Logic, Language, and Calculus

Logical Understanding I

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Abstract

The difference between object-language and metalanguage is crucial for logical analysis, but has yet not been examined for the field of computer science. In this paper the difference is examined with regard to inferential relations. It is argued that inferential relations in a metalanguage (like a calculus for propositional logic) cannot represent conceptual relations of natural language. Inferential relations govern our concept use and understanding. Several approaches in the field of Natural Language Understanding (NLU) and Natural Language Inference (NLI) take this insight in account, but do not consider, how an inference can be assessed as a good inference. I present a logical analysis that can assess the normative dimension of inferences, which is a crucial part of logical understanding and goes beyond formal understanding of metalanguages.

1 Introduction

1.1 Context – Commitments

Language is not only descriptive, it is also normative. Just by describing language and how it is used, i.e. creating models from language use, it will not be possible to recreate an ability that yields understanding. That language is normative does not only mean that language use is guided by norms, but it also implies a certain assessment of what is correct and what is incorrect language use. Dictionaries and grammar books encapsulate the correct use of a natural language, so does logic for artificial as well as for natural languages.

1.2 Problem

Approaches in the field of natural language understanding focus on the descriptive part of language use. They describe how language is used. It can lead to a certain understanding or better representation of the language usage of a group and depict maybe also to a certain degree a semantic component, but it will not yield a more complete understanding of semantic significance, unless also the normative component is considered. Normativity is a key part of meaning.

We play different language games that are governed by different norms. Our assessment of the normative significance guides our understanding. Normative
assessment of meaning is something exceptionally done by humans, but can it be also computed or to a certain degree expressed in mathematical models? First and foremost it has to be examined, how it can be expressed logically in order to see how it can be implemented in a mathematical model.

1.3 Motivation – The Status of Logic

It is important to distinguish between the status of the forms of languages that are used. The distinction between object-language and metalanguage is crucial, because the metalanguage allows the possibility to talk about the object-language and the concepts that cannot be expressed in the object-language. Paul Hoyningen-Huene calls the metalanguage of statement logic (object-language) “metalogic of statement logic”. It allows one to express the concepts of “logical truth” and “valid inference” for statement logic. Both concepts are determined by the use of the operators in statement logic and are purely syntactical. The logical truths of statement logic are therefore also only tautologies. It is possible to build a calculus of statement logic as a metalanguage.

Scientific examinations are written in natural language, but it is not directly a metalanguage in the logical sense, because it describes examined objects, although e.g., you can write an examination about German grammar in English. English is then your metalanguage that you use to describe German: the object-language. The difference between metalanguage and object-language can also be used to clarify, about what one talks, i.e. what is the object: an object/entity or an expression (e.g. the name of a city or the city itself). But when it comes to describe and assess the logic of language, this distinction faces several problems.

Logic is not just a formal tool that formulates a calculus of an object-language in an artificial metalanguage, it is part of the use of language. The attempt of this paper is to understand the status of logic in language use. I believe that making explicit this status will not only lead to a deeper understanding of logic and language, but will also enable us to build mathematical models of natural language that can be used in machine learning.

2 Philosophical Background

Kurt Gödel writes that “every precisely formulated yes-and-no question in mathematics must have a clear-cut answer.” Gödel was a Platonist and believed that there is a universe of all possible discourse, when it comes to mathematical objects. This ontological part has to be considered. Many problems about the foundations of mathematics in the first half of the 20th century were centered around the connection of set theory and logic as the fundament of mathematics. Logic gives in a certain way the syntactical or formal structure and set theory the semantical or ontological component. It depends in a way then on one’s “ontological commitment” to mathematical objects and how big

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1 The disjunction “or” should not mean that these terms can be used as synonyms. This connection is much more complicated.
the ontological realm should be, i.e. what it all includes, e.g. physical objects, mathematical objects, and so forth. Willard Van Orman Quine sees them as “myths” that are “good and useful”, because they simplify our theories.\[4\] The problem is that these mathematical objects should serve as truth-makers of mathematical propositions and if there is a possible universe of all mathematical objects, then this implies that the propositions are decidable. It seems like Gödel had the idea that such a decidability for mathematics could be possible\[3\], despite his incompleteness theorems. – It is not the case that there exists a universe of all possible discourse with truth-makers for propositions or concepts. We do not discover the meaning of them and there can also not be some kind of “metaphysical glue”\[5\] (109) that attaches a word to a referent or object that conveys or gives meaning to the word or also to a proposition.

Gödel’s incompleteness theorems stem from the phenomenon of self-reference, i.e. a system is powerful enough to talk about itself or at least to name objects. The system or mathematical model is able to formalize mathematical objects via natural numbers (the so called “Gödel numbering”). Every arithmetic formula is coded as a number and is assigned a definite Gödel number. The Gödel numbering is now used to formulate formulas that are expressively powerful enough to define concepts like “provable formula” and then also the negation of this concept.\[6\] \(175/176\) It is sometimes expressed in the following way:

\[
F = \text{“I am not provable.”} \quad \text{[7]}
\]

Gödel writes that we have a sentence that asserts its own unprovability.\[6\] (175) “Suppose, \(F\) is wrong. Then \(F\) can be proven and has therefore shown that \(F\) is not provable. This is a contradiction. Thus, \(F\) is true and therefore not provable.”\[7\]

Of course, it is a mix up of the predicates “true” and “provable”. Gödel plays with the similarity of the meaning “provable” and “true”. The former is a property of formulas and the latter a property of sentences. Gödel also mentions that these ideas are cognate with the liar paradox, where it is just about the truth of statements.\[6\] Tarski developed this problem further in the field of semantics towards a formalized theory of truth.\[8\] Both contributions to logic lie on the possibility of building a metalanguage to speak about an object-language.

It is problematic to hang the reflexivity of language on the difference between metalanguage and object-language, because the metalanguage consists of (abstract) names (metavariables) for the object-language, where it is about the use of language. One of the consequences of Gödel’s incompleteness theorems is that the construction of a metalanguage with well-defined names for predicates like “provable” or “true” is not possible, because it leads to contradictions. It has not been possible to develop a well-defined metalogical or metamathematical model with well-defined names for “provable” or “true” and despite that the mathematicians use proofs and despite that the philosophers use the predicate “true”, so it does not seem to confound us in our use of these words or better in the language games that we play. – I believe that the reflexivity of language stems from an epistemic gap between holding something for true
and that something is true (or between seeming right and being right). The possibility to err or to be wrong opens up the language game that we play. (How this is connected with modal logic, will be explained below.)

Artificial languages and formalization help to disambiguate the meaning of sentences and to understand the important distinction between metalanguage and object-language. This logical distinction can be sometimes seen as pedantic, but it is important to not get lost in the debates in linguistics, computer science, mathematics, and philosophy. It has to be clear, if it is just a name or if the word is used in a language and also how it is used. Naming something is also using a word (like e.g. predicating). Peter Geach pointed out that there is a difference between calling a thing “P” and predicating of a thing “P”. It may be that the predication “P” of a thing is embedded in an if-then-clause or in a disjunctive proposition and then it is embedded in a logical context.

“To say, ‘If the policeman’s statement is true, the motorist touched 60 mph’ is not to call the policeman’s statement true; to say, ‘If gambling is bad, inviting people to gamble is bad’ is not to call either gambling or invitations to gamble ‘bad.’” (223)

True is used here as a descriptive predicate. It makes the content judgeable and does not solely take it as a speech act (ascription). The if-then-clause specifies also under which circumstances it should be correct to call the policeman’s statement true. It gives to the descriptive content a normative assessment, because “true” is a normative concept and not a descriptive concept. If one commits oneself to the antecedent, one also has to commit oneself to the consequent. Committing oneself is a normative doing along with being entitled to a claim.

If we want to understand what meaning means, we cannot just look at the manipulation of symbols. This would be just manipulating strings of names connected by logical operators. At least this is the valid insight of John Searle’s thought experiment about the chinese room. The outcome of the argument is that understanding natural languages is something more than to follow rules or instructions. But then how can the trick be done? This question is not answered by Searle. He believes that we first need to understand the brain to build something like a strong artificial intelligence that understands natural languages. It might be also a valid way to first look at technological advancements, to understand how the trick is done. Therefore, one should look into the field of “Natural Language Understanding” (NLU) in computer science.

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2 Another example that reflects the ideas of this paper is: If there is not a universe of all possible discourse with meaningful facts, that warrants objectivity, then meaning does not have to be subjective.

3 Brandom emphasizes this point specifically for the connection of philosophy of language, cognitive science, and artificial intelligence in the chapter “How Analytic Philosophy Has Failed Cognitive Science”.

4 Commitment and entitlement are deontic concepts. See [13]
3 Literature Review

3.1 General Outlook – Natural Language Understanding

There are different forms of understanding that have to be considered. There are approaches that group together expressions that have similar meanings. This can be done by vector space models of semantics. (For a general overview of these models see Turney et al. (2010).) Another form of understanding is to be able to answer queries about a text. Hermann et. al. (2015) present a model that can identify objects (expressions) within a text as answers to questions. Evans et al. (2018) propose a model for “recognizing entailment between logical formulas”. Of course, they also state that there is a difference between recognizing entailment between logical formulas and “recognizing entailment between natural language sentences”. “Evaluating an entailment between natural language sentences requires understanding the meaning of the non-logical terms in the sentence.” They seem to have the idea that first the formal logical understanding of the model has to work, before one can apply it in natural language.

A crucial point is, how do they now that the entailments are correct or valid? The write that “[e]ntailment is primarily a semantic notion: $A$ entails $B$ if every model in which $A$ is true is also a model in which $B$ is true.” This is a definition in the metalanguage of the calculus of propositional logic and in order to “test if $A \models B$,” they “test whether $A \land \neg B$ is satisfiable.” In the metalanguage of the calculus it is tested, whether there is an inconsistency or not. The normative assessment is solely based on the principle of avoiding inconsistency and establishing a consistent calculus.

3.2 Natural Language Inference

In 2006 Dagan et. al wrote one of the first papers in the field of Natural Language Inference (NLI). They stated “that textual entailment recognition is a suitable generic task for evaluating and comparing applied semantic inference models.” And also hoped that “[e]ventually, such efforts can promote the development of entailment recognition ‘engines’ which may provide useful generic modules across applications.”

Bowman et al. (2015a) elaborate a neural network model of a “relational conception of semantics” as a counterpart to distributed semantic representation. The meaning is governed by the inferential connections:

“For instance, turtle is analyzed, not primarily by its extension in the world, but rather by its lexical network: it entails reptile, excludes chair; is entailed by sea turtle, and so forth. With generalized notions of entailment and contradiction, these relationships can be defined for all lexical categories as well as complex phrases, sentences, and

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5 We believe that isolating the purely structural sub-problem will be useful because only networks that can reliably predict entailment in a purely formal setting, such as propositional (or first-order) logic, will be capable of getting these sorts of examples consistently correct.”
The Stanford Natural Language Inference (SNLI) corpus\cite{SNLI} contains 570k sentence pairs with the labels entailment, contradiction, and neutral and is used e.g. also by Rocktäschel et al. (2015)\cite{Rocktaschel2015} and Cases et al. (2017)\cite{Cases2017}. Nie et al. (2019) focus on another aspect of NLI: the elaboration of a data collection method.\cite{Nie2019} Geiger et al. (2018) propose a “method for generating artificial data sets in which the semantic complexity of individual examples can be precisely characterized”. The “method is built around an interpreted formal grammar that generates sentences containing multiple quantifiers, modifiers, and negations”.\cite{Geiger2018} For an approach that relies on artificial data sets see also Geiger et al. (2019)\cite{Geiger2019}. There have been also studies about monotonic inferences through “interactions of entailment reasoning with negation”.\cite{Geiger2020} Geiger et. al. (2020) call it “Monotonicity NLI”\cite{Geiger2020}.

4 Problems of Reasoning

The SNLI corpus from Bowman et al. (2015) contains statements that are based on descriptions of images. Entailment, contradictory, and neutral statements were compiled and verified, but it is hard to reason with inferential relations that were made like the ones in the corpus, because logical connectors govern or should govern the correctness of inferences. The database is a descriptive representation of language and does not include a normative assessment of the inferential relations per se. It is something different to follow the rule and to be able to assess the correctness of the rule. That is why Ludwig Wittgenstein says that rule-following is a practice, because the mastery of a practice is something else than just blindly following a rule.

If one claims that e.g. x is a turtle, then it might make sense to claim that this claim excludes the claim that x is a chair (a contradictory statement). And it makes sense to infer that, if x is a turtle, it is an animal (entailment statement). One could now “infer” that a chair is also not an animal. That is correct, but if the contradictory statement would be the claim that x is a bird, one could make the inference that x is not an animal, which is not correct.

That a turtle is an animal is a form of inductive reasoning. It cannot be just inferred whether a bird is an animal or not. It relies on other inferences and facts – or on a semantic web that yet has not been established, but should be learned. In deductive reasoning semantic relations can be established more easily. If animal is contradictory to furniture, then all that is incompatible with animal will also be incompatible with the more specific concepts (like e.g. bird and turtle) that fall under the more general concept. But for deductive reasoning also a semantic web has to be already established.

\footnote{We would like to determine whether a system can actually reason about lexical entailment and, furthermore, whether it has learned that negation is downward monotone (roughly, that A entails B if, and only if, not-B entails not-A, for all A and B).\cite{Geiger2020} (2)}
Robert Brandom introduces the idea of an incompatibility semantics that is more precise with the logical vocabulary. Contradiction in formal logic has a different sense than the use of contradiction in natural language inferences in models of machine learning. Contradiction and entailment have already a (logical) meaning that governs their use. One needs to be precise of the use of concepts on a logical level otherwise the use of non-logical vocabulary is even more difficult. Of course, in natural language we often understand the ambiguous use of words or even the strange use of non-logical vocabulary, because we are mastering the practice of speaking and understanding the language and we can (at least most of the time) make sense of it. – I believe that the logic, that governs inferences, can be made explicit by modal vocabulary, which hopefully can be represented in mathematical models.

4.1 Modal Logic

Modal logic is an extension of propositional logic. The first extension is the so-called system $\mathbf{K}$ (after Saul Kripke) and the next extension is the system $\mathbf{T}$. This system introduces the following axiom:

$$\Box \varphi \rightarrow \varphi$$

With this axiom the following theorem can be proven:

$$\varphi \rightarrow \Diamond \varphi$$

Statements or compounded/combined statements are possible, if they are statements. This marks an important difference to propositional logic, because to take a statement as a possible statement, is taking it as a move within a language game of giving and asking for reasons by making explicit e.g. entailments of the statement that can serve as reasons. This is different to the usual semantic approaches to modal logic that rests on the notion of possible worlds.

Kripke introduces the idea of possible worlds to represent the semantics of modal logic. It is possible that a statement is true, if there exists at least one possible world in which the statement is the case. The possible worlds can stand in relations, which means that they are epistemically accessible. We could take e.g. our world as the initial possible world and think of counterfactual situations that represent other epistemically accessible possible worlds. The system $\mathbf{T}$ has an important characteristic, if it is understood within the framework of a possible world semantics. The system $\mathbf{T}$ is reflexive, i.e. that the possible world is accessible to “itself” ($wRw$) or as Kripke writes: “It is clear that every world

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7 Brandom shows, how normative and deontic vocabulary (commitment and entitlement) can make modal vocabulary explicit. Participating in practices of giving and asking for reasons (as Wilfrid Sellars puts it) for the language games that we play. We are being held responsible for our claims as commitments and have to justify, why and how we are entitled to these commitments. This is a crucial part of a pragmatic approach to language and human understanding, but it goes beyond what can be represented in mathematical models.  

8 I use greek letters for formulas, which belong to the metalanguage of propositional calculus. Latin letters are for statements of the object-language.
H is possible relative to itself; for this simply says that every proposition true in H is also possible in H." [27] (70)

An analogy of a card game might be helpful to explain the idea of possible worlds and the epistemic accessibility. [28] A player knows her own cards, but does not know the cards of all the other players, although it might be, that the player knows some cards of other players (epistemically accessible worlds) and it is possible for the player to imagine some cards of the other players (counterfactual possible worlds). If she knows her own cards, then this could also count as a certain kind of consciousness of her own (epistemic) states. This kind of reflexivity is different to the difference of metalanguage and object-language, that was mentioned above, which only originates from the possibility of naming things in a metalanguage.

For statement or propositional logic a calculus can be developed by purely syntactical means. [1] We have therefore a metalanguage that is purely syntactical. The metalanguage contains words like proofability and derivability, but these concepts have clearly also a modal background. Rudolf Carnap tried to construct a syntactical metalanguage for modal logic [29] (250/151), while e.g. Quine argued that modal logic opens an intensional context that is opaque. [30] Therefore, a calculus in a syntactical metalanguage cannot be developed. I will here not argue for one or the other side, but propose a different approach.

It is an important step in understanding that statements are only possibly true and if they are possibly true that there is at least one possible world that is epistemically accessible. Language is a social practice and different players interact to know which statement is true. Introducing modal vocabulary is a step in realizing that one is a player in that game with certain epistemic states (possible worlds). The epistemic states stand in possible relations with each other. This relations can be expressed as incompatibility and compatibility relations.

4.2 Incompatibility

The modal vocabulary operates in the scope of nonmontonic inferences, like e.g. material inferences. They express good inferences based on the principle of material incompatibility. The claim, if p then q, is incompatible with the claim that it is possible that p and not-q. [12]

\[ p \rightarrow q \text{ is incompatible with } 
\Diamond (p \land \neg q) \]

\[ p \rightarrow q \text{ is compatible with } 
\Diamond (\neg p \lor q) \]

\[ p \leftarrow q \text{ is compatible (equivalent) with } 
\Diamond (p \lor \neg q) \]

Another logical equivalence is the law of contraposition \((p \rightarrow q) = (\neg q \rightarrow \neg p)\). This law expresses a certain similarity to one tautology in statement logic: the *modus tollens* \(((p \rightarrow q) \land \neg q) \rightarrow \neg p\). So, if you know that not-q is the case, you can infer that not-p is the case, which expresses incompatibility relations of entailments that go from a specific claim to a more general claim, e.g. if x is a turtle, then it is an animal. And what is not an animal is also not a turtle.
With these material incompatibilities it can be seen, why chair is a better contradictory statement to turtle than bird, because it supports more good inferential relations. It lies also on a similar conceptual level. Chair entails furniture and everything that is incompatible with furniture is also incompatible with chair. What is incompatible with animal is also incompatible with turtle. The only problem is that a chair has (often) four legs and so does a turtle. It is a common property (of course there are different kinds of chairs, but let us leave that aside, because we cannot assume that an algorithm "knows" something like that). So, it cannot be that everything is incompatible in this case.

4.3 Entailment

What does now entailment mean? It has to be distinguished between different kinds of entailments – there is e.g. the \textit{modus ponens} (metalanguage) or the implication (object-language) – to clarify the concept of entailment. The \textit{modus ponens} is a deductive form of reasoning:

\begin{align*}
\varphi \text{ and } \varphi \Rightarrow \psi \text{ therefore } \psi
\end{align*}

The \textit{modus ponens} is sometimes called the “implication elimination”, because it allows to eliminate the implication and to detach the consequent of the implication, i.e. to assert it. With regard to Gerhard Gentzen and his calculus of natural deduction it can be represented the following way\textsuperscript{31} (186):

\begin{align*}
\varphi, \varphi \Rightarrow \psi \therefore \psi
\end{align*}

Deductive reasoning is certainly a sound way of reasoning, but the \textit{status} of it can nevertheless be questioned. Lewis Carroll’s story of the tortoise and Achilles gives an insight about the difference of metalanguage and object-language and the specific status that the \textit{modus ponens} has. The tortoise does not accept the consequent in a deductive inference and is not convinced by the application of the rule of inference to detach the consequent. The tortoise adds another premise that represents the rule of inference in the object-language ("If A and B are true, Z must be true."), but does still not accept the consequent and goes on to add more premises that supposedly should represent the rule of inference to finally detach the consequent.\textsuperscript{32} Bertrand Russell discusses these ideas by distinguishing between the assertion of propositions and the meaning of “\textit{therefore}” in the metalanguage on the one side and the meaning of “\textit{implies}” in the object-language on the other side.\textsuperscript{33} (§ 38) According to Gilbert Ryle there is a difference between applying a rule of inference, which the tortoise does not do (a kind of knowing how) or refuses to do and the acknowledgement of the propositional content (a kind of knowing that).\textsuperscript{34} (It seems that, for Ryle, to know how to do something or to apply it is prior to the knowledge of the rules or the propositional content.) The difference between metalanguage and object-language has to be considered to understand the \textit{status} of the inferences. I believe, it is therefore wrong to directly go from the \textit{modus ponens} to implications like e.g. Friedrich Kambartel and Pirmin Stekeler-Weithofer suggest\textsuperscript{35} (212/213):
The first formulation (1) is a rule of inference in the metalanguage, while the other formulation (3) is within the object-language of propositional logic. Even the attempt to write it similar like the rule of inference does fall short of the difference in scope and use, because the sentence or proposition $q$ cannot be detached and the implication cannot be eliminated in the object-language. The correct formulation would be:

$\textbf{(4)} \quad (p \rightarrow q) \land p \rightarrow q$

It is a compounded sentence in the object-language that cannot be dissolved in separate propositions. It is one proposition. The usage of the rule of inference in a meta-language, like e.g. the calculus of propositional logic, allows to use the detached and derived consequent as a premise and add it to the “true” or valid “set” of propositions ($\Sigma$). The requirement for propositional logic is then that a contradiction cannot be derived, because this would lead to an inconsistent or incoherent “set” of propositions. At least for propositional logic such a calculus can be developed.

A sentence only has meaning, if it is embedded in a logical context by the logical connector. The inferential relations show which role the sentences plays, if it is a premise or a consequent. In a metalanguage the derived consequent could also serve as a premise, but that would be to abstract from the original role it played and to not consider the context or circumstances under which it was implied. The propositions in a meta-language are just names of propositions (hence the greek letters to make that difference). They are not used and have no meaning. They are abstract and empty placeholders. Only in the logical context of an object-language they can have meaning. Propositions cannot be detached from their conditions, otherwise it would also not be an inferential semantics. In a way one needs to consider their place in the right language game (cf. Ludwig Wittgenstein).

5 Evaluation of Inferential Relations

Models that learn inferential relations are the foundation for reasoning. They represent a knowledge of the world. (The statements are descriptions of pictures like in the SNLI corpus.) An important questions is, whether the inferential relations are representing good inferences. How can one contradictory statement be better than another one? Or how can one entailment statement be better than another one? If we would take this into account, we could introduce more fine grained logical connectors, but then one might have to introduce an infinite number of logical connectors. The approach that I propose is based on the web of inferential relations that can be represented.

Meaning is not only represented in one inferential relation it is part of a whole web of inferences. It is within a possible space of reasoning and that includes more possible premises and consequences of the statements. Evans et. al.
(2018) introduce a model that learns to recognize relations of possible “entailment between logical formulas”.\cite{17} It would be an interesting task to combine this model with the model of Bowman et al. \citeyear{19} that learns inferential relations of natural language to widen the logical space of possible relations. It could make explicit further inferences. This inferences can lead to further statements, like mentioned above. It is correct to state that a chair is not an animal, while it is incorrect to state that a bird is not an animal. Of course, it relies also on the entailment that birds are animals. Meaning can only be understood within a web of inferences. This would allow the model to self-assess the goodness of inferences and make it possible to discard bad inferences.

Deductive reasoning fails to give an adequate account of (human) reasoning. Inductive reasoning has strengths and with a lot of data and statistical methods powerful tools have been developed, but does it give an account of (human) reasoning and what could be considered as a good explanation of something. To understand the goodness of inferences means to assess the normative part of reasoning.

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