The Central Temperature of the Sun can be Measured via the 

$^7$Be Solar Neutrino Line

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(June 13, 2018)

Abstract

A precise test of the theory of stellar evolution can be performed by measuring the difference in average energy between the neutrino line produced by $^7$Be electron capture in the solar interior and the corresponding neutrino line produced in a terrestrial laboratory. The high temperatures in the center of the sun broaden the line asymmetrically, FWHM = 1.6 keV, and cause an average energy shift of 1.3 keV. The width of the $^7$Be neutrino line should be taken into account in calculations of vacuum neutrino oscillations.

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The purpose of this letter is to describe the observational implications of a precise, new prediction of the theory of stellar evolution and to stimulate thinking about possible experimental tests. The predicted quantity is the energy profile of the neutrino line produced by $^7$Be electron capture in the solar interior. Standard solar models imply that the average energy of the $^7$Be neutrino line profile is shifted from its laboratory value by 1.3 keV and is broadened to a full-width at half-maximum of 1.6 keV. The predicted energy shift may be measurable with future solar neutrino experiments. The details of the calculations that lead to these results will be given elsewhere [1]; only the physical ideas and the principal observational consequences will be summarized here (for background material, see [2]).

The reaction in question is

$$e^- + ^7\text{Be} \rightarrow ^7\text{Li} + \nu_e .$$

Reaction (1) occurs in about 15% of the terminations of the proton-proton chain fusion reactions in standard solar models. In the interior of the sun, most of the electrons are captured from continuum states. The reaction produces a line because the recoiling nucleus takes up a significant amount of momentum but only a negligible amount of energy. I consider here only the experimentally more-accessible transition to the ground state of $^7$Li.

The average energy of the solar neutrino line is larger than for the corresponding laboratory decay because electrons and $^7$Be ions move with appreciable kinetic energies at the high temperatures ($\approx$ 1 keV) characteristic of the solar interior. The average energy difference between neutrinos emitted in solar and in laboratory decays reflects the temperature profile in the center of the sun and is determined primarily by the center-of-momentum thermal energy of the continuum electrons and of their capturing $^7$Be nuclei, by the Doppler shifts of the $^7$Be ions, by the fraction of electron captures that occur from bound orbits rather than from continuum orbits, and by the difference in atomic binding energies between solar and laboratory conditions.

The average shift, $\Delta$, in the neutrino energy, $q$, between $^7$Be neutrinos emitted in the sun and $^7$Be neutrinos emitted in the laboratory is [1].
\[ \Delta \equiv \left\langle q - q_{\text{lab}} \right\rangle = (1.29 \pm 0.01) \text{ keV} \]  

for three precise, recently-calculated solar models (constructed with and without including helium diffusion).

The energy shift, \( \Delta \), is approximately equal to the average temperature of the solar interior weighted by the fraction of \( ^7\text{Be} \) neutrinos that are produced at each temperature, i.e. \( \int T d\phi(\text{Be}, T) / \int T d\phi(\text{Be}, T) \), where \( d\phi(\text{Be}, T) \) is the flux of \( ^7\text{Be} \) neutrinos produced at the temperature \( T \). The \( ^7\text{Be} \) neutrinos are produced in the inner few percent of the solar mass, 75\% in the region \((0.04 \pm 0.03) M_\odot \). Therefore, a measurement of the energy shift is a measurement of the first moment of the central temperature distribution of the sun.

Figure I shows the calculated line profile derived by including the known physical effects. The neutrino energy at the profile peak is \( q_{\text{peak}} = 862.27 \text{ keV} \); the laboratory energy is \( q_{\text{peak}} = 861.84 \text{ keV} \). The line profile is asymmetric. Doppler shifts caused by thermal velocities of the \( ^7\text{Be} \) nuclei symmetrically broaden the line and determine the Gaussian shape below the peak. The low-energy side of the line profile, \( q_{\text{obs}} < q_{\text{peak}} \), is produced by \( ^7\text{Be} \) nuclei that are moving away from the observer. The profile at higher energies is determined by the center-of-momentum kinetic energies: \[ \text{Spectrum}_{\text{solar}}(q_{\text{obs}}) \propto \exp \left[ - (q_{\text{obs}} - q_{\text{peak}}) / kT_{\text{eff}} \right] \]. Since kinetic energies are always positive, the exponential tail is present only for \( q_{\text{obs}} > q_{\text{peak}} \).

On the low-energy side, the standard solar model with helium diffusion predicts a half-width at half-maximum of 0.56 keV, and on the high-energy side, the half-width at half-maximum is 1.07 keV. The effective temperature of the high-energy exponential tail is \( 15 \times 10^6 \text{ K} \). The effects of the electrostatic energy of the screening charge around the \( ^7\text{Be} \) and \( ^7\text{Li} \) nuclei, of gravitational redshifts on the neutrinos, and of collisional broadening of the line are much smaller than the direct effects of the thermal kinetic energies and Doppler shifts.

The precision with which the central thermal structure of the sun is determined by standard solar models and the fundamental and unique character of the prediction justifies
a special experimental effort to measure $\Delta$. For the standard solar models computed by the author and his colleagues over the past decade, the central temperature has varied over a total range of $\pm 0.5\%$, $T_c = (15.58 \pm 0.08) \times 10^6$ K. For a heterogeneous set of nine recently-calculated solar models \[4\], computed by different groups for different purposes using different input data and generally not required to have the highest-attainable precision, the central temperature varied by $\pm 1\%$, $T_c = (15.55 \pm 0.15) \times 10^6$ K.

Detailed calculations show \[1\] that the characteristic modulation of the shape of the $^7$Be neutrino line that would be caused either by vacuum neutrino oscillations or by matter-enhanced (MSW) neutrino oscillations is small. Other frequently-discussed weak interaction solutions to the solar neutrino problem, such as rotation of the neutrino magnetic moment, matter-enhanced magnetic moment transitions, and neutrino decay, will also not change significantly the line profile. The basic reason for the smallness of all these effects is that the ratio of the width of the line to the typical neutrino energy is only $\approx 0.001$, although the influence of fine-tuning must also be calculated.

The energy profile of the $^7$Be neutrino line should be included in precise calculations of what is expected from vacuum neutrino oscillations. It has become standard \[5\] to take account of the variation of the distance between the point of creation of the neutrinos and the point of detection. The variation in the point of creation corresponds to a phase-change, $\delta \phi$, of order $10^{-4}$, in the phase-angle, $\phi$, that determines the probability of observing an electron-type neutrino on earth (probability $\propto \sin^2 \phi$). (The ratio of the solar radius to the earth-sun distance is about 0.005 and $^7$Be neutrinos are produced in a region of about $\pm 0.025 R_\odot$.) Therefore, the change in phase, $\approx 10^{-3}$, due to the energy-width of the neutrino line is an order of magnitude larger than the phase-change caused by averaging over the region of production (and is comparable to the phase change due to seasonal variations).

A number of experiments have been proposed that would measure the $^7$Be neutrino flux with detectors that are based upon neutrino-electron scattering \[6,7\]. Radiochemical \[8,9\] and electronic detectors \[10\] of the total $^7$Be neutrino flux have also been proposed. The BOREXINO experiment \[6\] is the most advanced of these proposals and can, if recent
estimates of backgrounds are correct, measure the total flux of $^7$Be neutrinos. The $\nu - e$ scattering experiments will probably not be able to measure $\Delta$ since for most scattering events the neutrino and the electron share the final state energy (which is much larger than $\Delta$, see Figure 8.5 of [2]). Some experiments that measure the total flux at earth in the $^7$Be neutrino line should be performed before the experiments proposed here are attempted, since the total flux may be reduced (with respect to the prediction of the standard solar model) by neutrino oscillations or by other new physics.

The most direct way to study the $^7$Be energy profile may be to detect neutrino absorption by nuclei, which leaves an electron and a recoiling nucleus in the final state. Nearly all of the initial neutrino energy is transferred to the final-state electron (the nuclear recoil energy being small).

Lithium detectors, which have been discussed [3] as attractive detectors of solar neutrinos, deserve further study since the absorption cross section for this special case depends sensitively upon $\Delta$. The calculated absorption cross section for the reaction $\nu_e + ^7$Li $\rightarrow ^7$Be $+ e^-$, where $\nu_e$ is produced by $^7$Be electron capture in the sun, depends upon the assumed energy profile of the solar neutrinos. Neutrinos cannot be absorbed in this reaction if their energies lie below the energy threshold of 861.96 keV. The location of the threshold within the line profile determines the fraction of emitted neutrinos that can be absorbed. The absorption cross section for solar-produced $^7$Be neutrinos incident on a laboratory detector of $^7$Li is

$$\langle \text{Spectrum}_{\nu_e} (q_{\text{obs}}) \sigma_{\text{abs}} (q_{\text{obs}}) \rangle \simeq 19 \times 10^{-46} \text{ cm}^2,$$

assuming neutrinos do not change flavor after their creation. As usual, Eq. (3) includes a correction for the fact that only 89.7% of the $^7$Be neutrinos are produced in ground-state to ground-state transitions. The cross section given in Eq. (3) is almost a factor of two larger than obtained previously [1], which should make the contemplated experiments somewhat easier than previously considered. The earlier treatments neglected the difference in electron binding energies of solar and laboratory $^7$Be atoms as well as Doppler shifts of the $^7$Be nuclei, and did not average over the temperature profile of the sun. If the flux of $^7$Be
electron-type neutrinos were measured in an independent experiment, the total absorption rate in a lithium detector could be used to determine $\Delta$.

For absorption detectors, an energy resolution ($\Delta E/E$) of order 0.1% to 1.0% is desirable to measure the 1.3 keV (0.15%) energy shift. Detectors have been developed [12] for a variety of applications, including dark matter searches, the observation of double beta decay, and x-ray astronomy, that have energy resolutions of better than 1%, but their surface areas are not yet large enough for a practical solar neutrino detector. Consider, for specificity, a conceivable cryogenic experiment [13] that might be performed on $^{81}$Br with an energy resolution of 1% and with a total of $10^3$ measured neutrino events. The energy released to the recoil electron would be about 400 keV (the reaction threshold is about 450 keV), so the average neutrino energy would be measured to an accuracy of about 0.1 keV. With the experimental parameters assumed, a calibrated $^{81}$Br detector could measure the central temperature of the sun to an accuracy of about 10%.

The requirements for a practical experiment may be achievable since solar neutrino detectors currently under development are designed to detect several thousand events per year, although not yet with the energy resolution required to measure $\Delta$. It might be possible to calibrate the solar results by studying an intense laboratory source of $^7$Be neutrinos with the same detector as used in the solar observations [7].

This work was supported by NSF grant #PHY92-45317.
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FIGURES

FIG. 1. The Energy Profile for the 862 keV line. The probability for the emission of a neutrino with energy $q_{\text{obs}}$ in the laboratory frame is shown as a function of $q_{\text{obs}} - q_{\text{lab}}$, where $q_{\text{lab}} = 861.84$ keV and $q_{\text{peak}} = 862.27$ keV. The line profile was computed by averaging the probability distribution at a fixed temperature over the Bahcall-Pinsonneault standard solar model with helium diffusion.