Revealing the Evolution of Non-thermal Electrons in Solar Flares Using 3D Modeling

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Received 2018 February 19; revised 2018 March 24; accepted 2018 March 26; published 2018 May 17

Abstract

Understanding non-thermal particle generation, transport, and escape in solar flares requires detailed quantification of the particle evolution in the realistic 3D domain where the flare takes place. Rather surprisingly, apart from the standard flare scenario and integral characteristics of non-thermal electrons, not much is known about the actual evolution of non-thermal electrons in the 3D spatial domain. This paper attempts to begin to remedy this situation by creating sets of evolving 3D models, the synthesized emission from which matches the evolving observed emission. Here, we investigate two contrasting flares: a dense, “coronal-thick-target” flare SOL2002-04-12T17:42, that contained a single flare loop observed in both microwaves and X-rays, and a more complex flare, SOL2015-06-22T17:50, that contained at least four distinct flaring loops needed to consistently reproduce the microwave and X-ray emission. Our analysis reveals differing evolution patterns for the non-thermal electrons in the dense and tenuous loops; however, both patterns suggest that resonant wave–particle interactions with turbulence play a central role. These results offer new constraints for theory and models of the particle acceleration and transport in solar flares.

Key words: acceleration of particles – diffusion – Sun: flares – Sun: radio radiation – Sun: magnetic fields – turbulence

Supporting material: animations

1. Introduction

Creating a complete picture of particle acceleration in solar flares requires detailed knowledge of where the particles are accelerated and how they evolve in the actual three-dimensional (3D) domain where the flare happens. However, such information is not routinely available because it can neither easily be derived from the available data sets, nor be modeled from first principles. Indeed, most information about non-thermal electrons is derived from the X-ray emission that is produced due to collisions of non-thermal electrons with target particles (mainly, protons) via bremsstrahlung. For example, the Reuven Ramaty High Energy Solar Spectroscopic Imager (RHESSI, Lin et al. 2002) produces both spectral and imaging information, and this is the most appropriate instrument with which to study electron acceleration and transport. However, the limited dynamic range of RHESSI makes it hard or impossible to detect hard X-ray (HXR) emission from the coronal part of the flaring loops against much brighter footpoints. An exception is dense, so-called “coronal-thick-target” flares, where HXR emission comes from the corresponding coronal flaring loop (Xu et al. 2008; Guo et al. 2012a, 2012b, 2013; Lee et al. 2013). But even in such cases, the RHESSI imaging is typically performed over rather long time intervals (e.g., half a minute or longer), so detailed dynamics cannot be captured. In addition, study of the electron acceleration in the coronal-thick-target flares is biased by cases of rather dense sources, which cannot help with revealing a complete picture of electron acceleration and transport for the range of tenuous and dense flaring flux tubes, although both tenuous and dense acceleration regions were reported (Fleishman et al. 2011, 2016a). A viable enhancement of the non-thermal electron probing comes from microwave imaging spectroscopy (Gary et al. 2013). But even the immediate outcome of a microwave data analysis will be line-of-sight (los) integrated distributions of the parameters in the image plane, rather than the true 3D distribution. Therefore, in addition to the observational data set, revealing the true 3D structure must rely on realistic data-constrained modeling.

In this paper we attempt to constrain the flare dynamics in the 3D domain by creating evolving 3D models capable of producing synthetic microwave emission that matches the observed emission during a time range. To do so we start from “master” models developed using GX Simulator (Nita et al. 2015) for single time frames and validated by comparison with all available microwave and X-ray constraints (Fleishman et al. 2016c; Kuroda et al. 2018). Then, assuming that the basic flare topology does not change drastically over some time range, we use these master models and alter some of their physical parameters to derive model sets, each of which is capable of producing the synthesized microwave spectrum that matches the observed one at a given time frame. This set of “snapshot” models is then adopted to represent an evolving 3D model of the flare. It is apparent that the evolving flare model obtained this way is not unique, while a family of models could be consistent with the applied observational constraints. In particular, the magnetic structure of the evolving model is fixed by construction, which is of course an approximation of reality. However, given that the magnetic loop structure is fixed, the evolving distribution of the non-thermal electrons offers one of the likely evolutionary patterns of the physical parameter variation, the main trends of which truthfully reproduce the actual parameter trends.

We note that, to address the evolution, we only attempt to match the microwave spectra, rather than both spectral and imaging data, because the spectra are known to have a higher cadence than the images. In addition, we do not see any substantial fast evolution in the images, which would justify changing the magnetic topology in the model (e.g., altering the central, “reference” field line defining the flaring flux tube).
However, after advancing our model over a certain time range (typically, 2–3 minutes) it was not possible to obtain a good match with the microwave data within the fixed topology; we interpret this as an indication of the magnetic topology change. Once the evolving models have been obtained, it is instructive to straightforwardly analyze trends in evolving parameters and link these trends with possible acceleration models and transport regimes, which we discuss in some detail in Section 4.

We consider two flares with rather contrasting properties. The first flare, a dense, “coronal-thick-target” flare, SOL2002-04-12T17:42, showed a rather simple, single-source morphology in both X-ray and microwave images. This flare has been successfully modeled using a single asymmetric flare loop (Fleishman et al. 2016c, in fact, two slightly different flaring loops were needed to model two distinct time frames: one at the rise phase and the other one at the peak phase). The other one, SOL2015-06-22T17:50, appeared to be much more complex and contained at least four distinct flaring loops needed to consistently reproduce the microwave and X-ray emission. Only two of those four loops contributed noticeably to the microwave emission, with the main contribution coming from a large, tenuous, “overarching” loop that served as an efficient trap for the non-thermal electrons accelerated presumably somewhere lower in the corona, as follows from the X-ray morphology.

In what follows we discuss the trends of the physical parameters in both cases, comparing the trends, and formulating new constraints on the electron acceleration models and transport regimes.

2. Single-loop Flare

A list of coronal-thick-target flares was identified and studied by Xu et al. (2008), although a few examples of such flares were reported by Veronig & Brown (2004). These flares appear as single, loop-shaped sources at HXR and sometimes also observed in microwave (Lee et al. 2013; Fleishman et al. 2016c). In particular, Fleishman et al. (2016c) studied one of the flares from the Xu et al. (2008) list, SOL2002-04-12T17:42, with X-ray and microwave data augmented by 3D modeling using GX Simulator (Nita et al. 2015), and found that both radio and X-ray data are well reproduced within a 3D model involving only one asymmetric loop, although the loop itself evolved from the rise to peak phase of the flare.

2.1. Overview of SOL2002-04-12T17:42

SOL2002-04-12T17:42 was observed by RHESSI in X-rays and OVSA in microwaves. Fleishman et al. (2016c) analyzed the flare in detail and produced 3D models for two time frames of this flare, one at the rise phase, ~17:41:58 UT, and the other at the peak phase, ~17:45:10 UT. In both cases, a linear force-free field extrapolation has been performed to create a flaring flux tube, to populate it with a thermal plasma and non-thermal electrons, and to validate the model via comparison of the synthetic X-ray and microwave images and spectra. It turned out that the magnetic structure evolved noticeably between the two time frames, so two distinct magnetic flux tubes were needed to model those two time frames.

Another interesting finding was that the microwave images were displaced compared with the X-ray ones for both time frames. However, 3D modeling proved that this displacement is due only to the flaring loop asymmetry such that one leg of the loop was brighter in the X-rays, while the other one was brighter in the microwaves. It is also interesting that the flaring flux tube was noticeably thicker (the loop cross-sectional radius was bigger) at the peak time compared with the rise phase.

Fleishman et al. (2016c) modeled the main spectral peak of the microwave emission with the mentioned flaring flux tubes, while the microwave spectrum displayed a secondary spectral peak at low frequencies, $f = 1.2–2$ GHz, which was ignored at that time. It is now established that such a secondary low-frequency spectral peak, if not a coherent plasma emission, can either be associated with a distinct large-scale “plume” (Fleishman et al. 2016b, 2017b) or produced by resonant transition radiation (RTR) due to interaction of non-thermal electrons with plasma density inhomogeneities (Fleishman et al. 2005; Nita et al. 2005).

To choose between those possibilities, here we have produced the flare images at low microwave frequencies, $f = 1.2–2$ GHz, and plotted them along with other microwave and HXR images on top of a SoHO/MDI LOS magnetogram; see Figure 1. It is apparent from the figure that the low-frequency source projects onto the same solar area as the HXR and higher-frequency microwave images. This spatial relationship and the modest size of the low-frequency source favor the RTR nature of this low-frequency component (cf. Nita et al. 2005), rather than the gyrosynchrotron origin from a large loop, as in Fleishman et al. (2016b, 2017b). We will return to implication of this finding later.

2.2. Rise Phase of SOL2002-04-12T17:42

We started our modeling from the master model developed by Fleishman et al. (2016c) for a time frame of ~17:41:58 UT at the rise phase of this flare. Given the central role of the microwave spectra for our modeling, we inspected the quality of the corresponding OVSA data$^4$ a few minutes before and after this master time frame. We found that the three highest-frequency channels show large fluctuations in time, while the low-frequency spectral range, $f = 1.2–2$ GHz, displays an additional spectral component. Therefore, to objectively use the model-to-data residual, computed within the GX Simulator tool, we removed both the low- and high-frequency OVSA channels to create the new reference file containing 31 frequencies spanning the 2.4–14 GHz range; for consistency we have done so for both the rise and peak (Section 2.3) phases of this event. After removing those outliers, it became possible to further fine-tune the model parameters to get a closer model-to-data match than in the Fleishman et al. (2016c) paper, so we use this slightly modified model as the master one, and advance it backward and forward in time.

An important question is what physical parameters are to be adjusted to ensure, in accordance with observations, the desired modification of the microwave spectrum. Here, we consider the main factors affecting the optically thin and thick parts of the microwave spectrum. For example, changes of the optically

$^4$ Available in the form of .ref-files directly importable by GX Simulator from the same 3D GX model repository at http://www.ioffe.ru/LEA/S_AR/models/3models.html, or in various formats, from the official OVSA website directly.
images were obtained over 64 s starting at 17:45:48 UT: the 12–25 keV image was obtained over one minute starting at 17:45:48 UT (green), and the frequency-time synthesis microwave images were obtained over 64 s starting at 17:45:46 UT at 2.8–5.6 GHz (violet) and 1.2–1.8 GHz (white). Their projected locations are consistent with the emission coming from the same magnetic loop.

Figure 1. HXR and microwave images on top of an MDI LOS magnetogram obtained at ∼17:36 UT: the 12–25 keV image was obtained over one minute starting at 17:45:48 UT (green), and the frequency-time synthesis microwave images were obtained over 64 s starting at 17:45:46 UT at 2.8–5.6 GHz (violet) and 1.2–1.8 GHz (white). Their projected locations are consistent with the emission coming from the same magnetic loop.

Thick intensity require (see Equation (1) in Fleishman et al. 2017b) either a change of the source area (due to non-thermal electron redistribution within a given flux tube or change of the cross-sectional radius of the flux tube) or of the effective energy of the emitting non-thermal electrons (due to a change of the low-energy cutoff, spectral index, or the magnetic field, which, for a preselected magnetic flux tube, can be emulated by a redistribution of the non-thermal electrons in the loop), while the optically thin part is primarily controlled by the non-thermal electron energy spectrum (the spectral index and high-energy cutoff). While fine-tuning the parameters, we attempted to minimize the number of free parameters to change and look for coherent patterns in the parameter behavior.

At the rise phase we managed to successfully model a 4-minute time interval (2 minutes before and 2 minutes after the “master” time frame) with the 4 s cadence, during which two groups of physical parameters were adjusted—the 3D spatial distribution of the non-thermal electrons and the energy distribution of the non-thermal electrons, which we describe sequentially here, even though both groups affect the resulting spectrum simultaneously.

The adjustments in the 3D spatial distribution were needed primarily to change the source area required to account for the change (increase) of the microwave flux in the optically thick part of the spectrum. Whenever possible, we achieved the required change by a redistribution of the non-thermal electrons along the existing modeling loop with a fixed cross-sectional reference radius.

The spatial distribution of the non-thermal electrons along the loop is described by a Gaussian function

\[ n_{nth}(s) = n_0 \exp \left\{ -\left[q_0 \left( \frac{s - s_0}{l} + q_2 \right) \right]^2 \right\}, \]  

where \( l \) represents the length of the loop central field line, the parameter \( q_0 \) controls the width of the Gaussian, and \( q_2 \) indicates where the Gaussian has a maximum relative to a preselected \( s_0 \) reference point (e.g., the loop top).

Figure 2 displays the evolution of the adjusted parameters used in the modeling, along with some derived parameters. Figures 2(a), (d), and (e) display parameters of the non-thermal electron Gaussian distribution along the central field line for the selected magnetic flux tube, according to Equation (1). Panel (d) shows that the Gaussian width stays almost constant for more than two minutes of the rise phase and then slightly drops (the electron distribution becomes slightly more uniform along the loop spine). As shown by panel (e), the location of the spatial peak of the distribution, controlled by \( q_2 \), varies within \( s_{peak}/l = s_0/l - q_2 \approx 0.35 \pm 0.04 \). It does not display any monotonic trend but rather relatively modest variation around a “preferred” location at the middle of the flux tube. To provide an overview of the evolution of the non-thermal electron spatial distribution visualized in animated Figure 3, the evolution of Gaussian non-thermal electron distribution along the spine of the flaring flux tube is illustrated at a few selected time frames in Figure 4.

However, it turns out that such redistributions alone are insufficient to account for the observed build-up of optically thick microwave emission at the rise phase; thus we are forced to increase the source area by increasing the cross-sectional radius of the flaring flux tube. Figure 2(f) shows an almost monotonic rise of this radius from 0.9 to 1.6 grid points, which, for our model resolution of two arcseconds (\( dr = 1.45 \times 10^3 \) cm), corresponds roughly to a range from 1300 to 2300 km, which implies an increase of the flaring loop volume by a factor of three.

The information from panels (a) and (d)–(f) is used to compute the total number and total energy of the non-thermal electrons \( N_{nth} \) in the model flux tube shown in panel (b). Then, combining the inputs of panels (a) and (b) we compute an effective flux tube volume as \( V_n = N_{nth}/n_{nth} \), shown in panel (c). Finally, panel (h) displays the evolution of the model-to-data standard deviation, which characterizes the goodness of the model.

The apparent spectral evolution of the optically thin microwave emission (softening and then hardening) during roughly the first two minutes of our 4-minute time interval requires a corresponding “hard-soft-hard” evolutionary pattern in the non-thermal electron spectrum. Given that the model spectrum is described by a double power-law with a breakup at about 36 keV, the mentioned spectral evolution pertains to the higher-energy electrons (spectral index \( \delta_2 \) that produce the microwave emission. Figure 2(g) shows that \( \delta_2 \) gets softer from 3.4 at the very beginning of the interval to 5.5 and then comes back to 3.4; it stays at 3.4 for the two remaining minutes of this interval. The low-energy spectral index, \( \delta_1 \) stays constant almost all the time, \( \delta_1 = 7.3 \), but slightly hardens to \( \delta_1 = 7 \) during a few time frames at the middle of the interval. Although this change does improve the match with the data, we do not think it is a significant trend, so we are not discussing it in any detail.

5 The adopted convention is that the coordinate \( s \) along the central (reference) field line of the flaring flux tube is zero, where the magnetic field is minimal by the absolute value, \( s = s_{max} < 0 \) at the footpoint of the positive magnetic field, while \( s = s_{max} > 0 \) at the footpoint of the negative magnetic field; \( s_{max} - s_{min} = l \), where \( l \) is the field line length.
The number density of the non-thermal electrons (above 18 keV adopted to be the low-energy cutoff in the spectrum) shows a pattern correlated with the $\delta_2$ evolution during the first two minutes, which is expected: the softer spectrum implies much fewer high-energy electrons for the fixed $E_{\text{min}}$ and $E_{\text{break}}$; thus to compensate for that given the comparable microwave

Figure 2. Rise phase of the SOL2002-04-12T17:42 flare. The evolution of the modeled parameters describes the spatial and energy non-thermal electron distributions around the local peak time 17:41:58 UT, which is marked by vertical dashed lines in each panel. (a) Evolution of the Gaussian distribution full maximum $\theta_{\text{peak}}$ in Equation (1); (b) total number $N_{\text{nth}}$ and total energy of the non-thermal electrons; (c) effective volume of the flux tube; (d) normalized FWHM (2$\sqrt{\ln 2} / q_0$) evolution, controlled by parameter $q_0$ in Equation (1); (e) evolution of the Gaussian peak location normalized to the loop length ($s_{\text{peak}} / l$), controlled by parameter $q_2$ in Equation (1); (f) evolution of the flaring flux tube radius $R$ at the reference point $s_0 / l = -0.024$ (Equation (1)), normalized by the extrapolation grid size, $dr = 1.45 \times 10^8$ cm; the dotted line shows the corresponding linear fit; (g) evolution of the $\delta_1$ and $\delta_2$ spectral indices of the double power-law non-thermal electron distribution; and (h) standard deviations of the synthesized microwave spectra relative to the microwave spectra observed by OVSA. A median standard deviation of 12.4% is shown by the horizontal dotted line.
flux level, one has to increase the non-thermal electron distribution $n_{nth}$. It is instructive, however, to take a closer look at the $n_{nth}$ (Figure 2(a)) evolution during the last two minutes of the interval, when no spectral evolution is detected (both $\delta_1$ and $\delta_2$ are constant in time). In spite of the rise of the radio flux, the peak (over the source) number density goes down by a factor of two. This result, however, is not surprising given that the source volume, and thus the source area, goes up during the same time; thus the optically thick emission slowly goes up, while the optically thin one stays roughly at the same level.

The quality of the forward-fitted models is quantified by the relative model-to-data standard deviations displayed in Figure 2(h), which range from 7% to 37%, with a median value of 12%. This value shows a systematic decrease (the quality of the fit increases) with time, which is a direct outcome of the rising signal-to-noise ratio as the burst level goes up.

2.3. Peak Phase of SOL2002-04-12T17:42

For the peak time, $\sim$17:45:10 UT, we used the Fleishman et al. (2016c) master model without any adjustment and advanced it backward in time by 88 s and forward in time by 2 minutes. It is interesting that adjusting only two parameters, the location of the spatial peak of the non-thermal electron distribution along the loop spine, $q_2$, and the number density, is sufficient to successfully model the emission rise phase just prior to the spectral peak. In the decay phase, just after the peak, the same parameters evolve as well, but adjusting one more parameter, namely $E_{min}$, which affects the microwave spectral shape at the optically thick part of the spectrum, improves the model-to-data match remarkably; see Figure 5. This figure also shows a few derived parameters, such as the total number of non-thermal electrons, their total energy, and the effective volume of the flaring flux tube.

Interestingly, over the 1.5 minutes of the rise phase, the number density increases by a factor of 20, from $4 \times 10^6$ to $8 \times 10^7$ cm$^{-3}$, while the peak microwave flux rises by only a factor of 3. This happens because the non-thermal electron cloud moves gradually toward a flux tube region with a weaker magnetic field; see the animations for Figures 6 and 7. At the decay phase, the parameter trends revert. However, in addition to these trends, the value of the low-energy cutoff, $E_{min}$, goes

Figure 3. Rise phase of the SOL2002-04-12T17:42 flare. The evolution of the number density of the non-thermal electron distributions is shown with the same color scale for all the panels in the figure, along with the modeled flux tube in 3D. The animated version of this figure displays the evolution of the 3D distribution of non-thermal electrons shown in the perspective (top left) and top (top right) views, along with the running microwave spectrum and its fit (middle, left) and the corresponding residual (middle right), as well as the dynamic spectrum, where the moving vertical solid line shows the time frame currently displayed above it. The animation starts at 2002 April 12 at 17:41:57.9 and ends at 17:43:57.9. The duration is 31 s.

(An animation of this figure is available.)
up roughly from 20 to 40 keV at the decay phase. Other than that, no significant spectral evolution is demanded by the data.

3. Multi-loop Flare

The M6.5 class SOL2002-04-12T17:42 flare (Jing et al. 2016; Liu et al. 2016; Wang et al. 2017) has been observed by a new generation of high-resolution instruments, including the Goode Solar Telescope at BBSO and Expanded OVSA (EOVSA; D. E. Gary et al., in preparation). This flare represents an example contrasting the dense SOL2002-04-12T17:42 flare, given that it contained a relatively large flux tube dominating low-frequency radio emission similar to a few other recently reported flares (Fleishman et al. 2016b, 2017b).

3.1. Overview of SOL2015-06-22T17:50

Kuroda et al. (2018) performed a detailed 3D modeling of a single time frame corresponding to a local peak, 18:05:32 UT, of this flare using a magnetic data cube obtained from a nonlinear force-free field (NLFFF) reconstruction as a framework. Using this data cube, the flux tubes needed to fulfill all constraints available from X-ray (RHESSI) and microwave (EOVSA) observations were identified. It is interesting that three distinct flux tubes were needed to reproduce a rather complicated spatial distribution of X-ray images at different energies. However, even with these three loops it was impossible to reproduce the microwave low-frequency emission; neither the spectrum, nor the interferometric data were reproduced. This forced Kuroda et al. (2018) to add one more loop, a large “overarching” loop, making the main contribution to the microwave spectrum. Presumably, this large loop was magnetically connected with other flaring loop(s) and served as a particle trap.

It is important to note that NLFFF reconstruction ensures a measurably better approximation to the reality than other modeling means such as linear force-free or potential field reconstruction; thus for the first time, it became possible to build those various flaring loops using one single data cube, rather than employing various cubes as in previous studies (see, e.g., Fleishman et al. 2016b, 2017b). However, given that the NLFFF data cube is only an approximation of reality, which contains noticeable uncertainties (De Rosa et al. 2009; Fleishman et al. 2017a), these uncertainties propagate to the final model and do not permit a perfect match of all observables. Specifically, Kuroda et al. (2018) had to trade off between reproducing the microwave spectrum and relative visibilities (interferometric spatial information; see Kuroda et al. 2018 for more detail). To match the microwave spectrum, two distinct sources were needed, while the relative visibilities suggested that there was either only a single source or the two sources must have projected at the same place. Kuroda et al. (2018) managed to obtain an apparently correct spatial structure of two sources that matched the relative visibility constraints at the expense of noticeable spectral distortion. We believe that this close spatial association between the two microwave producing loops is not a random coincidence but rather comes from a connectivity between these loops. Such a connectivity is, however, not captured by the static NLFFF model, which cannot be fully correct, especially at the areas of loop interaction during the flare. Thus, having certain mismatches between models and data is not surprising. Given that in this study we are guided by solely the evolving microwave spectral shape, we modified the original Kuroda et al. (2018) model to get the best possible spectral match at 18:05:32 UT. Although the modification is relatively minor, the new model contains slight spatial displacement between the two microwave sources, which results in measurably worse model relative visibilities.

3.2. Peak Phase of SOL2015-06-22T17:50

Starting with this modified master model at 18:05:32 UT, we investigated 6.5 minutes of the flare evolution, 2.5 minutes before and 4 minutes after the master time frame with 5 s cadence (the data are available with a higher, 1 s cadence, but we did not see any fast variation of the data, which would warrant that high cadence in the modeling). Although two of the four flaring flux tubes (Loop 1 and 4; see Figure 8) contribute to the microwave emission, only the parameters of the overarching loop (Loop 4) show significant evolution, while in the other loop (Loop 1) only one parameter, the number density of the non-thermal electrons, evolves; see Figure 9. We do not show the total electron energy in this case because the value of the low-energy cutoff is not well constrained in the large loop; on top of that, this loop is only one of four flaring loops, which are present in this flare, so information about total non-thermal energy evolution in this flare cannot be obtained from our modeling. Similar to the peak time of the SOL2002-04-12T17:42 flare, the evolutionary patterns at the rise and decay phases are different from each other, which is not surprising.
At the rise phase, the modeling, visualized in animated Figure 8 and further quantified in Figure 10, shows a significant spatial evolution of the non-thermal electron population in Loop 4, which can be interpreted as a gradual filling of this loop by non-thermal electrons coming from another place, presumably the acceleration region of the flare. The number density of non-thermal electrons in Loop 4 increases at the beginning of the rise phase, but then stays relatively constant and even starts to go down before the peak time. However, given the motion of the non-thermal electron cloud toward the loop top, which has a bigger cross-sectional area, the effective area of the microwave source associated with Loop 4 continues to grow. It is also interesting that the high-energy cutoff in the electron spectrum grows at the beginning of the rise phase from 2.1 MeV up to 3.4 MeV and then stays constant. This may indicate that the acceleration of electrons up to MeV energies takes longer than that for HXR-producing deka-keV electrons. We cannot, however, distinguish if this acceleration happens at Loop 4 or if these high-energy electrons arrive there from the acceleration region.

In contrast, in the decay phase we almost do not see any spatial evolution of the non-thermal electron population and no spectral evolution, but rather a gradual decrease of the number density (and thus total number) of non-thermal electrons in Loop 4, which we interpret as a gradual loss of the trapped electrons from this loop, while the supply from the non-thermal electron source gets weaker. This is consistent with that the number density in Loop 1 also goes down, while showing some

![Figure 8 and 10](image)

**Figure 5.** Peak phase of the SOL2002-04-12T17:42 flare. The evolution of the modeled parameters describes the spatial and energy non-thermal electron distributions around the main peak time at 17:45:10 UT, which is marked by vertical dashed lines in each panel. (a) Evolution of the Gaussian distribution full maximum ($\sigma_{\text{max}}$ in Equation (1)); (b) total number $N_{\text{nth}}$ and total energy of the non-thermal electrons; (c) effective volume of the flux tube; (d) evolution of the Gaussian peak location normalized to the loop length ($s_{\text{peak}}/l$), controlled by parameter $q_2$ in Equation (1); (e) evolution of the minimum non-thermal electron energy cutoff; (f) standard deviations of the synthesized microwave spectra relative to the microwave spectra observed by OVSA. The median standard deviation of 7.8% is shown by the horizontal dotted line.
variations at the decay phase. This consideration shows that in contrast to significant spatial complexity, this flare displays a relatively simple time evolution.

4. Discussion

In this section we compare the identified properties of these two contrasting flares and use the revealed quantitative parameter trends to constrain possible mechanisms of electron acceleration and the regimes of their transport.

4.1. Comparison between Dense and Tenuous Flaring Flux Tubes

The flare SOL2002-04-12T17:42 displayed only one flux tube in both X-rays and microwaves. The thermal density of this loop proved to be rather high, \( \sim 10^{11} \text{ cm}^{-3} \). This estimate comes from analysis of the X-ray source sizes at different energies (Xu et al. 2008), from the emission measure determined from the X-ray spectral fit, and it is also consistent with joint 3D X-ray and microwave modeling performed by Fleishman et al. (2016c). In addition, the high plasma density is also supported by the presence of a distinct low-frequency radio component that is co-spatial (at least, in the projection) with the flaring loop seen in X-rays and microwaves; see Figure 1. This low-frequency component is likely produced by RTR (cf. Nita et al. 2005) around the plasma frequency; having the plasma frequency around 2 GHz implies an electron number density of about \( 5 \times 10^{10} \text{ cm}^{-3} \). In that high plasma density, the Coulomb loss time is very short, \( \lesssim 1 \text{ s} \), for the deka-keV electrons responsible for HXR emission. This means that the electrons must be energized inside the loop; thus this dense loop represents (or inscribes) the acceleration region of this coronal-thick-target flare. This conclusion is confirmed by the source modeling performed by Xu et al. (2008). Therefore, the trends of the flare parameters revealed in Section 2 pertain, in fact, to evolution of the acceleration region in this flare. In contrast, most of the dynamics revealed in the case of the SOL2015-06-22T17:50 flare pertains to the large, “overarching” flux tube; thus in this case we primarily study the evolution of the electron trapping, rather than the acceleration. From this perspective, and also given the highly different sizes

Figure 6. Peak phase of the SOL2002-04-12T17:42 flare. The evolution of the number density of the non-thermal electron distributions is shown with the same color scale for all the panels in the figure, along with the modeled flux tube in 3D. The animated version of this figure displays the evolution of the 3D distribution of non-thermal electrons shown in the perspective (top left) and top (top right) views, along with the running microwave spectrum and its fit (middle, left) and the corresponding residual (middle right), as well as the dynamic spectrum, where the moving vertical solid line shows the time frame currently displayed above it. The animation starts at 2002 April 12 at 17:43:25.9 and ends at 17:47:13.9. The duration is 60 s.

(An animation of this figure is available.)
of the flux tubes and the corresponding number densities, it is not at all surprising that these two flares demonstrate different evolutionary patterns.

4.2. Dynamics of Electron Acceleration in the Dense Flare

At the early rise phase of the SOL2002-04-12T17:42:42 dense flare (Figures 2–4), the main finding is related to how the flare emission builds up as the flare progresses. Specifically, we found that the increase of the microwave flux happens because of the rising volume of the flaring loop, rather than the evolution of the non-thermal number density. The number density of the non-thermal electrons can even go down but the microwave source area goes up proportionally. It is reasonable to assume that the increase of the flaring loop volume occurs because of the accumulation of the newly reconnected magnetic flux tubes. Likely, more non-potential magnetic subdomains become unstable earlier, and then trigger adjacent, less non-potential subdomains. This might explain why the number density of the accelerated electrons goes down during this process: less free energy density implies a lower number density of the accelerated electrons. Some spatial and spectral evolution of the non-thermal electron population occurs during this build-up; however, this evolution is modest. The spectral evolution is over by the end of this early rise phase.

A relatively slow early rise of the radio flux gives way to a much faster rise at about 17:43:42 UT, when the evolutionary pattern significantly alters compared with that at the early rise phase. Indeed, no change of the flaring loop topology or size is demanded by the data at this stage (recall that the master models for these two time intervals are different because of some topology change over the course of the flare), while the rise of the flaring emission between 17:43:42 UT and the peak at 17:45:10 UT is solely due to an increase of the non-thermal electron number density by a factor of 20. However, this number density increase resulted in only a factor of 3–4 increase of the microwave flux, because the newly accelerated electrons occupied an area with an accordingly weaker magnetic field. The fact that no significant topological changes happen at this stage implies that the flaring loop itself contains a sufficient amount of free energy to support the acceleration of electrons.

After the peak at 17:45:10 UT, the trends in the parameter behavior revert: the number density of the non-thermal electrons goes down, while the peak of their spatial distribution returns to the portion of the loop with a larger magnetic field. In addition, the low-energy cutoff in the electron energy spectrum goes up from roughly 20 to 40 keV, over a 2 minute interval in this decay phase. A naive interpretation that this trend is due to the collisional loss does not work because the collisional loss time is about 1 s or less at this energy range. We instead propose that this behavior indicates a change in the balance between the acceleration and losses, as follows. At the rise phase, a yet unspecified acceleration agent has sufficient energy to overcome the Coulomb losses at essentially all non-thermal energies, which leads to the growth of the non-thermal population. At some point, however, the free energy available for particle acceleration goes down. Thus, the acceleration efficiency goes down such that it cannot overcome the Coulomb losses at low energies any longer, but it is still sufficient to compensate the Coulomb losses of the electrons above some threshold energy. The lower the acceleration efficiency, the higher the threshold energy below which the acceleration is no longer efficient.

4.3. Casting of Acceleration Mechanisms in the Dense Flaring Flux Tube

Let us consider the corresponding implications for the acceleration mechanism. The main groups of the acceleration mechanisms are acceleration by (i) DC electric field, (ii) resonant turbulence, and (iii) non-resonant turbulence. Acceleration due to DC electric fields is known to be efficient at relatively low energies, say \( \lesssim 100 \text{ keV} \) (Holman 1985). However, electrons with much higher energies are involved in the generation of the microwave emission of the flare; thus we will concentrate here on stochastic electron acceleration by turbulence. It is reasonable to expect that the acceleration efficiency is higher in sub-volumes with accordingly higher free energy, and thus in locations with a relatively strong magnetic field. For this reason, in our estimates we place the center of the acceleration region at \( s/l \approx -0.8 \), where \( B = 7 \times B_0 \approx 327 \text{ G} \), given that \( B_0 = 46.7 \text{ G} \). This is consistent with the location of the RHESSI X-ray source interpreted as the acceleration region by Guo et al. (2012a, 2012b, 2013) and Xu et al. (2008).

4.3.1. Non-resonant Acceleration

Stochastic acceleration by a large-scale, non-resonant turbulence requires that the non-thermal particles are linked to a “fluid element” of the plasma due to efficient particle scattering by a small-scale, resonant turbulence (Bykov & Fleishman 2009; Fleishman & Toptygin 2013, Section 11.5). This implies that the local mean free path (mfp) \( \lambda \) of the non-thermal electrons, which is controlled by electron scattering by resonant turbulence, is much shorter than the main, energy-containing scale \( L_\varphi \); \( \lambda < L_\varphi \). In such a case, the particles are being pumped up with the energy of the large-scale turbulent pulsations. Given that the source of

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**Figure 7.** Peak phase of the SOL2002-04-12T17:42 flare, showing the variation of the absolute magnetic field along the central field line (left vertical axis) and the evolution of the longitudinal non-thermal electron distribution (right vertical axis) described by Equation (1). The horizontal axis, \( \lambda_i(l) \), indicates the signed distance from the loop apex \( (s=0) \) normalized to the total length of the central field line, \( l = 4.77 \times 10^3 \text{ cm} \). The absolute magnetic field line (black solid line) was normalized to the minimum value corresponding to the loop apex, \( B_0 = 46.70 \text{ Gauss} \). For easier direct comparison, the Gaussian longitudinal density distributions \( n_0(i,s) \) were power-law-scaled as \( n_0(i,s) \). The evolution of the density distribution is coded by color and line style, with the dashed and solid lines respectively representing time frames before and after the peak time, which occurred at 17:45:10.
Figure 8. The SOL2015-06-22T17:50 flare. The evolution of the number density of the non-thermal electron distributions is shown with the same color scale for all the panels in the figure, along with the modeled flux tube in 3D. The large “overarching” loop is Loop 4, and the one beneath it is Loop 1. The animated version of this figure displays the evolution of the the 3D distribution of non-thermal electrons shown in the perspective (top left) and top (top right) views, along with the running microwave spectrum and its fit (middle, left) and the corresponding residual (middle right), as well as the dynamic spectrum, where the moving vertical solid line shows the time frame currently displayed above it. The animation starts at 2015 June 22 at 18:03:07.0 and ends at 18:09:27.0. The duration is 89 s. (An animation of this figure is available.)

the non-thermal electrons controlled by resonant scattering by small-scale turbulence is much shorter than $L_c$; $\lambda < L_c$. However, the required $L_c$ is already very short; thus having an even shorter local mfp requires a rather high level of the resonant turbulence, which itself can effectively accelerate electrons. We also note that having that short mfp, $\lambda < 10^3$ cm, may be in conflict with the observed sizes of the X-ray source, which is likely controlled by collisional transport (Xu et al. 2008), rather than wave–particle interaction. These considerations do not favor the non-resonant stochastic acceleration in the given event; thus we now focus on the resonant stochastic acceleration by small-scale turbulence.

4.3.2. Resonant Acceleration

To be specific, here we consider stochastic acceleration by whistler turbulence following Hamilton & Petrosian (1992), Fleishman & Toptygin (2013, Section 11.2.1). In this case, the diffusion coefficient $D(E)$, and thus the acceleration time, depend on the electron energy, $D(E) \propto \beta(\beta \gamma)^q - 2$, where $\beta = v/c$ is the particle velocity normalized by the speed of light, $\gamma$ is the Lorentz-factor, and $q$ is the spectral index of the resonant turbulence. For our estimates we adopt that the
Figure 9. The SOL2015-06-22T17:50 flare. The evolution of the modeled parameters describes the spatial and energy non-thermal electron distributions around the main peak time at 18:05:32 UT, which is marked by vertical dashed lines in each panel. (a) Evolution of the Gaussian distribution full maximum ($n_{\text{th}}$ in Equation (1)); (b) total number of non-thermal electrons $N_{\text{nth}}$; (c) effective volume of the flux tube; (d) evolution of the Gaussian peak location normalized to the loop length ($s_{\text{peak}}/l_s$) controlled by parameter $q_2$ in Equation (1); (e) evolution of the maximum non-thermal electron energy cutoff; (f) standard deviations of the synthesized microwave spectra relative to the microwave spectra observed by EOVSA. The median standard deviation of 5.7% is shown by the horizontal dotted line.

Figure 10. The SOL2015-06-22T17:50 flare. The evolution of the longitudinal non-thermal electron distributions occurs along two of the four modeled flux tubes.
whistler spectrum is present between the proton and electron gyrofrequencies, $\omega_{\text{pl}}$ and $\omega_{\text{hy}}$; thus the main scale is $L_c \sim v_A/\omega_{\text{hy}} \approx 120$ cm, while its energy density $W_e$ is a fraction $\zeta$ of the magnetic field energy density, $W_B = \zeta B^2/(8\pi)$. Figure 11(a) demonstrates the dependence of the acceleration time on the electron energy for $q = 3.5$ and two different levels $\zeta$ of the selected whistler energy density, such as the acceleration time to exactly match the Coulomb loss time at 18 and 40 keV, respectively. This plot shows that the evolution of $E_{\text{min}}$ from 18 to 40 keV implies that the whistler turbulence decays by a factor of 9 (to be compared with a factor of 3 determined above for the non-resonant stochastic acceleration). The electron mfp due to scattering by the whistler waves $\lambda(18$ keV) $= 4.9 \times 10^6$ cm for $\zeta = 10^{-3}$ and $\lambda(18$ keV) $= 4.3 \times 10^8$ cm for $\zeta = 1.16 \times 10^{-4}$ are comparable to the collisional loss length $\lambda_c(18$ keV) $= 3.2 \times 10^9$ cm, in agreement with the currently established understanding of the coronal-thick-target flares.

Figure 11(b) displays the corresponding range of the whistler turbulence energy density as a function of the turbulence spectral index $q$. A strong dependence of the needed energy density on the spectral index simply follows from the fact that the electrons are accelerated by resonant waves whose wavelengths are about the electron Larmor radius $R_e \sim 1-2$ cm for the energy range of interest, while the energy density at these scales is a $(R_e/L_e)^{q-1}$ fraction of the total energy density $W_e$. Note that the width of the shaded area in Figure 11(b) corresponds to roughly one order of magnitude in $\zeta$ variation regardless of the spectral index value.

### 4.3.3. A Consistency Check

Now recall that the whistler mode is a high-frequency continuation of the fast mode (see, e.g., Chapter 3 in Fleishman & Toptygin 2013), which contains fluctuations of the plasma density along with fluctuations of the magnetic and electric fields. Interaction of the non-thermal electrons with such density fluctuations results in the generation of a so-called transition radiation (TR, Platonov & Fleishman 2002). Resonant TR (RTR) arising at frequencies close to the plasma frequency has been reported in a number of solar flares (Lee et al. 2003; Fleishman et al. 2005; Nita et al. 2005). In Section 2.1, we have already noted that the radio burst displays a distinct decimetric continuum component, which we are inclined to interpret as RTR. According to Platonov & Fleishman (2002), the functional form of the RTR intensity varies depending on the relationship between the spectral index of the density fluctuation spectrum, $\nu_c$ and the (low)-energy index in the non-thermal electron spectrum, $\nu_l$. In our case, $\nu_l = 0.8$ and $\nu_c = 2$, so that $\nu_c < \nu_l$ takes place and the RTR flux density is specified by Equation (13) in Fleishman et al. (2005):

$$F \approx 2 \cdot 10^8 \frac{\Delta f_0}{1 \text{ GHz}} \frac{\langle \Delta n_e^2 \rangle}{n_0} \left( \frac{v_0}{c} \right)^{\nu+2} \times \int \frac{n_{\text{nth}}(\geq E_{\text{min}}) dV}{\int_0^{\infty} f_n dV} \left( \frac{f}{f_0} \right)^{\nu+2} \times \exp \left( - \frac{(f - f_0)^2}{\Delta f_0^2} \right) \text{ sfu},$$

where $\langle \Delta n_e^2 \rangle$ is the mean square of the small-scale density inhomogeneities at the scales $L < L_e = c/f_p \approx 17$ cm, $n_0$ is the mean thermal plasma number density, $v_0(E_{\text{min}})$ is the minimum velocity in the non-thermal electron distribution that corresponds to the low-energy cutoff $E_{\text{min}}$. $N_{\text{nth}} = \int n_{\text{nth}}(\geq E_{\text{min}}) dV$ is the total, integrated over the volume $V$, number of non-thermal electrons at the source above the low-energy cutoff $E_{\text{min}}$. $f_0$ is the mean plasma frequency at the source, $\Delta f_0$ is the standard deviation of the plasma frequencies at the source that reflects the large-scale source non-uniformity, and $v_T$ is the thermal velocity of plasma electrons.

Remarkably, we can solve Equation (2) for $\langle \Delta n_e^2 \rangle / n_0^2$, given that all but one ($\nu$) input parameter of this equation are known from either the model or data directly. Specifically, from the model we know the evolution of the non-thermal electron population: $v_T$ is known from the plasma temperature $T = 21$ MK determined from the X-ray spectral fit (see Table 1 in...
4.4. Dynamics of Non-thermal Electron Trapping in a Large Tenuous Flux Tube

In contrast, in the SOL201506-22T17:50 flare we mainly recover the dynamics of the trapping in a large loop: the trapped population of non-thermal electrons grows in space during the rise phase, with decays in number at the decay phase. Only the very beginning of the rise phase shows an indication of the electron acceleration seen as an increase of the high-energy cutoff. It is unclear, however, if this acceleration takes place in the large loop itself, or in the loop interaction region, lower in the corona.

Let us consider the non-thermal electron evolution in the large loop. In the rise phase, the non-thermal electron cloud injected low at the eastern loop leg, presumably from the region of the loop interaction, slowly fills up the large loop, over roughly two minutes of the flare evolution. Kuroda et al. (2018) showed that electrons with reasonably high energies, $\gtrsim 300$ keV, are responsible for microwave emission from this loop, whose time of flight through the large loop with the length $L_{\text{loop}} = 1.5 \times 10^4$ cm is only a fraction of a second. Thus, the observed evolution, which is two orders of magnitudes slower, requires the diffusive transport of non-thermal electrons. Given the definition of the diffusion time, $\tau_d = \frac{3L^2}{a} / (\nu \Delta f)$, where $L \sim 0.3L_{\text{loop}}$ is the size of interest, we can easily estimate the required mfp as $a \sim 1.5 \times 10^3$ cm at the energies 300–600 keV. Now, we consider what level of magnetic turbulence is needed to provide the corresponding diffusion time of $\sim 100$ s. We adopt “generic” magnetic irregularities with a power-law spectrum

$$\langle \delta B^2 \rangle_k = (\nu - 1) \langle \Delta B^2 \rangle_k \frac{k^{\nu - 1}}{k^\nu},$$

where $\langle \Delta B^2 \rangle_k$ is the square of the rms magnetic field value at all $k > k_c = 2\pi / L_c$. In this case, the particle mfp is estimated as (Fleishman & Toptygin 2013, Equation 7.106)

$$\lambda \approx \left( \frac{R_0}{L_c} \right)^{2-\nu} \frac{L_c}{\zeta_B},$$

where $\zeta_B = \langle \Delta B^2 \rangle / B^2$. Figure 13(a) displays the diffusion time as a function of the electron kinetic energy for different levels of the random magnetic field indicated in the panel, assuming that $L_c = 10^3$ cm and $\nu = 5/3$ (Kolmogorov spectrum). The required diffusion time of $\sim 100$ s is achieved for $\zeta_B \sim 10^{-3}$.

Figure 13(b) displays the results of a similar exercise, but this time for the case of whistler turbulence, again bounded to the frequency range between $\omega_{pp}$ and $\omega_{pc}$. Given the thermal plasma density in the large loop, the main scale $L_c = v_A/\omega_{pp} \equiv c/\omega_{pp}$, where $\omega_{pp}$ is the proton plasma frequency, is $\approx 500$ cm. Here, we adopted $\zeta = 10^{-4}$ and considered dependencies for various spectral indices $q$ of the whistler turbulence. Thus, $\zeta = 10^{-4}$ requires $q \approx 2$. Given that the curves scale linearly with $\zeta$, for $q = 5/3$, as in panel (a), we need only $\zeta \approx 3 \times 10^{-5}$ to ensure the diffusion time is of the order of 2 minutes, as observed.
Then, in the decay phase, the apparent evolution pattern changes: there is no big change in the spatial distribution of the non-thermal electrons, while the total number of the non-thermal electrons slowly declines, which is most likely a result of the same regime of strong diffusion but after the electron injection is off.

5. Concluding Remarks

The study performed here reveals an important evolution of the physical parameters in two highly different flares—a dense single-loop flare and a multi-loop flare that involved both dense and tenuous loops. It is interesting that in the rise phase of the dense flare, the main evolution comes from the changing source geometry, primarily an increase of the flaring loop volume. In contrast, around the peak phases of both flares, the most significant evolution occurs in the non-thermal electron population, while the flux tubes do not show any noticeable change.

It is further instructive, that the observed parameter evolution requires wave–particle interaction of the electrons with turbulence, perhaps composed of the whistler waves. In the dense flare, a modest level of turbulence is needed to support a non-thermal electron population against rapid Coulomb losses in the dense loop. The same turbulence might be responsible for the decimetric component of the radio burst, likely produced by the RTR emission mechanism. Remarkably, the model of stochastic electron acceleration by whistler turbulence is quantitatively consistent with all available observational constraints.

In the multi-loop flare, the considered evolution of the microwave emission constrained mainly the non-thermal electron behavior in only one (large loop; loop #4) of the four flaring loops. Therefore, we obtained detailed 3D information about the non-thermal electron trapping in this large loop. We found that a strong diffusion regime with the non-thermal electron mfp $\lambda \sim 1.5 \times 10^5$ cm is needed to interpret a relatively slow spatial evolution of the non-thermal electron cloud in this loop.

Therefore, our study confirms the central role of turbulence in electron acceleration and transport in solar flares.

This work was supported in part by NSF grants AGS-1817277 and AST-1615807; and NASA grants NNX14AC87G, 80NSSC18K0015, and 80NSSC18K0667 to the New Jersey Institute of Technology. We acknowledge the support of the 2017 NJIT-CSTR Summer High School Research Internship Program provided by Harris Geospatial Solutions, which provided free IDL licenses to all participants for the duration of the program.

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