Constraints on modern microscopic equations of state

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Abstract. We compare a set of equations of state derived within microscopic many-body approaches, and study their predictions as far as phenomenological data on nuclei from heavy ion collisions, and astrophysical observations on neutron stars are concerned. All the data, taken together, put strong constraints not easy to be fulfilled accurately. However the results provide an estimate of the uncertainty on the theoretical prediction at a microscopic level of the nuclear equation of state.

1. Introduction

In the past years several efforts have been made in order to constrain the equation of state (EOS). The experimental data on heavy ion collisions and astrophysical observations of compact objects have been used for exploring the density dependence of the EOS, the symmetry energy and its isospin dependence. Recently, a set of equations of state based on the phenomenological Skyrme [1] force and on microscopic many-body methods was tested against the available experimental data [2]. We will present the analysis in this paper, which is based on the results published in ref.[2].

Empirical properties of infinite nuclear matter can be calculated using different microscopic theoretical approaches, the only input being a realistic free nucleon-nucleon (NN) interaction with parameters fitted to NN scattering phase shifts in different partial wave channels, and to properties of the deuteron [3]. In the following we discuss the non relativistic Brueckner-Hartree-Fock (BHF) method [4] and its relativistic counterpart, the Dirac-Brueckner-Hartree-Fock (DBHF) approximation [5] and the variational method [6]. The Brueckner-Bethe-Goldstone (BBG) theory is based on a linked cluster expansion of the energy per nucleon of nuclear matter The basic ingredient in this many-body approach is the Brueckner reaction matrix $G$, which is the solution of the Bethe-Goldstone equation

$$G[\rho;\omega]=v+\sum_{k<k_b}^{k_F} \frac{|a_{k_b}|}{\omega - e(k_a) - e(k_b)} G[\rho;\omega],$$

where $v$ is the bare NN interaction, $\rho$ is the nucleon number density, and $\omega$ the starting energy. The single-particle energy (assuming $\hbar=1$) $e(k) = e(k;\rho) = k^2 / 2m + U(k;\rho)$, and the Pauli operator $Q$ determine the propagation of intermediate baryon pairs. $U(k;\rho)$ is the single-particle potential, $U(k;\rho) = \text{Re} \sum_{k'\leq k_F} \langle kk' | G[\rho;e(k)+e(k')] | kk' \rangle_\alpha$, where the subscript “$\alpha$” indicates antisymmetrization of the matrix element. In the BHF approximation the energy per nucleon is

$$\frac{E}{A} = \frac{3}{5} \frac{k_F^2}{2m} + \frac{1}{2\rho} \sum_{k,k'\leq k_F} \langle kk' | G[\rho;e(k)+e(k')] | kk' \rangle_\alpha.$$

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In this scheme, the only input quantity we need is the bare NN interaction $v$ in the Bethe-Goldstone equation (1). It is well known that in order to reproduce the correct point of saturation for nuclear matter we must introduce Three Body Forces (TBF’s) in non-relativistic many-body methods. In this work we will illustrate results for two different approaches to the TBF’s, i.e., a phenomenological and a microscopic one. The phenomenological approach is based on the so-called Urbana model, which consists of an attractive term due to two-pion exchange with excitation of an intermediate $\Delta$ resonance, and a repulsive phenomenological central term [7]. The microscopic approach has been proposed using the same meson-exchange parameters as the underlying NN potential. Results have been obtained with the Argonne $v_{18}$, the Bonn B, and the Nijmegen 93 potentials [8].

This paper is organized as follows. In section 2 we review the heavy ion phenomenology. In section 3 we present the results regarding astrophysical observation. Section 4 contains our conclusions.

2. Heavy ion phenomenology

In the last two decades intensive studies of heavy ion collisions (HIC) have been performed. The main goal has been the extraction from the data of the gross properties of the nuclear EoS. In principle the dynamics of the collisions should be connected with the properties of the nuclear medium EoS and its viscosity. In the so-called “multifragmentation” regime, the transverse flow, which is strongly affected by the matter compression during the collision, can be measured.

Based on numerical simulations, in Ref.[9] a phenomenological range of densities was proposed where any reasonable EoS for symmetric nuclear matter should pass through in the pressure vs. density plane. The plot is reproduced in Fig.1, where a comparison with the microscopic calculations is made. The green dashed box represents the results of the numerical simulations of the experimental data discussed in Ref.[9], and the brown filled region represents the experimental data on kaon production [10]. We notice that the EoS calculated with the BHF and the variational methods including UVIX three-body (labelled by APR [11]) forces appear to be in agreement with the data in the full density range. On the other hand, the BHF EoS’s obtained using microscopic TBF’s are only marginally compatible with the experimental data, as well as the DBHF EoS [10], showing that they are too repulsive already at density $\rho \geq 3\rho_0$ if the Bonn potentials are used. Though, it has to be stressed that all EoS are compatible with the data around the saturation density; i.e., their incompressibility is as soft as required by the data. However, the values of the incompressibility do not characterize completely the EoS, since it is density dependent, but in any case the analysis indicates that the EoS at low densities must be soft.
A further constraint on the EoS is given by the symmetry energy, which has been extensively studied both from the theoretical and experimental points of view in Ref.[12]. The symmetry energy is displayed in Fig.2 as a function of the nucleon density. The green region is the result of a recent analysis performed by Danielewicz and Lee on the isobaric analog states (IAS) in nuclei [13]. This stems from the charge independence of nuclear interactions; i.e., strong interactions between nucleons in the same state do not depend on whether the nucleons are protons or neutrons. Therefore the energy difference between the ground state of a nucleus with $N > Z$ and the isobaric analogs of the ground states of neighboring isobars are given by the symmetry energy, and the Coulomb contributions to the binding energy can be determined using the IAS. Many such states have been identified, and by fitting the available data on the IAS, Danielewicz and Lee obtained the constraint shown as a green region in Fig.2. We observe that all EoS’s give results in very good agreement with the experimental data, except the ones of BHF with microscopic TBF at densities below the saturation density.

In Fig.3 we display the slope $L$ of the symmetry energy as a function of the symmetry energy at saturation $S_0$, which has been widely discussed in Ref.[12]. Several experimental data sets are displayed. The blue band represents experimental data from HIC, obtained from the neutron and proton spectra from central collisions at 50 MeV/A [14]. At the same incident energy, isospin diffusion was investigated depending on the different $N/Z$ asymmetry of the involved projectiles and targets [15, 16]. The full green circle shows the results from isospin diffusion observables measured for collisions at a lower beam energy of 35 MeV per nucleon [17]. Transverse collective flows of hydrogen and helium isotopes as well as intermediate mass fragments with $Z < 9$ have also been measured at incident energy of 35 MeV/A for $^{70}$Zn, $^{64}$Zn and $^{64}$Ni reactions and compared to transport calculations. The analysis yielded values denoted by the full green squares [18]. The box labeled FRDM (finite-range droplet model) represents a refinement of the droplet model [19], and includes microscopic ”shell” effects and the extra binding associated with $N = Z$ nuclei. In Fig.3 the other boxes represent experimental data obtained from measurements of the neutron skin thickness. The measurement of the neutron skin thickness made on the stable nucleus $^{208}$Pb reported a value $\delta R_{np} = 0.211^{+0.054}_{-0.063}$ fm. From the experiments constraints on the symmetry energy were derived, and these are plotted in Fig.3 as the short-dashed blue rectangular.
Figure 4. (Color online) The mass-radius (left panel) and the mass-central density (right panel) relations are plotted for the EoS’s discussed. Boxes are boundaries extracted from observations, see Ref.[24].

box labelled Pb(\vec{p}, \vec{j})). Last, we mention the experimental data on the Pygmy Dipole Resonance (PDR) in very neutron-rich nuclei such as \(^{68}\text{Ni}\) and \(^{132}\text{Sn}\), which peaks at excitation energies well below the Giant Dipole Resonance (GDR) [20]. Those constraints are shown as a long-dashed rectangle in Fig.3 with the label PDR. The predictions of the different EoS’s are also reported in Fig.3 as full symbols. They are distributed within a large region and they span a wide interval in the values of the parameter L. However, the various phenomenological data are at best marginally compatible, and it is difficult to put well definite constraints on the EoS.

3. Astrophysics

A neutron star is bound by gravity, and it is kept in hydrostatic equilibrium only by the pressure produced by the compressed nuclear matter. It is then apparent that the nuclear matter EoS is the main medium property that is relevant in this case, as can be seen in the celebrated Tolman-Oppenheimer-Volkoff [21] equations, valid for spherically symmetric NS. The value of the maximum mass depends on the nuclear EoS, so that the observation of a mass higher than the maximum one allowed by a given EoS simply rules out that EoS. The considered microscopic EoS’s are compatible with the largest masses observed up to now, that is the mass of the pulsar PSR J1614-2230, \(1.97 \pm 0.04 \, M_\odot\) [22], plotted in the figure, and the mass of the pulsar PSR J0348+0432, \(2.01 \pm 0.04 \, M_\odot\) [23]. This is clearly shown in Fig.4, where the mass-radius (left panel) and mass-central-density relations (right panel) are plotted for all the considered EoS’s as thick lines. It looks unlikely that this value is indeed the largest possible NS mass, and therefore future observational data on NS masses could overcome this limit and strongly constrain the nuclear EoS.

In Fig.4 a sample of observational data taken from Ref.[24] is displayed by closed thin lines for different sources, measured in quiescence and from thermonuclear bursts. It turns out that the current measurements are consistent with radii in the range 8-12 km and disfavor neutron stars with R ~15 km. Those measurements are consistent with the recent observation of the neutron star in SAX J1748.9-2021, which points to the neutron star radius in the 8-11 km range [25].

Additional tentative constraints on the nuclear EoS were obtained in a recent analysis of the data on six NS’s based on Bayesian statistical framework [26]. Depending on the hypothesis made on the structure of the NS, the results are slightly different, as shown in Fig.5, where the quantity \(r_{\text{ph}}\) is the photosphere radius. In the left panel \(r_{\text{ph}}\) is comparable to the neutron star radius R, whereas in the right panel a substantial expansion of the photosphere during an x-ray burst is assumed to occur. Among the different EoS’s, only the one calculated with BHF and phenomenological Urbana model appear to be
Figure 5. (Color online) Pressure as a function of the mass-energy density in neutron star matter. The shaded areas are taken from ref.[26]. See text for details.

Figure 6. (Color online) The gravitational mass is plotted as a function of the baryon mass for several EoS’s. The boxes indicate the boundaries from the simulations.

compatible with the extracted observational constraints over the whole density range. These boundaries obtained from astrophysical data are complementary to the ones obtained from heavy ion reactions. In fact, in heavy ion collisions the tested matter is essentially symmetric, while in a NS the matter is highly asymmetric. Considered together, the two types of constraints probe the density dependence of the symmetry energy.

In relation to the high density region of the nuclear EoS, an additional test is on the speed of sound $c_S$, which is required to be smaller than the speed of light $c$ (causality condition). The speed of sound is directly connected with the incompressibility and the energy density, and therefore it depends on the matter energy density and asymmetry. We found that at large enough density most of the EoS’s show a superluminal speed of sound, except the one calculated with BHF with phenomenological TBF’s which doesn’t become superluminal at central density corresponding to the maximum mass, unlike the other EoS’s.

A further additional constraint on the neutron star EoS is provided by the observation of the double pulsar J0737-3039, and the interpretation given by Podsiadlowski [27]. In fact, the gravitational mass of Pulsar B is very precisely known, $M_G = 1.249 \pm 0.001\, M_\odot$, whereas estimates of the baryonic mass depend upon its detailed mode of formation. As modeled by Podsiadlowski et al., if the pulsar B was formed from a white dwarf with an O-Ne-Mg core in an electron-capture supernova, assuming no or negligible loss of baryonic mass during the collapse, the newly born neutron star will have the same baryonic mass as the precollapse core of the progenitor star, i.e., $M_B \simeq 1.366 - 1.375\, M_\odot$. This result is displayed in Fig.6 as a black box. Though, taking into account the uncertainty in the EoS and the small mass loss during the collapse, Kitaura et al. [28] made another simulation which gave $M_B = 1.360 \pm 0.002\, M_\odot$, which is shown in Fig.6 by the green box. We have calculated for each neutron star matter EoS the relation between the gravitational and baryonic mass, and these are displayed in Fig.6 by the straight curves. We notice that the results of all microscopic EoS’s agree very well with the result of Podsiadlowski, at variance with the calculations based on the phenomenological Skyrme forces discussed in Ref.[1], where agreement was found with the result of Kitaura et al.[28], which assumed small mass loss during the collapse.

4. Conclusions

We have presented a systematic confrontation of the nuclear equation of state, obtained within different microscopic many-body methods, with the available constraints coming from phenomenology. Both nuclear structure and heavy ion collisions data were considered, along the same lines of the analysis on the Skyrme forces reported in Ref.[1]. Astrophysical observational data included the measurements
of NS masses, some hints on the mass-radius relation from Ref.[24, 25], and the constraints on the EoS presented in Ref [26], obtained from the analysis of transient phenomena in six NS’s. If one takes literally all the constraints, among the considered microscopic EoS’s only one passes all the tests, namely the BHF with the two-body potential Av18 and the UVIX model for the three-body forces [29]. This shows that the considered phenomenological constraints are well suited in selecting the microscopic EoS. However, before drawing a strong conclusion, it would be appropriate to have a firm estimate of the uncertainties affecting the data. Secondly, from the analysis it appears that one can explain reasonably well all the data with a microscopic EoS that includes only nucleonic degrees of freedom; in particular no exotic components in NS’s are needed. Indeed, if a NS with mass much larger than 2 solar masses will be observed, this will rule out most of the considered microscopic EoS’s and will introduce a serious and fundamental issue in the physics of NS’s. On the other hand, it has been shown that exotic matter such as hyperons or quarks should appear in NS’s [30], which will strongly affect the EoS. In the future one can expect that the interplay between theory and observations will continue to play a major role in the worldwide effort of determining the nuclear EoS.

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