Ultra-large-scale continuous-variable cluster states multiplexed in the time domain

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Quantum computers promise ultrafast performance for certain tasks1. Experimentally appealing, measurement-based quantum computation2 requires an entangled resource called a cluster state3, with long computations requiring large cluster states. Previously, the largest cluster state consisted of eight photonic qubits4 or light modes5, and the largest multipartite entangled state of any sort involved 14 trapped ions6. These implementations involve quantum entities separated in space and, in general, each experimental apparatus is used only once. Here, we circumvent this inherent inefficiency by multiplexing light modes in the time domain. We deterministically generate and fully characterize a continuous-variable cluster state7,8 containing more than 10,000 entangled modes. This is, by three orders of magnitude, the largest entangled state created to date. The entangled modes are individually addressable wave packets of light in two beams. Furthermore, we present an efficient scheme for measurement-based quantum computation on this cluster state based on sequential applications of quantum teleportation.

Originally formulated as a demonstration as to why quantum mechanics must be incomplete in the famous 1935 Einstein–Podolsky–Rosen (EPR) paradox7, entanglement is now recognized as a signature feature of quantum physics8,9, and it plays a central role in various quantum information processing (QIP) protocols10–11. For example, the bipartite entangled state known as an EPR state9 is a resource for quantum teleportation, whereby a quantum state is transferred from one location to another without physical transfer of the quantum information12–14. Measurement-based quantum computation (MBQC)2,7,8,15–18, which is based on the quantum teleportation of information and logic gates, requires the special class of multipartite entangled resource states known as cluster states3. The number of entangled quantum entities and their entanglement structure (represented by a graph) determine the resource space available for computation. Ultra-large-scale QIP (which could be based on MBQC) will require ultra-large-scale entangled states2,7,9.

In the vast majority of optical experiments, quantum modes are distinguished from each other by their spatial location. This leads to an inherent lack of scalability as each additional entangled party requires an increase in laboratory equipment and dramatically increases the complexity of the optical network19,20. Furthermore, due to the probabilistic nature of photon pair generation, demonstrations involving the post-selection of photonic qudits14,15,16 suffer from dramatically reduced event success rates with each additional qubit.

One method to overcome this problem of scalability is to deterministically encode the states within a beam. Entanglement between quadrature-phase amplitudes in continuous-wave laser beams has been deterministically created and exploited in QIP5,13,14,17–19,21–23, even though the quantum correlations are finite. Previous attempts to deterministically create cluster states within one beam have exploited the spatial21 or spectral22–24 orthogonality of quantum modes. Although such methods are potentially scalable, current experimental results are limited to generating entangled states of just a few modes each21–23. A novel method proposed in ref. 25 lets quantum modes propagate within the same beam—distinguished and ultimately made orthogonal by their separation in time. The time-domain multiplexing approach allows each additional quantum mode to be manipulated by the same optical components at different times, which is a powerful concept, as found in atomic ensemble quantum memories26.

Here, we demonstrate the deterministic generation of ultra-large-scale entangled states consisting of more than 10,000 entangled wave packets of light, multiplexed in the time domain. The generated states, which we call extended EPR (XEPR) states, are equivalent up to local phase shifts to topologically one-dimensional continuous-variable (CV) cluster states27 and are therefore a universal resource for single-mode MBQC with continuous variables9. Fully universal multimode MBQC is achievable simply by combining two XEPR states with differing time delays on two additional beamsplitters28. Note that in our time-domain multiplexed demonstration only a small part of the entire XEPR state exists at each instant of time. This is the reason why we could achieve such an efficient set-up for the creation and verification of an ultra-large number of entangled modes.

The XEPR states are generated by entangling together sequentially propagating EPR states contained within two beams. This can be viewed as four distinct steps, as illustrated in Fig. 1a. First, two continuous-wave squeezed light beams are generated from two optical parametric oscillators (OPOs) (step i). We divide the squeezed light beams into time bins of time period T, where 1/T is sufficiently narrower than the bandwidths of the identical OPOs. Wave packets of light in each of the time bins represent mutually independent (orthogonal) squeezed states. Second, a series of EPR states separated by time interval T are deterministically created by combining the two squeezed light beams on the first balanced beamsplitter (step ii). The quantum correlations that manifest from the beamsplitter interaction are represented by links between the nodes (coloured spheres). The nodes here represent the orthogonal wave packets. Third, the bottom-rail node
of the EPR state is delayed for the duration $T$ after passing through a fibre delay line (step iii). After the delay, the top-rail node of each EPR state is synchronized in time with the bottom-rail node of the previous EPR state. By combining the staggered EPR states on the second balanced beamsplitter, each EPR state interacts with the previous EPR state. By combining the staggered EPR states on the second balanced beamsplitter, each EPR state interacts with the previous and successive EPR states (step iv). This leads to all the second balanced beamsplitter, each EPR state interacts with all the previous EPR states on the first balanced beamsplitter, each EPR state interacts with the previous EPR state. By combining the staggered EPR states on the first balanced beamsplitter, each EPR state interacts with the previous EPR state. By combining the staggered EPR states on the first balanced beamsplitter, each EPR state interacts with the previous EPR state.

Ideal quadrature-entangled states are simultaneous eigenstates of particular linear combinations of the quadrature operators, called nullifiers\textsuperscript{27,28}. For example an ideal EPR state $|\text{EPR}\rangle$, which is the ideal state approximated by the Gaussian state at step ii in Fig. 1a, is specified by the following nullifier relations:

$$
(\hat{x}^A - \hat{x}^B)|\text{EPR}\rangle = 0
$$

$$
(\hat{p}^A + \hat{p}^B)|\text{EPR}\rangle = 0
$$

where superscripts $A$ and $B$ denote two wave packets. In our set-up, they refer to the top-rail and bottom-rail wave packets, respectively. $\hat{x}^Q$ and $\hat{p}^Q$ are the quadrature operators of a wave packet $Q$, which do not commute for the same wave packet: $[\hat{x}^Q, \hat{p}^Q] = i\hbar\delta_{Q, R}/2$, where $\delta_{Q, R}$ is the Kronecker delta, and $\hbar$ is normalized to 1/2. Although $\hat{x}$ and $\hat{p}$ of a single wave packet cannot be determined simultaneously due to the Heisenberg uncertainty principle, equation (1) shows that the correlations between the two wave packets are

Figure 2 | XEPR states for sequential quantum teleportation. a. Circuit of standard sequential quantum teleportation. b. Circuit of sequential quantum teleportation through the XEPR state. 50:50 BS, balanced beamsplitter; $|\phi\rangle$, arbitrary and unknown quantum state as the input of the quantum teleporter; $\hat{x}$ or $\hat{z}$, phase-space displacement operation based on measurement outcomes.

Figure 1 | Schematic of the experimental set-up and ultra-large-scale XEPR state. a, Generation of ultra-large-scale cluster-state entanglement. Squeezed temporal modes are mixed on a beamsplitter to create EPR states, which are then staggered by a fibre delay line. The staggered EPR-state wave packets are then mixed to generate a continuous graph structure, where each wave packet is entangled with neighbouring wave packets. Nodes (coloured spheres) and links between them represent optical wave packets and entanglement, respectively. Independent quantum states exist in each temporally localized wave packet with time duration $T$. OPO, optical parametric oscillator; 50:50 BS, balanced beamsplitter; HD, homodyne detector; LO, local oscillator. b. The extended EPR state, equivalent to a CV cluster state up to local phase shifts (redefinition of quadrature operators). Supplementary Section 2 presents full details and graphical representation.
perfectly determined. More precisely, the quadrature amplitudes $\hat{x}$ of the two wave packets are perfectly correlated ($x^A = x^B$), and the amplitudes $\hat{p}$ are perfectly anticorrelated ($p^A = -p^B$).

In its ideal form, the XEPR state (XEPR) generated in our experiment (Fig. 1b) is specified by

$$\langle (\hat{x}^A + \hat{x}^B + \hat{x}^A_{k+1} - \hat{x}^B_{k+1}) \rangle_{\text{XEPR}} = 0$$

$$\langle (\hat{p}^A + \hat{p}^B - \hat{p}^A_{k+1} + \hat{p}^B_{k+1}) \rangle_{\text{XEPR}} = 0$$

(2)

where $A$ and $B$ denote the two independent beams (top and bottom rails, respectively), and $k = 1, 2, \ldots$ represents the temporal index. We consider the XEPR state to be a natural extension of EPR states because the nullifiers are composed of either $\hat{x}$ or $\hat{p}$ quadratures, but not both. We see that the addition of two quadratures $\hat{x}^A + \hat{x}^B$ (respectively, $\hat{p}^A + \hat{p}^B$) at any given time index $k$ demonstrates a negative (positive) correlation with the difference of the two quadratures $\hat{x}^A_{k+1} - \hat{x}^B_{k+1}$ (respectively, $\hat{p}^A_{k+1} - \hat{p}^B_{k+1}$) at subsequent time index $k + 1$. The XEPR state is entirely equivalent to a CV cluster state under an appropriate redefinition of quadrature operators ($\hat{x} \rightarrow \hat{p}$, $\hat{p} \rightarrow -\hat{x}$) for every other temporal mode.$^{28}$ Such a redefinition is completely passive and does not change the resource requirements for MBQC using this state, but we choose to call it an XEPR state rather than a CV cluster state because our verification procedure takes advantage of the fact that the ideal nullifiers only involve $\hat{x}$ and $\hat{p}$ values.

In reality, the generated XEPR states—and therefore the nullifiers—have excess noise due to the unphysical nature of infinite squeezing. Despite this, the full inseparability of the state can be shown when the resource squeezing level is high enough. The sufficient conditions for fully inseparable entanglement$^{29}$ are given by the variances as

$$\langle (\hat{x}^A + \hat{x}^B + \hat{x}^A_{k+1} - \hat{x}^B_{k+1})^2 \rangle < \frac{1}{2}$$

$$\langle (\hat{p}^A + \hat{p}^B - \hat{p}^A_{k+1} + \hat{p}^B_{k+1})^2 \rangle < \frac{1}{2}$$

(3)

for all $k$, where bracketed $\langle \hat{O} \rangle$ denotes the expectation value of operator $\hat{O}$.

The experimental quadrature amplitudes $\hat{x}^A$, $\hat{p}^A$, $\hat{x}^B$, and $\hat{p}^B$ of the first 50 wave packets are plotted in Fig. 3a–d. We see they are randomly distributed around zero. Linear combinations of the quadrature amplitudes at neighbouring times exhibit quantum correlations as in equation (2) (shown in Fig. 3e,f). The amplitudes overlap almost perfectly, showing strong anticorrelations and correlations in both quadrature combinations.

To quantify the quantum correlations, we repeated the single-shot generation of the entire XEPR state more than 3,000 times, allowing us to measure the variances at each temporal position. We then evaluated the multiparticle inseparability criteria given in equation (3). The measurement results are shown in Fig. 4. The variances for the XEPR states are shown by traces (i). The bound of inseparability given in equation (3) corresponds to $-3.0$ dB, shown by the dashed lines (iii). We see that the experimental XEPR state is clearly below the bound of inseparability in the entangled region for more than 5,000 temporal positions. Given the dual-rail structure of the XEPR state, the two beams $A$ and $B$ each contain the same number of wave packets, so the 5,000 temporal positions correspond to 10,000 wave packets.
This line demonstrates entanglement. Quantum computation will also require efficient encoding and actuation of ultra-large-scale entangled states in a deterministic manner not corrupted by disturbances from the environment. In summary, we have experimentally demonstrated the generation of ultra-large-scale entangled states in a deterministic fashion. More than 10,000 wave packets of light are shown to be fully inseparable in a CV cluster-state configuration. Fault-tolerant quantum computation will also require efficient encoding and error correction. Compared to the largest entangled states previously engineered, the number of entangled modes here is larger by three orders of magnitude. Owing to their sheer size, regular structure, deterministic method of creation, we fully expect that these ultra-large-scale states will enable other QIP applications in addition to MBQC.

The squeezing levels of our OPOs were \(-6 \text{ dB}\) and \(-5 \text{ dB}\), respectively. Note that absolutely no corrections for any losses are performed.

The variances of the XEPR state steadily increase with time (and therefore the entanglement degrades), for technical reasons related to our control scheme. During the data acquisition process we switched off all active feedback control for the optical set-up to avoid any unwanted noise arising from the feedback that would degrade the measurements. The increase in variance is therefore explained by the relative phase drifts of the entangled state caused by disturbances from the environment.

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**Author contributions**
S.Y., R.U. and S.A. planned and designed the experiment under the supervision of J.Y., H.Y. and A.F., based on the proposal by N.C.M. The experimental set-up were designed by S.Y., and the optical set-up was constructed by C.S., T.K. and S.Y. The fibre alignment system was built by S.S. The theory was formulated by R.U., S.A., N.C.M. and J.Y. R.U. designed and constructed the data acquisition system. S.A. designed and constructed the digital control system. R.U., S.A., T.K. and S.Y. conducted the data analysis. H.Y. assisted in noise analysis. S.Y. and S.A. wrote the manuscript with assistance from the team.

**Additional information**
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**Competing financial interests**
The authors declare no competing financial interests.