Predictability of social interactions

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Abstract. The ability to predict social interactions between people has profound applications including targeted marketing and prediction of information diffusion and disease propagation. Previous work has shown that the location of an individual at any given time is highly predictable. This study examines the predictability of social interactions between people to determine whether interaction patterns are similarly predictable. I find that the locations and times of interactions for an individual are highly predictable; however, the other person the individual interacts with is less predictable. Furthermore, I show that knowledge of the locations and times of interactions has almost no effect on the predictability of the other person. Finally I demonstrate that a simple Markov chain model is able to achieve close to the upper bound in terms of predicting the next person with whom a given individual will interact.

Keywords: predictability, social, interaction, human dynamics, entropy

1 Introduction

One of the most important questions in the emerging field of human dynamics concerns predictability: to what extent is human behavior predictable, and how does predictability vary across the population? Recent technological advances have led to the development of wearable human sensors, which are capable of continuously collecting data on an individual’s movement, activities, and interactions among other features. These sensors could allow us to make predictions about many aspects of human behavior including social interactions. The ability to make such predictions has profound applications such as targeted marketing using an individual’s social network to predicting how diseases transmitted through human contact propagate over time.

This study is motivated by previous work on the predictability of human mobility. Using cell phone data from 50,000 individuals, Song et al. [5] studied the predictability of individuals’ locations using the closest cellular tower each time an individual used his phone. To capture the predictability of an individual’s location over time, the authors estimated the entropy rate of the time series of his locations and found that the real uncertainty in an individual’s location at any given time is fewer than two locations! Furthermore, there was found to be

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surprisingly little variability in the estimated entropy rate among the population, suggesting that individuals’ locations over time are, in general, highly predictable.

The main question behind this study is as follows: to what extent are individuals’ social interactions predictable? I investigate the extent to which three aspects of an individual’s interactions are predictable: the physical location and time of an interaction and the other person with whom the individual interacts. Similar to Song et al. [5], I estimate the entropy rate to capture predictability. In addition, I use a Markov chain model for social interaction to evaluate actual prediction performance on two real data sets.

2 Methodology

2.1 Entropy rates

To capture the predictability of a time series, I utilize the entropy rate. First we have the notion of entropy, which measures the amount of uncertainty in a random variable. The entropy of a single random variable X is defined as

$$H(X) = -\sum p(x_i) \log_2 p(x_i),$$

where the summation is over all possible outcomes \{x_i\}, and \(p(x_i)\) denotes the probability of outcome \(x_i\). For two random variables \((X, Y)\), we have the notions of joint entropy \(H(X, Y)\) and conditional entropy \(H(X|Y)\). The joint entropy measures the uncertainty associated with both random variables, while the conditional entropy measures the uncertainty in one random variable given that the value of the other random variable has been observed. The joint and conditional entropies are related through the equation

$$H(X|Y) = H(X, Y) - H(Y).$$

The entropy rate was first introduced by Shannon [4] and generalizes the notion of entropy to sequences of dependent random variables. For a stationary stochastic process \(X = \{X_i\}\), the entropy rate is defined as

$$H(X) = \lim_{n \to \infty} \frac{1}{n} H(X_1, X_2, \ldots, X_n) = \lim_{n \to \infty} H(X_n|X_1, X_2, \ldots, X_{n-1})$$ (1)

where the first equality holds for all stochastic processes, but the second requires stationarity of the process. The quantity on the right side of (1) leads to the interpretation of entropy rate as the uncertainty in a quantity at time \(n\) having observed the complete history. The entropy rate denotes the average per-variable entropy of each random variable in the stochastic process. Joint and conditional entropy rates can similarly be defined. In this study, I use the entropy rate to characterize the average uncertainty of a quantity at any given time.

2.2 Lempel-Ziv complexities

To calculate the entropy of a random variable \(X\), one needs to know the probability of each possible outcome \(p(x_i)\). When these probabilities are not known, one can estimate the entropy by replacing the probabilities with relative frequencies from observed data. Estimating the entropy rate of a stochastic process...
Predictability of social interactions

is more involved because the random variables are, in general, dependent on one another. A suitable estimator of the entropy rate for general stationary stochastic processes is the Lempel-Ziv complexity. Similar to Song et al. [3], I use the following Lempel-Ziv complexity to estimate the entropy rate of a time series:

$$\hat{H}(X) = \frac{n \log_2 n}{\sum_i A_i}, \quad (2)$$

where $n$ denotes the length of the time series, and $A_i$ denotes the length of the shortest substring starting from time $i$ that has not yet been observed prior to time $i$, i.e. from times 1 to $t-1$. It is known that for stationary ergodic processes, $\hat{H}(X)$ converges to the entropy rate $H(X)$ almost surely as $n \to \infty$.

To estimate the joint entropy rate, one can extend (2) to two time series, with $A_i$ denoting the length of the shortest substring of ordered pairs from both time series. This joint Lempel-Ziv complexity $\hat{H}(X, Y)$ also converges to the joint entropy rate as $n \to \infty$ [6]. I obtain an estimate of the conditional entropy rate using the conditional Lempel-Ziv complexity $\hat{H}(X|Y) = \hat{H}(X, Y) - \hat{H}(Y)$.

3 Results

I investigate the predictability of social interactions on two data sets. The first is the Reality Mining data [2], which provides location (via nearest cellular tower) and interaction (via Bluetooth proximity) data for 94 individuals at 5-minute intervals over a year. The second is the Friends and Family data [1], which provides only interaction (via Bluetooth proximity) data for 146 individuals at 6-minute intervals over 9 months. Similar to Song et al. [5], I compare the estimated entropy rate $\hat{H}$ (using the Lempel-Ziv complexity) with the estimated entropy rates of an iid sequence with the same marginal probabilities as the observed sequence $\hat{H}_{iid}$ and an iid sequence of uniformly likely outcomes $\hat{H}_{unif}$.

I begin with the Reality Mining data. The estimated entropy rates for the locations of interactions are shown in Fig. 1 (left). The mean of $\hat{H}_{loc}$ is about 1.1, indicating that the actual uncertainty in the location of an interaction is about $2^{1.1} = 2.1$ locations. This is similar to the finding of Song et al. [3] that the estimated entropy rate of an individual’s location peaks at about 0.8. Thus I conclude that the locations of an individual’s interactions are highly predictable. The estimated entropy rate $\hat{H}_{loc}$ is much lower than the iid entropy rate $\hat{H}_{iid}$, indicating that the temporal sequence is highly dependent. The estimated entropy rates for the times between interactions are shown in Fig. 1 (right). Similar to locations, the times of interactions are also highly predictable.

The estimated entropy rates for the person an individual interacts with are shown in Fig. 2 (left). Unlike with locations and times, the mean of $\hat{H}_{pers}$ is about 3.1, suggesting that the actual uncertainty is about $2^{3.1} = 8.5$ individuals. Thus it appears that the person an individual interacts with is significantly less predictable than the location or time! The estimated entropy rate $\hat{H}_{pers}$ is still much lower than the iid entropy rate $\hat{H}_{iid}$, so some temporal dependency is still present in the time series.
Fig. 1. Distributions of the estimated entropy rate $\hat{H}$, iid entropy rate $\hat{H}_{iid}$, and uniform entropy rate $\hat{H}_{unif}$ for locations (left) and times (right) of interactions. The low $\hat{H}_{loc}$ and $\hat{H}_{time}$ indicate that locations and times of interactions are highly predictable.

Fig. 2. Left: Distributions of the estimated entropy rate $\hat{H}_{pers}$, iid entropy rate $\hat{H}_{iid}^{pers}$, and uniform entropy rate $\hat{H}_{unif}^{pers}$. $\hat{H}_{pers}$ is much higher than $\hat{H}_{loc}$ and $\hat{H}_{time}$ (see Fig. 1), indicating that the person an individual interacts with is much less predictable than location or time. Right: Distributions of the estimated entropy rate $\hat{H}_{pers}$ and conditional entropy rates given locations $\hat{H}_{loc}^{pers}$ and times $\hat{H}_{time}^{pers}$. There is little difference between the three distributions, indicating that knowledge of times and locations does not provide any significant benefit in predicting the person an individual interacts with.

Perhaps the person an individual interacts with may be more predictable if one is given the locations or times of interactions. The predictability given this additional information is captured by the conditional entropy rate, which I estimate using the conditional Lempel-Ziv complexity $H(X|Y) = H(X,Y) - H(Y)$. Somewhat surprisingly, I find that the estimated conditional entropy rates given locations $\hat{H}_{loc}^{pers}$ or times $\hat{H}_{time}^{pers}$ do not differ significantly from the estimated unconditional entropy rates, as shown in Fig. 2 (right). Thus I conclude that knowing the locations or times does not add much predictive value when trying to predict the person with whom an individual interacts.

1 Note that the true conditional entropy rate $H(X|Y)$ must always be less than the true unconditional entropy rate $H(X)$, but this is not necessarily true for the estimated conditional and unconditional entropy rates due to finite sample size.
Fig. 3. Left: Graphical representation of Markov chain state transition matrix for a selected individual. Edge width is proportional to transition probability. Right: Distributions of the estimated entropy rate $\hat{H}_{\text{pers}}$ and Markov chain entropy rate $\hat{H}_{\text{pers}}^{\text{MC}}$. The estimated entropy rates of the Markov chains are only slightly higher than the rates of the actual sequences, suggesting that the Markov chain can achieve close to the upper bound for predicting the person an individual will interact with next.

Entropy rates provide only an upper bound on predictability. I now consider the problem of actually modeling the sequence of people an individual interacts with over time. A simple model consists of a Markov chain, which assumes that the person an individual interacts with next depends only on the person she is currently interacting with (or the last person she interacted with, if she is not currently interacting with anyone). I learn a stationary Markov chain for each individual, such as the one pictured in Fig. 3 (left), and estimate the entropy rates of these chains by substituting relative frequencies for probabilities. As shown in Fig. 3 (right), the estimated entropy rates of the Markov chains $\hat{H}_{\text{pers}}^{\text{MC}}$ are only slightly higher than those of the actual sequences $\hat{H}_{\text{pers}}$. Specifically, the mean $\hat{H}_{\text{pers}}^{\text{MC}}$ is 3.2 compared to the mean $\hat{H}_{\text{pers}}$ of 3.1. This suggests that the Markov chain is able to achieve close to the upper bound for predicting the next person an individual interacts with!

To measure how well the Markov chain works in practice, I learn the model on the first week of data and attempt to predict the most likely (top-1) and 5 most likely (top-5) people an individual will interact with next for each interaction in the second week. I then update the model based on the interactions in the second week and repeat the process until I reach the end of the data. Overall, the predictions from the Markov chain achieved a top-1 accuracy of 19% and top-5 accuracy of 49%. These results, while not spectacular, do appear to be reasonable given that I previously found the uncertainty to be about eight people.

I repeated the previous experiments on the Friends and Family data set. Location data is not available, but all of my findings from the Reality Mining data not involving location hold also for the Friends and Family data. The person an individual interacts with is slightly more predictable, with the mean $\hat{H}_{\text{pers}} = 2.3$.

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$^2$ The true entropy rate is always lower than the Markov chain entropy rate, but this is not necessarily true for the estimated rates, again, due to finite sample size.
corresponding to uncertainty of about $2^{2.3} = 5.0$ people. The learned Markov chain model achieves mean $\hat{H}_{\text{pers}}^{MC} = 2.7$, which is again close to the mean $H_{\text{pers}}$, although the larger gap compared to the Reality Mining data suggests that the effects of higher-order dependencies is stronger in the Friends and Family data.

The predictions from the Markov chain achieved top-1 and top-5 accuracies of 21% and 59%, respectively, which are also higher than in the Reality Mining data and agree with the lower entropy rate of the Friends and Family data.

4 Conclusions

This study examined the predictability of social interactions, an important question in the emerging area of human dynamics. My main findings are threefold:

1. The locations and times of interactions for an individual are highly predictable, but not the other person with whom the individual interacts.
2. Even if the locations and times of interactions are known, there is almost no effect on the predictability of the other person.
3. A simple Markov chain model achieves close to the upper bound for predicting the next person with whom an individual will interact.

I believe these findings have several key implications. Being able to predict the next person an individual will interact with could allow for indirect targeted marketing through this person. However, I found that there is significant uncertainty in who the next person is (roughly five to eight people), suggesting that one may need target a group of people rather than a single person. On a more positive note, the simplicity of the Markov chain model enables us to perform rigorous mathematical analyses that would not be possible with more complicated models. From the findings of this study, I believe that such a model is appropriate for making predictions about dynamic processes over social networks such as information diffusion and disease propagation.

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