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A Ten-Qubit Solid-State Spin Register with Quantum Memory up to One Minute

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Spins associated with single defects in solids provide promising qubits for quantum-information processing and quantum networks. Recent experiments have demonstrated long coherence times, high-fidelity operations, and long-range entanglement. However, control has so far been limited to a few qubits, with entangled states of three spins demonstrated. Realizing larger multiqubit registers is challenging due to the need for quantum gates that avoid cross talk and protect the coherence of the complete register. In this paper, we present novel decoherence-protected gates that combine dynamical decoupling of an electron spin with selective phase-controlled driving of nuclear spins. We use these gates to realize a ten-qubit quantum register consisting of the electron spin of a nitrogen-vacancy center and nine nuclear spins in diamond. We show that the register is fully connected by generating entanglement between all 45 possible qubit pairs and realize genuine multipartite entangled states with up to seven qubits. Finally, we investigate the register as a multiqubit memory. We demonstrate the protection of an arbitrary single-qubit state for over 75 s—the longest reported for a single solid-state qubit—and show that two-qubit entanglement can be preserved for over 10 s. Our results enable the control of large quantum registers with long coherence times and therefore open the door to advanced quantum algorithms and quantum networks with solid-state spin qubits.

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I. INTRODUCTION

Electron and nuclear spins associated with single defects in solids provide a promising platform for quantum networks and quantum computations [1,2]. In these hybrid registers, different types of spins fulfill different roles. Electron spins offer fast control [3–13] and high-fidelity readout [7,14,15] and can be used to control and connect nuclear spins [15–21]. Furthermore, electron-electron couplings enable on-chip connectivity between defects [19,22,23], while coupling to photons [12,24–28] allows for the realization of long-range entanglement links [29–31]. Nuclear spins provide additional qubits with long coherence times that can be used to store and process quantum states [16,17,21,24,32–35].

Recent experiments have demonstrated various schemes for high-fidelity two-qubit gates [34,36–41], as well as basic quantum algorithms [36,42] and error-correction codes [16,17]. However, to date, these systems have been restricted to few-qubit registers: The largest reported entangled state contains three qubits [16,17,43]. Larger quantum registers are desired for investigating advanced algorithms and quantum networks [44–46]. Such multi-qubit registers are challenging to realize due to the required gates that selectively control the qubits and at the same time decouple unwanted interactions in order to protect coherence in the complete register.

In this work, we develop a novel gate scheme based upon selective phase-controlled driving of nuclear spins interleaved with decoupling sequences on an electron spin. These gates enable high-fidelity control of hitherto inaccessible nuclear spin qubits. We combine these gates with previously developed control techniques [14,16,47] to realize a ten-qubit register composed of a diamond nitrogen-vacancy (NV) center, its 14N nuclear spin, and eight 13C...
spins (Fig. 1). We show that the register is fully connected by preparing entangled states for all possible pairs of qubits. Furthermore, by also decoupling nuclear-nuclear interactions through echo sequences, we generate $N$-qubit Greenberger-Horne-Zeilinger (GHZ) states and witness genuine multipartite entanglement for up to seven spins. Finally, we investigate the coherence properties of the register. We measure coherence times up to $63(2)$ s and show that an arbitrary single-qubit state can be protected for over $75$ s. Furthermore, we demonstrate that two-qubit entanglement can be preserved for over $10$ s.

II. TWO-QUBIT GATES: THEORY

We consider a NV center in diamond and surrounding $^{13}$C nuclear spins. To realize a multiqubit register, we design single-qubit gates and electron-nuclear two-qubit gates to control the NV $^{14}$N spin and several individual $^{13}$C spins. The key challenges in these hybrid systems of multiple coupled spins are to maintain coherence on the electron-spin qubit and to avoid unwanted cross talk. In particular, the electron spin continuously couples to all $^{13}$C spins through the hyperfine interaction, and the dynamics of the electron spin and nuclear spins typically occur on very different timescales [36]. To address these issues, a variety of decoherence-protected gates in which decoupling sequences on the electron spin are combined with nuclear spin control have been investigated [36,37,39,40,48–52]. Here we develop and demonstrate a novel electron-nuclear two-qubit gate based upon phase-controlled radiofrequency (rf) driving of nuclear spins interleaved with dynamical decoupling (DD) of the electron spin. We refer to this scheme as a DDrf gate. Our scheme enables the control of additional $^{13}$C spins while offering improved flexibility in dynamical decoupling to optimize the electron-spin coherence and avoid unwanted cross talk.

To design a selective two-qubit gate, we utilize the hyperfine interaction which couples each nuclear spin to the electron spin. As this interaction depends on the relative position of the spin to the NV, different nuclear spins can be distinguished by their precession frequencies [48–50]. In the interaction picture with respect to the electron energy splitting, and neglecting nonsecular terms, the Hamiltonian describing the electron and a single $^{13}$C nuclear spin is given by [48–50]

$$H = \omega_L I_z + A_{||} S_z I_z + A_{\perp} S_z I_x,$$

(1)

where $\omega_L = \gamma B_z$ is the nuclear Larmor frequency set by the external magnetic field $B_z$, $\gamma$ is the $^{13}$C gyromagnetic ratio, $S_{||}$ and $I_z$ are the spin-1 and spin-$1/2$ operators of the electron and nuclear spins, respectively, and $A_{||}$ and $A_{\perp}$ are the parallel and perpendicular hyperfine components, respectively.

To control the nuclear spin, we apply rf pulses of Rabi frequency $\Omega$, phase $\phi$, and frequency $\omega$. To target a specific nuclear spin, we set $\omega = \omega_1$, where $\omega_1 = \sqrt{(\omega_L - A_j)^2 + A_{\perp}^2}$ is the nuclear spin-precession frequency when the electron is in the $m_z = -1$ spin projection. In the following, we assume $(\omega_1 - \omega) \gg \Omega$, such that driving of the nuclear spin is negligible while the electron is in the $m_z = 0$ spin projection, and set $A_{\perp} = 0$ for simplicity (see the Supplemental Material [53] for the general case). Considering only the $m_z = \{0, -1\}$ subspace, with the addition of rf driving and in a rotating frame at the rf frequency, the Hamiltonian of Eq. (1) becomes [36,53]

$$H = |0\rangle\langle 0| \otimes (\omega_L - \omega_1) I_z + |1\rangle\langle 1| \otimes \Omega [\cos(\phi) I_x + \sin(\phi) I_z],$$

(2)

where $|0\rangle$ ($|1\rangle$) indicates the electron $m_z = 0$ ($m_z = -1$) spin projection. In this picture, for the electron in state $|0\rangle$, the nuclear spin undergoes precession around the $\hat{z}$ axis at frequency $(\omega_L - \omega_1) = A_j$. Conversely, while the electron is in the state $|1\rangle$, the nuclear spin is driven around a rotation axis in the $\hat{x}$-$\hat{z}$ plane defined by the phase of the rf field $\phi$.

To simultaneously decouple the electron spin from the environment, we interleave the rf pulses in a sequence of the form $(\tau - \pi - 2\tau - \pi - \tau)^N/2$, where $\pi$ is a $\pi$ pulse on the electron spin, $2\tau$ is the interpulse delay, and $N$ is the total number of electron decoupling pulses [Fig. 2(a)] [48–50]. We consider the evolution of the nuclear spin during this sequence separately for the two initial electron eigenstates: $|0\rangle$ and $|1\rangle$ [48–50]. We label each successive rf pulse by integer $k = 1, \ldots, K$, where $K = N + 1$ is the total number of rf pulses. If the initial electron-spin state is $|0\rangle$, only the even-$k$ rf pulses will be resonant and drive the nuclear spin [Fig. 2(b)]. Conversely, for initial state $|1\rangle$, the odd-$k$ pulses are resonant. The desired nuclear spin evolution can now be created by setting the phases $\phi_k$ of the rf pulses.

We construct both an unconditional rotation (single-qubit gate) and a conditional rotation (two-qubit gate).
antiparallel along the equator; therefore, the gate is a maximally 

electron starts in the state

path shows the nuclear spin evolution for the case where the

\( \omega \) (even \( k \)) and \( \phi \) are resonant with the electronic spin.

\( \Omega \) (in this work, \( \Omega = \pi/2 \)) and \( \tau \) are rotation angle

\( \theta \) and \( \phi \). Setting \( N \Omega \tau = \pi/2 \), a maximally entangling two-

ular spin gate is achieved, equivalent to a controlled-

\( r \) is restricted to a specific resonance condition for each spin,

making multiqubit control challenging. Third, because our

our method does not rely on an average frequency shift over

the two-electron-spin states \[37\], our gates can also be used

for selective control of nuclear spins coupled to spin-1/2
defects (such as the negatively charged group-IV color

centers \[7,9,13,20,28,54,55\]) and via a contact hyperfine

coupling, such as for donor spins in silicon \[34\] and SiMOS

quantum dots \[21\]. Finally, because control is achieved

through the rf field, a multitude of avenues for future

investigation are opened up, such as parallelizing gates by

frequency multiplexing and using shaped and composite

cycles to mitigate dephasing and cross talk \[38,56,57\].

FIG. 2. (a) Illustration of the pulse sequence employed to

realize a DDrf gate. Dynamical decoupling pulses on the electronic

spin (purple) are interleaved with rf pulses (yellow), which

selectively drive a single nuclear spin. (b) Illustration showing

that the initial state of the electronic spin determines which rf pulses

are resonant with the nuclear spin. If the electronic spin starts in \( |1\rangle \)

(\( m_s = -1 \)), the odd rf pulses (red) are resonant. For initial

electronic state \( |0\rangle \) (\( m_s = 0 \)), the even (blue) rf pulses are resonant.

The phase of each rf pulse is adapted to create the desired electronic

spin evolution, accounting for periods of free precession according

to Eq. (3). (c) Nuclear spin trajectory on the Bloch sphere for

a conditional rotation with \( N = 8 \) electronic decoupling pulses.

Starting from the initial nuclear state \( |\uparrow\rangle \) (yellow), the red (blue)

path shows the nuclear spin evolution for the case where the

electron starts in the state \( |1\rangle \) (\( |0\rangle \)). The final state vectors are

antiparallel along the equator; therefore, the gate is a maximally

entangling two-qubit gate. (d) Top-down view of (c).

To ensure that the sequential rf rotations build up constructively, the phases of each rf pulse should be set to

account for the periods of nuclear spin precession between

them, which build up in integer multiples of \( \phi_r = (\omega L - \omega I) \tau \).

For the case where the electronic starts in the state \( |0\rangle \)

(even \( k \)), the required sequence of phases is \( \phi_r, 3\phi_r, 5\phi_r, \ldots \),

while for the case where the electronic starts in the state \( |1\rangle \)

(odd \( k \)), we require the sequence \( 0, 2\phi_r, 4\phi_r, \ldots \). The

required phases are therefore given by \[53\]

\[
\phi'_k = \begin{cases} 
(k - 1)\phi_r + \pi, & \text{k odd,} \\
(k - 1)\phi_r, & \text{k even,}
\end{cases}
\]

where the (optional) \( \pi \) phase shift for the odd-\( k \) sequence

converts the unconditional rotation into a conditional rotation. By adding a further phase \( \varphi \) to all pulses, we

can also set the rotation axis of the gate. The rf pulse phases

are thus summarized by \( \phi_k = \varphi + \phi'_k \).

With this choice of phases, the total evolution of the

two-qubit system is given by \( V = V_z V_{\text{CROT}} \). Here, \( V_z \) is an

unconditional rotation of the nuclear spin around \( z \) \[53\], and \( V_{\text{CROT}} \)

is a conditional rotation of the nuclear spin depending on the electronic state, given by

\[
V_{\text{CROT}} = |0\rangle\langle 0| \otimes R_\varphi(N\Omega\tau) + |1\rangle\langle 1| \otimes R_\varphi(-N\Omega\tau),
\]

where \( R_\varphi(\theta) = e^{-i\theta[\cos(\varphi)I_x + \sin(\varphi)I_y]/\hbar} \). \( V_{\text{CROT}} \) describes a

controlled rotation of the nuclear spin with tuneable rotation angle (set by \( N, \Omega \), and \( \tau \)) and rotation axis (set by \( \varphi \)). Setting \( N\Omega\tau = \pi/2 \), a maximally entangling two-

ubit operation is achieved, equivalent to a controlled-

NOT gate up to local rotations. Example dynamics for a nuclear

spin evolving under such a sequence are shown in Figs. 2(c) and 2(d).

Our design has several advantages. First, the gate allows

nuclear spins with small or negligible \( A_\perp \) to be controlled,

thereby increasing the number of accessible nuclear spin

qubits. Second, because the targeted dynamics are achieved

by setting the rf phases and amplitudes, the interpulse delay

\( \tau \) of the decoupling sequence can be freely optimized to

protect the electronic coherence. This is in contrast to the
gates described in van der Sar et al. \[36\], for which \( \tau \) is

restricted to a specific resonance condition for each spin,

making multiqubit control challenging. Third, because our

method does not rely on an average frequency shift over

the two-electron-spin states \[37\], our gates can also be used

for selective control of nuclear spins coupled to spin-1/2
defects (such as the negatively charged group-IV color

centers \[7,9,13,20,28,54,55\]) and via a contact hyperfine

coupling, such as for donor spins in silicon \[34\] and SiMOS

quantum dots \[21\]. Finally, because control is achieved

through the rf field, a multitude of avenues for future

investigation are opened up, such as parallelizing gates by

frequency multiplexing and using shaped and composite

cycles to mitigate dephasing and cross talk \[38,56,57\].

III. TWO-QUBIT GATES: EXPERIMENT

Our experiments are performed at 3.7K using a single NV

center in diamond with natural abundance of carbon isotopes

(1.1% \( ^{13}\text{C} \)). Further details of the sample and experimental

setup can be found in the Supplemental Material \[53\]. As a

starting point, we use the DDrf gate to identify and

characterize \( ^{13}\text{C} \) nuclear spin qubits surrounding the NV

center. If the electron spin is prepared in a superposition

state, and the rf frequency is resonant with a nuclear spin in

the environment, the entangling interaction [Eq. (4)] deco-

heres the electron spin. Therefore, varying the rf frequency

(\( \omega \)) performs spectroscopy of the nuclear spin environment.

Figure 3 shows that multiple dips in the electron coherence

can be observed, indicating selective interactions with

several individual nuclear spins. Importantly, like other rf-

based approaches \[39,52\], the DDrf sequence is sensitive to

nuclear spins with small or negligible \( A_\perp \). In addition to

extending the number of qubits that can be controlled with a

single NV center, this capability also enables the detection of

additional spins when using the NV as a quantum sensor,
which we exploit in parallel work to realize 3D imaging of large spin clusters [58].

To verify the control offered by the DDrf two-qubit gate, we first demonstrate high-fidelity ancilla-based initialization and readout by preparation and tomography of a maximally entangled electron-nuclear state. To test the gate, we select a $^{13}$C spin (spin C1, Fig. 3) with a strong parallel hyperfine component of $A_1/2\pi = 213.154(1)$ kHz but a weak perpendicular hyperfine component $A_\perp/2\pi = 3.0(4)$ kHz [53]. We exploit the freedom in choosing the interpulse delay by setting $\tau$ to an integer multiple of the $^{13}$C Larmor period $\tau_L = 2\pi/\omega_L$, so that unwanted interactions between the electron spin and other $^{13}$C spins in the environment are effectively decoupled [47,59]. The choice of Rabi frequency $\Omega$ is a trade-off between obtaining faster gate speeds, maintaining frequency selectivity, and minimizing additional noise from the electronic hardware [53].

The sequence to perform the state preparation and tomography experiment is shown in Fig. 4(a) [16,37]. We first initialize the electron spin in the state $|0\rangle$ by resonant optical excitation [14]. We then swap the state of the electron spin onto the $^{13}$C spin and reset the electron spin. Next, we prepare the electron in a superposition state before performing the DDrf controlled-rotation gate, ideally preparing the electron-nuclear Bell state $|\psi_{\text{Bell}}\rangle = (|0+\rangle + |1-\rangle)/\sqrt{2}$, where $|\pm\rangle = (|\downarrow\rangle \pm |\uparrow\rangle)/\sqrt{2}$.

To perform quantum-state tomography on the two-qubit state, we first measure the electron spin along a chosen axis by appropriate basis rotations followed by $Z$-basis optical readout [14]. To mitigate potential dephasing of the nuclear spin induced by the electron-spin measurement, we make the electron-spin measurement nondestructive by using a short, weak laser pulse and conditioning progression of the sequence on the outcome $|0\rangle$, i.e., the detection of a photon [16,42] (see Supplemental Material for all readout parameters [53]). Following appropriate basis rotations, we then use the electron spin to measure the nuclear spin in the $X$ basis [16]. In this measurement, the electron is read out in a single shot with average fidelity 0.945(2) [14]. We independently characterize the nuclear spin readout, which is then used to correct for readout infidelities in subsequent measurements [53]. In order to reconstruct the full electron-nuclear state, we perform the sequence with and without an additional electron $\pi$ pulse before the first readout [53].

The reconstructed density matrix from quantum-state tomography is shown in Fig. 4(b). The prepared state $\rho$ exhibits a fidelity with the target Bell state of $F_{\text{Bell}} = \langle \psi_{\text{Bell}} | \rho | \psi_{\text{Bell}} \rangle = 0.972(8)$. Based upon a simple depolarizing noise model, we estimate the two-qubit gate fidelity to be $F_{\text{gate}} = 0.991(9)$ [53]. Additional characterization measurements in combination with numerical simulations indicate that the remaining infidelity can be mostly attributed to electron-spin dephasing due to noise from the electronic hardware [53].

**IV. A TEN-QUBIT SOLID-STATE SPIN REGISTER**

We now show how the combination of our DDrf gate with previously developed gates and control techniques [16,37] enables high-fidelity control of a ten-qubit hybrid spin register associated with a single NV center. Our register is composed of the electron and $^{14}$N spins of the NV center, along with eight $^{13}$C nuclear spins (Fig. 1). Our quantum register is connected via the central electron spin. To demonstrate this, we first show that all nuclear spins can be entangled with the electron spin by following the protocol shown in Fig. 4(a). For the case of the nitrogen spin, initialization is performed by a measurement-based scheme which heralds the preparation in a particular...
and perform operations analogous to those for the 13\textsuperscript{C} nuclear spins, including the two-qubit gates using the DDrf scheme. Genuine entanglement is probed by measuring the necessary expectation values in order to confirm bipartite entanglement [Fig. 5(b)].

Next, we show that the register is fully connected by preparing entangled states for all possible pairs of spins. To prepare nuclear-nuclear entanglement, we implement a probabilistic measurement-based scheme [62], as shown in Fig. 5(a). We first prepare a three-qubit GHZ state comprising the electron and two nuclear spins \( \text{GHZ}_3 = \frac{\{0++\} + \{1--\}}{\sqrt{2}} \) before performing a nondestructive \( X \)-basis measurement on the electron spin. The measurement ideally prepares the Bell state \( |\Phi^+\rangle = \frac{\{++\} + \{--\}}{\sqrt{2}} \) on the targeted pair of nuclear spins. Finally, we measure the necessary expectation values in order to reconstruct the nonzero matrix elements of this state and confirm bipartite entanglement [Fig. 5(b)].

The measured Bell state fidelities ranging from 0.63(3) to 0.97(1) are shown in Fig. 6. We attribute the variations in the measured values to differences in the two-qubit gate fidelities for each spin. In particular, the lower values measured for \( ^{13}\text{C} \) spins C7 and C8 are due to short coherence times in combination with long two-qubit gate durations necessitated by close spectral proximity to other spins [53]. All data are measured using a single set of gate parameters and using a single hardware configuration rather than separately optimizing for each pair of qubits.

V. GENERATION OF N-QUBIT GHZ STATES

Quantum-information processing tasks such as computations and error correction require the execution of complex algorithms comprising a large number of qubits. An important requirement for a quantum processor is thus the ability to perform operations on many of its constituents within a single algorithm. We test this capability by generating \( N \)-qubit GHZ-type states defined as

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**FIG. 4.** (a) Experimental sequence to prepare an electron-nuclear Bell state and determine the expectation value of the two-qubit operator \( ZX \). A series of single- and two-qubit gates are used to initialize the nuclear spin [16,37]. A subsequent \( \pi/2 \) rotation and two-qubit gate generate the Bell state \( |\psi_{\text{Bell}}\rangle = \frac{\{0+\} + \{1-\}}{\sqrt{2}} \). A measurement of the electron spin in the \( Z \)-basis is followed by an \( X \)-basis measurement of the nuclear spin through the electron spin. These measurements are separated by a nuclear spin echo, which is implemented to mitigate dephasing of the nuclear spin. The entire sequence is applied with and without an additional electron \( \pi \) pulse (dashed box) before the first electron readout in order to reconstruct the electron state while ensuring that the measurement does not disturb the nuclear spin state [16,42]. (b) Density matrix of the electron-nuclear state after applying the sequence shown in (a) to qubit C1, reconstructed with state tomography. We correct for infidelities in the readout sequence characterized in separate measurements [53]. The fidelity with the target Bell state is measured to be \( F_{\text{Bell}} = 0.972(8) \). Lighter blue shading indicates the density matrix for the ideal state \( |\psi_{\text{Bell}}\rangle \).
To generate such states, we follow the sequence shown in Fig. 7(a). First, \( N - 1 \) nuclear spins are initialized in the state \( |\uparrow\rangle \). Next, we prepare the electron spin in a superposition state and perform sequential controlled rotation gates between the electron and nuclear spins. Characterizing the full quantum state for a system of this size is an expensive task due to the dimensionality of the associated Hilbert space. However, we can determine if the state exhibits genuine multipartite entanglement of all \( N \) qubits using an entanglement witness with a reduced subset of measurement bases [61]. For a GHZ state with system size \( N \), there exist \( 2^N \) operators from which the nonzero elements of the density matrix can be reconstructed by linear inversion and from which a fidelity with the target state can be calculated. Negativity of the entanglement witness \( W_{GHZ} = 1/2 - |\langle GHZ_N|GHZ_N\rangle| \) heralds genuine multipartite entanglement [61]. We determine the required expectation values of products of Pauli operators on the register via the electron spin. In these experiments, the readout sequence is modified slightly. Prior to the readout of the electron-spin state, we rotate the nitrogen spin such that the desired measurement basis is mapped to the \( Z \) basis. This ensures that the population in the measurement basis is protected from dephasing during the optical readout of the electron spin, which is caused by the large electron-nitrogen hyperfine coupling in the excited state [53,63].

As the number of qubits is increased, a new challenge arises: The total sequence time becomes comparable to, or even exceeds the natural dephasing times \( T_2^* \) of the nuclear spins. In order to preserve the nuclear spin coherence, we insert spin-echo pulses (rf \( \pi \) pulses) into the state \( |\uparrow\rangle \).
sequence to refocus each spin at the point of the next operation performed upon it. In the Supplemental Material [53], we derive a general solution that can be used to algorithmically construct echo sequences that avoid any overlap in gates and that minimize idle time with the electron spin in a superposition state.

FIG. 7. (a) Experimental sequence to prepare a seven-qubit GHZ state $|\text{GHZ}_7\rangle$ (purple) and determine the expectation value of the seven-qubit operator $XYYYYZZ$ (orange). The measurement sequence is broken down into basis rotations (BR 1,2), an electron readout (RO), nuclear spin echoes (echo 1,2), and a multiqubit readout of the nuclear spins. All operations are applied sequentially (in the same way as shown in Fig. 5), but some are shown in parallel for clarity. (b),(c) Bar plots showing the measured expectation values (nonzero terms of the ideal state only) after preparing the five-spin (b) and seven-spin (c) GHZ states. The colors indicate the number of qubits involved, i.e., the number of (nonidentity) operators in the expectation value (inset). Gray bars show the ideal expectation values. See the Supplemental Material [53] for the operator corresponding to each bar. The fidelity with the target state is 0.804(6) (b) and 0.589(5) (c), confirming genuine multipartite entanglement in both cases. (d) Plot of GHZ state fidelity against the number of constituent qubits. A value above 0.5 confirms genuine $N$-qubit entanglement. The blue points are the measured data, while the green points are theoretical predictions assuming a simple depolarizing noise model whose parameters are extracted from single- and two-qubit experiments. Numerical values are given in the Supplemental Material [53].
In Figs. 7(b) and 7(c), we show measurements for \( N = 5 \) and \( N = 7 \) qubits. In Fig. 7(d), we present the measured fidelities with the target GHZ states for two to eight qubits, along with theoretical values as predicted by a depolarizing-noise model based on the individual two-qubit gate fidelities [53]. The growing discrepancy between the measured and predicted values for larger \( N \) suggests residual cross talk between the qubits, which is not taken into account in the model. For registers comprising up to seven spins, we observe negativity of the witness \( \mathcal{W}_{\text{GHZ}} \), revealing genuine \( N \)-qubit entanglement of up to seven qubits with high statistical significance.

VI. A LONG-LIVED QUANTUM MEMORY

The nuclear spin qubits surrounding the NV center are promising candidates for quantum memories with long coherence times [32,64]. Here we investigate the coherence properties of the register under dynamical decoupling and show that an arbitrary single-qubit state can be protected for over 75 s. Furthermore, we show that two-qubit entanglement can be preserved beyond 10 s.

We first investigate the coherence of individual nuclear spin qubits under dynamical decoupling. After initializing the nuclear spin in the state \( |+\rangle \), we prepare the electron in the state \( |1\rangle \) [electron \( T_1 = 3.6(3) \times 10^3 \) s [47]]. This has two effects. First, it allows us to perform selective rf \( \pi \) pulses on the target nuclear spin. Second, the magnetic field gradient imposed by the electron-nuclear hyperfine interaction induces a frozen core, which suppresses flip-flop interactions between nuclear spins [65,66] and thereby reduces the noise the spins are exposed to.

The observed spin-echo coherence times \( T_2^\alpha=1 \), with \( \alpha \) the number of rf pulses, vary between 0.26(3) and 0.77(4) s for the eight \( ^{13}\text{C} \) spins. For the \( ^{14}\text{N} \) spin, we find 2.3(2) s, consistent with the smaller gyromagnetic ratio by a factor of 3.4. The range of coherence times observed for the \( ^{13}\text{C} \) spins is likely caused by differences in the microscopic environment of each spin. In particular, \( ^{13}\text{C} \) spins close to the NV center are in the heart of the frozen core and generally tend to couple predominantly to the part of the spin environment for which the dynamics are also suppressed most strongly. Spins farther from the NV tend to couple more strongly to the spin environment outside the frozen core. This explanation is consistent with the observation that the spin with the longest \( T_2^\alpha=1 \) of 0.77(4) s is located closest [C1, \( r = 0.53(5) \) nm [58]] to the vacancy lattice site, while the shortest \( T_2^\alpha=1 \) of 0.26(3) s is found for a spin at a larger distance [C8, \( r = 1.04(4) \) nm [58]].

As expected, increasing the number of decoupling pulses leads to an increase in the measured coherence times. For \( \alpha = 256 \) pulses, the decay time of C5 reaches \( T_2^{\alpha=256} = 12.9(4) \) s, while for the \( ^{14}\text{N} \) spin, we measure \( T_2^{\alpha=256} = 63(2) \) s (see Supplemental Material Fig. 2 [53]). For the other \( ^{13}\text{C} \) spins for which we measure \( T_2^{\alpha=256} \), we find a range of values from 4(1) to 25(4) s [53].

To confirm that arbitrary quantum states can be protected, we prepare the six cardinal states and measure the average state fidelity under dynamical decoupling. The measured decay curves for spin C5 and the \( ^{14}\text{N} \) spin are shown in Figs. 8(a) and 8(b), where \( \alpha \) is varied from 1 to 256. With 256 pulses, we measure a state fidelity exceeding the classical memory bound of \( \frac{4}{5} \) at a time of 16.8 s for C5 and at a time of 75.3 s for the \( ^{14}\text{N} \) spin.

The coherence times demonstrated here are the longest reported for individual qubits in the solid state and exceed values for isolated nuclear spin qubits in isotopically purified materials [24,32,33]. More importantly, however, in our register we realize these long coherence times while maintaining access to ten coupled spin qubits.

We exploit the multiqubit nature of the register to investigate the protection of entangled states of two \( ^{13}\text{C} \) spin qubits. After preparing the state \( |\Phi^+\rangle = (|+\rangle + |-\rangle)/\sqrt{2} \) following the sequence shown in Fig. 5(a), we again prepare the electron in the state \( |1\rangle \). We then measure the Bell state fidelity as a function of total evolution time for \( \alpha = 1 \) to \( \alpha = 256 \) pulses. Note that since \( |\Phi^+\rangle \) is an eigenstate of ZZ, its evolution is not affected by the coupling between the two qubits, which is predominantly dipolar and of the form \( C I_1 I_2 \), with \( C = 1.32(4) \) Hz [58]. The measured fidelities are plotted in Fig. 8(c). For \( \alpha = 256 \) decoupling pulses, we confirm the preservation of entanglement for \( > 10 \) s, as quantified by a fidelity exceeding 0.5 with the desired Bell state.

With the capability to protect multiqubit quantum states, it becomes important to consider additional effects that may affect their coherence, such as the presence of correlated noise. As a first experimental step towards understanding such effects, we use entangled states of nuclear spins to explore spatial correlations within the noise environment. We perform experiments on two pairs of \( ^{13}\text{C} \) spins. We prepare two Bell states for each pair, one exhibiting even ZZ parity, which, written in the \( Z \) basis, is given by \( |\Phi^+\rangle = (|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle)/\sqrt{2} \), and another exhibiting odd ZZ parity, \( |\Psi^-\rangle = (|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle)/\sqrt{2} \). The difference in the coherence times of these two states gives an indication of the amount of correlation in the noise experienced by the two spins [67]. In the case of perfectly correlated noise, one would expect the state \( |\Phi^+\rangle \) to decay at four times the single-qubit decay rate (superdecoherence), while the state \( |\Psi^-\rangle \) would form a decoherence-free subspace [68,69]. In contrast, for completely uncorrelated noise, the coherence times for the two states would be identical.

We measure the coherence times for the two Bell states, varying the total evolution time for the case of a single spin-echo pulse (\( \alpha = 1 \)) with the electron spin prepared in the state \( |1\rangle \). In Fig. 8(d), we plot the normalized coherence
A statistically significant difference between the decay curves for the two Bell states is found for both pairs, where the odd-parity state $|\Psi^-\rangle$ decays more slowly than the even-parity state $|\Phi^+\rangle$, indicating partly correlated noise in the system. We can relate the size of the effect to the distance between the spins in the pairs, which has been characterized in separate work.[58]. This reveals that the pair with a smaller separation [C1 and C6, distance 0.96(3) nm] shows more correlation than the pair with a larger separation [C5 and C2, 1.38(7) nm]. This observation is consistent with the idea that spatially close spins tend to couple to the same nuclear spin environment, and therefore experience correlated noise, although large deviations from this rule are expected to be possible for specific cases.[67]. Characterizing such correlated noise provides new opportunities to investigate the physics of decoherence in spin baths[67] and to develop and test quantum-error-correction schemes that are tailored for specific correlated noise[70,71].

VII. CONCLUSION

In conclusion, we develop a novel electron-nuclear two-qubit gate and apply these gates to realize a ten-qubit solid-state spin register that can protect an arbitrary single-qubit state for over one minute. The techniques developed in this work can be readily implemented for multiqubit control in a variety of other donor and defect platforms, including spin-1/2[7,9,13,20,28,54,55] and contact hyperfine[21,34] systems, for which many previous gate designs are challenging to apply[37,48–50]. Further improvements in selectivity and fidelity of the gates are anticipated to be
possible by (optimal) shaping of the rf pulses \cite{38,56,57} and by reducing electronic noise. Additionally, the use of direct rf driving opens the possibility to perform gates in parallel on multiple qubits. Combined with already demonstrated long-range optical entanglement \cite{29–31}, our multiqubit register paves the way for the realization of rudimentary few-node quantum networks comprising tens of qubits. This will enable the investigation of basic error-correction codes and algorithms over quantum networks \cite{44–46}. Finally, looking beyond quantum information, the gate sequences developed here also enable new quantum-sensing methods \cite{58}.

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