Isoscalar g-factors of odd-odd $N = Z$ nuclei

S. Yeager$^1$, S. J. Q. Robinson$^2$, L. Zamick$^{1(a)}$ and Y. Y. Sharon$^1$

$^1$Department of Physics and Astronomy, Rutgers University - Piscataway, NJ 08854, USA
$^2$Department of Physics, Millsaps College - Jackson, MS 39210, USA

Abstract – It is known that the measured and calculated $g$-factors of $2^+_1$ states in even-even $N = Z$ nuclei have values of $g \approx 0.5$. Both the collective model and the $LS$ limit of the shell model (with $L = 2$, $S = 0$) can explain this. Here we note that the isoscalar $g$-factors of low-lying $1^+, 3^+$, and $5^+ T = 0$ states in medium-mass odd-odd $N = Z$ nuclei have also been measured to have values close to 0.5. An up-to-date compilation of the experimental values is presented. Also included are the results of large-scale shell model calculations which agree very well with the experimental data. We show that the isoscalar $g$-factors in $N = Z$ nuclei also approach 0.5 in the large-$l$ limit for a single-$j$ shell model.

Introduction. – The current availability of radioactive beams makes it possible to study the properties of nuclei far away from the stability line, including the neutron-deficient heavier $N = Z$ nuclei. These are of special interest because with both the valence protons and neutrons in the same shells, one can test the applicability of the shell model to unstable nuclei and investigate possible changes in the ordering and values of the single-particle levels and energies. Studying the lighter $N = Z$ nuclei provides a perspective for what to expect in the heavier ones. Recently the doubly magic $^{108}$Sn nucleus has been investigated [1].

We use the isotopic spin, $T$, concept, which treats the proton and neutron as two states of a $T = \frac{1}{2}$ doublet; $T = \frac{1}{2}$, $T_z = +\frac{1}{2}$ for the neutron, $T = \frac{1}{2}$, $T_z = -\frac{1}{2}$ for the proton. The proton-proton system has $T = 1$, $T_z = -1$; the neutron-neutron system has $T = 1$, $T_z = +1$. The proton-neutron $T = 0$ system can have $T = 1$ (isovector case) or $T = 0$ (isoscalar case).

Characterizing nuclear states by their $T$ values can provide insights into their excitation energies, moments, decays, and other properties. The magnetic moment operator $\mu$ can be written [2] in terms of isoscalar ($\mu_0$) and isovector ($\mu_1$) parts as $\mu = \mu_0 - \tau\mu_1$, where $\tau = -1$ for a proton and +1 for a neutron. Thus $\mu_0 = \frac{1}{2}(\mu_{unger} + \mu_{neutron})$ and $\mu_1 = \frac{1}{2}(\mu_{proton} - \mu_{neutron})$. For the free neutron-proton system $\mu_0 = 0.4398\mu_n$ and $\mu_1 = 2.35296\mu_n$, where $\mu_n$ is the nuclear magneton. In the present paper we investigate the isoscalar magnetic moments $\mu$ of some low-lying $T = 0$ states of odd-odd $N = Z$ nuclei. Measuring magnetic moments is of particular interest as they provide information on the proton and neutron configurations and on the single particle or collective nature of the nucleus.

Reference [3] summarized that the measured and calculated isoscalar $g$-factors (where $g = \frac{\mu}{I}$, with $I$ being the total angular momentum) of $2^+_1$ states in even-even $N = Z$ nuclei have values of $\approx 0.5$ and offered an explanation. This can be explained by either the collective model [4], where $g_{\text{collective}} = \frac{2}{5} = 0.5$, or the $LS$ limit of the shell model when $L = 2$ and $S = 0$ [5].

In the present paper we note that the isoscalar $g$-factors of the low-lying $I = 1^+$, $3^+$, and $5^+ T = 0$ states in the odd-odd medium-mass $N = Z$ nuclei have also been measured to have values $\approx 0.5$. Our large-scale shell model calculations yield similar values. Understanding these results could be important as $g$-factors of the heavier odd-odd $N = Z$ nuclei may be measured in the near future, utilizing radioactive beams.

Isoscalar magnetic moments. – In general isoscalar magnetic moments have much smaller deviations from the Schmidt (single-particle) values than the isovector ones [6]. However, despite being small, these deviations exhibit a systematic behavior, as was noted by Talmi [7]. Arima [5] related the smallness of these deviations to

(a)E-mail: lzamick@physics.rutgers.edu
the aforementioned isoscalar spin coupling (0.43981) being small compared to the isovector spin coupling (2.35296).

In “Theories of Nuclear Moments” Blin-Stoyle [8] discussed magnetic moments of odd-A and odd-odd nuclei but he did not consider magnetic moments of the excited states of even-even nuclei due to a lack of data at the time for such nuclei. Blin-Stoyle attributed the excited states of even-even nuclei due to a lack of data at the time for such nuclei. Blin-Stoyle attributed the excited states of even-even nuclei due to a lack of data for heavier nuclei and we will later emphasize that for heavier nuclei one must consider the jj limit. Furthermore, we also note that the LS eqs. (4) and (5) do not yield a value $g \approx 0.5$ for the very light $N = Z$ odd-odd nuclei. In table 1 are some selected results including the experimental values and the results of calculations using both the LS and jj limits.

For the $1^+$ state in $^6$Li, the LS result for $L = 0, S = 1$ is the same as that for the deuteron. In $^{10}$B, the $J = 3^+$ state has a unique configuration—the same in LS and jj—hence the same g-factor in both; it also equals the g-factor for $^6$Li in the jj limit, due to particle-hole relationships. For the deuteron, the LS and jj results are identical.

The LS result for $^{14}$N is for $L = 2, S = 1$, which is closer to the true wave function than the simple jj wave function $p_1/2(n)p_1/2(p)$. A conspiracy of the spin-orbit and tensor interactions results in an almost good LS configuration [10]. This is surprising because in general the spin-orbit force works against good LS wave functions. Bertsch and Baroni [11] point out that the short-range nuclear attraction is stronger in the isoscalar channel than in the isovector channel as evidenced by the deuteron’s existence. They note that pairing is visible in a few light odd-odd $N = Z$ nuclei with a $J = 1$ ground state.

Experimental and calculated g-factor results for odd-odd $N = Z$ nuclei. — Recent experimental and calculated results by the groups of Speidel and Benzcer-Koller [12–14] confirm earlier data [3,15] for $^{20}$Ne and $^{32}$Si and expand the $g(2J^+)$ $\approx 0.5$ even-even $N = Z$ picture to encompass also $^{36}$Ar and $^{44}$Ti.

In the present work (see table 2) we show that similar $g \approx 0.5$ results pertain to the low-lying $T = 0$ states of medium-mass (and presumably heavier) odd-odd $N = Z$ nuclei. The states considered have total

| Nucleus | State | $LS$ | $jj$ | exp |
|---------|------|-----|-----|-----|
| $^2$H   | $J = 1^+$ | 0.88 | 0.88 | 0.8574 |
| $^6$Li  | $J = 1^+$ | 0.88 | 0.627 | 0.8221 |
| $^{10}$B | $J = 3^+$ | 0.627 | 0.627 | 0.600 |
| $^{14}$N | $J = 1^+$ | 0.31 | 0.373 | 0.403 |

The magnetic moments of specific light nuclei in both the jj and LS couplings have been calculated by Talmi [9] and Blin-Stoyle [8].

The LS limit eqs. (4) and (5) can be generalized to $N = Z$ even-even nuclei. For the $2^+$ state of such nuclei, the pure LS configuration with $S = 0$ and $L = I$ will yield a g-factor of 0.5. This is because if in eq. (4) $l_p = l_n$ then $g_s = \frac{1}{2}$ and for $S = 0$, $g_s = 0$. This was used by Arima [5] to explain why the g-factors are so close to 0.5 for $2^+$ states of the $N = Z$ nuclei $^{36}$Ne, $^{24}$Mg, $^{28}$Si, and $^{32}$S.

However, it is known the LS picture does not work well for heavier nuclei and we will later emphasize that for heavier nuclei one must consider the jj limit. Furthermore, we also note that the LS eqs. (4) and (5) do not yield a value $g \approx 0.5$ for the very light $N = Z$ odd-odd nuclei. In table 1 are some selected results including the experimental values and the results of calculations using both the LS and jj limits.

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**Table 1: Selected LS and jj calculated results for the g-factors of light odd-odd $N = Z$ nuclei.**
Isoscalar $g$-factors of odd-odd $N=Z$ nuclei

Table 2: $g$-factors of $T=0$ states in odd-odd $N=Z$ nuclei.

| Nuclei | $J$ | Measured $g$-factors | Large-scale shell model | Single-$j$ model |
|--------|----|----------------------|-------------------------|-----------------|
| $^2$H  | 1$^+$ | 0.857438228(9) | 0.88$^a$ | 0.88 (s$_1/2$) |
| $^6$Li | 1$^+$ | 0.8220473(6) | 0.87$^b$ | 0.63 (p$_3/2$) |
| $^{10}$B | 3$^+$ | 0.00021493(2) | 0.61$^c$ | 0.63 (p$_3/2$) |
| $^{14}$N | 1$^+$ | 0.63(12) | 0.77$^d$ | 0.63 (p$_3/2$) |
| $^{18}$F | 3$^+$ | 0.59(4) | 0.62$^e$ | 0.58 (d$_5/2$) |
| $^{22}$Na | 3$^+$ | 0.582(1) | 0.59$^f$ | 0.58 (d$_5/2$) |
| $^{26}$Al | 5$^+$ | 0.561(8) | 0.57$^g$ | 0.58 (d$_5/2$) |
| $^{38}$K | 3$^+$ | 0.457(2) | 0.41$^h$ | 0.42 (d$_5/2$) |
| $^{46}$V | 3$^+$ | 0.55(1) | 0.58$^i$ | 0.55 (f$_7/2$) |
| $^{58}$Cu | 1$^+$ | 0.52(8) | 0.63$^j$ | 0.63 (p$_3/2$) |

$^a$ See table 1.  
$^b$ With PIII interaction [16]; full $p$ shell.  
$^c$ With USDA interaction [17]; full $sd$ shell.  
$^d$ With GXPF1 interaction [18]; full $fp$ shell.  
$^e$ With GXPF1 interaction, up to 4 particles excited from $f_{7/2}$ orbit.

angular-momentum values of $I = 1^+$, $3^+$, or $5^+$; some are ground states, for which the uncertainty in the $g$-factor measurement is very small. The table of Stone [19] was very useful in providing the measured values; the recent $^{58}$Cu result is from [20]. For each case, we provide the appropriate single-$j$ orbit in table 2.

Except for the very light nuclei $^2$H and $^6$Li, the measured $g$-factors are always within 0.13 of the $g = 0.5$ value. Also, with the exception of the $1^+$ states of $^{58}$Cu (where the shell model space was truncated) and $^{10}$B, the corresponding measured and calculated shell model $g$-factors in table 2 differ by less than 0.09.

Table 2 shows the proximity to $g = 0.5$ for both the measured values and the results of large-scale shell model calculations for the isoscalar states of $N=Z$ odd-odd nuclei. For the $2^+_1$ states in even-even $N=Z$ nuclei the proximity to $g = 0.5$ is often associated with the $g_{\text{collective}} = \frac{2}{3}$ for a $K=0$ band. In the next section we demonstrate that the single-$j$ shell model also predicts the $g \approx 0.5$ for the isoscalar states of both odd-odd and even-even $N=Z$ nuclei.

$g$-factors in the single-$j$ shell model. – In the single-$j$ shell model, the complicated general $g$-factor equations simplify for $N=Z$ nuclei, both even-even and odd-odd. We will use generalized expressions from McCullen et al. [21] for $N \neq Z$ to show how the general results simplify for $N=Z$.

Consider any nucleus for which all the active protons and neutrons are in the same $j$ shell. The lighter titanium isotopes with a closed $^{40}$Ca core are an example. There, the lower-energy states have all the valence protons and neutrons in the $f_{7/2}$ shell, with $j=7/2$ and $l=3$. In this case, the general expression [21] for the $g$-factor of a state of total angular momentum $I$ is

$$ g = \frac{g_{JP} + g_{JN}}{2} + \frac{g_{JP} - g_{JN}}{2} \times \sum_{J_P,J_N} |D^I(J_P, J_N)|^2 \frac{[J_P(J_P+1) - J_N(J_N+1)]}{I(I+1)}. $$

(6)

Here, $g_{JP}$ and $g_{JN}$ are the proton and neutron Schmidt $g$-factors in the $j$ shell under consideration. The second term is somewhat complicated [21], involving the factor $D^I(J_P, J_N)$ which is the probability that, in a state of total angular momentum $I$, the protons couple to an angular momentum $J_P$ and the neutrons to $J_N$.

However, for an $N=Z$ nucleus, due to charge symmetry, $D^I(J_P, J_N) = \pm D^I(J_N, J_P)$; this is true regardless of the isospin of the states. The sign does not matter as only the square of $D^I$ enters into eq. (6). These facts, along with the factor $[J_P(J_P+1) - J_N(J_N+1)]$, make the second term drop out for $N=Z$ nuclei. The result is the simple expression (7), where the $g_j$'s, as noted above, are the nucleon Schmidt $g$-factors for the $j$ shell under consideration:

$$ g = \left( \frac{g_{JP} + g_{JN}}{2} \right). $$

(7)

This expression is the same as eq. (2), but arrived at from a different viewpoint. However, it applies to all $N=Z$ nuclei, whereas eq. (2) was a special case of Blin-Stoyle's equation (9.2) for odd-odd nuclei only [8]. For any one $N=Z$ nucleus in the $jj$ case, all the states of that nucleus will have the same calculated $g$-factor given by eq. (7). This is certainly not true in the $LS$ limit.

Equation (7) is the de-Shalit and Talmi result (eq. (33.20) in [22]) for even-even $N=Z$ nuclei. However, due to lack of data, no comparison was made with experiment. Even in later textbooks (e.g. Bohr and Mottleson [4], Lawson [23], or Talmi [24]) there are no discussions of the magnetic moments of excited states of $N=Z$ nuclei. The 1993 Talmi book has a very brief discussion of mirror pairs. Bohr and Mottleson show data from measurements of $g$-factors of excited states for $N \neq Z$ nuclei. These have very large error bars. We are here examining isoscalar moments of excited states of $N=Z$ nuclei for which the data are much more sparse.

Equation (7) holds for all $N=Z$ nuclei in the single-$j$ shell approximation. It is true for states of any isospin $T$ and any total angular momentum $I$. Another general result for an $N=Z$ nucleus in the single-$j$ shell model is that, even with configuration mixing, the expectation value of the isovector magnetic moment in any state is zero. This is because this expectation value contains the isospin Clebsch-Gordon coefficient (1T00|T0) which vanishes for all integer $T$.  

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and (10) is always 1. Furthermore, in the limit of large \( l \) and \( j \), these two expressions converge to \( g_0 = 0.5 \), from above and from below, respectively. Even for \( l \) as small as 3, for \( f_{7/2} \) and \( f_{5/2} \), each of the two corresponding isoscalar \( g \)-factors given by (9) and (10) already differs from 0.5 by less than 11\%. The last column of table 2 provides, for each nucleus, the assumed single-\( j \) shell orbital and the corresponding isoscalar \( g \)-factor. For the heavier \( N = Z \) nuclei, which possibly could be accessed through radioactive beams, higher values of \( l \) may play a relatively important role.

Expressions (8), (9), and (10) apply to odd-odd as well as even-even nuclei.

We plot in fig. 1 the results from table 2 and expressions (9) and (10) for the odd-odd \( N = Z \) nuclei. Figure 1 indicates that there is overall good agreement between the predicted values from the single-\( j \) shell model expressions (9) and (10) (dashed lines), the measured values (solid symbols), and the calculated values obtained in large-scale shell model calculations (open symbols). Only two nuclei, \(^{14}\text{N}\) (with \( p_{1/2} \)) and \(^{38}\text{K}\) (with \( d_{3/2} \)), correspond to the \( j = l - \frac{1}{2} \) case and appear in the lower part of the figure.

For \(^6\text{Li}\) the experimental result is well accounted for by the more extensive calculations (table 2) but not by the simple expression (9). For the \( 1^+ \) state of \(^{10}\text{B}\) the situation is reversed. The measured \( g \)-factor values of the \( 1^+ \) states in \(^{58}\text{Cu}\) and \(^{22}\text{Na}\) are closer to 0.5 than eq. (9) predicts, for \( p_{3/2} \) and \( d_{5/2} \), respectively. The former has a large experimental uncertainty while the latter agrees with the large-scale shell model results.

Figure 1 indicates, for the odd-odd \( N = Z \) nuclei, the usefulness of their analysis in terms of a single-\( j \) shell model. Both fig. 1 and table 2 indicate that, aside from the \( 1^+ \) states of \(^6\text{Li}\) and \(^{10}\text{B}\), the single-\( j \) and the large-scale shell models’ results are surprisingly close. The approach of the data to \( g_0 = 0.5 \) as \( l \) increases is clear and this enables us to predict results for heavier \( N = Z \) odd-odd nuclei.

We have shown that in many cases for \( N = Z \) nuclei, the single-\( j \) shell model and the collective model both predict similar values of \( g \approx 0.5 \) for the isoscalar \( g \)-factors. One therefore cannot conclude details about the nuclear structure from the proximity to 0.5 of the experimental isoscalar \( g \)-factor results for \( N = Z \) nuclei. Such is usually not the case for isovector \( g \)-factors. The isoscalar \( g \)-factors are also of special interest because of their renormalization properties [2, 6, 25]. It was shown by Mavromatis et al. [25] that for a closed \( LS \) shell plus or minus one nucleon, there are no corrections to the magnetic moment in first-order perturbation theory and that only the tensor interaction contributes to the renormalization of the isoscalar \( g \)-factors in second-order perturbation theory. Core-polarization corrections to the Schmidt values are larger for the isovector case. Furthermore, the renormalization of \( g_1 \) due to one-pion exchange affects only the isovector orbital term but not the isoscalar one.

In closing, we have shown that the isoscalar \( g_0 \approx 0.5 \) results apply not only to even-even but also to odd-odd \( N = Z \) nuclei. For some light even-even \( N = Z \) nuclei, this is due to being close to the \( LS \) limit. But even in the \( LS \) limit one does not get a value of 0.5 for odd-odd \( N = Z \) nuclei and furthermore the \( LS \) limit is not valid for large \( A \). For large \( A \), the \( jj \) limit gives nearly the same answer of \( g_0 \approx 0.5 \). The combination of the \( LS \) and \( jj \) limits thus accounts for this result throughout the periodic table, supplementing the \( g = \frac{\pi}{2} \) perspective provided by the collective model.

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