Pion freeze-out as seen through HBT correlations in heavy ion collisions from FAIR/AGS to RHIC energies

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Abstract

We perform a systematic analysis of several HBT parameters in heavy ion collisions from \(E_{\text{beam}} = 2\) AGeV to \(\sqrt{s_{\text{NN}}} = 200\) GeV within the UrQMD transport approach and compare the results to experimental data where available. We find that the 'lifetime' of the emission source as calculated from \(\tau \sim \sqrt{R_O^2 - R_S^2}\), is larger than the experimentally observed values at all investigated energies. The calculated volume of the pion source \((V_f)\) is found to increase monotonously with increasing beam energy and the experimentally observed decrease of the measured \(V_f\) at AGS is not seen. Finally, we calculate the mean free path \(\lambda_f = 0.5 - 1\) fm of pions at freeze-out and find a good description of the experimental data above the AGS energy region, supporting the suggestion of a universal kinetic decoupling criterion up to the highest RHIC energies.

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In order to create the theoretically predicted deconfined phase of Quantum Chromodynamics (QCD) heavy ions have been collided with energies from less than $\sqrt{s} \sim 2.5$ GeV (SIS/FAIR energy regime), $2.5-20$ GeV (FAIR/AGS and SPS) up to $20-200$ GeV (RHIC). Indeed, it seems that some nontrivial signals - such as charmonium suppression, relative strangeness enhancement, etc. - of the (phase) transition to the deconfined phase have been observed in heavy ion collisions (HICs) at SPS energies [1, 2, 3, 4, 5, 6]. Additional information about the matter created in such collisions can be obtained from the investigation of the space-time structure of the particle emission source (the region of homogeneity). The established tool to extract this information is known as Femtoscopy [7] or originally as Hanbury-Brown-Twiss interferometry (HBT) [8, 9, 10]. Experimentally this technique is quite often used to extract the information on the spatio-temporal evolution of the particle source, which has been scanned thoroughly by several separate experimental collaborations over the whole discussed energy region [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 35]. However, so far the measured excitation function of HBT parameters shows no obvious discontinuities within the large span of explored beam energies [7].

A comprehensive theoretical investigation on the excitation function of the HBT parameters is thus highly required but still absent so far [7]. Recently, based on the Ultra-relativistic Quantum Molecular Dynamics (UrQMD, v2.2) transport model (employing hadronic and string degrees of freedom) (for details, the reader is referred to Refs. [23, 24, 25, 26]) and the program CRAB (v3.0β) [27, 28, 29], we have investigated the transverse momentum, system-size, centrality, and rapidity dependence of the HBT parameters $R_L$, $R_O$, $R_S$ (dubbed as HBT radii or Pratt radii), and the cross term $R_{OL}$ of pion source at AGS [30], SPS [31] and RHIC [32] energies, respectively. In general, the calculations are satisfying and well in line with the experimental data although discrepancies are not negligible. Such as, I), the calculated $R_L$ and $R_S$ values for Au+Au collisions at low AGS energies are visibly smaller than the data if the default UrQMD version 2.2 (cascade mode) is adopted. II), the HBT-'puzzle' with respect to the 'duration time' of the pion source, is present at all energies. In order to understand the origin of this HBT-'puzzle', some efforts were made, however, a complete understanding of this effect is still lacking.

In this paper, we present pion interferometry results on the 'duration-time' related quantity $\sqrt{R_O^2 - R_S^2}$, the freeze-out volume $V_f$, and derive the mean free path $\lambda_f$ of pions at freeze-out. The analysis is based on the comprehensive comparison of the excitation func-
tion of calculated HBT radii $R_L$, $R_O$, and $R_S$ at small transverse momenta with data, in the FAIR/AGS, SPS and RHIC energy regime. The standard UrQMD v2.2 in cascade mode is employed throughout this paper to serve as a benchmark for further discussions [47].

To calculate the two-particle correlator, the CRAB program is based on the formula:

$$C(k, q) = \int d^4x_1 d^4x_2 g(x_1, p_1)g(x_2, p_2)|\phi(q, r)|^2 \int d^4x_1 g(x_1, p_1) \int d^4x_2 g(x_2, p_2).$$

(1)

Here $g(x, p)$ is the probability for emitting a particle with momentum $p$ from the space-time point $x = (r, t)$. $\phi(q, r)$ is the relative two-particle wave function with $r$ being their relative position. $q = p_2 - p_1$ and $k = (p_1 + p_2)/2$ are the relative momentum and the average momentum of the two particles. Due to the underlying quantum statistics, this correlator is larger than unit at small $q$ for bosons and can be fitted approximately by a Gaussian form. Using Pratt’s three-dimensional convention (the LCMS system), the standard parametrisation of the correlation function in Gaussian form reads

$$C(q_O, q_S, q_L) = 1 + \lambda \exp(-R^2_L q^2_L - R^2_O q^2_O - R^2_S q^2_S - 2R^2_{OL} q_O q_L).$$

(2)

Here $q_i$ and $R_i$ are the components of the pair momentum difference $q$ and the homogeneity length (HBT radii) in the $i$ direction, respectively. The pre-factor $\lambda$ is the incoherence parameter and lies between 0 (complete coherence) and 1 (complete incoherence) in realistic HICs. The term $R^2_{OL}$ is called cross-term and vanishes at mid-rapidity for symmetric systems, while it deviates from zero at large rapidities [31, 33, 34].

We compare our calculations of the Pratt parameters of the pion source with experimental data for the following central collisions of heavy nuclei:

1. Au+Au at the AGS beam energies $E_b = 2, 4, 6$, and 8A GeV ($< 11\%$ of the total cross section $\sigma_T$), a rapidity cut $|Y_{cm}| < 0.5$ ($Y_{cm} = \frac{1}{2}\log\frac{E_{cm} + p_\parallel}{E_{cm} - p_\parallel}$, $E_{cm}$ and $p_\parallel$ are the energy and longitudinal momentum of the pion meson in the center-of-mass system) is employed. The experimental (E895) data are taken from [12].

2. Au+Au at the AGS beam energy 11.6A GeV (the $< 5\%$ most central collisions), a rapidity cut $|Y_{cm}| < 0.5$ is employed. The experimental (E802) data are taken from [13].

3. Pb+Pb at the SPS beam energies $E_b = 20, 30, 40, 80$, and 160A GeV ($< 7.2\%\sigma_T$ of most central collisions), a pion-pair rapidity cut $|Y_{\pi\pi}| < 0.5$ ($Y_{\pi\pi} = \frac{1}{2}\log\frac{E_1 + E_2 + p_{\parallel 1} + p_{\parallel 2}}{E_1 + E_2 - p_{\parallel 1} - p_{\parallel 2}}$)
is the pair rapidity with pion energies $E_1$ and $E_2$ and longitudinal momenta $p_{\parallel 1}$ and $p_{\parallel 2}$ in the center of mass system) is employed. The experimental (NA49) data are taken from [14, 15].

4. Pb+Au at the SPS beam energies $E_b = 40$, 80, and 160A GeV (the < 5% most central collisions), the pion-pair rapidity cut $Y_{\pi\pi} = -0.25 \sim 0.25$, $-0.5 \sim 0$, and $-1.0 \sim -0.5$ are chosen. The experimental (CERES) data are taken from [16].

5. Au+Au at the RHIC nucleon-nucleon center-of-mass energies $\sqrt{s_{NN}} = 30$ (< 15%$\sigma_T$), 62.4 (< 15%$\sigma_T$), 130 (< 10%$\sigma_T$), and 200 GeV (< 5%$\sigma_T$). Here a pseudo-rapidity cut $|\eta_{cm}| < 0.5$ ($\eta_{cm} = \frac{1}{2} \log \left( \frac{p + p_{\parallel}}{p - p_{\parallel}} \right)$, ($p$ is the momentum of the pion) is employed. The experimental (PHOBOS, STAR, and PHENIX) data are taken from [17, 18, 19, 20, 21].

Fig. 1 shows the excitation function of the calculated HBT radii $R_L$ [in (a)], $R_O$ [(b)], $R_S$ [(c)], and the duration-time related quantity $\sqrt{R_O^2 - R_S^2}$ [(d)] at $k_T = 100$ MeV (full lines, black) and 200 MeV (dotted lines, red). The experimental data within this transverse momentum region are shown for comparison. Since the experimental data from NA49 [14, 15] and from CERES [16] collaborations overlap at beam energies 40, 80, and 160A GeV, we show the calculations and data with respect to CERES energies separately as dashed-dotted lines and open symbols.

The calculated $R_L$ (Fig. 1(a)) increases faster than $R_O$ and $R_S$ with increasing energies, which is also observed in data. Meanwhile, with increasing beam energies, the splitting of $R_L$ with different $k_T$ becomes stronger, which can be attributed to the flow-dominated freeze-out scenario. This observation is also in line with previous results [32], where it was found that the decrease of $R_L$ with increasing $k_T$ at RHIC energies is stronger for central reactions than for peripheral ones. It should also be noted that the small $k_T$ behaviour of the correlation function is also affected by some other factors, such as the decay of resonances, potential interactions and/or the treatment of resonance life-times and widths in the medium.

Fig. 1(b) shows the calculations on $R_O$ in comparison to the experimental data. Here we find that model calculation in data agree fairly well at AGS and SPS-NA49 but deviations from data are observable at RHIC energies. The comparison to the CERES data (with the appropriate cuts) however shows a rather large discrepancy between calculation and data. This deviations was also report in a previous study [31] and might hint to systematic differences (apart from different experimental centralities and cuts) between the NA49 analysis.
FIG. 1: (Color Online) Excitation function of HBT radii $R_L$ [(a)], $R_O$ [(b)], $R_S$ [(c)], and the quantity $\sqrt{R_O^2 - R_S^2}$ [(d)]. The calculations are shown at $k_T = 100 \pm 50$ MeV (full line) and $200 \pm 50$ MeV/c (dotted line), respectively. The gray areas between the $k_T = 100$ MeV and $k_T = 200$ MeV lines are shown for better visibility. The data are at $k_T \sim 150$MeV/c for reactions at $E_b = 2, 4, 6, 8 A$ GeV (AGS-E895) \cite{12} and $20, 30, 40, 80, 160 A$ GeV (SPS-NA49) \cite{15}, at $k_T \sim 170$MeV/c for reaction at $\sqrt{s_{NN}} = 130$ GeV \cite{19, 21}, at $k_T \sim 200$MeV/c for reactions at $E_b = 11.6 A$ GeV (AGS-E802) \cite{13}, $E_b = 40, 80, 160 A$ GeV (SPS-CERES) \cite{16}, and $\sqrt{s_{NN}} = 62.4, 200$ GeV \cite{17, 18, 20}.
and the results measured by the CERES collaboration. The source of this difference remains therefore unclear. Recently, the NA57 collaboration published the $K_T$ dependence of the HBT radii (with $K_T$ up to 1.2 GeV/$c$) in Pb+Pb collisions at 40A GeV, and it was found that the NA57-data, especially the $R_O$, are in line with the NA49-data [35].

In Fig. 1 (c), we notice that the calculated sideward radii $R_S$ are in qualitative agreement with the data but seem to be 15% smaller for almost all energies. Furthermore, the increase of the measured $R_S$ at low AGS energies can not be reproduced [30] and might be due to the omission of potential interactions that gain importance at low beam energies. Detailed investigations will be presented in a future publication. The excitation function of the HBT radii from several systems (from light to heavy) inspired by the NA49-future collaboration [4] might also provide new insights into this problem, and predictions are in progress.

The HBT duration time "puzzle", i.e. the fact of the theoretical quantity $\sqrt{R_O^2 - R_S^2}$ being larger than extracted from the data, is present at all investigated energies (see Fig. 1 (d)): The calculated values of $\sqrt{R_O^2 - R_S^2}$ are about 3.5 $\sim$ 5 fm while the measured ones are 1.5 $\sim$ 4 fm. Many efforts have been put forward over the last years to clarify this issue [30, 31, 32, 36, 37, 38, 39, 40]. E.g., in our previous works [30, 31], we suggested that a sizeable amount of interactions between particles at the early stage of the reaction either on the mean field and/or (non-perturbative) partonic level seem to be relevant to understand the phenomenon. In fact, all straightforward cascade transport approaches (also those with partonic interactions with perturbative QCD cross sections) fail to describe the quantity $\sqrt{R_O^2 - R_S^2}$, as well as the elliptic flow [41], over the whole energy range.

Fig. 2 (a) shows the excitation function of the pion source volume $V_f$ at freeze-out, calculated as:

$$V_f = (2\pi)^{\frac{3}{2}} R_L R_S^2.$$  \hspace{1cm} (3)

Note that the radius $R_O$ is not considered to calculate the pion freeze-out volume since it contains the contribution of the temporal extent of the pion source. Fig. 2 (a) shows clearly that the UrQMD cascade calculations do provide a reasonable freeze-out volume for the pion source at RHIC energies. At SPS energies, the agreement is fine with CERES data while it slightly underpredicts those of NA49. Towards even lower energies, the model underpredicts the measured freeze-out volume due to the omission of the strong interaction potential and other in-medium effects. E.g. at $E_b = 2A$ GeV, the measured $V_f$ is about 2 $\sim$ 3 times larger than calculated value. As studied in [30], a mass-dependent lifetime of resonances accounts
FIG. 2: (Color Online) (a): Excitation function of the pion freeze-out volume $V_f$ (according to Eq. 3) at transverse momenta between $k_T = 100$ MeV and 200 MeV (gray area), compared with data in this $k_T$-region. (b): Excitation function of the mean free path $\lambda_f$ of pions at freeze-out (according to Eq. 4) at the same transverse momenta. The experimental value for $\lambda_f$ at $\sqrt{s_{NN}} = 200$ GeV is obtained with the help of recent $dN/dy$ data in [43], at all other energies the $\lambda_f$ data are taken from [42], for an improvement of the HBT-radii at small $k_T$ and hence reproduce the data better. From the discussions above, it is clear that the major part of this difference is related to the sideward radius $R_S$.

We are now ready to estimate the mean free path $\lambda_f$ of the pions at freeze-out from the following expression [42]

$$\lambda_f = \frac{V_f}{N\sigma} = \frac{V_f}{N_N\sigma_{NN} + N_\pi\sigma_{\pi\pi}}.$$
with the averaged pion-nucleon cross section $\sigma_{N\pi} = 72$ mb and the averaged pion-pion cross section $\sigma_{\pi\pi} = 13$ mb (Note that within the present model calculations these values are slightly energy dependent. However, here we have adopted the explicit numbers from Ref. [42] to compare to the results presented there). The nucleon and pion multiplicities $N_N$ and $N_\pi$ are calculated as

$$N_N = y_{th} \cdot \sqrt{2\pi} \cdot \left. \frac{dN_{\text{nucleons}}}{dy} \right|_{y_{\text{mid}}},$$

and

$$N_\pi = y_{th} \cdot \sqrt{2\pi} \cdot \left. \frac{dN_{\text{pions}}}{dy} \right|_{y_{\text{mid}}}.$$

using the assumption of a thermal equilibrated system at freeze-out with a temperature $T_f = 120$ MeV. Here, $y_{th}$ is the estimated thermal homogeneity scale in rapidity at a certain $k_T$ and $T_f$, and is given by the expressions: $y_{th} = \text{arctanh}(\langle \beta_{th} \rangle)$, with $\langle \beta_{th} \rangle = \sqrt{1 + \langle \gamma \rangle^2 / \langle \gamma \rangle}$ and $\langle \gamma \rangle = 1 + 1/3 (K_1(m_T/T_f)/K_2(m_T/T_f) - 1) + T_f/m_T$. Here $K_n(z)$ is the modified Bessel function of order $n$ and $m_T = \sqrt{m_\pi^2 + k_T^2}$. At $k_T = 100$ MeV and 200 MeV, the calculated homogeneity lengths in rapidity $y_{th}$ are 0.98 and 0.81, respectively. $dN/dy|_{y_{\text{mid}}}$ is the rapidity density of pion (nucleons) at mid-rapidity [48]. Recent calculations using the present UrQMD transport model [25], have shown that the calculated pion and nucleon yields are reasonably in agreement with data.

Fig. 2 (b) shows the excitation function of $\lambda_f$ of pions at freeze-out. It is seen that the theoretical $\lambda_f$ value increases gradually from $\sim 0.5$ to $\sim 1$ fm from AGS to highest RHIC energies with a weak dependence on $k_T$. The experimental values of $\lambda_f$ are also between 0.5 - 1 fm. The calculated $\lambda_f$ below the AGS energy deviates from the data due to the smaller calculated sideward radii (resulting in a too small freeze-out volume).

Certainly, the assumption of a constant kinetic freeze-out temperature $T_f = 120$ MeV in the calculations of $y_{th}$ over the whole energy range is not justified, especially for the collisions at low beam energies. By inserting an energy dependent temperature (assuming a kinetic freeze-out temperature of 70% of the chemical freeze-out temperature shown in [44]) into Eqs. 5 and 6, one finds that at lower AGS energies, the number of nucleons and pions in the pion source volume is visibly reduced due to the decreased $T_f$. As a result $\lambda_f$ increases and a flatter distribution of $\lambda_f$ as a function of the beam energy is obtained in the present calculation.
The observation (both experimentally and theoretically) of a nearly energy independent mean free path on the order of 0.7 fm at pion freeze-out is rather surprising. Physically it has been interpreted as a rather large opaqueness of the pion source at break-up \[45, 46\].

To summarize, we survey the excitation function of the HBT radii \(R_L\), \(R_O\), and \(R_S\), the quantity \(\sqrt{R_O^2 - R_S^2}\), the volume \(V_f\), and the mean free path \(\lambda_f\) of pions at freeze-out for heavy systems with energies ranging from lowest AGS to the highest RHIC energies. Generally, the model calculations with UrQMD v2.2 (cascade mode) are in line with the data over the whole inspected energy range. Although discrepancies especially in the lower AGS energy region are found and have to be resolved. Especially the HBT parameter \(R_S\) and the HBT duration-time related ”puzzle” deserve further attention. Finally we re-interpret the measured and calculated radii in terms of a mean free path for pions and kinetic freeze-out. Here we find a nearly constant mean free path for pions on the order of \(\lambda_f = 0.7\) fm indicating a significant opaqueness of the source.

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We have found the treatment of the mass-dependent resonance lifetimes can better describe the HBT radii for AGS energies (at small transverse momenta), meanwhile, the HBT time-related puzzle can be better understood with the consideration of a potential interaction. These effects are, however, not taken into account in this paper for the sake of clarity.

In Ref. [42], the experimental $N_{\text{nucleons}}$ is set to two times of total amount of emitted proton and anti-proton yields and $N_{\text{pions}}$ is three times of negatively charged pion number. In our calculations, $N_{\text{nucleons}}$ represents the sum of protons and neutrons, as well as anti-nucleons. Note that this introduces a small difference into the analysis due to iso-spin effects, especially at very low energies.