Heavy neutrinos from gluon fusion

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Heavy neutrinos, a key prediction of many standard model extensions, remain some of the most
searched-for objects at collider experiments. In this context, we revisit the premise that the gluon
fusion production mechanism, \( gg \rightarrow Z^*/h^* \rightarrow N \nu \), is phenomenologically irrelevant at the CERN
LHC and report the impact of soft gluon corrections to the production cross section. We resum
threshold logarithms up to next-to-next-to-next-to-leading logarithmic accuracy (N^3LL), thus
capturing the dominant contributions to the inclusive cross section up to next-to-next-to-leading order
(N^2LO). For \( m_N > 150 \text{ GeV} \) and collider energies \( \sqrt{s} = 7 - 100 \text{ TeV} \), corrections to the Born
rates span +160 to +260%. At \( \sqrt{s}=14 \text{ TeV} \), the resummed channel is roughly equal in size to the widely-
believed-to-be-dominant charged current Drell-Yan process and overtakes it outright at \( \sqrt{s} \approx 20 - 25 \text{ TeV} \).
Results are independent of the precise nature/mixing of \( N \) and hold generically for other low-
.scale seesaws. Findings are also expected to hold for other exotic leptons and broken axial-vector
currents, particularly as the \( Z^* \) contribution identically reduces to that of a pseudoscalar.

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INTRODUCTION

In stark contrast to the standard model (SM) of particle physics, neutrinos have nonzero mass, and misaligned
flavor and mass eigenstates [1, 2]. Hence, the origins of their sub-eV masses and large mixing angles are two of
the most pressing open questions in particle physics today. In light of recent evidence for the Higgs mechanism’s
role in generating charged lepton masses [3, 4], we argue that the existence of neutrino Dirac masses comparable
to other elementary fermions’ Dirac masses is an increasingly likely prospect. If this is the case, then observed
neutrino phenomenology can be accommodated by low-scale seesaw mechanisms, such as the Inverse [5–7] or
Linear [8, 9] seesaw models.

In such seesaw scenarios, TeV-scale heavy neutrinos’ mass eigenstates (\( N \)) can couple to electroweak (EW)
bosons with sizable [10, 11] active-sterile mixing, but at the same time do not decouple from Large Hadron
Collider (LHC) phenomenology [12, 13] due to their pseudo-Dirac nature. Subsequently, low-scale seesaw
mechanisms can be tested at the LHC with O(100 – 1000) fb\(^{-1}\) [14–19], demonstrating the sensitivity and complemenarity of collider and oscillation experiments.

Hadron collider investigations of heavy \( N \) typically rely on the charged current (CC) Drell-Yan (DY) process [20],
shown in Fig. 1(a) and given by

\[
q\bar{q} \rightarrow W^{\pm(*)} \rightarrow N\ell^{\pm}, \quad q \in \{u, c, d, s, b\}, \tag{1}
\]
or the sizable vector boson fusion (VBF) channel [22–25],

\[
q\gamma \xrightarrow{W^+ \rightarrow N\ell} N\ell^{\pm} q'. \tag{2}
\]

As seen in Fig. 1(c), VBF is driven by the \( W\gamma \rightarrow N\ell \) subprocess and receives longitudinal \( W \) enhancements for

\( m_N \gg M_W \) [24]. Notable is the renewed interest [25, 26] in the gluon fusion (GF) process [25–28], shown in
Fig. 1(b),

\[
gg \rightarrow Z^*/h^* \rightarrow N\nu. \tag{3}
\]

Variants of this process have been studied recently in [29–35]. While GF proceeds anomalously through off-shell
\( Z^*/h^* \) bosons and is formally an \( \mathcal{O}(\alpha_s^2) \) correction to the neutral current (NC) DY process, \( q\bar{q} \rightarrow Z^* \rightarrow N\nu \), the
channel’s cross section is known to surpass the DY and VBF rates for collider energies \( \sqrt{s} \gtrsim 30 – 40 \text{ TeV} \) [25–27].
At 14 TeV, GF is factors smaller than the DY channels. These conclusions are noteworthy, as they rely on the GF rate at leading-order (LO) accuracy being a good estimate of the total cross section. However, at LO, the GF rates for the SM Higgs boson [36–42], heavy scalars [38], and pseudoscalars [43–45] are greatly underestimated.

In light of this, we report, for the first time, the impact of soft gluon corrections to heavy \( N \) production in GF. We resum threshold logarithms up to next-to-next-to-next-to-leading logarithmic accuracy (N^3LL).

For GF, this captures the leading contributions to the inclusive cross section (\( \sigma \)) up to next-to-next-to-leading order (N^2LO) [46]. Our findings have immediate impact on searches at hadron colliders, and thereby challenge the paradigm that GF is phenomenologically irrelevant for the discovery and study of heavy \( N \) at the LHC.

For \( m_N = 150 – 1000 \text{ GeV} \), \( \sqrt{s} = 7 – 100 \text{ TeV} \), and scale choices comparable to the hard process, we report

\[
K^{N^{3\text{LL}}} = \frac{\sigma^{N^{3\text{LL}}}}{\sigma^{\text{LO}}} : \begin{cases} 
2.6 - 3.6, \\
2.3 - 3.0. 
\end{cases} \tag{4}
\]

We find that GF dominates over the DY-like processes of Eq. (1) for \( m_N = 500 – 1000 \text{ GeV} \) at \( \sqrt{s} \gtrsim 20 – 25 \text{ TeV} \).
The corrections exhibit perturbative convergence and are consistent with those for Higgs and heavy (pseudo)scalar production \[36-45\]. Our results are independent of the precise nature/mixing of \(N\) and hold generically for other low-scale seesaws. Results are also expected to hold for other exotic leptons, e.g. triplet leptons in the Type III seesaw and other colorless, axial-vector currents.

This report continues as follows: We first describe our phenomenological heavy \(N\) model, then present the resummation formalism employed, emphasizing a new treatment of the \(Z^*\) current. After summarizing our computational setup, we present our results and conclude.

**HEAVY NEUTRINO MODEL**

Throughout this study, we adopt the neutrino mixing formalism of \[47\]: For \(i (m) = 1, \ldots, 3\) left-handed (light) states and \(j (m') = 1, \ldots, n\) right-handed (heavy) states, chiral neutrinos can be rotated into mass eigenstates by

\[

\begin{pmatrix}
\nu_i
\end{pmatrix}_{N_R^j} = \begin{pmatrix}
U_{3\times3} & V_{3\times n}
\end{pmatrix}
\begin{pmatrix}
\nu_m
\end{pmatrix}_{N_{R'}^{m'}}.
\]

(6)

After further rotating the charged leptons into the mass basis, the flavor state \(\nu_\ell\) in the mass basis is explicitly

\[

\nu_\ell = \sum_{m=1}^3 U_{\ell m} \nu_m + \sum_{m' = 1}^n V_{\ell m'} N_{m'}^{m'}. \quad (7)
\]

\(U_{\ell m}\) is the observed light neutrino mixing matrix and \(V_{\ell m'}\) parametrizes active-heavy mixing. For EW-scale \(N_{m'}\), the latter is constrained by precision EW data to be \(|V_{\ell N}| \lesssim 10^{-2} - 10^{-1}\) \[10, 11\]. For simplicity, we consider only the lightest heavy state, denoted \(N\).

In the mass basis, the EW interaction Lagrangian is

\[

\mathcal{L}_{\text{int}} = -\frac{g}{\sqrt{2}} W^{\mu\nu}_{\ell \, \ell' \, 6} \left( \sum_{m=1}^3 \bar{\nu}_m U_{\ell m} \gamma^\mu P_L \nu_{\ell'} \right)

- \frac{g}{\sqrt{2}} \left( \sum_{m=1}^3 \bar{\nu}_m N_{m}^\nu P_L \nu_{\ell} \right)

- \frac{g}{2 \cos \theta_W} Z_{\mu} \left( \sum_{m=1}^3 \bar{\nu}_m U_{\ell m}^* \gamma^\mu P_L \nu_{\ell} \right)

\]

\[\]  

FIG. 1: Born diagrams for heavy \(N\) production via the (a) DY, (b) GF, and (c) VBF processes. Drawn using JaxoDraw \[21\].

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the likelihoods of observing parton \( i \) in \( p \) carrying longitudinal momentum \( p_i^z = \xi p_i^z \gg p_T \), when DGLAP-evolved [53–55] to a factorization scale \( \mu_f \), generating the partonic scale \( \sqrt{s} = \sqrt{\xi_1 \xi_2 s} \). \( \Delta_{ij}^{\beta, FO}(z) = \delta(1 - z) + \mathcal{O}(\alpha_s) \) accounts for soft gluons carrying a momentum fraction \((1 - z)\), with \( z = Q^2/s \), emitted in the \( ij \to A \) transition \((\vec{\alpha}^B)\) via a color/Lorentz structure labeled as \( \beta \). Above, \( \tau_0 = \min\{Q^2/s, \xi_1 \xi_2 z \} \) is the kinematic threshold below which \( ij \to A \) is kinematically forbidden, and \( \tau = Q^2/s = \xi_1 \xi_2 z \) is similarly the hard threshold.

For the \( g g \to Z^+/h^+ \to N\bar{\nu}_\ell \) process, with \( \beta \in \{Z, h\} \), \( Q^2 = (p_N + p_{\nu})^2 > m_h^2, M_Z^2 \) and \( \tau_0 \approx m_h^2/s \), the hard partonic-level Born cross sections are\(^2\) [26, 27]

\[
\hat{\sigma}^Z = G_F^2 \frac{\alpha_s^2(\mu_f) |V_{LN}|^2}{244(4\pi)^3} m_N^2(1 - r_N)^2 F_Z(Q^2)^2, \tag{12}
\]

\[
\hat{\sigma}^h = G_F^2 \frac{\alpha_s^2(\mu_f) |V_{LN}|^2}{244(4\pi)^3} m_N^2 Q^4 \frac{(1 - r_N)^2}{4(Q^2 - m_h^2)^2} F_h(Q^2)^2, \tag{13}
\]

where \( r_X = m_X^2/Q^2 \). For quarks with weak isospin charge \( T_{LQ}^2 = \pm 1/2 \), the \( Z/\bar{h} \) one-loop form factors are

\[
F_Z(Q^2) = \sum_{q=1,\ldots,b} 2(T_{LQ}^2 q - 2 r_q f(r_q)), \tag{14}
\]

\[
F_h(Q^2) = \sum_{q=1,\ldots,b} 2 r_q [2 + (1 - 4 r_q) f(r_q)], \tag{15}
\]

\[
f(r) = \begin{cases} 2 \left( \sin^{-1} \frac{1}{2 r} \right)^2, & r > \frac{1}{2}, \\ -\frac{1}{4} \left[ \log \left( 1 + \sqrt{1 - 4 r} \right) - i \pi \right]^2, & r \leq \frac{1}{2}. \end{cases} \tag{16}
\]

A few remarks: (i) While we use the fully integrated \( \hat{\sigma}^\beta \), the resummation formalism we employ [56, 57] operates in momentum space. Hence, phase-space cuts on an \( n \)-body final state can be implemented if one starts from the differential \( d\hat{\sigma}^\beta \). (ii) The \( Z^+/h^+ \) contributions add incoherently due to the (anti)symmetric nature of the \( (Z^+)/h^+ \) coupling [27]. (iii) The similarity of \( \hat{\sigma}^Z \) and \( \hat{\sigma}^h \) follows from the fact that, after summing over SU(2)_L doublet constituents, the net \( Z^+ \) contribution is a pseudoscalar-like coupling proportional to quark Yukawa couplings. This is in accordance with the Goldstone Equivalence Theorem, pointed out first for the \( gg \to N\bar{\nu}_\ell \) process in Ref. [25]. Subsequently, this asymptotic behavior means one can further simplify the original expressions of Refs. [26, 27] to those above.

The axial-vector-pseudoscalar correspondence, however, is more general: For a massive, colorless vector \( V(\mu_f) \) participating in the loop process \( gg \to V^* \), the most general current-propagator contraction (in the unitary gauge) is of the form \( \Gamma_{\mu\nu} \sim (g_{V^*} \gamma^\mu + g_{A^*} \gamma^\mu \gamma^5)(g_{\mu\nu} - q_{\mu} q_{\nu}/M_V^2) \). By C-symmetry (Furry’s theorem), the vector current \( g_{V^*} \gamma^\mu \) vanishes; by angular momentum conservation (Landau-Yang theorem), the transverse polarization \( g_{\mu\nu} \) does not contribute. Hence, \( \Gamma_{\mu\nu} \sim g_{A^*} \gamma^\mu \gamma^5 q_{\mu} q_{\nu}/M_V^2 \). After decomposing quark propagators in the triangle loop via spinor completeness relations and exploiting the Dirac equation, one finds \( \Gamma_{\mu\nu} \sim \gamma^5(2m_f q_{\mu}/M_V^2) \). That is, a pseudoscalar coupling proportional to the quark mass \( m_f \).

Moreover, emissions of soft gluons off fermions do not change the loop’s structure due to soft factorization. Therefore, one may approximate soft QCD corrections to the \( gg \to V^* \) subprocess for \( V^* \) possessing axial-vector couplings to fermions with those corrections for a pseudoscalar. This is a main finding of this work and was not observed in previous resummations of \( gg \to Z^* \).

As \( Q \) approaches the partonic threshold \( \sqrt{s} \), accompanying gluon radiation is forced to be soft, with \( \epsilon_{\gamma} \sim \sqrt{1 - z} \). This generates numerically large phase-space logarithms of the form \( \log(1 - z) \) that spoil the perturbative convergence of Eq. (11). Threshold logarithms, however, factorize and can be resummed to all orders in \( \alpha_s \log(1 - z) \) via exponentiation [58–62].

We perform this resummation by working in the Soft-Collinear Effective Theory (SCET) framework [63–65]. This permits Eq. (11) to be factorized directly in momentum space [56, 57] by segmenting and regulating divergent regions of phase space with hard and soft scales, \( \mu_h \) and \( \mu_s \). (This is unlike perturbative QCD where one works in Mellin space [58–60]). Scale invariance of physical observables then implies that factored components can also be independently renormalization group (RG)-evolved and matched via exponentiation [58–62]. Thus, numerically large quantities are replaced with perturbative ones regulated by \( \mu_h \) and \( \mu_s \) and with RG-evolution coefficients that run \( \mu_s, \mu_h \) to \( \mu_f \) and \( Q \).

In practice, the resummation procedure reduces to replacing the soft coefficient function \( \Delta_{ij}^{\beta, FO}(z) \) in Eq. (11): \( \sigma_{FO} \to \sigma_{Res} : \Delta_{ij}^{\beta, FO}(z) \to \Delta_{ij}^{\beta, Res}(z) \).

\[
\Delta_{\gamma\gamma}^{\beta, Res}(z) = \left| C_\beta(Q^2, \mu_f^2) \right|^2 U(Q^2, \mu_f^2, \mu_s^2, \mu_h^2, \mu_f^2) \times \left( \frac{\sqrt{z}}{1 - z} \right)^{z - \eta} \text{Higgs} \left( \frac{Q^2}{\mu_f^2} + \delta_{\eta, \mu_s}, \frac{\mu_s}{2\eta} \right) \tag{18}
\]

\[
C_\beta \text{ is the process-dependent, so-called hard function and accounts for (hard) virtual corrections to the hard process. For } \beta = h, \text{ the function is given by the two-step SCET matching coefficients } C_{\ell} \text{ and } C_S \text{ of [67], with}
\]

\[
C_{\ell}(Q^2, \mu_h^2) \equiv C_{\ell}(m_h^2, \mu_h^2) C_S(-Q^2, \mu_h^2), \text{ and } \tag{19}
\]

\[
C_S(Q^2, \mu_s^2) = \sum_{n=0}^{\infty} C_S^{(n)}(Q^2, \mu_s^2) \left( \frac{\alpha_s(\mu_f)}{4\pi} \right)^n, \tag{20}
\]

\( ^2 \) We note that the expression for \( \hat{\sigma}^h \) in [26] contains typographic errors.
where $X \in \{t, S\}$. The product of $C_t$ and $C_S$, which can be expanded individually as power series in $(\alpha_s/4\pi)$, is equivalent to a one-step SCET matching procedure when setting $\mu_t = \mu_h$ [68]. For $\beta = Z$, the one-step matching hard function can also be expanded as a power series. In the notation of [45, 69], this is

$$
C_Z(Q^2, \mu_h^2) = C_g^{A, \text{eff}}(Q^2, \mu_h^2) \sum_{n=0}^\infty C_{g, n}^{A, \text{eff}}(Q^2, \mu_h^2) \left( \frac{\alpha_s(\mu_h)}{4\pi} \right)^n. \tag{21}
$$

We briefly note that the $\log(\mu_h^2/m_t^2)$ term that appears in $C_{A, 2}^{A, \text{eff}}$ of [69] should be replaced with $\log(Q^2/m_t^2)$ in order to preserve the scale independence of the total cross section, a physical observable [45]. With this modification, both $C_h$ and $C_Z$ satisfy the evolution equation,

$$
\frac{d}{d\log \mu} C_\beta(Q^2, \mu^2) = \left[ \Gamma_{\text{cusp}}^A \log \left( \frac{Q^2}{\mu^2} \right) + \gamma^S + \gamma^t \right] C_\beta(Q^2, \mu^2), \tag{22}
$$

for anomalous dimensions $\Gamma_{\text{cusp}}^A, \gamma^{S, A, S}$, as given in [66, 67].

The soft scalar function $\hat{s}_{\text{Higgs}}$ describes (soft) radiation off incoming gluons and hence is universal for scalars and pseudoscalars. The derivatives in $\hat{s}_{\text{Higgs}}$ are regular partial derivatives that act to the right, before $\eta \equiv 2\alpha_t(\mu_t^2, \mu_f^2)$ is evaluated numerically.

We include an additional factor of $\sqrt{z}$ in Eq. (19) with respect to [66, 67]. As noted in [66, 67, 70, 71], the inclusion of the factor accounts precisely for power corrections that are manifest in the traditional QCD/Mellin-space resummation formalism. Numerically, we find this increases our total normalization by $\mathcal{O}(5-10\%)$ at $N^3$LL and our residual scale dependence by $\mathcal{O}(5\%)$.

RG-running is described by the evolution function,

$$
U(Q^2, \mu_t^2, \mu_h^2, \mu_s^2, \mu_f^2) = \frac{\alpha_s^2(\mu_f^2)}{\alpha_s^2(\mu_t^2)} \left[ \frac{\beta(\alpha_s(\mu_t^2)/\alpha_s(\mu_f^2))}{\beta(\alpha_s(\mu_h^2)/\alpha_s(\mu_f^2))} \right]^2 \left( \frac{Q^2}{\mu_t^2} \right)^{-2\alpha_t(\mu_t^2, \mu_s^2)} \left| e^{4s(\mu_s^2, \mu_f^2)-2a_{\gamma S}(\mu_s^2, \mu_f^2)+a_{\gamma S}^{\mu_s}(\mu_s^2, \mu_f^2)} \right|, \tag{23}
$$

where $\mu_s = \mu_t = \mu_h$ for $\beta = h$ ($Z$).

For definitions and explicit expressions of the quantities in Eqs. (19)–(23) up to $\mathcal{O}(\alpha_s^3)$, see [66, 67]. Mappings between $N^3$LL accuracy and required ingredients can be found in [66, 72]. At $N^3$LL, one needs at two loops: $C_\beta$ for both pseudoscalar [69] and scalar [67, 73], as well as $\hat{s}_{\text{Higgs}}$ [66, 67, 74]. Note that while the results of [67, 69, 73] are derived in the heavy top limit, $\mathcal{O}(Q^2/m_t^2)$ corrections to inclusive (pseudo)scalar cross sections are known to be $\mathcal{O}(1-10\%)$ [38, 73], even for $Q^2 \gg m_t^2$, justifying their use in our calculation.

**COMPUTATIONAL SETUP**

For the DY and VBF channels, we use the methodology of [25] to compute inclusive cross sections and uncertainties at NLO in QCD, but with the following exceptions: we use the NNPDF 3.0 QED NLO PDF set [76, 77] and do not apply phase-space cuts to the CC DY process. Scale choices and regulating VBF cuts are unchanged. For GF, we adopt the additional SM inputs [78]:

$$
m_h = 0 \text{ GeV}, \ m_t = 173.2 \text{ GeV}, \ m_h = 125.7 \text{ GeV}.
$$

To best match the accuracy of the resummation calculation, we use the NNPDF 3.0 NNLO+NNLL PDF set [79]; while the set’s uncertainties are sizable, the use of a FO PDF set would formally double-count initial-state gluons. Cross sections are calculated using in-house code with Monte Carlo integration performed via the CUBA libraries [80], and checked at LO against [25, 26]. The soft coefficient function $\Delta_{qg}^{\text{Res}}$ is checked against [45, 72, 81].

To minimize the numerical impact of missing QCD corrections, we follow [66, 67] and choose the scale scheme

$$
\mu_r, \mu_f, \mu_t, \mu_h = Q \quad \text{and} \quad \mu_s = Q/(1-\tau)/(1+7\tau), \tag{24}
$$

for both the Born and resummed GF calculations. For GF, we report the scale dependence associated with simultaneously varying $\mu_f, \mu_r$, and $\mu_s$ over $0.5 \leq \mu_s/\mu_{\text{Default}} \leq 2$. While the $\mu_s$ dependence itself is numerically small, we vary it jointly with $\mu_f$ to ensure that the subtraction terms required for numerical evaluation lead sufficiently to numerical convergence; see [66, 67] for more details. Missing FO terms that would otherwise stabilize $\mu_f$ represents the largest source of uncertainty. Indeed, we find other scale uncertainties to be relatively small owing to our high logarithmic accuracy.

In the following, we report only residual scale dependence. For studies on PDF uncertainties in heavy $N$ production, see [24, 25], and for threshold-improved PDF uncertainties, see [82, 83]. PDFs and $\alpha_s(\mu)$ are evaluated using the LHAPDF 6 libraries [84].

**RESULTS**

At $\sqrt{s} = 14$ TeV and as a function of heavy $N$ mass, we show in Fig. 2 (L) the inclusive $N$ production cross section (divided by active-heavy mixing $|V_{he}|^2$) for the CC DY and VBF processes at NLO, for GF at LO, and for GF at $N^3$LL. The thickness of each curve corresponds to the residual scale dependence; no scale dependence is shown for GF at LO. In the lower panel is the ratio of the resummed and Born GF rates. We quantify the impact of QCD corrections with a $K$-factor, defined generically as

$$
K^{N^3\text{LO}+N^3\text{LL}} = \sigma^{N^3\text{LO}+N^3\text{LL}}/\sigma^{\text{LO}}. \tag{25}
$$
For $m_N = 150 - 1000$ GeV, cross sections span roughly

\begin{align*}
\text{CC DY NLO} & : 3.5 - 5400 \text{ fb}, \\
\text{GF N}^3\text{LL} & : 1.9 - 280 \text{ fb}, \\
\text{GF LO} & : 0.73 - 110 \text{ fb}, \\
\text{VBF NLO} & : 4.4 - 37 \text{ fb}.
\end{align*}

(26) - (29)

For GF, $K$-factors and uncertainties span approximately

\begin{align*}
\text{GF N}^3\text{LL} & : K = 3.07 - 3.14 \text{ with } \delta\sigma/\sigma = \pm 8 - 13\%, \\
\text{GF N}^2\text{LL} & : K = 2.59 - 2.66 \text{ with } \delta\sigma/\sigma = \pm 6 - 9\%, \\
\text{GF NLL} & : K = 1.00 - 1.06 \text{ with } \delta\sigma/\sigma = \pm 25 - 27\%.
\end{align*}

(30) - (32)

These rates should be compared with DY (VBF) $K$-factors of $K_{\text{DY}}^{\text{VBF}} = 1.15 - 1.25$ (0.98 - 1.06) and uncertainties of $(\delta\sigma/\sigma)_{\text{DY}}^{\text{VBF}} = ±1 - 5 (5 - 11)\%$.

For the mass range under consideration, one observes unambiguously that the resummed GF rates at N$^3$LL and N$^3$LL are markedly larger than the LO rate, with $K \gtrsim 2 - 2.5$ and notably independent of $m_N$. This is unlike NLL, where $K \sim 1$, since one essentially runs only $\alpha_s(\mu)$ and $C_\beta$, $\delta_{\text{Higgs}} \sim 1$; here, the uncertainty simply corresponds to varying $\alpha_s$. We find $\sigma^{N^3\text{LL}}/\sigma^{N^2\text{LL}} \sim 1.1 - 1.2$, indicating convergence of the perturbative series.

As previously stated, the residual uncertainty at N$^3$LL stems from missing FO contributions. Such terms, likely positive definite [46], consist of hard, initial state radiation (ISR) with $p_T^f \gtrsim \mu_f = Q$, which are not, by construction, included in the DGLAP-evolution of the PDFs. The sizes of the N$^2$LL and N$^3$LL corrections are, in part, due to our scale choices and the desire to minimize the importance of missing QCD corrections. Choosing alternative, less intuitive scales for the Born process can, of course, lead to smaller $K$-factors, but also to larger ones. At both N$^2$LL and N$^3$LL, the size of the uncertainty band is due to a residual $\mu_f$ dependence, and requires matching to hard ISR from FO contributions to be reduced. Moreover, these corrections are in line with those for Higgs and heavy (pseudo)scalar production [36-39, 42-45].

In comparison with other heavy $N$ production modes, we find for $m_N \gtrsim 300$ GeV that the GF rate is now comparable to the CC and NC DY (not shown for clarity) rates. When basic fiducial cuts are applied on the charged lepton in the CC processes the combined GF+NC DY rate is slightly larger than the combined VBF+CC DY channel. For $m_N \lesssim 600$ GeV, i.e., masses that are most relevant for LHC phenomenology due to mixing-suppression [24, 47], the GF channel is factors larger than the VBF mechanism, indicating its potential importance at the LHC and its upgrades/successors.

We briefly note that the relative importance of the VBF mechanism found in Fig. 2 is considerably less than what has been found in previous investigations, e.g., Ref. [23] and follow-up works by the same authors. It was shown in Ref. [24] that the findings of Ref. [23] were qualitatively and quantitatively incorrect: Their claimed “t-channel enhancement” are in reality due to several poorly/unregulated QCD and QED collinear divergences. Numerically, their cross sections were overestimated by 100× in some instances. We refer readers to Refs. [24, 25, 48] for correct, all-orders/resummed treatments of these contributions; to Ref. [15, 24] for a quantitative assessment of $W\gamma$ scattering in heavy $N$.
TABLE I: Heavy N production cross sections via the GF mode at various accuracies, divided by active-heavy mixing \(|V_{iN}|^2\), scale dependence (\%), and K-factor, for representative \(m_N\) and \(\sqrt{s}\).

| \(\sqrt{s}\) | \(m_N\) | \(\sigma / |V_{iN}|^2\) [fb] | \(\sigma / K\) [fb] | \(\sigma / K\) [fb] | \(\sigma / K\) [fb] |
|-------|-------|--------|--------|--------|--------|
|       |       |       |       |       |       |
| \(300\) GeV | \(300\) GeV | \(300\) GeV | \(300\) GeV | \(300\) GeV | \(300\) GeV |
| \(500\) GeV | \(500\) GeV | \(500\) GeV | \(500\) GeV | \(500\) GeV | \(500\) GeV |
| \(100\) TeV | \(100\) TeV | \(100\) TeV | \(100\) TeV | \(100\) TeV | \(100\) TeV |
| \(14\) TeV | \(14\) TeV | \(14\) TeV | \(14\) TeV | \(14\) TeV | \(14\) TeV |
| \(33\) TeV | \(33\) TeV | \(33\) TeV | \(33\) TeV | \(33\) TeV | \(33\) TeV |
| \(100\) TeV | \(100\) TeV | \(100\) TeV | \(100\) TeV | \(100\) TeV | \(100\) TeV |

\[ K_{\text{N}^3\text{LL}} = \sigma_{\text{N}^3\text{LL}} / \sigma_{\text{LO}} : 2.6 - 3.6, \]  
\[ K_{\text{N}^2\text{LL}} = \sigma_{\text{N}^2\text{LL}} / \sigma_{\text{LO}} : 2.3 - 3.0. \]  

We find that GF dominates over DY-like processes of Eq. (1) for \(m_N = 500 - 1000\) GeV at \(\sqrt{s} \gtrsim 20 - 25\) TeV. Corrections exhibit perturbative convergence and are consistent with Higgs and heavy (pseudo)scalar production. Moreover, our results are independent of the precise nature/mixing of \(N\), and are expected to hold for other exotic leptons as well as other colorless, broken axial-vector currents one finds in other seesaw scenarios.

**SUMMARY AND CONCLUSION**

The existence of tiny neutrino masses and large mixing is unambiguous evidence for physics beyond the SM. In light of Higgs boson data, the prospect of neutrino Dirac masses existing is increasingly likely. Low-scale seesaw models with TeV-scale heavy neutrinos that couple appreciably to EW bosons are scenarios that can accommodate these seemingly contradictory observations, and still give rise to LHC phenomenology.

In this context, we have evaluated, for the first time, soft corrections to the GF production mode \(gg \rightarrow Z^*/h^* \rightarrow N\nu\). This was made possible by a new treatment of the \(gg \rightarrow Z^*\) subprocess. For \(m_N = 150 - 1000\) GeV and \(\sqrt{s} = 7 - 100\) TeV, we report:

This drop in \(K\) is again due to an increasing importance of hard ISR, and similarly leads to a sizable residual \(\mu_t\) dependence. In checks against heavy (pseudo)scalar production [45, 72, 81], we find similar results, and that the importance of FO corrections is \(O(\pm10\%)\). Such corrections would likely push net \(K\)-factors for the \(gg \rightarrow N\nu\) process to \(K \sim 3\), confirming the conjecture of [26]. We summarize our results in Table I.

Due to the severe model-dependence of \(V_{iN}\) as well as the associated phenomenology, an investigation into which is well beyond the scope of this study, we defer further interpretation of our results to future studies.

**Usage**: For the use of these results in studies, we advocate LO+parton shower event generation following [25]. Total inclusive rates should then be normalized to those tabulated in Tables II and III. The flatness of the re-summed \(K\)-factors means interpolation to unlisted \(m_N\) is reliable.

**Appendix**

In Tables II-III, we list total hadronic cross sections for \(gg \rightarrow Z^*/h^* \rightarrow N\nu\) at various accuracies, divided by active-heavy mixing \(|V_{iN}|^2\), for representative collider energies \(\sqrt{s} = 13, 14, 33,\) and \(100\) TeV.
| \(m_N \; [\text{GeV}]\) | \(\sigma^{\text{LO}} / |V_{tN}|^2 \; [\text{fb}]\) | \(\sigma^{\text{NLL}} / |V_{tN}|^2 \; [\text{fb}]\) | \(\sigma^{\text{N2LL}} / |V_{tN}|^2 \; [\text{fb}]\) | \(\sigma^{\text{N3LL}} / |V_{tN}|^2 \; [\text{fb}]\) |
|---|---|---|---|---|
| 150 | 0.9097E+02 | 0.9131E+02 | 0.2419E+03 | 0.2867E+03 |
| 175 | 0.8811E+02 | 0.8866E+02 | 0.2334E+03 | 0.2770E+03 |
| 200 | 0.8510E+02 | 0.8560E+02 | 0.2243E+03 | 0.2665E+03 |
| 225 | 0.8020E+02 | 0.8075E+02 | 0.2110E+03 | 0.2504E+03 |
| 250 | 0.7330E+02 | 0.7379E+02 | 0.1924E+03 | 0.2282E+03 |
| 275 | 0.6451E+02 | 0.6496E+02 | 0.1690E+03 | 0.2008E+03 |
| 300 | 0.5467E+02 | 0.5507E+02 | 0.1430E+03 | 0.1697E+03 |
| 325 | 0.4500E+02 | 0.4515E+02 | 0.1171E+03 | 0.1388E+03 |
| 350 | 0.3629E+02 | 0.3640E+02 | 0.9423E+02 | 0.1117E+03 |
| 375 | 0.2922E+02 | 0.2947E+02 | 0.7613E+02 | 0.9010E+02 |
| 400 | 0.2370E+02 | 0.2396E+02 | 0.6185E+02 | 0.7328E+02 |
| 450 | 0.1593E+02 | 0.1612E+02 | 0.4147E+02 | 0.4910E+02 |
| 500 | 0.1089E+02 | 0.1106E+02 | 0.2838E+02 | 0.3355E+02 |
| 550 | 0.7617E+01 | 0.7747E+01 | 0.1984E+02 | 0.2351E+02 |
| 600 | 0.5413E+01 | 0.5518E+01 | 0.1410E+02 | 0.1670E+02 |

\(\sqrt{s} = 13 \text{ TeV}\)

| \(m_N \; [\text{GeV}]\) | \(\sigma^{\text{LO}} / |V_{tN}|^2 \; [\text{fb}]\) | \(\sigma^{\text{NLL}} / |V_{tN}|^2 \; [\text{fb}]\) | \(\sigma^{\text{N2LL}} / |V_{tN}|^2 \; [\text{fb}]\) | \(\sigma^{\text{N3LL}} / |V_{tN}|^2 \; [\text{fb}]\) |
|---|---|---|---|---|
| 150 | 0.1065E+03 | 0.1069E+03 | 0.2824E+03 | 0.3348E+03 |
| 175 | 0.1039E+03 | 0.1043E+03 | 0.2733E+03 | 0.3239E+03 |
| 200 | 0.1006E+03 | 0.1012E+03 | 0.2641E+03 | 0.3132E+03 |
| 225 | 0.9522E+02 | 0.9567E+02 | 0.2490E+03 | 0.2952E+03 |
| 250 | 0.8711E+02 | 0.8776E+02 | 0.2279E+03 | 0.2702E+03 |
| 275 | 0.7693E+02 | 0.7746E+02 | 0.2008E+03 | 0.2380E+03 |
| 300 | 0.6538E+02 | 0.6585E+02 | 0.1704E+03 | 0.2018E+03 |
| 325 | 0.5386E+02 | 0.5425E+02 | 0.1401E+03 | 0.1659E+03 |
| 350 | 0.4377E+02 | 0.4395E+02 | 0.1133E+03 | 0.1341E+03 |
| 375 | 0.3557E+02 | 0.3565E+02 | 0.0917E+04 | 0.1105E+03 |
| 400 | 0.2900E+02 | 0.2913E+02 | 0.0784E+04 | 0.0881E+04 |
| 450 | 0.1951E+02 | 0.1975E+02 | 0.0506E+04 | 0.0589E+04 |
| 500 | 0.1351E+02 | 0.1367E+02 | 0.0349E+04 | 0.0418E+04 |
| 550 | 0.0949E+01 | 0.0963E+01 | 0.0245E+04 | 0.0289E+04 |
| 600 | 0.0680E+01 | 0.0692E+01 | 0.0176E+04 | 0.0208E+04 |

\(\sqrt{s} = 14 \text{ TeV}\)

TABLE II: Total hadronic cross sections for \(gg \rightarrow Z^*/h^* \rightarrow N\nu\ell\) at various accuracies, divided by active-heavy mixing \(|V_{tN}|^2\), for representative collider energies \(\sqrt{s}\).
| $m_N$ [GeV] | $\sigma^{\text{LO}} / |V_{IN}|^2$ [fb] | $\sigma^{\text{NLL}} / |V_{IN}|^2$ [fb] | $\sigma^{\text{NNLL}} / |V_{IN}|^2$ [fb] | $\sigma^{\text{NNNLL}} / |V_{IN}|^2$ [fb] |
|------------|---------------------------------|-----------------|-----------------|-----------------|
| 150        | 0.5547E+03                      | 0.5510E+03      | 0.1408E+04      | 0.1658E+04      |
| 200        | 0.5672E+03                      | 0.5663E+03      | 0.1428E+04      | 0.1674E+04      |
| 300        | 0.4146E+03                      | 0.4144E+03      | 0.1033E+04      | 0.1209E+04      |
| 400        | 0.2132E+03                      | 0.2127E+03      | 0.5250E+03      | 0.6134E+03      |
| 500        | 0.1147E+03                      | 0.1146E+03      | 0.2806E+03      | 0.3273E+03      |
| 600        | 0.6606E+02                      | 0.6615E+02      | 0.1608E+03      | 0.1873E+03      |
| 700        | 0.4011E+02                      | 0.4025E+02      | 0.9747E+02      | 0.1135E+03      |
| 800        | 0.2550E+02                      | 0.2564E+02      | 0.6183E+02      | 0.7191E+02      |
| 900        | 0.1681E+02                      | 0.1690E+02      | 0.4066E+02      | 0.4733E+02      |
| 1000       | 0.1141E+02                      | 0.1147E+02      | 0.2753E+02      | 0.3204E+02      |
| 1100       | 0.7916E+01                      | 0.7981E+01      | 0.1912E+02      | 0.2228E+02      |
| 1200       | 0.5625E+01                      | 0.5685E+01      | 0.1360E+02      | 0.1582E+02      |
| 1300       | 0.4066E+01                      | 0.4115E+01      | 0.9831E+01      | 0.1144E+02      |
| 1400       | 0.2985E+01                      | 0.3031E+01      | 0.7234E+01      | 0.8425E+01      |
| 1500       | 0.2225E+01                      | 0.2261E+01      | 0.5389E+01      | 0.6281E+01      |

$\sqrt{s} = 100$ TeV

| $m_N$ [GeV] | $\sigma^{\text{LO}} / |V_{IN}|^2$ [fb] | $\sigma^{\text{NLL}} / |V_{IN}|^2$ [fb] | $\sigma^{\text{NNLL}} / |V_{IN}|^2$ [fb] | $\sigma^{\text{NNNLL}} / |V_{IN}|^2$ [fb] |
|------------|---------------------------------|-----------------|-----------------|-----------------|
| 150        | 0.3230E+04                      | 0.3223E+04      | 0.7967E+04      | 0.9244E+04      |
| 200        | 0.3516E+04                      | 0.3507E+04      | 0.8569E+04      | 0.9922E+04      |
| 300        | 0.2839E+04                      | 0.2832E+04      | 0.6825E+04      | 0.7876E+04      |
| 400        | 0.1648E+04                      | 0.1645E+04      | 0.3909E+04      | 0.4493E+04      |
| 500        | 0.9911E+03                      | 0.9913E+03      | 0.2330E+04      | 0.2674E+04      |
| 600        | 0.6330E+03                      | 0.6322E+03      | 0.1473E+04      | 0.1685E+04      |
| 700        | 0.4242E+03                      | 0.4220E+03      | 0.0775E+03      | 0.1119E+04      |
| 800        | 0.2937E+03                      | 0.2938E+03      | 0.6768E+03      | 0.7728E+03      |
| 900        | 0.2101E+03                      | 0.2101E+03      | 0.4817E+03      | 0.5498E+03      |
| 1000       | 0.1541E+03                      | 0.1538E+03      | 0.3513E+03      | 0.4010E+03      |
| 1100       | 0.1152E+03                      | 0.1153E+03      | 0.2624E+03      | 0.2996E+03      |
| 1200       | 0.8800E+02                      | 0.8830E+02      | 0.2006E+03      | 0.2291E+03      |
| 1300       | 0.6822E+02                      | 0.6839E+02      | 0.1550E+03      | 0.1771E+03      |
| 1400       | 0.5369E+02                      | 0.5371E+02      | 0.1214E+03      | 0.1384E+03      |
| 1500       | 0.4272E+02                      | 0.4278E+02      | 0.9653E+02      | 0.1102E+03      |

Table III: Total hadronic cross sections for $gg \to Z'/h' \to N\nu$ at various accuracies, divided by active-heavy mixing $|V_{IN}|^2$, for representative collider energies $\sqrt{s}$. 

