A network application for modeling a centrifugal compressor performance map

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Abstract: The approximation of aerodynamic performance of a centrifugal compressor stage and vaneless diffuser by neural networks is presented. Advantages, difficulties and specific features of the method are described. An example of a neural network and its structure is shown. The performances in terms of efficiency, pressure ratio and work coefficient of 39 model stages within the range of flow coefficient from 0.01 to 0.08 were modeled with mean squared error 1.5 %. In addition, the loss and friction coefficients of vaneless diffusers of relative widths 0.014-0.10 are modeled with mean squared error 2.45 %.

Nomenclature

\( \bar{h}_1 \)  
inlet relative blade height

\( \bar{h}_2 \)  
outlet relative blade height

\( \bar{b}_3 \)  
VLD relative width an inlet

\( \bar{b}_4 \)  
VLD relative width an outlet

\( \bar{b}_5 \)  
inlet relative vane height

\( \bar{b}_6 \)  
outlet relative vane height

\( \bar{D}_p \)  
impeller inlet relative diameter

\( \bar{D}_o \)  
outlet of stage relative diameter

\( \bar{D}_v \)  
impeller blade cascade relative inlet diameter

\( \bar{D}_2 \)  
impeller diameter

\( \bar{D}_3 \)  
non-dimensional diameter in an inlet VLD

\( \bar{D}_4 \)  
VLD outlet relative diameter

\( \bar{D}_5 \)  
return channel vane cascade relative inlet diameter

\( \bar{D}_6 \)  
return channel vane cascade relative outlet diameter

\( \bar{D}_7 \)  
hub ratio

\( \bar{D}_s \)  
seal relative diameter
1. Introduction
The complexity of the aerothermodynamic processes in centrifugal compressors does not allow defining their performances analytically. Until recently, any simple way to create highly effective designs of turbocompressors was very expensive and relied on experiments.

With the accumulation of experimental data and the development of math models, the mean line modeling tools started [1-3]. Nowadays these tools have already been developed and are provided by many scientific centers [4-7].

The mathematical modeling of centrifugal compressors is reduced to the identification of the models derived from experimental data. These models have rather complicated analytic representation defined by every researcher in view of the operating process and the selected physical model. For example, the authors Lunev A., Vyakhilev O., Drozdov Y. [4-5] use hydraulic analogues. The school of modeling [6] uses concepts of the boundary layer theory. The Universal modeling method [1-3] operates with loss coefficients of blades, vane cascades, and vaneless channels. Arguments of algebraic equations are tangential and normal velocity gradients.

The response surfaces of the objective functions are multi-extremal, thus the process of identification of models becomes complicated. The largest problems were related to the necessity of calculating the flow in the air-gas channel, the a priori assignment of the generalized analytical dependencies of the various losses on the velocity distribution characteristics, the assignment of the initial vector of the desired parameters and the search area. These problems are connected with need of calculation of a current for flow part, with an aprioristic task of the generalized analytical types of
dependences of various losses on characteristics of distribution of velocities, a task of an initial vector of required parameters and areas of search of values. Despite that, scientists managed to develop mathematical models for calculating gas dynamic performances. It opens a way to create computing methods of design and to offer producers high effective centrifugal compressors without long and expensive physical experiments. This gave confidence to researchers but the above difficulties of math modeling didn't manage to be overcome yet.

Software packages of neural networks application provide solutions to overcome difficulties of math model development and identification. Neural networks provide the universal mechanism of approximation adequate to multidimensional data files, are capable to be trained and arranged at changing conditions, can generalize the gained knowledge on the basis of what is considered as systems of artificial intelligence. The basis of functioning of neural networks is made by the algorithms of training allowing optimizing process of search of decisions [8-9].

2. Object and aim

Neural networks are used in various fields of science. They successfully solve tasks such as function approximation and optimization, management and forecasting processes. These capabilities are already being used in aerospace, military, medical, industrial, financial and banking fields, data transmission. Due to its efficiency and broad range of functions neural networks are becoming more common and useful for more complex problems.

Neural networks play a role of an universal approximator of function of several variables which realizes nonlinear function of a type \( y = F(x) \), where \( x \) – an entrance vector (perhaps, an array), and \( y \) – the realized function of several variables. Many problems of modeling, identification and data processing can be solved by approximation with neural networks.

The process of approximation by means of neuronets consists of selecting the weight coefficients of defining the degree of importance of the communications between the neurons and this process is called “learning of neural networks”. For an assessment of quality of training mean squared error is applied to minimize deviations from the entered experimental values.

Using of gradient methods became possible thanks to the introduction of the sigmoid activation function \( f(x) = (1 + e^{-\beta x})^{-1} \), which is continuous, unipolar and differentiated. In practice it is most often used \( \beta = 1 \). But a user can independently select values \( \beta \) that influences a sigmoida form – at small \( \beta \) function is flat, at \( \beta \rightarrow \infty \) it turns into step function. Such possibility of a variation increases flexibility of a neural network in general (Figure 1).

![Figure 1. Sigmoid activation function of neurons: a) \( \beta = 0.5 \), b) \( \beta = 1 \), c) \( \beta = 2 \)]
Considered the simplest option of a multilayered neural network (Figure 2) with a direct signal transmission. The network consists of two layers, in the first layer - two neurons, in the second - one. Activation function of neurons is a logical sigmoid function.

Two arguments come to an input of the first layer come, each one is multiplied by weight $w_{ij}$, the weighed values are transmitted to the adder $\sum$ together with bias of $b$. Networks with bias allow to form more difficult communications between inputs and exits, than networks without bias, and to provide a nonzero exit of neurons. The resultant sum of $n_i$ serves as argument for function of activation of $f^{-1}$: $n_i^1 = w_{i1}x_1 + w_{i2}x_2 + b_i^1$, $n_i^2 = w_{i1}x_1 + w_{i2}x_2 + b_i^2$.

Exits of neurons of the first layer are passed to the second layer, which are described by the neuron equation with bias: $a_i^1 = \frac{1}{1 + e^{-\beta(n_i^1)}} = \frac{1}{1 + e^{-\beta(w_{i1}x_1 + w_{i2}x_2 + b_i^1)}}$.

The scalar $a_i^1$ and $a_i^2$ as increased by weight coefficients of a layer of $w_{ij}$ move on the adder of neuron of the second layer and finally are received at the exit of the second layer: $a^2 = \frac{1}{1 + e^{-\beta(w_{i1}x_1 + w_{i2}x_2 + b_i^2)}}$ by knowing $a_i^1$ and $a_i^2$ we can write as: $y = f(x_1, x_2) = a^2$, where: $a^2 = 1 / 1 + e^{\beta \left( \frac{1}{1 + e^{-\beta(w_{i1}x_1 + w_{i2}x_2 + b_i^1)}} + \frac{l_{w_{i1}}}{1 + e^{-\beta(w_{i1}x_1 + w_{i2}x_2 + b_i^1)}} + \frac{l_{w_{i2}}}{1 + e^{-\beta(w_{i1}x_1 + w_{i2}x_2 + b_i^1)}} + b_i^2 \right)}$. It is noticed that the exit of the second layer of $a_2$ can be designated as $y$, i.e. the exit of the last layer is a network exit. The type of total function is deprived of presentation, therefore neural networks are often called "a black box".

During learning procedure there is a control of the network and the weight-matrix $W^{11}$, which define extent of influence of this argument on a calculated value of function is formed: $W^{11} = \begin{bmatrix} w_{i1} & w_{i2} \\ w_{i2} & w_{i2} \end{bmatrix}$, where $w_{i2} = W(1,2)$ - coefficient on which the second element of an entrance of $x_2$ is multiplied by transfer of value by the first neuron.
Also, during training process, the matrix of the weight coefficients of the output of the layer \( LW^{21} \) is formed, the indexes show interrelation between structural elements.

\[
LW^{21} = \begin{bmatrix} h_{w_{11}} & h_{w_{12}} \end{bmatrix} - \text{coefficients are multiplied on which exits of the first layer before processing by neurons of the second layer.}
\]

The above example of equations shows inconvenience of use of analytical equations even for a simple case (two arguments, two layers). Addition of arguments, neurons or layers of a network repeatedly complicates analytical representation of mathematical model. If we add the third neuron in the first layer and one more argument of \( x_3 \), total number of weight coefficients will increase with 6 to 12. Therefore in practical use of neural network models people do not work with the equations, but with the files represented by computer programs.

The neural network can function in two modes:

1. **Training**, when weight adjustment occurs so that output signals most precisely correspond to the experimental;
2. **Operating**, when network takes input values and produces outputs.

The model is approached as much as possible to experimental data which are entered for training, but not to real process as the data, which are absolutely precisely reflecting real process, can’t be obtained. Also at a stage of creation of mathematical model identification it is not possible to avoid data-entry errors except as increase the vigilance. Naturally "human" factor reduces the accuracy of mathematical model.

The main advantages of application of neural networks for modeling of characteristics of centrifugal compressors are:

1. Creation of a model is simpler and quicker than methods, used earlier;
2. Creation of structure of model is completely formalized and doesn’t demand acceptance of preliminary hypotheses of the type of those or those dependences;
3. Flexibility of model creation and possibility after learning in process of receipt of new data;
4. The level of knowledge, necessary for successful application of neural networks, is significantly less than the level for traditional methods of modeling;
5. Possibility of analysis of parameters influence on the studied characteristics.

Since 1970th, Y. Galerkin and his group created a number of math models of stages and compressors [10-11]. The Method of Universal modeling had been successfully applied in numerous designs of centrifugal compressors with power up to 32 MWt and delivering pressure up to 12.5 MPa (more than 400 compressors with total power 5 000 MWt). New models, versions 5th and 6th, demonstrate high precision of performance modeling [10], but the process of constant improvements points out the necessity of alternatives at the same time.

Measured gas dynamic of specially developed model stages of centrifugal compressors in databases are presented in the form of tables or in a graphic form. For use in the computerized gas dynamic calculations the performances have to be presented in analytical form. The methods based on artificial neural networks are an appropriate tool for this task. Their flexible structure and big functionality allow generalizing experimental data in a form, convenient for use [2-3, 8] (Figure 1).

Neural models of performances were developed for centrifugal compressor stages “impeller + vanless diffuser + return channel” (Figure. 3). Information is provided by R&D Laboratory “Gas dynamics of turbomachines”. Data on 39 stages was the object of neural model. Range of geometry parameters of stages is presented in Table 1.

At a preliminary stage some types of models were created with different structure of a neural network and parameters of training, from which the network with the smallest mean squared error is chosen.
For modeling of values of polytrophic efficiency and work coefficient was used the network consisting of two layers, 20 neurons in the first layer and 2 neurons in the second layer. The function that activates neuron layers in models was called «logsig» (a logical sigmoid).

Measured and calculated performances of efficiency and pressure coefficient are presented in Figure. 4-10 (KPD - measured efficiency, KPD NS – calculated efficiency, PSI – measured pressure coefficient, PSI NS – calculated pressure coefficient). Sample of $Mu$ influence on the performances are depicted in Figure. 7.

The mean squared error for all 567 measured points is 2.5%, for pressure coefficient - 3%. The largest inaccuracy is for maximum flow coefficient (about 15%). Compressors never operate at these flow rates. For other points of performances the mean squared error for efficiency and for pressure coefficient is within 1.5%.

![Figure 3. Meridional projection of a stage of the centrifugal compressor](image)

| Table 1. Range of geometry parameters of stages |
|-----------------------------------------------|
| Symbol | Min | Max | Symbol | Min | Max |
| $\bar{D}_h$ | 0,25 | 0,3916 | $\bar{D}_4$ | 1,41 | 1,56 |
| $\bar{D}_{seal}$ | 0,436 | 0,592 | $\bar{b}_5$ | 0,0056 | 0,069 |
| $z_{imp}$ | 13 | 21 | $\bar{b}_4$ | 0,0056 | 0,069 |
| $\bar{\delta}_{bl}$ | 0,007 | 0,017 | $\bar{D}_5$ | 1,41 | 1,56 |
| $\bar{D}_0$ | 0,425 | 0,573 | $\bar{D}_6$ | 0,577 | 0,818 |
| $\bar{D}_1$ | 0,502 | 0,668 | $\bar{b}_5$ | 0,033 | 0,085 |
| $\bar{b}_1$ | 0,0063 | 0,124 | $\bar{b}_6$ | 0,033 | 0,085 |
| $\bar{b}_2$ | 0,0063 | 0,069 | $\bar{D}_0$ | 0,475 | 0,625 |
| $\beta_{bl1}$ | 24,39° | 37,61° | $\bar{R}_3$ | 0,0716 | 0,159 |
| $\beta_{bl2}$ | 22,5° | 85,5° | $\bar{R}_4$ | 0,032 | 0,074 |
| $\bar{R}_{s1}$ | 0 | 0,128 | $\beta_{bl5}$ | 8,6 | 37 |
One more sample of neural network application is related to vaneless diffuser performance. Numerical investigation by CFD (ANSYS CFX) of diffusers with relative width 0.014–0.10 at different Mach and Reynolds numbers is published in [12]. Diffusers were considered as isolated channels. Diffuser loss coefficient is presented as function of an inlet flow angle.

The object of modeling is a friction loss coefficient $\lambda$ introduced by authors of [1] and used by authors of [12]. The loss coefficient $\zeta$ as calculated by ANSYS CFX is connected with friction loss coefficient [1] as follows:

$$
\lambda = \left( \frac{\zeta b_2}{D_2} \left( \frac{1}{D_1} - \frac{1}{D_2} \right) \right) \times 4 \sin \alpha_2.
$$

**Figure 4.** Performances of the stage: a) $b_2=0.0484$, $\beta_{N2}=37^\circ$, $Mu=0.36$.

b) $b_2=0.039$, $\beta_{N2}=52.1^\circ$, $Mu=0.794$.

**Figure 5.** Performances of the stage: a) $b_2=0.0345$, $\beta_{N2}=48.5^\circ$, $Mu=0.785$.

b) $b_2=0.010$, $\beta_{N2}=35^\circ$, $Mu=0.589$. 

| $\bar{R}_{s_2}$ | 0 | 0.0653 | $z_{rch}$ | 16 | 32 |
|-----------------|---|--------|-----------|----|----|
| $\bar{D}_{3}$ | 1.01 | 1.05 | $\bar{\delta}_{rch}$ | 0.028 | 0.057 |
Figure 6. Performances of the stage: a) $\beta_2 = 0.063$, $\beta_{2s} = 22.5^\circ$. $Mu = 0.36$

b) $\beta_2 = 0.056$, $\beta_{2s} = 85.5^\circ$. $Mu = 0.92$

For identification of models of friction coefficient $\lambda$ and loss coefficient $\zeta$ data on VLD with the relative radius 1.60 were chosen. CFD performance calculations were made at velocity coefficient (compressibility criterion) 0.39; 0.64 and 0.82. Corresponding Reynolds numbers are $6 \times 10^6$, $9.2 \times 10^6$ and $10.4 \times 10^6$. The flow angle $\alpha$ changed with a step 5°. The maximum and minimum values of the arguments used when forming selection of basic data are specified in table 2.

Figure 7. Performance of the stage $\beta_2 = 0.063$, $\beta_{2s} = 37^\circ$ at different Mach numbers.

a) $Mu = 0.36$, b) $Mu = 0.648$, c) $Mu = 0.864$
Figure 8. a) Friction loss coefficient \( \lambda = f(\alpha_z, \frac{b_2}{D_2}, M, \text{Re}) \) \( M = 0.39, \text{Re} = 6.2 \times 10^6 \),

b) Friction loss coefficient \( \lambda = f(\alpha_z, \frac{b_2}{D_2}, M, \text{Re}) \) \( M = 0.82, \text{Re} = 10.42 \times 10^6 \)

Thus, for training of mathematical models, a sample of 308 value vectors was formed. At an early stage some models with various configurations (1-3 layers, 10-30 neurons) were tested. After training networks with the minimum mean square error were chosen. These are two-layer networks, with one neuron in an output layer and 10 neurons in the hidden layer (for \( \lambda = f(\alpha_z, \frac{b_2}{D_2}, M) = 25 \text{ neurons} \)), wherein function of activation of neurons is a logical sigmoid.

Calculation results of friction coefficient \( \lambda \) are presented in Figure 8 a, b. The mean squared error is 2.15%. Mathematical models for coefficient of loss coefficient \( \zeta \) were similarly developed, the mean squared error is 2.45%. Results of modeling are shown in Figure 9 a, b.

| Table 2. Range of VLD parameters | Parameter | Minimum | Maximum |
|----------------------------------|-----------|---------|---------|
| \( \bar{b}_2 \) | 0.014 | 0.1 |
| \( \alpha_z \) | 10 | 45 |
| \( \lambda \) | 0.0125 | 0.0367 |
| \( \zeta \) | 0.0263 | 0.4968 |
Figure 9. a) VLD loss coefficient $\zeta = f(\alpha_2, \frac{b_2}{D_2}, M_{c2}, Re_{c2}) \ M = 0.64, \ Re = 9.2 \times 10^5$;  
b) VLD loss coefficient $\zeta = f(\alpha_2, \frac{b_2}{D_2}, M_{c2}, Re_{c2}) \ M = 0.39, \ Re = 6.2 \times 10^5$.

Coefficients of weight and bias show interrelation between neurons and extent influence of parameters on function, characterizing model. Tables of coefficients are not presented due to their bulkiness, but only the data of the analysis of weights of the received models are shown. The weights averages on all neurons for each entrance argument are presented in table 3.

Table 3. Average values of weights for input parameters

| Function | Average weight on all neurons for parameter |
|----------|--------------------------------------------|
|          | Re$_{c2}$ | $M_{c2}$ | $\bar{b}_2$ | $\alpha_2$ |
| $\lambda = f(\alpha_2, \frac{b_2}{D_2}, M_{c2}, Re_{c2})$ | 2.57 | 2.08 | 3.39 | 2.22 |
| $\zeta = f(\alpha_2, \frac{b_2}{D_2}, M_{c2}, Re_{c2})$ | 2.27 | 1.97 | 4.17 | 1.44 |

Average weight coefficients are of the same order. The weight coefficient for $\bar{b}_2$ is the biggest.

Conclusion

Practical application of neural models for the calculations of the compressor characteristics is reduced to the data input in model in the same format and an order which was set at identification. Input of the basic data can be made for calculations manually or by loading of previously prepared tables. The values of friction coefficient calculated based on model and the coefficient of losses can be shown also in the form of tables or matrices. These data can be kept or imported for further work with them in other programs.
The obtained data on the calculation errors by means of the neural networks allows drawing a conclusion about the practical importance of the received mathematical models, as a tool for generalization and the analysis of the data.

Thus, the adapted and trained, artificial neural networks represent the paralleling systems, capable of training by the analysis the positive and negative impacts. Experiments with neural networks, in relation with the modeling of characteristics of centrifugal compressors, showed the encouraging results that speak about prospects of this direction, especially at the description of processes with a large number of arguments. The authors are ready to provide the interested persons with the above-described neural network models for use in calculations.

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