Further understanding the nature of $\Omega(2012)$ within a chiral quark model

Hui-Hua Zhong, Ru-Hui Ni, Mu-Yang Chen *, Xian-Hui Zhong ‡
Department of Physics, Hunan Normal University, and Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, Changsha 410081, China and
Synergetic Innovation Center for Quantum Effects and Applications (SICQEA), Hunan Normal University, Changsha 410081, China

Ju-Jun Xie ‡
Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China
School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing 100408, China and School of Physics and Microelectronics, Zhengzhou University, Zhengzhou, Henan 450001, China

In our previous works, we have analyzed the two-body strong decays of the low-lying $\Omega$ baryon states within a chiral quark model. The results show that the $\Omega(2012)$ resonance favors the three-quark state with $J^P = 3/2^-$ classified in the quark model. With this assignment, in the present work we further study the three-body strong decay $\Omega(2012) \rightarrow \Xi(1530)\bar{K} \rightarrow \Xi\pi\bar{K}$ and coupled-channel effects on $\Omega(2012)$ from nearby channels $\Xi\bar{K}$, $\Omega\eta$ and $\Xi(1530)\bar{K}$ within the chiral quark model as well. It is found that the $\Omega(2012)$ resonance has a sizeable decay rate into the three-body final state $\Xi\pi\bar{K}$. The predicted ratio $R_{\Xi\pi\bar{K}}^{\Xi(1530)\bar{K}} = B[\Omega(2012) \rightarrow \Xi(1530)\bar{K} \rightarrow \Xi\pi\bar{K}] = 12\%$ is close to the up limit $11\%$ measured by the Belle Collaboration in 2019, however, our predicted ratio is too small to be comparable with the recent data $0.97 \pm 0.31$. Furthermore, our results show that the coupled-channel effects on the $\Omega(2012)$ is not large, its components should be dominated by the bare three-quark state, while the proportion of the molecular components is only $\sim 16\%$. To clarify the nature of $\Omega(2012)$, the ratio $R_{\Xi\pi\bar{K}}^{\Xi(1530)\bar{K}}$ is expected to be tested by other experiments.

I. INTRODUCTION

In 2018, Belle Collaboration observed a new excited hyperon $\Omega(2012)$ decaying into $\Xi^0\bar{K}^-$ and $\Xi^-\bar{K}^0$ with a mass of $2012.4 \pm 0.7(stat) \pm 0.6(syst)$ MeV and a width of $\Gamma = 6.4^{+2.5}_{-2.0}(stat) \pm 1.6(syst)$ MeV [1]. In 2021, evidence of $\Omega(2012)$ was observed in the $\Omega$ weak decay process $\Omega^- \rightarrow \pi^+\Omega(2012)^- \rightarrow \pi^-(\Xi^0\bar{K}^-)$ at Belle [2] following the proposal in Ref. [3]. According to the previous mass spectrum predictions in various models and methods, such as the Skyrme model [4], quark model [5–14], lattice gauge theory [15, 16] and so on [17–22], the newly observed $\Omega(2012)$ may be a good candidate of the first orbital (1P) excitations of $\Omega(1672)$.

The discovery of $\Omega(2012)$ immediately attracted a great deal of attention from the hadron physics community in the literature. Our group analyzed the Okubo-Zweig-Iizuka (OZI)-allowed two body strong decays of the low-lying $P$- and $D$-wave $\Omega$ baryon states within the chiral quark model [23, 24] and $^3P_0$ model [25] by combining the mass spectrum analysis, and found that the $\Omega(2012)$ resonance favors the assignment of the 1P-wave state with $J^P = 3/2^-$. Furthermore, the newly measured ratio $B[\Omega^- \rightarrow \Omega(2012)\pi^+ \rightarrow (\Xi^0\bar{K}^-)\pi^+]$ at Belle can be well understood as well within the three quark picture in a recent work [26]. Our conclusion is consistent with that based on the framework of QCD sum rules [27, 28], the constituent quark model by including relativistic corrections [29], and the flavour SU(3) analysis [30].

However, in the literature the $\Omega(2012)$ was interpreted as a hadronic molecule [31–40]. In the hadronic molecular picture, the decay rate into the three-body final state $\Xi\pi\bar{K}$ is predicted to be similar to that of the two-body final state $\Xi\bar{K}$. While in the three-quark picture, the decay rates for OZI-allowed two-body final state $\Xi\bar{K}$ should be dominant, the decay rate of $\Omega(2012)^- \rightarrow \Xi(1530)\bar{K} \rightarrow \Xi\pi\bar{K}$ should be suppressed by the intermediate hadron state $\Xi(1530)$.

To test the decay properties of $\Omega(2012)$ predicted within different pictures, the three-body channel $\Xi\pi\bar{K}$ through the decay process $\Omega(2012)\rightarrow \Xi(1530)\bar{K} \rightarrow \Xi\pi\bar{K}$ was observed by the Belle Collaboration. The first observation was carried out in 2019 [41]. The collaboration observed no significant $\Omega(1530)$ signals in the three-body final state $\Xi\bar{K}$, and set an upper limit at 90 confidence level on the branching fraction ratio $R_{\Xi\pi\bar{K}}^{\Xi(1530)\bar{K}} = B[\Omega(2012)\rightarrow \Xi(1530)\bar{K} \rightarrow \Xi\pi\bar{K}] / B[\Omega(2012)\rightarrow \Xi(1530)\bar{K} \rightarrow \Xi\bar{K}] < 11\%$, which is consistent with expectation in the three-quark picture. However, in the recent search, they observed significant $\Omega(2012)$ signals in the three-body decay process $\Omega(2012)^- \rightarrow \Xi(1530)\bar{K}^- \rightarrow \Xi\pi\bar{K}^-$ [42]. A rather large ratio $R_{\Xi\pi\bar{K}}^{\Xi(1530)\bar{K}} = 0.97 \pm 0.24 \pm 0.07$ was extracted from the observations, which seems to be consistent with expectation in the hadronic molecular picture.

In the previous works [23, 24], we did not study the three-body decay process $\Omega(2012)^- \rightarrow \Xi(1530)\bar{K}^- \rightarrow \Xi\pi\bar{K}$. It is unclear how large the decay rate of the $\Xi\pi\bar{K}$ mode within our three-quark picture. Furthermore, the $\Omega(2012)$ as a conventional three-quark state, it may strongly couple to the $\Xi(1530)\bar{K}$ channel, then the $\Omega(2012)$ may contain significant molecular components due to coupled-channel effects. To uncover these puzzles and better understand the nature of $\Omega(2012)$, in the three quark picture we further study the three-body decays of $\Omega(2012) \rightarrow \Xi\pi\bar{K}$ and the coupled-channel

*E-mail: muyang@hunnu.edu.cn
†E-mail: zhongtx@hunnu.edu.cn
‡E-mail: xiejunj@impcas.ac.cn
Table I: The baryon wave functions involved in our calculations. The \( \psi_{\mu_l \mu_n}^{(\rho) \lambda} \) and \( \phi_{S \pi, \varnothing}^{(\omega) \lambda} \) represent the radial wave functions of \( \rho \)-mode and \( \lambda \)-mode, respectively, while the \( \chi_{3/2}^+ \) and \( \varnothing_{0,0,0} \) expressed as spin and flavor wave functions of the baryon system, respectively. The details can be found in Ref. [49].

\[
\begin{align*}
\Omega^+ | 1^2 P_{3/2}^+ \rangle &= \frac{1}{\sqrt{2}} \left( \psi_{00}^{(0)} \psi_{00}^{(0)} \chi_{3/2}^+ \phi_{S \pi}^{(0)} + \psi_{00}^{(0)} \psi_{00}^{(0)} \chi_{3/2}^+ \phi_{\varnothing_{0,0,0}} \right) \\
\Omega^+ | 1^2 S_{1/2}^+ \rangle &= \psi_{00}^{(0)} \psi_{00}^{(0)} \chi_{3/2}^+ \phi_{S \pi}^{(0)} \\
\Xi^+(1530) | 1^2 S_{1/2}^+ \rangle &= \psi_{00}^{(0)} \psi_{00}^{(0)} \chi_{3/2}^+ \phi_{S \pi}^{(0)} + \psi_{00}^{(0)} \psi_{00}^{(0)} \chi_{3/2}^+ \phi_{\varnothing_{0,0,0}} \right)
\end{align*}
\]

Effects from nearby channels \( \Xi K, \Omega \eta \) and \( \Xi^+(1530) K \). For self-consistency, we adopt the same framework, i.e., the chiral quark model [43], as our previous works for the \( \Omega(2012) \) studies [23, 24]. In this model an effective chiral Lagrangian is introduced to account for the quark-meson coupling at the baryon-meson interaction vertex. The light pseudoscalar mesons, i.e., \( \pi, K, \eta \), are treated as Goldstone bosons. Since the quark-meson coupling is invariant under the chiral transformation, some of the low-energy properties of QCD are retained [44-46].

This paper is organized as follows. In Sec. II, a brief review of the chiral quark model is given, then by using this model the two-body and three body strong decays of \( \Omega(2012) \) are estimated. In Sec. III, the coupled-channel effects on the \( \Omega(2012) \) are evaluated by combining the simplest version of the coupled-channel model with the chiral quark model. Finally, we give a short discussion and summary in Sec. IV.

Table II: Masses (MeV) of the hadrons adopted in this work.

| Hadron | \( J^P \) | Mass (MeV) |
|--------|----------|------------|
| \( \pi^0, \pi^\pm \) | 0+ | 135.0, 139.6 |
| \( K^0, K^\pm \) | 0+ | 497.6, 493.6 |
| \( \eta \) | 0+ | 547.9 |
| \( \Xi^-, \Xi^0 \) | 1/2+ | 1321.7, 1314.9 |
| \( \Xi^-, \Xi^0 \) | 3/2+ | 1535.0, 1531.8 |
| \( \Omega^- \) | 3/2- | 1671.7 |
| \( \Omega(2012) \) | 3/2- | 2012.4 |

II. STRONG DECAY

A. Two-body decay

In the chiral quark model [43], the low-energy quark-pseudoscalar-meson interactions in the SU(3) flavor basis are represented by the effective Lagrangian [44-46]

\[
\mathcal{L} = \sum_{j f m} \frac{\delta}{\sqrt{2}} \overline{\psi}_{j f m} \gamma_\mu \gamma_5 \psi_j \phi_{f m},
\]

where \( \delta \) is the pseudoscalar meson decay constant. To match the nonrelativistic wave functions of the initial and final hadron states, we adopt the nonrelativistic form of the Lagrangians in following form,

\[
\mathcal{H}_m^{\text{eff}} = \delta \sqrt{(E_f + M_f)(E_i + M_i)} \sum_{j f m} \left( \frac{\omega_m}{E_f + M_f} \sigma_j \cdot P_f + \frac{\omega_m}{E_i + M_i} \sigma_j \cdot P_i - \sigma_j \cdot \frac{\omega_m}{2 \mu_q} \sigma_j \cdot P_i \right) I_{j f m},
\]

where \( \omega_m \) and \( q_m \) are the energy and three momentum of the final state pseudoscalar meson, respectively; \( E_i \) and \( M_i \) are the energy and mass of the initial heavy hadron, respectively; \( E_f \) and \( M_f \) represent the energy and mass of the final state heavy hadron, respectively; \( P_f \) is the internal momentum operator of the \( j \)th quark in the baryon system rest frame; \( \sigma_j \) is the spin operator for the \( j \)th quark of the baryon system; and \( \mu_q \) is a reduced mass expressed as \( 1/\mu_q = 1/m_j + 1/m'_j \) with \( m_j \) and \( m'_j \) for the masses of the \( j \)th quark in the initial and final baryons, respectively. The plane wave part of the emitted light meson is \( \varnothing_m = e^{-iu_\varnothing \cdot x} \), and \( I_{j f m} \) is the flavor operator defined for the transitions in the SU(3) flavor space [45, 47, 48]. \( \delta \) is a global parameter accounts for the strength of the quark-meson couplings. Here, we take the same value as that determined in Refs. [23, 24, 49], i.e., \( \delta = 0.576 \).

For an excited baryon state, within the chiral quark model its two-body OZI-allowed strong decay amplitudes can be described by

\[
M [B \to B'M] = \langle B' | \mathcal{H}_m^{\text{eff}} | B \rangle,
\]

where \( |B \rangle \) and \( |B' \rangle \) are the wave functions of the initial and final baryons, respectively. With derived decay amplitudes from Eq.(3), the partial decay width for the \( B \to B'M \) process can be calculated with

\[
\Gamma = \frac{1}{8\pi} \frac{1}{M_i^2} \frac{1}{2J_i + 1} \sum_{J_i j_i} |M_{J_i J_f j_i,j_f}|^2,
\]

where \( J_i \) and \( J_f \) represent the third components of the total angular momenta of the initial and final baryons, respectively.

The OZI-allowed two-body strong decay for the \( \Omega \) resonances have been evaluated under the frame of the chiral quark model in our previous works [23, 24]. It is found that the newly observed \( \Omega(2012) \) resonance favors the assignment of the \( 1P \)-wave state with \( J^P = 3/2^- \) (i.e., \( \Omega^+ |1P_{3/2}^- \rangle \) listed in Table I) in the \( \Omega \) baryon family [23]. Considering the \( \Omega(2012) \) as \( \Omega^+ |1P_{3/2}^- \rangle \), the \( \Xi K \) is the only OZI allowed two-body decay channel, which is shown in Fig. 1, while the \( \Omega(2012) \to \Xi^+(1530) K \) decay process is forbidden since the mass of \( \Omega(2012) \) lies below the \( \Xi^+(1530) K \) threshold. Thus, the total width should be nearly saturated by the \( \Xi K \) channel.

By using the wave function calculated from the potential model [23], within the chiral quark model the partial width of \( \Omega(2012) \to \Xi K \) are predicted to be

\[
\Gamma_{\Xi K} = \Gamma_{\Xi K^0} + \Gamma_{\Xi K^0} \approx (3.0 + 2.7) \text{ MeV},
\]

which is consistent with the data \( \Gamma_{\text{exp}} = 6.4^{+2.5}_{-2.0} \pm 1.6 \text{ MeV} \) [50]. It should be mentioned that in the calculations,
to consist with the mass spectrum predictions in the potential model [23], the constituent quark masses of $u/d$ and $s$ quarks are adopted to be $m_u = m_d = 350$ MeV and $m_s = 600$ MeV. The meson decay constants for $\pi$, $K$ and $\eta$ are taken with $f_\pi = 132$ MeV and $f_K = f_\eta = 160$ MeV. The masses of the final and initial states are taken the measured values from experiments [50], which have been collected in Table II. The spatial wave functions of the baryons appearing in the initial and final states are adopted the harmonic oscillator form. For the $\Xi$ and $\Xi^*$ states, the harmonic oscillator strength parameter $\alpha_\rho$ is taken as $\alpha_\rho = 400$ MeV, while the parameter $\alpha_4$ for the $\lambda$ oscillator is related to $\alpha_\rho$ with $\alpha_4 = \sqrt{3} m_\rho/(2 m_s + m_u) \alpha_\rho$. For the $\Omega^*$ state, we take $\alpha_\rho = \alpha_4 = 411$ MeV, which is fitted by reproducing the root-mean-square radius of the $\rho$-mode excitations from the potential model [23].

![FIG. 1: The OZI-allowed two-body decay process $\Omega(2012) \to \Xi K$ at (i) the hadronic level and (ii) the quark level.](image1)

![FIG. 2: The cascade three-body decay process $\Omega(2012) \to \Xi^*(1530) K \to [\Xi \pi] K$ at (i) the hadronic level and (ii) the quark level.](image2)

| Channel                  | $\Xi^* \pi^+ K^- \Xi^0 \pi^0 K^- \Xi^0 \pi^0 K^0$ | Sum     |
|-------------------------|-----------------------------------------------|--------|
| $\Gamma_i$ (MeV)        | 0.25                                          | 0.18   |
| $\Gamma_f$ (MeV)        | 0.07                                          | 0.17   |
| $\Gamma$ (MeV)          | 0.67                                          | 0.64   |

### B. three-body decay

Recently, the Belle Collaboration observed a rather large three-body decay rate in the $\Omega(2012)^- \to \Xi^*(1530)^0K^- \to \Xi^0\pi^+K^-$ process. This cascade decay process can be evaluated in the chiral quark model with the same parameter set as well. For the cascade decay process $\Omega(2012) \to \Xi^*(1530) K \to [\Xi \pi] K$ as shown in Fig. 2, the decay amplitude can be expressed as

$$M_{\Omega(2012) \to \Xi \pi K} = \frac{M_{\Omega(2012) \to \Xi^* K} M_{\Xi^* \to \Xi \pi}}{2 M_{\Xi^*} \left(M_{\Xi^*} - M_{\Xi^0} + i f_{\Xi^*} \to \Xi \pi / 2\right)}$$

where $M_{\Omega(2012) \to \Xi^*(1530) K}$ and $M_{\Xi^0 \to \Xi \pi}$ are the amplitudes defined in Eq. (3). It should be emphasized that the $\Xi^*(1530)$ baryon mass $M_{\Xi^*}$ in $M_{\Omega(2012) \to \Xi \pi K}$ and $M_{\Xi^0 \to \Xi \pi}$ is replaced with the invariant mass $M_{\Xi^0}$ of $\Xi \pi$ system since the unstable baryon $\Xi^*(1530)$ can be slightly off-shell. Adopting the wave functions listed in Table I, one can work out the $M_{\Omega(2012) \to \Xi^*(1530) K}$ and $M_{\Xi^0 \to \Xi \pi}$ within the chiral quark model. Moreover, $\Gamma_{\Xi^0 \to \Xi \pi}$ also depends on the invariant mass of $M_{\Xi^0}$, and it can be obtained by Eq. (4).

Then the three-body decay width of the $\Omega(2012)$ resonance in their rest frame was obtained as

$$d\Gamma_{\Xi^0 \to \Xi \pi K} = \frac{M_{\Omega(2012) \to \Xi^* K}^2}{M_{\Omega(2012) \to \Xi^* K}^2 - 4 M_{\Omega(2012) \to \Xi^* K}^2 \left(M_{\Xi^0} - M_{\Xi^*} + \frac{i f_{\Xi^*} \to \Xi \pi / 2}{2}\right)} dM_{\Xi^0}.$$  

where the absolute value of the momentum for the final $K$ meson is give by

$$|q_K| = \sqrt{\left(M_{\Omega(2012) \to \Xi^* K}^2 - (m_K + m_{\Xi^0})^2\right) \left(M_{\Omega(2012) \to \Xi^* K}^2 - (m_K - m_{\Xi^0})^2\right)}.$$  

Integrating the Eq. (7) from $M_{\Xi^0} + m_\pi$ to $M_{\Xi^0} - m_K$, one can easily work out the three-body decay width of the cascade decay process $\Omega(2012) \to \Xi^*(1530) K \to [\Xi \pi] K$. Our results have been listed in Table III.

The $\Omega(2012)$ as the $\Omega||1P_{3/2}^7$ state has a sizeable partial decay width into the three-body final state $[\Xi \pi K]$

$$\Gamma_{\Xi \pi K} \approx 0.67 \text{ MeV},$$

combined it with the two-body decay width predicted in Eq. (5), the total width is predicted to be

$$\Gamma_{\text{total}} \approx 6.37 \text{ MeV},$$

TABLE III: The partial widths (MeV) of three-body final states for the $\Omega(2012)$ resonance. $\Gamma_i$ stands for the results by considering the isospin breaking effects of the final states. While $\Gamma_f$ stands for the results without the isospin breaking effects, the mass of a hadron in the final states is adopted the average value of different charged states.
which is in good agreement with the data $\Gamma_{exp} = 6.4^{+2.5}_{-2.6} \pm 1.6$ MeV [50]. The predicted ratio

$$R_{\Xi^* K}^{\Xi K} = \frac{\mathcal{B}[\Omega(2012) \rightarrow \Xi^*(1530) K \rightarrow \Xi \pi K]}{\mathcal{B}[\Omega(2012) \rightarrow \Xi K]} \approx 12\%, \quad (11)$$

is slightly larger than the previous data $(6.0 \pm 3.7 \pm 1.3)\%$ measured by the Belle Collaboration in 2020 [41], however, is a factor of $\sim 4 - 8$ smaller than their recent data $0.97 \pm 0.31$ with improved selection criteria [42]. In the recent work [29], the three body decay of $\Omega(2012) \rightarrow \Xi \pi K$ was also studied within a constituent quark model by including relativistic corrections, a small ratio $R_{\Xi^* K}^{\Xi K} \approx 4.5\%$ is obtained with the $\Omega^+|1P_{3/2}\rangle$ assignment. It should be mentioned that the isospin breaking effects on the partial widths of the three-body decays are obvious. The results in Table III show that there are significant differences between the partial widths with isospin breaking effects and that without isospin breaking effects.

The recent measured ratio $R_{\Xi^* K}^{\Xi K}$ at Belle [42] seems to be compatible with the molecular interpretation for the $\Omega(2012)$ proposed in Refs. [37-40] where similar branching fractions $\Omega(2012)$ decays into $\Xi \pi K$ and $\Xi K$ were predicted.

In the following, we will further explore the molecular components of $\Omega(2012)$ due to coupled-channel effects.

### III. COUPLED-CHANNEL EFFECTS

First, we give a brief review of the coupled-channel model adopted in this work. A bare baryon state $|A\rangle$ predicted in the quark model can couple to the two-hadron continuum $BC$ via hadronic loops, as shown in Fig. 3. In the simplest version of the coupled-channel model [51–55], the wave function of the physical state is given by

$$|\Psi\rangle = c_A|A\rangle + \sum_{BC} c_{BC}|p\rangle d^3|p|BC, p\rangle, \quad (12)$$

where $p = p_B = -p_C$ is final two-hadron relative momentum in the initial hadron static system, $c_A$ and $c_{BC}|p\rangle$ denote the probability amplitudes of the bare valence state $|A\rangle$ and $|BC, p\rangle$ continua, respectively.

The coupling between the bare state $|A\rangle$ and the continuum components sectors $|BC, p\rangle$ is achieved by creating light quark pairs. This coupling, as an effective coupling for the quark-meson interactions, can be described with the chiral interactions given in Eq.(2) in the chiral quark model. Thus, the full Hamiltonian of the physical state $|\Psi\rangle$ can be written as

$$\mathcal{H} = \begin{pmatrix} \mathcal{H}_0 & \mathcal{H}_m^\alpha \\ \mathcal{H}_m^{\alpha \dagger} & \mathcal{H}_c \end{pmatrix}, \quad (13)$$

where $\mathcal{H}_0$ is the Hamiltonian of the bare state $|A\rangle$ in the potential model, while $\mathcal{H}_c$ is the Hamiltonian for the continuum state $|BC, p\rangle$. Neglecting the interaction between the hadrons $B$ and $C$, one has

$$\mathcal{H}_c|BC, p\rangle = E_{BC}|BC, p\rangle, \quad (14)$$

where $E_{BC} = \sqrt{m_B^2 + p^2 + m_C^2 + p^2}$ represents the energy of $BC$ continuum.

The Schrödinger equation of a mixed system can be written

$$\begin{pmatrix} \mathcal{H}_0 & \mathcal{H}_m^\alpha \\ \mathcal{H}_m^{\alpha \dagger} & \mathcal{H}_c \end{pmatrix} \begin{pmatrix} c_A|A\rangle \\ \sum_{BC} c_{BC}|p\rangle d^3|p|BC, p\rangle \end{pmatrix} = M \begin{pmatrix} c_A|A\rangle \\ \sum_{BC} c_{BC}|p\rangle d^3|p|BC, p\rangle \end{pmatrix}. \quad (15)$$

From Eq.(15), we have

$$\langle A|\mathcal{H}|\Psi\rangle = c_A M \quad \Rightarrow \quad c_A M_A + \sum_{BC} c_{BC}|p\rangle d^3|p\rangle \langle A|\mathcal{H}_m^\alpha|BC, p\rangle, \quad (16)$$

$$\langle BC, p|\mathcal{H}|\Psi\rangle = c_{BC}(p) M \quad \Rightarrow \quad c_{BC}(p) E_{BC} + c_A(BC, p)|\mathcal{H}_m^\alpha|A\rangle. \quad (17)$$

Deriving $c_{BC}(p)$ from Eq.(17), and substituting it into Eq.(16), we get a coupled-channel equation

$$M = M_A + \Delta M(M), \quad (18)$$

where the mass shift $\Delta M(M)$ is given by

$$\Delta M(M) = \text{Re} \sum_{BC} \int_0^\infty \frac{|\langle BC, p|\mathcal{H}_m^\alpha|A\rangle|^2}{(M - E_{BC})} p^2 dp d\Omega, \quad (19)$$

$$= \text{Re} \sum_{BC} \int_0^\infty \frac{|M_{A-BC}(p)|^2}{(M - E_{BC})} p^2 dp, \quad (20)$$

and $M_A$ is the bare mass of the baryon state $|A\rangle$ obtained from the potential model. From Eqs.(18) and (19), the physical mass $M$ and the bare state mass shift $\Delta M$ can be determined simultaneously. It should be mentioned that when we calculate the mass shift $\Delta M$ by using the Eq.(19), the contribution in the higher $p$ region may be nonphysical because the quark
pair production rates via the non-perturbative interaction $\mathcal{H}^{\text{w}}_{\text{int}}$ should be strongly suppressed [51]. In the chiral quark model, the chiral interaction $\mathcal{H}^{\text{w}}_{\text{int}}$ is only applicable to the low $p$ region. To eliminate the nonphysical contributions, in our calculations, we cut off the momentum $p$ at the inflection point of $\Delta M(p)$ function as followed by our previous work [56]. It should be noted that due to the various locations of the inflection point, the cut-off momentum for each channel varies.

It is found that, the coupled-channel effects on the $\Omega(2012)$ resonance as a conventional $1P$-wave state $\Omega(1P_{3/2})$, there are sizeable decay rates into the three-body final state $\Xi\pi\bar{K}$. The partial width and branching fraction is predicted to be $\Gamma_{\Xi\pi\bar{K}} \approx 0.67$ MeV and $\mathcal{B}(\Omega(2012)\rightarrow \Xi\pi\bar{K}) \approx 11\%$. The predicted ratio $R_{\Xi\pi\bar{K}} = \mathcal{B}(\Omega(2012)^* \rightarrow \Xi\pi\bar{K}) / \mathcal{B}(\Omega(2012)^* \rightarrow \Xi\bar{K}) = 0.12$ is close to the previous data (6.0 ± 3.7% ± 1%) measured by the Belle Collaboration in 2019 [41], however, is too small to be comparable with recent data 0.97 ± 0.31 [42].

The newly measured ratio $R_{\Xi\pi\bar{K}} = 0.97 \pm 0.31$ at Belle seems to be consistent with the molecular interpretation for the $\Omega(2012)$ proposed in Refs. [37–40]. However, our results show that the coupled-channel effects on the $\Omega(2012)$ are not large, if it originates from the $1P$-wave three-quark state $\Omega^*(1P_{3/2})$. In this case, the components of $\Omega(2012)$ are dominated by the bare quark model state $\Omega^*(1P_{3/2})$, its proportion can reach up to about 84%; the proportion of the continuum components is only $\sim 16\%$. In other words if the $\Omega(2012)$ resonance is a molecular dominant state, it cannot originate from the three quark state $\Omega^*(1P_{3/2})$.

### IV. DISCUSSION AND SUMMARY

In our quark model study, with the $\Omega^*(1P_{3/2})$ assignment, both the observed mass and width of $\Omega(2012)$ can be well explained [23–25]. Furthermore, the newly measured ratio $\mathcal{B}[\Omega(2012)^* \pi^- \rightarrow (\Xi\bar{K})^- \pi^+]/\mathcal{B}([\Omega(2012)^* \rightarrow \Omega \pi^+)]$ at Belle can be well understood as well in our recent work [26].

### Acknowledgement

This work is supported by the National Natural Science Foundation of China (Grants No.12175065, No. 12005060, No. 12075288). Ju-Jun Xie is also supported by the Youth Innovation Promotion Association CAS.
in a chiral quark model, Phys. Rev. D 78, 014029 (2008).
[49] L. Y. Xiao and X. H. Zhong, Ξ baryon strong decays in a chiral quark model, Phys. Rev. D 87, 094002 (2013).
[50] P. A. Zyla et al. [Particle Data Group], Review of Particle Physics, PTEP 2020, 083C01 (2020).
[51] D. Morel and S. Capstick, Baryon meson loop effects on the spectrum of nonstrange baryons, [arXiv:nucl-th/0204014 [nucl-th]].
[52] Y. S. Kalashnikova, Coupled-channel model for charmonium levels and an option for X(3872), Phys. Rev. D 72, 034010 (2005).
[53] Y. Lu, M. N. Anwar and B. S. Zou, How Large is the Contribution of Excited Mesons in Coupled-Channel Effects?, Phys. Rev. D 95, 034018 (2017).
[54] J. F. Liu and G. J. Ding, Bottomonium Spectrum with Coupled-Channel Effects, Eur. Phys. J. C 72, 1981 (2012).
[55] Y. Lu, M. N. Anwar and B. S. Zou, Coupled-Channel Effects for the Bottomonium with Realistic Wave Functions, Phys. Rev. D 94, 034021 (2016).
[56] R. H. Ni, Q. Li and X. H. Zhong, Mass spectra and strong decays of charmed and charmed-strange mesons, Phys. Rev. D 105, 056006 (2022).