Abstract
Automated data-driven decision systems are ubiquitous across a wide variety of online services, from online social networking and e-commerce to e-government. These systems rely on complex learning methods and vast amounts of data to optimize the service functionality, satisfaction of the end user and profitability. However, there is a growing concern that these automated decisions can lead, even in the absence of intent, to a lack of fairness, i.e., their outcomes have a disproportionally large adverse impact on particular groups of people sharing one or more sensitive attributes (e.g., race, sex). In this paper, we introduce a flexible mechanism to design fair classifiers in a principled manner, by leveraging a novel intuitive measure of decision boundary (un)fairness. We instantiate this mechanism on two well-known classifiers, logistic regression and support vector machines, and show on real-world data that our mechanism allows for a fine-grained control of the level of fairness, often at a minimal cost in terms of accuracy.

1 Introduction
Decision making processes in online services have become increasingly automated and data-driven. By automatically analyzing vast amounts of their users’ online interactions, online services are able to improve their functionality, increase their users’ satisfactions, and ultimately be more profitable. For example, social networking sites rely on large-scale classifiers to detect spammers and e-commerce sites leverage recommender systems to personalize products, services, information and ads to match their users’ interests. Remarkably, automated data-driven decision making is also increasingly used by organizations and governments to detect and eliminate systemic biases in past human-decision making, be it when setting goals, recruiting people or selecting strategies. However, as automated data analysis replaces human supervision in decision making, and the scale of the analyzed data becomes “big”, there is growing concerns from civil organizations (EFF, 2005), governments (Podesta et al., 2014), and researchers (Hardt, 2014) about potential loss of transparency, accountability, and fairness. For example, classifiers used in online services act as large black boxes that leverage thousands of features to achieve high accuracy. As a consequence, it is difficult, if not impossible, to understand which features the classifiers use and to ensure that the classifiers do not discriminate particular groups of people.

Existing anti-discrimination laws in different countries recognize and define two distinct notions of discrimination when evaluating the fairness of a decision making process (Barocas & Selbst, 2016; Biddle, 2005):

(i) disparate treatment: the decision making process is (partly) based on information about sensitive attributes (e.g., gender, race) of subjects; and,
(ii) disparate impact: the outcomes of the decision making process have a disproportionally large adverse impact on particular groups of people sharing certain sensitive attributes, often called protected groups.

Open-source code implementation of our scheme is available at: https://github.com/mbilalzafar/fair-classification
Note that avoiding disparate treatment when making decisions does not necessarily prevent disparate impact in outcomes—since automated decision-making systems are often trained on historical data, if this data shows correlation with sensitive attributes, this bias (or discrimination) will persist in future label predictions by means of indirect discrimination (Pedreschi et al., 2008). Moreover, avoiding disparate impact in outcomes by using information about a subjects’ sensitive attributes in the decision process would constitute disparate treatment (also referred to as reverse discrimination (Zliobaite et al., 2011)).

In our work, we aim to design classifiers that avoid both disparate impact and disparate treatment. Our notion of disparate impact leverages the “80% rule” (Biddle, 2005), which states that the ratio between the percentage of subjects in the protected group selected as positive in the decision making and the percentage in the non-protected group also selected as positive should be no less than 80:100. However, since it is very challenging to directly incorporate this rule in the formulation of several well-known classifiers, we instead introduce a novel intuitive measure of decision boundary (un)fairness as a tractable proxy to the rule—the covariance between the sensitive attributes and the distance from the feature vectors to the decision boundary of the corresponding classifier, evaluated on the training set. Remarkably, our measure automatically allows for fair classifiers that also avoid disparate treatment, since they do not require the values of the sensitive attributes during the decision making process. Moreover, it satisfies several additional desirable properties: (i) for a wide variety of linear and nonlinear classifiers, it is convex and can be readily incorporated to their formulation without increasing their complexity; (ii) it provides a clear mechanism to trade-off fairness and accuracy; and, (iii) it can be used to ensure fairness with respect to several sensitive attributes. Experiments with two well-known classifiers—logistic regression and support vector machines—using both synthetic and real-world data show that our fairness measure allows for a fine-grained control of the level of fairness, often at a minimal cost in terms of accuracy, and provides more flexibility than alternatives (Kamiran & Calders, 2010; Kamishima et al., 2011).

Related Work. Pedreschi et al. (Pedreschi et al., 2008) were the first to consider discrimination in data-driven decision making by introducing a measure of discrimination for rule-based classifiers. Since then, there has been an increasing interest on controlling discrimination in data-driven decision making in the context of classification (Romei & Ruggieri, 2014). These studies have typically adopted one of the two following strategies:

The first strategy consists of pre-processing (or massaging) the training data to limit discrimination (Dwork et al., 2012; Feldman et al., 2015; Hajian & Domingo-Ferrer, 2012; Hajian et al., 2011; Kamiran & Calders, 2009, 2010). This typically means either changing the value of the sensitive attributes; switching the class labels of individual items in the data; or mapping the input data to a transformed space in which the dependencies with the sensitive attributes disappear. However, these approaches treat the learning algorithm as a black box and, as a consequence, the pre-processing can lead to unpredictable losses in accuracy. Moreover, previous methods implementing this strategy have been often restricted to rule-based classifiers (Hajian & Domingo-Ferrer, 2012; Hajian et al., 2011), support only one sensitive attribute (Kamiran & Calders, 2009, 2010), work only on numerical non-sensitive features (Feldman et al., 2015), or are not easily generalizable to a wider set of supervised learning tasks.

The second strategy consists of modifying existing classifiers to limit discrimination (Calders & Verwey, 2010; Kamiran et al., 2012; Kamishima et al., 2011, 2013; Pedreschi et al., 2009). The work by Kamishima et al. (Kamishima et al., 2011) is the most closely related to ours: it introduces a regularization term to penalize discrimination in the formulation of the classifier. However, it trains a different classifier for each sensitive attribute value and, as a consequence, it leads to disparate treatment. Moreover, all previous methods implementing this strategy share one or more of the following limitations: they only allow for one sensitive attribute; they are not easily generalizable to different classifiers; they need sensitive attribute values during testing; and, more importantly, they apply a different classifier for each sensitive attribute value, which results in reverse discrimination.

Recently, Zemel et al. (Zemel et al., 2013) combined both strategies by jointly learning a fair representation of the data and the classifier parameters. Their approach avoids both disparate treatment and impact, however, it suffers from two serious limitations: i) it leads to a non-convex optimization problem, i.e., it does not guarantee optimality; and ii) the accuracy of the classifier

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1 Since different scenarios may lead to different definitions of what is a disproportionally large adverse impact, we leave the task of specifying the exact value of the ratio to the system administrator.
depends on the dimension of the fair representation, however, there is no precise way to select it and it needs to be chosen blindly.

2 Fairness in Classification

In a binary classification task\(^2\) one needs to find a mapping function \(f(x)\) between user feature vectors \(x \in \mathbb{R}^d\) and class labels \(y \in \{-1, 1\}\). This is done by utilizing a training set, \(\{(x_i, y_i)\}_{i=1}^N\), to construct a mapping that works \emph{well} on an \emph{unseen} test set. There are many methods, called classifiers, to construct this mapping, \emph{e.g.}, logistic regression, SVMs, or deep neural networks. Finding the mapping usually reduces to building a decision boundary in feature space that separates users in the training set according to their class labels. One typically looks for a decision boundary, defined by a set of parameters \(\theta\), that achieves the greatest classification accuracy in a test set, by minimizing a loss function over a training set \(L(\theta)\), \emph{i.e.}, \(\theta^* = \arg\min_\theta L(\theta)\). Then, given an \emph{unseen} feature vector \(x_i\) from the test set, the classifier predicts \(f_\theta(x_i) = 1\) if \(d_\theta(x_i) \geq 0\) and \(f_\theta(x_i) = -1\) otherwise, where \(d_\theta(x)\) denotes the signed distance from the feature vector \(x\) to the decision boundary.

However, whenever the labels in the training set, \(\{y_i\}_{i=1}^N\), are correlated with one or more sensitive attributes \(\{z_i\}_{i=1}^N\) (\emph{e.g.}, gender, race, age), the decision boundary may (un)intentionally end up separating users according to their sensitive attribute values. As a consequence, the percentage of users sharing a particular sensitive attribute value on one side of the decision boundary may differ dramatically from the percentage of users not sharing this sensitive attribute value on the same side (\emph{i.e.}, the classifier suffers from disparate impact), leading to user discrimination and lack of fairness. Note that this may happen even if those sensitive attributes are not explicitly used to construct the decision boundary but are correlated with one or more of the user features, by means of indirect discrimination (Pedreschi et al., 2008).

2.1 Fairness Definition

As discussed in Section\(^1\) our definition of decision boundary fairness leverages the “80% rule” (Biddle, 2005), an application of the doctrine of disparate impact. More formally, a decision boundary satisfies the “80% rule” (or more generally the “\(p\)% rule”), if the ratio between the percentage of users sharing a particular sensitive attribute value that lie on one side of the decision boundary and the percentage of users not sharing that value lying on the same side is no less than 80:100 (\(p:100\)). For example, given a binary sensitive attribute \(z \in \{z_1, z_2\}\), one can write the \(p\)% rule as

\[
\min \left( \frac{P(d_\theta(x) > 0 | z_1)}{P(d_\theta(x) > 0 | z_2)}, \frac{P(d_\theta(x) > 0 | z_2)}{P(d_\theta(x) > 0 | z_1)} \right) \geq \frac{p}{100}. \tag{1}
\]

This notion of fairness is closely related to the Calder-Verwer (CV) score (Calders & Verwer, 2010), which have been used by previous works (Kamiran & Calders, 2009; Kamishima et al., 2011; Zemel et al., 2013). In particular, the CV score is defined as the (absolute value of the) difference of the above probabilities, \emph{i.e.}, \(|P(d_\theta(x) > 0 | z_2) - P(d_\theta(x) > 0 | z_1)|\). In our experiments, we adopt the \(p\)% rule as a measure of fairness since the CV score may underestimate discrimination whenever there is class imbalance (and some of the probabilities above are small). However, we obtain qualitatively similar results when considering the CV score (see Appendix\(^2\)).

Unfortunately, it is very challenging to directly incorporate the \(p\)% rule in the formulation of several well-known classifiers\(^2\). First, it is a nonconvex function of the classifier parameters \(\theta\) and, therefore, it would lead to nonconvex formulations, which are difficult to solve efficiently. Second, as long as the user feature vectors lie on the same side of the decision boundary, the \(p\)% rule is invariant to changes in the decision boundary. As a consequence, the ratio in the \(p\)% rule is a function with many saddle points, which would complicate the optimization procedure even further (Dauphin et al., 2014).

To overcome these challenges, we next introduce a novel measure of decision boundary (un)fairness which can be used as a proxy to efficiently design classifiers satisfying a given \(p\)% rule, as shown in Section\(^4\).

\(^2\)For simplicity, we consider binary classifiers, however, our ideas can be easily extended to m-ary classification.

\(^1\)The CV score suffers from similar limitations.
3 Our Approach

In this section, we first introduce our well-behaved measure of decision boundary (un)fairness, the decision boundary covariance. We then show how to incorporate it in a wide variety of fair classifiers, which can be trained efficiently and, in practice, satisfy a given \( p\% \) rule. We derive two complementary formulations. The first formulation maximizes accuracy subject to fairness constraints, which ensures compliance with a nondiscrimination policy or law. The second maximizes fairness subject to accuracy constraints, which guarantees fulfilling certain business needs. In the remainder of the paper, we denote the set of sensitive features by \( z \) and the set of non-sensitive features by \( x \), and assume these two sets to be disjoint. For conciseness, we also assume \( x_i = [-1, x_i] \).

3.1 Decision Boundary Covariance

Our measure of decision boundary (un)fairness is defined as the covariance between the users’ sensitive attributes, \( \{z_i\}_{i=1}^N \), and the signed distance from the users’ feature vectors to the decision boundary, \( \{d_\theta(x_i)\}_{i=1}^N \). In practice, the covariance is estimated empirically using the features and sensitive attributes from training set, i.e.,

\[
\text{Cov}(z, d_\theta(x)) = E[(z - \bar{z})d_\theta(x)] - E[(z - \bar{z})]d_\theta(x) \approx \frac{1}{N} \sum_{i=1}^N (z_i - \bar{z}) d_\theta(x_i), \tag{2}
\]

where \( E[(z - \bar{z})]d_\theta(x) \) cancels out since \( E[(z - \bar{z})] = 0 \). Remarkably, in linear models for classification, such as logistic regression or linear SVMs, the decision boundary is simply the hyperplane defined by \( \theta^T x = 0 \), and therefore, Eq. \( 2 \) reduces to \( \frac{1}{N} \sum_{i=1}^N (z_i - \bar{z}) \theta^T x_i \).

In contrast with the \( p\% \) rule, defined by Eq. \( 1 \), the decision boundary covariance is a convex function with respect to the decision boundary parameters, \( \theta \). Hence, it does not suffer from saddle points, its unique (global) maximum can be found efficiently using many well-known methods, and it can be easily included in the formulation of a wide-range of classifiers without increasing the complexity of their training. Moreover, note that, if a decision boundary satisfies the “100% rule”, then the covariance will be approximately zero for sufficiently large training set. Finally, we find that, in practice, the smaller the covariance of the decision boundary, the larger the \( p\% \) rule it satisfies (refer to Figure \( 2 \)).

3.2 Maximizing Accuracy Under Fairness Constraints

In this section, we design classifiers that maximize accuracy subject to fairness constraints and thus may be used to ensure compliance with a nondiscrimination policy or the law. To this end, we find the decision boundary parameters \( \theta \) by minimizing the corresponding loss function over the training set under fairness constraints, i.e.,

\[
\begin{align*}
\text{minimize} & \quad L(\theta) \\
\text{subject to} & \quad \frac{1}{N} \sum_{i=1}^N (z_i - \bar{z}) d_\theta(x_i) \leq c, \\
& \quad \frac{1}{N} \sum_{i=1}^N (z_i - \bar{z}) d_\theta(x_i) \geq -c, \tag{3}
\end{align*}
\]

where \( c \) is the covariance threshold, which specifies an upper bound on the covariance between each sensitive attribute and the signed distance from the feature vectors to the decision boundary. In this formulation, the constant \( c \) trade-offs fairness and accuracy, such that as we decrease \( c \) towards zero, the resulting classifier will satisfy a larger \( p\% \) rule but will potentially suffer from a larger loss in accuracy. It is important to note that the distance to the margin \( d_\theta(x) \) only depends on the non-sensitive features \( x \) and, therefore, the sensitive features \( z \) are not needed in the decision making process (on the test set). In other words, we account for disparate treatment (or direct discrimination), by removing the sensitive features from the decision making process and, for disparate impact (or indirect discrimination), by adding fairness constraints during the training process of the classifier. Finally, note that the constrained optimization problem in Eq. \( 3 \) can also be written as a regularized optimization problem by making use of its dual form, in which the fairness constraints are moved to the objective and the corresponding Lagrange multipliers act as regularizers.

Next, we particularize the formulation given by Eq. \( 3 \) for two well-known classifiers (Bishop 2006): logistic regression classifier and linear as well as nonlinear support vector machines (SVMs).
where the parameter vector $\theta$ is obtained by solving a maximum likelihood problem over the training set, i.e., $\theta^* = \arg \max_{\theta} \sum_{i=1}^{N} \log p(y_i | x_i, \theta)$. Thus, the corresponding loss function is given by

$$L(\theta) = -\sum_{i=1}^{N} \log p(y_i | x_i, \theta),$$

and Eq. [3] adopts the form

$$\begin{cases}
\text{minimize} & -\sum_{i=1}^{N} \log p(y_i | x_i, \theta) \\
\text{subject to} & \frac{1}{N} \sum_{i=1}^{N} (z_i - \bar{z}) \theta^T x_i \leq c, \\
& \frac{1}{N} \sum_{i=1}^{N} (z_i - \bar{z}) \theta^T x_i \geq -c,
\end{cases}$$

where $\theta$ and $\xi$ are the variables, $\|\theta\|^2$ corresponds to the margin between the support vectors assigned to different classes, and $C \sum_{i=1}^{n} \xi_i$ penalizes the number of data points falling inside the margin.

### III. Nonlinear SVM

In a nonlinear SVM, the decision boundary takes the form $\theta^T \Phi(x) = 0$, where $\Phi(\cdot)$ is a nonlinear transformation that maps every feature vector $x$ into a higher dimensional transformed feature space. Similarly as in the case of a linear SVM, one may think of finding the parameter vector $\theta$ by solving a constrained quadratic program similar to the one defined by Eq. [6]. However, the dimensionality of the transformed feature space can be large, or even infinite, making the corresponding optimization problem difficult to solve. Fortunately, we can leverage the **kernel trick** (Schölkopf & Smola, 2002) both in the original optimization problem and the fairness inequalities, and resort instead to the dual form of the problem, which can be solved efficiently. In particular, the dual form is given by

$$\begin{cases}
\text{minimize} & \sum_{i=1}^{N} \alpha_i + \sum_{i=1}^{N} \alpha_i y_i (g_{\alpha}(x_i) + h_{\alpha}(x_i)) \\
\text{subject to} & \sum_{i=1}^{N} \alpha_i y_i = 0, \\
& \frac{1}{N} \sum_{i=1}^{N} (z_i - \bar{z}) g_{\alpha}(x_i) \leq c, \\
& \frac{1}{N} \sum_{i=1}^{N} (z_i - \bar{z}) h_{\alpha}(x_i) \geq -c,
\end{cases}$$

where $\alpha$ are the dual variables, $g_{\alpha}(x_i) = \sum_{j=1}^{N} \alpha_j y_j k(x_i, x_j)$ can still be interpreted as a signed distance to the decision boundary in the transformed feature space, and $h_{\alpha}(x_i) = \sum_{j=1}^{N} \alpha_j y_j \delta_{ij}$, where $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$, otherwise. Here, $k(x_i, x_j) = \langle \phi(x_i), \phi(x_j) \rangle$ denotes the inner product between a pair of transformed feature vectors and is often called the kernel function.

### 3.3 Maximizing Fairness Under Accuracy Constraints

In the previous section, we designed classifiers that maximize accuracy subject to fairness constraints. However, if the underlying covariation between the class labels and the sensitive attributes in the training set is high, those classifiers may have an underwhelming performance in terms of classification accuracy and thus be unacceptable in terms of business objectives. As a consequence, in such scenarios, one may prefer instead to maximize fairness subject to accuracy constraints. To this end,
we find the decision boundary parameters $\theta$ by minimizing the corresponding (absolute) decision boundary covariance over the training set under constraints on the classifier loss function, \textit{i.e.},
\[
\begin{align*}
\text{minimize} & \quad |\frac{1}{N} \sum_{i=1}^{N} (z_i - \bar{z}) d_\theta(x_i)| \\
\text{subject to} & \quad L(\theta) \leq (1 + \gamma)L(\theta^*),
\end{align*}
\]
where $L(\theta^*)$ denotes the optimal loss over the training set provided by the unconstrained classifier and $\gamma \geq 0$ specifies the maximum additional loss with respect to the loss provided by the unconstrained classifier. Here, we can ensure maximum fairness without loss in accuracy by setting $\gamma = 0$. Similarly as in Section 3.2, it is possible to particularize Eq. [8] for the same classifiers as therein and show that the formulation remains convex. For example, in the case of logistic regression, $L(\theta) = -\sum_{i=1}^{N} \log p(y_i|x_i, \theta)$, and $L(\theta^*) = \min_\theta L(\theta)$ is the maximum likelihood solution to the unconstrained logistic regression classifier.

\textbf{Fine-Grained Accuracy Constraints.} In most classifiers, including logistic regression classifiers and SVMs, the loss function (or the dual of the loss function) is additive over the points in the training set, \textit{i.e.}, $L(\theta) = \sum_{i=1}^{N} L_i(\theta)$, where $L_i(\theta)$ is the individual loss function associated to the $i$-th point in the training set. Moreover, the individual loss $L_i(\theta)$ typically tells us how close the predicted label $f(x_i)$ is to the true label $y_i$, by means of the signed distance to the decision boundary. Therefore, one may think of incorporating loss constraints on individual users (or groups of users) and, in this way, for example, protect individual users in the positive class of being misclassified. To do so, we find the decision boundary parameters $\theta$ by solving the following optimization problem:
\[
\begin{align*}
\text{minimize} & \quad |\frac{1}{N} \sum_{i=1}^{N} (z_i - \bar{z}) \theta^T x_i| \\
\text{subject to} & \quad L_i(\theta) \leq (1 + \gamma_i)L_i(\theta^*) \quad \forall i \in \{1, \ldots, N\},
\end{align*}
\]
where $L_i(\theta^*)$ is the individual loss associated to the $i$-th user in the training set provided by the unconstrained classifier and $\gamma_i \geq 0$ is her allowed additional loss. Again, if we particularize the above formulation for the logistic regression classifier, we obtain:
\[
\begin{align*}
\text{minimize} & \quad |\frac{1}{N} \sum_{i=1}^{N} (z_i - \bar{z}) \theta^T x_i| \\
\text{subject to} & \quad \log p(y_i|x_i, \theta) \geq (1 + \gamma_i) \log p(y_i|x_i, \theta^*) \quad \forall i \in \{1, \ldots, N\},
\end{align*}
\]
where $\theta^* = \arg\min_\theta \sum_{i=1}^{N} - \log p(y_i|x_i, \theta)$. In this case, the individual losses have a probabilistic interpretation: if we set $\gamma_i = 0$ we are enforcing that the probability of the $i$-th user to be mapped in the positive class to be equal or higher than in the original unconstrained logistic regressor.

\section{Evaluation}

We evaluate our framework on several synthetic and real-world datasets. We first experiment with our first formulation and show that it allows for fine-grained fairness control on several well-known classifiers, often at a minimal loss in terms of accuracy, and provides more flexibility than...
alternatives (Kamiran & Calders 2010) (Kamishima et al. 2011). Then, we validate our second formulation, which, to the best of our knowledge, is the first that allows maximizing fairness while avoiding the misclassification of certain protected individual users or group of users, enabling the system administrator to fulfill certain business needs.

4.1 Experiments on Synthetic Data

Fairness constraints vs accuracy constraints. First, we generate several synthetic datasets with different levels of correlation between sensitive attributes and class labels, and then train two types of logistic regression classifiers: one type maximizes accuracy subject to fairness constraints (Section 3.2), and another type maximizes fairness under fine-grained accuracy constraints (Section 3.3). More in detail, we generate 4,000 user binary class labels uniformly at random and assign a 2-dimensional user feature vector per label by drawing samples from two different gaussian distributions: \( p(x|y=1) = \mathcal{N}([2; 2], [5; 1; 5]) \) and \( p(x|y=-1) = \mathcal{N}([-2; -2], [10; 1; 3]) \). Then, we draw each user’s sensitive attribute \( z \) from a Bernoulli distribution with probability \( p(z = 1) = p(x'|y = 1)/(p(x'|y = 1) + p(x'|y = -1)) \), where \( x' = [\cos(\phi) - \sin(\phi); \sin(\phi) \cos(\phi)]x \) is simply a rotated version of the user’s feature vector. We experiment with different values for the parameter \( \phi \), which controls the correlation value between the sensitive attributes and the class labels to generate datasets with different levels of correlation. Here, the closer the value of \( \phi \) to zero, the higher the correlation.

Finally, we trained the above mentioned classifiers in each dataset.

Fig. 1a shows the decision boundaries provided by the classifiers that maximize accuracy under fairness constraints for two different correlation values and two covariance threshold values \( \epsilon \). Here, we compare against the decision boundary provided by the unconstrained classifier (solid, light blue line). As one may have expected, given the data generation process, fairness constraints map into a rotation of the decision boundary (dashed lines), which is greater as we decrease threshold value \( \epsilon \) or increase the correlation in the original data (e.g., \( \phi = \pi/8 \)). This gives empirical evidence that our fairness constraints are successfully reverse engineering the mechanism we used to generate the values of the sensitive attributes. Fig. 1b shows the decision boundaries provided by the classifiers that maximize fairness under fine-grained accuracy constraints. Here, the fine-grained accuracy constraints ensure that the users in the non-protected group classified as positive by the unconstrained classifier (green circles on the top of the solid, light blue decision boundary) are not labeled as negative by the fair classifier. Here, the decision boundaries provided by this formulation, in contrast with the previous one, are shifted and rotated versions of the decision boundary provided by the unconstrained classifier. Similarly, we illustrate how the decision boundary of a non-linear classifier, a SVM with radial basis function (RBF) kernel, changes under fairness constraints in Appendix A.1.

4.2 Experiments on Real Data

Experimental Setup. We experiment with two real-world datasets: The Adult income dataset (Adult (1996)) and the Bank marketing dataset (Bank (2014)). The Adult dataset contains a total of 45,222 subjects, each with 14 features (e.g., age and educational level) and a binary label, which indicates whether their incomes are above (positive class) or below (negative class) 50K USD. For this database, we consider gender and the race as, respectively, binary and non-binary sensitive attributes. The Bank dataset contains a total of 41,188 subjects, each with 20 attributes (e.g., marital status) and a binary label, which indicates whether the client has subscribed (positive class) or not (negative class) to a term deposit. In this case, we consider age as (binary) sensitive attribute, which is discretized to indicate whether the user’s age is between 25 and 60 years. In our experiments, to obtain more reliable estimates of accuracy and fairness, we repeatedly split each dataset into a training (70%) and test (30%) set 5 times. Here, we adopt the \( \rho/\% \) rule as our true measure of fairness but we obtain similar results for the CV score (Calders & Verwer 2010). Appendix A.2 shows these results jointly with more detailed statistics of the datasets.

Maximizing accuracy under fairness constraints. First, we experiment with a single binary sensitivity attribute, gender and age, for respectively, the Adult and Bank data. For each dataset, we train several logistic regression and SVM classifiers (denoted by ‘C-LR’ and ‘C-SVM’, respectively), each subject to fairness constraints with different values of covariance threshold \( \epsilon \) (Section 3.2), and then empirically investigate the correspondence between their achieved covariance and \( \rho \) value in the \( \rho/\% \) rule, computed on the training set. Fig. 2a summarizes the results, which show that, as desired: i) the lower the covariance, the higher the \( \rho/\% \) rule the classifiers satisfy and (ii) a 100%
Figure 2: [Maximizing accuracy under fairness constraints] Panels in (a) show the correspondence between the empirical covariance in Eq. 2 and the \( p\% \) rule for classifiers trained under fairness constraints for the Adult (top) and Bank (bottom) datasets. Panels in (b) show the accuracy against \( p\% \) rule value (top) and the percentage of protected (dashed) and non-protected (solid) users in the positive class against the \( p\% \) rule value (bottom). Panels in (c) show the accuracy (top) and users in positive class (bottom) against a multiplicative covariance factor \( a \in [0, 1] \) such that \( c = ac^* \), where \( c^* \) denotes the unconstrained classifier covariance.

rule maps to zero covariance. Then, we compare our approach to a well-known competing method from each of the two categories discussed in Section 2, the preferential sampling approach (Kamiran & Calders, 2010), applied to logistic regression (‘PS-LR’) and SVM (‘PS-SVM’), as an example of data preprocessing, and the regularized logistic regression classifier (‘R-LR’) (Kamishima et al., 2011), as an example of modifying a classifier to limit discrimination. Fig. 2b summarizes the results: the top panel shows the average accuracy and the bottom panel the percentage of protected (dashed lines) and non-protected (solid lines) users in the positive class against the average \( p\% \) rule, as computed on the test sets. We observe that: i) the performance of our classifiers (C-LR and C-SVM) and the regularized logistic regression (R-LR) is comparable, ours are slightly better for the Adult data (left column) while slightly worse for the Bank data (right column), however, the regularized logistic regression uses sensitive attribute values from the test set to make predictions, allowing for reverse discrimination (Zliobaite et al., 2011); ii) the preferential sampling presents the worst performance and always achieves \( p\% \) rules under 80%; and, (iii) in the Adult data, all classifiers move non-protected users (males) to the negative class and protected users (females) to the positive class to achieve fairness, in contrast, in the Bank data, they only move non-protected (young and old) users originally labeled as positive to the negative class since it provides a smaller accuracy loss. However, the latter can be problematic: from a business perspective, a bank may be interested in finding potential subscribers rather than losing existing customers. This last observation motivates our second formulation (Section 3.3), which we experiment with in a later section.

Finally, we experiment with non-binary sensitive attributes (race) and with several sensitive attributes (gender and race) in the Adult dataset. We do not compare with competing methods since they cannot handle non-binary or several sensitive attributes. Fig. 2c summarizes the results by showing the accuracy and the percentage of subjects sharing each sensitive attribute value classified as positive against a multiplicative covariance factor \( a \in [0, 1] \) such that \( c = ac^* \), where \( c^* \) denotes the unconstrained classifier covariance (note that the \( p\% \) rule is only defined for a binary sensitive feature). As expected, as the value of \( c \) decreases, the percentage of subjects in the positive class sharing different sensitive attribute values become nearly equal while the loss in accuracy is modest.

Maximizing fairness under accuracy constraints. Next, we demonstrate that our second formulation (Section 3.3) can maximize fairness while precisely controlling accuracy loss. To this aim, we first train several logistic regression classifiers (denoted by ‘\( \gamma\)-LR’), which minimize the decision boundary covariance subject to accuracy constraints over the entire dataset by solving Eq. 8 with increasing values of \( \gamma \). Then, we train several logistic regression classifiers (denoted by ‘Fine-\( \gamma\)-LR’)

\( ^4 \) The scarce representation of the race value ‘Other’ (only 0.8% of the data) prevents from an accurate estimation of the decision boundary covariance and, as a result, the classifier does not reach perfect fairness with respect to this sensitive attribute value.
that minimize the decision boundary covariance subject to fine-grained accuracy constraints by solving Eq. [10]. For the latter, we prevent the non-protected users that were classified as positive by the unconstrained logistic regression classifier from being classified as negative by computing, for each of those users, the maximum $\gamma_i$ that ensures that she does not cross the decision boundary, and then consider increasing values of $\gamma_i = \gamma$ for the remaining users. In both cases, we increased the value of $\gamma$ until we reach a 100% rule during training. Fig. 3 summarizes the results for both datasets, by showing (a) the average accuracy (solid curves) and $p\%$ rule (dashed curves) against $\gamma$, and (b) the percentage of non-protected (N-P, solid curves) and protected (P, dashed curves) users in the positive class against $\gamma$. Here, we observe that, as we increase $\gamma$, the classifiers that constrain the overall training loss ($\gamma$-LR) remove non-protected users from the positive class and add protected users to the positive class, in contrast, the logistic regression classifiers that prevent the non-protected users that were classified as positive in the unconstrained classifier from being classified as negative (Fine-$\gamma$-LR) add both protected and non-protected users to the positive class. As a consequence, the latter achieves lower accuracy for the same $p\%$ rule value.

5 Conclusions

In this paper, we introduced a novel measure of decision boundary fairness, which enables us to guarantee fairness with respect to one or more sensitive attributes, both in terms of disparate impact and disparate treatment, in a wide variety of linear and nonlinear classifiers. Moreover, we leverage this definition to derive two complementary formulations: one that maximizes accuracy subject to fairness constraints, which may ensure compliance with a nondiscrimination policy or the law; and another one that maximizes fairness subject to accuracy constraints, which may guarantee fulfilling certain business needs and, to the best of our knowledge, is novel.

Our framework opens many venues for future work. For example, one could include fairness constraints in other supervised learning problems, such as regression or recommendation, and unsupervised learning problems, such as set selection or ranking problems. Also, it would be interesting to perform a theoretical analysis of the generalization error of the proposed fair classifiers. Finally, in this work, we assume that the class labels in the training and test set are true labels and thus is desirable to maximize the classifiers accuracy. However, in some scenarios, these labels may have been given by humans, biased and thus adversarial. Therefore, it would be interesting to augment our framework to allow for adversarial labels.

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A Additional Experiments

A.1 Experiments on Non-linear Synthetic Data

Here, we illustrate how the decision boundary of a non-linear classifier, a SVM with radial basis function (RBF) kernel, changes under fairness constraints. To this end, we generate 4,000 user binary class labels uniformly at random and assign a 2-dimensional user feature vector per label by drawing samples from $p(x|y=1, \beta) = \beta N([2; 2], [5 1; 1 5]) + (1 - \beta) N([-2; -2], [10 1; 1 3])$ if $y = 1$, and $p(x|y=-1, \beta) = \beta N([4; -4], [4 4; 2 5]) + (1 - \beta) N([-4; 6], [6 2; 2 3])$ otherwise, where $\beta \in \{0, 1\}$ is sampled from $\text{Bernoulli}(0.5)$. Then, we generate each user’s sensitive attribute $z$ by applying the same rotation as detailed in Section 4.1.

Figure 4 shows the decision boundaries provided by the SVM that maximizes accuracy under fairness constraints with $c = 0$ for two different correlation values, set by $\phi = \pi/4$ and $\phi = \pi/8$, in comparison with the unconstrained SVM. We observe that, in this case, the decision boundaries provided by the constrained SVMs are very different to the decision boundary provided by the unconstrained SVM, not simple shifts or rotations of the latter, and successfully reverse engineer the mechanism we used to generate the class labels and sensitive attributes.

![Figure 4: Decision boundaries for SVM classifier with RBF Kernel trained without fairness constraints (left) and with fairness constraints ($c = 0$) on two synthetic datasets with different correlation value between sensitive attribute values (crosses vs circles) and class labels (red vs green).](image)

A.2 Experiments on Real Data

Additional Data Statistics. In this section, we show the distribution of sensitive features and class labels in our real-world datasets.

| Sensitive Attribute | $y \leq 50K$ | $> 50K$ | Total |
|---------------------|-------------|---------|-------|
| Males               | 20,988      | 9,539   | 30,527|
| Females             | 13,026      | 1,669   | 14,695|
| Total               | 34,014      | 11,208  | 45,222|

(a) Adult dataset

| Sensitive Attribute | No | Yes | Total |
|---------------------|----|-----|-------|
| 25 $\leq$ age $\leq$ 60 | 35,240 | 3,970 | 39,210 |
| 25 $>$ age $> 60$ | 1,308 | 670 | 1,978 |
| Total               | 36,548 | 4,640 | 41,188 |

(b) Bank dataset

Table 2: Adult dataset (Non-binary sensitive attribute: race)

| Sensitive Attribute | $y \leq 50K$ | $> 50K$ | Total |
|---------------------|-------------|---------|-------|
| Am-In-Eskimo        | 382         | 53      | 435   |
| Asian-Pac-Is        | 934         | 369     | 1,303 |
| White               | 28,696      | 10,207  | 38,903|
| Black               | 3,694       | 534     | 4,228 |
| Other               | 308         | 45      | 353   |
| Total               | 34,014      | 11,208  | 45,222|

CV score as fairness measure. As mentioned in Section 2.1, the definitions of $p\%$ rule and CV score are similar in nature and either of them could be used to quantify fairness of a decision boundary. However, since legal definitions of fairness are often provided in terms of $p\%$ rule [Biddle 2005], we
adopted it as our measure of fairness to evaluate and compare the performance of different schemes in Section 4. In this section, we show that using CV score (instead of \( p\% \) rule) as a measure of fairness would yield similar results.

First, we show that constraining the covariance between users’ sensitive attributes (Figure 5a), and the signed distance from the decision boundary, corresponds to a decreasing CV score (a more fair decision boundary). Next, we show the performance of different methods based on the CV score (Figures 5b and 6). The results shown in Figures 5a, 5b and 6 correspond to the ones shown in Figures 2a, 2b and 3, where we took \( p\% \) rule as the measure of fairness. Notice that according to the definitions provided in Section 2.1, a decreasing CV score corresponds to an increasing \( p\% \) rule (and hence, a more fair decision boundary).

![Graphs showing the relationship between covariance and CV score for different datasets](image)

Figure 5: [Maximizing accuracy under fairness constraints] Panels in (a) show the correspondence between the empirical covariance in Eq. 2 and the CV score for classifiers trained under fairness constraints for the Adult (top) and Bank (bottom) datasets. Panels in (b) show the accuracy against CV score value (top) and the percentage of protected (dashed) and non-protected (solid) users in the positive class against the CV score value (bottom). Panels in (c) show the accuracy (top) and users in positive class (bottom) against a multiplicative covariance factor \( \gamma \in [0,1] \) such that \( c = \gamma c^* \), where \( c^* \) denotes the unconstrained classifier covariance.

![Graphs showing the relationship between accuracy and CV score for different datasets](image)

Figure 6: [Maximizing fairness under accuracy constraints] Panels in (a) show the accuracy (solid) and CV score value (dashed) against \( \gamma \). Panels in (b) show the percentage of protected (P, dashed) and non-protected (N-P, solid) users in the positive class against \( \gamma \).