New pole contribution to $P_{h\perp}$-weighted single-transverse spin asymmetry in semi-inclusive deep inelastic scattering

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Abstract

In this paper, we discuss the new hard pole contribution to the $P_{h\perp}$-weighted single-transverse spin asymmetry in semi-inclusive deep inelastic scattering. We perform the complete next-to-leading order calculation of the $P_{h\perp}$-weighted cross section and show that the new hard pole contribution is required in order to obtain the complete evolution equation for the Qiu-Sterman function derived by different approaches.
1 Introduction

The origin of the single transverse-spin asymmetries (SSAs) in various hard processes has been a longstanding problem for almost 40 years since the unexpected large asymmetries were observed in mid-1970s [1, 2]. Many theoretical works in recent decades found that twist-3 framework in collinear factorization approach is a possible extended framework which can provide a systematic description of the large SSA in perturbative QCD. The twist-3 framework has been well developed in leading-order (LO) accuracy [3-17] in recent decades. Started with the pioneering work by Efremov and Teryaev [3], the more systematic calculation was presented by Qiu and Sterman [4-6]. While the formalism was applied to SSAs in other processes [7, 8, 9], the solid foundation was finally provided in [11] to provide the gauge-invariant twist-3 cross section formula in terms of the complete set of the twist-3 distribution functions. The phenomenological analysis [10, 18] showed that the twist-3 distribution effect of the transversely polarized proton can give a reasonable description of the experimental data and therefore it is widely believed that this effect is one of possible sources of the large SSA.

In usual perturbative QCD calculation, higher-order corrections are often not negligible compared to a leading order contribution. Those corrections bring the logarithmic energy-scale dependence of nonperturbative function which is described by the evolution equation. Systematic treatment of the scale dependence of the twist-3 functions is essential to a quantitative description of the SSA. The twist-3 distribution effect of the transversely polarized proton is embodied as the so-called Qiu-Sterman (QS) function in the spin-dependent cross section formula. The scale evolution equation of the QS function was discussed by using several different approaches so far [19-26]. One of the approaches is the next-to-leading-order (NLO) calculation of the transverse momentum $P_{h\perp}$-weighted cross section. Based on this approach, some part of the evolution equation was first derived in the study of the Drell-Yan process [21]. Subsequently the authors of [26] examined the so-called hard-pole (HP) contribution in the semi-inclusive deep inelastic scattering (SIDIS) and identified an extra term in the evolution equation which had been derived by other approaches [22, 23, 24, 25], while the complete agreement for the whole evolution equation was not yet achieved. In the meanwhile the authors of [27] found the new HP contribution in the study of the $P_{h\perp}$-differential cross section for SSA in SIDIS. In this paper, we include this new HP contribution for the NLO $P_{h\perp}$-weighted cross section. We shall show that this new HP contribution yields extra collinear singularity and its factorization reproduce the correct evolution of the QS function found in [22, 24, 25]. We shall also present the complete NLO cross section for the twist-3 $P_{h\perp}$-weighted cross section for SSA.

The remainder of the paper is organized as follows: in Sec. 2 we introduce the twist-3 distribution functions for the transversely polarized proton. Next, in Sec. 3 we discuss the contribution of the real-emission diagrams in NLO $P_{h\perp}$-weighted cross section. In Sec. 4 we introduce the LO and NLO virtual-correction contributions which were already calculated in the previous work and present the complete NLO cross section formula. Finally, in Sec. 5 we summarize our work.

2 Twist-3 distribution functions for transversely polarized proton

Here we introduce twist-3 functions relevant to our study. The F-type twist-3 functions are defined as

$$M_{F_{ij}}^{\alpha}(x_1, x_2) = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS_\perp | \bar{\psi}_j(0) gF_\alpha^{\alpha n}(\mu n) \psi_i(\lambda n) | pS_\perp \rangle$$
where $F^{\alpha n}$ is a gluon's field strength tensor and we used the simplified notation $F^{\alpha \beta n}_{\beta}$ and $\epsilon^{\alpha \beta p n} \equiv \epsilon^{\alpha \beta \rho \sigma p} n_{\rho} n_{\sigma}$. The anti-symmetric tensor is defined as $\epsilon^{0123} = -1$. We introduced the nucleon mass $M_N$ in order to define the dimensionless functions. From the Hermiticity and $PT$-invariance, one can show the following symmetry properties:

$$G_F(x_1, x_2) = G_F(x_2, x_1), \quad \tilde{G}_F(x_1, x_2) = -\tilde{G}_F(x_2, x_1).$$

In this paper, we discuss the evolution equation of the QS function $G_F(x_1, x_2)$ at $x_1 = x_2$.

3 Contribution of real-emission diagrams to next-leading order cross section

We consider the SSA for light-hadron production in SIDIS,

$$e(\ell) + p(p, S_\perp) \rightarrow e(\ell') + h(P_h) + X.$$  

Within the collinear factorization framework, the SSA can be described by the twist-3 effects. In this process, the SSA receives two types of twist-3 contributions, the distribution effect of the transversely polarized proton and the fragmentation effect of the light-hadron. We focus on the former contribution in this study to derive the evolution equation of the Qiu-Sterman function $G_F(x, x)$. In the case of SIDIS, the cross section formula can be expressed in terms of the following Lorentz invariant variables,

$$S_{ep} = (p + \ell)^2, \quad Q^2 = -q^2, \quad x_B = \frac{Q^2}{2 p \cdot q}, \quad z_h = \frac{p \cdot P_h}{p \cdot q},$$

where $q = (\ell - \ell')$ is the momentum of the virtual photon. We choose the hadron frame [17] for the calculation,

$$\ell = \frac{Q}{2} (\cosh \psi, \sinh \psi \cos \phi, \sinh \psi \sin \phi, -1),$$

$$q = (0, 0, 0, -Q), \quad P^\mu = \left( \frac{Q}{2 x_B}, 0, 0, \frac{Q}{2 x_B} \right), \quad S_\perp^\mu = (0, \cos \Phi_S, \sin \Phi_S, 0)$$

$$P_h = \frac{z_h Q}{2} \left( 1 + \frac{P^2_{\perp h}}{z_h Q^2} \cos \chi, \frac{2 P_{\perp h}}{z_h Q} \sin \chi, \frac{P^2_{\perp h}}{z_h Q^2} - 1 \right),$$

where $\cosh \psi = \frac{2 x_B S_{ep}}{Q^2} - 1$. In this paper, we discuss the NLO $P_{h \perp}$-weighted polarized cross section defined as

$$\frac{d^4 \langle P_{h \perp} \Delta \sigma \rangle}{dx_B dQ^2 dz_h d\phi} \equiv \int d^2 P_{h \perp} \epsilon^{S_\perp P_{h \perp} p n} \left( \frac{d^6 \Delta \sigma}{dx_B dQ^2 dz_h dP^2_{h \perp} d\phi d\chi} \right).$$

First we consider the real-emission diagrams in NLO contribution. The NLO real-emission diagrams in $P_{h \perp}$-weighted cross section are the same as the LO diagrams in $P_{h \perp}$-differential...
The calculation technique to derive a twist-3 cross section for 2 → 2 scattering has been well developed in recent decades and a systematic way to derive the gauge-invariant cross section was established in [11]. We briefly discuss the derivation below. The cross section for SIDIS was presented in [17, 28] as

$$\frac{d^6 \Delta \sigma}{dx_B dQ^2 dz_1 dP^2_\perp d\phi d\chi} = \frac{\alpha_{em}^2}{128\pi^4 z_2 x_B S_{em}^2 Q^2} L_{\mu\nu} W^{\mu\nu}.$$  \hspace{1cm} (9)

where \(\alpha_{em} = \frac{e^2}{4\pi}\) is the QED coupling constant and \(L_{\mu\nu} = 2(\ell_\mu \ell'_\nu + \ell_\nu \ell'_\mu) - Q^2 g_{\mu\nu}\) is the leptonic tensor. Since we are interested in the twist-3 effect of the transversely polarized proton, we introduce the usual twist-2 fragmentation function \(D(z)\) for fragmentation part as

$$W^{\mu\nu} = \int \frac{dz}{z^2} D(z) u^{\mu\nu}$$  \hspace{1cm} (10)

The hadronic tensor \(u^{\mu\nu}\) describes a scattering of the virtual photon and the transversely polarized proton. We consider a “general” diagram given by

$$u^{\mu\nu} = \int d^4 \xi \int d^4 \eta \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} e^{i k_1 \cdot \xi} e^{i k_2 \cdot \eta} \langle PS_{\perp} | \bar{\psi}(0) g A_\alpha(\eta) \psi(x) | PS_{\perp} \rangle$$

$$\times \left( S_{ji}^\alpha(k_1, k_2) + \tilde{S}_{ji}^\alpha(k_1, k_2) \right),$$ \hspace{1cm} (11)

which represents the scattering of the virtual photon and the polarized proton graphically shown in Fig.1. We suppressed the Lorentz indices \(\mu\) and \(\nu\) of the hard parts \(S_{ji}^\alpha(k_1, k_2)\) and \(\tilde{S}_{ji}^\alpha(k_1, k_2)\) for simplicity. Within the collinear factorization framework, a complex phase required for the naively \(T\)-odd SSA can be provided by a pole contribution associated with a internal propagator. In SIDIS case, the pole contributions can be classified into four types as soft-gluon-pole(SGP), soft-fermion-pole(SFP), hard-pole(HP) and another hard-pole(HP2) which are respectively shown in Fig. 2-5. We would like to emphasize that the HP2 contribution was not considered in previous studies for the \(P_{h\perp}\)-weighted cross section and this contribution is essential to obtain the consistent evolution equation of \(G_F(x_1, x_2)\) with the results in different approaches [22, 24, 25]. We can check that the hard part \(S_{ji}^\alpha(k_1, k_2)\) with the pole contribution satisfies the Ward identity

\[(k_2 - k_1) \alpha S_{ji}^\alpha(k_1, k_2) = 0,\] \hspace{1cm} (12)

and associated relations

\[(x_2 - x_1) \frac{\partial}{\partial k_2^\nu} S_{ji}^\alpha p(k_1, k_2) \bigg|_{k_i = x_i p} = -S_{ji}^\alpha(x_1 p, x_2 p),\] \hspace{1cm} (13)

\[(x_2 - x_1) \frac{\partial}{\partial k_1^\mu} S_{ji}^\alpha p(k_1, k_2) \bigg|_{k_i = x_i p} = S_{ji}^\alpha(x_1 p, x_2 p).\] \hspace{1cm} (14)

For SFP and HP contributions, the above relations give

\[\frac{\partial}{\partial k_2^\nu} S_{ji}^\alpha p(k_1, k_2) \bigg|_{k_i = x_i p} = -\frac{\partial}{\partial k_1^\mu} S_{ji}^\alpha p(k_1, k_2) \bigg|_{k_i = x_i p},\] \hspace{1cm} (15)
Figure 1: Diagrammatic description for the hadronic tensor $w^{\mu\nu}$. The upper diagrams and the lower diagrams respectively represent $S_{ji}^\alpha(k_1,k_2)$ and $\tilde{S}_{ji}^\alpha(k_1,k_2)$.

and we can find the same relation for SGP contribution with a direct inspection. Another hard part $S_{ji}^{\text{pole}} p(k_1,k_2)$ also has the same relations. To extract the twist-3 $O(k_\perp)$ contribution from the general contribution (11), we perform the collinear expansion for the hard parts as

$$S_{ji}^{\text{pole}} \alpha(k_1,k_2) = S_{ji}^{\text{pole}} \alpha((k_1 \cdot n)p,(k_2 \cdot n)p) + \frac{\partial}{\partial k_1^\alpha} S_{ji}^{\text{pole}} p(k_1,k_2) \bigg|_{k_1=(k,n)p} \omega^\alpha k_1^\beta$$

$$+ \frac{\partial}{\partial k_2^\alpha} S_{ji}^{\text{pole}} p(k_1,k_2) \bigg|_{k_2=(k,n)p} \omega^\beta k_2^\beta$$

$$= S_{ji}^{\text{pole}} \alpha((k_1 \cdot n)p,(k_2 \cdot n)p) + \frac{\partial}{\partial k_1^\alpha} S_{ji}^{\text{pole}} p(k_1,k_2) \bigg|_{k_1=(k,n)p} \omega^\alpha (k_2^\beta - k_1^\beta), \quad (16)$$

where $\omega^\alpha = g^\alpha_\beta - p^\alpha n_\beta$ and we used the relation (15). And we separate the Lorentz components of the gluon field,

$$A^\alpha = A^n p^\alpha + \omega^\alpha A^\beta. \quad (17)$$

Then we pick up subleading contributions in (11) and construct the F-type correlator (1) as

$$w^{\mu\nu} = \int d^4 \xi \int d^4 \eta \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} e^{i k_1 \cdot \xi} e^{i (k_2 - k_1) \cdot \eta} \langle PS_\perp | \bar{\psi}_j(0) g A^n(\eta) \psi_i(\xi) | PS_\perp \rangle$$

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We express the hard parts in terms of each pole contribution as

\[ S_{ji}^{\text{pole} \alpha}(k_1, k_2) = \mathcal{H}_{L_{ji}}^{SGP \alpha}(k_1, k_2) \left\{ -i\pi \delta \left( \left( \frac{P_h}{z} - (k_2 - k_1) \right)^2 \right) \right\} (2\pi) \delta \left( (k_2 + q - \frac{P_h}{z})^2 \right) \]

\[ + \mathcal{H}_{L_{ji}}^{HP \alpha}(k_1, k_2) \left\{ -i\pi \delta \left( (k_1 + q)^2 \right) \right\} (2\pi) \delta \left( (k_2 + q - \frac{P_h}{z})^2 \right) \]

\[ + \mathcal{H}_{L_{ji}}^{SFP \alpha}(k_1, k_2) \left\{ -i\pi \delta \left( \left( \frac{P_h}{z} - (k_2 - k_1) - q \right)^2 \right) \right\} (2\pi) \delta \left( (k_2 + q - \frac{P_h}{z})^2 \right) \]

+ mirror diagrams 

\[ \tilde{S}_{ji}^{\text{pole} \alpha}(k_1, k_2) = \tilde{\mathcal{H}}_{L_{ji}}^{HP2 \alpha}(k_1, k_2) \left\{ i\pi \delta \left( (k_2 + q)^2 \right) \right\} (2\pi) \delta \left( (k_2 - k_1 + q - \frac{P_h}{z})^2 \right) \]
Figure 3: Diagrammatic description for HP diagrams $H_{\tiny Lji}^{HP\alpha}(k_1, k_2)$. The third gluon line with momentum $k_2 - k_1$ which comes from the transversely polarized proton attaches to one of the black dots in each diagram.

$$+\tilde{H}_{\tiny Lji}^{SFP\alpha}(k_1, k_2)\left\{i\pi\delta\left((\frac{P_h}{z} - k_2 - q)^2\right)\right\}(2\pi)\delta\left((k_2 - k_1 + q - \frac{P_h}{z})^2\right)$$

+ mirror diagrams

We can find that the SFP contributions $\tilde{H}_{\tiny ji}^{SFP\rho}(k_1, k_2)$ and $\tilde{H}_{\tiny ji}^{SFP\rho}(k_1, k_2)$ are the topologically same and then exactly cancel each other. After a little computation, we can obtain the formula for the hadronic tensor $W^{\mu\nu}$ as follows.

$$W^{\mu\nu} = \frac{M_N\pi^2}{2} \int \frac{dz}{z} D(z) \int \frac{dx}{x} \delta\left((xp + q - \frac{P_h}{z})^2\right) \left[-2e^p \epsilon_p S_\perp \frac{d}{dx} G_F(x, x) \left(\hat{s} + Q^2 \right) \frac{t}{t\bar{u}} \text{Tr}[x\gamma H(xp)]\right]$$

$$-2e^p \epsilon_p S_\perp G_F(x, x) \left\{Q^2 \left(\frac{\partial}{\partial \hat{s}} - \frac{\partial}{\partial Q^2}\right) \text{Tr}[x\gamma H(xp)]\right\}$$

$$+ G_F(x, x_B) \frac{1}{\hat{x} - 1} \frac{\hat{x}}{Q^2} \epsilon_p S_\perp \left(\text{Tr}[x\gamma H_{\tiny L}^{HP\alpha}(x_Bp, xp)] + \text{Tr}[x\gamma H_{\tiny R}^{HP\alpha}(xp, x_Bp)]\right)$$

$$- \tilde{G}_F(x, x_B) \frac{1}{\hat{x} - 1} \frac{\hat{x}}{Q^2} i S_\perp \left(\text{Tr}[\gamma_5 x\gamma \hat{H}_{\tiny L}^{HP\alpha}(x_Bp, xp)] - \text{Tr}[\gamma_5 x\gamma \hat{H}_{\tiny R}^{HP\alpha}(xp, x_Bp)]\right)$$

$$+ G_F(x_B, x_B - x) \frac{\hat{x}}{Q^2} \epsilon_p S_\perp \left(\text{Tr}[x\gamma H_{\tiny L}^{HP2\alpha}((x_B - x)p, x_Bp)]\right)$$

$$+ \text{Tr}[x\gamma H_{\tiny R}^{HP2\alpha}(x_Bp, (x_B - x)p)]$$

$$- \tilde{G}_F(x_B, x_B - x) \frac{\hat{x}}{Q^2} i S_\perp \left(\text{Tr}[\gamma_5 x\gamma \hat{H}_{\tiny L}^{HP2\alpha}((x_B - x)p, x_Bp)]\right)$$

$$- \text{Tr}[\gamma_5 x\gamma \hat{H}_{\tiny R}^{HP2\alpha}(x_Bp, (x_B - x)p)]\right\],$$

where we used the Mandelstam variables

$$\hat{s} = (xp + q)^2 = \frac{1 - \frac{\hat{x}}{x}}{Q^2},$$

$$\hat{s} = (xp + q)^2 = \frac{1 - \frac{\hat{x}}{x}}{Q^2}, \quad (21)$$

$$\hat{s} = (xp + q)^2 = \frac{1 - \frac{\hat{x}}{x}}{Q^2}, \quad (22)$$
Figure 4: Diagrammatic description for SFP diagrams. The upper diagrams and the lower diagrams respectively represent $H^{SFP}_j(k_1,k_2)$ and $\tilde{H}^{SFP}_j(k_1,k_2)$.

\[ \hat{t} = (p_c - q)^2 = \frac{1 - \hat{z}}{\hat{x}} Q^2, \]  
\[ \hat{u} = (xp - p_c)^2 = \frac{\hat{z}}{\hat{x}} Q^2, \]  

where $p_c = \frac{P_h}{z}$. We used the Ward identity \( \text{(13)} \) for hard-pole contributions. For the SGP contribution, we used master formula \cite{28,29}

\[ \frac{\partial}{\partial k_2^\beta} \text{Tr}[x_1 \psi S^{SGP}_j p(k_1,k_2)] \big|_{k_1 = x_1 p} = -i\pi \delta(x_1 - x_2) \frac{d}{dp_c^\beta} \text{Tr}[x_1 \psi S(x_1 p)] \]

\[ = 2i\pi \delta(x_1 - x_2) \hat{s} + \frac{Q^2}{u} p_c^\beta \frac{\partial}{\partial t} \text{Tr}[x_1 \psi S(x_1 p)], \]  

where $S(xp)$ is the $2 \rightarrow 2$ scattering cross section without the third gluon line comes from the transversely polarized proton (but the color factor is the same as $S^{SGP}_j$). In this paper, we consider the metric contribution,

\[ L_{\mu\nu} W^{\mu\nu} \rightarrow (-g_{\mu\nu} W^{\mu\nu}), \]  

\[ \frac{d^4(P_{h\perp} \Delta \sigma)^{\text{real}}}{dx_B dQ^2 d\phi} = \frac{\alpha_{em}^2}{32\pi^2 z_h x_B^2 s_{p\perp}^2 Q^2} \int dzzD(z) \int \frac{d^2p_{\perp}}{(2\pi)^2} c s_{p\perp} s_{p\perp} \left(-g_{\mu\nu} w^{\mu\nu}\right), \]  

and the metric should be normalized as $g_{\mu\nu} \rightarrow \frac{1}{1 - \epsilon} g_{\mu\nu}$ with $\epsilon = 2 - D/2$ in $D$-dimensional calculation. We can compute the $P_{h\perp}$-weighted cross section for NLO real-emission diagrams in
Figure 5: Diagrammatic description for HP2 diagrams $H_{L2}^{\alpha}(k_1, k_2)$. These diagrams were first found in [27] in the study of $P_{h\perp}$-differential SSA but was not considered in previous studies of the $P_{h\perp}$-weighted SSA.

D-dimension as follows.

$$
\frac{d^4\langle P_{h\perp}\Delta\sigma\rangle^{\text{real}}}{dx_BdQ^2dz_hd\phi}
= \pi M_N\alpha_s^2m_{\pi}\alpha_s\sum_q e_q^2 \int dz D^q(z)\mu^{2x} \int \frac{d^2-2\epsilon p_{c\perp}}{(2\pi)^{2-2\epsilon}} \left[ \int \frac{dx}{x} \delta\left(p_{c\perp}^2 - \frac{(1 - \hat{x})(1 - \hat{z})\hat{z}}{\hat{x}}\right) Q^2 \right] 
\times \frac{1}{1 - \epsilon} \left[ \frac{d}{dx} G_F^q(x, x) H_D + G_F^q(x, x) H_{ND} + G_F^q(x, x_B) H_{HP} + \tilde{G}_F^q(x, x_B) H_{HPT} 
+ G_F^q(x_B, x_B - x) H_{HP2} + \tilde{G}_F^q(x_B, x_B - x) H_{HPT2} \right],
$$

(28)

where $q$ denotes the quark flavor, $\alpha_s$ is the QCD coupling constant and we used the symmetry for $p_{c\perp}$-integral

$$
\int d^2-2\epsilon p_{c\perp} p_{c\perp\alpha}p_{c\perp\beta}\epsilon^{S_{\perp}\alpha\perp\beta\perp\alpha} = - \int d^2-2\epsilon p_{c\perp} \frac{1}{2(1 - \epsilon)} p_{c\perp}^2 g_{\perp\alpha\beta} \epsilon^{S_{\perp}\alpha\perp\beta\perp\alpha} 
= - \int d^2-2\epsilon p_{c\perp} \frac{1}{2(1 - \epsilon)} \frac{(1 - \hat{x})(1 - \hat{z})\hat{z}}{\hat{x}} Q^2,
$$

(29)

and the hard cross sections can be computed as

$$
H_D = \frac{1}{2N} \left\{ 1 - 2\hat{x} - \hat{z} + \epsilon(1 - 2\hat{x} + \hat{z}) + \frac{1 + \hat{x}^2 - \epsilon(1 - \hat{x})^2}{1 - \hat{z}} \right\}
$$

(30)

$$
H_{ND} = \frac{1}{2N} \left[ - \frac{2}{(1 - \hat{x})(1 - \hat{z})} + \frac{1 + \hat{z} + \epsilon(1 - \hat{z})}{1 - \hat{x}} 
+ \frac{(1 - \hat{x})(1 + 2\hat{x}) - \epsilon(1 - \hat{x})(2\hat{x} - 1)}{1 - \hat{z}} - 2(1 + \epsilon)(1 - \hat{x}) \right]
$$

(31)

$$
H_{HP} = \left( \hat{z}C_F + \frac{1}{2N} \right) \left[ \frac{2}{(1 - \hat{x})(1 - \hat{z})} - \frac{1 + \hat{z} + \epsilon(1 - \hat{z})}{1 - \hat{x}} - \frac{1}{1 - \hat{z}} + (1 + \hat{z} + \epsilon) \right]
$$

(32)

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\[ H_{HPT} = \frac{1}{1 - \epsilon} \left( z C_F + \frac{1}{2N} \right) \left[ - \frac{1 + \hat{z} - 2\epsilon + e^2(1 - \hat{z})}{1 - \hat{z}} - \frac{1 + \epsilon}{1 - \hat{z}} + (1 + \hat{z} + \epsilon \hat{z} + \epsilon^2) \right] \tag{33} \]

\[ H_{HP2} = \frac{1}{2N} \left[ \frac{1 - 2\hat{x}}{1 - \hat{z}} - (1 - 2\hat{x})(1 + \hat{z} + \epsilon) \right] + \frac{1}{1 - \epsilon/2} \left[ (1 - 2\hat{x})(2\hat{z}^2 - 2\hat{z} + 1 - \epsilon) \right] \tag{34} \]

\[ H_{HPT2} = \frac{1}{1 - \epsilon} \frac{1}{2N} \left[ \frac{1 + \epsilon}{1 - \hat{z}} - (1 + \hat{z} + \epsilon \hat{z} + \epsilon^2) \right] - \frac{1}{1 - \epsilon/2} \left[ 1 - 2\hat{z} - \epsilon \right], \tag{35} \]

where \( N = 3 \) is a number of colors and \( C_F = \frac{N^2 - 1}{2N} \). The \( p_{c \perp} \)-integral can be calculated in \( D \)-dimension as

\[
\int \frac{d^2 p_{c \perp}}{(2\pi)^{2-2\epsilon}} \delta \left( p_{c \perp}^2 - \frac{(1 - \hat{x})(1 - \hat{z})\hat{z}}{\hat{x}} Q^2 \right) = \frac{1}{(2\pi)^{2-2\epsilon}} \int dp_{c \perp} \int d\Omega_{2-2\epsilon} (p_{c \perp})^{1-2\epsilon} \delta \left( p_{c \perp}^2 - \frac{(1 - \hat{x})(1 - \hat{z})\hat{z}}{\hat{x}} Q^2 \right) = \frac{1}{4\pi} \frac{4\pi}{Q^2} \frac{1}{\Gamma(1 - \epsilon)} \left( \frac{(1 - \hat{x})(1 - \hat{z})\hat{z}}{\hat{x}} \right)^{-\epsilon}, \tag{36} \]

where \( \Omega_{2-2\epsilon} \) is a solid angle

\[
\int d\Omega_{2-2\epsilon} = \frac{2\pi^{1-\epsilon}}{\Gamma(1 - \epsilon)}. \tag{37} \]

We carry out the \( \epsilon \)-expansion for the phase-space integral as follows.

\[
\hat{z}^{-\epsilon} \simeq 1 - \epsilon \ln \hat{z}, \quad \hat{x}^\epsilon \simeq 1 + \epsilon \ln \hat{x}, \tag{38} \]

\[
(1 - \hat{z})^{-1-\epsilon} \simeq -\frac{1}{\epsilon} \delta(1 - \hat{z}) + \frac{1}{(1 - \hat{z})_+} - \epsilon \left( \frac{\ln(1 - \hat{z})}{1 - \hat{z}} \right)_+, \tag{39} \]

\[
(1 - \hat{x})^{-1-\epsilon} \simeq -\frac{1}{\epsilon} \delta(1 - \hat{x}) + \frac{1}{(1 - \hat{x})_+} - \epsilon \left( \frac{\ln(1 - \hat{x})}{1 - \hat{x}} \right)_+, \tag{40} \]

Then the cross section formula reads

\[
\frac{d^4\langle P_{h \perp} \Delta \sigma \rangle_{\text{real}}}{dx_B dQ^2 d\hat{z}_h d\phi} = -\frac{\pi M_{Nqem}}{4x_B^2 S_{qF}^2 2\pi} \frac{\alpha_s}{Q^2} \frac{4\pi^2}{Q^2} \frac{1}{\Gamma(1 - \epsilon)} \sum_q e_q^2 \left[ \int dz D^q(z) \int dx \frac{d}{dx} G_f^q(x, x) \hat{\sigma}_D + G_f^q(x, x) \hat{\sigma}_{ND} + G_f^q(x, x) \hat{\sigma}_{HP} + G_f^q(x, x) \hat{\sigma}_{HPT} + G_f^q(x, x) \hat{\sigma}_{HPT2} \right], \tag{41} \]

\[
\hat{\sigma}_D = \frac{1}{2N} \left[ -\frac{1}{\epsilon} (1 + \hat{x}^2) \delta(1 - \hat{z}) + (1 - \hat{z}) + \frac{(1 - \hat{x})^2 + 2\hat{z} \hat{x}}{(1 - \hat{z})_+} \right]. \]
\[-\delta(1 - \hat{z})(1 + \hat{\delta}^2)\ln\frac{\hat{x}}{1 - \hat{x}} + 2\hat{x})\]  

(42)

\[\hat{\sigma}_{ND} = \frac{1}{2N} \left[ (-\frac{2}{\epsilon^2})\delta(1 - \hat{x})\delta(1 - \hat{z}) + \left(-\frac{1}{\epsilon}\right)(2\delta(1 - \hat{x})\delta(1 - \hat{z}) - \frac{1 + \hat{z}^2}{(1 - \hat{z})_+}\delta(1 - \hat{x}) \right.\]

\[+ \left. \frac{2\hat{x}^3 - 3\hat{x}^2 - 1}{(1 - \hat{x})_+}\delta(1 - \hat{z}) \right] - 2\delta(1 - \hat{x})\delta(1 - \hat{z}) + \frac{2\hat{x}^3 - 3\hat{x}^2 - 1}{(1 - \hat{x})_+}\delta(1 - \hat{z}) \]

\[+ \frac{1 + \hat{z}}{(1 - \hat{x})_+} - 2(1 - \hat{x}) + \delta(1 - \hat{z}) \left(-\delta(1 - \hat{x})(1 + 2\hat{x})\log\frac{\hat{x}}{1 - \hat{x}} - 2\left(\frac{\ln(1 - \hat{x})}{1 - \hat{x}}\right)_+ \right.\]

\[+ \left. \frac{2}{(1 - \hat{x})_+} - 2(1 - \hat{x}) + \frac{2\ln\hat{x}}{(1 - \hat{x})_+} \right) + \delta(1 - \hat{x}) \left((1 + \hat{z})\ln\hat{z}(1 - \hat{z}) - 2\frac{\ln\hat{z}}{(1 - \hat{z})_+} \right) - 2\left(\frac{\ln(1 - \hat{z})}{1 - \hat{z}}\right)_+ \]

(43)

\[\hat{\sigma}_{HP} = \left(\hat{\sigma}_C + \frac{1}{2N}\right) \left[ \frac{2}{\epsilon^2}\delta(1 - \hat{x})\delta(1 - \hat{z}) + \frac{1}{\epsilon}(2\delta(1 - \hat{x})\delta(1 - \hat{z}) - \frac{1 + \hat{z}^2}{(1 - \hat{z})_+}\delta(1 - \hat{x}) \right.\]

\[+ \left. \frac{1 + \hat{z}}{(1 - \hat{x})_+}\delta(1 - \hat{z}) \right] + 2\delta(1 - \hat{x})\delta(1 - \hat{z}) + \frac{1 + \hat{z}^2}{(1 - \hat{x})_+}\delta(1 - \hat{z}) \]

\[+ \delta(1 - \hat{z}) \left( \log\frac{\hat{x}}{1 - \hat{x}} + 2\left(\frac{\ln(1 - \hat{x})}{1 - \hat{x}}\right)_+ - \frac{2\ln\hat{x}}{(1 - \hat{x})_+} \right) + \left. \frac{1 + \hat{z}}{(1 - \hat{x})_+} \right) \]

\[+ \delta(1 - \hat{x}) \left( -(1 + \hat{z})\ln\hat{z}(1 - \hat{z}) + 2\left(\frac{\ln(1 - \hat{z})}{1 - \hat{z}}\right)_+ + \frac{2\ln\hat{z}}{(1 - \hat{z})_+} - \frac{2\hat{z}}{(1 - \hat{z})_+} \right) \]

(44)

\[\hat{\sigma}_{HPT} = \left(\hat{\sigma}_C + \frac{1}{2N}\right) \left[ \frac{\delta(1 - \hat{z})}{\epsilon^2} - \frac{1 - \hat{z}^2}{(1 - \hat{z})_+}\delta(1 - \hat{z}) \left(\ln\frac{\hat{x}}{1 - \hat{x}} + 3\right) \right] \]

(45)

\[\hat{\sigma}_{HP2} = \frac{1}{2N} \left[ -\frac{1}{\epsilon}(1 - 2\hat{x})\delta(1 - \hat{z}) + \frac{1 - \hat{x}^2}{(1 - \hat{z})_+} - \delta(1 - \hat{z})(1 - 2\hat{x})(\ln\frac{\hat{x}}{1 - \hat{x}} + 1) \right] \]

\[+ \frac{1}{2}(1 - 2\hat{x})(1 - \hat{z}^2) + \hat{z}^2 \]

(46)

\[\hat{\sigma}_{HPT2} = \frac{1}{2N} \left[ -\frac{1}{\epsilon}\delta(1 - \hat{z}) + \frac{\hat{z}^2}{(1 - \hat{z})_+} - \delta(1 - \hat{z})(\ln\frac{\hat{x}}{1 - \hat{x}} + 3) \right] - \frac{1}{2}(1 - 2\hat{z}) \]

(47)

where we used the antisymmetric property \( \tilde{G}_F(x, x_B)\delta(1 - \hat{x}) = \tilde{G}_F(x, x)\delta(1 - \hat{x}) = 0 \). Finally we can derive the contribution of real-emission diagrams as

\[
\frac{d^4\langle P_{h\perp}\Delta\sigma\rangle_{\text{real}}}{dx_BdQ^2dz_Hd\phi}
\]
\[-z_{h}M_{N} \alpha_{s}^{2} \frac{\alpha_{s}}{4 \pi^2} \frac{e^{4 \pi^2}}{2 \pi^2} \left( \frac{Q^2}{\hat{Q}^2} \right) \left( \frac{Q^2}{\hat{Q}^2} \right) \sum_{q} e_{q}^{2} \left( C_{F} \frac{2 \hat{G}_{q}^{q}(x_{B}, x_{B})}{e_{q}^{2}} D_{q}^{q}(z_{h}) \right) \]

\[+ \left( -\frac{1}{\epsilon} \right) \left\{ D_{q}^{q}(z_{h}) \left[ \int_{x_{B}}^{1} \frac{dx}{x} \left[ C_{F} \frac{1 + \hat{z}^{2}}{1 - \hat{x}^{2}} G_{q}^{q}(x, x) + N \frac{(1 + \hat{x}) G_{q}^{q}(x, x) - (1 + \hat{x}) G_{q}^{q}(x, x)}{(1 \cdots \hat{\delta}(1 - \hat{x}) \delta(1 - \hat{z}) \right. \right. \]

\[+ \delta(1 - \hat{z}) \left( (1 + \hat{x}) \ln \hat{\gamma} + 2 \hat{x} \right) \left. \right] + G_{q}^{q}(x, x) D_{q}^{q}(z) \frac{1}{2N\hat{z}} \left[ 1 - \hat{z} + \frac{(1 - \hat{x})^{2} + 2\hat{x}\hat{z}}{(1 - \hat{z})_{+}} \right. \right. \]

\[-2 \left( \frac{\ln(1 - \hat{x})}{1 - \hat{x}} \right) + \frac{2}{(1 - \hat{x})_{+}} - 2(1 - \hat{x}) + 2 \frac{\ln \hat{x}}{(1 - \hat{x})_{+}} \right) + \delta(1 - \hat{x}) \left( 1 + \hat{z} \ln \hat{\gamma}(1 - \hat{z}) \right. \]

\[+ \frac{1 + \hat{x}^{2}}{(1 - \hat{z})_{+}} + \delta(1 - \hat{z}) \left[ \log \frac{\hat{x}}{1 - \hat{x}} + 2 \frac{\ln(1 - \hat{x})}{1 - \hat{x}} \right. \right. \]

\[+ \frac{1 + \hat{x}}{(1 - \hat{x})_{+}} - 2(1 - \hat{x}) \left. \right] \left. \right] + G_{q}^{q}(x, x) D_{q}^{q}(z) \left( C_{F} + \frac{1}{2N\hat{z}} \right) \left[ 2\delta(1 - \hat{x}) \delta(1 - \hat{z}) \right. \]

\[+ \delta(1 - \hat{x}) \left. \right] \left. \right] + G_{q}^{q}(x, x) D_{q}^{q}(z) \left( C_{F} + \frac{1}{2N\hat{z}} \right) \left[ \frac{1 - \hat{z}^{2}}{(1 - \hat{z})_{+}} + \delta(1 - \hat{z}) \left( \ln \frac{\hat{x}}{1 - \hat{x}} + 3 \right) \right. \]

\[+ G_{q}^{q}(x, x B - x) D_{q}^{q}(z) \left. \right] \left. \right] + G_{q}^{q}(x, x B - x) D_{q}^{q}(z) \left[ \frac{1}{2N\hat{z}} \left( \frac{(1 - 2\hat{x})^{2} + \hat{z}^{2}}{(1 - \hat{z})_{+}} - \delta(1 - \hat{z})(1 - 2\hat{x}) \left( \ln \frac{\hat{x}}{1 - \hat{x}} + 1 \right) \right) \right. \]

\[+ \frac{1}{2\hat{z}} \left( (1 - \hat{z})^{2} + \hat{z}^{2} \right) + G_{q}^{q}(x, x B - x) D_{q}^{q}(z) \left[ \frac{1}{2N\hat{z}} \left( \frac{\hat{z}^{2}}{(1 - \hat{z})_{+}} \right) \right. \}

\[- \delta(1 - \hat{z}) \left( \ln \frac{\hat{x}}{1 - \hat{x}} + 3 \right) - \frac{1}{2\hat{z}} \left( 1 - 2\hat{x} \right) \right] \right\}, \quad (48)
where we performed partial integral,
\[
\int_{x_B}^{1} \frac{dx}{x} G_F(x, x)(1 + \hat{x}^2) = \int_{x_B}^{1} \frac{dx}{x} G_F(x, x)(2\hat{x}^2 - 2\delta(1 - \hat{x})),
\]
and we used \( G_F(x, x_B)\delta(1 - \hat{x}) = G_F(x, x)\delta(1 - \hat{x}) \). The boundary condition of the integrals is determined by the condition \( 0 < P_{h\perp} < \sqrt{\frac{2^{2}(1-\hat{x})(1-\hat{x})^2}{x}} Q^2 < P_{h\perp}^{\max} \). The hard cross sections (45)-(47) associated with \( \tilde{G}(x, x_B), G(x_B, x_B - x) \) and \( \tilde{G}(x_B, x_B - x) \) are new results derived in this study and, in particular, the latter two contributions came from the HP2 contribution [27] which was not discussed in previous studies of the \( P_{h\perp} \)-weighted SSA. We should not neglect these new contributions to demonstrate the cancellation of the collinear singularities. Other contributions (42)-(44) agree with those derived in the previous study [26].

4 LO cross section and virtual-correction contribution in NLO cross section

In this section, we introduce the results of the LO cross section and virtual-correction contribution in NLO cross section already derived in [26]. Both contributions can be represented with \( 2 \rightarrow 1 \) scattering cross section. The phase-space integral should be changed from \( 2 \rightarrow 2 \) scattering as follows.

\[
\frac{d^3p_c}{(2\pi)^3 2p_c^0} \frac{d^3p_d}{(2\pi)^3 2p_d^0} (2\pi)^4 \delta^4(xp + q - p_c - p_d) \\
= \frac{d^3p_c}{(2\pi)^3 2p_c^0} (2\pi)^4 \delta^4((xp + q - p_c)^2) \\
\rightarrow \frac{d^3p_c}{(2\pi)^3 2p_c^0} (2\pi)^4 \delta^4(xp + q - p_c)
\]

In this case, we perform \( P_{h\perp} \)-integration before the collinear expansion as

\[
\int d^2 P_{h\perp} \epsilon^{\alpha\beta\rho m} S_{\perp\alpha} P_{h\perp\beta} \left( S_{L\gamma}(k_1, k_2) \delta^2(k_{2\perp} - \frac{P_{h\perp}}{z}) + S_{R\gamma}(k_1, k_2) \delta^2(k_{1\perp} - \frac{P_{h\perp}}{z}) \right)
\]
\[ e^{\alpha \beta m} S_{\perp \alpha} e^{\beta} \left( k_{2 \perp \beta} S_{L}(k_1, k_2) + k_{1 \perp \beta} S_{R}(k_1, k_2) \right), \tag{51} \]

where we used the fact the virtual photon doesn’t have transverse momentum in hadron frame. We can find the following relation for LO diagram shown in Fig. 6.

\[ S_{Lp}(x_1 p, x_2 p) = -S_{Rp}(x_1 p, x_2 p) \equiv S_p(x_1 p, x_2 p). \tag{52} \]

Since the \( P_{\perp} \) integration brought \( O(k_\perp) \) term, the leading term of the collinear expansion gives twist-3 contribution. We can construct the gluon’s field strength tensor as follows.

\[
\int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} e^{ik_1 \cdot x} e^{i(k_2 - k_1) \cdot n} A(n)(k_2 - k_1) S_p((k_1 \cdot n)p, (k_2 \cdot n)p)
\]

\[ = i \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} e^{ik_1 \cdot x} e^{i(k_2 - k_1) \cdot n} A(n)(k_2 - k_1) S_p((k_1 \cdot n)p, (k_2 \cdot n)p)
\]

\[ + \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} e^{ik_1 \cdot x} e^{i(k_2 - k_1) \cdot n} A(n)(k_2 - k_1) S_p((k_1 \cdot n)p, (k_2 \cdot n)p). \tag{53} \]

The last term vanishes due to the SGP delta function. Then we can use the following formula for LO contribution

\[
\frac{d^4 \langle P_{\perp} \Delta \sigma \rangle^{\text{LO}}}{dx_B dQ^2 dz_h d\phi} = \frac{\alpha^2_{em}}{2z_h x_B^2 S_{ep} Q^2} \int dz dD(z) \int dx_1 \int dx_2 \epsilon^\alpha \sum_{ij} F_{ij} \langle x_1, x_2 \rangle \times \left( -g_{\mu\nu} H_{ij}^{\mu\nu}(x_1, x_2) \right) \frac{2x_1}{u Q^2} \delta(x_1 - x_2) \delta(1 - x_2) \delta(1 - z), \tag{54} \]

which agrees with the corresponding formula in \[21, 26\]. LO and NLO contributions in SIDIS were already calculated in previous work \[26\]. We just introduce their results in our notation below.

\[
\frac{d^4 \langle P_{\perp} \Delta \sigma \rangle^{\text{LO}}}{dx_B dQ^2 dz_h d\phi} = -\frac{z_h \pi M N \alpha^2_{em}}{4 x_B^2 S_{ep} Q^2} \sum_q e_q^2 G_q(x_B, x_B) \frac{Q^2}{D^q(z_h)} \tag{55} \]

\[
\frac{d^4 \langle P_{\perp} \Delta \sigma \rangle^{\text{virtual}}}{dx_B dQ^2 dz_h d\phi} = -\frac{z_h \pi M N \alpha^2_{em}}{4 x_B^2 S_{ep} Q^2} \frac{\alpha_s}{2\pi} \sum_q e_q^2 G_q(x_B, x_B) \frac{Q^2}{D^q(z_h)} \times \left[ C_F \left( \frac{4\pi \mu^2}{Q^2} \right) \frac{1}{\Gamma(1 - \epsilon)} \left( \frac{2}{\epsilon^2} \frac{3}{2} - 8 \right) \right] \tag{56} \]

Combining (48), (55) and (56), we obtain the following complete formula for NLO \( P_{\perp} \)-weighted cross section.

\[
\frac{d^4 \langle P_{\perp} \Delta \sigma \rangle^{\text{LO} + \text{NLO}}}{dx_B dQ^2 dz_h d\phi} = -\frac{z_h \pi M N \alpha^2_{em}}{4 x_B^2 S_{ep} Q^2} \sum_q e_q^2 \left[ G_q^q(x_B, x_B) \frac{Q^2}{D^q(z_h)} + \frac{\alpha_s}{2\pi} \left( \frac{4\pi \mu^2}{Q^2} \right) \frac{1}{\Gamma(1 - \epsilon)} \left( \frac{1}{\epsilon} \right) \right] \]
\[
\times \left\{ D^q(\zeta) \left\{ \int_{x_B}^{x} \frac{dx}{x} \left[ P_{qq}(\hat{x})G^q_F(x,x) + \frac{N}{2} \left( \frac{(1 + \hat{x})G^q_F(x_B,x) - (1 + \hat{x}^2)G^q_F(x,x)}{(1 - \hat{x})^+} + \tilde{G}^q_F(x_B,x) \right) \right] \right\} \\
- NG^q_F(x_B,x_B) + \frac{1}{2N} \int_{x_B}^{1} \frac{dx}{x} \left( (1 - 2\hat{x})G^q_F(x_B,x_B - x) + \tilde{G}^q_F(x_B,x_B - x) \right) \right\} \\
+ G^q_F(x_B,x_B) \int_{z_h}^{1} \frac{dz}{z} P_{qq}(\hat{z})D^q(z) \right\} \\
\right. \\
+ \frac{\alpha_s(4\pi^2)\epsilon}{2\pi Q^2} \frac{1}{\Gamma(1 - \epsilon)} \int_{x_B}^{1} \frac{dx}{x} \int_{z_h}^{1} \frac{dz}{z} \left\{ \frac{dx}{x} \left[ xG^q_F(x,x)D^q(z) \frac{1}{2Nz} \left[ 1 - \hat{z} + \frac{(1 + \hat{x})^2 + 2\hat{x}\hat{z}}{(1 - \hat{z})^+} \right] \right] \\
- \delta(1 - \hat{z}) \left( (1 + \hat{x}^2) \ln \frac{\hat{x}}{1 - \hat{x}} + 2\hat{x} \right) \right\} + G^q_F(x,x)D^q(z) \frac{1}{2Nz} \left[ -2\delta(1 - \hat{x})\delta(1 - \hat{z}) \right] \\
+ \frac{2\hat{x}^3 - 3\hat{x}^2 - 1}{(1 - \hat{x})^+(1 - \hat{z})^+} + \frac{1 + \hat{z}}{(1 - \hat{x})^+} - 2(1 - \hat{x}) + \delta(1 - \hat{z}) \left( (1 - \hat{x})(1 + 2\hat{x}) \ln \frac{\hat{x}}{1 - \hat{x}} \right) \\
- 2\left( \ln \frac{1 - \hat{x}}{1 - \hat{z}} \right) + \frac{2}{(1 - \hat{x})^+} - 2(1 - \hat{x}) + 2\left( \ln \frac{\hat{x}}{1 - \hat{x}} \right) + \delta(1 - \hat{z}) \left( (1 + \hat{x}) \ln \frac{\hat{x}}{1 - \hat{x}} \right) \\
- 2\left( \ln \frac{1 - \hat{x}}{1 - \hat{z}} \right) - 2\left( \ln \frac{1 - \hat{z}}{1 - \hat{z}^2} \right) + \frac{2\hat{z}}{(1 - \hat{z})^+} + G^q_F(x,x_B)D^q(z) \left( C_F + \frac{1}{2Nz} \right) \left( 2\delta(1 - \hat{x})\delta(1 - \hat{z}) \right) \\
+ \frac{1 + \hat{x}^2}{(1 - \hat{x})^+(1 - \hat{z})^+} + \delta(1 - \hat{z}) \left( \ln \frac{\hat{x}}{1 - \hat{x}} + 2\left( \ln \frac{1 - \hat{x}}{1 - \hat{z}} \right) + 2\left( \ln \frac{\hat{x}}{1 - \hat{x}} \right) + 2\left( \ln \frac{1 - \hat{z}}{1 - \hat{z}^2} \right) + \frac{2\hat{z}}{(1 - \hat{z})^+} \right) \right\} \\
+ G^q_F(x,x_B)D^q(z) \left( C_F + \frac{1}{2Nz} \right) \left( -\frac{1 - \hat{x}^2}{(1 - \hat{x})^+(1 - \hat{z})^+} + \delta(1 - \hat{z}) \left( \ln \frac{\hat{x}}{1 - \hat{x}} + 3 \right) \right) \right\} \\
G^q_F(x_B,x_B - x)D^q(z) \left[ \frac{1}{2Nz} \left( \frac{(1 - 2\hat{x})\hat{z}^2}{(1 - \hat{z})^+} - \delta(1 - \hat{z})(1 - 2\hat{x})(\ln \frac{\hat{x}}{1 - \hat{x}} + 1) \right) \right] \\
+ \frac{1}{2\hat{z}} (1 - 2\hat{x}) \left( (1 - \hat{z})^2 + \hat{z}^2 \right) + G^q_F(x_B,x_B - x)D^q(z) \left[ \frac{1}{2Nz} \left( \frac{\hat{z}^2}{(1 - \hat{z})^+} \right) \right] \\
- \delta(1 - \hat{z}) \left( \ln \frac{\hat{x}}{1 - \hat{x}} + 3 \right) - \frac{1}{2\hat{z}} (1 - 2\hat{x}) \right] - 8C_F \delta(1 - \hat{x}) \delta(1 - \hat{z}) \right\} \right\},
\]
The double pole terms $\frac{2}{\epsilon} \delta(1-\hat{x})\delta(1-\hat{z})$ are cancelled between the real cross section and the virtual cross section. The single pole term in virtual cross section $2 \times \frac{3}{2} \delta(1-\hat{x})\delta(1-\hat{z})$ is incorporated into the splitting functions. The collinear singularities associated with the twist-3 functions can be subtracted with the following renormalization.

$$G_F(x_B, x_B)$$

$$= G_F^{(0)}(x_B, x_B) + \frac{\alpha_s}{2\pi} \left( -\frac{1}{\epsilon} \right) \left\{ \int_{x_B}^{1} \frac{dx}{x} \left[ P_{qq}(\hat{x})G_F(x, x) \right. \\
+ \frac{N}{2} \left( \frac{(1+\hat{x})G_F(x_B, x) - (1+\hat{x}^2)G_F(x, x)}{(1-\hat{x})_+} + \tilde{G}_F(x_B, x) \right) \right\} - NG_F(x_B, x_B)$$

$$+ \frac{1}{2N} \int_{x_B}^{1} \frac{dx}{x} \left( (1-2\hat{x})G_F(x_B, x_B - x) + \tilde{G}_F(x_B, x_B - x) \right),$$

where we adopted the $\overline{\text{MS}}$-scheme

$$\frac{1}{\epsilon} = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi.$$

(59)

These collinear singularities are the same as those in F-type correlator [11] at 1-loop order [22,24,25]. Then the collinear singularities are consistently subtracted and we can obtain the infrared-safe NLO cross section as follows.

$$\frac{d^4\langle P_{q\perp} \Delta\sigma \rangle_{\text{LO+NLO}}}{dx_B dQ^2 dz_B d\phi}$$

$$= -\frac{\pi\alpha_s}{4\pi} \frac{M_N}{S_{xy} Q^2} \sum_{q} e_q^2 \left[ G_F^q(x_B, x_B, \mu)D^q(z_h, \mu) \right.$$

$$+ \frac{\alpha_s}{2\pi} \ln \left( \frac{Q^2}{\mu^2} \right) \left\{ D^q(z_h, \mu) \left\{ \int_{x_B}^{1} \frac{dx}{x} \left[ P_{qq}(\hat{x})G_F^q(x, x, \mu) \right. \\
+ \frac{N}{2} \left( \frac{(1+\hat{x})G_F^q(x_B, x, \mu) - (1+\hat{x}^2)G_F^q(x, x, \mu)}{(1-\hat{x})_+} + \tilde{G}_F^q(x_B, x, \mu) \right) \right\} \\
- NG_F^q(x_B, x_B, \mu) + \frac{1}{2N} \int_{x_B}^{1} \frac{dx}{x} \left( (1-2\hat{x})G_F^q(x_B, x_B - x, \mu) + \tilde{G}_F^q(x_B, x_B - x, \mu) \right) \right\}$$

$$+ G_F^q(x_B, x_B, \mu) \int_{x_B}^{1} \frac{dz}{z} P_{qq}(\hat{z})D^q(z, \mu) \right\}$$

$$+ \frac{\alpha_s}{2\pi} \int_{x_B}^{1} \frac{dx}{x} \int_{z_h}^{1} \frac{dz}{z} \left\{ \frac{dx}{x} G_F^q(x, x, \mu)D^q(z, \mu) \frac{1}{2N\hat{z}} \left[ 1 - \hat{z} + \frac{(1-\hat{x})^2 + 2\hat{x}\hat{z}}{(1-\hat{z})_+} \right. \\
\left. - \delta(1-\hat{z}) \left( (1+\hat{x}^2) \ln \frac{\hat{x}}{1-\hat{x}} + 2\hat{x} \right) \right] + G_F^q(x, x, \mu)D^q(z, \mu) \frac{1}{2N\hat{z}} \left[ -2\delta(1-\hat{x})\delta(1-\hat{z}) \right.$$
\[
+ \frac{2\hat{x}^3 - 3\hat{x}^2 - 1}{(1 - \hat{x})_+(1 - \hat{z})_+} + \frac{1 + \hat{z}}{1 - \hat{x}} + 2(1 - \hat{x}) + \delta(1 - \hat{z})(-(1 - \hat{x})(1 + 2\hat{x}) \log \frac{\hat{x}}{1 - \hat{x}} \\
- 2\left(\frac{\ln(1 - \hat{x})}{1 - \hat{x}} + \frac{2}{(1 - \hat{x})_+} - 2(1 - \hat{x}) + 2\frac{\ln \hat{x}}{(1 - \hat{x})_+}\right) + \delta(1 - \hat{x})\left((1 + \hat{z}) \ln \hat{z}(1 - \hat{z})ight) \\
- 2\frac{\ln \hat{z}}{(1 - \hat{z})_+} - 2\left(\frac{\ln(1 - \hat{z})}{1 - \hat{z}} + \frac{2\hat{z}}{(1 - \hat{z})_+}\right)
\]

\[+G_F^q(x, x_B, \mu)D^q(z, \mu)\left(C_F + \frac{1}{2N} \right)\left[2\delta(1 - \hat{x}) \delta(1 - \hat{z}) + \frac{1 + \hat{x}\hat{z}^2}{(1 - \hat{x})_+(1 - \hat{z})_+} + \delta(1 - \hat{x})\left((1 + \hat{z}) \ln \hat{z}(1 - \hat{z})\right)\right] \\
+\delta(1 - \hat{z})\left(\ln \frac{\hat{x}}{1 - \hat{x}} + 2\left(\frac{\ln(1 - \hat{x})}{1 - \hat{x}} + \frac{2\ln \hat{x}}{(1 - \hat{x})_+} - \frac{1 + \hat{z}}{(1 - \hat{z})_+}\right)\right) \\
+\delta(1 - \hat{x})\left(- (1 + \hat{z}) \ln \hat{z}(1 - \hat{z}) + 2\left(\frac{\ln(1 - \hat{z})}{1 - \hat{z}} + \frac{2\ln \hat{z}}{(1 - \hat{z})_+} - \frac{2\hat{z}}{(1 - \hat{z})_+}\right)\right) \\
+G_F^q(x, x_B, \mu)D^q(z, \mu)\left(C_F + \frac{1}{2N}\right)\left[- \frac{1 - \hat{x}\hat{z}^2}{(1 - \hat{x})_+(1 - \hat{z})_+} + \delta(1 - \hat{z})\left(\ln \frac{\hat{x}}{1 - \hat{x}} + 3\right)\right] \\
+G_F^q(x_B, x_B - x, \mu)D^q(z, \mu)\left[\frac{1}{2N}\left(\frac{1 - 2\hat{x}}{(1 - \hat{z})_+} \delta(1 - \hat{z})(1 - 2\hat{x}) \left(\ln \frac{\hat{x}}{1 - \hat{x}} + 1\right)\right) \\
+\frac{1}{2\hat{z}}(1 - 2\hat{x})\{(1 - \hat{z})^2 + \hat{z}^2\}\right] + \hat{G}_F^q(x_B, x_B - x, \mu)D^q(z, \mu)\left[\frac{1}{2N}\left(\frac{\hat{z}^2}{(1 - \hat{z})_+}\right) \\
- \delta(1 - \hat{z})(\ln \frac{\hat{x}}{1 - \hat{x}} + 3) - \frac{1}{2\hat{z}}(1 - 2\hat{x})\right] - 8C_F\delta(1 - \hat{x})\delta(1 - \hat{z})\right\} + O(\alpha_s^2), \quad (61)
\]

where the scale dependence of $G_F(x, x, \mu^2)$ was introduced so that the cross section doesn’t depend on the artificial scale $\mu$. Then we can derive the scale evolution equation of $G_F(x, x, \mu^2)$ as

\[
\frac{\partial}{\partial \ln \mu^2} \frac{d^4(P_{h+1}\Delta \sigma)^{\text{LO+NLO}}}{dx_B dQ^2 d\eta d\phi} = 0
\]

\[
\rightarrow \frac{\partial}{\partial \ln \mu^2} G_F(x_B, x_B, \mu^2) = \frac{\alpha_s}{2\pi} \left\{ \int_{x_B}^1 \frac{dx}{x} \left[ P_{qq}(\hat{x}) G_F(x, x, \mu^2) \right. \right. \\
+ \frac{N}{2} \left( \frac{(1 + \hat{x})G_F(x_B, x, \mu^2) - (1 + \hat{x})^2G_F(x_B, x, \mu^2)}{(1 - \hat{x})_+} + \hat{G}_F(x_B, x, \mu^2) \right) \left. \right. \\
+ \frac{1}{2N} \int_{x_B}^1 \frac{dx}{x} \left( (1 - 2\hat{x})G_F(x_B, x_B - x, \mu^2) + \hat{G}_F(x_B, x_B - x, \mu^2) \right) \left. \right\} + O(\alpha_s^2), \quad (62)
\]

which completely agrees with the results in [22, 24, 25].
5 Summary

We added the new hard pole contribution to the $P_{h\perp}$-weighted single-spin asymmetry in semi-inclusive deep inelastic scattering. Since the new pole contribution brings some collinear singularities at one-loop order, we should not neglect it for the exact cancellation of the collinear singularities. Our result showed that the NLO $P_{h\perp}$-weighted cross section has the same collinear singularities with the F-type correlator at one-loop order and then the singularities can be subtracted consistently. In addition, our calculation provided the scale evolution equation of the Qiu-Sterman function which completely agrees with the corresponding results in different approaches.

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References

[1] R. D. Klem, J. E. Bowers, H. W. Courant, H. Kagan, M. L. Marshak, E. A. Peterson, K. Ruddick and W. H. Dragoset et al., Phys. Rev. Lett. 36, 929 (1976).

[2] G. Bunce et al., Phys. Rev. Lett. 36, 1113 (1976).

[3] A. V. Efremov and O. V. Teryaev, Sov. J. Nucl. Phys. 36, 140 (1982) [Yad. Fiz. 36, 242 (1982)]; Phys. Lett. B 150, 383 (1985).

[4] J.-w. Qiu and G. F. Sterman, Phys. Rev. Lett. 67, 2264 (1991).

[5] J.-w. Qiu and G. F. Sterman, Nucl. Phys. B 378, 52 (1992).

[6] J.-w. Qiu and G. F. Sterman, Phys. Rev. D 59, 014004 (1999) [hep-ph/9806356].

[7] Y. Kanazawa and Y. Koike, Phys. Rev. D 64, 034019 (2001) [hep-ph/0012225].

[8] X. Ji, J.-w. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. D 73, 094017 (2006) [hep-ph/0604023].

[9] X. Ji, J.-w. Qiu, W. Vogelsang and F. Yuan, Phys. Lett. B 638, 178 (2006) [hep-ph/0604128].

[10] C. Kouvaris, J. W. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. D 74, 114013 (2006) [arXiv:hep-ph/0609238].

[11] H. Eguchi, Y. Koike and K. Tanaka, Nucl. Phys. B 763, 198-227 (2007) [arXiv:hep-ph/0610314].

[12] Z. B. Kang and J. W. Qiu, Phys. Rev. D 78, 034005 (2008) [arXiv:hep-ph/0806.1970]; Z. B. Kang, J. W. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. D 78, 114013 (2008) [arXiv:hep-ph/0810.3333]
[13] F. Yuan and J. Zhou, Phys. Rev. Lett. 103, 052001 (2009) [arXiv:0903.4680 [hep-ph]].

[14] Z. B. Kang, F. Yuan and J. Zhou, Phys. Lett. B 691, 243 (2010) [arXiv:1002.0399 [hep-ph]].

[15] H. Beppu, Y. Koike, K. Tanaka and S. Yoshida, Phys. Rev. D 82, 054005 (2010) [arXiv:1007.2034 [hep-ph]].

[16] A. Metz and D. Pitonyak, Phys. Lett. B 723, 365 (2013) [arXiv:1212.5037 [hep-ph]].

[17] K. Kanazawa and Y. Koike, Phys. Rev. D 88, 074022 (2013) [arXiv:1309.1215 [hep-ph]].

[18] K. Kanazawa and Y. Koike, Phys. Rev. D 82, 034009 (2010) [arXiv:1005.1468 [hep-ph]]; Phys. Rev. D 83, 114024 (2011) [arXiv:1104.0117 [hep-ph]].

[19] Z. B. Kang and J. W. Qiu, Phys. Rev. D 79, 016003 (2009) [arXiv:0811.3101 [hep-ph]].

[20] J. Zhou, F. Yuan and Z. T. Liang, Phys. Rev. D 79, 114022 (2009) [arXiv:0812.4484 [hep-ph]].

[21] W. Vogelsang and F. Yuan, Phys. Rev. D 79, 094010 (2009) [arXiv:0904.0410 [hep-ph]].

[22] V. M. Braun, A. N. Manashov and B. Pirnay, Phys. Rev. D 80, 114002 (2009) [arXiv:0909.3410 [hep-ph]].

[23] A. Schafer and J. Zhou, Phys. Rev. D 85, 117501 (2012) [arXiv:1203.5293 [hep-ph]].

[24] J. P. Ma and Q. Wang, Phys. Lett. B 715, 157 (2012) [arXiv:1205.0611 [hep-ph]].

[25] Z. B. Kang and J. W. Qiu, Phys. Lett. B 713, 273 (2012) [arXiv:1205.1019 [hep-ph]].

[26] Z. B. Kang, I. Vitev and H. Xing, Phys. Rev. D 87, 034024 (2013) [arXiv:1212.1221 [hep-ph]].

[27] Y. Koike and K. Tanaka, [arXiv:0907.2797 [hep-ph]].

[28] Y. Koike, K. Tanaka and S. Yoshida, Phys. Rev. D 83, 114014 (2011) [arXiv:1104.0798 [hep-ph]].

[29] Y. Koike and K. Tanaka, Phys. Lett. B 646, 232 (2007) [Erratum-ibid. B 668, 458 (2008)] [hep-ph/0612117].