Note on renormalization of the spin-1 resonance propagator at one loop order

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We study various aspects of the renormalization of the Resonance Chiral Theory at the one-loop level using a spin-one resonance propagator as a concrete example. We calculate explicitly the one-loop self-energy within the antisymmetric tensor field formalism, briefly discuss the general structure of the corresponding propagator obtained by means of the Dyson re-summation and give a classification of the propagating degrees of freedom. We find that additional pathological poles (negative norm ghosts or tachyons) are unavoidably generated and various scenarios according to their position are possible. We also briefly comment on the eventual dynamical generation of the opposite parity resonances which are frozen at the tree level and discuss the role of appropriate symmetry which could prevent such a scenario.

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1. Introduction

Recently, effective theories have become a very efficient tool in particle physics. As far as the strong interactions are concerned, the Chiral perturbation theory (χPT) [1], [2], [3] as a low energy effective theory of QCD provides us with a rigorously defined simultaneous double expansion of the Green functions of the quark currents in powers (and logarithms) of external momenta and quark masses which is valid in the energy range $E \ll \Lambda_H \sim 1$ GeV. The scale $\Lambda_H$ corresponds to the mass of the lowest resonances which are separated from the relevant low energy QCD degrees of freedom, namely the (pseudo)Goldstone bosons (PGB) which are identified with the members of the lightest pseudoscalar octet, by a mass gap. Thanks to this mass gap the dynamics of the higher energy degrees of freedom $E \gtrsim \Lambda_H$ can be taken into account effectively and parametrized by means of the low energy constants (LEC). The corresponding low energy expansion is therefore possible and well behaved. The formal structure and the technical aspects of χPT are perfectly understood and the recent calculations reached the two-loop level which corresponds to the order $O(p^6)$ within the chiral power counting [4].

However, an extension of this successful method to the intermediate energy region $\Lambda_H \leq E < 2$ GeV is more problematic. The set of relevant degrees of freedom enlarges and contains not only the PGB but also the low lying resonances. These are not separated by a mass gap from the rest of the spectrum and therefore the formal expansion in the spirit of χPT (i.e. a simultaneous expansion in the momenta and both quark and resonance masses) cannot be expected to be well-founded. Fortunately, another type of effective Lagrangian description of this region exists which is based on the large $N_C$ expansion as well as on the high energy constraints derived from the operator product expansion (OPE). This was introduced in the pioneering works [5], [6] and now it is known as as a Resonance Chiral Theory (RχT). It becomes extremely useful for the estimates of the LEC in terms of the resonance parameters [5], [7], [8], which is necessary in order to connect the recent $O(p^4)$ predictions of χPT with physical data (cf. also [9]). The theory is organized according to the large $N_C$ expansion: the interaction vertices are accompanied with the appropriate power of $1/\sqrt{N_C}$ for each meson field, the leading order contributions to the Green function are given by the tree graphs and each loop brings about one additional power of $1/N_C$. Taking just one resonance multiplet for each channel and matching such a truncated theory in the UV region with the OPE (which corresponds to the so-called Minimal Hadronic Ansatz) and with χPT in the infrared was proved to be sufficient to saturate the values of the $O(p^4)$ LEC successfully at the leading order in $1/N_C$ [5].

Such a leading order matching suffers from the fact that the LEC depend on the renormalization scale. Therefore, this scale has to be fixed at some value (the saturation scale) at which the renormalization scale independent results of tree level RχT are sewed with χPT. This is one of the reasons why to go beyond the leading order and match χPT with the one-loop RχT. Also from the phenomenological point of view, the loops are inevitable in order to preserve (perturbative) unitarity and to generate finite resonance widths. It is therefore desirable to investigate the one-loop RχT in more details [10].

However, because the Weinberg formula [1] (according to which the loop calculations are organized within χPT) cannot be straightforwardly generalized to the case of RχT, new aspects of the renormalization procedure are expected. Namely, because of the presence of a new scale...
corresponding to the mass of the resonances and as a result of the nontrivial structure of the higher-spin resonance propagators, we can encounter mixing of the usual chiral orders in the process of the renormalization. Also, higher than expected chiral order of counterterms might be necessary already at the one-loop level. Furthermore, because the spin-one particles are described using fields transforming under reducible representation of the rotation group, new degrees of freedom (which were frozen at the tree-level) can come back to the game due to the loop corrections.

In this paper we would like to concentrate on a particular example of the renormalization of the one-loop spin-one resonance self-energy and the construction of the corresponding resonance propagator using a concrete interaction Lagrangian. We will use the antisymmetric tensor field for definiteness, since in such a formalism all the above aspects of the renormalization procedure can be illustrated. In addition we will briefly discuss the problems connected with the appearance of additional poles in the propagator obtained by means of Dyson re-summation of the one-particle irreducible insertions. Because the one loop corrections to the self-energy might be relatively large, these additional poles might lie near the region for which we assume $R\chi T$ to be valid. Moreover, the self-energy has higher order growth in the UV region than usual and therefore some of these poles could be negative norm ghosts or tachyons [11]. This might introduce well known problems with the physical interpretation of the theory due to the violation of unitarity or causality. Due to the lack of appropriate symmetry, some of the poles correspond to one particle states with opposite parity than the original degrees of freedom of the tree Lagrangian. We will also briefly discuss the possibility to interpret the non-pathological poles as dynamically generated higher resonances, as was done in [12].

2. The Lagrangian of Resonance Chiral Theory

We are going to work in the framework of Chiral perturbation theory where the Lagrangian is formulated in terms of external sources and pseudoscalar mesons. They transform as an octet under the group $SU(3)_V$. We define the chiral building block

$$ u(\phi) = \exp \left( i \frac{\phi}{\sqrt{2}F_0} \right), \quad (2.1) $$

where $\phi = \phi^a T^a$ with $T^a = \lambda^a / \sqrt{2}$ and

$$ \phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}} \eta & \sqrt{2} \pi^+ & \sqrt{2} K^+ \\ \sqrt{2} \pi^- & -\pi^0 + \frac{1}{\sqrt{3}} \eta & \sqrt{2} K^0 \\ \sqrt{2} K^- & \sqrt{2} K^0 & -\frac{2}{\sqrt{3}} \eta \end{pmatrix} \quad (2.2) $$

is the matrix describing the pseudoscalar mesons fields. The Goldstone bosons are parametrized by the elements $u(\phi)$ of the coset space $SU(3)_L \times SU(3)_R / SU(3)_V$, transforming as

$$ u(\phi) \rightarrow V_R u(\phi) h(g, \phi)^{-1} = h(g, \phi) u(\phi) V_L^{-1} \quad (2.3) $$

under a general chiral rotation $g = (V_L, V_R) \in G$ in terms of the $SU(3)_V$ compensator field $h(g, \phi)$.

The Resonance Chiral Theory enlarges the number of degrees of freedom of $\chi PT$ by including also massive multiplets. Let us now restrict ourselves to the octet of vector resonances $1^{--}$ which is
the subject of our interest. There are several possibilities how to choose corresponding interpolating
fields for them \[5, \[6, \[13\]. In this article we use the antisymmetric tensor field which can be written as
\[
V_{\mu \nu} = \begin{pmatrix}
\frac{1}{\sqrt{2}} \rho^0 + \frac{1}{\sqrt{6}} \omega^0 + \frac{1}{\sqrt{3}} \omega_0 \\
\rho^+ \\
K^{++} \\
\frac{1}{\sqrt{6}} \omega_0 + \frac{1}{\sqrt{3}} \omega_3 \\
\frac{2}{\sqrt{6}} \omega_8 + \frac{1}{\sqrt{3}} \omega_0
\end{pmatrix} \mu \nu.
\] (2.4)
The fields \(V_{\mu \nu}\) transform in the nonlinear realization of the \(U(3)_L \times U(3)_R\) according to the prescription
\[
V_{\mu \nu} \mapsto h(g, \phi) V_{\mu \nu} h(g, \phi)^{-1}.
\]
The Lagrangian for these field is then
\[
\mathcal{L}_\text{V} = -\frac{1}{2} \langle \nabla_\mu V^{\mu \nu} \nabla^\alpha V_{\alpha \nu} \rangle + \frac{1}{4} M^2 \langle V_{\mu \nu} V^{\mu \nu} \rangle + \mathcal{L}_\text{int},
\] (2.5)
where the relevant interaction part (contributing to the renormalization of the resonance self-energy
which is of our interest) is
\[
\mathcal{L}_\text{int} = \frac{i G_V}{2 \sqrt{2}} \langle V^{\mu \nu} [\mu_\mu, u_\nu] \rangle + d_1 \epsilon_{\mu \nu \alpha \sigma} \langle \{V^{\mu \nu}, V^{\alpha \beta}\} \nabla_\beta u_\sigma \rangle \\
+ d_3 \epsilon_{\alpha \beta \lambda} \langle \{\nabla_\nu V^{\mu \nu}, V^{\alpha \beta}\} u^\lambda \rangle + d_4 \epsilon_{\rho \sigma \mu \alpha} \langle \{\nabla^\alpha V^{\mu \nu}, V^{\rho \sigma}\} u_\nu \rangle + \ldots
\] (2.6)
3. Structure of poles

Let us briefly recall the basic properties of the Lagrangian for spin-1 fields and of the corre-
sponding propagator within the antisymmetric tensor field formalism [5]. We start with the most
general free Lagrangian
\[
\mathcal{L}_\text{V} = \frac{\alpha}{2} \langle \partial_\mu V^{\mu \nu} \partial^\alpha V_{\alpha \nu} \rangle + \frac{\beta}{4} \langle \partial_\alpha V^{\mu \nu} \partial^\alpha V_{\mu \nu} \rangle + \frac{1}{4} M^2 \langle V_{\mu \nu} V^{\mu \nu} \rangle,
\] (3.1)
which leads to the propagator
\[
\Delta^{V}_{\mu \nu \rho \sigma}(p) = \frac{2}{(\alpha + \beta) p^2 + M^2} \Pi^T_{\mu \nu \rho \sigma} + \frac{2}{\beta p^2 + M^2} \Pi^L_{\mu \nu \rho \sigma},
\] (3.2)
where \(\Pi^T\) and \(\Pi^L\) are projectors
\[
\Pi^T_{\mu \nu \alpha \beta} = \frac{1}{2} \left( g_{\mu \alpha} g_{\nu \beta} - g_{\mu \beta} g_{\nu \alpha} \right) - \frac{1}{2 p^2} \left( g_{\mu \alpha} p_{\nu \beta} - g_{\mu \beta} p_{\nu \alpha} - g_{\nu \beta} p_{\mu \alpha} - g_{\nu \alpha} p_{\mu \beta} \right),
\]
\[
\Pi^L_{\mu \nu \alpha \beta} = \frac{1}{2 p^2} \left( g_{\mu \alpha} p_{\nu \beta} - g_{\mu \beta} p_{\nu \alpha} + g_{\nu \beta} p_{\mu \alpha} - g_{\nu \alpha} p_{\mu \beta} \right).
\] (3.3)
The propagator \(\Delta^{V}_{\mu \nu \rho \sigma}(p)\) has two poles in general. In order to obtain just one pole (and assuming
\(M^2 > 0\) we have to fix \(\alpha = -1, \beta = 0\) (which is in agreement with (2.5), see [5] for further details)
that leads to the Lagrangian
\[
\mathcal{L} = -\frac{1}{2} \langle \partial_\alpha V^{\mu \nu} \partial^\alpha V_{\alpha \nu} \rangle + \frac{1}{4} M^2 \langle V_{\mu \nu} V^{\mu \nu} \rangle.
\] (3.4)
This Lagrangian is generally used for the description of any spin-1 resonances in $R \chi T$. From (3.4) we get the usual propagator
\[
\Delta_{\mu\nu\rho\sigma}(p) = -\frac{2}{p^2 - M^2} \Pi_{\mu\nu\rho\sigma}^L + \frac{2}{M^2} \Pi_{\mu\nu\rho\sigma}^T.
\] (3.5)

Provided that under parity and charge conjugation $V_{\mu\nu} \mapsto -V_{\mu\nu}$ and $V_{\mu\nu} \mapsto -V_{\mu\nu}^T$ respectively, the pole in $\Pi^L$ sector corresponds to a $1^{-+}$ resonance. Let us show that the possible pole of $\Delta_{\mu\nu\rho\sigma}(p)$ in the $\Pi^T$ sector corresponds to an opposite parity $1^{++}$ resonance.

The case of $1^{++}$ resonances was studied in detail in [14]. Starting with the field $B_{\mu\nu}$ describing a $1^{++}$ resonance (now $B_{\mu\nu} \mapsto -B_{\mu\nu}$ under parity) we can write the same Lagrangian (3.4) (just replacing $V_{\mu\nu} \rightarrow B_{\mu\nu}$). Now, we can introduce a field $U_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} B_{\alpha\beta}$ which has the same transformation properties with respect to parity as the field $V_{\mu\nu}$. Rewriting the Lagrangian (3.4) in terms of $U_{\mu\nu}$ we find
\[
\mathcal{L} = \frac{1}{12} \langle H_{\alpha\mu\nu} H_{\alpha\mu\nu} \rangle - \frac{1}{4} M^2 \langle U_{\mu\nu} U_{\mu\nu} \rangle, \quad \text{where} \quad H_{\alpha\mu\nu} = \partial_{[\alpha} U_{\mu\nu]_{\text{cycl}}}
\] (3.6)

and the corresponding propagator is
\[
\Delta_{\mu\nu\rho\sigma}(p) = -\frac{2}{M^2} \Pi_{\mu\nu\rho\sigma}^L + \frac{2}{p^2 - M^2} \Pi_{\mu\nu\rho\sigma}^T.
\] (3.7)

Because the $U_{\mu\nu}$ fields describe $1^{++}$ resonances and have the same quantum numbers as $V_{\mu\nu}$, the poles in the $\Pi^T$ sector found in the propagator for $V_{\mu\nu}$ indicate the presence of $1^{++}$ resonances. For the free field case there is nothing like that because we fixed $\alpha, \beta$ to have just one pole in the $\Pi^L$ sector. If we take $\beta \neq 0$ in (3.1) then the additional pole (in the $\Pi^T$ sector) is a ghost (for $\beta < 0$) or a tachyon (for $\beta > 0$). Therefore, we can not have both types of poles at the tree level.

4. Renormalization of the propagator

In the general case when the loop corrections are taken into account one obtains
\[
\Delta_{\mu\nu\rho\sigma}(p) = -\frac{2}{p^2 - M^2 - \Sigma_L(p^2)} \Pi_{\mu\nu\rho\sigma}^L + \frac{2}{M^2 + \Sigma_T(p^2)} \Pi_{\mu\nu\rho\sigma}^T,
\] (4.1)

where $\Sigma_L(p^2)$ and $\Sigma_T(p^2)$ are the self-energies which are determined at the one-loop level by the following Feynman graphs:

where the thick lines denote resonances and dashed lines Goldstone bosons. We use the interaction Lagrangian (2.6) that keeps just leading order operators (in the number of derivatives) with no more than two resonances. The counterterm Lagrangian (terms with up to six derivatives are needed) is
\[
\mathcal{L}_{\text{ct}} = \mathcal{L}_{\text{ct}}^{(0)} + \mathcal{L}_{\text{ct}}^{(2)} + \mathcal{L}_{\text{ct}}^{(4)}
\] (4.2)
shown that (provided we fix the coupling be arranged to correspond to the original 1 sectors. These additional poles decouple in the α and expected originally the validity of however for actual values of the couplings they might lie near or even inside the region where we presence of poles also in the α and 417 resonance states in both 1 and 417 poles can be treated as a prediction of the theory and identified as dynamically generated higher 

\[ \Sigma_L = \frac{M^2}{4\pi F} \left[ \sum_{i=0}^{3} \alpha_i x^i - \frac{1}{2} \left( \frac{G_V}{F} \right)^2 x^2 \hat{B}(x) + \frac{40}{9} d_1^2 (x^2 - 1)^2 \hat{f}(x) \right], \]

\[ \Sigma_T = \frac{M^2}{4\pi F} \left[ \sum_{i=0}^{3} \beta_i x^i + \frac{20}{9} \left( 2d_3^2 + (d_3^2 + 6d_3 d_4 + d_4^2)x + 2d_4^2 x^2 \right) (x - 1)^2 \hat{f}(x) \right], \]

where \( x = p^2/M^2 \) and \( \hat{B}(x), \hat{f}(x) \) are loop functions:

\[ \hat{B}(x) = 1 - \ln(-x), \quad \hat{f}(x) = \frac{1}{x} \left[ 1 - \left( 1 - \frac{1}{x} \right) \ln(1 - x) \right] \]

and \( \alpha_i, \beta_i \) are renormalization scale independent combinations of the couplings and logs, e.g.

\[ \alpha_0 = \left( \frac{4\pi F}{M} \right)^2 Z_M(\mu) - \frac{40}{3} d_1^2 \ln \frac{M^2}{\mu^2} - \frac{20}{9} (3d_1^2 - d_3^2). \]

The complete result can be found in [15].

We see that \( \Sigma_T(x) \) has generally non-trivial \( x \) dependence, therefore, we can expect the possible presence of poles also in the \( \Pi^T \) sector.

As was indicated in [13] the spectrum of the propagator poles is very diverse. One of them can be arranged to correspond to the original 1 resonance we have started with. However, it can be shown [15] that (provided we fix the coupling \( d_3 \) according to the OPE for the \( VVP \) correlator [16]) there exists a nonzero minimal number of additional poles in both sectors irrespective of the actual values of the other couplings in the interaction and the counterterm Lagrangians (2.5) and (4.3). For general values of resonance couplings we could obtain bound states, virtual states or resonances and also at least one of the pathological poles like ghosts or tachyons in both the \( \Pi^T \) and the \( \Pi^L \) sectors. These additional poles decouple in the \( N_C \to \infty \) limit when the interaction is switched off, however for actual values of the couplings they might lie near or even inside the region where we expected originally the validity of \( R\chi T \). We can then assume several possible scenarios.

In the most optimistic one, all the additional poles are far enough and we can treat them as harmless. Then the theory effectively (i.e. when we consider it in the energy region of its validity) describes the same number of degrees of freedom as we started with on the tree level.

Within another possible scenario only the pathologies are situated far away from the range of the assumed validity of \( R\chi T \) (this condition is tricky to satisfy). Then the non-pathological poles can be treated as a prediction of the theory and identified as dynamically generated higher resonance states in both 1 and 1+ channels. This mechanism is the same as used in [17], where
the scalar resonances were identified as the poles of the propagator (dressed with the pseudoscalar loops) of the bare quark-antiquark "seed".

The worst variant arises when some of the pathological poles appear within the region of assumed applicability of $R\chi T$; in such a case the theory will suffer from inconsistences like the loss of unitarity or acausality.

Let us add several brief remarks concerning the second scenario. Suppose e.g. that $1^{+-}$ resonances are really generated. The question then is which processes the $\Pi T$ sector of the propagator can really affect. We can easily find that in the most common cases of $VV$ correlator, pion-vector formfactor or $\pi\pi$ scattering it completely decouples. However, for other processes like $\rho \rightarrow \pi^+\pi^-\gamma$ or $\pi\gamma \rightarrow \pi\gamma$ we could obtain some nonzero contribution from these dynamically generated resonances due to the Feynman graphs

\[
\begin{align*}
&\text{(the thick lines denote resonances, dashed lines Goldstone bosons and wavy lines ending with the cross vertex indicate the insertion of the QED current)}
\end{align*}
\]

respectively. Generally, if the vertices in the Lagrangian that couples to the propagator are invariant under the transformation

\[
V^{\mu\nu} \rightarrow V^{\mu\nu} + \epsilon^{\mu\nu\alpha\beta} \partial_\alpha a_\beta
\]

then the $\Pi T$ sector of the propagator does not affect a given process. This transformation corresponds to an accidental symmetry that some of the vertices posses.

We can also invert our point of view. In the Lagrangian (2.5) we started with the description of $1^{--}$ resonance and after renormalization we can get also dynamically generated $1^{+-}$ ones. So, for this reason or another one we can ask the question: Which symmetry does prevent the dynamical generation of the $1^{+-}$ at the one-loop level, i.e. when is $\Sigma^T(p^2) = 0$? As an answer we obtain the same symmetry as above. If the operators contributing to the one loop renormalization of the propagator are invariant under (4.7) then $\Sigma^T(p^2) = 0$ and there are no $1^{+-}$ resonances. This can be easily seen in the path integral formalism [15]. The price for this is that we have to throw away many terms in the interaction Lagrangian and finally we may lose the chiral symmetry. Note also that having the symmetry (4.7) or any other mechanism (which would preserve in an ideal case the chiral symmetry) that freezes the $1^{+-}$ channel, one has to still care about the self-consistency for the $1^{--}$ degree of freedom, i.e. one has to still face the three above-mentioned scenarios.

5. Conclusions

We have presented here the results of the calculation of the spin-1 resonance self-energy within Resonance Chiral Theory in the antisymmetric tensor formalism at one-loop. We have found that
additional poles appear in the corresponding Dyson re-summed propagator some of which are pathological. We have also briefly discussed various scenarios for their position and consequences for the physical interpretation of the theory.

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