Lattice Extraction of $\epsilon'/\epsilon$ using (PQ) Chiral Perturbation Theory

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Results at NLO in (PQ)ChPT for extracting $\epsilon'/\epsilon$ from lattice data are discussed.

1. Introduction

There are renewed attempts to calculate $K \to \pi\pi$ and $\epsilon'/\epsilon$ on the lattice [1,2]. Recently we have shown [3] that all the amplitudes of interest for the (8,1)'s can be obtained to NLO [4] in full ChPT when one uses lattice computations from $K \to |0\rangle$, $K \to \pi$ and $K \to \pi\pi$ at the two unphysical kinematics points $\text{UK1} [5] \Rightarrow m_K = m_\pi$, $\text{UK2} [6] \Rightarrow m_K = 2m_\pi$. In [7] we extended our previous results for the full theory; we also presented results in the partially quenched case. In the full theory, we showed that none of the quantities considered in [3] require 3-momentum insertion, i.e., it is sufficient to consider nondegenerate quark masses in $K \to \pi$ amplitudes. We also demonstrated how to get the low energy constants (LEC’s) needed to construct $K \to \pi\pi$ for the (8,8)’s and (8,1)’s to NLO in the partially quenched theory. (Note that for the (8,1)’s one can go to NLO only for the PQS implementation, as discussed below.) There have been a few important revisions in [7] since the lattice conference. Though our general conclusion that one can get all of the needed combinations of NLO LEC’s in the full and partially quenched theory still holds, there are difficulties in calculating (8,1), $K \to \pi\pi$ amplitudes at UK1 ($m_K = m_\pi$) that must be circumvented by considering other unphysical kinematics in order to get all the NLO constants.

There were three main additions to [7] since the time of the lattice conference. First, we have demonstrated that the NLO LEC’s needed to construct $K \to \pi\pi$ for the (8,8)’s can be obtained in the partially quenched case from $K \to \pi$ with degenerate quark masses without momentum insertion and from $K \to 0$. When the number of dynamical flavors is three, the LEC’s of PQChPT are those of the full theory [8].

The second addition to the paper was a discussion of the power subtractions that are needed when one calculates $\Delta I = 1/2$ amplitudes. The effects of the subtraction are considered at NLO for the (8,8) and (8,1) amplitudes of interest. This subtraction makes use of the $\Theta^{(3,3)}$ operator introduced in [9], and is needed to perform the $K \to 0$ subtraction of $\Delta I = 1/2$ amplitudes as described in [2]. It was discovered that the operator subtraction for the (8,1), $K \to \pi\pi$ amplitude has a problem when the quark masses are degenerate, and in this case, the subtraction is expected to be quite difficult.

The third main addition to [7] was an inclusion of results at other unphysical kinematics points for (8,1), $K \to \pi\pi$ amplitudes. These kinematics points are such that all mesons are at rest, and the weak operator inserts energy to ensure 4-momentum conservation. We call this set of unphysical kinematics UKX, since UK1 and UK2 are just two special cases of UKX. If lattice calculations at UK1 prove to be difficult or impossible, one can use UKX as an alternative way to get all of the necessary NLO LEC’s for the (8,1)’s.

Since the time of the conference there appeared a paper by Lin, et al., [10], where they have made some important observations regarding our attempts to obtain $K \to \pi\pi$ to NLO. The full implications for our work are discussed below, but we mention here that our general conclusion that one can obtain all of the NLO LEC’s in the partially quenched theory needed for (8,1), $K \to \pi\pi$ amplitudes remains true, at least in principle. We also
review the ambiguity in embedding the partially quenched theory in (P)QChPT as first discussed by Golterman and Pallante [11].

2. (8,8), $K \rightarrow \pi\pi$ amplitudes with degenerate quark masses

The following expressions are for (8,8) $K \rightarrow \pi$ amplitudes with degenerate valence quark masses and no momentum insertion,

$$\langle \pi^+ | O^{(8,8),(3/2)}_{ct} | K^+ \rangle = \frac{4\alpha_{ss}}{f^2} + \frac{4}{f^2} \left[ (c_1^r + c_2^r) m^2 + 4c_1^r + 4c_2^r \right] + 2c_6^r N m_{SS}^2, \quad (1)$$

$$\langle \pi^+ | O^{(8,8),(1/2)}_{ct} | K^+ \rangle = \frac{8\alpha_{ss}}{f^2} + \frac{4}{f^2} \left[ (c_1^r - c_2^r) m^2 - 2c_1^r + 8c_1^r + 8c_2^r \right] + 4c_6^r N m_{SS}^2. \quad (2)$$

One could, in principle, determine $\alpha_{ss}$ from either leading order term, but in practice it is safer to use the 3/2 amplitude because the 1/2 term has a power divergence, and the subtracted amplitude could receive some residual chiral symmetry breaking contribution unless one uses a discretization that has exact chiral symmetry. By varying the valence and sea quark masses for $K \rightarrow \pi$ one can determine three linear combinations of NLO LEC’s, $c_6^{\pi}, c_1^r, c_2^r + 4c_2^r + 4c_3^r$, and $c_1^r - c_2^r - 2c_3^r + 8c_3^r + 8c_4^r$ from fits using Eqs (1,2), as well as the logarithmic parts given in Appendix D of [7]. One can verify that these combinations are sufficient to determine the physical NLO $K \rightarrow \pi\pi$ expressions, Eqs (42,43) of [7].

Note that when $N = 3$, the LEC’s are those of the full theory, but it is still necessary to vary the (degenerate) sea and valence masses independently in order to get all of the needed constants. Also note that for the $\Delta I = 1/2$ amplitude, Eq (2), the operator subtraction has been performed. For details, see [7]. Finally, for additional redundancy in the determination of the (8,8), NLO LEC’s, one can use nondegenerate valence quark masses in the $K \rightarrow \pi$ amplitudes.

3. PQS vs PQN

According to [11], there are at least two ways of embedding the left-right QCD penguins into the partially quenched theory. We have dubbed these the PQS and PQN methods, based on the notation in [11]. For the PQS method one assumes the right handed part of the penguin operator transforms as a singlet under the extended symmetry group of the penguin operator; hence the name partially quenched singlet (PQS) method. In the PQN method, the right handed part of the QCD penguin contains only valence quarks, so that the PQChPT at leading order becomes the sum of two operators, one of which is just the singlet operator of the PQS method, while the other does not transform as a singlet under the extended symmetry group; hence the name partially quenched non-singlet (PQN) method.

For the left-right QCD penguins we have considered only the PQS method to NLO because the PQN method requires a two-loop calculation (for insertions of the non-singlet operator) to the same order. Only in the full theory do the two methods coincide, and the contributions of the non-singlet operator vanish. Even if one believes that the PQN method is closer to the full theory because it keeps “eye diagram” contractions that would be discarded in the PQS method (see the discussion in [7]), it would still be useful to implement the PQS method, since this method can be carried out to NLO. Of course, an $N = 3$ calculation would resolve the ambiguity.

4. (8,1), $K \rightarrow \pi\pi$ amplitudes

The ingredients necessary to construct (8,1), $K \rightarrow \pi\pi$ amplitudes to NLO are $K \rightarrow 0$, $K \rightarrow \pi$ and $K \rightarrow \pi\pi$ at UKX, all of which require nondegenerate quark masses but no 3-momentum insertion. For a detailed description of how to obtain all of the necessary constants from the various amplitudes, see Section 8 of [7]. Here, a brief description of the operator subtraction is given. The power divergence in the $\Delta I = 1/2$ matrix elements reduces to an effective quark bilinear times a momentum independent coefficient [2],

$$\Theta^{(3,3)} \equiv \bar{\sigma}(1 - \gamma_5) d = \alpha^{(3,3)} \text{Tr}(\lambda_6 \Sigma), \quad (3)$$
to lowest order in ChPT. The subtraction makes use of the ratio of \( K \to 0 \) amplitudes, which to NLO is

\[
\frac{\langle 0 \vert \mathcal{O}^{(8,1)} \vert K^0 \rangle}{\langle 0 \vert \mathcal{G}^{(3,3)} \vert K^0 \rangle} = 2 \frac{\alpha_2}{\alpha^{(3,3)}} (m_K^2 - m_\pi^2) \\
+ 2 \frac{\alpha_1}{\alpha^{(3,3)}} (\text{logs}) + \frac{4}{\alpha^{(3,3)}} (m_K^2 - m_\pi^2) \frac{1}{2} \left( e^r_{1,\text{rot}} - e^r_{5,\text{rot}} \right) m_K^2 \\
+ \left( e^r_{2,\text{rot}} \right) N m_{S\bar{S}}^2 .
\]

(4)

The subscript, “rot,” accounts for the effect of the subtraction on the \( (8,1) \) LEC’s to NLO. They are transformed to new linear combinations involving the Gasser-Leutwyler coefficients [12], with the power divergences removed.

Using the ratio determined from the \( K \to 0 \) amplitudes, the power divergences can be subtracted from \( K \to \pi\pi \) amplitudes using the following formula,

\[
\langle \pi^+\pi^- \vert \mathcal{O}^{(8,1)}_{\text{sub}} \vert K^0 \rangle = \left( \pi^+\pi^- \vert \mathcal{O}^{(8,1)} \vert K^0 \rangle \right) \\
- 2 \frac{\alpha_2}{\alpha^{(3,3)}} (m_K^2 - m_\pi^2) \\
\times \langle \pi^+\pi^- \vert \mathcal{G}^{(3,3)} \vert K^0 \rangle .
\]

(5)

From the leading order expression for the \( \Theta^{(3,3)} \) operator,

\[
\langle \pi^+\pi^- \vert \Theta^{(3,3)} \vert K^0 \rangle = \frac{i}{f^3} \alpha^{(3,3)} \frac{2m_\pi - m_K}{m_K - m_\pi} .
\]

(6)

one can see there is a problem when \( m_K = m_\pi \) (UK1) where the amplitudes blow up, even though the final subtracted amplitude, Eq (5), is finite at UK1.

5. Enhanced Finite Volume Corrections

Lin, et al., [10] have done a calculation of the relevant finite volume Euclidean correlation functions for \( K \to \pi\pi \). They have discovered that the case of the \( \Delta I = 1/2 \), \( K \to \pi\pi \) amplitudes at degenerate quark masses (UK1) has difficulties in the full theory, and is intractable in the partially quenched theory because of enhanced finite volume effects, even at the special kinematics, \( m_{\text{sea}} = m_{u,d} \). According to the results of [10], these difficulties do not afflict the more general kinematics of UKX for \( m_K \) strictly greater than \( m_\pi \) in the two flavor dynamical case (at the special value of \( m_{\text{sea}} = m_{u,d} \)). In our revised paper we show that one could use this set of kinematics points instead of UK1 to obtain all of the NLO LEC’s in the two flavor theory. It was also pointed out by [10] that for \( \Delta I = 1/2 \), \( K \to \pi\pi \) amplitudes in the three flavor dynamical case, in order to avoid enhanced finite dynamical case, one must work in the full theory.

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