Indexing Temporal Relations for Range-Duration Queries

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ABSTRACT
Temporal information plays a crucial role in many database applications; however, support for queries on such data is limited. We present an index structure, termed RD-INDEX, to support range-duration queries over interval timestamped relations, which constrain both the range of the tuples’ positions on the timeline and their duration. RD-INDEX is a grid structure in the two-dimensional space, representing the position on the timeline and the duration of timestamps, respectively. Instead of using a regular grid, we consider the data distribution for the construction of the grid in order to ensure that each grid cell contains approximately the same number of intervals. RD-INDEX features provable bounds on the running time of all the operations, allowing for a simple implementation, and supports very predictable query performance. We benchmark our solution on a variety of datasets and query workloads, investigating both the query rate and the behavior of the individual queries. The results show that RD-INDEX performs better than the baselines on range-duration queries, for which it is explicitly designed. Furthermore, it outperforms specialized indexes also on workloads containing queries constraining either only the duration or the range.

1 INTRODUCTION
Temporal information plays a crucial role in many database applications; in fact, many database management systems and the SQL standard [27] provide automated version control of the data and time travel facilities, allowing to efficiently access past history. Past research mainly concentrated on efficient solutions for important temporal operators, such as temporal aggregation [9, 23, 29, 33], temporal joins [11, 15, 34], and time travel [32] queries. All these approaches consider only the position of intervals along the timeline, ignoring another important aspect, namely the duration of intervals. As a result, index structures to support more general selection queries that constrain both the duration and the position in time of intervals have been missing [8, 14, 27] until recently [4]. In many application domains, however, both aspects of temporal information are useful to formulate queries.

Example 1.1. As a concrete use case, consider the use of antibiotics in healthcare. Antibiotic resistance is a world challenge, and the emergence of new resistance factors is very difficult to monitor and to predict due to the diversity of antibiotic usage and events (e.g., environment, species evolution, medical practices, etc.) [19, 28]. Selecting the most appropriate antibiotic and the appropriate treatment duration is an essential step to reduce antibiotic resistance [19]. Thus, defining guidelines for the duration of antibiotic treatments, measuring the adherence to these guidelines, and developing stewardship tools regarding antibiotics usage can help clinicians in choosing the optimal treatment considering the patient’s medical history [28, 39]. Such measures should be implemented at a national level in order to monitor and audit antibiotic resistance on a larger scale. In this context, the following types of temporal queries are frequent:

- **Q1:** “Find all antibiotics prescriptions from October 1, 2016 to March 31, 2017.”
- **Q2:** “Find all antibiotics prescriptions with a treatment duration between 5 and 8 days.”
- **Q3:** “Find all antibiotics prescriptions from October 1, 2016 to March 31, 2017, with a treatment duration between 1 and 2 weeks.”

The first query Q1 retrieves tuples based on the position of the events on the timeline; we call it range query. In contrast, query Q2 imposes constraints on the duration of matching events, and we call it duration query. Finally, query Q3 constrains both types of information; we call it range-duration query. This type of queries can be found and have been reported as a primitive operation in other application scenarios that deal with interval data, e.g., in air traffic analysis [5, 37, 38], event detection for video surveillance [32], or the analysis of clinical data [6].

Existing index structures typically support only one of the two aspects, either the position of the interval on the timeline or the duration of the interval. For instance, the well-known relational interval tree [25] is optimized for efficiently determining temporal relationships between intervals but not interval lengths. In the worst case, the entire index tree must be traversed if a query solely contains restrictions on the interval length. Alternatively, the duration of the intervals can be indexed straightforwardly using a classic data structure such as a B-tree. In this case, however, queries constraining only the range of the intervals will need to traverse the entire tree. Combining the two indexes is typically inefficient since a query can have different selectivities in the two dimensions. Therefore, to efficiently support workloads involving a mix of all three types of queries mentioned above we seek a new index structure that supports both dimensions at the same time.

In this paper, we introduce RD-INDEX, a novel two-dimensional data structure that indexes time intervals both on their position on the timeline and their duration. Our index structure partitions the intervals in a grid according to their start times and durations. Rather than constructing a regular grid, the boundaries between the grid cells are...
determined by taking into account the data distribution. Such a strategy ensures that each cell contains approximately the same number of intervals, with the exception of some edge cases if the distribution of the intervals is extremely skewed. The uniform distribution of the data over all grid cells allows to obtain very predictable query times, which are proportional to the selectivity of the query. We prove that the time for answering a range-duration query with RD-INDEX is \( O\left(\frac{n}{s} \log\frac{n}{s} + \frac{n}{s} + s^2 + k\right) \), where \( n \) is the size of the input relation, \( k \) is the number of intervals matching the query predicate, and \( s \) is the page size. The index can be constructed in \( O(n \log n) \) time. The page size \( s \) is the only parameter of our index structure, and it is independent of the data distribution. The index structure also lends itself to a rather simple implementation. While being explicitly designed to address range-duration queries, RD-INDEX also supports range-only and duration-only queries efficiently.

We present the results of a detailed experimental evaluation. The results show that the overhead introduced by the data structure is indeed negligible and that the running time in practice is largely proportional to the selectivity of the query. This is in contrast to the competitors we compare to. On range-duration queries we find that RD-INDEX clearly outperforms the competitors. On mixed workloads comprising all three types of queries, we find that RD-INDEX outperforms the competitors in the vast majority of the workloads, even for cases for which specialized solutions exist.

Our contributions can be summarized as follows:

- We describe RD-INDEX, a novel index structure that supports temporal queries involving both the duration and the range of time intervals.
- We prove bounds on the performance of RD-INDEX, which can be tuned with a single page size parameter \( s \).
- We provide an extensible open source implementation, which we benchmark against state-of-the-art competitors, showing significantly better performance across several workloads. In particular, we show how RD-INDEX can efficiently handle mixed workloads where all three types of queries coexist.

The rest of the paper is organized as follows. In Section 2 we review the state of the art, before laying out the fundamental concepts underlying our approach in Section 3. Our data structure is introduced in Section 4, and the complexity of all operations is analyzed in Section 5. Experimental results are presented in Section 6, before drawing our conclusions in Section 7.

2 RELATED WORK

The type of selection queries we are studying in this paper are queries with a conjunctive predicate, where one predicate in the conjunction restricts the position of intervals on the time line and the second the duration of intervals. In this section, we review indexing structures that are (partially) suitable for such selection queries, and also review structures similar to our approach that are used for interval joins.

There are several approaches devoted to indexing interval timestamped data. Edelsbrunner’s Interval Tree [17] is one of the most popular indexing structure for intervals. It is asymptotically optimal for selection queries involving the overlap predicate, and there exists an implementation using standard relational database technology based on B-tree indexes [25]. A shortcoming of the interval tree is that, in contrast to our indexing structure, it does not provide a mechanism to restrict the duration of intervals, and thus can only solve one part of a range-duration query. A similar indexing structure is the segment tree [7]. It builds disjoint segments over intervals at the leaf level using all start and end points in a relation, and recursively merges segments in intermediate nodes of the tree. This data structure was originally designed for point queries over intervals (also known as time travel queries), i.e., for retrieving all intervals overlapping a given time point. The segment tree also supports selection queries with the overlap predicate given a query interval, albeit in this case a duplicate elimination step for intervals retrieved multiple times is required. Another index structure that support time travel queries is the timeline index [21, 22]. The timeline index stores the start and end points of intervals in an event list in sorted order and allows to retrieve all tuples that overlap a given time point by scanning through the event list and discarding tuples that ended before the given time point. To avoid scanning through the entire event list, the index maintains regular checkpoints that store all tuples that overlap the time point of the checkpoint. Similarly to the interval tree, the segment tree and timeline index do not support to restrict the duration of intervals.

In a two dimensional space, intervals can be represented as 2D points, where one dimension is the start point and the second dimension is the end point or the duration of an interval. In such a space, a selection query with the overlap predicate corresponds to a selection query over a 2D area. For this, multidimensional indices can be used. R-trees [1, 3] are multidimensional indices that group objects in a multidimensional space using minimum bounding (hyper) rectangles. Quadtrees and octrees [18, 36, 40] recursively divide the space into partitions and place objects into the best fitting partition according to some criteria. All of the aforementioned indexes are linked data structures, which suffer from poor locality, both when implemented in-memory and on disk. In contrast, our RD-INDEX can be implemented by means of simple arrays, and thus enjoys high cache locality.

Another multidimensional index structure is the grid file [31]. In the context of time intervals, the idea would be to partition the span of durations and of starting times into cells of equal width, thus allowing efficient access to both dimensions. The main drawback of this data structure is that in case of skewed data distributions the load of the cells is unequal, which might significantly harm the performance.

A recent approach to multidimensional indexing is that of learned indexes [24]: the proposition is that index structures are models mapping keys to records. Therefore machine learning models can be used to provide this mapping, in lieu of the classic data structures. In particular, Flood and Tsunami [16, 30] use Recursive Model Index [24] and a variant of decision trees to model the position of records in the database, adapting to the distribution of the data and of the query workload. Our approach shares some ideas with this line of work, namely adapting to the data distribution by means of the conditional cumulative distribution function. However, while our index supports both insertions and deletions, both Flood and Tsunami are tailored at read-only workloads. Furthermore, we prove bounds on the worst case running times for all the operations, while [16, 30] provide an empirical evaluation. Finally, our approach is arguably simpler, in that it is based just on sorting and iterating through records.
Very recently, Behrend et al. [4] proposed an index, named PERIOD-INDEX•, that explicitly supports range-duration queries. The index partitions the time domain in buckets. An interval is assigned to all buckets it intersects with. Within each bucket, intervals are further partitioned in levels according to their duration, with the minimum duration indexed within each level decreasing geometrically. To support efficient indexing along the start time dimension, each level is further partitioned in the time domain. This data structure is adaptive to the distribution of start times, while it assumes a Zipf-like distribution for the duration of the intervals. Our index structure removes this assumption, thus supporting datasets with arbitrary distributions of the tuples’ duration. Furthermore, our index features only one data-independent parameter, instead of the two data-dependent parameters of PERIOD-INDEX•, and it allows to control whether to index first by duration or time. Moreover, we do not replicate intervals in the index, yielding a significantly smaller structure, thereby avoiding the consequent possible performance degradation. Finally, PERIOD-INDEX• does not support updates of the index.

In recent years, algorithms for interval joins, which can be seen as a sequence of range queries, have been actively studied. Approaches based on the timeline index [22] process sets of intervals as sorted event lists of their start and end points. The interval join is computed by scanning these event lists in an interleaved fashion, thereby keeping and joining sets of active intervals, i.e., intervals whose start has been encountered but not the end point. To improve the performance of the original linked list data structure for storing active intervals, a gapless hash map has been proposed in [34] that provides a higher performance for scanning active intervals. The same idea has also been extended for joins using general Allen’s predicates [35] rather than only overlap predicates. The works in [10, 11, 15] compute an interval join using sorting and backtracking. First, the input relations are sorted by start time and then an interleaving merge-join is performed to compute the temporal join between the two relations. Other approaches [12, 13] for the interval join are based on partitioning intervals according to their position and then produce the join result by joining relevant partitions. While all these approaches for the interval join provide mechanisms to join overlapping intervals, in contrast to our work they are not applicable for general selection queries as they always require to read the entire relations. Moreover, the duration of intervals is not considered at all in these works.

We assume a linearly ordered, discrete time domain, $\Omega^T$. A time interval is a set of contiguous time points, and $t = [t_s, t_e]$ denotes the closed-open interval of points from $t_s$ to $t_e$. We use $|t| = t_e - t_s$ to denote the duration of time interval $t$ and $t \cap t'$ to denote the set of time points shared by two intervals $t$ and $t'$, which, if not empty, is itself an interval. The schema of a temporal relation is given by $R = (A_1, \ldots, A_m, T)$, where $A_1, \ldots, A_m$ are the non-temporal attributes with domains $\Omega_i$ and $T$ is the time interval attribute with domain $\Omega^T \times \Omega^T$ representing, for instance, the tuple’s valid time. A temporal relation $r$ with schema $R$ is a finite set of tuples, where each tuple has a value in the appropriate domain for each attribute in the schema. We use $r.A_i$ to denote the value of attribute $A_i$ in tuple $r$, and $r.T = (r.T_s, r.T_e)$ to refer to its time interval.

The index we propose efficiently supports the three following types of temporal queries (defined as in [4]).

**Definition 3.1 (Range query).** Given a temporal interval $t = [t_s, t_e]$ and a temporal relation $r$, a range query is defined as

$$Q(r, t) = \{ r \in r : r.T \cap t \neq \emptyset \}$$

**Definition 3.2 (Duration query).** Given a duration interval $d = [d_{\min}, d_{\max}]$ and a temporal relation $r$, a duration query is defined as

$$Q(r, d) = \{ r \in r : |r.T| \in [d_{\min}, d_{\max}] \}$$

**Definition 3.3 (Range-duration query).** Given a temporal interval $t = [t_s, t_e]$, a duration interval $d = [d_{\min}, d_{\max}]$, and a temporal relation $r$, a range-duration query is defined as

$$Q(r, t, d) = \{ r \in r : r.T \cap t \neq \emptyset \land |r.T| \in [d_{\min}, d_{\max}] \}$$

A range query retrieves all tuples whose time interval intersects with the query range $t$. A duration query retrieves all tuples whose time interval has a duration that is between $d_{\min}$ and $d_{\max}$. A range-duration query is a combination of the former two.

**Example 3.4.** As a running example, we consider real-world drug prescriptions from the MIMICIII open source database [20] (cf. use case in Example 1.1). It stores antibiotic prescriptions, characterized by a start date and an end date of the prescription and the duration of the treatment. An excerpt of four tuples is shown in Figure 1a. Consider the following range-duration query: Retrieve all prescriptions in the period from June 15 to July 15 with a treatment duration between 5 and 15 days. Figure 1b shows a graphical representation of the relation and the query, where the time intervals are drawn by thick solid horizontal lines. The red area indicates the constraint on the position. The red segment below of each timestamp interval denotes the duration constraint, where the dotted red line indicates the range of minimum and maximum duration. Hence, an interval satisfies the range constraint if it intersects with the red area, and it satisfies the duration constraint if its end point is within the dotted red line. Tuple $r_1$ satisfies only the duration constraint, tuple $r_2$ satisfies neither constraint, tuple $r_3$ satisfies only the range constraint, and tuple $r_4$ satisfies both.

## 4 THE RD-INDEX STRUCTURE

### 4.1 Overview

An interval can be completely described by its starting time and duration, alternatively, to its starting and ending time. Therefore, a set of temporal intervals can be represented as a set of points in a two dimensional space, with one coordinate being the start time and the other the duration. A range-duration query $Q(r, t, d)$ with $t = [t_s, t_e]$ and $d = [d_{\min}, d_{\max}]$ in such a space is represented by a polygon containing all tuples $r \in r$ such that

$$|r.T| \in [d_{\min}, d_{\max}]$$

$$r.T_e > t_s$$

$$r.T_s < t_e$$

**Example 4.1.** Figure 2 shows a sample of 31 192 drug prescriptions from the MIMICIII dataset in the two dimensional space, where
each point represents a timestamp interval of a tuple. The red polygon indicates the query of Example 3.4, i.e., the prescriptions from June 15 to July 15 with a duration between 5 and 15 days. Note that the points are not uniformly distributed, hence efficiently indexing this dataset requires to be adaptive in both the start time and duration dimension.

The RD-INDEX we are presenting is a two-dimensional grid, partitioning the tuples into disjoint buckets according to the start time and the duration of the intervals. The boundaries between cells are defined by using the empirical cumulative distribution function of the tuples' duration and the starting times, so that each cell contains approximately the same number of intervals, which corresponds to the page size of our index. This allows the index to adapt to the distribution of the input and to different scenarios. In a main memory scenario, the parameter $s$ could be set such that a cell fits in a cache line. In an external memory setting, it might be set to the disk block size.

To simplify the presentation, in the following we will focus on the timestamp interval attribute $T$ of relation $r$ in the discussion we assume that each interval being inserted in the index is associated with a reference to the tuple in $r$ it belongs to.

### 4.2 Index Construction

The grid structure of the RD-INDEX partitions an array of tuples first along either the start time or duration dimension of the intervals, and then along the other. The choice of which dimension to index first may impact the performance of the index, depending on the query workload and the data distribution (cf. Section 6). In the following we assume that the start time dimension is partitioned first, followed by the duration dimension. All the descriptions, considerations, and proofs also hold with the dimensions swapped.

Before describing the algorithm to build the index, we present the subroutine $\text{NEXTSUBSEQ}$, which partitions an array of tuples that is sorted according to a given key function. We will use this subroutine to determine columns and cells of the grid structure, using first the start time and then duration as keys. Given an index $h$ and a size parameter $b$, $\text{NEXTSUBSEQ}$ returns a subsequence starting at $h$ that either contains at most $b$ tuples, or contains tuples that all share the same key. Additionally, all the tuples with the same key are part of the same subsequence. The pseudocode is reported in Algorithm 1 and works as follows. Starting from index position $h$, if there are fewer than $b$ elements after $h$ then we return all the tuples from $h$ onwards. Otherwise, we look at the tuple at position $h'$, and $h'$ have the same key, then we scan forward until the first tuple with a different key occurs (lines 4–6). Otherwise, we scan backward until two consecutive tuples have different keys (lines 7–9). In both cases, the rationale is to avoid splitting runs of same-key tuples between different subsequences.

Note that Algorithm 1 might return a subsequence with more than $b$ elements if and only if all share the same key. In such case we deem the returned subsequence heavy, otherwise we deem it light.

**Example 4.2.** Figure 3 depicts three invocations of $\text{NEXTSUBSEQ}$ on a sequence of sorted start times, with parameter $b = 9$. The first jump by 9 positions would split the run of intervals with start time 13. Therefore, the algorithm iterates back until the first start time < 13. The second invocation would again split the same run since it contains more than 9 intervals with start time 13. This time, since the endpoints of the jump have the same value, the algorithm iterates forward until the last interval with the same start time, thus finding a heavy subsequence. The last jump defines the third group.

We are now ready to describe the index construction procedure $\text{BUILDINDEX}$, which is shown in Algorithm 2. Let $s$ be the page size parameter, and $r$ be the relation to be indexed. First, we sort $r$ by increasing start time. Then, we repeatedly invoke $\text{NEXTSUBSEQ}$ (line 10) to divide the relation into columns of $s^2$ tuples each, based on their start times. Defining columns in this way allows, intuitively, to then further partition each column in $s$ cells of $s$ elements each.

As a special case, if there are more than $s^2$ tuples with the same start time, $\text{NEXTSUBSEQ}$ will assign them to the same column, which we then deem heavy. We also keep track of the minimum start time in each column using an auxiliary array $\text{col_minstart}$. Similarly,

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**Algorithm 1: $\text{NEXTSUBSEQ}(r, h, b, k)$**

**Input:** Relation $r$ sorted according to $k$; current index position $h$; subsequence size $b$; key function $k$

**Output:** Subsequence of $r$ starting from position $h$, either of size $\leq b$ or with all tuples having the same key

1. if $h + b \geq |r|$ then
   2. return subsequence $(r_h, \ldots, r_{|r|-1})$;
3. $h' \leftarrow h + b$;
4. if $k(r_h) = k(r_{h'})$ then
   5. while $h' < |r| \land k(r_h) = k(r_{h'})$ do
      6. $h' \leftarrow h' + 1$
   7. else
      8. while $k(r_{h'-1}) = k(r_{h'})$ do
      9. $h' \leftarrow h' - 1$
10. return subsequence $(r_h, r_{h+1}, \ldots, r_{h'-1})$;

---

1. Therefore $h'$ is the end index of the subsequence, non-inclusive.
2. We could use two separate parameters to control the number of partitions in the two dimensions, but using $s$ and $s^2$ makes for a simpler exposition, analysis and implementation.
Indexing Temporal Relations for Range-Duration Queries

Figure 2: Drug prescription dataset in the start time/duration plane. The gray dots are tuples. The red polygon is the range-duration query of Example 3.4.

Figure 3: Partitioning a sequence of intervals sorted by start time using NEXTSUBSEQ with parameter $b = 9$, with the start time being the key function.

Figure 4: Instantiation of RD-INDEX with page size $s = 70$ on the example dataset. Above the plot we report the col_minstart and col_maxend arrays.

The array col_maxend stores the cumulative maximum end time in the columns, i.e., the maximum end time found so far in the relation. Both of these arrays will be used at query time: the first to find the first column to inspect for a given query, the second to determine when to stop iterating through columns. Note that col_maxend records the cumulative maximum end time of columns. This ensures that no interval in columns $\leq i$ ends after col_maxend[i], which will be useful at query time.
Each column is further partitioned in cells in a similar way. First
we sort the tuples in the column by increasing duration. Then we
define cells of size \( s \) by repeatedly invoking NEXTSUBSEQ (line 17).
Again, if there are more than \( s \) tuples with the same duration,
they will all be assigned to the same cell, which will then be deemed
heavy. Similarly to columns, also cells are complemented by two
arrays of ancillary information: cell_mindur stores the minimum
duration in each cell to be used at query time to find the first cell
to inspect, while cell_maxdur stores the maximum duration in
each cell, which at query time will determine when to stop iterating
through cells.

Finally, tuples in each cell are sorted by the end time. This is
useful at query time, since it allows to stop queries early, as we shall
discuss in the proof of Theorem 5.3.

We now formally define light and heavy columns and cells, since
they play a key role in the proof of the performance of our index
structure.

**Definition 4.3.** For a given page size \( s \), a heavy column (resp.
cell) contains \( > s^2 \) (resp. \( > s \)) intervals. Conversely, a light column
(resp. cell) contains \( \leq s^2 \) (resp. \( \leq s \)) intervals.

**Example 4.4.** Figure 4 shows the grid constructed by Algorithm 2
on our example relation from Figure 2, with page size parameter
\( s = 70 \). Note that the columns, which contain \( s^2 = 4900 \) tuples each,
span different ranges of start times, adapting to the density of the
points. Within each column, the points are partitioned according to
the distribution of durations. Since many drug prescriptions have
the same short durations, the cells at the bottom of the columns are
heavy (or span only a few different duration values). In this setting,
a uniform grid would suffer from a high imbalance in the number of
tuples in each cell.

### 4.3 Querying the Index

Given a range-duration query with time range \( t = [t_\ell, t_e) \) and
duration interval \( d = [d_{\min}, d_{\max}] \), recall that a tuple \( r \in \rho \) satisfies
the query if the conditions (1), (2), and (3) specified in Section 4.1 are
met. We report the conditions here for convenience:

\[
[r.T] \in [d_{\min}, d_{\max}] \quad (1) \\
\min[r.T] < t_\ell \quad (2) \\
\max[r.T] < t_e \quad (3)
\]

The pseudocode for querying the index structure is reported in
Algorithm 3. First, we seek the index of the last column that might
contain matching tuples. To this end, we perform a binary search on
the array col_minstart to find the last column \( i \) such that the
minimum start time in the column (which is the column bound) is
strictly less than the query end time \( t_e \) (line 2). This ensures that the
column contains at least one tuple satisfying condition (3). Then, we
iterate backwards through columns until column \( i \) cannot possibly
contain tuples satisfying the query. For this, we use the support array
col_maxend: if \( \text{col_maxend}[i] \leq t_\ell \) then we know that all
the columns at index \( \leq i \) contain tuples that stop earlier than the start
of the query range. Hence, we can avoid inspecting them because of
condition (2).

For each column that we consider, a binary search on the array
cell_mindur[i] is performed, looking for the last cell \( j \) such that
the minimum duration in the cell (which is the cell bound) is \( \leq \)
to the maximum duration \( d_{\max} \) specified in the query (line 4). Doing
so ensures that at least one tuple in the cell satisfies the upper bound
of condition (1). Then, we iterate backwards through the cells until
we reach a cell whose maximum duration is less than the minimum
duration \( d_{\min} \) of the query. At this point we stop since condition (1)
can no longer be satisfied.

Finally, we iterate through the tuples of each considered cell by
decreasing end time and stop as soon as condition (2) is no longer
satisfied (line 7). All the intervals that satisfy the query are returned
in the result.

### 4.4 Updating the Index

Our index data structure can be extended to support both insertion
and removal of tuples.

**Interval Insertion.** To insert a tuple \( r \), we query the index for the
start time \( r.T_s \) and the duration \( |r.T| \) to identify the cell that should
Indexing Temporal Relations for Range-Duration Queries

Algorithm 3: QUERY (grid, [t₀, tₑ], [d_min, d_max])

Input: A range duration query with time range [t₀, tₑ] and duration range [d_min, d_max]; An index grid with the ancillary arrays col_minstart, cell_mindur, col_maxend, and cell_maxdur

1. res ← ∅;
2. i ← argmax_{j} col_minstart[i] < tₑ;
3. while i ≥ 0 ∧ tᵢ < col_maxend[i] do
4.     j ← argmax_{j} cell_mindur[i][j] ≤ d_max;
5.     while j ≥ 0 ∧ cell_maxdur[i][j] ≥ d_min do
6.         for r ∈ grid[i][j] do
7.             if r.τₑ ≤ tᵢ then
8.                 break;
9.             if [r.τ] ∈ [d_min, d_max] ∧ r.τₑ < tₑ then
10.                res ← res ∪ {r};
11.         j ← j − 1;
12.     i ← i − 1;
13. return res

contain r. Inserting new intervals into cells might make them grow too large to be able to maintain the performance guarantees on the query time. Luckily, as we shall see in Section 5, heavy columns and cells do not present issues upon insertions by virtue of containing intervals that all share either the same start time or the same duration. If a light column exceeds size s², we replace it with two new columns. Similarly, if a light cell exceeds size s, we replace it with two new cells.

Splitting a column entails to consider all the intervals it contains, using NEXTSUBSEQ with b = s²/2 + 1 to find the breakpoint at which to split (this way, we balance the size of the new columns). For each of the two new columns that replace the original column, we apply NEXTSUBSEQ to split them into cells, exactly as in the inner loop of the index construction. The array col_minstart and col_maxend are updated to reflect the replacement of the old column with the new ones.

Similarly, to split a cell that exceeds size s in column i we use NEXTSUBSEQ with b = s/2 + 1 to find a new breakpoint and replace the cell with two new cells. The auxiliary structures cell_mindur[i] and cell_maxdur[i] are updated accordingly.

Both in the case of column and cell splitting, we sort all the intervals in the newly created columns by end time.

Interval Removal. As for the removal of a tuple r from the index, we query the index to find the cell grid[i][j] that contains r.T and remove the interval from the cell. As a consequence, the cell might contain fewer than s/2 items. As we shall see with Lemma 5.2, it is crucial for the performance of the index that cells contain at least s/2 intervals.

Therefore, upon removal of an element from a cell, we check whether the sum of elements of the cell and either of the adjacent ones is less than s. In such case, we merge the two cells, i.e., we replace them with a single cell where all intervals are then sorted by decreasing end time. After cells are merged, the arrays cell_minstart[i] and cell_maxend[i] are updated as well to reflect the changes.

Similarly, a removal might cause a column to have fewer than s²/2 elements. We then apply a similar reasoning. If the sum of the number of items in the column and either adjacent ones is smaller than s²/2, we merge the two columns, i.e., the two columns are replaced by a single one. NEXTSUBSEQ is then called to find the breakpoints to divide the newly created column into cells. After the two columns are merged, the arrays col_minstart and col_maxend are updated to reflect the changes.

5 ANALYSIS

In this section, we provide guarantees on the time required by all the operations supported by our index structure. We assume that the index is built by partitioning first in the time dimension, and then in the duration dimension. The asymptotic results presented in this section hold for both dimension orderings.

5.1 Querying the Index

Before stating our main result, we establish the following fundamental facts about heavy columns and cells.

Lemma 5.1. A heavy cell in a heavy column contains only copies of the same interval.

Proof. By construction, a heavy column contains only intervals with the same start time. Similarly, a heavy cell contains only intervals with the same duration. Therefore, a heavy cell in a heavy column contains intervals with the same start time and duration, i.e., multiple copies of the same interval.

Lemma 5.2. The RD-index with parameter s over n intervals has O \left( \frac{n}{s^2} \right) columns, each having O \left( \frac{n}{s} \right) cells.

Proof. To upper bound the number of columns into which a relation can be partitioned, we first devise a set of intervals that will force RD-index to use the maximum number of columns. Consider a relation such that exactly s²/2 + 1 intervals have start time 1, s²/2 intervals have start time 2, s²/2 + 1 intervals have start time 3, and so on. RD-index will have to build a separate light column for each distinct start time. Note that if the subsets of intervals with the same start time were any smaller, RD-index would create columns containing more than one start time, resulting in fewer columns overall. Therefore, each column has size ≥ s²/2, which implies that there are O \left( \frac{n}{s^2} \right) columns.

As for cells, the worst case occurs if all intervals fall in the same column. With a reasoning similar to the above argument, we have that these n intervals in one column are partitioned in O(n/s) cells.

Theorem 5.3. Given an index over a set of n intervals and a page size s, the time for answering a range-duration query is

O \left( \frac{n}{s^2} \log \frac{n}{s} + \frac{n}{s} + s^2 + k \right)

where k is the number of intervals matching the query predicate.

Proof. Let t = [t₀, tₑ] be the query interval and d = [d_min, d_max] be the duration range of the query. Recall that the index construction
The cell’s durations all satisfy the query constraints, but not all. Only some of the cell’s intervals satisfy both constraints. In this case we cannot be in a heavy column, since it implies a partition of the end times. Furthermore, duration constraints are typically much more selective than range constraints. Therefore, range queries can benefit from a partition of the duration dimension, which contains a single start time) and thus the column contains all its durations do so. The cell must be light, otherwise all its durations would be equal and would need to satisfy the duration constraint, contradicting the assumption. Therefore, the algorithm has to evaluate \( O(s) \) intervals.

The cell’s start times all satisfy the query constraints, but not all its durations do so. The cell must be light, otherwise all its durations would be equal and would need to satisfy the duration constraint, contradicting the assumption. Therefore, the algorithm has to evaluate \( O(s) \) intervals.

The cell’s durations all satisfy the query constraints, but not all its start times do. In this case we cannot be in a heavy column (which contains a single start time) and thus the column contains at most \( s^2 \) intervals.

Only some of the cell’s intervals satisfy both constraints. In this case neither the column nor the cell can be heavy, therefore the algorithm must only evaluate \( O(s) \) intervals.

All the cells of type (1) account for at most \( O(k) \) in terms of running time. As for cells of type (2) there are at most \( O(n/s^2) \) of them. Overall they account for \( O(\frac{n}{s^2}) \) interval evaluations. Cells of type (3) account for \( O(s^2) \) interval evaluations overall. Finally, there is a constant number of cells of type (4) (which are at the corners of the query polygon). These cells therefore account for at most \( O(s) \) interval evaluations in total. Overall, the time to evaluate the intervals in the cells is \( O(\frac{n}{s^2} + s^2 + k) \).

Combining this last result with the time to find the range of cells to query, we have that the overall query time is

\[
O\left(\frac{n}{s^2} \log \frac{n}{s} + \frac{n}{s} + s^2 + k\right).
\]

The above theorem exposes a fundamental tradeoff of our data structure: using a smaller page size allows to improve the precision of the data structure (by looking at fewer intervals that are not part of the query output), while at the same time increasing the number of columns that need to be queried. In Section 6.5 we investigate experimentally the effect of \( s \) on the performance, finding that the best performance is attained for \( n/s^2 \in [50, 500] \).

The choice of the order of partitioning has no impact on the theoretical complexity results, however it affects the practical performance as we will discuss in Section 6 (Figure 10). It turns out that generally it is better to index first the duration and then the start time, as summarized in the following observation.

**Observation 1.** While changing the order in which dimensions are indexed does not change the asymptotic behavior of RD-INDEX, the choice of such order affects the practical performance. In fact range and duration constraints play different roles in queries. When querying for duration, the position of start times on the timeline does not affect the outcome of a query. Conversely, a range query can be rephrased in terms of start times and durations. Therefore, range queries can benefit from a partition of the duration dimension, since it implies a partition of the end times. Furthermore, duration constraints are typically much more selective than range constraints. Therefore, indexing first by duration might imply that a query has to iterate through fewer columns, compared to the case where start times are indexed first.

### 5.2 Index Construction and Update

**Theorem 5.4.** Given a set of \( n \) intervals, building the index requires time \( O(n \log n) \).

**Proof.** Computing column boundaries for the duration requires time \( O(n \log n) \) for sorting the tuples, while invoking Algorithm 1 requires time \( O(n) \) for iterating through the sorted intervals to find the boundaries: in fact, each of the \( n \) intervals is visited at most once. Let \( n_i \) be the number of intervals in the \( i \)-th column. Sorting the column requires time \( O(n_i \log n_i) \). Then, the invocation of Algorithm 1 to find cell boundaries requires time \( O(n_i) \), following the same argument as above. Similarly, let \( n_{ij} \) be the size of cell \( ij \). Sorting its tuples by end time requires time \( O(n_{ij} \log n_{ij}) \). Therefore, the time for running the inner loop is \( O(n \log n) \) overall, and the theorem follows.

**Theorem 5.5.** Inserting a tuple \( r \) into the index requires time \( O(\log n/s + n/s + s^2 \log s) \).

**Proof.** Finding the cell that should contain \( r.T \) requires performing first a binary search over the \( O(\frac{n}{s^2}) \) columns and then over the \( O(\frac{n}{s^2}) \) cells of the resulting column (by Lemma 5.2), for a total time \( O(\log n/s) \).

Note that a heavy column or cell does not need to be split, since by definition they can contain more than \( s^2 \) (resp. \( s \)) intervals, as long...
Indexing Temporal Relations for Range-Duration Queries

as they all have the same start time (resp. duration). Therefore we consider only the case of splitting a light column or light cell. Splitting a column with \( s^2 + 1 \) intervals requires to first apply Algorithm 1 (which takes time \( O(s^2) \), sort the newly created columns (requiring time at most \( O(s^2 \log s) \)) and then applying again Algorithm 1 on the new columns to find the cells (for an overall time of \( O(s^2) \)). Furthermore, inserting the new column boundaries in order in the sorted array of boundaries requires \( O\left(\frac{n}{s}\right) \) time. By similar reasoning, splitting a cell with \( s + 1 \) intervals requires time \( O(s \log s) \), and updating cell boundaries by inserting in-order requires \( O\left(\frac{n}{s}\right) \).

In a similar way we can prove the following theorem.

**Theorem 5.6.** Removing a tuple \( r \) from the index requires time \( O(\log n/s + n/s) \).

### 6 EXPERIMENTAL EVALUATION

In this section, we evaluate experimentally our proposed index. To frame this evaluation, we consider the following data structures as baselines: The implementation of B-TREE provided by the Rust standard library, which is optimized for CPU cache usage; intervals are indexed by duration in this case. The INTERVAL-TREE index [25], which we implemented ourselves, indexing intervals by their position on the timeline. The GRID-Filename [31] and PERIOD-INDEX [4], which we also implemented, and which index both start times and durations. We also consider the R-TREE as a baseline, specifically the R*-TREE [3]: intervals are mapped to points identified by the start time and duration of the interval, and then indexed by the R*-TREE. Our proposed data structure will be denoted with RD-INDEX-TD when start time is indexed before duration, and with RD-INDEX-DT otherwise. In cases where the order of the dimensions is not relevant to the discussion, we will use RD-INDEX instead. To account for the potential shortcomings of our implementations, we will also evaluate the relative performance of the data structures with implementation-independent metrics [26].

We aim to answer the following questions:

(Q1) How robust are the indices over different workloads? (§ 6.2)
(Q2) How do indices compare on mixed workloads? (§ 6.3)
(Q3) How do query times relate with the query selectivity? (§ 6.4)
(Q4) How does the page size parameter affect the performance of the index? (§ 6.5)
(Q5) How does the performance scale with respect to the size of the dataset? (§ 6.6)
(Q6) What is the performance of updating the index? (§ 6.7)

### 6.1 Setup and Datasets

We implemented our index and the baseline competitors in Rust 1.44.1, using a configurable and extensible framework [2]. Code and data are available at https://github.com/Cecca/temporal-index. The experiments presented in this section were run on a machine equipped with 94 GB of memory and a Intel®Xeon®CPU E5-2667 v3 @ 3.20GHz processor.

As a benchmark we consider the following datasets and workloads:

*Flight:* A set of 701,353 flights, identified by their takeoff and landing time at the granularity of minute, covering August 2018.

Query ranges on this dataset are generated at random. Time range durations are uniformly distributed between one and 31 days, and duration ranges are uniformly distributed between one minute and one day.

*Webkit:* 1,547,419 file edits in the Webkit source code repository. Intervals represent the timespan between successive edits to a file. Query ranges on this dataset are generated at random. Time range durations are uniformly distributed between one minute and one year, and duration ranges are uniformly distributed between up to three years.

*MimicIII:* 4,134,909 drug prescriptions from the open MimicIII database [20]. Each prescription is characterized by its start and end day. Queries, generated at random, span the entire domain of times and durations: the former take values \( \in [1,40,251] \), the latter \( \in [1,200] \). The large span of start times (110 years) is due to the anonymization procedure applied to the database.

**Synthetic:** Randomly generated datasets with 10 million intervals by default. The interval start times are uniformly distributed in \([1, n]\), where \( n \) is the size of the dataset; interval durations follow a Zipf distribution with \( \beta = 1 \).

Figure 5 reports the distribution of start times and durations of the three real-world datasets. Note that the start times of the Flight dataset are approximately uniformly distributed, with durations geometrically distributed. The MimicIII dataset has similarly distributed start times, and duration values which are very concentrated. Finally, the Webkit dataset has a distribution of durations similar to Flight, but the distribution of start times is skewed towards recent times, meaning that more file edits have been done recently.

In the experiments we run a count query subject to range and duration constraints. Evaluating a count query rather than reporting the matching tuples, allows to measure the time to retrieve the tuples, rather than the time to print them to screen or write them to a file.

### 6.2 Robustness of Index Structures Across Different Workloads

In the first set of experiments, we consider both real-world and synthetic datasets (with 10 million intervals). Table 1 reports, under different combinations of dataset/query workload, an overview on the performance of different index structures on three indicators: the queries per second, the time to build the index, and the size of the index. The latter is measured in terms of bytes per interval, i.e., the number of bytes that the index uses for each input interval. Since we are representing intervals as pairs of 64-bits unsigned integers, 16 bytes per interval are required just to represent the data, and thus are a lower bound on this performance metric. Dark blue and light blue cells denote, respectively, the best and second-best performing data structures in terms of queries per second. For index structures that take parameters, we report on the best configuration. In particular, we defer the discussion of the effect of different parameterizations of RD-INDEX to Section 6.5.

We remark that the overall difference in throughput for different workloads is due to the different output sizes: duration-only queries are in general less selective than range-duration queries, hence it takes more time to iterate through the output. This explains why, in
Table 1: Performance attained by the tested indices on different workloads. Performance is characterized in terms of three numbers: the number of queries per second (larger is better), the time to build the index in milliseconds (smaller is better), and the index size (smaller is better) expressed as the number of bytes used for each indexed interval. Configurations with a dark blue background are the ones with the best throughput for a dataset/queryset combination, light blue cells denote the second-best data structure. The fastest index construction times are underlined, and the smallest indices are highlighted in bold.

| dataset | query       | RD-INDEX-TD | RD-INDEX-DT | GRID-FIFE | PERIOD-INDEX* | R*-TREE | INTERVAL-TREE | B-TREE |
|---------|-------------|--------------|-------------|-----------|--------------|---------|---------------|--------|
| Synthetic | range-only | 415 | 11024 | 16.1 | 602 | 889 | 16.1 | 468 | 445 | 27.7 | 242 | 2392 | 57.1 | 12 | 3358 | 80.0 | 272 | 3636 | 50.1 | 8 | 2950 | 31.4 |
|          | duration-only | 842 | 11030 | 16.1 | 734 | 785 | 17.1 | 77 | 1983 | 24.1 | 69 | 6139 | 178.1 | 106 | 3988 | 80.0 | 12 | 3636 | 50.1 | 43 | 1038 | 2469 | 31.4 |
|          | range-duration | 11737 | 11052 | 16.1 | 140085 | 829 | 16.5 | 1581 | 1857 | 24.1 | 1403 | 6566 | 178.1 | 2591 | 3402 | 80.0 | 215 | 3674 | 50.1 | 502 | 2529 | 31.4 |
| Flight   | range-only | 54945 | 142 | 17.1 | 57471 | 16.1 | 49505 | 20 | 23.2 | 15015 | 180 | 40.0 | 4921 | 195 | 97.9 | 51813 | 139 | 44.5 | 2798 | 39 | 23.3 |
|          | duration-only | 4182 | 144 | 16.1 | 4218 | 131 | 21.3 | 3791 | 20 | 23.2 | 538 | 111 | 62.9 | 378 | 205 | 97.9 | 683 | 150 | 44.5 | 714266 | 43 | 23.3 |
|          | range-duration | 212598 | 45 | 16.7 | 208333 | 39 | 16.6 | 188679 | 20 | 23.2 | 29940 | 95 | 62.9 | 21786 | 193 | 97.9 | 43860 | 139 | 44.5 | 13514 | 47 | 23.3 |
| Webkit   | range-only | 416 | 116 | 16.1 | 467 | 115 | 16.1 | 445 | 36 | 23.0 | 83 | 540 | 119.0 | 30 | 468 | 97.7 | 384 | 274 | 48.3 | 47 | 252 | 36.4 |
|          | duration-only | 2544 | 112 | 16.1 | 2520 | 104 | 16.9 | 1938 | 32 | 23.0 | 57 | 559 | 124.4 | 234 | 469 | 97.7 | 157 | 274 | 48.3 | 3393 | 255 | 36.4 |
|          | range-duration | 1357 | 112 | 16.1 | 2333 | 99 | 16.9 | 2322 | 51 | 23.0 | 80 | 737 | 322.6 | 258 | 468 | 97.7 | 256 | 274 | 48.3 | 758 | 252 | 36.4 |
| MimicIII | range-only | 372 | 240 | 16.0 | 201 | 213 | 16.0 | 391 | 143 | 23.5 | 381 | 1451 | 24.3 | 38 | 1112 | 81.2 | 347 | 717 | 46.6 | 127 | 134 | 21.2 |
|          | duration-only | 3560 | 275 | 16.0 | 2876 | 202 | 17.5 | 2075 | 184 | 27.7 | 520 | 718 | 41.3 | 452 | 1175 | 81.2 | 76 | 733 | 46.6 | 2500000 | 141 | 21.2 |
|          | range-duration | 10254 | 241 | 16.0 | 8020 | 201 | 17.5 | 5227 | 185 | 27.7 | 1779 | 720 | 41.3 | 1384 | 1081 | 81.2 | 232 | 721 | 46.6 | 3347 | 133 | 21.2 |
We investigate the relationship between the selectivity of queries (i.e., workloads where duration-only queries are the majority, where the workloads. We observe that RD-INDEX is rather robust to the ordering of indexing dimensions. For instance, the center of each triangle corresponds to a workload composed in equal parts by the three types of queries. Portions of the triangles are colored according to the best-performing index for the corresponding workloads. We observe that RD-INDEX is the best performing index structure in the vast majority of workloads. The exception is for workloads where duration-only queries are the majority, where the B-TREE outperforms RD-INDEX. On mixed workloads, we observe that the performance of RD-INDEX is rather robust to the ordering of indexing dimensions. This can be verified in Figure 13 in Appendix A, which reports ternary plots of the throughput of all algorithms on all datasets for all mixed workloads. The online supplemental material provides an interactive tool to explore the performance of all different index structures on any mixed workload.

6.4 Distribution of Query Times Against Selectivity

We investigate the relationship between the selectivity of queries (i.e., the fraction of the input that satisfies them) and the time taken by different data structures to answer them. Given that range-duration queries constrain both the time and the duration ranges, for a query we can define the \( \text{time selectivity} \) as the fraction of the input that satisfies the time range constraint of the query, and the \( \text{duration selectivity} \) as the fraction of the input satisfying the duration constraint of the query.

For a given dataset, we build a query set such that queries are uniformly distributed on the \( \text{time} \times \text{duration} \) selectivity plane: this way we have queries that are very selective in only one dimension, very selective in both dimensions, or not selective at all. The goal is to investigate the behavior of each index structure for queries with different characteristics. To account for the overhead of measuring time and to level out the effect of the CPU cache, we run each query 100 consecutive times and report the average. For RD-INDEX, we set the page size to \( s = 200 \), which is a good parameter choice of all the datasets we consider.

Figure 7 reports the results for such a setup, with 1024 queries arranged in a \( 32 \times 32 \) grid, running on a synthetic dataset with 10 million intervals, with uniformly distributed start times and Zipf distributed durations. Colors encode the fraction of the dataset inspected by each query. This metric allows a more implementation-independent assessment of the relative performance of different data structures. When reading Figure 7, remember that less selective queries require more time just to iterate through the output. Interestingly, different data structures exhibit different patterns in this plot, as a result of how they access data.

The INTERVAL-TREE plot exhibits vertical bands. The data structure is able to select intervals only based on their position on the timeline. Therefore, for a fixed time selectivity of the query the fraction of intervals inspected (and thus the time to answer the query) does not depend on the selectivity in the duration dimension, since all the candidate intervals need to be examined. For similar reasons, the B-TREE shows horizontal bands. Data structures that explicitly index both dimensions, instead, tend to exhibit a more diagonal pattern, in particular RD-INDEX, with a milder effect for GRID-FILE and R*-TREE. The pattern exhibited by PERIOD-INDEX★ tends to be more similar to the INTERVAL-TREE. This means that this index is more responsive to queries that are selective in the time dimension. This is to be expected since PERIOD-INDEX★ is adaptive to the distribution of start times in the dataset. Its worse performance compared to RD-INDEX and GRID-FILE for queries of a given selectivity is explained by the fact that some intervals might be represented multiple times in the index.

Figure 8 reports the performance against the overall selectivity of the same queries. The performance is assessed in terms of the fraction of the input inspected by each query, which are represented as dots whose position along the \( x \)-axis encodes their overall selectivity. Ideally, a data structure should answer queries by inspecting just the tuples which are part of the output. This behavior is represented by the black diagonal in Figure 8. We observe that RD-INDEX indeed shows the ideal behavior. As for the GRID-FILE, since the efficiency in answering a query depends on the density of the cells being considered, the performance is in many cases far from ideal. This can be seen from the fact that several queries are far away from the ideal diagonal. The B-TREE indexes intervals by their duration. As such, duration-only queries are answered most efficiently: In Figure 8 such queries lie on the ideal diagonal. On the other hand,
Figure 7: Heatmaps of the performance of index structures against selectivity in the time and duration dimensions. The top row reports the time, in milliseconds, required to answer a query with a given time and duration selectivity. The plots report the fraction of intervals (over the total $n = 10^7$) examined by each query. A fraction larger than 1 means that a query examined the same interval more than once. For readability the color scales are binned every 5 percentiles, therefore the scales are non linear. In both plots, lighter is better.

Figure 8: Number of examined intervals against selectivity of the query. Each point represents a query: red dots • are range-only queries, blue dots • are duration-only queries, black dots • are range-duration queries. The black line represents the ideal behavior, in which only matching intervals are examined. Note that all the plots except for the one related to the PERIOD-INDEX★ share the y axis: in all plots the black diagonal has slope 1.

range-only queries are answered by simply enumerating the entire dataset, thus scoring 1 on Figure 8. Similar considerations hold for the INTERVAL-TREE, with range-only queries performing the best and duration-only queries performing the worst. Finally, PERIOD-INDEX★ inspects the same tuples multiple times for the majority of queries, which are thus very far away from the ideal diagonal line in the plot.
Indexing Temporal Relations for Range-Duration Queries

So far, we have reported on the performance of a good configuration (i.e. $s=200$) of RD-INDEX, which gives a behavior close to optimal. Figure 9 reports, in the same scenario of Figure 8, the performance of RD-INDEX with a time-duration ordering and page size $s=10$. As we will see in the next section, this configuration is not ideal (the page size is too small): indeed, for several queries we have to visit more intervals than the precision of the query. This is especially true for duration-only queries (blue dots). Nevertheless, even under this non-optimal setting, RD-INDEX still performs better than the other approaches, i.e., points are more concentrated towards the diagonal.

6.5 Dependency on the Page Size and the Indexing Order

The aim of this section is to investigate the influence of the page size and the order in which the two dimensions are indexed. We use two datasets with 10 million intervals each: the first has uniform start times and skewed durations (Zipf distribution), the second has skewed start times and uniformly distributed durations. As for the query workload, batches of 10 000 range-duration, range-only, and duration-only queries are considered. The page size is varied between 1 and 10 000.

Due to the size of the dataset, any page size $s$ above $\sqrt{n} \approx 3162$ results in a degenerate configuration, where there is a single column (of size $s^2 = n$) that contains all the intervals. In this case the intervals are partitioned according to a single dimension, with the number of cells controlled by the page size parameter. This situation might also occur at lower values of the page size parameter, depending on the number of distinct values in the dimension being partitioned. On the other hand, for page size 1, each cell of the grid contains only intervals with the same start time and duration, and it contains all of them. Querying the data structure in this case amounts to perform binary searches directly on the values of the domains of start times and durations.

Figure 10 reports the results of this experiment in terms of queries per second. To ease the comparison between the plots, we rescale the throughput by the highest value for each combination of dataset and query workload. Degenerate configurations resulting in a single column are reported as triangles rather than dots. We observe very different trends for different query workloads.

Consider first the dataset with skewed durations. For range-duration queries, both orderings of dimensions exhibit a similar behavior. The peak performance is reached by intermediate values around the page size. If we consider duration-only queries the profile changes. Indexing first by duration (blue line) slightly favors smaller page sizes, which imply smaller columns and thus a more fine grained access to the data. Indexing first by time, instead, requires a duration-only query to traverse all the columns. In this scenario high page sizes are favored, since they translate into fewer columns to be iterated through. For range-only queries we observe a symmetric pattern.

When data has skew on the start times, the patterns exhibited by the two indexing orders are rather different. First, we note that the gap between the best configurations of the two indexing orders for range-duration queries is much wider than with the other dataset. Second, while indexing first by duration exhibits a similar pattern on both datasets, indexing first by start time performs best in the degenerate cases of a single column, i.e., with no partitioning of the start times at all. This is a consequence of Observation 1. In particular, having skewed start times exacerbates the difference in selectivity between the range and duration constraints: when start times are very concentrated on the timeline, the range constraint of a query is satisfied by either most of the intervals or by almost none.

Overall, we observe that indexing first by duration has either better or comparable performance compared to indexing first by time. Therefore, we recommend to choose the former ordering of dimensions when building an RD-INDEX.
6.6 Scalability with Respect to the Input Size

To test the scalability of the index structures, we consider range-duration queries and datasets of increasing size, while maintaining the output size constant. This allows to assess the influence of the input size on the performance without conflating the results with the time required to iterate over larger outputs.

We consider the three real-world datasets, along with a synthetic one with uniform start times and Zipf-distributed durations, which is representative of the distribution of many real-world datasets. Then, we artificially increase their size as follows. Given a dataset and a scale parameter $\eta$, let $\bar{t}$ be the span of time covered by all the intervals in the dataset. We make $\eta$ copies of each interval and shift copy $i$ in time by $i \cdot \bar{t}$, for $i \in (0, \eta)$. The underlying idea is to repeat the temporal patterns of the dataset on a longer time scale, simulating the scenario in which the relation grows over time.

Figure 11 reports for each scale factor the performance of the best configuration of each algorithm. First, we note that in general the relative performance of the data structures does not change at different dataset scales. There are some notable exceptions. The performance of B-TREE degrades by a factor $\approx 10$ from scale 1 to scale 10. The reason is that the B-TREE indexes the durations, and under our synthetic construction the number of intervals associated to each duration increases by the same scale of the dataset. For similar reasons, the performance of RD-INDEX-DT degrades, albeit in a much less pronounced way.

On Webkit, the performance of GRID-FILE and PERIOD-INDEX☆ increases with the scale as the dataset: the effect of our synthetic construction in this case is to compensate for the skew in the start times, giving to both data structures the chance of better partitioning the time dimension.

6.7 Insertion Performance

We now focus on the insertion operation, considering RD-INDEX, B-TREE, R*-TREE, and INTERVAL-TREE. We omit from the comparison PERIOD-INDEX☆, which does not support updates, and GRID-FILE, which is a static data structure that requires to know the range of the data beforehand.

For each of the four datasets we considered in the previous sections, we insert intervals into initially empty indices. The expectation is that the insertion performance degrades as the index grows larger. To measure this effect, we insert the intervals in batches of 50,000, measuring the time for each batch in order to be able to estimate the throughput of the insertions as the size of the index grows. We perform two sets of experiments. In the first set of experiments, the intervals are inserted in random order. In the second set of experiments, intervals are inserted by increasing start time, which simulates a natural append only scenario for time-related data.

Figure 12 reports the results of these experiments. The x-axis reports the fraction of the dataset that has been inserted into the index. The y-axis reports, in logarithmic scale, the number of insertions per second. In most cases, the best performing data structure for the insertion workload is the B-TREE, both when data is presented in random and sorted order. RD-INDEX follows on the second place for most datasets, with the ordering first by time and then by duration usually performing better. For randomly-ordered insertions, we note that the performance of RD-INDEX tends to slightly decrease as the index size increases. In the more realistic append only scenario, instead, the insertion throughput of RD-INDEX is more stable and tends to remain constant over time. This is expected, since in such a setting only the last column (when the start time is the first dimension being indexed, otherwise the last cell of each column) is ever restructured, requiring very little data to be moved. Furthermore, the performance in this scenario improves compared to the random order of insertions, in particular on the MimicIII dataset, and is on par with the B-TREE on all datasets.

7 CONCLUSIONS

RD-INDEX is an index data structure for temporal intervals that allows to answer efficiently range-duration queries. Our approach, which has provable theoretical guarantees, lends itself to a simple and efficient implementation. In particular, its ability to adapt to the distribution of the input data makes it compare favorably with the state of the art on a variety of workloads. In particular, RD-INDEX has superior performance on a vast array of mixed workloads.

A direction of future work is to extend the RD-INDEX to support interval joins [35], thus addressing several needs with a single index. Furthermore, the favorable comparison with the R*-TREE suggests that a promising research direction is to extend the ideas on which RD-INDEX is based to the case of multidimensional spatial data.

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Indexing Temporal Relations for Range-Duration Queries

Figure 12: Performance of the insertion for different index structures. The left group of plots (a) reports the throughput of insertions when intervals are added in random order. The right group of plots (b) focuses on insertions by increasing start time, i.e. a append only scenario.

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Figure 13 reports a collection of ternary plots, each focusing on a particular combination of dataset (columns) and index structure (rows). In order to accommodate on a readable color scale the wide range of throughputs encompassed by indices, we color-code the throughput divided by percentiles. Therefore, brighter colors correspond to high throughputs, dark colors correspond to low throughputs. Furthermore, these throughput percentiles are computed per dataset.

Therefore, we can read Figure 13 as follows. For a given dataset (say Random) we can take two different algorithms (e.g. RD-INDEX and B-TREE) and compare the color in any given area of the two corresponding ternary plots. For instance focusing on the bottom-right corner of the ternary plots, corresponding to workloads comprised mostly of range-duration queries, we observe that the plot for RD-INDEX features a brighter color than the one for B-TREE. This means that the throughput of RD-INDEX is higher than the one of B-TREE for those workloads.

From Figure 13 we can therefore notice that the behavior of RD-INDEX-DT and RD-INDEX-TD is very similar, across all datasets. Furthermore, we notice that with the exception of workloads where the vast majority of queries are duration-only, RD-INDEX enjoys a higher throughput than other indices.

A ADDITIONAL FIGURES

In this appendix we report on the complete results for mixed workloads of which Figure 6 in Section 6.3 is a compact summary.
Indexing Temporal Relations for Range-Duration Queries

Figure 13: Performance of all index structures on all considered datasets, for different mixed workloads.