Extraction of nuclear matter properties from nuclear masses by a model of equation of state
K. C. Chung, C. S. Wang and A. J. Santiago
Instituto de Física, Universidade do Estado do Rio de Janeiro, Rio de Janeiro-RJ 20559-900, Brazil

Abstract

The extraction of nuclear matter properties from measured nuclear masses is investigated in the energy density functional formalism of nuclei. It is shown that the volume energy \( a_1 \) and the nuclear incompressibility \( K_0 \) depend essentially on \( \mu_n N + \mu_p Z - 2E_N \), whereas the symmetry energy \( J \) and the density symmetry coefficient \( L \) as well as symmetry incompressibility \( K_s \) depend essentially on \( \mu_n - \mu_p \), where \( \mu_p = \mu_p - \partial E_C / \partial Z \), \( \mu_n \) and \( \mu_p \) are the neutron and proton chemical potentials respectively, \( E_N \) the nuclear energy, and \( E_C \) the Coulomb energy. The obtained symmetry energy is \( J = 28.5 MeV \), while other coefficients are uncertain within ranges depending on the model of nuclear equation of state.

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1 Introduction

The main measured quantities which are used to extract the properties of standard nuclear matter are the nuclear mass \( M \) and nuclear isoscalar giant monopole resonance energy \( E_M \). By nuclear matter we mean the uncharged nucleon system distributed uniformly in the space, and by standard nuclear matter we mean the groundstate nuclear matter with equal neutron and proton numbers. By properties of standard nuclear matter we mean the volume energy coefficient \( a_1 \), the symmetry energy coefficient \( J \), the incompressibility coefficient \( K_0 \), the density symmetry coefficient \( L \), and the symmetry incompressibility coefficient \( K_s \). Nowadays the most interesting quantity for heavy ion collision studies as well as for supernova explosion calculations is the nuclear incompressibility \( K_0 \).

In the earlier stage of the nuclear matter study, the nuclear incompressibility \( K_0 \) was determined in a compressible model of nuclei, e.g., the droplet model, from nuclear masses \( M \). A previous calculation by this approach based on Seyler-Blanchard interaction gave \( K_0 = 306 MeV \), and a recent investigation by using the droplet model based on the generalized Seyler-Blanchard interaction yielded \( K_0 = 234 MeV \). On the other hand, in a collective model of the nuclear breathing mode, e.g., the model based on the scaling assumption, \( K_0 \) ...

\(^1\)E-mail address: chung@uerj.br
\(K_0\) can be extracted from nuclear monopole resonance energies \(E_M\). An earlier attempt in this line based on Skyrme interaction gave \(K_0 = 210\text{MeV}\). However, a later study claimed that more accurate data of monopole resonance energy lead to a higher value \(K_0 \approx 300\text{MeV}\).

The staggering of the values of \(K_0\) stimulated many works on this topic. The purpose of this paper is to study the problem of extracting the nuclear incompressibility \(K_0\) from nuclear masses \(M\) as directly as possible. In addition, the extraction of the volume energy \(a_1\), the symmetry energy \(J\), the density symmetry coefficient \(L\), and the symmetry incompressibility \(K_s\) is discussed simultaneously. In Sec.2 the theory used in the present work is formulated. A model of nuclear equation of state is presented in Sec.3, while the approximation by leptodermous expansion is given in Sec.4. In Sec.5 a short discussion and summary are addressed.

2 Theory

The total energy \(E(A, Z)\) of a nucleus with nucleon number \(A\) and proton number \(Z\) can be written as

\[
E(A, Z) = E_N + E_C + E_{res},
\]

where \(E_N\) is the nuclear energy, \(E_C\) the Coulomb energy, and \(E_{res}\) the residual energy which includes the shell correction, the even-odd energy and the congruence energy. The nuclear energy \(E_N\) can be expressed as

\[
E_N = \int d^3r \mathcal{E}_N,
\]

where the nuclear energy density functional \(\mathcal{E}_N\) can be written with enough generality as

\[
\mathcal{E}_N = \rho(r) e(\rho, \delta) + \frac{1}{2} Q_1 (\nabla \rho)^2 + Q_2 \left[ (\nabla \rho_n)^2 + (\nabla \rho_p)^2 \right].
\]

In above equation, \(\rho_n\) and \(\rho_p\) are the neutron and proton densities, respectively, \(\rho = \rho_n + \rho_p\) the nucleon density,

\[
\delta = \frac{\rho_n - \rho_p}{\rho},
\]

the relative neutron excess, \(e(\rho, \delta)\) the nuclear equation of state, and \(Q_1, Q_2\) the model parameters related to the finite size and surface effects of nuclei.

The Euler-Lagrange equations obtained by the minimization of \(E(A, Z)\) are

\[
e + \rho \frac{\partial e}{\partial \rho} + (1 - \delta) \frac{\partial e}{\partial \delta} - Q_1 \nabla^2 \rho - 2Q_2 \nabla^2 \rho_n - \mu_n = 0,
\]
\[ e + \rho \frac{\partial e}{\partial \rho} - (1 + \delta) \frac{\partial e}{\partial \delta} - Q_1 \nabla^2 \rho - 2Q_2 \nabla^2 \rho_p - \pi_p = 0, \quad (6) \]

\[ \pi_p = \mu_p - \frac{\partial E_C}{\partial Z}, \quad (7) \]

where \( \mu_n \) and \( \mu_p \) are the neutron and proton chemical potentials, respectively.

Eqs. (6) and (7) can be reduced to

\[ e + \rho \frac{\partial e}{\partial \rho} - \delta \frac{\partial e}{\partial \delta} - Q_2 \nabla^2 \rho = \frac{1}{2}(\mu_n + \pi_p), \quad (8) \]

\[ \frac{\partial e}{\partial \delta} - Q_2 \nabla^2 (\rho \delta) = \frac{1}{2}(\mu_n - \pi_p), \quad (9) \]

where \( Q = Q_1 + Q_2 \). By substituting \( \partial e/\partial \delta \) solved from Eq. (9) into Eq. (8), it results

\[ e + \rho \frac{\partial e}{\partial \rho} - Q \nabla^2 \rho - Q_2 \delta \nabla^2 (\rho \delta) = \frac{1}{2}[(1 + \delta) \mu_n + (1 - \delta) \pi_p]. \quad (10) \]

It is worthwhile to note that, if the nuclear surface and the Coulomb energies are omitted and the equilibrium condition \( \partial e/\partial \rho = 0 \) is assumed, the above equation is reduced to

\[ e = \frac{1}{2}[(1 + \delta) \mu_n + (1 - \delta) \mu_p]. \quad (11) \]

This is the so-called generalized Hugenholtz-Van Hove theorem [12] which is used successfully in nuclear mass data fit [13].

By integrating Eqs. (8) and (10) over the nucleon density, the following relationships can be obtained:

\[ \int d^3r \rho \left[ \frac{\partial e}{\partial \delta} - Q_2 \nabla^2 (\rho \delta) \right] = \frac{1}{2}(\mu_n - \pi_p) A, \quad (12) \]

\[ \int d^3r \rho \left[ -e + \rho \frac{\partial e}{\partial \rho} \right] = \mu_n N + \pi_p Z - 2E_N, \quad (13) \]

which are the basic equations used in the present work. In obtaining Eq. (13), Eqs. (8) and (10) are used.

The nuclear energy \( E_N \) can be calculated by the following formula in MeV [3]:

\[ E_N = \Delta M - 8.071431 N - 7.289034 Z + 0.00001433 Z^2.39 - E_C - E_{res}, \quad (14) \]

where the measured mass excess \( \Delta M \) can be taken from the table given in Ref. [14], the residual energy can be calculated by the empirical formulas of Ref. [14], while the Coulomb energy can be determined by the following formula [13]:

\[ E_C = \frac{3}{5} \frac{e^2 Z^2}{R} \left[ 1 - \frac{5}{2} \left( \frac{b}{R} \right)^2 \right] + 3.0216 \left( \frac{b}{R} \right)^3 + \left( \frac{b}{R} \right)^4 \]. \quad (15) \]
In above equation, \( b \) is the Süssmann width which is related to the surface diffuseness \( d \) as \( d = \sqrt{3}b/\pi \), and \( R = r_0 A^{1/3} \), \( r_0 \) the nuclear radius constant. The Coulomb exchange energy is not included in this equation, since we want to make a comparison with the Myers-Swiatecki’s Thomas-Fermi calculation\(^3\) where this contribution is not considered. However, the inclusion of the Coulomb exchange energy in the present scheme is easy and we have checked that it gives no significant change in the numerical result.

The chemical potentials \( \mu_n \) and \( \mu_p \) can be obtained from nuclear masses by the following formulas\(^13\):

\[
\mu_n = \frac{1}{2} [E(A + 1, Z) - E(A - 1, Z)], \quad (16)
\]

\[
\mu_p = \frac{1}{2} [E(A + 1, Z + 1) - E(A - 1, Z - 1)]. \quad (17)
\]

Similarly, \( \partial E_C/\partial Z \) can be calculated as

\[
\frac{\partial E_C}{\partial Z} = \frac{1}{2} [E_C(A + 1, Z + 1) - E_C(A - 1, Z - 1)]. \quad (18)
\]

Thus, the righthand sides of Eqs.\( (12) \) and \( (13) \) are known from the measured mass excess \( \Delta M \) and the calculated Coulomb energy \( E_C \) as well as the empirical residual energy \( E_{res} \). Therefore, these two equations can be used to determine the equation of state and thus the nuclear matter properties if the nucleon density \( \rho(r) \) is known also from experiment.

The \( \mu_n - \mu_p \) obtained by this way is shown in Fig.1 versus the nuclear asymmetry \( I = (N - Z)/A \), while \( \mu_n - \mu_p \) versus \( I \) is shown in Fig.2. It can be seen that there is an approximate linear correlation between \( \mu_n - \mu_p \) and \( I \), but the correlation between \( \mu_n - \mu_p \) and \( I \) is not so simple. This correlation will be explained by the leptodermous expansion given in Sec.4. In addition, Fig.3 shows the data of \( (\mu_n N + \mu_p Z - 2E_N)/A \) obtained by this way versus \( A^{-1/3} \). The total number of masses is 1654, while the total number of \( \mu_n - \mu_p \) and \( (\mu_n N + \mu_p Z - 2E_N)/A \) data obtained from these masses is 1140 each one.

### 3 Model of nuclear equation of state

It is shown that for a variety of nuclear interactions the nuclear equation of state can be written with enough generality as

\[
e(\rho, \delta) = T \left[ D_2(\delta) \left( \frac{\rho}{\rho_0} \right)^{2/3} - D_3(\delta) \left( \frac{\rho}{\rho_0} \right)^{3/3} + D_{\gamma}(\delta) \left( \frac{\rho}{\rho_0} \right)^{\gamma/3} \right], \quad (19)
\]

where \( \rho_0 = 3/4\pi r_0^3 \) is the standard nuclear matter density, \( T \) an appropriate constant with dimension of energy such that \( D_2(\delta), D_3(\delta) \) and \( D_{\gamma}(\delta) \) are dimensionless, and \( \gamma \) a model parameter. It is convenient to choose \( T \) as the Fermi
energy of the standard nuclear matter,  

\[ T = \left( \frac{\hbar^2}{2m} \right) \left( 3\pi^2 \rho_0 / 2 \right)^{2/3} \]  

(See more details in Ref. [11].)

For standard nuclear matter, we have \( \partial e / \partial \rho |_{0} = 0 \) at \( \rho = \rho_0 \) and \( \delta = 0 \), the following relationship among \( D_{20}(0), D_{32}(0), D_{\gamma}(0) \) and \( \gamma \) should be satisfied:

\[ 2D_{20}(0) - 3D_{32}(0) + \gamma D_{\gamma}(0) = 0. \]  

(20)

The following formulas of nuclear matter properties can be obtained:

\[ a_1 = -e(\rho_0, 0) = -\frac{T}{3} \left[ D_{20} - (\gamma - 3) D_{\gamma} \right], \]  

(21)

\[ K_0 = 9\rho_0^2 \frac{\partial^2 e}{\partial \rho^2} |_{0} = T \left[ -2D_{20} + \gamma(\gamma - 3) D_{\gamma} \right], \]  

(22)

\[ J = \frac{1}{2} \frac{\partial^2 e}{\partial \delta^2} |_{0} = T \left[ D_{22} - D_{32} + D_{\gamma} \right], \]  

(23)

\[ L = \frac{3}{2} \rho_0 \frac{\partial^2 e}{\partial \rho \partial \delta^2} |_{0} = T \left[ 2D_{22} - 3D_{32} + \gamma D_{\gamma} \right], \]  

(24)

\[ K_s = \frac{9}{2} \rho_0^2 \frac{\partial^4 e}{\partial \rho^2 \delta^2} |_{0} = T \left[ -2D_{22} + \gamma(\gamma - 3) D_{\gamma} \right], \]  

(25)

where

\[ D_{i0} = D_i(0), \quad D_{i2} = \frac{1}{2} \frac{\partial^2 D_i}{\partial \delta^2} |_{0}, \quad i = 2, 3, \gamma, \]  

(26)

and the relation (20) is used in obtaining Eqs. (21) and (22).

Since the nuclear interaction is symmetric in proton and neutron, \( D_i(\delta) \) are even functions of \( \delta \). The specific discussion for these dependences is beyond the scope of this paper and will be given in a forthcoming paper. Hence, the coefficients \( D_i(\delta) \) can be written approximately as linear function of \( \delta^2 \) for the integrals involved in Eqs. (12) and (13). In addition, for stable nuclei, where \( \delta \) is small, we have:

\[ D_i(\delta) = D_{i0} + D_{i2}\delta^2, \quad i = 2, 3, \gamma. \]  

(27)

Due to the equilibrium condition (20), there are only 2 independent coefficients among 3 \( D_{i0} \), they can be chosen as \( D_{20} \) and \( D_{\gamma} \). Therefore, there are 5 independent coefficients within the above approximation (27): \( D_{20}, D_{\gamma}, D_{22}, D_{32} \) and \( D_{\gamma \gamma} \). Correspondingly, all of the 5 nuclear matter properties \( a_1, K_0, J, L \) and \( K_s \) are independent each other.

Eqs. (12) and (13) can be fitted to nuclear masses, by using \( D_{20}, D_{\gamma}, D_{22}, D_{32} \) and \( D_{\gamma \gamma} \) as free adjustable parameters in the equation of state, while the following two-parameter Fermi distribution is assumed for nucleon distribution \( \rho(\mathbf{r}) \):

\[ \rho(\mathbf{r}) = \frac{\rho A}{1 + e^{-(r/a)T}}, \]  

(28)
where
\[ C = R \left[ 1 - \frac{1}{3} \left( \frac{\pi d}{R} \right)^2 + O \left( \left( \frac{d}{R} \right)^6 \right) \right] \] (29)
is the nuclear half density radius\(^\text{[15]}\). In the numerical calculation, \( b = 1.0 \text{fm} \) and \( r_0 = 1.14 \text{fm} \) are used\(^\text{[16]}\).

In addition, the following approximations are assumed:
\[ \int d^3r \rho(r) \delta \left( \frac{\rho}{\rho_0} \right)^{i/3} \approx I \int d^3r \rho(r) \left( \frac{\rho}{\rho_0} \right)^{i/3}, \quad i = 2, 3, \gamma, \] (30)
\[ \int d^3r \rho(r) \delta^2 \left( \frac{\rho}{\rho_0} \right)^{i/3} \approx \xi_i I^2 \int d^3r \rho(r) \left( \frac{\rho}{\rho_0} \right)^{i/3}, \quad i = 2, 3, \gamma, \] (31)
where the \( \xi_i \)'s are adjustable parameters to be fitted to measured data. These approximations are reasonable, because \( \rho/\rho_0 \approx 1 \) in the core and Eq.(30) is exact for \( \rho/\rho_0 = 1 \). Furthermore, the surface region gives negligible contribution to the integral. A study based on an approximation to Eq.(4), using two-parameter Fermi distribution for \( \rho_n \) and \( \rho_p \), shows that both Eqs. (30) and (31) are good approximation.

On the other hand, for the integral involving the Laplacian of \( \rho \delta \) in Eq.(12), the following approximation is employed:
\[ \int d^3r \rho \nabla^2 (\rho \delta) \approx -\frac{\rho_C t^4}{4R^2 A}, \] (32)
where \( \rho_C = \rho(0) \), \( t = (C_n - C_p)/d \), \( C_n \) and \( C_p \) are the neutron and proton half density radii respectively. It can be seen that this term is much smaller than other term in Eq.(12); \( t \) can be assumed approximately as an averaged constant in the data fit.

Numerically, \( D_{22}, D_{32}, \) and \( D_{\gamma 2} \) can be determined by the data fit of Eq. (12), and then the variables \( J, L, \) and \( K_s \) can be calculated from these fitted parameters by Eqs. (23)-(25). On the other hand, \( D_{20} \) and \( D_{\gamma 0} \) can be determined by the data fit of Eq. (13), and then the variables \( a_1 \) and \( K_0 \) can be calculated from these fitted parameters by Eqs. (21) and (22). Therefore, the volume energy \( a_1 \) and nuclear incompressibility \( K_0 \) depend essentially on \( \mu_n N + \mu_p Z - 2E_N \), while the symmetry energy \( J \), the density symmetry coefficient \( L \) and the symmetry incompressibility \( K_s \) depend essentially on \( \mu_n - \mu_p \). In this way, the present data fit is separated into two steps, \( a_1 \) and \( K_0 \) are extracted from data of \( \mu_n N + \mu_p Z - 2E_N \), while \( J, L, \) and \( K_s \) are extracted from data of \( \mu_n - \mu_p \).

The result obtained by this data fit depends on the choice of \( \gamma \). \( \text{No } 1 \) in Table 1 gives the result with \( \gamma = 5 \). For comparison, the results obtained by this data fit corresponding to Myers-Swiatecki equation of state\(^\text{[8]}\) and Tondeur equation of state\(^\text{[17]}\) are shown as No 2 and No 3 in Table 1 respectively. For Myers-Swiatecki equation of state, we have \( \gamma = 5 \), and the following restriction can be derived:
\[ -\frac{6(3D_{22} - 1)}{5D_{20} - 3} + \frac{9D_{52}}{5D_{50}} = 2. \] (33)
For Tondeur equation of state, in addition to $\gamma = 4$, we have $D_{20} = 3/5$ and $D_{32} = D_{32} = 0$ which gives $L = 2J$ and $K_s = -L$. The original values given in Refs.\cite{3} and \cite{17} are shown in parentheses of $\mathbb{N}_2$ and $\mathbb{N}_3$ in Table 1 respectively. Note the difference between the present results and the original ones. That difference can be understood, since the present data fit is separated into two independent steps which is different from what is done in Refs.\cite{3} and \cite{17}, in addition to some approximation used in the present data fit.

Fig.4 plots the coefficients $a_1$ (circles), $J$ (full dots), and $\xi_1$ (circled crosses) versus $\gamma$. As $\gamma$ increases from 3.1 to 25, the symmetry energy $J$ increases from 28.50 to 28.52 MeV, the volume energy $a_1$ increases from 15.86 to 17.47 MeV, while the density symmetry coefficient $L$ decreases from 61.76 to 50.97 MeV. Similarly, Fig.5 shows $K_0$ and $K_s$ versus $\gamma$. As $\gamma$ increases from 3.1 to 25, the nuclear incompressibility $K_0$ increases from 199.2 to 673.9 MeV, and the symmetry incompressibility $K_s$ decreases from $-119.7$ to $-716.4$ MeV.

It is interesting to see what will be obtained if the present model and approximations \cite{30} and \cite{31} are applied to the data fit based on mass formula. In present formulation, Eq.(1) together with Eqs.(2), (3) and (19) is equivalent to the mass formula. Using the same approximations, the following expression can be written:

$$
\frac{1}{2}Q_1(\nabla \rho)^2 + Q_2\left[(\nabla \rho_0)^2 + (\nabla \rho_p)^2\right] = \frac{1}{2}(Q' + Q'_2\delta^2)(\nabla \rho)^2, \quad (34)
$$

where $Q' = Q$ and $Q'_2 = Q_2$ when $(\nabla \delta)^2$ and $\nabla \rho \cdot \nabla \delta$ can be neglected, and are the parameters to be fitted to $E_N$ data. In this case, the independent parameters to be fitted are: $D_{20}, D_{30}, D_{32}, D_{32}, D_{32}, D_{32}, D_{32}$, and $Q'$, and $\xi_1$ where $\xi_1$ is a parameter introduced in an approximation similar to Eq.(31). With the fitted values of these parameters, the total energy $E(A, Z)$ and thus the nuclear mass can be calculated for given $A$ and $Z$. In the same time, the nuclear volume energy $a_1$ and incompressibility $K_0$ can be calculated also from the fitted $D_{20}$ and $D_{30}$. The result with $\gamma = 5$ is shown as $\mathbb{N}_4$ in Table 1. In this data fit, the total of 1654 $E_N$ data, which are the same data used in the data fit based on Thomas-Fermi calculation with a standard deviation 0.655 MeV\cite{3}, are fitted. It is interesting to note that, when $\gamma$ increases from 3.1 to 25, $a_1$ varies from 16.034 to 16.042 MeV with a minimum 16.028 MeV at $\gamma = 11$, $K_0$ increases monotonically from 181 to 647 MeV.

Therefore, neither in the traditional data fit to nuclear energies $E_N$ nor in the present data fit to both $\mu_n - \mu_p$ and $\mu_n N + \mu_p Z - 2E_N$, the nuclear incompressibility $K_0$ cannot be determined from nuclear masses alone without additional assumption on the model of equation of state. In this aspect, we can make a model assumption on $\gamma$, as given in the Seyler-Blanchard type interaction\cite{3}, or make a specific choice of $\gamma$, as given in the Tondeur interaction\cite{17} or the Skyrme interaction\cite{18}. In addition, we can make some approximation on the equation of state, as the leptodermous expansion which will be given in the next
4 Leptodermous Expansion

For the nucleon distribution of finite nuclei, there is an approximate flat plateau in the core and a steep decrease in the surface region. In this leptodermous distribution of nucleons $\rho(r)$, the surface region gives a very small contribution to the integrals on the left-hand side of Eqs. (12) and (13). Therefore, the following expansion of equation of state is expected to be good enough for these integrals:

$$e(\rho, \delta) = -a_1 + \frac{1}{18} \left(K_0 + K_s \delta^2\right) \left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \left[J + \frac{L}{3} \left(\frac{\rho - \rho_0}{\rho_0}\right)\right] \delta^2. \tag{35}$$

In addition, the Fermi distribution (28) is a good approximation for the nucleon density.

By using the leptodermous expansion (35) for lefthand side of Eqs. (12) and (13), and assuming the Fermi distribution (28) for nucleon density $\rho(r)$, we can obtain the following approximate formulas:

$$2JI \left[1 - \frac{Ld}{JR}\right] = \frac{1}{2} (\mu_n - \mu_p), \tag{36}$$

$$a_1 - \frac{K_0 d}{4R} \left[1 - 2d + 3R\right] - J \left[1 - \frac{L}{3J}\right] I^2 = \frac{1}{A} (\mu_n N + \mu_p Z - 2E_N). \tag{37}$$

In obtaining Eq. (36), the second term in the integrand on the lefthand side of Eq. (12) has been neglected, as its integral is a small quantity of order of $(d/R)^2$ (see Sec. 3). Equation (37) is approximately valid for nuclei with small relative neutron excess $\delta$, as the terms proportional to $t$ and $t^2$ are neglected in the derivation, and $t$ is proportional to $\delta$ [19].

Equation (36) can be reduced to

$$\mu_n - \mu_p \approx 4JI, \tag{38}$$

if the term $Ld/JR$ is neglected comparing to 1. This relation gives an intuitive geometric picture for the symmetry energy $J$ in Fig. 2, i.e., $4J$ is the slope of the curve approximately. Numerically, the symmetry energy $J$ and density symmetry coefficient $L$ can be extracted by fitting Eq. (36) to the $\mu_n - \mu_p$ data. This fit to 1140 data of nuclei gives $J = 28.16 \text{MeV}$, $L = 70.69 \text{MeV}$, and the result is shown in Fig. 6 as $(\mu_n - \mu_p)/(1 - Ld/JR)$ versus $I$. The slope of the straight line in this figure is $4J$.

These fitted $J$ and $L$ can be substituted into Eq. (37), and then the volume energy $a_1$ and the nuclear incompressibility $K_0$ can be extracted by fitting Eq. (37) to the data of $(\mu_n N + \mu_p Z - 2E_N)/A$ with small $|\delta|$. The fitted result is shown in Fig. 7 as $(\mu_n N + \mu_p Z - 2E_N)/A + (J - L/3) I^2$ versus $A^{-1/3}$. It can be seen from Eq. (37) that the extrapolation of the curve to infinite nuclei will give...
the volume energy $a_1$, while the minus slope of the curve on the lefthand side will be approximately proportional to nuclear incompressibility $K_0$. So Eq. (37) gives an intuitive geometric picture for $a_1$ and $K_0$ in this plot. This fit to 222 data of nuclei with $|\delta| < 0.1$ gives $a_1 = 15.29 \text{MeV}$ and $K_0 = 225.7 \text{MeV}$, as shown as N=5 in Table I.

5 Discussion and Summary

The present way of data fit is similar to what is proposed in Ref. [13] in two aspects. At first, the data fit is separated into two steps instead of a unique one. In the present work, the first step gives $J$, $L$ and $K_s$ when the model of equation of state is assumed, based on the data of $\mu_n - \mu_p$, while the second step gives $a_1$ and $K_0$ based on the data of $\mu_n N + \mu_p p - 2E_N$. Therefore, it is easy to see how good the fit is in a more direct and intuitive way. Actually, it can be seen from Figs.2 and 3, or the corresponding Figs.6 and 7, that the fitted $J$ is more accurate than the fitted $a_1$ and $K_0$ in the present data fit. Secondly, what is used in the data fit is not only the nuclear masses but also the separated energies $\mu_n$ and $\mu_p$ (in the present work), or $\mu_p$ (in Ref. [13]). This means that the correlations among nuclear masses are taken into account in this kind of data fit. Even the shell effect is not treated consistently in this way as was done in Ref. [13] but rather is calculated by the empirical formula of Ref. [14], the present result is still comparable with Ref. [13]. It is noteworthy for that their $a_1 = 16.11 \text{MeV}$ is very close to what we obtained with $\gamma = 5$.

It can be seen from Figs. 4 and 5 as well as Table 1 that there is almost no model dependence in the symmetry energy $J$, while the model dependence in the volume energy $a_1$ and density symmetry coefficient $L$ is weak. However, there is strong model dependence in the nuclear incompressibility $K_0$ and symmetry incompressibility $K_s$. In this aspect, the choice of $\gamma$ should be considered as an ingredient of the model. This model dependence is unescapable whenever $K_0$ can not be measured directly from the definition $K_0 = 9\rho_0^2 \partial^2 \epsilon / \partial \rho^2 |_0$. It is worthwhile to note that the value of $K_0$ obtained from different nuclear measurements and astrophysical observations are spread over a large range from 180 to 800 MeV [24], and the value of $K_s$ is between $-566 \pm 1350$ to $34 \pm 159 \text{MeV}$ obtained by data fit to breathing-mode energies [21] while between $-400 \text{MeV}$ to 466 MeV estimated by various types of nuclear force calculations [22].

This indeterminancy of $K_0$ can be understood from Fig.7. As the nuclear incompressibility $K_0$ is approximately proportional to minus slope of the curve on the lefthand side, it is hard to be determined due to the wide spread of the mass data, even if based on selected 222 points instead of all 1654 points as shown in Fig.3. This spread of mass data supports reasonably the understanding that $K_0$ cannot be determined from nuclear masses alone, and some additional measurement such as breathing-mode energies is required also. As the model of equation of state is essential in the formulation of the model of nucleon
interaction, indeterminacy of $K_0$ means that the model of nucleon interaction can not be determined from nuclear masses alone, and this is well understood in the field.

The model of equation of state is the same for $N_1$ and $N_4$ in Table 1, with same value of $\gamma = 5$, and the fitted $a_1$ and especially $K_0$ are also very close each other. This means that the present data fit, separated into two steps, is consistent with the data fit based on the nuclear mass formula. Therefore, the present model provides also an appropriate nuclear mass formula, if it is fitted to nuclear masses.

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Figure Captions

Figure 1. $\mu_n - \mu_p$ obtained from nuclear masses versus nuclear asymmetry $I = (N - Z)/A$.

Figure 2. $\mu_n - \mu_p$ versus nuclear asymmetry $I = (N - Z)/A$. It can be seen that there is an approximate linear correlation for $\mu_n - \mu_p$, but the correlation between $\mu_n - \mu_p$ and $I$ is not so simple.

Figure 3. $(\mu_n N + \mu_p Z - 2E_N)/A$ versus $A^{-1/3}$.

Figure 4. The coefficients $a_1$(circles), $J$(full dots), and $L$(circled crosses) versus $\gamma$.

Figure 5. $K_0$ and $K_s$ versus $\gamma$.

Figure 6. $(\mu_n - \mu_p)/(1 - Ld/JR)$ versus $I$, where $R = r_0A^{1/3}$, $b = 1.0\,fm$ and $r_0 = 1.14\,fm$ are used. The slope of the straight line in this figure is $4J$. The data fit gives $J = 28.16\,MeV$, $L = 70.69\,MeV$.

Figure 7. $(\mu_n N + \mu_p Z - 2E_N)/A + (J - L/3)I^2$ versus $A^{-1/3}$. It can be seen from Eq. $[\text{[7]}]$ that the intersection of the curve at $A^{-1/3} = 0$ gives the
volume energy $a_1$, while the minus slope of the curve on the left-hand side is approximately proportional to nuclear incompressibility $K_0$. The data fit gives $a_1 = 15.29\,\text{MeV}$ and $K_0 = 225.7\,\text{MeV}$.

Table 1: The fitted values of $a_1$, $K_0$, $J$, $L$, and $K_s$, all in $\text{MeV}$. N\textsuperscript{2} 1 is the result of Eqs.(12) and (13) with $\gamma = 5$. N\textsuperscript{2} 2 is the result corresponding to Myers-Swiatecki equation of state, while N\textsuperscript{2} 3 is the result corresponding to Tondeur equation of state. N\textsuperscript{2} 4 is the result of Eq.(12) together with Eqs.(3), (19), (34), and $\gamma = 5$. The values shown in parentheses in N\textsuperscript{2} 2 and N\textsuperscript{2} 3 are given by Refs.[3] and [17] respectively. N\textsuperscript{2} 5 is the result of the leptodermous expansion. See the text for details.

| No. | $a_1$ | $K_0$ | $J$  | $L$  | $K_s$ | $\gamma$ |
|-----|-------|-------|------|------|-------|---------|
| 1   | 16.10 | 237.9 | 28.50| 60.33| -157.9| 5       |
| 2   | 16.10 | 237.9 | 28.82| 81.81| 40.4  | 5       |
|     | (16.24)| (234)| (32.65)| (49.9)|       |         |
| 3   | 16.63 | 243.9 | 27.52| 55.05| -55.0 | 4       |
|     | (15.978)| (235.8)| (32.12)|       |       |         |
| 4   | 16.03 | 237.8 |      |      |       | 5       |
| 5   | 15.29 | 225.7 | 28.16| 70.69|       |         |
\[ \frac{\left( \mu_N + \mu_p \right)}{A} \]

\[ = \frac{\mu_N + \mu_p Z - 2E_N^*}{A} \]

\[ (\text{MeV}) \]
\[ \frac{\mu_n - \bar{\mu}_p}{1 - L_d/J_R} \text{ (MeV)} \]
