Fry (1996) showed that galaxy bias has the tendency to evolve towards unity, i.e. in the long run, the galaxy distribution tends to trace that of matter. Generalizing slightly Fry's reasoning, we show that his conclusion remains valid in theories of modified gravity (or equivalently, complex clustered dark energy). This is not surprising: as long as both galaxies and matter are subject to the same force, dynamics would drive them towards tracing each other. This holds, for instance, in theories where both galaxies and matter move on geodesics. This relaxation of bias towards unity is tempered by cosmic acceleration, however: the bias tends towards unity but does not quite make it, unless the formation bias were close to unity. Our argument is extended in a straightforward manner to the case of a stochastic or nonlinear bias. An important corollary is that dynamical evolution could imprint a scale dependence on the large scale galaxy bias. This is especially pronounced if non-standard gravity introduces new scales to the problem: the bias at different scales relaxes at different rates, the larger scales generally more slowly and retaining a longer memory of the initial bias. A consistency test of the current (general relativity + uniform dark energy) paradigm is therefore to look for departure from a scale independent bias on large scales. A simple way is to measure the relative bias of different populations of galaxies which are at different stages of bias relaxation. Lastly, we comment on the possibility of directly testing the Poisson equation on cosmological scales, as opposed to indirectly through the growth factor.

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I. INTRODUCTION

In a classic paper, Fry [1] showed that galaxies, once formed, would evolve to eventually trace mass. This result was subsequently extended to the case of a stochastic bias by Tegmark and Peebles [2]. Similar results were found in numerical simulations [3]. The linear bias paradigm has been applied to a variety of cosmological models, including ACDM and quintessence [4]. Both [1] and [2] assumed Newtonian/Einstein gravity from the start. One of our principal aims is to show that relaxing this assumption does not alter the basic conclusions. In the process, we will uncover an interesting, unanticipated corollary.

The basic reasoning is simple: both galaxies and (dark) matter can be regarded as test particles in some gravitational field; since their dynamics are identical, they are expected to trace each other given sufficient time. This holds regardless of the origin of the gravitational field. Departures from Newtonian/Einsteinian gravity on large scales have been suggested as a possible explanation for the observed cosmic acceleration. Most of these theories of modified gravity, for instance DGP [5] or scalar-tensor theories [6], are metric theories, and matter (or galaxy) particles move on geodesics defined by the metric. This is true as long as matter and galaxies are minimally coupled to gravity in the relevant frame [7]. The statement that matter and galaxies should evolve to trace each other does not rely on how the metric comes about. All that matters is that they have the same dynamics under the metric. More generally, as we will see, our conclusions on bias evolution remain valid as long as both galaxies and matter are subject to the same force.

Note that the same conclusion holds if, instead of modifying gravity, one introduces additional sources of fluctuations, for instance clustered dark energy. The presence of dark energy perturbations modifies the relation between matter perturbations and the metric, or the gravitational potential (just as modifying gravity does). But once this gravitational potential is given, both galaxies and matter particles respond to it in the same way, therefore guaranteeing eventual evolution towards a trivial biasing relation between them. This is consistent with the fact that modifying gravity can be viewed as some complicated form of dark energy - no surprise given that one can always shift terms from the left hand side of the Einstein equation to the right [8]. Modifications of gravity are indistinguishable from complex dark energy, at least at the linear level, unless constraints are put on the nature of dark energy, such as spatial homogeneity or zero coupling to matter (e.g. [8, 10] and references therein).

Why are we interested in proving these general but in a sense rather obvious statements about bias evolution? First, as we will see, there is an interesting and probably not so obvious corollary: that bias evolution can imprint a scale dependence even if the initial bias were scale independent. The effect is especially large if one's non-standard theory of gravity/dark energy introduces new scales to the problem, making the growth factor scale dependent. A promising way to test the current (general relativity + uniform dark energy) paradigm is therefore to look for a scale dependent bias. Our second (and orig-
nal) motivation is to address the issue of directly testing the Poisson equation. Most observations, such as gravitational lensing and the microwave background, tell us the metric fluctuations or gravitational potential, the left hand side of the Poisson equation. There are very few observations that tell us directly what sources the gravitational potential, the right hand side of the Poisson equation. The galaxy distribution provides one such handle, but it is complicated by the uncertain galaxy bias. One might hope that the tendency of the galaxy bias to relax towards unity could help bridge the gap between galaxies and mass. However, as we will see, in an accelerating universe, the bias relaxation process is quite slow. Modifications of gravity that predict cosmic acceleration tend to slow it further. The slow relaxation process makes it difficult to argue that galaxies that formed at high redshifts tend to have a trivial bias today, unless they formed with a bias close to unity. On the other hand, as we will see, the slow bias relaxation is a boon to the scale dependence test (motivation number one).

Two clarifying remarks are in order. First, throughout this paper, the galaxy bias is defined with respect to the matter fluctuations. It is important to bear this in mind since, in models that allow clustered dark energy, there are different kinds of galaxy bias: with respect to matter, dark energy or the sum. From the point of view of dynamical evolution, the bias with respect to matter is the natural one to consider. Secondly, an important assumption in \cite{1} as well as here is that the galaxy number is conserved, just as mass is. In practice, galaxies are of course created (they form) and destroyed (they merge). A complete theory of galaxy bias needs to fold in these processes. We will have more to say about this in \textsection VI. Dynamical evolution is in any case an important part of the story.

This paper is organized as follows. The basic equations are reviewed in \textsection II. The implications for the linear galaxy bias are worked out in \textsection III. The extension to a (linear) stochastic bias is presented in \textsection IV and the extension to a nonlinear bias is given in \textsection V. The interesting corollary of a possible scale dependent imprint on the galaxy bias is derived and discussed at the end of both \textsection III and \textsection IV. In \textsection VI we conclude with some remarks about testing theories of modified gravity or complex dark energy.

\section{Basics}

According to the current structure formation paradigm, the fundamental equations governing the dynamics of sub-Hubble matter fluctuations are \cite{12}:

\begin{equation}
\delta_m' = -\nabla \cdot [(1 + \delta_m) v_m] \tag{1}
\end{equation}

\begin{equation}
v_m' + (v_m \cdot \nabla) v_m + \frac{a'}{a} v_m = -\nabla \phi \tag{2}
\end{equation}

\begin{equation}
\nabla^2 \phi = 4\pi G \rho_m a^2 \delta_m \tag{3}
\end{equation}

where $\delta_m \equiv (\rho_m - \bar{\rho}_m)/\bar{\rho}_m$ is the matter overdensity, with $\rho_m$ being the matter mass density and $\bar{\rho}_m$ its spatial average, $v_m$ is the matter peculiar velocity, $a$ is the scale factor, $\phi$ is the gravitational potential or metric perturbation, $G$ is Newton’s constant, $'$ denotes a derivative with respect to conformal time $\eta$, and $\nabla$ denotes a derivative with respect to comoving position $x$.

Equations (1) and (2) express respectively mass and momentum conservation. Equation (3) is the Poisson equation, which tells us how matter fluctuations source the gravitational potential according to Newtonian/Einsteinian gravity.

Galaxies (or more precisely, some population thereof), once formed, obey the following equations on sub-Hubble scales:

\begin{equation}
\delta_g' = -\nabla \cdot [(1 + \delta_g) v_g] \tag{4}
\end{equation}

\begin{equation}
v_g' + (v_g \cdot \nabla) v_g + \frac{a'}{a} v_g = -\nabla \phi \tag{5}
\end{equation}

where $\delta_g \equiv (n_g - \bar{n}_g)/\bar{n}_g$ is the galaxy overdensity, with $n_g$ being the galaxy number density and $\bar{n}_g$ its spatial average, and $v_g$ is the galaxy peculiar velocity. Equation (4) crucially assumes number conservation, a subject to which we will return in \textsection VI. Equation (5) is momentum conservation, or the equation of motion, for the galaxies. It is worth noting that we do not assume $v_m = v_g$, as is often done. This is in part because we want to make explicit the importance of subjecting the galaxies and matter to the same force. If they are not subject to the same force, $v_m$ and $v_g$ would diverge from each other; if they are, the two velocities tend to converge, as we will see.

Equations (1) to (3) constitute the starting point for \cite{1, 2} on the evolution of bias. Our key observation is that Eq. (3) is unnecessary, at least on a qualitative level.

This is interesting in light of the recent interest in theories of modified gravity. By and large, these theories respect mass and momentum conservation; particles (matter or galaxy) move on geodesics. The only equation that is modified in these theories is therefore the Poisson equation (3). Similarly, in theories where dark energy has non-trivial clustering behavior, dark energy can act as an additional source for gravitational perturbations, and therefore the simple form of the Poisson equation given in (3) is again modified.

Henceforth, our derivation relies on Eqs. (1), (2), (4) and (5), but not on (3). In other words, our only assumptions are: that mass and number are conserved, and that matter and galaxies are subject to the same force.

\section{Linear Bias}

Keeping only terms linear in fluctuations, one can see from Eqs. (2) and (4) that

\begin{equation}
(v_g - v_m)' + \frac{a'}{a} (v_g - v_m) = 0 \tag{6}
\end{equation}

which implies

\begin{equation}
(v_g - v_m) = u_0/a \tag{7}
\end{equation}
where $\textbf{u}_0$ is independent of time but dependent on position. Note that in deriving this result, we do not need the right hand sides of Eqs. (2) and (3) to be the gradient of some potential; all we need is that they are identical i.e. matter and galaxies are subject to the same force. Equation (7) tells us that if $v_g = v_m$ initially, it will remain so; any initial velocity bias, should it exist, decays away.

This holds in the context of linear perturbation theory (i.e. large scales) [11]. In fact, [1, 2] set the galaxy and so; any initial velocity bias, should it exist, decays away.

Similarly, linearizing Eqs. (1) and (4) and taking the difference, we obtain

$$\left(\delta - \delta_m\right)' = -\nabla \cdot \left(v_g - v_m\right) = -\nabla \cdot \textbf{u}_0/a .$$  \hspace{1cm} (8)

This can be integrated to give

$$\delta - \delta_m = -\nabla \cdot \textbf{u}_0 \int \frac{da}{a^3} H + \Delta_0$$  \hspace{1cm} (9)

where $\Delta_0$ is independent of time but spatially dependent, and $H$ is the Hubble parameter. The integral over $a$ generally gives a decaying term in contrast with the constant $\Delta_0$ term, and so to leading order, the galaxy bias evolves as:

$$b(a) = \frac{\delta_g(a)}{\delta_m(a)} = 1 + \frac{\Delta_0}{\delta_m(a)}$$  \hspace{1cm} (10)

where we have inserted the argument $a$ in each quantity that is time dependent (while space dependence is kept implicit). The quantity $\Delta_0$ is independent of time, and can be alternatively expressed as

$$\Delta_0 = (b_0 - 1)\delta_m(a_0)$$  \hspace{1cm} (11)

where $b_0$ is the initial bias at some early time corresponding to scale factor $a_0$. From Eq. (10), we can see that as long as the matter fluctuation $\delta_m$ grows, the galaxy bias $b$ evolves towards unity. Exactly how rapidly it evolves depends on the growth rate of $\delta_m$, which cannot be determined unless the modification to Eq. (3), or lack thereof, is specified. For small fluctuations, it is not unreasonable to expect that $\delta_m$ grows roughly on the Hubble time scale.

There is an interesting corollary that follows from Eq. (10). Suppose the galaxy bias is initially scale independent. This means $\delta_g(a_0, x) = b_0\delta_m(a_0, x)$, where $b_0$ is the initial bias which is independent of position, and $a_0$ denotes the scale factor at some early time. Equations (10) and (11) then tell us for $a \geq a_0$:

$$b(a, x) = 1 + (b_0 - 1)\frac{\delta_m(a_0, x)}{\delta_m(a, x)}$$  \hspace{1cm} (12)

which means $b$ is independent of position, or is scale independent, only if $\delta_m(a, x)$ preserves exactly the initial spatial dependence encoded in $\delta_m(a_0, x)$. We know this holds in general relativity (plus uniform dark energy) at the level of linear theory in the sub-Hubble limit. One can define the bias $b$ in Fourier space as well, in which case Eq. (12) takes the form:

$$b(a, k) = 1 + (b_0 - 1)\frac{\delta_m(a_0, k)}{\delta_m(a, k)} .$$  \hspace{1cm} (13)

In the standard paradigm of general relativity plus uniform dark energy, different sub-Hubble Fourier modes of $\delta_m$ grow with the same (k independent) growth factor, hence an initially scale independent bias ($b_0$) will remain scale independent ($b$). Note that according to the standard paradigm, the initial linear bias is expected to be scale independent [13, 14, 15].

However, it is entirely possible that modifications to the Poisson equation (3) take a scale dependent form, which would make the perturbation growth scale dependent. General relativity itself introduces such corrections, but they are generally suppressed by $(aH/k)^2$ and are therefore very small except close to the Hubble scale where cosmic variance is large [10].

A less trivial example is a Yukawa-like modification of the Poisson equation: $\nabla^2 \rightarrow \nabla^2 - \mu^2$, where $1/\mu$ defines some new scale [12, 13]. Scalar-tensor theories introduce modifications of this kind where $\mu^2$ is related to the second derivative of the scalar potential [19], in addition to the better known effect of giving $G$ a time dependence [20, 21, 22]. In these cases, the linear growth factor $\delta_m(a, k)/\delta_m(a_0, k)$ will be $k$ dependent. As a concrete example, we show in the bottom two panels of Fig. 1 the linear bias as a function of redshift and wavenumber $k$ for two different populations of galaxies, with the following modification to the Poisson equation:

$$(\nabla^2 - \mu^2)^2 \phi = 4\pi G\rho a^2 \delta_m$$  \hspace{1cm} (14)

where $\mu$ is a constant and $a^2\mu^2$ plays the role of $\mu^2$. We use $\mu \propto a$ such that the Yukawa correction becomes small in the early universe [18], although some other time dependence is possible. There is also the question of how such a scalar-tensor theory can be made consistent with solar system constraints: this depends on the full Lagrangian of the theory - the kinetic term, the potential and the coupling to curvature can always be chosen to satisfy such constraints [23]. In Fig. 1 we adopt $m = 0.05$ h/Mpc which appears to be consistent with current observations [17, 18]. A flat cosmology of $\Omega_m = 0.27$ and $\Omega_L = 0.73$ is assumed in computing the background expansion rate.

From Eq. (14), one can see that the high $k$ modes ($k/a \gg m$) are insensitive to the Yukawa correction. Hence, the high $k$ end of Fig. 1 shows a bias evolution that is consistent with Einsteinian/Newtonian gravity. It is worth stressing that while the bias wants to evolve towards unity, it does not quite make it to unity. This is the result of cosmic acceleration: perturbation growth is much suppressed once the universe starts to accelerate, and the bias relaxation process is stalled. In fact, even if we continue the evolution into the future, the bias will not become very close to unity, unless its initial value were close to unity to begin with. According
FIG. 1: The evolution of the linear bias and relative bias as a function of wavenumber $k$. In the bottom panel is $b^A$, the linear bias of galaxies that formed at redshift 5 with an initial scale independent bias of $b_0 = 2$. In the middle panel is $b^B$, the linear bias of galaxies that formed at redshift 2 with an initial scale independent bias of $b_0 = 0.7$. The top panel shows the ratio of the two. The number labeling each curve is the corresponding redshift. This makes use of the Yukawa modification to the Poisson equation: Eq. (14).

FIG. 2: Analog of Fig. 1 except that a DGP motivated modification of growth rate is used: Eq. (15). Note that this does not account for the scale dependence arising from the transition from the scalar-tensor regime to the general relativity regime, the so-called $r_*$ effect.

We note in passing that the rather popular $f(R)$ theory is a scalar-tensor theory, and is therefore expected to exhibit a scale dependent bias qualitatively similar to that shown in Fig. 1. It appears, however, that the particular $f(R)$ theories usually considered in the literature have an effective mass $m$ (or $\mu$) that is rather small [6], which makes for a weak scale dependence. There is no fundamental reason, though, to focus only on these particular realizations of a scalar-tensor theory.

Another example of a modified gravity theory is DGP [5], which is expected to introduce scale dependence to the growth factor as well. There are two sources of scale dependence. One is that DGP is expected to give corrections to the growth factor of the order of $aH_0/k$, arising from its peculiar nonlocal character from the 4D perspective (Scoccimarro, private communication). The other is that DGP introduces a special scale, often referred to as $r_*$, where gravity makes a transition from the general relativity regime to a scalar-tensor regime [24]. One expects the structure growth to have a feature around $r_*$. Its precise value is however uncertain, because it depends on the exact nature and characteristics of the fluctuations. None of the existing calculations of structure growth in DGP gravity address growth in this transition regime [25]. The general expectation is that $r_*$ is close to the nonlinear scale, but its actual value could well be larger. As an illustration, in Fig. 2 we sidestep the $r_*$ issue, and simply modulate the DGP growth by an $aH_0/k$.
correction (see also [16]):

$$\delta_{m}^{\text{DGP}}(a, k) = \delta_{m}^{\text{Poisson}}(a, k)(1 - aH_0/k)$$

(15)

where $\delta_{m}^{\text{Poisson}}$ denotes the matter fluctuation according to the Poisson equation, with a background expansion chosen to match the expected DGP growth on small scales (but larger than $r_\ast$) [26]. Here, the scale dependence of the bias is much weaker than in the Yukawa example, though one must remember that this ignores the $r_\ast$ effect. As mentioned before, general relativity itself introduces corrections of this sort, but they come in at quadratic order $(aH/k)^2$ and are even weaker.

Lastly, instead of modifying the Poisson equation [3] on the left hand side, one could also modify it on the right hand side by having dark energy that clusters. This would introduce new scales to the problem, such as the Jeans scale, and one expects the growth of perturbations, and therefore the bias relaxation, to become scale dependent.

We should add that in theories where gravity is modified, or clustered dark energy is introduced, it is quite possible that the galaxies are born with a scale dependent bias on large scales, contrary to expectations according to the standard excursion set theory of halos [13, 14]. It would be very surprising if any such initial scale dependence conspires to cancel the scale dependence that develops dynamically. Checking for departure from a scale independent bias on large scales therefore constitutes an interesting test of (scale dependent) modifications to the Poisson equation [3]. One way to perform this test is to compare the galaxy power spectrum and the mass power spectrum from lensing (the latter requires the Poisson equation to convert from metric to mass fluctuations). A simpler test is to compare the large scale bias of different populations of galaxies, which are at different stages of dynamical bias relaxation. For instance, labeling the two populations of galaxies by $A$ and $B$, their relative bias is

$$b^A(a, k) = 1 + (b^A_0 - 1)\delta_m(a^A, k)/\delta_m(a, k)$$

$$b^B(a, k) = 1 + (b^B_0 - 1)\delta_m(a^B, k)/\delta_m(a, k)$$

(16)

for $a \geq a_0$. A ratio of this kind can be obtained from the ratio of the observed power spectra of $A$ and $B$. The top panels of Fig. 1 and 2 illustrate this ratio for two particular populations of galaxies. It is worth noting that the best chance for observing a scale dependent relative bias is to compare overbias (large) galaxies with underbias (small) galaxies. This is because modified gravity theories tend to predict a higher bias for the overbiased galaxies, and a lower bias for the underbiased galaxies. This effect gets stronger at lower $k$'s; the scale dependence is most visible in their relative bias. So far, there is no observational evidence for departure from scale independence on large (i.e. linear) scales [27].

IV. (LINEAR) STOCHASTIC BIAS

Having a stochastic bias means one introduces an additional parameter into the problem, following Dekel & Lahav [28]. In addition to $b$, defined by

$$b^2 = \langle \delta_g^2 \rangle / \langle \delta_m^2 \rangle$$

(17)

one introduces $r$, defined by

$$br = \langle \delta_g \delta_m \rangle / \langle \delta_m^2 \rangle$$

(18)

Here, one can think of the ensemble averages in several different ways. For instance, $\langle \delta_g \delta_m \rangle$ can be thought of as the ensemble average of the product of $\delta_g$ and $\delta_m$, each smoothed on some given scale $R$. It can also be thought of as the two point correlation function of $\delta_g$ and $\delta_m$ at some given comoving separation $\Delta x$. Another alternative is to think of it as the power spectrum at a wavenumber $k$: $\propto [\langle \delta_g(k) \delta_m^\ast(k) \rangle + \langle \delta_g^\ast(k) \delta_m(k) \rangle]/2$. Depending therefore on the definition, $b$ and $r$ are in general functions of $R$, $\Delta x$ or $k$ respectively. It is worth noting that cross-correlation measurements of galaxies suggest $r$ is consistent with unity on large scales [27].

Squaring Eq. (9), and keeping only the leading contribution to the ensemble average, we have

$$b^2 - 2br + 1 = \langle \Delta_0^2 \rangle / \langle \delta_m^2 \rangle$$

(19)

where $\langle \Delta_0^2 \rangle$ is time independent, but $\langle \delta_m^2 \rangle$ is not. Alternatively, multiplying Eq. (9) by $\langle \delta_m^2 \rangle$, and keeping only the leading contribution:

$$br - 1 = \langle \Delta_0 \delta_m \rangle / \langle \delta_m^2 \rangle$$

(20)

The evolution of the linear galaxy bias $b$ and the cross-correlation coefficient $r$ is therefore given by:

$$b = \left[ 1 + 2\langle \Delta_0 \delta_m \rangle / \langle \delta_m^2 \rangle + \langle \Delta_0^2 \rangle / \langle \delta_m^2 \rangle \right]^{1/2}$$

(21)

$$r = \frac{1}{b} \left[ 1 + \langle \Delta_0 \delta_m \rangle / \langle \delta_m^2 \rangle \right].$$

(22)

Recall that $\Delta_0$ is independent of time (Eq. 9). Assuming the matter fluctuation $\delta_m$ grows, we can see that at late times:

$$\langle \Delta_0^2 \rangle / \langle \delta_m^2 \rangle < \langle \Delta_0 \delta_m \rangle / \langle \delta_m^2 \rangle \ll 1$$

(23)

and therefore at late times:

$$b \sim 1 + \langle \Delta_0 \delta_m \rangle / \langle \delta_m^2 \rangle$$

(24)

$$r \sim 1 - \frac{1}{2} \langle \Delta_0 \rangle^2 / \langle \delta_m^2 \rangle + \frac{1}{2} \left[ \langle \Delta_0 \delta_m \rangle / \langle \delta_m^2 \rangle \right]^2$$

(25)

and so, $b \to 1$ and $r \to 1$, as expected, with $r$ tending to unity at a faster rate than $b$, although, as emphasized
before, cosmic acceleration slows the approach to unity considerably.

Our remarks in §III concerning the possible development of a scale dependence to the large scale galaxy bias apply to $b$ and $r$ here as well. In other words, the quantities that control the evolution of $b$ and $r$, $(\Delta_0 \delta_m^0)/(\delta_m^0)^2$ and $(\Delta_0^2)/(\delta_m^2)$, can develop a scale dependence (on $R$, $\Delta x$ or $k$), due to possible scale dependent modifications of the Poisson equation.

V. NONLINEAR BIAS

Fry & Gaztañaga [29] introduced a systematic expansion that relates the galaxy and matter fluctuations:

$$\delta_g = b \delta_m + \frac{b_2}{2} \delta_m^2 + \ldots$$

To find the evolution of $b_2$, we need to keep terms up to second order in fluctuations. Assuming gradient flow i.e. $v_m = \nabla \phi_m^v$ and $v_g = \nabla \phi_g^v$, Eqs. (2) and (5) tell us that

$$\left( \phi_g^v - \phi_m^v \right) + \frac{\alpha'}{a} \left( \phi_g^v - \phi_m^v \right) + \frac{1}{2} \nabla (\phi_g^v - \phi_m^v) \cdot \nabla (\phi_g^v + \phi_m^v) = 0.$$  \hspace{1cm} (27)

To linear order, this equation implies what we already know from Eq. (4), i.e. $(\phi_g^v - \phi_m^v)^{(1)}$ decays like $1/a$, where the superscript $(1)$ denotes the order we are considering. Whether $(\phi_g^v - \phi_m^v)^{(2)}$ decays depends on the behavior of $(\phi_g^v + \phi_m^v)^{(1)}$ which depends on how the Poisson equation is modified. It can be shown that $(\phi_g^v - \phi_m^v)^{(2)}$ decays away if $(\phi_g^v + \phi_m^v)^{(1)}$ grows slower than $\alpha'$. Henceforth, we will assume so, following Fry & Gaztañaga who essentially assumed $v_g = v_m$ from the beginning.

Equation (1) can be rewritten, up to second order, as

$$- \nabla \cdot v_m(x) = \delta_m(x)$$

$$- \int \frac{d^3k}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} e^{i(k \cdot x)} \frac{i \cdot k}{k^2} \delta_m(k) \delta_m(\hat{k} - \hat{k}) $$

where all time dependence is left implicit, and $k$ and $\hat{k}$ denote wavevectors. We have abused the notation a bit by using $\delta_m$ to stand for both the real space and Fourier space counterparts, distinguishing them purely by their arguments. A similar equation holds for the galaxies as well. Assuming the difference $v_g = v_m$ decays away to second order (or is zero to begin with), we have therefore

$$[\delta_g(x) - \delta_m(x)]' = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} e^{i(k \cdot x)} \frac{i \cdot k}{k^2} \delta_g(k) \delta_g(\hat{k} - \hat{k})$$

$$[\delta_g(k) \delta_g(\hat{k} - \hat{k}) - \delta_m(k) \delta_m(\hat{k} - \hat{k})]$$

which holds to second order. Expanding $\delta_g$ as in Eq. (26), the first order terms in the above expression tell us that $(b - 1)$ decays like $1/\delta_m^{(1)}$, consistent with the finding in §III. The second order terms go like

$$[\frac{b_2}{2} \frac{\delta_m^{(1)}}{\delta_m^{(2)}}]^2 + O[1]$$

where we have suppressed the $x$ dependence on the left hand side, and the time dependence on both sides. Here, $\Delta_0^{(1)}$ is the Fourier counterpart of $\Delta_0$ in Eq. (10), and is therefore a first order quantity, and denoted as such. Note also that it is time independent.

It is straightforward to see that, to leading order:

$$b_2 = O[\delta_m^{(2)}/\delta_m^{(1)}] + O[1]$$

One generally expects that $\delta_m^{(2)} \sim \delta_m^{(1)}$, and therefore $b_2$ tends towards zero at late times as long as $\delta_m^{(1)}$ grows. The same caveat applies as before: cosmic acceleration significantly slows the relaxation of $b_2$.

It is noteworthy that $b_2$ develops a scale dependence if it started off scale independent. This is true even if the Poisson equation holds. The stability of the relaxation of $b_2$.

VI. DISCUSSION

Our results on the evolution of bias are summarized by Eq. (11), (12), or (13) for the linear bias, by Eqs. (21) and (22) for the (linear) stochastic bias, and by Eq. (31) for the nonlinear bias. The Poisson equation is not assumed in deriving these results, therefore leaving open the possibility that gravity is modified or new sources of fluctuations (such as clustered dark energy) exist. The only assumptions made are mass and number conservation, and that matter and galaxies are subject to the same force. For the linear bias and the stochastic bias, it is not even necessary that the force be the gradient of some potential.

Following these assumptions, our main conclusions are: (1) the bias tends to relax towards unity, but this relaxation is significantly slowed by the diminishing structure growth once cosmic acceleration commences; (2) the correlation coefficient $\gamma$ tends towards unity faster than the bias $b$ does; (3) the large scale (linear) bias dynamically develops a scale dependence if modifications to the Poisson equation are scale dependent.

Conclusion (1) implies, everything being equal, galaxies that formed at higher redshifts tend to have a more relaxed bias today. The stalling of bias relaxation by cosmic acceleration, however, implies we cannot automatically conclude that such galaxies have close to unity bias today, unless they had an initial bias that was fairly close to unity. In other words, one cannot freely approximate $\delta_m \sim \delta_g$ by appealing to bias relaxation alone.
This is unfortunate, because $\delta_m$ is about the only observational handle we have on $\delta_m$ without assuming the Poisson equation. Other observations, such as the microwave background and gravitational lensing, generally probe the metric perturbations, i.e. the left hand side of the Poisson equation [31]. A direct test of the Poisson equation requires independent knowledge of $\delta_m$, i.e. the right hand side. It appears one has to be content with indirect tests via the linear growth factor, such as those widely discussed in the literature [32]. They test for how $\phi$ changes with time, which the system of equations (1), (2) and (3) predicts. The relation between $\phi$ and $\delta_m$ is tested only in so far as the prediction for linear growth from this whole system of equations is tested. To go beyond this, one would need some independent method for constraining the galaxy bias. Standard methods to do so carry additional baggage. Methods using higher moments, for instance, assume Gaussian initial conditions as well as the Poisson equation itself.

Conclusion (3) is, in our view, the most interesting one. It suggests a simple way to test for scale dependent modifications to the Poisson equation, which are expected in many existing theories of modified gravity or clustered dark energy (see [33]). One can measure the relative large scale (linear) bias of two different populations of galaxies which are at different stages of bias relaxation (Eq. [16]). The standard paradigm of general relativity + uniform dark energy predicts a scale independent linear bias (both at birth and in its subsequent evolution). Checking for departures from scale independence therefore constitutes a test of the paradigm. As illustrated in the top panels of Fig. 1 and 2, this test is best carried out by comparing overbiased and underbiased galaxies. It is also important to focus on large scales, since the bias is expected to develop scale dependence on nonlinear scales.

It was recently noted by [34] that primordial non-Gaussianity could give rise to a scale dependent large scale bias as well. Distinguishing this possibility from modifications to the Poisson equation should be feasible since there are other more direct ways to test for primordial non-Gaussianity, such as from the microwave background anisotropies [34].

As emphasized in [1] our calculation of the bias evolution assumes galaxy number conservation, just as in [1]. Ultimately, one would like to go beyond the treatment in this paper, and formulate a complete theory of halo bias accounting for formation, mergers and dynamics, extending the excursion set theory [13, 14] beyond the standard (general relativity + uniform dark energy) paradigm. Such a theory would depend on the details of how the Poisson equation is modified. It is interesting to note that the standard excursion set theory predicts a bias evolution that is consistent with the one derived here: $b(a) - 1 \propto 1/D(a)$ where $D$ is the linear growth factor (the halos are identified at some scale factor $a_0 < a$). This suggests our simple dynamical calculation already captures much of the relevant physics. An additional bonus of extending the excursion set theory beyond the standard paradigm is that one could address the issue of whether galaxies/halos could be born with a scale dependent bias in non-standard theories. We hope to pursue these issues in the future.

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These observations actually measure a combination of the two different scalar perturbations $\Phi$ and $\Psi$, which are related to the gravitational potential $\phi$ only if additional assumptions are made about their relationship. Here, the metric is $ds^2 = a^2[-(1 + 2\Psi)d\eta^2 + (1 - 2\Phi)|dx|^2]$. Note that $\phi = \Phi$ in the Poisson equation, while $\phi = \Psi$ in the momentum conservation equation.

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