Inflation from Tachyon Condensation, Large $N$ Effects

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Abstract: Using only general properties of the tachyon potential we show that inflation may be generic when many branes and anti-branes become coincident. Inflation may occur because of: (1) the assistance of the many diagonal tachyon fields; (2) when the tachyons condense in a staggered fashion; or (3) when some of them condense very late. We point out that such inflation is in some sense a stringy implementation of chaotic inflation and may have important applications for “regularizing” a lopsided or singular cosmological compact surface.
1. Introduction

Inflation corresponds to an instability – a period of accelerated growth of the universe. Field theoretic tachyons likewise correspond to instabilities and similarly arise whenever string theoretic instabilities appear. For example, branes oriented at generic angles are unstable and possess tachyons [1]. Systems with non-coincident $Dp$ and $D(p-2)$ branes are unstable and possess tachyons [2]. Also, non-BPS branes and brane anti-brane pairs are unstable and are tachyonic [3]. Such tachyons generate the dynamics of the systems but, eventually disappear once the systems have flowed to a stable and supersymmetric state. The correspondence between tachyons and rapid time dependent behaviour is
suggestive of some cosmological importance and the challenge posed by this paper is to link inflationary instabilities with tachyonic ones. Some previous attempts are [4]. We find that configurations with \( N \) coincident \( Dp - Dp \) branes and anti-branes may generate inflation if \( N \gg 1 \).

The early universe was very hot and thus a stringy understanding of cosmological time-dependence will require a study of string instabilities at high temperature. As discussed in [5], there exists evidence, at least in the case of non-BPS branes and brane anti-brane systems, that tachyons disappear at high temperature. Essentially, tachyons in such a system correspond to negative curvature of a “tachyon” potential. At high temperature, the curvature becomes positive as symmetry of the potential is restored in the usual thermal field theory sense. Hence, the tachyons disappear. However, once the universe expands, the temperature falls (if the expansion is adiabatic). At a critical temperature \( T_c \), the tachyon potential changes shape and tachyons appear.

As shown in [6] these tachyons can lead to the production of D-brane remnants and any leftover high dimensional defects need to be diluted away. We will show that tachyons which condense late may dilute away problematic defects.

The central result of this paper is: inflation may be generic in non-supersymmetric string theories. Whenever, a patch of the universe is covered with many coincident branes and anti-branes or non-BPS branes, tachyons appear and inflation may result. In some sense tachyonic inflation is a stringy implementation of chaotic inflation.

This kind of inflation may not generate appropriate density perturbations. However, the goal of Planckian inflation shouldn’t necessarily be the solution of all of cosmology’s problems. It would be extraordinary if one quantum gravity mechanism at \( 10^{19}\text{GeV} \) were responsible for structure formation at a few eV via the generation of acceptable density perturbations. A more realistic aim of Planckian inflation is the prevention of early collapse of the universe, i.e. a solution of the flatness problem.

Later bouts of inflation can generate acceptable density perturbations and cure the ills prior episodes of inflation did not, or other mechanisms such as a curvaton
mechanism [3] can be used to generate perturbations. There exists some prejudice that inflation should occur only once. However, we take the view that if inflation can happen once, it can probably happen many times during the history of the universe. No inflation after the first bout means that $a \leq 0$ forever. It is difficult to believe that any quantity which can be positive or negative will forever choose one sign over the other. Strong no-go theorems would be required and none exist. It is true that normal matter obeying the strong energy condition does not give $\ddot{a} > 0$. However, the universe may contain all sorts of matter, e.g. topological defects, vacuum energy, etc. In fact, the universe now seems to be filled with some sort of quintessential vacuum energy causing it to accelerate today [8].

This paper is organized as follows. We review some generalities about the tachyon potential from the vacuum string field theory point of view in section 2. We show how the potential changes for a large number of branes in section 3. Then we discuss inflationary scenarios in section 4. There we describe what happens when: (1) all the tachyons condense at the same time and show that this is an example of assisted inflation as in [9]; (2) what happens when they condense in a staggered fashion, which we call staggered inflation; (3) finally, what happens when some tachyons condense very late. Next, we briefly discuss density perturbations in section 5. Finally, we end with several comments, criticisms and make the analogy between chaotic and tachyonic inflation.

2. Tachyon Potential

String theory possesses many extra scalar and gauge fields. These fields are not fixed, and their dynamics are uncertain because their potentials are unknown. Even when a non-flat potential is written down – it is never more than an inspired guess of the non-perturbative physics. Tachyon physics is rather different, as the form of the potential is known up to two derivative terms. This is rather striking, as the tachyon is an
intrinsically stringy field and its potential is *background independent*. Recall, that the potential takes the uniform form, where \( t \) is the tachyon field and \( \tau \) is time,

\[
V(t) = 2\tau_p(1 + v(t))
\]  

(2.1)

where \( v(t) \) is a universal function and is the same for D-branes wrapped on cycles of an internal compact manifold, D-branes in the presence of a background metric, or anti-symmetric tensor field, etc. Only the multiplicative factor, \( \tau_p \) (the brane tension) is model dependent.

The tachyon potential truncated at level (2,4) for a non-BPS brane system is [10]

\[
V(t) = \tau_p(1 - 0.87 t^2 + 0.21 t^4)
\]

(2.2)

This is similar to the potential of a brane anti-brane system this is the form of the potential which we shall use. The potential is characterized by the presence of a global minimum at a finite distance from \( t = 0 \) in field space, and its double well shape.

The above potential is not completely sufficient to describe the time dependence of the tachyon field. Once the tachyon starts to roll down from \( t = 0 \), it will couple to a countably infinite number of other string fields, complicating the analysis of the time dependence. In fact, an appropriate treatment of time dependence would entail knowledge of an infinite number of time derivatives of the string field. This can be seen by noting that the Witten star product *, and its generalizations, e.g. \( \hat{*} \), are related to the non-local Moyal product [11], \( *_M \), which acting between two functions \( f \) and \( g \) acts as

\[
f *_M g(y) = \exp \left( \frac{i}{2} \theta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y^\nu} \right) f(x)g(y) \bigg|_{x=y}
\]

(2.3)

where the \( \theta_{\mu\nu} \) are constants. Thus, the Moyal product involves an infinite number of derivatives, and an infinite number of time derivatives for \( \theta_{0\mu} \neq 0 \). Hence, by analogy the string field theory action, which involves the Witten star product, also involves an
infinite number of time derivatives. For example, in the bosonic case, every three point vertex is accompanied by a factor \( \exp\left(-\ln\left(4/3\sqrt{3}\alpha'\partial^2\right)\right) \), and thus involves an infinite number of derivatives.

Hence, a string field theory description of the time dependent tachyon condensation will involve an infinite number of time derivatives, and specific cosmological consequences may depend on higher derivative corrections. Nevertheless, as our discussions will depend only on the gross features of the tachyon potential and not on its particular shape, we will use a potential like \( \ref{2.2} \).

We will not use a DBI effective action because: (1) the questionable validity of the DBI action when the tachyon is very near the top of its potential – the region for \( t \) which we are most interested in; (2) we are interested in multiple coincident branes and there is ambiguity on how to write the DBI effective actions of non-Abelian theories.

3. Tachyon Condensation on \( N Dp - \bar{D}p \) Brane Anti-brane Pairs

A system of \( N Dp - \bar{D}p \) brane anti-brane pairs will possess a \( U(N) \times U(N) \) gauge symmetry. The tachyon connecting the branes to the anti-branes transforms in the bifundamental \( (N, N) \) representation of the gauge group. Thus, such a system generally possesses \( N^2 \) tachyons \( t_{ij} \) where \( 1 \leq i, j \leq N \).

The potential for such a configuration will be similar to the potential in \( \ref{2.2} \) with \( t^2 \) replaced by \( tt^\dagger \) and \( t \) replaced by an \( N \times N \) matrix. The tachyon potential can then be written as the truncated effective potential

\[
V(t) = 2\tau_p - c_1\text{tr}(tt^\dagger) + c_2(\text{tr}(tt^\dagger))^2 + O(|t_i|^6)
\]  

which mimics the shape of \( V(t) \) in \( \ref{2.2} \) and possesses a minimum at \( t_0 \).

Using the \( U(N) \times U(N) \) gauge symmetry the tachyon can be diagonalized at any point in time, such that for \( u \) and \( v \) in the first and second \( U(N) \)'s of the \( U(N) \times U(N) \) symmetry we have
\[ utv = \text{diag}(t_1, \ldots, t_N) \]  

(3.2)

where the \( t_i \) are complex scalar fields. The potential then becomes

\[ V(t_1, \ldots, t_N) = N\tau - c_1 \sum_{i=1}^{N} |t_i|^2 + c_2 \sum_{i=1}^{N} |t_i|^4 + \mathcal{O}(|t_i|^6) \]  

(3.3)

At the minimum of the potential \( V(t_1 = t_0, \ldots, t_N = t_0) = 0 \), and the individual \( t_i \) satisfy the same equation as that satisfied by a single brane anti-brane pair tachyon at the minimum of the potential,

\[ 2\tau_p - c_1|t_i|^2 + c_2|t_i|^4 + \mathcal{O}(|t_i|^6) = 0 \]  

(3.4)

Thus the tachyons are non-interacting and behave like the tachyon of a single \( Dp - \bar{D}p \) brane anti-brane pair. However, to diagonalize the tachyon at every point in its evolution, would require time dependent \( u(\tau) \) and \( v(\tau) \) matrices. This would not necessarily yield a continuous trajectory for the \( N \) diagonal tachyons \( t_i \). Thus, it is impossible to isolate the tachyonic degrees of freedom in \( N \) tachyonic fields which are differentiable throughout their entire trajectories. One can however from the outset restrict the tachyon to an Abelian part of \( U(N) \times U(N) \) by setting the off-diagonal terms to zero. Then, although the branes and anti-branes are coincident, the worldvolume tachyons will not interact. Hence, \( t \) will then take the form

\[ t = \text{diag}(t_1, \ldots, t_N) \]  

(3.5)

and the potential will take the form \( (3.3) \) and the kinetic term will also be diagonal.

A generic tachyon matrix will possess off-diagonal terms, and hence the restriction is somewhat unphysical. However, if \( N \gg 1 \) large diagonal blocks may exist in the \( N \) by \( N \) tachyon matrix.

In [12] the existence of a two parameter set of solutions of a single time dependent (rolling) tachyon field was demonstrated. Here, we show that a similar \( 2N \) parameter
set of solutions for $N$ independent tachyons exists. We can show that time dependent solutions with (3.5) exist by solving the cubic string field theory equations of motion at level $(0,0)$.

At level 0, the string field is

$$|\Psi\rangle = c_1|0\rangle_g \otimes t|0\rangle_m$$  \hspace{1cm} (3.6)

where $c_1$ is a $c$ ghost, $|0\rangle_g$ and $|0\rangle_m$ are the ghost and matter parts of the vacuum state. The equation of motion of $t$ is

$$Q_B|\Psi\rangle = 0 \Rightarrow (\Box + m^2)t = 0 \Rightarrow (\Box + m^2)t_i = 0$$ \hspace{1cm} (3.7)

where $Q_B$ is the BRST operator. Equation (3.7) has a solution of

$$\tilde{t}_i = A_i e^\tau + B_i e^{-\tau}$$ \hspace{1cm} (3.8)

for spatially homogeneous tachyons. Here $\tau$ is the physical time; $A_i, B_i$ are set by the initial conditions of the tachyons $t_i(0)$ and $dt_i/d\tau|_0$. As in [12], we can then construct a two parameter family of solutions for each time dependent tachyon labeled by the initial position of the tachyon and its initial velocity.

Now, whenever a system possesses $N$ scalar fields, $N - 1$ fields can usually be set to zero, and only one field be allowed to roll to the bottom of the potential. For example, in the system given by the action with an $O(N)$ symmetry

$$S = \frac{1}{2} \int d^2 x \left( \sum_{i=1}^{N} (\partial \phi_i)^2 - \left( \sum_{i=1}^{N} \phi^2_i - \mu^2 \right)^2 \right)$$ \hspace{1cm} (3.9)

any path of steepest descent given by $(\phi_1(t), ..., \phi_n(t))$ can be rotated by an $O(N)$ rotation into the path $(\phi_1(t), 0, ..., 0)$. However, for this to occur, $\phi_1$ must have have support over $[0, \mu]$.

Similar arguments do not apply to the tachyon case because one tachyon mode cannot lead to the annihilation of all $N$ $Dp - \bar{D}p$ brane anti-brane pairs. Each tachyon...
mode is defined only over \([0, t_0]\), and can lead only to the annihilation of only one brane anti-brane pair; it is not defined over the interval \([0, Nt_0]\).

Thus tachyon condensation genuinely involves the rolling of \(N\) independent tachyon fields.

4. Inflationary Scenarios

We now describe several scenarios where inflation occurs because of tachyon condensation. The amount of inflation is heavily dependent on the manner in which the tachyons condense – whether (1) the all condense at once; (2) in a staggered fashion; and (3) whether there is a group of tachyons which condense very late.

As is well known, sufficient conditions for inflation are: (1) the energy density is potential dominated; (2) the motion is over-damped; (3) the potential is sufficiently flat such that the energy density is roughly constant

\[
\left( \frac{\dot{a}}{a} \right)^2 \sim V(t) \sim V(0); \quad \ddot{\epsilon} \ll 1 \quad \Rightarrow \quad a(\tau) \sim a(0) \exp \sqrt{V(0)} \tau \quad (4.1)
\]

where \(t\) is a scalar field generating inflation. The potential will be flat and the energy will be potential dominated if \(\ddot{\epsilon}\) is small and \(V\) doesn’t strongly vary with \(t\), i.e. if the slow roll parameters, \(\eta, \epsilon\) are small

\[
\eta \equiv m_p^2 \left| \frac{V''}{V} \right| \ll 1; \quad \epsilon \equiv m_p^2 \left| \frac{V'}{V} \right|^2 \ll 1 \quad (4.2)
\]

If the slow roll conditions are satisfied, then it can be shown that inflation will occur for \(n\) efolds, where

\[
n = \int_{\tau_i}^{\tau_f} H d\tau \sim - \int \frac{V(t_i)}{V'(t_i)} dt_i. \quad (4.3)
\]

Although near \(t_i \approx 0\) the tachyon potential is flat \((V'(0) = 0)\), an individual tachyon field will satisfy the slow roll conditions for only a small portion of time. Furthermore,
vacuum fluctuations will displace the field by a magnitude $H/(2\pi)$, causing the field to start “rolling” at $t \sim H/(2\pi)$ and not at $t_i = 0$. Because tachyon condensation occurs at an energy scale $E \sim m_s$, where $m_s$ is typically the string scale, we will have $H \sim m_s$, and the minimum of the tachyon potential will be $t_0 \sim m_s$. Hence, perturbations will typically displace the tachyon field far enough away from $V(0)$ to make any inflation very short-lived. Thus, it is unlikely that a single tachyon will lead to the necessary number of efolds for inflation to be successful (about 60 efolds for 4D physics). However, the situation may change for $N \gg 1$ tachyon fields.

Suppose that $N$ independent tachyons exist. Because of their independence they may condense at different times, and will condense whenever a perturbation dislodges one of them away from the maximum of the tachyon potential. Alternatively, they could all condense approximately simultaneously. These alternatives give rise to different inflationary scenarios, which we elucidate below.

4.1 Simultaneous Condensation: Assisted Inflation

Suppose that the tachyons condense at roughly the same time. Then the tachyons will follow identical trajectories, which for convenience we identify as $t_1$. Then we can write

$$\sum_{i=1}^{N} V(t_i) = NV(t_1) \equiv \tilde{V}(\tilde{t})$$

(4.4)

where we have written the potential in terms of a single field $\tilde{t}$ which is related to $t_1$ as

$$\tilde{t} = \sqrt{N} t_1 \quad \tilde{c}_1 = c_1 \quad \tilde{c}_2 = \frac{c_2}{\sqrt{N}}$$

(4.5)

The slow roll parameters for this $N$ field configuration, $\epsilon_N$ and $\eta_N$ are then related to the slow roll parameters of a single tachyon, $\epsilon$ and $\eta$ as

$$\epsilon_N = \frac{\epsilon}{N}; \quad \eta_N = \frac{\eta}{N}$$

(4.6)
This $1/N$ suppression is a well known feature of assisted inflation \cite{13}. Thus, for large $N$, the slow-roll parameters are shrunk dramatically, and multi-tachyon inflation becomes feasible when single tachyon inflation is not.

We now qualify what we mean by simultaneous condensation. If it takes a time $\Delta \tau$ to condense, then by “roughly condense at the same time,” we mean that all the tachyons condense in a time $\kappa \Delta \tau$ where $\kappa \sim O(1)$ is a constant of order one. This means that if a small number of tachyons have condensed while the remainder are still condensing that slow-roll is not violated. In essence, slow roll is insensitive to small numbers of tachyons (small relative to $N$) condensing before the bulk of them have condensed. This is because the energy density in the early tachyons is small relative to the energy density of the rest of the tachyons.

However, if the tachyons start condensing at widely differing times, then as in other assisted inflation scenarios, no general attractor solution pushing the tachyons toward identical trajectories exists. Only the trivial attractor $t_i = t_j = t_0$ at the bottom of the potential exists. For example, defining $\psi_i \equiv t_i - t_1$

$$\ddot{t_i} + (D - 1)H \dot{t_i} = \frac{\partial V}{\partial t_i}; \quad \ddot{t_j} + (D - 1)H \dot{t_j} = \frac{\partial V}{\partial t_j}; \quad \Rightarrow \quad \ddot{\psi_i} + (D - 1)H \dot{\psi_i} = 2 \frac{\partial V}{\partial \psi_i} \quad (4.7)$$

Because, $\psi_i(t) \neq 0, \forall t$, initially different tachyon profiles will remain different until late times. Only at late times, when the tachyons have reached the minimum of the potential $t_0$, are the tachyons equal because,

$$\frac{\partial V}{\partial \psi_i} = 2c_1 \psi_i + 12c_2 t_1 \psi_i (\psi_i + t_1) = 0 \quad \Rightarrow \quad \psi = 0 \quad \text{at} \quad t_i = t_0 \quad (4.8)$$

In addition to vacuum fluctuations, thermal fluctuations will kick the tachyons up and down their potentials. In our scenario, tachyon condensation begins at the end of a second order phase transition at $T = T_c$. However, tachyon profiles will not get frozen in until the temperature drops to the Ginzburg temperature $T_G < T_c$. Large thermal fluctuations may persist while $T_G < T < T_c$. For example, for a Mexican hat
type potential like (2.2), \( V(t) = g_s(|t|^2 - |t_0|^2)^2 \), the high temperature effective thermal potential will be [14]

\[
V_{\text{eff}}(t, T) = m^2(T)|t|^2 + g_s|t|^4; \quad m^2(T) \sim g_s(T^2 - T_c^2).
\] (4.9)

Minimizing the potential gives \(|t|^2 \sim (T_c^2 - T^2)\). The characteristic length of the system is \( r_c \sim |1/m| \). When fluctuations in the potential energy, \( \delta E_{\text{pot}} \sim r^d\delta V \), where \( d \) is the number of spatial dimensions, are comparable to the thermal energy, \( \delta E_{\text{thermal}} \sim T \), then for fluctuations on a scale of \( r_c \), in \( 3 + 1 \) dimensions we find

\[
r_c^3m^2t\delta t \sim T \quad \Rightarrow \quad \delta t \sim \frac{g_s T}{m}
\] (4.10)

The Ginzburg temperature is defined by \( g_s T = m \). Thus for \( T_G < T < T_c \) fluctuations may be large. Using the second half of (4.9), one can show that the \( T_G \sim (1-g_s)T_c \). If \( g_s \) is not very small, then \( T_c \) and \( T_G \) may be significantly different, allowing sufficient time for large thermal fluctuations to cause \( t_i \) and \( t_j \) to lose coherence.

### 4.2 Staggered Condensation: Staggered Inflation

We now analyse the case when the tachyons do not condense at the same time. If the tachyons condense at widely differing times, they can be thought of as condensing serially; i.e. in a sufficiently short time interval, \( \tau^* \), at most one tachyon condenses. This means that the \( N \) tachyons condense according to a Poisson distribution. If the time interval \( \tau^* \) is sufficiently long such that one tachyon will very likely have condensed in an interval, \( \tau^* \), then we can say that one tachyon will condense on average in a time interval of \( \sim \tau^* \), and all the tachyons will have condensed within a time \( \sim N\tau^* \). Then in the case of a brane anti-brane system the potential will decrease by \( 2\tau_p \) in a time \( \sim \tau^* \). The potential in this case then takes the form

\[
V(\tau) = 2\tau_p(N - \frac{\tau}{\tau^*})
\] (4.11)
Let us rewrite the slow roll parameters in a way more amenable to this time dependent situation. For a time dependent potential $V(\tau)$,

$$V(\tau) = V(0) + V'(0)\tau + V''(0)\frac{\tau^2}{2} + \mathcal{O}(\tau^3)$$

(4.12)

If $\tau$ is a time scale of order $\sim 1/m_p$, then the slow-roll conditions can loosely be written as

$$\eta \equiv \frac{1}{m_p^2} \left| \frac{V''}{V} \right| \ll 1; \quad \epsilon \equiv \frac{1}{m_p^2} \left| \frac{V'}{V} \right| \ll 1$$

(4.13)

which for the potential (4.11) gives

$$\epsilon_N \sim \frac{1}{N(\tau)m_p\tau^*}; \quad \eta \sim 0$$

(4.14)

Hence, for $\tau^*$ no smaller than a Planck time $1/m_p$, there is a $1/N(\tau)$ suppression in the slow roll parameters. (Note, since $N$ is the number of brane anti-brane pairs left over at time $\tau$, $N$ is a function of time). Thus, inflation would seem to naturally occur, although because of the staggered nature of the tachyon condensation the density perturbations may be large.

Suppose instead that the tachyons condense more frequently such that in a time $\tau^*$ many tachyons may condense. For definiteness, suppose that in a time step $\tau^*$ each tachyon has a probability of $1/2$ of condensing. Then the tachyons will condense via a binomial distribution. For example, in the first time step half of the tachyons will condense, in the second time step one fourth will condense, etc. The time dependent potential will then be

$$V(\tau) = 2N\tau_p 2^{-\tau/\tau^*} = 2N\tau_p \exp \left(-\frac{\tau}{\tau^*} \ln 2 \right)$$

(4.15)

which is an exponential potential. It gives rise to the slow roll parameters

$$\eta_N \sim \left| \frac{\ln 2}{m_p\tau^*} \right|; \quad \epsilon \sim \left| \frac{\ln 2}{m_p\tau^*} \right|^2$$

(4.16)
which is small for sufficiently large $\tau^*$. This is similar to power law inflation arising from an exponential potential in that the slow roll parameters are constant. However, unlike power law inflation, any extant inflation will end when all the tachyons have condensed.

4.3 Late Condensation

We now analyse what happens when a group of tachyons condense late. If $N$ is large, then in any scenario, it is likely that there will always be some very late condensing tachyons. This can happen as follows. Each tachyon will start condensing once it is kicked off the top of the tachyon potential. The rms value of such a kick by a vacuum fluctuation is $H/2\pi$. Since $H \sim m_s$, most tachyons will receive a very large kick. If the size of this kick is given by a Gaussian probability distribution, then for $N \gg 1$, there may be some tachyons out in the wings of the distribution which receive very small kicks. Hence, they will start rolling very close to the top of the potential, as opposed to say a third of the way down as for the other tachyons. The top of the potential is necessarily flat $\epsilon = 0$ (because of the need for $U(1)$ invariance). The curvature $V''$ is not zero and hence $\eta \neq 0$ at the top. However, if the number of late simultaneously condensing branes is small, but greater than one, say three or four, then $\eta$ may also be suppressed. Thus, for $N \gg 1$, a few tachyons might start to slow roll and condense long after the other tachyons have “finished” condensing. This is very interesting as such late condensing tachyons may dilute defects left behind by earlier tachyons. In order for this to work, these late tachyons must condense only to a tachyon gas (and eventually radiation) and not create any defects by themselves. This is likely, as entropic arguments in [3] suggest that tachyons prefer condensing to vacuum rather than lower dimensional $D$-branes. Dilution will not require a great deal of inflation, only perhaps $10^{-25}$ efolds. Such inflation by late condensing fields would be philosophically similar to thermal inflation whereby late rolling inflatons dilute troublesome inhomogeneities at energies much lower than $m_{GUT}$. 

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5. Density Perturbations

If all the tachyons condense at the same time, then the density perturbations will be the same as of a single redefined tachyon field \( \tilde{t} \) and a Hubble constant defined by the redefined potential \( \tilde{V}(\tilde{t}) \). The curvature perturbations and spectral index will then be

\[
P(k) = \left( \frac{H}{2\pi} \right) \frac{H^2}{\dot{\tilde{t}}} \quad 1 - n = 6\epsilon_N - 2\eta_N 
\]

(5.1)

Isocurvature perturbations will be produced if the tachyons oscillate transversely to their classical path. Because, each tachyon is a complex field and possesses a Mexican hat type potential, the orthogonal direction is the angular direction. Oscillatory behaviour may thus occur if the \( U(1) \) symmetry of the vacuum manifold is broken and a mass, \( m^2_\theta \) is given to the angular excitations. However, the \( U(1) \) is spontaneously broken and the angular tachyon mode is eaten up to make all gauge fields very massive. Thus no oscillatory behaviour occurs.

Even if isocurvature perturbations are produced, they will not contribute to the curvature perturbation. This is because the path in field space is not curved, i.e.

\[
\dot{\theta}_{ij} = 0 \quad \text{where} \quad \tan \theta_{ij} = \frac{\dot{t}_j}{\dot{t}_i}
\]

(5.2)

and hence, the isocurvature perturbations do not source curvature perturbations\(^1\). The path is not curved because in the assisted case, \( t_1 = t_2 = \cdots = t_N \). Because, isocurva-

\(^1\)For example consider the case of two fields \( t_1 \) and \( t_2 \). Isocurvature perturbations, \( \delta s \), orthogonal to the classical trajectory, can source adiabatic fluctuations, \( \delta \sigma \), which are tangent to the inflaton field trajectory because the evolution equations for adiabatic fluctuations, \( \delta \sigma \), may depend on \( \delta s \). For example [15],

\[
3H\dot{\delta} \sigma + \left( V_{\sigma \sigma} - \dot{\phi}^2 \right) \delta \sigma = 2 \left( \dot{\phi} \delta s \right) - 2 \left( \frac{V_{\sigma}}{\sigma} + \frac{H}{H} \right) \dot{\theta} \delta s 
\]

(5.3)

where \( V_{\sigma \sigma} \) and \( V_\sigma \) are the first and second derivatives of the potential in the direction tangential to the trajectory. Most importantly, isocurvature perturbations will only act as a source if \( \dot{\theta} \neq 0 \) where \( \tan \theta = \dot{t}_1/\dot{t}_2 \); i.e. if the trajectory is curved.
ture perturbations are suppressed outside the horizon and are difficult to observe, we conclude that they are not particularly relevant in our case.

If the string scale is not much smaller than the Planck scale, then density perturbations due to gravitational waves may be very large [16]. However, if all of the tachyons condense simultaneously, then as shown by [17] the amplitude of gravitational waves may be suppressed by a factor of $1/N$.

However, if the tachyons do not simultaneously condense and late condensing tachyons exist, the observable curvature perturbations and spectral index may change. The tachyons which are most important are the last slowly rolling tachyons. If the last tachyon to condense does so after all the others have condensed, the perturbations will largely be that of a lone tachyon during an era with a smaller Hubble constant.

6. Comments, Criticisms, Chaotic Inflation and Conclusions

We now list some potential criticisms and discuss how our scheme may provide a stringy implementation of chaotic inflation and comment on how the large $N$ scenarios may be combined with a more complete description of brane cosmology.

Criticisms: Some criticisms of our schemes are that they use large $N$ effects to obtain inflation and rely on a finite brane tension as a vacuum energy source.

The assisted inflation version partially relies on a non-zero brane tension. To understand its effect, suppose that $t_{in}$ is the initial value of the inflaton field and that there is no initial energy ($V(t_{in}) = 0$). One still obtains the $\epsilon_N = \epsilon/N, \eta_N = \eta/N$ suppression. However, $\epsilon$ and $\eta$ may then be rather large:

$$\epsilon \equiv \frac{m_p^2}{2} \left( \frac{V'(t_{in})}{t_{in} V''(t_{in})} \right)^2 = \frac{m_p^2}{2 t_{in}^2}, \quad \eta \equiv \frac{m_p^2}{t_{in}^2} \left( \frac{V''(t_{in})}{t_{in} V''(t_{in})} \right) = \frac{m_p^2}{t_{in}^2} \quad (6.1)$$

The initial tachyon field value is $t_{in} \approx 0$, making $\epsilon$ and $\eta$ diverge. This is overcome by taking $N$ to be large and not taking $t_{in} = 0$. For example, due to vacuum fluctuations, it is much more reasonable to take something like $t_{in} = 1/10$ in string units.
Thus, $\epsilon, \eta \sim 100$ which is not very large and can be suppressed without taking $N$ to be exorbitantly large.

For the staggered case, a non-zero brane tension would mean that no background energy existed and thus no staggered inflation would occur. This is somewhat troublesome, because in realistic cases the brane tensions may vanish. For example, in Type I theory, the negative energy of orientifolds may cancel the positive brane energy.

From a philosophical point of view, using a non-dynamical vacuum energy to generate inflation is not always prudent. There are many types of vacuum like the susy breaking vacuum energy, electroweak phase transition breaking vacuum energy, etc., and one must carefully argue why a chosen vacuum energy is relevant while others are not. We argue that because branes are physical objects with an energy density, their vacuum energy is much more relevant than a vacuum energy resulting from a not yet understood cancellation of zero point energies, etc.

Furthermore, using brane tensions to generate inflation is analogous to a feature in virtually all inflationary models - assuming the inflaton is miraculously displaced significantly up the potential to create an initial vacuum energy $V(\phi_{in})$. In this respect, brane tension inflation is similar to other models of inflation.

Another possible criticism is our use of the cubic string field theory tachyon potential (2.2) for ease of use. Instead we could have used the tachyon potential exact up to two derivative terms [18]. The exact two derivative action possesses a non-canonical kinetic term, making it harder to work with. The exact tachyon potential is $e^{-T^2/4}$, where $T$ is the tachyon field in boundary string field theory formalism. In the case of a $U(N) \times U(N)$ symmetry, $T$ is a matrix. If we choose $T$ to be diagonal, such that $T = \text{diag}(T_1, ..., T_N)$ then the potential will be $V = \exp(-\sum_i T_i^2 / 4) = \prod_i \exp(-T_i^2 / 4)$. This means the $T_i$ are interacting, and assisted inflation fails whenever the component fields interact. However, by moving to the canonically normalized case via the field redefinition $t = \text{erf}(T)$, and expanding the potential about $T = 0$, the potential takes the form [19]
\[ V(t = \text{erf}(T)) = a_0 - a_1 t^2 + a_2 t^4 + \cdots \] (6.2)

where \( a_0, a_1, a_2 \) are constants. This potential is of the same form as (2.2), and happily there appears to be no mixing of the \( t_i \), allowing assisted inflation to occur.

Unbroken supersymmetry is often needed by high energy inflation models to keep the potential flat. For example \( H \) dependent terms can appear increasing the mass of the fields. Supersymmetry breaking is not so disastrous for our tachyonic inflation models. The assumptions of large \( N \) decreases the need for the individual tachyons to have flat potentials. Also, because of background independence the shape of the tachyon potential is fixed. Except for the value of the brane tension, the potential is the same in a curved (cosmological) background as it is in a flat background; extra terms spoiling flatness such as \( H^2 \) terms do not contribute to the potential.

**Chaotic Inflation:** Chaotic inflation gets around the problem of displacing the inflaton, \( \phi \), by arguing that it will take different values in different Hubble regions. If the number of Hubble regions is large, then thermal interactions may cause some regions to have a suitably displaced \( \phi > m_p \), which allows the vacuum energy \( V(\phi > m_p) \) to source inflation. Tachyonic inflation requires branes to provide vacuum energy to source inflation. As described in [20], branes may be thermally produced because of the diverging free energy of their open string gases. They may wrap various cycles of an initial compact manifold and are analogous to the displaced field values of the inflaton in chaotic inflation. Hence, if a (compact) patch of spacetime finds itself wrapped by many thermally produced non-BPS branes or branes and anti-branes, it will inflate just as a patch with \( \phi > m_p \) will inflate in chaotic inflation. This provides a stringy mechanism for implementing chaotic inflation. This is particularly attractive if the brane vacuum energy source eventually disappears via tachyon condensation (no stable lower dimensional remnant branes created), or if small numbers of tachyons condense late, diluting any remaining defects. (The late tachyons are themselves unlikely to
condense to anything but vacuum as shown in [5]).

**Some Comments:** This leads to an interesting brane world scenario. Various dimensional branes and anti-branes wrap an initially compact spacetime. Higher dimensional branes if not coincident attract each other and find each other first as shown in [21]. This leads to tachyon condensation and sometimes inflation of the cycles they were wrapping. The spacetime which inflates will inevitably contain embedded $D3$ branes and $\bar{D}3$ anti-branes which did not manage to find each other before the higher dimensional branes in which they were embedded annihilated and eventually inflated. Thus after inflation a spacetime will be created in which the predominant species of branes are $D3$ (or $\bar{D}3$) braneworlds and $D1$ and $\bar{D}1$ strings. Thus, although observers on a braneworld will see only their braneworld and none others and hence believe that their universe is very finely tuned to produce only one braneworld; in reality it will not finely tuned. Other branes do exist, but are simply too far off to see.

Finally, we reiterate the motivation for tachyonic inflation. Although, density perturbations may be suppressed in the assisted inflation case, even if they are excessively large tachyonic inflation is still very interesting. Early bouts of tachyonic inflation may cure many cosmological problems (horizon, brane remnant, etc) and prevent the early universe from collapsing. Later bouts may provide the appropriate density perturbations, etc.

Inflation depends only on the sign of $\ddot{a}$. Thus if the sign can be positive at one instant of time during the evolution of the universe, it is conceivable it may be positive at many instants in time. Otherwise a selection rule must exist explaining why $\ddot{a}$ always chooses one sign and not the other.

We finish with one speculative comment. One situation where one may very realistically obtain many coincident branes and anti-branes is at points where some cycles

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2 We thank Ian Kogan and Steve Abel for related discussions.

3 Note, in principle because of interactions with other branes (assume no supersymmetry) an observer on a braneworld would be able to feel the presence of other branes. However, it is likely that if other branes are close enough to be felt by an observer on a braneworld that the density of branes is large enough to cause the universe to collapse. Thus, in viable scenarios, an observer should not be able to easily detect the presence of other branes.
of a compact manifold shrink or become degenerate. Generic compact manifolds will possess “pinching” singularities like the point \( P \) in figure 1 and more generally the volume of the cycles of the compact surface will not be constant and may vary. Branes and anti-branes wrapping cycles of the manifold will minimize their energy and will try to move toward points where the homology cycles have minimum volume. Figure 1 shows the case where one cycle degenerates to a point. Such singularities or points of minimum volume will thus act as a sink for branes and anti-branes and once the branes reach them (assuming it takes a finite time \[22\]) they will be parallel and coincident. It is not inconceivable that these coincident brane and anti-brane pairs may then inflate, resolving singularities like \( P \) by literally blowing them up. This has important applications to any compact initial state of the universe. As generic manifolds are singular and not regular, it is likely that the early universe’s compact manifold possessed degenerate cycles and cycles with varying volume. Our inflationary mechanism is possibly a means for removing singularities or points of small cycle volume, and making the early universe more regular.

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