Covariant Description of Flavor Violation at the LHC

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Abstract

A simple formalism to describe flavor and CP violation in a model independent way is provided. Our method is particularly useful to derive robust bounds on models with arbitrary mechanisms of alignment. Known constraints on flavor violation in the $K$ and $D$ systems are reproduced in a straightforward and covariant manner. Assumptions-free limits, based on top flavor violation at the LHC, are then obtained. In the absence of signal, with 100 fb$^{-1}$ of data, the LHC will exclude weakly coupled (strongly coupled) new physics up to a scale of 0.6 TeV (7.6 TeV), while at present no general constraint can be set related to $\Delta t = 1$ processes. $\Delta F = 2$ contributions will be constrained via same-sign tops signal, with a model independent exclusion region of 0.08 TeV (1.0 TeV). However, in this case, stronger bounds are found from the study of CP violation in $D-\bar{D}$ mixing with a scale of 0.57 TeV (7.2 TeV). We also apply our analysis to supersymmetric and warped extra dimension models.

I. INTRODUCTION

The standard model (SM) has a unique way of incorporating CP violation (CPV) and suppressing flavor changing neutral currents (FCNCs). Till today no deviation from the SM predictions related to quark flavor violation has been observed. Regarding the first two generations, models which do not include some sort of degeneracies or flavor alignment (that is, when new physics contributions are diagonal in the quark mass basis) are bounded to a high energy scale. Moreover, contributions involving only quark doublets cannot be simultaneously aligned with both the down and the up mass bases, hence even alignment theories are constrained by measurements. However, the hierarchy problem is not triggered by the light quarks, but rather by the large top Yukawa, where almost any natural new physics (NP) model consists of an extended top sector. Ironically, the top flavor sector is the least understood one, and at present no model independent bound on its coupling is known to exist.

In this work, we formulate a simple and model independent formalism for studying flavor constraints in the quark sector (recent related work about algebraic flavor invariants can be found in [1, 2]). We start with a two generations analysis, where a natural geometric interpretation can be applied. It allows us to straightforwardly reproduce known results [3]. We then consider the three generations case, where a dramatic improvement in the measurements related to the top sector is expected at the LHC. The combination of data from the down and the up sectors is used to robustly constrain models including arbitrary mechanisms of alignment.

In the absence of Yukawa interactions, the SM quark sector possesses a global $G_{SM} = U(3)_Q \times U(3)_U \times U(3)_D$ flavor symmetry, where $Q$, $U$ and $D$ stand for quark doublets, up and down type quark singlets, respectively. $G_{SM}$ is broken by the Yukawa couplings $Y_u$ and $Y_d$, which transform as ($3$, $\bar{3}$, $1$) and ($\bar{3}$, $1$, $3$), respectively, under the flavor group. The spurions $Y_u Y_u^\dagger$ and $Y_d Y_d^\dagger$ are then both in the ($8$+$1$, $1$, $1$) representation. Since the trace of these matrices does not affect flavor changing processes, it is useful to remove it, and work with $(Y_u Y_u^\dagger)_{\alpha\beta}$ and $(Y_d Y_d^\dagger)_{\alpha\beta}$, adjoints of $SU(3)_Q$. For simplicity of notation, we denote these objects as

$$\mathcal{A}_u \equiv (Y_u Y_u^\dagger)_{\alpha\beta}, \quad \mathcal{A}_d \equiv (Y_d Y_d^\dagger)_{\alpha\beta}. \quad (1)$$

II. TWO GENERATIONS

Any hermitian traceless $2 \times 2$ matrix can be expressed as a linear combination of the Pauli matrices. This combination can be naturally interpreted as a vector in 3D real space, which applies to $\mathcal{A}_d$ and $\mathcal{A}_u$. We can then define a length of such a vector, a scalar product, a cross product and an angle between two vectors, all of which are basis independent:

$$|\vec{A}|^2 \equiv \frac{1}{2} \text{tr}(A^2), \quad \vec{A} \cdot \vec{B} \equiv \frac{1}{2} \text{tr}(AB), \quad \vec{A} \times \vec{B} \equiv \frac{i}{2} [B, A],$$

$$\cos \theta_{AB} \equiv \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}, \quad \sin \theta_{AB} = \left| \frac{\vec{A} \times \vec{B}}{|\vec{A}||\vec{B}|} \right|, \quad (2)$$

where the two angle definitions are equivalent. This allows for an intuitive understanding of the flavor and CPV induced by a NP source. Consider a dimension six $SU(2)_L$-invariant operator, involving only quark doublets,

$$\frac{z_1}{\Lambda_{NP}^2} O_1 = \frac{1}{\Lambda_{NP}^2} \left[ \overline{Q}_i (X_Q)_{ij} \gamma_\mu Q_j \right] \left[ \overline{Q}_i (X_Q)_{ij} \gamma^\mu Q_j \right], \quad (3)$$

where $\Lambda_{NP}$ is some high energy scale and $z_1$ is the Wilson coefficient. $X_Q$ is a traceless hermitian matrix, transforming as an adjoint of $SU(3)_Q$ (or $SU(2)_Q$ for two generations).
The contribution to $\Delta c, s = 2$ transitions due to $X_Q$ is given by the misalignment between it and $A_{u,d}$, and it is easy to see that this is equal to

$$|z^{D,K}_1| = |X_Q \times A_{u,d}|^2 / |A_{u,d}|^2 = |X_Q \times \hat{A}_{u,d}|^2,$$

where $\hat{A}_{u,d} \equiv A_{u,d} / |A_{u,d}|$. This result is manifestly invariant under a change of basis. Next we move to CPV

$$\text{Im}(z^{K,D}_1) = 2 \left( X_Q \cdot \hat{J} \right) \left( X_Q \cdot \hat{J}_{u,d} \right),$$

where $\hat{J} \equiv A_d \times A_u / |A_d \times A_u|$ and $\hat{J}_{u,d} \equiv \hat{A}_{u,d} \times \hat{J} / |\hat{A}_{u,d} \times \hat{J}|$. The above spurions and observables are easily described geometrically, say in the $\hat{A}_d - \hat{J} - \hat{J}_d$ space, as shown in Fig. 1. To derive the weakest bound,

![Diagram](image)

**FIG. 1:** Flavor violation in the Kaon system induced by $X_Q$. The overall contribution to $K^0 - \overline{K^0}$ mixing is given by the solid blue line. The CPV contribution, $\text{Im}(z^K_1)$, is twice the product of the two solid orange lines, which are the projections of $X_Q$ on the $\hat{J}$ and $\hat{J}_d$ axes. Note that the angle between $\hat{A}_d$ and $\hat{A}_u$ is twice the Cabibbo angle, $\theta_C$.

we express $X_Q$ in terms of its components

$$X_Q = X^{u,d} \hat{A}_{u,d} + X^{d} \hat{J} + X^{J_{u,d}} \hat{J}_{u,d},$$

where we have $X^u = \cos 2\theta_C X^d - \sin 2\theta_C X^{J_d}$, $X^{J_u} = -\sin 2\theta_C X^d - \cos 2\theta_C X^{J_d}$ and $X^{J}$ remains invariant. Plugging the expression for $X_Q$ from Eq. (6) into Eqs. (1) and (5), one easily reproduces the results of [3] derived in a specific basis.

A new condition for CPV is implied, exclusively related to $\Delta c, s = 2$ processes and not to $\Delta c, s = 1$ ones:

$$X^{J_{u,d}} \propto \text{tr} \left( X_Q [A_{u,d}, [A_d, A_u]] \right) \neq 0,$$

while $X^J \neq 0$ provides a necessary condition for all types of two generations CPV [3]. The conditions are physically transparent and involve only observables, where the weakest bound on NP is derived for the ratio $X^d / X^{J_d}$ given a fixed amount of CPV, $X^J$. Note, however, that this new condition in Eq. (7) is only applicable to either the down or the up sector, while $X^J \neq 0$ is universal.

### III. THREE GENERATIONS

For three generations, a simple 3D geometric interpretation does not naturally emerge anymore, as the relevant space is characterized by the eight Gell-Mann matrices. A useful approximation appropriate for third generation flavor violation is to neglect the eight Gell-Mann matrices, while the breaking of the flavor symmetry is characterized by $[U(3)/U(2)]^2$ [4]. It is especially suitable for the LHC, where it would be difficult to distinguish between light quark jets of different flavor. In this limit the CKM matrix is reduced to a real matrix with a single rotation angle between an active light flavor (say, the 2nd one) and the 3rd generation,

$$\theta \equiv \sqrt{\theta^2_{13} + \theta^2_{23}},$$

where $\theta_{13}$ and $\theta_{23}$ are the corresponding CKM mixing angles. The other generation (the first one) decouples, and is protected by a residual $U(1)_Q$ symmetry [5].

As before, we wish to analyze the flavor violation induced by $X_Q$ in a covariant form. The new contributions to $\Delta t, b = 1$ transitions are characterized by

$$\text{BR}(Q_3 \to Q_1) \propto \frac{4}{3} X_Q \times \hat{A}_{u,d},$$

where $Q_1$ stands for light doublets. We stick to the same definitions as in the two generation part, Eq. (6). In the $[U(3)/U(2)]^2$ limit we can covariantly identify four independent directions out of the eight generators space: $\hat{A}_d, \hat{J}, \hat{J}_d$ and an additional one, $\hat{J}_Q \equiv -2\hat{A}_d + \sqrt{3}\hat{J} \times \hat{J}_d$. Since $\hat{A}_d$ and $\hat{J}_Q$ are not orthogonal, we replace the former with $\hat{A}_Q \equiv \hat{J} \times \hat{J}_d$ (and a similar expression for the up sector). Note that $\hat{J}_Q$ corresponds to the conserved $U(1)_Q$ generator, so it commutes with both $\hat{A}_d$ and $\hat{A}_u$, and takes the same form when interchanging $d \leftrightarrow u$ ($\hat{J}$ also remains the same in both up and down bases, as in the two generations case). There are four additional directions, collectively denoted as $\hat{D}$, which transform as a doublet of the CKM (2-3) rotation, and do not mix with the other directions.

### IV. APPLICATION – THIRD GENERATION DECAYS

We next use measurements of down type FCNC and LHC projection for top FCNC to derive a model independent bound on the corresponding NP scale. We focus on the following operator

$$O_{LL}^p = i \left[ \overline{Q}_i \gamma^\mu (X_Q^{[F=1]})_{ij} Q_j \right] \left[ H^\dagger \overline{D}_\mu H \right] + \text{h.c.},$$

(10)
which contributes at tree level to both top and bottom decays \[1\]. We adopt the weakest limits on the coefficient of this operator, \( C_{LL}^b \), derived in \[1\]:

\[
\text{Br}(B \to X_s \ell^+ \ell^-) \to |C_{LL}^b|_b < 0.018 \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2,
\]

\[
\text{Br}(t \to (c, u) Z) \to |C_{LL}^b|_t < 0.18 \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2,
\]

(11)

where the latter is based on the prospect for the LHC bound in the absence of signal, with 100 fb\(^{-1}\), and we define \( r_{tb} = |C_{LL}^b|_b / |C_{LL}^b|_t \).

The NP contribution can be decomposed in the covariant bases

\[
X_Q = X^{tu \cdot d} \hat{A}_{u \cdot d} + X^J \bar{j} + X^{J_{u \cdot d}} \bar{J}_{u \cdot d} + X^{J_{b \cdot d}} \bar{J}_b + X^{\bar{D} \bar{D}}.
\]

(12)

The weakest bound is obtained, for a fixed length \( L \equiv |X_Q| \), by finding a direction of \( X_Q \) that minimizes the contributions to \( |C_{LL}^b|_b \) and \( |C_{LL}^b|_t \). It is clear, however, that directions that contribute to first two generations flavor and CPV at \( O(\lambda_C) \) (\( \lambda_C \sim 0.23 \)) are strongly constrained. Thus, the resulting bounds would not correspond to the best alignment case. For example, when only \( X^{J_Q} \neq 0 \), no third generation flavor violation is induced. However, switching back on the light quark masses, \( X^{J_Q} \) (more precisely, a combination of \( X^{J_{Q}} \) and \( X^{J_{d}} \)) does induce flavor violation between the first two generations. At best it can be aligned with the down mass basis, so that it contributes to \( \Delta c = 1 \) transition at \( O(\lambda_C) \). The corresponding bound is \[5\]

\[
L < 0.59 \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2; \quad \lambda_{NP} > 1.7 \text{ TeV},
\]

(13)

where the latter is for \( L = 1 \). Similarly, it can be shown that \( X^\bar{D} \) yields \( 2 \to 1 \) transitions when the contributions to third generation decays are minimized. These cases, therefore, do not represent the best alignment scenario.

The induced flavor violation is then given by

\[
\frac{4}{3} \left| X_Q \times \hat{A}_{u \cdot d} \right|^2 = (X^J)^2 + (X^{J_{u \cdot d}})^2,
\]

(14)

and

\[
X^{J_{d}} = \cos 2\theta X^{J_d} + \sin 2\theta X^{J_d},
\]

(15)

From the above relations it is clear that \( X^J \) contributes the same to both rates, so it should be set to zero for optimal alignment. Thus the best alignment is obtained by varying \( \alpha \), defined by

\[
\tan \alpha \equiv X^{J_d} / X^d,
\]

(16)

where \( X^d \) is the coefficient of \( \hat{A}_d \), which is the generator that does not produce flavor violation among the first two generations to leading order (up to \( O(\lambda_C^5) \)). We now consider two possibilities: (i) complete alignment with the down sector; (ii) the best alignment satisfying the bounds of Eq. (11), which gives the weakest unavoidable limit. The bounds for these cases are

i) \( \alpha = 0 \), \( L < 2.5 \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2 \); \( \lambda_{NP} > 0.63 \text{ (7.9) TeV} \),

(17)

ii) \( \alpha = \frac{\sqrt{3} \theta}{\sqrt{1 + \theta^2}}, \quad L < 2.8 \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2 \); \( \lambda_{NP} > 0.6 \text{ (7.6) TeV} \),

as shown in Fig. 2 where in parentheses we give the strong coupling bound, in which the coefficient of the operators in Eqs. (3) and (10) is assumed to be \( 16\pi^2 \).

Note that these are weaker than the bound in Eq. (13).

![FIG. 2: Upper bounds on L as a function of α, coming from the measurements of flavor violating decays of the bottom and the top quarks, assuming \( \lambda_{NP} = 1 \text{ TeV} \).](image)

It is important to mention that the optimized form of \( X_Q \) generates also \( c \to u \) decay at higher order in \( \lambda_C \), which might yield stronger constraints than the top decay. In \[5\] it is shown that the bound from the former is actually much weaker than the one from the top, as a result of a \( \lambda_C^2 \) suppression. Therefore, the LHC is indeed projected to strengthen the model independent constraints.

V. THIRD GENERATION \( \Delta F = 2 \) TRANSITIONS

Next we analyze \( \Delta F = 2 \) processes, where for simplicity, we only consider complete alignment with the down sector,

\[
X_Q^{\Delta F = 2} = L \hat{A}_d,
\]

(18)

as the constraints from this sector are much stronger. This generates in the up sector \( D^0 - \bar{D}^0 \) mixing and top flavor violation. Yet, there is no top meson, so we analyze instead the process \( uu \rightarrow tt \), which is most appropriate

\footnote{It is important to note that a given new physics model might generate different higher-dimensional operators via different types of processes. Therefore \( X_Q \) is in general different for each operator, so we denote it specifically as \( X_Q^{\Delta F = 1} \) for the current case.}
for the LHC (and related to mixing by crossing symmetry). This process was studied in the literature in the context of different models (see e.g. [7, 9] and refs. therein). It is observed through the dilepton mode, in which two same-sign leptons are produced from the top quarks. We emphasize that in this case the parton distribution functions of the proton strongly break the approximate $U(2)$ symmetry of the first two generations. Thus, a useful bound is obtained only from the operator involving up (and not charm) quarks.

In order to estimate the prospect for the LHC bound on same-sign tops production, we calculated the $uu \rightarrow tt$ cross section using MadGraph/MadEvent [10], as a $t$ (or $u$) channel process mediated by a heavy vector boson, matched onto the operator in Eq. [3]. We then used the fact that the cross section times the integrated luminosity must be lower than 3 for a 95% exclusion, in the absence of signal [11]. Adding an assumption of 1% signal efficiency [7], after background reduction, we have

$$z_{tt}^D < 7.1 \times 10^{-3} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2 ,$$

for 100 fb$^{-1}$ at a center of mass energy of 14 TeV. The experimental constraint from CPV in the D system is [12]

$$\text{Im}(z^D) < 1.1 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2 .$$

The contribution of $X_Q^{DF=2}$ to these processes is calculated by applying a simple CKM rotation, and then taking $\text{Im} \left( (X_Q^{DF=2})^2 \right)$ for CPV in $D$ mixing and $\left| (X_Q^{DF=2})_{12} \right|^2$ for $uu \rightarrow tt$. The resulting bounds are

$$L < 1.8 \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right) ; \quad \Lambda_{NP} > 0.57 \text{ (7.2) TeV} ,$$

$$L < 12 \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right) ; \quad \Lambda_{NP} > 0.08 \text{ (1.0) TeV} ,$$

for $D$ mixing and $uu \rightarrow tt$, respectively. Note that the latter bound depends on the quartic root of the cross section that was evaluated above, thus it is only mildly sensitive to that calculation and to the efficiency assumption. Anyway, in this case the existing bound is stronger than the one which will be obtained at the LHC for top quarks, as opposed to $\Delta F = 1$ case considered above.

VI. SUPERSYMMETRY

We now consider the application of our formalism to $\Delta F = 2$ transitions in supersymmetry (constraints from $\Delta F = 1$ processes are more involved, due to a richer operator structure, and discussed in [5]). We use the approximation of quasi degenerate squark masses (see e.g. [13]), and consider the leading order in the expansion $\Delta \tilde{m}^2_{Q_2Q_1}$, $(\Delta \tilde{m}^2_{Q_2Q_1})$, is the mass-squared difference between the $i$th and $j$th squarks), where the level of degeneracy is much stronger [3]. We arrive at the following expression for the length of $X_Q$

$$L = \frac{\alpha_s}{18} \sqrt{\frac{g(x)}{2}} \frac{\Delta \tilde{m}^2_{Q_2Q_1}}{m_Q^2} ,$$

where $x = m^2 / \tilde{m}_Q^2$, $m_Q$ is the gluino mass and $g(x)$ is a known kinematic function [13]. Taking for concreteness, $\tilde{m}_Q = \left( 2m_{Q_1} + m_{Q_3} \right) / 3$ (appropriate for models with only weak degeneracy [14]), $\tilde{m}_Q = 100$ GeV and $m_Q \approx \tilde{m}_Q$, which implies $g(1) = 1$, we find

$$\left| \frac{m^2_{Q_2} - m^2_{Q_1}}{2m_{Q_1} + m_{Q_3}} \right| < 0.45 \left( \frac{\tilde{m}_Q}{100 \text{ GeV}} \right)^2 .$$

VII. WARPED EXTRA DIMENSION

Another example for a concrete model that is constrained by measurements is the Randall-Sundrum (RS) framework [15]. When the fermions are allowed to propagate in the bulk, their localization yields mass hierarchies and mixing angles, thus addressing the flavor puzzle. The $\Delta F = 2$ process is induced at tree level by a Kaluza-Klein (KK) gluon exchange. The $\Delta F = 1$ operator in Eq. [10] is generated, among others, via mixing between the SM Z and its KK excitations, which results in a non-diagonal coupling in the mass basis [16, 17]. For simplicity, we only focus below on these contributions, as the others are of the same order [17]. For the $\Delta F = 2$ case we have

$$m_{KK} = \Lambda_{NP} , \quad X_Q \cong \frac{g_{ss}}{\sqrt{6}} \text{ diag}(f^2_{Q_1}, f^2_{Q_2}, f^2_{Q_3}) ,$$

before removing the trace, where $g_{ss}$ is the dimensionless 5D coupling of the gluon ($g_{ss} \approx 3$ at one loop [18]) and the $f_{Q_i}$’s are the values of the quark doublets on the IR brane. These are related to each other through the CKM elements $-f_{Q_i}Q^2_i/f_{Q_3} \sim V_{ub}, V_{cb}$. The resulting limit is

$$m_{KK} > 0.4f^2_{Q_3} \text{ TeV} ,$$

where $f_{Q_3}$ is typically in the range of $0.4-\sqrt{2}$. For the $\Delta F = 1$ process we find

$$X_Q \cong g_{Zs} \delta g_Z \text{ diag}(f^2_{Q_1}, f^2_{Q_2}, f^2_{Q_3}) ,$$

where $g_{Zs}$ is the dimensionless 5D coupling of the Z to left-handed up type quarks ($g_{Zs} \approx 1.2$ at one loop) and $\delta g_Z \cong \log(M_{NP}/\text{TeV}) (m_Z/m_{KK})^2$ describes the non-universal coupling coming from mixing between the different Z states. The bound that stems from this is

$$m_{KK} > 0.33f^2_{Q_3} \text{ TeV} .$$
VIII. CONCLUSIONS

We find that projected LHC bounds on $\Delta t = 1$ processes enable us to provide a new model independent constraint on the strength of left-handed quarks flavor violation, even in the presence of general flavor alignment mechanisms. The projected bound on $\Delta t = 2$ transitions from same sign tops production at the LHC is also studied. In this case a surprising result is that a stronger robust bound already exists due to the experimental constraint on CP violation in $D - \bar{D}$ mixing. We use our analysis to obtain new limits on supersymmetric and warped extra dimension models of alignment, which are rather weak (as a result of the weaker experimental constraints, compared to the first two generations – see e.g. in [3]), but replacing practically non-existing current bounds.

Acknowledgments G.P. is supported by the Israel Science Foundation (grant #1087/09), EU-FP7 Marie Curie, IRG fellowship and the Peter & Patricia Gruber Award.

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