D-Brane Recoil and Supersymmetry Breaking as a Relaxation Process

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Abstract

We propose a new mechanism for the formation of conical singularities on D-branes by means of recoil resulting from scattering of closed string states propagating in the (large) transverse dimensions. By viewing the (spatial part of the) four-dimensional world as a 3-brane with large transverse dimensions the above mechanism can lead to supersymmetry obstruction at the TeV scale. The vacuum remains supersymmetric while the mass spectrum picks up a supersymmetry obstructing mass splitting. The state with “broken” supersymmetry is not an equilibrium ground state, but is rather an excited state of the D-brane which relaxes to the supersymmetric ground state asymptotically in (cosmic) time.
The issue of supersymmetry breaking remains unresolved; while supersymmetry provides a natural explanation for the vanishing of the cosmological constant (vacuum energy) and offers a resolution of the hierarchy problem it is not a symmetry of the low-energy world. Softly broken supersymmetry can still control the Higgs mass and resolve the hierarchy problem, but radiative corrections to Standard Model processes measured in precision electroweak experiments at LEP coupled with direct Higgs and sparticle searches require that the scale of supersymmetry breaking (i.e. the masses of the lightest sparticles) be no more than a few TeV without necessitating unnatural fine tuning of the Standard Model parameters \[1\]. Some years ago a novel scenario for generating massive sparticles while maintaining the vanishing cosmological constant was proposed \[2\]. The scenario referred to (2+1)-dimensional supergravity theories and, instead of breaking supersymmetry, involved the obstruction of supersymmetry by massive states in the spectrum of supergravity living in a spacetime with conical singularities. In fact, as pointed out in reference \[2\] based on work in reference \[3\], any state in (2+1) dimensions which has non-zero energy produces a geometry which is asymptotically conical. In such spacetimes there are no covariantly constant spinors; given that in supersymmetry the unbroken supercharges are spinor fields which are covariantly constant at infinity, this suggests that there are no unbroken supersymmetries in (2+1)-dimensional supergravity theories. The Bose–Fermi degeneracy in the massive spectrum is lifted in proportion to the deficit angle \(\delta\) of the conical geometry,

\[
\delta = 2\pi G_3 \delta m
\]

where \(m\) is the mass splitting, and \(G_3\) is Newton’s constant in three dimensions \[8\]. However, the vacuum energy (cosmological constant) remains zero, given that the vacuum state remains supersymmetric.

An explicit realization of the above phenomenon in the context of a specific supergravity model in three spacetime dimensions has been provided in reference \[4\], while conical singularities and supersymmetry obstruction in the context of \(\mathcal{N}=1\) supergravity in four spacetime dimensions have been discussed in \[3\]. The presence of conical singularities on any Einstein manifold \(X\) will in general lead to a complete breaking of supersymmetry, except for special choices of the manifold \(X\), in which case some supersymmetries may survive. Their number depends on the number of covariantly constant spinors (or equivalently the number of Killing spinors) on the manifold \(X\). For certain geometries a classification of the unbroken supersymmetries is complete \[3\]. In the present work we shall be interested in the case where no supersymmetries are left unobstructed; in particular we shall be interested in four-dimensional \(\mathcal{N}=1\) supergravity models, viewed as low-energy field theories of some string (or D-brane) theories.

The point of this article is to describe what is in our opinion a novel way to generate conical singularities on the four-dimensional world by adopting the modern view \[7–9\] that the four-dimensional spacetime we observe is actually a D-3-brane living in a higher-(ten or eleven)-dimensional universe. Only closed string states (gravitons) propagate in the dimensions transverse to the brane while gauge and matter fields are described by open string states ending on the brane. Consistent embeddings of this idea in detailed string models have recently been discussed in references \[8, 9\].

Despite the fact that in such a picture the detailed dynamics of the bulk higher-dimensional spacetime is not fully known, several non-trivial predictions for the (low-energy)
physics on four-dimensional spacetime can emerge [7]. In this article we shall point out yet another prediction, that of possible supersymmetry obstruction at the TeV scale.

We first note an important ingredient of closed-string scattering: that of the resulting recoil of the D-brane [10,13]. This has been ignored in most discussions of the scattering process, such as emission of closed string states into the bulk and/or absorption by the brane. From a worldsheet viewpoint the recoil is described [10] by deformations of the pertinent $\sigma$-model which obey a logarithmic conformal algebra [14]. Field theories described by such algebras lie on the border between conformal field theories and general renormalizable two-dimensional quantum field theories. The fact that recoil of a D-brane is described not by an ordinary conformal field theory but by a logarithmic one is associated with the fact that the recoil process describes a change of state in the $\sigma$-model background, and as such is a non-equilibrium process. This is reflected [11,13] in the logarithmic operator algebra itself.

It has been argued recently [15] that when properly taken into account the recoil may lead to non-trivial phenomena on the brane such as stochastic fluctuations in the arrival times of photons propagating on the brane. Given that such effects are considerably larger in size than the string scale, it is evident that they are further enhanced in the picture of reference [7] where the string scale in the bulk is of the order of a TeV. In reference [15] this has been used to place bounds on consistent string models of large extra dimensions [9].

In the present article the recoil process will be discussed in conjunction with another phenomenon that characterizes such theories, namely supersymmetry obstruction on the brane. Our objective is to discuss the appearance and nature of the conical singularities due to the recoil process, mentioned in references [10,13], and then to estimate the order of magnitude of the induced supersymmetry obstruction [2].

As discussed in references [10,11,13] in the case of D-brane string solitons, their recoil after interaction with a closed string (graviton) state is characterized in a worldsheet context by a $\sigma$-model deformed by pairs of logarithmic operators [14]:

$$ C^I_\epsilon \sim \epsilon \Theta_\epsilon (X^I), \quad D^I_\epsilon \sim X^I \Theta_\epsilon (X^I), \quad I \in \{0, \ldots, 3\} $$

(2)
defined on the boundary $\partial \Sigma$ of the string worldsheet. Here $X^I$ obey Neumann boundary conditions on the string worldsheet, and denote the brane coordinates. The remaining $y^i, i \in \{4, \ldots, 9\}$ denote the transverse directions. In the case of D-particles, examined in references [10,13], $I$ takes the value 0 only, in which case the operators (2) act as deformations of the conformal field theory on the worldsheet: the operator

$$ U_i \int_{\partial \Sigma} \partial_n X^i D^I_\epsilon $$
describes the shift of the D-brane induced by the scattering, where $U_i$ is its recoil velocity, and

$$ Y_i \int_{\partial \Sigma} \partial_n X^i C^I_\epsilon $$
describes quantum fluctuations in the initial position $Y_i$ of the D-particle. It has been shown [13] that energy-momentum is conserved during the recoil process. We also note that

$$ U_i = g_s P_i, \quad \text{where} \quad P_i = \text{momentum and} \quad g_s = \text{string coupling}, $$

which is assumed here
to be weak enough to ensure that D-branes are very massive, with mass \( M_D = 1/(\ell_s g_s) \), where \( \ell_s \) is the string length. From these one obtains \( U_i = \ell_s g_s (k_1^i + k_2^i) \), where \( k_1 \) (\( k_2 \)) is the momentum of the propagating closed string state before (after) the recoil \([13]\).

In the case of D-p-branes, the pertinent deformations are slightly more complicated. As discussed in reference \([10]\), the deformations are given by

\[
\sum_I g_{\alpha I} \int_{\partial \Sigma} \partial_n X^i D_I^i \quad \text{and} \quad \sum_I g_{\gamma I} \int_{\partial \Sigma} \partial_n X^i C_I^i.
\]

The 0i component of the “tensor” couplings \( g_{\alpha I}^i, \alpha \in \{1, 2\} \) include the collective momenta and coordinates of the D-brane as in the D-particle case above, but now there are additional couplings \( g_{\gamma I}^i, I \neq 0 \), describing the “folding” of the D-brane under the emission of a closed string state propagating in a transverse direction, as shown schematically in Fig. [4]. Intuitively it is clear that this emission and the resulting recoil results in a wedge-shaped “folded” conical space, like a surface tension effect on the higher-dimensional analogue of an elastic membrane. In the following we will verify this for the D-particle case after correctly Liouville-dressing the deformation operators.

\[
\text{FIG. 1. Schematic representation of the recoil effect: the surface of the D-brane D1 is distorted by the conical singularity that results from closed string emission into the bulk. The dashed line on D1 represents the (disturbed) trajectory of a matter particle living on the brane.}
\]

The correct specification of the logarithmic pair in equation \([2]\) entails a regulating parameter \( \epsilon \to 0^+ \), which appears inside the \( \epsilon \)-regularized \( \Theta_\epsilon(t) \) operator:

\[
\Theta_\epsilon(X^i) = \int \frac{d\omega}{2\pi} \frac{1}{\omega - i\epsilon} e^{i\omega X^i}.
\]

In order to realize the logarithmic algebra between the operators \( C \) and \( D \), one takes \([10]\):

\[
\epsilon^{-2} \sim \ln[L/a] \equiv \Lambda,
\]

where \( L \) (\( a \)) are infrared (ultraviolet) worldsheet cutoffs. The recoil operators \([2]\) are slightly relevant, in the sense of the renormalization group for the worldsheet field theory, having small conformal dimensions \( \Delta_\epsilon = -\epsilon^2/2 \). Thus the \( \sigma \)-model perturbed by these operators is not conformal for \( \epsilon \neq 0 \), and the theory requires Liouville dressing \([11, 12, 16, 17]\).
To determine the effect of such dressing on the spacetime geometry, it is essential to write the boundary recoil deformations as bulk worldsheet deformations by partial integration

\[ \int_{\partial \Sigma} g_{\alpha i}^1 X^I \Theta_\epsilon (X^I) \partial_\alpha X^i = \int_{\Sigma} \partial_\alpha (g_{\alpha i}^1 X^I \Theta_\epsilon (X^I) \partial^\alpha X^i) \]  

These operators can be made marginal on a curved worldsheet by Liouville-dressing. One Liouville-dresses the (bulk) integrand by multiplying it by a factor \( e^{\alpha_i \phi} \), where \( \phi \) is the Liouville field and \( \alpha_i \) is the gravitational conformal dimension, which is related to the flat worldsheet anomalous dimension \( -\epsilon^2/2 \) by

\[ \alpha_i = -\frac{Q_b}{2} + \sqrt{\frac{Q_b^2}{4} + \frac{\epsilon^2}{2}} \]  

where \( Q_b \) is the central-charge deficit of the bulk worldsheet theory. In the recoil problem at hand, as discussed in reference [12], \( Q_b^2 \sim \epsilon^4/g_1^b \) for weak folding deformations \( g_1^b \). This yields \( \alpha_i \sim -\epsilon \) to leading order in perturbation theory in \( \epsilon \), to which we restrict ourselves here.

We next remark that, as the analysis of reference [11] indicates, the \( X^I \)-dependent field operators \( \Theta_\epsilon (X^I) \) scale as follows with \( \epsilon \): \( \Theta_\epsilon (X^I) \sim e^{-\epsilon X^I} \Theta (X^I) \), where \( \Theta (X^I) \) is the normal (not \( \epsilon \)-regularized) Heaviside step function without any field content, evaluated in the limit \( \epsilon \to 0^+ \). The bulk deformations, therefore, yield the following \( \sigma \)-model terms:

\[ \epsilon g_{\alpha i}^1 X^I e^{\epsilon (\phi(0) - X^I(0))} \Theta (X^I(0)) \int \partial^\alpha X^I \partial^\alpha X^i \]  

where the subscripts \( (0) \) denote worldsheet zero modes. Upon the interpretation of the Liouville zero mode \( \phi(0) \) as target time \( t \), the deformations (6) yield spacetime metric deformations in a \( \sigma \)-model sense, which were interpreted in reference [11] as expressing the distortion of the spacetime surrounding the recoiling D-brane soliton.

We choose to work in a region of spacetime on the D-3-brane such that \( \epsilon (\phi - X^I) \) is finite in the limit \( \epsilon \to 0^+ \). The resulting spacetime distortion is therefore described by the metric elements

\[ G_{0i} = \epsilon g_{\alpha i}^1 X^I \Theta_\epsilon (X^I) + \mathcal{O}(\epsilon^2). \]

The presence of the \( \Theta (X^I) \) functions indicate that the induced spacetime is piecewise continuous.

To see how the Liouville dressing above leads to conical spacetimes, it is sufficient to restrict attention to the D-particle case [12]. In that case, the resulting spacetime resembles flat Minkowski spacetime to \( \mathcal{O}(\epsilon^2) \) [12], upon making the following transformation for \( t > 0 \):

\[ \text{The important implications for non-thermal particle production and decoherence for a spectator low-energy field theory in such spacetimes were discussed in [11, 12], where only the D-particle recoil case was considered.} \]
\[ \tilde{X}^i = X^i + \frac{1}{2} \epsilon U^i t^2, \quad \tilde{t} = t. \]  

(8)

This implies that the spacetime induced by the recoiling D-brane resembles, for \( t \gg 0 \) and to order \( O(\epsilon^2) \), a Rindler wedge space with (non-uniform \( [12] \)) ‘acceleration’ \( \epsilon U^i \). This space is well known to produce a conical singularity when the (Euclidean) time is compactified over an inverse ‘temperature’ interval, and has bulk deficit angle

\[ \delta_{0i} \sim 2\pi \left( 1 - \frac{1}{\epsilon |U^i|} \right). \]  

(9)

Such conical singularities lead to supersymmetry breaking in the bulk, not on the D-particle’s world-line. An interesting realization of this D-particle case is that of a D-0-brane defect embedded in a D-p-brane, i.e. dimensionally reduced intersecting D-branes. In the deficit above the rôle of the ‘temperature’ is played by the ‘acceleration’ \( \epsilon |U^i| \) \[18\]. Indeed, the conical spaces described here have a similar effect to non-zero temperatures which can be computed using a thermal superspace formalism \[18\]: there are thermal mass splittings between superpartners, proportional to \( T \) (for particles massless at \( T = 0 \) like the graviton and photon and their supersymmetric partners). Our discussion above refers, rather generically, to mass splittings on the D-p-brane (specifically a D-3-brane). However, for scattering events like the one depicted in Figure 1, the formalism also implies similar “thermal” mass splittings in the bulk. Let the compactification volume (in units of the (bulk) string scale \( M_s^{-1} \)) of the extra \( n \) dimensions be denoted by \( \Omega = (\Lambda M_s)^n \), where \( \Lambda \) is the radius of the extra dimensions (assumed, for simplicity, to be of equal size). The scale \( \Lambda \) is related to the Planck scale \( M_P \) on the D-p-brane by \[12\]: \( M_P = M_s/g_s = (\Lambda M_s)^{n/2} \). Then a (“thermal”) mass splitting on the D-3-brane, \( \delta M_{D3} \), can be estimated by naive dimensional reduction from the corresponding one in the bulk geometry, \( \delta M_{b} \sim M_s \epsilon |U^i| \), as:

\[ \delta M_{D3} \sim \delta M_{b} \Omega = M_P (1/g_s) \epsilon |U_i|. \]

Taking into account energy-momentum conservation during the recoil process \[13\] one has:

\[ U_i = g_s \Delta P_i/M_s, \]

where \( \Delta P_i \) is a typical momentum transfer in the bulk, along the direction of recoil. In such a case,

\[ \delta M_{D3} \sim M_P \epsilon (\Delta P_i/M_s). \]

As we shall discuss below, if the scattering occured at very early times, then a typical bulk scattering energy would be of order \( M_s \), which implies that the induced supersymmetry-obstructing mass splittings on the D-3-brane would be of order \( \epsilon M_P \).

When the above formalism is extended to the full recoiling D-p-brane case the deformations arising from localized emissions (e.g. closed strings or heavy D-particles) into the bulk induce a wedged or conical (for the deformation should be symmetric around the axis of recoil) world-volume for the brane also; see Figure 1. This excited state of the brane is expected to lead to mass-splittings proportional to the properly renormalized recoil couplings \( g_{dI} \), being proper generalizations of the D-particle operators \( Y^i \) and \( U^i \). In the following we will show how careful interpretation of the Liouville-dressing, and proper identification of the parameter \( \epsilon \) as the physical time, lead to the possibility of supersymmetry obstruction well below the natural scale \( M_P \).
To this end, we recall that the worldsheet two-point correlation functions of the recoil operators have the following form [10]:

\[
\langle C_\epsilon(z)C_\epsilon(0) \rangle \overset{\epsilon \to 0}{\sim} 0 + O(\epsilon^2)
\]

\[
\langle C_\epsilon(z)D_\epsilon(0) \rangle \overset{\epsilon \to 0}{\sim} \frac{\pi}{2} \sqrt{\frac{2}{\epsilon^2 \Lambda}} \left(1 + 2 \epsilon^2 \log |z/a|^2\right)
\]

\[
\langle D_\epsilon(z)D_\epsilon(0) \rangle = \frac{1}{\epsilon^2} \langle C_\epsilon(z)D_\epsilon(0) \rangle \overset{\epsilon \to 0}{\sim} \frac{\pi}{2} \sqrt{\frac{2}{\epsilon^2 \Lambda}} \left(\frac{1}{\epsilon^2} + 2 \log |z/a|^2\right)
\]

which in the limit \( \epsilon \to 0^+ \) gives the logarithmic algebra [14] modulo the leading divergence in the \( \langle D_\epsilon D_\epsilon \rangle \) recoil correlation function.

The identification (3) turns out to be very important for our purposes here. As discussed in references [10, 20], under worldsheet scaling transformations parametrized by variations of the cutoff

\[
L \mapsto L' = Le^t \quad \Rightarrow \quad \epsilon^2 \mapsto \epsilon^2 = \frac{\epsilon^2}{1 + 2\epsilon^2 t},
\]

then as a result of the logarithmic algebra (10) the operators \( C \) and \( D \) transform as

\[
D_\epsilon \mapsto D_\epsilon' = D_\epsilon + tC_\epsilon,
\]

\[
C_\epsilon \mapsto C_\epsilon' = C_\epsilon
\]

which implies a similar transformation for the couplings. In particular, the the \( g^2_{i_1} \) bending couplings are shifted as

\[
g^2_{i_1} \mapsto (g^2_{i_1})' = g^2_{i_1} + g^1_{i_1}t,
\]

while the \( g^1_{i_1} \) couplings remain invariant. From this and the fact that \( g^2_{0i} \) are interpreted respectively as the collective coordinates and momenta of the recoiling D-brane, one observes that the scale \( \epsilon^{-2} \) may be interpreted as a Galilean time for the (heavy) defect system.

It is important to understand the connexion of this time with the physical time as measured by an observer on the brane. To answer this question we remark that the worldsheet renormalization group scale \( \ln |L/a|^2 \) may be associated with the zero-mode of the Liouville field [16, 17], which in turn is identified with the target time \( t \) on the brane, as justified in detail in references [11, 13]. From this one immediately has the identification

\[
\epsilon^{-2} \longleftrightarrow \eta t,
\]

where the time \( t \) is measured in string units \( t_s = \ell_s/c \), where \( \ell_s \) is the string length. The constant of proportionality \( \eta \) can be determined as follows: the Liouville field used to dress and hence restore conformal invariance [16, 17] in the non-conformal \( \sigma \)-model perturbed by the recoil operators [14] has a kinetic term in the \( \sigma \)-model of the form:

\[
\int \Sigma d^2z Q^2(\partial \phi)^2
\]

which is integrable in the \( \sigma \)-model.
where $\Sigma$ denotes a (closed string) worldsheet surface. The central charge deficit $Q^2 = C[g] - C^*$ is written in terms of the running central charge $C[g]$ given by the Zamolodchikov $C$-function which can in turn be defined by an appropriate combination of the two-point worldsheet correlation functions of the stress tensor for the $\sigma$-model $\langle T_{\alpha\beta}T_{\gamma\delta} \rangle$ (the indices run over worldsheet coordinates). For closed string excitations the worldsheet is assumed to be a sphere, with Euler characteristic $\chi = 2$. This implies that the worldsheet correlation functions entering in the expression for the $C$-function will have a prefactor of $1/g_s^2$. For the weakly coupled string theories in which we are interested, and for which the recoil formalism of references [10, 13] applies, the detailed analysis of reference [13] demonstrated that the identification of the Liouville mode $\phi$ with the target time $t$ leads to a consistent interpretation of the central charge deficit $Q^2$ in the deformed $\sigma$-model. The recoil is described by an effective Lagrangian in target space of Born–Infeld type as a result of the identification:

$$Q^2 \sim \frac{1}{g^2_s} \sqrt{1 - |g_{Ii}^1|^2 + \ldots}$$

to leading orders in worldsheet perturbation theory.

The set of bending couplings $g_{Ii}^1 \equiv g_{Ii}$, $I \in \{0, \ldots, p\}$, $i \in \{p + 1, \ldots, 9\}$, are relevant couplings with a worldsheet renormalization-group $\beta$-function of the form

$$\beta_g = \frac{d}{dt} g_{Ii} = -\frac{1}{2t} g_{Ii}, \quad t \sim \epsilon^{-2}$$

which implies that one may construct an exactly marginal set of couplings $\overline{g}_{Ii}$ by redefining

$$\overline{g}_{Ii} \equiv \frac{g_{Ii}}{\epsilon}$$

The renormalized couplings $\overline{g}_{Ii}$ in [13] play the rôle of the physical recoil velocity of the D-brane, while the remaining $\overline{g}_{Ii}$, $i \neq 0$, describe the bending of the D-$p$-brane, $p \neq 0$.

By following an analysis similar to that for the D-particle case in reference [13] it can easily be shown that the renormalized bending couplings $\overline{g}_{Ii}$ are related to the sum of momenta of the closed string states along the transverse directions $k_i^{1,2}$ as follows:

$$\overline{g}_{Ii} \sim \frac{g_s}{M_s} (k_i^1 + k_i^2), \quad I \in \{0, \ldots, p\}, \quad i \in \{p + 1, \ldots, 9\}$$

where $M_s \sim \ell_s^{-1}$ is the string scale (in units where $\hbar = c = 1$).

In this way we find that $\overline{g}_{Ii} = O(1)$ for closed string excitations with Planckian energies of order $M_s/g_s$, in which case $Q^2 \to 0$. For any other low-energy state $E \ll M_s/g_s$ $Q^2 \sim 1/g_s^2$. We now notice that from a target spacetime point of view such a kinetic term contributes to the $G_{00}$ temporal component of the metric. In order to obtain a Robertson–Walker type of target-space universe under the above identifications, it is crucial that we rescale the Liouville mode $\phi$ to $Q\phi$ before identifying it with the (observable) cosmic time $t_{\text{phys}}$. In this way, from (3), one obtains

$$\epsilon^{-2} = g_s t_{\text{phys}}$$

where $t_{\text{phys}}$ is the physical (cosmic) time in string units $t_s$, pertaining to a Robertson–Walker universe.
The identification (15) implies that the recoil/bending deformation operators \( g_i \int_{\partial \Sigma} D \) can be expressed in terms of the marginal couplings \( \mathcal{g}_i \) through (reinstating the string timescale \( t_s \))

\[
g_i \rightarrow \frac{g_s}{M_s} (k_i^1 + k_i^2) \left( \frac{t_s}{g_s t_{\text{phys}}} \right)^{1/2}
\]

which in turn implies that the conical singularities on the D-brane due to these deformations will have a deficit which will be decaying with time as \( t_{\text{phys}}^{-1/2} \), as the system relaxes towards its ground state. Therefore if we view the world as a D-brane living in a higher-dimensional string (or M-theory) universe, then a scattering process whereby a closed string state propagates in the transverse extra dimensions will excite the D-brane through recoil. This will create a conical singularity at the time of the scattering event whose formation can be described by deforming the worldsheet \( \sigma \)-model by recoil operators. It is crucial to identify the worldsheet renormalization-group scale with the target time for the mathematical consistency of the logarithmic algebra \([11,13]\). This identification naturally implies a relaxation process for the recoiling brane.

During this relaxation process the presence of a conical singularity with a deficit implies supersymmetry obstruction with mass splittings given by a formula analogous to (1), where now \( G \) should be Newton’s constant on the D-brane. The latter is related to the four-dimensional Planck mass scale \( M_P^{(4)} \sim M_s/g_s \sim 10^{19} \text{ GeV} \). Thus the induced supersymmetry-obstructing mass splitting \( \delta m \) would be of the form:

\[
\delta m \sim g_i M_P^{(4)} \sim M_P^{(4)} g_s |k_i^1 + k_i^2| \left( \frac{t_s}{g_s t_{\text{phys}}} \right)^{1/2}
\]

The result (17) has to be interpreted with care. First of all one should note that asymptotically in time \( t_{\text{phys}} \rightarrow \infty \) the splitting tends to zero, implying the restoration of supersymmetry in the spectrum. This is natural from the point of view adopted here, i.e. that the world we live on is a 3-brane in a recoiling excited state after scattering with a closed string state. The low-energy supersymmetry obstruction today is a result of the relaxation of the brane.

In this scenario, at early times \( t_{\text{phys}} \sim t_s \), the D-brane had a small size, of order of the string length \( \ell_s \). Therefore the closed string states trapped on it should have uncertainties in energies of order \( M_P^{(4)} \), implying that at times \( t_{\text{phys}} \sim t_s \) after the initial scattering event, the recoiling D-brane would have experienced obstructed supersymmetry with mass splittings

\[
\delta m(t_s) \sim M_P^{(4)}
\]

This initial splitting diminishes as the time elapses according to

\[
\delta m \sim M_P^{(4)} \left( \frac{t_s}{g_s t_{\text{phys}}} \right)^{1/2},
\]

One may arrange for the present day supersymmetry obstruction to be of order a few TeV by selecting appropriately the “frequency” of the scattering events, i.e. the quantities \( t_{\text{phys}} \).
$g_s$ and $t_s$. For instance, for a scattering event occurring at early cosmological times, e.g. at the time of last scattering $t_{\text{phys}} \sim 10^{26}/c$, for “large” string sizes $t_s \simeq 10^{-27} \text{ s}$, as in the scenario of reference [4], and small couplings $g_s \simeq 10^{-14}$, as required for a consistent string theory embedding of such scenarios in type IIB closed string theories [9], one obtains from (19) a supersymmetry obstruction scale today of order of a few TeV.

In the type I' open string case [8] there exist D-3-branes as solutions in the model, with the restriction that only gravitational closed string states can propagate in the bulk, exactly as required by the picture of reference [7]. In this model the string coupling $g_s$ is given by the Yang–Mills fine structure constant at the string scale,

$$g_s = 4\alpha_G(M_s).$$

According to reference [15] this results in stochastic fluctuations in the arrival times of photons with energy $E$ and travelling a distance $L$ of

$$\Delta t \sim \alpha_G \frac{LE}{M_s}.$$  

Astrophysical data on gamma ray bursters are sensitive [21] to $\Delta t \sim LE/M_{QG}$ with $M_{QG} \sim 10^{15} \text{ GeV}$, whence the type I' model seems incompatible with a low (TeV) string scale [15], and hence also with TeV scale supersymmetry obstruction by the mechanism described here.

In this article we have presented a mechanism by which supersymmetry can be obstructed on our world at the TeV scale as a result of D-brane recoil within the large extra dimension picture of reference [7]. Supersymmetry is obstructed by a Planck scale mass splitting on the brane as a result of the formation of a conical singularity. As this excited state of the brane relaxes back to the ground state the scale of supersymmetry obstruction is lowered. We stress that this is a non-equilibrium process, and that the relaxation we have described differs fundamentally from the “slow rolling” by which a false vacuum decays to a true vacuum. In our case supersymmetry is obstructed [2] so that the vacuum remains supersymmetric, and hence the cosmological constant on the brane vanishes, and it is only the matter spectrum which does not respect supersymmetry as a result of the excited state of the brane.

As stressed above, this scenario for supersymmetry obstruction is based on the Liouville (non-critical-string) approach to D-brane recoil, which involves the identification of the target time with the Liouville mode. Moreover, for the scenario to yield viable phenomenological predictions, it is essential that the extra dimensions are relatively “large”, compared with the Planck scale, and that the emission of closed string states from the brane into the bulk is a very rare event. Such rare emissions, although compatible with the depletion of closed string states due to inflation, are still far from being understood at a satisfactory level of mathematical rigour in the context of the Liouville-string approach to D-brane recoil. This would require a detailed knowledge of the underlying non-perturbative D-brane dynamics, which is still lacking. Nevertheless, we believe that the scenario for supersymmetry obstruction presented here is of sufficient interest to motivate further detailed studies of such issues.
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