The nature of the $X(3915)/X(3930)$ resonances from a coupled-channels approach

Pablo G. Ortega
Grupo de Física Nuclear and Instituto Universitario de Física Fundamental y Matemáticas (IUFFyM), Universidad de Salamanca, E-37008 Salamanca, Spain
E-mail: pgortega@usal.es

Jorge Segovia
Institut de Física d’Altes Energies (IFAE) and Barcelona Institute of Science and Technology (BIST), Universitat Autònoma de Barcelona, E-08193 Bellaterra (Barcelona), Spain
E-mail: jsegovia@ifae.es

The positive parity $\chi_{cJ}(2P)$ charmonium states are expected to lie around the 3.9 GeV/$c^2$ energy region, according to the predictions of quark models. However, a plethora of states with difficult assignment and unconventional properties have been discovered over the years, i.e., the $X(3872)$, $X(3940)$, $Y(3940)$, $X(3915)$, $X(3860)$ and the $X(3930)$ resonances, which complicates the description of this intriguing region.

In this work we analyze the $0^{++}$ and $2^{++}$ sectors, employing a coupled-channels formalism successfully applied to the $1^{++}$ sector, where the $X(3872)$ was described as a $D\bar{D}^* + h.c.$ molecule with a sizable $c\bar{c} (2^3P_1)$ component. This coupled-channels formalism is based on a widely-used Constituent Quark Model, which describes the quark-quark interactions, and the $^3P_0$ quark pair creation mechanism, used to couple the two and four quark sectors.

The recent controversy about the quantum numbers of the $X(3915)$ state, the properties of the $X(3930)$ one and the nature of the new $X(3860)$ resonance are analyzed in a unified theoretical framework, being all the parameters completely constrained from previous calculations in the low-lying heavy quarkonium phenomenology.
1. Introduction

The theoretical interest in the 3.9 GeV/c² energy region of the positive-parity charmonium spectra began in 2003, after the discovery of the X(3872) by the Belle Collaboration [1]. This state, with $J^{PC} = 1^{++}$, decays through the $J/\psi \rho$ and $J/\psi \omega$ channels which are, respectively, forbidden and OZI-suppressed for a $c\bar{c}$ configuration. Since then, many other structures have been observed in this energy region, like the $Y_{c}$ forbidden and OZI-suppressed for a $c\bar{c}$ configuration. Since then, many other structures have been observed in this energy region, like the $Y_{c}(3940)$ one, discovered by Belle [2], the $X(3940)$ [3] or the $X(3930)$, a $J^{PC} = 2^{++}$ peak measured in 2006 by the same Collaboration in the mass spectrum of the $D\bar{D}$ mesons produced by $\gamma\gamma$ fusion, rapidly assigned to the $\chi_{c2}(2P)$ state at the PDG.

Additionally, a recent controversy has appeared concerning the $X(3915)$ resonance [4, 5], traditionally assigned to a $J^{PC} = 0^{++}$ state. Recent works favor a $J^{PC} = 2^{++}$ nature for the latter, which as a side effect would imply a large non-$q\bar{q}$ component for the $X(3915)$ state [6, 7]. Furthermore, a new charmonium-like state decaying to $D\bar{D}$, called $X(3860)$, has been recently reported by the Belle Collaboration [8], with a mass of $3862^{+26+40}_{-32-13}$ MeV/c² and a width of $201^{+154+86}_{-87-2}$ MeV. In this case, the $J^{PC} = 0^{++}$ option is preferred over the $2^{++}$ one, but its quantum numbers are not determined yet.

To study this abundance of states, all belonging to the same energy region, it is necessary to include all the nearby thresholds. Based on previous calculations of the $J^{PC} = 1^{++}$ sector by Refs. [9, 10], which used a coupled-channels approach including the bare $c\bar{c}(2^{3}P_{1})$ and $D\bar{D}^{*} + h.c.$ components, in this work we will perform a calculation for the $0^{++}$ and $2^{++}$ channels [11] including the $c\bar{c}(2^{3}P_{0,2})$ bare charmonium states, which are coupled to the closest thresholds. Recently, this formalism has also been successfully applied to the $c\bar{s}$ [12] and $b\bar{s}$ sectors [13].

2. Coupled-channels model

In order to describe mixed molecular-$c\bar{c}$ states we need two ingredients. On the one hand, a model for the $q-q$ interactions to describe the naive charmonium spectra and the hadron-hadron interactions. On the other hand, a formalism to describe the coupling between Fock spaces must be developed, through a mechanism that couples the two and four quark systems.

Regarding the first ingredient, the constituent quark model (CQM) employed in this work has been extensively detailed in Ref. [14], thus only its most relevant aspects will be briefly described. Further details of the model and explicit expressions for these interactions are given in the latter reference. This CQM has been successful in describing the meson [14, 15, 16] and baryon phenomenology [17, 18]. The present CQM is based on the assumption that quarks acquire a dynamical mass as a consequence of the spontaneous chiral symmetry breaking. In order to compensate the mass term in a chiral-invariant QCD Lagrangian we must include chiral fields. Then, the simplest Lagrangian that fulfills the previous properties can be written as

$$\mathcal{L} = \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - M(p^{2})U^{\mu}_{\nu})\psi$$

(2.1)

where $U^{\mu}_{\nu} = e^{i\frac{p^{\mu}}{2}\phi}$ is the Goldstone boson fields matrix and $M(p^{2})$ the dynamical constituent mass. The Goldstone boson field matrix $U^{\mu}_{\nu}$ can be expanded in terms of boson fields, from where one-boson exchange interaction between quarks naturally emerges.

Above the chiral symmetry breaking scale the interaction through Goldstone bosons does not act. However, quarks still interact through gluon exchange diagrams. Besides, the model is com-
poles with the confinement, a nonperturbative QCD effect. This interaction can be modeled with a
screened potential [19], which takes into account the saturation of the potential at some interquark
distance due to the spontaneous creation of light-quark pairs.

About the second ingredient, the aforementioned CQM is insufficient to describe the interplay
between charmonium and meson-meson channels, hence, we need to develop a complete coupled-
channels formalism. For that purpose we will follow the formalism of Ref. [10]. That way, the
influence of the \( c\bar{c} \) states on the dynamics of the two meson states is studied. This allows to
generate new states through hadron-hadron interaction, described by the coupling with heavy quarkonium
states and the underlying \( q\bar{q} \) new states through hadron-hadron interaction, described by the coupling with heavy quarkonium

The global hadronic state, combination of heavy quarkonium and meson-meson channels, is

\[
|\Psi\rangle = \sum_{\alpha} c_\alpha |\psi_\alpha\rangle + \sum_\beta \chi_\beta(P)|\phi_{M_1}\phi_{M_2}\rangle
\]  

being \( |\psi_\alpha\rangle \) the \( c\bar{c} \) eigenstates of the two body Hamiltonian and \( \phi_{M_i} \) the \( c\bar{n} \) (or \( \bar{c}n \)) \(^1\) eigenstates
describing the \( M_i \) mesons. Moreover, \( |\phi_{M_1}\phi_{M_2}\rangle \) is the two meson state with \( \beta \) quantum numbers
coupled to total \( J^{PC} \) quantum numbers and \( \chi_\beta(P) \) is the relative wave function between the two
mesons \( M_1M_2 \) in the molecule.

The description of hadron-hadron interaction from the fundamental \( q\bar{q} \) interactions is ob-
tained from Resonating Group Method (RGM). The wave functions of the mesons (either \( c\bar{c} \) or \( c\bar{n} \)) are calculated by solving the Schödinger equation for the quark-antiquark problem using the
Gaussian Expansion Method (GEM). We employ Gaussian trial functions whose ranges are in
generic progression [20], which helps to optimize the ranges using few free parameters. This
choice generates a dense distribution of Gaussians at small range, which is adequate for short-range
potentials.

The coupling between the \( c\bar{c} \) and the \( c\bar{n} - n\bar{c} \) channels requires the creation of a light quark pair
\( q\bar{q} \). We will use the quark pair creation \(^3\) \( P_0 \) model [21], which describes the strong decay processes
from simple principles, giving equivalent results compared to microscopic calculations [22].

Accordingly, the transition potential \( h_{\beta\alpha}(P) \) describing the \( c\bar{c} \rightarrow M_1M_2 \) process within the \(^3\) \( P_0 \) model can be defined as

\[
\langle \phi_{M_1}\phi_{M_2}\beta|T|\psi_\alpha\rangle = P h_{\beta\alpha}(P) \delta^{(3)}(\vec{P}_{\text{CM}}),
\]

being \( P \) the relative momentum of the two meson state and \( T \) the transition operator, non-relativistic reduction of the \(^3\) \( P_0 \) pair
creation Hamiltonian.

Now, the coupled-channels system for the bare charmonium states and the meson-meson channels
can be expressed as an Schrödinger-type equation [9] where, apart from the potential coming
from \( q\bar{q} \) interactions described by the CQM, we have an effective potential coding the coupling
with \( c\bar{c} \) states,

\[
V^{\text{eff}}_{\beta\beta}(P', P; E) = \sum_\alpha h_{\beta\alpha}'(P') h_{\alpha\beta}(P) \frac{1}{E - M_\alpha}.
\]

In order to describe states above thresholds, a more suitable formalism is to describe resonances as poles in the complex \( T(\vec{p}, \vec{p}', E) \) matrix, solution of the Lippmann-Schwinger equation.

Employing the total potential \( V_T(P', P; E) = V(P', P) + V^{\text{eff}}(P', P; E) \), where \( V(P', P) \) is the potential
from CQM and \( V^{\text{eff}}(P', P; E) \) the effective potential of Eq. 2.3, the \( T \)-matrix can be factorized

\(^1\)In this work we will denote \( n \) as the light quarks \( \{u,d,s\} \)
shows the masses and widths of the states found in the coupled-channels calculation \cite{PoS(Hadron2017)113} expected to be essential to describe the $0^{++}$ because it is the only threshold which couples to the $\bar{D}J$ the threshold energies for each channel are indicated in parenthesis. For \textbf{3. Results} vertex dressed by the RGM meson-meson interaction. 

The nature of the $X(3915)/X(3930)$ resonances

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
Model & $J^{PC}$ & Mass & Width & $\mathcal{P}[c\bar{c}]$ & $\mathcal{P}[D\bar{D}]$ & $\mathcal{P}[D\bar{D}^*]$ & $\mathcal{P}[\omega J/\psi]$ & $\mathcal{P}[D_s\bar{D}_s]$ & $\mathcal{P}[D_s^*\bar{D}_s^*]$ \\
\hline
A & $0^{++}$ & 3890.3 & 6.7 & 44.1\% & 21.6\% & – & 28.4\% & 2.6\% & 3.3\% \\
 & $2^{++}$ & 3925.6 & 19.0 & 42.2\% & 11.3\% & 37.0\% & 4.0\% & 0.4\% & 5.1\% \\
\hline
B & $0^{++}$ & 3889.0 & 11.8 & 43.5\% & 27.3\% & – & 20.4\% & 3.8\% & 4.9\% \\
 & $2^{++}$ & 3915.1 & 19.8 & 37.8\% & 14.1\% & 36.4\% & 5.12\% & 0.4\% & 6.1\% \\
\hline
\end{tabular}
\caption{Mass and decay width, in MeV, and probabilities of the different channels, for models A and B.}
\end{table}

as

$$T^{\beta\beta}(P', P; E) = T^{\beta\beta}_V(P', P; E) + \sum_{\alpha, \alpha'} \phi^{\beta\alpha'}(P'; E) \Delta_{\alpha'\alpha}(E) \phi^{\alpha\beta}(P; E)$$

(2.4)

where $T^{\beta\beta}_V(P', P; E)$ the $T$ matrix of the CQM potential excluding the coupling to the $c\bar{c}$ pairs and $\Delta_{\alpha'\alpha}(E) = (E - M_\alpha) \delta^{\alpha'\alpha} + g^{\alpha'\alpha}(E)$ is the propagator of the mixed state, where $M_\alpha$ is the bare mass of the heavy quarkonium and $g^{\alpha'\alpha}(E)$ the complete mass-shift. Then, resonances emerge as zeros of the latter propagator ($|\Delta_{\alpha'\alpha}(E)| = 0$). Additionally, the $\phi^{\beta\alpha}(P; E)$ functions are the $^3P_0$ vertex dressed by the RGM meson-meson interaction.

\textbf{3. Results}

For the present calculation, the bare $c\bar{c}(2^3P_{0,2})$ meson states will be coupled to $D\bar{D}$ (3734 MeV/c$^2$), $\omega J/\psi$ (3880 MeV/c$^2$), $D_s\bar{D}$ (3937 MeV/c$^2$) and $D_s^*\bar{D}^*$ (4017 MeV/c$^2$) channels, where the threshold energies for each channel are indicated in parenthesis. For $J^{PC} = 2^{++}$ we add the $D\bar{D}^* + h.c.$ (3877 MeV/c$^2$) channel, which is closed for $0^{++}$ sector. The $D_s^*\bar{D}^*$ channel is included because it is the only threshold which couples to the $c\bar{c}(2^3P_0)$ in $S$ wave and, hence, its effect is expected to be essential to describe the $0^{++}$ sector. Furthermore, Heavy Quark Spin Symmetry tells us that the $D_s^*\bar{D}^*$ in the $2^{++}$ sector has the same dynamics as the $D\bar{D}^* + h.c.$ channel in the $1^{++}$ sector, which is fundamental in describing the $X(3872)$.

Table 1 shows the masses and widths of the states found in the coupled-channels calculation using the original parameters of Ref. \cite{9} (denoted as \textit{model A}). In addition, to explore the sensitivity of the results, a second calculation (named \textit{model B}) is shown in the same table, where the bare mass of the $c\bar{c}(2^3P_{0,2})$ pairs has been slightly changed (0.25\%) and the value of the $^3P_0$ strength parameter $\gamma$ has been taken from Ref. \cite{23}. Such changes in the initial parameters are within the uncertainties of the model.

The same number of states is predicted by both models: two resonances with $J^{PC} = 0^{++}$, a broad and a narrow one, and one with $J^{PC} = 2^{++}$. The mass and width of the $J^{PC} = 2^{++}$ state is compatible with those measured for the $X(3930)$ for model A, but its mass moves closer to the mass region of the $X(3915)$ for model B, so its assignment remains unclear.

In the $J^{PC} = 0^{++}$ region both models show a similar phenomenology. On the one hand, the mass of the narrow state is more compatible with the new $X(3860)$ resonance than to the $X(3915)$. However, its predicted width is smaller than the experimental one, which is most likely connected to the position of the node in the $c\bar{c}(2^3P_0)$ bare wave function, which makes the width sensitive to small variations in the wave function structure or, equivalently, to the mass of the $X(3860)$. 

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The second $J^{PC}=0^{++}$ state is more intriguing. Its mass suggests a preliminary assignment to the $Y(3940)$ resonance, but, as already mentioned, its width is far from the experimental value. Alternatively, one can claim the second $0^{++}$ state is actually the $X(3860)$, as it is a broad state whose width agrees with the experimental one and its mass is close to the experiments, whose value reaches up to 3900 MeV. Within this picture, the first state seems too narrow, which can complicate its measurement in the experiments of Ref. [8]. What appears to be clear is that none of the $J^{PC}=0^{++}$ states seems to favor a $X(3915)$ assignment, neither by mass nor by width.

Nevertheless, the predicted $J^{P}=2^{+}$ state is compatible with both $X(3915)$ and $X(3930)$, within the uncertainties of the CQM and the value of the $3P_{0}$ quark-pair creation parameter. So, in order to further analyze the properties of the $X(3915)$ resonance, we can explore its decay properties. Under a $J^{PC}=0^{++}$ hypothesis for the $X(3915)$ state we are unable to describe the experimental branching fractions. In contrast, the product of the two-photon decay width and the branching fraction to $\omega J/\psi$ and $D\bar{D}$ channels can be calculated assuming the $X(3915)$ and $X(3930)$ are two faces of the same $J^{PC}=2^{++}$ resonance. Such results are quoted in Table 2, where the decay to the $D\bar{D}^{*}$ channel is also shown. The results for both models A and B are very similar, not far from the experimental values. Consequently, our results suggest that both the $X(3915)$ and $X(3930)$ resonances fit within a $J^{PC}=2^{++}$ assignment, pointing to the conclusion that they are two faces of the same state, which agrees with Ref. [24].

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References

[1] BELLE collaboration, S. K. Choi et al., Observation of a narrow charmonium - like state in exclusive $B^{+} \rightarrow K^{+} \, \pi^{+} \, \pi^{-} \, J/\psi$ decays, Phys. Rev. Lett. 91 (2003) 262001.

[2] BELLE collaboration, K. Abe et al., Observation of a near-threshold omega J/psi mass enhancement in exclusive $B \rightarrow K \, \omega \, J/\psi$ decays, Phys. Rev. Lett. 94 (2005) 182002.

[3] BELLE collaboration, K. Abe et al., Observation of a new charmonium state in double charmonium production in $e^{-}e^{+}$- annihilation at $s^{*}(1/2)-10.6$-GeV, Phys. Rev. Lett. 98 (2007) 082001.

[4] BELLE collaboration, S. Uehara et al., Observation of a charmonium-like enhancement in the gamma gamma $\rightarrow \omega \, J/\psi$ process, Phys. Rev. Lett. 104 (2010) 092001.

[5] BABAR collaboration, J. P. Lees et al., Study of $X(3915) \rightarrow J/\psi \omega$ in two-photon collisions, Phys. Rev. D86 (2012) 072002.

Table 2: Product of the two-photon decay width and the branching fraction (in eV) for the $J^{PC}=2^{++}$ sector for each model, compared with Belle and BaBar Collaboration experimental results [4, 5, 25, 26]

|                        | Belle          | BaBar         | model A | model B |
|------------------------|---------------|---------------|---------|---------|
| $\Gamma_{\gamma\gamma}\times\mathcal{B}(2^{++}\rightarrow\omega J/\psi)$ | 18(5)(2)      | 10.5(1.9)(0.6) | 20.9    | 24.9    |
| $\Gamma_{\gamma\gamma}\times\mathcal{B}(2^{++}\rightarrow D\bar{D})$ | 180(50)(30)   | 249(50)(40)   | 75.4    | 81.4    |
| $\Gamma_{\gamma\gamma}\times\mathcal{B}(2^{++}\rightarrow D\bar{D}^{*})$ | -             | -             | 196.0   | 151.9   |
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[6] S. L. Olsen, Is the $X(3915)$ the $\chi_{c0}(2P)$?, Phys. Rev. D91 (2015) 057501.

[7] Z.-Y. Zhou, Z. Xiao and H.-Q. Zhou, Could the $X(3915)$ and the $X(3930)$ Be the Same Tensor State?, Phys. Rev. Lett. 115 (2015) 022001.

[8] BELLE collaboration, K. Chilikin et al., Observation of an alternative $\chi_{c0}(2P)$ candidate in $e^+e^- \rightarrow J/\psi DD$, Phys. Rev. D95 (2017) 112003.

[9] P. G. Ortega, J. Segovia, D. R. Entem and F. Fernandez, Coupled channel approach to the structure of the $X(3872)$, Phys. Rev. D81 (2010) 054023.

[10] P. G. Ortega, D. R. Entem and F. Fernandez, Molecular Structures in Charmonium Spectrum: The XYZ Puzzle, J. Phys. G40 (2013) 065107.

[11] P. G. Ortega, J. Segovia, D. R. Entem and F. Fernandez, Charmonium resonances in the 3.9 GeV$^2$ energy region and the $X(3915)/X(3930)$ puzzle, 1706.02639.

[12] P. G. Ortega, J. Segovia, D. R. Entem and F. Fernandez, Molecular components in P-wave charmed-strange mesons, Phys. Rev. D94 (2016) 074037.

[13] P. G. Ortega, J. Segovia, D. R. Entem and F. Fernandez, Threshold effects in P-wave bottom-strange mesons, Phys. Rev. D95 (2017) 034010, [1612.04826].

[14] J. Vijande, F. Fernandez and A. Valcarce, Constituent quark model study of the meson spectra, J. Phys. G31 (2005) 481.

[15] J. Segovia, D. R. Entem and F. Fernandez, Is chiral symmetry restored in the excited meson spectrum?, Phys. Lett. B662 (2008) 33–36.

[16] J. Segovia, P. G. Ortega, D. R. Entem and F. Fernandez, Bottomonium spectrum revisited, Phys. Rev. D93 (2016) 074027.

[17] A. Valcarce, H. Garcilazo, F. Fernandez and P. Gonzalez, Quark-model study of few-baryon systems, Rept. Prog. Phys. 68 (2005) 965–1042.

[18] P. G. Ortega, D. R. Entem and F. Fernandez, p anti-p $\rightarrow$ Lambda anti-Lambda depolarization and spin transfer in a constituent quark model, Phys. Lett. B696 (2011) 352–358.

[19] J. D. Born, E. Laermann, N. Pirch, T. F. Walsh and P. M. Zerwas, Hadron Properties in Lattice QCD With Dynamical Fermions, Phys. Rev. D40 (1989) 1653–1663.

[20] E. Hiyama, Y. Kino and M. Kamimura, Gaussian expansion method for few-body systems, Prog. Part. Nucl. Phys. 51 (2003) 223–307.

[21] L. Micu, Decay rates of meson resonances in a quark model, Nucl. Phys. B10 (1969) 521–526.

[22] E. S. Ackleh, T. Barnes and E. S. Swanson, On the mechanism of open flavor strong decays, Phys. Rev. D54 (1996) 6811–6829.

[23] J. Segovia, D. R. Entem and F. Fernandez, Scaling of the $^3P_0$ Strength in Heavy Meson Strong Decays, Phys. Lett. B715 (2012) 322–327.

[24] T. Branz, R. Molina and E. Oset, Radiative decays of the $Y(3940)$, $Z(3930)$ and the $X(4160)$ as dynamically generated resonances, Phys. Rev. D83 (2011) 114015.

[25] BELLE collaboration, S. Uehara et al., Observation of a chi-prime(c2) candidate in gamma gamma $\rightarrow D$ anti-D production at BELLE, Phys. Rev. Lett. 96 (2006) 082003.

[26] BABAR collaboration, B. Aubert et al., Observation of the $\chi_{c2}(2p)$ Meson in the Reaction $\gamma\gamma \rightarrow D\bar{D}$ at BaBar, Phys. Rev. D81 (2010) 092003.