Stückelino Dark Matter in Anomalous $U(1)'$ Models

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Abstract

We study a possible dark matter candidate in the framework of a minimal anomalous $U(1)'$ extension of the MSSM. It turns out that in a suitable decoupling limit the Stückelino, the fermionic degree of freedom of the Stückelberg multiplet, is the lightest supersymmetric particle (LSP). We compute the relic density of this particle including coannihilations with the next to lightest supersymmetric particle (NLSP) and with the next to next to lightest supersymmetric particle (NNLSP) which are assumed almost degenerate in mass. This assumption is needed in order to satisfy the stringent limits that the Wilkinson Microwave Anisotropy Probe (WMAP) puts on the relic density. We find that the WMAP constraints can be satisfied by different NLSP and NNLSP configurations as a function of the mass gap with the LSP. These results hold in the parameter space region where the model remains perturbative.
1 Introduction

A great deal of work has been done recently to embed the standard model of particle physics (SM) into a brane construction [1, 2, 3, 4]. This research is part of the effort, initiated in [5], to build a fully realistic four dimensional vacuum out of string theory. While the original models were formulated in the framework of the heterotic string, the most recent efforts were formulated for type II strings in order to take advantage of the recent work on moduli stabilization using fluxes. Such brane constructions naturally lead to extra anomalous $U(1)$’s in the four dimensional low-energy theory and, in turn, to the presence of possible heavy $Z'$ particles in the spectrum. These particles should be among the early findings of LHC and besides for the above cited models they are also a prediction of many other theoretical models of the unification of forces (see [6] for a recent review). It is then of some interest to know if these $Z'$ particles contribute to the cancelation of the gauge anomaly in the way predicted from string theory or not. In [7] some of the present authors have studied a supersymmetric (SUSY) extension of the minimal supersymmetric standard model (MSSM) in which the anomaly is canceled à la Green-Schwarz. The model is only string-inspired and is not the low-energy sector of some brane construction. The reason of this choice rests in our curiosity to explore the phenomenology of these models keeping a high degree of flexibility, while avoiding the intricacies and uncertainties connected with a string theory construction. For previous work along these lines we refer to [8]-[15]. In this work we perform a consistency check of our model [7] by evaluating the thermal relic density and comparing it against the WMAP data.

The cancelation of the $U(1)'$ anomaly in our model requires the introduction of an extra complex scalar field whose supersymmetric partner is called the Stückelino. We will see in the following that if the latter is the lightest supersymmetric particle (LSP) its interactions are such to define it as an XWIMP (i.e. a weakly interacting particle with couplings at least one order of magnitude less than the standard weak interactions): in fact it will also turn out to be a cold relic. If the Stückelino is the lightest supersymmetric particle (LSP), its relic density turns out to be too high with respect to the experimental data. This is why, following [16], we favor a next to lightest supersymmetric particle (NLSP) with a mass close to the LSP. We show that an interesting scenario arises also for three particle coannihilation processes. In these two cases, the model is consistent with the experimental data. Moreover in the three particle case we find configurations in which the LSP and the NLSP do not need to be nearly degenerate in mass. In this case the mass gap between the two can be of the order of 20%. This is the plan of the paper: in Section 2 we describe our model. In Section 3 and 4 we find the LSP and study the Stückelino interactions. Finally in Section 5 we compute the relic density. Section 6 is a
2 Model Setup

In this section we briefly discuss our theoretical framework. We assume an extension of the MSSM with an additional abelian vector multiplet $V^{(0)}$ with arbitrary charges. The anomalies are canceled with the Green-Schwarz (GS) mechanism and with the Generalized Chern-Simons (GCS) terms. All the details can be found in [7]. All the MSSM fields are charged under the additional vector multiplet $V^{(0)}$, with charges which are given in Table 1, where $Q_i, L_i$ are the left handed quarks and leptons respectively while $U^c_i, D^c_i, E^c_i$ are the right handed up and down quarks and the electrically charged leptons. The superscript $c$ stands for charge conjugation. The index $i = 1, 2, 3$ denotes the three different families. $H_u, d$ are the two Higgs scalars.

|       | SU(3)$_c$ | SU(2)$_L$ | U(1)$_Y$ | U(1)' $Q_i$ |
|-------|-----------|-----------|-----------|-------------|
| $Q_i$ | 3         | 2         | 1/6       | $Q_Q$       |
| $U^c_i$ | 3         | 1         | $-2/3$    | $Q_{U^c}$   |
| $D^c_i$ | 3         | 1         | $1/3$     | $Q_{D^c}$   |
| $L_i$  | 1         | 2         | $-1/2$    | $Q_L$       |
| $E^c_i$ | 1         | 1         | 1         | $Q_{E^c}$   |
| $H_u$  | 1         | 2         | 1/2       | $Q_{H_u}$   |
| $H_d$  | 1         | 2         | $-1/2$    | $Q_{H_d}$   |

Table 1: Charge assignment.

The key feature of this model is the mechanism of anomaly cancelation. As it is well known, the MSSM is anomaly free. In our MSSM extension all the anomalies that involve only the $SU(3), SU(2)$ and $U(1)_Y$ factors vanish identically. However, triangles with $U(1)'$ in the external legs in general are potentially anomalous. These anomalies are\(^5\)

\[
\begin{align*}
U(1)' - U(1)' - U(1)' & : \mathcal{A}^{(0)} \\
U(1)' - U(1) - U(1)' & : \mathcal{A}^{(1)} \\
U(1)' - SU(2) - SU(2) & : \mathcal{A}^{(2)} \\
U(1)' - SU(3) - SU(3) & : \mathcal{A}^{(3)} \\
U(1)' - U(1)' - U(1)' & : \mathcal{A}^{(4)}
\end{align*}
\]

\(^5\)We are working in an effective field theory framework and we ignore throughout the paper all the gravitational effects. In particular, we do not consider the gravitational anomalies which, however, could be canceled by the Green-Schwarz mechanism.
All the remaining anomalies that involve $U(1)'s$ vanish identically due to group theoretical arguments (see Chapter 22 of [17]). Consistency of the model is achieved by the contribution of a St"uckelberg field $S$ and its appropriate couplings to the anomalous $U(1)'$. The St"uckelberg lagrangian written in terms of superfields is [15]

$$\mathcal{L}_S = \frac{1}{4} \left( S + S^\dagger + 4b_3 V^{(0)} \right)^2 \left| \frac{1}{\theta}\partial^2 \theta \right| - \frac{1}{2} \left\{ \sum_{a=0}^3 b_2^{(a)} S \text{Tr} \left( W^{(a)} W^{(a)} \right) + b_2^{(4)} S W^{(1)} W^{(0)} \right\} + h.c. \right.$$  

(6)

where the index $a = 0, \ldots, 3$ runs over the $U(1)'$, $U(1)_Y$, $SU(2)$ and $SU(3)$ gauge groups respectively. The St"uckelberg multiplet is a chiral superfield

$$S = s + i\sqrt{2}\theta \psi_S + \theta^2 F_S - i\theta \sigma^\mu \bar{\theta} \partial_\mu s + \frac{\sqrt{2}}{2} \theta^2 \bar{\theta} \sigma^\mu \partial_\mu \psi_S - \frac{1}{4} \theta^2 \bar{\theta}^2 \Box s$$

(7)

The lowest component of $S$ is a complex scalar field $s = \alpha + i\phi$. In our scenario the scalar $\phi$ is eaten up in the St"uckelberg mechanism to give mass to the gauge field. On the other end, the scalar $\alpha$ is the dilaton of string theory and its value must be determined somehow. We will not investigate the way in which this happens, but we will just retain the final result: $\alpha$ drops out of our effective lagrangian. In the string literature (see for instance [7]-[16]) the fields $\phi$, $\alpha$ and $\psi_S$ are respectively known as axion, saxion and axino.

In this paper, to avoid confusion with the much better known QCD axion/axino system (see for instance [18]-[22]), we will adopt the convention of [23] and, from now on, we will refer to $s$ as the St"uckelberg scalar and to $\psi_S$ as the St"uckelino.

The St"uckelberg multiplet $S$ transforms under $U(1)'$ as

$$V^{(0)} \rightarrow V^{(0)} + i \left( \Lambda - \Lambda^\dagger \right)$$

$$S \rightarrow S - 4i b_3 \Lambda$$

(8)

where $b_3$ is a constant related to the $Z'$ mass. In our model there are two mechanisms that give mass to the gauge bosons: (i) the St"uckelberg mechanism and (ii) the Higgs mechanism. In this extension of the MSSM, the mass terms for the gauge fields for $Q_{H_u} = -Q_{H_d} = 0^6$ are given by

$$\mathcal{L}_M = \frac{1}{2} \begin{pmatrix} V^{(0)}_\mu & V^{(1)}_\mu & V^{(2)}_{3\mu} \end{pmatrix} M^2 \begin{pmatrix} V^{(0)}_\mu \\ V^{(1)}_\mu \\ V^{(2)}_{3\mu} \end{pmatrix}$$

(9)

with $M^2$ being the gauge boson mass matrix

$$M^2 = \begin{pmatrix} M_{V^{(0)}} & 0 & 0 \\ \cdots & g_1^2 v^2 & -g_1 g_2 v^2 \\ \cdots & \cdots & g_2^2 v^2 \end{pmatrix}$$

(10)

We impose this condition to simplify our computations and to give analytical expressions of limited dimensions. There are no obstructions to set $Q_{H_u} = -Q_{H_d} \neq 0$. 

3
where $M_{V(0)} = 4b_3g_0$ is the mass parameter for the anomalous $U(1)$ and it is assumed to be in the TeV range. The lower dots denote the obvious terms under symmetrization. After diagonalization, we obtain the eigenstates

$$A_{\mu} = \frac{g_2 V_{\mu}^{(1)} + g_1 V_{3\mu}^{(2)}}{\sqrt{g_1^2 + g_2^2}}$$

$$Z_{0\mu} = \frac{g_2 V_{3\mu}^{(1)} - g_1 V_{\mu}^{(1)}}{\sqrt{g_1^2 + g_2^2}}$$

$$Z_{\mu}' = V_{\mu}^{(0)}$$

and the corresponding masses

$$M_{\gamma}^2 = 0$$

$$M_{Z_0}^2 = \frac{1}{4}(g_1^2 + g_2^2) v^2$$

$$M_{Z'}^2 = M_{V(0)}^2$$

Finally the rotation matrix from the hypercharge to the photon basis is

$$\begin{align*}
\left( \begin{array}{c}
Z_{\mu}' \\
Z_{0\mu} \\
A_{\mu}
\end{array} \right) &= O_{ij} \left( \begin{array}{c}
V_{\mu}^{(0)} \\
V_{\mu}^{(1)} \\
V_{3\mu}^{(2)}
\end{array} \right) =
\begin{pmatrix}
1 & 0 & 0 \\
0 & -\sin \theta_W & \cos \theta_W \\
0 & \cos \theta_W & \sin \theta_W
\end{pmatrix}
\begin{pmatrix}
V_{\mu}^{(0)} \\
V_{\mu}^{(1)} \\
V_{3\mu}^{(2)}
\end{pmatrix}
\end{align*}$$

where $i, j = 0, 1, 2$.

We now give the expansion of the lagrangian piece $L_S$ defined in (6) in component fields only for the part that is needed in the following sections. Using the Wess-Zumino gauge we get

$$L_{\text{Stückelino}} = \frac{i}{4} \psi_S \sigma^\mu \partial_\mu \bar{\psi}_S - \sqrt{2}b_3 \psi_S \lambda^{(0)} - \frac{i}{2\sqrt{2}} \sum_{a=0}^{2} b_2^{(a)} \text{Tr} \left( \lambda^{(a)} \sigma^\mu \bar{\sigma}^\nu F_{\mu\nu}^{(a)} \right) \psi_S$$

$$- \frac{i}{2\sqrt{2}} b_2^{(4)} \left[ \frac{1}{2} \lambda^{(1)} \sigma^\mu \bar{\sigma}^\nu F_{\mu\nu}^{(0)} \psi_S + (0 \leftrightarrow 1) \right] + \text{h.c.}$$

As it was pointed out in [8], the Stückelberg mechanism is not enough to cancel all the anomalies. Mixed anomalies between anomalous and non-anomalous factors require an additional mechanism to ensure consistency of the model: non-gauge invariant GCS terms must be added. In our case, the GCS terms have the form [10]

$$L_{\text{GCS}} = -d_4 \left[ (V^{(1)} D^\alpha V^{(0)} - V^{(0)} D^\alpha V^{(1)}) W^{(0)}_\alpha + \text{h.c.} \right]_{\theta^2 \bar{\theta}^2} +$$

$$+ d_5 \left[ (V^{(1)} D^\alpha V^{(0)} - V^{(0)} D^\alpha V^{(1)}) W^{(1)}_\alpha + \text{h.c.} \right]_{\theta^2 \bar{\theta}^2} +$$

$$+ d_6 \text{Tr} \left[ (V^{(2)} D^\alpha V^{(0)} - V^{(0)} D^\alpha V^{(2)}) W^{(2)}_\alpha + \text{n.a.c + h.c.} \right]_{\theta^2 \bar{\theta}^2}$$

(19)
where \( n.a.c. \) refers to non-abelian completion terms. The \( b \) constants in (6) and the \( d \) constants in (19) are fixed by the anomaly cancelation procedure (for details see [7]).

For a symmetric distribution of the anomaly, we have

\[
\begin{align*}
    b_2^{(0)} b_3 &= - \frac{\mathcal{A}^{(0)}}{384\pi^2} \\
    b_2^{(1)} b_3 &= - \frac{\mathcal{A}^{(1)}}{128\pi^2} \\
    b_2^{(2)} b_3 &= - \frac{\mathcal{A}^{(2)}}{64\pi^2} \\
    b_2^{(4)} b_3 &= - \frac{\mathcal{A}^{(4)}}{128\pi^2} \\
    d_4 &= - \frac{\mathcal{A}^{(4)}}{384\pi^2} \\
    d_5 &= \frac{\mathcal{A}^{(1)}}{192\pi^2} \\
    d_6 &= \frac{\mathcal{A}^{(2)}}{96\pi^2}
\end{align*}
\]

(20)

It is worth noting that the GCS coefficients \( d_{4,5,6} \) are fully determined in terms of the \( \mathcal{A} \)'s by the gauge invariance, while the \( b_2^{(a)} \)'s depend only on the free parameter \( b_3 \), which is related to the mass of the anomalous \( U(1) \).

The soft breaking sector of the model is given by

\[
\mathcal{L}_{\text{soft}} = \mathcal{L}_{\text{soft}}^\text{MSSM} - \frac{1}{2} \left( M_0 \lambda^{(0)} \lambda^{(0)} + \text{h.c.} \right) - \frac{1}{2} \left( \frac{M_S}{2} \psi_S \psi_S + \text{h.c.} \right)
\]

(21)

where \( \mathcal{L}_{\text{soft}}^\text{MSSM} \) is the usual soft susy breaking lagrangian while \( \lambda^{(0)} \) is the gaugino of the added \( U(1)' \) and \( \psi_S \) is the Stückelino. The Stückelino soft mass term deserves some comment: from [24] we know that a fermionic mass term for a chiral multiplet is not allowed in presence of Yukawa interactions in which this chiral multiplet is involved. But in the classical Lagrangian the Stückelberg multiplet cannot contribute to superpotential terms given that the gauge invariance given from our \( U(1)' \) symmetry (8) requires non-holomorphicity in the chiral fields. In fact in our model both the Stückelino and the scalar \( \phi \) couple only through GS interactions. It is worth noting that a mass term for the scalar \( \phi \) is instead not allowed since it transforms non trivially under the anomalous \( U(1)' \) gauge transformation (8).

3 Neutralino Sector

Assuming the conservation of R-parity the LSP is a good weak interacting massive particle (WIMP) dark matter candidate. As in the MSSM the LSP is given by a linear combination of fields in the neutralino sector. The general form of the neutralino mass matrix is given in [7]. Written in the interaction eigenstate basis \( (\psi^0)^T = (\psi_S, \lambda^{(0)}, \lambda^{(1)}, \lambda^{(2)}_3, \tilde{h}^0_d, \tilde{h}^0_u) \) it is a six-by-six matrix. From the point of view of the strength of the interactions the two extra states are not on the same footing with respect to the standard ones. The Stückelino and the extra gaugino \( \lambda^{(0)} \) dubbed primeino are in fact extremely weak interacting massive particle (XWIMP). Thus we are interested in situations in which the extremely weak sector is decoupled from the standard one and the LSP belongs to this sector. This can be achieved at tree level with the choice

\[
Q_{H_u} = Q_{H_d} = 0
\]

(22)
The neutralino mass matrix $M_N$ becomes

$$
M_N = \begin{pmatrix}
\frac{M_S}{2} & \frac{M_{V(0)}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\
\cdots & M_0 & 0 & 0 & 0 & 0 \\
\cdots & \cdots & M_1 & 0 & -\frac{g_1 v_d}{2} & \frac{g_2 v_u}{2} \\
\cdots & \cdots & \cdots & M_2 & \frac{g_2 v_d}{2} & -\frac{g_2 v_u}{2} \\
\cdots & \cdots & \cdots & \cdots & 0 & -\mu \\
\cdots & \cdots & \cdots & \cdots & \cdots & 0
\end{pmatrix}
$$

where $M_S$, $M_0$, $M_1$, $M_2$ are the soft masses coming from the soft breaking terms (21) while $M_{V(0)}$ is given in (10). It is worth noting that the D terms and kinetic mixing terms can be neglected in the tree-level computations of the eigenvalues and eigenstates.

Moreover, we make the assumption that

$$M_0 \gg M_S, M_{V(0)}$$

so that the Stückelino is the LSP. This assumption is motivated by the interaction strengths of the two extra states: the Stückelino interacts via the vertex shown in Fig. 1b which can easily be seen (from (18)) to be proportional to the coefficient $b_2^{(a)}$ of (20) which, in turn, is inversely proportional to $b_3$ that is the $Z'$ mass given that $M_{Z'} = M_{V(0)} = 4b_3g_0$. Given these considerations the vertex shown in Fig. 1b is then of order $\sim g_0 g_2^2 / M_{Z'}$. The primeino interacts via the vertex in Fig. 1a which is of order $\sim g_a$ that is the standard strength of weak interactions. Assuming the two decoupling relations (22) and (24) we will see in the following sections that the dominant contribution in the (co)annihilation processes is that of the primeino, which is of the type of a standard gaugino interaction.

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Figure 1: (a) Gaugino-fermion-sfermion interaction vertex. (b) Stückelino-gaugino-vector interaction vertex.
4 Stückelino Interactions

The Stückelino interactions can be read off from the interaction lagrangian (18). The relevant Stückelino-MSSM neutralino interaction term, written in terms of four components Majorana spinors\(^7\), is given by:

\[
L = i\sqrt{2}g_1^2 b_1(1) \tilde{\Lambda}^{(1)} \gamma_5 [\gamma^{\mu}, \gamma^{\nu}](\partial_\mu V^{(1)}_{\nu})\Psi_S + i\frac{\sqrt{2}}{2} g_2^2 b_2(2) \tilde{\Lambda}^{(2)} \gamma_5 [\gamma^{\mu}, \gamma^{\nu}](\partial_\mu V^{(2)}_{3\nu})\Psi_S
\]  

where the \(b_2^{(a)}\) coefficients are given in (20). The related interaction vertex Feynman rule is

\[
C^{(a)} \gamma_5 [\gamma^{\mu}, \gamma^{\nu}]i k_\mu
\]

where \(k_\mu\) is the momentum of the outgoing vector and the \(C^{(a)}\)'s are

\[
C^{(1)} = \sqrt{2}g_1^2 b_1(1) \\
C^{(2)} = \frac{\sqrt{2}}{2} g_2^2 b_2(2)
\]  

The interaction Lagrangian (25) expressed in the mass eigenstates basis is

\[
L = i \sum_j \tilde{N}_j (\cos \theta_W C^{(1)} N_{(1)j} + \sin \theta_W C^{(2)} N_{(2)j}) \gamma_5 [\gamma^{\mu}, \gamma^{\nu}](\partial_\mu A_\nu)\Psi_S + \\
i \sum_j \tilde{N}_j (-\sin \theta_W C^{(1)} N_{(1)j} + \cos \theta_W C^{(2)} N_{(2)j}) \gamma_5 [\gamma^{\mu}, \gamma^{\nu}](\partial_\mu Z_{0\nu})\Psi_S
\]  

where \(\tilde{N}_j\) is a generic neutralino and \(N_{ij}\) is the matrix that diagonalizes (23). We remind that the Stückelino \(\Psi_S\) is directly a mass eigenstate because of the decoupling (24). We stress that these are only interactions between the Stückelino and the MSSM neutralinos. All the Stückelino interactions in (18) include also analogous interactions involving the charged wino or the primeino.

The factors in (27) are of naive dimension higher than four. They then contain the parameters \(b_2^{(a)}\) which are inversely proportional to the mass of the \(Z'\) (see (20)). Given these interactions, our Stückelino will be an extremely weak interacting particle, according to the definition we gave in the introduction. The simplifying assumption (22) has now decoupled our Stückelino and primeino from the standard MSSM sectors and, at tree level, the relevant diagrams are given in Figs. 2 and 3. We can now give a rough estimate of the effective interaction, comparing with the standard Fermi coupling, \(G_F\), of weak interactions: given a mass of the \(Z'\) boson of the order of the TeV, the effective coupling in Fig. 2 is \(G'_F \approx 10^{-4} G_F\) and that in Fig. 3 is \(G'_F \approx 10^{-2} G_F\). Let us go back now to

\(^7\)The gamma matrices \(\gamma^{\mu}\) are in the Weyl representation. We use capital letters for four components spinors and lower case letters for two components spinors.
Figure 2: Annihilation of two St"uckelinos into two gauge vectors via the exchange of a gaugino.

Fig. 2, where we denoted with \( p_1 \) and \( p_2 \) the incoming momenta of the St"uckelinos while \( k_1 \) and \( k_2 \) are the two outgoing momenta of the gauge bosons in the final state. We will concentrate on the case with two photons in the final state. In this case the result for the differential cross section is given by

\[
\frac{d\sigma}{d\Omega} = \frac{4M_S^2\omega_1}{16\pi^2(\omega_1 + \omega_2)^2(\sqrt{M_S^2 - E_2^2})} \sum_{i,j=1}^{2} M_iM_j^* \tag{29}
\]

where \( \omega_1 \) and \( \omega_2 \) are the energies of the two outgoing photons. Each amplitude \( M_i \) is proportional to the relative coefficient \( (C'(a))^2 \) whose generic form is given in (27). In our scenario the St"uckelino annihilations alone, being the cross section (29) extremely weak, cannot give a relic density in the WMAP preferred range. Thus, in the scenario of an XWIMP St"uckelino, we are forced to consider coannihilations between the St"uckelino and the NLSP [16]. Several scenarios can be considered for the NLSP. We can split them into two major classes: one in which the NLSP is either a pure bino or a pure wino, and thus a coannihilation with a third MSSM particle is needed in order to recover the WMAP result, and one in which the NLSP is a generic MSSM neutralino with a non-negligible bino and/or wino component. In both classes in order to have effective coannihilations, the NLSP (and eventually the other MSSM particle involved in the coannihilation process) must be almost degenerate in mass. Furthermore, in the most common applications, the cross sections for the annihilations of the LSP and the NLSP and that of the coannihilations between the LSP and NSLP are roughly of the same order of magnitude. In our case this last condition will be stretched and the cross section we are going to discuss will differ for some orders of magnitude. This situation is not completely new in literature: already in [25] in which the NLSP is the stop these differences are of order \( 10^{-2} \). In [16] differences of order \( \leq 10^{-4} \) are considered, while in [26] \( 10^{-4} \div 10^{-5} \)
Figure 3: Coannihilation of a Stückelino and a bino into a $f\bar{f}$ pair via the exchange of a photon or a $Z_0$.

\[ \Psi_S(p_1) \rightarrow f p_3 \]
\[ \lambda(p_2) \rightarrow f p_4 \]
\[ \gamma/Z_0 \]

differences are found\(^8\). But what assures us that the two species are still in thermal equilibrium and do not decouple separately? The existence of interactions of the type \( LSP + MSSM1 \rightarrow NLSP + MSSM2 \) is then required to keep the LSP in equilibrium [27]. \( MSSM1, MSSM2 \) are two MSSM particles. Furthermore, \( MSSM1 \) better be relativistic so that its abundance is much larger of any cold particle to foster the above reaction. The above reaction will then keep the LSP at equilibrium and the formalism of coannihilation can be safely employed. As we will see in the next section, all of these requirements are met in our scenario.

As a first example we then consider a pure bino as the NLSP. The allowed coannihilation processes with the Stückelino are those which involve an exchange of a photon or a $Z_0$ in the intermediate state and with a SM fermion-antifermion pair, Higgses and $W$’s in the final state. The diagram with the fermion-antifermion in the final state is sketched in Fig. 3. The differential cross section in the center of mass frame has the following general form

\[ \frac{d\sigma}{d\Omega} \propto \frac{1}{s} \frac{p_f}{p_i} |\mathcal{M}|^2 \]  
\(\text{(30)}\)

where \(s\) is the usual Mandelstam variable and \(p_{f,i}\) is the spatial momentum of the outgoing (incoming) particles. On dimensional ground \(|\mathcal{M}|^2\) has at least a linear dependence on \(p_f\) and this implies that the dominant contribution comes from the diagram with the SM fermion-antifermion pair \(f\) and \(\bar{f}\) in the final state:

\[ \Psi_S \lambda^{(a)} \rightarrow f \bar{f} \]  
\(\text{(31)}\)

The resulting differential cross section, computed in the center of mass frame, is

\(^8\)This can be extracted from Fig.4 of the previous reference after an appropriate rescaling.
\[
\frac{d\sigma}{d\Omega} = \sum f c_f \frac{\sqrt{(E_3 - m_f)^2}}{64\pi^2 (E_1 + E_2)^2 \sqrt{(E_i^2 - M_S^2)}} (M_\gamma^2 + M_{Z_0}^2 + M_{\gamma Z_0}^2 + M_{\gamma Z_0}^2) \quad (32)
\]

where the sum is extended to all the SM fermions (with mass \(m_f\)) while \(c_f\) is a color factor. Details of the amplitude computation can be found in Appendix A.

5 Stückelino Relic Density

In this section we compute the relic density of the Stückelino. The case of the Stückelino as a cold dark matter candidate has been studied for the first time in [22]. As we said in the previous section we study two scenarios: the first in which the Stückelino coannihilates with only one NLSP degenerate in mass (a generic MSSM neutralino), the second in which there is an additional supersymmetric particle (either a chargino or a stau) involved in the coannihilation process with the Stückelino and the NLSP. In the following we will be largely following [16]. Since this is a first study and given also the simplifying choice (22) we will defer a complete analysis to a future work and will content ourselves with showing that our model can accommodate for WMAP data. Then, following this philosophy we will not solve the Boltzmann equation numerically but, in agreement with [16], we will argue that, if the ratio between the thermally averaged cross sections of the coannihilation of Fig. 3 and that of a typical neutralino annihilation is much less than one, a relic density satisfying the WMAP requirements can be found.

Just to fix the notation we briefly review the relic density computation for \(N\) interacting species [25, 26, 27]. The requirements discussed in the previous section are met by our model given the lagrangian (18) and the condition of (22). In this case all channels are open to interactions and the Stückelino has an interaction with the photon and the bino of strength \(b_2^{(1)}\).

The Boltzmann equation for \(N\) particle species is given by:

\[
\frac{dn}{dt} = -3Hn - \sum_{i,j=1}^{N} (\sigma_{ij}v_{ij})(n_in_j - n_{eq_i}n_{eq_j}) \quad (33)
\]

where \(n_i\) denotes the number density per unit of comoving volume of the species \(i = 1, \ldots, N\) (\(i = 1\) refers to the LSP, \(i = 2\) refers to the NLSP, and so on), \(n = \sum_i n_i\), \(H\) is the Hubble constant, \(\sigma_{ij}\) is the annihilation cross section between a species \(i\) and a species

\[\text{\footnotesize \(\int d^2\theta \int d^2\bar{\theta} \log(S + S^\dagger + V^{(0)})/M_S (LH_u/M_S^2 + H_u^\dagger L^\dagger/M_S^2)\) can be safely added to the lagrangian. This term induces a vertex between the Stückelino, neutrino and Higgs field which can be a viable candidate to keep the Stückelino in thermal equilibrium.} \]
$j$, $v_{ij}$ is the modulus of the relative velocity while $n_i^{eq}$ is the equilibrium number density of the species $i$ given by:

$$
\frac{n_i^{eq}}{n_i^{eq}} = \frac{g_i (1 + \Delta_i)^{3/2} e^{-\Delta_i x_f}}{\sum_i g_i (1 + \Delta_i)^{3/2} e^{-\Delta_i x_f}}
$$

(34)

where $g_i$ are the internal degrees of freedom, $\Delta_i = (m_i - m_1)/m_1$. $x_f = m_1/T$ is known once (33) is solved. A preliminary estimate of $x_f$ can be obtained by solving the equation

$$
x \simeq \ln \left( x^{1/2} M_p m_{LSP} \langle \sigma v \rangle \right)
$$

(35)

which can be obtained from the decoupling condition. This can be done using the estimate\(^{10}\)

$$
\langle \sigma v \rangle \simeq G^2 m_{LSP}^2 x^{-5/2}
$$

(36)

where the effective coupling $G$ can be the $G_F, G_F'$ introduced in Section 4. In the mass range $m_{LSP} = 10 \div 1000$ GeV, by plugging in (36) $G = G_F$ we would get $x_f = 25 \div 30$ (if we would take our Stückelino as a separate species, that is we would use $G = G_F'$, we would get those values divided by half).

Eq. (33) can be rewritten in a useful way by defining the thermal average of the effective cross section

$$
\langle \sigma_{eff} v \rangle \equiv \sum_{i,j=1}^{N} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{eq} n_j^{eq}}{n_i^{eq} n_j^{eq}}
$$

(37)

obtaining

$$
\frac{dn}{dt} = -3Hn - \langle \sigma_{eff} v \rangle (n^2 - (n^{eq})^2)
$$

(38)

where $n^{eq} = \sum_i n_i^{eq}$. It is sensible to use these approximations when the LSP is kept in equilibrium by a relativistic particle in the thermal background as discussed in the previous section. Given the typical values of $x_f$ discussed above, the ratio between the number density per comoving volume of this relativistic species and that of a cold relic is $n_{rel}^{eq} / n_{cold}^{eq} = 10^4 \div 10^6$, which is sufficient to keep the Stückelino coupled until the end of coannihilations.

As a rule of thumb [28] a first order estimate of the relic density is given by

$$
\Omega_\chi h^2 \simeq \frac{10^{-27} \text{cm}^3 \text{s}^{-1}}{\langle \sigma_{eff} v \rangle}
$$

(39)

\(^{10}\)To check this crude estimate in the case of our Stückelino, we have also solved numerically $\langle \sigma v \rangle$ sweeping the temperatures range $1 \div 100$ GeV. We have then fit the results to recover a function in good agreement with (36).
To give a rough idea of the role played by coannihilations we plotted in Fig. 4 the relic density estimate (39) induced by an electro-weak cross section ((36) with $G = G_F$), a St"uckelino cross section ((36) with $G = G'_F$) and two coannihilations cross sections estimations. We can see that the St"uckelino annihilations cannot give a relic density in the WMAP data range, while coannihilations can do it. Moreover we can see that the effect of coannihilations on the MSSM is to decrease the efficiency of the MSSM annihilations (while they increase the St"uckelino one) and to increase the LSP mass value (according to an increasing mass gap) in order to agree with WMAP data. We stress that Fig. 4 does not take into account several MSSM parameters such as the sfermion masses, neutralino composition etc., so it is just a rough estimate that, however, clarifies the role played by the coannihilations.

In the following we will now give a better estimation of $\langle \sigma_{\text{eff}} v \rangle$ in the two cases $N = 2$ and $N = 3$ using what we have learnt in our scenario and the coannihilation cross section of Fig. 3 presented in detail in Appendix A.

- $N = 2$ case. Assuming that the relative velocities are all equal $v_{ij} \equiv v$ we get:

$$
\langle \sigma_{\text{eff}}^{(2)} v \rangle = \frac{\langle \sigma_{11} v \rangle / \langle \sigma_{22} v \rangle}{1 + Q^2} \left(1 + 2 \frac{\langle \sigma_{12} v \rangle / \langle \sigma_{22} v \rangle Q + Q^2}{\langle \sigma_{22} v \rangle} \right)
$$

(40)

where $Q = n_{1}^{eq} / n_{2}^{eq}$. The first term in the numerator can be neglected because the St"uckelino annihilation cross section is suppressed by a factor $(C^{(a)})^4$ with respect to the MSSM neutralino annihilations (see the previous section) and thus $\langle \sigma_{11} v \rangle \ll \langle \sigma_{22} v \rangle$. The second term involves the coannihilation cross section. Let us consider the case in which the NLSP is a generic MSSM neutralino (a linear combination
of $\lambda^{(1)}$, $\lambda^{(2)}_3$, $\tilde{h}^0_d$, $\tilde{h}^0_u$) with a non-vanishing bino or wino components. As we saw in the previous section each amplitude is generically proportional to $C^{(a)} g_i$ with $i = 1, 2$. Without loss of generality we consider the diagram which involves the bino component $\Psi S \lambda^{(1)} \rightarrow f \bar{f}$ and a photon exchange in the intermediate channel, i.e. the $M^2_\gamma$ amplitude in (32). We get

$$C^2_\gamma = (C^{(1)} \cos \theta_W)^2 = 2(b_2^{(1)})^2 g_i^4 \cos^2 \theta_W$$  \hspace{1cm} (41)$$

From the expression of the mixed $U(1)' - U(1)_Y - U(1)_Y$ anomaly (see [7]) and from the (20) we have the following relation

$$b_2^{(1)} = \frac{3(3Q_Q + Q_L)}{256 \pi^2 b_3}$$  \hspace{1cm} (42)$$

where $b_3 = M_{Z'}/4g_0$. With the assumption $M_{Z'} = 1$ TeV as in [7] we finally get

$$\frac{C^2_\gamma}{e^2} \simeq 5.76 \times 10^{-12} (3g_0Q_Q + g_0Q_L)^2 \text{GeV}^{-2}$$  \hspace{1cm} (43)$$

where $e$ is the electric charge. We get similar expressions for the other three terms in (32). This result has to be compared to the typical weak cross section $\langle \sigma_22v \rangle \simeq 10^{-9} \text{GeV}^{-2}$. As long as the charges and the coupling constant of the extra $U(1)$ satisfy the perturbative requirement

$$g_0^2 \cdot (3Q_Q + Q_L)^2 < 16$$  \hspace{1cm} (44)$$

the following upper bound is satisfied:

$$\frac{\langle \sigma_{12}v \rangle}{\langle \sigma_{22}v \rangle} \lesssim 10^{-6}$$  \hspace{1cm} (45)$$

in the case of a pure bino, while

$$\frac{\langle \sigma_{12}v \rangle}{\langle \sigma_{22}v \rangle} \lesssim 10^{-5}$$  \hspace{1cm} (46)$$

in the case of a pure wino. Accordingly to eqs. (39), (40), (45) and (46) the relic density gets rescaled as [16]

$$\langle \Omega h^2 \rangle^{(2)} \simeq \left[ \frac{1 + Q}{Q} \right] \langle \Omega h^2 \rangle^{(1)}$$  \hspace{1cm} (47)$$

We performed a random sampling of MSSM models in which the NLSP is a pure bino or a mixed bino-higgsino (the case of a pure wino falls back into the $N = 3$ case due to the wino-chargino mass degeneracy) and we computed the relic density in presence of coannihilations using the DarkSUSY package [29]. These two situations are easily realized in some corners of the mSUGRA parameter space. Thus
in our scan we assumed this scenario in order to fix the pattern of the supersymmetry breaking parameters at weak scale. We emphasize here that this choice is completely arbitrary, and it is assumed only for simplicity, since in our model [7] the supersymmetry breaking mechanism is not specified. In the former case there is no model which satisfies the WMAP constraints [30]:

\[
0.0913 \leq \Omega h^2 \leq 0.1285
\]  

(48)

since the annihilation cross section of a pure bino is too low and the rescaling (47) is not enough to get the right relic density. In the latter case the higgsino component tends to increase the annihilation cross section and thus we find models which satisfy the WMAP constraints. The results are summarized in Fig. 5 for \( \Delta_2 = 1\% \) and \( \Delta_2 = 5\% \). In order to fulfill the WMAP data (red (darker) points in the plot (5)) the Stückelino mass must be in the range 50 GeV \( \lesssim M_S \lesssim 700 \) GeV in the limit \( \Delta_2 \to 0 \), where the lowest bound is given by the current experimental constraints [31].

• \( N = 3 \) case. This is the case in which there is a third MSSM particle almost degenerate in mass with the LSP and the NLSP. Typical situations of this kind arise when the NLSP and the next to next to lightest supersymmetric particle (NNLSP) are respectively the bino and the stau or the wino and the lightest chargino. Expanding
in an explicit way all the terms in the sum (37) we get:

\[ \langle \sigma_{\text{eff}}^{(3)} v \rangle = \langle \sigma^{(3)} v \rangle = \frac{1}{m_{\text{eff}}} v (1 + \Delta_i)^{3/2} e^{-x_i \Delta_i} \quad \text{for } i = 2, 3 \]  

(50)

where \( g_i \) are the internal degrees of freedom of the particle species, \( x_f = m_1 / T \) and \( \Delta_i = (m_i - m_1) / m_1 \), we obtain

\[ \langle \sigma_{\text{eff}}^{(3)} v \rangle \approx \frac{\langle \sigma_{22} v \rangle Q_2^2 + 2 \langle \sigma_{23} v \rangle Q_2 Q_3 + \langle \sigma_{33} v \rangle Q_3^2}{(1 + Q_2 + Q_3)^2} \]  

(51)

Under the assumption \( (m_3 - m_2) / m_1 \ll 1 / x_f \), \( Q_3 / Q_2 \approx g_3 / g_2 \) we finally get

\[ \langle \sigma_{\text{eff}}^{(3)} v \rangle \approx \frac{Q_2^2}{1 + \left(1 + \frac{g_3}{g_2}\right) Q_2^2} \langle \sigma_{\text{MSSM}} v \rangle \]  

(52)

where

\[ \langle \sigma_{\text{MSSM}} v \rangle = \langle \sigma_{22} v \rangle + 2 \frac{g_3}{g_2} \langle \sigma_{23} v \rangle + \left(\frac{g_3}{g_2}\right)^2 \langle \sigma_{33} v \rangle \]  

(53)

In order to compute the rescaling factor between the relic density of our model and the MSSM relic density we have to express \( \sigma_{\text{MSSM}} \) in terms of a two coannihilating species effective cross section. This is given by

\[ \langle \sigma_{\text{eff}}^{(2)} v \rangle = \frac{\langle \sigma_{22} v \rangle \left(n_{21}^{eq} \right)^2 + 2 \langle \sigma_{23} v \rangle n_{21}^{eq} n_{31}^{eq} + \langle \sigma_{33} v \rangle \left(n_{31}^{eq} \right)^2}{\left(n_{\text{eff}} \right)^2} \]  

(54)
where
\[ Q_{23} = n_e^q / n_e^i = \frac{g_3}{g_2} \left( 1 + \frac{m_3 - m_2}{m_2} \right)^{3/2} e^{-x_f \frac{m_3 - m_2}{m_2}} \]
\[ \simeq \frac{g_3}{g_2} \]  
(55)
since \((m_3 - m_2)/m_1 \ll 1/x_f\) and \(m_2 > m_1\) then \((m_3 - m_2)/m_2 \ll 1/x_f\). We remind the reader that the values of \(n_e^q, n_e^i\) and \(n_e^a\) are different with respect to those in the former case since now there are only two species in the thermal bath. We then find
\[ \langle \sigma_{\text{eff}}^{(2)} v \rangle \simeq \frac{\langle \sigma_{22} v \rangle + 2 \frac{g_3}{g_2} \langle \sigma_{23} v \rangle + \left( \frac{g_3}{g_2} \right)^2 \langle \sigma_{33} v \rangle}{\left( 1 + \frac{g_3}{g_2} \right)^2} \]
\[ \simeq \frac{\langle \sigma_{\text{MSSM}} v \rangle}{\left( 1 + \frac{g_3}{g_2} \right) Q_2} \]  
(56)
and inserting back this relation into (52) we obtain
\[ \langle \sigma_{\text{eff}}^{(3)} v \rangle \simeq \left[ \frac{\left( 1 + \frac{g_3}{g_2} \right) Q_2}{1 + \left( 1 + \frac{g_3}{g_2} \right) Q_2} \right]^2 \langle \sigma_{\text{eff}}^{(2)} v \rangle \]  
(57)
The rescaling factor between the three and two particle species relic density is given by the following relation
\[ (\Omega h^2)^{(3)} \simeq \left[ \frac{1 + \left( 1 + \frac{g_3}{g_2} \right) Q_2}{1 + \left( 1 + \frac{g_3}{g_2} \right) Q_2} \right]^2 (\Omega h^2)^{(2)} \]  
(58)
We performed a random sampling of MSSM models with bino-stau and wino-chargino coannihilations. The first situation is realized in some corners of the mSUGRA parameter space\(^\text{11}\) while the second situation is naturally realized in anomaly mediated supersymmetry breaking scenarios. For each model we computed the relic density \((\Omega h^2)^{(2)}\) for the two coannihilating species with the DarkSUSY package [29]. We finally computed \((\Omega h^2)^{(3)}\) using (58). The bino-stau models which satisfy the WMAP constraints have a Stückelino mass in the range 100 GeV \(\lesssim M_S \lesssim 350\) GeV in the limit \(\Delta_2 \to 0\). As the mass gap increases the number of allowed models drastically decreases and eventually vanishes for \(\Delta \simeq 5\%\). In the wino-chargino case, models which satisfy the WMAP constraints are shown in Fig. 6 for four reference values of \(\Delta_2\). The space of parameters with \(\Delta_2 \lesssim 5\%\) and a Stückelino mass \(M_S \gtrsim 700\) GeV is favored while as the mass gap increases lower Stückelino masses become favored, e.g. \(100\) GeV \(\lesssim M_S < 200\) GeV \((\Delta_2 \simeq 20\%)\).

\(^{11}\)Or in the so called Constrained MSSM (CMSSM).
6 Conclusions

We studied a possible dark matter candidate in the framework of our minimal anomalous $U(1)'$ extension of the MSSM [7]. In the decoupling limit (22) and under the assumption $M_0 \gg M_S, M_{V(0)}$ the Stückelino turns out to be the LSP. Being an XWIMP the Stückelino annihilation cross section is suppressed with respect to the typical weak interaction cross sections. This implies that in order to satisfy the WMAP constraints on the relic density we must have at least a NLSP almost degenerate in mass with the Stückelino. We considered the case with two and three coannihilating particles and we found some configuration which satisfies the WMAP constraints. The results depend on the mass gap between the Stückelino and the NLSP. In the exact degeneracy limit $\Delta_2 \to 0$ the allowed models have a Stückelino mass in the range $50 \text{ GeV} \lesssim M_S \lesssim 700 \text{ GeV}$ for the bino-higgsino coannihilation case while $900 \text{ GeV} \lesssim M_S \lesssim 2 \text{ TeV}$ for the wino-chargino coannihilation case. When the mass gap is $\Delta_2 \simeq 20\%$ the allowed models are those with wino-chargino coannihilations and a Stückelino mass of $100 \text{ GeV} \lesssim M_S < 200 \text{ GeV}$. Finally let us comment on the differences between our scenario and that studied in the work [16]. In our framework the

Figure 6: Stückelino relic density in the case in which the NLSP is a wino while the NNLSP is the lightest chargino. Red (darker) points denote models which satisfy WMAP data. Upper left panel: $\Delta_2 = 20\%$. Upper right panel: $\Delta_2 = 10\%$. Lower left panel: $\Delta_2 = 5\%$. Lower right panel: $\Delta_2 = 1\%$. 
$U(1)'$ does not arise from a hidden sector and thus all the MSSM fields can be charged under this extra abelian gauge group. This is the most relevant feature which could also be detected experimentally (see for example [6]). Moreover, in our scenario the Stückelino interactions are suppressed with respect to the weak interactions due to the GS couplings while in [16] the mechanism to suppress the couplings and give an XWIMP is provided by the kinetic mixing between the $U(1)'$ and $U(1)_Y$.

### A Amplitude for $\lambda_1 + \psi_S \rightarrow f \bar{f}$

In this Appendix we give some details of the amplitude computation for the process $\lambda_1 + \psi_S \rightarrow f \bar{f}$,

$$\mathcal{M} = -ik^\mu \bar{v}_S \gamma_5 [\gamma_\mu, \gamma_\nu] u_1 \left[ e q_f C_\gamma \frac{\eta^\nu p}{k^2} \bar{u}_f \gamma_\nu v_f + \frac{g Z_0}{2} C_{Z_0} \frac{\eta^\nu p}{k^2} \bar{u}_f \gamma_\nu (v_f^{Z_0} - a_f^{Z_0} \gamma_5) v_f \right]$$  

(59)

where $q_f$ denote the electric charges, $v_f^{Z_0}$ and $a_f^{Z_0}$ are the vectorial and axial couplings with $Z_0$, $C_\gamma = C^{(1)} \cos \theta_W$, $C_{Z_0} = -C^{(1)} \sin \theta_W$ while $k^2 = s$ is the momentum of the intermediate gauge boson. The corresponding square modulus is

$$|\mathcal{M}|^2 = -64 \left[ T_a \left( \frac{a_f C_{Z_0} g Z_0}{k^2 - M_{Z_0}^2} \right)^2 + T_v \left( \frac{2 C_\gamma q_f}{k^2} + \frac{C_{Z_0} g Z_0 v_f}{k^2 - M_{Z_0}^2} \right)^2 \right]$$  

(60)

with

\[T_v = m_f^4 (p_{\lambda_1} p_S) + M_1 M_S \left[ 2 m_f^4 + 3 (p_f p_f) m_f^2 + (p_f p_f)^2 \right] +
- (p_f p_f) \left[ (p_{\lambda_1} p_f) (p_f p_S) + (p_{\lambda_1} p_f) (p_f p_f) \right] + m_f^2 \left[ (p_{\lambda_1} p_S) (p_f p_f) +
-2 (p_{\lambda_1} p_f) (p_f p_S) - (p_{\lambda_1} p_f) (p_f p_f) - 2 (p_{\lambda_1} p_f) (p_f p_f) \right]
\]

\[T_a = \left[ (p_{\lambda_1} p_f) (p_f p_S) + (p_{\lambda_1} p_f) (p_f p_f) \right] m_f^2 - M_1 M_S \left[ m_f^4 - (p_f p_f)^2 \right] +
- (p_f p_f) \left[ (p_{\lambda_1} p_f) (p_f p_S) + (p_{\lambda_1} p_f) (p_f p_S) \right]
\]

(61)

where $p_{\lambda_1}, p_S, p_f$ and $p_{\bar{f}}$ are the bino, Stückelino and SM fermions 4-momenta respectively. Writing all the momenta in function of $s$ and integrating over the solid angle we get

$$\sigma = c_f \left( \frac{g_1^2 b_2^{(1)}}{g_1^2 b_2^{(2)}} \right)^2 \sqrt{s - 4 m_f^2} \times$$

$$\times \left[ - 2 M_1^4 + (4 M_2^2 + s) M_1^2 - 6 M_2 s M_1 - 2 M_2^2 + s^2 + M_2^2 \right] \times$$

$$\times \left[ \sqrt{2} \left( 2 \cos \theta_W q_f (M_2^2 - s) + \sin \theta_W g Z_0 v_f \right)^2 + \left( \sin \theta_W g Z_0 a_f \right)^2 s^2 (s - 4 m_f^2) \right]$$

(62)

18
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