Higher-Twist Effects in the Drell-Yan Angular Distribution

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Abstract

We study the Drell-Yan process $\pi N \rightarrow \mu^+\mu^-X$ at large $x_F$ using perturbative QCD. A higher-twist mechanism suggested by Berger and Brodsky is known to qualitatively explain the observed $x_F$ dependence of the muon angular distribution, but the predicted large $x_F$ behavior differs quantitatively from observations. We have repeated the model calculation taking into account the effects of nonasymptotic kinematics. At fixed-target energies we find important corrections which improve the agreement with data. The asymptotic result of Berger and Brodsky is recovered only at much higher energies. We discuss the generic reasons for the large corrections at high $x_F$. A proper understanding of the $x_F \rightarrow 1$ data would give important information on the pion distribution amplitude and exclusive form factor.

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The Drell-Yan process of inclusive muon pair production in hadronic collisions [1] is the most important means of determining the valence parton content of hadrons other than the nucleon. The kinematics are defined by the longitudinal-momentum fraction of the muon pair, $x_F$, the squared invariant mass of the pair, $Q^2$, and the transverse momentum, $Q_{\perp}$, of the pair. In the Drell-Yan model the reaction is described as the annihilation of a massless quark and an antiquark into a virtual photon which decays into a muon pair. Because of rotational invariance and parity conservation, the polar angle distribution of massless muons in the muon pair rest frame must be of the form

$$1 + \lambda \cos^2 \theta. \quad (1)$$

In the Drell-Yan model $\lambda = 1$ corresponding to the fact that the annihilation of on-shell spin-$\frac{1}{2}$ particles produces a transversely polarized virtual photon. This expectation is maintained within small corrections in the presence of order $\alpha_s(Q^2)$ perturbative QCD radiative corrections [2].

Experimentally, a transverse photon polarization is observed in most of the kinematic region. At large $x_F$, however, the photon appears to become longitudinally polarized [3, 4]. In the following, we shall study a higher-twist mechanism leading to longitudinal polarization, within a model originally suggested by Berger and Brodsky (BB) [5].

To produce a large $x_F$ muon pair, it is necessary that $x_a$, the momentum fraction carried by the annihilating projectile parton, is close to 1. Fock states $F$ of the projectile $H$ where one parton carries most of the momentum have large energy,

$$E_H - E_F = \Delta E \simeq -\frac{k_{\perp}^2 a}{2p(1 - x_a)} \quad (2)$$

where $-k_{\perp}$ and $(1 - x_a)p$ are the transverse and longitudinal momenta of the
soft parton(s) in the Fock state. In the limit \( x_a \to 1 \), the lifetime of the Fock state, \( \tau_F = 1/\Delta E \propto 1-x_a \) is short, and perturbation theory is applicable. In the BB model, the leading contribution with a pion projectile is then given by the diagrams of Fig. 1, where the spectator valence quark transfers its momentum to the active quark via single gluon exchange. Thus the whole pion state participates in the scattering, implying longitudinal polarization of the virtual photon due to helicity conservation in the pion-virtual photon interaction.

As a first approximation, BB took the nonperturbative wave function of the pion to be \( \delta(z - \frac{1}{2}) \), i.e., both valence quarks carry half of the pion momentum. In the limit

\[
x_a \to 1, \quad k_{\perp a}^2/Q^2 \to 0 \quad \text{with} \quad k_{\perp a}^2/Q^2 \sim (1-x_a)^2,
\]

the differential cross section is proportional to

\[
(1-x_a)^2(1+\cos^2\theta) + \frac{4k_{\perp a}^2}{9Q^2}\sin^2\theta.
\]

This gives a simple expression for the parameter \( \lambda \) in the angular distribution,

\[
\lambda = \frac{(1-x_a)^2 - 4k_{\perp a}^2/9Q^2}{(1-x_a)^2 + 4k_{\perp a}^2/9Q^2}.
\]

It is clear that if \( (1-x_a)^2 \) vanishes faster than \( k_{\perp a}^2/Q^2 \), the angular distribution turns into \( \sin^2\theta \), and the parameter \( \lambda \) approaches \(-1\). Experimentally, a drop in \( \lambda \) at large \( x_F \) is observed, but the data lie well above the curve derived from the asymptotic expression. The purpose of this letter is to show that there are important nonasymptotic corrections to the limiting expression at present energies. The BB model evaluated with exact kinematics is, in fact, in rather good agreement with the data.

\footnote{This assumption was relaxed and the BB model generalized in Ref.}
The muon pair production cross section resulting from the model of Fig. 1, without taking the limit (3) and after integrating over the azimuthal angle of the $\mu^+$, is

\[
\frac{Q^2 d\sigma_{\pi^{-}N\rightarrow \mu^{+}\mu^{-}X}}{dQ^2 d^2Q_{\perp} dx_F d\cos \theta} \propto (\alpha_s \psi_{\pi}(r_{\perp} = 0))^2 |p| \int \frac{dx_a dx_b d^2k_{\perp a} d^2k_{\perp b}}{\sqrt{k_{\perp a}^2 + (1 - x_a)^2 p^2}} 
\times \left[ e_u f_{u/N}(x_b, k_{\perp b}) + e_d f_{d/N}(x_b, k_{\perp b}) \right] \frac{1}{st^2} \left[ \varrho_{11}(1 + \cos^2 \theta) + \varrho_{00} \sin^2 \theta \right] 
\times \delta(Q^2 + Q_{\perp}^2 + x_F^2 p^2 - |p| + \sqrt{k_{\perp b}^2 + x_b^2 p^2} - \sqrt{k_{\perp a}^2 + (1 - x_a)^2 p^2}) 
\times \delta(x_F - x_a + x_b) \delta(Q_{\perp} - k_{\perp a} - k_{\perp b}) \tag{6}
\]

where $p$ is the pion momentum in the hadron center-of-mass frame, and we have expressed the invariants at the $\pi$-parton level as

\[
s = 2p \cdot p_b = 2|p| (\sqrt{k_{\perp b}^2 + x_b^2 p^2} + x_b |p|) \tag{7}
\]

\[
t = -2p \cdot p_1 = -2|p| (\sqrt{k_{\perp a}^2 + (1 - x_a)^2 p^2} - (1 - x_a) |p|), \tag{8}
\]

neglecting all masses but $Q^2$. In terms of $s$ and $t$, the diagonal density matrix elements $\varrho_{MM} = \varrho_{-M,-M}$ for the production of a virtual photon with spin projection $M$ in the Gottfried–Jackson frame are

\[
\varrho_{11} = \frac{2}{(s + t)^2(2Q^2 - s - t)^2} \left[ s^5 + 3s^4 t + 3s^4 Q^2 + 4s^3 t^2 + 6s^3 t Q^2 + 4s^2 t^3 - 2s^2 t^2 Q^2 - 4s^2 Q^6 + 3s t^4 - 10s t^3 Q^2 + 8s^2 Q^4 + t^5 - 5t^4 Q^2 + 8t^3 Q^4 - 4t^2 Q^6 \right], \tag{9}
\]

\[
\varrho_{00} = -\frac{8stQ^2}{(s + t)^2}, \tag{10}
\]

In the following, we will neglect the target parton transverse momentum $k_{\perp b}$ in Eq. (3). In terms of the density matrix elements above, the polarization parameter $\lambda$ of Eq. (11) then becomes:

\[
\lambda = \frac{\varrho_{11} - \varrho_{00}}{\varrho_{11} + \varrho_{00}}. \tag{11}
\]
In Fig. 2 we show $\lambda$ as a function of $x_F$. Our result (solid line) takes the exact kinematics into account. We compare this to the BB limit of Eq. (1) (dashed line) and to the E615 data [4]. We fix $k_{\perp a}^2 = 0.8$ GeV$^2$, $Q = 4.5$ GeV and $p_{\text{beam}} = 252$ GeV for both calculations, in accordance with the data [4]. There is a sizeable difference between the solid and dashed curves, indicating the importance of nonasymptotic kinematics at present energies. Furthermore, the agreement with the data is clearly better using the exact kinematics. At this value of $k_{\perp a}^2$, we find that the general expression (6) gives values of $\lambda$ consistent within $\Delta \lambda = 0.1$ with the asymptotic limit (4) for $x_F \leq 0.9$ when $Q^2$ and $s$ are scaled by a common factor $> 25$. Since the mean value of $k_{\perp a}^2$ should increase with $Q^2$, the approach to the limit (5) will in reality be even slower.

There are generic reasons for the large finite energy corrections at high $x_F$. This can be seen, e.g., from the definition of $t$ in (8). For $|p| \to \infty$ at fixed $x_a$ and $k_{\perp a}$, the leading term of $O(p^2)$ in $t$ cancels, giving $t \simeq -k_{\perp a}^2/(1 - x_a)$. On the other hand, for $x_a \to 1$ at fixed (albeit large) $|p|$ we get the very different result $t \simeq -2|p||k_{\perp a}|$. A similar sensitivity to the order in which the limits are taken can be seen in the argument of the first delta function in Eq. (1). This is the constraint which fixes the relation between the BB model variables $x_a$, $x_b$ and the measurable quantities $x_F$, $Q^2$. In Ref. [4], the data was compared to the BB model as a function of $x_a$, using the relation $x_a = \frac{1}{2}(x_F + \sqrt{x_F^2 + Q^2/|p|^2})$, which holds only in the asymptotic limit. In Fig. 2, we chose the physical quantity $x_F$ instead, to avoid ambiguities related to model kinematics.

At present fixed target energies, $|p| = O(10$ GeV), hence $(1 - x_F)|p| = O(1$ GeV) $\simeq \langle |Q_{\perp}| \rangle$ for $x_F = 0.9$. Thus it is not surprising to find large deviations due to finite energy effects. The improved agreement with data
obtained in Fig. 2 when such effects are taken into account is of course encouraging. However, it is also an indication that one should carefully reconsider the applicability of the twist expansion of QCD, on which the BB model is based.

In the usual, leading twist QCD approach, the time scale $\tau_I \approx 2p/Q^2$ of the hard interaction (here $q\bar{q} \to \mu\mu$) is much shorter than the lifetime of the Fock state, $\tau_F \approx 2p(1 - x_a)/\k_{\perp a}^2$ (c.f. Eq. (2)). In the BB model [5], the limit (3) was taken such that $\tau_I/\tau_F = O(1 - x_a)$ is vanishing. Hence the factorization between the wave function dynamics and that of the hard scattering subprocess is still valid. On the other hand, in the limit [7]

$$x_a \to 1, \quad Q^2 \to \infty \text{ with } \k_{\perp a}^2/Q^2 \sim (1 - x_a) \quad (12)$$

we have $\tau_I \sim \tau_F$ and the dynamics of the subprocess scattering is inseparable from that of the Fock state. In this limit the twist expansion breaks down and the two new diagrams shown in Fig. 3 contribute at leading order to the muon pair production process.

At finite energies, it is not obvious which (if any) of the asymptotic limits (3), (12) is more appropriate. Although one may, as we have done here, take into account the exact kinematics within a given model, any application of perturbative QCD must still depend on an idealized high energy limit. It would be worthwhile to systematically study how the effects of finite energy corrections may be minimized, since they reflect the inherent uncertainties of the approach. In the present case, we did check numerically that including the diagrams of Fig. 3 does not significantly change the prediction for $\lambda$ given by the solid curve in Fig. 2.

Muon pair production is a good test case for studying large $x_F$ QCD dynamics, since the number of diagrams is relatively small, and data is avail-
able. This reaction is moreover used to determine the structure function of the pion, as well as the shadowing effects of antiquarks in nuclei [8]. The reliability of these determinations at large $x_F$ depend on a proper understanding of the dominant reaction mechanism. In the limits (3), (12) the production cross section is not given by the projectile structure function but rather by the distribution amplitude, *i.e.*, the valence wave function at vanishing transverse distance between the partons [9], as indicated in Eq. (6). The pion distribution amplitude is in itself of considerable current interest. It is not conclusively settled whether the exclusive pion form factor data can, at present energies, be described using the asymptotic hard QCD dynamics of Ref. [9] or whether the nonperturbative “Feynman” mechanism [10] is more appropriate. A better understanding of large $x_F$ muon pair production could help resolve this issue, by yielding information on the distribution amplitude.

While completing this work, we learned of a related study of the muon angular distribution at high $x_F$ [11]. These authors are considering the effects of terms where the transverse momentum $Q_\perp^2$ of the pair is comparable to $Q^2$. In the high energy limit where this ratio is nonvanishing, the lifetime $\tau_F$ of the Fock state is actually much shorter than the time scale $\tau_I$ of the hard scattering. This illustrates a further possibility of taking limits, not covered by Eqs. (3) and (12).

To summarize, we have found that there are general reasons to expect large finite energy corrections when applying QCD expressions that are valid in $x_F \to 1$ limits to current data. As a case study, we considered the model proposed by Berger and Brodsky [3], and found that this model agrees with data on the angular distribution of the muon pair only when the kinematics are treated exactly.
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Figure Captions

**Fig. 1.** The relevant Feynman graphs for the Berger and Brodsky higher-twist model of Drell-Yan production.

**Fig. 2.** The polarization parameter $\lambda$ of Eq. (1) as a function of $x_F$. Our calculation (11) with exact kinematics is shown by the solid line, the prediction (4) of the Berger-Brodsky limit (3) by the dashed line, and the data (4) by the boxes. We used $p_{\text{beam}} = 252$ GeV, $Q = 4.5$ GeV, $k^2_{\perp a} = 0.8$ GeV$^2$ and $k^2_{\perp b} = 0$.

**Fig. 3.** The two higher-twist diagrams which, together with the diagrams of Fig. 1, contribute at leading order in the limit (12).
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