Compressive spectral imaging enables to reconstruct the entire three-dimensional (3D) spectral cube from a few multiplexed images. Here, we develop a novel compressive spectral imaging technique using diffractive lenses. Our technique uses a coded aperture to spatially modulate the optical field from the scene and a diffractive lens such as a photon-sieve for dispersion. The coded field is passed through the diffractive lens and then measured at a few planes using a monochrome detector. The 3D spectral cube is then reconstructed from these highly compressed measurements through sparse recovery. A fast sparse recovery method is developed to solve this large-scale inverse problem. The imaging performance is illustrated at visible regime for various scenarios with different compression ratios through numerical simulations. The results demonstrate that promising reconstruction performance can be achieved with as little as two measurements. This opens up new possibilities for high resolution spectral imaging with low-cost and simple designs. © 2019 Optical Society of America

Spectral imaging is a fundamental diagnostic technique in physical sciences with application in diverse fields such as physics, chemistry, biology, medicine, astronomy, and remote sensing. Conventional techniques rely on a scanning process to build up the 3D spectral cube from a series of 2D measurements [1]. One important disadvantage is that higher number of scans is needed with increased spatial and spectral resolutions [2]. This may lead to low light throughput, increased hardware complexity, and long acquisition times, resulting in temporal artifacts in dynamic scenes [3]. Moreover, the temporal, spatial, and spectral resolutions are inherently limited as they are purely determined by the physical systems involved.

Compressive spectral imaging provides an effective way to overcome these limitations by passing on some of the burden to a computational system. It enables to reconstruct the entire spectral cube from a few multiplexed measurements via sparsity-based reconstruction. This is made possible by compressive sensing (CS) which relies on two principles: sparsity of the spectral images in some transform domain and incoherence of the measurements. It is widely known that spectral images exhibit both spatial and spectral correlations, which allow sparse representations [2]. For the incoherence of the measurements, different compressive spectral imaging techniques have been proposed such as coded aperture snapshot spectral imaging (CASSI) [2, 4, 5] and compressive hyperspectral imaging by separable spectral and spatial operators (CHISS) [6].

In this letter, we develop a novel compressive spectral imaging technique named compressive spectral imaging with diffractive lenses (CSID). CSID uses a coded aperture to spatially modulate the optical field from the scene and a diffractive lens such as a photon sieve [7, 8] for dispersion. The coded field is first passed through the diffractive lens and then measured at a few planes using a moving detector. A new fast sparse recovery method is developed to reconstruct the spectral cube from these compressive measurements. The performance is also illustrated numerically for various settings. Different than the earlier works that use diffractive lenses for spectral imaging [9–11], here we utilize them for the first time in a compressive modality.

Figure 1 illustrates the CSID system, which has a simple optical configuration consisting of (1) an imaging lens, (2) a coded mask, (3) a diffractive lens (such as a photon sieve [7, 8]), and (4) a monochrome detector [12]. First the image of the scene is formed on the plane of the coded mask. Then the coded field is passed through the diffractive lens. Since the diffractive lens has a wavelength-dependent focal length, each spectral component is exposed to a different amount of focus. As a result, each measurement is a superposition of differently blurred and coded spectral bands. Using a moving detector, a total of K such measurements can be recorded at different distances from the diffractive lens.

The measurements obtained with the CSID system can be related to the intensity of each spectral component as follows:

$$y_k(u, v) = \int (f_\lambda(u, v) * h_{d,\lambda}(u, v)) b(\lambda) d\lambda.$$  \hspace{1cm} (1)

Here $y_k(u, v)$ represents the kth measurement obtained at dis-
tance \( d_k \), \( f_k(u,v) = x_h \left( \frac{d}{d_x} u, -\frac{d}{d_y} v \right) \) is the coded and scaled intensity of the spectral field \( s_h(u,v) \) with the coded aperture \( c_h(u,v) \). Assuming an ideal imaging lens with unit magnification, the coded and scaled intensity \( x_h(u,v) \) is convolved with the incoherent point-spread function (PSF) of the diffractive lens, \( h_{d,h}(u,v) \), which has a closed-form expression given elsewhere [13]. Lastly, \( b(\lambda) \) denotes the spectral response of the detector. Note that although the terms \( x_h \left( \frac{d}{d_x} u, -\frac{d}{d_y} v \right) \)'s involve different scaling for different \( k \)'s, when \( d_k \) is much larger than \( d_h \) this scaling can be effectively taken as the same.

We discretize the spectral field into \( S \) spectral bands, and \( x_s(u,v) \) represents the intensity of the \( s \)-th band with central wavelength \( \lambda_s \). This spectral component is modulated with the coded mask which has pattern \( c_s(u,v) \) at wavelength \( \lambda_s \). The patterns \( c_s(u,v) \) are the same for all wavelengths \((s = 1, \ldots, S)\) if an uncolored (traditional block-unblock) mask is used; however, the mask patterns will be different if a colored coded aperture [14] is used instead. The coded aperture \( c_s(u,v) = \sum c_s(m,n) \text{rect}(\frac{m}{\Delta}, \frac{v}{\Delta} - n) \) is a pixelated array with a pixel size of \( \Delta_s \), and \( c_s(m,n) \) denotes the value of the coded aperture at pixel \((m,n)\).

After discretizing the spectral field along the spectral dimension, discretization along the spatial dimensions is also needed to arrive at a discrete model. By replacing each spatially continuous function with its discretized version, we can obtain the following discrete forward model:

\[
y_k[m,n] = \sum_{s=1}^{S} (x_s[m,n]c_s[m,n]) \ast h_{d,h_s}[m,n] b_s. \tag{2}
\]

Here, \( y_k[m,n] \) denotes the \( k \)-th measurement obtained over \( N_x \times N_y \) detector pixels, and corresponds to the samples of \( y_k(u,v) \), i.e. \( y_k[m,n] = y_k(m\Delta, n\Delta) \). The sampling interval \( \Delta \) corresponds to the pixel size of the detector. The coded aperture pixel size can be chosen as an integer multiple of \( \Delta \) to avoid the need for subpixel positioning accuracy. Here, we choose \( \Delta_s \) for simplicity. Moreover, \( x_s[m,n] \) and \( h_{d,h_s}[m,n] \) are the uniformly sampled versions of their continuous counterparts with the same sampling interval \( \Delta \). Lastly, \( b_s \) represents the coefficient resulting from the response of the detector at the central wavelength \( \lambda_s \).

The above discrete forward model can be expressed in matrix-vector form as follows:

\[
y = HCx + n, \tag{3}
\]

where \( y = [y_1^T, \ldots, y_K^T]^T \in \mathbb{R}^{KN} \) is vertically concatenated measurement vector with \( N \triangleq N_x N_y \) where \( y_k \in \mathbb{R}^N \) represents the \( k \)-th measurement vector. Similarly, \( x_k \in \mathbb{R}^N \) is the vector corresponding to the spectral image with wavelength \( \lambda_s \), and \( x = [x_1^T, \ldots, x_K^T]^T \in \mathbb{R}^{KN} \) is the concatenated image vector. The \( KN \times SN \) matrix \( H \) consists of \( N \times N \) convolution matrices representing the convolutions with PSFs \( h_{d,h_s}[m,n] \). The diagonal matrix \( C \in \mathbb{R}^{SN \times SN} \) represents the overall coding operation, and has values 0 or 1 along its diagonal. Finally, the vector \( n = [n_1^T, \ldots, n_K^T]^T \) denotes the measurement noise, which is often white Gaussian. In our setting, the number of measurements \( K \) is smaller than the number of spectral bands \( S \), which results in an under-determined system of equations.

In the inverse problem, the goal is to reconstruct the unknown spectral images, \( x \), from their compressive superimposed measurements, \( y \), which involve their coded and blurred versions. This problem is inherently ill-posed. There are a variety of approaches to solve such ill-posed linear inverse problems. Here, to exploit the sparsity of the spectral images after some transformation \( \Phi \), we formulate the inverse problem as the following constrained optimization problem:

\[
\min_x \| \Phi x \|_1 \quad \text{subject to } \| y - HCx \|_2 \leq \epsilon, \tag{4}
\]

where \( \epsilon \geq 0 \) is a parameter that depends on noise variance. Here the \( \ell_1 \)-norm enforces the sparsity of the spectral cube after transformation with \( \Phi \), as motivated by the CS theory [15].

To solve the resulting optimization problem, we convert our constrained problem to an unconstrained problem by adding the constraint to the objective function as a penalty function:

\[
\min_x \| \Phi x \|_1 + t \| (y - HCx) \|_2 \tag{5}
\]

where the indicator function \( t \| (y - HCx) \|_2 \) takes value 0 if the constraint is satisfied, and \(+\infty\) otherwise. We solve this problem by developing a fast reconstruction algorithm that is based on alternating direction method of multipliers (ADMM) [16]. After variable-splitting, this results in the following problem:

\[
\min_{x,z^{\{1\}},z^{\{2\}}} \| \Phi z^{\{1\}} \|_1 + t \| (y - z^{\{2\}}) \|_2 \tag{6}
\]

subject to \( z^{\{1\}} = x \), \( z^{\{2\}} = HCx \)

where \( z^{\{1\}}, z^{\{2\}} \) are the auxiliary variables in the ADMM framework. After expressing the problem in 6 in augmented Lagrangian form [16], minimization over \( x, z^{\{1\}} \), and \( z^{\{2\}} \) is needed. Here, we minimize them in an alternating fashion.

For minimization over \( x \), we face a least-squares problem which has the following normal equation:

\[
(I + C^H H^H HC)x_{k+1} = (z^{\{1\}} + d^{\{1\}} + C^H H^H (z^{\{2\}} + d^{\{2\}})) \tag{7}
\]

with \( d \) denoting the dual variable in the ADMM framework. A direct matrix inversion approach for solving the linear system in Eq. (7) is not feasible for large-scale spectral cubes. Here, we solve this iteratively using the conjugate-gradient method. For this iterative process, forming any of the matrices is not required, which provides huge savings for the memory as well as the computation time. Specifically, multiplications with matrices \( H \) and \( H^T \) correspond to summation of some convolutions. That is, for multiplication with \( H^T \) matrix, we can simply take 2D Fourier transforms of underlying PSFs \( h_{d,h}(m,n), \ldots, h_{d,h}(m,n) \) and the spectral images \( x_1[m,n], \ldots, x_K[n,m] \), multiplying them elementwise, and then summing all the results. For multiplication with \( H^T \) matrix, a similar operation is performed using the PSFs \( h_{d,h}(m,n), \ldots, h_{d,h}(m,n) \). Lastly, the required multiplications with \( C \) and \( C^T \) in the iterative process reduce to simple elementwise multiplications with coded aperture functions \( c_s[m,n] \).

For minimization over \( z^{\{1\}} \), we need to perform the following operation involving soft-thresholding:

\[
z_{k+1}^{\{1\}} = \Phi^{-1}(\text{soft}(\Phi(x_{k+1} - d^{\{1\}}))) \frac{1}{\tau}, \tag{8}
\]

where \( \text{soft}(w, \tau) \) denotes the soft-thresholding operation and is component-wise computed as \( w_i \rightarrow \text{sign}(w_i) \max(|w_i| - \tau, 0) \) for all \( i \), with \( \text{sign}(w_i) \) taking value 1 if \( w_i > 0 \) and \(-1\) otherwise [16]. That is, the solution in Eq. (8) can be obtained by first transformation with \( \Phi \), followed by soft-thresholding with parameter \( 1/\tau \), and inverse transformation operation \( \Phi^{-1} \).
For minimization over \( z^{(2)} \), a projection of \( s \triangleq \left( \text{HCx}_{k+1} - d_{k}^{(2)} \right) \) onto \( \epsilon \)-radius hypersphere centered at \( y \) is required [16]. This projection has the following form:

\[
z_{k+1}^{(2)} = \begin{cases} 
  y + e \frac{s - y}{\|s - y\|_2}, & \text{if } \|s - y\|_2 > \epsilon \\
  s, & \text{if } \|s - y\|_2 \leq \epsilon.
\end{cases}
\]  

As a result, we have three update steps resulting from the ADMM formulation, i.e., x-update, \( z^{(1)} \)-update, and \( z^{(2)} \)-update. The overall algorithm is summarized in Table 1.

### Table 1. Reconstruction algorithm for CSID

| Input: Compressive measurements \( y \) obtained using Eq. (3). |
|---|
| Initialization: Initialize iteration count \( k = 0 \), choose \( \mu > 0, \epsilon, z_{0}^{(1)}, z_{0}^{(2)}, d_{0}^{(1)}, d_{0}^{(2)} \). |
| Main Iteration: Increment \( k \) by 1 and repeat the following steps until some stopping criterion is satisfied. |
| 1. Calculate spectral images \( x_{k+1} \) by solving Eq. (7) using conjugate-gradient algorithm. |
| 2. Calculate \( z_{k+1}^{(1)} \) using soft-thresholding in Eq. (8). |
| 3. Calculate \( z_{k+1}^{(2)} \) using projection in Eq. (9). |
| 4. Update \( d_{k+1}^{(1)} \) as \( d_{k+1}^{(1)} = d_{k}^{(1)} - (x_{k+1} - z_{k+1}^{(1)}) \). |
| 5. Update \( d_{k+1}^{(2)} \) as \( d_{k+1}^{(2)} = d_{k}^{(2)} - (\text{HCx}_{k+1} - z_{k+1}^{(2)}) \). |
| Output: Spectral images \( x \). |

We now present numerical simulations to illustrate the performance of the developed imaging technique. We consider a spectral scene of size \( 256 \times 256 \times 10 \) (10 wavelengths from the range 530–620 nm with 10 nm interval), which was taken from an online hyperspectral database [17]. For the diffractive lens, we consider a photon sieve design with an outer diameter of 3.45 mm and a smallest hole diameter of 15 \( \mu \)m. This results in a focal length of \( f_{0} = 9 \) cm at the central wavelength \( \lambda_{0} = 575 \) nm and Abbe’s spatial resolution of 15 \( \mu \)m. Pixel size of the detector, \( \Delta, \) is chosen as 7.5 \( \mu \)m to match the expected spatial resolution. Moreover, the expected spectral resolution is \( 4\delta^2 / f_{0} = 10 \) nm, as given by the spectral bandwidth of the diffractive lens [18][ Chap. 9]. Note that this expected spectral resolution matches to the chosen spectral sampling interval, i.e., 10 nm.

The compressive measurements are simulated using the model in Eq. (3) with additive Gaussian noise. In each measurement, the system applies the masking operation on individual spectral bands using colored coded apertures. The entries of these apertures are drawn from a Bernoulli distribution. A sample mask pattern is shown in Fig. 2. After the coded field passes through the photon sieve, we capture measurements at different planes. A sample compressive measurement is shown in Fig. 2 together with the true spectral cube superimposed (integrated) along the spectral dimension. As seen, the measurements involve not only the superposition of all spectral images but also significant amount of blur and degradation.

We consider different compressive scenarios with 2, 3, 4, and 5 measurements taken at different planes with equidistant points from the central focal plane. These correspond to compression ratios (CRs), \( K/S \), of 20%, 30%, 40%, and 50%, respectively. Different measurement SNRs of 20, 30, and 40 dB are also considered for the additive noise. Reconstructions are obtained from these compressive and noisy measurements using the algorithm in Table 1. Similar to previous compressive spectral imaging approaches [2], we enforce sparsity in a Kronecker basis \( \Phi = \Phi_1 \otimes \Phi_2 \) where \( \Phi_1 \) is the basis for 2D Symblet-8 wavelet and \( \Phi_2 \) is the 1D discrete cosine (DCT) basis. This transformation with \( \Phi \) is computed by first taking the 2D Symblet-8 transform of each spectral image and then the 1D DCT along the spectral dimension. One reconstruction takes approximately 35 seconds on a computer with 16 GB of RAM and i7 7700K 4.20 GHz CPU.

Table 2 shows the average reconstruction PSNRs for the considered measurement scenarios. It can be seen that reconstruction PSNR is above 30 dB at 30% compression ratio and 30 dB input SNR, which demonstrates the high-quality reconstruction of the spectral cube in practical imaging scenarios. In addition, the performance degrades gracefully with decreasing input SNR and increasing compression ratio. That is, the imaging performance is also robust to high noise and compression levels.

To visually evaluate the results, we present in Fig. 3 the reconstructed spectral images at 30 dB input SNR and different compression ratios, together with the true spectral images. We can see that the image details and edges, as well as the spectral variations, are well preserved in the reconstructions. In the left of the figure, superimposed reconstructions along the spectral dimension are also shown, which are indistinguishable from the true one. Hence, the results demonstrate successful reconstruction of the spectral image cube from compressive measurements with compression ratios of as low as 20%.

To further demonstrate the successful recovery along the spectral dimension, we select three representative spatial points with different spectral characteristics. These points P1, P2, and P3 are shown in Fig. 3. The reconstructed spectra at these points are plotted in Fig. 4, together with the ground truth. It can be seen that the spectrum is recovered almost perfectly in all compression ratios for P1. At points P2 and P3, the reconstruction with 20% compression ratio slightly deviates from the ground truth, while other compression ratios provide good reconstructions.

To also numerically evaluate the spectrum recovery performance, the normalized mean squared error (NMSE) for the spectrum is computed at these selected points, using \( \left\| x \otimes x^* \right\|^2 / \left\| x \right\|^2 \) where \( x \) is the ground truth and \( x^* \) is the reconstructed spectrum. The resulting NMSE values are given in Table 3. These values also support the successful recovery of the spectrum with a typical NMSE value of less than 1%.

In the presented results, the pixel size of the detector, as well as the reconstruction grid, are chosen to match the expected spatial resolution of the diffractive lens. Because the developed imaging modality is a computational imaging technique and the compression is performed along the spectral direction, effective spectral resolution not only depends on the spectral bandwidth of the diffractive lens, but also on the scene content (mainly, its spectral correlation). Although the imaging performance appears to be robust to higher compression levels and noise, clearly increasing the number of measurements improves the
reconstructions. However, this comes with the cost of increased acquisition time, which may be undesirable for dynamic scenes. The reconstructions can be further improved with the optimization of coded apertures and learning-based recovery. In summary, we have presented a novel compressive spectral imaging modality that relies on a simple optical configuration involving a coded aperture and a diffractive lens. The developed technique offers promising imaging performance with compressed data. Since the system takes compressive measurements along the spectral dimension, successful reconstructions can be achieved for spectrally-correlated scenes at visible and infrared regime. Different than the earlier compressive spectral imaging techniques that rely on prisms or gratings to disperse the optical field and many lenses to form images, here a single diffractive lens is used for both purposes. Calibration of the system also appears to be simpler because the imaging system is shift-invariant and measuring the PSFs is sufficient, instead of the system response for each voxel. Hence the presented work opens up new possibilities for high resolution spectral imaging with low-cost and simple designs.

**Funding:** Scientific and Technological Research Council of Turkey (TUBITAK), 3501 Research Program, 117E160.

**Table 2.** Comparison of average reconstruction PSNRs (dB) for different compressive measurement scenarios and SNRs.

| SNR (dB) | 20% CR | 30% CR | 40% CR | 50% CR |
|----------|--------|--------|--------|--------|
| 20       | 25.34  | 27.43  | 28.11  | 28.54  |
| 30       | 27.65  | 31.52  | 32.61  | 33.07  |
| 40       | 29.14  | 34.56  | 36.29  | 37.00  |

Table 3. NMSE of the spectrum at the selected points.

| Point # | 20% CR | 30% CR | 40% CR | 50% CR |
|---------|--------|--------|--------|--------|
| 1       | 1.14%  | 0.33%  | 0.86%  | 0.18%  |
| 2       | 0.67%  | 0.15%  | 0.06%  | 0.06%  |
| 3       | 6.92%  | 0.55%  | 0.52%  | 0.26%  |

**REFERENCES**

1. T. Okamoto and I. Yamaguchi, Opt. Lett. 16, 1277 (1991).
2. G. Arce, D. Brady, L. Carin, H. Arguello, and D. Kittle, IEEE Signal Process. Mag. 31, 105 (2014).
3. R. Willett, M. Duarte, M. Davenport, and R. Baraniuk, IEEE Signal Process. Mag. 31, 116 (2014).
4. A. Wagadarikar, R. John, R. Willett, and D. Brady, Appl. Opt. 47, B44 (2008).
5. Y. Wu, I. O. Mirza, G. R. Arce, and D. W. Prather, Opt. Lett. 36, 2692 (2011).
6. Y. August, C. Vachman, Y. Rivenson, and A. Stern, Appl. Opt. 52, D46 (2013).
7. F. S. Oktém, F. Kamalabadi, and J. M. Davila, “High-resolution computational spectral imaging with photon sieves,” in IEEE ICIP (IEEE, 2014), pp. 5122–5126.
8. G. Andersen, Opt. Lett. 30, 2976 (2005).
9. P. Wang and R. Menon, JOSA A 35, 189 (2018).
10. F. D. Hallada, A. L. Franz, and M. R. Hawks, Opt. Eng. 56, 081811 (2017).
11. M. Nimmer, G. Steidl, R. Riesenberg, and A. Wuttig, Opt. Express 26, 28335 (2018).
12. O. F. Kar, U. Kamaci, F. C. Akyon, and F. S. Oktém, “Compressive photon-sieve spectral imaging,” in Computational Optical Sensing and Imaging (Optical Society of America, 2018), pp. CTu5D–8.
13. F. S. Oktém, F. Kamalabadi, and J. M. Davila, Opt. Express 26, 32259 (2018).
14. H. Arguello and G. R. Arce, IEEE Trans. Image Process. 23, 1896 (2014).
15. D. L. Donoho, IEEE Trans. Inf. Theory 52, 1289 (2006).
16. M. V. Alonso, J. M. Bioucas-Dias, and M. A. T. Figueiredo, IEEE Trans. Image Process. 20, 681 (2011).
17. S. M. Nascimento, F. P. Ferreira, and D. H. Foster, JOSA A 19, 1484 (2002).
18. D. Attwood, *Soft x-rays and extreme ultraviolet radiation: principles and applications* (Cambridge University Press, 2000).
FULL REFERENCES

1. T. Okamoto and I. Yamaguchi, “Simultaneous acquisition of spectral image information,” Opt. Lett. 16, 1277–1279 (1991).
2. G. Arce, D. Brady, L. Carin, H. Arguello, and D. Kittle, “Compressive coded aperture spectral imaging: An introduction,” IEEE Signal Process. Mag. 31, 105–115 (2014).
3. R. Willett, M. Duarte, M. Davenport, and R. Baraniuk, “Sparsity and structure in hyperspectral imaging: Sensing, reconstruction, and target detection,” IEEE Signal Process. Mag. 31, 116–126 (2014).
4. A. Wagadarikar, R. John, R. Willett, and D. Brady, “Single disperser design for coded aperture snapshot spectral imaging,” Appl. Opt. 47, B44–B51 (2008).
5. Y. Wu, I. O. Mirza, G. R. Arce, and D. W. Prather, “Development of a digital-micromirror-device-based multishot snapshot spectral imaging system,” Opt. Lett. 36, 2692–2694 (2011).
6. Y. August, C. Vachman, Y. Rivenson, and A. Stern, “Compressive hyperspectral imaging by random separable projections in both the spatial and the spectral domains,” Appl. Opt. 52, D46–D54 (2013).
7. F. S. Oktem, F. Kamalabadi, and J. M. Davila, “High-resolution computational spectral imaging with photon sieves,” in IEEE Int. Conf. on Image Processing (ICIP), (IEEE, 2014), pp. 5122–5126.
8. G. Andersen, “Large optical photon sieve,” Opt. Lett. 30, 2976–2978 (2005).
9. P. Wang and R. Menon, “Computational multispectral video imaging,” JOSA A 35, 189–199 (2018).
10. F. D. Hallada, A. L. Franz, and M. R. Hawks, “Fresnel zone plate light field spectral imaging,” Opt. Eng. 56, 081811 (2017).
11. M. Nimmer, G. Steidl, R. Riesenberg, and A. Wuttig, “Spectral imaging based on 2d diffraction patterns and a regularization model,” Opt. Express 26, 28335–28348 (2018).
12. O. F. Kar, U. Kamaci, F. C. Akyon, and F. S. Oktem, “Compressive photon-sieve spectral imaging,” in Computational Optical Sensing and Imaging, (Optical Society of America, 2018), pp. CTu5D–8.
13. F. S. Oktem, F. Kamalabadi, and J. M. Davila, “Analytical fresnel imaging models for photon sieves,” Opt. Express 26, 32259–32279 (2018).
14. H. Arguello and G. R. Arce, “Colored coded aperture design by concentration of measure in compressive spectral imaging,” IEEE Transactions on Image Process. 23, 1896–1908 (2014).
15. D. L. Donoho, “Compressed sensing,” IEEE Transactions on Inf. Theory 52, 1289–1306 (2006).
16. M. V. Afonso, J. M. Bioucas-Dias, and M. A. T. Figueiredo, “An augmented lagrangian approach to the constrained optimization formulation of imaging inverse problems,” IEEE Trans. Image Process. 20, 681–695 (2011).
17. S. M. Nascimento, F. P. Ferreira, and D. H. Foster, “Statistics of spatial cone-excitation ratios in natural scenes,” JOSA A 19, 1484–1490 (2002).
18. D. Attwood, Soft x-rays and extreme ultraviolet radiation: principles and applications (Cambridge University Press, 2000).