From DeWitt initial condition to cosmological quantum entanglement

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Abstract

At the classical level, a linear dilaton action offers an eternal inflation evolution governed by the unified (cosmological constant plus radiation) equation of state $\rho - 3P = 4\Lambda$. At the quantum mechanical mini-superspace level, a ‘two-particle’ variant of the no-boundary proposal, notably ‘one-particle’ energy dependent, is encountered. While a (gravity–anti-gravity) GaG-odd wave function can only host a weak Big Bang (BB) boundary condition, albeit for any $k$, a strong BB boundary condition requires a GaG-even entangled wave function, and singles out $k = 0$ flat space. The locally maximal values for the cosmological scale factor $a$ and the inverse Newton constant $\phi$ form a grid $\{a^2, \phi\} \sim \sqrt{4n_1 + 1} \pm \sqrt{4n_2 + 1}$.

Keywords: mini superspace, dilaton, quantum cosmology, quantum entanglement, DeWitt initial condition

Introduction

The idea that the Newton constant may have changed its sign during the very early universe is not new. Originally, it was presented [1] as a possible consequence of the interaction of geometry with electro/nuclear matter. With the early universe in mind, it has been recently suggested [2] that anti-gravity may have played a crucial role in taming the classical Big Bang (BB) singularity by curing the geodesic incompleteness problem. A central ingredient in this theory is the underlying conformal Weyl symmetry (see [3] for remarks). The fact is, however, that anti-gravity can be regarded as a natural companion of gravity in almost any scalar-tensor theory [4]. Such an intimate relation can even be elevated to the level of a gravity ↔ anti-gravity (GaG) discrete symmetry [5]. The latter originally includes a metric change of sign, which is hereby traded for a cosmological scale factor change of sign, causing our

\[\ldots\]
variant GaG symmetry to make its appearance only at the level of the mini superspace (and the classical equations of motion).

Starting from a linear dilaton gravity Lagrangian, we first establish the classical mirror cosmology, and demonstrate how spatially symmetric mirror-gravity deflation from anti-gravity. Reflecting the fact that the Ricci scalar serves as a constant of motion, the emerging classical equations of state signal the presence of a mandatory (not put ad hoc) radiation companion to the cosmological constant. In the quantum mechanical picture, at least within the framework of the mini-superspace model, we encounter a ‘two-particle’ variant of the Hartle–Hawking no-boundary proposal [6]. While the Hamiltonian constraint is fully respected, the scheme still allows for a ‘one-particle’ energy dependence, highly resembling Vilenkin’s tunnelling wave function approach [7] with radiation included.

Quantum mechanically, the BB can never stay out of the game, but here, within the framework of a singularity-free boundary proposal, it is accompanied by a non-trivial interplay. While a GaG-odd/BB-even entangled wave function can only host a weak DeWitt boundary condition [8] at the BB, albeit for any spacial curvature $k$, a strong DeWitt boundary condition requires a GaG-even/BB-odd entangled wave function, and furthermore singles out the $k = 0$ flat space option. The classically problematic (under anisotropic fluctuations [9]) GaG transition does not seem to leave pathological imprints on the outgoing Wheeler–DeWitt (WDW) wave function. The highlight of our analysis is the emergence of a grid structure for the locally most probable cosmological scale factor and the inverse Newton constant.

Dilaton cosmology preliminaries

In its simplest form, devoid of a Brans–Dicke [10] kinetic term ($\omega_{BD} = 0$), dilation gravity is formulated by means of the action principle

$$I = -\int (\phi R + V(\phi)) \sqrt{-g} \, d^4x.$$  \hspace{1cm} (1)

The gravity ($\phi > 0$) anti-gravity ($\phi < 0$) transition, depending on the potential, is not necessarily singular. The GR limit, when it exists [11], is associated with $\phi(x)$ developing a positive vacuum expectation value $(8\pi G)^{-1}$. The cosmological field equations which govern the Friedmann–Lemaître–Robertson–Walker (FLRW) metric

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$  \hspace{1cm} (2)

can be compactly arranged into

$$\frac{\dot{a}}{a} \phi + \frac{\ddot{a}^2 + k}{a^2} \phi = \frac{1}{6} V(\phi), \hspace{1cm} (3a)$$

$$\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + k}{a^2} = \frac{1}{6} V'(\phi). \hspace{1cm} (3b)$$

A deeper physical insight can be gained by reconstructing (by substituting equations $(3a)$ and $(3b)$ into the time derivative of equation $(3a)$) the Klein–Gordon equation

$$\ddot{\phi} + 3\frac{\dot{a}}{a} \dot{\phi} + V'_{\text{eff}}(\phi) = 0,$$  \hspace{1cm} (4a)
\[ V_{\text{eff}}(\phi) = \frac{1}{3} \int (\phi V'(\phi) - 2V(\phi))d\phi. \]  

\( V_{\text{eff}}(\phi) \) is not sensitive to \( V(\phi) \rightarrow V(\phi) + \lambda \phi^2 \). The inclusion of a Brans–Dicke kinetic term would only modify its scale \( 1/3 \rightarrow 1/(3 + 2\omega_{BD}) \).

**Inflation/radiation interplay**

Let our starting point be a particular dilaton gravity theory characterized by a linear scalar potential, namely

\[ V(\phi) = 4A\phi, \quad A > 0. \]  

(5)

Note that such a dilaton theory does not have an analogous \( f(R) \) gravity theory [12] because the dictionary usually used to extract \( f(R) \), namely \( R + V'(\phi) = 0 \), turns a constraint in this case. Associated with the linear potential equation (5) is the quadratic ‘wrong sign’ (no ground state) effective potential

\[ V_{\text{eff}}(\phi) = -\frac{2}{3}A\phi^2; \]  

(6)

which governs the Klein–Gordon evolution of the dilaton field. It constitutes a vital ingredient for a realistic spontaneously induced GR theory (for comparison, Starobinsky’s \( R + R^2 \) type gravity [13] comes with \( V_{\text{eff}} \sim + (\phi - \nu)^2 \)).

The scale factor evolution can be directly derived from the now \( \phi \)-independent nonlinear differential equation (3b). The bouncing solution

\[ a^2(t) = \frac{3k}{2A} = A^2 \cosh aot, \]  

\[ a(t)\phi(t) = B^2 \sinh aot, \quad a^2 = \frac{4}{3}A, \]  

(7a)

(7b)

attracts particular attention because of the special case \( A^2 = \frac{3k}{2A} \) which has been used by Hartle–Hawking in the no-boundary proposal [6] formulation (see [14] for stringy and loop bouncing solutions). The other solution, characterized by a BB singularity, is obtained by switching the hyperbolic trigonometric functions. While the Ricci scalar serves as a constant of motion

\[ R = -4A, \]  

(8)

all other curvature scalars are non-singular solely for the bouncing solution provided

\[ a_0^2 = \frac{3k}{2A} + A^2 > 0. \]

It is convenient to recast the gravitational Jordan frame field equations into the standard general relativistic format \( R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu} \), squeezing all floating around pieces into an effective energy/momentum tensor on the rhs. The corresponding FLRW cosmology would then involve an effective energy density \( \rho = 3(H^2 + k/a^2) \) accompanied by an effective pressure \( P \). Together, these two close upon the energy/momentum conservation law

\[ \dot{\rho} + 3H(\rho + P) = 0. \]

A closer inspection reveals that the entire cosmic evolution, from the BB to the de-Sitter limit, is governed by the unified equation of state

\[ \rho - 3P = 4A, \]  

(9)

being a direct consequence of the linear potential dilaton field equation. The differential version of the equation of state, namely \( d\rho - 3dP = 0 \), signals the presence of an evolving
radiation component. To be specific, associated with the scale factor equation (7a) is the total energy density

$$\rho(t) = \Lambda \left(1 + \frac{9k^2}{4A^2} - A^4 \right)$$

(10)

As long as the underlying Lagrangian is GaG-odd, that is $V(-\phi) = -V(\phi)$, the corresponding effective potential is by construction GaG-even $V_{\text{eff}}(-\phi) = V_{\text{eff}}(\phi)$. In turn, the equations of motion stay intact under $\phi \rightarrow -\phi$.

Such a symmetry, unaffected by the possible presence of a Brans–Dicke kinetic term, suggests itself as the tool for synchronizing the BB creation with the gravity–anti-gravity transition. With this in mind, mirror gravity is perhaps a better name for the GaG antisymmetric gravity companion. After all, an optical/electromagnetic and even a gravitational [15] mirror image is a reflected duplication of an object that appears identical but reversed.

**Bifurcated mini superspace**

Mirror gravity admits an elegant diagonalization at the mini-superspace level, where the GaG odd linear potential equation (5) leaves a distinctive fingerprint. Following the standard procedure of integrating out over the maximally symmetric space, that is

$$\int \mathcal{L} \sqrt{-g} \, dt \, d^3x \rightarrow \int \mathcal{L}_{\text{mini}} \, dt,$$

(11)

and up to a total derivative, the mini-superspace Lagrangian is given by

$$\mathcal{L}_{\text{mini}} = -\frac{6}{n} \left( a \phi a^2 + a^2 \dot{a} \phi \right) + n \phi \left( 6ka - 4a^3 \Lambda \right).$$

(12)

Note that the lapse function $n(t)$ has been revived to keep track of the underlying diffeomorphism. By supplementing $\phi \rightarrow -\phi$ by $a \rightarrow -a$ the GaG symmetry has made its appearance at the mini-superspace level. A new pair of GaG parity conjugate variables, namely

$$x_\pm = \frac{a}{2} (a \pm \phi),$$

(13)

is then invoked to diagonalize the quadratic kinetic term ($\phi$ is dimensionless in our $8\pi G = 1$ notations). Consequently, the above GaG-odd mini-superspace Lagrangian, with $\phi \rightarrow -\phi$ (or $a \rightarrow -a$) reading now $x_+ \leftrightarrow x_-$, gets decomposed into $\mathcal{L} = \mathcal{L}_+ - \mathcal{L}_-$. Note that such a decomposition is not possible in the presence of a kinetic Brans–Dicke term $\omega_{\text{ND}} \neq 0$. Using the more convenient Hamiltonian language, we face

$$\mathcal{H}_{\text{mini}} = n \left( \mathcal{H}(x_+, p_+) - \mathcal{H}(x_-, p_-) \right).$$

(14)

The upside-down harmonic oscillator building block

$$\mathcal{H}(x, p) = \frac{p^2}{12} + U(x), \ U(x) = 6kx - 4\Lambda x^2,$$

(15)
resembles in a way the Hartle–Hawking Hamiltonian
\[ H_{HH}(a, p) \propto \frac{p^2}{4} + ka^2 - \frac{1}{3} \Lambda a^4. \] (16)

The two almost decoupled ‘one-particle’ physical systems communicate with each other solely by means of the overall Hamiltonian constraint \( H_{\text{min}} = 0 \). Their associated conserved energies, \( E_+ \) and \( E_- \) respectively, must therefore obey \( E_+ - E_- = 0 \). Thus, in contrast with the Hartle–Hawking prescription, a bit closer to the Vilenkin approach, a non-vanishing energy parameter \( E_+ = E_- = E \) makes an appearance via
\[ H(x_+, p_+) = H(x_-, p_-) = E. \] (17)

The corresponding mechanical analogue involves two identical non-interacting point particles, carrying the one and the same energy \( E \), whose one-dimensional motion is governed by the \( U(x) \) potential.

The most general classical solution of the associated field equations is given by
\[ \begin{bmatrix} x_+(t) \\ x_-(t) \end{bmatrix} = \frac{9k^2}{4\Lambda} + \begin{bmatrix} c_+^+ \\ c_-^+ \end{bmatrix} \begin{bmatrix} e^{+\omega t} \\ e^{-\omega t} \end{bmatrix}. \] (18)

While fixing the ‘one-particle’ energy
\[ E = \frac{9k^2}{4\Lambda} - 16\Lambda c_+^+ c_-^- = \frac{9k^2}{4\Lambda} - 16\Lambda c_+^- c_-^+ \] (19)
counts for two relations among the various \( c \)-parameters, fixing the origin of time is translated into scaling \( c_+^+ \) and inversely scaling \( c_-^- \), thus leaving the \( c_+^-/c_-^+ \) ratio as the second physical parameter.

The bouncing solution specified by equations (7a) and (7b) is associated with an energy level
\[ E = A \left( \frac{9k^2}{4\Lambda^2} - A^4 \right) + \Lambda B^4. \] (20)

The first term is immediately recognized as the strength of the radiation energy density equation (10). A positive radiation energy density guaranties \( E \geq 0 \). If furthermore \( A^2 \geq B^2 \), both point particles, each carrying energy \( E < U_c = \frac{9k^2}{4\Lambda} \), move from right to left towards the

![Figure 1. The motion of two identical non-interacting point particles, carrying a total energy \( E \) each, is governed by the one-particle potential \( U(x) \). While their center of mass location represents the cosmological scale factor squared \( a^2(t) \), their crossing point marks the gravity–anti-gravity transition.](image)
potential hill. As they hit the turning point, the first at some time \( t_1 \) and the other a bit later at \( t_2 \), they must cross each other in between. Such a crossover, with \( x_+ - x_- \) changing sign, signals the GaG transition. If on the other hand \( B^2 > A^2 \), the two point particles, each carrying now a larger energy \( E > U_c \), move in opposite directions above the potential hill. Like before, their unavoidable crossing point fixes the minimal \( a^2_0 \) at the GaG transition. In both cases, the center of mass location stays positive definite, causing no classical BB singularity.

The fact that the energy formula is only sensitive to \( A^4 \), rather than to \( A^2 \), implies that the two classically disconnected solutions \( a^2_0(t) = \frac{3k}{2\lambda} \pm A^2 \cosh \omega t \) share the one and the same energy \( 0 < E < U_c \). The \( a_+ (t) \) branch describes a BB created baby universe that does not have a chance to grow forever. Quantum mechanically, however, the story is by far more interesting as the Euclidean sector \( a_E(t) \) connects the embryonic and the expanding Lorentzian regions to eventually constitute the piecewise glued sharp edge manifold depicted in figure 2. The existence of an embryonic stage in the quantum history was originally introduced by Vilenkin, who supplemented the Hartle Hawking proposal by an ad hoc radiation energy density. At any rate, the quantum mechanical picture is far richer. The \( E \) degree of freedom allows to (i) go beyond the Euclidean regime, (ii) let \( k \leq 0 \) into the game, and most importantly, (iii) consistently generalize the DeWitt boundary condition.

### Quantum entanglement and a quantum grid

The Hamiltonian constraint destroys the explicit time dependence of the universe wave function. For a time independent wave eigenfunction, of the separate variables form \( \psi_E(x_+, x_-) = \psi_E(x_+)\psi_E(x_-) \), the corresponding WDW equation splits into

\[
\left[ -\frac{\hbar^2}{12} \frac{\partial^2}{\partial x^2} + U(x) - E \right] \psi_E(x) = 0 \quad \text{for} \quad x = x_\pm.
\]

Each individual differential equation is not a legitimate Schrödinger equation by its own rights since the (absence of) time evolution is controlled by the full system.

The one-particle Schrödinger equation admits two independent solutions. They are

\[
R(x) = D_{-\frac{i}{2}(1-\omega)} \left[ \frac{2}{\sqrt{\hbar}}(i - 1)(3\Lambda)^{\frac{3}{4}} \left( x - \frac{3k}{4\Lambda} \right) \right].
\]
\[
L(x) = D_{-\frac{1}{2}(1+i)} \left[ \frac{2}{\sqrt{\theta}} (i + 1)(3A)^{\frac{1}{2}} \left( x - \frac{3k}{4A} \right) \right]. \tag{23}
\]

\(D_k(x)\) stands for the Weber parabolic cylinder function, and \(s(E) = \frac{1}{\sqrt{\theta}} \left( E - \frac{9\Delta^2}{4\theta} \right)\). On simplicity grounds we stick to fixed \(E\), but keep in mind wave packet solutions \([16]\) as well. Regarding boundary conditions, primarily invoked to relax the classical BB singularity, two options arise, both leading to cosmological entangled states \([17]\):

(i) *Weak DeWitt initial condition* \(\psi(0, 0) = 0\): as the cosmological scale factor approaches zero, \(\psi \to 0\) for any finite value of \(\phi\). Such a requirement can be naturally embedded, notably for any value of \(k\), and in accord with the global symmetries of the mini-superspace Hamiltonian, within the strong GaG boundary condition \(\psi(x, x) = 0\). Up to a normalization factor, the entangled WDW wave function takes the real by construction form

\[
\Psi_{\rho}(x_+, x_-) = \frac{1}{\sqrt{2}} \left( R(x_+)L(x_-) - R(x_-)L(x_+) \right). \tag{24}
\]

Reflecting the GaG-odd nature of \(\Psi_{\rho}\), the anti-gravity sector can be eliminated. Roughly speaking, the classical analog is the \([a^2(t) \sim \sinh \omega t, \ a(t)\phi(t) \sim \cosh \omega t]-\)type solution. The \(|\Psi_{\rho}|^2\) contour plot is depicted in figure 3.

(ii) *Strong DeWitt initial condition* \(\psi(x, -x) = 0\): here we further require that, as the cosmological scale factor approaches zero, \(\psi \to 0\) for any finite value of \(a\phi\) (infinite \(\phi\) included). Following the above prescription, one constructs the GaG-even BB-odd entangled combination

\[
\Psi_{\sigma}(x_+, x_-) = \frac{1}{\sqrt{2}} \left( R(x_+)L(-x_-) - R(-x_-)L(x_+) \right) \tag{25}
\]
but immediately realizes that this is a solution of the WDW equation only provided $k = 0$ (for which $U(x) = U(x)$). It is remarkable how the strong DeWitt initial condition actually singles out the flat space configuration. Contrary to the previous case, it is now the $a^2 < 0$ sector which can be dismissed. The classical analog here is the \( a^2(t) \sim \cosh \omega t, \quad a(t)\phi(t) \sim \sin \omega t \)-type solution. The \(|\Psi_5|^2\) contour plot is depicted in figure 4.

The first mathematical feature observed is the two-dimensional grid formed by the local maxima of \( |\Psi_{0,2}|^2 \). In what follows we analyze the grid structure of \( \Psi_5 \). We start with the simplest special case $E = 0$, and hence $s(0) = 0$ by virtue of $k = 0$, for which the large-$x_\pm$ behavior in the $x_+ > 0$ region is proportional to

\[
\Psi_5 \propto \sin \left[ \frac{\sqrt{3A}}{\hbar} \left( x_+^2 + x_-^2 \right) \right] + \frac{1}{\sqrt{2}} \cos \left[ \frac{\sqrt{3A}}{\hbar} \left( x_+^2 - x_-^2 \right) \right]
\]

subject to an overall $(x_+x_-)^{-1/2}$ suppression factor. The density of zeroes grows $\sim \sqrt{N}$, thus rapidly approaching the classical regime. The local extrema are located at

\[
x_\pm^2 \approx \frac{\hbar (n_\pm + \frac{1}{4})}{2\sqrt{3A}}, \quad (27)
\]

where $n_\pm$ are non-negative integers. In turn, the locally most probable values for the scale factor and the inverse Newton constant are well approximated by

\[
\left\{ \begin{array}{l}
a^2 \\
a\phi
\end{array} \right\} \approx \frac{\hbar}{2\sqrt{3A}} \left( n_+ + \frac{1}{4} \pm n_- + \frac{1}{4} \right).
\]

Numerically, the highest peak is the closest one to the origin, located at $a_0 = 0.874\sqrt{\hbar}/\Lambda^{-1/4}$ along the GaG axis. In the other $x_+x_- < 0$ region, owing to the different asymptotic behavior
the grid turns one-dimensional, exhibiting an approximated $a^3\phi$-periodicity. Once the energy parameter $E$ is switched on, and hence $s(E) = \frac{E}{2\sqrt{\Lambda}}$, only mild changes are experienced. The major change has to do with a logarithmic modification of the approximate periodicity $\Delta x^2$, that is

$$\Delta x^2 + \frac{s(E) \log 2(3 \Lambda)^{2x}}{4\sqrt{3} \Lambda} = \frac{\hbar}{\sqrt{3} \Lambda}. \quad (29)$$

Finally we note that the most general WdW wave function, subject to the strong/weak DeWitt initial conditions, is of the form \( \Psi_{SW}(E) \int \frac{\Psi}{f(E)} \) involving some weight function $f(E)$.

**Epilog**

Recalling the underlying upside down effective potential $V_{\text{eff}}(\phi) = -\frac{2}{3}\Lambda \phi^2$, the quantum mechanical background associated with the BB-odd GaG-even WdW wave function $\Psi_5$ constitutes a perfect setup for a spontaneous GaG symmetry breaking mechanism following a quantum creation at the top of the hill. Appreciating the fact that a GaG-odd $V(\phi)$ gets generically translated into a GaG-even $V_{\text{eff}}(\phi)$, one may envision a cosmological Klein–Gordon evolution (entropy oriented arrow of time?) governed by a double well $V_{\text{eff}}(\phi)$, with general relativity formulated around the dilaton vacuum expectation value $\langle \phi \rangle = (8\pi G)^{-1}$. This holds for any $E$ in the spectrum, with potentially far reaching consequences for the multiverse hypothesis.

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