Tunable pinning of a superconducting vortex a by a magnetic vortex

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The interaction between a straight vortex line in a superconducting film and a soft magnetic
nanodisk in the magnetic vortex state in the presence of a magnetic field applied parallel to the
film surfaces is studied theoretically. The superconductor is described by London theory and the
nanodisk by the Landau-Lifshitz continuum theory of magnetism, using the approximation known as
the rigid vortex model. Pinning of the vortex line by the nanodisk is found to result, predominantly,
from the interaction between the vortex line and the changes in the nanodisk magnetization induced
by the magnetic field of the vortex line and applied field. In the context of the rigid vortex model,
these changes result from the displacement of the magnetic vortex. This displacement is calculated
analytically by minimizing the energy, and the pinning potential is obtained. The applied field can
tune the pinning potential by controlling the displacement of the magnetic vortex. The nanodisk
magnetization curve is predicted to change in the presence of the vortex line.

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I. INTRODUCTION

Pinning of vortices in superconducting films by arrays of soft nanomagnets placed in the vicinity of the film is a
subject that has not been much studied in the literature. Most experimental [1] and theoretical [2, 3, 4, 5, 6, 7]
work carried out so far deals with permanent nanomagnets. For both soft and permanent nanomagnets, pin-
ning results from the action of the inhomogeneous magnetic field created by their magnetization in the super-
conductor. However, in the case of soft nanomagnets, the magnetization depends on the magnetic fields act-
ing on it, including the field caused by the vortex itself. The question is then how this modifies the pinning po-
tential. Recently, this question was investigated for vortices interacting with arrays of soft magnetic nanodisks
using a very simple model in which the nanodisks are approximated by point magnetic dipoles with magnetic
moments free to rotate [8]. This model was suggested by the results of Cowburn et. al. [9], which show that in
small nanodisks the magnetic state is a single domain one, with the magnetization of each nanodisk behaving
like a giant magnetic moment free to rotate. The calculations of Ref [9] show that the pinning potential differs
considerably from that for a permanent dipole, and that it can be tuned by a magnetic field applied parallel to the
film surfaces. Similar properties are expected for the pinning potential due to soft nonomagnets in general. This
paper studies the pinning of vortices by nanodisks in another magnetic state: the magnetic vortex. This state
is found in larger nanodisks [10, 11], and is characterized by the magnetization vector circulating around a
nucleus of small dimensions compared with the nanodisk radius. The motivation to consider this particular state
is that it contains the basic physics of nanomagnets in general, but its theoretical description can be approxi-
imated by a simple analytic model. Here, the interaction between one straight vortex line and one nanodisk is cal-
culated using the Landau-Lifshitz continuum theory of magnetism [12] for the nanodisk and London theory for
the superconductor [13]. The magnetic vortex state is described by the rigid vortex model introduced by Usov and
Peschany [14], and by Guslienko and collaborators [15]. These authors obtained the nanodisk equilibrium mag-
netization using the variational principle to minimize the energy. Usov and Peschany [14] proposed an analytic ex-
pression for the trial magnetization at zero applied field, with the magnetic vortex core at the nanodisk center, and
the core radius as the variational parameter. Guslienko and collaborators [15] extended it to finite fields paral-
lel to the nanodisk faces. They assumed that the only effect of the applied field is to displace the magnetic vor-
tex rigidly, with the core moving away from the nanodisk center. The new trial magnetization is the zero field one
extrapolated using the variational principle to minimize the energy. Peschany and collaborators [14] extended it to
finite fields parallel to the nanodisk faces. They assumed that the only effect of the applied field is to displace the magnetic vortex rigidly, with the core moving away from the nanodisk center. The new trial magnetization is the zero field one displaced rigidly with the core, and the core displacement vector is a variational parameter. The predictions of the rigid vortex model are in reasonable agreement with numerical calculations for small displacements compared to the nanodisk radius [9, 11, 16].

This paper assumes the validity of the rigid vortex model for the nanodisk placed in the vicinity of the su-
perconducting film. The total energy of the nanodisk-superconductor system is obtained exactly as a function
of the magnetic vortex displacement vector. The calculations carried out in the paper assume small displacements
compared to the nanodisk radius, and approximate the total energy by its expansion to second order in the
displacement. The equilibrium displacement of the magnetic vortex is obtained by minimizing the approximate
total energy. It is found that, contrary to what happens for the isolated nanodisk, the equilibrium displacement
is a non-linear function of the vortex line and applied
fields. The pinning potential for the vortex line is also calculated from the approximate total energy. The most important contribution to the pinning potential is found to come from the changes in the magnetization caused by the displacement of the magnetic vortex. There is also a contribution from the interaction between the vortex line and the magnetization in the magnetic vortex core, but it is negligible, due to the small dimensions of the core. It is found that the pinning potential can be tuned by the applied field. The mechanism is that the field controls the magnetic vortex displacement which, in turn, modifies the pinning potential. Applications to systems of experimental interest are considered, and the conclusions of the paper are stated. The Appendix gives the mathematical treatment of the model parameters. The results suggest that the pinning potential for these systems can be estimated by applying the linear response theory of elementary magnetism to the nonoidik. The magnitude of the applied field necessary to annihilate the magnetic nanodisk is briefly reviewed. Then, in Sec. II B, the nanodisk-superconductor system is developed in Sec. II. The superconductor-nanodisk system is shown schematically in Fig. 1. The superconducting film is planar, of thickness $d$, isotropic and has penetration depth $\lambda$. Its surfaces are parallel to the $x-y$ plane. One straight vortex line, with vorticity $q = \pm 1$, is present in the film at a position defined by $\rho_c$ with respect to an origin in the film top surface. The vortex line core radius is $\xi$. A thin magnetic nanodisk, of radius $R$ and thickness $L_z \ll R$, is located above the film, with the faces parallel to the film surfaces, and center located above the film at $(x = 0, y = 0, z = z_0 > 0)$. A magnetic field, $\mathbf{H}$, is applied along the $x$-axis, that is parallel to the nanodisk faces and to the film surfaces.

II. PINNING POTENTIAL

The superconductor-nanodisk system is shown schematically in Fig. 1. The superconducting film is planar, of thickness $d$, isotropic and has penetration depth $\lambda$. Its surfaces are parallel to the $x-y$ plane. One straight vortex line, with vorticity $q = \pm 1$, is present in the film at a position defined by $\rho_c$ with respect to an origin in the film top surface. The vortex line core radius is $\xi$. A thin magnetic nanodisk, of radius $R$ and thickness $L_z \ll R$, is located above the film, with the faces parallel to the film surfaces, and center located above the film at $(x = 0, y = 0, z = z_0 > 0)$. A magnetic field, $\mathbf{H}$, is applied along the $x$-axis, that is parallel to the nanodisk faces and to the film surfaces.

A. isolated nanodisk

First, the rigid vortex model for the isolated nanodisk is briefly reviewed. The magnetization is written as $\mathbf{M}(r) = M_s\hat{s}(r)$, $M_s$ being the saturation magnetization. The nanodisk energy is given by

$$E_D = \frac{M_s^2 R_0^2}{2} \int d^3r \sum_{\alpha=x,y,z} |\nabla s_\alpha(r)|^2 + \frac{1}{2} \int d^2r \int d^2r' \frac{\sigma(r)\sigma(r')}{|r-r'|} - \mathbf{m} \cdot \mathbf{H}.$$  (1)

In Eq. (1), the first term is the exchange energy, where $R_0$ is the exchange length. The second is the magneto-static energy, with $\sigma(r) = M_s\hat{s}(r) \cdot \hat{n}$ being the density of magnetic charges at the surface of the nanodisk, and $\hat{n}$ is the unit vector in the direction normal to the nanodisk surface. The last term is the energy of interaction with the external field, with $\mathbf{m} = \int d^3r \mathbf{M}(r)$. It is assumed that the nanodisk is isotropic, and that there are no volume magnetic charges, that is $\nabla \cdot \mathbf{M} = 0$. The rigid vortex model is a variational minimization of Eq. (1). The trial magnetization is written in the cylindrical coordinate system with origin at the nanodisk center and $z$-axis perpendicular to the nanodisk faces shown in Fig. 2. For $H = 0$ Usov and Peschany [14] proposed the following expression

$$\hat{s}(\rho) = \begin{cases} 0 & (a < \rho \leq R) \\ \hat{n} - \frac{2a\rho}{a^2 + \rho^2}\hat{\theta} \pm \frac{\rho^2}{a^2 + \rho^2}\hat{z} & (0 < \rho \leq a) \end{cases},$$  (2)
where \( \hat{\theta} = \hat{z} \times \hat{\rho} \). This equation describes the magnetization vector curling around the nanodisk center (Fig. 2a), with counterclockwise (clockwise) rotation for \( q_m = +1 \) (−1). The variational parameter \( \alpha \) is the radius of the magnetic vortex core. The magnetization outside the core is parallel to the nanodisk faces. Inside it has a \( z \)-component, which can be either positive or negative. The equilibrium value of \( \alpha \), obtained by Usov and Peschany, is \( \alpha = 0.7(R_0^3L_z)^{1/3} \). What is important for the purpose of this paper is not the precise value of \( \alpha \), but the fact that it is small compared to \( R \). This follows from the assumption that \( R_0, L_z \ll R \). For \( H \neq 0 \) the trial magnetization proposed by Guslienko and collaborators is that in Eq. (2) displaced rigidly by \( \delta \) from the nanodisk center, that is \( \hat{\delta}(|\rho - \delta|) \) (Fig. 2b). The equilibrium value of \( \delta \) is determined by minimizing the energy. The vortex core radius is unchanged as long as the displaced core remains within the nanodisk, that is for \( \delta < R - \alpha \). The simplest treatment of the rigid vortex model assumes that the magnetic vortex displacement is small, and obtains \( E_D \) to second order in \( \delta/R \). The result is

\[
E_D(\delta) = E_D(0) + M_s^2V_D\left[ \frac{\delta_x^2 + \delta_y^2}{2\chi R^2} - \frac{q_m H\Delta_{s}}{M_s R} \right],
\]

where \( V_D = \pi R^2 L_z \) is the nanodisk volume, and \( \delta_x \) and \( \delta_y \) are, respectively, the components of \( \delta \) parallel and perpendicular to \( H \). In Eq. (3) the term quadratic in \( \delta \) comes from both the exchange and magnetostatic energies, whereas the term linear in \( \delta \) comes from the interaction with the applied field. The constant \( \chi \) is the nanodisk linear magnetic susceptibility, and is given by \[ \chi = \frac{1}{M_s^2} \left( \frac{R_0^3}{R^2} + 2\frac{L_z}{R} \ln \frac{8R}{L_z} - \frac{1}{2} \right). \]

The first term in Eq. (4) is the contribution from the exchange energy, and is small because \( R \gg R_0 \). The second term comes from the magnetostatic energy of the magnetic charge density generated at the nanodisk edges by the vortex displacement. The density of these charges is given by

\[
\sigma = -q_m M_s \frac{(\hat{z} \times \hat{\rho}) \cdot \hat{\rho}}{(R^2 - 2R\delta \cdot \hat{\rho} + \delta^2)^{1/2}}.
\]

The interaction between the magnetic charges in the vortex core and in the nanodisk edges is neglected. To calculate the magnetostatic energy to second order in \( \delta/R \), it is sufficient to use \( \sigma \) to first order, that is

\[
\sigma \approx \sigma^{(1)} = -q_m M_s \frac{(\hat{z} \times \hat{\delta}) \cdot \hat{\rho}}{R}.
\]

The energy of interaction with the applied field also comes from \( \sigma^{(1)} \). The magnetic moment generated by the displacement of the magnetic vortex is related to \( \sigma \), Eq. (6), by

\[
m(\delta) = \int d^2r \sigma \rho,
\]

where the integral is over the nanodisk edge surface. To first order in \( \delta/R \), the magnetic moment is obtained by replacing \( \sigma \) by \( \sigma^{(1)} \) in Eq. (7). The result is

\[
m^{(1)}(\delta) = -q_m M_s V_D \frac{(\hat{z} \times \delta)}{R}.
\]

By symmetry \( m(\delta) \) does contain terms of second order in \( \delta/R \).

The equilibrium vortex displacement, obtained by minimizing \( E_D \) with respect to \( \delta \), is

\[
\delta_{eq} = q_m \chi \frac{\hat{z} \times H}{M_s}.
\]

In equilibrium, the magnetic moment is

\[
m_{eq} = m^{(1)}(\delta_{eq}) = \chi V_D H.
\]

This result shows that \( \chi \) is the nanodisk susceptibility. According to Eq. (9), \( \delta_{eq} \) is perpendicular to \( H \), and depends only on \( H \) scaled by \( M_s/\chi \). As will be shown in Sec. II B, \( M_s/\chi \) is also the scale for the combined effects of the applied field and vortex line field on the magnetic vortex. The approximation to second order in \( \delta/R \) to \( E_D \) is valid only for \( \chi H/M_s < 1 \). However, it has been used outside this range to estimate the vortex annihilation field, \( H_{an} \), at which the magnetic vortex is destroyed, and the nanodisk switches to the saturated state (see Eq. (10)). This estimate assumes that at \( H = H_{an} \) the magnetic vortex reaches the nanodisk edge, that is \( \delta_{eq} = R \). Thus, according to Eq. (9), \( H_{an} = M_s/\chi \). In this case the nanodisk magnetic moment is the saturation moment, \( M_s V_D \) in the direction of \( H \) (see Eq. (10)). This estimate for \( H_{an} \) is in reasonable agreement with experiments and numerical simulations.

### B. Nanodisk-superconductor interaction

Now the nanodisk is assumed to be in the proximity of the superconducting film, as shown in Fig. 1. The equilibrium displacement of the magnetic vortex is recalculated by minimizing the total energy of the superconducting film-nanodisk system, \( E_T(\rho_s, \delta) \), to second order in \( \delta/R \). In the London limit

\[
E_T(\rho_s, \delta) = E_D(\delta) + E_{MS}(\delta) + E_{VM}(\rho_s, \delta),
\]

where \( E_{MS} \) is the energy of interaction of the nanodisk with the superconducting film in the absence of vortices, and \( E_{VM} \) is the interaction energy of the vortex line and the nanodisk. The influence of \( H \) on the superconductor is neglected. This is justifiable as long as \( H \) is smaller than the lower critical field parallel to the film surfaces.

The energy \( E_{MS} \) results from the interaction between the nanodisk and the magnetic field of the screening current generated by it in the superconductor. As shown in the Appendix, \( E_{MS} \) can be written as an interaction between magnetic charges,

\[
E_{MS} = \frac{1}{2} \int d^2r \int d^2r' \chi(r)\chi(r')U_{MS}(r; r')\,
\]

[Original reference page: 3]
In Fig. 3, a) Interaction potential induced by superconducting film between two magnetic charges separated by $\rho$, and at the same height $z$, compared with the Coulomb potential. a) $\lambda U_{MS}$ vs. $\rho$ for $d = \lambda$ and two $z$ values. Inset: magnetic charges above superconducting film. b) Ratio $U_{MS}$ to Coulomb potential, $\rho U_{MS}$, vs. $\rho$ for $z = 0.2\lambda$ and several $d$.

Thus, in the presence of the superconducting film, the interaction between magnetic charges is modified by the addition of $U_{MS}$ to the Coulomb potential (see Eq. (1)). In Fig. 3, $U_{MS}$ is shown for two magnetic charges separated by $\rho$ and at the same height $z$, and compared with the Coulomb potential. For large distances, $U_{MS}$ coincides with the Coulomb potential. Large distances mean $\rho \gg \lambda = 2\lambda^2/d$ for thin films ($d \ll \lambda$) and $\rho \gg \lambda$ for films with $d \sim \lambda$. For short distances, $U_{MS}$ is considerably smaller than the Coulomb potential. The effects of $E_{MS}$ on the magnetic vortex are to change the vortex core radius, $a$, and nanodisk susceptibility, $\chi$. However, these modifications are small. The reason is that the dominant contributions to $a$ and $\chi$ come from short distances $d < \lambda$, where $U_{MS}$ is much smaller than the Coulomb interaction. For instance, the contribution to $\chi^{-1}$ from $E_{MS}$ is found to be about one order of magnitude smaller than that from the Coulomb interaction, $\chi^{-1}$ (see Eq. (A.3)). Hereafter $E_{MS}$ is neglected.

The energy $E_{VM}$ is given by

$$E_{VM} = -\int d^3r' M(r') \cdot b(r' - \rho_c) + \frac{1}{2} \int d^2r' \sigma(r') \Phi(r' - \rho_c).$$

(13)

where $b(r) = -\nabla \Phi(r)$ is the magnetic field created by the vortex line outside the film. There are two contributions to $E_{VM}$: one from the magnetic vortex core, and another from the magnetic charge density at the nanodisk edges, $\sigma$, Eq. (4), denoted by $E_{VM}^{(c)}$. The contribution to $E_{VM}$ from the core is negligible, as justified later. In order to obtain $E_{VM}^{(c)}$ to second order in $\delta/R$ it is necessary to use $\sigma$ to the same order. The first order term is $\sigma^{(1)}$, Eq. (4). The second order term is, according to Eq. (5), given by

$$\sigma^{(2)} = \sigma^{(1)} \delta \cdot \frac{\hat{r}}{R}. $$

(15)

The contribution from $\sigma^{(1)}$ to $E_{VM}$, denoted $E_{VM}^{(c)}$, is obtained by noting that $\sigma^{(1)}$ can be interpreted as resulting from the uniform magnetization

$$M^{(1)} = -q_m M_a \hat{z} \times \frac{\delta}{R}. $$

(16)

Using Eq. (16) it follows that

$$E_{VM}^{(c)} = -V_D M^{(1)} \cdot B_{\perp} (-\rho_c), $$

(17)

where

$$B_{\perp} (-\rho_c) = \frac{1}{V_D} \int_{disk} d^3r' b_{\perp} (r' - \rho_c) = -qB_{\perp} (\rho_c) \hat{r}, $$

(18)

is the average over the nanodisk volume of the component of the vortex field perpendicular to the $z$-direction (parallel to the nanodisk faces), $b_{\perp}$. The argument of $B_{\perp}$ in Eq. (18) is $-\rho_c$ because $\rho_c$ is defined with respect to the origin shown in Fig. 1 whereas in the definition of the vortex line field the origin is at the vortex line (see Eq. (A.1)). Thus

$$E_{VM}^{(c)} = q_m M_a V_D q B_{\perp} (\rho_c) \frac{\delta_T}{R}, $$

(19)

where $\delta_T$ denotes the component of $\delta$ perpendicular to $B_{\perp} (\rho_c)$ (parallel to $-\hat{z} \times \hat{r}$), see Fig. 1. The contribution from $\sigma^{(2)}$ is calculated in the Appendix. The result is

$$E_{VM}^{(c)} = q_m M_a V_D q B_1 (\rho_c) \frac{\delta_L \delta_T}{R^2}, $$

(20)

where $\delta_L$ denotes the component of $\delta$ parallel to $B_{\perp} (\rho_c)$ (Fig. 1), and $B_1 (\rho_c)$ has dimension of magnetic field, and is given by Eq. (A.12).

Now the equilibrium displacement of the magnetic vortex is calculated neglecting the contribution from the vortex core to $E_{VM}$. The total energy is thus

$$E_T (\rho_c, \delta) = E_D (0) + M_a^2 V_D \left[ \frac{\delta_T^2 + \delta_L^2}{2 \chi R^2} + q m \frac{q B_1 (\rho_c) \delta_L \delta_T}{M_a^2 R^2} \right]. $$

(21)
The components of $\delta$ parallel and perpendicular to $H$, $\delta_x$ and $\delta_y$, respectively, are related to $\delta_L$ and $\delta_T$ by (see Fig. 4).

$$
\begin{align*}
\delta_x &= -\delta_L \cos \theta + \delta_T \sin \theta \\
\delta_y &= -\delta_L \sin \theta - \delta_T \cos \theta .
\end{align*}
$$

Minimizing $E_T$ with respect to $\delta$, with the vortex line held fixed at $\rho_v$, it follows that

$$
\begin{align*}
\delta_{T,eq}(\rho_v) &= \frac{q m}{R} \frac{q B_1 - H \cos \theta + q m q B_1 H \sin \theta}{1 - B_1^2} , \\
\delta_{L,eq}(\rho_v) &= -\frac{q m}{R} \frac{H \sin \theta + q m q B_1 (q B_1 - H \cos \theta)}{1 - B_1^2} , \\
\tilde{B}_1 &= \frac{\chi B_1 (\rho_v)}{M_s} ,
\end{align*}
$$

Thus $\delta_{eq}$ depends only on the vortex line and applied fields scaled by $M_s/\chi$, and is non-linear in these fields. The non-linearity results from the inhomogeneity of the vortex field over the nanodisk volume, which is responsible for $B_1$ being non-zero. This solution is only valid if $B_1^2 < 1$. Otherwise $\delta_{eq}$, Eq. (23), does not correspond to a minimum of $E_T(\rho_v, \delta)$, Eq. (22). The consequences of Eq. (23) are discussed in detail in Sec. III.

Now the neglect of the core contribution to $E_{VM}$, denoted $E_{VM}^{(c)}$, is justified. Since $a$ is small compared to $R$, $E_{VM}^{(c)}$ can be estimated by approximating the core by a point dipole located at the nanodisk center with the total magnetic moment of the core. According to Eq. (24), it has only the $z$-component $m_z^{(c)} = \pm M_s a^2 L_z (2 \ln 2 - 1)$. Thus, $E_{VM}^{(c)} \approx -m_z^{(c)} b_z (\delta - \rho_v + z_0 \hat{x})$. The basic reason why $E_{VM}^{(c)}$ can be neglected is that it is proportional to $a^2$, which is already a small quantity. One effect of $E_{VM}^{(c)}$ is to modify the vortex core. Since it depends on $a$, its contribution must be added to $E_D$ in order to obtain the equilibrium value of $a$. However, the effect is a small one because $E_D \sim M_s^2 a^4 / L_z$ [14], so that $E_{VM}^{(c)} / E_D \sim (L_z / a)^2 (b_z / M_s)$, and $(b_z / M_s)$ is small, as shown in Sec. III. Another effect of $E_{VM}^{(c)}$ is to displace the magnetic vortex from the nanodisk center. This displacement can be estimated as follows. The force exerted by the superconducting vortex on the magnetic vortex core is $| \nabla E_{VM}^{(c)} | \sim M_s^2 V_D (a / R)^2 b_z / M_s \ell$, where $\ell$ is the typical scale for variations of $b_z$, namely $\ell \sim z_0$ for thin films ($d \ll \lambda$) and $\ell \sim \lambda$ for films with $d \sim \lambda$. This force must be balanced by the elastic force $M_s^2 V_D b_z / M_s \ell$, which gives $\delta / R \sim (a / R)^2 (R / \ell) \chi b_z / M_s$, and is smaller than the vortex displacement caused by $E_{VM}^{(c)}$ at least by a factor $a / R$.

The pinning potential for the vortex line is defined as the equilibrium total energy for the vortex line held fixed at $\rho_v$. That is $U_p(\rho_v) = E_T(\rho_v, \delta_{eq})$. Thus

$$
U_p(\rho_v) = -\frac{\chi V_D}{2(1 - B_1^2)} [B_1^2 - 2 q B_1 H \cos \theta] + 2 q m q B_1 H \sin \theta (q B_1 - H \cos \theta) .
$$

FIG. 5: a) Fields $B_\perp$ and $B_\parallel$, in units of $\phi_0 / \lambda^2$, vs. $\rho_v$ for $R = 2.0 \lambda$. b) Maximum values of $B_\perp$ and $B_\parallel$ in units of $\phi_0 / \lambda^2$ vs. $R$. The right-hand scale in Gauss corresponds to left-hand one for $\lambda = 75 \text{nm}$. Parameters: $\xi = 0.1 \lambda$, $z_0 = 0.2 \lambda$, $L_z = 0.1 \lambda$.

This result shows that the pinning potential can be tuned by the applied field. The mechanism is that, according to Eq. (23), $H$ controls the equilibrium displacement of the magnetic vortex which, in turn, modifies $U_p$.

In order to evaluate the consequences of the above results it is necessary to attribute values to the model parameters. This is done next.

III. DISCUSSION

The results of Sec. III are applicable to arrays of soft magnetic nanodisks placed on top of low-$T_c$ superconducting films, provided that the nanodisks are sufficiently far apart to neglect dipole-dipole interactions between them, and the vortex density is low enough to neglect vortex-vortex interactions. Realistic values for the model parameters are the following [1, 9, 10, 11]. Permalloy nanodisks: $100 \text{nm} < R < 500 \text{nm}$, $L_z \sim 10 \text{nm}$, $M_s = 0.8 \text{kG}$, and $R_0 \sim 15 \text{nm}$. Superconducting films at zero temperature: $\lambda = 75 \text{nm}$, $\xi = 0.1 \lambda$, $d \sim 0.2 - 2 \lambda$. With this value for $\lambda$, the scale for the vortex fields is $\phi_0 / \lambda^2 = 3.6 \text{kG}$. The distance from the nanodisk center to the film surface is chosen as the smallest possible, namely $z_0 = 0.2 \lambda = 2 \xi$. This value takes into account the existence of an insulating layer between the film and the nanodisk, of thickness $\sim \xi$, to avoid the proximity effect. It is found that for these parameter values the vortex fields $B_\perp$ and $B_\parallel$ are small compared to $M_s / \chi$. As consequences, $B_1$ is negligible in Eq. (23), the magnetic vortex displacement is small compared with and nanodisk radius, and its dependence on the vortex field is linear. This is discussed in detail next.

The fields $B_\perp$ and $B_\parallel$ are shown in Fig. 5. They depend only on scaled variables, with $\lambda$ as the length scale and $\phi_0 / \lambda^2$ as the magnetic field scale (see Eqs. (A.4), (A.5)). Their dependencies on $\rho_v$ are shown in Fig. 5a. Both vanish for $\rho_v = 0$, and have a maximum at $\rho_v \sim R$, with $B_\parallel$ smaller than $B_\perp$ by a factor $\sim 3$. The maximum values of $B_\perp$ and $B_\parallel$ depend on $R$ as shown in Fig. 5b.
This result is, up to a constant, the magnetostatic energy of interaction between the magnetic moment induced by \( \mathbf{H}_T \) in the nanodisk, \( \mathbf{m}_{eq} = \chi V_D \mathbf{H}_T \), and \( \mathbf{H}_T \) itself. That is

\[
U_p = -\frac{1}{2} \mathbf{m}_{eq} \cdot \mathbf{H}_T + \frac{\chi V_D H^2}{2}.
\]  

This simple result is the just the linear response approximation from elementary macroscopic magnetism \( \chi \) applied to the nanodisk. It depends only on the nanodisk susceptibility, \( \chi \), and on the macroscopic vortex field acting on it, \( \mathbf{B}_\perp (-\mathbf{p}_v) \). Results for the pinning potential \( U_p(\mathbf{p}_v) \), based on Eq. \( (24) \), are shown in Fig. 7 as two-dimensional plots for characteristic values of \( H \). For \( H = 0 \), \( U_p \) has circular symmetry, and is the same for vortices \( (q = 1) \) and anti-vortices \( (q = -1) \), with a degenerate minimum located in a circle of radius \( \rho_v \sim R \) (Fig. 7a). For \( H \neq 0 \), \( U_p \) has a non-degenerate minimum and also a maximum along the direction of \( \mathbf{H} \). The plot in Fig. 7b corresponds to \( \mathbf{H} \parallel \mathbf{B}_\perp \), whereas that in Fig. 7c is for \( \mathbf{H} \parallel \mathbf{B}_\perp \). In the latter case the pinning potential reduces to \( U_p(\mathbf{p}_v) = q V_D \chi B_\perp(\mathbf{p}_v) H \cos \theta \), which is identical to that for a nanodisk with permanent uniform magnetization \( \chi \mathbf{H} \). Since \( B_\perp \ll M_s/\chi \), the range of \( H \) values for which the vortex line field influences the pinning potential is small. Thus, except for small \( H \), the spatial dependence of the pinning potential is given by \( B_\perp(\mathbf{p}_v) \cos \theta \), and the magnitude is proportional to \( H \). This allows for continuous tuning of \( U_p \) over a wide range. It is useful to compare the magnitude of \( U_p \) to that for the pinning potential for a nanodisk with permanent magnetization, equal to the saturation one, \( U \sim M_s V_D B_\perp \). For \( H < M_s/\chi \), \( U_p \) is smaller than \( U \), since for \( \mathbf{H} \parallel \mathbf{B}_\perp \), \( | U_p/U | \sim \chi B_\perp/M_s \ll 1 \), and for \( M_s/\chi > H > B_\perp \), \( | U_p/U | \sim \chi H/M_s \ll 1 \). Using Eq. \( (24) \) to extrapolate \( U_p \) to the region \( H > M_s/\chi \), it follows that \( U_p \sim U \) only at \( H \sim M_s/\chi \).

The linear response approximation is now used to estimate the effect of the vortex line on the magnetic vortex annihilation field, \( H_{an} \). As in Sec. II A, \( H_{an} \) is estimated as the value of \( H \) for which \( \delta_{eq} = R \). In the linear response approximation this occurs for \( H_T = M_s/\chi \). Since \( B_\perp \ll M_s/\chi \), \( H_{an} \) differs little from that predicted for isolated magnetic vortex, that is \( H_{an} \approx M_s/\chi \). Consequently, the nanodisk magnetization curve is expected to differ little from that for the isolated nanodisk.

In summary then, for the parameter values described earlier, the main conclusions are: i) the vortex line field causes only a small displacement of the magnetic vortex; ii) the pinning potential is the energy of interaction between the magnetic moment induced in the nanodisk by the vortex line and applied fields and the fields themselves, calculated according to the laws of macroscopic linear magnetism, iii) the vortex line has little effect on the nanodisk magnetization curve.

Now the non-linear dependence of \( \delta_{eq} \) on the vortex line field and applied fields, predicted by Eq. \( (23) \), is considered. The question is whether or not there are parameter values for which \( B_\perp \) is sufficiently large that the non-linearity is important and what are its consequences. One limitation on \( B_\perp \) is that Eq. \( (23) \) only
FIG. 7: Pinning potential for a vortex line by a magnetic nanodisk in the vortex state, in units of \( \epsilon_0 \lambda = \phi_0^2/(16\pi^2 \lambda) \), for \( d = 2.0 \lambda \), \( z_0 = 0.2 \lambda \), \( \xi = 0.1 \lambda \), \( R = 1.5 \lambda \), \( L_z = 0.1 \lambda \), \( M_s = 0.1 \phi_0/\lambda^2 \). a) \( H = 0 \). b) \( H = 0.01 \phi_0/\lambda^2 \). c) \( H = 0.1 \phi_0/\lambda^2 \).

makes sense if \( \delta/R < 1 \). This effectively restricts \( \tilde{B}_1 \) to relatively small values. One reason is that the denominators in Eq. \( \Delta \) cannot be too small. Another reason is that \( B_1 \) and \( B_1^\perp \) are not independent (see Eqs. \( \Delta \Delta \), \( \Delta \Delta \)), so that if \( \tilde{B}_1 \) is not sufficiently small, \( \tilde{B}_1^\perp \) is large enough to make \( \delta/R > 1 \). It is found that, in general, \( B_1 < B_1^\perp \), as exemplified in Fig. \( \Delta \). To investigate this quantitatively, the equilibrium displacements of the magnetic vortex for \( H = 0 \) is calculated using a new set of model parameters. These are chosen in order to give displacements larger than the ones obtained with the parameters mentioned at the beginning of this Section. For the superconducting film the parameters are the same as before (\( \xi = 0.1 \lambda \), \( z_0 = 0.2 \lambda = 2 \xi \)), except for \( \lambda \) which is now chosen as \( \lambda = 50 \text{nm} \). In this case the fields \( B_1 \) and \( B_1^\perp \) are unchanged in units of \( \phi_0/\lambda^2 \), but their values in Gauss change because now \( \phi_0/\lambda^2 = 8 \text{kG} \). The new parameters for the nanodisk are chosen as \( R = 3.0 \lambda \), \( L_z = 0.1 \lambda \), \( M_s = 0.4 \text{kG} \). In this case \( \chi = 3.0 \) and \( M_s/\chi = 0.13 \text{kG} \). The results for the magnetic vortex displacements are shown in Fig. \( \Delta \), and compared with those obtained from Eq. \( \Delta \Delta \) with \( \tilde{B}_1 = 0 \). The differences between the curves for \( \delta_{\perp,eq} \) with \( \tilde{B}_1 = 0 \) and \( \tilde{B}_1 = 0 \) are due to \( B_1^2 \) in the denominator of Eq. \( \Delta \). The results show that the differences are small, even when \( \delta_{\perp,eq} \) is a considerable fraction of \( R \). For \( \delta_{L,eq} \), only the curves for \( \tilde{B}_1 \neq 0 \) are shown, because \( \delta_{L,eq} \) vanishes for \( \tilde{B}_1 = 0 \). The non-zero \( \delta_{L,eq} \) shown in Fig. \( \Delta \), results from the non-linearity of Eq. \( \Delta \), with the most important contribution coming from the numerator. Comparing the curves for \( \delta_{L,eq} \) and \( \delta_{\perp,eq} \) in Fig. \( \Delta \), and using \( \delta_{L,eq}/\delta_{\perp,eq} = \tilde{B}_1 \), it follows that \( \tilde{B}_1 \lesssim 0.3 \). This shows that the validity of Eq. \( \Delta \) requires small values of \( \tilde{B}_1 \).

The non-linear effects in the pinning potential \( U_p \), Eq. \( \Delta \), for \( H = 0 \) come only from \( B_1^2 \) in the denominator, and are also small according to the above discussion. The properties of \( U_p \) for the new set of parameters are similar to those described earlier.

The results shown in Fig. \( \Delta \) also have consequences for the magnetic vortex annihilation field, \( H_{an} \). When the vortex line displaces the magnetic vortex by a significant fraction of the nanodisk radius it takes only a small applied field to annihilate the magnetic vortex by further displacing it to the nanodisk edge. In this case \( H_{an} \) differs significantly from that for the isolated nanodisk. To estimate \( H_{an} \), Eq. \( \Delta \), with \( \tilde{B}_1 = 0 \) is used. In this case, as discussed above, the magnetic vortex is annihilated, when \( H_T \sim M_s/\chi \). This estimate for \( H_{an} \) is shown in Fig. \( \Delta \). A strong dependence of \( H_{an} \) on the vortex line position results. The absolute minimum of \( H_{an} \) occurs when the vortex line is at the pinning potential minimum for \( H = 0 \), located at \( y_v = 0 \) and \( x_v \sim -R \), since in this case the displacement of the magnetic vortex is maximum. Thus, when the vortex line is in equilibrium with the nanodisk \( H_{an} \) is minimum. This result indicates that the vortex line in equilibrium with the nanodisk can significantly modify the magnetization curve.

To conclude then, the magnitude of the displacement of the magnetic vortex caused by the vortex line depends on the particular values of the model parameters. In the cases where the displacement is a significant fraction of the nanodisk radius, effects of the non-linear relationship between the displacement and the vortex line field are felt, but are small, and the magnetization curve for the nanodisk in equilibrium with the vortex line is predicted to differ from that for the isolated nanodisk. In all cases, the displacement of the magnetic vortex and

FIG. 8: a) Equilibrium displacements of the magnetic vortex ( \( q_m = 1 \) ) for \( H = 0 \). b) Magnetic vortex annihilation field for vortex line at position \( (x_v, y_v) \) vs. \( x_v \) for \( y_v \) constant. Parameters: \( \xi = 0.1 \lambda \), \( z_0 = 0.2 \lambda \), \( R = 3.0 \lambda \), \( L_z = 0.1 \lambda \), \( M_s = 0.05 \phi_0/\lambda^2 \).
the pinning potential can be estimated by applying the linear response theory of elementary magnetism to the nanodisk. This suggests that the pinning potential for nanomagnets with other geometrical forms for which the magnetic vortex state has been reported \[18\] can be likewise estimated.

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APPENDIX: LONDON THEORY RESULTS

Here some results of London theory for the superconducting film, obtained in Refs \[13\] \[14\] are reviewed. The objective is to write out the mathematical expressions needed to carry out the calculations mentioned in Sec. \[11\].

1. nanodisk-screening current interaction

In the arrangement shown in Fig. \[11\] the nanodisk generates in the superconducting film a screening current which, in turn, creates a magnetic field at the nanodisk. The energy of interaction of the nanodisk with this field is given by

\[ E_{MS} = -\frac{1}{2} \int d^3r \ M(r) \cdot b_{sc}(r) , \]  

(A.1)

where \( b_{sc}(r) \) is the field of the screening current outside the film. The field \( b_{sc} \) is can be written as

\[ b_{sc}(r) = -\nabla \Phi_{sc}(r) , \]
\[ \Phi_{sc}(r) = \int d^3r' (M(r') \cdot \nabla') U_{MS}(r; r') , \]  

(A.2)

where \( U_{MS} \) is given by

\[ U_{MS}(r; r') = \int d^2k e^{ik \cdot (r - r')} e^{-kz} g(k) , \]
\[ g(k) = \frac{\sinh \tau d}{k [e^{-\tau d} (k - \tau)^2 - e^{\tau d} (k + \tau)^2]} , \]
\[ \tau = \sqrt{k^2 + \lambda^2} , \]  

(A.3)

where \( r \perp \) denotes the component of \( r \) perpendicular to the \( z \)-direction. Assuming that \( \nabla \cdot M = 0 \), and integrating by parts, Eq. \( A.1 \) can be written as Eq. \( A.2 \).

2. vortex line field

The field created by the vortex line outside the film is given by

\[ \mathbf{b}(r) = -\nabla \Phi(r) , \]
\[ \Phi(r) = -\int \frac{d\phi_0}{\lambda^2} \int \frac{d^2k}{(2\pi)^2} e^{ik \cdot r} e^{-kz} F_1(k) , \]  

(A.4)

where \( r \) is the position vector with respect to an origin at the vortex line, and

\[ F_1(k) = \frac{e^{-2\xi^2 k^2 [(k + \tau)^2 e^{\tau d} + (k - \tau)^2 e^{-\tau d} - 2k]}}{k\tau k^2 (k - \tau)^2 e^{-\tau d} - (k + \tau)^2 e^{\tau d}} . \]  

(A.5)

Integrating Eq. \( A.3 \) over the direction of \( k \), \( \Phi \) can be written as

\[ \Phi(r_{\perp}, z) = -q \int_0^\infty \frac{dk}{2\pi} k J_0(k) e^{-kz} F_1(k) , \]  

(A.6)

where \( J_0 \) is the Bessel function of first kind. Thus, the vortex field can be written as

\[ \mathbf{b}(r) = b_{\perp}(r_{\perp}, z) \hat{r}_{\perp} + b_z(r_{\perp}, z) \hat{z} , \]
\[ b_{\perp} = -\frac{\phi_0}{\lambda^2} \int_0^\infty \frac{dk}{2\pi} k^2 J_0(k) e^{-kz} F_1(k) , \]
\[ b_z = -\frac{\phi_0}{\lambda^2} \int_0^\infty \frac{dk}{2\pi} k^2 J_0(k) e^{-kz} F_1(k) . \]  

(A.7)

Using Eq. \( A.7 \) and Eq. \( A.8 \), it follows that

\[ B_{\perp}(\rho_{\perp}) = -\frac{\phi_0}{\lambda^2} \int_0^\infty \frac{dk}{2\pi} S_1(k) e^{-k\rho_{\perp}} F_1(k) , \]
\[ S_1(k) = \frac{2J_1(kR)}{kR} \frac{2\sinh(kL_z/2)}{kL_z} . \]  

(A.8)

3. superconducting vortex-magnetic vortex interaction

Here the contribution from \( \sigma^{(2)} \) to the vortex line - nanodisk interaction is calculated. Using Eqs. \( A.1 \), and \( A.2 \), \( E_{V_M}^{(e2)} \) can be written as

\[ E_{V_M}^{(e2)} = \int_{z_0 - L_z/2}^{z_0 + L_z/2} dz' \int_0^{2\pi} \frac{d\theta'}{2\pi} \sigma^{(2)}(\theta') \]
\[ (-\frac{\phi_0}{\lambda^2}) \int \frac{d^2k}{(2\pi)^2} e^{ik \cdot (\rho_{\perp} - \rho_{\perp}')} e^{-kz} F_1(k) , \]  

(A.9)

where \( |\rho_{\perp}'| = R \). According to Eq. \( A.11 \), \( \sigma^{(2)} \) is given by

\[ \sigma^{(2)}(\theta') = q_m M_s (\frac{\delta}{R})^2 \sin(\beta' - \theta') \cos(\beta' - \theta') \]  

(A.10)

where \( \beta' \) and \( \theta' \) are, respectively, the angles between \( \delta \) and \( k \), and between \( \rho_{\perp}' \) and \( k \). Integrating over \( z' \), \( \theta' \) and \( \beta' \), it follows that

\[ E_{V_M}^{(e2)} = q_m M_s V_D q B_1(\rho_{\perp}) \frac{\delta^2 \cos \gamma \sin \gamma}{R^2} , \]  

(A.11)
where
\[ B_1(\rho_v) = -\frac{\phi_0}{\lambda^2} \int_0^\infty \frac{dk}{2\pi} S_2(k) k^2 J_2(k\rho_v) e^{-kz_0} F_1(k), \]
\[ S_2(k) = \frac{2J_2(kR)}{kR} \frac{2\sinh (kL_z/2)}{kL_z}, \] (A.12)
and \( \gamma \) is the angle between \( \delta \) and \( \rho_v \) (Fig. 4). Using
\[ \delta_L = \delta \cos \gamma \quad \text{and} \quad \delta_T = \delta \sin \gamma, \]
Eq. (A.11) is identical to Eq. (20).

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