Tight focusing of second-order cylindrical vector beam by Mikaelian lens

S.S. Stafeev\textsuperscript{1,2}, E.S. Kozlova\textsuperscript{1,2}, A.G. Nalimov\textsuperscript{1,2}, V.V. Kotlyar\textsuperscript{1,2}

\textsuperscript{1}Image Processing Systems Institute of the RAS – Branch of FSRC "Crystallography & Photonics" of the RAS, Molodogvardejskaya street 151, Samara, Russia, 443001
\textsuperscript{2}Samara National Research University, Moskovskoe Shosse 34A, Samara, Russia, 443086

Abstract. In this paper we simulated the focusing of a second-order cylindrical vector beam by Mikaelian microlens. By using FDTD-method it was shown that the gradient lens produces a region of the energy backflow near its shadow surface. Presence of a microhole in the center of the lens allows localize the direct energy inside the lens material and to concentrate the energy backflow in free space.

1. Introduction
Cylindrical vector beams (CVBs) are the beams, which have spatially nonuniform polarization distribution [1]. Such beams can be generated by simple optical elements as multi-sector phase plate [2] or spatial light modulator [3]. However, to produce the arbitrary CVBs quite complex optical schemes are needed [4-6]. Nowadays, they attracted research interests due to their unique optical properties, and they have perspective application in optical manipulation [7-9], optical microscopy [10], telecommunication [11] and quantum information [12-13].

A large number of scientists are interested in the optical properties of CVB in a sharp focus [14-16]. In [14] it was shown by analytical and numerical study that compact focal spot can be formed while focusing of Gaussian vortex beam with high-order cylindrical polarization which order is equal to the vortex phase singularity. In [16] authors used numerical simulation based on Richards-Wolf equations to show that radially and azimuthally polarized beams with topological charges equal to 0, 1, 2 and 10 can form dark channel which is useful for particle acceleration.

Another interesting phenomena which can be observed in the tight focus of optical vortices is the energy backflow [17-20]. In [18] authors theoretically and numerically studied formation of a reverse energy flux in the focus of a linearly polarized field with the binary phase. In [19] it was shown that tightly focused \textit{n}-th order CVBs with a vortex phase and the topological charge of \textit{m} which is higher than the beam order by two \((m = n + 2)\) formed in the focal plane near the optical axis an area of energy backflow. Similar effects have been obtained while focusing of optical vortices with a topological charge of 2 [20] and second order CVBs [21]. Using of optical lenses with a high numerical aperture (NA) allow to make the magnitude of the reverse energy flow comparable to the magnitude of the direct energy flow. The disadvantage of the methods in [19-21] is that the region of the energy backflow is close to the area of the direct energy flow. It will be more suitable for an optical trapping or optical manipulation if these areas would be isolated from each other. The separation of these flows could be reached by focusing of the incident radiation near the interface of the element and the...
medium, for example, when the direct energy flow is inside the element and the reverse flow region is outside it. Gradient lenses, such as the Mikaelian lens [22], Luneberg lens [23], Maxwell's fisheye [24], focus light near the boundary of two media. They could be used to separate the areas of flows. Moreover, photonic-crystal lenses which are analogues lenses with continuously changing refractive index and are based on the variation of the effective refractive can be used [25-27].

In this paper, the tight focusing of a second-order CVB by a Mikaelian lens with a length of 10 μm, a radius of 6 μm, and a refractive index on the axis of 1.5 is studied. We considered three cases: an ordinary Mikaelian lens, Mikaelian lens covered by metal film and a Mikaelian lens with a small hole in the center of the shadow side. The influence of metal film and presence of a hole on a localization of energy flows was investigated. The optimal parameters of the lens design which allows to separate the revers and direct energy flows were found.

2. Mikaelian lens
The Mikaelian lens can be described by its refractive index which varies in accordance with the formula [22,28]:

\[ n(r) = n_0 \left[ ch \left( \frac{\pi r}{2L} \right) \right]^{-1} \]

where \( n_0 \) is the refractive index on the optical axis, \( r \) is the radial coordinate, \( L \) and \( R \) are the length and the radius of the lens, respectively. The NA of the proposed lens is equal to \( NA = \sqrt{n_0^2 - 1} \) [29].

Using finite difference time domain (FDTD) method implemented in FullWAVE package we simulated process of tight focusing of second order CVB with wavelength of 0.633 μm and Jones vector equal to \( \left( -\sin(2\varphi), \cos(2\varphi) \right) \) (\( \varphi \) is an azimuthal angle) by this lens. The wavefront of the incident light was flat (the amplitude and phase distribution of the beam was the same as the distribution of the amplitude and phase of the field in [32]). In our simulation it was assumed that \( n_0 = 1.5, L = 10 \mu m, R = 6 \mu m \) (\( n(R) = 1, \) previously it was demonstrated the manufacturing of metamaterials with an effective refractive index close to unity [30,31]). Spatial steps of the grid was equal to 0.015 and the time step was chosen in accordance with Courant's condition. Fig. 1 shows calculated distribution of the Poynting vector longitudinal component \( S_z = \text{Re}(E \times H^*) \). The dashed line indicates the lens border.

![Figure 1](image-url)

**Figure 1.** The Poynting vector longitudinal component \( S_z \) in XZ-plane (a) produced by ordinary Mikaelian lens, and an enlarged fragment of calculated field near the focus (b).
It can be seen from Fig. 1 that the proposed lens forms a focus in the form of a cylinder with a radius of about ~ 400 nm, and height of 500 nm. There is a reverse energy flow also in the form of a cylinder with a diameter of 0.3 μm inside this focus. The maximum of absolute values of $S_z$ in the regions of the forward and reverse flows are comparable in order of magnitude, although in the region of the forward flow the maximum is $|S_z|$ and exceeds the maximum $|S_z|$ in the reverse flow area by about 1.5 times. The distribution of $S_z$ is asymmetrical along the Z-axis in contrast to the of a conventional lens which produced a symmetrical energy backflow [19–21].

3. Mikaelyan lens with metallic layer

In previous section, it was shown that the region of backward flow is surrounded by the forward energy flow region similar to the case of using lenses with high NA. In this section we proposed thin metal cover on shadow side of Mikaelyan lens to isolate the reverse flow region [33]. The thickness of chromium film was 70 nm thick and there is an uncoated area of 0.3 μm wide in the center of the lens. Figure 2 shows the distribution of the calculated Poynting vector longitudinal component $S_z$.

![Figure 2](image.png)

Figure 2. The Poynting vector longitudinal component $S_z$ in XZ-plane (a) produced by Mikaelyan lens with chromium layer, and an enlarged fragment of calculated field near the focus (b).

Figure 2 shows that the direct energy flow is concentrated inside the lens, however, the region of the reverse energy flow also does not extend beyond the boundary of the lens.

4. Mikaelyan lens with a hole

Another way to separate the regions of the reverse and direct energy flows (Fig. 1) is to cut a hole in the center of the shadow side of the proposed lens with a simultaneous increase of the lens thickness. We simulated a lens with a height of 0.5 μm higher than the previously simulated lens (Fig. 1) and with a cylindrical hole on the axis. The diameter and height of the hole were 0.3 μm and 0.8 μm, respectively. The result of focusing is shown in Fig. 3.

![Figure 3](image.png)

It can be seen from Fig.3 that it is possible to localize the region with the direct energy flow (dark areas in Fig. 3) inside the element. At the same time reverse energy flow is in the air. It should be noted that in comparison with Fig. 1 reverse flow decreased – maximum of absolute value $|S_z|$ in Fig. 3 in the direct flow region is 4.7 times higher than the maximum of $|S_z|$ in the reverse flow area (versus 1.5 in Fig. 1).
**Figure 3**. The Poynting vector longitudinal component $S_z$ in XZ-plane (a) produced by Mikaelyan lens with a small hole in the center, and an enlarged fragment of calculated field near the focus (b).

5. Conclusion

In this paper, the tight focusing of a CVB by three different design of Mikaelyan lens with a radius of 6 μm, and a refractive index on the axis of 1.5 was investigated. Conventional Mikaelyan lens, Mikaelyan lens with chromium layer on the shadow side and Mikaelyan lens with a small hole in the center part of the shadow side were observed. The presence of the reverse energy flow near the lens shadow surface was demonstrated for all proposed configurations. Analysis of three design of proposed lens showed that the most appropriate way to separate areas of direct and reverse energy flow is producing of a small hole with simultaneously increasing of the lens length. It was shown, that it is possible to localize the region of the direct energy flow inside the lens, and the region with the reverse energy flow into free space while using Mikaelyan lens with a length of 10.5 μm and with a cylindrical hole which diameter and depth are 0.3 μm and 0.8 μm, respectively.

Acknowledgments

The work was funded by the Russian Science Foundation under grant # 18-19-00595 (in part "Mikaelyan lens"), by the Russian Foundation for Basic Research under grant #18-07-01122 (in part "Mikaelyan lens with a hole"), grants # 18-07-01380 and #18-29-20003 (in part "Mikaelyan lens with metallic layer"), and by the RF Ministry of Science and Higher Education within a state contract with the "Crystallography and Photonics" Research Center of the RAS (in part "Introduction").

References

[1] Zhan Q 2009 *Adv. Opt. Photonics* 1 1-57
[2] Khonina SN, Ustinov AV, Fomchenkov SA, Porfirev AP 2018 *Sci. Rep.* 8 14320
[3] Kumar P, Nishchal NK 2020 *Imaging and Applied Optics Congress, OSA JTh2A.22*
[4] Liua J, Chen X, He Ya, Lua L, Ye H, Chaia G, Chen Sh, Fan D 2020 *Results in Physics* 19, 103455
[5] Zhou Yu, Li X, Cai Ya, Zhang Ya, Yan Sh, Zhou M, Li M, Yao B 2020 *Appl. Opt.* 59 8932-8938
[6] He Ya, Ye H, Liu J, Xie Zh, Zhang X, Xiang Yu, Chen Sh, Li Yi, Fan D 2017 *IEEE Phot. J.* 9 1-10
[7] Zhang Yu, Shen J, Min Ch, Jin Yu, Jiang Yu, Liu J, Zhu S, Sheng Yu, Zayats AV, Yuan X 2018 *Nano Lett.* 18 5538-5543
[8] Shi P, Du L, Yuan X 2018 *Opt. Express* 26, 23449-23459
[9] Rui G, Wang X, Cui Y 2015 *Opt. Express* 23 25707-25716
[10] Xie X, Chen Y, Yang K, Zhou J 2014 Phys. Rev. Lett. 113 263901
[11] Fang J, Xie Zh, Lei T, Min Ch, Du L, Li Zh, Yuan X 2018 ACS Photonics 5 3478–3484
[12] Ndagano B, Nape I, Cox MA, Rosales-Guzman C, Forbes A 2018 Journal of Lightwave Technology 36 2292-301
[13] Fickler R, Lapkiewicz R, Ramelow S, Zeilinger A 2014 Phys. Rev. A 89 4172-4183
[14] Khonina SN 2019 Appl. Phys B 125 100
[15] Meng P, Man Zh, Konijnenberg AP, Urbach HP 2019 Opt. Express 27 35336-35348
[16] Xiaoqiang Zh, Ruishan Ch, Anting W 2018 Opt. Commun. 414 10-15
[17] Yuan G, Rogers ETF, Zheludev NI 2019 Light: Science & Applications 8 2
[18] Khonina SN, Ustinov AV 2019 Opt. Lett. 44 2008-2011
[19] Kotlyar VV, Stafeev SS and Kovalev AA 2019 Opt. Express 27 16689–16702
[20] Kotlyar VV, Kovalev AA and Nalimov AG 2018 Opt. Lett. 43 2921–2924
[21] Stafeev SS, Kotlyar VV, Nalimov AG and Kozlova ES 2019 IEEE Photonics J. 11 4500810
[22] Mikaelian AL 1951 Dokl. Akad. Nauk SSSR 81 569–571
[23] Zentgraf T, Liu Y, Mikkelsen MH, Valentine J and Zhang X 2011 Nat. Nanotechnol. 6 151–155
[24] Born M and Wolf E 2013 Principles of optics: electromagnetic theory of propagation, interference and diffraction of light (Elsevier)
[25] Fathollahi Khalkhali T, Alipour-Beyraghi M, Lalenejad M and Bananej A 2018 Opt. Commun. 435 202–211
[26] Gilarlue MM, Badri SH, Rasooli Saghai H, Nourinia J and Ghobadi C 2018 Photonics Nanostructures - Fundam. Appl. 31 154–159
[27] Xia F, Li S, Zhang K, Jiao L, Kong W, Dong L and Yun M 2018 Phys. B Condens. Matter 545 233–236
[28] Stafeev SS, Kozlova ES and Nalimov AG 2020 Computer Optics 44 29-33, 2020
[29] Kotlyar VV, Stafeev SS and Nalimov AG 2019 Sharp Focusing of Laser Light (CRC Press)
[30] Zhang XA, Bagal A, Dandley EC, Zhao J, Oldham CJ, Wu BI, Parsons GN and Chang CH 2015 Adv. Funct. Mater. 25 6644-6649
[31] Kwon DH, Werner DH 2017 Opt. Express 15 9267-9272
[32] Kotlyar VV, Nalimov AG, Stafeev SS and O’Faolain L 2019 J. Opt. 21 055004
[33] Novitsky AV and Novitsky DV 2007 J. Opt. Soc. Am. A 24 2844–2849