The influence of the emitting bodies surface boundary conditions on particle streams in their vicinity

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Abstract. The problem of the influence of boundary conditions on the surface of emitting bodies on particle flows in their vicinity is studied in this paper. The solution to this problem is actual and relevant both for studying the behavior of electron plasma in a closed space (diodes and multi-electrode devices) and in open space (studying electron flows in the vicinity of natural and artificial celestial bodies). This research is theoretical. The emitting particles are electrons. Moreover, it is assumed that the emission of electrons occurs in vacuum. The electric field potential on the surface of the emitting bodies is considered constant (the first boundary condition). The second condition is determined by setting the field strength on the body surface. Four types of boundary conditions are considered: 1) fully screened electric field; 2) mono-speed flow from the surface; 3) monoenergetic isotropic flow; 4) the starting particles are distributed according to the Maxwell’s law. In the second of the above cases, the desired dependence is found analytically. In the third and fourth cases, the dependences of the electric potential strength are found numerically using the kinetic approach. The calculation results are presented in the graphs.

1. Introduction

To determine the electrophysical properties of materials and, above all, their emission properties, it is important to know the potentials of self-consistent electric fields, the values of which are largely determined by environmental features and boundary conditions set on surfaces of emitting bodies in rarefied plasma. The solutions of this kind problems are widely used both in theoretical works in the field of radio electronics applied research, radio engineering, in space research, in the study of the natural and technogenic origin microparticles evolution of microparticles in the conditions of near and far space, and also in the development of new nanotechnologie [1–5].

In this work, we study the effect of boundary conditions for electric and gas-dynamic quantities on the electron flux density both near and large distances from the surface of bodies. The study was carried out on the basis of solutions obtained for the potential of a self-consistent electric field for a one-dimensional rectilinear motion of particles forming the flow, at different boundary conditions on the surface of emitting bodies. The influence of four types of such conditions is analyzed.
2. Calculation of the electric current density dependence on the electric field potential and on the conditions of particle motion.

2.1. The first type of boundary conditions

For fully screening electric field the desired dependence is given by the Child-Langmuir formula (law of "degree 3/2")

\[ j = \frac{1}{9\pi} \sqrt{\frac{2e}{m_e} \frac{\varphi_a^{3/2}}{d^2}}, \]  

where \( \varphi_a \) is the potential of the electric field at some equipotential surface remote from the surface of the body at a distance \( d \).

Formula (1) was obtained under the assumption that on the surface of the emitting body the potential of the electric field, the strength of the electric field are equal to zero, as well as the velocities of particles starting from the surface.

In this formulation, the electric current density at a distance \( d \) from the emitting surface is proportional to the field potential at this point, raised to the power 3/2. Describing well enough the dependence of the electric current density \( j(\varphi_a) \) for small \( \varphi_a \) this formula is not applicable for large \( \varphi_a \), in view of the unlimited growth of \( j \).

2.2. The second type of boundary conditions

In this case, plotting the density dependence electric current from the specified boundary conditions is constructed using so-called quasisolutions, first applied to emission problems in the work [6]. In this case, it was assumed that the potential of the electric fields on the surface \( \varphi|_S = \varphi_0 = \text{const} \), the electric field strength near the surfaces are nonzero, the escape velocity of all emitting particles is the same.

In this formulation, the dependence \( j(E_0, x) \) (\( E_0 \) is the electric field strength on the body surface) was studied at distances from the body surface greater than several Debye radii.

Note that the quantity \( E_0 \) and the related dimensionless parameter \( R_E = \sqrt{E_0^2/8\pi n_0 m_e v_0^2} \) determine the flow regimes. For \( R_E \geq 0 \) and \( E_0 > 0 \) the self-consistent electric field will have an accelerating character. For \( 0 \leq R_E \leq 1 \) and \( E_0 < 0 \), the field will be decelerating, but the return particles in it will be absent.

For \( R_E \geq \sqrt{2} \) and \( E_0 < 0 \), there is a return the flow of charged particles reflected by the field. For \( 1 < R_E < \sqrt{2} \) and \( E_0 < 0 \), there are no “classical” (that is, continuous and non-discontinuous derivatives) solutions to the problem of determining the potential of a self-consistent electric field.

However, a group of so-called ”quasi-solutions” can be introduced. In this case, quasisolutions are such dependencies \( \varphi(x), \rho(x), v(x) \), which, firstly, satisfy the boundary conditions on the emitting surface, and, secondly, they are solutions of the original equations of the posed problem almost everywhere. In our case, ”almost everywhere” will mean: everywhere, except for one point. The construction of quasisolutions can be carried out by introducing a new parameter \( p \), which determines the probability of passing the emitted particles through a surface, which in radio electronics is called a surface virtual cathode. Although in theoretical works the concept of ”quasi-solutions” is not was introduced, they correspond to certain physical states in the field-flow space. One of these states is the so-called virtual real cathode state.

For \( x = d \), all the listed functions (in particular, \( \varphi(x) \)) will take certain values. The sought dependence \( j(\varphi) \) was found by comparing these values with the flux density of emitting particles. It can also be interpreted as the current-voltage characteristic of the diode under the specified conditions.

Dependency graphs \( j_1(\varphi_1) \) are shown in Figure 1 and Figures 2 for the values \( x_1 = 3 \) and \( x_1 = 5 \) respectively. The index ”1” indicates that the considered value is taken in dimensionless form. \( \varphi_1 = 2e\varphi/(m_e v_0^2) \), \( x_1 = 3\sqrt{2e^2 n_0\pi/(m_e v_0^2)} \).

Let’s note the following features of the found dependence.
First, the electric current density is limited in magnitude.
Secondly, the investigated dependence looks different for different $x_1$.
Thirdly, the dependency of $j_1$ on $\varphi_1$ has two branches. One branch - a straight line parallel to the abscissa axis, it corresponds to real solutions. The second branch is built using quasi-solutions.

The current-voltage characteristic has a branch point. Around this point, the solution equations for the self-consistent electric potential may turn out to be unstable.

2.3. The third type of boundary conditions
This case is based on the assumption that the electron emission from the body surface has a monoenergetic isotropic character (this emission corresponds to photoemission from the surface of the body under uniform irradiation by radiation of a certain wavelength of ultraviolet or X-ray range). In this case, the result based on the solution of the kinetic the Vlasov equation for a one-component cold electron gas was obtained [7]. Implicit dependence of $j(\varphi)$ in dimensionless form looks like this:

$$j_1 = j_{10} = 1, \text{ for } E_{10} > 0; \quad (2)$$

and

$$j_1 = \varphi_{1\text{min}}(\varphi_1), \text{ for } E_{10} < 0, \quad (3)$$

In dimensional form, formulas (2) and (3) can be written as follows:

$$j = j_0 = \frac{n_0 V}{2}, \text{ for } E_0 > 0; \quad (4)$$

$$j = \varphi_{\text{min}}(\varphi), \text{ for } E_0 < 0, \quad (5)$$

in these formulas: $V$ is the escape speed of emitting particles, $\varphi_{\text{min}}$ is minimum value for potential at a given tension value electric field on the surface of the body. Index "1", as before, indicates that the corresponding values are taken in dimensionless form.

The graph of dependence $j_1(\varphi_1)$ is shown in Figure 3.

Calculations and in this case were also carried out for $x_1 = 3$. $\varphi_1 = 2e\varphi/(m_e V^2)$,

$$x_1 = 3x\sqrt{2e^2n_0\pi/(m_e V^2)}.$$
2.4. The fourth type of boundary conditions

The main assumptions that were made in this case:

1. Gas-dynamic and electric fields are one-dimensional. There is no magnetic field. All quantities depend only on the distance from the point under consideration to the plane, defining the surface of the body. This distance is denoted by $x$.

2. The emitted particles are distributed according to the Maxwellian law. Spread over velocity is characterized by the parameter $T$, which can be taken as the temperature electron gas.

3. The electron gas flow is stationary.

4. Collisions between particles are not counted.

5. Charged particles returning to the surface of the body are absorbed by it.

6. The strength of the electric field on the surface of the body is considered as given magnitude.

The initial equations governing the behavior of the self-consistent electric field under the accepted assumptions are as follows:

$$\vec{v} \cdot \nabla f + \frac{e}{m_e} \vec{E} \cdot \nabla \vec{v} f = 0 ; \quad (6)$$

$$\Delta \varphi = -4 \pi \rho_e ; \quad (7)$$

$$\rho_e = e \int f \, dv ; \quad (8)$$

where (6) is the Vlasov equation for the electron velocity distribution function; (7) - Poisson’s equation for the potential of a self-consistent electric field; (8) - expression for determining the volumetric density of the electric current, obtained with the kinetic approach.

Let us introduce a Cartesian coordinate system with the $Ox$ axis directed along the outward normal to the surface of the body; the axes $Oy$ and $Oz$ are placed on the surface of the body in such a way that all axes are mutually orthogonal and form a right triplet. The distribution function on the body surface are defined as follows:

$$f|_S = n_0 \left( \frac{me}{2 \pi kT} \right)^{3/2} \exp \left( - \frac{me}{2kT} (v_x^2 + v_y^2 + v_z^2) \right) \text{ for } v_x \geq 0 ; \quad f|_S = 0 \text{ for } v_x < 0 . \quad (9)$$

The integrals of motion in this case have the form:

$$\frac{m_e v_x^2}{2} - e \varphi(x) = \frac{m_e v_{x0}^2}{2} - e \varphi(0) ; \quad v_y = v_{y0} ; \quad v_z = v_{z0} . \quad (10)$$

We will assume that $\varphi(0) = \varphi_0 = 0$. Then

$$f(x, \vec{v}) = n_0 \cdot \left( \frac{me}{2 \pi kT} \right)^{3/2} \exp \left( - \frac{me}{2kT} (v_x^2 + v_y^2 + v_z^2) + \frac{e\varphi}{kT} \right) . \quad (11)$$
It is known that the bulk density of the electric current is determined by the formula

$$\vec{j}_e = \int_{\Omega(x)} e\bar{v} f(x, \bar{v}) \, d\bar{v},$$  \hspace{1cm} (12)$$

where $\Omega(x)$ is a region in the velocity space that defines the set valid values $\bar{v}$. $\Omega(x)$ depends on the coordinate $x$. The projections of $\vec{j}_e$ on the coordinate axes $Oy$ and $Oz$ are equal to zero ($j_y = j_z = 0$). For the projection of $\vec{j}_e$ onto the $Ox$ axis, we have:

$$j_x = n_0 e \cdot \left( \frac{m_e}{2 \pi kT} \right)^{3/2} \int_{\Omega(x)} v_x \exp \left( - \frac{m_e}{2kT} \left( v_x^2 + v_y^2 + v_z^2 \right) + \frac{e\varphi}{kT} \right) \, d\bar{v}. \hspace{1cm} (13)$$

The integrand in (13), considered as the function $v_x, v_y, v_z$ is multiplicative with respect to these variables, therefore the result of integration over these variable is the product of the values of the integrals over these variables. Integrating, we get

$$j_x = e n_0 \cdot \left( \frac{m_e}{2 \pi kT} \right)^{1/2} \exp \left( \frac{e\varphi}{kT} \right) \int_{\Omega(x)} \exp \left( - \frac{m_e}{2kT} v_x^2 \right) \cdot \frac{kT}{m_e} \exp \left( \frac{e\varphi}{2kT} \right) =$$

$$= en_0 \left( \frac{kT}{2\pi m_e} \right)^{3/2} \int_{\Omega_1(x)} \exp(-s) \, ds,$$ \hspace{1cm} (14)

where $s = m_e v_x^2 / 2kT$.

The configuration of the $\Omega(x)$ region depends on the structure of the electron gas flow near the surface of the body.

As already mentioned, the flow regimes for a monoenergetic flow can be conditionally divided into the following: 1) flow with an accelerating electric field, 2) flow with a decelerating electric field, provided that there are no return particles outside, 3) purely return flows, and 4) so-called quasi-solutions.

If the boundary conditions are specified in the form (9), then the number of modes is reduced to two: 1) with an accelerating field and 2) a partially decelerating field with the presence of return particles.

Let’s continue the calculation of $j_x$ now, assuming that the field is accelerating. For an accelerating field, the dependence of the potential on the coordinate is monotonic increasing. As a consequence, the particle velocity in flow (this follows from formulas (10)). Minimum value of $v_x(x)$ will be equal to

$$v_{x,min}(x) = \left( v_{x0}^2 + \frac{2e}{m_e} \varphi(x) \right)^{1/2} \bigg|_{v_{x0}=0} = \left( \frac{2e}{m_e} \varphi(x) \right)^{1/2}. \hspace{1cm} (15)$$

It follows from formula (15) that the domain $\Omega(x)$ is a half-space lying to the right of the plane $v_x = \left( \frac{2e}{m_e} \varphi(x) \right)^{1/2}$ and therefore:

$$j_x = en_0 \left( \frac{kT}{2\pi m_e} \right)^{1/2} \exp \left( \frac{e\varphi}{kT} \right) \int_{2\varphi(x)/m_e}^{+\infty} \exp(-s) \, ds = \frac{en_0}{4} v_T = j_0,$$ \hspace{1cm} (16)

where $v_T = (8kT/\pi m)^{1/2}$ is the average or thermal velocity of the particles.

If the field near the emitting surface is decelerating, then the region $\Omega(x)$ is a half-space located (in the velocity space) to the right of the plane $v_x = 0$. In this case, integration leads to the following expression for $j_x$: 

\[ j_x = \exp\left(\frac{e\varphi}{kT}\right) j_0. \]  

(17)

As the x coordinate for the decelerating field increases, the potential \( \varphi \) decreases from zero to some \( \varphi_{\text{min}} \), the value of \( j_x \) will decrease (in modulus) from \( j_0 \) to some value:

\[ j_c = \exp\left(\frac{e\varphi_{\text{min}}}{kT}\right) j_0. \]  

(18)

where \( j_c \) is the saturation current for the given integral curve.

To calculate \( \varphi_{\text{min}} \), it is required to integrate the equation for the potential of the electric field, which requires knowing the dependence of the volumetric density of the electric charge on the coordinate. The latter is given by formula (8). Formula (8) in expanded form takes the form:

\[ \rho_e = e \int f \, d\vec{v} = e \, n_0 \cdot \left( \frac{m_e}{2\pi kT} \right)^{3/2} \int_{\Omega(x)} \exp \left( - \frac{m_e}{2kT}(v_x^2 + v_y^2 + v_z^2) + \frac{e\varphi}{kT} \right) \, d\vec{v}. \]  

(19)

For accelerating potential

\[ \rho_e = e \, n_0 \frac{1}{\sqrt{\pi}} \exp(e\varphi/kT) \int_{\sqrt{e\varphi(x)/kT}}^{+\infty} \exp(-s^2) \, ds. \]  

(20)

For a decelerating field, formulas containing quadratures are not written out explicitly due to their bulkiness. The dependence \( j_1(\varphi_1) \) was found by the method of numerical integration of the equations (6) - (8).

The results of the calculations are shown in Figure 4.

Calculations were also carried out for \( x_1 = 3 \).

![Figure 4](image)

**Figure 4.** The dependence \( j_1(\varphi_1) \).

The velocities of the emitted particles are distributed according to Maxwell’s law.

In this Figure \( \varphi_1 = \frac{e\varphi}{kT}, x_1 = x \sqrt{\frac{4\pi e^2 n_0}{kT}} \).

The numerical results obtained in this case in qualitative agreement with the data experiments on photoelectron emission.

3. Conclusions

1. The boundary conditions for both the electric field potential and the equations describing the motion of particles significantly affect the dependence of the emission current density on the electric current potential at the point under consideration. Boundary conditions are divided into four types.
2. For boundary conditions of the first type (the condition of complete screening of the electric field on the surface of the body), the current density grows indefinitely with increasing potential.

3. For boundary conditions of the second type, the dependence graph $j_1 = j_1(\varphi_1)$ has two branches and a branch point. The presence of a branch point indicates a possible instability of the electrons motion in its vicinity.

4. For boundary conditions of the third and fourth types (the description of the motion of electrons is carried out at the kinetic level), dependences are obtained that smoothly reach the saturation mode and are in good agreement with the experimental data on thermionic and photoelectron emission.

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