Flat bands and strongly correlated Fermi systems

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Abstract
Many strongly correlated Fermi systems including heavy-fermion (HF) metals and high-\textit{T}_c superconductors belong to that class of quantum many-body systems for which Landau Fermi-liquid (LFL) theory fails. Instead, these systems exhibit non-Fermi-liquid properties that arise from violation of time-reversal (\textit{T}) and particle-hole (\textit{C}) invariance. Measurements of tunneling conductance provide a powerful experimental tool for detecting violations of these basic symmetries inherent to LFLs, which guarantee that the measured differential conductivity \(\text{d}I/\text{d}V\), where \(I\) is the current and \(V\) the bias voltage, is a symmetric function of \(V\). Thus, it has been predicted that the conductivity becomes asymmetric for HF metals such as CeCoIn\textsubscript{5} and YbRh\textsubscript{2}Si\textsubscript{2}. In these systems, the background electron liquid is considered to undergo a transformation that renders a portion of its excitation spectrum dispersionless, giving rise to so-called flat bands. The presence of a flat band indicates that the system is close to a special quantum critical point, namely a \textit{topological fermion-condensation quantum phase transition}. An essential aspect of the behavior of a system hosting a flat band is that application of a magnetic field can restore its normal Fermi-liquid properties, including \textit{T}- and \textit{C}-invariance, with the differential conductivity again becoming a symmetric function of \(V\). This behavior has been observed in recent measurements of tunneling conductivity in both YbRh\textsubscript{2}Si\textsubscript{2} and graphene. Also within the FC framework, we describe and explain recent empirical observations of scaling properties related to universal linear-temperature resistivity for a large number of strongly correlated high-temperature superconductors. We show that the observed scaling is explained by the emergence of flat bands formed by fermion condensation.

Keywords: flat bands, strongly correlated Fermi systems, tunneling conductance

(Some figures may appear in colour only in the online journal)

Introduction

Many strongly correlated Fermi systems, notably those electronic solids identified as heavy-fermion (HF) metals and high-\textit{T}_c superconductors (HTSC), belong to a new class of materials for which standard Landau Fermi-liquid (LFL) theory no longer applies. Instead, the systems demonstrate so-called non-Fermi-liquid (NFL) properties, typically reflecting violation of time-reversal and particle-antiparticle invariances (respectively \textit{T}-invariance and \textit{C}-invariance). Measurement of tunneling conductance provides a powerful experimental technique for detecting these symmetry violations. Conservation of \textit{T} and \textit{C} symmetries is inherent in the LFL theory, which implies that the differential conductivity \(\sigma_{\text{fl}}(V) = \text{d}I/\text{d}V\) and differential resistance \(\rho_{\text{fl}}(V) = \text{d}V/\text{d}I\) formed from the current \(I\) and bias voltage \(V\) are necessarily symmetric functions of \(V\). It has been predicted that the conductivity becomes asymmetric for HF metals such as CeCoIn\textsubscript{5} and YbRh\textsubscript{2}Si\textsubscript{2}. As an underlying mechanism, it is postulated in these materials the electronic system undergoes a special kind of transformation: a portion of its single-particle spectrum becomes dispersionless, forming a so-called flat band [1–5]. Emergence of a flat band implies that the system
possesses a quantum critical point representing a topological instability. This instability induces a topological quantum phase transition (FCQPT) [3, 4] involving the phenomenon of fermion condensation (FC). Importantly, the application of a magnetic field $B$ restores the usual Fermi-liquid properties and the different conductivity becomes a symmetric function of $V$ [6] due to the the $T$- and $C$-invariances reenact [1–3]. Such a suppression of the asymmetric part of $dI/dV$ has been observed in the HF metal YbCu$_{1-x}$Al$_x$ (for $x = 1.5$) under the application of magnetic fields up to 22.5 T [6]. This behavior has been detected in recent measurements of the tunneling conductivity on YbRh$_2$Si$_2$ [7, 8] and, importantly, in measurements of the tunneling resistance on graphene possessing flat band [9]. Analysis of the data [8, 9] provides an explicit demonstration of the restoration of the symmetry in YbRh$_2$Si$_2$ at $B \approx 9$ T and in graphene in tiny magnetic fields $B \approx 140$ mT.

In many common superconductors, at temperature $T = 0$ the density $n_s$ of superconducting (SC) electrons is equal to the total electron density $n_t$, as a manifestation of both the well-known BSC theory and the theorem of Leggett [10, 11]. However, recent measurements on overdoped copper HTSC oxides have demonstrated the putatively anomalous behavior $n_s \ll n_t$, awaiting explanation. In other words, the density of paired (superfluid) charge carriers turns out to be much lower than that predicted by the standard theory of Bardeen, Cooper, and Schrieffer (BCS) for conventional superconductors, within which $n_s$ is directly proportional to the critical temperature $T_c$ over a wide doping range [12, 13]. Broadly speaking, if no conventional explanation of this behavior can be given, see e.g. [14], such a departure from BCS theory may occur because the electronic fluid within the material is not a normal Fermi liquid in the sense of Landau FL theory [12, 13]. More specifically, the system may exhibit a FCQPT, beyond which some charge carriers form a fermion condensate (FC) having very special properties [3, 15]. In particular, the Leggett theorem no longer applies even for uniform electron liquid [16], since both $T$- and $C$-invariances are violated in systems with NFL behavior [3, 15, 17].

Also of special interest as possible expressions of new physics are observations involving resistivity $\rho$ and at low temperatures, $T \to 0$, the London penetration depth $\lambda_0$ that indicate a universal scaling property

$$\frac{d\rho}{dT} \propto \lambda_0^2$$

for a large number of strongly correlated high-temperature superconductors [18]. This scaling relation spans several orders of magnitude in $\lambda_0$, attesting to the robustness of the empirical law (1); indeed, the behavior is similar to that documented in [19] and explained in [20]. We shall show the observed scaling is simply explained by the emergence of flat bands formed by fermion condensation [21, 22].

**Asymmetric conductivity and the NFL behavior**

Direct experimental studies of quantum phase transitions in HTSC and HF metals are of great importance for understanding the underlying physical mechanisms responsible for their anomalous properties. However, such studies of HF metals and HTSC are difficult because the corresponding critical points are usually concealed by the proximity to other phase transitions, commonly antiferromagnetic (AF) and/or SC. Recently, extraordinary properties of tunneling conductivity in the presence of a magnetic field were observed in a graphene preparation having a flat band [9], as well as in HTSC’s and the HF metal YbRh$_2$Si$_2$ [7, 8]. Measuring and analyzing these properties will shed light on the nature of the quantum phase transitions occurring in these substances.

Most of the experiments on HF metals and HTSC’s explore their thermodynamic properties. However, it is equally important to determine other properties of these strongly correlated systems, notably quasiparticle occupation numbers $n(p, T)$ as a function of momentum $p$ and temperature $T$. These quantities are not linked directly to the density of states (DOS) $N(\varepsilon = 0)$ determined by the quasiparticle energy $\varepsilon$ or to the behavior of the effective mass $M^*$. Scanning tunneling microscopy [23–25] and point contact spectroscopy [26], being sensitive to both the DOS and quasiparticle occupation numbers, are ideal tools for exploring the effects of $C$ and $T$ symmetry violation. When $C$ and $T$ symmetries are not conserved, the differential tunneling conductivity and dynamic conductance are no longer symmetric functions of the applied voltage $V$.

Indeed, if under the application of bias voltage $V$, the current of electrons with the charge $-e$, traveling from HF to a common (i.e. ‘non-HF’) metal changes the sign of a charge carrier to $+e$, then current character and direction alters. Namely, now the carriers are holes with the charge $+e$ traveling from the common to HF metal. Turning this around, one can obtain the same current of electrons provided that $V$ is changed to $-V$. The resulting asymmetric differential conductivity $\Delta \sigma(0) = \sigma(0) - \sigma(-0)$ becomes nonzero, as it is seen from figure 2. On the other hand, if time $t$ is changed to $-t$ (but charge is kept intact), the current changes its direction only. The same result can be achieved by $V \to -V$, and we conclude that $T$ symmetry is broken, provided that $\Delta \sigma(0) \neq 0$. Thus, we detect the presence of $\Delta \sigma(0) \neq 0$ signals under both $C$ and $T$ symmetries violation. At the same time, the change of both $e \to -e$ and $t \to -t$ returns the system to its initial state so that CT symmetry is conserved. We keep in mind that the same consideration is true when analyzing $\rho_d(V)$. Note that the parity symmetry $P$ is conserved and the well-known CPT symmetry is not broken in the considered case. On the other hand, the time-reversal invariance and particle-hole symmetry remain intact in normal Fermi systems, the differential tunneling conductivity and dynamic conductance are symmetric functions of $V$. Thus, a conductivity asymmetry is not observed in conventional metals at low temperatures.

To determine the tunneling conductivity, we first calculate the tunneling current $I(V)$ through the point contact between two metals. This is done using the method of Harrison [23–25], based on the observation that $I(V)$ is proportional to the particle transition probability introduced by Bardeen [27]. Bardeen considered the probability $P_{12}$ of a
particle (say an electron) making a transition from a state 1 on one side of the tunneling layer to a state 2 on the other side. This quantity has the behavior \( P_{12} \sim |t_{12}|^2 N_1(0)N_2(0) (1 - n_2) \) in terms of the DOS \( N_2(0) \) (at \( \varepsilon = 0 \)) in state 2, the electron occupation numbers \( n_{1,2} \) in these states, and a transition matrix element \( t_{12} \). The total tunneling current \( I \) is then proportional to the difference between the current from 1 to 2 and that from 2 to 1, with the result taking the form

\[
I \approx P_{12} - P_{21} \sim |t_{12}|^2 N_1(0)N_2(0) \\
\times [n_1(1 - n_2) - n_2(1 - n_1)] = |t_{12}|^2 N_1(0)N_2(0)(n_1 - n_2).
\]

Harrison applied the WKB approximation to calculate the matrix element [23–25], \( t_{12} = \tau (N_1(0)N_2(0))^{-1/2} \), where \( \tau \) denotes the resulting transition amplitude. Multiplication of expression (2) by 2 to account for the electron spin and integration over the energy \( \varepsilon \) leads to the expression for total (or net) tunneling current [23–25]:

\[
I(V) = 2|t|^2 \int [n_1(\varepsilon - \mu - V) - n_2(\varepsilon - \mu)]d\varepsilon.
\]

Here \( n_\varepsilon(\varepsilon) \) is the electron occupation number for a metal in the absence of a FC, and we have adopted atomic units \( e = m = \hbar = 1 \), where \( e \) and \( m \) are the electron charge and mass, respectively. Since temperature is low, \( n_\varepsilon(\varepsilon) \) can be approximated by the step function \( \theta(\varepsilon - \mu) \), where \( \mu \) is the chemical potential.

It follows from equation (3) that quasiparticles with single-particle energies \( \varepsilon \) in the range \( \mu \leq \varepsilon \leq \mu + V \) contribute to the current, while \( I(V) = c_1V \) and \( \sigma_d(V) \equiv dI/dV = c_1 \), with \( c_1 = \text{const} \). Thus, in the framework of LFL theory the differential tunneling conductivity \( \sigma_d(V) \), being a constant, is a symmetric function of the voltage \( V \), i.e., \( \sigma_d(V) = \sigma_d(-V) \). In fact, the symmetry of \( \sigma_d(V) \) holds provided \( C \) and \( T \) symmetries are observed, as is customary for LFL theory. The symmetry of \( \sigma_d(V) \) is therefore quite obvious and common in case of contact of two ordinary metals (without a FC), regardless whether they are a normal or SC state. We note that a more rigorous consideration of the densities of states \( N_1 \) and \( N_2 \) entering equation (2) for \( \varepsilon \approx \mu \) requires their inclusion in the integrand of equation (3) [28–30]. For example, see equation (7) of [30], where this refinement has been carried out for the system of a magnetic adatom and scanning tunneling microscope tip. However, this complication does not break the \( C \) symmetry in LFL case. On the other hand, it will be seen below that if the system hosts a FC, the presence of the density-of-states factors in the integrand of equation (3) acts to promote the asymmetry of tunneling spectra, for the DOS strongly depend on \( \varepsilon \approx \mu \), see figure 1. Indeed, the situation becomes quite different in the case of a strongly correlated Fermi system in the vicinity of FCQPT that engenders a flat band [21, 22], and violates the \( C \) symmetry [3, 15]. We note that as we have seen above, the violation of \( C \) symmetry entails the violation of \( T \) symmetry. Panel (a) of figure 1 illustrates the resulting low-temperature single-particle energy spectrum \( \varepsilon(k, T) \). Panel (b), which displays the momentum dependence of the occupation numbers \( n(k, T) \) in such a system, shows that the flat band induced by the FCQPT, as we have seen above, in fact, violates \( T \) symmetry as well. The broken \( C \) symmetry is reflected in the asymmetry of the regions occupied by particles (labeled \( p \)) and holes (labeled \( h \)). We note that a system in its SC state and located near FCQPT exhibits asymmetrical tunneling conductivity, for the \( C \) symmetry remains broken in both the SC and the normal states. This observation is in accordance with experimental facts [2, 3], as it is seen from figure 2.

It is seen from figure 1 that at low temperatures the electronic liquid of the system has two components. One is an exotic component made up of heavy electrons occupying a range of momenta \( p_1 < p < p_2 \) surrounding the Fermi volume near the former Fermi surface \( p = p_F \). This component is characterized by the SC order parameter \( \kappa(p) = \sqrt{n(p)(1 - n(p))} \). The other component is made up of normal electrons occupying the momentum range \( 0 \approx p \approx p_1 \); it maintains LFL properties [3, 16]. This unusual momentum-space distribution cannot be described within standard BCS theory; nor can its distinctive properties. In particular, the density of paired charge carriers that form the superfluid density is no longer equal to the total particle density \( n_0 \) represented by paired and unpaired charge carriers. This violation of Leggett’s theorem is to be expected, since both \( C \) and \( T \) invariances are violated in the NFL state of some HF metals and compounds [1, 3, 15].

We are proposing that for the strongly correlated many-fermion systems in question, the approximate equality \( n_s \approx n_{el} \) that would normally be expected for a real system...
the tunneling current becomes finite and we obtain \[ \Delta \sigma_f(V) = \sigma_f(V) - \sigma_f(-V) \]
becomes finite and we obtain [1–3, 5, 15]

\[ \Delta \sigma_f(V) \approx c \left( \frac{V}{2T} \right)^{p_f - p_i} \]

(5)

where \( p_F \) is the Fermi momentum and \( c \) is a constant of order unity. It is worthy noting that equation (5) is also valid if the DOS \( N_1 \) and \( N_2 \) are taken into account, for that only changes \( c \). Note that the conductivity \( \Delta \sigma_d(V) \) remains asymmetric also in the SC phase of both HTSC and HF metals. In such cases it is again the occupation number \( n(p) \) that is responsible for the asymmetric part of \( \Delta \sigma_d(V) \), since this function is not appreciably disturbed by the superconductive pairing. This is because superconductive pairing is usually weaker than that of the Landau interaction in forming the function \( n(p) \) [3]. As a result, \( \Delta \sigma_d(V) \) remains approximately the same below the SC \( T_c \) [3, 35]. It is seen from equation (5) and figure 2 that with raising temperatures the asymmetry diminishes and finally vanishes at \( T \geq 40 \) K. Such a behavior has been observed in measurements on the HF metal CeCoIn\(_5\) [36], displayed in figure 2. Under application of a magnetic field \( B \) at sufficiently low temperatures \( k_B T \ll \mu_B B_c \), where \( k_B \) and \( \mu_B \) are the Boltzmann constant and the Bohr magneton, the strongly correlated Fermi system transits from the NFL to the LFL regime [3, 37]. As we have seen above, the asymmetry of the tunneling conductivity vanishes in the LFL state [1–3, 5]. We surmise that \( \Delta \sigma_f(V) \) seen in figure 2 should vanish in the normal state at sufficiently high magnetic fields applied along the easy axis and low temperatures \( k_B T \ll \mu_B B_c \) with the critical field \( B_c \approx 5 \) T. Under this condition the system transits from the NFL behavior to the LFL, and obtains the LFL behavior with the resistance \( \rho \) becoming a quadratic function of temperature, \( \rho(T) \propto T^2 \) [3]. The examples of suppression of the asymmetric parts of differential conductivity and resistance under the application of magnetic field, are shown in figures 4 and 6 respectively.

Figure 3 shows the differential conductivity \( \sigma_d \) observed in measurements on YbRh\(_2\)Si\(_2\) [7, 8]. It is seen that its asymmetry diminishes with elevation of the magnetic field \( B \), as the minima of the curves shift to the \( V = 0 \) point, see also figure 4 for details. The magnetic field is applied along the hard magnetization direction, \( B || c \), with \( B_c \approx 0.7 \) T [8], where \( B_c \) is the critical field suppressing the AF order [38]. The asymmetric part of the tunneling differential conductivity, \( \Delta \sigma_d(V) \), is displayed in figure 4, being extracted from measurements shown in figure 3. It is seen that \( \Delta \sigma_d(V) \) decreases as \( B \) increases. We predict that application of the magnetic field in the easy magnetization plane, \( B \perp c \) with \( B_c \approx 0.06 \) T, leads to a stronger suppression of the asymmetric part of the conductivity, observing that in this case the magnetic field effectively suppresses the AF order and the NFL behavior. Indeed, the experimental data show that low-temperature electrical resistivity \( \rho(T) \) of the HF metal YbRh\(_2\)Si\(_2\) measured at \( T \approx 20 \) mK, under the application of magnetic field \( B \geq 75 \) mT along easy magnetization plane, exhibits the LFL behavior \( \rho(T) \propto T^2 \), while at \( B \approx 60 \) mT it demonstrates the NFL behavior, \( \rho(T) \propto T \). At the same time, magnetic field \( B \)}
applied along the hard magnetization direction makes the resistivity show the LFL behavior at much higher $B \approx 0.8 \, \text{T}$ \cite{38}. The same transition from the NFL behavior to LFL one is observed in measurements of the thermodynamic, transport and relaxation properties, see e.g. \cite{3,15,38}. We surmise that the asymmetric part $\Delta \sigma_d(V)$ vanishes as soon as $YbRh_2Si_2$ enters its AF state, exhibiting LFL behavior $T \approx 2 \mu \text{K}$ at $B = 0$ and $T < 70 \, \text{mK}$.

On measuring the differential resistance $\rho_d(V) = dV/dI$ as a function of current $I$, one finds that its symmetry properties are the same as those of $\sigma_d(V)$; namely, under the application of a magnetic field, the asymmetry of the differential resistance vanishes as the system transits to the LFL state. The differential resistance $\rho_d(V)$ of graphene as a function of a direct current $I$ for different magnetic fields $B$ is reported in figure 5 \cite{9}. The asymmetric part of the differential resistance $A_s(I) = \rho_d(V) - \rho_d(-V)$ is seen to diminish with increasing magnetic field, vanishing near $B \approx 140 \, \text{mT}$. Such behavior is extremely compelling, since the strongly correlated graphene sample has a perfect flat band, implying that the FC effects should be clearly manifested in this material \cite{9}. Thus, in accordance with prediction \cite{1-3,5}, the asymmetric part tends to zero at tiny magnetic fields of $140 \, \text{mT}$, as it is seen from figure 6. The asymmetry persists in the SC state of graphene \cite{9} and is suppressed at $B \approx 80 \, \text{mT}$.

Disappearance of the asymmetric part of the differential conductivity in figure 6 indicates that as the magnetic field increases, graphene transits from the NFL to the LFL state. To support this statement, we surmise that the resistance $\rho(T)$ should exhibit linear dependence $\rho(T) \propto T$ in the normal state at zero magnetic field, as generally the case in other strongly correlated Fermi systems. While at higher magnetic fields and low temperatures $k_B T \ll \mu_B B$, the system transits from the NFL behavior to the LFL one and the resistance becomes a quadratic function of temperature $\rho(T) \propto T^2$ \cite{3,15,39}.
Universal scaling relation

Another experimental result [18] providing insight into the NFL behavior of strongly correlated Fermi systems is the universal scaling relation, which can also be explained using the flat band concept. The authors of [18] measured the temperature dependence \(d\rho/dT\) of the resistivity \(\rho\) for a large number of HTSC substances for \(T > T_c\). (Among these were LSCO and the well-known HF compound CeCoIn5; see table 1 of [18].) They discovered quite remarkable behavior: for all substances considered, \(d\rho/dT\) shows a linear dependence on \(\lambda_0^2\). All of the superconductors considered belong to the London type, for which \(\lambda_0 \gg \xi_0\), where \(\xi_0\) is the zero-temperature coherence length (see, e.g. [17]).

It has been shown in [18], that the scaling relation

\[
\frac{d\rho}{dT} \propto \frac{k_B}{\hbar} \lambda_0^2
\]

remains valid over several orders of magnitude in \(\lambda_0\), signifying its robustness. At the phase transition point \(T = T_c\), relation (6) yields the well-known Holmes law [18] (see also [40] for its theoretical derivation):

\[
\sigma T_c \propto \lambda_0^{-2},
\]

in which \(\sigma = \rho^{-1}\) is the normal state dc conductivity. It has been shown by Kogan [40] that Holmes law applies even for the oversimplified model of an isotropic BCS superconductor. Within the same model of a simple metal, one can express the resistivity \(\rho\) in terms of microscopic substance parameters [41]: \(e^2n\rho \approx p_F/(\tau v_F)\), where \(\tau\) is the quasiparticle lifetime, \(n\) is the carrier density, and \(v_F\) is the Fermi velocity. Taking into account that \(p_F/v_F = M^*\), we arrive at the equation

\[
\rho = \frac{M^*}{n\tau}. \tag{8}
\]

We note that equation (8) formally agrees with the well-known Drude formula. It has been shown in [17] that good agreement with experimental results [12] is achieved when the effective mass and the superfluid density are attributed to the carriers in the FC state only, i.e. \(M^* \equiv M_{FC}\) and \(n \equiv n_{FC}\). Keeping this in mind and utilizing the relation \(1/\tau = k_B T/\hbar\) (see [42] and chapter 9 of [15]), we obtain

\[
\rho = \frac{M_{FC}}{e^2 n_{FC}} \frac{k_B T}{\hbar} \equiv 4\pi \lambda_0^2 \frac{k_B T}{\hbar}, \tag{9}
\]

i.e. \(d\rho/dT\) is indeed given by the expression (6). Equation (9) demonstrates that fermion condensation can explain all the above experimentally observed universal scaling relations. It is important to note that the FC approach presented here is insensitive to and transcends the microscopic, non-universal features of the substances under study. This is attributed to the fact that the FC state is protected by its topological structure and therefore represents a new class of Fermi liquids [15, 22].

In particular, consideration of the specific crystalline structure of a compound, its anisotropy, its defect composition, etc do not change our predictions qualitatively. This strongly suggests that the FC approach provides a viable theoretical framework for explaining universal scaling relations similar to those discovered in the experiments of Božović et al [12] and Hu et al [18]. In other words, the fermion condensation of charge carriers in the considered strongly correlated HTSC’s, engendered by a quantum phase transition, is indeed the primary physical mechanism responsible for their observable universal scaling properties. This mechanism can be extended to a broad set of substances with a very different microscopic characteristics, as discussed in detail in [3, 15].

Conclusions

The central message of the present paper is that if the electronic spectrum of a substance happens to feature a dispersionless part, or ‘flat bands,’ it is just this aspect that is responsible for measured properties that depart radically from those of familiar condensed-matter systems described by the LFL theory. This is the case irrespective of varied microscopic details characterizing these substances such as crystal symmetry and defect structure. The explanation for this finding lies in the fact that FC most readily occurs in substances hosting flat bands. Experimental manifestations of FC phenomena are varied, which implies that different experimental techniques are most suitable for detecting and analyzing them. We have shown that one such technique is scanning tunneling microscopy, which has the advantage of being sensitive both to the DOS and quasiparticle occupation numbers. The reason for this dual sensitivity is that this technique is a well suited to studying effects related to the violation of particle-hole symmetry and time-reversal invariance. Their violation leads to asymmetry of the differential tunneling conductivity and that of resistance to the applied voltage \(V\) or current \(I\). Based on the recent experimental results, we have demonstrated that the asymmetric part of both the conductivity and the resistivity vanishes under the application of a magnetic field, as predicted in [1, 2, 5]. To support our statement regarding the role of broken \(T\)-invariance, we have analyzed and discussed recent challenging measurements in overdoped cuprates by Božović and coauthors [12] within the fermion condensation framework. Also within the FC framework, we have described and explained otherwise purely empirical observations of scaling properties [18]. Finally, our study of such recent experimental results strongly suggests that the topological FCQPT is an intrinsic feature of many strongly correlated Fermi systems and can be viewed as the universal agent of their non-Fermi-liquid behavior.

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