Transeversely Rough Narrow Width Tapered Pad Bearing

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Abstract

Objective: The article is aim to analyse the behaviour of a transversely rough narrow width tapered pad bearing.

Design/methodology/approach: The stochastic model of Christensen and Tonder has been utilized to find the effect surface roughness. The associated stochastically averaged Reynolds type equation is discussed to attain the pressure distribution, load carrying capacity and friction. Findings: The findings suggest that the effect of transverse surface roughness remains adverse. However, the situation gets better in the case of negatively skew roughness. This positive effect improves when variance ( -ve) occurs. Application: There was no results on the effect of transverse roughness on the performance of narrow width tapered pad bearing. Although there were some discussion on the effect of longitudinal roughness. Therefore, it was decided to evaluate the effect of transverse roughness in general.

Keywords: Friction, Load Carrying Capacity, Pressure, Surface Roughness, Tapered Pad Bearing

Introduction

Tapered pad bearings are the easiest and least expensive to make. They are usually used in conjunction with other types of thrust bearings. They carry 10 to 20% of the overall axial load. Bearing surface sometimes incorporated with oil grooves that help store and distribute oil over the surface. Under such conditions, some metal-to-metal contact arises and hence suitable oil additives and bearing material must be utilized to reduce wear and heat generation.

In¹ investigated various types of thrust bearing and showed that opposed parallel surfaces, under certain conditions of operation, had a load carrying capacity approaching to that of tilting pad bearings of the Michell type and of the same bearing area. In² the effects of varying the oil feed rate on bearing temperature and power loss were discussed. Some observations on the laminar to turbulent transition region were included. In³ a new running torque formula of a tapered roller bearing under axial load was proposed and good agreement with actual bearing torque was confirmed.

In⁴ an operating tilting-pad thrust bearing generated a fore-region which was responsible for maintaining, at the bearing entrance, a pressure which was higher than the ambient pressure. In⁵ the effect of surface roughness on the performance of hydrodynamic slider bearings was studied. The results were obtained for the general lubricant film shape in integral form which were numerically computed for the shapes under consideration. The results were presented both graphically as well as in tabular form. In⁶ discussed a solution of Reynolds equation by using Finite Difference Method (FDM) on the surface of the tilting pad to evaluate the pressure distribution in the lubricant oil film. In⁷ discussed that the occurrence of angular misalignment could considerably change bearing characteristics. Under the constant force preload, the running toques constantly decreased with an increase in angular misalignment. In⁸ conducted an experimental theory of hydrodynamic thrust bearing device and its
application to the study of a tapered-land thrust bearing. Experimental results with respect to the real geometry of the bearings were discussed with both processes being correlated. In discussed experimental and numerical studies of cavitation effects in a tapered land thrust bearing.

Most of the above studies neglected the effect of surface roughness by considering the bearing surface to be smooth. But there are some studies regarding the effect of longitudinal surface roughness on the performance of slider bearings. In observed the shape of lubricant film for the optimal performance of a longitudinally rough slider bearing. It was found that all the three roughness parameters affected the bearing performance significantly. In analysed longitudinally rough slider bearing with magnetic squeeze films. This study established that for an enhanced bearing performance the role standard deviation associated with roughness remain prominent. In described the adiabatic model of a lateral slide bearing with a floating ring bearing. The evaluation criteria of a theoretical model were presented in regard to results from empirical studies.

2. Analysis

The geometrical configuration of the bearing system is presented in Figure 1.

![Figure 1. Configuration of the bearing system.](image)

where

- \( h_1 \) = thickness of fluid film at entry, mm
- \( h_0 \) = thickness of fluid film at exist, mm
- \( U \) = velocity of lower pad in the \( x \)-direction
- \( B \) = bearing breadth

In a narrow width tapered pad bearing the width ‘B’ of bearing in \( Z \)-direction is short. In such bearing there is a considerable flow of fluid in \( Z \)-direction. In narrow width tapered pad bearings, the pressure gradient in \( Z \)-direction \( \frac{\partial \rho}{\partial z} \) is much higher than the pressure gradient in \( X \)-direction \( \frac{\partial \rho}{\partial x} \). Hence in such bearings, the pressure gradient in \( X \)-direction is neglected that is \( \frac{\partial \rho}{\partial x} = 0 \). The tapered pad bearings with width to length ratio (B/L ratio) less than 0.25 are considered as narrow width tapered pad bearings.

Following the method of the thickness \( h(x) \) is considered as

\[
h(x) = \bar{h}(x) + h_s
\]

where \( \bar{h} \) is the mean film thickness and \( h_s \) is the deviation from the mean film thickness characterizing the irregular roughness of the bearing roughness. Here \( h_s \) is assumed to be stochastic in nature and governed by the probability density function \( F(h_s) \), which is defined by the relationship

\[
F(h_s) = \begin{cases} \frac{32}{3\sqrt{b}} \left(1 - \frac{h_s^2}{b^2}\right)^3 & -b \leq h_s \leq b \\ 0 & \text{elsewhere} \end{cases}
\]

The mean \( \alpha \), the standard deviation \( \sigma \) and the parameter \( \varepsilon \) which the measure of symmetry associated with the random variable \( h_s \) are determined by the relations

\[
\alpha = E(h_s)
\]

\[
\sigma^2 = E\left[(h_s - \alpha)^2\right]
\]

and

\[
\varepsilon = E\left[(h_s - \alpha)^3\right]
\]

where \( E \) denotes the expected value defined by

\[
E(R) = \int_{-\infty}^{\infty} Rf(h_s)dh_s
\]

The details regarding the characterization of the roughness aspects can be had from

It is well known that the Reynold’s equation for two dimensional fluid flow is given by

\[
\frac{\partial}{\partial x}\left[h_1 \frac{\partial \rho}{\partial x}\right] + \frac{\partial}{\partial z}\left[h_1 \frac{\partial \rho}{\partial z}\right] = 6\mu U \frac{dh}{dx}
\]
\[ \mu \] being the viscosity of lubricant.

For narrow width tapered pad bearing,
\begin{equation}
\frac{\partial p}{\partial x} \approx 0
\end{equation}
which gives
\begin{equation}
\frac{\partial}{\partial z} \left[ k \frac{\partial p}{\partial z} \right] = 6 \mu U \frac{dh}{dx}
\end{equation}
In view of the stochastic averaging method of 13, 14, this leads to
\begin{equation}
\frac{\partial}{\partial z} \left[ g(h) \frac{\partial p}{\partial z} \right] = 6 \mu U \frac{dh}{dx}
\end{equation}
\[ g(h) = h^3 + 3 \sigma^2 h + 3 \alpha^2 h + 3 h^2 \alpha + 3 \sigma^2 \alpha + \varepsilon + \alpha^3 \]
As the fluid film thickness 'h' ceases to be a function of z, equation (3) transforms to
\begin{equation}
\frac{d^2 p}{dz^2} = \frac{6 \mu U \frac{dh}{dx}}{g(h)}
\end{equation}
Integration of (5) with boundary conditions:
\[ z = \pm B \]
\[ p = 0 \]
\[ z = 0 \]
\[ \frac{dp}{dz} = 0 \]
results in the expression for pressure distribution as
\begin{equation}
p = -\frac{3 \mu U}{(h^3 + 3 \sigma^2 h + 3 \alpha^2 h + 3 h^2 \alpha + 3 \sigma^2 \alpha + \varepsilon + \alpha^3)} \left[ z^2 - \frac{B^2}{4} \right]
\end{equation}
as \[ h = h_i - \theta x \]
Introduction of dimensionless quantities:
\[ \sigma^* = \frac{\sigma}{h} \quad \alpha^* = \frac{\alpha}{h} \quad z^* = \frac{z}{B} \quad \varepsilon^* = \frac{\varepsilon}{h^3} \quad p^* = \frac{ph^3}{3 \mu UB^2} \]
leads to the expression for the pressure distribution in the dimensionless form:
\begin{equation}
p^* = -\frac{\theta}{(1 + 3 \sigma^* + 3 \alpha^* + 3 \sigma^* \varepsilon^* + \alpha^* \varepsilon^*)} \left[ z^* - \frac{1}{4} \right]
\end{equation}
The load carrying capacity of narrow width tapered pad bearing is given by,
\begin{equation}
w = \frac{9}{2} \int L \ p \ dz
\end{equation}
Substituting the value of \( p \) from equation (6) in equation (8), one gets to
\begin{equation}
w = \frac{9}{2} \int L \ p \ dz
\end{equation}
where \( h = h_i/\theta \)
Then the dimensionless load carrying capacity is found to be
\begin{equation}
w = \frac{4 \mu U L^2}{3 \mu UB^2} \left(1 + 3 \sigma^* + 3 \alpha^* + 3 \sigma^* \varepsilon^* + \alpha^* \varepsilon^*\right) \left[ \frac{1}{n^2} \right]
\end{equation}
Now the frictional force on moving plate is given by
\begin{equation}
F = \frac{\theta}{\mu U L} \int L \ p \ dz
\end{equation}
On simplifications this yields
\begin{equation}
F = \frac{\theta}{\mu U L} \int L \ p \ dz
\end{equation}
Lastly,
\begin{equation}
f = \frac{F}{W}
\end{equation}
\[ f = \frac{F}{W} \]
\[ 4 h_i \sigma^* \varepsilon^* n \left(1 + 3 \sigma^* + 3 \alpha^* + 3 \sigma^* \varepsilon^* + \alpha^* \varepsilon^*\right) n \left(1 + 3 \sigma^* + 3 \alpha^* + 3 \sigma^* \varepsilon^* + \alpha^* \varepsilon^*\right) \]
which is the expression of non-dimensional frictional force on moving plate for finite width tapered pad bearings.

3. Result and Discussion

It is easily seen that the non-dimensional pressure distribution is obtained from equation 9, while equation 11 determines the non-dimensional load carrying capacity. Further the non-dimensional frictional force is computed from equation 13. In the absence of roughness this investigation reduces to the discussion of 14.

The variation of non-dimensional load carrying capacity is presented in Figures 2-7. From Figure 2 one can observe that the effect of skewness on load carrying
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capacity with respect to $\sigma^*$ remains nominal when $\sigma^*$ exceed 4. The negatively skewed roughness makes the situation better. From Figures 3-5 we can say that the variance follows the path of skewness so far as the trends of load carrying capacity are concerned. From Figure 4 it is clear that the $\alpha^*$ affects the bearing performance considerably by reducing the load. The friction profile is presented in Figures 8-13. From Figure 8 it is interesting to note that effect of skewness fails to have significant effect on the frictional force with respect to standard deviation while the effect of variance is more sharp. It is observed that either the friction remains nominal up to certain extent or it rises.

Figure 2. Non dimensional load carrying capacity $W$ versus $\sigma^*$ for different values of $\varepsilon^*$ $n = 2$ $\alpha^* = -0.1$ $z^* = 0.2$.

Figure 3. Non dimensional load carrying capacity $W$ versus $\sigma^*$ for different values of $\alpha^*$ $z^* = 0.2$ $n = 2$ $\varepsilon^* = -0.05$.

Figure 4. Non dimensional load carrying capacity $W$ versus $\sigma^*$ for different values of $n$ $\varepsilon^* = -0.05$ $\alpha^* = -0.1$ $z^* = 0.2$.

Figure 5. Non dimensional load carrying capacity $W$ versus $\varepsilon^*$ for different values of $\alpha^*$ $z^* = 0.2$ $n = 2$ $\sigma^* = 0.2$.

Figure 6. Non dimensional load carrying capacity $W$ versus $\varepsilon^*$ for different values of $n$ $\sigma^* = 0.2$ $\alpha^* = -0.1$ $Z^* = 0.2$.

Figure 7. Non dimensional load carrying capacity $W$ versus $\alpha^*$ for different values of $n$ $\varepsilon^* = -0.05$ $Z^* = 0.2$. $\sigma^* = 0.2$.

Figure 8. Non dimensional frictional force $f$ versus $\sigma^*$ for different values of $\varepsilon^*$ $n = 2$ $\alpha^* = -0.1$ $z^* = 0.2$. $\sigma^* = 0.2$. 
4. Conclusion

In general the effect of transverse roughness is adverse. Therefore, the investigation suggests that the roughness parameter must be discussed while designing the bearing system. This is necessary from bearing’s life period point of view. Further, the positive effect of negatively skewed roughness may be channelized to improve the bearing performance when variance (-ve) occurs.

5. Nomenclature

\( \alpha \): Variance  
\( \sigma \): Standard deviation  
\( \varepsilon \): Skewness  
\( \alpha^* \): Dimensionless variance  
\( \sigma^* \): Dimensionless standard deviation  
\( \varepsilon^* \): Dimensionless skewness  
\( \theta \): Angle of inclination of the tapered pad  
\( p \): Lubricant Pressure (N/mm\(^2\))  
\( P \): Dimensionless Pressure  
\( w \): Load carrying capacity (N)  
\( W \): Dimensionless load carrying capacity  
\( F \): Dimensionless friction force  
\( f \): Coefficient of friction  
\( e \): Eccentricity (mm)

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