Autonomous In-Tank Localization Monte Carlo Error Analysis

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Abstract. In this paper, we address the problem of localization of swimming robots inside liquid storage tanks. Our proposed localization technique uses two or more proximity sensors with one or more angular orientation sensors while exploiting the cylindrical geometry of tanks. For the planar case, we study the map of the sensor error to the robot position error using a Monte Carlo error optimization analysis. We study the case of two, three and four proximity sensors while varying the relative orientation placement of these sensors. We conclude that the use of one orientation sensor, and four proximity sensors spaced 45 degrees apart give the least error in computing the robot position inside the tank from sensor measurements.

1. Introduction

Large storage tanks are used in a variety of petrochemical applications. These include temporary storage at production facilities, in-transit storage at pipeline facilities, and long term storage at end-use facilities. These tanks can be quite large (50 m in diameter and 20 m tall or more) and contain hundreds of thousands of gallons of potentially toxic materials. The tanks are typically constructed of carbon steel, and are therefore susceptible to corrosion caused by the contents and by the ambient environmental conditions. Therefore, it is important that the tanks be periodically inspected to locate potential defects before such defects can reach a critical stage.

The tank walls are accessible from outside the tank, and therefore it is possible to inspect them using external sensors. The tank floors rest on solid foundations and therefore require internal access to perform inspection. The large number of these tanks and relatively easy floor landscape makes inspection robots an attractive means to perform the frequently needed floor inspections. In order for inspection results to be meaningful, it is necessary for the robot to accurately record its position (perform localization) as it inspects the tank. This paper performs a Monte Carlo analysis of an autonomous, self-contained localization approach.

2. Background

Previous researchers [1] investigated the localization problem using fixed beacons mounted inside the tank and cooperating navigation equipment fixed on the inspection robot. Although this technique is feasible, it presents operational difficulties. The beacons must be installed in each tank prior to inspection requiring either potentially expensive permanent installation or time consuming mounting each time a tank is inspected. The beacons must themselves be localized in the tank prior to operation. These difficulties motivate the development of an autonomous, self-contained localization technique.

The present study consists of posing the problem as an error optimization problem. Specifically, this study hypothesizes a localization system employing two, three, or four commercial ultrasonic (UT) sensors and one commercial magnetoresistive (MR) sensor. In particular, this study is intended to determine the effects of varying the number of range sensors and the effects of varying the orientation angles among the sensors for sets of two, three, and four range sensors, for various robot positions on the tank floor.

2.1 Localization geometry

Using the MR sensor and a single UT sensor, the measurement vector produces an arc of position. Note that we assume that we know the tank inside radius \( R \) and that the tank is perfectly cylindrical. If we mount the UT sensors at fixed angular displacements, one MR sensor measurement and two UT sensor measurements produce two arcs of position as shown in...
Figure 1. With perfect measurements, the robot position is the intersection of the two arcs of position.

The errors inherent in the measurements introduce uncertainty, as illustrated in Figure 2. Owing to uncertainty, the measurements define location bands rather than arcs. The intersections of the two location bands defines a region in which the robot is located rather than a point.

Owing to the uncertainties in the range measurements, the measurement arcs of position will not coincide at a single point if we employ more than two range sensors. Therefore, given the fact of uncertainty, we cannot compute location directly. A practical solution is to estimate robot location \((x, y)\) by minimizing an error measure.

Fix the origin of a Cartesian coordinate system in the plane at the center of the tank of radius \(R\). Referring to Figure 3, for a single measurement \((w_i, \theta_i)\), for a robot at \((x, y)\), the measurement vector intersects the shell of the tank at \((\bar{x}_i, \bar{y}_i)\). Hence

\[
\bar{x}_i = x + w_i \cos \theta_i \quad (1)
\]

\[
\bar{y}_i = y + w_i \sin \theta_i
\]

giving

\[
(x + w_i \cos \theta_i)^2 + (y + w_i \sin \theta_i)^2 = R^2
\]

For a single angular orientation measurement \(\theta_i\), range measurement \(w_i\), and position estimate \((\bar{x}_i, \bar{y}_i)\), the square of the position estimate error is

\[
e_i^2 = (\bar{x} + w_i \cos \theta_i)^2 + (\bar{y} + w_i \sin \theta_i)^2 - R^2 \quad (2)
\]

where \(R\) is the tank inside radius. We can then estimate the position by minimizing the sum of the error squared of two or more measurements with respect to the position estimate \((\bar{x}, \bar{y})\). That is, minimize

\[
f(\bar{x}, \bar{y}) = \sum e_i^2
\]
with respect to \((\xi, \eta)\). Here, we perform the minimization using the Modified Quasilinearization Algorithm (MQA) [2]. Briefly, MQA is a Newton-type algorithm with a variable step size. The objective is to minimize a scalar function \(f\) of vector \(x\). The algorithm proceeds as follows:

- Starting at a nominal point \(\bar{x}\), compute numerically the gradient of \(f\) with respect to \(x\) \((f_{\bar{x}})\), for example using central differences.
- Compute
  \[ Q = f^T f_x \]
- Compute numerically the Hessian of \(f\) with respect to \(x\) \((f_{xx})\).
- Compute vector \(A\) such that
  \[ f_{xx} A + f_x = 0 \]
- The variation in \(x\) is
  \[ \Delta x = \alpha A \]
where \(\alpha\) is the stepsize, with an initial trial value of 1 (each iteration). Then the varied point

\[ \bar{x} = \bar{x} + \alpha A \]

- Compute
  \[ \bar{Q} = Q(\bar{x}) \]
and bisect \(\alpha\) until

\[ \bar{Q} < Q \]

Iterate until \(Q\) is less than some small preselected convergence criterion.

3. Study Approach

The UT sensors used in this study exhibit a maximum error of approximately 1.5\% [3] of the measurement and the MR sensors exhibit a maximum error of approximately 1.5° [4]. Error statistics are not specified for either sensor, so we conservatively assume uniform distributions in the maximum error bands.

Using a Monte Carlo technique implemented in MATLAB [5], we compute mean squared measurement error as a function of angular separation of the sensors for configurations using two, three, and four UT sensors. Briefly, the Monte Carlo method used is as follows:

- For a given sensor configuration (two, three, four sensors), and robot position \((x, y)\), specify a range of values of angular spacing between the sensors (using 1° increments).
  - For the case of two sensors, use a spacing ranging from 1° to 179°.
  - For the case of three sensors, use a spacing ranging from 1° to 178°.
  - For the case of four sensors, use a spacing ranging from 1° to 90°.
- For each 1° increment
  - Compute using (1) exact values of measurement \((w_i, \theta_i)\)
  - Generate a large number of pseudo-random trial points around \((w_i, \theta_i)\), using the MATLAB random number generator `rand`.
  - At each trial point, compute using MQA the estimated position \((\hat{x}, \hat{y})\)
  - Compute the mean-squared error
We applied the Monte Carlo algorithm for the cases of two, three, and four range sensors at three representative locations in the tank, namely the points (5, 5), (7, 1), and (1, 0.5) in a tank of radius 10.

4. Study Results

Figures 4 and 5 show representative plots for two and four sensors. Similar results were obtained for sensors located near the center of the tank (1, 0.5) and on a diagonal (5, 5).

Examination of the plots shows that increasing the number of sensors provides a modest improvement in accuracy, although it also adds robustness in case of sensor failures. Angular spacing is not critical, but spacing at approximately 90° is best for two sensors, 120° is best for three sensors, and 45° is best for four sensors.

5. Conclusions and Future Work

This paper explored the question of determining the location of a swimming robot inside liquid storage tanks. Localization methods using fixed beacons inside or outside the tank are difficult to implement. We proposed a localization technique that exploits the geometry of the tank while using one or more orientation sensors and two or more proximity sensors. We also considered the relative placement of the proximity sensors. We performed a Monte Carlo error optimization analysis and concluded that the localization approach is viable.

Future work will entail implementation of the algorithm on a real-time processor and testing in storage tanks.

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