Combined identification of constitutive parameters and in-situ stress in layered rock mass formulated by Cosserat theory

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Abstract. This paper focuses on the identification of constitutive parameters and in-situ (initial) stress in the layered rock mass formulated by Cosserat theory. The direct problem is modeled by FEM (Finite Element Method), providing a platform at which the sensitivity analysis can be implemented, and the inverse problem is solved via the Gauss-Newton technique. The effects of initial guesses and noisy data on the solutions are investigated, and satisfactory results are exhibited in the numerical verification.

1. Introduction

Occurrence of a stratified (layered) rock mass for a mineral deposit is not uncommon in mining practice and results in highly anisotropic strength and deformation characteristics. This makes it necessary to include effects of joints into the mathematical formulations describing the load-deflection behavior. In addition to the discontinuous modeling method by which the joints in the rock mass and intact rock layers are modeled individually [1,2], another kind of modeling method is the continuum one by which the whole layered rock mass with joints is represented by an equivalent continuum model providing a reasonable large scale (average) description of the material response to loading especially when closely spaced joints occur in large numbers[3,4], and make the discontinuous modeling impossible.

One of the effective equivalent continuum models is that based on the couple stress theory (Cosserat theory). In a mechanical point of view, Reference[5] and its cited papers illustrated the justification for the use of the Cosserat theory in layered rock mass, and provided numerical evidence that the Cosserat model is capable of accurately reproducing complex load-deformation patterns via a comparison with calculations conducted using the discrete joint model.

The couple stress theory (Cosserat theory) can be traced back to 1887 when Voigt assumed the existence of couple stress. In 1909, the Cosserat brothers first set up a framework of couple stress theory which has been further developed by [6,7,8]. Zvolinskii and Shkhinek [9] applied the Cosserat theory to formulate a model of layered material with elastic coupling between the layers and later Bogan [10] confirmed the mathematical viability of this model. Extensive development of the Cosserat theory in relation to layered rock mass has been carried out by Mühlhaus [11], Adhikary and A.V. Dyskin [5,12,13].

The study of this paper is motivated by a question that if a continuum couple stress theory is adopted for modeling the layered rock mass, how to determine those equivalent constitutive
coefficients? The question becomes more challenging if the in-situ stress state also needs to be determined at the same time.

Experimental means (e.g. plate-bearing tests, hydraulic fracturing tests, and borehole stress-relief tests etc.) can be employed to determine above parameters. However, the implementation of these experimental means does not to be an easy task since:

1) The mechanical parameters at various locations around the site vary greatly due to the complexity of the geological conditions, a large number of field tests may be required to describe the rock parameters adequately [14].

2) Due to the influences of joints or fissures, hydraulic fracturing tests and borehole stress-relief tests are often costly and time-consuming with dispersive data covering small test domain [15].

The issue of identification can be treated as an inverse problem with unknown constitutive parameters and initial stress, and can be investigated under the framework of inverse problems in elasticity for which a comprehensive review was given by Bonnet [16]. If the sufficient ‘measurement’ message, such as the displacements, strains etc. could be provided, all the unknowns would be determined analytically or numerically. In the rock engineering, the identification based upon the measured displacement is often used to determine classical elastic constitutive parameters or/and in-suit stress of rock mass [14,15,16,17,18,19,20,21,22]. In comparison with the previous work based on the classical elasticity, the parameters identification of the inverse couple stress problem includes both constitutive parameters appearing in the classical model and those additional items describing the constitutive relationship of couple stress. To the best of the authors’ knowledge, it seems there are no reports directly relevant to this matter.

Since the displacement that is usually reliable and is easily measured [15], this paper suggests exploiting the displacements of rock masses induced by excavation to determine the unknown constitutive parameters and in-situ stress for the layered rock mass formulated by Cosserat theory. We propose a numerical model that consists of two parts, one is for the description of direct problem formulated by FEM, two key issues in this part include the effect of excavation and implementation of sensitivity analysis; Another one is for the description of inverses problem that is treated as an optimization problem solved by the Gauss-Newton technique, the major issues concerned in this part include the combined identification, regional in-homogeneity, and computing accuracy with the consideration of noisy ‘measurement’ message.

2. The Description of Governing Equation

The equilibrium equation of couple stress problem can be written in a tensor form [23]

$$\sigma \cdot \nabla + f = 0, \quad \mu \cdot \nabla + m - \varepsilon \sigma = 0$$

(1)

in which $\varepsilon$ is the alternating tensor and $\nabla$ is the Hamiltonian differential operator. $f$ and $m$ are the body force and the body couple respectively. $\sigma$ and $\mu$ denote the stress tensor and the couple stress tensor respectively. The second part of equation (1) indicates that the stress tensor $\sigma$ generates an equivalent body couple $- \varepsilon \sigma$ acting together with $m$ to maintain the equilibrium of the continuum [23].

In order to express motion of a particle in a Cosserat continuum, we need to introduce the micro-rotation $\omega$ in addition to the well-known displacement vector $u$. The relationships of strain-displacement, rotation-displacement and curvature-rotation are described by [23]

$$\varepsilon = 1/2 (u \otimes V + V \otimes u), \kappa = 1/2 (\omega \otimes V + V \otimes \omega), \omega = - \varepsilon; 1/2 u \otimes V$$

(2)

where the infinitesimal strain tensor $\varepsilon$ and the micro-curvature strain tensor $\kappa$ are conjugated to $\sigma$ and $\mu$, respectively[23].

The boundary conditions are specified by [23]

$$\begin{cases}
\sigma \cdot n = \sigma_0, & x \in \Gamma_\sigma, \\
\mu \cdot n = \mu_0, & x \in \Gamma_\mu,
\end{cases}
$$

(3)

where $u_0$ and $\omega_0$ are the prescribed values of $u$ and $\omega$ on the $\Gamma_\sigma$ and $\Gamma_\mu$, $\sigma_0$ and $\mu_0$ are the
prescribed vectors of traction and moment on the boundary of $\Gamma^\sigma$, $n$ denotes the unit outside normal on the boundary, $\Gamma^\sigma + \Gamma^\nu = \Gamma$ designates the boundary of domain $\Omega$, $x$ represents a vector of coordinates.

For the 2-D problem, the general stress and strain vectors are given by
$$[\sigma, \mu] = [\sigma_{xx}, \sigma_{xy}, \sigma_{yy}, \mu_{xx}, \mu_{yy}, \mu_{yx}, \mu_{xy}, \mu_{xy}, \mu_{yx}] \quad [\varepsilon, \kappa] = [\varepsilon_{xx}, \varepsilon_{xy}, \varepsilon_{yy}, \kappa_{zz}, \kappa_{yy}, \kappa_{xy}, \kappa_{yx}, \kappa_{xy}, \kappa_{yx}]$$

(4)

Under a coordinate system in which the direction of $x$ axis is identical with that of the layer, the 2-D Cosserat constitutive equation for the layered rock is given by [12]
$$[\sigma, \mu]^T = D [\varepsilon, \kappa]^T$$

(5)

where $D$ is a symmetric matrix given by
$$D = \begin{bmatrix}
A_{11} & A_{12} & 0 & 0 & 0 & 0 \\
A_{12} & A_{22} & 0 & 0 & 0 & 0 \\
G_{11} & G_{12} & 0 & 0 & 0 & 0 \\
G_{12} & G_{22} & 0 & 0 & 0 & 0 \\
sym & B_1 & 0 & 0 & 0 & 0 \\
\end{bmatrix}$$

(6)

$$A_{11} = \frac{E}{1 - \nu^2 - \nu^2(1 + \nu)^2/(1 - \nu^2 + \frac{E}{b k_n})}, \quad A_{12} = \frac{\nu/(1 - \nu)}{1 - \nu^2 - \nu^2(1 + \nu)^2/(1 - \nu^2 + \frac{E}{b k_n})}, \quad G_{11} = G_{12} = G_{22}$$

(7)

$$A_{22} = \frac{1}{12} \left(\frac{E b^2}{(1 - \nu^2)(G + G_{11})} + \frac{1}{b k_n} \right), \quad G_{22} = G_{12} + G_{11} + G_{12}, \quad B_1 = \frac{E b^2}{12(1 - \nu^2)} \left(\frac{G - G_{11}}{G + G_{11}}\right)$$

(8)

where $E$, $\nu$ and $G$ represent the Young’s modulus, Poisson’s ratio and shear modulus of the intact rock layers, respectively, $b$ is the layered thickness (i.e. joint spacing), $k_n$ and $k_s$ are the normal and shear stiffness of joint, respectively.

### 3. Implementation of FEM on the direct elastic Cosserat problem with the consideration of excavation

Based on the principle of virtual work for elastic couple stress problem, it follows that [23]
$$\int_{\Omega} (\{\delta \varepsilon\}^T \cdot \{\sigma\} + \{\delta \kappa\}^T \cdot \{\mu\}) dV = \int_{\Gamma} (\{\delta u\}^T \cdot \{\sigma_0\} + \{\delta \omega\}^T \cdot \{\mu_0\}) d\Gamma$$
$$+ \int_{\Omega} (\{\delta u\}^T \cdot \{f\} + \{\delta \omega\}^T \cdot \{m\}) dV$$

(9)

where $\{\sigma\}$ stands for the vector of stress, $\{\mu\}$ designates the vectors of couple stress, $\{\sigma_0\}$ and $\{\mu_0\}$ represent the vectors of traction and moment on the boundary, respectively; $\{f\}$ and $\{m\}$ are the vectors of body force and the body couple, respectively, $\{\delta u\}$, $\{\delta \varepsilon\}$, $\{\delta \omega\}$ and $\{\delta \kappa\}$ denote the vectors of virtual displacement, strain, rotation, and curvature, respectively.

The distribution of displacements and rotations within a finite element are approximated by
$$\begin{bmatrix}
\{u\} \\
\{\omega\}
\end{bmatrix} = \psi \begin{bmatrix}
\{\bar{u}\} \\
\{\bar{\omega}\}
\end{bmatrix}, \quad \begin{bmatrix}
\{\delta u\} \\
\{\delta \omega\}
\end{bmatrix} = \psi \begin{bmatrix}
\{\delta \bar{u}\} \\
\{\delta \bar{\omega}\}
\end{bmatrix}$$

(10)

where $\{u\}$ and $\{\omega\}$ represent the vectors of displacement and rotation, respectively; $\psi$ stands for a matrix of shape functions, $\{\bar{u}\}$ and $\{\bar{\omega}\}$ denote the nodal vectors of $\{u\}$ and $\{\omega\}$ respectively, $\{\delta \bar{u}\}$ and $\{\delta \bar{\omega}\}$ are the vectors of virtual nodal displacement and rotation, respectively.
Substituting equation (10) into (9) and assembling all elements over the domain then yields

\[ KU = \int_{\Omega} \psi^{T} \left\{ \sigma_{0} \right\} d\Gamma + \int_{\Omega} \psi^{T} \left\{ f \right\} dV = F \]  

(11)

where \( F \) and \( U \) refer to the general nodal vectors of load and displacement, \( K \) stands for the general stiffness matrix. In the 2D case, \( K = \sum_{\Omega} B^T D B t dxdy \), \( t \) stands for the thickness of the material, \( D \) is as defined as in equation (6)

\[
B = \begin{bmatrix}
\psi_{,x} & 0 & \psi_{,y} & 0 & 0 & 0 & 0 & 0
\end{bmatrix}^T
\]  

(12)

Finite element formulation of the Cosserat continuum is, in general, the same as that of a conventional continuum with some differences in the strain matrix (\( B \)-matrix) and the elastic stiffness moduli (\( D \)-matrix) [12].

We assume that the distribution of in-situ stress in the field is specified by [19]

\[
\sigma_{xx} = \sigma_{x}^{a} + \sigma_{x}^{b}, \quad \sigma_{yy} = \sigma_{y}^{a} + \sigma_{y}^{b}, \quad \sigma_{xy} = \sigma_{yx}, \quad \mu_{ij} = 0
\]  

(13)

where \( \sigma_{ij}^{a} \) and \( \mu_{ij} \) stand for the in-situ stress and couple stress, respectively, \( \sigma_{ij}^{b} \), \( \tau \) refer to the constants, respectively, \( y \) denotes the coordinate along the vertical direction.

Figure 1 illustrates an equivalent process of excavation, \( u^E = u^R - u^I \)

Figure 1  An illustration of equivalent process of excavation, \( u^E = u^R - u^I \)

4. The description of inverse problem

The determination of unknown constitutive parameters or/and in-situ stresses can be realized by minimizing the following functional

\[
\Pi = \frac{1}{2} \left\| LU - U^* \right\|^2 = \frac{1}{2} (LU - U^*)^T (LU - U^*) = \frac{1}{2} R^TR
\]  

with constrains

\[ K(\lambda)U = F(\lambda) \]  

(16)
where \( \lambda_1 = \{ E, v, b, k_1, k_2 \}^T \), \( \lambda_2 = \{ \sigma_{11}^b, \sigma_{11}^b, \sigma_{22}^b, \tau_1^b, \tau_2^b \}^T \), \( U^* \) stands for the vector of measured or simulated displacement along the excavated boundary, \( L \) is a transformation matrix mapping the relationship of location between \( U^* \) and \( \lambda \).

The minimization of equation (15) with constrain (16) can be carried out by utilizing Gauss-Newton method with following iterative process

\[
\lambda_i^{k+1} = \lambda_i^k + \Delta \lambda_i^k, \quad i = 0, 1, 2, \ldots \text{ till } \max \left\{ \left| \Delta \lambda_i^k \right| \right\} \leq \varepsilon \quad \text{is satisfied} \quad (k = 1, 2)
\]

\[
\left\{ \begin{array}{c}
G(\lambda_i^k, G(\lambda_2^k)) [G(\lambda_i^k), G(\lambda_2^k)] \Delta \lambda_i^k = [G(\lambda_i^k), G(\lambda_2^k)]^T R, \\
G(\lambda_i^k) = -LK^{-1} \frac{\partial K}{\partial \lambda_i^k} U
\end{array} \right.
\]

where \( \Delta \lambda_i^k \) stands for the \( i \)-th iteration, \( \varepsilon \) is the error bound.

5. Numerical verification and remarks

Consider an inverse problem of constitutive parameters and in-situ stress for a underground rectangular opening (20m×25m) in the layered rock mass as shown in Figure 2 that also gives a description of FE mesh with 168 8-node isoparametric elements. The bulk density is 25 kNm\(^3\). The computing domain sizes 200m×250m, and consists of 2 regions divided by the right diagonal. The constitutive parameters in these two regions are different. 8 'measurement' points locate at 4 corners and midpoints of 4 sides of the rectangular opening.

![Finite element mesh](image)

Figure 2 Finite element mesh

The above inverse problem is solved by means of the simulated excavation displacement \( U^* \) (\( U^* \) is provided by solving equation (16) with the exact constitutive parameters and in-situ stress). The noisy data is injected in the form [24]

\[
U_{\text{noise}} = (1 + \xi \cdot \Theta) U^*
\]

where \( U_{\text{noise}} \) represents a displacement vector containing the noisy data, the product of \( \xi \cdot \Theta \) stands for a relative deviation, \( \xi \) is a random variable that follows a normal distribution with zero mean and unit standard deviation, \( \Theta \) denotes a constant. For each fixed value of \( \Theta \), 40 groups of results are obtained with 40 \( \xi \) produced randomly. The mean values are taken as the final results of identification, and the confidence interval is evaluated by

\[
\bar{x} \pm t_{(\alpha/2, N'-1)} \times s / \sqrt{N'}
\]

where \( \bar{x} \) represents the mean of the identified parameters, \( s \) is a standard deviation of the identified parameters, \( t \) denotes a \( t \) distribution with the degree of freedom \( (N' - 1) \), \( N' \) is the capability of samples, and the confidence level is \( 1 - \alpha \).
Two cases are presented to illustrate the performance of proposed numerical model, and the impact of noisy data and initial guesses on the solution are investigated.

Case 1 considers a single identification of constitutive parameters

The injection of noisy data with $\Theta = 0.01$ and $0.05$ results in a $11.4\%$ maximum relative error of the additional information. Tables 1 and 2 exhibit the solutions with different noisy data and initial guesses, and indicate that

1) The majority of confidence intervals are significantly magnified with the increase of level of noisy data except for the Poison’s ratio and layers thickness which seem not as sensitive as others. The relative errors of all mean values are within the range of $1\%$ at the $95\%$ confidence level.

2) Different initial guesses lead to different times of iterations, but the process converges to a same solution. In this example the maximum times of iterations is 7.

**Table 1.** The effects of noisy data on the results

| Constitutive parameters | $\Theta = 0.01$ | $\Theta = 0.05$ | Actual values of constitutive parameters |
|--------------------------|-----------------|-----------------|----------------------------------------|
| $E_i (Pa)$               | $(1.00 \pm 0.003) \times 10^9$ | $(9.98 \pm 0.14) \times 10^0$ | $1.0 \times 10^{10}$ |
| $\nu_l$                  | $0.20 \pm 8.84 \times 10^{-15}$ | $0.20 \pm 4.60 \times 10^{-13}$ | $0.2$ |
| $b_l (m)$                | $0.75 \pm 4.65 \times 10^{-15}$ | $0.75 \pm 2.00 \times 10^{-13}$ | $0.75$ |
| $k_{11} (Pam^{-1})$      | $(2.00 \pm 0.006) \times 10^8$  | $(2.00 \pm 0.03) \times 10^8$  | $2.0 \times 10^8$  |
| $k_{21} (Pam^{-1})$      | $(1.50 \pm 0.05) \times 10^9$   | $(1.50 \pm 0.02) \times 10^9$   | $1.5 \times 10^9$   |
| $E_i (Pa)$               | $(1.80 \pm 0.005) \times 10^9$   | $(1.80 \pm 0.02) \times 10^9$   | $1.8 \times 10^9$   |
| $\nu_2$                  | $0.25 \pm 6.58 \times 10^{-14}$  | $0.25 \pm 4.88 \times 10^{-12}$  | $0.25$  |
| $b_2 (m)$                | $1.20 \pm 8.67 \times 10^{-14}$  | $1.20 \pm 5.75 \times 10^{-12}$  | $1.2$  |
| $k_{22} (Pam^{-1})$      | $(3.51 \pm 0.01) \times 10^8$   | $(3.49 \pm 0.05) \times 10^8$   | $3.5 \times 10^8$   |
| $k_{22} (Pam^{-1})$      | $(2.00 \pm 0.06) \times 10^9$   | $(2.00 \pm 0.03) \times 10^9$   | $2.0 \times 10^9$   |

**Table 2.** The effects of initial guesses on the results

| Constitutive parameters | 1 Results of identification | 2 Results of identification | Actual values of constitutive parameters |
|--------------------------|-----------------------------|-----------------------------|----------------------------------------|
| $E_i (Pa)$               | $1.0 \times 10^9$ | $1.0 \times 10^9$ | $1.0 \times 10^9$ |
| $\nu_l$                  | $3.0 \times 10^{-3}$ | $2.0 \times 10^{-1}$ | $2.0 \times 10^{-1}$ |
| $b_l (m)$                | $1.5 \times 10^0$ | $7.5 \times 10^{-1}$ | $4.5 \times 10^{-1}$ |
| $k_{11} (Pam^{-1})$      | $1.0 \times 10^8$ | $2.0 \times 10^8$ | $1.4 \times 10^7$ |
| $k_{21} (Pam^{-1})$      | $9.0 \times 10^8$ | $1.5 \times 10^9$ | $3.0 \times 10^8$ |
| $E_i (Pa)$               | $9.0 \times 10^9$ | $1.8 \times 10^{10}$ | $1.8 \times 10^9$ |
| $\nu_2$                  | $2.0 \times 10^{-1}$ | $2.5 \times 10^{-1}$ | $2.5 \times 10^{-1}$ |
| $b_2 (m)$                | $1.4 \times 10^0$ | $1.2 \times 10^0$ | $2.2 \times 10^0$ |
| $k_{22} (Pam^{-1})$      | $7.0 \times 10^8$ | $3.5 \times 10^8$ | $2.5 \times 10^7$ |
| $k_{22} (Pam^{-1})$      | $1.0 \times 10^9$ | $2.0 \times 10^9$ | $4.0 \times 10^8$ |
Case 2 considers a combined identification of in-situ stress and Young’s modulus. The injection of noisy data with $\Theta = 0.01$ and $0.05$ results in a 12% maximum relative error of the additional information. Tables 3 and 4 show the solutions with different noisy data and initial guesses.

By comparison with the single identification in cases 5.1, the combined identification makes more deviation from mean values, and leads to more computing cost with 10 round iterations.

Table 3. The effects of noisy data on the results

| Constitutive parameters and initial stress | $\Theta = 0.01$ | $\Theta = 0.05$ | Actual values of initial stress and constitutive parameters |
|-------------------------------------------|-----------------|-----------------|----------------------------------------------------------|
| $E_1 (Pa)$                                | $(9.98 \pm 0.02) \times 10^9$ | $(9.95 \pm 0.01) \times 10^9$ | $1.0 \times 10^{10}$ |
| $\nu_1$                                   | $0.20 \pm 1.56 \times 10^{-14}$ | $0.20 \pm 2.34 \times 10^{-12}$ | 0.2 |
| $b_1 (m)$                                 | $0.75 \pm 3.35 \times 10^{-15}$ | $0.75 \pm 1.37 \times 10^{-13}$ | 0.75 |
| $k_{x1} (Pam^{-1})$                       | $(2.00 \pm 0.004) \times 10^8$ | $(2.00 \pm 0.02) \times 10^8$ | $2.0 \times 10^8$ |
| $k_{z1} (Pam^{-1})$                       | $(1.50 \pm 0.003) \times 10^9$ | $(1.50 \pm 0.02) \times 10^9$ | $1.5 \times 10^9$ |
| $E_2 (Pa)$                                | $(1.80 \pm 0.004) \times 10^{10}$ | $(1.80 \pm 0.02) \times 10^{10}$ | $1.8 \times 10^{10}$ |
| $\nu_2$                                   | $0.25 \pm 6.46 \times 10^{-14}$ | $0.25 \pm 9.65 \times 10^{-12}$ | 0.25 |
| $b_2 (m)$                                 | $1.20 \pm 1.12 \times 10^{-14}$ | $1.20 \pm 7.31 \times 10^{-13}$ | 1.2 |
| $k_{x2} (Pam^{-1})$                       | $(3.50 \pm 0.008) \times 10^8$ | $(3.48 \pm 0.04) \times 10^8$ | $3.5 \times 10^8$ |
| $k_{z2} (Pam^{-1})$                       | $(2.00 \pm 0.004) \times 10^9$ | $(1.99 \pm 0.02) \times 10^9$ | $2.0 \times 10^9$ |
| $\sigma^{a}_{xx} (Pa)$                    | $(-1.20 \pm 0.003) \times 10^6$ | $(-1.21 \pm 0.01) \times 10^6$ | $-1.2 \times 10^6$ |
| $\sigma^{b}_{xx} (Pa)$                    | $(1.00 \pm 0.002) \times 10^4$ | $(1.01 \pm 0.009) \times 10^4$ | $1.0 \times 10^4$ |
| $\sigma^{a}_{yy} (Pa)$                    | $(-3.00 \pm 0.006) \times 10^6$ | $(-3.02 \pm 0.03) \times 10^6$ | $-3.0 \times 10^6$ |
| $\sigma^{b}_{yy} (Pa)$                    | $(2.50 \pm 0.005) \times 10^4$ | $(2.51 \pm 0.02) \times 10^4$ | $2.5 \times 10^4$ |
| $\tau (Pa)$                               | $(8.50 \pm 0.02) \times 10^5$ | $854745 \pm 7263$ | $8.5 \times 10^5$ |

Table 4. The effects of initial guesses on the results

| Initial stress and constitutive parameters | Initial guesses | Results of identification | Initial guesses | Results of identification | Actual values of initial stress and constitutive parameters |
|--------------------------------------------|-----------------|---------------------------|-----------------|---------------------------|----------------------------------------------------------|
| $\sigma^{a}_{xx} (Pa)$                     | $-6.0 \times 10^5$ | $-1.2 \times 10^6$ | $-8.4 \times 10^5$ | $-1.2 \times 10^6$ | $-1.2 \times 10^6$ |
| $\sigma^{a}_{yy} (Pa)$                     | $4.0 \times 10^4$ | $1.0 \times 10^4$ | $0.5 \times 10^4$ | $1.0 \times 10^4$ | $1.0 \times 10^4$ |
| $\sigma^{b}_{yy} (Pa)$                     | $-1.8 \times 10^4$ | $-3.0 \times 10^6$ | $-3.6 \times 10^6$ | $-3.0 \times 10^6$ | $-3.0 \times 10^6$ |
| $\sigma^{b}_{xy} (Pa)$                     | $2.0 \times 10^4$ | $2.5 \times 10^4$ | $5.0 \times 10^4$ | $2.5 \times 10^4$ | $2.5 \times 10^4$ |
| $\tau (Pa)$                               | $1.1 \times 10^6$ | $8.5 \times 10^5$ | $4.3 \times 10^5$ | $8.5 \times 10^5$ | $8.5 \times 10^5$ |
| $E_1 (Pa)$                                 | $1.0 \times 10^9$ | $1.0 \times 10^{10}$ | $2.0 \times 10^{10}$ | $1.0 \times 10^{10}$ | $1.0 \times 10^{10}$ |
| $E_2 (Pa)$                                 | $9.0 \times 10^9$ | $1.8 \times 10^{10}$ | $1.8 \times 10^9$ | $1.8 \times 10^{10}$ | $1.8 \times 10^{10}$ |
6. Conclusions

This paper presents a numerical model to identify the unknown equivalent constitutive parameters and in-situ stress in elastic layered rock mass formulated by Cosserat theory singly/simultaneously. In comparison with the identification procedure for the classical elastic model, the major difference of the proposed model comes from the increase of unknowns with the appearance of constitutive parameters describing couple stress. Numerical example indicates that such a difference seems not to cause a distinct impact on the identification process conducted using a conventional Gauss-Newton technique. The results obtained encourage authors to make further effort to improve on the proposed model for its real application in practical engineering.

Acknowledgements

The research leading to this paper is funded by the oversea returnee’s initiating fund [1999-363], the key project fund [99149], and backbone faculty fund [2000-65], all of which came from National Education Department of P. R. China. The research is also funded by NSF (10421002), NSF (10472019), NSF (10772035), NSF(10721062), NKBRSF [2005CB321704], and the fund of disciplines leaders of young and middle age faculty in colleges of Liaoning Province.

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