A Bayesian estimation-based uncertainty quantification of flaws in steel welds detected by ultrasound phased array

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Abstract. Phased array ultrasonic scanning is currently the most advanced non-destructive test for detecting flaws in a steel welding. However, large uncertainty exists in the detection results. Technically, this uncertainty is induced by the sparsity of detection resolution compared to the scale of the flaws. Consequently, it is essential to enrich the resolution of detection results and quantify the corresponding statistical uncertainty before evaluating the number and equivalent of the flaws in the steel welding. In this paper an uncertainty quantification framework is built with Bayesian compressive sensing method from which the equivalent of the flaws in the steel welding with certain degree of confidence can finally be given.

1. Introduction

Phased array ultrasonic scanning is widely used in the non-destructive test for the steel welding to measure the equivalent of the flaws and evaluate the quality of the steel welding \cite{1, 2}. Even though it is the most advanced technique to detect the flaws in steel welding, the measurements made by this equipment are still sparse with respect to the scale of the flaws \cite{3}. Therefore, the measurement needs to be enriched which will inevitably introduce statistical uncertainty. The degree of confidence of the estimation is of great importance especially in the quality control of the engineering products \cite{4, 5}. Hence it is essential to quantify the statistical uncertainty in the enrichment procedure and give a rational evolution of the quality of the steel weld.

The equivalent of defect in a steel welding have notable spatial variability and spatial correlation, therefore, it can be modeled by a random field \cite{6}. In this study, the Bayesian compressive sensing \cite{7} is utilized to enrich the measured data and quantify the statistical uncertainty as this method is widely used in the quantification of uncertainty of random fields \cite{8, 9}. The Bayesian compressive sensing is an extension of the compressive sensing method, which is firstly proposed by Donoho \cite{10} with the insight of the ubiquitous compressibility in measurements. By setting up the compressive sensing theory in the framework of Bayesian inference, the information of statistical uncertainty is preserved in the procedure of enriching the measurements \cite{7, 11}. Because the preserved probability information contains the spatial correlation of the enriched data, this method is suitable of solving the uncertainty quantification of random fields.

In this paper, the Bayesian compressive sensing is firstly revisited in section 2, afterwards, the engineering problem of uncertainty in detecting flaws of steel welds by phased array ultrasonic is described and the Bayesian compressive sensing is herein applied to quantify the uncertainty. Finally, some concluding remarks are drawn.
2. Statistical uncertainty quantification based on Bayesian compressive sensing

The Bayesian compressive sensing (BCS) is a method that can enrich a sparse measurement and quantify the corresponding statistical uncertainty. For example, consider a vector \( f \in \mathbb{R}^N \) as an accurate measurement of a sample of a one-dimensional random field \( F \) \( x \), where \( x \) is the one-dimensional spatial coordinate. In general, for a base matrix \( B \in \mathbb{R}^{N \times N} \) which satisfies \( BB^T = I_N \), where \( ^T \) represents the transpose of the matrix and \( I_N \in \mathbb{R}^{N \times N} \) is the identity Matrix, \( f \) can be decomposed as:

\[
 f = B\omega
\]  

(1)

where \( \omega \in \mathbb{R}^N \) is the generalized coordinate of \( f \) in the space stretched by \( B \). Assume the number of non-trivial terms in \( \omega \) is \( M \), and set the \( N - M \) trivial terms in \( \omega \) to 0, for the resultant vector \( \hat{\omega} \), there is:

\[
 \hat{f} = B\hat{\omega}
\]  

(2)

From equation (2), the compressibility of a set of data \( f \) is described as the sparsity of vector \( \omega \). When \( N \gg M \) and the relative error \( \varepsilon \) defined in equation (3) is neglectable, the vector \( f \) is considered as compressible [10].

\[
 \varepsilon = \frac{\| f - \hat{f} \|}{\| f \|}
\]  

(3)

where \( \| \cdot \| \) denotes certain type of norm (e.g. 2-norm in Euclidean space) of the inner vector.

It has been proved in many researches that the compressibility of measured data is ubiquitous [10, 12, 13], hence for a sparse measurement \( g \in \mathbb{R}^K \) of \( f \):

\[
 g = \Phi f = \Phi B^T \omega = \Lambda \omega
\]  

(4)

where \( \Phi \in \mathbb{R}^{K \times N} \) represents the measurement procedure and \( K < N \). Matrix \( \Lambda = \Phi B^T \) is defined for convenience.

In practice, \( g \) and \( \Lambda \) are known. Since \( K < N \), \( \omega \) is hard to accurately estimate, consequently, \( g \) is decomposed into two parts:

\[
 g = \Lambda \omega = \Lambda \omega_i + n
\]  

(5)

where \( \omega_i \in \mathbb{R}^N \) is the estimated generalized coordinate and \( n \in \mathbb{R}^N \) represents the corresponding error. In general, \( n \) can be considered as random vector with its components being independent identical distributed zero-mean \( \sigma^2 \)-variance Gaussian random variables. Hence a Gaussian likelihood model can be built [7, 14]:

\[
 p( g | \omega_i, \sigma^2 ) = 2\pi \sigma^2^{-K/2} \exp \left( -\frac{1}{2\sigma^2} \| g - \Lambda \omega_i \|^2 \right)
\]  

(6)

To control the sparsity of the estimation, a hierarchical Gaussian prior function [15, 16] is assumed for \( \omega_i \):

\[
 p( \omega_i | \alpha ) = \prod_{i=1}^{N} N( \omega_{\alpha_i}, 0, \alpha_i^{-1} )
\]  

(7)

where \( \omega_{\alpha_i} \) is the \( i \)th components of \( \omega_i \) and hyperparameter \( \alpha_i \) is the reciprocal of the variance of \( \omega_{\alpha_i} \).

\( N \left[ m_z, v_z \right] \) denotes a Gaussian probability density function for random variable \( Z \) which has a \( m_z \)-mean and \( v_z \)-variance. Assume the hyperparameters \( \alpha = \alpha_1, \alpha_2, \ldots, \alpha_N \) satisfy a Gamma prior function:
\[ p(\alpha | a, b) = \prod_{i=1}^{N} \Gamma(\alpha_i | a, b) \]  

(8)

where \( \Gamma(\alpha_i | a, b) \) denotes a Gamma probability density function for random variable \( \alpha_i \). \( a \) and \( b \) are the distribution parameters that is to be estimated.

The overall prior for \( \omega_s \) is then derived by marginalizing the hyperparameters:

\[ p(\omega_s | a, b) = \prod_{i=1}^{N} \int_{0}^{\infty} N(\omega_{s,i} | 0, \alpha_i^{-1}) \Gamma(\alpha_i | a, b) \ d\alpha_i \]  

(9)

The overall posterior distribution of \( \omega_s \) can be written as:

\[ p(\omega_s | g, \alpha, \sigma^2) = \frac{p(g | \omega_s, \sigma^2) p(\omega_s | \alpha)}{p(g | \alpha, \sigma^2)} \]  

(10)

where \( p(g | \alpha, \sigma^2) \) is used for normalization, it can be calculated by the integral:

\[ p(g | \alpha, \sigma^2) = \int p(g | \omega_s, \sigma^2) p(\omega_s | \alpha) \ d\omega_s \]  

(11)

Finally, the posterior of \( \omega_s \) is analytically derived as:

\[ p(\omega_s | g, \alpha, \sigma^2) = 2\pi^{-N+1/2} |\Sigma_{ws}|^{-1/2} \exp\left(-\frac{1}{2} (\omega_s - \mu_{ws})^T \Sigma_{ws}^{-1} (\omega_s - \mu_{ws})\right) \]  

(12)

where \( \mu_{ws} \in \mathbb{R}^N \) and \( \Sigma_{ws} \in \mathbb{R}^{N \times N} \) are the corresponding mean vector and variance matrix of \( \omega_s \). They can be written as:

\[ \mu_{ws} = \sigma^2 \Sigma_{ws} \Phi^T g \]
\[ \Sigma_{ws} = \sigma^2 \Phi^T \Phi + A^{-1} \]  

(13)

where \( A = \text{diag} \ \alpha \). The estimation of \( f \) is represented by the mean value vector \( \mu_f \) and covariance matrix \( \Sigma_f \):

\[ \mu_f = B\mu_{ws} \]
\[ \Sigma_f = BS_{ws}B^T \]  

(14)

To this end, the Bayesian inference framework is built and converted into the optimization of \( \alpha \) and \( \sigma^2 \). This problem can be solved by a Relevance Vector Machine (RVM) [14] and the solving procedure is not presented in this paper for conciseness.

The BCS is also suitable for solving two dimensional problems by simply introducing a 4th rank tensor (4-dimensional matrix) \( B \) as the base matrix. This part is also omitted in this paper and the reader can see detailed derivation in literatures such as [17].

In the matrix \( \Sigma_f \), the diagonal components represents the variance of the estimated data \( f \), while the off-diagonal components represents the covariance of \( f \). Hence the BCS is able to generate the whole first 2 orders of moment information of the statistical uncertainty of the estimation which is enough to recover the total probability information under the Gaussian distribution assumption. Further, by integral the probability density function on each spatial point, the confidence interval of the certain level can be estimated.
3. The uncertainty quantification of the ultrasound phased array test for detecting the flaws in the steel welds

3.1. Description of the problem
In engineering practice, the steel structures are assembled with steel components by bolts and welds. The safety of the welds is essential to most of the joints. Hence the quality of the steel welds should be taken into serious consideration. However, even by applying the most advanced equipment such as ultrasound phased array [5], the measured data (which is the $g$ in equation (4)) is still sparse, and the reliability of detecting the flaws is still low. As shown in Figure 1, two tests result for the same steel weld differ a lot.

![Figure 1](image1.png)

(a) Test #1 vertical view.

(b) Test #2 vertical view.

Figure 1. Two test results for the same weld.

From the two measurements in Figure 1, the location and equivalent of flaws share some similarity yet still different in general, hence the flaws can’t be quantified due to the large uncertainty.

Clearly, the location and equivalent of the flaws are spatially variated and correlated, the spatial correlation is an important property of a measurement. By quantifying the spatial correlation, the agglomeration degree of the flaws is estimated which will be an important reference for determining the credibility of measured flaws.

3.2. Statistical uncertainty quantification of flaws in steel weld based on BCS
The statistical uncertainty is quantified for the enrichment of the measured data in the above subsection. In this process, the vertical view of the two measurements of the steel weld is averaged and the Bayesian compressive sensing (BCS) is herein performed.

![Figure 2](image2.png)

Figure 2. The mean of the estimated result.

The mean of the estimation $\mu_f$ is firstly given in Figure 2. This estimation is much refined compared with the original data. By averaging, some of the peaks that appeared in a single test result vanished. The BCS can well preserve the spatial correlation information hence the area around a peak will also be considered as weight to decide the confidence of the occurrence of the flaw.

Afterwards, the mean along with the 95% confident interval are shown in Figure 3. Figure 3 (a) is the mean and the 95% confident interval along the weld for several positions of the width direction of the weld. Figure 3 (b) is the result for a representative position.
(a) The cross-section view of estimated results for several positions on the width of the weld.

In Figure 3 (b), there is a high uncertainty for the peak around 220mm while the uncertainty for the peak around 200mm is relatively low. This clearly shows the effectiveness of the spatial correlation for estimating the flaws. The resultant flaws with different confident levels (95%, 99%) are shown in the binomial state fields in Figure 4.

(b) The estimated result for a representative position.

**Figure 3.** The mean and 95% confident interval of estimated result.

By an integral along space, the probability of area of the flaws in vertical view can be evaluated, as shown in Figure 5.
Figure 5. The probability of area of the flaws in vertical view.

From the probability information of the area of flaws in Figure 5, the quality of the weld can be accessed with ease, and for different levels of confidence, the spatial distribution of the flaws can also be evaluated by Figure 4.

4. Concluding remarks
In this paper, the Bayesian compressive sensing is introduced, followed by the uncertainty quantification of the flaws in steel weld. From this study, the following concluding remarks are drawn.

(1) The Bayesian compressive sensing is an extension of the compressive sensing and it can be applied to quantify the statistical uncertainty of the engineering problems.

(2) The Bayesian compressive sensing is able to capture the spatial correlation information of the measured data.

(3) The ultrasound phased array is able to measure the flaws in the steel weld, but large uncertainty exists in the measurement. By applying the Bayesian compressive sensing, the spatial correlation of the data is preserved and the statistical uncertainty can be quantified. Different levels of confidence for the equivalent of flaws can be drawn for evaluation of the quality of steel weld.

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