Menu-size Complexity and Revenue Continuity of Buy-many Mechanisms

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We study the multi-item mechanism design problem where a monopolist sells $n$ heterogeneous items to a single buyer. In recent work, Chawla et al. [2] advocated studying revenue maximization of multi-item mechanisms under the so-called “buy-many” constraint. Informally, a mechanism is buy-many if the buyer is allowed to participate in the mechanism any number of times. For example, a buyer interested in purchasing a subset of items may purchase the components of this subset individually. Viewing the mechanism as a function that assigns prices to allocations, the buy-many constraint is essentially equivalent to a subadditivity constraint over the prices.

The buy-many constraint is a natural property that most real-world mechanisms satisfy. All of the simple classes of mechanisms studied in the literature such as item pricing, grand bundle pricing, two part tariffs, etc. also satisfy this property. As such, buy-many mechanisms are a worthy object of study. Chawla et al. asked whether buy-many mechanisms exhibit structural properties that arbitrary mechanisms do not. In this paper we study two such properties: menu-size complexity and revenue continuity. We discuss these two properties, their significance, and our results.

Menu-size Complexity. The menu size of a mechanism, defined as the number of different outcomes the seller offers to the buyer, was first introduced by Hart and Nisan [4]. It has been studied extensively in the literature as a measure of complexity for single-buyer mechanisms (see, e.g., [1, 3, 5]). For the unconstrained setting, it is known that even for selling two items to an additive buyer, the menu-size complexity of the optimal buy-one mechanism can be infinite [4]. The same is true for any mechanism that achieves a bounded approximation to the optimal revenue. Positive results for getting near-optimal revenue using mechanisms with finite menu-size complexity have only been established for buyers with subadditive valuation functions and independent values for each item [1, 5].

For buy-many mechanisms, we define their menu-size complexity to be similar to the “additive menu size” introduced by Hart and Nisan [4], which corresponds to the number of “basic” options the buy-many mechanism offers. Can we approximate the optimal revenue obtained by buy-many mechanisms with bounded menu-size? We give an affirmative answer to this question.
Theorem 1. For any distribution $\mathcal{D}$ over arbitrary valuation functions, there exists a buy-many mechanism $M$ generated by $(n/\epsilon)^{O(n)}$ menu entries, such that $\text{Rev}_\mathcal{D}(M) \geq (1 - \epsilon) \text{BuyManyRev}(\mathcal{D})$.

The theorem above implies that a mechanism with finite menu-size complexity can get near-optimal revenue obtained by any buy-many mechanism. Such doubly-exponential dependency on $n$ is tight since no mechanism with sub-doubly-exponential description complexity can get $o(\log n)$-approximation in revenue.

Theorem 2. There exists a distribution over XOS valuation functions for which no mechanism with description complexity at most $2^{o(n^{1/4})}$ can obtain a $o(\log n)$ fraction of the optimal revenue obtained by buy-many mechanisms.

Revenue Continuity. We are interested in understanding the extent to which the optimal revenue changes if the value distribution is perturbed slightly. Formally, let $\mathcal{D}$ be a distribution over valuation functions, and let $\mathcal{D}'$ be another distribution obtained by taking each valuation function in the support of $\mathcal{D}'$ and changing each component of this function multiplicatively by some factor in $[1 - \epsilon, 1 + \epsilon]$ for some small $\epsilon > 0$. Can we then show that $\text{Rev}_{\mathcal{D}'} \geq (1 - \epsilon') \text{Rev}_\mathcal{D}$ where $\epsilon'$ goes to 0 as $\epsilon$ goes to 0? We call such a property revenue continuity.

While revenue continuity is inherently interesting, it also has practical implications. Continuity implies that revenue estimates established on the basis of market analysis will be robust to errors in estimating demand. Furthermore, the accuracy of these estimates will improve directly with a reduction in measurement error. From an algorithmic standpoint, revenue continuity allows discretizing the values to their most significant digits through a sufficiently fine multiplicative grid. This is possible to do without a significant drop in revenue.

A surprising result based on an example by Psomas et al. [6] shows that optimal revenue obtained by buy-one mechanisms does not exhibit revenue continuity, even for additive buyers. It is even possible that the optimal revenue is infinite before the perturbation, but finite afterward. However, in sharp contrast to the buy-one setting, such revenue discontinuity does not happen to buy-many mechanisms. Denote by $\text{BuyManyRev}(\mathcal{D})$ the optimal revenue obtained by buy-many mechanisms for a buyer with value distribution $\mathcal{D}$. We state the result in the following theorem.

Theorem 3. Let $\mathcal{D}$ be a distribution over arbitrary valuation functions, and $\mathcal{D}'$ a $(1 \pm \epsilon)$-multiplicative-perturbation of $\mathcal{D}$. Then $\text{BuyManyRev}(\mathcal{D}') \geq (1 - \text{poly}(n, \epsilon)) \text{BuyManyRev}(\mathcal{D})$.

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