Oscillons from Pure Natural Inflation

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We examine the oscillon formation in a recently proposed inflation model of the pure natural inflation, where the inflaton is an axion that couples to a strongly-coupled pure Yang-Mills theory. The plateau of the inflaton potential, which is favored by recent observations, drives the fragmentation of the inflaton and can produce spatially localized oscillons. We find that the oscillons are formed for $F \lesssim O(0.1)M_{pl}$, with $F$ the effective decay constant of the model. We also comment on observational implications of the oscillons.

I. INTRODUCTION

Recently a new inflation model, called the pure natural inflation, has been introduced in Ref. [1]. This is a very simple model where the role of the inflaton is played by an axion, which couples to a strongly-coupled pure Yang-Mills theory. The inflaton here is a pseudo Nambu-Goldstone boson for the shift symmetry of the axion, and this naturally explains the flatness of the potential [2, 3].

The natural question is then if we can test this model in future observations. In Ref. [1], the predictions for the values of the spectral index $n_s$ and the tensor-to-scalar ratio $r$ have been worked out. The result is in impressive agreement with current observational constraints. This is in contrast with the natural inflation [2, 3] with the cosine potential, which has been extensively studied so far but is being disfavored by recent results by Planck [4] and BICEP/Keck Array [5].

The predictions for the values of $r$ and $n_s$ depend crucially on the model parameter $F$, which plays the role of the effective decay constant of the axion. When the value of $F$ is large, namely of the order the Planck scale $F \gtrsim O(m_{pl})$ ($m_{pl} \approx 1.22 \times 10^{19}\text{GeV}$), the value of $r$ can be of order $r \sim O(0.1)$, which will be within the reach of future CMB experiment [6]. When $F$ is smaller and is of order $\lesssim O(0.1)m_{pl}$, however, $r$ becomes smaller and a detection of tensor modes becomes more challenging. It is therefore an interesting question to see if there is other distinctive features of the model, when $F \lesssim O(0.1)m_{pl}$.

In this paper we point out that for $F \lesssim O(0.1)m_{pl}^2$ we find interesting new phenomenon absent for larger values of $F$—the generation of the spatially-inhomogeneous profile of the inflaton field, known as oscillon/I-ball [8-17].

It is known that when the scalar field $\phi$ oscillates in the potential that is shallower than quadratic, $\phi$ fragments into quasi-stable lumps, oscillons/I-balls. These can arise in the context of inflation as the fragmentation of the inflaton field after the inflation, whose phenomenon is extensively studied in various models of inflation with shallow potentials [12, 14, 15, 17, 18]. Since the small $F$ in our case leads to a flatter potential, as we will see later, the non-linear effect becomes more effective, especially overcoming the damping effect due to the cosmic expansion. The inflaton fragmentation can have phenomenological implications including the generation of gravitational waves, and spatially localized reheating, which essentially come from the fact the inflaton dominates the universe after the inflation. We will discuss these issues in the later sections.

We first begin with the linear analysis of the growth of the inhomogeneous modes, where in particular we estimate the threshold of $F$ for the instability to overcome the cosmic expansion (Sec. II), and next present a full non-linear analysis from numerical lattice simulations (Sec. IV). We comment on the observational consequences of oscillons in Sec. V. The final section (Sec. VI) is devoted to conclusions. We include in App. A a review of the I-ball solution.

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1 See [1] for discussion on theoretical lower bound on the value of $F$.
2 As we will see later, the actual threshold is estimated as a somewhat smaller value: $F \lesssim O(0.1)M_{pl}$, with $M_{pl} = (8\pi)^{-1/2}m_{pl} \approx 2.44 \times 10^{18}\text{GeV}$ the reduced Planck mass, which we will mainly use throughout this paper.
II. MODEL

In this section let us summarize salient features of the pure natural inflation \[\text{[11]}.\]

The potential \(V(\phi)\) for the inflaton \(\phi\) is given by\(^3\)

\[
V(\phi) = M^4 \left[1 - \left(1 + \left(\frac{\phi}{F}\right)^2\right)^{-p}\right].
\]

(1)

Here \(M\) determines the overall size of the potential, and \(F\) is the effective decay constant mentioned above. The power \(p\) is included here to parametrize our ignorance of the strongly-coupled gauge theories, and can be determined by improved lattice gauge theory computations in the near future \[\text{[17]}\]. For the analysis of this paper the precise value of \(p \sim O(1 - 10)\) is not important, and in this paper we use the value \(p = 3\), as suggested by holographic computations in the large \(N\) limit \[\text{[19]}\].

The potential \[\text{[1]}\] is quadratic \(V(\phi) \sim \frac{1}{2} m^2 \phi^2\) near the bottom of the potential \(\phi \sim 0\), where the value of the inflaton mass \(m\) is constrained by the observed size of the primordial density perturbations:

\[
m = \sqrt{2p} M^2 \sim 10^{-5} M_{\text{pl}},
\]

(2)

where \(M_{\text{pl}}\) is the reduced Planck mass. In the opposite limit of \(\phi\) large, the potential becomes flat and has a plateau, an attractive feature in light of the observed values of \(r\) and \(n_s\) by Planck \[\text{[14]}\] and BICEP/Keck Array \[\text{[5]}\]. This plateau, which becomes more important as \(F\) becomes smaller, will be the origin of the oscillons discussed in this paper.

III. LINEAR INSTABILITY ANALYSIS

Let us study the growth of the spatial inhomogeneities of the inflaton, first at the linear level. The question is if the inhomogeneities as produced by the resonance effect can be strong enough to overcome the cosmic expansion.

We divide the inflaton field \(\phi\) into the background \(\phi_0(t)\) (independent of spatial coordinates \(x\)) and the fluctuation \(\delta \phi(x, t) = \phi(x, t) - \phi_0(t)\). Their equations of motion are given as follows:

\[
\ddot{\phi}_0 + 3H \dot{\phi}_0 + V'(\phi_0) = 0,
\]

(3)

\[
\ddot{\delta \phi}_k + 3H \dot{\delta \phi}_k + \left[\frac{k^2}{a^2} + V''(\phi_0)\right] \delta \phi_k = 0,
\]

(4)

where \(\delta \phi_k(t)\) is the Fourier modes for the fluctuation \(\delta \phi(x, t)\), and the dot denotes the derivative with respect to the time \(t\). If \(\phi\) is small enough, the background is dominated by harmonic oscillation:

\[
\phi_0 \simeq \Phi_0 \cos(mt),
\]

(5)

where \(\Phi_0\) is a constant and \(m = \sqrt{6M^2/F}\) is the inflaton mass (see Eq. \[\text{[2]}\], recall we have chosen \(p = 3\)). Then Eq. \[\text{[4]}\] for the potential of Eq. \[\text{[1]}\] is rewritten as

\[
\ddot{\delta \phi}_k + 3H \dot{\delta \phi}_k + \left[\frac{k^2}{a^2} + m^2 - 6m^2 \left(\frac{\Phi_0}{F}\right)^2\right] \delta \phi_k \simeq 0,
\]

(6)

where we assumed \(\Phi_0 \lesssim F\) and expanded the potential up to a quartic order.

If we temporarily ignore the cosmic expansion, this becomes the Mathieu equation, which for \(\Phi_0 \lesssim F\) has a narrow instability band at

\[
\frac{k^2}{a^2} + m^2 - 6m^2 \left(\frac{\Phi_0}{F}\right)^2 \simeq m^2,
\]

(7)

hence

\[
\frac{k}{ma} \simeq \frac{\sqrt{6}\Phi_0}{F} \lesssim O(1).
\]

(8)

The maximal growth rate of the instability \(\mu_{\text{max}}\) is given as

\[
\mu_{\text{max}} \simeq \frac{3m}{2} \left(\frac{\Phi_0}{F}\right)^2,
\]

(9)

from the analysis of the Mathieu equation \[\text{[20]}\].

Of course, we need to take into account the cosmic expansion, which quickly damps \(\phi_0\). To overcome this damping effect, we need to have sufficiently large growth rate, which can be attained by a small \(F\), and we should obtain an upper bound for such a value of \(F\). The condition for the instability to become non-linear gives

\[
1 \sim \frac{\delta \phi}{\dot{\phi}} \simeq e^{\mu t} \times \frac{\delta \phi_{\text{initial}}}{\dot{\phi}_{\text{initial}}},
\]

(10)

where \(\mu\) is the growth rate. If we use the observational value for initial fluctuation \(\delta \phi_{\text{initial}}/\dot{\phi}_{\text{initial}} \sim O(10^{-5})\), Eq. \[\text{[10]}\] leads to the following condition:

\[
\frac{\mu}{H} \sim 10.
\]

(11)

Since the inflaton at the end of inflation \(\phi_{\text{end}}\) is not that different from \(F\) in this model, we set \(\Phi_0 \sim \phi_{\text{end}} \sim F\). Then, using Eqs. \[\text{[9]}\], \[\text{[11]}\] and also \(H \sim m\Phi_0/M_{\text{pl}}\), we find that the maximal \(F\) for strong resonance is about \(O(0.1)M_{\text{pl}}\).

\(^3\) This potential is the same potential studied in Ref. \[\text{[14]}\], which reference also studied oscillons for \(p < 0\). We emphasize, however, the potential of Eq. \[\text{[1]}\] here arises dynamically from strong-coupling effects of the Yang-Mills gauge field, and predicts a specific sign \(p > 0\).
That the strong resonance occurs for $F \lesssim 0.1M_{pl}$ can also be checked independently by numerically solving Eq. (4). In Fig. 1, we show an example of instability band for $F = 0.08M_{pl}$. Here we set the initial amplitude of the background oscillation as $\phi_i = \phi_{\text{end}} \simeq 0.11M_{pl}$. We note that the instability band is in good agreement with our estimation given in Eq. (8).

IV. NON-LINEAR ANALYSIS: LATTICE SIMULATIONS

We expect that the small inhomogeneities produced in the linear analysis of the previous section will grow into oscillons—the domination of the quadratic piece of the inflaton potential, when combined with the shallower higher order corrections, will eventually help to stabilize fragmented modes of the inflaton into quasi-stable objects, namely oscillons.

Since this requires us to solve the non-linear dynamics, we performed lattice simulations by modifying the program LatticeEasy [21], which is a C++ program designed for simulating the evolution of the scalar fields in an expanding universe with the homogeneous FRW metric. We integrate the equation of motion using the leapfrog method of second order, and approximate the spatial derivatives through the Central-Difference formulas of second order. We confirmed that the numerical error for our lattice (and time) spacing, which will be presented below, did not affect the order-estimation of the threshold value for the oscillon formation, and the formation time, etc., by performing additional simulations with smaller spacing. We set the initial amplitude of the background oscillation as $\phi_i = \phi_{\text{end}} \simeq 0.11M_{pl}$, which is rather a conservative choice as we mentioned in the previous section. We set the initial scale factor $a$ as unity, and defined the Hubble parameter as

$$H = \sqrt{\frac{\langle \rho \rangle}{3M_{pl}^2}},$$

where $\langle \cdot \rangle$ is the spatial average and $\rho$ denotes the energy density of the inflaton:

$$\rho = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} a^2 (\nabla \phi)^2 + V(\phi).$$

We estimated the formation time as $t_{\text{form}} \sim \mathcal{O}(100)m^{-1}$. The reheating temperature without the oscillons is estimated as $T_R \sim 10^9$ GeV [1], whose time scale is much later than the formation time, hence the branching ratio into the other sectors during the formation is negligible.

In Table I we present the parameters used in the simulations, including the number of grids, box size, and time step, which are denoted as $N_{\text{grid}}, L, \Delta t$ respectively. The rescaled program variables are defined as follows:

$$\phi_{pr} \equiv \phi/\phi_{\text{end}}, \quad V_{pr} \equiv V/(m\phi_{\text{end}})^2, \quad t_{pr} \equiv mt, \quad x_{pr} \equiv mx.$$ 

In Fig. 2 we illustrate the result of 1D simulations, where we plot energy density $\rho$ after the fragmentation. We estimated the formation time as $t_{\text{form}} \sim \mathcal{O}(100)m^{-1}$. The reheating temperature without the oscillons is estimated as $T_R \sim 10^9$ GeV [1], whose time scale is much later than the formation time, hence the branching ratio into the other sectors during the formation is negligible.

We also present the results of 2D and 3D simulations in Figures 3 and 4 respectively. In Fig. 3 we plot the iso-surfaces of the energy density at $a^3\rho = 3.4 \times 10^5m^4$ and $a^3\rho = 1.1 \times 10^8m^4$, where we can indeed identify the localized spherical oscillons.

It is natural to identify the profile of the resulting oscillons as I-balls, solutions minimizing energy under the conserving law of the adiabatic invariant (see Sec. A for summary). In particular the energy density as derived from 1D simulations is in good agreement with the analytic profile of Eq. (10), confirming the identification

$\text{TABLE I:}$ The parameters used in the simulations including the number of grids, box size, and time step.

| $N_{\text{grid}}$ | $Lm$ | $m\Delta t$ |
|------------------|------|-------------|
| 1D               | 1024 | 50          |
| 2D               | 3$2^2$ | 40      | 0.11       |
| 3D               | 128$^3$ | 30      | 0.14       |

$^4$ Strictly speaking, the amplitude of oscillation is larger than $\phi_{\text{end}}$, since the velocity is non-zero: $\dot{\phi}_{\text{end}} \neq 0$. However, $\phi$ damps to $\phi_{\text{end}}$ due to the expansion in a short time scale compared to that of the fragmentation of $\phi$: $\mathcal{O}(100)m^{-1}$. Since the fragmentation is more efficient for a larger amplitude, our choice of initial condition is rather conservative.

$^5$ Since the analytic profiles are valid for Minkowski spacetime, we compared the numerical results at rather short time, for which the cosmic expansion is negligible.
A. Gravitational Waves

Inflaton fragmentation into oscillons can cause back-reaction to the gravitational modes, and hence produce the gravitational waves (GWs) through the anisotropic stress tensor. It is therefore an interesting question to compute the frequency $f$ of the such GWs, and see if it is in the currently observable range (see \[22–24\]) for related recent discussions.

Since $O(1-10)$ I-balls are formed per horizon, a naive expectation is that the frequency corresponds to the Hubble scale:

$$f \sim H_{\text{end}} \sim \frac{M^2}{M_{\text{pl}}} \sim 10^{-5} M_{\text{pl}} \left(\frac{F}{M_{\text{pl}}}\right) \lesssim 10^{37} \text{ Hz}, \quad (15)$$

where we used the relation $F \lesssim O(M_{\text{pl}})$ \[1\]. Then, it is redshifted until present, which can be estimated as follows:

$$f_0 = \frac{a_{\text{end}}}{a_0} f = \frac{a_{\text{end}}}{a_R} \frac{a_R}{a_0} f \sim 5.5 \times 10^{-32} \left(\frac{M_{\text{pl}}}{\Gamma}\right)^{1/2} \left(\frac{H_{\text{end}}}{\Gamma}\right)^{-2/3} H_{\text{end}}, \quad (16)$$

$$\lesssim 10^8 \text{ Hz} \times \left(\frac{\Gamma}{10^{-5} M_{\text{pl}}}\right)^{1/6},$$

where $\Gamma$ is the decay rate of inflaton (oscillon) and $a_0$ and $a_R$ represent the scale factors at present and the time of reheating, respectively, and we used $a_R/a_0 \sim 5.5 \times 10^{-32} \sqrt{M_{\text{pl}}/\Gamma}$ and $a_R/a_{\text{end}} \sim (\rho_R/\rho_{\text{end}})^{-1/3} \sim (H_{\text{end}}/\Gamma)^{2/3}$. This is an extremely high frequency compared to the currently observable frequency range \[25–27\] (this result is consistent with those of Ref. \[22\], which discussed inflaton potential \[1\] for $p < 0$). Since the result is not highly sensitive to the inflation scale: $f_0 \propto H_{\text{end}}^{1/3}$, lowering the frequency by changing the inflation scale will be difficult. Hence we conclude that it is quite generally difficult to produce the GWs in the observable range, through the inflaton fragmentation.
B. Reheating

Oscillon formation may alter the reheating process after inflation. If the oscillons are formed, the universe is reheated by the decay of the spatially-localized oscillons—only after the locally generated radiation diffusionally diffuses throughout the space, we get the spatially-homogeneous radiation era, as is often assumed in the standard treatment of reheating.

Let us here define the reheating temperature $T_R$ to be the temperature when such spatially-homogeneous radiation is first created. We find that there is an upper bound on this temperature.

To derive such a constraint, let us assume that we indeed have reached an era of homogeneous radiations. Then the diffusion length $l_d$ of the photons at that time should be larger than the distance $l_{o.o}$ between the localized oscillons:

$$l_d \gtrsim l_{o.o}. \quad (17)$$

The diffusion length $l_d$ during the time $t \sim H^{-1}$ is estimated to be

$$l_d \sim l_{mf} \sqrt{N_i} \sim \frac{1}{\sqrt{n\sigma H}} \sim \frac{1}{\sqrt{\alpha^2 TH}}. \quad (18)$$

Here $l_{mf} \equiv 1/n\sigma$ is the mean free path, with $n \sim T^3$ the number density of the plasma and $\sigma$ the cross section, which we assume as $\sigma \sim \alpha^2/T^2$, with $\alpha$ the structure constant for the radiation. $N_i \equiv t/l_{mf}$ is the number of collision during $t$. The distance $l_{o.o}$ between oscillons is given by

$$l_{o.o} \sim N_o^{-1/3} H^{-1} \sim N_o^{-1/3} \left( \frac{H}{H_f} \right)^{1/3} H^{-1}, \quad (19)$$

where $N_o$ denotes the total number of oscillons in the horizon, and the index $f$ refers to the quantities at the time of formation of the oscillons. Plugging in Eqs. (18) and (19) into Eq. (17) and using $H \sim T_R^2/M_{pl}$, we thus obtain the upper bound on the reheating temperature:

$$T_R \lesssim 10^8 \text{ GeV} \alpha^{-6} N_{o,f} \left( \frac{H_f}{10^{13} \text{ GeV}} \right)^2. \quad (20)$$

Now, $H_f$ has the same order of magnitude $H_{end}$, as expected from the flatness of the potential (we have also checked this in the numerical simulations). Then, using $H_{end} \sim M^2/M_{pl} \sim m_F/M_{pl}$ (recall Eq. (2)), Eq. (20) reduces to

$$T_R \lesssim 10^8 \text{ GeV} \alpha^{-6} N_{o,f} \left( \frac{m_\phi}{10^{13} \text{ GeV}} \right)^2 \left( \frac{F}{M_{pl}} \right)^2. \quad (21)$$

Hence, for small $F$ and for $\alpha \sim 1$, the upper bound on the temperature for the usual radiation domination becomes severe: For instance, $F \lesssim 0.01 M_{pl}$ gives a rather stringent constraint $T_R \lesssim 10^4 \text{ GeV}$.\(^6\)

We emphasize again that the value of the reheating temperature $T_R$ as defined here is the temperature at the onset of the standard homogeneous radiation era; the cosmological scenario before this time can potentially be altered significantly from the standard scenarios, due to the localization of the radiation originating from oscillons. For example, the temperature at the energy core, which will coincide with the location of oscillons, may be higher than predicted by usual perturbative decay rate of the inflaton, which in turn may lead to localized thermal leptogenesis. We will discuss these phenomenological consequences in detail in the upcoming paper [28].

VI. CONCLUSIONS

We studied inflaton fragmentation in a recently proposed inflation model, called a pure natural inflation. The small model parameter $F$ gives the flatness to the potential, which drives strong resonance. We found that for $F \lesssim 0.1 M_{pl}$, the resonance becomes strong enough to overcome the cosmic expansion, and the inflaton fragments into localized quasi-stable objects called oscillons/I-balls. We confirmed the agreement with the analytical profiles.

We pointed out that the reheating through the oscillon decay may localize the radiation, which must sufficiently diffuse in order to realize the radiation era in the usual sense. We gave an upper bound on the temperature of the beginning of the radiation era by requiring the sufficiently long diffusion length at that temperature. We found this upper bound for the standard homogeneous radiation domination can be stringent, e.g., we obtained $T_R \lesssim 10^4 \text{ GeV}$ for $F \lesssim 0.01 M_{pl}$. The localization of the radiation can be important since the high temperature phenomena such as thermal leptogenesis may occur in a localized manner, which we will pursue in the future work.

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We would like to thank Fuminori Hasegawa and Yasunori Nomura for helpful discussion. This work is supported by WPI Initiative, MEXT, Japan. MK is supported in part by MEXT KAKENHI Grant No. 6 The decay into the other sectors is negligible during the formation of the oscillons, as mentioned in the previous section. However, they may decay through the self-interactions, before the decay into radiations. The lifetime of the oscillons is not well-understood in the literature. We confirmed that the oscillons are stable up to $t \sim O(10^4) m^{-1}$ in the 2D simulation with the same setting, and here we simply assume that they are also persistent until the low reheating.
and we used the time-averaged quantities, which are given as

\[ \overline{\dot{\phi}^2} \approx \frac{1}{2} \Phi^2, \quad (A5) \]

\[ \overline{\ddot{\phi}^2} \approx \frac{1}{2} \omega^2 \Phi^2, \quad (A6) \]

\[ V(\phi) \approx \frac{3M^4}{2F^2} \Phi^2 - \frac{6M^4}{8F^4} \Phi^4 \]

\[ \approx \frac{1}{4} \omega^2 \Phi^2 - \frac{6M^4}{8F^4} \Phi^4. \quad (A7) \]

Note that we approximated the potential by keeping terms up to quartic order in the field \( \phi \). By varying Eq. (A4) with respect to \( \Phi \), we obtain the following equation for the amplitude

\[ \Delta \Phi - \left(2 - \frac{\ddot{\omega}}{\omega}\right) \omega^2 \Phi + \frac{18M^4}{F^4} \Phi^3 = 0, \quad (A8) \]

which defines I-balls.

For 1+1 dimension we find the following analytic solution for 1+1 dimension:

\[ \Phi(x) = \Phi(0) \text{sech} \left[ \left(\sqrt{3M/F}\right) \Phi(0)x \right], \quad (A9) \]

where we used the expression for \( \omega \approx m \) given in Eq. 2 (with \( p = 3 \)), and \( \ddot{\omega} \) is traded for a constant \( \dot{\Phi}(0) \).

This profile is consistent with that obtained by solving the equation of motion in small amplitude approximation \([29, 30] \), whose method is especially useful when the potential is asymmetric and the anharmonic correction becomes important \([17, 24] \). In Sec. IV we compare this analytic profile to that obtained from 1D lattice simulations, which gives a strong evidence that oscillons are identified with I-balls.

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[1] Y. Nomura, T. Watari, and M. Yamazaki (2017), 1706.08522.
[2] K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. Lett. 65, 3233 (1990).
[3] F. C. Adams, J. R. Bond, K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. D47, 426 (1993), hep-ph/9207245.
[4] P. A. R. Ade et al. (Planck), Astron. Astrophys. 594, A20 (2016), 1502.02114.
[5] P. A. R. Ade et al. (BICEP2, Keck Array), Phys. Rev. Lett. 116, 031302 (2016), 1510.09217.
[6] P. Creminelli, D. L. López Nacir, M. Simonović, G. Trisvan, and M. Zaldarriaga, JCAP 1511, 031 (2015), 1502.01983.
[7] Y. Nomura and M. Yamazaki, to appear.
[8] I. L. Bogolyubsky and V. G. Makhankov, Pisma Zh. Eksp. Teor. Fiz. 24, 15 (1976).
[9] M. Gleiser, Phys. Rev. D49, 2978 (1994), hep-ph/9308279.
[10] E. J. Copeland, M. Gleiser, and H. R. Muller, Phys. Rev. D52, 1920 (1995), hep-ph/9503217.
[11] S. Kasuya, M. Kawasaki, and F. Takahashi, Phys. Lett. B559, 99 (2003), hep-ph/0209358.
[12] M. A. Amin, R. Easther, and H. Finkel, JCAP 1012, 001 (2010), 1009.2505.
[13] M. Gleiser, N. Graham, and N. Stamatopoulos, Phys. Rev. D83, 096010 (2011), 1103.1911.
[14] M. A. Amin, R. Easther, H. Finkel, R. Flauger, and M. P. Hertzberg, Phys. Rev. Lett. 108, 241302 (2012), 1106.3335.
[15] K. D. Lozanov and M. A. Amin, Phys. Rev. D97, 023533 (2018), 1710.06851.
[16] P. Salmi and M. Hindmarsh, Phys. Rev. D85, 085033 (2012), 1201.1934.
[17] F. Hasegawa and J.-P. Hong (2017), 1710.07487.
[18] N. Takeda and Y. Watanabe, Phys. Rev. D90, 023519 (2014), 1405.3830.
[19] S. Dubovsky, A. Lawrence, and M. M. Roberts, JHEP 02, 053 (2012), 1105.3740.
[20] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. Lett. 73, 3195 (1994), hep-th/9405187.
[21] G. N. Felder, Comput. Phys. Commun. 179, 604 (2008), 0712.0813.
[22] S.-Y. Zhou, E. J. Copeland, R. Easther, H. Finkel, Z.-G. Mou, and P. M. Saffin, JHEP 10, 026 (2013), 1304.6094.
[23] S. Antusch, F. Cefala, and S. Orani, Phys. Rev. Lett. 118, 011303 (2017), 1607.01314.
[24] S. Antusch, F. Cefala, S. Krippendorf, F. Muia, S. Orani, and F. Quevedo (2017), 1708.08922.
[25] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 116, 061102 (2016).
[26] B. S. Sathyaprakash and B. F. Schutz, Living Rev. Rel. 12, 2 (2009), 0903.0338.
[27] C. J. Moore, R. H. Cole, and C. P. L. Berry, Class. Quant. Grav. 32, 015014 (2015), 1408.0740.
[28] F. Hasegawa, J.-P. Hong, M. Kawasaki, and M. Yamazaki, in progress.
[29] J. P. Boyd, *Weakly nonlocal solitary waves and beyond-all-orders asymptotics: generalized solitons and hyper-asymptotic perturbation theory* (Dordrecht ; Boston : Kluwer Academic Publishers, 1998), ISBN 0792350723 (hardcover ; alk. paper), includes index.
[30] G. Fodor, P. Forgacs, Z. Horvath, and A. Lukacs, Phys. Rev. D78, 025003 (2008), 0802.3525.