Optimum and performance of absorber with zero damping

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Abstract. This paper considers optimum parameters tuned mass damper (TMD) as a vibration absorber with zero damping. Generally, tuned absorbers with damping have been considered to be applied to the structures. While adjusting damping constant produces additional complexity in real applications, this paper investigates the use of absorber with zero damping. The properties of the absorber to be designed then is reduced to only its spring constant, while the damping is set to zero throughout the design process. To optimize its spring constant, genetic algorithm (GA) with real coding is utilized. With the optimized spring properties obtained from the optimization, the performance of structures is then investigated under several earthquake excitations. The performance of the optimum absorber with zero damping is also compared with the one when arbitrary damping presents in the absorber. The simulation results show that the performance of the absorber with zero damping is smaller compared to the one of absorber with damping. However, the response of absorber with damping tends to decrease when time increases.

1. Introduction
The use of absorbers or tuned mass dampers (TMDs) for reducing structural vibrations have been widely proposed. There include the use of TMD in several structures such as in Taipei 101 building a 509 m building in Taipei, Taiwan, where a 728 ton pendulum type TMD has been suspended from the 92nd to the 87th floor. The TMD is made of a ball shape steel, where as many as eight viscous dampers are also attached to act as shock absorbers. Other installation of TMD were in Citicorp Center building (278 m building, 370 tons TMD), Trump World Tower (262.4 m building, 600 tons TMD), John Hancock building (457 m building, 2 x 360 tons TMD), etc.

The application of TMD as an absorber dated back to Frahm who obtained a US patent in 1909 in dampening out a single degree of freedom system with spring constant only by using a relatively small mass with spring constant. It can be mathematically proofed that the main system is able to totally dampen when subject to a sinusoidal load in the main system. Warburton [1] presented an undamped SDOF subject to various loadings, such as harmonic and white noise random loading, applied at the main system and at the support. Numerical study has been discussed for systems with damping constant at the main system.

Several other researchers has also discussed optimization TMD by using metaheuristic optimization methods, such as genetic algorithms [2, 3], harmony search method [4], and particle swarm optimization [5].

In most cases usually the mass ratio of TMD is specified and the properties of TMD are optimized, i.e., the damping and spring constant of the TMD. However, for simple application, the inclusion of
damping of the TMD is normally not usually available. Therefore, the exclusion of damping in the TMD is a viable way to solve the problem.

2. Equation of motion

The equation of motion of a SDOF structure with the TMD in general can be written as:

\[ \mathbf{M}_s \ddot{U}_s + \mathbf{C}_s \dot{U}_s + \mathbf{K}_s U_s = -\mathbf{M}_s \mathbf{1}_s \ddot{u}_g \]  

(1)

where \( \mathbf{M}_s \), \( \mathbf{C}_s \), and \( \mathbf{K}_s \) are mass, damping, and stiffness matrix, respectively; \( U_s \) is displacement with respect to the ground, \( u_g \) is ground displacement, \( \mathbf{1}_s \) is a vector that contain 1, and the dot(.) represents derivative with respect to time. In general:

\[
\begin{align*}
\mathbf{M}_s &= \begin{bmatrix} m_s & 0 \\ 0 & m_d \end{bmatrix} \\
\mathbf{C}_s &= \begin{bmatrix} (c_s + c_d) & -c_d \\ -c_d & c_d \end{bmatrix} \\
\mathbf{K}_s &= \begin{bmatrix} (k_s + k_d) & -k_d \\ -k_d & k_d \end{bmatrix}
\end{align*}
\]

(2a-c)

When the damping of the absorber is not present as shown in figure 1, the damping matrix becomes:

\[
\mathbf{C}_s = \begin{bmatrix} c_e & 0 \\ 0 & 0 \end{bmatrix}
\]

(2d)

\[\text{Figure 1. Structure with absorber.}\]

The equation of motion can be written as state space equation as:

\[ \dot{\mathbf{Z}} = \mathbf{AZ} + \mathbf{Ew} \]  

(3)

where

\[
\begin{align*}
\mathbf{Z} &= \begin{bmatrix} \mathbf{U}_s \\ \dot{\mathbf{U}}_s \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M}_s^{-1} \mathbf{K}_s & -\mathbf{M}_s^{-1} \mathbf{C}_s \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{1}_s \end{bmatrix}, \quad \text{and} \quad w = \ddot{u}_g
\end{align*}
\]

(4a-d)
3. Optimization
To optimize the absorber properties, a metaheuristic algorithm was used, where in this paper a real coded genetic algorithm (GA) was utilized. GA starts by initializing the candidates of design variables and measures the fitness by using the inverse of the objective function. The objective function in this paper is taken as the displacement of the main structure. The candidates of the design variables experience mutation and crossover with a certain degree of mutation and crossover. The fitness of new created design variables are then evaluated, and some of design variables are also chosen again for mutation and crossover. This process are done generation per generation. At the final generation, the candidate that has highest fitness is considered as the design variable.

Because the damping of the absorber is taken equal to zero, only the spring constant is to be determined. Note that the boundary of the design variable is unknown. Real coded genetic algorithm similar to the one in [3] is used so that it can explore the unknown boundary of the design variables.

In order to obtain the maximum displacement of the structure, the simulations were carried out. Two cases are considered. The first case is when the objective function as the maximum displacement of the structure when subjected to four recorded ground accelerations due to El Centro 1940, Kobe 1995, Hachinohe 1968, and Northridge 1994 earthquakes. The simulation is taken for 10 seconds time history only. While the second case is when the structure is subjected to a harmonic ground excitation with the frequency is the same as the frequency of the structure.

4. Results
To optimize the spring constant of the absorber, the properties of the structure is taken as follow: \( m_s = 10 \text{ t} \), \( c_s = 5.4 \text{ kN-s/m} \), and \( k_s = 63 \text{ kN/m} \). The mass of the absorber is taken as \( m_d = 0.0262 m_s \). GA with the following properties is used: the number of population = 20, probability of crossover = 0.8, probability of mutation = 0.10, and maximum generation = 500. In each generation as many as 10% of the population are randomly replaced by a new individuals in order to increase the variability of the population.

4.1. Case 1: Minimizing displacement due to 4 recorded earthquakes
As many as 4 recorded earthquake, i.e., El Centro 1940, Kobe 1995, Hachinohe 1968, and Northridge 1994 earthquakes are used to obtain the displacement of the structure. The performance index is the sum of the peak displacement of the structure subject to each earthquake. The record is taken for 10 seconds, as shown in figure 2. GA is used to optimize the absorber spring constant. The history of the objective function generation per generation is depicted in figure 3, with the resulting \( k_d = 1.4436 \text{ kN/m} \).
Figure 2. A 10 s earthquake records for simulations.

Figure 3. Objective function.

The result of the optimization is used to simulate the structure subject to El Centro 1940, Kobe 1995, Hachinohe 1968, and Northridge 1994 earthquakes. The time history responses are shown in figure 4. The peak displacements are presented in table 1.
### Table 1. Peak displacements case 1.

| Displacement | El Centro | Kobe | Hachinohe | Northridge |
|--------------|-----------|------|-----------|------------|
| w/o TMD (m)  | 0.2111    | 0.3149 | 0.2257    | 0.5498     |
| With TMD (m) | 0.1826    | 0.3078 | 0.1742    | 0.5354     |
| $\xi_d = 0.10$ (m) | 0.1902 | 0.3094 | 0.1860    | 0.5372     |

#### Case 1:

**Figure 4.** Displacement response case 1.

4.2. *Case 2: Minimizing displacement due to a harmonic ground acceleration*

For the second case, a harmonic ground excitation, where the frequency is the same with the frequency of the structure is taken as the input acceleration. The duration of the ground acceleration is taken as 30 s. The same GA is utilized, the resulting $k_d = 1.4703$ kN/m. The time history response due to El Centro 1940, Kobe 1995, Hachinohe 1968, and Northridge 1994 earthquakes are presented in figure 5. Peak displacements are presented in table 2.

In order to see the effect of damping to the current result, an arbitrary damping is defined for the absorber, i.e., $\xi_d = 0.10$. The structure is simulated with the same ground excitations. The results are presented in table 2 and figure 6.
Case 2

![Displacement response case 2.](image)

**Figure 5.** Displacement response case 2.

**Table 2.** Peak displacement case 2.

| Displacement | El Centro | Kobe | Hachinohe | Northridge |
|--------------|-----------|------|-----------|------------|
| w/o TMD (m)  | 0.2111    | 0.3149| 0.2257    | 0.5498     |
| With TMD (m) | 0.1824    | 0.3079| 0.1746    | 0.5353     |
| $\xi_d = 0.10$ (m) | 0.1902 | 0.3095 | 0.1863 | 0.5371 |

5. **Discussions**

From the results of simulation in table 1, it can be seen that the peak displacements are reduced, although not very significant. From figure 4 and table 1, the largest response reduction is achieved for the structure subject to El Centro 1940 and Hachinohe 1968 earthquakes. From figure 4, it can be seen that, although the peak response is reduced, when the time increases, the response tends to be larger than the response without absorber. These conditions also occur for case 2, when the objective function is obtained from the harmonic ground acceleration. When the damping is provided to the TMD, the peak displacement increases for all cases regardless of the ground accelerations as can be seen from tables 1 and 2. However, as can be seen from figure 6, when the time increases the response tends to be minimum.
6. Conclusions
The optimum spring constant of the absorbers obtained by minimizing the displacement of the structure subject to a series ground motions of recorded earthquakes. When the damping of the absorber is set to zero, the peak displacement is reduced, although the response reduction is not very significant. When the damping is added to the absorber, it is seen that the response is dampening out when the time increases.

References
[1] Warburton G B 1982 Earthq Engng and Struct Dyn 10 381–401
[2] Hadi M N S and Arfiadi Y 1998 J. of Struct Engng ASCE 124 1272–80
[3] Arfiadi Y and Hadi M N S 2001 Comp and Struct 79(17) 1625–34
[4] Bekdas G and Nigdeli S M 2011 Engrg Struct 33 2716–23
[5] Leung A Y T, Zhang H, Cheng C C and Lee Y Y 2005 Earthq Engng Struct Dyn 37 1223–46