A Secure Quantum Protocol for Anonymous One-Vote Veto Voting

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ABSTRACT Voting is one of the most prominent components of democracy. The one-vote veto, which requires the voting result to be only ‘‘yes’’ or ‘‘no’’, is a particularly noteworthy type of voting with widespread application. When obtaining a ‘‘no’’ result, no useful information about the number of ‘‘no’’ votes and who votes ‘‘no’’ is disclosed. In this paper, we introduce a protocol for anonymous one-vote veto utilizing qubits and local Pauli operations $\text{Z}$ and $\text{X}$, building simple processes for easy implementation under the current technology. For better elaboration, we give two examples: all voters cast ‘‘yes’’ votes and only one voter casts a ‘‘no.’’ Then, the corresponding experiment tests are conducted on the simulated IBM quantum computer to verify their feasibility. We also show that the proposed protocol has the desirable properties of privacy, fairness, verifiability and robustness. Furthermore, we analyze the proposed protocol’s security against cheating from eavesdroppers, a semi-honest server and malicious voters. This work is the first attempt to illustrate how qubits can be useful for building a secure anonymous one-vote veto strategy.

INDEX TERMS Quantum anonymous voting, one-vote veto, privacy, security.

I. INTRODUCTION

Voting is a daily social activity in modern society. Often, the adoption of significant decisions and the conduct of democratic elections depend on voting systems. Initially, voting systems require voters to cast votes at designated places, followed by manual tallying of votes under supervision. One common example is ballot box voting. Each voter is assigned one blank ballot paper, and he (she) writes his (her) vote on it. Then, the voters place their filled ballot papers into predetermined ballot boxes. Some authorized servers collect all votes and publicize the voting results. This method achieves specific functions, but some limitations affect security. For example, voters can cheat by tracing marked ballot papers without being detected. Because of time and geographical barriers, voters’ difficulty engaging in real-time and face-to-face voting has led to relatively harsh conditions for voting and poor operability.

With the rapid development of information technology and the popularity of the Internet, electronic voting technology arose to meet the demands of the times and has gradually replaced primary voting. Since Chaum presented the first private electronic election in 1981 [1], various distinctive electronic voting protocols have been proposed [2]–[4]. One common characteristic of most of them is that the security is supported by the computing complexity of some difficult problems, such as discrete logarithmic problems and many factorization problems.

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[36], quantum private query (QPQ) [21]–[23], quantum secure multiparty computation (QSMC) [24], [25] and others [26]–[28], have been proposed.

In 2007, Vaccaro et al. presented quantum anonymous voting based on a multiparticle entangled state [29], which inspired many researchers to explore the advantages of quantum mechanics in solving anonymous voting problems. Subsequently, based on different quantum mechanics principles, many significant quantum anonymous voting protocols have been proposed [30]–[33]. The one-vote veto is a particular and meaningful type of voting. In the voting activity of the one-vote veto, the voted content will be denied as long as there is a negative vote. The use of the one-vote veto system can effectively protect the minority’s specific power from being violated and prevent “majority tyranny”. For example, in an investment company, some investors have veto power on certain voting matters related to their interests to protect their vital profits. Internationally, the UN Security Council’s permanent members own veto power, preventing conflict among countries. To date, quantum anonymous one-vote veto protocols have rarely been considered. In 2015, Rahaman and Kar propose the first quantum anonymous one-vote veto protocol (QAV) [34] with prior entanglement of Greenberger-Horne-Zeilinger (GHZ) state |GHZ⟩ = 1/√2(|0⟩⊗n + |1⟩⊗n). At the cost of reduced privacy, such as even votes of “yes” or odd votes of “no,” the proposed protocol can perfectly achieve the voting result. However, an essential challenge in their protocol is how to distribute GHZ states among distrustful voters. As is well-known, entanglement is a valuable resource. When the number of particles n is very large, |GHZ⟩ is quite difficult to prepare in experiments. Furthermore, entangled particles are prone to lose their coherence due to external interference. Hence, in practice, the QAV based on GHZ states can be implemented neither efficiently nor economically. This motivates us to research QAV based on qubits, which has better stability and operability.

In this paper, we explore the first QAV protocol based on qubits and Pauli operations Z and X (QAVSP in short). Our QAVSP protocol satisfies the following properties.

1. Privacy: For each of the voters, no one else can obtain any useful information about his (her) vote. The information about the partial tally of the votes in any voters’ set is only computable by all remaining voters’ cooperation.

2. Fairness: Each voter cannot obtain any other voters’ voting information before his (her) voting to ensure that the voters’ willingness is protected from other objective factors.

3. Verifiability: According to the nature of a one-vote veto, every voter who votes “no” can predict that the voting result is rejected or his (her) option is uncounted.

4. Robustness: If any voter refuses to follow the QAVSP protocol, then the voting result is rejected.

The remainder of the paper is organized as follows. In section 2, we present the detailed process of our QAVSP protocol. In section 3, we show two examples of three voters and perform experimental tests on the online quantum computing environment. Each |d⟩ (1 ≤ j ≤ 5) is randomly chosen from the set of detection particles d1, · · · , d5, where δ is a security strength depending on the actual application environment. Each |d⟩ (1 ≤ j ≤ 5) is randomly chosen from the set of detection particles d1, · · · , d5, where δ is a security strength depending on the actual application environment. Each |d⟩ (1 ≤ j ≤ 5) is randomly chosen from the set of detection particles d1, · · · , d5, where δ is a security strength depending on the actual application environment. Each |d⟩ (1 ≤ j ≤ 5) is randomly chosen from the set of detection particles d1, · · · , d5, where δ is a security strength depending on the actual application environment. Each |d⟩ (1 ≤ j ≤ 5) is randomly chosen from the set of detection particles d1, · · · , d5, where δ is a security strength depending on the actual application environment. Each |d⟩ (1 ≤ j ≤ 5) is randomly chosen from the set of detection particles d1, · · · , d5, where δ is a security strength depending on the actual application environment. Each |d⟩ (1 ≤ j ≤ 5) is randomly chosen from the set of detection particles d1, · · · , d5, where δ is a security strength depending on the actual application environment. Each |d⟩ (1 ≤ j ≤ 5) is randomly chosen from the set of detection particles d1, · · · , d5, where δ is a security strength depending on the actual application environment. Each |d⟩ (1 ≤ j ≤ 5) is randomly chosen from the set of detection particles d1, · · · , d5, where δ is a security strength depending on the actual application environment. Each |d⟩ (1 ≤ j ≤ 5) is randomly chosen from the set of detection particles d1, · · · , d5, where δ is a security strength depending on the actual application environment.

II. PRODUCE OF THE QAVSP PROTOCOL

In this paper, the quantum voting network is controlled and managed by a server that has sufficient capacity to prepare quantum states and tally votes. Suppose that a voting activity involves n voters V0, V1, · · · , Vn−1, and each of them privately votes “yes” or “no” on one proposal. Once at least one “no” is received, the proposal is rejected. In practice, voting activities usually occur between partially distrusted voters or even between competitors. Here, the server is semi-honest and would like to perform any attacks constrained by quantum mechanics, except bribing by malicious voters and conspiring with any of them. There are no attack limits on voters. Under quantum mechanics, an active malicious voter can take any possible aggressive actions by himself (herself) or conspire with other evil voters. In theory, we assume that the classical and quantum channels are authenticated and the implemented environment is ideal, such as noiseless, no lost particles, and perfect equipment performance. The diagram of the entire voting process of the QAVSP protocol is given in Figure 1.

A. SHARING BINARY BIT KEY SEQUENCES

The server shares two private binary strings:

\[ k_{1,1} = \{ k_{1,1}^1, \cdots, k_{1,1}^i, \cdots, k_{1,1}^m \} \]

\[ k_{1,2} = \{ k_{1,2}^1, \cdots, k_{1,2}^i, \cdots, k_{1,2}^m \} \]

where \( k_{1,1}^i, k_{1,2}^i \in \{0, 1\} \) for \( 1 \leq i \leq n \) and \( 1 \leq j \leq m \), with each voter \( V_i \) based on some secure manners, such as QKD [5]–[8] and private face-to-face sharing technologies.

Furthermore, the server randomly generates two key strings \( l_{1,1}, l_{1,2} \) of length \( m \) with binary bit elements \( l_{1,1}, l_{1,2} \) for \( j = 1, 2, \cdots, m \), and privately retains them.

B. PREPARING QUANTUM VOTING STATES

The server prepares one sequence \( S_1 \) of \( m \) qubits, \( p_1, \cdots, p_1, \cdots, p_m \), according to the following rule.

For \( l_{1,1} = 0 \), the voting carrier is prepared on a rectilinear basis as follows:

\[ p_l = \begin{cases} |0\rangle, & \text{if } l_{1,2} = 0; \\ |1\rangle, & \text{if } l_{1,2} = 1. \end{cases} \]  

(2)

For \( l_{1,1} = 1 \), the voting carrier is prepared on a diagonal basis as follows:

\[ p_l = \begin{cases} |+\rangle, & \text{if } l_{1,2} = 0; \\ |–\rangle, & \text{if } l_{1,2} = 1. \end{cases} \]  

(3)

The server prepares \( \delta \) detection particles \( d_1, \cdots, d_{\delta} \), where \( \delta \) is a security strength depending on the actual application environment. Each \( d_j (1 \leq j \leq 5) \) is randomly chosen from the set of detection particles d1, · · · , d5, where δ is a security strength depending on the actual application environment.
in four states \{\{0\}, \{1\}, \{+\}, \{-\}\} with a uniform probability of \(\frac{1}{4}\). Then, the server inserts each detection particle \(d^j_t\) in the sequence \(S_{t,1}\) in a random position, and the new sequence is \(S_{t,2}\) with a length of \(m + \delta\).

C. TRANSMITTING QUANTUM VOTING STATES

The server sends the sequence \(S_{t,2}\) to voter \(V_1\) through a regular quantum authentication channel. After \(V_1\) receives the complete sequence \(S_{t,2}\), they cooperate to perform a security check test to ensure that the quantum authentication channel between the server and \(V_1\) is secure. To do so, the server first publishes the positions and corresponding bases, rectilinear basis or diagonal basis \([[\{0\} \{0\}, \{1\} \{1\} \{+\} \{+\}, \{-\} \{-\}]]\), respectively, of all detection particles. Then, \(V_1\) measures each detection particle in the correct basis and records its classical measurement result. Next, \(V_1\) announces all classical measurement results of \(\delta\) detection particles, and the server announces their initial states. According to the open information, \(V_1\) and the server set check whether the initial states and the measurement results are consistent. For example, if the initial state is \(|+\rangle\), the corresponding classical measurement result should be 1 when measuring in basis \([[\{0\}; \{+\}, \{-\}\]\). When all \(\delta\) detection particles are consistent, the test passes, and the protocol continues; otherwise, the test fails, and they \((V_1\) and the server) abort the protocol.

D. CASTING VOTES

If \(V_1\) votes “yes”, he (she) applies a random operation \(U^j_1 = X^{k_{1,1}} Z^{k_{1,2}}\) on the \(j\)th voting carrier according to the keys \(k_{1,1}\) and \(k_{1,2}\). Here,

\[
X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{4}
\]

\(X\) can turn \(0\) to \(1\) and \(1\) to \(0\), \(Z\) can protect \(0\) from being changed, and \(1\) turns to \(-1\). Otherwise, he (she) applies a random operation \(U^j_1 = X^{|x_{1,1}|} Z^{x_{1,2}}\) on the \(j\)th voting carrier \(p^j_1\), where \(x_{1,1}, x_{1,2}\) are randomly chosen in the set \(\{0, 1\}\).

\(V_1\) arbitrarily disturbs the order of voting carriers. To achieve this, \(V_1\) randomly selects a permutation \(C_1 = c_1^j c_1^{j+1} \cdots c_1^m\) from the set \(P_m\) with a uniform probability, where \(P_m\) is constructed by all permutations of \(\mathbb{Z}_m := \{1, \ldots, m\}\) [35]. Taking \(C_1\) as the private address index, voting carrier \(p^j_1\) temporarily moves from the \(j\)th position to the \(c^j_1\)th position. The renumbered voting carrier sequence is \(S_{1,1}\). Hereafter, as the server does, \(V_1\) prepares \(\delta\) detection particles \(d_1^1, \ldots, d_1^k\) and inserts each detection particle \(d_1^j\) into \(S_{1,1}\) randomly. The newly combined sequence is \(S_{1,2}\); furthermore, all the qubits are labeled as \(s_{1,2}^1, s_{1,2}^2, \ldots, s_{1,2}^m\).

\(V_1\) sends sequence \(S_{1,2}\) to the next voter \(V_2\) through a regular quantum authentication channel. Using all \(\delta\) detection particles, \(V_1\) and \(V_2\) first cooperate to perform a security check test. If the test passes, \(V_1\) announces the original order of all voting carriers, and \(V_2\) resets the voting carriers into the original order arrangement. The following voting process for \(V_2\) is the same as that for \(V_1\).

Other voters, individually, take the same actions as \(V_1\) and \(V_2\), including performing security check tests, casting votes and transmitting particle sequences. Finally, all traveling voting carriers \(p^1_1, \ldots, p^j_1, \ldots, p^m_1\) return to the server securely. A special note that should be mentioned is that if any voter does not strictly follow the established procedure of the protocol, the voting result is “no”, which means the proposal is rejected.

E. DETERMINING THE VOTING RESULT

For each voting carrier \(p^j_1\) \((1 \leq j \leq m)\), the server measures it in its original basis, rectilinear basis or diagonal basis, encoded by \(t_{1,1}\), obtaining the classical measurement result \(r_j\). Then, the server compares whether \(r_j\) is consistent with \(U^n_{1,1} \cdots U^n_{1,1}\). For all voting carriers \(p^1_1, p^2_1, \ldots, p^m_1\), if there is
any inconsistency, the server rejects the proposal. Otherwise, the proposal is accepted.

F. VERIFYING THE “NO” VOTES
Whenever the proposal passes, any voter \( V_i \) who votes “no” can use anonymous quantum communication technology [20], [36] to broadcast the abort signal “1”. Once “1” appears, the protocol aborts.

III. EXAMPLE AND EXPERIMENT
Thus far, we have shown the details of the QAVSP protocol. For a better understanding, we set the number of voters \( n \) to be 3 and give two simple examples for two cases, where all voters cast “yes” votes and only one voter casts a “no” vote. Then, we perform two experimental tests of the examples on the online quantum computer, (i.e., ibmq_qasm_simulator), of the IBM Corporation placed on the cloud [37]. For simplicity, we present only the core of the encoding and decoding votes process and ignore some parts of the state distribution and security check tests.

A. EXAMPLE 1: ALL VOTERS CAST “YES” VOTES
Following the QAVSP protocol, the server shares two private binary strings \( k_{1,1} \) and \( k_{1,2} \) with \( V_1 \), \( k_{2,1} \) and \( k_{2,2} \) with \( V_2 \), and \( k_{3,1} \) and \( k_{3,2} \) with \( V_3 \). Assume that

\[
\begin{align*}
    k_{1,1} &= \{0, 0, 0, 0, 0\} ; \\
    k_{1,2} &= \{1, 0, 0, 0, 1\} ; \\
    k_{2,1} &= \{0, 0, 0, 1, 1\} ; \\
    k_{2,2} &= \{0, 1, 0, 0, 0\} ; \\
    k_{3,1} &= \{1, 1, 1, 0, 0\} ; \\
    k_{3,2} &= \{1, 0, 1, 0, 0\} .
\end{align*}
\]

Suppose the 5 voting carriers are

\[
    p_i^1, p_i^2, p_i^3, p_i^4, p_i^5 = \{0, 1\}, \{+\}, \{-\}, \{0\}. \tag{5}
\]

Since all voters cast “yes” votes, according to \( k_{1,1} \) and \( k_{1,2} \) for \( i = 1, 2, 3 \), \( V_1 \) applies \( X^{k_{1,1}}Z^{k_{1,2}} \) on \( p_i \). That is, \( V_1 \) applies the unitary operations \( Z, I, X, I, Z \) on voting carriers \( p_i^1, p_i^2, p_i^3, p_i^4, p_i^5 \), respectively. Successively, \( V_2 \) applies \( I, Z, I, X, X \) and \( V_3 \) applies \( XZ, X, XZ, I, X \) on the five voting carriers. Thus, the casting votes process can be described as follows:

\[
\begin{align*}
    |1\rangle &= XZ \cdot I \cdot Z|0\rangle; \\
    |0\rangle &= X \cdot Z \cdot I\{1\}; \\
    \{\}_+ &= XZ \cdot I \cdot \{+\}; \\
    \{\}_- &= I \cdot X \cdot \{\}_+; \\
    |0\rangle &= X \cdot X \cdot Z|0\rangle. \tag{6}
\end{align*}
\]

In step 2.5, the server measures each voting carrier \( p_{ij} \) (1 \( \leq j \leq 5 \)) in its original basis decided by \( k_{ij} \). In other words, the server measures \( p_{11}^i, p_{12}^i, p_{14}^i \) on a rectilinear basis and \( p_{13}^i, p_{15}^i \) on a diagonal basis. From the foregoing equation (7), the measurement outcomes \( |1\rangle, |0\rangle, |{\_}\_\_, |{\_}\_\_, |0\rangle \) entirely follow the rule of all voters casting “yes” votes. Therefore, the server accepts the proposal, which is exactly what all voters expect.

To verify the realization of the protocol, we perform an experimental test of the above example on the online IBM quantum computer (ibmq_qasm_simulator).

In Figure 2, the five voting carriers \( p_{11}^1, p_{12}^1, p_{14}^1, p_{15}^1 \) are denoted by \( q_0, q_1, q_2, q_3, q_4 \), respectively. Part I is the stage of preparing quantum voting states. Since the initial states are all \( |0\rangle \) in the IBM quantum computer, the process of preparing five voting carriers can be described as follows:

\[
\begin{align*}
    p_0 &: |0\rangle; \\
    p_1 &: |1\rangle = X|0\rangle; \\
    p_2 &: |+\rangle = H|0\rangle; \\
    p_3 &: |\_\_\_\_\_\_\_\rangle = H \cdot X|0\rangle; \\
    p_4 &: |0\rangle; \tag{7}
\end{align*}
\]

where \( H \) gate is in the form of

\[
    H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \tag{8}
\]

\( H \) can turn \( |0\rangle \) into \(|+\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \) and \( |1\rangle \) into \(|\_\_\_\_\_\_\_\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \).

Part II is the vote casting stage, and the measuring states and collecting votes stages are in part III.

In Figure 3, after running 2048 shots on ibmq_qasm_simulator and measuring \( p_{11}^1, p_{12}^1, p_{14}^1 \) in basis \(|0\rangle, |1\rangle\), \( p_{13}^i, p_{15}^i \) in basis \(|+\rangle, |\_\_\_\_\_\_\_\rangle\), the classical measurement results of \( p_{11}^1, p_{12}^1, p_{14}^1, p_{15}^1 \) are 1, 0, 1, 1, 0 with a probability of 100%. Obviously, the experimental results are consistent with the theoretical results deduced in equation (7). Therefore, the proposal is accepted as all voters expect.

B. EXAMPLE 2: ONLY ONE VOTER CASTS A “NO” VOTE
We suppose \( k_{i,j} \) (1 \( \leq i \leq 3, 1 \leq j \leq 2 \)) is the same as equation (5) and \( p_{ij}^i \) (1 \( \leq j \leq 5 \)) is the same as equation (7).
Without any loss of generality, suppose that $V_3$ is the voter who casts “no”. Thus, for each voting carrier $p_j^i$ ($1 \leq j \leq 5$), $V_3$ randomly applies one unitary operation in $\{I, X, Z, XZ\}$ on each $p_j^i$. Suppose that $V_3$ chooses $X, Z, I, Z, XZ$ for $p_1^1, p_2^2, p_3^3, p_4^4, p_5^5$, respectively. As usual, controlled by $k_1, k_1, k_1, k_1, k_2, k_2, k_2, V_1$ applies $Z, I, I, I, I, Z$ and $V_2$ applies $I, Z, I, X, X$ on the five voting carriers in turn. The voting process can be given as follows:

\[
\begin{align*}
|1\rangle &= X \cdot Z \cdot Z |0\rangle; \\
|1\rangle &= Z \cdot Z \cdot I |1\rangle; \\
|+\rangle &= I \cdot I \cdot |+\rangle; \\
|+\rangle &= Z \cdot X \cdot |1\rangle; \\
|0\rangle &= XZ \cdot X \cdot Z |0\rangle.
\end{align*}
\]

(10)

As above, in Figure 4, part I is the stage of preparing quantum voting states. Part II describes how each voter is voting. The voting results are opened in part III.

In Figure 5, after running 2048 shots on ibmq_qasm_simulator and measuring $p_1^1, p_2^2, p_5^5$ in basis $\{|0\rangle, |1\rangle\}$, $p_1^3, p_3^4$ in basis $\{|+\rangle, |−\rangle\}$, the classical measurement results of $p_1^1, p_2^2, p_3^3, p_4^4, p_5^5$ are 1, 1, 0, 0, 0 with a probability of 100%. Obviously, the experimental results are different from the theoretical results 1, 0, 1, 1, 0. These results indicate that there is at least one voter who is voting “no”. The server rejects the proposal while protecting the privacy of each voter. $V_3$ can anonymously verify that the voting result is the same as his (her) wish, which means his (her) vote “no” is correctly counted.

IV. ANALYSIS

First, we analyze the correctness of our protocol. The voting result is successfully obtained, with a small probability of failure approaching 0 as $m$ increases. Then, we analyze the desirable properties of reliability, privacy, fairness and verifiability.

A. CORRECTNESS

Theorem 1: If the voters all agree to the proposal, then the protocol achieves the perfect result with a probability of $1$. If there exists at least one voter who disagrees with the proposal, then the probability of success of the protocol is at least $1 - \frac{1}{2^m}$.

Proof: If all the voters vote “yes”, they honestly implement the protocol as planned, and the final state obtained by the server is in $U_1^1 \cdots U_2^2 U_3^3$ for $j = 1, 2, \cdots, m$ without any deviation. Obviously, the proposal passes with a probability of 1.

If at least one voter rejects the proposal, then the voting result should be “no”. Without any loss of generality, we assume that $V_j$ is one of the voters voting “no”. For each voting carrier, $V_j$ applies an operation that is random in $\{I, X, Z, XZ\}$ with a uniform probability of $\frac{1}{4}$, leading to the final voting carrier obtained by the server being a state in the set of $\{|0\rangle, |1\rangle\}$ or $\{|+, |−\rangle\}$ with probability $\frac{1}{4}$. When the server measures the voting state in its original basis, the disturbance of each voting carrier escapes detection successfully with a probability of $\frac{1}{2}$. For $m$ voting carriers, the error rate of passing the proposal is $\frac{1}{2^m}$. Hence, the probability of rejecting the proposal is $1 - \frac{1}{2^m}$, which approaches 1 as $m$ increases to a sufficiently large number.

What needs to be pointed out is that the above correctness is considered under the ideal situation. Apparently, when some attackers exist in the transmission process, they might disturb voting carriers, leading to an incorrect voting result without being detected.

In our protocol, the voting carriers and detection particles are thoroughly mixed and transmitted in the form of quantum blocks. For any outside attacker, each transmitted particle is randomly encoded in two mutually unbiased orthogonal bases. Through theoretical guessing, each $s_{i,j}^{l}$ for $i = t, 1 \leq i \leq t, 1 \leq j \leq m + 8$ is in the rectilinear basis $\{|0\rangle, |1\rangle\}$ or diagonal basis $\{|+, |−\rangle\}$ with a uniform probability of $\frac{1}{2}$; thus, the density matrix of each transmitted particle $s_{i,j}^{l}$ for any outside attackers is always

\[
\rho_{s_{i,j}^{l}} = \frac{1}{4} (|0\rangle\langle 0| + |1\rangle\langle 1| + |+\rangle\langle +| + |−\rangle\langle −|)
\]

\[
= \frac{1}{2} I.
\]

(11)

Since all transmitted particles are in the maximally mixed states, the attackers cannot distinguish the detection particles from the voting carriers. Actually, the security of every quantum authenticated channel between two participants is ensured by a security check test, which is similar to that of the BB84 protocol [5]. Directly referring to the security proof of the BB84 protocol, we know that regardless of the type of attack an attacker utilizes, his (her) active action will inevitably disturb some of the detection states and be detected.
by honest participants, thus failing the protocol. Hence, it is impossible for attackers to affect the voting results.

B. RELIABILITY

Theorem 2: In the ideal model, if all quantum authentication channels are secure, no legal voter can abort the voting process.

Proof: Under the ideal environment hypothesis, the process of security check tests can ensure that all quantum states traveling among all participants are secure. If any voter $V_i$ $(1 \leq i \leq n)$ refuses to implement the protocol as it is planned, such as terminating the transmission of the quantum states, the voting result is rejected.

C. PRIVACY

Theorem 3: For each voter, the value of his (her) vote is kept secret from all others. After the voting protocol, the output is only agreement or rejection. No useful information regarding how many voters vote “no” is leaked whenever the voting result is rejected.

Proof: In our protocol, the adversaries might be outside eavesdroppers and malicious participants, i.e., malicious voters and semi-honest servers. The next part will focus on how to achieve privacy and defend against potential adversaries.

As we mentioned above, each transmitted particle is in the maximally mixed state for an outside eavesdropper, and she is unable to acquire any useful information for breaking privacy without being detected by the server or honest voters. Hereafter, we mainly discuss participant attacks. Generally, participant attacks are a much more powerful threat in multiparty quantum protocols [38, 39]. In participant attacks, the participants have an unparalleled advantage in reasonably obtaining valuable resources, such as intermediate data $k_i, k_i$, voting carriers $p_i$ and detection particles $d_i$. Here, we analyze two possible cases of participant attacks in detail as follows.

Case 1 (Attacks by a Semi-Honest Server):

During transmission, the voting carriers and detection particles are intensively mixed. Even with two private binary strings $k_i, k_i$, the server still cannot separate voting carriers from detection particles. Actually, if he (she) makes some active attacks on the transmitted particles, he (she) will be detected by honest voters as an outside eavesdropper in testing detection particles. In addition, the number of traveling voting carriers is disrupted, and it is impossible for the server to obtain the original order when the security check test fails. Hence, compared with outside eavesdroppers, the server has no advantages in breaking privacy.

Case 2 (Attacks by Malicious Voters):

Taking $V_i$ as an example, suppose that $V_{i+1}$ wants to attack $V_i$ by eavesdropping on the transmitted voting carriers $p_1, \ldots, p_{i-1}, p_i, \ldots, p_m$. Following the protocol process, $V_{i+1}$ does not know the exact state of $p_1, \ldots, p_{i-1}, p_i, \ldots, p_m$ received by $V_i$. To ensure how $V_i$ is voting, $V_{i+1}$ should determine the Pauli operations applied by $V_i$. Without any loss of generality, assume that the compound systems of all transmitted voting carriers $p_1, \ldots, p_i, \ldots, p_m$ are in state $\rho^A$ when $V_i$ just receives them from $V_{i-1}$.

This occurs because the set of $2^m$ unitary matrices $\{a_1, b_2 \in \{0, 1\}^m\}$ forms an orthogonal basis. Expanding any message state, $\rho^A$ in $\{a_1, b_2 \}$ base is given [40):

$$\rho^A = \sum_{a_i, b_i} l_{a_i, b_i} a_1^{a_i} b_1^{b_i},$$

where $l_{a_i, b_i} = \frac{\sqrt{\mu(a_1^{a_i} b_1^{b_i})}}{2^n}$.

If $V_i$ is voting “yes”, $V_i$ applies the operation $U_i = a_1^{a_i} b_1^{b_i}$, and the state turns to $\rho^A$. Using this formalism, we can easily determine $\rho^A$. However, for $V_{i+1}$, he (she) does not know the exact unitary operation that is applied by $V_i$. That is, $U_i$ is a random element in the set $\{a_1, b_2 \in \{0, 1\}^m\}$ with the same probability of $\frac{1}{2^m}$ for $V_{i+1}$. The above evolution can be described as follows:

$$\rho^A = \sum_j P_{U_j}(U_i \otimes \mathbb{I}) \rho^A(U_i \otimes \mathbb{I})^\dagger$$

$$= \frac{1}{2^m} \sum_{k_1, k_2} (X^{k_1, k_2} \otimes \mathbb{I}) \rho^A(X^{k_1, k_2} \otimes \mathbb{I})^\dagger$$

$$= \frac{1}{4^m} \sum_{k_1, k_2} (X^{k_1, k_2} \otimes \mathbb{I}) (\sum_{a_i, b_i} l_{a_i, b_i} a_1^{a_i} b_1^{b_i}) (X^{k_1, k_2} \otimes \mathbb{I})^\dagger$$

$$= \frac{1}{4^m} \sum_{k_1, k_2} l_{a_i, b_i} a_1^{a_i} b_1^{b_i} (X^{k_1, k_2} \otimes \mathbb{I}) a_1^{a_i} b_1^{b_i}$$

$$= \frac{1}{4^m} \sum_{k_1, k_2} l_{a_i, b_i} a_1^{a_i} b_1^{b_i} (X^{k_1, k_2} \otimes \mathbb{I}) (1 + \delta_{a_i, 0} \delta_{b_1, 0}) a_1^{a_i} b_1^{b_i}$$

$$= \frac{1}{2^m} \mathbb{I}. \ (13)$$

If $V_i$ is voting “no”, $V_i$ randomly applies an operation in the set of $I, X, Z, XX$ on each traveling voting carrier $p_i$ $(1 \leq i \leq m)$. Similar to the above analysis, for $V_{i+1}$, the evolutionary process is as follows:

$$\rho^N = \sum_j P_{U_j}(U'_i \otimes \mathbb{I}) \rho^N(U'_i \otimes \mathbb{I})^\dagger$$

$$= \frac{1}{2^m} \mathbb{I}. \ (14)$$

Regardless of what $V_i$ votes, each voting carrier is always in the maximum mixed state. Thus, $V_{i+1}$ cannot determine how $V_i$ is voting. Therefore, the privacy is maintained.

As described above, strictly following the protocol, it is impossible for $V_{i+1}$ to break the privacy of the vote of $V_i$. If $V_{i+1}$ wants to eavesdrop some useful information, he (she)...
must take some active actions under quantum mechanics. \( V_{i+1} \) has the right to possess \( \rho^A \), and he (she) needs to attack \( \rho^A \). Since the transmission process of \( \rho^A \) is similar to the BB84 protocol’s process, any effective attack will inevitably disturb the detection particles and be discovered by \( V_{i-1} \) and \( V_i \) in the security check test among them. Therefore, the most predictably successful attack is that \( V_{i-1} \) and \( V_{i+1} \) are malicious and conspire together. By lying to \( V_i \), \( V_{i-1} \) and \( V_{i+1} \) can easily escape the security check tests of \( V_{i-1} \) and \( V_i \) and of \( V_i \) and \( V_{i+1} \). In such circumstances, \( V_{i-1} \) and \( V_{i+1} \) can easily determine the operation \( U_i \) applied by \( V_i \). For example, \( V_{i-1} \) prepares fake voting carriers \( f_1^i, \ldots, f_{j-1}^i, \ldots, f_{1}^m \) and sends them to \( V_i \) instead of \( p_1^i, \ldots, p_{j-1}^i, \ldots, p_{1}^m \). Then, \( V_i \) performs the unitary operation \( U_i \) on the fake voting carriers to complete his (her) vote casting process. When \( V_{i+1} \) receives the fake voting carriers from \( V_i \), \( V_{i-1} \) collaborates with \( V_{i+1} \) to measure each voting carrier in his (her) initial basis and compare it to the initial state, thus easily obtaining \( U_i \). Since \( k_{i,1} \) and \( k_{i,2} \) are private binary strings shared between the server and \( V_i \), \( V_{i-1} \) and \( V_{i+1} \) have no idea about them. Benefiting from one-time pad encryption technology [40], without \( k_{i,1} \) and \( k_{i,2} \), \( V_{i-1} \) and \( V_{i+1} \) still cannot determine how \( V_i \) is voting just with \( U_i \). Privacy is still maintained even if \( V_{i-1} \) conspires with some other malicious voters. Therefore, each voter can maintain the privacy of his (her) vote.

D. FAIRNESS

Theorem 4: In our protocol, no individual can obtain a partial votes tally or any useful information about someone else’s vote before he (she) is voting. He (she) decides to vote entirely on his (her) initial wishes.

Proof: As we all know, if a voter determines some useful information about some other votes beforehand, he (she) might change his (her) vote. In our protocol, the voters encrypt their votes twice. First, each voter \( V_i \) (0 ≤ \( i \) ≤ \( n - 1 \)) encrypts his (her) vote using the classical one time pad technique with secret keys \( k_{i,1} \) and \( k_{i,2} \). In addition to the server and \( V_i \), no individual can decode the vote from ciphertext. However, the server cannot cooperate with any voter. Second, \( V_i \) encodes the ciphertext by performing the unitary operations \( (I, X, Z, ZX)^m \) on the voting carriers \( p_1^i, \ldots, p_{j-1}^i, \ldots, p_{1}^m \). Since the density matrix of each voting carrier is in a maximum mixed state and invariant under the encoding operations in the entire procedure of the protocol, no useful information about the vote of \( V_i \) is leaked. Therefore, no voter can determine how the other voters are voting, and each voter casts a vote based on his (her) initial wishes. The fairness of our protocol can be maintained.

E. VERIFIABILITY

Theorem 5: Any voter in our protocol who votes “no” can verify whether his (her) vote has been correctly counted.

Proof: Directly from the analysis of the correctness, our protocol almost achieves correctness if it completes the implementation as it is planned. Whenever there is an error, any voter who rejects the proposal but finds an inconsistency in the voting results can anonymously declare that his (her) vote has not been counted correctly and abort the protocol in step 2.6. Actually, the verifiability of each voter will be eclipsed when all voters agree to the proposal and the proposal is rejected. To resist this potential threat, another authority center can be introduced to monitor the behavior of the server in step 2.6.

V. CONCLUSION

We investigate how qubits can be useful for anonymous one-vote veto activity with strong privacy at the expense of the protocol’s error rate with respect to \( m \). Any malicious voter cannot arbitrarily abort the protocol, thus achieving reliability. During the entire voting process, each voter votes completely according to his (her) wish without affecting other voters’ votes; therefore, the protocol has the property of fairness. Verifiability provides an approach in which any voter who votes “no” can check whether the voting result is consistent with his (her) vote. With the explosive progress of quantum technology, classical one-vote veto protocols are facing serious threats. Our QA VSP protocol is a significant attempt under the current technology. There is one interesting and valuable open question that deserves further investigation in the future. Our QA VSP protocol is just preliminary theoretical research. Future research can assess how to handle the effects of device noise, which is very common in NISQ [41]; and realize the process in a real-world situation.

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