Boson Mapping and Nonlinear Response of Type-II Superconductors

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ABSTRACT

The vortices in a high-Tc superconductor with strong correlated pinning centers have been studied numerically using the mapping to charged bosons in two-dimensions(2D) and the Monte-Carlo algorithm. Considering the viscous dissipation of moving vortices we derived a nonlinear voltage response expression which describes different regimes and their crossover uniformly. This equation accords with experimental results.

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I. INTRODUCTION

The static and dynamic properties of vortices in the mixed state of high-Tc superconducors (HTSC) have been intensively studied both experimently and theoretically in recent years. For applying superconductors in external magnetic fields, it is important to minimize the dissipative loses of moving flux lines by improving the flux pinning with some kinds of strong correlated disorder (material inhomogeneities). The vortex dynamics with strong correlated pinning can be studied efficiently by exploiting the mapping between vortices and 2D bosons.

Similar to the physics of flux lines in a pure system, the statistical mechanics of vortices interacting with columnar pinning centers which are aligned parallel to the magnetic field may be mapped into the quantum mechanics of charged bosons in two-dimensions (2D). Table I summarizes the analogy between the vortices system, with the tilt modules $\tilde{\varepsilon}_1$ and thickness $L$ (length of vortex), and the corresponding 2D charged bosons system.

Table I Boson analogy applied to vortex transport

| Charged bosons | Mass | $h$ | $h/T$ | Pair potential | Charge | Electric field | Current |
|----------------|------|-----|-------|----------------|--------|--------------|--------|
| Vortices       | $\tilde{\varepsilon}_1$ | T   | L     | $2\varepsilon_0 K_0(r/\lambda)$ | $\phi_0$ | $\vec{z} \times \vec{J}/c$ | E(J)    |

In the Bose-Glass phase, the linear resistivity vanishes for low external current $J \ll J_c$, and the most important mechanics for vortex transport is “tunneling” between different columnar effect sites via the deformation of a pair of “superkinks”. This is very closely related to variable-range-hopping (VRH) transport of charged carriers in disordered semiconductors, and leads to the highly nonlinear expression.

By further exploring the analogy to two-dimensions (2D) Boses localized at randomly distributed defect sites, it leads to a “coulomb” gap in the distribution of the pinning energies $g(\varepsilon)$ near the chemical potential $\mu$ which separates the filled and empty energy levels. In the limit of infinitely long-range interaction, $\lambda \rightarrow \infty$, one would expect $g(\varepsilon)$
to vanish near the chemical potential according to a power law

$$g(\varepsilon) = |\varepsilon - \mu|^s.$$  

(1)

This distribution affects the vortex transport properties. An inplane current $\vec{J} \perp \vec{B}$ induces a Lorentz force per unit length $\vec{f}_L$ perpendicular to $\vec{J}$, acting on all the flux lines:

$$\vec{f}_L = \frac{\phi_0}{c} \hat{z} \times \vec{J}$$

(2)

which modifies the free energy of vortices system. In the boson picture this additional term represent an electric field $\vec{E} = \hat{z} \times \vec{J}/c$ acting on the particles carrying charge $\phi_0$. In the spirit of the thermally assisted flux-flow (TAFF) model of vortex transport, the superconducting resistivity $\rho = E/J$ may be written as

$$\rho \approx \rho_f \exp[-U_B(J)/kT]$$

(3)

where $\rho_f$ is a characteristic flux-flow resistivity, and $U_B$ represents an effective barrier height which is of the type $U_B(J) = U_0 (J_0/J)^p$.\[2\]

Besides this inverse power-law $U_B(J)$, some other types have also been suggested, such as the Anderson-Kim model $U_B(J) = U_c (1 - J_c/J)$\[3,4\] and the logarithmic barrier $U_B(J) = U_c \ln(J_0/J)$\[5,6\].

In a system with moving vortices, the total current density $J = J_s + J_n$, where the normal component

$$J_n \equiv E(J)/\rho_f$$

(4)

with $\rho_f \equiv \rho_n(B/B_{c2})$ which gives rise to dissipation and viscous drag\[4\]. Thus the effect of Lorentz force on the effective barrier $U_B$(i.e., the saddel-point free-energy price for a flux line to leave its columnar pin) should be attributed to the supercurrent component of the total current density

$$J_s \equiv J - E(J)/\rho_f.$$  

(5)
This paper is organized as follows. In the subsequent section we briefly describe the model and the Monte-Carlo simulation procedure which have been successfully used \[4\]. In section III we derived the current-voltage characteristics with the consideration of the viscous dissipation of moving vortices. In section IV we present results of simulation and compare them with some experimental results. Finally, a short summary concludes this work.

**II MODEL AND SIMULATION**

**A. Model**

For describing a system which has \(N_D\) columnar defect sites randomly distributed on the \(xy\) plane, we use the model in Ref.\[4\] with two-dimensional effective Hamiltonian

\[
H = \frac{1}{2} \sum_{i \neq j} n_i n_j V(r_{ij}) + \sum_{i=1}^{N_D} n_i t_i
\]  

(6)

and its grand-canonical counterpart

\[
\tilde{H} = H - \mu \sum_{i=1}^{N_D} n_i.
\]  

(7)

Here \(i, j = 1, 2, ..., N_D\) denote the defect sites. \(n_i = 0, 1\) represents the corresponding site occupation number\(n_i = 1\) if a flux line is bound in columnar defect \(n_i\), \(\sum_{i=1}^{N_D} n_i\) is the number of the flux lines. \(V(r) = 2\varepsilon_0 K_0(r/\lambda)\) represents the repulsive interaction between the lines; the modified Bessel function \(K_0(r/\lambda)\) describes a screened logarithmic interaction and \(\varepsilon_0 = (\phi_0/4\pi\lambda)^2\) with \(\lambda\) the London penetration length. We have also included a random site energy \(t_i\), originating in the variation of pin diameters. Their distribution \(P\) can be chosen to be centered at \(< t > = 0\), with width \(w\). For simplicity we assume a flat distribution

\[
P(t_i) = \theta(w - |t_i|)/2w
\]  

(8)

[\(\Theta(x)\) denotes the Heaviside step function].

For the interacting system\[3\], we define single particle site energies \(\varepsilon_i\) as follows:

\[
\varepsilon_i \equiv \frac{\partial H}{\partial n_i} = \sum_{j \neq i}^{N_D} n_j V(r_{ij}) + t_i.
\]  

(9)
For filled sites \( (n_i = 1) \), \( \varepsilon_i \) is the energy required to remove the particle at sites \( i \) to infinity, for empty sites \( (n_i = 0) \), correspondingly \( \varepsilon_i \) is the energy needed to introduce an additional particle from infinity to sites \( i \). In the thermal equilibrium, the chemical potential \( \mu \) separates the occupied and empty states. With the intervortex repulsion taken into account, the distribution of pinning energies \( g(\varepsilon) \) can be viewed as an interacting single-particle density of states and may be obtained from the statistics of the energy levels \( \varepsilon_i \). One would expect that the normalization of the energy distribution is

\[
\int_{-\infty}^{+\infty} g(\varepsilon) d\varepsilon = N_D/A = 1/d^2 \tag{10}
\]

with \( A \) the area of the system.

**B. Simulation**

Using a zero-temperature Monte-Carlo algorithm minimizing the total energy with respect to all possible one-vortex transfers\(^4\), we reproduce the results of Ref.\[4\] including the spatial configurations on the ground states and the distribution of the pinning energy \( g(\varepsilon) \). We have performed extensive studies for the cases \( N_D = 200, 400 \). In order to study the size effect, we simulate with \( N_D = 800 \) too. We have reproduced the result of Ref.\[4\] of the size effect. Fig.1 shows our result of the energy distribution, where one finds the “Coulomb” gap.

**III CURRENT-VOLTAGE CHARACTERISTICS**

Consider first the intermediate current regime \( J_1 < J < J_c \) \((J_1 \equiv U_0/\phi_0 d, \text{ and} U_0 = < U_k > \text{ the average of pinning energy} \)\(^4\)) in which the motion of a single vortex is unaffected by the other vortex in the sample\[^5\]. Driven by the external current \( J \), a vortex will start to leave its columnar pin by detaching a segment of length \( Z \) into the defect-free region, thereby forming a half-loop of transverse size \( R \). Considering the dissipation loses of moving flux lines, free-energy price of forming a half-loop of transverse size \( R \) by detaching a flux line segment \( Z \) into the defect-free region is approximatively

\[
\delta F(R, Z) \approx \tilde{\varepsilon}_1 R^2/Z + U_0 Z - f_L R Z + E(J)\phi_0 R Z/c\rho_f \tag{11}
\]
From Eq.\((\text{11})\) we estimate the saddle-point free-energy \(\delta(F_1)^*\) and find the current-voltage relationship

\[
E(J) = \rho_f \exp\left[-(E_k/kT)(J_1/J_s)\right]
\]

where \(E_k = d\sqrt{\varepsilon_1 U_0}\) and \(J_s\) is the supercurrent component described in Eq.\((\text{5})\).

For \(J_L < J < J_1(J_L\) is the current which satisfies \(R^*(J_L) = L\), we have to consider the configurational limitation imposed by other vortex. The most important thermally activated excitation will now be a double superkink\(^4\). This is the vortex analog of variable-range-hopping charge transport in disordered semiconductor\(^6\). The cost in free energy for such a configuration of transverse size \(R\) and extension \(Z\) along the magnetic-field direction will consist of three terms: (i) the double-superkink energy \(2E_k R/d\) stemming from the elastic term, (ii) the difference in pinning energies of the highest-energy occupied site, \(\varepsilon_i \approx \mu\) and the empty site at distance \(R\) with \(\varepsilon_i = \mu + \Delta(R)\), and (iii) the viscous dissipation of the vortex stemming from the motion of the current around the vortex kernel in the external magnetic field \(\vec{B}\), which is \(E(J)\phi_0 RZ/\rho_f\). Thus the free-energy difference with respect to the situation without kinks and external current is

\[
\delta f \approx 2E_k R/d + Z\triangle(R) - f_L RZ + E(J)\phi_0 RZ/\rho_f
\]

The concentration available states as a function of \(R\) with \(D\) dimensions transverse to \(\vec{B}\) (here \(D = 2\)) on the one hand equals \(d^D \int_{\mu}^{\mu+\Delta(R)} g(\varepsilon) d\varepsilon\), and on the other hand is simply given by \(\approx (d/R)^D\), thus \(\triangle(R)\) is to be determined from the equation\(^4\).

\[
\int_{\mu}^{\mu+\Delta(R)} g(\varepsilon) d\varepsilon = R^{-D}.
\]

Optimizing first for vanishing current \(J = 0\) gives the longitudinal extent \(Z^*\) of the superkink as a function of its transverse size \(R^*\),

\[
Z^* \approx -\frac{2E_k/d}{(\partial \Delta/\partial R)_{R^*}}.
\]
Upon balancing the last term in Eq. (13) against the optimized sum of the first two, one arrives at

\[
(J - E(J)/\rho_f)\phi_0/c = J_0\phi_0/c \approx \Delta(R^*)/R^*
\]  
\[\text{(16)}\]

which through inversion yields a typical hopping range \( R^*(J) \). Inserting back into Eq. (13) finally yields the result for the optimized free-energy barrier for jump,

\[
\delta F^*(J) \approx (2E_k/d)R^*(J)
\]  
\[\text{(17)}\]

which we identify with the current-dependent activation energy in Eq. (3)

\[
E(J) \approx \rho_f J \exp[-(2E_k/kT d)R^*(J)].
\]  
\[\text{(18)}\]

Considering a power-law form for the distribution of pinning energies

\[
g(\varepsilon) = \kappa|\varepsilon - \mu|^s
\]  
\[\text{(19)}\]

one obtains

\[
\delta F^*(J) = 2E_k(J_0/J_s)^p
\]  
\[\text{(20)}\]

where the transport exponent \( p \)

\[
p = \frac{s+1}{D+s+1}
\]  
\[\text{(21)}\]

and the current scale

\[
J_0 \approx c/\phi_0 \kappa^{1/(s+1)}d^{1/p}.
\]  
\[\text{(22)}\]

Thus we get

\[
E(J) = \rho_f J \exp[-(E_k/kT)(J_0/J_s)^p]
\]  
\[\text{(23)}\]

with \( J_s \) described by Eq. (5).

Eq. (23) can also be expressed in a general form

\[
y = x \exp[-\gamma(1 + y - x)^p]
\]  
\[\text{(24)}\]
with
\[
\gamma \equiv 2p \ln \frac{J_L}{J_{Lf}} = 2p \left( \frac{E_k}{kT} \right) \left( \frac{J_0}{J_L - J_{Lf}} \right)^p \\
\approx 2p \left( \frac{E_k}{kT} \right) \left( \frac{J_0}{J_L} \right)^p \\
x \equiv \frac{1}{2} \left( \frac{E_k}{kT} \right)^{-\frac{1}{p}} \left( \ln \frac{J_L}{J_{Lf}} \right)^{\frac{1}{p}} \left( \frac{J}{J_0} \right) = \frac{1}{2} \left( \frac{J}{J_L - J_{Lf}} \right) \\
\approx \frac{J}{2J_L} \\
y \equiv \frac{1}{2} \left( \frac{E_k}{kT} \right)^{-\frac{1}{p}} \left( \ln \frac{J_L}{J_{Lf}} \right)^{\frac{1}{p}} \left( \frac{E(J)}{J_0 \rho_f} \right) \\
= \frac{1}{2} \left( \frac{E(J)}{(J_L - J_{Lf}) \rho_f} \right) \\
\approx \frac{E(J)}{2J_L \rho_f}
\]

where \(J_{Lf} \equiv E(J_L)/\rho_f\) which is much smaller than \(J_L\).

Because of the size of the sample and the periodic boundary condition, the transverse size \(R^*(J)\) cannot be greater than the sample width \(L\). For \(J < J_L\), one would expect that the current-voltage characteristics will become Ohmic. This regime is also described by Eq. (24).

It is interesting to note, that the general form Eq. (24) is also compatible with other type of suggested \(U_B(J)\). For instance, an \(E(J)\) equation for type-II superconductors has been derived in connection to the Anderson-Kim model with replacing the total transporting current density \(J\) in \(U_B(J) = U_c(1 - J_c/J)\) with \(J_s\) of Eq. (3) \([12]\), as

\[
0E(J) = J \rho_f \exp[(-U_c - W_v + W_L)/kT] \\
\]

where \(W_v = E(J) \cdot B \cdot \mathcal{A}/\rho_f\) is the viscous dissipation term of flux motion, \(W_L\) is the energy due to Lorentz driving force, \(W_L = J \cdot B \cdot \mathcal{A}\), the parameter \(\mathcal{A}\) is a product of the volume of moving flux bundles and the range of force action.

Eq. (24) can be expressed with a reduced form

\[
y = x \exp[-\gamma(1 + y - x)] \\
\]
with $\gamma \equiv U_c/kT$, $x \equiv W_L/U_c$ and $y \equiv W_v/U_c$, which corresponds with Eq.(24) with $p = 1$.

IV RESULTS AND DISCUSSION

For simplicity, we assume that $E_k/k = 1$. Thus temperature has unit $E_k/k$. Moreover, in this paper the energy is normalized by $\varepsilon_0$, current is normalized by $\phi_0 d/\varepsilon_0 c$ and voltage is normalized by $\rho_f$. Using the relations (14)-(18) and the Monte-Carlo result $g(\varepsilon)$, we can numerically evaluate the current-voltage characteristics for any form of $g(\varepsilon)$. Results derived from the distribution of interacting pinning energies in Fig.1 are shown in Fig.2 which has highly nonlinear characteristics and a superconducting phase. The double-logarithimic plots of $E(J)$ as the function of the current $J$ in the inset of Fig.2 is consistent with the result in Ref.[13].

From Eq.(18), we have

$$
\sigma \equiv \frac{d \ln(E/J)}{d \ln J} = -(2E_k/kTd) \frac{dR^*(J)}{dJ}.
$$

(27)

This immediately leads to

$$
\sigma_{\text{max}} = \frac{dR^*(J)}{dJ} / \left[ \frac{dR^*(J)}{dJ} \right]_{\text{max}}.
$$

(28)

From Eq.(16)-Eq.(18), we see that temperature will have little effect near the maximal slope point.

On the other hand, we get from the analytical equation Eq.(24)

$$
S \equiv \frac{d\ln y}{d\ln x} = 1 + \sigma = \frac{1 + p\gamma x(1 + y - x)^{p^{-1}}}{1 + p\gamma y(1 + y - x)^{p^{-1}}}.
$$

(29)

At the maximal slope point $(x_i, y_i)$, we have

$$
p^2 \gamma^2 xy(1 + y - x)^{(p-1)} = 1 - p(x - y).
$$

(30)

As we have mentioned in the previous section that the Eq.(24) is compatible with different types of suggested $U_B(J)$, one would expect its general agreement with experimental results of various kinds of type-II superconductors.
In Fig.3 we compare the current dependency of the slope $\sigma$ derived from Eq.(24) and Eq.(30) with the wide range $V \sim I$ data observed by Repaci et al. on the YBCO films\cite{15}. The agreement is rather well.

We also compared our Eq.(24) with the scaling of isothermal $E(J)$ curves observed by Koch et al.\cite{16}. Taking simple trial form $\gamma = 10(1 - T/T_c)^{\delta} \cdot T/T_c$ with $\delta = 0.5$ and $p = 0.6$, we get from Eq.(24) one hundred $E(J)$ isotherms near the $T_g \approx 0.84T_c$(as observed in Ref.\cite{17}) with $T$ ranging from 0.74$T_c$ to 0.94$T_c$. All the isotherms collapsed nicely onto two curves ($T > T_g$ and $T < T_g$), consistent with the scaling of $\nu = 1.7$, $z = 4.8$ as shown in Fig.4. The similar scaling result of Ref.\cite{16} is shown in the inset of Fig.4 which has the same scaling exponents of $\nu = 1.7$ and $z = 4.8$.

V SUMMARY

Using the Monte-Carlo simulation method introduced by Ref.\cite{1}, we studied the current-voltage relation of type II superconductors which have columnar defect sites. Considering the viscous dissipation of moving vortices, we find that the three regime’s current-voltage relation can be described by a unified equation Eq.(24). This equation is consistent with different types of $U_B(J)$ so far suggested by different models and agrees with experimental observed $E(J)$ data of wide range of temperature and current density. With proper trial effective barrier function $U_B(T)$ it gives $E(J)$ isotherms which can be collapsed onto two curves ($T > T_g$ and $T < T_g$) with scaling exponents found from experimental data.

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Figure Caption

FIG.1. Normalized distribution of pinning energies $g(\varepsilon)$ as function of the single-particle energies $\varepsilon$ which is normalized by $\varepsilon_0$. Another parameters is $w = 0.2$. $g(\varepsilon)$ vanished near $\mu$ according to a power law $|\varepsilon - \mu|^3$, thus the power $p$ in Eq(19) is about $2/3$.

FIG.2. The nonlinear current-voltage curve derived from FIG.1.$E(J)$. Voltage is normalized by $\rho_f$ while current is normalized by $\phi_0d/\varepsilon_0c$.

FIG.3. (a) The original experimental result of Ref.[15]. (b) The result $\sigma/\sigma_{\text{max}} \sim \ln(I/I_i)$ plotted from the results of Ref.[15] are compared with analytical Eq.(24) and Eq.(30) (solid lines). The parameter $p$ is taken as 1.8 and $\gamma$ ranges from 0.5 to 5.0 in 0.5 intervals. $\sigma(I_i) \equiv \sigma_{\text{max}}$.

FIG.4. The collapsed data derived from Eq.(12), with $\nu = 1.7$, $z = 4.8$ and $T_g/T_c = 0.84$. 100 curves are plotted for $T > T_g$ and $T < T_g$. Inset is the original experimental result of Ref.[16].
\( \nu = 1.7 \quad z = 4.8 \)

\( T_g = 0.84 \quad T_c = 1 \)

\( \delta = 0.5 \)