The effect of contact friction forces on the stress - strain state of an inclined base during the construction of buildings on complex terrain

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Abstract. The paper considers the interaction of contact friction forces in the system “base - inclined foundation - holding structure” and their influence on the stress-strain state of the soil base in the zone of the foundation structure’s adjacency to the holding structure. The main features of the contact friction forces’ distribution, the places of their application and the calculation algorithm are determined. The design scheme of the inclined foundation is developed taking into account the occurrence of a generalized total friction force. Based on the obtained dependences, the additions and refinements to the existing methodology for calculating foundations on an inclined soil base using a variable stiffness coefficient have been made.

Introduction
In recent years, much attention has been paid to the development of the Black Sea coast in Russia - this large unique resort region with a healing climate, excellent conditions for the recreational development. However, one of the many problems that complicate the construction development in this area is the lack of horizontal areas suitable for new buildings. The sites with complex terrain, when they are built up with typical buildings using the usual constructive foundations’ solutions, require leveling and terracing of the territory with a large amount of excavation and construction of special engineering structures to stabilize the inclined foundations [1,2,3]. All this leads to significant expenses and increases the construction costs.

Therefore, the use of the building foundations’ effective structural solutions in the form of a cross-beam system, which fits onto the leveled day surface of an inclined base without undercutting, is extremely urgent [4,5]. At the same time, the land surface suitable for recreational and agricultural use is not involved, but the plots with complex terrain are used.

Formulation of the problem
The methods for calculating the cross foundations on the slopes built on the experimental studies have been developed in sufficient detail taking into account almost all the force factors with the contact pressure plots’ construction, which is the external load on the foundation beams [6].

However, some features of this type of work with the foundation during its interaction with the retaining structure, in particular, the appearance of a region with an altered stress state of the soil
associated with the presence of several contact friction forces, require a more thorough theoretical
calculation and experimental refinement.

Design scheme

Let us consider the design scheme of the cross-beam foundation on the slope, taking into account
the existing contact friction forces. The inclined foundation beams in the lower part of the slope abut against
a recessed retaining structure in the form of an operated structure. Vertical focused efforts \( G_i \) from the
external load on the building are applied at the intersection of the foundation tapes. Depending on the
inclination angle of the base \( \alpha \) they are decomposed into two components: force \( F_i \) normal to the surface
of the slope and force \( N_i \) parallel to the slope’s surface. In this case, the horizontal shear component \( T \)
vertical load \( G_i \) is completely perceived by the retention structure. Under the external load action,
vertical movement \( S_i \) of the foundation axis \( Z \), and moving \( \delta T \) along the slope direction from the action
of the shear component of the force \( G_i \) occurs due to the retaining structure’s displacement.

**Figure 1.** The design scheme of the building on a slope with cross-beam foundations: 1-
underground retaining structure; 2- longitudinal inclined foundation tape; 3- inclined base of the slope;
4- diagram of normal contact pressures from the dead weight of the building and external forces

In this case, three contact friction forces arise: \( F_{bb} \) - the friction force of concrete on concrete at the
contact of the foundation beam and the retaining structure when they are interacted; it arises as a result
of the interaction of the reinforced concrete foundation beam and the wall of the retaining structure with
the vertical movement of the beam along the axis \( Z \); \( F_{gb} \) - the friction force of concrete on the ground
at the contact point of the inclined base and the foundation tape when it moves down the slope occurs
when the inclined longitudinal tape of the foundation and base interact; \( F_{gw} \) - ground friction force on
concrete of a vertical wall when moving the foundation tape down the axis \( Z \). The \( F_{gw} \) forces friction
occurs as a result of the soil pressure on the retaining structure’s vertical wall and the effect of the base
vertical deformation. The friction forces act on the site of the compressible stratum of soil \( N \) in the
contact plane of the wall and the soil base, and are led in the direction opposite to the direction of
movements causing them. In addition, in an inclined base under the normal component \( F_i \) action a plot
of normal contact pressures \( P_{co} \) is formed; its form and its characteristic features have been sufficiently
studied.

The first two friction forces are recommended to be determined based on the characteristics of the
foundation tapes’ interaction with an inclined base and a holding structure, taking into account the
corresponding friction coefficients according to the following relationships:
where $G_i$ is the vertical nodal load, kN; $\alpha$ is the base angle; $f_{bb}^{bb}$ defines the coefficient of concrete friction on concrete, taken equal to 0.6; $f_{bb}^{gb}$ is the coefficient of concrete friction on the soil, taken equal to 0.45.

To determine the friction force $F_{gb}^{gw}$, the solution proposed in [7] is used for a half-plane, inside of which a vertical force $F$ is applied, directed opposite to the acting soil movements.

Taking the fact that the friction force will manifest itself directly below the base of the foundation and at a certain depth $H$ in the core, where inelastic deformations are concentrated, into account, it is recommended to find its resultant by integrating the functions obtained in [7] within the core $H$ according to the formula (3):

$$F_{fr} = \int_0^H \frac{2p_{av} f_{gw}^w \Delta z}{\pi} \left( \arctg \frac{L_b}{z_i} - \frac{L_{gb} z_i}{z_i^2 + L_b^2} \right) dz = -\frac{2p_{av} f_{gw}^w \xi \Delta z}{\pi} \left( H \arctg \frac{L_{gb}}{H} \right)$$

where $p_{av}$ defines the uniformly distributed load from the external load on the foundation beam; $f_{fr}^w$ is the coefficient of the soil friction along the vertical wall of the retaining structure is taken equal to 0.4; $\Delta z$ is the size of the elementary section into which the compressible thickness of the soil is divided; $H$ is the compressible depth (core); $\xi$ defines the correlation coefficient $\Delta z$ from the number of elementary plots. It is found by the empirical formula $\xi = 0.0692 - 0.0648 n$, $n$ is the number of sections into which the compressible stratum is divided, $L_b$ is the foundation strip length; $z_i$ is the distance from the soil surface to the point where the elementary friction force is determined.

The compressible stratum thickness can be determined by various methods and depends on many factors. In our opinion, for the problem being solved under the inclined base conditions, the dependence proposed in [8] is most suitable. In this case, the lower boundary of the base compressible thickness is determined by the formula:

$$H_c = k \cdot b$$

where $b$ is the foundation tape width, m; $k$ is the coefficient, which is taken according to Table 5 BC 22.13330.2016 [9].

From the consideration of the acting friction forces’ force triangle (Figure 2), the generalized total friction force is found by the following dependence:

$$F_{fr, tot} = \sqrt{\left( F_{fr}^{bb} + F_{fr}^{gw} \right)^2 + \left( F_{fr}^{gb} \right)^2 - 2 \left( F_{fr}^{bb} + F_{fr}^{gw} \right) \cdot F_{fr}^{gb} \cos(90 + \alpha)}$$

**Figure 2.** Force triangle of the friction forces’ contact: $D$ - distance from the resultant friction forces to the point of contact of the beam and the wall of the retaining structure; $\gamma$ – angle of the retaining structure resulting from the vertical wall; $\beta$ – resultant angle to the inclined base.
Then, normal contact pressures in the inclined base will be determined from solving the equilibrium equations of all the forces acting on the inclined foundation beam, when designing them on the local coordinate axes OZ and the moment relative to the contact point of the inclined beam and the holding structure, taking into account the total resultant friction forces. In this case, they will have the following form:

\[ \sum F_i = \sum G_i \cos \alpha + \sum G_i \cos \alpha \sin \alpha - F_{p,fr} \sin \beta - \frac{P_1 + P_2}{2} \times L \times b = 0 \] (6)

\[ \sum M_i = \sum G_i \cos \alpha x_i - \frac{P_1 + P_2}{2} \times L \times b \times \left[ L - \frac{L}{3} \left( \frac{2P_1}{P_2} + \frac{P_2}{P_1} \right) \right] - F_{fr, tot} D = 0 \] (7)

To determine the stress-strain state of the inclined base, we analyze the characteristics of the normal contact pressure diagram. At the same time, we assume that due to the sufficient rigidity of the foundation beam and the base work at the initial stage of the elastic stage, the soil is modeled before the formation of plastic zones by the Winkler model with a variable stiffness coefficient.

In this case, to determine the boundary values of contact pressures, it is recommended to use the conditions for their determination, taking into account the coefficient \( \Delta K \) [10], which establishes the relationship of the boundary stiffness coefficients of the base and the contact pressures. Based on the foregoing, we will have:

\[ P_1 = 2 P_{cr} \Delta K \] (8)

\[ P_2 = 2 P_{cr} (1 - \Delta K) \] (9)

\[ P_{cr} = \frac{P_1 + P_2}{2} \leq R_e \] (10)

where \( \Delta K = \frac{K_{cr}}{K_1 + K_2} \) is the coefficient determining the base boundary stiffness coefficients ratio at the initial stages of structural loading, \( \delta_i \) is the coefficient taking into account the uneven distribution of sediments along the length of the slope [10], \( R_e \) is the standard calculated soil resistance.

To determine the coefficient \( \Delta K \) we use the equilibrium equations (6) and (7), in which the generalized total friction force is explicitly present \( F_{fr, tot} \). Then a new expression for \( \Delta K \) can be defined as:

\[ \Delta K = \frac{\frac{2}{3} P_{cr} L^2 b - \sum G_i \cos \alpha x_i - F_{fr, tot} D}{\frac{1}{3} P_1 L^2 b} \] (11)

To calculate an unknown distance \( D \) from the resultant friction force to the point of the inclined beam’s contact with the retaining structure, we use the following assumptions:

1. Generalized total frictional force \( F_{p,fr} \) will pass through the gravity center of the triangle formed by the corresponding components application points of the friction forces \( F_{p,v}^{fr}, F_{p,b}^{fr}, F_{p}^{bb} \) at appropriate angles: \( \gamma \) - to the vertical wall of the retaining structure and \( \beta \) - to the inclined base. They are determined on the basis of the force triangle consideration by the following dependencies

\[ \sin \gamma = \frac{F_{p,v}^{fr} \sin(90 + \alpha)}{F_{fr, tot}} ; \sin \beta = \frac{(F_{p,b}^{bb} + F_{p}^{bb}) \sin(90 + \alpha)}{F_{fr, tot}} \] (12)

2. Taking into account the action nature of the friction forces \( F_{p,v}^{fr}, F_{p,b}^{fr}, F_{p}^{bb} \) they are applied respectively at the points determined by the following geometric characteristics (Figure 3): friction force \( F_{p,v}^{fr} \) - on the distance \( \psi H \) from the contact point of the foundation tape and the wall of the retaining
structure; friction force $F_{fr}^{bb}$ - at the point of contact between the foundation tape and the wall of the retaining structure; friction force $F_{fr}^{by}$ - on the distance $0.5L$ from the contact point of the foundation tape and the wall of the retaining structure, i.e. under the middle of the inclined foundation beam.

Then, based on these assumptions and considering the above-mentioned scheme, the distance from the generalized total resultant friction forces to the point of beam contact and the retaining structure wall is determined by the following dependence:

$$D = \frac{x_0 z_0}{\sqrt{x_0^2 + z_0^2}}$$  \hspace{1cm} (13)

where: $x_0 = x_{cg} - z_{cg} \tan \gamma$; $z_0 = \frac{x_{cg}}{\tan \gamma} - z_{cg}$;

$x_{cg}$, $z_{cg}$ are the gravity center coordinates of the triangle AOB (Figure 3).

![Figure 3. The geometric scheme for calculating the point of the generalized total friction force application.](image)

An analysis of the change nature in the resultant inclination angle on the basis of the dependence (12) shows a certain pattern in the change in their values. It is established that with an increase in the base inclination angle $\alpha$ from $15^0$ to $45^0$ and the constant external load $G_i$ on the foundation, as well as the constant geometric parameters of the inclined beam $L$ and $b$, the angle $\beta$ to the inclined base maintains a stable value within $34^0-38^0$. In this case, the angle $\gamma$ to the vertical wall of the retaining structure decreases from $38^0$ to $11^0$. This is due to a decrease in normal loads and an increase in shear, which leads to the transformation and redistribution of the existing friction forces.

The calculations also found that the values of the regional pressures under the foundation base under the action of the total resultant friction forces decrease with an increase in the base inclination angle, the stable external load and the geometry of the foundation. At the same time, their unevenness decreases, which leads to a more stable load on the foundation and a decrease in bending moments by 10-15% compared to the known calculation methods, which leads to a 5-7% reduction in the material consumption for the reinforcement foundation.

**Summary**
1. The theoretical and practical approaches for determining the real contact frictional forces acting in the “base - inclined foundation - holding structure” system have been established. They create a special stress-strain state of the inclined base in the contact zone between the holding structure and the inclined foundation. Under the influence of the friction acting contact forces, a generalized total
force is formed at the base, which acts at certain angles to the vertical surface of the retaining structure wall and the inclined base and distance $D$ from the inclined base intersection and the wall of the retaining structure. It was found that its absolute value depends on the level of load on the foundation, the length and width of the inclined foundation tape and the base inclination angle.

2. An algorithm for calculating the effective contact friction forces and their resultant, taking into account the features of the real work of the foundation on an inclined soil base limited by a rigid holding structure, has been developed.

3. The theoretical calculations by the above-described methodology showed that the revealed contact friction forces have a positive effect on the contact pressures distribution in the base when using the contact model of the base stress-strain state with a variable stiffness coefficient, which leads to a decrease in its material consumption.

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