The growth and hydrodynamic collapse of a protoplanet envelope

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ABSTRACT
We have conducted three-dimensional self-gravitating radiation hydrodynamical models of gas accretion onto high mass cores (15-33 M$_\oplus$) over hundreds of orbits. Of these models, one case accretes more than a third of a Jupiter mass of gas, before eventually undergoing a hydrodynamic collapse. This collapse causes the density near the core to increase by more than an order of magnitude, and the outer envelope to evolve into a circumpalnetary disc. A small reduction in the mass within the Hill radius ($R_H$) accompanies this collapse as a shock propagates outwards. This collapse leads to a new hydrostatic equilibrium for the protoplanetary envelope, at which point 97 per cent of the mass contained within the Hill radius is within the inner 0.03 $R_H$ which had previously contained less than 40 per cent. Following this collapse the protoplanet resumes accretion at its prior rate. The net flow of mass towards this dense protoplanet is predominantly from high latitudes, whilst at the outer edge of the circumplanetary disc there is net outflow of gas along the midplane. We also find a turnover of gas deep within the envelope that may be caused by the establishment of convection cells.

Key words: planets and satellites: formation – methods: numerical – hydrodynamics – radiative transfer

1 INTRODUCTION

The ideas discussed in this paper begin with [Perri & Cameron (1974)], who stated that “when the mass of the core becomes sufficiently great, the surrounding gaseous envelope will become hydrodynamically unstable against collapse onto the planetary core”. This process is controlled by the battle between gravity that acts to contract an envelope onto the core and the gas pressure which acts to support it. [Mizuno (1980)] performed stability calculations to determine the combinations of core masses and opacities that would make a protoplanetary envelope unstable to collapse. Work of his contemporaries considering the structure of giant planets in the solar system suggested that each such planet (Jupiter, Saturn, Uranus, & Neptune) possessed a solid core with a mass of order 10 M$_\oplus$ ([Slattery 1977] [Hubbard & Macfarlane 1980]). Using these values as a target, and assuming a fixed accretion rate, Mizuno concluded that a grain opacity of $\kappa \approx 1$ cm$^2$ g$^{-1}$ was required in the envelope material during formation to trigger a collapse when the core mass was $\approx 10$ M$_\oplus$. Lower opacities were found to lead to envelope collapse at lower core masses. Following the envelope collapse [Mizuno] states that continued accretion is likely required for the protoplanet’s to attain their final masses.

It was suggested later by [Bodenheimer & Pollack (1986)], who performed evolution calculations that included evolution beyond the attainment of the critical core mass, that rather than a dynamical collapse, the envelope may quasi-statically contract onto the solid core. They suggested that if a sufficient mass of molecular hydrogen in the envelope was apt to undergo dissociation, then this would remove enough energy from the contraction to bring about a dynamical collapse. However, their models indicate that such dissipative regions possess an insufficient fraction of the envelope mass for this to occur.

In the early 1990s [Wuchterl] wrote a series of papers exploring the evolution of a protoplanetary envelope through the hydrostatic phases approaching the critical core mass, and in what he found to be a subsequent hydrodynamic phase [Wuchterl (1991)]. Solving the equations of radiation hydrodynamics in one-dimension, [Wuchterl] indicated that following a period of quasi-static contraction, during which the envelope heats up, the transport of this heat out through the envelope by convection and radiation perturbs the hydrostatic equilibrium. In particular Wuchterl cites the $\kappa$- mechanism as the means of exciting the dynamical waves that destabilise the envelope. The result of this hydrodynamic evolution was the ejection of a large fraction of the envelope mass, rather than an inwards collapse as had previously been supposed by [Perri & Cameron (1974)].

Pollack et al. (1996) continued performing models assuming a quasi-hydrostatic contraction, and suggested that further work was required to consider the hydrodynamic evolution of young protoplanets to establish the proper evolution scenario. A step in this direction was taken by [Tajima & Nakagawa (1997)] who performed a stability analysis of a growing protoplanetary envelope using a distinct numerical code to that of previous authors. Their aim was to determine whether quasi-static contraction, or an envelope instabil-
ity akin to that suggested by Wuchterl was the more likely evolutionary course for a growing protoplanet. They perturbed the envelope at intervals during its evolution to see if such action might push a marginally stable system towards instability. They concluded that quasi-static contraction was viable for a protoplanet growing all the way to a Jupiter mass.

There has since been a substantial amount of work considering the gas accretion rates that cores might achieve in circumstellar discs with a variety of properties. Koma et al. (2000) performed quasi-static evolutionary models to determine the dependence of gas accretion upon core mass, grain opacity, and the core’s accretion history, finding that these factors were strongly, moderately, and weakly significant respectively. Bryant et al. (1999) and Lubow et al. (1999) performed locally-isothermal hydrodynamics simulations of discs containing planetary cores, the latter finding that the accretion rate drops off as the protoplanet mass becomes very large; a result of the broadening disc gap that it forms. Further hydrodynamic models with more realistic thermodynamics were performed by D’Angelo et al. (2003a), Klahr & Kley (2006), Paardekooper & Mellema (2008), and Ayliffe & Bate (2009a), finding similar turn overs in the accretion rate with increasing mass, and illustrating the impact of grain opacity on accretion. These models have generally had limited resolution in the vicinity of the protoplanet, and though Ayliffe & Bate (2009a) achieved high resolution, the evolutionary period was extremely short due to the computational demands of the calculations.

In this paper we report results from three-dimensional self-gravitating radiation hydrodynamics calculations that resolve the protoplanet’s envelope, whilst modelling its hydrodynamic evolution and growth within a section of a circumstellar disc. Using high mass discs (though still stable; Toomre Q >> 1), and assuming low opacities, we achieve accretion rates that allow significant envelope growth in only a few hundred orbits, allowing us to examine their development. Our computational method is described in Section 2 followed by our results in Section 3 and a discussion of their relationship with previous results in Section 4.

## 2 Computational Method

The calculations discussed in this paper were performed using a three-dimensional SPH code. This SPH code is derived from a code first developed by Benz (1990) which has undergone substantial modification in subsequent years. Energy and entropy are conserved to timestepping accuracy by use of the variable smoothing length formalism of Springel & Hernquist (2002) and Monaghan (2002), where our particular implementation is described in Price & Monaghan (2007). Gravitational forces are calculated and particle neighbours are found using a binary tree. Radiative transfer is modelled using the flux-limited diffusion approximation, employing the method developed by Whitehouse, Bate & Monaghan (2005) and Whitehouse & Bate (2006). Integration of the SPH equations is achieved using a second-order Runge-Kutta-Fehlberg integrator with particles having individual timesteps (Bate 1995). Gas within the models is subject to an artificial viscosity, implemented in a parameterised form as developed for SPH by Monaghan & Gingold (1983), and modified to deal with high Mach number shocks by Monaghan (1992). The code has been parallelised by M. Bate using OpenMP and MPI.

### 2.1 Model Setup

The calculations we have performed were conducted by modelling a small section of a circumstellar disc, centred upon a protoplanetary core and corotating with its orbit. The protoplanet is modelled as a gravitating mass with a ‘surface’. It attracts gas from the disc, building up an atmosphere which is supported by the surface. The disc section measures $r = 1 \pm 0.15 r_p$ (5.2 $\pm$ 0.78 AU), and $\phi = \pm 0.15$ radians. The protoplanet is sited at a radius of $r_p$, which in all cases is equivalent to 5.2 AU, and orbits about a star of 1 $M_\odot$. The disc has a surface density profile of $\Sigma \propto r^{-1/2}$, and a temperature profile of $T_s \propto r^{-1}$ (giving a scaleheight of $H/r = 0.05$), where these profiles are equivalent to those used in our previous work, and were originally chosen to match work of Lubow et al. (1999) and Bate et al. (2003). The initial temperature at $r_p$ is $\approx 73$ K, where this temperature is taken to principally result from stellar irradiation. Near the midplane of the disc, this initial temperature may increase due to viscous heating as the model evolves if the opacity is sufficient to insulate the region against rapid radiative cooling. However, at the upper and lower radiation boundaries (see section 2.3 below), where the medium is always optically thin, an assumption of temperatures dominated by the stellar irradiation is reasonable. The surface density at $r_p$ has an unperturbed value of $750$ g cm$^{-2}$, which gives a relatively massive disc, comprising 0.1 $M_\odot$ of gas within 25 AU. We use a high disc surface density because Ayliffe & Bate (2009a) found that increasing the disc surface density resulted in somewhat faster accretion rates onto embedded protoplanets and the goal of this paper is to investigate the three-dimensional evolution of protoplanets that accrete massive gaseous envelopes.

Further details of our model setup are given below, but the method is identical to that employed in Ayliffe & Bate (2009a), and that paper contains somewhat more extensive information, including resolution tests. We note that the most interesting model discussed in this paper, a case which results in a hydrodynamic envelope collapse, required more than 6 CPU years to reach its final state. It is this extensive calculation time which has limited the number of models that is has been possible to perform.

### 2.2 Disc Sections

Within the disc section being modelled, the particles are initially distributed according to the underlying density profile, and their velocities are Keplerian. The protoplanet is orbiting its star in an anticlockwise direction, and in the corotating frame of the modelled section this leads particles at $r < r_p$ to orbit anticlockwise, and those at $r > r_p$ to orbit in a clockwise fashion. Particles encountering the boundary of the section are removed from the calculation, whilst a number of ghost particles beyond the domain of the calculation, along its boundaries, act to replicate the pressure and viscous forces expected from a continuous disc. To prevent the depletion of gas within the disc section, particles are injected along the boundaries where the material should flow in. For the anticlockwise orbiting gas this input is along the $\phi = -0.15$ radian boundary, between $r = 0.85 \pm 1.0 r_p$ and the clockwise flowing gas is injected along the $\phi = 0.15$ radian boundary, between $r = 1.0 \pm 1.15 r_p$. The injection scheme does more than simply replace gas which leaves the section. The velocity and density structure of the injected gas is obtained from three-dimensional global simulations of protoplanets embedded in discs performed using ZEUS (Bate et al. 2003), such that a gap is opened in the disc which corresponds to the mass of the embedded protoplanet. The models of Bate et al. used a surface density of 75 g cm$^{-2}$, but the particles injected in the calcula-
tions discussed here have their masses scaled to reflect the chosen surface density of 750 g cm$^{-2}$. As the protoplanet grows the gap width is suitably increased by interpolation through the protoplanet mass range provided by [Bate et al. 2003] ($1 M_\odot$–$1 M_\text{Jupiter}$). The injected particles come to dominate the section after less than five orbits, ensuring the structure is consistent with the presence of the protoplanet.

2.3 Thermodynamics

All the calculations discussed in this paper were performed using radiation hydrodynamics, employing a two temperature (gas and radiation) radiative transfer scheme using a flux-limited diffusion approximation (as described by Whitehouse et al. 2005 and Whitehouse & Bate 2006). Work and artificial viscosity act to increase the thermal energy of the gas, and work done on the radiation field increases the radiative energy, which can be transported via flux-limited diffusion. The energy transfer between the gas and radiation fields is dependent upon their relative temperatures, the gas density, and the gas opacity.

The gas is treated using an ideal gas equation of state $p = \rho T_s \mu m_g$ where $T_s$ is the gas constant, $\rho$ is the density, $T_s$ is the gas temperature, and $\mu$ is the mean molecular mass. The thermal evolution takes into account the translational, rotational, and vibrational degrees of freedom of molecular hydrogen (assuming a 3:1 mix of ortho- and para-hydrogen; see Boley et al. 2007). Also included are molecular hydrogen dissociation, and the ionizations of hydrogen and helium. The hydrogen and helium mass fractions are $X = 0.70$ and $Y = 0.28$, respectively, whilst the contribution of metals to the equation of state and the thermal evolution is neglected.

The flux-limited diffusion scheme transfers energy between SPH particles, which does not enable it to radiate into a vacuum. In order that the disc can cool from its upper and lower surfaces, a boundary is applied that maintains the initial temperature profile in the high atmosphere of the disc. This boundary is situated at a height above/below the midplane that corresponds to the edge of the optically thick region, that is where the optical depth ($\tau$) from outside the disc to that depth is $\tau \approx 1$. SPH particles comprising the boundary regions evolve normally, but their energies are set according to the initial radial profile, allowing them to act as energy sinks.

2.4 Opacity treatment

We use the opacity tables of Pollack et al. 1985 to provide grain opacities, whilst the tables of Alexander 1975 (the UVa King model) provide the gas opacities at higher temperatures when the grains have sublimated. The former table gives interstellar grain opacities (IGO) for solar metallicity molecular gas, but in this work we reduce these opacities by orders of magnitude below this nominal level; we divide by factors of 100 and 1000. The justification for this comes of the likely agglomeration or sublimation of grains in the vicinity of a forming protoplanet (Podolak 2003, Movshovitz et al. 2010). We do not modify the gas opacities, but enforce a minimum for the grain opacities at the interface between the two regimes that corresponds to the gas minimum ensuring a smooth transition (see Ayliffe & Bate 2009a for more details).

| Model | Core mass [M$_\odot$] | Core radius [r$_c$] | $\times$ Physical core size | Opacity [% IGO] |
|-------|----------------------|---------------------|-----------------------------|----------------|
| A     | $4.54 \times 10^{-5}$ | $2.2 \times 10^{-4}$ | 10                          | 0.1            |
| B     | $6 \times 10^{-5}$   | $6.6 \times 10^{-5}$ | 3                           | 1              |
| C     | $6 \times 10^{-5}$   | $6.6 \times 10^{-5}$ | 3                           | 1              |
| D     | $6 \times 10^{-5}$   | $2.2 \times 10^{-4}$ | 10                          | 0.1            |
| E     | $6 \times 10^{-5}$   | $2.2 \times 10^{-4}$ | 10                          | 1              |
| J     | $1 \times 10^{-4}$   | $2.54 \times 10^{-4}$ | 10                          | 0.1            |

Table 1. Properties of the various models described in this paper, all of which were performed in a disc with a surface density of 750 g cm$^{-2}$ at $r_p$. The opacity is given as a percentage of the interstellar grain opacity (IGO) assumed in the opacity tables we employ. Core radii are all based on the models of Seager et al. 2007 multiplied by factors of 3 and 10 as marked.

2.5 Planetary Cores

The planetary cores in these simulations are modelled by a gravitational potential, and a surface potential that yields an opposing force upon gas within one core radius of the core’s surface. The combination of the gravitational and surface forces takes the form of a modification to the usual gravitational force as

$$F_r = -\frac{GM_c}{r^2} \left( 1 - \left( \frac{2R_c - r}{R_c} \right)^4 \right)$$ (1)

for $r < 2 R_c$ where $r$ is the radius from the centre of the planetary core, $R_c$ is the radius of the core, and $M_c$ is the mass of the core (see Ayliffe & Bate 2009a for further details). This equation yields zero net force between a particle and the planetary core at the surface radius $R_c$, whilst inside of the core’s radius the force is outwards and increases rapidly with decreasing radius. Gas particles therefore come to rest very close to the core radius, though the equilibrium position is slightly inward of this value due to the pressure exerted by the gas that accumulates on top of the inner most layer of particles. Seager et al. 2007 calculate core radii for solid exoplanets, amongst which are cases comprising of 75 per cent water, 22 per cent silicates, and 3 per cent iron. We use these models to determine the realistic sizes of protoplanetary cores that correspond to the masses used in this paper. These core radii were employed for a number of calculations in Ayliffe & Bate 2009a, but these cases were not evolved for many orbits due to the short timesteps required deep within the planetary potential. To follow the evolution over longer periods it has been necessary to perform calculations using larger core radii. To this end we scale up these previously used core radii by factors of 3 or 10 to reduce the required computation time. We also performed new models using the realistic core radii, but these calculations were too slow to give useful results and thus are not reported here. The properties of our different models are given in Table 1. The rate at which the planetary core accretes gas may be affected by the form of the surface potential described by equation 1 and this is explored briefly in Section 5.1. A smooth start to the calculations is provided by shrinking exponentially towards the desired $R_c$ from an initial radius of 0.01 $r_p$ over the course of the first orbit.
2.6 Measuring the gas accretion rates onto the planetary cores

We measure the gas accretion rates by calculating the rate at which gas passes into the self-consistently calculated Hill sphere of the protoplanet given by

$$R_H = \sqrt[3]{\frac{M_p}{3M_\ast r_p}}$$  \hspace{1cm} (2)

where $M_\ast$ is the protoplanet mass which is the sum of the core mass ($M_c$) and the accreted mass ($M_{acc}$), where accreted mass comprises all the gas within $R_H$. The gas mass is discretised amongst the SPH particles, allowing iteration through equation 2 until such time as the addition of a particle’s mass to $M_{acc}$ no longer increases $R_H$ sufficiently to encompass the next available particle.

The net flux through the Hill radius corresponds to the growth rate of the envelope and/or circumplanetary disc, which are the only repositories for gas that fails to remerge from this region. Our use of the Hill radius to measure the mass growth is arbitrary, but reasonable, since any protoplanet must be smaller than the Hill radius. It is important to note that the Hill radius does not define the extent of the envelope, which tends to be smaller (e.g. $\sim 0.25 R_H$) (Lissauer et al. 2009). However, the difference between the mass accretion rates (and total accreted masses) as measured at the Hill radius and at $0.3 R_H$ is small except at the start of the calculation (Section 3.2).

3 RESULTS

We have performed three-dimensional radiation hydrodynamics models of the accretion of gas onto planetary cores (or embryos) with a range of masses, over hundreds of orbits, in discs of varying opacity. This work extends upon models we performed in Ayliffe & Bate (2009a) where we followed the accretion for a relatively short period. Moreover, by using discs with highly reduced opacities, the models presented here include the accretion of much more significant envelope masses. In one case the accreted mass is sufficient to trigger a hydrodynamic collapse of the envelope (Section 3.2), followed by a return to steady gas accretion.

3.1 Envelope accretion

We performed calculations starting with various core masses ranging from 15 – 33 M⊕ (Table 1). The principal property that controls the accretion rate of a protoplanet of a given mass is the opacity of its envelope. A lower opacity enables more rapid radiative cooling, allowing the envelope to contract more quickly and so accrete gas at a faster rate (Hubickyj et al. 2005; Papaloizou & Nelson 2005; Ayliffe & Bate 2009a). The models presented in this work exploit this dependence to accelerate the growth process by adopting reduced opacities. The impact of opacity on the growth rate of a protoplanet is demonstrated by comparing Models E and D in Figs. 1 and 2. Model E employs an opacity ten times larger than D, which results in a much slower rate of accretion for the former.

In each case we introduced a bare core with no preexisting envelope. This results in a rapid initial gas accretion rate to form a quasi-static envelope, followed by a period of slowing accretion. This initial growth phase is not realistic for a protoplanet that forms in situ and concurrently accretes both solids and gas during the core formation phase (e.g. Pollack et al. 1996; Alibert et al. 2005). In models that account for both the core and envelope growth, the gas accretion rate tends to have an extended pe-
In our new models, Model J is similar to the model from Ayli & Bate (2009a), but with a much lower opacity and a higher disc surface density. This produces higher accretion rates; this marks the transition from the thermally-dominated to gravitationally-dominated accretion regime.

As mentioned in Section 2.3, in order to make these three-dimensional models computationally viable, we were forced to adopt non-realistic core radii for the planetary surfaces. In Ayli & Bate (2009a) we compared the accretion rates achieved in models using different core radii and found that for high opacities the accretion rates did not depend significantly on the core radius that was used, but for low opacities (1 per cent and 0.1 per cent interstellar grain opacities) the accretion rates obtained with smaller cores were significantly lower than for equivalent models adopting larger cores. However, the earlier models were only evolved for 10 orbits, which is still during the initial phase of envelope creation when the accretion rate is decreasing. Fig. 2 illustrates the growth of 20 M$_\oplus$ cores embedded in a disc of either 0.1 or 1 per cent interstellar grain opacity (as marked). The solid lines present models with planetary cores 10 times the realistic core radii, and the dashed lines 3 times.

Over the course of the initial settling period the different core radii in the 0.1 per cent opacity models lead to some divergence in the growth, as is evident. However, measuring the accretion rates of the two calculations at equivalent masses beyond the initial 50 orbits, it is found that these rates deviate by no more than 20 per cent from one another. In the 1 per cent opacity case, the results obtained using the two different core radii are indistinguishable from each other. We believe that measuring the accretion rates at longer times, when the models are more established is responsible for the relatively small differences seen with different core radii. However in this case we are only varying the core radius by just over a factor of 3, whilst in Ayli & Bate (2009a) the comparison was made spanning more than a factor of 12. At the present time we do not have any models with which we can ascertain to what extent the accretion rate differences measured in this previous work are due to the large factor difference in the core radius, and what fraction came of the early times at which the measurements were made. As such, the enlarged radii of the cores that we have adopted here should be kept in mind.

In the rest of this paper we focus on one particular case, Model J, that accretes a very significant envelope in a short period of time that undergoes a dynamic collapse. The other calculations discussed in this section are ongoing, and will eventually allow us to explore the evolution of envelopes that are built up under less extreme conditions (i.e. with slower accretion rates).

### 3.2 Hydrodynamical collapse - Model J

Model J accretes the most significant mass of any of our models, as is to be expected given its favourable conditions. The 33 M$_\oplus$ core is the most massive that we employ, and in this case is coupled with a large 10 times realistic core radius, and a 0.1 per cent interstellar...
The accretion history of the gaseous envelope in model J, where the accreted mass is that contained within the self-consistently calculated Hill radius (solid line). Also shown is the mass evolution with time within radii of 0.3 (dashed), 0.1 (dot-dash), and 0.03 $R_H$ (dots-dash). The envelope undergoes a hydrodynamical collapse after around 190 orbits when its mass is $\approx 0.375 M_{\text{Jupiter}}$, with the resulting shock pushing material out of the Hill radius, whilst the remaining mass becomes more centrally condensed. This phase is shown in the inset panel, which illustrates the central condensation by the increasing mass within small radii as the overall mass falls. Following the collapse, accretion resumes, replacing the mass lost due to the shock propagation.

Figure 4. The accretion history of the gaseous envelope in model J, where the accreted mass is that contained within the self-consistently calculated Hill radius (solid line). Also shown is the mass evolution with time within radii of 0.3 (dashed), 0.1 (dot-dash), and 0.03 $R_H$ (dots-dash). The envelope undergoes a hydrodynamical collapse after around 190 orbits when its mass is $\approx 0.375 M_{\text{Jupiter}}$, with the resulting shock pushing material out of the Hill radius, whilst the remaining mass becomes more centrally condensed. This phase is shown in the inset panel, which illustrates the central condensation by the increasing mass within small radii as the overall mass falls. Following the collapse, accretion resumes, replacing the mass lost due to the shock propagation.

consistent at nearly all radii, indicating that gas is falling directly onto the new denser envelope, rather than becoming suspended in a more extended structure as was the case prior to collapse where the accretion rates were different at different radii.

The process of envelope collapse is presented in Fig. 5, the left hand panels of which shows cross-sections in density in the Z-X plane through the centre of the planet, from a time preceding the collapse in the uppermost panel, and at various stages during the collapse in subsequent panels. In the first panel the density contours are near spherical at small radii, elliptical at 0.5 $R_H$, and pinched in towards the planet’s poles at large radii. The second panel illustrates the strong shock propagating outwards from near the planet’s core, which at this point has not altered the gas structure near the Hill radius (marked with a dotted line). However, it is possible to see that the collapse has been greater at the poles, deforming the previously elliptical contours. The third panel demonstrates the continued propagation of the shock, which is elongated along the vertical axis due to the lower density of material above and below the plane of the disc, which allows the shock to propagate more rapidly in this direction. Contours which had been pinched in at the planet’s poles are forced outwards as the shock passes, but as can be seen, inside they are already resuming their pinched structure. The bottom panel illustrates the resettled state of the protoplanet and its surroundings. The pinching at the poles, which in the top panel was found beyond 0.5 $R_H$, now extends down to $\approx 0.1 R_H$, and the surrounding medium now forms a circumplanetary disc, having previously existed as an ellipsoidal envelope. The central density of the protoplanet, that is the gas density near the core, was $\approx 5 \times 10^{-17}$ g cm$^{-3}$ in the top panel, and has increased to $\approx 5.5 \times 10^{-13}$ g cm$^{-3}$ in the structure shown in the bottom panel following the collapse.

Returning to the formation of a circumplanetary disc, Fig. 6 illustrates the midplane and vertical density structures about the protoplanet, emphasising the altered state of the envelope. The divergence from a spherically-symmetric density distribution, defined here as a difference of greater than 10 per cent between the midplane and vertical distributions, occurs at a radius 5 times smaller in the post collapse state than in the pre-collapse state. This illustrates that the envelope undergoes a more significant change in its structure vertically, than it does in the plane of the disc, and is similar to the results seen in Ayli & Bate (2009). This was investigated further in Ayli & Bate (2009b), where it was found that circumplanetary discs formed around massive protoplanets, and that these discs tended to be thick, with dimensionless scale heights generally larger than 0.2. We measure the circumplanetary disc scale height by taking radial bins within the region with $r < R_H/3$ (measured from the protoplanet). In each radial bin, we take the SPH particle densities (from all azimuthal angles) and fit Gaussian profiles to the resulting vertical density distributions. A Levenberg–Marquardt algorithm is used to perform the fit, allowing the scale height to vary. This gives a measure of disc scale height versus radius, which in this case yields values for $H/r$ that increase from 0.02 to 0.5 over the radial range of 0.05–0.3 $R_H$ over which the disc extends. This radial extent of the circumplanetary disc can be seen in Fig. 7, in which the disc edge is taken to be at the location where the peak of the specific angular momentum occurs. Beyond this radius, the specific angular momentum decreases with radius since, in the frame rotating with the protoplanet, the material in the circumstellar disc is counter-rotating relative to the gas captured by the protoplanet. Over the radial range 0.07–0.3 $R_H$ the specific angular momentum is on average 0.65 of the Keplerian values (marked by the dashed line). The displacement is due to the pressure support within the thick circumplanetary disc, and the degree of this displacement can
Figure 5. Cross-section plots illustrating the envelope collapse about a 33 $M_\oplus$ ($\approx 0.1M_{Jupiter}$) core. The Hill radius is marked as a dotted line. The accreted mass (gas within $R_H$) is $\approx 0.375 M_{Jupiter}$ prior to the collapse, reducing by $\approx 1.3$ per cent as the shock pushes material away from the core. The x and z axes are given in units of $r_p$. Left panels: Cross sections in density illustrate the envelopes collapse, the shock propagation, and the formation of a circumplanetary disc. The central density increases by an order of magnitude from the top panel to the last. Right panels: Temperature cross-sections at equivalent times to the density panels. The peak temperature increases by 2100K to 6800K from the first to the last panel. The protoplanetary disc scaleheight is $\approx 0.05 r_p$, thus there is little material obstructing the vertical propagation of the shock front. As a result, the shock propagates more easily in the vertical direction than through the denser midplane, yielding the non-spherical propagation most clearly shown in the third righthand panel.
be used to approximately calculate the disc scale height as another check on the values measured above. Using the ratio of the specific angular momentum ($j$) to the Keplerian value ($j_k$) we obtain a scale height of 0.55. This is obtained using equation (3) (see appendices B & C of Laibe et al. 2012), where we have used values of 1/2 and 7/10 for the surface density and temperature exponents ($p$ and $q$) for the circumplanetary disc, typical values from Ayliffe & Bate (2009); the calculated scale height is not enormously sensitive to variations in these values within a reasonable range.

$$H \approx \frac{2(1 - j/j_k)}{p + q/2 + 3/2}$$

This resulting scale height is somewhat larger than that which we directly measured, but has been calculated assuming a vertically isothermal disc and a lack of self-gravity, and is thus only approximate. The specific angular momentum distribution further supports our assertion that a circumplanetary disc has formed as a result of the envelope collapse.

The right hand panels of Fig. 6 shows the temperature structure in a $Z$-$X$ slice at equivalent times to those shown in the density panels to the left, allowing us to see the temperature evolution during the collapse and shock propagation. A hot front associated with the shock can be seen expanding away from the core with time, and the vertical elongation of the shock can be clearly seen in the third panel. Near the protoplanet’s core, the peak temperature has increased by 2100K to more than 6800K in the post-collapse state shown in the final panel, and continues to increase for the remainder of the calculation when accretion has recommenced. The change between 50 and 100 orbits of the calculation when accretion has recommenced.

Fig. 7 depicts the changing distribution of mass in the inner envelope from its pre-collapse state, to its post-collapse state. The total time span shown is 229 days, the time between pre and post collapse within the inner envelope, with these states marked using dashed lines. However, the most significant changes occur over less than 22 days at this scale, and this period is broken down in Fig. 9 into 4.3 day increments which are marked with solid lines. The Hill radius just prior to the collapse is equal to 0.054 $r_p$, which taking Lissauer et al. (2009)'s estimate of a $0.25 R_H$ envelope radius, gives a size of 0.0135 $r_p$. This radius is in reasonable agreement with the region over which the mass is significantly redistributed.

**Figure 6.** The density distribution along the x-axis (solid lines) and along the z-axis (dashed lines) through the protoplanet before (top panel) and after (bottom panel) collapse. The plus symbols mark the radius and density at which the midplane and vertical density distributions differ by more than 10 per cent. The vertical dotted line marks the protoplanet’s Hill radius. The density distribution is modified significantly in both the midplane and vertically following collapse, but the point at which these distributions diverge moves inwards by a factor of 5 in radius.

**Figure 7.** The specific angular momentum of the gas surrounding the protoplanet that comprises the circumplanetary disc. The vertical dot-dashed line marks $R_H/3$, the analytically expected edge of the disc, and this matches the measured turnover very well. The dashed line marks the Keplerian orbital velocity based on the mass within the associated radius. Pressure support within the disc means that a sub-Keplerian orbital velocity is expected, though with a similar gradient if the disc is rotating about the planet. This gradient matches reasonably between 0.07–0.3 $R_H$. 

150 orbits (the dashed lines) is relatively small, increasing as would be expected for a growing protoplanet. Arriving at the pre-collapse state (thin solid line) at around 190 orbits, at which point the accretion rate is at its maximum, the density and temperature maxima have increased more over 40 orbits than in the preceding 100 orbits. Moreover, the temperature structure shows a marked change in its form. However the most significant changes occur during the collapse and 5 further orbits of evolution (thick solid line). The envelope’s collapse occurs very rapidly, and only stops when the new structure of the envelope is able to reestablish hydrostatic equilibrium. For this to occur the density structure changes to deliver a much steeper gradient away from the planet’s solid surface. This leads to a much steeper pressure gradient in this region, as shown in the bottom panel of Fig. 8, eventually satisfying the requirement $\nabla P = -\rho \nabla \phi$, where $\phi = GM(r)/r$. As such, the envelope is able to resume steady accretion as the structure is able to bear the increasing weight; as mentioned previously, accretion resumes at the pre-collapse rate.
Figure 8. Spherically averaged density (upper panel), temperature (middle panel), and pressure (lower panel) distributions about the protoplanet. The dashed lines are at times of 50, 100, and 150 orbits, their order ascending up the left hand axis. The solid lines show the pre-collapse state (thinner line), and the post-collapse state (thicker line). The pre-collapse state is equivalent to that in Fig. 9 whilst the post-collapse state is instead taken at the very end of the calculation in this case, when accretion has resumed its pre-collapse rate.

Figure 9. Cumulative mass distribution calculated outwards from the core of Model J, where $M_{\text{acc}} = 0.375 M_{\text{Jupiter}}$. The mass distribution is shown over times ranging from just before the collapse, to after the structure stabilises. The collapse proceeds very rapidly, as shown by the solid lines with cover a period of less than 22 days from first (lowest) to last (highest).

Figure 10. Fraction of atomic hydrogen versus radius within the inner region of the protoplanetary envelope before it collapses (dashed line), and after it collapses (solid line). The higher temperatures that develop within the deep envelope when it collapses lead to a higher dissociation fraction of molecular hydrogen, whilst this process of dissociation will absorb energy, reducing the maximum temperature that is achieved. Prior to collapse the fraction of atomic hydrogen peaks at 0.33, whilst in the immediate aftermath it is as high as 0.48.

during the collapse, which can be seen in Fig. 9 to be $0.01 r_p (\approx 0.2 R_H)$.

Collapse of the protoplanetary envelope leads to a substantial increase in the temperature near the protoplanetary core, as discussed above. A result of this temperature rise is an increase in the dissociation fraction of molecular hydrogen about the core, and an
enlargement of the region within which hydrogen is substantially dissociated; this can be seen in Fig. [10]. It is not the dissociation that triggers the collapse of the envelope, despite the process acting as an energy sink; the capacity of dissociation to absorb energy does lead to a lower final temperature within the envelope than might otherwise have been achieved.

3.3 Accretion flow

Whilst we find that a protoplanet envelope increases in mass throughout its evolution, excepting a brief period following a dynamical collapse (seen at radii of 1, 0.3, and 0.1 $R_H$ in Fig. [3]), it is not obvious that this accretion is a spherically symmetric process. Machida et al. (2008) and Tanigawa et al. (2012) have performed three-dimensional calculations of protoplanet growth, and find that gas flows outwards along the midplane from a growing protoplanet, such that accreted material must be delivered vertically. In Ayliffe & Bate (2009b) we found that mass predominantly entered the Hill sphere along the midplane, but this analysis was simplistic in that it only considered inflow, failing to consider the possibility of outflow, and we worked solely with the azimuthally integrated values. Here we make a more thorough assessment of the mass flow, and are able to look at this flow in a large extended envelope that has not undergone collapse, as well as in the dense envelope formed subsequent to such a collapse. It is the latter case which most closely resembles the principle model of Tanigawa et al. (2012) for a high mass protoplanet.

Before and after the envelope collapse, we see material flowing in and out at the Hill radius in an alternating pattern, as can be seen in the first panels of Figs. [11] & [12] where negative values correspond to inflow, and the solid contour line denotes a flux of zero. This flow is easily explained as the passage of gas passing the planet and being deflected by the spiral shocks that form due to the gravitational perturbation provided by the protoplanet. Another contribution comes from gas following horseshoe orbits which also enter and leave the Hill sphere at broadly similar longitudes about the planet. Fig. [13] illustrates the vector field that results in the mass flow observed at the Hill radius, which is marked in this figure with a dashed line. The data presented in Figs. [11] & [12] was constructed by considering the flow over a period of 4 orbits preceding and following the collapse respectively. The stability of the gas flow about the protoplanet over these periods leads to the regular pattern that is seen at the Hill radius in both figures. In the pre-collapse case the pattern persists down to 0.3 $R_H$ as the second panel of Fig. [11] illustrates. At smaller radii the flow is inwards across the spherical shell, with a larger flux at smaller radii as is expected for a consistent mass flowing across a decreased surface area. In the pre-collapse state the mass flow at small radii does not appear to possess any latitudinal dependence, rather it flows in almost spherically symmetrically. Note that the figures are noisy at the poles as a result of the spherical polar grid used to calculate the flux, which leads to very small bins at high latitudes.

In the post-collapse case, shown in Fig. [12] the mass flux at the Hill sphere is little changed from the pre collapse case, and in both cases it is reminiscent of the structures seen in Fig. 5 of Tanigawa et al. (2012); note that we are plotting a time average flux, whilst Tanigawa et al. plot an instantaneous flux (i.e., $\dot{\rho}v_i$). Unlike Tanigawa et al. (2012), who see these alternating structures persisting down to very small radii, our model has already lost any sign of the in-out flow pattern at a radius of 0.3 $R_H$. Instead, at 0.3 $R_H$ we find an outflow along the circumplanetary disc midplane at every longitude, though this outflow shows a slightly alternating magnitude, with more significant outflow at longitudes of $\sim$ 20 and 200 degrees. This outflow originates at a radius of $\approx$ 0.17 $R_H$, which is around half the radius of the circumplanetary disc, inside of this radius the flow is inwards. Meanwhile at 0.3 $R_H$, mass is flowing inwards at higher latitudes. The relative fluxes are such that the net flux at each radius is negative, enabling the protoplanet to continue to grow. This growth is corroborated by Fig. [3] which shows the mass evolution of Model J within the 4 radii considered here, and indicates that the mass consistently increases within 0.3 $R_H$, where the flow is only found to be inwards, signalling that material within this radius is truly bound to the protoplanet. This is of particular interest because of the gas flow found at 0.03 $R_H$, and shown in the final panel of Fig. [12]. At this small radius there are significant flows of material, pushing outwards at intermediate latitudes ($\sim$ 45 – 70 degrees), and pouring inwards again along the midplane. This is indicative of significant circulation of the bound material below 0.1 $R_H$, and will be discussed further in Section [4.2].

4 DISCUSSION

4.1 Hydrodynamic collapse

From the earliest suggestion of Perri & Cameron (1974) it has been thought that a giant planet might form through the hydrodynamic collapse of a gaseous envelope onto a solid core which caused it to assemble. This was followed by numerous models that effectively sought for hydrostatic solutions to various combinations of properties to establish when such a collapse might occur (Mizuno et al. 1978; Mizuno 1980; Sasai 1989). The first models that attempted to model giant planet growth from the initial core formation, through to the envelope growth were performed by Bodenheimer & Pollack (1986). These models revealed that a protoplanetary envelope would gradually contract as the planet grew, leading to a quasi-static contraction beyond previously calculated values for the critical mass, as long as the envelope did not effectively detach from the protoplanetary disc (that is there was a sufficiently rapid supply of material from the latter to the former). Under these conditions, their was no evidence to suggest that the hydrostatic balance should reach some limit beyond which a collapse was inevitable, and later semi-analytic models originating from these earlier works, such as Pollack et al. (1996), Hubickyj et al. (2005), and Lissauer et al. (2009), suggest no need for a dynamic collapse. Our Model J follows a pattern of stable growth for the vast majority of its history, though not evidently undergoing any significant contraction, and resolves this pattern of growth subsequent to its envelope collapse.

The models presented in this article are performed using a three-dimensional hydrodynamics code that include self-gravity, and radiative transfer, but which omit the core formation phase, and the deposition of energy due to planetesimal accretion that are included in the semi-analytic works discussed. However, at the time of interest around the envelope collapse, the energy release is utterly dominated by the contraction of the gaseous envelope, such that solids accretion energy may be regarded as negligible. The metallicity of the envelope might be significantly modified by the
Figure 11. Mass flux through shells of various radii (marked in panels) surrounding the protoplanet core in Model J prior to its envelope collapse. At one Hill radius, the flow takes on a form that is similar to that seen in Fig. 5 of Tanigawa et al. (2012), and reflects the combined effect of horseshoe orbits and bent flow lines passing the protoplanet. Material both enters and leaves the sphere predominantly at the midplane where densities are highest; the marked contour line denotes a flux of zero such that the area within the contour marks the outflow. For ease of comparison with Tanigawa et al. (2012) the solar and anti-solar points are at longitudes of 0 and 180 degrees respectively, and inflowing material is shown by a negative flux. At the smallest radii the flow is largely inwards across the shell, with an average flux of $-1 \times 10^{-2}$ code units ($-5.5 \times 10^{-5}$ g cm$^{-2}$ s$^{-1}$).
Figure 12. Identical to Fig. 11 except illustrating the gas flow after Model J undergoes its envelope collapse. The flow at one Hill radius is very similar to the pre-collapse case, however at radii between $0.1 - 0.3 \, R_{\text{H}}$ the outflow is concentrated along the midplane, whilst inflow occurs at higher latitudes, reflecting the vertical accretion seen by Machida et al. (2008). At the smallest radii, there is evidence of a gas turn over, where material is flowing in along the midplane and at the poles, and out in a range of moderate to high latitudes (see Section 4.2).
Figure 13. Velocity vectors in the plane of the disc, illustrating the gas flow about the Hill radius (marked with a dashed line) that leads to the alternating pattern of in and out flow seen in the first panels of the mass flux plots shown in Figs. [11] & [12]. This vector field is plotted for a time preceding the envelope collapse, but a very similar field exists after the shock associated with the collapse has passed out of the region.

ablation and evaporation of grains that have been accreted over the planets history, and we make no attempt to account for this. The opacity in Model J is reduced by a fixed factor of $10^3$, at the lowest end of the range suggested by Movshovitz et al. (2010), who found such opacities due to grain settling and coagulation in regions of the envelope.

It is difficult to disentangle the causes and effects of the very rapid collapse we find, for example the surge in temperature leads to a higher fraction of dissociated hydrogen. However, as stated in Section 3.2 it does not appear that the dissociation of molecular hydrogen acts to trigger the collapse, as the fraction remains steady in the preceding period. There is also no evidence of the Kappa-mechanism acting within the protoplanetary atmosphere in our model, as was found by Wuchterl (1991) to cause a dynamic collapse. Wuchterl also found that this collapse led to a significant ejection of material from the protoplanet, whilst we find only a small drop of $\approx 1.3$ per cent in the mass within the Hill radius, and a rapid increase at and below 0.1 $R_H$ as the protoplanet structure shifts to its new state.

It appears that our Model J protoplanet reaches the hydrostatic limit for its formative structure, and that the internal pressure gradient can no longer accommodate the addition of mass by a small adjustment. This is illustrated in Fig. [14] which shows the ratio of the pressure force to the gravitational force against radius at a number of stages of the envelope collapse. From an initial state, in which the protoplanetary atmosphere is in hydrostatic equilibrium out to a radius of 0.0067 $r_p$ (0.12 $R_H$), the atmosphere rapidly begins to restructure, pushing the hydrostatic region down to a radius of $\approx 0.0013$ $r_p$ (0.025 $R_H$). This initial stage leaves the form of the graph otherwise relatively unchanged, but as the central concentration of mass continues, a shock begins to form as the pressure near the core surges. The maximum pressure within 0.001 $r_p$ has increased by an order of magnitude between the first panel and the third, leading to a somewhat steeper gradient over this region. However, at this point in time the gradient between 0.001 – 0.002 $r_p$ steepens much more rapidly, forming a shock front, and this front marks the new radial limit of hydrostatic equilibrium as can be seen in the third panel. The subsequent panel shows the shock prop-

Figure 14. Each panel shows the ratio of the pressure force to the gravitational force in the envelope and beyond, with the first and last panels corresponding in time to the pre and post-collapse cumulative mass distributions shown in Fig. [9]. A ratio of one indicates that the material is in hydrostatic equilibrium, which before the envelope collapse applies to a region out to 0.0067 $r_p$ (0.12 $R_H$, top panel, marked with vertical dotted line), but which post-collapse reaches out to just 0.0009 $r_p$ (0.016 $R_H$). By the final panel the shock wave has cleared the inner 0.01 $r_p$, leaving the environment internal to this in a new hydrostatic equilibrium, whilst its mass continues to increase, as shown in Fig. [11].
agating outwards, whilst the hydrostatic core shrinks a little. By the final panel the inner envelope has resettled and the structure has stabilised, whilst the shock’s propagation continues outwards, eventually moving beyond the limits of the modelled region.

It is possible that this envelope collapse only occurs due to the high accretion rates achieved due to our selected disc conditions. The low opacity assumed promotes very rapid planet growth, and it may be that this rapidity that prevents the envelope from adjusting its structure more gradually to accommodate the increasing mass. As such, it may be that the hydrodynamical collapse of a planetary atmosphere can only occur if that planet if accreting very rapidly. Further models will be required to determine whether or not this is the case. We note however that the accretion rate measured in Model J, both just before and after the collapse, is still not as rapid as would be found using a locally-isothermal equation of state, despite the large reduction in opacity. Lissauer et al. (2009) present accretion rates in their fig. 3, where these rates were obtained by D’Angelo et al. (2003b) using three-dimensional hydrodynamical models with a locally-isothermal equation of state. Applying our disc conditions to their results yields an accretion rate of $8 \times 10^{-4} \, \text{M}_{\text{Jup}} \, \text{year}^{-1}$, which is twice the rate measured in our radiation hydrodynamics models prior to collapse.

At this juncture we note that the results given in Lissauer et al. (2009) show a viscosity dependence, where higher viscosities lead to more rapid accretion. In our calculations viscosity is not a constant, but is proportional to the spatial resolution of the SPH method. Thus, the viscosity is lower in regions of higher density, and these differences mean the above comparison is only approximate. Further, we note that in the absence of a protoplanet, the unperturbed circumstellar disc in our models has a viscosity of $\alpha \approx 4 \times 10^{-3}$, consistent with the fixed viscosity global models of Bate et al. (2003) that are used to inject gas at the boundaries of the disc section. It is these boundaries that determine the rate at which gas is supplied to the disc section in these local models. As such, once the disc is perturbed, the spatially varying viscosity of the SPH calculations leads to to an inconsistency with the global models, and so the boundaries. A further caveat arising from the boundary implementation is that the injected material comprises gas on both circulating and librating orbits (Lubow et al. 1999), orbits that are modified as the gas passes through the modelled disc section. However, these modifications are lost when the gas leaves the section, and new gas is injected without these modifications, leading to a further inconsistency.

4.2 Atmospheric turn over

As briefly mentioned in Section 3.3 in reference to the third panel of Fig. 12 there appears to be significant motion of the bound gas besides rotation in the plane of the disc. An apparent rolling motion is indicated by the flow through the surface at 0.03 $R_{\text{H}}$, demonstrating a significant thermal gradient against radius. The 4 distinct cells revealed in the velocity field might well be indications of convection.
4.3 Gas accretion

Tanigawa et al. (2012) have recently examined gas flow onto and within circumplanetary discs. They found, in agreement with the work of Machida et al. (2008), that gas flowed onto a circumplanetary disc predominantly in the vertical direction. Moreover, their works suggests that material is flowing out along the midplane of a circumplanetary disc. Once the envelope collapses in Model J of this work, and a circumplanetary disc forms, we also find that material is flowing outwards along the midplane beyond a radius of 0.17 $R_H$, which is at around half the outer radius of the circumplanetary disc (Quillen & Trilling 1998; Ayliiffe & Bate 2009b; Martin & Lubow 2011). Fig. 16, which is equivalent to Fig. 6 in Tanigawa et al. (2012), illustrates the longitudinally (equivalently, azimuthally) integrated mass flux for the pre (solid line) and post (dashed line) collapse flows shown in Figs. 11 & 12. Before the envelope collapses, at 0.3 $R_H$ the outflow seen in two places at the midplane is counterbalanced by the associated midplane inflows and the inflow at higher latitudes, yielding a net inflow, as shown by the solid line in the second panel of Fig. 16. However, post collapse the consistent midplane outflow seen for this radius at all longitudes results in a peak of outflowing material in a region ±30 degrees latitude.

About the planet, at one Hill radius the flow is dominated by the streams of material passing in and out of the region due to the form of their circumstellar orbits (see Fig. 13). As such, it is unsurprising that the fluxes we obtain, and those of Tanigawa et al. (2012) scaled to our model, are similar at this radius; note we make the following comparisons using our post-collapse case which better resembles Tanigawa et al.’s models. At 1 $R_H$ they obtain a peak outflow along the midplane of $1.8 \times 10^{-7}$ g cm$^{-2}$ s$^{-1}$, whilst we obtain a value of $2.5 \times 10^{-7}$ g cm$^{-2}$ s$^{-1}$. The form is also similar, with wings of inflow at higher latitudes of similar magnitude; at the highest latitudes, our normalisation by area leads to a non-zero inflow. However, at smaller radii the mass fluxes we find are considerably larger than at the Hill radius, which differs substantially from Tanigawa et al.’s results, where the peak flux at all radii differ by less than an a factor of 4. At 0.3 $R_H$ the peak outward flux has grown to $2.2 \times 10^{-6}$ g cm$^{-2}$ s$^{-1}$, and at 0.1 $R_H$ the inflow has ceased, but the inflow flux at high latitudes has increased by another factor of around 5. The final panel of Fig. 16 reveals the mass flow resulting from the formation of convection cells in the deep atmosphere post collapse.
5 SUMMARY

We have performed three-dimensional self-gravitating radiation hydrodynamical models of planet growth that, subsequent to an extended period of growth, result in a dynamical collapse. A series of models were performed using different core masses, core radii, and opacities, that extend the range of accreted mass achieved in

Ayliete & Bate (2009a). We present the first results from a three-dimensional hydrodynamical model of planet growth by core accretion that has been found to produce a hydrodynamic collapse. The result of this collapse is a very centrally-condensed protoplanet, surrounded by a circumplanetary disc, that continues to accrete. The inner reaches of the envelope have undergone significant dissociation of molecular hydrogen, and appear to possess convection-like cells of gas turn over, whilst the inflow of new gas occurs near vertically at high latitudes.

The circumplanetary disc, with radius $R_\text{H}/3$ and dimensionless scaleheight of $0.4$ – $0.5$, exhibits a reversal in the direction of mass flow along the midplane at around 50 per cent of its radius; that is to say there is inflow only within the inner $0.17 R_\text{H}$. The degree of central condensation in the post-collapse state leads the model to show good agreement with previous calculations considering mass flow that have presumed a pre-existing high mass core of relatively small size (Tangawa et al. 2012). Conversely, before the collapse, the extended protoplanetary envelope exhibits a more spherically symmetric inflow of material.

To achieve rapid growth in these models we have adopted very favourable conditions, particularly a low opacity of just 0.1 per cent of the interstellar grain opacity. It may be that the hydrodynamic collapse found in this work is a result of the very rapid growth of the envelope, promoted by these disc conditions, though this cannot be said definitively without performing further models in less favourable discs.

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