Exotic phases of finite temperature $SU(N)$ gauge theories

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Abstract

We calculate the phase diagrams at high temperature of SU(N) gauge theories with massive fermions by minimizing the one-loop effective potential. Considering fermions in the adjoint (Adj) representation at various $N$ we observe a variety of phases when $N_f \geq 2$ Majorana flavours and periodic boundary conditions are applied to fermions. Also the confined phase is perturbatively accessible. For $N = 3$, we add Fundamental (F) representation fermions with antiperiodic boundary conditions to adjoint QCD to show how the $Z(3)$-symmetry breaks in the confined phase.

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1. Introduction

The phase diagram of pure $SU(N)$ gauge theories is defined by a confined phase in which the $Z(N)$ center symmetry is preserved, and a deconfined phase in which the center symmetry is spontaneously broken. Lattice simulations have been particularly important in the study of the confined phase because it is in a region of strong coupling where perturbation theory is not valid. In this paper we discuss theories which are QCD-like, given by two $Z(N)$-invariant extensions to pure Yang-Mills theory: 1) center-stabilized Yang-Mills theory, which introduces multiply wound adjoint Polyakov loops to Yang-Mills theory, and 2) adjoint QCD [QCD(Adj)], which is QCD with adjoint representation fermions rather than fundamental.

In $^1$ we performed lattice simulations with an adjoint Polyakov loop extension to Wilson action. The action has the form:
\[ S = S_W + \sum_x H_A \text{Tr}_A P(x); \quad S_W(\beta, U) = -\beta \sum_p \left( 1 - \frac{1}{N} \text{Re} \text{Tr}_F U_p \right), \quad (1) \]

where \( \sum_x \) is over all spatial sites. Figure 1 shows the resulting phase diagram for \( SU(3) \) as a function of the inverse coupling \( \beta = 2N/g^2 \) and the tunable parameter \( H_A \). Two important features of this phase diagram are that the confined phase is accessible in a region of weak coupling, and that there is a region of the parameter space in which a new, ”skewed” phase is favoured over both the confined and deconfined phases. Both of these features were recently confirmed in [2] in simulations using a demon algorithm. Simulations of a similar theory in [3] also confirm that the confined phase becomes accessible beyond the deconfinement temperature of the pure gauge theory.

2. Center-stabilized Yang-Mills theory

The simulation results using eq. (1) are in good agreement with high-temperature perturbation theory [4]. However, for \( SU(4) \) both lattice results and perturbation theory agree that the confined phase is not observed in the weak-coupling limit, unlike in \( SU(3) \). In order to perturbatively obtain the confined phase for arbitrary \( N \) we introduced in [4] an extension in terms of the Polyakov loop \( P = \text{diag}\{e^{iv_1}, e^{iv_2}, ..., e^{iv_N}\} \) to the boson contribution [5] from pure Yang-Mills theory

\[ V_{CYM}(P) = -\frac{2}{\pi^2\beta^4} \sum_{n=1}^{\infty} \frac{1}{n^4} |\text{Tr}_A (P^n)| + \frac{1}{\beta} \sum_{n=1}^{N/2} \text{Tr}_F (P^n) \text{Tr}_F (P^\dagger_n), \quad (2) \]

where \( |N/2| \) is the integer part of \( N/2 \). This is the minimum number of terms required to obtain the confined phase for some value of the \( a_n \) parameters. This potential was recently extensively studied in [6] and we have adopted their notation and the nomenclature "center-stabilized Yang-Mills theory". We minimized \( V_{CYM} \) with respect to the eigenvalue angles \( v_i \) to obtain the phase diagram for a range of values of the \( a_n \).

3. Adjoint QCD

In order to have a renormalizable theory we also study adjoint QCD where periodic boundary conditions (PBC) are applied to fermions in the adjoint representation. The lattice action in eq. (1) and the center-stabilized theory of eq. (2) are both approximations to adjoint QCD with PBC. The one-loop effective potential for \( N_f \) Majorana flavours \( (N_{f,Dirac} = \frac{1}{2}N_f) \) of fermions in representation \( R \) and with finite mass \( m \) is [7]
Fig. 2. $N = 3$: (Left) $V_{ADJ(+)}$, $N_f = 2$ Majorana flavours; (Right) $V_{CY M}$ vs. $a_1$

$$V_{1\text{-loop}} = \frac{1}{\beta V_3} \left[ -N_f \ln \det \left( -D_R^2 (P) + m^2 \right) + \ln \det \left( -D_{adj}^2 (P) \right) \right]$$

$$= \frac{m^2 N_f}{\pi^2 \beta^2} \sum_{n=1}^{\infty} \frac{(\pm 1)^n}{n^2} \text{Re} \left[ \text{Tr}_R (P^n) \right] K_2 (n \beta m) - \frac{2}{\pi^2 \beta^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \text{Tr}_A (P^n).$$  (3)

We minimize $V_{1\text{-loop}}$ with respect to the Polyakov loop eigenvalue angles $v_i$ to obtain the phase diagram as a function of $m \beta$. The resulting phase diagram of adjoint QCD is quite rich in the case of $N_f \geq 2$ Majorana flavours ($N_f, \text{Dirac} \geq 1$) and PBC on fermions [8]. The phase diagram for $N = 3$ and $N_f = 2$ is shown in Figure 2 (L). The dots correspond to the results of the numerical minimization of $V_{1\text{-loop}}$ in eq. (3). Similarly, the phase diagram of the center-stabilized theory is shown in Figure 2 (R). The phase curves in Figure 2 can be classified according to the Polyakov loop eigenvalue angles: $v = \{v_1, v_2, \ldots, v_N\}$. For $N = 3$ the confined phase is defined by $v = \{0, \frac{2 \pi}{3}, \frac{4 \pi}{3}\}$. The deconfined phases are defined by $v = \{0, 0, 0\}$ and $Z(3)$ rotations. The skewed phases are $SU(2) \times U(1)$ phases defined by $v = \{0, \pi, \pi\}$ and $Z(3)$ rotations.

4. QCD with fermions in the adjoint and fundamental representations

The perturbative accessibility of the confined phase in both the center-stabilized theory and in adjoint QCD for $N_f \geq 2$ Majorana flavours is useful for studying how the $Z(3)$ symmetry is broken when adding fundamental fermions with antiperiodic boundary conditions (ABC). We opt to add the fundamental fermions to adjoint QCD to have a renormalizable theory. For $N_F = 3$ Dirac flavours of fundamental fermions asymptotic freedom is maintained for up to $N_A = 4$ Majorana flavours of adjoint fermions.

Figure 3 (L) shows the phase diagram for $N = 3$ from minimizing eq. (3) for $N_A = 4$ Majorana flavours of adjoint fermions and PBC. Figure 3 (R) shows effect of adding $N_F = 3$ Dirac flavours of fundamental fermions with antiperiodic boundary conditions to adjoint QCD. Here the mass of the adjoint fermions is fixed at $m_A = 1$. As the mass of fundamental fermions $m_F$ is brought down from infinity the eigenvalues of the confined phase shift so that the $Z(3)$ symmetry is broken. $m_F = \infty$ corresponds to adjoint QCD and the confined phase is given by $v = \{0, -\phi, \phi\}$ where $\phi = \frac{2 \pi}{3}$ and $\text{Tr}_F P = 0$. The contours in the confined phase of Figure 3 (R) represent intervals of $\Delta \phi = \pi/36$ away
Fig. 3. \( N = 3 \): (Left) Phase diagram of adjoint QCD with \( N_A = 4 \) Majorana flavours. (Right) The phase diagram of \( V_{QCD(F(-A_{+(+})} \) at high temperature as it varies with \( m_F \) \( (N_F = 3, N_A = 4, m_A = 1) \).

from \( \nu = \{0, -2\pi/3, 2\pi/3\} \). As \( m_F \to 0 \), \( \phi \) decreases towards roughly \( \pi/3 \), depending on \( T \), and \( Tr_P \) goes out from 0 along the real axis towards 2, keeping the confined phase distinguishable at any observed \( m_F \) from the deconfined phase which has \( Tr_P = 3 \).

5. Conclusions

We study three \( Z(N) \)-invariant Polyakov loop extensions to Yang-Mills theory that offer confinement in a perturbatively accessible regime, as well as additional phases under certain conditions. An adjoint Polyakov loop extension gives perturbative confinement for \( N = 3 \). Center-stabilized Yang-Mills theory includes the minimum number of powers of adjoint Polyakov loop terms needed to get perturbative confinement for all \( N \). As well this theory contains various other phases depending on the values of the \( a_n \) parameters. Adjoint QCD for \( N_A = 2 \) or more Majorana flavours and PBC on fermions also gives perturbative confinement for all \( N \), and small \( m/\beta \), in addition to other phases contained in the center-stabilized model. Adding fundamental representation fermions to adjoint QCD, while being careful to preserve asymptotic freedom, shows that in the confined phase the \( Z(3) \) symmetry is broken in a predictable way. The degree of symmetry breaking depends on the mass of fundamental fermions \( m_F \) and the temperature, but the confined phase remains distinguishable from the deconfined phase for all observed \( m_F/\beta \).

References

[1] J. C. Myers and M. C. Ogilvie, Phys. Rev. D 77 (2008) 125030 [arXiv:0707.1869 [hep-lat]].
[2] C. Wozar, T. Kastner, B. H. Wellegehausen, A. Wipf and T. Heinzel, arXiv:0808.4046 [hep-lat].
[3] A. Dumitru and D. Smith, Phys. Rev. D 77 (2008) 094022 [arXiv:0711.0868 [hep-lat]].
[4] M. C. Ogilvie, P. N. Meisinger and J. C. Myers, PoS LAT2007 (2007) 213 [arXiv:0710.0649 [hep-lat]].
[5] D. J. Gross, R. D. Pisarski and L. G. Yaffe, Rev. Mod. Phys. 53 (1981) 43.
[6] M. Unsal and L. G. Yaffe, arXiv:0803.0344 [hep-th].
[7] P. N. Meisinger and M. C. Ogilvie, Phys. Rev. D 65 (2002) 056013 arXiv:hep-ph/0108026.
[8] J. C. Myers and M. C. Ogilvie, arXiv:0809.3964 [hep-lat].