1. Introduction

Aging of components in chemical and power plant facilities is a crucial factor affecting their maintenance. Elbows are main components of nuclear power plant facilities and present the following three degradation problems.

1) Inner surfaces of elbows are susceptible to local wall thinning due to flow-accelerated corrosion and liquid-droplet-impingement (LDI) erosion.

2) Deformation due to transient phenomena is greater in elbows than in straight pipes.

3) Multi-axial stress occurs during deformation at elbows (Takahashi et al., 2014).

Therefore, establishing an accurate low-cycle fatigue-life-evaluation method is necessary to evaluate the structural integrity of elbow pipes.

To investigate the fracture behavior of elbow piping, many repeated load tests of elbow piping have been conducted. (Sakakida et al., 2000; Nuclear Power Engineering Corp, 2001; Otani et al., 2011; Takahashi et al., 2009; Takahashi et al., 2010).

In general, the low-cycle fatigue life of steel specimens under tension-compression loading can be accurately predicted by using Manson’s universal slope method (Manson, 1965). However, the accuracy of the predicted values of the low-cycle fatigue lives of elbows using Manson’s universal slope method is not high, where the experimental fatigue lives of elbows were shown to be approximately 20% of the values predicted by the method. To overcome this problem, a revised Manson's universal slope method was proposed as a prediction equation for the fatigue life of elbows and that considers the multi-axial stress field (Takahashi et al., 2014). Furthermore, the fatigue behavior of carbon steel elbows...
was evaluated by considering the effects of the shape and position of local wall thinning, and it became clear that low-cycle fatigue lives of elbows could be predicted with high accuracy using the revised method (Urabe et al. 2015; Nagamori and Takahashi, 2017). However, verification of the revised Manson's universal slope method on materials other than carbon steel has yet to be conducted. In this study, fatigue tests on both carbon and stainless steel elbows with local wall thinning were performed to investigate the effects of difference in materials on the low-cycle fatigue behaviors as well as on the utility of the revised method.

2. Experiments
2.1 Experimental method

The elbow specimens used in this study conformed to Japanese Industrial Standard (JIS) G3456 STPT410 (carbon steel pipes to withstand high temperature) and JIS G3459 SUS304 (stainless steel pipes). The mechanical properties of both materials are shown in Table 1.

Fig. 1(a) shows the shape and dimensions of elbow specimens. Two straight pipes were welded to the two ends of a 90° elbow. The outer diameter of the pipes was 114.3 mm, the wall thickness was 6.0 mm, and the radius of curvature of the elbow was 154.2 mm. Fig. 1(b) shows details of wall thinning. Sound elbows and two types of local wall thinned elbows having different eroded angles 2θ were used. The local wall thinning was machined on the inner surface of the crown to simulate wall thinning caused by LDI erosion. The length and eroded depth were 50 mm and 4.8 mm, respectively. Their eroded angles were 90° or 180°. As shown in Fig. 1(a), fatigue tests were conducted by applying a displacement of ± 20 mm using a universal testing machine. An internal pressure of 9 MPa was applied to the elbows using water, and the fatigue life \( N_f \) was defined as the number of loading cycles until fatigue cracks propagated and water began to leak from the specimen.

Table 1 Mechanical properties of elbow specimen.

| Material   | Yield Stress \( \sigma_Y \) [MPa] | Young’s modulus \( E \) [GPa] | Tensile strength \( \sigma_B \) [MPa] | Reduction of area R.A. [%] | True fracture stress \( \sigma_f \) [MPa] | True fracture strain \( \epsilon_f \) [-] |
|------------|----------------------------------|------------------------------|----------------------------------|---------------------------|---------------------------|---------------------|
| STPT410    | 340                              | 206                          | 504                              | 72.7                      | 1250                      | 1.13                 |
| SUS304     | 272                              | 197                          | 633                              | 79.4                      | 1673                      | 1.58                 |

![Fig. 1(a) Elbow specimens](image1)

![Fig. 1(b) Detail of local wall thinning](image2)

Fig. 1 Shape and geometry of elbow specimens with local wall thinning [mm].

2.2 Experimental results

Table 2 lists the test results observed under each test condition. Fig. 2 shows the typical fracture behavior of elbows. In all specimens, axial fatigue cracks initiated from the inside of the crown and propagated outward. Thus, local wall thinning introduced in this study did not affect the fatigue behavior. The fatigue lives were 103 to 129 cycles for the
STPT 410 elbows and 321 to 430 cycles for the SUS 304 elbows. The fatigue lives of the SUS 304 elbows were approximately three to four times longer than those of STPT 410 elbows. It was noted that the fatigue lives with local wall thinning were longer than those of sound elbows. In addition, the fatigue lives tended to increase with increases in the eroded angle. This peculiar trend is explained in Section 3.3.

Table 2 Test conditions and results.

| Material  | Test conditions | Eroded angle $\theta$ [°] | Direction of crack | Location of crack | Fatigue life $N_{f_{\text{exp}}}$ [cycles] |
|-----------|-----------------|---------------------------|-------------------|------------------|------------------------------------------|
| STPT410   | Sound           | -                         | -                 | -                | 103                                      |
|           | Extrados        | 90                        | -                 | Axial            | 107                                      |
| SUS304    | Sound           | 90                        | -                 | Axial            | 129                                      |
|           | Extrados        | 180                       | -                 | Crown            | 321                                      |
|           |                 |                           |                   |                  | 400                                      |
|           |                 |                           |                   |                  | 430                                      |

![Image of test conditions and results](image)

Fig. 2 Typical fracture behavior of elbows.

3. Analysis

3.1. Finite element analysis method

For each specimen condition, a finite element analysis (FEA) was performed to simulate the low-cycle fatigue behavior by using commercial FEA code ANSYS version 14.5. Fig. 3 shows an example of the FEA model. With the symmetry of elbow pipes considered, the FEA model was set to the 1/4 model. 20-node solid elements were used for the
model. The data obtained by multilinear approximation of true stress-true strain curves of STPT410 and SUS304 obtained from the uniaxial tensile test were used as the material properties for FEA. Fig. 4(a) and (b) show the true stress-true strain curves for STPT410 and SUS304, respectively. The black solid line shows the stress-strain curve for FEA and the red dotted lines show the curves approximated by the Ramberg-Osgood equation (Ramberg & Osgood, 1943) as follows:

$$\left( \frac{\varepsilon}{\sigma_Y/E} \right) = \left( \frac{\sigma}{\sigma_Y} \right) + \lambda \left( \frac{\sigma}{\sigma_Y} \right)^n$$  \hspace{1cm} (1)

where $\varepsilon$ is the true strain [-], $\sigma$ is the true stress [MPa], $\sigma_Y$ is the yield stress [MPa], $E$ is the Young’s modulus [GPa], and $\lambda$, $n$ are material constants [-].

The material constants obtained from the figures were $\lambda = 9.2$, $n = 3.3$ for STPT 410 and $\lambda = 27.4$, $n = 2.3$ for SUS 304. The von Mises yield condition and linear kinematic hardening rule were applied as a plasticity option. An internal pressure of 9 MPa was applied to the inner surface of the FEA model, and an elastoplastic analysis was conducted for up to 10 cycles under the displacement control condition.

![Finite element model of elbows.](image)

**Fig. 3** Finite element model of elbows.

![Approximated true stress-strain curve by Ramberg-Osgood](image)

**Fig. 4** Approximated true stress-strain curve by Ramberg-Osgood (STPT410: $\lambda = 9.2$, $n = 3.3$; SUS304: $\lambda = 27.4$, $n = 2.3$).

### 3.2. Estimation of fatigue life

Manson's universal slope method (Manson, 1965) is known as a prediction equation for low-cycle fatigue life of metals and is expressed by the following equation:

$$\Delta \varepsilon = \frac{3.5 \sigma_B}{E} N_t^{-0.12} + \varepsilon_f^{0.6} N_f^{-0.6}$$  \hspace{1cm} (2)

where $N_t$ is the fatigue life [cycles], $\Delta \varepsilon$ is the strain range [-], $\sigma_B$ is the tensile strength [MPa], and $\varepsilon_f$ is the true
fracture strain [-]
The values of $\varepsilon_f$ are obtained using the following equation:

$$
\varepsilon_f = \ln\left(\frac{100}{100 - R.A.}\right)
$$

where R.A. is the reduction of area in a tensile test [%]. The values of R.A. and $\varepsilon_f$ are shown in Table 1.

In (2), fatigue life can be predicted from $\Delta \varepsilon$ and material properties obtained from the uniaxial tensile test. However, it is known that the low-cycle fatigue lives of elbow pipes are approximately 20% of the value obtained in a uniaxial tensile-compression fatigue test. Manson's universal slope method is not suitable for predicting the fatigue lives of elbow pipes because it predicts excessively on the non-safety side. Therefore, with a focus on the multi-axial stress state of the elbow pipe, the revised universal slope method was proposed. This method is expressed using the following equation (Takahashi et al., 2014).

$$
\Delta \varepsilon = \frac{3.5\sigma_f}{E} N^{-0.12} + \varepsilon_{mf}^{0.6}N_f^{-0.6}
$$

where $\Delta \varepsilon$ is the principal strain range [-], $\sigma_f$ is the true fracture stress [MPa], and $\varepsilon_{mf}$ is the true fracture strain under a multi-axial stress state [-].

The revised universal slope method is different from Manson’s universal slope method in that $\sigma_f$, which is a material property at the location of the fracture, is used. In addition, $\varepsilon_{mf}$ is used considering a decrease in the true fracture strain resulting from a multiaxial stress state. $\varepsilon_{mf}$ is calculated from the following equation proposed by Miyazaki et al. (2002).

$$
\varepsilon_{mf} = \left(\frac{\omega \sigma_f}{\sigma_Y}\right) + \lambda \left(\frac{\omega \sigma_f}{\sigma_Y}\right)^n \varepsilon_f
$$

$$
m = \sqrt{(1 + \alpha + \beta)^2 - 3(\alpha + \beta + \alpha \beta)}
$$

$$
\omega = \frac{1}{1 + \alpha + \beta}
$$

$$
\alpha = \frac{\sigma_2}{\sigma_1} \quad \text{and} \quad \beta = \frac{\sigma_3}{\sigma_1}
$$

where $\sigma_1$, $\sigma_2$, and $\sigma_3$ are principal stresses and $\sigma_1 > \sigma_2 > \sigma_3$.

In this study, the following assumptions were made to predict both the location of crack initiation and the initial crack propagation direction:

(a) Fatigue cracks initiate where the principal strain range $\Delta \varepsilon$ is maximum.

(b) Fatigue cracks propagate perpendicular to the direction of the maximum strain range $\Delta \varepsilon$.

3.3 Numerical Results

Table 3 shows the numerical results. In all specimen conditions, it was predicted that cracks initiated inside the crown parts of elbows and propagated in the axial direction. Therefore, the predictions agreed with the test results. Fig. 5 shows the relationship between the biaxial stress factor $\alpha$ ($= \sigma_2/\sigma_1$) and $\varepsilon_{mf}$. The figure shows that the values of $\alpha$ at the predicted crack initiation points were approximately 0.6 regardless of the different materials and wall thinning conditions. The figure further shows that the values of $\varepsilon_{mf}$ at these points were nearly constant and approximately 0.15 for STPT410 and 0.4 for SUS304. Therefore, it can be said that the $\varepsilon_{mf}$ of SUS304 was approximately 2.6 times greater than that of STPT410. Fig. 6 presents a comparison of fatigue lives between STPT410 and SUS304. The fatigue life of SUS304 predicted by (4) was four to five times greater than that of STPT410. Eq. (4) shows a similar tendency to the experimental results. Thus, the difference in the value of $\varepsilon_{mf}$ has a great effect on the fatigue life.

In section 2.2, we showed that fatigue life of elbow piping increase with the increase of $2\theta$. Therefore, we examined this tendency by comparing the strain distributions. Fig. 7 shows the distribution of the hoop strain range $\Delta \varepsilon_{hoop}$ on the inner surface of the elbow center section obtained from FEA. In all specimen conditions, a peak of $\Delta \varepsilon_{hoop}$ appeared at
the crown of the elbow. The maximum values of \( \Delta \varepsilon_{\text{hoop}} \) decreased with increasing \( 2\theta \) values. In addition, as the values of \( 2\theta \) increased, the values of \( \Delta \varepsilon_{\text{hoop}} \) between the extrados and crown area increased. As a result, the strain concentration at the crown decreased. Thus, as described in Section 2.2, the fatigue lives increased with an increase in the \( 2\theta \).

Table 3  Numerical results by FEM.

| Material | Test conditions | Numerical result by FEM | Fatigue life \( N_{f, \text{pre}} \) [cycles] |
|----------|----------------|-------------------------|---------------------------------|
| STPT410  | Location of wall thinning | Direction of crack | Location of crack | Bi-axial stress factor \( \alpha = \sigma_2 / \sigma_1 \) [-] | Principal strain range \( \Delta \varepsilon \) [%] | |
| Sound    | Eroded angle \( 2\theta \) [°] | Axial | Crown | 0.59 | 3.97 | 66 |
| Extrados | 90 |  |  | 0.63 | 3.79 | 69 |
| 180 |  |  |  | 0.56 | 4.34 | 56 |
| SUS304   | Sound    | - |  | 0.56 | 3.30 | 329 |
| Extrados | 90 | Axial | Crown | 0.61 | 3.30 | 314 |
| 180 |  |  |  | 0.55 | 3.65 | 258 |

Fig. 5  Relationship between fracture strain and multi-axial stress factor.

Fig. 6  Comparison between experimental results and revised universal slope method (\( \alpha = 0.6 \)).
3.4. Comparison of Numerical Results with Experimental Results

The fatigue lives predicted using Manson's universal slope method (Eq. (2)) and the revised universal slope method (Eq. (4)) were compared with the experimental results. Fig. 8 shows a comparison between predicted and actual fatigue lives derived from the experiments. The solid line shows the agreement between experimental and predicted fatigue lives, and the short dashed lines show the error range of 50–200%. The estimated fatigue lives predicted using Manson's universal slope method were on the non-conservative side with respect to the experimental results. However, the estimated fatigue lives predicted using the revised universal slope method were within the error range of 50–200% with respect to the experimental results and were highly accurate. This comparison reveals that fatigue life prediction using the revised universal slope method is useful not only for STPT410 elbows but also for SUS304 elbows.
4. Conclusion

In this study, fatigue tests on stainless steel and carbon steel elbows with local wall thinning and FEA were conducted to evaluate the effects of differences of elbow materials on low-cycle fatigue behavior and the utility of the revised Manson’s universal slope method. The following results were obtained.

1) The fatigue lives of SUS 304 elbows were three to four times longer than those of STPT 410 elbows. The primary explanation for this is that the true fracture strain under the multi-axial stress of SUS304 was higher than that of STPT410.

2) The fatigue lives increased with an increase in the eroded angle at extrados. This is because the strain concentration at the crown decreased with an increase in the eroded angle.

3) The low-cycle fatigue lives of SUS304 elbows could be predicted with an improved accuracy and conservatively by using the revised universal slope method. The results of STPT410 were similar in this regard.

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