\( \mathcal{N} = 1 \) supersymmetric three-dimensional QED in the large-\( N_f \) limit and applications to super-graphene

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Abstract: We study \( \mathcal{N} = 1 \) supersymmetric three-dimensional Quantum Electrodynamics with \( N_f \) two-component fermions. Due to the infra-red (IR) softening of the photon, \( \varepsilon \)-scalar and photino propagators, the theory flows to an interacting fixed point deep in the IR, \( p_E \ll e^2 N_f/8 \), where \( p_E \) is the euclidean momentum and \( e \) the electric charge. At next-to-leading order in the \( 1/N_f \)-expansion, we find that the flow of the dimensionless effective coupling constant \( \tilde{\alpha} \) is such that:
\[
\tilde{\alpha} \to \frac{8}{N_f(1 + C/N_f)} \approx \frac{8}{N_f}(1 - 0.4317/N_f)
\]
where \( C = 2(12 - \pi^2)/\pi^2 \). Hence, the non-trivial IR fixed point is stable with respect to quantum corrections. Various properties of the theory are explored and related via a mapping to the ones of a \( \mathcal{N} = 1 \) model of super-graphene. In particular, we derive the interaction correction coefficient to the optical conductivity of super-graphene, \( C_{sg} = (12 - \pi^2)/(2\pi) = 0.3391 \), which is six times larger than in the non-supersymmetric case, \( C_g = (92 - 9\pi^2)/(18\pi) = 0.0561 \).
1 Introduction

Three-dimensional Quantum Electrodynamics (QED$_3$) is an archetypal relativistic
gauge-field theory model describing strongly interacting planar fermions. In Minkowski
space, it is described by the action:

$$S_{\text{QED}_3} = \int d^3x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \slashed{D} \psi \right),$$  \hspace{1cm} (1.1)
where \(D_\mu = \partial_\mu + ieA_\mu\), \(\psi^j\) are \(N_f\) flavours \((j = 1, \cdots, N_f)\) of two-component massless Dirac fermions and the coupling constant \(e\) has positive mass dimension \((|e| = 1/2)\) making the theory super-renormalizable.

This model has been studied during more than four decades now. An original motivation \([1, 2]\) came from the fact QED\(_3\) might serve as a prototype for three-dimensional Quantum Chromodynamics with its large-\(N_f\) limit being calculable in the infra-red (IR). Since then, there has been extensive focus on dynamical (flavour) symmetry breaking and fermion mass generation in this model \([3–14]\) (see recent progress in \([15–19]\)). Another original motivation was to study, in the small-\(N_f\) limit, the fate of IR singularities that are ubiquitous to super-renormalizable models \([20–24]\) (see recent progress in \([25–27]\)). In the last three decades, a revived interest in QED\(_3\) also arose in relation with condensed matter physics system exhibiting Dirac-like low-energy excitations such as high-\(T_c\) superconductors \([28–30]\), planar antiferromagnets \([31]\) and graphene \([32]\) (for graphene, see reviews in refs. \([33–36]\)).

In this paper, we will focus on a variant of QED\(_3\), namely (minimal) \(\mathcal{N} = 1\) supersymmetric three-dimensional QED (SQED\(_3\)) with \(N_f\) two-component fermions. Originally, SQED\(_3\) was considered with the hope that the characteristic softer ultra-violet (UV) behaviour of supersymmetric theories will help in solving theoretical issues related to, e.g., dynamical flavour symmetry breaking. Note that the latter may occur without violating supersymmetry (SUSY) in agreement with general arguments \([37]\) stating that SUSY is not broken in SQED. In a seminal paper, Pisarski \([3]\) focused on extended \(\mathcal{N} = 2\) SQED\(_3\) which is constructed by dimensional reduction from four-dimensional SQED (SQED\(_4\)); the large-\(N_f\) limit (see \([38]\) for a review on large-\(N_f\) techniques) of this model was found to be (potentially) tractable with flavour symmetry breaking taking place for all values of \(N_f\). Later, it was argued \([39]\) that a non-perturbative non-renormalization theorem actually forbids dynamical mass generation in \(\mathcal{N} = 1\) SQED\(_4\) \([40]\) which therefore extends by dimensional reduction to \(\mathcal{N} = 2\) SQED\(_3\). Further evidence for the absence of dynamical mass generation in \(\mathcal{N} = 2\) SQED\(_3\) came from numerical simulations \([41]\) and a refined analytic treatment \([42]\).

The situation in \(\mathcal{N} = 1\) SQED\(_3\) is more subtle because of the absence of non renormalization theorems in this case. The model was first considered by Koopmans and Steringa \([39]\) along the lines set by Appelquist et al. for standard QED\(_3\) \([5]\). Their leading-order (LO) in the \(1/N_f\)-expansion Schwinger-Dyson equations approach resulted in a critical fermion flavour number, \(N_{f,cr}\), which, in the Landau gauge \((\xi = 0)\), takes the value \(N_{f,cr}(\xi = 0) = 32/\pi^2 = 3.242\). This value is such that for \(N_f \leq 2\) (parity-invariant) mass is generated. Evidence for possible dynamical mass generation in \(\mathcal{N} = 1\) SQED\(_3\) was also given in \([43]\). However, to the best of our knowledge, there was no improvement of the solution proposed by \([39]\) in the last two decades.

At this point, let us note that of crucial importance for the study of QED\(_3\) (with respect to, e.g., dynamical mass generation) is the existence of a non-trivial interacting IR fixed point as first noticed by \([5]\). Indeed, in such a super-renormalizable theory, one can
define the following dimensionless effective charge:

\[ \overline{\alpha}(p_E) = \frac{e^2}{p_E^2 (1 - \Pi_{1\text{QED}}(p_E^2))} = \begin{cases} \frac{e^2}{p_E} & p_E \gg e^2 N_f / 16 \\ \frac{16}{N_f} & p_E \ll e^2 N_f / 16 \end{cases} \tag{1.2} \]

where \( p_E = \sqrt{-p^2} \) is the Euclidean momentum, \( \Pi_{1\text{QED}}(p_E^2) = -e^2 N_f / (16p_E) \) is the one-loop polarization operator of (non-SUSY) QED\(_3\) and the mass scale \( e^2 \) fixes the separation between the UV and IR regimes. The corresponding beta function:

\[ \beta_{\text{QED}}(\overline{\alpha}) = \frac{d\overline{\alpha}}{dp_E} = -\overline{\alpha} \left( 1 - \frac{N_f}{16} \overline{\alpha} \right) \tag{1.3} \]

displays two stable fixed points: an asymptotically free UV fixed point (\( \overline{\alpha} \to 0 \)) and an interacting IR fixed point: \( \overline{\alpha} \to \overline{\alpha}_{\text{QED}} = 16/N_f \) at the LO of the \( 1/N_f \)-expansion. The latter has some similarity with the Banks-Zaks fixed point [44] known in non-abelian gauge field theories. For the present abelian case, it originates from the softening of the QED\(_3\) photon propagator in the IR limit \( p_E \ll e^2 N_f / 16 \) [1, 2]. This in turn cures QED\(_3\) from its IR singularities, exchanges the dimensionful coupling \( e^2 \) for the dimensionless one \( \overline{\alpha} \), and provides QED\(_3\) with a four-dimensional-like power counting making it effectively renormalizable. Extending these arguments to the next-to-leading order (NLO) of the \( 1/N_f \)-expansion yields [45]:

\[ \overline{\alpha}_{\text{QED}} = \frac{16}{N_f} \left( 1 - \frac{C_{\text{QED}}}{N_f} + O(1/N_f^2) \right), \quad C_{\text{QED}} = \frac{4(92 - 9\pi^2)}{9\pi^2} = 0.1429. \tag{1.4} \]

The striking feature about this result is that the smallness of the coefficient \( C_{\text{QED}} \) preserves the stability of the IR fixed point with respect to radiative corrections. In the following, we shall refer to QED\(_3\) at the IR fixed point as the large-\( N_f \)-limit of QED\(_3\).

On the SUSY side, it is known that \( \mathcal{N} = 2 \) and \( \mathcal{N} = 4 \) SQED\(_4\) also display IR fixed points and that they are present for all values of \( N_f \), see the review [47] (for \( \mathcal{N} = 4 \) SQED\(_4\) it is even one-loop exact). In relation with the IR structure of three-dimensional supersymmetric gauge field theories, the cases \( \mathcal{N} = 2 \) and \( \mathcal{N} = 4 \) were extensively studied by the string theory community for more than two decades now starting from the seminal papers of Seiberg and Witten [48] and Intriligator and Seiberg [49], see also [50] for a review. The more subtle \( \mathcal{N} = 1 \) case is still under active scrutiny, see, e.g., [51–56]. These works are part of a recent revival of interest in three-dimensional quantum field theories (both supersymmetric and non-supersymmetric) in the context of the study of IR dualities, see, e.g., references in [54]. The existence of an IR fixed point in SQED\(_3\) at the LO of the \( 1/N_f \)-expansion was readily assumed in [39] as well as in the most recent work [57] that fits in the modern framework just described, see also [58] for a review.

At the interface with condensed matter physics, there have been proposals that SUSY may emerge in the low-energy limit of various lattice models, see, e.g., [59–65]. At this point, let’s recall that there is no experimental evidence so far that SUSY is realized in

\[ \text{The result } C_{\text{QED}} \text{ was also derived in refs. [9, 10, 46]. Though it does not appear explicitly in these publications, the knowledge of } C_{\text{QED}} \text{ was required to perform the calculations carried out in these papers.} \]
Nature. An emergent SUSY should certainly be difficult to detect in the lab [66]. But this still opens the interesting possibility that some observable quantity in a particular material might be affected by the IR SUSY-invariant fixed point. Alternatively, some condensed-matter physics models describing planar relativistic Dirac fermions where supersymmetrized. Such is the case for the super-graphene model that has been introduced in [67]. Notice that, similarly to the large-\(N_f\) limit of QED\(_3\), the (non-SUSY) relativistic model of graphene is a conformally-invariant field theory that corresponds to the IR Lorentz-invariant fixed point of (non-relativistic) graphene [68]. With fermions localized in (2 + 1)-dimensions while photons propagate in the bulk, such a (conformal) brane-world model and its variants has attracted significant interest in the last years, see, e.g., [69–77]. A universal quantity of interest to compute in graphene-like systems is the optical conductivity in the collisionless regime. Presently, in the non-SUSY case, it is known at two-loop order in the (dimensionless) fine structure constant, \(\alpha_g = e^2/(4\pi)\), and the result reads [68, 74, 78]:

\[
\sigma_g = \sigma_{0g} \left(1 + C_g \alpha_g + O(\alpha_g^2)\right), \quad C_g = \frac{92 - 9\pi^2}{18\pi} = 0.0561,
\]

where \(\sigma_{0g} = N_f e^2/16\) is the minimal conductivity of graphene and \(C_g\) the interaction correction coefficient. This universal (flavour independent) coefficient can actually be related to \(C_{\text{QED}_3}\) of (1.4) with the help of a mapping [71]. Moreover, while the most recent NLO results show that (non-SUSY) QED\(_3\) has a gauge-invariant \(N_{f\text{cr}} = 5.694\) [15, 17, 19], there is no sign of dynamical mass generation for the corresponding (non-SUSY) graphene model [71].

Having set the background material together with the contemporary frame of activity, our primary concern in this paper will be related to the stability of the IR fixed point in SQED\(_3\) with respect to radiative corrections. Focusing on the NLO of the \(1/N_f\)-expansion, we will therefore derive a formula analogous to (1.4) in the SUSY case. In order to achieve this, we will first provide a detailed study of the properties of SQED\(_3\) at the LO of the \(1/N_f\)-expansion. This will lead us to reconsider the problem of dynamical mass generation in this model following [39] but in the modern formulation initiated by [9, 10] and [15] and successfully used for (non-SUSY) QED\(_3\) in [15, 17, 19]. Following [71], we will also find a mapping between SQED\(_3\) and a model of super-graphene which will allow us to transcribe all the results obtained for SQED\(_3\) to the case of super-graphene. In particular, we will derive a formula analogous to (1.5) that will give us access to the universal interaction correction coefficient to the optical conductivity of \(\mathcal{N} = 1\) super-graphene.

This paper is organized as follows. In section 2, we describe the \(\mathcal{N} = 1\) SQED\(_3\) model and its symmetries, followed by the dimensional reduction regularization scheme, the perturbative set-up and the mapping to \(\mathcal{N} = 1\) super-graphene. In section 3, we first obtain the effective gauge multiplet propagators which show a softer momentum dependence in the IR. We utilize these to calculate the field and mass anomalous dimensions of the matter fields. With the help of these results we re-examine dynamical mass generation in \(\mathcal{N} = 1\) SQED\(_3\). All of these LO results are then mapped to analogous quantities for \(\mathcal{N} = 1\) super-graphene. Next, in section 4, we compute the next to leading order corrections to the
gauge multiplet self-energy functions. This is used to examine the stability of the infrared fixed point of this theory. These NLO results are then mapped to the optical conductivity of $N=1$ super-graphene. In section 5, we conclude. Finally, in appendix A we present our notations and conventions, in appendix B the Feynman rules of SQED$_3$, in appendix C the master integrals relevant to our calculations and in appendix D exact formula for all the computed NLO diagrams.

2 The model

2.1 The massless $N=1$ SQED$_3$ action

The degrees of freedom of $N=1$ super QED$_3$ are the $N_f$ matter multiplets $\{\phi^j, \psi^j, F^j\}$ and a gauge multiplet $\{A_{\mu}, \lambda\}$. Here, $\phi^j$ are complex pseudo-scalars, $\psi^j$ are two-component Dirac fermions and $F^j$ are complex auxiliary scalar fields without any dynamics. The gauge multiplet, after choosing the Wess-Zumino gauge, has the $U(1)$ gauge field $A_{\mu}$ and its superpartner the photino $\lambda$, which is a two component Majorana field, and without any auxiliary field (unlike in $3+1$ dimensions). Following [39] and the notation already used in the Introduction, the microscopic action of $N=1$ massless SQED$_3$ is then given by

$$S = \int d^3x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \bar{\lambda} \partial F + |D_{\mu}\phi|^2 + i\bar{\psi} \partial \psi + |F|^2 - i\epsilon (\bar{\psi} \lambda \phi - \bar{\lambda} \psi \phi^*) \right).$$

(2.1)

Similarly to QED$_3$, this theory is super-renormalizable. Moreover, the residual gauge degree of freedom associated to $A_{\mu}$ can be partially fixed by adding to the above action a linear covariant gauge fixing term $L_{gf} = -(1/2\xi) (\partial_{\mu} A^\mu)^2$.

The supersymmetry transformations for the fields are given by

$$\delta_{\epsilon} \phi = \bar{\epsilon} \psi, \quad \delta_{\epsilon} \psi = -(i\partial \phi + F) \epsilon, \quad \delta_{\epsilon} F = i\epsilon (\partial \psi - e \lambda \phi),$$

(2.2a)

$$\delta_{\epsilon} A_{\mu} = i\bar{\epsilon} \gamma_{\mu} \lambda, \quad \delta_{\epsilon} \lambda = \frac{1}{2} \gamma_{\mu\nu} \epsilon F^{\mu\nu},$$

(2.2b)

where the parameter of the transformation $\epsilon$, is an anticommuting Majorana spinor (see appendix A for our conventions). In the absence of the gauge-fixing term, these transformations leave the Lagrangian associated to (2.1) invariant up to a total derivative

$$\delta_{\epsilon} L = \partial_{\mu} \left( -\frac{i}{4} \bar{\epsilon} \gamma^{\mu\rho\sigma} \lambda F_{\rho\sigma} - \frac{i}{2} \bar{\psi} \gamma^{\mu} \epsilon F + (D^{\mu} \phi)^* \bar{\psi} + D^{\mu} \phi \bar{\psi} \epsilon - (D_{\nu} \phi)^* \bar{\epsilon} \gamma^{\nu\mu} \psi \right),$$

(2.3)

With respect to [39], we have augmented the transformation of the auxiliary field $F$ by the term $-ie \bar{\epsilon} \lambda \phi$ (in agreement with the result of [79]). Next, we have to check that the transformations (2.2) provide a representation of the $M=1$ supersymmetry algebra, $\{Q_\alpha, Q_\beta\} = 2(\gamma^\mu C)_{\alpha\beta} P_\mu$ ($\alpha = 1, 2$), where $Q_\alpha$ are the supercharges and $C$ is the charge conjugation matrix. To this end, we evaluate the following commutators

$$[\delta_{\epsilon_2}, \delta_{\epsilon_1}] X = -2i\bar{\epsilon}_1 \gamma^{\mu} \epsilon_2 D_{\mu} X, \quad X \in \{\phi, \psi, F\},$$

(2.4a)

$$[\delta_{\epsilon_2}, \delta_{\epsilon_1}] \lambda = -2i\bar{\epsilon}_1 \gamma^{\mu} \epsilon_2 \partial_{\mu} \lambda, \quad [\delta_{\epsilon_2}, \delta_{\epsilon_1}] A_{\mu} = -2i\bar{\epsilon}_1 \gamma^{\rho} \epsilon_2 \partial_{\rho} A_{\mu} + \partial_{\mu} (2i\bar{\epsilon}_1 \gamma^{\rho} \epsilon_2 A_{\rho}),$$

(2.4b)
which result in a covariant translation (translation combined with a gauge transformation). Such closure of the algebra up to field-dependent gauge transformations (with the gauge function $\Lambda = 2i \bar{\epsilon}_1 A \epsilon_2$) is expected of supersymmetric gauge theories in the Wess-Zumino gauge.

The supersymmetry invariance of (2.1) under the two supercharges $Q_\alpha$, together with its $U(N_f)$ flavour symmetry and its invariance under parity and time-reversal are actually enough to fix the Lagrangian once the purely bosonic part is written down. This enforces the (already assumed) equality of the gauge coupling $e$ appearing in the cubic and quartic interaction terms as well as the equality of any possible mass terms which might be added to the model.

### 2.2 Dimensional reduction scheme

While computing perturbative amplitudes in a supersymmetric gauge theory using the usual dimensional regularization (DREG) scheme in a $d$-dimensional space, one immediately encounters an apparent problem that can be understood essentially in terms of the mismatch of degrees of freedom in the gauge multiplet: between the gauge field (which has $d - 1$ components) and the photino (which retains all of its components). To remedy this, Siegel [80] introduced the dimensional reduction (DRED) scheme as a manifestly supersymmetric regulator. We shall be following this scheme along with modified minimal subtraction ($\overline{\text{DR}}$) and briefly describe it (see [81] for an early pedagogical treatment and [82–84] for more recent reviews) for the special case of SQED in $2 + 1$ dimensions that we are interested in.

In DRED, the continuation from the physical space-time dimension 3 to $d = 3 - 2\varepsilon < 3$ dimensions can be interpreted as a compactification which preserves the three-dimensional character of the fields. Indeed, while coordinates and momenta are $d$-dimensional, the fermion fields still have a 3-dimensional nature and are two-component objects, whereas the gauge field $A_{\mu}$ (formally) splits as $A_{\mu} = A_{\tilde{\mu}} + A_{\tilde{\mu}}$. Here $A_{\tilde{\mu}}$ are referred to as the $\varepsilon$-scalars and correspond to the $2\varepsilon$ amount of scalars obtained; the hatted indices refer to the usual $d$ dimensions. The $\varepsilon$-scalars thus account for the degrees of freedom lost by the gauge bosons (see appendix A for more on the conventions we use).

Let’s note that some potential technical inconsistencies may arise at higher loop orders as pointed out by Siegel [85]. These can be overcome by using certain infinite dimensional quasi-spaces as introduced in [86] and later clarified in [87]. Such a consistent formulation of DRED was proved to be supersymmetric at low orders of perturbation theory and is presently the most convenient regularization scheme for practical calculations in the component formalism. The preserved three-dimensional nature of the fields is supposed to insure the validity of supersymmetric Ward-Takahashi or Slavnov-Taylor identities while the $d$-dimensional nature of space-time coordinates regularizes, just as in DREG, divergent loop integrals, see [83] for a detailed review on these developments in the frame of four-dimensional theories. Without entering any related mathematical construction, we will assume that they also hold in the present three-dimensional case and that, as a consequence, the $\overline{\text{DR}}$ scheme is a consistent supersymmetric regulator for SQED$_3$, at least at the low orders of perturbation theory that we shall consider in the following.
At the level of the Lagrangian, the decomposition of the gauge field yields:

\[ \mathcal{L}_{d=3} = \mathcal{L}_{3-2\varepsilon} + \mathcal{L}_{\varepsilon\text{-scalars}}, \]  

(2.5)

where \( \mathcal{L}_{3-2\varepsilon} \) takes the same form as the original Lagrangian but with all vector indices restricted to \( d = 3 - 2\varepsilon \) dimensions, and

\[ \mathcal{L}_{\varepsilon\text{-scalars}} = -\frac{1}{2} \partial_{\mu} A_{\rho} \partial^{\mu} A^{\rho} - e A_{\rho} \bar{\psi} \gamma^{\rho} \psi + e^2 A_{\rho} |\phi|^2. \]  

(2.6)

Notice that the \( \varepsilon \)-scalar part of the Lagrangian, \( \mathcal{L}_{\varepsilon\text{-scalars}} \), does not appear in usual dimensional regularization (DREG) and is specific to the DRED scheme. Thus, \( \varepsilon \)-scalars give rise to additional cubic and quartic couplings; the equality of the matter couplings to \( A^{\hat{\mu}} \) and \( A_{\bar{\nu}} \) is a consequence of supersymmetry [88, 89] and has been assumed.

2.3 Perturbation theory setup

From eq. (2.5), the momentum-space bare propagators of the model are given by:

\[ S_0(p) = \langle \psi(x) \bar{\psi}(0) \rangle_{\text{F.T.}} = \frac{i}{\not{p}}, \]  

(2.7a)

\[ D_{0}^{\hat{\mu}\hat{\nu}}(p) = \langle A^{\hat{\mu}}(x) A^{\hat{\nu}}(0) \rangle_{\text{F.T.}} = -\frac{i}{p^2} \left( g^{\hat{\mu}\hat{\nu}} - (1 - \xi) \frac{p^\mu p^\nu}{p^2} \right), \]  

(2.7b)

\[ \mathcal{E}_{0}^{\hat{\mu}\hat{\nu}}(p) = \langle A^{\hat{\mu}}(x) A^{\hat{\nu}}(0) \rangle_{\text{F.T.}} = -\frac{i g^{\hat{\mu}\hat{\nu}}}{p^2}, \]  

(2.7c)

\[ \sigma_0(p) = \langle \lambda(x) \bar{\lambda}(0) \rangle_{\text{F.T.}} = \frac{i}{\not{p}}, \]  

(2.7d)

\[ \Delta_0(p) = \langle \phi(x) \phi^\dagger(0) \rangle_{\text{F.T.}} = \frac{i}{p^2}, \]  

(2.7e)

where F.T. stands for Fourier transform. These propagators are part of a set of Feynman rules (which also include vertices) that are displayed in appendix B and allow for a systematic study of SQED3 in a loop-expansion in the dimensionless parameter \( \alpha/\sqrt{-p^2} \).

Upon turning on interactions, the dressed propagators and vertex functions will satisfy a set of coupled Schwinger-Dyson equations. Focusing on the propagators, the general solutions of these equations take the form (in the massless case):

\[ S(p) = \frac{i}{\not{p} - \frac{1}{1 - \Sigma^\psi_{\bar{\nu}}(p^2)}}, \]  

(2.8a)

\[ D^{\hat{\mu}\hat{\nu}}(p) = \frac{-i}{p^2} \left( g^{\hat{\mu}\hat{\nu}} - (1 - \xi) \frac{p^\mu p^\nu}{p^2} \right) \frac{1}{1 - \Pi^\gamma(p^2)}, \]  

(2.8b)

\[ \mathcal{E}^{\hat{\mu}\hat{\nu}}(p) = -\frac{i g^{\hat{\mu}\hat{\nu}}}{p^2} \frac{1}{1 - \Pi^\gamma(p^2)}, \]  

(2.8c)

\[ \sigma(p) = \frac{i}{\not{p} - \Pi^\lambda(p^2)}, \]  

(2.8d)

\[ \Delta(p) = \frac{i}{p^2 - \Sigma^\phi_{\bar{\nu}}(p^2)}, \]  

(2.8e)
where, anticipating the study of the IR fixed point, a usual non-local gauge has been adopted for the photon propagator [6, 90, 91]. The following parametrization has been used for the (1-particle irreducible) self-energies entering the massless eqs. (2.8):

\[ \Sigma_\psi(p^2) = \not{p} \Sigma_\psi(p^2), \]
\[ \Sigma_\psi V(p^2) = \frac{\text{Tr}[\not{p} \Sigma_\psi(p^2)]}{2p^2}, \]
\[ \Pi_\gamma(p^2) = \not{p} \Pi_\gamma(p^2), \]
\[ \Pi_\gamma(p^2) = \frac{\Pi_\mu_\mu(p)}{(d-1)p^2}, \]
\[ \Pi_\lambda(p^2) = \not{p} \Pi_\lambda(p^2), \]
\[ \Pi_\lambda V(p^2) = \text{Tr}[\not{p} \Pi_\lambda(p^2)] \]
\[ \Sigma_\phi(p^2) = \not{p} \Sigma_\phi S_\phi(p^2). \]

As will be shown in section 3, the photon, \( \varepsilon \)-scalar and photino propagators get IR softened already at the LO of the \( 1/N_f \)-expansion, i.e.,

\[ D_\gamma^{\hat{\mu}\hat{\nu}}(p) = i \frac{a}{2p^2} \left( g^{\hat{\mu}\hat{\nu}} - (1 - \xi) \frac{p^{\hat{\mu}} p^{\hat{\nu}}}{p^2} \right), \]
\[ \varepsilon_\gamma^{\hat{\mu}\hat{\nu}}(p) = i \frac{g^{\hat{\mu}\hat{\nu}}}{2a \sqrt{-p^2}}, \]
\[ \sigma(p) = \frac{-i p^{\hat{\mu}}}{2a \sqrt{-p^2}}, \]

where \( a = N_f e^2/16 \). Replacing (2.7b), (2.7c) and (2.7d) by eqs. (2.10) allows for a systematic study of SQED\(_3\) at the IR fixed point in a \( 1/N_f \)-expansion.

At this point, let’s remark that some of the above self-energies and related anomalous dimensions might be constrained by Ward-Takahashi or Slavnov-Taylor identities. From the study of four-dimensional abelian gauge field theories, see [92, 93], let us mention in particular two important identities that will be of interest to us: \(^2\)

(i) the polarization functions in the gauge multiplet (see (2.9)) must all be equal:

\[ \Pi_\gamma(p^2) = \Pi_\epsilon(p^2) = \Pi_\lambda V(p^2). \]

(ii) the mass anomalous dimensions of the selectron and the electron must coincide.

We do not propose here any formal proof of these identities in the three-dimensional case. However, as will be explicitly shown in the following, both of them hold for SQED\(_3\) at the orders considered in this paper (LO for the anomalous mass dimensions and NLO for the polarization functions).

\(^2\) As noted in [93], supersymmetry is seen to realize only on the physical Hilbert space and hence unphysical quantities such as field anomalous dimensions for the matter fields (which are gauge dependent) are not constrained. In our calculations, we find that the field anomalous dimensions of the selectron and the electron do not coincide at LO, see (3.19) and (3.25).
2.4 Mapping to \( \mathcal{N} = 1 \) super-graphene model

The results that we will obtain for the large-\( N_f \) limit of \( \mathcal{N} = 1 \) SQED3 can be used to study \( \mathcal{N} = 1 \) (suspended) super-graphene. The latter may be thought of as the minimal supersymmetric extension of the model of (suspended) graphene at its IR Lorentz invariant fixed point \([68]\). It is characterized by \( N_f \) electron fields localized on a three dimensional membrane and interacting via photons that are allowed to propagate in the full four dimensional space-time. Our model for super-graphene (henceforth denoted by sg) differs from the boundary model considered in \([67]\) in that our boundary is a transparent interface while the model of \([67]\) considers a purely reflecting boundary (graphene on a substrate).

The action for \( \mathcal{N} = 1 \) (suspended) super-graphene can be written as \( S = S_{\text{bdry}} + S_{\text{bulk}} \) where

\[
S_{\text{bdry}} = \int d^3x \left( |D_\mu \phi|^2 + i \bar{\psi} D \psi + |F|^2 - i e_\rho (\bar{\psi} \lambda \phi - \lambda \bar{\psi} \phi^*) \right), \tag{2.12a}
\]

\[
S_{\text{bulk}} = \int d^4x \left( -\frac{1}{4} F_{ab} F^{ab} + \frac{i}{2} \bar{\lambda} \partial \Lambda + \frac{1}{2} D^2 \right), \tag{2.12b}
\]

where \( e_\rho \) denotes the (dimensionless) coupling constant of graphene. In (2.12a), the notation for the fields is the same as in (2.1). In the bulk action (2.12b), the indices \( a \) and \( b \) take values \( 0, ..., 4 \), \( \theta = \Gamma^a \partial_a \) where \( \Gamma^a \) are \( 4 \times 4 \) gamma matrices, \( \Lambda \) denotes a four-component Majorana field and \( D \) is a real auxiliary field. We can add a (four-dimensional) bulk gauge fixing term \( L_{gf} = -1/2 \xi (\partial_\alpha A^\alpha)^2 \) to write down the bulk propagators:

\[
\langle A_a(p)A_b(-p) \rangle = \frac{i}{p^2} \left( g_{ab} - \frac{(1 - \xi) p_a p_b}{p^2} \right), \tag{2.13a}
\]

\[
\langle A_a(p)\bar{\Lambda}_\beta(-p) \rangle = \frac{i p_a \Gamma^a_{\alpha\beta}}{p^2}. \tag{2.13b}
\]

At this point, we perform a dimensional reduction that introduces hatted and barred objects as for SQED3 together with an additional propagator for the (bulk) \( \varepsilon \)-scalar. Furthermore, by integrating these propagators over the bulk degrees of freedom \(^3\) it is possible to derive effective gauge propagators on the three-dimensional membrane. This results in:

\[
D_{0,\mu\nu}^{\text{sg}}(p) = \frac{i}{2} \frac{d_{\mu\nu}}{\sqrt{-p^2}}, \quad D_{\mu\nu}^{\text{LO,SQED3}}(p) = \frac{8i}{N_f e^2 \sqrt{-p^2}} d_{\mu\nu}(p, \eta), \tag{2.14a}
\]

\[
\sigma_0^{\text{sg}}(p) = -\frac{i p^\mu}{2 \sqrt{-p^2}}, \quad \sigma_{\mu\nu}^{\text{LO,SQED3}}(p) = -\frac{8i p^\mu}{N_f e^2 \sqrt{-p^2}}, \tag{2.14b}
\]

\[
\xi_0^{\text{sg}}(p) = \frac{i}{2} \frac{g_{\mu\nu}}{\sqrt{-p^2}}, \quad \xi_{\mu\nu}^{\text{LO,SQED3}}(p) = \frac{8i}{N_f e^2 \sqrt{-p^2}} g_{\mu\nu}, \tag{2.14c}
\]

where \( d^{\mu\nu}(\eta) = g^{\mu\nu} - \eta p^\mu p^\nu / p^2, \eta = 1 - \xi, \bar{\eta} = 1 - \bar{\xi} \) and we have also added the IR softened propagators of SQED3 (2.10a) for comparison. Similarly to the non-supersymmetric case \(^3\)In the case of the bulk four-component photino, \( \Lambda \), this procedure is accompanied by projecting out two of its components to identify it with the boundary two-component photino \( \lambda \).
we see from (2.14) that there is a mapping between large-$N_f$ SQED$_3$ and the super-graphene model which is related to the fact that the gauge multiplet propagators in both models have the same form. Explicitly, the mapping reads:  

\[
\frac{1}{\pi^2 N_f} \rightarrow g = \frac{\alpha_g}{4\pi}, \quad \eta \rightarrow \frac{\eta}{2} \left( \xi \rightarrow \frac{1 + \xi}{2} \right),
\]

(2.17)

where $\alpha_g = e^2 / (4\pi)$ is the (dimensionless) fine-structure constant of graphene and $g$ the corresponding reduced coupling constant.

The mapping (2.17) will be used in the following to transcribe all the results obtained for large-$N_f$ SQED$_3$ to the case of super-graphene.

## 3 Leading order analysis

In this section we present the LO analysis of the model. We will show how several propagators get IR softened. We will also compute wave function renormalizations of the electron and scalar fields as well as their mass anomalous dimensions. Finally, the dynamical generation of a parity-even mass term will be studied.

### 3.1 IR softened photon, epsilon-scalar and photino propagators

We consider first the photon propagator, eq. (2.8b), and compute the LO photon polarization function

\[
\Pi^{\hat{\mu}\hat{\nu}}_1(p) = \Pi^{\hat{\mu}\hat{\nu}}_{1a}(p) + \Pi^{\hat{\mu}\hat{\nu}}_{1b}(p),
\]

(3.1)
which is parameterized as in (2.9b) and consists of the sum of the two diagrams displayed on figure 1. These diagrams are defined as:

\[ i \Pi_{1a}^{\hat{\mu}\hat{\nu}}(p) = -\mu^2 N_f \int [d^d k] \text{Tr} \left[ (-ie\gamma^\hat{\mu})S_0(k-p)(-ie\gamma^\hat{\nu})S_0(k) \right], \tag{3.2a} \]

\[ i \Pi_{1b}^{\hat{\mu}\hat{\nu}}(p) = \mu^2 N_f \int [d^d k] (ie(2k-p)^\hat{\mu}) \Delta_0(k-p)(ie(2k-p)^\hat{\nu}) \Delta_0(k), \tag{3.2b} \]

where \( \mu \) is the renormalization scale and the bare propagators are given in (2.7) (see also appendix B). These diagrams are straightforward to compute using techniques of massless Feynman diagram calculations, see, e.g., [94] for a review. The final results, expressed in the DR scheme, read:

\[ \Pi_{1a}^\gamma(p^2) = -\frac{N_f e^2}{16 \sqrt{-p^2}} \left( 1 - (1 - 2 \log 2 + L_p) \varepsilon + O(\varepsilon^2) \right), \tag{3.4a} \]

\[ \Pi_{1b}^\gamma(p^2) = -\frac{N_f e^2}{16 \sqrt{-p^2}} \left( 1 + (1 + 2 \log 2 - L_p) \varepsilon + O(\varepsilon^2) \right), \tag{3.4b} \]

where \( L_p = \log(-p^2/\bar{p}^2) \), we see that both diagrams equally contribute in the limit \( \varepsilon \to 0 \).

From these results, the total photon polarization function at LO is given by:

\[ \Pi_1^\gamma(p^2) = -\frac{N_f e^2}{(4\pi)^{3/2} \sqrt{-p^2}} \left( \frac{\bar{p}^2}{-p^2} \right)^{-\varepsilon} e^{\gamma \varepsilon \varepsilon} G(d, 1, 1), \tag{3.5} \]

and its expression in strictly \( d = 3 \) reads:

\[ \Pi_1^\gamma(p^2) = -\frac{2a}{\sqrt{-p^2}}, \tag{3.6} \]

\(^5\)Notice that, because we work in DRED, all massless tadpoles are zero. We shall therefore neglect them all, both at the LO and at the NLO of the 1/N_f-expansion.
Figure 3: Leading order corrections to the photino propagator. The indicated momenta flow in the anti-clockwise sense.

where $a = N_f e^2 / 16$. Interestingly, eq. (3.6) is simply twice the value for QED$_3$ [1, 2] which coincides with the earlier result given in ref. [39], but now obtained using dimensional reduction. Substituting (3.6) in (2.8b), the photon propagator IR softens in the large-$N_f$ limit, $p_E \ll 2a = N_f e^2 / 8$ (where $p_E = \sqrt{-p^2}$ is the Euclidean momentum), and takes the form already advertised in (2.10a).

Next, we proceed in the same way for the $\varepsilon$-scalar propagator, eq. (2.8c), and compute the LO $\varepsilon$-scalar polarization function which is parameterized as in (2.9c) and consists of a single non-vanishing diagram displayed on figure 2. This diagram is defined as:

$$i \Pi_1^{\varepsilon\bar{\varepsilon}}(p) = -\mu^{2\varepsilon} N_f \int [d^d k] \text{Tr} \left[ (-i e \gamma^\mu) S_0(k-p)(-i e \gamma^\nu) S_0(k) \right] ,$$

and its computation yields:

$$\Pi_1^{\varepsilon}(p^2) = -\frac{N_f e^2}{(4\pi)^{3/2} \sqrt{-p^2}} \left( \frac{p^2}{-p^2} \right)^{\varepsilon} e^{\gamma E \varepsilon} G(d, 1, 1) ,$$

which is exactly equal to the one-loop photon polarization function, (3.5). Exactly in $d = 3$ we therefore have:

$$\Pi_1^{\varepsilon}(p^2) = -\frac{2a}{\sqrt{-p^2}} .$$

Substituting (3.9) in (2.8c), the $\varepsilon$-scalar propagator IR softens in the large-$N_f$ limit, $p_E \ll N_f e^2 / 8$, and takes the form already advertised in (2.10b).

Lastly, we proceed in the same way for the photino propagator, eq. (2.8d), and compute the LO photino self-energy which is parameterized as in (2.9d) and consists of two non-vanishing diagrams with opposite fermion number flows (or charge flows) displayed on figure 3. As we briefly explain at the end of appendix B, in order to evaluate these diagrams (and other diagrams involving Majorana fermions) we need to use specific Feynman rules that involve assigning a continuous fermion flow in addition to the fermion number flow. Such a procedure reveals that the two diagrams of figure 3 are equal and the resulting contribution
Figure 4: One loop electron self energy diagrams. The photon, ε-scalar and photino propagators are the IR softened ones.

is defined as:

\[ -i \Pi_1^\gamma(p) = 2 \mu^{2 \varepsilon} N_f \int [d^d k] (e^+ S_0(k)(-e) \Delta_0(k-p)) . \]  

(3.10)

Evaluating the integral yields:

\[ \Pi_{1V}^\lambda(p^2) = -\frac{N_f e^2}{(4\pi)^{3/2} \sqrt{-p^2}} \left( \frac{\mu^2}{-p^2} \right)^{\varepsilon} e^{7\varepsilon\varepsilon} G(d,1,1), \]  

(3.11)

which is exactly equal to both the one-loop photon (3.5) and ε-scalar (3.8) polarization functions. So, exactly in \( d = 3 \) we have:

\[ \Pi_{1V}^\lambda(p^2) = -\frac{2a}{\sqrt{-p^2}}. \]  

(3.12)

Substituting (3.12) in (2.8d), the photino propagator IR softens in the large-\( N_f \) limit, \( p_E \ll N_f e^2/8 \), and takes the form already advertised in (2.10c).

Summarizing, we find that the photon, ε-scalar and photino self-energies are all equal at the LO of the \( 1/N_f \)-expansion:

\[ \Pi_1^\gamma(p^2) = \Pi_1^\varepsilon(p^2) = \Pi_{1V}^\lambda(p^2) = -\frac{2a}{\sqrt{-p^2}}, \]  

(3.13)

in agreement with the identity discussed at the end of section 2.3 and that the corresponding propagators soften in the IR. Analogously to the case of QED\(_3\) discussed in the Introduction, this softening gives rise to the non-trivial IR fixed point at which we have an (IR-safe) interacting \( \mathcal{N} = 1 \) superconformal field theory.

3.2 Wave function renormalizations

We start with the electron propagator, eq. (2.8a), and compute the LO electron self-energy

\[ \Sigma_1^\psi(p) = \Sigma_{1a}^\psi(p) + \Sigma_{1b}^\psi(p) + \Sigma_{1c}^\psi(p), \]  

(3.14)

\[ ^6 \text{For the diagrams of figure 3, we assign a fermion flow to the fermion chain going from left to right. In case of diagram 3a, the fermion flow, fermion number flow and momentum flow are all in the same direction. Moreover, the vertices are proportional to the unit matrix. Therefore, there is no reversed propagator or vertex. In the case of diagram 3b, the fermion flow is opposite to the fermion number flow so that the Dirac propagator gets reversed. However, the fermion number flow is also opposite to the momentum flow so that the momentum gets an additional − sign. All together, the Dirac propagator remains unchanged (as \( S(k) \rightarrow S((-k)) \)) and diagram 3b is therefore equal to diagram 3a.} \]
which consists of the sum of the three diagrams displayed on figure 4. They are defined as:

\[-i \Sigma_1^\psi(p) = \mu^2 e^{\int [d^d k] (-e^\gamma k) S_0(k)(-ie^\gamma k) D_{LO}(p-k)\}, (3.15a)\]

\[-i \Sigma_2^\psi(p) = \mu^2 e^{\int [d^d k] (-e^\gamma k) S_0(k)(-ie^\gamma k) E_{LO}(p-k)\}, (3.15b)\]

\[-i \Sigma_3^\psi(p) = \mu^2 e^{\int [d^d k] (+e) \Delta_0(k)(-e) \sigma_{LO}(p-k)\}, (3.15c)\]

where \(\gamma\) vanishes in the Landau gauge. This is to be contrasted with the non-supersymmetric LO electron anomalous dimension (expressed in terms of \(N_f\) 2-component spinors):

\[\gamma_{LO} = \frac{8}{N_f^2} \frac{1}{4\pi^2} \left( \frac{2}{3} \right) \gamma_{LO} = \frac{8}{N_f^2} \left( \frac{2}{3} \right) \gamma_{LO} \]  

that vanishes in the so-called Nash gauge \[6\], \(\xi = 2/3\).

We proceed in a similar way for the scalar propagator, eq. (2.8e), and compute the LO scalar self-energy

\[\Sigma_1^\phi(p) = \Sigma_1^\phi + \Sigma_1^\phi(p)\], (3.20)
Figure 5: One loop scalar self energy diagrams. The photon and photino propagators are the IR softened ones. The momenta flow from left to right.

which consists of the sum of the two diagrams displayed on figure 5. They are defined as:

\[ -i \Sigma_{1a}^\phi(p) = \mu^{2\varepsilon} \int [d^d k] \left( -ie(p + k)^\mu \right) \Delta_0(k)(-ie(p + k)^\nu)D_{\text{LO}} \hat{\mu}\hat{\nu}(p - k) , \]  
(3.21a)

\[ -i \Sigma_{1b}^\phi(p) = -\mu^{2\varepsilon} \int [d^d k] \text{Tr} \left[ (-e)S_0(k)(+e)\sigma_{\text{LO}}(k - p) \right] , \]  
(3.21b)

where the photon and photino propagators are the IR softened ones and \( \Sigma_{1b}^\phi \) is a fermion loop. Computing these diagrams with the parametrization (2.9e) yields:

\[ \Sigma_{1S,a}^\phi(p^2) = \frac{8}{(4\pi)^{3/2} N_f} \left( \frac{\mu^2}{-p^2} \right) ^\varepsilon \left( 4 \frac{(d - 1)(d - 2)}{2d - 3} - (2d - 5) \xi \right) e^{\gamma E \varepsilon} G(d, 1, 1/2) , \]  
(3.22a)

\[ \Sigma_{1S,b}^\phi(p^2) = -\frac{16}{(4\pi)^{3/2} N_f} \left( \frac{\mu^2}{-p^2} \right) ^\varepsilon \frac{d - 2}{2d - 3} e^{\gamma E \varepsilon} G(d, 1, 1/2) . \]  
(3.22b)

The total scalar self-energy is given by:

\[ \Sigma_{1S}^\phi(p^2) = \frac{8}{(4\pi)^{3/2} N_f} \left( \frac{\mu^2}{-p^2} \right) ^\varepsilon \left( 2(d - 2) - (2d - 5) \xi \right) e^{\gamma E \varepsilon} G(d, 1, 1/2) , \]  
(3.23)

which, in \( \varepsilon \)-expanded form, reads:

\[ \Sigma_{1S}^\phi(p^2) = \frac{2}{\pi^2 N_f} \left( \frac{\mu^2}{-p^2} \right) ^\varepsilon \left( \frac{2 - \xi}{\varepsilon} + 4(1 - \log 2) + 2\xi \log 2 + O(\varepsilon) \right) . \]  
(3.24)

From this result, we extract the LO scalar wave-function renormalization:

\[ \gamma_\phi = \mu \frac{d\Sigma_{1S}^\phi(p^2)}{d\mu} = \frac{4(2 - \xi)}{\pi^2 N_f} + O(1/N_f^2) , \]  
(3.25)

which is not equal to the one of the electron (3.19). 

\footnotemark[1] For the diagram of figure 5b, we assign a counter-clockwise fermion flow to the fermion loop consisting of the Dirac and Majorana fermions. For the Dirac propagator, the fermion flow, fermion number flow and momentum flow all have the same orientation. On the other hand, for the Majorana propagator, the fermion flow is opposite to the momentum flow. Hence, the Majorana propagator gets reversed: \( \sigma_{\text{LO}}(p - k) \rightarrow \sigma_{\text{LO}}(-(p - k)) \).
3.3 Mass anomalous dimensions

So far we have considered a model of massless particles, (2.1). In this subsection, we add a bare mass \( m_f \) to the electron in order to extract its mass anomalous dimension. Because of SUSY invariance, an equal mass should be given to the selectron, \( m_s = m_f = m \). Moreover, we would like to preserve the parity invariance of our Lagrangian, (2.5). So we will assume that these masses are parity-even.

Let’s then assume that both the Dirac fermion and complex scalar fields are massive and momentarily change their bare propagators accordingly:

\[
\tilde{S}_{0\pm}(p) = \frac{i (p \pm m_f)}{p^2 - m_f^2}, \quad \tilde{\Delta}_0(p) = \frac{i}{p^2 - m_s^2},
\]

(3.26)

where we take \( m_f \) and \( m_s \) as arbitrary masses for the moment (as will be shown below, we shall recover the mass degeneracy constraint from our calculations). The electron and selectron self-energies are then parameterized as:

\[
\tilde{\Sigma}^\psi_0(p) = m\Sigma^\psi_V(p^2) \pm m_f \Sigma^\psi_S(p^2), \quad \tilde{\Sigma}^\phi_0(p) = p^2 \Sigma^\phi_V(p^2) + m_s^2 \Sigma^\phi_S(p^2),
\]

(3.27)

from which we define the corresponding mass anomalous dimensions:

\[
\gamma_{m\psi} = \mu \frac{d (\tilde{\Sigma}^\psi_S)'(p^2)}{d \mu}, \quad (\tilde{\Sigma}^\psi_S)'(p^2) = \frac{1 + \Sigma^\psi_S(p^2)}{1 - \Sigma^\psi_V(p^2)},
\]

(3.28a)

\[
\gamma_{m\phi} = \frac{1}{2} \mu \frac{d (\tilde{\Sigma}^\phi_S)'(p^2)}{d \mu}, \quad (\tilde{\Sigma}^\phi_S)'(p^2) = \frac{1 + \Sigma^\phi_S(p^2)}{1 - \Sigma^\phi_V(p^2)},
\]

(3.28b)

where \((\tilde{\Sigma}^\psi_S)'(p^2)\) and \((\tilde{\Sigma}^\phi_S)'(p^2)\) are gauge invariant combinations. Because we are interested in anomalous dimensions, only the singular part of these self-energies will be computed. We already know \(\Sigma^\psi_V(p^2)\) and \(\Sigma^\phi_V(p^2)\) from the previous subsection. So we shall focus now on the computations of \(\Sigma^\psi_S(p^2)\) and \(\Sigma^\phi_S(p^2)\).

We first focus on the LO \(\Sigma^\psi_S(p^2)\) which consists of three contributions corresponding to the scalar parts of the diagrams displayed on figure 4 and defined as in (3.15) but now with massive Dirac fermion and scalar propagators:

\[
-i \tilde{\Sigma}^\psi_1\pm(p) = \mu^{2\varepsilon} \int [d^d k] (-ie\gamma^\mu) \tilde{S}_{0\pm}(k)(-ie\gamma^\nu) D_{LO}\tilde{\Delta}\tilde{\Delta}\tilde{\Delta}(p - k),
\]

(3.29a)

\[
-i \tilde{\Sigma}^\psi_1\pm(p) = \mu^{2\varepsilon} \int [d^d k] (-ie\gamma^\mu) \tilde{S}_{0\pm}(k)(-ie\gamma^\nu) E_{LO}\tilde{\Delta}\tilde{\Delta}\tilde{\Delta}(p - k),
\]

(3.29b)

\[
-i \tilde{\Sigma}^\psi_1\pm(p) = \mu^{2\varepsilon} \int [d^d k] (+e) \tilde{\Delta}_0(k)(-e)\sigma_{LO}(p - k),
\]

(3.29c)

where the photon, \(\varepsilon\)-scalar and photino propagators are still the IR softened ones. The scalar integrals are projected out from (3.29) with the help of:

\[
\Sigma^\psi_S(p^2) = \pm \frac{1}{2m_f} \text{Tr}[\tilde{\Sigma}^\psi_S(p)].
\]

(3.30)

\(^8\)The \(\pm\) signs in the fermion propagator are related to the different fermion flavours according to (3.40) which is a parity-even mass operator.
These integrals are then computed in the zero-mass limit \((m_f = m_s = 0)\) which is enough to extract their UV singular structure as known from IR rearrangement \([95]\). The computations yield:

\[
\Sigma_{1S_a}(p^2) = \frac{8}{(4\pi)^{3/2} N_f} \left( \frac{\overrightarrow{p}^2}{-p^2} \right)^\varepsilon (d - 1 + \xi) e^{\gamma_E \varepsilon} G(d, 1, 1/2), \tag{3.31a}
\]

\[
\Sigma_{1S_b}(p^2) = -\frac{8}{(4\pi)^{3/2} N_f} \left( \frac{\overrightarrow{p}^2}{-p^2} \right)^\varepsilon (d - 3) e^{\gamma_E \varepsilon} G(d, 1, 1/2), \tag{3.31b}
\]

\[
\Sigma_{1S_c}(p^2) = 0, \tag{3.31c}
\]

where the contribution of \(\Sigma_{1S_b}\) is finite in \(d = 3\) while that of \(\Sigma_{1S_c}\) vanishes identically.

Hence, in the limit \(d \to 3\), the LO contribution to \(\Sigma_{1S}\) is given by:

\[
\Sigma_{1S}(p^2) = \frac{2(2 + \xi)}{\pi^2 N_f} \left( \frac{\overrightarrow{p}^2}{-p^2} \right)^\varepsilon \left( \frac{1}{\varepsilon} + O(1) \right). \tag{3.32}
\]

Together with (3.18) and (3.28a), the fermion mass anomalous dimension therefore reads:

\[
\gamma_{m\psi} = \frac{8}{\pi^2 N_f} + O(1/N_f^2). \tag{3.33}
\]

We now consider \(\Sigma_{1S_m}(\mu^2)\) which consists of two contributions corresponding to the scalar parts of the diagrams displayed on figure 5 and defined as in (3.21) but now with massive Dirac fermion and scalar propagators:

\[
-i \tilde{\Sigma}_{1a}(p) = \mu^{2\varepsilon} \int [d^d k] \left( -ie(p + k)\hat{\mu} \right) \tilde{\Delta}_0(k)(-ie(p + k)^\hat{\nu}) D_{LO}\hat{\mu}\hat{\nu}(p - k), \tag{3.34a}
\]

\[
-i \tilde{\Sigma}_{1b}(p) = -\mu^{2\varepsilon} \int [d^d k] \text{Tr} \left[ (-e)\tilde{S}_0\pm(k)(+e)\sigma_{LO}(k - p) \right], \tag{3.34b}
\]

where the photon and photino propagators are the IR softened ones. As an IR rearrangement, we can extract the UV singular part of the above integrals by reducing them to massive tadpoles. This amounts to simply set the external momentum to zero, \(p = 0\), in (3.34). Together with the parametrization (3.27), this yields:

\[
\Sigma_{1S_m a}(0) = \frac{8\xi}{(4\pi)^{3/2} N_f} \left( \frac{\overrightarrow{p}^2}{m_s^2} \right)^\varepsilon e^{\gamma_E \varepsilon} B(d, 1, 1/2), \tag{3.35a}
\]

\[
\Sigma_{1S_m b}(0) = \frac{m_f^2}{m_s^2} \frac{16}{(4\pi)^{3/2} N_f} \left( \frac{\overrightarrow{p}^2}{m_f^2} \right)^\varepsilon e^{\gamma_E \varepsilon} B(d, 1, 1/2), \tag{3.35b}
\]

where \(B(\alpha, \beta)\) is the semi-massive tadpole integral defined in (C.4). In expanded form, we have:

\[
\Sigma_{1S_m a}(0) = \frac{2\xi}{\pi^2 N_f} \left( \frac{\overrightarrow{p}^2}{m_s^2} \right)^\varepsilon \left( \frac{1}{\varepsilon} + O(1) \right), \tag{3.36a}
\]

\[
\Sigma_{1S_m b}(0) = \frac{m_f^2}{m_s^2} \frac{4}{\pi^2 N_f} \left( \frac{\overrightarrow{p}^2}{m_f^2} \right)^\varepsilon \left( \frac{1}{\varepsilon} + O(1) \right). \tag{3.36b}
\]
At this point, we recall that in minimal subtraction schemes (including DR), renormalization constants and anomalous dimensions cannot depend on momenta and masses [96]. This enforces mass degeneracy:

\[ m_f = m_s = m, \]  

(3.37)

which is also a requirement of SUSY invariant theories. Hence, in the limit \( d \to 3 \), the LO contribution to \( \Sigma_{1Sm}^0 \) is given by:

\[
\Sigma_{1Sm}^0(0) = \frac{2(2 + \xi)}{\pi^2 N_f} \left( \frac{\mu^2}{m^2} \right)^\varepsilon \left( \frac{1}{\varepsilon} + \text{O}(1) \right). 
\]

(3.38)

Together with (3.24) and (3.28b), the scalar mass anomalous dimension therefore reads:

\[
\gamma_{m\phi} = \frac{8}{\pi^2 N_f} + \text{O}(1/N_f^2), 
\]

(3.39)

and is equal to the fermion mass anomalous dimension (3.33). Hence, not only the bare fermion and scalar masses are the same, the corresponding renormalized masses are also equal. This also agrees with general constraints arising from SUSY invariance as discussed at the end of section 2.3. Here, we have explicitly checked that such constraints do hold at the LO of the 1/N_f-expansion for SQED_3.

3.4 Critical coupling for dynamical mass generation

In the previous section, bare (parity-even) mass terms for the electron and selectron fields were added to the Lagrangian. Starting from a model with zero bare masses \( m_f = m_s = 0 \), it is possible that, at strong coupling, masses be dynamically generated. Again because of SUSY invariance, if the electron acquires a dynamical mass then the selectron has to acquire an equal dynamical mass. Let’s recall that parity-odd masses cannot be dynamically generated [97]. On the other hand, a parity-even but flavour breaking mass can be generated. This corresponds to the condensation of the following operator (expressed in terms of 2-component spinors): 9

\[
\sum_{i=1}^{[N_f/2]} \left( \bar{\psi}_i \psi_i - \bar{\psi}_i^{[N_f/2]} \psi_i^{[N_f/2]} \right), 
\]

(3.40)

which indeed breaks \( U(N_f) \) to \( U(N_f/2) \times U(N_f/2) \). Such a breaking may take place for large enough values of the coupling which, deep in the IR, corresponds to the dimensionless \( 1/(\pi^2 N_f) \), i.e., for small values of \( N_f \). Of importance is to determine the critical flavour fermion number, \( N_{f\text{cr}} \), which is such that dynamical symmetry breaking takes place at \( N_f < N_{f\text{cr}} \). We therefore focus here on an estimation of \( N_{f\text{cr}} \) at the LO of the 1/N_f-expansion.

In order to compute \( N_{f\text{cr}} \), it is enough to focus on the critical properties of SQED_3, i.e., to work at the non-trivial IR fixed point. We then have two ways to proceed further. The first one is via Schwinger-Dyson (SD) equations for either the electron or the selectron

---

9A parity-odd mass term would correspond to the condensation of: \( \sum_{i=1}^{N_f} \bar{\psi}_i \psi_i \).
Figure 6: The three diagrams contributing to the electron self energy in terms of dressed propagators and vertices. The photon, \( \varepsilon \)-scalar and photino propagators are also intended to be dressed quantities.

(because of SUSY invariance, both sets of SD equations yield the same critical coupling for mass generation). This has already been considered in [39] but we will follow a simpler and hopefully clearer approach following [9, 10], see also [19] and references therein for more recent works. Another approach is based on using anomalous mass dimensions of either the electron or selectron (they are equal as we found in the last subsection) following [15].

We consider first the SD equation approach. Following the notations of the previous subsection, we set the bare masses to zero: \( m_f = m_s = m = 0 \), and parameterize the electron and scalar self-energies as:

\[
\Sigma^\psi_\pm(p^2) = p^2 \Sigma^\psi_S(p^2), \quad \Sigma^\phi(p) = p^2 \Sigma^\phi_S(p^2) + \Sigma^\phi_m(p^2),
\]

(3.41)

where \( \Sigma^\psi_S(p^2) \) and \( \Sigma^\phi_m(p^2) \) are dynamically generated, i.e., they are obtained as non-trivial solutions of the SD equations. The dressed electron and selectron propagators then read:

\[
S^\pm(p) = \frac{1}{1 - \Sigma^\psi_\pm(p^2)} \frac{i}{\rho \mp (\Sigma^\psi_S)'(p^2)}, \quad (\Sigma^\psi_S)'(p^2) = \frac{\Sigma^\psi_S(p^2)}{1 - \Sigma^\psi_\pm(p^2)},
\]

(3.42a)

\[
\Delta(p) = \frac{1}{1 - \Sigma^\phi_S(p^2)} p^2 - (\Sigma^\phi_S)'(p^2), \quad (\Sigma^\phi_S)'(p^2) = \frac{\Sigma^\phi_S(p^2)}{1 - \Sigma^\phi_S(p^2)},
\]

(3.42b)

where \( (\Sigma^\psi_S)' \) and \( (\Sigma^\phi_S)' \) are analogous to the ones defined in (3.28) but in the absence of a bare mass. We then consider the SD equations for the electron propagator (similar derivations hold for the selectron SD equations). They read:

\[
-i \Sigma^\psi_{\alpha \pm}(p) = \mu^{2 \epsilon} \int [d^4k] \frac{\left(-ie\gamma^\mu\right)}{\rho \mp (\Sigma^\psi_S)'(p^2)} S^\pm(k)(-ie\Gamma^\nu)D_{\mu\nu}(p-k),
\]

(3.43a)

\[
-i \Sigma^\phi_{\alpha}(p) = \mu^{2 \epsilon} \int [d^4k] \frac{\left(-ie\gamma^\mu\right)}{\rho \mp (\Sigma^\phi_S)'(p^2)} S^\pm(k)(-ie\Gamma^\nu)\epsilon_{\mu\nu}(p-k),
\]

(3.43b)

\[
-i \Sigma^\psi_\circ(p) = \mu^{2 \epsilon} \int [d^4k] \frac{(1)}{\rho \mp (\Sigma^\phi_S)'(p^2)} \Delta(k)(+ef(p,k))\sigma(p-k),
\]

(3.43c)

corresponding to the three diagrams displayed on figure 4 in terms of dressed propagators and vertices. The unknown function \( (\Sigma^\phi_S)' \) may further be parameterized as:

\[
(\Sigma^\phi_S)'(p^2) = B(-p^2)^{-\alpha},
\]

(3.44)
In the LO approximation to the \(1/N_f\)-expansions, we take the photon, \(\varepsilon\)-scalar and photino propagators as the IR softened ones and \(\Sigma^\psi_1 = 0\), \(\Gamma^\nu = \gamma^\nu\), \(\Gamma^\bar{\nu} = \gamma^\bar{\nu}\) and \(f(p,k) = 1\). Together with (3.44), the scalar part of eqs. (3.43) in the linearized approximation significantly simplify and read:

\[
\begin{align*}
\Sigma^\psi_1aS(p^2) &= B(-p^2)^{-\alpha} \frac{2 + \xi}{\pi^2N_f \alpha(1/2 - \alpha)}, \\
\Sigma^\psi_1bS(p^2) &= B(-p^2)^{-\alpha} \frac{3 - d}{\pi^2N_f \alpha(1/2 - \alpha)}, \\
\Sigma^\psi_1cS(p^2) &= 0,
\end{align*}
\]  

(3.45a)-(3.45c)

where \(\Sigma_{1bS}\) vanishes in the limit \(d \to 3\) and \(\Sigma_{1cS}\) vanishes identically. The total LO scalar self-energy is therefore given by \(\Sigma^\psi_{1aS}\) which is equal to (3.44). From this identity, we deduce the LO gap equation:

\[
\alpha(1/2 - \alpha) = \frac{2 + \xi}{\pi^2N_f}.
\]  

(3.46)

Solving the gap equation yields two values for the index \(\alpha\):

\[
\alpha_{\pm} = \frac{1}{4} \left( 1 \pm \sqrt{1 - \frac{16(2 + \xi)}{\pi^2N_f}} \right).
\]  

(3.47)

Dynamical symmetry breaking takes place for complex values of \(\alpha\), i.e., for \(N_f < N_{fer}\) with

\[
N_{fer} = \frac{16(2 + \xi)}{\pi^2}.
\]  

(3.48)

In the Landau gauge (\(\xi = 0\)), we recover from (3.48) the result first obtained in [39].

The gauge-dependence of (3.48) and (3.58) is not satisfactory especially that critical couplings are physical observables. Following [15], we therefore consider an alternative derivation based on using the anomalous mass dimensions of either the electron or the selectron. This approach is based on noting that these self-energies have two asymptotic behaviours [15, 98] as a function of the external momentum \(p\):

\[
\Sigma(p) \sim m(-p^2)^{-\gamma_m/2} + m_{dyn}(-p^2)^{-\gamma_m/2},
\]  

(3.49)

where \(m\) is the bare mass (that we have momentarily restored for the sake of generality) and \(m_{dyn}\) the dynamically generated one. In particular, eq. (3.49) shows that in \(d = 3\) and in the limit of large euclidean momenta, \(p^2_E = -p^2 \to \infty\), the bare mass dominates the asymptotic behaviour for \(\gamma_m < 1/2\) while the dynamical mass dominates the asymptotic behaviour for \(\gamma_m > 1/2\). Actually, \(\gamma_m\) is related to the scaling dimension of the quartic fermion operator: \(\Delta[(\bar{\psi}\psi)^2] = 2d - 2 - 2\gamma_m = 4 - 2\gamma_m\) where the last equality is valid in \(d = 3\). For \(\gamma_m < 1/2\), we have \(\Delta[(\bar{\psi}\psi)^2] > 3\), e.g., the four-fermion operator is irrelevant. On the other hand, for \(\gamma_m < 1/2\) we have \(\Delta[(\bar{\psi}\psi)^2] < 3\), e.g., the four-fermion operator is relevant. These arguments [99] therefore relate the relevancy of the four-fermion operator...
to the regime where the dynamical mass dominates the asymptotic behaviour of the fermion propagator. Assuming that the bare mass is zero, the critical regime separating the massless and (dynamically) massive phases occurs when the four-fermion operator is marginal. In order to find the marginality crossing relation, let’s further note that for $d = 3$ and $m = 0$, we have

$$\Sigma(p) \sim m_{\text{dyn}}(-p^2)^{-\frac{1}{2} - \gamma_m}, \quad (3.50)$$

which is similar to (3.44) and implies a relation between $\alpha$ and $\gamma_m$:

$$\alpha_+(N_f) = \frac{1}{2} (1 - \gamma_m(N_f)) = \frac{1}{2} - \alpha_-(N_f), \quad (3.51)$$

where the $N_f$ dependence has been explicited. From (3.47), we saw that dynamical symmetry breaking takes place for complex values of $\alpha$ and in particular for: $(\alpha - 1/4)^2 < 0$. In terms of $\gamma_m$, this relation reads: $(\gamma_m - 1/2)^2 < 0$. Therefore, the marginality crossing relation is given by:

$$\left(\gamma_m(N_{f\text{cr}}) - \frac{1}{2}\right)^2 = 0. \quad (3.52)$$

Within a $1/N_f$-expansion, this equation has to be truncated. In particular, the LO critical coupling constant is given by:

$$\gamma_{1m}(N_{f\text{cr}}) = \frac{1}{4} \quad \Rightarrow \quad N_{f\text{cr}} = \frac{32}{\pi^2} = 3.242, \quad (3.53)$$

where either (3.33) or (3.39) can be used and the result is gauge-invariant because $\gamma_m$ is a gauge-invariant quantity. Interestingly, (3.53) coincides with the result of (3.48) in the Landau gauge which was advocated by [39]. This is not so surprising because for SQED$_3$ the Landau gauge is the “good gauge” where to work as it is the gauge where the LO wave function renormalization of the fermion vanishes, see (3.19).

### 3.5 Application to $N = 1$ super-graphene

All the results so far derived for $N = 1$ SQED$_3$ can be mapped to $N = 1$ super-graphene as discussed in section 2.4.

With the help of (2.17), the mapping of the results (3.19) and (3.25) to the ones valid for super-graphene yields:

$$\gamma_\psi^{(\text{sg})} = -2g(1 + \xi) + O(g^2). \quad (3.54a)$$

$$\gamma_\phi^{(\text{sg})} = +2g(3 - \xi) + O(g^2). \quad (3.54b)$$

Similarly, from (3.33) and (3.39) we obtain:

$$\gamma_{m\psi}^{(\text{sg})} = \gamma_{m\phi}^{(\text{sg})} = 8g + O(g^2). \quad (3.55)$$

As for dynamical mass generation, in the case of super-graphene, the dimensionless coupling is the (reduced) fine structure constant so dynamical mass generation may take place for $g > g_{\text{cr}}$ and of interest is to compute the critical reduced coupling constant, $g_{\text{cr}}$. 


This can be done either via SD equations or via the mass anomalous dimension. In the former case, from (2.17), the gap equation (3.46) is mapped to:

$$\alpha \left( \frac{1}{2} - \alpha \right) = \frac{g}{2} (5 + \xi) .$$  \hspace{1cm} (3.56)

Solving this gap equation yields two values for the index $\alpha$:

$$\alpha_{\pm} = \frac{1}{4} \left( 1 \pm \sqrt{1 - 8g (5 + \xi)} \right) .$$  \hspace{1cm} (3.57)

Dynamical symmetry breaking takes place for complex values of $\alpha$, i.e., for $g > g_{cr}$ with

$$g_{cr} = \frac{1}{8 (5 + \xi)} .$$  \hspace{1cm} (3.58)

In the gauge $\xi = -1$ (corresponding to the Landau gauge for the three-dimensional gauge fixing parameter $\xi$), the critical reduced coupling is given by $g_{cr} = 1/32$. A fully gauge-invariant answer can be obtained by mapping (3.53) which yields:

$$\gamma_{1m}^{(sg)} (g_{cr}) = \frac{1}{4} \Rightarrow g_{cr} = \frac{1}{32} = 0.031 ,$$  \hspace{1cm} (3.59)

where (3.55) was used. Comparing (3.59) to (3.58), we see that the gauge-invariant result coincides with the result in the result for $\xi = -1$ which is the “good gauge” in which the LO wave function renormalization of the fermion vanishes, see (3.54a). Moreover, we notice that the fine structure constant of super-graphene is given by: $\alpha_g = 1/137$ which yields a reduced fine structure constant $g = \alpha_g/(4\pi) = 0.00058$. This value is negligible in comparison with the gauge-invariant value $g_{cr} = 0.031$ found in (3.59). Hence, already at the LO of the $1/N_f$-expansion, we find that no dynamical mass is generated in super-graphene.

4 Next to leading order analysis

In this section, we study how the IR softening of the photon, $\varepsilon$-scalar and photino propagators obtained at the LO of the $1/N_f$-expansion in section 3 is affected by NLO corrections. The results will be applied to the study of the stability of the IR interacting fixed point and to the computation of the interaction correction coefficient to the optical conductivity of super-graphene.

4.1 Photon polarization function

At the NLO of the $1/N_f$-expansion, 20 Feynman diagrams contribute to the photon polarization function. Taking into account of the fact that mirror conjugate graphs take the same value, we are left with 11 distinct graphs to evaluate. This can be done exactly for all of the diagrams and the final result for the total NLO photon polarization function in the DR scheme reads:

$$\Pi_2^{\gamma}(p^2) = \frac{e^2}{(4\pi)^3} \frac{\nu^2}{\sqrt{-p^2}} \left( \frac{\nu^2}{p^2} \right)^{2\epsilon} \frac{8}{2d - 5} e^{2\gamma_E\epsilon} \left\{ (d - 1)G(d, 1, 1, 1, 1, 1/2) 
+ 6 (3d - 7) G(d, 1, 1/2) G(d, 1, (3 - d)/2) \right\} ,$$  \hspace{1cm} (4.1)
where the 2-loop master integral $G(d, 1, 1, 1, 1, 1/2)$ is defined in appendix C and its expansion is also given there. The result is finite and, in $d = 3$, simplifies to:

$$\Pi_2^\gamma(p^2) = \frac{e^2}{8\sqrt{-p^2}} \frac{2(12 - \pi^2)}{\pi^2}. \quad (4.2)$$

Together with the LO result of (3.6), the total contribution to the photon polarization function up to NLO in $d = 3$ reads:

$$\Pi_3^\gamma(p^2) = \Pi_1^\gamma(p^2) \left(1 + \frac{C}{N_f} + O(1/N_f^2)\right), \quad C = \frac{2(12 - \pi^2)}{\pi^2} = 0.4317. \quad (4.3)$$

This result will be commented on in section 4.4.

As for the calculational details, the exact results for the 11 distinct diagrams contributing to (4.1) are all given in appendix D. For clarity, we here provide an explicit view on the graphs together with their $\varepsilon$-expansion with accuracy $O(\varepsilon)$ (we use the notations of (D.1)):

\[\text{Diagram:} \quad \hat{\Pi}_{2a}^\gamma = -\frac{4(2 + \xi)}{\pi^2} + O(\varepsilon), \quad (4.4a)\]

\[\text{Diagram:} \quad \hat{\Pi}_{2bode}^\gamma = \frac{16}{3\pi^2} \left(\frac{1}{\varepsilon} + \frac{19}{3} + \frac{3\xi}{2} + O(\varepsilon)\right), \quad (4.4b)\]

\[\text{Diagram:} \quad \hat{\Pi}_{2f}^\gamma = -\frac{2}{3\pi^2} \left(\frac{8 - 3\xi}{\varepsilon} + \frac{128}{3} - 9\xi + O(\varepsilon)\right), \quad (4.4c)\]

\[\text{Diagram:} \quad \hat{\Pi}_{2h}^\gamma = -\frac{2}{\pi^2} \left(\frac{\xi}{\varepsilon} + \frac{70}{9} - \frac{\pi^2}{2} + 5\xi + O(\varepsilon)\right), \quad (4.4d)\]

\[\text{Diagram:} \quad \hat{\Pi}_{2ij}^\gamma = -\frac{2}{3\pi^2} \left(\frac{2 - 3\xi}{\varepsilon} + \frac{14}{3} - 6\xi + O(\varepsilon)\right), \quad (4.4e)\]

\[\text{Diagram:} \quad \hat{\Pi}_{2k}^\gamma = \frac{2}{3\pi^2} \left(\frac{2 - 3\xi}{\varepsilon} - \frac{32}{3} + \frac{3\pi^2}{2} - 6\xi + O(\varepsilon)\right), \quad (4.4f)\]

\[\text{Diagram:} \quad \hat{\Pi}_{2lm}^\gamma = \frac{4}{3\pi^2} + O(\varepsilon), \quad (4.4g)\]
\[ \hat{\Pi}^\gamma_{2_n} = -\frac{4}{3\pi^2} + O(\varepsilon), \quad (4.4h) \]

\[ 2 \times \quad \hat{\Pi}^\gamma_{2_op} = \frac{4}{3\pi^2} \left( \frac{1}{\varepsilon} + \frac{13}{3} + O(\varepsilon) \right), \quad (4.4i) \]

\[ 2 \times \quad \hat{\Pi}^\gamma_{2_qr} = \frac{4}{3\pi^2} \left( \frac{1}{\varepsilon} + \frac{19}{3} + O(\varepsilon) \right), \quad (4.4j) \]

\[ 2 \times \quad \hat{\Pi}^\gamma_{2_st} = -\frac{8}{3\pi^2} \left( \frac{1}{\varepsilon} + \frac{11}{3} + O(\varepsilon) \right). \quad (4.4k) \]

From these results, we first notice that individual graphs may diverge and/or depend on the gauge fixing parameter. Though the sum of all these contributions is finite and gauge-invariant, in agreement with (4.2) and general properties of the polarization function in \( d = 3 \), a proper use of the DRED scheme is crucial to regularize the divergences which take the form of \( 1/\varepsilon \) poles.

In order to further clarify the diagrammatic analysis, we can classify the 11 distinct diagrams into 3 distinct groups depending on the nature of the internal propagators entering the graph. Accordingly, we refer to the first group as the “scalar QED”-like diagrams and it consists of the first 4 diagrams in (4.4). Summing them yields a finite and gauge-invariant result:

\[ \hat{\Pi}^\gamma_{2\text{ scalar}} = \hat{\Pi}^\gamma_{2a} + \hat{\Pi}^\gamma_{2bcd} + \hat{\Pi}^\gamma_{2fg} + \hat{\Pi}^\gamma_{2h} = \frac{9\pi^2 - 164}{9\pi^2} + O(\varepsilon). \quad (4.5) \]

Upon further using (D.1), we have (in \( d = 3 \)):

\[ \Pi^\gamma_{2\text{ scalar}}(p^2) = \Pi^\gamma_1(p^2) \frac{C_{\text{scalar}}}{N_f}, \quad C_{\text{scalar}} = \frac{164 - 9\pi^2}{9\pi^2} = 0.8463. \quad (4.6) \]

The next 2 diagrams are referred to as the “spinor QED”-like diagrams. Their sum is also finite and gauge-invariant

\[ \hat{\Pi}^\gamma_{2\text{ spinor}} = \hat{\Pi}^\gamma_{2ij} + \hat{\Pi}^\gamma_{2k} = \frac{9\pi^2 - 92}{9\pi^2} + O(\varepsilon). \quad (4.7) \]

Upon further using (D.1), we have (in \( d = 3 \)):

\[ \Pi^\gamma_{2\text{ spinor}}(p^2) = \Pi^\gamma_1(p^2) \frac{C_{\text{spinor}}}{N_f}, \quad C_{\text{spinor}} = \frac{92 - 9\pi^2}{9\pi^2} = 0.0357. \quad (4.8) \]

At this point, we may perform a useful check of our results by relating them to the ones of (non-SUSY) spinor QED\(_3\) with \( N_f \) two-component fermions. In order to do that, we
have two elements to take into account. Firstly, as we saw in the previous section, the LO polarization operator of SQED$_3$ is twice the value for QED$_3$. Secondly, such difference in LO softening for the two models in turn affects the internal photon lines of the NLO diagrams and implies that the NLO contribution to the polarization operator of SQED$_3$ is twice smaller than the corresponding contribution for QED$_3$. Hence, altogether, the (non-SUSY) spinor QED$_3$ coefficient is actually given by: $C_{\text{QED}_3} = 4C_{\text{spinor}} = 0.1429$, in perfect agreement with the coefficient given in (1.4). Moreover, from the mapping between (non-SUSY) QED$_3$ and (non-SUSY) graphene at its IR Lorentz invariant fixed point [71], we have that $10 C_g = (\pi/8)C_{\text{QED}_3} = (\pi/2)C_{\text{spinor}} = 0.0561$, also in perfect agreement with the coefficient given in (1.5). Hence, reassuringly, from our SQED$_3$ results, we recover the results of [45] and [68] for (non-supersymmetric) spinor QED$_3$ in the large-$N_f$ limit and (non-supersymmetric) graphene at its IR Lorentz invariant fixed point, respectively.

The next 2 diagrams are referred to as the “$\varepsilon$-scalar QED”-like diagrams. Their sum vanishes (in $d = 3$)

$$
\hat{\Pi}^\gamma_{2 \varepsilon-\text{scalar}} = \hat{\Pi}^\gamma_{2 \, \ell m} + \hat{\Pi}^\gamma_{2 \, n} = O(\varepsilon),
$$

so:

$$
C_{\varepsilon-\text{scalar}} = 0.
$$

The last 3 diagrams are referred to as the “photino QED”-like diagrams. Summing these gauge-invariant contributions again yields a finite result:

$$
\hat{\Pi}^\gamma_{2 \, \text{photino}} = \hat{\Pi}^\gamma_{2 \, \ell p} + \hat{\Pi}^\gamma_{2 \, qp} + \hat{\Pi}^\gamma_{2 \, st} = \frac{40}{9\pi^2} + O(\varepsilon).
$$

Upon further using (D.1), we have (in $d = 3$):

$$
\Pi^\gamma_{2 \, \text{photino}}(p^2) = \Pi^\gamma_{1 \, \ell a}(p^2) \frac{C_{\text{photino}}}{N_f}, \quad C_{\text{photino}} = -\frac{40}{9\pi^2} = -0.4503,
$$

which is opposite in sign with respect to both $C_{\text{scalar}}$ and $C_{\text{spinor}}$.

Hence, the following decomposition holds:

$$
C = C_{\text{scalar}} + C_{\text{spinor}} + C_{\varepsilon-\text{scalar}} + C_{\text{photino}},
$$

where $C_{\text{scalar}}$ has an overwhelming contribution to the screening which is counter-balanced by the anti-screening brought by $C_{\text{photino}}$. On the other hand $C_{\varepsilon-\text{scalar}}$ does not have any contribution while the one of $C_{\text{spinor}}$ is negligible.

### 4.2 Epsilon-scalar polarization function

At the NLO of the $1/N_f$-expansion, 9 Feynman diagrams contribute to the $\varepsilon$-scalar polarization function. Taking into account of the fact that mirror conjugate graphs take the same value, we are left with 6 distinct graphs to evaluate. Proceeding along the same
way as for the photon polarization function, the NLO \( \varepsilon \)-scalar polarization function in the DR scheme reads:

\[
\Pi_2^{\varepsilon}(p^2) = \frac{e^2}{(4\pi)^3 \sqrt{-p^2}} \left( \frac{\Pi^2}{-p^2} \right)^{2\varepsilon} \frac{8}{2d-5} e^{2\gamma_E \varepsilon} \left\{ (d-1)G(d,1,1,1,1/2) + 6(3d-7)G(d,1,1/2)G(d,1,(3-d)/2) \right\},
\]

(4.14)

which is exactly equal to (4.1). Hence, in \( d = 3 \), we have:

\[
\Pi^{\varepsilon}(p^2) = \Pi_1^{\varepsilon}(p^2) \left( 1 + \frac{C}{N_f} + O\left(\frac{1}{N_f}^2\right) \right),
\]

(4.15)

where (3.9) has been used and the coefficient \( C \) is identical to the one in (4.3).

The exact results for the 6 distinct diagrams contributing to (4.14) are all given in appendix D. In the following, we will display the corresponding graphs together with their \( \varepsilon \)-expansion with accuracy \( O(\varepsilon) \). Let’s note that, contrary to the case of the photon polarization function, there is no advantage here in grouping diagrams according to the nature of the internal propagators entering each graph. This is because there is no complete cancellation of singularities and/or gauge dependence within each group.

The diagrams are given by:

\[
\begin{align*}
\text{\begin{tikzpicture}[baseline={(-3,0)}, scale=0.5]
  \draw[thick, dashed, double] (0,0) -- (1.5,1.5) -- (3,0) -- (1.5,-1.5) -- (0,0);
  \end{tikzpicture}} & : \hat{\Pi}_2^{\varepsilon} = -\frac{8}{\pi^2} + O(\varepsilon), \\
\text{\begin{tikzpicture}[baseline={(-3,0)}, scale=0.5]
  \draw[thick] (0,0) -- (1.5,1.5) -- (3,0) -- (1.5,-1.5) -- (0,0);
  \end{tikzpicture}} & \times 2 : \hat{\Pi}_2^{\varepsilon} = \frac{8}{3\pi^2} + O(\varepsilon), \\
\text{\begin{tikzpicture}[baseline={(-3,0)}, scale=0.5]
  \draw[thick] (0,0) -- (1.5,1.5) -- (3,0) -- (1.5,-1.5) -- (0,0);
  \end{tikzpicture}} & : \hat{\Pi}_2^{\varepsilon} = \frac{8}{\pi^2} \left( \frac{1}{\varepsilon} + 2 + O(\varepsilon) \right), \\
\text{\begin{tikzpicture}[baseline={(-3,0)}, scale=0.5]
  \draw[thick] (0,0) -- (1.5,1.5) -- (3,0) -- (1.5,-1.5) -- (0,0);
  \end{tikzpicture}} & \times 2 : \hat{\Pi}_2^{\varepsilon} = \frac{8}{\pi^2} \left( \frac{2-3\xi}{3\varepsilon} + \frac{20}{9} - 3\xi + O(\varepsilon) \right), \\
\text{\begin{tikzpicture}[baseline={(-3,0)}, scale=0.5]
  \draw[thick] (0,0) -- (1.5,1.5) -- (3,0) -- (1.5,-1.5) -- (0,0);
  \end{tikzpicture}} & : \hat{\Pi}_2^{\varepsilon} = \frac{8}{\pi^2} \left( \frac{2+\xi}{\varepsilon} + 10 - \frac{\pi^2}{2} + 3\xi + O(\varepsilon) \right), \\
\text{\begin{tikzpicture}[baseline={(-3,0)}, scale=0.5]
  \draw[thick] (0,0) -- (1.5,1.5) -- (3,0) -- (1.5,-1.5) -- (0,0);
  \end{tikzpicture}} & : \hat{\Pi}_2^{\varepsilon} = \frac{8}{3\pi^2} \left( \frac{1}{\varepsilon} + \frac{16}{3} + O(\varepsilon) \right), \\
\text{\begin{tikzpicture}[baseline={(-3,0)}, scale=0.5]
  \draw[thick] (0,0) -- (1.5,1.5) -- (3,0) -- (1.5,-1.5) -- (0,0);
  \end{tikzpicture}} & \times 2 : \hat{\Pi}_2^{\varepsilon} = \frac{8}{\pi^2} \left( \frac{1}{\varepsilon} + 2 + O(\varepsilon) \right).
\end{align*}
\]
Summing all of these contributions and using (D.1) yields back the NLO term in (4.15).

4.3 Photino self-energy

At the NLO of the $1/N_f$-expansion, 14 Feynman diagrams contribute to the photino self-energy. Taking into account of the fact that mirror conjugate graphs take the same value, we are left with 7 distinct graphs to evaluate. Proceeding along the same way as for the photon and $\varepsilon$-scalar polarization functions, the NLO photino self-energy in the $\overline{\text{DR}}$ scheme reads:

\[
\Pi_\lambda^{V}(p^2) = \frac{\epsilon^2}{(4\pi)^3 \sqrt{-p^2}} \left( \frac{p^2}{-p^2} \right)^{2\epsilon} \frac{8}{2d-5} e^{2\epsilon \epsilon^\varepsilon} \left\{ (d - 1) G(d, 1, 1, 1, 1, 1/2) + 6 (3d - 7) G(d, 1, 1/2) G(d, 1, (3 - d)/2) \right\},
\]

which is exactly equal to both (4.1) and (4.14). Hence, in $d = 3$, we have:

\[
\Pi_\lambda^{V}(p^2) = \Pi_\lambda^{V} \left( 1 + \frac{C}{N_f} + O(1/N_f^2) \right),
\]

where (3.12) has been used and the coefficient $C$ is identical to the one in (4.3).

The exact results for the 7 distinct diagrams contributing to (4.17) are all given in appendix D. We display the corresponding graphs together with their $\varepsilon$-expansion with accuracy $O(\epsilon)$:

\[
\begin{align*}
2 \times \includegraphics[width=0.2\textwidth]{graph1} : & \quad \hat{\Pi}_2^{V_{ab}} = -\frac{2}{3\pi^2} \left( \frac{8 - 3\xi}{\epsilon} + \frac{80}{3} - 3\xi + O(\epsilon) \right), \\
2 \times \includegraphics[width=0.2\textwidth]{graph2} : & \quad \hat{\Pi}_2^{V_{cd}} = -\frac{2}{3\pi^2} \left( \frac{2 - 3\xi}{\epsilon} + \frac{8}{3} - 3\xi + O(\epsilon) \right), \\
2 \times \includegraphics[width=0.2\textwidth]{graph3} : & \quad \hat{\Pi}_2^{V_{ef}} = -\frac{4}{\pi^2} \left( \frac{\xi}{\epsilon} + 6 - \frac{\pi^2}{2} + \xi + O(\epsilon) \right), \\
2 \times \includegraphics[width=0.2\textwidth]{graph4} : & \quad \hat{\Pi}_2^{V_{gh}} = \frac{4}{3\pi^2} + O(\epsilon), \\
2 \times \includegraphics[width=0.2\textwidth]{graph5} : & \quad \hat{\Pi}_2^{V_{ij}} = \frac{4}{3\pi^2} \left( \frac{1}{\epsilon} + \frac{13}{3} + O(\epsilon) \right),
\end{align*}
\]
\[ 2 \times \circ \quad : \quad \hat{\Pi}_{2Vkl}^\Lambda = \frac{4}{3\pi^2} \left( \frac{1}{\varepsilon} + \frac{10}{3} + O(\varepsilon) \right), \quad (4.19f) \]

\[ 2 \times \circ \quad : \quad \hat{\Pi}_{2Vmn}^\Lambda = \frac{4}{\pi^2} \left( \frac{1}{\varepsilon} + 2 + O(\varepsilon) \right). \quad (4.19g) \]

Summing all of these contributions and using (D.1) yields back the NLO term in (4.18).

### 4.4 Stability of the IR fixed point

From eqs. (4.3), (4.15) and (4.18), we see that

\[ \Pi^\gamma(p^2) = \Pi^\epsilon(p^2) = \Pi^\lambda V(p^2) \equiv \Pi(p^2), \quad (4.20) \]

where

\[ \Pi(p^2) = -\frac{N_f e^2}{8\sqrt{-p^2}} \left( 1 + \frac{C}{N_f} + O(1/N_f^2) \right), \quad C = \frac{2(12 - \pi^2)}{\pi^2} = 0.4317. \quad (4.21) \]

The equality (4.20) is in agreement (up to and including NLO corrections) with the identity discussed at the end of section 2.3. Moreover, eq. (4.21) shows that, for \( N_f = 1 \), the NLO contribution shifts the LO result by \( \sim 40\% \). In the case of a parity-even theory, only even values of \( N_f \) are allowed which further reduces this shift to \( \sim 20\% \) for \( N_f = 2 \). Therefore, the IR softening of the photon, \( \varepsilon \)-scalar and photino propagators is only weakly affected by NLO corrections.

By analogy with QED\(_3\), (1.2), a single effective dimensionless coupling constant can then be defined for SQED\(_3\) as follows

\[ \overline{\alpha}(p_E) = \frac{e^2}{p_E^2 (1 - \Pi(p_E^2))} = \begin{cases} \frac{e^2/\rho_E}{8/(N_f (1 + C/N_f))} & \rho_E \gg e^2N_f/8 \\ \frac{e^2/\rho_E}{8/(N_f (1 + C/N_f))} & \rho_E \ll e^2N_f/8 \end{cases}, \quad (4.22) \]

where we have used (4.21). As a result and again similarly to the QED\(_3\) result, (1.3), the corresponding beta function:

\[ \beta(\overline{\alpha}) = \rho_E \frac{d\overline{\alpha}(p_E)}{dp_E} = -\overline{\alpha} \left( 1 - \frac{N_f}{8} (1 + C/N_f) \overline{\alpha} \right) \quad (4.23) \]

displays two stable fixed points: an asymptotically free UV fixed point \( (\overline{\alpha} \to 0) \) and an interacting IR fixed point \( (\overline{\alpha} \to \overline{\alpha}^*) \) such that:

\[ \overline{\alpha}^* = \frac{8}{N_f} \left( 1 - \frac{0.4317}{N_f} + O(1/N_f^2) \right). \quad (4.24) \]

Hence, the NLO corrections only weakly shift the non-trivial IR fixed point SQED\(_3\) albeit with a coefficient \( C \) which is approximately 3 times larger than \( C_{\text{QED}_3} \), see (1.4).
4.5 Optical conductivity of $\mathcal{N}=1$ super-graphene

The coefficient $C = 0.4317$ in SQED$_3$ can also be related to the interaction correction coefficient affecting the optical conductivity of $\mathcal{N}=1$ super-graphene. In order to do this, we again use (2.17) which allows to map (4.3) to the corresponding two-loop photon polarization function of $\mathcal{N}=1$ super-graphene:

$$\Pi_{sg}^\gamma(p^2) = \Pi_{1sg}^\gamma(p^2) \left( 1 + C_{sg} \alpha_g + O(\alpha_g^2) \right), \quad C_{sg} = \frac{\pi}{4} C. \quad (4.25)$$

This can be related to the optical conductivity via:

$$\sigma(q_0) = \lim_{\vec{q} \to 0} \frac{iq_0}{|\vec{q}|^2} \Pi^{00}(q_0,\vec{q}), \quad (4.26)$$

where $q^\mu = (q_0, \vec{q})$. So:

$$\sigma_{sg} = \sigma_{0sg} \left( 1 + C_{sg} \alpha_g + O(\alpha_g^2) \right), \quad C_{sg} = \frac{12 - \pi^2}{2\pi} = 0.3391, \quad (4.27)$$

where $\sigma_{0sg} = N_f e^2/8$ is the minimal conductivity of super-graphene which is twice larger than in the non-supersymmetric case. The interaction correction coefficient $C_{sg}$ is also seen to be significantly larger than in the non-supersymmetric case, $C_{sg} \approx 6C_g$, see (1.5).

5 Summary and Conclusion

In this paper, we have presented a detailed study of massless $\mathcal{N}=1$ supersymmetric QED$_3$ with $N_f$ matter flavours. Due to its super-renormalizable nature, we have studied it in the $1/N_f$-expansion in the IR limit, where an effective scale invariance emerges. One of our goals has been to analytically investigate the stability of this IR fixed point in the presence of quantum corrections.

Our analysis has shown that this is indeed the case, up to NLO in the $1/N_f$-expansion. To this end, we have employed the dimensional reduction with modified minimal subtraction (DR) scheme, which also introduces 2\epsilon amount of real scalars called $\epsilon$-scalars into the gauge multiplet. The gauge multiplet propagators soften their divergent behaviour from $1/p^2$ to $1/\sqrt{-p^2}$ already at the leading order, due to the bubble summation. We have utilized these softened propagators for studying the corrections to the matter multiplet propagators at LO and the gauge multiplet propagators at the NLO.

Our work extended the analytic results for gauge multiplet to NLO and calculated anomalous dimensions for matter fields to LO. Although the choice of the Wess-Zumino gauge combined with the $R_\xi$ covariant gauge fixing breaks the supersymmetry of the model, there are still constraints that follow from supersymmetric Slavnov-Taylor identities analogous to the ones known for SQED$_4$ (see discussion at the end of subsection 2.3). For an abelian gauge field theory, they predict that the gauge multiplet propagators must all receive identical quantum corrections and that the renormalized masses in each multiplet must be the same.
Indeed, we have demonstrated that the polarization functions in the gauge multiplet \( \Pi^\gamma(p^2), \Pi^\varepsilon(p^2) \) and \( \Pi^\lambda_V(p^2) \) for the photon, \( \varepsilon \)-scalars and the photino, respectively, all coincide in SQED\(_3\), up to NLO as given in (4.20) and (4.21). Our focus has been on the propagators of the theory, since they are sufficient to extract the quantities of interest to us. At leading order, we have verified that the mass anomalous dimensions for the selectron \( \gamma_{m\phi} \) (3.39) and the electron \( \gamma_{m\psi} \) coincide (3.33), whereas the field anomalous dimensions \( \gamma_\phi \) (3.25) and \( \gamma_\psi \) (3.19) do not. The effective coupling at the IR interacting fixed point was found to shift as in (4.24) suggesting that it remains stable.

Furthermore, we have explored the possibility of dynamical mass generation in our theory by closely examining the Schwinger-Dyson equations at LO. This led to a gauge parameter dependent value for the critical flavour number generalizing the result of [39]. Interestingly, a refined analysis based on mass anomalous dimensions (following the modern approach of [15], see also [57]) has yielded a gauge-invariant value: \( N_{f,cr} \approx 3.342, \) (3.53), that is in agreement with the Landau gauge (\( \xi = 0 \)) value from the former method. As previously stated, this is not so surprising because for SQED\(_3\) the Landau gauge is the “good gauge” where to work as it is the gauge where the LO wave function renormalization of the fermion vanishes, see (3.19). Though this suggests a mass generation, higher order corrections may strongly affect this result, as we shall remark below.

Along the way, we were able to study the critical behaviour of an \( N = 1 \) super-graphene model by relating it to the large-\( N_f \) limit of SQED\(_3\), see (2.17). This allowed us to obtain the anomalous dimensions for the matter multiplet and a critical coupling for dynamical mass generation. We have found that super-graphene does not exhibit mass generation at the leading order. An interesting outcome of this mapping has been to analytically obtain the 2-loop interaction correction coefficient of our super-graphene model, which turns out to be \( C_{sg} \approx 0.3391, \) see (4.27). Interestingly, while SUSY is generally expected to reduce quantum corrections, we see from equations (4.24) and (4.27) that interaction corrections in SQED\(_3\) are actually more important than in QED\(_3\).

In closing, we make some brief remarks about possible extensions of our study. First, as we just summarized above, our explicit calculations have revealed that supersymmetric Slavnov-Taylor identities analogous to the ones known for SQED\(_4\) do hold at the lowest orders of perturbation theory for SQED\(_3\). Because they represent stringent constraints on present and future calculations, it would be worth demonstrating that such identities do hold (to all orders in perturbation theory) in the present three-dimensional case (possibly along the lines of the four-dimensional proof [92, 93]). Second, it would be quite interesting to reformulate (and extend) the present study using the three-dimensional superfield formalism [100, 101] which fully utilizes supersymmetry. This could throw light on the additional structures that may arise. Third, it would be interesting to calculate the critical flavour number at the NLO of the \( 1/N_f \)-expansion, either in the component or the superfield formalism. This could help one clarify whether flavour symmetry breaking is indeed forbidden as per the duality between \( N_f = 2 \) SQED\(_3\) and the so called 7-field Wess-Zumino model [54, 55, 57, 58]. We leave these tasks for future investigations.
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A Notations, conventions and useful identities

We work in a three-dimensional Minkowski space with metric $g^{\mu\nu} = \text{diag}(+, -, -)$. The three $2 \times 2$ Dirac $\gamma$-matrices satisfy the usual Clifford algebra: \{\(\gamma^\mu, \gamma^\nu\}\} = 2g^{\mu\nu}$. As an explicit representation, we take the Majorana basis where all $\gamma$-matrices are purely imaginary:

\[
\gamma^0 = \sigma_2, \quad \gamma^1 = i\sigma_1, \quad \gamma^2 = i\sigma_3. \quad (A.1)
\]

The following relations are useful in practice:

\[
\gamma^\mu \gamma^\nu = g^{\mu\nu} + i\varepsilon^{\mu
u\rho}\gamma^\rho, \quad (A.2)
\]
\[
\gamma^{\mu\nu} = +i\varepsilon^{\mu
u\rho}\gamma^\rho, \quad (A.3)
\]
\[
\gamma^\mu \gamma^\nu \gamma^\rho = \gamma^{\mu\nu\rho} + g^{\mu\nu} \gamma^\rho + g^{\nu\rho} \gamma^\mu - g^{\mu\rho} \gamma^\nu, \quad (A.4)
\]

where $\gamma^{\mu\nu} = [\gamma^\mu, \gamma^\nu]/2$ and we take $\varepsilon^{012} = +1$.

The identity matrix, $1$, together with the three $\gamma^\mu$ ($\mu = 0, 1, 2$) span the vector space of $2 \times 2$ matrices and we have the following useful (three-dimensional) Fierz rearrangement identity

\[
\psi_1 \bar{\psi}_2 = -\frac{1}{2} \bar{\psi}_2 \psi_1 - \frac{1}{2} (\bar{\psi}_2 \gamma_\mu \psi_1) \gamma^\mu, \quad (A.5)
\]

which holds for two (Dirac or Majorana) spinors $\psi_1$ and $\psi_2$.

Majorana spinors are defined as:

\[
\psi = C\bar{\psi}^\top = \psi^*, \quad \bar{\psi} = \psi^\dagger \gamma^0 = -\psi^\top C, \quad (A.6)
\]

where the charge conjugation matrix is anti-symmetric: $C^\top = -C$, and such that: $CC^\dagger = 1$, $\gamma_\mu^\top = -C^{-1} \gamma_\mu C$ and $C = -\gamma^0$. For two Majorana spinors $\epsilon_1$ and $\epsilon_2$, we have the following Majorana flip identities:

\[
\bar{\epsilon}_1 \epsilon_2 = \bar{\epsilon}_2 \epsilon_1, \quad \bar{\epsilon}_1 \gamma^\mu \epsilon_2 = -\bar{\epsilon}_2 \gamma^\mu \epsilon_1, \quad \bar{\epsilon}_1 \gamma^{\mu\nu} \epsilon_2 = -\bar{\epsilon}_2 \gamma^{\mu\nu} \epsilon_1, \quad \bar{\epsilon}_1 \gamma^{\mu\nu\rho} \epsilon_2 = \bar{\epsilon}_2 \gamma^{\mu\nu\rho} \epsilon_1. \quad (A.7)
\]

Let us also note that the field strength satisfies the Bianchi identity

\[
\partial_{[\mu} F_{\nu\rho]} = 0. \quad (A.8)
\]
In the DRED scheme, the metric tensor and \( \gamma \)-matrices are decomposed as (we follow the notations of [83]):

\[
g^{\mu\nu} = g^{\bar{\mu}\bar{\nu}} + g^{\bar{\mu}\bar{\nu}} \quad (g^{\bar{\mu}\bar{\nu}} = 0), \quad \gamma^\mu = \gamma^{\mu} + \gamma^\mu, \quad (A.9)
\]

and the following properties hold:

\[
g^\mu_\mu = 3, \quad g^{\hat{\mu}^\hat{\mu}} = d, \quad g^{\bar{\mu}\bar{\mu}} = 2\varepsilon, \quad (A.10a)\]

\[
\{\gamma^\mu, \gamma^\nu\} = 2g^{\bar{\mu}\bar{\nu}}, \quad \{\gamma^\mu, \gamma^\nu\} = 2g^{\bar{\mu}\bar{\nu}}, \quad \{\gamma^\mu, \gamma^\nu\} = 0. \quad (A.10b)
\]

where \( d = 3 - 2\varepsilon \). The following trace formula is useful in practice:

\[
\text{Tr}[\gamma^\rho_1 \ldots \gamma^\rho_n \gamma^\beta_1 \ldots \gamma^\beta_m] = \frac{1}{2} \text{Tr}[\gamma^\rho_1 \ldots \gamma^\rho_n] \text{Tr}[\gamma^\beta_1 \ldots \gamma^\beta_m], \quad (A.11)
\]

with \( \text{Tr}[1] = 2 \). Some simple examples of contraction and trace identities include:

\[
\gamma^\mu \gamma^\mu = d, \quad \gamma^\mu \gamma^\mu \gamma^\nu = -(d - 2) \gamma^\nu, \quad \gamma^\mu \gamma^\alpha \gamma^\mu = -2\varepsilon \gamma^\alpha, \quad (A.12a)
\]

\[
\text{Tr}[\gamma^\mu \gamma^\nu] = 2g^{\mu\nu}, \quad \text{Tr}[\gamma^\mu \gamma^\nu \gamma^\beta \gamma^\nu] = 2 \left( g^{\mu\bar{\alpha}} g^{\beta\bar{\nu}} - g^{\mu\bar{\nu}} g^{\beta\bar{\nu}} + g^{\mu\bar{\alpha}} g^{\beta\bar{\nu}} \right). \quad (A.12b)
\]

### B Feynman rules

In this appendix, we present the Feynman rules (in Minkowski space) that follow from eq. (2.5) before and after taking the large \( N_f \) limit. As known from the main text, the propagators of eqs. (B.2), (B.4) and (B.6) (see below) are subject to an IR softening. They are given by (2.7b), (2.7c) and (2.7d), respectively, before taking the large \( N_f \) limit and by (2.10) after taking the large-\( N_f \) limit where the softening takes place. For simplicity, we will not make any graphical distinction between the plain propagators and the IR softened ones. It will be clear within the main text which ones are used. Moreover, we will drop the flavour indices, \( i \) and \( j \), in the following since they always lead to a factor of \( \delta_{ij} \) whenever they appear.

The rules are given by:

(i)

\[
S^\beta_0\alpha(p) = \frac{i \gamma^\beta}{p^2} \quad \begin{array}{c} \beta \\ p \end{array} \quad \alpha \quad (B.1)
\]

(ii)

\[
D^\mu_0(p) = \frac{-i}{p^2} \left( g^{\hat{\mu}\hat{\nu}} - (1 - \xi) \frac{p^{\mu} p^{\nu}}{p^2} \right) \quad \begin{array}{c} \hat{\mu} \hphantom{\nu} \\ p \end{array} \quad \hat{\nu} \quad (B.2)
\]

\[
D^\hat{\nu}_{\hat{\mu}}(p) = \frac{i}{2a \sqrt{-p^2}} \left( g^{\hat{\mu}\hat{\nu}} - (1 - \xi) \frac{p^{\mu} p^{\nu}}{p^2} \right) \quad \begin{array}{c} \hat{\mu} \\ p \end{array} \quad \hat{\nu} \quad (B.3)
\]
(iii)\[
\mathcal{E}_0^{\bar{\mu}\bar{\nu}}(p) = \frac{-ig^{\bar{\mu}\bar{\nu}}}{p^2} = \bar{\mu} \cdots \cdots \cdots \cdots \bar{\nu} \tag{B.4}
\]

\[
\mathcal{E}_{\text{LO}}^{\bar{\mu}\bar{\nu}}(p) = \frac{ig^{\bar{\mu}\bar{\nu}}}{2a\sqrt{-p^2}} = \bar{\mu} \cdots \cdots \cdots \cdots \bar{\nu} \tag{B.5}
\]

(iv)\[
\sigma_0^{\beta\alpha}(p) = \frac{i\,p^{\beta\alpha}}{p^2} = \beta \underbrace{}_{p} \alpha \tag{B.6}
\]

\[
\sigma_{\text{LO}}^{\beta\alpha}(p) = \frac{-i\,p^{\beta\alpha}}{2a\sqrt{-p^2}} = \beta \underbrace{}_{p} \alpha \tag{B.7}
\]

(v)\[
\Delta_0(p) = \frac{i}{p^2} = \underbrace{}_{p} \tag{B.8}
\]

(vi)\[
\hat{\mu} \begin{array}{c}
\beta \\
\alpha
\end{array} = -ie^{\gamma}_{\beta\alpha} \hat{\mu} \tag{B.9}
\]

(vii)\[
\hat{\mu} \begin{array}{c}
\beta \\
\alpha
\end{array} k = -ie(p + k)^{\beta} \tag{B.10}
\]

(viii)\[
\hat{\mu} \begin{array}{c}
\beta \\
\alpha
\end{array} = +2ie^2g_{\mu\nu} \tag{B.11}
\]

(ix)\[
\hat{\mu} \begin{array}{c}
\beta \\
\alpha
\end{array} = -ie^{\gamma}_{\beta\alpha} \hat{\mu} \tag{B.12}
\]
Additionally, notice that we have adopted the compact Feynman rules of [102, 103] that are based on assigning a fermion flow to each graph along fermion lines and involve only one kind of propagator together with vertices without explicit charge-conjugation matrices for Majorana fermions. We find that these rules allow for a rather simple evaluation of Feynman diagrams with Majorana fermions and are helpful to fix some sign ambiguities. They may be contrasted with those of [104, 105] that lead to multiple propagators and vertices as compared to the purely Dirac fermion case.

C Master integrals

We consider an Euclidean space of dimension $d$. Following the notations of the review [94], the one-loop massless propagator-type master integral is given by:

\[
J(d, p, \alpha, \beta) = \int \frac{[d^dk]}{k^{2\alpha} (k - p)^{2\beta}} = \frac{(p^2)^{(d/2 - \alpha - \beta)}}{(4\pi)^{d/2}} G(d, \alpha, \beta),
\]

where $[d^dk] = d^dk/(2\pi)^d$, $\alpha$ and $\beta$ are the so-called indices of the propagators and the dimensionless function $G(d, \alpha, \beta)$ has a simple expression in terms of Euler $\Gamma$-functions:

\[
G(d, \alpha, \beta) = \frac{a(\alpha) a(\beta)}{a(\alpha + \beta - d/2)}, \quad a(\alpha) = \frac{\Gamma(d/2 - \alpha)}{\Gamma(\alpha)}.
\]

Our calculations will also require the knowledge of the one-loop semi-massive tadpole integral which is defined as:

\[
M(d, m, \alpha, \beta) = \int \frac{[d^dk]}{(k^2 + m^2)^{\alpha} k^{2\beta}} = \frac{(m^2)^{(d/2 - \alpha - \beta)}}{(4\pi)^{d/2}} B(d, \alpha, \beta),
\]
with
\[ B(d, \alpha, \beta) = \frac{\Gamma(\alpha + \beta - d/2) \Gamma(d/2 - \beta)}{\Gamma(d/2) \Gamma(\alpha)} . \]  

The two-loop massless propagator-type master integral is given by:
\[ J(d, p, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) = \int \frac{[d^d k_1] [d^d k_2]}{(k_1 - p)^{2\alpha_1} (k_2 - p)^{2\alpha_2} k_2^{2\alpha_3} k_1^{2\alpha_4} (k_1 - k_2)^{2\alpha_5}} = \frac{(p^2)^{d - \sum_{i=1}^5 \alpha_i}}{(4\pi)^d} \frac{1}{G(d, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)} , \]  

where \( G(d, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) \) is dimensionless and unknown for arbitrary indices \( \{\alpha_i\}_{i=1-5} \). In our calculations, we encountered two master integrals at NLO that we shall denote as:
\[ T_{2,1} = e^{2\pi \varepsilon} G(d, 1, 1/2) G(d, 1, (3 - d)/2) , \quad T_{2,2} = e^{2\pi \varepsilon} G(d, 1, 1, 1, 1, 1/2) , \]  

where \( T_{2,1} \) is a product of two simple one-loop \( G \)-functions (C.2). The more interesting \( T_{2,2} \) is part of a class of complicated two-loop master integrals of the type (C.5) (with up to 3 arbitrary indices) that have been computed in [106]. With the help of the result of [106], \( T_{2,2} \) can be expanded to high orders in \( \varepsilon \) (\( \varepsilon = (3 - d)/2 \)) and we only provide here the first terms of this expansion for completeness:
\[ \frac{T_{2,2}}{\pi^3} = 1 + \left( \frac{63}{\pi^2} \zeta_3 - 2 \ln 2 \right) \varepsilon + \left( \frac{144}{\pi^2} \text{Li}_4(1/2) + \frac{143}{10} \zeta_2 + \frac{6}{\pi^2} \log^4 2 - 4 \log^2 2 \right) \varepsilon^2 + O(\varepsilon^3) , \]  

where \( \zeta_n = \text{Li}_n(1) \) and \( \text{Li}_n \) are polylogarithms.

## D  Feynman diagrams at NLO

In this appendix, we provide the interested reader with the exact expressions (valid for any \( d \)) of all the NLO diagrams we have computed. They were explicitly displayed in the main text in section 4.

We introduce the notation:
\[ \Pi^X_{2a_1 \cdots a_n}(p^2) = \frac{e^2}{8\sqrt{-p^2}} \left( \frac{-\pi^2}{-p^2} \right)^{2\varepsilon} \tilde{\Pi}^X_{2n_1 \cdots n_2} , \]  

where \( X = \{\gamma, \varepsilon, \lambda\} \) and \( a_1, \cdots, a_n \) is a collection of \( n \) indices referring to graphs taking the same value; hence: \( \Pi^X_{2a_1 \cdots a_n}(p^2) = n \Pi^X_{a_i}(p^2) \) (\( i = 1, \cdots, n \)) (it is also understood that \( \Pi^X_{2a_1 \cdots a_n}(p^2) \equiv \Pi^X_{2V_{a_1 \cdots a_n}}(p^2) \)). For example, \( \Pi^\gamma_{2f g}(p^2) = 2\Pi^\gamma_{2f}(p^2) \) because, for the photon polarization function, NLO diagrams (f) and (g) are mirror conjugate graphs and take the same value. The same notation holds for the momentum-independent function \( \tilde{\Pi}^X_{2n_1 \cdots n_2} \). The results below will further be expressed in terms of the two master integrals that were introduced in (C.6).
For the 11 distinct photon polarization diagrams, our results read:

\[
\hat{\Gamma}_{2a}^\gamma = \frac{4}{\pi^3} \left[ 1 + \frac{\xi}{d-1} \right] T_{2,1},
\]

\[
\hat{\Gamma}_{2b}^{\psi} = \frac{8}{\pi^3} \left[ \frac{2(d-2)}{(d-3)(2d-3)} - \frac{\xi}{d-1} \right] T_{2,1},
\]

\[
\hat{\Gamma}_{2f}^{\psi} = \frac{2}{\pi^3} \left[ \frac{4(d-1)(d-2)}{(d-3)(2d-3)} - \frac{(2d-5)\xi}{d-3} \right] T_{2,1},
\]

\[
\hat{\Gamma}_{2h}^{\gamma} = \frac{2}{(d-1)\pi^3} \left[ \frac{16d^4 - 80d^3 + 106d^2 + 21d - 83}{(2d-3)^2(2d-5)} - \frac{2d^2 - 9d + 11}{d-3} \xi \right] T_{2,1}
\]
\[\quad + \frac{(5d-9)}{(d-1)(2d-3)(2d-5)\pi^3} T_{2,2},\]

\[
\hat{\Gamma}_{2i}^{\gamma} = -\frac{4}{\pi^3} \left[ \frac{(d-2)^3}{d-3} - \frac{1}{2d-3} \right] T_{2,1},
\]

\[
\hat{\Gamma}_{2j}^{\gamma} = -\frac{1}{\pi^3} \left[ \frac{2(d-2)}{(d-1)(d-3)} \right] \left[ \frac{12d^5 - 192d^4 + 1077d^3 - 2786d^2 + 3405d - 1596}{(2d-3)^2(2d-5)} \right]
\]
\[\quad + \frac{12d^6 - 93d^5 + 204d^4 - 147}{(d-1)(2d-3)(2d-5)\pi^3} T_{2,1} + \frac{(d-2)(2d^5 - 15d^4 + 41d - 36)}{(d-1)(2d-3)(2d-5)\pi^3} T_{2,2},\]

\[
\hat{\Gamma}_{2k}^{\gamma} = -\frac{4(d-2)^3}{(d-1)(2d-3)\pi^3} T_{2,1},
\]

\[
\hat{\Gamma}_{2l}^{\gamma} = \frac{2(d-2)(14d^3 - 93d^2 + 204d - 147)}{(d-1)(2d-3)^2\pi^3} T_{2,1} - \frac{(d-3)^2(d-2)}{(d-1)(2d-3)\pi^3} T_{2,2},
\]

\[
\hat{\Gamma}_{2m}^{\gamma} = -\frac{4(d-2)^3}{(d-1)(2d-3)\pi^3} T_{2,1},
\]

\[
\hat{\Gamma}_{2n}^{\gamma} = -\frac{4(d-2)^3}{(d-3)^3T_{2,1},}
\]

\[
\hat{\Gamma}_{2o}^{\gamma} = -\frac{4(d-2)^3}{(d-3)(2d-3)^3\pi^3} T_{2,1},
\]

\[
\hat{\Gamma}_{2p}^{\gamma} = -\frac{8(d-2)^3(14d^2 - 63d + 69)}{(d-1)(d-3)(2d-5)^3\pi^3} T_{2,1} + \frac{8(d-3)(d-2)}{(d-1)(2d-3)(2d-5)^3\pi^3} T_{2,2}.
\]

For the 6 distinct \(\varepsilon\)-scalar polarization diagrams, our results read:

\[
\hat{\Gamma}_{2a}^{\varepsilon} = \frac{4}{\pi^3} T_{2,1},
\]

\[
\hat{\Gamma}_{2bc}^{\varepsilon} = -\frac{4(d-2)^2}{(2d-3)\pi^3} T_{2,1},
\]

\[
\hat{\Gamma}_{2d}^{\varepsilon} = \frac{2(d-1)(7d^2 - 34d + 41)}{(d-3)(2d-5)^3\pi^3} T_{2,1} - \frac{(d-1)(d-3)}{(2d-5)\pi^3} T_{2,2},
\]

\[
\hat{\Gamma}_{2e}^{\varepsilon} = \frac{4(d-2)^2}{(d-3)\pi^3} \left( \frac{\xi - d-1}{2d-3} \right) T_{2,1},
\]

\[
\hat{\Gamma}_{2f}^{\varepsilon} = -\frac{2(d-2)}{(d-3)\pi^3} \left( \frac{3d^2 - 10d + 7}{2d-5} + 2(d-2)\xi \right) T_{2,1} + \frac{(d-1)(d-2)}{(2d-5)\pi^3} T_{2,2},
\]

\[
\hat{\Gamma}_{2hi}^{\varepsilon} = \frac{4(d-2)(d-1)}{(d-3)(2d-3)\pi^3} T_{2,1}.
\]
For the 7 distinct photino self-energy diagrams, our results read:

\[ \hat{\Pi}_{\lambda V_{ab}}^{2} = -\frac{2(d-2)}{(d-3)\pi^3} \left[ \frac{4(d-2)(d-1)}{2d-3} - (2d-5)\xi \right] T_{2,1}, \quad (D.4a) \]

\[ \hat{\Pi}_{\lambda V_{cd}}^{2} = -\frac{2(d-2)(2d-5)}{(d-3)\pi^3} \left[ \frac{d-1}{2d-3} - \xi \right] T_{2,1}, \quad (D.4b) \]

\[ \hat{\Pi}_{\lambda V_{ef}}^{2} = \frac{4(d-2)}{\pi^3} \left[ 4d-9 \right] \left[ \frac{2d-5}{d-3} \right] \xi T_{2,1} + \frac{2(d-2)}{(2d-5)\pi^3} T_{2,2}, \quad (D.4c) \]

\[ \hat{\Pi}_{\lambda V_{gh}}^{2} = -\frac{2(d-2)(2d-5)}{(2d-3)\pi^3} T_{2,1}, \quad (D.4d) \]

\[ \hat{\Pi}_{\lambda V_{ij}}^{2} = \frac{4(d-2)^2}{(d-3)(2d-3)\pi^3} T_{2,1}, \quad (D.4e) \]

\[ \hat{\Pi}_{\lambda V_{kl}}^{2} = \frac{2(d-1)(2d-5)}{(d-3)(2d-3)\pi^3} T_{2,1}, \quad (D.4f) \]

\[ \hat{\Pi}_{\lambda V_{mn}}^{2} = \frac{2(3d-7)(3d-8)}{(d-3)(2d-5)\pi^3} T_{2,1} - \frac{d-3}{(2d-5)\pi^3} T_{2,2}. \quad (D.4g) \]

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