Signal amplification in a nanomechanical Duffing resonator via stochastic resonance

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We experimentally study stochastic resonance in a nonlinear bistable nanomechanical resonator. The device consists of a PdAu doubly clamped beam serving as a nanomechanical resonator excited capacitively by an adjacent gate electrode and its vibrations are detected optically. The resonator is tuned to its bistability region by an intense pump near a point of equal transition rates between its two metastable states. The pump is amplitude modulated, inducing modulation of the activation barrier between the states. When noise is added to the excitation, the resonator’s displacement exhibits noise dependent amplification of the modulation signal. We measure the resonator’s response in the time and frequency domains, the spectral amplification and the statistical distribution of the jump time.

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Stochastic resonance (SR) is a phenomenon in which an appropriate amount of noise is used to amplify a periodic signal acting on a bistable nonlinear system. SR has been demonstrated experimentally in electrical, optical, superconducting, and neuronal systems. SR could be used for amplification in nanomechanical devices in order to improve force detection sensitivity. Nanomechanical resonators operating in their nonlinear regime exhibit the well known Duffing bistability. In a Duffing oscillator, above a critical excitation amplitude, the response becomes a multi-valued function of the frequency in some finite frequency range, and the system becomes bistable (with a low amplitude state $S_l$ and a high amplitude one $S_h$) with jump points in the frequency response. In the presence of noise, the oscillator can occasionally overcome the activation barrier and hop between the states. When an oscillator is excited in the bistability region near a point of equal transition rates between its states, an amplitude modulation (AM) of the force could be amplified by noise when the noise dependent transition rate is comparable to twice the modulation frequency. This type of SR, where the bistability property depends on the driving force, is usually referred to as high frequency SR.

In this paper we demonstrate high frequency SR in our nanomechanical resonator and measure the noise dependent amplification. Our study extends previous work by characterizing the SR by spectral amplification, and by measuring the statistical distribution of the jump time at SR condition. The system under study consists of a nonlinear doubly clamped nanomechanical PdAu beam, excited capacitively by an adjacent gate electrode. The device is shown in the inset of Fig. 2. The bistability region of the device is found by exciting the resonator with a harmonic pump signal, sweeping its amplitude upward and then downward for constant pump frequency, calculating the difference between the two responses, and repeating for a range of frequencies. The result is shown in Fig. 1a. An example of a pump amplitude hysteresis loop for a constant pump frequency of 520.58kHz (the broken line in Fig. 1a) is shown in Fig. 1b. When the pump is amplitude modulated without additional noise, the resonator will respond with small amplitude oscillation in the respective hysteresis branch (vertical black line in Fig. 1b). To bring the system into SR, the resonator is tuned to its bistability region by an intense pump near a point of equal transition rates between its states. Next, the pump is amplitude modulated, inducing thus modulation of the activation barrier between the states and modulating the transition rates $\Gamma_1$ and $\Gamma_2$ of the noise-driven transitions $S_l \rightarrow S_h$ and $S_h \rightarrow S_l$ respectively. When an appropriate amount of noise is added, the resonator will hop from one state to the other in synchronization with the modulation signal and with large amplitude (vertical red line in Fig. 1b). The working point (pump amplitude and frequency) is determined such that $\Gamma_1 \approx \Gamma_2$.

The nonlinear dynamics of the fundamental mode of a doubly clamped beam excited by an external force per unit mass $F(t)$ can be described by a Duffing oscillator equation for a single degree of freedom $x$

$$\ddot{x} + 2\mu \dot{x} + \omega_0^2 (1 + \kappa x^2) x = F(t),$$

where $\mu$ is the damping constant, $\omega_0/2\pi$ is the resonance frequency of the fundamental mode of the oscillator, and $\kappa$ is the cubic nonlinear constant. Our resonator has a quality factor $Q = \omega_0/2\mu \approx 2000$ (at $10^{-5}$ torr) and its fundamental mode resonance frequency is $\omega_0/2\pi \approx 520.4$kHz. The resonator is excited by an applied force $F(t) = f_p (1 + A_{mod} \cos \Omega t) \cos (\omega_p t) + F_n(t)$ composed of an amplitude modulated pump signal with amplitude $f_p$, frequency $\omega_p$, modulation frequency $\Omega$, and modulation depth $A_{mod} < 1$, and $F_n(t)$ is a zero-mean Gaussian white noise with autocorrelation function $\langle F_n(t) F_n(0) \rangle = 2D \delta(t)$ and noise intensity $D$. This is achieved by applying a voltage of the form $V = V_{dc} + V_p (1 + A_{mod} \cos \Omega t) \cos (\omega_p t) + V_n(t)$ where $V_{dc}$ is a DC bias (employed for tuning the resonance frequency), $V_p$ is the pump amplitude, and $V_n(t)$ is the applied voltage noise. The voltage noise intensity is $I \equiv \langle V_n^2(t) \rangle^{1/2}$ and $V_p, I \ll V_{dc}$. 

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The displacement spectral density can be expressed as

\[ S_x(\omega) = \sum_{k=\pm\infty} A_k(D) \delta(\omega_p + k\Omega) + S_{nx}(\omega), \]  

(2)

composed of delta peaks at the mixing products \( \omega_p + k\Omega, k = 0, \pm 1, \pm 2, \ldots \), with noise dependent amplitudes \( A_k(D) \), and a background spectral density of the noise denoted by \( S_{nx}(\omega) \). In order to characterize the noise dependent amplification, we define a spectral amplification parameter \( \eta_k \) by

\[ \eta_k(D) = \frac{A_k(D)}{A_k(D=0)}. \]  

(3)

A schematic diagram of the experimental setup employed for measuring SR is depicted in Fig. 2. The resonator is excited by two sources (pump and noise) and its vibrations are detected optically using a knife-edge technique. The device is located at the focal point of a lensed fiber which is used to focus laser light at the beam and to collect the reflected light back to the fiber and to a photodetector (PD). To measure the response, the PD signal is amplified, mixed with a local oscillator (LO), and low pass filtered. The spectrum around \( \omega_p \) of the amplified PD signal is measured using a spectrum analyzer. The measurement is done in vacuum (10^{-5} \text{ torr}) and at room temperature. The resonator length is 100 \text{ µm}, width 600 nm, and thickness 250 nm. The gap separating the doubly clamped beam and the stationary side electrode is 4 \text{ µm} wide. The device is fabricated using bulk nano-machining process together with electron beam lithography.18

Typical results of SR measured in the time and frequency domains are shown at the left and right sides respectively of Fig. 3 for five voltage noise intensities (panels (a)-(e)). Here \( \Omega = 20 \text{ Hz}, A_{\text{mod}} = 10\% \), and \( V_{\text{dc}} = 25 \text{ V} \). The blue dotted line drawn in the time domain represents the modulation signal. The voltage noise intensities (a) 1 mV and (b) 349 mV correspond to low noise levels below the value corresponding to SR. Panel (a) shows response without jumps. Panel (b) shows a response containing few arbitrary jumps. The voltage noise intensity (c) 464 mV corresponds to SR condition where every half cycle, the resonator jumps to the other metastable state. The voltage noise intensities (d) 530 mV and (e) 600 mV are higher than the the value corresponding to SR. In panel (d), as in Ref.11 the resonator stays in the \( S_l \) state with few jumps to the \( S_h \) state. In panel (e), the high noise almost completely screens the signal. In the frequency domain displayed at the right side of Fig. 3, the fundamental frequency and the mixing products can be seen. At SR, the spectrum contains high order mixing products.

The dependence of the spectral amplification \( \eta_k(D) \)
represents the modulation signal. The intensity is increased. The dotted line in the time domain (right) as the input voltage noise shots of the resonator’s response in the time domain (left) and in the frequency domain (right) as the input voltage noise intensity is increased. The dotted line in the time domain represents the modulation signal.

FIG. 3: (Color online). Panels (a) – (c) exhibit typical snapshots of the resonator’s response in the time domain (left) and in the frequency domain (right) as the input voltage noise intensity is increased. The dotted line in the time domain represents the modulation signal.

FIG. 4: (Color online). (a),(b) Spectral amplification $\eta_k(D)$ ($k = \pm 1$ and $k = \pm 3$) vs. voltage noise intensity. (c) The spectral amplification $\eta_3(D)$ vs. noise intensity for three AM frequencies. (d) Measurement of the probability density $f(\tau)$, where $\tau$ is the difference between the time of the transition $S_i \rightarrow S_h$ and the time at which the modulation amplitude gets its maximal value.

The dependence of the spectral amplification $\eta_3(D)$ on $I$ for three AM frequencies $\Omega = 20\, \text{Hz}, 30\, \text{Hz},$ and $40\, \text{Hz}$ is shown in Fig. 4c for $A_{\text{mod}} = 10\%$. As predicted theoretically, amplification is monotonically decreasing with $\Omega$.

Finally, we demonstrate the method proposed in (20) for extracting transition rates from SR measurements. Near the maximum (minimum) points of the AM signal, the rate $\Gamma_1 (\Gamma_2)$ obtains its largest value, which is denoted by $\Gamma_{m1}$ ($\Gamma_{m2}$). Let $\tau$ be the difference between the time of the transition $S_i \rightarrow S_h$ and the time at which the modulation amplitude gets its maximal value (namely, the time at which $\Gamma_1 = \Gamma_{m1}$). The probability density of the random variable $\tau$, which is denoted by $f(\tau)$, is experimentally derived from 1000 modulation cycles sampled in the time domain (see Fig. 4d). The solid line represents a Gaussian function fitted to the measured probability density. The rate $\Gamma_{m1}$ can be estimated from the expectation value $\mu_\tau$ and the variance $\sigma_\tau^2$ of $\tau$ by $\Gamma_{m1} = -\mu_\tau/\sigma_\tau^2$, yielding $\Gamma_{m1} = 10.5\, \text{kHz}$.

In conclusion, stochastic resonance has been demonstrated in a nanomechanical resonator. The resonator was tuned to its bistability region by an intense pump near a point of equal transition rates between its states. An amplitude modulation is used to modulate the activation barrier between the states. When noise is injected, the resonator’s response exhibits noise dependent amplification. We measure the resonator’s displacement in the time and frequency domains, the spectral amplification and statistics of the jumps time. SR could be very useful in nanomechanical devices as a mean to implement on-chip mechanical amplification and to increase the signal to noise ratio.

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