Cryptanalysis and improvement of several quantum private comparison protocols

Zhao-Xu Ji, Pei-Ru Fan, Huan-Guo Zhang†

Key Laboratory of Aerospace Information Security and Trusted Computing, Ministry of Education, School of Cyber Science and Engineering, Wuhan University, Wuhan 430072, China

†liss@whu.edu.cn

Abstract

We find a serious information leakage problem in several existing QPC protocols, such as the ones proposed by Ji and Ye [Commun. Theor. Phys. 65, 711, (2016) and Int. J. Theor. Phys. 56, 1517, (2017)]. A common feature of these protocols is to encrypt participants’ data with many times, such as generating multiple sets of keys with quantum key distribution to encrypt the data with one-time-pad encryption, which can not only protect data privacy but also make a third party unable to obtain the final comparison results. Nevertheless, we find that the encryption and decryption methods adopted in these protocols are insecure: a participant can easily obtain the others’ data through his own data and the final comparison result. In response to this problem, we give an improved method and make some comments.

Keywords: Information security; Quantum cryptography; Quantum private comparison; Information leakage problem

1 Introduction

Quantum cryptography is widely concerned because of its unconditional security [1–5]. The difference between quantum cryptography and classical cryptography is that its security is based on the principles of quantum mechanics rather than mathematical complexity assumptions. A fascinating feature of quantum cryptography is that it allows users to detect whether there is a eavesdropper in quantum channels during communications, which can not be done by classical cryptography [2, 5]. With the rapid development of quantum computers and quantum algorithms, the security of classical cryptography has been severely challenged, which makes the role of quantum cryptography in modern cryptography more and more important [2, 5].

Since the birth of quantum cryptography, quantum key distribution (QKD) has been one of the main research directions in the field of quantum cryptography [2]. Indeed, the first quantum cryptography protocol is the QKD protocol proposed by Bennett in 1984, which is the famous BB84 protocol. QKD aims to generate random shared keys between different users; combined with one-time pad encryption, it can provide unconditional security for users. Moreover, the decoy photon technology derived from QKD has become one of the effective means for eavesdropping checking [4, 5].

Quantum private comparison (QPC), originated from the famous “millionaires problem” [5–7], aims to judge whether the date of at least two users who do not trust each other are the same or not while maintaining data privacy using some quantum mechanics laws, such as quantum noncloning theorem and Heisenberg’s uncertainty principle. In fact, the comparison of data equality is widely used in real life, including secret bidding and auctions, secret ballot elections, e-commerce and data mining. One of the common applications is the identification of a system for users, which aims to judge whether the user’s secret information (e.g., password, fingerprint) is the same as the secret information stored in the system. QPC can also solve the “Tercé problem”, which is also known as the “socialist millionaires’ problem” [9].

After about ten years of development, QPC has attracted extensive attention in academia. Many protocols have been proposed based on different quantum states or different quantum technologies [9–35]. In terms of the number of participants, they can be divided into two-party protocols and multi-party protocols. From the quantum states adopted, they can be roughly divided into single-particle-based protocols and entangled-state-based protocols, among which the quantum states include Bell states, GHZ states etc. The quantum techniques used in QPC protocols include quantum measurements, unitary operations, entanglement swapping, etc. Generally, a QPC protocol needs to meet the following two conditions: (1) fairness: at the end of the protocol, all users need to get the comparison result at the same time, without any particular order; (2) security: all users’ data are confidential, and there is no information leakage problem; only when all users’ data are the same, can all users know each other’s secret data.

Although many important results have been achieved in the theoretical research of QPC, designing a secure and efficient protocol still faces many challenges. After all, there is always a trade-off between the security and efficiency of a quantum cryptography protocol, but in no case efficiency cannot be pursued at the cost of reducing security. Unfor-
fortunately, information leakage often occurs; many existing quantum cryptography protocols have been proved to be insecure, and QPC is no exception. A typical information leakage problem is that the two-party QPC protocol without a third party has been proved insecure. Therefore, a third party (conventionally called TP) must be introduced into the protocol to help users achieve the purpose of private comparison. The third party is usually assumed to be semi-honest, which is widely adopted in QPC. Specifically, TP faithfully executes protocol processes and does not collude with any user, but he can steal the user’s data in all possible ways. For example, he may record the results of his calculations from which he tries to infer the user’s data.

In this paper, we will point out that there is a serious information leakage problem in the QPC protocols presented in Refs. [36–42]. These protocols all use some additional encryption means to prevent the third party from obtaining the final comparison result. Indeed, it works, but we find that it also makes it easy for one user to steal the data from others. We will propose a solution to this problem and make some comments. The rest of the paper is arranged as follows: in Sec. 2, we review briefly the protocol proposed by Ji et al., and then introduce the solution and give our comments in Sec. 3. We summarize this paper in Sec. 4.

2 Information leakage problem

In this section, we would first like to review briefly Ji and Ye’s protocol, and point out the information leakage problem in the protocol. We will then point out that this problem also exists in several other QPC protocols.

2.1 Review on Ji et al.’s protocol

Let us start by reviewing Ji et al.’s protocol. The highly entangled six-qubit genuine state has the form

\[
|\Psi\rangle = \frac{1}{\sqrt{32}}\left[ |000000\rangle + |011111\rangle + |111000\rangle + |111100\rangle + |001010\rangle + |011010\rangle + |001100\rangle + |100110\rangle + |010110\rangle + |101110\rangle + |011101\rangle + |101011\rangle + |010111\rangle + |100100\rangle + |010010\rangle + |100111\rangle + |010111\rangle + |101011\rangle + |011010\rangle + |001001\rangle + |001101\rangle + |010100\rangle + |010001\rangle + |010101\rangle + |001010\rangle + |001100\rangle + |001001\rangle + |000101\rangle + |000110\rangle + |000010\rangle + |000001\rangle \right],
\]

which is rewritten as

\[
|\Psi\rangle = \frac{1}{\sqrt{4}}\left[ (|0000\rangle - |0101\rangle - |1010\rangle + |1111\rangle) \otimes |\phi^+\rangle \\
+ (|0001\rangle + |0100\rangle + |1011\rangle + |1110\rangle) \otimes |\phi^-\rangle \\
+ (|0110\rangle - |0011\rangle - |1001\rangle + |1100\rangle) \otimes |\phi^+\rangle \\
+ (|0010\rangle + |0111\rangle - |1000\rangle - |1111\rangle) \otimes |\phi^-\rangle \right],
\]

(2)

where

\[
|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \quad |\phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle).
\]

are four Bell states. The prerequisites of the protocol are:

1. Suppose that Alice and Bob have the secret data X and Y respectively, and that the binary representations of X and Y are \(x_1, x_2, \ldots, x_N\) and \(y_1, y_2, \ldots, y_N\) respectively, where \(x_j, y_j \in \{0, 1\}\) \(j \in \{1, 2, \ldots, N\}\), hence \(X = \sum_{j=1}^{N} x_j 2^{j-1}, Y = \sum_{j=1}^{N} y_j 2^{j-1}\).

2. Alice (Bob) divides the binary representation of \(X(Y)\) into \([N/2]\) groups:

\[
G_A^1, G_A^2, \ldots, G_A^{[N/2]} \ (G_B^1, G_B^2, \ldots, G_B^{[N/2]}).
\]

Each group \(G_A^i(G_B^i)\) includes two bits, where \(i = 1, 2, \ldots, [N/2]\) throughout this protocol. If \(N\) mod 2 = 1, Alice (Bob) adds one 0 into the last group \(G_A^{[N/2]_1}(G_B^{[N/2]_1})\).

3. Alice and Bob generate the shared key sequences \(\{K_A^1, K_A^2, \ldots, K_A^{[N/2]}\}\) and \(\{K_B^1, K_B^2, \ldots, K_B^{[N/2]}\}\) through a QKD protocol, where \(K_A^i, K_B^i \in \{00, 01, 10, 11\}\). Similarly, Alice(Bob) and TP generate the shared key sequence \(\{K_{AC}^1, K_{AC}^2, \ldots, K_{AC}^{[N/2]}\}\) and \(\{K_{BC}^1, K_{BC}^2, \ldots, K_{BC}^{[N/2]}\}\), where \(K_{AC}, K_{BC} \in \{00, 01, 10, 11\}\).

4. Alice, Bob, and TP agree on the following coding rules: \(0\leftrightarrow 0, |0\rangle \leftrightarrow 1, |\psi^+\rangle \leftrightarrow 00, |\psi^-\rangle \leftrightarrow 11, |\phi^+\rangle \leftrightarrow 01, \) and \(|\phi^-\rangle \leftrightarrow 10\).

The steps of the protocol are as follows:

1. TP prepares \([N/2]\) copies of the highly entangled six-qubit genuine state \(|\Psi\rangle\), and marks them by

\[
|\Psi(p_1^1, p_1^2, p_1^3, p_1^4, p_1^5, p_1^6), |\Psi(p_2^1, p_2^2, p_2^3, p_2^4, p_2^5, p_2^6), \ldots, |\Psi(p_{N/2}^1, p_{N/2}^2, p_{N/2}^3, p_{N/2}^4, p_{N/2}^5, p_{N/2}^6)\rangle,
\]

(5)

in turn to generate an ordered sequence, where the subscripts 1, 2, …, \([N/2]\) denote the order of the highly entangled six-qubit genuine states in the sequence, and the superscripts 1, 2, 3, 4, 5, 6 denote six particles in one state. Then TP takes the first two particles marked by \(p_1^1, p_1^2\) out from \(|\Psi(p_1^1, p_1^2, p_1^3, p_1^4, p_1^5, p_1^6)\rangle) to construct the new sequence

\[
p_1^1, p_2^2, p_2^3, \ldots, p_{N/2}^1, p_{N/2}^2.
\]

(6)
and denotes it as $S_A$. Similarly, he takes out the second and third particles to construct another new sequence

$$p_1^3, p_1^4, p_2^3, p_2^4, \ldots, p_{[N/2]}^3, p_{[N/2]}^4,$$

and denotes it as $S_B$. The remaining particles construct another new sequence

$$p_1^5, p_1^6, p_2^5, p_2^6, \ldots, p_{[N/2]}^5, p_{[N/2]}^6,$$

denoted as $S_C$.

2. TP prepares two sets of decoy photons in which each decoy photon is randomly chosen from four single-particle states $|0\rangle, |1\rangle, |+\rangle, |−\rangle (|±\rangle = 1/\sqrt{2}(|0\rangle \pm |1\rangle))$. Then he inserts randomly the two set of decoy photons into $S_A$ and $S_B$, respectively, and records the insertion positions. Finally, he denotes the two new generated sequences as $S'_A$ and $S'_B$, and sends them to Alice and Bob, respectively.

3. After receiving $S'_A$ and $S'_B$, TP and Alice(Bob) use the decoy photons in $S'_A$ and $S'_B$ to judge whether eavesdroppers exist in quantum channels. The error rate exceeding the predetermined threshold will lead to the termination and restart of the protocol, otherwise the protocol proceeds to the next step.

4. Alice(Bob) measures the two particles marked by $p_1^1, p_1^2, p_2^1, p_2^2$ in $S_A(S_B)$ with Z basis ($|0\rangle, |1\rangle$), and denotes the binary numbers corresponding to the measurement results as $M_1^A(M_1^B)$. Then, Alice(Bob) calculates $G_A^i \oplus M_1^A \oplus K_{AC}^i \oplus K_{BC}^i \oplus K_{1B}^i \oplus K_{BC}^i \oplus K_{1B}^i$, and marks the calculation results by $R_A^i(R_B^i)$. Finally, Alice(Bob) announces $R_A^i(R_B^i)$ to TP.

5. After receiving $R_A^i(R_B^i)$, TP performs Bell measurements on the particles marked by $p_1^0, p_1^1, p_2^0, p_2^1$, and marks the binary numbers corresponding to the measurement results by $M_2^C$. Then, TP calculates $R_i = R_A^i \oplus R_B^i \oplus K_{AC}^i \oplus K_{BC}^i \oplus M_2^C$, and announces $R_i$ to Alice and Bob.

6. After receiving $R_i$, Alice and Bob calculate $R_i \oplus K_{1}^{i} \oplus K_{1B}^{i}$, respectively, and marks the calculation results by $R'_i$. If $R'_i = 00$ (i.e., each classical bits in $R'_i$ is 0), they conclude that their data X and Y are the same. Otherwise, they conclude that X and Y are different and stop the comparison.

2.2 The problem

At the end of the protocol, both Alice and Bob obtain $G_A^i \oplus G_B^i$, that is,

$$R'_i = R_A^i \oplus R_B^i \oplus K_{AC}^i \oplus K_{BC}^i \oplus M_2^C$$

and

$$= (G_A^i \oplus M_1^A \oplus K_{AC}^i \oplus K_{BC}^i \oplus M_2^C) \oplus (K_{AC}^i \oplus K_{BC}^i \oplus K_{1B}^i \oplus K_{1B}^i)$$

$$= (G_A^i \oplus G_B^i) \oplus (M_1^A \oplus M_1^B \oplus M_2^C)$$

$$= G_A^i \oplus G_B^i.$$

In this case, Alice and Bob can easily steal each other’s data. Specifically, Alice(Bob) can calculate $R'_A \oplus G_A^i \oplus G_B^i$, thus she/he can get $G_A^i(G_B^i)$, that is, $R'_A \oplus G_A^i \oplus G_B^i = G_A^i(R'_A \oplus G_A^i \oplus G_B^i)$. In fact, for a cryptography protocol, except that the keys generated in the protocol is confidential, the process, prerequisites, and coding rules of the protocol are all public. Therefore, Alice and Bob, as participants, will surely know that the final comparison result is $G_A^i \oplus G_B^i$.

We find that the protocols in Refs. [14, 13] also have such information leakage problem. In these protocols, Alice and Bob get $G_A^i \oplus G_B^i$ at the end of the protocol, thus they can easily know each other’s data.

3 Improvement and comment

In this section, we will propose a simple solution to the information leakage problem in the protocol, and then make some relevant comments.

3.1 Improvement

Let us now present the solution. For simplicity and clarity, we change directly the steps 5 and 6 of the protocol as follows:

5. After receiving $R_A^i(R_B^i)$, TP performs Bell measurements on the particles marked by $p_1^3, p_1^4$, and marks the binary numbers corresponding to the measurement results by $M_1^C$. Then, TP calculates $R_i = R_A^i \oplus R_B^i \oplus K_{AC}^i \oplus K_{BC}^i \oplus M_1^C$, and marks the calculation results by $a_i^0a_i^1$ (e.g., if $R_i = 01$, then $a_i^0a_i^1 = 01$). TP calculates

$$\sum_{i=1}^{[N/2]} \sum_{j=1}^{2} a_i^j,$$

and marks the calculation result by $S$. Finally, TP announces $S$ to Alice and Bob.

6. After receiving $S$, Alice and Bob calculate $K_{AC}^i \oplus K_{1B}^i$, respectively, and marks the calculation results by $b_i^0b_i^1$. Then they calculate

$$\sum_{i=1}^{[N/2]} \sum_{j=1}^{2} b_i^j,$$

and marks the calculation result by $R'$. Finally, they calculate $S - S'$. If $S - S' = 0$, they can conclude that their data X and Y are the same. Otherwise, they conclude that X and Y are different.

After the improvement, the correctness of the protocol is easy to verify. In Step 5, TP computes $R_i = R_A^i \oplus R_B^i \oplus K_{1AC}^i \oplus K_{1BC}^i \oplus M_1^C$, hence we get

$$R_i = R_A^i \oplus R_B^i \oplus K_{1AC}^i \oplus K_{1BC}^i \oplus M_1^C$$

and

$$= (G_A^i \oplus M_1^A \oplus K_{AC}^i \oplus K_{1B}^i) \oplus (G_B^i \oplus M_1^B \oplus K_{1B}^i \oplus K_{BC}^i) \oplus K_{AC}^i \oplus K_{BC}^i \oplus M_1^C$$

$$= G_A^i \oplus G_B^i \oplus K_{1AC}^i \oplus K_{1BC}^i.$$
Obviously, iff \( G_A = G_B, S = \sum_{i=1}^{N/2} b_i \) (i.e., \( S = S' \)). Otherwise, \( S \neq S' \). Note here that \( K_A' \) and \( K_B' \) are random keys generated by QKD, thus \( K_A' \) and \( K_B' \) are not all the same (the probability that they are all the same can be ignored because it is very small).

Similar improvements can be made to the protocols in Refs. [44–48]. For simplicity, we would not like to review these protocols and describe their amendments.

3.2 Comment

In fact, in a classical private comparison protocol, the data of Alice and Bob are confidential but the final comparison result is public. QPC, as the generalization of classical private comparison in quantum mechanics, does not need to keep the final comparison result secret. On the one hand, adding this requirement will make the protocol more complex. On the other hand, it will consume more expensive quantum resources. At present, most QPC protocols allow the third party to publish the final comparison result. Of course, if a protocol needs to keep the comparison result secret from the third party, one can design the protocol in a similar way according to the proposed improved scheme.

4 Conclusion

We have found a serious information leakage problem in several QPC protocols, and propose a simple and effective solution. In addition, we have made some comments on this problem. We believe that the solution and comments are constructive to the design of a QPC protocol.

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