On Radiation Pressure and the Poynting-Robertson Effect for Fluffy Dust

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Abstract. Equation of motion for real dust particle under the action of electromagnetic radiation is more general than equation of motion corresponding to standardly used Poynting-Robertson effect (P-R effect). As a consequence, orbital evolution of particles may significantly differ from that corresponding to the P-R effect. The paper discusses recently published (Icarus, June 2002) derivation of equation of motion, which is in contradiction with known relativistically covariant formulation. The “new” derivation does not respect fundamental physical laws (law of conservation of energy, law of conservation of momentum) which must hold in any frame of reference. Application of the derived “general” equation of motion to the special case treated by Einstein in 1905 yields result which is not consistent with Einstein’s result. Correct solution is presented.

Key words: relativity theory, cosmic dust

1. Introduction

Relativistically covariant equation of motion for arbitrarily shaped (dust) particle under the action of electromagnetic radiation was derived by Klačka (2000a). A more simple derivations to the first order in \( v/c \) (\( v \) is velocity of the particle, \( c \) is the speed of light)
were presented by Klačka (2000b), Klačka and Kocifaj (2001a), where also application to orbital evolution of micron-sized meteoroids can be found. Application of the general equation of motion to larger bodies (e. g., meteor-sized bodies, asteroids) can be found in Klačka (2000c). As for review papers of various derivations (including relativistically covariant forms) we refer to Klačka (2001) and Klačka (2002a).

Important property of the above discussed derivations and presentations of general equation of motion is that it covers not only Poynting-Robertson effect (P-R effect; Robertson 1937, Klačka 1992), but also Einstein’s example (Einstein 1905) as special cases – as it is required for more general theory. Equation of motion is expressed in terms of particle’s optical properties standardly used in optics for stationary particles – various optical properties can be taken into account.

Application of the general equation of motion for orbital evolution of meteoroids was presented in Klačka and Kocifaj (2001a, 2001b, 2002), Kocifaj and Klačka (2002a, 2002b).

In spite of easy accessible of at least some of the above referenced papers a new paper by Kimura et al. (2002) was published, now. Kimura’s paper derives and presents a “new” derivation and completely ignores the above referenced papers, most fundamental of which has been known to Kimura et al. New paper in Icarus may generate an impression that the older presentations are incorrect and Kimura’s paper is right. In order to enable orientation in physics of more general equation of motion than that corresponding to the P-R effect, it is inevitable to make comments on Kimura’s presentation, corresponding to presentations by Klačka and Kocifaj (1994) and Kocifaj et al. (2000); as for some optical properties of dust particles, see also Kimura (2000).

2. Equation of motion for arbitrarily shaped dust particle

Relativistically covariant equation of motion may be expressed as

\[
\frac{dp^\mu}{d\tau} = \frac{w^2}{c^2} A' \sum_{j=1}^{3} Q_j' \left( c b_j^\mu - u^\mu \right),
\]

where \( p^\mu \) is four-vector of the particle of mass \( m \)

\[
p^\mu = m u^\mu,
\]

four-vector of the world-velocity of the particle is

\[
u^\mu = (\gamma c, \gamma \mathbf{v}), \quad \gamma = \frac{1}{\sqrt{1 - \mathbf{v}^2/c^2}},
\]

four-vectors

\[
b_j^\mu = \left( \frac{1}{w_j}, 1, e_j \right), \quad j = 1, 2, 3.
\]
\( \mathbf{e}_1 \) is unit vector of the incident radiation, the system of unit vectors \( \{ \mathbf{e}_j', j = 1, 2, 3 \} \) measured in the rest frame of the particle forms an orthogonal basis, \( S \) is flux density of the incident radiation energy, \( A' \) is geometrical cross section of a sphere of volume equal to the volume of the particle \( (A' = \pi a^2, a \) is “effective radius”), \( w \equiv w_1 \),

\[
w_j = \gamma (1 - \mathbf{v} \cdot \mathbf{e}_j/c) , \quad j = 1, 2, 3 ,
\]

(5)
effective factors are given by relations

\[
Q'_1 = Q'_{\text{ext}} - \langle \cos \vartheta' \rangle \quad Q'_{\text{sca}} ,
\]

\[
Q'_2 = - \langle \sin \vartheta' \cos \varphi' \rangle \quad Q'_{\text{sca}} ,
\]

\[
Q'_3 = - \langle \sin \vartheta' \sin \varphi' \rangle \quad Q'_{\text{sca}}
\]

(6)
and form components of the vector of radiation pressure efficiency factor; see references mentioned above. P-R effect immediately follows from Eq. (1): \( Q'_2 = Q'_3 = 0 \). Application of general equation Eq. (1) to Einstein’s special case is presented in Klačka (2002b).

2.1. Equation of motion and its application to Solar System

Within the accuracy to the first order in \( \mathbf{v}/c \), Eqs. (1) and (4) yield

\[
\frac{d \mathbf{v}}{d t} = \frac{S A'}{m c} \sum_{j=1}^{3} Q'_j \left[ \left( 1 - 2 \frac{\mathbf{v} \cdot \mathbf{e}_1}{c} + \frac{\mathbf{v} \cdot \mathbf{e}_j}{c} \right) \mathbf{e}_j - \frac{\mathbf{v}}{c} \right] ,
\]

\[
\mathbf{e}_j = (1 - \mathbf{v} \cdot \mathbf{e}_j'/c) \mathbf{e}_j' + \mathbf{v}/c , \quad j = 1, 2, 3 .
\]

(7)
(We want to stress that values of \( Q' \)-coefficients depend on particle’s orientation with respect to the incident radiation – their values are time dependent.)

In the case of the most simple application to Solar System, gravitational force of the Sun has to be considered. Equation of motion of a particle in the gravitational and electromagnetic radiation fields of the Sun is

\[
\frac{d \mathbf{v}}{d t} = - \frac{4 \pi^2}{r^2} \mathbf{e}_1 + \beta \frac{4 \pi^2}{r^2} \sum_{j=1}^{3} \frac{Q'_j}{Q'_1} \mathbf{X}_j ,
\]

\[
\mathbf{X}_j \equiv \left( 1 - 2 \frac{\mathbf{v} \cdot \mathbf{e}_1}{c} + \frac{\mathbf{v} \cdot \mathbf{e}_j}{c} \right) \mathbf{e}_j - \mathbf{v}/c ,
\]

\[
\beta = \frac{0.02868}{12 \pi} \frac{Q'_1 A' [m^2]}{m [kg]} ,
\]

(8)
if length is measured in astronomical unit (AU) and time in years; \( \mathbf{e}_1 \equiv \mathbf{r}/r \).

3. Fundamental Physics

One has to take into account relativistic physics when he wants to correctly understand processes with electromagnetic radiation. Although it may seem that some electromagnetic phenomena can be completely understood on the basis of Newtonian physics, it is
not true. On the basis of fundamental work of Planck and Einstein we know that the speed of light in vacuum is $c$ for all observers, concentration of photons depends on the frame of reference. Thus, it has no sense to say that some astronomically important phenomena can be explained by violation of the above statements and affirm that they are based on Newtonian physics (see also Klačka 1992, 1993).

We have relativity theory in disposal for almost one century. We have to respect its laws if we want to discuss interaction of a particle with electromagnetic radiation. As a consequence, Eq. (1) is obtained as equation of motion for the particle under the action of electromagnetic radiation. Relativistically covariant formulation ensures that “all inertial observers are equivalent”, which corresponds to the first postulate of special relativity. This equivalence of all observers also means that if one inertial observer carries out some experiments and discovers a physical law, then any other observer performing the same experiments must discover the same law (d’Inverno 1992). Applying to our problem, we can affirm: conservations of energy and momentum in one frame of reference ensures the conservations also in any other frame of reference.

Relativistic requirement for covariant formulation of equation of motion leads to Eq. (1) together with three four-vectors $b^\mu_1$, $b^\mu_2$, $b^\mu_3$ defined by Eq. (4). Accuracy to the first order in $v/c$ reduces Eqs. (1) and (4) to Eq. (7).

### 4. Kimura’s access and our result – comparison

Kimura’s access (Kimura et al. 2002) is based on definitions of two sets of orthonormal vectors – primed and unprimed. On the basis of these definitions Kimura et al. (2002) obtain their equation of motion.

Eq. (7) shows that orthonormality of the primed unit vectors does not admit orthonormality of the unprimed unit vectors.

Our access is based on definition of the only one set of orthonormal vectors – primed frame of reference (see text below Eq. (4)). We do not need any another definition! Moreover, relativity principles do not enable any other definition. Relativity principles strictly determine the expressions for unprimed unit vectors!

Thus, strict requirement for fulfilling relativity principles leads to equation of motion which differs from that presented by Kimura et al. (2002). Since man’s knowledge does not doubt about the requirement for fulfilling relativity principles, Eq. (1) has to be considered as the right equation of motion. Any other form of equation of motion, not consistent with Eq. (1), has to be rejected. On this basis one has to reject Kimura’s equation of motion (Eqs. (10), (13) in Kimura et al. 2002).
5. Physics and Kimura’s access

We have discussed why equation of motion presented in Kimura et al. (2002) has to be rejected. We will present physical arguments for the rejection of Kimura’s equation of motion in a more simple way, now.

Covariant formulation of equation of motion ensures fulfillment of the principles of relativity theory. As a consequence: i) if law of conservation of energy is fulfilled in one frame of reference, then the law conservation of energy is fulfilled in any other frame of reference; ii) if law of conservation of momentum is fulfilled in one frame of reference, then the law conservation of momentum is fulfilled in any other frame of reference.

Since equation of motion presented in Kimura et al. (2002) cannot be formulated in relativistically covariant form, the first principle of relativity is violated. Thus, an observer in a frame of reference determines that the laws of conservation of energy and momentum hold during the process of interaction of the electromagnetic radiation with the particle, while any other observer not in the same frame of reference will state that the laws of conservation of energy and momentum do not hold.

Another even more simple argument exists why equation of motion presented in Kimura et al. (2002) has to be rejected. It is well accepted that a new and more general theory must contain an older theory as a special case. In other words, more general theory has to be reducible to the older theory. As we know, two special cases corresponding to equation of motion for a particle under the action of electromagnetic interaction, were presented. The first one was presented by Einstein (1905), the second one by Robertson (1937). Application of equation of motion presented in Kimura et al. (2002) to the special case treated by Einstein yields result which is not consistent with Einstein result – for more details see Klačka (2002b).

6. Equation of motion and practical calculations

We can write for continuous distribution of density flux of energy as a function of frequency

\[
\frac{d p'}{d \tau} = \frac{A'}{c} \sum_{j=1}^{3} \int_{0}^{\infty} c \ h \ \nu' \ \partial n' \ \frac{\partial \nu'}{\partial \nu'} Q_j'(\nu') \ d\nu' \ e^{i \nu} .
\]  

(9)

Taking into account that concentration of photons fulfills \( n' = w \ n \) and \( \nu' = w \ \nu \), we have \( \partial n' / \partial \nu' = \partial n / \partial \nu \). Lorentz transformation finally yields

\[
\frac{d p^\mu}{d \tau} = \frac{w^2 A'}{c^2} \sum_{j=1}^{3} (c \ b_j^\mu - w^\mu) \ \int_{0}^{\infty} c \ h \ \frac{\partial n}{\partial \nu} \ \nu \ Q_j'(w \ \nu) \ d\nu .
\]  

(10)
6.1. Correct equation of motion

Instead of incorrect Eq. (13) in Kimura et al. (2002), we write the right form of equation of motion:

\[
\frac{d\mathbf{v}}{dt} = -\frac{G M_{\odot}}{r^2} \mathbf{e}_1 + \frac{G M_{\odot}}{r^2} \sum_{j=1}^{3} \beta_j \left[ \left( 1 - 2 \frac{\mathbf{v} \cdot \mathbf{e}_1}{c} + \frac{\mathbf{v} \cdot \mathbf{e}_j}{c} \right) \mathbf{e}_j - \frac{\mathbf{v}}{c} \right],
\]

\[
\mathbf{e}_j = \left( 1 - \frac{\mathbf{v} \cdot \mathbf{e}_j}{c} \right) \mathbf{e}_j' + \frac{\mathbf{v}}{c}, \quad j = 1, 2, 3,
\]

(11)

where

\[
\beta_1 = \frac{\pi R_{\odot}^2}{G M_{\odot} m c} \int_0^\infty B_\odot(\lambda) \left\{ C'_{\text{ext}}(\lambda/w) - C'_{\text{sca}}(\lambda/w) g'_1(\lambda/w) \right\} d\lambda,
\]

\[
\beta_2 = \frac{\pi R_{\odot}^2}{G M_{\odot} m c} \int_0^\infty B_\odot(\lambda) \left\{ - C'_{\text{sca}}(\lambda/w) g'_2(\lambda/w) \right\} d\lambda,
\]

\[
\beta_3 = \frac{\pi R_{\odot}^2}{G M_{\odot} m c} \int_0^\infty B_\odot(\lambda) \left\{ - C'_{\text{sca}}(\lambda/w) g'_3(\lambda/w) \right\} d\lambda,
\]

\[
w = 1 - \frac{\mathbf{v} \cdot \mathbf{e}_1}{c},
\]

(12)

\( R_{\odot} \) denotes the radius of the Sun and \( B_\odot(\lambda) \) is the solar radiance at a wavelength of \( \lambda \); \( G, M_{\odot}, \) and \( r \) are the gravitational constant, the mass of the Sun, and the distance of the particle from the center of the Sun, respectively. \( C'_{\text{ext}} \) and \( C'_{\text{sca}} \) denote the usual extinction and scattering cross sections, the asymmetry parameter vector \( \mathbf{g}' \) is defined by \( \mathbf{g}' = \int \mathbf{n}'(dC'_{\text{sca}}/d\chi')d\chi' \), where \( d\chi' \) is the element of solid angle, \( \mathbf{n}' \) is a unit vector in the direction of scattering, \( dC'_{\text{sca}}/d\chi' \) is the differential scattering cross section; cartesian coordinates \( (g'_1, g'_2, g'_3) \) describe the asymmetry parameter vector \( \mathbf{g}' \) as \( \mathbf{g}' = g'_1 \mathbf{e}_1' + g'_2 \mathbf{e}_2' + g'_3 \mathbf{e}_3' \). See Kimura et al. (2002) for further details.

Eq. (11) corresponds to Eq. (8): \( \beta_j = \beta Q'_j / Q'_1, \quad j = 1, 2, 3 \). Eq. (12) is equivalent to Eq. (23) in Klačka and Kocifaj (2001a). As a consequence of the fact that unit vectors \( \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \) do not form orthonormal set, we may mention that equation \( \mathbf{v} = (\mathbf{v} \cdot \mathbf{e}_1)\mathbf{e}_1 + (\mathbf{v} \cdot \mathbf{e}_2)\mathbf{e}_2 \) is not fulfilled.

7. Conclusion

Equation of motion presented by Kimura et al. (2002 – Eqs. (10), (13)) does not respect basic principles of relativity. Kimura et al. (2002) use physically unacceptable definitions. As a consequence, application of equation of motion presented by Kimura et al. (2002) to the special case treated by Einstein yields result which is not consistent with Einstein result. All these facts lead to the conclusion: equation of motion presented in Kimura et al. (2002) has to be rejected.

Correct equation of motion is presented in Klačka (2000a, 2000b, 2000c, 2001, 2002a), Klačka and Kocifaj (2001a) and in some other later papers of these authors – all these
papers present and use equation of motion consistent with Eq. (1) of this paper. If one wants to use correct equation of motion which contains the quantities used by Kimura et al. (2002), Eqs. (11) and (12) of our paper are correct.

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