Factorization vs. Flavor SU(3) in Charmless $B$ Decays

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Abstract

Two types of predictions for charmless $B$ decays are compared. One involves estimates based on factorization and models for form factors, while the other involves the use of flavor SU(3), sometimes with assumptions about the smallness of certain amplitudes. After a comparison of some factorization predictions with recent data, specific decays of $B$ mesons to two charmless mesons are discussed.

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1 Introduction

The weak decays of hadrons are simpler when the weak current couples to a lepton pair than when it couples to a hadron, which can re-interact with the rest of the system. However, the effects of this re-interaction in some cases can be neglected or evaluated, permitting the calculation of the decay rate and individual helicity amplitudes. In such cases one is employing the factorization hypothesis. An early version of this hypothesis has recently been justified for certain classes of decays.

We shall contrast applications of factorization, which often require models for form factors, with a more general approach based on flavor SU(3) in which data are used to evaluate a set of reduced matrix elements but no assumptions are made about form factors. We review in Section 2 some successes of factorization in final states containing a heavy meson. Present data are compared with early predictions, and found consistent with them. The successful predictions involve color-favored cases, in which weak currents couple to a quark pair ending up in the same meson. By contrast, when the weak current produces a pair of quarks which end up in different mesons (color-suppressed processes), as well as for penguin diagrams, we shall question the applicability of factorization.

We compare factorization and flavor SU(3) for $B$ meson decays to charmless final states: $B \to PP$ in Section 3 and $B \to PV$ in Section 4, where $P$ is a light pseudoscalar meson ($\pi$, $K$, $\eta$, or $\eta'$) and $V$ is a light vector meson ($\rho$, $\omega$, $K^*$, or $\phi$). For $B \to PP$ we discuss the possible origin of the small $B \to \pi^+\pi^-$ branching ratio. We then mention recent applications of flavor SU(3) to $B \to PP$ decays, and remark upon the large branching ratios for the decays $B \to K\eta'$. For $B \to PV$ we compare an updated flavor-SU(3) analysis with others more dependent upon factorization and explicit form factors. We conclude in Section 5.

2 Some successes of factorization

2.1 Semileptonic decays

Semileptonic $b \to \ell\nu u$ or $b \to \ell\nu c$ decays are tractable for several reasons. (1) The weak current is a color singlet, so its effect “factorizes.” The leptons
to which it couples can be treated in isolation from the rest of the problem.

(2) When both the initial and final hadrons are heavy, the decays are char-
acterized by a single universal (“Isgur-Wise”) form factor [10]. (3) The form
factors are measurable via the effective mass distribution of the lepton pair
and the angular distributions of the decay products.

2.2 Nonleptonic decays

Nonleptonic decays involve final-state interactions between the hadronic sys-
tems comprising the two interacting weak currents. The corresponding form
factors are not always measurable. Nevertheless, in some cases these nonlep-
tonic decays can be treated by a factorization hypothesis.

Color-favored nonleptonic decays of a $bq$ meson involve such subprocesses
as $b \rightarrow \pi^+ \bar{c}$ or $b \rightarrow \pi^+ \bar{u}$. The coupling of the weak current to the $\pi^+$ (or
another light meson) is described by a directly-measurable decay constant,
while the amplitude for the $\bar{c}$ or $\bar{u}$ quark to form a hadron with the spectator
quark $q$ is described by one or more measurable form factors. The interaction
of the light meson with the rest of the system is an effect of order $\Lambda_{QCD}/m_b$
and may be neglected for a lowest-order estimate; a framework for calculating
corrections has recently been established [2].

Color-favored decays with current coupling to heavy mesons satisfy fac-
torization (at least for $D_s$ and $D_s^*$ production), though there is no corre-
sponding theoretical justification. In contrast to the case in which the weak
current produces a light meson, the heavy meson produced by the weak
current does not escape the interaction rapidly enough to avoid significant
interaction with the rest of the system [1, 2].

Color-suppressed or penguin amplitudes are not of leading order in $1/N_c$,
where $N_c$ is the number of quark colors, or involve the application of pertur-
bative QCD under questionable circumstances. The corresponding factoriza-
tion predictions for helicity amplitudes are not obeyed, and the correspond-
ing form factors are not directly measurable since they involve an effective
flavor-changing neutral current.

2.3 Application to $D^{(*)}\ell\nu$ and $D^{(*)}\pi$ decays

The decays $B \rightarrow \bar{D}^{(*)}\ell\nu$ and $B \rightarrow \bar{D}^{(*)}\pi$ involve a universal [10] form factor
which is a function of the dimensionless variable $z^2 \equiv (v-v')^2$, where $v$ and $v'$
are the four-velocities of the initial and final heavy meson. [One often speaks of the variable \( w = v \cdot v' \), related to \( z \) by \( z^2 = 2(1 - w) \).] Defining the four-momentum of the lepton pair or hadron to which the weak current couples as \( q \), and the dimensionless variable \( y = q^2/m_B^2 \), we can write, adopting a single-pole \( [3] \) universal form factor:

\[
   z^2 = \frac{q^2 - (m_B - m_D)^2}{m_B m_D} , \quad \frac{d\Gamma(B \to D^{(*)}\ell\nu)}{dy} \sim \text{kinem. factor} \left[ \frac{1}{1 - (z^2/z_0^2)^2} \right] , \quad (1)
\]

where \( z_0 \) is related to the slope parameter \( \rho^2 \) (see, e.g., \([11]\)) by \( \rho^2 = 2/z_0^2 \).

In Fig. 1 we plot some predictions \([3]\) of rates for \( B \to D^{(*)}\ell\nu \) and \( B \to D^{(*)}\pi \) decays as a function of \( \rho^2 \). Horizontal bands denote \( \pm 1\sigma \) error bars based on present averages \([12]\). The corresponding vertical bands are consistent with a value of \( \rho^2 \) around 2 or \( z_0 \) around 1, not far from the value \( z_0 = 1.12 \pm 0.17 \) found in Ref. \([3]\) (where the variable we now call \( z \) was denoted by \( w \)). Also shown is a recent branching ratio, \( \mathcal{B}(B^0 \to D^{*-}\ell^+\nu) = (5.66 \pm 0.29 \pm 0.33)\% \), and a value \( \rho^2 = 1.67 \pm 0.11 \) reported by the CLEO Collaboration \([13]\).

### 2.4 Current producing \( D_s^{(*)} \) in \( B^0 \) decays

In Table 1 we compare some factorization predictions \([3]\) of light-meson and \( D_s^{(*)} \) production with experiment \([12, 14]\). The \( D_s^{(*)} \) predictions are as well obeyed as those for the light mesons.

An additional prediction involving heavy meson production by the weak current \([3]\) is that \( \mathcal{B}(B^0 \to D^{*+}D^{*-})/\mathcal{B}(B^0 \to D^{*+}D_s^-) = 0.13(f_D/f_{D_s})^2 \simeq 0.09 \), where \( f_D \) and \( f_{D_s} \) are the decay constants for the nonstrange and strange \( D \) mesons. The experimental value for this ratio \([13]\) is \( 0.06_{-0.03}^{+0.04} \).

The decays of spinless particles to two vector mesons are describable \([16]\) by amplitudes \( A_0 \) (longitudinal polarization), \( A_\parallel \) (linear parallel polarization) and \( A_\perp \) (linear perpendicular polarization), normalized such that \( |A_0|^2 + |A_\parallel|^2 + |A_\perp|^2 = 1 \). Factorization predicts \( (|A_0|^2, |A_\parallel|^2, |A_\perp|^2) = (88, 10, 2)\% \) for \( \overline{B}^0 \to D^{*+}D^{*-} \) and \( (55, 39, 6)\% \) for \( \overline{B}^0 \to D^{*+}\rho^- \). Experimental values are only quoted for \( |A_0|^2 \): \( (87.8 \pm 3.4 \pm 3.0)\% \) for \( \overline{B}^0 \to D^{*+}D^{*-} \) \([13]\) and \( (50.6 \pm 13.9 \pm 3.6)\% \) for \( \overline{B}^0 \to D^{*+}\rho^- \) \([17]\). These agree with the predictions, as does the intermediate case of \( \rho' \) production \([18]\).
2.5 Color-suppressed and penguin amplitudes

The subprocess $\bar{b} \to \bar{c}c \bar{s} \to J/\psi \bar{s}$ is an example of color-suppression since the $\bar{c}c$ pair is not automatically produced in a color singlet. It is responsible for the decays $B \to J/\psi K^*$. The application of factorization to such decays is risky. (1) There is no independent measurement of the $B \to K^*$ form factor, which would involve a flavor-changing neutral current. (2) Factorization does not predict the helicity amplitudes properly in $B \to J/\psi K^*$ [13]. (3) Final-state phases observed between different helicity amplitudes for $B \to D^* \rho$ [17] and $B \to J/\psi K^*$ [20] are not predictable by factorization, and may indicate the importance of non-perturbative effects. (4) QCD corrections to color-suppressed amplitudes are important. For example, the amplitude for $B^0 \to D^- \pi^+$ is purely color-favored, while that for $B^- \to D^0 \pi^-$ contains also a color-suppressed contribution which interferes constructively with the color-favored amplitude. This is in contrast to charmed particle decays, where the color-suppressed and color-favored decays interfere destructively.

Similar cautionary remarks apply to the use of factorization for penguin amplitudes. (1) Perturbative calculations of penguin contributions to processes such as $B \to K\pi$, where they seem to be dominant, fall short of actual measurements [21]. One possible explanation is the presence of a $c\bar{c}$ loop with substantial enhancement from on-shell states, equivalent to strong rescattering from such states as $D_s \bar{D}$ to charmless meson pairs. In this case, penguin amplitudes could have different final-state phases from tree amplitudes, enhancing the possibility of observing direct CP violation. (2) Other hints that $c\bar{c} \to q\bar{q}$ rescattering may be important include the suppression of the $B$ semileptonic branching ratio with respect to theoretical expectations, the deficit of charmed particles in $B$ decays, and the large rate for inclusive and exclusive $\eta'$ production [22].

2.6 Further $B \to VV$ information

For decays of a spinless particle to two vector mesons, the amplitudes $A_0$ and $A_\parallel$ are linear combinations of partial waves $\ell = 0, 2$, while the amplitude $A_\perp$ corresponds to $\ell = 1$. In decays to CP eigenstates, the even- and odd-$\ell$ partial waves correspond to opposite CP-parities: e.g., for $B_s \to J/\psi \phi$, to even and odd CP, respectively. The decay $B \to J/\psi K^*$ is related to $B_s \to J/\psi \phi$ by
flavor SU(3) [16], so the helicity structures of the two decays should be the same. In Table 2 we compare CDF information on helicities for both decays [20] with CLEO [23] and BaBar [24] results on $B \to J/\psi K^*$. In all cases the parity-odd fraction $|A_\perp|^2$ is small, indicating that $B_s \to J/\psi \phi$ occurs mostly from the CP-even mixture of $B_s$ and $\bar{B_s}$. This will help in searching for lifetime differences between the CP-even and CP-odd states [16, 25].

A 1998 CLEO analysis [17] suggested non-zero relative final state phases between partial waves in $B \to D^* \rho$. Such final-state phases are of interest in the more general context of final-state interactions, which are usually thought to be small at a c.m. energy of $m_b c^2$. The existence of three partial waves (S,P,D) for such $B \to VV$ decays, as well as for final states with light mesons such as $B^0 \to \phi K^{*0}$, means that helicity analyses can detect the presence of final-state interactions which could be relevant to the question of rescattering and final-state interactions in decays such as $B \to PP$ [22].

3 Decays to two light pseudoscalars

A flavor-SU(3) decomposition of the decays $B \to PP$, where $P$ denotes a light pseudoscalar meson, is given in Table 3 [26]. Here $T$, $C$, $P$, and $P_{EW}$ denote color-favored tree, color-suppressed tree, penguin, and color-favored electroweak penguin (EWP) amplitudes, respectively. We omit exchange, annihilation, and color-suppressed EWP amplitudes. The “singlet-penguin” term $S'$ is needed whenever one final meson (such as $\eta, \eta'$) has a flavor-singlet component. These amplitudes describe the decays in Table 4, based on reports by CLEO [27, 28], BaBar [14], and BELLE [29] at the 2000 Osaka Conference, and some earlier values [30]. Remarks:

(1) The $K\pi$ rates should be dominated by penguin amplitudes. Expanding to lowest order in the remaining amplitudes yields the sum rule [31]

$$B(K^+\pi^-) + \frac{\tau^0}{\tau^+}B(K^0\pi^+) = 2 \left[ B(K^0\pi^0) + \frac{\tau^0}{\tau^+}B(K^+\pi^0) \right] ,$$

where $\tau^0/\tau^+ = 0.94 \pm 0.03$ is the ratio of $B^0$ and $B^+$ lifetimes. The left-hand side is $32 \pm 4$, while the right-hand side is $57 \pm 11$. The rates involving $\pi^0$ production may have been overestimated experimentally; little could go wrong with the sum rule unless penguin dominance turns out to have been a very poor approximation.
For now, complete penguin dominance of the amplitudes for $B \rightarrow K\pi$ is as good as the sum rule (2). One doesn’t see any compelling pattern of the subsidiary (tree and EWP) amplitudes.

(3) The ratio $\Gamma(B^+ \rightarrow K^0\pi^+)/2\Gamma(B^+ \rightarrow K^+\pi^0)$ is $0.69 \pm 0.22$. Its value can provide a constraint on the weak phase $\gamma = \text{Arg}(V_{ub}^*)$.

(4) The ratio $\Gamma(B^0 \rightarrow K^+\pi^-)/\Gamma(B^+ \rightarrow K^+\pi^0)$ is $0.92 \pm 0.24$, compatible with 1. If it were less than 1, one could establish an upper bound on $\sin^2\gamma$.

Several $B \rightarrow PP$ decays are amenable to a flavor-SU(3) treatment.

3.1 Tree-penguin interference in $B^0 \rightarrow \pi^+\pi^-$?

We shall quote all rates in units of $(B^0$ branching ratio $\times 10^6)$. Thus, the average of $B^0 \rightarrow \pi^+\pi^-$ branching ratios in Table 5 implies (updating [30])

$$|T|^2 + |P|^2 - 2|TP| \cos \alpha \cos \delta = 5.6 \pm 1.3$$ (3)

where $\alpha$ is a CKM phase and $\delta$ is a strong phase difference between tree and penguin amplitudes. From $B^+ \rightarrow \pi^+\pi^0$ one infers $|T + C|^2/2 = (4.6 \pm 2.0)(\tau^0/\tau^+)$ (see Tables 3 and 4), and with $\text{Re}(C/T) = 0.1$ (cf. [2]) one estimates $|T| = 2.7 \pm 0.6$. Meanwhile the penguin amplitude can be estimated from $B^+ \rightarrow K^0\pi^+$: $|P'|^2 = (17.9 \pm 4.1)(\tau^0/\tau^+)$, $|P'| = 4.1 \pm 0.5$, $|P| = \lambda |P'| = 0.9 \pm 0.1$, where $\lambda \simeq 0.22$ is a parameter describing the hierarchy of CKM matrix elements. Combining these results, we find $\cos \alpha \cos \delta = 0.5 \pm 0.7$, less than 1$\sigma$ evidence for destructive interference. Our conclusion is thus more guarded than some based on factorization and explicit form factors [5, 6].

3.2 Information on $K\eta'$ decays

Given estimates of $|P'|$ and $|P'_{EW}|$, and constructive $S'\pi-P'$ interference, one needs a modest singlet-penguin contribution $|S'| = (0.6 \pm 0.2)|P'|$ to account for the large branching ratios for $B^+ \rightarrow K^+\eta'$ and $B^0 \rightarrow K^0\eta'$. Its size may be a problem for explicit models (e.g., [7, 36]), but not for flavor SU(3). The $K\eta'$ rate is also large as a result of constructive interference between nonstrange and strange quark contributions of $\eta'$ to the ordinary penguin amplitude $P'$. 

6
Both the singlet penguin and ordinary penguin contributions are much smaller for the $K\eta$ final states \[26\], which are so far not observed.

3.3 Measuring $\gamma$ with $K\pi$ decays

The Fermilab Tevatron and the CERN Large Hadron Collider will produce large numbers of $\pi^+\pi^-$, $\pi^\pm K^\mp$, and $K^+K^-$ pairs from neutral $B$ mesons. (1) The processes $B^0 \rightarrow K^+K^-$ and $B_s \rightarrow \pi^+\pi^-$ involve only the spectator-quark amplitudes $E$ and $PA$, and thus should be suppressed. They are related to one another by a flavor SU(3) “U-spin” reflection $s \leftrightarrow d$ \[38\]. (2) The decays $B^0 \rightarrow \pi^+\pi^-$ and $B_s \rightarrow K^+K^-$ also are related to each other by a U-spin reflection. Time-dependent studies of both processes allow one to distinguish strong and weak interaction information and to measure the angle $\gamma$ \[39\]. This appears to be a promising method for Run II at the Fermilab Tevatron \[40\]. (3) The decays of non-strange and strange neutral $B$ mesons to $K^\pm\pi^\mp$ provide another source of information on $\gamma$ when combined with information on $B^+ \rightarrow K^0\pi^+$ \[41\]. An error of 10° on $\gamma$ seems feasible, and a relation between the CP-violating rate asymmetries for the strange and nonstrange decays to $K^\pm\pi^\mp$ provides a check of the flavor-SU(3) assumption.

4 Decays to one pseudoscalar and one vector meson

4.1 Flavor SU(3) and $B \rightarrow PV$ decays

For decays $B \rightarrow PV$ there are twice as many amplitudes as for $B \rightarrow PP$ since the spectator quark can end up either in the pseudoscalar meson or the vector meson. We label amplitudes with a subscript $P$ or $V$ corresponding to the meson containing the spectator, using small letters to denote amplitudes corrected for EWP contributions. Present data from CLEO \[28\], \[43\], BaBar \[14\], and BELLE \[29\]) provide partial information on individual amplitudes. One neglects special amplitudes associated with the flavor-singlet components of the $\omega$ and $\phi$. In contrast to the $\eta$ and $\eta'$, these mesons couple to other matter only through connected quark diagrams, respecting the Okubo–Zweig–Iizuka (OZI) rule. The decay $B^+ \rightarrow \pi^+\omega$ is then dominated by $t_V$, while both $B \rightarrow \phi K$ charge states are dominated by $p'_P$ (including
a non-negligible EWP contribution). If the penguin amplitude involves an intermediate state including only a quark-antiquark pair, Lipkin has argued that one must have $p_V = -p_P$ \cite{30, 42}. In factorization models $p_V$ tends to be smaller in magnitude than this estimate. This is a key difference between the flavor-SU(3) and factorization approaches.

It is harder to learn $t_P$, which is expected to dominate $B^0 \to \rho^+\pi^-$. That decay must be distinguished via flavor-tagging from $\bar{B}^0 \to \rho^+\pi^-$, dominated by $t_V$. Present data quote only the sum of the two modes \cite{28, 43}. Nonetheless, within wide errors it is possible to estimate $|t_P|^2$ and $|t_V|^2$ separately.

4.2 Comparison of data and predictions

In Tables 5 and 6 we compare $B^+ \to PV$ and $B^0 \to PV$ data with predictions of the flavor-SU(3) scheme \cite{30} and factorization models \cite{9}. The range of SU(3) predictions is given neglecting tree-penguin interference, but rates can exceed or be less than the italicized values if $\gamma > 90^\circ$ or $\alpha < 90^\circ$ and strong final-state phase differences are small. In the opposite cases of $\gamma < 90^\circ$ or $\alpha > 90^\circ$ the rates can exceed or be less than the bold-faced values.

The data must improve in accuracy to distinguish flavor-SU(3) predictions from explicit models. One needs to separate $B^0 \to \rho^+\pi^-$ from $\bar{B}^0 \to \rho^+\pi^-$ in order to separate $t_P$ from $t_V$ adequately. Both approaches agree on which “signals” should be real and which are statistical fluctuations.

There is hope of learning whether tree and penguin amplitudes are interfering constructively (e.g., in $B^0 \to K^{*+}\pi^-$ and $B^+ \to K^{*+}\eta$) or destructively in specific processes, but one cannot yet do this reliably. Let us see where present data stand on the first of these.

4.3 Tree-penguin interference in $B^0 \to K^{*+}\pi^-$

The branching ratio for $B^0 \to K^{*+}\pi^-$ quoted in Table 6 implies that

$$|p_P'|^2 + |t_P'|^2 - 2|t_P' p_P'| \cos \delta \cos \gamma = 22^{+9}_{-8} > 12 \ (90\% \ c.l.)$$

(4)

where we are using units of $(b.r. \times 10^6)$. At the same time, averaging charged and neutral modes, $B(B \to \phi K)$ implies

$$|p_P' - \frac{1}{6} p_P'|^2 = 6.2^{+2.0+0.7}_{-1.8-1.7}, \ |p_P'| = 3.0^{+0.5}_{-0.5} \ ,$$

(5)
where the term $-(1/6)p'_{P}$ is an estimate of EWP effects $^{30, 44}$. To estimate $|t'_{P}|$ we must use $B^{+} \rightarrow (\rho^{0}, \omega)\pi^{+}$ and $B^{0} \rightarrow \rho^{\pm}\pi^{\pm}$ $^{30}$, finding $|t_{P}| \leq 4.5$, $|t'_{P}| \leq 1$. Thus the inequality (4) weakly favors constructive $t'_{P}-p'_{P}$ interference in $B^{0} \rightarrow K^{*+}\pi^{-}$. For $\cos\delta > 0$ this would require $\cos\gamma < 0$, as has been claimed in several factorization-based calculations $^{5, 6}$. Within the more general flavor-SU(3) treatment a firm conclusion requires many of the input branching ratios to be better measured.

5 Conclusions

- Naïve factorization works well for color-favored processes, including some for which it is “not proven.”

- Be wary of “factorization” results for color-suppressed or penguin amplitudes. They actually contain considerable phenomenological input.

- Flavor SU(3), supplemented with electroweak penguin calculations and assumptions about rescattering (such as the neglect of exchange and annihilation contributions) can lead to many useful relations, e.g., between $B \rightarrow \pi\pi$ and $B_{s} \rightarrow K\bar{K}$, and between $B \rightarrow K\pi$ and $B_{s} \rightarrow K\pi$.

- There is no problem in describing $B \rightarrow K\eta'$ decays as long as one allows a singlet penguin amplitude, whose magnitude is probably a nonperturbative effect.

- $B \rightarrow PV$ decays are consistent with flavor SU(3) but more data will be needed to test the predictions incisively and to compare them with those of factorization and form-factor models.

- Some interferences (e.g., in $B^{0} \rightarrow \pi^{+}\pi^{-}$, $B^{0} \rightarrow K^{*+}\pi^{-}$, and $B \rightarrow \eta K^{*}$) suggest $\gamma > 90^\circ$ or $\alpha < 90^\circ$ if final-state phases are small, but the pattern is not yet compelling.

The wealth of forthcoming data from experiments at Cornell, SLAC, KEK, and the hadron machines will make substantial progress on these questions in the next few years. At the same time we look forward to more progress on proving the validity and limits of factorization.
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References

[1] J. D. Bjorken, in New Developments in High-Energy Physics, Proc. IV International Workshop on High-Energy Physics, Orthodox Academy of Crete, Greece, 1–10 July 1988, edited by E. G. Floratos and A. Verganelakis, Nucl. Phys. B Proc. Suppl. 11 (1989) 325.

[2] M. Beneke, G. Buchalla, M. Neubert, and C. Sachrajda, Phys. Rev. Lett. 83 (1999) 1914; CERN report CERN-TH-2000-159, hep-ph/0006124.

[3] J. L. Rosner, Phys. Rev. D 42 (1990) 3732.

[4] M. Gronau and J. L. Rosner, Phys. Rev. D 61 (1999) 073008.

[5] X.-G. He, W.-S. Hou, and K. C. Yang, Phys. Rev. Lett. 83 (1999) 1100.

[6] W.-S. Hou, J. G. Smith, and F. Würthwein, hep-ex/9910014.

[7] Z. Xiao, C. S. Li, and K.-T. Chao, Peking University preprint, hep-ph/0010326 (unpublished).

[8] H.-Y. Cheng and K.-C. Yang, Phys. Rev. D 59 (1999) 092004.

[9] H. Y. Cheng and K. C. Yang, National Taiwan University report, hep-ph/9910291 v2, March, 2000; M.-Z. Yang and Y.-D. Yang, Ochanomizu University report OCHA-PP-161, hep-ph/0007038 v2 (unpublished).
[10] N. Isgur and M. B. Wise, Phys. Lett. B 232 (1989) 113; 237 (1990) 527.

[11] T. Coleman, M. G. Olsson, and S. Veseli, University of Wisconsin report MADPH-00-1189, hep-ph/0009103 (unpublished).

[12] Particle Data Group, D. E. Groom et al., Eur. Phys. J. C 15 (2000) 1.

[13] CLEO Collaboration, J. P. Alexander et al., CLEO-CONF 00-3, presented at XXX International Conference on High Energy Physics, Osaka, Japan, July 27 – August 2, 2000.

[14] BaBar Collaboration, D. Hitlin, Osaka Conf. [13].

[15] CLEO Collaboration, M. Artuso et al., Phys. Rev. Lett. 82 (1999) 3020.

[16] A. S. Dighe, I. Dunietz, H. J. Lipkin, and J. L. Rosner, Phys. Lett. B 369 (1996) 144.

[17] CLEO Collaboration, CLEO report CLEO CONF 98-23, ICHEP98 852, submitted to XXIX International Conference on High Energy Physics, Vancouver, BC, Canada, July 1998.

[18] CLEO Collaboration, M. Artuso et al., CLEO report CLEO CONF 00-01, ICHEP00-78, hep-ex/0006018, presented at Osaka Conf. [13].

[19] See the discussion and references in CLEO Collaboration, C. P. Jessop et al., Phys. Rev. Lett. 79 (1997) 4533. Some later discussions (e.g., [8]) use helicity ratios as an input and thus cannot be regarded as predictive.

[20] CDF Collaboration, T. Affolder et al., CDF report CDF/PHYS/ BOT-TOM/PUBLIC/5281, hep-ex/0007034 (unpublished).

[21] See, e.g., M. Ciuchini et al., Nucl. Phys. B501 (1997) 271; B512 (1998) 3; B531 (1998) 656(E); Nucl. Instr. Meth. A 408 (1998) 28; hep-ph/9909530, to be published in Kaon Physics, edited by J. L. Rosner and B. Weinstein, University of Chicago Press, 2000.

[22] J. L. Rosner, Phys. Rev. D 60 (1999) 074029.

[23] Jessop et al. [19].
[24] BaBar Collaboration, SLAC report SLAC-PUB-8679, hep-ph/0010067, presented by G. Raven at Osaka Conf. [13].

[25] J. L. Rosner, in Proceedings of the TASI-2000 Summer School, Boulder, CO, June 5–30, 2000, to be published by World Scientific.

[26] M. Gronau and J. L. Rosner, Phys. Rev. D 53 (1996) 2516; A. S. Dighe, M. Gronau, and J. L. Rosner, Phys. Lett. B 367 (1996) 357; 377 (1996) 325; Phys. Rev. Lett. 79 (1997) 4333; A. S. Dighe, Phys. Rev. D 54 (1996) 2067.

[27] CLEO Collaboration, D. Cronin-Hennessy et al., Phys. Rev. Lett. 85 (2000) 515.

[28] CLEO Collaboration, D. Cinabro, Osaka Conf. [13].

[29] BELLE Collaboration, H. Aihara, Osaka Conf. [13].

[30] M. Gronau and J. L. Rosner, Phys. Rev. D 61 (2000) 073008.

[31] H. J. Lipkin, Phys. Lett. B 445 (1999) 403.

[32] M. Neubert and J. L. Rosner, Phys. Lett. B 441 (1998) 403; Phys. Rev. Lett. 81 (1998) 5076; M. Neubert, JHEP 9902 (1999) 014.

[33] R. Fleischer and T. Mannel, Phys. Rev. D 57 (1998) 2752.

[34] M. Gronau and J. L. Rosner, Phys. Rev. D 57 (1998) 6843.

[35] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.

[36] A. Ali, G. Kramer, and C.-D. Lu, Phys. Rev. D 58 (1998) 094009.

[37] H. J. Lipkin, Phys. Rev. Lett. 46 (1981) 1307; Phys. Lett. B 254 (1991) 247; Phys. Lett. B 415 (1997) 186; 433 (1998) 117.

[38] M. Gronau, CERN report CERN-TH-2000-250, hep-ph/0008292.

[39] R. Fleischer, Phys. Lett. B 459 (1999) 306; DESY preprint DESY 00-014, hep-ph/0001253. See also I. Dunietz, Proceedings of the Workshop on B Physics at Hadron Accelerators, Snowmass, CO, 1993, p. 83; D. Pirjol, Phys. Rev. D 60 (1999) 054020.
[40] F. Würthwein and R. Jesik, talks presented at Workshop on B Physics at the Tevatron – Run II and Beyond, Fermilab, February 2000 (unpublished).

[41] M. Gronau and J. L. Rosner, Phys. Lett. B 482 (2000) 71.

[42] H. J. Lipkin, private communication.

[43] CLEO Collaboration, C. P. Jessop et al., Cornell University report CLNS 99-1652, hep-ex/0006008, submitede to Phys. Rev. Lett.

[44] A. J. Buras and R. Fleischer, in Heavy Flavours II, edited by A. J. Buras and M. Lindner (World Scientific, Singapore, 1998), p. 65, and references therein.
Table 1: Comparison of predictions for ratios of decay rates in light-meson and $D_s^{(*)}$ production by the weak current in $\bar{B}^0$ decays.

| Subprocess                  | $b \to c\bar{u}d$ | $b \to c\bar{c}s$ |
|-----------------------------|-------------------|-------------------|
| Ratio                       | $D^{**}\pi^-/D^+\pi^-$ | $D^{**}D_s^-/D^+D_s^-$ |
| Experiment                  | 0.93 ± 0.14       | 1.2 ± 0.5         |
| Prediction                  | 1                 | 1                 |
| Ratio                       | $D^+\rho^-/D^{**}\pi^-$ | $D^+D_s^-/D^{**}D_s^-$ |
| Experiment                  | 2.8 ± 0.5         | 0.97 ± 0.54       |
| Prediction                  | 1.9               | 1                 |
| Ratio                       | $D^{**}\rho^-/D^{**}\pi^-$ | $D^{**}D_s^-/D^{**}D_s^-$ |
| Experiment                  | 3.4 ± 0.8         | 2.2 ± 0.6         |
| Prediction                  | 2.2               | 2.6               |

Table 2: Comparison of helicity amplitudes for $B^0 \to J/\psi K^{*0}$ and $B_s \to J/\psi \phi$.

| Amp.       | CLEO ($B^0$)         | CDF ($B^0$)         | CDF ($B_s$)         |
|------------|----------------------|---------------------|---------------------|
| $|A_0|^2$   | 0.52 ± 0.08          | 0.59 ± 0.06 ± 0.01  | 0.61 ± 0.14 ± 0.02  |
| $|A_\perp|^2$ | 0.16 ± 0.09          | 0.13±0.09 ± 0.06    | 0.23 ± 0.19 ± 0.04  |
| Amp.       | BaBar ($B^0$)        | CDF ($B_s$)         |
| $|A_0|^2$   | 0.60 ± 0.06 ± 0.04   | 0.61 ± 0.14 ± 0.02  |
| $|A_\perp|^2$ | 0.13 ± 0.06 ± 0.02   | 0.23 ± 0.19 ± 0.04  |
Table 3: Flavor-SU(3) decomposition of some amplitudes for $B \to PP$, where $P$ denotes a light pseudoscalar meson. Unprimed amplitudes denote strangeness-preserving decays; primed amplitudes denote strangeness-changing decays.

| Mode         | Amplitude                                           |
|--------------|-----------------------------------------------------|
| $\pi^+\pi^-$ | $-(T + P)$                                          |
| $\pi^+\pi^0$ | $-(T + C + P_{EW})/\sqrt{2}$                       |
| $K^+\pi^-$   | $-(T' + P')$                                        |
| $K^+\pi^0$   | $-(T' + P' + C' + P'_{EW})/\sqrt{2}$               |
| $K^0\pi^+$   | $P'$                                                |
| $K^0\pi^0$   | $(P' - C' - P'_{EW})/\sqrt{2}$                     |
| $K^+\eta'$   | $(3P' + 4S' + T' + C' - \frac{1}{3}P'_{EW})/\sqrt{6}$ |
| $K^0\eta'$   | $(3P' + 4S' + C' - \frac{1}{3}P'_{EW})/\sqrt{6}$   |

Table 4: Branching ratios for some $B \to PP$ decays, in units of $10^{-6}$.

| Mode         | CLEO       | BaBar      | BELLE      | Average |
|--------------|------------|------------|------------|---------|
| $\pi^+\pi^-$ | $4.3^{+1.0}_{-1.4} \pm 0.5$ | $9.3 \pm 2.8$ | $6.3 \pm 4.0$ | $5.6 \pm 1.3$ |
| $\pi^+\pi^0$ | $5.4 \pm 2.6$ |            | $3.3 \pm 3.2$ | $4.6 \pm 2.0$ |
| $K^+\pi^-$   | $17.2 \pm 2.7$ | $12.5 \pm 3.2$ | $17.4 \pm 5.9$ | $15.4 \pm 2.0$ |
| $K^0\pi^+$   | $18.2 \pm 4.6$ |            | $16.6 \pm 9.1$ | $17.9 \pm 4.1$ |
| $K^+\pi^0$   | $11.2 \pm 3.2$ |            | $18.8 \pm 5.7$ | $13.0 \pm 2.8$ |
| $K^0\pi^0$   | $14.6 \pm 6.2$ |            | $21.0 \pm 8.9$ | $16.7 \pm 5.1$ |
| $K^+\eta'$   | $80 \pm 12$ | $62 \pm 20$ |            | $75 \pm 10$ |
| $K^0\eta'$   | $89 \pm 19$ |            |            | $78 \pm 9$ (a) |

(a) Average for $K^+\eta'$ and $K^0\eta'$ modes.
Table 5: Comparison of $B^+ \rightarrow PV$ data with predictions of flavor SU(3) and factorization models. Numbers denote predicted branching ratios in units of $10^{-6}$. See text for explanation of italicized and bold entries.

| Mode           | CLEO     | Flavor SU(3) | Fact. models |
|----------------|----------|--------------|--------------|
| $\rho^+\pi^0$  | $< 43$   | $4 - 15$     | 10–13        |
| $\rho^0\pi^+$  | $10.4^{+3.3}_{-3.4} \pm 2.1$ | $8 - 17$ | 9–13         |
| $\omega\pi^+$  | $11.3^{+3.3}_{-2.9} \pm 1.4$ | Input | 10–11        |
| $\rho^- K^0$   | $< 48$   | $< 48$       | 5–12         |
| $\rho^0 K^+$   | $8.4^{+4.0}_{-3.4} \pm 1.8$ | $1 - 2$ | $\sim 1$    |
| $\omega K^+$   | $< 7.9$  | $< 7.9$      | 1–2          |
| $\phi K^+$     | $6.4^{+2.5}_{-2.1 - 2.0}$ | Input |            |
| $K^{*0}\pi^+$  | $7.6^{+3.5}_{-3.0} \pm 1.6$ | $5 - 12$ | $\sim 4$ |
| $K^{*+}\pi^0$  | $< 31$   | $4 - 7$      | 4–6          |
| $K^{*+}\eta$   | $26.4^{+9.6}_{-8.2} \pm 3.3$ | $13 - 22$ |            |

Table 6: Comparison of $B^0 \rightarrow PV$ data with predictions of flavor SU(3) and factorization models. Numbers denote predicted branching ratios in units of $10^{-6}$. See text for explanation of italicized and bold entries.

| Mode           | CLEO     | Flavor SU(3) | Fact. models |
|----------------|----------|--------------|--------------|
| $\rho^-\pi^+$  | $27.6^{+8.4}_{-7.4} \pm 4.2$ | $15 - 32$ | $\sim 14$  |
| $\rho^+\pi^-$  | (combined b. r.) | $6 - 30$ | $\sim 18$  |
| $\rho^- K^+$   | $16.0^{+7.6}_{-6.4} \pm 2.8$ | $6 - 10$ | 1–4         |
| $\rho^0 K^0$   | $< 27$   | $< 27$       | 6–14         |
| $\omega K^0$   | $10.0^{+5.4}_{-4.2} \pm 1.5$ | $2 - 3$ | 0.4–2       |
| $\phi K^0$     | $5.9^{+4.0}_{-2.9 - 0.9}$ | Input |            |
| $K^{*+}\pi^-$  | $22^{+8.4}_{-6.5}$ | $5 - 10$ |            |
| $K^{*0}\pi^0$  | $< 3.6$  | $< 3.6$      | 1–2          |
| $K^{*0}\eta$   | $13.8^{+5.5}_{-4.4} \pm 1.7$ | Input |            |
Figure 1: Factorization predictions for branching ratios (in percent) for some $B$ decays based on the universal form factor (1), plotted as functions of $\rho^2 = 2/z_0^2$. Solid line: $10B(B^0 \to D^{(*)-}\pi^+)$; dashed line: $B(B^0 \to D^{*-}\ell^+\nu)$; dotted line: $B(B^0 \to D^-\ell^+\nu)$. Horizontal bands denote $\pm 1\sigma$ experimental limits; vertical bands denote corresponding range of $\rho^2$. Plotted point denotes value from Ref. [13].