Fragmentation of electric dipole strength in $N = 82$ isotones

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Fragmentation of the dipole strength in the $N = 82$ isotones $^{140}$Ce, $^{142}$Nd and $^{144}$Sm is calculated using the second random-phase approximation (SRPA). In comparison with the result of the random-phase approximation (RPA), the SRPA provides the additional damping of the giant dipole resonance and the redistribution of the low-energy dipole strength. Properties of the low-energy dipole states are significantly changed by the coupling to two-particle-two-hole (2p2h) states, which are also sensitive to the correlation among the 2p2h states. Comparison with available experimental data shows a reasonable agreement for the low-energy $E1$ strength distribution.

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The low-energy dipole states, often referred to as the pygmy dipole resonance (PDR), have attracted recent experimental [11] and theoretical interests [7,12] (see also the recent review [13] and references therein). It is of significant astrophysical interest, since the low-energy dipole strengths close to the neutron threshold strongly affect the astrophysical r-process nucleosynthesis [14].

The quasiparticle random-phase approximation based on the Hartree-Fock-Bogoliubov ground state (HFB+QRPA) has been extensively used to study the PDR as well as the giant dipole resonances (GDR). Recent systematic calculations [15] for the Nd and Sm isotopes show that although the HFB+QRPA nicely reproduces characteristic features of the shape phase transition in the GDR, it fails to produce the low-energy dipole strengths at $E_x = 5.5 \sim 8$ MeV, observed in the $N = 82$ isotones, $^{142}$Nd and $^{144}$Sm [1,8]. The disagreement suggests that the coupling to complex configurations, such as multi-particle-multi-hole states, are required to study the PDR in these nuclei. In fact, the quasiparticle-phonon model (QPM), which takes into account coupling to multi-phonon states, successfully reproduces the low-energy dipole strengths in the $N = 82$ nuclei [2,4]. A similar approach based on the relativistic mean-field model has also been used to study the PDR in the tin and nickel isotopes [10]. These models assume the multi-phonon characters of the complex states and violate the Pauli principle. Thus, it is desirable to study properties of the PDR with a method complementary to these phonon-coupling approaches. In this work, we present studies for the dipole excitations in the $N = 82$ isotones, with the second random-phase approximation (SRPA) (Ref. [17] and references therein). The SRPA explicitly incorporates the two-particle-two-hole (2p2h) states instead of “two-phonon” states, and respects the Pauli principle in the 2p2h configurations. Recently, the low-energy dipole states in $^{40,48}$Ca have been studied with the SRPA [18], which suggests that the coupling between one-particle-one-hole (1p1h) and 2p2h configurations enhances the electric dipole ($E1$) strength in the energy range from 5 to 10 MeV. We investigate whether a similar effect can be observed in the isotones of $N = 82$. Since there are many dipole states with small $E1$ strengths in the energy region below 8 MeV, it is difficult to compare property of each state with the experiment. Thus, we perform the comparison of integrated properties at low energies.

The SRPA equation is written in the matrix form [17]

$$
\begin{pmatrix}
a & c \\
b & d
\end{pmatrix}
\begin{pmatrix}
x^\mu \\
x^\mu
\end{pmatrix} = \omega_\mu
\begin{pmatrix}
x^\mu \\
x^\mu
\end{pmatrix},
$$

where $x^\mu$ and $X^\mu_{pp'h'h'}$ ($p \leftrightarrow h$) are the 1p1h and 2p2h transition amplitudes for an excited state with an excitation energy $\omega_\mu$. The explicit expression for the matrices $a$, $b$, $c$, and $d$ are given in Ref. [19].

The Skyrme interaction of the SIII parameter set is used to calculate the Hartree-Fock single-particle states. The continuum states are discretized by confining the single-particle wave functions in a sphere of radius of 20 fm. Single-particle states with the angular momenta $j_\alpha \leq 15/2$ up to 30 MeV in energy ($\epsilon_\alpha < 30$ MeV) are adopted for the 1p1h space ($x_{hh}'$ and $x_{hp}'$), both for protons and neutrons. This roughly amounts to one hundred single-particle states. For the 2p2h amplitudes ($X^\mu_{pp'h'h'}$ and $X^\mu_{pp'h'h'}$), we truncate the space into the one made of the single-particle states near the Fermi level, the 2p3/2, 2p1/2, 1g9/2, 1g7/2, 2d5/2, 2d3/2, 3s1/2, 1h11/2, and 1h9/2 orbits for protons and the 2d5/2, 2d3/2, 1h11/2, 1h9/2, 2f7/2, and 1i13/2 orbits for neutrons. The proton orbits up to the 1g7/2 orbit are assumed to be fully occupied in the ground state of $^{140}$Ce, while the proton 2d5/2 orbit is to be partially occupied in the ground states of $^{142}$Nd and $^{144}$Sm. The numbers of 1p1h and 2p2h amplitudes in the SRPA are about 800 and 9000, respectively.

For calculation of the SRPA matrix elements, we employ a residual interaction of the $t_0$ and $t_3$ terms of the SIII interaction. Since the residual interaction is not fully consistent with the one used in the calculation of the single-particle states, it is necessary to adjust the strength of the residual interaction so that the spurious mode corresponding to the center-of-mass (COM) motion comes at zero excitation energy in the RPA. This condition determines the renormalization factor $f$ for the resid-
profile are better described by a slightly larger value of $\Gamma = 0.5$ MeV in the SRPA as well. Thus, we use this interaction for the calculation of the matrices $a$, $b$, and $c$ in Eq. (1). For the residual interaction for the matrix $d$, following a prescription in Ref. [19], we introduce a zero-range interaction $v_0 \delta^3(r-r')$ in addition to the original $t_0$ and $t_3$ terms, then, fix the parameter $v_0$ by approximately reproducing the excitation energy of the lowest $1^-$ state in $^{142}$Nd ($v_0 = -570$ MeV fm$^3$). With these residual interactions in the given model space, the spurious mode appears at a small imaginary energy ($\omega^2 \approx -1$ MeV$^2$) in the SRPA.

We first show the results for the GDR. The $E1$ strength functions, $S(E) = \sum_{\alpha} |\langle n| f_{1\mu} |0 \rangle|^2 \delta(E - E_{\alpha}) = dB(E1; 1^- \rightarrow 0^+_\alpha) / dE$, calculated in the SRPA (solid line) and RPA (dotted line) for $^{140}$Ce, $^{142}$Nd, and $^{144}$Sm are shown in Figs. 1, 2, and 3 respectively. We use the $E1$ operator with the recoil charges, $N/e/A$ for protons and $-Ze/A$ for neutrons, for the calculation of $S(E)$. The obtained discrete strength functions are smoothed with a small width ($\Gamma = 0.5$ MeV) of Lorentzian. The energy-weighted strength summed up to 50 MeV exhausts 87% of the energy-weighted sum-rule value including the enhancement term arising from the momentum-dependent parts of the Skyrme interaction. The strength distributions of the GDR in the SRPA are broadened, compared to the RPA, due to the coupling to the $2p2h$ states. In the inset of Fig. 2, the total photoabsorption cross section (solid line) calculated in the SRPA is compared with the experimental data [20]. The shape of the GDR depends on the parameter $f$, whereas it is little affected by the parameter $v_0$. The GDR peak position and the profile are better described by a slightly larger value of $f$ (See the dotted line in the inset of Fig. 2). Our calculation indicates that the coupling to the $2p2h$ induces an additional broadening due to the spreading width, however, the peak position is close to that obtained in the RPA calculation. This is very different from the recent SRPA calculation for $^{16}$O in Ref. [21], which indicates a large shift of the GDR peak energy (more than 5 MeV) but almost no broadening. At present, we do not fully understand the origin of this discrepancy. More quantitative analysis of the GDR require an improvement of the present calculation, especially, a self-consistent treatment of the residual interaction and the enlargement of the $2p2h$ space.

Next, let us discuss the low-energy $E1$ strengths. In contrast to the GDR at high energy, the truncation of the $2p2h$ configurations is supposed to be less serious. The $E1$ strengths, $B(E1) \downarrow$, below 10 MeV in $^{140}$Ce,
142Nd, and 144Sm are shown in Figs. 4, 5, and 6, respectively. In the RPA calculation, there is very little E1 strength in the energy region below 8 MeV, which agrees with the result of the QRPA calculation [15]. However, this is different from the experimental findings [1–4]. In the SRPA calculation, the coupling to the 2p2h configurations leads to a considerable E1 strength in this energy region. To make a quantitative comparison with experiment, the mean excitation energies and the summed $B(E1)$ values for low-energy dipole states are calculated in the same way as the experiment [3]: The mean energy is defined as $\bar{E} = \sum E B(E1)/\sum B(E1)$, in which the summation is performed for the dipole states below 7.7 MeV for 140Ce, those below 7.1 MeV for 142Nd, and below 7.0 MeV for 144Sm. The lowest 1$_1^-$ states are excluded in the summation. The result is tabulated in Table I. For comparison, the RPA values, which include the lowest 1$^-$ state, are listed in the table, but no 1$^-$ state is predicted below 7.1 MeV for 142Nd and 144Sm. Although the calculated mean energies are slightly larger than the observed values, their isotope dependence is consistent with the experiment and the summed transition probabilities are comparable to the experimental values [3].

In the RPA calculation, the neutron excitations are dominant in the low-lying states [15]. The present RPA calculation also indicates, for instance in 142Nd, that the largest components of the low-lying dipole states located at $E_x = 7.36, 8.64, 9.15,$ and 9.55 MeV are $(2p_{\uparrow}/2d_{\downarrow}/2)\pi, (3s_{\uparrow}/2 \rightarrow 3p_{\downarrow}/2)\nu, (3s_{\uparrow}/2 \rightarrow 3p_{\downarrow}/2)\nu,$ and $(3s_{\uparrow}/2 \rightarrow 3p_{\downarrow}/2)\nu,$ respectively. In the SRPA, we see a significant fragmentation of the dipole strength into the energy range of 5 < $E$ < 8 MeV, in addition to the emergence of the lowest 1$_1^-$ state at $E \approx 3.5$ MeV. Many of these low-lying dipole states consist of proton 2p2h characters, such as $[(1g_{\uparrow}/2d_{\downarrow}/2)^{5+} \rightarrow [1h_{31/2}/2d_{3/2}]/^7\pi]$ and $(1g_{\uparrow}/2d_{5/2}/2)^{5+} \rightarrow [1h_{11/2}/2s_{1/2}]/^5\pi).$ These proton 2p2h configurations come down to the lower energy because of the coupling to the 2p2h configurations consisting of the neutron 1p1h transition from the 1$h_{11/2}$ orbit to the 1$h_{9/2}$ orbit and the proton 1p1h transitions from the 1$g_{\uparrow}$ orbit (or 2$d_{\downarrow}$ orbit) to the 1$h_{11/2}$ orbit; $\pi 1g_{\uparrow}/2u 1h_{11/2} \rightarrow \pi 1h_{11/2}/2h_{9/2}$ and $\pi 2d_{\downarrow}/2u 1h_{11/2} \rightarrow \pi 1h_{11/2}/2h_{9/2}$ and. We have confirmed the importance of these proton-neutron 2p2h configurations by performing the SRPA calculation in a smaller 2p2h space. The SRPA calculation with the neutron 1h orbits qualita-

![Fig. 4](image1)

**Fig. 4.** Low-energy E1 strength distributions, $B(E1; 1^- \rightarrow 0^+)$ calculated in the SRPA (solid line) and RPA (dotted line) for 140Ce.

![Fig. 5](image2)

**Fig. 5.** Same as Fig. 4 but for 142Nd.

![Fig. 6](image3)

**Fig. 6.** Same as Fig. 4 but for 144Sm.

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### Table I

Mean energies $\bar{E}$ and summed $B(E1)$ values for the low-energy dipole states. The experimental values are taken from Ref. [3]. See text for details.

| Nucleus | $E_{x}$ [MeV] | $\sum B(E1)$ [e²fₘ] |
|---------|-------------|-------------------|
| $^{140}$Ce | 7.53 | 0.021, 0.219, 0.308 |
| $^{142}$Nd | 6.31 | 0.0 | 0.224, 0.184 |
| $^{144}$Sm | 6.04 | 0.0, 0.233, 0.208 |
Finally, let us discuss the property of the lowest $1^-$ state. The excitation energies and the reduced transition probabilities $B(E1) \uparrow$ of the $1^-$ states in $^{140}$Ce, $^{142}$Nd and $^{144}$Sm are compared with the experimental values [3] in Table II. The calculated excitation energies decrease with increasing proton number, which is consistent with the experiment. However, the SRPA calculations overestimate the $B(E1) \uparrow$ values by a factor of 2.7 – 3.5. The structure of the $1^-$ states in these nuclei is supposed to be predominantly of the two-phonon quadrupole-octupole character $2^+ \otimes 3^- [1, 3]$. However, in the present SRPA calculation, the $2p2h$ configuration $[\pi g_{7/2} \nu 1h_{11/2}]^2 \rightarrow [\pi 2h_{11/2} \nu 1h_{9/2}]^{2\uparrow}$ is dominant in these $1^-$ states, which differs from the two-phonon $2^+ \otimes 3^-$ character. The pairing correlation, which is not taken into account in the present calculation, may play an important role for a better description of the two-phonon character of the $1^-$ states, because they are essential in the description of the lowest quadrupole and octupole states. Furthermore, it has been known that the SRPA fails to describe the collectivity of the two-phonon states [22]. This is due to the fact that the next-leading terms in the two-phonon state are missing in the SRPA. These missing terms beyond the SRPA can be taken into account by introducing $X_{\text{php'k'}}$ amplitudes in Eq. (1). A general equation for the extended RPA formalism with the ground-state correlation is given in Ref. [23]. Another possible method to improve the description of the two-phonon states is the dressed-four-quasi-particle approach proposed in Ref. [21]. These are beyond the scope of the present work, but of significant interest in future.

In summary, the fragmentation of the dipole strength in the $N = 82$ isotones, $^{142}$Nd, $^{142}$Sm, was studied using the second random-phase approximation (SRPA). The SRPA successfully produces the same result.

Note that the energy of the $1^-$ state was overestimated in the SRPA, indicating the necessity of a more elaborate treatment for the states with the two-phonon character. The calculation based on the extended RPA with the ground-state correlations is of great interest and currently under progress.

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