Effects of supersymmetric grand unification scale physics on $\Gamma(b \to s\gamma)$

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ABSTRACT

Although calculations of the $b \to s\gamma$ rate in supersymmetric grand unified models have always either ignored the gluino mediated contribution or found it to be negligible, we show that taking universal supersymmetry breaking masses at the Planck scale, rather than at the gauge unification scale as is customary, leads to the gluino contribution being more significant and in fact sometimes even larger than the chargino mediated contributions when $\mu > 0$ and $\tan \beta$ is of order 1. The impact is greatest felt when the gluinos are relatively light. Taking the universal boundary condition at the Planck scale also has an effect on the chargino contribution by increasing the effect of the wino and higgsino-wino mediated decays. The neutralino mediated contribution is found to be enhanced, but nevertheless it remains relatively insignificant.
The flavor changing decay $b \to s\gamma$ is often an important test of new physics because it is rapid enough to be experimentally observable although it appears first at the one loop level in the standard model (SM), thus allowing new physics to add sizeable corrections to it. For example, the decay is useful to limit parameter space in the minimal supersymmetric standard model (MSSM) \cite{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}. This is an especially useful tool if certain constraints have already been placed on the MSSM. Since the decay vanishes in the limit of unbroken supersymmetry, the relevant constraints pertain to the terms in the Lagrangian that softly break supersymmetry (SUSY). The general soft SUSY breaking scalar interactions for squarks and sleptons in the MSSM are of the following form:

$$V_{\text{soft}} = Q(A_U \lambda_U) U^c H_2 + Q(A_D \lambda_D) D^c H_1 + L(A_E \lambda_E) E^c H_1 + \text{h.c.} + Q^c \mathbf{m}_Q^2 Q + U^c \mathbf{m}_U^2 U^c + D^c \mathbf{m}_D^2 D^c + L^c \mathbf{m}_L^2 L + E^c \mathbf{m}_E^2 E^c,$$  

where $Q$ and $L$ are squark and slepton doublets and $U^c$, $D^c$, and $E^c$ are squark and slepton singlets. It is most frequently assumed that soft breaking operators are induced by supergravity, and that these operators have a universal form which is generation symmetric and CP conserving. Usually this assumption is coupled with that of grand unification to provide further constraints on the model’s parameters. For purposes of simplicity the universal boundary condition is traditionally taken at the grand unification scale, even though the soft-breaking operators would be present up to near the Planck scale. The universal soft SUSY-breaking boundary condition is described by the following parameters: the scalar mass $m_0$, the gaugino mass $m_{1/2}$, and the trilinear and bilinear scalar coupling parameters $A$ and $B$, respectively.

Despite convention, some recent papers \cite{12, 13, 14, 15} have demonstrated important implications of taking the boundary conditions at the Planck scale $M_P$ and evolving them down to the scale $M_G$ where grand unification is broken. The important difference between taking the universal boundary condition at the Planck scale versus at the GUT scale is due to that the top quark mass is known to be large ($\sim 174\,\text{GeV}$) and grand unification causes some fields to feel the effects of the top coupling by unifying them into the same multiplet with the top. For example in $SU(5)$ grand unification, $Q$, $U^c$, and $E^c$ together transform as the $10$-representation. Taking the universal boundary condition at the GUT scale is ignoring the fact that the soft-breaking parameters run above the GUT scale. One may at first think that any effects of grand unification would be supressed by powers of $1/M_G$, but it has been demonstrated that such effects rather depend on $\ln (M_P/M_G)$ \cite{16}. One surprising result, which is given in refs. \cite{13, 15}, is that the predicted rate for the lepton flavor violating decay $\mu \to e\gamma$ may be only one order of magnitude beneath current
experimental limits. This is found even in SU(5) grand unification, where only neutralinos are available to mediate the decay at the one-loop order. Although due to the soft-breaking masses being assumed flavor blind and the unitarity of the CKM mixing matrix one might naively expect the partial width to vanish, this does not happen because operators with strength of the top coupling cause the third generation scalar fields contained in the 10-representation, in SU(5), to be considerably lighter than the corresponding fields of the other two generations [13]. Below the grand unification scale, the only squarks effected by the top Yukawa coupling are the top singlet and the third generation SU(2) \_L squark doublet.

We will apply these facts to demonstrate that in some of the same regions of parameter space where chargino corrections are important, one may also expect sizeable corrections from gluinos. Previous calculations of \( b \to s\gamma \) in the MSSM have routinely either ignored the contributions mediated by gluinos [12, 13, 14, 15, 16, 17] or found them to be less important [1, 2, 9, 11] than we will find them to be when we take the universal SUSY soft breaking boundary condition at the Planck scale rather than at the GUT breaking scale, as is customary. This is due to the fact, as explained above, that taking the universal boundary condition at the Planck scale leads to a different low energy SUSY spectrum than taking it at the GUT breaking scale does. For some parameter space, not only is the contribution from the gluinos important, but it is also comparable to that of the charginos. The fact that the top squark soft breaking masses are lighter when the universal boundary condition is taken at the Planck scale rather than the grand unification scale is, of course, likewise felt by the wino and higgsino-wino mediated decays.

The standard model amplitude for \( b \to s\gamma \) has been derived in refs. [17, 18], and expressions for the additional MSSM amplitudes have been derived in refs. [1, 2, 3, 4], with ref. [5] containing the first and most complete derivation. The leading-order QCD corrected partial width is given by its ratio to the inclusive semi-leptonic decay width in the following form [13]:

\[
\frac{\Gamma (b \to s\gamma)}{\Gamma (b \to c \nu \ell)} = \frac{6 \alpha}{\pi g(z)} \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} |c_7 (m_b)|^2 ,
\]

(2)

where \( \alpha \) is the electromagnetic coupling, \( g(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \ln z \) with \( z = m_c/m_b = 0.316 \pm 0.013 \) is the phase-space factor, and the inclusive semi-leptonic branching ratio is \( BR(b \to c \nu \ell) = 0.107 \). The ratio of the CKM matrix entries, for which we will use the experimental mid-value, is \( |V_{ts}^* V_{tb}|^2 / |V_{cb}|^2 = 0.95 \pm 0.04 \). The QCD corrected amplitude \( c_7 (m_b) \) is given as

\[
c_7 (m_b) = \eta^{16/23} \left[ c_7 (M_W) - \frac{8}{3} c_8 (M_W) \left[ 1 - \eta^{-2/23} \right] \right] + \sum_{i=1}^{8} a_i \eta^{b_i} ,
\]

(3)
with \(a_i\) and \(b_i\) being given in ref. [19], \(\eta = \alpha_s(M_W)/\alpha_s(m_b)\), for which we will use \(\eta = 0.548\). The terms \(c_7(M_W)\) and \(c_8(M_W)\) are respectively \(A_\gamma\), the amplitude for \(b \rightarrow s\gamma\) evaluated at the scale \(M_W\) and divided by the factor \(A_\gamma^0 \equiv 2G_F\sqrt{\alpha/8\pi^3}V_{ts}^*V_{tb}m_b\) and \(A_g\), the amplitude for \(b \rightarrow sg\) divided by the factor \(A_0^\gamma \sqrt{\alpha_s/\alpha}\). The effective interactions for \(b \rightarrow s\gamma\) and \(b \rightarrow sg\) are given by

\[
L_{\text{eff}} = \frac{A_0^\gamma}{2} (A_\gamma \sigma^{\mu \nu} P_R b F_{\mu \nu} + A_g \sigma^{\mu \nu} P_R b G_{\mu \nu}) + \text{h.c.}
\]  

(4)

In calculating \(c_7(M_W)\) and \(c_8(M_W)\), we will however use the conventional approximation of taking the complete MSSM to be the correct effective field theory all the way from the scale \(M_G\) down to \(M_W\), and hence ignore threshold corrections.

As previously stated, normally \(A_\gamma\) is taken to be approximately the sum of \(A_W^\gamma\), \(A_{H^-}^\gamma\), and \(A_{\tilde{\chi}^-}^\gamma\). In such a case, the charged Higgs contribution adds constructively to the SM amplitude. On the other hand, the chargino amplitude may combine either constructively or destructively with the other two, and in some cases may even cancel the charged Higgs amplitude. Even though the squarks strongly couple to the gluino, the contribution from the gluino mediated diagrams are considered negligible because the three generations of down squarks \(\tilde{d}_L\) belonging to \(\tilde{Q}_L\) are conventionally assumed to have degenerate soft-breaking masses at the GUT breaking scale. However, the mass parameter \(m_{\tilde{Q}_3L}^2\) is reduced by a small amount relative to \(m_{\tilde{Q}_iL}^2\) for the first two generations in running the mass parameters down from the GUT scale, and b-squark mass matrix has off-diagonal entries proportional to \(m_b\) as given in the following equation:

\[
m_b^2 = \left( m_{\tilde{Q}_3L}^2 + m_b^2 - \frac{1}{6} \left( 2M_W^2 + M_Z^2 \right) \cos 2\beta \right) m_b (A_b + \mu \tan \beta) + m_b^2 (A_b + \mu \tan \beta) + \frac{1}{3} \left( M_W^2 - M_Z^2 \right) \cos 2\beta \),
\]  

(5)

where \(\tan \beta = v_2/v_1\) is the ratio of Higgs vacuum expectation values and \(\mu\) is the coefficient of the Higgs superpotential interaction \(\mu H_1 H_2\). These two effects make the b-squark eigenvalues somewhat different from the down squark masses of the other two generations, however the total effect is insignificant compared to the mass splitting that takes place in the stop sector due to the size of the top quark mass. (See, for example, Fig. 8 in ref. [20].) For this reason, and because the chargino contribution includes an often highly significant higgsino mediated decay, the chargino contribution to \(b \rightarrow s\gamma\) is found to be very important for some regions of parameter space, while the gluino and neutralino contributions are conventionally either found or assumed to be of little significance when the universal boundary condition is taken at the scale \(M_G\).

Now, we will perform the calculation with the universal boundary condition taken at the Planck scale and run the soft breaking parameters from there down to the
weak scale. In the following discussion we will use the one loop renormalization group equations for the gauge couplings, top Yukawa coupling, and soft breaking masses (See refs. [12, 21]). We will also use the exact analytic solutions, in the form derived in ref. [15], to these one loop equations. We will use the conventions for the sparticle mass matrices and the trilinear coupling parameter $A_i$ as found in ref. [SUSY]. ($A_i \rightarrow -A_i$ in the RGEs and RGE solutions of ref. [15].) We will use $\alpha_s(M_Z) = 0.12$. For the purpose of illustration, we will consider the specific case where the Planck scale trilinear scalar coupling $A_0 = 0$, $\tan\beta = 1.5$, $\lambda_t(M_G) = 1.4$, and the grand unification model is the minimal SUSY SU(5) model. In another paper [22], we will look at larger regions of parameter space, including large $\tan\beta$, for SO(10) grand unification. If $\lambda_t(M_G)$ is reduced significantly then so also would be the effects we are discussing. For our chosen values of $\alpha_s(M_Z)$, the $M_G$ scale top coupling, and $\tan\beta$, the top quark pole mass is 168 GeV and the SM branching ratio is about $3.0 \cdot 10^{-4}$. For larger $\tan\beta$ the gluino and neutralino contribution would be greatly increased [2], but at the same time this would tend to occur for sparticle masses where the chargino contribution to $b \rightarrow s\gamma$ is large enough to rule out the region of parameter space.

Since the off diagonal terms in the b-squark mass matrix are much smaller than the diagonal ones and give relatively only a small contribution to the mass splitting between the b-squark mass eigenstates, we choose for simplicity to take the b-squark mass eigenstates to be the soft breaking masses. (See ref. [14].) The two types of diagrams that contribute to $b \rightarrow s\gamma$ and $b \rightarrow sg$ in SU(5) with $\tilde{d}_{iL}$ running in the internal loop are shown in Fig. 1. The internal fermion line represents either a gaugino or a neutralino propagator. To derive the contributions to $A_\gamma$ or $A_g$, one must sum the graphs with an external photon or gluon, respectively, attached in all possible ways. It is possible and simplest to work in a basis in which $\lambda_U$, the soft breaking squark masses and the trilinear couplings $A$ are always diagonal in generation space [14]. The masses of the first two generations of squarks $\tilde{d}_{iL}$ are essentially equal to their soft breaking masses, which receive renormalization effects only from gaugino loops, and are hence degenerate. The soft breaking mass of $\tilde{b}_L$ is much smaller than that of the other two generations, as we will see, since the $\tilde{b}_L$ belongs to the same multiplet as the top above $M_G$. Noting that there is no mixing at the $b_R-\tilde{b}_R$-gaugino vertex introduced by SU(5) grand unification and using the unitarity of the CKM matrix $V$, one may express the contributions to $A_\gamma$ by the gluinos [1, 2, 15] as follows:

$$A^b_\gamma = -C(R)e_D M_W^2 \frac{\alpha_s}{\alpha_2} \frac{g_1 \left( m_{Q_{3L}}^2 / M_3^2 \right) - g_1 \left( m_{Q_{1L}}^2 / M_3^2 \right)}{M_3^2} + \eta_b^{-1} (A_d + \mu \tan\beta) \left[ G \left( m_{b_R}^2, m_{Q_{3L}}^2 \right) - G \left( m_{b_R}^2, m_{Q_{1L}}^2 \right) \right]$$

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\[ + \eta_b^{-1} (A_b - A_d) G \left( m_{bR}^2, m_{\tilde{Q}_{3L}}^2 \right) \], \quad (6)

where \( G \) is given by
\[ G \left( m_1^2, m_2^2 \right) = \frac{1}{M_3} \frac{g_2 \left( \frac{m_1^2}{M_3^2} \right) - g_2 \left( \frac{m_2^2}{M_3^2} \right)}{m_1^2 - m_2^2}, \quad (7)\]
with
\[ g_1(r) = \frac{1}{6(r - 1)^4} [2 + 3r - 6r^2 + r^3 + 6r \ln r], \quad (8) \]
\[ g_2(r) = \frac{-1}{2(r - 1)^3} [r^2 - 1 - 2r \ln r], \quad (9) \]
where we have used \( e_D = -1/3, C(R) = 4/3, \) and \( \eta_b \equiv m_b(m_b)/m_b(M_W), \) which we take to be \( \eta_b = 1.5. \) The analogous expression for neutralinos may be obtained from the above expressions by working in the neutralino mass eigenbasis and noting that the first term in Eq. (6) comes from the diagram of Fig. 1a with the bino propagator and the other terms come from Fig. 1b with the bino-wino propagator. (See, for example, ref. [13].) We have not included the diagram with the higgsino-wino propagator. Because neutralinos do not interact with gluons, one finds that simply \( A^0_g = A^0_{\tilde{\chi}_0}/e_D. \) On the other hand because a gluon can attach to the gluino propagator, one finds
\[ A_g = -\frac{C(G)}{C(R)} \frac{1}{2e_D} A_g^g [g_1 \to h_1, g_2 \to h_2] + \frac{1}{2e_D} (2 - C(G)/C(R)) A_g^g, \quad (10) \]
with \( C(G) = 3 \) and
\[ h_1(r) = (1 - 6r + 3r^2 + 2r^3 - 6r^2 \ln r)/6(r - 1)^4, \quad (11) \]
\[ h_2(r) = -(1 + 4r - 3r^2 + 2r^2 \ln r)/2(r - 1)^3. \quad (12) \]

Notice that the gluino contribution to \( b \to sg \) can be highly significant.

The mass of \( \tilde{Q}_{3L} \) may be expressed in terms of the first generation squark mass \( m_{\tilde{Q}_{1L}}^2 \) as
\[ m_{\tilde{Q}_{3L}}^2 = m_{\tilde{Q}_{1L}}^2 - \left( I_G + \frac{I_Z}{2} \right), \quad (13) \]
where \( I_G = (3/8\pi^2) \int_{M_G}^{M_W} \frac{A_t}{\lambda^2} \left( m_{\tilde{t}_1}^2 + 2m_{\tilde{b}_1}^2 + A_t^2 \right) d \ln \mu, \) and \( I_Z/2 \) is the analogous contribution obtained from running the scale down from \( M_G \) to \( M_W. \) The integrals \( I_G \) and \( I_Z \) may be obtained from the analytic one loop solutions in terms of \( m_{\tilde{Q}_{1L}}^2 \) and \( M_3(M_W) \) as follows:
\[ I_G \approx 0.80 m_{\tilde{Q}_{1L}}^2 - 0.71 M_3^2, \quad I_Z \approx 0.19 m_{\tilde{Q}_{1L}}^2 + 0.17 M_3^2, \quad (14) \]
where we have taken $M_P = 2.4 \cdot 10^{18}\text{GeV}$. One can also find that the relevant weak scale trilinear scale couplings are $A_b \approx -1.38 M_3$ and $A_d \approx -1.55 M_3$. We calculate the parameter $\mu$ at the tree level and find $\mu^2 \approx 1.0 m_{Q_{1L}}^2 - 0.038 M_3^2 - 4200 \text{GeV}^2$.

To illustrate the relative sizes of the separate contributions to the $b \to s\gamma$ amplitude, we plot

$$r_{A_i} \equiv \frac{A_i}{A_W}$$

versus gluino mass for different values of $m_{Q_{1L}}$ in Fig. 2 for the case $\mu > 0$. Notice that the gluino contribution to $A_{\gamma}$ is sometimes comparable to that of the charginos. This happens when the gluino mass is light. Note that the neutralino contribution is insignificant as usual. In Fig. 3, we plot the resulting branching ratio for $b \to s\gamma$. The dotted line corresponds to the calculation neglecting gluino and neutralino mediated contributions, while the solid line represents the full calculation. In both cases, we have included the SM, charged Higgs and chargino corrections as found in [3], which work very well for low $\tan\beta$. When the gluino mass is 150 GeV, the gluino contribution can increase the branching ratio by as much as about 20-percent. This happens when there is strong destructive interference between the charged Higgs and chargino mediated amplitudes, and the gluino mediated amplitude then contributes to the branching ratio mainly through its cross term with the SM amplitude. In Fig. 4 and 5, we show the analogous situation with the universal boundary condition taken at the GUT scale. Notice that when gluino masses are light, the gluino contributions are about one-third the size as when the boundary condition is taken at the Planck scale. The effect of taking the universal boundary condition at the Planck scale has only a small effect on the total chargino contribution for the parameter space shown here, however we find that for other nearby regions, for example with $\tan\beta = 2$, the effect of making the chargino contribution more positive, but smaller in magnitude, is more apparent. In Fig. 6a and 6b, we plot the branching ratio as a function of the $M_G$ scale gaugino mass $M_{5G}$ for curves of constant $m_0$ for the cases where the universal boundary condition is taken at the Planck scale and at the scale $M_G$, respectively. Notice that for the curves with $m_0 > 0$, the branching ratios for the complete calculation in the two cases differ by about 10-percent when $M_{5G} = 60 \text{GeV}$, which corresponds to a not very light gluino mass of about 170 GeV. When $\mu < 0$ and $\tan\beta$ is of order 1, we find the contribution to be much less important due to a strong destructive interference between the two diagrams in Fig. 1.

In conclusion, if one is to calculate the decay rate for the flavor changing process $b \to s\gamma$ in a SUSY GUT with SUSY breaking communicated by gravity above the GUT breaking scale in the form of soft breaking mass terms, it is essential to include the GUT scale renormalization group effects. An important result of including these
renormalization effects is that the gluino contribution to the decay rate can now no longer be neglected when the gluino mass is relatively light.

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References

[1] S. Bertolini, F. Borzumati, A. Masiero, and G. Ridolfi, Nucl. Phys. B353, 591 (1991).

[2] N. Oshimo, Nucl. Phys. B404, 20 (1993).

[3] R. Barbieri and G. F. Giudice, Phys. Lett. B309, 86(1993)

[4] V. Barger, M. Berger, P. Ohmann, and R. J. N. Phillips, Phys. Rev. D51, 2438 (1995), and references therein.

[5] R. Arnowitt and Pran Nath, CTP-TAMU-65/94, NUB-TH-3111/94.

[6] J. L. Hewett, SLAC-PUB-6521.

[7] M. A. Diaz, Phys. Lett. B322, 207 (1994).

[8] R. Garisto and J. N. Ng, Phys. Lett. B315, 372 (1993).

[9] F. Borzumati, Z. Phys. C63, 291 (1994).

[10] J. L. Lopez, D. V. Nanopoulos, and K. Yuan, Phys. Lett. B267, 219 (1991); S. Kelly, J. L. Lopez, D. V. Nanopoulos, H. Pois, and K. Yuan, Phys. Rev. D47, 2461 (1993).

[11] S. Bertolini and F. Vissani, SISSA 40/94/EP.

[12] N. Polonsky and A. Pomarol, Phys. Rev. Lett. 73, 2292 (1994); N. Polonsky and A. Pomarol, UPR-0627T.

[13] R. Barbieri and L. J. Hall, Phys. Lett. B338 212, (1994).

[14] S. Dimopolous and L. J. Hall,Phys. Lett. B344, 185 (1995).

[15] R. Barbieri, L. J. Hall, and A. Strumia, Nucl. Phys. B445 219 (1995).

[16] L. J. Hall, V. A. Kostelecky, and S. Raby, Nucl. Phys.B267 415, (1986).

[17] T. Inami and C. S. Lim, Prog. Theor. Phys. 65, 297 (1981).

[18] N. G. Deshpande and G. Eilam, Phys. Rev. D26, 2463 (1982).

[19] A. J. Buras, M. Misiak, M. Munz, and S. Pokorski, Nucl. Phys. B424, 374 (1994).
[20] V. Barger, M. Berger, and P. Ohmann, Phys. Rev. D49 4908 (1994).

[21] K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, Prog. Theo. Phys. 68 927 (1982).

[22] B. Dutta, E. Keith, and T. V. Duong, Univ. of Ca., Riverside preprint UCRHEP-T154 and University of Oregon preprint OITS-591 (1995).
Figure captions

Fig. 1: The two types of diagrams with $\tilde{d}_{iL}$ running in the internal loop that can contribute significantly to $b_R \rightarrow s_L \gamma$ and $b_R \rightarrow s_L g$. One must sum the graphs with an external photon or gluon attached in all possible ways.

Fig. 2: Plots of $r_{A_i} \equiv A_i^\chi/A_\chi^W$ versus gluino mass for different values of $m_{Q_{iL}}$ for the case $\mu > 0$ and $\tan \beta = 1.5$ with the universal boundary condition taken at the scale $M_P$. The curves correspond to squark masses $m_{\tilde{d}_L} = 200, 300, 400,$ and $500$ GeV. The gluino masses for each curve range from $150$ GeV to the corresponding value of $m_{\tilde{d}_L}$. For example, $m_{\tilde{d}_L} = 200$ GeV corresponds to the curve for which the gluino mass ranges from $120$ GeV to $200$ GeV. Figs. 2a, 2b, 2c, and 2d correspond to the charged Higgs, chargino, gluino, and neutralino contributions, respectively.

Fig. 3: Plots of the branching ratio of $b \rightarrow s \gamma$ for the case of Fig. 2. The solid lines represent the calculation including SM, charged Higgs, chargino, gluino, and neutralino contributions. The dashed lines represent the calculation using only the SM, charged Higgs, and chargino contributions. The curves represent the same squark masses as have been used in Fig. 2.

Fig. 4: Same as Fig. 2, but with the universal boundary condition taken at the $M_G$ scale.

Fig. 5: Same as Fig. 3, but with the universal boundary condition taken at the $M_G$ scale as in Fig. 4.

Fig. 6: Plots of the branching ratio for the case of $\mu > 0$ and $\tan \beta = 1.5$ as a function of the $M_G$ scale gaugino mass $M_{5G}$ for curves of constant $m_0$. Fig. 6a corresponds to the universal boundary condition taken at the Planck scale. Fig. 6b corresponds to the universal boundary condition taken at the GUT breaking scale. The solid lines represent the calculation including SM, charged Higgs, chargino, gluino, and neutralino contributions. The dashed lines represent the calculation using only the SM, charged Higgs, and chargino contributions. The curves represent, in descending order in the two plots, $m_0 = 0, 250$ GeV, and $500$ GeV.
Fig. 2

Fig. 3
**Fig. 4**

- (a) $r_{AH}$ vs. $M_3/\text{GeV}$
- (b) $r_{A_{\text{Charg.}}}$ vs. $M_3/\text{GeV}$
- (c) $r_{A_{\text{Neut.}}}$ vs. $M_3/\text{GeV}$
- (d) $r_{A_{\text{g}}}$ vs. $M_3/\text{GeV}$

**Fig. 5**

- BR [$10^{-4}$] vs. $M_3/\text{GeV}$
Fig. 6