Unitarity at Infinity and Topological Holography

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ABSTRACT

Recently it has been suggested that non-gaussian inflationary perturbations can be usefully analysed in terms of a putative dual gauge theory defined on the future conformal infinity generated by an accelerating cosmology. The problem is that unitarity of this gauge theory implies a strong constraint [the “Strominger bound”] on the matter fields in the bulk. We argue that the bound is just a reflection of the equation of state of cosmological matter. The details motivate a discussion of the possible relevance of the “dS/CFT correspondence” to the resolution of the Big Bang singularity. It is argued that the correspondence may require the Universe to come into existence along a non-singular spacelike hypersurface, as in the theories of “creation from nothing” discussed by Firouzjahi, Sarangi, and Tye, and also by Ooguri et al. and others. The argument makes use of the unusual properties of gauge theories defined on topologically non-trivial spaces.
1. dS/CFT: Limitations and Applications

Efforts to connect the AdS/CFT correspondence [1] with cosmology lead naturally to the idea of a dS/CFT correspondence, in which the physics of accelerating spacetimes is related to a Euclidean CFT defined on spacelike conformal infinity [2][3]. In the form used by Maldacena [4][5][6], this version of holography has recently been revived [7] [see also [8]], in the hope of establishing a new understanding of the improved observational data. There has also been a revival of interest in the theoretical aspects of the correspondence. Thus for example Polchinski [9] has recently discussed the idea of emergent time in asymptotically de Sitter spacetimes, comparing it with “emergent gauge symmetries” of a possible gauge theory at future infinity.

There are, however, some doubts as to whether a precise de Sitter analogue of the AdS/CFT correspondence can actually be constructed. It is generally agreed that a de Sitter phase can only be metastable in string theory [10][11]; generically it would decay to a cosmological spacetime with a Crunch; so the future boundary on which the dual theory is to be defined may not exist. This could simply mean that the dual theory can only be defined on some spacelike hypersurface at large but finite proper time, or that the dual theory can only be formulated in the Euclidean version of the spacetime [as in the work of Maldacena and Maoz [12]], or it may imply more serious limitations. A concrete proposal for understanding the limitations of dS/CFT was suggested by van der Schaar [6], who argued that dS/CFT is effective only at the level of a coarse-graining in which each lattice site of the field theory corresponds to an entire static patch in the de Sitter bulk. One way of stating the case would be to suggest that, on the bulk side of the correspondence, dS/CFT is primarily relevant to specifically “cosmological” features of spacetime and its matter content, not to their detailed structure.

In this spirit, we wish to make an observation regarding a curious and apparently unphysical feature of the dS/CFT correspondence: it appears to impose a severe upper bound on the masses of particles. In detail, the limit of a massive p-form field amplitude [for a p-form field \( \varphi \) of mass \( m_\varphi \)] in de Sitter spacetime defines a CFT two-point function at de Sitter conformal infinity. In the \( p = 0 \) case, the conformal weight corresponding to the boundary operator defined by \( \varphi \) is given, in four spacetime dimensions, by

\[
h_+ = \frac{1}{2} [3 + \sqrt{9 - 4L^2m_\varphi^2}]. \tag{1}
\]

One sees immediately that the weight will be complex unless \( m_\varphi \) satisfies

\[
m_\varphi^2 \leq 9/(4L^2). \tag{2}
\]

Presumably a violation of this bound would therefore signal a failure of unitarity in the field theory. We stress that while this conclusion is derived for scalar matter, it applies to all kinds of p-form matter: see the discussion of the analogous extension in the AdS case in [13].

This “Strominger bound” [2][4][15] on particle masses is analogous to the well-known Breitenlohner-Freedman bound [16] on the masses of p-form fields in anti-de Sitter spacetime; but, being an upper bound, it is at first sight much less reasonable. For some purposes, it can be ignored: it has been argued by Seery and Lidsey [7] that only very
low-mass scalars could give rise to non-trivial perturbations at late times, so the Strominger bound is not a problem for the recent studies of inflationary perturbations from the dS/CFT point of view. However, one would certainly prefer to have a more precise demonstration that the Strominger bound really does not impose unreasonable conditions on any of the matter fields which are relevant on the scales to which the dS/CFT correspondence applies.

We shall suggest a solution of this problem inspired by the arguments of van der Schaar [6] mentioned earlier: we shall assume that dS/CFT should only be expected to yield useful information at the cosmological level, and not on smaller scales. The relevant forms of matter at this level, apart from the dark energy/inflaton [both of which we represent by a cosmological constant] are zero-pressure [non-relativistic] matter, radiation, and, in the pre-inflationary era, “fluids” representing strings and other extended objects. The question is then whether there is some sense in which these specifically “cosmological” forms of matter do in fact satisfy the Strominger bound. If this is not the case, then dS/CFT would imply that the apparently reasonable demand of unitarity on the part of the field theory imposes unphysical conditions on bulk physics, and this would call the whole approach into question.

We argue here that “cosmological matter” does [barely] satisfy the Strominger bound: interestingly, zero-pressure matter actually saturates the bound. We conclude that the bound is not unreasonable, provided that the limits to the validity of dS/CFT, of the kind pointed out by van der Schaar, are kept in mind. The agreement of the demand of unitarity on the boundary with the requirement that the bulk field reproduce the classical equation of state is rather remarkable.

One of the major hopes for the AdS/CFT correspondence is that it will allow us to probe the singularities in black holes [17][18][19]. Similarly, one might hope to use AdS/CFT or dS/CFT to probe the Big Bang singularity [20]. Unfortunately, even in the AdS case, it is extremely difficult to do this in an even partially realistic way. For some recent examples: in [21] an ingenious attempt is made to study de Sitter space by embedding it inside an AdS black hole, but the resulting cosmology has no Big Bang; conversely, the AdS/CFT correspondence has been used to study Bang singularities in certain cosmological models [22], but these are not asymptotically de Sitter to the future. For dS/CFT the case is still worse, since so little is known about the putative boundary theory. In the concluding section we draw attention to the fact that the topologically non-trivial spacetime structure assumed here [we take the spatial sections to be tori] can be expected to give rise to some unfamiliar physics, of the kind discussed some time ago by Krauss and Wilczek [23] and by Preskill and Krauss [24]. If the field theory at infinity is of this kind, in which parallel transport around non-contractible loops can cause discrete transformations, then this can give rise to very puzzling behaviour deep in the bulk, near to the initial singularity. We speculate that this kind of “topological holography” could have some role to play in resolving that singularity, when the dS/CFT correspondence is better understood.
2. Representing Cosmological Matter by a Massive Field

In this section, we show how to represent the familiar forms of cosmological matter in a way that can be connected with the Strominger bound. At the same time, we must incorporate the effect of such matter on the spacetime geometry.

We begin with the version of de Sitter spacetime with flat spatial sections; we compactify these sections to copies of the [cubic] three-torus $T^3$. It has been argued elsewhere that this is the most suitable global structure for investigations of quantum cosmology \[25\][26][27]. One of the many virtues of this toral version of de Sitter spacetime is that its boundary at infinity is compact and connected, so that we can avoid the subtle issues \[2\] which arise when the dS/CFT correspondence is constructed on the spatially spherical version of de Sitter spacetime, which has two spheres at infinity. Here, instead, we have one torus at [future] infinity. Henceforth, then, our working hypothesis is that the spacetime topology is $\mathbb{R} \times T^3$ [or possibly some non-singular quotient].

Thus the metric, before we introduce any matter, is that of Spatially Toral de Sitter:

$$g(\text{STdS}) = dt^2 - K^2 e^{(2t/L)} [d\theta_1^2 + d\theta_2^2 + d\theta_3^2],$$

where $\theta_1, \theta_2, \theta_3$ are angular coordinates on a cubic three-torus, where $K$ defines the spatial length scale, $L$ is the scale defined by the cosmological constant, and we use $(+ − − −)$ signature.

Now we wish to introduce into this spacetime the familiar cosmological matter fields --- non-relativistic matter, radiation, cosmic strings and so on. A common property of all these forms of matter is that they have a constant equation-of-state parameter $w$, where $w$ specifies the ratio of pressure to density in the homogeneous case. In order to determine whether the Strominger bound is satisfied, we have to find a formal representation of these matter fields by some p-form field with a definite mass. In view of the isotropy and homogeneity of cosmological matter, the obvious choice is to try to do this using a scalar field \[p = 0\].

The solution is as follows. We take STdS spacetime, and introduce into it a homogeneous scalar field $\varphi$ with the usual kinetic term and with a potential

$$V(\varphi, \epsilon) = -\frac{3}{8\pi L^2} [1 - \frac{1}{6} \epsilon] \sinh^2(\sqrt{2\pi \epsilon} \varphi);$$

here $\epsilon$ is a positive constant. Bear in mind that STdS spacetime in this signature has a negative cosmological constant $-3/L^2$ which gives rise to a positive energy density $3/(8\pi L^2)$. The precise numerical factor in the potential is chosen for later convenience\(^1\).

We now make the following claim: we assert that, as far as the spacetime geometry is concerned, the field $\varphi$ exactly mimics the effects on STdS spacetime of a homogeneous matter field with positive energy density and a constant equation-of-state parameter related to $\epsilon$ by

$$w_\varphi = \frac{1}{3} \epsilon - 1.$$  

Thus for example if we insert non-relativistic matter [zero pressure, hence $w = 0$] into the STdS spacetime and allow it to act on the spacetime geometry, this will have the same

\(^1\)This choice normalizes the cosmological scale factor in a convenient way.
effect as introducing $\varphi$ with $\epsilon = 3$, while $\varphi$ with the value $\epsilon = 4$ mimics the effects of radiation; $\epsilon = 1$ corresponds to a static network of planar domain walls, increasing to 1.5 for the scaling regime \cite{28}; $\epsilon = 2$ for a static network of strings, and so on. We stress that we are not primarily interested in using this field to violate the Strong Energy Condition — asymptotically at least, the acceleration is due to the negative contribution made by the STdS cosmological constant to the total pressure. Instead, $\varphi$ just represents any form of homogeneous cosmological matter, with a constant equation-of-state parameter $w_{\varphi} \geq -1$, which is to be superimposed on the STdS spacetime.

We now proceed to justify these claims. We shall consider Friedmann cosmological models with metrics of the form

$$g = dt^2 - K^2 a(t)^2 [d\theta_1^2 + d\theta_2^2 + d\theta_3^2];$$

(6)

this generalizes the STdS metric in an obvious way. Adding the energy density of the $\varphi$ field to that of the initial STdS space, we have a Friedmann equation of the form

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} \left[ \frac{1}{2} \dot{\varphi}^2 - V(\varphi, \epsilon) + \frac{3}{8\pi L^2} \right].$$

(7)

The equation for $\varphi$ itself is

$$\ddot{\varphi} + 3 \frac{\dot{a}}{a} \dot{\varphi} - \frac{dV(\varphi, \epsilon)}{d\varphi} = 0.$$  

(8)

Surprisingly, these equations have very simple solutions: one finds that [with natural initial conditions] $\varphi$ is given by

$$\varphi = \frac{1}{\sqrt{\pi \epsilon/2}} \tanh^{-1}(e^{-\epsilon t/2L}),$$

(9)

and the metric is\footnote{The reader who wishes to undertake the task of verifying these solutions will find the following simple fact helpful: if $A$ and $B$ are quantities related by $\tanh(A) = e^{-B}$, then $\cosh(2A) = \coth(B)$.}

$$g(\epsilon, K, L) = dt^2 - K^2 \sinh^{(4/\epsilon)}(\frac{\epsilon t}{2L}) [d\theta_1^2 + d\theta_2^2 + d\theta_3^2].$$

(10)

From these results one can compute the energy density and pressure of the $\varphi$ field alone:

$$\rho_{\varphi} = \frac{3}{8\pi L^2} \text{cosech}^2 \left( \frac{\epsilon t}{2L} \right),$$

(11)

$$p_{\varphi} = \frac{3}{8\pi L^2} \left[ \frac{1}{3} \epsilon - 1 \right] \text{cosech}^2 \left( \frac{\epsilon t}{2L} \right),$$

(12)

from which equation (5) above is immediate. Note that the density decays more rapidly with increasing time for larger values of $\epsilon$.

Since we are using the standard kinetic term, $\varphi$ automatically satisfies the Null Energy Condition. To see the conditions under which the Strong Energy Condition is satisfied, we have to determine whether the quantity $\Pi = \rho + 3p$ is non-negative, where $\rho$ and $p$ are the total energy density and total pressure [including the contributions due to the
background cosmological constant, which total to \(-3/4\pi L^2\) respectively. For the metrics we are concerned with here, \(\Pi\) is a function of \(\epsilon, L,\) and \(t\) given by

\[
\Pi(\epsilon, L, t) = \frac{-3}{4\pi L^2} \rho_\phi + 3\, p_\phi = \frac{-3}{4\pi L^2} \left[ 1 + \left(1 - \frac{\epsilon}{2}\right) \cosh^2\left(\frac{\epsilon t}{2L}\right) \right].
\] (13)

We see that the SEC is always violated eventually, but that it is satisfied in the early Universe provided \(\epsilon > 2;\) otherwise it is violated at all times, just as it is in de Sitter spacetime. In physical terms, this means that, for all \(\epsilon > 2\) the Universe decelerates in its earlier [post-inflationary] stages, and only later begins to accelerate: this is the case for the post-inflationary phase of our Universe. For \(\epsilon \leq 2,\) by contrast, the spacetime always accelerates, and in fact for \(\epsilon < 2\) the acceleration actually diverges as \(t = 0\) is approached. We shall have more to say about this below.

If \(\epsilon = 3,\) we should have the local metric for a spacetime containing non-relativistic matter and a de Sitter cosmological constant, and indeed \(g(3, K, L)\) gives — purely locally — the classical Heckmann metric [see [29] for a recent discussion]. In the general case it agrees [again locally] with the results reported in [30], where it is obtained by postulating a linear equation of state [without giving a matter model]. For large \(t\) we have

\[
g(\epsilon, 2^{3/\epsilon} K, L) \approx dt^2 - K^2 e^{2t/L}[d\theta_1^2 + d\theta_2^2 + d\theta_3^2],
\] (14)

which is the STdS metric given in equation (3); notice that \(\epsilon\) effectively drops out. Thus our metric is “asymptotically STdS”, for all \(\epsilon.\)

All of the metrics in (10) appear to be singular at \(t = 0,\) but we remind the reader that such appearances can be deceptive: for example, consider the metric

\[
g(SHdS_4) = dt^2 - \sinh^2(t/L)[dt^2 + L^2 \sinh^2(r/L)\{d\theta^2 + \sin^2(\theta) \, d\phi^2\}],
\] (15)

with hyperbolic spatial sections [which we do not compactify here]. This appears to be singular in the same way, but in fact this is just a version of de Sitter spacetime, the [non-compactified] “spatially hyperbolic” version [27], which is, of course, entirely non-singular. The question as to which members of (10) are really singular is important, and we shall return to it below.

We have shown how to represent cosmological matter in a way that is relevant to the Strominger bound. Let us now show that the bound is in fact satisfied by such matter.

### 3. Cosmological Matter Satisfies the Strominger Bound

At late times, the metric \(g(\epsilon, K, L)\) [equation (10)] is locally indistinguishable from that of de Sitter spacetime, and, furthermore, \(\varphi\) is very small [equation (9)]; hence we see, from (4), that \(\varphi\) can be regarded as a scalar field of squared mass

\[
m_\varphi^2 = \frac{3}{2L^2} \epsilon \left[1 - \frac{1}{6} \epsilon\right]
\] (16)

propagating on a local de Sitter background. By computing

\[
\frac{9}{4L^2} - m_\varphi^2 = (\epsilon - 3)^2/4L^2 \geq 0,
\] (17)
we see at once that the Strominger bound is automatically — but only barely — satisfied for all values of $\epsilon$. The conformal weight is real: from (11) we have

\[ h_+ = \frac{\epsilon}{2}, \quad \epsilon \geq 3 \]
\[ = 3 - \left(\frac{\epsilon}{2}\right), \quad \epsilon \leq 3. \]  

(18)

Notice that by varying $\epsilon$, either from 0 to 3 or from 3 to 6, one can obtain all values of the weight allowed by the Strominger bound [that is, all values between 3/2 and 3]: in fact, apart from 3/2, all allowed values of the weight can be obtained in two different ways. Thus for example $h_+ = 2$ can be obtained either by choosing $\epsilon = 2$ or by taking $\epsilon = 4$.

From (17) we see that the Strominger bound is most nearly violated when $\epsilon = 3$; it is saturated in this case, that is, in the case of zero-pressure, non-relativistic matter. This has a deep mathematical significance, as follows. Because de Sitter spacetime and Euclidean hyperbolic space have the same isometry group, the relevant representation theory has been extensively developed \[31\][32]. The scalar representations fall into three families, the principal, complementary, and discrete series. The principal representations are those which, under contraction of the de Sitter group to the Poincaré group, correspond to the familiar flat space representations. They are of two kinds, which in fact are classified precisely by the weight $h_+$ which appears in the dS/CFT correspondence. The first kind is the case where $h_+$ is complex: these of course violate the Strominger bound. The only other principal representation corresponds precisely to the case where the bound is saturated.

The complementary series consists of representations where the Strominger bound is satisfied but not saturated\(^3\); these representations have no flat-spacetime analogue. [The discrete series corresponds to the special, massless case.] We conclude that $\epsilon = 3$ is the only value which corresponds to a well-defined flat space representation. It is interesting that $\epsilon = 3$ is singled out in this way, for this corresponds to the kind of matter that, in classical cosmology, is the most important at late times. It is also interesting that the field we have used to represent asymptotically de Sitter cosmology with parameter $\epsilon$ corresponds to precisely the same de Sitter representation as the cosmology with parameter $6 - \epsilon$.

We conclude that the Strominger bound is in fact satisfied by all kinds of “cosmological matter”. We have shown this by representing cosmological matter by fields which, except in the case of non-relativistic matter, correspond exactly to the complementary series of representations of the de Sitter group. The perfect agreement here — note the fact that the most familiar kind of cosmological matter just exactly saturates the bound — is rather remarkable, in that the Strominger bound is required by a quantum condition on the boundary [unitarity of the field theory] while the form of the potential (4) is dictated by the demand that the classical equation of state of the bulk matter should be reproduced\(^4\).

Our discussion shows that the cosmological models with metrics given by equation (10) do define a well-behaved CFT at infinity. Since these models are both asymptotically de

\(^3\)The relevant Hermitian form in the complementary case \[31\], page 518] involves a gamma function which is ill-defined if the Strominger bound is saturated, so the $\epsilon = 3$ representation certainly does not belong to this series.

\(^4\)In this regard it may be of interest to examine the status of the Strominger bound when more complex equations of state are considered; see for example \[33\].
Sitter and [unlike de Sitter spacetime itself] have large energy densities and pressures in the remote past, they are more realistic than de Sitter spacetime, and they are the natural models to consider if one hopes to use the dS/CFT correspondence to try to understand the distant past of the Universe. We conclude with some observations regarding these spacetimes and the possible relevance of dS/CFT to the problem of the initial singularity.

4. Comments on The Initial Singularity and dS/CFT

The recent work of Seery and Lidsey [7] is based on the idea that the dS/CFT correspondence may be able to teach us something about the inflationary era. It is natural to extend this idea to ask whether the correspondence may be brought to bear on the pre-inflationary era, about which very little is known. The most important problem which arises here is that of the initial singularity, a problem which is not solved by Inflation [34]. In the case of AdS/CFT, good behaviour [such as unitarity] on the part of the boundary field theory can be used to argue [17] for good behaviour in the bulk [avoidance of information loss in black hole evaporation]. Ultimately one hopes to construct an analogous argument in the dS/CFT case.

Unfortunately, very little is understood about the field theory at infinity in this case, so we cannot yet hope to replicate the progress that has been made in the AdS case. However, the spacetimes we have been discussing do have some unusual properties which may suggest novel approaches to the problem of the initial singularity.

First let us consider the field theory on the boundary. The key point to note here is that the conformal boundary is geometrically very simple [it is flat] but topologically non-trivial: it is either a torus or perhaps some non-singular quotient of a torus [35][36]. The fundamental group is infinite, that is, there are non-contractible curves which wind around the space arbitrarily many times.

Gauge theories on such spaces can have a number of very interesting properties, going far beyond the need to impose periodic boundary conditions. For example, extending the work of Krauss and Wilczek [23] on the theory of “discrete gauge hair” on black holes [6], Preskill and Krauss [24] were led to study the extraordinary phenomena arising when gauge theories with discrete or disconnected gauge groups are constructed on spaces with non-trivial fundamental groups. These occur very naturally when gauge groups are embedded in larger groups, as in Grand Unified theories. Let us take a concrete example, though we stress that phenomena of the kind we are about to describe can occur in very many other ways.

When the SU(3) of QCD is embedded in the Spin(10) Grand Unified Group [usually known, not quite accurately, as “SO(10)”], one finds that there are many disconnected subgroups of Spin(10) with SU(3) as identity component. An interesting example of such a subgroup has the following form: it may be expressed as a semi-direct product of $\mathbb{Z}_4$ with SU(3):

$$SU(3) \ltimes \mathbb{Z}_4 = SU(3) \bigcup \gamma \cdot SU(3) \bigcup (-1) \cdot SU(3) \bigcup (-\gamma) \cdot SU(3),$$  \hspace{1cm} (19)

\[5\] The boundary can of course have a non-trivial fundamental group even in the locally spherical case, but this group is always finite in that case.

\[6\] See [37] for a comprehensive review of this subject.
where $-1$ is the element of Spin(10) which is factored out to obtain SO(10), and where $\gamma$ is a certain Spin(10) element with $\gamma^2 = -1$, such that conjugation by $\gamma$ has the effect of complex conjugation on SU(3). Conjugation by $\gamma$ also has the effect of complex conjugation on the electromagnetic U(1) embedded in Spin(10); hence it is related to charge conjugation.

Such disconnected groups are of physical interest if one can prove that there is a gauge connection with a holonomy group — the group generated by gauge parallel transport around closed loops — which contains an element representing $\gamma$. It is in fact possible to prove \cite{38} that, with topology $T^3$, there do exist $SU(3) \triangleleft \mathbb{Z}_4$ gauge connections with holonomy groups isomorphic to the $\mathbb{Z}_4$ generated by $\gamma$. This is analogous to parallel transport on a [flat] Klein bottle: despite the fact that the metric is flat, parallel transport around certain non-contractible loops on the Klein bottle can give rise to a transformation which reverses orientation. The [linear] holonomy group of the Klein bottle is just $\mathbb{Z}_2$, generated by an element of the orthogonal group $O(2)$ which does not lie in the connected component of the identity.

If we had a gauge connection on the torus at infinity with holonomy group $\mathbb{Z}_4$ generated by $\gamma$ as above, then moving an object around certain closed, non-contractible curves would cause particles to transform to anti-particles, and vice versa: the system is “non-orientable” with respect to charge conjugation instead of parity. The very remarkable properties of such gauge theories are described in detail by Preskill and Krauss in \cite{24}, to which the reader is referred for further details. Here we shall not need to consider a specific gauge group like $SU(3) \triangleleft \mathbb{Z}_4$: all we need is to understand that we should expect parallel transport around certain non-contractible loops on the torus to implement discrete transformations, analogous to [but probably different from] charge conjugation.

There are three specific points to be made here. First, with spatial topology $T^3$, there is nothing artificial about gauge connections with holonomy groups of this kind; indeed, such behaviour should be regarded as generic when gauge theories are constructed on the torus, provided that a disconnected gauge group arises. Second, in the cosmological context, one might require a gauge field to have vanishing field strengths; but phenomena of the kind discussed by Preskill and Krauss can still be present even in this case, in the manner of the Aharonov-Bohm effect\textsuperscript{7}. Third, such gauge fields are in no sense pathological; that is, we have no reason whatever to expect any kind of unphysical behaviour on the part of the boundary field theory, though the details may be unfamiliar — for example, conservation of charge [or whatever conservation law is relevant to the discrete symmetry in question] is enforced in a very unusual way, through “Cheshire charges” \cite{24}. In fact, non-trivial holonomies of this kind appear routinely in string theory in the form of “Wilson lines”, which have been used recently in string cosmology \cite{40}.

We conclude that, if anything resembling the AdS/CFT correspondence is valid here, then it should be possible to construct a model of bulk physics which remains non-pathological even when the boundary theory fully incorporates effects due to the toral topology. As we shall see, this leads us in a very interesting direction.

In order to give a more concrete basis for discussion, let us construct a very simple explicit model of the pre-inflationary era. In fact, some of the spacetimes discussed above are ideally suited to this. For all of them evolve naturally to an “inflationary” state,

\footnote{See \cite{39} for a recent discussion of such effects in cosmology.}
that is, they are all asymptotically de Sitter to the future, and they all have a [single] torus as their conformal boundaries. On the other hand, some of them can be used to set up a semi-realistic model of conditions in the very early Universe, which may well have contained networks of extended objects of the kind encountered in, for example, string gas cosmology [11]. Such networks correspond to matter fields with energy densities that decay relatively slowly; in fact, using equation (5), one can show that the relevant range of $\epsilon$ is $\epsilon \leq 2$. We stress that it is the pre-inflationary era that is being discussed here; the usual lore regarding the cosmology of extended objects is not relevant.

For example, take the case of a spacetime of topology $\mathbb{R} \times T^3$ containing dark energy and a static network of planar domain walls, described by (10) with $\epsilon = 1$:

$$g(1, K, L) = dt^2 - K^2 \sinh^4(\frac{t}{2L}) [d\theta^2_1 + d\theta^2_2 + d\theta^2_3].$$

(20)

This appears to be singular at $t = 0$, but, as we mentioned earlier, this could be deceptive. In the metric given in (15), what is contracting to zero size is not a spatial section, but rather the separation of neighbouring members of the congruence of timelike curves which defines the given coordinates. The same might happen here. Physically, one can argue very strongly that the metric in (20) should not be singular, as follows.

The metrics in (10) with $\epsilon > 2$ are in fact singular; but this is just what we would expect. For we saw [equation (13)] that in those cases the Strong Energy Condition holds in the early Universe, and a singularity is then demanded by the classical singularity theorems. In physical terms, gravity is attractive when the SEC holds, and so the presence of a singularity is to be expected. [Since the boundary theory is assumed to be well-behaved, this presumably just means that such matter fields are not relevant in the very early Universe — which is reasonable.] By contrast, the dark energy corresponding to a de Sitter cosmological constant generates gravitational repulsion, and this is how we understand the fact that pure de Sitter spacetime is not singular.

By this logic, it would not make sense for the spacetime with metric $g(1, K, L)$ to be singular: here, gravitation is always repulsive, just as it is in the case of the de Sitter spacetime [equation (3)] of which it is a deformation. For the latter, the quantity $p + 3\rho$, which [when negative] measures the extent of violation of the Strong Energy Condition, is given by the negative constant $-3/4\pi L^2$; whereas here [from (13)] we have

$$\Pi(1, L, t) = -\frac{3}{4\pi L^2} \left[ 1 + \frac{1}{2} \cosech^2\left(\frac{t}{2L}\right) \right].$$

(21)

Evidently the function $\Pi(1, L, t)$ is always negative, and in fact it is more negative than in the case of de Sitter spacetime. As this quantity measures “gravitational repulsion”, there is in fact even less reason for the spacetime to be singular here than in the de Sitter case. It is true that $\Pi(1, L, t)$ tends to negative infinity as $t$ approaches zero, but this could be due to a singularity of the congruence of timelike curves being used here, and not of the spacetime. We saw that this accounted for the apparent singularity in the metric in equation (15). Our discussion leads us to expect something similar here.

It is therefore very surprising to find that $g(1, K, L)$ is in fact singular. The scalar curvature of this metric is

$$R(g(1, K, L)) = -\frac{12}{L^2} - \frac{9}{L^2} \cosech^2\left(\frac{t}{2L}\right).$$

(22)
so indeed we have a curvature singularity at $t = 0$; in fact, all of the metrics with $\epsilon \leq 2$ are singular. To appreciate fully what this means, take this metric and define it on the interval $(-\infty, 0)$, so that it describes a contracting cosmology. The “force of gravitational repulsion” increases without limit as the Universe contracts, and yet it manages to contract to zero size. This appears to be self-contradictory behaviour.

One’s first reaction to this extraordinary situation is to suggest that the singularity is a result of the special boundary conditions or special symmetries of the metric, and that a more realistic model would “bounce”, just as the spatially spherical version of de Sitter spacetime does, and for the same reason. *This is not correct, however:* Andersson and Galloway [42][43] have proved results which essentially imply that that every future asymptotically de Sitter metric defined on a manifold of topology $\mathbb{R} \times T^3$ must be geodesically incomplete in the past if the Null Energy Condition and the Einstein equations hold at all times. Thus boundary conditions and symmetries cannot be “blamed” for the fact that $g(1, K, L)$ is singular.

The next suggestion is that the metrics with $\epsilon \leq 2$ might perhaps be ruled out by some internal contradiction in dS/CFT itself, and indeed it was this possibility that inspired the present investigation. But we have seen that the relevant forms of matter define a well-behaved unitary field theory at infinity; matter with, for example, $\epsilon = 2$ is precisely as far from violating unitarity at infinity as ordinary radiation [$\epsilon = 4$], and indeed both are represented formally in our analysis by fields in the same representation of the de Sitter group. Hence there is no hint that the spacetimes with $\epsilon \leq 2$ are unacceptable from a holographic point of view. Furthermore, it has been shown [26] that these spacetimes are stable against known [44] perturbative and non-perturbative instabilities in string theory.

The Andersson-Galloway results imply that it is the topology of conformal infinity that is to be “blamed” for the singularities in these metrics, *as long as the Einstein equations are assumed to hold everywhere.* But we have stressed that, while the topology of the boundary can be expected to influence the boundary gauge theory in important ways, it certainly should not give rise to any unphysical effects. We conclude that if anything like a dS/CFT correspondence is valid, the bulk singularity in the metrics with $\epsilon \leq 2$ will be resolved, and this will happen because the boundary topology will enforce a suitable modification of the Einstein equations\(^9\). This is certainly a reasonable prediction since, as we saw, it is not physically reasonable for the metrics with $\epsilon \leq 2$ to be singular in the first place.

The boundary topology can have physical consequences in a variety of ways, but we wish to propose that the strange effects discussed by Preskill and Krauss [24], which directly link the boundary topology to gauge theory physics, are relevant here. How can these effects enforce a non-singular spacetime structure? Only a much more detailed understanding of the correspondence can tell us exactly how this is accomplished, but a further examination of the metrics in (10) does reveal some striking hints.

Let us assume that that the field theory at infinity has a discrete or disconnected gauge group, so that parallel transport around non-contractible loops can impose some discrete

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\(^8\)See [26] for a discussion.

\(^9\)The only alternative is a violation of the Null Energy Condition, which we consider to be implausible; see [24][43][46] for discussions of this.
transformation. For the sake of clarity we shall refer to this discrete transformation as “charge conjugation”, but we stress that charge conjugation is just an example of the kind of transformation that can arise in this way.

If a dS/CFT correspondence is valid, we can expect a global “particle/antiparticle” ambiguity at infinity to be inherited by the spatial sections of the bulk. At the present time, and indeed by the end of Inflation, this would be physically irrelevant, since circumnavigations of the Universe along non-contractible curves are of course by that time not possible. They may well, however, have been possible in the pre-inflationary era, and this we now investigate.

There is in fact a major difference between the metrics of the form \(g(\epsilon, K, L)\) with \(\epsilon \leq 2\) [which we originally hoped to use to describe the pre-inflationary era] and those with \(\epsilon > 2\); this may be seen as follows. As \(\sinh^{(2/\epsilon)}(\frac{L}{2\epsilon})\) is approximately proportional to \(t^{(2/\epsilon)}\) when \(t\) is small, it follows that the Lorentzian [angular] conformal time \(\eta\) [defined by \(d\eta = dt/K\sinh^{(2/\epsilon)}(\frac{L}{2\epsilon})\)] converges as \(t\) tends to zero, provided that \(\epsilon > 2\). On the other hand, if \(t\) is large, \(\sinh^{(2/\epsilon)}(\frac{L}{2\epsilon})\) resembles a multiple of \(e^{t/L}\), and so \(\eta\) always converges as \(t\) tends to infinity. Since the spatial sections are always finite, it follows that, in the case \(\epsilon > 2\), our spacetime is conformal to a piece of Minkowski space which is finite in both space and time.

The Penrose diagram for this case is shown in Figure 1. Future conformal infinity is spacelike, and there is a Big Bang singularity which is also spacelike. The horizontal direction represents only half of the range of any angular coordinate on the torus, namely the half corresponding to angles between zero and \(\pi\). Thus the vertical lines represent the origin and the point which in this direction is most distant from the origin. [The reader may find it helpful to compare this with the diagrams in [47].]

The width of the diagram is given by \(\pi\). Its height, in the \(\epsilon > 2\) case, is given by

\[
H(L/K, \epsilon) = \frac{2L}{K\epsilon} \int_{0}^{\infty} \frac{dx}{\sinh^{(2/\epsilon)}(x)},
\]

that is, it is proportional to \(L/K\), and it is related to \(\epsilon\) in a more complicated way. For example, for \(\epsilon = 3\) [non-relativistic matter], the height is approximately \(2.81 \times L/K\).

The shape of this diagram determines whether the Universe can be circumnavigated. For example, with the choice of parameters leading to the particular shape shown in

![Figure 1: Penrose diagram for \(g(\epsilon, K, L)\) spacetime, \(\epsilon > 2\).](image-url)
Figure 1, it is clear that no particle or signal can circumnavigate the spatial torus, even in the infinite proper time available. However, if we allow $\epsilon$ to descend towards 2, the integral in equation (23) becomes steadily larger. For values just above 2, $H(L/K, \epsilon)$ will be much larger than $\pi$, even if $L$ and $K$ are of similar magnitudes. In that case, the Penrose diagram will resemble Figure 1, but it will be much taller than it is wide. In this case, circumnavigation is easy; in fact, for very tall diagrams, a generic worldline will wrap around the torus$^{10}$.

Circumnavigations of the early Universe can be of interest for a variety of reasons. For example, they can help to preserve any initial homogeneity, a fact which is the basis of Linde’s model of low-scale Inflation $^{48}$.$^{49}$. They are also of interest to us here, however, because we are assuming that circumnavigations can convert particles to anti-particles [or cause some other similar discrete transformation]. Suppose that $\epsilon$ is just above 2, so that the Penrose diagram is much taller than it is wide; then a generic timelike worldline, representing the history of [say] a quark, will execute a definite, non-zero but finite number of circumnavigations as we trace it back to the singularity, and so we can specify whether this particle emerged from the singularity as a quark or as an antiquark. But now let us ask what happens when $\epsilon \leq 2$, as we wish to assume here.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{penrose_diagram.png}
\caption{Penrose diagram for $g(\epsilon, K, L)$ spacetime, $\epsilon \leq 2$.}
\end{figure}

If $\epsilon \leq 2$, then conformal time still converges as $t$ tends to infinity, but it diverges as $t$ tends to zero. In this case, the Penrose diagram has an unusual structure pictured in Figure 2. The triangle OAC represents the lower half of the Minkowski conformal diagram, and the line EFC represents a typical spacelike hypersurface defined by a fixed value of $t$.

$^{10}$Recall again that we are dealing with the situation before Inflation; by the end of Inflation, circumnavigations will no longer be possible.
The point F represents the two-sphere of radius $\pi K \sinh^{(2/\epsilon)}\left(\frac{\epsilon t}{2}\right)$ in that spacelike surface. This two-sphere can be enclosed in a cube of side length $2\pi K \sinh^{(2/\epsilon)}\left(\frac{\epsilon t}{2}\right)$, and this cube can be allowed to expand or contract as we follow the sphere either into the future or the past along the geodesic OFB. We now perform the usual identifications of the faces of these cubes, to obtain tori. The effect on the Penrose diagram is [to a good approximation] to cut away all parts of the original triangle which lie to the right of the chosen geodesic OFB; that is, we cut away the dotted region in Figure 2. Note that conformal infinity is also compactified: it is represented by the solid part, AB, of the upper horizontal line, and of course it is also a torus. The singularity is represented by a single point, O.

Now note that we do not expect holography to resolve the singularity in Figure 1, at least not directly. Instead, the boundary theory must in some way inform us that values of $\epsilon$ greater than 2 are not appropriate for describing the pre-inflationary era. For, as we discussed earlier, the matter content of those spacetimes is such that gravity is attractive in the earliest era, and so a singularity is natural. The singularity should be directly resolved only in those cases where the unusual matter content of the early universe [dominated by extended objects such as strings] is such as to lead us to expect that a singularity is not physically reasonable. That is, holography should work in a way so that the singularity at O in Figure 2, but not the one in Figure 1, is revealed to be unphysical. How can the boundary physics distinguish the singularities in Figures 1 and 2?

Consider a timelike geodesic extending to the past from the point D. In the diagram it appears to “bounce” off the line OFB, because it re-enters the torus on the opposite side when it passes beyond the most distant point from the origin; it then proceeds towards the line OA [the worldline of the origin], and again appears to “bounce” as it passes through the origin. It reaches the singularity at O in a finite proper time, but it must circumnavigate the torus a literally infinite number of times in order to do this. It is not entirely clear what this statement means, and it is particularly confusing if we recall that circumnavigations can have the effect of some discrete transformation such as charge conjugation, so that this transformation is being applied an infinite number of times in a finite period of proper time. It follows, for example, that it simply does not make sense to ask whether a given object, such as a quark, was initially a quark or an anti-quark. We find ourselves getting entangled in the traditional paradoxes associated with infinity.

This is the classical way of stating the case. In reality, quantum effects would render this situation unphysical well before the singularity is reached: quantum uncertainties, combined with the effects of parallel transport, would make it impossible to specify whether a given object is a particle or an anti-particle for some interval of time after the singularity. The nature of the system and the physics of its interactions cannot be properly formulated under such conditions.

These paradoxes simply indicate that we are dealing with a procedure that cannot actually be realized: that is, the singularities in the metrics with $\epsilon \leq 2$ cannot really be attained by any actual particle or antiparticle, and they should be regarded as a mathematical fiction. The question then is: what takes the place of the region near O in Figure 2?

The most straightforward assumption is that the paradoxes we have been discussing are avoided because the spatial sections of the Universe are never in fact smaller than a certain size; this is also compatible with the ideas of string gas cosmology [11]. That
is, the Universe must be born along some spacelike hypersurface large enough so that the status of a given object is always well-defined modulo a finite number of discrete transformations\textsuperscript{11}.

This is just the “creation from nothing” scenario \textsuperscript{51,52}, recently revived and improved in various ways by Tye and co-workers \textsuperscript{53,54,55} and by Ooguri et al \textsuperscript{56}; see also \textsuperscript{57} for similar approaches. However, if we wish to avoid violating the Null Energy Condition, it is difficult to arrange for the Universe to be created smoothly along a three-torus; in fact, it can only be done by modifying the Einstein equations \textsuperscript{27}. A simple way of performing such a modification, using the ideas of Gabadadze and Shang \textsuperscript{58,59}, was explained in \textsuperscript{26}; the resulting “Creation From Nothing on a Torus” metric, defined for $t \geq 0$, is given by

$$g_{\text{CFNT}}(K, L) = dt^2 - K^2 \cosh^{(2/3)} \left( \frac{3t}{L} \right) [d\theta_1^2 + d\theta_2^2 + d\theta_3^2].$$

\textsuperscript{(24)}

It is completely non-singular and smooth.

In this roundabout way, we find that the topology of the boundary, expressed by the effects discovered by Krauss, Wilczek, and Preskill, does seem to have profound consequences for bulk physics in the case depicted in Figure 2 [and only in that case]. In fact, the effects due to non-trivial gauge holonomy become more pronounced as we penetrate deeper into the bulk, that is, farther into the past and closer to the apparent initial singularity. By requiring that the bulk geometry should account for the boundary gauge holonomy group in a reasonable way, we may be able to resolve the singularity portrayed in Figure 2. One might call this an example of \textit{topological holography}.

Obviously this is a mere sketch of a novel way in which dS/CFT might guide us towards a resolution of the initial singularity. What this discussion really tells us is that as we work towards a better understanding of de Sitter holography, we should be aware of the possibility that the gauge group may be disconnected or discrete, since this may well have profound consequences both for the boundary field theory and for the bulk.

### 5. Conclusion

We have seen that, once its limitations are understood, the dS/CFT correspondence may have much to teach us about cosmology. We found that the apparently unreasonable Strominger bound is seen to be perfectly reasonable when it is related to the equation of state of cosmological matter. We then asked whether dS/CFT can have any bearing on the problem of the initial singularity in cosmology: can a gauge theory at future infinity influence the physics of the deep bulk? While we were not able to answer this question definitively, we have uncovered some hints suggesting that the answer may be positive.

The specific examples of gauge theories and spacetimes we have discussed here are of course over-simplified. They should be regarded as suggestions as to the lines along which one might work when the dS/CFT correspondence is better understood. What this discussion does clearly show is that the \textit{topology} of the future conformal boundary has a strong influence \textit{both} on the gauge theory at infinity \textit{and} on bulk physics. On one side, non-trivial topology leads to unusual physics due to holonomy effects, while on the other

\textsuperscript{11}Alternatively, the Universe could be asymptotic in the past to some quasi-static state, as in \textsuperscript{50}.
it may well enforce a modification of the Einstein equations, leading to a replacement of
the singularity shown in Figure 2 by “creation from nothing”. Clarifying the connections
between these two ideas may help us to understand both the dS/CFT correspondence
and the problem of the initial singularity.

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