1 Introduction

Everything is a network – whenever we look at the interactions between things, a network is formed implicitly. In the areas of data mining, machine learning, information retrieval, etc., networks are modeled as graphs. Many, if not most problem types can be applied to graphs: clustering, classification, prediction, pattern recognition, and others. Networks arise in almost all areas of research, commerce and daily life in the form of social networks, road networks, communication networks, trust networks, hyperlink networks, chemical interaction networks, neural networks, collaboration networks and lexical networks. The content of text documents is routinely modeled as document–word networks, taste as person–item networks and trust as person–person networks. In recent years, whole database systems have appeared specializing in storing networks. In fact, a majority of research projects in the areas of web mining, web science and related areas uses datasets that can be understood as networks. Unfortunately, results from the literature can often not be compared easily because they use different datasets. What is more, different network datasets have slightly different properties, such as allowing multiple or only single edges between two nodes. In order to provide a unified view on such network datasets, and to allow the application of network analysis methods across disciplines, the KONECT project defines a comprehensive network taxonomy and provides a consistent access to network datasets. To validate this approach on real-world data from the Web, KONECT also provides a large number (180+) of network datasets of
Figure 1: All networks in KONECT arranged by the size (the number of nodes) and the average number of neighbors of all nodes. Each network is represented by a two- or three-character code. The color of each code corresponds to the network category as given in Table 3.

different types and different application areas.

KONECT, the Koblenz Network Collection, contains 168 network datasets as of April 2013. In addition to these datasets, KONECT consists of Matlab code to generate statistics and plots about them, which are shown on the KONECT website. KONECT contains networks of all sizes, from small classical datasets from the social sciences such as Kenneth Read’s Highland Tribes network with 16 vertices and 58 edges (HT), to the Twitter social network with 52 million nodes and 1.9 billion edges (TF). Figure 1 shows a scatter plot of all networks by the number of nodes and the average degree in the network. Each network in KONECT is represented by a unique two- or three-character code which we write in a sans-serif font, and is indicated in parentheses as used previously in this paragraph. The full list of codes is given online.

This handbook first describes the different network types covered by KONECT in Section 2 gives important mathematical definitions in Section 3 lists the numerical network statistics in Section 4 lists the plot types in Section 5 describes KONECT’s file format in Section 7 and gives a short overview of the Matlab

\[\text{konect.uni-koblenz.de}\]
\[\text{konect.uni-koblenz.de/networks}\]
Table 1: The network formats allowed in KONECT. Each network dataset is exactly of one type.

| Type | Edge partition | Edge types | Internal name |
|------|----------------|------------|---------------|
| U    | Undirected     | Unipartite | Undirected    |
| D    | Directed       | Unipartite | Directed      |
| B    | Bipartite      | Bipartite  | Undirected    |

Throughout the handbook, we will use margin notes to give the internal names of various parameters.

### 2 Taxonomy of Networks

Datasets in KONECT represent networks, i.e., a set of nodes connected by links. Networks can be classified by their format (directed/undirected/bipartite), by their edge weight types and multiplicities, by the presence of metadata such as timestamps and node labels, and by the types of objects represented by nodes and links. The full list of networks is given online.[3]

The format of a network is always one of the following. The network formats are summarized in Table 1.

- **In undirected networks** (U), edges are undirected. That is, there is no difference between the edge from $u$ to $v$ and the edge from $v$ to $u$; both are the edge $\{u, v\}$. An example of an undirected network is the social network of Facebook (Ow), in which there is no difference between the statements “A is a friend of B” and “B is a friend of A.”

- **In a directed network** (D), the links are directed. That is, there is a difference between the edge $(u, v)$ and the edge $(u, v)$. Directed networks are sometimes also called *digraphs* (for *directed graphs*), and their edges *arcs*. An example of a directed social network is the follower network of Twitter (TF), in which the fact that user A follows user B does not imply that user B follows user A.

- **Bipartite networks** (B) include two types of nodes, and all edges connect one node type with the other. An example of a bipartite network is a rating graph, consisting of the node types *user* and *movie*, and each rating connects a user and a movie (M3). Bipartite networks are always undirected in KONECT.

The edge weight and multiplicity types of networks are represented by one of the following six types. The types of edge weights and multiplicities are summarized in Table 2.

[3] konect.uni-koblenz.de/networks
• An **unweighted network** (−) has edges that are unweighted, and only a single edge is allowed between any two nodes.

• In a **network with multiple edges** (=), two nodes can be connected by any number of edges, and all edges are unweighted. This type of network is also called a multigraph.

• In a **positive network** (+), edges are annotated with positive weights, and only a single edge is allowed between any node pair. The weight zero identified with the lack of an edge and thus, we require that each edge has a weight strictly larger than zero.

• In a **signed network** (±), both positive and negative edges are allowed. Positive and negative edges are represented by positive and negative edge weights. Many networks of this type have only the weights ±1, but in the general case we allow any nonzero weight.

• **Rating networks** (∗) have arbitrary real edge weights. They differ from positive and signed networks in that the edge weights are interpreted as an interval scale, and thus the value zero has no special meaning. Adding a constant to all edge weights does not change the semantics of a rating network. Ratings can be discrete, such as the one-to-five star ratings, or continuous, such as a rating given in percent. This type of network allows only a single edge between two nodes.

• **Networks with multiple ratings** (∗∗) have edges annotated with rating values, and allow multiple edges between two nodes.

• **Dynamic networks** (⇄) are networks in which edges can appear and disappear. They are always temporal. Individual edges are not weighted.

Metadata of networks are further properties that go beyond the formats and weights listed above.

• **Temporal networks** (⊙) include a timestamp for each edge, and thus the network can be reconstructed for any moment in the past.

• **Networks with loops** (⟳) are unipartite networks in which edges of the form {u,u} are allowed, i.e., edges connecting a node with itself.

Finally, the network categories classify networks by the type of data they represent. An overview of the categories is given in Table 3.

**Affiliation networks** are bipartite networks denoting the membership of actors in groups. Groups can be defined as narrowly as individual online communities in which users have been active (FG) or as broadly as countries (CN). The actors are mainly persons, but can also be other actors such as musical groups. Note that in all affiliation networks we consider, each actor can be in more than one group, as otherwise the network cannot be connected.
Table 2: The edge weight and multiplicity types allowed in KONECT. Each network dataset is exactly of one type. Note that due to historical reasons, networks with multiple unweighted edges have the internal name positive, while positively weighted networks have the internal posweighted.

| Type           | Multiple edges | Edge weight range | Edge weight scale | Internal name |
|----------------|----------------|-------------------|-------------------|---------------|
| - Unweighted   | No             | {1}               | -                 | unweighted    |
| = Multiple unweighted | Yes             | {1}               | -                 | positive      |
| + Positive weights | No             | (0, +∞)           | Ratio scale      | posweighted   |
| ± Signed       | No             | (−∞, +∞)          | Ratio scale      | signed        |
| * Rating       | No             | (−∞, +∞)          | Interval scale   | weighted      |
| * Rating       | Yes            | (−∞, +∞)          | Interval scale   | multiweighted |
| ⇄ Dynamic      | Yes            | {1}               | -                 | dynamic       |

Table 3: The network categories in KONECT. Each category is assigned a color, which is used in plots, for instance in Figure 1. The property symbols are defined in Table 2. U: Undirected network, D: Directed network, B: Bipartite network. The given dataset counts are current as of April 2013.

| Category         | Vertices          | Edges      | Properties      | Count |
|------------------|-------------------|------------|-----------------|-------|
| Affiliation      | Actors, groups    | Memberships| B − =           | 8     |
| Authorship       | Authors, works    | Authorships| B − =           | 18    |
| Co-occurrence    | Items             | Co-occurrences| U D −             | 2     |
| Communication    | Persons           | Messages   | U D − =         | 8     |
| Contact          | Persons           | Interactions| U D =          | 4     |
| Features         | Items, features   | Properties | B −             | 5     |
| Folksonomy       | Users, tags, items| Tag assignments| B =          | 17    |
| Interaction      | Persons, items    | Interactions| B − =          | 14    |
| Lexical          | Words             | Lexical relationships| U D B − =    | 5     |
| Physical         | Various           | Physical connections| U D − = | 13     |
| Ratings          | Users, items      | Ratings    | B − * *         | 11    |
| Reference        | Documents         | References | D − =           | 28    |
| Semantic         | Entities          | Relationships| D =           | 1     |
| Social           | Persons           | Ties       | U D − = + ± *   | 29    |
| Text             | Documents, words  | Occurrences| B =             | 5     |
Authorship networks are unweighted bipartite networks consisting of links between authors and their works. In some authorship networks such as that of scientific literature (Pa), works have typically only few authors, whereas works in other authorship networks may have many authors, as in Wikipedia articles (en).

Co-occurrence networks represent the simultaneous appearance of items. Co-occurrence networks are unipartite and unweighted. An example is the co-purchase network of Amazon (AM) indicating which persons have purchased the same articles. Note that in most cases, such networks can be derived from another network using a two-mode projection. For instance, a user–user co-purchase network can be derived from a user–item purchase network. In cases where the underlying bipartite network is known, we do not include the two-mode network, as its properties can be derived from the corresponding properties of the original network. As an example, the eigenvalues of the two-mode network’s adjacency matrix are the squares of the singular values of the bipartite network’s biadjacency matrix. Only when the underlying data is unknown as for the Amazon dataset do we include a co-occurrence network.

Communication networks contain edges that represent individual messages between persons. Communication networks are directed and allow multiple edges. Examples of communication networks are those of emails (EN) and those of Facebook messages (Ow). Note that in some instances, edge directions are not known and KONECT can only provide an undirected network.

Contact networks consist of people and interactions between them. Contact networks are unipartite and allow multiple edges, i.e., there can always be multiple interactions between the same two persons. They can be both directed or undirected. Examples are people that meet each other (RM), or scientists that write a paper together (Pc).

Feature networks are bipartite, and denote any kind of feature assigned to entities. Feature networks are unweighted and have edges that are not annotated with edge creation times. Examples are songs and their genres (GE).

Folksonomies consist of tag assignments connecting a user, an item and a tag. For folksonomies, we follow the 3-bipartite projection approach and consider the three possible bipartite networks, i.e., the user–item, user–tag and item–tag networks. This allows us to apply methods for bipartite graphs to hypergraphs, which is not possible otherwise. Items that are tagged in folksonomies include bookmarks (Dui), scientific publications (Cui) and movies (Mui).

Interaction networks are bipartite networks consisting of people and items, where each edge represents an interaction. In interaction networks, we
always allow multiple edges between the same person–item pair. Examples are people writing in forums (UF), commenting on movies (Fc) or listening to songs (Ls).

**Lexical networks** consist of words from natural languages and the relationships between them. Relationships can be semantic (i.e., related to the meaning of words) such as the synonym relationship (W0), associative such as when two words are associated with each other by people in experiments (EA), or denote co-occurrence, i.e., the fact that two words co-occur in text (SB). Note that lexical co-occurrence networks are explicitly not included in the broader Co-occurrence category.

**Physical networks** represent physically existing network structures in the broadest sense. This category covers such diverse data as physical computer networks (TO), transport networks (OF) and biological food networks (FD).

**Rating networks** consist of assessments given to items by users, weighted by a rating value. Rating networks are bipartite. Networks in which users can rate other users are not included here, but in the Social category instead. If only a single type of rating is possible, for instance the “favorite” relationship, then rating networks are unweighted. Examples of items that are rated are movies (M3), songs (YS), jokes (JE), and even sexual escorts (SX).

**Reference networks** consist of citations or hyperlinks between various types of documents. Reference networks are directed. Examples are hyperlinks between pages on the World Wide Web (W3), citations between scientific publications (CS), and citations among patents (PC).

**Semantic networks** are generic networks of entities connected by relationships. Our dataset collection contains a single semantic network, DBpedia (DB), containing data extracted from the English Wikipedia, in which entities are individual lemmas and relationships are inferred from infoboxes.

**Social networks** represent ties between persons. Certain social networks allow negative edges, which denote enmity, distrust or dislike. Examples are Facebook friendships (FSG), the Twitter follower relationship (TF), and friends and foes on Slashdot (SZ). Note that some social networks can be argued to be rating networks, for instance the user–user rating network of a dating site (LI). These networks are all included in the Social category.

**Text networks** consist of text documents containing words. They are bipartite and their nodes are documents and words. Each edge represents the occurrence of a word in a document. Document types are for instance newspaper articles (TR) and Wikipedia articles (EX).

Note that the category system of KONECT is in flux. As networks are added to the collection, large categories are split into smaller ones.
We do not include certain kinds of networks that lack a complex structure. This includes networks without a giant connected component, in which most nodes are not reachable from each other, and trees, in which there is only a single path between any two nodes. Note that bipartite relationships extracted from n-to-1 relationships are therefore excluded, as they lead to a disjoint network. For instance, a bipartite person–city network containing was-born-in edges would not be included, as each city would form its own component disconnected from the rest of the network. On the other hand, a band–country network where edges denote the country of origin of individual band members is included, as members of a single band can have different countries of origin. In fact the Countries network (CN) is of this form. Another example is a bipartite song–genre network, which would only be included in KONECT when songs can have multiple genres. As an example of the lack of complex structure when only a single genre is allowed, the degree distribution in such a song–genre network is skewed because all song nodes have degree one, the diameter cannot be computed since the network is disconnected, and each connected component trivially has a diameter of two or less.

3 Definitions

The areas of graph theory and network analysis are young, and many concepts within them do not have a single established notation. The notation chosen for KONECT represents a compromise between familiarity with the most common conventions, and the need to use an unambiguous choice of letters and symbols.

Graphs will be denoted as $G = (V,E)$, in which $V$ is the set of vertices, and $E$ is the set of edges \cite{Bol98}. Without loss of generality, we assume that the vertices $V$ are consecutive natural numbers, i.e.,

$$V = \{1, 2, 3, \ldots, |V|\}.$$  

(1)

Edges $e \in E$ will be denoted as sets of two vertices, i.e., $e = \{u, v\}$. We say that two vertices are adjacent if they are connected by an edge; this will be written as $u \sim v$. We say that an edge is incident to a vertex if the edge touches the vertex.

We also allow loops, i.e., edges of the form $\{u, u\} = \{u\}$. Loops appear for instance in email networks, where it is possible to send an email to oneself, and therefore an edge may connect a vertex with itself. Most networks however do not contain loops, and therefore networks that allow loops are annotated in KONECT with the #loop tag, as described in Section 7.

Most of the time, we work with only one given graph, and therefore it is unambiguous with node and edge set are meant by $V$ and $E$. When ambiguity is possible, we will however use the notation $V[G]$ and $E[G]$ to denote the vertex and edge sets of a graph $G$. This notation may occasionally be extended to other graph characteristics.

In directed networks, edges are pairs instead of sets, i.e., $e = (u, v)$. In directed networks, edges are sometimes called arcs; in KONECT, we use the
In bipartite graphs, we can partition the set of nodes $V$ into two disjoint sets $V_1$ and $V_2$, which we will call the left and right set respectively. Although the assignment of a bipartite network’s two node types to left and right sides is mathematically arbitrary, it is chosen in KONECT such that the left nodes are active and the right nodes are passive. For instance, a rating graph with users and items will always have users on the left since they are active in the sense that it is they who give the ratings. Such a distinction is sensible in most networks [Ops12]. The number of left and right nodes will be denoted $n_1 = |V_1|$ and $n_2 = |V_2|$.

Networks with multiple edges will be written as $G = (V,E)$, where $E$ is a multiset. The degree of nodes in such networks takes into account multiple edges. Thus, the degree does not equal the number of adjacent nodes but the number of incident edges. When $E$ is a multiset, it can contain the edge $\{u,v\}$ multiple times. Mathematically, we may write $\{u,v\}_1$, $\{u,v\}_2$, etc. Note that we will be lax with this notation. In expressions valid for all types of networks, we will use sums such as $\sum_{\{u,v\} \in E}$ and understand that the sum is over all edges.

In positively weighted networks, we define $w$ as the weight function, returning the edge weight when given an edge. In such networks, the weights are not taken into account when computing the degree.

In a signed network, each edge is assigned a signed weight such as $+1$ or $-1$. In such networks, we define $w$ to be the signed weight function. In the general case, we allow arbitrary nonzero real numbers, representing degrees of positive and negative edges.

In rating networks, we define $r$ to be the rating function, returning the rating value when given an edge. Note that rating values are interpreted to be invariant under shifts, i.e., adding a real constant to all ratings in the network must not change the semantics of the network. Thus, we will often make use of the mean rating defined as

$$\mu = \frac{1}{|E|} \sum_{e \in E} r(e).$$  \hspace{1cm} (2)

For consistency, we also define the edge weight function $w$ for unweighted and rating networks:

$$w(e) = \begin{cases} 1 & \text{when } G \text{ is unweighted} \\ r(e) - \mu & \text{when } G \text{ is a rating network} \end{cases}$$ \hspace{1cm} (3)

We also define a weighting function for node pairs, also denoted $w$. This function takes into account both the weight of edges and edge multiplicities. It is defined as $w(u,v) = 0$ when the nodes $u$ and $v$ are not connected and if they
are connected as

\[ w(u, v) = \begin{cases} 
1 & \text{when } G = - \\
|\{k \mid \{u, v\} \in E\}| & \text{when } G = + \\
w(\{u, v\}) & \text{when } G = \pm \\
r(\{u, v\}) - \mu & \text{when } G = * \\
\sum_{\{u, v\} \in E} [r(\{u, v\}) - \mu] & \text{when } G = \ast 
\end{cases} \]  

(4)

Dynamic networks are special in that they have a set of events (edge addition and removal) instead of a set of edges. In most cases, we will model dynamic networks as unweighted networks \( G = (V, E) \) representing their state at the latest known timepoint. For analyses that are performed over time, we consider the graph at different time points, with the graph always being an unweighted graph.

In an unweighted graph \( G = (V, E) \), the degree of a vertex is the number of neighbors of that node

\[ d(u) = \{v \in V \mid \{u, v\} \in E\}. \]  

(5)

In networks with multiple edges, the degree takes into account multiple edges, and thus to be precise, it equals the number of incident edges and not the number of adjacent vertices.

\[ d(u) = \{\{u, v\} \in E \mid v \in V\} \]  

(6)

In directed graphs, the sum is over all of \( u \)'s neighbors, regardless of the edge orientation. Note that the sum of the degrees of all nodes always equals twice the number of edges, i.e.,

\[ \sum_{v \in V} d(u) = 2|E|. \]  

(7)

In a directed graph we define the outdegree \( d_1 \) of a node as the number of outgoing edges, and the indegree \( d_2 \) as the number of ingoing edges.

\[ d_1(u) = \{v \in V \mid (u, v) \in E\} \]  

(8)

\[ d_2(u) = \{v \in V \mid (v, u) \in E\} \]  

(9)

The sum of all outdegrees, and likewise the sum of all indegrees always equals the number of nodes in the network.

\[ \sum_{u \in V} d_1(u) = \sum_{u \in V} d_2(u) = |E| \]  

(10)

We also define the weight of a node, also denoted by the symbol \( w \), as the sum of the absolute weights of incident edges

\[ w(u) = \sum_{\{u, v\} \in E} |w(\{u, v\})|. \]  

(11)

The weight of a node coincides with the degree of a node in unweighted networks and networks with multiple edges. The weight of a node may also be called its strength [OAS10].
3.1 Graph Transformations

Sometimes, it is necessary to construct a graph out of another graph. In the following, we briefly review such constructions.

Let $G = (V, E, w)$ be any weighted, signed or rating graph, regardless of edge multiplicities. Then, $\bar{G}$ will denote the corresponding unweighted graph, i.e.,

$$\bar{G} = (V, E).$$

(12)

Note that the graph $\bar{G}$ may still contain multiple edges.

Let $G = (V, E, w)$ be any graph with multiple edges. We define the corresponding unweighted simple graphs as

$$\bar{G} = (V, \bar{E}),$$

(13)

where $\bar{E}$ is the set underlying the multiset $E$. For simple graphs, we define $\bar{G} = G$.

Let $G = (V, E, w)$ be a signed or rating network. Then, $|G|$ will denote the corresponding unsigned graph defined by

$$|G| = (V, E, w')$$

$$w'(e) = |w(e)|.$$  

(14)

Let $G = (V, E, w)$ be any network with weight function $w$. The negative network to $G$ is then defined as

$$-G = (V, E, w')$$

$$w'(e) = -w(e).$$

(15)

This construction is possible for all types of networks. For unweighted and positively weighted networks, it leads to signed networks.

3.2 Characteristic Matrices

A very useful representation of graph is using matrices. In fact, a subfield of graph theory, *algebraic graph theory*, is devoted to this representation [GR01]. When a graph is represented as a matrix, operations on graphs can often be expressed as simple algebraic expressions. For instance, the number of common friends of two people in a social network can be expressed as the square of a matrix.

An unweighted graph $G = (V, E)$ can be represented by a $|V|$-by-$|V|$ matrix containing the values 0 and 1, denoting whether a certain edges between two nodes is present. This matrix is called the adjacency matrix of $G$ and will be denoted $A$. Remember that we assume that the vertices are the natural numbers $1, 2, \ldots, |V|$. Then the entry $A_{uv}$ is one when $\{u, v\} \in E$ and zero when not. This makes $A$ square and symmetric for undirected graphs, generally asymmetric (but still square) for directed graphs.
For a bipartite graph \( G = (V_1 \cup V_2, E) \), the adjacency matrix has the form
\[
A = \begin{bmatrix}
  B & \mathbf{B}^T
\end{bmatrix}.
\] (16)

The matrix \( \mathbf{B} \) is a \(|V_1|\)--by--\(|V_2|\) matrix, and thus generally rectangular. \( \mathbf{B} \) will be called the biadjacency matrix.

In weighted networks, the adjacency matrix takes into account edge weights. In networks with multiple edges, the adjacency matrix takes into account edge multiplicities. Thus, the general definition of the adjacency matrix is given by
\[
A_{uv} = w(u, v).
\] (17)

The degree matrix \( \mathbf{D} \) is a diagonal \(|V|\)--by--\(|V|\) matrix containing the absolute weights of all nodes, i.e.,
\[
D_{uu} = |w(u)|.
\] (18)

Note that we define the degree matrix explicitly to contain node weights instead of degrees, to be consistent with the definition of \( \mathbf{A} \).

The normalized adjacency matrix \( \mathbf{N} \) is a \(|V|\)--by--\(|V|\) matrix given by
\[
N = \mathbf{D}^{-1/2} \mathbf{A} \mathbf{D}^{-1/2}.
\] (19)

Finally the Laplacian matrix \( \mathbf{L} \) is an \(|V|\)--by--\(|V|\) matrix defined as
\[
L = \mathbf{D} - \mathbf{A}.
\] (20)

Note that in some disciplines the Laplacian matrix may be defined as \( \mathbf{A} - \mathbf{D} \), making it negative-semidefinite.

Other matrices used in KONECT include the normalized Laplacian matrix, the stochastic adjacency matrix and the signless Laplacian.

The normalized Laplacian \( \mathbf{Z} \) is a normalized version of the Laplacian matrix \( \mathbf{L} \). Just as the ordinary Laplacian, \( \mathbf{Z} \) capture aspects of the graph that are useful for clustering.
\[
Z = \mathbf{I} - \mathbf{N} = \mathbf{D}^{-1/2} \mathbf{L} \mathbf{D}^{-1/2}
\] (21)

The equation \( \mathbf{Z} = \mathbf{I} - \mathbf{N} \) shows that \( \mathbf{Z} \) has the same eigenvectors as \( \mathbf{N} \), and its eigenvalues are those of \( \mathbf{N} \), but shifted and inverted.

The consideration of random walks on a graph leads to the definition of the stochastic adjacency matrix \( \mathbf{P} \). Imagine a random walker on the nodes of a graph, who can walk from node to node by following edges. If, at each edge, the probability that the random walker will go to each neighboring node with equal probability, then the random walk can be described be the transition probability matrix defined as
\[
P = \mathbf{D}^{-1} \mathbf{A} = \mathbf{D}^{-1/2} \mathbf{N} \mathbf{D}^{1/2}.
\] (22)
The matrix $P$ is right stochastic, since its row sums are one.

A further variant of Laplacian matrix is the signless Laplacian $K$.

$$K = D + A.$$  

(23)

The signless Laplacian $K$ corresponds to the ordinary Laplacian $L$ of the graph with inverted edge weights, i.e., $K[G] = L[-G]$.

Note that in most cases, we work on just a single graph, and it is implicit that the characteristic matrices apply to this graph. In a few cases, we may need to consider the characteristic matrices of multiple graphs. In these cases, we will write

$$A[G], D[G], L[G], \ldots$$

to denote the characteristic matrices of the graph $G$.

4 Statistics

A network statistic is a numerical value that characterizes a network. Examples of network statistics are the number of nodes and the number of edges in a network, but also more complex measures such as the diameter and the clustering coefficient. Statistics are the basis of most network analysis methods; they can be used to compare networks, classify networks, detect anomalies in networks and for many other tasks. Network statistics are also used to map a network’s structure to a simple numerical space, in which many standard statistical methods can be applied. Thus, network statistics are essential for the analysis of almost all network types. All statistics described in KONECT are real numbers.

This section gives the definitions for the statistics supported by KONECT, and briefly reviews their uses. All network statistics can be computed using the KONECT toolbox using the function `konect_statistic()`. Each statistic has an internal name that must be passed as the first argument to `konect_statistic()`. The internal names are given in the margin in this section. Additionally, the KONECT toolbox includes functions named `konect_statistic.<NAME>()` which compute a single statistic `<NAME>`.

The values of selected statistics are shown for the KONECT networks on the website[1].

4.1 Basic Network Statistics

Some statistics are simple to define, trivial to compute, and are reported universally in studies about networks. These include the number of nodes, the number of edges, and statistics derived from them such as the average number of neighbors a node has.

[1]konect.uni-koblenz.de/statistics
The size of a network is the number of nodes it contains, and is almost universally denoted $n$. The size of a graph is sometimes also called the order of the graph.

$$ n = |V| \quad (24) \quad \text{size} $$

In a bipartite graph, the size can be decomposed as $n = n_1 + n_2$ with $n_1 = |V_1|$ and $n_2 = |V_2|$. The size of a network is not necessarily a very meaningful number. For instance, adding a node without edges to a network will increase the size of the network, but will not change anything in the network. In the case of an online social network, this would correspond to creating a user account and not connecting it to any other users – this adds an inactive user, which are often not taken into account. Therefore, a more representative measure of the size of a network is actually given by the number of edges, giving the volume of a network.

The volume of a network equals the number of edges and is defined as

$$ m = |E|, \quad (25) \quad \text{volume} $$

Note that in mathematical contexts, the number of edges may be called the size of the graph, in which case the number of nodes is called the order. In this text, we will consistently use size for the number of nodes and volume for the number of edges.

The volume can be expressed in terms of the adjacency or biadjacency matrix of the underlying unweighted graph as

$$ m = \begin{cases} \frac{1}{2} \|A[\bar{G}]\|_F^2 & \text{when } G \text{ is undirected} \\ \|A[G]\|_F^2 & \text{when } G \text{ is directed} \\ \|B[\bar{G}]\|_F^2 & \text{when } G \text{ is bipartite} \end{cases} \quad (26) $$

The number of edges in a network is often considered a better measure of the size of a network than the number vertices, since a vertex unconnected to any other vertices may often be ignored. On the practical side, the volume is also a much better indicator of the amount of memory needed to represent a network.

We will also make use of the number of edges without counting multiple edges. We will call this the unique volume of the graph.

$$ \bar{m} = m[\bar{G}] \quad (27) \quad \text{uniquevolume} $$

The weight $w$ of a network is defined as the sum of absolute edge weights. For unweighted networks, the weight equals the volume. For rating networks, remember that the weight is defined as the sum over ratings from which the overall mean rating has been subtracted, in accordance with the definition of the adjacency matrix for these networks.

$$ w = \sum_{e \in E} |w(e)| \quad (28) \quad \text{weight} $$
The average degree is defined as

\[ d = \frac{1}{|V|} \sum_{u \in V} d(u) = \frac{2m}{n} \]  

The average degree is sometimes called the density. We avoid the term density in KONECT as it is sometimes used for the fill, which denotes the probability that an edge exists. In bipartite networks, we additionally define the left and right average degree

\[ d_1 = \frac{1}{|V_1|} \sum_{u \in V_1} d(u) = \frac{m}{n_1} \]  

\[ d_2 = \frac{1}{|V_2|} \sum_{u \in V_2} d(u) = \frac{m}{n_2} \]

Note that in directed networks, the average outdegree equals the average indegree, and both are equal to \( m/n \).

The fill of a network is the proportion of edges to the total number of possible edges. The fill is used as a basic parameter in the Erdős–Rényi random graph model [ER59], where it denotes the probability that an edge is present between two randomly chosen nodes, and is usually called \( p \), which is the notation we also use in KONECT.

\[ p = \begin{cases} 
2m/\lfloor n(n+1) \rfloor & \text{when } G \text{ is undirected} \\
m/n^2 & \text{when } G \text{ is directed} \\
m/(n_1 n_2) & \text{when } G \text{ is bipartite} 
\end{cases} \]

In the undirected case, the expression is explained by the fact that the total number of possible edges is \( n(n+1)/2 \) including loops. The corresponding numbers for directed and bipartite networks are \( n^2 \) and \( n_1 n_2 \). The fill is sometimes also called the density of the network, in particular in a mathematical context, or the connectance of the network.

The maximum degree equals the highest degree value attained by any node.

\[ d_{\text{max}} = \max_{u \in V} d(u) \]

In a directed network, the reciprocity equals the proportion of edges for which an edge in the opposite direction exists, i.e., that are reciprocated.

\[ \rho = \frac{|\{(u,v) \in E \mid (v,u) \in E\}|}{m} \]

The reciprocity can give an idea of the type of network. For instance, citation networks only contain only few pairs of papers that mutually cite each other. On the other hand, an email network will contain many pairs of people who have sent emails to each other. Thus, citation networks typically have low reciprocity, and communication networks have high reciprocity.
In networks that allow negative edges such as signed networks and rating networks, we may be interested in the proportion of edges that are actually negative. We call this the \textit{negativity} of the network.

\[
\frac{\left| \{ e \in E \mid w(e) < 0 \} \right|}{m}
\]

4.2 Connectivity Statistics

Connectivity statistics measure to what extent a network is connected. Two nodes are said to be connected when they are either directly connected through an edge, or indirectly through a path of several edges. A connected component is a set of vertices all of which are connected, and unconnected to the other nodes in the network. The largest connected component in a network is usually very large and called the giant connected component. When it contains all nodes, the network is connected.

The size of the largest connected component is denoted \(N\).

\[
N = \max_{F \subseteq C} |F|
\]

\(C = \{ C \subseteq V \mid \forall u, v \in C : \exists w_1, w_2, \ldots \in V : u \sim w_1 \sim w_2 \sim \cdots \sim v \}\)

In bipartite networks, the number of left and right nodes in the largest connected components are denoted \(N_1\) and \(N_2\), with \(N_1 + N_2 = N\).

The relative size of the largest connected component equals the size of the largest connected component divided by the size of the network.

\[
N_{rel} = \frac{N}{n}.
\]

In directed networks, we additionally define the size of the largest strongly connected component \(N_s\). A strongly connected component is a set of vertices in a directed graph such that any node is reachable from any other node using a path following only directed edges in the forward direction. We always have \(N_s \leq N\).

4.3 Count Statistics

The fundamental building block of a network are the edges. Thus, the number of edges is a basic statistic of any network. To understand the structure of a network, it is however not enough to analyse edges individually. Instead, larger patterns such as triangles must be considered. These patterns can be counted, and give rise to count statistics, i.e., statistics that count the number of occurrences of specific patterns.

Table \[\] gives a list of fundamental patterns in networks, and their corresponding count statistics.

A star is defined as a graph in which a node is connected to \(k\) other nodes. A \(k\)-star is defined as a star in which the central node is connected to \(k\) other nodes.
Table 4: Patterns that occur in networks. Each pattern can be counted, giving rise to a count statistic.

| Pattern | Name(s) | Count statistic | Internal name |
|---------|---------|----------------|---------------|
| ![Edge](image1) | Edge, 1-path, 1-star, 2-clique | Volume $m$ | volume |
| ![Wedge](image2) | Wedge, 2-star, 2-path | Wedge count $s$ | twostars |
| ![Claw](image3) | Claw, 3-star | Claw count $z$ | threestars |
| ![4-star](image4) | 4-star | 4-star count $x$ | fourstars |
| ![Triangle](image5) | Triangle, 3-cycle, 3-clique | Triangle count $t$ | triangles |
| ![Square](image6) | Square, 4-cycle | Square count $q$ | squares |

Thus, a 2-star consists of a node connected to two other nodes, or equivalently two incident edges, or a path of length 2. 2-stars are also called wedges. The number of 2-stars can be defined as

$$s = \sum_{u \in V} \left( \frac{d(u)}{2} \right) = \sum_{u \in V} \frac{1}{2} d(u)(d(u) - 1),$$

(38) **twostars**

where $d(u)$ is the degree of node $u$.

The number of triangles defined in the following way is independent of the orientation of edges when the graph is directed. Loops in the graph, as well as edge multiplicities, are ignored.

$$t = |\{u, v, w \mid u \sim v \sim w \sim u\}|$$

(39) **triangles**

Three-stars are defined analogously to two-stars, and their count denoted $z$. Three-stars are also called claws.

$$z = \sum_{u \in V} \left( \frac{d(u)}{3} \right) = \sum_{u \in V} \frac{1}{6} d(u)(d(u) - 1)(d(u) - 2)$$

(40) **threestars**

A square is a cycle of length four, and the number of squares in a graph will be denoted $q$.

$$q = |\{u, v, w, x \mid u \sim v \sim w \sim x \sim u\}|$$

(41) **squares**

Multiple edges are ignored in these count statistics, and edges in patterns are not allowed to overlap.
Triangles and squares are both cycles – which we can generalize to $k$-cycles, sequences of $k$ distinct vertices that are cyclically linked by edges. We denote the number of $k$-cycles by $C_k$. For small $k$, we note the following equivalences:

\[
C_1 = 0 \\
C_2 = m \\
C_3 = t \\
C_4 = q
\]

for graphs without loops. Cycles of length three and four have special notation: $C_3 = t$ and $C_4 = q$ and are called triangles and squares.

A cycle cannot the same node twice. Due to this combinatorial restriction, $C_k$ is quite complex to compute for large $k$. Therefore, we may use tours instead, defined as cyclical lists of connected vertices in which we allow several vertices to overlap. The number of $k$-tours will be denoted $T_k$. For computational convenience, we will define labeled tours, where two tours are not equal when they are identical up to shifts or inversions. We note the following equalities:

\[
T_1 = 0 \\
T_2 = 2m \\
T_3 = 6t \\
T_4 = 8q + 4s + 2m
\]

Again, these are true when the graph is loopless. The last equality shows that trying to divide the tour count by $2k$ to count them up to shifts and inversions is a bad idea, since it cannot be implemented by dividing the present definition by $2k$.

As mentioned before, counting cycles is a complex problem. Counting tours is however much easier. The number of tours of length $k$ can be expressed as the trace of a power of the graph’s adjacency matrix, and thus also as a moment of the adjacency matrix’s spectrum when $k > 2$.

\[
T_k = \text{Tr}(A^k) = \sum_i \lambda_i [A]^k
\]

This remains true when the graph includes loops.

### 4.4 Degree Distribution Statistics

The distribution of degree values $d(u)$ over all nodes $u$ is often taken to characterize a network. Thus, a certain number of network statistics are based solely on this distribution, regardless of overall network structure.

The power law exponent is a number that characterizes the degrees of the nodes in the network. In many circumstances, networks are modeled to follow a degree distribution power law, i.e., the number of nodes with degree $n$ is taken to be proportional to the power $n^{-\gamma}$, for a constant $\gamma$ larger than one \[BA99\].
This constant $\gamma$ is called the power law exponent. Given a network, its degree distribution can be used to estimate a value $\gamma$. There are multiple ways of estimating $\gamma$, and thus a network does not have a single definite value of it. In KONECT, we estimate $\gamma$ using the robust method given in [New06, Eq. 5]

$$\gamma = 1 + n \left( \sum_{u \in V} \ln \frac{d(u)}{d_{\min}} \right)^{-1},$$

(43) power

in which $d_{\min}$ is the minimal degree.

The Gini coefficient is a measure of inequality from economics, typically applied to distributions of wealth or income. In KONECT, we apply it to the degree distribution, as described in [KP12]. The Gini coefficient can either be defined in terms of the Lorenz curve, a type of plot that visualizes the inequality of a distribution, or using the following expression. Let $d_1 \leq d_2 \leq \cdots \leq d_n$ be the sorted list of degrees in the network. Then, the Gini coefficient is defined as

$$G = \frac{2}{n} \sum_{i=1}^{n} i d_i - \frac{n + 1}{n}.$$  (44) gini

The Gini coefficient takes values between zero and one, with zero denoting total equality between degrees, and one denoting the dominance of a single node.

The relative edge distribution entropy is a measure of the equality of the degree distribution, and equals one when all degrees are equal, and attains the limit value of zero when all edges attach to a single node [KP12]. It is defined as

$$H_{ev} = \frac{1}{\ln n} \sum_{u \in V} \frac{d(u)}{2m} \ln \frac{d(u)}{2m}.$$  (45) dentropyn

Another statistic for measuring the inequality in the degree distribution is associated with the Lorenz curve (see Section 3.3), and is given by the intersection point of the Lorenz curve with the antidiagonal given by $y = 1 - x$ [KP12]. By construction, this point equals $(1 - P, P)$ for some $0 < P < 1$, where the value $P$ corresponds exactly to the number “25%” in the statement “25% of all users account for 75% of all friendship links on Facebook”. By construction, we can expect $P$ to be smaller when $G$ is large.

The analysis of degrees can be generalized to pairs of nodes: What is the distribution of degrees for pairs of connected edges? In some networks, high-degree nodes are connected to other high-degree nodes, while low-degree nodes are connected to low-degree nodes. This property is called assortativity. Inversely, in a network with dissortativity, high-degree nodes are typically connected to low-degree and vice versa. The amount of assortativity can be measured by the Pearson correlation $\rho$ between the degree of connected nodes.

4.5 Clustering Statistics

The term clustering refers to the observation that in almost all networks, nodes tend to form small groups within which many edges are present, and such that
only few edges connected different clusters with each other. In a social network for instance, people form groups in which almost every member known the other members. Clustering thus forms one of the primary characteristics of real-world networks, and thus many statistics for measuring it have been defined. The main method for measuring clustering numerically is the clustering coefficient, of which there exist several variants. As a general rule, the clustering coefficient measures to what extent edges in a network tend to form triangles. Since it is based on triangles, it can only be applied to unipartite networks, because bipartite networks do not contain triangles.

The number of triangles $t$ itself as defined in Section 4.3 is however not a statistic that can be used to measure the clustering in a network, since it correlates with the size and volume of the network. Instead, the clustering coefficients in all its variants can be understood as a count of triangles, normalized in different ways in order to compare several networks with it.

The local clustering coefficient $c(u)$ of a node $u$ is defined as the probability that two randomly chosen (but distinct) neighbors of $u$ are connected \cite{WS98}.

\[
c(u) = \begin{cases} 
\{v, w \in V \mid u \sim v \sim w \sim u\} & \text{when } d(u) > 1 \\
\{v, w \in V \mid u \sim v \neq w \sim u\} & \text{when } d(u) \leq 1 
\end{cases}
\]  

The global clustering of a network can be computed in two ways. The first way defines it as the probability that two incident edges are completed by a third edge to form a triangle \cite{NWS02}. This is also called the transitivity ratio, or simply the transitivity.

\[
c = \frac{\left| \left\{ u, v, w \in V \mid u \sim v \sim w \sim u \right\} \right|}{\left| \left\{ u, v, w \in V \mid u \sim v \neq w \sim u \right\} \right|} = \frac{3t}{s} \tag{47} \text{clusco}
\]

This variant of the global clustering coefficient has values between zero and one, with a value of one denoting that all possible triangles are formed (i.e., the network consists of disconnected cliques), and zero when it is triangle free. Note that the clustering coefficient is trivially zero for bipartite graphs. This clustering coefficient is however not defined when each node has degree zero or one, i.e., when the graph is a disjoint union of edges and unconnected nodes. This is however not a problem in practice.

The second variant of the clustering coefficient uses the average of the local clustering coefficients. This second variant was historically the first to be defined. It was defined in 1998 \cite{WS98} and precedes the first variant by four years.

\[
c_2 = \frac{1}{|V|} \sum_{u \in V} c(u) \tag{48} \text{clusco2}
\]

This second variant of the global clustering coefficient is zero when a graph is triangle-free, and one when the graph is a disjoint union of cliques of size at least three. This variant of the global clustering coefficient is defined for all graphs, except for the empty graph, i.e., the graph with zero nodes. A slightly
different definition of the second variant computes the average only over nodes with a degree of at least two, as seen for instance in [BKM08].

Because of the arbitrary decision to define \( c(u) \) as zero when the degree of \( c \) is zero or one, we recommend to use the first variant of the clustering coefficient. In the following, the extensions to the clustering coefficient we present are all based on the first variant, \( c \).

For signed graphs, we may define the clustering coefficient to take into account the sign of edges. The signed clustering coefficient is based on balance theory [KLB09]. In a signed network, edges can be positive or negative. For instance in a signed social network, positive edges represent friendship, while negative edges represent enmity. In such networks, balance theory stipulates than triangles tend to be balanced, i.e., that three people are either all friends, or two of them are friends with each other, and enemies with the third. On the other hand, a triangle with two positive and one negative edge, or a triangle with three negative edges is unbalanced. In other words, we can define the sign of a triangle as the product of the three edge signs, which then leads to the stipulation that triangles tend to have positive weight. To extend the clustering coefficient to signed networks, we thus distinguish between balanced and unbalanced triangles, in a way that positive triangles contribute positively to the signed clustering coefficient, and negative triangles contribute negatively to it. For a triangle \( \{u, v, w\} \), let \( \sigma(u, v, w) = w(u, v)w(v, w)w(w, u) \) be the sign of the triangle, then the following definition captures the idea:

\[
c_s = \frac{\sum_{u,v,w \in V} \sigma(u,v,w)}{|\{u,v,w \in V \mid u \sim v \neq w \sim u\}|}
\]  

(49)

Here, the sum is over all triangles \( \{u, v, w\} \), but can also be taken over all triples of vertices, since \( w(u, v) = 0 \) when \( \{u, v\} \) is not an edge.

The signed clustering coefficient is bounded by the clustering coefficient:

\[
|c_s| \leq c
\]

(50)

The relative signed clustering coefficient can then be defined as

\[
c_r = \frac{c_s}{c} = \frac{\sum_{u,v,w \in V} \sigma(u,v,w)}{|\{u,v,w \in V \mid u \sim v \sim w \sim u\}|}
\]

(51)

which also equals the proportion of all triangles that are balanced, minus the proportion of edges that are unbalanced.

4.6 Distance Statistics

The distance between two nodes in a network is defined as the number of edges needed to reach one node from another, and serves as the basis for a class of network statistics.

A path in a network is a sequence of incident edges, or equivalently, a sequence of nodes \( P = (u_0, u_2, \ldots, u_k) \), such that \( (u_i, u_{i+1}) \in E \) for all \( i \in \)
\{0, \ldots, k - 1\}. The number \(k\) is called the length of the path, and will also be denoted \(l(P)\). A further restriction can be set on the visited nodes, defining that each node can only be visited at most once. If the distinction is made, the term path is usually reserved for sequences of non-repeating nodes, and general sequence of adjacent nodes are then called walks. We will not make this distinction here.

Paths in networks can be used to model browsing behavior of people in hyperlink networks, navigation in transport networks, and other types of movement-like activities in a network. When considering navigation and browsing, an important problem is the search for shortest paths. Since the length of a path determines the number of steps needed to reach one node from another, it can be used as a measure of distance between nodes of a network. The distance defined in this way may also be called the shortest-path distance to distinguish it from other distance measures between nodes of a network.

\[
d(u, v) = \begin{cases} 
\min \{l(P) \mid P = (u, \ldots, v)\} & \text{when } u \text{ and } v \text{ are connected} \\
\infty & \text{when } u \text{ and } v \text{ are not connected}
\end{cases}
\]

(52)

In the case that a network is not connected, the distance is defined as infinite. In practice, only the largest connected component of a network may be used, making it unnecessary to deal with infinite values. The distribution of all \(|V|^2\) values \(d(u, v)\) for all \(u, v \in E\) is called the distance distribution, and it too characterizes the network.

The eccentricity of a node can then be defined as the maximal distance from that node to any other node, defining a measure of non-centrality:

\[
\epsilon(u) = \max_{v \in E} d(u, v)
\]

(53)

The diameter \(\delta\) of a graph equals the longest shortest path in the network. It can be equivalently defined as the largest eccentricity of all nodes.

\[
\delta = \max_{u \in E} \epsilon(u) = \max_{u, v \in E} d(u, v)
\]

(54)\text{ diam}

Note that the diameter is undefined (or infinite) in unconnected networks, and thus in numbers reported for actual networks in KONECT we consider always the diameter of the network’s largest connected component. Due to the high runtime complexity of computing the diameter, it may be estimated by various methods, in which case it is noted noted \(\hat{\delta}\).

A statistic related to the diameter is the radius, defined as the smallest eccentricity

\[
r = \min_{u \in E} \epsilon(u) = \min_{u \in E} \max_{v \in E} d(u, v)
\]

(55)\text{ radius}

The radius is bounded from above by the diameter, and from below by half the diameter, i.e.,

\[
\frac{1}{2} \delta \leq r \leq \delta
\]
The radius and the diameter are not very expressive statistics: Adding or removing an edge will, in many cases, not change their values. Thus, a better statistic that reflects the typical distances in a network in given by the mean and average distance.

The mean path length $\delta_m$ in a network is defined as as the mean distance over all node pairs, including the distance between a node and itself:

$$
\delta_m = \frac{1}{n^2} \sum_{u \in V} \sum_{v \in V} d(u, v)
$$

The mean path length defined in this way is undefined when a graph is disconnected.

Likewise, the median path length $\delta_M$ is the median length of shortest paths in the network. In KONECT, both the median and mean path lengths are computed taking into account node pairs of the form $(u, u)$.

Both the mean and median path length can be called the characteristic path length of the network.

A related statistic is the 90-percentile effective diameter $\delta_{0.9}$, which equals the number of edges needed on average to reach 90% of all other nodes.

### 4.7 Algebraic Statistics

Algebraic statistics are based on a network’s characteristic matrices. They are motivated by the broader field of spectral graph theory, which characterizes graphs using the spectra of these matrices [Chu97].

In the following we will denote by $\lambda_k[X]$ the $k$th dominant eigenvalue of the matrix $X$. For the adjacency matrix $A$, the dominant eigenvalues are the largest absolute ones; for the Laplacian $L$, they are the smallest ones.

Also, the matrix $L$ will only be considered for the network’s largest connected component.

The spectral norm of a network equals the spectral norm (i.e., the largest absolute eigenvalue) of the network’s adjacency matrix

$$
|\lambda_1[A]| = \|A\|_2.
$$

The spectral norm can be understood as an alternative measure of the size of a network.

The algebraic connectivity equals the second smallest nonzero eigenvalue of $L$ [Fie73]

$$
a = \lambda_2[L].
$$

The algebraic connectivity is zero when the network is disconnected – this is one reason why we restrict the matrix $L$ to each network’s giant connected component. The algebraic connectivity is larger the better the network’s largest connected component is connected.

In signed and ratings networks, i.e., networks in which the weights of node pairs can be negative, the smallest eigenvalue of $L$ can be larger than zero. (In
other networks, it is always zero.) The algebraic conflict equals this smallest eigenvalue

\[ \xi = \lambda_1 |L| \]. \hspace{1cm} (59) conflict

The algebraic conflict measures the amount of conflict in the network, i.e., the tendency of the network to contain cycles with an odd number of negatively weighted edges.

4.8 Bipartivity Statistics

Some unipartite networks are almost bipartite. Almost-bipartite networks include networks of sexual contact [LEA+01] and ratings in online dating sites [BP07, KGG12]. Other, more subtle cases, involve online social networks. For instance, the follower graph of the microblogging service Twitter is by construction unipartite, but has been observed to reflect, to a large extent, the usage of Twitter as a news service [KLP10]. This is reflected in the fact that it is possible to identify two kinds of users: Those who primarily get followed and those who primarily follow. Thus, the Twitter follower graph is almost bipartite. Other social networks do not necessarily have a near-bipartite structure, but the question might be interesting to ask to what extent a network is bipartite. To answer this question, measures of bipartivity have been developed.

Instead of defining measures of bipartivity, we will instead consider measures of non-bipartivity, as these can be defined in a way that they equal zero when the graph is zero. Given an (a priori) unipartite graph, a measure of non-bipartivity characterizes the extent to which it fails to be bipartite. These measures are defined for all networks, but are trivially zero for bipartite networks. For non-bipartite networks, they are larger than zero.

A first measure of bipartivity consists in counting the minimum number of frustrated edges [HLEK03]. Given a bipartition of vertices \( V = V_1 \cup V_2 \), a frustrated edge is an edge connecting two nodes in \( V_1 \) or two nodes in \( V_2 \). Let \( f \) be the minimal number of frustrated edges in any bipartition of \( V \), or, put differently, the minimum number of edges that have to be removed from the graph to make it bipartite. Then, a measure of non-bipartivity is given by

\[ F = \frac{f}{|E|} \]. \hspace{1cm} (60) frustration

This statistic is always in the range \([0, 1/2]\). It attains the value zero if and only if \( G \) is bipartite.

The minimal number of frustrated edges \( f \) can be approximated by algebraic graph theory. First, we represent a bipartition \( V = V_1 \cup V_2 \) by its characteristic vector \( x \in \mathbb{R}^{|V|} \) defined as

\[ x_u = \begin{cases} +1/2 & \text{when } u \in V_1 \\ -1/2 & \text{when } u \in V_2 \end{cases} \]
Note that the number of edges connecting the sets $V_1$ and $V_2$ is then given by
\[
\{\{u,v\} \mid u \in V_1, v \in V_2\} = \frac{1}{2} x^T K[\bar{G}] x = \frac{1}{2} \sum_{(u,v) \in E} (x_u + x_v)^2,
\]
where $K[\bar{G}] = D[\bar{G}] + A[\bar{G}]$ is the signless Laplacian matrix of the underlying unweighted graph. Thus, the minimal number of frustrated edges $f$ is given by
\[
f = \min_{x \in \{\pm 1/2\}^{|V|}} \frac{1}{2} x^T K[\bar{G}] x.
\]
By relaxing the condition $x \in \{\pm 1/2\}^{|V|}$, we can express $f$ in function of $K[\bar{G}]$’s minimal eigenvalue, using the fact that the norm of all vectors $x \in \{\pm 1/2\}^{|V|}$ equals $\sqrt{|V|}/4$, and the property that the minimal eigenvalue of a matrix equals its minimal Rayleigh quotient.
\[
\frac{2f}{|V|/4} \approx \min_{x \neq 0} \frac{x^T K[\bar{G}] x}{||x||^2} = \lambda_{\min}[K[\bar{G}]]
\]
We can thus approximate the previous measure of non-bipartivity by
\[
\hat{F} = \frac{|V|}{8|E[\bar{G}]|} \lambda_{\min}[K[\bar{G}]]
\]
(61) anticonflict

The eigenvalue $\lambda_{\min}[K[\bar{G}]]$ can also be interpreted as the algebraic conflict in $G$ interpreted as a signed graph in which all edges have negative weight.

A further measure of bipartivity exploits the fact that the adjacency matrix $A$ of a bipartite graph has eigenvalues symmetric around zero, i.e., all eigenvalues of a bipartite graph come in pairs $\pm \lambda$. Thus, the ratio of the smallest and largest eigenvalues can be used as a measure of non-bipartivity
\[
b_\lambda = 1 - \frac{\lambda_{\min}[A[\bar{G}]]}{\lambda_{\max}[A[\bar{G}]]},
\]
(62) nonbip

where $\lambda_{\min}$ and $\lambda_{\max}$ are the smallest and largest eigenvalue of the given matrix, and $\bar{G}$ is the unweighted graph underlying $G$. Since the largest eigenvalue always has a larger absolute value than the smallest eigenvalue (due to the Perron–Frobenius theorem, and from the nonnegativity of $A[\bar{G}]$), it follows that this measure of non-bipartivity is always in the interval $[0, 1)$, with zero denoting a bipartite network.

Another spectral measure of non-bipartivity is based on considering the smallest eigenvalue of the matrix $N[\bar{G}]$. This eigenvalue is $-1$ exactly when $G$ is bipartite. Thus, this value minus one is a measure of non-bipartivity. Equivalently, it equals two minus the largest eigenvalue of the normalized Laplacian matrix $Z$.
\[
b_N = \lambda_{\min}[N[\bar{G}]] + 1 = 2 - \lambda_{\max}[Z[\bar{G}]]
\]
(63) nonbipn
5 Plots

Plots are drawn to visualize a certain aspect of a dataset. These plots can be used to compare several network visually, or to illustrate the definition of a certain numerical statistic.

As a running example, we show the plots for the Wikipedia elections network (EL). Plots for all networks (in which computation was feasible) are shown on the KONECT website. The KONECT Matlab toolbox contains code for generating these plot types.

5.1 Temporal Distribution

The temporal distributions shows the distribution of edge creation times. It is only defined for networks with known edge creation times. The X axis is the time, and the Y axis is the number of edges added during each time interval.

![Temporal Distribution](image)

Figure 2: The temporal distribution of edges for the Wikipedia elections network.

5.2 Edge Weight and Multiplicity Distribution

The edge weight and multiplicity distribution plots show the distribution of edge weights and of edge multiplicities, respectively. They are not generated for unweighted networks. The X axis shows values of the edge weights or multiplicities, and the Y axis shows frequencies. Edge multiplicity distributions are plotted on doubly logarithmic scales.

5.3 Degree Distribution

The distribution of degree values \( d(u) \) over all vertices \( u \) characterizes the network as a whole, and is often used to visualize a network. In particular, a power

\[ d(u) \sim n^{-\gamma} \]

\[ d(u) \sim n^\gamma \]

\[ d(u) \sim n^{-\gamma} \]

\[ d(u) \sim n^\gamma \]

\[ d(u) \sim n^{-\gamma} \]

\[ d(u) \sim n^\gamma \]

\[ d(u) \sim n^{-\gamma} \]

\[ d(u) \sim n^\gamma \]

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law is often assumed, stating that the number of nodes with \( n \) neighbors is proportional to \( n^{-\gamma} \), for a constant \( \gamma \) \cite{BA99}. This assumption can be inspected visually by plotting the degree distribution on a doubly logarithmic scale, on which a power law renders as a straight line. KONECT supports two different plots: The degree distribution, and the cumulative degree distribution. The degree distribution shows the number of nodes with degree \( n \), in function of \( n \). The cumulative degree distribution shows the probability that the degree of a node picked at random is larger than \( n \), in function of \( n \). Both plots use a doubly logarithmic scale.

Another visualization of the degree distribution supported by KONECT is in the form of the Lorenz curve, a type of plot to measure inequality originally used in economics (not shown).

Figure 3: The distribution of (a) edge weights for the MovieLens rating network (M2) and (b) edge multiplicities for the German Wikipedia edit network (de).

Figure 4: The degree distribution and cumulative degree distribution for the Wikipedia election network (EL).
The Lorenz curve is a tool originally from economics that visualizes statements of the form “X% of nodes with smallest degree account for Y% of edges”. The set of values \((X, Y)\) thus defined is the Lorenz curve. In a network the Lorenz curve is a straight diagonal line when all nodes have the same degree, and curved otherwise \([KP12]\). The area between the Lorenz curve and the diagonal is half the Gini coefficient (see above).

![Lorenz curve for Wikipedia election network](EL)

**Figure 5:** The Lorenz curve for the Wikipedia election network (EL).

### 5.4 Clustering Coefficient Distribution

In Section 4.5 we defined the clustering coefficient of a node in a graph as the proportion of that node’s neighbors that are connected, and proceeded to define the clustering coefficient as the corresponding measure applied to the whole network. In some case however, we may be interested in the distribution of the clustering coefficient over the nodes in the network. For instance, a network could have some very clustered parts, and some less clustered parts, while another network could have many nodes with a similar, average clustering coefficient. Thus, we may want to consider the distribution of clustering coefficient. This distribution can be plotted as a cumulated plot.

### 5.5 Spectral Plot

The eigenvalues of a network’s characteristic matrices \(A, N\) and \(L\) are often used to characterize the network as a whole. KONECT supports computing and visualizing the spectrum (i.e., the set of eigenvalues) of a network in multiple ways. Two types of plots are supported: Those showing the top-\(k\) eigenvalues computed exactly, and those showing the overall distribution of eigenvalues, computed approximately. The eigenvalues of \(A\) are positive and negative reals, the eigenvalues of \(N\) are in the range \([-1, +1]\), and the eigenvalues of \(L\) are all nonnegative. For \(A\) and \(N\), the largest absolute eigenvalues are used, while for \(L\) the smallest eigenvalues are used. The number of eigenvalue shown \(k\) depends
Local clustering coefficient ($c$)

Figure 6: The clustering coefficient distribution for Facebook link network (Ol).

on the network, and is chosen by KONECT such as to result in reasonable runtimes for the decomposition algorithms.

Figure 7: The top-$k$ eigenvalues of $A$ and the cumulative spectral distribution of $N$ for the Wikipedia election network (EL). In the first plot (a), positive eigenvalues are shown in green and negative ones in red.

Two plots are generated: the non-cumulative eigenvalue distribution, and the cumulative eigenvalue distribution. For the non-cumulative distribution, the absolute $\lambda_i$ are shown in function of $i$ for $1 \leq i \leq k$. The sign of eigenvalues (positive and negative) is shown by the color of the points (green and red). For the cumulated eigenvalue plots, the range of all eigenvalues is computed, divided into 49 bins (an odd number to avoid a bin limit at zero for the matrix $N$), and then the number of eigenvalues in each bin is computed. The result is plotted as a cumulated distribution plot, with boxes indicating the uncertainty of the computation, due to the fact that eigenvalues are not computed exactly, but only in bins.
5.6 Complex Eigenvalues Plot

The adjacency matrix of an undirected graph is symmetric and therefore its eigenvalues are real. For directed graphs however, the adjacency matrix $A$ is asymmetric, and in the general case its eigenvalues are complex. We thus plot, for directed graphs, the top-$k$ complex eigenvalues by absolute value of the adjacency matrix $A$.

Three properties can be read off the complex eigenvalues: whether a graph is nearly acyclic, whether a graph is nearly symmetric, and whether a graph is nearly bipartite. If a directed graph is acyclic, its adjacency matrix is nilpotent and therefore all its eigenvalues are zero. The complex eigenvalue plot can therefore serve as a test for networks that are nearly acyclic: the smaller the absolute value of the complex eigenvalues of a directed graph, the nearer it is to being acyclic. When a directed network is symmetric, i.e., all directed edges come in pairs connecting two nodes in opposite direction, then the adjacency matrix $A$ is symmetric and therefore all its eigenvalues are complex. Thus, a nearly symmetric directed network has complex eigenvalues that are near the real line. Finally, the eigenvalues of a bipartite graph are symmetric around the imaginary axis. In other words, if $a + bi$ is an eigenvalue, then so is $-a + bi$ when the graph is bipartite. Thus, the amount of symmetric along the imaginary axis is an indicator for bipartivity. Note that bipartivity here takes into account edge directions: There must be two groups such that all (or most) directed edges go from the first group to second. Figure 8 shows two examples of such plots.

(a) Wikipedia elections

(b) UC Irvine messages

Figure 8: The top-$k$ complex eigenvalues $\lambda_i$ of the asymmetric adjacency matrix $A$ of the directed Wikipedia election (EL) and UC Irvine messages (UC) networks.
5.7 Distance Distribution Plot

Distance statistics can be visualized in the distance distribution plot. The distance distribution plot shows, for each integer $k$, the number of node pairs at distance $k$ from each other, divided by the total number of node pairs. The distance distribution plot can be used to read off the diameter, the median path length, and the 90-percentile effective diameter (see Section 4.6). For temporal networks, the distance distribution plot can be shown over time.

The non-temporal distance distribution plot shows the cumulated distance distribution function between all node pairs $(u,v)$ in the network, including pairs of the form $(u,u)$, whose distance is zero.

The temporal distance distribution plot shows the same data in function of time, with time on the X axis, and each colored curve representing one distance value.

![Distance distribution plot](a) Distance distribution plot ![Temporal distance distribution plot](b) Temporal distance distribution plot

Figure 9: The distance distribution plot and temporal distance distribution plot of the Wikipedia election network (EL).

5.8 Graph Drawings

A graph drawing is a representation of a graph, showing its vertices and edges laid out in two (or three) dimensions in order for the graph structure to become visible. Graph drawings are easy to produce when a graph is small, and become harder to generate and less useful when a graph is larger.

Given a graph, a graph drawing can be specified by the placement of its vertices in the plane. To determine such a placement is a non-trivial problem, for which many algorithms exist, depending on the required properties of the drawing. For instance, each vertex should be placed near to its neighbors, vertices should not be drawn to near to each other, and edges should, if possible, not cross each other. It is clear that it is impossible to fulfill all these requirements at once, and thus no best graph drawing exists.

In KONECT, we show drawings of small graphs only, such that vertices and edges remain visible. The graph drawings in KONECT are spectral graph
In particular, KONECT included graph drawings based on the adjacency matrix $A$, the normalized adjacency matrix $N$ and the Laplacian matrix $L$. Let $x$ and $y$ be the two chosen eigenvectors of each matrix, then the coordinate of the node $u \in V$ is given by $x_u$ and $y_u$.

For the adjacency matrix $A$ and the normalized adjacency matrix $N$, we use the two eigenvectors with largest absolute eigenvalue. For the Laplacian matrix $L$, we use the two eigenvectors with smallest nonzero eigenvalue. Examples for the Zachary karate club social network $\text{(ZA)}$ are shown in Figure 10.

### 6 Matlab Toolbox

The KONECT Matlab Toolbox\footnote{konect.uni-koblenz.de/toolbox} is a set of functions for the Matlab programming language\footnote{www.mathworks.com/products/matlab} containing implementations of statistics, plots and other network analysis methods. The KONECT Matlab Toolbox is used to generate the numerical statistics and plots in this handbook as well as on the KONECT website.
Installation  The toolbox is provided as a directory containing *.m files. The directory can be added to the Matlab path using addpath() to be used.

Usage  All functions have names beginning with konect_.

6.1 Variables

Naming variables can be quite complicated and hard to read in Matlab. Therefore KONECT code follows these rules.

Long variable names (containing full words) are in all-lowercase. Words are separated by underscore. When referring to a variable in comments, the variable is written in all-uppercase. Short variable names (letters) are lowercase for numbers and vectors, and uppercase for matrices.

6.1.1 Strings

Table 5 shows common variable names used for string variables.

6.1.2 Scalars

Table 6 shows variable names used for scalar values.

6.1.3 Matrices

Table 7 shows variable names used for matrix-valued variables.

Note that when the adjacency matrix of an undirected graph is stored in a variable, each edge is usually stored just once, instead of twice. In other words, the variable $A$ for undirected networks does not equal the matrix $A$, instead the expression $A + A'$ does.

6.1.4 Compound Types

A struct containing elements whose names are of a specific type are named [VALUETYPE]s_[KEYTYPE]. For instance, a struct with labels used for methods is named as follows:

```
labels_method.('auc') = 'Area under the curve';
```

Note:

- The first element is the name of the content type.
- The plural is used only for the content type.
Table 5: Long variable names of string type used in KONECT.

| Variable    | Description |
|-------------|-------------|
| network     | The internal network name, e.g., “advogato”. The internal network name is used in the names of files related to the network. |
| decomposition | The internal of a matrix decomposition, as passed to the function `konect_decomposition()`, e.g., “sym”, “asym” and “lap”. |
| format      | The network format in lower case as defined in the function `konect_consts()`, e.g., “sym” and “bip”. |
| weights     | The edge weight type as defined in the function `konect_consts()`, e.g., “unweighted” and “signed”. |
| code        | The 1/2/3-character code for a network, e.g., “EN” for Enron. |
| statistic   | The internal name of a network statistic, e.g., “power” and “alcon”. |
| curve       | The internal name of a curve fitting method. |
| method      | The internal name of a link prediction method. |
| measure     | The internal name of a measure of link prediction accuracy, e.g., “map” and “auc”. |
| label       | The readable name of things used in plots, tables, etc. |
| class       | The internal name for a set of networks, e.g., “test”, “1”, “2”, “3”. The class “N” includes the $10 \times N$ smallest networks. |
| filename    | A filename. |
| type        | The internal name of the computation type. This can be “split” or “full”. This decides which version of a network gets used, in particular for time-dependent analyses. |
| centrality  | The internal name of a centrality measure, e.g., “degree” and “decomp.sym”. |

6.1.5 IDs

Variables named `method`, `decomposition`, etc. are always strings. If a method, decomposition or any other type is represented as an integer (e.g., as an index into an array), then `_id` is appended to the variable name. For instance:

```python
decomposition = 'sym'; decomposition_id = 2;
```

This means that an array of values by ID of keys is called for instance:

```python
labels_decomposition_id{1} = 'Eigenvalue decomposition';
labels_decomposition_id{2} = 'Singular value decomposition';
```
Table 6: Variable names used for scalars in KONECT.

| Variable | Description |
|----------|-------------|
| n, n1, n2 | Row/column count in matrices, left/right vertex count |
| r | Rank of a decomposition |
| m | Edge count |
| i, j | Vertices as integer, i.e., indexes in rows and columns. |
| prediction | A link prediction score, i.e., a value returned by a link prediction algorithm for a given node pair. |
| precision | The prediction accuracy value, typically between 0 and 1. |
| means | Values used for additive (de)normalization, as a structure. |

7 File Formats

Due to the ubiquity of networks in many areas, there are a large number of file formats for storing graphs and graph-like structures. Some of these are well-suited for accessibility from many different programming languages (mostly line-oriented text formats), some are well-suited for integration with other formats (semantic formats such as RDF and XML-based ones), while other formats are optimized for efficient access (binary formats). In KONECT, we thus use three file formats covering the three cases:

- **Text format**: This format is text-based and uses tab-separated values. This is the main KONECT data format from which the two others are derived. The format has the advantage that it can be read easily from many different programming languages and environment.

- **RDF format**: Datasets are also available as RDF. This is intended for easy integration with other datasets.

- **Matlab format**: To compute statistics and plots and perform experiments, we use Matlab’s own binary format, which can be accessed efficiently from within Matlab.

In the following, we describe KONECT’s text format. Each network `$\text{NETWORK}$` is represented by the following files:

- **out.$\text{NETWORK}$**: The edges stored as tab-separated values (TSV). The file is a text file, and each line contains information about one edge. Each line contains two, three or four numbers represented textually, and separated by spaces. The first two columns are mandatory and contain the source and destination node ID of the edge. The third column is optional and contains the edge weight. When the network is dynamic, the third column contains +1 for added edges and −1 for removed edges. For unweighted, non-temporal networks, multiple edges may be aggregated into a single line containing, in the third column, the number of aggregated edges. The
Table 7: Variable names used for matrices in KONECT.

| Variable | Description |
|----------|-------------|
| A        | \((n \times n)\) Adjacency matrix (in code where the adjacency and biadjacency matrix are distinguished) |
| A        | \((n \times n)\) or \((n_1 \times n_2)\) Adjacency or biadjacency matrix (in code where the two are not distinguished) |
| B        | \((n_1 \times n_2)\) Biadjacency matrix (in code where the adjacency and biadjacency matrix are distinguished) |
| D        | \((r \times r)\) Central matrix; e.g., eigenvalues; as matrix |
| dd       | \((r \times 1)\) Diagonal of the central matrix |
| L        | \((n \times n)\) Laplacian matrix |
| M, N     | Normalized (bi)adjacency matrix |
| T, At    | \((m \times 2)\) or \((m \times 3)\) or \((m \times 4)\) Compact adjacency matrix, as stored in \texttt{out.*} files, and such that it can be converted to a sparse matrix using \texttt{konect_spconvert()}. First column: row IDs Second column: column IDs Third column (optional): edge weights (1 if not present) Fourth column (optional): timestamps in Unix time |
| U        | \((n \times r)\) or \((n_1 \times r)\) Left part of decomposition; e.g., left eigenvectors |
| V        | \((n \times r)\) or \((n_2 \times r)\) Right part of decomposition; e.g., right eigenvectors |
| X        | \((r \times r)\) Central matrix, when explicitly nondiagonal |
| Z        | \((n \times n)\) Normalized Laplacian matrix |

The fourth column is optional and contains the edge creation time, and is stored as UNIX time, i.e., the number of seconds since 1 January 1970. If the fourth column is present, the third column must also be given. The beginning of the file contains up to three comment lines with the following information:

\[
\text{% FORMAT WEIGHTS RELATIONSHIP-COUNT SUBJECT-COUNT OBJECT-COUNT}
\]

where \texttt{FORMAT} is the internal name for the format as given in Table 1, \texttt{WEIGHTS} is the internal name for the weight types as given in Table 2, \texttt{RELATIONSHIP-COUNT} is the number of data lines in the file, and \texttt{SUBJECT-COUNT} and \texttt{OBJECT-COUNT} both equal the number of nodes \(n\) in unipartite networks, and the number of left and right nodes \(n_1\) and \(n_2\) in bipartite networks. The first line is mandatory; the second line is optional.

- \texttt{meta.$NETWORK}: This file contains metadata about the network that is independent of the mathematical structure of the network. The file is a text file coded in UTF-8. Each line contains one key/value pair, written as the key, a colon and the value. The following metadata are used:

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- **name**: The name of the dataset (usually only the name of the source, without description the type or category, e.g., “YouTube”, “Wikipedia elections”). The name uses sentence case. For networks with the same name the source (e.g., the conference) is added in parentheses. Within each category, all names must be distinct.

- **code**: The short code used in plots and narrow tables. The code consists of two or three characters. The first two characters are usually uppercase letters and denote the data source. The last character, if present, usually distinguishes the different networks from one source.

- **url**: The URL(s) of the data sources, as a comma separated list. Most datasets have a single URL.

- **category**: The name of the category, as given in the column “Category” in Table 3.

- **description**: A short description of the form “User–movie ratings”. Note that the file should contain an actual en dash, coded in UTF-8.

- **cite**: The bibtex code(s) for this dataset, as a comma separated list. Most dataset have a single bibtex entry.

- **fullname**: (optional) A longer name to disambiguate different datasets from the same source, e.g., “Youtube ratings” and “Youtube friendships”. Uses sentence case. All networks must have different full-names.

- **long-description**: (optional) A long descriptive text consisting of full sentences, and describing the dataset in a verbose way. HTML markup may be used sparingly.

- **entity-names**: (optional) A comma-separated list of entity names (e.g., “user, movie”). Unipartite networks give a single name; bipartite networks give two.

- **relationship-names**: (optional) The name of the relationship represented by edges, as a substantive (e.g., “friendship”).

- **extr**: (optional) The name of the subdirectory that contains the extraction code for this dataset.

- **timeiso**: (optional) A single ISO timestamp denoting the date of the dataset or two timestamps separated by a slash(/) for a time range. The format is: YYYY[-MM[-DD]][/YYYY[-MM[-DD]]], e.g., “2005-10-08/2006-11-03” or “2007”.

- **tags**: (optional) A space-separated list of hashtags describing the network. The following tags are used:

  * **#incomplete**: The network is incomplete, i.e., not all edges or nodes are included. This implies that for instance its degree distribution is not meaningful.
* #kcore: The network contains only nodes with a certain minimal degree \( k \). In other words, the nodes with degree less than a certain number \( k \) were removed from the dataset. This changes a network drastically, and is called the “\( k \)-core” of a network. The is sometimes done to get a less sparse network applications that do not perform well on sparse networks. This tag implies the #incomplete tag.

* #missingorientation: This tag is used for undirected networks which are based on an underlying directed network. For instance, in a citation network, we may only know that the documents A and B are linked, but not which one cites the other. In such a case, the network in KONECT is undirected, although the underlying network is actually directed.

* #loop: The network may contain loops, i.e., edges connecting a vertex to itself. This tag is only allowed for unipartite networks. When this tag is not present, loops are not allowed, and the presence of loops will be considered an error by analysis code.

* #regenerate: The network can be regenerated periodically and may be updated when a more recent dataset becomes available.

- n3-*: (optional) Metadata which is used for the generation of RDF files. The symbol \( \{n\} \) in the name of the meta key represents an order by unique, sequential numbers starting at 1.

  * n3-add-prefix\{n\} (optional): Used to define additional N3 prefixes. The default prefixes are specified in this way.
  * n3-comment\{n\} (optional): Add commentary lines which are placed at the beginning of the N3 file.
  * n3-edgedata\{n\} (optional): Additional N3-data, to be displayed with each edge.
  * n3-nodedata-m\{n\} (optional): Additional N3-data, to be displayed with the first occurrence of the source ID.
  * n3-nodedata-n\{n\} (optional): Additional N3-data, to be displayed with the first occurrence of the target ID.
  * n3-prefix-m: N3-prefix for the source IDs.
  * n3-prefix-n (optional): N3-prefix for the target IDs. If this field is left out, the value of \( \{n3\text{-prefix-m}\} \) is used.
  * n3-prefix-j (optional): Additional prefix which can be used with the source id, if there is an entity to be represented with the same id.
  * n3-prefix-k (optional): Additional prefix which can be used with the target id, if there is an entity to be represented with the same id. This is used for example in meta.facebook-wosn-wall for the representation of users walls.
  * n3-prefix-l (optional): N3-prefix for the edges, if they are to be represented by some N3-entity.
* n3-type-l (optional): RDF-type for the edges.
* n3-type-m: RDF-type for source IDs.
* n3-type-n (optional): RDF-type for target IDs.

The following symbols are used in the n3-expressions for edgedata and nodedata:

$m$: n3-prefix-m + source ID
$n$: n3-prefix-n (or n3-prefix-m if the other is undefined) + target ID
$j$: source ID
$k$: target ID
$l$: edge ID
$\text{timestamp}$: edge timestamp

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A Glossary of Terms

The area of graph theory and network analysis is still recent enough that there is no unified glossary across the literature. The choices made in this work are those of the authors, and were chosen to reflect best practices, and to avoid confusion.

**Arc** A directed edge.
Central matrix  The matrix $X$ in the decomposition $UXV^T$, not necessarily diagonal or symmetric; a generalization of the diagonal eigenvalue matrix

Decomposition  In KONECT the word decomposition is used to denote the combination of a characteristic graph matrix (e.g. the adjacency matrix or Laplacian) with a matrix decomposition. As an extension, some other constructions are also called decomposition, such as LDA.

Density  This word is avoided in KONECT. In the literature, it may refer to either the fill (probability that a node exists), or to the average degree. The former definition is used in mathematical contexts, while the latter is used in computer science contexts.

Edge  A connection between two nodes.

Feature  A node feature. I.e., a number assigned to each node. Equivalently, a node vector.

Fill  The probability that two randomly chosen nodes are connected. Also called the density, in particular in a mathematical context.

Half-adjacency matrix  The adjacency matrix $A$ of an undirected graph contains two nonzero entries for each edge $\{i, j\}$: $A_{ij}$ and $A_{ji}$. To avoid this, KONECT code uses the half-adjacency matrix, which contains only one of the two nonzero entries. The half-adjacency matrix is therefore not unique. In code, the half-adjacency matrix is denoted $A$.

Score  A numerical value given to a node pair. Usually used for link prediction, but can also measure distance or similarity between nodes.

Size  The number of nodes in a network.

Volume  The number of edges in a network.

B  Glossary of Mathematical Symbols

The following symbols are used in mathematical expressions throughout KONECT. Due to the large number of different measures used in graph theory and network analysis, many common symbols for measures overlap. For many measures, there is more than one commonly-used notation; the following tables show a reasonable balance between using established notation when it exists, and having distinct symbols for different measures.
a algebraic connectivity
b non-bipartivity
c global clustering coefficient
c(u) local clustering coefficient
d average degree
d(u) degree of a vertex
d(u,v) shortest-path distance
e edge
m volume, edge count
n size, node count
p fill
q number of squares
r rank of a decomposition
r rating value
r radius of a graph
s wedge count
t number of triangles
u, v, w vertices
w edge weight
w network weight
w(…) weight function
x 4-star count
z claw count
γ power law exponent
δ diameter
ε eccentricity
λ eigenvalue
μ average edge weight
ρ assortativity
ξ algebraic conflict
σ singular value
Ck number of k-cycles
E edge set
F frustration
G graph
G Gini coefficient
H entropy
N size of largest connected component
S_k k-star count
T_k k-tour count
V vertex set
| Symbol | Description                          |
|--------|-------------------------------------|
| A      | adjacency matrix                    |
| B      | biadjacency matrix                  |
| D      | degree matrix                       |
| K      | signless Laplacian matrix           |
| L      | Laplacian matrix                    |
| M      | normalized biadjacency matrix       |
| N      | normalized adjacency matrix         |
| P      | stochastic adjacency matrix         |
| U, V   | eigenvector matrices                |
| Z      | normalized Laplacian matrix         |
| Λ      | eigenvalue matrix                   |
| Σ      | singular value matrix               |
| ¯G     | unweighted graph                    |
| ¯G     | graph with unique edges             |
| |G|  | unsigned graph                      |