The simulation study of damage assessment of a surface-to-air missile system attacked by an anti-radiation missile and the optimization of fuze-warhead coordination

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Abstract. The final damage effectiveness of warhead is the most important objective in missile system design. To obtain the damage probability of a certain anti-radiation missile warhead against a typical field surface-to-air missile system and find the optimal fuze-warhead coordination strategy, the studies of modelling, simulation and optimization are carried out. First, the vulnerability of the target is analysed and the target vulnerability model is established. Secondly, the calculation models of fragments field and over-pressure field are established to realize the calculation of encounter between warhead and target. Finally, the damage assessment model is established to calculate the damage probability of the parts and the whole system. Through Monte Carlo simulation, the single-shot damage probability of the missile is obtained when the parameters of the encounter between warhead and target are randomly selected according to a certain probability distribution. And the optimal fuze control strategies of the missile at different entrance angles are obtained through optimization calculation. If the entrance angle of the missile cannot be identified, the optimal fuze initiation strategy of the anti-radiation missile in this paper is a delay of 13ms. The results show that the research method of this paper can be applied to the missile weapon system with the function of entrance angle identification to maximize the damage efficiency.

1. Introduction

A certain field surface-to-air missile system is an advanced weapon system for short-range medium-low altitude air defense. The system is highly integrated and includes a full suite of functional units, including transport, missile launch, and radar. The system can complete air defense operations independently and has the ability to quickly respond and launch missiles in motion. Because of the serious threat of this system to low altitude targets, it becomes the key target of the anti-radiation missiles.

There have been many studies on radar damage assessment of anti-radiation missile warheads. Webster. R. D. from the USA described the radar targets with cuboid units, established the encounter model for the warhead and the radar target, and established the damage probability model of the shock wave and fragments of the single-shot warhead on the single radar target. The Monte-Carlo method was used to calculate the damage probability, and the influence of fuze-warhead coordination strategy was studied [1]. Zhang Yi et al. established a single-shot damage probability model of a certain air-to-
ground anti-radiation missile, analyzed the missile's inefficacy killing zone, and calculated the single-shot damage probability with systematic error [2]. Lu Hongpeng et al. studied the damage probability of missile and target encounter based on fuze warhead coordination, focused on key technologies such as coordinate system and coordinate conversion and Monte-Carlo algorithm, and designed a simulation system for the encounter of missile and target [3]. Liu Yan et al. used the analytic hierarchy process and fuzzy comprehensive evaluation method to establish the damage effect evaluation model and damage evaluation index system of the warhead attacking to phased array radar [4]. Based on the structural and functional analysis of radar, Li Chao et al. established the damage assessment model of fragmentation warhead attacking to phased array radar and calculated the vulnerability of radar in each direction [5]. However, the above studies only involve the radar antenna and the square cabin, not the field full-function surface-to-air missile launching vehicle mentioned in this paper. On the other hand, there are few studies on fuze-warhead coordination strategy.

In this paper, the vulnerability analysis of a certain field surface-to-air missile system was carried out for first time. The calculation models of the fragmentation field and over-pressure field were established, and the encounter calculation of a certain anti-radiation missile warhead and the surface-to-air system was realized. Through Monte-Carlo stochastic simulation, the single-shot damage probability of the missile against the target and the optimal control strategy of the missile fuze were obtained.

2. Target analysis of the surface-to-air missile system

A field surface-to-air missile system is targeted as shown in figure 1. The system is highly integrated and includes both target searching, tracking and missile launch functions in one vehicle. The chassis of the system is the caterpillar type. The two radars responsible for search and fire control are integrated with the radar square cabin, which can rotate horizontally around the central axis of the vehicle body. The system uses the vertical launch module of 8 missiles, which is installed at the rotating shaft in the center of the square cabin.

Due to the complex structure and functions of the weapon system and the Extremely limited public information, this paper only takes the radar antenna and the square cabin module as the main target for analysis, while the vehicle chassis as the secondary target for simplified modeling.

Figure 1. Schematic diagram of a surface-to-air missile system with dimensions in meters.

2.1. Analysis and modelling of the surface-to-air missile system

The whole surface-to-air missile system is mainly divided into the vehicle body, square cabin, search radar and fire control radar for modeling.

Firstly, a cartesian coordinate system is established to be fixed with the vehicle body as the coordinate system of the whole anti-aircraft system. Both the search radar and the fire control radar can be rotated, so it is necessary to establish their respective cartesian coordinate systems and then
combine them with the cartesian coordinate systems of the square cabin through coordinate transformation. The square cabin can also rotate relative to the vehicle body, and the coordinate transformation method is also used to connect the vehicle body.

The whole system is composed of quadrilateral surface elements of a certain thickness, as shown in figure 1. The vehicle body is divided into body, caterpillar band, and missile box. The square cabin is divided into the housing, exhaust pipe, and circuit boards. The exhaust pipe is the main part of the air conditioning system, while the circuit boards are vertically arranged in the square cabin, which is the key part of the search and fire control radar system. The search radar is divided into the lower bracket, support leg, upper bracket, housing and phased array antenna, which is responsible for searching the airspace, warning and finding targets. Fire control radar is divided into bracket, housing and phased array antenna, which is responsible for locking the target and guiding the anti-aircraft missile to the target. Phased array antenna is composed of a large number of small elements arranged according to certain rules, with 20% redundancy. The above parts are modeled separately, and an equivalent duralumin thickness shall be set for each surface element for subsequent calculation, as shown in table 1.

In this paper, the vulnerability coefficient $\zeta$ is used to reflect the vulnerability characteristics of different parts of the surface-to-air missile system under the action of fragments' penetration or explosion shock wave. The vulnerability coefficient represents the ratio of the volume of the vulnerable part in the equivalent geometry to the total volume. The vulnerability coefficients used in each part of the system are shown in table 1.

Table 1. The vulnerability coefficient and equivalent duralumin thickness of each part of the surface-to-air missile system.

| Part Name                                             | Equivalent Duralumin Thickness | Vulnerability Coefficient |
|-------------------------------------------------------|-------------------------------|----------------------------|
| Shell of square cabin                                 | 10 mm                         | 0.3                        |
| Exhaust pipe                                          | 12 mm                         | 0.4                        |
| Bracket of fire control radar                         | 10 mm                         | 0.6                        |
| Lower bracket of search radar                         | 10 mm                         | 0.5                        |
| Support leg of search radar                           | 10 mm                         | 0.3                        |
| Missile box                                           | 15 mm                         | 0.5                        |
| Circuit boards of square cabin                        | 5 mm                          | 0.9                        |
| Vehicle body                                          | 15 mm                         | 0.2                        |
| Caterpillar band                                      | 100 mm                        | 0.1                        |
| Upper bracket of search radar                         | 10 mm                         | 0.5                        |
| Housing of search radar                               | 7 mm                          | 0.4                        |
| Phased array elements of search radar                 | 3 mm                          | 0.9                        |
| Housing of fire control radar                         | 7 mm                          | 0.4                        |
| Phased array elements of fire control radar           | 3 mm                          | 0.9                        |

2.2. The surface-to-air missile system function, damage level and damage tree

The phased array antennas used in the search radar and the fire control radar are key components of the whole radar system, which uses the electromagnetic wave to obtain the target information. The radar square cabin mainly contains the electronic equipment of the radar system, which is mainly responsible for the analysis and transmission of radar antennas’ signals. The vehicle body carries the radar system, while the missile box is a weapon subsystem containing eight missiles.

After each subsystem and its components are attacked by fragments and shock wave over-pressure, the characteristics of the surface-to-air missile system and the damage mode, degree, and the time to troubleshoot and recover the combat capability are the main factors affecting the combat effectiveness of the whole system, reflecting the damage degree of the target. Based on the above analysis, the damage of the surface-to-air missile system is divided into three levels [6]:

1) The M-class, or mission class damage, the surface-to-air missile system is suppressed. To eliminate the damage to the system, maintenance personnel need 1 to 24 hours;
2) The F-class, or fire control class damage. The entire system lost workability, and unable to follow the scheduled procedure, unable to track the target or indicate missile emission, and the radar lost its early warning and control interception capabilities. It takes 1-7 days and nights to remove obstacles by specialized maintenance organizations.

3) The K-class, or kill class damage. The surface-to-air missile system is completely damaged beyond repair or it is not economically feasible to repair the damage.

According to the above three classes, the damage situation of the surface-to-air missile system was judged, and the structure, function and vulnerability characteristics of each key component were analyzed, as well as the relationship between them. The influence of damage of key components on the whole system function is also analyzed. Finally, the M, F and K class damage trees were established respectively as shown in figure 2.

In this paper, it is considered that the damage of caterpillar band, exhaust pipe, vehicle body and search radar leg are M-class, the damage of square cabin housing, search radar antenna housing and fire control radar housing are F-class, and the damage of missile box, search radar phased array elements, fire control radar phased array elements, and square cabin circuit board are K-class.

The ‘+’ in figure 2 represents the logical relationship of ‘or’. Only two radar phased array elements in the damage tree have redundancy.
3. Damage assessment model of warhead against the surface-to-air missile system

The anti-radiation missile studied in this paper uses prefabricated fragments warhead, and the calculation models of the corresponding fragmentation field and the over-pressure field must be established to realize the calculation of the encounter between the warhead and the target, thus establishing the damage evaluation model.

3.1. Fragmentation field and over-pressure field

The establishment of warhead fragmentation and over-pressure field requires certain input data, including explosive charging parameters, charging structure and prefabricated fragments structure. Firstly, based on these parameters, the initial velocity, dispersion angle and dispersion direction angle of the fragments after the static detonation, as well as the over-pressure field of the detonation center are calculated, and the spatial analytical model under the cylindrical coordinate system is established. Then, the model generates the fragmentation field and the over-pressure field from the explosion of the warhead in flight state, which is combined with the data of blast point, attitude, and velocity of the missile in the target coordinate system. And the two fields are converted into a cartesian coordinate system, which can be used for missile and target encounter calculation and damage assessment.

In the warhead's fragmentation field, the trajectory of each fragment is simulated by ray, and each ray contains the mass, shape, initial velocity and other information of the fragment. The formulas in the references [5,7] and [8] are used to calculate the fragmentation dispersion angle, initial velocity and velocity attenuation in the establishment of the fragmentation field.

There are the following calculation methods for the peak over-pressure and positive pressure specific impulse of the shock wave [9]:

1) Calculating the dose converted from charge with case to exposed charge. For cylindrical shell charge, there is:

$$\omega_e = \omega \left[ \frac{\alpha}{2-\alpha} \frac{2(1-\alpha)}{r_0/r_f} (r_f/r_0)^{2(\gamma-1)} \right]$$  \hspace{1cm} (1)

Where $\omega$ is the dose of charge with case; $\alpha$ is the charge coefficient; $r_0/r_f$ is the ratio of the initial radius of the shell to the rupture radius; $\gamma$ is the polytropic exponent of detonation products.

2) The TNT equivalent $m_e$ is calculated based on the dose of exposed charge obtained from the conversion of formula 1, and then the shock wave peak over-pressure of charge is calculated. The formula is as follows:

$$\Delta p_m = \frac{1.379}{r / \sqrt[3]{m_e}} + \frac{0.543}{(r / \sqrt[3]{m_e})^2} - \frac{0.035}{(r / \sqrt[3]{m_e})^3} - \frac{0.0006}{(r / \sqrt[3]{m_e})^4} \left( r / \sqrt[3]{m_e} \leq 0.3 \right)$$  \hspace{1cm} (2)

$$\Delta p_m = \frac{0.607}{r / \sqrt[3]{m_e}} + \frac{0.032}{(r / \sqrt[3]{m_e})^2} - \frac{0.209}{(r / \sqrt[3]{m_e})^3} \left( 0.3 \leq (r / \sqrt[3]{m_e}) \leq 1 \right)$$  \hspace{1cm} (3)

$$\Delta p_m = \frac{0.065}{r / \sqrt[3]{m_e}} + \frac{0.397}{(r / \sqrt[3]{m_e})^2} - \frac{0.322}{(r / \sqrt[3]{m_e})^3} \left( 1 \leq (r / \sqrt[3]{m_e}) \leq 10 \right)$$  \hspace{1cm} (4)

Where $\Delta p_m$ is the peak over-pressure of the shock wave when the charge is exploded in the infinite air, and the unit is MPa; $m_e$ is the TNT equivalent, and the unit is kg; $r$ is the distance from the detonation center, and the unit is m.

3) The positive pressure specific impulse of TNT charge is calculated by the following formula:

$$i = \frac{C}{(r / \sqrt[3]{m_e})^{\frac{2}{3}} \omega}$$  \hspace{1cm} (5)
Where \( C \) is a dimensionless constant, and the unit of \( i \) is \( \text{Pa} \cdot \text{s} \).

According to the above calculation method, the average peak over-pressure of the shock wave and the specific impulse of the over-pressure at each surface element of the surface-to-air missile system can be obtained.

### 3.2. Calculation model for the encounter of warhead and target

Assuming that the surface-to-air missile system is stationary on the ground, to establish the model of warhead and target encounter, the initiation point must be determined according to the position relationship between the anti-radiation missile and target, fuze warhead coordination conditions, missile velocity vector, missile attitude, and other conditions. Then, the initiation point in the fragmentation field is moved to the initiation point in the target coordinate system through coordinate transformation, and the velocity vectors of the fragments relative to the target are updated according to the velocity vector of the warhead relative to the target. The trajectory ray of the fragments can be drew as shown in figure 3. The different colours of the rays represent different types of fragments, and the length of the rays are proportional to the corresponding fragments speed.

![Figure 3](image-url)

**Figure 3.** The track ray of the fragments and the point where penetration occurs (blue dots).

Based on the fragment trajectory ray, which surface elements of the target intersect with a fragment are calculated, then the order in which the fragment arrives at these intersection points and whether the fragment can penetrate the surface elements are determined. If penetration is possible, the residual velocity of the fragment is calculated, and the specific calculation method is detailed in the literature [5]. The residual velocity of the fragment as it travels a distance to the next intersection point can be calculated similarly, and whether the fragment can penetrate the next surface is determined. Finally, a series of penetration points can be obtained, as shown in figure 3 in the blue dots on the radar square cabin.

### 3.3. Calculation of damage probability

The damage probability of the whole surface-to-air missile system is calculated by the damage probability of each component according to the damage tree. When calculating the damage probability of some components, all fragments penetrating on all surface elements of a component are taken as the effective damage fragments of the component, such as the support leg of search radar, etc., and the formula is as follows:

\[
p = 1 - e^{-\zeta \cdot n}
\]

Where the \( \zeta \) is the coefficient of vulnerability, and \( n \) is the total number of fragments penetrating surface elements.
In some components, each surface is not redundant, that is, as long as one of the surface elements that make up the component is damaged, the whole component is considered damaged, such as the square cabin circuit board. The following formula is used to calculate the damage probability of these components:

\[ p = 1 - \prod_{i=1}^{N} (1 - p_i) \]  

(7)

Where \( p_i \) is the damage probability of a surface element.

Other components, such as phased array elements of search radar and fire control radar, etc., are considered damaged if 20% of the surface elements that make up these components are damaged.

The damage probability of the above components needs to take into account damage of both fragments’ penetration and shock wave over-pressure. When the over-pressure at a surface element is greater than a certain threshold value, the element’s over-pressure damage probability is 0.95, and the damage probability of fragments’ penetration is calculated by the formula (6) or (7). Finally, the maximum value of both is the damage probability of this surface element.

The damage of shock wave over-pressure can be described by the standard damage law of peak over-pressure \( \Delta P \) and specific impulse \( I \) [10]:

\[ \left( \Delta P - P^* \right) \left( I - I^* \right) = K \]  

(8)

Where the \( \Delta P \) and \( I \) are the peak over-pressure and the specific impulse of the shock wave, respectively. \( P^* \) and \( I^* \) are the critical over-pressure and specific impulse of the target, respectively. \( K \) is a constant that depends on the vulnerability of the target. So, if the \( \Delta P \) and \( I \) of a surface element meet the following formula:

\[ \left( \Delta P - P^* \right) \left( I - I^* \right) \geq K \]  

(9)

Then the surface element is considered to have a 95% probability of being destroyed.

In this paper, the target vulnerability data of the search radar and the fire control radar under over-pressure are set as follows: \( P^* = 0.03 \) MPa, \( I^* = 250 \) Pa·s, \( K = 23 \). At the same time, the appropriate \( K \) values are set for other components such as the vehicle body. According to the above formulas, it is determined whether each surface element is damaged under the over-pressure of the shock wave.

After calculating the damage probability of each part of the system, the overall damage probability can be calculated according to the following formula [5]:

\[ P = 1 - \prod_{i=1}^{N} (1 - P_{or,i}) \times \left( 1 - \prod_{j=1}^{M_1} P_{and,j} \right) \times \cdots \times \left( 1 - \prod_{j=1}^{M_k} P_{and,j} \right) \]  

(10)

Where \( N \) is the number of non-redundant components, \( P_{or,i} \) is the damage probability of each non-redundant component; \( M_1, M_2, \ldots, M_k \) is the number of redundant components of each component group respectively; \( P_{and,j} \) is the damage probability of components in the redundant components group.

4. Study of the damage assessment based on Monte-Carlo method

Based on MATLAB software programming to achieve the damage probability calculation of anti-radiation missile attacking the surface-to-air missile system. The experiments are run on a workstation with two Intel(R) Xeon(R) Gold 5120 CPUs, and 128GB of memory.

4.1. The input parameters

In this paper, a typical example is used to evaluate the single-shot damage probability of an anti-radiation missile to the surface-to-air missile system. The main data to be entered in the calculation are:

1) The number and structure of prefabricated fragments and the explosive charge parameters etc. The TNT equivalent of the warhead in this paper is 3.6 kg and the charging coefficient is 0.33.
According to the theoretical calculation results, the fragments are divided into four groups, and the number, average mass, initial velocity, flying direction angle, angle of dispersion, and velocity attenuation coefficient in the air are set respectively;

2) The terminal attitude parameters of the missile. For example, the missile's attack angle in this paper is 3°, the sideslip angle is 2°, the terminal velocity is 200 m/s~300 m/s, and the included angle between the velocity vector and the ground is less than 10°;

3) The guidance accuracy of the missile, namely the circular error probability (CEP≤1 m). And the target point of the guidance is set as the vehicle body center. Then, when the z-direction coordinates of the initial position of the missile are randomly selected according to the normal distribution, the mean value is 3.5m and the variance is 1/1.1774, while the miss distance of the missile is also randomly selected according to the normal distribution, and the variance is 1/1.1774;

4) Reaction and judgment time after the missile fuze detects the target (0.9ms for the missile in this paper) and initiation delay time (the delay times of the missile in this paper are between 3ms and 23ms with 7 values which are average distribution).

4.2. The analysis of the calculation results and the discussion on the fuze warhead matching

In this paper, there are two variables for each calculation: the entrance angle of 8 positions and 7 delay times, so there are 56 conditions. Based on Monte-Carlo random sampling method, the single-shot damage probability of the missile was calculated. The more the sample size, the closer the results are to the real situation. In this paper, 1000 samples were sampled for each condition.

The image of a sampling calculation result when the entrance angle is around 90° and the initiation delay time is 11ms is shown in figure 4, and figure 5 is the top view corresponding to figure 4.

**Figure 4.** Results of a sampling calculation at 90° entrance angle and 11ms initiation delay time.

**Figure 5.** A top view of figure 4.

In figure 4 and figure 5, the black sector represents the range of sight of laser detection, and the corresponding position is the position of the missile when the target is detected, while the thick black
The yellow, red, blue, and green surfaces in the figures represent the velocity vectors of different fragments groups, and the central position is the location of the missile initiation point. In the figure, the blue scattered points on the search radar, the square cabin, and the vehicle body represent the holes on the corresponding surface that are penetrated by fragments, while the red surface elements represent the surfaces with damage probability greater than 90%.

The above images show that under this condition, in the random sampling calculation of Monte-Carlo, the missile caused relatively high damage probability to the search radar, vehicle body, square cabin, and other parts. Result data show that the surface-to-air missile system's M-class, F-class and K-class damage probability are all 1.

The average probabilities of all 56 conditions calculated through 1,000 Monte-Carlo random samplings are listed in table 2.

**Table 2.** The average damage probabilities of 56 conditions which are calculated by Monte-Carlo random sampling.

| 3 ms | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° |
|------|----|-----|-----|------|------|------|------|------|
| M-class | 0.087 | 0.137 | 0.176 | 0.105 | 0.031 | 0.137 | 0.170 | 0.185 |
| F-class | 0.109 | 0.191 | **0.566** | 0.138 | 0.027 | 0.170 | **0.546** | 0.219 |
| K-class | 0.002 | 0.008 | 0.785 | 0.019 | 0.000 | 0.006 | 0.737 | 0.011 |

| 8 ms | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° |
|------|----|-----|-----|------|------|------|------|------|
| M-class | **0.205** | 0.242 | 0.082 | 0.240 | 0.196 | 0.181 | 0.085 | 0.191 |
| F-class | **0.539** | 0.544 | 0.360 | 0.578 | 0.396 | **0.553** | 0.359 | **0.620** |
| K-class | 0.253 | 0.328 | **0.793** | 0.302 | 0.128 | 0.302 | 0.773 | 0.381 |

| 11 ms | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° |
|-------|----|-----|-----|------|------|------|------|------|
| M-class | 0.186 | 0.177 | 0.157 | 0.189 | 0.178 | 0.156 | 0.150 | 0.190 |
| F-class | 0.512 | **0.559** | 0.368 | **0.587** | **0.558** | 0.506 | 0.391 | 0.565 |
| K-class | 0.548 | 0.633 | 0.720 | 0.580 | 0.453 | 0.547 | 0.701 | 0.641 |

| 13 ms | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° |
|-------|----|-----|-----|------|------|------|------|------|
| M-class | 0.183 | 0.193 | **0.210** | 0.144 | 0.182 | 0.145 | **0.231** | 0.172 |
| F-class | 0.462 | 0.455 | 0.419 | 0.440 | 0.541 | 0.494 | 0.376 | 0.450 |
| K-class | 0.641 | 0.704 | 0.620 | 0.677 | 0.618 | 0.724 | 0.591 | 0.745 |

| 16 ms | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° |
|-------|----|-----|-----|------|------|------|------|------|
| M-class | 0.183 | 0.184 | 0.188 | 0.118 | 0.148 | 0.129 | 0.191 | 0.168 |
| F-class | 0.445 | 0.456 | 0.289 | 0.396 | 0.471 | 0.410 | 0.367 | 0.427 |
| K-class | **0.784** | **0.824** | 0.350 | **0.789** | 0.735 | **0.794** | 0.352 | **0.793** |

| 18 ms | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° |
|-------|----|-----|-----|------|------|------|------|------|
| M-class | 0.187 | 0.158 | 0.135 | 0.116 | 0.155 | 0.173 | 0.129 | 0.189 |
| F-class | 0.416 | 0.441 | 0.280 | 0.414 | 0.447 | 0.462 | 0.266 | 0.381 |
| K-class | 0.757 | 0.761 | 0.268 | 0.771 | **0.753** | 0.763 | 0.265 | 0.771 |

| 23 ms | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° |
|-------|----|-----|-----|------|------|------|------|------|
| M-class | 0.133 | 0.123 | 0.073 | 0.144 | 0.145 | 0.141 | 0.075 | **0.196** |
| F-class | 0.380 | 0.312 | 0.156 | 0.318 | 0.393 | 0.339 | 0.123 | 0.327 |
| K-class | 0.563 | 0.467 | 0.120 | 0.517 | 0.605 | 0.528 | 0.074 | 0.470 |

The bold numbers in table 2 indicate the maximum damage probability of each level corresponding to different delay times when the missile attacks the target at a certain entrance angle. In general, the damage probability of K-class is the highest when the entrance angle is 45° and the delay time is 16ms, which can reach 0.824.

If the missile can choose the entrance angle, then the 45° attack direction is optimal. If the missile can only identify the entrance angle but cannot choose it, then from the data in the table that, except for the two entrance angles of 90° and 270°, it is relatively optimal to choose the 16ms delay time. Because the 16ms and 18ms delay times at the entrance angle of 180° have similar damage probability. In fact, under the two entrance angles of 90° and 270°, the target presents the largest area in the field of view of the missile seeker, which is also the largest possibility in actual combat. Therefore, the delay time of 8ms is the optimal choice.

If the missile cannot identify the entrance angle and can only set a fixed delay, then it can be found that in general, when the delay time increases from 3ms to 23ms, the damage probabilities of K-class
increase first and then decrease, but the optimal delay time of the 90° and 270° entrance angles are inconsistent with other cases. Therefore, 13ms delay time is a better choice after comprehensive consideration. In this case, the average flight distance of the missile is about 3m, and the K-class damage probability corresponding to all the entrance angles is at a high level.

5. Conclusion
This paper analyzes the structure and function of a field surface-to-air missile system and establishes its target vulnerability model. Firstly, the relationship between the damage of key components and the damage of the whole surface-to-air missile system is described by the method of damage tree, and the damage trees of each level of the target were established. Then, the calculation models of the fragments field and the over-pressure field are established, and the encounter calculation of the warhead and the target is also realized. The damage effect of shock wave over-pressure is also analyzed, and the calculation method of corresponding damage probability is established. Then the calculation methods of three kinds of damage probability are established for each part. Finally, the damage probability of the whole system is obtained by combining fragments and over-pressure damage. MATLAB software programming is used to realize the calculation process. The Monte-Carlo random sampling method is used to calculate the single-shot damage probability of the missile to the target under the combined conditions of different entrance angles and different delay times, which provide suggestions for the optimization of missile initiation control.

The results indicate that the single-shot damage probability of the anti-radiation missile attacking a field surface-to-air missile system is up to a maximum of 0.824. And if the entrance angle of the missile cannot be identified, the best initiation strategy of the fuze is a 13ms delay. The research method in this paper can be applied to the missile with the function of entrance angle identification to maximize the damage probability.

Acknowledgements
This research is supported by “National Natural Science Foundation of China” (NSFC11702140) and the authors would like to thank the support from xi’an modern control technology research institute.

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