Double protection of the Higgs potential

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Abstract

A mechanism of double protection of the Higgs potential, by supersymmetry and by a global symmetry, is investigated in a class of supersymmetric models with the $SU(3)_C \times SU(3)_W \times U(1)_X$ gauge symmetry. In such models the electroweak symmetry can be broken with no fine-tuning at all.

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Stabilizing the Higgs potential is the central motivation for most extensions of the standard model. In softly broken supersymmetry the quadratic sensitivity of the Fermi scale to the ultraviolet (UV) cutoff is removed to all orders in perturbation theory. In generic supersymmetric models the Higgs potential mass parameter depends quadratically on the soft supersymmetry breaking scale $M_{\text{soft}}$ and logarithmically on the cut-off $\Lambda_{\text{UV}}$. For example, in the MSSM the one-loop corrections which lead to the electroweak symmetry breaking are dominated by the top sector contribution and one approximately has:

$$m_H^2 \approx m_0^2 - \frac{3}{8\pi^2} y_t^2 M_{\text{soft}}^2 \ln \frac{\Lambda_{\text{UV}}^2}{M_{\text{soft}}^2},$$

where $y_t$ is the top quark Yukawa coupling. This mechanism of radiative electroweak symmetry breaking strongly links the electroweak scale with $M_{\text{soft}}$. Indeed, the tree-level term $m_0^2$ contains a supersymmetry breaking contribution of order $M_{\text{soft}}^2$ (and the $\mu$-term contribution, which should be of the same order as $M_{\text{soft}}$), and for $\Lambda_{\text{UV}}$ close to the GUT scale the one loop term is also of that order. However, in view of the existing experimental constraints, such relation appears to be unsatisfactory. The lower limit on the Higgs boson mass and the precision electroweak data put a lower bound on $M_{\text{soft}}$ of order of 1 TeV. In consequence, since $m_H^2 \approx -M_Z^2/2$, the cancellation between the tree-level and one-loop terms in the Higgs potential must be at least 1 part to 100 - the fact known as the “supersymmetric fine-tuning problem”.

The following two features would be welcome to improve this picture. The underlying physics should forbid the tree-level Higgs mass parameter. This would also have an advantage of avoiding the $\mu$-problem of the MSSM. Secondly, $\Lambda_{\text{UV}}$ in (1) should be replaced with another scale of order $M_{\text{soft}}$. This would lead to $m_H^2 \sim M_{\text{soft}}^2/16\pi^2$ and the correct value of $m_H^2$ for $M_{\text{soft}}$ of order 1 TeV.

The supersymmetric fine-tuning problem has stimulated several authors to look for alternatives to supersymmetry. The little Higgs models [1] revive the idea of the Higgs doublet being a pseudo-Goldstone boson of some global symmetry spontaneously broken at a scale $\mathcal{O}(1 \text{ TeV})$. However, the scale of the global symmetry breaking is usually linked to the mass scale of new gauge bosons, $W^{\pm}$ and/or a $Z'$, and is constrained by precision electroweak data. In consequence, the fine-tuning in the Higgs potential is at least as large as in supersymmetric models [2, 3].

In this paper we explore the idea of double protection of the Higgs potential. This mechanism operates in supersymmetric models in which the Higgs doublet is a pseudo-Goldstone boson of an approximate global symmetry spontaneously broken at $\mathcal{O}(1 \text{ TeV})$. One role of the additional symmetry is to forbid the tree-level Higgs mass term. In addition, its interplay with supersymmetry removes also the logarithmic dependence of $m_H^2$ on $\Lambda_{\text{UV}}$ at one loop: $\Lambda_{\text{UV}}$ gets replaced there with the mass scale of additional vector-like quark multiplets which is of the order of the spontaneous global symmetry breaking scale $f$. As a result, the dominant one-loop
contribution to the Higgs potential mass parameter is finite and takes the form:

\[ m_H^2 \approx -\frac{3}{8\pi^2} y_t^2 \left[ (M_{\text{soft}}^2 + f^2) \ln(M_{\text{soft}}^2 + f^2) - M_{\text{soft}}^2 \ln M_{\text{soft}}^2 - f^2 \ln f^2 \right] . \] (2)

This vanishes in the limit of unbroken supersymmetry \( M_{\text{soft}} \to 0 \) as well as for unbroken global symmetry \( f \to 0 \).

The idea of the double protection has been explored in ref. [4] in a model proposed in ref. [5]. However the unattractive feature of this model is that the scale of the \( SU(3) \) symmetry breaking is linked to the mass of the \( Z' \) boson. The allowed parameter space is then very limited and the fine-tuning remains as large as in the MSSM, although for different reasons. In this paper we discuss the mechanism of double protection in a class of models in which the global \( SU(3) \) is a natural consequence of a \( SU(3) \) gauge symmetry. Furthermore, the scale \( f \) of spontaneous breaking of the global symmetry is not related to the scale \( F \) of spontaneous gauge symmetry breaking and the experimental limits on the masses of new gauge bosons do not constrain \( f \). For \( f < 1 \) TeV \( \ll F \), the electroweak symmetry can be broken with no fine-tuning at all.

We consider a class of supersymmetric models with \( SU(3)_C \times SU(3)_W \times U(1)_X \) gauge symmetry. The electroweak \( SU(2)_W \times U(1)_Y \) group is a subgroup of \( SU(3)_W \times U(1)_X \) and the matter and Higgs fields are extended to \( SU(3)_W \) multiplets. Several models of this kind exist in the literature [6,7] and several others can be constructed. They differ in the assignment of particles to \( SU(3)_W \times U(1)_X \) representations as well as in existence of additional exotic matter multiplets. The assignment can be such that all anomalies cancel [6,7]. In this letter we concentrate only on the most universal features of such models.

We shall require that the Higgs sector has global symmetry \( SU(3)_1 \times SU(3)_2 \) whose diagonal subgroup is the gauge \( SU(3)_W \) group. This can be easily achieved if the Higgs sector consists of two pairs of Higgs multiplets:

\[ \Phi_D, \Phi_U, \quad \text{and} \quad \mathcal{H}_d, \mathcal{H}_u, \] (3)

where \( \Phi_D \) and \( \Phi_U \) transform as a triplet and an antitriplet under the global \( SU(3)_1 \) while \( \mathcal{H}_d \) and \( \mathcal{H}_u \) are a triplet and an antitriplet under \( SU(3)_2 \). The Higgs multiplets should acquire vacuum expectation values (vevs) aligned in such a way that the \( SU(2)_W \) gauge symmetry remains unbroken

\[ \langle \Phi_D \rangle = \begin{pmatrix} 0 \\ 0 \\ F_D \end{pmatrix}, \quad \langle \mathcal{H}_d \rangle = \begin{pmatrix} 0 \\ 0 \\ f \cos \beta \end{pmatrix}, \] (4)

\[ \langle \Phi_U \rangle = (0, 0, F_U), \quad \langle \mathcal{H}_u \rangle = (0, 0, f \sin \beta) . \]

For \( F \equiv \sqrt{(F_D^2 + F_U^2)/2} \gg f \) we get then the following picture. The \( SU(3)_W \) gauge symmetry and the global \( SU(3)_1 \) symmetry are broken down to \( SU(2) \) at the
scale $F$, while the global $SU(3)_2$ survives and is spontaneously broken only at the scale $f$. This pattern of gauge and global symmetry breaking leads to ten Goldstone bosons, five of which become longitudinal components of the massive gauge bosons corresponding to broken $SU(3)_W$ generators. For $F \gg f$ the five physical Goldstone bosons are dominantly linear combinations of the components of $H_u$ and $H_d$. They can be conveniently parametrized as follows:

\[
H_u = f \sin \beta \left( \frac{H^\dagger}{|H|} \sin \left( \frac{|H|}{f} \right), \ e^{i\eta} \cos \left( \frac{|H|}{f} \right) \right), \quad H_d = f \cos \beta \left( \frac{-H^\dagger}{|H|} \sin \left( \frac{|H|}{f} \right), \ e^{-i\eta} \cos \left( \frac{|H|}{f} \right) \right).
\] (5)

Here, as in [4], $H$ is a weak $SU(2)_W$ doublet, which is identified with the SM Higgs doublet, and $|H| = \sqrt{H^\dagger H}$. The remaining Goldstone boson $\eta$ is a SM singlet. We ignore it in most of the following discussion, yet we will comment on its physical effects at the end of the paper.

As we outlined in the introduction, we are interested in a scenario in which the global symmetry breaking scale $f$ is $f \sim 1$ TeV. Obviously, supersymmetry should not be broken spontaneously at the scale $F$. The required pattern can be obtained by choosing the following superpotential for the Higgs sector [8]

\[
W = \kappa_1 N_1 \left( \Phi_U \Phi_D - \mu^2 \right) + \kappa_2 N_2 H_u H_d + \frac{1}{3} \lambda_2 N_2^3.
\] (6)

where we introduced singlet superfields $N_{1,2}$ (superfields, and their scalar components are denoted by the same letters) and $\mu$ is a mass parameter. Note that the terms $\Phi_U H_d$ and/or $H_u \Phi_D$ are not present by construction as they would break the global $SU(3)_1 \times SU(3)_2$ symmetry. Of course, in supersymmetric models the form of the superpotential is stable with respect to radiative corrections due to the non-renormalization theorem, even if it is not the most general one consistent with the gauge symmetry. Its constrained form could be, in principle, a consequence of the local symmetry structure of the high energy completion or of some discrete symmetries, but in this paper we do not construct any explicit models which would ensure this.

The superpotential (6) is a simple choice which has the necessary qualitative features. In the limit of unbroken supersymmetry the scalar potential resulting from (6) has its minimum for

\[
F_U = F_D = \mu
\] (7)

and vanishing vacuum expectation values of the other fields. When the soft masses are taken into account,

\[
\mathcal{L}_{\text{soft}} = -M_U^2 |\Phi_U|^2 - M_D^2 |\Phi_D|^2 - M_N^2 |N_1|^2 - m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_d|^2 - m_{N_2}^2 |N_2|^2,
\] (8)

the vevs $F_U, F_D$ are shifted by terms of order $M_{\text{soft}}^2 / F$, and $F_U^2 - F_D^2 \sim M_U^2 - M_D^2$. In the limit $F \gg f \sim M_{\text{soft}}$, in order to study the dynamics of light fields we can first
decouple the heavy components of $\Phi_U$, $\Phi_D$ and $N_1$ with masses of order $F$. This procedure yields the effective potential

$$V_{\text{eff}} = |\kappa_2 N|^2 (|H_u|^2 + |H_d|^2) + |\kappa_2 H_u H_d + \lambda_2 N_2|^2$$

$$+ \frac{\kappa^2}{8} \left( H_d^\dagger \lambda^i H_d - H_u \lambda^i H_u^\dagger \right)^2 + \frac{\kappa^2}{2} \left( H_d^\dagger \lambda^2 H_d - H_u \lambda^2 H_u^\dagger \right)^2$$

$$+ m_u^2 |H_u|^2 + m_d^2 |H_d|^2 + m_N^2 |N_2|^2 + \delta V_{\text{soft}} + O(1/F^2)$$

(9)

$$\delta V_{\text{soft}} \sim (M_D^2 - M_U^2) \left[ v_1 (H_d^\dagger \lambda^8 H_d - H_u \lambda^8 H_u^\dagger) + v_2 (H_d^\dagger \lambda^2 H_d - H_u \lambda^2 H_u^\dagger) \right]$$

(10)

Here $\lambda^i = \text{diag}(1/2, 1/2, 0)$, $\lambda^a$ denote the Gell-Mann matrices, $i = 1 \ldots 3$, $g$ and $g_x$ are the gauge couplings of $SU(3)_W \times U(1)_X$, $g_y = \frac{g g_x}{\sqrt{g^2 + g_x^2}}$ is the hypercharge coupling and $v_1$, $v_2$ are irrelevant numerical factors. We have neglected possible trilinear soft terms. In general, the effective potential contains the soft masses that do not respect the global $SU(3)_2$ [9], and would give a large tree-level mass to the Higgs doublet. To avoid this, we require $M_D^2 \approx M_U^2$ at the scale $F$. This is possible, for example, in models with universal soft masses at the supersymmetry breaking mediation scale, as long as non-universal contribution from renormalization group running down to $F$ are small enough. The second line in (9) is the D-term potential corresponding to the unbroken gauge group $SU(2)_W \times U(1)_Y$ (the D-terms corresponding to the broken generators of $SU(3)_W \times U(1)_X$ cancel out, when the heavy fields are properly integrated out). For $M_D^2 = M_U^2$ these D-terms are the only $SU(3)_2$ breaking terms in the tree-level effective Higgs potential below $F$. Thus, at tree-level we get only the quartic term (and no soft mass term) for the Higgs doublet.

Soft masses may induce vevs of the the electroweak singlet components of $\mathcal{H}$, so that $f \sim M_{\text{soft}}$. For large $\tan \beta$ we find

$$f^2 \approx -\frac{m_n^2}{\kappa_2^2}, \quad \langle N_2 \rangle^2 \approx -\frac{m_u^2}{\kappa_2^2}, \quad \tan \beta \approx \frac{\kappa_2 m_n^2 + m_u^2 - m_d^2}{\lambda_2 - m_u^2},$$

(11)

where $m_n$, $m_u$ and $m_d$ are soft masses of $N_2$, $H_u$ and $H_d$, respectively. For generic soft masses, large enough $\tan \beta$ is obtained for $|\kappa_2| \gg |\lambda_2|$. The necessary negative masses squared may result from renormalization group running. Indeed, similarly as in the MSSM, the triplet $H_u$ mass is driven negative by the large top Yukawa coupling, while the singlet soft mass also acquires large negative contribution from Yukawa interactions as long as $|\kappa_2| \sim 1$.

The $SU(3)_1 \times SU(3)_2$ symmetry must only be approximate so that the Higgs doublet $H$ is rather a pseudo-Goldstone boson. As we mentioned, the gauge interactions below the scale $F$ do not respect the global symmetry, and the corresponding $D$-terms generate the quartic Higgs potential. We require that another source of the explicit breaking comes from the supersymmetric interactions in the top sector. In such case, the Higgs doublet can acquire negative mass parameter at one-loop level. However due to an interplay between the approximate global symmetry and supersymmetry these radiative corrections are finite at one loop [4]. Logarithmic divergences are cut-off by the contribution of the additional top quarks, whose presence
is required by the approximate $SU(3)$. This double protection mechanism alleviates the supersymmetric fine-tuning problem, as we demonstrate in the following.

We shall illustrate our point in a specific model, which is a straightforward supersymmetrization of "the simplest little Higgs" model of ref. [6] and later comment on the generality of our results. The relevant for us chiral fermion superfields are the $SU(3)_W$ triplet $\Psi_Q = (Q, T)^T$, and the $SU(3)_W$ quark singlets $t^c$ and $T^c$. The superpotential is given by

$$W = y_1 \Phi_U \Psi_Q T^c + y_2 \mathcal{H}_u \Psi_Q t^c .$$

(12)

As for the Higgs fields, this is not the most general choice consistent with the gauge symmetry. Once the $SU(3)_W$ gauge symmetry is broken at the scale $F$, the second term in eq. (12) preserves the global $SU(3)_2$ symmetry, while the first term breaks it explicitly.

Inserting the parametrization (5) of $\mathcal{H}_{u,d}$ we can read the top matrix $L = -(t, T) \mathcal{M}_{\text{top}}(t^c, T^c)$ as a function of the vev of $H$. The mass matrix squared reads

$$\mathcal{M}^\dagger_{\text{top}} \mathcal{M}_{\text{top}} = \begin{pmatrix} y_1 F^2 & y_1 y_2 f \sin \beta \cos(|H|/f) \\ y_1 y_2 f \sin \beta \cos(|H|/f) & y_2^2 f^2 \sin^2 \beta \end{pmatrix} .$$

(13)

For $|H| \ll f$ the matrix (13) has two hierarchical eigenvalues corresponding to the standard model top quark and its heavy $SU(3)_W$ partner:

$$m_t \approx y_t |H| , \quad y_t = \frac{y_1 y_2 F}{m_T} \sin \beta ,$$

$$m_T \approx \sqrt{y_1^2 F^2 + y_2^2 f^2 \sin^2 \beta} .$$

(14)

At this point it is useful to summarize the orders of magnitudes of the model parameters needed for a coherent picture. The scale $F$ cannot be too high because soft terms must be approximately $SU(3)$ symmetric at the TeV scale. For definiteness we will assume $F \sim 10$ TeV. On the other hand, for the sake of minimizing the fine-tuning we will need $m_T \sim 1$ TeV, so that we need $y_1 F \sim y_2 f \lesssim 1$ TeV. Note that the smallness of $y_1$ is consistent with our assumption that renormalization group effects do not generate a large splitting between $M_U^2$ and $M_D^2$. For given values of $F$ and $f$ the Yukawa couplings $y_1$ and $y_2$ can be chosen such that the Standard Model top Yukawa coupling $y_t$ has the correct value and $m_T$ is in the desired range. Relation (14) implies then a lower limit $m_T > 2 y_t f$.

We move to the determination of the Higgs potential

$$V = \delta m^2_H |H|^2 + (\lambda_0 + \delta \lambda)|H|^4 + \ldots$$

(15)

The tree-level quartic term comes from $SU(2)_W \times U(1)_Y$ $D$-terms. Its form is analogous as in the MSSM and, in the limit $\tan \beta \gg 1$, reads

$$\lambda_0 = \frac{g^2 + g_Y^2}{8} .$$

(16)
The one-loop corrections $\delta m_H^2$ and $\delta \lambda$ can be computed from the 1-loop effective potential:

$$\Delta V_{1\text{-loop}} = \frac{1}{64\pi^2} \text{STr} \left\{ \mathcal{M}^4 \left( \ln \frac{M^2}{\Lambda^2_{UV}} - \frac{3}{2} \right) \right\}. \tag{17}$$

By computing $\text{STr}\mathcal{M}^4$ it can be seen that there is no logarithmically divergent contribution from the top and stop sector to the Higgs potential mass parameter $\delta m^2_H$. This can be understood by a simple dimensional analysis. The coefficient of a logarithmically divergent term would have to break both supersymmetry and the approximate global symmetry, but in the top-stop sector there is no such dimensionful parameter. The conclusion holds for any stop soft masses and trilinear terms, as long as their tree-level values respect the $SU(3)_2$ global symmetry. Furthermore, in a simplified situation when all stops have approximately the same soft mass squared $m_Q$ and the mixing between left and right-chiral stops is negligible we obtain a simple formula for the Higgs mass parameter:

$$\delta m_H^2 \approx -\frac{3}{8\pi^2} y_t^2 \left[ (m_Q^2 + m_T^2) \ln(m_Q^2 + m_T^2) - m_Q^2 \ln m_Q^2 - m_T^2 \ln m_T^2 \right] + \Delta. \tag{18}$$

Here $\Delta$ stands for contributions from other sectors of the theory. For example, the SM gauge interactions contribute

$$\Delta \supset \frac{3g_2^2 M_2^2 + g_y^2 M_y^2}{8\pi^2} \ln \frac{F}{M_{\text{soft}}}, \tag{19}$$

where $M_2$ and $M_y$ are soft gaugino masses. The cut-off is given by the scale at which the $SU(3)_W$ gauge symmetry is restored.

The top contribution in (18) has a remarkable property that it vanishes for both $m_Q \to 0$ and $m_T \to 0$. As advertised, the double protection mechanism leads to the softening of the UV sensitivity of the Higgs potential. For a given $m_T$, the top contribution is minimized for $m_Q = m_T$, while for $m_Q > m_T$ it increases only as $\ln(m_Q/m_T)$.

The dominant contribution to the Higgs potential quartic coupling (needed to evaluate the Higgs boson mass) is given by

$$\delta \lambda \approx \frac{3}{16\pi^2} y_t^4 \left[ \ln \left( \frac{m_Q^2 m_T^2}{(m_Q^2 + m_T^2) m_t^2} \right) + \frac{3}{2} - 2 \frac{m_Q^2}{m_T^2} \ln \left( \frac{m_Q^2 + m_T^2}{m_Q^2} \right) \right]. \tag{20}$$

For $m_Q \sim m_T$ it behaves as $(3/8\pi^2) y_t^4 \ln(m_Q/m_t)$, very much like in the MSSM. Therefore, the physical Higgs boson mass given by

$$M_h^2 = 2(\lambda_0 + \delta \lambda) v^2 = M_Z^2 + 2\delta \lambda v^2 \tag{21}$$

takes similar values as in the MSSM with analogous stop masses.

We are now ready to estimate the level of fine-tuning of the electroweak breaking. The value of $\delta m_H^2$ is tied to the electroweak scale by the relation $v^2 = -\delta m_H^2/(\lambda_0 +$
\[ \delta \lambda, \text{ where } v = 246 \text{ GeV. We can always obtain the correct } v \text{ by arranging for appropriate } \Delta \text{ (for example, by tuning the gaugino masses), but large cancellations between the top contribution and } \Delta \text{ are unnatural. We can quantify the fine-tuning as follows:} \]

\[ \text{FT} = \left| \frac{\Delta - |\delta m^2_H|}{|\delta m^2_H|} \right| = \left| \frac{\Delta - \frac{1}{2} M_H^2}{\frac{1}{2} M_H^2} \right| \quad (22) \]

In fig. 1 we show the dependence of the Higgs boson mass and the fine-tuning on the input parameters of our model. The Higgs boson mass is plotted as a function of \( m_T \) for \( m_Q/m_T = 1 \) (the dependence on the \( m_Q/m_T \) ratio is very weak in this case). This is compared with the direct search limit from LEP2, \( M_h > 114.4 \text{ GeV} \). We have used the value of the top mass \( m_t = 172.7 \pm 2.9 \text{ GeV} \) [10] and the corresponding MS central value \( 164 \pm 3 \text{ GeV} \). The effect of varying the top mass within the \( 2\sigma \) limit is also displayed. The fine-tuning as a function of \( m_T \) is plotted for several values of the ratio \( m_Q/m_T \geq 1 \) (the formulae are of course symmetric under interchange of \( m_Q \) and \( m_T \)). We conclude that for \( 0.8 \text{ TeV} < m_T < 1 \text{ TeV} \) and \( m_Q \sim m_T \) the electroweak symmetry can be broken with no fine-tuning at all, while the LEP2 Higgs mass bound can be respected. For \( m_T > 1 \text{ TeV} \) the fine-tuning is of order \( 10\% \). Note that even when either \( m_Q \) or \( m_T \) are of order \( 10 \text{ TeV} \) fine-tuning is not worse than in the MSSM with TeV scale stop masses.

We leave the detailed analysis of phenomenological properties of the double protection mechanism for a future publication. Here we just point out its main experimental signatures. One of them is the existence of a vector-like top quark with mass around \( 1 \text{ TeV} \). New gauge bosons are expected to be much heavier, not within the...
reach of the LHC. The gauginos should have masses at most around 1 TeV. Furthermore, if the mechanism of spontaneous global symmetry breaking at the scale $f$ is linked to the soft supersymmetry breaking parameters, as in the mechanism discussed in this paper, squark masses are around 1 TeV, too. However, one can perhaps think about other mechanisms of generating the scale $f$. It is worth noting that merely from the point of view of the electroweak symmetry breaking with little fine tuning of the parameters, the squark masses are very weakly constrained.

We now comment on the pseudoscalar singlet $\eta$ in the parametrization (3). It is a true massless Goldstone boson corresponding to the Peccei-Quinn $U(1)$ symmetry acting on the $SU(3)$ partners of the SM weak doublets. As such, it is subject to experimental, cosmological and astrophysical constraints on light bosons [11]. However, $\eta$ couples to the ordinary matter only via mixing of the latter with their $SU(3)$ partners. For the first two generation such mixing can be very small, as long as the corresponding $SU(3)$ partners have masses of order $f$. Therefore, processes like energy loss in stars [11] put on $f$ only a weak lower bound of order 100 GeV. At the nucleosynthesis epoch, the background $\eta$’s, decoupled from the thermal bath would contribute to the energy density of the Universe. The cosmological effect of such a scalar is equivalent to the one of 0.57 neutrino generation. The conservative estimates of $N_\nu$ from nucleosynthesis still allow for $1.4 < N_\nu < 4.9$ [12], and do not exclude the presence of $\eta$.

The model studied above can be extended so as to accommodate all three generations of quark and leptons and their masses [6]. In this case, anomaly cancellation implies that the assignment to $SU(3)_W \times U(1)_X$ representations cannot be generation universal. Furthermore, it is not possible to embed this spectrum in a simple unified group. However one can consider models with a different spectrum, which ensure the double protection mechanism for the Higgs potential. For example, with the $SU(3)_C \times SU(3)_W \times U(1)_X$ gauge symmetry the following matter content can be chosen (the first two generations can be introduced analogously):

$$\Phi_U, \mathcal{H}_u : (1, 3)_{1/3} \quad \Phi_D, \mathcal{H}_d : (1, 3)_{-1/3}$$

$$\psi_Q = \begin{pmatrix} Q_3 \\ D \end{pmatrix} : (3, 3)_0 \quad T^c : (3, 3)_{-1/3} \quad T : (3, 3)_{1/3} \quad t^c : (3, 1)_{-2/3} \quad (23)$$

$$b_1^c, b_2^c : (3, 1)_{1/3} \quad \psi_L = \begin{pmatrix} L_3 \\ E \end{pmatrix} : (1, 3)_{-2/3} \quad \tau_1^c, \tau_2^c : (1, 1)_1$$

with the superpotential

$$W = y_1 \Phi_U \Psi_Q T^c + y_2 \mathcal{H}_u T t^c + \mu_T T T^c$$

$$+ y_{b1} \psi_Q \mathcal{H}_d b_1^c + y_{b2} \psi_Q \Phi_D b_2^c + y_{r1} \psi_L \mathcal{H}_d \tau_1^c + y_{r2} \psi_L \Phi_D \tau_2^c. \quad (24)$$

The top sector here is slightly more complicated. It contains a vector like triplet pair $T$ and $T^c$ with a supersymmetric mass term $\mu_T$. Still, for $F \gg f$ the picture is qualitatively and quantitatively the same as in the supersymmetric version of the model of ref. [6] discussed in this paper. In particular, the formulae (18) and (20) for the parameters of the Higgs potential still hold, with $y_1 F$ replaced by $\mu_T$ in eq. (14).
This indicates that the structure of the effective Higgs potential at one loop is a general feature of models in which the double protection mechanism is realized.

In conclusion, the double protection of the Higgs potential: by supersymmetry softly broken at the TeV scale and by a global symmetry which is spontaneously broken at the scale $\lesssim 1 \text{ TeV}$ may be a mechanism allowing to understand the origin and the stability of the Fermi scale.

Note added: Shortly after our paper appeared, models with double protection of the Higgs potential were discussed by Roy and Schmaltz in ref. [13]. These authors consider a similar model like the one in our paper and conclude it is not viable. In the model of ref. [13] the conditions $M_U^2 = M_D^2$ and $m_u^2 = m_d^2$ are imposed and the scale $f$ is generated by supersymmetric terms in the potential (and not by soft terms, as in our model). Under these assumptions $F_U = F_D$ and $\tan \beta = 1$, which ensure that the D-term contributions to the Higgs mass parameter are absent. Obviously, for $\tan \beta = 1$ the Higgs boson is not heavy enough. However, it has been overlooked in [13] that for $F \gg f$ the condition $\tan \beta = 1$ is not needed to avoid the D-term contributions to the Higgs mass parameter. This is discussed in our paper below eq. (S).

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