Tool life modelling in continuous variable speed turning

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Abstract In this paper, a generalized tool life model that considers non-stationary cutting was developed. In particular, the model was conceived for predicting the life of the tool when Spindle Speed Variation $SSV$, one of the most effective techniques for suppressing regenerative chatter vibrations, is used. The proposed formulation takes into account the main cutting parameters and the parameters associated to the $SSV$. A dedicated experimental campaign of turning tests was carried out and the data were used to develop the model. A proper validation was even carried out performing additional tool life tests. It was found that the generalized formulation can be used for predicting the tool life both at constant spindle machining $CSM$ and adopting $SSV$ within the maximum estimating error of 6%.

Keywords tool life modeling · spindle speed variation · non stationary cutting · chatter suppression

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1 Introduction

The occurrence of high vibrations in machining limits the achievable Material Removal Rate $MRR$ [1], the surface quality and the life of the tool $TL$ [2]. Vibrations are typically due to regenerative effects that bring cutting process to instability. Spindle Speed Variation (SSV) is one of the available techniques for suppressing chatter vibrations [3]. It is based on a continuous modulation of the spindle speed that aims at repressing the regenerative effect and thus the growth of undesired vibrations [4, 5, 6]. Sinusoidal spindle speed variation SSSV is the most studied approach for modulating the spindle speed. Over the years, SSV has been developed and tested both in turning [5, 7, 8] and milling applications [6, 9]. From the quantitative perspective, the effects of SSV on the stability lobes diagrams $SLD$ were analyzed in different research works through analytical [7, 10], numerical [8] and experimental approaches [9]. It was proven that stability enhancement due to SSV is effective and robust especially in the high-order lobes region of the stability maps [8, 9]. The SSV, thanks to the the stabilization capabilities, reduces the risk of tool chipping and too early failure that typically occur when cutter vibrations affect the cutting [11]. Conversely, Albertelli et al. [12] found that SSSV has a negative impact on the life of the tool, fostering the formation of cracks that tend to progressively detach the coating and thus increasing the wear rate. In this study, the achieved results were obtained comparing SSSV machining to constant speed machining $CSM$, both in stable conditions. Chiappini et al. in [13] used a finite element model $FEM$ for simulating SSSV cutting. It was proven that the modulation of the spindle speed is the cause of an additional mechanical-thermal load that could be the origin of the cracks formation. Although both the research works gave an interesting interpretation of the involved phenomena, a quantification of the $TL$ reduction, according to the adopted cutting parameters, was not provided. Being able to quantify the $TL$ reduction would be extremely useful for assessing the potentialities and the limitations of the SSSV, especially in terms of industrial applicability. Although variable speed machining $VSM$ was first conceived in the 70's, its wide diffusion in real applications has not been registered so far. For instance, the use of such technique to stabilize machining operations that otherwise could be regularly carried out with stiffer, but even more expensive machines, needs to be properly analyzed. In such way, a trade off between tooling and machine costs could be investigated. In order to bridge this gap, in this paper a generalization of the Taylor’s model for estimating the $TL$ in turning, when both $CSM$ and $SSSV$ are indiscriminately used, was developed. So far, the literature that has been dealt with the tool wear under unstationary cutting conditions is rather poor [14, 15]. One of the first studies that presented the difficulties related to the wear modelling when different cutting conditions are set was carried out by Jemielniak et al. in [16]. Lin in [17] and Pálmai in [18] proposed a cumulative wear model for taking into considerations different spindle speed steps. To the authors’ knowledge, no specific modelling approaches for estimating the $TL$ when the cutting speed is continuously modulated have been developed. The
paper was structured as follows. In section 2, a more exhaustive description of the research goals together with the explanation of the conceived approach were provided. The experimental set-up, the designed experimental campaign and the preliminary tests were also presented. In section 3 the results of the experimental tests, the development of the modelling formulations and their validation were reported and critically analysed. In section 4, the conclusions were also outlined.

**Nomenclature**

- $\beta$ regression coefficients
- $\hat{\beta}$ regression coefficient estimations
- $L$ least square function
- $X$ regression variables matrix
- $y$ response variables vector
- $\chi$ primary tool lead angle
- $\Delta A_\Omega \%$ percentage tracking error with respect to the set $A_{\Omega \text{ set}}$
- $\Delta RVA \%$ percentage tracking error with respect to the set $RVA_{\text{set}}$
- $T L_s$ $TL$ estimation carried out with the $s^{th}$ formulation
- $\Omega$ Spindle speed
- $\Omega_0$ Nominal spindle speed
- $a_p$ radial depth of cut
- $A_\Omega$ amplitude of the sinusoidal modulation of the spindle speed
- $A_{B_w}$ area of the wear land on the tool flank
- $CI$ confidence interval
- $C T_{i(p+1)}$ cutting time associated to the average flank width $V B B_{i(p+1)}$
- $C T_{ip}$ cutting time associated to the average flank width $V B B_{ip}$
- $D F_j$ degrees of freedom of the factor $j$
- $f$ feed per revolution
- $F-\text{value}_j$ Fisher tests for the factor $j$
- $freq$ frequency of the sinusoidal spindle variation $SSSV$
- $H_{0j}$ null hypothesis for the $j^{th}$ paired CSM-SSSV test
- $l_{B_{ik}}$ length of the wear area used for the $V B B_{i\text{ik}}$ computation
- $m$ number of considered factors in the $2^{m}$ experimental plans
- $M S_j$ Mean Squares of the factor $j$
- $n_{rc}$ number of replicates of the central points
- $n_r$ number of replicates of tests carried out at the corner points
- $P-\text{value}_j$ $P$ value of the test for the factor $j$
- $R^2$ coefficient of determination of the regression
- $R^2_{\text{adj}}$ adjusted coefficient of determination of the regression
- $RVA$ non-dimensional amplitude variation of $SSSV$: sinusoid amplitude/$\Omega_0$
- $SE$ standard error
- $SS_{j}^{III}$ $SS_{(j|k,\ldots,r)}$ adjusted sum of squares (type III) of the factor $j$
\( SS_{(j,k,\ldots,r)} \) sequential sum of square (type I) considering the factors \( j, k, \ldots, r \) in the model

\( t \) time

\( TL \) tool life

\( TL_i \) Tool Life \( TL \) of the \( i^{th} \) tested cutting edge

\( TL_{s-error} \) percentage errors in the \( TL \) estimation adopting the \( s^{th} \) formulation

\( v_c \) cutting speed

\( VBB \) flank wear average width

\( VBB_t \) flank wear width threshold

\( VBB_{ik} \) local measurement of the flank width \( VBB \) after the \( k^{th} \) stop of the \( i^{th} \) wear test

\( VBB_{ik} \) is the \( k^{th} \) measurement of the average flank width associated to the \( i^{th} \) wear test

\( X_n \) radial coordinate - distance from the external surface (micro-hardness measurements)

### 2 Material and Methods

According to [8], even in this research experimental tool wear tests (ISO 3685, [19]) were performed both in CSM and VSM regimes. The definition of the tests was carried out adopting a Design of Experiments DOE approach. More specifically, in both the cases a full factorial scheme was used. Additional information were provided in section 2.1. The \( TL \) data were analyzed and regression models were developed. Exploiting the achieved results, a generalized \( TL \) model was developed and then properly validated.

#### 2.1 Design of Experiments

A steel turning application was selected. Indeed, for the cutting speeds typically adopted in steel machining and considering the limiting eigenmodes generally associated to tool-holder systems (70-160 Hz), it was demonstrated that SSSV can assure effective chatter suppression properties [8]. Moreover, such application is rather widespread in most of the shop-floors.

To limit the overall number of runs, two levels full factorial designs \( 2^m \) (\( m \) is the number of the considered factors) were conceived for the tests performed at CSM and for the tests that involved the sinusoidal spindle speed modulation SSSV. For both the experimental plans, the cutting velocity \( v_c \) and the feed per revolution \( f \) were the main analyzed factors. Since the SSSV can be described by Eq. 1, two additional factors were considered: RVA and \( \text{freq} \). RVA is the non-dimensional amplitude variation parameter while \( \text{freq} \) is the frequency parameter. It is worth noting that, since the spindle modulation makes the chip thickness to continuously vary [8, 13], the \( f \) parameter was even considered in the experimentation at CSM although, according to the Taylor’s theory, the effect of such parameter should be less relevant than \( v_c \).
For all the analyzed factors \((v_c, f, RVA, \text{freq})\) two levels were considered. Moreover, in order to capture possible deviations from a linear behaviour (i.e. curvatures due to second order effects) or to enhance the model error estimation as well as, center points were added to the 2\textsuperscript{nd} factorial design scheme. More specifically, to restrict the experimental effort, it was decided to carry out one single test \(n_r = 1\) for each corner condition \([20]\) while \(n_{rc} = 5\) replicates were set for the intermediate cutting conditions.

\[
\Omega(t) = \Omega_0 (1 + RVA \cdot \sin (2\pi \cdot \text{freq} \cdot t)) ; \quad A_\Omega = RVA \cdot \Omega_0
\]

The analyzed factors and the corresponding selected values, both for CSM and SSSV, were resumed in Table 1 and Table 2, respectively. The values of the analyzed factors were chosen according to \([21]\) and considering the technological limitations associated to the adopted tool (\(v_c\) and \(f\)) and lathe. Indeed, the set parameters allow continuously changing the cutting speed in the SSSV tests. Preliminary turning tests were carried out in order to check the feasibility of both the DOE plans. Moreover, it was demonstrated in \([8]\) that the selected parameters (\(RVA\) and \(\text{freq}\)) assured relevant chatter suppression properties if unstable cut occurred. It is worth noting that all the tests in this experimental campaign were executed in stable conditions. For this purpose, a radial depth of cut \(a_p = 3\ mm\) was set. All the cutting tests (both at CSM and adopting SSSV) were completely randomized. More details on the experimental set-up were reported in Section 2.2.

| Level   | \(v_c\) [m/min] | \(f\) [mm/rev] |
|---------|-----------------|----------------|
| High    | 220             | 0.3            |
| Center  | 190             | 0.2            |
| Low     | 160             | 0.1            |

Table 1 Design of experiments - CSM, factors and values

| Level   | \(v_c\) [m/min] | \(f\) [mm/rev] | \(RVA\) | \(\text{freq}\) [Hz] |
|---------|-----------------|----------------|---------|---------------------|
| High    | 220             | 0.3            | 0.3     | 1.5                 |
| Center  | 190             | 0.2            | 0.2     | 1                   |
| Low     | 160             | 0.1            | 0.1     | 0.5                 |

Table 2 Design of experiments - SSSV, factors and values

2.2 Experimental set-up

The flank wear average width \(VBB\) was monitored during the cutting tests. The flank wear threshold \(\bar{VBB}_t = 0.15\ mm\) was used as the end of the...
tool − life criteria. The cutting time that corresponds to the considered wear threshold was the main process response $TL$ for all the tested conditions. Steel bars (material 39NiCrMo3, with hardness 255 HB ($UNI$ 7845 − 78 [22])) hardened and tempered were used to perform the wear tests. The tool life tests were performed following the standard $ISO$3685 [19]. A Stereomicroscope $Optika SZN − T$ with a Motic SMZ − 168T was used to measure flank wear width $VBB$ during the cutting tests. More details on the performed wear measurements can be found in Albertelli et al. [21]. A carbide tool with a lead angle $\chi = 95^\circ$ was adopted ($ISO$ code $TNMG220404 − M5$ 5615 with a $Al2O3−TiCN$ coating) and fixed on tool holder, $ISO$ code $MTJNL2525M22$. Cutting fluid (oil-water emulsion with 5% of $Hot$ Cut 795 SC) was used in order to reproduce realistic industrial machining conditions. The lubricant was injected to the cutting zone through a flexible and adjustable nozzle, visible also in Fig. 1. It is worth noting that a proper control unit was specifically developed (adopting National Instruments hardware and software) for performing the cutting tests in VSM. Specifically in this research, a $\Omega_{set} − RVA_{set} − freq_{set}$ parameter combination can be set to the controller to perform cutting tests with the $SSSV$. The conceived solution was integrated with the drives and the numerical controller of the lathe. A preliminary version of such control unit was even described in Albertelli et al. [21].

![Fig. 1 Experimental set-up: lathe and developed control unit](image)

2.3 Preliminary verifications

Before executing the wear tests, some preliminary checks were carried out:

− lathe tracking performance verification
− workpiece hardness analysis

The first verification was performed in order to exclude any side effects of the tracking performance of the developed control unit of the spindle system,
especially when the SSSV was adopted, on the results of the experimentation. More specifically, since the spindle available torque is limited, the SSSV cannot be arbitrary implemented. Moreover, the maximum achievable speed modulation (in terms of combination of RVA and freq) depends on the nominal cutting speed. Several spindle speed tracking tests were carried out to assess the limitation of the adopted equipment, Fig. 1. All the tracking tests in the SSSV regime were executed with workpiece hold by the spindle in order to reproduce as much as possible the real cutting conditions. For each selected combination of SSSV parameters (RVA set and freq set), the imposed spindle speed $\Omega_{\text{set}}$ was progressively increased. During each run, the actual spindle speed $\Omega_{\text{meas}}(t)$ was acquired through the spindle encoder and the developed acquisition system. A Fast Fourier Transform (FFT) was carried out in order to estimate the average spindle speed $\bar{\Omega}_{\text{meas}}$, the resulted modulation $A_{\Omega_{\text{meas}}}$ and the corresponding RVA meas. In Table 3, some results were reported. The percentage tracking errors ($\Delta A_{\Omega_{\text{meas}}} \%$ and $\Delta \text{RVA}_\%$) in terms of deviation from the nominal values (respectively $A_{\Omega_{\text{set}}}$ and $\text{RVA}_{\text{set}}$) were computed. Even in Fig. 2, it can be observed that up to 1000 rpm the tracking errors are negligible while just starting from 1200 rpm they become unacceptable. Several additional tests were carried out with different combinations of RVA set-freq set.

| $\Omega_{\text{set}}$ [rpm] | RVA set | freq set [Hz] | $A_{\Omega_{\text{set}}}$ [rpm] | $A_{\Omega_{\text{meas}}}$ [rpm] | $\Delta A_{\Omega_{\text{meas}}} \%$ | RVA meas [rpm] | $\Delta \text{RVA}_\%$ |
|-----------------------------|---------|----------------|----------------|----------------|------------------|----------------|----------------|
| 700                         | 0.3     | 1              | 225            | 225.2          | +0.08            | 0.3001         | +0.03          |
| 1000                        | 0.3     | 1              | 300            | 304.6          | +1.53            | 0.3047         | +1.58          |
| 1200                        | 0.3     | 1              | 360            | 320.5          | -10.97           | 0.2707         | -9.8           |
| 1500                        | 0.3     | 1              | 450            | 271.6          | -39.65           | 0.1892         | -36.9          |

Table 3 Tracking performance verification - example ($\text{RVA} = 0.3$, $freq = 1 \text{ Hz}$)

The second verifications was carried out for investigating if the workpiece hardness changes according to the machining region. Indeed, a non homogeneous material property could have affected the reliability of the tool life tests introducing a possible bias. In order to exclude such effect, several hardness measurements were performed. Both macro-hardness and micro-hardness measures were carried out on different workpiece locations as reported in Fig. 3. Macro-hardness measurements were executed on the lateral part of the workpiece ($\varnothing = 132 \text{ mm}$) both on the external raw surface (zone C) and on the internal turned part (zone B of $\varnothing = 130 \text{ mm}$). The macro-hardness data were statistically analyzed and it was confirmed that the hardness measured in the zone C cannot be be considered different from the one measured in zone B. An additional analysis was carried out investigating the dependence of the hardness on the radial coordinate $X$: three micro-hardness repetitions (durometer Future − Tech FM − 700, Vikers, loading 1 kg, dwell time 15 s) were performed increasing the distance $X_n$ from the external surface zone B. The micro-hardness measurements were executed on the cross-section zone A, properly prepared through multiple polishing steps. The obtained results were
Fig. 2  tracking errors $RVA = 0.3$  $freq = 1\, Hz$: $A_{\Omega\, set}$ and $A_{\Omega\, meas}$

reported in Fig.4. It was statistically demonstrated that the hardness cannot be considered affected by the radial coordinate $X$.

Both the performed verifications allowed to adequately plan the experimental campaign for the wear tests. Since the experimented tracking limitations and the fact that the bars to be machined can be considered homogeneous it was decide to partially randomize the tool life tests in order to limit the wasted material. Indeed, the cutting tests at high velocity $v_c$ were carried out machining the external parts of the bars as far as the tracking limitations occurred and the remaining part of the workpieces were used for the low cutting speed tests.

Fig. 3  workpiece hardness analysis - effects of the position
2.4 Tool wear analysis

For the $i^{th}$ tested insert, a set of $n$ VBB measurements is available $VBB_i = \{VBB_{i1}, VBB_{i2}, \ldots, VBB_{ik}, \ldots, VBB_{ip}, VBB_{i(p+1)}, \ldots, VBB_{in}\}$ where $VBB_{ik}$ is the generic $k^{th}$ measurement of the average flank width (performed after the $k^{th}$ stop). According to ISO 3685 [19], the $VBB_{ik}$ was computed using Eq. 2 where $VBB_{ik,j}$ is the local measurement of the flank width (see Fig. 5). Integrating the $VBB_{ik,j}$ over the length $l_{Bik}$ of the analyzed region, the area $A_{Bik}$ of the wear land and the $VBB_{ik}$ can be computed. The $TL_i$ can be estimated linearly interpolating two subsequent cutting times ($CT_{ip}$ and $CT_{i(p+1)}$) that were associated respectively at the $p^{th}$ and the $(p+1)^{th}$ stops. Moreover, $VBB_{ip}$ and $VBB_{i(p+1)}$ fulfill the relationship reported in Eq. 4.
\[ VBB_{ik} = A_{B_{ik}}/l_{B_{ik}} = 1/l_{B_{ik}} \cdot \int_{0}^{l_{B_{ik}}} VBB_{ik} \, dl \quad (2) \]

\[ TL_i = CT_{ip} + \frac{CT_{i(p+1)} - CT_{ip}}{VBB_{i(p+1)} - VBB_{ip}} \cdot (VBB_t - VBB_{ip}) \quad (3) \]

\[ VBB_{ip} < VBB_t < VBB_{i(p+1)} \quad (4) \]

Fig. 6 Example of the evolution of the average flank wear evolution \( VBB_{ik} \) of the \( i^{th} \) test

2.5 Tool life modelling and validation

All the acquired data were statistically analyzed through the Analysis of Variance ANOVA. For what concerns the modelling approach, a multiple linear regression was adopted [20]. In the present research, the \( TL \) data or a transformation of them, were considered in the response variables vector \( y = [y_1, y_2, \ldots, y_n] \) while the main analyzed factors, a sub-combinations or a transformation of them \( (j, k, \ldots, r) \), were considered as the regression variables matrix, (i.e. \( X = [1, x_j, x_k, \ldots, x_r] \)). \( \beta = [\beta_0, \beta_j, \beta_k, \ldots, \beta_r] \) is the vector of the regression coefficients. The matrix notation was reported in Eq. 5.

\[ y = X \cdot \beta + \epsilon \quad (5) \]
The estimation of the unknown regression coefficients $\hat{\beta}$ was carried out through the least squares minimization. Indeed, Eq. 6 brings to Eq. 7.

$$L = \epsilon' \cdot \epsilon = (y - X\beta)' \cdot (y - X\beta)$$

$$\hat{\beta} = (X'X)^{-1} X'y$$

The developed models were also validated assessing the predicting capabilities on additional wear tests, executed for this specific purpose.

3 Results and Discussion

In this section the analysis of the performed wear tests was presented. The achieved tool life $TL$ data were reported in Table 4. As already explained in the previous paper section, at the beginning, the results of the cutting tests performed at $CSM$ and adopting the $SSSV$ were separately analysed. The idea underpinning the conceived experimental campaigns was based on the suspicion that not all the parameters describing the speed modulating law really affect the life of the tool $TL$. If it is the case, the data coming from each plan could be grouped and can be analysed allowing the development of a general modelling. Before doing the analysis, a natural logarithmic transformation was performed on the following factors: $TL$ (expressed in min), $v_c$, $f$, $RVA$, $freq$.

3.1 CSM modelling

The transformed data $\ln TL$, obtained in the $CSM$ experimental session, were analyzed through the ANOVA [23]. For sake of generality, according to Eq. 8, the Fisher’s statistics $F$-value$_j$ and the associated $P$-value$_j$ were computed for each generic factor $j$ considered in the model. $SS_{II}^j$ is the adjusted (type III) sum of squares for the factor $j$, obtained removing any possible confounding due to the the remaining factors added to the model, Eq. 9. $SS_{j,k,...,r}$ is the sequential sum of squares (type I) of the model considering the $j,k,...,r$ factors while $SS_{k,...,r}$ is the sum of squares (type I) considering a model with a subset of factors ($k,...,r$), thus excluding the one under investigation [23]. $MS_j$ is the resultant mean of squares ($MS_j \equiv SS_{II}^j / DF_j$) while $DF_j$ is the correspondent degrees of freedom. As expected, the main affecting factors were $\ln v_c$ and $\ln f$ while their interaction is not statistically relevant. The ANOVA results considering the main factors were reported in Table 5. For sake of completeness, the analyses performed on the residuals were summarize in Fig. 7. In addition, the regression equation (Eq. 10) was even outlined through the least square $L_{CSM}$ minimization, Eq. 6. All the estimated regression coefficients ($\hat{\beta}_{-CSM}$, $\cdots$, $\hat{\beta}_{r-CSM}$) and the corresponding 95% confidence intervals.
CI were reported in Table 6. As suggested in [23], the CI were computed estimating the standard error $SE_{CSM} = \sqrt{\text{diag} \left( \text{Cov} (\hat{\beta}_{CSM}) \right)}$. The last two columns in Table 6 reported the $t$-student $t$-test statistic values and the corresponding $P$-values that measure how the model parameters are usefulness in the proposed formulation. The coefficient of determination $R^2_{CSM} = 99.26\%$ and the adjusted statistic $R^2_{adj-CSM} = 96.92\%$.

$$F-value_j = \frac{SS_{IIIj}/DF_j}{SS_{error}/DF_{error}}$$  \hspace{1cm} (8)

$$SS_{IIIj} = SS_{(j|k,...,r)} = SS_{(j,k,...,r)} - SS_{(k,...,r)}$$  \hspace{1cm} (9)

$$y_{CSM} \equiv \ln TL_{CSM} = \hat{\beta}_0_{CSM} + \hat{\beta}_{lnv_c-CSM} \cdot \ln v_c + \hat{\beta}_{lnf-CSM} \cdot \ln f$$  \hspace{1cm} (10)

### 3.2 SSSV modelling

The described methodology (Section 3.1) was adopted even for analysing the transformed $\ln TL$ data obtained from the experimental campaign performed.
in the SSSV regime. The ANOVA results, considering the main factors (\(\ln v_c\), \(\ln f\), \(\ln RVA\) and \(\ln freq\)) and the 2-ways interactions, were reported in Table 7.

As can be observed, the relevant factors to be considered in the SSSV formulation are respectively \(\ln v_c\), \(\ln f\), \(\ln RVA\) while \(\ln freq\) seems not affecting the life of the tool. The effect of SSSV cutting can be also observed in Fig. 8 where the \(VBB_{ik}\) associated to the tests carried out in both the center points (Fig. 10, left side) were reported. Five repetitions for each cutting condition (CSM or SSSV) were performed. Although the results were affected by the process variability, it is quite evident that the tools adopted for the SSSV

### Table 5 CSM: ANOVA results

| Source       | DF | SST | MS | F-value | P-value |
|--------------|----|-----|----|---------|---------|
| Model        | 2  | 2.21858 | 1.10929 | 403.05 | 0.000   |
| Linear       | 2  | 2.21858 | 1.10929 | 403.05 | 0.000   |
| \(\ln v_c\)  | 1  | 1.43110 | 1.43110 | 519.97 | 0.000   |
| \(\ln f\)    | 1  | 0.76357 | 0.76357 | 277.43 | 0.000   |
| Error        | 6  | 0.01651 | 0.00275 |        |         |
| Curvature    | 1  | 0.00481 | 0.00481 | 2.06   | 0.211   |
| Lack of Fit  | 1  | 0.00570 | 0.00570 | 3.80   | 0.123   |
| Pure Error   | 4  | 0.00600 | 0.00150 |        |         |
| Total        | 8  | 2.23509 |        |        |         |

### Table 6 CSM: regression results

| Model Term CSM | \(\hat{\beta}_{CSM}\) | \(S\hat{\epsilon}_{CSM}\) | \(95\% CI\) | t-value | P-value |
|----------------|------------------------|---------------------------|-------------|---------|---------|
| Constant       | \(\hat{\beta}_0-\hat{\beta}_{CSM}\) = 21.035 | 0.867 | (18.914; 23.155) | 24.27   | 0.000   |
| \(\ln v_c\)   | \(\hat{\beta}_{\ln v_c-\hat{\beta}_{CSM}}\) = -3.750 | 0.164 | (-4.153; -3.348) | -22.8   | 0.000   |
| \(\ln f\)     | \(\hat{\beta}_{\ln f-\hat{\beta}_{CSM}}\) = -0.7807 | 0.0469 | (-0.8954; -0.6660) | -16.66  | 0.000   |

Fig. 7 CSM: residuals analysis
Table 7 SSSV: ANOVA results

| Source              | DF  | SSI** | MSJ | F-value | P-value |
|---------------------|-----|-------|-----|---------|---------|
| Model               | 11  | 8.22306 | 0.74755 | 77.97  | 0.000   |
| Linear              | 4   | 8.06582 | 2.01646 | 210.31 | 0.000   |
| ln $c$              | 1   | 4.63263 | 4.63263 | 483.19 | 0.000   |
| ln $f$              | 1   | 0.25841 | 0.25841 | 339.81 | 0.000   |
| ln RVA              | 1   | 0.17168 | 0.17168 | 17.91  | 0.002   |
| ln freq             | 1   | 0.00319 | 0.00319 | 0.33   | 0.578   |
| 2 way inter.        | 6   | 0.08812 | 0.01469 | 1.15   | 0.271   |
| ln $c$ · ln $f$     | 1   | 0.04813 | 0.04813 | 5.02   | 0.052   |
| ln $c$ · ln RVA     | 1   | 0.00001 | 0.00001 | 0.00   | 0.982   |
| ln $f$ · ln freq    | 1   | 0.01371 | 0.01371 | 1.19   | 0.304   |
| ln RVA · ln freq    | 1   | 0.00331 | 0.00331 | 0.03   | 0.861   |
| Curvature           | 1   | 0.00873 | 0.00873 | 0.91   | 0.365   |
| Error               | 9   | 0.08629 | 0.00959 |        |         |
| Lack of Fit         | 5   | 0.06329 | 0.01266 | 2.20   | 0.232   |
| Pure Error          | 4   | 0.02300 | 0.00575 |        |         |
| Total               | 20  | 8.30936 |      |        |         |

**Fig. 8** $VBB_{ik}$ evolution: effect of SSSV on $TL$ in the intermediate cutting conditions

cutting showed a reduced $TL$. The ANOVA residuals analysis was reported in Fig. 9. Even in this case, the regression equation was obtained (Eq. 11) as well as $R^2_{SSSV} = 97.93\%$ and the adjusted statistic $R^2_{adj-SSSV} = 97.69\%$. 
For sake of simplicity, it was decided to consider in the model formulation the amplitude modulation \( RVA \) instead of its logarithmic transformation.

\[
y_{SSSV} = \ln TL_{SSSV} = \hat{\beta}_0 - SSSV + \hat{\beta}_{\ln v_c} - SSSV \cdot \ln v_c + \hat{\beta}_{\ln f} - SSSV \cdot \ln f + \hat{\beta}_{RVA-SSSV} \cdot RVA
\]  

(11)

\[
\begin{array}{cccccc}
\text{Model Term} & \hat{\beta}_{SSSV} & SE_{SSSV} & t-value & P-value \\
\text{Constant} & 19.075 & 0.052 & 22.39 & 0.000 \\
\ln v_c & -3.376 & 0.162 & -20.9 & 0.000 \\
\ln f & -0.8186 & 0.0465 & -17.61 & 0.000 \\
RVA & -1.036 & 0.257 & -4.02 & 0.001 \\
\end{array}
\]

Table 8 SSSV: regression results

3.3 General model development

Since the frequency of the speed modulation \( freq \) resulted not to be a significant factor, therefore the unreplicated design in four factors conceived for performing the SSSV campaign can be considered a two times replicated plan with three factors: \( \ln v_c, \ln f \) and \( \ln RVA \), Fig. 10. According to the CSM and SSSV models, the velocity and feed were found to be significant factors in both the cases, while for the SSSV case even \( RVA \) resulted to be significant. Since the experimentation performed at CSM can be considered a specific realization of the SSSV formulation (setting \( RVA = 0 \)), it is interesting to
develop a general model considering the whole data-set. For sake of clarity, the resultant graphical representation was reported in Fig. 10. Moreover, the test conditions used for the model formulations validation (see section 3.4) were underlined in Fig. 10.

Before doing that, the test of equality of means was performed on the coefficients present in by both CSM (Section 3.1) and SSSV (Section 3.2) formulations: $H_0$-constant : $\hat{\beta}_0$-CSM = $\hat{\beta}_0$-SSSV, $H_0$-$\ln v_c$ : $\hat{\beta}_{\ln v_c}$-CSM = $\hat{\beta}_{\ln v_c}$-SSSV, $H_0$-$\ln f$ : $\hat{\beta}_{\ln f}$-CSM = $\hat{\beta}_{\ln f}$-SSSV. All the tests confirmed that the two proposed formulations shared the fitted parameters (the null hypotheses $H_0$ cannot be refused) and this confirmed the adequacy of a general model for interpreting all the experimental data. For sake of completeness, the ANOVA results and the residual analysis were respectively reported in Table 9 and Fig. 11.

![Diagram](image_url)

**Fig. 10** General model scheme: initial experimental scheme (left) - resulted test representation and validation test conditions (right)

| Source      | DF | $SS^{III}$ | $MS_j$ | F-value | P-value |
|-------------|----|------------|--------|---------|---------|
| Model       | 3  | 10.3793    | 3.45977| 410.21  | 0.000   |
| Linear      | 3  | 10.3793    | 3.45977| 410.21  | 0.000   |
| $\ln v_c$   | 1  | 6.0553     | 6.05527| 717.95  | 0.000   |
| $\ln f$     | 1  | 4.0659     | 4.06592| 482.08  | 0.000   |
| $\ln RVA$   | 1  | 0.2913     | 0.29135| 34.54   | 0.000   |
| Error       | 26 | 0.2913     | 0.00843|         |         |
| Curvature   | 1  | 0.0037     | 0.00374| 0.43    | 0.516   |
| Lack of Fit | 9  | 0.0995     | 0.01105| 1.52    | 0.221   |
| Pure Error  | 16 | 0.1161     | 0.00725|         |         |
| Total       | 29 | 10.5986    |        |         |         |

Table 9 general: ANOVA results
The general model equation and the corresponding identified terms were reported respectively in Eq. 12 and Table 10.

\[ y_{\text{gen}} \equiv \ln TL_{\text{gen}} = \hat{\beta}_0 - \text{gen} + \hat{\beta}_{\ln v_c - \text{gen}} \cdot \ln v_c + \hat{\beta}_{\ln f - \text{gen}} \cdot \ln f + \hat{\beta}_{\text{RVA - gen}} \cdot \text{RVA} \]  

(12)

| Model Term  | \( \hat{\beta}_{\text{gen}} \) | \( SE_{\hat{\beta}_{\text{gen}}} \) | %95 CI          | t-value | P-value |
|-------------|-------------------------------|------------------|----------------|----------|---------|
| Constant    | \( \hat{\beta}_{0 - \text{gen}} = 19.438 \) | 0.679            | (18.042; 20.844) | 28.63    | 0.000   |
| \( \ln v_c \) | \( \hat{\beta}_{\ln v_c - \text{gen}} = -3.452 \) | 0.129            | (-3.717; -3.188) | -26.79   | 0.000   |
| \( \ln f \) | \( \hat{\beta}_{\ln f - \text{gen}} = -0.8121 \) | 0.0370           | (-0.8882; -0.7361) | -21.96   | 0.000   |
| RVA         | \( \hat{\beta}_{\text{RVA - gen}} = -0.842 \) | 0.143            | (-1.136; -0.547) | -5.88    | 0.000   |

Table 10 General model: regression results

3.4 Model Validation

In order to validate the model, additional experimental tests were carried out. More specifically, new cutting conditions (both in CSM and SSSV) were tested, Fig.10. The adopted parameters and the obtained tool life \( TL \) data were reported in Table 11. In Table 12, the percentage errors \( TL_{\text{error}} \) obtained adopting the \( s \)th formulation for the tool life estimation \( TL_s \) were reported. It can be observed that the developed general model shows limited errors \( TL_{\text{gen-error}} \) (less than 6%) and a lower \( SE_{\hat{\beta}_{\text{gen}}} \) for all the shared \( \hat{\beta}_k \) with respect to the other formulations maybe because it was developed exploiting the full data set. These results even confirmed the validity of the conceived
general modelling approach. \textit{SSSV} formulation shows similar performances in terms of estimating errors (less than 5\%) while \textit{CSM} modelling exhibits a worse performance (although the error is limited to 10\%) if compared to the other modelling approaches.

\[ TL_{\text{error}} = 100 \cdot \frac{TL - \hat{TL}}{TL} \quad (13) \]

| Cutter | Test | Mode | \( v_c \) [m/min] | \( f_z \) [mm/tooth] | \( RVA \) | \( \text{freq} \) | \( TL \) [s] |
|--------|------|------|----------------|----------------|--------|---------|--------|
| 33     | 31   | \textit{SSSV} | 205 | 0.25 | 0.2 | 1 | 439.1 |
| 34     | 32   | \textit{CSM} | 205 | 0.25 | 0  | 0 | 559.2 |
| 31     | 33   | \textit{CSM} | 205 | 0.25 | 0  | 0 | 566.7 |
| 32     | 34   | \textit{SSSV} | 205 | 0.25 | 0.2 | 1 | 469.1 |

\textbf{Table 11} Tool life: tested conditions for the model validation

| Cutter | Test | Mode | \( TL_{\text{CSM}} \) [s] | \( TL_{\text{CSM}-\text{exp}} \) [s] | \( TL_{\text{SSSV}} \) [s] | \( TL_{\text{SSSV}-\text{exp}} \) [s] | \( TL_{\text{gen}} \) [s] | \( TL_{\text{gen}-\text{exp}} \) [s] |
|--------|------|------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 34     | 32   | \textit{CSM} | 518.2 | 7.92 | 535.5 | 4.44 |
| 31     | 33   | \textit{CSM} | 518.2 | 9.37 | 535.5 | 5.84 |
| 32     | 34   | \textit{SSSV} | 457.8 | 2.46 | 452.5 | 3.67 |

\textbf{Table 12} predicting modelling errors

\section*{4 Conclusions}

In this paper a generalized model for the prediction of the tool duration in steel turning when the stabilizing technique based on the sinusoidal modulation of the spindle speed \textit{SSSV} is adopted was defined and presented. The model was outlined exploiting tool wear tests performed in different cutting conditions. Since the conceived tool life model takes into account the detrimental effects of spindle speed on tool life \( TL \), it could be used to widely analyse the economic feasibility of the technique in different cutting scenarios and applications. The following results can therefore be summarised:

- The statistical analysis performed on the experimental data confirmed that the continuous modulation of the spindle speed, while keeping the feed constant, negatively affects the achievable useful life of the tool.
- From the experimental session carried out with the sinusoidal spindle speed modulation it was found that the modulating frequency \( \text{freq} \) does not affect the tool duration.
- A specific tool life model formulation for \textit{SSSV} was developed exploiting the experimental test results. The normalized amplitude of the spindle modulation \( RVA \), together the other affecting parameters (cutting speed \( v_c \) and feed \( f \)), was included in the model.
According to the previous reported finding, it was possible to develop a generalized tool life model formulation that can indifferently be used both in regular cutting regime CSM and when the SSSV is adopted.

The generalized formulation was validated performing additional tool life tests. It was found that it capable of predicting the useful tool life within an maximum estimating error of 6%.

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