}u, dds \bar{d}d$ | 1 | $\Sigma^*_s \Delta, \Delta \Sigma^*_s$ | 2085 | 340 |
| $uus \bar{u}u, dds \bar{d}d$ | 2 | $\Sigma^*_s \Delta, \Delta \Sigma^*_s$ | 2091 | 334 |
| $uuu \bar{u}u, uuu \bar{u}d$ | 1 | $\Sigma^*_s \Delta, \Delta \Sigma^*_s$ | 2091 | 526 |
| $uuu \bar{u}u, uuu \bar{u}d$ | 2 | $\Sigma^*_s \Delta, \Delta \Sigma^*_s$ | 2002 | 33 |
| $uuu \bar{u}u, uuu \bar{u}d$ | 1 | $\Sigma^*_s \Delta, \Delta \Sigma^*_s$ | 2004 | 317 |
| $uud \bar{u}u, uud \bar{u}d$ | 2 | $\Sigma^*_s \Delta, \Delta \Sigma^*_s$ | 2004 | 509 |
| $uud \bar{u}u, uud \bar{u}d$ | 1 | $\Sigma^*_s \Delta, \Delta \Sigma^*_s$ | 2004 | 509 |
| $uud \bar{u}u, uud \bar{u}d$ | 2 | $\Sigma^*_s \Delta, \Delta \Sigma^*_s$ | 2004 | 509 |

Quark masses $m_q = 410 \text{MeV}$, $m_c = 557 \text{MeV}$.
TABLE II: $qgQgQgQ$, $q = u,d$, $Q = s$. Parameters of model: cutoff $\Lambda = 11.0$, $\Lambda_{q,s,s} = 9.17$, gluon coupling constant $g = 0.314$. Quark masses $m_{u,d} = 410$ MeV, $m_s = 557$ MeV.

| Quark content | $I_3$ | $J$ | Baryonium | Mass (MeV) | Binding energy (MeV) |
|---------------|-------|-----|------------|------------|---------------------|
| $uus \bar{u}u$, $dds \bar{d}d$; $uus \bar{d}d$, $dds \bar{u}u$ | 0; 2  | 0   | $\Sigma_0 \Sigma_0$ | 2180        | 206                 |
|               |       | 1   | $\Sigma_0 \Sigma_0$ | 2179        | 207                 |
|               |       |     | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2179        | 399                 |
|               |       |     | $\Sigma_0^* \Sigma_0$ | 2179        | 591                 |
|               |       | 2   | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2189        | 389                 |
|               |       |     | $\Sigma_0^* \Sigma_0$ | 2189        | 581                 |
| $uus \bar{u}s$, $dds \bar{d}s$; $uds \bar{u}s$, $uds \bar{d}s$ | 1; 0  | 0   | $\Sigma_0 \Sigma_0$ | 2190        | 196                 |
|               |       |     | $\Sigma_0^* \Sigma_0$ | 2190        | 580                 |
|               |       | 1   | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2190        | 119                 |
|               |       |     | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2190        | 538                 |
|               |       |     | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2190        | 119                 |
|               |       |     | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2190        | 580                 |
|               |       | 2   | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2205        | 373                 |
|               |       |     | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2205        | 296                 |
|               |       |     | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2205        | 565                 |
| $uds \bar{u}s$ | 0; 0  | 0   | $\Sigma_0 \Sigma_0$ | 2200        | 186                 |
|               |       |     | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2200        | 109                 |
|               |       |     | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2200        | 22                 |
|               |       |     | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2200        | 570                 |
|               |       | 1   | $\Sigma_0 \Sigma_0$ | 2200        | 186                 |
|               |       |     | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2200        | 109                 |
|               |       |     | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2200        | 378                 |
|               |       |     | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2200        | 301                 |
|               |       |     | $\Sigma_0 \Sigma_0$, $\Sigma_0^* \Sigma_0$ | 2200        | 32                  |
|               |       |     | $\Sigma_0 \Sigma_0$ | 2200        | 570                 |

TABLE III: The vertex functions and coefficients of Chew-Mandelstam functions.

| $i$ | $G_i^2 (s_{kl})$ | $\alpha_i$ | $\beta_i$ | $\delta_i$ |
|-----|-----------------|------------|-----------|------------|
| 0+  | $4g^2$          | $\frac{3}{3}$ | $\frac{3}{4}$ | 0         |
| 1+  | $2g^2$          | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| 0−  | $8g^2$          | $\frac{1}{3}$ | $\frac{1}{4}$ | 0         |
| 1−  | $4g^2$          | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |