Amplitude and Phase Estimation for Absolute Calibration of Massive MIMO Front-Ends

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Abstract—Massive multiple-input multiple-output (MIMO) promises significantly higher performance relative to conventional multi-user systems. However, the promised gains of massive MIMO systems rely heavily on the accuracy of the absolute front-end calibration, as well as quality of channel estimates at the base station (BS). In this paper, we analyze user equipment-aided calibration mechanism to estimate the amplitude scaling and phase drift at each radio-frequency chain connected to the BS array. Assuming a uniform linear array at the BS and Ricean fading, we obtain the estimation parameters with moment-based (amplitude, phase) and maximum-likelihood (phase-only) estimation techniques. In stark contrast to previous works, we mathematically articulate the equivalence of the two approaches for phase estimation. Furthermore, we rigorously derive a Cramér-Rao lower bound to characterize the accuracy of the two estimators. Via numerical simulations, we evaluate the estimator performance with varying dominant line-of-sight powers, dominant angles-of-arrival, and signal-to-noise ratios.

I. INTRODUCTION

Fifth-generation (5G) systems are being deployed into commercial networks [1]. The standardization efforts have resulted in a new radio access framework, known as Third Generation Partnership Project Release 15 (and beyond) [2]. A fundamental technology contributing to the spectral and energy efficiency targets of 5G systems is massive multiple-input multiple-output (MIMO). By scaling up the number of antennas at cellular base stations (BSs), massive MIMO sharply increases the beamforming gain of the system, and enhances the ability to provide uniformly good service to each user equipment (UE) [3]–[5]. This has resulted in an order-of-magnitude increase in the average spectral efficiency of 5G systems relative to their fourth-generation counterpart [4]. Since its inception in 2010, a vast amount of literature has developed around characterizing different performance aspects of massive MIMO systems (see e.g., [7], [8], [18] for a summary). Nevertheless, the promised gains of massive MIMO greatly hinge on two key factors: (1) the knowledge of the channel state information at the BS and UE, (2) calibration quality (precise definition presented later in the text). According to the related literature, massive MIMO calibration approaches are generally classified into two categories: namely, reciprocity calibration [10]–[14] and absolute calibration [15]–[17], [21]–[24]. Reciprocity calibration is required in massive MIMO to ensure that the downlink channel is reciprocal to the uplink. The concept of the relative reciprocity calibration was first introduced in [10]. Extending this, a high-level network protocol of UE synchronization and reciprocity-based calibration was presented in [11]. Moreover, the authors of [12], [13] derived several practical approaches for reciprocity calibration and validated the results in real-time via the Lund University massive MIMO testbed. A taxonomy of the existing reciprocity calibration methods with an antenna grouping strategy is proposed in [14] to shorten the calibration time. In contrast to reciprocity calibration, absolute calibration is required for angle-of-arrival (AOA) estimation and positioning. Absolute calibration exploits the amplitude and phase spectra across the BS array, as shown in [15], [16]. Approaches such as intra-array and UE-aided calibration are discussed in [17], [21], [22]. The authors of [23] combine array calibration with AOA estimation, while the authors of [24] propose mutual coupling-based methods for estimating the phase and amplitude relationships between each radio-frequency (RF) chain at the BS.

The intra-array based calibration can be implemented either with or without transmission lines between antenna elements. The later case outperforms the former in terms of interconnect flexibility at the cost of calibration accuracy, since its performance degrades with increasing electrical distance between successive antennas [17]. For UE-aided calibration, a better trade-off between the flexibility and accuracy is expected, and is therefore worth further investigation. To our best knowledge, prior works on UE-aided calibration only consider simple additive white Gaussian noise (AWGN) channels, which naturally do not reflect the physics of wave propagation. To this end, we analyze UE-aided absolute calibration over a Ricean fading channel, often used to model dominant line-of-sight (LOS) components in addition to diffuse multipath components [18], [19]. We provide a methodology to analyze two types of practical estimators (described later in the text) and derive the corresponding Cramér-Rao lower bound (CRLB) for evaluating the quality of amplitude and phase estimates.

Our main contributions are as follows: For an uplink single-user massive MIMO system, assuming a uniform linear array (ULA) at the BS and Ricean fading propagation, we establish two general, yet practical, analytical approaches to estimate the amplitude scaling and phase drift associated with each RF chain. The first approach is based on moment-based estimation of the aforementioned parameters, while the second is based on maximum-likelihood estimation (MLE), for obtaining phase estimates. We mathematically show that both estimators have an equivalent form when estimating the phase of the RF chains, and back up the mathematical findings with the
required physical intuition. For evaluating the accuracy of both estimators, we derive the CRLB to characterize the fundamental lower limits on error of the estimated phase and amplitude scaling coefficients across the array. To the best of our knowledge, this has been missing from the literature. We evaluate the derived estimator performance on a ULA-based numerical framework. We show that under the presence of dominant LOS conditions, the variance of the phase estimates rapidly converges to the predicted CRLB for both estimator types. In addition, the amplitude and phase estimation accuracies of both approaches significantly improve with growing LOS powers and signal-to-noise ratios (SNRs).

II. SYSTEM MODEL

We consider the uplink of a single-user massive MIMO system, which has $M$ antenna elements configured in a ULA at the BS. We assume reciprocity-based operation in the time-division duplex mode where the UE sends uplink pilot signals, which are used to estimate the calibration parameters at the BS RF chains interfacing with the receive antennas. The overall system model is depicted in Fig. 1. We assume narrowband propagation between the UE and the BS, with uniform power allocation. More specifically, we employ the use of a general Ricean fading model, where the small-scale fading impulse response is an amalgamation of a dominant LOS component, in addition to the diffuse multipath components. The LOS component is governed by the far-field array steering vector in a given direction, and the diffuse components are modeled as complex Gaussian random variables (exact definition later in the text). The use of such model is rather popular in massive MIMO performance evaluation, particularly in urban scenarios where many diffuse paths are expected with some dominant LOS components [25–27]. Considering this, the received signal observation vector, $y_{t} \in \mathbb{C}^{M \times 1}$ during time $t$ can be written as

$$y_{t} = \sum_{s_{1}} \gamma_{t} D_{t} a(\phi_{t}) p_{t} + \sum_{\omega_{t}} h_{t} p_{t} + n_{t},$$

where $\gamma_{t}$ and $p_{t}$ are scalar quantities which denote the large-scale LOS power and the pilot transmitted by the UE at time $t$. The power contained in $p_{t}$ is normalized to unity, such that $|p_{t}|^2 = 1$, over all values of $t = 1, 2, \ldots, T$. Since we consider a ULA, the array steering vector is a known function of the azimuth AOA, which is denoted as $a(\phi_{t}) \in \mathbb{C}^{M \times 1}$ with an incoming angle $\phi_{t}$. In addition, the vectors $h_{t} \in \mathbb{C}^{M \times 1}$ and $n_{t} \in \mathbb{C}^{M \times 1}$ denote the diffuse multipath components and the AWGN at time $t$, such that $h_{t} \sim \mathcal{C}\mathcal{N}(0, \sigma^2)$ and $n_{t} \sim \mathcal{C}\mathcal{N}(0, N_0/2)$. To this end, the mean of the diffuse components is zero and the variance (power) is $\sigma^2$ across all $t = 1, 2, \ldots, T$. Likewise, the mean of the AWGN at the BS is zero and variance is $N_0/2$. Following this, the SNR at time $t$ is given by $|p_{t}|^2/(N_0/2)$. The diagonal matrix, $D_{t} \in \mathbb{C}^{M \times M}$, contains the $M$ amplitude scaling and the phase drift entries for each RF chain. This matrix models the random phase and amplitude changes introduced by phase jitter at the local oscillators, and RF signal conditioning units such as low-noise amplifiers and active bandpass filters. We note that $D_{t} = \text{diag}(d_{1} e^{j\phi_{1}}, d_{2} e^{j\phi_{2}}, \ldots, d_{M} e^{j\phi_{M}})$. We further assume that $h_{t}$ and $n_{t}$ are statistically independent and $n_{t}$ is uncorrelated over $t = 1, 2, \ldots, T$. With the above in mind, the auto-correlation at time $t$, $\mathbb{E}\{\omega_{t}\omega_{t}^{H}\}$, can be evaluated as

$$\mathbb{E}\{\omega_{t}\omega_{t}^{H}\} = \mathbb{E}\{(D_{t} h_{t} p_{t} + n_{t}) (D_{t} h_{t} p_{t} + n_{t})^{H}\} = N_0 I_{M} + D_{t} |p_{t}|^2 \sigma^2 D_{t}^{H},$$

(2)

where $I_{M}$ denotes the $M \times M$ identity matrix. Moreover, by definition, the cross-correlation between two time intervals, namely $t = 1$ and $t = 2$, can be expressed as

$$\mathbb{E}\{\omega_{t=1}\omega_{t=2}^{H}\} = \sigma^2 D_{1} D_{2}^{H}.$$  

(3)

Between multiple time instances, the channel, $h_{t}$, is assumed to be changing in accordance with its definition. This can be caused by small changes in the UE position, or mobility of objects in the propagation environment. For simplicity, from here onward, we drop the subscript $t$ used in the right-hand side of (2), and assume that all further computations are performed at a given time instance $t$. Therefore, the received vector $y_{t}$ follows a complex Gaussian distribution given by

$$y_{t} \sim \mathcal{C}\mathcal{N}(\gamma D a(\phi) p_{t}, N_{0} I_{M} + D |p_{t}|^2 \sigma^2 D^{H}).$$  

(4)

Given the model in (1)–(4), $y_{t}, p_{t}$ and $a(\phi)$ are assumed to be known by the BS. Other parameters such as $\gamma$, $\sigma^2$, $D$, $h$, $n_{t}$ are assumed to be unknown, which is the case in practice. Observing over $T$ intervals, the composite received signal is given by stacking all $y_{t}$ across $t = 1, 2, \ldots, T$ obtaining

$$y \sim \mathcal{C}\mathcal{N}(\mu(\xi) \otimes \gamma p D a(\phi), \Gamma_{T} \otimes I_{M} N_{0} + \Gamma_{T} \otimes \sigma^2 D D^{H}),$$

(5)

where $\mu(\xi)$ is a column vector of unit entries, $[1, 1, \ldots, 1]^{T}$, while $\Gamma_{T} = 1(1)^{T}$ is a matrix containing unit entries. In addition, $\otimes$ is the Kronecker product operation, $\mu(\xi)$ denotes the mean vector and $C(\xi)$ denotes the variance over all $T$ time intervals. Note that the vector argument $\xi$ contains the unknown quantities in $d, \alpha, \sigma^2$ and $\gamma$, respectively. That is, $\xi = [d_{1}, d_{2}, \ldots, d_{M}, \alpha_{1}, \ldots, \alpha_{M}, \sigma^2, \gamma]^{T}$.

With this setup, the subsequent section of the paper discusses the phase and amplitude estimation techniques with the aim to calibrate $M$ RF chains at $M$ antennas of the BS. 

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Fig. 1. A single-user uplink massive MIMO system with pilot transmission from the UE to the $M$ BS antennas, which are interfaced with $M$ RF chains.
In order to perform absolute calibration, one needs to estimate the subspace spanned by the vector \( \mathbf{v} = \{d_1e^{j\alpha_1}, d_2e^{j\alpha_2}, \ldots, d_Me^{j\alpha_M}\} \). According to this requirement, we analyze two estimators, namely the moment-based estimator and the MLE estimator for estimating the phase drift vector \( \mathbf{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_M]^T \) and a moment-based estimator for estimating the vector space \( \mathbb{R}^M \). Later on, we prove that the analytical expression of the moment-based phase drifting estimator coincides with the MLE-based estimator.

### A. Phase Estimation

1. **Moment-Based Estimator:** From (5), we can see that the information regarding the RF phase drifts is only embedded in the first-order statistics of the composite received signal. Enlightened by this, we analyze the moment-based estimator which computes the expectation of the composite vector \( \mathbf{y} \) before estimating the phase drifts. The phase vector \( \mathbf{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_M]^T \) is estimated by

\[
\hat{\mathbf{\alpha}} = \arg\left\{ \sum_{t=1}^T \mathbf{y}_t \right\} - \arg\{\mathbf{a}(\phi)\} - \arg\{\mathbf{1}_p\}.
\] (6)

**Sketch of Proof:** Via some straightforward algebra, one can show that \( \mathbb{E}\{\mathbf{y}_t\} = \gamma \mathbf{D} \mathbf{a}(\phi) \). With a large total observation time in \( T \), one can expect that the empirical probability distribution of \( \mathbf{y}_t \) converges almost surely to its true probability distribution. Using this fact allows us to accurately approximate the moments (first-order only, since second-order contains no utilizable information) of the empirical distribution with the true distribution, such that \( \frac{1}{T}\sum_{t=1}^T \mathbf{y}_t = \mathbb{E}\{\mathbf{y}_t\} \). Taking \( \arg\left\{ \sum_{t=1}^T \mathbf{y}_t \right\} = \arg\{\mathbf{a}(\phi)\} + \arg\{\mathbf{1}_p\} + \mathbf{\alpha} \), and solving for \( \mathbf{\alpha} \) yields the desired phase estimate.

2. **MLE:** If \( \mathbf{y} \) has the probability distribution function \( \tilde{p}(\xi, \mathbf{y}) \), then the MLE formulates an optimization problem on the maximization of the log-likelihood function. That is

\[
\hat{\mathbf{\alpha}} = \arg\max_{\mathbf{\alpha}} \{ \tilde{p}(\xi, \mathbf{y}) \}
\]

\[
\begin{align*}
\quad & \overset{(a)}{=} \min_{\mathbf{\alpha}} \left\{ \ln\det(\mathbf{A}) + \beta^H \mathbf{C}^{-1}(\xi) \beta \right\},
\end{align*}
\] (7)

where \( \beta = \mathbf{y} - (1 \otimes \mathbf{D} \mathbf{a}(\phi)) \), \( \mathbf{C}(\xi) = \mathbf{Q}^H \mathbf{A} \mathbf{Q} \), with \( \mathbf{Q} = \mathbf{Q} \otimes \mathbf{I}_M \) and \( \mathbf{Q} \in \mathbb{C}^{T \times T} \) is defined as the normalized discrete Fourier transform (DFT) matrix. Moreover, \( \mathbf{A} \) is defined as

\[
\mathbf{A} = \sum_{t=1}^T \mathbf{y}_t \otimes \mathbf{y}_t^* + \text{vecdiag} \left[ \sum_{t=1}^T \sum_{t' = 1, t' \neq t}^T \tilde{\mathbf{C}}(\xi)_{t,t'} \right],
\] (10)

and is a \( MT \times MT \) matrix. In (7), (a) is a result of equivalently minimizing the argument of the exponential function in \( \tilde{p}(\xi, \mathbf{y}) \). Substituting \( 1 \otimes \mathbf{D} \mathbf{a}(\phi) = \sqrt{T} \mathbf{Q}^H \mathbf{I}_M \) in (7) and simplifying yields

\[
\hat{\mathbf{\alpha}} = \min_{\mathbf{\alpha}} \left\{ \ln\det \mathbf{A} + \gamma^H \mathbf{C}^{-1}(\xi) \mathbf{y} \right. \\
- 2\gamma^H \mathbf{R}[\mathbf{a}(\phi)]^{-1} \mathbf{D}^{-1} \mathbf{Q} \mathbf{y} \\
+ \gamma^H \mathbf{D}^{-1} \mathbf{Q} \mathbf{y} \left. \right\}
\] (8)

Note that \( \mathbf{R}[\cdot] \) denotes the real component of a complex quantity. From (8), it is clear that the phase information is only contained in the term \( \mathbb{R}[\mathbf{a}(\phi)]^{-1} \mathbf{D}^{-1} \mathbf{Q} \mathbf{y} \). Thus, can derive the MLE of \( \mathbf{\alpha} \) as

\[
\hat{\mathbf{\alpha}} = \arg \left\{ \mathbb{R}[\mathbf{a}(\phi)]^{-1} \mathbf{D}^{-1} \mathbf{Q} \mathbf{y} \right\}
\]

\[
= \arg \left\{ \sum_{t=1}^T \mathbf{y}_t \right\} - \arg\{\mathbf{a}(\phi)\} - \arg\{\mathbf{1}_p\}.
\] (9)

The above result is mathematically equivalent to the one derived from the moment-based estimator in (6). The intuition behind this equivalence can be explained as follows: The received vector \( \mathbf{y} \) follows complex Gaussian distribution, and hence the first and second-order statistics of \( \mathbf{y} \) contain the vast majority of its underlying information. To this end, the optimal solution can be found by exploiting the first and second-order statistics [28]. For the moment-based estimator, since the second-order statistics do not contribute to the phase estimates, the first-order statistics can be used to derive an optimal estimator, which is identical to the MLE.

### B. Amplitude Estimation

We now analyze the moment-based estimator for deriving the amplitude scaling coefficients of the \( M \) RF chains. We refrain from utilizing the MLE for amplitude estimation as the presence of higher-order terms makes maximization of the log-likelihood function a computationally complex task. We estimate the vector space spanned by \( \mathbf{d} \). Unlike for phase estimation, since both the first and second-order statistics of \( \mathbf{y} \) contain useful information, it is necessary to estimate the covariance matrix \( \mathbf{C}(\xi) \), which we denote as \( \hat{\mathbf{C}}(\xi) \). We observe that the upper and lower triangular block diagonal sub matrices of \( \hat{\mathbf{C}}(\xi) \) contain the relevant terms for \( \sigma^2 \mathbf{d} \otimes \mathbf{d} \), which can be extracted for estimation.

We therefore provide a closed-form solution for the moment-based amplitude estimator as

\[
\hat{\mathbf{d}} = \sqrt{\sum_{t=1}^T \mathbf{y}_t \otimes \mathbf{y}_t^* + \text{vecdiag} \left[ \sum_{t=1}^T \sum_{t' = 1, t' \neq t}^T \hat{\mathbf{C}}(\xi)_{t,t'} \right]},
\] (10)

where * represents the complex conjugate operation and “vecdiag” is an operation which extracts and stacks the diagonal elements of a matrix into a vector. Also, \( \hat{\mathbf{C}}(\xi)_{t,t'} \) represents the \( (t, t') \)-th sub-matrix of \( \hat{\mathbf{C}}(\xi) \).

### IV. CRLB Analysis

We derive the Fisher Information Matrix (FIM), followed by the analytical squarred estimation error bound for evaluating the accuracy of the estimators in the previous section.

#### A. FIM

The derivation of FIM starts from equation (5), according to [29], the FIM of the unknown vector \( \xi \) is given by

\[
\mathbf{I}(\xi)_{ij} = \text{Tr} \left[ \frac{\partial \mathbf{C}(\xi)}{\partial \xi_i} \mathbf{C}^{-1}(\xi) \frac{\partial \mathbf{C}(\xi)}{\partial \xi_j} \mathbf{C}^{-1}(\xi) \right]
+ 2\mathbb{R} \left[ \frac{\partial \mathbf{H}(\xi)}{\partial \xi_i} \mathbf{C}^{-1}(\xi) \frac{\partial \mathbf{\mu}(\xi)}{\partial \xi_j} \right].
\] (11)
We exercise a slight abuse of notation here when we denote the FIM as $I(\xi)$, since a $M \times M$ identity matrix is denoted by $I_M$. We note that $I(\xi)\in \mathbb{C}^{(2M+2)\times(2M+2)}$. Furthermore, $\text{Tr}[.]$ denotes the matrix trace operator. According to (11), the FIM $I(\xi)$ is an addition of two matrices, namely, $I(\xi)_C$ and $I(\xi)_M$, where $[I(\xi)_C]_{i,j}$ is defined as the $(i,j)-\text{th}$ element of $\text{Tr}[(\partial C(\xi))/\partial \xi_i (\partial C(\xi))/\partial \xi_j]^{-1}$. Likewise, $[I(\xi)_M]_{i,j}$ is defined as the $(i,j)-\text{th}$ element of $2\text{Re}((\partial \mu^H(\xi)/\partial \xi_i) C^{-1}(\xi) (\partial \mu(\xi)/\partial \xi_j))$.

First we evaluate $I(\xi)_C$, which begins with calculating the derivative of the $C(\xi)$ with respect to elements in $\xi$. That is,

$$\frac{\partial C(\xi)}{\partial \gamma} = 0 \quad \text{and} \quad \frac{\partial C(\xi)}{\partial \sigma^2} = \tilde{I}_T \otimes DD^H.$$ (12)

For every RF chain, $m = 1,2,\ldots,M$,

$$\frac{\partial C(\xi)}{\partial \alpha_m} = 0 \quad \text{and} \quad \frac{\partial C(\xi)}{\partial d_m} = \tilde{I}_T \otimes \sigma^2 \tilde{D}_m.$$ (13)

Here $\tilde{D}_m = \text{diag}(0,\ldots,2d_m,\ldots,0)$ denotes a diagonal matrix. Closely observing (12) and (13), one can see that $I(\xi)_C$ contains all zero elements except for the sub-matrix blocks of $I(\xi)_C|_{(\alpha^2,\sigma^2)}$, $I(\xi)_C|_{(d,d)}$, and $I(\xi)_C|_{(\sigma^2,d)}$, respectively. To derive these three quantities, it is necessary to perform eigenvalue decompositions of $\partial C(\xi)/\partial \sigma^2$ and $\partial C(\xi)/\partial d_m$ via $Q$, which leads to the following representation:

$$\frac{\partial C(\xi)}{\partial \sigma^2} = Q^H \begin{bmatrix} \text{TDD}^H & 0 & \cdots & 0 \end{bmatrix} Q,$$ (14)

and

$$\frac{\partial C(\xi)}{\partial d_m} = Q^H \begin{bmatrix} \sigma^2 \tilde{D}_m T & 0 & \cdots & 0 \end{bmatrix} Q.$$ (15)

Leveraging the unitary property of $Q$ and the cyclic property of the $\text{Tr}[.]$ operation,

$$[I(\xi)_C]_{\sigma^2,\sigma^2} = \text{Tr} \left[ \frac{\partial C(\xi)}{\partial \sigma^2} C^{-1}(\xi) \frac{\partial C(\xi)}{\partial \sigma^2} C^{-1}(\xi) \right] = \sum_{m=1}^{M} \frac{T^2d_m^4}{(\sigma^2 T d_m^2 + N_0)^2}.$$ (16)

Following a similar methodology, one can compute

$$[I(\xi)_C]_{d,d} = \frac{4\sigma^4 T^2 d_m^2}{(\sigma^2 T d_m^2 + N_0)^2} \delta_{mk},$$ (17)

and

$$[I(\xi)_C]_{\sigma^2,d_m} = \frac{2\sigma^2 T^2 d_m^3}{(\sigma^2 T d_m^2 + N_0)^2},$$ (18)

where $\delta_{mk} = 1$ only if $m = k$. Due to space constraints, we avoid presenting the full calculation of (17) and (18), respectively. Following this, we derive $I(\xi)_M$. We begin by taking the derivative of $\mu(\xi)$ with respect to elements in $\xi$.

Doing this yields the following results

$$\frac{\partial \mu(\xi)}{\partial \gamma} = 1 \otimes \mathbf{a}(\phi), \quad \frac{\partial \mu(\xi)}{\partial d_m} = 1 \otimes e^{j\alpha_m} \mathbf{E}_{mm} \gamma \mathbf{a}(\phi),$$

$$\frac{\partial \mu(\xi)}{\partial \sigma^2} = 0, \quad \text{and} \quad \frac{\partial \mu(\xi)}{\partial \alpha_m} = 1 \otimes j d_m e^{j\alpha_m} \mathbf{E}_{mm} \mathbf{a}(\phi).$$ (19)

Note that $E_{mm}$ is the elementary matrix which has unit value only at the intersection of the $m-$th row and $m-$th column, and zeros elsewhere. In accordance with (19), it is trivial that $I(\xi)_M|_{(\sigma^2,\sigma^2)}$, $I(\xi)_M|_{(\sigma^2,\alpha)},$ and $I(\xi)_M|_{(\sigma^2,\gamma)}$ are all 0, since the first two quantities are zero vectors, while the second two quantities are zero scalars. To derive the remaining sub-matrices of $I(\xi)_M$, we express a unit vector as $1 = \sqrt{T} Q^H \eta$, where $\eta$ denoted as a $T \times 1$ column vector $[1,0,\ldots,0]^T$. Based on the properties of the unitary matrix and the mixed-product property of the kronecker operation, we can express $I(\xi)_M|_{d,d}$ as

$$I(\xi)_M|_{d,d} = 2 \text{Re} \left[ \frac{\partial \mu^H(\xi)}{\partial d_m} C^{-1}(\xi) \frac{\partial \mu(\xi)}{\partial d_k} \right] = 2 \text{Re} \left[ \kappa^H (Q \otimes I_M)^{-1} \Lambda^{-1} (Q \otimes I_M) \kappa \right]$$

$$= \frac{2T}{N_0 + \sigma^2 T d_m^2} \delta_{mk},$$ (20)

where $(a)$ contains $\kappa = \sqrt{\frac{T}{2}} Q \otimes \mathbf{I}_M$.

Due to space limitation, we avoid presenting the exact calculations, however we quote the final results below:

$$[I(\xi)_M]_{\gamma}\gamma = \sum_{m=1}^{M} \frac{2d_m^2 T}{\sigma^2 T d_m^2 + N_0},$$ (21)

$$[I(\xi)_M]_{\alpha_m}\gamma = \frac{2T d_m^2}{\sigma^2 T d_m^2 + N_0},$$ (22)

$$[I(\xi)_M]_{\alpha_m,\alpha_k} = \frac{2T d_m^2 \gamma}{N_0 + \sigma^2 T d_m^2} \delta_{mk},$$ (23)

$$[I(\xi)_M]_{\gamma}\alpha_m = 0 \quad \text{and} \quad [I(\xi)_M]_{\alpha_m,\alpha_k} = 0.$$ (24)

Adding $I(\xi)_M$ with $I(\xi)_C$, the closed-form FIM $I(\xi)$ is given by (25) presented on top of the following page.

B. Inverse of FIM

To compute the CRLB, one is generally required to invert the FIM. We check the invertibility of $I(\xi)$ by computing its determinant numerically, and ensuring that the result is non-zero. In order to perform parameter estimation for absolute calibration, we are only interested in the following two terms of the FIM: $I(\xi)_M^{-1}$ and $I(\xi)_C^{-1}$. This is since only these terms contain the necessary information for the amplitude scaling and phase shifts associated with each RF chain. The other terms do not need to be inverted, since they contain information relating to $\gamma$ and $\sigma^2$ which denote the LOS power and power of the diffuse multipath components which do not need to be estimated. Enlightened by this, we provide the following analysis which begins by splitting $I(\xi)$ into four parts for mathematical convenience. Specifically,

$$I(\xi) = \begin{bmatrix} \mathbf{X} & \mathbf{H} \end{bmatrix} \psi w,$$ (26)

where the scalar $w = I(\xi)_M^{-1}([\gamma,\gamma])$, the vector $\psi$ is given by $(I(\xi)_M^{-1})_{[d,d]} 0_{1 \times M}$, $I(\xi)_C^{-1}([\alpha,\gamma])_{T}$ and $\mathbf{X}$ for the rest of $I(\xi)$. The remaining sub-matrices of $I(\xi)_C$ are the elementary matrices which contain only non-zero elements in the $m-$th row and $m-$th column.
Using block matrix inversion theorem \[30\], (30) can be analyzed, which is as follows: Supposing that the receive antennas at the BS. An interesting special case of the problem which holds for any SNR value and any number of receive antennas. Thus, we can extract the resulting sub-matrix after striking out the \(i\)-th row and column of \(I(\xi)\). Since only the diagonal elements of \(X\) contain the phase shift and amplitude scaling estimation parameters of interest, the range of \(i = 1, 2, \ldots, M, M + 2, M + 3, \ldots, 2M + 1\). Then, by applying the definition of adjugate matrix and Schur complement, one can calculate the \(i\)-th diagonal element of \(I^{-1}(\xi)\) as \[30\]

\[
\begin{align*}
I^{-1}(\xi)_{ii} & = \frac{\det(X_{ii})}{\det(X)} \left( \frac{w - \eta^H X_i^{-1} \eta}{w - \eta^H X^{-1} \eta} \right),
\end{align*}
\]

where \(X_{ii}\) can be obtained by striking the \(i\)-th row and column, while \(\eta_i\) can be extracted from the column vector \(\eta\) by striking the \(i\)-th row. We now present the full analytical form of \(I^{-1}(\xi)_{ii}\). For convenience, we let vectors \(\phi \in \mathbb{R}^M, \zeta \in \mathbb{R}^M\) denote the diagonal elements of the matrices \(I(\xi)_{\mu|\sigma(d,d)} + I(\xi)_{\zeta|\sigma(d,d)}\) and \(I(\xi)_{\mu|\sigma(\alpha,\alpha)}\) respectively. Furthermore, we let \(\varphi \in \mathbb{R}^M\) and \(\theta \in \mathbb{R}^M\) represent vectors \(I(\xi)_{\zeta|\sigma(\delta,d)}\) and \(I(\xi)_{\mu|\gamma(\delta,d)}\) respectively. In addition, we define scalars \(\rho = I(\xi)_{\varphi,\varphi}^{-1}, \chi_i = \det(X_{ii})/\det(X)\) and \(\chi' = (w - \eta^H X_i^{-1} \eta)/(w - \eta^H X^{-1} \eta)\).

With the aid of Gaussian elimination, we can calculate \(\chi_i\) as \[30\]

\[
\chi_i = \begin{cases} 
\frac{\rho - \sum_{j \neq i}^M \varphi_j^2}{\varphi_i (\rho - \sum_{j=1}^M \varphi_j^2)} ; & i = 1, 2, \ldots, M \\
\frac{1}{\zeta_i} ; & i = M + 2, M + 3, \ldots, 2M + 1.
\end{cases}
\]

Using block matrix inversion theorem \[30\],

\[
\chi_i = \begin{cases} 
\frac{w - \left( \sum_{j \neq i}^M \varphi_j^2 / \varphi_i^2 \right) - f_2^{-1} \left( \sum_{j \neq i}^M \varphi_j \varphi_j^2 \right) - \sum_{j \neq i}^M \varphi_j^2 / \varphi_i^2}{w - \left( \sum_{j=1}^M \varphi_j^2 / \varphi_j^2 \right) - f_1^{-1} \left( \sum_{j=1}^M \varphi_j \varphi_j^2 \right) - \sum_{j=1}^M \varphi_j^2 / \varphi_j^2} ; & i = 1, 2, \ldots, M \\
1 ; & i = M + 2, M + 3, \ldots, 2M + 1,
\end{cases}
\]

where scalars \(f_1\) and \(f_2\) are defined as \(f_1 = \rho - \sum_{j=1}^M (\varphi_j^2 / \varphi_j), f_2 = \rho - \sum_{j \neq i}^M (\varphi_j^2 / \varphi_j)\), respectively.

Note that this is a very general solution to a complex problem which holds for any SNR value and any number of receive antennas at the BS. An interesting special case of \[30\] can be analyzed, which is as follows: Supposing that the system is operated in the high SNR regime, implying that \(N_0\) is much smaller than \(\sigma^2 T d_i^2\), when \(1 \leq i \leq M\). \(\chi'_i\) can be approximated as:

\[
\chi'_i \approx \frac{\varepsilon \left[ \sigma^2 M (2 \varepsilon - \gamma^2) + 2 \sigma^4 + \gamma^2 \varepsilon + 2 \gamma^2 \sigma^2 \right]}{\sigma^2 (2 \varepsilon - \gamma^2) (\varepsilon + \sigma^2)},
\]

where \(\varepsilon\) is defined as \(\varepsilon = 2 T \gamma^2 + (4 T^2 - 1) \sigma^2\). If \(M\) is much larger than \(\sigma^2\), which is typically the case for massive MIMO systems, then \(\chi'_i\) can be approximated as \(1\) since the numerator and denominator of \(31\) both scale linearly with \(M\) resulting in a cancellation. Relative to \(M\), the other variables do not significantly influence the result of \(31\) and hence are less dominant. Based on \[28\]-\[30\], in high SNR conditions, for a massive MIMO system, the diagonal elements of \(I^{-1}(\xi)\) can be revealed in a rather elegant form, which demonstrate the CRLBs of the amplitude and phase estimations. These are

\[
\begin{align*}
I^{-1}(\xi)_{ii} & = \frac{\sigma^2 d_i^2}{\sigma^2} ; & i & = 1, 2, \ldots, M \\
I^{-1}(\xi)_{ii} & = \frac{\sigma^2}{2 \gamma^2} ; & i & = M + 2, M + 3, \ldots, 2M + 1.
\end{align*}
\]

From \[32\], the CRLB of the phase estimation is proportional to \(\sigma^2\) and inversely proportional to \(\gamma^2\). As a special case, if the system is operating with pure non LOS propagation environment (i.e., \(\gamma = 0\)), the CRLB relating to phase drifts goes to infinity, and it is impossible to estimate the phase drifts in this situation. However, for UE-aided absolute calibration, it is common that the UE will be in close proximity to the BS and hence will almost surely have a dominant LOS component, along with other multipath components. In contrast, for the amplitude estimations, both \(\gamma\) and \(\sigma^2\) can contribute to the enhancement of the FIM. Therefore, it is possible to find a solvable estimator, even when there is no LOS component.

V. Numerical Results

The ultimate aim of our work is to implement the aforementioned calibration parameter estimation techniques into a real-time massive MIMO testbed. To this end, as a first step in this direction, we evaluate the estimation performance via Monte-Carlo simulations. Our simulation framework caters to a 100 element ULA connected to 100 individual RF chains. We assume that the physical distance between the electrical phase centers of successive antenna elements is \(d = \lambda_f / 2\), where \(\lambda_f\) is the wavelength corresponding to the operating carrier frequency. Considering this, the overall steering vector can be written as

\[
\mathbf{a}(\phi) = \left[ 1, e^{-j2\pi d \cos(\phi)}, \ldots, e^{-j2\pi d(M-1) \cos(\phi)} \right].
\]

Consistent with \[14\], the ground truth of the magnitudes of the RF chain coefficients are assumed to be unity, while the phases are assumed to be distributed uniformly between \([-\pi, \pi]\). This
In this paper, we consider UE-aided amplitude and phase estimation for absolute calibration of massive MIMO front-ends. Assuming a Ricean fading channel model, for a single-user massive MIMO system, we analyze the performance of

![Graph](image.png)

**Fig. 2.** MLE (and moment-based) estimator performance as a function of LOS powers for phase calibration with varying SNRs. The phase estimation CRLBs are shown for comparison purposes.

Figures 3 and 4 depict the phase estimation results of the two estimators as a function of LOS powers, for different dominant LOS AOAs at SNR=3 dB. It can readily be observed that the resulting phase estimates for different AOAs are essentially the same. This is since each element of the steering vector has a constant amplitude of one for all of the incoming angles between $[-\pi/2, \pi/2]$. To this end, the change of dominant AOAs will have no influence on the estimator performance for a given LOS power and a given SNR. Although not shown here, the same trends hold for the moment-based amplitude estimator following the same phenomena.

**VI. Conclusion**

In this paper, we consider UE-aided amplitude and phase estimation for absolute calibration of massive MIMO front-ends. Assuming a Ricean fading channel model, for a single-user massive MIMO system, we analyze the performance of
moment-based and MLE estimators for estimating the relative amplitude scalings and phase shifts associated with each RF chain. Our analysis assumes no knowledge of the LOS power, diffuse multipath component power, amplitudes and phase shifts. The derived estimators only need knowledge of the received array’s steering vectors and transmitted pilots by the UE. We mathematically prove that for phase estimation, both MLE and moment-based estimators have the same form, and hence perform equally well. To evaluate the performance of respective estimators, we investigate the CRLB via the analysis of the FIM, where we draw several important insights. We show that the presence of a dominant LOS component is mandatory for phase drift estimation, while not necessary for the amplitude estimation. Our numerical results indicate that the variance of the phase estimates converge to the corresponding CRLBs with increasing LOS powers and SNRs. Likewise, the variance of the amplitude estimates converge to the corresponding CRLBs with increasing SNRs. Different dominant LOS AOA tends to make almost no difference to the phase and amplitude estimator performance, since the amplitude of each entry in the steering vector remains constant over all considered angles. In the future, our aim is to implement the estimators for absolute calibration on the Lund University massive MIMO testbed.

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