Weak-interaction rates in stellar conditions

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Abstract. Weak-interaction rates, including β-decay and electron captures, are studied in several mass regions at various densities and temperatures of astrophysical interest. In particular, we study odd-\(A\) nuclei in the \(pf\)-shell region, which are involved in presupernova formations. Weak rates are relevant to understand the late stages of the stellar evolution, as well as the nucleosynthesis of heavy nuclei. The nuclear structure involved in the weak processes is studied within a quasiparticle proton-neutron random-phase approximation with residual interactions in both particle-hole and particle-particle channels on top of a deformed Skyrme Hartree-Fock mean field with pairing correlations. First, the energy distributions of the Gamow-Teller strength are discussed and compared with the available experimental information, measured under terrestrial conditions from charge-exchange reactions. Then, the sensitivity of the weak-interaction rates to both astrophysical densities and temperatures is studied. Special attention is paid to the relative contribution to these rates of thermally populated excited states in the decaying nucleus and to the electron captures from the degenerate electron plasma.

1. Introduction
Weak \(\beta\)-decay and electron-capture (EC) have long been recognized [1] as key processes to understand the late stages of the stellar evolution. In particular, the presupernova stellar structure, as well as the nucleosynthesis of heavy nuclei, are determined to a large extent by those mechanisms. \(pf\)-shell nuclei are specially important in these scenarios because they are the main constituents of the stellar core in presupernova formations leading to core-collapse (type II) or thermonuclear (type Ia) supernovae.

From the theoretical side, the first extensive calculations of stellar weak rates in relevant ranges of densities (\(\rho\)) and temperatures (\(T\)) were done in Ref. [2] under severe assumptions on the energy distribution of the Gamow-Teller (GT) strength. Concerning the nuclear structure involved, different approaches are found in the literature that can be roughly divided into shell model (SM) [3] and proton-neutron quasiparticle random-phase approximation (QRPA) [4] categories. Although QRPA calculations cannot reach the detailed spectroscopic accuracy achieved by present state-of-the-art SM calculations, the global performance of QRPA is quite satisfactory. Moreover, one clear advantage of the QRPA method is that it can be extended to heavier nuclei, which are beyond the present capability of full SM calculations, without increasing much the complexity of the calculation.

It is also important to realize that there are clear differences between terrestrial and stellar decay rates caused by the effect of high \(\rho\) and \(T\) conditions. One effect of \(T\) is directly
related to the thermal population of excited states in the decaying nucleus, accompanied by the corresponding depopulation of the ground states. The weak rates of excited states can be significantly different from those of the ground state and a case by case consideration is needed. Another distinctive effect comes from the fact that atoms in stellar scenarios are completely ionized, and consequently electrons are no longer bound to the nuclei, but forming a degenerate plasma that obeys a Fermi-Dirac distribution. This opens the possibility for continuum EC, in contrast to the orbital EC caused by bound electrons in the atom under terrestrial conditions. These two effects make weak-interaction rates in the stellar interior sensitive functions of T and \( \rho \).

In the case of even-even nuclei the eventual contributions from excited states could be safely neglected because of the high energy excitation of the first excited states, typically \( 2^+ \) states beyond 1 MeV that can hardly be excited within the range of temperatures considered in this work. However, low-lying excited states would contribute to the rates in the case of even-even well deformed nuclei [5], where the rotational states drop easily below 1 MeV, as well as in odd-\( A \) nuclei where quasiparticle states are found at very low excitation energies. In this work we study contributions to the weak rates coming from excited states in odd-\( A \) nuclei. To perform this study, a set of nuclei has been chosen according to their interest in presupernova models. Here we focus on \(^{55}\)Mn and \(^{55}\)Co.

2. Theoretical Formalism

The weak-interaction rate of a nucleus can be expressed as follows,

\[
\lambda = \sum_i \lambda_i P_i(T); \quad P_i(T) = \frac{2J_i + 1}{G} e^{-E_i/(k_B T)},
\]

where \( P_i(T) \) is the probability of occupation of the excited state \( i \) in the parent nucleus. Assuming thermal equilibrium, it is given by a Boltzmann distribution. \( G = \sum_i (2J_i + 1) e^{-E_i/(k_B T)} \) is the partition function and \( J_i(E_i) \) is the angular momentum (excitation energy) of the parent nucleus state \( i \). In principle, the sum extends over all excited states in the parent nucleus up to the proton separation energy. However, because of the range of temperatures considered in this work (T=1-10 GK), only a few low-lying excited states are expected to contribute significantly. The scale of excitation energies to consider is determined by \( k_B T \), which for maximum T around 10 GK is given by \( k_B T = 0.862 \) MeV. The weak-interaction rate corresponding to the parent state \( i \) is given by

\[
\lambda_i = \sum_f \lambda_{if} = \frac{\ln 2}{D} \sum_f B_{if} \Phi_{if}(\rho, T),
\]

where the sum extends over all the states in the final nucleus that can be reached in the process and \( D = 6146 \) s. This expression is decomposed into a phase space factor \( \Phi_{if} \), which is a function of \( \rho \) and T and a nuclear structure part \( B_{if} \) that contains the transition probabilities for allowed GT transitions.

In the astrophysical scenarios of our study, nuclei are fully ionized and continuum ECs from the degenerate electron plasma are possible. The phase space factor for continuum EC is given by

\[
\Phi_{EC}^{if} = \int_{\omega_{\min}}^{\infty} \omega p(Q_{if} + \omega)^2 F(Z, \omega) \times S_e(\omega) \left[ 1 - S_\nu(Q_{if} + \omega) \right] d\omega.
\]

and the corresponding phase space factor for \( \beta^+ \) decay is given by

\[
\Phi_{\beta^+}^{if} = \int_{1}^{Q_{if}} \omega p(Q_{if} - \omega)^2 F(-Z + 1, \omega) \times \left[ 1 - S_{e^+}(\omega) \right] \left[ 1 - S_\nu(Q_{if} - \omega) \right] d\omega.
\]
In these expressions \( \omega \) is the total energy of the electron, \( p = \sqrt{\omega^2 - 1} \) is the momentum, and \( Q_{if} \) is the total energy available

\[
Q_{if} = \frac{1}{m_e c^2} (M_p - M_d + E_i - E_f), \tag{5}
\]

written in terms of the nuclear masses of parent \( (M_p) \) and daughter \( (M_d) \) nuclei and their excitation energies \( E_i \) and \( E_f \), respectively. \( F(Z, \omega) \) is the Fermi function that takes into account the distortion of the electron wave function due to the Coulomb interaction.

\( S_e, S_{e^+}, \) and \( S_{\nu} \), are electron, positron, and neutrino distribution functions, respectively. Its presence inhibits or enhances the phase space available. In the stellar scenarios considered here the commonly accepted assumption is that \( S_{e^+} = 0 \) because electron-positron pair creation becomes important only at higher energies and \( S_{\nu} = 0 \) because neutrinos and antineutrinos can escape freely from the interior of the star at these densities. The electron distribution is described by a Fermi-Dirac distribution

\[
S_e = \frac{1}{\exp[(\omega - \mu_e)/(k_B T)] + 1}, \tag{6}
\]

where \( \mu_e \) is the chemical potential. The phase space factor for EC in Eq. (3) is therefore a sensitive function of both \( \rho \) and \( T \), through the electron distribution \( S_e \). On the other hand, the phase space factor for \( \beta^+ \) decay in Eq. (4) under the assumptions \( S_{e^+} = S_{\nu} = 0 \), does not depend on \( \rho \) and \( T \). The only dependence of the positron decay rates to \( \rho \) and \( T \) appears indirectly through the population of excited states.

The nuclear structure part of the problem is described within the QRPA formalism. The method starts with a selfconsistent deformed Hartree-Fock mean field calculation with Skyrme interactions including pairing correlations. The single-particle energies, wave functions, and occupation probabilities are generated from this mean field. The Skyrme force SLy4 is used in this work. The solution of the HF equation is found by assuming time reversal and axial symmetry. The single-particle wave functions are expanded in terms of the eigenstates of an axially symmetric harmonic oscillator in cylindrical coordinates, using twelve major shells. The method also includes pairing between like nucleons in BCS approximation with fixed gap parameters for protons and neutrons, which are determined phenomenologically from the odd-even mass differences involving the experimental binding energies. Calculations for GT strengths are performed for the equilibrium shape of each nucleus, that is, for the configuration that minimizes the energy.

To describe GT transitions, a spin-isospin residual separable interaction is added to the Skyrme mean field and treated in a deformed proton-neutron QRPA. This interaction contains a \( ph \) and a \( pp \) part with coupling strengths \( \chi_{GT}^{ph} \) and \( \kappa_{GT}^{pp} \). In previous works [6] we studied the sensitivity of the GT strength distributions to the various ingredients contributing to the deformed QRPA calculations, namely to the nucleon-nucleon effective force, to pairing correlations, and to residual interactions. We found different sensitivities to them. In this work, all of these ingredients have been fixed to the most reasonable choices found previously. In particular we use the coupling strengths \( \chi_{GT}^{ph} = 0.10 \text{ MeV} \) and \( \kappa_{GT}^{pp} = 0.05 \text{ MeV} \). The method has been successfully applied in the past to the study of the decay properties in different mass regions from proton-rich [5] to stable [7, 8], and to neutron-rich nuclei [9].

The GT strength for a transition from an initial state \( i \) to a final state \( f \) is given by

\[
B_{if}(GT^\pm) = \frac{1}{2J_i + 1} \left( \frac{\beta_A}{g_V} \right)^2 \sum_j A f \sum_j \langle f| \sum_j \sigma_j f_j^\pm |i \rangle^2, \tag{7}
\]
where \((g_A/g_V)_{\text{eff}} = 0.7(g_A/g_V)_{\text{bare}}\) is an effective ratio of axial and vector coupling factors that takes into account in an effective manner the observed quenching of the GT strength.

When the parent nucleus has an odd nucleon, the ground state can be expressed as a one quasiparticle state in which the odd nucleon occupies the single-particle orbit of lowest energy. Quasiparticle excitations correspond to configurations with the odd nucleon in an excited state. Then, two types of transitions are possible. One type is due to phonon excitations in which the odd nucleon acts only as a spectator. These are three-quasiparticle states (3qp). In the intrinsic frame, the transition amplitudes are basically the same as in the even-even case, but with the blocked spectator excluded from the calculation. The other type of transitions are those involving the odd nucleon state. These are one-quasiparticle states (1qp), which are treated by taking into account phonon correlations in the quasiparticle transitions in first order perturbation. The transition amplitudes for the correlated states can be found in Refs. [4, 6].

3. Results

![Figure 1](image1.png) **Figure 1.** QRPA GT strength distributions \(B(GT^+)\) for the transition of the ground state 5/2\(^-\) and excited state 7/2\(^-\) of \(^{55}\)Mn going to \(^{55}\)Cr, plotted versus the excitation energy of the daughter nucleus. Experimental data extracted from \((n,p)\) are from Ref. [10].

![Figure 2](image2.png) **Figure 2.** Same as in Figure 1, but for the ground state 7/2\(^-\) and excited state 3/2\(^-\) of \(^{55}\)Co going to \(^{55}\)Fe.

We start with the study the GT strength distributions corresponding to the ground state and the first excited states. In the case of \(^{55}\)Mn (Figure 1) the profile of the ground state shows a broad peak centered a 5 MeV with a two-peaked substructure containing somewhat more strength in the lower one. The GT distribution of the excited state is close to that one with the strength shifted to the peak at higher energy. The distributions in the ground and excited states of \(^{55}\)Co (Figure 2) are quite similar with a peak centered at about 9 MeV.

In the following figures we present the weak-interaction rates for these nuclei as a function of the temperature and for various densities. The range of T considered varies from \(T_0 = 1\) up to \(T_0 = 10\), in \(T_0\) (GK) units, whereas the range in \(\rho Y_e\) varies from \(\rho Y_e = 10^6\) mol/cm\(^3\) up to \(\rho Y_e = 10^{10}\) mol/cm\(^3\). This grid of \(\rho\) and \(T\) includes those ranges relevant for astrophysical scenarios related to the silicon-burning stage in a presupernova star (\(\rho Y_e = 10^7\) mol/cm\(^3\) and \(T_0 = 3\)), as well as scenarios related to pre-collapse of the core and thermonuclear runaway type Ia supernova (\(\rho Y_e = 10^9\) mol/cm\(^3\) and \(T_0 = 10\)).
Figure 3. Weak-interaction rates as a function of T in $^{55}$Co, decomposed into their EC and $\beta^+$ contributions for various densities.

Figure 4. Total weak-interaction rates for $^{55}$Mn as a function of T for various densities.

Figure 5. Same as in Figure 4, but for $^{55}$Co.

At these ranges of densities and temperatures, weak-interaction rates are dominated by EC. $\beta^+$-decays will always contribute for sufficiently high $\rho$ and T because of thermal population of excited states beyond $Q_{EC}$ energies. However, for $\rho$ and T values in this work, positron-decay contributions can be neglected. The most favored cases for $\beta^+$ contributions are those with positive $Q_{EC}$ values, such as $^{55}$Co ($Q_{EC} = 3.451$ MeV). We can see in Figure 3 for $^{55}$Co the decomposition of the total weak-interaction rates into their EC and $\beta^+$ components. At $\rho Y_e = 10^6$ there is a competition between EC and $\beta^+$ at low T. At higher T, EC clearly dominates because it increases while $\beta^+$ remains almost constant. At higher densities, $\rho Y_e = 10^8 - 10^{10}$, $\beta^+$ is always very small in comparison with EC and can be safely neglected. The dominant EC determines the total rates in practically all the cases. Therefore, only the total rates will be shown in the following figures.

Figures 4 and 5 show the total rates as a function of the temperature for various densities from $\rho Y_e = 10^6$ up to $\rho Y_e = 10^{10}$ mol/cm$^3$. In these figures one can see that the rates always increase when either $\rho$ or T increases. This is a simple consequence of the phase factors that increase accordingly. The sensitivity of the EC rates to the environmental ($\rho, T$) conditions is obvious. At
Figure 6. Weak-interaction rates for $^{55}$Mn from SLy4-QRPA calculations as a function of the temperature $T$ (GK) for various densities. Separate contributions from the ground and excited states are shown.

Figure 7. Same as in Figure 6, but for $^{55}$Co.

low $\rho$ (low Fermi energies) and low $T$ (sharp shape of the energy distribution of the electrons $S_e$), the rates are low and very sensitive to details of the GT strength of the low-lying excitations and therefore to nuclear structure model calculations. On the other hand, when $\rho$ increases (larger Fermi energies) and $T$ increases (smearing of the electron distribution functions), EC rates also increase and become sensitive to all the spectrum. Then, the whole description of the GT strength distribution is more important than a detailed description of the low-lying spectrum. This is also the reason why all the rates become flat and roughly independent of $T$ at high $\rho$. The dependence on the $Q_{EC}$ values is also apparent. These energies determine the lower integration limit of the EC phase factor in Eq. (3) and thus, the minimum energy of electrons in the stellar plasma to suffer a nuclear capture. The effect is more pronounced at low densities and one can roughly distinguish two types of patterns. Those that start with very low rates at low $T$ and increase rapidly with $T$, such as the rates in $^{55}$Mn and those exhibiting a rather flat behavior with much higher rates, as in the case of $^{55}$Co. This global behavior can be understood in terms of the $Q_{EC}$ energies. Large negative values, as in the cases of $^{55}$Mn lead to very low rates. On the other hand, positive values of $Q_{EC}$ as in $^{55}$Co cause much higher rates.

In the next figures, Figures 6 - 7, the relative contributions of the ground and excited states to the total rates are studied. In addition, total rates and rates obtained from the ground state with full population labeled ‘gs only’ in the figures, are included. The relative importance of the various excited states is determined by their thermal population at a given $T$, by the phase
factor, and by the structure of the GT strength distribution.

4. Conclusions
We have evaluated the weak-interaction rates of two odd-\(A\) \(pf\)-shell nuclei (\(^{55}\)Mn and \(^{55}\)Co), which are representative of the constituents in presupernova formations. The nuclear structure involved in the calculation of the energy distribution of the Gamow-Teller strength is described within a selfconsistent deformed HF+BCS+QRPA formalism with density-dependent effective Skyrme interactions and spin-isospin residual interactions.

First, GT strength distributions for ground states and excited states have been studied in those nuclei and compared with the available data from charge-exchange reactions. Secondly, weak-interaction rates have been calculated at densities and temperatures holding in astrophysical scenarios of interest, such as silicon burning and Ia supernova pre-collapse stages. Although ECs are always dominant, positron decays have been also included because some nuclei are unstable \(Q_{EC} > 0\) and because excited states above \(Q_{EC}\) values can be thermally populated.

The contributions of the excited states to the rates depend certainly on the density and temperature through the population of these states and their phase factors, but depend also on their GT structure. All in all, we find quite similar results for the total rates and for the rates considering only the contribution from the ground states fully populated. The reason is that population of excited states leads to a depopulation of ground states and both effects are compensated to some extent when the GT strength distributions are not very different from each other.

Acknowledgments

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