Distributed K-means over Compressed Binary Data

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Abstract—We consider a network of binary-valued sensors with a fusion center. The fusion center has to perform K-means clustering on the binary data transmitted by the sensors. In order to reduce the amount of data transmitted within the network, the sensors compress their data with a source coding scheme based on LDPC codes. We propose to apply the K-means algorithm directly over the compressed data without reconstructing the original sensors measurements, in order to avoid potentially complex decoding operations. We provide approximated expressions of the error probabilities of the K-means steps in the compressed domain. From these expressions, we show that applying the K-means algorithm in the compressed domain enables to recover the clusters of the original domain. Monte Carlo simulations illustrate the accuracy of the obtained approximated error probabilities, and show that the coding rate needed to perform K-means clustering in the compressed domain is lower than the rate needed to reconstruct all the measurements.

I. INTRODUCTION

Networks of sensors have long been employed in various domains such as environmental monitoring, electrical energy management, and medicine [1]. In particular, inexpensive binary-valued sensors are successfully used in a wide range of applications, such as traffic control in telecommunication systems [2], self-testing in nanoelectric devices [3], or activity recognition on home environments [4]. In this paper, we consider a network of \( J \) binary-valued sensors that transmit their data to a fusion center.

In such network, the sensors potentially collect a large amount of data. The fusion center may realize complex data analysis tasks by aggregating the sensors measurements and by exploiting the diversity of the collected data. Clustering is a particular data analysis task that consists of separating the data in a given number of classes with similar characteristics. One of the most popular clustering methods is the K-means algorithm [5] due to its simplicity and its efficiency. The K-means algorithm groups the \( J \) measurement vectors into \( K \) clusters so as to minimize the average distance between vectors in a cluster and the cluster center. If the measurements are real-valued, K-means usually considers the Euclidian distance [5], while in case of binary measurements, K-means relies on the Hamming distance [6].

In our context, the \( J \) sensors should send their measurements to the fusion center in a compressed form in order to greatly reduce the amount of data transmitted within the network. Low Density Parity Check (LDPC) codes, initially introduced in the context of channel coding [7], have been shown to be very efficient for distributed compression in a network of sensors [6]. However, the standard distributed compression framework [8] considers that the fusion center has to reconstruct all the measurements from all the sensors. Here, in order to avoid potentially complex decoding operations, we would like to perform K-means directly over the compressed data. Distributed K-means over compressed data raises three questions: (i) How should the data be compressed so that the fusion center can perform K-means without having to reconstruct all the measurements? (ii) How good is clustering over compressed data compared to clustering over the original data? (iii) Is the rate needed to perform K-means lower than the rate needed to reconstruct all the data?

Regarding the first two questions, [9], [10] considered real-valued measurement vectors compressed from Compressed Sensing (CS) techniques, and proposed to apply the K-means algorithm directly in the compressed domain. CS consists of computing \( M \) random linear combinations of the \( N \) components of the measurement vectors, with \( M \leq N \). The results of [9] show that if \( M \) is large enough, the compression preserves the Euclidian distances with high probability, which enables to perform K-means in the compressed domain. Other than K-means, detection and parameter estimation can be applied over compressed real-valued data [11]–[13], but also over binary data compressed with LDPC codes [14], [15]. However, none of these works consider the K-means algorithm over compressed binary data.

Regarding the third question, information theory has recently addressed data analysis tasks such as distributed hypothesis testing [16], [17] or similarity queries [18], [19] over compressed data. These works provide analytic expressions of the minimum rates that need to be transmitted in order to perform the considered tasks, although these general analytic expressions can be difficult to evaluate for particular source models. However, to the best of our knowledge, the K-means algorithm has not been studied yet with information theory.

In this paper, we consider binary measurement vectors and we assume that the compression is realized from LDPC codes. We want to determine whether the K-means algorithm can be applied directly over the compressed data in order to recover the clusters of the original data. In the following, after describing the source coding system (Section II), we propose a formulation of the K-means algorithm over binary data in the compressed domain (Section III). We then carry a theoretical analysis of the performance of the K-means algorithm in the compressed domain (Section IV). We in particular derive approximated error probabilities of each of the two steps of the K-means algorithm. The theoretical analysis shows that applying the K-means algorithm in the compressed domain permits to recover the clusters of the original domain. Monte Carlo simulations confirm the accuracy of the obtained approximated error probabilities, and show that the effective rate needed to perform K-means over compressed data is lower than the rate that would be needed to reconstruct all the sensors measurements (Section V).
II. SYSTEM DESCRIPTION

In this section, we first introduce our notations and assumptions for the binary measurement vectors collected by the sensors. We then present the source coding technique based on LDPC codes that is used in the system.

A. Source Model

The network is composed by $J$ sensors and a fusion center. Each sensor $j \in [1,J]$ performs $N$ binary measurements $x_{j,n} \in \{0,1\}$ that are stored in a vector $x_j$ of size $N$.

Consider $K$ different clusters $C_k$ where each cluster is associated to a centroid $\theta_k$ of length $N$. The binary components $\theta_{k,n}$ of $\theta_k$ are independent and identically distributed (i.i.d.), with $P(\theta_{k,n} = 1) = p_c$. We assume that each measurement vector $x_j$ belongs to one of the $K$ clusters.

The cluster assignment variables $e_{j,k}$ are defined as $e_{j,k} = 1$ if $x_j \in C_k$, $e_{j,k} = 0$ otherwise. Let $\Theta = \{\theta_1, \ldots, \theta_K\}$ and $E = \{e_{1,1}, \ldots, e_{J,K}\}$ be the sets of centroids and of cluster assignment variables, respectively. Within cluster $C_k$, each vector $x_j \in C_k$ is generated as

$$x_j = \theta_k \oplus b_j,$$

where $\oplus$ represents the XOR componentwise operation, and $b_j$ is a vector of size $N$ with binary i.i.d. components such that $P(b_{j,n} = 1) = p$. In the following, we assume that the cluster assignment variables $e_{j,k}$, the centroids $\theta_k$, and the parameter $p$ and $p_c$ are unknown. This model is equivalent to the model presented in [20] for K-means clustering with binary data. Some instances of the K-means algorithm have been proposed to deal with an unknown number of clusters $K$ [5]. However, here, as a first step, $K$ is assumed to be known in order to focus on the compression aspects of the problem.

Each sensor has to transmit its data to the fusion center in order to recover the cluster assignments $E$ and the centroids $\Theta$. We now describe the source coding technique that is used in our system in order to greatly reduce the amount of data transmitted to the fusion center.

B. Source Coding with Low Density Parity Check Codes

In [8], it is shown that LDPC codes are very efficient to perform distributed source coding in a network of sensors, and in [14], [15] it is shown that they allow parameter estimation over the compressed data. Denote by $H$ the binary parity check matrix of size $N \times M$ ($M < N$) of an LDPC code. Denote by $d_c \ll M$ the number of non-zero components in any row of $H$, and denote by $d_c \ll N$ the number of non-zero components in any column of $H$. In our system, each sensor $j$ transmits to the fusion center a binary vector $u_j$ of length $M$, obtained as

$$u_j = H^T x_j.$$  \hspace{1cm} (2)

For each sensor, the coding rate is given by $r = \frac{M}{N} = \frac{d_c}{d}$. The set of all the possible vectors $x_j$ is called the original domain and it is denoted as $\mathcal{X}^N = \{0,1\}^N$. The set of all the possible vectors $u_j$ is called the compressed domain and it is denoted by $\mathcal{U}^M \subset \{0,1\}^M$. The compressed domain $\mathcal{U}^M$ depends on the considered code $H$.

As in [8], we assume that the vectors $u_j$ are transmitted reliably to the fusion center. We consider this assumption in order to focus on the source coding aspects of the problem, and we do not describe the channel codes that should be used in the system in order to satisfy this assumption. The source coding technique described by (2) was initially proposed in [8] in a context where the fusion center has to reconstruct all the measurement vectors $x_j$. However, reconstructing all the sensor measurements usually requires complex decoding operations and may need a higher coding rate $r$ than simply applying K-means over the compressed data. Hence, in the following, we propose to apply the K-means algorithm directly over the compressed vectors $u_j$, without having to reconstruct the original vectors $x_j$.

III. K-MEANS ALGORITHM

The K-means algorithm for clustering binary vectors $x_j$ in the original domain $\mathcal{X}^N$ was initially proposed in [6] and it makes use of the Hamming distance. In this section, we restate the K-means algorithm of [6] in the compressed domain $\mathcal{U}^M$.

The Hamming distance between two vectors $a, b \in \mathcal{U}^M$ in the compressed domain is defined as $d(a, b) = \sum_{m=1}^M a_m \oplus b_m$. Denote $\psi_k = H^T \theta_k$, and $\Psi = \{\psi_1, \ldots, \psi_K\}$ the compressed versions of the centroids $\theta_k$. Applying the K-means algorithm in the compressed domain corresponds to minimizing the objective function

$$F(\Psi, E) = \sum_{j=1}^J \sum_{k=1}^K e_{j,k} d(u_j, \psi_k).$$  \hspace{1cm} (3)

with respect to the compressed centroids $\psi_k$ and to the cluster assignment variables $e_{j,k}$.

We initialize the K-means algorithm with $K$ compressed centroids $\psi_k^{(0)}$ that may be either selected at random among the set of input vectors $u_j$, or obtained from the K-means++ procedure [21]. Denote by $L$ the number of iterations of the K-means algorithm. In the following, exponent $\ell$ always refers to a quantity obtained at the $\ell$-th iteration of the algorithm. At iteration $\ell \in [1,L]$, K-means proceeds in two steps. First, from the centroids $\psi_k^{(\ell-1)}$ obtained at iteration $\ell-1$, it assigns each vector $u_j$ to a cluster as

$$\forall j, \forall k, e_{j,k}^{(\ell)} = \begin{cases} 1 & \text{if } d(u_j, \psi_k^{(\ell-1)}) = \min_{k' \in [1,K]} d(u_j, \psi_{k'}^{(\ell-1)}), \\ 0 & \text{otherwise.} \end{cases}$$  \hspace{1cm} (4)

Second, the algorithm updates the centroids as follows:

$$\forall j, \forall n, \psi_{k,n}^{(\ell)} = \begin{cases} 1 & \text{if } \sum_{j=1}^J e_{j,k}^{(\ell)} u_{j,n} \geq \frac{1}{2} J_k^{(\ell)}, \\ 0 & \text{otherwise.} \end{cases}$$  \hspace{1cm} (5)

where $J_k^{(\ell)}$ is the number of vectors assigned to cluster $k$ at iteration $\ell$. The cluster assignment step (4) assigns each vector $u_j$ to the cluster with the closest compressed centroid $\psi_k^{(\ell)}$. The centroid computation step (5) is a majority voting operation which can be shown to minimize the average distances.
between the centroid $\psi_k^{(l)}$ and all the vectors $u_j$ assigned to cluster $k$ at iteration $l$.

Following the same reasoning as for K-means in the original domain [6], it is easy to show that when applying K-means in the compressed domain, the objective function $F(\Psi^{(l)}, E^{(l)})$ is decreasing with $\ell$ and at least converges to a local minimum. However, this property does not guarantee that the cluster assignment variables $c_{j,k}^{(l)}$ obtained from the algorithm in the compressed domain will correspond to the correct cluster assignments in the original domain. In order to justify that the K-means algorithm applied in the compressed domain can recover the correct clusters of the original domain, we now propose a theoretical analysis of the two steps of the algorithm.

IV. K-MEANS PERFORMANCE EVALUATION

In this section, in order to assess the performance of the K-means algorithm in the compressed domain, we evaluate each step of the algorithm individually. We provide an approximated expression of the error probability of the cluster assignment step in the compressed domain, assuming that the compressed cluster centroids $\hat{\psi}_k$ are perfectly known. In the same way, we provide an approximated expression of the error probability of the centroid computation step in the compressed domain, assuming that the cluster assignment variables $c_{j,k}$ are perfectly known. Although evaluated in the most favorable cases, these error probabilities will enable us determine whether it is reasonable to apply K-means in the compressed domain in order to recover the clusters of the original domain.

The expressions of the error probabilities we derive rely on two functions $B_M$ and $f$ defined as

\[ B_M(m,p) = \binom{M}{m} p^m (1-p)^{M-m}, \]

\[ f(d,p) = \frac{1}{2} - \frac{1}{2} (1-2p)^d. \]  

A. Error Probability of the Cluster Assignment Step

The following proposition evaluates the error probability of the cluster assignment step [4] applied to the compressed centroids $\hat{\psi}_k$.

**Proposition 1.** Let $\hat{e}_{j,k}$ be the cluster assignments obtained when applying the cluster assignment step [4] to the true compressed centroids $\psi_k$. The error probability $P_a = P(\hat{e}_{j,k} = 0 | x_j \in C_k)$ can be approximated as

\[ P_a \approx (K-1) \sum_{m_1=0}^{M} \sum_{m_2=m_1}^{M} B_M(m_1, q_1) B_M(m_2, q_2) \]

with $q_1 = f(d_e, p)$, $q_2 = f(d_c, (1-p) f(2, p_e))$.

**Proof:** We first evaluate the error probabilities $P_{a,k'} = P(\hat{e}_{j,k'} = 1 | x_j \in C_k), \forall k' \neq k$. Let

\[ a_k = u_j \oplus \psi_k = H^T b_j, \]

\[ a_{k'} = u_j \oplus \psi_{k'} = H^T (\theta_k \oplus \theta_{k'} \oplus b_j). \]

and define $A_k = \sum_{m=1}^{M} a_{k,m}, A_{k'} = \sum_{m=1}^{M} a_{k',m}$. According to the cluster assignment step [4], the error probability $P_{a,k'}$ can be expressed as $P_{a,k'} = P(A_{k'} \geq A_k)$ which can be approximated as

\[ P_{a,k'} \approx \sum_{u=0}^{M} \sum_{v=u}^{M} P(A_k = u) P(A_{k'} = v). \]  

In order to get (10) we implicitly assume that the random variables $A_k$ and $A_{k'}$ are independent, and hence (10) is only an approximation of $P_{a,k'}$. Assuming that the $a_{k,m}$ and $a_{k',m}$ are all independent, we get $P(A_k = u) \approx B_M(u, q_1)$ and $P(A_{k'} = v) \approx B_M(v, q_2)$. To finish, the error probability of the cluster assignment step is given by $P_a = \sum_{k' \neq k} P_{a,k'} = (K-1)P_{a,k'}$ since $P_{a,k'}$ does not depend on $k'$.

It can be seen from [8] that the approximated error probability $P_a$ does not depend on the considered cluster $k$. The expression (8) is only an approximation of the error probability of the cluster assignment step, since it assumes that the components of the vector $H^T b_j$ are independent, which is not true in general. However, it is shown in [13, 22] that this assumption is reasonable for parameter estimation over LDPC codes. In Section V we verify the accuracy of the approximation by comparing the values of (8) to the error probabilities measured from Monte Carlo simulations.

B. Error Probability of the Centroid Computation Step

The following proposition evaluates the error probability of the centroid computation step in the compressed domain.

**Proposition 2.** Let $\hat{\psi}_k$ be the estimated compressed centroids obtained after applying the centroid estimation step [5] to the true cluster assignment variables $c_{j,k}$. The error probability $P_{c,k} = P(\hat{\psi}_{k,m} \neq \psi_{k,m})$ for cluster $k$ can be approximated as

\[ P_{c,k} \approx \sum_{j=\lceil\frac{J_k}{2}\rceil}^{J_k} B_{J_k}(j, p_d) \]

where $J_k$ is the number of vectors in cluster $k$, and $p_d = f(d_e, p)$.

**Proof:** From the model defined in Section II a codeword $u_j (j \in C_k)$, can be expressed as

\[ u_j = H^T (\theta_k \oplus b_j) = \psi_k \oplus a_j, \]

where $a_j = H^T b_j$ is such that $P(a_{j,m} = 1) = p_d$. Let $A_j = \sum_{j=1}^{J_k} a_{j,m}$. The error probability of the centroid computation step can be evaluated as

\[ P_{c,k} = P\left( A_j \geq \frac{J_k}{2} \right) \approx \sum_{j=\lceil\frac{J_k}{2}\rceil}^{J_k} B_{J_k}(j, p_d). \]

The approximation comes from the fact that (11) assumes that the $a_{j,m}$ are independent. It can be seen from (11) that the approximated error probability $P_{c,k}$ only depends on the considered cluster $k$ through the number $J_k$ of vectors in cluster $C_k$. The expression (11) is only an approximation of the error probability of the centroid assignment step for the same reasons as for the cluster assignment step. We will also verify the accuracy of this approximation in Section V.
V. Simulation Results

In this section, we evaluate through simulations the performance of the K-means algorithm in the compressed domain. We first consider each step of the algorithm individually, and we verify the accuracy of the approximated error probabilities obtained in Section IV. We then assess the performance of the full algorithm and we evaluate the rate needed to perform K-means over compressed data. Throughout the section, we set $J = 200$, $K = 4$, $p_c = 0.1$ and we consider two LDPC codes of length $N = 1000$ with $d_v = 2$. The two codes are constructed from the Progressive Edge Growth algorithm [23]; the first code is of rate $r = 1/4$ with $M = 250$ and $d_c = 8$, and the second one is of rate $r = 1/2$, with $M = 500$ and $d_c = 4$. We set $d_v = 2$ for the two considered codes, since it can be shown from (8) and (11) that the error probabilities $P_a$ and $P_{c,k}$ are increasing with $d_v$ ($d_c$ is necessarily greater than 2).

A. Accuracy of the error probability approximations

We compare the approximated expressions $P_a$ (4) and $P_{c,k}$ (11) with the effective error probabilities measured from Monte Carlo simulations for each step of the algorithm for the two considered codes over $Nt = 10000$ simulations. Figure 1(a) represents the obtained error probabilities for the cluster assignment step, while Figure 1(b) represents the centroid estimation step. We see that for the two considered codes, the theoretic error probabilities $P_a$ and $P_{c,k}$ are close to the measured error probabilities for the two steps of the algorithm, which shows the accuracy of the proposed approximations. Figure 1(a) also illustrates that the cluster assignment step in the compressed domain can indeed recover the correct clusters of the original domain, since it is possible to reach error probabilities from $10^{-3}$ to $10^{-7}$. The same conclusion holds for Figure 1(b) for the centroid estimation step.

B. K-means algorithm and rate evaluation

In addition to analyzing the performance of the K-means algorithm in the compressed domain, we also want to compare the coding rate $r$ needed to perform K-means to the rate that would be needed to reconstruct all the sensors measurements.

From [24], the minimum coding rate (per symbol normalized by the number of sources $J$) the fusion center should receive to reconstruct all the $x_j$ is given by $R_d = \frac{1}{2}H(X_1, \ldots, X_J)$, where $H(X_1, \ldots, X_J)$ is the joint entropy of the sources $(X_1, \ldots, X_J)$. Since no closed-form expression of $R_d$ exist, we evaluate the rate $R_d$ from Monte-Carlo simulations.

We run over $Nt = 10000$ simulations the K-means algorithm in the compressed domain initialized with the K-means++ procedure and $L = 10$ iterations. In order to evaluate the performance of the algorithm, we measure the error probability of the cluster assignments decided by the algorithm in the compressed domain with respect to the correct clusters in the original domain. Figure 1(c) represents the error probabilities with respect to $p$ obtained for the two considered codes. As expected, the error probability is increasing with $p$ and is decreasing with the coding rate $r$.

We then compare the rate needed to perform K-means over compressed data to the rate needed to reconstruct all the sensors measurements. For $p_c = 0.1$ and $p = 0.1$, we get $R_d = 0.68$ bits/symbol, and, for $p_c = 0.05$ and $p = 0.1$, we obtain $R_d = 0.43$ bits/symbol. The results of Figure 1(c) show that for $p_c = 0.1$ and $p = 0.1$, the code of rate $r = 1/2 < 0.68$ enables to perform K-means with an error probability lower than $10^{-6}$. For $p_c = 0.1$ and $p = 0.05$, the code of rate $r = 1/4 < 0.43$ also enables to perform K-means with a low error probability $P_e = 10^{-7}$. This shows that the rate needed to perform K-means is lower than the rate needed to reconstruct all the sensors measurements, which justifies the use of the method presented in the paper.

VI. Conclusion

In this paper, we considered a network of sensors which transmit their compressed binary measurements to a fusion center. We proposed to apply the K-means algorithm directly over the compressed data, without reconstructing the sensor measurements. From a theoretical analysis and Monte Carlo simulations, we showed the efficiency of applying K-means in the compressed domain. We also showed that the rate needed to perform K-means on the compressed vectors is lower than the rate needed to reconstruct all the measurements.
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