How low-scale Trinification sheds light in the flavour hierarchies, neutrino puzzle, dark matter and leptogenesis

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We propose a low-scale renormalizable trinification theory that successfully explains the flavour hierarchies and neutrino puzzle in the Standard Model (SM) as well as provide a Dark Matter candidate and contains necessary means for efficient leptogenesis. The proposed theory is based on the trinification $SU(3)_C \times SU(3)_L \times SU(3)_R$ gauge symmetry, which is supplemented with an additional flavor symmetry $U(1)_X \times \mathbb{Z}_2^{(1)} \times \mathbb{Z}_2^{(2)}$. In the proposed model the top quark and the exotic fermions acquire tree-level masses, whereas the lighter SM charged fermions gain masses radiatively at one-loop level. In addition, the light active neutrino masses arise from a combination of radiative and type-I seesaw mechanisms, with the Dirac neutrino mass matrix generated at one-loop level.

I. INTRODUCTION

Despite the huge success of Standard Model (SM) as a theory of fundamental interactions, it has several open issues, which include the lack of explanation of the SM flavor structure, in particular, the fermion masses and mixing, the origin of Dark Matter (DM), as well as the source of parity violation in electroweak (EW) interactions. Besides, the SM features drawbacks such as the absence of sufficient CP violation and a strong departure from thermal equilibrium, both necessary for explaining the cosmological baryon asymmetry of the Universe. In this paper, we would like to address all these issues on the same footing in a single new framework, based on the trinification symmetry $SU(3)_C \times SU(3)_L \times SU(3)_R$ (see e.g. Refs. [1–17]) supplemented with an additional flavor symmetry, $U(1)_X \times \mathbb{Z}_2^{(1)} \times \mathbb{Z}_2^{(2)}$, with the spontaneously broken $U(1)_X \times \mathbb{Z}_2^{(1)}$ symmetry (we refer to this model as to 3331 framework in what follows). We have included a preserved $\mathbb{Z}_2^{(2)}$ symmetry to implement the one-loop level radiative seesaw mechanisms that generate the light active neutrino masses as well as the masses for the SM charged fermions below the top quark mass. The spontaneously broken $\mathbb{Z}_2^{(1)}$ symmetry allows to separate the scalar bi-triplet that gives tree-level masses for the top and exotic up-type quarks from the one that produces the exotic down-type quark masses. We also investigate the potential implications of this scenario for DM and leptogenesis.

Our 3331 model is the most economical theory of trinification that can naturally explain the masses and hierarchies for the SM fermions, since unlike the other existing trinification-based models such as the one presented in Ref. [15], it does not rely on the inclusion of a large variety of additional representations such as scalar anti-sextets in order to generate the light active neutrino masses in the SM. In our case, the actual mechanism for the light neutrino mass generation is provided by a one-loop level radiative seesaw. Moreover, our model is capable of simultaneously explaining the hierarchy of charged SM fermion masses, which is not considered in earlier works.

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II. THE MODEL

Our model is based on the SU(3)_C × SU(3)_L × SU(3)_R × U(1)_X gauge symmetry which can be motivated by a high-scale ultraviolet (UV) completion based on the embedding of trinification as a subgroup of E_8 [18–20], whereas an additional flavor symmetry can be inspired by the coset of the E_8 → [SU(3)]^3 reduction [21–25]. Indeed, in the framework of a super-string inspired Z_n orbifolding procedure of E_8 reduction [26], one of the possible E_8 breaking patterns features the following scheme

\[ E_8 \rightarrow SU(3)_C \times SU(3)_L \times SU(3)_R \times [U(1)]^2, \]  

(1)

where the rank-reduction [U(1)]^2 → U(1)_X can occur via a Higgs mechanism. All the subsequent breaking steps may in principle take place at any energy scale between the E_8, or alternatively E_6, (M_{GUT}) and the EW (M_{EW}) breaking scales. Here, we consider a particularly attractive opportunity of the trinification breaking at a relatively low scale compared to the E_8 breaking one, M_{GUT} ≫ M_3 ≈ 100 TeV, i.e. not too far from the reach of future collider measurements.

In fact, both supersymmetric and non-supersymmetric realizations of this model involving the E_6 symmetry would require a very high scale for both the E_6 and the trinification breaking scales, above 10^{16} GeV, due to strong constraints on the E_6 gauge mediated proton decay. In order to relax this constraint, we explore a non-supersymmetric version of the model, without a manifest embedding of its particle content into representations of E_6 or even E_8, such that the scale of the trinification symmetry breaking can indeed be within the reach of future colliders such as the 100 TeV proton-proton Future Circular Collider (FCC).

Our model realizes the following particular symmetry breaking scheme:

\[ G \equiv SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_X \times Z_{2}^{(1)} \times Z_{2}^{(2)} \times \mathbb{C}, \]

\[ SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times Z_{2}^{(2)} \times \mathbb{C}, \]

\[ SU(3)_C \times SU(2)_L \times U(1)_{Y} \times Z_{2}^{(2)} \times \mathbb{C}, \]

\[ SU(3)_C \times U(1)_Q \times Z_{2}^{(2)} \times \mathbb{C}, \]

(2)

where the different symmetry breaking scales fulfill the following hierarchies:

\[ v \ll w_X \ll v_X^{(k)} \sim v_X^{(4)} \sim M_3, \quad k = 1, 2. \]

(3)

Here, v = 246 GeV is the EW symmetry breaking scale. We assume that the trinification breaking scales v_X^{(k)} and v_X^{(4)} are of the order of 100 TeV, which would make our model potentially testable at the future FCC 100 TeV collider.

In our model the exotic particles carry the SM electric charge which is defined in terms of the trinification symmetry generators as follows:

\[ Q = T_{3L} + T_{3R} + \beta(T_{SL} + T_{SR}) + X = T_{3L} + T_{3R} + \frac{1}{2}(B - L), \]

(4)

where \( \beta = -1/\sqrt{3} \), \( B - L = 2[\beta(T_{SL} + T_{SR}) + X] \).

(5)

We have chosen such particular value of \( \beta \) in order to have the third component of the leptonic triplet to be electrically neutral, which allows for a consistent implementation of a low-scale seesaw mechanism for light active neutrino masses generation.

The scalar sector of our model is composed only of three scalar bi-triplets and one SU(3)_L scalar triplet that feature the following patterns of the vacuum expectation values (VEVs)

\[ \langle \chi_1 \rangle = \begin{pmatrix} \frac{v}{\sqrt{2}} & 0 & w_X^0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{v^{(1)}}{\sqrt{2}} \end{pmatrix}, \]

\[ \langle \chi_2 \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & v^{(2)}_X \end{pmatrix}, \]

\[ \langle \chi_3 \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \]

\[ \langle \chi_4 \rangle = \begin{pmatrix} 0 & 0 & v^{(4)}_X \end{pmatrix}, \]

(6)
where \( \chi_3 \) does not acquire a VEV since it is charged under the preserved \( \mathbb{Z}_2^{(2)} \) symmetry. The latter has been incorporated in order to implement the one-loop level radiative seesaw mechanisms for producing the light active neutrino masses, as well as the masses for the SM charged fermions lighter than the top quark.

A justification of the chosen structure of the scalar sector is as follows. The scalar bi-triplet \( \chi_1 \) is needed to generate tree-level masses for the top and exotic up-type quarks, whereas the scalar bi-triplet \( \chi_2 \) is required to produce tree-level masses for the exotic down-type quarks. We include the spontaneously broken \( \mathbb{Z}_2^{(1)} \) symmetry in order to separate the scalar bi-triplets \( \chi_1 \) and \( \chi_2 \) in the mass spectrum. Furthermore, the inclusion of the scalar bi-triplet \( \chi_3 \) is needed for the implementation of the one-loop level radiative seesaw mechanism that generates the masses for the SM charged fermions below the top quark mass scale. Note that the presence of potentially light pseudo-Goldstone CP-odd state in the scalar spectrum, our model offers a long-lived scalar candidate for warm DM known in the literature as the Majoron – an appealing feature of the considered model further discussed below in Sec. VI. In addition, the scalar bi-triplet \( \chi_3 \) is crucial to generate the masses for the light active neutrinos. Besides, the scalar triplet \( \chi_4 \) is necessary to generate the quark mixing angles in the 13 and 23 planes.

It is worth mentioning that the lightest 125 GeV SM-like Higgs boson is mainly composed of the CP-even neutral component of \((\chi_1)_{11}\). Moreover, our model can naturally accommodate its alignment limit since all the other scalar states are typically very heavy and thus are decoupled from the SM in the mass spectrum, as detailed below in Sec. V. This also implies the absence of tree-level flavour changing neutral currents (FCNCs) for the SM-like Higgs boson state, while such contributions from the heavier scalars are strongly suppressed by their large mass scale.

In addition, the fermion sector, which is motivated by conventional 3331 models and is obtained after the Left-Right (LR) symmetry is imposed, has the following structure:

\[
Q_{n(L,R)} = \left( \begin{array}{c} d_n \\ -u_n \\ J_n \end{array} \right)_{L,R}, \quad Q_{3L} = \left( \begin{array}{c} u_3 \\ d_3 \\ U \end{array} \right)_{L,R}, \quad L_{\alpha(L,R)} = \left( \begin{array}{c} \nu_\alpha \\ e_\alpha \\ N_\alpha \end{array} \right)_{L,R}, \quad n = 1, 2, \quad \alpha = 1, 2, 3. \tag{7}
\]

The transformation properties of the scalar and fermionic fields under the symmetries of the model are shown in Tables I and II.

With the previously specified particle content and symmetries, the Lagrangian for Yukawa interactions reads as:

\[
-\mathcal{L}_Y^{(Q)} = y_3^{(Q)\alpha Q_3 L \chi_1} Q_{3 \alpha R} + \sum_{n=1}^2 \sum_{m=1}^2 y_{nm}^{(Q)\alpha Q_{nL \chi_2}^2} Q_{m R} + \sum_{n=1}^2 y_{n3}^{(Q)\alpha Q_{nL \chi_3}^3} Q_{3 R} + \text{h.c.}, \tag{8}
\]

\[
-\mathcal{L}_Y^{(f)} = 3 \sum_{\alpha=1}^3 \sum_{\beta=1}^3 (x_\chi)_{\alpha \beta} L_{\alpha L \chi_2 L_{\beta R}} + \sum_{\alpha=1}^3 \sum_{n=1}^2 (x_\psi)_{\alpha n} L_{\alpha L \chi_4} \Psi_{n R} + \sum_{n=1}^2 \sum_{m=1}^2 (m_\psi)_{nm} \Psi_{n R} \bar{\Psi}_{m R} + \text{h.c.}. \tag{9}
\]

If follows from (8) and (9) that the top and exotic up- and down-type quarks get tree-level masses which are directly proportional to \( v, v^{(1)}_\chi \) and \( v^{(2)}_\chi \), respectively. The mixing between the top and the exotic \( U \) quark is controlled by the \( w_\chi \) VEV. In addition, after the first step of the trinification symmetry breaking, the exotic, vector-like, quarks remain singlets under the LR \( \text{SU}(2)_L \times \text{SU}(2)_R \) symmetry group. Furthermore, the SM charged fermions, which are significantly lighter than the top quark, get one-loop masses via a radiative seesaw mechanism mediated by heavy exotic fermions and heavy scalars running in the internal lines of the loop, as shown in Fig. 1. Notice, as shown in Fig. 1, that the one-loop masses for the light SM charged fermions receive contributions from \( v^{(1)}_\chi \) and \( v^{(2)}_\chi \). However, it is worth mentioning that the loop-generated masses for the up, charm, down and strange quarks and the SM charged leptons are mainly fixed by \( v^{(2)}_\chi \) VEV, whereas the bottom quark mass is mainly determined by \( v^{(1)}_\chi \) VEV. This is due to the fact that the charged exotic fermions that induce such one-loop masses are proportional to those VEVs. In addition, the Cabbibo mixing is mainly fixed by \( v^{(2)}_\chi \), since it is generated by the one-loop contribution mediated by the exotic down-type quarks. Notice that the Cabbibo mixing receives contributions from both up- and down-type quark sectors, whereas the down-type quark sector helps to generate the remaining mixing angles. The VEVs that are crucial for generating the quark mixing angles in the 13 and 23 planes are both \( v^{(2)}_\chi \) and \( v^{(4)}_\chi \), as indicated in Fig. 1. As seen from Fig. 1, where the one loop Feynman diagrams have Trinification VEVs in the external legs, it follows that the dominant contribution for the masses of such SM-like charged fermions is not due the Electroweak Symmetry Breaking (EWSB) mechanism but mostly due to the trinification breaking. Despite of this issue, the interactions of the 125 GeV SM like Higgs boson \( h \) with SM fermion-antifermion pairs, such as \( hbb, hcc \) can be effectively generated at one loop level via the quartic scalar interaction insertions \((\chi_1 \chi_1)\) and \((\chi_1 \chi_1)(\chi_2 \chi_2)\) where the EW scale VEV
and the 125 GeV SM like Higgs boson $h$ are in the external legs, in addition to the Trinification VEVs. Some of the one-loop Feynman diagrams contributing to the $hff$ interactions are shown in Fig. 2. Furthermore, despite of the fact that the dominant contribution for the masses of the SM-like charged fermions lighter than the top quark is mostly due to the trinification breaking, the masses of such SM charged fermions can be successfully reproduced by having appropriate values of the masses of non SM scalars and exotic fermions running in the internal lines of the loops and of the quartic scalar couplings and Yukawa couplings. For instance, to successfully explain the GeV scale value of the bottom quark and tau lepton masses, from Fig. 1, we have that such masses can be estimated as:

\[
m_b \sim m_{\tau} \sim \frac{y^2}{16\pi^2} \lambda \frac{v^2}{M_F},
\]

where $M_F$ is the mass scale of the exotic fermions, $y$ the SM fermion-exotic fermion Yukawa coupling and $\lambda$ the quartic scalar coupling. Taking $v_\chi \sim M_F \sim \mathcal{O}(100)$ TeV and $y \sim \lambda \sim \mathcal{O}(0.1)$, Eq. (10) takes the form $m_b \sim m_{\tau} \sim 10^{-5} v_\chi \sim \mathcal{O}(1)$ GeV, thus showing that our model naturally explain the smallness of the bottom and tau masses with respect to the EWSB scale. Furthermore, despite the fact that the masses of the light SM charged fermions (below the top quark mass) are generated at one-loop level, the hierarchy between such masses can be accommodated by having some deviation from the scenario of universality of the Yukawa couplings in both quark and lepton sectors. This would imply some moderate tuning among the Yukawa couplings. However, such a situation is considerably better compared to that of the SM, where a significant Yukawa parameter tuning is required.

In addition, the Cabibbo mixing together with the quark mixing in the 13 and 23 planes are generated at one-loop level too. For this to happen, the $Z_2^{(2)}$ symmetry has to be softly broken in the scalar sector which is achieved by the trilinear $f_{\chi 3 \chi 3}$ interaction term in the scalar potential (see Sec. V).

It is worth mentioning, as follows from Eqs. (8) and (9), that the Yukawa interactions and mass terms are given in terms of the following parameters: $y_Q^\alpha$, $y_m^\alpha$, $y_{\kappa_3}^\alpha$, $(x_\chi)_{\alpha \beta}$, $(x_\Psi)_{\alpha n}$, $(m_\Psi)_{nm}$ ($n, m = 1, 2$). Assuming all these parameters to be real except two of them amounts for a total of 28 parameters, from which 8 correspond to the quark Yukawa couplings, 16 – to the lepton Yukawa couplings, and 4 – to exotic lepton mass parameters. Apart from these, there are 8 additional parameters useful to fit the SM fermion masses and mixing angles, which correspond to the scalar boson mass terms, $m_{(\chi_\phi^3)^2}$, $m_{(\chi_\phi^3)^2}$, $m_{(\chi_\phi^3)^2}$, $m_{(\chi_\phi^3)^2}$, $m_{(\chi_\phi^3)^2}$, $m_{(\chi_\phi^3)^2}$ and $m_{(\chi_\phi^3)}$ for the scalars in the internal lines of the Feynman diagrams in Fig. 1 and in the first diagram in Fig. 3. This gives a total of 36 input parameters allowing for enough parametric freedom to accommodate the experimental values for the 10 and 9 physical observables of the quark and lepton sectors, respectively.

| $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|-------|-------|-------|-------|
| SU(3)$_C$ | 1 | 1 | 1 | 1 |
| SU(3)$_L$ | 3 | 3 | 3 | 3 |
| SU(3)$_R$ | 3 | 3 | 3 | 1 |
| U(1)$_X$ | 0 | 0 | 1/3 | $-1/3$ |
| $Z_2^{(1)}$ | $-1$ | 1 | $-1$ | 1 |
| $Z_2^{(2)}$ | 1 | 1 | $-1$ | 1 |

Table I: Scalar assignments under SU(3)$_C \times$ SU(3)$_L \times$ SU(3)$_R \times$ U(1)$_X \times$ $Z_2^{(1)} \times Z_2^{(2)}$ symmetry.

| $Q_\alpha L$ | $Q_3 L$ | $Q_\alpha R$ | $Q_3 R$ | $L_\alpha L$ | $L_\alpha R$ | $\Psi_{\alpha R}$ |
|--------------|----------|-------------|----------|-------------|-------------|------------------|
| SU(3)$_C$    | 3       | 3           | 3        | 3          | 1           | 1               |
| SU(3)$_L$    | 5       | 3           | 1        | 1          | 3           | 1               |
| SU(3)$_R$    | 1       | 1           | 3        | 3          | 1           | 3               |
| U(1)$_X$     | 0       | 1/3         | 0        | 1/3        | $-1/3$      | 0               |
| $Z_2^{(1)}$  | 1       | 1           | $-1$     | 1          | 1           | $-1$            |
| $Z_2^{(2)}$  | 1       | $-1$        | 1        | $-1$       | 1           | 1               |

Table II: Fermion assignments under SU(3)$_C \times$ SU(3)$_L \times$ SU(3)$_R \times$ U(1)$_X \times$ $Z_2^{(1)} \times Z_2^{(2)}$. Here $n = 1, 2$ and $\alpha = 1, 2, 3$. 
Figure 1: One-loop Feynman diagrams contributing to the entries of the SM charged fermion mass matrices. Here, \( n, m, k, r, s = 1, 2 \) and \( \alpha, \beta, \gamma, \delta = 1, 2, 3 \).
Figure 2: Some one-loop Feynman diagrams contributing to the $hff$ interactions. Here, $n, m = 1, 2$, $h$ is the 125 GeV SM-like Higgs boson and $f$ a SM fermion lighter than the top quark.

III. THE NEUTRINO SECTOR

From the neutrino Yukawa interactions, we obtain the following mass terms:

$$-\mathcal{L}_{\text{mass}}^{(\nu)} = \frac{1}{2} \mathbf{\Delta}^T M_\nu \mathbf{\Delta} + \text{h.c.},$$

where the neutrino basis and neutrino mass matrix (at one-loop order) are given by, respectively,

$$\mathbf{\Delta} = \begin{pmatrix}
\nu_{\alpha L}^c \\
\nu_{\alpha R} \\
N_{\alpha L}^c \\
N_{\alpha R} \\
\Psi_{\alpha R}
\end{pmatrix},$$

$$M_\nu = \begin{pmatrix}
\nu_{\beta L}^c & \nu_{\beta R} & N_{\beta L}^c & N_{\beta R} & \Psi_{\beta R} \\
M_{\mu L \nu_{\beta L}} & M_{\mu L \nu_{\beta R}} & M_{\mu L \nu_{\beta L}} & v_{\beta L} & \frac{\sqrt{2}}{2} (x_\chi)_{\alpha \beta} \\
M_{\mu R \nu_{\beta R}} & M_{\mu R \nu_{\beta R}} & M_{\mu R \nu_{\beta R}} & v_{\beta R} & \frac{\sqrt{2}}{2} (x_\Psi)_{\alpha \beta} \\
M_{\mu L \nu_{\beta L}} & 0 & 0 & v_{\beta L} & \frac{\sqrt{2}}{2} (x_\chi)_{\alpha \beta} \\
M_{\mu R \nu_{\beta R}} & 0 & 0 & v_{\beta R} & \frac{\sqrt{2}}{2} (x_\Psi)_{\alpha \beta} \\
\Psi_{\alpha R} & 0 & 0 & 0 & \frac{\sqrt{2}}{2} (x_\Psi)_{\alpha \beta} \\
\Psi_{\alpha R} & 0 & 0 & 0 & (m_\Psi)_{\alpha \beta}
\end{pmatrix},$$
with some of the sub-matrices generated at one-loop level from the Feynman diagrams in Fig. 3. The sub-matrices appearing in Eq. (13) are given by:

\[
M_{\alpha L \nu^c \beta L} = \frac{2}{16\pi^2} \sum_{n=1}^{2} \frac{(x_{\chi})_{\alpha n} (x_{\chi})_{\beta n} m_{\psi_n}}{m_{\psi_n}^2} \left( \left( m_{\psi_n}, m_{\text{Re}(x_{\chi})_n}, m_{\text{Im}(x_{\chi})_n} \right), \quad \alpha, \beta = 1, 2, 3, \right) \tag{14}
\]

\[
M_{\alpha L \nu^c \beta R} = \frac{3}{16\pi^2} \sum_{\gamma=1}^{3} (x_{\chi})_{\alpha \gamma} (x_{\chi})_{\beta \gamma} m_{N_{\gamma}} \left( \left( m_{N_{\gamma}}, m_{\text{Re}(x_{\chi})_{13}}, m_{\text{Im}(x_{\chi})_{13}} \right), \quad m_{N_{\gamma}} = (x_{\chi})_{\gamma} \frac{v_{\chi}}{\sqrt{2}}, \right) \tag{15}
\]

\[
M_{\nu_{\alpha R} N_{\beta L}} = \frac{\kappa_{(\chi)}^{(1)} v_{\chi}^{(2)} (x_{\chi})_{\alpha \beta}}{2 \sqrt{2} m_{(\chi)_3}} \tag{16}
\]

\[
M_{\nu_{\alpha R} N_{\beta R}} = \frac{\kappa_{(\chi)}^{(1)} v_{\chi}^{(2)} (x_{\chi})_{\alpha \beta}}{2 \sqrt{2} m_{(\chi)_3}} \tag{17}
\]

\[
f \left( m_F, m_R, m_I \right) = \left[ \frac{m_R^2}{m_R^2 - m_F^2} \ln \left( \frac{m_R^2}{m_F^2} \right) - \frac{m_I^2}{m_I^2 - m_F^2} \ln \left( \frac{m_I^2}{m_F^2} \right) \right] \tag{18}
\]

where \( \alpha = 1, 2, 3 \) and \( n = 1, 2 \). In addition, the entries denoted by \( X \) and \( y \) are generated at tree- and one-loop levels, respectively.

The light active neutrino masses arise from a combination of radiative and type-I seesaw mechanisms (with the Dirac neutrino mass matrix generated at one-loop order). This mechanism in a more general setup has been recently proposed in Ref. [27]. Thus, the mass matrix for the light neutrinos takes the form

\[
\tilde{M}_\nu = M_{\nu L} v_L - A M_S^{-1} A^T, \tag{20}
\]

where the matrices \( M_{\nu L} \) and \( A \) are generated at one-loop level whereas \( M_S \) receives tree-level and one-loop contributions. The matrices \( \tilde{A} \) and \( M_S \) are found as follows

\[
A = \begin{pmatrix} M^T_{\nu_{\alpha R} \nu^c \beta R} & M_{\nu_{\alpha R} N_{\beta L}} & 0 & 0 \\ 0 & M^T_{\nu_{\alpha R} N_{\beta L}} & \frac{v_{\chi}^{(2)}}{\sqrt{2}} (x_{\chi})_{\alpha \beta} & \frac{v_{\chi}^{(4)}}{\sqrt{2}} (x_{\chi})_{\alpha m} \\ 0 & \frac{v_{\chi}^{(2)}}{\sqrt{2}} (x_{\chi})_{\alpha \beta} & 0 & 0 \\ \frac{v_{\chi}^{(4)}}{\sqrt{2}} (x_{\chi})_{\alpha m} & 0 & \left( m_{\psi} \right)_{nm} \end{pmatrix} \tag{20}
\]

Considering \( v \ll w_{\chi} \ll m_{(\chi)_3} \sim v_{\chi}^{(1)} \sim v_{\chi}^{(2)} \ll m_{\psi} \), from the previous relations it follows that the entries of the light active neutrino mass matrix can be approximated as

\[
\left( \tilde{M}_\nu \right)_{\alpha \beta} \approx M_{\nu_{\alpha L} \nu^c \beta L} = \sum_{n=1}^{2} \frac{(x_{\chi})_{\alpha n} (x_{\chi})_{\beta n} m_{\psi_n}}{16\pi^2} \left( \left( m_{\psi_n}, m_{\text{Re}(x_{\chi})_n}, m_{\text{Im}(x_{\chi})_n} \right), \quad m_{\psi_n} \sim O(1) \text{ TeV}, \right) \tag{19}
\]

In order to get an approximate expression for the physical sterile neutrino mass matrices, we assume \( w_{\chi} \sim O(10^3) \) TeV and consider \( v_{\chi}^{(1)} \sim v_{\chi}^{(2)} \sim v_{\chi}^{(4)} \sim O(10^2) \) TeV. Taking the Yukawa couplings of order unity, we thus recover the following estimates:

\[
M_{\nu_{\alpha R} N_{\beta L}} \sim O(1) \text{ TeV}, \quad M_{\nu_{\alpha L} N_{\beta L}} \sim O(10) \text{ TeV}.
\]

In the limit when the remaining sterile neutrinos are very heavy, i.e. when their mass matrices approach \( \pm x_{\chi} v_{\chi}^{(2)}/\sqrt{2} \) and \( m_{\psi} \), the lightest physical sterile neutrino mass matrix can be approximated as \( M_{\nu_{\alpha R} \nu^c \beta R} \):

\[
IV. \text{ IMPLICATIONS FOR QUARKS AND CHARGED LEPTONS}
\]

Expanding the Lagrangian in Eq. (8) and considering, for simplicity, that

\[
y_{nm}^{(Q)} \equiv y \quad \text{for} \ m = n \tag{21}
\]
Figure 3: Feynman diagrams contributing to the entries of the neutrino mass matrix in Eq. 13. Here, $n, m, k = 1, 2$ and $\alpha, \beta, \gamma = 1, 2, 3$. 
and

\[ y^{(Q)}_{nm} \equiv h \quad \text{for } m \neq n, \quad (22) \]

one obtains five massless quarks, corresponding to all down-type quarks as well as the first and second generation up-type ones. The top quark mass is readily generated at tree level,

\[ m_t^2 = \frac{1}{2} y_3^2 v^2 \frac{v^2}{v^2 + \omega^2}. \quad (23) \]

The model also predicts three heavy vector-like quarks with the following masses

\[ m_{V_1}^2 \simeq \frac{1}{2} y_3^2 (v^2 + \omega^2), \quad m_{V_2}^2 = \frac{1}{2} v^2 (h + y)^2, \quad m_{V_3}^2 = \frac{1}{2} v^2 (h - y)^2. \quad (24) \]

Since the top Yukawa coupling \( y_3 \) is large, \( V_1 \) will always be of order 100 TeV and can only be probed at a future FCC facility. However, if the couplings \( h \) and \( y \) are somewhat smaller, say of order 0.01, then both \( V_2 \) and \( V_3 \) can be at a TeV scale and hence at the reach of the LHC. In a third scenario, if both \( h \) and \( y \) are of order one and not far off each other, then only \( V_3 \) can become light enough to be probed at the LHC. Furthermore, as follows from Eq. (8), the exotic quarks can decay into a SM quark and either a neutral or a charged scalar and can be pair-produced at the LHC via Drell-Yan and gluon fusion processes, mediated by charged gauge bosons and gluons, respectively. A detailed study of the collider phenomenology of the model is beyond the scope of this paper and is left for future dedicated explorations.

For the charged lepton sector we follow a similar approach. First, we expand the first term in Eq. (9) and set

\[ (x\chi)_{\alpha\beta} = x \quad \text{for } \alpha = \beta, \quad (25) \]

and

\[ (x\chi)_{\alpha\beta} = k \quad \text{for } \alpha \neq \beta. \quad (26) \]

The tree-level mass matrix yields six massless leptons. Their Yukawa couplings are of radiative origin, thus are naturally small in comparison to the top quark. The model also contains three heavy vector-like leptons with the following mass spectrum:

\[ m_{E_1}^2 = \frac{1}{2} v^2 (2k + x)^2, \quad m_{E_2}^2 = m_{E_3}^2 = \frac{1}{2} v^2 (k - x)^2. \quad (27) \]

Similarly to the quark sector, if both \( k \) and \( x \) are of order 0.01 or slightly smaller, then the model predicts three generations of vector-like leptons at the TeV scale or even below. On the other hand, if such Yukawa couplings are of order one, then only \( E_2 \) and \( E_3 \) can become sufficiently light to be at the reach of the LHC measurements.
V. THE SCALAR SECTOR

We start our discussion of the scalar sector by considering the first breaking step in the chain (2). The scalar potential of the low-scale trinification theory, which is invariant under the transformations specified in Tab. I, reads

\[
V(\chi_a) = \sum_{a=1}^{2} \mu_{(\chi)a}^2 (\chi_a)_l^l (\chi_a)_r^l + \frac{1}{2} \mu_{(\chi)3}^2 (\chi_3)_l^l (\chi_3)_r^r + \frac{1}{2} \mu_{(\chi)4}^2 (\chi_4)_l^l + \frac{1}{2} \mu_{(\chi)5}^2 (\chi_5)_l^l + \frac{1}{2} \mu_{(\chi)6}^2 (\chi_6)_l^l
\]

\[
+ \frac{1}{4} \sum_{b \geq a} \kappa_{ab} (\chi_a)_l^l (\chi_b)_l^l (\chi_b)_l^r (\chi_b)_r^r + \kappa_{a3} \chi_3^l \chi_3^l + \frac{1}{2} \kappa_{a4} \chi_4^l \chi_4^l + \frac{1}{2} \kappa_{a5} \chi_5^l \chi_5^l + \frac{1}{2} \kappa_{a6} \chi_6^l \chi_6^l
\]

\[
+ \frac{1}{4} \kappa_{33} \chi_3^l \chi_3^l + \frac{1}{4} \kappa_{44} \chi_4^l \chi_4^l + \frac{1}{4} \kappa_{55} \chi_5^l \chi_5^l + \frac{1}{4} \kappa_{66} \chi_6^l \chi_6^l
\]

\[
+ \frac{1}{2} \kappa_{12} \chi_1^l \chi_1^l + \frac{1}{2} \kappa_{13} \chi_1^l \chi_1^l + \frac{1}{2} \kappa_{14} \chi_1^l \chi_1^l + \frac{1}{2} \kappa_{15} \chi_1^l \chi_1^l + \frac{1}{2} \kappa_{16} \chi_1^l \chi_1^l
\]

\[
+ \frac{1}{2} \kappa_{23} \chi_2^l \chi_2^l + \frac{1}{2} \kappa_{24} \chi_2^l \chi_2^l + \frac{1}{2} \kappa_{25} \chi_2^l \chi_2^l + \frac{1}{2} \kappa_{26} \chi_2^l \chi_2^l
\]

\[
+ \left( \mu_{12}^2 \chi_1^l \chi_1^l + \mu_{13}^2 \chi_1^l \chi_1^l + \mu_{14}^2 \chi_1^l \chi_1^l + \mu_{15}^2 \chi_1^l \chi_1^l + \mu_{16}^2 \chi_1^l \chi_1^l + \mu_{23}^2 \chi_2^l \chi_2^l + \mu_{24}^2 \chi_2^l \chi_2^l + \mu_{25}^2 \chi_2^l \chi_2^l + \mu_{26}^2 \chi_2^l \chi_2^l + \mu_{34}^2 \chi_3^l \chi_3^l + \mu_{35}^2 \chi_3^l \chi_3^l + \mu_{36}^2 \chi_3^l \chi_3^l + \mu_{45}^2 \chi_4^l \chi_4^l + \mu_{46}^2 \chi_4^l \chi_4^l + \mu_{56}^2 \chi_5^l \chi_5^l + \mu_{65}^2 \chi_6^l \chi_6^l \right)
\]

where \( l \) and \( r \) denote SU(3)_L and SU(3)_R indices, respectively, and \( a \) is a scalar flavour index. Fundamental and anti-fundamental SU(3)_L x SU(3)_R indices are written in superscript and subscript, respectively. Note that the potential \( V(\chi_a) \), in the limit \( f_2 \rightarrow 0 \), is invariant under an accidental global U(1)_{acc} phase rotation, which can be defined as

\[
\chi_{1,2} \rightarrow e^{-i q_1, q_2} \chi_{1,2} \, ,
\]

and where the global charges of the \( \chi_1 \) and \( \chi_2 \) fields can be defined as \( q_1 = \frac{1}{2} \) and \( q_2 = -1 \), respectively. Therefore, in the limit \( f_2 \rightarrow 0 \), the vacuum of the theory, after the breaking SU(3)_L x SU(3)_R x U(1)_X x U(1)_{acc} \rightarrow SU(2)_L x SU(2)_R x U(1)_{B-L} , features 11 Goldstone bosons, where one of them becomes physical since it corresponds to the breaking of the global U(1)_{acc} generator. In particular, such a Goldstone boson, which we will denote as \( A \) in what follows, is a CP-odd scalar resulting from a combination of the imaginary parts of the \( (\chi_{1,2})^3 \) components. Furthermore, while \( f_2 \neq 0 \) violates U(1)_{acc}, contractions with the Levi-Civita symbols in both \( f_2 \) and \( f_{12} \) terms imply that the \( (\chi_{1,2})^3 \) components are still protected from acquiring mass. On the other hand, as we have mentioned above, a complete description of the quark mixing, and in particular of the small Cabibbo–Kobayashi–Maskawa (CKM) matrix elements in the 13 and 23 planes in the model under consideration, requires a small cubic interaction of the form \( f_{234} \chi_2 \chi_3 \chi_4 \), which is forbidden by \( Z_2^{(2)} \).

Interestingly, one notices that introducing a small explicit violation of both \( Z_2^{(1)} \) and \( Z_2^{(2)} \) by the following soft-breaking terms

\[
V_{soft}^{Z_2} = \mu_{(\chi)12}^2 (\chi_1)_l^l (\chi_2)_l^l (\chi_2)_l^r (\chi_2)_r^r (\chi_4)_l^l + \text{h.c.}
\]

we are not only allowing for the generation of small entries in the CKM matrix, but also softly breaking U(1)_{acc} by means of \( \mu_{(\chi)12}^2 \) term. The latter promotes the CP-odd scalar to a pseudo-Goldstone boson with mass

\[
m_A^2 = -2 \mu_{(\chi)12}^2.
\]
typically referred to as Majoron. This can be understood from a comparison with the conventional Majoron models with type-I seesaw mechanism [30–38]. There, a complex SM-singlet scalar couples directly to active Majorana neutrinos. However, in our model, in place of a complex singlet we have a bi-triplet, \( \chi \), where the Majoron \( A \), belonging to the \((\chi_2)^3\) component, only couples to the sterile neutrinos in the third entry of both \( L_{L,R} \). In turn, this means that tree-level couplings to the EW gauge bosons are always suppressed by a tiny mixing with active neutrinos suppressing the loop-induced Majoron decays into photons, \( A \to \gamma \gamma \). This implies that a trinification Majoron can become a light DM candidate if its mass is below a MeV scale. This will further be discussed in Sec. VI. Note that due to unbroken CP-symmetry in the scalar sector, this particle only forms quadratic and quartic interactions in the scalar potential of the low-energy effective theory. It is also worth mentioning that (30) prevents the formation of a neutralino, typically referred to as Majoron. This can be understood from a comparison with the conventional Majoron models

\[
\begin{align*}
\mu^2_{(\chi_1)1} &= \mu^2_{(\chi_2)2} \gg \mu^2_{(\chi_1)12}, \quad \kappa_{11}^{(x)} = \kappa_{22}^{(x)} = \kappa_{1}^{(x)} = \kappa_{2}^{(x)} = \kappa_{13}^{(x)} = \kappa_{23}^{(x)} = \kappa_{14}^{(x)} = \kappa_{24}^{(x)} = \kappa_{15}^{(x)} = \kappa_{25}^{(x)} = \kappa_{16}^{(x)} = \kappa_{26}^{(x)} = \kappa_{17}^{(x)} = \kappa_{27}^{(x)}, \\
\kappa_{12}^{(x)} &= \kappa_{13}^{(x)} = \kappa_{14}^{(x)} = \kappa_{15}^{(x)} = \kappa_{16}^{(x)} = \kappa_{17}^{(x)}, \quad \kappa_{21}^{(x)} = \kappa_{22}^{(x)} = \kappa_{23}^{(x)} = \kappa_{24}^{(x)} = \kappa_{25}^{(x)} = \kappa_{26}^{(x)} = \kappa_{27}^{(x)}.
\end{align*}
\]  

Note that such conditions, together with the transformation properties in Tab. I, imply that

\[
v^{(1)} = v^{(2)} = v_x. 
\]  

Solving the tadpole equations with respect to the \( v_x \) and \( v^{(4)}_x \) VEVs we get

\[
\mu^2_{(\chi_1)1} = -\frac{1}{2} \left[ v_x^2 \left( \kappa_{11}^{(x)} + 2 \kappa_{12}^{(x)} + 2 v^{(4)}_x \kappa_{14}^{(x)} + 4 \mu^2_{(\chi_2)12} \right) v^{(4)}_x \right], \quad \mu^2_{(\chi_3)3} = -\frac{1}{2} \left( 2 v_x^2 \kappa_{14}^{(x)} + v^{(4)}_x \kappa_{4}^{(x)} \right). 
\]  

Before proceeding, and in order to understand how the gauge structure and scalar mixing splits the trinification scalar representations, let us first note that we can decompose the \( \chi_x \) in a total of three SU(2)_R × SU(2)_L bi-doublets, three SU(2)_R doublets, denoted as R-doublets in what follows, four SU(2)_L doublets, which we will call L-doublets, as well as eight singlets corresponding to the \((\chi_{1,2,3})_3^3\) and \((\chi_4)_3^3\) components. Provided that the Goldstone bosons correspond to one L-doublet, one R-doublet as well as two real singlets, the physical fields can be decomposed in three bi-doublet blocks, two R-doublet and three L-doublet blocks, and six singlets. Now, one should note that in the vacuum of the theory, the \( f_{234} \) cubic coupling splits two of the bi-doublets into four L-doublets, while the single bi-doublet left in the scalar spectrum results from the fact that there are no VEVs in \( \chi_3 \). For the same reason, out of the six singlets, four are real and two form a complex one charged under U(1)_{B–L}. In summary, we can list the physical scalars after the breaking of the trinification symmetry as

- 1 bi-doublet \( \Sigma \),
- 7 L-doublets \( L_{1,\ldots,7} \),
- 2 R-doublets \( R_{1,2} \),
- 1 complex singlet \( \sigma \),
- 3 real CP-even singlets \( \varphi_{1,2,3} \),
- 1 Majoron \( A \).

1. A minimal light scalar sector

To visualize the model’s behaviour at low-energy scales it is instructive to look at a numerical example. Here, we will use our freedom to set numerical values that: 1) enable us to sufficiently split the mass spectrum in order to obtain a minimal viable low-energy effective theory, and 2) ensure that such a scenario is simple enough to clearly highlight the most important features of the model under consideration. First, we set the following scales

\[
\begin{align*}
\mu^2_{(\chi_1)1} &= -\left( 2.3442 \, \text{TeV} \right)^2, \quad f = 0.1 \, \text{TeV}, \quad f_{234} = 0.01 \, \text{TeV}, \\
v_x &= 160 \, \text{TeV}, \quad v^{(4)}_x = 150 \, \text{TeV}, \quad \mu^{(3)}_{(\chi_3)} = 150 \, \text{TeV}, \quad \mu^{(3)}_{(\chi_3)} = 150 \, \text{TeV}.
\end{align*}
\]
For the quartic couplings, in addition to the simplifying assumptions in Eq. (32), we have also considered that quartic interactions involving one single scalar flavour provide the leading contributions and are of order $O(1)$, while the remaining ones are below $O(0.1)$. Such a behaviour can typically be explained with flavour symmetries engineered to forbid tree-level couplings between different representations of a UV complete theory. For our benchmark example, we choose for the quartics the following sizes:

$$
\begin{align*}
\kappa_1^{(\chi)} &= 1.1, & \kappa_4^{(\chi)} &= 0.95, & \kappa_7^{(\chi)} &= -2.0 \times 10^{-3}, & \kappa_{12}^{(\chi)} &= -6.5 \times 10^{-3}, \\
\kappa_{13}^{(\chi)} &= 8.0 \times 10^{-2}, & \kappa_{14}^{(\chi)} &= -4.0 \times 10^{-2}, & \kappa_{34}^{(\chi)} &= 4.9 \times 10^{-2}.
\end{align*}
$$

(36)

If we now choose a criterion to denote \textit{light states} whenever their mass is below the 10 TeV threshold, the input values in Eqs. (35) and (36) result in a light L-doublet, a light R-doublet and a Majoron with masses

$$
m_{L_1} \approx m_{R_1} \approx 9.7 \text{ TeV} \quad \text{and} \quad m_A \approx 3.3 \text{ TeV},
$$

(37)

respectively. We also obtain five \textit{next-to-light} L-doublets, i.e. with masses between 10 and 20 TeV, whose values read

$$
m_{L_2} \approx 14.2 \text{ TeV}, \quad m_{L_3} \approx 14.4 \text{ TeV}, \quad m_{L_4} \approx 14.6 \text{ TeV}, \quad m_{L_5} \approx 16.2 \text{ TeV}, \quad \text{and} \quad m_{L_6} \approx 19.2 \text{ TeV}.
$$

(38)

Finally, in a third category, we group those states that we denote as \textit{heavy} whose masses read

$$
m_{L_7} \approx 111 \text{ TeV}, \quad m_{R_2} \approx 112 \text{ TeV}, \quad m_{\Sigma} \approx 109 \text{ TeV}, \quad m_\sigma \approx 117 \text{ TeV},
$$

$$
m_{\tilde{\varphi}_1} \approx 103 \text{ TeV}, \quad m_{\tilde{\varphi}_2} \approx 118 \text{ TeV}, \quad m_{\tilde{\varphi}_3} \approx 119 \text{ TeV}.
$$

(39)

Inspired by the numerical example above, we consider a minimal scenario for the SU(2)$_L \times$ SU(2)$_R \times$ U(1)$_{B-L}$ theory, typically referred to as the LR symmetric theory, where the scalar content can be reduced to L$_1$, R$_1$ and A. Note that other parameter choices may provide a low-energy limit with a richer SU(2)$_L$ L-doublet content. In what follows, we recast our L- and R-doublets as L and R respectively. The most generic renormalizable scalar potential can be written as

$$
V_{LR} = \mu_L^2 L^\dagger L + \mu_R^2 R^\dagger R + \mu_A^2 A^2 + \lambda_L |L^\dagger L|^2 + \lambda_R |R^\dagger R|^2 + \lambda_A A^4 \\
+ \lambda_{LR} (L^\dagger L) (R^\dagger R) + \lambda_{AL} A^2 (L^\dagger L) + \lambda_{AR} A^2 (R^\dagger R).
$$

(40)

It follows from Eq. (6) that both the LR and the EW symmetries can be broken by the vacuum assignment

$$
\langle L \rangle = \left( \frac{v}{\sqrt{2}} \ 0 \right), \quad \langle R \rangle = \left( \frac{\omega_\chi}{\sqrt{2}} \ 0 \right),
$$

(41)

where the solutions of the tadpole equations are given by

$$
\mu_L^2 = -\frac{1}{4} \left( 2v^2 \lambda_L + \omega^2 \lambda_{LR} \right), \quad \mu_R^2 = -\frac{1}{2} \left( 2\omega^2 \lambda_R + v^2 \lambda_{LR} \right).
$$

(42)

The two neutral CP-even scalar masses read

$$
m_{H,h}^2 = v^2 \lambda_L + \omega^2 \lambda_R \pm \sqrt{v^4 \lambda_L^2 + v^2 \omega^2 (\lambda_{LR}^2 - 2\lambda_L \lambda_R) + \omega^4 \lambda_R^2},
$$

(43)

where $h$ is the SM-like Higgs boson state, while the CP-odd scalar mass receives extra contributions through the \textit{portal couplings} $\lambda_{AL}$ and $\lambda_{AR}$, acquiring the form

$$
m_A^2 = 2 \left( v^2 \lambda_{AL} + \omega^2 \lambda_{AR} + \mu_A^2 \right).
$$

(44)

Once again, let us provide a numerical estimate, taking a purely classical field theory approach in the sense that the values of the LR theory quartic couplings are directly extracted from the unification scalar potential at tree level. Note that both tree-level and one-loop matching, as well as the Renormalisation Group evolution (RGE) effects, are beyond the scope of this study and will be considered in a future work.

We first consider that the $\omega_\chi$ VEV is developed at the same scale as $\mu_R$. Thus we fix it to

$$
\omega_\chi = 9 \text{ TeV} \quad \text{while} \quad v = 246 \text{ GeV}.
$$

(45)
The quartic couplings of the LR theory, at our level of accuracy, depend solely on the $\kappa_1^{(x)}$, $\kappa_1^{(x)}$, and $\kappa_1^{(x)}$, as well as on the scalar mixing angles of the trinification theory. Using Eq. (36) we get

$$\lambda_L \approx 0.164, \quad \lambda_R \approx 0.135, \quad \lambda_{LR} \approx 0.137, \quad \lambda_{AL} \approx \lambda_{AR} \approx 0.068.$$ (46)

Taking $\mu_A^2 = \mu_{(x)12}^2$ in this example, the scalar masses become

$$m_h \approx 125 \text{ GeV}, \quad m_H \approx 4.7 \text{ TeV}, \quad m_A \approx 184 \text{ GeV},$$ (47)

suggesting that our model is compatible with the Higgs sector of the SM, and offers a new heavy CP-even scalar as well as a Majoron state.

Note that different choices for the size of the $\mathbb{Z}_2^{(1)}$ and U(1)$_{\text{acc}}$ soft breaking parameter $\mu_{(x)12}^2$ yield distinct Majoron masses. We show in Fig. 4 the allowed values of $m_A$ as a function of the size of the soft-breaking parameter $\mu_A$, while keeping all other parameters fixed as in the example above.

![Figure 4: The Majoron mass as a function of the size of the accidental U(1)$_{\text{acc}}$ soft-breaking term.](image)

Note that the numerical example that we have outlined above is simply indicative of the key properties of the model, and a full phenomenological analysis is left for a future work. Let us also mention that, in addition to the SM-like gauge bosons, the model also contains new $W'$ and $Z'$ gauge bosons. Their masses can be either at the $\omega$ scale, if the corresponding gauge couplings are of order unity, or at the TeV scale if such gauge couplings are of order 0.1. Therefore, the gauge sector of our model also offers interesting prospects for the LHC Run-III, which is scheduled to start in 2021.

While a detailed analysis of the FCNC constraints goes beyond the scope of the current work, we can make a generic statement about non-existence of the tree-level FCNCs in our model based upon the Glashow-Weinberg-Paschos theorem \[28, 29\]. This theorem states that there will be no tree-level FCNC processes coming from the scalar sector if all right-handed fermions of a given electric charge couple to only one of the L-doublets. As was demonstrated above the minimal low-energy LR SU(2)$_L \times$ SU(2)$_R \times$ U(1)$_{B-L}$ symmetric scenario in the considered trinification model features the scalar sector composed of one SU(2)$_L$ doublet, one SU(2)$_R$ doublet and one pseudo-Goldstone state. As follows from from Eqs. (8) and (9), the condition of the Glashow-Weinberg-Paschos theorem is automatically satisfied in this case. Possible FCNC contributions would emerge at loop level only rendering the model safe with respect to the corresponding phenomenological constraints.

**VI. DARK MATTER**

The $\mathbb{Z}_2^{(1)}$ symmetry is exact and remains unbroken by any of the VEVs in (6). Therefore, the lightest neutral particle which carries an odd-$\mathbb{Z}_2^{(1)}$ number can be a candidate for DM. The particles carrying the $\mathbb{Z}_2^{(1)}$-odd charge are the components of the scalar bi-triplets $\chi_3$ and the third quark generation, while only the neutral components of $\chi_3$ noted as $\chi_3^{12}, \chi_3^{21}, \chi_3^{23}, \chi_3^{32}$ can potentially contain a DM candidate.
However, each component of the $\chi_3$ bi-triplet couples to a pair of quarks via Yukawa interactions, given in Eq. (8). Namely, the $\chi_3^{12}, \chi_3^{21}$ couple to a pair of light SM quarks whereas the other neutral components of $\chi_3$ bi-triplet couple to a pair of quarks including a light SM quark and an exotic heavy (vector-like) quark. In fact, both $\chi_3^{12}, \chi_3^{21}$ decay into a pair of light quarks and thus cannot serve as DM candidates.

In order to ensure a stability of the remaining neutral components of $\chi_3$ bi-triplet, one should assume that their masses are below the mass of the exotic quarks in order to prohibit their fast tree-level decays. Then, the lightest $\chi_3$ makes it difficult to prevent the heavy scalar components of $\chi_3$ bi-triplet from decaying and hence to stabilise the heavy DM candidate without a significant fine-tuning of the model parameters.

However, in order to get small quark-mixing angles in the 13 and 23 planes at one-loop level one should break softly the $Z_2^{(2)}$ symmetry in the scalar sector by introducing the trilinear $f_{234}\chi_3\chi_3\chi_4$ interaction term. The latter interaction makes it difficult to prevent the heavy scalar components of $\chi_3$ bi-triplet from decaying and hence to stabilise the heavy DM candidate without a significant fine-tuning of the model parameters.

As mentioned in Sec. (V), our model also predicts a CP-odd pseudo-Goldstone Majoron whose mass can vary greatly as shown in Fig. (4). Interestingly, such a state can play a role of light DM candidate under certain conditions in full analogy to the existing Majoron DM scenarios.

Indeed, starting from Yukawa interactions Lagrangian, the couplings of the Majoron to the right-handed neutrinos read

$$ -\mathcal{L}^{(l)} \supset \sum_{a=1}^{3} \sum_{\beta=1}^{3} (x_{\chi})_{a\beta} \bar{L}_a L \chi_3 \bar{L}_R + \text{h.c.} \supset i \nu_L x_{\chi} \left( \frac{M_\nu v^2}{M_S} \right)^2 \nu_R A + \text{h.c.}. $$

(48)

In the seesaw regime, the Majoron can decay into the light neutrinos with partial widths proportional to $m_j^4$,

$$ \Gamma(A \to \nu\nu) \simeq \frac{m_A}{8\pi f^2} \sum_{j=1}^{3} m_j^4 \simeq \frac{1}{3} \times 10^{22} \left( \frac{m_A}{1\text{MeV}} \right) \left( \frac{10^3 \text{GeV}}{f} \right)^4 \left( \frac{\sum_{i=1}^{3} m_i^4}{10^{-6} \text{eV}^4} \right), \quad f \equiv \text{Tr} \left[ \frac{M_S}{\sqrt{\chi}} \right]. $$

(49)

It is straightforward to notice that a light Majoron can easily be long-lived enough to be a DM candidate for typical seesaw scales, assuming that $A \to \nu\nu$ is the main Majoron decay channel.

Just as in the singlet Majoron model [41, 42], the Majoron couples also to the charged fermions, $g_{A\ell\ell}$ through the EW one-loop diagrams, due to the mixing between the new neutral fermions, $N_{a}$, and the active neutrinos $\nu_{a}$. The coupling to quarks is induced by a one-loop $A-Z^0$ mixing with neutrinos being inside the loop, and the coupling to the charged leptons is obtained by an analogue $Z^0$ exchange diagram and additional $V$ exchange graphs, see Refs. [41, 49]. Due to a small coupling of the Majoron to neutrinos, which is suppressed by $(M_{\nu}/M_S)^2$, these diagrams give a rather small contribution to the coupling strength $g_{A\ell\ell}$ [41]. However, the Majoron couples to the exotic quarks and neutral fermions $N_{a}$ that interact with new heavy gauge bosons, $Z_{LR}^L, Z_{R}^L, X_{LR}^{0.04}, \gamma_{LR}^{\pm}$. The additional diagrams of the Majoron coupling to the fermions are predicted such as the $A-Z_{LR}^L, A-Z_{R}^L$ mixing graphs with $N_{a}$ in the loop and the graphs that are mediated by the exotic quarks and the $\gamma^{\pm}$ gauge boson are suppressed by a factor $m_W^4/M_{\gamma_{LR}^{\pm}}^2$. If the new physics scale is of the order of 100 TeV, then the contribution to the effective coupling of the Majoron to the charged leptons is not small enough to ensure that the Majoron’s lifetime is larger than the age of the Universe. Therefore, to prevent the Majoron decays into charged fermions, we need to impose an upper limit on the Majoron mass $m_{A} < 2m_{\tau} \sim 1 \text{ MeV}$ yielding a tantalising possibility for warm DM in our model.

The Majoron would only be considered as a successful DM candidate if its relic density is consistent with cosmological observations [43]. In this sense, the coupling of the Majoron to the SM Higgs plays an important role in order to determine the DM relic density [44, 46, 47]. In the considered model, the Majoron has a quartic coupling with the SM Higgs boson,

$$ V_{LR} \supset \lambda_{LR} A^2 L^L \supset \lambda_{LR} A^2 h + \frac{\lambda_{LR}}{2} A^2 h^2. $$

(50)

Thus, the light Majoron can be produced by the SM Higgs decay, $h \to AA$. The corresponding decay rate is given by [47]

$$ \Gamma(h \to AA) = \frac{1}{16\pi} \frac{\lambda_{LR}^2}{m_h} \frac{v^2}{m_h} \sqrt{1 - 4 \frac{m_h^2}{m_{AA}^2}}. $$

(51)

There are two production mechanisms for DM known as the freeze-out and freeze-in mechanisms. The Majoron cannot be produced by the freeze-out mechanism due to strong constraints from the direct detection measurements and the
LHC bounds on the invisible decay of the SM Higgs \cite{46–48}, while the freeze-in mechanism can efficiently produce the correct DM density. For such a scenario, the Majoron relic density is determined by

\[
\Omega_A h^2 \simeq 2 \frac{1.09 \times 10^{27} m_A \Gamma(h \rightarrow AA)}{g_\ast^2/\sqrt{s}} m_h^2,
\]

where \( g_\ast \) and \( g_c \) are the numbers of degrees of freedom contributing to the entropy and energy densities when the Majoron decouples. To obtain the corrected relic density given by Ref. \cite{43}, we can derive the constraint from Eq. (52) as follows

\[
\lambda_{LR} \simeq 2 \times 10^{-10} \sqrt{\frac{1 \text{MeV}}{m_A}} > 2 \pi \times 10^{-9}.
\]

Note that with a quartic interaction of the Majoron with a SM-like Higgs doublet a possibility for collider searches of DM in the invisible Higgs decay channel is opened. In this case, the DM signature can emerge as missing energy in the production processes at the LHC. Another possibility is via indirect DM detection channels through the relic Majoron scattering off nucleons via \( t \)-channel exchange of the SM Higgs boson. For more detail, see Refs. \cite{44, 45}.

On the other hand, the Majoron couples to two gauge bosons via two-loop diagrams. A detailed analysis of these two-loop contributions has not been performed in this work. However, based upon the results given in Ref. \cite{49} and the new contributions to the effective one-loop couplings specific to the considered model, we estimate the coupling of the Majoron to photons to be very small. This implies that the estimated decay rate \( A \rightarrow \gamma\gamma \) is more suppressed than the corresponding decay into a neutrino pair, \( A \rightarrow \nu\nu \). We conclude that effective Majoron-photon coupling is consistent with astrophysical limits \cite{50} in the considered case of light Majoron, \( m_A < 1 \text{ MeV} \).

\[ \text{VII. LEPTOGENESIS} \]

In this section, we will discuss the implications for leptogenesis of the considered trinification theory. In our model, the right-handed neutrinos carry one unit of \( B-L \) charge and acquire Majorana masses (two units of lepton number) via a radiative correction after the spontaneous \( U(1)_{B-L} \) breaking. This constitutes a source for lepton asymmetries, which must be produced entirely during or after the \( B-L \) symmetry breaking. Therefore, the lepton asymmetry can be realized due to CP-violating decays of the heavy right-handed Majorana neutrinos to one of the SM charged leptons \( l_i \) \((i = 1, 2, 3)\) and the electrically charged Higgs boson, \((\chi_2)_{12}\).

The relevant Yukawa interactions are given by

\[- \mathcal{L}_Y \ni (x_\chi)_{\alpha\beta} \tilde{e}_\alpha L \nu_{\beta R} (\chi_2)_{12} + \text{h.c.}, \]

We assume a normal mass hierarchy for the heavy right-handed Majorana neutrinos, thus implying that the final lepton asymmetry is given only by the CP-violating decay of the lightest one, \((\nu_{R1})\). The CP-asymmetry \( \epsilon_1 \) comes from the interference of tree-level contribution, self-energy correction and the one-loop radiative corrections via diagrams involving the heavier Majorana neutrinos \( \nu_{R2}, \nu_{R3} \). Thus, it can be written as

\[
\epsilon_1 = \frac{1}{16 \pi (x_\chi x_\chi)_{11}} \sum_{j \neq 1} \Im \left[ (x_\chi x_\chi)_{1j}^2 \right] \frac{g(\xi_{j1})}{g(\xi)}, \]

where \( \xi_{j1} = M_j^2/M_{\chi}^2 \), and

\[
g(\xi) = \sqrt{\xi} \left[ \frac{2}{1-\xi} + 1 - (1+\xi) \ln \frac{1+\xi}{\xi} \right].
\]

We would like to note that

\[
\sum_{j \neq k} \Im \left[ (x_\chi x_\chi)_{1j}^2 \right] \sqrt{\xi_{j1}} = \frac{\kappa}{M_{R1}} \sum_{\alpha, \beta} \Im \left[ (x_\chi^* x_\chi^*)_{11} (x_\chi^* x_\chi^*)_{12} M_{\nu R \nu R} \right]
\]

with the assumption

\[
\kappa = 16 \pi^2 \frac{M_{R2}}{m_{N_2} f(m_{N_2}, m_{R(\chi_2)_{13}}, m_{\text{Im}(\chi_2)_{13}})} = 16 \pi^2 \frac{M_{R3}}{m_{N_3} f(m_{N_3}, m_{R(\chi_2)_{13}}, m_{\text{Im}(\chi_2)_{13}})}.
\]
Let us now consider the Dirac term of the neutrino mass matrix

\[ M_{\nu L \nu R} = x_N M_N^D x_T, \quad M_N^D = \text{Diag}(h_{N1}, h_{N2}, h_{N3}) \frac{v_X}{\sqrt{2}} , \quad h_{N_i}^D = \frac{1}{16\pi^2} (x_N \gamma_5 f(m_{N_1}, m_{Re(\chi_2)_{13}}, m_{Im(\chi_2)_{13}})) . \]  

We also assume that all complex scalars acquire complex VEVs, namely, \( v_X = v_X e^{i\theta} \). Thus, we find the diagonalizing matrices \( U_L = O_L U^R_{\text{phase}}, U_R = O_R U^R_{\text{phase}} \) such that

\[ U_L^\dagger M_{\nu L \nu R} U_R = D_{m_{\nu L \nu R}} = \text{Diag}(m_{\nu_1}^D, m_{\nu_2}^D, m_{\nu_3}^D) . \]

If we choose \( U^L_{\text{phase}} = U^R_{\text{phase}} = e^{-i\frac{\pi}{2}} \), and other couplings are real, the matrix \( D_{m_{\nu L \nu R}} \) can be real and written as

\[ D_{m_{\nu L \nu R}} = O_L \left( x_N \text{Diag}(h_{N1}, h_{N2}, h_{N3}) \frac{v_X}{\sqrt{2}} \right) O_R . \]

On the other hand, we assume that \( O_L O^R_{\text{I}} = \text{Diag}(1,1,1) \), which implies that the CP-asymmetry \( \epsilon_1 \) can be rewritten as

\[ \epsilon_1 \simeq \frac{\kappa}{16\pi M_{R_i}} \sum_i \left( x_X^T O_L \right)_{i1} m_{\nu_i}^D \left( O_R^\dagger x_X \right)_{i1} \sum_j g(\xi_{i1}) \sqrt{\xi_{j1}} e^{i\theta} . \]

Therefore, the upper bound on the CP-asymmetry is given by

\[ \epsilon_1^{\text{max}} \simeq \frac{\kappa}{16\pi M_{R_i}} \sum_i m_{\nu_i}^D \sum_j g(\xi_{i1}) \sqrt{\xi_{j1}} . \]  

The lepton asymmetry is related to the observed baryon asymmetry of the universe, given in terms of the baryon number \( n_b \) to entropy \( s \) ratio as follows

\[ \frac{n_b}{s} = -1.38 \times 10^{-3} \epsilon_1 \eta . \]  

Here, the efficiency factor \( \eta \) measures the number density of the right-handed neutrinos with respect to the equilibrium value, the out-of-equilibrium condition at the decay, as well as the thermal corrections to the asymmetry. This factor depends on the effective mass,

\[ \tilde{m} = \frac{D_{m_{\nu L \nu R}} D^T_{m_{\nu L \nu R}}}{M_R} \]  

denoted \( M_{1R} \) in our model, predicted to be around \( 1 \div 10 \) TeV. For \( \tilde{m}_i \simeq (10^{-4} \div 10^{-3}) \) eV, \( \eta \) can be as large as \( O(10^1 \div 10^2) \) \[51\].

We displayed in Fig. (5) the region of parameter space \( M_{R_i} \times \) the values of function \( f(m_{N_i}, m_{Re(\chi_2)_{11}}, m_{Im(\chi_2)_{11}}) \) that yields the right baryon number asymmetry,

\[ \frac{n_b}{s} = (0.87 \pm 0.04) \times 10^{-10} \]  

The allowed value of function \( f(m_{N_i}, m_{Re(\chi_2)_{11}}, m_{Im(\chi_2)_{11}}) \) strongly depends on the efficiency factor \( \eta, \tilde{m}_i \), as well as on the ratio \( m_{N_i}/m_{\nu R_i} \). Comparing the results shown in the left and right panels of Fig. (5), we notice that the allowed value of the function \( f(m_{N_i}, m_{Re(\chi_2)_{11}}, m_{Im(\chi_2)_{11}}) \) decreases sharply as the efficiency factor \( \eta \) or a ratio \( m_{N_i}/m_{\nu R_i} \) decreases. The results show that the model allows to generate the baryon asymmetry if the lightest right-handed neutrino mass, \( M_{R_i} \), is of \( \sim O(1) \div O(10) \) TeV, and the value of the function \( f(m_{N_i}, m_{Re(\chi_2)_{11}}, m_{Im(\chi_2)_{11}}) \) varies from a few units up to a few hundred units.

VIII. CONCLUSIONS

We have built a renormalizable trinification gauge theory with an additional flavor symmetry \( U(1)_X \times Z^2_2 \times Z^2_2 \) at a 100 TeV energy scale, i.e. at a much lower scale than the conventional Grand- Unified field theories imply. The
Figure 5: The value of function $f(m_{N_j}, m_{\nu_{\chi_2}^{(1)\pm}}, m_{\nu_{\chi_2}^{(1)\mp}})$ versus the lightest right-handed neutrino mass, $M_{R_1}$, for different values of the ratio $m_{N_j}/M_{R_j}$, which yields the sufficient baryon number asymmetry $n_b/s = (0.87 \pm 0.04) \times 10^{-10}$. In the left plot, $\tilde{m}_i = 10^{-3}$ eV, $\eta = 10^2$, while in the right plot, $\tilde{m}_i = 10^{-4}$ eV, $\eta = 10$. In both plots, $\xi_{j1} = 10$ is fixed.

low-energy spectra of this theory are shown to be consistent with the SM charged fermion mass hierarchy and the tiny values for the light active neutrino masses. Besides, the model predicts a light Majoron Dark Matter candidate in the mass range below a MeV scale and provides essential means for efficient leptogenesis.

As the main appealing feature of the considered model, the top quark, as well as the exotic heavy fermions, obtain tree-level masses, whereas the SM charged fermions lighter than the top quark get one-loop level masses. The light active neutrino masses are generated from a combination of radiative and type-I seesaw mechanisms, with the Dirac neutrino mass matrix generated at one-loop level. The model yields one naturally light SM-like Higgs boson strongly decoupled from the other heavy scalars as well as the absence of tree-level FCNC processes mediated by the light Higgs state rendering the model safe against existing flavor physics bounds.

The suggested flavoured trinification model can be potentially probed at the Future Circular proton-proton Collider through a discovery of $O(10)$ TeV scale vector-like fermions, scalars and gauge bosons of trinification, while some of the next-to-lightest states in a TeV range can also be probed by future High-Luminosity/High-Energy LHC upgrades.

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