Spin relaxation in multiple (110) quantum wells

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We consider theoretically the relaxation of electron spin component parallel to the growth direction in multiple (110) GaAs quantum wells. The sources of spin relaxation are the random Rashba spin-orbit coupling due to the electric field of donors and spin-flip collisions of electrons from different quantum wells. We show that the screening of the Coulomb forces at low temperatures leads to a very strong enhancement of the spin relaxation time. In a degenerate electron gas the Pauli blocking suppresses the electron-electron collisions, and the leading spin relaxation mechanism comes from the field of donors. If the electron gas is nondegenerate the electron-electron collisions and scattering by the ionized donors give similar contributions to the relaxation rate.

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I. INTRODUCTION

The ability to produce semiconductor systems with sufficiently long spin relaxation time and spin diffusion length is expected to lead to a design of reliable spin transport devices.1,2 Also, the problem of theoretical description of systems with a very long relaxation time and understanding the physics of the limits for the spin relaxation rate is of a fundamental interest. The major ingredient required for the spin relaxation is spin-orbit coupling, which usually has the form of the wavevector dependent effective magnetic field. In quantum well structures there are two contributions to this field, namely, linear in the in-plane momentum Rashba1,4 and Dresselhaus5 terms. One of the fundamental aspects in this understanding is that the very low relaxation rates for some spin directions are related to the specific symmetry of the spin-orbit coupling Hamiltonian.6,7

One of the hopes at a possible realization of a very long spin relaxation time system is related to the (110) GaAs quantum wells. In these structures the form of the bulk inversion asymmetry (Dresselhaus) spin-orbit coupling prevents relaxation of the spin component parallel to the growth axis [110]. Here the effective field is always collinear with this axis and, therefore, the spin, oriented along it, experiences no torque. However, the experiment clearly demonstrates finite spin coherence lifetimes8–10 causing the discussion of its origin. A possible explanation can be related to the effects of intersubband scattering of electrons and intersubband spin-orbit coupling11,12 which become efficient at relatively high temperatures and depend strongly on the disorder in the quantum well. Calculations show the dephasing times being on the order of ten nanosecond at \( T = 100 \) K.14 With the temperature decrease, the relaxation time rapidly increases, and for the values of disorder, corresponding to the measured mobility, exceeds the observed one by more than two orders of magnitude.14 Therefore, other causes for the experimentally observed dephasing rates should be looked for.

The possible mechanism can be related to the intrinsic disorder in the Rashba coupling. If the carriers in the well come from the donors, the electric field of the ionized donors causes a random Rashba field15,16. The corresponding effective magnetic field is directed in the structure plane, and, therefore, can influence the growth-direction component of the spin. Its randomness causes random spin precession and leads to the finite spin relaxation rate. If the Coulomb field of the donors can be reduced, the relaxation time can be made longer. This can be achieved by using multiple quantum wells, bringing about two new physical effects not considered in previous works to the best of our knowledge. First, Coulomb forces can be strongly reduced by the screening due to the presence of conduction electrons. Second effect in these structures is the spin-flip scattering of electrons from different quantum wells. In these collisions spin of one electron remains constant while the spin of the other flips. This non-conservation arises due to the spin-orbit coupling effect in the scattering process. It is very similar to the spin-flip scattering by charged donors: screened electric field of electron in one well induces a Rashba field acting on the electron spin in the other well. We focus on these two spin relaxation processes, inherent for multiple quantum wells and limiting the spin relaxation time there.

We demonstrate here that in symmetric multiple quantum well structures grown along the [110] axis, extremely long spin lifetimes, up to \( \sim 100 \) ns can be achieved at low temperatures. These times exceed significantly the experimental values reported in the literature. We discuss the origin of such a discrepancy and find that even a very weak asymmetry of the quantum well structure can bring the spin lifetimes to the experimentally observed values.

This paper is organized as follows. In the second Section we introduce the quantum well structure of interest and study the role of the screening due to the presence
of several quantum wells filled with mobile electrons. In the third Section we study the spin relaxation due to the random Rashba field of screened donors as a function of temperature. In the fourth Section we investigate the temperature-dependent effect of collisions of electrons from different quantum wells on the spin relaxation and compare the spin relaxation rates due to both effects. In Conclusions we summarize the results and discuss possible experimental observations of the effects considered in this paper.

II. QUANTUM WELL STRUCTURE AND THE COULOMB SCREENING

In Fig. 1(a) we present a scheme of a typical multiple quantum well structure: relatively thin conducting layers separated by wide barriers. Such a structure is similar to the one studied in Ref. [12]. To reduce the structure asymmetry-induced Rashba field, each quantum well is located in a macroscopically symmetric environment. The electrons in the conducting layers come from ionized donors located between them. A typical building element of a multiple quantum well is a three-well structure presented in Fig. 1. The central and the side quantum wells are separated by the distance \( L \), typically on the order of 50-100 nm. The donor layers are inserted symmetrically at the distance \( z_{\text{ext}} \) to prevent the appearance of the macroscopic Rashba field in the central well, so that the structure in Fig. 1 maintains macroscopic symmetry with respect to the \( z \leftrightarrow -z \) reflection. The central and the side wells are filled with conduction electrons. The screening due to the redistribution of these electrons strongly reduces the electric field of the donor ions in the wells. We assume that the quantum wells are sufficiently narrow and can be considered as zero-width conducting layers.

The fluctuating Rashba field caused by the ionized donors and the electrons in the side wells is proportional to the \( z \)-component of the electric field caused by the charges. As a result, it depends strongly on how this field is screened. Electrons are free to move in the quantum well plane but cannot move along the growth direction, making screening of the \( z \)-axis and lateral field components drastically different. In a narrow single quantum well the \( z \)-component of the donor-induced field is not screened at all. In multiple quantum wells this component is reduced due to the redistribution of the electron density in the side wells. In order to evaluate the screening, we find the static potential \( U (r) \) by solving the Poisson equation:

\[
\kappa \Delta U (r) = -4\pi \left[ \rho_{\text{ext}} (r) + \rho_{\text{ind}} (r) \right],
\]

where \( \kappa \) is the background dielectric constant, and \( \rho_{\text{ext}} (r) \) is the external charge density. The corresponding induced charge density \( \rho_{\text{ind}} (r) \) at the layers numbered by index \( l \) is given by:

\[
\rho_{\text{ind}} (r) = -\frac{\kappa}{2\pi} \sum_{l} q_{s}^{[l]} U (r) \delta (z - z_{l}),
\]

where summation is taken over all the layers \( l \) in the system. The concentration- and temperature-dependent two-dimensional Thomas-Fermi screening wavevector \( q_{s}^{[l]} \) can vary from well to well and is given by:

\[
q_{s}^{[l]} = \frac{2}{a_{B}} \frac{1}{1 + \exp (-\mu^{[l]}/T)}.
\]

Here \( a_{B} \) is the effective Bohr radius, corresponding to the electron effective mass and the background dielectric constant, and \( \mu^{[l]} \) is the chemical potential of the electron gas in a given well. Due to the translational invariance of the system in the lateral direction \( \mathbf{p} \), it is convenient to use the Fourier-transformed potential \( U (q, z) \) corresponding to the in-plane wavevector \( q \). For an external
point charge $q_{\text{ext}} (r) = e\delta (r_{\text{ext}}) \delta (z - z_{\text{ext}})$, where $e$ is the elementary charge, selected in Fig. 1, the solution of the Poisson equation \( U (q, z) \) can be presented in the form:

\[
U (q, z) = \frac{2\pi e}{\kappa} \left( \frac{1}{q} e^{-q |z - z_{\text{ext}}|} + \sum g_l (q) e^{-q |z - z_l|} \right). \tag{4}
\]

Everywhere the normalization area is set to unity. Equation for the components $g_l (q)$ can be obtained by a conventional method \cite{27,28}, keeping only the singular parts in the second derivative of $U (q, z)$ in Eq. (4). As a result, we obtain the system of linear equations:

\[
\sum_l \left( e^{-q |z_m - z_l|} + \frac{q}{q^2} \delta_{ml} \right) g_l (q) = -\frac{1}{q} e^{-q |z_m - z_{\text{ext}}|}. \tag{5}
\]

\( \xi \) from this system one obtains the components $g_l (q)$ and, therefore, the field in the entire space. In order to calculate the $z$-component of the electric field in the central well, we take into account that the lateral redistribution of the electron density there does not cause net $z$-component of the electric field acting on electrons in this well. Therefore, this redistribution does not contribute in the screening of the $z$-component. For this reason, we can express the solution in terms of $g_l (q)$ with $l \neq 0$ only.

In what follows we focus on the three-well structure, our results do not change qualitatively for a multiple well systems. Then, for a point charge we obtain the Fourier-component of the electric field in the layer $l = 0$:

\[
E_z (q, z = 0) = -\text{sign}(z_{\text{ext}}) 2\pi e \kappa \left\{ e^{-q |z_{\text{ext}}|} + e^{-qLq} [g_{-1} (q) - g_1 (q)] \right\} e^{iqx_{\text{ext}}}.
\]

Below, for simplicity, we omit the central layer coordinate and for $x_{\text{ext}} = 0$ obtain from Eqs. (5), (6):

\[
E_z (q) = -\text{sign}(z_{\text{ext}}) 2\pi e \kappa \frac{e^{-q |z_{\text{ext}}|} - q_s e^{-2qL + q |z_{\text{ext}}|}}{q + q_s (1 - e^{-2qL})}, \tag{7}
\]

where we assumed that the screening wave vectors are the same in side wells, $q_{\text{in}}^{-1} = q_{\text{in}}^1 \equiv q_s$, otherwise the structure is asymmetric and one has to allow for the regular Rashba field as well. The field $E_z (q)$ has the following important features. First, if the charge is close to the central quantum well, that is $|z_{\text{ext}}| \ll L$, and $qL \gg 1$, the induced field of the side layers can be neglected, and, therefore, the screening does not play a significant role. Second, if the external charge is close the side plane, that is $L - |z_{\text{ext}}| \ll L$, and the screening is strong enough, such as $(L - |z_{\text{ext}}|) q_s \gg 1$, the resulting field can be to a good approximation described by a dipole formed by the external charge and the opposite charge induced at the nearest side plane.

### III. Spin Relaxation in the Random Rashba Field of Donors

Fluctuations in the donor density form in the central well a local Rashba spin-orbit coupling proportional to their local field $E_z (\rho)$. These fluctuations cause a random precession of the spin, and, as a result, spin relaxation. The spin-orbit Hamiltonian for the system under study can be presented as the sum of the regular Dresselhaus and a random Rashba terms as:

\[
H_{\text{so}} = H_D + H_R, \tag{8}
\]

\[
H_D = \frac{\hbar}{2} (\Omega_D \cdot \sigma), \quad H_R = \frac{\hbar}{2} (\Omega_R (\rho) \cdot \sigma), \tag{9}
\]

\[
\Omega_D = \frac{2}{\hbar} \alpha_D (0, 0, k_x), \quad \Omega_R (\rho) = \frac{2}{\hbar} \alpha_R (\rho) (k_y, -k_x, 0), \tag{10}
\]

where $k$ is the in-plane momentum, and $\sigma$ is the vector composed of the Pauli matrices. Here $\alpha_D$ is the Dresselhaus coupling constant, dependent on the quantum well width, and the axes are chosen as: $x \parallel [100]$, $y \parallel [001]$, and $z \parallel [110]$. The precession rate due to the Rashba coupling $\Omega_R (\rho)$ is a random function of the lateral coordinate, providing the only contribution leading to the relaxation of the $z$-component of averaged electron spin. We assume here the classical spin-orbit potential, that can be done for $k |z_{\text{ext}}| \gg 1$, neglecting non-commutativity of the momentum and coordinate-dependent Rashba field.\cite{27,28} We take the Rashba parameter being proportional to the local electric field:

\[
\alpha_R (\rho) = \xi eE_z (\rho), \tag{11}
\]

where $\xi$ is the structure-dependent constant.\cite{27,28}

We begin with the reminder of the spin relaxation mechanism due to the donors producing random electric field. The Fermi golden rule shows that the relaxation rate for the $z$-component of the spin $\Gamma^{[s]}_d (k)$ for electron with wavevector $k$ can be expressed as the integral of the precession rates correlator\cite{19} in the random Rashba field:

\[
\Gamma^{[s]}_d (k) = \int_0^\infty \left\langle \Omega_R (t) \Omega_R (0) \right\rangle \, dt. \tag{12}
\]

For the straightforward ballistic motion of electron with $\rho = vt$, which can be used if the electron free path is much larger than $|z_{\text{ext}}|$, the relaxation rate can be expressed using Eqs. (10), (11) as:

\[
\Gamma^{[s]}_d (k) = \frac{4}{\hbar^2} \xi^2 e^2 k^2 \int \langle EE \rangle_q dq^2 \int_0^\infty e^{iqv \xi} \, dt = \frac{2}{\pi \hbar^2} \xi^2 e^2 k^2 \int_0^\infty \langle EE \rangle_q dq, \tag{13}
\]

where the Fourier-transform of the correlator $\langle E_z (\rho) E_z (0) \rangle$ entering Eq. (12) is written as:

\[
\langle E_z (\rho) E_z (0) \rangle \equiv \int \langle EE \rangle_q e^{iqx} \frac{dq^2}{(2\pi)^2}. \tag{14}
\]
FIG. 2: (Color online). Plot of dimensionless correlator $F$ as a function of donor layer position $|z_{\text{ext}}|/L$ for different system parameters. Dashed line (1) $F = L/(2|z_{\text{ext}}|)$ corresponds to the absence of the screening, lines (2) and (3) correspond to $q_sL = 1.0$, and $q_sL = 10$, respectively, and dashed-dot line (4) presents the field of the dipole formed by the point donor charge and the induced charge in the side well, which corresponds to $q_sL \rightarrow \infty$. At $|z_{\text{ext}}|/L \ll 1$ lines (1),(2), and (3) converge, demonstrating that the screening becomes unimportant in this regime.

The resulting spin relaxation rate for electron with wavevector $k$ can be presented as:

$$
\Gamma^{[s]}_d(k) = 8\pi \frac{\xi^2k_m}{h^3} \frac{e^4}{\pi^2L} n_d F,
$$

(15)

with $n_d$ being the concentration of donors per layer, and the dimensionless integrated correlator:

$$
F = L \int_0^\infty f_k dq,
$$

(16)

with

$$
f_k = \left[ \frac{(q + q_s)e^{-q|z_{\text{ext}}|} - q_s e^{-2q|z_{\text{ext}}|}}{q + q_s(1 - e^{-2qL})} \right]^2.
$$

(17)

Here we assume the white noise $\langle n(\rho_1)n(\rho_2) \rangle = n_d \delta(\rho_1 - \rho_2)$ random distribution of the ion density.

It is clearly seen in Eq. (15) that the spin relaxation due to the fluctuating Rashba field occurs even at straightforward electron motion and does not require, in general, the momentum scattering. The relaxation rate can be estimated as a product of squared fluctuation of the random spin precession frequency, $\Omega^2_k$, and the passage time of the correlated domain of spin-orbit coupling, making spin and momentum relaxation processes independent, by contrast to the conventional Elliott-Yafet mechanism.

Eq. (15) shows that the spin flip rate, being quadratic in the strength of the Coulomb interaction, is proportional to the donor density, the wave vector $k$ and the form-factor $F$. The dependence of numerically integrated correlator $F$ on the distance between the donor layer and the central well is presented in Fig. 2 for different values of $q_s$. As one can see in the Figure, the relaxation rate rapidly decreases and almost vanishes when the donors become close to the side well, where the screening becomes efficient. The role of the screening can be seen from a comparison of the dashed (no screening) and solid (with the screening) lines: at $|z_{\text{ext}}| > L/2$ all solid lines are located well below the dashed one.

The temperature dependence of the relaxation rate for the Fermi gas is given by the formula:

$$
\frac{1}{\tau_d^{[s]}} = \frac{2}{\tau_d} \langle \Gamma^{[s]}_d(k) \rangle = \frac{2\sum_k |\Gamma^{[s]}_d(k)|}{\sum_k |\partial f_k/\partial E|},
$$

(18)

where $f_k$ is the Fermi-Dirac distribution, and factor 2 in front corresponds to the sum of contributions of two donor layers in the relaxation rate. The result can be presented in the form:

$$
\frac{1}{\tau_d^{[s]}} = 16\pi \frac{\xi^2k_F m}{h^3} \frac{e^4}{\pi^2L} \frac{n_d}{k_F} F,
$$

(19)

where the angular brackets stand for the averaging similar to Eq. (18). There are two main causes for the $T$-dependence of $1/\tau_d^{[s]}$. First, $\Gamma^{[s]}_d(k)$ depends linearly on the electron momentum. This dependence will give a factor in the spin relaxation rate proportional to the expectation value $\langle k \rangle$. In addition, the screening wavevector $q_s$ decreases with the temperature, as given by Eq. (3). The calculated dependence of $(\langle k \rangle/k_F)$ $F$ on $T$ is presented in Fig. 3. At low temperatures $T \ll E_F$ spin relaxation rate is very weakly temperature-dependent and is given by $2\Gamma^{[s]}_d(k)$ from Eq. (15) with $k = k_F$, and zero-temperature $q_s = 2/a_B$. At high temperatures $T \gg E_F$ where the screening becomes unimportant, $q_s$ is small, we obtain: $q_s = e^{-2q|z_{\text{ext}}|}$ and $F = L/(2|z_{\text{ext}}|)$. Therefore, the mean spin relaxation rate in this regime is:

$$
\frac{1}{\tau_d^{[s]}} = \frac{4\pi^{3/2} \xi^2k_F m}{h^3} \frac{e^4}{\pi^2|z_{\text{ext}}|} n_d,
$$

(20)

where $k_F = \sqrt{2mT/h}$, and $1/\tau_d^{[s]}$ increases as $T^{1/2}$. The temperature at which the screening effects become weaker can be estimated with condition $q_sL/2 = 1$ for typical $z_{\text{ext}} = L/2$. Taking into account that usually $q_s(0)L$ is on the order of ten, the role of the screening is sufficiently reduced only in a nondegenerate gas at $T \gg E_F$: since in this regime $q_s \sim q_s(0) \times (E_F/T)$, temperatures $T > 10E_F$ are required.
vectors of the second electron, \( q \) respectively. The Fourier transform of the spin-operators of the first and second electron, respectively can be interpreted as the random spin precession of a given electron in the fluctuating spin-orbit field caused by the other electron.

The corresponding matrix element of the spin-flip electron-electron interaction can be written following Refs. \([22,23]\):

\[
\hat{M} = \delta_{k+k',p+p'} \frac{\xi e E_z(q)}{2} \times \\
\left\{ [\hat{\sigma}^{(1)} \times (p + k)]_z - [\hat{\sigma}^{(2)} \times (p' + k')]_z \right\},
\]

where \( k, p \) are the initial and final wave vectors of the first electron and \( k', p' \) are the initial and the final wave vectors of the second electron, \( q = |k - p|, \hat{\sigma}^{(1)}, \hat{\sigma}^{(2)} \) are the spin-operators of the first and second electron, respectively. The Fourier transform of the \( z \)-component of the electric field \( E_z(q) \) is described by Eq. \( 16 \) with the inter-well distance \( L \) instead of the distance to the donor layer \( |z_{ext}| \). Hence, the electric field of electrons is weaker than the field of the ions and it is considerably reduced by the screening. Note, that the collisions within the same well do not lead to the spin component relaxation because \( E_z = 0 \) in this case, and bulk inversion asymmetry contribution to \( \hat{M} \) is proportional to \( \sigma_z^{(1)} \), \( \sigma_z^{(2)} \) in \([110]\)-grown quantum wells. The calculation of the spin relaxation rate \( 1/\tau_{ee}^{[z]} \) is carried out by using the kinetic equation for the spin density matrix. The electron-electron collision integral for the scattering of electrons from different wells is expressed via the properly antisymmetrized matrix element \( \hat{M} \) using Keldysh technique.\([22,23]\)

where the dimensionless factor \( R \) depends on the temperature, electron density, screening wavevector, and inter-well distance.

At low temperatures, \( T \ll E_F \), the electron gas is degenerate and the electron-electron scattering is strongly suppressed by the Pauli exclusion principle. The temperature dependence of the constant \( R \) is shown in Fig. \( 5(a) \). There are two regimes of electron-electron scattering for degenerate gas, different by the possible momentum transfer at the scattering process. The first regime, when the momentum transfer is much smaller than \( 1/L \) is realized at very low temperatures \( T \ll E_F/(k_F L) \), and a typical Fermi-liquid behavior:

\[
R = a \times \left( \frac{T}{E_F} \right)^2.
\]

is restored. In this regime one can take the \( q \to 0 \) limit in Eq. \( 16 \), and obtain the coefficient \( a \) in the form \( c/(1 + 2 q s L)^2 \) with the constant \( c \approx 8.0 \) as obtained by the fitting of the numerical results to this simple analytic expression [see Fig. \( 5(b) \)]. At higher temperatures \( E_F/(k_F L) \ll T \ll E_F \) the wave vector transferred in the process of the Coulomb collision cannot exceed \( \sim 1/L \) which reduces the phase space for the final state. Therefore \( R \) becomes linear in the temperature, as can be seen in Fig. \( 5(a) \), where the slope of \( d\ln R/d\ln T \) gradually changes from 2 to 1 with the temperature increase.

At the temperatures close to the Fermi energy the transition to the non-degenerate gas occurs and the screening reduces, leading to the enhancement of the electron-electron scattering. At high temperatures \( T \gg E_F \) where
FIG. 5: (Color online). (a) Temperature dependence of the reduced spin relaxation rate $R$ due to the electron-electron scattering for different screening parameters marked near the plots. (b) Calculated values of the coefficient $a$ as a function of screening (circles). Dashed line is $8.0/(1 + 2q_sL)^2$. $k_F L = 10$ for both panels.

the screening vanishes with $q_s = 0$, the calculations can be carried out analytically. The spin relaxation rate in this regime is given by

$$
\frac{1}{\tau_{ee}} = \frac{6\sqrt{2}}{\sqrt{3}} \frac{\xi^2 k_T m}{\hbar^3} \frac{e^4}{\pi^2 L} n_{el},
$$

where $n_{el}$ is the concentration of electrons per single side well, where we assume for simplicity equal electron filling in all wells. This situation is very similar to the scattering by charged donors, and $1/\tau_{ee}$ increases as $T^{1/2}$. The ratio of the relaxation rates due to donors and electron-electron collisions tends, therefore, to a system-dependent constant:

$$
\frac{\tau_{ee}^{[s]}}{\tau_d^{[s]}} = \frac{2\sqrt{2}}{3} \frac{L}{|z_{ext}|} n_d n_{el},
$$

and is very close to 1 when the electron and ion densities are equal and $L = |z_{ext}|$. Therefore, the donors and electron-electron collisions give contributions of the same order of magnitude in a nongenerate gas.

As discussed above, at low temperatures, where the electron gas is degenerate, one can expect a less efficient contribution of spin-flip electron-scattering compared to the random Rashba fields of the donors. To provide an example, if donors are close to the middle of the interwell barriers, at $T = E_F$, typical contribution of electron-electron collisions is two orders of magnitude weaker than that of the random Rashba field of donors. With the decrease in the temperature, the relative effect of collisions compared to the role of donors decreases.

V. DISCUSSION AND CONCLUSIONS

We investigated the electron spin relaxation in a symmetric three-layer GaAs (110) quantum well structure, which can be considered as a building element for larger multiple quantum well structures. Since the Dresselhaus spin-orbit term in these systems is proportional to $\sigma_z$, the $z$-component of the spin demonstrates a very long lifetime. The spin relaxation can appear due to the random Rashba spin-orbit coupling resulting from the electric field of donors providing electrons for the quantum wells filling. The screening of the Coulomb interaction by the degenerate gas of conduction electrons strongly suppresses the field of the ions and the corresponding random Rashba field. As a result, the spin relaxation time for an appropriately chosen geometry increases by at least an order of magnitude compared to a single symmetric quantum well.

Another effect important in these structures is the non spin conserving electron-electron collision, having the same physical reason as the spin-flip scattering by charged donors, that is the local Rashba field induced by the electric field of a charge located near a quantum well. The contribution of electron-electron collisions is suppressed by the Pauli principle at low temperatures, where the electron gas is strongly degenerated, being much smaller than the contribution of the ions. At high temperatures, the scattering by the donors and by the electrons give contributions to the spin relaxation rate of the same order of magnitude.

Here we concentrated on the spin relaxation in the symmetric central well. Two other wells can be in a strongly asymmetric environment, and, therefore, demonstrate a conventional relatively fast spin relaxation due to the regular Rashba field. In the multiple quantum wells structures containing many single layers, only edge wells are in the asymmetric environment. Therefore, very long spin relaxation times can be achieved for the most of the electrons in the system.

These times can be estimated taking into account that for the low temperatures the main contribution comes from the ions. Eq. (15) with the typical pa-
rameters of GaAs-based quantum wells, and $\xi = 5$ Å$^2$ (Refs. 22, 27, 28) yields the value of $\tau_0$ close to 400 nanosecond at $|z_{\text{ext}}| = L/2$. A comparison of lines (1) and (3) in Fig. 2 shows that such a long time is considerably attributed to the screening; in a single well geometry with the same $|z_{\text{ext}}|$ it would be smaller by approximately a factor of 2. The experiment reveals the times an order of magnitude shorter. The difference can be related to the nonprecise information about the material and structure-dependent parameters and possible structure asymmetry resulting in a very weak but regular Rashba field on the spatial scale comparable or larger than the electron free path. For the single-electron momentum relaxation time $\tau_{\text{ext}}^*$ on the order of 1 ps, it is sufficient to have a very small spin splitting at the Fermi level on the order of 0.01 meV to ensure the times observed in the experiment. The contributions of the random and the regular Rashba mechanisms can be separated by studying the temperature dependence of the relaxation rate at the temperatures $T \ll E_F$, where, however, the single electron momentum relaxation time is already determined by the electron-electron collisions rather than by the disorder. In this case, the pure random Rashba field mechanism would result in a very weak $T$-dependence, while the regular one would lead to a $1/T^2$ decrease.

At these momentum relaxation times $\tau_{\text{ext}}^*$, typical for $T$ on the order of 10-20 K, the electron free path is larger than five interwell distances. Therefore, if the expectation value of the Rashba field is zero, at these temperatures electron spin indeed experiences only fluctuations of the Rashba field around its zero mean value. With the temperature increase, free path decreases and becomes comparable to the interwell distance, being however still much smaller than the spin diffusion length. At this temperature the electron gas demonstrates crossover to the conventional Dyakonov-Perel’ mechanism of spin relaxation with the effective averaged coupling $\langle \alpha F(\rho) \rangle$ (Ref [31]). A microscopic model of spin relaxation in the presence of nonvanishing Rashba coupling in [110] systems was developed in Refs [32].

We conclude from our analysis that by improving quality and design of multiple (110) quantum wells one can still achieve spin relaxation times an order of magnitude longer than the longest times observed so far.

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\[ R = \pi \langle k / k_F \rangle F / (2 k_F L) \]

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