Decay of the charged Higgs boson and the top quark in two-Higgs-doublet model at NNLO in QCD

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Abstract: We present numerical calculations of the partial width of the charged Higgs boson decay into a top quark, $H^+ \rightarrow \bar{t} + b + X$, and the partial width of the top quark decay into a light charged Higgs boson $t \rightarrow H^+ + b + X$ at next-to-next-to-leading order (NNLO) in QCD, based on a factorization formula of the jet mass. The NNLO corrections significantly reduce the renormalization scale dependence of the partial decay width in both cases. We show relative size of the NNLO corrections for different charged Higgs boson masses and for different renormalization scales. The NNLO corrections are about 16% (1%) of the leading order widths for the charged Higgs boson mass of 200 GeV (2000 GeV), while it is quite small for the top quark decay. Our analyses are independent of the detailed structure of the Yukawa couplings, and can be applied to various new physics models, as demonstrated by the decay branching ratio in different types of the two-Higgs-doublet models.

Keywords: Higher-Order Perturbative Calculations, Multi-Higgs Models

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1 Introduction

The discovery of the Higgs boson by the ATLAS and CMS experiments [1, 2] at the Large Hadron Collider (LHC) makes a milestone in particle physics. Further measurements at the LHC have shown that properties of the 125 GeV Higgs boson are consistent with the predictions of the standard model (SM) up to current accuracy. Nevertheless, there are numerous motivations suggesting that the scalar sector is likely to be non-minimal. For example, the existence of extended Higgs sectors may help to explain origins of the neutrino masses, dark matter, and matter-antimatter asymmetry. Extra Higgs sectors are also needed in such as the supersymmetric models, the Peccei-Quinn model [3] etc. An interesting feature of the extended Higgs sectors is the possibility of one or more charged scalars (also known as the charged Higgs boson), which is the topic of this study. One of the simplest models that contain an extra Higgs sector is the two-Higgs-doublet models...
In a 2HDM, there are three neutral scalars (one of them is a pseudo-scalar if CP is conserved) and two charged scalars \( H^\pm \). The study on properties of the charged Higgs boson can be essential to distinguish different types of 2HDM [5, 6].

In this work, we refer to the charged Higgs boson with mass \( m_H \) larger/less than the top quark mass as heavy/light charged Higgs boson. Due to the large couplings between the charged Higgs boson and the heavy fermions in new physics models, such as the 2HDM, one of the promising production channel at the LHC for heavy charged Higgs boson is \( gg \to t\bar{b}H^+ \), while light charged Higgs boson can be produced via top quark pair production with one of the top quark decaying into a charged Higgs boson. Both the ATLAS and CMS collaborations have searched for the charged Higgs boson, and these measurements can be classified by how the resonant charged Higgs boson decays, such as the \( H^\pm \to t\bar{b} \) channel [7–10], the \( H^\pm \to cs/cb \) channel [11, 12], the \( H^\pm \to \tau \nu \) channel [13, 14], or the bosonic decay channels [15–18]. The couplings and mass of the charged Higgs boson in specific new physics models can also be constrained indirectly by e.g., \( \bar{B} \to X_s \gamma, B \to \tau \nu \) [19].

On the side of theoretical calculations, the next-to-leading order (NLO) QCD corrections to the decay widths of both the top quark \( t \to H^+ + b + X \) and the charged Higgs boson \( H^- \to \bar{t} + b + X \) in the limit of \( m_b \to 0 \) have been known for a long time [20–25]. In this paper, we are dedicated to the numerical calculations of the partial decay width of the heavy charged Higgs boson \( H^- \to \bar{t} + b + X \), and of the top quark decaying into a light charged Higgs boson \( t \to H^+ + b + X \) at next-to-next-to-leading order (NNLO) in QCD, using a phase space slicing method [26].

In recent years, there have been enormous advances in the higher-order QCD calculations. As to the higher-order corrections to the decay of scalar bosons, the partial width of \( H \to b\bar{b} \) is known up to the next-to-next-to-next-to-leading order (N\(^4\)LO), in the limit where the mass of the bottom quark is neglected [27–29]. The partial width for \( H \to gg \) has been calculated to the N\(^3\)LO [30] and N\(^4\)LO [29] in the heavy top-quark limit. We refer the readers to [31, 32] for a complete list of relevant calculations. The fully differential cross sections for \( H \to b\bar{b} \) have been calculated to NNLO in [33, 34] and N\(^3\)LO in [35] for massless bottom quarks, and to NNLO in [36–39] with massive bottom quarks. On the other hand, there is a long history for calculation for higher-order corrections to heavy quark decays. The NLO QCD corrections to the top quark decay width were calculated in [40–42]. The NNLO corrections were calculated in the large top quark mass limit in refs. [43–45], and the full NNLO corrections were given in [26, 46]. The top quark decay width via model-independent flavor-changing neutral current couplings was calculated to NLO in QCD [47, 48]. The QCD corrections to the decay \( b \to c\ell\nu \) have been known to N\(^3\)LO [49].

Very recently there have been several implementations towards matching hadronic decays of the Higgs boson at NNLO with parton shower. Ref. [50] presents the matched results for the Higgs boson decaying into massless bottom quarks within POWHEG framework [51] by the MnNLO method [52]. Ref. [53] presents the calculation for Higgs decays to massless bottom quarks as well as to gluons within the GENEVA framework [54]. Furthermore, in ref. [55] the matching on decays has been extended to including massive bottom quarks by merging of samples with different jet multiplicities.
The rest of our paper is organized as follows. In section 2, we present the framework of our fixed-order calculations. Section 3 gives the numerical results including NNLO partial decay width for $H^- \rightarrow \bar{t} + b + X$ and $t \rightarrow H^+ + b + X$ for different charged Higgs boson masses, together with applications to benchmark scenarios in type-II and type-X 2HDM. We conclude in section 4.

2 Framework

2.1 Effective operator

The interactions between the charged Higgs boson and quarks can be expressed as

$$\mathcal{L}_{H^\pm} = \sum_{i,j} \bar{u}_i(Y_{ij}^{(1)} P_L + Y_{ij}^{(2)} P_R) d_j H^+ + \text{h.c.},$$

(2.1)

where $u_i \in \{u, c, t\}$ and $d_j \in \{d, s, b\}$ are up-type and down-type quarks, respectively. $P_L (P_R)$ is the left(right)-handed projection operators, and $Y_{ij}^{(1,2)}$ are the corresponding complex matrix elements of Yukawa couplings.

Potential applications of this effective operator in phenomenology can be found in the studies of two-Higgs-doublet models (2HDM) without tree-level flavor-changing neutral currents [5]. For example, in a type-II 2HDM, the corresponding Yukawa couplings read

$$Y_{ij}^{(1)} = \frac{\sqrt{2}}{v} V_{u_i d_j} m_{u_i} \cot \beta,$$

$$Y_{ij}^{(2)} = -\frac{\sqrt{2}}{v} V_{u_i d_j} m_{d_j} \tan \beta,$$

where $V$’s are CKM matrix elements, $v$ is the vacuum expectation value, $\beta$ is the rotation angle which diagonalizes the mass-squared matrices of the charged Higgs bosons and of the pseudo-scalars.

In this work, we focus on the interaction between charged Higgs bosons and third-generation quarks. We assume the bottom quark to be massless except for its non-vanishing Yukawa coupling,\(^1\) then the cross section is proportional to $|Y_{33}^{(1)}|^2 + |Y_{33}^{(2)}|^2$. So our calculation is essentially independent of the details of the Yukawa coupling, and our results can be applied to different types of two-Higgs-doublet models. Note, however, the renormalization of the Yukawa coupling matters in our work, which can be found in appendix A. Since the detailed structure of $\gamma^{(i)}$ is beyond the scope of our paper, we will mainly focus on the ratio of the NNLO corrections to that of the LO contribution (as known as the $K$ factors). For both LO and NNLO contributions given in our work, the Yukawa couplings run at three-loop and match at two-loop near thresholds, such that the Yukawa couplings are canceled out in these $K$ factors. As a result, the $K$ factors given in our work will be independent of the details of the Yukawa coupling, and may be applied to different types of two-Higgs-doublet models.

\(^1\)For example, in type-II 2HDM, the $m_b \cot \beta$ term may be competitive with or even larger than the $m_t \cot \beta$ term if $\tan \beta \gg 1$, which is the region that experimental results prefer.
2.2 Phase space slicing method

Our calculation of the NNLO decay width of charged Higgs boson or top quark is based on the phase space slicing method [26]. For both top quark decay \( t \to H^+ + b + X \) and charged Higgs boson decay \( H^- \to \bar{t} + b + X \), where \( X \) are massless partons or bottom quarks, we cluster all the partons in the final state into a single jet, the mass of which is defined as

\[
m_J^2 \equiv (p_t - p_H)^2.
\]

According to the cutoff parameter

\[
\rho \equiv \frac{m_J m_t}{m_H^2 - m_t^2},
\]

the phase space can be divided into two regions, the resolved part where \( \rho > \rho_{\text{cut}} \), and the unresolved part where \( \rho \leq \rho_{\text{cut}} \), with \( \rho_{\text{cut}} \ll 1 \). Then the NNLO partial decay width can be rewritten as,

\[
\Gamma = \int_0^{\rho_{\text{cut}}} \frac{d\Gamma}{d\rho} d\rho' + \int_{\rho_{\text{cut}}}^{\rho_{\text{max}}} \frac{d\Gamma}{d\rho} d\rho' \\
\equiv \Gamma_{\text{unres}} + \Gamma_{\text{res}}.
\]

In the unresolved region, the contribution can be obtained approximately at \( \mathcal{O}(\rho_{\text{cut}}^0) \) by factorization in soft-collinear effective theory (SCET), and in the resolved region it is calculated up to NNLO by Monte-Carlo simulation. The details are given in section 2.3.

2.3 Fixed-order calculation

Firstly, let us consider the evaluation of decay width in the resolved region (including three-body and four-body phase spaces), taking heavy charged Higgs decay \( H^- \to \bar{t} + b + X \) as an example. The NLO corrections to \( \Gamma_{\text{res}} \) are given by the tree level contribution of \( H^- \to \bar{t} + b + g \), which has no divergence in the \( \rho > \rho_{\text{cut}} \) region. The NNLO corrections to \( \Gamma_{\text{res}} \) consist of two parts, the NLO corrections of \( H^- \to \bar{t} + b + g \), denoted by \( \Gamma_{\text{res}}^{(2)} \), and the LO contribution \( \Gamma_{\text{res}}^{(1)} \) of \( H^- \to \bar{t} + b + j + j \), where \( jj \) may take \( gg, q\bar{q} \) or \( b\bar{b} \).

Note that both \( \Gamma_{\text{res}}^{(2)} \) and \( \Gamma_{\text{res}}^{(1)} \) contain infrared (IR) divergences. The key point is that for \( H^- \to \bar{t} + b + j + j \), the cut \( \rho > \rho_{\text{cut}} \) in four-body phase space forbids the appearance of double unresolved partons, that is, there is at least one resolved parton. So we may regard the sum of \( \Gamma_{\text{res}}^{(2)} \) and \( \Gamma_{\text{res}}^{(1)} \) as the NLO corrections to \( H^- \to \bar{t} + b + g \), which are IR safe. The IR divergences of the two ingredients can be removed individually by introducing appropriate dipole subtraction terms [56–59], then they can be calculated numerically by Monte-Carlo event generators.

We neglect the masses of light quarks. The mass of bottom quark is also omitted except for its non-vanishing Yukawa coupling. The external gluon and quark fields are renormalized with on-shell (OS) scheme. The Yukawa coupling is renormalized in the \( \overline{\text{MS}} \) scheme. The renormalization of QCD coupling is carried out in 5-flavor \( \overline{\text{MS}} \) scheme, with \( \alpha_s^{(N_f=5)}(m_Z) = 0.1181 \). For completeness we present ingredients of fixed-order calculation in appendix A. The NLO amplitudes of three-body decays is generated by FeynArts [60].
They are further simplified and reduced to scalar integral in the Passarino-Veltman reduction scheme [61] by F{\textsc{eyncalc}} [62].

The NLO decay widths in QCD for both top quark decay \( t \rightarrow H^+ + b + X \) [20, 21] and charged Higgs decay \( H^- \rightarrow \bar{t} + b + X \) [22–25] in the \( m_t \rightarrow 0 \) limit have been known for a long time, and we present them here for completeness.

\[
\Gamma_{t \rightarrow H^+ + b + X}^{\text{NLO}} = \frac{1}{32\pi} m_t y_{\text{MS}}^2 \left(1 - \frac{m_H^2}{m_t^2}\right)^2 \left[1 + \frac{16}{3} a_s \Delta_t + 2a_s C_F \left(3 \ln \frac{\mu^2}{m_t^2} + 4\right)\right],
\]

\[
\Delta_t = -2 \text{Li}_2(z) + \frac{z}{z - 1} \ln z + \left(1 - \frac{5}{2}\right) \ln(1 - z) - \ln z \ln(1 - z) - 2\zeta_2 + \frac{9}{4},
\]

\[
\Gamma_{H^- \rightarrow \bar{t} b X}^{\text{NLO}} = \frac{N_c}{16\pi} m_H y_{\text{MS}}^2 \left(1 - \frac{m_t^2}{m_H^2}\right)^2 \left[1 + \frac{16}{3} a_s \Delta_H + 2a_s C_F \left(3 \ln \frac{\mu^2}{m_t^2} + 4\right)\right],
\]

\[
\Delta_H = 2 \text{Li}_2(z) + \ln z \ln(1 - z) + \left(z - \frac{5}{2}\right) \ln \left(\frac{1 - z}{z}\right) + \frac{1}{z - 1} \ln z + \frac{9}{4},
\]

where \( z = m_t^2/m_H^2 \) and \( m_t^2/m_H^2 \) for the top and charged Higgs decays, respectively. \( m_H \) is the mass of charged Higgs boson, \( N_c = 3 \) and \( C_F = 4/3 \) are color factors, \( y_{\text{MS}}^2 \) denotes the renormalized \( |Y_{33}^{(1)}|^2 + |Y_{33}^{(2)}|^2 \) in the MS scheme, and \( a_s = \alpha_s(\mu) = \frac{a_s^2}{4\pi} \) is the strong coupling constant at renormalization scale \( \mu \), \( \zeta_2 = \frac{\pi^2}{6} \). \text{Li}_2 \) is polylogarithm of order 2.

### 2.4 QCD factorization and singular distribution

As is mentioned above, the decay width in the unresolved region \( \Gamma_{\text{unres}} \) defined in eq. (2.4) is calculated with the help of the factorization formula in the threshold limit \( m^2_j \rightarrow 0 \)

\[
\frac{1}{\Gamma_{i}^{\text{LO}}} \frac{d\Gamma_i}{dm^2_j} = H_i(m_H, m_t, \mu) \int dp_J^2 d\omega J(p_J^2, \mu) S_i(\omega, \mu) \delta(m^2_j - p_J^2 - 2E_J^i \omega),
\]

where the subscript \( i \) takes \( t, H \) for top quark decay \( t \rightarrow H^+ + b + X \) and charged Higgs decay \( H^- \rightarrow \bar{t} + b + X \) respectively. \( \Gamma_{i}^{\text{LO}}, H_i, S_i \) are the corresponding LO partial decay width, hard function and soft function, respectively. The jet energy \( E_J^i \) in the threshold limit is given by

\[
E_J^t = \frac{m_t^2 - m_H^2}{2m_t} \quad \text{for top quark decay},
\]

\[
E_J^H = \frac{m_H^2 - m_t^2}{2m_H} \quad \text{for charged Higgs decay}.
\]

The factorization formula eq. (2.7) is valid up to the leading power in power expansion of \( \rho_{\text{cut}} \) [63–65]. So the cut-off \( \rho_{\text{cut}} \) should be small enough such that the power corrections may be safely omitted for phenomenological applications. In this work, an empirical choice \( \rho_{\text{cut}} = 3 \text{GeV}/m_H \) is used. The dependence of the decay width on \( \rho_{\text{cut}} \) is discussed in section 3.
The heavy-to-light soft functions in eq. (2.7) for top quark decay and charged Higgs decay, denoted by $S_t$ and $S_H$ respectively, read

$$S_t(\omega) = \frac{1}{N_c} \sum_X \text{Tr}(0)|\bar{Y}^i_V(0)Y_a(0)|X\langle X|Y^{a+i}(0)\bar{Y}_e(0)|0\rangle\delta(\omega - n \cdot p_X),$$

$$S_H(\omega) = \frac{1}{N_c} \sum_X \text{Tr}(0)|\bar{Y}^i_V(0)Y_a(0)|X\langle X|Y^{a+i}(0)Y_e(0)|0\rangle\delta(\omega - n \cdot p_X),$$

(2.9)

where the Tr is trace over color indices, $n^\mu$ is the light-like vector in the jet direction. Both quark jet function and soft functions have been known up to three loops [66–69]. More details about the quark jet function and the soft functions are presented in appendix B.

The hard function $H_i$ is the square of the Wilson coefficient of the $H^+\bar{t}b$ operator, determined by matching from QCD to SCET. In practice, they are the virtual corrections to on-shell amplitude squares. The NNLO Wilson coefficient for charged Higgs decay is the same as the form factor for Goldstone boson ‘decay’, which can be found in [70]. Because to on-shell amplitude squares. The NNLO Wilson coefficient for charged Higgs decay is the same as the form factor for Goldstone boson ‘decay’, which can be found in [70]. Ward identity is applied to extract the hard function of $t \to \bar{b} + H^+$ from the two-loop virtual corrections to $b \to u$ decay [71]. The obtained hard function is expanded in $\alpha_S^{(6)}$, which is then expressed in terms of $a_s^{(5)} \equiv \alpha_S^{(5)}/(4\pi)$ as

$$H = 1 + a_s^{(5)}(\mu)H_1 + (a_s^{(5)})^2H_2 + \mathcal{O}((a_s^{(5)})^3),$$

(2.10)

with the help of the decoupling relation

$$\alpha_s^{(N+1)} = \alpha_s^{(N)} \left[ 1 + \frac{8}{3} T_F \frac{\alpha_s^{(N)}}{4\pi} \left( - \frac{1}{2} L_\mu + \epsilon \left( \frac{L_\mu^2}{4} + \frac{1}{24}\pi^2 \right) \right) \right],$$

(2.11)

where $N_l = 5$ is the number of light quark flavor, $T_F = 1/2$ for SU(3)$_c$, and $L_\mu = \ln(m_t^2/\mu^2)$.

Combining all these ingredients, NNLO $\rho$ distribution at leading power is given by

$$\frac{1}{\Gamma_0} \frac{d\Gamma}{dp} = \delta(\rho) + a_s C_F \left\{ (4L^2 + 10L + 7 - 7\zeta_2 + H_1/C_F)\delta(\rho) - 14L_0(\rho) - 16L_1(\rho) \right\}$$

$$+ a_s^2 C_A C_F \left\{ \frac{1}{648} \delta(\rho) \left[ 6336L^3 - 144(36\zeta_2 - 299)L^2 - 24(1008\zeta_2 + 1188\zeta_3 - 2935)L ight. 
$$

$$+ 50521 - 45324\zeta_2 - 22536\zeta_3 + 162000\zeta_4 \Big] + \frac{1}{9} L_0(\rho)(-924L - 905 + 408\zeta_2 + 72\zeta_3)$$

$$+ \frac{4}{9} L_1(\rho)(-264L + 95 + 72\zeta_2) + 176L_2(\rho) \right\}$$

$$+ a_s^2 C_F \left\{ \delta(\rho) \left[ 8L^4 + 40L^2 + \frac{1}{4} (312 - 112\zeta_2)L^2 + \frac{1}{2} (-188\zeta_2 + 96\zeta_4 + 146)L ight. 
$$

$$+ \frac{205}{8} - 94\zeta_2 + 22\zeta_3 + \frac{401\zeta_4}{4} \Big] + L_0(\rho)(-56L^2 - 140L - 101 + 66\zeta_2 - 16\zeta_3)$$

$$+ 4L_1(\rho)(-16L^2 - 40L + 21 + 12\zeta_2) + 336L_2(\rho) + 128L_3(\rho) \right\}$$

$$+ a_s^2 C_F N_l \left\{ \frac{1}{324} \delta(\rho) \left[ -576L^3 - 3600L^2 + 24(72\zeta_2 - 251)L - 4073 + 2628\zeta_2 - 72\zeta_3 \right] 
$$

$$\left. - \frac{2}{9} L_0(\rho)(-84L - 85 + 24\zeta_2) + \frac{8}{9} (24L - 13)L_1(\rho) - 32L_2(\rho) \right\}$$

$$+ a_s^2 H_2 \delta(\rho) + a_s^2 C_F H_1 \left[ (4L^2 + 10L + 7 - 7\zeta_2)\delta(\rho) - 14L_0(\rho) - 16L_1(\rho) \right],$$

(2.12)
where \( L = \ln \frac{\mu m_t}{m_{\nu} - m_t} \), \( L_n \) are plus distributions defined as

\[
L_n \equiv \left\lfloor \frac{\ln n}{\rho} \right\rfloor_+, \quad n \geq 0.
\]

Note that eq. (2.12) is valid for both top quark decay \( t \to H^+ + b + X \) and charged Higgs decay \( H^- \to \bar{t} + b + X \), which is a merit of the proper choice of the slicing parameter \( \rho \) defined in eq. (2.3).

3 Numerical result

In the numeric calculations, we use 3-loop running of \( \alpha_s \) with \( \alpha_s^{(N_f=5)}(m_Z) = 0.1181 \) [72]. The pole masses of the top quark and bottom quark are set to 172.5 GeV and 4.78 GeV respectively. For the Yukawa coupling, the \( \overline{\text{MS}} \) masses of top and bottom quarks run at three-loop and match at two-loop near the flavor threshold [73]. The vacuum expectation value takes \( v = 246.22 \text{ GeV} \). We set the renormalization scale to \( \mu_R = \frac{1}{2} m_H \) for charged Higgs boson decay \( H^- \to \bar{t} + b + X \) and to \( \mu_R = m_t \) for top quark decay \( t \to H^+ + b + X \) unless specified, as will be justified later.

We use the VEGAS [74] algorithm implemented in CUBA library [75] to perform the numerical integration. The polylogarithm and harmonic polylogarithm functions appeared in the hard function are calculated by HANDYG library [76]. The scalar integrals in the NLO corrections of the three-body decay are numerically calculated with QCDLoop [77]. The LO matrix elements of the four-body decay are calculated with HELAS library [78].

3.1 Charged Higgs boson decay

In this subsection, a charged Higgs boson with mass greater than the top quark is considered. In the first part, the dependence of our predictions on the cut-off of the phase space slicing variable and on the renormalization scale is demonstrated. By comparison with the analytical results available at NLO, we justify the consistency of our results for small cut-offs. We then show our NNLO predictions with various charged Higgs boson masses in the second part.

3.1.1 Validation of the calculations

In figure 1 we plot the dependence of the NLO and the NNLO partial decay width on the cut-off of the phase space slicing variable \( \rho \) for a 300 GeV (left) or a 1500 GeV (right) charged Higgs boson decaying into a top quark and jets. All predictions are normalized to the LO partial width for simplicity. The blue (orange) scatter points with error bars represent our NLO (NNLO) calculations with Monte-Carlo statistical uncertainties, and the green horizontal lines represent the analytical NLO predictions shown in eq. (2.6).

In both panels, as \( \rho \) decreases, our NLO results clearly approach the analytical NLO predictions. The deviations of our NLO results from the genuine predictions for large cut-off \( \rho_{\text{cut}} \) are due to power corrections. For \( \rho_{\text{cut}} \) below 0.03 (0.005) for a 300 (1500) GeV charged Higgs boson, however, these differences are within the Monte-Carlo statistical errors and
Figure 1. Dependence of the partial decay width on the phase space slicing parameter cut-off $\rho_{\text{cut}}$ for a 300 GeV (left) or a 1500 GeV (right) charged Higgs boson decaying into a top quark and jets. The $\rho$ parameter is defined in eq. (2.3). The green horizontal lines represent the analytical predictions of the NLO partial decay width. The blue (orange) scatter points with error bars represent our NLO (NNLO) predictions. The renormalization scale is set to the half of the charged Higgs boson mass. All predictions are normalized to the LO partial width $\Gamma_{H \to tb}$ with $m_H = 300$ GeV (left) or 1500 GeV (right).

are less than one per mille. The power corrections in these regions can therefore be safely neglected. In spite of the consistency between our numerical and the analytical results, an increase in the Monte-Carlo uncertainties manifests for very small cut-offs, which is especially marked for $m_H = 1500$ GeV at NNLO. Our NNLO results tend to be stable when $\rho_{\text{cut}}$ is small. The Monte-Carlo uncertainties instead grows by multiple times, but still keep small in absolute values. Furthermore, a clear distinction between the soft or collinear region and the fixed order region is needed for definite predictions. Empirically, we take $\rho_{\text{cut}} = 3 \text{GeV}/m_H$ in the following analyses.

The dependence on renormalization scale for the partial decay width of a 300 GeV charged Higgs boson is shown in figure 2. In this figure, predictions at LO, NLO and NNLO are plot in dot-dashed blue, dashed orange and green lines, respectively. All results are normalized to the LO partial width at central scale $\mu_R = m_H$. As we can see, the LO partial decay width varies by about $-17\% \sim 24\%$ as the renormalization scale changes by a factor of 4. The dependence at LO is completely due to the running of the $\overline{\text{MS}}$ Yukawa couplings. The NLO partial decay width has an uncertainty of about $-13\% \sim 13\%$. The renormalization scale uncertainty is further suppressed to about $-7\% \sim 3\%$ of the LO width with the inclusion of the NNLO QCD corrections. Besides, from figure 2 it indicates that taking $\mu_R = m_H/2$ or $\mu_R = m_H/4$ leads to better convergences of the perturbative series. The optimal renormalization scale actually depends on the mass of the charged Higgs boson. As is mentioned above, a general setting of $\mu_R = m_H/2$ is used for $200\text{GeV} \leq m_H \leq 3000\text{GeV}$ in our work, unless otherwise specified.
Figure 2. Renormalization scale dependence of the LO (dot-dashed blue line), NLO (dashed orange line) and NNLO (green line) partial decay width of a 300 GeV charged Higgs boson decaying into a top quark and jets. All results are normalized to the LO partial width at central scale $\mu_R = m_H$. Monte-Carlo uncertainties are rather small and are not shown here.

3.1.2 NNLO partial width for different $m_H$

In figure 3, we demonstrate the dependence of the partial decay width on the mass of the charged Higgs boson for its decay to a top quark and jets. In this figure, the NLO (NNLO) partial decay width is plotted in dashed black (dot-dashed red) line. The green band is bounded by the NLO partial width at three different renormalization scales, more clearly, it is between the minimum and the maximum of $\{\Gamma^{\text{NLO}}(\mu_R = \mu) / \Gamma^{\text{LO}}(\mu_R = m_H/2)\}$, for $\mu = m_H/4$, $m_H/2$ or $m_H$, and the yellow band is the NNLO counterpart. These two bands give estimations of the residual perturbative uncertainties. Though the possibility exists that the ratios locate outside the green (yellow) band for certain $\mu \in [m_H/4, m_H]$. All these results are normalized to $\Gamma^{\text{LO}}(\mu_R = m_H/2)$.

Compared with the NLO results, the uncertainties from renormalization scale are significantly reduced in entire ranges of the charged Higgs boson masses considered once the NNLO corrections are included. For a charged Higgs boson with moderate mass of about 400 GeV, this reduction can be as large as 80%. Moreover, the NNLO corrections are most sizable for charged Higgs boson with low masses. For a charged Higgs boson with mass of 200 GeV, the corrections reach up to 10%, and the corrections decrease to about 1% for a mass of 3000 GeV.

We also summarize ratios of the NNLO (NLO) to the LO predictions using the same renormalization scale in both the numerator and the denominator, for a variety of the charged Higgs boson masses in table 1. As we already commented at the end of section 2.1, such ratios are independent of the detailed structure of the Yukawa couplings, and may be applied to different types of two-Higgs-doublet models. The renormalization scale is chosen as either the mass of the charged Higgs or the half of that. At $\mu_R = m_H$, the NNLO corrections decrease monotonically with the mass of the charged Higgs boson, from 12% for a mass of 200 GeV to about 1% for 3000 GeV. The change of the scale from $m_H$ to $m_H/2$ has little impact on this trend, and leads to smaller corrections in general.
Figure 3. The NLO (dashed black line) and NNLO (dot-dashed red line) partial decay width of charged Higgs boson decaying into a top quark and jets for different charged Higgs boson masses. The green (yellow) band is bounded by the maximum and minimum of NLO (NNLO) partial width at three renormalization scales $\mu_R = m_H, \frac{1}{2} m_H, \frac{1}{4} m_H$. All the results are normalized to the LO partial width at $\mu_R = \frac{1}{2} m_H$.

Table 1. Ratios $\Gamma^{(N)NLO}(\mu_R)/\Gamma^{LO}(\mu_R)$ for $\mu_R = m_H/2$ or $m_H$. Monte-Carlo statistical uncertainties at NNLO are small and not shown.

| $m_H$/GeV | 200 | 250 | 300 | 400 | 500 | 700 | 1000 | 1500 | 2000 | 3000 |
|-----------|-----|-----|-----|-----|-----|-----|------|------|------|------|
| NLO($\frac{1}{2} m_H$) | 1.394 | 1.291 | 1.239 | 1.183 | 1.153 | 1.124 | 1.104 | 1.091 | 1.085 | 1.079 |
| NNLO($\frac{1}{2} m_H$) | 1.559 | 1.365 | 1.279 | 1.196 | 1.156 | 1.119 | 1.096 | 1.082 | 1.076 | 1.070 |
| NLO($ m_H$) | 1.451 | 1.355 | 1.306 | 1.253 | 1.224 | 1.194 | 1.173 | 1.158 | 1.150 | 1.142 |
| NNLO($ m_H$) | 1.670 | 1.481 | 1.394 | 1.308 | 1.265 | 1.222 | 1.195 | 1.175 | 1.166 | 1.156 |

Additionally, throughout our calculation, a five-flavor strong coupling is used and the Yukawa coupling is renormalized in $\overline{MS}$ scheme. Though the Yukawa coupling is canceled out in the ratio $\Gamma^{NNLO(NLO)}(\mu_R)/\Gamma^{LO}(\mu_R)$, the numeric value of this ratio will be quite different if an on-shell Yukawa coupling is taken. The relation between Yukawa couplings under these two schemes can be found in appendix A.

### 3.2 Top quark decay

In this subsection, a charged Higgs boson with mass smaller than the top quark is considered. Conversely to the case in subsection 3.1, the light charged Higgs boson now turns out to be the decay product of the top quark. Analogous discussions to that subsection are performed.

#### 3.2.1 Validation of the calculations

Given a light charged Higgs boson, the top quark can decay into the charged Higgs boson along with jets. In figure 4, we show the dependence of NLO (blue) and NNLO (orange) partial decay width on the cut-off of the phase space slicing parameter, $\rho_{cut}$, for a 100 GeV...
Figure 4. Dependence of the partial decay width on the phase space slicing parameter cut-off $\rho_{\text{cut}}$ for the top quark decaying into a 100 GeV (left) or a 140 GeV (right) charged Higgs boson and jets. The $\rho$ parameter is defined in eq. (2.3). The green horizontal lines represent the analytical predictions of the NLO partial decay width. The blue (orange) scatter points with error bars represent our NLO (NNLO) predictions. The renormalization scale is set to the pole mass of the top quark. All predictions are normalized to the LO partial width $\Gamma_{t\to Hb}^{\text{LO}}$ with $m_H = 100$ GeV (left) or 140 GeV (right).

(left) or a 140 GeV (right) charged Higgs boson. The green horizontal lines represent the analytical NLO predictions given in eq. (2.5). All predictions are normalized to the LO partial width.

As with the case in subsection 3.1, for small enough cut-offs, the deviations of our NLO predictions from the analytical NLO calculations are negligible. The empiric choice of $\rho_{\text{cut}} = 3$ GeV $/m_H$ is, however, no longer valid in current scenarios. For a charged Higgs mass of 100 GeV, the empiric value is 0.03 which is away from the stable region. Considering the range of the charged Higgs boson mass studied is limited, in the following analyses we choose a constant value of $\rho_{\text{cut}} = 0.01$ which is small enough to give a stable result.

Under this convention, we can find that the difference between the genuine results and our predictions at NLO is within a per mille. At NNLO, instead, the relative fluctuations in the stable region, and the Monte-Carlo uncertainties seem to be large especially for a charged Higgs boson with mass of 100 GeV, which is due to the fact that higher-order corrections are small, while the Monte-Carlo uncertainties are only relevant to the size of generating samples. Considering the absolute deviations, we can safely neglect the Monte-Carlo errors as well as the power corrections.

In figure 5, we show the renormalization scale dependence of the partial width of the top quark decaying into a 100 GeV charged Higgs boson. Results at LO, NLO and NNLO are plotted in dot-dashed blue, dashed orange and green lines, respectively. All the results are normalized to the LO partial width at a central scale $\mu_R = m_t$. As can be seen, the introduction of higher-order corrections significantly reduces the renormalization scale dependence already at NLO. The supplement of NNLO corrections further stabilize the predictions for $\mu_R \in \left[ \frac{1}{4} m_t, 4 m_t \right]$ to within 1%, which indicates the importance of higher-
Figure 5. Renormalization scale dependence of the LO (dot-dashed blue line), NLO (dashed orange line) and NNLO (green line) partial decay width of a top quark decaying into a 100 GeV charged Higgs boson and jets. All the results are normalized to the LO partial width at central scale $\mu_R = m_t$. The Monte-Carlo uncertainties are very small and are not shown here.

| $m_H$/GeV | NLO($\frac{1}{2}m_t$) | NLO($m_t$) | NNLO($\frac{1}{2}m_t$) | NNLO($m_t$) |
|-----------|-----------------|-----------|-----------------|-----------|
| 70        | 0.895           | 1.000     | 0.890           | 0.997     |
| 80        | 0.896           | 1.001     | 0.890           | 0.998     |
| 100       | 0.901           | 1.006     | 0.893           | 1.002     |
| 120       | 0.911           | 1.015     | 0.899           | 1.009     |
| 125       | 0.915           | 1.018     | 0.901           | 1.013     |
| 130       | 0.920           | 1.022     | 0.905           | 1.017     |
| 140       | 0.932           | 1.033     | 0.915           | 1.028     |
| 150       | 0.951           | 1.051     | 0.932           | 1.047     |

Table 2. Ratios $\Gamma^{(N)LLO}(\mu_R)/\Gamma^{LO}(\mu_R)$ for $\mu_R = m_t/2$ or $m_t$. Monte-Carlo statistical uncertainties at NNLO are small and not shown.

order corrections in this situation. From figure 5 it indicates the optimal renormalization scale choice is $m_t$ which shows a very good perturbative convergence.

### 3.2.2 NNLO partial width for different $m_H$

The variation of the partial decay width with the charged Higgs boson mass, as well as bands of scale variations, are shown in figure 6, with all results normalized to $\Gamma^{LO}(\mu_R = m_t)$. Our NLO (NNLO) results are plotted in dashed black (dot-dashed red) line. On contrary to the tens of percent level corrections to the LO results for a heavy charged Higgs boson, QCD corrections now are at most a few percents. Nevertheless, as we have shown in figure 5, the supplement of these higher-order corrections significantly reduces the scale uncertainties. In most regions, the incorporation of NNLO corrections can further reduce the NLO scale uncertainties by at least 70%.

Analogously, the ratios of the NNLO or the NLO to the LO predictions at the renormalization scale of $m_t$ or $m_t/2$ are summarized in table 2. Unlike the case of a heavy charged Higgs boson, the NNLO corrections are rather small. At $\mu_R = m_t$, NNLO corrections over the entire range of the charged Higgs boson mass are at per mille level. On the other hand,
difference due to the change of scale can be sizable, which mainly originates from the large
cscale uncertainties of the denominator $\Gamma_{\text{LO}}(\mu_R)$. This can be deduced in combination with
figure 5 in which the running coupling of the denominator is fixed at scale $m_t$. Also, it is
worthy noting that, all scale variances at LO are from the running of the Yukawa couplings.

3.3 Phenomenological implications

In this subsection, we show some phenomenological implications of our results for typical
two-Higgs-doublet models. The Higgs sector of the two-Higgs-doublet models consists of
two SU(2)$_L$ scalar doublets $\Phi_i (i = 1, 2)$ with hyper-charge $Y = 1/2$ [5],

$$\Phi_i = \left( \frac{\phi_i^+}{v_i + \phi_i^0 + i G_i^0} / \sqrt{2} \right),$$

(3.1)

where the $\phi_i^+, \phi_i^0$ and $G_i^0 (i = 1, 2)$ are the parametrized component fields. $v_i (i = 1, 2)$ are
the vacuum expectation values of the doublets after the electroweak symmetry breaking, satisfying
$\sqrt{v_1^2 + v_2^2} = v = 246\text{GeV}$. There are eight degrees of freedom with the two
complex scalar SU(2)$_L$ doublets. Three of those give mass to the $W^+, W^-$ and $Z^0$ gauge
bosons from the Higgs mechanism, the remaining five are physical scalar (‘Higgs’) fields.
There are two charged scalars, two neutral scalars, and one pseudoscalar.

The Lagrangian for the Higgs sector of the two-Higgs-doublet models is written as following:

$$\mathcal{L} = \sum_i |D_\mu \Phi_i|^2 - V(\Phi_1, \Phi_2) + \mathcal{L}_{\text{Yuk}},$$

(3.2)
Table 3. Four types of assignments for the $Z_2$ charges of the $\Phi_{1,2}$ and SM fermions. The last two columns indicate the non-zero Yukawa coupling of each scalar doublets $\Phi_{1,2}$.

\[
\begin{array}{cccccccc}
\text{Types} & \Phi_1 & \Phi_2 & u_R & d_R & l_R & Q_L, L_L & \Phi_1 & \Phi_2 \\
\text{Type-I} & + & - & - & - & - & + & u, d, l \\
\text{Type-II} & + & - & - & + & + & d, l & u \\
\text{Type-X} & + & - & - & - & + & + & l, u, d \\
\text{Type-Y} & + & - & - & - & + & + & d, u, l \\
\end{array}
\]

where $D_\mu$ is the covariant derivative and $V(\Phi_1, \Phi_2)$ is the scalar potential. The Yukawa sector of the two Higgs doublets is given by

\[
-\mathcal{L}_{\text{Yuk}} = Y_u \bar{Q}_L \tilde{\Phi}_u u_R + Y_d \bar{Q}_L \Phi_d d_R + Y_e \bar{L}_L \Phi_e e_R + h.c.,
\]

(3.3)

where $\tilde{\Phi} = i\sigma^2 \Phi^*$; $Q_L$ and $L_L$ are the quark and the lepton left-hand doublet; $u_R, d_R, e_R$ are the right-hand singlet; $Y_{u,d,e}$ are Yukawa coupling constants; $\Phi_{u,d,e}$ are either $\Phi_1$ or $\Phi_2$. To avoid tree-level flavor-changing-neutral-currents, a discrete $Z_2$ symmetry is imposed \[79\]. There are four possible choices for the charge assignment of the fermions under $Z_2$, corresponding to type-I, type-II, type-X and type-Y respectively. We summarize the charge assignment of the four types of 2HDM in table 3, along with the non-zero Yukawa couplings for each $\Phi$.

In the following we focus on the type-II and type-X of the two-Higgs-doublet models. We set the model parameters to be $\tan \beta = 20$, $\sin(\beta - \alpha) = 0.995$ and $\cos(\beta - \alpha) > 0$ according to the benchmark point in ref. \[80\]. In addition, we set the mass of charged Higgs boson ($H^+$) and the neutral Higgs boson ($A, H$) to be the same, varying from 200 GeV to 1500 GeV. Moreover, the renormalization scale is chosen as the mass of the Higgs boson. We study the branching ratio of the charged Higgs boson decaying into the top quark and the anti-bottom quark, which can be expressed as

\[
BR(H^+ \rightarrow t\bar{b}) = \frac{\Gamma_{tb}}{\Gamma_{tb} + \Gamma_{\text{rest}}},
\]

(3.4)

where $\Gamma_{tb}$ can be the LO, NLO and NNLO decay width of the channel $H^+ \rightarrow t\bar{b}$ presented earlier. $\Gamma_{\text{rest}}$ represents partial width from all other decay channels which we calculated with 2HDMC-1.8.0 \[81\], where LO QCD corrections are included if the charged Higgs boson decays to two massless quarks, while all other channels only include the Born level contributions.

In figure 7, we show the branching ratio as a function of the Higgs boson mass at different orders of QCD for the type-II model. One can find that the branching ratio firstly increases then decreases. The increase of the branching ratio is because that the phase space of the channel $H^+ \rightarrow t\bar{b}$ is increasing, the Yukawa coupling of the charged Higgs boson with the top and bottom quark is large. The branching ratio decreases at large values of the charged Higgs boson mass due to the contribution of the charged Higgs boson decaying into the W boson and the SM Higgs boson ($H^+ \rightarrow W^+ h$) to the total
width. The branching ratio is 60% and 63% at the NLO and NNLO respectively, for a charged Higgs boson mass of 200 GeV. The observed (expected) 95% confidence level (CL) upper limits of $\sigma(pp \to tbH^+) \times B(H^+ \to tb)$ range from 3.6(2.6)pb at $m_{H^+} = 200$GeV to 0.036(0.019)pb at $m_{H^+} = 2$TeV [8]. It shows that the observed limits are improved by 5% to 70% depending on the mass of the charged higgs boson. And the relative accuracy between the NLO and NNLO predictions given by our results is 5% at $m_{H^+} = 200$GeV. The NLO and NNLO predictions are both about 45% when the mass is 1500 GeV. Overall, one can find that the NNLO corrections are most significant when the charged Higgs mass is between 200 GeV and 500 GeV, and fade away while the mass increases. The NNLO corrections are much smaller than the NLO corrections indicating a good convergence of the perturbative calculations.

We present results of the branching ratio for the type-X model in figure 8. We can see that the branching ratio shows a similar dependence on the charged Higgs boson mass but is much smaller. It is because the Yukawa coupling constant is different between for the two models. The branching ratio is 2.1% and 2.4% at the NLO and NNLO respectively, for
a charged Higgs boson mass of 200 GeV. The NLO and NNLO predictions are both about
1.6% when the mass is 1500 GeV.

The 95% confidence level upper limits of $\sigma(pp \to tbH^+) \times \mathcal{B}(H^+ \to tb)$ is observed
by the ATLAS collaboration, ranging from 3.6 pb at $m_{H^+} = 200$ GeV to 0.036 pb at
$m_{H^+} = 2$ TeV [8]. Compared to the previous ATLAS search, the limits is improved by 5%
to 70% depending on the mass of the charged higgs boson.

4 Summary

In this work, we present the calculations of NNLO partial width of the charged Higgs
boson decay $H^- \to t + b + X$ and the top quark decay $t \to H^+ + b + X$ using a phase
space slicing method with the jet mass as a slicing parameter. The ratios of the NNLO
partial decay width to the LO prediction for a variety of charged Higgs boson mass are
given. They are independent of the detailed structure of the Yukawa couplings, thus for
a specific new physics model, one can easily get the corresponding NNLO partial decay
width by rescaling with the LO prediction (the Yukawa coupling should be renormalized
in $\overline{\text{MS}}$ scheme). The renormalization scale dependence is studied in details for $m_H =
300$ GeV (charged Higgs boson decay) and $m_H = 100$ GeV (top quark decay) respectively.
For both cases, the renormalization scale dependence is significantly reduced by the NNLO
corrections. The renormalization scale dependence of the NNLO partial decay width of the
charged Higgs boson is very weak for $\mu_R = m_H/4 \sim m_H/2$, while for the top quark decay
t $t \to H^+ + b + X$, the renormalization scale uncertainty is within 1%.

For the heavy charged Higgs boson decay, the NNLO corrections with $\mu_R = m_H/2$
are about 16% (1%) of the LO widths for the charged Higgs boson mass $m_H = 200$ GeV
(2000 GeV). On the other hand, the NNLO corrections for the top quark decaying into a light
charged Higgs boson with $\mu_R = m_t$ are quite small, at about 0.3% (0.4%) for $m_H = 70$ GeV
(150 GeV). Note the size of the QCD corrections can have a large renormalization scale
dependence due to the running of the Yukawa couplings. We also show some phenomenology
results on the impact of our calculations to the branching ratio of the charged Higgs boson
decay in two typical 2HDM. The NNLO corrections increase the decay branching ratio
moderately for smaller charged Higgs boson masses.

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A Ingredients of the fixed-order calculation

The strong coupling constant $\alpha_s$ is generally renormalized in the $\overline{\text{MS}}$ scheme, and is related to the bare coupling constant $\alpha_0$ by

$$\alpha_0 = S_\epsilon^{-1} \mu^{2\epsilon} Z_\alpha \alpha_s$$

$$= \alpha_s S_\epsilon^{-1} \mu^{2\epsilon} \left[ 1 + \left( -\frac{\beta_0}{\epsilon} \right) \frac{\alpha_s(\mu)}{4\pi} + \left( \frac{\beta_1}{2\epsilon} - \frac{\beta_2}{\epsilon^2} \right) \left( \frac{\alpha_s}{4\pi} \right)^2 + O(\alpha_s^3) \right], \quad (A.1)$$

where $\epsilon = \frac{4-d}{2}$ is the dimensional regulator, $S_\epsilon = e^{\epsilon(4\pi - \gamma_E)}$, and the expansion coefficients for the QCD beta function up to three-loop order are

$$\beta_0 = \frac{11}{3} C_A - \frac{4}{3} N_f T_F,$$

$$\beta_1 = \frac{34}{3} C_A^2 - \frac{20}{3} C_A N_f T_F - 4 C_F N_f T_F,$$  \quad (A.2)

$$\beta_2 = \frac{2857}{54} C_A^3 + \left( 2C_F^2 - \frac{205}{9} C_F C_A - \frac{1415}{27} C_A^2 \right) T_F N_f + \left( \frac{44}{9} C_F + \frac{158}{27} C_A \right) T_F^2 N_f^2,$$

where $N_f$ is the number of active quark flavors, $C_A = 3, C_F = 4/3, T_F = 1/2$ for QCD. We also use $\alpha_s = \alpha_s(\mu)/(4\pi)$ for simplicity. In this work, the partial decay width is expanded in a five-flavor strong coupling constant, which is related to the $\overline{\text{MS}}$ strong coupling constant with $N_f = 6$ by the decoupling relation eq. (2.11).

The Yukawa coupling is renormalized in $\overline{\text{MS}}$ scheme throughout our calculation. The corresponding renormalization constant and anomalous dimension can be expanded in $\alpha_s^{(N_f)}$ as [82–84]

$$Z_y = 1 - 3 \frac{C_F}{\epsilon} \frac{\alpha_s}{4\pi}$$

$$+ \left[ C_F^2 \left( \frac{9}{2\epsilon^2} - \frac{3}{4\epsilon} \right) + C_F C_A \left( \frac{11}{2\epsilon^2} - \frac{97}{12\epsilon} \right) + C_F N_f \left( \frac{1}{\epsilon} - \frac{5}{6\epsilon} \right) \right] \frac{\alpha_s}{4\pi}^2$$

$$+ \left[ C_F^3 \left( \frac{-9}{2\epsilon^3} + \frac{9}{4\epsilon^2} - \frac{43}{2\epsilon} \right) + C_F^2 C_A \left( -\frac{33}{2\epsilon^2} + \frac{313}{12\epsilon^2} + \frac{43}{4\epsilon} \right) \right] \frac{\alpha_s}{4\pi}^3$$

$$+ C_F C_A N_f \left( \frac{1679}{54\epsilon^2} - \frac{11413}{324\epsilon} \right) + C_F N_f \left( \frac{3}{\epsilon^3} - \frac{29}{6\epsilon^2} + \frac{1}{\epsilon} \left( \frac{23}{3} - 8\zeta_3 \right) \right)$$

$$+ C_F N_f^2 \left( \frac{-4}{9\epsilon^3} + \frac{10}{27\epsilon^2} + \frac{35}{81\epsilon} \right) \left( \frac{\alpha_s}{4\pi} \right)^3 + O(\alpha_s^4), \quad (A.3)$$

$$\gamma_y(\alpha_s(\mu)) = \frac{d \ln y(\alpha_s(\mu))}{d \ln \mu} = \sum_{n=0}^{\infty} \left( \frac{\alpha_s}{4\pi} \right)^{n+1} \gamma_n^y,$$

$$\gamma_0^y = -6 C_F,$$

$$\gamma_1^y = -3 C_F + \frac{97}{3} C_A - \frac{20}{3} N_f T_F.$$
$$\gamma_2^y = -C_F\left\{129C_F^2 - \frac{129}{2}C_AC_F + \frac{11413}{54}C_A^2 + (48\zeta_3 - 46)C_FN_f - \left(\frac{556}{27} + 48\zeta_3\right)C_AN_f - \frac{70}{27}N_f^2\right\}. $$

Here we briefly comment on the situation where the overall Yukawa coupling is renormalized in on-shell (OS) scheme. Numerically, between these two schemes, the NNLO partial widths differ only by higher-order (N$^3$LO here) corrections, while K factors, instead, are quite different. The conversion from one scheme into the other can be performed through relation [85]

$$\frac{y_{MS}}{y_{OS}} = 1 + \left(\frac{\alpha_s(N_l+1)}{\pi}\right) d_1 + \left(\frac{\alpha_s(N_l+1)}{\pi}\right)^2 d_2 + O(\alpha_s^3),$$

(A.4)

$$d_1(m,\mu) = -C_F\left(1 + \frac{3}{4}L\right),$$

$$d_2(m,\mu) = C_F^2 \left[\frac{7}{128} - \frac{3}{4}\zeta_3 + 3\zeta_2 \ln 2 - \frac{15}{8}\zeta_2 + \frac{21}{32}L + \frac{9}{32}L^2\right]$$

$$+ C_A CF \left[-\frac{1111}{384} + \frac{3}{8}\zeta_3 + \frac{1}{2}\zeta_2 - \frac{3}{2}\zeta_2 \ln 2 - \frac{185}{96}L - \frac{11}{32}L^2\right]$$

$$+ C_FT_FN_f \left[\frac{71}{96} + \frac{1}{2}\zeta_2 + \frac{13}{24}L + \frac{1}{8}L^2\right] + C_FT_F \left[\frac{143}{96} - \zeta_2 + \frac{13}{24}L + \frac{1}{8}L^2\right],$$

with $L = \ln(\mu^2/m^2)$. $N_l = 5$ is the number of light quarks.

B Ingredients of the factorization formula

B.1 The jet function

The light quark jet function in SCET was introduced in [86], can be defined in terms of QCD fields as [66]

$$J_q(p^2,\mu) = \frac{1}{\pi N_c} \text{Im} \int d^4x e^{-ip\cdot x} \times \left\{ 0 \bigg| T \text{Tr} \left[ \frac{i}{4} W^\dagger(0) \psi(0) \tilde{\psi}(x) W(x) \right] 0 \bigg| \right\}$$

(B.1)

where $T$ is the time-ordering operator, the trace is over color and spinor indices,

$$W(x) = P \exp \left[ ig_s \int_{-\infty}^{0} ds \bar{n} \cdot A(x + s\bar{n}) \right]$$

denotes a $n$-collinear Wilson line, with $n^\mu$ being the lightlike jet direction, and $\bar{n}$ being the lightlike vector subjected to $\bar{n} \cdot n = 2$.

The renormalized one-loop quark jet function in momentum space reads

$$J_q(p^2,\mu) = \frac{1}{\mu^2} \left\{ \delta(x) + C_F a_s \left[ 4 \left[ \frac{\ln x}{x} \right]_+ - 3 \left[ \frac{1}{x} \right]_+ + (7 - \pi^2)\delta(x) \right] + O(a_s^2) \right\} , x \equiv \frac{p^2}{\mu^2}.$$

However, it is often more convenient to transform the renormalized jet function in the Laplace space

$$\tilde{J}_q \left( L \equiv \ln \frac{1}{e^{\gamma_E} \kappa\mu^2}, \mu \right) = \int dp_2 e^{-\kappa p^2} J_q(p_2^2, \mu^2) .$$

(B.2)
The quark jet function is known up to three-loop \[66, 67, 87–89\]. Here we show the results up to two loops due to limited space,

\[
\tilde{j}^q(L, \mu) = \sum_{n=0} \left( \frac{\alpha_s(\mu)}{4\pi} \right)^n \tilde{j}^{q(n)}(L),
\]

\(\tilde{j}^{q(0)} = 1\),

\(\tilde{j}^{q(1)} = C_F \left( 2L^2 - 3L - \frac{2\pi^2}{3} + 7 \right)\),

\(\tilde{j}^{q(2)} = C_A C_F \left\{ -\frac{22L^3}{9} + \left( \frac{367}{18} - \frac{2\pi^2}{3} \right) L^2 + L \left( 40\zeta_3 + \frac{11\pi^2}{9} - \frac{3155}{54} \right) - 18\zeta_3 \right\} - \frac{155\pi^2}{36} - \frac{37\pi^2}{180} + \frac{53129}{648} \right\} + C_F^2 \left\{ 2L^4 - 6L^3 + \left( \frac{37}{2} - \frac{4\pi^2}{3} \right) L^2 \right. \\
+ L \left( -24\zeta_3 + 4\pi^2 - \frac{45}{2} \right) - 6\zeta_3 + \frac{61\pi^2}{90} + \frac{205}{8} - \frac{97\pi^2}{12} \right\} \\
+ C_F N_f \left\{ \frac{4L^3}{9} - \frac{29L^2}{9} + \left( \frac{247}{27} - \frac{2\pi^2}{9} \right) L + \frac{13\pi^2}{18} - \frac{4057}{324} \right\}.
\]

B.2 The soft function

The heavy-to-light soft functions for top quark decay and charged Higgs decay, denoted by \(S_t\) and \(S_H\) respectively, are defined in eq. (2.9) as

\[
S_t(\omega) = \frac{1}{N_c} \sum_X \text{Tr}(0|\bar{Y}_v(0)Y_n(0)|X)(X|Y^+_n(0)\bar{Y}_v(0)|0)\delta(\omega - n \cdot p_X),
\]

\[
S_H(\omega) = \frac{1}{N_c} \sum_X \text{Tr}(0|\bar{Y}_v(0)Y_n(0)|X)(X|Y^+_n(0)Y_v(0)|0)\delta(\omega - n \cdot p_X).
\]

The soft Wilson lines are defined as

\[
\bar{Y}_v(x) \equiv \text{P exp} \left[ ig_s \int_{-\infty}^0 v \cdot A(x + sv)ds \right],
\]

\[
Y_v(x) \equiv \text{P exp} \left[ -ig_s \int_{0}^\infty v \cdot A(x + sv)ds \right],
\]

\[
Y^+_n(x) \equiv \text{P exp} \left[ ig_s \int_{0}^\infty n \cdot A(x + sn)ds \right],
\]

where \(g_s\) is the strong coupling constant, \(A_\mu(x) := A^a_\mu(x)T^a\) is the (ultra)soft gluon field in the SCET, \(v^\mu \equiv p^\mu/n_t\) is the 4-velocity of the top quark in the rest frame of the parent particle, \(\text{P} (\text{P}^\dagger)\) is the (anti-)path ordering operator. Note that the renormalized soft functions \(S_t\) and \(S_H\) are related to each other by

\[
S_H(\omega, \mu) = \frac{m_t}{m_H} S_t \left( \omega \rightarrow \frac{m_t}{m_H} \omega, \mu \right).
\]

They have been calculated up to three-loop order \[68, 69\]. The one-loop results are given by

\[
S_t(\omega, \mu) = \frac{1}{\mu} \left[ \delta(x) + a_s C_F \left( -\zeta_2 \delta(x) - 4 \left( \frac{1}{x} \right)_+ - 8 \left( \frac{\ln x}{x} \right)_+ \right) \right], \quad x = x_t = \frac{\omega}{\mu},
\]

\[
S_H(\omega, \mu) = \frac{m_t}{m_H} S_t \left( \frac{m_t}{m_H} \omega, \mu \right).
\]
B.3 The hard function

The Wilson coefficient renormalized in $\overline{\text{MS}}$ scheme at NLO QCD for both charged Higgs decay and top quark decay, is given by

$$C_{qQ}^{(RN)}(m_t, m_H, \mu) = \frac{y_{\overline{\text{MS}}}(\mu)}{y_{\text{LO}}(\mu_H)} \left\{ 1 + a_s C_F \left[ - \frac{1}{2} \ln^2 \frac{m_t^2}{\mu^2} + \left( 2 \ln \frac{m_t^2}{m_t^2 - m_H^2} - \frac{1}{2} \right) \ln \frac{m_t^2}{\mu^2} \right. 
$$
$$\left. \left. - \ln^2(1 - \lambda) + 2 \frac{1}{\lambda} \ln(1 - \lambda) + 2 \text{Li}_2 \left( \frac{\lambda}{\lambda - 1} \right) - \frac{\pi^2}{12} \right] \right\} ,$$

$$\lambda \equiv \frac{m_H^2}{m_t^2} + i0^+ .$$

For top quark decay, the Wilson coefficient is real. For charged Higgs decay, the physical branching is chosen by

$$m_t^2 \to m_t^2 - i0^+ .$$

The corresponding hard functions are given by

$$H_i(m_H, m_t, \mu) = |C_i(m_H, m_t, \mu)|^2, i = H, t .$$

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