Fulling-Davies-Unruh effect and spontaneous excitation of an accelerated atom interacting with a quantum scalar field

Zhiying Zhu\textsuperscript{2} and Hongwei Yu\textsuperscript{1,2,*}

\textsuperscript{1}CCAST(World Lab.), P. O. Box 8730, Beijing, 100080, P. R. China
\textsuperscript{2}Department of Physics and Institute of Physics, Hunan Normal University, Changsha, Hunan 410081, China \dag

Abstract

We investigate, from the point of view of a coaccelerated frame, the spontaneous excitation of a uniformly accelerated two-level atom interacting with a scalar field in a thermal state at a finite temperature $T$ and show that the same spontaneous excitation rate for the uniformly accelerated atom in the Minkowski vacuum obtained in the inertial frame can only be recovered in the coaccelerated frame assuming a thermal bath at the Fulling-Davies-Unruh temperature $T_{FDU} = a/2\pi$ for what appears to be the Minkowski vacuum to the inertial observer. Our discussion provides another example of a physical process different from those examined before in the literature to better understand the Fulling-Davies-Unruh effect.

PACS numbers: 04.62.+v, 03.70.+k, 42.50.Lc,
I. INTRODUCTION

It is well known that the particle content of a quantum field theory is observer dependent. According to the works of Fulling, Davies and Unruh (FDU), for a uniformly accelerated observer, the Minkowski vacuum is seen to be equivalent to a thermal bath of Rindler particles at a temperature $T_{\text{FDU}} = a/2\pi$ (FDU effect \[1\]). The existence of the FDU effect has been shown to be mandatory for the consistency of quantum field theory in various cases, where it has been demonstrated that the way to obtain agreement between physical observables calculated from the inertial and coaccelerated frame point of view is by assuming the FDU effect \[2, 3, 4, 5, 6\]. For the weak decay of a uniformly accelerated proton, the same lifetime can be obtained in the inertial frame and the coaccelerated frame considering the FDU effect \[3\]. For the bremsstrahlung effect associated with a uniformly accelerated point charge, Higuchi, Matsas and Sudarsky \[4\] have shown that the same response rate of the uniformly accelerated point charge to the Larmor radiation can be obtained in both the inertial and coaccelerated frame assuming the FDU thermal bath with an infinite number of zero-energy Rindler photons in the Rindler wedge, and the same has been shown to be true in the presence of boundaries \[3\]. Meanwhile the generalization to the quantum bremsstrahlung effect for a uniformly accelerated point scalar source has also been made\[6\]. As another example, besides the weak decay of the proton and the bremsstrahlung effect, demonstrating the necessity of the FDU effect for the consistency of quantum field theory, we will compute, from the point of view of a coaccelerated frame, the spontaneous excitation rate of a uniformly accelerated two-level atom interacting with a massless scalar field, and show that equality can only be established between the excitation rates obtained in the inertial and the coaccelerated frames by assuming the FDU effect.

Let us note that the spontaneous excitation of a uniformly accelerated atom interacting with a massless scalar field \[7, 8\] and with electromagnetic fields \[8\] in a Minkowski vacuum has already been studied in the inertial frame \[9\] using the formalism proposed by Dalibard, Dupont-Roc and Cohen-Tannoudji \[10, 11\], which demands a symmetric operator ordering of atom and field variables and allows one to separately calculate the contributions of vacuum fluctuations and radiation reaction to the spontaneous excitation rate of the accelerated atom. It is found that for a uniformly accelerated atom transition from ground state to excited states becomes possible even in vacuum. This phenomenon provides a physically appealing interpretation of the FDU effect, since it gives a transparent illustration for why an accelerated detector clicks (See Ref. \[12\] for a discussion in a different context and Ref. \[13\] for a non-perturbative approach to study the interaction of a uniformly accelerated detector, modelled by a harmonic oscillator which may be regarded as a simple version of an atom, with a quantum field in (3+1) dimensional spacetime).

The purpose of the present paper is to show that the rate of change of the atomic energy for a uniformly accelerated atom interacting with a massless scalar field in vacuum obtained in the inertial frame for both cases with and without boundaries \[7, 8\] can be recovered in the coaccelerated frame by using Fulling’s quantization in conjunction with the fact that the
Minkowski vacuum is a thermal state of Rindler particles at a temperature \( T_{FDU} = a/2\pi \). So, we will consider a uniformly accelerated two-level atom interacting with a massless scalar field in the coaccelerated frame both in space with and without the presence of boundary, and evaluate the rate of change of the mean atomic energy of the atom assuming a thermal bath of Rindler particles at a finite temperature \( T \). Since the rate of change of the atomic energy is a scalar, the same value of this observable must be obtained in the inertial and coaccelerated frame. We will show that only when the temperature equals the FDU value, i.e., \( T = a/2\pi \), can agreement in both frames be obtained.

II. THE GENERAL FORMALISM FOR THERMAL FLUCTUATIONS AND RADIATION REACTION

We consider a two-level atom in interaction with a massless scalar field, and assume the atom has a constant proper acceleration \( a \) along the \( x \) direction in the Minkowski coordinates. In the Rindler wedge, the line element [14] can be described by

\[
d s^2 = e^{2a\xi}(d\tau^2 - d\xi^2) - dy^2 - dz^2 .
\]

The Rindler coordinates are related to the usual Minkowski coordinates by

\[
t = \frac{e^{a\xi}}{a} \sinh a\tau , \quad x = \frac{e^{a\xi}}{a} \cosh a\tau .
\]

In the Rindler coordinates our accelerated atom is stationary on the trajectory \( x = (\tau, \xi, y(\tau), z(\tau)) \). The stationary trajectory guarantees that the undisturbed atom has stationary states, \(|-\rangle\) and \(|+\rangle\), with energies \(-\frac{1}{2} \omega_0\) and \(+\frac{1}{2} \omega_0\) and a level spacing \( \omega_0 \). The Hamiltonian that governs the time evolution of the atom with respect to the proper time \( \tau \) is written in Dick’s [15] notation

\[
H_A(\tau) = \omega_0 R_3(\tau) ,
\]

where \( R_3 = \frac{1}{2} |+\rangle \langle +| - \frac{1}{2} |-\rangle \langle -| \). The free Hamiltonian of the quantum scalar field with respect to \( \tau \) is

\[
H_F(\tau) = \int d\omega \int dk_y \int dk_z \omega b_{\omega,k_y,k_z}^\dagger(\tau) b_{\omega,k_y,k_z}(\tau) ,
\]

where \( b_{\omega,k_y,k_z}^\dagger \) and \( b_{\omega,k_y,k_z} \) denote the creation and annihilation operators for a Rindler particle with transverse momentum \( k_y \) and \( k_z \) and frequency \( \omega \). The quantization of the field in the Rindler wedge can be carried out by the expansion in terms of annihilation and creation operators as

\[
\phi(x) = \int d\omega \int dk_y \int dk_z [ b_{\omega,k_y,k_z}(\tau) \nu_{\omega,k_y,k_z}(x) + H.c.] ,
\]

3
where $\nu_{\omega,k_y,k_z}$ are the Rindler modes. Let us note that the positive acceleration $a$ of the atom makes only the Rindler wedge $R^+$ ($x > |t|$) completely accessible to a coaccelerated observer [16]. Thus in the following sections, we will only consider the Rindler modes in $R^+$. Following Ref. [7], the Hamiltonian that describes the interaction between the atom and the quantum field can be written

$$H_I(\tau) = \mu R_2(\tau) \phi(x).$$

(6)

Here $\mu$ is a coupling constant which we assume to be small, and $R_2 = \frac{1}{2} i (R_+ - R_-)$, where $R_+ = |+\rangle\langle-|$ and $R_- = |-\rangle\langle+|$. The coupling is effective only on the trajectory of the atom. The Heisenberg equations of motion for the dynamical variables of the atom and the field can be derived from the Hamiltonian $H = H_A + H_F + H_I$: 

$$\frac{d}{d\tau} R_\pm(\tau) = \pm i \omega_0 R_\pm(\tau) + i \mu \phi(x) [R_2(\tau), R_\pm(\tau)],$$

(7)

$$\frac{d}{d\tau} R_3(\tau) = i \mu \phi(x) [R_2(\tau), R_3(\tau)],$$

(8)

$$\frac{d}{d\tau} b_{\omega,k_y,k_z}(\tau) = -i \omega b_{\omega,k_y,k_z}(\tau) + i \mu R_2(\tau) [\phi(x), b_{\omega,k_y,k_z}(\tau)].$$

(9)

We can split the solutions of these equations of motion into the “free” and “source” parts.

Let us now assume that the system is in a thermal bath of arbitrary temperature $T$ as seen by the coaccelerated observer (Rindler observer), so that the scalar field is in a thermal state (as opposed to a vacuum state in the inertia frame) and the density matrix is given by $\rho = e^{-\beta H_F} = e^{-H_F/T}$, and the atom is in the state $|a\rangle$. Our aim is to identify and separate the contributions of quantum thermal fluctuations (as opposed to vacuum fluctuations in the inertial frame) and radiation reaction to the rate of change of the mean atomic energy. For this purpose, we choose a symmetric ordering between the atom and field variables, and separate the two contributions of thermal fluctuations and radiation reaction to the rate of change of $H_A$ (cf. Ref. [7, 10, 11]),

$$\frac{dH_A(\tau)}{d\tau} = \left( \frac{dH_A(\tau)}{d\tau} \right)_{TF} + \left( \frac{dH_A(\tau)}{d\tau} \right)_{RR},$$

(10)

where

$$\left( \frac{dH_A(\tau)}{d\tau} \right)_{TF} = \frac{1}{2} i \omega_0 \mu (\phi^f(x) [R_2(\tau), R_3(\tau)] + [R_2(\tau), R_3(\tau)] \phi^f(x)), $$

(11)

representing the contribution of thermal fluctuations and

$$\left( \frac{dH_A(\tau)}{d\tau} \right)_{RR} = \frac{1}{2} i \omega_0 \mu (\phi^s(x) [R_2(\tau), R_3(\tau)] + [R_2(\tau), R_3(\tau)] \phi^s(x)), $$

(12)

representing that of radiation reaction.
We can separate $R_2$ and $R_3$ into their free part and source part, and take the expectation value in the field’s and atom’s states. We obtain, up to order $\mu^2$,

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle^R_{TF} = 2i\mu^2 \int_{\tau_0}^{\tau} d\tau' C^F_\beta(x, x') \frac{d}{d\tau} \chi^A(\tau, \tau') , \quad (13)$$

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle^R_{RR} = 2i\mu^2 \int_{\tau_0}^{\tau} d\tau' C^F_\beta(x, x') \frac{d}{d\tau} C^A(\tau, \tau') , \quad (14)$$

where the superscript $R$ denotes the calculation performed in the Rindler coordinates and $|\rangle = |a, \beta \rangle$ representing the atom in the state $|a\rangle$ and the field in the thermal state $|\beta\rangle$. The statistical functions $C^F_\beta(x, x')$ and $\chi^F_\beta(x, x')$ of the field at a nonzero temperature $T = 1/\beta$ are defined as

$$C^F_\beta(x, x') = \frac{1}{2} \langle \{ \phi^f(x), \phi^f(x') \} \rangle^F_\beta = \frac{1}{2} \text{tr}(\rho \{ \phi^f(x), \phi^f(x') \})/\text{tr}(\rho) \, , \quad (15)$$

$$\chi^F_\beta(x, x') = \frac{1}{2} \langle [ \phi^f(x), \phi^f(x') ] \rangle^F_\beta = \frac{1}{2} \text{tr}(\rho [ \phi^f(x), \phi^f(x') ])/\text{tr}(\rho) \, , \quad (16)$$

and those of the atom as

$$C^A(\tau, \tau') = \frac{1}{2} \langle a | R^f_2(\tau), R^f_2(\tau') \rangle |a\rangle \, , \quad (17)$$

$$\chi^A(\tau, \tau') = \frac{1}{2} \langle a | [ R^f_2(\tau), R^f_2(\tau') ] |a\rangle \, . \quad (18)$$

$C^F_\beta (C^A)$ is called the symmetric correlation function of the scalar field at a finite temperature $T = 1/\beta$ (atom), $\chi^F_\beta (\chi^A)$ is its linear susceptibility. The explicit forms of the statistical functions of the atom are given by

$$C^A(\tau, \tau') = \frac{1}{2} \sum_b |\langle a | R^f_2(0) |b\rangle|^2 \left( e^{i\omega_{ab}(\tau-\tau')} + e^{-i\omega_{ab}(\tau-\tau')} \right) , \quad (19)$$

$$\chi^A(\tau, \tau') = \frac{1}{2} \sum_b |\langle a | R^f_2(0) |b\rangle|^2 \left( e^{i\omega_{ab}(\tau-\tau')} - e^{-i\omega_{ab}(\tau-\tau')} \right) , \quad (20)$$

where $\omega_{ab} = \omega_a - \omega_b$ and the sum extends over a complete set of atomic states. In order to get the statistical functions for the field, we consider firstly the two point function for the field at a nonzero temperature $T = 1/\beta$.

$$\langle \phi^f(x)\phi^f(x') \rangle^F_\beta = \text{tr}[\rho \phi^f(x)\phi^f(x')] / \text{tr}(\rho) \, . \quad (21)$$

From Eq. (5), one finds

$$\langle \phi^f(x)\phi^f(x') \rangle^F_\beta = \int_0^\infty d\omega \int_{-\infty}^\infty dk_y \int_{-\infty}^\infty dk_z \left[ \sum_{n=0} (n+1) e^{-n\omega/T} u_{\omega,k_y,k_z}(x) u^*_{\omega,k_y,k_z}(x') \right]$$

$$+ \sum_{n=1} n e^{-n\omega/T} u_{\omega,k_y,k_z}(x) u^*_{\omega,k_y,k_z}(x') \right] / \sum_{n=0} e^{-n\omega/T} \, . \quad (22)$$
Here we assume that there exist an infinite number of Rindler particles in the thermal bath. We can obtain

\[
\langle \phi^f(x)\phi^f(x') \rangle_\beta = \int_0^\infty d\omega \int_{-\infty}^\infty dk_y \int_{-\infty}^\infty dk_z \left[ \frac{e^{i\omega/\beta}}{e^{\omega/\beta} - 1} u_{\omega,k_y,k_z}(x) v_{\omega,k_y,k_z}^*(x') + \frac{1}{e^{\omega/\beta} - 1} v_{\omega,k_y,k_z}^*(x) u_{\omega,k_y,k_z}(x') \right].
\]

(23)

The statistical functions of the field can be calculated using (23).

III. EXCITATION RATE IN THE COACCELERATED FRAME IN THE UNBOUNDED SPACE

In this section, we will apply the previously developed formalism to calculate, in the coaccelerated frame, the rate of change of the mean atomic energy for a uniformly accelerated atom interacting with a massless free scalar field at finite temperature \( T = 1/\beta \). In the Rindler coordinates, the atom is static and its trajectory can be described by

\[
\tau, \quad \xi = y(\tau) = z(\tau) = 0.
\]

(24)

The wave equation for the scalar field in the Rindler coordinates is given by

\[
\left[ e^{-2a\xi} \left( \frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right] \phi(x) = 0.
\]

(25)

It can be shown that the orthonormal mode solution with a positive frequency is

\[
v_{\omega,k_y,k_z}(x) = \frac{1}{2\pi^2 \sqrt{a}} \sinh^{1/2} \left( \frac{\pi\omega}{a} \right) K_{\omega/a} \left( \frac{k_\perp}{a} e^{a\xi} \right) e^{ik_y y + ik_z z - i\omega \tau},
\]

where \( k_\perp = \sqrt{k_y^2 + k_z^2} \) and \( K_{\nu}(x) \) is the Bessel function of imaginary argument. Now we can evaluate the two point function (23) for the trajectory (24), and get

\[
\langle \phi^f(x)\phi^f(x') \rangle_\beta = \frac{1}{4\pi^4a} \int_0^\infty d\omega \int_{-\infty}^\infty dk_y \int_{-\infty}^\infty dk_z \sinh \left( \frac{\pi\omega}{a} \right) K_{\omega/a}^2 \left( \frac{k_\perp}{a} \right) \times \left[ \frac{e^{i\omega/\beta}}{e^{\omega/\beta} - 1} e^{-i\omega(\tau - \tau')} + \frac{1}{e^{\omega/\beta} - 1} e^{i\omega(\tau - \tau')} \right].
\]

(27)

With the help of the following integral,

\[
\int_{-\infty}^{\infty} dk_y \int_{-\infty}^{\infty} dk_z K_{\omega/a}^2 \left( \frac{k_\perp}{a} \right) = \frac{\pi \omega^2}{\sinh \left( \frac{\pi\omega}{a} \right)},
\]

we obtain

\[
\langle \phi^f(x)\phi^f(x') \rangle_\beta = \frac{1}{4\pi^2} \int_0^\infty d\omega \left( \frac{e^{i\omega/\beta}}{e^{\omega/\beta} - 1} e^{-i\omega(\tau - \tau')} + \frac{1}{e^{\omega/\beta} - 1} e^{i\omega(\tau - \tau')} \right). \quad (29)
\]
The statistical functions of the field, (15) and (16), can now be written
\[ C^F_{\beta}(x, x') = \frac{1}{8\pi^2} \int_0^\infty d\omega \omega \left( 1 + \frac{2}{e^{\omega/T} - 1} \right) \left( e^{-i\omega(\tau-\tau')} + e^{i\omega(\tau-\tau')} \right), \quad (30) \]
\[ \chi^F_{\beta}(x, x') = \frac{1}{8\pi^2} \int_0^\infty d\omega \omega \left( e^{-i\omega(\tau-\tau')} - e^{i\omega(\tau-\tau')} \right). \quad (31) \]

Note that \( \chi^F_{\beta}(x, x') \) has no temperature dependence and agrees with the linear susceptibility in vacuum. This is a result of the fact that only the field commutator appears in (16) and the linear susceptibility thus does not depend on the state of the field. With a substitution \( u = \tau - \tau' \), the contributions Eq. (13) and Eq. (14) to the rate of change of the atomic energy can be evaluated to get
\[
\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle^R_{TF} = -\frac{\mu^2}{8\pi^2} \sum_b \omega_{ab}|\langle a | R^f_2(0) | b \rangle|^2 \int_0^\infty d\omega \omega \left( 1 + \frac{2}{e^{\omega/T} - 1} \right) \times \int_0^\infty du (e^{-i\omega u} + e^{i\omega u})(e^{-i\omega_{ab} u} + e^{i\omega_{ab} u}) \quad (32)
\]
\[
\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle^R_{RR} = -\frac{\mu^2}{8\pi^2} \sum_a \sum_b \omega_{ab}|\langle a | R^f_2(0) | b \rangle|^2 \int_0^\infty d\omega \omega \times \int_0^\infty du (e^{-i\omega u} - e^{i\omega u})(e^{-i\omega_{ab} u} - e^{i\omega_{ab} u}) \quad (33)
\]

Here we have extended the range of integration to infinity for sufficiently long times \( \tau - \tau_0 \). After the evaluation of the integrals we obtain
\[
\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle^R_{TF} = -\frac{\mu^2}{2\pi} \left[ \sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a | R^f_2(0) | b \rangle|^2 \left( \frac{1}{2} + \frac{1}{e^{\omega_{ab}/T} - 1} \right) \right] - \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a | R^f_2(0) | b \rangle|^2 \left( \frac{1}{2} + \frac{1}{e^{\omega_{ab}/T} - 1} \right) \quad (34)
\]
for the contribution of the thermal fluctuations to the rate of change of atomic excitation energy, and
\[
\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle^R_{RR} = -\frac{\mu^2}{2\pi} \left( \sum_{\omega_a > \omega_b} \frac{1}{2} \omega_{ab}^2 |\langle a | R^f_2(0) | b \rangle|^2 + \sum_{\omega_a < \omega_b} \frac{1}{2} \omega_{ab}^2 |\langle a | R^f_2(0) | b \rangle|^2 \right) \quad (35)
\]
for that of radiation reaction. A comparison with the spontaneous excitation rate of an accelerated atom calculated in the inertial frame [7] shows that the contribution of radiation reaction to the rate of change of the atomic energy is the same as that calculated in the inertial frame, and thus is observer independent and temperature independent. While the contributions of vacuum fluctuations are equal only when the temperature equals the FDU value, i.e., \( T = a/2\pi \). Hence the same total rate of change of the atomic energy can be obtained both in the inertial and coaccelerated frames, only when we assume a thermal state of Rindler particles at a temperature \( T = a/2\pi \) to a coaccelerated observer for what appears to be a vacuum state to an inertial observer. This is the FDU effect.
IV. A SPACE-TIME WITH A REFLECTING PLANE BOUNDARY

It is well known that the presence of boundaries modifies the quantum fluctuations of fields both in a vacuum state and a thermal state and can lead to a lot of novel effects, such as the Casimir effect \[17\], the light-cone fluctuations \[18\], the Brownian (random) motion of test particles caused by quantum fluctuations in vacuum and quantum fluctuations at finite temperature \[19\], and so on. Therefore, the investigation of spontaneous excitation rate of accelerated atoms interacting with the scalar field in the presence of a reflecting plane boundary is an interesting issue and has been carried out in the inertial frame \[8\]. The results obtained there show that, with the presence of a reflecting boundary, a uniformly accelerated atom in a vacuum does not have to behave as if it were static in a thermal bath at a temperature \( T = a/2\pi \), in the sense that the spontaneous excitation rate is changed, by the presence of the boundary, in such a way that it is different from what one would expect for an inertial atom in a thermal bath in the same space with the boundary. We will, however, show that the same excitation rate can be obtained in the coaccelerated frame assuming the FDU effect.

We assume that a perfectly reflecting boundary for the scalar field is located at \( z = 0 \) in space, the atom is being uniformly accelerated in the \( x \) direction with a proper acceleration \( a \) at a distance \( z \) from the boundary, and the whole system is in a thermal bath at a temperature \( T = 1/\beta \). The atom’s trajectory is now described in the Rindler coordinates by

\[
\tau , \quad \xi = y(\tau) = 0 , \quad z(\tau) = z .
\] (36)

To satisfy the Dirichlet boundary condition, \( \phi(x)|_{z=0} = 0 \), the normalized Rindler mode function with a positive frequency for the scalar field becomes

\[
u_{\omega,k_y,k_z}(x) = \frac{1}{\pi^2\sqrt{2a}} \sinh^{1/2} \left( \frac{\pi \omega}{a} \right) K_{i\omega/a} \left( \frac{1}{a} k_\perp e^{a\xi} \right) \sin(k_z z) e^{ik_y y - i\omega \tau} .
\] (37)

From Eq. (23) we can calculate the two-point function of the field at a finite temperature \( T = 1/\beta \) for the trajectory (36) to get

\[
\langle \phi^f(x)\phi^f(x') \rangle_\beta = \frac{1}{4\pi^4 a} \int_0^\infty d\omega \int_{-\infty}^\infty dk_y \int_{-\infty}^\infty dk_z \sinh \left( \frac{\pi \omega}{a} \right) K_{i\omega/a} \left( \frac{k_\perp}{a} \right) [1 - \cos(2k_z z)]
\times \left( \frac{e^{\omega/T}}{e^{\omega/T} - 1} e^{-i\omega(\tau - \tau')} + \frac{1}{e^{\omega/T} - 1} e^{i\omega(\tau - \tau')} \right) .
\] (38)

With the help of the following integral

\[
\int_{-\infty}^\infty dk_y \int_{-\infty}^\infty dk_z K_{i\omega/a}^2 \left( \frac{k_\perp}{a} \right) [1 - \cos(2k_z z)] = \frac{a \pi^2}{\sinh(\pi \omega/a)} \left[ \omega - \frac{\sin(2\omega \sin^{-1}(a z))}{2z\sqrt{1 + a^2 z^2}} \right] ,
\] (39)
we find
\[
\langle \phi(x) \phi(x') \rangle = \frac{1}{4\pi^2} \int_0^\infty d\omega \left[ \omega - \sin\left(\frac{2\omega \sinh^{-1}(az)}{2z\sqrt{1 + a^2z^2}}\right) \right] \times \left( \frac{e^{\omega/T}}{e^{\omega/T} - 1} e^{-i\omega(\tau - \tau')} + \frac{1}{e^{\omega/T} - 1} e^{i\omega(\tau - \tau')} \right).
\]

(40)

From the general form (15) and (16), we can obtain the corresponding statistical functions of the field
\[
C^F_{\beta}(x, x') = \frac{1}{8\pi^2} \int_0^\infty d\omega \left[ \omega - \sin\left(\frac{2\omega \sinh^{-1}(az)}{2z\sqrt{1 + a^2z^2}}\right) \right] \left( 1 + \frac{2}{e^{\omega/T} - 1} \right) \left( e^{-i\omega(\tau - \tau')} + e^{i\omega(\tau - \tau')} \right),
\]

(41)

\[
\chi^F_{\beta}(x, x') = \frac{1}{8\pi^2} \int_0^\infty d\omega \left[ \omega - \sin\left(\frac{2\omega \sinh^{-1}(az)}{2z\sqrt{1 + a^2z^2}}\right) \right] \left( e^{-i\omega(\tau - \tau')} - e^{i\omega(\tau - \tau')} \right).
\]

(42)

Again \(\chi^F_{\beta}(x, x')\) has no temperature dependence as expected, it however does rely on the acceleration. With a substitution \(u = \tau - \tau'\) and extending the range of integration to infinity for sufficiently long times \(\tau - \tau_0\), we find the contributions of thermal fluctuations (13) and radiation reaction (14) to the rate of change of the atomic energy,
\[
\left\langle \frac{dH_A(\tau)}{dt} \right\rangle_{TF}^R = -\frac{\mu^2}{2\pi} \left[ \sum_{\omega_0 > \omega_b} \frac{1}{2} \omega^2_{ab} |\langle a| R^f_2(0)|b \rangle|^2 f(\omega_{ab}, a, z) \left( \frac{2}{e^{\omega_{ab}/T} - 1} \right) \right. \\
\left. - \sum_{\omega_0 < \omega_b} \frac{1}{2} \omega^2_{ab} |\langle a| R^f_2(0)|b \rangle|^2 f(\omega_{ab}, a, z) \left( \frac{2}{e^{\omega_{ab}/T} - 1} \right) \right],
\]

(43)

and
\[
\left\langle \frac{dH_A(\tau)}{dt} \right\rangle_{RR}^R = -\frac{\mu^2}{2\pi} \left[ \sum_{\omega_0 > \omega_b} \frac{1}{2} \omega^2_{ab} |\langle a| R^f_2(0)|b \rangle|^2 f(\omega_{ab}, a, z) \right. \\
\left. + \sum_{\omega_0 < \omega_b} \frac{1}{2} \omega^2_{ab} |\langle a| R^f_2(0)|b \rangle|^2 f(\omega_{ab}, a, z) \right],
\]

(44)

where
\[
f(\omega_{ab}, a, z) = 1 - \frac{1}{2\omega_{ab} z \sqrt{1 + a^2z^2}} \sin\left(\frac{2\omega_{ab} \sinh^{-1}(az)}{a}\right).
\]

(45)

Comparing the above results with Eqs. (34) and (35), one can see that the function \(f(\omega_{ab}, a, z)\) gives the modification induced by the presence of the boundary. Letting \(T \to 0\), we obtain the contribution of vacuum fluctuations
\[
\left. \left\langle \frac{dH_A(\tau)}{dt} \right\rangle_{TF}^R \right|_{T \to 0} = -\frac{\mu^2}{2\pi} \left[ \sum_{\omega_0 > \omega_b} \frac{1}{2} \omega^2_{ab} |\langle a| R^f_2(0)|b \rangle|^2 f(\omega_{ab}, a, z) \right. \\
\left. - \sum_{\omega_0 < \omega_b} \frac{1}{2} \omega^2_{ab} |\langle a| R^f_2(0)|b \rangle|^2 f(\omega_{ab}, a, z) \right],
\]

(46)
Adding up the contributions of vacuum fluctuations (46) and radiation reaction (44) to the Rindler atom yields the total spontaneous excitation rate,

$$
\left. \langle \frac{dH_A(\tau)}{d\tau} \rangle^R \right|_{\text{tot}} \bigg|_{T \to 0} = -\frac{\mu^2}{2\pi} \sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a | R_2(0) f | b \rangle|^2 f(\omega_{ab}, a, z). \tag{47}
$$

This indicates that if no FDU thermal bath were assumed to exist, the Rindler atom would be stable in its ground state with no excitation. Yet this is different from the behavior of an inertial atom (cf. Eq. (23) in Ref. [8]), because of the appearance of the acceleration $a$ in function $f(\omega_{ab}, a, z)$, revealing that without assuming the FDU effect what a coaccelerated observer would see will be different from an inertial observer. This is in contrast to the case without the presence of the boundary.

Now, let us recall the spontaneous excitation rate of a uniformly accelerated atom in the inertial frame, with the presence of a plane boundary [8]. The contributions of vacuum fluctuations and radiation reaction are

$$
\left. \langle \frac{dH_A(\tau)}{d\tau} \rangle^{\text{Inertial}} \right|_{\text{V F}} = -\frac{\mu^2}{2\pi} \left[ \sum_{\omega_a > \omega_b} \frac{1}{2} \omega_{ab}^2 |\langle a | R_2(0) | b \rangle|^2 f(\omega_{ab}, a, z) \left(\frac{1}{2} + \frac{1}{e^{2\pi\omega_{ab}/a} - 1}\right) \right. \\
\left. - \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a | R_2(0) | b \rangle|^2 f(\omega_{ab}, a, z) \left(\frac{1}{2} + \frac{1}{e^{2\pi\omega_{ab}/a} - 1}\right) \right], \tag{48}
$$

and

$$
\left. \langle \frac{dH_A(\tau)}{d\tau} \rangle^{\text{Inertial}} \right|_{\text{RR}} = -\frac{\mu^2}{2\pi} \left[ \sum_{\omega_a > \omega_b} \frac{1}{2} \omega_{ab}^2 |\langle a | R_2(0) | b \rangle|^2 f(\omega_{ab}, a, z) \right. \\
\left. + \sum_{\omega_a < \omega_b} \frac{1}{2} \omega_{ab}^2 |\langle a | R_2(0) | b \rangle|^2 f(\omega_{ab}, a, z) \right]. \tag{49}
$$

A comparison with Eq. (43) and Eq. (44) shows that, the same as in the unbounded case, the contribution of radiation reaction is again observer independent, however, only when we assume a thermal bath of Rindler particles at the temperature $T = a/2\pi$ for the coaccelerated observer, is Eq. (43) equal to Eq. (48) and the two observers agree on the total rate of the change of the atomic energy for the accelerated atom. Let us note that, due to the appearance of $f(\omega_{ab}, a, z)$, the total spontaneous excitation rate of a uniformly accelerated atom in the presence of a reflecting boundary deviates from the pure thermal rate in the unbounded case. However, this discrepancy does not imply that the exact final thermal equilibrium is not achieved [20]. As a matter of fact, since function $f(\omega_{bd}, z, a)$ is an even function of $\omega_{bd}$, one can show, by the same argument as that in Ref [20], that exact thermal equilibrium will be established at the FDU temperature $T_{FDU} = a/2\pi$, even though the accelerated atom radiates and absorbs differently from an inertial atom immersed in the thermal bath.
V. CONCLUSIONS

In conclusion, from the point of view of the coaccelerated frame, we have examined the spontaneous excitation of a uniformly accelerated two-level atom interacting with a massless quantum scalar field both in a free space and in a space with a reflecting plane boundary and separately calculated the contributions of thermal fluctuations and radiation reaction to the rate of change of the mean atomic energy, assuming a thermal bath of Rindler scalar particles at a finite temperature $T$. Our results show that the same spontaneous excitation rate for a uniformly accelerated atom in the Minkowski vacuum obtained in the inertial frame can be recovered in the coaccelerated frame ONLY when assuming a thermal bath at the Fulling-Davies-Unruh temperature $T_{FDU} = a/2\pi$ for what appears to be the Minkowski vacuum to the inertial observer.

As an example of a different physical process, our results give further support to endeavors already made by other authors in other different contexts [3, 4, 5, 6] in clarifying the confusion on what the Fulling-Davies-Unruh effect means. The Fulling-Davies-Unruh effect implies that a uniformly accelerated atom (observer) interprets as a thermal bath of Rindler particles at the temperature $T_{FDU} = a/2\pi$ what an inertial observer sees as a vacuum devoid of particles, or in different words, the physical observables calculated in the inertial frame can be recovered in the coaccelerated frame by using Fulling’s quantization in conjunction with the fact that the Minkowski vacuum is a thermal state of Rindler particles at a temperature $T_{FDU} = a/2\pi$.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grants No.10375023 and No.10575035, and the Program for New Century Excellent Talents in University (NCET, No. 04-0784).

[1] S. A. Fulling, Phys. Rev. D 7, 2850 (1973); P. C. W. Davies, J. Phys. A 8, 609 (1975); W. G. Unruh, Phys. Rev. D 14, 870 (1976).
[2] W. G. Unruh and R. M. Wald, Phys. Rev. D 29, 1047 (1984).
[3] G. E. A. Matsas and D. A. T. Vanzella, Phys. Rev. Lett 87, 151301 (2001).
[4] A. Higuchi, G. E. A. Matsas and D. Sudarsky, Phys. Rev. D 45, R3308 (1992); Phys. Rev. D 46, 3450 (1992).
[5] Danilo T. Alves and Luís C. B. Crispino, Phys. Rev. D 70, 107703 (2004).
[6] Hai Ren and Erick J. Weinberg, Phys. Rev. D 49, 6526 (1994).
[7] J. Audretsch and R. Müller, Phys. Rev. A 50, 1755 (1994).
[8] H. Yu and S. Lu, Phys. Rev. D 72, 064022 (2005); ibid, D 73, 109901 (2006).
[9] Z. Zhu, H. Yu and S. Lu, Phys. Rev. D 73, 107501 (2006).
[10] J. Dalibard, J. Dupont-Roc and C. Cohen-Tannodji, J. Phys. (France) 43, 1617 (1982).
[11] J. Dalibard, J. Dupont-Roc and C. Cohen-Tannodji, J. Phys. (France) 45, 637 (1984).
[12] T. Padmanabhan, Class. Quan. Grav., 2, 117 (1985).
[13] S. -Y. Lin and B. L. Hu, Phys. Rev. D 73, 124018 (2006); eprint, gr-qc/0610024.
[14] N. D. Birrell and P. C. W. Davies, Quantum Field Theory in Curved Space (Cambridge Univ. Press, Cambridge, 1982).
[15] R. H. Dicke, Phys. Rev. 93, 99 (1954).
[16] D. G. Boulware, Ann. Phys. (N. Y.) 124, 169 (1980).
[17] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. 51, 793 (1948).
[18] H. Yu and L. H. Ford, Phys. Rev. D 60, 084023 (1999); Phys. Lett. B 496, 107 (2000); e-print gr-qc/0004063; H. Yu and P.X. Wu, Phys. Rev. D 68, 084019 (2003).
[19] H. Yu and L. H. Ford, Phys. Rev. D 70, 065009 (2004); H. Yu and J. Chen, ibid 70, 125006 (2004); H. Yu, J. Chen and P. Wu, JHEP 0602, 058 (2006).
[20] B. Garbrecht and T. Prokopec, Class. Quantum. Grav. 21, 4993 (2004).