ON THE ROAD TO PERCENT ACCURACY: THE REACTION WAY

MATTEO CATANEO
Argelander-Institut für Astronomie, Auf dem Hügel 71, D-53121 Bonn, Germany

Taking advantage of the unprecedented statistical power of upcoming cosmic shear surveys will require exquisite knowledge of the matter power spectrum over a wide range of scales. Analytical methods can achieve such precision only up to quasi-linear scales. For smaller non-linear scales we must resort to N-body and hydrodynamical simulations, which despite recent technological advances and improved algorithms remain computationally expensive. Over the past decade machine learning and the advent of emulators have propelled our ability to hit the target accuracy with impressive computing efficiency. Yet, realistically these techniques will be able to produce high-precision non-linear matter power spectra only for a restricted sub-set of extensions to the “vanilla” ΛCDM cosmology. I will present a promising alternative to alleviate these shortcomings that draws strength from the combination of halo model, perturbation theory and emulators—the reaction framework. I will show how a power spectrum evolved in the standard cosmology can be readily adjusted to account for physics beyond ΛCDM, and discuss the accuracy of the reaction for well-known modifications to gravity, dark energy parametrizations and massive neutrino cosmologies.

1 Introduction

The last two decades have seen the rise of the Λ-Cold Dark Matter (ΛCDM) paradigm to Standard Model of Cosmology—a status granted for its ability to explain with only a handful of parameters a diverse array of observations tracing a staggering 13.8 billion years of cosmic evolution (for recent analyses see, e.g., Planck collaboration\(^1\), eBOSS collaboration\(^2\), DES\(^3\) and KiDS\(^4\) collaborations). The ever-increasing volume of data continues to challenge the ΛCDM model, and tantalizing tensions are now emerging between early- and late-time cosmological probes\(^5,6\) potentially exposing cracks in the theory. When taken together with our largely phenomenological (rather than fundamental) understanding of the content of the Universe, this fact encourages the exploration of alternative models in an attempt to find (most likely, constrain) deviations from the standard picture.

Central to many cosmological analyses is the 2-point correlation function of the matter density field, or equivalently its Fourier transform, the matter power spectrum. In the era of Stage IV surveys, accurate predictions for this statistic down to small non-linear scales will be essential to derive tight as well as robust constraints on parameters informing us whether new physics is at work\(^7,8\), be it yet unknown interactions, undetected particles or particle properties. As the clustering of matter can react quite dramatically to changes to General Relativity (GR), testing gravity on cosmological scales is high up on the agenda of all science collaborations\(^9,10,11,12\). Modelling the non-linear power spectrum in modified gravity scenarios is the focus of this talk, but I will also discuss how our framework can capture the impact of massive neutrinos—particularly relevant for its degeneracy with fifth force effects\(^13\)—and evolving dark energy (i.e., \(w \neq -1\)).
2 Non-linear matter power spectrum predictions for ΛCDM extensions

$N$-body simulations are the gold standard for the study of non-linear structure formation, and they provide the means to directly estimate the statistical properties of the matter density field, including the power spectrum. On the flip side is their considerable computational cost. Unfortunately, the complexities of new physics in ΛCDM extensions only worsen this problem\textsuperscript{14}. Methods to predict the non-linear power spectrum based on (semi-)analytical or machine learning approaches can provide a fast alternative to simulations, but their validity is restricted to particular domains (e.g., quasi-linear regime, certain class of cosmologies etc.)

By capitalizing on the strengths of well-established techniques (perturbation theory, halo model and emulators), the reaction method described below provides a framework for predicting the non-linear matter power spectrum of a broad class of cosmologies while meeting the accuracy requirements set by Stage IV surveys. Moreover, its small computation time is an especially attractive feature for likelihood analyses.

3 The Reaction framework

In this framework the non-linear matter power spectrum of our target cosmology (which we call the ‘real’ cosmology) can be obtained from that of a particular reference cosmology (the ‘pseudo’ cosmology) as\textsuperscript{15}

\begin{equation}
P_{\text{real}}(k, z) = \mathcal{R}(k, z) \times P_{\text{pseudo}}(k, z),
\end{equation}

where $\mathcal{R}$ is a scale- and time-dependent quantity called reaction. The linear growth of the pseudo cosmology follows that of a flat and massless neutrino ΛCDM cosmology, with the important difference that the shape and amplitude of its linear power spectrum are specified by the real cosmology, i.e.

\begin{equation}
P_{L}^{\text{pseudo}}(k, z) = P_{L}^{\text{real}}(k, z).
\end{equation}

The reason for choosing the pseudo cosmology in such a way is twofold: (i) it ensures $\mathcal{R} \rightarrow 1$ on linear scales; (ii) since to a good approximation the spherical collapse threshold is only weakly dependent on cosmology, from the Press-Schechter formalism we expect the halo mass functions of the pseudo and real cosmology to be rather similar. We shall see that it is this similarity that enables us to predict the reaction with the required accuracy.

To take advantages of the complementarity between different techniques, we use flexible semi-analytical methods for the reaction together with an accurate simulation-based route for the pseudo cosmology non-linear power spectrum, so that errors in the predicted beyond-ΛCDM non-linear power spectrum will be mostly associated with inaccuracies in the reaction modelling. The challenge consists in computing the reaction at percent level for all wavenumbers $k \lesssim 10 \ h/\text{Mpc}$, and at the same time efficiently evaluate $P_{\text{pseudo}}(k, z)$ for a broad class of linear power spectrum shapes.

3.1 Pseudo cosmology emulator

Thanks to the concept of pseudo cosmology the reaction framework can substantially reduce our reliance on beyond-ΛCDM simulations, therefore placing the construction of a wide scope emulator within the realm of possibility. The advantage is that the computing time for a pseudo cosmology simulation is similar to that for a ΛCDM run. However, compared to conventional emulators the pseudo cosmology emulator must capture a variety of linear power spectrum shapes that cannot be reproduced by the standard cosmological parameters. Giblin et al. (2019)\textsuperscript{16} developed a proof-of-concept emulator precisely with this idea in mind. In addition to the five ΛCDM parameters they introduced a new set of effective parameters, $\Delta\Omega_{\nu_{1-8}}$, describing the ratio $S(k) \equiv P_{L}^{\text{pseudo}}(k)/P_{L}^{\Lambda\text{CDM}}(k)$ for a large class of cosmologies converging to ΛCDM on
linear scales. By using Halofit as a proxy for the $N$-body simulations, they constructed a 13-dimensional Gaussian Process emulator trained on 500 parameter combinations distributed in a Latin Hypercube, and showed that this initial setup can already predict the pseudo non-linear power spectrum of their test cosmologies at 2% level.

### 3.2 The reaction

To model the reaction we resort to the halo model (HM) and standard perturbation theory (SPT). For modified gravity and dark energy cosmologies with a fraction, $f_\nu$, of the total matter density ($\rho$) in massive neutrinos ($\nu$) we have\(^\text{17}\)

$$\mathcal{R}(k, z) \approx \frac{P_{\text{HM}}^{\text{real}}(k, z)}{P_{\text{HM}}^{\text{pseudo}}(k, z)} = \frac{(1 - f_\nu)^2 P_{\text{HM}}^{(cbw)}(k, z) + 2 f_\nu (1 - f_\nu) P_{\text{HM}}^{(cbw)}(k, z) + f_\nu^2 P_{L}^{(\nu)}(k, z)}{P_{\text{HM}}^{(\nu)}(k, z)} + P_{L}^{(\nu)}(k, z),$$\(^\text{3}\)

with $P_{\text{HM}}^{(cbw)}(k, z)$ and $P_{\text{HM}}^{(cb)}(k, z)$ being, respectively, the cross power spectrum of the massive neutrinos and the CDM+baryons (cb), and the predicted halo model non-linear power spectrum for the CDM+baryons component. To lighten the notation the designation ‘real’ has been omitted for all terms in the numerator of Eq. 3. The expressions for the pseudo and real 1-halo terms follow the standard halo model prescription with halo mass functions and halo profiles adapted to the specific cosmologies. The left panel of Fig. 1 shows that when the halo profiles are not fully corrected for fifth force effects the reaction predictions can match well the simulation measurements up to $k \sim 1\, h/\text{Mpc}$. The right panel, instead, illustrates the strong connection enabled by the reaction framework between the halo properties (abundance and profiles) and the matter power spectrum: after calibrating the halo mass functions of the pseudo and real cosmologies with simulations, our power spectrum predictions can achieve sub-percent accuracy up to $k \sim 1\, h/\text{Mpc}$ (orange line); including also information from the halo profiles improves the agreement with the simulations deep in the non-linear regime (green line).

Recently Bose et al. (2021)\(^\text{17}\) have extended the reaction framework to include a class of interacting dark energy cosmologies, and packaged this formalism in ReACT, a publicly available C++ code and Python wrapper valuable for likelihood analyses of cosmic shear data\(^\text{19}\).
4 Future directions

This research programme can be expanded in many ways: (i) the reaction framework uses the same halo mass function to predict both the matter power spectrum and the abundance of massive halos. This link has the potential to break parameter degeneracies in joint analyses of cosmic shear data and cluster number counts; (ii) our formalism is adaptable to a larger class of dark energy and modified gravity cosmologies falling under the umbrella of the Horndeski’s theory. By generalizing the spherical collapse dynamics we will be able to capture the phenomenology of these models too. (iii) AGN feedback redistributes the gas content of massive halos \(M_h \gtrsim 10^{13.5} M_\odot/h\) thus changing their overall density profiles. This effect can be readily incorporated into the reaction formalism, which together with multi-wavelength observations of groups and clusters of galaxies will put us in a better position to break degeneracies between astrophysical systematics and cosmology, currently a major limiting factor in cosmic shear analyses; (iv) the reaction framework enables the simultaneous treatment of various physics beyond the standard gravity-only paradigm, and thanks to the growing library of models implemented in ReACT it will be easier in the future to study the combined effect of exotic dark matter particles, new interactions, gravity beyond GR and astrophysics on the non-linear matter power spectrum; (v) finally, Cataneo et al. (2021)\textsuperscript{20} found that for sufficiently large smoothing scales the real and pseudo modified gravity probability distribution functions of the matter density field are remarkably similar. This bodes well for potential applications of the reaction to higher-order statistics, e.g. the matter bispectrum, which will be essential to access the non-Gaussian information stored in the large-scale structure.

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