Short range correlations in nuclei and nuclear matter

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Abstract. Short range correlations in nuclei and nuclear matter arise from the strong repulsion of the nuclear interaction at short distances. The existence of these correlations has been confirmed in a variety of nuclear experiments with electroweak and hadronic probes. The theoretical and experimental efforts in the last decade have focused on quantifying short-range correlations and, in particular, their isospin dependence. Here, I will analyze and summarise some of the recent conclusions in the literature using as a baseline a theoretical method, based on many-body Green’s functions theory, and a single quantity, the one-body momentum distribution.

1. Introduction

Microscopic interactions between two nucleons that account realistically for the scattering properties and for the deuteron properties have been constructed based on a variety of models [1]. These models differ in their underlying physics ingredients, but also share common features. Among these common attributes, one identifies a repulsive short-distance repulsive core, that accounts for the strong energy dependence of the singlet channel scattering phase-shifts, and a tensor component that can describe the quadrupole moment of the deuteron [2] and other light nuclei properties [3]. A short range core has also been identified in the nucleon-nucleon (NN) interaction with lattice QCD calculations, as the pion mass approaches its natural value [4, 5].

A natural consequence of the repulsive short-range core is the existence of correlations at short distances in the many-body nuclear wavefunction [6, 7]. Short-range correlations (SRCs) are also responsible for a substantial fragmentation of single-particle strength and for the population of high-momentum states. Further non-perturbative effects associated to the tensor component of the NN force also generate SRCs [9], in a way that is difficult to model theoretically in standard mean-field or shell model approaches.

SRCs have been explored in a variety of nuclear physics experiments and facilities. Pioneering (\(e, e'p\)) experiments on stable isotopes drew a picture of nuclear structure that is, to a certain extent, mass-independent [10]. In a mean-field or shell-model picture, orbitals below (above) the Fermi surface are fully occupied (completely empty). In the picture arising from electron scattering, instead, single-particle strength is fragmented, and valence nucleons right below the Fermi surface are 70% occupied. The remaining strength is shifted above the Fermi surface by SRCs. Different kinematical coverages allowed for a new generation of (\(e, e'p\)) studies in the early 2000s that yielded a direct measurements of nuclear strength in the correlated regions [11].

Several theoretical methods have been used over time to explain this behaviour, including variational [8] [12, 13], diagrammatical [14] and, recently, quantum Monte Carlo [15] approaches.
Theoretical work in SRCs is hampered by the very non-perturbative nature of NN forces but also by the difficulties associated to identifying unique signatures of beyond mean-field correlations that are scale- or model-independent [16]. All of these complications bring in a series of systematic uncertainties that need to be explored in detail [17].

The last decade has admittedly seen a renaissance of SRCs in nuclear physics. This has been driven by several experimental developments that have focused on the microscopic origin and the quantitative nature of SRCs, based on two-nucleon knock-out experiments at Jefferson Laboratory. A series of initial experiments both in proton- and electron-induced reactions at high momenta have unambiguously demonstrated that neutron-proton (np) pairs, rather than proton-proton or neutron-neutron pairs, are preferentially knocked out of carbon targets at high energies and momenta above the Fermi momentum [18, 19]. These pairs have small center-of-mass momentum, emitted in back-to-back kinematics and account for about 80% of the two-nucleon knock-out events. Further electron scattering research in heavier targets confirmed this result [20]. The observed np dominance of SRCs has important consequences for the physics of isospin-imbalanced nuclei [21, 22]. It also provides support for a tensor-mediated origin of SRCs [3] with implications across the nuclear chart [23] and into neutron-star physics [24, 25, 26].

In this contribution, I will quantify briefly the effect of SRCs in nuclear systems, by looking at specific calculations in infinite nuclear matter. The results obtained in this system mimic those in nuclei and serve as a useful guidance tool for the understanding of the observed experimental effects. I will focus the discussion around the isospin dependence of SRCs and draw some conclusions that can be relevant for a wide range of nuclear physics phenomena, ranging from structure to astrophysics.

2. Short range correlations and momentum distributions

The first problem one meets when considering SRCs either theoretically or experimentally is their formal definition. On most theoretical work, correlations are customarily defined with respect to a given reference state that provides a starting point, an “uncorrelated state” which offers a simplified view of the many-body problem. In this sense, behaviours that depart from the reference picture can be characterised as “correlations”. In experiments, the task of identifying correlated states or regions is even more difficult, as there is no equivalent of a “reference” baseline state. In this sense, theoretical guidance is key to experiments in order to identify regimes, observables and analysis schemes that can quantify SRCs adequately.

In the following, I use an operational definition of SRCs in order to focus the discussion. In my view, SRCs are any phenomena occurring at sub-femtometer scales (or at momenta approximately above \( \approx 400 \text{ MeV} \)) that cannot be naturally explained in an independent particle or mean-field picture. I will illustrate my key points using calculations based on infinite nuclear matter rather than finite nuclei. Infinite matter is an idealized theoretical system consisting of nucleons at fixed density, \( \rho \), interacting via the strong force only. This idealised system contains all the physically relevant ingredients of SRCs, but is easier to tackle from a microscopic perspective. First, in an infinite system, momentum is a good quantum number and the single-particle problem, composed of plane waves, is automatically solved for. The discussion can thus be neatly focused around momentum space and the corresponding regimes below or above the Fermi momentum, \( k_F \). Second, theoretical and experimental studies demonstrate that SRCs scale mildly with mass number, so the quantitative properties of SRC in the infinite bulk system are very similar to those in a finite nucleus [8] [27]. Third, infinite matter can be studied as a function of number density, \( \rho \), and isospin asymmetry, \( \eta = \frac{\rho_n - \rho_p}{\rho} \), providing direct access to
Figure 1. Momentum distribution at saturation density obtained with different NN interactions in the SCGF approach. The left panel shows the distribution in a linear scale as a function of \( k/k_F \), with \( k_F \) the Fermi momentum. The right panel shows the same results in a logarithmic scale and as a function of momentum, \( k \), in units of MeV.

Isovector effects that are not influenced by surface degrees of freedom [17, 21]. Finally, when approaching the \( \eta \sim 1 \) regime of very neutron-rich matter, theoretical calculations can be used to predict the corresponding properties of neutron-star cores [25, 28], of relevance for astrophysics.

The single-particle momentum distribution in infinite matter, \( n(k) = \langle a_k^{\dagger} a_k \rangle \), provides a particularly clean theoretical measure of SRCs. In a non-interacting free Fermi gas (FFG) or even in a mean-field approach the momentum distribution \( n(k) \) is 1 (0) below (above) the Fermi surface. The left panel of Figure 1 shows \( n(k) \) as a function of the momentum over the Fermi momentum, \( k_F = (3\pi^2/2)^{1/3} \). The dashed double-dotted (red) line illustrates the FFG behaviour. The remaining curves correspond to calculations performed with one specific many-body method - the so-called self-consistent Green’s functions (SCGF) approach. This incorporates diagrammatically beyond-mean-field correlations that account for nucleon scattering in the medium, including both short-range-core and tensor effects [17, 27, 29]. For the calculations displayed in Fig. 1 finite temperature simulations at a constant fixed density of \( \rho = 0.16 \text{ fm}^{-3} \) have been extrapolated to zero temperature [30]. Two sets of results correspond to the traditional mid-1990s high-quality phase-shift equivalent phenomenological potentials Av18 (solid line) [31] and CD-Bonn (dashed line) [32]. The plot also shows results for the N3LO Entem-Machleidt chiral potential [33] (dotted line). One of the key advantages of modern chiral interactions is that they can consistently predict associated many-body forces [34]. The results provided in the dashed-dotted line in Fig. 1 have been obtained incorporating three-nucleon forces in the SCGF method [35, 36].

The two panels in Fig. 1 provide a summary of the features of the typical momentum distribution in nuclear matter and nuclei. Compared to the FFG, the distribution is depleted below the Fermi surface to about 85 – 90%, depending on the interaction. This depletion is broadly speaking momentum independent, and only close and below the Fermi surface \( n(k) \) has a visible momentum dependence that brings \( n(k) \) further down. The discontinuity of \( n(k) \) at \( k = k_F \) is a characteristic of normal fermionic systems [37]. Right above \( k_F \), the momentum distribution decays with momentum in a power-law-like fashion. This is highlighted in the right panel of Fig. 1 which shows a semilogarithmic plot of \( n(k) \). The high-momentum tail extends to high momenta, into the GeV region, with components that reflect the associated short-range

\(^2\) These two models serve as “reference” states in this case.
structure of the NN potentials. Hard semilocal potentials like Av18 yield more strength at high momenta than softer non-local forces, like CDBonn. N3LO is constructed with an explicit cutoff at $\Lambda = 500$ MeV in relative momentum, which translates into a fast decay of $n(k)$ above $k \approx 600$ MeV. While the structure of $n(k)$ reflects the NN force under consideration, it is still an open question whether any physical observable can be directly associated to such momentum distributions which are, strictly speaking, not an observable [38].

The momentum distribution $n(k)$ reflects the available single-particle strength at any given momentum, $k$. This single-particle strength is however fragmented over a range of energies by SRCs, and this fragmentation can be directly described in terms of the so-called spectral function, $A_k(\omega)$ [37, 39], which provides a measure of the probability of finding a nucleon with a given momentum and energy. Fig. 2 shows a density plot of $A_k(\omega)$ computed using the SCGF method at $\rho = 0.16$ fm$^{-3}$ for the Av18 interaction. The single-particle strength extends to regions at high momenta and both positive (addition) and negative (removal) energies. These regions of spectral strength have been directly accessed with $(e,e'p)$ high-momentum transfer and parallel-kinematics experiments in $^{12}$C [11], and in heavier isotopes [40]. The strength measured experimentally compares well with theoretical calculations based on other many-body models.

In an independent-particle or mean-field reference state, there is only a one-to-one correspondence between single-particle energy, $\omega$, and momentum, $k$, described in terms of a so-called dispersion relation, $\omega = \varepsilon_k$. This relation is illustrated by the solid (red) line in Fig. 2. In such reference states, for a given value of momentum, all the single-particle strength is concentrated on a single value of energy. Consequently, the experimental observation of single-particle strength in an otherwise inaccessible kinematical regions is direct evidence of the existence of SRCs in the nuclear many-body wavefunction.

3. Isospin dependence of short-range correlations
The momentum distribution $n(k)$ describes qualitatively the effects of fragmentation on single-particle orbitals observed in nuclear experiments [10]. Recent efforts in electron scattering experiments have veered towards the analysis of the isospin dependence of SRCs by looking at nucleon knock-out on nuclei that have non-zero isospin asymmetry, $\eta \neq 0$. Data analysed by the CLAS collaboration has found substantial effects in both the majority neutrons and minority protons in neutron-rich nuclear systems [20, 41, 42]. Fig. 3 shows the momentum distribution of...
neutrons (left) and protons (right panel) in infinite matter at a fixed density of $\rho = 0.16 \text{ fm}^{-3}$. The isospin asymmetry $\eta$ changes from $\eta = 0$ (signalling an $N = Z$ symmetric system) to $\eta = 1$ (neutron matter) at this fixed density. Again, these results are obtained with the SCGF method using an N3LO NN force. The 3NFs used here are an extension of the work in Refs. [35, 36] to isospin asymmetric systems. Other calculations employing different many-body techniques and nuclear interactions achieve quantitatively similar results [43, 44].

The results of neutrons in the left panel show two distinctive features. On the one hand, the discontinuity at the Fermi surface of $n(k)$ is found at lower momenta as $\eta$ increases, in agreement with the expectations of a FFG. On the other hand, the depletion of neutron momentum distributions at low momenta decreases with asymmetry. In other words, at fixed density, as systems become more neutron-rich, neutrons become less correlated. Whereas in symmetric matter neutrons are depleted by $\approx 10\%$, in neutron matter the depletion is closer to 5%. This is expected on the basis of the decrease of available tensor-correlated np pairs $\eta$ increases [17, 45]. The effects for protons are complementary. The minority species becomes more correlated as the imbalance increases. At high momenta, the tails of both momentum distributions (not shown here) are proportional to the density of the species.

The dominant role of np pairs in correlated configurations of two nucleons has been identified clearly in electron two-nucleon knock-out experiments in stable isotopes, in the so-called np-dominance picture [20, 42]. Integrating knock-out cross section ratios over regions of energy and momenta, one has access to probability ratios in given kinematic regions. These, in turn, can be interpreted as integrals over specific momentum regions of the corresponding momentum distributions [40]. The recent analysis by the CLAS collaboration indicates a strong linear dependence with isospin imbalance of the low-momentum strength, qualitatively (but not yet quantitatively) consistent with the variation of strength displayed in Fig. 3. In contrast, the mild dependence on isospin asymmetry is reminiscent to the experimental results obtained in knock-out in quasifree kinematics [47], although a theoretical connection between these two types of experiments is still not developed [48].

A direct comparison of the results in Fig. 3 to experimental data is also hampered for two reasons. First, the results are computed at a fixed density, thus ignoring both the changes in density within a single nucleus, and the evolution of local density for different nuclei. Second, while an average isospin asymmetry is easily defined in nuclei, local regions can have isospin asymmetries that differ substantially from the average value, like neutron skins. This non-trivial interplay between density and isospin asymmetry across the interior of nuclei may play an important role in linking theory and experiments. There are very few existing calculations...
that consistently consider these two aspects in the literature [8, 46, 49, 50].

A different consideration has to do with the typical size of $\eta$ in nuclei. Recent conclusions on the isospin dependence of SRCs have been drawn by analysing the evolution of experimental data from a balanced system, like $^{12}$C ($\eta = 0$), to stable, heavy, but only mildly isospin imbalanced systems, like $^{208}$Pb ($\eta = 0.2$). One would like to access systems with higher values of $\eta$, to explore fully the isospin dependence of SRCs. There are two different ways to do this. On the one hand, radioactive beam facilities have the ability to produce both proton and, especially, neutron-rich isotopes. The challenge there is to find experimental telltale signs of SRCs in such short-lived isotopes. Hadronic knock-out reactions have been the candidates of choice so far [47, 51]. On the other hand, the interior of neutron stars are likely to be the most neutron-rich environments in nature. Global properties of neutron stars, like masses and radii, are relatively insensitive to SRCs [28]. There are other observations of neutron stars, however, that depend sensitively on nuclear in-medium properties. Both glitching phenomena and neutrino cooling are determined by superfluid pairing gaps, for instance [52]. These can decrease substantially when single-particle strength around the Fermi surface is fragmented [30, 53]. Working astrophysical models incorporating these microscopic effects are missing, but future comparison with astrophysical data may shed light on SRCs in extremely neutron-rich environments.

4. Future outlook

The field of SRCs in nuclear physics is driven by experimental results. Electron scattering experiments were key in the initial stages of the field, and helped characterise the evolution of single-particle strength with mass number [10, 54]. Recent electron two-nucleon knock-out experiments have also helped characterise our understanding of the isospin dependence of SRCs [9]. The continuation of the successful experiments at the upgraded Jefferson Laboratory facility will no doubt provide further insight, by extending the isotopic reach of the experiments.

While hadronic reactions are more difficult to interpret theoretically, they bear the promise of reaching a much more exotic set of isotopes than traditional electron scattering experiments. The R3B program at FAIR [47, 55] and recent ($p, pd$) experiments at Osaka [56] are examples of recent insightful results. The future of these experimental programs is essential to continue to understand the evolution and structure of SRC in stable and exotic isotopes.

SRCs permeate all of nuclear physics. Efforts in understanding quantitatively the properties of SRCs are essential, because several domains are directly affected by the associated uncertainties. These include traditional nuclear physics endeavours, like spectroscopy and reaction theory, but a quantitative understanding of SRCs is also needed in the calculation of nuclear matrix elements of relevance for neutrinoless double decay [57]; neutrino scattering for oscillation experiments [58, 59] and, in some cases, dark matter studies [60]. SRCs can also affect our understanding of high-energy proton-ion and heavy-ion reactions [61] and are also connected to fundamental physics problems like the EMC effect [62]. Most of the time, it is difficult to isolate their effects because they depend on the scale and computational scheme used to describe theoretical models. Disentangling their relevance in experimental data is a challenge facing nuclear theory in the next few years.

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