Influence of Imperfection on Buckling Resistance of Perforated Thin-Walled Bar with Very Low Slenderness – FEM Analysis and Comparison with Experimental Results

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Abstract. In this paper, the results of modelling of compression test for the perforated thin-walled bars of very low slenderness are shown and confronted with the experimental results. Three sets of imperfections as well as a model without imperfections were analysed and compared. Application of initial imperfections improved the results in most cases, but not always in the same extent, there were also some cases, where they worsened the compliance with experiment instead.

1. Introduction
Using thin-walled bars allows to greatly reduce the weight of the steel structure, but on the other hand can be difficult and may increase the risk in case of a mistake in design or realisation because of the high vulnerability for various buckling effects [1-3]. For this reason, thin-walled structures were used only when absolutely needed, and more traditional solutions dominated civil engineering for a long time. Because of the progress in computational techniques and rising understanding of thin-walled structures due to experience and research [4-12], they became much more popular in recent years and are even domineering in some kinds of structures (car, aircraft and civil engineering). This in turn rises the importance of researching them even more [13].

The paper presents results of modelling of certain perforated thin-walled bars of very low slenderness compressed through assumption of the displacement type boundary conditions (the compressing force is interpreted as the reaction force). Results obtained from FEM modelling [14] are confronted with experiment results. This is a continuation of work presented in articles [15-20]. In the previous articles, numerical estimation of critical forces, understood as maximum forces acquired during the test, was correct for elements shorter or equal to 20 cm. For longer elements (tested up to 50 cm length) calculations gave higher values than observed in reality. One of the possible reasons for that, considered in aforementioned papers, was using idealized model without any imperfections. Therefore, this time, imperfections [21-26] according to [27] were taken into account [28-30].

2. Description of elements, experimental tests and numerical analysis
Tested elements are short fragments (5 to 50 cm length, what corresponds to slenderness from 1 to 11) of storage system columns. The columns are made from steel and their cross-section is similar to the
rectangular letter Ω without the horizontal lines on the bottom part of the letter. The height of this letter is about 80 mm and its width is 120 mm. The Young modulus of the steel is 230 GPa and the plasticity limit is about 450 MPa. There are four rows of perforations, some of them circular and some trapezoidal. For the detailed description and pictures see [15-20].

The bars were tested using fixed compressing plates with 5 mm depth fixtures in universal testing machine. The test was displacement-controlled and for collecting results both built-in sensors and external digital image correlation system [31] were used. After unloading, some linear dimensions of the elements were measured. For details, see [18-20].

The finite element method shell model [32] from article [20] was used. Aside from the procedure described in the aforementioned paper, three more procedures, considering imperfections, were created. The assumed imperfections are presented in figure 1. From now on, the first kind of imperfection in the figure will be referenced as “bow imperfection” and the second kind as “sway imperfection”.

![Figure 1. Imperfections according to standard [27]](image)

The sway imperfection was realised with adding an extra step before compressing. In that step the end of the bar was moved in the direction perpendicular to the axis of the bar. If we name the cross-section axes x and y, and the bar axis z, in that step the sway was realized in both xz and yz planes, which obviously results in summary sway being larger than on the picture. Two versions of that procedure were implemented, the difference between them lying in the direction of displacement: in the first version – in the direction of closed side of the cross-section, and in the second one – in the direction of open side of the cross-section. From now on the first version will be called “1a” and the second “1b”.

The bow imperfection was implemented differently, in a two-step procedure. First, the buckling procedure was used to find possible buckling modes of the model. Second, the buckling mode most similar to experimental results was used as pre-defined state of the model, with the scaling factor chosen according to figure 1. Therefore, the so-called “bow imperfection” is in reality the sum of global bow imperfection and local imperfections. From now on, results obtained using this procedure will be marked with number 2.

The third procedure is a sum of the two procedures above. Firstly, imperfections are imported. Secondly, swaying is realised. Thirdly, the compressing is conducted. Since the sway procedure exists in two versions, as a result this one will also have versions a and b.

The summary of these three procedures is written in the table 1. From now on, symbols written in the first column will be used to identify the results obtained through this procedures on all of the graphs.
Table 1. Procedures used in FEM analysis

| No. | Name | Step 0                      | Step 1                                      | Step 2 |
|-----|------|-----------------------------|---------------------------------------------|--------|
| A0  | simple | boundary conditions         | compression                                 | x      |
| A1a | sway a | boundary conditions         | sway to the closed side                     | compression |
| A1b | sway b | boundary conditions         | sway to the open side                       | compression |
| A2  | bow   | boundary conditions, importing imperfection | compression | x |
| A3a | sum a | boundary conditions, importing imperfection | sway to the closed side | compression |
| A3b | sum b | boundary conditions, importing imperfection | sway to the open side | compression |

3. Results and discussions

Comparison of numerical and experimental equilibrium paths for elements of different lengths is presented in figures 2-4. In the left column average strains were calculated as the displacements obtained from the machine divided by initial lengths, and in the right column they were acquired from digital image correlation system.

Figure 2. Equilibrium paths: E – experimental results, A – numerical results according to table 1, (50) – initial length 50 mm, (100) – initial length 100 mm, (a) – experimental results obtained directly from the machine, (b) – experimental results obtained with the use of digital image correlation system.
Figure 3. Equilibrium paths: E – experimental results, A – numerical results according to table 1, 
(150) – initial length 150 mm, (200) – initial length 200 mm, (250) – initial length 250 mm, 
(300) – initial length 300 mm, (a) – experimental results obtained directly from the machine, 
(b) – experimental results obtained with the use of the digital image correlation system
As it can be seen in the left column of the figures above, calculating the average strains from the displacements obtained from the machine is resulting in big differences between numerical and experimental results even in the elasticity range. Replacing them with strains acquired from digital image correlation system gives much better results, as can be seen in the right column, although this method also generates some errors, especially in the post-critical stage for the longest elements. The reason for that is, the boundary conditions are not ideal fixed boundary conditions in reality, and act as intended only for very short bars. In case of the longer elements, the ends of the bars move out of the fixtures, causing i.a. slowing, stopping, or even temporary inversion of the direction of movement of the points on the observed surface of the elements.

From figures 2, 3 and 4 it can be seen, that elastic behaviour is generally correctly predicted. In longer elements, using bow imperfections gives the best estimation of the stiffness, other procedures give slightly higher estimations. In shorter elements, all procedures give similar, correct results. Modelling sway imperfections resulted in improving results of critical behaviour predictions in all analysed cases, adding bow imperfection improves results in longer elements, but worsens them for the shorter ones. Plastic behaviour estimation is still different from experiment, there is quite good quality compliance, but force values are incorrect – adding imperfections did not change much in that aspect. The reasons for that lie both in the experiment (boundary conditions, difficulties in measuring average strains) and calculations (better modelling of plasticity is needed). As shown in figure 5, using different solver procedure (Riks instead of Newton-Raphson) or allowing free rotation of the end nodes (linear displacements are still controlled) didn’t result in any noticeable changes. Since these modifications of numerical procedures did not result with many changes in results, they are therefore not included in figures 2-4.
Experimental and numerical results of critical force in the function of slenderness are presented in figure 6. It can be seen, that the relation for shorter bars (up to 20 cm length) is correctly predicted even without modelling imperfections, using sway imperfections doesn’t change much in that aspect, but adding bow imperfections or using both kinds in the same time, instead worsens the compliance, resulting in too small critical forces. However, it should be noticed, that these results are not completely wrong – some small number of experimental tests also resulted in noticeably smaller critical forces than the average. For longer bars, adding imperfections always improves numerical results, the best compliance can be obtained by using both sway and bow imperfections at the same time.

4. Conclusions
Methods presented in this paper and in previous works [19,20] correctly predict the elastic behaviour of tested elements and buckling modes. Adding imperfections to numerical analysis solved the problems...
encountered in [19,20], especially values of critical forces are now correctly predicted also for longer elements. For shorter ones, adding too many imperfections in the same time often leads to too small critical forces. The possible reason for that is, the imperfection values proposed in [27] are too big for such short elements that in reality were closer to ideal shape than it was assumed. As for plastic behaviour, although displacements, strains and stresses show good quality compliance with experimental results [18-20], the quantity compliance is not so good. To improve it, better modelling of existing boundary conditions, as well as conducting experiments with more unambiguous boundary conditions, are needed.

Comparing results presented in the right and left column in the figures 2-4 confirm that DICS or extensometers should be used for obtaining strains in the elastic phase of experimental tests. On the other hand, using displacement data from the testing machine gives better picture of global plastic behaviour of the element. For data in certain points or areas of course DICS or extensometers are better solution.

Further research is planned in the future: FEM modelling of imperfections assumed according to different buckling modes, improving boundary conditions in the FEM models, carrying out calculations according to analytical formulas from [27], conducting experiments and modelling for hinged boundary conditions, improving calculations of average strains from DICS, trying different models of plastic behaviour of the material and better determination of parameters for them.

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