The Knee and the Second Knee of the Cosmic-Ray Energy Spectrum

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Abstract
The cosmic ray flux measured by the Telescope Array Low Energy Extension (TALE) exhibits three spectral features: the knee, the dip in the $10^{16}$ eV decade, and the second knee. Here the spectrum has been measured for the first time using fluorescence telescopes, which provide a calorimetric, model-independent result. The spectrum appears to be a rigidity-dependent cutoff sequence, where the knee is made by the hydrogen and helium portions of the composition, the dip comes from the reduction in composition from helium to metals, the rise to the second knee occurs due to intermediate range nuclei, and the second knee is the iron knee.

1. Introduction

The spectrum recently measured by the TALE fluorescence detector of the Telescope Array experiment covers the energy range, $10^{15.3} \text{ eV} < E < 10^{18.3} \text{ eV}$, and represents the first time that fluorescence telescopes have observed this low in energy. In this energy range there are three spectral features. In the TALE spectrum the knee appears as a broad maximum centered at $10^{15.6} \text{ eV}$, there is a broad dip centered at $10^{16.2} \text{ eV}$, and the second knee occurs at $10^{17.04} \text{ eV}$. The energy scale of TALE is the same as that of the entire TA experiment, and is consistent with the energy of the GZK cutoff which is observed at $10^{19.75} \text{ eV}$ [4]. The energy resolution

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of the TALE fluorescence detector is about 15% and is constant as a function of energy. This represents the first time that this energy range has been observed calorimetrically, with excellent energy resolution.

The Kascade experiment [5] was the first to see the systematic change in composition as the energy increases above the knee of the spectrum. This observation was made using ground array detectors sensitive to the muonic and electromagnetic components of cosmic ray air showers. Although the specific changes in composition they described were model-dependent, the interpretation was widely accepted as representing a rigidity-dependent cutoff sequence at the end of the galactic cosmic ray spectrum. Their interpretation was that the hydrogen knee was at about $10^{15.5}$ eV. For a rigidity-dependent cutoff sequence, this would put the iron knee at $10^{16.9}$ eV. However the Kascade experiment could not make reliable measurements this high in energy.

Subsequent experiments have seen the three spectral features, with different energy scales. Figure 1a shows the spectra of TALE [1], Telescope Array (TA) [6], HiRes [7], Pierre Auger Observatory [8], IceTop [9], and Kascade-Grande [10]. Some of the differences in the spectra seem to be due to different energy scales of experiments. Figure 1b shows the same spectra, but with the energy scale of IceTop reduced by 9.2% and the Auger energy scale increased by 10.2%. Here the IceTop and TALE spectra agree up to about $10^{17.5}$ eV, the TA and Auger spectra agree up to about $10^{19.5}$ eV. TALE, HiRes, and Kascade-Grande agree without energy scale adjustment. Thus there is a broad consensus that there exist features called the knee, the dip in the $10^{16}$ eV decade, and the second knee, and that we have a good idea of the energies at which the features occur.

In what follows we argue that the second knee is the iron knee, and the spectrum is consistent with a rigidity-dependent cutoff sequence. First we argue on general grounds, then we present three models that illustrate fits to the spectrum.

### 2. The TALE Experiment

The TALE experiment consists of two sets of detectors of cosmic rays: a fluorescence detector (FD) consisting of 10 telescopes that look high in the sky (at elevations from 31° to 59°), and a surface detector consisting of 103 scintillation counters forming an infill array in front of the FD. The TALE spectrum in reference [1] was based on observations made with the FD alone. Events seen by the FD below about $10^{17}$ eV were dominated by Cherenkov light from cosmic ray shower particles, but above $10^{17.5}$ eV, fluorescence light was the largest contribution to the signal. In the intermediate energy range, a mixture of Cherenkov and fluorescence light pertained. The
energy resolution of the TALE FD is about 15%, independent of energy, and the spectrum measured is independent of models.

3. The Rigidity-dependent Cutoff Sequence

The basic idea of a rigidity dependent cutoff sequence is that in a cosmic ray accelerator a moving magnetic field accelerates particles. The maximum energy achieved by the accelerator depends upon the strength of the magnetic field, its speed of movement, and the time duration that particles are in contact with it. In this situation the maximum energy of nuclei will be proportional to their charge; i.e., the cutoff rigidity (energy/charge) is the same for all nuclear species. Thus if the maximum energy of hydrogen nuclei is \( E \), for helium it is \( 2 \times E \), carbon is \( 6 \times E \), etc. Practically speaking, the sequence ends with iron, at an energy of \( 26 \times E \). If we start with the interpretation of the Kascade experiment as a guide, but using the TALE spectrum energy scale, the second knee can be identified as the iron knee, at \( 10^{17.04} \) eV. Dividing by 26, the proton maximum of the spectrum would be \( 10^{15.6} \) eV, and the helium maximum would be at \( 10^{15.9} \) eV. The broad maximum of the knee is then identified as the result of H and He coming to their maximum energies.
abundance of metals is considerably lower than H and He, so a dip in the spectrum should occur at higher energies. In all cosmic scenarios, the abundance of Li, Be, and B is very low, which enhances the dip. Although the CNO group is more abundant, it is much less so than H and He. Thus, there should be a broad minimum in the spectrum of a rigidity-dependent cutoff sequence, with a rise near the carbon location of $10^{16.4}$ eV. This is what is seen in the TALE spectrum. Due to intermediate weight nuclei the spectrum rises (on an $E^3 J$ plot), then peaks at Fe.

4. The Extragalactic Contribution

One detail that must be taken into account is the low energy end of the extragalactic cosmic ray flux. All experiments with fluorescence detectors that can measure the depth of shower maxima indicate that between $10^{18.0}$ and $10^{18.5}$ eV the composition is very light, and probably protonic [11] [12] [13] [14]. If these cosmic rays originated within our galaxy, there would be considerable anisotropy in their arrival directions, but this is not seen either in the northern [15] or southern [16] hemispheres. Hence, one expects these protons to be of extragalactic origin. This is a general result. Using galactic magnetic field models, one limit [15] has been put that (at 95% confidence level) $< 1.6$% of cosmic rays are of galactic origin. By extension, an extragalactic flux of light composition must extend down to energies lower than $10^{18}$ eV. One can estimate the contribution of extragalactic protons in the $10^{17}$ eV decade using measurements of Xmax, the depth of shower maximum. The HiRes-MIA [17] and Auger [18] measurements indicate that at $10^{17}$ eV the mean of Xmax is midway between what is expected for hydrogen and iron. Perhaps half of cosmic rays at this energy are extragalactic protons.

5. Three Specific Models of the $10^{15} - 10^{18}$ Decades

Here we wish to present three models of the rigidity-dependent cutoff sequence in comparison with the TALE spectrum: the H4a model by T. Gaisser [19], a model by T. Gaisser, T. Stanev, and S. Tilav (GST) [20], and a model taken from direct measurements of composition in the $10^{13}$ eV decade [21] and extrapolated to higher energies by 2.5 decades. The H4a and GST models include an extragalactic component, but in the direct measurement model we have supplied an estimate of the extragalactic flux. The H4a model assigns the features of the spectrum to three populations of cosmic rays, two of galactic origin and the third of extragalactic origin. The first population forms the knee, and the second population makes the second knee. The dip in the $10^{16}$ eV decade seems to be poorly expressed in this model. Figure 2 shows the comparison of the H4a model to the TALE spectrum. Figure 3
shows the comparison of the GST model (which also uses three populations) to the TALE spectrum. The GST model might have a higher energy scale than the TALE data.

In the third model, we attempt to recreate the observed spectrum from extrapolating the measured composition at lower energies, and applying a rigidity-dependent cutoff for the termination of the dominant galactic components. We also assume a very simple phenomenological model for an extragalactic component at higher energies. Hence this is a model of 2 populations, one galactic and one extragalactic.

The known lower-energy data is obtained by extrapolation from the Particle Data Group’s compilation given in Figure 29.1 of the Particle Data Book [21]. Assuming the 11 nuclei displayed are the dominant species, we interpolated relative abundances at $10^{13}$ eV. These values (normalized to one) are shown in Table 1. Other species are neglected in our model. In this energy regime, all species shown in Figure 29.1 appear to follow a common power law $E^{-\alpha}$ with an index of $\alpha = 2.8$. We assume this value in our model.

We attempt to explain the occurrence of the knee and the second knee as the result of terminations in the acceleration of the 11 nuclei. The flux observed on Earth could be dominated by one local and recent galactic source, or a class of sources. In such a scenario, if protons are accelerated up to a cut-off energy $E_p$, then the cut-off energies for heavier species should be given by $ZE_p$. However, an abrupt, step-function cut-off is clearly unphysical. Cut-offs are often modeled as an exponential

![Figure 2: Cosmic Ray spectrum measured by TALE between $10^{15.3}$ eV and $10^{18.3}$ eV, overlaid with the H4A model described in this section.](image)
Figure 3: Cosmic Ray spectrum measured by TALE between $10^{15.3}$ eV and $10^{18.3}$ eV, overlaid with the GST model described in this section.

decay above the break, resulting from, for instance Bohm diffusion. However, it is possible that there are additional, weaker sources that extend to higher energies still. We therefore model the break by a broken power law where the slope of the spectrum in each case changes from $\alpha = 2.8$ to a larger (steeper) value $\beta$.

The extra-galactic component is assumed to follow a simple power law with a log-exponential cut-off represented by

$$\log_{10}[J_{XG}(E)] = B + (1 - e^{\frac{-E}{c}}) - \gamma x$$

(1)

where $x = \log_{10} E$, and the $-\gamma x$ term represents the extrapolation of the piece-wise power-law spectrum previously seen below the ankle feature. The low-energy cut-off for the extra-galactic component is assumed to occur at $10^c$ eV, smoothed by a broadening of the cut-off with width $d$. This form for the cut-off was chosen for its simplicity.

Figure 4 shows the TALE energy spectrum overlaid with the simple model described in the previous paragraphs. Table 2 lists the parameter values that combine to give a total flux that gives good agreement to the shape of TALE data. The group contributions of H+He, C+O, and Fe are shown separately in addition to the total galactic component.
Table 1: Normalized abundances, at $10^{13}$ eV, of the 11 nuclei from the Particle Data Book

| Element  | Z | fraction at $10^{13}$ eV |
|----------|---|-------------------------|
| hydrogen | 1 | 0.3019                  |
| helium   | 2 | 0.4104                  |
| carbon   | 6 | 0.0388                  |
| oxygen   | 8 | 0.0745                  |
| neon     | 10| 0.0153                  |
| magnesium| 12| 0.0293                  |
| silicon  | 14| 0.0308                  |
| sulfur   | 16| 0.0082                  |
| argon    | 18| 0.0043                  |
| calcium  | 20| 0.0070                  |
| iron     | 16| 0.0800                  |

Table 2: Model parameters used to calculate the TALE spectrum

| symbol | value | explanation                                      |
|--------|-------|--------------------------------------------------|
| $\alpha$ | 2.80  | galactic spectral power index below cut-off energy |
| $\beta$ | 4.20  | galactic spectral power index above cut-off energy |
| $E_p$ | $10^{15.60}$ eV | cut-off energy for protons                      |
| $\gamma$ | 3.24  | extra-galactic spectral index                    |
| $d$     | 0.55  | width of extra-galactic cut-off                  |
| $c$     | 16.4  | log-Energy of cut-off of extra-galactic flux      |

6. Conclusions

The TALE spectrum shows three spectral features which resemble the effects of a rigidity-dependent cutoff sequence, which could occur at the high energy end of the galactic cosmic ray spectrum. The general features of such a sequence are: a broad knee made from H and He, a drop in spectrum (a dip) caused by the much lower abundance of metals, and a rise to an iron knee marking the very end of the galactic spectrum. Identifying the second knee seen in the TALE spectrum with the iron knee, the energies of the features seen correspond closely with this general picture. One can also compare specific models of the rigidity-dependence sequence to the TALE spectrum. The H4a model uses three populations to form the knee, the second knee, and an extragalactic population, but overshoots the dip. The GST model has a stronger dip. A model using the PDG compendium as a starting point can form an acceptable fit to the spectrum when one adds an extragalactic component. In all
Figure 4: Cosmic Ray spectrum measured by TALE between $10^{15.3}$ eV and $10^{18.3}$ eV, overlaid with the phenomenological model described in this section. The galactic component, extrapolated from Figure 29.1 of the Particle Data Book, is shown by the red dot-dashed curve. The proton acceleration cut-off is placed at $10^{15.6}$ eV. Pre-break post-break power indices of $\alpha = 2.80$ and $\beta = 4.20$ are assumed, respectively, for all species. The galactic component is described by a power law of index $\gamma = 3.24$, with a cut-off at $10^{16.4}$ eV and width of $c = 0.55$.

In these cases we identify the second knee at $10^{17.04}$ eV as the rigidity-dependent cutoff of iron.

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