Numerical Investigation of Pressure Fluctuation and Cavitation inside a Francis Turbine Draft Tube with Air Admission through a Fin

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Abstract. The stabilizer installation and air admission have been proved to be two effective ways to alleviate the pressure fluctuation in the Francis turbine draft tube. In this research, a combined method of these two ways, i.e. air admission through a fin (one type of the stabilizer) is applied, and its effectiveness, especially air admission’s effects, on pressure fluctuation suppression has been investigated through numerical calculation. Two cases of different air amount (Q*=0.01 and 0.02) are investigated. Results show that cavitation inside the draft tube is suppressed and the size of vortex rope is also slightly decreased after air is admitted. As a result, pressure fluctuation inside the draft tube is generally alleviated compared with no air admission case. The maximum amplitude of pressure fluctuation decreases by 9.2% with 0.01 air admission, and by 26.3% with 0.02 air admission. In addition, the position where the maximum amplitude of pressure fluctuation occurs also moves downstream, which should be due to the more violent interaction between the vortex rope in the main stream and a small vortex rope attached to the fin. Consequently, a slight increase of pressure fluctuation downstream the fin can be observed. Although air admission through the fin can suppress pressure fluctuation in the draft tube, it would also lead to turbine efficiency decrease. Therefore, the application of both fin and air admission should be carefully considered to achieve better performance for Francis turbines.

1 Introduction

Hydropower is widely used to supply electric energy for the power grid and plays an important role in power-frequency control, which means the hydro turbine, the core component of hydropower, usually works in a wide range of operating conditions. Through experimental data from many prototype and
model hydro turbines, Jacob [1, 2] pointed out that when working at off-design conditions, especially at part-load operation, significant pressure fluctuations appeared in the draft tube of a Francis turbine. Such pressure fluctuation, of which the frequency is relatively low, can harm stable operation of a Francis turbine and even cause its structural troubles.

Causes of pressure fluctuation in the draft tube have been under discussion since this phenomenon was explained by Rheingans [3]. The basic understanding on this problem is that pressure fluctuation at part-load operation is closely related to the rotation of the corkscrew-like flow structure (vortex rope). The US Bureau of Reclamation [4] conducted experimental research on the swirling flow in the draft tube and focused on the behavior of vortex rope at different swirl ratios. When the swirl ratio reached a value, the asymmetrical flow field led to the appearance of corkscrew-like vortex rope in the draft tube [4, 5]. Rotation of the eccentric vortex rope makes the draft tube wall experience pressure vibration in its one rotating cycle, which is observed as pressure fluctuation. Based on abundant previous studies, it is found that in modern Francis turbines, pressure fluctuation at part-load operation typically prevails over a range of the turbine relative flow rate (with respect to the design flow rate) between approximately 0.5 and 0.85. And the relative frequency ($f/f_n$) of the main component caused by vortex rope ration is between 0.2 and 0.4 [1, 6, 7]. In addition, under cavitation cases, the appearance of cavity inside the vortex rope would enlarge the scale of vortex rope and also magnify the pressure fluctuation amplitude [5]. A small increase in the procession frequency of the vortex rope can also be observed [8].

There has been a lot of methods developed to suppress pressure fluctuation in the draft tube at part-load operation. Air admission and the installation of stabilizer are two of them. Nishi has done a lot of work on the effect of fin installation on pressure fluctuation suppression, including different types of fin [9] and also fin’s number and location [10]. Fin can effectively reduce the eccentric distance of vortex rope and thus suppress pressure fluctuation. But it should be also noted that too many fins may not be a good choice. Air admission through the center hole of turbine shaft is also an effective way [11, 12, 13, 14]. Air admission can decrease pressure gradient on the horizontal section of draft tube, and sufficiently reduce cavity volume under cavitation cases and the eccentric distance of vortex rope. In addition, Nishi [15] also experimentally investigated the effect of air admission through small ports on the fin under cases that fin can’t alleviate pressure fluctuation. Results showed that a small amount of air can be sufficient.

In this research, a fin derived from Nishi’s work [9, 10] is installed on the draft tube wall and its top is cut to form a flat surface for air admission. The effects of the combined method of fin installation and air admission on pressure fluctuation are investigated through numerical calculation. As a successive study of our previous work, the present paper focuses on the discussion for the influence of air admission from the fin on the internal flow and pressure fluctuation in draft tube. Besides, a cavitation case is chosen as the working condition. Therefore, the effect of air admission on the cavitating flow will also be discussed.

2 Turbine Geometry and Simulation Methods

2.1 Geometry of model Francis turbine and fin
The whole flow passage of the test Francis turbine is shown in figure 1, including the volute casing, stay vane, guide vane, runner and draft tube. Geometrical parameters of the turbine are listed in table 1. The fin used in this paper was shaped from the fin of Type-B investigated by Nishi et al. [8].

| Parameter                        | Value  |
|----------------------------------|--------|
| Runner inlet diameter $D_r$ (mm) | 420    |
| Runner blade number $Z_b$        | 17     |
| Guide vane number                | 24     |
| Stay vane number                 | 23     |
| Dimensionless guide vane height $b_0 = b / D_r$ | 0.183 |

![Turbine model](image1)

![Fin geometry](image2)

**Figure 1.** Computational domain and fin geometry.

### 2.2 Simulation setup

The simulation method is based on homogeneous assumption, where the mixture of water and vapor is the flowing fluid with spatially and timely dependent density and viscosity. The basic governing equations for the flow are expressed as follows:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0 \tag{1}
\]

\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \left( \mu + \mu_t \right) \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + f_i \tag{2}
\]

where $u$ and $p$ represent velocity and pressure, respectively. $\mu$ and $\mu_t$ are the molecular and turbulent viscosity, respectively. $f_i$ is the body force, i.e. the gravity.

For closure of the turbulence in governing equations, the standard $k-\omega$ SST turbulence model is adopted to calculate $\mu_t$, which is computed from the turbulent kinetic energy and turbulent frequency:

\[
\mu_t = \rho \frac{k}{\omega} \tag{3}
\]

For calculation of cavitation, the Zwart cavitation model [16] based on Rayleigh-Plesset
The equation is applied. The source terms due to vaporization and condensation in the cavitation model are represented as equation (4) and (5).

\[
\dot{m}^+ = C_e \frac{3\rho_v(1 - \alpha_v)\alpha_{nuc}}{R_b} \sqrt[3]{\frac{2\max(p_v - p, 0)}{\rho_l}} \tag{4}
\]

\[
\dot{m}^- = C_c \frac{3\rho_v\alpha_v}{R_b} \sqrt[3]{\frac{2\max(p - p_v, 0)}{\rho_l}} \tag{5}
\]

where the subscripts v and l mean vapor and liquid respectively. \(C_e\) and \(C_c\) denote two empirical coefficients, whose values are 50 and 0.01 respectively. \(\alpha_{nuc}\) is the volume fraction of nucleation sites and the value of 5e-4 is used. \(\alpha_v\) is the vapor volume fraction.

Numerical simulations have been carried out using ANSYS CFX. Total pressure is assigned at the inlet of volute casing, and static pressure is assigned at the draft tube outlet. All solid walls have the nonslip boundary condition. As for computational domains, the runner is set to rotate at the speed of \(n\) and the other domains are stationary. The runner rotational frequency is \(f_n = n/60 = 15.12\) Hz, which corresponds to one period of runner rotation i.e. \(T_n\). At the beginning, the time step is set as \(T_n/60\) (six degrees per time step). After several runner rotations, a finer time step of \(T_n/120\) (three degrees per time step) is set for the subsequent calculation. It is noted that the structured mesh is generated for the whole flow passage. After verification of the grid independence, the final mesh has 3568347 nodes.

Guide vane opening of \(\alpha_0 = 12.175\) mm is selected as a typical part-load operating point, where the unsteady flow in the test turbine admitted with different amount of air is analyzed for the cavitation condition. Cavitation number \(\sigma\) given by equation (6) is used to represent the condition and \(Q^*\) represents the relative amount of air admitted into the draft tube. Those operating parameters are listed in table 2.

\[
\sigma = \frac{p_{out} - p_v - \rho g H_s}{\rho g H} \tag{6}
\]

\[
Q^* = \frac{Q_a}{Q_l} \tag{7}
\]

where \(p_{out}\) and \(p_v\) denote the static pressure at the outlet of draft tube and the saturated vapor pressure respectively. \(H_s\) and \(H\) are the permissible suction head and head of the turbine respectively. \(\rho\) is water density and \(g\) is the gravitational acceleration. \(Q_a\) and \(Q_l\) are the flow rate of air and working fluid respectively.

| Parameter                  | Value  |
|----------------------------|--------|
| Guide vane opening \(\alpha_0\) (mm) | 12.175 |
| Head \(H\) (m)              | 30.07  |
| Rotational speed \(n\) (r/min) | 907.4  |
| Flow rate \(q\) (L/s)       | 266.1  |
| \(\sigma\)                  | 0.04   |
| \(Q^*\)                     | 0.01, 0.02 |

2.3 Validation of simulation results

To validate the simulation methods used in this paper and make sure simulation results are acceptable
for further analysis, comparison of hydraulic performance and pressure fluctuation component $f_1$ (caused by vortex rope rotation) are made between the experiment and simulation. The experiment is conducted under non-cavitation case ($\sigma = 0.04$) without fin or air admission, as shown in table 3. There is small difference of 3.6% in the efficiency and the frequency of $f_1$ can also be predicted accurately. The difference in the amplitude of $f_1$ is relatively high, with a relative error of 5.1%. Considering that the working condition is part load operation which is usually not predicted very accurately, it is not irrational to treat the simulation as acceptable.

Table 3. Validation of simulation results ($\sigma = 0.04$, without fin or air admission)

| Parameter                  | Experimental value | Numerical value | Relative error |
|----------------------------|--------------------|-----------------|----------------|
| Efficiency (%)             | 91.05              | 94.37           | 3.6%           |
| Frequency of $f_1$ ($f_1 / f_3$) | 0.204              | 0.200           | 2.0%           |
| Normalized amplitude$^2$ of $f_1$ | 0.59               | 0.62            | 5.1%           |

$^1 f_1$: The monitor point is located in the conical part, 0.3$D_2$ away from the draft tube inlet. $D_2$ is the draft tube inlet diameter, $D_2 = 0.359$ mm.

$^2$ Normalized amplitude: Pressure fluctuation amplitude normalized by $\rho g H$.

3 Results and Discussion

3.1 Comparison of hydraulic performance
Taking methods to alleviate pressure fluctuation in the draft tube is the focus of our research. But apart from this, the effect of these methods on hydraulic performance should also be taken into consideration, as it is connected with the balance between costs and benefits. Table 4 shows the efficiency of the Francis turbine under different cases, i.e. adopting different methods for pressure fluctuation suppression.

Table 4. Comparison of turbine efficiency under different cases.

| $Q^*$ | Without fin | With a fin | With a fin | With a fin |
|-------|-------------|------------|------------|------------|
| $\eta$ | 90.95%      | 90.97%     | 90.57%     | 89.85%     |

Fin’s installation mainly changes local flow and usually has no obvious impact on hydraulic performance. A suitable selection of fin’s number and location may even increase the performance of the draft tube, including the pressure recovery and total pressure loss [17]. This coincides with the efficiency comparison between no fin and with fin cases in table 4. Unfortunately, there is a small drop in the turbine efficiency (about 1% under the air admission case of $Q^*$=0.02) after air is admitted from the fin. Thus, the combination of fin installation and air admission should be further optimized to obtain better performance, e.g. the suitable air amount control to satisfy pressure fluctuation suppression without efficiency drop for Francis turbine.

3.2 Pressure fluctuation in the draft tube
In order to conduct comparison and consideration of internal flow characteristics and pressure
fluctuation, five monitoring points and four reference cross-sections are set in the draft tube. They are marked with “P1”, “P2”, “P3”, “P4”, “P5” and “SV”, “SC” respectively, whose locations are shown in figure 2. It is noted that “P2” and “P3” are located at the head and the tail of the fin respectively.

**Figure 2.** Reference sections and monitoring points.

Figure 3 shows pressure fluctuation at different monitoring points from FFT analysis, for which the pressure data cover a time range of 35 runner rotating cycles. It is clear that there is only one main component of pressure fluctuation $f_1$ (accompanied by its harmonics) and its frequency is about $0.23f_n$. The frequency and amplitude of $f_1$ at different points are listed in table 5.
Table 5. Frequency and amplitude of $f_1$ at different monitoring points.

|       | P1  | P2  | P3  | P4  | P5  |
|-------|-----|-----|-----|-----|-----|
| $Q^*=0$ |     |     |     |     |     |
| Frequency ($f/f_n$) | 0.227 | 0.226 | 0.227 | 0.227 | 0.227 |
| Normalized Amplitude (%) | 0.633 | 0.567 | 0.772 | 0.655 | 0.302 |
| $Q^*=0.01$ |     |     |     |     |     |
| Frequency ($f/f_n$) | 0.227 | 0.227 | 0.228 | 0.228 | 0.228 |
| Normalized Amplitude (%) | 0.551 | 0.397 | 1.166 | 0.675 | 0.322 |
| $Q^*=0.02$ |     |     |     |     |     |
| Frequency ($f/f_n$) | 0.228 | 0.228 | 0.229 | 0.228 | 0.229 |
| Normalized Amplitude (%) | 0.498 | 0.266 | 0.627 | 0.763 | 0.367 |

In Table 5, air admission scarcely changes the frequency of $f_1$, or properly speaking, there is a slight increase in frequency (about 0.5%~1%). In contrast, air admission has changed the amplitude a lot. From P1 to P2, the amplitude of $f_1$ significantly decreases with air admission and for present research, in the range of air amount between 0 and 0.02, the larger the air amount is, the better the effect gets. But on P5 or further downstream, the amplitude of $f_1$ becomes larger. This may be connected with the more complex flow with air admission, which will be discussed in section 3.3. On P3 and P4, there is no obvious regularity in amplitude change. So for further analysis, the variation of amplitude of $f_1$ on Z direction is plotted in figure 4, where $D_z$ is the distance between the monitoring point and the draft tube inlet on Z direction.

![Figure 4. Variation of amplitude of $f_1$ on Z direction.](image)

From figure 4, it is obvious that the maximum value of amplitude decreases after air is admitted, which implies that roughly pressure fluctuation in the draft tube is suppressed. Here the decrease in the maximum amplitude of $f_1$ is considered as the measure of air-admission’s effect. In this sense, with 0.01 air admitted, pressure fluctuation amplitude decreases by 9.2%, and with 0.02 air admitted, it is 26.3%. The trend in the amplitude over $D_z$ is similar with different air amount. If some small turning points in the curves are neglected, it is easily found that with $D_z/D_2$ increasing, the amplitude first increases and then decreases. In addition, the position where maximum amplitude occurs moves downstream due to air admission. As a result, from P1 to P2, the amplitude is suppressed while in the elbow, it is the opposite.
3.3 Internal flow in draft tube

As mentioned above, pressure fluctuation is mainly caused by rotation of the vortex rope in the draft tube, which derives from the swirling flow inside. Therefore, to understand the reason for air admission to alleviate pressure fluctuation, it is necessary to give an insight into the internal flow in the draft tube. Figure 5 shows the vortex rope and air in the draft tube, of which the iso-surface is drawn based on $Q$-criterion (1500 $1/s^2$) and air volume fraction (0.1) respectively.

In figure 5, there are two vortex ropes in the draft tube under all conditions. The rotation of the bigger one in mainstream is the main reason for component $f_1$. The smaller one is caused by the fin and attached to it. It is more stable and thus decreases pressure vibration caused by rotation of the mainstream vortex rope. This means that the attached vortex rope plays the role to stabilize the pressure around the fin. The two vortexes interact with each other and after air is admitted, such interaction becomes more violent, accompanied by many small, broken pieces of vortex rope. Although there is a lack of direct evidence, it can be inferred that air will raise the smaller vortex rope up and weaken its stabilizing effect. Therefore, the position where most severe pressure fluctuation occurs under cases with air admission is pushed downstream compared with that without air admission. More clues can also be found in figure 6, which shows pressure distribution on SV and SC. In addition, after air is admitted, the size of vortex rope slightly decreases and somewhat accounts for pressure fluctuation suppression. This will be discussed next.

![Figure 5. Vortex rope and air in the draft tube (blue: the vortex rope; gray: air).](image)

Pressure distribution in the draft tube is also important for understanding pressure fluctuation in the draft tube. In figure 6, after air is admitted, the low pressure area decreases both in the vertical section and horizontal section. The direct consequence is that pressure fluctuation in the draft tube will be suppressed in general. In addition, attention should be paid to a small region, which is marked with red circles in figure 6. After air is admitted, relatively high pressure in this region disappears and low pressure takes its place. This means that the vortex rope derived from the fin loses its stabilizing effect there. As a result, the amplitude of pressure fluctuation downstream the fin increases, as shown in figure 4.
Figure 6. Pressure distribution on SV and SC sections.

Air admission also affects cavitation inside the draft tube. Figure 7 shows the vapor volume oscillation in the draft tube. As a direct result of the decrease of low pressure area, air admission reduces vapor volume a lot in the draft tube, which is also a reason for pressure fluctuation suppression.

Figure 7. Vapor volume oscillation in the draft tube.

3.4 Analysis based on vorticity transport equation
The swirling flow at the draft tube inlet is the cause of the vortex rope, which is closely linked to the evolution of the vortex in the draft tube. The swirling flow in the draft tube is analyzed based on vorticity transport equation, which is expressed as equation (8).
\[
\frac{D\omega}{Dt} = (\omega \cdot \nabla)V - \omega(\nabla \cdot V) + \frac{\nabla \rho_m \times \nabla p}{\rho_m^2} + \nu(\nabla^2 \omega)
\]  
(8)

where \(\omega\) is vorticity.

There are four terms on the right side of equation (8), and they express the stretching term, expansion term, baroclinic term, which caused by density gradient and pressure gradient, and viscosity term respectively. The combined effects of the four terms lead to the evolution of vortex.

Compared with the other three terms, the effect of viscosity is very small in the draft tube [14]. Figure 8 demonstrates the distribution of the stretching term, expansion term and baroclinic term on the section of SC.

Figure 8 shows that on SC, air’s effects on vorticity distribution and transportation are limited to the region close to the fin. In the case without air admission, there are mainly two high value regions of vorticity, which correspond to the two vortex ropes in the draft tube respectively. Air doesn’t change vorticity distribution in mainstream but leads to another high value region of vorticity close to the fin, whose direction is opposite to the previous one. As the air amount increases, the additional high vorticity area expands. This is the result of air interacting with water flow. Air enters the draft tube and forms multiphase flow with water, whose density varies with time and space. Meanwhile, air also changes water flow and forms more complex vortex rope interaction, as shown in figure 5. Thus velocity gradient and pressure gradient also become much larger than that without air. As a result, as shown in figure 8, the stretching term and baroclinic term under air admission cases are much larger than without-air case. High value regions of the stretching term and baroclinic term are located close to the fin, where air exists, and are mainly negative, causing the appearance of negative vorticity region close to the wall. This also somewhat confirms air’s breaking and rolling-up effects on the vortex rope.

**Figure 8.** Distributions of the stretching term, expansion term and baroclinic term on SC.

4 Conclusions
Based on the present study, several concluding remarks can be drawn as follows:

1. Air admission through a fin can decrease the amplitude of pressure fluctuations upstream the fin, but cannot decrease pressure oscillation downstream the fin. Actually, air admission through the fin would make the position with the maximum amplitude of pressure fluctuation moves downstream.

2. Air admission decreases the low pressure area in the draft tube and suppresses pressure fluctuation. It is noted that the maximum amplitude of pressure fluctuation in the draft tube decreases by 9.2% with 0.01 air admission, and by 26.3% with 0.02 air admission.

3. Cavitation inside the draft tube is suppressed by air admission, which also contributes to pressure fluctuation alleviation.

Acknowledgements
This work was financially supported by National Key R&D Program of China (2018YFB0606101), the National Natural Science Foundation of China (Grant Nos 91852103 and 51776102), Beijing Natural Science Foundation (3182014), a grant from the Institute for Guo Qiang, Tsinghua University (Project No. 2019GQG1019), and the Tsinghua National Laboratory for Information Science and Technology.

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