Student difficulties in word problems of derivatives: A multisemiotic perspective

N Fatmanissa¹, Kusnandi² and D Usdiyana²

¹Departemen Pendidikan Matematika, Sekolah Pascasarjana, Universitas Pendidikan Indonesia, Jl. Dr. Setia Budhi No. 229, Bandung 40154, Indonesia
²Departemen Pendidikan Matematika, Universitas Pendidikan Indonesia, Jl. Dr. Setia Budhi No. 229, Bandung 40154, Indonesia

"namirahf@student.upi.edu"

Abstract. This study aims to describe findings on students’ difficulties in solving derivatives word problems from the perspective of multiple semiotic systems. A test in the form of word problems was constructed, validated to mathematics expert, and then given to 69 students of grade 11. Their works were analyzed from the perspective of multiple semiotic systems by classifying each obtained case into six sub-features, i.e. transforming symbol into visual representation, visual representation into symbol, language into symbol, symbol into language, language into visual representation, and visual representation into language. It was found that transforming language into visual representation was the most found sub-feature of difficulties. This study concludes that the difficulties due to multiple semiotic systems are real. Students faced these difficulties during word problem solving of derivatives in diverse cases involving transforming meaning from one system to another.

1. Introduction
The topic of derivative is one of calculus topics (besides limit and integration) given to senior high school students in Indonesia. The topic covers the definition of derivative, rules of derivatives, and application of derivatives in real life. Derivative is given to grade 11 and is to be taught with assumption that students have obtained several prerequisite materials such as algebra, function, and limit. As the content standard of high school education (launched by Indonesia Ministry of Education and Culture) mentions, students are required to master the topic and should be able to apply it to real life problems [1].

The representation of application of derivatives is word problem, a problem that is situated in the context of daily life [2]. Word problems had been considered difficult as it requires not only the understanding of the derivative topic, but also skill to make meaning from the text of the problem and to transform it to mathematical or visual representation [3]. The difficulties related to word problems of derivatives had been studied both in and outside Indonesia. In a study conducted in Indonesia, students mostly found it difficult to determine what was known in the problem or to write the conclusion of the numerical solution being obtained [4]. This fact is supported by the study of students’ perspectives upon derivatives application [5]. The study found that the difficulties were in transforming the context written in the problem into mathematical expression of appropriate function.
It cannot be denied that the existence of context in word problems of derivatives should be paid more attention. The context in the word problems, written in daily language, demands students to transform it to either mathematical representation (written with mathematical symbol) or to visual representation (drawn or illustrated into pictures).

The inter-related systems of language, symbol, and visual representation, i.e. multiple semiotic (multisemiotic) systems, has been the language character of word problems solving [6]. The work of O’Halloran in describing multiple semiotic systems as the character of mathematical discourse uses the fusion of mathematics and linguistics [6, 7]. It is stated that there are three systems in mathematical discourse (as it is in word problem) which are language, symbol, and visual representation. Those three systems become the main features constructing the information and the understanding of those affects the way students transform information from one system to another.

The inter-related systems require students to inter-play appropriate transformations in order to construct accurate solution the problem. The transformation types (or sub-features) among the systems, given in Table 1, are used as codes to analyze the difficulties of solving word problems. By using this character as the framework of analysis, it is hoped that the difficulties of solving word problems of derivatives can be more systematically learnt. Thus, the aim of this study is to describe students’ difficulties from the perspectives of multiple semiotic systems.

| Code   | Sub-features                        | Code   | Sub-features                        |
|--------|-------------------------------------|--------|-------------------------------------|
| MSS1   | transforming symbol into visual     | MSS4   | transforming symbol into language   |
| MSS2   | transforming visual representation   | MSS5   | transforming language into visual    |
| MSS3   | transforming language into symbol   | MSS6   | transforming visual representation   |

2. Methods
A test consisted of six essay items was given to 69 students of 11th grade. The three items were in the form of word problems with the topic of conic sections. The test items were constructed with the aim to check whether the sub-features of difficulties appeared. The test items readability and clarity were consulted to two mathematics education experts. The test was written in Indonesian language; its English translation is given in Table 2. The test topic and plan has been announced to students a day before the test. As the test tried to picture students difficulties related to multiple semiotic systems in it, any tool to help those calculating numbers was not forbidden. The test was given and invigilated by the researcher. Student’s work was analyzed by first giving code(s) of its appropriate sub-feature of multiple semiotic systems. The codes were “MSS1” (for transforming symbol into visual representation), “MSS2” (for transforming visual representation into symbol), and so on (as displayed in Table 1). The codes were recorded and students’ works were classified based on it. After being classified, distinct cases of difficulty in each code were further explored through interview. Eight students were interviewed to see their cognitive processes deeper.
Table 2. Test items.

| No | Word problem                                                                 | No | Word problem                                                                 |
|----|-------------------------------------------------------------------------------|----|-------------------------------------------------------------------------------|
| A  | The cost to make $x$ unit of good is $\left( \frac{1}{2}x^2 - 10x + 10 \right)$ rupiahs.  
The selling price of each unit is $(20 - \frac{1}{2}x)$. Draw the graph representing the selling profit. Determine the number of unit that should be sold to get maximum profit. | E  | A piece of rectangular paper is 8 cm long and 3 cm wide. Its corners are cut in the form of similar square. After being cut, the remaining paper is flipped and formed into a box without top. Give an illustration of the paper explained before. Determine the maximum volume of the box. |
| B  | A wire is 10 meter long and will be made into three rectangles as given in the picture below (Note: not to scale). Determine the maximum area of the bordered region. | F  | Below is the graph of the function of displacement of a moving object with time $x$ in second, $x \geq 0$. Explain the velocity of the object 2 seconds before, exactly at, and 2 seconds after its maximum displacement. |
| C  | There are 120 marbles in two boxes. If the number of marbles in one box is doubled and the result is multiplied by the number of marbles in another box, then the final result is maximum.  
Determine the number of marbles in each box. |    |                                                                              |
| D  | A bullet is shot upward vertically. The relation between its height $(h)$ in meter and time $(t)$ in second with $0 \leq t \leq 60$ is defined by $h(t) = 300t - 5t^2$. Explain the bullet’s height 5 seconds before, exactly at, and 5 seconds after its maximum height. |    |                                                                              |

3. Results and discussion

The discussion in this section will be about the general findings first, then followed by the discussion of difficulties found in each sub-feature. There are 69 students participated in the test. Table 3 depicts the numerical data on students’ difficulties for each sub-feature. Column 2 and 5 show the number of students who showed respective sub-feature while column 3 and 6 show the percentage of difficulties among all difficulties found. The case was counted if it showed mistakes during the transformation process. All sub-features of multiple semiotic systems were found from students’ work. The most found cases were the difficulties in transforming language into visual representation.

Table 3. Number of difficulties found.

| Sub-feature of difficulties               | n  | %     | Sub-feature of difficulties               | n  | %     |
|-------------------------------------------|----|-------|-------------------------------------------|----|-------|
| transforming symbol into visual representation | 12 | 7.5   | transforming symbol into language         | 18 | 11.3  |
| transforming visual representation into symbol | 12 | 7.5   | transforming language into visual representation | 56 | 35.2  |
| transforming language into symbol         | 49 | 30.8  | transforming visual representation into language | 12 | 7.5   |

The difficulty in transforming symbol into visual representation was found in student’s work on item A. This difficulty refers to the difficulty of transforming information in the form of mathematical symbol into visual representation needed to solve the problem. Item A required student to transform the given information in the form of mathematics expressions into a function of profit. Most students could not give the correct function of profit, thus could not draw the correct graph.

The failure in constructing the correct function was caused by the failure of understanding the meaning behind the expressions given. For example, one student thought that the sentence “The cost to make $x$ unit of good is $\left( \frac{1}{2}x^2 - 10x + 10 \right)$ rupiahs” was given to make him calculate the cost of 1 unit of good. As shown in Figure 1, he substituted $x = 1$ into each expression and treated the result as coordinates of points to be put in the graph. He failed to recognize that the profit function being asked
was obtained by subtracting the cost from the total selling price. However, from the example, it also can be seen that the student was not able to correctly put the points in the graph, resulting in incorrect graph as well. In this case, interpreting the symbol used in “The cost to make \( x \) unit of good is \( \frac{1}{2}x^2 - 10x + 10 \) rupiahs” and “The selling price of each unit is \( 20 - \frac{1}{2}x \)” plays important role in transforming them into correct mathematical representations.

Students’ difficulties in doing the opposite transformation, i.e. from visual representation to symbol, were also found. This difficulty was shown in students’ work on item B. This item demands students to firstly determine the function representing area of the figure given. In order to do that, students will need to transform the information of side’s length into algebraic expression of perimeter.

![Figure 1](image1.png)

**Figure 1.** Example of difficulty in transforming (a) symbol into visual representation; (b), (c) visual representation into symbol.

Some students simply used arithmetic operation from the given wire length of 10 meters (Figure 1b), while some others failed to make use of wire length information into the equation of area (Figure 1c). Student in Figure 1b presumed that in order to get maximum area, he needed to divide the wire length equally, and then predicted particular side’s length to satisfy the given perimeter of each rectangle. By
paying attention to the picture given, as visual representation, student should had been aware that the wire length should not be divided equally as there are common sides constructing different rectangles. On the other hand, student in Figure 1c could correctly write that the area of the rectangles equals \(3(a \times b)\). However, she failed to incorporate the fact that the sides of three rectangles, represented by \(a\) and \(b\), made up to 10 meters.

Interesting findings of students’ difficulties were also found during process of transforming language into symbol and vice versa. In transforming language into symbol, students’ difficulties lie in finding the correct mathematical model for the given written information. For example, in item C, students were asked to find the number of marbles in each box so that the result of the process being demanded maximum. The common process of students in answering this problem was determining the number of each box right after knowing that their sum is 120. As shown in Figure 2a, although student correctly write an equation \(x + y = 120\), she decided to simply divided 120 by 2 and concluded that the number of marble in each box was 60.

![Figure 2](image_url)

**Figure 2.** Example of difficulty in transforming (a) language into symbol; (b) symbol into language.

After doing so, student got puzzled by the meaning of the word “dilipatgandakan” (i.e. Indonesian word originally printed in the test sheet which means “being doubled”) as shown by her dual processes. The first process showed transformation of the word into “being doubled”, written as \(2x.y = z\); while the second process transform it into “being squared”, written as \(x^2.y = z\). Student’s confusion was evidenced further from the interview result below.

“For example “dilipatgandakan” means being doubled or squared? It is like there is another thing to consider, each person has different perspective, right? So, I gave two options (ways) here.”
Student’s opinion showed that she found it difficult to determine the exact model for the sentence and believed that this question could be answered differently based on different perspectives in understanding the word “dilipatgandakan”. This example shows a student who was capable in doing the mathematical process, yet because she vaguely grasped the meaning of the text, the problem solution became partially correct.

In doing the opposite process, i.e. transforming symbol into language, student commonly found it difficult to explain the symbol into daily language. In item D, students were expected to explain the height of a bullet represented by a given function of time. The function, which is quadratic, was actually considered routine function for grade 11 students. However, while being asked to explain it in three exact times (i.e. 5 seconds before, exactly at, and 5 seconds after its maximum height), students surprisingly struggled. Representative example of this case was given in Figure 2b where student substituted \( t = -5, t = 0, \) and \( t = 5 \) into the function to answer the question. Interview excerpt showed the reason behind this choice.

“This problem stated (started reading the problem) “Explain the bullet’s height 5 seconds before, exactly at, and 5 seconds after its maximum height”. So, I think time of 5 seconds before being launched must be negative. And time of 5 seconds after must be more than 0.”

The excerpt evidenced the stronger attention on symbol (i.e. number “5”) than on the phrase “its maximum height”, as student kept replacing it with “being launched” when being asked to explained his process and substituted \( t = 5 \) regardless what “5” means in the text. It can be inferred that the difficulty in transforming the symbol into the correct language transformation lies in whether student can incorporate information either language or symbol equally.

Different visual representations were created by students when solving item E, displayed difficulties in transforming written information into representative visual representation. Item E required students to draw the picture of the paper in order to determine its maximum volume. The description of the paper was given in the first two sentences of the problem. In fact, various visual misrepresentations of this description were found (representative examples are given in Figure ). Some drew multiple cut sides in each corner (Figure a); while some did get confused by what it was meant by “corners” (Figure b). The process of understanding the sentence “Its corners are cut in the form of similar square” was indeed the main difficulty in transforming it to the correct visual representation.

![Figure 3](image.jpg)

(a) Figure 3. Example of difficulties in transforming (a), (b) language into visual representation; (c) visual representation to language.
The difficulty in transforming visual representation into language was found in students’ work on item F. In item F, students were asked to explain the velocity of the object in the given times. Similar to item D, in order to solve this problem, students should know the exact time when the object reached its maximum displacement. However, in item F, students were given a graph representing the displacement of the object. The difficulties found in this item was seen when students ignored or did not make use of the graph to answer the question. A representative example in Figure c shows how student did not make use of the graph to find the maximum displacement. If students made use of the graph, it was clearly shown that the maximum displacement happened when $t = 3$, thus they could have used this fact to get to the solution. In the figure, it is shown that the student paid attention to the phrase “2 seconds” without paying attention to the graph (which is actually similar case of difficulty in item D discussed before).

Student difficulties in doing transformation from one system to another are found even in high school where students are supposedly more advanced in terms of language understanding. Transforming information, either in the form of written language or visual representation, into its representative mathematical symbols is shown to be difficult for students. Students habitually prefer to find the numerical solution directly from the first information they read rather than to construct its algebraic expression. This is also evidenced by their strong attention on numbers and symbols compared to texts and pictures. These cases are supported by the study of Wijaya, et al which found that students tend to directly jump into calculation, without carefully read or observe what is given in the problem [8].

It can also be inferred that visual representation and written texts are less considered by students. Regarding visual representation, it may be said that some students unconsciously treat pictures excluded from the problem (although it is printed alongside the problem). This case is explained by the study of Eisenberg where students are more comfortable in solving word problems through symbols given in the text, than through making use of the picture provided [9]. Another explanation is mentioned by Chan,
who said that as picture (i.e. visual representation) gives lower linguistic demand, it is considered to be easy to understand, thus lures students to ignore it [10]. Those facts lead to the urgency of giving more chance for students to exercise their transformations skill and pay equal attention to each system.

4. Conclusions
The difficulties in solving derivatives word problems caused by multiple semiotic systems as language feature of mathematics are faced by students even in senior high school level. The difficulties found were in transforming the mathematical systems involved in the problem. While student has difficulty in understanding one system, he/she will also find it difficult to transform it to another system. However, it does not have to be that way. Sometimes, student has understood one system (i.e. knowing its meaning in that system only), but does not know how to transform it to another system. In this process of going back and forth from one system to another lies the challenge given to student in order to solve word problem correctly.

As the difficulties lie in how students manage to understand one system and how to transform it to another system, it is suggested that teacher pay more attention in how information makes meaning in the three systems. This emphasis in meaning-making can be done in word problem solving by routinely discuss what the information is, what the meaning is, and what other forms (i.e. system) that are equivalent to that meaning. Making students exercise transforming information from one system to another will help them to sharpen both their skill and understanding in order to solve word problems.

It is not claimed that all possible difficulties are found and discussed in this study due to its specific mathematics topic and particular word problems given to students. However, it is hope that this study gives more insights on difficulties and their possible causes from the perspective of language. In the future, the investigation of difficulties may be expanded by investigating a particular strategy to strengthen students’ skill in transforming information to solve word problems.

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References
[1] Regulation of Ministry of Education and Culture Regulation No. 21 Year 2016 about Content Standard of Elementary and Middle Education
[2] Verschaffel L van Dooren W Greer B and Mukhopadhyay S 2010 "Die Rekonzeptualisierung von Textaufgaben als Übungen in mathematischer Modellierung," J. fur Math. 31 1 p 9–29
[3] Ryan J and Williams J 2007 Children ’ s Mathematics: Learning from Errors and Misconceptions (Berskhire: McGraw Hill)
[4] Wahyuningtyas W and Amin S M 2014 "Peningkatan Kemampuan Pemecahan Masalah Matematika Siswa pada Materi Turunan Fungsi Melalui Diskusi Kelompok," J. Ilm. Pendidik. 3 1 p 1–8
[5] Klymchuk S Zverkova T Gruenwald N and Sauerbier G 2010 "University students’ difficulties in solving application problems in calculus: Student perspectives," Math. Educ. Res. J. 22 2 p 81–91
[6] O’Halloran K L 2005 Mathematical Discourse: Language, Symbolism, and Visual Images (London: Continuum)
[7] O’Halloran K L 2015 "The language of learning mathematics: A multimodal perspective," J. Math. Behav. 40 p 63–74
[8] Wijaya A Heuvel-panhuizen M Van Den Doorman M and Robitzch A 2014 "Difficulties in solving
context-based PISA mathematics tasks: An analysis of students' errors," Math. Enthus. 11 3 p. 555–584
[9] Eisenberg T 2002 Advanced Mathematical Thinking, D. Tall, Ed. (Netherlands: Kluwer Academic Publishers) p 297
[10] Chan S 2015 "Linguistic challenges in the mathematical register for EFL learners: linguistic and multimodal strategies to help learners tackle mathematics word problems," Int. J. Biling. Educ. Biling. 18 3 p 306–318