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ABSTRACT
Flow-induced vibration (FIV) of a flexible cylinder with an upstream wake interference at a subcritical Reynolds number is numerically investigated in this study. Two cylinders are installed in a tandem arrangement with the tandem separation between the cylinder centers set at 5.0 diameters. The downstream cylinder is flexible and placed in the wake of the stationary rigid upstream cylinder. A quasi-three-dimensional fluid-structure interaction (FSI) numerical methodology that couples the strip theory-based Lagrangian discrete vortex method with the finite-element method (FEM) for structural dynamics is developed to simulate the FIV response of the flexible cylinder with the upstream wake interference. The vortex-induced vibration (VIV) of an identical isolated cylinder is also numerically simulated as a contrast. This numerical study characterizes the dynamic response of the cylinder FIV with the upstream wake interference and sheds light on the FSI mechanisms responsible for the structural dynamic response. With the upstream wake interference, the cylinder FIV response shows two features distinct from the isolated VIV response: the vibration of large amplitude during the modal resonance branch transition and the extension of the modal resonance branch. The hydrodynamic coefficients database is constructed by the rigid cylinder forced vibration experiment to help explain the FSI properties of the FIV dynamic response. The lower added mass coefficient for the FIV with the upstream wake interference than the VIV of the isolated cylinder guarantees the synchronization between the vortex shedding frequency and the "true" natural frequency of the structure persisting to higher reduced velocity in a certain modal resonance response branch. The excitation coefficient distribution indicates that the cylinder FIV with the upstream wake interference reaches higher amplitude at high reduced velocity, instead of ceasing resonance as the isolated cylinder. The numerical wake visualization is shown and used to explain the correlation between the distribution of hydrodynamic coefficients along the cylinder span and the wake vortex mode. It is found that the upstream wake interference effect is strongly correlated with the vortex-structure interaction pattern between the upstream wake vortices and the downstream motion. When the upstream vortex impinges on the downstream cylinder and splits into subvortices, the effect of the upstream wake interference acting on the downstream cylinder reduces. When the downstream cylinder enters the gap between the upstream vortices over the entire vibration process, the upstream wake has a stronger interference effect on the downstream FIV response.

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I. INTRODUCTION
When a current flows across the flexible marine structures of blunt shape, the alternate shedding vortex behind both sides of the body will excite the vibration of the structures. If the vortex shedding frequency approaches the structural natural frequency, the "lock-in" phenomenon will occur and the vibration will be enhanced, which is called "vortex-induced vibration" (VIV). The possible stresses and fatigue damage resulting from VIV on these structures require a careful calculation and prediction of the responses. In the last few decades, most studies of VIV have been carried out with an isolated cylindrical structure. However, the configurations of multiple cylinders with parallel axes closely placed in tandem, side-by-side, or stagger arrangements are more encountered in marine structures, and a representative
application is the array of production riser tubes used in tension leg platform (TLP). When circular cylinders are immersed in the wake of other cylinders, their dynamic responses and wake pattern become rather different from what would be expected if isolated due to an occurrence of the wake interference mechanism.

For current flows around an isolated stationary cylinder, the wake is characterized by Karman vortex street that forms a certain frequency. For the flow past the structure consisting of multiple cylinders, the occurrence of wake interference between cylinders would produce more complicated fluid morphologies compared with the wake pattern generated from an isolated cylinder. Based on the flow structure developed in the cylinder gap and wake pattern behind the cylinders, several comprehensive literatures proposed different classification criteria that categorized the flow pattern into several primary interference regimes (Zdravkovich, 1987; Sumner et al., 2008; Hu and Zhou, 2008; and Alam and Meyer, 2011).

If the cylinder subjected to the current is flexibly mounted, its dynamic response will be characterized by VIV. A typical VIV response of an isolated cylinder finds its maximum displacement amplitudes in a limited reduced velocity region where the vortex shedding frequency is close to the natural frequency of the structure (Khalak and Williamson, 1996; Govardhan and Williamson, 2000, and Williamson, and Govardhan, 2004). However, if the cylinder free to oscillate in cross-flow direction is placed in the wake of an upstream stationary cylinder, its dynamic response of flow-induced vibration (FIV) is characterized by a buildup of oscillation amplitude persisting to high reduced velocities. Moreover, unlike the frequency lock-in of typical VIV response of the isolated cylinder, the frequency of the downstream cylinder is not locked to a specific value close to the structural natural frequency, but linearly increases with the reduced velocity (Bokaian and Geoola, 1984; Brika and Laneville, 1999; Hover and Triantafyllou, 2001; Assi et al., 2010; Li et al., 2018; Zhu et al., 2019; and Lee et al., 2021). Such a characteristic response is referred to as “wake-induced vibration” (WIV). Different from VIV of an isolated cylinder as a result of interactions between the body and the vortex shedding behind it, WIV of the downstream cylinder is excited by the unsteady vortex–structure interaction between the downstream body and the wake of the upstream body instead of its own wake (Assi et al., 2010 and 2013).

Moving a step further in the complexity of the problem, the FIV interference response with two elastically mounted rigid cylinders in tandem arrangement is widely studied in recent years. Understanding from these studies, the FIV interference responses of the two cylinders can be generally characterized for two interference regimes, including the proximity–wake interference regime where the shear layers of the upstream cylinder reattach on the downstream body and the full-wake interference regime where the shear layers of the upstream cylinder fully roll up and produce wake vortices in the cylinder gap. For the FIV response within the proximity–wake interference regime (Kim et al., 2009; Borazjani and Sotiropoulos, 2009; Jiang and Lin, 2016; Griffith et al., 2017; and Hu et al., 2020), the upstream and downstream cylinders response in a similar way. Both cylinders exhibit galloping-like responses manifested as large-amplitude persisting to high reduced velocities. For the FIV within the full-wake interference region (Mittal and Kumar, 2004; Papaioannou et al., 2008; Prasanth and Mittal, 2009; Bao et al., 2012; Xu et al., 2019; and Lin et al., 2020b), the upstream cylinder shows typical VIV-like responses as those of an isolated cylinder. However, when placed in the wake of the oscillating upstream cylinder, the downstream cylinder shows VIV-like responses instead of the typical WIV response that is expected in the case of the stationary upstream cylinder. The downstream response is characterized by the larger oscillation amplitude and broader synchronization region compared with the typical VIV response of an isolated cylinder.

For long flexible cylinders that have a countable sequence of natural frequencies, there can be many differences between their vibratory response and the response of the short, elastically mounted rigid cylinders, due to distributed vortex-induced forces may excite several natural frequencies instead of single natural frequency as the case of rigid cylinder. Here we find a few studies with FIV of multiple flexible cylinders with wake interference. Allen and Henning (2003) conducted experiments with two tandem flexible cylinders. The length of the cylinder was 17.83 m, with length-to-diameter ratio $L/D = 112.8$ and Reynolds number varying from 25,000 to 65,000. The cylinder was installed in the water channel with the upper 20% of the length submitted to the oncoming flow and the rest of the length immersed in the still water. The tandem spacing between the cylinders ranges from 3.0 to 12.5D. The analysis of the cylinder dynamic response reported two primary conclusions in terms of the cylinder frequency and amplitude responses, respectively. The conclusion of frequency explained that the downstream cylinder consistently experiences bimodal responses, as one modal frequency from the upstream vortex shedding, and another modal frequency possibly due to its own vortex shedding. The conclusion of amplitude explained that the downstream amplitude response is much greater than the upstream cylinder amplitude, but in general decreases with increasing upstream cylinder amplitude.

The FIV interference responses of two flexible cylinders in tandem with near- and far-wake interference are experimentally studied by Huera-Huarte and Bearman (2011) and Huera-Huarte and Gharib (2011), respectively. The cylinder models in the two studies were identical, with external diameters of 16 mm, total lengths of 1.5 m, and Reynolds numbers up to 12,000. The cylinders were partially immersed in the water flume, with the lower 45% of the total length exposed to the current. In the study with near-wake interference, the tandem spacing varies from 2 to 4D. For the wake interference study, the spacing varies from 4 to 8D. The distinction between the two interference modes is the existence or not of vortices in the gap region. The analysis of the dynamic response of the cylinders indicated that the near-wake interference affects the FIV response of both two cylinders, resulting in larger amplitudes of the upstream oscillation than those of the downstream cylinder as well as large oscillations outside the expected lock-in region. With far-wake interference, the upstream cylinder responses in the typical VIV manner of an isolated cylinder and the downstream cylinder shows large amplitudes of vibration at reduced velocities over the lock-in region. Due to the low modal density of the cylinder model, only the first mode of the cross-flow vibration was excited in their experiments.

Laboratory measurement of the FIV interference response of two flexible cylinders in tandem arrangement was also carried out by Liu et al. (2016). With a total length of 6.2 m and a length-to-diameter ratio of 310, the flexible risers were vertically placed in a water pool. The upper 19.4% of the cylinder was placed in the current with the rest of the length placed in still water. The first to third modes of the cross-flow vibration were observed in their experiments. They
reported that the downstream riser undergoes vibration of large amplitudes even during the FIV mode transition due to the upstream wake interference effect. By analyzing the modal weight of the cross-flow vibration, it was found that the downstream riser experiences multimode vibrations with a strong lower mode contribution. The combination of the resonance at several modes was also reported by Huera-Huarte et al. (2016), where the FIV response of the downstream riser with the wake interference of a stationary upstream riser was experimentally studied.

The latest laboratory test of FIV interference response of two long flexible cylinders in tandem was conducted by Xu et al. (2018). With a total length of 5.6 m and an aspect ratio of length-to-diameter of 350, the riser models were horizontally towed along the tank. Five tandem separations ranging from 4 to 16D were tested in the experiments. Their experimental results showed that the FIV behaviors of the upstream cylinder are similar to an isolated flexible cylinder within all the separations tested, the downstream cylinder undergoes vibration of multifrequencies.

The previous studies with FIV interference response have been mostly accomplished through laboratory experiments. These experimental studies have provided rich and high-quality data on the dynamic behavior of two tandem flexible cylinders subjected to FIV interference, at subcritical Reynolds number. There are very scarce studies for the FIV of two interfering flexible cylinders, which have been carried out with numerical simulations. Communicating a finite-volume method (FVM) solver of Star-CCM+ for the fluid dynamics and a finite-element (FEM) solver of Abaqus for the structural dynamics, Gonzalez et al. (2015) presented a computational fluid dynamics (CFD) simulation on the FIV response of a flexible cylinder with wake interference, which was also experimentally studied by Huera-Huarte et al. (2016). Their simulation reproduced a qualitative description of the physical phenomena but was lack of a precise comparison with the experimental data. Using ANSYS MFX multifield solver, Wang et al. (2017) numerically studied the FIV of two tandem flexible cylinders at a constant Reynolds number of 500. Three tandem separation ratios ranging from 2.5 to 5 and seven reduced velocities ranging from 4 to 10 were examined. As the cylinders mainly vibrate in the first mode and without mode transition, the FIV response of two flexible cylinders resembles the response in the cases of two elastically mounted rigid cylinders. Apart from the dynamic response, the aspects that were rarely reported in the previous experimental studies, involving the correlation lengths and three-dimensional flow structures, were discussed.

Up to now, the problem of the FIV interference response of two flexible cylinders is approached by laboratory experiments mostly. The focus of these studies has been for the most part on characterizing the dynamic response of the structures but extent little to the flow structure and fluid-structure interaction (FSI) mechanisms responsible for the structural response of the FIV. Numerical simulation is an effective approach to obtain more details concerning the flow structure and FSI properties. However, due to a high-precision grid-dependent computation of the three-dimensional flow domain around the flexible structures of the FSI interference problem, the numerical study with such a problem is still limited. In this study, we develop a quasi-three-dimensional (Q3D) FSI simulation methodology that couples a grid-independent strip theory-based discrete vortex method (SDVM) for the flow computation with the finite-element method (FEM) of structural dynamics to simulate the FIV interference response of multiple flexible cylinders.

With the FIV response of two flexible cylinders in the full-wake interference regime, the previous studies have shown that upstream cylinder responds as an isolated cylinder and showing the classical VIV response, but the downstream cylinder exhibits distinctive FIV response properties due to the upstream wake interference effect. With an intention to address the following questions: (a) what the difference between the flexible cylinder FIV response with an upstream wake interference and the isolated flexible cylinder VIV response is; (b) how the difference between the FIV response with the upstream wake interference and the isolated cylinder VIV response comes about, this study focus on the FIV response of the downstream cylinder. Therefore, we hold the upstream cylinder stationary and allow the downstream cylinder to oscillate in the cross-flow direction in simulations. The tandem separation between cylinder centers is held at 5 diameters, which guarantees the FIV getting into the full-wake interference regime.

II. SDVM-FEM COUPLED METHOD

In the FSI simulation with flow past the structures of slender geometric profile where hydrodynamic components of the radiation and diffraction potentials vary slowly along the length of the structure, strip theory is a practical approach that simplifies the three-dimensional flow computation into the calculations of a series of axially arranged two-dimensional flow “strips.” The two-dimensional simulation is implemented on each of flow “strips” (Wilden and Graham, 2001; Bao et al., 2016; and Lin and Wang, 2019). In this study, a Q3D numerical algorithm that combines the strip theory and a two-dimensional Lagrangian CFD model of discrete vortex method (DVM) are used in the flow calculation of the FIV of two flexible cylinders. Figure 1 shows the numerical model of the flow field based on the strip theory. DVM simulates the unsteady, high Reynolds number flow past a bluff body by discretizing the continuously distributed vorticity-carrying regions into a number of vortex elements and tracking the computational elements in the Lagrangian frame. It provides automatic grid adaptivity and devotes little computational effort to regions devoid of vorticity. Moreover, the particle treatment of the convective terms is free of numerical dissipation. Benefiting from these aspects, this method has been well used in many previous fluid dynamics simulations.

In DVM simulations, vorticity ω is defined by the curl of velocity field ∇ × u is the primary physical quantity of flow field. The evolution of vorticity field in a two-dimensional viscous flow is described by the Navier–Stokes (N–S) equations that can be equivalently expressed in a velocity–vorticity formulation as

$$\frac{\partial \omega}{\partial t} = -u \cdot \nabla \omega + \nu \Delta \omega,$$  \hspace{1cm} (1)

where t is time and \( \nu \) is fluid kinematic viscosity. The evolution of the vorticity field is considered in discrete time steps. In each time step, the vorticity field is convected (according to \( u \cdot \nabla \omega \)) and diffused (according to \( \nu \Delta \omega \)). The algorithm of viscous splitting consists of substeps in which the convective and the diffusive motions are considered successively. The two-step viscous slitting algorithm then is expressed as

Convection

$$\frac{\partial \omega}{\partial t} + u \cdot \nabla \omega = 0.$$  \hspace{1cm} (2)

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33, 065104-3
The basic ideal of DVM is to discretize the continuous vorticity field into a number of scattered vortex elements with certain circulations. The flow is simulated by solving a system of ordinary differential equations that describe vortex element motion in a Lagrangian frame, instead of solving the Navier–Stokes equations. The viscous splitting algorithm proposed by Chorin (1973) as the "vortex element" model. The vorticity field and velocity field then may be expressed in a discrete form as

\[ \omega(x_p, t) = \sum_{p=1}^{N} \Gamma_p \delta(x - x_p), \]  

(9)

\[ u(r, t) = U_\infty + \sum_{p=1}^{N} \Gamma_i \frac{K \times (x - x_p)}{2\pi (x - x_p)^2 + \sigma_p^2}. \]  

(10)

where \( \Gamma_p \) and \( \sigma_p \) denote the circulation and core radii of vortex blob, respectively, \( K \) is the unit vector orthogonal to the flow plane and \( N \) is the number of vortex elements. There are several alternative approaches for the viscous diffusion effect taking into account. For example, "random-walk" method in which the viscosity is included in the vortex elements motion by adding independent Brownian motions (Chorin, 1973); "vortex-in-cell" method in which the circulation of vortex elements should be recalculated at each time step (Wilden and Graham, 2001); and "vortex-spread" method where the core radii of vortex blob are growing at each time step (Yamamoto et al., 2004). In this study, the "vortex-spread" method is introduced to simulate the diffusion motion of the vorticity field.

In the flow past a bluff body, the vorticity is initially generated from the shear layers of the body. In the numerical implementation of DVM, the vortex elements are created on the surface of the body to simulate the shear layers. The circulation of the newly created vortex elements is calculated by the enforcement of no-slip boundary condition in terms of stream function as

\[ \psi_{i+1} - \psi_i = -U_c \cdot n \Delta S, \]  

(11)

where \( \psi \) is the stream function value at \( i \)th control point on the body surface, \( \Delta S \) is the length between two control points, and \( U_c \) is the velocity vector of the body. Figure 2 shows the body boundary discretization and vortex elements generation scheme for two tandem-arranged circular cylinders in DVM simulation.

The flexible cylinder is modeled as an extensible Bernoulli–Euler beam in which the shear deformation may be ignored compared to the bending deformation in the VIV response. The structural dynamics of the cylinder are governed by the following equation:

\[ m \frac{\partial^2 x(z, t)}{\partial t^2} + c \frac{\partial x(z, t)}{\partial t} + \frac{\partial^2}{\partial z^2} \left( EI(z) \frac{\partial^2 x(z, t)}{\partial z^2} \right) - \frac{\partial}{\partial z} (T(z, t) \frac{\partial x(z, t)}{\partial z}) = f_x(z, t), \]  

(12)

where \( x \) and \( z \) are the displacements of deformation and axial locations of the cylinder; \( m \) and \( c \) are the mass per unit length and structural damping ratio; \( EI(z) \) and \( T(z, t) \) are the bending stiffness and tension in section, respectively; and \( f_x(z, t) \) is the hydrodynamic forces. Applying the weak formulation of the Galerkin method to Eq. (12), the finite-element equation of structural dynamics is obtained as

\[ \Delta u = -\nabla \times \omega. \]  

(8)
\begin{equation}
[M]\dddot{x} + [B]\ddot{x} + [K_l + K_g]x = [Q(t)],
\end{equation}

where \([M] = m \int N_i(z)N_j(z)dz\) is the mass matrix, \([K_E] = EI \int N_i(z)N_j'(z)dz\) is the bending stiffness matrix, \([K_G] = T(z) \int N_i(z)N_j'(z)dz\) is the geometric stiffness matrix, \([Q(t)] = \int f(z,t)N^T(z)dz\) is the load matrix, and \(N(z)\) is the matrix of shape functions. The damping matrix \([B]\) is the linear combination of the mass matrix and stiffness matrix, based on the Rayleigh damping model.

The coupled fluid-structure system of FIV is numerically solved by a loose coupling (LC) method. The calculation of the flow field and dynamic response of the cylinder is explicitly implemented separately. The hydrodynamic forces are calculated by the strip theory-based DVM and then are passed into the load item of the structural finite-element equation. The dynamical equation is then solved in the time domain by the Newmark method. The structural displacement and velocity responses are then returned to the computation of the flow field as new boundary conditions in the next time step. The main computational parameters are included as follows: the generation rate of the discrete vortex element is set to 256 per time step, where 128 vortex elements are generated for each cylinder per time step; the number of vortex element on each flow “strip” is kept below 50,000; the computational domain extends the distance of 40 cylinder-diameters downstream of the downstream cylinder beyond which the vortex element is eliminated from computations; and the time step, nondimensionalized by the cylinder diameter and inflow speed, is set at 0.1. The convergence of these parameters has been addressed in the previous work of Lin and Wang (2019).

### III. NUMERICAL VALIDATION

In our previous study (Lin et al., 2020b), the fluid solver has been validated for the FIV simulation of two tandem elastically mounted rigid cylinders in two dimension. In this section, we further demonstrate the validity of the SDVM-FEM coupled method in the FIV simulation with the long and flexible cylinders by applying it to simulate an experimental case, which is relevant to the problem interest in this work. The numerical model is constructed by reference to the configurations that were experimentally studied by Huera-Huarte and Bearman (2009). The cylinder is installed with both ends connected to a supporting structure through a universal joint. The lower 40% length of the cylinder is exposed to the water current in the flume. The main parameters of the cylinder model are summarized in Table I. One may refer to the original study for more details (Huera-Huarte and Bearman, 2009). The modal analysis is carried out on the flexible cylinder model. The first eight-order modal natural frequencies of the cylinder are provided in Table II.

#### TABLE I. Main parameters of the cylinder model.

| Properties               | Values | Units |
|--------------------------|--------|-------|
| Length \(L\)             | 1.5    | m     |
| Submerged length \(L_w\) | 0.585  | m     |
| External diameter \(D\)  | 0.016  | m     |
| Aspect ratio \(L/D\)     | 93.75  |       |
| Flexural stiffness \(EI\)| 6.04   |       |
| Axial stiffness \(EA\)   | 1.84 × 10^6 | N m^2 |
| Mass in air \(m\)        | 0.362  | kg/m  |
| Mass ratio \(m^*\)       | 1.8    |       |
| Water density \(\rho_w\) | 1000.0 | kg/m^3 |
| Flow speed \(U_w\)       | 0.2–1.0 | m/s  |
| Reynolds number \(Re\)   | 3600–12 000 |       |
| Damping ratio \(\xi\)    | 1.75   | %     |
| Top tension \(T\)        | 15.0   | N     |
| First natural frequency \(f_1\) | 3.0 | Hz    |
Figures 3(a) and 3(b) present the structural cross-flow deflection displacement amplitude and dominant frequency response, respectively. A direct comparison is made between the results of the present simulation with those of Huera-Huarte and Bearman measured from the experiment. The experimental amplitude result shown in Fig. 3(a) indicates that an upper branch of the VIV response occurs at reduced velocities in the range of 5–8 with the amplitude peaking at 0.7 diameters, which is caused by the synchronization between the frequency of the wake vortex mode and the first-order modal frequency of the cylinder, which is also defined as first-order modal resonance branch. It is also clear that instead of the first-order modal resonance branch, the buildup of the second branch of the amplitude response has been observed by the experiment. A second-order modal resonance branch would be desired if a higher reduced velocity is reached. The simulated frequency result shows good agreement with the result of the experimental in the reduced velocity range of lock-in branches, the critical reduced velocity between two branches, and the value of oscillation frequency.

Figure 3(b) describes the frequency response of the cylinder VIV. The vibration frequency has been normalized by the first-order modal natural frequency. In the first-order modal resonance branch, the normalized oscillation frequency remains close to unity. The black dashed line indicates the vortex shedding frequency of a static cylinder obeying the Strouhal number. The departure of the oscillation frequency from the Strouhal number indicates the lock-in phenomenon. With reduced velocity increasing, the lock-in of the first-order modal resonance ceases. The frequency response turns into the lock-in branch of the second-order modal resonance. The simulated frequency result shows good agreement with the result of the experimental in the reduced velocity range of lock-in branches, the critical reduced velocity between two branches, and the value of oscillation frequency.

Figure 4 shows two representative plots of the instantaneous deflection shapes of the cylinder over two cross-flow vibration periods in the first-order and second-order modal resonance, respectively. In the case of $U/f_1D = 7.3$ shown in Fig. 4(a), the exact first harmonic vibration dominates the entire cylinder. High repetition can be observed between two motion periods. Coming to Fig. 4(b) for $U/f_1D = 17.7$, the deflected shape of the cylinder is dominated by the second-order modal shape, as two “anti-nodes” (maxima of the displacement envelope) remain constant along the cylinder span. However, the deflected shapes exhibit deviation between two oscillation periods. Figure 5 presents the spanwise distributions of the temporal power spectra of the cross-flow displacement for two reduced velocities. That the cylinder vibrates at a single frequency that corresponds to the specific natural frequency along the entire span is observed in two plots of power spectral density (PSD). A narrow-banded frequency is observed in the PSD plot for $U/f_1D = 7.3$ and a broad-banded frequency is observed in the PSD plot for $U/f_1D = 17.7$. This corresponds to the situation that the motion of the cylinder for $U/f_1D = 7.3$ is more regular than that for $U/f_1D = 17.7$.

It is evident from the above discussion that our numerical approach yields result in a good agreement with the previous experiment, and that captures all qualitative trends of the flexible cylinder VIV response as observed in the previous experimental studies. In addition to the experimental model of Huera-Huarte and Bearman (2009), the SDVM-FEM coupled method also has been validated for

| Modal order $n$ | Natural frequency $f_n$ (Hz) | Natural frequency ratio $f_n/f_1$ |
|-----------------|-------------------------------|----------------------------------|
| 1               | 3.11                          | 1.0                              |
| 2               | 10.26                         | 3.3                              |
| 3               | 23.33                         | 7.5                              |
| 4               | 40.74                         | 13.1                             |
| 5               | 64.07                         | 20.6                             |
| 6               | 91.43                         | 29.4                             |
| 7               | 124.09                        | 39.9                             |
| 8               | 162.65                        | 52.3                             |

Table II. The first eight-order modal natural frequencies of the cylinder.
the FIV simulation of the other experimental model in Lin and Wang (2019). In that study, we reported more validation work.

IV. FIV SIMULATION OF THE CYLINDER WITH AN UPSTREAM WAKE INTERFERENCE

In this part, the numerical simulation is conducted with the FIV response of a flexible cylinder with an upstream wake interference using the validated SDVM-FEM solver. The flexible cylinder model is identical to the model considered in Sec. III. The flexible cylinder is constrained to oscillate in the cross-flow direction and immersed in the wake of a stationary rigid cylinder of identical diameter and length in tandem with the center-to-center separation equal to 5 diameters. The cylinder is also exposed to the uniform oncoming flow with the lower 40% of its length.

A. Dynamic response of the cylinder FIV response

We first present the instantaneous deflection shapes of the FIV of the cylinder with upstream wake interference, as well as the deflection

![Instantaneous deflection shapes](image)

FIG. 4. Envelopes of the instantaneous deflection shapes of the cylinder in the first mode for (a) $U/f_1D = 7.3$ and the second mode for (b) $U/f_1D = 17.7$.

![Temporal spectral analysis](image)

FIG. 5. Temporal spectral analysis of the VIV displacement of the cylinder for (a) $U/f_1D = 7.3$ and (b) $U/f_1D = 17.7$. 
shapes of the VIV of the isolated cylinder as a comparison, at four representative reduced velocities in Fig. 6. A direct comparison between the deflection shape envelopes of the VIV of an isolated cylinder and the cylinder FIV with the upstream wake interference indicates distinct a difference in the cases of $U/f_1D = 11.5$ and $U/f_1D = 15.6$. In the case of $U/f_1D = 11.5$, the deflection displacement of the cylinder with upstream wake interference reaches a larger amplitude than that of the isolated cylinder, although both the responses are dominated by the

FIG. 6. Comparisons of the instantaneous deflection shapes envelope between [(a)–(d)] the VIV of an isolated cylinder and [(e)–(h)] the FIV of the cylinder with upstream wake interference for [(a) and (e)] $U/f_1D = 7.3$, [(b) and (f)] $U/f_1D = 11.5$, [(c) and (g)] $U/f_1D = 15.6$, and [(d) and (h)] $U/f_1D = 17.7$. 

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first-modal shape. In the case of $U/f_1D = 15.6$, the VIV deflection of the isolated cylinder is dominated by the second-order modal shape, while the FIV deflection of the cylinder with an upstream wake interference resembles the first-order modal shape. In the other two cases, the VIV and the FIV deflections show similarity in the amplitude value and the dominant modal shape. From a brief comparison between the deflection shapes of VIV of the isolated cylinder and FIV of the cylinder with upstream wake interference, it is found that the deflection
amplitude and dominant modal shape of the cylinder can be affected by the upstream wake interference.

To make a full comparison, the maximum cross-flow deflection amplitude of FIV of the cylinder with upstream wake interference vs reduced velocity is plotted against the amplitude of VIV of the isolated cylinder and shown in Fig. 7. It is found that the deflection amplitude of the FIV with the upstream wake interference is distinctly larger than that of the VIV of the isolated cylinder in the reduced velocity range from 9 to 15, which is the region between the two modal resonance branches. In this region, the amplitude of the FIV is roughly above 0.3 diameters while the VIV amplitude is about 0.1 diameters. In the first- and second-order modal resonance branches, the peak amplitude of the FIV is about equal to the value of the VIV amplitude. Of course, the upstream wake affects the amplitude response of the downstream cylinder in the reduced velocity range between two sequential modal resonance branches. This phenomenon is different from the FIV of an elastically mounted cylinder with an upstream wake interference in which the cylinder displacement amplitude builds up with increasing reduced velocity (Hover and Triantafyllou, 2001 and Assi et al., 2010). This is owing to that the elastically mounted rigid cylinder has a single natural frequency, while the flexible model has infinite-order modal natural frequencies.

We perform a spectral analysis on the cross-flow motion of the cylinder FIV with the upstream wake interference and show the frequency PSD plots for the various reduced velocities in Fig. 8. The first review of these plots raises a point: for all reduced velocities, the cylinder vibrates in one single frequency. In the cases of reduced velocity range from 5.2 to 15.6, two frequency branches can be identified from the PSD plots. In the cases of reduced velocity range from 16.7 to 18.8, three frequency branches are identified. For reduced velocity larger than 18.8, the FIV responses in two frequencies. Based on the variation of the frequency branch vs reduced velocity, the frequencies are categorized into three groups that are connected by a different colored line.

To determine which vibration mode the different frequency group corresponds to, the spanwise distributions of the temporal power spectra of the vibration are presented for four representative reduced velocities in Fig. 9. Based on the number and length of the cell highlighted on the contour plot, the vibration mode corresponding to the frequency branch can be identified. For \( U/f_1D = 7.3 \), both the two frequency branches comprising one cell indicate that the two frequencies correspond to the first-order modal vibration. In this way, we can find that the two frequencies of \( U/f_1D = 11.5 \) correspond to the first-order modal vibration; the two frequencies of \( U/f_1D = 15.6 \) correspond to the first-order and second-order modal vibrations, respectively, and the three frequencies of \( U/f_1D = 17.7 \) correspond to a first-order and two second-order modal vibrations. Return to Fig. 8, the red line connected frequency branch indicates the component of first modal vibration, the blue line connected frequency branch indicates the second modal vibration component. As to the green line connected frequency branch, it varies with reduced velocity following in the line of Strouhal number equal to 0.2. This frequency branch equal to the upstream wake vortex frequency indicates that this frequency component is directly induced by the upstream vortex force.

In Fig. 10, the frequency response of the cylinder with the upstream wake interference is plotted against the frequency response of the isolated cylinder. The critical reduced velocity of the first-order modal resonance branch jumping to the second order is about 12.5 for the VIV response of the isolated cylinder. However, lock-in of the first-order modal resonance is persisting to reduced velocity \( U/f_1D = 18.6 \) for the FIV response with the upstream wake interference. On the other hand, there is a fluid force that is induced by the upstream wake vortices acting on the downstream body. Due to the vortex shedding frequency in the upstream wake is directly depending on the oncoming flow speed, this upstream vortex force component excites the downstream cylinder into the resonance branch in the same way as the isolated cylinder VIV response. The combined action of the extension of the first-order modal resonance branch and the upstream wake vortex-induced second-order modal resonance result in the modal resonance overlapping and the several modal vibrations contributing to the FIV response.

In order to quantify the contribution of each modal vibration in the cylinder response, a modal decomposition method that was used in Huera-Huarte and Bearman (2009) is applied to compute the modal amplitude. The modal decomposition is based on the fact that the cylinder response \( y \) can be expressed in the matrix form as a linear combination of its modes

\[
y(z, t) = \Phi(z)y_w(t),
\]

where \( \Phi = [\phi_1(z), \ldots, \phi_n(z)] \) is the matrix composed of the modal shape vectors from the first-order to the \( n \)th-order, \( y(z, t) = [y_1(z_1, t), \ldots, y_n(z_n, t)]^T \) is the matrix of the time series of the displacement at the heights from \( z_1 \) to \( z_n \) along the cylinder span. The time series of each mode’s weight \( y_w(t) = [w_1(t), \ldots, w_n(t)]^T \) is calculated by

\[
y_w(t) = \Phi^{-1}(z)y(z, t).
\]

Figure 11 shows the amplitude of the first-order and second-order modal vibrations of the cylinder FIV response with the upstream wake interference and the isolated cylinder VIV response as a...
Concerning the VIV response of the isolated cylinder, the amplitude of the second-order modal vibration increases with reduced velocity, and that gets larger than the first-order modal amplitude when reduced velocity exceeds 13. The second-order modal vibration dominates the response with little contribution of the first-order modal vibration. Moving to the FIV response of the cylinder with upstream wake interference, the variation of the amplitude of the second-order modal vibration vs reduced velocity is similar to the isolated cylinder VIV response in the beginning of the second-order modal resonance branch. However, it is strikingly observed that the...
first-order modal vibration amplitude remains large in this region and overtakes that of the first-order modal vibration until reduced velocity 16. Due to the contribution of the first-order modal vibration persisting to high reduced velocities, the dominance of the second-order modal vibration in the response is delayed.

B. Hydrodynamic coefficients of the cylinder FIV response

In Sec. IV A, we characterize the cylinder FIV response with the upstream wake interference and find two distinct features of the structural response: the vibration of large amplitude during the resonance branch transition and the extension of modal resonance branch. In this section, we analyze the hydrodynamic coefficients for the FIV response of the cylinder with upstream wake interference to investigate the mechanism responsible for the two phenomena.

In the previous studies with VIV of the isolated cylinder, apart from the free vibration test where the cylinder is elastically mounted and free to react to the fluid forces, one approach to an understanding and possible prediction of VIV has been to undertake the forced vibrations of a cylinder in the flow. With the elastically mounted rigid cylinder, the consistency of the hydrodynamic coefficients and the wake vortex pattern measured from the forced vibration and the free VIV response has been verified by Hover et al. (1998) and Carberry et al.
In further studies, Fan et al. (2019b) and Wang et al. (2021) found a good agreement between the hydrodynamic coefficients measured from the rigid cylinder forced vibration and the sectional hydrodynamic coefficients reconstructed from the flexible cylinder vibration. They concluded that the hydrodynamic coefficient obtained from forced rigid cylinder vibration can be used to predict the distributed forces of the flexible cylinder accurately.

In Lin et al. (2020a), we performed the forced vibration experiment on a rigid cylinder placed in the wake of a stationary rigid cylinder in tandem and obtained the database of hydrodynamic coefficients of the steady drag coefficient, the excitation coefficient, and the added mass coefficient. Correspondence between hydrodynamic coefficients from forced vibration and free vibration was verified for the FIV response of an elastically mounted rigid cylinder with the upstream wake interference. In this study, we refer to the database given in Lin et al. (2020a) and plot the contours of excitation coefficient and added mass coefficient for the separation identical to the FIV simulation in this study and the data of the isolated cylinder as a comparison in Figs. 12 and 13, respectively. Given a displacement amplitude $A_y/D$ and
FIG. 13. Contours of added mass coefficient $C_{my}$ measured from the forced-vibration tests of (a) the isolated cylinder and (b) the cylinder with upstream wake interference (Lin et al., 2020a).

FIG. 14. (a) Effective added mass coefficient and (b) excitation coefficient distribution along the span of the flexible cylinder and the predicted value from the forced vibration test of the rigid cylinder for $U/f_vD = 7.3$. 
reduced frequency \( f_r \) (defined by \( f_r D/U \)), the corresponding coefficients value are obtained.

The excitation coefficient derived from the lift coefficient in phase with the velocity is used to quantify the average energy transfer between fluid and structural motion. The positive value means the positive energy transfer from the fluid to the structure, and the negative value means that the fluid takes energy out of the structural motion. From an energy balance point of view, for a stable FIV response of the cylinder, the average FSI energy transfer should be balanced for a dynamic equilibrium. If the structural damping is negligible, the excitation coefficient should be equal to zero, in other words, the FIV amplitude and frequency response should roughly follow the line of \( C_{lv} = 0 \). In Fig. 12(a), we highlight the line of \( C_{lv} = 0 \) for the isolated cylinder and the cylinder with upstream wake interference. The line of \( C_{lv} = 0 \) in Fig. 12(a) indicates that the response of isolated cylinder is of large amplitude in a certain reduced velocity range. If reduced frequency is less than 0.12 (corresponding to reduced velocity greater than 8.3), the resonance will disappear. The line of \( C_{lv} = 0 \) in Fig. 12(b) indicates that the response amplitude of the cylinder with upstream wake interference increases with decreasing reduced frequency continually. The increase in the cylinder response amplitude persists to higher reduced velocity than the isolated cylinder response.

Going back to the FIV amplitude response of the cylinder with upstream wake interference shown in Fig. 7, one may observe that the amplitude drops to a relatively low level, instead of increasing with reduced velocity, for reduced velocity greater than 8.3. Such a situation is due to the contribution of the first-order modal resonance response decreases as Fig. 11(a) shows. So, it is concluded that the vibration of large amplitude during the FIV modal resonance branch transition can be explained by the distribution of excitation coefficient and the contribution of the modal resonance response.

For a tensioned long flexible cylinder, the modal natural frequency of it \( \text{in vacuo} \) can be estimated by Eq. (16), based on the assumption of the Euler–Bernoulli beam

\[
\frac{p}{2EI} \sqrt{\frac{m}{m} \left( n^4 + \frac{n^2 T^2 L^2}{\pi^2 E I} \right)^{1/2}},
\]

where \( L \) is the cylinder length, \( m \) is the cylinder mass, \( T \) is the mean tension along the cylinder length, \( E I \) is the cylinder bending stiffness, and \( n \) is the modal order. In the FIV response of the cylinder placed in water, the cylinder vibrates in its true natural frequency which considers the added mass effect. The true modal natural frequency of the cylinder in water can be formulated by Eq. (17).
\[ f_w^n = \frac{\pi}{2L^2} \sqrt{\frac{EI}{m(1 + C_{my})}} \left( n^2 + \frac{n^2 TL^2}{\pi^2 EI} \right)^{1/2}, \]  

where \( C_{my} \) is added mass coefficient. The added mass coefficient takes the value 1.0 in quiescent fluid. But in the cylinder FIV response, the actual added mass coefficient varies with the amplitude and frequency of the cylinder oscillation. Considering that the mass, length, bending stiffness and mean tension of the cylinder are constant in the simulation, the true natural frequency \( f_w^n \) depends on the added mass coefficient and modal order. In a certain modal resonance region, as reduced velocity
increases, the vortex shedding frequency tends to increase. To maintain a stable FIV response in the resonance region, the true natural frequency of the cylinder has to be synchronized with the vortex shedding frequency. And the added mass coefficient is adjusted to match $f_w$ with the increasing vortex shedding frequency in the FSIs. Hence, given a certain modal resonance branch, the added mass coefficient decreases with the increasing reduced velocity to match $f_w$ with the increasing vortex shedding frequency. When the added mass coefficient reaches or is close to the minimum, the order of the modal resonance response will change from $n$ to $n + 1$, which is modal resonance transition (Fan et al., 2019b).

Figure 13 shows the added mass coefficient as a function of the oscillation amplitude and reduced frequency for the isolated cylinder and the cylinder with the upstream wake interference. It is common to the isolated cylinder and the cylinder with the upstream wake interference that the added mass coefficient decreases as the reduced frequency decreases (corresponding to the increasing reduced velocity). However, the difference comes in the minimum added mass coefficient that the isolated cylinder and the cylinder with the upstream wake interference reaches. The minimum added mass coefficient of the isolated cylinder is −0.5. But the minimum added mass coefficient of the cylinder with the upstream wake interference reaches −1.0, and the minimum value is achieved at lower reduced frequency (corresponding to larger reduced velocity) than the isolated cylinder. Return to Eq. (17), the smaller added mass coefficient that the cylinder can attain indicates that the synchronization of the actual natural frequency of the cylinder with the vortex shedding frequency can extend to the higher reduced velocities with the modal order of resonance $n$ remaining unchanged. Therefore, it is concluded that the upstream wake interference results in the lower added mass coefficient that the cylinder vibrates with and extends the modal resonance region to a higher reduced velocity.

C. Wake vortex structure of the cylinder FIV response

With the FIV response of the flexible cylinder with upstream wake interference, the previous studies little extend into the flow vortex structure of the cylinder FIV response. In this section, we provide results of wake visualization obtained through the numerical simulation for the isolated cylinder VIV response and the cylinder FIV response with the upstream wake interference to investigate the wake vortex structure responsible for the difference in the hydrodynamic coefficients between the VIV and FIV responses. The following figures are categorized into four groups which correspond to the cases of the isolated cylinder VIV response at $U/f_D = 7.3$ (Figs. 14–16), the isolated cylinder VIV response at $U/f_D = 11.5$ (Figs. 17–19), and the cylinder FIV response with the upstream wake interference at $U/f_D = 7.3$ (Figs. 20–22), the cylinder FIV response with the upstream wake interference at $U/f_D = 15.6$ (Figs. 23–25). Each group contains three sets of subfigures: the
FIG. 18. Instantaneous wake vortex pattern along the flexible cylinder span for the VIV of isolated cylinder at (a) $t = 0$ and (b) $t = 0.5 \, T$ for $U/f_1D = 11.5$.

FIG. 19. Instantaneous wake vortex pattern over one period of cross-flow oscillation at two representative span-wise locations $z = 0.11 \, m$ and $z = 0.44 \, m$ for the VIV of isolated cylinder at (a) $t = 0$ and (b) $t = 0.5 \, T$ for $U/f_1D = 11.5$. 
distribution of the hydrodynamic coefficients along the cylinder span, the wake vortex pattern along the cylinder span, and the instantaneous sectional wake vortex pattern over one oscillation period at the representative heights.

For the vibrating flexible cylinder placed in the flow, the FSI process is distributed along its length. The hydrodynamic coefficients vary significantly along the cylinder span. We present the distribution of the excitation coefficient and the added mass coefficient along the cylinder span. In addition, the coefficient distribution that is calculated from the flexible cylinder is plotted together with that is reconstructed from the database of forced vibration database, as a direct evidence that the distributed hydrodynamic forces of the cylinder FIV response with the upstream wake interference can be predicted by the forced vibration.

For the isolated cylinder VIV at $U/f_1D = 7.3$, the added mass coefficient increases as the height increases and then decreases as the height exceeds the value about 0.2 m. In the simulated wake vortex pattern figures, the red and blue points represent the positive and negative vortex elements, respectively. Based on the distribution of vortex elements, the clockwise and counterclockwise vortex structures are identified by the gathering of the vortex elements in blue and red, respectively. In Fig. 15, it is observed that the wake vortex shedding presents different modes along the cylinder span, as the vortex structure at low height resembles the typical Karman vortex street and the wake vortex structure at high height is observed to be stretched and split into sub-vortices. The instantaneous wake vortex pattern at two representative heights over one oscillation period is plotted in Fig. 16. At the height of $z = 0.11$ m, two single vortices of the opposite sense of rotation are shed from the cylinder in one oscillation period, which is so-called $2S$ mode. At the height of $z = 0.44$ m, it is observed that a single vortex structure is stretched and split into two sub-vortices due to the body motion of large amplitude. Two vortex pairs form over one period of the cylinder motion, which is so-called $2P$ mode. The increase in the added mass coefficient is correlated with the “$2S$” mode, while a “$2P$” mode tends to reduce the added mass coefficient. On the other hand, as the cylinder moves toward the negative (positive, respectively) maximum position, a counterclockwise (clockwise, respectively) vortex forms behind the body. This newly formed vortex induces a flow in favor of the cylinder motion, which results in the energy input to the structure. Such a situation explains that the excitation coefficient remains positive along the entire span.

At $U/f_1D = 11.5$ where the modal resonance branch of the isolated cylinder VIV is about to change, the added mass coefficient along the span decreases to the value of about $-0.4$ which is close to the minimum value in the database of the forced vibration. Due to the VIV response of small amplitude, the distribution of the added mass coefficient and the excitation coefficient along the cylinder length has little variation. Figure 18 shows that the wake vortex patterns along the
cylinder span resemble the "2S" mode. Coming to the sectional wake vortex pattern at the heights of \( z = 0.11 \) and \( 0.44 \) m, as the cylinder moves toward the negative maximum position, a clockwise vortex that induces a flow opposite to the cylinder motion forms at the height of \( z = 0.11 \) m, and a counterclockwise vortex that induces a flow aligned with the cylinder motion direction forms at the height of \( z = 0.44 \) m. Such a wake pattern corresponds to the fact that the excitation coefficient varies from negative to positive with the increasing sectional height.

In the cylinder FIV response with the upstream wake interference, vortex–structure interactions occur not only between the cylinder and the vortices from its wake, but also between the cylinder and the vortices from the upstream wake. The distribution of the hydrodynamic coefficients and the wake vortex pattern become different from the case of the isolated cylinder. Figure 20 shows the distribution of the added mass coefficient and the excitation coefficient along the cylinder length for the FIV with the upstream wake interference at \( U/f_1 D = 7.3 \). Comparing with the hydrodynamic coefficient distribution of the isolated cylinder, the added mass coefficient experiences a sharp variation at the height of about \( z = 0.45 \) m, and the excitation coefficient gets larger than that of the isolated cylinder within the height range from 0.3 to 0.5 m. The span-wise wake vortex pattern shown in Fig. 21 indicates that the upstream wake vortices and the downstream cylinder interaction pattern at different sectional heights vary a lot.

Figure 22 shows the instantaneous wake vortex pattern at two representative heights of \( z = 0.11 \) and \( 0.44 \) m over one oscillation period for the FIV at \( U/f_1 D = 7.3 \). It is observed that the wake vortex structure near the upstream cylinder resembles the typical Karman vortex street which indicates that the upstream vortex shedding is little affected by the downstream cylinder. However, when the upstream vortex is close to the downstream cylinder, the moving body may impinge on the upstream vortex and result in the vortex structure splitting around the body with a portion passing by the inner side and the rest passing by the outside of the cylinder, as shown in Fig. 22(c) of \( z = 0.11 \) m. However, there is a phase difference between the downstream motion and the upstream vortex shedding that guarantees the downstream cylinder crossing the gap between the upstream vortices over the entire oscillation process, as shown in the plots of \( z = 0.44 \) m. In addition to that, at the height of \( z = 0.44 \) m, as the cylinder moves.
toward the negative maximum position, there is a clockwise upstream vortex that moves to the upside of the downstream cylinder, and a counterclockwise upstream vortex moves to the downside of the cylinder. There is a contrast situation when the cylinder moves toward the positive maximum position. A vortex structure induces two effects on the structure: the force that is induced by low pressure of the vortex draws the body toward the vortex center; the rotating flow caused by the vortex aligned with or opposite to the motion direction of the

FIG. 22. Instantaneous wake vortex pattern over one period of cross-flow oscillation at two representative span-wise locations \( z = 0.11 \) m and \( z = 0.44 \) m for the FIV with upstream wake interference at (a) \( t = 0 \), (b) \( t = 0.25T \), (c) \( t = 0.5T \), (d) \( t = 0.75T \) for \( U/\Omega = 7.3 \).
structure. When the upstream vortex impinges on the cylinder and splits into two sub-vortices of the same direction at the two sides of the cylinder, the forces induced by the low pressure will be balanced, and the direction of the rotating flow acting on the cylinder will be opposite. In such circumstances, the cross-flow motion of the cylinder is little affected, the effect of the upstream wake interference acting on the downstream cylinder is weak. However, when the cylinder moves to the gap between the upstream vortices, the forces that draw the body toward the vortex center may be balanced, but the rotating flow around the vortices will be in favor of or damp the body cross-flow motion. In such circumstances, the upstream wake has a strong effect on the downstream motion. Such a vortex–structure interaction pattern corroborates the fact that the hydrodynamic coefficient distributions of the isolated cylinder VIV and the cylinder FIV with the upstream wake interference show similarity within the region around the height of \( z = 0.11 \) m but exhibit significant difference within the region around the height of \( z = 0.44 \) m.

At \( U/f_1 D = 15.6 \) where the first-order modal resonance response of the FIV with the upstream wake interference is about to finish, the added mass coefficient along the span decreases to about \(-0.6\), and that shows little variation within the height range from 0.3 to 0.5 m. The excitation coefficient generally remains positive along the entire cylinder span. The spanwise instantaneous wake vortex pattern shows that the vortex–structure interaction between the upstream wake and downstream cylinder is different at different sectional heights. It is observed that the upstream wake merges with the shear layers of the downstream body, developing into a combined wake vortex street. At the heights of \( z = 0.11 \) and \( 0.44 \) m, the wake vortex patterns over one oscillation period show that when the cylinder moves toward the negative maximum position, a counterclockwise vortex forms at the forward side of the downstream cylinder. The forward vortex induces a flow in favor of the cylinder motion and the downside vortex induces a force that draws the cylinder to the vortex center. Due to the cylinder moving within the width of the upstream wake, the vortex force is always in favor of the cylinder motion. Such a wake pattern illustrates the point that the excitation coefficient of the FIV with the upstream wake interference is larger than the isolated cylinder VIV excitation coefficient.

V. CONCLUSION

We numerically investigate the FIVs of a flexible cylinder with an upstream wake interference in this work. A Q3D FSI numerical method that couples the strip theory-based DVM and the FEM of structural dynamics is developed to simulate the FIV of the flexible cylinder. A major finding in this study answers two questions: what the difference between the flexible cylinder FIV response with an upstream wake interference and the isolated flexible cylinder VIV...
response is; and how the difference between the FIV response with the upstream wake interference and the isolated cylinder VIV response comes about.

Compared with the flexible isolated cylinder VIV response, the FIV response of the flexible cylinder with the upstream wake interference is characterized by the vibration of larger amplitude during the modal resonance branch transition and the extension of the modal resonance branch. Comparing with the isolated cylinder VIV response, the cylinder FIV with the upstream wake interference in larger amplitude during the modal resonance branch transition. On the other hand, the modal resonance response of the cylinder FIV with the upstream wake interference extends to higher reduced velocity, compared to the isolated cylinder VIV response. The upstream wake interference delays the end but little affects the beginning of a local modal resonance branch. Due to the extension of the former modal resonance branch, the resonance overlapping between the later mode and the former mode occurs, and the cylinder vibrates in the combination of multiple modal resonances.

To reveal the FSI mechanisms responsible for the phenomena of the FIV dynamic response, the database of the added mass coefficient and the excitation coefficient for the cylinder with the upstream wake interference are obtained by the forced vibration experiments on a rigid cylinder. Based on a comparison between the hydrodynamic coefficients measured through rigid cylinder forced oscillation, and the sectional fluid coefficients reconstructed from the flexible cylinder vibration, we verify the correlation between the free and forced vibration for the cylinder FIV response with the upstream wake interference. Such a result indicates that the sectional FSI properties of a flexible cylinder model can be predicted by the forced vibration on a rigid cylinder undergoing the same prescribed motion. With the upstream wake interference, it is found that the added mass coefficient of the cylinder decreases with the increasing reduced velocity and reaches lower negative value than the isolated cylinder VIV response. The lower added mass coefficient results in that the synchronization between the vortex shedding frequency and the true natural frequency of the structure persists to the higher reduced velocity in a certain modal resonance branch. The excitation coefficient distribution indicates that the cylinder with the upstream wake interference reaches high amplitude at high reduced velocity, instead of ceasing resonance as the isolated cylinder. However, with the reduced velocity increasing, the contribution of the modal resonance in the FIV reduces. This situation leads to the amplitude of the flexible cylinder FIV not
increasing continually with reduced velocity as the WIV response of the elastically mounted rigid cylinder.

The numerical spanwise wake vortex pattern and the sectional instantaneous wake vortex structure explain the correlation between the distribution of the hydrodynamic coefficients along the cylinder span and the wake vortex mode. For the isolated cylinder VIV response, it is found that the added mass coefficient is strongly correlated with the vortex shedding mode and the excitation coefficient is more correlated with the phase difference between the cylinder motion and the vortex shedding. With the cylinder FIV response with the upstream wake interference, the vortex-structure interactions between the upstream wake vortices, and the downstream cylinder motion has a significant effect on the distribution of the added mass coefficient and the excitation coefficient. When the upstream vortex impinges on the downstream body and splits into the sub-vortices, the effect of the upstream wake interference acting on the downstream cylinder reduces. When the downstream cylinder enters the clearance between the upstream vortices over the entire oscillation process, the upstream wake results in a stronger interference effect on the downstream FIV response.

The results found herein have implications for applications, especially in the field of marine structures, such as multiple risers and cable systems placed in the ocean current. The findings of this study are expected to provide insight for both the development of semi-empirical modeling in predicting the FIV response of multiple cylinders with the wake interference effect, especially for the downstream response, as well as the elaboration of simplified low-order models for fatigue damage quantification and offshore structure design of the multiple-cylindrical structures.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request. The hydrodynamic coefficient database of the cylinder FIV with the upstream wake interference in this paper can be downloaded from the link of “https://github.com/hyperpotato/Hydrodata-Tandem-Cylinders.”

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