Radiation from Secondary Planar Surfaces Sources in Quantum Field Theory

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Abstract. In quantum optics (optical coherence) theorem, many sources are employed in the laboratory are secondary planar sources. A source of this kind is usually an aperture in an opaque planar surface screen, illuminated either directly or via an optical system with primary sources. The expression following into the formulation of the radiation in radiant intensity which, analytical solution has not given the time evolution of amplitude. In this research, we consider this problem in quantum field theory (QFT) viewpoint. By using the method of study based on a configuration space for explaining characteristic of the complete process, which begins and ends with the vacuum state so-called vacuum-to-vacuum transition amplitude between emitters and detectors. Propose in this research, we attend and explain the radiation for radiant intensity with time evolution process. To calculation amplitude transition of massive quantum particles propagator stimulated emission by secondary planar surface sources in space-time. Finally, we use the mathematical program for corresponding numerical evaluations between quantum optics and quantum field theory situation.

1. Introduction

In quantum optics (optical coherence) theorem, many sources are employed in the laboratory are secondary planar sources. A source of this kind is usually an aperture in an opaque planar surface screen, illuminated either directly or via an optical system with a primary source [3].

![Secondary planar surfaces sources on opaque screen](image)

(a) Circular aperture (b) Rectangular aperture (c) Crossed aperture

Figure 1: Secondary planar surfaces sources on opaque screen (a) Circular aperture (b) Rectangular aperture (c) Crossed aperture
Representation of the source distribution as a correlation Green’s function in the space-frequency domain $\tilde{G}(\vec{r} - \vec{r}', \nu)$, is given by

$$W(\vec{r}, \nu) = \int J(\vec{r}, \vec{r}') \tilde{G}(\vec{r} - \vec{r}', \nu) d^3 r', \quad (1)$$

where

$$\tilde{G}(\vec{r} - \vec{r}', \nu) = \frac{k^2}{\pi^2} \left( \sin \frac{k(\vec{r} - \vec{r}')}{c} \right), \quad k = \frac{2\pi\nu}{c}. \quad (2)$$

The expression following into the formulation of the radiation in radiant intensity which, analytical solution has not given the time evolution of amplitude [5].

Propose in this research focus on radiation from secondary planar surface sources of a crossed aperture. We attend and explain the radiation for radiant intensity together with Feynman's so-called spin-zero electron are technically called Klein-Gordon particles of finite mass [2, 12]. To calculation amplitude transition of the massive spin zero particle propagator stimulated emission by secondary planar surface sources of a crossed aperture in space-time. Finally, we use the mathematical program for corresponding numerical evaluations between quantum optics and quantum field theory situation.

2. Vacuum-to-Vacuum Transition Amplitude in The Presence of Source

We already obtained the transition amplitude

$$\langle q_f, t_f | q_i, t_i \rangle = N \int Dq \exp \left[ \frac{i}{\hbar} \int_{t_i}^{t_f} dt L(q, \dot{q}) \right] \quad (3)$$

with the boundary conditions $q(t_f) = q_f$, $q(t_i) = q_i$. However, what we really want to know in quantum mechanics is that particles are first created, interaction each other and finally destroyed by the measurement. Therefore, using "source" to create particles, we need vacuum-to-vacuum transition amplitude in the presence of a source [1]. If we note the vacuum state with a source by $|0, t\rangle$, we define the vacuum-to-vacuum transition amplitude as

$$Z[J] \equiv \langle 0, \infty | 0, -\infty \rangle. \quad (4)$$

We assume the source $J(t)$ is non-zero only between and $t' \ (t < t')$. We introduce $T$ and $T'$ for $T < t$ and $t' > T'$ the amplitude becomes

$$\langle Q'T' | QT \rangle^J = \int dq'dq \langle Q'T' | q' \rangle \langle q' \rangle | q \rangle \langle q | QT \rangle \quad (5)$$

$$\langle Q'T' | q' \rangle = \sum_m \phi_m(Q') \phi^*_m(q') \exp \left[ \frac{i}{\hbar} \frac{E_m(t' - T')}{} \right] \quad (6)$$

and

$$\langle q | QT \rangle = \sum_n \phi_n(q) \phi^*_n(Q) \exp \left[ -\frac{i}{\hbar} \frac{E_n(t - T)}{} \right] \quad (7)$$

Taking the limit $T' \to \infty$ and $T \to -\infty$ in equation (5) we found only the ground state (vacuum) contributes to $\langle Q'T' | QT \rangle^J$.
\[
Z[J] \equiv \langle 0, i\infty | 0, -i\infty \rangle^J = \lim_{T' \to -\infty, T \to \infty} \frac{\langle QT' | QT \rangle^J}{\phi_i^*(Q) \phi_i(Q') \exp \left[ -\frac{i}{\hbar} E_i(T' - T) \right]}
\]

\[
Z[J] = \int dq d\varepsilon \phi_i^*(q', t') \langle q' | q | t \rangle^J \phi_i(q, t)
\]

(8)

Where \( dq \) is a differential of components in unitary space time given development is then given completely by the propagator or Green’s function [4] thus

\[
G(q', q t) \equiv \langle q' t | q t \rangle^J
\]

(9)

3. The Configuration Space for Explaining Characteristic of the Space-Time Domain

In quantum field theory (QFT) operates is based on concept in special relativity (SR). We characterize space-time by an event, which is something that happens in time \( t \) and location \( (x, y, z) \). The speed of light \( c \) can serve in a role as a conversion factor, transforming time into space. Space and time, therefore, from a configuration space and we denote coordinates by \((ct, x, y, z)\). We take \( x_{ct} = 0 \) and \( (x, y, z) \to (x', x^2, x^3) \). Then an event in space-time is labelled by the coordinates of a vector \( x^\mu = (x^0, x^1, x^2, x^3) \).

The quantum mechanics characterization of the complete process, which begins and ends with the vacuum state has the typical for m vacuum-to-vacuum transition amplitude in the presence of the external source \( Z[K(x)] \) (the effect of external forces acting on the particle) [6].

\[
Z[K(x)] \equiv \langle 0, 0 \rangle^{K(x)} = \exp \left[ \frac{i}{2} \int (dx)(dx') K^*(x) G(x, x') K(x') \right]
\]

where \( (dx) = dx^0 dx^1 dx^2 dx^3 \) and \( G(x, x') \) is the Green’s function. The exponential structure is derived from the physical independence of sufficiently remote individual acts of emission propagation and detection, which are embodied in the form of an equation (9).

Let \( K(x) = K_1(x) + K_2(x) \), where the source \( K_2(x) \) is switched on after the source \( K_1(x) \) is switched off. \( K_1(x) \) will be identified with the emitter and \( K_2(x) \) with the detector.

We start from the Green’s function of the massive spin zero particles is emitted from an external source \( K(x) \equiv \delta(x-x') \) satisfies to the differential equation of Schrödinger’s equation.

\[
\left[ -i\hbar \frac{\partial}{\partial ct} + \frac{\hbar^2 \nabla^2}{2m} \right] G(x, x') = \delta(x-x')
\]

(11)

Also, the solution of the differential equation from the equation (11) is given by

\[
G(x, x') = \begin{cases} 
\int \frac{dk}{(2\pi)^3} e^{i(x-x')} \frac{e^{-i\hbar^{2}k^{2}/2m}}{-\hbar k^{0} + \hbar^{2}k^{2} - i\varepsilon}, & \text{for } x^{0} > x'^{0} \\
0, & \text{for } x^{0} < x'^{0}
\end{cases}
\]

(12)

where \( dk = (dk^{0}, d\vec{k}) \) be differential of components of energy-momentum four vector description in unitary space [7-11]. The problem is simplified as non-relativistic and half-space boundary conditions on the behavior of the massive spin zero particle. Integrate the equation (12) over \( k^{0}, \vec{k} \) to obtain time evolution process \( (x^{0} > x'^{0}) \) is then given completely by the Green’s function in the space-time domain from the equation (9).
\[
\left\langle \tilde{x}, x^0 \left| \tilde{x}', x^0' \right. \right\rangle = \left( \frac{m}{2i \pi \hbar T} \right)^{\frac{1}{2}} \exp \left\{ \frac{im|\tilde{r} - \tilde{r}'|^2}{2hT'} \right\} \exp \left\{ \frac{im(z - z')^2}{2hT} \right\} - \exp \left\{ \frac{im(z + z')^2}{2hT} \right\}
\]

where, \( x = (\tilde{x}, x^0) = (\tilde{r}, z, x^0), x' = (\tilde{x}', x^0') = (\tilde{r}', z', x^0') \) and \( T = x^0 - x^0' \).

### 4. Experimental situation

Let \( x^0' \) be any fixed value on the secondary planar surface sources on opaque screen. Suppose that a point source (in Figure 2), situated at \( x' \), emits a particle and the latter was determined to reach and through the secondary planar surface sources in an opaque screen situated at, at the time \( T_i \) and finally was determined to reach a detector, situated at \( x \), at time \( T_2 \) and then we get the completeness relation to describe the situation given by

\[
\left\langle \tilde{x}, x^0 \left| \tilde{x}', x^0' \right. \right\rangle = \left( \frac{m}{2i \pi \hbar (T_2 + T_1)} \right)^{\frac{1}{2}} \exp \left\{ \frac{im|\tilde{r} - \tilde{r}'|^2}{2hT_2} \right\} \exp \left\{ \frac{im|\tilde{r}'' - \tilde{r}'|^2}{2hT_1} \right\} \times \left\{ \exp \left[ \frac{im}{2hT_2} \left( z - z'^* \right)^2 \right] \exp \left[ \frac{im}{2hT_1} \left( z^* - z' \right)^2 \right] \right\} - \exp \left[ \frac{im}{2hT_2} \left( z + z'^* \right)^2 \right] \exp \left[ \frac{im}{2hT_1} \left( z^* + z' \right)^2 \right] \right\}
\]

where \( x^0 = (\tilde{r}^*, z^*, x^0'), T_1 = x^0 - x^0' \) and \( T_2 = x^0 - x^0'' \)

The transition amplitude that a particle propagate from a point source at space-time coordinate, reach the secondary planar surface sources in an opaque screen at space-time coordinate \( x^0 = (\tilde{x}^*, x^0) \) and end up in an opaque screen at space-time coordinate \( x = (\tilde{x}, x^0) \) the exponential structure is described by vector analysis to be

\[
\left\langle \tilde{x}, x^0 \left| \tilde{x}', x^0' \right. \right\rangle = \left( \frac{m}{2i \pi \hbar (T_2 + T_1)} \right)^{\frac{1}{2}} \exp \left\{ \frac{im}{2hT_2} \left[ \tilde{r} - \tilde{r}' \right]^2 + (z - z'^*)^2 \right\} \exp \left\{ \frac{im}{2hT_1} \left[ \tilde{r}^* - \tilde{r}' \right]^2 + (z^* - z')^2 \right\} \times \left\{ \exp \left[ \frac{im}{2hT_2} \left( \tilde{r} - \tilde{r}' \right)^2 + (z - z'^*)^2 \right] \right\} - \exp \left[ \frac{im}{2hT_1} \left( \tilde{r}^* - \tilde{r}' \right)^2 + (z^* - z')^2 \right] \right\}
\]

\[
\left\langle \tilde{x}, x^0 \left| \tilde{x}', x^0' \right. \right\rangle = \left( \frac{m}{2i \pi \hbar (T_2 + T_1)} \right)^{\frac{1}{2}} \exp \left\{ \frac{im}{2hT_2} \left[ T_1 \tilde{A}^2 + T_2 |\tilde{B}|^2 \right] \right\} - \exp \left[ \frac{im}{2hT_1} \left( T_1 \tilde{C}^2 + T_2 |\tilde{D}|^2 \right) \right] \right\}
\]
Figure 2: The following experimental situation of radiation from secondary planar surface sources of a crossed aperture

\[
\langle \tilde{x}, x^0 \mid \tilde{x}', x'^0 \rangle = \left( \frac{m(2)^{\frac{3}{2}}}{2\pi\hbar(T_2 + T_1)} \right)^{\frac{3}{2}} \times \left[ \exp \left[ i\alpha(T_1|\tilde{A}|^2 - (-T_2|\tilde{B}|^2) \right] - \exp \left[ -i\alpha(T_1|n\tilde{A}|^2 - (-T_2|n\tilde{B}|^2) \right) \right] \right]^{\frac{1}{2}}
\]

(18)

With the functional exponential is defined by \( \cos(x) = \frac{e^{ix} - e^{-ix}}{2} \)

\[
\langle \tilde{x}, x^0 \mid \tilde{x}', x'^0 \rangle = \left( \frac{m(2)^{\frac{3}{2}}}{2\pi\hbar(T_2 + T_1)} \right)^{\frac{3}{2}} \cos \left[ \alpha(T_1|n\tilde{A}|^2 - (-T_2|n\tilde{B}|^2) \right] \]

(19)

where \( \alpha = \frac{m}{2hT_1T_2} \) and \( n \) is an integer

\[
\langle \tilde{x}, x^0 \mid \tilde{x}', x'^0 \rangle = \left( \frac{m(2)^{\frac{3}{2}}}{2\pi\hbar(T_2 + T_1)} \right)^{\frac{3}{2}} \cos \left( \beta_1|\tilde{A}|^2 - \beta_2|\tilde{B}|^2 \right)
\]

(20)

where \( \beta_1 = n\alpha T_1 \) and \( \beta_2 = -n\alpha T_2 \)

We can write of the probability of the general expression for transition amplitude

\[
|\langle \tilde{x}, x^0 \mid \tilde{x}', x'^0 \rangle|^2 \approx \cos \left( \beta_1|\tilde{A}|^2 - \beta_2|\tilde{B}|^2 \right)^2
\]

(21)
5. Results and Discussion

Development of the Green’s function in the space-time domain description of the propagation of massive spin zero particles in equation (15) was based on the configuration space and a time evolution process where particle propagates between emission source and detector. We use the mathematical program for corresponding numerical evaluations between the experimental situation in quantum optics (Figure: 3) and the probability of the general expression in equation (21) for a transition amplitude in quantum field theory situation (Figure: 4).

Figure: 3 Radiation from secondary planar surface sources of a crossed aperture. The diffraction pattern of a crossed aperture arrangement illustrating the spatial coherence of a free-electron laser. (Source: DESY in Hamburg; http://mpsd.cfel.de/images/content/e208/e209/index_eng.html).

Figure: 4 Radiation from secondary planar surface sources of a crossed aperture. The diffraction pattern of a crossed aperture arrangement illustrating the spatial coherence of the Maxima calculation program (equation (21)).
6. Summary
The expression in quantum optics following into the formulation of the radiation in radiant intensity which, analytical solution has not given the time evolution of amplitude. This research was involved with the careful analysis of the propagation of a massive quantum particle with corresponding numerical evaluation in space time as a time evolution process dealing with amplitudes of transitions in the presence of the external source $Z[K(x)]$. By using Feynman’s so-called spin-zero electron are technically called Klein-Gordon particles of finite mass between different points in the presence of the emission source and detection source.

We attend and explain the radiation for radiant intensity in space-time domain for the vacuum-to-vacuum transition amplitude. To calculation amplitude transition of the massive spin zero particle propagator stimulated emission by secondary planar surface source in the space-time domain in equation (14). In development of transition amplitude in equation (15) to describe in detail of experimental situations by the diffraction pattern of a crossed aperture arrangement illustrating the spatial coherence of a free-electron laser in quantum optics situation (Figure: 3). Finally, a comparison between our result and experimental in the quantum optics situation by using the mathematical program for corresponding numerical evaluations by the probability of the general expression for a transition amplitude in the equation (21) from quantum field theory situation (Figure:4).

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