Vehicle handling improvement by fuzzy explicit nonlinear tire forces parametrization
Said Mammar, Andre Benine-Neto, Sebastien Glaser, Naima Ait Oufroukh

To cite this version:

Said Mammar, Andre Benine-Neto, Sebastien Glaser, Naima Ait Oufroukh. Vehicle handling improvement by fuzzy explicit nonlinear tire forces parametrization. Chinese Control and Decision Conference (CCDC 2011), May 2011, Mianyang, China. pp.1560-1565, 10.1109/CCDC.2011.5968442. hal-00654099

HAL Id: hal-00654099
https://hal.science/hal-00654099v1
Submitted on 5 Jul 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Distributed under a Creative Commons Attribution 4.0 International License
Vehicle Handling Improvement by Fuzzy Explicit Nonlinear Tire Forces Parametrization

Saïd Mammar, André Benine-Neto, Sébastien Glaser, Naima Ait Oufroukh

Abstract—This paper presents the design and the simulation test of a Takagi-Sugeno (TS) fuzzy output feedback for yaw motion control. The control synthesis is conducted on a nonlinear model in which tire-road interactions are modeled using Pacejka’s magic formula. Using sector approximation, a TS fuzzy model is obtained. It is able to handle explicitly the nonlinear Pacejka lateral tire forces including the decreasing or saturated region. The controller acts through the steering of the front wheels and the differential braking torque generation. The computation of the controller takes into account the constraints that the trajectories of the controlled vehicle remain inside an invariant set. This is achieved using quadratic boundedness theory and Lyapunov stability. Some design parameters can be adjusted to handle the trade-off between safety constraints and comfort specifications. The solution to the associated problem is obtained using Linear and Bilinear Matrix Inequalities (LMI-BMI) methods. Simulation tests show the controlled car is able to well achieve standard maneuvers such as the ISO3888-2 transient maneuver and the sine with dwell maneuver.

Index Terms—Vehicle handling, Fuzzy control, Output feedback, LMI, BMI.

I. INTRODUCTION

Ground vehicles experience instabilities that are difficult for the driver to control. In fact, bifurcation analysis has shown that the stability region, given for example in the sideslip angle - yaw rate phase plane is limited [1]. In addition its size is function of the driver input on the steering speed, the road adhesion and the longitudinal speed. Notice that instabilities are mainly due to lateral tire-road forces saturation. It is thus important to help the driver in maintaining control of the vehicle is extreme dynamics and even prevent that the vehicle enters them.

In this context, electronic stability control systems (ESC) is the subject of intense research while solutions are already available and become more and more popular on commercial vehicles in Europe. They have largely contributed during the last decade to accident and death reduction [12]. Today’s systems act on the vehicle lateral dynamics mainly through independent wheel braking. Recent studies have demonstrated that differential braking may have a better effect on yaw dynamics than independent active wheel braking [13], [14]. Optimal strategies for braking forces allocation have been explored in [15]. In parallel, vehicle handling has been also investigated through active steering [5], [7]. Even if the mechanical linkage between them is still a limiting factor, solutions have been already implemented in series production [17]. In such systems, the additional steering angle is limited and it is expected that real gain from active steering will come with steer-by-wire systems which will offer additional freedom-factors for the controller intervention [16].

Using both steering angle rate and differential braking, this paper proposes a dynamic fuzzy output feedback. The Takagi-Sugeno [8] fuzzy formalism allows modeling of the nonlinear behavior of the lateral forces described by the pacejka formula [6]. The nonlinearity includes the decreasing region. The dynamic output feedback formulation considered in this paper presents three main advantages: the use of only the yaw rate and the steering angle as controller input, better flexibility to formulate the stabilization conditions and the ability to handle input or state constraints and bounded disturbances. This controller uses the property of quadratic boundedness and invariant set [4]. This allows the constraints that the trajectories of the controlled vehicle remain inside an invariant set. In fact, during control intervention, it is important to ensure a good safety level by bounding state variables. The chosen strategy fulfills this requirement and consists of building an invariant set for the system state. It guarantees that each trajectory that starts in the invariant set will not exceed it, hence the trajectories will be bounded inside it [9]. Some design parameters can be adjusted to handle the trade-off between safety constraints and comfort specifications. The solution to the associated problem obtained using Linear and Bilinear Matrix Inequalities (LMI-BMI) methods.

The paper is organized as follows: the next Section gives a description of the developed vehicle lateral dynamics Takagi-Sugeno model of the vehicle. The fuzzy output feedback synthesis, including the requirements concerning the quadratic boundedness, the state constraints and control limitation are then presented in Section 3. In Section 4, simulation results for the ISO 3888-2 and the sine with dwell transient maneuvers which excite the nonlinear tire dynamics are provided. The conclusions wrap up the paper.

II. VEHICLE LATERAL DYNAMICS T-S MODEL

As lateral control is concerned, a simple nonlinear model of a vehicle is obtained by neglecting the roll and pitch motions. This model includes the lateral translational motion and the yaw motion (Fig. 1). The two wheels of each axle are lumped into one located at its center. This leads to the vehicle bicycle model. The lateral forces between each tire
and the road surface are added at each axle leading to two resulting forces \( f_f(\alpha_f) = f_{f1} + f_{f2} \) and \( f_r(\alpha_r) = f_{r3} + f_{r4} \) at the front and rear wheels of the bicycle model respectively. These forces which will be detailed below are function of the front and rear tires sideslip angle, denoted \( \alpha_f \) and \( \alpha_r \) respectively.

The lateral translation and rotational yaw motion equations written in the vehicle fixed frame take the following form

\[
\begin{bmatrix}
  m(v + \beta + r) \\
  J_f \dot{\beta} + r
\end{bmatrix} =
\begin{bmatrix}
  1 & 1 & 0 \\
  l_f & -l_r & 1
\end{bmatrix}
\begin{bmatrix}
  f_f(\alpha_f) \\
  f_r(\alpha_r)
\end{bmatrix}
\]

where \( \beta \) is the vehicle side slip angle, \( \psi = r \) is the yaw rate and \( T \) is the yaw moment input applied by differential wheel braking. \( m \) is the vehicle mass while \( J \) is the vehicle moment of inertia. The vehicle center of gravity is located at a distance \( l_f \) from the front axle and a distance \( l_r \) from the rear axle. The vehicle parameters values are listed in Table I in the Appendix.

Assuming that the angles remain small, the front and the rear sideslip angles are given by:

\[
\begin{align*}
\alpha_f &= \delta_f - \left( \beta + \frac{v}{\omega} r \right) \\
\alpha_r &= -\beta + \frac{v}{\omega} r
\end{align*}
\]

A. Lateral tire forces model

Several types of models of the forces of tire-pavement interaction have been proposed in the literature [6]. They are usually derived from experimental data, as for the Pacejka model, and have as parameters the adhesion, the speed \( v \) and the vertical load \( f_{vi} \). The shape of the lateral force is often similar from one model to another. A first linear domain for small sideslip angle allows to define a slope factor called the tire cornering stiffness coefficient. When the sideslip angle increases, the tire enters a nonlinear operating zone where the lateral force saturates. The maximum value defines the limit of the vehicle maneuverability, resulting in a loss of controllability that can cause an understeering phenomenon or an unusual oversteering which may surprise the driver.

Here, the Pacejka magic formula [10], [11] is used to represent the efforts exerted on each tire. This model is based on the mathematical representation of the tire dynamic behavior using analytical functions having a particular structure. Lateral forces of front and rear tires are function of the side slip angle \( \alpha \) at the tire-road contact location. The effect of the camber angle is neglected. Here, the index \( i \) stands for \( f \) (front) or \( r \) (rear):

\[
f_i(\alpha) = d_i \sin(c_i \cdot \tan^{-1}(b_i(1 - e_i)\alpha + e_i \cdot \tan^{-1}(b_i\alpha)))
\]

Notice that the adhesion coefficient and the normal force acting on each tire are embedded inside the parameters \( b_i, c_i, d_i \) and \( e_i \). See [7] for further details. The definition and the value of the above parameters are described in the appendix at the end of the paper.

The goal now is to achieve a Takagi-Sugeno fuzzy model which covers the entire operating domain (linear and nonlinear) of the forces [2].

B. Four rules Takagi-Sugeno vehicle fuzzy model

The nonlinear vehicle model is transformed into a four rules Takagi-Sugeno (T-S) fuzzy model according to the values of the front and rear cornering stiffnesses:

- if \( |\alpha_f| \) is \( m_1 \) and \( |\alpha_r| \) is \( n_1 \) then \[
  \begin{align*}
  f_f &= c_{f1} \alpha_f \\
  f_r &= c_{r1} \alpha_r
  \end{align*}
\]
- if \( |\alpha_f| \) is \( m_2 \) and \( |\alpha_r| \) is \( n_1 \) then \[
  \begin{align*}
  f_f &= c_{f2} \alpha_f \\
  f_r &= c_{r1} \alpha_r
  \end{align*}
\]
- if \( |\alpha_f| \) is \( m_1 \) and \( |\alpha_r| \) is \( n_2 \) then \[
  \begin{align*}
  f_f &= c_{f1} \alpha_f \\
  f_r &= c_{r2} \alpha_r
  \end{align*}
\]
- if \( |\alpha_f| \) is \( m_2 \) and \( |\alpha_r| \) is \( n_2 \) then \[
  \begin{align*}
  f_f &= c_{f2} \alpha_f \\
  f_r &= c_{r2} \alpha_r
  \end{align*}
\]

The membership functions \( m_i \) and \( n_i \) \((i = 1, 2)\) are determined by the approximation method of nonlinear function by linear sectors. Coefficients \( c_{fi} \) and \( c_{ri} \) \((i = 1, 2)\) represent the tire cornering stiffnesses associated to each sector. In fact they represent also the slope of the limits of the sectors which include the tire forces (Fig. 2). For example, given two coefficients \( c_{f1} \) and \( c_{f2} \), chosen according to the expected road adhesion and driving conditions, one can determine
the membership functions \( m_1(\alpha_f) \) and \( m_2(\alpha_f) \). The evolution of the two functions \( m_1 \) and \( m_2 \) as functions of the sideslip angle are shown in Figure 3. They are obtained with numerical values: \( c_{f1} = 1.2c_f \) and \( c_{f2} = 0.6c_f \). It is important to outline that this sector representation is an exact approximation of the nonlinear system.

![Figure 3. Membership functions \( m_1 \) and \( m_2 \) associated to the front tire contact forces.](image)

The membership functions \( n_1 \) and \( n_2 \) for the rear tire forces are obtained by the same procedure. Finally, one can write:

\[
\begin{align*}
&f_j = [(h_1 + h_3) c_{f1} + (h_2 + h_4) c_{f2}] \alpha_f \\
&f_r = [(h_1 + h_2) c_{r1} + (h_3 + h_4) c_{r2}] \alpha_r
\end{align*}
\]

with \( h_1 = m_1 \times n_1, \ h_2 = m_2 \times n_1, \ h_3 = m_1 \times n_2 \) and \( h_4 = m_2 \times n_2 \).

In order to have the front and the rear sideslip angles as state vector components, let us define the state \( \tilde{x} = [\alpha_f, \alpha_r, \delta_f] \) and the control input \( u = [\delta_f, T_c]^T \), the fuzzy system takes the form:

\[
\dot{x} = 4 \sum_{i=1}^{4} h_i (\alpha_f, \alpha_r) \tilde{A}_i \tilde{x} + Bu
\]

where

\[
\tilde{A}_i = \begin{bmatrix} a_{11i} & a_{12i} & a_{13i} \\ a_{21i} & a_{22i} & a_{23i} \\ 0 & 0 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 1 & -l_f & 0 \\ 0 & \frac{l_f}{l_r} & 0 \end{bmatrix}
\]

where

\[
\begin{align*}
&a_{11i} = -\frac{v}{l_f + l_r}, \quad -\frac{1}{v} \left( \frac{1}{m} + \frac{l_f}{l_r} \right) c_{f1}, \\
&a_{12i} = \frac{v}{l_f + l_r}, \quad -\frac{1}{v} \left( \frac{1}{m} - \frac{l_f}{l_r} \right) c_{r1}, \\
&a_{21i} = -\frac{v}{l_f + l_r}, \quad -\frac{1}{v} \left( \frac{1}{m} + \frac{l_f}{l_r} \right) c_{f2}, \\
&a_{22i} = \frac{v}{l_f + l_r}, \quad -\frac{1}{v} \left( \frac{1}{m} - \frac{l_f}{l_r} \right) c_{r2}, \\
&a_{13i} = \frac{v}{l_f + l_r}, \\
&a_{23i} = \frac{v}{l_f + l_r}, \quad -\frac{1}{v} \left( \frac{1}{m} + \frac{l_f}{l_r} \right) c_{r1}, \\
&a_{23i} = \frac{v}{l_f + l_r}, \quad -\frac{1}{v} \left( \frac{1}{m} - \frac{l_f}{l_r} \right) c_{r2}
\end{align*}
\]

where \( c_{f1} = c_f \) for \( i = 1, 3 \) and \( c_{f2} = c_f \) for \( i = 2, 4 \). Similarly, \( c_{r1} = c_r \) for \( i = 1, 2 \) and \( c_{r2} = c_r \) for \( i = 3, 4 \).

### C. Reference yaw rate tracking

Ideally, the vehicle should respond to driver’s steering angle \( \delta_d \) as a speed dependent yaw rate reference steady state value with almost constant settling time. Let \( T_0 \) be the desired transfer function between \( \delta_d \) and \( r \). In order to ensure at nominal speed, the same steady state value for the controlled and the conventional car, the reference model is chosen as a first order transfer function with the same steady state gain as the conventional car. It is of the form \( r_j = \frac{K(j\omega)}{\tau} \delta_d \). The speed dependent steady state gain is \( K_j(v) \), derived from the nominal linear bicycle model, and \( \tau = 0.2 \) sec.

In order to ensure that the yaw rate reference value is achieved in steady state, the integral \( z \) of the yaw rate tracking error is added as state a variable:

\[
\dot{z} = r - r_d = \frac{\delta_d + \alpha_r - \alpha_f}{l_f + l_r} - v - r_d
\]

This variable is thus added to the previous third order model while the desired yaw rate is considered as a disturbance. The fuzzy model is finally discretized at a sample time of \( T = 0.005 \) sec. The final fuzzy model is of the form:

\[
x(t + 1) = \sum_{i=1}^{4} h_i (\alpha_f, \alpha_r) A_i x(t) + Bu(t) + Ew(t)
\]

\[
y(t) = C x(t) + Dw(t)
\]

where \( x = [\alpha_f, \alpha_r, \delta_d, z]^T \) and \( y = [r, z]^T \). The disturbance \( w(t) = r_d(t) \in \mathcal{E}_Q = \{ w \in \mathbb{R}^4 \mid Qw \leq 1 \} \) is bounded. Matrices \( A_i \) and \( B \) can be easily derived from equations (6) and (7). This discrete time fuzzy system is characterized by common \( B \), \( E \) and \( C \) matrices. This property simplifies drastically the stability and performance conditions as only simple summations are involved.

### III. DYNAMIC OUTPUT FEEDBACK FUZZY CONTROLLER

In the following, a dynamic output feedback fuzzy controller is sought. It has the form:

\[
x_c(t + 1) = \sum_{i=1}^{4} h_i (\alpha_f, \alpha_r) A_i' x_c(t) + B u(t)
\]

\[
u(t) = C_c x_c(t) + D_c y(t)
\]

where \( x_c \in \mathbb{R}^4 \) is the controller state; \( \{ A_i', B_c, C_c, D_c \} \) are matrices to be designed.

This controller uses the parallel distributed compensation (PDC) concept of the fuzzy system control. In this concept, each control rule is distributively designed for the corresponding rule of a T-S fuzzy model. Linear control theory can then be used to design controllers for each of the consequent part of the fuzzy system while ensuring the same properties for the fuzzy system.

As pointed out in [4], \( D_c \) is an important parameter for stabilization, and the controller structure is able to handle constraints on the input and the state. By combining (8) and (9), the augmented closed-loop fuzzy model is given by

\[
\ddot{x}(t + 1) = \sum_{i=1}^{4} h_i (\alpha_f, \alpha_r) \Phi_i \ddot{x}(t) + \Gamma w(t).
\]
Consider the reachable set \( \Lambda \). This means that every trajectory that starts inside \( A \) is an invariant set for the system (10) with all allowable \( w \). Assume that there exists a quadratic function \( \Phi \) satisfying (10), \( V(\tilde{x}) \leq 1 \) implies \( V(\tilde{x}) \leq 1 \), the condition [3]:

\[
V(\tilde{x} + 1) \leq V(\tilde{x})
\]

Consider the reachable set \( \Lambda \) defined by:

\[
\Lambda \triangleq \{ \tilde{x}(T) | \tilde{x}, w \text{ satisfying } (10), \tilde{x}(0) = 0, w^T Q w \leq 1, T \geq 0 \}
\]

The set \( \varepsilon P \) defined by:

\[
\varepsilon P = \{ \tilde{x}(t) \in \mathcal{R}^n | \tilde{x}(T)^T P \tilde{x}(t) \leq 1 \}
\]

is an invariant set for the system (10) with \( w \in \mathcal{R}^n, w^T Q w \leq 1 \). This means that every trajectory that starts inside \( \varepsilon P \) remains inside it for \( t \to \infty \).

The existence of such a function \( V(\tilde{x}) \) means that the set \( \varepsilon P \) is an outer approximation of the reachable set \( \Lambda \).

The set \( \varepsilon P \) is also an outer approximation of the reachable set

\[
\Lambda^* \triangleq \{ \tilde{x}(T) | \tilde{x}, w \text{ satisfying equation } (10), \tilde{x}(0) \in \varepsilon P, w^T Q w \leq 1, T \geq 0 \}
\]

In this section the control law and the invariant set \( \varepsilon P \) are synthesized. This is achieved using BMI (Bilinear Matrix Inequalities) optimization method such that the system without the disturbance is asymptotically stable and at the same time, the reachable set for an initial state values inside the invariant set is contained in this invariant set.

### B. Invariant set - quadratic boundedness

According to the previous considerations, the closed loop linear system \( \tilde{x}(t+1) = \Phi \tilde{x}(t) + \Gamma w(t) \) is strictly quadratically bounded with a common Lyapunov matrix \( P > 0 \) for all allowable \( w(t) \in \varepsilon Q \). For \( T > 0 \), if \( \tilde{x}(t)^T P \tilde{x}(t) > 1 \) implies \( \Phi \tilde{x}(t) + \Gamma w(t) \leq \tilde{x}(t)^T P \tilde{x}(t) \), for any \( w \in \varepsilon Q \).

The corresponding condition is obtained using the \( S \)-procedure and invoking the Schur complement, that the satisfaction of \( w \in \varepsilon Q \) and \( \tilde{x}^T P \tilde{x} \geq 1 \) implies \( w^T Q w \leq \tilde{x}^T P \tilde{x} \).

Defining \( P = \begin{bmatrix} P_1 & P_2^T \cr P_2 & P_3 \end{bmatrix} \) and \( P^{-1} = \begin{bmatrix} M_1 & M_2^T \\
M_2 & M_3 \end{bmatrix} \). Assuming that \( P_2 \) and \( M_2 \) are full rank matrices and setting:

\[
\begin{align*}
\hat{D}_c &= D_c \\
\hat{C}_c &= D_c M_1 + C_c M_2 \\
\hat{B}_c &= P_2 BD_c + P_2^T B_c \\
\hat{A}_c &= P_2 A_c M_1 + P_2 BD_c C_c + P_2^T B_c C_c + P_1 BC_c M_2 + P_2^T A_c M_2 \\
\end{align*}
\]

the existence of the controller is ensured if there exist matrices \( P_1, P_2, M_1, M_2 \) and a positive scalar \( \alpha \) such that the following condition holds

\[
\sum_{i=1}^{4} h_i (\alpha_f, \alpha_r) Y_i \geq 0
\]

where

\[
Y_i = \begin{bmatrix} (1-\alpha) P_1 & 0 & 0 & 0 \\
(1-\alpha) I & A_1 + B \hat{D}_c C & A_2 M_2 + B \hat{C}_c & BD_c D + E & \end{bmatrix}
\]

Notice that the matrices \( M_1, M_2, P_1 \) and \( P_2 \) verify:

\[
M_2^T P_2 = I - M_1 P_1
\]

In addition, it is possible to handle constraints on the control signal and the state:

\[
-\bar{u} \leq u(t) \leq \bar{u}, \quad -\bar{Y} \leq \Psi x(t+1) \leq \bar{Y}, \quad \forall t \geq 0
\]

where \( \bar{u} = [\bar{u}_1, \bar{u}_2]^T \) with \( \bar{u}_1 > 0, \bar{u}_2 > 0 \) and \( \bar{Y} := [\bar{Y}_1, \bar{Y}_q]^T \) with \( \bar{Y}_j > 0, j = 1, \ldots, q \). \( \Psi \in \mathcal{R}^{2 \times 4} \) and \( q \) is the number of imposed constraints. Notice that the bounds are provided separately on each state variables or a combination of state variables.

The control input limitation is verified if for a pre-specified scalar \( \eta \in (0, 1] \), the additional inequality

\[
\begin{bmatrix} \eta P_1 & * & * & * \\
\eta I & \eta M_1 & * & * \\
0 & 0 & Q & * \\
\sqrt{2D_c} & \sqrt{2C_c} & \sqrt{2D_c} & \bar{Z} \end{bmatrix} \geq 0
\]

holds with \( Z \in \mathcal{R}^{2 \times 2} \) is such that \( Z_{11} \leq \bar{u}_1^2 \) and \( Z_{22} \leq \bar{u}_1^2 \).

A similar procedure can be applied for the constraints on the state variables. One can achieve from the convex property conditions (21):

\[
\sum_{i=1}^{4} h_i (\alpha_f, \alpha_r) \bar{Y}_i \geq 0, \quad \forall t \geq 0,
\]

where \( \Xi \) is a symmetric matrix and

\[
\bar{Y}_i = \begin{bmatrix} \eta P_1 & * & * & * \\
\eta I & \eta M_1 & * & * \\
0 & 0 & Q & * \\
\bar{Y}_{i41} & \bar{Y}_{i42} & \bar{Y}_{i43} & \Xi \end{bmatrix}
\]
C. Controller synthesis

Under the proposed modeling approach, the desired yaw rate could be seen as an input disturbance under which the closed-loop system should remain stable with bounded values for the state vector components. More generally, the state variables should not exceed the bounds of a “safety zone”, namely \( |α_f| \leq α_f^M, |α_r| \leq α_r^M \) and \( |δ_f| \leq δ_f^M \). Thus, the state vector \( x \) has to be confined to a hypercube \( L(2^M) \) defined by the above bounds. Finally, the control input, the steering angle rate and the yaw moment, have to be bounded \( |δ_f| \leq δ_f^M \) and \( |T_e| \leq T_e^M \).

According to the equation (19), control limitation is given by \( u = [δ_f^M, T_e^M] \), while state limitation is given by \( \Psi = [α_f^M, α_r^M, δ_f^M]^T \) and \( \Psi = [I_3 \ 0] \).

The PDC output feedback controller was synthesized with the following numerical values:

\[
α = 0.02, \quad η = 0.02, \quad δ_f^M = 100 \text{deg/s}, \quad T_e^M = 10KN \quad α_f^M = α_r^M = 13 \text{deg} \quad δ_f^M = 6 \text{deg},
\]

These design parameters could be adjusted to handle the trade-off between safety constraints and comfort specifications.

The achieved \( Q \) is 5, which ensures that the constraints are verified for a disturbance of a magnitude less than 0.447rad/s at the considered longitudinal speed of 20m/s. In fact, the maximum value is constrained by [18]:

\[
r_{d_{\text{max}}} = 0.85 \frac{g}{v}
\]

IV. SIMULATION TESTS

In order to proof the assistance ability to maintain the dynamic vehicle stability in extreme conditions, several type of maneuvers have been defined to test the ESC systems. Among them, double lane-change manoeuvre defined in ISO 3888-2 standard and the sine with dwell transient maneuver. The controller is tested below for the two maneuvers.

A. Testing for the ISO 3888-2 maneuver

The ISO3888-2 double lane-change maneuver setup is depicted in Figure 4-a. The maneuver is carried out with and without the controller at the same speed of 80km/h. During the maneuver, the throttle is released.

The driver initiates the maneuver by applying the steering angle shown in dashed line in Figure 4-b. Figures 4-a highlights that the controlled vehicle is able to perform the maneuver (solid line) while the uncontrolled vehicle fails (dashed line). Figure 4-d shows that the controller shares the effort on the steering angle rate at the yaw moment, respectively. In this situation the driver applied steering angle is too high (dashed line in Figure 4-b) while the the steering angle of the controlled vehicle is limited to the admissible safety value of few degrees, as shown by the solid line in Figure 4-b for the angle value. The steering angle rate is depicted in the top plot of Figure 4-d and is limited. The yaw moment handles the main effort as shown in the bottom plot of Figure 4-d. Figure 4-c shows that the controlled car yaw rate is closer to the reference one than the yaw rate of the uncontrolled vehicle (dashed line). The contribution of each sub-controller to according to the actual vehicle dynamics are shown in Figure 5. Finally, Figures 6-a and 6-b provide the developed sideslip angles at the front and rear tires. The corresponding front and rear forces are shown in Figures 6-c and 6-d. It is clear that the saturation zones are reached by the uncontrolled vehicle during the maneuver while the controller avoids that these zones are reached.

B. Sine with dwell maneuver

The sine with dwell is a transient maneuver considered by NHTSA (National Highway Traffic Safety Administration) for electronic stability control evaluation. Such type of maneuver is suited for the excitation of the vehicle oversteer response. The maneuver is conducted at the same speed of 80km/h. It corresponds to a 0.7Hz frequency sine wave form with dwell steering angle of 500ms. During the maneuver, the throttle is released. Figure 7 shows both the control sharing...
between the steering and the differential braking. It also shows that the realized relative yaw angle is higher which means that the vehicle is more able to change direction.

V. Conclusion

In this paper the design and the test of an integrated steering and differential braking control for yaw moment generation has been described. Controlled vehicle trajectories are confined inside an invariant set. The nonlinear behavior of the vehicle dynamics are modeled using a fuzzy Takagi-Sugeno approach. An output feedback fuzzy controller constituted by four sub-controllers handles constraints on the state variables and the control inputs. Simulation tests have shown that the controlled vehicle is able to achieve the ISO 3888-2 and the sine with dwell transient maneuvers where the uncontrolled vehicle fails.

APPENDIX

TABLE I

| Parameter   | Value         |
|-------------|---------------|
| m           | Vehicle total mass 1600 kg. |
| c_f         | Front cornering stiffness 40000 N/rad. |
| c_r         | Rear cornering stiffness 35000 N/rad. |
| J           | Vehicle yaw moment of inertia 2454 kg·m². |
| l_f         | Distance CG to front axle 1.22m. |
| l_r         | Distance CG to rear axle 1.44m. |
| v           | Longitudinal velocity. |

TABLE II

| Tire   | h_i | c_i | d_i | e_i |
|--------|-----|-----|-----|-----|
| Front  | 8.3278 | 1.1009 | 4536.0 | -1.661 |
| Rear   | 11.6590 | 1.1009 | 3671.6 | -1.542 |

REFERENCES

[1] H.D. Tuan, E. Ono, S. Hosoe, S. Doi, Bifurcation in Vehicle Dynamics and Robust Front Wheel Steering Control. IEEE Transactions on Control Systems Technology, 6, pp. 412-421, 1998.
[2] T. Takagi, M. Sugeno, Fuzzy identification of systems and its application to modeling and control, IEEE Trans. Systems Man Cybern. 15, pp. 116-132, 1985.
[3] A. Alessandri, M. Baglietto, G. Battistelli, Design of state estimators for uncertain linear systems using quadratic boundedness. Automatica, 42, 497-502, 2006.
[4] B. Ding, Quadratic boundedness via dynamic output feedback for constrained nonlinear systems in Takagi–Sugeno’s form, Automatica, vol. 45, N° 9, pp. 2093-2098, 2009.
[5] J. C. Gerdes et E. J. Rossetter A Unified Approach to Driver Assistance Systems Based on Artificial Potential Fields Authors. Journal of Dynamic Systems, Measurement and Control, Vol. 123, No. 3, pp. 431-438, 2001.
[6] E. Bakker, H. B. Pacejka, and L. Lidner. A new tire model with an application in vehicle dynamics studies. SAE paper, 1989.
[7] S. Mammar and D. Koenig. Vehicle Handling improvement by Active Steering, Vehicle Sys. Dyn. Journal, vol 38, No3, pp. 211-242, 2002.
[8] K. Tanaka, H.O. Wang, Fuzzy Control Systems Design and Analysis, Wiley Inc, New York, 2001.
[9] N. Minoiu, M. Netto, S. Mammar, B. Lusetti, Driver steering assistance for lane departure avoidance, Control Engineering Practice, Vol. 17, No 6, pp. 642-651, 2009.
[10] Hans B. Pacejka, *Tyre and Vehicle Dynamics*, p 511- 562 , Delft University of Technology, 2002.
[11] W.D. Versteeden, Improving a tyre model for motorcycle simulations, Masters Thesis, Technical University of Eindhoven, June 2005.
[12] Grands thèmes de la sécurité routière en France. ONISR, Observatoire national interministériel de sécurité routière, février 2007.
[13] A.T.V. Zanten, R. Erhart, and G. Pfaff, VDC, The Vehicle Dynamics Control System of Bosch ., SAE Technical Paper No. 95759 , 1995.
[14] M.J. Hancock, R.A. Williams, T.J. Gordon, M.C. Best, A comparison of braking and differential control of road vehicle yaw-sideslip dynamics,Proceedings of IMechE., Part D: Automotive Engineering, V.219, pp. 309-327, 2005.
[15] J. Tjoennes and T. A. Johansen, Adaptive Optimizing Dynamic Control Allocation Algorithm for Yaw Stabilization of an Automotive Vehicle using Brakes, in Control and Automation, 2006. MED ’06. 14th Mediterranean Conference on, 2006, pp. 1-6.
[16] E. Ono, Y. Hattori, Y. Muragishi, K. Koibushi, Vehicle Dynamics Integrated Control for Four-Wheel-Distributed Steering and Four-Wheel-Distributed Traction/Braking Systems, Vehicle System Dynamics, V 44, No.2, pp. 139-151, 2006.
[17] P. Kohen and M. Ecric, Active Steering - The BMW Approach Towards Modern Steering Technology, SAE Technical Paper No. 2004-01-1105, 2004.
[18] Rajamani R., Vehicle Dynamics and Control, Springer, New-York, 2006.