Correlations among symmetry energy elements in Skyrme models

C. Mondal,1,2 B. K. Agrawal,1,3 J. N. De,1 and S. K. Samaddar1

1Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
2Departament de Física Quàntica i Astrofísica and Institut de Ciències del Cosmos (ICCUB), Facultat de Física, Universitat de Barcelona, Martí i Franquès 1, E-08028 Barcelona, Spain
3Homi Bhabha National Institute, Anushakti Nagar, Mumbai 400094, India.

Motivated by the interrelationships found between the various symmetry energy elements of the energy density functionals (EDF) based on the Skyrme forces, possible correlations among them are explored. A total of 237 Skyrme EDFs are used for this purpose. As some of these EDFs yield values of a few nuclear observables far off from the present acceptable range, studies are done also with a subset of 162 EDFs that comply with a conservative set of constraints on the values of nuclear matter incompressibility coefficient, effective mass of the nucleon and the isovector splitting of effective nucleon masses to see the enhancement of the correlation strength, if any. The curvature parameter $K_{\text{sym}}^{0}$ and the skewness parameter $Q_{\text{sym}}^{0}$ of the symmetry energy are found to be very well correlated with the linear combination of the symmetry energy coefficient and its density derivative $L_{0}$. The isovector splitting of the effective nucleon mass, however, displays a somewhat meaningful correlation with a linear combination of the symmetry energy, its slope and its curvature parameter.

I. INTRODUCTION

Lack of accurate knowledge of the density dependence of nuclear symmetry energy hinders the understanding of the neutron-rich nuclei near drip-line [1]. This knowledge is also of seminal importance in astrophysical context. The interplay of gravitation with the pressure of nuclear matter (related to the density derivative of symmetry energy) is a key determining factor in the radii of neutron stars [2]. The dynamical evolution of the core-collapse of a massive star and the associated explosive nucleosynthesis depend sensitively on the density content of the symmetry energy [3, 5]. Density dependence of symmetry energy controls the nature and stability of different phases within a neutron star, its critical composition, thickness, frequencies of crustal vibration [3, 6] and also determines the feasibility of direct Urca cooling processes within its interior [3, 6, 8]. The density dependence of symmetry energy $C_{2}(\rho)$ around the saturation density can be well expressed in terms of its slope $L_{0}$, curvature $K_{\text{sym}}^{0}$ and skewness $Q_{\text{sym}}^{0}$ evaluated at the saturation density ($\rho_{0} \approx 0.16$ fm$^{-3}$). These quantities have received a great deal of attention in recent times [3, 5]. Another quantity of topical interest is the isovector splitting of effective nucleon masses, $\Delta m_{0}^{*}$ (measure of the difference between neutron and proton effective masses) for asymmetric nuclear matter defined at $\rho_{0}$. The value of $\Delta m_{0}^{*}$ is very uncertain [10, 11, 13, 14, 29, 30]. There is even not much clarity about its sign. Whereas microscopic ab-initio models consistently predict $\Delta m_{0} > 0$ in neutron-rich matter [20, 22], parameters of recently suggested 'best-fit' Skyrme energy density functionals (EDF) [23] obtained from constraints provided by properties of nuclear matter, of doubly magic nuclei and microscopic calculations of low-density neutron matter are found to yield negative values for the isospin-splitted nucleon effective mass.

The value of $C_{2}^{0}$ ($= C_{2}(\rho_{0})$) is known to lie in the range $\sim 32 \pm 2$ MeV [11, 24, 28]. Extensive efforts have also been made in the last decade or so to constrain the value of $L_{0}$ [4, 11, 13, 14, 29, 30]. The uncertainties in the values increase further for higher order density derivatives of symmetry energy i.e. $K_{\text{sym}}^{0}$ or $Q_{\text{sym}}^{0}$. The value of $K_{\text{sym}}^{0}$ ranges from $\sim -700$ MeV to 400 MeV and $Q_{\text{sym}}^{0}$ from $\sim -800$ MeV to 1500 MeV [31, 32] across a few hundred models of mean-field energy density functionals (EDF). There is also enormous diversity in the predicted values of $\Delta m_{0}^{*}$ [11, 22, 33, 35]. Comprehensive understanding of the isovector part of nuclear interaction is thus hindered. The uncertainties in these nuclear matter constants can, however, be reduced if one can express them in terms of quantities that are known in better constraints. Searching for correlated structures among different symmetry energy elements like $C_{2}^{0}$, $L_{0}$, $K_{\text{sym}}^{0}$, $Q_{\text{sym}}^{0}$ and $\Delta m_{0}^{*}$ thus seems highly desirable. The values of $C_{2}^{0}$ and $L_{0}$ being relatively better established, in recent years, attempts are made to look for the correlation between $K_{\text{sym}}^{0}$ and $L_{0}$ [39, 42]. Apparently, the correlation shows some degree of model dependence. In a nearly model independent framework it was, however, analytically shown that $K_{\text{sym}}^{0}$ is very neatly tied to $(3C_{2}^{0} - L_{0})$ [38]. A strong correlation among them was found using a total of 500 EDFs based on Skyrme functionals, EDFs based on realistic interactions and relativistic mean field (RMF) models. The correlation of $Q_{\text{sym}}^{0}$ with $(3C_{2}^{0} - L_{0})$ was, however, found to be poor.

The so-found correlation or its absence calls for the need to bring into the focus the analytical relationship among the different symmetry elements in the EDFs so used. For the EDFs based on realistic interactions...
or those based on RMF, finding analytical relationship between different symmetry elements is not easy, the structure of the Skyrme EDF, however, makes it more amenable to the aim. We try to find that out in this paper. Once that is done, we explore the solidity of the correlation between the symmetry energy coefficient and its higher order derivatives and examine in what context the correlation is better established. Towards that purpose, initially a total of 237 Skyrme EDFs compiled by Dutra et al. [31] are used, this is later followed by a restricted set selected out of them that has compliance with some conservative constraints on the Skyrme EDFs to see how the nature of the manifested correlation is affected.

The paper is organized as follows. We present the analytical relations for various symmetry energy parameters obtained within the Skyrme formalism in Sec. II. The results for correlations among various symmetry energy elements are discussed in Sec. III. Conclusions are drawn in Sec. IV.

II. FORMALISM

The energy per particle \( e(\rho, \delta) \) of asymmetric nuclear matter (ANM) with density \( \rho \) and isospin asymmetry \( \delta = (\rho_+ - \rho_-)/\rho \) is given by,

\[
e(\rho, \delta) \simeq e(\rho, \delta = 0) + C_2(\rho)\delta^2 ,
\]

where \( e(\rho, \delta = 0) \) is the energy per particle for symmetric nuclear matter (SNM) and \( C_2(\rho) \) is the symmetry energy defined as,

\[
C_2(\rho) = \frac{1}{2} \left[ \frac{\partial^2 e(\rho, \delta)}{\partial \delta^2} \right]_{\delta = 0} .
\]

Energy per particle for SNM has a minimum at the saturation density \( \rho_0 \) around which it can be expanded as,

\[
e(\rho, 0) \simeq e_0 + \frac{1}{2}K_0\epsilon^2 + \frac{1}{6}Q_0\epsilon^3 ,
\]

where \( \epsilon = \frac{\rho - \rho_0}{\rho_0} \) and \( e_0 \) the energy per particle of SNM at \( \rho_0 \). The incompressibility parameter \( K_0 \) and stiffness parameter \( Q_0 \) are defined at \( \rho_0 \) as,

\[
K_0 = \left. 9\rho^2\frac{\partial^2 e(\rho, 0)}{\partial \rho^2} \right|_{\rho_0} ,
\]

\[
Q_0 = \left. 27\rho^3\frac{\partial^3 e(\rho, 0)}{\partial \rho^3} \right|_{\rho_0} .
\]

Similarly, the symmetry energy coefficient \( C_2(\rho) \) can be expanded around the saturation density \( \rho_0 \) in terms of different symmetry energy elements as,

\[
C_2(\rho) \simeq C_2^0 + L_0\epsilon + \frac{1}{2}K_{sym}^0\epsilon^2 + \frac{1}{6}Q_{sym}^0\epsilon^3 ,
\]

where the symmetry energy parameters \( L_0, K_{sym}^0 \) and \( Q_{sym}^0 \) are related to different density derivatives of \( C_2(\rho) \) as,

\[
L_0 = 3\rho \left. \frac{\partial C_2(\rho)}{\partial \rho} \right|_{\rho_0} ,
\]

\[
K_{sym}^0 = 9\rho^2 \left. \frac{\partial^2 C_2(\rho)}{\partial \rho^2} \right|_{\rho_0} ,
\]

\[
Q_{sym}^0 = 27\rho^3 \left. \frac{\partial^3 C_2(\rho)}{\partial \rho^3} \right|_{\rho_0} .
\]

In the standard Skyrme parametrization one can write the expression for energy per particle of asymmetric nuclear matter of density \( \rho \) and asymmetry \( \delta \) as [13],

\[
e(\rho, \delta) = \frac{3\hbar^2}{5 \cdot 2m} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{2/3} F_{5/3} + \frac{1}{8}t_0\rho[2(x_0 + 2) - (2x_0 + 1)F_2]
\]

\[
+ \frac{1}{48}t_3\rho^{\alpha^0+1}[2(x_3 + 2) - (2x_3 + 1)F_2]
\]

\[
+ \frac{3}{40} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{\alpha^0+1/2} \left\{ [t_1(x_1 + 2) + t_2(x_2 + 2)]F_{5/3}
\]

\[
+ \frac{1}{2}[t_2(2x_2 + 1) - t_1(2x_1 + 1)]F_{8/3} \right\} ,
\]

where \( F_l(\delta) = \frac{1}{2} \left[ (1 + \delta)^l + (1 - \delta)^l \right] \). All the parameters \( t_i \), \( x_i \), etc. can be expressed in terms of nuclear matter properties. Doing that one observes that the parameters \( t_0, t_3, \alpha \) are completely determined by the bulk properties of SNM. On the other hand, the other parameters \( x_0, x_1, x_2, x_3, t_1, t_2 \) are connected to isovector elements of asymmetric nuclear matter [14].

Following the expression for \( C_2(\rho) \) in Eq. (2) and the definitions of different symmetry energy elements in Eq. (4) one can write the expressions for them in the Skyrme formalism as (see Appendix A),

\[
C_2^0 = \frac{1}{3}E_F^0 - \frac{1}{8}t_0(2x_0 + 1)\rho_0
\]

\[
- \frac{1}{24} \left( \frac{3\pi^2}{2} \right)^{2/3} \left( 3t_1x_1 - t_2(4 + 5x_2) \right)\rho_0^{5/3}
\]

\[
- \frac{1}{48}t_3(2x_3 + 1)\rho_0^{\alpha^0+1} ,
\]

\[
L_0 = \frac{2}{3}E_F^0 - \frac{3}{8}t_0(2x_0 + 1)\rho_0
\]

\[
- \frac{5}{24} \left( \frac{3\pi^2}{2} \right)^{2/3} \left( 3t_1x_1 - t_2(4 + 5x_2) \right)\rho_0^{5/3}
\]

\[
- \frac{1}{16}(\alpha + 1)t_3(2x_3 + 1)\rho_0^{\alpha^0+1} ,
\]

\[
K_{sym}^0 = -5(3C_2^0 - L_0) + E_F^0
\]

\[
+ \frac{1}{16}(2 - 3\alpha)t_3(2x_3 + 1)\rho_0^{\alpha^0+1} ,
\]
\[ Q_{sym}^0 = (3\alpha + 2)K_{sym}^0 + 15(\alpha + 1)(3C_2^0 - L_0) - (3\alpha + 1)E_F^0. \] (11)

Here, \( E_F^0 \) is the Fermi energy of SNM at \( \rho_0 \) given by,
\[
E_F^0 = \frac{h^2}{2m} \left( \frac{3\alpha^2}{2} \right)^{2/3} \rho_0^{2/3}.
\]

The effective mass \( m^*_q \) of a nucleon \([q=n/p \ (neutron/proton)]\), at density \( \rho \) in the Skyrme formalism is given from the relation \[45\]
\[
\frac{h^2}{2m^*_q} = \frac{h^2}{2m} + \frac{1}{8}[t_1(2 + x_1) + t_2(2 + x_2)]\rho_0 - \frac{1}{8}[t_1(1 + 2x_1) - t_2(1 + 2x_2)]\rho_q.
\] (12)

The isospin-splitted effective nucleon mass is defined as the difference between the neutron and proton effective masses, at \( \rho_0 \) it is
\[
\Delta m^*_\alpha = \left[ \frac{m^*_n - m^*_p}{m} \right]_{\rho_0} / \delta.
\] (13)

In Skyrme formalism it is written in terms of the symmetry elements \( C_2, L_0 \) etc. as
\[
\Delta m^*_\alpha = \left( K_{sym}^0 + 3(1 + \alpha)(3C_2^0 - L_0) + (1 - 3\alpha)E_F^0 \right)
+ \frac{2}{3}(3\alpha - 2)\frac{m}{m_0}E_F^0 \left/ \left[ (3\alpha - 2)E_F^0 \left( \frac{m}{m_0} \right)^2 \right] \right.. \] (14)

Some details for the derivation of Eqs. \[10\]-\[13\] are given in the Appendix A.

III. RESULTS AND DISCUSSIONS

Using relations among different thermodynamic state functions, without any specific form of the nuclear interaction but with only viable approximations on its nature, in a recent work \[38\] it was shown that a nearly universal correlation exists between \( K_{sym}^0 \) and \( (3C_2^0 - L_0) \). Imposing a general constraint that the neutron energy per particle should be zero at zero density of neutron matter, a plausible explanation of such a correlation was recently given \[10\]. The structure of the Eqs. \[10\]-\[14\] suggests that such a correlated structure among the different symmetry energy elements might also exist in the Skyrme EDF framework. With this in mind, we have performed an analysis using all the Skyrme EDFs compiled by Dutra et. al. \[31\] except the ones namely ZR3a, ZR3b and ZR3c \[47\], where the symmetry energy \( C_2^0 \) is negative. We present the results obtained using all 237 Skyrme EDFs, referred to as ‘ALL’ hereafter. Some of these EDFs, however, yield values of nuclear constants like the nuclear incompressibility \( K_0 \) and the nucleon effective mass \( m^*_0 \) of SNM beyond present acceptable range. Estimates of \( K_0 \) obtained from analyzes related to isoscalar giant monopole resonances (ISGMR)

\[48\-52\] is now well constrained to \( K_0 = 230 \pm 30 \text{ MeV} \). The effective mass \( m^*_0 \) varies between \( \sim 0.7m \) \[47\-50\] to \( \sim 1.1m \) \[57\-59\]. Experimental and theoretical studies of isoscalar giant quadrupole resonance (ISGQR) \[18\-60\-62\] suggest a value of \( m^*_0/m = 0.90 \), but more experimental data may be needed for a better quantification. We choose to constrain it at \( m^*_0/m = 0.85 \pm 0.15 \). Imposing a further constraint on the isovector splitting of effective mass \( |\Delta m^*_\alpha| < 1 \) (which more than covers the values from the limited experimental data \[18\-13\-60\] and recent theoretical values on it), the restricted set of Skyrme EDFs is downsized to 162 in number. This is referred to as the ‘SELECTED’ set. Calculations are performed with this selected set also to see how the correlations are affected.

![Image](fig1.png)

**FIG. 1:** (Color online) The correlation of \( K_{sym}^0 \) with \( L_0 \) and with \( (3C_2^0 - L_0) \) are depicted in the right and left panels, respectively. Results for 237 Skyrme EDFs (‘ALL’) are displayed in the upper panels, the lower panels contain the results for a selected subset of 162 models (‘SELECTED’) (see text for details). The inner(outer) colored regions around the best-fit straight line in upper left panel depict the loci of 95% confidence (prediction) bands of the regression analysis.

Correlations between second and higher order density derivatives of symmetry energy with the slope parameter \( L_0 \) have earlier been studied in the literature \[39\-41\-42\] with some Skyrme EDFs. The results are mixed, the degree of correlation is found to depend on the subjective choice of selection of models. Eq. \[10\] however shows that \( K_{sym}^0 \) may be better correlated with \( (3C_2^0 - L_0) \). The correlation plot between them for all the 237 Skyrme EDFs is displayed in the upper left panel of Fig. 1. A linear correlation as suggested in Eq. \[10\] is observed, the correlation coefficient is \( r = -0.926 \). As mentioned earlier, \( K_{sym}^0 \) is a poorly determined quantity, existence of this correlation enables one to determine its value from
relatively better known isovector quantities \( C^2_0 \) and \( L_0 \) by using the best fit straight line \( K^0_{sym} = a(3C^2_0 - L_0) + c \) with \( a = -5.51 \pm 0.15 \) and \( c = 106.84 \pm 3.37 \) MeV. One notes that \( a \) is not too far from -5, the coefficient of \((3C^2_0 - L_0)\) in Eq. 10. In Ref. 41, for a restricted set of EDFs, the correlation of \( K^0_{sym} \) with \( L_0 \) was studied and reported to be stronger. For the set ‘ALL’ of Skyrme EDFs, we find it to be weaker \((r=0.808)\). This correlation is shown in the upper right panel of Fig. 1. The correlation between \( K^0_{sym} \) and \((3C^2_0 - L_0)\) is then tested for the ‘SELECTED’ set. It increases marginally, however, a marked improvement in the correlation between \( K^0_{sym} \) and \( L_0 \) is observed. These correlations are displayed in the bottom panels of Fig. 1. The correlation of \( K^0_{sym} \) with \((3C^2_0 - L_0)\) is thus seen to be more robust compared to that with \( L_0 \).

That is why the correlation between \( Q^{0*}_{sym} \) and \((3C^2_0 - L_0)\) improves significantly for the ‘SELECTED’ set. Even the correlation of \( Q^{0*}_{sym} \) with \( L_0 \) shows a marked gain in this case as displayed in the right bottom panel; it is nearly the same as with \((3C^2_0 - L_0)\). Correlation of \( Q^{0*}_{sym} \) with a linear combination of \( K^0_{sym} \) and \((3C^2_0 - L_0)\) is seen to be quite robust for both the sets, with correlation coefficient \( r \sim 0.95 \) (not shown in the figure). This robustness points to the fact that even though a strong correlation exists between \( K^0_{sym} \) and \((3C^2_0 - L_0)\), a minute deviation from exact correlation may affect the correlation with higher order derivatives.

The plots of \( Q^{0*}_{sym} \) as a function of \((3C^2_0 - L_0)\) and of \( L_0 \) for the set ‘ALL’ is shown in the upper panels of Fig. 2. \( Q^{0*}_{sym} \) is seen to be poorly correlated with \( L_0 \). The situation does not improve significantly for the correlation of \( Q^{0*}_{sym} \) with \((3C^2_0 - L_0)\). Eq. 11 shows \( Q^{0*}_{sym} \) to be nearly a linear combination of \( K^0_{sym} \) and \((3C^2_0 - L_0)\) and \( K^0_{sym} \) is seen to be well correlated with \((3C^2_0 - L_0)\); one may thus expect \( Q^{0*}_{sym} \) to be well correlated with \((3C^2_0 - L_0)\). A weak correlation is found though between \( Q^{0*}_{sym} \) and \((3C^2_0 - L_0)\) with all the models with a correlation coefficient \( r = 0.6 \). However, with the subset of models (‘SELECTED’), substantial improvement in the correlation between \( Q^{0*}_{sym} \) and \((3C^2_0 - L_0)\) can be observed \((r = 0.864)\). Due to imposed constraints on \( K_0 \) and \( m_0^\ast \), the values of \( a \) for the Skyrme models (as used in the formalism) get limited to a narrower range.

FIG. 2: (Color online) Same as Fig 1 but for \( Q^{0*}_{sym} \). The confidence bands of regression analysis are given only for the subset of models (‘SELECTED’) in the lower left panel.

FIG. 3: (Color online) The correlation of \( \Delta m_0^* \) with \((3C^2_0 - L_0)\) is depicted in the upper panel and with \( aK^0_{sym} + b(3C^2_0 - L_0) + c \) in the lower panel with selected set of models (‘SELECTED’). The values of \( a, b \) and \( c \) are given in the last row of Table II. The 95% confidence (prediction) band is also depicted by brown (grey) region in the lower panel.

The symmetry element \( \Delta m_0^* \) shows practically no correlation with either \( L_0 \) or with \((3C^2_0 - L_0)\) for the complete set of EDFs ‘ALL’. The situation does not improve with the ‘SELECTED’ set which can be noted from the upper panel of Fig. 3. We therefore looked for a correlation between \( \Delta m_0^* \) and a linear combination of \( K^0_{sym} \) and \((3C^2_0 - L_0)\) as suggested in Eq. 14. Even then, a meaningful correlation could not be found for the full EDF set \((r = 0.286)\). The reason behind this is that \( \Delta m_0^* \) is a sum of small positive and negative numbers (see Eq. 14). Small errors in those quantities may shadow the correlation. That is why, the selected set with reasonable constraints on few nuclear matter properties pulls the correlation up to a somewhat significant value \((r = 0.790)\). This is shown in the lower panel of Fig. 3.

Results of the correlation analyses are presented in Table II. One notices from the table that the values of the
TABLE I: The fitted expressions for $K_0^0$, $Q_0^0$ and $\Delta m_0^*$ along with the fitted parameters are listed. In the third column ‘A’ means ‘ALL’ the 237 EDFs which are employed for the analysis and ‘S’ means the ‘SELECTED’ set that are chosen where $K_0 = 230 \pm 30$ MeV, $\Delta m_0^* = 0.85 \pm 0.15$ and $|\Delta m_0^*| < 1$. The units of the coefficients $a$, $b$ and $c$ are such that they yield the values of $K_0^0$, $Q_0^0$ in MeV and $\Delta m_0^*$ comes out to be dimensionless. The correlation coefficient ‘r’ is listed in the seventh column. The last column shows the estimates of $K_0^0$, $Q_0^0$ and $\Delta m_0^*$ along with their uncertainties once $C_2^0$ and $L_0$ are given. The numbers in the parentheses depict the estimates once the dispersions in $C_2^0$ and $L_0$ are included.

| Quantity | fitted expression | Set | $a$ | $b$ | $c$ | $r$ | estimate |
|----------|-------------------|-----|-----|-----|-----|-----|----------|
| $K_0^0$  | $a(3C_2^0 - L_0) + c$ | A   | $-5.51 \pm 0.15$ | 106.84 $\pm$ 3.37 | -0.926 | $-97.0 \pm 6.5$ (86.8) |
|          | $a(3C_2^0 - L_0) + c$ | S   | $-4.56 \pm 0.14$ | 71.80 $\pm$ 2.70 | -0.931 | $-96.9 \pm 5.8$ (71.9) |
| $Q_0^0$  | $a(3C_2^0 - L_0) + c$ | S   | $10.72 \pm 0.49$ | $-97.38 \pm 8.84$ | 0.864 | 299.4 $\pm$ 20.2 (169.2) |
|          | $aK_0^0 + b(3C_2^0 - L_0) + c$ | A   | $3.51 \pm 0.07$ | 22.21 $\pm$ 0.42 | $-115.87 \pm 6.03$ | 0.952 | 365.4 $\pm$ 28.0 (147.3) |
|          | $aK_0^0 + b(3C_2^0 - L_0) + c$ | S   | $2.65 \pm 0.05$ | 18.88 $\pm$ 0.38 | $-86.87 \pm 3.73$ | 0.956 | 354.5 $\pm$ 22.3 (139.1) |
| $\Delta m_0^*$ | $aK_0^0 + b(3C_2^0 - L_0) + c$ | S   | $-0.0094 \pm 0.0003$ | $-0.0363 \pm 0.0012$ | $0.3958 \pm 0.0202$ | 0.790 | $-0.034 \pm 0.081$ (0.260) |

Constants appearing in the best-fit equations connecting one symmetry element with others with significant correlation are in near harmony with those that appear in Eq. (10). For instance, the slope ‘$a$’ in the best fit equation $K_0^0 = a(3C_2^0 - L_0) + c$ is found to be $a = -5.51 \pm 0.15$ for all models and $a = -4.56 \pm 0.14$ for the selected set of models; this is compatible with the coefficient of $(3C_2^0 - L_0)$ in Eq. (10). Similar is the case for the coefficients ‘$a$’ and ‘$b$’ in the expressions $Q_0^0 = aK_0^0 + b(3C_2^0 - L_0) + c$ and for $\Delta m_0^* = aK_0^0 + b(3C_2^0 - L_0) + c$ with $a \sim 0.2$, $E_p^0 \sim 36$ MeV and $m_0^* \sim 0.85$. From meaningful correlation coefficients as listed in Table I with fiducial values of $C_2^0 = 32$ MeV and $L_0 = 59$ MeV [16], the values of $K_0^0$ and $Q_0^0$ are seen to be nearly independent of the set chosen. Their values are reported in the last column of Table I. The errors in the calculated quantities arise due to lack of perfect correlation between the symmetry coefficients. Besides the imperfect correlation, the uncertainties in the values of $C_2^0(32 \pm 2$ MeV) and $L_0(59 \pm 15$ MeV) cause a considerably large dispersion in the values of the symmetry elements. They are also shown in the parentheses in the last column of Table I. The value of symmetry incompressibility $K_\ast = (K_{sym} - 6L_0 - Q_{0LM})$ can be estimated provided the value of the skewness parameter $Q_0$ is known. There is no experimental knowledge on $Q_0$; with the constraint on the value of $K_0(230 \pm 30$ MeV), in the selected set of Skyrme EDFs, $Q_0$ is seen to lie in a very narrow range ($Q_0 = -370 \pm 25$ MeV). The value of $K_\ast$ then turns out to be $K_\ast = -356 \pm 93$ MeV, matching quite well the recent theoretical estimates [40, 61, 62]. The isospin-splitted effective mass $\Delta m_0^*$ comes out to be slightly negative. This is, however, found to be very sensitive on the value of $(3C_2^0 - L_0)$ indicating that the present knowledge in the accuracy of $L_0$ may be incapable of extracting $\Delta m_0^*$ reliably. Its value may also partly depend on the definition of effective mass (Eq. 15). In astrophysical context, terms beyond the linear in density have been suggested [63] for the evaluation of the nuclear effective mass, however, at the saturation density $\rho_0$, their effect on $\Delta m_0^*$ are found to be not very significant.

IV. CONCLUSIONS

Calculations on the correlation between the symmetry coefficients presented in this paper are done in the ambit of the Skyrme EDFs. Undeniable model dependence in the conclusions arrived so far thus can not be ruled out. However, knowing that the isovector wing of the nuclear interaction is not yet very precise, constraining it through a structural relationship among the nuclear symmetry elements bearing its imprint is highly relevant even in a model, particularly when the model (Skyrme) has been extremely successful in explaining diverse experimental data. With this objective, analytical expressions for the different symmetry energy elements are obtained for the standard Skyrme EDFs and cast in forms suggestive of correlation between the lower and higher order density derivatives of symmetry energy. To this purpose we have employed 237 Skyrme class of energy density functionals[31]. Calculations reveal that there is a robust correlation between $K_0^0$ and $(3C_2^0 - L_0)$. The calculations were repeated for a set of restricted EDFs selected with imposition of a conservative set of empirical constraints on the values of nuclear matter incompressibility, effective mass of the nucleon and the isospin-split nucleon effective mass. The insignificant change in the correlation coefficient reinforces the robustness of the correlation between $K_0^0$ and $(3C_2^0 - L_0)$. Even if the value of $K_0^0$ varies widely in the Skyrme EDFs, it can be better bound with provision for good empirical knowledge of $C_2^0$ and $L_0$. $Q_{sym}^0$ is also reasonably correlated with $(3C_2^0 - L_0)$ but with only the restricted set of EDFs. A strong correlation of $Q_{sym}^0$ with linear combination of $K_0^0$ and $(3C_2^0 - L_0)$ is also observed. For $\Delta m_0^*$ a somewhat meaningful correlation is found with linear combination of $K_0^0$ and $(3C_2^0 - L_0)$ subject to the restricted set of EDFs. To the best of our knowledge, strong correlation of $Q_{sym}^0$ and even moderate correlation of $\Delta m_0^*$ with other symmetry...
energy parameters have not been reported earlier. The symmetry energy coefficient $C_2^0$ is a somewhat well estimated quantity extracted from different experimental observations. Though, $L_0$ is not that well determined as $C_2^0$, progress in constraining it is going on for some years. Experiments like PREX-II [67] give promises for a better determination of $L_0$ in a model independent way. Thus exploiting the correlated structures we have presented, one can estimate the higher order symmetry energy derivatives like $K_{sym}$ and $Q_{sym}^0$ in good bounds. It is not that certain for $\Delta$ symmetry energy coefficient $\delta$ parameters have not been reported earlier. The energy derivatives like $\delta$ can be obtained respectively as,

$$
\frac{\partial \delta}{\partial \rho} = \frac{1}{2} \left[ \frac{\partial^2 e (\rho, \delta)}{\partial \delta^2} \right]_{\delta=0}
$$

Using the expression of (3A1) in Eq. (A8) one can write

$$
K_{sym}^0 = -5(3C_2^0 - L_0) + E_F^0 + \frac{1}{16} \alpha(2 - 3\alpha)t_3(2x_3 + 1)\rho^{\alpha+1}.
$$

Here, $E_F^0$ is the Fermi energy of the system at $\rho_0$ given by, $E_F^0 = \frac{\hbar^2}{2m} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho_0^{2/3}$.

Taking third derivative of $C_2(\rho)$ given in Eq. (A1) one can arrive at the expression for $Q_{sym}(\rho)$ as,

$$
Q_{sym}(\rho) = 27\rho \left( \frac{\partial C_2(\rho)}{\partial \rho} \right)
$$

After simplification one can express $Q_{sym}$ in terms of nuclear matter properties at $\rho_0$ as,

$$
Q_{sym}^0 = (3\alpha + 2)K_{sym}^0 + 15(\alpha + 1)(3C_2^0 - L_0) - (3\alpha + 1)E_F^0.
$$

To find the expression for $\Delta n_0^*$ we take recourse to Eq. (12). Defining $m_0^*$ as the effective mass for SNM, from Eq. (12), one obtains

$$
\frac{\hbar^2}{2m_0^*} - \frac{h^2}{2m} = \frac{3h^2}{2m_p^*} \left( \frac{\hbar^2}{2m_n^*} - \frac{\hbar^2}{2m_p^*} \right)
$$

Replacing RHS of Eq. (A8) from Eq. (A4) one can write

$$
3C_2^0 - L_0 = E_F^0 - \frac{m}{3m_0^*} E_F^0 + E_F^0 \left( \frac{m_n^* - m_p^*}{m_n^* m_p^*} \right)
$$

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Appendix A: Symmetry energy parameters in Skyrme Formalism

Following the definition (Eq. (2)), the symmetry energy $C_2(\rho)$ is obtained from Eq. (4) as [43],

$$
C_2(\rho) = \frac{1}{2} \left[ \frac{\partial^2 e (\rho, \delta)}{\partial \delta^2} \right]_{\delta=0}
$$

$$
= \frac{1}{32m} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{2/3} - \frac{1}{8} t_0(2x_0 + 1)\rho
$$

$$
- \frac{1}{24} \left( \frac{3\pi^2}{2} \right)^{2/3} \left( 3t_1x_1 - t_2(4 + 5x_2) \right)\rho^{5/3}
$$

$$
- \frac{1}{48} t_3(2x_3 + 1)\rho^{\alpha+1}.
$$

Similarly, taking first and second order derivatives of $C_2(\rho)$ with respect to $\rho$, expressions for $L(\rho)$ and $K_{sym}(\rho)$ can be obtained respectively as,

$$
L(\rho) = 3\rho \left( \frac{\partial C_2(\rho)}{\partial \rho} \right)
$$

$$
= \frac{2}{3} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{2/3} - \frac{3}{8} t_0(2x_0 + 1)\rho
$$

$$
- \frac{5}{24} \left( \frac{3\pi^2}{2} \right)^{2/3} \left( 3t_1x_1 - t_2(4 + 5x_2) \right)\rho^{5/3}
$$

$$
- \frac{1}{16} (\alpha + 1)t_3(2x_3 + 1)\rho^{\alpha+1},
$$

$$
K_{sym}(\rho) = 9\rho^2 \left( \frac{\partial^2 C_2(\rho)}{\partial \rho^2} \right)
$$

$$
= -\frac{2}{3} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{2/3}
$$

$$
- \frac{5}{12} \left( \frac{3\pi^2}{2} \right)^{2/3} \left( 3t_1x_1 - t_2(4 + 5x_2) \right)\rho^{5/3}
$$

$$
- \frac{3}{16} \alpha(\alpha + 1)t_3(2x_3 + 1)\rho^{\alpha+1}.
$$

From Eqs. (A1) and (A2) one obtains the expression of $(3C_2^0 - L_0)$ as

$$
(3C_2^0 - L_0) = \frac{1}{32m} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{2/3}
$$

$$
+ \frac{1}{12} \left( \frac{3\pi^2}{2} \right)^{2/3} \left( 3t_1x_1 - t_2(4 + 5x_2) \right)\rho^{5/3}
$$

$$
+ \frac{1}{16} \alpha t_3(2x_3 + 1)\rho^{\alpha+1}.
$$

Using the expression of $(3C_2^0 - L_0)$ in Eq. (A4) one obtains the expression for $K_{sym}$ at $\rho_0$ as,

$$
K_{sym}^0 = -5(3C_2^0 - L_0) + E_F^0 + \frac{1}{16} \alpha(2 - 3\alpha)t_3(2x_3 + 1)\rho^{\alpha+1}.
$$

$\Delta m_0^*$ is not that well determined $\delta$ is not that certain for $\Delta$ symmetry energy coefficient $\delta$ parameters have not been reported earlier. The energy derivatives like $\delta$ can be obtained respectively as,
Making the approximation \( m^n_0 \approx m_0^n \), one can write the expression of \( \Delta m_0 \) as:

\[
\Delta m_0 = \left( 3C_2^0 - L_0 - E_F^0 + \frac{2}{3} \frac{m_0}{m} E_F^0 \right)
- \frac{1}{16} t_3 (2x_3 + 1)^{\rho^{\alpha+1}}. \tag{A9}
\]

Using the expression of \( K_{sym}^0 \) (see Eq. \( \text{A5} \)) to eliminate \( t_3 \) and \( x_3 \) in Eq. \( \text{A10} \), one obtains:

\[
\Delta m_0 = \left( K_{sym}^0 + 3(1 + \alpha)(3C_2^0 - L_0) + (1 - 3\alpha)E_F^0 \right)
+ \frac{2}{3} (3\alpha - 2) \frac{m_0}{m} E_F^0 \left( \frac{m}{m_0} \right)^2 \tag{A11}
\]

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