Solution of loose medium movement problem on a curvilinear sifting surface

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Abstract. The paper presents the results of movement simulation of loose medium stratum on a curvilinear sifting surface using the program realizing a finite element method. The indicial equations represent an analogue describing fluid stratum movement under the theory of “shallow water”. The allocation of a part movement speed rate in the cross-section of a stratum is obtained. The presence of a braking underlayer on a sifting surface lowering efficiency of sifting process is detected.

1. Introduction

The loose material consisting of a great number of particles is a difficult object for studying. Each particle of a stratum, not having its own steady characteristics, for example, mechanical ones, saves all qualities of substance, from which the given loose material is obtained. Besides in different physical processes loose material behaves differently. Analytical, probability methods and similitude methods were used to describe its movement. The solution of loose medium movement problem in a stratum on a curvilinear sifting surface in strict mathematical setting is difficult enough and allows us to receive only qualitative rate in solving differential equations. The given paper presents a method of numerical solution of loose medium movement problem on a curvilinear sifting surface. For simulation of movement conditions, software package Ansys was used, which can implement numerical solution systems of differential equations by a finite element method.

2. Mathematical simulation of the process

Consisting of particles and hollows, the loose medium is considered as a continuous medium in solving the given problem, and the oscillations of a continuity of a mass, which can take place during movements, are insignificant. Taking into consideration the given assumption made by the authors of the paper [1] the function of current was obtained:

\[ f = \alpha J_1 + J_2^{k\frac{1}{2}} = k, \]  

where \( k \) and \( \alpha \) - positive constants in each point of the loose environment, \( J_1 \) - the sum of main powers equals to:

\[ J_1 = \sigma_x + \sigma_y \equiv \sigma_{11} + \sigma_{22}, \]
\( J_2 \) - second invariant deviator of powers equals to:

\[
J_2 = \frac{1}{2} s_{ij} s_{ij},
\]

where \( s_{ij} = \sigma_{ij} - \left( \frac{J}{3} \right) \delta_{ij} \) deviator stress, \( \delta_{ij} \) - delta of the Kronecker.

According to the theory of a plastic potential and accepted function of current (1) relation of strains and deformations submits to the equation:

\[
\varepsilon_{ij}^p = \lambda \frac{\partial f}{\partial \sigma_{ij}}, \quad (2)
\]

where - \( \varepsilon_{ij}^p \) degree of a residual plastic deformation, - \( \lambda \) positive factor of proportionality.

which can accept different values for different particles.

Substituting the equation (1) in the expression (2) we obtain a relation of strains and deformations as follows:

\[
\varepsilon_{ij}^p = \lambda \left[ \alpha \delta_{ij} + \frac{s_{ij}}{2J_2^{0.5}} \right]. \quad (3)
\]

The function of current (1) for a flat deformed state allows us to spot quantity of strain

\[
s_{ij} = -2\alpha J_2^{0.5}, \quad (4)
\]

then

\[
J_1 = \frac{3}{2} \left( \sigma_{11} + \sigma_{22} \right) - 3\alpha J_2^{0.5} \quad (5)
\]

and

\[
J_2 = \left[ \frac{\left( \sigma_x - \sigma_y \right)^2 + \tau_{xy}^2}{\left(1 - 3\alpha^2 \right)} \right]. \quad (6)
\]

By substitution of expressions (5) and (6) in function of current (1) we finally find its expression for a flat deformable state of loose medium.

\[
f = 3\alpha \frac{\sigma_x + \sigma_y}{2} + (1 - 3\alpha^2) J_2^{0.5} = k \quad (7)
\]

The received solution makes it possible to explore movement of a stratum on a curvilinear surface [1]. In the given operation the offered mathematical model was implemented for the solution of two-dimensional problem of loose material stratum current on the basis of finite elements method.

3. Forming of an array of input dates for a numerical solution of material stratum movement problem on a curvilinear sifting surface

The dynamic of systems, consisting of set of bodies, differs by rough behavior of many parameters, for example, leaps of velocities at collision of bodies, failure of dry friction law etc. These features should be considered during the development of numerical methods of dynamic systems simulation.

The statement of a problem is executed with reference to technological process of material classification. The width of moving stratum varies from the peak value during the inlet into working space to the minimum value on exit. In its absolute values the width of material stratum is much
smaller than the basic geometrical sizes of a sifting surface, therefore change of stratum width on its length should be referred to quantities of the second order of smallness. On this basis we shall introduce an assumption that the free surface of moving material stratum also takes the form of a circle arc. The minimal stratum width on exit can be accepted equal to size class greater by one of mesh gleam sifting surface. Then the peak width of a stratum on inlet at given classification efficiency and grain-size analysis data of a source material is determined by simple recalculation. Thus, medium movement area is generated [2].

A mathematical model of a system whose behavior must be analyzed is a collection of discrete regions (elements) interconnected at a finite number of points (nodes). The main unknowns are the degrees of freedom of the nodes of the finite element model. The degrees of freedom include: movements, rotational displacements, pressure, velocity. In accordance with the degrees of freedom, stiffness and resistance matrices are formed for each element of the system. These matrices lead to a system of linear equations that are processed by operators that determine the exact solution of the system of equations.

With reference to the requirements of the problem set, the whole area of the flow was divided into 920 elements (Figure1).
The starting conditions for the solution of a viewed problem were represented by velocity of particles on inlet in working space $v_y = 1.22 \text{ m/s}$ and $v_x = 0 \text{ m/s}$ and density of medium. It makes $3 \times 10^{-2} \text{ MPa}$. The coefficients of interior friction of medium and its friction with sifting surface are equal respectively 0.65 and 0.5. The radius of sifting surface curvature consisted 0.35 m, and height difference in working space of arch screen - 0.19 m. Due to the stated initial conditions we established boundary conditions, including three surfaces of medium current area (figure 2). The solution of combined equations (8) was realized step by step, in 100 iterations. The allocation of pressure in a stratum of loose material is shown in figure 2, and on a sifting surface - in figure 3. The peak pressure magnitude of a stratum equal to $107.28 \times 10^{-5} \text{ MPa}$ was scored from input in working space on 2/3 of whole length of stratum.

Velocity field of particles movement is also very variegated. Continuously growing from $1.22 \text{ m/s}$ and reaching $1.293 \text{ m/s}$ on a free stratum surface, velocity reduces on exit from working space and particles movement reduces to $0.1437 \text{ m/s}$. Velocity changing function on sifting surface of arch screen looks even more complicated (figure 6). Roughly slowing down till $0.652 \text{ m/s}$ on input as a result of particles hitting a sifting surface, under the influence of gravity these particles accelerate up to $0.794 \text{ m/s}$. This process is also supported by the deflux of material through a sifting surface.

Reduction of particles energy by friction caused by the interaction of particles with the surface leads to the reduction of velocity on exit from working space. It’s important to notice that distribution maximums of pressure and velocity on sifting surface are equal (figure 4 and figure 6).

The distribution of particles velocities in transversal section of material stratum is also variegated. It’s established that the minimum of particles movement velocity is inside of material stratum closer to a sifting surface, but not on the surface. This underlayer is one of the reasons of lowering efficiency of classification process. It does not let material particles which are above to get closer to a sifting surface, and for particles located below it, it is an additional suppressor of their movement energy. This underlayer is formed on an input (figure 5) and exists throughout the working space, dividing all material in two areas. It is possible to reduce harmful influence of the detected effect, approaching the given underlayer to a sifting surface on distance smaller, than size of its mesh on a gleam.

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**Figure 2** - Allocation of pressure in loose material stratum, Pa

**Figure 3** - Allocation of pressure on a sifting surface
4. Conclusion

Thus, from the above we can draw the following conclusions.

- The proposed method of calculation using the finite element method adequately reflects the processes occurring in the layer of loose material as it passes along a curvilinear sifting surface.
- By numerical modeling, we revealed the underlayer decelerating sifting process, and found it is necessary to bring it as close as possible to the sifting surface to reduce the negative effect of the underlayer.

References

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