Effects of dynamo magnetic fields on observational properties of Accreting Millisecond X-ray Pulsars

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Abstract In this paper, we have investigated the accreting millisecond X-ray pulsars, which are rapidly rotating neutron stars in low-mass X-ray binaries. These systems show coherent X-ray pulsations that arise when the accretion flow is magnetically channeled to the stellar surface. Here, we have developed the fundamental equations for an accretion disc around accreting millisecond X-ray pulsars in the presence of a dynamo generated magnetic fields in the inner part of the disc and we have also formulated the numerical method for the structure equations in the inner region of the disc and the highest accretion rate is enough to make the inner region of the disc which is overpowered by radiation pressure and electron scattering. Finally, we have examined our results with the effects of dynamo magnetic fields on accreting millisecond X-ray pulsars.

Key words: Accretion disc, neutron stars, pulsars, millisecond X-ray pulsars

1 INTRODUCTION

Low-mass X-ray binaries (LMXBs) consists of accreting millisecond X-ray pulsars (AMXPs) from a low mass evolved star (a degenerate dwarf star), which has rapidly rotating neutron stars (NSs). Hence, AMXPs are distinguished from the group of ordinary rotation-powered pulsars by their small spin periods Becker 2001. In these systems, the accreting matter may spin up the NS. Here, one of the possible endpoints of the evolution of a LMXB is expected to be millisecond pulsar Strohmayer 2001. Most of the LMXB does not show coherent pulsation in their light curves due to that they are still under debate; it could be due to the alignment of the effect of the magnetic field on NS with its rotational axis. The only subclass of LMXB in which coherent pulsations have been observed is that the AMXPs.

A more observationally inclined review of accreting a millisecond pulsar is given by Wijnands et al. 2005. It is a transient system in which the outburst stage associated with the matter falling onto the NS surface and spin up its period in the order of a millisecond. Also, the first real AMXPs was studied by Wijnands & Van der klis 1998, in which the spin frequencies range from 182 up to 599 Hz Falanga et al. 2013. Among these, the fastest accreting millisecond pulsar is IGR J 0029 + 5934 with the period of just 1.67ms Shaw et al. 2005; Falanga et al. 2005. It shows pulse frequency variations. These observations are very important for the understanding of the evolution of the NSs in LMXBs Poutanen 2006.

Here, in AMXPs, we have considered that the rapidly rotating NS has a weak magnetic dipole moments \( \sim 10^{15} \text{Tm}^3 \) in the inner region than the ordinary X-ray pulsars. In these systems, the
accretion disc will be extended near to the NS and the temperature becomes more in which their opacity can be overpowered by radiation pressure and electron scattering Lasota 2016. The magnetic fields are important for the transportation of angular momentum in these systems. As it was studied by Tessema & Torkelsson 2010 the region of the accretion disc which is located in the inner part of corotation radius supply spin-up torque on the NS while the outer part of the accretion disc brakes the NS. The resultant torque is investigated by the inner region of the disc position, which is displaced inwards as the accretion rate increases.

AMXPs become important in many areas of astrophysical research. Here, It shows a very high average mass transfer rate \( \dot{M} = 10^{14} \text{kg s}^{-1} \) in the inner region and exhibit persistent X-ray pulsations with less than 10ms and weak magnetic fields. Many authors have been studied accretion disc in different models for example Shakura & Sunyaev 1973; Ghosh & Lamb 1979. Hence, they didn’t address an accretion disc in these systems particularly in the inner region of the disc. However, Tessema & Torkelsson 2010 were tried to study accretion disc around magnetized stars using pure models of magnetohydrodynamics (MHDs), but the present study will focus on the accretion disc in AMXPs in the inner region of the disc using analytical and numerical solutions.

In this study, we develop the fundamental equations of an accretion disc for a dynamo generated an accretion disc around AMXPs, particularly we investigate the solution of these equations in the inner part of the disc using surface density, temperature, and radial velocity as a function of the radius and finally, we present the numerical solution for the structure equations in the inner region of the disc.

This paper is organized as follows: In section (2) we investigate the fundamental equations of an accretion disc and we present the numerical method for structure equation in the inner region of the disc. Hence, their results and discussion were incorporated in section (3) and finally, we summarize our results in section (4).

2 FUNDAMENTAL EQUATIONS OF ACCRETION DISC

2.1 Basic Assumptions

In this study, we consider the accretion disc around AMXPs with NS of mass \( 1.4M_\odot \), radius 10km and magnetic dipole moment \( 10^{15} T m^3 \).

Here, we have considered that the scale height of the disc, \( H \) is much smaller than the radial extension of the disc, \( R \). Shakura & Sunyaev 1973. The gas in the disc rotates at Keplerian velocity and the orbital kinetic energy is transformed into radiation by viscosity of an accretion disc, \( v \), while the angular momentum is transported outward.

\[
v = \alpha_{ss} c_s H.
\]

where \( \alpha_{ss} \sim 10^{-2} \) is turbulence stress of the disc that describes the transport of angular momentum and that numerical simulations suggested by Hawley et al. 1995 and \( c_s \) is the speed of sound in the gas.

2.2 Conservation of mass

The law of conservation of mass or principle of mass conservation states that for any system closed to all transfers of matter and energy, the mass of the system must remain constant over time, as the system’s mass cannot change, so the quantity can not be added nor removed. Hence, the quantity of mass is conserved over time. Then, the conservation of mass is ensured by the continuity equation:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0.
\]

Then, from Eq. (2) we have

\[
\nabla \cdot (\rho v) = 0,
\]
due to steady-state and where $\rho$ is the density and $v = (v_R, v_\phi, v_z)$ of the systems. Here, from the axisymmetric disc we have

$$\frac{1}{R} \frac{\partial}{\partial R}(R\Sigma v_R) = 0,$$

(4)

where $\Sigma$ is the surface density and for a steady state, Eq. (4) yields an accretion rate as:

$$\dot{M} = -2\pi R\Sigma v_R = \text{constant},$$

(5)

### 2.3 Angular momentum conservation

Assuming a steady-state, the Navier-Stoke’s equation can be expressed as:

$$\rho(v, \nabla)v = -\nabla p + \rho \nabla \phi + J \times B + \nabla.(\rho v(v - \frac{2}{3}(\nabla \cdot v)),$$

(6)

where $p$ is pressure, $v$ kinematic viscosity, $\phi$ the gravitational potential $J = \frac{1}{\mu_0}(\nabla \times B) = (J_R, J_\phi, J_z)$ the current density and $B = (B_R, B_\phi, B_z)$ the magnetic field. Here, we only consider the azimuthal component of the Navier-Stoke’s equation, which is given by:

$$\Sigma \left( \frac{\partial v_\phi}{\partial t} + \frac{v_R}{R} \frac{\partial}{\partial R}(RB_\phi) \right) = \frac{B_R}{\mu_0} \frac{1}{R} \frac{\partial}{\partial R}(RB_\phi) + \frac{B_z}{\mu_0} \frac{\partial B_\phi}{\partial z} + \frac{1}{R^2} \frac{\partial}{\partial R} \left( R^3 \Sigma \frac{\partial}{\partial R}(v_\phi - \frac{R^2}{3}(\nabla \cdot v)) \right),$$

(7)

Here, we neglect $\frac{B_R}{\mu_0} \frac{\partial}{\partial R}(RB_\phi)$ and for steady-state disc $\frac{\partial}{\partial t} = 0$. By integrating Eq. (7) and multiplying both sides by $R$, we get the angular momentum conservation:

$$\Sigma \left( v_R \frac{dl}{dR} \right) = \left[ \frac{B_z B_\phi}{\mu_0} \right] \frac{\mu}{R} + \frac{1}{R^2} \frac{d}{dR} \left[ R^3 \Sigma \frac{d}{dR}(\frac{l}{R^2}) \right],$$

(8)

where $l = Rv_\phi \propto R^{1/2}$ is the specific angular momentum. Then, the magnetic field of the NS in the Wang 1995 is given by:

$$B_z = -\frac{\mu}{R^3}.$$

(9)

where $\mu$ is the magnetic dipole moment.

Here, from Eq. (8) we have two sources of magnetic fields, $B_\phi$, such as shear magnetic field, $B_{\phi, \text{shear}}$ and dynamo generated magnetic field, $B_{\phi, \text{dyn}}$. As it was proposed by Wang 1995 the magnetosphere is nearly force free, and reconnection takes place outside the disc.The ratio of vertical and azimuthal field strengths is related to the shear between the disc and the magnetic field. Then, this ratio can be expressed in the form of Livio & Pringle 1992:

$$\frac{B_{\phi, \text{shear}}}{B_z} \sim -\gamma(\frac{\Omega_k - \Omega_s}{\Omega_k}),$$

(10)

where $\Omega_k$ and $\Omega_s$ represents the Keplerian angular velocity at the inner radius of the disc and the angular velocity of the star, respectively. The subscript k- denotes the keplarian. By rearranging Eq. (10) we obtain:

$$B_{\phi, \text{shear}} = -\gamma B_z \left( \frac{\Omega_k - \Omega_s}{\Omega_k} \right),$$

(11)

where $\gamma$ is a dimensionless parameter of a system Ghosh & Lamb 1979. The dynamo magnetic field, $B_{\phi, \text{dyn}}$, generated by magnetohydrodynamical turbulence in accretion disc through the dynamo action Balbus et al. 1998, which is given by:

$$B_{\phi, \text{dyn}} = \epsilon(\alpha_s \mu_0) \gamma_{\text{dyn}} P(r)^1/2,$$

(12)

where the subscript dyn - is stands for the dynamo generated magnetic field and $P(r)$ is the radiation pressure. From Eq. (12) $\gamma_{\text{dyn}}$ is the order of 10 that is given by Brandenburg et al. 1995 and $\epsilon$ is a
dynamo parameter that describes the direction of the magnetic field in the range of \(-1 \leq \epsilon \leq 1\). Then, substituting Eqs. (9), (11) and (12) into Eq. (8) and from Tessema & Torkelsson 2010 we have obtained:

\[
\Sigma \left( v_R \frac{dl}{dR} \right) = 2\varepsilon \frac{(\mu R^{-3})}{\mu_0} (\alpha_{ss} \mu_0 \gamma_{\text{dyn}} P_r(r))^{1/2} R - 2\gamma \left( \frac{\mu R^{-3}}{\mu_0} \right) \left( \frac{\Omega_k - \Omega_s}{\Omega_k} \right) R + \frac{d}{dR} \left( R^2 \Sigma \frac{d}{dR} \left( \frac{l}{R^2} \right) \right).
\]

Eq. (13) shows an ordinary differential equation for an accretion disc of the angular momentum conservation.

2.4 Hydrostatic vertical balance

We now consider the structure of the disc in the vertical z-direction. Hence, the angular momentum conservation is reduced to hydrostatic equilibrium condition if the net flow of gas along the vertical direction is zero. Then, the hydrostatic equilibrium equation is given by

\[
\frac{1}{\rho} \frac{\partial P}{\partial z} = \frac{\partial}{\partial z} \left( \frac{GM}{(R^2 + z^2)} \right)^{1/2},
\]

in the limit \(z \ll R\) and neglecting the self-gravity of the disc, Eq. (14) becomes:

\[
\frac{1}{\rho} \frac{\partial P}{\partial z} = -\frac{GMz}{R^3}.
\]

where \(G, M\) are the universal gravitational constant and a mass of the accreting star, respectively.

As a consequence, from Eq. (1) and the approximation of vertical pressure gradient we express \(\frac{\partial P}{\partial z} \sim \frac{P}{H}\) and \(z \sim H\). Then, Eq. (15) yields:

\[
\frac{P}{\rho} = c_s^2,
\]

Thus, from Eq. (15) and Eq. (16) we find the \(H\) as:

\[
H \cong c_s R \left( \frac{R}{GM} \right)^{1/2}.
\]

For a thin accretion disc, the local Kepler velocity should be highly supersonic. In general, we can define a central disc density approximately by

\[
\rho = \frac{\Sigma}{H} \quad \text{and} \quad H = c_s \left( \frac{R}{v_\phi} \right),
\]

where \(v_\phi\) is given by

\[
v_\phi = \sqrt{\left( \frac{GM}{R} \right)}.
\]

The speed of sound can be expressed by using Eq. (18) and (19) as:

\[
c_s = \frac{H}{R} \left( \frac{GM}{R} \right)^{1/2},
\]

As it was proposed by Tessema & Torkelsson 2010 we can write the gas and radiation pressure as:

\[
P = \frac{\rho K_B T_c}{\mu m_p} + \frac{4\sigma}{3c} T_c^4
\]
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where $\sigma$ is the Stefan-Boltzmann constant, $m_p$ is the mass of a proton, $T_c^4$ central temperature, the subscript c- denotes values in the central plane, $c$ is the speed of light, $\mu$ is the mean molecular weight for ionized gas and $K_B$ is the Boltzmann’s constant. Then, we can write the pressure for hydrostatic equilibrium using Eqs. (15) and (18) as:

$$P = \Sigma \left( \frac{HGM}{R^3} \right).$$

(22)

Here, for a newtonian accretion disc, the $f_{R\phi}$ component of the viscous stress tensor is given by:

$$f_{R\phi} = -\frac{3}{2} \rho v \Omega = \alpha_{ss} P(r),$$

(23)

where

$$\Omega = \left( \frac{GM}{R^3} \right)^{1/2}.$$  

(24)

Substituting Eq. (24) into Eq. (23) we obtain:

$$f_{R\phi} = -\frac{3}{2} \rho v \left( \frac{GM}{R^3} \right)^{1/2} = \alpha_{ss} P(r),$$

(25)

where, $\rho = \frac{\Sigma}{2\pi}$. Then, the viscous stress tensor, $f_{R\phi}$, can be expressed by:

$$f_{R\phi} = \frac{3\Sigma v}{4H} \left( \frac{GM}{R^3} \right)^{1/2} = \alpha_{ss} P(r).$$

(26)

From Eqs. (16), (22) and (26) we obtained the gas density of the NS as:

$$\rho = \frac{3\Sigma v}{4\alpha_{ss} H^3} \left( \frac{GM}{R^3} \right)^{-1/2}.$$  

(27)

Also, we can express the scale height of the disc in terms of the total pressure as:

$$H = \left( \frac{\rho k_BT_c R^3}{m_p\mu GM} + \frac{4\sigma T_c^4 R^3}{3c\rho GM} \right)^{1/2}.$$  

(28)

The local viscous dissipation is determined by radiative losses when the matter flow through an optical disc is low. Then, we have the $T_c$, $v$, $\Sigma$, $M$ and $R$ relations:

$$\frac{4\sigma}{3\tau} T_c^4 = \frac{9}{8} \rho \Sigma \frac{GM}{R^3}.$$  

(29)

Here, the optical depth of the disc, $\tau$, is given by

$$\tau = \int_0^H k_R \rho dz = \rho H k_R,$$  

(30)

where $K_R = k_{es} + k_{ff}$, in the inner region of the disc the temperature is high the approximation of $k_R \approx k_{es}$ is valid because this region is dominated by electron scattering opacity. Then, from Eqs. (29) and (30) we obtain the central temperature

$$T_c^4 = \frac{27}{32\sigma} \rho \Sigma^2 k_R \frac{GM}{R^3}.$$  

(31)
As it was investigated by Tessema & Torkelsson 2010; Frank et al. 2002; Shapiro et al. 1983:

\[ R_A = \left( \frac{2\pi^2 \mu_A^4}{GM^2 \mu_0^2} \right)^{\frac{1}{2}} \simeq 1.4 \times 10^4 \dot{M}_{14}^{\frac{8}{7}} M_1^{\frac{7}{4}} \mu_{15}^{\frac{2}{7}} m. \]  \hspace{1cm} (32)

\[ R_{co} = \left( \frac{GM P_{spin}^2}{4\pi^2} \right)^{\frac{\frac{1}{2}}{7}} \simeq 1.5 \times 10^6 P_{spin}^{\frac{2}{3}} M_1^{\frac{1}{7}} m, \]  \hspace{1cm} (33)

where \( P_{spin} = \frac{2\pi}{14} \) and \( M_1 = \frac{M}{M_\odot} \). Let us introduce a parameter \( y = \Sigma v \) in order to solve an ordinary differential equation for an accretion disc. Then, from Eqs. (13), (22), (27), (32) and (33) we have obtained:

\[ y' = \frac{\dot{M}}{6\pi r} - \frac{y}{2r} - \epsilon D_1 (Gm)^{\frac{5}{8}} R_A^{\frac{7}{8}} - D_2 R_A^{\frac{7}{8}} \left[ 1 - \left( \frac{R_A}{R_{co}} \right)^{\frac{3}{2}} \right]. \]  \hspace{1cm} (34)

where,

\[ D_1 = \sqrt{\frac{4\mu_2^2 \gamma_{dyn} y}{3\mu_0 HR_A^{1/2}}} \quad \text{and} \quad D_2 = \frac{4\mu_2^2 \gamma}{3\mu_0 (Gm)^{1/2}}, \]  \hspace{1cm} (35)

which is a differential equation of \( y \) for accretion disc around accreting millisecond X-ray pulsars. At large radii the solution of Eq. (34) approaches the Shakura- Sunyaev solution, which giving us the boundary condition \( y \to \Lambda \dot{M} \) as \( R \to \infty \). Here, we need to transform Eq. (34) by introducing dimensionless quantities \( \Lambda \) and \( r \), so that

\[ y = \Lambda \dot{M} \]  \hspace{1cm} (36)

where \( \Lambda \) is a dimensionless parameter for accretion disc and

\[ R = rR_A \]  \hspace{1cm} (37)

where \( r \) is a dimensionless radial coordinate and \( R_A \) is the Alfvén radius, which is a characteristic radius at which magnetic stresses dominate the flow in the accretion disc.

As noted by Elsner & Lamb 1977 we have \( \omega_s \) as:

\[ \omega_s = \left( \frac{R_A}{R_c} \right)^{\frac{2}{7}} = 0.36 M_{14}^{\frac{8}{7}} \dot{M}_{14}^{\frac{5}{7}} \mu_{15}^{\frac{2}{7}} \left( \frac{P_{spin}}{4.8 \text{ms}} \right)^{-1}, \]  \hspace{1cm} (38)

Finally, using Eq. (36), Eq. (37) and Eq. (38) we get the differential equation of an accretion disc from Eq. (34) which is given by:

\[ \Lambda' = \frac{1}{6\pi r} - \frac{\Lambda}{2r} - \epsilon D_3 (Gm)^{\frac{5}{8}} R_A^{\frac{7}{8}} r^{\frac{5}{8}} - D_4 r^{\frac{7}{8}} \left( 1 - \omega_s r^{\frac{2}{7}} \right). \]  \hspace{1cm} (39)

where \( D_3 = \sqrt{\frac{4\mu_2^2 \gamma_{dyn} \Lambda}{3\mu_0 HR}} \) and \( D_4 = \frac{4\mu_2^2 \gamma}{3\mu_0 (Gm)^{1/2}} R_A^{\frac{7}{8}} \). This equation is the new analytical solution for an accretion disc around accreting millisecond X-ray pulsars.
2.5 The structure of the disc

Here, to analyze the dynamics of an accretion disc, we emphasize on the inner region of the disc, in which the radiation pressure is much higher than the gas pressure and the accretion rate is large. In this region, Compton scattering occurs more frequently than free-free absorption. To solve Eq. (39) numerically we have to determine scale height in the inner region of the disc, which is given by:

\[ H = \frac{9}{8c} k_{cs}(M\Lambda). \]  

(40)

Then, the shear magnetic field is given by:

\[ B_{\phi,\text{shear}} = -4 \times 10^3 \gamma M_1^2 \dot{M}_1^{1/2} \mu_{15}^{2/3} \left(1 - \omega_s r^2\right) r^{-3/2} T, \]  

(41)

In the inner region of the disc the radiation pressure is larger than the gas pressure, we have that:

\[ \Sigma = 9.57 \times 10^1 \alpha_{ss} 1 \dot{M}_1^{-5/7} \dot{M}_1^{14/7} \mu_{15}^{6/7} \Lambda(r)^{-1} r^{-3/2} \text{kgm}^{-2}, \]  

(42)

\[ \rho_c = 3.18 \times 10^{-2} \alpha_{ss} 1 \dot{M}_1^{-5/7} \dot{M}_1^{17/7} \mu_{15}^{6/7} \Lambda(r)^{-2} r^{-3/2} \text{kgm}^{-3}, \]  

(43)

\[ v_R = 1.18 \times 10^7 \alpha_{ss} \dot{M}_1^{6/7} \dot{M}_1^{14/7} \mu_{15}^{10/7} \Lambda(r)^{3/2} r^{-3/2} \text{m}^{-1}, \]  

(44)

\[ T_c = 1.86 \times 10^8 \alpha_{ss} \dot{M}_1^{5/28} \dot{M}_1^{3/28} 1 M_1^{12/14} \mu_{15}^{10/7} \Lambda(r)^{3/2} r^{-3/8} \text{ks}, \]  

(45)

\[ v = 1.1 \times 10^{12} \alpha_{ss} \dot{M}_1^{5/7} \dot{M}_1^{17/7} \mu_{15}^{6/7} \Lambda(r)^{2} r^{-3/2} \text{m}^{-1}, \]  

(46)

\[ \tau_{es} = 1.8 \alpha_{ss} \dot{M}_1^{-5/7} \dot{M}_1^{10/7} \mu_{15}^{6/7} \Lambda(r)^{3/2} r^{-3/2} \text{ms}^{-1}, \]  

(47)

The transition radius in the inner region of the disc is estimated by approximating \( \Lambda = 1/3\pi \).

\[ r_{IM} = 12.5 \mu_{15}^{8/21} \alpha_{ss}^{2/21} \dot{M}_1^{10/21} \dot{M}_1^{22/21} \mu_{15}^{4/7}, \]  

(48)

Here, the accretion disc outside of Alfven radius is overpowered by radiation pressure only if

\[ \mu_{15} < 8.256 \mu_{15}^{2/3} \alpha_{ss}^{-1/10} \dot{M}_1^{5/6} \dot{M}_1^{11/6}, \]  

(49)

This circumstance is not satisfied for ordinary X-ray pulsar with a magnetic dipole moment of \( \sim 10^{20}\text{Tm}^3 \) White & Stella 1988, though it can be satisfied for AMXPs.

As we have incorporated so far about the equations of an accretion disc around accreting millisecond X-ray pulsars, then we applied some parameters and investigate Eq. (39) in the inner region of the disc. The inner region in which the radiation pressure is overpowered and electron scattering is the most important source of opacity Shakura & Sunyaev 1973. As a result, in the innermost regions, the emitted spectrum of the disc cannot be approximated by a black-body spectrum, whereas if the accretion rates are high, radiation pressure towards the inner edge of the accretion disc exceeds the thermal pressure. Using the appropriate selection of the magnetic field and accretion rate, then the inner region solution is found below.

3 RESULT AND DISCUSSION

3.1 Global Solutions

Here, as it was studied by Tessema & Torkelsson 2011 we integrate Eq. (39) for the inner region inwards from very small radius \( \sim 12.5 \) and \( \Lambda = 1/3\pi \). The dimensionless parameters \( \gamma, \gamma_{dyn} \) and \( \alpha_{ss} \) are 1, 10 and \( 10^{-2} \), respectively. Hence, the disc is overpowered by radiation pressure and electron scattering which \( R_A < R_{IM} \) by increasing accretion rate. In this region, it is possible to use the accretion rate up to Eddington limit, so that we take \( \dot{M} = 1.5 \times 10^{14}\text{kg}^{-1} \) for our calculation and we use \( \Lambda = \)
1/3\pi$ for the analytical solution of the disc, and solve Eq. (39) for the inner region of the disc starting from $r_{1M} = 12.5$. Our solution for the inner disc region of AMXPs with different dynamo parameters such as $\epsilon = 0.45, 0.15, 0, -0.15, -0.45$ are shown in Fig.1. This figure shows the variations of $\Lambda$ as a function of $r$ for all $\epsilon = 0.45, 0.15$ and 0 all solutions are case v inner boundaries except $-0.15, -0.45$

Fig.1 Result of $\Lambda$ as a function of $r$ for the AMXPs with accretion rate $\dot{M} = 1.5 \times 10^{14}\text{kg s}^{-1}$ and the different dynamo parameters are shown with $\epsilon = -0.45$ solid blue line, $\epsilon = -0.15$ red dotted line, $\epsilon = 0$ solid green line, $\epsilon = 0.15$ black dotted line, and $\epsilon = 0.45$ solid black line. In fig.2 the $\Sigma$ is purely a decreasing function of $r$ for $\epsilon = 0, 0.45$ and increasing for $\epsilon = -0.45$.

Fig.2 Result of $\Sigma$ as a function of $r$ for AMXPs with accretion rate $\dot{M} = 1.5 \times 10^{14}\text{kg s}^{-1}$ and the different dynamo parameters are shown with $\epsilon = -0.45$ solid blue line, $\epsilon = 0$ solid green line, and $\epsilon = 0.45$ solid black line. In Fig.3 the high surface density results in a hot flow. Here, the mid-plan
temperature as a function of the radius in the inner region of the disc does not depend on $\Lambda$ so that as the radius decreases the temperature increases.

Fig.3 Result of $T_c$ as a function of $r$ for AMXPs with accretion rate $\dot{M} = 1.5 \times 10^{14}$kgs$^{-1}$ and the different dynamo parameters are shown with $\epsilon = -0.45$ solid blue line, $\epsilon = 0$ solid green line, and $\epsilon = 0.45$ solid black line.

In Fig.4 we investigated that the high surface density which is corresponding to a decreases in radial velocity and this radial velocity is dependent on $\Lambda$. On this figure, the inner edge of the accretion disc approaches to the surface of the star the radial velocity either goes to zero or infinite. Here, the radial velocity decreases as the radius increases.

Fig.4 Result of $V_R$ as a function of $r$ for AMXPs with accretion rate $\dot{M} = 1.5 \times 10^{14}$kgs$^{-1}$ and the different dynamo parameters are shown with $\epsilon = -0.45$ solid blue line, $\epsilon = 0$ solid green line, and $\epsilon = 0.45$ solid black line.
3.2 Accretion torques

The torques on a neutron star range from material to magnetic; it is obtained from Eq. (13) by multiplying $2\pi R$ and then integrating from the inner radius of the disc, $R_{\text{in}}$, to the outer edge of the disc, $R_{\text{out}}$, see, e.g., Kluzniak & Rappaport 2007; Tessema & Torkelsson 2010; Shi et al. 2015.

$$\dot{M}\sqrt{GM_{\text{in}}}-\dot{M}\sqrt{GM_{\text{out}}} = - \int_{R_{\text{in}}}^{R_{\text{out}}} \left[ \frac{4\pi(\mu R^{-3})}{\mu_0} (B_{\phi,dyn} + B_{\phi,shear}) \right] R^2 dR - \left[ 3\pi y(GMR)^{1/2} \right] \frac{R_{\text{out}}}{R_{\text{in}}},$$  \hspace{1cm} (50)

Note that the two expressions on the LHS of Eq. (50) show the rate at which angular momentum is transported past the inner and outer edges of the accretion disc, while the RHS shows the implications of magnetic and viscous torques to the angular momentum balance. In this case, the material, magnetic and viscous torque can be expressed in Eqs. (51), (52), (53), (54), (55) and (56): Thus, the material torque of the inner edge of the disc on the neutron star is given by

$$N_{\text{in}} = \dot{M} (GM_{\text{in}})^{1/2} = 1.4 \times 10^{26} \frac{\mu_2}{M_{15}^{3/7}} \frac{M_{14}^{6/7}}{r_{\text{in}}^{1/2}},$$  \hspace{1cm} (51)

Here, in Fig. 5 we investigate the inner accretion torque on the disc in the inner region of the disc. This material torque increases as the accretion rate and the inner radius increases.

![Fig. 5](image-url)

Fig. 5 shows the material torque on AMXPs with different accretion rates by varying the inner region radius, $r_{\text{in}}$, of the disc. The magnetic torque is the result of the coupling between the vertical magnetic field of the star and the toroidal magnetic field in the disc. Hence, the torque acting on the lower surface of the disc can be written Ghosh & Lamb 1979 as:

$$N_{\text{mag}} = -4\pi \int_{R_{\text{in}}}^{R_{\text{out}}} \left( \frac{\mu R^{-3}}{\mu_0} \right) (B_{\phi,dyn} + B_{\phi,shear}) R^2 dR,$$  \hspace{1cm} (52)

This magnetic torque is separated into the shear and dynamo generated magnetic torque. Then, the shear magnetic torque, $N_{\text{mag,shear}}$, is given by:

$$N_{\text{mag,shear}} = \int_{R_{\text{in}}}^{R_{\text{out}}} \frac{4\pi (\mu R^{-3}) B_{\phi,shear}}{\mu_0} R^2 dR \approx 4 \times 10^{26} \frac{\gamma}{\mu_{15}} M_{14}^{3/7} \int_{r_0}^{\infty} \left[ r^{-4}(1-\omega_s r^{3/2}) \right] dr,$$  \hspace{1cm} (53)

![Diagram](image-url)
and the dynamo generated magnetic torque on the neutron star is:

\[
N_{\text{mag,dyn}} = - \int_{R_{\text{in}}}^{R_{\text{out}}} 4\pi \frac{(\mu R^3) B_{\phi,dyn}}{\mu_0} R^2 dR. \tag{54}
\]

Here, the dynamo generated magnetic torque in the inner region of the disc is given by:

\[
N_{\text{dyn,inner}} = 7 \times 10^{26} \epsilon \gamma_{\text{dyn}}^{1/2} A^{7/14} M_1^{5/14} \dot{M}_{14}^{3/14} \int_{\text{inner}} r^{-7/4} dr. \tag{55}
\]

On the other hand, the viscous torque in the inner region of the disc is given by:

\[
N_{\text{vis}} = -3\pi y R_{\text{in}} (G M R_{\text{in}})^{1/2} = -1.3 \times 10^{27} \mu_{15}^{2/7} M_1^{3/7} \dot{M}_{14}^{6/7} A(r_0) r_{1/2}^{1/2}. \tag{56}
\]

Moreover, as it was investigated by Tessema & Torkelsson 2011 the standard accretion disc solution has a case D inner region boundary when the viscous torque is neglected in accretion disc theory. Hence, the angular momentum is transported from the neutron star to the disc when it is in case V inner region boundary.

Except if \( \epsilon = 0 \) the dynamo magnetic torque importantly greater than the shear magnetic torque, and both are greater for \( \epsilon = 0.15 \) than for \( \epsilon = 0.45 \). Because of this effect the central hole of the disc grows too large when \( \epsilon = 0.45 \). The overpowered torque at \( \epsilon = 0.45 \) is the viscous torque at this region, which is ignored as show in Fig.6 below.

Fig.6 shows the variation of viscous torque as a function of radius for AMXPs with different masses and radius in the inner region of the disc.

### 3.3 Comparison with observational Results

There is a large variation in the accreting rates among the accreting millisecond X-ray pulsars. The well studied system IGR J00291+5934 is accreting at a rate of at least \( \sim 10^{14} \text{kg s}^{-1} \) based on its x-ray flux Burderi et al. 2007, while in some other systems, for instance, SAX J1808.4-3658 Bildsten & Chakrabarty 2001, the neutron star is accreting at a rate below \( 10^{12} \text{kg s}^{-1} \) from a brown dwarf companion. There is also a great doubtful in the spin variations that have been reported for the AMXPs. For instance, Burderi et al. 2006 reported \( \dot{\nu} \) vs these spin variations between \( -7.6 \times 10^{-14} \) and \( 4.4 \times 10^{-13} \text{Hz s}^{-1} \) for SAX J1808.4 − 3658. But Hartman et al. 2008 noted that the measurements of
this source are plagued by more variations in the pulse shape, and put an upper limit of $2.5 \times 10^{-14} \text{Hz s}^{-1}$ on the spin variations and found along-term spin down $\dot{\nu} = -5.6 \times 10^{-16} \text{Hzs}^{-1}$.

On the other hand Burderi et al. 2007 reported that IGR J00291 + 5934 is spinning up at $\sim 10^{-13} \text{Hzs}^{-1}$ during the December 2004 outburst. The more spin variations that have been observed in some AMXPs depends on the accreting torque, which is given by:

$$N = 2\pi \dot{\nu} I,$$

(57)

where $I$ is in $\text{Kg m}^2$, and $\dot{\nu}$ is in $\text{Hz s}^{-1}$.

\section*{4 CONCLUSION}

In this paper, we have studied the interaction between the accreting millisecond X-ray pulsars and the inner region of the disc, which is supported by the dynamo generated magnetic field. Hence, we found that the fundamental equations of an accretion disc around accreting millisecond X-ray pulsars gives the more stable system than the previous study. We have made an effort to find an analytical solution by using a numerical method for an accretion disc around AMXPs in the inner region of the disc, in which the accretion rate is high and the disc overpowered by radiation pressure and electron scattering region. Here, the analytical solution of Eq. (39) at higher accretion rate in the inner region of the accretion disc is greater than the radius of the neutron star for different values of dynamo parameters, $\epsilon$ and we observed the behavior of these solutions in the inner region in Fig. 1. We have formulated the numerical method for structure equation in the inner region of the disc and the highest accretion rate is sufficient to make the innermost region of the accretion disc to be overpowered by radiation pressure and electron scattering. We have observed that the relationship between surface density and radius in Fig. 2. Then, on this figure, the surface density decreases with the radius. The viscous torque in the inner region of the disc is ignored, which is shown in Fig. 6. Hence, the viscous torque on AMXPs decreases for different masses and radius of the accretion disc. The accretion torque is important in explaining the observed variations in the spin frequency of AMXPs like IGR J00291+5934. Thus, we have found that the spin derivatives for accretion rate $1.5 \times 10^{14} \text{kg/s}$ in this model are in agreement with RXTE observed data in accreting millisecond X-ray pulsars are consistently explained by this model.

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