Multi-agent economic analysis in coping with novel coronavirus pneumonia

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Abstract. This paper aims to find a way of allocating masks appropriately during the outbreak of COVID-19. By considering the goals of different perspectives such as mask producers, mask consumers and countries’ government, this paper constructs some mathematical models in order to maximize each group’s objective function. The model takes an assumption that the masks produced are all allocated without friction cost. Based on the model, it is demonstrated that the government plays an important role in mask allocation only after the initial stage of the outbreak of Covid-19. It is also shown that the optimal allocation plan of the mask market is influenced by the government action even if we do not consider direct interference of the government on the market, such as taxes, or price ceiling.

1. Introduction
In 2020, an outbreak of infectious coronavirus occurs worldwide. The world health organization (WHO) later identified it as a new type of coronavirus, which is named COVID-19. It is a disease that causes an infection mainly in a person's respiratory tract, and can spread in a rapid spread through person - to - person contact. Although patients may only feel as if they have caught a cold, the disease actually “kills” people. The rapid spread of it has aroused countries’ attention around the world including health organizations such as WHO.

According to WHO’s advice for the public, aa the virus can enter people’s body quickly through mouth, nose and eyes, wearing masks is one of the simple precautions to reduce chances of getting infected or spreading it [1].

This fact turns masks as a necessary resource to protect one’s health, which causes a huge demand for it worldwide. From an economist's point of view, the fundamental problem of economics is that people's unlimited wants and needs are not able to be satisfied by the world's scarce resources. Since the consequences of not wearing a mask can be causing more infections and thus more deaths, the appropriate allocation of masks becomes extremely more significant than usual times.

To make an appropriate allocation of masks under such special situations as COVID-19, three important groups of people needed to be considered: the government, mask producers, and mask consumers. However, the goal of different people under such situation can be varied which makes an appropriate allocation of masks a hard thing to do.

The responsibility of a government is to ensure its citizens and the persons in their territory are secure against threats to their well-being and the stability of their society. Thus, the purpose of a government...
is to maximize public protection. Under COVID-19, the achievement of the goal can be expressed by minimizing both the number of people who get infected by the disease and the number of deaths caused by the virus.

On the other hand, from a mask producer's point of view, the purpose of a businessman is to maximize the enterprise's profits. Thus, a firm is more likely willing to produce masks with minimum costs and try to set the price at the level that can bring the enterprise a considerable profit.

The last group of people worth discussing is countries' citizens around the world. Since the masks are considered to be a necessity during the outbreak of coronavirus, it is reasonable to assume that most of the people are willing to buy masks in order to protect themselves. So, the consumers of masks should be nearly all the people living on the Earth.

Since the epidemic of the virus may cause the consumers to have a concern of the safety of themselves, they may want to have a stock of masks in order to acquire a feeling of security. At the same time, following the assumption in economics, consumers should be willing to buy masks at a price as low as possible. High price of masks will cause less people to be able to buy masks.

The conflicting aims of different viewpoints increase the difficulty of resource allocation. Hence, this investigation breaks into two aims, the first one is setting suitable mathematical models of the goal of each group of people listed above, and the second one is to find the most appropriate way to allocate masks during COVID-19 through the mathematical models.

In summary, main contributions in this study are listed as follows: (1) Reasonable mathematical models have been created by considering each group’s goal. (2) The most appropriate way of masks allocation under the outbreak should be founded through those models. (3) Detailed evaluations of the method to allocate resources.

2. Assumptions of the analysis
The government, mask producers, mask consumers, have different aims.

| Groups          | Assumptions                                                                 | Symbols of the components that will be used in the paper |
|-----------------|-----------------------------------------------------------------------------|--------------------------------------------------------|
| Government      | Aim: minimizing the number of people to get infected and the number of deaths Other factors: The political reasons for decision making are ignored. | A: The number of people get infected by the virus B: The number of deaths caused by the virus O: Overall populations in a country |
| Mask producers  | Aim: lowering costs of production, and setting a high price Other factors: The possibilities of making decisions due to morality is ignored. | C: A mask enterprise’s revenue D: Costs of production E: A mask enterprise’s profit F: the cost to produce a single mask P: price of individual mask in the market X: the average number of masks per person provided by producers |
| Mask consumers  | Aim: have enough stock of masks in order to acquire a feeling of security. | FOS: A feeling of security |

Now we proceed to formulate these assumptions with the above symbols:
2.1. Objective function of Government

Since government wants to minimize the number of people that are infected or dead because of COVID-19, we need an objective function that is negatively correlated to the value A and value B. Also, note that the number of deaths is much more serious, and thus should take a higher weight. Therefore, we use the objective function

\[ G_{obj}(A, B, O) = 1 - \frac{2A + 8B}{O} \]  

(1)

2.2. Objective function of the mask producer

It is straightforward to assume that the mask producer aims to maximize profit. There are numerous mask producers in the market, and we assume that the market is a perfect competitive market and every producer offers the mask with a single price. Thus, we can view the collection of all mask producers as a single abstract mask producer. Based on our symbols, the total revenue is just the product of the masks sold and the price. To determine the total number of masks sold, we make a further assumption that the market can adjust to equilibrium point instantly. Therefore, the total number of masks produced always equals to the total number of masks bought by the public. This is because if there are masks produced but not sold, the producer can produce less masks or reduce the price of mask quickly to hit the equilibrium of the market.

Under such assumption, the total revenue of the mask producer is

\[ C = O \times X \times P \]  

(2)

whereas the total cost of the mask producer is

\[ D = O \times X \times F \]  

(3)

Thus, the total profit is

\[ E = C - D = O \times X \times (P - F) \]  

(4)

We note that there is also a relation between price P and the total number of masks sold \( O \times X \). We assume a negative and linear relation between price and the number of masks sold. Since the total population \( O \) is constant, we can write

\[ X = Q - \alpha P \]  

(5)

Note that the value \( Q \) and \( \alpha \) can be determined with the market data later. Now the objective function of the mask producer can be rewritten as

\[ E = O \times X \times (P - F) \]  

\[ = O \times (Q - \alpha P)(P - F) \]  

(6)

Note this objective function is a quadratic function of price P, and takes a maximum value at
\[ p = \frac{\alpha F + Q}{2\alpha} \]  

(7)

2.3. Objective function of the public (mask consumers)

We assume the aim of the public is to store as many masks as possible to obtain a sense of security. However, we also assume diminishing marginal benefit overall. That is when someone obtains enough masks, he or she has reached a maximum sense of security that additional masks will not increase the security any more. Before the maximum sense of security is reached, we assume a positive linear relation between the number of the masks owned and the feeling of security:

\[ POS = \min(1, 0.1X) \]  

(8)

As \( X \) is the average number of masks per person used in the previous objective function, the objective function \( POS \) describes the average feeling of security for the public.

3. Developing a reasonable model by analyzing the behaviors of a government at different times during Covid-19 outbreak

As stated previously, the aim of a government is to minimize the number of people who get infected and the number of deaths caused by COVID-19. In this section, the paper will analyze the actions and decisions that a government might make during the epidemic’s three different time periods which are the initial phase of the outbreak, the peak of the outbreak, and the time when the trend in the number of deaths begins to decline in order to develop a reasonable mathematical model.

3.1. At the starting stage of the outbreak

At the starting stage of the outbreak, people only have a few information about the virus, which may cause a rapid spread of COVID-19 due to a lack of information about precautions. Therefore, the number of infected people and the number of deaths should be increasing at a rapid speed which can be shown by the curves from the websites in different countries.

Given that the objective function of the government assigns more weight to the number of deaths, and the number of deaths increased more rapidly at the starting stage, the government needs to spend more on curing the infected than on preventing the spread of the COVID-19. Therefore, at the starting stage of the outbreak, it is not recommended for the government to devote most portion of budget to mask distribution, but is recommended to increase the equipment at hospital instead.

3.2. At the peak of the outbreak

According to the data base, the weekly changed of COVID-19 confirmed cases and death cases are both oscillating within the range of 15% at the peak. Based on our objective (1) for the government, it seems that the government should devote all of the budget to reduce the number of death \( B \) at any time, since it has a higher weight. More specifically, let \( \Delta_A \) and \( \Delta_B \) be the number of infected cases and death cases respectively, which can be reduced by the government if the government spends budget on it. If we have

\[ \Delta_A + \Delta_B = Budget \]  

(9)

where \( Budget \) is a constant. Then we obviously \( G_{obj}(A, B, O) \) reaches the maximum at
\[
G_{obj}(A - \Delta_A, B - \Delta_B, O) = 1 - \frac{2(A - \Delta_A) + B(B - \Delta_B)}{O} \leq 1 - \frac{2A + B(B - \text{Budget})}{O}
\] (10)

When we devote all the budget to the cases of death. However, we note that we can use the assumption of diminishing benefit again. When the government first starts to spend the budget on preventing the spreading of the infected cases or reducing the number of the death cases, we can assume a linear effect: the more the government spends, the linear more of cases are reduced. However, when the government has hospital and necessary equipment to treat a person, to further increase the success rate, we assume the success rate has an inverse relation with the number of equipment and hospital available. This is on the ground that to treat a person already in the hospital, we need effort to search a number of hospital and equipment in order to find the best fit for that person in order to make further improvement [2]. Therefore, for every additional budget spent at this stage, the effect is inversely proportional to the budge already spent on the relevant procedure. This gives the following differential equation:

\[
\frac{dy}{dx} = \frac{1}{x}
\] (11)

where \(dy\) is the increase in the effect, and \(dx\) is the increase in budget on this target. This gives the utility function of the budget as \(y(x) = \ln(x)\). From the basic theory in mathematics analysis, we have \(\ln(x)\) approximates \(x - 1\) when \(x\) is small [3]. This is a subtle beauty of natural log. It successfully models the linearity at the start stage of budget spending and also model the later stage where the budget is overspent and has limited effect. Therefore, we decide to use natural log as the utility function for both budgets spent on preventing the spread of the infected cases and also the budget spent on curing the infected (i.e. reducing the death cases). Now, we get

\[
\Delta_A = \ln(\text{Budget}_A)
\] (12)

\[
\Delta_B = \ln(\text{Budget}_B)
\] (13)

\[
\text{Budget}_A + \text{Budget}_B = \text{Budget}
\] (14)

where Budget is a constant total funding government can spend on COVID-19. We also note that to maximize the objective (9) is equivalent to optimizing \(\frac{2(A - \Delta_A) + B(B - \Delta_B)}{O}\), which is equivalent to maximizing \(2\Delta_A + 8\Delta_B\) constraint to (10). We know calculate that

\[
2\Delta_A + 8\Delta_B = 2\ln(\text{Budget}_A) + 4\ln(\text{Budget}_B)
\]

\[
= \ln(\text{Budget}_A^2 \cdot \text{Budget}_B^2)
\] (15)

Since natural log is an increasing function, maximizing \(2\Delta_A + 8\Delta_B\) is equivalent to maximizing
\[ \begin{align*}
\text{Budget}_A^2 \ast \text{Budget}_B^4 & = \text{Budget}_A^2 \ast (\text{Budget} - \text{Budget}_A)^4 \\
& \leq 16 \left( \frac{2 \ast \text{Budget}}{6} \right)^5
\end{align*} \] (16)

The final inequality is based on inequality of arithmetic and geometric means and the equality [4] is hold when

\[ \text{Budget}_A = \frac{\text{Budget} - \text{Budget}_A}{2} \] (17)

That is when one third of the government budget is spent on preventing the spread of the COVID-19 cases, and the other two thirds of the budget is spent on curing the infected.

3.3. At the time when the number of deaths is decreasing

At this stage, we can make the assumption that virtually all the patients of COVID-19 can access necessary medical care [5]. Therefore, at this stage, we assume very limited effect and utility increase if we devoted additional budget on hospital equipment. We note that even when the number of deaths is constantly decreasing at this stage, the number of infected cases is still increasing steadily at some range. From the latest data of WHO [6], the number of confirmed cases begin to increase in the recent 3 months at a monthly rate of 10 percent. Therefore, from the above statistics, the government should devote most of the budget on infection prevention. Based on recent research, face mask can reduce the spread of COVID-19 with more than 94% efficiency [7][8]. Hence, our conclusion is that most of the budget of the government should be spent on face mask production and distribution to the public in order to optimize the objective function at this stage.

4. Developing a reasonable model by analyzing the behaviors of mask market

For mask producers and mask consumers, we build the model by analyzing their objectives together. Since both producers and consumers play an important role in reaching the equilibrium of the mask market.

4.1. At the starting stage of the outbreak

At the beginning, masks are in shortage worldwide [9] and most people in the public consider masks a necessity. Therefore, we can assume an almost completely inelastic demand curve at the starting stage of the outbreak. Given that the profit of mask producer is Eq. (4) and we assume the demand to be perfectly inelastic, \( X \) is only determined by the production efficiency. We also note that at the starting stage of the outbreak, since the price tends to be high due to the shortage of face masks and inelastic property of the demand, governments from various countries use law to limit the price of the masks [7]. So, we can assume \( P \) to be the highest possible price allowed by the law. On the consumer side, we already define the objective function as the feeling of security, given by (8). Note this objective function defines the average benefit for mask consumers. Therefore, the total utility of the mask market can be calculated as

\[ U(X) = O \ast POS + E = O \ast (X \ast (P - F) + \min(1, 0.1X)) \] (18)

Note this utility is a monotonically increasing function of \( X \), so at the starting stage the mask market should focus entirely on how to open more mask industry to increase the mask production.
2. At the peak of the outbreak and at the time when the number of deaths is decreasing
We assume that after the starting stage, the mask market reaches an equilibrium and the demand of the mask becomes elastic to some extent. More specifically, we have (5) to quantify the number of masks sold at each price level. We already derive that the total profit for mask producer is (6). Combined with the total consumer benefit, we have the utility function

\[
U(X) = E + O \cdot POS \\
= O \cdot X \cdot (P - F) + O \cdot \min(1, 0.1X)
\]

(19)

Given that the total population \(O\) is fixed, we only need maximize

\[
X \cdot (P - F) + \min(1, 0.1X) \\
= X \cdot \left(\frac{Q - x}{\alpha} - F\right) + \min(1, 0.1X) \\
\leq -\frac{X^2}{\alpha} + \left(0.1 + \frac{Q}{\alpha} - F\right) \cdot X
\]

(20)

Therefore, the production of mask will reach the maximum utility in the mask market when the average mask per consumer reaches

\[
X = \frac{\alpha}{2} \cdot (0.1 + \frac{Q}{\alpha} - F)
\]

(21)

If this quantity is no greater than 10. If \(X > 10\), \(\min(1, 0.1X)\) is a constant, and the maximum utility in the mask market is reached at

\[
X = \frac{\alpha}{2} \cdot \left(\frac{Q}{\alpha} - F\right)
\]

(22)

5. Find a most appropriate way to allocate masks by using the models
In order to combine the above models altogether, we need to make several more assumptions. Firstly, we ignore the externality of wearing masks. Note that in reality, for every mask used by an individual, it brings not only the feeling of security to himself, but also bring the feeling of security to other people. Also, we make the assumption that the government and the market have no direct causal relationship. This means that the government will not try to influence the market directly except the price limit at the starting stage of the outbreak. Instead, both the government and the market aim to maximize their utility according to the status of the outbreak. Therefore, the behavior of the government and the market may be (actually is very likely to be) correlated since they are influenced by the same event and the same stage of the event. Also note that we model the government behavior by deciding the partition of the government’s budget. To relate this with the market model, we simply assume that there is a linear increase in the number of masks produced proportional to the government’s budget on the prevention of spread of COVID-19.

5.1 At the starting stage of the outbreak
Based on our model, the government allocates almost all of its budget on curing the infected rather than preventing the spread of the COVID-19. Therefore, the mask allocation is entirely decided by the market. The only intervention here is the price limit set on the market price of the mask by the government.
Based on our market model, the mask produced and allocated to the public is solely determined by the mask production rate at this stage. Therefore, all of the capital and element of production should be devoted to the mask production in the mask industry.

5.2. At the peak of the outbreak
At this stage, our government model gives that one third of the government budget is spent on the COVID-19 spread prevention. We assume a linear increase in the number of masks sponsored by the government to the market, and we get

$$\Delta x = \frac{1}{3} G_x$$  \hspace{1cm} (23)

Where $G_x$ is the total number of increases in mask production and allocation if all the government budget is spent on COVID-19 spread prevention. This factor is assumed to be constant for now, since it is only proportional to the number of the government budget available. Now the objective of function of the market can be rewritten as

$$U(X) = E + O \ast POS$$

$$= O \ast X \ast (P - F) + \min\{1, 0.1(X + \Delta x)\}$$

$$= X \ast \left(\frac{O - X}{\alpha} - F\right) + \min\left(1, 0.1\left(X + \frac{G_x}{3}\right)\right)$$

$$\leq -\frac{X^2}{\alpha} + \left(0.1 + \frac{Q}{\alpha} - F\right) \ast X + \frac{G_x}{30}$$  \hspace{1cm} (24)

The influence of the government on the market seems negligible here since it turns out to be a constant increase in the market objective function. However, we note that the increase $\frac{G_x}{3}$ can help $0.1 \left(X + \frac{G_x}{3}\right)$ reaches a value of 1 faster, and thus turn the recommended allocation of masks through market from

$$X = \frac{\alpha}{2} \ast \left(0.1 + \frac{Q}{\alpha} - F\right)$$  \hspace{1cm} (25)

to

$$X = \frac{\alpha}{2} \ast \left(\frac{Q}{\alpha} - F\right)$$  \hspace{1cm} (26)

In the case when $\frac{G_x}{3}$ is greater than 10, the market model is determined to have (26) as the optimal number of masks to produce and allocate regardless of the value of X itself.
5.3. At the time when the number of deaths is decreasing

At this stage, our government model analysis concludes that we need to devote the complete budget to the mask market. This has a similar effect on the allocation of mask in the mask market, except we have a larger constant increase:

\[
U(X) = E + O \cdot POS \\
= O \cdot X \cdot (P - F) + \min\left(1.01(X + \Delta x)\right) \\
= X \cdot \left(\frac{Q - X}{\alpha} - F\right) + \min\left(1.01(X + G_{\alpha})\right) \\
\leq -\frac{X^2}{\alpha} + \left(0.1 + \frac{Q}{\alpha} - F\right) \cdot X + \frac{G_{\alpha}}{10}
\]  

(27)

From here we know that the market will switch from allocation of \(\frac{\alpha}{2} \cdot \left(0.1 + \frac{Q}{\alpha} - F\right)\) masks per person to \(\frac{\alpha}{2} \cdot \left(0.1 - F\right)\) masks per person when \(G_{\alpha} + X \geq 10\).

6. Discussion on the way of mask allocations

In this section, we analysis the way of mask allocations in our models. Firstly, we make the assumption that the number of masks produced is equal to the number of masks allocated, so our model mostly focuses on the production of masks by the market and government, which is more direct to calculate. However, we note that in reality, there can be a certain amount of inefficiency between the masks production and allocation, which is ignored by our models. Our model also simplifies the way government can interfere with the mask allocation. Since we assume that allocation equals production, all the government budget is considered only in the mask production market.

Our model is really good at getting the optimal value of mask production in the market at the stage of the peak of the outbreak, and also at the later stage of the outbreak. Therefore, our model can be used to provide a guide for mask industry on how to divide the resource and element of production based on the optimal value from the mask market. Unlike the starting stage of the outbreak where all of the resource is recommended to be devoted to mask production, in the later stage if the industry has resource to produce masks greater than

\[
O \cdot X = O \cdot \frac{\alpha}{2} \cdot \left(0.1 + \frac{Q}{\alpha} - F\right)
\]

(28)

It is recommended for the industry to devote the additional resource in advocating the importance of wearing mask, and driving the demand of face masks.

7. Conclusion

In conclusion, we build our models on two most important assumptions. The first assumption is that we define all masks produced are allocated perfectly efficiently. This implies optimizing allocation is equivalent to optimizing production. The second assumption is that there is no direct interference from government on the mask market. In other word, government will not set tax, quota, price ceiling or floor on the mask market. Though government is allowed to decide how much budget is spent on the mask production. Our finding is that government’s spending on the mask production has a direct influence on
the optimal pricing and quantity for the mask industry. Therefore, without direct price constraint, government still has sufficient control over the mask market and is able to influence the structure of mask allocation. The other finding is that the objective function of mask production has a unique optimal point. This implies given the government spending structure, there always exists one and only one optimal price and quantity that the mask industry will try to reach. The uniqueness of the optimal point is due to the convexity property of quadratic objective function which the paper constructs. We conclude that, whenever government changes the budget structure, the mask industry will always find a corresponding optimal point. However, further research needs to be conducted to determine the speed at which the mask industry changes from one optimal equilibrium to another. Also in the future, we can relax the assumption that the conversion from mask production to allocation is perfectly efficiently.

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