Scale Invariant Power Spectrum in Hořava-Lifshitz Cosmology without Matter

Bin Chen*, Shi Pi† and Jin-Zhang Tang‡

1Department of Physics, and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China (Dated: August 7, 2009)

Abstract

In this paper we investigate the physical implications of the dynamical scalar mode in pure Hořava-Lifshitz gravity on cosmology. We find that it can produce a scale-invariant power spectrum in UV era if the detailed balance condition on the action is relaxed. This indicates that the physical scalar mode may seed the large scale structure and Hořava-Lifshitz cosmology could be a qualified alternative to inflationary scenario.

PACS numbers: 98.80.Cq

* Electronic address: bchen01@pku.edu.cn
† Electronic address: spi@pku.edu.cn
‡ Electronic address: alberttjz@163.com
I. INTRODUCTION

Recently, inspired by the quantum critical phenomena in condensed matter physics[1], Hořava proposed a renormalizable gravitation theory with dynamical critical exponent $z = 3$ in UV limit[2, 3]. In this Hořava-Lifshitz theory, the time and space will take different scaling behavior as

$$x \to bx, \quad t \to b^z t, \quad (1)$$

where $z$ is the dynamical critical exponent characterizing the anisotropy between space and time. In the action, the kinetic term is still quadratic in the first derivatives of the metric of spatial slice, but the potential terms may have higher order spatial derivatives. This fact help us avoid the breaking of unitarity, which happens in other higher derivative gravity theory. Due to the existence of higher spatial derivative terms, the UV behavior of the theory is much improved. In fact, from power counting, it seems that the theory is renormalizable in the UV. By turning on relevant perturbation, the theory flows to an IR fix point where it behaves like standard Einstein general relativity. As a price one has to pay, the Lorentz symmetry is broken by construction and it will only emerge as an accidental symmetry in the IR.

Since its discovery, many authors have attempted to apply this Hořava-Lifshitz gravity to the study of early cosmology[4, 5]. Especially it was pointed out that the Hořava-Lifshitz gravity may be a candidate for theory of the early universe instead of inflation. The causality and flatness problem could be solved under this framework as in inflationary model. By considering the scalar field coupled to gravity, it was shown in [4, 5, 6], that the scale-invariant power spectrum can be produced, with a new dispersion relation at UV

$$E \sim k^6. \quad (2)$$

The key point here is that the matter scalar action may have higher derivative terms, allowed by the scaling (1). Such scalar field theory could be taken as field theory at Lifshitz points without detailed balance condition[7]. The general field theory at a Lifshitz point was discussed in [8]. The other applications of Hořava-Lifshitz gravity to cosmology and black hole physics could be found in [10].

On the other hand, one important feature of Hořava-Lifshitz gravity is that due to anisotropy, the usual diffeomorphism is broken to foliation-preserving diffeomorphisms. As a result, there exist an extra dynamical scalar degree of freedom. This fact has been shown both for the perturbations about flat background[3] and about FRW cosmology[11]. This means that we can have a scalar mode from the perturbation of gravitational field in the early universe, besides the other two tensor modes of gravitational waves. It would be interesting to investigate the physical implication of such scalar mode.

In this paper, we consider the possibility if we may interpret the scalar mode of the gravity as the origin of the large scale structure and cosmic microwave background anisotropy in the IR era, without any additional matter field. If this is case, we need neither the inflaton nor curvaton in the early universe, and gravity itself can produce the seed of the structure. As the first step, we need to check if this scalar mode could produce the scale-invariant spectrum. But unfortunately, in [11], the dispersion relation of the gravitational scalar is no more (2), but instead

$$E^2 \sim k^4. \quad (3)$$

Thus the power spectrum will be proportional to $k$ which is not consistent with the CMB observation. The essential point is that the relation (3) stems from the fact that the terms
of sixth order derivatives in the equation of motion are absent due to the detailed balance condition, which was used originally by Hořava to simplify the form of the action of gravity. We will show that after we relax the detailed balance condition, we can recover the scale invariant spectrum for the gravitational scalar.

The rest of this paper is organized as follows. In section II we will take a brief review of the Hořava-Lifshitz gravity, while in section III we study the cosmological implication of the gravitational scalar. After relaxing the detailed balance condition and doing analytic continuation of two parameters, we obtain the action of the gravity, which are slightly different from the original action but allows us to have sixth order spatial derivative in the equation of motion of scalar mode. We derive the equation of motion of the gravitational scalar without detailed balance condition and calculate the scalar power spectrum. In section IV we will end with some discussions.

II. REVIEW OF HOŘAVA-LIFSHITZ GRAVITY WITH DETAILED BALANCE CONDITION

In this section, we give a brief review of the Hořava-Lifshitz gravity. Due to the anisotropy of time and space, it is more convenient to work with the ADM metric,

\[ ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt). \]

The classical scaling dimensions of the fields are

\[ [N] = 0, \quad [N_i] = z - 1, \quad [g_{ij}] = 0. \]

From this metric, we can construct an action of gravity from the scaling rule (1) and the invariance under foliation-preserving diffeomorphisms. It turns out that the kinetic term is quadratic in the second fundamental form. And before considering the relevant terms of lower scaling dimension, we consider the marginal potential term which dominate on the high-energy limit. In \( z = 3, D = 3 \) case, it could contain the higher derivatives of the metric up to sixth order [3]. The terms quadratic in \( g_{ij} \) could be

\[ \nabla_i R_{jk} \nabla^i R^{jk}, \quad \nabla_i R_{jk} \nabla^k R^{ij}, \quad R \nabla^2 R, \quad R_{ij} \nabla^2 R^{ij}. \]

The other combinations are either identical up to a total derivative or related by Bianchi identity or other symmetries. Although there are many constraints, the relative magnitudes of these terms are arbitrary. The other marginal terms are cubic in curvature. For the relevant terms, there are even more possibilities. To reduce these uncertainties Hořava suggest to impose the detailed balance condition to confine the action further more. Under this condition, the potential term including relevant perturbations are determined by

\[ S_v = \frac{\kappa^2}{8} \int dt d^3 x \sqrt{g} E^{ij} G_{ijkl} E^{kl}, \]

where

\[ \sqrt{g} E^{ij} = \frac{\delta W[g_{kl}]}{\delta g_{ij}} \]

for some action \( W \) in three-dimension and \( G_{ijkl} \) is the De Witt metric defined by

\[ G_{ijkl} = \frac{1}{2} (g^{ik} g^{jl} + g^{il} g^{jk}) - \lambda g^{ij} g^{kl}. \]
The action $W$ could be of the form

$$W = \frac{1}{\omega^2} \int \omega_3(\Gamma) + \mu \int d^3x \sqrt{g}(R - 2\Lambda_W), \quad (10)$$

where $\omega_3$ is the gravitational Chern-Simons term and $\omega$ is a dimensionless coupling constant. $\mu$ is coupling constant with dimension $[\mu] = 1$ and $\Lambda_W$ is effectively a cosmological constant with dimension $[\Lambda] = 2$.

Finally the action of gravity with detail balance can be written as

$$S_g = \int dtd^3x \sqrt{g}N \left\{ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) - \frac{\kappa^2}{2\omega^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2\omega^2} \epsilon^{ijk} \sqrt{g} R_{il} \nabla_j R^l_k - \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} + \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left( \frac{1}{4} - \frac{4\lambda}{3} \right) R^2 + \Lambda_W R - 3\Lambda^2_W \right\}, \quad (11)$$

where $K_{ij}$ is the second fundamental form, or the extrinsic curvature, of the spatial slice; $C_{ij}$ is the Cotton tensor which is used to construct the action preserving the detailed balance condition; and $R_{ij}$ is the Ricci tensor in the three dimensional space. Their definitions are

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i), \quad (12)$$

$$C_{ij} = \frac{\epsilon^{ikl}}{\sqrt{g}} \nabla_k \left( R^l - \frac{1}{4} \delta^l_i \right). \quad (13)$$

The first parenthesis in (11) involving only the extrinsic curvature is the kinetic term, while the others are potential terms. $\lambda$ is the coupling constant in kinetic term, and runs to $\lambda = 1$ in IR era such that the kinetic term goes back to the case in general relativity. $\kappa$ is something like the Newton’s gravitational constant $G_N$, and indeed there is a proportional relation between them

$$G_N = \frac{\kappa^2}{32\pi c} \quad (14)$$

where the speed of light is

$$c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda_W}{1-3\lambda}}. \quad (15)$$

Note that in UV era, the dominant potential term will be the integral of

$$C_{ij} C^{ij} = (\nabla_i R_{jk} \nabla^i R^{jk} - \nabla_i R_{jk} \nabla^j R_{ik} - \frac{1}{8} \nabla_i R \nabla^i R), \quad (16)$$

which is the only term involving sixth order derivatives of the metric after partial integrations. The Cotton tensor $C_{ij}$ is symmetric and traceless, and is covariantly conserved. Most of all, it is conformal,

$$g_{ij} \rightarrow \exp(2\Omega(x))g_{ij}, \quad C_{ij} \rightarrow \exp(-5\Omega(x))C_{ij}. \quad (17)$$

And it is the successor of Weyl tensor in three dimensional space to play the role of the criterion of conformal flatness.
III. THE POWER SPECTRUM WITHOUT DETAIL BALANCE

In this section we will calculate the equation of motion of the extra scalar mode in Hořava-Lifshitz gravity in the early universe without matter, but the detailed balance condition will be relaxed. We begin with perturbed Friedmann-Robertson-Walker metric in ADM form after gauge fixing \([11]\)

\[
 ds^2 = -c^2 dt^2 + a^2 (1 - 2\psi) \delta_{ij} (dx^i + a^{-1} \partial^i B dt) (dx^j + a^{-1} \partial^j B dt), 
\]

where \(\delta_{ij}\) is the metric on the three dimensional leaf of the foliation. Here we have assumed the background universe is flat. From (18), we have

\[
 N^2 = c^2, 
\]

\[
 N_i = ca \partial_i B 
\]

\[
 g_{ij} = a^2 (1 - 2\psi) \delta_{ij} \simeq a^2 e^{-2\psi} \delta_{ij}, 
\]

\[
 g^{ij} \simeq a^{-2} (1 + 2\psi) \delta^{ij} \simeq a^{-2} e^{2\psi} \delta^{ij}, 
\]

where \(\simeq\) means equalization accurate up to first order, and we adopt the exponential expression of the scalar \(\psi\) for convenience, as in \([12]\). From this metric we have the Ricci tensor and scalar,

\[
 R_{ij} = \left[ \partial^2 \psi - (\partial \psi)^2 \right] \delta_{ij} + \partial_i \partial_j \psi + \partial_i \psi \partial_j \psi, 
\]

\[
 R = 2a^{-2} e^{2\psi} \left[ 2\partial^2 \psi - (\partial \psi)^2 \right]. 
\]

More importantly the spatial slice is conformal flat. This fact leads to vanishing Cotton tensor and the absence of six derivative terms in the equation of motion of \(\psi\). The same thing happens for the perturbation around the flat spacetime background. In the latter case, the equation of motion of physical scalar mode contains terms with spatial derivatives up to fourth order. As a result, the ultraviolet behavior is not good enough. This stems from the detailed balance condition.

As pointed out in the original paper \([3]\), there is no first principle at this moment to decide which kind of combination of terms in (6) is more physical. The coefficients of these terms could be arbitrary, only being constrained by the requirements of stability and unitarity of the quantum theory. Just from simplicity, the detailed balance condition was imposed. The very interesting relation to three-dimensional massive gravity from detailed balance is another story, being very little to do with the renormalizability of the theory. Therefore, one can expect that without detailed balance condition the theory is still UV well-defined. In the following we will take this philosophy and consider the more general action without detailed balance.

Our starting point is the following action of the pure Hořava-Lifshitz gravity

\[
 S = S_K + S_V^{(UV)} + S_V^{(inter)} + S_V^{(IR)}, 
\]

where \(S_K\) is the kinetic term, \(S_V^{(UV)}\), \(S_V^{(inter)}\) and \(S_V^{(IR)}\) are terms dominant in UV, intermediate
and IR era, respectively, being of the forms

\[ S_K = \int dt d^3x \sqrt{g} N \alpha (K_{ij} K^{ij} - \lambda K^2), \]

\[ S_V^{(UV)} = \int dt d^3x \sqrt{g} N \beta (\beta_1 \nabla_i R_{jk} \nabla^i R^{jk} + \beta_2 \nabla_i R_{jk} \nabla^j R^{ik} + \beta_3 \nabla_i R \nabla^i R), \]

\[ S_V^{(inter)} = \int dt d^3x \sqrt{g} N \left\{ \gamma \frac{\epsilon^{ijk}}{\sqrt{g}} R_{il} \nabla_j R^l_k + \zeta R_{ij} R^{ij} + \eta R^2 \right\}, \]

\[ S_V^{(IR)} = \int dt d^3x \sqrt{g} N (\xi R + \sigma), \]

with the coupling coefficients

\[ \alpha = \frac{2}{\kappa^2}, \quad \beta = \frac{\kappa^2}{2\omega^4}, \]

\[ \gamma = -\frac{\kappa^2 \mu}{2\omega^2}, \quad \zeta = \frac{\kappa^2 \mu^2}{8}, \quad \eta = -\frac{\kappa^2 \mu^2}{32} \frac{1 - 4\lambda}{1 - 3\lambda}, \]

\[ \xi = -\frac{\Lambda_W \kappa^2 \mu^2}{8(1 - 3\lambda)}, \quad \sigma = \frac{3\Lambda_W \kappa^2 \mu^2}{8(1 - 3\lambda)}. \]

The essential difference from the action in [3] comes from \( S_V^{(UV)} \). The dimensionless parameters \( \beta_1, \beta_2 \) and \( \beta_3 \) in it are the ones dominating the UV behavior. In detailed balance case, from [14], we have \( \beta_1 = 1, \beta_2 = -1, \beta_3 = -1/8 \). But here when discarding the condition, these three parameters can be arbitrary. Furthermore, we have made the following analytic continuation

\[ \mu \rightarrow i\mu, \quad \omega^2 \rightarrow -i\omega^2 \]

as suggested in [9] in order to have a physical evolution. As a result, the emergent speed of light is

\[ c = \frac{\kappa^2 \mu}{4 \sqrt{\frac{\Lambda_W}{3\lambda - 1}}}, \]

which requires \( \Lambda_W \) being positive for \( \lambda > \frac{1}{3} \).

The above action is not the most general one we can have. For simplicity, we still keep most of the terms in the Hořava-Lifshitz gravity, but only modify the marginal terms which are quadratic in curvature and of two spatial derivatives.

The first task is to derive the constraint equation. Write down Hamiltonian [2] and take the variations with respect to the momentum conjugate to \( N_i \) and \( N \), we get respectively two equations. One of them corresponding to \( N_i \) gives so called super-momentum constraint:

\[ 0 = \nabla_i N (K^i j - \lambda K \delta^i j) \]

which lead to the relation on \( B \) up to the first order

\[ c \partial^2 B = \frac{1 - 3\lambda}{\lambda - 1} \dot{\psi}. \]

The constraint corresponding to \( N(t) \) is subtler. Since we require projectivity condition, we cannot get the local super-Hamiltonian constraint. For the classical evolution, the isotropy
and homogeneity of the background allow us to reduce the integral constraint to a local one, which is just the Friedman equation on the scale factor:

$$\left(\frac{\dot{a}}{a}\right)^2 = c^2 \frac{\sigma}{3\alpha(1 - 3\lambda)} = \frac{c^4 \Lambda_{W}}{3\lambda - 1}. \quad (34)$$

From now on, our discussion will base on the gravity action with above analytic continued parameters. As a consequence, the Hubble “constant” $H = \dot{a}/a$ is really a constant if we just focus on classical evolution and the universe is undergoing a exponentially expansion no matter whatever $\lambda$ is. In other words, the universe is in a de-Sitter phase once $\lambda$ is fixed. This means that the early universe is very much like in an exponentially expanding inflation period. The key difference is that we do not need a scalar field with slow roll potential to drive the inflation.

Another important feature of the evolution is that the acceleration is related to the emergent speed of light. At UV, the speed of light could be large such that the acceleration is quite large. But at IR, the acceleration is the same as the late time acceleration in Einstein gravity with a positive cosmological constant.

Using (33) and (34), we calculate the action of second order.

$$\begin{align*}
(2) S_K &= c^{-1} \alpha \int dt d^3 x a^3 (1 - 3\lambda) \left[ \frac{2}{3(1 - \lambda)} \dot{\psi}^2 + 6H \dot{\psi} \psi + \frac{9}{2} H^2 \psi^2 + \frac{H^2}{2} B \partial^2 B \right], \\
(2) S_V^{(UV)} &= -2c\beta \int dt d^3 x \frac{1}{a^3} (3\beta_1 + 2\beta_2 + 8\beta_3) \dot{\psi} \partial^6 \psi, \\
(2) S_V^{(inter)} &= 2c(3\zeta + 8\eta) \int d^3 x \frac{1}{a} \partial^4 \psi, \\
(2) S_V^{(IR)} &= c \int dt d^3 x \left[ -2a \xi \dot{\psi} \partial^2 \psi + \frac{9}{2} a^3 \sigma \dot{\psi}^2 - \frac{1}{2} a^3 \sigma B \partial^2 B \right].
\end{align*} \quad (35)$$

When adding together, the terms in (2) $S_V^{(IR)}$ involving $\sigma$ can be converted into terms of $H^2$ by virtue of (34), and can be combined with those in (2) $S_K$. Therefore, the total action of second order is

$$\begin{align*}
(2) S &= \int dt d^3 x \left\{ 3\alpha a^3 (1 - 3\lambda) \left[ \frac{2}{3(1 - \lambda)} \dot{\psi}^2 + 6H \dot{\psi} \psi + \frac{9}{2} H^2 \psi^2 \right] \\
&\quad - \frac{2\beta}{a^3} c^2 (3\beta_1 + 2\beta_2 + 8\beta_3) \dot{\psi} \partial^6 \psi + \frac{2}{a} c^2 (3\zeta + 8\eta) \psi \partial^4 \psi - 2c^2 a \xi \dot{\psi} \partial^2 \psi \right\}. \quad (39)
\end{align*}$$

From the second order action we can derive the equation of motion,

$$\begin{align*}
\ddot{\psi} + 3H \dot{\psi} + \frac{9}{2} (1 - \lambda) \dot{H} \psi + \frac{\kappa^4}{4\omega^4} c^2 (3\beta_1 + 2\beta_2 + 8\beta_3) \frac{1 - \lambda}{1 - 3\lambda} a \dot{\partial}^6 \psi \\
&\quad + \frac{\kappa^4}{16} c^2 \left( \frac{1 - \lambda}{1 - 3\lambda} \right)^2 \frac{1}{a^4} \partial^4 \psi + \frac{\kappa^4}{16} c^2 \Lambda_{W} \frac{1 - \lambda}{(1 - 3\lambda)^2} \frac{1}{a^2} \partial^2 \psi = 0. \quad (40)
\end{align*}$$

It is remarkable that when $\lambda = 1$, the above equation reduces to the one in standard Einstein theory, in which case $\psi$ is not a real dynamical field. In other words, when the theory flows to IR fixed point where the Einstein theory and the diffeomorphism are partially recovered, the scalar field is just a gauge artifact due to the extra gauge degrees of freedom.
This formula coincides with the equation of motion of the gravitational scalar in [11], except for the $\partial^6 \psi$ dependent term here. We note, that the equation in [11] is derived from Hořava’s original action with detailed balance condition, where $\beta_1 = 1, \beta_2 = -1$ and $\beta_3 = -1/8$, thus $3\beta_1 + 2\beta_2 + 8\beta_3 = 0$, which just makes the $\partial^6 \psi$ term vanishing. Then there’s no $(2) S_{\psi}^{(UV)}$ term any more, and $(2) S_{\psi}^{(inter)}$ will be dominant in UV era, when the dispersion relation will be $E^2 \sim k^4$, and spectrum will be $P_k \sim k$. On the other hand, if we discard the detailed balance condition, the $k^6$-dependence in dispersion relation will emerge. Moreover, one has to care about the stability and unitarity at UV, which requires that $3\beta_1 + 2\beta_2 + 8\beta_3 > 0$. Finally, following the treatment in [5, 6] we can obtain the power spectrum. In UV limit, the equations of motion reads

$$\ddot{\psi} + \Omega^2 \frac{a^6}{a^6} \psi = 0, \quad \Omega^2 = \frac{k^4 c^2}{4 \omega^4} (3\beta_1 + 2\beta_2 + 8\beta_3) \frac{1 - \lambda}{3\lambda - 1} k^6$$ (41)

which can be solved under WKB approximation as

$$\psi \simeq \frac{1}{\sqrt{2\Omega}} \exp \left(-i\Omega \int \frac{dt}{a^3}\right).$$ (42)

And we can use (42) to calculate the scale-invariant power spectrum,

$$P_\psi(k) = \frac{k^3}{2 \pi^2} |\psi|^2 = \frac{2}{\pi^2} \frac{\omega^2}{k^4 \mu} \frac{3\lambda - 1}{\Lambda_W (3\beta_1 + 2\beta_2 + 8\beta_3) (1 - \lambda)}.$$ (43)

We see, that the power spectrum produced by the scalar mode of the gravitational field at UV is scale-invariant, sharing the same property of temperature fluctuations in CMBR observation. This is a hint, that the gravitational scalar may be the seed of the large scale structure of the universe and the origin of the anisotropy of the cosmic microwave background radiation.

It seems that the power spectrum is divergent when $\lambda \to 1$. This is an illusion. The above spectrum corresponds to the fluctuations generated at the scale much higher than IR fixed point. And the $\lambda$ in (43) corresponds to the matching point between UV and IR, roughly at the horizon crossing, at which $\lambda$ is not equal to 1.

The fluctuations of the scalar mode would be frozen after horizon crossing. And after IR fixed point, the scalar mode is not physical anymore, instead it changes to the Newton potential. In fact, to respect the projectivity condition, the perturbed FRW metric was set to (18) in terms of Painleve-Gullstrand coordinates, which is equivalent to the standard scalar perturbed FRW metric. Moreover, the higher-derivative terms are much suppressed after RG flows to $\lambda = 1$ so that we can trust the usual treatment in general relativity. As usual, the scalar perturbation may combine with matter perturbation to form a conserved curvature perturbation. Unfortunately due to the ignorance of the coupling of matter with gravity with $z \neq 1$, we do not know if there exist a similar quantity before RG flowing to $\lambda = 1$. It is an important issue which we would like to address in the future.

**IV. DISCUSSIONS**

In this paper we investigated the possibility that the scalar perturbation of Hořava-Lifshitz gravity, instead of matter perturbation, seeds the large scale structure of our
universe. As the first step, we showed that the power spectrum of the gravitational scalar at UV is scale invariant. It has been argued that in Hořava-Lifshitz gravity the inflation is not necessary. Our study suggest that we do not even need matter scalar field and gravity itself provide the natural scalar mode for us.

We found that in order to have scale invariant spectrum, we had to relax the detailed balance condition and have more general gravity action. This is not a drawback. From our investigation, the absence of Cotton tensor allows the equation of motion of scalar mode to have spatial derivatives up to six order. Similarly this happens for the fluctuations around the flat spacetime background. This actually improve the ultraviolet behavior of the scalar mode. On the other hand, the discarding of the detailed balance condition open many other possibilities to construct the model. Some of the possibility may have interesting physical implication. For example, the marginal term as $R^3, R^i_k R^j_l, RR_{ij} R^{ij}$ may induce the cubic interaction terms of the scalar mode, leading to non-Gaussianity\cite{13}.

In Hořava-Lifshitz gravity, to have a physical evolution requires a positive cosmological constant from beginning. At the very early stage, the speed of sound is very large and drive a very fast exponentially expansion. This stage could plays the role of inflation and help us to solve the well-known problems in big-bang cosmology. At very late time stage, we recover the accelerating expansion due to a small positive cosmological constant.

Even though the idea that Hořava-Lifshitz cosmology can be an alternative to the inflationary scenario, there are lots of questions waiting to be answered. Due to our ignorance of the exact RG flow of the theory, it is not clear when and how the Einstein gravity is recovered and the scalar mode disappear. It is also an interesting issue to study how the gravitational scalar is related to the temperature perturbation. More fundamentally, it is essential to study the ultraviolet behavior of the Hořava-Lifshitz gravity, especially considering so many marginal terms and relevant terms without detail balance. Besides the requirement of stability and unitarity, does there exist other principle to determine the parameters? It could be possible that there exist UV fixed points characterized by a few parameters. The RG flows from UV fixed points to IR by various relevant terms are essential for us to understand the Hořava-Lifshitz cosmology.

Acknowledgments

The work was partially supported by NSFC Grant No.10535060, 10775002, NKBPRPC (No. 2006CB805905) and RFDP. We would like to thank Qing-guo Huang for stimulating questions and comments.

[1] R.H. Mornreich, M. Luban and S. Shtrikman, “Critical Behavior at the Onset of $\vec{k}$-space Instability on the $\lambda$ Line”, Phys. Rev. Lett. 35 (1975)1678;
S. Sachdev, “Quantum Phase Transitions”, Cambridge U.P. (1999);
E. Ardonne, P. Fendley and E. Fradkin, “Topological Order and Conformal Quantum Critical Points”, Annals Phys. 310 (2004)493-551, [cond-mat/0311466].

[2] P. Horava, “Membranes at Quantum Criticality,” JHEP 0903, 020 (2009) arXiv:0812.4287 [hep-th]].
[3] P. Horava, “Quantum Gravity at a Lifshitz Point,” Phys. Rev. D 79, 084008 (2009) [arXiv:0901.3775 [hep-th]].
[4] G. Calcagni, “Cosmology of the Lifshitz universe,” arXiv:0904.0829 [hep-th].
[5] E. Kiritsis and G. Kofinas, “Horava-Lifshitz Cosmology,” arXiv:0904.1334 [hep-th].
[6] S. Mukohyama, “Scale-invariant cosmological perturbations from Horava-Lifshitz gravity without inflation,” arXiv:0904.2190 [hep-th].
[7] E.M. Lifshitz, “On the Theory of Second-Order Phase Transitions I & II”, Zh. Eksp. Teor. Fiz 11 (1941)255 & 269.
[8] B. Chen and Q. G. Huang, “Field Theory at a Lifshitz Point,” arXiv:0904.4565 [hep-th].
[9] H. Lu, J. Mei and C. N. Pope, “Solutions to Horava Gravity,” arXiv:0904.1595 [hep-th].
[10] H. Nastase, “On IR solutions in Horava gravity theories,” arXiv:0904.3604 [hep-th].
R. Brandenberger, “Matter Bounce in Horava-Lifshitz Cosmology,” arXiv:0904.2835 [hep-th].
H. Nikolic, “Horava-Lifshitz gravity, absolute time, and objective particles in curved space,” arXiv:0904.3412 [hep-th]. R. G. Cai, L. M. Cao and N. Ohta, “Topological Black Holes in Horava-Lifshitz Gravity,” arXiv:0904.3670 [hep-th]. R. G. Cai, Y. Liu and Y. W. Sun, “On the z=4 Horava-Lifshitz Gravity,” arXiv:0904.4104 [hep-th]. Y. S. Piao, “Primordial Perturbation in Horava-Lifshitz Cosmology,” arXiv:0904.4117 [hep-th].
E. O. Colgain and H. Yavartanoo, “Dyonic solution of Horava-Lifshitz Gravity,” arXiv:0904.4357 [hep-th]. U. H. Danielsson and L. Thorlacius, “Black holes in asymptotically Lifshitz spacetime,” JHEP 0903, 070 (2009) arXiv:0812.5088 [hep-th].
P. Horava, “Spectral Dimension of the Universe in Quantum Gravity at a Lifshitz Point,” arXiv:0902.3657 [hep-th]. T. Takahashi and J. Soda, “Chiral Primordial Gravitational Waves from a Lifshitz Point,” arXiv:0904.0554 [hep-th]. X. Gao, “Cosmological Perturbations and Non-Gaussianities in Horava-Lifshitz Gravity,” arXiv:0904.4187 [hep-th].
G.E. Volovik, “z=3 Lifshitz-Horava model and Fermi-point scenario of emergent gravity”, arXiv:0904.4113 [gr-qc].
T. Sotiriou, M. Visser, S. Weinertner, “Phenomenologically viable Lorentz-violating quantum gravity”, arXiv:0904.4464 [hep-th]. S. Mukohyama, K. Nakayama, F. Takahashi and S. Yokoyama, “Phenomenological Aspects of Horava-Lifshitz Cosmology”, arXiv:0905.0055 [hep-th]. Y. S. Myung and Y.W. Kim, “Thermodynamics of Hoˇrava-Lifshitz black holes”, arXiv:0905.0179 [hep-th]. T. Nishioka, “Horava-Lifshitz Holography”, arXiv:0905.0473 [hep-th]. Songbai Chen, Jiliang Jing, “Quasinormal modes of a black hole in the deformed Horava-Lifshitz gravity”, arXiv:0905.1409 [gr-qc].
R.B. Mann, “Lifshitz Topological Black Holes”, arXiv:0905.1136 [hep-th].
Yun Soo Myung, “Thermodynamics of black holes in the deformed Horava-Lifshitz gravity”, arXiv:0905.0957 [hep-th].
Ahmad Gho-ods, “Toroidal solutions in Horava Gravity”, arXiv:0905.0836 [hep-th].
Rong-Gen Cai, Li-Ming Cao, Nobuyoshi Ohta, “Thermodynamics of Black Holes in Horava-Lifshitz Gravity”, arXiv:0905.0751 [hep-th].
S.Kalyana Rama, “Anisotropic Cosmology and (Super)Stiff Matter in Horava’s Gravity Theory”, arXiv:0905.0700 [hep-th].
Alex Kehagias, Konstadinos Sfetsos, “ The black hole and FRW geometries of non-relativistic gravity”, arXiv:0905.0477 [hep-th].
R.A. Konoplya, “Towards constraining of the Horava-Lifshitz gravities”, arXiv:0905.1523 [hep-th].
Robert H. Brandenberger, “Processing of Cosmological Perturbations in a Cyclic Cosmology”, arXiv:0905.1514 [hep-th].
D. Orlando and S. Reffert, “On the Renormalizability of Horava-Lifshitz-type Gravities”, arXiv:0905.0301 [hep-th].
J. Kluson, “Branes at Quantum Criticality”, arXiv:0904.1343 [hep-th].
J. Kluson, “Stable and Unstable D-Branes at Criticality”, arXiv:0905.1483 [hep-th].
[11] R. G. Cai, B. Hu and H. B. Zhang, “Dynamical Scalar Degree of Freedom in Horava-Lifshitz Gravity,” arXiv:0905.0255 [hep-th].
[12] J. M. Maldacena, “Non-Gaussian features of primordial fluctuations in single field inflationary models,” JHEP 0305, 013 (2003) [arXiv:astro-ph/0210603].

[13] B. Chen, S. Pi and J.Z. Tang, Work in progress.