Comments on 4–point functions in the CFT/AdS correspondence

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Abstract

We study the four–point function of chiral primaries corresponding to the
dilaton–axion sector in supergravity in the $AdS_5/CFT_4$ correspondence. We
find relations between some of the supergravity graphs and compute their lead-
ing singularities. We discuss the issue of logarithmic singularities and their
significance for the OPE structure of the CFT.
1 Introduction

Recently it has been proposed that IIB string theory on $AdS_5 \times S^5$ is dual to a CFT: $\mathcal{N} = 4$ supersymmetric $SU(N)$ Yang-Mills on the boundary of $AdS_5$ \cite{1, 2, 3}. Using this correspondence the 2-point functions \cite{2}–\cite{5} and 3-point functions \cite{4}–\cite{15} of primary operators in the CFT were computed in the limit $N \to \infty$, $g_{YM} \to 0$, $g_{YM}^2 N \to \infty$, where the string theory computations reduce to tree level supergravity calculations.

Several interesting physical issues arise when we move to the study of 4-point functions. We will focus on the limit $N \to \infty$, $g_{YM} \to 0$, $g_{YM}^2 N \to \infty$ mentioned above. In the CFT the scaling dimensions of the chiral primary operators (and their superconformal descendents) are protected, while the dimensions of fields corresponding to massive string states are infinite in this limit. Does there exist a ‘complete’ set of fields and an operator product expansion (OPE) structure that allows us to obtain 4-point functions much the same as in the case of 2-D CFT? If so, do the chiral primaries and their descendents form the complete set, or do we need other fields in the CFT? Is there a connection between supergravity fields propagating in the internal leg of a supergravity graph, and the contribution of a specific chiral primary (plus descendents) in the OPE expansion of the corresponding CFT correlator? Preliminary results on these questions were presented in \cite{16} and \cite{17}.

To address such issues we study in this letter some simple supergravity graphs corresponding to 4-point functions in the CFT. We consider the dilaton ($\phi$) and axion ($C$) sector. (This sector has also been studied in \cite{17}, and, while we use similar methods, we arrive at somewhat different conclusions).

2 4-point functions in the dilaton-axion sector

The relevant part of the $AdS_5 \times S_5$ supergravity action is

$$S = \frac{1}{2\kappa^2} \int_{AdS_5} d^5x \sqrt{g} \left[ -R + \frac{1}{2} (\partial \phi)^2 + \frac{1}{2} e^{2\phi} (\partial C)^2 \right]$$

$$= \frac{1}{2\kappa^2} \int_{AdS_5} d^5x \sqrt{g} \left[ -R + \frac{1}{2} (\partial \phi)^2 + \frac{1}{2} (\partial C)^2 + a(\partial C)^2 + b\phi^2 (\partial C)^2 + \ldots \right] \quad (1)$$
where $a = 1, b = 1$. We use coordinates where the (Euclidean) $AdS$ space appears as the upper half space ($z_0 > 0$) with metric:

$$ds^2 = \frac{1}{z_0^2} [dz_0^2 + \sum_{i=1}^{d} dx_i dx_i] \quad (2)$$

The $AdS$ space has dimension $d + 1$; thus in our present case $d = 4$.

First consider the CFT correlator $\langle O_\phi(x_1)O_C(x_2)O_\phi(x_3)O_C(x_4) \rangle$. In the AdS calculation we encounter the supergravity graphs shown in Figure 1. The s-channel amplitude is

$$s = -(4a^2) I^s_{\phi C \phi C}(x_1, x_2, x_3, x_4) \quad (3)$$

$$I^s_{\phi C \phi C}(x_1, x_2, x_3, x_4) \equiv \int \frac{d^5 z}{z_0^5} \frac{d^5 w}{w_0^5} w_0^2 K(z, x_1) \partial_{z_{\mu}} K(z, x_2) \partial_{z_{\mu}} \partial_{w_{\nu}} G(z, w) K(w, x_3) \partial_{w_{\nu}} K(w, x_4) \quad (4)$$
where

\[ K_\Delta(z, x) = \frac{\Gamma(\Delta)}{\pi^{d/2} \Gamma[\Delta - d/2]} \left( \frac{z_0}{z_0^2 + (z - x)^2} \right)^\Delta \]  

(5)

is the normalized boundary to bulk propagator for scalar fields in supergravity corresponding to primary operators in the CFT of scaling dimension \( \Delta \) [3, 4]. We have \( d = 4 \) and note that for both \( \phi \) and \( C \) we have \( \Delta = 4 \). For this case we will simply write \( K \) without subscript. \( G(z, w) \) is the bulk to bulk propagator in the AdS\(_5\) space for massless scalar fields, satisfying

\[ 2^\Delta z G(z, w) = \delta(z, w) \]  

(6)

We will not need the explicit form of \( G(z, w) \).

The quartic graph is

\[ q = - (4b I^q_{\phi C, \phi C}(x_1, x_2, x_3, x_4) \]  

(7)

\[ I^q_{\phi C, \phi C}(x_1, x_2, x_3, x_4) \equiv \int \frac{d^5z}{z_0^5} z_0^2 K(z, x_1) \partial_{z^\mu} K(z, x_2) K(z, x_3) \partial_{z^\mu} K(z, x_4) \]  

(8)

The combinatoric factors in (3), (7) can be obtained either from Feynman perturbation theory of supergravity or directly from the fourth variation of the supergravity action [1] with respect to boundary values of the fields.

In [17] a nice manipulation was given which relates \( I^s \) to a 4-point contact graph:

\[ \int \frac{d^5z}{z_0^5} \frac{d^5w}{w_0^5} z_0^2 w_0^2 K(z, x_1) \partial_{z^\mu} K(z, x_2) \partial_{z^\mu} \partial_{w^\nu} G(z, w) K(w, x_3) \partial_{w^\nu} K(w, x_4) \]

\[ = \int \frac{d^5z}{z_0^5} \frac{d^5w}{w_0^5} z_0^2 w_0^2 \partial_{z^\mu} K(z, x_1) K(z, x_2) \partial_{z^\mu} \partial_{w^\nu} G(z, w) K(w, x_3) \partial_{w^\nu} K(w, x_4) \]

\[ = \frac{1}{2} \int \frac{d^5z}{z_0^5} \frac{d^5w}{w_0^5} z_0^2 w_0^2 [K(z, x_1) K(z, x_2)] \partial_{z^\mu} \partial_{w^\nu} G(z, w) K(w, x_3) \partial_{w^\nu} K(w, x_4) \]

\[ = \frac{1}{2} \int \frac{d^5z}{z_0^5} \frac{d^5w}{w_0^5} K(z, x_1) K(z, x_2) \delta(z, w) \partial_{w^\nu} K(w, x_3) \partial_{w^\nu} K(w, x_4) \]

\[ = \frac{1}{2} \int \frac{d^5z}{z_0^5} z_0^2 K(z, x_1) K(z, x_2) \partial_{z^\mu} K(z, x_3) \partial_{z^\mu} K(z, x_4) \]  

(9)

\(^1\)We assume \( \Delta > d/2 \). The case \( \Delta = d/2 \) saturates the unitarity bound and requires a special normalisation [3].

\(^2\)In [17] the notation is instead \( \Delta z G(z, w) = -\delta(z, w) \).
where we have integrated by parts (noting that surface terms vanish), used the fact that $\Delta_z K(z, x) = 0$, and used (6). Thus we see that

$$
I_{\phi C \phi C}(x_1, x_2, x_3, x_4) = \frac{1}{2} I_{C \phi \phi C}(x_1, x_2, x_3, x_4) 
$$

(10)

$$
I_{\phi C \phi C}(x_1, x_2, x_3, x_4) = \frac{1}{2} I_{C \phi \phi C}(x_1, x_2, x_3, x_4) 
$$

(11)

Note that the RHS of (10) or (11) is not the same as the quartic graph in Figure 1(q) since the derivatives act on different variables.

It is easy to see by using integration by parts that

$$
I_{\phi \phi C C}(x_1, x_2, x_3, x_4) = I_{C C \phi \phi}(x_1, x_2, x_3, x_4) 
$$

(12)

$$
I_{\phi \phi C C}(x_1, x_2, x_3, x_4) + I_{\phi C \phi C}(x_1, x_2, x_3, x_4) + I_{C \phi \phi C}(x_1, x_2, x_3, x_4) = 0 
$$

(13)

Thus we find that the contributions to $\langle O_C(x_1)O_C(x_2)O_C(x_3)O_C(x_4) \rangle$ from the $s$, $u$ and quartic graphs add up to

$$
-4a^2 \frac{1}{2} I_{\phi \phi C C}(x_1, x_2, x_3, x_4) - 4a^2 \frac{1}{2} I_{C \phi \phi C}(x_1, x_2, x_3, x_4) - 4b I_{\phi C \phi C}(x_1, x_2, x_3, x_4) 
$$

$$
= (-4b + 2a^2) I_{\phi C \phi C}(x_1, x_2, x_3, x_4) 
$$

(14)

Putting $a = 1, b = 1$ we see that the coefficient on the RHS is not zero. In the next section we show that the function $I_{\phi C \phi C}(x_1, x_2, x_3, x_4)$ is nonzero by computing its leading singularities.

The 4-point function of the primary operator corresponding to the axion field $\langle O_C(x_1)O_C(x_2)O_C(x_3)O_C(x_4) \rangle$ is given by the AdS graphs in Figure 2. Using (13) we see that the sum of the three dilaton exchange graphs sums to zero, though each of these graphs will not separately vanish.
3 Singularities in 4-point graphs

We have seen that the s and u graphs of Figure 1 reduce to the form of an $I^q$ integral. In the function $I^q_{\phi\phi CC}(x_1, x_2, x_3, x_4)$ there are two independent short distance limits to be considered:

(a) $x_{12} \equiv |x_1 - x_2| \to 0$.
(b) $x_{13} \equiv |x_1 - x_3| \to 0$.

(From (12) we see that $x_{34} \to 0$ is similar to $x_{12} \to 0$ etc.).

We first observe the identity

$$\int \frac{d^{d+1}z}{z_0^2} K_{\Delta_1}(z, x_1) K(z, x_2) \Delta_2 \partial_{z_\mu} K(z, x_3) \Delta_3 \partial_{z_\mu} K(z, x_4) \Delta_4$$

$$= \Delta_3 \Delta_4 J_{\Delta_1, \Delta_2, \Delta_3, \Delta_4}(x_1, x_2, x_3, x_4)$$

$$- 2(\Delta_3 - \frac{d}{2})(\Delta_4 - \frac{d}{2})x_{34}^2 J_{\Delta_1, \Delta_2, \Delta_3 + 1, \Delta_4 + 1}(x_1, x_2, x_3, x_4)$$

where

$$J_{\Delta_1, \Delta_2, \Delta_3, \Delta_4}(x_1, x_2, x_3, x_4) \equiv \int \frac{d^{d+1}z}{z_0^2} K_{\Delta_1}(z, x_1) K(z, x_2) \Delta_2 K(z, x_3) \Delta_3 K(z, x_4) \Delta_4$$

This identity can be derived by methods similar to those in [5] (translating $x_3$ to the origin, performing an inversion $z_\mu = \frac{x'_\mu}{(x')^2}, x_i = \frac{x''_i}{(x'')^2}$, evaluating the derivatives and inverting back).

This manipulation reduces the calculation of an integral of the type $I^q$ to computing the quartic graph with no derivatives on any of the legs. A special case of this latter calculation (with all $\Delta_i = \Delta$) was given in [6]; we make a straightforward extension of their calculation to the case with arbitrary $\Delta_i$:

$$J_{\Delta_1, \Delta_2, \Delta_3, \Delta_4}(x_1, x_2, x_3, x_4) =$$

$$\frac{1}{2\pi^{3d/2}} \frac{\Gamma[-\frac{d}{2} + \sum_i \Delta_i]\Gamma[-\Delta_4 + \sum_i \Delta_i]\Gamma[-\Delta_3 + \sum_i \Delta_i]\Gamma[\Delta_3]\Gamma[\Delta_4]}{\Gamma[\sum_i \Delta_i] \Gamma[\Delta_i - \frac{d}{2}] \prod_i \Gamma[\Delta_i - \frac{d}{2}]$$

$$\int_0^\infty d\beta_2 \frac{\beta_2}{\beta_2} (\beta_2 x_{24}^2 + x_{14}^2)^{-\Delta_4} (\beta_2 x_{12}^2)^{\Delta_4 - \sum_i \Delta_i} \frac{x_{34}^2}{(\beta_2 x_{24}^2 + x_{14}^2)^2} \beta_2^{2\Delta_2}$$

$$2 F_1[-\Delta_4 + \sum \frac{\Delta_i}{2}, \Delta_3, \sum \frac{\Delta_i}{2}, 1 - \alpha]$$

where

$$\alpha = \frac{(\beta_2 x_{23}^2 + x_{13}^2)(\beta_2 x_{24}^2 + x_{14}^2)}{\beta_2 x_{12}^2 x_{34}^2}$$

(17)
and \( _2F_1 \) is the hypergeometric function. For the estimates below it is helpful to use the integral representation:

\[
_2F_1[\alpha, \beta; \gamma, z] = \frac{1}{B[\beta, \gamma - \beta]} \int_0^1 t^{\beta-1}(1-t)^{\gamma-1}(1-tz)^{-\alpha} dt
\]

(19)

where \( B[\alpha, \beta] \) is the Beta function.

From (17) and (19) we find that as \( x_{12} \to 0 \):

\[
I^q_{\phi CC}(x_1, x_2, x_3, x_4) \to \frac{6^4 4}{\pi^6 21} \frac{1}{x_{13}^8 x_{14}^8} \ln \frac{x_{13} x_{14}}{x_{12}^2}
\]

(20)

As \( x_{13} \to 0 \):

\[
I^q_{\phi CC}(x_1, x_2, x_3, x_4) \to \frac{6^4 2}{\pi^6 21} \frac{1}{x_{12}^8 x_{14}^8} \ln \frac{x_{12} x_{14}}{x_{13}^2}
\]

(21)

Note that the strengths of the singularities in (20) and (21) are such that they respect the identity (13).

In [17] it was argued that each of the s,u and quartic graphs given in Figure 1 vanishes separately, while we have reached a somewhat different conclusion.\(^3\) We have not evaluated the graviton exchange graph, which was speculated to vanish in [17], but we discuss in the next section our expectations for its contribution.

### 4 Discussion

We know that the \( \mathcal{N} = 4 \) SYM theory is exactly conformal. Consider a 4-point function \( \langle O_1(x_1)O_2(x_2)O_3(x_3)O_4(x_4) \rangle \) in the limit \( x_1 \to x_2, x_3 \to x_4 \). We might try to expand\(^4\)

\[
O_1(x_1)O_2(x_2) = \sum_n \frac{\alpha_n O_n(x_1)}{(x_1 - x_2)^{\Delta_1 + \Delta_2 - \Delta_n}}, \quad O_3(x_3)O_4(x_4) = \sum_m \frac{\beta_m O_m(x_3)}{(x_3 - x_4)^{\Delta_3 + \Delta_4 - \Delta_m}}
\]

and get

\[
\langle O_1(x_1)O_2(x_2)O_3(x_3)O_4(x_4) \rangle = \sum_{n,m} \frac{\alpha_n \beta_m \langle O_n(x_1)O_m(x_3) \rangle}{(x_1 - x_2)^{\Delta_1 + \Delta_2 - \Delta_m}(x_3 - x_4)^{\Delta_3 + \Delta_4 - \Delta_n}}
\]

(23)

In a non-conformal theory, where a mass scale \( m \) would be available, we could also have, for instance, \( O_{\Delta_1}(x_1)O_{\Delta_2}(x_2) \sim \log(m|x_1 - x_2|)O_{\Delta_1 + \Delta_2}(x_1) \), but in a conformal

\(^3\)The resubmitted version (v4) of [17] appears to agree with our conclusions.

\(^4\)See also [18] for discussions of conformal OPEs and the the contribution of a given primary operator and its descendents to the CFT 4-point function.
theory such a term should not arise. Thus if the sums in (23) are to converge, we expect that the limit $x_{12} \to 0$ in the correlator would have no term in $\log(x_{12})$. Individual graphs from supergravity, however, are generically expected to have such logarithmic singularities and (20),(21) are examples of this fact. Thus either the logs all cancel when the supergravity graphs are summed, or a naive OPE summation of the form (23) is invalid.

We now proceed to discuss our results for 4-point functions in the dilaton-axion sector in the light of the questions of cancellation of logs and expectations for power singularities. For the correlator $\langle O_\phi O_C O_\phi O_C \rangle$ we found in (14) that the sum of s,u and quartic graphs is proportional to the contact amplitude and contains logarithmic singularities. We have not evaluated the t-channel graviton exchange graph, which is quite difficult, but which could contain logarithms that cancel those in the sum s+u+quartic. Note that if such a cancellation occurs for the $AdS_5 \times S_5$ supergravity theory then it would certainly fail to occur for an arbitrary choice of couplings between the fields. Thus a generic theory in $AdS$ would not give a boundary theory which would possess a convergent local OPE.

In the $\langle O_C O_C O_C O_C \rangle$ correlator we found a cancellation among 3 $\phi$-exchange graphs which each have a log singularity. The t-channel graviton exchange diagram in this correlator is the same as the t-channel graviton exchange in $\langle O_\phi O_C O_\phi O_C \rangle$. Suppose that this latter graph does contain the cancelling logarithms discussed above. It is then a simple consequence of (12) and (13) that the sum of log singularities in the t,s, and u channel graviton exchange diagrams will also cancel in $\langle O_C O_C O_C O_C \rangle$.

Although we have not evaluated the graviton exchange graphs in Figs. 1 and 2, it does appear on physical grounds that they are non-vanishing and have a strong singularity $\sim 1/x^4$ for $x \to 0$, where $x$ is the separation of any two boundary operators connected to the same internal vertex. Part of this physical intuition stems from the fact that the 3-point functions $\langle O_C(x_1)O_C(x_2)T_{ij}(x_3) \rangle$ and $\langle O_\phi(x_1)O_\phi(x_2)T_{ij}(x_3) \rangle$, where $T_{ij}$ is the stress-energy tensor, are different from zero, so that we expect from the leading term of the OPE the singularity $\sim 1/x^{\Delta_1+\Delta_2-\Delta_3}$, where all $\Delta_i = 4$. This would imply that the t-channel graph in Fig.1 is more singular as $x_{13} \to 0$ than any of the other graphs, so that the overall sum of all diagrams contributing to $\langle O_\phi O_C O_\phi O_C \rangle$ is not expected to vanish. One can state the same physical expectation in the language of the boundary $\mathcal{N} = 4$ SYM theory, in which $O_\phi = TrF^2$ and $O_C = TrF\tilde{F}$, and the 2- and 3-point functions of these operators are exactly given by their free-field
values due to superconformal non-renormalization theorems. It is easy to calculate the free field OPE’s and see that $\text{Tr} F^2(x)\text{Tr} F^2(y)$ and $\text{Tr} \tilde{F}(x)\text{Tr} \tilde{F}(y)$ contain the stress tensor with expected $1/(x-y)^4$ singularity. Thus physical considerations within the boundary CFT lead us to expect a non-vanishing t-channel contribution to $\langle O_\phi O_C O_\phi O_C \rangle$.

It is also easy to understand on physical grounds why the naively expected $1/(x_1^2)^4$ singularity of the s-channel graph for $\langle O_\phi O_C O_C \rangle$ is not present. First, one can use the formulae of [5] to show that $\langle O_\phi O_C O_C \rangle = 0$ (The AdS integral $\int \frac{dz_z z_0^2}{z_0} K \partial z_\mu K \partial z_\mu K$ vanishes even though the action (3) contains the vertex $\phi(\partial C)^2$.) Second, one can compute the free field OPE $\text{Tr} F^2(x)\text{Tr} \tilde{F}(y)$ and see that there is no $1/(x-y)^4$ singularity (although we expect a weaker singularity from operators of dimension greater than 4).

We comment on the relation between supergravity graphs and OPE’s. Consider a 4-point correlator of chiral primaries, $\langle O_1(x_1)O_2(x_2)O_3(x_3)O_4(x_4) \rangle$. In the expansion (23), let us consider the sum over chiral primaries and their conformal descendents. The $SO(6)$ symmetry of the $N = 4$ SYM theory allows only a finite number of chiral primaries to appear in this expansion. The same symmetry of the $AdS_5 \times S^5$ supergravity theory allows only a finite number of fields to propagate in the internal lines of the corresponding $AdS$ graphs. It is thus tempting to seek a relation between, say, the s-channel $AdS$ graph whose internal line corresponds to a specific primary operator $O(x)$ and the contribution of $O(x)$ and its descendents (i.e. derivatives) in the double OPE (23). Consider the limit $x_{12}$ small, $x_{34}$ small, $x_{13}$ large. The s–channel supergravity graph has two 3-point vertices in the interior of $AdS$. Generically, we expect large contributions from two distinct domains of integration in the space of $z$ and $w$: (a) $z$ is near $\vec{x}_1, \vec{x}_2$, while $w$ is near $\vec{x}_3, \vec{x}_4$; (b) both $z$ and $w$ are near $\vec{x}_1, \vec{x}_2$ (or both near $\vec{x}_3, \vec{x}_4$).

In region (a) the bulk supergravity propagator goes from near one pair to near the other pair, so this contribution might correspond to the double OPE (23). A toy example to study this hypothesis was presented in [16]. The $CFT$ and $AdS$ calculations were compared to fourth order in $\frac{x_{12}}{x_{13}}$ and $\frac{x_{34}}{x_{13}}$, and exact agreement was obtained. Recently, in [17] it was argued that a generic s–channel supergravity graph exactly matches the corresponding OPE contribution. However the argument relied on an implicit assumption of analyticity (in order to separate terms with physical and shadow singularities) which is not satisfied if there are logarithmic singularities.
Thus the identification of \(s\)-channel graphs and double OPE contributions may not be exact. For example, since the 3-point function \(\langle O_\phi O_C O_C \rangle \) vanishes, the double OPE for the correlator \(\langle O_\phi O_C O_\phi O_C \rangle \) would also be naively expected to vanish. However, we showed explicitly in Section 3 that the corresponding supergravity \(s\)-channel graph (Fig.1,s) has a leading singularity which is logarithmic. It is an important problem for future work to determine the exact circumstances under which logarithmic singularities occur. This will require detailed input from the \(AdS_5 \times S_5\) bulk supergravity theory, since \(s\)-channel graphs formed from derivative and non-derivative \(\phi^3\) vertices may have different analytic properties.

We finally would like to make some comments on the issues of duality both on the supergravity and the CFT side. Supergravity graphs are not expected to be dual, indeed in the \(\phi C\phi C\) example we found that the \(s\) and \(u\) channels are manifestly different since they exhibit different singularities. Operator product expansions are instead dual by definition under the assumption of their convergence. It appears unlikely that \(\mathcal{N} = 4, d = 4 \) \(SU(N)\) SYM in the \(N \to \infty, g_{YM}^2 N \to \infty\) limit possesses a convergent OPE in terms of only chiral primaries and their descendents, if one assumes the validity of the AdS/CFT correspondence. Consider again \(\langle O_\phi(x_1) O_C(x_2) O_\phi(x_3) O_C(x_4) \rangle \). The only chiral primary that could enter the double OPE \((23)\) is \(O_C\), but the coupling is zero since \(\langle O_\phi O_C O_C \rangle = 0\). Hence in this way of doing the OPE we expect a zero answer from the chiral sector. However, using the OPE to expand \(O_\phi(x_1) O_\phi(x_3)\) and \(O_C(x_2) O_C(x_4)\), only the stress-energy tensor \(T_{ij}\) can enter as an intermediate chiral operator, and the coupling is this time non-zero since \(\langle O_\phi O_\phi T_{ij} \rangle\) and \(\langle O_C O_C T_{ij} \rangle\) do not vanish as shown in \([3]\). We thus see that the assumption of a convergent OPE in terms of only chiral operators appears to lead to a contradiction. It would be interesting to find out the minimum set of operators needed in the theory to allow duality of the OPE expansion for chiral field correlators.

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