The Spin of the Nucleon in Effective Models

H. Weigel†

Institute of Theoretical Physics, Tübingen University
Auf der Morgenstelle 14, D–72076 Tübingen, Germany

The three flavor soliton approach for baryons is utilized to discuss effects of flavor symmetry breaking in the baryon wave–functions on axial current matrix elements. The flavor content of the singlet axial current matrix elements, that parameterizes the quark spin contribution to the total angular momentum, is disentangled and studied as a function of the effective flavor symmetry breaking. Here the nucleon and the Λ–hyperon are considered.

1 Introduction

Even though the fundamental theory for the strong interaction processes of hadrons, Quantum Chromodynamics (QCD), is well established, hadron properties can unfortunately not be computed directly. However, QCD contains a hidden expansion parameter, the number \((N_C)\) of color degrees of freedom, that is beneficial for model building. For arbitrarily large \(N_C\), QCD becomes equivalent to a theory of weakly interacting mesons \([1]\). That is, the meson interaction strengths scale like \(1/N_C\) while baryon masses and radii scale like \(N_C\) and \(N_C^0\), respectively \([2]\). Meson Lagrangians may possess localized solutions to the field equations with finite field energy: solitons. Their energies scale inversely with the meson coupling and their extensions approach constants as the coupling increases. These analogies lead to the conjecture that baryons emerge as solitons in the effective meson theory that is equivalent to QCD \([2, 3]\). Although this meson theory cannot be derived from QCD, low–energy meson phenomenology provides sufficient constraints to build sensible models. Especially chiral symmetry and its breaking in the vacuum introduce non–linear interactions for the pions, the (would–be) Goldstone bosons of chiral symmetry. Then effective Lagrangians are constructed from the chiral field \(U = \exp (i\vec{\tau} \cdot \vec{\pi}/f)\) that are invariant under global chiral transformations \(U \rightarrow L U R^\dagger\). As \(U^\dagger U = 1\), at least two derivatives are required

\[
\mathcal{L}_0 = \frac{f^2}{4} \text{tr} \left( \partial_\mu U \partial^\mu U^\dagger \right).
\]  

Extracting the axial current \(A_\mu^a = f \partial_\mu \pi^a + \mathcal{O}(\vec{\pi}^3)\) from \(\mathcal{L}_0\) provides the electroweak coupling and determines the pion decay constant \(f = f_\pi = 93\text{MeV}\). Having established a chiral model, a finite energy soliton solution must be obtained and quantized to describe baryon states. I will outline this approach in section 2. In section 3 I will consider three flavor extensions thereof with special emphasis on the role of flavor symmetry breaking \([4]\). I will employ these methods to compute axial current matrix elements of baryons in section 4. These matrix elements are major ingredients for the description of the nucleon spin structure \([5]\) as they reflect its various quark flavor contributions \([6]\) and they parameterize hyperon beta–decay. The effects of flavor symmetry breaking will be essential to discuss the strange quark contribution. Section 5 contains some concluding remarks.

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†Heisenberg–Fellow
2 Baryons as Chiral Solitons

Scaling considerations show that the model (1) does not contain stable soliton solutions. Therefore Skyrme added a stabilizing term [3]

\[ \mathcal{L} = \frac{f_{\pi}^2}{4} \text{tr} \left[ \partial_\mu U \partial^\mu U^\dagger \right] + \frac{1}{32\pi} \text{tr} \left( \left[ U^\dagger \partial_\mu U, U^\dagger \partial_\nu U \right] \left[ U^\dagger \partial^\rho U, U^\dagger \partial^\sigma U \right] \right), \]  

(2)

that is of fourth order in the derivatives. There are other stabilizing extensions of \( \mathcal{L}_0 \), as e.g. including vector mesons [7, 8]. Although such extensions appear physically more motivated, I will stick to the Skyrme model for pedagogical reasons when explaining the soliton picture for baryons.

The soliton solution to (2) assumes the famous hedgehog shape

\[ U_H (\vec{r}) = \exp \left( i \vec{r} \cdot \hat{r} F(r) \right). \]  

(3)

The equations of motion become an ordinary second order differential equation for the chiral angle \( F(r) \) that is obtained by extremizing the classical energy

\[ E_{\text{cl}} = E_{\text{cl}}[F] = \int d^3r \left\{ \frac{f_{\pi}^2}{2} \left( r^2 F'^2 + 2 \sin^2 F \right) + \frac{\sin^2 F}{2c^2} \left( 2F'^2 + \sin^2 F \right) \right\}. \]  

(4)

It can be argued [9] that the baryon number equals the winding number of the mapping (3), i.e. \( B = [F(\infty) - F(0)]/\pi \). Hence the boundary conditions \( F(0) = -\pi \) and \( F(\infty) = 0 \), that correspond to unit baryon number, determine the chiral angle uniquely. This soliton does not yet describe states of good spin and/or flavor as the ansatz (3) does not possess the corresponding symmetries. Such states are generated by restoring these symmetries through collective coordinates \( A(t) \)

\[ U(\vec{r}, t) = A(t) U_H(\vec{r}) A^\dagger(t). \]  

(5)

and subsequent canonical quantization thereof [11]. This introduces right \([A, R_i] = A\tau_i/2 \) and left generators \([A, L_i] = \tau_i A/2 \). While the isospin interpretation \( I_i = L_i \) is general, the identity \( J_i = -R_i \) for the spin is due to the hedgehog structure (3) as is the relation \( |\vec{I}| = |\vec{J}| \). Quantizing the collective coordinates yields a Hamiltonian in terms of spin (isospin) operators

\[ H_{\text{coll}} = E_{\text{cl}} + \frac{f_{\pi}^2}{2c^2} = E_{\text{cl}} + \frac{\vec{f}^2}{2c^2}. \]  

(6)

The moment of inertia is also a functional of the above determined chiral angle

\[ \alpha^2[F] = \frac{2}{3} \int d^3r \sin^2 F \left[ f_{\pi}^2 + \frac{1}{c^2} \left( F'^2 + \sin^2 F \right) \right]. \]  

(7)

Matching the mass difference \( M_{\Delta} - M_{N} = \frac{3}{2c^2} \sim 300 \text{MeV} \) fixes the undetermined parameter \( c \approx 4.0 \).

3 Extension to Three Flavors

The generalization to three flavors is carried out straightforwardly by taking \( A(t) \in SU(3) \) with the hedgehog (3) embedded in the isospin subgroup. However, the Lagrangian acquires two essential extensions. The first one is the Wess–Zumino–Witten term [9]. Gauging it for local hedgehog (3) embedded in the isospin subgroup. However, the Lagrangian acquires two essential extensions. The first one is the Wess–Zumino–Witten term [9]. Gauging it for local

\[ \mathcal{L}_{SB} = \frac{f_{\pi}^2 m_{\pi}^2 - f_{K}^2 m_{K}^2}{2\sqrt{3}} \text{tr} \left\{ \lambda_8 \left( U + U^\dagger \right) \right\} + \frac{f_{K}^2 - f_{\pi}^2}{4\sqrt{3}} \text{tr} \left\{ \lambda_8 \left( \partial_\mu U \partial^\mu U^\dagger + \text{h.c.} \right) \right\}. \]  

(8)
The explicit form of $\mathcal{L}_{\text{SB}}$ is model dependent, however, the techniques to study its effects on baryon properties are general. The $SU(3)$ collective coordinates are parameterized by eight “Euler–angles”

$$A = D_2(\hat{I}) e^{-i\nu\lambda_4} D_2(\hat{R}) e^{-i(\rho/\sqrt{3})\lambda_8},$$

where $D_2$ denote rotation matrices of three Euler–angles for each, rotations in isospace ($\hat{I}$) and coordinate–space ($\hat{R}$). Substituting the ansatz $\mathcal{F}$ into $\mathcal{L} + \mathcal{L}_{\text{SB}}$ and canonical quantization of the collective coordinates $A$ yields

$$H = H_s + \frac{3}{4} \gamma \sin^2 \nu.$$  \hspace{1cm} (10)

The symmetric piece of this Hamiltonian only contains Casimir operators that may be expressed in terms of the $SU(3)$–right generators $R_a$ ($a = 1, \ldots, 8$):

$$H_s = E_{\text{cl}} + \frac{1}{2\alpha^2} \sum_{i=1}^{3} R_i^2 + \frac{1}{2\beta^2} \sum_{a=4}^{7} R_a^2.$$ \hspace{1cm} (11)

While $\beta^2$ is a moment of inertia similar to $\alpha^2$ in eq (9), $\gamma$ originates from symmetry breaking

$$\gamma = \gamma[F] = \frac{2\pi}{3} \int d^3r \left[ (f^2 K_m^2 - f^2_\pi m^2_\pi) (1 - \cos F) + \frac{f^2_\pi - f^2 K}{2} \cos F \left( F^2 r^2 + 2 \sin^2 F \right) \right].$$

The generators $R_a$ can be expressed in terms of derivatives with respect to the ‘Euler–angles’. The eigenvalue problem $H \Psi = \epsilon \Psi$ reduces to sets of ordinary second order differential equations for isoscalar functions which only depend on the strangeness changing angle $\nu$ [11]. Only the product $\omega^2 = \frac{2}{3} \gamma \beta^2$ appears in these differential equations that are integrated numerically. Thus $\omega^2$ is interpreted as the effective strength of the flavor symmetry breaking. A value in the range $5 \leq \omega^2 \leq 8$ is required to obtain reasonable agreement with the empirical mass differences for the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons [1]. The eigenstates of the symmetric piece [11] are members of definite $SU(3)$ representations, e.g. the octet (8) for the low–lying $\frac{1}{2}^+$ baryons. Upon flavor symmetry breaking, states of different representations are mixed. At $\omega^2 = 6$ the nucleon amplitude contains a $23\%$ contamination of the state with nucleon quantum numbers in the $10$ representation. This clearly shows a strong deviation from flavor covariant wave–functions.

4 Axial Current Matrix Elements

The effect of the derivative type symmetry breaking terms is mainly indirect. They provide the splitting between the various decay constants and thus increase $\gamma$ because of $f^2_\pi m^2_\pi - f^2 K m^2 K \approx 1.5 f^2_\pi (m^2_\pi - m^2 K)$. Otherwise the $(f^2_\pi - f^2 K)$–terms may be omitted. Whence there are no symmetry breaking terms in current operators and the non–singlet axial charge operator is parameterized as

$$\int d^3r A_i^{(a)} = c_1 D_{ai} - c_2 D_{a8} R_i + c_3 \sum_{a,\beta=4}^{7} d_{i\alpha\beta} D_{\alpha a} R_\beta, \quad c_i = c_i[F],$$ \hspace{1cm} (12)

Table 1: The empirical values for the $g_A/g_V$ ratios of hyperon beta–decays [12]. For $\Sigma \to \Lambda$ only $g_A$ is given. Also the flavor symmetric predictions are presented using the values for $F$ & $D$ of Ref. [13].

|   | $n \to p$ | $\Lambda \to p$ | $\Sigma \to n$ | $\Xi \to \Lambda$ | $\Xi \to \Sigma$ | $\Sigma \to \Lambda$ |
|---|----------|----------------|---------------|-----------------|----------------|----------------|
| emp. | $1.267 \pm 0.004$ | $0.718 \pm 0.015$ | $0.340 \pm 0.017$ | $0.25 \pm 0.05$ | $1.287 \pm 0.158$ | $0.61 \pm 0.02$ |
| $F$ & $D$ | $1.26 = g_A$ | $0.725 \pm 0.009$ | $0.339 \pm 0.026$ | $0.19 \pm 0.02$ | $1.26 = g_A$ | $0.65 \pm 0.01$ |
Figure 1: The predicted decay parameters for the hyperon beta–decays using $\omega_{\text{fix}}^2 = 6.0$. The errors originating from those in $\Delta \Sigma_N$ are indicated.

where $D_{ab} = \frac{1}{2} \text{tr} (\lambda_a A \lambda_b A^\dagger)$, $a = 1, \ldots, 8$ and $i = 1, 2, 3$. When integrating out strange degrees of freedom, $\omega^2 \to \infty$ the strangeness contribution to the nucleon axial charge should vanish. The eigenstates of (10) parametrically depend on $\omega^2$ and for $\omega^2 \to \infty$ the singlet current

$$\int d^3 r A^{(0)}_i = -2\sqrt{3} c_2 R_i, \quad (i = 1, 2, 3) \quad (13)$$

yields a vanishing nucleon matrix element of the strangeness projection, $A^{(s)}_i = (A^{(0)}_i - 2\sqrt{3} A^{(8)}_i)/3$. The identity of $c_2$ in eqs (12) and (13) goes beyond group theoretical arguments. Actually all model calculations in the literature [14, 15] are consistent with (13). To completely describe the hyperon beta–decays I demand matrix elements of the vector charges that are obtained from the operator

$$\int d^3 r V^{(a)}_0 = \sum_{b=1}^{8} D_{ab} R_b = L_a. \quad (14)$$

The values for $g_A$ and $g_V$ (only $g_A$ for $\Sigma^+ \to \Lambda e^+ \nu_e$) are obtained from the matrix elements of respectively the operators in eqs (12) and (14), sandwiched between the eigenstates of the full Hamiltonian (10). I choose $c_2$ according to $\langle N \uparrow | \int d^3 r A^{(0)}_3 | N \uparrow \rangle = \sqrt{3} c_2 = \Delta \Sigma = 0.2 \pm 0.1$ and subsequently determine $c_1$ and $c_3$ at $\omega_{\text{fix}}^2 = 6.0$ such that the empirical values for the nucleon axial charge, $g_A$ and the $g_A/g_V$ ratio for $\Lambda \to p e^- \bar{\nu}_e$ are reproduced. This predicts the other decay parameters and describes their variation with symmetry breaking as shown in figure 1. The dependence on flavor symmetry breaking is very moderate\footnote{Here the problem of the too small model prediction for $g_A$ will not be addressed but rather the empirical value $g_A = 1.26$ will be used as an input to fix the $c_n$.} and the results can be viewed as

\footnote{However, the individual matrix elements entering the ratios $g_A/g_V$ vary strongly with $\omega^2$ [14].}
reasonably agreeing with the empirical data, cf. table 1. The two transitions, \( n \to p \) and \( \Lambda \to p \), which are not shown in figure 1, exhibit a similar negligible dependence on \( \omega^2 \). Hence these predictions are not sensitive to the choice of \( \omega^2 \). Comparing the results in figure 1 with the data in table 1 shows that the calculation using the strongly distorted wave–functions agrees equally well with the empirical data as the established \([13]\) flavor symmetric \(F&D\) fit.

Figure 2 shows the flavor components of the axial charge of the \( \Lambda \) hyperon. Again, the various contributions to the axial charge of the \( \Lambda \) exhibit only moderate dependences on \( \omega^2 \). The non–strange component, \( \Delta U_\Lambda = \Delta D_\Lambda \) slightly increases in magnitude. The strange quark piece, \( \Delta S_\Lambda \) grows with symmetry breaking since \( \Delta \Sigma_\Lambda \) is kept fixed. These results nicely agree with an \( SU(3) \) analysis applied to the data \([17]\).

The observed independence on \( \omega^2 \) does not occur for all matrix elements of the axial current. A prominent exemption is the strange quark component in the nucleon, \( \Delta S_N \). For \( \Delta \Sigma = 0 \), say, it is significant at zero symmetry breaking, \( \Delta S_N = -0.131 \) while it decreases (in magnitude) to \( \Delta S_N = -0.085 \) at \( \omega^2 = 6.0 \).

This far I have only considered the general structure of the current operators without computing the constants \( c_i \) from a model soliton, though I had the Skyrme model in mind. However, this model is too simple to be realistic. For example, it improperly predicts \( \Delta \Sigma = 0 \) \([4]\). More complicated models must be utilized, as e.g. the vector meson model that has been established for two flavors in ref \([7]\). Later it has been generalized to three flavors and been shown to fairly describe hyperon beta–decay \([14]\). To account for different masses and decay constants a minimal set of symmetry breaking terms is included \([13]\) that add symmetry breaking pieces to the axial charge operator,

\[
\delta A_i^{(a)} = c_4 D_{a8} D_{8i} + c_5 \sum_{\alpha,\beta=4}^7 d_{i\alpha\beta} D_{a\alpha} D_{8\beta} + c_6 D_{ai}(D_{88} - 1) ,
\delta A_i^{(0)} = 2\sqrt{3} c_4 D_{8i} .
\]

The coefficients \( c_1, \ldots , c_6 \) are functionals of the soliton and can be computed once the soliton is constructed \([6]\). As the model parameters cannot be completely determined in the meson sector \([7]\) I use the small remaining freedom to accommodate baryon properties in three different ways, see table 2. The set denoted by ‘masses’ refers to a best fit to the baryon mass differences. It predicts the axial charge somewhat on the low side, \( g_A = 0.88 \). The set named ‘mag.mom.’ refers to parameters that yield magnetic moments of the \( \frac{1}{2}^+ \) baryons close to the respective empirical data (with \( g_A = 0.98 \)) and finally the set labeled ‘\( g_A \)’ reproduces \([14]\) the axial charge of the nucleon as well as the hyperon beta–decay data. As presented in table 2, the predictions for the axial properties of the \( \Lambda \) are insensitive to the model parameters. The singlet matrix element of the \( \Lambda \) hyperon is smaller than that of the nucleon. Sizable polarizations of the \( up \) and \( down \) quarks in the \( \Lambda \) are again
Table 2: Spin content of the Λ in the realistic vector meson model. For comparison the nucleon results are also given. Three sets of model parameters are considered, see text.

|        | Λ     | N     |
|--------|--------|--------|
|        | ∆U = ∆D | ∆S    | ∆Σ    | ∆U    | ∆D    | ∆S    | ∆Σ    |
| masses | -0.155 | 0.567  | 0.256  | 0.603  | -0.279 | -0.034 | 0.291  |
| mag. mom. | -0.166 | 0.570  | 0.238  | 0.636  | -0.341 | -0.030 | 0.265  |
| g_A    | -0.164 | 0.562  | 0.233  | 0.748  | -0.476 | -0.016 | 0.256  |

predicted. They are slightly smaller in magnitude but nevertheless comparable to those obtained from the SU(3) symmetric analyses. [17]

5 Conclusions

In this talk I utilized the picture that baryons emerge as solitons in an effective meson theory to compute various baryon matrix elements. Here I focused on the effects of flavor symmetry breaking in the baryon wave–functions and showed that despite of strong deviations from flavor covariant wave–functions the empirical parameters for hyperon beta–decay are reproduced. Effective symmetry breaking is treated as a parameter and consistency with the the two–flavor limit (infinitely heavy strange quarks) relates singlet and octet axial currents beyond group theory. With this I showed that chiral soliton models explain the proton spin puzzle, i.e. the smallness of the observed axial singlet current matrix element. Furthermore flavor symmetry breaking in the nucleon wave–function significantly reduces the polarization of the strange quarks inside the nucleon.

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