The Stability Region of the Two-User Degraded Gaussian Broadcast Channel

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Abstract

The stability region of the two-user degraded Gaussian broadcast channel is characterized in this paper. Two simple power allocation schemes are considered, namely fixed and queue-aware power control. We then provide convexity/concavity conditions for the stability region and we investigate the aggregate stable throughput of the network.

I. INTRODUCTION

A fundamental question to which Information Theory aims to provide an answer is how to maximize the use of a communication channel between a transmitter and a receiver. In other words, its main objective is to characterize the maximum achievable rate of information that can be reliably transmitted over a communication channel, which is called the channel capacity. In contrast to point-to-point channels, if the channel is shared among multiple nodes (multiuser channel), the goal is to find the capacity region, i.e. the set of all simultaneously achievable rates. One of the main assumptions in the information-theoretic formulation of the capacity region is that the maximum achievable rate is obtained under infinitely backlogged users. However, the bursty nature of the sources in communication networks gave rise to the development of a different concept of “capacity region”, which is the maximum stable

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throughput region or the stability region [1]. Understanding the relationship between the information-theoretic capacity region and the stability region has received considerable attention in recent years and some progress has been made primarily for multiple access channels. Interestingly, the aforementioned regions are not in general identical and general conditions under which they coincide are known only in very few cases [2].

In this study, we consider the two-user degraded Gaussian broadcast channel [3], which models the simultaneous communication of information from one source to multiple receivers. Marton in [4] derived an inner bound, which is the best known achievable information-theoretic capacity region for a general discrete memoryless broadcast channel. Fayolle et al. [5] provided a theoretical treatment of some basic problems related to the packet switching broadcast channel. The work in [6] provides a partial characterization of the capacity region of the two-user Gaussian fading broadcast channel. Caire and Shamai in [7] investigated the achievable throughput of a multi-antenna Gaussian broadcast channel. In [8], scheduling policies in a broadcast system are considered and general conditions that cover a class of throughput optimal scheduling policies are obtained. In [9], the authors characterize the stability regions of two-user Gaussian fading multiple access and broadcast networks with centralized scheduling under the assumption of infinite backlogged users. In [10], the capacity region of the two-user broadcast erasure channel is characterized, algorithms based on linear network coding are constructed and the stability region of these algorithms is also provided.

Superposition Coding (SC) [3] is one of the fundamental coding schemes in Information Theory. The objective of SC is to simultaneously encode two messages into a single signal. The receiver with the “better” channel can recover the signal by applying successive interference cancelation while the other receiver treats interference as noise. In [11], SC with conventional frequency division in a Poisson field of interferers is analyzed. Furthermore, in [12], the authors provide a software-radio based design and implementation of SC. Their results show that SC can provide substantial spectral efficiency gains compared to orthogonal schemes, such as time division multiplexing. The stability region of the two-user interference channel is derived in [13], where the case of successive interference cancelation is also considered.

In this paper, the stability region of the two-user degraded Gaussian broadcast channel is obtained.
We assume that the user with the better channel uses SC scheme and we consider two cases for the transmission power: i) the assigned power remains fixed, and ii) the power is adapted to the state of the queues. Furthermore, for the obtained stability region we provide convexity/concavity conditions; their importance lies in the inferiority/superiority of the SC compared to scheduled access. Finally, we provide the maximum aggregate stable throughput of the system.

II. SYSTEM MODEL

We consider a two-user broadcast channel, as depicted in Fig. 1, in which a single transmitter having two different queues intends to communicate with two receivers. In the first (resp. second) queue, the packets (messages) that are destined to receiver $D_1$ (resp. $D_2$) are stored. The packet arrival processes at the first and the second queue are assumed to be independent and stationary with mean rates $\lambda_1$ and $\lambda_2$, respectively. Both queues have infinite capacity to store incoming packets and $Q_i$ denotes the size in number packets of the $i$-th queue. The transmission rates of packets from the first and the second queue are fixed at $R_1$ and $R_2$, respectively.

Time is assumed to be slotted and each source transmits a packet in a timeslot if its queue is not empty; otherwise it remains silent. The transmission of one packet requires one timeslot and we assume that receive acknowledgements (ACKs) are instantaneous and error-free.

The signal $y_i$ received at user $D_i$ at a timeslot $t$ is given by $y_i^t = h_i^t x^t + n_i^t$, $i = 1, 2$, where $n_i^t$ is the additive white Gaussian noise at timeslot $t$ with zero mean and unit variance. The channel gain from the transmitter to $D_i$ is denoted by $h_i^t$ at $t$, and the transmitted signal is $x^t$. A block fading channel
model is considered here with Rayleigh fading, i.e. the fading coefficients $h_i$ remain constant during one timeslot, but change independently from one timeslot to another. In the transmission phase (downlink), the transmitter assigns power $P_i$ for messages (packets) of queue $i$ with $P_1 + P_2 = P$. We assume that each receiver $D_i$ knows each channel $h_i$ (perfect CSIR) and that the transmitter has perfect channel state information (CSIT), i.e. it knows $h_i$, $\forall i$. Each receiver $i$ decodes separately its data using the received signal $y_i$. The distance between the transmitter and $D_i$ is denoted by $d_i$ and $\alpha$ is the pathloss exponent. The signal-to-interference-plus-noise ratio (SINR) threshold for receiver $i$ is $\gamma_i$.

Without loss of generality, we assume that receiver $D_1$ has a better channel than $D_2$. Thus, a superposition coding scheme can be used [14], i.e. the transmitted signal is the superposition of the signals of the two receivers. At the receiver’s side, $D_2$ treats the message of $D_1$ as noise and decodes its data from $y_i$. Receiver $D_1$, which has a better channel, performs successive decoding, i.e. it decodes first the message of $D_2$, then it subtracts it from the received signal, and afterwards decodes its message. Note that in the broadcast channel with superposition coding, the decoding order is different from SIC in the interference channel, in which the signal with the strongest channel is decoded first. The successive decoding is feasible at the first receiver if

$$\left\{ \frac{P_2|h_1|^2 d_1^{-\alpha}}{1 + P_1|h_1|^2 d_1^{-\alpha}} \geq \gamma_2, \quad P_1|h_1|^2 d_1^{-\alpha} \geq \gamma_1 \right\}. \quad (1)$$

The second receiver is able to decode its intended packet if and only if the received SINR is greater than $\gamma_2$. Thus, the achieved rates $R_1$ and $R_2$ by $D_1$ and $D_2$ respectively, are

$$R_i = \log_2 (1 + \gamma_i) \text{ bits/sec/Hz, } i = 1, 2. \quad (2)$$

Let $D_{i/T}$ denote the event that $D_i$ is able to decode the packet transmitted from the $i$-th queue of the transmitter given a set of non-empty queues denoted by $T$ i.e. $D_{1/1,2}$ denotes the event that the first receiver can decode the packet from the first queue when both queues are not empty ($T = \{1, 2\}$). When only the $i$-th queue is non-empty, the event $D_{i/i}$ is defined as

$$D_{i/i} \triangleq \left\{ R_i \leq \log_2 (1 + |h_i|^2 d_i^{-\alpha} P_i) \right\}, \quad (3)$$

which is equivalent to $D_{i/i} = \{2^{R_i} - 1 \leq |h_i|^2 d_i^{-\alpha} P_i\}$. 
We define SNR_i \triangleq |h_i|^2 d_i^{-\alpha} P_i and \gamma_i \triangleq 2^{R_i} - 1. The probability that the link between the transmitter and D_i is not in outage when only the i-th queue is non-empty is given by (Ch. 5.4 in [15]):

$$\Pr(D_{i|i}) = \Pr\{\text{SNR}_i \geq \gamma_i\} = \exp\left(-\frac{\gamma_i d_i^\alpha}{P_i}\right).$$  (4)

Furthermore, if the following condition is satisfied

$$P_2 > P_1 \frac{\gamma_2(1 + \gamma_1)}{\gamma_1},$$  (5)

then the link success probability for D_1 when both queues are non-empty is given by

$$\Pr(D_{1/1,2}) = \Pr(D_{1/1}) = \exp\left(-\frac{\gamma_1 d_1^\alpha}{P_1}\right).$$  (6)

The probability that the link between the transmitter and D_2 is not in outage when both queues are non-empty is given by

$$\Pr(D_{2/1,2}) = \mathbb{1}\left\{P_2 > \frac{\gamma_2}{1 + \gamma_2}\right\} \exp\left(-\frac{\gamma_2 d_2^\alpha}{(1 + \gamma_2)P_2 - \gamma_2}\right),$$  (7)

where \mathbb{1}\{\} is the indicator function. The proof is omitted due to space limitations.

We use the following definition of queue stability [16]:

**Definition 1.** Denote by Q_t^i the length of queue i at the beginning of time slot t. The queue is said to be stable if \lim_{t \to \infty} \Pr[Q_t^i < x] = F(x) and \lim_{x \to \infty} F(x) = 1. If \lim_{x \to \infty} \lim_{t \to \infty} \inf \Pr[Q_t^i < x] = 1, the queue is substable. If a queue is stable, then it is also substable. If a queue is not substable, then we say it is unstable.

Loynes’ theorem [17] states that if the arrival and service processes of a queue are strictly jointly stationary and the average arrival rate is less than the average service rate, then the queue is stable. If the average arrival rate is greater than the average service rate, then the queue is unstable and the value of Q_t^i approaches infinity almost surely. The stability region of the system is defined as the set of arrival rate vectors \lambda = (\lambda_1, \lambda_2) for which the queues in the system are stable.
III. Stability Region

In this section, we derive the stability region of the two-user Gaussian degraded broadcast channel. We consider two schemes regarding the transmission power at each receiver’s packets. The first scheme is the case where we have fixed transmit power $P_i$ for the $i$-th receiver, such that $P_1 + P_2 = P$. The second scheme comes naturally whenever a user is inactive, i.e. has no packets to receive. We consider that the transmitter adapts the power considering the queue state of each receiver, i.e. if the queue $Q_i$ is empty, then all power $P$ is allocated to the $j$-th queue, $(i \neq j)$.

A. Fixed Power Scheme

We assume that the transmitter assigns fixed power $P_1$ (resp. $P_2$) at the $D_1$ (resp. $D_2$) on every timeslot. The service rate seen by the first queue is given by

$$\mu_1 = \Pr (Q_2 > 0) \Pr (D_{1/1,2}) + \Pr (Q_2 = 0) \Pr (D_{1/1}) .$$  \hfill (8)

Since constant transmitting power $P_1$ is used and $D_1$ has better channel than $D_2$, from (6) we have that $\Pr (D_{1/1,2}) = \Pr (D_{1/1})$. Thus, we have

$$\mu_1 = \Pr (D_{1/1}) .$$  \hfill (9)

From Loyne’s criterion for stability [17], the first queue is stable if and only if $\lambda_1 < \mu_1$. From Little’s theorem (Ch. 3.2 in [18]), we have that

$$\Pr (Q_1 > 0) = \frac{\lambda_1}{\Pr (D_{1/1})} .$$  \hfill (10)

The service rate for the second queue is given by

$$\mu_2 = \Pr (Q_1 > 0) \Pr (D_{2/1,2}) + \Pr (Q_1 = 0) \Pr (D_{2/2}) .$$  \hfill (11)

After substituting (10) into (11) we obtain

$$\mu_2 = \Pr (D_{2/2}) + \frac{\Pr (D_{2/1,2}) - \Pr (D_{2/2}) \Pr (D_{1/1})}{\Pr (D_{1/1})} \lambda_1 .$$  \hfill (12)
From Loyne’s criterion we have that the second queue is stable if and only if $\lambda_2 < \mu_2$. The stability region for the degraded broadcast channel is given by (13) is depicted by Fig. 2

$$\mathcal{R} = \left\{ (\lambda_1, \lambda_2) : \frac{\lambda_2}{\Pr (D_{2/2})} + \frac{\Pr (D_{2/2}) - \Pr (D_{2/1,2})}{\Pr (D_{1/1}) \Pr (D_{2/2})} \lambda_1 < 1, \lambda_1 < \Pr (D_{1/1}) \right\}$$  \hspace{1cm} (13)

The success probability $\Pr (D_{2/1,2})$ is given by (7).

**B. Variable Power Scheme based on Queue State**

In this part, we consider a simple adaptive scheme regarding the power allocation for each receiver’s packets. The power allocation is performed as follows: when both queues are not empty, the transmit power for the first and second queue is $P_1$ and $P_2$, respectively, satisfying $P_1 + P_2 = P$. However, when the queue of $i$-th receiver is empty, the total transmit power $P$ is used for transmitting the packets from for the $j$-th (where $j \neq i$) receiver.

The average service rate of the first queue, $\mu_1$, is given by

$$\mu_1 = \Pr (Q_2 > 0) \Pr (D_{1/1}) + \Pr (Q_2 = 0) \Pr (D_{1/1}) .$$  \hspace{1cm} (14)

Respectively, the average service rate of the second queue, $\mu_2$, is given by

$$\mu_2 = \Pr (Q_1 > 0) \Pr (D_{2/1,2}) + \Pr (Q_1 = 0) \Pr (D_{2/2}) .$$  \hspace{1cm} (15)
The success probabilities $\Pr(D_{i/i})$ for $i = 1, 2$ are given by

$$\Pr(D_{i/i}) = \exp\left(-\frac{\gamma_i d_i^a}{P}\right),$$  \hspace{1cm} (16)$$

since when a queue is empty, the transmitter assigns all power to the other queue, and can be obtained from (4).

The success probability $\Pr(D_{1/1,2})$ is given by

$$\Pr(D_{1/1,2}) = \exp\left(-\frac{\gamma_1 d_1^a}{P_1}\right).$$ \hspace{1cm} (17)$$

In the above scheme it is evident that $\Pr(D_{1/1}) \neq \Pr(D_{1/1,2})$, and as a consequence, there is coupling between the queues. Since the average service rate of each queue depends on the queue size of the other queues, it cannot be computed directly. Therefore, we apply the stochastic dominance technique [1], i.e. we construct hypothetical dominant systems, in which one of the sources transmits dummy packets when its packet queue is empty, while the other transmits according to its traffic.

1) **First Dominant System:** The first queue transmits dummy packets: In the first dominant system, when the first queue empties, then the source transmits a dummy packet for the $D_1$, while the second queue behaves in the same way as in the original system. All other assumptions remain unaltered in the dominant system. Thus, in this dominant system, the first queue never empties, thus the service rate for the second queue is given by $\mu_2 = \Pr(D_{2/1,2})$.

Then, we can obtain stability conditions for the second queue by applying Loyne’s criterion [17]. The queue at the second source is stable if and only if $\lambda_2 < \mu_2$, thus $\lambda_2 < \Pr(D_{2/1,2})$.

Thus, the service rate is non-zero if $P_2 > \frac{\gamma_2}{1+\gamma_2}$ from (7). If $\mu_2 = 0$, then the second queue is unstable and consequently the system is unstable.

If $P_2 > \frac{\gamma_2}{1+\gamma_2}$, then we can obtain the probability that the queue of the second transmitter is empty by Little’s theorem and is given by

$$\Pr(Q_2 = 0) = 1 - \frac{\lambda_2}{\Pr(D_{2/1,2})}. \hspace{1cm} (18)$$

After replacing (18) into (14), we obtain that the service rate for the first queue in the first dominant
system is

\[ \mu_1 = \Pr (D_{1/1}) - \Pr (D_{1/1}) - \Pr (D_{1/1,2}) \lambda_2. \quad (19) \]

The first queue is stable if and only if \( \lambda_1 < \mu_1 \). The stability region \( \mathcal{R}_1 \) is obtained from the first dominant system when \( P_2 > \frac{\gamma_1}{1 + \gamma_2} \) is given in (23) after replacing (7), (17) and (16), otherwise the system is unstable and the region is empty, i.e. \( \mathcal{R}_1 = \emptyset \).

2) Second Dominant System: The second queue transmits dummy packets: In the second dominant system, when the second queue empties then the source transmits a dummy packet for the \( D_2 \) while the first queue behaves in the same way as in the original system. In this dominant system, the second queue never empties, so the service rate for the first queue is

\[ \mu_1 = \Pr (D_{1/1,2}). \quad (20) \]

The first is stable if and only if \( \lambda_1 < \mu_1 \). The probability that \( Q_1 \) is empty is

\[ \Pr (Q_1 = 0) = 1 - \frac{\lambda_1}{\Pr (D_{1/1,2})}. \quad (21) \]

The service rate of the second queue, after substituting (21) into (15) is

\[ \mu_2 = \Pr (D_{2/2}) - \frac{\Pr (D_{2/2}) - \Pr (D_{2/1,2})}{\Pr (D_{1/1,2})} \lambda_1. \quad (22) \]

The stability region if \( P_2 > \frac{\gamma_2}{1 + \gamma_2} \) is given by \( \mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2 \) where \( \mathcal{R}_1 \) and \( \mathcal{R}_2 \) are given by (23) and (24) respectively and is depicted by Fig. 3

\[ \mathcal{R}_1 = \left\{ (\lambda_1, \lambda_2) : \frac{\lambda_1}{\exp \left( -\frac{\gamma_1 r_1^2}{P_1} \right)} + \exp \left( -\frac{\gamma_1 r_1^2}{P_1} \right) - \exp \left( -\frac{\gamma_2 r_2^2}{P_1} \right) - \exp \left( -\frac{\gamma_2 r_2^2}{(1+\gamma_2)P_2 - \gamma_2} \right) \lambda_2 < 1, \right. \]

\[ \left. \lambda_2 < \exp \left( -\frac{\gamma_2 r_2^2}{(1+\gamma_2)P_2 - \gamma_2} \right) \right\} \quad (23) \]
\[ R_2 = \left\{ (\lambda_1, \lambda_2) : \frac{\lambda_2}{\exp\left(-\frac{\gamma_2^2}{P}\right)} + \frac{\exp\left(-\frac{\gamma_2^2}{P}\right) - \exp\left(-\frac{\gamma_2^2}{P(1+\gamma_2)}\right)}{\exp\left(-\frac{\gamma_2^2}{P}\right)} \lambda_2 < 1, \lambda_1 < \exp\left(-\frac{\gamma_1^2}{P_1}\right) \right\} \] (24)

If power \( P_2 < \frac{\gamma_2}{1+\gamma_2} \) the region is given by (25), thus, the stability region in this case is \( R = \emptyset \cup R_2 \).

\[ R_2 = \left\{ (\lambda_1, \lambda_2) : \frac{\lambda_2}{\Pr\left(D_{2/2}\right)} + \frac{\lambda_1}{\Pr\left(D_{1/1,2}\right)} < 1, \lambda_1 < \Pr\left(D_{1/1}\right) \right\} \] (25)

An important observation made in [1] is that the stability conditions obtained by the stochastic dominance technique are not only sufficient but also necessary conditions for the stability of the original system. The indistinguishability argument [1] applies to our problem as well. Based on the construction of the dominant system, it is easy to see that the queues of the dominant system are always larger in size than those of the original system, provided they are both initialized to the same value. Therefore, given \( \lambda_2 < \mu_2 \), if for some \( \lambda_1 \), the queue at \( S_1 \) is stable in the dominant system, then the corresponding queue in the original system must be stable. Conversely, if for some \( \lambda_1 \) in the dominant system, the queue at node \( S_1 \) saturates, then it will not transmit dummy packets, and as long as \( S_1 \) has a packet to transmit, the behavior of the dominant system is identical to that of the original system because dummy packet transmissions are eliminated as we approach the stability boundary. Therefore, the original and the dominant system are indistinguishable at the boundary points.

3) Convexity/Concavity conditions: The stability region \( R \), if \( P_2 > \frac{\gamma_2}{1+\gamma_2} \) is convex/concave if

\[ \frac{\Pr\left(D_{1/1,2}\right)}{\Pr\left(D_{1/1}\right)} + \frac{\Pr\left(D_{2/1,2}\right)}{\Pr\left(D_{2/2}\right)} \geq 1, \] (26)

or after replacing the probabilities with their expressions

\[ \frac{\exp\left(-\frac{\gamma_1 d_1^2}{P_1}\right)}{\exp\left(-\frac{\gamma_1 d_1^2}{P}\right)} + \frac{\exp\left(-\frac{\gamma_2 d_2^2}{(1+\gamma_2)P_2-\gamma_2}\right)}{\exp\left(-\frac{\gamma_2 d_2^2}{P}\right)} \geq 1. \] (27)

Concavity of the stability region means that we have a broader region, hence better performance as compared to scheduled access. Scheduled access on the other hand provides a broader region, when \( R \)
is convex. In the latter case, it is preferable to have scheduled access (no interference between receivers) instead of allowing parallel concurrent transmissions.

IV. AGGREGATE STABLE THROUGHPUT

In addition to the stability region, another important performance metric is the maximum aggregate stable throughput, i.e. the sum of the arrivals rates such that both queues are stable as stated by

\[
\text{Maximize } \lambda_1 + \lambda_2 \\
\text{subject to } (\lambda_1, \lambda_2) \in \mathcal{R}
\]

The above maximization problem is a linear program, hence the optimal solution lies at an extreme point (corner point of the stability region). In the following, we provide the corner points for each case.

A. Fixed Power Scheme

The corner points of this scheme are \( (0, \Pr(D_{2/2})) \) and \( (\Pr(D_{1/1}) + \Pr(D_{2/1,2})) \), which give \( \Pr(D_{2/2}) \) and \( \Pr(D_{1/1}) + \Pr(D_{2/1,2}) \) aggregate stable throughput, respectively.

B. Variable Power Scheme based on Queue State

Two cases need to be considered in this scheme, regarding the transmission power to the second receiver. If \( P_2 > \frac{\gamma_2}{1+\gamma_2} \), then we have three corner points to examine. The points are \( (0, \Pr(D_{2/2})) \), \( (\Pr(D_{1/1}) + \Pr(D_{2/1,2})) \) and \( (\Pr(D_{1/1}), 0) \), which have \( \Pr(D_{2/2}) \), \( \Pr(D_{1/1}) + \Pr(D_{2/1,2}) \) and \( \Pr(D_{1/1}) \) aggregate stable throughput, respectively.

If \( P_2 < \frac{\gamma_2}{1+\gamma_2} \), there are two corner points, the \( (0, \Pr(D_{2/2})) \) and \( (0, \Pr(D_{1/1,2}), 0) \). The achievable aggregate stable throughput for these points is \( \Pr(D_{2/2}) \) and \( \Pr(D_{1/1,2}) \), respectively.

V. CONCLUSIONS

In this work, we derived the stability region for the two-user degraded Gaussian broadcast channel under two power allocation policies, namely fixed and adaptive power scheme based on the queue state. Furthermore, we obtained the convexity/concavity conditions for the stability region and investigated the aggregate stable throughput of the network.
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