Higher order Symmetric Cumulants

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Study the Properties of Quark-Gluon Plasma

- State of matter with deconfined quarks and gluons
- Present in
  - our early Universe?
  - the core of Neutron Stars?
Study the Properties of Quark-Gluon Plasma

- State of matter with deconfined quarks and gluons
- Present in
  - our early Universe?
  - the core of Neutron Stars?

Better understanding of the Universe by studying the properties of QCD matter at extreme conditions
Direct observation of QGP not possible
Many probes to study QGP in heavy-ion collisions:
- Anisotropic flow
- Jet quenching
- ...

Study of heavy-ion collisions can provide access to the properties of the QGP
Anisotropies in the Initial and Final States

- Volume of interacting matter initially anisotropic in coordinate space.

- Anisotropic flow: Transfer of the initial anisotropy into anisotropy in momentum space via the thermalized medium.
Anisotropies in the Initial and Final States

Described by Fourier series:

\[ f(\varphi) = \frac{1}{2\pi} \left[ 1 + 2 \sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)] \right] \]

S. Voloshin and Y. Zhang, Z. Phys. C 70 (1996) 665

\[ v_n \text{ and } \Psi_n \text{ related to } \varphi : \]

\[ \langle \cos[n_1\varphi_1 + \cdots + n_k\varphi_k] \rangle = v_{n_1} \cdots v_{n_k} \cos[n_1\Psi_{n_1} + \cdots + n_k\Psi_{n_k}] \]

R.S. Bhalerao, M. Luzum and J.-Y. Ollitrault, PRC 84, 034910 (2011)

φ: Azimuthal angle

\( v_n \): Flow harmonic

\( \Psi_n \): Symmetry-plane
Flow and Properties of Quark-Gluon Plasma

- Measurements of $v_n$ sensitive to properties of QGP
- Correlations between two harmonics probed by:
  \[ SC(m, n) = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle \]
- New and independent set of constraints on the system

ALICE Collaboration, PRL 117, 182301 (2016)
Cindy Mordasini - Higher order Symmetric Cumulants - INPC 2019
Flow and Properties of Quark-Gluon Plasma

- Combinations of $v_n$ and $\Psi_n$ sensitive to properties of QGP
- Correlations between two harmonics probed by:
  \[ \text{SC}(m, n) = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle \]
- New and independent set of constraints on the system

Sensitivity to $\eta/s$ of QGP (not accessible with only one harmonic)

ALICE Collaboration, PRL 117, 182301 (2016)
Generalisation to three Different Harmonics

Are there genuine correlations between more than two harmonics?
→ Possible new constraints for the system
(Not accessible with $SC(m, n)$ or only one harmonic)

“Higher order Symmetric Cumulants”
CM, A. Bilandzic, D. Karakoç and S. F. Taghavi, arXiv:1901.06968

Example: 3-harmonic Symmetric Cumulants:

$$SC(k, l, m) = \langle v_k^2 v_l^2 v_m^2 \rangle - \langle v_k^2 v_l^2 \rangle \langle v_m^2 \rangle - \langle v_k^2 v_m^2 \rangle \langle v_l^2 \rangle - \langle v_l^2 v_m^2 \rangle \langle v_k^2 \rangle + 2 \langle v_k^2 \rangle \langle v_l^2 \rangle \langle v_m^2 \rangle$$

$$SC_\epsilon(k, l, m) = \langle \epsilon_k^2 \epsilon_l^2 \epsilon_m^2 \rangle - \langle \epsilon_k^2 \epsilon_l^2 \rangle \langle \epsilon_m^2 \rangle - \langle \epsilon_k^2 \epsilon_m^2 \rangle \langle \epsilon_l^2 \rangle - \langle \epsilon_l^2 \epsilon_m^2 \rangle \langle \epsilon_k^2 \rangle + 2 \langle \epsilon_k^2 \rangle \langle \epsilon_l^2 \rangle \langle \epsilon_m^2 \rangle$$
Validation with Toy Monte Carlo Simulations

- Less than 3 correlated harmonics $\rightarrow SC(2,3,4) = 0$

- 3 correlated harmonics $\rightarrow SC(2,3,4) \neq 0$
From Toy Simulations to Realistic Models

➢ Toy Monte Carlo simulations good verification tool…

… But what are the predictions for more realistic models?
From Toy Simulations to Realistic Models

➢ Toy Monte Carlo simulations good verification tool…
… But what are the predictions for more realistic models?

➢ Predictions from iEBE-VISHNU
  ✷ Event generator for heavy-ion collisions
  ✷ Describes the evolution of the collision
  ✷ Simulations → Access to $SC_\epsilon(k, l, m)$ and $SC(k, l, m)$
  ✷ 14000 simulated events per centrality bin → Feasibility test

Initial state from MC-Glauber model
After 0.6 fm/c, flow from the hydrodynamic evolution with $\eta/s = 0.08$
Predictions with iEBE-VISHNU

- System smaller for peripheral collisions $\rightarrow$ Harder to transfer the initial anisotropy to the final state
Predictions with iEBE-VISHNU

- System smaller for peripheral collisions $\rightarrow$ Harder to transfer the initial anisotropy to the final state

Cannot compare the initial and final states due to different scales
Normalised Symmetric Cumulants

\[ \text{NSC}(k, l, m) = \frac{SC(k, l, m)}{\langle v_k^2 \rangle \langle v_l^2 \rangle \langle v_m^2 \rangle} \quad \text{and} \quad \text{NSC}_\epsilon(k, l, m) = \frac{SC_\epsilon(k, l, m)}{\langle \epsilon_k^2 \rangle \langle \epsilon_l^2 \rangle \langle \epsilon_m^2 \rangle} \]

- How much of the final particle correlations come from the initial state?
- Sign change between NSC(2,3,4) and NSC_\epsilon(2,3,4)
  - SC(4,2) > 0
  - SC(3,2) < 0
Summary and Outlook

- Introduced the generalisation to higher order Symmetric Cumulants
- Validated SC($k,l,m$) with Toy Monte Carlo simulations
- Showed predictions from iEBE-VISHNU

- **Next step?** Analyses of SC($k,l,m$) on ALICE Pb-Pb data from RUN 1 and RUN 2
Backups
Parametrisations for $\eta/s$

H. Niemi, K. J. Eskola, R. Paatelainen, Phys. Rev. C 93, 024907 (2016)
Cumulants

- Cumulant $\langle \ldots \rangle_c$ describes the genuine correlations between all the elements in the correlator
  - Term unique in the decomposition
- Cumulants not directly measurable $\rightarrow$ Must be computed from the correlations
- 2-particle cumulant: $\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$
- In 3-particle cumulant:
  $$\langle X_1 X_2 X_3 \rangle_c = \langle X_1 X_2 X_3 \rangle - \langle X_1 X_2 \rangle \langle X_3 \rangle - \langle X_1 X_3 \rangle \langle X_2 \rangle - \langle X_2 X_3 \rangle \langle X_1 \rangle + 2 \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle$$
  - Recursive method for higher order cumulants
- Study correlations
  - Different k-particle correlators: $X_j = e^{i n \phi_j}$ (N. Borghini, P. M. Dinh and J.-Y. Ollitrault, PRC 63, 054906 (2001))
  - Different harmonics: e.g. Symmetric Cumulants
Cumulants

- Cumulants not directly measurable ⇒ Must be computed from the correlations
- 2-particle cumulant: $\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$
- In 3-particle cumulant:
  \[
  \langle X_1 X_2 X_3 \rangle_c = \langle X_1 X_2 X_3 \rangle - \langle X_1 X_2 \rangle \langle X_3 \rangle - \langle X_1 X_3 \rangle \langle X_2 \rangle - \langle X_2 X_3 \rangle \langle X_1 \rangle + 2 \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle
  \]
- Recursive method for higher order cumulants

![Diagram]
Requirements to be a Symmetric Cumulant

1) **No built-in trivial contribution**: if $v_k$ constant $\rightarrow$ $SC(k,l,m) = 0$

2) **Genuine 3-harmonic correlations**: $SC(k,l,m) \neq 0$ only for 3 correlated harmonics

3) **Symmetry**: $SC(k,l,m) = SC(l,m,k) = SC(k,m,l) = ...$

4) **Cleanliness**: no dependence on symmetry planes in SC by definition

5) **Isotropy**: all correlators used in SC must be isotropic

6) **Uniqueness**: no ambiguity in the combination of correlators used to get the final expression

7) **Robustness against nonflow**: if only nonflow in the system $\rightarrow$ expected scaling of $SC(k,l,m)$

8) **Multiplicity weights**: which weights to use to go from $\langle ... \rangle$ to $\langle \langle ... \rangle \rangle$?

➔ **How to check if $SC(k,l,m)$ follows the requirements?** *Toy Monte Carlo simulations*
Toy Monte Carlo Setup

➢ Controlled environment to test the influence of the initial parameters

➢ Setup implemented for this study:
  ❖ Set the flow harmonics $v_n$ in the Fourier series
  ❖ Uniform sampling of azimuthal angles $\varphi$ for $M$ particles
  ❖ Compute the Q-vectors $Q_n$ for the event
  ❖ Compute the needed correlators $\langle \ldots \rangle$ from the Q-vectors for the event
  ❖ Loop over $N$ events to compute $\langle \langle \ldots \rangle \rangle$
  ❖ Compute $SC(k,l,m)$

Initial: $v_n, M, N$

For one event: $\varphi, Q_n, \langle \ldots \rangle$

Loop over events

For N events: $\langle \langle \ldots \rangle \rangle$

Final: $SC(k,l,m)$
1) No built-in Trivial Contribution?

- $v_2, v_3$ and $v_4$ constant $\Rightarrow$ SC(2,3,4) = 0
- No built-in contribution
2) Genuine 3-harmonic Correlations?

- SC(2,3,4) compatible with 0 for all multiplicities

- SC(k,l,m) not sensitive to less than 3 correlated harmonics
2) Genuine 3-harmonic Correlations?

- Simple model of correlations among 3 harmonics:
  \[ f(v_2, v_3, v_4) \text{ in: } <v_2^a v_3^b v_4^c> = \iiint v_2^a v_3^b v_4^c f(v_2, v_3, v_4) dv_2 dv_3 dv_4 \]

- Compute all terms of SC(2,3,4)

- Simulations in agreement with theory for all multiplicities

- Sensitive to 3-harmonic correlations only

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6) Uniqueness in SC($m,n$)?

- Ambiguity in $\langle v_m^2 v_n^2 \rangle$ from $\langle \cos[n_1 \psi_1 + \cdots + n_k \psi_k] \rangle = v_{n_1} \cdots v_{n_k} \cos[n_1 \psi_{n_1} + \cdots + n_k \psi_{n_k}]$

- SC($m,n$) can be written as
  - Usual: $\langle \cos(m(\phi_1 - \phi_2) + n(\phi_3 - \phi_4)) \rangle$
  - $\langle \cos(m(\phi_1 - \phi_2)) \rangle \langle \cos(n(\phi_1 - \phi_2)) \rangle$
  - Alternative:
    $\langle \cos(m(\phi_1 - \phi_2)) \rangle \langle \cos(n(\phi_1 - \phi_2)) \rangle$

- From mathematical derivation:
  - $\langle \cos(m(\phi_1 - \phi_2) + n(\phi_3 - \phi_4)) \rangle$ works
  - Autocorrelations in $\langle \cos(m(\phi_1 - \phi_2)) \rangle \langle \cos(n(\phi_1 - \phi_2)) \rangle$
6) Uniqueness?

- As for SC(m,n), many expressions lead in theory to the same final expression for SC(k,l,m)
  - Again from \( \langle \cos[n_1 \varphi_1 + \cdots + n_k \varphi_k] \rangle = v_{n_1} \ldots v_{n_k} \cos[n_1 \Psi_{n_1} + \cdots + n_k \Psi_{n_k}] \)

- Example for the first term: \( \langle v_k^2 v_l^2 v_m^2 \rangle \)
  - \( \langle \cos(k(\varphi_1 - \varphi_2) + l(\varphi_3 - \varphi_4) + m(\varphi_5 - \varphi_6)) \rangle \)?
  - \( \langle \cos(k(\varphi_1 - \varphi_2)) \rangle \langle \cos(l(\varphi_1 - \varphi_2)) \rangle \langle \cos(m(\varphi_1 - \varphi_2)) \rangle \)?
  - \( \langle \cos(k(\varphi_1 - \varphi_2) + l(\varphi_3 - \varphi_4)) \rangle \langle \cos(m(\varphi_1 - \varphi_2)) \rangle \)?

- Mathematical derivation too tedious for 3 different harmonics
  - Use of the Toy Monte Carlo setup for 3 different expressions
6) Uniqueness?

- **Usual:** \( SC(k, l, m) = \langle \langle 6 \rangle \rangle_{k,l,m,-k,-l,-m} - \langle \langle 4 \rangle \rangle_{k,l,-k,-l} \langle \langle 2 \rangle \rangle_{m,-m} - \langle \langle 4 \rangle \rangle_{k,m,-k,-m} \langle \langle 2 \rangle \rangle_{l,-l} - \langle \langle 4 \rangle \rangle_{l,m,-l,-m} \langle \langle 2 \rangle \rangle_{k,-k} + 2 \langle \langle 2 \rangle \rangle_{k,-k} \langle \langle 2 \rangle \rangle_{l,-l} \langle \langle 2 \rangle \rangle_{m,-m} \)

- **Alternative:** \( SC(k, l, m) = \langle \langle 2 \rangle \rangle_{k,-k} \langle \langle 2 \rangle \rangle_{l,-l} \langle \langle 2 \rangle \rangle_{m,-m} - \langle \langle 2 \rangle \rangle_{k,-k} \langle \langle 2 \rangle \rangle_{l,-l} \langle \langle 2 \rangle \rangle_{m,-m} - \langle \langle 2 \rangle \rangle_{k,-k} \langle \langle 2 \rangle \rangle_{m,-m} \langle \langle 2 \rangle \rangle_{l,-l} - \langle \langle 2 \rangle \rangle_{l,-l} \langle \langle 2 \rangle \rangle_{m,-m} \langle \langle 2 \rangle \rangle_{k,-k} + 2 \langle \langle 2 \rangle \rangle_{k,-k} \langle \langle 2 \rangle \rangle_{l,-l} \langle \langle 2 \rangle \rangle_{m,-m} \)

- **Mixed:** \( SC(k, l, m) = \langle \langle 4 \rangle \rangle_{k,-l,-k,-l} \langle \langle 2 \rangle \rangle_{m,-m} - \langle \langle 2 \rangle \rangle_{k,-k} \langle \langle 2 \rangle \rangle_{l,-l} \langle \langle 2 \rangle \rangle_{m,-m} - \langle \langle 2 \rangle \rangle_{k,-k} \langle \langle 2 \rangle \rangle_{m,-m} \langle \langle 2 \rangle \rangle_{l,-l} - \langle \langle 2 \rangle \rangle_{l,-l} \langle \langle 2 \rangle \rangle_{m,-m} \langle \langle 2 \rangle \rangle_{k,-k} + 2 \langle \langle 2 \rangle \rangle_{k,-k} \langle \langle 2 \rangle \rangle_{l,-l} \langle \langle 2 \rangle \rangle_{m,-m} \)
6) Uniqueness?

- Simulation with two anticorrelated harmonics
  - Expected: zero for all multiplicities
- Usual: ok
- Alternative and mixed:
  - Multiplicity dependence
  - Presence of autocorrelations
- Unique way to write $SC(k,l,n)$ in terms of azimuthal angles
7) Robustness against Nonflow?

- \( SC(k, l, m) = \langle v_k^2 v_l^2 v_m^2 \rangle - \langle v_k^2 v_l^2 \rangle \langle v_m^2 \rangle - \langle v_k^2 v_m^2 \rangle \langle v_l^2 \rangle - \langle v_l^2 v_m^2 \rangle \langle v_k^2 \rangle + 2 \langle v_k^2 \rangle \langle v_l^2 \rangle \langle v_m^2 \rangle \)

- Expected behavior: \( \delta_{SC(k,l,m)} \sim \frac{\alpha}{M^{6-1}} + \frac{\beta}{(M^{4-1})M^{2-1}} + \frac{\gamma}{(M^{2-1})^3} \)

- Simple nonflow with strong 2-particle correlations
  - Maximises nonflow

- \( \chi^2/\text{ndf} = 0.7 \)

- Behavior as expected
  - \( \beta, \gamma \) compatible with zero
  - Dominant contribution to nonflow from 6-particle correlator
7) Robustness against Nonflow?

Toy Monte Carlo, $N_{\text{events}} = 1 \times 10^8$

$v_2, v_3, v_4 = 0, M_{\text{final}} = 2M_{\text{initial}}$

$\alpha/M^5 + \beta/M^4 + \gamma/M^3$
8) Multiplicity Weights?

- Which weights to pass from $\langle \ldots \rangle$ to $\langle \langle \ldots \rangle \rangle$?
  - Number of combinations for a k-particle correlator: $\prod_{i=0}^{k-1} (M - i)$
  - Multiplicity of the event $M$
  - Unit weight
- $M$ uniformly sampled in (50, 500)
- Constant harmonics
  - Smallest statistical spread for number of combinations
    - Same conclusion as for $\text{SC}(m,n)$
Realistic Monte Carlo studies

- Used two Monte Carlo models: HIJING and iEBE-VISHNU
- Heavy-Ion Jet INteraction Generator (HIJING):
  - Model to study particle and jet production in nuclear and heavy-ion collisions
  - Description of phenomena with correlations between few particles ➔ Only nonflow
  - Used to test the robustness of SC$(k,l,m)$ against nonflow
- iEBE-VISHNU:
  - Heavy-ion collision event generator based on hydrodynamics calculations
  - Describes heavy-ion collision evolution
  - Contains both flow and nonflow, only flow used in this work
- For both, simulations of Pb-Pb collisions with $\sqrt{s} = 2.76$ TeV
HIJING Simulations

- SC($k,l,m$) compatible with zero for head-on and mid-central collisions ➔ Not sensitive to nonflow
Cross-Check of VISHNU with ALICE Data

- Comparison of ALICE 2010 Pb-Pb data with VISHNU simulations
- Sensitivity of SC on the lower limit of the $p_T$ range
  - $0.28 < p_T < 4$ GeV: better qualitative agreement with data
    - ALICE $p_T$ range: $0.2 < p_T < 5$ GeV
- Dependence of SC(3,2) on $\eta/s$ not studied here
  - Already studied in ALICE Collaboration, PRL 117, 182301 (2016)
Comparison between HIJING and VISHNU

- Values from HIJING smaller than the ones from VISHNU
- Results from VISHNU not a systematic bias of HIJING

Any nonzero results of SC in real data ➔ Collective flow effects