Simulation of the electromagnetic field in a rectangular multimode chamber of a 14GHz ion source

E A Orozco¹, A Estupiñán¹ and V D Dugar-Zhabon¹
¹ Universidad Industrial de Santander, Bucaramanga, Colombia
E-mail: eaorozco@uis.edu.co, alex.estupinan@saber.uis.edu.co

Abstract. Electron cyclotron ion sources producing multicharged ion beams of high intensity are widely used in scientific and industrial technologies. To multiply ionize heavy atoms, the electron component of a plasma confined in a magnetic trap is energized by microwaves in the cyclotron resonance conditions. The design of the existing sources does not permit determining the microwave field characteristics which are responsible for the electron component heating and consequently for the performance of the source. In this work we present a numerical study of the microwaves propagation from a magnetron generator to a multimode rectangular chamber through a rectangular waveguide and an aperture. The electromagnetic energy from a magnetron with frequency of 14GHz works as microwave source is coupled by a rectangular waveguide through an aperture. The evolution of the stationary microwave field in the chamber is calculated using the Yee’s method through a subrouting based on the numerical solution of Maxwell’s equations which uses the Yee’s method. The results show that the spatial distribution of the electromagnetic field is appropriate for the plasma heating, due to the uniformity of electric field maximums on the resonance surface. The precision of the numerical calculation is verified through the calculation of the divergences of both the electric and magnetic fields.

1. Introduction
In the year of 1972, Richard Geller [1], and independently Wiesemann [2], reported the fabrication of the first electron cyclotron resonance Ion source (ECRIS) to heat a plasma confined by a minimum-B magnetic trap [3]. The ECRIS are used for heating plasma, in the optical emission spectrum, particle accelerators, ion implantations, and ion engines among others [4,5]. The ECRIS has been subjected to various theoretical and experimental studies [6,7]. Since the study of the plasma as a many particles system, heated under ECR conditions in a minimum-B magnetic trap presents insuperable difficulties for a theoretical study, related to non-linearity of the problem and the lack of clarity in physical phenomena for analytical studies which are related with that occurs within the plasma, the system is studied through computer simulation. Such simulations relate the laws and physical principles with real experiments. Numerical simulations of the ECRIS plasma presents a very good alternative for theoretical analysis clarifying the details of the plasma collective as well the individual particle behaviour. [8,9]. In an ECRIS, the discharge chamber is in the cylindrical shape [10]; however, it could also be in the rectangular shape. For numerical studies of an ECRIS, such geometrical shape present advantages respect to the cylindrical one to define the boundary conditions of both, the electromagnetic fields and...
the particles forming the plasma. The last one is a very important aspect to consider in studies of the interaction plasma-wall.

It is evident that before starting a plasma heating simulation, the microwave field in the chamber can be calculated. We have not come across so far the works devoted to the study of process of the microwaves propagation from a generator to the chamber and the establishment of the multimode regime. The chamber is rectangular of $7.4cm \times 7.4cm \times 10cm$.

2. Theoretical formalism

A physical model scheme for the proposed ECRIS is shown in Figure 1. The ECR plasma is confined by the minimum-B magnetic trap that consists of two current coils-1 and the octupole permanent magnet system-2. The chamber-5 is feeded by microwave energy from a $14GHz$ magnetron (not shown) through a rectangular waveguide-4, coupled to the chamber by the window of $0.7 \times 1.4cm^2$ in size-3.

In order to model numerically the microwave field in the discharge chamber-5 and the microwave energy injection through the waveguide-4 (See Figure 1), in the simulation model the microwaves propagation occurs in the perfect electric boundary conditions (PEC) for both the discharge chamber and the waveguide. To avoid non physical reflections on microwave port, we use the Uniaxial Perfectly Matched Layer (UPML) method. The Maxwell equations expressed in a finite difference time domain form are solved numerically applying the Yee method [11].

In the UPML method, the Maxwell equations describing the evolution of the electric and magnetic fields of frequency $\omega$ in a finite medium are expressed in a phasor-like form as [12]:

$$ \nabla \times \hat{H} = j\omega\varepsilon S \hat{E} $$

and

$$ \nabla \times \hat{E} = -j\omega\mu S \hat{H} $$

Where:

$$ S = \begin{bmatrix} S_x^{-1} & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & S_z \end{bmatrix} \begin{bmatrix} S_y & 0 & 0 \\ 0 & S_z^{-1} & 0 \\ 0 & 0 & S_x \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_z & 0 \\ 0 & 0 & S_y \end{bmatrix} = \begin{bmatrix} S_y S_x S_z^{-1} & 0 & 0 \\ 0 & S_x S_y S_z^{-1} & 0 \\ 0 & 0 & S_x S_y S_z^{-1} \end{bmatrix} $$

Figure 1. Physical model scheme of the proposed ECRIS with a rectangular discharge chamber: 1-current coils, 2-octupole magnetic system, 3-window, 4-waveguide, 5-rectangular discharge chamber.
in which, 

\[ S_x = k_x + \frac{\sigma_x}{j\omega \varepsilon}; \quad S_y = k_y + \frac{\sigma_y}{j\omega \varepsilon}; \quad S_z = k_z + \frac{\sigma_z}{j\omega \varepsilon} \tag{4} \]

Where, \( k_x, k_y \) and \( k_z \) are dimensionless real constants; while \( \sigma_x, \sigma_y \) and \( \sigma_z \) are electric conductivities functions used to simulate the perfectly matched layers (at the microwave port in our case) and \( \varepsilon = \varepsilon_0 \).

From the equations (1)-(3) and applying the Fourier inverse transform, the next set of equations for the electromagnetic field in the time domain form is obtained:

\[ \partial_t H_j - \partial_j H_i = \partial_t (K_i D_k) + (\sigma_i / \epsilon_0) D_k \quad (i, j, k = 1, 2, 3) \tag{5} \]

and

\[ \partial_t E_j - \partial_j E_i = -\partial_t (K_i B_k) - (\sigma_i / \epsilon_0) B_k \tag{6} \]

Where \( i, j \) and \( k \) indicate cartesian indexes which permute cyclically.

\( D_i \) and \( H_i \) (\( i = 1, 2, 3 \)) are obtained in a similar way from the constitutive relations in the fasor-like form:

\[ D_x = \epsilon_0 \frac{S_z}{S_x} E_x, \quad D_y = \epsilon_0 \frac{S_x}{S_y} E_y, \quad D_z = \epsilon_0 \frac{S_y}{S_z} E_z \tag{7} \]

and

\[ B_x = \mu \frac{S_z}{S_x} H_x, \quad B_y = \mu \frac{S_x}{S_y} H_y, \quad B_z = \mu \frac{S_y}{S_z} H_z \tag{8} \]

The set of equations (5)-(8) is written in a finite difference scheme using the Yee method and solved in the traditional way (See the Figure 2) [11]. For the \( x \) component, it can be written:

\[ E_x^{n+1}(i+1/2, j, k) = \frac{1 - \sigma_x / \epsilon_0 K_x(i+1/2)}{1 + \sigma_x / \epsilon_0 K_x(i+1/2)} E_x^n(i+1/2, j, k) \]

\[ + \frac{K_x(i+1/2)}{\epsilon_0 K_x(i)} \left[ 1 - \frac{\sigma_x / \epsilon_0 K_x(i+1/2)}{2 \epsilon_0 K_x(i+1/2)} \right] D_z^{n+1}(i+1/2, j, k) \]

\[ - \frac{K_x(i+1/2)}{\epsilon_0 K_x(i)} \left[ 1 - \frac{\sigma_x / \epsilon_0 K_x(i+1/2)}{2 \epsilon_0 K_x(i+1/2)} \right] D_z^n(i+1/2, j, k) \tag{9} \]

and

\[ D_x^{n+1}(i+1/2, j, k) = \frac{1 - \sigma_x / \epsilon_0 K_y(i)}{1 + \sigma_x / \epsilon_0 K_y(i)} D_x^n(i+1/2, j, k) + \frac{\Delta t}{K_y(i)} \left[ 1 + \frac{\sigma_x / \epsilon_0 K_y(i+1/2)}{2 \epsilon_0 K_y(i+1/2)} \right] \]

\[ \left[ H_x^{n+1/2}(i+1/2, j+1/2, k) - H_x^{n+1/2}(i+1/2, j-1/2, k) \right] \]

\[ \frac{\Delta y}{\Delta z} \]

\[ - \frac{D_x^{n+1/2}(i+1/2, j, k+1/2) - D_x^{n+1/2}(i+1/2, j, k-1/2)}{\Delta y} \tag{10} \]
Here $n$ denote the index for the time $t_n = n\Delta t$. Similarly, equations for the evolution of the magnetic field components are obtained. In our simulations we choose $k_x = k_y = k_z = 1$ and $\sigma_x = \sigma_y = \sigma_z = 0$ in all space except of the microwave port where $\sigma_z = (z/d)^3\sigma_z^{\text{max}}$. $d = 10\Delta z$ is the width of the perfectly matched layer (PML), being $\Delta z$ the spatial step in $z$ direction, where $z$ is the distance from the border towards the interior of the PML and $\sigma_z^{\text{max}} = 0.8(m + 1)/(\Delta z\sqrt{\mu_0/\varepsilon_0})$ with $m = 3$ when 10 cells are used in the PML zone, as in the present case.

**Figure 2.** (a) Flowchart used for simulate the electromagnetic field evolution inside the chamber. (b) Elemental Yee’s cell showing the mutual disposition of the different electric and magnetic fields components.

### 3. Results and discussion

The numerical simulation of the microwave field inside the chamber is made under the above-mentioned physical parameters. To excite a microwave field of $1kV/cm$ tension, an input power of $2.9kW$ is injected into the chamber. The simulations are fulfilled on a rectangular 3D mesh at the spatial steps $\Delta x = \Delta y = 0.07cm$, $\Delta z = 0.2cm$ at a time step $\Delta t = 2.07ps$, that are chosen in accordance with the Courant stability condition [12].

Figures 3 and 4 shows the electric field strength evolution, for the instants $t = 3500 + 1/8$, $3500 + 3/8$, $3500 + 5/8$ and $3500 + 7/8$ $hf$ cycles, at the cross section $z = L_c/2$ (being $L_c$ the
chamber length) and at the plane $x = 0$, respectively. Considering the central location of the waveguide (See Figure 1), as expected the obtained pattern shows similar behaviors in each quadrant (See Figures 3 and 4). Besides, we can see a distribution of maxima in a ring-shaped region, with an inner radius of 1.6 cm and an outer radius of 2.6 cm, approximately; in which the ECR surface can be located for an effective heating of the plasma.

The precision obtained by numerical calculation for electromagnetics fields inside of the chamber was verified through the calculation of $\nabla \cdot \vec{B} = 0$ and $\nabla \cdot \vec{E} = 0$, in a finite difference scheme, which was found no worse than $10^{-7}$.

Figure 3. The electric field strength in the cross section $z = L_c/2$ for different times: (a) $t = 3500 + 1/8 \, h_f \, \text{Cycles}$, (b) $t = 3500 + 3/8 \, h_f \, \text{Cycles}$, (c) $t = 3500 + 5/8 \, h_f \, \text{Cycles}$ and (d) $t = 3500 + 7/8 \, h_f \, \text{Cycles}$. 
Figure 4. The electric field strength in the plane $x = 0$ for different times: (a) $t = 3500 + 1/8\, hf$ Cycles, (b) $t = 3500 + 3/8\, hf$ Cycles, (c) $t = 3500 + 5/8\, hf$ Cycles and (d) $t = 3500 + 7/8\, hf$ Cycles.

4. Conclusions
A multimode microwave field distribution inside the rectangular chamber was obtained. The electric field maxima exhibits the microwave field distribution which can be fitted to the ECR surface for an efficient plasma heating. In the near future, the numerical simulation of plasma heating in this system will will be fulfilled through conservative electromagnetic Particle-In-Cell (PIC) method.

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