Optical-vortex diagnostics via Fraunhofer slit diffraction with controllable wavefront curvature

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Far-field slit-diffraction of circular optical-vortex (OV) beams is efficient for measurement of the topological charge (TC) magnitude but does not reveal its sign. We show that this is because in the common diffraction schemes the diffraction plane coincides with the incident OV waist plane. With explicit involvement of the incident beam spherical wavefront and based on the examples of Laguerre-Gaussian modes we show that the far-field profile possesses an asymmetry depending on the wavefront curvature and the TC sign. These features enable simple and efficient ways for the simultaneous diagnostics of the TC magnitude and sign, which can be useful in many OV applications, including the OV-assisted metrology and information processing.

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Optical vortices (OV) are among the most interesting and attractive objects of structured light physics [1–3]. In paraxial fields, an OV appears as an isolated point of the beam cross section with zero amplitude and indeterminate phase (phase singularity); upon a round trip near this point, the field phase changes by $2\pi m$ where the integer $m$ is the topological charge (TC) of the OV. Accordingly, the beam wavefront near an OV is helical, and the OV core (zero-amplitude point) is a center for the local transverse energy circulation being the source of the orbital angular momentum (OAM) [1–4]. Due to their unique topological and singular properties, beams with OVs find many useful applications associated with the sensitive optical diagnostics and metrology [5–7], micromanipulation [8–10] and information processing [11,12].

For all fields of the OV application, rapid and reliable recognition of its rotational characteristics (determined by the magnitude and the sign of its TC) is imperative. Usually, the rich and non-trivial rotational structure of a circular OV is hidden due to its symmetry and can be revealed only in some indirect way. Standard approaches to the OV diagnostics are based on the interference with non-singular reference beams or beams with the known singular properties [1–3] but such schemes are generally complicated and cumbersome. In many situations, referenceless methods are more appropriate. For example, when a circular OV beam undergoes the astigmatic transformation, its transverse intensity distribution acquires a characteristic deformation with distinct “fingerprints” of the initial OV structure [13–15].

To the best of our knowledge, the most flexible and universal approaches exploit specific features of the OV diffraction in which the helical properties of an OV and its OAM-related circulatory nature are explicitly manifested. The simplest edge diffraction schemes [16–18] provide spectacular demonstration of the transverse energy circulation but a reliable detection of the OV “strength” (TC magnitude |m|) requires additional time-consuming and precise procedures. More efficient methods enabling the “full” (TC magnitude + sign) OV diagnostics are based on the traditional approaches employing a single or double slit [19,20] and strip [21,22] Fresnel diffraction. However, the most suitable and universal means for the OV detection involve the far-field (Fraunhofer) diffraction [23–28]. The far-field scheme is, generally, less sensitive to inevitable misalignments, provides advantages of a well defined and stable reference frame as well as a considerable freedom in the choice of the registration plane, and can be easily implemented even in the ultra small-scale experimental environment. Actually, the far-field diffraction approaches are realized in the recently reported techniques adapted to the nanoscale OV diagnostics [29–32].

Despite the diversity of specific practical schemes, the interpretation of the OV-diffraction results relies on some common principles: as a rule, the immediately observable diffraction pattern (DP) contains a set of bright (dark) spots whose number is associated with the TC magnitude, and the overall pattern asymmetry indicates its sign (for example, the far-field diffraction by a triangular aperture [23–25]). However, for the most suitable cases of slit or strip diffraction, the far-field intensity pattern appears to be symmetric [24,27,28] and the “full” OV diagnostics becomes unavailable or requires additional observations.

In this Letter, based on the typical example of the Laguerre-Gaussian (LG) beams [1–3], we analyze the reasons of this
deficiency and propose the simple way for its elimination thus enabling the full OV diagnostics by the far-field slit (strip) diffraction. Additionally, the proposed procedure may contribute to the better visibility of informative details of the DP (e.g., its peripheral bright lobes).

We start with a brief theoretical examination. Let a paraxial monochromatic light beam be described by the usual model with the electric field distribution expressed as

\[ E(x, y, z) = \text{Re}\left[u(x, y, z) \exp(ikz - i\omega t)\right] \]

where \(\omega\) is the light frequency, \(k = c\omega/c\) is the wavenumber (with \(c\) standing for the speed of light), and \(u(x, y, z)\) is the slowly varying complex amplitude (CA) [1,2].

The beam propagates along axis \(z\) and the transverse plane is parameterized by the \((x, y)\) Cartesian frame (see Fig. 1a). The diffraction obstacle (slit) is situated in the plane \(z = 0\), and its special role is highlighted by the special transverse coordinates notation \((x_o, y_o)\); the slit is adjusted symmetrically with respect to the beam axis \(z\).

We consider the incident LG_{0m} beams with zero radial index for which the incident CA distribution in the diffraction plane is described by

\[ u_{OVM}(x_o, y_o, z = 0) \equiv u_{OVM}(x_o, y_o) = \left(\frac{x_o + i\sigma y_o}{b}\right)^{\text{|\sigma|}} \exp\left(-\frac{x_o^2 + y_o^2}{2b^2}\right) \]

where \(\sigma = \pm 1\) is the sign of the OV TC (the winding handedness of the screw wavefront). This expression implies that the diffraction plane coincides with the incident beam waist plane (which is usual in the OV-diffraction studies [16,19–20]), and \(b\) is the Gaussian envelope waist radius. Then, if the slit width equals to \(2\delta\) (see Fig. 1a), the DP in the observation plane is calculated via the Fresnel-Kirchoff integral [33]

\[ u(x, y, z) = \frac{k}{2\pi iz} \int_{-\infty}^{\infty} dx_o \int_{-\infty}^{\infty} u_{OVM}(x_o, y_o) \exp\left(\frac{ik}{2z}\left[(x-x_o)^2 + (y-y_o)^2\right]\right) dy_o \]

or in the far-field conditions,

\[ u(\xi, \eta, z) = \frac{k}{2\pi iz} \exp\left(\frac{ikz^2 + \eta^2}{2}\right) \]

\[ \times \int_{-\infty}^{\infty} dx_o \int_{-\infty}^{\infty} u_{OVM}(x_o, y_o) \exp\left[-ik(\xi x_o + \eta y_o)\right] dy_o \]

which follows from (2) when \(z \to \infty\). In Eq. (3), the dimensionless far-field coordinates are introduced according to relations \(x/z \to k, y/z \to k, k = \eta\). In case of the strip diffraction, the results can be easily obtained from (2) and (3) through the Babinet principle [33].

In Fig 2, we present the far-field intensity patterns

\[ I(\xi, \eta) \propto |u(\xi, \eta, z)|^2, \]

calculated via Eq. (3) for the diffraction scheme depicted in Fig. 1a with \(\sigma = 0.5b\) and the incident LG beams described by Eq. (1). In full agreement with known results [27,28,32], the far-field slit-DP formed by the incident OV beam with the TC \(m\) contains exactly \(|m| + 1\) bright lobes, but, due to its rectangular symmetry, is quite identical for the oppositely charged OV beams. This symmetry is a direct consequence of Eqs. (1) and (3), (4) which dictate that \(I(\xi, -\eta) = I(-\xi, \eta) = I(\xi, \eta)\). However, it does not hold for the Fresnel DPs determined by Eq. (2); in agreement with other similar situations [16–22], examples of Fig. 1b calculated by Eq. (2) show the distinct asymmetry directly related to the internal energy circulation and to the sign of the incident beam TC.

![Fig. 1](image1.png)

Fig. 1. (a) Geometrical conditions of the OV diffraction. (b) DPs observed at a distance \(z = 15kb\) behind the slit with \(\sigma = 0.5b\) for the incident LG_{02} beams (1).

![Fig. 2](image2.png)

Fig. 2. (1st row) Far-field intensity patterns calculated for the incident LG_{0m} beams (1) of different TCs (indicated above each column), \(\sigma = 0.5b\) (see Fig. 1a); the far-field coordinate frame, common for all images, is indicated in the upper left image. (2nd row) The corresponding intensity plots (in units relative with respect to the maximum) along the \(\xi\) axis; values of \(\xi\) are given in units of the divergence angle of the Gaussian envelope \(\gamma = 1/(kb)\).

Another drawback inherent in the multi-lobe DPs of Fig. 2 is that only few central lobes are practically distinguishable. The intensities of the peripheral lobes rapidly decay with the off-axis distance: in case of \(|m| = 3\) the side-lobe intensities are approximately 10% of the central maximum, for \(|m| = 4\) the peripheral peaks \(P_{+2}\) and \(P_{-2}\) hardly reach 2% of the central peak \(P_0\) and for higher \(|m|\) the side-lobe intensities progressively
decrease. Normally, in presence of noise, this essentially restricts the maximum detectable TCs via the slit-DP.

Now the problem is to unite the above-mentioned practical advantages of the far-field scheme with the ability of immediately detecting the TC sign inherent in the Fresnel diffraction (see Fig. 1b). It can be solved based on the known fact [34]: if the beam with initial CA distribution \( u(x, y, z) \) produces the diffracted field described by \( u(x, y, z) \) (cf. Fig. 1a), the modified initial beam with the CA distribution

\[
u_{\mu}(x, y, z) = u(x, y, z) \exp \left( \frac{ik x^2 + y^2}{2R} \right)
\]

will produce the diffracted field described by

\[
u_{\mu}(x, y, z) = \frac{1}{1 + \frac{z}{R}} \exp \left[ -\frac{ik x^2 + y^2}{2(z + R)} \right] u \left( \frac{x}{1 + z/R}, \frac{y}{1 + z/R}, \frac{z}{1 + z/R} \right).
\]

Indeed, the transformation (5) is nothing but addition of a spherical component to the beam wavefront with preserving the same intensity profile, which can be readily performed, e.g., by usual focusing (defocusing) schemes. In turn, Eq. (6) means that the far-field \((z \to \infty)\) intensity distribution created by diffraction of the modified beam (5) is proportional to \([u \left( R\xi, R\eta, R \right)]^2\), that is, reproduces (in a changed scale) the DP which could be observed with the non-modified initial beam \( u(x, y, z) \) at a certain finite distance \( z = R \) behind the screen. In application to the OV beams of Eq. (1) this means that the DP asymmetry indicating the TC sign can be observed in the Fraunhofer plane once the diffraction plane (cf. Fig. 1a) deviates from the incident beam waist plane.

The “quality” of the resulting DP is determined by its convenience for the TC diagnostics, which includes not only the asymmetry but also sufficient visibility of the side lobes. For a given incident beam [cf. Eq. (1)] this quality depends on the introduced wavefront curvature characterized by the relative parameter

\[R_s = R/\left( kb^2 \right)\]

and the on the relative slit width \( \delta/\beta \). Fig. 3 shows the best examples chosen from a series of far-field DPs calculated for different \( R_s \) and \( \delta/\beta \). It explicitly demonstrates the \(|m| + 1\) bright lobes and, additionally, the asymmetry which indicates the OV rotational properties and the sign of its TC; when \( R_s > 0 \) (diverging incident beam), the multi-lobe DP “rotates” in agreement with the incident-beam energy circulation, when \( R_s > 0 \) (the case of Fig. 3), the rotation is opposite. Additionally, the side lobes of the DPs are much more intense (in comparison to the central ones) than those presented in the 2nd row of Fig. 2, which is profitable for practical measurements.

In experiment, we used a laser beam with the wavelength \( \lambda = 405 \text{ nm} \) \((k = 1.55 \times 10^3 \text{ cm}^{-1})\) focused by the convex lens with the focal length \( f_1 = 50 \text{ cm} \) (see Fig. 4). At the lens input, an LG beam was formed with the Gaussian envelope radius \( b_0 \approx 340 \mu\text{m} \) and slightly convex wavefront so that the focused LG beam converged to the waist cross section at a distance \( z \approx 60 \text{ cm} \) behind the lens, with the Gaussian envelope radius \( b_0 = 0.125 \text{ mm} \). The slit width and position can be adjusted to different focused-beam cross sections with desirable local beam size \( b \) and the wavefront curvature radius \( R \).

![Fig. 3. (1st row) Far-field slit-diffraction patterns (see Fig. 1a) of LG0m beams (1) modified by the transformation (5). The TCs, slit half-widths and the wavefront curvature radii (7), accepted upon calculation, are indicated above each column. (2nd row) Corresponding intensity distributions along the \( \xi \)-axis, the scale marks are in units of \( \mu \).](image)

![Fig. 4. The experimental setup including two lenses with focal distances \( f_1 \) and \( f_2 \) slit and the CCD camera. The registering unit (slit + lens \( f_2 \) + CCD) can be adjusted along the longitudinal \( z \)-direction.](image)
probably, due to the non-linear response of the CCD device. There are additional bright fringes on both sides of the DPs; however, this “ripple structure” emerging due to stray diffraction is distinctly different from the “main” lobes and practically does not deteriorate the diagnostic possibilities.

In case of strip diffraction, practically the same DP may be masked by the strong incident-beam radiation. Nevertheless, the strip diffraction can be equally suitable for the OV diagnostics if the incident beam is efficiently screened by appropriate spatial filters or stops. An analogue of such a scheme was recently realized in the nanoscale [32]. In the subwavelength situation, the vectorial nature of the optical field is essential, and the full scattering theory [33] should be applied rather than the scalar diffraction approach employing Eqs. (2), (3). However, qualitatively, the results of [32] (the scattering asymmetry observed when the incident beam is focused onto the nanowire) can be well explained by the diffraction arguments. The diffraction obstacle (nanowire) is not small compared to the longitudinal inhomogeneity of the strongly focused incident beam, so the coincidence of the waist cross section with the “diffraction plane” can only be occasional in [32]. An essential part of the incident light “meets” the obstacle in locations where the incident beam possesses a significant spherical wavefront component, which causes the DP asymmetry.

In conclusion, both the theoretical analysis and experimental verification have persuasively shown that the far-field slit-diffraction with controllable wavefront curvature can be efficient means for the “full” OV diagnostics. The TC magnitude \(|m|\) and sign can be immediately seen from the number of bright lobes \((|m| + 1)\) and the asymmetry of the intensity distribution. These properties are common with the known Fresnel diffraction techniques but the far-field approach provides practical advantages of a well defined and stable reference frame and is less sensitive to the system misalignments. As a side note, we have proposed a method by which one can reproduce the Fresnel DP, characteristic for arbitrary distance behind the diffraction obstacle (slit), in the Fraunhofer (far-field) plane. A judicious choice of the slit width and the local wavefront curvature may be used for optimization of the DP bright-lobes’ positions and visibilities.

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