A correct renormalization procedure for the electroweak chiral Lagrangian

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We perform a systematic one-loop renormalization to the standard model with a nonlinearly realized Higgs boson field and the electroweak chiral Lagrangian up to $O(p^2)$ order. We find even in the nonlinear form, the Higgs model of the standard model is renormalizable in our procedure. We also examine $\beta$ functions of gauge couplings for the standard model and $\beta$ functions for anomalous couplings in the electroweak chiral Lagrangian up to $O(p^2)$ order, and find our results agree with all well-known results. Based on these observations, we conclude that our calculation procedure could be a correct one to systematically construct the divergences of the electroweak chiral Lagrangian when $O(p^2)$ operators are included.

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I. INTRODUCTION

How to perform the systematic renormalization of the electroweak chiral Lagrangian (EWCL) \[1, 2, 3, 4, 5, 6\] including dimensionless anomalous couplings is a long-time unsolved puzzle \[7, 8, 9, 10, 11, 12, 13, 14, 15\]. In order to obtain the correct answer, the correct renormalization procedure is very crucial. Otherwise, it seems confusing that starting with one unique theory, the EWCL, we can have different answers, as shown in literatures.

While according to statements of the effective field theory method \[16, 17, 18\], even with the nonrenormalizable theory, in principle, the radiative correction should be well-defined by using the renormalization group method. Based on the validity and great success of the effective field theory method in B physics \[19\] and low energy hadronic physics \[20, 21\], etc, we believe that there exists a unique correct answer about the radiative corrections of anomalous couplings in the EWCL.

It is well-known that if a theory is not renormalizable, in order to construct the renormalization group equations (RGE), a correct renormalization procedure which can reliably extract divergences is very crucial and important. In a gauge theory, the divergences of renormalization constants are gauge dependent, but the one-loop $\beta$ functions of the gauge couplings are gauge independent. Therefore, even in a nonrenormalizable theory (say, the EWCL), we expect that we can construct gauge independent one-loop $\beta$ functions for anomalous couplings. We are fully aware of the fact that gauge choosing is very important for gauged nonlinear sigma models, as pointed out by Einhorn \[22, 23\] and as we learn from the history of renormalization of the massive Yang-Mills theories \[5, 21\] and the standard model \[25\]. For example, the calculations in unitary gauge \[13, 14\] (a nonrenormalizable gauge \[25, 26\]) by using the dimensional regularization scheme with $O(p^2)$ operators can not produce the same divergences of those obtained in Landau gauge \[3, 4, 27, 28, 29\], and therefore have the difficulty to produce the “screening effects” of a decoupled Higgs, as dubbed by Veltman \[30\]. Therefore we believe that a correct systematic renormalization procedure should choose a renormalizable gauge and at least pass the following criteria: to demonstrate the renormalizability of the SM, to reproduce the correct divergences of $O(p^2)$, to generate the correct $\beta$ functions for the gauge couplings, etc.

This paper is devoted to investigate our renormalization procedure and examine whether it can pass these criteria. Before launching on the systematic one-loop renormalization of the EWCL including all dimensionless anomalous couplings \[31\], we regard that it is a necessary and constructive step for us to first examine two simple cases: 1) the nonlinearly realized Higgs model and 2) the EWCL up to $O(p^2)$ operators. We will explore two types of power counting rules in our calculation: Weinberg’s derivative power counting rule \[16\] and Georgi-Manohar mass dimension power counting rule (naive dimension analysis) \[32\]. We will also introduce the superficial divergence power counting rule which is important for the construction of divergences.

By using the background field method \[33, 34\], the Stueckelberg transformation \[35, 36\], path integral, heat kernel \[37, 38\] and dimensional regularization method, and $\overline{\text{MS}}$ renormalization scheme, we show how to perform the systematic renormalization at one-loop level with a nonlinearly realized Higgs boson and the EWCL in $O(p^2)$ operators without a Higgs boson. New points in this paper include: 1) Compared with calculation in the momentum space \[29\], some nonperturbative contributions can be reliably extracted out in coordinate space for the nonlinearly realized Higgs model; 2) With the help of the classic equation of motions of the vector and Higgs bosons and with a proper parameterization of the Goldstone fields, the nonlinearly realized standard model with a Higgs boson is still renormalizable in one-loop level at least; this point is not trivial, there is no reference which has shown us that the standard model with a nonlinear realized standard model is renormalizable. 3) Weinberg’s derivative power counting rule and Georgi-Manohar’s mass dimensional analysis are compatible with each other in the nonlinearly realized SM with a Higgs boson; 4) Gauge fixing term for vector bosons can be defined in either mass eigenstates or weak interaction eigenstates due to the global symmetry in ghost
sector. 5) The $\beta$ functions of the gauge couplings in our calculation procedure agree with well-known results.

6) Our calculation procedure reproduces the well-known divergences of $O(p^2)$ operators at one-loop level.

7) We also show how to construct the ghost BRST transformations in the background field method with and without a nonlinearly realized Higgs fields, which might be of special importance to understand the higher order renormalizability of the nonlinearly realized SM with a Higgs boson and the EWCL up to $O(p^2)$ order.

Based on these points, we conclude that our renormalization procedure is suitable and trustable in constructing divergences of the EWCL, even though such a theory is a nonrenormalizable theory in the perturbation expansion due to the nonlinearly realized Goldstone bosons.

As far as we know, all the techniques, methods, and concepts demonstrated in this paper have been used in the hadronic chiral Lagrangian, up to $O(p^4)$ order, and up to $O(p^6)$ order calculations, two loop renormalization in gravity in 4 dimension, three loop $\beta$ functions in Yang-Mills theory, the one-loop renormalization in the $\pi - \rho$ system, and the gauged nonlinear $SU(2)$ sigma model including dimensionless anomalous couplings.

For the sake of consistency and simplicity, we avoid the Wick rotation in the loop integral and simply work in the Euclidean coordinate space to extract divergences. We have checked our formula for extracting divergences in the coordinate space with those provided in, and found agreement.

II. THE $SU(2) \times U(1)$ STANDARD MODEL WITH NONLINEARLY REALIZED HIGGS BOSON

The classic Lagrangian of the standard $SU(2) \times U(1)$ gauge theory without including Fermions can be formulated as

$$\mathcal{L} = -H_1 - H_2 - (D\phi) \cdot (D\phi) + \mu^2 \phi^\dagger \phi - \frac{\lambda}{4} (\phi^\dagger \phi)^2, \quad (1)$$

$$H_1 = \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu}, \quad (2)$$

$$H_2 = \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \quad (3)$$

where the $W$ and $B$ are the vector bosons of $SU_L(2)$ and $U_Y(1)$ gauge groups, respectively. The $\phi$ is the Higgs field, a weak doublet. The $\mu^2$ and $\lambda$ are two variables of the Higgs potential, which determine the spontaneous breaking of symmetry. The definitions of other quantities are standard and listed as

$$W'^{\alpha}_{\mu\nu} = \partial_\mu W'^{\alpha}_{\nu} - \partial_\nu W'^{\alpha}_{\mu} + g f^{abc} W'^{b}_{\mu} W'^{c}_{\nu}, \quad (4)$$

$$B_{\mu\nu} = \partial_\mu B_{\nu} - \partial_\nu B_{\mu}, \quad (5)$$

$$D_{\mu}\phi = \partial_\mu \phi - ig W'^{\alpha}_{\mu} \tau^{\alpha} + i g' B_{\mu} T^3 \phi, \quad (6)$$

$$\phi^\dagger = (\phi_1^*, \phi_2^*), \quad (7)$$

where $T^a$ are the operators of the Lie algebra of $SU(2)$ gauge groups, and $a = 1, 2, 3$. The $\gamma$ charge of the field $\phi$ is given as $y_\phi = 1$. The $g$ and $g'$ are the couplings of the corresponding gauge interactions, respectively.

The spontaneous symmetry breaking is realized by taking the mass square $\mu^2$ to be positive. The vacuum expectation value of Higgs field is solved from the Higgs potential as $|\langle \phi \rangle| = v/\sqrt{2}$. According to the Goldstone theorem, there are three Goldstone bosons which break break $SU_L(2) \times U_Y(1)$ down to $U_{EM}(1)$ symmetry. And by eating the corresponding Goldstone bosons, the vector bosons $W$ and $Z$ obtain their masses, while the vector bosons $A$ of the unbroken $U(1)_{em}$ gauge symmetry are still massless.

The Lagrangian given in Eq. (1) with the Higgs mechanism can be reformulated in its nonlinear form by changing the variable $\phi$

$$\phi = \frac{1}{\sqrt{2}} (v + h) U, \quad U = \exp (2i \xi a T^a), \quad v = 2 \sqrt{\frac{\mu^2}{\lambda}}, \quad (8)$$

where the $h$ is the Higgs scalar, $v$ is the vacuum expectation value. The $U$ is a phase factor, and the $\xi^a$, $a = 1, 2, 3$ are the corresponding phases which can be parameterized as the Goldstone bosons, as prescribed by the Goldstone theorem. The exact form for this parameterization will be given latter.

As we know, the change of variables in Eq. (8) induces a determinant factor in the functional integral $Z$

$$Z = \int Dw^a DwD\xi^b \exp \left(S'[W, h, \xi] \right) \times \det \left\{ (1 + \frac{h}{v}) \delta(x - y) \right\}, \quad (9)$$

and correspondingly modifies the Lagrangian density to

$$\mathcal{L} = -H_1 - H_2 - \frac{(v + h)^2}{4} tr[DU^\dagger, DU] - \frac{1}{2} \partial h \cdot \partial h + \frac{\mu^2}{2} (v + h)^2 - \frac{\lambda}{16} (v + h)^4 + \delta^4(0) ln \left\{ 1 + \frac{h}{v} \right\}. \quad (11)$$

As pointed out by several references, this determinant containing quartic divergences is indispensable and crucial to cancel exactly the quartic divergences brought into by the longitudinal part of vector bosons, and is important in verifying the renormalizability of the Higgs model in the $U$-gauge and the equivalence of $U$-gauge to other gauges.
There are also quartic divergent terms induced by $\zeta$ as shown in [36], here we have omitted them.

The Lagrangian $\mathcal{L}$ given in Eq. (11) is invariant under the following local $SU(2)_L \times U(1)_Y$ chiral transformation

$$
U \rightarrow g_L U g_R^\dagger, \\
W_\mu \rightarrow g_L^\dagger W_\mu g_L + i g R g_L, \\
W_{\mu\nu} \rightarrow g_L^\dagger W_{\mu\nu} g_L, \\
B_\mu \rightarrow B_\mu + i g R g_L, \\
B_{\mu\nu} \rightarrow B_{\mu\nu}, \\
v + h \rightarrow v + h,
$$

where the gauge transformation factors $g_L$ and $g_R$ are defined as

$$
g_L = \exp \left\{ - i g a^a L T^a \right\},
$$

$$
g_R = \exp \left\{ - i g' \beta R T^3 \right\}.
$$

Here $a^a$ and $\beta_R$ are parameters of gauge group transformations.

In order to contact with the standard nonlinear realization formalism [52 53], we can define a covariant differential operator

$$
V_\mu = U^\dagger \partial_\mu U - igU^\dagger W_\mu U + ig' B_\mu,
$$

therefore the mass term $tr[DU^\dagger \cdot DU]$ can be formulated as

$$
tr[DU^\dagger \cdot DU] = - tr[V \cdot V].
$$

The minus sign is from the fact that $(D_\mu U^\dagger) = -U^\dagger (D_\mu U) U^\dagger$.

Compared with the nonlinear sigma model, we are fully aware of that there exist two types of power counting rules which can be introduced: 1) Weinberg’s derivative power counting rule [12]. In order to make all these operators as $O(p^2)$ order, we must assign

$$
[v]_p = [h]_p = [W]_p = [B]_p = [\xi]_p = 0, \\
[\partial]_p = [g]_p = [g']_p = 1, \ [\mu^2]_p = [\lambda]_p = 2.
$$

This power counting rule is self-consistent in the assignment of $[v]_p = 0$, which is determined as $v = \sqrt{\mu^2}/\lambda$. With this power counting rule, the one-loop divergence terms should be counted as $O(p^4)$, which can be served as self-consistency checking for this rule. 2) Georgi-Manohar mass dimension power counting rule [52] without Fermion is equivalent to count the mass dimension of fields and couplings, we have

$$
[g]_m = [g']_m = [\lambda]_m = 0, \ [v]_m = [h]_m = 1, \\
[W]_m = [B]_m = [\xi]_m = [\partial]_m = 1, \ [\mu^2]_m = 2. (17)
$$

Due to the fact that the theory is renormalizable when formulated with a linear form, so the one-loop divergence terms are expected to just repeat those tree-level operators while the couplings and mass parameter are expected to shift in order to absorb the UV divergences, which is exactly the meaning of the renormalizability.

These two power counting rules also serve as an important guidance for us to check our intermediate and final results.

### III. The Nonlinearly Realized SM in the Background Field Method

The Higgs sector breaks the local symmetry $SU_L(2) \times U_Y(1) \rightarrow U_{EM}(1)$ and fixes the mixing in the vector sector from weak interaction eigenstates to the mass eigenstates. The transformation from the weak eigenstates to the mass eigenstates is determined by the following relations

$$
W^\pm = \frac{1}{\sqrt{2}} (W^1 \mp iW^2), \\
Z = \sin \theta_W B - \cos \theta_W W^3, \\
A = \cos \theta_W B + \sin \theta_W W^3,
$$

where $\theta_W$ is Weinberg angle.

Correspondingly, the rotation in the vector fields induces the corresponding rotation in the gauge group parameters

$$
\alpha^\pm = \frac{1}{\sqrt{2}} (\alpha_1 \mp i\alpha_2), \\
\alpha_Z = \sin \theta_W \beta_Y - \cos \theta_W \alpha_3, \\
\alpha_A = \cos \theta_W \beta_Y + \sin \theta_W \alpha_3.
$$

We observe that these orthogonal rotations do not change neither the Lagrangian nor the functional measure nontrivially. Here, the rotation in the gauge group parameters is matched with the rotation of the vector fields, which is necessary to guarantee the well-defined ghost sector. We will address the issue of matching in the next section.

In the spirit of the background field gauge quantization [20 33 34], we can decompose the Goldstone field into the classic part $\bar{U}$ and quantum part $\xi$ as

$$
U \rightarrow \bar{U}, \ \bar{U} = \exp\left\{ \frac{i2\xi}{(v + h)} \right\}. (20)
$$

Here we parameterize the Goldstone phase in $\xi/(v + h)$. To parameterize the quantum Goldstone field in the above form is to simplify the presentation of the standard form of quadratic terms of Goldstone bosons. With this parameterization, the D’Alambert operator of Goldstone bosons enjoys the advantage that the quartic divergences are well-organized [49].
Compared with the standard parameterization of the Goldstone's degree of freedom in the hadronic chiral Lagrangian \[20\], our parameterization modifies the definition of the Goldstone

\[
\xi = \frac{v + h}{v} \tilde{\xi},
\]

where the \( \tilde{\xi} \) is the standard Goldstone field, or \( \pi \) fields in the hadronic chiral Lagrangian. This parameterization also induces a new quartic term in the Lagrangian and modifies it as \( \delta^4(0) \ln(1 + (\tilde{h}/(v + h))) \). Such a modification in the parameterization of the Goldstone fields is justified by the equivalence theorem \[54\]. However, such a parameterization of Goldstone fields becomes invalid when the system has singular solutions with \( v + h = 0 \) (vortex solutions) \[55\]. However, we only care about the perturbation and UV divergences around the regular solution \( v + h \neq 0 \), therefore such a parameterization can be acceptable.

The vector boson field are split as

\[
V^\mu \to \tilde{V}_\mu + \hat{V}_\mu, \tag{22}
\]

where \( \tilde{V}_\mu = (\tilde{W}_\mu, \tilde{B}_\mu) \) represents the classic background vector fields and \( \hat{V}_\mu = (\hat{W}_\mu, \hat{B}_\mu) \) represents the quantum vector fields.

By using the Stueckelberg transformation \[32, 30\] for the background vector fields,

\[
\tilde{W}^i \to \bar{U}^i \tilde{W} U + iU^i \partial U \bar{W}, \quad \tilde{B}^i \to \bar{B}, \quad \hat{W}^i \to \bar{U}^i \hat{W} U, \quad \hat{B}^i \to \hat{B}, \tag{23}
\]

the background Goldstone fields can be completely absorbed by redefining the background vector fields, and will not appear in the one-loop effective Lagrangian. By formulating the term \( \text{tr}[D U U^\dagger \cdot DU] \) into the standard nonlinear chiral realization form \( -\text{tr}[V \cdot V] \) \[52, 53\], we observe that in the Stueckelberg transformation, only \( SU(2) \) weak bosons are modified while the \( U(1) \) boson is unaffected.

The Stueckelberg fields is invariant under the gauge transformation of the background gauge fields, such a property guarantees that the following computation is gauge invariant under the classic gauge transformation from the beginning if we can express all effective vertices into the Stueckelberg fields. Such a fact is more apparently by examining the partition functional \( Z \), which is invariant under the classic gauge transformation. After the loop calculation, by using the reversed Stueckelberg transformation, the Lagrangian can be restored back to the form represented by its low energy degree of freedom \[30\]: the transverse vector bosons and the longitudinal Goldstone bosons.

In effect, the combination of the background Goldstone boson field and background vector boson field into the Stueckelberg fields is equivalent to take the unitary gauge for the classic fields. In this case, the mass eigenstates of \( A \) and \( Z \) are understood as the mixing between the third component of the dressed \( SU(2) \) vector fields and the undressed \( U(1) \); the mass eigenstates of \( W^\pm \) as the mixing between the first and second components of the dressed \( SU(2) \) vector fields. Since only the quantum fields need to be quantized, we can take different gauges for the classic fields and the quantum fields \[52, 53\], which is one of the advantages of the background field method.

Similarly to the vector bosons, the Higgs boson is split as

\[
h = \bar{h} + \hat{h}, \tag{24}
\]

where \( \bar{h} \) and \( \hat{h} \) are classic and quantum Higgs boson, respectively.

\section*{IV. The EOM, Quantization and the Gauge Fixing Terms}

The equation of motion of the background classic vector fields (Stueckelberg field, below we omit the * in both the classic and quantum fields) is determined by the Euler-Lagrange variation method, which can be simply formulated as

\[
D_\mu \bar{W}^{\mu \nu} = -\sigma_{0, \nu} V V^\nu, \tag{25}
\]

with \( \bar{W}^{\mu \nu} = \{ A^{\mu \nu} + i e F_{Z}^{\mu \nu}, Z^{\mu \nu} - i g^2 F_{Z}^{\mu \nu}, W^{+\mu \nu} - i g F^{+ \mu \nu}, W^{-\mu \nu} - i g F^{- \mu \nu} \} \). Here and below, we omit the overline on the classic field for the simplicity of representation. The \( V^T \) means the \( (A, Z, W^+, W^-) \). The relevant definition are given as

\[
A_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \\
Z_{\mu \nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu, \\
W^{\pm}_{\mu \nu} = d_\mu W^\pm_{\nu} - d_\nu W^\pm_{\mu}, \\
F^Z_{\mu \nu} = W^+_{\mu \nu} - W^-_{\mu \nu}, \\
F^W_{\mu \nu} = \pm(Z W^+_{\mu \nu} - W^\mp_{\mu \nu}), \\
d_\mu W^\pm_{\nu} = \partial_\mu W^\pm_{\nu} - i(\pm 1) e A_\mu W^\pm_{\nu}. \tag{26}
\]

The covariant operator \( D \) will be given when we define the D’Alambert operator of vector boson fields.

The EOM of vector bosons induces the following relations

\[
\partial \ln(v + h) \cdot Z = -\frac{1}{2} \partial \cdot Z, \tag{27}
\]

\[
\partial \ln(v + h) \cdot W^+ = -\frac{1}{2} d \cdot W^+ + i \frac{g^2}{2} Z \cdot W^+, \tag{28}
\]

\[
\partial \ln(v + h) \cdot W^- = -\frac{1}{2} d \cdot W^- - i \frac{g^2}{2} Z \cdot W^- , \tag{29}
\]

which can also be obtained by extracting the linear terms of Goldstone fields. According to the our calculation, it seems that it is not necessary to impose the condition \( \partial A = 0 \).
The equation of motion of the background Higgs field is given as

$$\partial^2 h = (v + h)\left[ \frac{G^2}{4} Z \cdot Z + \frac{g^2}{2} W^+ \cdot W^- \right] - \mu^2 + \frac{\lambda}{4} (v + h)^2].$$

(30)

These equations of motion satisfy both power counting rules. In the Weinberg’s derivative power counting rule, both sides of the EOM of photon and Higgs fields are counted as 2. In the Georgi-Manohar mass dimension power counting rule, both sides of the EOM of vector and Higgs fields are counted as 3 (The EOM of photon is different, only left hand side is counted as 3, while the right hand side is counted as 0).

The quantization of the dynamics system is selected to be performed around the classic gauge transformation given in Eq. (29). The quantum degree of freedoms include quantum vector bosons, quantum Goldstone bosons, and quantum Higgs bosons, and the partition functional without gauge fixing terms can be formulated as

$$Z(V, h) = N \int \mathcal{D} \alpha \mathcal{D} \bar{\alpha} \mathcal{D} h \mathcal{D} \bar{h} \mathcal{D} \xi \cdot \cdot \cdot ,$$

(31)

where the vector and Goldstone bosons are formulated in mass eigenstates. The vector quantum fields, $\bar{V}$, represent $\bar{A}$, $\bar{Z}$, $\bar{W}^+$, and $\bar{W}^-$. In the Goldstone bosons, unlike in the gauged nonlinear $SU(2)\sigma$ model, there is no $SU_c(2)$ global symmetry any more. And the Goldstone bosons must be parameterized as $\xi Z$, $\xi_+$, and $\xi_-\cdot$

In order to separate the infinite volume of gauge group parameter, we use the conventional background gauge fixing \[34\]. Follow the standard quantization procedure, the infinite gauge parameter volume is separated by imposing gauge fixing to the quantum fluctuations, as given as

$$Z(V, h) = \int \mathcal{D} \alpha Z_G[V, h],$$

(32)

where $Z_G$ is the fixed partition functional, which can be expressed as

$$Z_G[V, h] = N' \int \mathcal{D} \bar{V} \mathcal{D} B \mathcal{D} h \mathcal{D} \xi \mathcal{D} \bar{h} \mathcal{D} u \cdot \cdot \cdot ,$$

(33)

Quantization procedure introduces two types of real ghost fields, $v$ and $u$, into the quantum degree of freedoms. Here we follow the convention of references \[56\], \[57\], \[58\] to label these two types of ghost differently.

The Lagrangian of $Z_G$ is given as

$$\mathcal{L} = \mathcal{L}(V, h; \bar{V}, \xi, \bar{h}) + \mathcal{L}_{NL},$$

(34)

where the first term contains the classic vector bosons (in U-gauge) and the classic Higgs bosons, the quantum fields. And the second term contains gauge fixing terms. In the Nakanishi-Lautrup auxiliary field form, the gauge fixing and ghost terms \[51\] can be formulated as

$$\mathcal{L}_{NL} = i F^a B^a - \frac{1}{2} B^a \Gamma^{abc} B^b$$

$$-i v^a M^{ab} \delta F^b_{\alpha \beta} \frac{1}{\delta \alpha^\beta} u^\delta \cdot \cdot \cdot .$$

(35)

Here we have deliberately used the index $\delta$ to label $u$ type ghost and the gauge group parameter index to distinguish the ghost and anti-ghost. Due to the structure of the semi-simple group, the matrices $\Gamma^{abc}$, $M^{ab}$, and $M^{\bar{a} \bar{b}}$, are introduced to accommodate the rotation of $F^a$, $v^a$, and $u^\delta$. The matrices $M^{ab}$ and $M^{\bar{a} \bar{b}}$ are related with the ambiguity in exponentializing the Faddeev-Popov determinant to the Lagrangian. Such an ambiguity could occur if the det $M^{ab}$ and det $M^{\bar{a} \bar{b}}$ do not vanish, the following relation can be justified

$$\det(\delta F^a_{\alpha \beta}) = \frac{\det(M^{ab} \delta F^a_{\alpha \beta})}{\det(M^{ab}) \det(M^{\bar{a} \bar{b}})}.$$  

(36)

If the $F^a$ (here the index $a$ is related with quantum field index) rotate to $a'$, in order to have well-defined canonical ghost fields, the $\alpha^a$ must rotate accordingly. For example, for the gauge fixing given in Eq. \[36\] where the field index have changed from $a = (Y, 3, 1, 2)$ to $a' = (A, Z, W^+, W^-)$, if we do not defined $\alpha^A$, $\alpha^Z$, $\alpha^W$ as given in Eq. \[36\], while keep using the $\alpha_Y$ and $\alpha_3$, we will find that the ghost kinetic term have the following form

$$-u^\delta \partial^2 (\sin \theta W v^\delta + \cos \theta W u^\delta)$$

$$- u^2 \partial^2 (\sin \theta W v^\delta - \cos \theta W u^\delta) + \cdot \cdot \cdot .$$

(37)

In the Hamiltonian form with the canonical ghost fields and its conjugate fields, and the canonical commutation relation and the propagators can not be well-defined, therefore the quantization procedure has problems. So we emphasize here, that the matching between the rotations of gauge fields and gauge group parameters are important for the ghost term.

To represent ghost fields and anti-ghost differently is for the correct hermiticity assignment as pointed out by \[57\], \[58\], and is well-known in the conformal field theory \[56\] where $b$ and $c$ type real ghosts are introduced. Here we would like to emphasize that the rotations of ghost and anti-ghost are independent. The rotation of $v$ type ghosts can be related with the change of $F^a$ fields, and the rotation of $u$ type ghosts are related with the rotation of the gauge parameter $\alpha$. After diagonalizing of Nakanishi-Lautrup auxiliary fields $B^a$, the eigenvalues of the matrix $\Gamma$ are required to be real and positive. The Landau gauge and the unitary gauge, which correspond to the case that some of the eigenvalues of the matrix $\Gamma$ is zero and infinite, respectively. These two extreme cases should be treated carefully in the functional integral, as shown in \[61\] and \[62\], respectively.
The quantization of Yang-Mills fields can be performed in a more strict and sophisticated way by incorporating BRST global symmetry in the formalism, as shown in \[ 62 \]. However, for our purpose, here we provide a simple and direct understanding on the quantization, the gauge fixing, and gauge parameter independence for the S-matrix. The quantization and gauge fixing can be understood as a procedure to couple the vector dynamic system to an auxiliary system. Even after the covariant Lorentz gauge fixing, in the SU(N) theories for instance, the auxiliary system can still enjoy an isospin-like global invariance which is related with the ghost number content of parameters of these matrices.

The real ghost fields enjoy a scaling transformation invariance which is related with the ghost number conservation law.

\[
\begin{align*}
v & \rightarrow e^{\lambda}v \\
u & \rightarrow e^{-\lambda}u,
\end{align*}
\]

\(v\) is identified as anti-ghost field, while \(u\) is identified as ghost field. This ghost number conservation scaling transformation is consistent with the hermiticity assignment \[ 57, 58 \]. This scaling group is the subgroup of general matrix \(M\) and \(M^\dagger\), both of which are defined with each component is a complex number.

The gauge fixing term for the quantum fields are chosen in the mass eigenstates, and are given as below

\[
\begin{align*}
\mathcal{L}_{GF,A} & = -\frac{1}{2} (\partial \cdot \hat{A}) \\
& + i e (\hat{W}^- \cdot W^+ - \hat{W}^+ \cdot W^-))^2, \\
\mathcal{L}_{GF,Z} & = -\frac{1}{2} (\partial \cdot \hat{Z} - 1/2 G(v + h) \xi Z \\
& + i g^2 G (\hat{W}^- \cdot W^+ - \hat{W}^+ \cdot W^-))^2, \\
\mathcal{L}_{GF,W} & = -(d \cdot \hat{W}^+ + 1/2 g (v + h) \xi W^+) + i g^2 G Z \cdot \hat{W}^+ \\
& - i g^2 G W^+ \cdot \hat{Z} + i e W^- \cdot \hat{A}) \\
& + (d \cdot \hat{W}^- + 1/2 g (v + h) \xi W^- - i g^2 G Z \cdot \hat{W}^- \\
& + i g^2 G W^- \cdot \hat{Z} - i e W^+ \cdot \hat{A}).
\end{align*}
\]

To determine the gauge parameters, we use the following guidances: 1) The D’Alambert operator of quantum vector field must have the form as given in Eq. 14. When gauge potential is neglected, this correspond to the Feynman-‘t Hooft gauge in the momentum space. The terms with background fields in the gauge fixing term have the function to eliminate extra Lorentz structures in the vector sector, and guarantee a well-defined exact vector propagator in coordinate space. 2) Although we can introduce nonlinear terms of \(\xi\), up to one-loop level, we only consider the linear term in the gauge fixing. This linear term induces mass term of the corresponding ghost fields. The nonlinear terms might be useful when we consider beyond one-loop calculation, and these nonlinear terms also induce nonlinear interactions between Goldstone and ghost. Since we only consider one-loop renormalization, we confine to the linear term. 3) There is no mixing between massive vector fields and their corresponding Goldstone bosons, and there is no mixing between massless photon fields and chargeless Goldstone \(\xi_Z\); which is a typical feature of Feynman-‘t Hooft gauge. 4) The classic \(U_{EM}(1)\) symmetry must be fulfilled explicitly.

These gauge fixing terms will fulfill two important functions when we consider to extract divergences: 1) to guarantee that the exact vector propagator have well-controlled divergence structure. 2) to eliminate the kinetic mixing term between the vector and Goldstone fields and to make the expansion of the vector and Goldstone mixing term \(Tr \ln(1 - \bar{X}_I \hat{\Box}^{-1} \bar{X}_I \hat{\Box}^{-1})\) to stop in producing UV divergences to a specific term. We avoid in using the Landau gauge as being used in \[ 3, 4, 27, 28, 29 \], which has the advantage to identify the massless poles to the Goldstone particles in the asymptotic region and to eliminate the ghost-Goldstone interactions. The reason is that it makes the expansion of vector-Goldstone mixing term complicated.

By using the arbitrariness in choosing \(\Gamma\) and two \(M\) matrices, we can rotate these gauge fixing terms back to weak interaction form (the background vector fields and gauge group parameters also rotate accordingly), which have the following form

\[
\begin{align*}
F_Y & = \partial \cdot \hat{B} - \frac{g'}{2}(v + h) \xi Z, \\
F_W & = (\delta^{ij} \partial + g f^{ijk} W^j) \cdot \hat{W}^k + \frac{g}{2}(v + h) \xi^i.
\end{align*}
\]

Such a fact demonstrates we can put gauge fixing terms either in mass eigenstates or in weak interaction eigenstates for vector bosons, and they are equivalent with each other and do not affect the S-matrix, as indicated by \(\Gamma\) and two \(M\) matrices in the \(\mathcal{L}_{NL}\) given in Eq. \[ 35 \]. Gauge fixing terms in the weak interaction eigenstate basis have the advantage to explicitly show that our gauge fixing terms are covariant background field gauge fixings.

V. THE QUADRATIC FORMS OF THE ONE-LOOP LAGRANGIAN

Since we only care about the one-loop renormalization, we can only concentrate on quadratic terms (bilinear terms) of quantum fields and neglect those trilinear terms, quartic terms, and higher order terms in the Lagrangian. Those terms neglected are necessary when we consider higher order renormalization.

In order to make the one-loop renormalization easier to handle, we cast the quadratic quantum fluctuation terms
of the one-loop Lagrangian into a standard form, which reads
\[
\mathcal{L}_{\text{quad}} = \left\{ \frac{1}{2} \mathcal{D}_{\mu} a^{\mu,ab} \mathcal{D}^\mu b + \frac{1}{2} s \mathcal{D}_{\mu} \xi \xi^j \right\} \\
+ \frac{1}{2} \mathcal{D}_{\mu} \xi \xi^j + \frac{1}{2} \mathcal{D}_{\mu} \xi \xi^j \mathcal{D}^\mu \xi \xi^j.
\]

Here we would like to emphasize that our Hermitian transformation is equal to the charge conjugate and does not require the transpose transformation. Here we would like to mark that such a convention is widely taken in conformal and string theories. Considering the remnant \( U_{EM}(1) \) symmetry, here we have modified this hermiticity assignment simply as charge conjugate. Historically, the ghost fields are introduced as complex fields. For the perturbation calculation, for all existed examples as far as we know, such an assignment makes no difference. However, we find when \( O(p^4) \) operators are included, such a hermiticity assignment indeed makes difference [31]. Anyhow, for the current two cases, such a hermiticity assignment makes no difference.

The matrix \( \sigma_{2,VV} \) contains Lorentz tensor structure and corresponds to the dipole magnetic term of vector bosons due to the fact that vector bosons are spin-one particles. It is given as

\[
\sigma_{2,\mu,ab}^{\mu,\nu,\alpha,\beta} = \begin{pmatrix}
0 & 0 & \sigma_{2,AW}^{\mu} & \sigma_{2,W}^{\mu} \\
0 & 0 & \sigma_{2,ZW}^{\mu} & 0 \\
\sigma_{2,W}^{\mu} & \sigma_{2,W}^{\mu} & 0 & 0 \\
\sigma_{2,ZW}^{\mu} & \sigma_{2,ZW}^{\mu} & 0 & 0
\end{pmatrix},
\]

and each of its components reads as

\[
\sigma_{2,AW}^{\mu} = -\sigma_{2,W}^{\mu} = 2ie\tilde{W}^{-,\mu}, \\
\sigma_{2,ZW}^{\mu} = -\sigma_{2,ZW}^{\mu} = -2ie\tilde{W}^{+,\mu}, \\
\sigma_{2,W}^{\mu} = -\sigma_{2,W}^{\mu} = -2ie\tilde{W}^{-,\mu}, \\
\sigma_{2,ZW}^{\mu} = -\sigma_{2,ZW}^{\mu} = 2ie\tilde{W}^{+,\mu}.
\]

Here we would like to mark that \( \sigma_{V} \) is Hermitian. Under the Hermitian transformation, the Lorentz indices should be exchanged as \( \mu \rightarrow \nu (\nu \rightarrow \mu) \), and this makes the invariance of \( \sigma_{V} \) explicitly. The \( \sigma_{V} \) does not depend on the background Higgs field \( h \).

Goldstone bosons are spin-zero particles, but the matrix \( \sigma_{2,\xi,\xi} \) does not vanish and has scalar structure in the similar to the case without spontaneous symmetry breaking.
Lorentz index. It is given as

$$\sigma^{ij}_{2,\xi\xi} = \begin{pmatrix} \sigma_{2,\xi\xi\xi\xi} & \sigma_{2,\xi\xi\xi+} & \sigma_{2,\xi\xi\xi-} \\ \sigma_{2,\xi\xi\xi+} & \sigma_{2,\xi\xi+\xi} & \sigma_{2,\xi\xi+\xi-} \\ \sigma_{2,\xi\xi\xi-} & \sigma_{2,\xi\xi+\xi-} & \sigma_{2,\xi\xi-\xi-} \end{pmatrix},$$

and each of its components reads as

$$\sigma_{2,\xi\xi\xi\xi} = \frac{g^2}{2} W^+ \cdot W^- - \frac{\partial^2 h}{v + h},$$

$$\sigma_{2,\xi\xi\xi+} = \frac{g^2}{4} W^- \cdot W^-,$$

$$\sigma_{2,\xi\xi\xi-} = -\frac{g^2}{4} W^+ \cdot W^+,$$

$$\sigma_{2,\xi\xi\xi+} = \sigma_{2,\xi\xi\xi-} = \frac{gG}{4} W^- \cdot Z,$$

$$\sigma_{2,\xi\xi\xi-} = \sigma_{2,\xi\xi+\xi} = \frac{gG}{4} W^+ \cdot Z,$$

$$\sigma_{2,\xi\xi+\xi-} = \frac{g^2}{4} W^+ \cdot W^- + \frac{G^2}{4} Z \cdot Z - \frac{\partial^2 h}{v + h}. \quad (52)$$

The $\sigma_{2,\xi\xi}$, under the Hermitian transformation, is explicitly invariant. The term $\frac{\partial^2 h}{v + h}$ in the $\sigma_{2,\xi\xi\xi\xi}$ and $\sigma_{2,\xi\xi\xi+}$ is not trivial, and it represents a nonperturbative expansion on the h field. By using the momentum expansion and simply extracting Feynman rules, we can not obtain such a compact form.

In order to explicitly trace the function of the EOM of the Higgs field to eliminating extra divergence structures, we have restricted from substituting $\partial^2 h$ term in $\sigma_{2,\xi\xi}$ by the equation Eq. 39.

The ghost fields are spin-zero particles, their propagators have no nontrivial $\sigma$ structure, like ordinary scalar particles. However, we observe that the ghost fields are different from the Goldstone fields in this aspect.

The $\sigma_{hh}$ is determined as

$$\sigma_{hh} = \frac{1}{4} (G^2 Z \cdot Z + 2g^2 W^+ \cdot W^-)$$

$$= \frac{\lambda}{4} v^2 - \frac{3}{4} \lambda (v + h)^2. \quad (53)$$

And the Hermitian transformation invariance is apparently. We observe that the $\sigma_{hh}$ depends on the background vector fields.

The mass matrices have the form $\sigma^b_{0,VV} = \text{dia}\{0, G^2 (v + h)^2 / 4, g^2 (v + h)^2 / 2, g^2 (v + h)^2 / 4\}$ and $\sigma^{ij}_{0,\xi\xi} = \text{dia}\{G^2 (v + h)^2 / 4, g^2 (v + h)^2 / 4, g^2 (v + h)^2 / 4\}$. The background Higgs field $h$ contributes to the mass matrices. Due to the gauge fixing term we have chosen, we observe that the vector boson, its corresponding Goldstone and ghost fields, have the same mass term, similar to the Feynman–Hooft gauge. The $m^2_h$ is the mass of Higgs boson, which is determined as $m^2_h = \lambda v^2 / 2$.

The mixing term between the vector and Goldstone bosons is a $4 \times 3$ matrix, and is determined as

\[ -X^\mu_{-} = \begin{pmatrix} 0 \\ -G \partial^\mu h \\ -iG \frac{g^2}{G} (g - g^2) (v + h) W^- \cdot W^- \\ -g \partial^\mu h - \frac{1}{2} i g G (v + h) Z^\mu \\ 0 \\ -g \partial^\mu h + \frac{1}{2} i g G (v + h) Z^\mu \end{pmatrix}, \]

while the matrix $X^\mu_{-\xi}$ is a $3 \times 4$ matrix and is just the rearrangement of the $X^\mu_{-\xi}$ due to our Hermitian transformation (charge conjugate and transpose transformation), and here we do not rewrite it explicitly. The mixing term between vector and Higgs bosons is determined as

\[ -X^\mu_{-h} = \begin{pmatrix} 0, -\frac{1}{2} G^2 (v + h) Z^\mu, \\ -\frac{1}{2} g^2 (v + h) W^+ \cdot W^- - \frac{1}{2} g^2 (v + h) W^- \cdot W^- \end{pmatrix}, \]

and Goldstone bosons are determined as

\[ X^\alpha_{h,0} = \begin{pmatrix} (-G Z^0, g W^+ v - G W^- v), \end{pmatrix}, \]

\[ X^\alpha_{h,0} = \begin{pmatrix} - \frac{G}{2} \partial \cdot Z, \frac{g}{2} d \cdot W^- - \frac{1}{2} \left( g^2 - G \right) W^- \cdot Z, \\ \frac{g}{2} d \cdot W^+ + \frac{1}{2} \left( g^2 - G \right) W^+ \cdot Z \end{pmatrix}. \]

The terms $X^\alpha_{h,0}$ and $X^\alpha_{\xi,0}$ are also the rearrangement of $X^\alpha_{h,0}$ and $X^\alpha_{\xi,0}$, and are omitted here. To extract the mixing terms between Higgs and Goldstone bosons, we have used the EOM of vector fields given in Eqs. 27–29.

It is an interesting fact that in both Weinberg’s derivative power counting rule and Georgi-Manohar’s power counting rule, those propagators and mixing matrices given in Eq. 52 are counted as 2;
There is one remarkable feature about the ghost term: in the Feynman’'t Hooft gauge we take, the $U_{EM}(1)$ ghost does not decouple from interaction with other ghost fields. This is different from the common knowledge with pure Abelian gauge theories where ghosts are expected to decouple. The underlying reason is that the $U_{EM}(1)$ ghost field is a mixture of the $U_Y(1)$ and $SU_L(2)$ ghosts and it couples to charged ghost fields while compatible with the principle of gauge invariance. Such a nondecoupled behavior also help to cure the ambiguity in the gauge fixing when a theory with a semi-simple group symmetry has $U(1)$ groups.  

VI. EVALUATING THE TRACES AND LOGARITHMS

By diagonalizing the quantum vector fields, Goldstone fields, and Higgs fields successively, we can integrate out all quantum fluctuations by using the Gaussian integral. And the $L_{1-loop}$ can be concisely expressed as the traces and logarithms

$$Tr\Box_{\nu\nu} - \frac{1}{2} \left[ Tr \ln \Box_{VV} + Tr \ln \Box_{\xi\xi} + Tr \ln \Box_{hh} \right], \quad (57)$$

where

$$\Box_{\xi\xi}^{ij} = \Box_{\xi\xi}^{ij} - \frac{1}{2} \Box_{\xi\xi}^{-1} \Box_{\xi\xi} \Box_{\xi\xi}^{ij} \Box_{\xi\xi}^{-1} \Box_{\xi\xi}, \quad (58)$$

Here the $Tr$ means to sum over space-time points, Lorentz indices, and group indices, respectively. Thanks to the property of the $Tr$, the above equation clearly shows that divergences are independent of the sequence of integrating-out quantum fields. Even when finite terms are considered, this is also true. This is a pleasant result which confirms that the sequence to integrate out quantum fields has no physical meaning.

Since we only care about divergence structure, and we omit the finite terms by $\cdots$. We would like to remark that each term in Eq. (57) corresponds to a series of Feynman diagrams. In principle, we can make a correspondence between Feynman diagrams and terms in Eq. (57), though it is cumbersome.

In both power counting rule, these terms can be counted as 0. So we must extract divergence structure from these terms, which should be counted as $p^4$ terms in the Weinberg’s power counting rule and dimension 4

$$\Box_{hh}^{'} = \Box_{hh} - \frac{1}{2} \Box_{VV}^{-1} \Box_{\xi\xi} X_{\xi} X_{\xi}, \quad (59)$$

$$\Box_{hh}^{''} = \Box_{hh} - \frac{1}{2} \Box_{VV}^{-1} \Box_{\xi\xi} X_{\xi} X_{\xi}, \quad (60)$$

Terms $Tr \ln \Box_{\xi\xi}$ and $Tr \ln \Box_{hh}$ are dependent on the sequence as how to integrate quantum fields, and are determined by the following relations

$$Tr \ln \Box_{\xi\xi} = Tr \ln \Box_{\xi\xi} + Tr \ln (1 - \frac{1}{2} \Box_{\xi\xi}^{-1} \Box_{\xi\xi} X_{\xi} X_{\xi}) \Box_{\xi\xi}^{-1} \Box_{\xi\xi}, \quad (63)$$

$$Tr \ln \Box_{hh} = Tr \ln \Box_{hh} + Tr \ln (1 - \frac{1}{2} \Box_{\xi\xi}^{-1} \Box_{\xi\xi} X_{\xi} X_{\xi}) \Box_{\xi\xi}^{-1} \Box_{\xi\xi}). \quad (64)$$

In order to extract all divergences at one-loop level, we need to expand the Eq. (57) to exact propagator terms and interaction terms. Thanks to the gauge fixing terms given in Eqs. (59–61), we find that only the following terms can contribute the divergences

$$\int_x L_{1-loop} = Tr\Box_{\nu\nu} - \frac{1}{2} \left[ Tr \ln \Box_{VV} + Tr \ln \Box_{\xi\xi} + Tr \ln \Box_{hh} - Tr(\Box_{\xi\xi}^{-1} \Box_{\xi\xi} X_{\xi} X_{\xi}) - Tr(\Box_{\xi\xi}^{-1} \Box_{\xi\xi} X_{\xi} X_{\xi}) + \cdots \right]. \quad (65)$$

in the Georgi-Manohar power counting rule.

VII. DIVERGENCES AND THE RENORMALIZABILITY AT ONE-LOOP LEVEL

To extract divergences, we need to introduce the third counting rule to count the superficial divergence degree of freedom, so that we can know which terms in the traces and logarithms contain divergences. When we calculate with the Schwinger proper time method in coordinate space, we introduce the following superficial divergence power counting rule

$$[\partial_{\mu}, d] = -1, \quad [x_{\mu}, d] = 1, \quad [\tau, d] = 2, \quad (66)$$

where the $\partial_{\mu}$ means the differential partial operator which might appear in the vertices, the $x_{\mu}$ means the vector in the coordinate space, the $\tau$ means the Schwinger
proper time. The total divergence degree of a term depends on the number of integrals over the coordinate space, $n_x$, the number of proper time integrals, $n_\tau$, the power index of the differential partial operators, $v_d$, the power index of coordinate vectors, $v_x$, and the power index of proper times, $v_\tau$. For example, the number of integrals over the coordinate space $\int_x \int_y$ is counted as 2, the number of integrals over the proper time integrals $\int_\tau$ is counted as 1, the power index of $\partial_\tau \partial_\eta$ is counted as 2, the power index of $x_1^\mu x_2^\nu$ is counted as 2, while the power index of $\tau_1 \tau_2^\nu$ is counted as 3. At one-loop, after expressing each term given in Eq. (65 with exact propagators in the coordinate space, we observe the following superficial power counting rule

$$\Omega = 4 - (n_x - 1)d - (2 - d/2) n_\tau + v_d - 2 v_\tau - v_x.$$  \hspace{1cm} (67) Here $d$ means the dimension of the space-time. There is only one integral in the coordinate space is taken as 4, while others integrated out are taken as $d$. Only positive $\Omega$ means divergence.

With this superficial divergence power counting rule, the traces and logarithms in Eq. (60) are badly divergent. However, the terms with $\Omega = 4$ contribute to the vacuum and can be removed by a global normalization factor in path integral, and the terms with $\Omega = 2$ can be reduced to logarithmic divergences in the dimensional regularization method. Therefore there are only logarithmic divergences which are physically meaningful.

Then by using the heat kernel method directly, we obtain the following divergence structures from the contributions of $Tr \ln \Box$ in the Eq. (65)

$$\frac{1}{2} Tr \ln \Box_{VV} = - \frac{20 g^2}{3} H_1 - \frac{(2g^4 + G^4)}{16} (v + h)^4,$$  \hspace{1cm} (68)

$$\frac{1}{2} Tr \ln \Box_{\xi\xi} = + \frac{g^2}{12} H_1 + \frac{g^2}{12} H_2 + \frac{1}{24} L_1 - \frac{1}{24} L_2 - \frac{1}{24} L_3 - \frac{1}{12} L_4 - \frac{1}{24} L_5$$
$$+ \frac{G^2 g^2}{16} (v + h)^2 \mathbf{Z} \cdot \mathbf{Z} + \frac{g^2 (G^2 + g^2)}{32} (v + h)^2 \mathbf{W}^+ \cdot \mathbf{W}^-$$
$$- \frac{3}{4} \frac{(\partial^2 h)^2}{(v + h)^2} + \frac{\partial^2 h}{4(v + h)} (2g^2 W^+ \cdot W^- + G^2 \mathbf{Z} \cdot \mathbf{Z}) - \frac{1}{8} (2g^2 + G^2)(v + h) \partial^2 h$$
$$- \frac{1}{64} (G^4 + 2g^4)(v + h)^4,$$  \hspace{1cm} (69)

$$\frac{1}{2} Tr \ln \Box_{hh} = - \frac{1}{16} L_5 + \frac{1}{32} \lambda [v^2 - 3(v + h)^2] (G^2 \mathbf{Z} \cdot \mathbf{Z} + 2g^2 W^+ \cdot W^-)$$
$$+ \frac{3}{32} \lambda^2 v^2 (v + h)^2 - \frac{9}{64} \lambda^2 (v + h)^4 - \frac{1}{16} \lambda^2 v^4,$$  \hspace{1cm} (70)

$$- \bar{\epsilon} Tr \ln \Box_{vu} = - \frac{2g^2}{3} H_1 + \frac{G^4 + 2g^4}{32} (v + h)^4,$$  \hspace{1cm} (71)

where $16 \pi^2 / \bar{\epsilon} = (2/\epsilon - \gamma_E + \ln(4\pi^2))$, $\gamma_E$ is the Euler constant, and $\epsilon = 4 - d$. For the convenience to construct counter terms, we have deliberately add a sign in each term in the Eq. (65). In the Eq. (69), we would like to mark terms proportional to $(\partial^2 h)$, which are impossible to be extracted if we work in the momentum space.

The divergence terms from the mixing terms with two propagators are calculated by using the technique shown in [49] and are given as

$$- \bar{\epsilon} Tr \left( \tilde{X}_\xi \Box^{-1} \tilde{X}_\xi \tilde{X}_\xi^{-1} \right) = \frac{g^2 g^2}{8} Z \cdot Z (v + h)^2 - \frac{g^2 + G^2}{8} (v + h)^2 (G^2 Z \cdot Z + 2g^2 W^+ \cdot W^-)$$
$$- \frac{1}{2} (2g^2 + G^2) \partial h \cdot \partial h,$$  \hspace{1cm} (72)
\[-\frac{\bar{\epsilon}}{2}Tr(\bar{X}_h \square_{V,V}^{\frac{1}{2}}X_h \square_{hh}^{-1}) = -\frac{g^2}{8}Z \cdot Z(v+h)^2 - \frac{g^2}{8}(v+h)^2(G^2Z \cdot Z + 2g^2W^+ \cdot W^-), \tag{73}\]

\[-\frac{\bar{\epsilon}}{2}Tr(X_h \square_{\xi,\xi}^{-1}X_{\xi h} \square_{hh}^{-1}) = -\frac{1}{2}(t_{BB1} + t_{BB2} + t_{BC} + t_{CC}), \tag{74}\]

\[t_{BB1} = \frac{g^2}{6}H_1 - \frac{g^2}{6}H_2 + \frac{1}{6}L_1 - \frac{1}{6}L_2 + \frac{1}{6}L_3 + \frac{1}{6}L_4 - \frac{1}{6}L_5\]
\[-\frac{1}{2}(\frac{g^2}{G^2} - 1)^2L_6 + \frac{1}{2}(\frac{g^2}{G^2} - 1)^2L_10 + \frac{G^2}{4}(\partial \cdot Z)^2 - \frac{g^2}{2}(d \cdot W^+)(d \cdot W^-) \]
\[-i\frac{g^2}{G}[(d \cdot W^+)(W^- - Z) - (d \cdot W^-)(W^+ - Z)], \tag{75}\]

\[t_{BB2} = -\frac{1}{4}L_2 - \frac{1}{4}L_3 - \frac{1}{2}L_4 + \frac{1}{4}L_5\]
\[-\frac{1}{16}(v + h)^2(2g^2W^+ \cdot W^- + G^2Z \cdot Z) - \frac{3\lambda}{16}(v + h)^2(2g^2W^+ \cdot W^- + G^2Z \cdot Z) \]
\[+ \frac{\lambda}{16}v^2(2g^2W^+ \cdot W^- + G^2Z \cdot Z) - \frac{1}{4(v + h)}(2g^2W^+ \cdot W^- + G^2Z \cdot Z)\partial^2h, \tag{76}\]

\[t_{BC} = (\frac{g^2}{G^2} - 1)^2L_6 + (\frac{g^2}{G^2} - 1)^2L_10 + \frac{G^2}{2}(\partial \cdot Z)^2 + g^2(d \cdot W^+)(d \cdot W^-) \]
\[-i\frac{g^2}{G}[(d \cdot W^+)(W^- - Z) - (d \cdot W^-)(W^+ - Z)], \tag{77}\]

\[t_{CC} = \frac{1}{2}(\frac{g^2}{G^2} - 1)^2L_6 + \frac{1}{2}(\frac{g^2}{G^2} - 1)^2L_10 - \frac{G^2}{4}(\partial \cdot Z)^2 - \frac{g^2}{2}(d \cdot W^+)(d \cdot W^-) \]
\[+ i\frac{g^2}{G}[(d \cdot W^+)(W^- - Z) - (d \cdot W^-)(W^+ - Z)]. \tag{78}\]

Here the term proportional to $\partial^2h$ in Eq. comes from the $\sigma_{\xi,\xi}$, and it is impossible to find such a compact divergence term if we work in momentum space.

Due to the derivative coupling between the Goldstone and Higgs boson, the term with four propagators (two Goldstone and two Higgs propagators) also contributes divergences, which can be extracted out and given as

\[-\frac{\bar{\epsilon}}{4}Tr(X_{h \xi} \square_{\xi,\xi}^{-1}X_{h \xi} \square_{hh}^{-1}X_{h \xi} \square_{\xi,\xi}^{-1}X_{h \xi} \square_{hh}^{-1}) = -\frac{1}{12}L_4 - \frac{1}{24}L_5. \tag{79}\]

In the Weinberg’s power counting rule, these divergences are counted as $p^4$ due to the proper momentum assignment on the dimensionless couplings. While in Georgi-Manohar’s power counting rule, these divergences are counted as dimension 4. In these intermediate results, there are extra divergence structures as shown by those $L_i$. These extra intermediate divergence structures are defined as

\[L_1 = \frac{gg'}{2}B_{\mu \nu}tr(TW^{\mu \nu}), \tag{80}\]

\[L_2 = \frac{g^2}{2}B_{\mu \nu}tr(T[V^\mu, V^\nu]), \tag{81}\]

\[L_3 = itr(W_{\mu \nu}[V^\mu, V^\nu]), \tag{82}\]
\[ \mathcal{L}_4 = [tr(V_\nu V_\nu)]^2, \quad (83) \]
\[ \mathcal{L}_5 = [tr(V_\nu V^\mu)]^2, \quad (84) \]
\[ \mathcal{L}_6 = tr(V_\nu tr(\mathcal{T}V^\mu))tr(\mathcal{T}V^\nu), \quad (85) \]
\[ \mathcal{L}_7 = tr(V_\nu V^\mu)[tr(\mathcal{T}V^\nu)]^2, \quad (86) \]
\[ \mathcal{L}_8 = \frac{g^2}{4}[tr(\mathcal{T}W_{\mu\nu})]^2, \quad (87) \]
\[ \mathcal{L}_9 = i\frac{g}{2}tr(\mathcal{T}W_{\mu\nu})tr(\mathcal{T}[V^\mu, V^\nu]), \quad (88) \]
\[ \mathcal{L}_{10} = \frac{1}{2}[tr(\mathcal{T}V_\mu)tr(\mathcal{T}V_\nu)]^2. \quad (89) \]

These operators have been used to construct the set of complete operators of the EWCL when C, P, T, and CP discrete symmetries are conserved [3, 4].

To sum over all contributions yields the following total divergence structures as

\[ \tilde{\epsilon}_{D_{\text{tot}}} = -\frac{43g^2}{6}H_1 + \frac{g^2}{6}H_2 - \frac{3}{16}\mathcal{L}_5 - \frac{3}{32}(2g^2 + G^2)(v + h)^2(G^2Z \cdot Z + 2g^2W^+ \cdot W^-) \]
\[ + \frac{3}{8(v + h)}\partial^2 h(G^2Z \cdot Z + 2g^2W^+ \cdot W^-) - \frac{3}{64}(2g^4 + G^4 + 3\lambda^2)(v + h)^4 \]
\[ - \frac{3\lambda^2}{64}v^4 + \frac{3\lambda^2}{32}v^2(v + h)^2 - \frac{1}{2}(2g^2 + G^2)\partial h \cdot \partial h - \frac{1}{8}(2g^2 + G^2)(v + h)\partial^2 h - \frac{3(\partial^2 h)^2}{4(v + h)^2}. \quad (90) \]

It is remarkable that terms related with the EOM of classic vector fields are exactly eliminated out, and there is no trace of it at all. However, here we observe there are some extra divergences, like \( \mathcal{L}_5 \), \( \partial^2 h(G^2Z \cdot Z + 2g^2W^+ \cdot W^-) \), and \( (\partial^2 h)^2 \), etc., which can not be eliminated. Terms proportional to \( \partial^2 h \) are beyond the reach of the perturbation calculation method in the momentum space.

If we stop here, then it seems that our calculation procedure can not demonstrate the renormalizability of the renormalizable theory, therefore it should be rejected as a proper method to construct divergences of EWCL. Fortunately, in the background field method, we can use the EOM of classic fields. By using the linear realized Higgs model as a guide and using the EOM of the Higgs field given in Eq. (30) carefully, the total divergence structure can be reformulated as

\[ \tilde{\epsilon}_{D_{\text{tot}}} = -\frac{43g^2}{6}H_1 + \frac{1}{6}g^2H_2 - \frac{2g^2 + G^2}{8}(v + h)^2(G^2Z \cdot Z + 2g^2W^+ \cdot W^-) \]
\[ - \frac{2g^2 + G^2}{2}\partial h \cdot \partial h + \frac{1}{32}(6\lambda + 2g^2 + G^2)\lambda v^2(v + h)^2 \]
\[ - \frac{1}{64}[(6g^4 + 3G^4) + (2g^2 + G^2)\lambda + 12\lambda^2](v + h)^4 - \frac{1}{16}\lambda^2v^4. \quad (91) \]

We observe that all the extra divergence structures \( \mathcal{L}_i \) and \( \partial^2 h \) are eliminated out. This just indicates that the extra divergence structure just cancel out exactly with each other, and even the terms like \( (\partial Z)^2 \) do not appear in the total divergence structures. This is exactly the meaning of the renormalizability. Even no extra gauge fixing term of the background fields should be added to the Lagrangian, and the equations of motion of the background fields are sufficient. This is a pleasant result and really support our renormalization procedure.

By using the reverse Stueckelberg transformation [30], the background Goldstone fields (the longitudinal component of the vector bosons) can be restored in the Lagrangian. Therefore the Lagrangian contains the correct dynamic degrees of freedom at low energy region. It is interesting to observe that these divergence terms (counter terms) are counted as \( p^4 \) in the Weinberg’s power counting rule and simply dimension 4 operators in the Georgi-Manohar’s power counting rule. In Weinberg’s power counting rule, these \( p^4 \) divergences can be absorbed by refining operator’s renormalization constants, which induce the anomalous dimension matrix among operators.
while in the Georgi-Manohar’s power counting rule, these divergences of dimension 4 operators can be absorbed by redefining the renormalization constants of fields, couplings and mass parameter, like the standard renormalization procedure.

Here we would like to point out that in order to justify the perturbation method, in the realistic consideration, even we can define both these two power counting rules in our theory, we must require that all couplings in the loop expansion should be weak couplings in order to guarantee the validity of the perturbation expansion.

There are terms which contribute to the vacuum. After using the normalization of the partition functional, these terms can be eliminated out.

We would like to mention that the coefficients of $H_1$ and $H_2$ have the correct value which contribute to the systematic renormalization of the EWCL.

According to the well known results [39, 40, 41], the results given in Eq. (91) agree with these well-known results.

One interesting observation is that the Goldstone boson, another half comes from the Goldstone-Higgs mixing contribution, as shown in Eq. (69). In the nonlinear realization of Abelian gauge theory can be expressed as

$$\frac{g^3}{16\pi^2} \left( -\frac{11}{3} \times 2 + \frac{1}{3} \times \frac{1}{2} \right). \quad (92)$$

While the $\beta$ function of Abelian gauge theory can be expressed as

$$\frac{g^3}{16\pi^2} \left( \frac{1}{3} \times \frac{1}{2} \right). \quad (93)$$

Our results given in Eq. (91) agree with these well-known results.

After taking into account the quantization in the background field method, the active degree of freedom of the system includes quantum vector boson, quantum Goldstone boson, and ghost fields. By working in the mass eigenstates, the gauge fixing terms are determined as

$$\mathcal{L}_{GF,A} = -\frac{1}{2}(\partial \cdot \hat{A})$$

$$-ie(\hat{W}^+ \cdot W^- - \hat{W}^- \cdot W^+) \mathcal{L}_{GF,W} = -(d \cdot \hat{W}^+ + \frac{1}{2}g_v \xi_v^+ + ig^2 Z \cdot \hat{W}^+)$$

$$-i g^2 W^+ \cdot \hat{Z} + ieW^+ \cdot \hat{A})$$

$$(d \cdot \hat{W}^- + \frac{1}{2}g_v \xi_v^- - ig^2 Z \cdot \hat{W}^-)$$

$$+i g^2 W^- \cdot \hat{Z} + ieW^- \cdot \hat{A}). \quad (97)$$

VIII. DIVERGENCES OF $O(p^2)$ WITHOUT A HIGGS BOSON

When the Higgs field is assumed to be decoupled from low energy phenomenology and only $O(p^2)$ operators (in Weinberg’s power counting rule) are considered, the classic Lagrangian is modified as

$$\mathcal{L} = -H_1 - H_2 + \frac{1}{4} tr[V \cdot V]. \quad (94)$$

After setting $h = 0$. These gauge fixing terms are obtained from Eqs. [39—41] by setting $h = 0$. These gauge fixing terms are covariant background field gauges when we rotate they back to weak interaction eigenstate basis. The bilinear terms of the quantum fluctuations can be expressed as

$$\mathcal{L}_{quad} = \left\{ \begin{array}{c} \frac{1}{2} V_{\mu V} \ln \xi_{\xi^j} \xi^j + \frac{1}{2} \xi^{ij} \xi^{ij} \\ +i[a \cdot b_{\nu}] \xi^j + i[a \cdot b_{\nu} \xi^j] + \frac{1}{2} \xi^{ij} X_{\xi} \xi^{ij} \end{array} \right\}. \quad (98)$$

These operators can be obtained from Eq. (43) by setting $h = 0$. While other parts, like the gauge potential, etc, are not modified. After performing the path integral to integrate out all quantum fluctuations, the one-loop effective Lagrangian can be expressed as

$$\int \mathcal{L}_{1-loop} = Tr \ln \xi_{\xi^j} + \frac{1}{2} \left[ Tr \ln \xi_{\xi^j} + Tr \ln \xi_{\xi^j} \right]$$

$$-Tr\left( X_{\xi} \square_{\xi}^{\frac{1}{2}} X_{\xi} \right) \cdots. \quad (99)$$
After extracting divergences, we obtained the following divergence structure
\[
\varepsilon D_{\text{tot}} = -\frac{29}{4} g^2 H_1 + \frac{1}{12} g^2 H_2 \\
+ \frac{1}{12} \mathcal{L}_1 - \frac{1}{24} \mathcal{L}_2 - \frac{1}{24} \mathcal{L}_3 - \frac{1}{12} \mathcal{L}_4 - \frac{1}{24} \mathcal{L}_5 \\
+ 3 (G^2 + g^2) \times \frac{v^2}{4} \text{tr}[V \cdot V] \\
- \frac{3 g^2}{8} \times \frac{v^2}{4} \left( \text{tr}[T V] \right)^2 \\
- \frac{3}{64} (2 g^4 + G^4) v^4 .
\] (100)

We observe that without the help of the Higgs field, those divergences can not be eliminated any more. Here we mark two facts: 1) The \( \beta \) functions of the gauge couplings are correct when we count the active degree of freedom.

The \( \beta \) function of \( g \) is given as
\[
\beta_g = \frac{g^3}{16\pi^2} \left( -\frac{11}{3} \times 2 + \frac{1}{6} \times \frac{1}{2} \right) .
\] (101)

The \( \beta \) function of \( g' \) is given as
\[
\beta_{g'} = \frac{g'^3}{16\pi^2} \left( \frac{1}{6} \times \frac{1}{2} \right) .
\] (102)

The \( \beta \) function of the \( v^2 \) is given as
\[
\beta_{v^2} = \frac{v^2}{8\pi^2} \times \left( 3 (G^2 + g^2) \right) / 8 .
\] (103)

2) The \( \beta \) functions for the anomalous couplings are completely determined by the evaluation of the exact Goldstone propagator, which can be formulated as
\[
\beta_\beta = \frac{1}{8\pi^2} \times (-3g^2) ,
\] (104)
\[
\beta_{\alpha_1} = \frac{1}{8\pi^2} \times \frac{1}{12} ,
\] (105)
\[
\beta_{\alpha_2} = \frac{1}{8\pi^2} \times (-\frac{1}{24}) ,
\] (106)
\[
\beta_{\alpha_3} = \frac{1}{8\pi^2} \times (-\frac{1}{24}) ,
\] (107)
\[
\beta_{\alpha_4} = \frac{1}{8\pi^2} \times (-\frac{1}{12}) ,
\] (108)
\[
\beta_{\alpha_5} = \frac{1}{8\pi^2} \times (-\frac{1}{24}) .
\] (109)

By solving these RGE of the anomalous couplings, the logarithmic contributions of the heavy Higgs can be found and reproduce the "screening" effects of Higgs to low energy phenomenology.

Due to our definition of the covariant differential operator given in Eq. [143] and Euclidean space, we observe several sign differences in our divergences when compared with the well-known results [3, 4, 27, 28, 29]. Compared with the well-known paper given by Appelquest and Wu [8], our \( \beta \) parameter has a sign difference due to the Euclidean space, and our triple anomalous couplings have the opposite signs due to the definition of the covariant differential operator given in [8] has the following form
\[
V_\mu = U^\dagger \partial_\mu U + iU^\dagger W_\mu U - iB_\mu .
\] (110)

After realizing these differences, we find complete agreements between our results and with these well-known results.

As matter of fact, sign differences existed in literatures are due to the fact there exist discrete symmetries in the definition of the partition functional
\[
g \to \pm g , \ g' \to \pm g' , \ v \to \pm v ,
\] (111)

which can explain why there exist arbitrariness in the definition of the covariant differential operator. In the path integral, vector boson and Goldstone both are assumed to be real fields, there exists another type of discrete symmetries in the field definition which corresponds to the above discrete symmetries in couplings (here we regard \( v \) as a coupling)
\[
W \to \mp W , \ B \to \mp B , \ \xi \to \mp \xi .
\] (112)

These symmetries cause convention problems in the anomalous couplings at \( O(p^2) \) order.

The fact that our renormalization procedure can reproduce the divergences of these \( O(p^2) \) operators reinforces our belief that our renormalization procedure is correct.

**IX. BRST TRANSFORMATIONS**

The BRST global transformation [59] is very important to formal proof of the renormalizability of the spontaneous symmetry breaking models and gauge independence of the S-matrix. Here we ask the question: in our calculation procedure with nonlinear realization, is there such a global transformation? The answer is yes. Below we explicitly construct the BRST transformation in the BFM formalism in the nonlinearly realized Higgs model and in \( O(p^2) \). We have referred the BRST transformation without spontaneous symmetry breaking [54] and in the linearly realized Higgs doublet [65] for our construction.

We start with the BRST transformation with a linearly realized Higgs field. Due to the fact that in the background field method we can choose different gauges for the classic and quantum fields, the background Goldstone fields can be eliminated from the Lagrangian by such a global transformation? The answer is yes. Below we explicitly construct the BRST transformation in the BFM formalism in the nonlinearly realized Higgs model and in \( O(p^2) \). We have referred the BRST transformation without spontaneous symmetry breaking [54] and in the linearly realized Higgs doublet [65] for our construction. We start with the BRST transformation with a linearly realized Higgs field. Due to the fact that in the background field method we can choose different gauges for the classic and quantum fields, the background Goldstone fields can be eliminated from the Lagrangian by choosing the unitary gauge for the background vector fields. So the Higgs field can be parameterized as
\[
\Phi = \frac{1}{\sqrt{2}} \left( 1(v + \rho + \tilde{\rho}) + i\tilde{g}^i T^i \right) .
\] (113)

The gauge fixing terms in this linearly realized Higgs field can be expressed as
\[
F_Y = \partial \cdot B + g'(v + \rho)\delta^3 ,
\]
\[
F_W^i = \left( \delta^{ij} \partial + g f^{ijk} W^j \right) \cdot \dot{W}^k + g(v + \rho) \delta^i .
\] (114)
Here we notice that in these gauge fixing terms the $SU_c(2)$ isospin symmetry originated from $SU(2)$ has been explicitly broken by $F_Y$, so we can not rotate $\phi^1, \phi^2,$ and $\phi^3 (= \phi^4)$ any more. Here we observe that the quantum fluctuation of Higgs field, $\tilde{\rho}$, does not enter into the gauge fixing terms.

For both the renormalizable linearly realized Higgs model and the EWCL up to $O(p^2)$ operators, there exists a universal rotation matrix for all the gauge group parameters, quantum vector fields, ghost fields, which can be expressed as

\[
\begin{pmatrix}
    c_w & s_w & 0 & 0 \\
    s_w & -c_w & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1 
\end{pmatrix}
\]

Then we observe that this rotation changes the functional measure with a trivial constant number, which is innocent and does not affect physics. After this rotation, the index of vector fields $a$ changes from $\{Y, 3, 1, 2\}$ to $\{A, Z, W^+, W^-\}$. While there are only three Goldstone, their index can be represented as $i$, which is changed from the remnant $U_{EM}(1)$ symmetry. While the group structure constants $f_{ijk}'$ of $SU(2)$ also are also changed and the new group structure constants $g_{ijk}$ with indices as $i = \{Z, W^+, W^-\}$ are determined by the commutation relations of generators, $T^2, T^+, T^-$.

\[
[T^+, T^-] = T^3, \quad [T^3, T^+] = T^+, \quad [T^-, T^3] = T^-, \quad (115)
\]

We have $g^{++-} = g^{3++} = g^{3-3} = -g^{-+3} = -g^{-+3} = 1$.

The BRST transformation of the quantum system in the mass eigenstate basis can be formulated as

\[
\begin{align*}
\delta_B \hat{V}_{\mu} &= -D_\mu^a (\nabla + \hat{V}) u^b, \\
\delta_B u^a &= \frac{1}{2} \eta^{ai} g_{ijk} \omega^j \omega^k u^b u^c, \\
\delta_B (\phi^i)^j &= -(v + \hat{\rho} + \hat{\rho})(u^l)^a \eta^{ai}, \\
\delta_B \hat{\rho} &= \frac{1}{4} (u^l)^a \eta^{ai} \phi^i, \\
\delta_B v^a &= B^a, \\
\delta_B B^a &= 0. \quad (116)
\end{align*}
\]

The BRST transformation of quantum fluctuations in mass eigenstate basis is modified as

\[
\begin{align*}
\delta_B \hat{V}_{\mu} &= -D_\mu^a (\nabla + \hat{V}) u^b, \\
\delta_B u^a &= \frac{1}{2} \eta^{ai} g_{ijk} \omega^j \omega^k u^b u^c, \\
\delta_B (\phi^i)^j &= -\frac{1}{2} (v + h + \hat{h}) (u^l)^a \eta^{ai}, \\
\delta_B \hat{\rho} &= \frac{1}{2} (v + h + \hat{h}) \hat{\phi}^i \sin \zeta. \quad (121)
\end{align*}
\]

Where $\zeta = \sqrt{\hat{\zeta}^2}$, and $\hat{\phi}^i = \zeta \phi^i / \zeta$. The $\zeta^i$ are the phase angle which parameterize the Goldstone fields with $\zeta^i = 2\zeta^i / (v + h)$.

By checking the gauge transformation of Higgs fields, the BRST transformation of quantum fluctuations in mass eigenstate basis is modified as

\[
\begin{align*}
\delta_B \hat{V}_{\mu} &= -D_\mu^a (\nabla + \hat{V}) u^b, \\
\delta_B u^a &= \frac{1}{2} \eta^{ai} g_{ijk} \omega^j \omega^k u^b u^c, \\
\delta_B (\phi^i)^j &= -\frac{1}{2} (v + h) (u^l)^a \eta^{ai}, \\
\delta_B \hat{\rho} &= 0, \\
\delta_B v^a &= B^a, \\
\delta_B B^a &= 0. \quad (122)
\end{align*}
\]

In order to guarantee the derivative power counting rule, the BRST transformation $\delta_B$ is assumed to change the power by the unit $+1$, and ghost fields are assumed to be $[v]_p = [u]_p = 0$. The matrix $\eta$ contains the gauge couplings and $[\eta]_p = 1$ which guarantees that this power counting rule can hold. While for the mass dimension power counting rule, $\delta_B$ is also assumed to change the power by $+1$, and ghost fields are assumed to be $[v]_m = [u]_m = 1$. The matrix $\eta$ is dimensionless in mass dimension power counting rule.

In both these two power counting rule, the matrix $\omega$ is dimensionless.

However, both these two assignments can not be consistent for the last equation. Anyway, after integrating out the auxiliary field $B^a$ and using the on-shell condition for the $u$ type ghost fields given as

\[
\frac{\delta F^a}{\delta \eta^b} u^b = 0, \quad (117)
\]

the Lagrangian enjoys the on-shell BRST transformation and can be consistent with both these two power counting rules.

As shown in [59], the ghost-BRST is sufficient for the demonstration on the perturbation renormalizability and gauge independence of S-matrix.

The relation between the linearly realized and nonlinearly realized Higgs and Goldstone fields are given as [36]

\[
(v + \nabla + \hat{\rho}) + \phi^i T^i = (v + h + \hat{h}) U^k C \sqrt{\gamma^T}, \quad (118)
\]

with the relations between the component fields given as

\[
\begin{align*}
\tilde{\eta} &= (v + h) \cos \zeta - v, \\
\tilde{\rho} &= \sqrt{\hat{\zeta}^2}, \\
\tilde{\phi}^i &= (v + h + \hat{h}) \hat{\phi}^i \sin \zeta. \quad (121)
\end{align*}
\]

The $\eta^i$ is a $4 \times 3$ matrix, which is given as

\[
\begin{pmatrix}
    e & 0 & 0 \\
    -\frac{s_w}{c_w} & 0 & 0 \\
    0 & g & 0 \\
    0 & 0 & g
\end{pmatrix}.
\]
The reason for the \( \delta_B \hat{h} = 0 \) can be easily understood: in the nonlinear form, \( \hat{h} \) field is a \( SU(2) \) singlet. But the \( \delta_B \hat{h} \) poses a new dilemma for both power counting rules if we want to assign powers to BRST transformation, compared with the case in the linear form given in Eq. (116), where both these two power counting rules sustain with on-shell ghost fields. However, the Lagrangian with a nonlinearly realized Higgs fields is consistent with the relation \( \delta_B \hat{h} = 0 \).

When the Higgs scalar decouples or there is no Higgs at all, the BRST transformation of quantum fluctuations in mass eigenstate basis in the EWCL up to \( O(p^2) \) is modified as

\[
\begin{align*}
\delta_B \hat{V}_\mu & = -D^a_{\mu}(\hat{V} + \hat{\omega})u^b, \\
\delta_B u^a & = -\frac{1}{2} \hat{g}^{ai} \hat{g}^{jk} \hat{\omega}^{b} u^{k} u^{b} c, \\
\delta_B (\xi^1)^i & = \frac{1}{2} \hat{\nu}(u^i)^a \eta^{ai}, \\
\delta_B V^a & = B^a, \\
\delta_B B^a & = 0. \quad (123)
\end{align*}
\]

These constructions just show that there is no difficulty to construct the BRST transformation even with nonlinear realization. However, the BRST transformation does not necessarily guarantee the renormalizability of the nonlinear realization of the spontaneous symmetry breaking for any higher loop order. Even in the nonlinearly realized Higgs boson, the Feynman rules of Goldstone bosons and the superficial divergence power counting rule make the renormalizability of the model obscure.

X. DISCUSSIONS AND CONCLUSIONS

In this paper, by using the dimensional regularization (a symmetry preserving regularization scheme) and Feynman-‘t Hooft gauge in the background field method, we show how our calculation procedure can demonstrate the renormalizability of the nonlinearly realized Higgs in the SM, can produce correct \( \beta \) functions of gauge couplings, and can reproduce well-known results on the divergences of the \( O(p^2) \) operators in the EWCL. Compared with those calculations with Landau gauge \[3, 4, 27, 28, 29\], our results show that those divergences are gauge independent if a renormalizable gauge are taken. In the \( R_\xi \) gauge, the ordering of performing loop integrals and taking the limit \( \xi \rightarrow \infty \) can not be exchanged. This may explain why the divergences extracted by using the unitary gauge can not agree with other renormalizable gauges. We conclude that our method can serve as a reliable method to extract the divergences of the EWCL.

Below we would like remark on the regulation method existed in literatures. Reference \[3, 4\] introduces a Higgs field as a regulator, after renormalization the effective Lagrangian the Higgs was taken to the decoupling limit. This procedure can not produce the correct \( \beta \) functions of gauge couplings and anomalous couplings of the EWCL without a Higgs, since the basic degree of freedoms are different before and after the decoupling limit. A careful matching must be made between the EWCL with a Higgs and the EWCL without a Higgs. Only after this matching procedure, the \( \beta \) functions for anomalous couplings can be correctly obtained.

In the Slavnov’s scheme \[66\], higher dimensional covariant operator are introduced as a regulator. Such a regularization scheme has been used by J. J. van der Bij and B. Kastening \[15\] to study the radiative correction of two point functions in the EWCL. By using the standard superficial divergence counting rule, it is obviously that this regularization method can improve the divergence power counting behavior but can not lead to a consistent treatment to all divergences in the theory \[67\]. The biggest trouble is that it can not produce the correct \( \beta \) function for the gauge couplings in the renormalizable gauge theory, as pointed out in \[67, 68\], which also means the breaking of unitarity of the S-matrix. For us, to use it as a regulator to nonlinear gauged sigma model seems to be more problematic.

Recently, Y.L. Wu has proposed \[69\] a new symmetry preserving regularization scheme. Whether this scheme can be used to perform systematic renormalization of low energy QCD chiral Lagrangian or EWCL needs further study.

About our renormalization procedure, there are two remarks on it:

1) Although we have explicitly demonstrate the renormalizability at one-loop level, and have constructed the BRST transformation formally, it is reasonable to ask whether the renormalizability in this calculation is accidental or necessary, like the one-loop renormalizability for the gravity? Can it work at two-loop level or higher? These questions are worthy of our future study.

2) Due to the semi-simple structure of the \( SU(2) \times U(1) \) symmetry, and the symmetry breaking pattern \( SU(2) \times U(1) \rightarrow U(1) \), in the vector sector and ghost sector, we can have both the weak interaction eigenstates and mass eigenstates. It is natural to ask whether our calculation procedure in the weak interaction eigenstates are also viable. In weak interaction eigenstate basis, the BRST of \( SU(2) \times U(1) \) can be more easily constructed. In principle, although massless vector fields theory has the problem of infrared divergences, when we only consider the ultraviolet divergences, it is possible to calculate in the weak interaction eigenstate basis. The computation in the weak interaction eigenstates is worthy of examination in our future work.

By using these concepts and methods, we will explore the systematic renormalization of the EWCL in our future works \[51\].
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