Electroweak Precision Physics
at $e^+e^-$ Colliders with RACOONWW †

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Abstract

We present precise predictions for the processes $e^+e^- \rightarrow WW \rightarrow 4f(\gamma)$ at LEP2 and future Linear-Collider (LC) energies obtained with the Monte Carlo generator RACOONWW. The program RACOONWW includes the complete $O(\alpha)$ electroweak radiative corrections to $e^+e^- \rightarrow WW \rightarrow 4f$ in the double-pole approximation (DPA). While the virtual corrections are treated in DPA, the calculation of the bremsstrahlung corrections is based on the full lowest-order matrix elements to the processes $e^+e^- \rightarrow 4f + \gamma$. This asymmetric treatment of virtual and real photons requires a careful matching of the arising infrared and collinear singularities. We also take into account higher-order initial-state photon radiation via the structure-function method. Here, we briefly describe the RACOONWW approach, give numerical results for the total W-pair production cross sections, confront them with LEP2 data, and study the impact of the radiative corrections on angular and W-invariant-mass distributions at LEP2 and LC energies.

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Abstract. We present precise predictions for the processes $e^+e^- \to WW \to 4f(\gamma)$ at LEP2 and future Linear-Collider (LC) energies obtained with the Monte Carlo generator RACOONWW. The program RACOONWW includes the complete $O(\alpha)$ electroweak radiative corrections to $e^+e^- \to WW \to 4f$ in the double-pole approximation (DPA). While the virtual corrections are treated in DPA, the calculation of the bremsstrahlung corrections is based on the full lowest-order matrix elements to the processes $e^+e^- \to 4f + \gamma$. This asymmetric treatment of virtual and real photons requires a careful matching of the arising infrared and collinear singularities. We also take into account higher-order initial-state photon radiation via the structure-function method. Here, we briefly describe the RACOONWW approach, give numerical results for the total W-pair production cross sections, confront them with LEP2 data, and study the impact of the radiative corrections on angular and W-invariant-mass distributions at LEP2 and LC energies.

INTRODUCTION

The precise measurements of the W-boson mass and the W-pair production cross sections as well as the direct observation of triple-gauge-boson couplings [1] at LEP2 provide further important precision tests of the Standard Model of electroweak interactions. To match the experimental accuracy at LEP2 and at a future linear collider (LC), the knowledge of the observed cross sections beyond leading order in perturbation theory is crucial [2–4]. For instance, at LEP2 the total W-pair production cross sections need to be known theoretically to better than 1% [4], which requires to go even beyond universal radiative corrections such as the running of the electromagnetic coupling, corrections connected to the $\rho$ parameter, the Coulomb singularity, and leading photonic corrections.

The full treatment of the processes $e^+e^- \to 4f$ at the one-loop level is of enormous complexity [5] and poses severe gauge-invariance problems when introducing
the finite W-boson width. Fortunately, for W-pair production at LEP2 and at not too high LC Center-of-Mass (CM) energies \(^1\), it is sufficient for the envisioned theoretical precision to take into account only radiative corrections to those contributions that are enhanced by two resonant W propagators. The use of this so-called double-pole approximation (DPA) for the calculation of the pair production of unstable particles has been proposed in [6]. Recently different versions of the DPA for the calculation of radiative corrections to off-shell W-pair production have been described in the literature [7–12].

Here we present results of the first complete calculation of the \(\mathcal{O}(\alpha)\) corrections for off-shell W-pair production in the DPA that has been implemented in a Monte Carlo (MC) generator. The resulting MC program is called RACOONWW, and numerical results for the W-pair production cross sections and various distributions at LEP2 and LC CM energies are provided in [4,11,12] and [13], respectively. In [11] we also presented a detailed description of the RACOONWW approach, discussed the intrinsic theoretical uncertainty of our version of the DPA, and compared our results with existing calculations (see also [4]).

In the following sections we give a brief overview of the RACOONWW approach and provide numerical results for the total W-pair production cross sections, angular and W-invariant-mass distributions at the typical LEP2 and LC CM energies \(\sqrt{s} = 200\) GeV and \(500\) GeV, respectively. For the same setup as described in [4], we also present a comparison of RACOONWW results with the results of the Monte Carlo generator YFSWW3 [8,9] at \(\sqrt{s} = 500\) GeV.

THE RACOONWW APPROACH

The radiative corrections to the processes \(e^+e^- \rightarrow WW \rightarrow 4f\) consist of virtual corrections as well as of real corrections originating from the processes \(e^+e^- \rightarrow WW \rightarrow 4f + \gamma\). In RACOONWW only the virtual one-loop corrections are treated in DPA, including the full set of factorizable and non-factorizable \(\mathcal{O}(\alpha)\) corrections to \(e^+e^- \rightarrow WW \rightarrow 4f\) for massless fermions. The factorizable contributions comprise those virtual one-loop corrections that can either be assigned to the W-pair production or to the W-decay subprocesses. The corresponding matrix element in DPA

\[
\mathcal{M}_{\text{DPA}}^{e^+e^-\rightarrow WW\rightarrow 4f} = \frac{R(k_{W+}^2 = M_W^2, k_{W-}^2 = M_W^2)}{(k_{W+}^2 - M^2)(k_{W-}^2 - M^2)}
\]

is defined by the gauge-invariant residue \(R(M_W^2, M_W^2)\) and the W propagators with complex poles at \(M^2 = M_W^2 - iM_W\Gamma_W\). The one-loop corrections to this residue can be deduced from the known results for on-shell W-pair production [14] and W decay [15]. The non-factorizable corrections [16] connect the production and

\(^1\) Above 0.5–1 TeV at least the leading electroweak logarithms at the two-loop level should be taken into account.
decay processes. In DPA, they comprise all doubly-resonant contributions that are not already included in the factorizable corrections. Only those diagrams contribute where a photon with energy \( E_\gamma \lesssim \Gamma_W \) is exchanged between the production and decay subprocesses; all other non-factorizable diagrams are negligible in DPA. A typical diagram contributing to the non-factorizable corrections is shown in Fig. 1. The calculation of the real \( \mathcal{O}(\alpha) \) corrections is based on the full lowest-order matrix element to \( e^+e^- \rightarrow 4f + \gamma \) for massless fermions as described in [17]. In this way, we avoid potential problems with a definition of a DPA for real photons with energies \( E_\gamma \sim \Gamma_W \). However, since we treat virtual and real photons differently, the former in DPA but the latter completely, a careful matching of the arising infrared (IR) and collinear singularities is needed: first the singularities are extracted from the real photon contribution by using either a subtraction [18] or the phase-space-slicing method. Both methods are implemented in RacoonWW. As usual, these IR- and collinear-singular contributions factorize into the Born cross section and a factor containing the IR- and collinear-singular logarithms, where the dependence on the photon phase space has been partially integrated out analytically. The remaining part of the real corrections is free of singularities and can be evaluated with the usual MC techniques. Then, we extract those singular parts from the IR- and collinear-singular contributions that exactly match the singular parts of the virtual corrections, replace the full Born cross section by the DPA Born cross section, and add this modified part to the virtual corrections. This modification only introduces ambiguities that are smaller than the accuracy of the DPA and leads to a proper matching of IR and collinear singularities.

Beyond \( \mathcal{O}(\alpha) \), RacoonWW includes soft-photon exponentiation and leading higher-order initial-state radiation (ISR) up to \( \mathcal{O}(\alpha^3) \) via the structure-function approach (see e.g. [3] and references therein). Also the leading-order effects from \( \Delta\rho \) and \( \Delta\alpha \) are taken into account by using the \( G_\mu \) scheme.

By default, QCD corrections are taken into account by considering a multiplicative factor \((1 + \alpha_s/\pi)\) for each hadronically decaying W boson. This affects the W-decay width in the W propagators and the amplitudes to \( e^+e^- \rightarrow 4f \) for quarks in the final state when the full phase space for gluons is integrated over. In the total cross section these so-called “naive” QCD factors cancel. If the gluon phase space is not integrated over, the virtual and real QCD corrections to \( e^+e^- \rightarrow 4f \)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{A typical diagram for virtual non-factorizable corrections (the blob denotes the lowest-order W-pair production process).}
\end{figure}
have to be calculated, which is technically similar to the calculation of the photonic corrections. This option is also supported by RACOONWW.

NUMERICAL RESULTS

For the numerical results we use the following parameters:

\[
\begin{align*}
G_\mu &= 1.16637 \times 10^{-5} \text{GeV}^{-2}, & \alpha &= 1/137.0359895, \\
M_W &= 80.35 \text{GeV}, & \Gamma_W &= 2.08699 \ldots \text{GeV}, \\
M_Z &= 91.1867 \text{GeV}, & \Gamma_Z &= 2.49471 \text{GeV}, \\
m_t &= 174.17 \text{GeV}, & M_H &= 150 \text{GeV}, \\
m_e &= 510.99907 \text{keV}, & \alpha_s &= 0.119.
\end{align*}
\]

We work in the fixed-width scheme and fix the weak mixing angle by using \( c_w = M_W/M_Z, \ s_w^2 = 1 - c_w^2 \). These parameters are over-complete but self-consistent. Instead of \( \alpha \) we use \( G_\mu \) to parametrize the lowest-order matrix element, i.e. we use the effective coupling

\[
\alpha G_\mu = \frac{\sqrt{2} G_\mu M_W^2 s_w^2}{\pi} \tag{3}
\]

in the lowest-order matrix element. This parameterization has the advantage that all higher-order contributions associated with the running of the electromagnetic coupling from \( q^2 = 0 \) to \( q^2 = M_W^2 \) and the leading universal two-loop \( m_t \)-dependent corrections are already absorbed in the Born cross section. In the relative \( \mathcal{O}(\alpha) \) corrections, on the other hand, we use \( \alpha = \alpha(0) \), since in the real corrections, which yield the bulk of the remaining corrections, the scale of the real photon is zero. The W-boson width given above is calculated including the electroweak and QCD one-loop corrections.

In the following, for the lowest-order contributions only the CC03 diagrams are taken into account. Our “best” results comprise the CC03 Born cross sections and the corrected cross sections including all the radiative corrections briefly described in the previous section (more details can be found in [11]), i.e. the electroweak one-loop corrections in DPA, the exact \( \mathcal{O}(\alpha) \) photon radiation based on the full matrix element for \( e^+e^- \rightarrow 4f\gamma \), higher-order ISR up to \( \mathcal{O}(\alpha^3) \), and “naive” QCD \( \mathcal{O}(\alpha_s) \) corrections.

Figure 2 shows a comparison of the results of RACOONWW for the total CC03 W-pair production cross section (see also [4]) and of other Monte Carlo predictions with recent LEP2 data, as given by the LEP Electroweak Working Group [19] for the Winter 2000 conferences. The data are in good agreement with the predictions of RACOONWW and YFSWW3 [8,9]. At the time of the conferences in Winter 2000 the predictions of YFSWW3 were about 0.5–0.7% larger than those of RACOONWW, which is somewhat larger than the intrinsic DPA ambiguity. Meanwhile, however, the main source of this discrepancy was found [4], and the new
YFSWW3 [9] results differ from the ones of RACOONWW only by about 0.3% at LEP2 CM energies. More details on the conceptual differences of the two generators, as well as a detailed comparison of numerical results for LEP2 energies, can be found in [4] and below. Figure 2 also includes the prediction provided by GENTLE [20], which is 2–2.5% larger than those from RACOONWW and YFSWW3. This difference is due to the neglect of non-leading, non-universal $O(\alpha)$ corrections in GENTLE. In summary, the comparison between SM predictions and LEP2 data reveals evidence of non-leading electroweak radiative corrections beyond the level of universal effects.

In the following we study the impact of the radiative corrections on the W-invariant-mass distributions and angular distributions and use the separation and recombination cuts of [4,11]:

1. All photons within a cone of 5 degrees around the beams are treated as invisible, i.e. their momenta are disregarded when calculating angles, energies, and invariant masses.

2. Next, the invariant masses $M_{f\gamma}$ of the photon with each of the charged final-state fermions are calculated. If the smallest one is smaller than a certain cutoff $M_{\text{rec}}$ or if the energy of the photon is smaller than 1 GeV, the photon is combined with the charged final-state fermion that leads to the smallest $M_{f\gamma}$, i.e. the momenta of the photon and the fermion are added and associated with the momentum of the fermion, and the photon is discarded.

![Figure 2](image-url)  
**FIGURE 2.** Total W-pair production cross section at LEP2, as given by the LEPEWWG [19].
3. Finally, all events are discarded in which one of the charged final-state fermions is within a cone of 10 degrees around the beams. No other cuts are applied.

We consider two recombination cuts: $M_{\text{rec}} = 5$ GeV (bare) and $M_{\text{rec}} = 25$ GeV (calo).

As mentioned before, RACOONWW contains two branches for the treatment of IR and collinear singularities, one following the subtraction method of [18] and one the phase-space-slicing method. While these two branches use the same matrix elements, the Monte Carlo integration is performed completely independently, thus providing us with a powerful numerical check of RACOONWW. In the following we present numerical results of both branches of RACOONWW. In Table 1 we provide the CC03 Born and “best” results for the total W-pair production cross sections for the semileptonic process $e^+e^- \rightarrow u\overline{d}\mu^-\overline{\nu}_\mu(\gamma)$ at the CM energies $\sqrt{s} = 200$ GeV and 500 GeV with the bare or calo cuts applied. The results obtained when using the phase-space-slicing method agree well with those of the subtraction method, i.e. within the statistical errors of 0.05–0.09\%.

In Figs. 3, 4, and 5 we show the relative corrections $d\sigma/d\sigma_{\text{Born}} - 1$ (in per cent), where $d\sigma_{\text{Born}}$ and $d\sigma$ denote the CC03 Born and “best” predictions, respectively, for a variety of distributions for $e^+e^- \rightarrow u\overline{d}\mu^-\overline{\nu}_\mu(\gamma)$ at the LEP2 CM energy $\sqrt{s} = 200$ GeV. The relative corrections to the W-invariant-mass distributions are shown in Figs. 3 and 4 for the bare and calo recombination cuts. The invariant masses are obtained from the four-momenta of the decay fermions of the W bosons after eventual recombination with the photon four-momentum. They are particularly sensitive to the treatment of the photons, and thus a strong dependence of the relative corrections on the recombination cut can be observed. The observed distortion of the W-invariant-mass distributions is of particular interest when the W-boson mass is reconstructed from the W-decay products. A detailed discussion of the distortion of the invariant-mass distributions of the W bosons and their origin has been given in [12]. The distributions obtained from the phase-space-slicing and subtraction methods are compatible with each other. The integration errors are larger further away from the W resonance because of the decreasing statistics. For the angular distributions, we define all angles in the laboratory system, which is the CM system of the initial state. The relative
corrections to the distributions in the cosines of the $W^+$ production angle $\theta_{W^+}$ and of the decay angle $\theta_{W^-\mu^-}$ are shown in Fig. 5 when bare cuts are applied. In contrast to the invariant-mass distributions, the angular distributions hardly depend on the recombination procedure apart from very large angles where the cross section is small. Thus, similar results are obtained when the calo recombination cut is used [11]. The distributions obtained with the subtraction and the phase-space-slicing branches of RACOONWW agree with each other within the integration errors, i.e. in general to better than 0.5%. The increase of the integration errors for large
FIGURE 5. Relative corrections to the distributions in the cosines of the W\(^+\) production angle (l.h.s) and the \(\mu^-\) scattering angle with respect to the W\(^-\) direction (r.h.s.) for \(e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu\) at \(\sqrt{s} = 200\) GeV when bare cuts are applied.

decay angles is due to the smallness of the corresponding cross section.

Comparison with YFSWW3 at \(\sqrt{s} = 500\) GeV

In [4] a tuned comparison of RACOONWW and YFSWW3 of predictions for \(e^+e^- \rightarrow WW \rightarrow 4f(\gamma)\) observables at LEP2 CM energies has been performed. This comparison together with a study of the intrinsic uncertainty of the DPA with RACOONWW, as well as a comparison with a semi-analytical calculation [7], enabled the assignment of an overall theoretical uncertainty of 0.4\% to the state-of-the-art MC predictions for the total W-pair production cross sections at 200 GeV [4]. This confirms the naively expected DPA accuracy of about 0.5\% for \(\sqrt{s} \sim 180\) GeV for the total CC03 cross sections. In [11] we studied the intrinsic ambiguities of the \(\mathcal{O}(\alpha)\) corrections in DPA also at 500 GeV and found them to be 0.1\% for the total W-pair production cross section and a few per mil whenever W-pair production dominates the considered observable. A comparison of RACOONWW and YFSWW3 results for the total W-pair production cross sections at 200 GeV and 500 GeV, when no or bare cuts are applied, is given in Table 2. The relative differences are at the 0.3\% level, which is of the order of the intrinsic ambiguity of any DPA implementation.

In Figs. 6, 7, and 8 we show the predictions of the subtraction-method branch of RACOONWW together with the predictions of YFSWW3 (scheme A) [9] for the relative corrections to the W-invariant-mass and cos\(\theta_{W^+}\) distributions to \(e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu(\gamma)\) at 500 GeV when bare or calo cuts are applied. In contrast to the LEP2 case, the radiative corrections now mostly increase the Born cross sections...
TABLE 2. Born and “best” predictions for the total CC03 cross-sections for $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu(\gamma)$ obtained with RACOONWW (subtraction-method branch) and YFSWW3 (scheme A) [9] at $\sqrt{s} = 200$ GeV and 500 GeV when no or bare cuts are applied. The numbers in parentheses are statistical errors corresponding to the last digits.

| no cuts | $\sqrt{s} = 200$ GeV | $\sqrt{s} = 500$ GeV |
|---------|----------------------|----------------------|
|         | Born | best | Born | best |
| YFSWW3  | 659.64(07) | 622.71(19) | 261.377(34) | 279.086(97) |
| RACOONWW| 659.51(12) | 621.06(14) | 261.400(70) | 280.149(86) |
| ($Y-R)/Y$ | 0.02(2)% | 0.27(4)% | -0.01(3)% | -0.38(5)% |

| bare cuts | $\sqrt{s} = 200$ GeV | $\sqrt{s} = 500$ GeV |
|-----------|----------------------|----------------------|
|           | Born | best | Born | best |
| YFSWW3  | 627.18(07) | 592.68(19) | 181.507(33) | 197.933(84) |
| RACOONWW| 627.22(12) | 590.94(14) | 181.507(63) | 198.696(76) |
| ($Y-R)/Y$ | -0.01(2)% | 0.29(4)% | 0.00(4)% | -0.39(6)% |

and are especially pronounced in the angular distributions. The dramatic increase of the relative corrections to the $W^+$-production-angle distribution of Fig. 8 is due to initial-state hard photon radiation which causes a redistribution of events to a phase-space region where the Born cross sections are very small. A detailed discussion of the invariant-mass and angular distributions obtained with RACOONWW at 500 GeV is given in [13].

The invariant-mass distributions in Figs. 6 and 7 obtained by RACOONWW and YFSWW3 are statistically compatible with each other for both recombination procedures, i.e. they agree to better than 1%. The predictions of RACOONWW
and YFSWW3 for the distributions in the cosine of the $W^+$ production angle, shown in Fig. 8, agree within the statistical errors for small angles but differ by up to several per cent for intermediate and large angles where the cross sections become small. This behaviour is independent of the applied recombination cuts.

**FIGURE 7.** Relative corrections to the $W^-$ invariant-mass distributions for $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ at $\sqrt{s} = 500$ GeV when bare (l.h.s) or calo (r.h.s) cuts are applied.

**FIGURE 8.** Relative corrections to the cosine of the $W^+$-production-angle distributions for $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ at $\sqrt{s} = 500$ GeV when bare (l.h.s) or calo (r.h.s) cuts are applied.
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