Abstract. We use recently introduced Rasmussen invariant to find knots that are topologically locally-flatly slice but not smoothly slice. We note that this invariant can be used to give a combinatorial proof of the slice-Bennequin inequality. Finally, we compute the Rasmussen invariant for quasipositive knots and show that most of our examples of non-slice knots are not quasipositive and, to the best of our knowledge, were previously unknown.

1. Rasmussen invariant and the slice-Bennequin inequality

In [17] Jacob Rasmussen used the theory of knot (co)homology developed by Mikhail Khovanov [6] and results of Eun Soo Lee [10] to introduce a new invariant $s$ of knots in $S^3$. This invariant takes values in even integers. Its main properties are summarized as follows.

1.A. Theorem (Rasmussen [17] Theorems 1–4). Let $K$ be a knot in $S^3$. Then

(1) $s$ gives a lower bound on the slice (4-dimensional) genus $g_s(K)$ of $K$:

$$|s(K)| \leq 2g_s(K);$$

(2) $s$ induces a homomorphism from $\text{Conc}(S^3)$, the concordance group of knots in $S^3$, to $\mathbb{Z}$;

(3) If $K$ is alternating, then $s(K) = \sigma(K)$, where $\sigma(K)$ is the classical knot signature of $K$;

(4) If $K$ can be represented by a positive diagram $D$, then

$$s(K) = 2g_s(K) = 2g(K) = n(D) - O(D) + 1,$$

where $n(D)$ and $O(D)$ are the number of crossings and Seifert circles of $D$, respectively, and $g(K)$ is the ordinary (3-dimensional) genus of $K$.

1.B. Corollary ([17, Corollary 4.3]). Let $K_+$ and $K_-$ be two knots that are different at a single crossing that is positive in $K_+$ and negative in $K_-$. Then

$$s(K_-) \leq s(K_+) \leq s(K_-) + 2.$$

Equality (1.2) can be easily generalized to arbitrary knots. It becomes an inequality then.

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of an oriented surface (without closed components) smoothly embedded in $D^4$. The theory developed by Khovanov, Lee, and Rudolph \cite{KHL} using the gauge theory. The theory developed by Khovanov, Lee, and Rudolph \cite{KHL} using the gauge theory. Hence, the inequality is preserved.

When a positive crossing of $D$ is changed into a negative one, the right hand side of \eqref{eq:1.4} decreases by 2, while the left hand side decreases by at most 2 because of \eqref{eq:1.3}. Hence, the inequality is preserved. \hfill \Box

Lemma \ref{lem:1.C} implies the slice-Bennequin inequality that was originally proved by Rudolph \cite{Rudolph} using the gauge theory. The theory developed by Khovanov, Lee, and Rasmussen provides the first purely combinatorial proof of this inequality.

\textbf{1.D. Corollary (Slice-Bennequin Inequality, cf. \cite{KHL}).} Let $\beta$ be a braid with $k$ strands and let $\widehat{\beta}$ be its closure. Denote by $\chi_s(\widehat{\beta})$ the greatest Euler characteristic of an oriented surface (without closed components) smoothly embedded in $D^4$ with boundary $\widehat{\beta}$. Then

\begin{equation}
\chi_s(\widehat{\beta}) \leq k - w(\beta).
\end{equation}

\textbf{Proof.} If $\widehat{\beta}$ is a knot, then \eqref{eq:1.5} and \eqref{eq:1.2} imply that

\begin{equation}
g_s(\widehat{\beta}) \geq \frac{w(\beta) - k + 1}{2}.
\end{equation}

It remains to notice that $\chi_s = 1 - 2g_s$ for knots.

Assume now that $\widehat{\beta}$ is a link. Let $\beta^+$ be a braid obtained from $\beta$ by removal from the braid word representing $\beta$ of all the standard generators that appear with negative exponents. For example, if $\beta = \sigma_2\sigma_1^{-1}\sigma_2$, then $\beta^+ = \sigma_2^2$. Inserting a cancelling pair of generators $\sigma_i\sigma_i^{-1}$ into $\beta$ changes neither $w(\beta)$ nor $\chi_s(\widehat{\beta})$, but adds a crossing to $\beta^+$, so one can assume without a loss of generality that the closure $\widehat{\beta^+}$ of $\beta^+$ is a knot.

Since $\widehat{\beta^+}$ is a (positive) knot, \eqref{eq:1.5} holds true for it (in fact, it is an equality). Now, addition of a negative crossing to a braid increases the right-hand side of \eqref{eq:1.5} by exactly 1. On the other hand, the following Lemma shows that $\chi_s$ can not change by more than 1. This completes the proof. \hfill \Box

\textbf{1.E. Lemma.} Let $\beta$ and $\beta'$ be two braids such that $\beta = w_1w_2$ and $\beta' = w_1\sigma_i^\varepsilon w_2$, where $w_1$ and $w_2$ are some braid words, $\sigma_i$ is a standard braid group generator, and $\varepsilon = \pm 1$. Let $\widehat{\beta}$ and $\widehat{\beta'}$ be the corresponding closures. Then $|\chi_s(\widehat{\beta}) - \chi_s(\widehat{\beta'})| \leq 1$.

\textbf{Proof.} Let $S$ be an oriented surface (without closed components) smoothly embedded in $D^4$ with $\partial S = \widehat{\beta}$ and $\chi(S) = \chi_s(\widehat{\beta})$. Addition of a twisted band to $S$ at the place where a crossing is added to $\beta$ produces a smoothly embedded surface $S'$ with $\partial S' = \widehat{\beta'}$. Then $\chi(S') = \chi(S) - 1$ and $\chi_s(\widehat{\beta'}) \geq \chi_s(\widehat{\beta}) - 1$. On the other hand, $\beta = w_1\sigma_i^{-\varepsilon}\sigma_i^\varepsilon w_2$. Then $\beta = w_1\sigma_i^{-\varepsilon}w_2$ and $\beta' = w_1w_2^\varepsilon$ with $w_2^\varepsilon = \sigma_i^\varepsilon w_2$. Repeating the previous argument, one obtains that $\chi_s(\widehat{\beta}) \geq \chi_s(\widehat{\beta'}) - 1$. \hfill \Box

The slice-Bennequin inequality leads to a formula for the Rasmussen invariant of (strongly) quasipositive knots. We use this formula in section \ref{sec:2}.
If, moreover, \( \beta \) is quasipositive braid \( \beta \).

**Proposition.**

1. Let \( \sigma \) be a knot that can be represented as the closure of a braid \( w_1^{\sigma} \cdots w_p^{\sigma} \), where \( \sigma \) are the standard generators of the corresponding braid group, and \( w_i \) are braid words.

2. A knot \( K \) is said to be strongly quasipositive if it is the closure of a braid that has the form \( \sigma_{i_1,j_1} \cdots \sigma_{i_p,j_p} \), where \( \sigma_{i,j} = (\sigma_{i+1} \cdots \sigma_{j-2})\sigma_{j-1}(\sigma_{i+1} \cdots \sigma_{j-2})^{-1} \) for \( j \geq i + 2 \) and \( \sigma_{i,i+1} = \sigma_i \).

The \((-3,5,7)\)-pretzel knot depicted in Figure 1a is a strongly quasipositive knot. It is the closure of the braid \( \sigma_1 \sigma_2 \sigma_3 \sigma_4 \sigma_5 \sigma_6 \sigma_7 \sigma_8 \sigma_9 \).

**Definitions.**

1. A knot \( K \) is said to be quasipositive if it is the closure of a braid that has the form \( (w_1^{\sigma_1} w_1^{-1}) \cdots (w_p^{\sigma_p} w_p^{-1}) \), where \( \sigma \) are the standard generators of the corresponding braid group, and \( w_i \) are braid words.

2. A knot \( K \) is said to be strongly quasipositive if it is the closure of a braid that has the form \( \sigma_{i_1,j_1} \cdots \sigma_{i_p,j_p} \), where \( \sigma_{i,j} = (\sigma_{i+1} \cdots \sigma_{j-2})\sigma_{j-1}(\sigma_{i+1} \cdots \sigma_{j-2})^{-1} \) for \( j \geq i + 2 \) and \( \sigma_{i,i+1} = \sigma_i \).

The \((-3,5,7)\)-pretzel knot depicted in Figure 1a is a strongly quasipositive knot. It is the closure of the braid \( \sigma_1 \sigma_2 \sigma_3 \sigma_4 \sigma_5 \sigma_6 \sigma_7 \).

**Proposition.** Let \( K \) be a knot that can be represented as the closure of a quasipositive braid \( \beta \) with \( k \) strands and \( b \) bands, that is, \( \beta \) is a product of \( b \) factors of the form \( w_1^{\sigma_i} w_i^{-1} \). Then

\[
s(K) = 2g_s(K) = b - k + 1.
\]

If, moreover, \( \beta \) is strongly quasipositive, then

\[
s(K) = 2g(K) = 2g_s(K) = b - k + 1.
\]

**Proof.** It is clear that \( w(K) = b \) and \( O(K) = k \). Hence,

\[
b - k + 1 \leq s(K) \leq |s(K)| \leq 2g_s(K) \leq 2g(K)
\]

by (1.4) and (1.1). On the other hand, for a (strongly) quasipositive knot one can explicitly construct a surface \( S \) smoothly embedded in \( D^4 \) (respectively, \( S^3 \)) such that \( \partial S = K \) and the Euler characteristic \( \chi(S) \) of \( S \) is \( k - b \) (see Figure 1c that illustrates this construction in the strongly quasipositive case). Since \( S \) has a single boundary component, its genus equals \( \frac{1 - \chi(S)}{2} \). It follows that \( g_s(K) \) (respectively, \( g(K) \)) does not exceed \( \frac{k-b+1}{2} \). This finishes the proof.

**Remark.** One of the main applications of the \( s \)-invariant provided in [17] was a purely combinatorial proof of the Milnor Conjecture [13] that was originally proved by Kronheimer and Mrowka [9] using the gauge theory. More specifically, Rasmussen showed that the slice genus of a \((p,q)\)-torus knot with \( p,q > 0 \) is \( (p-1)(q-1)/2 \). In fact, the original question posed by Milnor and answered in [9] is more general (see [14], Remark 10.9). It asks whether the unknotting number \( u(K) \) of an algebraic knot \( K \) equals its genus \( g(K) \). Here, an algebraic knot is the knot associated to an isolated singular point of a complex algebraic curve in \( \mathbb{C}^2 \) by intersecting it with a 3-dimensional sphere of a sufficiently small radius centered at
the singularity. For example, a \((p, q)\)-torus knot corresponds to the singular curve 
\[ z^p + w^q = 0. \]

It is well-known that \( g_s(K) \leq u(K) \leq g(K) \) for any algebraic knot \( K \) and that it can be represented by a positive diagram obtained as a closure of a positive braid. The general Milnor Conjecture now follows from (1.2) straightforwardly. I am grateful to Sergei Chmutov for pointing out this fact to me.

Remark. Charles Livingston [11] used the Ozsváth-Szabó knot invariant \( \tau \) [15] to give new proofs to several results of Lee Rudolph [19, 20] on the slice genus, including the slice-Bennequin Inequality. Invariants \( s \) and \( \tau \) share many of their main properties and our approach is similar to the Livingston’s one. The key difference is that the Rasmussen invariant is defined combinatorially, while the Ozsváth-Szabó invariant is based on the theory of knot Floer homology. It was originally conjectured that \( s(K) = 2\tau(K) \) for every knot \( K \), but counter-examples were later found [3, 12].

Remark. Relation between the Rasmussen invariant and the slice-Bennequin inequality was independently observed by several other authors, including Olga Plamenevskaya [16] and Alexander Stoimenov [23]. After the original version of this paper was published, Tomomi Kawamura [5] used the Rasmussen invariant to prove a sharper slice-Bennequin inequality.

2. Sliceness of knots

In many cases one can easily compute \( s(K) \) from the Khovanov homology of \( K \). For a given knot \( K \), let \( h^{i,j}(K) = \dim \mathbb{Q}(H^{i,j}(K) \otimes \mathbb{Q}) \) be the ranks of its homology and \( Kh(K)(t, q) = \sum_{i,j} t^i q^j h^{i,j}(K) \) be the corresponding Poincaré polynomial in variables \( t \) and \( q \). Denote by \( hw(K) \) the homological width of \( K \), that is, the minimal number of adjacent diagonals \( 2i - j = \text{const} \) that support the homology of \( K \).

In was shown by Rasmussen [17, Proposition 5.2] that for all knots \( K \) with \( hw(K) \leq 3 \), one has

\[
Kh(K) = q^{s(K)-1}(1 + q^2 + (1 + tq^4)Kh'(K)),
\]

where \( Kh'(K) \) is some (Laurent) polynomial in \( t \) and \( q \) with non-negative coefficients. In fact, Rasmussen’s arguments can be applied to a more general case.

2.A. Let \( K \) be a knot. Assume that \( h^{i,j}(K)h^{i+1,j+4(n-1)}(K) = 0 \) for all \( i, j \), and \( n \geq 3 \) (this is automatically the case if \( hw(K) \leq 3 \)). Then (2.1) holds true for \( K \).

Proof. Construction of the Rasmussen invariant is based on a spectral sequence structure on the Khovanov chain complex that is due to Lee [10]. The differential \( d_n \) in this spectral sequence has bidegree \((1, 4(n-1))\). The condition on \( h^{i,j} \) implies that \( d_n \) is trivial for all \( n \geq 3 \). The rest of the arguments is the same as in [17, Proposition 5.2]. \( \square \)

It is possible for a knot to have homological width 4, but still satisfy the condition of 2.A (see Table 1). On the other hand, the knot 16_{6489} may theoretically have \( d_3^{1,-7} \neq 0 \), since \( h^{-1,-7} = h^{0,1} = 1 \) (see Table 2). Hence, its Rasmussen invariant can equal either 0 or \(-2\). Let us demonstrate that it is indeed the former.

\[1\] We enumerate knots according to the convention from Knotscape [4], due to Hoste and Thistlethwaite. For example, the knot 13_{1496} is a non-alternating knot number 1496 with 13 crossings.
|   | -9 | -8 | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|----|----|----|----|----|----|----|----|----|---|---|---|---|---|---|---|
| 9 |    |    |    |    |    | 1  |    |    |    |    |    |    |    |    |    |    |
| 7 |    |    |    |    |    | 2  |    |    |    |    |    |    |    |    |    |    |
| 5 |    |    |    |    |    | 2  | 1  |    |    |    |    |    |    |    |    |    |
| 3 |    |    |    |    |    | 2  | 3  | 2  |    |    |    |    |    |    |    |    |
| 1 |    |    |    |    |    | 2  | 3  | 2  |    |    |    |    |    |    |    |    |
| -1|    |    |    |    |    | 4  | 4  | 3  |    |    |    |    |    |    |    |    |
| -3|    |    |    |    |    | 4  | 5  | 3  |    |    |    |    |    |    |    |    |
| -5|    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| -7|    |    |    |    |    | 1  | 3  | 4  | 2  |    |    |    |    |    |    |    |
| -9|    |    |    |    |    | 1  | 2  | 3  | 1  |    |    |    |    |    |    |    |
| -11|   |    |    |    |    | 1  | 3  |    |    |    |    |    |    |    |    |    |
| -13|   |    |    |    |    | 1  | 1  | 2  |    |    |    |    |    |    |    |    |
| -15|   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| -17|   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| -19|   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |

**Table 1.** Homology ranks of the knot $16_{809057}$. The homological width is 4, but the differential $d_3$ of bidegree $(1,8)$ is trivial.

|   | -9 | -8 | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|----|----|----|----|----|----|----|----|----|---|---|---|---|---|---|---|
| 9 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 7 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 5 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 3 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 1 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| -1|    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| -3|    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| -5|    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| -7|    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| -9|    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| -11|   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| -13|   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| -15|   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| -17|   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| -19|   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |

**Table 2.** Homology ranks of the knot $16_{864894}$. Encircled entries show that $d_3^{-1,-7}$ can possibly be non-trivial.

For a given knot $K$, denote by $rk(K)$ and $\tilde{rk}(K)$ the total ranks of its standard and reduced (see \[2\] for the definition) Khovanov homologies, respectively. In particular, $rk(K) = \sum_{i,j} h^{i,j}(K) = Kh(K)(1,1)$. 
2.B. Lemma ([8], page 189). Let $K$ be a knot. If $rk(K) - \tilde{rk}(K) = 1$, then (2.1) holds true for $K$.

As it turns out, $rk(16_{864894}) = 66$ and $\tilde{rk}(16_{864894}) = 65$. Lemma 2.B implies that (2.1) holds true for this knot and its Rasmussen invariant can be computed to be equal 0.

Remark. Bar-Natan’s conjecture [1] that (2.1) holds true for every knot for some integer $s\{K\}$ is still open. Moreover, no examples are known with $s\{K\}$ different from $s(K)$.

Given a knot $K$, denote by $\Delta_K(t)$ its Alexander polynomial. M. Freedman proved [2] that if $\Delta_K(t) = 1$, then $K$ is topologically locally-flatly slice. We used Knotscape [4] to list all the non-alternating knots with up to 16 crossings that have $\Delta = 1$. In total, there are 699 such knots (not counting the mirror images). The first one of them has 11 crossings (see Table 3). We did not consider alternating knots, since the Rasmussen invariant equals the signature for them (see Theorem 1.A) and the signature is known to be 0 when $\Delta = 1$.

| Number of crossings | 11 | 12 | 13 | 14 | 15 | 16 |
|---------------------|----|----|----|----|----|----|
| Number of non-alternating knots | 185 | 888 | 5110 | 27436 | 168030 | 1008906 |
| among them with $\Delta = 1$ | 2 | 2 | 15 | 36 | 145 | 499 |
| among them with $s \neq 0$ | 0 | 0 | 1 | 1 | 15 | 65 |

Table 3. Number of non-alternating knots that have Alexander polynomial 1 and those among them that have non-zero Rasmussen invariant.

Next, we used KhoHo, a program for computing and studying Khovanov homology [21], to find the homology of all the knots with $\Delta = 1$. Fortunately for us, most of the knots considered (in particular, all knots with at most 15 crossings) have homological width 3. There are 42 knots with 16 crossings and homological width 4 that satisfy the condition of 2.A. Hence, one can use (2.1) to deduce the Rasmussen invariant of these knots from their homology.

There are only two knots with 16 crossings and $\Delta = 1$, for which the assumption of 2.A fails. They are $16_{864894}$ and $16_{825408}$. As it turns out, both of them satisfy the condition of Lemma 2.B and, hence, (2.1) is still applicable. Their Rasmussen invariants are both 0.

In total, 82 knots with up to 16 crossings have Alexander polynomial 1 and non-zero Rasmussen invariant. Theorem 1.A implies the following.

2.C. Proposition. All knots with up to 15 crossings have Alexander polynomial 1 and non-zero Rasmussen invariant. Theorem 1.A implies the following.

Among these 65 knots with 16 crossings have table numbers 2601, 4787, 10734, 15919, 35456, 38567, 54888, 55405, 63905, 64312, 85435, 88272, 95001, 100099, 146445, 196836, 201101, 205822, 211749, 213930, 225414, 231486, 233317, 247683, 247710, 249903, 253331, 271353, 281590, 287865, 322069, 345376, 355871, 359271, 367431, 380325, 383790, 412372, 418128, 432810, 446116, 464148, 470729, 487352, 499458, 528093, 538818, 542632, 548142, 591990, 596477, 637428, 644951, 696243, 707728,
Table 4. All knots with at most 15 crossings and two knots with 16 crossings that have Alexander polynomial 1 and non-zero Rasmussen invariant. The two 16-crossing knots or their mirror images can potentially be strongly quasipositive.

| knot K | s(K) | hw(K) | e(K) | E(K) | knot K | s(K) | hw(K) | e(K) | E(K) |
|--------|------|-------|------|------|--------|------|-------|------|------|
| 13^{2}_{1496} | 2 | 3 | 0 | 8 | 14^{7}_{7708} | 2 | 3 | 0 | 8 |
| 15^{9}_{28998} | 2 | 3 | -4 | 8 | 15^{9}_{89822} | 2 | 3 | -2 | 8 |
| 15^{5}_{40132} | 2 | 3 | 0 | 8 | 15^{5}_{1313775} | 2 | 3 | 2 | 12 |
| 15^{5}_{52282} | 2 | 3 | 0 | 6 | 15^{5}_{1332396} | 2 | 3 | 0 | 6 |
| 15^{5}_{54421} | -2 | 3 | -8 | 2 | 15^{5}_{139256} | 2 | 3 | 0 | 10 |
| 15^{5}_{58433} | 2 | 3 | -2 | 8 | 15^{5}_{135981} | -2 | 3 | -8 | 0 |
| 15^{7}_{58501} | 2 | 3 | 0 | 8 | 15^{7}_{165398} | 2 | 3 | 0 | 6 |
| 15^{7}_{65084} | -2 | 3 | -8 | 2 | 16^{12}_{955859} | 2 | 3 | 2 | 12 |

Stoimenow arranges all the knots with a given number of crossings into a single list. Hence, the knot $13^{2}_{1496}$ has number $1496 + 4878 = 6374$ in [22], where 4878 is the number of alternating knots with 13 crossings.
2.F. **COROLLARY.** Among all knots from Proposition 2.C, only knots $15_113775$, $16_955859$, and $16_{412372}$ can be strongly quasipositive (see Table 4). $15_113775$ is the $(-3,5,7)$-pretzel knot and is indeed strongly quasipositive (see Figure 4).

**Remark.** After the original version of this paper was published, Stoimenow demonstrated [24] that neither $16_{955859}$ nor $16_{412372}$ are in fact strongly quasipositive.

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**References**

[1] D. Bar-Natan, *On Khovanov’s categorification of the Jones polynomial*, Algebr. Geom. Topol. 2 (2002), 337–370; arXiv:math/0201043.
[2] M. Freedman, *A surgery sequence in dimension four; the relations with knot concordance*, Invent. Math. 68 (1982), no. 2, 195-226.
[3] M. Hedden and P. Ording, *The Ozsváth-Szabó and Rasmussen concordance invariants are not equal*, Amer. J. Math. 130 (2008), no. 2, 441–453; arXiv:math/0512348.
[4] J. Hoste and M. Thistlethwaite, *Knotscape — an interactive program for the study of knots*, http://www.math.utk.edu/~morwen/knotscape.html
[5] T. Kawamura, *The Rasmussen invariants and the sharper slice-Bennequin inequality on knots*, Topology 46 (2007), no. 1, 29–38.
[6] M. Khovanov, *A categorification of the Jones polynomial*, Duke Math. J. 101 (2000), no. 3, 359–426; arXiv:math/9908171.
[7] M. Khovanov, *Patterns in knot cohomology I*, Experiment. Math. 12 (2003), no. 3, 365–374; arXiv:math/0201306.
[8] M. Khovanov, *Link homology and Frobenius extensions*, Fund. Math. 190 (2006), 179–190; arXiv:math/0411447.
[9] P. Kronheimer and T. Mrowka, *Gauge theory for embedded surfaces, I*, Topology, 32 (1993), no. 4, 773–826.
[10] E. S. Lee, *An endomorphism of the Khovanov invariant*, Adv. Math. 197 (2005), no. 2, 554–586; arXiv:math/0210213.
[11] C. Livingston, *Computations of the Ozsváth-Szabó knot concordance invariant*, Geom. Topol. 8 (2004), 735–742; arXiv:math/0311036.
[12] C. Livingston, *Slice knots with distinct Ozsváth-Szabó and Rasmussen Invariants*, Proc. Amer. Math. Soc. 136 (2008), no. 1, 347–349; arXiv:math/0602631.
[13] J. Milnor, *Singular Points of Complex Hypersurfaces*, Ann. Math. Stud. 61, Princeton University Press, 1968.
[14] H. Morton, *Seifert circles and knot polynomials*, Math. Proc. Camb. Phil. Soc. 99 (1986), 107–109.
[15] P. Ozsváth and Z. Szabó, *Knot Floer homology and the four-ball genus*, Geom. Topol. 7 (2003), 615–639; arXiv:math/0301149.
[16] O. Plamenevskaya, *Transverse knots and Khovanov homology*, Math. Res. Lett. 13 (2006), no. 4, 571–586; arXiv:math/0412184.
[17] J. Rasmussen, *Khovanov homology and the slice genus*, Invent. Math. 182 (2010), no. 2, 419–447; arXiv:math.GT/0402131.
[18] J. Roberts and P. Teichner, *UCSD Quantum Topology in Dimension Four seminar*, Spring 2004, http://math.ucsd.edu/~justin/khovseminar.html
[19] L. Rudolph, *Quasipositivity as an obstruction to sliceness*, Bull Amer. Math. Soc. (N.S.) 29 (1993), no. 1, 51–59; arXiv:math/9307253.
[20] L. Rudolph, *An obstruction to sliceness via contact geometry and “classical” gauge theory*, Invent. Math. 119 (1995), no. 1, 155-163.
[21] A. Shumakovitch, *KhoHo — a program for computing and studying Khovanov homology*, https://github.com/AShumakovitch/KhoHo
[22] A. Stoimenow, *On polynomials and surfaces of variously positive links*, J. Eur. Math. Soc. (JEMS) 7 (2005), no. 4, 477–509; arXiv:math/0202226.

[23] A. Stoimenow, *Some examples related to knot sliceness*, J. Pure Appl. Algebra 210 (2007), no. 1, 161–175; arXiv:math/0412276.

[24] A. Stoimenow, private communications.

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