Does Lorentz Boost Destroy Coherence?*

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Abstract

It is shown that the time-energy uncertainty relation can be combined into the position-momentum uncertainty relation covariantly in the quark model of hadrons. This leads to a Lorentz-invariant form of the uncertainty relations. This model explains that the quark model and the parton model are two different manifestations of the same covariant model. In particular, this covariant model explains why the coherent amplitudes in the quark model become incoherent, after a Lorentz boost, in the parton model. It is shown that this lack of coherence is consistent with the present form of quantum mechanics.

I. INTRODUCTION

At this conference, there are a number of papers dealing with the time variable. The reason is very simple. We cannot do physics without this variable. Its role is well defined in Newton’s equation in classical mechanics. However, the time variable becomes complicated when we move to quantum mechanics and to relativity. In classical mechanics, we use the horizontal axis for the time variable and the vertical axis for the position or momentum. However, at this conference whose purpose is to address fundamental questions of quantum mechanics, we have seen a number of papers with space-time diagrams in which the vertical axis is used for the time variable.

Quantum mechanics is not the only place where we use the vertical axis for the time variable. We have been doing this in special relativity since it was formulated by Einstein in 1905. Does this mean that quantum mechanics is becoming closer to special relativity? The purpose of this report to say YES to this question. In quantum mechanics, where position and momentum are constrained by the uncertainty relation, we are still arguing about the whether there is an uncertainty relation between the time and the energy variables. As for the position-energy uncertainty relation, Heisenberg’s uncertainty relation is stated by the canonical commutation relations between position and momentum operators. For the time variable, we are not allowed to write down a commutation relation between the time and energy variables because there are no time-like excitations in the real world. However, the time-energy uncertainty clearly is clearly observed in the world. How to accommodate this

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space-time asymmetry into the space-time symmetric relativistic world has been one of the most outstanding problems in physics since 1927 [1].

These days, it is a routine laboratory procedure to accelerate the proton to the energy one thousand times higher than its rest mass. While we regard the proton as a bound state of quarks when it is at rest or slow [2], the question arises whether we can use the existing rules of quantum mechanics to understand the proton whose speed is very close to that of light. In 1969 [3], Feynman observed that the high-energy proton is a collection of free particles with a wide-spread momentum distribution. They appear to be incoherent when they interact with external signals. If then the high-energy proton is a Lorentz-boosted proton at from its rest frame, the coherence observed in the quark model is destroyed by Lorentz boost.

Does the Lorentz boost destroy the coherence? We shall study this question in connection with the task of combining bound-state quantum mechanics with special relativity. In Sec. II, we spell out this problem in terms of the known principles of quantum mechanics and special relativity. In Sec. III, the problem is formulated in terms of the covariant harmonic oscillators. In Sec. IV, it is explained how the peculiarities in the Feynman’s parton picture arise from the covariance of quantum mechanics. In Sec. V, we explain that the lack of coherence in the parton picture is consistent with the present form of quantum mechanics and therefore that the Lorentz boost does not destroy the coherence.

II. COVARIANCE AND QUANTUM MECHANICS

To physicists, Einstein’s $E = mc^2$ means $E = \sqrt{p^2 + m^2}$. This Lorentz-covariant quantity leads to $E = p^2 / 2m$ and $E = cp$ in the limits of low and high speeds respectively. In addition, relativistic particles have internal space-time degrees of freedom. For a massive particle, there is always a Lorentz frame in which the particle is at rest. In this Lorentz frame, the particle has the three-dimensional rotational degrees of freedom. The dynamical quantity associated with this degree of freedom is the intrinsic angular momentum called the spin. For a massless particle, however, there are no Lorentz frames where the particle is at rest. The particle in this case has the angular momentum either parallel or antiparallel to the momentum. It does not have the rotational symmetry the massive particle has in its rest frame. In addition, the massless particle has a gauge degree of freedom.

It is still one of the most fundamental questions in physics to ask whether the gauge degree of freedom can be regarded as a space-time transformation. This issue has a stormy history, but it has now been firmly established that the transverse rotational degrees of freedom become a gauge degree of freedom in the infinite-momentum/zero-mass limit. This feature is illustrated the second row of Table I.

In order to arrive at the conclusion of the second row, we have to study the little groups of the Poincaré group. The Poincaré group is the group of inhomogeneous Lorentz transformations, namely Lorentz transformations preceded or followed by space-time translations. In order to study this group, we have to understand first the group of Lorentz transformations, the group of translations, and how these two groups are combined to form the Poincaré group. The Poincaré group is a semi-direct product of the Lorentz and translation groups. The two Casimir operators of this group correspond to the (mass)$^2$ and (spin)$^2$ of a given particle. Indeed, the particle mass and its spin magnitude are Lorentz-invariant quantities.
TABLE I. Covariance of Relativistic Particles. In addition to the four-momentum, the particle has internal space-time symmetries. The spin and gauge symmetries are tabulated in the second row. The particle can also have a space-time extension, which manifests itself as the quark model and the parton model.

| Energy-Momentum | Massive, Slow | COVARIANCE | Massless, Fast |
|-----------------|--------------|-------------|---------------|
| $E = p^2/2m$    | Einstein’s   | $E = [p^2 + m^2]^{1/2}$ | $E = cp$ |
| Internal Space-time Symmetry | $S_3$       | Wigner’s Little Group | $S_3$ |
| $S_1, S_2$      |              |            | Gauge Trans. |

The next question is how to construct the representations of the Lorentz group which are relevant to physics. For this purpose, Wigner in 1939 studied the subgroups of the Lorentz group whose transformations leave the four-momentum of a given free particle invariant [4]. The maximal subgroup of the Lorentz group which leaves the four-momentum invariant is called the little group. This little group governs the internal space-time symmetries of relativistic particles. Wigner shows in his paper that the internal space-time symmetries of massive and massless particles are dictated by the $O(3)$-like and $E(2)$-like little groups respectively.

The group of Lorentz transformations consists of three boosts and three rotations. The rotations therefore constitute a subgroup of the Lorentz group. If a massive particle is at rest, its four-momentum is invariant under rotations. Thus the little group for a massive particle at rest is the three-dimensional rotation group. Then what is affected by the rotation? The answer to this question is very simple. The particle in general has its spin. The spin orientation is going to be affected by the rotation!

There are no Lorentz frames where a massless particle is at rest. If the massless particle moves along the $z$ direction, rotations around the $z$ momentum leave the four-momentum invariant. In addition, there are two generators of the Lorentz group which leave the momentum invariant. If we take the commutation relations of these three generators of the little group, they are exactly like those for the two-dimensional Euclidean group which we call $E(2)$. The group $E(2)$ consist of translations and rotations on a flat surface. It is not difficult to associate the rotational degree of freedom with the helicity of the massless particle. But the generators of the translation-like transformations have a stormy history. They generate gauge transformations when applied to the four-potential [5].
If the rest-particle is boosted along the $z$ direction, it will pick up a non-zero momentum component. The generators of the $O(3)$ group will then be boosted. The boost will take the form of conjugation by the boost operator. This boost will not change the Lie algebra of the rotation group, and the boosted little group will still leave the boosted four-momentum invariant. We call this the $O(3)$-like little group. The question then is whether the $O(3)$-like little group becomes the $E(2)$-like little group in the high-speed limit.

The question of Lorentz-boosted Poincaré group in the infinite-momentum limit was addressed first by Bacry and Chang in 1968 in connection with scattering problems [6]. Since the $O(3)$-like little group is a subgroup of the Poincaré group, Bacry and Chang in effect obtained the $E(2)$-like little group as the infinite-momentum limit of a subgroup of the Lorentz group. This high-speed contraction of the little group was later modeled after the Inonu-Wigner contraction of the $O(3)$ to $E(2)$ as a flat-surface approximation of a spherical surface with a large radius [7]. It was found later that the transverse rotation generators become contracted to the generators of the translation-like transformations [8]. Indeed, the rotations around the transverse directions become contracted to gauge transformations in the limit of infinite momentum and/or zero mass [9].

Next, let us summarize quantum mechanics. Quantum field theory has been quite successful in terms of perturbation techniques in quantum electrodynamics. However, this formalism is basically based on the S matrix for scattering problems and useful only for physically processes where free a set of particles becomes another set of free particles after interaction. Quantum field theory does not address the question of localized probability distributions and their covariance under Lorentz transformations.

The Schrödinger quantum mechanics of the hydrogen atom deals with localized probability distribution. Indeed, the localization condition leads to the discrete energy spectrum. Here, the uncertainty relation is stated in terms of the spatial separation between the proton and the electron. If we believe in Lorentz covariance, there must also be the time separation between the two constituent particles. This does not manifests itself in nonrelativistic quantum mechanics, but it exists. The time interval seems to be an important issue at this conference.

Indeed, we have to add a time dimension to spatial coordinates before getting into the relativistic world. As we noted in Sec. [1], there are many papers in this conference with space-time diagrams with the time coordinate as the vertical axis. This is precisely what we do in relativity. When we make a Lorentz boost along the $z$ direction, the transformation is written as

$$
\begin{pmatrix}
z' \\
t'
\end{pmatrix}
= 
\begin{pmatrix}
\cosh \eta & \sinh \eta \\
\sinh \eta & \cosh \eta
\end{pmatrix}
\begin{pmatrix}
z \\
t
\end{pmatrix},
\tag{2.1}
$$

This formula is well known, but it is not yet widely known that this is a squeeze transformation. In order to see this point, let us use the light-cone variables defined as [10]

$$
u = (z + t)/\sqrt{2}, \quad v = (z - t)/\sqrt{2}.
\tag{2.2}
$$

Then the boost transformation of Eq. (2.1) takes the form

$$
u' = e^{\eta}v, \quad v' = e^{-\eta}v,
\tag{2.3}
$$

where $\eta$ is the boost parameter and is $\tanh^{-1}(v/c)$. The $u$ variable becomes expanded while the $v$ variable becomes contracted. This is the squeeze mechanism discussed extensively in
the literature [11,12]. The Lorentz boost is a squeeze transformation. By now, the word “squeeze” is quite familiar to us from the squeezed states of light. Thus, the first step in making quantum mechanics covariant is to work out carefully a space-time picture of non-relativistic quantum mechanics in one Lorentz frame. The next step is to squeeze the space-time diagram.

Let us be more specific. Before 1964 [2], the hydrogen atom was used for illustrating bound states. These days, we use hadrons which are bound states of quarks. Let us use the simplest hadron consisting of two quarks bound together an attractive force. For the probability distribution, we can use the Gaussian form for spatial separation between the quarks. This spatial coordinates are quantized, and the position and momentum variables are q-numbers.

There is also the time-energy uncertainty relation applicable to the time separation between the quarks. Unlike Heisenberg’s uncertainty relation applicable to position and momentum, the time and energy separation variables are c-numbers, and we are not allowed to write down the commutation relation. On the other hand, the c-number time energy uncertainty relation allows to write down a time distribution function without excitations. If we use Gaussian forms for both space and time distributions, we can start with the expression

$$\exp\left\{-\frac{1}{2}(z^2 + t^2)\right\}, \quad (2.4)$$

where the $z$ and $t$ are the space and time separations respectively. The present form of quantum mechanics allows the excitations along the $z$ direction, but there are no excitations along the $t$ direction. Yet, we can start from a circular space-time distribution given by the above expression. We can then boost the distribution by squeezing it. We are therefore able to start from a hadron at rest, and boost it to the infinite-momentum frame.

For the third row in Table I, we propose to solve the following problem in high-energy physics and foundations of quantum mechanics. The quark model works well when hadrons are at rest or move slowly. However, when they move with speed close to that of light, they appear as a collection of infinite-number of partons [3]. As we stated above, we need a set of wave functions which can be Lorentz-boosted. How can we then construct such a set? In constructing wave functions for any purpose in quantum mechanics, the standard procedure is to try first harmonic oscillator wave functions. In studying the Lorentz boost, the standard language is the Lorentz group. Thus the first step to construct covariant wave functions is to work out representations of the Lorentz group using harmonic oscillators [13–15].

### III. COVARIANT HARMONIC OSCILLATORS

If we construct a representation of the Lorentz group using normalizable harmonic oscillator wave functions, the result is the covariant harmonic oscillator formalism [13]. The formalism constitutes a representation of Wigner’s $O(3)$-like little group for a massive particle with internal space-time structure. This oscillator formalism has been shown to be effective in explaining the basic phenomenological features of relativistic extended hadrons observed in high-energy laboratories. In particular, the formalism shows that the quark
model and Feynman’s parton picture are two different manifestations of one covariant entity \[15,16\]. The essential feature of the covariant harmonic oscillator formalism is that Lorentz boosts are squeeze transformations \[11,12\]. In the light-cone coordinate system, the boost transformation expands one coordinate while contracting the other so as to preserve the product of these two coordinate remains constant. We shall show that the parton picture emerges from this squeeze effect.

Let us consider a bound state of two particles. For convenience, we shall call the bound state the hadron, and call its constituents quarks. Then there is a Bohr-like radius measuring the space-like separation between the quarks. There is also a time-like separation between the quarks, and this variable becomes mixed with the longitudinal spatial separation as the hadron moves with a relativistic speed. There are no quantum excitations along the time-like direction. On the other hand, there is the time-energy uncertainty relation which allows quantum transitions. It is possible to accommodate these aspect within the framework of the present form of quantum mechanics. The uncertainty relation between the time and energy variables is the c-number relation \[1\], which does not allow excitations along the time-like coordinate. We shall see that the covariant harmonic oscillator formalism accommodates this narrow window in the present form of quantum mechanics.

For a hadron consisting of two quarks, we can consider their space-time positions \(x_a\) and \(x_b\), and use the variables

\[
X = \frac{(x_a + x_b)}{2}, \quad x = \frac{(x_a - x_b)}{2\sqrt{2}}.
\]

The four-vector \(X\) specifies where the hadron is located in space and time, while the variable \(x\) measures the space-time separation between the quarks. In the convention of Feynman et al. \[17\], the internal motion of the quarks bound by a harmonic oscillator potential of unit strength can be described by the Lorentz-invariant equation

\[
\frac{1}{2} \left\{ x^2 - \frac{\partial^2}{\partial x^2} \right\} \psi(x) = \lambda \psi(x).
\]

It is now possible to construct a representation of the Poincaré group from the solutions of the above differential equation \[15\].

The coordinate \(X\) is associated with the overall hadronic four-momentum, and the space-time separation variable \(x\) dictates the internal space-time symmetry or the \(O(3)\)-like little group. Thus, we should construct the representation of the little group from the solutions of the differential equation in Eq. (3.2). If the hadron is at rest, we can separate the \(t\) variable from the equation. For this variable we can assign the ground-state wave function to accommodate the c-number time-energy uncertainty relation \[1\]. For the three space-like variables, we can solve the oscillator equation in the spherical coordinate system with usual orbital and radial excitations. This will indeed constitute a representation of the \(O(3)\)-like little group for each value of the mass. The solution should take the form

\[
\psi(x, y, z, t) = \psi(x, y, z) \left( \frac{1}{\pi} \right)^{1/4} \exp \left( -t^2/2 \right),
\]

where \(\psi(x, y, z)\) is the wave function for the three-dimensional oscillator with appropriate angular momentum quantum numbers. Indeed, the above wave function constitutes a representation of Wigner’s \(O(3)\)-like little group for a massive particle \[13\].
Since the three-dimensional oscillator differential equation is separable in both spherical and Cartesian coordinate systems, \( \psi(x, y, z) \) consists of Hermite polynomials of \( x, y, \) and \( z \). If the Lorentz boost is made along the \( z \) direction, the \( x \) and \( y \) coordinates are not affected, and can be temporarily dropped from the wave function. The wave function of interest can be written as

\[
\psi^n(z, t) = \left( \frac{1}{\pi n! 2^n} \right)^{1/4} \exp \left( -t^2/2 \right) \psi_n(z),
\]

with

\[
\psi_n(z) = \left( \frac{1}{\pi n! 2^n} \right)^{1/2} H_n(z) \exp(-z^2/2),
\]

where \( \psi_n(z) \) is for the \( n \)-th excited oscillator state. The full wave function \( \psi^n(z, t) \) is

\[
\psi^n_z(z, t) = \left( \frac{1}{\pi n! 2^n} \right)^{1/2} H_n(z) \exp \left\{ -\frac{1}{2} \left( z^2 + t^2 \right) \right\}.
\]

The subscript 0 means that the wave function is for the hadron at rest. The above expression is not Lorentz-invariant, and its localization undergoes a Lorentz squeeze as the hadron moves along the \( z \) direction. 

The wave function of Eq.(3.6) can be written as

\[
\psi^n_0(z, t) = \psi^n_0(z, t) = \left( \frac{1}{\pi n! 2^n} \right)^{1/2} \frac{1}{\sqrt{2}} H_n \left( (u + v)/\sqrt{2} \right) \exp \left\{ -\frac{1}{2} (u^2 + v^2) \right\}.
\]

If the system is boosted, the wave function becomes

\[
\psi^n_\eta(z, t) = \left( \frac{1}{\pi n! 2^n} \right)^{1/2} H_n \left( (e^{-\eta}u + e^{\eta}v)/\sqrt{2} \right) \times \exp \left\{ -\frac{1}{2} \left( e^{-2\eta}u^2 + e^{2\eta}v^2 \right) \right\}.
\]

In both Eqs. (3.7) and (3.8), the localization property of the wave function in the \( uv \) plane is determined by the Gaussian factor, and it is sufficient to study the ground state only for the essential feature of the boundary condition. The wave functions in Eq.(3.7) and Eq.(3.8) then respectively become

\[
\psi_0(z, t) = \left( \frac{1}{\pi} \right)^{1/2} \exp \left\{ -\frac{1}{2} (u^2 + v^2) \right\}.
\]

If the system is boosted, the wave function becomes

\[
\psi_\eta(z, t) = \left( \frac{1}{\pi} \right)^{1/2} \exp \left\{ -\frac{1}{2} \left( e^{-2\eta}u^2 + e^{2\eta}v^2 \right) \right\}.
\]

We note here that the transition from Eq.(3.9) to Eq.(3.10) is a squeeze transformation. The wave function of Eq.(3.9) is distributed within a circular region in the \( uv \) plane, and thus in the \( zt \) plane. On the other hand, the wave function of Eq.(3.10) is distributed in an elliptic region. This ellipse is a “squeezed” circle with the same area as the circle on the \( zt \) plane.
IV. FEYNMAN’S PARTON PICTURE

It is safe to believe that hadrons are quantum bound states of quarks having localized probability distribution. As in all bound-state cases, this localization condition is responsible for the existence of discrete mass spectra. The most convincing evidence for this bound-state picture is the hadronic mass spectra which are observed in high-energy laboratories [17,15]. However, this picture of bound states is applicable only to observers in the Lorentz frame in which the hadron is at rest. How would the hadrons appear to observers in other Lorentz frames? More specifically, can we use the picture of Lorentz-squeezed hadrons discussed in Sec. III.

The radius of the proton is $10^{-5}$ of that of the hydrogen atom. Therefore, it is not unnatural to assume that the proton has a point charge in atomic physics. However, while carrying out experiments on electron scattering from proton targets, Hofstadter in 1955 observed that the proton charge is spread out [18].

In this experiment, an electron emits a virtual photon, which then interacts with the proton. If the proton consists of quarks distributed within a finite space-time region, the virtual photon will interact with quarks which carry fractional charges. The scattering amplitude will depend on the way in which quarks are distributed within the proton. The portion of the scattering amplitude which describes the interaction between the virtual photon and the proton is called the form factor.

Although there have been many attempts to explain this phenomenon within the framework of quantum field theory, it is quite natural to expect that the wave function in the quark model will describe the charge distribution. In high-energy experiments, we are dealing with the situation in which the momentum transfer in the scattering process is large. Indeed, the Lorentz-squeezed wave functions lead to the correct behavior of the hadronic form factor for large values of the momentum transfer [19].

While the form factor is the quantity which can be extracted from the elastic scattering, it is important to realize that in high-energy processes, many particles are produced in the final state. They are called inelastic processes. While the elastic process is described by the total energy and momentum transfer in the center-of-mass coordinate system, there is, in addition, the energy transfer in inelastic scattering. Therefore, we would expect that the scattering cross section would depend on the energy, momentum transfer, and energy transfer. However, one prominent feature in inelastic scattering is that the cross section remains nearly constant for a fixed value of the momentum-transfer/energy-transfer ratio. This phenomenon is called “scaling” [20].

In order to explain the scaling behavior in inelastic scattering, Feynman in 1969 observed that a fast-moving hadron can be regarded as a collection of many “partons” whose properties do not appear to be identical to those of quarks [3]. For example, the number of quarks inside a static proton is three, while the number of partons in a rapidly moving proton appears to be infinite. The question then is how the proton looking like a bound state of quarks to one observer can appear different to an observer in a different Lorentz frame? Feynman made the following systematic observations.

a). The picture is valid only for hadrons moving with velocity close to that of light.

b). The interaction time between the quarks becomes dilated, and partons behave as free independent particles.
c). The momentum distribution of partons becomes widespread as the hadron moves fast.

d). The number of partons seems to be infinite or much larger than that of quarks. Because the hadron is believed to be a bound state of two or three quarks, each of the above phenomena appears as a paradox, particularly b) and c) together. We would like to resolve this paradox using the covariant harmonic oscillator formalism.

For this purpose, we need a momentum-energy wave function. If the quarks have the four-momenta $p_a$ and $p_b$, we can construct two independent four-momentum variables \[ P = p_a + p_b, \quad q = \sqrt{2}(p_a - p_b). \tag{4.1} \]

The four-momentum $P$ is the total four-momentum and is thus the hadronic four-momentum. $q$ measures the four-momentum separation between the quarks.

We expect to get the momentum-energy wave function by taking the Fourier transformation of Eq.\((3.10)\):

\[
\phi_\eta(q_z, q_0) = \left(\frac{1}{2\pi}\right) \int \psi_\eta(z, t) \exp\{-i(q_z z - q_0 t)\} dx dt. \tag{4.2}\]

Let us now define the momentum-energy variables in the light-cone coordinate system as

\[
q_u = (q_0 - q_z) / \sqrt{2}, \quad q_v = (q_0 + q_z) / \sqrt{2}. \tag{4.3}\]

In terms of these variables, the Fourier transformation of Eq.\((4.2)\) can be written as

\[
\phi_\eta(q_z, q_0) = \left(\frac{1}{2\pi}\right) \int \psi_\eta(z, t) \exp\{-i(q_u u + q_v v)\} dudv. \tag{4.4}\]

The resulting momentum-energy wave function is

\[
\phi_\eta(q_z, q_0) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2} \left(e^{-2\eta q_u^2} + e^{2\eta q_v^2}\right)\right\}. \tag{4.5}\]

Because we are using here the harmonic oscillator, the mathematical form of the above momentum-energy wave function is identical to that of the space-time wave function. The Lorentz squeeze properties of these wave functions are also the same. This aspect of the squeeze has been exhaustively discussed in the literature [15,16].

When the hadron is at rest with $\eta = 0$, both wave functions behave like those for the static bound state of quarks. As $\eta$ increases, the wave functions become continuously squeezed until they become concentrated along their respective positive light-cone axes. Let us look at the $z$-axis projection of the space-time wave function. Indeed, the width of the quark distribution increases as the hadronic speed approaches that of the speed of light. The position of each quark appears widespread to the observer in the laboratory frame, and the quarks appear like free particles.

The momentum-energy wave function is just like the space-time wave function. The longitudinal momentum distribution becomes wide-spread as the hadronic speed approaches the velocity of light. This is in contradiction with our expectation from nonrelativistic quantum mechanics that the width of the momentum distribution is inversely proportional to that of the position wave function. Our expectation is that if the quarks are free, they
must have their sharply defined momenta, not a wide-spread distribution. This apparent contradiction presents to us the following two fundamental questions:

a). If both the spatial and momentum distributions become widespread as the hadron moves, and if we insist on Heisenberg’s uncertainty relation, is Planck’s constant dependent on the hadronic velocity?

b). Is this apparent contradiction related to another apparent contradiction that the number of partons is infinite while there are only two or three quarks inside the hadron?

The answer to the first question is “No”, and that for the second question is “Yes”. Let us answer the first question which is related to the Lorentz invariance of Planck’s constant. If we take the product of the width of the longitudinal momentum distribution and that of the spatial distribution, we end up with the relation

\[ <z^2> <q_z^2> = \frac{1}{4} \cosh(2\eta)^2. \]  

(4.6)

The right-hand side increases as the velocity parameter increases. This could lead us to an erroneous conclusion that Planck’s constant becomes dependent on velocity. This is not correct, because the longitudinal momentum variable \( q_z \) is no longer conjugate to the longitudinal position variable when the hadron moves.

In order to maintain the Lorentz-invariance of the uncertainty product, we have to work with a conjugate pair of variables whose product does not depend on the velocity parameter. Let us go back to Eq.(4.3) and Eq.(4.4). It is quite clear that the light-cone variable \( u \) and \( v \) are conjugate to \( q_u \) and \( q_v \) respectively. It is also clear that the distribution along the \( q_u \) axis shrinks as the \( u \)-axis distribution expands. The exact calculation leads to

\[ <u^2> <q_u^2> = 1/4, \quad <v^2> <q_v^2> = 1/4. \]  

(4.7)

Planck’s constant is indeed Lorentz-invariant.

Let us next resolve the puzzle of why the number of partons appears to be infinite while there are only a finite number of quarks inside the hadron. As the hadronic speed approaches the speed of light, both the x and q distributions become concentrated along the positive light-cone axis. This means that the quarks also move with velocity very close to that of light. Quarks in this case behave like massless particles.

We then know from statistical mechanics that the number of massless particles is not a conserved quantity. For instance, in black-body radiation, free light-like particles have a widespread momentum distribution. However, this does not contradict the known principles of quantum mechanics, because the massless photons can be divided into infinitely many massless particles with a continuous momentum distribution.

Likewise, in the parton picture, massless free quarks have a wide-spread momentum distribution. They can appear as a distribution of an infinite number of free particles. These free massless particles are the partons. It is possible to measure this distribution in high-energy laboratories, and it is also possible to calculate it using the covariant harmonic oscillator formalism. We are thus forced to compare these two results. Indeed, according to Hussar’s calculation [21], the Lorentz-boosted oscillator wave function produces a reasonably accurate parton distribution.
V. COHERENCE PROBLEMS

The most puzzling problem in the parton picture is that partons in the hadron appear as incoherent particles, while quarks are coherent when the hadron is at rest. Does this mean that the coherence is destroyed by the Lorentz boost? The answer is NO, and here is the resolution to this puzzle.

When the hadron is boosted, the hadronic matter becomes squeezed and becomes concentrated in the elliptic region along the positive light-cone axis. The length of the major axis becomes expanded by $e^\eta$, and the minor axis is contracted by $e^\eta$.

This means that the interaction time of the quarks among themselves become dilated. Because the wave function becomes wide-spread, the distance between one end of the harmonic oscillator well and the other end increases. This effect, first noted by Feynman [3], is universally observed in high-energy hadronic experiments. The period is oscillation is increases like $e^\eta$.

On the other hand, the interaction time with the external signal, since it is moving in the direction opposite to the direction of the hadron, it travels along the negative light-cone axis. If the hadron contracts along the negative light-cone axis, the interaction time decreases by $e^{-\eta}$. The ratio of the interaction time to the oscillator period becomes $e^{-2\eta}$. The energy of each proton coming out of the Fermilab accelerator is $900\, GeV$. This leads the ratio to $10^{-6}$. This is indeed a small number. The external signal is not able to sense the interaction of the quarks among themselves inside the hadron.

CONCLUDING REMARKS

The time variable plays many important roles in physics. Its place in Einstein’s special relativity is well known. In this paper, we noted that the time-separation variable together with the spatial separation can be combined into one covariant world in the quark-parton model of relativistic hadrons. The time separation variable plays also plays a pivotal role in the measurement process in the parton picture where the partons appear like incoherent entities. It is shown that the lack of coherence in the parton picture is perfectly consistent with special relativity.
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