A novel test of Lorentz violation in the photon sector with an LC circuit

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In the presence of an external magnetic field, the Carroll-Field-Jackiw term introduces a displacement current proportional to the Lorentz-violating background that induces a time-dependent magnetic field. Axion-like particles or hidden photons could generate an analogous signal, potentially detectable with the set-up suggested by Sikivie, Tanner and Sullivan – a sensitive magnetometer coupled to a superconducting LC circuit. We show that a similar set-up, but with an externally driven pick-up loop whose area varies harmonically at $\sim$ Hz, can be used to probe the spatial components of the Lorentz-violating background to the level of $\lesssim 10^{-31}$ GeV. This is eight orders of magnitude more sensitive than previous laboratory-based limits.

Introduction.— Some extensions of the Standard Model, like string theory, allow Lorentz symmetry to be violated [1]. In fact, Lorentz-symmetry violation (LSV) may be introduced in all sectors of the Standard Model, which is generalized in the so-called Standard Model Extension (SME) [2,3]. For experimental limits on its different sectors, see ref. [4] and references therein.

Carroll, Field and Jackiw (CFJ) proposed a CPT-odd, Chern-Simons-like Lagrangian in which the electromagnetic fields are coupled to the 4-vector $k_{AF}$ via [5]

$$\mathcal{L}_{CFJ} = \frac{1}{2} \epsilon_{\mu \nu \alpha \beta} (k_{AF})^\nu A^\mu F^{\alpha \beta},$$

where $A^\mu = (\phi, \mathbf{A})$ is the 4-potential and $F^{\mu \nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Note that $[k_{AF}] = \text{mass}$. This term is gauge invariant if $k_{AF}$ is non-dynamic, thus providing a preferred direction in space-time and breaking Lorentz invariance.

Maxwell’s electrodynamics is modified by eq. (1) and, since it has been very well tested, the CFJ background is equally well constrained. Besides theoretical investigations, many experimental tests of the CFJ model have been proposed. As discussed already in CFJ’s seminal work, the presence of this LSV term would induce a rotation of the polarization of the light from distant radio galaxies, whose observation led to the upper bound $k_{AF}^2 \lesssim 10^{-42}$ GeV [5, 6]. Current bounds from CMB polarization are one order of magnitude stronger [7, 8]. Laboratory tests looking for LSV-induced birefringence are usually not as stringent, reaching $k_{AF}^2 \lesssim 10^{-23}$ GeV, mostly due to the shorter optical path length in comparison to astrophysical sources, but nonetheless represent complementary tests of LSV in the photon sector [9].

In this Letter we discuss a new laboratory-based test of the CFJ model inspired by the proposal put forward by Sikivie, Sullivan and Tanner [10]. These authors considered photons coupled to axion-like particles (ALPs), which are well-motivated cold dark matter candidates [11, 12]. This coupling modifies the Maxwell equations and, in the presence of an external magnetic field, creates a displacement current serving as the source for an ALP-originated magnetic field. The flux of this field through a pick-up loop generates a current in a superconducting LC circuit that in turn induces a high-frequency magnetic field in a separate coil. This field could then be detected by a very sensitive magnetometer (e.g., a SQUID). A similar arrangement, but without the necessity of an external magnetic field, was proposed by Arias et al. to search for hidden photons, also as cold dark matter candidates [13].

Here we show that the CFJ Lagrangian [1] modifies the Maxwell equations analogously to the ALP-photon couplings discussed in ref. [10], but instead of light ALPs, it is the time component of the LSV background that couples to an external magnetic field. The same process as in the ALP case would take place and the resulting magnetic field could be detected by a magnetometer. However, due the much lower frequencies involved, the set-up discussed in ref. [10] would have to be modified to improve the sensitivity to the CFJ signal.

The CFJ electrodynamics.— If external sources $J^\mu = (\rho, \mathbf{J})$ are present, the equations of motion become

$$\partial_\mu F^{\mu \nu} = 4\pi J^\nu - 2 (k_{AF})_\mu \tilde{F}^{\mu \nu}$$

with the homogeneous equations left unchanged. In terms of electric and magnetic fields the inhomogeneous equations read

$$\nabla \cdot \mathbf{E} = 4\pi \rho + 2k_{AF} \cdot \mathbf{B}$$

$$-\partial_0 \mathbf{E} + \nabla \times \mathbf{B} = 4\pi \mathbf{J} + 2k_{AF}^0 \mathbf{B} - 2k_{AF} \times \mathbf{E},$$

where $F^{0i} = -\mathbf{E}_i$ and $F^{ij} = -\epsilon_{ijk} \mathbf{B}_k$. We note that, if external electric or magnetic fields are applied, the source terms in the Coulomb and Ampère-Maxwell laws acquire novel LSV contributions. Natural units ($c = \hbar = 1$, $\mu_0 = 1/\epsilon_0 = 4\pi$) are used throughout.

The CFJ model has a few interesting features. In momentum space, eq. (3) indicates that, in a charge-free region, the electric field is not transverse anymore. This also implies that the Poynting vector is not parallel to the wave vector, analogously to what is encountered in anisotropic materials [3, 14]. The presence of the background induces terms playing the role of a polarization or magnetization in empty space [5].
The stability, unitarity and causality of the CFJ model were extensively discussed in the literature. For instance, it can be shown that the Hamiltonian is not positive-definite for a time-like background, even if a small non-zero photon mass \( \dot{a} \) is introduced [15][17]. This means that, for such a background, the theory is unstable. The unitarity of the model is also not guaranteed [15]. Hence, in principle, only a space-like background would give rise to an acceptable field theory. Experimental searches should therefore find stringent upper limits, particularly on the time component of \( k_{AF} \).

It is important to note that the non-dynamic nature of the CFJ 4-vector is only evident in an inertial frame. Earth-bound experiments do not satisfy this requirement due to Earth’s sidereal and orbital motions. A convenient choice is the so-called Sun-centered frame (SCF) [18], which is connected to the laboratory frame by a Lorentz transformation given by

\[
\Lambda^0_T = 1, \quad \Lambda^0_j = -\beta^j, \quad \Lambda^i_T = -(R \cdot \beta)^i, \quad \Lambda^i_j = R^{ij}, \quad (5)
\]

where \( \beta \) is the velocity of the laboratory relative to the SCF and \( R^{ij} \) is a spatial rotation. Both are explicitly time dependent.

In the following we focus on the Ampère-Maxwell equation (4) with an external magnetic field. In this scenario the only relevant component is \( k_{AF} \), which is measured in the laboratory frame. By using the Lorentz transformations above we may write it in terms of the background in the SCF as

\[
k_0^{AF} \approx k_0^{AF} - \beta(t) \cdot k_{AF}. \quad (6)
\]

This shows that any LSV signal detected in an Earth-bound experiment would present a very broad time modulation given by Earth’s sidereal and orbital frequencies \( \omega_{\text{sid}} = 2\pi/\text{day} \approx 73 \, \mu\text{Hz} \) and \( \omega_{\text{orb}} = 2\pi/\text{year} \approx 0.2 \, \mu\text{Hz} \), respectively [18]. Incidentally, the second term in eq. \( (5) \) is suppressed relative to the time component by a factor \( |\beta| \approx 10^{-3} \), therefore further dampening the overall time dependence of \( k_0^{AF} \).

The Sikivie-Sullivan-Tanner set-up.— Let us now focus on eq. (4) in the presence of an external magnetic field \( B_{\text{ext}} \). Assuming that the apparatus is sufficiently well shielded from external electric fields, the time component of the CFJ background induces a current density

\[
J_{CFJ} = 2k_0^{AF}B_{\text{ext}}, \quad (7)
\]

whose time dependence is determined by eq. \( (6) \) in the case of a static external magnetic field.

In ref. [10] the authors consider ALPs coupled to the electromagnetic fields via \( \mathcal{L}_{\text{ALP}} = -g_{\alpha\gamma}a(t)E \cdot B \), where \( a(t) \) is the ALP field. With this extra term we obtain modified Maxwell equations that are analogous to eqs. \( (3) \) and \( (4) \), but with \( \Phi_{CFJ}^0 \rightarrow g_{\alpha\gamma}a(t) \), where \( \dot{a} \) is the time derivative of the ALP field. In the following we analyse the consequences of eq. \( (7) \) in analogy to the ALP discussion using the Sikivie-Sullivan-Tanner set-up detailed in ref. [10] (see also ref. [13]).

For ALPs with \( m_a \approx 10 \, \text{neV} \) we have \( \lambda_a \approx 20 \, \text{m} \), which is larger than the typical size of the experiment \( \sim \mathcal{O}(10 \, \text{m}) \), so that a magnetostatic regime may be assumed [19]. In our case the frequencies involved are much lower (\( \sim \mu\text{Hz} \)), so we are in the same regime. In this case, the current in eq. \( (7) \) serves as the source of a magnetic field satisfying

\[
\nabla \times \mathbf{B}_{CFJ} = \mathbf{J}_{CFJ}. \quad (8)
\]

This can be accomplished by tuning the capacitance \( C \), where \( \dot{\phi} \) is the unitary vector in the azimuthal direction and \( r \) the radial distance from the symmetry axis of the magnet bore.

Let us consider a large rectangular pick-up loop with length \( l_m \) and radius \( r_m \) conveniently placed inside the bore of a solenoid and connected to a small detection coil as illustrated in fig. 1. The loop is traversed by a flux \( \Phi_{CFJ} = 2V_M k_0^{AF} B_{\text{ext}} \), where \( V_M = l_m r_m^2 / 4 \), thus inducing a current \( \mathbf{J}_{CFJ} = -\Phi_{CFJ} / L \). Here \( L \approx l_m + l_d + l_c \) is the inductance of the circuit, where \( l_m, l_d \) and \( l_c \) are the inductances of the pick-up loop, of the detection coil and of the coaxial cable connecting the two, respectively. Note that \( L_d \) is frequency dependent [10].

The current \( \mathbf{J}_{CFJ} \) flows into the small detection coil of radius \( r_d \) with \( N_d \) turns and generates a magnetic field of magnitude

\[
B_d = 4\pi N_d V_M k_0^{AF} B_{\text{ext}} / r_d L. \quad (9)
\]

This is essentially the signal we wish to detect with the magnetometer in the Sikivie-Sullivan-Tanner set-up. In the following we discuss the sensitivity if their set-up is used, as well as modifications necessary for the detection of our particular signal.

Detection sensitivity.— The discussion above is based on the close analogy between the CFJ Lagrangian and the ALP-photon coupling. The Sikivie-Sullivan-Tanner set-up would then be useful not only to search for dark matter candidates, but also to look for LSV in the photon sector of the SME.

There is however one important difference between these applications: the frequency of the signal. The target in ref. [10] is to detect ALPs with masses \( \sim 10 \, \text{neV} \), corresponding to frequencies \( \sim 1 \, \text{MHz} \). In order to amplify the signal, the LC circuit is designed so that its resonance frequency \( \omega_r = 2\pi f_r = 1/\sqrt{LC} \) approximately matches the signal frequency given by the ALP’s mass. This can be accomplished by tuning the capacitance \( C \) of the system [20]. In this case, the current would be enhanced by \( Q \), the quality factor of the LC circuit.
The LSV signal is crucially determined by the flux of \( B_{\text{CFJ}} \) through the pick-up loop. A time dependence in the flux may be introduced through three factors: the CFJ background itself, the external magnetic field and the area of the pick-up loop. Let us consider each of them separately. As shown in eq. (6), the CFJ background varies very slowly as Earth moves relative to the SCF. Let us assume an inductance \( L \sim 10 \, \mu\text{H} \) and a capacitance \( C \sim 0.1 \, \mu\text{F} \) – a typical value for commercial capacitors. With these parameters, the LC circuit resonates at \( \sim \text{MHz} \), very far from the original signal frequency \( \sim \mu\text{Hz} \). Therefore, if the original Sikivie-Sullivan-Tanner set-up would be used and the only time dependence is that from eq. (6), the signal would be too far from resonance and would be strongly suppressed.

The second factor is the magnetic field to which the pick-up loop is exposed. Instead of a constant field, we could use alternating-current (AC) fields. Unfortunately, this approach has a number of disadvantages. High-frequency AC fields – up to \( \sim 0.5 \, \text{MHz} \) – with intensities of \( O(0.1 \, \text{T}) \) can only be produced in solenoids with bore volumes of a few cm\(^3\) \([21, 22]\). Furthermore, solenoids that produce such high frequencies and field intensities require very strong currents and thick, tightly wound coils that would generally experience significant ohmic losses. An alternative would be to place the pick-up loop inside a superconducting, high finesse resonant cavity designed to operate in the TE mode, where high frequencies and strong magnetic fields may be more easily produced in a larger volume. Both solutions would still critically suffer from the fact that strong AC magnetic fields would induce equally strong AC electric fields. These fields would interact with the wires in the pick-up loop and induce large background currents, effectively masking the LSV signal.

Finally, the flux depends on the area through which the LSV induced field \( B_{\text{CFJ}} \) flows. Keeping the external magnetic field static and ignoring the broad time modulation due to the CFJ background, we may induce an AC LSV current by mechanically varying the area of the pick-up loop along its shortest side, \( r_m \), cf. fig. 1. This can be achieved by state-of-the-art actuators used, for example, in modern optical lithography applications, where wafer stages must be repetitively positioned with sub-\(\mu\)m precision within ranges of a few cm, thereby reaching accelerations of up to \( 12 \, g \) – for a review, see ref. \([25]\). For a harmonic movement, the acceleration \( a \) and the maximal displacement \( x_{\text{max}} \) are connected to the driving frequency \( \nu_{\text{act}} \) via \( a = (2\pi \nu_{\text{act}})^2 x_{\text{max}} \), so that \( \nu_{\text{act}} \simeq 2 \, \text{Hz} \) can be achieved for \( a = 10 \, \text{m/s}^2 \) and \( x_{\text{max}} = 5 \, \text{cm} \).

With the strategy outlined above it is possible to raise the signal frequency from \( \mu\text{Hz} \) to a few Hz. In order to gain the enhancement from the quality factor, we need to increase the capacitance of the circuit, which for the high frequencies in ref. \([10]\) is of order \( \mu\text{F} \). This may be achieved by using so-called supercapacitors – potentially several combined – which may reach up to a few kF \([26]\). With inductances of a few \( \mu\text{H} \), this means that the resonance frequency of the LC circuit is \( \nu_{\text{r}} \simeq O(1 \, \text{Hz}) \), which is in the range of frequencies attainable with the external actuators described above.

For the rest of the discussion we assume that the original Sikivie-Sullivan-Tanner set-up can be modified to accommodate a pick-up loop with a varying area as outlined above. The signal resonates with a frequency \( \nu_{\text{r}} \) of a few Hz and is enhanced by the quality factor of the circuit. This adaptation is sketched in fig. 1 where the basic parameters of the pick-up loop and detection region are shown. The dashed line indicates the moving side of the loop and the arrows highlight the action of the actuator, which is isolated from the solenoid to avoid vibrations and thermal effects.

The placement of the pick-up loop in the magnet bore is very important. Unfortunately, just half of the diameter of the bore – typically cylindrical – may be used, otherwise the net flux is zero. Also, the external magnetic field must lie in the plane of the pick-up loop in order to avoid the induction of parasitic currents due to the flux of \( B_{\text{ext}} \) through the time-dependent area of the tilted plane of the loop. Due to Faraday’s law, \( B_{\text{CFJ}} \) will induce a small AC electric field pointing in the \( z \)-direction that turns on the term \( \sim k_{\text{AF}} \times E \) in eq. (4). This contribution is of second order in the CFJ background and can be therefore neglected. Moreover, the CFJ contribution to the Coulomb equation (3) may be ignored in comparison with the charge densities in the system.
The LSV current is given by \( I_{\text{CFJ}} \) multiplied by \( Q \), the quality factor of the circuit, and may be conveniently expressed as

\[
I_{\text{CFJ}} = 1.0 \times 10^{-3} A \left( \frac{V_m}{\text{cm}^3} \right) \left( \frac{\mu H}{L} \right) \cdot \left( \frac{Q}{10^2} \right) \left( \frac{B_{\text{ext}}}{T} \right) \left( \frac{L_0}{10^{-23} \text{GeV}} \right).
\]

(10)

The main noise sources were discussed in ref. 10, where it is shown that the magnetometer noise is typically much lower than the thermal noise, given by \( \delta I_T = \sqrt{4k_B T Q \Delta \nu / L \omega_r} \). Setting \( \omega_r = 2\pi \nu_r \), the signal to noise ratio SNR \( \approx I_{\text{CFJ}} / \delta I_T \) reads

\[
\text{SNR} \approx 3.5 \times 10^5 \left( \frac{V_m}{\text{cm}^3} \right) \left( \frac{Q}{10^2} \right)^{1/2} \left( \frac{\mu H}{L} \right)^{1/2} \left( \frac{B_{\text{ext}}}{T} \right) \cdot \left( \frac{mK}{T} \right)^{1/2} \left( \frac{\nu_r}{Hz} \right)^{1/2} \left( \frac{L_0}{10^{-23} \text{GeV}} \right),
\]

(11)

with the bandwidth \( \Delta \nu = 1 \text{ mHz} \) held fixed. The signal, whose frequency is now determined by the external driving actuator, is assumed to be coherent throughout the measurement time of, say, \( 10^3 \text{ s} \). In this case, the magnetometer may be sensitive to magnetic fields as low as \( \sim 10^{-18} \text{ T} \).

For the sake of concreteness, let us consider a few options of existing magnets in which our set-up could be implemented. From here on we assume that the frequency of the external actuator can be made to approximately match the resonance frequency of the circuit, i.e., \( \nu_r \approx \nu_{\text{ext}} \). The inductance of the pick-up loop is given by \( L_m \approx (\mu_0 / \pi) \ell_m \log (r_m / a_m) \), whereas the inductance of the small detection coil is \( L_d = \mu_0 r_d a_2 N_d^2 \) with \( c_d \approx \log (8r_d / a_d) \). Here \( a_m \) and \( a_d \) are the radii of the wires in the pick-up loop and detection coil, respectively. In the following we use \( r_d = 0.5 \text{ cm} \) and \( a_m = 0.1 \text{ mm} \) so that, for \( \ell_m \gg r_d \) and \( \nu_r \sim 1 \text{ Hz} \), we have \( L_m + L_c \gg L_d \).

Here we explicitly consider the magnets from the ADMX 29 and CMS 30 experiments, as well as the ultra wide-bore magnet at the NHMFL 31. For further options, see ref. 32. In order to estimate the sensitivities, we assume that the respective bore volumes can be efficiently cooled down to \( T = 0.4 \text{ mK} \) and that the superconducting circuit has \( Q = 10^4 \). Using \( \text{SNR} = 5 \) as a threshold 10 13, we find the sensitivities for \( k_{\text{AF}}^3 \) listed in table I, where the relevant parameters are summarized. Since the time component of the background in the SCF is in principle zero and \( |\beta| \approx 10^{-4} \), cf. eq. 7, the detection sensitivity to the spatial components is

\[
|k_{\text{SCF}}^3| \lesssim 10^{-31} \text{ GeV} ,
\]

(12)

which varies anually with \( \Omega_S \approx 0.2 \mu \text{Hz} \). Our set-up would be roughly eight orders of magnitude more sensitive than the best upper bounds from laboratory-based tests 4 9.

| Magnet | \( B_{\text{ext}}(T) \) | \( \ell_m (\text{m}) \) | \( r_m (\text{cm}) \) | \( L_m (\mu \text{H}) \) | \( k_{\text{AF}}^3 \) (GeV) |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|
| ADMX   | 8               | 1               | 10              | 1.8             | \( 6.9 \times 10^{-35} \) |
| CMS    | 4               | 13              | 10              | 24              | \( 3.4 \times 10^{-35} \) |
| NHMFL  | 21              | \( \sim 1 \)    | 5               | 1.6             | \( 9.8 \times 10^{-35} \) |

Table I. Basic parameters of the magnets considered in the text. In order to limit the necessary frequency of the varying side of the pick-up loop, we have restricted the ranges to \( x_{\text{max}} = 10 \text{ cm} \) for the ADMX and CMS magnets, and \( x_{\text{max}} = 5 \text{ cm} \) for the ultra wide-bore magnet at the NHMFL. We assumed that the coaxial cable connecting the pick-up loop to the detection circuit has an inductance \( L_c \approx 0.5 \mu \text{H} \).

**Concluding remarks.**— In this Letter we analysed the consequences of the Carroll-Field-Jackiw model in the context of classical electrodynamics. We focused on the displacement current \( J_{\text{CFJ}} \), eq. 7, in the presence of a strong external magnetic field. Its flux through a carefully placed superconducting pick-up loop with a varying area would generate a current, which in turn induces a magnetic field in a small detection coil. This field could be measured by a magnetometer (e.g., a SQUID).

The proposed modification of the Sikivie-Sullivan-Tanner set-up increases the signal frequency by means of an external actuator allowing the area of the pick-up loop and the flux to oscillate harmonically at \( \nu_r \sim 1 \text{ Hz} \). The frequency of the LC circuit is made to match that of the varying area of the pick-up loop by means of a very large capacitance, thus allowing the signal to resonate. It is worth noticing that, for this range of frequencies, the magnetostatic condition is easily fulfilled and stray capacitances do not limit the sensitivity. Under these circumstances, the attainable sensitivities for the spatial components may reach \( 10^{-31} \text{ GeV} \) for the magnets considered. Furthermore, if we allow a non-zero time component in the SCF and neglect the anisotropic part of \( k_{\text{AF}}^3 \), we find \( k_{\text{AF}}^3 \lesssim 10^{-35} \text{ GeV} \).

Though certainly challenging, dedicated experiments could provide the best laboratory-based limits on the CFJ background, helping to bridge the gap to the more indirect limits from astrophysical sources under controlled conditions.

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