Ratchet Effects in Active Matter Systems

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Abstract
Ratchet effects can arise for single or collectively interacting Brownian particles on an asymmetric substrate when a net dc transport is produced by an externally applied ac driving force or by periodically flashing the substrate. Recently, a new class of active ratchet systems has been realized through the use of active matter, which are self-propelled units that can be biological or non-biological in nature. When active materials such as swimming bacteria interact with an asymmetric substrate, a net dc directed motion can arise even without external driving, opening a wealth of possibilities such as sorting, cargo transport, or micromachine construction. We review the current status of active matter ratchets for swimming bacteria, cells, active colloids, and swarming models, focusing on the role of particle-substrate interactions. We describe ratchet reversals produced by collective effects and the use of active ratchets to transport passive particles. We discuss future directions including deformable substrates or particles, the role of different swimming modes, varied particle-particle interactions, and non-dissipative effects.
1. INTRODUCTION

When a particle is placed in a symmetric periodic potential and subjected to an ac drive, in general the particle simply rocks back and forth and undergoes no net dc motion. If an external dc drive $F_{dc}$ is imposed on a particle in a substrate, the particle has no net dc motion and remains pinned as long as $F_{dc} < F_c$, where $F_c$ is the critical depinning force for motion over the substrate. Once $F_{dc} > F_c$, the particle enters a sliding state (1). When the motion of the particle is overdamped, its dynamics can be described by the following equation of motion:

$$\eta \frac{dR}{dt} = F^s + F_n + F_{ext} + F_T$$  \hspace{1cm} (1)

where $\eta$ is the damping constant, $F^s$ is the force from the substrate, $F_n$ is the interaction force from the other particles, $F_{ext}$ is an externally applied driving force, and $F_T$ is a stochastic force representing thermal fluctuations.

The situation changes if the substrate is asymmetric. An example of a one-dimensional (1D) asymmetric substrate potential with periodicity $a$,

$$U(x) = U_0[\sin(2\pi x/a) + 0.25 \sin(4\pi x/a)]$$  \hspace{1cm} (2)

is illustrated in Fig. 1(a). In the absence of thermal fluctuations, the maximum pinning force exerted by the substrate on the particle for motion in the positive $x$-direction is $3\pi U_0/a$, while the maximum pinning force for motion in the negative $x$-direction has a smaller value of $3\pi U_0/2a$. As a result, if an externally applied ac driving force of the form $F_{ext} = F_{AC} \cos(\omega t)x$ is introduced, then the particle remains trapped and oscillates within a single potential minimum for $F_{AC} < 3\pi U_0/2a$. In contrast, when $3\pi U_0/a \leq F_{AC} < 3\pi U_0/2a$, the particle can overcome the substrate potential barrier for motion in the negative $x$-direction but cannot overcome the barrier for motion in the positive $x$-direction, so that during each ac cycle the particle undergoes a net dc transport in the negative $x$-direction. Such motion is termed a ratchet effect. The amount of dc translation that occurs per cycle depends on the amplitude and frequency of the ac drive. For fixed ac frequency, the efficiency of the ratchet is generally nonmonotonic, increasing with increasing $F_{AC}$ for $F_{AC} < 3\pi U_0/2a$ and then decreasing with increasing $F_{AC}$ at higher $F_{AC}$ (2).
When the mechanism responsible for producing a ratchet effect is an ac driving force, the system is known as a rocking ratchet. An alternative mechanism operates in a flashing ratchet, where thermally diffusing particles are placed on a substrate that is periodically flashed on and off, and where a maximum ratchet efficiency emerges as a function of flashing frequency. In the single particle limit for overdamped particles, the ratchet effect causes the particle to translate along the easy direction of the substrate regardless of the nature of the ratchet. When collective particle-particle interactions are introduced, however, it is possible to observe reversals of the ratchet effect, where the motion is along the hard direction of the substrate asymmetry. These reversals are often produced by commensurability effects between the interacting particles and the substrate, where emergent quasiparticle objects such as kinks or antikinks dominate the transport and behave as if they are moving through an inverted substrate potential. Examples of higher-dimensional asymmetric substrates on which ratchet effects can occur appear in Fig. 1(b), which shows a quasi-1D asymmetric potential with no structure transverse to the asymmetry direction, and in Fig. 1(c), which shows a linear array of V-shaped barriers. The asymmetry necessary for generating ratchet effects can also be introduced by placing individual symmetric pinning sites in an array that has a density gradient along one direction. In geometric or drift ratchets, which require a minimum of two spatial dimensions, particles driven over an asymmetric substrate by a dc drive undergo a net dc drift in the direction transverse to the applied dc drive. In transverse ratchets, which are a variation...
of drift ratchets, a net dc ratchet motion arises in the direction transverse to an applied ac driving force \(12, 13, 14\). Additional ratchet effects including ratchet reversals can arise when there are non-dissipative terms in the equations of particle motion, such as inertia \(15\) or a Magnus term \(16\).

Studies of ratchet effects have been performed for colloids interacting with asymmetric flashing substrates \(17\), vortices in type-II superconductors interacting with asymmetric periodic 1D \(5, 6\) and 2D substrates \(4, 12, 13, 14\), granular matter on sawtooth substrates \(18, 19\), cold atoms \(20, 21\), electron systems \(22, 23\), and domain walls moving over asymmetric substrates \(24, 25\). These systems all require application of some form of external driving in order to create the nonequilibrium conditions necessary for a ratchet effect to occur. In a different class of nonequilibrium systems, the dynamics is governed by internal driving or self-propulsion \(26, 27, 28, 29\). These active matter systems include flocking and swarming particles, pedestrian and traffic flow, swimming bacteria, crawling cells, and actin filaments. Active matter has been attracting growing attention due to the increasing availability of non-biological active systems such as artificial swimmers, self-driven colloids, robot swarms, and autonomous agents. Such systems exhibit a variety of phenomena including pattern formation \(30\), self-clustering \(31, 32, 33, 34\), and nonequilibrium phase transitions \(35\), and represent a rich and very active field of research.

2. RATCHET EFFECTS FOR SWIMMERS IN FUNNEL ARRAYS

One of the first demonstrations of an active matter ratchet was achieved by P. Galajda et al. \(7\) for a system of \textit{E. coli} swimming in a sample divided into two chambers by an array of funnel-shaped barriers similar to those illustrated in Fig. 1(c). The entire sample is 400\(\mu m\) on a side, while the funnel barriers are each 27\(\mu m\) long, form a 60° angle at their tips, and have a minimum separation of 3.8\(\mu m\) between adjacent funnels. If Brownian particles are placed at a uniform density in such a geometry, the density remains equal in both chambers of the sample. In contrast, when an equal number of swimming bacteria are introduced into each chamber, after roughly an hour there is a buildup of bacteria in the chamber that is on the easy-flow side of the funnel barriers, as illustrated in Fig. 2(a, b). In the figure, the easy-flow direction is to the right. In Fig. 2(c), the ratio \(A\) of the bacterial concentration \(\rho\) in the right and left chambers, \(A(t) = \rho_R(t)/\rho_L(t)\), has an initial value of \(A = 1.0\) at time \(t = 0\) and saturates to a value of \(A = 3.0\) at longer times, showing that a ratchet effect is occurring. When the same experiment is performed with chambers separated by symmetric holes instead of asymmetric funnels, then \(A(t) = 1.0\) at all times, indicating that the asymmetry of the funnel is a crucial ingredient for the occurrence of rectification. When a mixture of swimming and non-swimming bacteria is placed in the sample, the swimming bacteria are rectified while the non-swimming bacteria are not, showing that swimming is also important in producing the ratchet effect. Due to the complexity of bacterial systems, it was not obvious whether this initial observation of an active matter ratchet effect could be understood as being produced solely by the dynamics of the self-propulsion combined with bacteria-wall interactions, or whether more complex bacterial behaviors play a role in the ratcheting motion.

Wan et al. \(36\) subsequently studied a simple model of pointlike run-and-tumble particles moving in a 2D container with the same funnel barrier geometry used in the experiments of Ref. \(7\), as illustrated in Fig. 3. In this model, particle \(i\) obeys the following overdamped
Figure 2

(a,b) Experimental images of the concentration of bacteria, indicated by fluorescence, in two chambers separated by a series of funnel barriers resembling those illustrated in Fig. 1(c). The easy-flow direction through the barriers is toward the right chamber. (a) The initial uniform distribution of bacteria at time $t = 0$. (b) The distribution at $t = 80$ minutes showing that the bacteria concentration is significantly higher in the right chamber. (c) The ratio $A = \rho_R / \rho_L$ of the bacteria density $\rho$ in the right and left chambers versus time, showing that a rectification effect is occurring. Reprinted with permission from P. Galajda, J. Keymer, P. Chaikin, and R. Austin, J. Bacteriol. 189, 8704 (2007). Copyright 2007 by the American Society for Microbiology.

The equation of motion:

$$\eta d\mathbf{R}_i/dt = F_m^i(t) + F_T^i + F_B^i + F_S^i$$

(3)

where $\eta$ is the damping coefficient and $\mathbf{R}_i$ is the location of particle $i$. Instead of the external driving force used in Eqn. 1, Eqn. 3 contains a motor force $F_m^i(t)$ representing self-propulsion in a randomly-chosen direction that remains constant during a running time $\tau$. Every $\tau$ simulation time steps, particle $i$ undergoes an instantaneous tumbling event during which the motor force is reoriented into a new random direction. The particle then runs in this new direction during the next time period $\tau$ before tumbling again. The running speed $v$ is held constant. For an isolated particle in the absence of any barriers, the motor force produces a motion that is ballistic at short times and diffusive at very long times (29). If $F_m^i(t) = 0$ so that the particles experience only thermal fluctuations but no motor force, then no rectification occurs.

$F_S^i$ represents particle-particle interactions, while $F_B^i$ is the particle-barrier interaction term. The barriers and confining walls are modeled as exerting a short-range steric repulsion on particle $i$, and a particle interacting with a barrier or wall runs along the wall at a speed determined by the component of $F_m^i$ that is parallel to the wall until the particle reaches the end of the wall or undergoes a tumbling event that rotates its running direction away from the wall.

A simulated system in which particle-particle interactions and thermal fluctuations are both neglected, so that the particles experience only a motor force propulsion and interactions with the barriers and walls, is illustrated in Fig. 3(a,b). The sample is of size $L = 99a_0$ on a side, where $a_0$ is the unit of distance in the simulation, and contains 12 funnel barriers that each have a tip angle of $60^\circ$ and an arm length of $l_f = 5a_0$. The running length is $l_b = 40a_0$, so a particle can move much further than the length of a funnel arm during a single run time. The particle density is initially uniform, as shown in Fig. 3(a), but after a time $100\tau$, Fig. 3(b) shows that the particle density has increased in the upper chamber. The ratio of the particle density $\rho^{(1)}$ in the upper chamber and $\rho^{(2)}$ in the lower chamber to the initial density is significantly higher in the right chamber.
Figure 3
(a,b) Simulation images of noninteracting run-and-tumble particles moving between two chambers separated by a line of funnel-shaped barriers. The easy-flow direction of motion through the funnels is toward the top chamber. A particle striking a funnel or chamber wall runs along the wall until reaching the end of the wall or undergoing a tumbling event. (a) Initial uniformly dense condition for particles with a run length of $l_b = 40$ between tumbles. (b) After 100 tumbling events, the particle density in the upper chamber has increased. (c) Ratios $\rho^{(1)}/\rho_b$ (upper red line) and $\rho^{(2)}/\rho_b$ (lower blue line) of the particle density $\rho^{(1)}$ in the upper chamber and $\rho^{(2)}$ in the lower chamber to the initial particle density $\rho_b$ as a function of time. Solid lines are for a sample with a run length of $l_b = 180$ where there is a large rectification effect, and dashed lines are for a sample with a run length of $l_b = 0.01$ where the rectification effect is negligible. Adapted from M. B. Wan, C. J. Olson Reichhardt, Z. Nussinov, and C. Reichhardt, Phys. Rev. Lett. 101, 018102 (2008). Copyright 2008 by the American Physical Society.

The inset of Fig. 4 illustrates the ratchet mechanism. The left inset shows that under the wall-following rule of motion, when $l_b$ is sufficiently large, a particle approaching the funnel barrier from the top chamber becomes trapped at the funnel tip until a tumbling event allows it to escape back into the top chamber, while a particle approaching the funnel barrier from below is shunted along the barrier wall into the top chamber. In the right inset, a particle with a very short run time spends very little time interacting with the wall, similar to a purely Brownian particle, and is unable to sample the barrier asymmetry over the initial particle density $\rho_b$ is plotted as a function of simulation time steps in Fig. 3(c). The solid lines show that in a system with $l_b = 180$, the density in the upper chamber increases over time with a simultaneous decrease in the lower chamber density until the sample reaches a steady state. The dashed lines indicate that when the running length is very short, $l_b = 0.01$, the density in both chambers remains nearly constant at a value equal to the initial particle density. When $l_b$ is very small, the particle motion is Brownian-like, so no rectification can occur. The steady-state ratio $r = \rho^{(1)}/\rho^{(2)}$ is plotted as a function of $l_b$ in Fig. 4. For small $l_b$, $r = 1$, while for large $l_b$, $r$ reaches a value of nearly $r = 5.0$. The inset of Fig. 4 illustrates the ratchet mechanism. The left inset shows that under the wall-following rule of motion, when $l_b$ is sufficiently large, a particle approaching the funnel barrier from the top chamber becomes trapped at the funnel tip until a tumbling event allows it to escape back into the top chamber, while a particle approaching the funnel barrier from below is shunted along the barrier wall into the top chamber. In the right inset, a particle with a very short run time spends very little time interacting with the wall, similar to a purely Brownian particle, and is unable to sample the barrier asymmetry over.
a time scale long enough to bias its motion toward the upper chamber. The two ingredients that are required to produce a ratchet effect are the running of particles along the walls and the breaking of detailed balance by the motor force. In the same simulation study, the rectification effect is reduced when a finite temperature is added since this increases the chance that a particle will prematurely move away from a wall. The ratchet effect is also reduced by the inclusion of steric particle-particle interactions, since particle collisions effectively reduce the running length while a buildup of particles in the funnel tip reduces the ability of the funnels to trap particles approaching from the upper chamber.

Figure 4
The ratio \( r = \rho^{(1)} / \rho^{(2)} \) of the particle density in the upper chamber to that in the lower chamber for the system in Fig. 3 at steady state vs run length \( l_b \), showing that for small run lengths \( r = 1.0 \) indicating no rectification, while for large \( l_b \) the ratio \( r \) reaches a value of nearly \( r = 5.0 \). Inset: Schematic illustration of the rectification mechanism. On the left, an individual particle approaching the funnel barrier from above becomes trapped at the funnel tip, while a particle approaching from below is shunted by the funnel wall into the upper chamber. On the right, a Brownian particle spends very little time in contact with the funnel walls. Adapted from M. B. Wan, C. J. Olson Reichhardt, Z. Nussinov, and C. Reichhardt, Phys. Rev. Lett. 101, 018102 (2008). Copyright 2008 by the American Physical Society.

Figure 5(a) shows a snapshot from a simulation of run-and-tumble particles with steric particle-particle interactions moving between two chambers separated by funnel barriers. When the run length is sufficiently large \( l_b \), the inclusion of steric interactions produces two additional features: a build up of particles along the walls, and trapping of particles in the funnel tips. Both of these effects were observed in the experiments of Ref. (7). One question is whether the accumulation of particles along the walls in the experiments is produced by hydrodynamic effects or whether it arises purely due to the self-propulsion of the bacteria. Using numerical and analytical methods, Tailleur and Cates (38) investigated run-and-tumble particles confined by a spherical trap and found that the dynamics alone produce particle accumulation on the walls. Other studies of confined active matter where the motor force rotates diffusively also show a build up of particle density along the walls,
and indicate that this effect is enhanced near corners (39, 40). The tendency of active matter particles to accumulate in corners is one of the reasons the funnel barriers can produce a ratchet effect, as the increased trapping occurs on only one side of the funnels due to their curvature. The work in Ref. (41) also shows how the nature of the particle-wall interactions leads to rectification. Rectification by the funnel geometry is robust against changes in the running length distribution, as is observable by comparing the results of Wan et al. (36), who used only a single running length in each simulation, and of Tailleur and Cates (38), who considered a Poisson distribution of running lengths. Changes in the nature of the particle-wall interactions, however, can destroy the rectification. Four possible interaction rules are shown schematically in Fig. 5(b-e). Under Rule I, the motor force alignment is unchanged by contact with the wall and the particle moves according to the component of the motor force vector that is parallel to the wall. In Rule II, the motor force is realigned to be parallel with the wall upon contact. For Rule III, the particle is reflected by the wall, while in Rule IV, the particle collides elastically with the wall. In Ref. (35), when the motor force realigning Rule II is employed, efficient rectification of the particles occurs; however, if the elastic collisions of Rule IV are substituted, the rectification is lost, as shown in Fig. 6. An examination of all four wall rules in Ref. (37) shows that the running of the particle along the wall, as in Rules I and II, is essential for producing a ratchet effect. When the particle does not remain in contact with the wall, as in Rules III and IV, the rectification is lost.

Figure 5
(a) Simulation image of the buildup of sterically interacting run-and-tumble particles along the walls and in the funnel barrier tips for a system with a run length of \( l_b = 120 \). (b-e) Different possible particle-wall interaction rules. (b) Rule I: The particle maintains its original running direction and moves along the wall with the component of its velocity that is parallel to the wall. (c) Rule II: The particle realigns its running direction to be parallel with the wall. (d) Rule III: The particle reflects from the wall. (e) Rule IV: The particle undergoes an elastic collision with the wall. Ratchet effects occur for Rules I and II, with the strongest ratcheting for Rule II, but Rules III and IV produce no ratcheting. Adapted from C.J. Olson Reichhardt, J. Drocco, T. Mai, M.B. Wan, and C. Reichhardt, Proc. SPIE 8097, 80970A (2011).

Followup experiments by Galajda et al. (42) using self-propelled large scale swimming bathtub toys produced a rectification effect similar to that observed for the bacteria, while experiments by Hulme et al. (43) demonstrate how the ratchet effect can be used to fractionate motile \( E. coli \) using arrays of asymmetric chambers of the type illustrated in Fig. 7(a). The arrows in panels 1 to 5 of Fig. 7(b) show how bacteria moving in the positive \( x \)-direction are guided through the chamber array, while in panels 6 to 10, bacteria moving
in the negative $x$ direction are blocked by the chamber geometry. Variations on similar channel geometries have also been shown to produce guided motion of bacteria \cite{Kim05}. Kim et al. use similar microfabricated ratchet channels with asymmetry to concentrate \textit{E. coli} at particular locations \cite{Kim05}.

Numerous other shapes can produce rectification of bacteria, such as the multiple rows of U-shaped funnel barriers illustrated in Fig. 8(a) that produce rectification of particles with a motor force that rotates diffusively \cite{Berdakin11}. In Fig. 8(b), a plot of the rectification ratio $A_r$, given by the ratio of the density in the right chamber to that in the left chamber, versus the motor force $F_A$ for one, two, and three rows of barriers shows that $A_r$ increases linearly with $F_A$, and that there is a strong enhancement of the ratchet effect with the addition of more layers of funnels. In the same study, an examination of barriers made from open and closed triangles, U shapes, and half boxes indicates that barriers with an open shape always produce a larger rectification than barriers with closed shapes due to the trapping of particles within the tips of the open barriers.

Theoretical and numerical studies show that rectification effects are a general phenomenon that occurs for self-driven particles in the presence of asymmetric piecewise periodic potentials \cite{Berdakin11}. In other numerical studies, the directed transport that occurs for self-propelled particles interacting with arrays of nonsymmetric convex obstacles without cavities was shown to result from the fact that the particles tend to attach to solid surfaces \cite{Berdakin11}.

### 2.1. Sorting in Funnel Arrays

In numerical studies of the motion of active particles with a diffusively rotating motor force through funnel arrays, I. Berdakin et al. examined the effect of changing the swimming properties by varying the distribution of run lengths, the magnitude of the rotational diffusion, and the preservation of run orientation memory after a tumbling event \cite{Berdakin11}. They observe that there are optimal swimming properties that maximize the rectification as a
The motion of bacteria through an array of asymmetric chambers. (A) The geometry of a single chamber, indicating the easy flow direction. (B) Experimentally obtained trajectories of bacteria. Panels 1 through 5 show that bacteria can easily pass through the chambers for motion in the forward direction, while panels 6 through 10 show that motion in the reverse direction is blocked due to the chamber shape. Using this geometry it is possible to fractionate swimming bacteria that have different sizes or mobilities. Reproduced from E. Hulme, W.R. DiLuzio, S.S. Shevkoplyas, L. Turner, M. Mayer, H.C. Berg, and G.M. Whitesides, Lab Chip. 8, 1888 (2008), with permission of The Royal Society of Chemistry. http://dx.doi.org/10.1039/b809892a

function of the geometry of the funnels. An illustration of the simulated funnel geometry appears in Fig. 9(a) along with colored dots indicating the position of three different E. coli mutants modeled on the basis of characterizations performed by Berg and Brown [50]. Red particles, or species s1, represent CHeC497 with a mean run duration of 6.3 seconds and a speed of 20 \( \mu \text{m/s} \); blue particles, or species s3, represent the wild type AW405 with a mean run duration of 0.86 seconds and a speed of 14.2 \( \mu \text{m/s} \); and green particles, or species s4, represent Un602 which is the slowest with a mean run duration of 0.42 seconds and a speed of 14.2 \( \mu \text{m/s} \). The particles are initially placed along the leftmost wall, and after 12 minutes the red particles, which have the longest run time and the highest speed, have moved the furthest to the right, followed by the blue particles, which move less far to the right, and finally by the green particles, which have the least motion to the right. Figure 9(b) shows the number of bacteria as a function of time in the leftmost chamber (decreasing curves) and rightmost chamber (increasing curves), indicating that the different species become spatially separated over time. In follow-up studies, the same group observed that the use of
multiple arrays of funnels makes it possible to sort species that have only a small variation in a single swimming parameter (51).

### 2.2. Eukaryotic Cells and Swimming Animals in Funnel Arrays

The motion of biological cells through funnel geometries has been explored in several experiments. Kantner et al. (52) considered swimming *Chlamydomonas* in both simulations and experiments using four chambers separated by funnel barriers, and measured the rectification ratio \( R = \langle N_4 \rangle / \langle N_1 \rangle \), where \( \langle N_4 \rangle \) is the steady state number of cells in chamber 4 and \( \langle N_1 \rangle \) is the steady state number of cells in chamber 1. For funnels with a tip angle of 35°, \( R = 4.0 \), indicating strong rectification. In Ref. (53), experiments on human sperm cells in quasi-2D microchambers containing funnel barriers show that rectification occurs when the cells become trapped in the funnel tips. The same study showed that since the sperm are attracted to the boundaries, the ratchet effect can be optimized by using U-shaped barriers. Experiments with *C. elegans* moving through funnel barriers showed that the wild type, which moves along walls that it contacts, can be rectified by the barriers (54). In
Figure 9

(a) Snapshots of simulated bacteria positions at different times for a mixture of three E. coli mutants moving through a series of funnel barriers. Red: CheC497 or s1 (fastest), blue: AW405 or s3, green: Ucn602 or s4 (slowest). The bacteria begin along the leftmost wall at time $t = 0$. (b) The number of bacteria vs time in the leftmost chamber (solid symbols) and the rightmost chamber (open symbols), showing that the different bacteria species can be fractionated using the funnel arrays. Reprinted with permission from I. Berdakin, Y. Jeyaram, V. V. Moshchalkov, L. Venlen, S. Dierckx, S. J. Vanderleyden, A. V. Silhanek, C. A. Condat, and V. I. Marconi, Phys. Rev. E 87, 052702 (2013). Copyright 2013 by the American Physical Society.

contrast, mutant strains that lack touch sensory neurons in their body reverse motion upon contacting a wall instead of following the wall, and these mutants are not rectified by the barriers.

2.3. Other Active Ratchet Geometries

A variety of other ratchet geometries for active particles have been studied, including 2D corrugated asymmetric substrates (55) and 2D geometries where entropic effects are important (56). Other works focus on the effect of changing the active particle shape to swimming ellipses (57), or on including an additional external drift force to the active particles to produce an active drift ratchet (58). Kulic et al. proposed that ratchet effects can occur in a
variety of plants where the asymmetry of the roots, seeds or leaves is important for transport in the natural world where fluctuations may be present (59). Finally, in the case of symmetric substrates, it has been proposed that by periodically modulating the velocity of the active particles and then phase-shifting this velocity with respect to the substrate periodicity, enough symmetries are broken to permit a ratchet effect to occur (60).

2.4. Future Directions

In ratchets produced using funnel barriers, it would be interesting to examine additional modes of motion of the active particles such as particles that avoid walls, particles that reverse their motion if the density is too high, or particles that change swimming strategies as a function of local density or of time. Many of the active matter ratchet studies performed so far employ infinitely repulsive barriers or walls; however, many previous studies of non-active matter ratchet effects produced by external driving were performed for substrates containing pinning sites with a finite strength, so that a particle experiencing a sufficiently large driving force can pass through the pinning potential or barrier. It would be interesting to examine active matter particles moving through asymmetric finite-strength pinning arrays rather than obstacle arrays. Pinning arrays for colloidal particles can be readily created experimentally using optical means, so it should be possible to create optically generated pinning arrays for active colloidal particles or swimming bacteria. Additionally, optical traps introduce the possibility of creating flashing substrates, which could significantly enhance active matter ratchet effects. It was recently proposed that by using a light intensity pattern with a funnel or chevron shape to manipulate a dense assembly of sterically interacting active particles that are attracted by light, the patterned regions would become filled with trapped active particles that would then serve as an asymmetric substrate that can produce ratcheting motion of the remaining untrapped active particles (61). There has also been work examining how asymmetry in the curvature of the walls can produce different pressures on each side of the wall, producing a ratchet effect (62). This suggests that curvature is another route to explore in producing different types of active ratchet devices. Other possible systems to explore include crawling insects or animals. Studies that reveal boundary following behaviors in animals (63) suggest that asymmetry in the boundaries could be used to produce ratchet effects. Another avenue of study is the introduction of feedback effects to a ratchet system, which could be achieved through optical methods. Recent studies of active matter using feedback controls have produced a variety of behavior for very simple feedback rules (64, 65).

3. COLLECTIVE ACTIVE MATTER EFFECTS

3.1. Active Ratchet Reversals

In rocking and flashing ratchet systems, a rich variety of collective effects including ratchet reversals arise when particle-particle interactions are included (2, 3, 4, 5). For run-and-tumble active particles moving through funnel barriers, Wan et al. observed that the addition of steric particle-particle interactions produced only a monotonic reduction in the rectification effect (36). Drocco et al. (66) placed particles obeying a Vicsek model, modified to include particle-barrier interactions and steric particle-particle interactions with an exclusion radius $r_e$, in a funnel barrier geometry. For small $r_e$, the particles ratchet in the easy flow direction, and the magnitude of the ratchet effect depends on the noise term $\eta$. 

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in the Vicsek model. For small $\eta$ the system forms flocks, while for large $\eta$ the system is in a liquidlike state. In the funnel barrier system, the ratchet effect is strongly reduced or completely lost for large $\eta$, while for small $\eta$ where flocking occurs, a pronounced ratchet effect appears with density ratios in the two chambers approaching $r = 5.0$. Figure 10(b,c) highlights the enhancement of the ratchet effect due to the collective rectification of groups of active particles. In Fig. 10(b), a flock approaches the funnel barriers from below. The flock becomes compressed and moves through the barrier array as a unit, as illustrated in Fig. 10(c). A reversal of the ratchet effect, in which the particles collect in the bottom chamber in the hard flow direction of the funnel barriers, can occur when flocks are present. Figure 10(a) shows a plot of the particle density in the top chamber $\rho_{\text{top}}$ versus the particle exclusion radius $r_e$ for a system in which $\eta = 1.5$, the flocking radius is 1.0, the starting uniform density is $\rho = 0.4$, and the closest spacing between adjacent funnel barriers is $l_0 = 0.6$. For $0.0 < r_e < 0.1$, rectification occurs in the easy flow direction of the funnel barriers as illustrated in Fig. 10(b,c), but there is a reversal of the rectification into the hard flow direction for $0.1 \leq r_e < 0.3$. For $r_e \geq 0.3$, the diameter of the particles is larger than the closest spacing between funnels, so it is no longer possible for particles to pass between the upper and lower chambers, and the rectification effect is lost. The ratchet reversal is produced by a jamming effect that occurs during the compression of a flock of particles as it approaches the opening between adjacent funnel barriers from below. The compression causes the particles at the leading edge of the flock to form a rigid close-packed structure that is wider than the interfunnel spacing $l_0$, and makes it impossible for the flock to cross the line of funnel barriers. In contrast, when a flock approaches the funnel barriers from above, the barriers cleave the flock into smaller clusters, and during this process one or two particles can escape from the flock and pass between the funnel barriers to enter the lower chamber. This reversed motion process always occurs but cannot compete with the forward motion of flocks of particles moving into the top chamber for small $r_e$. When the forward motion is inhibited by jamming at larger $r_e$, the reversed motion dominates and a reversed ratchet effect appears. The reverse ratchet effect operates much more slowly than the forward ratchet, so a much longer time is required for the system to reach a steady state in the reverse ratchet regime than in the forward ratchet regime. If two species of particles with different exclusion radii of $r_e = 0.055$ (small) and $r_e = 0.22$ (large) are placed in the container, flocks of the different species ratchet in opposite directions and can be spatially separated, as shown in Fig. 11.

Ratchet reversals have also been observed for eukaryotic cells moving on asymmetric substrates. Mahmud et al. (67) examined cell motility on micropatterned ratchet surfaces. They considered cancerous B16F1 cells, which are a mouse epithelial-like melanoma cell, non-cancerous Rat2 fibroblast cells, and cancerous MDA-MB-231 human mammary gland cells. For the B16F1 and Rat2 cells, directed motion occurs along the easy flow direction of the funnel array. The addition of a protruding spike shape to the funnel shaped chambers does not affect the Rat2 cells, which still ratchet in the easy direction, but causes the B16F1 and MDA-MB-231 cells to perform a reversed ratchet motion in the hard flow direction. The differences in the direction of the ratchet motion for the different cells were attributed to the different crawling mechanisms of the cells. Rat2 cells use long protrusions that can grab the funnel edges or the ends of the spike shapes, while the other cells prefer to spread out in the open spaces as they move. Mahmud et al. also found that it is possible to use arrays of the spike shaped funnels to separate a mixture of different cells over time, and they propose the creation of a bidirectional ratchet, opening the possibility of creating cell traps.
Figure 10
Simulations of sterically interacting flocking particles in a funnel barrier array. (a) Steady state density $\rho_{\text{top}}$ of particles in the upper chamber vs the particle exclusion radius $r_e$. The dashed line indicates the uniform starting density with $\rho_{\text{top}} = \rho_{\text{bottom}} = 0.4$. The ratchet effect in the easy flow direction is maximized for small $r_e$. A reversal in the ratchet direction occurs for $0.1 < r_e < 0.3$, and the ratchet effect is lost for $r_e > 0.3$ when the particles become too large to fit between adjacent funnels. (b,c) Images of a small portion of the sample illustrating the forward rectification mechanism. (b) A flock approaches the funnel barriers from below. (c) The flock elongates and passes through the barrier array as a unit. Adapted from J.A. Drocco, C.J. Olson Reichhardt, and C. Reichhardt, Phys. Rev. E 85, 056102 (2012). Copyright 2012 by the American Physical Society.

Figure 11
Images from simulations of sterically interacting flocking particles with two different exclusion radii, $r_e = 0.055$ (small, blue) and $r_e = 0.22$ (large, red). (a) The initial configuration with uniform density in both top and bottom chambers, which are separated by a line of funnel barriers. (b) A representative steady state configuration at later time. The larger red particles undergo reversed rectification and collect in the bottom chamber, while the smaller blue particles undergo forward rectification and collect in the top chamber.
and ratchets to achieve various types of medical treatments. Other studies have shown that unidirectional cell motion can be achieved using asymmetric nanotopography \cite{68}.

3.2. Active and Passive Ratchet Mixtures

Self-propelled Janus colloids, whose direction of motion slowly diffuses rotationally, represent another type of experimental artificial active matter system. Gosh et al. \cite{69} performed simulations of active Janus particles in an asymmetric channel, and observed that rectification occurs in the easy flow direction of the channel and that the magnitude of the ratchet effect increases with increasing correlation time of the particle running direction, eventually saturating for large correlation times. They also considered a mixture of $N_m$ active particles and $N_p$ non-active particles in the same geometry, with the particle-particle interactions modeled as a repulsive harmonic force, and found that the active particles can induce a net rectification of the non-active particles. A particularly interesting effect is that the rectification efficiency for the non-active particles is non-monotonic as a function of particle density. For one choice of parameters, the maximum non-active particle ratchet efficiency occurs for a ratio of $N_p/N_m = 12$, indicating that even a very small fraction of active particles can efficiently rectify the passive particles. In the dense limit of $\phi = 1.0$ close to the jamming regime, Ghosh et al. find a small reversal of the ratchet effect.

In experiments with passive colloids immersed in a bath of swimming bacteria, Koumakis et al. showed that introducing an asymmetric substrate that causes the bacteria to undergo rectification also produces transport of the passive colloids into or out of enclosed regions \cite{70}. Figure 12(a) shows the time evolution of the number of passive colloids in each chamber of a sample containing three layers of ratchets for a geometry that concentrates the colloids in the central chamber, while Fig. 12(b) shows the same quantity for a geometry that ejects the colloids from the box. Koumakis et al. also found that curved ratchet geometries were not as efficient as flat boundaries in their device since the colloids would follow curved trajectories over the curved surfaces. These results show how collective effects in active ratchets could be used to achieve a variety of active pumping techniques. It may also be possible that biological systems such as cells already take advantage of such collective ratchet effects.

3.3. Future Directions

The field of collective effects in active matter systems is wide open. Since a wide variety of flocking models exist, it would be interesting to determine what types of ratchet effects occur for different flocking rules. This would also be an interesting question to examine in experiments on schools of fish or swarming insects which could be placed in funnel like channels, with a comparison between ratchet behaviors for schooling and non-schooling systems. Similar experiments could be performed with herding animals. It would also be interesting to explore systems with competing effects, such as a ratchet geometry that biases the motion of the system in one direction combined with some other competing force that biases the motion of the system in the opposite direction. In such a case, flocking or collective effects could be turned on or off to take advantage of or nullify the ratchet effect. There has already been some evidence for competing interactions that can overwhelm the ratchet effect, such as an experiment on bacteria in a funnel barrier ratchet geometry, where the bacteria form Fisher wave swarms that move against the ratchet direction \cite{71}.

Self-driven colloids have been shown to exhibit a phase separation transition into gaslike
Figure 12
Experimental demonstration of directed motion of passive colloids immersed in a bath of bacteria which interact with a ratchet geometry. The plots indicate the number of passive particles in each chamber or spatial region, color coded according to the inset. (a) A geometry that concentrates the passive particles at the center of the device. (b) A geometry that ejects the passive particles. Reprinted by permission from MacMillan Publishers Ltd: Nature Communications, N. Koumakis, A. Lepore, C. Maggi, and R. Di Leonardo, Nature Commun. 4, 2588 (2013), copyright 2013.

and clustered solidlike phases for high enough density or activity, so it would be interesting to examine how the ratchet efficiency is affected by the onset of self-clustering. Introduction of a substrate can modify the density or activity at which the self-clustering begins to occur, as well as the mean distance that individual particles can move. Collective ratchet effects should also depend strongly on the type of interactions between the particles. In the systems studied so far, short-range steric repulsion has been used for the particle-particle interactions; however, it is also possible to consider active particles with Lennard-Jones interactions, long-range repulsive interactions, competing long-range repulsion and short-range attraction, or many body interactions.

Another class of system is active membranes or sheets that could mimic cells or collections of tightly bound organisms. Swimming deformable droplets are one example of an extended active object of this type, and if such droplets were placed on an asymmetric...
surface, ratchet effects could occur. In this case, varying the substrate periodicity to be larger or smaller than the size scale of the active object could cause changes in the ratchet effect. There has already been a study demonstrating ratchet effects for active polymers in funnel barrier arrays (74). Other motion rules could also be introduced to model objects that have mechanics similar to cells, such as protrusions that push or pull to mimic the experiments in Ref. (67). There could also be ratchet effects for extended active objects that can expand or contract, such as a squirmer model or a system with inertial dynamics.

Collective ratchet effects can also be examined for groups of interacting rods coupled by molecular motors when an additional asymmetry is introduced, or active gel systems such as active nematics (75, 76) could be studied in the presence of an asymmetric substrate. Active nematics can also exhibit mobile defects (77, 78), so it may be possible to produce ratcheting motion of the defects.

4. RATCHETS IN CIRCLE SWIMMERS AND CHIRAL SYSTEMS

There are many examples of active biological and artificial systems that exhibit a chirality in their motion, such as biological circle swimmers (79, 80, 81) and artificial circle swimmers (82, 83, 84). In this case, ratchet effects can occur even in the absence of a substrate asymmetry since the chirality of the swimming introduces an asymmetry to the system. One of the first examples of a ratchet effect produced by chirality was observed for circularly moving particles on a 2D periodic substrate (85). As the substrate-free particle orbits are varied from circular to more complex Lissajous patterns, a series of transitions occurs between localized closed trajectories with no net translation to open trajectories that permit the particles to translate in a particular direction, producing a ratchet effect. Additional simulations have shown that directed motion can be achieved for circularly moving particles on periodic substrates, where various types of translating trajectories appear (87), while in experiments with circularly moving colloids on 2D substrates, directed or ratchet motion can be produced (87). Nourhani et al. (88) considered a model of counterclockwise circle swimmers on a 2D periodic substrate for both deterministic and stochastic swimmers. In the deterministic regime they found that the particles can either form closed orbits or translate at fixed angles with respect to the substrate periodicity. In the stochastic regime, the ratchet effect extends over a wider range of parameters since the particles can hop between different rectifying orbits even if they become temporarily trapped in a closed or nonrectifying orbit.

It is also possible to obtain ratchet effects for circle swimmers moving over asymmetric substrates (89). Figure 13(a) shows a counterclockwise swimming particle moving with a motor force amplitude of $A = 0.57$ over an array of L-shaped barriers. The particle translates one substrate lattice constant in the positive $y$ direction during every swimming cycle. For $A = 0.9$ in Fig. 13(b), the particle is confined to a closed non-translating orbit. In Fig. 13(c), the translating orbit at $A = 1.5$ includes particle motion in both the positive $x$ and positive $y$ directions. Figure 13(d) shows that at $A = 2.05$ the particle translates in the negative $x$-direction. The average drift velocity of the particle in the $y$ direction, $\langle V_y \rangle$, and in the $x$ direction, $\langle V_x \rangle$, is plotted as a function of $A$ in Fig. 14. A series of quantized orbits occur in which the particle translates in a fixed direction over a finite interval of $A$. Due to the asymmetry of the L-shaped barriers, the locations of the translating or ratcheting regimes differ for clockwise and counterclockwise swimming particles; however, in general, swimmers with different chirality move in different directions, permitting a
mixture of chiral swimmers to be separated by the barrier array. When thermal effects are included, the additional noise expands the range of values of $A$ over which rectification occurs; however, the efficiency of the maximum ratchet effect is reduced compared to the nonthermal or deterministic limit.

Figure 13
The particle orbits from a simulation of circle swimmers in an array of L-shaped barriers. (a) At a motor force of $A = 0.57$, translation occurs in the positive $y$ direction. (b) For $A = 0.9$ a non-ratcheting orbit appears. (c) At $A = 1.6$ the particle translates in the positive $x$ and $y$-directions. (d) At $A = 2.05$ the particle translates in the negative $x$-direction. Adapted from C. Reichhardt and C. J. Olson Reichhardt, Phys. Rev. E 88, 042306 (2013). Copyright 2013 by the American Physical Society.

Mijalkov and Volpe (90) considered the separation of different species of circle swimmers with different angular velocities by an asymmetric comb channel with a geometry that could be nanofabricated, as illustrated in Fig. 15(a). The particles are initially placed in the leftmost chamber, and as time progresses, the particles moving at 2.2 rad/s undergo the largest amount of ratcheting motion along the channel, as shown by the histograms of particle density at different times in Fig. 15(b-e). At the longest times of $t = 10000$s in Fig. 15(e) the spatial separation of the different particle species is clearly visible.

Ai et al. (91) considered chiral swimmers interacting with an array of half-circle barriers and found that particles with different chirality migrate to opposite sides of the sample due to interactions with the barriers. In a numerical study of chiral swimmers in a geometry containing M-shaped barriers on the bottom of the sample at $y = 0$ and periodic boundary conditions in the $x$ direction, the chirality of the swimmers leads to a rectification effect and makes it possible to achieve the transport of a larger passive particle (92). The direction in which the active particles ratchet depends on the chirality of the swimming, so that counterclockwise and clockwise swimmers move in opposite directions. One of the surprising results of this work is that the direction in which the passive particle drifts can, for
Figure 14
The net velocity of the circularly swimming particle from the system in Fig. 13 showing a series of ratchet orbits as a function of the motor force amplitude $A$. (a) $\langle V_y \rangle$ vs $A$. (b) $\langle V_x \rangle$ vs $A$. If the particle swimming direction is reversed from counterclockwise to clockwise, a different set of rectification phases occurs due to the asymmetry of the L-shaped barriers. Reprinted with permission from C. Reichhardt and C. J. Olson Reichhardt, Phys. Rev. E 88, 042306 (2013). Copyright 2013 by the American Physical Society.

Figure 15
Simulations of the separation of circle swimmers with different angular velocities by an asymmetric comb channel with the geometry illustrated in panel (a). The particles have angular velocities of 3.1 (black), 2.8 (yellow), 2.5 (blue), and 2.2 (red) rad/sec. (b,c,d,e) Histograms of the particle positions for increasing times. The particles rotating at 2.2 rad/s exhibit the largest amount of rectification. Reproduced from M. Mijalkov and G. Volpe, Soft Matter 9, 6376 (2013), with permission of The Royal Society of Chemistry.
some parameters, be opposite to the direction in which the swimmers are moving, and this directional reversal depends strongly on the packing fraction of the system, which again indicates that collective effects can introduce new behaviors.

5. VARIANTS ON RATCHET INDUCED TRANSPORT

Ratchet effects are generally studied in systems with fixed asymmetric substrates; however, if the substrate is mobile and can respond to the active particle, an active ratchet effect can occur that can be used to transport larger-scale asymmetric objects. Angelani et al. (93) examined a model of run-and-tumble bacteria interacting with large scale passive rotary objects that have asymmetric saw-tooth shapes. The bacteria accumulate in the corners of the sawteeth and generate a net torque that produces a persistent rotation of the object in one direction. Such active matter-induced gear rotation was subsequently confirmed in several experiments with swimming bacteria (94, 95, 96) as well as for gears interacting with small robots (97). Additionally, if an asymmetric object is placed in a bath of active particles, it is possible to generate enhanced transport of the object. Figure 16(a,b) shows images of asymmetric objects placed in a bath of rod-like bacteria to create microshuttles (98). The bacteria, which swim in the direction of their white tips, accumulate in the corners of each shuttle object and produce a net force \( f \) that causes the shuttle to translate. Figure 16(c) shows that the probability distribution of \( f \) shifts its weight to higher values of \( f \) as the shuttle length is increased, producing a larger biasing propulsion force.

Experiments on asymmetric wedges in bacterial baths reveal enhanced transport of these objects (99), and other studies have examined how the shape of the asymmetric passive object affects the transport of the object (100). Since active particles accumulate in corners or concave regions of a funnel shape, it is possible to use static (101) or moving wedges (102) to capture active particles. Many additional variants of using active matter to transport passive objects are possible. It would also be interesting to study systems in which the passive objects could change shape in response to the activity.

6. CONCLUSION

We have highlighted the recent results for a new class of ratchet system called active ratchets, which appear when the directed motion of self-propelled particles, which breaks detailed balance, is combined with an additional asymmetry in the system. Unlike rocking and flashing ratchets, active ratchets do not require the application of an external driving force to produce rectification. Active ratchet effects and variations upon them will be a growing field of research as the ability to fabricate additional types of artificial swimmers, nanobots, and other self-driven systems continues to improve, along with the possibility of applying simple motion rules to the active objects that can permit them to perform usable work. Due to the general nature of the conditions under which active ratcheting can occur, it is likely that nature may already be exploiting active ratchet effects in various biological systems, so it would be interesting to examine motility process in complex biological environments to understand whether active ratchet effects are present. Another area for future research is the collective effects that arise in assemblies of interacting active particles or in more complex objects such as cells. It has already been demonstrated that collective effects can produce a variety of novel ratcheting effects including ratchet reversals, which offers the possibility to utilize bidirectional ratchets for various medical technologies and other types
Figure 16
(a,b) Snapshots from simulations of asymmetric microshuttle objects (blue) interacting with self-propelled rods (red/white). The rods move in the direction of their white tips. (a) A shuttle composed of a single wedge shape moves when the active particles accumulate along the left side of the wedge. (b) Particles accumulate at each corner of a shuttle composed of three wedge shapes. (c) The probability distribution $p(f)$ of the time-averaged force $f$ exerted on the shuttle by the active particles for shuttles made from 1 (black), 2 (red), 3 (green), or 4 (blue) wedges, showing an increase in the effective driving force as the number of wedges increases. Reprinted with permission from L. Angelani and R. Di Leonardo, New J. Phys. 12, 113017 (2010). ©IOP Publishing & Deutsche Physikalische Gesellschaft. CC BY-NC-SA.[http://dx.doi.org/10.1088/1367-2630/12/11/113017](http://dx.doi.org/10.1088/1367-2630/12/11/113017)

of biological applications.

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