Relativistically generated asymmetry in the missing momentum distribution from the \((e, e'p)\) reaction

S. Gardner  
_Nuclear Theory Center and Department of Physics, Indiana University, Bloomington, IN 47505_

J. Piekarewicz  
_Supercomputer Computations Research Institute, Florida State University, Tallahassee, FL 32306_  
(August 5, 2021)

Abstract

We calculate the asymmetry in the missing-momentum distribution from the \((e, e'p)\) reaction in a relativistic formalism. Longitudinal and transverse response functions are evaluated in parallel kinematics as a function of the three-momentum transfer to the nucleus. Analytic expressions for the responses are obtained in a relativistic plane-wave impulse approximation. These expressions reveal a large asymmetry in the momentum distribution, at the plane-wave level, induced by the lower components of the bound-state wave functions. These relativistic effects contaminate any attempt to infer color transparency from a measurement of the asymmetry in the \((e, e'p)\) reaction.

PACS number(s): 25.30.Fj, 24.10.Jv
I. INTRODUCTION

The \((e, e'p)\) reaction constitutes an invaluable tool in the study of many diverse nuclear phenomena. The experimental setup enables one to determine the 4-momentum of both the virtual photon and the outgoing proton, so that the reaction kinematics are completely determined. As a result, it was originally believed that the \((e, e'p)\) cross section provides direct information about the nucleon momentum distribution in the nuclear medium [1]. A plane-wave, independent-particle description of the process suggests that the \((e, e'p)\) cross section should be given by a set of discrete peaks – each corresponding to knock-out from a given nuclear shell – with the momentum distribution obtained by integrating the strength under these peaks. This description is nearly adequate for states near the Fermi surface, yet additional study has revealed that the single-particle strength is strongly fragmented for deeply bound states [2,3]. Knock-out from states near the Fermi surface does support the notion of unfragmented single-particle strength, but the occupancy of these states is less than one [2]. These results show conclusively that the traditional picture of the nucleus, as a collection of particles moving independently in a mean-field potential, is too naive. Consequently, the \((e, e'p)\) reaction shows the limitations of an independent-particle picture, yet it provides a fruitful testing ground for various dynamical mechanisms, such as short-range correlations, that go beyond a simple mean-field approach.

The \((e, e'p)\) reaction has also been used to study possible medium modifications to the electromagnetic coupling of the nucleon [4]. That such effects exist has been suggested by a variety of studies. The original European-Muon-Collaboration (EMC) effect, for example, can be explained by a softening of the quark momentum distribution in the nuclear medium [3], which, in turn, modifies the nucleon’s electromagnetic coupling.

Interest in the \((e, e'p)\) reaction has been recently stimulated by suggestions that color transparency may manifest itself as an asymmetry [3,8] in the missing-momentum distribution, even in the absence of a modification of the total cross section [3]. “Color transparency” describes the hypothesis that hadrons produced with large laboratory momenta in certain exclusive reactions with nuclear targets interact in an anomalously weak manner with the residual nucleus [4,10]. We shall not concern ourselves directly with color transparency, yet part of our interest in considering conventional relativistic effects in the \((e, e'p)\) reaction is to the end of establishing a robust “baseline” calculation, against which possible novel effects may be inferred.

The possible existence of color transparency has been discussed in a great many different exclusive processes, yet the focus on the NE18 experiment at SLAC has made the quasielastic \((e, e'p)\) reaction the paradigm [11]. With the advent of CEBAF, the focus on this particular reaction is unlikely to diminish. Indeed, several \((e, e'p)\) experiments have been approved in the hope of identifying this novel behavior [12,14].

Our purpose is to explore the asymmetry in the missing momentum distribution in a conventional \((e, e'p)\) calculation. For concreteness, let us now quantify what we mean by the “asymmetry.” We consider the \((e, e'p)\) reaction in the kinematics where the three-momentum transfer \(\mathbf{q}\) and the transverse missing momentum \(p_t\) are fixed (in all that follows, “longitudinal” and “transverse” are defined as the components parallel and perpendicular to \(\mathbf{q}\), respectively). With this choice of kinematics, the longitudinal momentum distribution of the bound proton is scanned by varying the energy transfer to the nucleus – this results
in a varying momentum for the outgoing proton. The asymmetry is, then, defined as the integrated sum of events with positive longitudinal missing momentum relative to those events having negative missing momentum \[\text{[8]}\]. Here we shall consider the reaction specifically in “parallel kinematics”, that is, we shall take \(p_t = 0\), so that the knocked-out proton’s momentum is parallel to \(\mathbf{q}\), in all that follows.

Studies of the \((e, e'p)\) reaction at low momentum transfers \(|\mathbf{q}| \lesssim 1 \text{ GeV}\) yield missing-momentum \((p_m)\) distributions which are asymmetric about the \(p_m = 0\) point \[15\]. A calculation in a nonrelativistic plane-wave impulse approximation has no such asymmetry. The asymmetry at low momentum transfers is understood to arise from the momentum dependence of the distortions that the struck proton suffers in its exit from the nucleus \[16\]. In the momentum transfer regime beyond \(|\mathbf{q}| \sim 1 \text{ GeV}\), nuclear quasielastic electron scattering is essentially unexplored; this frontier provides a non-trivial testing ground for phenomenologies constructed in a lower energy regime. At large momenta, Glauber theory has been used to define the “baseline”. The attenuation of the ejected proton through the nucleus is given in this model by the inelastic proton-nucleon cross section. For proton momenta in excess of 3 GeV, the inelastic cross section is approximately constant, so that the asymmetry is essentially zero.

The virtue of the asymmetry as a signal of transparency rests wholly on the assumption that such an asymmetry is absent in any conventional (i.e., without transparency) calculation of the \((e, e'p)\) reaction at large momentum transfers. We shall show that this assumption is flawed. Relativistic effects, even in a plane-wave impulse approximation, serve to generate an asymmetry in the \((e, e'p)\) reaction. In a previous work, one of us examined the role of relativity on the spectral function \[17\]. In that context, relativistic effects, driven by particle-antiparticle mixing, are responsible for generating features, such as a fragmentation of the single-particle strength, which are associated nonrelativistically with physics beyond the mean-field level. These relativistic effects exist even if a mean-field ground state is used. As relativity generates effects similar to what a more complicated nonrelativistic calculation would generate, they can not be regarded as a definitive relativistic signature. Here we consider an effect which is absent in the conventional nonrelativistic formalism at the plane-wave level. Moreover, including distortions as in the Glauber theory also results in a small asymmetry at large momentum transfers. In contrast, relativistic corrections increase with the momentum transfer to the nucleus and can become large. As these effects are absent in the conventional nonrelativistic theory, they are interesting in their own right. Moreover, their existence limits the utility of the missing-momentum asymmetry as a signal of color transparency \[8\].

We have organized the paper as follows. In Sec. \[\text{II}\] we review some general properties of the \((e, e'p)\) reaction and develop analytic expressions for the nuclear response in a relativistic plane-wave impulse approximation. These plane-wave expressions identify the lower components of the bound-state spinors as the source of the asymmetry in the missing-momentum distribution. The only dynamical input required is the relativistic bound state wave function. Consequently, in Sec. \[\text{III}\], we use a \(^{16}\text{O}\) ground-state wave function obtained in a relativistic mean-field approximation to the Walecka model as an illustrative example. Finally, we offer our conclusions in Sec \[\text{IV}\].
II. FORMALISM

We begin by writing the inclusive hadronic tensor $W^{\mu\nu}(q)$ as

$$W^{\mu\nu}(q) = \sum_n \langle \Psi_n | J^{\mu}(q) | \Psi_0 \rangle \langle \Psi_n | J^{\nu}(q) | \Psi_0 \rangle^* \delta(\omega - \omega_n).$$

The nuclear electromagnetic current $J^{\mu}$ is responsible for inducing transitions between the exact nuclear ground state ($\Psi_0$) and an excited state ($\Psi_n$) with excitation energy $\omega_n = (E_n - E_0)$. In an independent-particle description of the ground state, the inclusive hadronic tensor can be expressed in terms of individual particle-hole transitions; i.e.,

$$W^{\mu\nu}(q) = \sum_{\alpha\beta} W^{\mu\nu}(q;\beta\alpha) \delta(\omega - E_{\beta\alpha}),$$

where

$$W^{\mu\nu}(q;\beta\alpha) \equiv J^{\mu}(q;\beta\alpha) J^{\nu}(q;\beta\alpha).$$

Note that $\omega$ is the energy transfer to the nucleus, and $E_{\beta\alpha} = (E_{\beta} - E_{\alpha})$ is the excitation energy from the initial single-particle state $\alpha$ to the final state $\beta$. The dynamical information about the process is contained in

$$J^{\mu}(q;\beta\alpha) = \int dx \overline{U}_{\beta}(x) e^{iq \cdot x} j^\mu(q) U_{\alpha}(x),$$

the transition matrix element between an occupied single-particle state $U_{\alpha}$ and an unoccupied state $U_{\beta}$. We assume the impulse approximation valid and employ an electromagnetic current with single-nucleon form factors parameterized from on-shell $eN$ scattering. That is,

$$j^\mu(q) = F_1(Q^2)\gamma^\mu + F_2(Q^2)i\sigma^{\mu\nu} q^\nu 2M, \quad (Q^2 \equiv q^2 - \omega^2)$$

where $F_1(Q^2)$ and $F_2(Q^2)$ are given as in Ref. [18]. Specifically, for the proton,

$$F_1(Q^2) = [1 + \tau (1 + \kappa_p)] (1 + 4.97\tau)^{-2}/(1 + \tau),$$
$$F_2(Q^2) = \kappa_p(1 + 4.97\tau)^{-2}/(1 + \tau).$$

Note that $\tau \equiv Q^2/4M^2$, and $\kappa_p = 1.793$ is the proton’s anomalous magnetic moment.

A. Relativistic plane-wave impulse approximation

The plane-wave limit of the $(e, e'p)$ reaction is useful in providing crude guidance into the nature of the reaction. Indeed, the plane-wave impulse approximation (PWIA) analysis leads one to believe that the $(e, e'p)$ reaction can be used to extract the spectral function and, thus, the nucleon momentum distribution [1]. This is only approximately correct. For example, final-state interactions (FSI) between the outgoing nucleon and the residual nucleus are known to modify the plane-wave picture. Of particular relevance to our present work is the symmetry in the missing-momentum distribution predicted in the nonrelativistic PWIA.
This symmetry is broken by final-state interactions. Yet, the inclusion of distortions in the Glauber formalism suggests that the symmetry in the missing-momentum distribution will become approximately restored at large momentum transfers. This observation supports the claim made in Ref. [8] that any measured asymmetry in the missing-momentum distribution is due to the formation of a compact state and, thus, constitutes evidence in support of color transparency.

In this section we will show that some notions developed in a nonrelativistic context no longer hold in a relativistic formalism. In particular, we will prove that the symmetry in the missing-momentum distribution is broken at the plane-wave level due to relativistic effects. Moreover, the asymmetry increases with the momentum transfer red to the nucleus – and as the momentum of the outgoing nucleon – is increased. Consequently, these relativistic “corrections” to the asymmetry contaminate any attempt to identify color transparency.

In a relativistic plane-wave impulse approximation (RPWIA) the outgoing proton with momentum \( p' \) and spin projection \( s' \) is described by a plane-wave Dirac spinor [19]:

\[
U_{p's'}(x) = e^{ip'\cdot x} \mathcal{U}(p', s') ,
\]

\[
\mathcal{U}(p', s') = \frac{\sqrt{E_{p'} + M^2}}{2E_{p'}} \left( \frac{1}{E_{p'} + M} \right) \chi_{s'} ,
\]

where the on-shell energy of the outgoing proton is given by

\[
E_{p'} = \sqrt{p'^2 + M^2} .
\]

Note that the normalization implied by Eq. (8) differs from the one in Ref. [19] in that

\[
\mathcal{U}^\dagger(p, s) \mathcal{U}(p, s') = \delta_{ss'} .
\]

In the following presentation, we shall use only the Dirac (or vector) piece of the electromagnetic current, for simplicity. Our final results, presented at the end of this section, include the anomalous piece of the current as well.

In a RPWIA the Dirac piece of the current matrix element for the knock-out of a bound-state nucleon is

\[
J^\mu_D(q, p', s'; \alpha) = \mathcal{U}^\dagger(p', s') \gamma^\mu \mathcal{U}_\alpha(p) ,
\]

where \( \mathcal{U}(p's') \) is the plane-wave state defined above and where the Fourier transform of the relativistic bound-state wave function is given by

\[
\mathcal{U}_\alpha(p) = \int dx \, e^{-ip\cdot x} \mathcal{U}_\alpha(x) .
\]

Note that we have also introduced the missing momentum

\[
p_m \equiv p = p' - q .
\]

The semi-inclusive hadronic tensor can now be written in terms of a Feynman trace,
\[ W^{\mu\nu}(q, p', s'; \alpha) = \left[ \mathcal{U}(p', s') \gamma^\mu \mathcal{U}_\alpha(p) \right] \left[ \mathcal{U}(p', s') \gamma^\nu \mathcal{U}_\alpha(p) \right]^* \]

\[ = \text{Tr} \left[ \gamma^\mu S_\alpha(p) \gamma^\nu \left( \frac{p' + M}{2E_{p'}} \right) \left( \frac{1 + \gamma^5 g'}{2} \right) \right], \tag{14} \]

involving the spin-dependent projection operator for the outgoing nucleon \[19\], as well as the bound-state “propagator”

\[ S_\alpha(p) = \mathcal{U}_\alpha(p) \overline{\mathcal{U}}_\alpha(p), \tag{15} \]

which satisfies the normalization condition

\[ \int \frac{d\mathbf{p}}{(2\pi)^3} \text{Tr} \left[ \gamma^0 S_\alpha(p) \right] = 1. \tag{16} \]

To proceed, we need an explicit form for \( S_\alpha(p) \). To do so, we recognize that in the presence of a spherically symmetric potential the eigenstates of the Dirac equation can be classified according to a generalized angular momentum \( \kappa \) and can be written in a two component representation; i.e.,

\[ \mathcal{U}_{E\kappa m}(\mathbf{x}) = \frac{1}{x} \left[ g_{E\kappa}(x) \mathcal{Y}_{+\kappa m}(\hat{\mathbf{x}}) \right] \left[ i f_{E\kappa}(x) \mathcal{Y}_{-\kappa m}(\hat{\mathbf{x}}) \right]. \tag{17} \]

The upper and lower components are expressed in terms of spin-spherical harmonics defined by

\[ \mathcal{Y}_{\kappa m}(\hat{\mathbf{x}}) \equiv \langle \hat{\mathbf{x}} | \frac{1}{2} j m \rangle; \quad j = |\kappa| - \frac{1}{2}; \quad l = \begin{cases} \kappa, & \text{if } \kappa > 0; \\ -1 - \kappa, & \text{if } \kappa < 0. \end{cases} \tag{18} \]

The Fourier transform of the above bound-state wave function can now be easily evaluated. That is,

\[ \mathcal{U}_{E\kappa m}(\mathbf{p}) = \frac{4\pi}{p} (-i)^l \left[ g_{E\kappa}(p) \mathcal{Y}_{+\kappa m}(\hat{\mathbf{p}}) \right] \left[ i f_{E\kappa}(p) \mathcal{Y}_{-\kappa m}(\hat{\mathbf{p}}) \right] = \frac{4\pi}{p} (-i)^l \left[ g_{E\kappa}(p) \right] \left[ i f_{E\kappa}(p) (\sigma \cdot \hat{\mathbf{p}}) \right] \mathcal{Y}_{+\kappa m}(\mathbf{p}), \tag{19} \]

where we have written the Fourier transforms of the radial wave functions as

\[ g_{E\kappa}(p) = \int_0^\infty dx \, g_{E\kappa}(x) j_l(px), \tag{20a} \]

\[ f_{E\kappa}(p) = (\text{sgn}\kappa) \int_0^\infty dx \, f_{E\kappa}(x) j_l(px). \tag{20b} \]

In the above expression we have introduced \( j_l(z) \), the Riccati-Bessel function, through the relation \( j_l(z) = z j_l'(z) \). By now making use of the following identities

\[ \sum_m \mathcal{Y}_{+\kappa m}(\mathbf{p}) \mathcal{Y}_{+\kappa m}^*(\mathbf{p}) = \sum_m \mathcal{Y}_{-\kappa m}(\mathbf{p}) \mathcal{Y}_{-\kappa m}^*(\mathbf{p}) = \frac{2j + 1}{8\pi}, \tag{21a} \]

\[ \sum_m \mathcal{Y}_{+\kappa m}(\mathbf{p}) \mathcal{Y}_{-\kappa m}^*(\mathbf{p}) = \sum_m \mathcal{Y}_{-\kappa m}(\mathbf{p}) \mathcal{Y}_{+\kappa m}^*(\mathbf{p}) = -\frac{2j + 1}{8\pi} (\sigma \cdot \mathbf{p}), \tag{21b} \]

we can express the bound-state propagator in closed form. That is,
\[ S_\alpha(p) \equiv S_{E\alpha}(p) = \frac{1}{2j + 1} \sum_m U_{E\alpha m}(p) \overline{U}_{E\alpha m}(p) = (\slashed{p}_\alpha + M_\alpha) , \tag{22} \]

where we have defined the above mass-, energy-, and momentum-like quantities as

\[ M_\alpha = \left( \frac{\pi}{p^2} \right) [g_\alpha^2(p) - f_\alpha^2(p)] , \tag{23a} \]
\[ E_\alpha = \left( \frac{\pi}{p^2} \right) [g_\alpha^2(p) + f_\alpha^2(p)] , \tag{23b} \]
\[ p_\alpha = \left( \frac{\pi}{p^2} \right) [2g_\alpha(p)f_\alpha(p)] . \tag{23c} \]

Note that they satisfy the “on-shell relation”

\[ p_\alpha^2 = E_\alpha^2 - p_\alpha^2 = M_\alpha^2 . \tag{24} \]

The above representation of the bound-state propagator, given entirely in terms of Dirac gamma matrices, enables one to evaluate the semi-inclusive hadronic tensor,

\[ W_{\mu\nu}^{DD}(q; p' s' \alpha) = \text{Tr} \left[ \gamma^\mu (\slashed{p}_\alpha + M_\alpha) \gamma^\nu \left( \frac{\slashed{p}' + M}{2E_{p'}} \right) \left( 1 + \gamma^5 \frac{\slashed{s'}}{2} \right) \right] , \tag{25} \]

with Feynman’s trace techniques. This relativistic plane-wave expression for the Dirac component of the hadronic tensor is completely general and does not assume any specific kinematical constraint. In the present work, however, we are interested in the missing-momentum asymmetry evaluated in parallel kinematics ($\hat{p}' \cdot \hat{q} = 1$). For this case, then, the charge, transverse, and longitudinal components of the hadronic tensor are given, respectively, by

\[ W_{DD}^{00}(q; p' s' \alpha) = \left( \frac{E_{p'} + M}{2E_{p'}} \right) \left( \frac{4\pi}{p^2} \right) \left[ g_\alpha(p) + (\hat{p}' \cdot \hat{p}') \xi_{p' \alpha}(p) \right] ^2 , \tag{26} \]
\[ W_{DD}^{11}(q; p' s' \alpha) = \left( \frac{E_{p'} + M}{2E_{p'}} \right) \left( \frac{4\pi}{p^2} \right) \left[ \xi_{p' \alpha}(p) - (\hat{p}' \cdot \hat{p}') f_\alpha(p) \right] ^2 , \tag{27} \]
\[ W_{DD}^{33}(q; p' s' \alpha) = \left( \frac{E_{p'} + M}{2E_{p'}} \right) \left( \frac{4\pi}{p^2} \right) \left[ \xi_{p' \alpha}(p) + (\hat{p}' \cdot \hat{p}') f_\alpha(p) \right] ^2 , \tag{28} \]

where we have defined

\[ \xi_{p'} \equiv \frac{|p'|}{(E_{p'} + M)} . \tag{29} \]

Two aspects of these results are particularly noteworthy. First, we have included the charge as well as the longitudinal components of the hadronic tensor because the electromagnetic current is not conserved at the plane-wave level. Second, the term $(\hat{p'} \cdot \hat{p})$ generates an asymmetry in the missing-momentum distribution. This has its origin in the relation

\[ \hat{p'} \cdot \hat{q} = \begin{cases} +1, & \text{if } |p'| > |q| ; \\ -1, & \text{if } |p'| < |q| . \end{cases} \tag{30} \]
Note, however, that in the absence of lower bound-state components (i.e., \( f_\alpha \equiv 0 \)), there is no asymmetry, in agreement with nonrelativistic expectations. Moreover, including distortions as in the Glauber theory does not significantly alter this result at large momentum transfers. The asymmetry at large momenta, then, is a genuinely relativistic effect. This simple result, namely, the relativistic generation of an asymmetry at the plane-wave level, is the main result of the present work.

One of the advantages of plane-wave analyses is that they suggest that the \((e, e'p)\) cross section can be factorized into an (off-shell) elementary electron-proton cross section times a spectral function containing all nuclear-structure information. In order to isolate the nuclear structure effects, we shall introduce a reduced cross section. This reduced cross section is obtained from the full \((e, e'p)\) cross section by dividing out an off-shell electron-proton cross section – with some kinematical factors. Unfortunately, the off-shell electron-proton cross section is not uniquely defined \([20]\). The choice made can even obscure the dynamical features one is trying to uncover. Indeed, an off-shell electron-proton cross section also generates an asymmetry in the missing-momentum distribution, but this asymmetry is unrelated to nuclear-structure effects. For our present purpose, we shall adopt the following procedure. First, we separate the full cross section into longitudinal and transverse response functions. Then, we divide each of these responses by an off-shell electric or magnetic single-nucleon form factor, as appropriate, to yield the reduced cross sections. This definition has the advantage that the nonrelativistic limit of the reduced cross section does, indeed, yield a symmetric missing-momentum distribution. This procedure provides, then, a baseline for comparing different theoretical mechanisms of breaking the symmetry, be they relativistic effects, or color transparency.

We shall now introduce longitudinal and transverse RPWIA reduced cross sections for the \((e, e'p)\) reaction. The reduced cross sections have been computed with the full electromagnetic current – including the anomalous piece – and are given by

\[
\rho_L(q, p'; \alpha) \equiv \frac{W^{00}(q, p'; \alpha)}{w^{00}(q, p')} = \left( \frac{4\pi}{p^2} \right) g_\alpha^2(p) \left[ 1 \pm \left( \frac{\xi_{p'}F_1 + \bar{q}F_2}{F_1 - \xi_{p'}\bar{q}F_2} \right) g_\alpha(p) \right]^2, \tag{31a}
\]

\[
\rho_T(q, p'; \alpha) \equiv \frac{W^{11}(q, p'; \alpha)}{w^{11}(q, p')} = \left( \frac{4\pi}{p^2} \right) g_\alpha^2(p) \left[ 1 \mp \left( \frac{\xi_{p'}F_1 + (\bar{q} - \xi_{p'}\bar{\omega})F_2}{\xi_{p'}F_1 + (\bar{q} - \xi_{p'}\bar{\omega})F_2} \right) g_\alpha(p) \right]^2. \tag{31b}
\]

The off-shell single-nucleon responses, \(w^{00}\) and \(w^{11}\), have been chosen as

\[
w^{00}(q, p') = \left( \frac{E_{p'} + M}{2E_{p'}} \right) \left[ F_1 - \xi_{p'}\bar{q}F_2 \right]^2 \longrightarrow \left( \frac{1 + \tau}{1 + 2\tau} \right) G^2_L(Q^2), \tag{32a}
\]

\[
w^{11}(q, p') = \left( \frac{E_{p'} + M}{2E_{p'}} \right) \left[ \xi_{p'}F_1 + (\bar{q} - \xi_{p'}\bar{\omega})F_2 \right]^2 \longrightarrow \left( \frac{\tau}{1 + 2\tau} \right) G^2_M(Q^2), \tag{32b}
\]

where the following dimensionless quantities have been introduced

\[
\tau \equiv \frac{Q^2}{4M^2}, \quad \bar{q} \equiv \frac{|q|}{2M}, \quad \bar{\omega} \equiv \frac{\omega}{2M}. \tag{33}
\]

In constructing the single-nucleon responses we have demanded that the resulting missing-momentum distribution be symmetric at the plane-wave level in the limit of a vanishing lower bound-state component. Note that in this limit
sensitive to physics beyond the mean-field picture. Note that when $f_\alpha$ is useful as it enables us to exclude those large missing momentum events that might be even if it exists, is due to the coherent interference of the distortions suffered by the outgoing

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The arrow in the previous expressions is used to indicate that the single-nucleon responses in the isolated nucleon limit (so that $p = 0$) become proportional to the electric and magnetic form factors of the nucleon. The off-shell single-nucleon responses have been defined to the end of establishing a baseline calculation for the asymmetry, against which relativistic effects may be assessed.

The longitudinal reduced cross section clearly shows that the symmetry in the missing momentum distribution is broken by relativistic effects. Consequently, the symmetry is restored when all momenta become small relative to the nucleon mass. The nonrelativistic limit of the transverse reduced cross section is, in contrast, not clearly defined. The relativistic “corrections” are of $O(1)$, so that the transverse response is an intrinsically relativistic observable.

In a recent publication, Bianconi, Boffi, and Kharzeev suggested studying the asymmetry in the missing-momentum distribution as a means of uncovering color transparency. In particular, they suggest comparing the number of events with positive missing momentum ($|\mathbf{p}'| > |\mathbf{q}|$) to those events with negative missing momentum ($|\mathbf{p}'| < |\mathbf{q}|$). In this work we follow their suggestion and define the following asymmetries. Here we define, however, separate longitudinal and transverse asymmetries:

\[
A_L(\mathbf{q}, p_{\text{max}}) = \frac{N^{(+)}_L(\mathbf{q}, p_{\text{max}}) - N^{(-)}_L(\mathbf{q}, p_{\text{max}})}{N^{(+)}_L(\mathbf{q}, p_{\text{max}}) + N^{(-)}_L(\mathbf{q}, p_{\text{max}})} \bigg|_{f_\alpha = 0} \to 0 ,
\]

\[
A_T(\mathbf{q}, p_{\text{max}}) = \frac{N^{(+)}_T(\mathbf{q}, p_{\text{max}}) - N^{(-)}_T(\mathbf{q}, p_{\text{max}})}{N^{(+)}_T(\mathbf{q}, p_{\text{max}}) + N^{(-)}_T(\mathbf{q}, p_{\text{max}})} \bigg|_{f_\alpha = 0} \to 0 .
\]

In all that follows, we will assume that we are studying single-particle transitions from a specific bound state $\alpha$ to the continuum, so that the $\alpha$ label is suppressed – both in the above and in the reduced cross sections. The asymmetries are obtained from the total number of longitudinal or transverse events having either positive or negative missing momentum:

\[
N^{(+)}_{L,T}(\mathbf{q}, p_{\text{max}}) = \int_{p_{\text{max}}}^{\infty} \frac{p^2 dp}{2\pi^2} \rho_{L,T}(\mathbf{q}, \mathbf{p}') \bigg|_{f_\alpha = 0} \to \frac{2}{\pi} \int_{p_{\text{max}}}^{\infty} dp \ g_\alpha^2(p) \bigg|_{p_{\text{max}} \to \infty} 1 ,
\]

\[
N^{(-)}_{L,T}(\mathbf{q}, p_{\text{max}}) = \int_{-p_{\text{max}}}^{0} \frac{p^2 dp}{2\pi^2} \rho_{L,T}(\mathbf{q}, \mathbf{p}') \bigg|_{f_\alpha = 0} \to \frac{2}{\pi} \int_{-p_{\text{max}}}^{0} dp \ g_\alpha^2(p) \bigg|_{p_{\text{max}} \to \infty} 1 .
\]

The arrows indicate two successive limits. In the first the lower bound-state component is set to zero, while in the second the cut-off $p_{\text{max}}$ goes to infinity. Introducing a cut-off is useful as it enables us to exclude those large missing momentum events that might be sensitive to physics beyond the mean-field picture. Note that when $f_\alpha = 0$ the number of events with positive and negative missing momenta become equal, so that the asymmetry vanishes. The symmetry is broken, however, by the inclusion of momentum-dependent final-state interactions – or by relativistic effects.

The symmetry may also be broken by color transparency. Color transparency, if it exists, is due to the coherent interference of the distortions suffered by the outgoing
proton and other, higher-mass, baryon states produced by the virtual photon – the strong interactions of the individual components conspire to cancel, at least in part. The initial and final electron 4-momenta fix \( x_B \equiv Q^2/2M\omega \) so that the longitudinal component of the struck proton momentum depends on whether a proton or a \( N^* \) is produced. For \( x_B > 1 \), the bound-state momenta \( p \) required to make a proton \( (N) \) or an excited baryon state \( (N^*) \) must be of the same sign. Satisfying the kinematic constraints for both \( N \) and \( N^* \) production at moderate momentum transfers is unlikely, as bound-nucleon momenta in the tail of the momentum distribution are required. For \( x_B < 1 \), the required momenta can be of opposite sign, so that the kinematic constraints can be satisfied at moderate \( Q^2 \) with a finite probability. This is the source of the suggested asymmetry effect [6]. Bianconi, Boffi, and Kharzeev [8] relate the proposed asymmetry in \( x_B \) to an asymmetry in the missing-momentum distribution; they find a positive asymmetry, as we shall find here for the longitudinal asymmetry. However, their arguments give no insight into how the longitudinal and transverse asymmetries may differ, nor is it helpful in disentangling off-shell form factor effects from the modification of the final-state interactions. Yet, the conventional Glauber analysis suggests that the asymmetry should be small at large \( Q^2 \), supporting the asymmetry as a signal of color transparency, as argued by the authors of Ref. [8].

We believe that relativistic effects modify this picture. Indeed, we have shown that relativistic effects generate an asymmetry in the missing-momentum distribution at the plane-wave level.

### III. RESULTS

We now proceed to show relativistic plane-wave results for the missing-momentum asymmetry. In addition, we estimate how the asymmetry is changed upon the inclusion of relativistic distortions which are essential for restoring electromagnetic gauge invariance. At the plane-wave level, the reduced cross sections, in Eq. (31), require only the bound-state wave functions as dynamical input. We have assumed that the single-nucleon form factors are not modified by the nuclear medium and have simply adopted the parameterization for the on-shell form factors given in Eq. (6). The bound-state wave functions have been obtained in a relativistic mean-field approximation to the Walecka model [21]. In the Walecka model nucleons interact via the exchange of neutral scalar (\( \sigma \)) and vector (\( \omega \)) mesons [22,23]. The parameters of the model are adjusted, at the mean-field level, to reproduce the binding energy and density of nuclear matter at saturation. With this input, the mean-field approximation to the Walecka model has been successful in describing the ground-state properties of finite nuclei – as well as the linear response of the ground state to a variety of probes [23]. This is realized, in part, because the Walecka model is characterized by strong scalar and (timelike) vector potentials that cancel to generate the observed weak binding energies, but combine to yield a strong spin-orbit splitting for the positive-energy states [23].

In Fig. 1 we present the charge response as a function of the missing momentum for the two valence orbitals in \(^{16}\text{O}\) for the three-momentum transfers of \( |q| = 1.5 \text{ GeV} \). Noting Eq. (36a), the response has been multiplied by the phase-space factors necessary to make the area under the curve directly proportional to the number of integrated events. In addition, we have scaled the response in order to compensate for the rapid falloff of the form factors,
since we have chosen to use the same scale here as in Fig. 2. These constant factors do not change the asymmetries – these are displayed in brackets next to the appropriate momentum-transfer labels.

The asymmetry in the charge response is driven by a combination of nuclear-structure and single-nucleon effects. As we have discussed, nuclear-structure effects generate an asymmetry in the momentum distribution due to the presence of lower components in the bound-state wave function. However, even in the absence of relativistic corrections, the charge response, at fixed three-momentum transfer, will display an asymmetry from single-nucleon effects. The latter asymmetry is driven by the \( Q^2 \) dependence of the form factor. The change in \( Q^2 \) itself depends on the magnitude and on the direction of the missing momentum. For fixed \( |p| \), the energy transfer to a nucleon moving parallel to \( \hat{q} \) must be larger than that to a nucleon moving antiparallel to \( q \), in order to yield an outgoing nucleon which is on its mass shell. Hence, the electromagnetic coupling is stronger for the nucleon moving parallel to \( q \) since its charge form factor is being probed at a lower value of \( Q^2 \). This single-nucleon effect is responsible for generating a positive asymmetry in the missing momentum distribution. Its contribution must, then, be eliminated from the response before one can identify the role of the nuclear medium in breaking the symmetry.

Eliminating the single-nucleon contribution from the asymmetry is, however, not free from ambiguity. Indeed, certain off-shell prescriptions could accentuate the effect. We believe that the off-shell prescription adopted here [Eq. (32a)] is an appealing choice for the study of the asymmetry. This choice yields, in particular, a symmetric missing-momentum distribution in the nonrelativistic plane-wave limit. This is an useful baseline against which interesting behavior may be established.

In Fig. 2 we present the longitudinal reduced cross section [Eq. (31a)] for the two valence orbitals in \(^{16}\)O for the three-momentum transfers of \( |q| = 1.5 \) GeV. In this case the area under the curve is directly proportional to the number of events \( N_L^{(±)} \) defined in Eq. (30). The reduced cross section clearly shows an asymmetric distribution. This asymmetry is entirely due to relativistic nuclear-structure effects. The total asymmetries are displayed in the appropriate brackets. In particular, we note that removing the single-nucleon contribution to the response yields asymmetries which are slightly smaller than those of Fig. 1.

In Fig. 3 we show the longitudinal asymmetry [Eq. (35a)] as a function of the three-momentum transfer \( |q| \) and the momentum cut-off \( p_{\text{max}} \). At small three-momentum transfers, the longitudinal asymmetry is negligible because the relativistic corrections are small \((\xi_{p'} \approx |p'|/2M \ll 1, \text{ and } \bar{q} = |q|/2M \ll 1)\). In contrast, at large momentum transfers, the relativistic “corrections” are large and so is the asymmetry. We can understand this by examining the high-energy limit of the longitudinal reduced cross section [Eq. (31a)]. At large three-momentum transfers \((\bar{q} = |q|/2M \gg 1)\) and small missing momenta \((|p|/2M \ll 1)\), we obtain the following limits for the other previously defined kinematical variables

\[
\xi_{p'} \equiv \frac{|p'|}{E_{p'} + M} \rightarrow 1 , \quad (37a)
\]
\[
\omega \equiv E_{p'} - E \rightarrow |p'| - M , \quad (37b)
\]
\[
\tau \equiv \frac{Q^2}{4M^2} \rightarrow \bar{q} . \quad (37c)
\]

These relations imply, in particular, that the following ratio of anomalous to Dirac form
factors becomes
\[
\frac{F_2}{F_1} = \frac{\bar{q}}{\bar{q}'} \frac{\kappa_p}{1 + \tau(1 + \kappa_p)} \to \frac{\kappa_p}{(1 + \kappa_p)},
\]  
so that one finds the following simple expression for the reduced cross section:
\[
\rho_L(q, p'; \alpha) \to \left(\frac{4\pi}{p^2}\right) g_\alpha^2(p) \left[1 \pm (1 + 2\kappa_p) \frac{f_\alpha(p)}{g_\alpha(p)}\right]^2.
\]  
(39)

This result indicates that the longitudinal asymmetry is enhanced not only by the in-medium wave function ratio of \((f_\alpha/g_\alpha)\) but also by an additional factor of \([1 + 2\kappa_p] \sim 4.6\].

In Fig. 4 we show, for completeness, the transverse asymmetry. The transverse response is an intrinsically relativistic observable, so that the transverse asymmetry is large even at small momentum transfers. The asymmetry, however, saturates rapidly, as reflected by the high-energy limit of the reduced cross section:
\[
\rho_T(q, p'; \alpha) \to \left(\frac{4\pi}{p^2}\right) g_\alpha^2(p) \left[1 \mp \frac{f_\alpha(p)}{g_\alpha(p)}\right]^2.
\]  
(40)

We conclude this section with a brief discussion of distortion effects. A nonrelativistic analysis of the \((e, e'p)\) reaction generates an asymmetry in the missing-momentum distribution when distortion effects are included, at low energies. At high energies, however, a Glauber analysis suggests that the symmetry should be restored. Relativistically, however, the energy dependence of the distorting potential depends strongly on its Lorentz character and can be substantially different from its nonrelativistic counterpart [24].

Distortions play an important role in the conservation of the electromagnetic current. Electromagnetic gauge invariance has been lost as a result of our plane-wave treatment, and distortions play a fundamental role in its restoration. The included distortions, however, must be consistent with the mean-field potential used to obtain the single-particle spectrum. Hence, in calculating the distorted wave for the outgoing proton, we have employed the same real and energy-independent potential used to generate the mean-field ground state. Unfortunately, this treatment implies that the imaginary part of the nucleon self-energy – which is physically important – has been suppressed. In spite of this, we have calculated the nuclear response in a relativistic distorted-wave impulse approximation (RDWIA), in order to show what impact the restoration of gauge invariance has on the asymmetry. As we shall see, the longitudinal asymmetry remains large in the presence of the included distortions. The incorporation of absorptive and energy-dependent effects in a calculation with electromagnetic gauge invariance remains an important open problem.

In Fig. 5 we show the longitudinal reduced cross section as a function of the missing momentum for the two valence orbitals in \(^{16}\text{O}\) at \(|q| = 1\text{ GeV}\). The dashed line is the relativistic plane-wave result also shown in Fig. 2. The symmetric distribution (dot-dashed line) results from neglecting the lower component of the bound-state wave function in the RPWIA calculation. This represents our best attempt in generating a nonrelativistic result. Finally, relativistic distorted-wave results are shown by the solid line. The positive shift in the RDWIA distribution relative to the plane-wave result is due to the repulsive character of the real distorting potential. The self-consistent, mean-field potential employed in
the RDWIA calculation becomes repulsive for a kinetic energy of $T_{\text{lab}} \simeq 150 - 200$ MeV ($|p'| \simeq 550 - 650$ MeV). This implies that at a momentum transfer of $|q| = 1$ GeV the distorting potential is repulsive over the entire range of missing momenta shown in the figure, thus, producing the observed shift. In Table I we display the sensitivity of the longitudinal asymmetry to distortion effects. The RDWIA asymmetry is consistently larger than the plane-wave results due to the added effect of the repulsive distorting potential.

**IV. CONCLUSIONS**

We have calculated the asymmetry in the missing-momentum distribution from the $(e, e'p)$ reaction in a relativistic plane-wave impulse approximation, using a mean-field approximation to the Walecka model. Longitudinal and transverse nuclear-response functions have been evaluated in parallel kinematics as a function of the three-momentum transfer to the nucleus. By dividing out off-shell, single-nucleon form factors from the response functions, we were able to define a nonrelativistic plane-wave limit with a symmetric momentum distribution. This provides a baseline for comparing different theoretical mechanisms of breaking the symmetry.

In a relativistic formalism the presence of a lower component in the Dirac bound-state wave function is sufficient to break the symmetry at the plane-wave level. We have developed analytic expressions for the plane-wave responses and have illustrated our findings by calculating the missing-momentum asymmetry using $^{16}$O as an example. The ground state for $^{16}$O was generated self-consistently in a mean-field approximation to the Walecka model. The mean-field approximation is characterized by the presence of strong (attractive) scalar and (repulsive) timelike vector potentials. The scalar potential leads to a reduction of the effective nucleon mass in the medium and is, ultimately, responsible for a substantial enhancement of the ratio of upper to lower components in the bound-state wave function. This dynamical enhancement of the ratio, together with the increasing importance of the relativistic corrections with momentum transfer, yield longitudinal asymmetries as large as $A_L \sim 0.8$ for a three-momentum transfer of $|q| \sim 10$ GeV. These nuclear-structure effects preclude the use of the asymmetry in the missing-momentum distribution as a definitive signature of color transparency. However, the missing-momentum asymmetry does provide interesting information about the role of relativity in the nuclear medium.

We have also investigated the effects of distortions. We include distortions to assess the impact of the restoration of gauge invariance on the asymmetry. The RDWIA calculations yield a larger longitudinal asymmetry than in the plane-wave case. This results from the repulsive character of the real distorting potential. We have stressed that a distorting potential consistent with the mean field used to generate the nuclear ground state is essential for the conservation of the electromagnetic current. These mean-field distortions ignore, however, some important physics – absorption and energy-dependence – associated with the in-medium propagation of the outgoing proton. We believe that a gauge-invariant treatment of the $(e, e'p)$ reaction that successfully describes the distortions of the outgoing proton is an important area for future work.
ACKNOWLEDGMENTS

This research was supported by the U.S. Department of Energy contract # DE-FG02-87ER40365 (S.G.), as well as by the Florida State University Supercomputer Computations Research Institute through the U.S. Department of Energy contracts # DE-FC05-85ER250000 and # DE-FG05-92ER40750 (J.P.).
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FIGURES

FIG. 1. Charge response in a RPWIA for the two valence orbitals in $^{16}\text{O}$ at the momentum transfers of $|q| = 1.5$ GeV. The quantities in brackets give the value of the missing-momentum asymmetry.

FIG. 2. Longitudinal reduced cross section in a RPWIA for the two valence orbitals in $^{16}\text{O}$ at the momentum transfers of $|q| = 1.5$ GeV. The quantities in brackets give the value of the missing-momentum asymmetry.

FIG. 3. Longitudinal missing-momentum asymmetry in a RPWIA as a function of the momentum transfer $|q|$ for the two valence orbitals in $^{16}\text{O}$. The integrals were carried out up to a maximum value of the missing momentum of 100 (solid), 200 (dashed), and 300 MeV (dot-dashed), respectively.

FIG. 4. Transverse missing-momentum asymmetry in a RPWIA as a function of the momentum transfer $|q|$ for the two valence orbitals in $^{16}\text{O}$. The integrals were carried out up to a maximum value of the missing momentum of 100 (solid), 200 (dashed), and 300 MeV (dot-dashed), respectively.

FIG. 5. Longitudinal reduced cross section for the two valence orbitals in $^{16}\text{O}$ at a momentum transfer of $|q| = 1$ GeV. The dashed(solid) line displays the result of a relativistic calculation obtained by ignoring(including) distortion effects. The dot-dashed line shows the results obtained when the lower component of the bound-state wave function in dashed calculation is turned off. The quantities in brackets give the value of the missing-momentum asymmetry.
TABLE I. Longitudinal numbers of events and missing-momentum asymmetries as a function of the three-momentum transfer with $p_{\text{max}} = 350$ MeV for the $1p^{1/2}$ state in $^{16}\text{O}$. The first(second) row of numbers are the results of a relativistic calculation that neglects(incorporates) distortion effects.

| $|q|$ (GeV) | $N_L^{(+)}$ | $N_L^{(-)}$ | $A_L$  |
|-----------|-------------|-------------|--------|
| 1.00      | 1.49        | 0.67        | 0.38   |
|           | 1.78        | 0.41        | 0.62   |
| 1.50      | 1.74        | 0.58        | 0.50   |
|           | 1.87        | 0.24        | 0.77   |
| 2.00      | 1.99        | 0.51        | 0.59   |
|           | 2.05        | 0.21        | 0.82   |
| 2.50      | 2.23        | 0.47        | 0.65   |
|           | 2.30        | 0.19        | 0.85   |
| 3.00      | 2.46        | 0.44        | 0.70   |
|           | 2.62        | 0.18        | 0.87   |
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