Adaptive robust cross-coupling position synchronization control of a hydraulic press slider-leveling

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Abstract
To achieve a high performance synchronized motion trajectory tracking of the hydraulic press slider-leveling electrohydraulic control system, an adaptive robust cross-coupling control strategy that incorporates the cross-coupling approach into adaptive robust control (ARC) architecture has been proposed. The primary objective of this study was to describe that the nonlinear ARC controller together with a cross-coupling control (CCC) controller was integrated to solve the slider-leveling synchronization control system using four axes. A discontinuous projection-based ARC controller was constructed. A robust control method with dynamic compensation type fast adaptation was introduced to attenuate the effects of parameter estimation errors, unmodeled dynamics and disturbances, and improved the transient tracking performance of the system. The stability of the controller was proven by Lyapunov theory and the trajectory tracking error asymptotically converges to zero. The simulation of a desired reference trajectory was included. The max tracking error of the proposed ARC controller of single axis was kept within 0.06 mm. The trajectory tracking error asymptotically converges to zero, which guaranteed the system would possess good transient behavior and confirmed the stability performance of the control system. The four axes synchronous errors of reference trajectory with cross-coupling controller indicated the maximum synchronization error of the proposed ARC + CCC controller between axis was within ±0.1 mm. The ARC together with a CCC controller for four hydraulic cylinders used parameter adaptation to obtain estimates of model parameters for reducing the extent of parametric uncertainties, and used a robust control law to attenuate the effects of parameter estimation errors, unmodeled dynamics, and disturbances. This study result shows that the proposed cross-coupling synchronization control scheme, together with the ARC law, provides excellent synchronization motion performance in a control system with four axes.

Keywords

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Adaptive robust control, cross-coupling control, position synchronization, hydraulic press slider-leveling, servo control

Introduction

As an efficient machine tool, hydraulic press is widely used in various kinds of industrial production and plays a strategic role in national economic development. With the rapid development of modern industry, the tonnage of hydraulic press and working surface is now getting bigger, the precision of part is getting higher and the shape is getting complicated. However, due to limitations of hydraulic equipment structure, uneven force and/or other factors to the work bed, the slider action is often affected by the off-center loading in the actual work process. The off-center loading of hydraulic press is due to the geometry of the part not being conducive to an even load. Many applications do not apply the load evenly over the slider and can wreak havoc on the press structure and tooling. Hydraulic presses, especially those with large slider and/or small tooling, can also be improperly loaded due to operator error, creating additional alignment problems. Therefore, it is necessary to equip hydraulic press with synchronization control to prevent slider from off-center loading, thus ensuring motion accuracy of the slider, which also improve loading status and extend the service life of the hydraulic press.

To research and design synchronization electrohydraulic control system of hydraulic press, the paper has proposed adaptive robust cross-coupling position synchronization control method which can be integrated into a hydraulic press to increase part quality, reduce die wear, and reduce complex setup time. This provides the ideal solution for manufacturers who need to counteract the effects off-center loading, significant breakthrough shock, and reverse shock loading situations.

The hydraulic press slider-leveling electrohydraulic control system (HPSECS) which belongs to a typical four axes synchronization control system is a key component of high-precision hydraulic press. As a typical multi-axis synchronous control technology, synchronization motion control has always been the focus of the scholars at home and abroad research. Many control algorithms, for example, PID control, disturbance observer, decoupling control, adaptive control, and the sliding mode control are applied to the multi-axis synchronous control of electrohydraulic system. Synchronous control method which is based on the equivalent mode or master-slave mode, can obtain a satisfactory performance on some occasions. The HPSECS belongs to the typical four axes synchronous control system. When any one axis suffers from strong nonlinearities and external disturbances synchronous precision immediately becomes poor so as to inaccurately track the motion trajectory. An exact system model is difficult to be satisfied for the HPSECS because of the existing two uncertainties. One is parametric uncertainties, for example, bulk modulus, square root relationship between pressure and flow, temperature and pressure dependent oil properties and friction, the other is uncertain nonlinearities, for example, friction and external disturbances. Hence, in order to attain high precision motion trajectory tracking performance, many
advanced control strategies have been applied. Ullah et al.\textsuperscript{16} and Asghar and Ullah\textsuperscript{17} has proposed fractional and integer order controllers, which are effective approaches that can be used to solve systems with parametric uncertainties and external disturbances. Yao has developed adaptive robust control (ARC) theory, combined with both adaptive control and robust control (such as sliding mode control), that can simultaneously solve systems with parameter uncertainties and uncertain nonlinearities.\textsuperscript{18–21}

Although the parameter estimation may hardly converge to their true values when the persistent exciting condition is not satisfied, the output tracking performance is guaranteed in terms of final tracking accuracy. Owing to the motion of hydraulic press slider is the instantaneous dynamic leveling system, the fast dynamic compensations are employed to ensure the transient tracking performance which may achieve better transient performance. The effectiveness of the approach has been verified in many literatures, for example, single-rod hydraulic actuators, hydraulic robotics arm, the control of pneumatic muscles driven parallel manipulators and pneumatic cylinders, etc.\textsuperscript{22–25}

To achieve a high performance synchronized motion trajectory tracking of the HPSECS, an adaptive robust cross-coupling control strategy that incorporates the cross-coupling approach into ARC architecture has been proposed.\textsuperscript{14,26,27,28–31} The current study establishes a mathematical model of HPSECS. ARC theory is based on a backstepping method that was designed for the HPSECS. The proposed controller guarantees robustness, transient performance and steady-state tracking accuracy. In the process of designing the controller, two Lyapunov functions were adopted to prove asymptotic convergence to zero of position tracking errors.

The rest of this paper is organized as follows. The second section describes problem formulation and dynamic models. The third section presents the proposed controller integrated ARC with cross-coupling controller. The fourth section presents the simulation setup and results. The final section draws conclusions.

\textbf{Problem formulation and dynamic models}

In the HPSECS, four hydraulic cylinders with same structure were set under the four corners of the base bolster of the hydraulic press. And four high-precision displacement sensors were also placed on the four hydraulic cylinders. Only in the procedure of clamping need the hydraulic cylinder to be in contact with the slider, and followed downward. If the slider deflected under partial load torque, the controller obeys the control algorithm and controls the opening of the HSPV, which is the backpressure of the piston-side chamber of the hydraulic cylinder. Oil of the piston-side chamber goes back to the tank by the opening of the HSPV.\textsuperscript{6} Four hydraulic cylinders control the accuracy of the slider, and simultaneously, parallelism should be guaranteed under the condition of four axes synchronous control. The hydraulic principle of the HPSECS is shown in Figure 1.

The load force balance equation of the \textit{i}th cylinder \((i = 1–4)\) can be described
where $m_i$ is the equivalent mass of the load, $x_{pl}$ is the displacement of the load, $p_i$ is the pressure inside piston-side chambers of the cylinder, $A_i$ is the area inside piston-side chambers of the cylinder, $b_i$ is the combined coefficient of modeled damping and viscous friction forces on the load, $F_{coli}$ is the modeled coulomb friction force, $G_i$ is the equivalent gravity of the load, $F_{Li}$ is the equivalent external load, and $\tilde{f}_i$ is the lumped model nonlinearities caused by unmodeled friction forces and external disturbances.

By neglecting both the internal and external leakage, the pressure dynamics can be expressed

$$\frac{V_{ih}}{\beta_{ei}} \dot{p}_i = A_i \ddot{x}_{pl} - Q_i$$  \hspace{1cm} (2)

The internal volume of the piston-side chamber of the cylinder may be written

$$V_{ih} = V_{i0} + A_i(L_i - x_{pl})$$  \hspace{1cm} (3)

where $Q_i$ is the flow through the piston-side chambers of the cylinder, $V_{ih}$ is the volume of the piston-side chambers, $V_{i0}$ is the initial volume of the piston-side chambers, $\beta_{ei}$ is the effective bulk modulus of the system, and $L_i$ is the hydraulic cylinder stroke.

The flow through the cylinder related to the high-response servo proportional valve (HSPV) can be described

$$Q_i = k_{iq}u_i \left[ s_g(u_i) \sqrt{p_s - p_i} + s_g(-u_i) \sqrt{p_i - p_t} \right]$$  \hspace{1cm} (4)

Now, by definition
where \( k_{iqv} \) is the HSPV flow gain coefficients, \( u_i \) is the HSPV control input voltage, \( p_s \) is the pump supply pressure, and \( p_t \) is the tank pressure.

Define \( u_i \) as the control input and define the state variables \( x_i \) form as:

\[
x_i = [x_{i1}, x_{i2}, x_{i3}]^T = [x_{pi}, \dot{x}_{pi}, p_i]^T.
\]

The entire system, equations (1) to (4) can be expressed in the state space form

\[
\begin{align*}
\dot{x}_{i1} &= x_{i2} \\
\dot{x}_{i2} &= \frac{1}{m_i} [G_i + F_{Li} - p_i A_i - b_i \dot{x}_{pi} - F_{coi} \tanh(\dot{x}_{pi})] + d_i \\
\dot{x}_{i3} &= \frac{\beta_{si}}{V_{ih}} (A_i x_{i2} - k_{iqv} g_i u_i)
\end{align*}
\]

in which

\[
g_i = s_g(u_i) \sqrt{p_s - x_{i3}} + s_g(-u_i) \sqrt{x_{i3} - p_t}
\]

Define the unknown parameter set \( \theta_{ij} = [\theta_{i1}, \theta_{i2}, \theta_{i3}]^T = [1/m_i, d_{mi}, \beta_{ei}]^T \) (\( j = 1 \sim 3 \)). The state-space equation (6) can be linearly parameterized in terms of \( \theta_{ij} \)

\[
\begin{align*}
\dot{x}_{i1} &= x_{i2} \\
\dot{x}_{i2} &= \theta_{i1} [G_i + F_{Li} - A_i x_{i3} - b_i x_{i2} - F_{coi} \tanh(x_{i2})] + \theta_{i2} + \tilde{d}_i \\
\dot{x}_{i3} &= h_i [\theta_{i3} (A_i x_{i2} - k_{iqv} g_i u_i)]
\end{align*}
\]

in which

\[
h_i = \frac{1}{V_{ih}}, d_i = \frac{\tilde{f}_i}{m_i} = \tilde{d}_i + d_{mi}
\]

where \( \tilde{f}_i \) represents the uncertain nonlinearities in the model, \( \tilde{d}_i \) is the time-varying portion of the modeling error and disturbance varies, and \( d_{mi} \) is the constant nominal value of the lumped uncertain nonlinearities.

The extent of parametric uncertainties and uncertain nonlinearities in equation (8) are known as

\[
\begin{align*}
\theta_{ij} = \Omega_{\theta_{ij}} \Delta \{ \theta_{ij} : \theta_{ij_{\min}} < \theta_{ij} < \theta_{ij_{\max}} \} \\
|d_i(t, x_{i1}, x_{i2})| &\leq \delta_{di}(x_{i1}, x_{i2}, t)
\end{align*}
\]

where \( \theta_{ij_{\min}} = [\theta_{i1_{\min}}, \theta_{i2_{\min}}, \theta_{i3_{\min}}]^T \), \( \theta_{ij_{\max}} = [\theta_{i1_{\max}}, \theta_{i2_{\max}}, \theta_{i3_{\max}}]^T \), and \( \delta_{di}(x_{i1}, x_{i2}, t) \) are known.

Let \( \hat{x}_{i2} \) and \( \hat{x}_{i3} \) represent the calculable part of \( \dot{x}_{i2} \) and \( \dot{x}_{i3} \) in equation (8), respectively, which are expressed as

\[
\begin{align*}
\hat{x}_{i2} &= \hat{\theta}_{i1} [G_i + F_{Li} - A_i x_{i3} - b_i x_{i2} - F_{coi} \tanh(x_{i2})] + \hat{\theta}_{i2} \\
\hat{x}_{i3} &= h_i [\hat{\theta}_{i3} (A_i x_{i2} - k_{iqv} g_i u_i)]
\end{align*}
\]
For the present analysis, a discontinuous projection mapping-based adaptation law was chosen. Here, \( \bullet_{ij} \) is used to represent the part calculated with estimated parameters, and let \( \bullet_{ij} \) denote the estimation error, defined as \( \hat{\theta}_{ij} = \theta_{ij} - \theta_{ij} \), where \( \bullet_{ij} \) represents the \( j \)th component of the vector \( \bullet_i \).

This simple discontinuous projection mapping has the form as

\[
\text{proj}_{\theta_{ij}}(\hat{\theta}_{ij}) = \begin{cases} 
0, & \hat{\theta}_{ij} = \theta_{ij_{\text{max}}} \bullet_{ij} > 0 \\
0, & \hat{\theta}_{ij} = \theta_{ij_{\text{min}}} \bullet_{ij} < 0 \\
\bullet_{ij}, & \text{other}
\end{cases}
\]

(12)

Under this projection mapping, the adaptation law is defined by

\[
\dot{\theta}_{ij} = \text{proj}_{\theta_{ij}}(\Gamma_i \tau_i)
\]

(13)

where \( \Gamma_i \) is the adaptation rate coefficient of \( \dot{\theta}_{ij} \), and \( \tau_i \) is the adaptation tuning function of \( \dot{\theta}_{ij} \).

This adaptation law used in equation (13) guarantees the conditions bound as shown in equation (14)

\[
\begin{cases}
\hat{\theta}_{ij} \in \tilde{\Omega}_{\theta_{ij}} \overset{\Delta}{=} \{ \hat{\theta}_{ij} : \theta_{ij_{\text{min}}} \leq \hat{\theta}_{ij} \leq \theta_{ij_{\text{max}}} \} \\
\tilde{\theta}_{ij}^T (\Gamma_i^{-1} \text{Proj}_{\theta_{ij}}(\Gamma_i \tau_i) - \tau_i) \leq 0, \forall \tau_i
\end{cases}
\]

(14)

Define the motion trajectory tracking error vector of the \( i \)th cylinder

\[
\dot{z}_{i1} = x_{i1} - x_{i1d}
\]

(15)

where \( x_{i1d} \) is the desired motion trajectory of the \( i \)th cylinder.

In the synchronized motion, in addition to \( \dot{z}_{i1} \rightarrow 0 \), the goal is also aimed at regulating the motion relationships among four axes during the tracking so that

\[
z_{11} = z_{21} = z_{31} = z_{41}
\]

(16)

Define synchronization errors of a subset of all possible pairs of the axis from the total of four axes in the following way\(^{32}\)

\[
\begin{align*}
\varepsilon_1 &= z_{11} - z_{21} \\
\varepsilon_2 &= z_{21} - z_{31} \\
\varepsilon_3 &= z_{31} - z_{41} \\
\varepsilon_4 &= z_{41} - z_{11}
\end{align*}
\]

(17)

where \( \varepsilon_i \) is the synchronization errors of the \( i \)th axis.

Four sub-goals such as \( \dot{z}_{i1} \) contain the relationship between the two actuators as

\[
z_{11} = z_{21}, z_{21} = z_{31}, z_{31} = z_{41}, z_{41} = z_{11}
\]

(18)

Obviously, if \( \varepsilon_i = 0 \) for all \( \dot{z}_{i1} \), the goal of synchronizing the four axes of equation (18) can be achieved.
Adaptive robust cross-coupling controller design

Step 1. Define a switching function-like quantity as

\[
\begin{align*}
    z_{i2} &= \dot{z}_{i1} + k_{i1} z_{i1} = x_{i2} - x_{i2eq} \\
    x_{i2eq} &= \dot{x}_{i2} - k_{i1} z_{i1}
\end{align*}
\]

(19)

where \( k_{i1} \) is the positive feedback gain. Since \( G(s) = 1/(s + k_{i1}) \) is a stable transfer function, making \( z_{i1} \) small or converging to zero, this is equivalent to making \( z_{i2} \) small or converging to zero. Differentiating equation (19) and noting equation (8), it is straightforward to obtain

\[
\begin{align*}
    \dot{z}_{i2} &= u_{i1} G_i + F_{Li} A_i x_{i3} - b_i x_{i2} - F_{col} \tanh(x_{i2}) + \theta_{i2} + \tilde{d}_i - \dot{x}_{i2eq} \\
    \dot{x}_{i2eq} &= \dot{x}_{i2} - k_{i1} \dot{z}_{i1}
\end{align*}
\]

(20)

Let \( p_{Li} = -A_i x_{i3} \) denotes the virtual control input and let \( z_{i3} = p_{Li} - \alpha_{2i} \) denotes the input discrepancy. In view of \( p_{Li} \), the expectation virtual control function \( \alpha_{2i} \) is designed as equation (21)

\[
\begin{align*}
    \alpha_{2i} &= \alpha_{2ai} + \alpha_{2si} \\
    \alpha_{2ai} &= \alpha_{2ai1} + \alpha_{2ai2}, \alpha_{2si} = \alpha_{2si1} + \alpha_{2si2} \\
    \alpha_{2ai1} &= b_i x_{i2} + F_{col} \tanh(x_{i2}) - G_i - F_{Li} + \frac{1}{\theta_{i1}}(\dot{x}_{2eq} - \hat{\theta}_2) \\
    \alpha_{2si1} &= -\frac{1}{\theta_{i1 \min}} k_{i2} z_{i2}
\end{align*}
\]

(21)

where \( \alpha_{2ai1} \) is the adjustable model compensation and represents the calculable parts of \( \alpha_{2i} \), and \( \alpha_{2ai2} \) is the fast dynamic compensation term synthesized below. \( \alpha_{2ai1} \) is used to stabilize the nominal system, which is chosen to be simple proportional feedback of \( z_{i2} \), and \( \alpha_{2si2} \) is a robust feedback term to be synthesized later. \( k_{i2} \) is the linear feedback gain function and \( k_{i2} > 0 \).

Substituting (21) to (20), \( \dot{z}_{i2} \) can be expressed by

\[
\begin{align*}
    \dot{z}_{i2} &= -\frac{\theta_{i1}}{\theta_{i1 \min}} k_{i2} z_{i2} + \hat{\theta}_{i1} \alpha_{2ai1} + \theta_{i1} \alpha_{2ai2} - \hat{\theta}_{i1} (\alpha_{2ai} + G_i + F_{Li} - b_i x_{i2} - F_{col}) \\
    &\quad - \tilde{\theta}_{i2} + \tilde{d}_i + \theta_{i1} z_{i3}
\end{align*}
\]

(22)

Let

\[
\begin{align*}
    d_{ci1} + \Delta_{i1}(t) &= -\hat{\theta}_{i1} (\alpha_{2ai} + G_i + F_{Li} - b_i x_{i2} - F_{col}) - \tilde{\theta}_{i2} + \tilde{d}_i
\end{align*}
\]

(23)

where \( d_{ci1} \) is a low-frequency component and \( \Delta_{i1}(t) \) is a high-frequency component.

The low-frequency component \( d_{ci1} \) can be compensated for through the following fast dynamic compensation as follows:

\[
\alpha_{2ai2} = -\frac{d_{ci1}}{\hat{\theta}_{i1}}
\]

(24)
where $\hat{d}_{c1}$ is the estimate of $d_{c1}$. $d_{c1 \text{ max}}$ represents any pre-set bound and use this bound to construct the following projection type adaptation law for $\hat{d}_{c1}$

$$\hat{d}_{c1} = \text{proj}_{\hat{d}_{c1}}(\gamma_{c1} z_{i2})$$

$$= \begin{cases} 
0 & \text{if } |\hat{d}_{c1}| = d_{c1 \text{ max}} \text{ and } \hat{d}_{c1}(t) z_{i2} > 0 \\
\gamma_{c1} z_{i2} & \text{else}
\end{cases}$$

(25)

with $|\hat{d}_{c1}(0)| \leq d_{c1 \text{ max}}$, where $\gamma_{c1} > 0$ is the adaptation rate. Such a projection type adaptation law guarantees that $|\hat{d}_{c1}(t)| \leq d_{c1 \text{ max}}, \forall t$.

Substituting equations (23) and (24) into equation (22), $\dot{z}_{i2}$ can be expressed by

$$\dot{z}_{i2} = -\frac{\theta_{i1}}{\theta_{i1 \text{ min}}} k_{i2} z_{i2} + \theta_{i1} \alpha_{2s2i} + \Delta_{i1}(t) - \tilde{d}_{c1} + \theta_{i1} z_{i3}$$

(26)

Define a positive semi-definite Lyapunov function

$$V_{i2} = \frac{1}{2} w_{i2} z_{i2}^2$$

(27)

where $w_{i2} > 0$ is a constant weighting factor.

Substituting equation (26) into equation (27) gives

$$\dot{V}_{i2} = w_{i2} \theta_{i1} z_{i2} z_{i3} - w_{i2} \frac{\theta_{i1}}{\theta_{i1 \text{ min}}} k_{i2} z_{i2}^2 + w_{i2} z_{i2} \left[ \theta_{i1} \alpha_{2s2i} + \Delta_{i1}(t) - \tilde{d}_{c1} \right]$$

$$= w_{i2} \theta_{i1} z_{i2} z_{i3} + \dot{V}_{i2}\big|_{a_{i2}}$$

(28)

The following condition is now chosen to satisfy the robust control function $\alpha_{2s2i}$ as follows

$$\begin{cases} 
I. \quad z_{i2} \left[ \theta_{i1} \alpha_{2s2i} + \Delta_{i1}(t) - \tilde{d}_{c1} \right] \leq e_{i2} \\
II. \quad z_{i2} \alpha_{2s2i} \leq 0
\end{cases}$$

(29)

The adaptation tuning function $\tau_{i2}$ is given by

$$\tau_{i2} = w_{i2} \phi_{i2} z_{i2}$$

(30)

where $\phi_{i2} = [\alpha_{2ai} - b_{i} z_{i2} - F_{co} \tanh(x_{i2}) + G_{i} + F_{Li}, 1, 0]^{T}$.

Step 2. Step 2 requires synthesizing a virtual control function such as $z_{i3}$ that converges to zero or has a small value with a guaranteed transient performance as follows. Differentiating $z_{i3}$ with respect to time yields

$$\dot{z}_{i3} = \dot{p}_{Li} - \dot{\alpha}_{2i}$$

$$= \theta_{i3} A_{i} h_{i} k_{ip} g_{i} u_{i} - \theta_{i3} A_{i}^{2} h_{ix_{i2}} - \dot{\alpha}_{2ci} - \dot{\alpha}_{2ai}$$

(31)
According to equations (8) and (21), one obtains
\[ \dot{\alpha}_{2i} = \frac{\partial \alpha_{2i}}{\partial x_{ij}} \dot{x}_{ij} + \frac{\partial \alpha_{2i}}{\partial x_{ij}} \ddot{x}_{ij} + \frac{\partial \alpha_{2i}}{\partial \theta_j} \ddot{\theta}_j + \frac{\partial \alpha_{2i}}{\partial t} \]  
(32)
in which
\[ \dot{\alpha}_{2ci} = \frac{\partial \alpha_{2i}}{\partial x_{ij}} x_{ij} + \frac{\partial \alpha_{2i}}{\partial x_{ij}} \dot{x}_{ij} + \frac{\partial \alpha_{2i}}{\partial x_{ij}} \ddot{x}_{ij} + \frac{\partial \alpha_{2i}}{\partial t} \]  
(33)
In equation (33), the calculable part is \( \dot{\alpha}_{2ci} \) since it is composed by the desired state variables, actual state variables and estimated parameters. \( \dot{\alpha}_{2ui} \) is the incalculable part because of the unknown terms, such as the estimated errors and uncertain non-linearity, and has to be dealt with by a certain amount of robust feedback.

Define a positive semi-definite Lyapunov function as
\[ V_{i3} = V_{i2} + \frac{1}{2} w_{i3} z_{i3}^2 \]  
(34)
Differentiating equation (34) and noting equation (28), one obtains
\[ \dot{V}_{i3} = \dot{V}_{i2} + w_{i3} z_{i3} \dot{z}_{i3} = \dot{V}_{i2}\bigg|_{\alpha_{2i}} + w_{i2} \theta_i \dot{\theta}_i z_{i2} + w_{i3} z_{i3} \dot{z}_{i3} \]  
(35)
where \( \dot{V}_{i2}\bigg|_{\alpha_{2i}} \) denotes \( \dot{V}_{i2} \) under the condition that \( p_{Li} = \alpha_{2i} \).

Noting equations (31) and (35), \( \dot{V}_{i3} \) can be obtained
\[ \dot{V}_{i3} = \dot{V}_{i2}\bigg|_{\alpha_{2i}} + w_{i2} \theta_i \dot{\theta}_i z_{i2}^2 z_{i3} + w_{i3} z_{i3} \dot{z}_{i3} \]  
(36)
Defining
\[ H_i = A_i h_i k_{iqi} g_i \]  
(37)
Substituting equation (37) into equation (36), we see that equation (36) is rewritten into
\[ \dot{V}_{i3} = \dot{V}_{i2}\bigg|_{\alpha_{2i}} + w_{i3} z_{i3} \left\{ \dot{\theta}_i H_i u_{ai} + \theta_{i3} H_i u_{ai} - \dot{\theta}_i H_i u_{ai} - \theta_{i3} h_i x_{i2} - \alpha_{2ci} - \alpha_{2ui} \right\} \]  
(38)
Let
\[ d_{ci2} + \Delta_{i2}(t) = -\hat{\theta}_{i3}H_iu_{ai} + \hat{\theta}_{i3}A_i^2h_ix_{i2} - \frac{w_{i2}}{w_{i3}}z_{i2}\tilde{\theta}_{i} - \hat{\alpha}_{2ai} + \frac{\partial \alpha_{2i}}{\partial \theta_{i}}\hat{\theta}_{i} \]  

(39)

As in Step 1, the fast dynamic compensation term \( u_{ai2} \) can be chosen as

\[ u_{ai2} = -\frac{\hat{d}_{ci2}}{\theta_{i3}H_i} \]  

(40)

where \( \hat{d}_{ci2} \) represents the estimate of \( d_{ci2} \) updated by

\[ \hat{d}_{ci2} = \text{proj}_{d_{ci2}}(\gamma_{ci2}z_{i3}) \]

\[ = \begin{cases} 
0 & \text{if } |\hat{d}_{ci2}| = \hat{d}_{ci2,\text{max}} \text{ and } \hat{d}_{ci2}(t)z_{i3} > 0 \\
\gamma_{ci2}z_{i3} & \text{else}
\end{cases} \]  

(41)

with \( |\hat{d}_{ci2}(0)| \leq \hat{d}_{ci2,\text{max}} \), where \( \gamma_{ci2} > 0 \) is the adaptation rate and \( \hat{d}_{ci2,\text{max}} > 0 \) is a preset bound. Such a projection-type adaptation law guarantees that \( |\hat{d}_{ci2}(t)| \leq \hat{d}_{ci2,\text{max}}, \forall t \).

Substituting equations (39) and (40) into equation (38) gives

\[ \dot{V}_{i3} = \dot{V}_{i2}\big|_{a2} - w_{i3}k_{i3}\theta_{i3}z_{i2}^2 + w_{i3}z_{i3}\left[\theta_{i3}H_iu_{ai2} + \Delta_{i2}(t) - \hat{d}_{ci2}\right] \]  

(42)

Similarly, \( u_{si2} \) is chosen to satisfy the following conditions:

\[ \begin{cases} 
I. & z_{i3}\left[\theta_{i3}H_iu_{ai2} + \Delta_{i2}(t) - \hat{d}_{ci2}\right] \leq e_{i3} \\
II. & z_{i3}\theta_{i3}u_{ai2} \leq 0
\]  

(43)

The following ARC control law, \( u_i \), is obtained as

\[ \begin{align*}
    u_i &= u_{ai1} + u_{ai2} + u_{si1} + u_{si2} \\
    u_{ai1} &= \frac{1}{\theta_{i3}H_i}\left(\hat{\theta}_{i3}A_i^2h_ix_{i2} + \hat{\alpha}_{2ci} - \frac{w_{i2}}{w_{i3}}\hat{\theta}_{i} + \frac{\partial \alpha_{2i}}{\partial \theta_{i}}\hat{\theta}_{i}\right) \\
    u_{ai2} &= -\frac{\hat{d}_{ci2}}{\theta_{i3}H_i} \\
    u_{si1} &= -\frac{1}{\theta_{i3}\min\left(H_i\right)}k_{i3}z_{i3}
\end{align*} \]  

(44)

where \( u_{ai1} \) is the adjustable model compensation and represents the calculable parts of \( u_i \), and \( u_{ai2} \) is the fast dynamic compensation term synthesized. \( u_{si1} \) is used to stabilize the nominal system, which is chosen to be simple proportional feedback of \( z_{i3} \), and \( u_{si2} \) is a robust feedback term to be synthesized later. \( k_{i3} \) is the linear feedback gain and \( k_{i3} > 0 \).

The adaptation tuning function \( \tau_{i3} \) is given by

\[ \tau_{i3} = w_{i3}\phi_{i3}z_{i3} \]  

(45)

where \( \phi_{i3} = \left[w_{i2}z_{i2} - \frac{\partial \phi_{i1}}{\partial z_{i2}}\right|_{G_i + F_{Li} - A_i\phi_{i3} - b_{3x_{i2}} - F_{cod} \tanh(x_{i2})}, -\frac{\partial \phi_{i1}}{\partial u_{ai}}, H_iu_{ai} - A_i^2h_ix_{i2}\right] \).

The system adaptation function, \( \tau_i \), is obtained
\[ \tau_i = w_i z_{i2}^2 + w_i z_{i3}^3 \]  \hspace{1cm} (46)

Substituting the first inequation of equation (44) into equation (43), one obtains

\[ \dot{V}_{i3} = \dot{V}_{i2} \big|_{a_{i2}} - w_{i2} k_{i2} \theta_{i2} z_{i2}^2 + w_{i3} \theta_{i3} [\theta_{i3} H(t) u_{i3} + \Delta_{i2}(t) - \tilde{d}_{i2}] \]
\[ \leq -w_{i2} k_{i2} z_{i2}^2 + w_{i2} \varepsilon_{i2} - w_{i3} k_{i3} z_{i3}^2 + w_{i3} \varepsilon_{i3} \]
\[ \leq -\lambda_{iV} V_{i3} + \varepsilon_{iV} \]  \hspace{1cm} (47)

**Theorem 1:** According to equation (8), if the control law satisfied with equation (10) is synthesized as equation (44) and with the adaptation law given by equation (13), the following conclusions can be obtained

I. The tracking error \( z_{i1} \), and its state variables, \( z_i = [z_{i1}, z_{i2}, z_{i3}]^T \), are bounded. Furthermore, the positive semidefinite Lyapunov function \( V_{i3}(t) \) in equation (42) is exponentially convergent as

\[ V_{i3}(t) \leq \exp(-\lambda_{iV} t) V_{i3}(0) + \frac{\varepsilon_{iV}}{\lambda_{iV}} [1 - \exp(-\lambda_{iV} t)] \]  \hspace{1cm} (48)

in which

\[ \lambda_{iV} = 2 \min\{k_{i2}, k_{i3}\} \]
\[ \varepsilon_{iV} = w_{i2} \varepsilon_{i2} + w_{i3} \varepsilon_{i3} \]  \hspace{1cm} (49)

II. If after a finite time \( t_0, \tilde{d}_i = 0 \), that is, in the presence of only parametric uncertainties, a system asymptotic track is also achieved. That is, \( t \to \infty; \quad z_{i1} \to 0 \).

Remark: Result I of Theorem 1 indicates that the final tracking errors will be bounded by \( \varepsilon_{iV}/\lambda_{iV} \) with the proposed controller, by choosing an adequately large converging rate, \( \lambda_{iV} \), or an adequately small controller parameter, \( \varepsilon_{iV} \). Result II of Theorem 1 implies that asymptotic position tracking will be finally achieved if the system is subjected to only linear parametric uncertainties.

**Cross-coupling controller design**

Figure 2 presents a cross-coupling control (CCC) block diagram based on an ARC approach. The cross-coupling approach based the adaptive robust motion controller makes the trajectory tracking error \( z_i \) convergence to zero and guarantees the stability of the system. The cross-coupling controller uses a PI controller, whose transfer function is expressed as equation (50)
Simulation setup and parameter settings

Simulation setup. MATLAB/Simulink was used and was based on the model illustrated in equation (8), whose structure is shown in Figure 3. The HPSECS’s control algorithm is carried out on the Simulink/AMESim co-simulation platform. The hydraulic press photograph is shown in Figure 4. The HPSECS is mainly composed of a frame, a return cylinder, two master cylinders, four leveling hydraulic cylinders and a base bolster. In the test rig, the leveling actuators with the dimensions of 140 mm × 100 mm × 100 mm were controlled by four HSPVs (REXROTH: 4WRREH6VB40L-1X/G24K0/B5M), had a bandwidth above 80 Hz with a ±100% control signal. The system state variables used in the controller, including actuator’s displacement $x_i$, and velocity $x_{i2}$, and pressures in the working chambers $x_{i3}$ was directly measured by pressure transducers. The supply pressure $p_s$ was regulated to 20 MPa for the control. The displacement and velocity of the actuator was directly obtained by the position transducers.

Parameter settings. The essential parameters of the HPSECS simulations are given in Table 1. The values of the essential parameters come from design, measurements, tests and the literature.

Figure 2. Block diagram based on adaptive robust cross-coupling approach.

$$G_{cccl}(s) = K_p + \frac{K_i}{s}$$ (50)
The uncertain parameters of the HPSECS simulations are given in Table 2. To illustrate the discrepancy of synchronization errors between the ARC controller with CCC controller, the ARC controller parameters of the reference trajectory for four axes are given in Tables 3. The CCC controller parameters are given in Table 4.

**Figure 3.** Block diagram of the hydraulic press slider-leveling electro-hydraulic control system.

**Figure 4.** Photograph of hydraulic press.

The uncertain parameters of the HPSECS simulations are given in Table 2. To illustrate the discrepancy of synchronization errors between the ARC controller with CCC controller, the ARC controller parameters of the reference trajectory for four axes are given in Tables 3. The CCC controller parameters are given in Table 4.
### Results and discussion

In the process of the simulation, a desired reference trajectory is shown in Figure 5. The max tracking error of the proposed ARC controller of single axis was kept within—0.06 mm in Figure 6. System parameter adaptive estimation curves are

**Table 1.** Values of essential parameters (*i* = 1–4).

| Parameter (unit) | Value       |
|------------------|-------------|
| \( m_i \) (kg)   | 7.5 × 10^2 |
| \( A_i \) (m^2)  | 1.5394 × 10^{-2} |
| \( b_i \) (N·s/m) | 1.0 × 10^5 |
| \( F_{col} \) (N) | 2.4 × 10^3 |
| \( V_{0i} \) (m^2) | 8.3695 × 10^{-7} |
| \( k_{eqi} \) (m^2/sV/\( p_o \)) | 3.5635 × 10^{-8} |
| \( G_i \) (N) | 7.5 × 10^3 |
| \( p_i \) (\( p_o \)) | 2.0 × 10^7 |
| \( p_i \) (\( p_o \)) | 0 |
| \( F_{Li} \) (N) | 9.0 × 10^3 |

**Table 2.** Extent of uncertain parameters (*i* = 1–4).

| Parameter (unit) | \( \theta_i \) norm | \( \theta_i \) min | \( \theta_i \) max |
|------------------|---------------------|-------------------|-------------------|
| \( \theta_{i1} \) (kg^-1) | 1.3333 × 10^{-3} | 1.3333 × 10^{-4} | 1.3333 × 10^{-2} |
| \( \theta_{i2} \) (N·kg^-1) | 0 | -10.0 | 10.0 |
| \( \theta_{i3} \) (\( p_o \)) | 7.0 × 10^8 | 1.4 × 10^8 | 3.5 × 10^9 |

**Table 3.** Values of controller parameters (*i* = 1–4).

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| \( \Lambda \) | 320.0 | \( \Gamma_{i1} \) | 1.0E-20 |
| \( k_{i2} \) | 5.6E6 | \( \Gamma_{i2} \) | 1.0E-11 |
| \( k_{i3} \) | 3.9E-7 | \( \Gamma_{i3} \) | 1.0E5 |
| \( K_c \) | 1.0 | \( \beta \) | 0.5 |

**Table 4.** Values of cross-coupling control controller parameter (*i* = 1–4).

| Parameter | Value |
|-----------|-------|
| \( k_{pi} \) | 3.5 × 10^3 |
| \( k_{ii} \) | 1.0 × 10^{-2} |
shown in Figure 7, which caused the controller to deal with parametric uncertainty. The four axes synchronous errors of reference trajectory with cross-coupling controller is shown in Figure 8, which indicates the maximum synchronization error of the proposed ARC + CCC controller between axis was within $\pm 0.1\,\text{mm}$. Figure 8 shows the contact between the slider and the leveling hydraulic cylinder which results in a collision impact at $t = 0.2\,\text{s}$. 

**Figure 5.** Reference trajectory.

**Figure 6.** Position error of HPSECS with single axis.

**Figure 7.** Parameter estimations of axis controller.
Figure 8. Synchronization errors with ARC + CCC.
Conclusion

In this paper, a detailed control model that was constructed to describe the HPSECS and an adaptive robust cross-coupling position synchronization control strategy for four axes was presented. The ARC controller used parameter adaptation to obtain estimates of model parameters for reducing the extent of parametric uncertainties, and used a robust control law to attenuate the effects of parameter estimation errors, unmodeled dynamics, and disturbances. A synchronization algorithm based on ARC together with a CCC controller for four hydraulic cylinders was proposed. The trajectory tracking error asymptotically converges to zero, which guaranteed the system would possess good transient behavior and confirmed the stability performance of the control system. The stability of the controller designed was verified through Lyapunov’s theory. The ARC controller of four axes electro-hydraulic control system solved the parameter uncertainties and uncertain nonlinearities problems. The four axes synchronous control systems integrated the ARC algorithm with the CCC approach for motion control could achieve excellent synchronization performance.

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