1 INTRODUCTION

The output member, which is usually controlled at hydrostatic drives, is a hydraulic motor. There are rotary or linear hydraulic motors. Output quantities, which can be controlled at these motors, are pressure and its derivation over time and force (torque). There are two main control methods in the case of the hydrostatic drives, i.e., volume and valve control. In the case of the volume drive control, the geometric displacement volume of the hydrostatic pump or the rotary hydraulic motor is changed. In the case of the valve drive control, flow rate or pressure are changed by means of a valve. This paper deals with the valve control of the output speed of the hydrostatic drive. There are many possibilities of the valve control of the speed of the rotary hydraulic motor. It is necessary to primarily control the flow rate in order to control speeds. For this reason, this paper is focused on the flow control valves. From a qualitative point of view, these valves are divided into classical valves (conventional), e. g. [Himr 2017], [Kozdera 2014], proportional valves or servo valves, e. g. [Luan 2018]. The speed control problem of the hydrostatic drive can be most clearly described on a classical flow control valve, in this case on the throttle valve.

2 THEORETICAL BACKGROUND

A model example of the hydrostatic drive with the rotary hydraulic motor is shown in Fig. 1. Efficiencies of the hydraulic pump HP and the hydraulic motor HM, pressure losses in pipes and pressure dependence of the pressure relief valve RV are neglected in this basic theoretical analysis. Speeds of the rotary hydraulic motor HM are controlled by the throttle valve TV. The pressure is realized by means of the unidirectional fixed displacement hydraulic pump HP and the pressure relief valve RV. Furthermore, the hydraulic system consists of the tank T.

The speed control method of the hydraulic motor HM by means of the throttle valve is based on the assumption of flow rate dividing. The pump HP delivers the constant flow rate Q_P at the constant speed n_p and the constant geometric displacement volume V_H. If the working liquid doesn’t flow through the pressure relief valve RV, the motor speed n_M is not controlled. The pressure relief valve RV is open at the moment when the working liquid pressure at the point A (see Fig. 1) reaches the pressure value, which is adjusted by the pressure relief valve RV, i.e. p_A = p_RV (see Fig. 2). It is possible to increase the pressure at the point A in two ways, i.e. by increasing the load torque T_M on the motor shaft or by closing of the throttle valve TV. The cross-section area A_TV is reduced due to the closing of the throttle valve.

Hydrostatic drives are in general considered to be flexible drives. It is caused by the fact that a load change (in this case the motor torque change T_M) leads to the speed change n_M. The dependence of the speed on the load torque for a constant cross-section area of the throttle valve A_TV is shown in Fig. 3. This dependence is called as the torque-speed characteristic.
It is possible to mathematically express the speed dependence on the load torque (see Fig. 3). It is possible to appear from the flow rate equality through the hydraulic motor HM and the throttle valve TV (see Fig. 1):

\[ Q_M = Q_{TV}. \]  
(1)

The motor flow rate \( Q_M \) can be defined as the product of the motor geometric displacement \( V_M \) and the motor speed \( \omega_M \):

\[ Q_M = V_M \cdot \omega_M. \]  
(2)

The flow rate through the throttle valve is given by the equation:

\[ Q_{TV} = \mu_{TV} \cdot A_{TV} \cdot \sqrt{\frac{2 \cdot \Delta p_{TV}}{\rho}}, \]  
(3)

where \( \mu_{TV} \) is the flow coefficient of the throttle valve, \( A_{TV} \) is the cross-section area of the throttle valve, \( \Delta p_{TV} \) is the pressure gradient through the throttle valve and \( \rho \) is the density of the working liquid.

It is possible to express the pressure gradient \( \Delta p_{TV} \) through the throttle valve by means of the pressure gradient \( p_{RV} \) adjusted on the pressure relief valve and the pressure gradient \( \Delta p_M \) on the motor:

\[ \Delta p_{TV} = p_{RV} - \Delta p_M. \]  
(4)

The pressure gradient \( \Delta p_M \) on the motor is given by the equation:

\[ \Delta p_M = \frac{2 \cdot \pi \cdot T_M}{V_M}. \]  
(5)

Substituting the equations 2 + 5 into the equation 1, it is possible to obtain the equation of the dependence of the motor speed on the torque \( n_M = f(T_M) \), see Fig. 3, eventually dependencies of the motor speed on the cross-section area of the throttle valve \( n_M = f(A_{TV}) \):

\[ n_M = \frac{\mu_{TV} \cdot A_{TV} \cdot \sqrt{2 \cdot \frac{2 \cdot \pi \cdot T_M}{V_M}}}{\rho}. \]  
(6)

Based on the equation (6), it is possible to plot the drive control characteristic \( n_M = f(A_{TV}) \), i.e. the dependence of the motor speed \( n_M \) on the cross-section area of the throttle valve \( A_{TV} \) at the constant motor torque \( T_M \) (see Fig. 4).

![Figure 4. Dependence of motor speed \( n_M \) on cross-section area of throttle valve \( A_{TV} \) (control characteristic)](image)

### 3 DESCRIPTION OF HYDRAULIC SYSTEM AND SPECIFICATION OF HYDRAULIC ELEMENTS

The hydraulic pump HP1, the pressure relief valve RV1, and the tank T1 are a pressure liquid source. The pressure relief valve RV1 is used to adjust the working pressure in the hydraulic system. The rotary hydraulic motor HM is driven from this source. The torque sensor TQS, the shaft speed sensor SS and the hydraulic pump HP2 are located on the shaft of the hydraulic motor. The hydraulic pump HP2 with the pressure relief valve RV2 and the tank T2 are used to generate the torque on the shaft of the hydraulic motor HM. The speed control of the hydraulic motor is realized by the throttle valve TV. The hydraulic pump HP1 allows changing the geometric displacement volume by means of an adjusting screw. The pressure gradient through the hydraulic motor HM was measured by means of the pressure sensors PS1 and PS2. The oil flow rate was measured by the flow sensor FS. Time dependencies of the measured quantities for the given adjustment were recorded using the MS 5060+ equipment during measurements. Average values of the quantities for a given measurement were evaluated from a record using Hydrowin software. The measurements were performed at a constant oil temperature, which is measured by the temperature sensor TS. The specification of the used hydraulic elements and sensors is shown in Tab. 1.

![Figure 5. Simplified schematic diagram of hydraulic system](image)

### Table 1. Specification of used hydraulic elements and sensors

| Symbol | Specification |
|--------|---------------|
| HP1    | Axial piston variable pump with manual setting angle of swashplate, PPAR 2-63 10AP (Vrchl), \( V_S = (0 \pm 51.22) \text{ cm}^3 \), \( n_{max} = 2600 \text{ min}^{-1} \), \( p_s = 250 \text{ bar} \) |
| HP2    | Bent axis axial piston fixed pump, HM 28 AV (Glentor), \( V_S = 28.5 \text{ cm}^3 \), \( n_{max} = 4800 \text{ min}^{-1} \), \( p_s = 250 \text{ bar} \) |
| HM     | Gear hydraulic motor, SM 05 / 10 (SM Hydro Group), \( V_S = 5.8 \text{ cm}^3 \), \( n_{max} = 2500 \text{ min}^{-1} \), \( p_s = 220 \text{ bar} \) |
| RV1    | Pressure relief valve, ARAM-20/350/70 (Atos), \( p_{max} = 350 \text{ bar} \), \( Q_{max} = 350 \text{ dm}^3\text{min}^{-1} \) |
| RV2    | Pressure relief valve, VP-20-2000 (Vrchl), \( p_{max} = 160 \text{ bar} \), \( Q_{max} = 100 \text{ dm}^3\text{min}^{-1} \) |
| TV     | Throttle valve, 9N12005 (Parker), \( p_{max} = 345 \text{ bar} \), \( Q_{max} = 95 \text{ dm}^3\text{min}^{-1} \) |
| T1     | Tank, \( V = 240 \text{ dm}^3 \) |
| T2     | Tank, \( V = 75 \text{ dm}^3 \) |
| PS1, PS2 | Pressure sensor, PR15 (Hydrotechnik), measuring range \( 0 \text{ bar} \) to \( 200 \text{ bar} \), measurement accuracy of 0.5 % |
| TS     | Temperature sensor, PT 100 (Hydrotechnik), measuring range \( -50 \text{ °C} \) to \( 200 \text{ °C} \), measurement accuracy of 1 % |
| TQS    | Torque sensor, T2250NM (HBM), measuring range \( 50 \text{ Nm} \) to \( 500 \text{ Nm} \), measurement accuracy of 0.5 % |
| SS     | Speed sensor, DS 03 (Hydrotechnik), signal cycle frequency max. 500 Hz |
| FS     | Flow sensor, Gear flow meter GF670 (Hydrotechnik), measuring range \( 0.7 \text{ to } 70 \text{ dm}^3\text{min}^{-1} \) |
| MS 5060+ | Measuring system, MS 5060+ (Hydrotechnik) |
| Oil    | Mineral oil VG46, \( to = (40 \pm 2 \text{ °C}) \), hydraulic power unit is equipped with cooler |

### 4 THEORETICAL CHARACTERISTICS OF ASSEMBLED HYDRAULIC SYSTEM

Theoretical characteristics of the appropriate hydraulic system, which is described in Chapter 3, will be created according to the theoretical equation in this chapter. It is possible to determine several dependencies of the motor speed on the torque \( n_M = f(T_M) \) for different cross-section areas \( A_{TV} \) of the throttle valve and different sizes of the torque \( T_M \) (see Fig. 6) based on the equation (6), where the flow coefficient of the throttle valve \( \mu_{TV} = 0.73 \), the geometric displacement volume of the hydraulic system

\[ M_M = \frac{1}{\pi} \cdot \frac{A_{TV}}{V_M} \cdot m \cdot \pi \cdot \mu_{TV} \cdot A_{TV} \cdot \sqrt{\frac{2 \cdot \Delta p_{TV}}{\rho}}. \]  
(7)
motor $V_M = 5.8 \text{ cm}^3$, the oil density $\rho = 868 \text{ kg} \cdot \text{m}^{-3}$ and the working pressure $p_{RV} = 150 \text{ bar}$.

5 MEASUREMENT OF CHARACTERISTICS OF HYDRAULIC DRIVE

5.1 Measurement of torque-speed characteristic of hydrostatic drive

The pressure $p_{RV} = 150 \text{ bar}$ is adjusted by the pressure relief valve RV1. The flow rate $Q_{P} = 8 \text{ dm}^3 \text{min}^{-1}$ is adjusted by the hydraulic pump HP1. The throttle valve TV is adjusted in such a way that a part of the flow rate is flowing through the pressure relief valve RV1. Subsequently, the speed $n_M$ of the hydraulic motor, the pressure gradient $\Delta p_m$ through the hydraulic motor, the oil temperature $t_0$ and the flow rate $Q_m$ through the hydraulic motor are measured. These measurements are performed for several load torques $T_M$ on the motor and the cross-section areas $A_{TV}$ of the throttle valve TV. An increasing of the load torque $T_M$ on the shaft of the hydraulic motor is realized by an increasing resistance on the output of the hydraulic pump HP2 by means of the pressure relief valve RV2. This measurement is repeated for several cross-section areas of the throttle valve. Since a specific cross-section area at a given adjustment of the throttle valve is not known, these cross-section areas are marked with the parameters $\phi_1 \div \phi_5$. The measured torque-speed characteristic $n_M = f(T_m)$ of the hydraulic drive for several adjustments of the cross-section area $A_{TV}$ of the throttle valve and several load torques $T_M$ is shown in Fig. 8.

Figure 6. Theoretical torque-speed characteristic $n_M = f(T_m)$ of hydraulic drive

It is evident from the dependencies (see Fig. 6) for single cross-section area $A_{TV}$ of the throttle valve that the motor speed $n_M$ is decreasing with increasing the torque $T_M$ on the hydraulic motor.

In addition, it is possible to determine the theoretical drive control characteristic $n_M = f(A_{TV})$, i.e. the dependence of the speed $n_M$ of the hydraulic motor on the cross-section area $A_{TV}$ of the throttle valve for several constant motor torques $T_M$, see Fig. 7.

Figure 7. Theoretical control characteristic $n_M = f(A_{TV})$ of hydraulic drive

It is evident from Fig. 7 that the motor speed $n_M$ is increasing with increasing the cross-section area $A_{TV}$ of the throttle valve at a constant motor torque $T_M$.

5.2 Measurement of control characteristic $n_M = f(T_m)$ of hydraulic drive

The pressure $p_{RV} = 150 \text{ bar}$ is adjusted by the pressure relief valve RV1. The flow rate $Q_{P} = 8 \text{ dm}^3 \text{min}^{-1}$ is adjusted by the hydraulic pump HP1. The flow rate $Q_m$ through the hydraulic motor is controlled by means of the cross-section area $A_{TV}$ of the throttle valve. Subsequently, the speed $n_M$ of the hydraulic motor, the pressure gradient $\Delta p_m$ through the hydraulic motor, the oil temperature $t_0$ and the flow rate $Q_m$ through the hydraulic motor are measured. These measurements are performed for several cross-section areas $A_{TV}$ of the throttle valve and a constant load torque $T_M$ on the motor. The constant load torque $T_M$ on the motor shaft is maintained by means of the pressure relief valve RV2, which is located on the output of the hydraulic pump HP2. This measurement is repeated for several load motor torques $T_M$. 

![Figure 8. Measured torque-speed characteristic $n_M = f(T_m)$ of hydraulic drive](image-url)
6 COMPARISON OF THEORETICAL AND MEASURED CHARACTERISTICS OF HYDROSTATIC DRIVE

There are compared measured characteristics with theoretical assumptions in this chapter. The comparison of the theoretical and measured torque-speed characteristics of the hydrostatic drive is shown in Fig. 10. It is visible that the dependence of the measured characteristics does not correspond to the theoretically determined dependencies. It is caused by the fact that the efficiencies of single hydraulic elements were not included in the theoretical analysis.

7 EFFICIENCIES OF ROTARY HYDRAULIC MOTOR

There are determined the efficiencies of the rotary hydraulic motor in this chapter, i.e. the volumetric, mechanical-hydraulic and total efficiencies, e.g. [Paszota 2010], [Hruzk 2014].

7.1 Volumetric efficiency

The volumetric efficiency of the rotary hydraulic motor HM is determined from the equation:

\[ \eta_{MV} = \frac{V_M \cdot n_M}{Q_M} \]  

(7)

The volumetric efficiencies depending on the pressure gradient \( \Delta p_M \) of the rotary hydraulic motor for several constant speeds are shown in Fig. 12. It is evident that the volumetric efficiency \( \eta_{MV} \) decreases with increasing the pressure gradient \( \Delta p_M \) and increases with increasing the speed \( n_M \).

7.2 Mechanical-hydraulic efficiency

Mechanical-hydraulic efficiency of the rotary hydraulic motor HM is expressed by the equation:

\[ \eta_{M, mh} = \frac{2 \cdot \pi \cdot T_M}{\Delta p_m \cdot V_M} \]  

(8)
The mechanical-hydraulic efficiencies depending on the pressure gradient $\Delta p_M$ of the rotary hydraulic motor for several constant speeds are shown in Fig. 13. It is evident that the mechanical-hydraulic efficiency $\eta_{M,mh}$ increases with increasing the pressure gradient $\Delta p_M$ and decreases with increasing the speed $n_M$.

\[ \Delta p_M = \frac{2 \cdot \pi \cdot T_M}{V_M \cdot \eta_{M,mh}} \]

where the mean value of the volumetric efficiency of the hydraulic motor from the measured range $\eta_{M,V} = 0.93$.

On the basis of the determined mechanical-hydraulic efficiency, it is possible to modify the equation (5) for the calculation of the pressure gradient through the hydraulic motor as follows:

\[ \Delta p_M = \frac{2 \cdot \pi \cdot T_M}{V_M \cdot \eta_{M,mh}} \]

8 INCLUSION OF EFFICIENCY OF HYDRAULIC MOTOR IN THEORETICAL CHARACTERISTIC

The efficiencies of the hydraulic motor determined in Chapter 7 aren’t constant. They have different values depending on the change of the speed $n_M$ (or the flow rate $Q_M$) or the torque $T_M$ (or the pressure gradient $\Delta p_M$). Thus, it is possible to substitute concrete values of the efficiencies for each steady state or to use the average values of these efficiencies in a simplified way. In this chapter, the theoretical characteristics will be extended by the average values of the volumetric and mechanical-hydraulic efficiencies of the hydraulic motor that were determined in Chapter 7.

On the basis of the determined volumetric efficiency, it is possible to modify the equation (2) for the calculation of the flow rate through the hydraulic motor as follows:

\[ Q_M = V_M \cdot n_M \cdot \frac{1}{\eta_{M,V}} \]

Fig. 15 shows the measured (points in the diagram), theoretical (continuous lines in the diagram) and theoretical (including the efficiency of the hydraulic motor – i.e. dashed lines in the diagram) torque-speed characteristics of the hydraulic motor. It is visible that these characteristics are closer to the measured dependencies when considering the efficiencies of the hydraulic motor into the theoretical equation for the calculation of the torque-speed characteristic.
9 MATHEMATICAL MODEL OF MEASURED HYDROSTATIC DRIVE

There are two possibilities for dynamic solving of this drive. Firstly, it is possible to neglect the effect of changes in the efficiencies of the hydraulic motor depending on the speed and the pressure gradient and to enter them as mean constant values. Or it is possible to change these efficiencies for each time depending on the speed \( n_M \) (or the flow rate \( Q_M \)) or the torque \( T_M \) (or the pressure gradient \( \Delta p_M \)). The latter variant can be realized using Matlab Fluids software for 1D mathematical simulation of fluid mechanisms. In this software, the volumetric and mechanical-hydraulic efficiencies are specified in the tabular form.

Data entered in this way corresponds to area dependencies of the volumetric and mechanical-hydraulic efficiencies on the motor pressure gradient \( \Delta p_M \) and the motor speed \( n_M \). The dependence of the volumetric efficiency on the pressure gradient \( \Delta p_M \) and the motor speed \( n_M \) is shown in Fig. 17. The dependence of the mechanical-hydraulic efficiency on the pressure gradient \( \Delta p_M \) and the motor speed \( n_M \) is shown in Fig. 18.

9.1 Description of mathematical model of experimental equipment

The mathematical model was created using Matlab Fluids software (see Fig. 19), e. g. [Mehennaoui 2015], [Jablonska 2017]. Steady states of the drive for different cross-section areas \( A_{TV} \) of the throttle valve and different torques \( T_M \) on the shaft of the hydraulic motor were simulated by means of this model.

The hydraulic pump HP1 is a pressure liquid source and is driven by the motor M. Mineral oil flows to the hydraulic motor HM, further through the flow sensor FS and the throttle valve TV to the tank T1. The hydraulic motor HM is loaded by means of the hydraulic pump HP2 and the pressure relief valve RV2. The system pressure is adjusted by means of the pressure relief valve RV1. Furthermore, the model consists of the block for the size adjustment ”\( c_p \)” of the cross-section area \( A_{TV} \) of the throttle valve, the blocks PS1 and PS2 for pressure measurement, the block SS for speed measurement, the block TQS for torque control and the blocks PS3 and PS4 for adjustment of the drive parameters. The mathematical model of the experimental equipment is shown in Fig. 20.
measurement and the solver block „Solver”. Parameters of the hydraulic oil are adjusted by the block „Oil”.

Figure 19. Schematic diagram of mathematical model of experimental equipment

The mathematical model of the throttle valve was defined by means of the $\Delta p$-$Q$ characteristics (see Fig. 20) that were measured for the single openings $\phi_1$ + $\phi_5$ of the throttle valve.

Figure 20. Measured $\Delta p$-$Q$ characteristics of throttle valve for single openings $\phi_1$ + $\phi_5$.

### 9.2 Simulation of torque-speed characteristic

The speed $n_M$ and the torque $T_M$ on the hydraulic motor for the given adjustment range of the pressure relief valve RV2, which loads the hydraulic pump HP2, were simulated for the single adjustments $\phi_1$ + $\phi_5$ of the throttle valve TV in the mathematical model (see Fig. 19). The result of the simulated torque-speed characteristics for the single openings of the throttle valve TV is compared in Fig. 21 with the experimentally determined characteristics. It is visible that the simulated characteristics almost correspond to the measured characteristics.

![Figure 21. Comparison of measured and mathematically simulated torque-speed characteristics](image)

### 10 CONCLUSIONS

This paper was focused on valve control of a hydrostatic drive. The theoretical analysis was performed on a system controlled by the throttle valve. The theoretical torque-speed and control characteristics were determined for the drive, where the pump and motor efficiencies, pressure losses and pressure dependence of the pressure relief valve are not taken into account. Subsequently, the torque-speed and control characteristics were experimentally measured on the assembled hydraulic drive. It was found by comparing the theoretical and measured characteristics that these characteristics are significantly different. These deviations are mainly caused by the efficiency of the hydraulic motor. Therefore the volumetric, mechanical-hydraulic and total efficiencies of the rotary hydraulic motor were experimentally determined. Mean values of these efficiencies were subsequently considered at the theoretical determination of the torque-speed and control characteristics. A greater agreement was achieved by comparing these characteristics with the measured characteristics in respect to the original theoretical characteristics without including the efficiencies of the hydraulic motor. The volumetric and mechanical-hydraulic efficiencies of the rotary hydraulic motor depend on the speed and the motor torque. The exact mathematical description of this hydraulic system was performed using Matlab Fluids software, which allows simulations with the efficiencies for instantaneous values of the speed and the torque on the hydraulic motor. The torque-speed characteristics, which were simulated in this manner, already closely correspond to the measured characteristics.

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