Electromagnetic Inverse-Scattering Reconstructions by an Inexact Newton Method: Numerical and Experimental Results

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Abstract. This paper deals with the inverse scattering problem for electromagnetic imaging. In particular, a reconstruction procedure based on the inexact Newton method is reviewed. A new formulation, developed in the $L^p$ functional Banach space, is also considered. Numerical and experimental results recently obtained by using the above mentioned reconstruction methods are discussed with specific reference to nondestructive evaluation of dielectric materials such as plastic and wood.

1. Introduction
In this paper, the application of the so-called inexact Newton method (INM) [1]-[6] to electromagnetic imaging is reviewed. The electromagnetic inverse scattering problem [7] represents the basic formulation for image-based methods devoted to the inspection of unknown bodies in several applications, ranging from nondestructive testing in industrial areas [8][9] to biomedical diagnostics [10]. When microwaves are used as interrogating waves, the field is scattered by the target and simplifying assumptions (e.g., ray propagation) cannot be applied. Consequently, the inverse scattering problem must be addressed starting from its “exact” equations [7][11]. Only in the case one knows some a priori information about the target some approximations can be applied (e.g., Born or Rytov approximations [12]). In general, however, the problem must be addressed by considering the nonlinear equations governing the scattering phenomenon.

In the following, the formulation of the considered inverse scattering problem will be briefly outlined in Section 2. We consider here the inspection of penetrable targets by using a two-dimensional tomographic configuration. In Section 3, the INM will be described, with particular emphasis to new developments in the $L^p$ functional Banach space [13]-[17].

Numerical and experimental results obtained with the described approaches will be reviewed in Section 3. Finally, some concluding remarks are reported in Section 4.

2. Problem formulation
The inspection of a target as the solution of an inverse scattering problem can be performed following various approaches. First of all, the problem can be faced in time domain (for example, by using efficient time reversal techniques [18]) or in the frequency domain. In the last case, integral equations are usually solved [7].
We consider here the inspection of dielectric targets in a tomographic configuration. The scatterer is assumed to be an infinite cylinder illuminated under transverse-magnetic (TM) conditions. The cross section of the cylinder is arbitrarily shaped and inhomogeneous. A \( \exp[i\omega t] \) time-dependence is assumed and omitted. The object cross section is represented by a contrast function

\[
c(r) = \varepsilon_r (r) - 1 - \frac{j\sigma(r)}{\omega \varepsilon_0}
\]

which describes the dielectric characteristics of the bounded investigation domain. It depends on the relative dielectric permittivity and on the electric conductivity [S/m] of the object (\( \mathbf{r} \) denotes the position vector in the transversal plane and \( \omega \) the angular frequency). The object is successively illuminated by incident waves and the field scattered is collected in a set of measurement points (the observation domain, \( D_{\text{meas}} \)). The inverse problem can be formulated by using the following two equations:

\[
e_{\text{scatt}} (\mathbf{r}) = -k_0^2 \int_{D_{\text{inv}}} c(\mathbf{r'}) e_{\text{inc}}(\mathbf{r'}) g_0(\mathbf{r}, \mathbf{r'}) d\mathbf{r'}, \quad \mathbf{r} \in D_{\text{meas}}
\]

\[
e_{\text{tot}} (\mathbf{r}) = e_{\text{inc}} (\mathbf{r}) + k_0^2 \int_{D_{\text{inv}}} c(\mathbf{r'}) e_{\text{inc}}(\mathbf{r'}) g_0(\mathbf{r}, \mathbf{r'}) d\mathbf{r'}, \quad \mathbf{r} \in D_{\text{inv}}
\]

where the involved field quantities refer to the incident, scattered, and total field components, which are polarized as the cylinder axis. The above equations are scalar ones, due to the assumptions about the problem geometry and illumination (the index of the multiple illumination is omitted here). Equation (2) is usually called the data equation, whereas equation (3) is called the state equation [7]. Both equations involve the Green’s function for free space \( g_0(\mathbf{r}, \mathbf{r'}) \) [12] and the propagation constant \( k_0^2 = \omega^2 \varepsilon_0 \mu_0 \). The integrals are performed over the investigation domain \( D_{\text{inv}} \). Equations (2) and (3) can be solved simultaneously or they can be combined together in a unique operator equation having the contrast function as the sole unknown term, i.e.,

\[
e_{\text{scatt}} (\mathbf{r}) = F(c) = G_{\text{data}} \tilde{c} ((I - G_{\text{state}} \tilde{c})^{-1} e_{\text{meas}})(\mathbf{r}), \quad \mathbf{r} \in D_{\text{meas}}
\]

where \( G_{\text{state}} \) and \( G_{\text{data}} \) denote the operators related to equations (1) and (2), respectively, and \( \tilde{c} \) is given by \( \tilde{c}(\cdot) = c(\cdot) \bar{c}(\cdot) \).

It should be mentioned that other formulations are possible, e.g., by using the equivalent current density

\[
j_{eq}(\mathbf{r}) = c(\mathbf{r}) e_{\text{inc}}(\mathbf{r})
\]

as an unknown. This approach is usually referred as the contrast source method [19]. In this case, it results that equation (2) is linear and some mitigation of the overall nonlinearity of the problem seems to be achievable [20]-[22].

Several numerical methods, both deterministic and stochastic ones have been devised to solve the above problem (see, for example, [23]-[41] and the references therein). In [42]-[48] the use of the INM has been proposed. The INM solves the nonlinear integral equations in a regularized way by “combined iterative processes with Newton’s method as primary method and some, as yet unspecified, secondary method for solving the [obtained] linear systems” [3]. In particular, the scheme proposed in [42] is an outer/inner iterative algorithm which can be summarized as follows:

1) Outer steps: the equations are linearized by means of a Newton’s first-order expansion involving the Fréchet derivative of the operator. The functional equation \( F(x) = y \) (e.g., equation
(4)) is iteratively solved by finding a function \( h \) such that \( F'_{u_k} h = y - F(x_k), \) \( k = 0,1,\ldots \) (until a stopping criterion is fulfilled) and by updating the solution with \( x_{k+1} = x_k + h . \) \( F'_{u_k} \) denotes the Fréchet derivative.

2) \textbf{Inner steps:} regularized solutions of the obtained linear equations are obtained by the truncated Landweber method [49]-[51]. In particular, this method is based on the following relation

\[
h_{i+1} = h_i - \tau_i F'_{u_k} (y_k - F'_{u_k} (h_i)), i = 1,2,\ldots \text{max}
\]

where \( 0 < \tau_i < 2\|F'_{u_k} F'_{u_k}\|_2^{-1} \) is the relaxation parameter, \( y_k = y - F(x_k), \) and \( h_0 = 0 . \)

The inner iterative method is a regularization algorithm, i.e., the components of the input data corrupted by noise, usually related to the highest frequencies Fourier components, are filtered out during the first iterations [52]. A regularized solution must be computed since any Newton step gives rise to an ill-posed linear equation. A similar outer–inner Newton–Landweber scheme for a general nonlinear operator equation has first been introduced and comprehensively analyzed in [53]. In applying the INM, we explore the possibility of keeping fixed the number of inner iterations. In this way, the only parameter controlling the so-called semiconvergence [52] is the number of outer iterations.

The Landweber method is an iterative method endowed with very high regularization capabilities [51]-[52]. Although it is not fast as other methods, the Landweber method yields very high stability [52] and becomes competitive for solving nonlinear and severely ill-posed problems [2].

Although several interesting results have been obtained by applying the approach in [42] by using both simulated and experimental data (as described in Section 3), the considered version of the algorithm, as most of other regularization methods currently applied in electromagnetic imaging, minimizes the residual function (in Hilbert space) \( R_p(x) = \|F(x) - y\|_2^2 . \) Recently, a different approach has been proposed [54]. In particular, a different cost functional is assumed, i.e., \( R_p(x) = \|F(x) - y\|_p \)

where \( \|\cdot\|_p \) is the norm of the \( L^p \) functional Banach space. The metric of the Banach space \( L^p \) emphasizes, for values of the constant \( 1 < p < 2, \) the points where the residual is small with respect to the classical \( L^2 \) norm. This choice gives rise to a substantial reduction of the ineffective smoothing in the restored solution \( x \) and has been proven to provide more accurate reconstructions. However, the INM required significant modifications, which are detailed in [54]. In particular, the inner loop is based on the following recursive relation \((h_0 = h_0^* = 0)\)

\[
\tilde{h}_{i+1}^* = \tilde{h}_i^* - \tau_i J_{\tau_i}^{L^p} (F_{u_k} h_{i+1}^* - y_k),
\]

\[
h_{i+1} = J_{\tau_i}^{L^p} (\tilde{h}_{i+1}^*), i = 0,1,\ldots \text{max}
\]

where \( J_{\tau_i}^{L^p} : L^p(D_{\text{meas}}) \to L^p(D_{\text{meas}}) \) and \( J_{\tau_i}^{L^p} : L^p(D_{\text{meas}}) \to L^p(D_{\text{meas}}) \) are the duality maps of \( L^p(D_{\text{meas}}) \) and \( L^p(D_{\text{meas}}) \) [13] and \( 0 < \tau_i < 2\|F'_{u_k} F'_{u_k}\|_p^{-1} \).

3. Review of numerical and experimental results.

The proposed approach has been intensively studied by using numerical methods. Results can be found in the mentioned references. Concerning in particular the improvement related to the new version developed in the \( L^p \) Banach space, it has been shown [54] that, for \( p = 1,2, \) we can expect a reduction of the mean reconstruction error on the contrast function inside the investigation domain of about 30% when compared with the same solution obtained in the Hilbert space (for a homogeneous cylinder with a triangular cross section of side 0.5\( \lambda \), \( \lambda \) being the wavelength of the incident radiation, \( c = 1.9\times 10^3 \), 8 line-current sources, 241 measurement points, and a signal to noise (SNR) ratio on the simulated scattered field of 20 dB). More relevant, the improvement becomes more and more evident when the noise on the data increases. In particular, the same error reduces of about 55% for SNR = 10 dB and about 60% for SNR = 5 dB. The new approach significantly affects the reconstruction of sharp scatterers, as the ones encountered, for example, in nondestructive testing applications. As shown in
[55], for a square cylinder with dielectric parameters similar to the ones of a cement paste, in which a circular crack is present, not only the overall quality is improved comparing with the standard approach, but the shape of the small defect is retrieved with a better accuracy, confirming that the metrics of the $L^p$ Banach space, for $p$ less than 2 (the reconstructed images reported in [56] concerns the case $p = 1.2$), allows for a certain edge-preserving effect, even if no regularizing terms of this kind (adopted by certain authors, e.g., in [56]) are used.

The INM has been also tested against experimental data. In [57], the measured data obtained at the Frénel Institute of Marseille, France, have been successfully inverted by using the standard approach [42]. For the configuration composed by two adjacent circular cylinders (radii: $R_1 = 30$ mm and $R_2 = 80$ mm; $\varepsilon_1 = 3.0$ and $\varepsilon_2 = 1.45$; frequency $f = 2$ GHz), the new approach developed in the $L^p$ Banach space has resulted in an improvement of more than 10% in the relative error on the reconstructed contrast function [54]. The original method has been also applied, in the framework of a joint research between DITEN, University of Genoa, Italy, and SUPSI, University of Lugano-Manno, Switzerland, to data measured by the microwave tomographic prototype described in [58]. In particular, wood and plastic (including hollowed cylinders filled with sand) objects have been inspected. Results can be also found in [59]-[61]. Quite surprisingly, good results have been recently obtained also in the case in which metallic inclusions are present in the dielectric structure (this fact has a significant importance in industry applications in which foreign bodies can affect the production process and even damage the used machinery) [62]. Although highly underestimated, the retrieved distribution of the electric conductivity allows one to precisely localize the inclusion, although no information about the high conductivity of the inspected target has been inserted into the model, Future work will be aimed at assessing the possible improvement that can be expected to these results when the new version of the INM is applied. In particular, it will be of interest to evaluate whether the same quality levels of the reported reconstructions can be obtained by a reduced number of views, with a corresponding reduction of the overall acquisition time.

4. Conclusions.
In this paper, the application of an inexact Newton method to the integral equations of the inverse scattering problem for electromagnetic imaging has been discussed. After a description of the considered tomographic configuration, the application of the inexact Newton method has been described, also with reference to a new formulation developed in the the $L^p$ functional Banach space. Considering the numerical and experimental results obtained by applying this regularization scheme, the approach can be considered quite satisfactory and further investigations are expected in the light of practical applications in the field of nondestructive testing. In particular, the results obtained by using the new formulation, although preliminary, seems to indicate that a significant improvement in the reconstruction accuracy can be obtained, especially for measurements performed in a noisy environment.

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