Effective cosmological constant from TeV–scale physics

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Abstract

It has been suggested previously that the observed cosmological constant $\Lambda$ corresponds to the remnant vacuum energy density of dynamical processes taking place at a cosmic age set by the mass scale $M \sim E_{ew}$ of ultramassive particles with electroweak interactions. Here, a simple modification of the nondissipative dynamic equations of $q$–theory is presented, which produces a remnant vacuum energy density (effective cosmological constant) of the correct order of magnitude. Combined with the observed value of $\Lambda$, a first estimate of the required value of the energy scale $E_{ew}$ ranges from 3 to 9 TeV, depending on the number of species of ultramassive particles and assuming a dissipative coupling constant of order unity. If correct, this estimate implies the existence of new TeV–scale physics beyond the standard model.

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I. INTRODUCTION

It has been argued by Arkani-Hamed et al. [1] that two fundamental energy scales, the electroweak scale $E_{\text{ew}} \sim 1$ TeV and the gravitational scale $E_{\text{Planck}} \sim 10^{15}$ TeV, suffice to explain the triple cosmic coincidence puzzle: why are the orders of magnitude of the energy densities of vacuum, matter, and radiation approximately the same in the present Universe? For this explanation to work, the parametric form of the effective cosmological constant (remnant vacuum energy density) must be

$$\Lambda \equiv \rho_{V,\text{remnant}} \sim \left(\frac{E_{\text{ew}}^2}{E_{\text{Planck}}}\right)^4 \sim (10^{-3} \text{ eV})^4.$$  \hspace{1cm} (1.1)

If true, formula (1.1) would be a remarkable explanation of the measured value from observational cosmology, which appears to be of order $10^{-29}$ g cm$^{-3}$ $\sim 10^{-11}$ eV$^4$ (setting $\hbar = c = 1$ and referring to, e.g., Refs. [2–4] and other references therein). However, (1.1) was not derived convincingly in Ref. [1], as an unknown adjustment mechanism needed to be invoked.

Subsequently, Volovik and the present author realized [6] that, in the framework of $q$–theory, there is the possibility of generating a vacuum energy density precisely of the form (1.1). Here, $q$–theory is a particular approach [7] to solving the first cosmological constant problem (CCP1): why is $|\Lambda| \ll (E_{\text{Planck}})^4$? The original references on the statics and dynamics of $q$–theory are [8] and [9], respectively. The second cosmological constant problem (CCP2) is the question addressed here, namely, the actual order of magnitude of $\Lambda$, if indeed nonzero.

The positive remnant vacuum energy density obtained in Ref. [6] relied crucially on Eq. (4.1) of that article. That particular equation was taken to describe the quantum-dissipative effects of the vacuum energy density, but was, in the end, purely hypothetical and disconnected from the previous $q$–theory discussion.

The question arises if it is at all possible to modify the previous $q$–theory equations [9] in such a way as to effectively recover the results of Ref. [6]. The answer is that this is indeed possible, even though the required modifications are quite subtle. The scope of the present article is restricted to finding these appropriate phenomenological equations, rather than establishing the relevant microscopic processes of the underlying theory.

In a way, it can be said that this whole article is about the proportionality constant implicit in (1.1) and that, within the framework of $q$–theory, the article gives an existence proof for a set of dynamical equations which produces a proportionality constant of order unity.

As the previous discussion makes clear, the present article is a direct follow-up of Ref. [6], to which the reader is referred for the original motivation and detailed analysis. This article

1 There have, of course, been many other explanations of the smallness of $\Lambda$ by an appropriate ratio of energy scales (see, e.g., Sec. X of Ref. [1]), but the relation (1.1) is special as it carries the ingredients to naturally give the correct orders of magnitude for the present matter and radiation energy densities [1].
is, by necessity, rather technical and it may be helpful to give the reader a road map. The material of this article is organized as follows:

**Track 1:** Secs. I, II, III A, III B, and V

**Track 2:** Secs. III C, III D, and IV

**Track 3:** Sec. III E and Appendix A

The basic idea and main results are presented in Track 1, the dimensionless differential equations and their numerical solution in Track 2, and a more detailed discussion and further refinement in Track 3 (the most realistic calculations are shown in the very last two figures and the very last table of Appendix A). In a first reading, it is possible to follow Track 1 and to add the other Tracks later.

**II. THEORETICAL FRAMEWORK**

This section reviews the main ingredients of the type of theory considered in this article (see Refs. [8, 9] for details). The particular $q$-theory realization used involves the so-called 4-form field strength [10, 11]. Very briefly, the theory is defined over a four-dimensional Lorentzian spacetime manifold and employs a 4-form field strength $F$ derived from a 3-form gauge field $A$. The corresponding rank-four tensor can always be written as

$$F_{\alpha\beta\gamma\delta}(x) = q(x) \sqrt{-g(x)} \epsilon_{\alpha\beta\gamma\delta}(x), \quad (2.1)$$

with the Levi–Civita tensor density $\epsilon_{\alpha\beta\gamma\delta}(x)$, the determinant of the metric $g(x) \equiv \det g_{\alpha\beta}(x)$, and the scalar field $q(x)$. The crucial point is that this scalar field $q(x)$ is nonfundamental, being built from the metric field $g_{\alpha\beta}(x)$ and the 3-form gauge field $A(x)$, as will become clear shortly. This 3-form gauge field $A(x)$ is considered to be one of the fields which characterize the quantum vacuum at the fundamental microscopic level [8].

The macroscopic effective action of the relevant high-energy fields (here, $A$) and the low-energy fields (here, $g$ and $\psi$) is taken to be of the following form [9]:

$$S_{\text{eff}}[A, g, \psi] = \int_{\mathbb{R}^4} d^4x \sqrt{-g} \left( K(q) R[g] + \epsilon(q) + \mathcal{L}^M[\psi, g] \right), \quad (2.2a)$$

$$F_{\alpha\beta\gamma\delta} = \nabla_\alpha A_{\beta\gamma\delta}, \quad (2.2b)$$

$$q^2 \equiv -\frac{1}{24} F_{\alpha\beta\gamma\delta} F^{\alpha\beta\gamma\delta}, \quad (2.2c)$$

where the effective gravitational coupling parameter $K$ is allowed to depend on $q$, $R[g]$ is the Ricci curvature scalar obtained from the metric $g_{\alpha\beta}$, $\nabla_\alpha$ denotes the standard covariant derivative, and the square bracket around spacetime indices stands for complete antisymmetrization. The energy density $\epsilon(q)$ is assumed to be a generic function of $q$, that is, a function different from the simple quadratic $\frac{1}{2} q^2$ corresponding to a Maxwell-type theory [10, 11]. The field $\psi$ in (2.2a) stands for a generic low-energy matter field with a scalar
Lagrange density $\mathcal{L}^M[\psi, g]$, which, for simplicity, is assumed to be without explicit $q$-field dependence (the dependence on the metric arises from the covariant derivatives).

Remark that the effective action (2.2a) corresponds to a Brans–Dicke-type action \cite{12}, but without kinetic term for the (nonfundamental) scalar $q$. For spacetime-independent $q$ [that is, $q(x) = \bar{q} =$ const], the effective action (2.2a) corresponds to the one of standard general relativity with a cosmological constant $\Lambda = \epsilon(\bar{q}) + \Lambda^M$, where $\Lambda^M$ refers to contributions to $\Lambda$ from the matter Lagrange density $\mathcal{L}^M$.

By taking variations of $A_{\alpha\beta\gamma}(x)$ and $g_{\alpha\beta}(x)$ in the effective action (2.2a), generalized Maxwell and Einstein equations can be derived. The generalized Maxwell equation can be solved explicitly and the solution depends on a constant of integration $\mu$. With the solution of the generalized Maxwell equation, the generalized Einstein equation reduces to the following field equation:

$$2K \left( R_{\alpha\beta} - g_{\alpha\beta} R/2 \right) = -2 \left( \nabla_\alpha \nabla_\beta - g_{\alpha\beta} \Box \right) K(q) + \rho_V(q) g_{\alpha\beta} - T^M_{\alpha\beta}, \quad (2.3)$$

where the combination

$$\rho_V(q) \equiv \epsilon(q) - \mu q \quad (2.4)$$

plays the role of the gravitating vacuum energy density rather than the single term $\epsilon(q)$ appearing in the effective action (2.2a). Furthermore, there is an equation remaining from the particular solution of the generalized Maxwell equation, which reads

$$\frac{d\rho_V}{dq} + R \frac{dK}{dq} = 0. \quad (2.5)$$

The final equations (2.3)–(2.5) can be specialized to the case of a spatially flat Friedmann–Robertson–Walker universe. The resulting cosmological equations have been studied in Refs. \cite{6, 9}, and it is the aim of the present article to find a modification of them which allows for the generation of a nonvanishing remnant vacuum energy density.

III. VACUUM DYNAMICS IN A FLAT FRW UNIVERSE

A. Basic idea

Following Ref. \cite{1}, assume the existence of ultramassive unstable particles (here, called ‘type 1a’) with masses $M$ of order $E_{\text{ew}} \sim 1$ TeV and electroweak interactions. Consider a spatially flat Friedmann–Robertson–Walker (FRW) universe and assume the type–1a particles to be effectively in thermal equilibrium at early enough times. Then, the masses of these particles start to affect the Hubble expansion rate $H(t)$ when the temperature drops to $T \sim E_{\text{ew}}$, corresponding to a cosmic age $t$ of order

$$t_{\text{ew}} \equiv E_{\text{Planck}}/(E_{\text{ew}})^2, \quad (3.1)$$

in terms of the reduced Planck energy,

$$E_{\text{Planck}} \equiv \sqrt{1/(8\pi G_N)} \approx 2.44 \times 10^{18} \text{ GeV}. \quad (3.2)$$
Note that the definition \((3.1)\) is motivated by the standard Friedmann equation \(H^2 \sim \rho/(E_{\text{Planck}})^2\) with \(H \sim 1/t_{\text{ew}}\) and \(\rho \sim T^4 \sim (E_{\text{ew}})^4\). In the following, the Friedmann equation will be modified, but the order of magnitude \((3.1)\) remains relevant. Throughout this article, natural units are used with \(\hbar = c = 1\).

Compared to the case of having only ultrarelativistic particles (these lighter particles are called ‘type 1b’ and can be thought to have masses of order \(M/10\)), the change of the expansion rate from the ultramassive type–1a particles can be modeled by a nonzero function \(\kappa_{M1}(t/t_{\text{ew}})\). In fact, this function \(\kappa_{M1}\) can be written in terms of the standard equation-of-state (EOS) parameter \(w_{M1} \equiv P_{M1}/\rho_{M1}\) as follows:

\[
\kappa_{M1} \equiv 1 - 3 w_{M1},
\]

which vanishes for ultrarelativistic particles \((w_{M1} = 1/3)\) and equals unity for pressure-less nonrelativistic particles \((w_{M1} = 0)\). By taking different unstable type–1 particles \((a, b, c, \ldots)\) it is possible to obtain an effective EOS function \(\kappa_{M1}(t/t_{\text{ew}})\) which peaks at \(t = t_{\text{ew}}\). Here, however, a particular form of \(\kappa_{M1}\) will simply be assumed.

The main conditions on this assumed EOS function \(\kappa_{M1}\) are that it peaks at \(t = t_{\text{ew}}\) and is nonzero only in a finite range around the maximum (this last condition is not essential but simplifies the discussion). Specifically, the conditions are taken to be:

\[
\begin{align*}
\kappa_{M1}(t/t_{\text{ew}}) &< \kappa_{M1}(1) \quad \text{for } t \neq t_{\text{ew}}, \\
\kappa_{M1}(t/t_{\text{ew}}) &\neq 0 \quad \text{for } t \in (t_{\text{start}}, t_{\text{end}}), \\
\kappa_{M1}(t/t_{\text{ew}}) &= 0 \quad \text{for } t \notin (t_{\text{start}}, t_{\text{end}}),
\end{align*}
\]

with \(0 < t_{\text{start}} < t_{\text{ew}} < t_{\text{end}} < \infty\), having set \(t = 0\) for the big bang where \(H(t)\) diverges. The physical picture corresponding to \((3.4)\) is that ultramassive type–1a particles are dominant at \(t \sim t_{\text{ew}}\), but, then, decay into lighter type–1b particles which are still ultrarelativistic for \(t\) not very much larger than \(t_{\text{ew}}\). The main goal is to study the effects of this prescribed EOS function \(\kappa_{M1}\), relegating the discussion of a more realistic EOS function to App. A.

A flat FRW universe containing only type–1 particles with a prescribed EOS parameter \((3.3)-(3.4)\) has a standard radiation-dominated Hubble expansion rate \(H(t) = (1/2) t^{-1}\) for \(t < t_{\text{start}}\) and \(t > t_{\text{end}}\). The expansion rate is changed, \(H(t) \neq (1/2) t^{-1}\), for times \(t\) between \(t_{\text{start}}\) and \(t_{\text{end}}\). The question, now, is what happens if this FRW universe also has a dynamical vacuum-energy-density component.

For the theory outlined in Sec. II two results were obtained in Ref. \([6]\). First, it was shown that there is an exact solution having \(\rho_V(t) = 0\) in the radiation-dominated phase with \(\kappa_{M1}(t) = 0\). Second, it was shown that the changed Hubble expansion from \(\kappa_{M1}(t) \neq 0\) kicks \(\rho_V(t)\) away from zero. Specifically, the following behavior was established \([6]\):

\[
\rho_V(t) \sim \kappa_{M1}^2(t) H(t)^4,
\]

which vanishes asymptotically as \(\kappa_{M1}\) drops to zero and the standard radiation-dominated expansion of the model universe resumes. At the moment of the kick, \(t \sim t_{\text{ew}}\), the vacuum
energy density (3.5) is of order \((t_{\text{ew}})^{-4} \sim \left(\frac{(E_{\text{ew}})^2}{E_{\text{Planck}}}\right)^4 \sim \left(\frac{E_{\text{ew}}}{E_{\text{Planck}}}\right)^4 (E_{\text{ew}})^4\), which is negligible compared to the matter energy density \(\rho_{\text{M1}} \sim (E_{\text{ew}})^4\). The vacuum energy density \(\rho_{\text{V}}(t)\), therefore, just responds to (is being kicked by) the Hubble expansion and does not affect the expansion substantially.

The result (3.5) has been obtained from the simplest time-reversible (nondissipative) version of \(q\)-theory, with field equations given by (2.3)–(2.5). It has been argued that quantum-dissipative effects (e.g., because of particle production in an expanding universe [13, 14]) may result in a freezing of the previous result (3.5) to a constant nonzero value.

As explained in Sec. I the aim of this article is to find a suitable modification of the “classical” \(q\)-theory equations, which produces a finite remnant vacuum energy density. In the approach followed here, there are three changes:

1. The matter energy-conservation equation is modified to include appropriate particle-production effects operating at a cosmic age \(t \sim t_{\text{ew}}\).

2. The reduced Maxwell equation is modified, so as to match the standard Einstein equation of an FRW universe with a nonzero effective cosmological constant at later times.

3. Different particle species are considered with ultramassive type–1a particles first decaying into lighter type–1b particles, which, in turn, decay into massless type–2 particles.

The first two modifications are essential for the generation of a nonvanishing remnant vacuum energy density. (The first modification has already been discussed in general terms in Sec. IV of Ref. [6]). The third modification allows for a possibly more realistic scenario, with type–1 particles corresponding to new TeV-scale physics and type–2 particles corresponding to the standard model of elementary particle physics (see also App. A).

The particle-production effects of point 1 above will be controlled by an effective coupling constant \(\zeta > 0\) and a particular type of dissipation function \(\gamma(t/t_{\text{ew}}; \zeta) \geq 0\). The reason for calling \(\gamma\) a “dissipation” function will become clear in Sec. III E. The main conditions on this function are as follows:

\[
\gamma(0; \zeta) = 1 ,
\]
\[
\forall t \geq t_{\text{freeze}} : \quad \gamma(t/t_{\text{ew}}; \zeta) = 0 ,
\]
\[
\lim_{\zeta \to 0} (t_{\text{freeze}})^{-1} = 0 ,
\]

with a particular time \(t_{\text{freeze}}\) that is of order \(t_{\text{ew}}\) for \(\zeta \sim 1\) and approaches infinity for \(\zeta \to 0\).

The coupling constant \(\zeta\) and the corresponding function \(\gamma\) are purely phenomenological. The vanishing of \(\gamma\) for large enough times will be seen to have two effects: first, to freeze the “classical” value (3.5) and, second, to switch to a standard FRW expansion with relativistic matter and a tiny value (1.1) for the remnant vacuum energy density. Further discussion of \(\zeta\) and \(\gamma\) will appear in the next subsection, after the differential equations have been presented.
B. Modified ODEs

For a spatially flat FRW universe and the 4-form realization of \(q\)-theory with a variable gravitational coupling parameter \(K(q)\), the cosmological differential equations have been derived in Ref. \[9\] and were already mentioned in Sec. II. The basic idea of the proposed modification of these ordinary differential equations (ODEs) has been discussed in the previous subsection. Specifically, the modified ODEs are given by:

\[
6 \frac{dK}{dq} \left( \frac{dH}{dt} + 2H^2 \right) = \left[ \gamma(t/t_{ew}) \right] \frac{d\rho_V}{dq} + \left[ 1 - \gamma(t/t_{ew}) \right] \frac{1}{K} \frac{dK}{dq} 2\rho_V ,
\]

\[
\frac{d\rho_{M1}}{dt} + \left[ 4 - \kappa_{M1}(t/t_{ew}) \right] H \rho_{M1} = -\frac{\zeta}{\gamma(t/t_{ew})} \frac{d}{dt} \left( \frac{d\rho_V}{dq} \right) - \frac{\lambda_{12}}{t_{ew}} \left[ 1 - \gamma(t/t_{ew}) \right] \rho_{M1} ,
\]

\[
\frac{d\rho_{M2}}{dt} + 4 H \rho_{M2} = +\frac{\lambda_{12}}{t_{ew}} \left[ 1 - \gamma(t/t_{ew}) \right] \rho_{M1} ,
\]

\[
6 \left( H \frac{dK}{dq} \frac{d}{dt} + K H^2 \right) = \rho_V + \rho_{M1} + \rho_{M2} ,
\]

where only the arguments of the functions \(\kappa_{M1}\) and \(\gamma\) have been shown explicitly. The four equations in (3.7), going from the top to the bottom, can be recognized as modified versions of the reduced Maxwell equation (2.5), the two matter energy-conservation equations, and the Friedmann equation [the standard Friedmann equation is recovered for \(K = K(q_0) = 1/(16\pi G_N) = \text{const}\)]. For \(\zeta = 0\) and \(\gamma(t) = 1\), the ODEs (3.7) correspond to Eqs. (4.12abc) of Ref. \[9\], supplemented by an equation for the adiabatic evolution of \(\rho_{M2}\).

The modified ODEs for \(\zeta > 0\) have a dependence on an external time scale, here taken to be \(t_{ew}\) from (3.1). The \(t_{ew}\) dependence enters implicitly through the EOS function \(\kappa_{M1} \geq 0\) and the dissipation function \(\gamma \geq 0\) discussed in the previous subsection and explicitly through the matter energy-exchange terms proportional to \(\lambda_{12}/t_{ew}\). Note that the dimension of the \(q\)-field in Eqs. (3.7) is irrelevant, which concords with the fact that this \(q\)-field may be realized in different ways \[8\].

The particular modification (3.7) of the cosmological ODEs from classical \(q\)-theory has two main ingredients: first, the function \(\gamma(t/t_{ew})\) with characteristics (3.6) and, second, the presence of a finite coupling constant \(\zeta\),

\[
\zeta = O(1) ,
\]

which enters directly on the right-hand side of (3.7b) and indirectly via condition (3.6c) for the dissipation function \(\gamma\). A finite value for the remnant vacuum energy density from the dynamical ODEs (3.7) requires both \(\zeta > 0\) and \(\zeta < \infty\), as will be explained in Sec. III E.

The rather simple structure of (3.7), combined with conditions (3.4) and (3.6), will be seen to allow for the generation of a nonzero remnant vacuum energy density.\(^2\) A detailed

\(^2\) A somewhat more general modification of the ODEs has \(2\rho_V\) in the last term on the right-hand side of (3.7a) replaced by \[2\rho_V + \kappa_{M1}(t/t_{ew}) \rho_{M1}/2\], but, for the case considered in this section and Sec. IV, the results are essentially unchanged, because \(\kappa_{M1}(t/t_{ew}) = 0\) for \(t \geq t_{end} > t_{ew}\) according to (3.4).
discussion of the modified ODEs is postponed until Sec. III E, after these equations have been established in dimensionless form.

Before embarking on this technical enterprise, it may be useful to recapitulate the basic assumptions. The first assumption is the existence of a particular type of vacuum variable \( q(x) \), namely, a variable which corresponds to a conserved relativistic quantity \( q_0 \) in flat Minkowski spacetime. Such a variable \( q(x) \) provides a possible solution of the main cosmological constant problem (CCP1) by explaining why \( \Lambda/E_{\text{Planck}}^4 \) is naturally zero in the equilibrium state.\(^3\) This vacuum variable \( q(x) \) is taken to have an effective action of the form of (2.2), where, in particular, the gravitational coupling constant \( K \) may carry a dependence on \( q \).

The second assumption is that the field equations from the effective action (2.2), specialized to a spatially flat FRW universe, are modified by the introduction of terms involving the coupling constant \( \zeta \). The crucial term is the first one on the right-hand side of (3.7b), whose physical motivation is that it reproduces the dissipative behavior suggested in Ref. [6] (this behavior is analogous to that of bulk viscosity in compressible material fluids [15]). The coupling constant \( \zeta \) and the corresponding function \( \gamma \) are purely phenomenological. As mentioned in Sec. I, ultimately \( \zeta \) and \( \gamma \) (or appropriate generalizations) need to be derived from the underlying microscopic theory, but that task lies outside the scope of the present article.

Clearly, the first assumption is better motivated than the second. But the second assumption may (or may not) gain in credibility depending on the success (or not) of producing a reasonable remnant vacuum energy density and predicting new TeV–scale physics.

C. Ansätze and dimensionless variables

Following Refs. [6, 9], take quadratic and linear Ansätze for the vacuum energy density and the gravitational coupling parameter:

\[
\rho_V(q) = \frac{1}{2} (q - q_0)^2, \tag{3.9a}
\]

\[
K(q) = \frac{1}{2} q. \tag{3.9b}
\]

These Ansätze imply the following equilibrium value for the \( q \)-field of theory (2.2):

\[
q_0 = 1/(8\pi G_N) \equiv (E_{\text{Planck}})^2, \tag{3.10}
\]

\(^3\) The \( q \)-theory approach to CCP1 provides only a possible solution, because it is not known for sure that the underlying microscopic theory does contain an appropriate \( q \)-type field. In addition, there remain other equally fundamental (perhaps related) questions, such as the nature of gravity and the origin of spacetime. The goal of the present article is relatively modest: to explore, in the framework of \( q \)-theory, a possible connection between the observed value of the effective cosmological constant and new TeV–scale physics.
where $E_{\text{Planck}}$ is the energy scale from (3.2). In this article, $q$ is considered to be realized by a 4-form field strength $F$ with mass dimension 2. With a proportionality constant in (3.9b) of order unity, the natural scale of the vacuum variable $q$ is then of order $(E_{\text{Planck}})^2$. However, the energy scale of $q$ in the cosmological ODEs (3.7) is, in principle, arbitrary, as noted already in the second paragraph of Sec. III B.

Next, recall the time scale $t_{\text{ew}}$ defined by (3.1), which corresponds to the age of the Universe at a temperature of order $E_{\text{ew}}$. For later use, also define the following number characterizing the hierarchy of energy densities:

$$\xi \equiv (E_{\text{Planck}}/E_{\text{ew}})^4, \quad (3.11)$$

which is of order $10^{60}$ for $E_{\text{ew}} \sim 1$ TeV. For such a large value of $\xi$, the cosmic time $t_{\text{ew}}$ considered in this article is large compared to the Planck time, $t_{\text{ew}} = \sqrt{\xi} (E_{\text{Planck}})^{-1}$. In addition, the relation $(t_{\text{ew}})^2 q_0 = \xi$ can be seen to hold, which will be used later for the derivation of the dimensionless ODEs.

With $t_{\text{ew}}$ and $\xi$, the following dimensionless variables can be defined for the cosmic time, the Hubble expansion rate, the energy densities, and the $q$ shift away from equilibrium:

$$\tau \equiv (t_{\text{ew}})^{-1} t, \quad h \equiv t_{\text{ew}} H, \quad (3.12a)$$

$$r_V \equiv (t_{\text{ew}})^4 \rho_V, \quad r_{M_n} \equiv \xi^{-1} (t_{\text{ew}})^4 \rho_{M_n}, \quad (3.12b)$$

$$x \equiv \xi \left(\frac{q}{q_0} - 1\right) \equiv \xi y, \quad (3.12c)$$

where $n$ stands for the matter-species label ($n = 1, 2$) and $y$ is the variable used previously in Refs. [6, 9]. Observe that $\rho_{M_n}$ has been rescaled by an extra factor $1/\xi$ but $\rho_V$ not.

At this moment, it is appropriate to give explicit examples for the EOS function $\kappa_{M1}$ and the dissipation function $\gamma$ discussed in Sec. III A. With central value $\tau_c$ and total width $\Delta \tau$, define the auxiliary variable $\sigma \equiv 2 (\tau - \tau_c)/\Delta \tau$ and take the EOS function to be given by

$$\kappa_{M1}(\tau) = \begin{cases} \kappa_c \sin^2 \left(\frac{\pi}{2} (1 + \sigma^2)\right) & \text{for } -1 \leq \sigma \leq 1, \\ 0 & \text{otherwise}. \end{cases} \quad (3.13)$$

In the main part of this article, $\kappa_c$ is set to 1 and the dynamic strength of the kick is controlled by the value of the initial energy-density ratio $\rho_{M1}/\rho_{M2}$ (see App. A for a more realistic EOS function).

Turning to the dissipation function $\gamma$, introduce the basic time scale $\tau_\infty$, define

$$\tau_{\text{freeze}} \equiv (1 + 1/\xi) \tau_\infty, \quad (3.14a)$$

and take the function to be given by

$$\gamma(\tau) = \begin{cases} \cos^2 \left[\left(\frac{\pi}{2}\right) \frac{\tau}{\tau_{\text{freeze}}\right] & \text{for } 0 \leq \tau \leq \tau_{\text{freeze}}, \\ 0 & \text{otherwise}. \end{cases} \quad (3.14b)$$

The two functions used will also be shown in the plots of the numerical results later on.
D. Dimensionless ODEs

Take, now, the Ansätze \((3.9a)-(3.9b)\) and assume small deviations of \(q\) away from the equilibrium value \(q_0\), i.e., \(|q/q_0 - 1| \ll 1\). Then, the ODEs \((3.7)\) reduce to the following four equations for the four dimensionless variables \(h(\tau), r_{M1}(\tau), r_{M2}(\tau),\) and \(x(\tau)\) from \((3.12)\):

\[
3 \left( \dot{h} + 2h^2 \right) = \gamma x + (1 - \gamma) \xi^{-1} x^2, \tag{3.15a}
\]

\[
\dot{r}_{M1} + (4 - \kappa_{M1}) hr_{M1} = -(\xi/\gamma) [\dot{x}] - \lambda_{12} (1 - \gamma) r_{M1}, \tag{3.15b}
\]

\[
\dot{r}_{M2} + 4 hr_{M2} = +\lambda_{12} (1 - \gamma) r_{M1}, \tag{3.15c}
\]

\[
3 \left( \xi^{-1} h \dot{x} + h^2 \right) = \xi^{-1} x^2/2 + r_{M1} + r_{M2}, \tag{3.15d}
\]

where the overdot stands for differentiation with respect to \(\tau\). Henceforth, the functions \(\kappa_{M1}(\tau)\) and \(\gamma(\tau)\) are considered to be given by the explicit expressions \((3.13)\) and \((3.14)\).

For the numerical calculation, the factor \([\dot{x}]\) on the right-hand side of \((3.15b)\) is to be replaced by the appropriate expression for \(\dot{x}\) obtained from \((3.15d)\). Recall, furthermore, that the dimensionless vacuum energy density \(r_V\) is given by \(x^2/2\) and that, according to \((3.12b)\), the dimensionless matter energy densities \(r_{M1}\) and \(r_{M2}\) include an extra numerical factor \(\xi^{-1}\) compared to \(r_V\).

For later use, also the ODEs for the special case \(\xi^{-1} = 0\) are needed. From \((3.15)\), the following system of equations can be derived for \(\xi^{-1} = 0\), with three ODEs:

\[
3 \left( \dot{h} + 2h^2 \right) = \gamma x, \tag{3.16a}
\]

\[
\dot{x} = -\xi^{-1} h \gamma \left( 2 \gamma x - \kappa_{M1} [3 h^2 - r_{M2}] \right), \tag{3.16b}
\]

\[
\dot{r}_{M2} + 4 hr_{M2} = +\lambda_{12} (1 - \gamma) [3 h^2 - r_{M2}], \tag{3.16c}
\]

and a single algebraic equation:

\[
3 h^2 = r_{M1} + r_{M2}. \tag{3.17}
\]

The derivation of \((3.16b)\) proceeds in three steps: first, take the time derivative of \((3.17)\); second, use \((3.16a)\) to eliminate \(\dot{h}\) in the resulting expression for \((\dot{r}_{M1} + \dot{r}_{M2})\); third, use the final expression for \((\dot{r}_{M1} + \dot{r}_{M2})\) in the sum of the two Eqs. \((3.15b)\) and \((3.15c)\) to get \((3.16b)\). All in all, the \(\xi^{-1} = 0\) equations consist of three ODEs \((3.16a)-(3.16c)\) for three variables \(h(\tau), x(\tau),\) and \(r_{M2}(\tau)\), with the energy density \(r_{M1}\) following from the Friedmann Eq. \((3.17)\). These ODEs will be used in Sec. IV B to get the value of the remnant vacuum energy density \(r_V\) for \(\xi \to \infty\).

Purely mathematically, there is another special case to consider for the ODEs \((3.15)\) as they stand, namely, the case \(\xi = 0\). From \((3.15a)\) and \((3.15d)\), together with the proper boundary condition \(x(0) = 0\), the \(\xi = 0\) solution can be seen to have \(x(\tau) = 0\), which implies \(r_{V}(\tau) = 0\). But, as said, this solution is not directly relevant for the physical situation considered.
E. Additional remarks

A few remarks may be helpful to better understand the proposed ODEs, given by (3.7) in the general form or by (3.15) in the specialized and dimensionless form.

First, note that the previous results [6] on the dynamics at \( \tau \sim 1 \) are readily recovered. From (3.15) for \( \zeta = 0, \gamma(\tau) = 1, \) and \( \xi r_{M1} \gg \xi r_{M2} \gg r_V, \) one immediately obtains at \( \tau \sim 1 \)

\[
x(\tau) \sim \frac{3}{2} \kappa_{M1}(\tau) h(\tau)^2,
\]

\[
r_V(\tau) \sim \frac{9}{8} \kappa_{M1}(\tau)^2 h(\tau)^4,
\]

which corresponds to Eqs. (3.1a) and (3.4a) of Ref. [6] apart from a trivial rescaling. With \( \kappa_{M1}(\tau)^2 h(\tau)^4 \) dropping to zero rapidly for large enough \( \tau, \) there is no sizable remnant vacuum energy density, at least, according to the unmodified ODEs given by (3.15) for \( \zeta = 0 \) and \( \gamma = 1. \)

For nonzero \( \zeta, \) however, the same approximations in the ODEs (3.15) give the following dissipation-type equation at \( \tau \sim 1: \)

\[
\dot{x}(\tau) \sim -\gamma_{\text{diss}}(\tau) \left[ \gamma(\tau) x(\tau) - \frac{3}{2} \kappa_{M1}(\tau) h(\tau)^2 \right],
\]

\[
\gamma_{\text{diss}}(\tau) \equiv 2 \zeta^{-1} h(\tau) \gamma(\tau),
\]

whose derivation parallels the one of (3.16b) in the previous subsection. Equation (3.19a) with boundary condition \( x(0) = 0 \) is the analogue of the crucial relation (4.1) of Ref. [6] that allows for a positive remnant vacuum energy density as discussed in Sec. IV of that article. Section IV of Ref. [6] contains, in fact, the analytic solution of (3.19a) for given functions \( \gamma(\tau), \kappa_{M1}(\tau), \) and \( h(\tau). \) For completeness, the dimensionful quantity corresponding to \( \gamma_{\text{diss}}(\tau) \) is given by \( \Gamma_{\text{diss}}(t) = 2 \zeta^{-1} H(t) \gamma(t/t_{\text{ew}}). \)

The dissipative ODE (3.19) and its analytic solution [6] make clear that a finite remnant vacuum energy density with \( \lim_{\tau \to \infty} x(\tau) \neq 0 \) requires both \( \zeta > 0 \) and \( \zeta < \infty. \) Indeed, for the case \( \gamma_{\text{diss}} \to \infty \) (or \( \zeta \to 0), \) the solution \( x(\tau) \) follows \( \kappa_{M1}(\tau) h(\tau)^2 \) which drops to zero rapidly for large times and, for the case \( \gamma_{\text{diss}} \to 0 \) (or \( \zeta \to \infty), \) the solution \( x(\tau) \) simply remains at the initial value, \( x(\tau) = x(0) = 0. \)

Second, it appears essential that the ODEs (3.7) and (3.15) are singular, with the coefficients of the first terms on the right-hand sides of (3.7b) and (3.15b) diverging for \( \tau > \tau_{\text{freeze}}; \) because of the \( \gamma \) condition (3.6b). In fact, the divergence of the coefficient \( \zeta/\gamma(\tau) \) for \( \tau \) above \( \tau_{\text{freeze}} \) forces \( q/q_0 - 1 \) to be strictly constant. It could very well be that the exact vanishing of \( \gamma \) for \( \tau > \tau_{\text{freeze}} \) in the cosmological context traces back to the existence of an energy threshold in the relevant particle reaction process. As noted in Ref. [6], the energies involved are tiny (of the order of meV), so that only sufficiently light neutrinos and gravitons can be expected to play a role. The first term on the right-hand side of (3.7b), as it stands, is not simply proportional to \( R^2 \) as for the well-known Zeldovich–Starobinsky result [13], but does involve \( R \) via its time derivative, as follows by use of (3.7a).
Third, the discussion of the two previous remarks suggests that the value of the remnant vacuum energy density can be at most of the order of the maximum possible “classical” value (i.e., the \( r_V \) peak from the nondissipative \( \zeta = 0 \) equations). The idea is that, in general, dissipation leads to reduction of the produced energy rather than enhancement. Specifically, the conjectured inequality is

\[
r_V(\tau_{\text{freeze}}) \lesssim \max_{\tau} \left[ \frac{9}{8} \kappa_{M1}(\tau)^2 \left( h(\tau)^2 - r_{M2}(\tau)/3 \right)^2 \right],
\]

(3.20)

which is based on the analytic result (3.18b) with \( h^4 \) on the right-hand side replaced by \( (h^2 - r_{M2}/3)^2 \), as suggested by (3.16b). It remains to sharpen the approximate upper bound (3.20) and to determine the corresponding conditions.

Fourth, having a constant nonzero value of \( x \propto (q/q_0 - 1) \) does not automatically allow for a standard de-Sitter universe, as the original ODE (3.15a) [with \( \gamma \equiv 1 \)] and the ODE (3.15d) are inconsistent for \( \dot{x} = \dot{h} = 0, x < \xi \), and \( r_{M1} = r_{M2} = 0 \). However, the modified Eq. (3.15a) [with \( \gamma(\tau) = 0 \) for \( \tau > \tau_{\text{freeze}} \)] has been designed to match the corresponding Einstein equation of a standard flat FRW model with ultrarelativistic matter and constant vacuum energy density, which asymptotically approaches a de-Sitter universe.\(^4\) This particular modification also makes clear that there must be more than just energy exchange between the vacuum and matter sectors. Rather, there must be a type of modulated interaction between the vacuum field and the nonstandard gravitational field, which can be seen as follows. Multiply (3.7a) by \( K dq/dK \) to get a modified FRW–Einstein equation,

\[
6K \left( \frac{dH}{dt} + 2H^2 \right) = \left[ \gamma(t/t_{\text{ew}}) \right] K \frac{d\rho_V}{dK} + \left[ 1 - \gamma(t/t_{\text{ew}}) \right] 2\rho_V,
\]

(3.21)

where the nonstandard term \( K d\rho_V/dK \) is switched off for large enough cosmic times by the factor \( \gamma(t/t_{\text{ew}}) \) going to zero.

Fifth, the ODEs (3.15), for given functions \( \kappa_{M1}(\tau) \) and \( \gamma(\tau) \) and fixed coupling constants \( \zeta \) and \( \lambda_{12} \), contain one last free parameter, the hierarchy parameter \( \xi \) defined by (3.11). Heuristically, it is to be expected that the precise value of \( \xi \gg 1 \) does not affect the resulting value \( r_V(\tau_{\text{freeze}}) \). But, as \( \xi \) fixes the ratio \( \rho_M/\rho_V \) at \( t \sim t_{\text{ew}} \), it does affect the later (standard)

---

\(^4\) The assumption, here, is that other contributions to the vacuum energy density generated at times later than \( t_{\text{ew}} \) would be self-adjusted away by appropriate \( q \)-type fields [8]. The prime example is the quantum-chromodynamics (QCD) vacuum energy density of order \((100 \, \text{MeV})^4\), which is expected to appear during the cosmological QCD transition at \( T \sim 100 \, \text{MeV} \). This huge contribution to the vacuum energy density \( \rho_V \) has been shown to self-adjust to zero [16, 17], as long as there is no term proportional to \( |H|E_{\text{QCD}}^3 \) contributing to \( \rho_V \). If there is such a nonanalytic term (cf. the discussion in Ref. [18]), then the final value of \( \Lambda \) could be a combination of electroweak and QCD effects. However, the experimental signatures of the electroweak vacuum energy density (effectively, a ΛCDM model, as explained in the next footnote) and the QCD vacuum energy density (an \( f(R) \) modified-gravity model) are different, in particular as regards the effective EOS parameter \( w_X \) discussed in Ref. [17]. But, for the moment, there is no definitive proof that the required nonanalytic term occurs in four-dimensional QCD.
evolution of the model universe and, in fact, determines the cosmic time $t_{\text{accel}}$ at which the matter energy density drops below that of the constant vacuum energy density, $t_{\text{accel}} \sim \sqrt{\xi} t_{\text{ew}}$. More precisely, the onset of acceleration $\ddot{a}/a > 0$ [having defined $h \equiv \dot{a}/a$ in terms of the scale factor $a(\tau)$] occurs at the energy-density ratio $\rho_V/\rho_{\text{Mtot}} = \overline{\sigma}$, with $\overline{\sigma} = 1$ for relativistic matter ($w_{M2} = 1/3$ and $\kappa_{M2} = 0$) or $\overline{\sigma} = 1/2$ for nonrelativistic matter ($w_{M2} = 0$ and $\kappa_{M2} = 1$). The model considered in the present article has $\overline{\sigma} = 1$, but can easily be adapted to give the value $\overline{\sigma} = 1/2$ which is more realistic.

IV. NUMERICAL SOLUTION

A. Numerical results for $\xi = 10^2$

The mathematical parameter $\xi$ entering the ODEs (3.15) is, first, considered to have the moderately large value of 100. The boundary conditions are taken from the epoch before the electroweak kick, when there was a standard radiation-dominated flat FRW universe. With the onset of the electroweak kick given by $\tau_{\text{start}}$ from (3.4b) and (3.13), the following boundary conditions on $h$, $x$, and $r_{\text{Mtot}} \equiv r_{M1} + r_{M2}$ hold at a time $\tau = \tau_{\text{min}} \leq \tau_{\text{start}} < \tau_{\text{ew}}$:

\begin{align}
    h(\tau_{\text{min}}) &= 1/2 (\tau_{\text{min}})^{-1}, \\
    x(\tau_{\text{min}}) &= 0, \\
    r_{\text{Mtot}}(\tau_{\text{min}}) &= 3 \left[ h(\tau_{\text{min}}) \right]^2.
\end{align}

This leaves only the initial ratio $[r_{M1}/r_{M2}(\tau_{\text{min}}) \equiv r_{M1}(\tau_{\text{min}})/r_{M2}(\tau_{\text{min}})$ to be determined, which is, for the moment, taken to be 1 (other initial ratios will be discussed shortly).

The corresponding numerical solutions of the ODEs (3.15) are shown in Figs. 1–3. Figure 1 illustrates the fact that the standard (nondissipative) dynamic equations for $\zeta = 0$ and $\gamma = 1$ do not produce a constant positive remnant vacuum energy density from the electroweak kick [the oscillatory effects in $r_V(\tau)$ are suppressed for larger values of the hierarchy parameter $\xi$; see the first figure called in Sec. 4.5]. However, as shown by Fig. 2, the modified (dissipative) dynamic equations for $\zeta = O(1)$ do produce a sizable remnant vacuum energy density.

The subsequent evolution of Fig. 2 is shown in Fig. 3. The content of this model universe for $\tau > 3$ is given by a constant vacuum energy density (effective cosmological constant) and two species of matter, with massive type–1 particles playing a role for the generation of the

\[5\] The adapted model contains an additional EOS function $\kappa_{M2}(t/t_{\text{eq}})$, which is a smoothed step function running from 0 to 1 as the cosmic time $t$ increases and which has a half-way time $t_{\text{eq}}$ where $\kappa_{M2}(1) = 1/2$. This matter-radiation-equality time $t_{\text{eq}}$ has a parametric form $\alpha^3 \xi^{1/2} t_{\text{ew}}$, with $\alpha$ the electromagnetic fine-structure constant (see Ref. 1 for further details). The adapted model then has an acceleration phase for $\rho_{V}/(\rho_{M1} + \rho_{M2}) \approx \rho_{V}/\rho_{M2} > 1/2$ and corresponds, for $t \gg t_{\text{ew}}$, to a particular $\Lambda$CDM model 4–19. As noted in Ref. 6, this EOS function $\kappa_{M2}$ can also be expected to perturb the vacuum energy density, but the magnitude involved is tiny compared to the one from the electroweak scale because $H(t_{\text{eq}})^4 \ll H(t_{\text{ew}})^4$. 13
vacuum energy density during the electroweak epoch ($\tau \sim 1$) and ultimately decaying into massless type–2 particles.

Similar results are obtained for initial ratios $[r_{M1}/r_{M2}](\tau_{\text{min}}) \gtrsim 1$. Table II gives the function values at cosmic time $\tau = \tau_{\text{freeze}}$, in particular, the values for $r_V$ which, by construction, stay constant for later times $\tau > \tau_{\text{freeze}}$. Remark that even for a relatively mild kick with initial ratio $[r_{M1}/r_{M2}] = 1/10$, the generated $r_V$ is still of order $10^{-3}$.

Returning to the boundary condition $[r_{M1}/r_{M2}](\tau_{\text{min}}) = 1$, Figs. 2–3 are seen to give a value $r_V(\tau_{\text{freeze}}) \approx 0.04$. By changing the model parameters and the model functions somewhat it is possible to get $r_V(\tau_{\text{freeze}})$ values in the range of $10^{-3}$ to 1. But it appears impossible to get a remnant $r_V$ much larger than unity, which agrees with the conjectured upper bound (3.20).

### B. Numerical results for $\xi \gg 10^2$

The parameter $\xi$ has been defined in physical terms by (3.11) and its mathematical role for the solution of the ODEs (3.15) has already been discussed in the last paragraph of Sec. III E. Here, numerical results are presented for large values of this parameter, ranging from $\xi = 10^4$ to $\xi = \infty$.

Numerical results for the standard nondissipative ($\zeta = 0$) dynamic equations at $\xi = 10^4$ are given in Fig. 4, which show reduced oscillatory effects of $r_V(\tau)$ compared to Fig. 1 and recover the smooth behavior of (3.18b). [Recall that the analytic approximation (3.18b) was derived for a negligible type–2 matter energy density and a better approximation has $h^4$ on the right-hand side replaced by $(h^2 - r_{M2}/3)^2$, as used already in (3.20).] Further numerical results for $\zeta = 2$ and $\xi = 10^4$ confirm the expectation from Sec. III E that the generation of the remnant vacuum energy density at $\tau \sim 1$ is qualitatively the same as for $\xi = 10^2$ (compare Fig. 5 with Fig. 2) and that the main effect of a larger value of $\xi$ is that of pushing the onset of the vacuum-dominated expansion to larger values of $\tau$, with $\tau_{\text{acc}} \propto \sqrt{\xi}$ (compare Fig. 6 with Fig. 3). It is also instructive to contrast the behavior of the vacuum variable $x(\tau)$ in Fig. 4 and that of Fig. 5 where the latter figure displays the “time-lag effect” because of the finite dissipative coupling constant $\zeta$ (cf. the heuristic discussion of the paragraph starting a few lines under Eq. (4.5) in Ref. [6]).

As the model evolution for $\tau > \tau_{\text{freeze}} = 3$ is perfectly standard (described by an FRW universe with ponderable matter and an effective cosmological constant), the crucial segment of the numerical solution is over the interval $[\tau_{\text{min}}, \tau_{\text{freeze}}]$. The numerical data for the function values are given in Table III, where the $\xi = \infty$ values refer to the solution of the equations (3.16) derived in Sec. III D. The functions from Table III are observed to converge for $\xi \gtrsim 10^4$. The convergence of the vacuum energy density results is also shown by Fig. 7. With $r_V(\tau_{\text{freeze}})$ values for parameters $\xi$ both below and above the “realistic” number $\xi = 10^60$, the following estimate is obtained by interpolation:

$$
\left. r_V(\tau_{\text{freeze}}) \right|_{\xi = 10^{60}} \approx 0.051,
$$

(4.2)
for the model parameters and boundary conditions mentioned in the caption of Table II.

The function values at $\tau = \tau_{\text{freeze}}$ calculated from the $\xi^{-1} = 0$ ODEs (3.16) are given in Table III for a wide range of values of the initial ratio $[r_{M1}/r_{M2}]$ at $\tau = \tau_{\text{min}} = 0.1$. These function values can be expected to approximate the physical $(\xi^{-1} \sim 10^{-60})$ values with an accuracy of one per mill or better, at least, for initial ratios $[r_{M1}/r_{M2}]$ of order unity.

As far as the dimensionless remnant vacuum energy density $r_V(\tau_{\text{freeze}})$ is concerned, the values quoted in Table III constitute the complete solution of the problem where the initial ratio $[r_{M1}/r_{M2}](\tau_{\text{min}})$ controls the relative strength of the kick at $t \sim t_{\text{ew}}$, assuming the validity of the phenomenological ODEs (3.7) for coupling constant $\zeta = O(1)$ and taking model functions $\kappa_{M1}(\tau)$ and $\gamma(\tau)$ from (3.13) and (3.14), respectively. The dimensionful remnant vacuum energy density $\rho_V(t_{\text{freeze}})$ requires knowledge of the absolute energy scale $E_{\text{ew}}$ used in the rescaling of the variables, as will be discussed further in the next section.

V. DISCUSSION

A. General case

In the scenario considered [1, 6], the theoretical value of the effective cosmological constant (remnant vacuum energy density) is given by

$$\Lambda^{\text{theory}} \equiv \lim_{t \to \infty} \rho_V^{\text{theory}}(t) = r_V^{\text{num}} (E_{\text{ew}})^8 / (E_{\text{Planck}})^4,$$

(5.1)

with $r_V^{\text{num}}$ a number obtained by numerically solving ODEs of the type of (3.7). Equating the theoretical value (5.1) with the experimental value $\Lambda^{\exp} \approx (2 \text{ meV})^4$ from observational cosmology [2–4], the following estimate for the required energy scale is obtained:

$$E_{\text{ew}} = (\Lambda^{\exp} / r_V^{\text{num}})^{1/8} (E_{\text{Planck}})^{1/2} \approx 3.2 \text{ TeV} \left( \frac{0.051}{r_V^{\text{num}}} \right)^{1/8} \left( \frac{\Lambda^{\exp}}{(2.0 \text{ meV})^4} \right)^{1/8}.$$

(5.2)

For the moment, the numerical value $r_V^{\text{num}} = 0.051$ in (5.2) is purely for illustrative purpose.

Clearly, the calculation of the present paper relies on many assumptions, but it appears that values of the order of unity for the dimensionless energy density $r_V^{\text{num}}$ are quite reasonable. The value $r_V^{\text{num}} = 1.0$ corresponds to $E_{\text{ew}} \approx 2.2$ TeV, according to (5.2). On the other hand, values $r_V^{\text{num}} \gg 1$ appear unlikely, at least, in the present framework [see the discussion in Sec. III E leading up to (3.20)]. Note that an $r_V^{\text{num}}$ value of order $10^{10}$ would be required in (5.2) to bring $E_{\text{ew}}$ down to the order of magnitude of standard-model particle masses, $m_{\text{SM}} = 10^2 \text{ GeV}$.

Taking $r_V^{\text{num}} \lesssim 1$ for granted, the correct reading of (5.2) is that of a lower bound,

$$E_{\text{ew}} \gtrsim 2 \text{ TeV},$$

(5.3)

since, without further input, the value of $r_V^{\text{num}}$ can be made arbitrarily small (for example, by taking a sufficiently small value for the initial energy density $r_{M1}$ in Table III). Indeed, the main uncertainty (apart from the unknown $E_{\text{ew}}$ value) is the dynamic importance at $t \sim t_{\text{ew}}$ of the nonrelativistic ($M \sim E_{\text{ew}}$) type–1 particles compared to that of the relativistic type–2 particles.
B. Special case

In order to get further predictions, the following three assumptions can be made. First, assume the type–1 and type–2 particles to have been ultrarelativistic and in thermal equilibrium for $T \gg E_{\text{ew}}$, so that their energy densities are given by

$$\rho_{Mn} = \left(\frac{\pi}{30}\right) N_{\text{eff},n} T^4,$$

with bosons ($b$) and fermions ($f$) of particle type $n = 1, 2$ contributing as follows:

$$N_{\text{eff},n} = \sum_b g_{n,b} + \left(\frac{7}{8}\right) \sum_f g_{n,f},$$

in terms of the numbers of degrees of freedom of the particles ($g_b = 2$ for the photon). Then, the relevant energy-density ratio $\rho_{M1}/\rho_{M2}$ before the kick starts is simply given by the ratio of the respective effective numbers of degrees of freedom, $N_{\text{eff},1}/N_{\text{eff},2}$.

Second, assume the type–1 particles to have approximately the same mass and a mass scale $M \sim E_{\text{ew}}$. (5.4c)

As discussed in Sec. [3.1A], these type–1 particles can be thought to consist of a mix of different unstable particles. What matters here, though, is their average thermodynamic properties as given by the prescribed EOS function $\kappa_{M1}$ from (3.13). See App. A for a realistic setup.

Third, assume the massless type–2 particles to correspond to those of the standard model ($m \lesssim m_{\text{SM}} \ll E_{\text{ew}}$), so that

$$N_{\text{eff},2} = N_{\text{eff,SM}} = 427/4 \sim 10^2.$$

(5.4d)

See, e.g., Ref. [20] for the count of the degrees of freedom in the standard model.

With these assumptions, there are only two unknowns: the numerical value of the energy scale $E_{\text{ew}}$ and the effective number $N_{\text{eff},1}$ of type–1 degrees of freedom. The first quantity, $E_{\text{ew}}$, sets, according to (3.1), the physical time $t \sim t_{\text{ew}}$ when the ‘kick’ of the vacuum energy density occurs (the kick mechanism [6] relies on the change of the Hubble expansion rate by type–1 mass effects). The second quantity, $N_{\text{eff},1}$, controls the initial energy-density ratio:

$$[r_{M1}/r_{M2}](\tau_{\text{min}}) = N_{\text{eff,1}}/N_{\text{eff,2}} \sim N_{\text{eff,1}}/10^2,$$

(5.5)

where the dimensionless cosmic time $\tau_{\text{min}}$ is taken before the kick starts and $r$ denotes the dimensionless energy density according to (3.12).

For a substantial number $N_{\text{eff},1} = 10^2$ of these ultramassive type–1 particles (possibly corresponding to partners of the standard-model particles from broken supersymmetry [21, 22]), the initial energy-density ratio is given by $[r_{M1}/r_{M2}](\tau_{\text{min}}) \sim 1$ and the dimensionless remnant vacuum energy density $r_{\text{num}}^V$ is found to be of order $5.1 \times 10^{-2}$ [see Table III].
This particular \( r_{\text{V}}^{\text{num}} \) value requires, according to (5.2), an \( E_{\text{ew}} \) value of order 3.2 TeV, in order to reproduce the experimental value of the cosmological constant. A similar \( E_{\text{ew}} \) value is obtained in App. A if the prescribed (artificial) EOS function (3.13) is replaced by a physically-motivated EOS function. Only for \( N_{\text{eff},1} = 1 \) (corresponding to a single ultramassive real scalar) does the remnant vacuum energy density drop to such a low value, \( r_{\text{V}}^{\text{num}} \sim 2.0 \times 10^{-5} \) [Table III], that the required energy scale becomes significantly larger, \( E_{\text{ew}} \sim 8.5 \) TeV.

Hence, if all particles have initially been in thermal equilibrium and if the type–1 particles with mass scale \( M \sim E_{\text{ew}} \) have an effective number of degrees of freedom \( N_{\text{eff},1} \gtrsim 1 \), the required energy scale \( E_{\text{ew}} \) from (5.2) lies in the following range:

\[
E_{\text{ew}} \bigg|_{\text{prescribed kick}} \mid_{N_{\text{eff},1} \geq 1, N_{\text{eff},2} = 10^2} \sim 3 - 9 \text{ TeV},
\]

assuming a dissipative coupling constant \( \zeta \) of order unity. The trend is, as expected from (5.1), that a smaller number \( N_{\text{eff},1} \) requires a larger energy scale \( E_{\text{ew}} \). Table IV presents the required energy scales \( E_{\text{ew}} \) for selected values of \( N_{\text{eff},1} \), with the understanding that the quoted numbers for \( E_{\text{ew}} \) are only indicative because of the many assumptions made along the way (some of which may be more reasonable than others). See App. A for further discussion of some of the systematic uncertainties involved (its very last table complements Table IV of this section).

C. Outlook

If the observed “cosmological constant” results from dynamics at cosmic temperatures of order \( M \sim E_{\text{ew}} \), then some set of differential equations must be relevant. In the framework of \( q \)-theory, a particular set of differential equations has been proposed. It appears to be impossible to have very much simpler differential equations which achieve the same result and the phenomenological equations used here can be expected to carry some of the essential ingredients. If so, the estimates from Table IV suggest the need for new physics with particle masses at the TeV–scale.

Particle-collider experiments are called upon to confirm or exclude the existence of these TeV–scale particles and, if confirmed, to determine their characteristics. Knowing the characteristics of the new TeV-scale particles (assuming their detection), the main task for theorists would be to derive the relevant particle-production effects contained in the simple phenomenological equations considered here or to find the appropriate generalizations of these equations.

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Appendix A: Dynamic kick

1. Introduction

The goal of this appendix is to present a calculation for the generation of the remnant vacuum energy density by a more or less realistic kick from dynamically generated ultramassive and unstable type–1 particles. The description of this dynamic kick is rather involved, but the final ODEs are similar to those of the main text for a prescribed kick, with the crucial role again being played by the phenomenological dissipation function $\gamma$ (the other functions entering the ODEs will now be given different notations, for clarity). The heuristics of these ODEs will be discussed in the last paragraph of Sec. A 4.

2. Mass spectrum and EOS functions

The massless type–2 particles from the model introduced in Sec. III A are considered to correspond to those of the standard model (mass scale $m_{\text{SM}} \ll E_{\text{ew}}$) and the round number $N_2 \equiv N_{\text{eff},2} = 10^2$ will be used. The ultramassive type–1 particles are considered to arise from broken supersymmetry [21, 22], so that $N_1 \equiv N_{\text{eff},1} = N_{\text{eff},2} = 10^2$, and they can be taken to be bosons (the standard-model particles being mostly fermions [20]). Just as discussed in Sec. VB, the type–1 and type–2 particles are assumed to have been in thermal equilibrium before the generation of the vacuum energy density starts.

A general type–1 mass spectrum is given by the effective numbers $n_i \geq 0$ of particles with dimensionless masses $m_i \equiv M_i/E_{\text{ew}}$, for which the following two constraints hold:

$$\sum_{i=a,b,c,...} n_i = N_1,$$  \hspace{1cm} (A1a)

$$\frac{1}{N_1} \sum_{i=a,b,c,...} n_i m_i = 1,$$  \hspace{1cm} (A1b)

where the last constraint ensures that the average dimensionful mass equals $E_{\text{ew}}$.

For simplicity, consider two cases: case A with two different mass values and case B with a single mass value, by definition equal to $E_{\text{ew}}$. (The generalization to a general type–1 mass spectrum will be obvious.) The specific numbers are chosen as follows:

\begin{align*}
\text{case A:} \quad & (n_a, m_a ; n_b, m_b) = (40, 2 ; 60, 1/3), \quad (A2a) \\
\text{case B:} \quad & (n_a, m_a ; n_b, m_b) = \quad (A2b)
\end{align*}

The case–B partition $\{n_a, n_b\}$ of 100 is arbitrary, at least, for the dynamic ODEs considered here. It will be seen, later on, that case A and case B give more or less the same remnant vacuum energy density, which even holds for the extreme version of case–A having 50 particles with $m_a = 2$ and 50 with $m_b = 0$.

Next, take, instead of the prescribed EOS function (3.13) used in the main text, the exact EOS function or, at least, a controlled approximation of it. In fact, the following rational
function of the variable $\theta_i \equiv T/M_i$ will be employed for type–1 subspecies $i \in \{a, b, c, \ldots \}$:

\[
\kappa_{M1i}(\theta_i) = 1 - 3 \overline{w}_{M1i}(\theta_i), \tag{A3a}
\]
\[
\overline{w}_{M1i}(\theta_i) = \frac{\theta_i^2 + \overline{\pi} \theta_i}{3 \theta_i^2 + \overline{\beta} \theta_i + \overline{\alpha}}, \tag{A3b}
\]
\[
(\overline{\pi}, \overline{\beta}) = (0.625, 1.91). \tag{A3c}
\]

Expression (A3b) with constants (A3c) gives an accurate approximation (better than 1 per mill) to the exact equilibrium result for the EOS parameter $w_{M1i} \equiv P_{M1i}/\rho_{M1i}$ of bosons at zero chemical potential (see, e.g., Sec. 3.3 of Ref. [19]). For the EOS parameter $w_{M1i}$ of fermions, the constants would be $(\alpha, \beta) = (0.770, 2.15)$, but the approximation would be somewhat less accurate (still better than 5 per mill).

As the type–2 particles are massless and adiabatically expanding in the early phase (for $\tau \leq \tau_{\text{min}} \ll 1$), their initial energy density can be used to normalize the temperature-over-mass variable $\theta_i$ used in (A3a),

\[
\theta_i = \frac{1}{m_i E_{\text{ew}}} \left( \frac{30}{\pi N_{\text{eff},2}} \rho_{M2}(t_{\text{min}}) \right)^{1/4} a(t_{\text{min}}) = \frac{1}{m_i} \left( \frac{30}{\pi N_{\text{eff},2}} \tau_{M2}(\tau_{\text{min}}) \right)^{1/4} a(\tau_{\text{min}})/a(\tau), \tag{A3d}
\]

where $m_i E_{\text{ew}}$ is the physical mass $M_i$ of the type–1 subspecies considered. As the scale factor $a(\tau)$ evolves with dimensionless cosmic time $\tau$, so do $\theta_i$ and $\kappa_{M1i}$ in the above equations. Recall that the scale factor $a(\tau)$ is defined by $h = \dot{a}/a$ and that the temperature of a noninteracting gas of “photons” drops as $T(\tau) \propto 1/a(\tau)$ in the expanding FRW universe [19].

### 3. Model functions

Because the EOS function $\kappa_{M1i}$ does not vanish for $\tau > \tau_{\text{freeze}}$, the appropriately modified Maxwell equation needs to be used, which has already been discussed in Ftn. 2. But $\kappa_{M1i}$ also does not vanish exactly in the early phase (as long as $a > 0$ or $T < \infty$). In order to have a standard radiation-dominated FRW universe in the early phase before $t_{\text{ew}}$, the initial density of type–1 particles must be taken to be strictly zero. The further assumption is, then, that type–1 particles are generated dynamically just before $t_{\text{ew}}$ and thermalized rapidly, so that the type–1 energy density relative to that of the type–2 particles is approximately given by $n_i/N_2$ before $\kappa_{M1i}$ starts to differ significantly from 0.

For the ODEs to be given shortly, the rapid energy transfer from type–2 particles to type–1 particles is governed by a coupling constant $\lambda_{21} \geq 0$ and a burst function $\hat{\omega}(\tau)$, which is taken to peak around $\tau = \tau_{21} \ll 1$. For simplicity, define this burst function $\hat{\omega}$ using the previous function $\gamma$ from (3.14b) for a fixed value of $\tau_{\text{freeze}}$:

\[
\hat{\omega}(\tau) = \hat{\gamma}_{21}(\tau) \left[ 1 - \hat{\gamma}_{21}(\tau) \right], \tag{A4a}
\]

\[
\hat{\gamma}_{21}(\tau) \equiv \hat{\gamma}(\tau/\tau_{21}), \tag{A4b}
\]

\[
\hat{\gamma}(\tau) \equiv \gamma(\tau) \bigg|_{\tau_{\text{freeze}} = 3}. \tag{A4c}
\]
The subsequent decay of type–1 particles into type–2 particles is governed by a coupling constant $\lambda_{12} \geq 0$ and a decay function $\hat{\nu}(\tau)$. For simplicity, this function is taken to have the following form:

$$\hat{\nu}(\tau) = 1 - \hat{\gamma}(\tau).$$  

(A4d)

The hats on the above functions indicate that they are independent of the value of the dissipative coupling constant $\zeta$, whereas $\gamma$ from (3.14) does depend on it. The model functions used will also be shown in the plots of the numerical results later on.

4. ODEs and boundary conditions

With a type–1 mass spectrum of the form (A2) and the dynamic EOS function $\pi_{M_{1i}}$ from (A3), the dimensionless ODEs are now taken to be the following:

$$3(\dot{h} + 2h^2) = \gamma x + (1 - \gamma) \left( \xi^{-1} x^2 + \frac{1}{2} \overline{\pi}_{M_{1a}} r_{M_{1a}} + \frac{1}{2} \overline{\pi}_{M_{1b}} r_{M_{1b}} \right), \quad \text{(A5a)}$$

$$\dot{r}_{M_{1a}} + (4 - \overline{\pi}_{M_{1a}}) h r_{M_{1a}} = (N_{1a}/N_1) \left( \lambda_{21} \hat{\omega} r_{M_{2}} - (\zeta/\gamma) [\dot{x}] \right) - \lambda_{12} \hat{\nu} r_{M_{1a}}, \quad \text{(A5b)}$$

$$\dot{r}_{M_{1b}} + (4 - \overline{\pi}_{M_{1b}}) h r_{M_{1b}} = (N_{1b}/N_1) \left( \lambda_{21} \hat{\omega} r_{M_{2}} - (\zeta/\gamma) [\dot{x}] \right) - \lambda_{12} \hat{\nu} r_{M_{1b}}, \quad \text{(A5c)}$$

$$\dot{r}_{M_{2}} + 4 h r_{M_{2}} = -\lambda_{21} \hat{\omega} r_{M_{2}} + \lambda_{12} \hat{\nu} (r_{M_{1a}} + r_{M_{1b}}), \quad \text{(A5d)}$$

$$3 (\xi^{-1} \dot{x} + h^2) = \xi^{-1} x^2/2 + r_{M_{1a}} + r_{M_{1b}} + r_{M_{2}}, \quad \text{(A5e)}$$

where the dissipation function $\gamma$ is given by (3.14) and the other model functions by (A4). The basic structure of these ODEs is identical to that of (3.15) with a prescribed EOS function (3.13).

As the physical hierarchy parameter $\xi$ can be expected to be very large (perhaps of order $10^{60}$), the $\xi^{-1} = 0$ ODEs are especially relevant:

$$3(\dot{h} + 2h^2) = \gamma x + (1 - \gamma) \left( \frac{1}{2} \overline{\pi}_{M_{1a}} [3h^2 - r_{M_{2}} - r_{M_{1b}}] + \frac{1}{2} \overline{\pi}_{M_{1b}} r_{M_{1b}} \right), \quad \text{(A6a)}$$

$$\dot{x} = -\xi^{-1} h \gamma^2 \left( 2 x - \overline{\pi}_{M_{1a}} [3h^2 - r_{M_{2}} - r_{M_{1b}}] - \overline{\pi}_{M_{1b}} r_{M_{1b}} \right), \quad \text{(A6b)}$$

$$\dot{r}_{M_{1b}} + (4 - \overline{\pi}_{M_{1b}}) h r_{M_{1b}} = (N_{1b}/N_1) \left( \lambda_{21} \hat{\omega} r_{M_{2}} - (\zeta/\gamma) [\dot{x}] \right) - \lambda_{12} \hat{\nu} r_{M_{1b}}, \quad \text{(A6c)}$$

$$\dot{r}_{M_{2}} + 4 h r_{M_{2}} = -\lambda_{21} \hat{\omega} r_{M_{2}} + \lambda_{12} \hat{\nu} [3h^2 - r_{M_{2}}], \quad \text{(A6d)}$$

with $r_{M_{1a}}$ following from the solution of these ODEs by the corresponding Friedmann equation,

$$r_{M_{1a}} = 3h^2 - r_{M_{2}} - r_{M_{1b}}. \quad \text{(A7)}$$
For both sets of ODEs, (A5) and (A6), the following boundary conditions on \( h, r_{M1i}, r_{M2}, \) and \( x \) are taken to hold at a time \( \tau = \tau_{\text{min}} \ll 1: \)

\[
\begin{align*}
    h(\tau_{\text{min}}) &= 1/2 \left(\tau_{\text{min}}\right)^{-1}, \\
r_{M1i}(\tau_{\text{min}}) &= 0, \\
r_{M2}(\tau_{\text{min}}) &= 3 \left[h(\tau_{\text{min}})\right]^2, \\
x(\tau_{\text{min}}) &= 0,
\end{align*}
\]  

(A8a) (A8b) (A8c) (A8d)

which correspond to a standard radiation-dominated FRW universe. It must be emphasized that there is essentially no free parameter in these boundary conditions, the precise value of \( \tau_{\text{min}} \) being irrelevant as long as it is sufficiently small compared to 1 (so that \( t_{\text{min}} \ll t_{\text{ew}} \)).

Before turning to the numerical solutions, it may be helpful to discuss very briefly the heuristics of the ODEs (A5), the discussion of Eqs. (A6) and (A7) being similar. Starting from the values (A8), the \( \lambda_{21} \hat{\nu} \) terms in (A5b) and (A5c) generate a nonzero value of the type–1 matter energy density \( r_{M1a} + r_{M1b} \), which then gives a nonzero value of the vacuum variable \( x \) from (A5e). The values of \( r_{M1i}(\tau) \) and \( x(\tau) \) peak around \( \tau = 1 \), which provides a dynamic version of the electroweak kick, improving upon the prescribed (artificial) kick discussed in Ref. [6] and the main text of the present article. Finally, the value of \( x \) approaches a constant positive value and so does the vacuum energy density \( r_V = \frac{1}{2} x^2 \), whereas \( r_{M1a} + r_{M1b} \) drops to zero because of the decay terms \( \lambda_{12} \hat{\nu} \) in (A5f) and (A5g).

5. Numerical solution

Numerical results for the case–A mass spectrum (A2a) and hierarchy parameter \( \xi = 10^2 \) are given in Fig. 8 for the standard (nondissipative) \( q \)--theory ODEs with \( \zeta = 0 \) and in Figs. 9 and 10 for the modified (dissipative) \( q \)--theory ODEs with \( \zeta = 2 \). [The coupling constant value \( \lambda_{21} = 18 \) has been chosen to give \( r_{M1a} + r_{M1b} \sim r_{M2} \) at \( \tau = 0.25 \), as would approximately correspond to thermal equilibrium. The value of \( \lambda_{12} = 2 \) has been chosen so that most type–1 particles have decayed by a cosmic time \( \tau = 3 \). But the results for \( r_V^{\text{num}} \) are more or less constant for \( \lambda_{12} \lesssim 20 \), which includes the unrealistic case \( \lambda_{12} = 0 \) of stable (massive) type–1 particles.] Figures 9 and 10 are seen to give a somewhat lower value of the dimensionless remnant vacuum energy density \( r_V^{\text{num}} = r_V(\tau_{\text{freeze}}) \) than Figs. 2 and 3, one reason being that the maximum value of \( \overline{r}_{M1} r_{M1} \) is less for the dynamic kick considered than for the prescribed kick (cf. Ftn. 6).

Numerical results for the case–A mass spectrum (A2a) and the physically relevant hierarchy parameter \( \xi = \infty \) are given in Fig. 11 with \( r_V^{\text{num}} \) of order \( 1.34 \times 10^{-2} \) for \( \zeta = 2 \).

\[ \sum_i \overline{r}_{M1i} r_{M1i} \]

\[ \sum_i \overline{r}_{M1i} r_{M1i} \]

---

6 It is clear from (A6b), in particular, that the kick of the vacuum energy density is driven by the combination \( \sum_i \overline{r}_{M1i} r_{M1i} \).
Similar results (Fig. 12) are obtained for the case–B mass spectrum (A2b), with $r_{V}^{\text{num}}$ of order $1.10 \times 10^{-2}$. As mentioned in Sec. II Figs. 11 and 12 present the best (i.e., most realistic) calculations of this article.

According to (5.2), an average value $r_{V}^{\text{num}} \sim 1.2 \times 10^{-2}$ has a required $E_{\text{ew}}$ value of order $3.8$ TeV, which is $20\%$ above the value $3.2$ TeV from Table IV. The mass values corresponding to $E_{\text{ew}} \sim 3.8$ TeV are $M_{1a} \sim 7.7$ TeV and $M_{1b} \sim 1.3$ TeV for the case–A spectrum and $M_{1a} \sim 3.8$ TeV for the case–B spectrum. With the dynamic kick as defined in this appendix, the obtained value $E_{\text{ew}} \sim 3.8$ TeV depends primarily on the assumption that the dissipation function $\gamma$ enters the ODEs (A6) in the way shown and that the relevant coupling constant $\zeta$ is of order unity.

Consider, then, the $\zeta$ dependence of the predicted $E_{\text{ew}}$ value. As explained in the fourth paragraph of Sec. III E the value of the remnant vacuum energy density $r_{V}^{\text{num}}(\zeta)$ can be expected to drop to zero for $\zeta \downarrow 0$ and $\zeta \to \infty$. From the numerical solution of ODEs (A6) for the case–A mass spectrum (A2a) and the same boundary conditions as in Fig. 11 the following rough fit of $r_{V}^{\text{num}}(\zeta)$ for $\zeta \geq 0.02$ is found: $r_{V}^{\text{fit}}(\zeta) = \zeta^{0.5}/(\tilde{\alpha}\zeta^{7} + \tilde{\beta})$ with constants $(\tilde{\alpha}, \tilde{\beta}) = (13.29, 680.3)$. More precisely, the $r_{V}^{\text{num}}(\zeta)$ value peaks at $1.34 \times 10^{-2}$ for $\zeta = 2$ and drops to $2.46 \times 10^{-7}$ for $\zeta = 0.2$ and to $5.82 \times 10^{-4}$ for $\zeta = 20$. [Note that, for $\zeta \ll 0.2$, the time scale $\tau_{\text{freeze}}$ from (3.14a) may become unreasonably large and that, for $\zeta \gg 20$, the particle-production effects on the right-hand side of (A6c), for given value of $\dot{x} \propto \dot{\rho}_{V}$, may be larger than can be expected from weakly-coupled particles.] The corresponding results for $E_{\text{ew}}$ are given in Table VI which, together with Table IV, gives an idea of the systematic uncertainties involved (compare the $E_{\text{ew}}$ values for $N_{\text{eff},1} = 10^{2}$ and $\zeta = 2$ from both tables).

For the dynamic kick considered in this Appendix, the estimate is thus

$$E_{\text{ew}} |_{N_{\text{eff},1} = N_{\text{eff},2} = 10^{2}, \text{ case–A}} \sim 4 - 15 \text{ TeV},$$

(A9)

assuming that the dissipation function $\gamma$ enters the ODEs (A6) in the way shown and that the coupling constant $\zeta$ lies between $0.2$ and $20$. The corresponding case–A mass values are $M_{1a} = 2E_{\text{ew}}$ and $M_{1b} = 1/3E_{\text{ew}}$. The case–B mass spectrum (A2b), with a unique mass value $M_{1a} = E_{\text{ew}}$, gives the same $E_{\text{ew}}$ estimate as in (A9).

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TABLE I: Function values of $h(\tau)$, $r_{M1}(\tau)$, $r_{M2}(\tau)$, and $r_V(\tau)$ at $\tau = \tau_{\text{freeze}} = 3$ from the numerical solution of the ODEs (3.15) for dissipative coupling constant $\zeta = 2$, hierarchy parameter $\xi = 10^2$, and various values of the boundary condition $r_{M1}/r_{M2}$ at $\tau = \tau_{\text{min}} = 0.1$. The other parameters and boundary conditions are given in the caption of Fig. 1. The numerical accuracy of the quoted function values is estimated to be equal to $\pm 1$ in the last digit.

| $\xi$ | $[r_{M1}/r_{M2}](\tau_{\text{min}})$ | $h$    | $r_{M1}$ | $r_{M2}$ | $r_V$  |
|-------|----------------------------------|--------|----------|----------|--------|
| $10^2$ | $10^{-1}$                        | 0.1702 | 0.01329  | 0.07357  | 0.001473 |
| $10^2$ | 1                               | 0.1849 | 0.06321  | 0.03893  | 0.04407 |
| $10^2$ | $10^1$                          | 0.1983 | 0.1007   | 0.01577  | 0.1441  |
| $10^2$ | $10^2$                          | 0.2008 | 0.1070   | 0.01220  | 0.1705  |

TABLE II: Same as Table I but now for a fixed boundary condition $[r_{M1}/r_{M2}](\tau_{\text{min}}) = 1$ and various hierarchy parameters $\xi$ ranging from 1 to $10^6$. The entry for $\xi = \infty$ has been calculated from the ODEs (3.16) and the algebraic equation (3.17).

| $\xi$ | $[r_{M1}/r_{M2}](\tau_{\text{min}})$ | $h$    | $r_{M1}$ | $r_{M2}$ | $r_V$  |
|-------|----------------------------------|--------|----------|----------|--------|
| 1     | 1                               | 0.1758 | 0.04279  | 0.04394  | 0.006026 |
| $10^3$| 1                               | 0.1817 | 0.05582  | 0.04069  | 0.02584 |
| $10^2$| 1                               | 0.1849 | 0.06321  | 0.03893  | 0.04407 |
| $10^4$| 1                               | 0.1859 | 0.06530  | 0.03841  | 0.05052 |
| $10^6$| 1                               | 0.1859 | 0.06533  | 0.03841  | 0.05061 |
| $\infty$ | 1                              | 0.1860 | 0.06533  | 0.03841  | 0.05061 |

TABLE III: Same as Table I but now from the $\xi^{-1} = 0$ ODEs (3.16) and Eq. (3.17).

| $\xi$ | $[r_{M1}/r_{M2}](\tau_{\text{min}})$ | $h$    | $r_{M1}$ | $r_{M2}$ | $r_V$  |
|-------|----------------------------------|--------|----------|----------|--------|
| $\infty$ | $10^{-3}$                        | 0.1667 | 0.0001577| 0.08322  | $2.058 \times 10^{-7}$ |
| $\infty$ | $10^{-2}$                        | 0.1671 | 0.001557 | 0.08219  | $2.021 \times 10^{-5}$ |
| $\infty$ | $10^{-1}$                        | 0.1704 | 0.01384  | 0.07330  | 0.001699 |
| $\infty$ | 1                               | 0.1860 | 0.06533  | 0.03841  | 0.05061 |
| $\infty$ | $10^1$                          | 0.1995 | 0.1036   | 0.01583  | 0.1647  |
| $\infty$ | $10^2$                          | 0.2020 | 0.1100   | 0.01243  | 0.1948  |
| $\infty$ | $10^3$                          | 0.2023 | 0.1107   | 0.01207  | 0.1983  |
TABLE IV: Estimates for the energy scale $E_{\text{ew}}$ of type–1 particles with mass scale $M \sim E_{\text{ew}}$ as a function of their effective number of degrees of freedom $N_{\text{eff,1}}$. The inverse of the hierarchy parameter (3.11) is taken to vanish, $\xi^{-1} = 0$, and the dissipative coupling constant is assumed to have a value of order unity, specifically $\zeta = 2$. The kick from the type–1 particles is modeled by the prescribed EOS function $\kappa_{M1}(\tau)$ from (3.13) with parameters listed in the caption of Fig. 1. Both ultramassive type–1 and massless type–2 particles are assumed to have been in thermal equilibrium for $T \gg E_{\text{ew}}$ and the effective number of type–2 particles is taken as $N_{\text{eff,2}} = 10^2$. The $E_{\text{ew}}$ estimates are obtained from (5.2), using the $r_V$ values from Table III for $[r_{M1}/r_{M2}](\tau_{\text{min}}) = N_{\text{eff,1}}/10^2$ and taking the measured value of the cosmological constant to be $\Lambda_{\text{exp}}^\text{exp} = (2 \text{ meV})^4$.

| $\zeta$ | $N_{\text{eff,1}}$ | $E_{\text{ew}}$^{prescribed kick} [TeV] |
|--------|-------------------|-----------------------------------|
| 2      | 1                 | 8.5                               |
| 2      | $10^1$            | 4.9                               |
| 2      | $10^2$            | 3.2                               |
| 2      | $10^3$            | 2.8                               |
| 2      | $10^4$            | 2.7                               |

TABLE V: Same as Table IV but now for the dynamic EOS function $\kappa_{M1}(\tau)$ of Appendix A. The total effective numbers of degrees of freedom are $N_{\text{eff,1}} = N_{\text{eff,2}} = 10^2$ and the type–1 mass spectrum is given by (A2a). The estimates for the energy scale $E_{\text{ew}}$ are given for $\xi^{-1} = 0$ and three values of the dissipative coupling constant $\zeta$, the other coupling constants being listed in the caption of Fig. 1.

| $\zeta$ | $N_{\text{eff,1}}$ | $E_{\text{ew}}$^{dynamic kick} [TeV] |
|--------|-------------------|----------------------------------|
| 0.2    | $10^2$            | 14.8                             |
| 2      | $10^2$            | 3.8                              |
| 20     | $10^2$            | 5.6                              |
FIG. 1: Numerical solution of the ODEs (3.15) with a prescribed EOS function $\kappa_{M1}$, for vanishing dissipative coupling constant $\zeta = 0$ and trivial function $\gamma(\tau) = 1$. Model parameters are $\{\xi, \lambda_{12}\} = \{100, 8/100\}$ and the model function $\kappa_{M1}(\tau)$ is defined by (3.13) with parameters $\{\kappa_c, \tau_c, \Delta \tau\} = \{1, 1, 18/10\}$. The ODEs are solved over the interval $[\tau_{\text{min}}, \tau_{\text{max}}] = [0.1, 3]$ with the following boundary conditions from (4.1) at $\tau = \tau_{\text{min}} = 0.1$: $\{x, h, a, r_{M1}, r_{M2}\} = \{0, 5, 0.3, 37.5, 37.5\}$.

FIG. 2: Same as Fig. 1 but now for nonvanishing dissipative coupling constant $\zeta = 2$ and nontrivial function $\gamma(\tau)$ defined by (3.14) with parameter $\tau_\infty = 2$, giving $\tau_{\text{freeze}} = 3$.

FIG. 3: Same as Fig. 2 but also evolved for $\tau > \tau_{\text{freeze}} = 3$ with the standard FRW equations [given by (3.15) for $\dot{x} = 0$, $\kappa_{M1} = 0$, and $\gamma = 0$].
FIG. 4: Same as Fig. 1 still with $\zeta = 0$ but now for a larger hierarchy parameter $\xi = 10^4$.

FIG. 5: Same as Fig. 2 still with $\zeta = 2$ but now for a larger hierarchy parameter $\xi = 10^4$.

FIG. 6: Same as Fig. 3 still with $\zeta = 2$ but now for a larger hierarchy parameter $\xi = 10^4$. 
FIG. 7: Remnant vacuum energy density $r_V(\tau_{\text{freeze}})$ from Table II plotted against the compact parameter $\eta \equiv \log_{10} \xi / (|\log_{10} \xi| + 3) \in [0, 1)$ defined in terms of the hierarchy parameter $\xi \in [1, \infty)$ of the theory. The realistic value $\xi = 10^{60}$ from (3.11) corresponds to $\eta = 20/21 \approx 0.9524$. 
FIG. 8: Numerical solution of the ODEs (A5) with the dynamic EOS function $\bar{\kappa}_{M1}$ from (A3), for vanishing dissipative coupling constant $\zeta = 0$ and trivial function $\gamma(\tau) = 1$. Model parameters are \(\{\xi, \lambda_{21}, \lambda_{12}\} = \{100, 18, 2\}\) and model functions are defined by (A4) with parameter $\tau_{21} = 1/10$ [the function $\hat{\omega}(\tau)$ peaks below $\tau = 0.3$ and $\hat{\nu}(\tau)$ rises from 0 to 1]. The case–A type–1 mass spectrum (A2a) has been assumed, with type–1a functions shown by full curves and type–1b functions by dashed curves. The ODEs are solved over the interval $[\tau_{\text{min}}, \tau_{\text{max}}] = [0.01, 3]$ with the following boundary conditions from (A8) at $\tau = \tau_{\text{min}} = 0.01$: \(\{x, h, a, r_{M1a}, r_{M1b}, r_{M2}\} = \{0, 50, 0.1, 0, 0, 7500\}\).

FIG. 9: Same as Fig. 8 but now for nonvanishing dissipative coupling constant $\zeta = 2$ and nontrivial function $\gamma(\tau)$ defined by (3.14) with parameter $\tau_{\infty} = 2$, giving $\tau_{\text{freeze}} = 3$.

FIG. 10: Same as Fig. 9 but also evolved for $\tau > \tau_{\text{freeze}} = 3$. 
FIG. 11: Same as Fig. 9 still for the case–A mass spectrum (A2a) but now solving the $\xi^{-1} = 0$ ODEs (A6) and using the algebraic Eq. (A7).

FIG. 12: Same as Fig. 11 still with $\xi^{-1} = 0$ but now for the case–B mass spectrum (A2b).