Toward a leading indicator of catastrophic shifts in complex systems: Assessing changing conditions in nation states

Leisha Vance, Tarsha Eason, Heriberto Cabezas, Michael E. Gorman

Abstract

The 20th century was characterized by substantial change on a global scale. There were multiple wars and unrest, social and political transitions, technological innovation and widespread development that impacted every corner of the earth. In order to assess the sustainability implications of these changes, we conducted a study of three advanced nations particularly affected during this time: France, Germany and the United States (USA). All three nation states withstood these changes and yet continued to thrive, which speaks to their resilience. However, we were interested in determining whether any of these countries underwent a regime shift during this period and if they did, whether there was advanced warning that transition was imminent. This study seeks to evaluate systemic trends in each country by exploring key variables that describe its condition over time. We use Fisher Information to assess changing conditions in the nation states based on trends in social, economic and environmental variables and employ Bayes Theorem as an objective means of determining whether declines in Fisher information provide early warning signals of critical transitions. Results indicate that while the United States was relatively stable and France experienced a great
deal of change during this period, only Germany appeared to undergo a regime shift. Further, each country exhibited decreasing Fisher information when approaching significant events (e.g., World Wars, Great Depression), and reflected unique mechanisms linked to dynamic changes in each nation state. This study highlights the potential of using trends in Fisher information as a sentinel for evaluating dynamic change and assessing resilience in coupled human and natural systems.

Keywords: Information science

1. Introduction

Numerous factors influence change in complex systems. In the context of nation states, the linkages between social, political, economic and environmental systems as well as their impact on stability and sustainability are of growing interest (Lubchenco, 1998; Goodland, 1995; Karunanithi et al., 2011). Hence, studies on trends in key indicators, and how they affect movement toward and away from sustainability are critical and highlight the need for methods that aid in assessing and managing patterns of growth and development (Kates and Parris, 2003). Sustainability relates to finding and maintaining a set of system conditions (i.e., a regime) that can support social and economic development in human and ecological systems without major, irrecoverable environmental consequences (Karunanithi et al., 2008). Accordingly, sustainability is impossible to maintain or achieve without resilience. Simply stated, resilience relates to the capacity of a system to withstand change and continue to function (Holling, 1973). Resilience research is a burgeoning area of study buoyed by the growing need for managing human and natural systems given the inevitability of change. Accordingly, assessing dynamic regimes and regime shifts is an important element in studying sustainability because it provides insight on the relationships and organizational dynamics necessary to support current and future generations (Eason and Cabezas, 2012).

There is great debate on how system condition and regime shifts should be assessed in time varying systems, particularly because these systems, by nature, are always changing, i.e. they typically experience periodic fluctuations (Scheffer et al., 2009). Hence, it is not obvious when a major event or a regime shift is in progress, principally during the early stages of the transition. A regime is typically characterized by observable patterns, which may fluctuate within some range of variation while maintaining an overall condition. We define an orderly dynamic system as one that has persistent observable patterns (Eason et al., 2016). More stable patterns represent a more orderly system.

If a dynamic system is perturbed to the point that it changes from one attractor to another with a different set of observable patterns (i.e. fundamental changes in the...
dynamics of the system and how it operates), the process typically involves traversing a threshold before shifting to a new regime governed by a new set of processes, patterns and structures. There is consensus that: (1) a system regime has distinguishing patterns and therefore characteristic order, (2) two different dynamic regimes of a system do not generally operate in line with the same patterns and accordingly, do not have the same degree of order, (3) the transition between two different regimes usually involves a period with lower order than either regime, and (4) dynamic systems must maintain some level of order to continue to function (Fath et al., 2003; Karunanithi et al., 2008). These principles have been demonstrated through numerous empirical studies for a variety of systems (Mayer et al., 2007; Karunanithi et al., 2008; Rico-Ramirez et al., 2010; Spanbauer et al., 2014; Eason et al., 2016; Sundstrom et al., 2017) and explored more fully in Gonzalez-Mejia et al. (2015). Natural processes such as volcanic eruptions, changes in water availability, as well as anthropogenic influences such as wars, hunting practices and pollution may cause regime shifts in multiple ways, sometimes decimating entire populations, e.g. the Anasazi Indians and the American Bison (Diamond, 2005; Records, 1995).

Regime shifts have often occurred in natural and human systems. For example, Haiti, experienced a regime shift when the supply of wood was exhausted. Consequently, what was once lush tropical forest is now dominated by a desolate infertile landscape unable to adequately support the Haitian population (Haggerty, 1989). It is suspected that the loss of water caused the disappearance of the Anasazi population from the American West (Gumerman, 1988). When a lake or river experiences a large, continuous influx of phosphorous from agricultural runoff, it is possible for the water body to reach a tipping point which causes the system to shift from an oligotrophic (high oxygen content and fish populations) to eutrophic state (i.e. algal overgrowth, impaired water quality adversely impacting fish and wildlife populations) (Carpenter, 2003; Maler, 2000). Reduced predator populations often have led to increased and overabundant prey populations, which in turn exhaust natural resources, forcing a regime shift (Holling Crawford, 1965). As experienced by Europeans during the Middle Ages, the Irish during the Great Potato Famine, and in the disintegration of the Mayan civilization, wars, famine, and disease seem to have caused regime shifts as well, oftentimes decimating entire human populations. (Hays, 2005; Kinealy, 1995; Diamond, 2005).

Human beings are now thought to live in the “Anthropocene,” a historical epoch where human activity affects every corner of the global ecosystem. Therefore, human and natural system perturbations are closely coupled, and linked to technological evolution (Allenby, 2013). Since regime shifts typically involve a loss of order and function potentially resulting in large ecological and economic consequences (e.g., declining fish stocks, decimated populations), early identification of warning signals is critically important.
Numerous traditional indicators have been proposed and studied as regime shift indicators, including variance, skewness, kurtosis, and critical slowing down (e.g. Kleinen et al., 2003; Oborny et al., 2005; Carpenter and Brock, 2006; Dakos et al., 2008; Biggs et al., 2009; Scheffer et al., 2012). They have often been applied to evaluate model or simplified real systems (i.e., those with few variables); however, researchers note that it is not clear whether these approaches are useful for evaluating real, complex multivariate systems (Scheffer et al., 2009; Dakos et al., 2012; Batt et al., 2013; Eason et al., 2014) because drivers often are unknown (Bestelmeyer et al., 2011; Brock and Carpenter, 2012). Moreover, Biggs et al. (2009) found that these approaches often do not provide warning of a shift until it is well underway and too late for management intervention.

Researchers have noted that characterizing the behavior of these systems requires simultaneously tracking numerous and often disparate variables over time to expose patterns that reveal change in the condition of the system (Eason and Cabezas, 2012). Fisher information, an information theory approach, provides a means of quantifying changes in the organizational dynamics of complex systems (Karunanithi et al., 2008) and may provide key insights for studying stability in complex systems. Fisher information has been used to assess a number of systems for multiple purposes. It has been used as a tool to derive many laws of nature (Frieden, 1998, 2001), and was adapted into a method for assessing dynamic order in model and real world systems, including a simple predator-prey model and multi-species model food webs, as well as real-life systems, e.g., the Bering Strait ecosystem, the western African savannah and the U.S.-Florida pine-oak ecosystem (Mayer and Pawlowski, 2006). It has also been used as a sustainability metric, employed as a tool for monitoring and managing environmental and regional systems (Shastri et al., 2008a, 2008b; Cabezas and Eason 2010; Eason and Cabezas, 2012; Gonzalez-Mejia et al., 2015) and used to study political instability in various nations throughout the world (Karunanithi et al., 2011). Further, it has been demonstrated as a method of assessing changing conditions in urban systems (Gonzalez-Mejia, 2011; Eason and Garmestani, 2012; Gonzalez-Mejia et al., 2012a, 2012b, 2014) and proposed as a leading indicator of temporal and spatial regime shifts (Karunanithi et al., 2008; Eason et al., 2014; Spanbauer et al., 2014; Eason et al., 2016; Sundstrom et al., 2017).

Eason et al. (2014) explored the relationship between Fisher information and traditional regime shift indicators, and comparatively assessed their performance and efficacy in evaluating both simple and complex systems. For simple systems (defined by one or few variables), their results indicated that Fisher information tended to identify the regime shifts synchronously or sometimes before the traditional indicators. However, the true power of the method is demonstrated when evaluating complex multivariate systems. Since it collapses the behavior of many variables into an index that can be tracked over time, it is able to easily handle
multivariate systems and identify patterns where traditional indicators seem to fail or render inconclusive results (Eason et al., 2014; Spanbauer et al., 2014). For example, rising variance may be present in one system variable but not others (Eason et al., 2014; Spanbauer et al., 2014). This underscores the sentiment by Scheffer et al. (2009) and Dakos et al. (2012) that more work is needed to determine if traditional indicators are useful for evaluating real, complex systems. Further, since declines in Fisher information signify loss of dynamic order and function (Karunanithi et al. (2008)), Eason et al. (2014) proposed exploring these declines as possible warning signals of an impending transition. However, it is critical that mechanisms be developed that afford the ability to determine the significance of declines in Fisher information and assess whether they are the function of normal variation or provide warning of major or disastrous events soon to come.

In order to determine if declines in Fisher information are significant and whether they provide appreciable warning signals, Bayes theorem was employed. Bayes Theorem was developed by Thomas Bayes (1763) and provides a means of assessing the likelihood of event A occurring if event B has already transpired. Here, we use a frequentist interpretation of the theorem to estimate whether a decline in Fisher information is likely to be significant.

In this study, we (1) applied Fisher information to evaluate France, Germany and the United States by exploring key variables which describe their condition over time and determine whether these nations experienced actual regime shifts during the 20th century, and (2) used Bayes theorem to assess the likelihood that developing trends captured by Fisher information provide warning signals of impending regime shifts. Using Fisher information trends, we define regime shifts as a transition from a period characterized by relatively stable non-zero Fisher information values to progressive declines in the index followed by increasing Fisher information and a recovery to a stable non-zero Fisher information afterwards. France, Germany and the United States were selected for this study because they are industrialized, well-established countries, with a strong history of recordkeeping during the study period (1900–2000). Accurate record keeping was important to ensure adequate data were available to support analyses and conclusions. Moreover, all three nations underwent several major and potentially catastrophic events during the 20th century, which makes them viable candidates for possible regime shift occurrences.

2. Methodology

2.1. Approach

Fisher Information is an information theory approach and provides a method of assessing and quantifying dynamic order (patterns) in both simple and complex systems (Cabezas and Fath, 2002). It was originally developed by statistician Ronald Fisher as a measure of the amount of information present in a data set that
could be used to estimate a parameter (Fisher, 1922; Fath et al., 2003). It has since been adapted to create an integrated metric (index) that monitors system variables in order to characterize changes in system condition, including regimes and regime shifts (Cabezas et al., 2003; Karunanithi et al., 2008). The form of Fisher information used in this study is defined in Eq. (1) as a function of the probability density \( p(s) \) of observing the system in a particular state \( s \):

\[
I = \int ds \frac{[dp(s)]^2}{p(s)}
\]  

(1)

Here, a state of the system \( s \) is defined as a point in a state space where the dimensions are the measurable variables of the system \( x_1, x_2, \ldots, x_n \) over time \( t \). The trajectory of the system is illustrated by plotting the \( n \) system variables in a series of points \( s(x_1(t), x_2(t), \ldots, x_n(t)) \) as a function of time. This is illustrated in Fig. 1 where the system trajectory is shown as a sequence of dots each representing a state of the system \( s(x_1(t), x_2(t), \ldots, x_n(t)) \). Now, because all measurements inherently contain uncertainty, a tolerance is applied to the definition of a state such that system variables are able to fluctuate within some range. In fact, any two points which are within measurement error of each other can be considered to be repeat observations of the same point. This means that the states of the system are not strictly points but more like boxes (or hyperboxes) as shown in Fig. 1, and the likelihood of observing a particular state of the system is then assumed to be

![Fig. 1. Illustration of the trajectory over time for a dynamic system with \( n \) variables where each point in the trajectory depends on the variables \( x_1, x_2, \ldots, x_n \) and time \( t \), and each state of the system \( s \) is shown as a box. The likelihood of observing a particular state of the system (box) depends on how many of the points lie within the state.](image-url)
proportional to how many observations (points) lie inside the state (box). In this manner, it is possible to estimate the probability density of observing any particular state (box) by counting the number of points inside the state and dividing by the total number of points within a particular window of evaluation. Once the probability density for observing states \( p(s) \) is computed, the index is calculated from the discrete form of Fisher information given by Eq. (2):

\[
I \approx 4 \sum_{s=1}^{m} [q(s) - q(s + 1)]^2
\]

(2)

where the sum is taken over all possible states \( m \) of the system, and \( q(s) \) is the amplitude \( q(s) \equiv \sqrt{p(s)} \) of the probability density function for states of the system.

Details on the theoretical aspects of Fisher information, subsequent derivations and computation methodology may be found in Mayer et al., 2007; Karunanithi et al., 2008; Cabezas and Eason, 2010.

Fisher information (henceforth referenced as FI) is calculated over time by separating the variable time series for the system into a sequence of overlapping windows. Each point in the window is then “binned” into a state of the system and a FI value is computed for each window. The window is then moved forward one time step and the process is repeated. The FI value for each time window is assigned to the last time step of each time window so that only “past” data is used in the computation (Gonzalez-Mejia et al., 2015). The basis for interpreting FI trends is understanding that stable regimes are denoted by distinct patterns which may experience natural fluctuations, but the overall condition does not change with time; hence, system regimes have relatively stable non-zero FI (i.e., \( dFI/dt \approx 0 \)).

Regime shifts are typically noted when there are declines in FI between two otherwise steady regimes denoted by stable FI over time (Karunanithi et al., 2008; Eason and Cabezas 2012). In this work, we examine a specific level of decline which would signal a regime shift: two standard deviations below the mean FI. The criteria of two standard deviations was established in accordance with Chebyshev’s theorem, a non-parametric mechanism for assessing statistical variation which states that at least \( 1-1/k^2 \) observations lie within \( k \) standard deviations of the mean (Lapin, 1975: p. 58). Hence, two standard deviations would encompass about 75% of the FI points and a regime shift point is denoted as a variation in FI outside of this range (Gonzalez-Mejia et al., 2012a). This criterion provides a more objective process for assessing the difference between random variation in dynamic order and unusual events (e.g. regime shifts). Once a system shifts, it may not return to the previous state. Since human and natural systems are typically quite resilient, they may reorganize into a new steady state with distinctive patterns, however, the new conditions may not be desirable (Eason and Cabezas, 2012). Note that while high FI denotes predictable patterns, it is most critical that desired system conditions remain intact and exhibit stability such that \( d(FI)/dt \approx 0 \) Accordingly, when evaluating multiple system regimes, the variation in FI (i.e. standard
deviation in FI: \( \sigma_{FI} \) is used as a measure of stability (Gonzalez-Mejia, 2011, Gonzalez-Mejia et al., 2012b).

### 2.1.1. Bayes theorem

As previously mentioned, this work uses a frequentist interpretation of Bayes Theorem in which the probability is determined through a proportion of outcomes (Berry and Stangl, 1996; Ellison, 1996). Eq. (3) provides the simplest form of Bayes Theorem:

\[
P(A|B) = \frac{P(A)P(B|A)}{P(B)}
\]

where \( P(A|B) \) is the probability of event \( A \), given the previous occurrence of event \( B \); \( P(B|A) \) is the probability of event \( B \), given the occurrence of event \( A \); \( P(A) \) is the probability of event \( A \), and \( P(B) \) is the probability of event \( B \). From a practical standpoint, the simple probabilities \( P(A) \) and \( P(B) \) are determined by counting the number of times event \( A \) (or \( B \)) occurred divided by the total number of events, however, the conditional probabilities \( P(A|B) \) and \( P(B|A) \) require further assumptions which will be discussed later.

To explore whether FI provides early signs of critical transitions, we extend the analysis to assess the probability and intensity of declines in FI. We presented a short case study of this approach in Gonzalez-Mejia et al. (2015), but here we expand the application to comparatively assess multiple systems and examine not only declines but slopes of declines. Bayes theorem is employed as a means of assessing the probability that a decline or series of declines in FI provides warning of an impending regime shift (RS). Eq. (4) is the Bayes theorem expression for assessing the likelihood of a regime shift (RS) if a single decline (\( D_1 \)) has occurred.

\[
P(RS|D_1) = \frac{P(RS)P(D_1|RS)}{P(D_1)}
\]

A decline in FI is defined as a decrease in FI from the initial point of comparison to the next FI value \( \Delta FI < 0 \). Many systems experience single declines, however, degrading systems often undergo a series of modest declines or sometimes a few sharp decreases in FI. Eq. (5) replaces \( D_1 \) in Eq. (4) to assess the probability of a regime shift if two consecutive declines (\( D_2 \)) have been observed.

\[
P(RS|D_2) = \frac{P(RS)P(D_2|RS)}{P(D_2)}
\]

To determine the probability that a nation state experienced a shift during the time period of the study, we established criteria to identify a regime shift when the FI value drops more than (1) one standard deviation below average FI, and (2) two standard deviations below average FI for the entire time period followed by a rise
or recovery in Fisher information. We decided to explore a one standard deviation drop below average as a less strict criterion that may represent a regime shift or at least a significant event.

Since it is possible for a system to “quickly” undergo a transition reflected by sharp declines in FI, Bayes theorem was also used to assess the probability that particular declines in FI are significant. In this case, a severe decline event (SDE) is defined as a decline slope that is more than one or two standard deviations greater than the average slope of all FI declines within the time period of study. The appropriate Bayes’ Theorem expression is given by Eq. (6) as:

\[
P(SDE|D1) = \frac{P(SDE) P(D1|SDE)}{P(D1)}
\]

which denotes the probability of a SDE if a single decline has occurred. While, these significant declines may or may not meet the regime shift criteria of two standard deviations from the mean FI, they quantify the fact that relatively abrupt change and rapid loss in the dynamic order in the system may represent important events.

2.2. Data

Time series data for demographic, labor force, agriculture (crops and livestock), industry, external trade, transport/communication, finance, prices, and education variables were compiled and used to represent conditions in France, Germany and the United States from 1900 to 2000. In order to ensure consistency, the same 34 variables (Table 1) were chosen and data was collected from the same data source, the International Historical Statistics series for Europe and the Americas (Mitchell, 2007a, 2007b). On occasions when gaps occurred within the time series (i.e., no data were reported), interpolation was performed using the PCHIP (piecewise cubic hermite interpolating polynomial) method included in MATLAB (Fritsch and Carlson, 1980; Kahaner et al., 1988).

The variables characterizing each system were grouped into system components consistent with the “Triple Bottom Line” concept of the sustainability paradigm: environmental, economic and social (Elkington, 1998). In this work, the social dimension was represented by the population, labor force and education variables, and the economic dimension was represented by industry (e.g. coal), external trade (imports), transport/communication (railway), finance (bank deposits/currency, government revenues), and price (CPI) variables. Agricultural variables were used as a proxy for the environmental dimension because they can be used to represent the carrying capacity of the environment, which is a measure of its ability to sustain human populations.
Table 1. Study variables.

| COMPONENT          | CATEGORY           | VARIABLES                                                                 |
|--------------------|--------------------|---------------------------------------------------------------------------|
| SOCIAL (SOC)       | Population         | Total Population – Census                                                 |
|                    | Labor Force        | Labor – Ag/Forestry/Fishing (000)                                         |
|                    |                    | Labor – Extractive (000)                                                  |
|                    |                    | Labor – Manufacturing (000)                                               |
|                    |                    | Labor – Construction (000)                                                |
|                    |                    | Labor – Commerce/Finance/etc. (000)                                       |
|                    |                    | Labor – Transport/Communications (000)                                    |
|                    |                    | Labor – Services (000)                                                   |
|                    |                    | Workers Involved (000)                                                   |
|                    | Education          | Pupils in Schools (000)                                                  |
|                    |                    | Students in Universities (000)                                            |
| ENVIRONMENTAL (ENV)| Agricultural Crops | Arable Cropland (000 ha) – wheat                                          |
|                    |                    | Arable Cropland (000 ha) – barley                                         |
|                    |                    | Arable Cropland (000 ha) – oats                                           |
|                    |                    | Arable Cropland (000 ha) – rye                                            |
|                    |                    | Arable Cropland (000 ha) – potatoes                                       |
|                    |                    | Crops (000 mt) – wheat                                                   |
|                    |                    | Crops (000 mt) – barley                                                  |
|                    |                    | Crops (000 mt)- oats                                                     |
|                    |                    | Crops (000 mt) – rye                                                     |
|                    |                    | Crops (000 mt) – potato                                                  |
|                    | Livestock          | Horses (000)                                                             |
|                    |                    | Cattle (000)                                                             |
|                    |                    | Pigs (000)                                                               |
|                    |                    | Sheep (000)                                                              |
| ECONOMIC (ECO)     | Industry           | Coal output Bituminous (000 mt)                                           |
|                    |                    | Crude Steel output (000 mt)                                              |
|                    |                    | Sulphuric Acid (000 mt)                                                  |
|                    | External Trade     | Imports (mill $)                                                         |
|                    | Transport/Communication | Length of open railroad lines (km)                                    |
|                    |                    | RR traffic (mil passenger km)                                            |
|                    | Finance            | Currency in Circulation (mill)                                           |
|                    |                    | Central govt. revenue total (mill)                                       |
|                    | Prices             | Consumer price indices                                                   |
2.3. Comparative assessment steps

Using the procedure outlined by Cabezas and Eason, (2010), FI values for Germany, France and the United States were computed using an eight year moving time window with an overlap of one year. A three-year average of the FI values was computed in order to focus on trends and smooth out minor fluctuations. Trends in the FI index were overlaid with information on major historical events for each country to provide some context to the FI results. In line with previous assessments of underlying drivers (Cabezas and Eason, 2010; Eason and Cabezas, 2012; Gonzalez-Mejia et al., 2012b), FI of the components of each system (i.e., environmental, economic and social) was computed separately as a means of determining possible drivers of change in each system and Spearman Rank Order Correlations (SROC) were used to assess which components of the system were possibly driving the dynamic changes in the system. Additionally, overall FI was compared among the 3 countries by examining trends in FI values, as well as, means and standard deviations in the index. This analysis used a One-way ANOVA and within group comparison tests which utilized Tukey’s honestly significant difference criterion. In addition, Bayes was employed to assess the likelihood that FI declines provide warning signals of dynamic regime shifts.

3. Results and discussion

3.1. France

From 1900 to 2000, FI trends fluctuated a great deal, but never exceeded two standard deviations from mean FI (Fig. 2). Accordingly, France did not undergo a regime shift. However, it is evident that during both World Wars, France experienced steep declines in system order (~1909-1921 and 1936–1945) and increases from 1923 to 1934. After these, there were relatively few detrimental events and subsequently, FI was relatively stable with a shallow increase from 1973 to the end of the period.

Over the time of the study, France participated in four wars (i.e., World War I, World War II, the Indochinese War, and the Algerian War), experienced extensive internal unrest and subsequently established and abolished three separate constitutions. The constitutions included the 3rd Republic which collapsed in 1940 with the German occupation of World War II, the 4th Republic which was established in 1946 and collapsed in 1958 during the Algerian War, and the 5th Republic which still stands. France was also characterized by political instability during this period (Rioux and Rogers, 1987), and many of the French colonies in North Africa and Asia were struggling for independence. The social unrest did not become less pronounced until near the end of the 20th century, reflected as a steady and consistent rise in FI (Fig. 2).
In comparing the FI of the components (i.e., environmental, economic and social, separately) to the overall system (all components), we note that each component was positively linked with trends in the overall system. Both the environmental and economic components were highly correlated with trends in the overall system ($\rho > 0.84$ for both, $p$-values < 0.05; Table 2). Moreover, the FI trends of the environmental component closely mimicked the overall system, yet reflected more dramatic increases and declines. Additionally, economic FI was moderately correlated with environmental and social FI ($\rho = 0.67$ and 0.52, $p$-values < 0.05; Table 2).

France’s environmental component appeared most adversely affected by the World Wars (Fig. 3). The FI of the environmental component dropped dramatically during these periods (declines of over three standard deviations) with the most significant change happening around World War I, prior to which FI was at its highest level in 1909 and declined nearly 50% afterwards. Similar behavior is exhibited during World War II. Looking at the underlying variables during this period, we found that the largest changes occurred in the crop and cropland variables. One can imagine that this could have been a consequence of major military operations on French soil, and the resulting toll on young agricultural workers and infrastructure. Social and economic dimensions also reflected the great decreases in FI during both world wars, but not nearly to the degree that environmental FI declined. In line with the political and social unrest, social FI reflected a gradual decline of approximately two and a half standard deviations from 1948 to 1975. The population increased steadily from 40.2 million to 52.6

Fig. 2. Overall Fisher Information for France.
million during this period, and there were notable changes in several labor variables including commerce, service, agricultural/forestry/fishing categories, as well as worker disputes and university enrollment variables.

### 3.2. Germany

Unlike France and the United States, Germany did not maintain its territorial integrity through the study period of 1900 to 2000. This created some complexity.

---

**Table 2.** Summary of Spearman Rank Order Correlation results. Statistically significant correlations (p-value < 0.05) are bolded.

|        | Overall | SOC     | ENV     | ECO     |
|--------|---------|---------|---------|---------|
| **France** |         |         |         |         |
| Overall | 1       |         |         |         |
| SOC     | 0.64 (0.0001) | 1       |         |         |
| ENV     | 0.84 (< 0.001) | 0.28 (0.1278) | 1       |         |
| ECO     | 0.86 (< 0.001) | 0.52 (0.0034) | 0.67 (0.0001) | 1       |

| **Germany** |         |         |         |         |
| Overall | 1       |         |         |         |
| SOC     | 0.26 (0.1527) | 1       |         |         |
| ENV     | 0.90 (< 0.001) | 0.01 (0.968) | 1       |         |
| ECO     | 0.65 (0.0001) | 0.11 (0.5702) | 0.56 (0.0012) | 1       |

| **United States** |         |         |         |         |
| Overall | 1       |         |         |         |
| SOC     | 0.49 (0.0055) | 1       |         |         |
| ENV     | 0.51 (0.0035) | -0.16 (0.3845) | 1       |         |
| ECO     | 0.76 (< 0.001) | 0.59 (0.0005) | 0.04 (0.8394) | 1       |
because Germany has represented different areas on the map of Europe over time. For purposes of this study, we refer to Germany as follows: from 1900 to the end of World War I, Germany is the territorially recognized area under the jurisdiction of the German Empire; from the end of World War I (WW I) to the end of World War II (WW II), Germany consists of the area under the control of the Weimar Republic and later the Third Reich, but it does not include Austria, the Sudetenland or other areas under German control during the war. From the end of WW II to the German reunification of the early 1990s, Germany is represented by the German Federal Republic, and it now consists of the combined territories formerly under the jurisdiction of the German Federal Republic and the German Democratic Republic.

Similar to France, Germany experienced steep declines in FI during both World Wars (Fig. 4). Particularly at the end of WW II, FI declined by two standard deviations below the mean FI for the study period. Partly the result of the devastation from WW II, this period (1947–52) appears to reflect a regime shift in the nation-state that included the partitioning of the country into two separate states. From 1952 to 1976, FI for Germany steadily increased, followed by a decreasing trend which became more pronounced in the late 1980s during the time of the reunification and the Maastricht Treaty.

The overall system FI positively correlated with the environmental ($\rho = 0.90$, $p$-value < 0.05) and economic components ($\rho = 0.65$, $p$-value < 0.05) and the environmental and economic components were moderately correlated with each other; however, neither environmental nor economic FI displayed significant correlations with social FI (Table 2).
Note that during both world wars, Germany experienced steep declines in FI, which continued for several years following these events (Fig. 4). However, after each decline, the FI for Germany recovered to near average FI levels within 10 years. Following WW I, Germany lost land and colonies to its neighboring states, and was forced to pay extensive war reparations. Beginning in 1919, the Weimar Republic was established, which faced political instability, high unemployment and unchecked inflation (Balderston, 2002). All of these issues most likely contributed to the continued decline in FI in the period following World War I.

In the aftermath of World War II, Germany was divided into the German Democratic Republic in the east and the German Federal Republic in the west (Fig. 4). For the first ten years following this division, a period of quick economic development took place in the German Federal Republic; hence, FI continued to increase, displaying higher levels of system order from 1949 to 1979. During this period, several significant historical events also took place which eventually served as a proactive strategy for orderly transition and recovery. While the German Democratic Republic joined the Warsaw Pact in 1955, the German Federal Republic joined NATO in 1955 and the European Economic Community in 1957. During this time, the Berlin Wall rose and fell, and Germany reunified with the new united German Federal Republic joining NATO in 1990. From 1987 to 1996, there was a nearly two standard deviation decrease in FI which may have reflected some of the internal disturbances associated with the process of reunification.
Fig. 5 shows that Environmental FI had similar trends and accordingly, the highest correlation with overall FI for Germany ($\rho = 0.90$, $p$-value $< 0.05$; Table 2). Upon examination, the primary variables used in calculating environmental FI reflected large decreases in barley, pig and potato (40%, 40%-60%, and 20%-40%, respectively) production during both world wars. Economic variable values such as deposit/currency and revenue variables for both war events displayed very large increases (a factor of four or more). For World War II, there were also very large decreases in industrial output variables ($\geq 50\%$) and railroad usage variables ($\geq 25\%$).

### 3.3. United States

FI results indicate that the United States was relatively stable and experienced no regime shifts during the period of the study (Fig. 6). While system order dropped during the periods in which major events occurred (i.e., World Wars, the Great Depression, and the Dust Bowl), those decreases were less than two standard deviations from the mean FI. Spearman rank order correlations indicated a strong positive correlation between overall FI and the FI for the economic component (Table 2). While the social and economic components were moderately correlated, the environmental and economic components were not (Table 2).

Although the United States did not display as large a change in FI as Germany and France during the World Wars, FI gradually decreased signifying changes in dynamic behavior from 1916 to 1946 (Fig. 6). System order steadily increased from 1946 to 1963, appeared to settle until 1990, increased from 1991 to 1996 and...
exhibited a slight decrease at the end of the study period. Even during the period of the Great Depression and the Dust Bowl, Overall FI for the United States only decreased approximately 1 SD from the average. The steepest increase in system order occurred around 1992, directly after the end of the Cold War.

Economic FI had the highest correlation with overall FI ($\rho = 0.76$, p-value $< 0.05$; Table 2) and displayed the most change in system order during the Great Depression/Dust Bowl period (Fig. 7). While there were large increases in the steel and sulphuric acid industrial outputs (106% and 67%), the initial increase during this period reflects less extreme fluctuation in many of the variable values from 1921–1930. Conversely, other than imports, changes in economic variables varied between 16% to 145% from 1930 to 1938, thus explaining the drop in FI.

### 3.4. Comparative assessment

A one-way analysis of variance indicated that there was a statistically significant difference between the overall FI result for the countries ($F(2,90) = 19.03$, p-value $= 1.28E-07$). Tukey’s HSD test revealed that while FI trends were statistically the same for the United States ($\mu FI = 3.83$, $\sigma FI = 0.52$) and France ($\mu FI = 3.58$, $\sigma FI = 0.69$), these results varied significantly from Germany’s FI ($\mu FI = 4.61$, $\sigma FI = 0.81$). Except for the periods immediately following the two world wars, the FI for Germany was consistently higher than those of the United States and France, and the standard deviation of the FI for Germany was 17% and 55% greater than that of France and the United States, respectively. As discussed in Section 2.2, the

![Fig. 6. Overall Fisher Information for the United States.](http://dx.doi.org/10.1016/j.heliyon.2017.e00465 2405-8440/© 2017 Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).)

---

**United States FI**  
($\mu FI = 3.83$, $\sigma FI = 0.52$)
deviation in FI is an indication of system stability; hence, analysis of standard deviation in FI ($\sigma_{FI}$) reveals that although Germany exhibited higher average FI, it was the least stable of the countries under study.

As previously noted, the FI results indicated that the United States did not undergo the amount or magnitude of change as displayed in Germany and France and accordingly, was the most stable nation in the study. There are several reasons why this may have occurred. First, the United States did not experience any war events on its own soil (with the possible exception of the Pearl Harbor attack), so it did not undergo the environmental, economic and social destruction suffered by the other countries. While, the environmental component (as characterized by agricultural variables) was highly correlated with overall trends in Germany and France, the United States was primarily driven by economics which appeared to be decoupled from the environmental component ($\rho = 0.04$, $p$-value > 0.05). Further, in relation to France and Germany, the United States is a much larger country comprised of greater land area and population. The United States has 15 times and 28 times more land area and a 3½ to 5 times larger population than France and Germany, respectively. Previous work on urban systems identified a significant correlation between population size, mean FI and decreased standard deviation in FI (Eason and Garmestani et al., 2012) and noted these phenomena as an indication of stability and resilience. Such a finding provides insight on why the United States system was likely to be more robust and resilient to the impact of major historical events.

Since the FI is computed using a time bracket (window) and is reported for the last year of each window, each FI value is based on “past” data. Accordingly, note that
the FI for France begins to steadily decline before the start of both World Wars. Likewise, declining FI for Germany coincides with the loss of German momentum during World War I around the time of the Battle of Verdun (1916), the beginning of World War II, and the initial phases of German reunification. Further, rises in the FI correspond with periods of social and economic recovery and relative stability and prosperity for France (1925–1929 and 1972–2000), Germany (1949–1975 and 1996–2000). While the United States was relatively stable and reasonably prosperous for most of the 20th century, there are still distinct changes in dynamic order that mark major national and global events, e.g. declines in the FI for the United States roughly coincide with the American entry into both World Wars.

3.5. Declines in FI as warning signals of impending regime shifts

As a follow-up to the proposal that declines in FI may serve as precursors to forthcoming shifts in system dynamics (Eason et al., 2014), we further explore FI using Bayes Theorem. Here we assess the rate and slope of declines in FI for each nation state. The goal is to determine whether there are detectable patterns in FI in advance of major events that can be tracked and used as a warning signal of regime shifts. Operationalizing Eq. (4) involves estimating the probability of: (1) a regime shift \( P(RS) \), (2) declines if a regime shift has occurred (e.g., \( P(D1|RS) \)) and (3) a decline event (e.g. \( P(D1) \)). Tables 3, 4, 5 summarize the statistics on declines, regime shifts \( (RS) \) and severe decline events \( (SDE) \) for each nation state.

We estimated \( P(RS) \) by counting the number of times FI met or exceeded the regime shift cut-off point (e.g. Regime shift cut-off point 1 \( (RS\_CP1) = \mu_{FI} - \sigma_{FI} \); Table 4) and dividing that by the total number of FI values (In this case study, \( N = 30 \)). Since FI typically declines prior to a regime shift (Mayer et al., 2007), \( P(D1|RS) = P(D2/RS) = 100\% \). Similarly, there is 100\% probability of a single decline if a severe decline event has been identified: \( P(D1/SDE) = 100\% \). The probability of decline \( (P(D1)) \) is computed by dividing the number of time steps with declining FI by the total number of possible declines. Since two data points are needed to assess single declines, there are 29 \( (N-1) \) possible single declines and as such, 28 \( (N-2) \) possible double declines \( (P_{NumD1} = 29, P_{NumD2} = 28) \). Additionally, the average slope of all declines was computed to assess the significance of decline events in relation to the slope of single declines \( (D’) \). The standard deviation \( (\sigma_{D'}) \) and mean slope \( (\mu_{D'}) \) of the decline were then used to evaluate the severity of declines in FI for subsequent investigation of the nation states.

Results indicate that there is nearly an equal probability of declines (14/29) and increases (15/29) in the overall FI for all three systems, indicating that single time step declines are essentially random events nearly as probable as single time step
increases. However, the probability of two sequential declines ($D_2$) and the intensity of the slopes of the declines distinguish the countries from each other.

### 3.5.1. France

To illustrate the use of Bayes Theorem for assessing the probability of a regime shift, we provide a step-by-step computation of the first equation for France. As previously noted, since regime shifts always involve a decline in FI, the probability of there being a single decline given that a regime shift has occurred is 100% ($P(D_1|RS) = 100\%$). FI declined below regime shift cut-off point 1 ($RS_{\text{CP1}} = \mu_{FI} - \sigma_{FI}$) three times during the period; hence, the probability of a regime shift ($P(RS)$) is $3/30$, and the probability of a single decline ($P(D_1)$) is $14/29$. Eq. (7) shows the computation for assessing the probability of regime shift ($RS$) given the occurrence of a single decline event ($D_1$):

$$P(RS@RS_{CP1}|D_1) = \frac{P(RS@RS_{CP1}P(D_1|RS))}{P(D_1)} = \frac{\left(\frac{3}{30}\right) x 1}{\left(\frac{14}{29}\right)} = 0.2071 \text{ (7)}$$

and there is a 20.7% probability of a regime shift given the occurrence of a single decline event ($D_1$) (Table 5). Using the same cut-off point for a double decline ($D_2$), the probability of an impending regime shift increased to 35% (Table 5).

In assessing the severity of the declines in FI, note that there were four FI declines with slopes greater than 1 standard deviation below the average slope of all declines during the time period. Therefore, the probability of observing a severe decline event given the occurrence of all single declines was 27.62% as shown in Equation (8):

$$P(SDE@SDE_{CP1}|D_1) = \frac{P(SDE@SDE_{CP1}P(D_1|SDE))}{P(D_1)} = \frac{\left(\frac{4}{30}\right) x 1}{\left(\frac{14}{29}\right)} = 0.2762 \text{ (8)}$$

Using the stricter criteria of 2 standard deviations below average decline slope ($P(SDE@SDE_{CP2}|D_1)$): Table 5), the probability decreases to 6.9%.

### Table 3. Decline Statistics.

| Statistic     | Description                               | France | Germany | USA  |
|---------------|-------------------------------------------|--------|---------|------|
| NumD1        | Number of single declines in FI           | 14     | 14      | 14   |
| NumD2        | Number of FI declines over two points sequentially | 8      | 8       | 5    |
| $\mu_D$      | Mean slope of declines                    | -0.1290| -0.1360 | -0.0950|
| $\sigma_D$   | Standard deviation of decline slopes      | 0.1260 | 0.1230  | 0.0870|
| $P(D_1)$     | Probability of a single decline: NumD1/NumD1 | 48.3%  | 48.3%   | 48.3%|
| $P(D_2)$     | Probability of a double decline: NumD2/NumD2 | 28.6%  | 28.6%   | 17.9%|
Table 4. Event Statistics.

**Regime Shifts**

| Statistic | Description | France | Germany | USA |
|-----------|-------------|--------|---------|-----|
| RS_CP1 | Regime shift cut-off point at μFI −σFI | 2.885 | 3.797 | 3.309 |
| RS_CP2 | Regime shift cut-off point at μFI −2σFI | 2.191 | 2.987 | 2.789 |
| NumRS_CP1 | Number of times FI declines below RS_CP1 | 3 | 2 | 1 |
| NumRS_CP2 | Number of times FI declines below RS_CP2 | 0 | 1 | 0 |

| Statistic | Description | Probability of regime shift based on CP1: P(RS@CP1) | Probability of regime shift based on CP2: P(RS@CP2) |
|-----------|-------------|--------------------------------------------------|--------------------------------------------------|
| | NumRS_CP1/NumFI | 10.0% | 6.7% | 3.3% |
| | NumRS_CP2/NumFI | 0.0% | 3.3% | 0.0% |

**Severe Decline Events**

| Statistic | Description | France | Germany | USA |
|-----------|-------------|--------|---------|-----|
| SDE_CP1 | Severe decline cut-off point based on slope of FI declines: μD1'−σD1' | -0.255 | -0.259 | -0.182 |
| SDE_CP2 | Severe decline cut-off point based on slope of FI declines: μD1'−2σD1' | -0.381 | -0.382 | -0.269 |
| NumSDE@SDE_CP1 | Number of times the decline slope is below SDE_CP1 | 4 | 3 | 2 |
| NumSDE@SDE_CP2 | Number of times the decline slope is below SDE_CP2 | 1 | 1 | 1 |

| Statistic | Description | Probability of a severe decline event based on SDE_CP1: NumSDE@SDE_CP1/NumFI | Probability of a severe decline event based on SDE_CP2: NumSDE@SDE_CP2/NumFI |
|-----------|-------------|--------------------------------------------------|--------------------------------------------------|
| | NumSDE@SDE_CP1/NumFI | 13.33% | 10.00% | 6.67% |
| | NumSDE@SDE_CP2/NumFI | 3.33% | 3.33% | 3.33% |
### Table 5. Likelihood of Significant Events: Application of Bayes' Theorem to FI Declines.

| Statistic | Description | France | Germany | USA |
|-----------|-------------|--------|---------|-----|
| P(RS@RS_CP1|D1) | Probability of regime shift at RS_CP1 if there are single declines: \( P(RS@RS_CP1) \times P(D1/RS@RS_CP1/D1)/P(D1) \) | 20.71% | 13.81% | 6.90% |
| P(RS@RS_CP2|D1) | Probability of regime shift at RS_CP2 if there are single declines: \( P(RS@RS_CP2) \times P(D1/RS@RS_CP2/D1)/P(D1) \) | 0.00% | 6.90% | 0.00% |
| P(RS@RS_CP1|D2) | Probability of regime shift at RS_CP1 if there are double declines: \( P(RS@RS_CP1) \times P(D2/RS@RS_CP1/D2)/P(D2) \) | 35.00% | 23.33% | 18.67% |
| P(RS@RS_CP2|D2) | Probability of regime shift at RS_CP2 if there are double declines: \( P(RS@RS_CP2) \times P(D2/RS@RS_CP2/D2)/P(D2) \) | 0.00% | 11.67% | 0.00% |
| P(SDE@SDE_CP1|D1) | Probability of severe decline event based on SDE_CP1 if there are single declines in FI: \( P(SDE@SDE_CP1) \times P(D1/SDE@SDE_CP1/D1)/P(D1) \) | 27.62% | 20.71% | 13.81% |
| P(SDE@SDE_CP2|D1) | Probability of severe decline event based on SDE_CP2 if there are single declines in FI: \( P(SDE@SDE_CP2) \times P(D1/SDE@SDE_CP2/D1)/P(D1) \) | 6.90% | 6.90% | 6.90% |

**Note:** If a regime shift has been identified, there is a 100% probability that a decline has occurred; hence, \( P(D1/RS) \) or \( P(D2/RS) \) at any cut-off point = 100%. Severe declines involve at least one decline event; hence the \( P(D1/SDE) = 100\% \) for at any SDE_CP.
3.5.2. Germany

Like France, Germany experienced 14 single declines \((D1)\) and 8 double declines \((D2)\) in FI during the 20th century. Germany had the highest mean and standard deviation in FI of the three nation-states and was the only country where FI fell more than 2 standard deviations below average FI, Signifying a regime shift. The probability for a regime shift given the occurrence of single declines in FI \((P(RS@RS_CP1\mid D1))\) was 13.81\% at one standard deviation below average FI and 6.9\% at two standard deviations below average FI (Table 5). Given the occurrence of double declines, the probability for regime shifts was 23.33\% and 11.67\% for \(RS_CP1\) and \(RS_CP2\). The probability of a severe decline event given the occurrence of single declines at one and two standard deviations less than average decline slope was 20.71\% and 6.9\% (Table 5) respectively.

3.5.3. United States

The United States experienced the same number of single declines as Germany and France \((D1=14)\), but only 5 double declines \((D2=5)\), which led to the lowest probability of a regime shift at one standard deviation below average FI for single (6.9\%) and double declines (18.67\%) (Table 5). As previously noted, the US did not undergo a regime shift, however, the probability of a severe decline event was 13.81\% and 6.9\% for the two cut-off points. Note that while there is a relatively significant possibility of successive decreases \((D2)\) in FI for France and Germany (28.6\% for both), that likelihood is lower (17.9\%) for the United States. Another important observation is that although the degree of decline in FI for France was much greater than that of the United States, it was much less than that of Germany. This correlates with our common understanding of the level of destruction in France relative to that in Germany during WW II.

4. Conclusions

Results of this study illustrate that changing conditions in each nation state was captured by Fisher information and trends corresponded well with historical events including world wars and economic crises. Further, it demonstrated that the methods presented here not only facilitate an objective assessment of dynamic changes and regime shifts in complex systems but are also useful in: (1) determining the severity of those changes, and (2) assessing the likelihood that declines in system order represent early warning signals of major events.

Detecting patterns of change in complex systems is useful and important for management. It affords the ability to identify trends that may warn of impending crises, and therefore provide time for intervention and mitigation. One of the key outcomes in this work is the development of an objective procedure for assessing trends in FI as a leading indicator of regime shifts. This approach was developed to
provide a useful tool for decision makers monitoring a system in real time. While, managers have no knowledge of future events, they typically have access to historical data. Note that FI values are computed based on “past” data; hence, the access to available data coupled with the methods described in this work may help decipher whether measurable changes in system condition represent random variation or are signals of major events to come. Further, the techniques illustrated may be used to identify both the underlying components and the distinct behaviors that differentiate the behavior of each nation state. Other FI studies have been conducted to devise additional methods of identifying variables that drive dynamic change in complex systems, and to explore how this knowledge can be applied in the management of systems toward sustainability and resilience (e.g. González-Mejia et al., 2015; Eason et al., 2016).

The post-hoc application of Fisher information and Bayes’ Theorem provides an objective means of classifying ongoing events. It hints at the potential for Fisher information to be used to forecast trends in coupled human and natural systems. González-Mejia et al. (2016) further explored this ability by utilizing the power of Fisher information to bound the forecasts and build a neural network for projecting trends in system variables based on patterns captured from historical data. These applications demonstrate that Fisher information in combination with other tools and methods, is useful for assessing regime shifts and forecasting trends, as well as, identifying warning signals of impending change. Such an approach may provide crucial time for intervention and facilitate guidance on suitable management actions to help circumvent undesirable conditions and outcomes.

Declarations

Author contribution statement

Leisha Vance, Tarsha Eason, Heriberto Cabezas, Michael Gorman: Conceived and designed the study; Analyzed and interpreted the results; Contributed analysis tools or data; Wrote the paper.

Funding statement

This work was supported by the U.S. EPA.

Competing interest statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.
Acknowledgements

The views expressed in this article are those of the authors and do not necessarily represent the views or policies of the U.S. Environmental Protection Agency.

References

Allenby, B.R., 2013. The theory and practice of sustainable engineering. Pearson Prentice Hall, Upper Saddle River, NJ.

Balderston, T., 2002. Economics and politics in the Weimar Republic. Cambridge University Press 45.

Batt, R.D., Brock, W.A., Carpenter, S.R., Cole, J.J., Pace, M.L., Seekell, D.A., 2013. Asymmetric response of early warning indicators of phytoplankton transition to and from cycles. Theor. Ecol. 6, 285–293.

Bayes, T., Price, M., 1763. An essay towards solving a problem in the doctrine of chances. by the late rev. mr. bayes, frs communicated by Mr. Price, in a letter to John Canton, amfrs. Philosophical Transactions (1683-1775), , pp. 285–293.

Berry, D.A., Stangl, D.K., 1996. Bayesian methods in health-related research. In: Berry, D.A., Stangl, D.K. (Eds.), Bayesian Biostatistics. Marcel Dekker, New York, pp. 3–66.

Bestelmeyer, B.T., Ellison, A.M., Fraser, W.R., Gorman, K.B., Holbrook, S.J., Laney, C.M., et al., 2011. Analysis of abrupt transitions in ecological systems. Ecosphere art129.

Biggs, R., Carpenter, S.R., Brock, W.A., 2009. Turning back from the brink: detecting an impending regime shift in time to avert it. Proc. Natl. Acad. Sci. 106, 826–831.

Brock, W.A., Carpenter, S.R., 2012. Early warnings of regime shift when the ecosystem structure is unknown. PLoS One 7, 1–10.

Cabezas, H., Fath, B., 2002. Towards a theory of sustainable systems. Fluid Ph. Equilibria 194-197, 3–14.

Cabezas, H., Pawlowski, C.W., Mayer, A.L., Hoagland, N.T., 2003. Sustainability: Ecological, social, economic and systems perspectives. Clean Technol. Environ. Policy, 167–180.

Cabezas, H., Eason, T., 2010. Fisher information and orderIn: Heberling, M.T., Hopton, M.E. (Eds.), San Luis Basin Sustainability Metrics Project: A Methodology for Assessing Regional Sustainability 182, pp. 163–222. EPA Number: EPA/600/R-10/182 (Last Accessed: 11/14/2017) http://nepis.epa.gov/Exe/ZyPURL.cgi?Dockey=P100BSKA.txt.
Carpenter, S., 2003. Regime shifts in lake ecosystems: pattern and variation. Excellence in Ecology Series, Ecology Institute, Oldendorf/Luhe, Germany 15.

Carpenter, S.R., Brock, W.A., 2006. Rising variance: a leading indicator of ecological transition. Ecol. Lett. 9, 311–318.

Dakos, V., Scheffer, M., van Nes, E.H., Brovkin, V., Petoukhov, V., Held, H., 2008. Slowing down as an early warning signal for abrupt climate change. Proc. Natl. Acad. Sci. 105, 14308–14312.

Dakos, V., Carpenter, S.R., Brock, W.A., Ellison, A.M., Guttal, V., Ives, A.R., Kéfi, S., Livina, V., Seekell, D.A., Van Nes, E.H., Scheffer, M., 2012. Methods for detecting early warnings of critical transitions in time series illustrated using simulated ecological data. PLoS One 7 (7) e41010.

Diamond, J., 2005. Collapse: How societies choose to fail or succeed. Penguin, Viking New York, NY.

Eason, T., Cabezas, H., 2012. Evaluating the sustainability of a regional system using Fisher information, San Luis Basin, Colorado. J. Environ. Manage. 94 (1), 41–49.

Eason, T., Garmestani, A.S., 2012. Cross-scale dynamics of a regional urban system through time Region et Developpement 45, 55–77. (Last Accessed: 11/12/2017) http://region-developpement.univ-tln.fr/fr/pdf/R36/4_EasonGarmestani.pdf.

Eason, T., Garmestani, A.S., Cabezas, H., 2014. Managing for resilience: early detection of catastrophic shifts in complex systems. Clean Technol. Environ. Policy 16, 773–783.

Eason, T., Garmestani, A.S., Stow, C.A., Rojo, C., Alvarez-Cobelas, M., Cabezas, H., 2016. Managing for resilience: an information theory based approach to assessing ecosystems. J. Appl. Ecol. 53, 656–665.

Elkington, J., 1998. Cannibals with Forks: The triple bottom line of 21st century business. New Society Publishers, Stony Creek, CT, pp. 416.

Ellison, A.M., 1996. An introduction to Bayesian inference for ecological research and environmental decision-making. Ecol. Appl. 64, 1036–1046.

Fath, B., Cabezas, H., Pawlowski, C., 2003. Regime changes in ecological systems: an information theory approach. J. Theor. Biol. 222, 517–530.

Fisher, R.A., 1922. On the mathematical foundations of theoretical statistics. Phil. Trans. R. Soc. London A 222, 309–368.

Frieden, B. Roy, 1998. Physics from Fisher Information: A Unification. Cambridge University Press, Cambridge, UK.
Frieden, B. Roy, 2001. Probability, statistical optics and data testing, Third ed. Springer-Verlag, New York.

Fritsch, Fred, Carlson, Ralph, 1980. Monotone piecewise cubic interpolation. SIAM J. Numer. Anal. 17, 238–246.

Gonzalez-Mejia, A.M., 2011. Fisher information – sustainability analysis of several US metropolitan statistical areas. University of Cincinnati, Cincinnati, OH, USA, pp. 109.

Gonzalez-Mejia, A.M., Eason, T.N., Cabezas, H., Suidan, M.T., 2012a. Assessing sustainability in real urban systems: the greater Cincinnati metropolitan area in Ohio, Kentucky, and Indiana. Environ. Sci. Technol. 46 (17), 9620–9629.

Gonzalez-Mejia, A.M., Eason, T.N., Cabezas, H., Suidan, M.T., 2012b. Computing and interpreting Fisher Information as a metric of sustainability: regime changes in the United States air quality. Clean Technol. Environ. Policy 14 (5), 775–788.

Gonzalez-Mejia, A.M., Eason, T.N., Cabezas, H., Suidan, M., 2014. Social and economic sustainability of urban systems: comparative analysis of metropolitan statistical areas in Ohio, USA. Sustainability Sci. 9 (2), 217–228.

Gonzalez-Mejia, A.M., Vance, L., Eason, T., Cabezas, H., 2015. Recent development in the application of Fisher information to sustainable environmental management. In: Klemes, J. (Ed.), Assessing and Measuring Environmental Impact and Sustainability.. Elsevier, New York.

González-Mejía, A.M., Eason, T.N., Cabezas, H., 2016. System learning approach to assess sustainability and forecast trends in regional dynamics: The San Luis Basin study Colorado, USA. Environ. Model. Software 81, 1–11.

Goodland, R., 1995. The concept of environmental sustainability. Annu. Rev. Ecol. Syst. 26 (1), 1–24.

The Anasazi in a changing environment. In: Gumerman, G. (Ed.), Cambridge CB2 IRP. Cambridge University Press, pp. 317.

Haiti: A Country Study In: Haggerty, R.A. (Ed.), Washington: GPO for the Library of Congress, , pp. 2012. Retrieved November 19, 2012 http://countrystudies.us/haiti/.

Hays, J., 2005. Epidemics and Pandemics: Their Impacts on Human History. ABC-CLIO, Inc., Santa Barbara, CA.

Holling Crawford, S., 1965. The functional response of predators to prey density and its role in mimicry and population regulation. Mem. Entomol. Soc. Can. 45, 1–160.
Holling, C.S., 1973. Resilience and stability of ecological systems. Annu. Rev. Ecol. Syst. 4 (1-23), 23. (Last Accessed: 11/12/2017) http://www.jstor.org/stable/2096802.

Kahaner, David, Moler, Cleve, Nash, Stephen, 1988. Numerical methods and software. Prentice Hall.

Karunanithi, A., Cabezas, H., Frieden, B.R., Pawlowski, C., 2008. Detection and assessment of ecosystem regime shifts from Fisher information. Ecol. Soc. 13 (1), 22.

Karunanithi, A., Garmestani, A., Eason, T., Cabezas, H., 2011. The characterization of socio-political instability, development and sustainability with Fisher information. Global Environ. Change 22 (1), 77–84.

Kates, R.W., Parris, T.M., 2003. Long term trends and a sustainability transition. Proc. Natl. Acad. Sci. USA 100 (14), 8062–8067.

Kinealy, C., 1995. This great calamity: The Irish famine 1845-52. Gill & Macmillan.

Kleinen, T., Held, H., Petschel-Held, G., 2003. The potential role of spectral properties in detecting thresholds in the earth system: application to the thermohaline circulation. Ocean Dyn. 53, 53–63.

Lapin, L., 1975. Statistics: Meaning and Method. Brace Jovanovich, Harcourt New York, New York, USA.

Lubchenco, J., 1998. Entering the century of the environment: a new social contract for science. Science 279 (5350), 491–497.

Maler, K., 2000. Development, ecological resources and their management: a study of complex dynamic systems. Eur. Econ. Rev. 44, 645–665.

Mayer, A., Pawlowski, C., 2006. Fisher Information and dynamic regime shifts in ecological systems. Ecol. Model. 195, 72–82.

Mayer, A., Pawlowski, C., Fath, B., Cabezas, H., 2007. Applications of Fisher information to the management of sustainable environmental systems. In: Frieden, B. Roy, Gatenby, Robert A. (Eds.), Exploratory data analysis using Fisher information. Springer-Verlag, London, pp. 217–244.

Mitchell, B.R., 2007a. International historical statistics: Europe. 1750-2005. Macmillan, Palgrave New York, NY.

Mitchell, B.R., 2007b. International historical statistics: The Americas. 1750-2005. Macmillan, Palgrave New York, NY.
Oborny, B., Meszena, G., Szabo, G., 2005. Dynamics of populations on the verge of extinction. Oikos 109, 291–296.

Records, L.S., 1995. Cherokee outlet cowboy: Recollections of Laban S. Records. University of Oklahoma Press, Norman, Oklahoma.

Rico-Ramirez, V., Reyes-Mendoza, P.A., Ortiz-Cruz, J.A., 2010. Fisher information on the performance of dynamic systems. Ind. Eng. Chem. Res. 49 (4), 1812–1821.

Rioux, J.P., Rogers, G., 1987. The Fourth Republic, 1944-1958. Cambridge University Press, Cambridge, pp. 350.

Scheffer, M., Bascompte, J., Brock, W.A., Brovkin, V., Carpenter, S.R., et al., 2009. Early-warning signals for critical transitions. Nature 461, 53–59.

Scheffer, M., Carpenter, S.R., Lenton, T.M., Bascompte, J., Brock, W., Dakos, V., et al., 2012. Anticipating critical transitions. Science 338, 344–348.

Shastri, Y., Diwekar, U., Cabezas, H., 2008a. Optimal control theory for sustainable environment management. Environ. Sci. Technol. 42, 5322–5328.

Shastri, Y., Diwekar, U., Cabezas, H., Williamson, J., 2008b. Is sustainability achievable? Exploring the limits of sustainability with model systems. Environ. Sci. Technol. 42, 6710–6716.

Spanbauer, T.L., Allen, C.R., Angeler, D.G., Eason, T., Fritz, S.C., Garmestani, A. S., et al., 2014. Prolonged instability prior to a regime shift. PLoS One 9, e108936.

Sundstrom, S.M., Eason, T., Nelson, R.J., Angeler, D.G., Barichievy, C., Garmestani, A.S., Nash, K.L., 2017. Detecting spatial regimes in ecosystems. Ecol. Lett. 20 (1), 19–32.