Chiral Symmetry and Meson Vertex Operators in QCD Strings

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Abstract

The worldline representation of the one loop fermionic effective action is used to obtain the vertex operator for the pion and the sigma in QCD strings. The vertex operator of the scalar sigma is distinguished from that of the pseudo-scalar pion by the presence of an additional operator $mV_0$ where $m$ is the current quark mass and $V_0$ is a vertex operator that would describe a tachyon in the open bosonic string theory. This leads to a relation between the sigma propagator and the pion propagator, when chiral symmetry is spontaneously broken this relation implies that the propagator constructed from $V_0$ must behave like a massless ghost state. The presence of this state ensures that the sigma is massive and no longer degenerate with the pion. The expectation value of $V_0$ in a string description is related to the vacuum expectation value of the chiral condensate in QCD. Our analysis emphasizes the need for boundary fermions in any string representation of mesons and is suggestive of world sheet supersymmetry.

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1 Introduction

The large $N$ limit of QCD, where $N$ is the number of colors, has provided various useful insights as to how QCD describes the strong interactions [1]. It has also suggested the possibility of a string description of QCD [2, 3]. In this letter we will further explore this possibility using the worldline representation of the one loop fermionic effective action (FEA) [4, 5]. The worldline representation of the one loop FEA has largely been used as a tool for simplifying perturbative calculation of one loop diagrams in the presence of external fields [6]. Here we will use the worldline representation as a bridge between the large $N$ QCD and its string representation.

Starting from the worldline formalism of the real part of the one loop FEA we will construct vertex operators appropriate for describing meson propagators in the corresponding string representation. An important feature of these vertex operators is that they explicitly take into account the spin degrees of freedom of the quarks and correspondingly involve worldline fermions.

In developing a string description of the strong interactions a natural question to ask is how the spontaneous breaking of chiral symmetry is reflected in this representation. This classic question has been reviewed in [7]. We will study it in the context of QCD strings and ask how the sigma and the pion vertex operator behave when chiral symmetry is spontaneously broken. We will find that what distinguishes the sigma vertex operator from the pion vertex operator is the presence of an additional term $mV_0$, where $m$ is the current quark mass and $V_0$ is a vertex operator that would describe a tachyon in the open Nambu-Goto bosonic string theory. This leads to a relation between the sigma and the pion propagator which in turn requires that when chiral symmetry is spontaneously broken the propagator for $mV_0$ must describe, unlike in the case of the Nambu-Goto strings, a massless ghost state. The presence of this ghost state ensures that the sigma is massive and no longer degenerate with the pion. It is worth noting that $V_0$ by itself does not correspond to any physical particle of the theory, but it’s expectation value is related to the chiral condensate of the quark anti-quark pairs in QCD vacuum.

The outline of the paper is as follows. In the next section we will remind ourselves how in the large $N$ limit the connected gauge invariant Green’s functions can be written using the functional average over the gauge fields of the one loop FEA. We will also in this section summarize in our notation the
worldline representation of the real part of one loop FEA obtained in [4, 5].

In section 3 we will construct meson vertex operators for describing processes involving even powers of pion field in the large $N$ limit of the theory. We will observe that these vertex operators are also appropriate for describing meson propagators in the string representation of the large $N$ QCD. In section 4 we will study how the spontaneous breaking of chiral symmetry is reflected in the behavior of the sigma and the pion vertex operator. We state our conclusions in the final section.

2 The Large $N$ Limit and the Worldline Formalism

We start with the familiar observation that in the large $N$ limit of QCD the gauge invariant Green's functions of quark bilinears can be obtained from the one loop FEA averaged over the gauge fields [8, 9]. Consider the Euclidean partition function for two quark flavors in the presence of a source for the sigma, $J_\sigma$, and a source for the pion triplet, $J_\pi$,

$$Z[J_\pi, J_\sigma] = \int DAD\bar{\Psi}D\Psi \exp\{-S_{YM}[A]\} \exp\left\{-\int x \bar{\Psi}O[A, J_\pi, J_\sigma]\Psi\right\}, \quad (1)$$

$S_{YM}[A]$ represents the Yang-Mills action for the $SU(N)$ gauge field $A$ while $\Psi$ and $\bar{\Psi}$ are the quark fields. The fermionic part of the action, using the Feynman slash notation, is given by

$$O[A, J_\pi, J_\sigma]_{ij}^{ab} = -i\not{\partial} + m)\delta_{ab}\delta_{ij} - A_{ab}(x)\delta_{ij} - i\delta_{ab}\delta_{ij} J_\sigma - \delta_{ab}\gamma_5 J_\pi(x)\tau_\alpha^{ij}, \quad (2)$$

where $a, b$ denotes the color indices while $i, j$ denotes the flavor indices, and $m$ is a flavor independent current quark mass. The generators of the flavor group $SU_f(2)$ are represented by $\tau^\alpha$ and we have defined

$$J_\pi = J_\pi^\alpha \tau^\alpha. \quad (3)$$

A formal integration over the quark fields leads to

$$Z[J_\pi, J_\sigma] = Z_{YM} < \exp\{-\Gamma[A, J_\pi, J_\sigma]\} >_A, \quad (4)$$

where $Z_{YM}$ is the partition function for the $SU(N)$ gauge theory in the absence of matter fields. $\Gamma[A, J_\pi, J_\sigma]$ is the one loop FEA. The averaging
over the gauge fields is defined by
\[
< \exp\{-\Gamma[A, J_\pi, J_\sigma]\} >_A = \frac{1}{Z_{YM}} \int DA \exp\{-S_{YM}[A]\} \exp\{-\Gamma[A, J_\pi, J_\sigma]\}.
\]
(5)

We define the connected pseudoscalar and scalar Green's function as
\[
\Delta_\alpha^\beta(x - y) = \left( \frac{\delta}{\delta J_\pi^\alpha(x)} \right) \left( \frac{\delta}{\delta J_\pi^\beta(y)} \right) < -\Gamma[A, J_\pi, J_\sigma] >_A \bigg|_{J_\pi = 0; J_\sigma = 0},
\]
(6)
\[
\Delta_\sigma(x - y) = \left( \frac{\delta}{\delta J_\sigma(x)} \right) \left( \frac{\delta}{\delta J_\sigma(y)} \right) < -\Gamma[A, J_\pi, J_\sigma] >_A \bigg|_{J_\pi = 0; J_\sigma = 0}.
\]
(7)

We will refer to above Green's functions as propagators implying that the long distance behavior of these two-point connected Green's functions are governed by the corresponding meson poles.

The worldline formalism expresses the one loop FEA as a sum over closed paths of a spin-half particle. A worldline path integral for the one loop FEA in the presence of a scalar and a pseudoscalar source has been derived in [4, 5]. In the rest of this section we will restate their result in a notation that will clarify the link between the worldline path integral and a possible string representation of mesons in the large $N$ QCD.

For the task at hand which involves Green's functions with even powers of $\gamma_5$ we will only need the worldline path integral for the real part of the one loop FEA [5] which we will denote by $\Gamma_R[A, J_\pi, J_\sigma]$. It's functional average over the gauge fields can be written as
\[
< -\Gamma_R[A, J_\pi, J_\sigma] >_A = \int_0^\infty \frac{dT}{T} \exp\{-m^2 T^2\} \times < \text{Tr}_f \hat{P} \exp\{-S[J_\pi, J_\sigma]\} >_{x, \psi_\mu, \psi_5, \psi_6}.
\]
(8)

where $\text{Tr}_f$ represents the trace over the flavor degrees of freedom while $\hat{P}$ is the path ordering operator for matrices, and the worldline average for an arbitrary functional $F[x, \psi_\mu, \psi_5, \psi_6]$ is defined by the path integral
\[
< F[x, \psi_\mu, \psi_5, \psi_6] >_{x, \psi_\mu, \psi_5, \psi_6} = -\frac{1}{4} N[T] \int Dx D\psi_\mu D\psi_5 D\psi_6 \exp\{-S_0\} \times < \mathcal{W}[x, \psi_\mu; A] >_A F[x, \psi_\mu, \psi_5, \psi_6].
\]
(9)

The worldline path integral can be thought of as sum over closed paths of length $T$. A path being described by the bosonic coordinates $x_\mu(\tau)$ and by
a set of fermionic coordinates $\psi_\mu(\tau), \psi_5(\tau), \psi_6(\tau)$ where $\tau$ parametrizes the worldline and the index $\mu$ takes values from 1 to 4. The bosonic coordinates satisfy periodic boundary condition while the fermionic coordinates satisfy anti-periodic boundary condition. The functional integral over the fermionic coordinates is related to taking the trace over gamma matrices in the usual formalism \[2\]. The action $S_0$ and the worldline action for the source term $S[J_\pi, J_\sigma]$ are given by

$$S_0 = \int_0^T d\tau \left\{ \frac{\dot{x}^2}{2} + \frac{1}{2} \psi_\mu \dot{\psi}_\mu + \frac{1}{2} \psi_5 \dot{\psi}_5 + \frac{1}{2} \psi_6 \dot{\psi}_6 \right\}, \quad (10)$$

$$S[J_\pi, J_\sigma] = \int_0^T d\tau \left\{ \frac{1}{2} J_\pi^2 + i \psi_\mu \psi_5 \partial_\mu J_\pi + m J_\sigma + \frac{1}{2} J_\sigma^2 + i \psi_\mu \psi_6 \partial_\mu J_\sigma \right\}. \quad (11)$$

a dot over a coordinate denotes a derivative with respect to the parameter $\tau$. The Wilson loop for a spin-half particle, $W[x, \psi_\mu; A]$, which we will refer to as the fermionic Wilson loop, is defined by

$$W[x, \psi_\mu; A] = \text{Tr}_c \hat{P} \exp \left\{ -i \int_0^T d\tau \{ \dot{x}_\mu A_\mu - \frac{1}{2} \psi_\mu F_{\mu\nu} \psi_\nu \} \right\}, \quad (12)$$

$\text{Tr}_c$ denotes the trace over the color indices, and $F_{\mu\nu}$ is the Yang-Mills field strength. $\mathcal{N}[T]$ in (9) is a normalization factor for the functional integral and is given by

$$\mathcal{N}[T] = \int D\Sigma \exp \left\{ -\frac{1}{2} \int_0^T d\tau p^2 \right\}. \quad (13)$$

In the next section we will use the above representation of the one loop FEA to describe mesons in the strings dual to the large $N$ QCD.

### 3 Vertex Operators for Mesons

One natural way of stating the string - QCD duality is in terms of the Wilson Loop \[2\]

$$< W[x; A] >_A = < \text{Tr}_c \hat{P} \exp \{ i \oint A_\mu dx_\mu \} >_A, \quad (14)$$

$$< \text{Tr}_c \hat{P} \exp \{ i \oint A_\mu dx_\mu \} >_A = \int D\Sigma \exp \{ -S_{CS}[\Sigma, x] \}, \quad (15)$$

where the functional integral is over the world-sheets whose boundary is the Wilson loop. The string action $S_{CS}$ is of course famously unknown, various
attempts towards discovering it have been reviewed in [10, 11]. One ex-


tends a similar string representation for the fermionic Wilson loop

\[
< W[x, \psi; A] >_A = \int D\Sigma \exp\{- (S_{CS}[\Sigma, x] + S_B[\Sigma, x, \psi])\},
\]

(16)

where \( S_B[\Sigma, x, \psi] \) is a boundary action describing the interaction between
the quark spin and the world-sheet degrees of freedom [12]. Assuming this
string-QCD duality and using the worldline path integral for the one loop
FEA allows us to obtain a string representation for the meson propagators.

Let us first consider the pion propagator, to obtain it’s string representa-
tion we substitute the worldline path integral of the one loop FEA (8) in
the definition of the pion propagator (6) to obtain

\[
\Delta_{\alpha\beta}(y_1 - y_2) = \delta_{\alpha\beta} \int_0^\infty \frac{dT}{T} \exp\{-m^2 \frac{T}{2}\} < V_5(y_1)V_5(y_2) >_{x,\psi_\mu,\psi_5,\psi_6}
\]

(17)

where the pion vertex function \( V_5 \) is given by

\[
\begin{align*}
\tau^\alpha V_5(y) &= \left( \frac{\delta}{\delta J_\pi^\alpha(y)} \right) \exp\{- S[J_\pi, J_\sigma] \} \bigg|_{J_\pi=0; J_\sigma=0}, \\
\tau^\alpha V_5(y) &= -i \tau^\alpha \int_0^T d\tau \{ \psi_\mu(\tau) \psi_5(\tau) \partial_\mu (\delta(x(\tau) - y)) \}.
\end{align*}
\]

(18)

(19)

In writing the pion propagator in terms of the vertex operator \( V_5 \) we have
neglected a contact term that only contributes when \( y_1 \) and \( y_2 \) coincide,
for we will be interested only in the relationship between the pion and the
sigma propagator and an identical contact term will appear in the case of
the sigma propagator too\(^1\). The vertex operator appropriate for describing
sigma is given by

\[
V_\sigma(y) = \left( \frac{\delta}{\delta J_\sigma(y)} \right) \exp\{- S[J_\pi, J_\sigma] \} \bigg|_{J_\pi=0; J_\sigma=0},
\]

(20)

using (11) for \( S[J_\pi, J_\sigma] \) leads to it’s explicit form as

\[
V_\sigma(y) = mV_0(y) + V_6(y),
\]

(21)

\(^1\) The contact term has the following form \( \int_0^T d\tau \delta(y_1 - x(\tau)) \delta(y_2 - x(\tau)). \)
where $V_0(y)$ and $V_6(y)$ are given by

$$
V_0(y) = -\int_0^T d\tau \delta(x(\tau) - y),
$$

(22)

$$
V_6(y) = -\int_0^T d\tau \{\psi_\mu(\tau)\psi_6(\tau)\partial_\mu(\delta(x(\tau) - y))\}.
$$

(23)

Again, as in the case of the pion, using the sigma vertex operator and substituting the worldline representation of the one loop FEA in the definition of the sigma propagator, Eq. (7), leads to

$$
\Delta_\sigma(y_1 - y_2) = \int_0^\infty \frac{dT}{T} \exp\{-m^2 T\} \langle V_\sigma(y_1) V_\sigma(y_2) \rangle_{x,\psi,\psi_5,\psi_6},
$$

(24)

where we have again neglected the above mentioned contact term.

What we have been able to do above is to map the interpolating fields for mesons to geometrical quantities, the vertex operators, which are defined in terms of the quark worldline. Our expressions for meson propagators represents a formal sum of all the planer diagrams which have one quark loop as their boundary. There has always been a hope that some string theory may provide a tractable way of summing the planer diagrams, Eq. (17) and Eq. (24) expresses that possibility. Importantly, using them we could identify the vertex operators that describe mesons in the unknown QCD string theory, but with an important caveat that the vertex operators so obtained can only be used for processes involving even number of pions in the leading approximation of the large $N$ limit. This is a consequence of the fact that they were derived from the real part of the one loop FEA. Though we will not need the imaginary part of the one loop FEA, it too can be written in the worldline formalism, but not in a unique manner [5].

4 Chiral Limit in the String Representation

The idea that the pion is an approximate Nambu-Goldstone boson of a spontaneously broken chiral symmetry has proved to be an extremely useful one [13]. If the large $N$ limit is a good approximation to QCD then chiral symmetry should be spontaneously broken for an arbitrary large value of $N$. A strong evidence in favor of this is provided by the Coleman-Witten theorem [14] which shows that under some reasonable assumptions not only
chiral symmetry is spontaneously broken in the large $N$ limit but also the pattern of breaking is the one that is observed in nature.

We would like to see how the spontaneous breaking of chiral symmetry is reflected in the meson vertex operators that we obtained in the previous section. For this purpose it will be convenient to work with the momentum space vertex operators and propagators and we will ignore the flavor index as it does not play any role in the leading term of the large $N$ expansion. Let us first consider the pion propagator in the chiral limit. The momentum space pion vertex operator is

$$V_5(k) = \int_0^T d\tau \{ k.\psi(\tau) \psi_5(\tau) \exp\{ik.y(\tau)\} \}, \quad (25)$$

and the momentum space pion propagator is given by

$$\Delta_\pi(k) = \int_0^\infty \frac{dT}{T} \exp\{-m^2 T/2\} < V_5(k) V_5(-k) >_{y,\bar{\psi}_5,\bar{\psi}_6} . \quad (26)$$

In the worldline functional integral we have eliminated the zero mode from the path $x(\tau)$. The resulting path $y(\tau)$ satisfies the condition

$$\int_0^T d\tau y(\tau) = 0. \quad (27)$$

In the chiral limit the pion is a Nambu-Goldstone boson therefore it’s propagator will have the following form

$$\lim_{m \to 0} \Delta_\pi(k) = \frac{F_\pi(k^2)}{k^2}, \quad (28)$$

where the unknown function $F_\pi(k^2)$ is regular at $k^2 = 0$. The key point is that the pion propagator is well defined as the quark mass goes to zero and has a pole at $k^2 = 0$. Now let us consider the momentum space propagator for the sigma in the chiral limit. The momentum space vertex operator for the sigma is given by

$$V_\sigma(k) = mV_0(k) + V_6(k), \quad (29)$$

where the vertex operator $V_0(k)$ and $V_6(k)$ are given by

$$V_0(k) = -\int_0^T d\tau \exp\{ik.y(\tau)\}, \quad (30)$$

$$V_6(k) = \int_0^T d\tau \{ k.\psi(\tau) \psi_6(\tau) \exp\{ik.y(\tau)\} \}. \quad (31)$$
Using these operators we can write the sigma propagator as

\[ \Delta_{\sigma}(k) = \int_{0}^{\infty} \frac{dT}{T} \exp\{-m^2 \frac{T}{2}\} < V_{\sigma}(k)V_{\sigma}(-k) >_{y,\psi_\mu,\psi_5,\psi_6} \]  

\[ = m^2 \Delta_0(k) + \Delta_6(k), \quad (33) \]

where we have defined \( \Delta_0(k) \) and \( \Delta_6(k) \) as

\[ \Delta_0(k) = \int_{0}^{\infty} \frac{dT}{T} \exp\{-m^2 \frac{T}{2}\} < V_0(k)V_0(-k) >_{y,\psi_\mu,\psi_5,\psi_6} \]  

\[ \Delta_6(k) = \int_{0}^{\infty} \frac{dT}{T} \exp\{-m^2 \frac{T}{2}\} < V_6(k)V_6(-k) >_{y,\psi_\mu,\psi_5,\psi_6} \]  

In writing Eq. (33) we have used the fact that the cross terms

\[ < V_6(k)V_0(-k) >_{y,\psi_\mu,\psi_5,\psi_6} = 0 \]  

because the integrand is odd in \( \psi_6(\tau) \). Next we notice that \( \Delta_{\pi}(k) \) and \( \Delta_6(k) \) are identical, for the right hand side of Eq. (15) is identical to the right hand side of Eq. (26) apart from a relabeling of the integration variable \( \psi_6 \) as \( \psi_5 \), thus

\[ \Delta_{\pi}(k) = \Delta_6(k). \]  

This immediately gives us a relation between the sigma and the pion propagator

\[ \Delta_{\sigma}(k) = m^2 \Delta_0(k) + \Delta_{\pi}(k), \]  

(38)

before considering the above relation in the chiral limit, let us note that it would not be modified even if we had included the contact term in the pion and the sigma propagator (see the comments after Eq. (13)), for the contact term for both the propagators are identical. Let us now look at this relation in the chiral limit, it implies that \( \Delta_0(k) \) cannot have a smooth limit as the current quark mass \( m \) goes to zero, otherwise the pion and the sigma will become degenerate and that will violate our assumption of the spontaneous breaking of chiral symmetry. In fact, we know from the phenomenology of the strong interactions that there is no evidence for a light scalar particle. Thus, the propagator \( \Delta_0(k) \) must have the following form

\[ \lim_{m \to 0} m^2 \Delta_0(k) = -\frac{F_{\pi}(0)}{k^2} + H(k^2), \]  

(39)

where the unknown function \( H(k^2) \) is regular at \( k^2 = 0 \). In other words when the chiral symmetry is spontaneously broken the state corresponding to the
vertex operator $V_0$ is a massless ghost state. We note that $V_0$ by itself does not correspond to any physical particle. It only appears as the part of the sigma vertex operator. We are now in a better position to understand Eq. (38) which seems to suggest that in the absence of quark mass the pion and sigma propagators are identical. In particular it seems to imply that the pion and the sigma have identical mass. This indeed would be the case if chiral symmetry was not spontaneously broken (see for e.g [16]). If chiral symmetry is spontaneously broken then one has to study the relation (38) in the limit of the current quark mass $m$ going to zero. Our analysis reveals that in this phase $m^2 \Delta_0(k)$ does not vanish as $m$ tends to zero. The pion and the sigma are no longer degenerate. In fact one can readily see that the vacuum expectation value of the chiral condensate is given by the expectation value of the vertex operator $mV_0$,

$$<\bar{\Psi}\Psi(k) >= \lim_{m \to 0} \delta(k) \int_0^\infty \exp\{-m^2 T \frac{T}{2}\} \frac{dT}{T} < -mV_0(k) >_{y,\psi,\bar{\psi},\bar{\psi}}.$$  (40)

The above equation can be regarded as another statement of the large QCD string duality, the left hand side is the expectation value of the quark field operators while the right hand side is a worldline - string functional integral.

By the same account if the chiral symmetry was not spontaneously broken then $m^2 \Delta_0(k)$ should vanish in the limit of $m$ going to zero and so would the expectation value of $mV_0$. The vertex operator $V_0$ is of course familiar in the open bosonic string theory where it describes a tachyon [15]. In the string theory dual to the large $N$ QCD it is unique in that it neither excites (boundary) fermions nor the world sheet degrees of freedom, but its behavior allows us to delineate the correct ground state of the theory.

5 Conclusions

The large $N$ expansion of QCD has given surprisingly many unique insights as to how QCD describes the strong interactions. They are surprising because they are obtained without being able to calculate even the leading term of the large $N$ expansion. In the present paper we have written these leading terms as a functional integral over the quark worldline together with suitable worldline vertex operators. Our motivation for this was to use the worldline formalism as an intermediate step towards the string description of the large $N$ QCD. In doing so we found that the worldline vertex operators for mesons
are also the vertex operators for mesons in their string representation, at least as far as describing processes involving even number of pion operators in the leading approximation of the large $N$ limit. An important feature of these vertex operators is that they involve the worldline or the boundary fermions. Thus the string theory that we seek must have at the least boundary fermions and plausibly even world-sheet fermions with some kind of world-sheet supersymmetry as suggested by Polyakov [10]. The relationship between the one component boundary fermions and the two component world sheet fermions, together with the vertex operators that we have obtained, may perhaps give us some clue as to the nature of the QCD strings.

Using these vertex operators we were also able to get some hint as to how the spontaneous breaking of chiral symmetry is reflected in the string description of the large $N$ QCD. In the spontaneously broken phase of the large $N$ QCD the vertex operator which would have described a tachyon in the Nambu-Goto open strings, instead describes a massless ghost state while it's expectation value gives the chiral condensate. Reassuringly, we also saw that this does not do any violence to the theory, this vertex operator by itself does not represent any particle of the theory but is a part of the vertex operator that describes the sigma. The occurrence of the ghost state simply ensures that the sigma is massive when chiral symmetry is spontaneously broken. There has always been an intuition that once the correct ground state of a string theory is identified then there should be no tachyons in it, for the strings describing the large $N$ QCD this indeed seems to be the case.

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