ACTIVE ACOUSTIC SOURCE TRACKING EXPLOITING PARTICLE FILTERING AND MONTE CARLO TREE SEARCH

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ABSTRACT

In this paper, we address the task of active acoustic source tracking as part of robotic path planning. It denotes the planning of sequences of robotic movements to enhance tracking results of acoustic sources, e.g., talking humans, by fusing observations from multiple positions. Essentially, two strategies are possible: short-term planning, which results in greedy behavior, and long-term planning, which considers a sequence of possible future movements of the robot and the source. Here, we focus on the second method as it might improve tracking performance compared to greedy behavior and propose a flexible path planning algorithm which exploits Monte Carlo Tree Search (MCTS) and particle filtering based on a reward motivated by information-theoretic considerations.

Index Terms— Active source tracking, particle filter, Monte Carlo tree search, path planning

1. INTRODUCTION

One of the main challenges in modern robotics is intuitive human-robot interaction for which reliable information about the location of the human is essential. If microphones are mounted on the robot, e.g., for the purpose of communication, acoustic data can be used for inferring localization information. Typically, the relative Angle of Arrival (AoA) between the robot and the source is estimated by, e.g., MUSIC or SRP-PHAT. Inferring distance information from microphone data is generally difficult in reverberant rooms and often relies on labeled training data, e.g., MUSIC.

An option to improve localization accuracy is by estimating multiple AoAs at different sensor positions in parallel and fusing them to obtain a position estimate. This can be achieved for example by exploiting several spatially distributed microphone arrays, i.e., an acoustic sensor network. Alternatively, AoAs can also be measured sequentially, e.g., by a microphone array-equipped robot that is moving in a room. The latter approach greatly complicates the task as the robot-source configuration will generally vary over time. Thus, sequential fusing algorithms have to be employed. On the bright side, however, sequential measurements allow to control the array movement as a whole and possibly internal array configuration, e.g., by moving parts of a robot, between two measurement instants such that the tracking accuracy may benefit. This idea is referred to as active sensing. For this problem, heuristic strategies have been suggested, e.g., moving a robotic head or its limbs furnished with microphones in the direction of the estimated source. Besides heuristic policies, different algorithms have been proposed to solve this task by maximizing a specific objective function. Those approaches can be categorized as greedy or long-term path planning algorithms. Greedy one-step algorithms, as proposed, e.g., in [9], model the objective to depend only on one future hypothesis. This short-term planning is prone to suboptimal tracking performance as it ignores measurements while moving the sensor and subsequent ones. Thus, long-term motion planning algorithms have been proposed in, e.g., [6] for active localization, i.e., static sources, where the objective function is evaluated over several hypothetical future robot-source configurations. Conditional independence of the objective function given prior movements is assumed to deal with the exponentially increasing amount of possible future hypotheses. In [10], an extension based on MCTS has been proposed which does not rely on the previously assumed conditional independence. Moreover, to circumvent the limitation of discrete source positions, the belief about the state is modelled by a mixture of Gaussians.

In this paper, we introduce a path planning algorithm for active tracking which is also based on MCTS but represents the belief by weighted particles. This leads to a very flexible and robust approximation of the state belief which is well-suited to deal with the nonlinear and non-Gaussian state-space models often used in acoustic source tracking. Based on the belief approximation by particles, an estimate of the differential entropy is used as planning objective [11]. Additionally, a new default policy for MCTS is proposed which incorporates the fundamental assumption that moving closer to the source enhances localization results. Furthermore, compared to [6] and [10], uncertainty of robotic movements is taken into account during planning, and dynamic sources are simulated. Finally, the effect of different planning depths is discussed based on simulation results.
2. ACOUSTIC SOURCE TRACKING

In the following section, the system is described in terms of a state-space model. Based on this, an appropriate tracking algorithm is discussed.

2.1. State-Space Model

The system state at time $t$, captured by the state vector $x_t = ((x^i_t)^T (x^q_t)^T)^T$, comprises the robotic, i.e., array, state $x^q_t$ and the source state $x^i_t$. For simplicity, the array, and the source are assumed to move in the same 2D plane. A state describing either the array or the source is given by $x^q_t = (x^q_t \ y^q_t \ \theta^q_t \ v^q_t)^T$, including position in Cartesian coordinates $(x^q_t \ y^q_t)^T$, orientation $\theta^q_t$ and speed $v^q_t$. At each time step the system can be influenced by a specific action $u_{q,t}$ modelling the angular speed. Following [12], the system dynamics is modelled by the nonlinear equations

$$\begin{align*}
\theta^q_t &= \text{wrap}_{[0,360]}((\theta^q_{t-1} + u_{q,t}\Delta T + w_1^q(\theta^q)), \ (1) \\
v^q_t &= v^q_{t-1} + w^q_v, \ (2) \\
(y^q_t \ x^q_t)^T &= \left(\begin{array}{c}
y^q_{t-1} \\
x^q_{t-1}
\end{array}\right) + \left(\begin{array}{c}
\cos(\theta^q_{t-1}) \\
\sin(\theta^q_{t-1})
\end{array}\right) \Delta T v^q_t + \left(\begin{array}{c}
w_{q,x}^q \\
w_{q,y}^q
\end{array}\right). \ (3)
\end{align*}$$

Hereby, $w_{1}^{q}(\cdot) \sim \mathcal{N}(0, \sigma^2_{w_{1}^{q}})$ denotes additive Gaussian noise to account for model uncertainties, $\text{wrap}_{[a,b]}(\cdot)$ a function which wraps its argument to the range $[a,b]$ and $\Delta T$ the sampling interval between successive observations. Note that only the robot can be controlled and thus $u_{a,t} = 0^\perp$. The state equations (1) - (3) do not just account for uncertain source movements but also model the array movement to be noisy. Inference about the states is drawn from observations as follows: While we assume the array state to be known, its movement is stochastic. Information about the latent source states is obtained by observing the sound field as sensed by the robot’s microphone array. Hence, under the assumption that the source is continuously active and located in the far-field, an estimate of the continuous AoA $\hat{z}_t$ between the robot and the source can be computed from the microphone measurements at each time step.

In this paper, the conditional measurement probability density function (PDF) of observing $\hat{z}_t$ while being in state $x_t$ is assumed to be a Gaussian $p(\hat{z}_t|x_t) = \mathcal{N}(\mu_m(x_t), \sigma^2_m(x_t))$ with mean $\mu_m(\cdot)$ and variance $\sigma^2_m(\cdot)$ being, typically nonlinear, functions of the state. These parameters have to be either modelled or learned. We suggest to learn them in a training phase. In [6], it was proposed that these parameters mainly depend on the distance $d(x) = \sqrt{(x^e-x^a)^2 + (y^e-y^a)^2}$ and the true AoA $\phi_{\text{true}}(x) = \text{wrap}_{[-180,180]}(\arctan(\frac{y^e-y^a}{x^e-x^a}) - \theta^r)$ between the robot and the source, with $\arctan(\cdot)$ denoting the four-quadrant inverse tangent. However, if the robot is equipped with a uniform linear array (ULA), it can additionally be assumed that sources which are located symmetrically with respect to the array axis inherit similar parameters. Thus, the observation model can be described by

$$p(\hat{z}_t|x_t) = \mathcal{N}(\mu_m(d(x_t), |\phi_{\text{true}}(x_t)|), \sigma^2_m(d(x_t), |\phi_{\text{true}}(x_t)|)).$$

This greatly simplifies the task as mean and variance have to be estimated only for varying distances and AoAs.

As most AoA estimators create discrete-valued measurements $z_t$ with a given angular resolution $\Delta \phi$, i.e., a finite uniform grid of AoA hypotheses $Z$, the question arises how to compute the probability $P(\cdot)$ of observing $z_t \in Z$ while being at state $x_t$ which can be calculated by integrating the underlying measurement PDF in the resolution range $P(z_t|x_t) = \int_{z_t-\frac{\Delta \phi}{2}}^{z_t+\frac{\Delta \phi}{2}} p(\hat{z}_t|x_t)dz_t$.

2.2. Bayesian Tracking

As the states of the system are partially hidden, we introduce the state posterior, denoted as belief, $p(x_t|h_t)$ representing the knowledge about the system, i.e., the states, given the vector of prior actions and observations $h_t = (u_{r,1} \ z_1 \ldots \ u_{r,t} \ z_t)^T$. This leads to the question how to fuse a new action observation vector $(u_{r,t+1} \ z_{t+1})^T$ into a given belief. Straightforward extensions of the Kalman filter update, i.e., extended or unscented Kalman filter, might not model accurately the belief due to the possibly multimodal state posterior which results for example from front-back ambiguity of the ULA [14]. A very general approach, approximating the ideal continuous Bayes filter [13], is given by sequential Monte Carlo methods, i.e., particle filtering [15]. Hereby, the state posterior is represented by a set of $I$ weights $w_i^{(t)}$ and state samples $x_i^{(t)}$ with $i = 1, \ldots, I$. In this paper a Sampling Importance Resampling (SIR) particle filter with systematic resampling is used to update the belief [15] [16] [17]. The source position is estimated as weighted mean of the respective particles.

3. PATH PLANNING

Paths are considered in this paper as consecutively executed control commands chosen from a finite set of $J$ discrete commands $u_j^t = u_j \in U$. Thus, path planning can be interpreted as a sequential decision making problem, i.e., selecting the then optimal action at each time step.

3.1. Rewards and Returns

For comparing different paths and actions the concept of rewards and returns, as described in [18], is adopted. Rewards $R_t$ can be interpreted as instantaneous feedback on the performance of a specific control command. Ideally one wants to minimize an error norm between the true state and the estimated state. However, the true state is not known. Thus, a common idea is to minimize the belief uncertainty which
has to be quantified. [9] proposed the negative differential entropy of the belief as intrinsic measure of uncertainty for active localization. We adopt this idea by utilizing the recursive weighted particle-based entropy estimator proposed in [11]

\[ \mathcal{H}[p(x_t|h_t)] = \log \left( \sum_{i=1}^{l} p(z_t|x_t^{(i)}w_t^{(i)}) \right) \]

\[-\sum_{i=1}^{l} \log(p(z_t|x_t^{(i)})\left( \sum_{j=1}^{l} p(x_t^{(j)}|x_{t-1}^{(j)}w_{t-1}^{(j)}) \right)w_t^{(i)}) \]

as reward \( R_t \).

As rewards can only account for a single action at a time, the concept of returns \( G_t = \sum_{k=1}^{K} \gamma^{k-1} R_{t+k} \) as cumulative discounted, i.e., weighted, rewards is introduced [13]. Hereby, \( \gamma \in [0,1] \) defines the discount factor which models the behavior of striving for early rewards within the planning horizon of K time intervals. As actions and observations are stochastic, the expected future return \( \mathbb{E}[V(h_t, u_{t+1}')] = \mathbb{E}[G_t|u_{t+1}', h_t] \), starting from history \( h_t \) and selecting action \( u_{t+1}' \), is introduced [13]. The action maximizing the expected future return is selected for path planning

\[ u_{t+1}^{\text{opt}} = \arg\max_{u_{t+1}' \in U} V(h_t, u_{t+1}'). \] 

(5)

### 3.2. Monte Carlo Tree Search Planning

As mentioned in the previous section, the path planning algorithm at time step \( t \) consists essentially of estimating the expected future returns when applying the actions \( u_{t+1}' \in U \). After planning, the robot moves by executing the optimal control command \( u_{t+1}^{\text{opt}} \), measures the real AoA \( z_{t+1} \) and plans again. This leads to the question of how to efficiently compute the expected future returns: An option is to use Monte Carlo simulation, i.e., simulating \( L \) returns corresponding to hypotheses of sequences of future actions and observations \( h_{t+k} = (h_t \ldots u_{t+k}z_{t+k})^T \) followed by arithmetic averaging. However, this approach struggles because the number of possible future hypotheses is growing exponentially with increasing planning horizon \( K \). To overcome this problem the MCTS technique [19] has been developed which builds up a partial tree \( T_{h_{t}} \) of future hypotheses and uses action selection to concentrate on promising paths, i.e., individual sequences with high return rates. The exploitation-exploration dilemma, i.e., deciding which paths to investigate, is hereby tackled by the Upper Confidence Bound (UCB) policy [20]. Algorithm [1] and [2] summarize the proposed path planning method, using MCTS and particle filtering. The path planner samples \( L \) times from the current belief \( x_t^{(i)} \sim p(x_t|h_t) \) and updates the estimated future return \( \mathbb{E}(\hat{V}(h_{t+1} \cdot), \cdot) \) by simulating a possible future hypothesis, computing the corresponding return and arithmetic averaging, i.e., l. 17 of Algorithm 2.

Algorithm 1 and 2 summarize the proposed path planning method. Using MCTS and particle filtering. The path planner samples \( L \) times from the current belief \( x_t^{(i)} \sim p(x_t|h_t) \) and updates the estimated future return \( \mathbb{E}(\hat{V}(h_{t+1} \cdot), \cdot) \) by simulating a possible future hypothesis, computing the corresponding return and arithmetic averaging, i.e., l. 17 of Algorithm 2. SearchTree is a recursive algorithm (Algorithm 2) which builds up a partial Tree \( T_{h_{t}} \) including the respective future movements, observations and estimated future returns. It initially checks if the maximum planning depth \( K \) is reached. In this case, it returns 0, otherwise, an action \( u_{t+1}' \) according to the UCB policy is computed (l. 3).

Hereby, \( n \) denotes the number of previous visits of the simulated hypothesis and \( n_i \) the number of previously simulated situations where action \( u_{t+1}' \) had been selected. The ratio of exploration and exploitation can be controlled via the hyperparameter \( c \) [19]. Then, a future state \( x_t^{(i)} \) is sampled (l. 4) from the state transition equations conditioned on the sampled particle and selected action. Finally, an observation \( z_{t+1} \) is sampled from the measurement process with \( m \) indexing the finite observation set \( Z \) (l. 5). Based on the selected action \( u_{t+1}' \) and sampled measurement \( z_{t+1} \) it is checked if the simulated future hypothesis \( h_{t+1} \) is part of the tree \( T_{h_{t}} \). If it is, the algorithm gets the previously computed reward \( R_t \) from \( T_{h_{t}} \) (l. 8), i.e., GETREWARD, and recursively calls itself with the new particle \( x_t^{(i)} \) and incremented depth \( k \) (l. 9). If \( h_{t+1} \) has not been simulated, it is added to the tree \( T_{h_{t}} \) (l. 13), the belief is updated by an SIR particle filter (l. 11), i.e., UPDATE, the reward \( R_t \) computed by (l. 12) and a default algorithm DEFAULT is applied until reaching the planning horizon \( K \) (l. 14) [19].

### Algorithm 1 Monte Carlo Path Planning at time step \( t \)

1: \textbf{for} \( l = 1 : L \) \textbf{do}
2:   \( x_t^{(i)} \sim p(x_t|h_t) \)
3:   \textbf{SEARCHTREE}(x_t^{(i)}, T_{h_{t}}, 0)
4:   \( u_{t+1}' = \arg\max V(h_t, u_{t+1}') \)
5: \textbf{end for}

### Algorithm 2 Monte Carlo Return Estimation

1: \textbf{function} \text{SEARCHTREE}(x_t^{(i)}, T_{h_{t}}, k)
2: \textbf{if} \( k == K \) \textbf{then return} 0
3:   \( u_{t+1}' = \arg\max V(h_t, u_{t+1}') + c \sqrt{\ln n / n_i} \)
4: \textbf{else}
5:   \( x_t^{(i)} \sim p(x_t^{(i)}, u_{t+1}', T_{h_{t}}) \)
6:   \( z_{t+1} \sim p(z_{t+1}|x_t^{(i)}, u_{t+1}') \)
7: \textbf{if} \( h_{t+1} \in T_{h_{t}} \) \textbf{then}
8:   \( R_t \leftarrow \text{GETREWARD}(T_{h_{t}}, h_{t+1}) \)
9:   \( G_t \leftarrow R_t + \gamma \text{SEARCHTREE}(x_t^{(i)}, T_{h_{t}}, k + 1) \)
10: \textbf{else}
11:   \( p(x_t|h_{t+1}) \leftarrow \text{UPDATE}(p(x_t|h_t), u_{t+1}', z_{t+1}) \)
12:   \( R_t \leftarrow \mathbb{H}[p(x_t|h_{t+1})] \)
13:   \( T_{h_{t}} \leftarrow T_{h_{t}} \cup \{h_{t+1}\} \)
14:   \( G_t \leftarrow R_t + \gamma \text{DEFAULT}(x_t^{(i)}, T_{h_{t}}, k + 1) \)
15: \textbf{end if}
16: \textbf{end if}
17: \textbf{return} \( G_t \)
default algorithm consists of computing an action according to a default policy, followed by sampling the transition and observation PDF to create possible future hypotheses. The default policy (l. 14) cannot rely on experience from previous simulations for action selection \[19\]. Thus, in \[10\] uniform random behavior was used. We propose to incorporate the prior knowledge that moving closer to the target enhances localization results. However, in order to not fully rely on this naive assumption, our proposed default policy randomly uses either a uniform random behavior or an informed action with equal probability. The informed decision is made by sampling the array transition equations \((1) - (3)\) with \(q = r\) for each possible action and selecting the one moving closest to the estimated source position. Subsequently, the default algorithm recursively calls itself until reaching the planning horizon and gives back the return \(G_t\). After simulating one future return \(G_t\) by recursively either calling SEARCHTREE or DEFAULT, \(n\) and \(n_{\text{UCB}}\) are incremented and the expected future return \(\hat{V}(h_t, n_{\text{UCB}}^{t+1})\) is updated by computing the mean of the previously simulated returns and the current return \(G_t\) (l. 17).

4. RESULTS

In this section, we evaluate the proposed path planning algorithm in a simulated environment. The sensor is modelled to move at a constant speed of \(v_t = 0.3\) m/s. The action set \(A_t\) consists of 4 different actions, namely staying in the current state, or changing the angular velocity \(u_t\) to \(-45°, 0°, 45°\). This change of orientation is modelled to be noisy by adding zero-mean Gaussian noise with variance \(\sigma_{r,\theta}^2 = 25°^2\) to account for uncertainty in angular sensor movements. The source is modelled to move at the same speed as the sensor with uncertainties \(\sigma_s, v = 0.02\) m and \(\sigma_s, \theta = 10°\). The remaining variances \(\sigma_{q, r}, \sigma_{q, \theta}\) for \(q \in \{r, s\}\) and \(\sigma_{s, v}^2\) are set to zero. The sampling interval \(\Delta T\) is set to 1 s. The particle filter model was chosen according to the simulated model and with \(I = 1000\). However, additional source position uncertainties are taken into account: \(\sigma_{s, r} = \sigma_{s, \theta} = 0.1\) m.

The sensor consists of a ULA with 6 microphones and a spacing of 4.2 cm. AoAs are estimated by SRP-PHAT \[3\] from microphone signals sampled at a sampling frequency of 8 kHz and observation length of 4096 samples per measurement. The observation parameters \(\mu_{\text{sr}}(\cdot, \cdot)\) and \(\sigma_w^2(\cdot, \cdot)\) are estimated in a training phase by simulating various AoA measurements, i.e., different sensor and source states, for each position of a uniform distance-AoA grid. Positions in between are computed by linear interpolation. Several rooms with dimensions \(7\) m \(\times 5\) m \(\times 3\) m, and varying reverberation time of \(.4\) s, \(.6\) s and \(.8\) s and SNR 10 dB, 20 dB and 30 dB, are simulated \([21, 22]\).

The path planning parameters are set to \(\gamma = .9\) and \(c = 1.75\). The rewards are normalised to lie in the interval \([-1, 0]\). The maximum number of simulated histories \(L\) is set to 500. For reliable results, 300 source paths and robot starting positions are created with uniformly distributed random initial positions. The initial source belief is modelled by particles whose positions and orientations are distributed uniformly in the enclosure. The first two actions are set to \(u_q = u_l = 0°\) to start path planning from an informed belief. The Euclidean distance between the true and the estimated source position is computed for each time step and averaged over all 2700 combinations of paths and rooms, i.e., reverberation times and SNRs. Figure 1 depicts median and standard deviation of error over time, starting from \(t = 2\), i.e., after the two fixed actions at the beginning, for varying planning depths, i.e., \(K = 1, 5\) and \(7\). Additionally, similar to \([6, 10]\) random action selection was simulated as a classical reference for movement behaviour. As one deduces from Figure 1 actively planning paths significantly increases the accuracy and reliability of position estimates, compared to selecting an action at random at each time step. The effect of planning depths can be seen by comparing \(K = 1\) and \(K = 5\). Both perform similar at the beginning, however drift apart in the long-term. This is consistent with the idea that some actions might pay off later. However, a saturation effect can be seen. Planning \(K = 7\) time steps ahead does not improve the results further which can be explained either by the increased uncertainty of future predictions or an insufficient number of simulated paths. As the same effect has also been observed for \(L = 1500\) we can expect this results from the increased uncertainty.

5. SUMMARY

In this paper we proposed a path planning algorithm based on MCTS \([10]\) and particle filtering. Due to the particle-based description of the belief, a differential entropy approximation has been used as planning objective. Additionally, a new default policy for MCTS has been introduced. The efficacy of the algorithm and the effect of different planning horizons were demonstrated by simulated data of dynamic speech sources for a variety of challenging acoustic scenarios.
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