Coherent transport in a system of periodic linear chain of quantum dots situated between two parallel quantum wires

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Abstract. We study coherent transport in a system of periodic linear chain of quantum dots situated between two parallel quantum wires. We show that the resonant-tunneling conductance between the wires exhibits a Rabi splitting of the resonance peak as a function of Fermi energy in the wires. This effect is an electron transport analogue of the Rabi splitting in optical spectra of two interacting systems. The conductance peak splitting originates from the anticrossing of Bloch bands in a periodic system that is caused by a strong coupling between the electron states in the quantum dot chain and quantum wires.

1. Introduction

During the past decade, strong coupling effects in the optics of nanostructures have been a subject of intense interest [1]. Optical interactions between excited dye molecules or excitons in semiconductor nanostructures, and resonant optical cavity modes or surface plasmons can lead to the mixed state with a dispersion characterized by an anticrossing gap (Rabi splitting) in the resonance region [2-25]. A strong coupling regime is established when the coherent energy exchange between two systems exceeds the incoherent losses through radiative or nonradiative mechanisms, while the Rabi splitting magnitude can vary in a wide range depending on the systems involved and coupling type.

In this paper we show that strong coupling effects can be observed in coherent electron transport in periodic semiconductor structures. It has long been known that many features of quantum transport in nanostructures bear substantial similarities to coherent optical processes [26]. Interference of propagating electron waves on the nanoscale [27-29] can give rise to transport counterparts of optical phenomena such as, e.g., Dicke superradiance in resonant tunneling [30], Fabri-Perrot interference in electron waveguides [31], or, more recently, extraordinary electron transmission through a QD lattice [32]. Here we demonstrate that an electron transport analogue of strong coupling regime in optical systems can be realized in a linear periodic quantum dot chain (QDC) [33,34] placed between two parallel semiconductor quantum wires (QW) (see figure 1). When the Fermi energy in QWs is brought into resonance with electron energy levels in QDs, the resonant tunneling conductance between QWs develops a pronounced Rabi splitting of the resonance peak due to strong coupling between electron
states in QDC and one-dimensional electron gas (1DEG) in QWs that causes anticrossing of Bloch bands in the system energy spectrum.

2. Theory

We consider resonant tunneling through a periodic chain of \( N \) QDs with coordinates \( y_j \) and lattice constant \( a \) separated by potential barriers of width \( d \) from the left and right QWs (see inset in figure 1). We adopt the tunneling Hamiltonian formalism [26] with the system Hamiltonian of the form

\[
H = \sum_j E_0 c_j^\dagger c_j + \sum_{k,a} E_k^a c_{ka}^\dagger c_{ka} + \sum_{\alpha j} \left( V_{\alpha j}^a c_j^\dagger c_{\alpha ka} + H.c. \right),
\]

where \( E_0 \) and \( c_j^\dagger (c_j) \) are, respectively, the energy and creation (annihilation) operators for QD localized states, \( E_k^a \) and \( c_{ka}^\dagger (c_{ka}) \) are those for 1DEG states with momentum \( k \) (\( \alpha = L, R \) stands for left/right QW), and \( V_{\alpha j}^a \) is a transition matrix element between localized and 1DEG states. We assume that direct tunneling between QDs is weak and do not include interdot coupling in Hamiltonian (1).

We restrict ourselves to single-electron picture of transport and disregard electron interaction effects due to a low probability of QD double occupancy in a long (\( N \gg 1 \)) chain.

The zero-temperature conductance through a system of \( N \) QDs is given by a standard expression [26]

\[
G = \frac{e^2}{\pi \hbar} \text{Tr} \left( \hat{G}^R \left( \frac{1}{E_F - E_0 - \Sigma} \right) \hat{G}^L \left( \frac{1}{E_F - E_0 - \hat{\Sigma}} \right) \right),
\]

where \( \Sigma_y = \Sigma^L_y + \Sigma^R_y \) is self-energy matrix of QD states due to coupling to left and right 1DEGs,

\[
\Sigma_y^\alpha = \sum_k \frac{V_{ik}^\alpha V_{kj}^\alpha}{E_F - E_k^a + i\gamma_\alpha} = \Delta_y^\alpha - i\frac{\Gamma_y^\alpha}{2},
\]

Here the principal and singular parts of \( \Sigma_y^\alpha \) determine the energy matrix \( \Delta_y^\alpha \) and the decay matrix \( \Gamma_y^\alpha \) respectively, and the trace is taken over QDC sites. The transition matrix element can be presented as [26,30] \( V_{\alpha j}^a = L^{-1/2} e^{i k_a y_j} t_a \) where \( t_a \) is tunneling amplitude between QD and 1DEG and \( L = Na \) is the normalization length. We assumed that the barrier is sufficiently high and so the dependence of \( t_a \) on energy is weak [30]. Then the self-energy (3) takes the form \( \Sigma_y^\alpha = \hat{t}_a^\dagger G_a \left( y_i - y_j \right) \) where \( G_a \left( y_i - y_j \right) \) is the electron Green function in QW.

Due to system periodicity, the coupling between QDC and QWs gives rise to a quasimomentum \( p \) along QDC direction that lies in the first Brillouin zone \( (\pi / a, \pi / a) \) and conserves across the QW/QDC/QW system [32]. In each QW, the 1DEG momentum space \( k \) splits into Bloch bands \( E_{g+p} \), where \( g = k_n \) denotes the Bloch bands \( (k_n = 2\pi / a \) is the reciprocal lattice vector, and \( n \) is an integer). The energy spectrum of QDC states can be obtained by performing the Fourier transform of the self-energy matrix equation (3) as \( \Sigma_y^\alpha = N^{-1} \sum_p e^{i p (y_i - y_j)} \Sigma_p^\alpha \), where \( \Sigma_p^\alpha \) is self-energy in momentum space,

\[
\Sigma_p^\alpha = \frac{\hat{t}_a^\dagger}{a} \sum_g G_{p+g}^\alpha = \frac{\hat{t}_a}{a} \sum_g \frac{1}{E_F - E_{g+p} + i\gamma_a}.
\]
Here $G_{g,p}^\alpha$ is the 1DEG Green function in momentum space for a band $g$ electron with quasimomentum $p$, dispersion $E_{g,p}^\alpha = \hbar^2 (g + p)^2 / 2m_\alpha$ ($m_\alpha$ is the electron mass), and scattering rate $\gamma_\alpha$. The self-energy is a complex function of $p$, $\Sigma_p = \Delta_p^\alpha - i \Gamma_p^\alpha$, whose real and imaginary parts determine, respectively, the QDC states’ dispersion, $E_p = E_0 + \Delta_p^L + \Delta_p^R$, and their decay rate, $\Gamma_p = \Gamma_p^L + \Gamma_p^R$, due to the coupling to left and right QWs.

By performing Fourier transform of equation (2), the conductance can be expressed in terms of system eigenstates as

$$G = NG_0 \int_\alpha dp G_p,$$

where

$$G_p = \Gamma_p^L S_p \Gamma_p^R S_p^\dagger = \Gamma_p A_p$$

is dimensionless partial conductance (transmission coefficient) of the state $p$, and

$$S_p = \frac{1}{E_p - E_p - i \frac{\Gamma_p}{2}}$$

is the QDC Green function. Here we separated out QDC and QW contributions to $G_p$ by introducing spectral function of QDC states, $A_p = -2 \text{Im} S_p$, and the effective rate associated with tunneling time across the system, $\Gamma_p = \left(1 / \Gamma_p^L + 1 / \Gamma_p^R \right)^{-1}$, which is determined by the overlap of QWs’ spectral functions (see below).

![Figure 1. QW/QDC/QW system schematics.](image-url)
3. Numerical Results and Discussions

We present the results of numerical calculations for symmetric configuration, i.e., QDC at the midpoint between similar QWs ($\gamma = \gamma$, $m_a = m$, $t_a = t$). The lattice constant was chosen to set $E_0 = 16E_a$, where $E_a = \hbar^2 k_a^2 / 2m$ is a geometric energy scale associated with the lattice, so that the transmission resonance $E_F = E_0$ occurs at the Fermi momentum $k_F \approx 4k_a$ (two values, $k_F / k_a = 4.0$ and $k_F / k_a = 4.25$, were used). The QW elastic scattering rate, $\gamma = \hbar v_F / l$, where $v_F$ is the Fermi velocity and $l$ is the scattering length, was varied in the range from $\gamma = 0.1E_a$ to $\gamma = 1.1E_a$ (or $\gamma / E_F$ in the range 0.006 to 0.07), yielding $l / a$ in the range from 1 to 10; for QDC period $a \sim 100$ nm, this corresponds to low-to-intermediate mobility in the range $10^4 - 10^6$ cm$^2$/Vs. The QDC tunnel coupling to QWs is characterized by the parameter $\Gamma_a = 2m a^2 / \pi^2 k_a$ related to single QD electron escape rate, $\Gamma_{k_F}$, by replacement of $k_a$ with $k_F$; the latter parameter was chosen to be $\Gamma_{k_F} / E_0 = 0.01$.

![Figure 2](image-url) Figure 2. Normalized conductance vs. Fermi momentum $k_F$ for resonance positions at $k_F / k_a = 4.0$ is shown along with the respective density plots in $(p, k_F)$ plane of joint tunneling rate $\Gamma_p$, in units of $ma^2$ [(b)], QDC spectral function in units of $\Gamma_a^{-1}$ [(c)], and partial conductivity $G_p$ [(d)].
Figure 3. Normalized conductance vs. Fermi momentum $k_F$ for resonance positions at $k_F/k_a = 4.25$ is shown along with the respective density plots in $(p,k_p)$ plane of joint tunneling rate $\Gamma_p$ in units of $ma_a$ [(b)], QDC spectral function in units of $\Gamma^{-1}$ [(c)], and partial conductivity $G_p$ [(d)].

In figure 2 and figure 3 we show conductance along with density plots of $\Gamma_p$, $A_p$, and $G_p$ for resonance positions at $k_F/k_a = 4.0$ (figure 2) and $k_F/k_a = 4.25$ (figure 3). Since the QDC electron escape rates to left/right QW, $\Gamma^{LR}_{p} = -2\text{Im}\Sigma^{LR}_{p}$, are proportional to the respective 1DEG spectral functions [see equation (4)], the effective rate $\Gamma'_p = \left(1/\Gamma_p + 1/\Gamma^{LR}_p\right)^{-1}$ depends on their product and, hence, is strongly enhanced in the regions of $p$-space where the spectral functions’ maxima overlap. In the symmetric case, the left and right 1DEG spectral functions coincide, and so the maxima of $\Gamma'_p$ indicate the Bloch bands in either QW [see panels (b)]. These Bloch bands are prominent in the QDC spectral function $A_p$ as well, shown in panels (c), since tunneling of a QDC electron with quasimomentum $p$ satisfying $E_p = h^2(g+p)^2/2m$ into QWs is greatly enhanced due to momentum conservation across the system. Remarkably, near the resonance $E_p = E_o$, the QDC energy spectrum shows anticrossing of Bloch bands which translates into Rabi splitting of the conduction peak. Note that if $E_o$ coincides with Bloch bands’ intersection [panel (c) in figure 2], there
is condensation of QDC states at exactly \( E_F = E_0 \), but for general QD level position [panel (c) in figure 3.] there are no such states within the anticrossing gap. The partial conductance \( G_p = \Gamma_p A_p \), shown in panels (d), represents, in fact, the map of conducting states in \( (k_F, p) \)-plane, and the normalized per QD conductance [panels (a)] is obtained by \( p \)-integration of \( G_p \) over the Brillouin zone. The conductance shows pronounced Bloch sidebands for values of \( E_F \) (or \( k_F \)) corresponding to Bloch bands’ intersections in \((p, k_F)\)-plane which reflect singularities of 1DEG energy spectrum due to periodic potential. The condensation of QDC states at \( E_0 \) for \( k_F \leq k_a = 4 \) (or any integer) results in a narrow resonance peak [panel (a) on figure 2.], which moves away as the resonance position is shifted [panel (c) on figure 2.]. A striking feature of the conductance lineshape is the emergence of two prominent peaks near the resonance with peak-to-peak separation equal to anticrossing gap in QDC energy spectrum. To estimate this Rabi splitting, e.g., for \( k_F \leq k_a = n \), where \( n \) is an integer, we note that, in this case, the main contribution to electron self-energy (4) comes from terms in the sum with \( g = \pm nk_a \). Keeping only these terms, the resonances in \( A_p \) (for \( p = 0 \)) can be straightforwardly found to occur at \( E_F = n^2 E_a \) (central peak) and \( E_F = n^2 E_a \pm \Delta_R / 2 \), where
\[
\Delta_R = 2 \sqrt{\frac{E_a \Gamma_a - \gamma^2}{\pi}}
\] (7)
is the Rabi splitting. When plotted against \( k_F \), the conduction peak splitting in figure 1(a) is accurately described by \( \sqrt{2m\Delta_R / \hbar} \) for the parameters used.

The underlying mechanism of strong coupling can be traced to conservation of quasimomentum \( p \) across the QW/QDC/QW system that forces a QDC electron only tunnel into QW states with the same \( p \). Since the QW spectral function peaks for \( g + p = k_F \) [panels (b)], QDC states can resonantly couple to Bloch band states in QWs leading to Bloch bands’ anticrossing [panels (c)] and, hence, to Rabi splitting of the conductance peak [panels (a)].

In summary, we have shown that resonant tunneling conductance through a linear periodic chain of quantum dots situated between two parallel quantum wires exhibits multiple Rabi splittings of the conductance peak as a function of Fermi energy that are caused by strong coupling between electronic states in the quantum dot chain and quantum wires.

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