Nanofiber-based atom trap created by combining fictitious and real magnetic fields

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Abstract
We propose a trap for cold neutral atoms using a fictitious magnetic field induced by a nanofiber-guided light field in conjunction with an external magnetic bias field. In close analogy to magnetic side-guide wire traps realized with current-carrying wires, a trapping potential can be formed when applying a homogeneous magnetic bias field perpendicular to the fiber axis. We discuss this scheme in detail for laser-cooled cesium atoms and find trap depths and trap frequencies comparable to the two-color nanofiber-based trapping scheme but with one order of magnitude lower power of the trapping laser field. Moreover, the proposed scheme allows one to bring the atoms closer to the nanofiber surface, thereby enabling efficient optical interfacing of the atoms with additional light fields. Specifically, optical depths per atom, $\frac{\sigma_0}{A_{\text{eff}}}$, of more than 0.4 are predicted, making this system eligible for nanofiber-based nonlinear and quantum optics experiments.

1. Introduction

Trapping and optically interfacing cold neutral atoms in the near field of nanophotonic structures have attracted considerable attention in recent years [1–4]. A successful and highly promising approach in this endeavor relies on the use of optical dipole forces of a blue- and a red-detuned nanofiber-guided light field in order to form a so-called two-color trap [5, 6]. This scheme has...
been demonstrated experimentally for laser-cooled cesium, storing the atoms at about 200 nm above the nanofiber surface [1, 2]. Other types of nanofiber-based traps for cold atoms have been discussed theoretically, relying on, e.g. the combination of an attractive potential of a red-detuned field and the repulsive potential of the centrifugal force [7], the interference of higher-order modes [8, 9], a diffracted laser field impinging perpendicularly to the fiber [10] or by modifying the two-color scheme to form a helical trapping potential [11].

In this paper, we propose a nanofiber-based trap for cold neutral atoms that relies on a Zeeman-state-dependent energy level shift induced by a combination of a nanofiber-guided light field and an externally applied magnetic bias field. Our work has close conceptual ties with optically induced hybrid traps on atom chips [12] and is focused on exploiting the advantages offered by optical nanofibers. One of the favorable aspects compared to chip-based trapping schemes comprises the option to efficiently interface the trapped atoms with additional guided light fields. Optical nanofibers allow one to simultaneously achieve a large optical depth per atom and an efficient coupling of many atoms to the same optical mode. Specifically, the trapping scheme proposed here features a lower required power of the guided trapping light compared to the two-color trap as well as a smaller distance between the atoms and the fiber surface. This enables a better coupling of the trapped atoms to additional light fields. Moreover, the proposed scheme is closely related to magnetic wire traps, allowing one to benefit from the well-established methods developed in this field.

2. Underlying concept

We consider a cylindrical dielectric waveguide of refractive index $n$ and radius $a$ surrounded by vacuum. For a sufficiently small radius $a$, typically a few hundred nanometers, this system acts as a single-mode waveguide for light. The only sustained mode of this optical nanofiber is the hybrid HE$_{11}$ mode [13]. Light propagating in optical nanofibers is strongly guided and, hence, exhibits a significant longitudinal polarization component. In addition, a large fraction of the total optical power $P$ of the guided field propagates outside of the nanofiber in the form of an evanescent wave [14]. We sketch a typical intensity distribution for a quasi-circularly polarized nanofiber-guided mode in figure 1(a).

Generally, the energy levels of an atom exposed to a light field are modified due to the ac-Stark effect. For alkali atoms in their electronic ground state, this light shift has two contributions arising from the scalar and vector polarizabilities of the atom. The scalar light shift is the same for all Zeeman states within one hyperfine manifold. Here, we make this contribution vanish by using a nanofiber-guided trapping field at a ‘tune-out’ wavelength [15]. For $^{133}$Cs atoms, this is achieved at $\lambda_{\text{trap}} = 880.25$ nm, which is in between the D1 line ($\approx 894$ nm) and the D2 line ($\approx 852$ nm) such that their contributions to the scalar shift exactly cancel. The remaining effect of the nanofiber-guided light field on the atomic levels arises from the vector light shift only. The latter depends on the magnetic quantum number $m_F$ and can be expressed as the effect of a light-induced fictitious magnetic field [16, 17]

$$\mathbf{B}^{\text{det}} = \frac{\alpha_{nJF}^\nu}{8\mu_B g_{nJF} E} [\mathbf{E}^* \times \mathbf{E}] .$$

(1)

Here, $\mu_B$ is the Bohr magneton, $g_{nJF}$ is the Landé factor for the hyperfine level $|nJF\rangle$, $\alpha_{nJF}^\nu$ is the conventional vector polarizability of the particular hyperfine level and $\mathbf{E}$ is the positive-frequency electric field envelope for a light field, which is related to the electric field $\mathbf{E}$ by
Figure 1. (a) Schematic representation of the setup including an illustration of the intensity distribution of a nanofiber-guided light field propagating in the $+z$ direction with quasi-circular polarization. The direction in which to apply the external homogeneous magnetic bias field ($-x$ direction) as required for trapping is also indicated (green arrows). (b) Schematic vector profile of the fictitious magnetic field $B^\text{fict}$ induced by the light field shown in (a) with counter-clockwise polarization. A cesium atom in the $6S_{1/2}$ ground state is assumed and the wavelength of the guided field lies between the cesium D-lines. (c) Radial dependence of the components $B^\text{fict}_\phi$ (blue curve) and $B^\text{fict}_z$ (red curve) of the fictitious magnetic field. Calculations are for a guided field at $\lambda_{\text{trap}} = 880.25$ nm of optical power $P = 1.2$ mW and a radius of the nanofiber of $a = 230$ nm.

\[ E = \frac{1}{2} (Ee^{-i\omega t} + \text{c.c.}) \]

The magnitude of the light-induced fictitious magnetic field $B^\text{fict}$ is proportional to the intensity of the light field, and the direction of $B^\text{fict}$ is determined by the sign of $\alpha'_{n J F}$ and by the cross product $i[E^* \times E]$. If the light field exhibits a polarization with nonzero ellipticity, the fictitious magnetic field points in a direction perpendicular to the plane defined by the circulating electric field vector. For linearly polarized light, $B^\text{fict} = 0$.

In figure 1(b), we show schematically the fictitious magnetic field $B^\text{fict}$ induced by a quasi-circularly counter-clockwise polarized light field guided in the HE$_{11}$ mode [18]. The fictitious field $B^\text{fict}$ can be decomposed into two components, azimuthal and axial. The azimuthal ($\varphi$-) component (blue arrows) has the same orientation as the magnetic field around a current-carrying wire. The axial ($z$-) component of $B^\text{fict}$ (red arrows) is anti-parallel to the direction of propagation of the guided field. This component has no equivalent in current-carrying wires. Furthermore, the radial ($r$-) dependence of $B^\text{fict}$ differs from the $1/r$-behavior encountered in the case of current-carrying wires. In figure 1(c), we plot the azimuthal and axial components.
of $B^{\text{fict}}$ as a function of the radial position. Analytically, $B^{\text{fict}}$ is given by

$$B^{\text{fict}} = \frac{\alpha v_n j_F}{4 \mu_B g_{nJF}} \text{Im}(E_z E^*_\phi) \hat{\phi} + \text{Im}(E_r E^*_\phi) \hat{\phi},$$

(2)

where $E = (E_r, E_\phi, E_z)$

$$E = A (\hat{r} e_r + \hat{\phi} e_\phi + \hat{z} e_z) \exp(i \beta z + i \phi).$$

(3)

Here, $(\hat{r}, \hat{\phi}, \hat{z})$ are the unit vectors and $e(r)$ is the unnormalized mode-profile vector function of the electric part of the fundamental guided mode HE$_{11}$. The components of the latter are given by $[13]

$$e_r = i [(1 - s) K_0(qr) + (1 + s) K_2(qr)],
$$

$$e_\phi = - [(1 - s) K_0(qr) - (1 + s) K_2(qr)],
$$

$$e_z = \frac{2q}{\beta} K_1(qr).$$

(4)

The above equations for $e(r)$ are valid for $r > a$, i.e. outside of the nanofiber. The parameter $s$ is defined as $s = (1/h^2 a^2 + 1/q^2 a^2)/[J'_n(ha)/ha J_n(ha) + K'_n(qa)/qa K_n(qa)]$ with $h = (n^2 k^2 - \beta^2)^{1/2}$ and $q = (\beta^2 - n^2 k^2)^{1/2}$. The notations $J_n$ and $K_n$ stand for the Bessel functions of the first kind and the modified Bessel functions of the second kind, respectively. Other parameters are the propagation constant $\beta$ of the guided field, and the free-space wave number $k$ of the field.

The normalization constant $A$ in equation (3) can be determined from the optical power of the nanofiber-guided light field.

For an atom in the electronic ground state, the fictitious magnetic field behaves in almost every respect like a real magnetic field $[16-18]$. In particular, both fictitious and real magnetic fields are pseudo-vectors. Thus, the fictitious magnetic field $B^{\text{fict}}$ can be vector-added to any static real magnetic field $B$, such that the atom is in total exposed to the effective magnetic field $B^{\text{eff}} = B^{\text{fict}} + B$. This has been found in early work by Cohen-Tannoudji and Dupont-Roc $[16]$, and further supported in various experiments, demonstrating, e.g. optically induced spin precession in an atomic beam $[19]$.

Based on these considerations and in analogy to the side-guide traps using current-carrying wires $[20-22]$, we propose to create a nanofiber-based magnetic trap for neutral atoms. For this purpose, we have to apply a real external homogeneous magnetic bias field $B_{\text{bias}}$ perpendicular to the axis of the nanofiber in figure 1. This can straightforwardly be achieved, e.g. by switching the coils of a magneto-optical trap from the anti-Helmholtz to the Helmholtz configuration. Then, similar to conventional side-guide wire traps, low-field-seeking atoms can be confined in the $(x, y)$-plane around a line of minimal magnetic field $B^{\text{eff}}$ that forms parallel to the nanofiber axis. The shift of the internal-state energy of a paramagnetic atom due to the effective magnetic field $B^{\text{eff}}$ is given by

$$U_{\text{mag}} = -\mu \cdot B^{\text{eff}},$$

(5)
Figure 2. Contour plot of the potential energy of a cesium atom in the nanofiber-based magnetic trap. The cross section of the nanofiber is marked as a solid gray disc at the bottom of the figure. The unit of the contour labels is $k_B \mu K$ where $k_B$ is Boltzmann’s constant. Parameters are as in figure 1(c); a magnetic bias field $B_{\text{bias}} = 22 \text{ G}$ is applied in the $-x$-direction. The van der Waals potential induced by the nanofiber is taken into account as $U_{\text{vdW}} = -C_3/(r-a)^3$ with $C_3 = 5.6 \times 10^{49} \text{ J m}^3$.

where $\mu$ is the magnetic moment of the atom. If $\mu$ can adiabatically follow the direction of the local effective magnetic field while the atom moves within the potential, then equation (5) simplifies to

$$U_{\text{mag}} = \mu B_{\text{g}} n_{gf} m_F |B_{\text{eff}}|.$$  

(6)

For the trap presented here, the separation between the atom and the nanofiber is on the order of a few hundred nanometers and surface effects have to be taken into account [23]. We include the latter into our calculations using a van der Waals potential of the form $U_{\text{vdW}} = -C_3/(r-a)^3$ with $C_3 = 5.6 \times 10^{49} \text{ J m}^3$ for cesium [6], which describes the potential of a cesium atom in front of a flat silica surface. When the distance of the atom with respect to the surface is much smaller than the radius of the fiber and the wavelengths of the dominant optical transitions of the atom, this potential constitutes a good approximation. For larger distances, this model is a conservative estimate in the sense that it over-estimates the true surface-induced potential. Therefore, if a trap can be formed assuming a flat-surface van-der-Waals potential, then this can also be accomplished taking into account the true potential instead. The total potential energy shift of the atom is then given by $U = U_{\text{mag}} + U_{\text{vdW}}$.

We now discuss the characteristics of the nanofiber-based magnetic trap. Specifically, we calculate the potential energy $U$ of a cesium atom in the state $|6S_{1/2}, F = 4, m_F = 4\rangle$. Figure 2 shows the trapping potential calculated for a silica nanofiber with refractive index $n = 1.45$, radius $a = 230 \text{ nm}$, a guided trap light field with a power of $1.2 \text{ mW}$ and a homogeneous magnetic bias field of $22 \text{ G}$, applied along the $-x$-direction. A local minimum of $U$ is formed about $150 \text{ nm}$ above the nanofiber surface at $(x = 0, y_0 \approx 150) \text{ nm}$. The depth of the potential $U_0/k_B \approx 300 \mu K$ in figure 2 is sufficient for storing laser-cooled cesium atoms. Note that, in contrast to conventional wire traps, the azimuthal component of the fictitious magnetic field and the applied bias field do not completely cancel each other at the minimum of the trapping potential. This is connected to the presence of the spatially varying $z$-component of the fictitious
magnetic field, $B_{\text{fict}}$. When neglecting surface effects, the local minimum of the potential in the fiber transverse plane, $U(r, \varphi)$, can be determined from the condition $\nabla |B_{\text{eff}}(r, \varphi)| = 0$. For a bias field oriented along the $-x$-direction, the minimum of the potential in the radial direction can be found by solving

$$0 = (B_{\text{fict}}^z(r) + B_{\text{bias}} \sin \varphi) \partial_r B_{\text{fict}}^z(r) + B_{\text{fict}}^\varphi(r) \partial_r B_{\text{fict}}^{\varphi}(r).$$

As opposed to conventional wire traps, the term $\partial_r B_{\text{fict}}^z(r)$ generally differs from zero in the case of the nanofiber-based magnetic trap.

3. Discussion of properties and parameters

We now investigate the dependence of the nanofiber-based magnetic trap on various parameters that can be readily varied experimentally. The distance between the trap minimum and the fiber surface can be influenced by altering the ratio of the strengths of the fictitious field and the bias field. In figure 3, the trapping potential $U(x = 0, y)$ is shown for a fixed optical power $P = 1.2 \text{ mW}$ and different strengths of the bias field in the range $B_{\text{bias}} = 10, \ldots, 34 \text{ G}$ for increments of $3 \text{ G}$. For each calculated potential, we add an energy offset such that $U(x = 0, y \to \infty) = 0$. For increasing $B_{\text{bias}}$, the separation between the trap minimum and the surface of the fiber reduces. For the trap configurations with $B_{\text{bias}} = \{25, 28, 31, 34\} \text{ G}$, the relative contribution of $U_{\text{vdW}}$ to the total trapping potential is so large that the potential opens toward the surface of the nanofiber. Changing the trap depth while keeping the position of the trap minimum approximately constant can be achieved by scaling $P$ and $B_{\text{bias}}$ proportionally. In figure 4, the trapping potential $U(x = 0, y)$ is shown for $P = \kappa 1.2 \text{ mW}$ and $B_{\text{bias}} = \kappa 22 \text{ G}$ with $\kappa = \{0.5, 1, 1.5, 2, 2.5 \text{ and } 3\}$.

Table 1 lists the trap depth $U_0$, the separation $y_0 - a$ between the trap minimum and the surface, and the radial and azimuthal trap frequencies ($\omega_r$, $\omega_\varphi$) for five example trapping configurations. The traps (a)–(d) have a depth that is suitable for storing laser-cooled atoms and exhibit trapping frequencies between a few ten kHz and a few hundred kHz. Configuration (e) exhibits a smaller trap depth, aimed for the storage of ultracold atoms. As trapping of ultracold clouds in close proximity to a hot nanofiber remains to be demonstrated experimentally,
Figure 4. Trapping potential $U(x=0,y)$ for $P = \kappa \times 1.2 \text{ mW}$ and $B_{\text{bias}} = \kappa \times 22 \text{ G}$ with $\kappa = \{0.5, 1, 1.5, 2, 2.5 \text{ and } 3\}$. The black solid line corresponds to $\kappa = 0.5$, the magenta solid line corresponds to $\kappa = 3$. The trap depth is changing while the position of the trap minimum is approximately constant. The common crossing point of all potential lines results from the chosen presentation of the data in conjunction with the proportional scaling of $P$ and $B_{\text{bias}}$ by the factor $\kappa$.

Table 1. Example configurations of the nanofiber-based trap and respective trap parameters: $P$ optical power of the guided trapping field, $B_{\text{bias}}$ magnetic bias field, $U_0/k_B$ trap depth, $y_0 - a$ separation between trap minimum and nanofiber surface, $\omega_r$ radial trap frequency, $\omega_\phi$ azimuthal trap frequency, $\sigma_0/A_{\text{eff}}$ optical depth per atom, $\Gamma_{\text{sf}}$ spin flip rate and $\Gamma_{\text{exc}}$ trapping-light-induced excitation rate.

| $P$ (mW) | $B_{\text{bias}}$ (G) | $U_0/k_B$ (µK) | $y_0 - a$ (nm) | $\omega_r/(2\pi)$ (kHz) | $\omega_\phi/(2\pi)$ (kHz) | $\sigma_0/A_{\text{eff}}$ (s$^{-1}$) | $\Gamma_{\text{sf}}$ (s$^{-1}$) | $\Gamma_{\text{exc}}$ (s$^{-1}$) |
|----------|----------------------|----------------|----------------|--------------------------|--------------------------|--------------------------|----------------|----------------|
| (a) 1.2   | 16                   | 210            | 189            | 247                      | 67                      | 0.20                     | $8 \times 10^{-7}$ | 16.0          |
| (b) 1.2   | 22                   | 310            | 148            | 307                      | 92                      | 0.30                     | $7 \times 10^{-8}$ | 23.4          |
| (c) 1.2   | 28                   | 150            | 115            | 357                      | 119                     | 0.43                     | $6 \times 10^{-9}$ | 32.1          |
| (d) 2.4   | 44                   | 608            | 150            | 433                      | 128                     | 0.30                     | $2 \times 10^{-13}$ | 46.0          |
| (e)$^a$ 0.165 | 2.6             | 7.8            | 227            | 43                       | 17                      | 0.14                     | $5 \times 10^{-10}$ | 1.6           |

$^a$ Indicates the presence of an additional magnetic offset field along $-z$ of 1 G.

(e) provides a means to study this problem. The configuration for ultracold atoms requires a lower trapping laser power of only $P = 165 \mu$W and yields trap frequencies of a few ten kHz. In all five cases, the separation between the trap minimum and the surface of the fiber is between about 100 and 230 nm.

A small distance between the trap minimum and the surface of the nanofiber allows one to efficiently interface the atoms with additional guided light fields. A simple and common measure for the strength of this coupling is given by the resonant optical depth per atom $\sigma/A_{\text{eff}}$ [1, 2, 24]. Generally, the on-resonance cross section of the atom is given by $\sigma = \hbar \omega \Gamma/(2I_{\text{sat}})$ with the decay rate of the excited state $\Gamma$ and the saturation intensity $I_{\text{sat}}$. Here, we approximate $\sigma$ as $\sigma_0 = 3\lambda^2/2\pi$, which is the on-resonance cross section of the atom driven on a cycling transition in free space. We note that this description neglects a possible increase of the spontaneous emission rate of the trapped atom due its close proximity to the fiber surface [25–27]. The effective mode area $A_{\text{eff}}$ of a guided probe field at the trap minimum $(x=0,y_0)$ is given by $A_{\text{eff}} = P_{\text{probe}}/I_{\text{probe}}$, with $P_{\text{probe}}$ being the power of the guided probe field.
and \( I_{\text{probe}} = c \varepsilon_0 \langle |E(t)|^2 \rangle_T \) the intensity of the probe field at the trap minimum. In the definition of the intensity, \( c \) is the speed of light in vacuum, \( \varepsilon_0 \) is the electric permittivity and \( \langle \ldots \rangle \) denotes the time average of the square of the absolute value of the electric field over one oscillation period. In table 1, \( \sigma_0/A_{\text{eff}} \) is evaluated for a quasi-linearly polarized probe light field with its plane of polarization orientated such that the intensity at the position of the atoms is maximized [14], i.e. with its main plane of polarization containing the fiber axis and the trap minima. The field is chosen to be resonant with the cesium D2 cycling transition \( F = 4 \rightarrow F' = 5 \) at a wavelength of 852 nm. All configurations show large values of \( \sigma_0/A_{\text{eff}} \), with the weakest coupling being 0.14 (e) and the strongest 0.43 (c). This significantly exceeds the optical depth per atom of 0.08 that has been obtained experimentally with the two-color nanofiber-based trapping scheme [2] and is a result of the smaller trap–surface distance that can be achieved in the case of the nanofiber-based magnetic trap.

Cold atoms trapped in an inhomogeneous magnetic field can undergo spin flips [28] when their magnetic moment does not adiabatically follow the orientation of the local magnetic field. An atom can, for example, undergo a transition from a trapped (low-field-seeking) state to an untrapped (magnetic-field-insensitive or high-field-seeking) state and be lost from the trap. This effect becomes significant if atoms move through regions of small or even vanishing magnetic field, e.g. in a linear or spherical magnetic quadrupole field. Spin flips can be efficiently suppressed by an offset magnetic field or by using time-averaged potentials. In order to quantify the effect of spin flips in our system, we calculate the spin flip rate \( \Gamma_{\text{sf}} \) for the trapping configurations (a)–(e) using the formula [28, 29]

\[
\Gamma_{\text{sf}} = \frac{\pi \omega_t}{2} \exp \left( -\frac{\pi E_0}{2\hbar \omega_t} \right),
\]

where \( \omega_t \) is the trap frequency, \( \hbar \) is the reduced Planck constant and \( E_0 = \mu_B g_n J F |B| \) is the energy gap between the potential branches for the different states. For all example configurations, we obtain negligible spin flip rates as summarized in table 1. In order to perform a conservative estimation, the larger of the two trap frequencies, \( \omega_y \), has been used for the calculations. Advantageously, the axial component of the fictitious magnetic field shown in figure 1 acts as an integrated offset field for the nanofiber-based magnetic trap, thereby intrinsically suppressing spin flips. This intrinsic offset field can be changed by varying the fiber radius \( a \), thereby changing the local polarization of the guided trapping laser field and, thus, the ratio between the azimuthal and axial components of \( B_{\text{local}} \). Additionally, the offset field can be modified using an axial component of the external magnetic field as it is typically done for conventional magnetic wire traps. This method was applied for example (e) in table 1.

An atom trapped in state \( |F = 4, m_F = 4\rangle \) may also undergo a change of its hyperfine- and/or Zeeman state due to spontaneous Raman scattering of trapping light. In table 1, we calculate the light-induced excitation rate \( \Gamma_{\text{exc}} \) according to (see the appendix)

\[
\Gamma_{\text{exc}} = I_{\text{trap}}(\eta_s + (-1)^{F-l+1/2} m_F C \eta_s),
\]

with \( I_{\text{trap}}(x = 0, y_0) \) being the intensity of the trapping field at the trap minimum, \( \eta_s = 0.2446 \) and \( \eta_s = 2.860 \times 10^{-2} \text{kHz cm}^2 \text{MW}^{-1} \) the scalar and vector scattering coefficients, \( F \) and \( I \) the conventional quantum numbers of the total spin and the spin of the atomic core, respectively, and \( C \) the ellipticity of component of trapping field in the fiber transverse plane. The rate \( \Gamma_{\text{exc}} \) is a worst-case estimate as spontaneous scattering does not necessarily change the atomic state. However, we expect inelastic scattering to be enhanced because the wavelength of the nanofiber-based magnetic trap lies in between the two D-lines [30]. The values obtained for
the configurations (a)–(e) are on the order of \(1–10 \text{ s}^{-1}\) which, despite the smaller detuning of the trapping field, is comparable to the rates for the two-color trap realized in [1]. This is due to the low intensity of the trapping light at the position of the atoms as compared to the two-color scheme. Note that optical-wire-trapping of alkali atoms other than cesium should be feasible, too. However, for the same trap depths, the scattering rate will be larger by a factor of about 3, 12 and 67 for rubidium, potassium and sodium, respectively [31].

The potentials presented above are translationally invariant along the fiber and thus form a guide for atoms parallel to the fiber axis. Additional axial confinement can be straightforwardly provided by means of an externally applied inhomogeneous magnetic field. Moreover, counter-propagating fiber-guided fields can yield a periodic axial modulation of the fictitious magnetic field [18] and, thus, might allow one to form a periodic array of trapping sites.

4. Conclusion

In summary, we proposed a novel nanofiber-based trapping scheme and analyzed important trap parameters for cesium atoms in detail. The scheme should be feasible for other alkalies, too, and generally for atoms that exhibit a zero scalar but finite vector polarizability of the ground state for the trapping light field at an appropriately chosen wavelength. Our nanofiber-based magnetic trap features depths which are sufficient to store laser-cooled atoms while using optical powers of the trapping field which are one order of magnitude smaller than these for a typical two-color nanofiber-based trap. Thanks to the close analogy of our scheme to conventional wire traps for paramagnetic atoms, established techniques from this field such as the adiabatic transformation of trapping potentials for trap loading [21], evaporative cooling of the atoms [32, 33] or the interrogation of the atomic ensemble using dispersive light fields [32, 34] should be applicable for the nanofiber-based magnetic trap. Our scheme can access trap parameters comparable to the steepest wire traps formed close to miniaturized wires (see [35, 36] for reviews) such as current-carrying carbon nanotubes [37–39], thereby opening up opportunities for studying surface physics [23] and one-dimensional matter wave dynamics [40, 41]. Additional magnetic trapping configurations might arise due to the fact that fictitious magnetic fields can also have a local maximum in free space which is not possible for real magnetic fields [42]. In contrast to conventional wire traps, surface-induced spin flips due to Johnson noise [43] will be negligible in our system [44], even for small separations of the atoms to the dielectric surface of the nanofiber. For typical configurations of the nanofiber-based magnetic trap, the local minima of the potential are about 100–230 nm away from the fiber surface which is a key advantage when it comes to optically interfacing trapped atoms with additional nanofiber-guided light fields. In our system, optical depths per atom of up to about 0.4 are accessible, opening a realm of applications in nanofiber-based nonlinear and quantum optics.

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Appendix A. Scattering of light of arbitrary polarization from an atom

A.1. Scattering of light from a two-level atom

We consider a two-level atom interacting with a far-off-resonance light field of arbitrary polarization. The electric component of the light field is

\[ E = \frac{1}{2} \mathcal{E} e^{-i\omega t} + \text{c.c.} = \frac{1}{2} \mathcal{E} u e^{-i\omega t} + \text{c.c.}, \]  

where \( \omega \) is the angular frequency and \( \mathcal{E} = \mathcal{E} u \) is the positive-frequency electric field envelope, with \( \mathcal{E} \) and \( u \) being the field amplitude and the polarization vector, respectively. In general, \( \mathcal{E} \) is a complex scalar and \( u \) is a complex unit vector.

Let \( |a\rangle \) and \( |b\rangle \) be the lower and upper eigenstates of the atom, respectively, with unperturbed energies \( E_a = \hbar \omega_a \) and \( E_b = \hbar \omega_b \), respectively. The interaction between the atom and the field is described, in the dipole approximation, by the operator \( H_{AF} = -\mathbf{E} \cdot \mathbf{d} \), that is,

\[ H_{AF} = -\frac{1}{2} (\mathcal{E} u e^{-i\omega t} + \mathcal{E}^* u^* e^{i\omega t}) \cdot (d_{ba} |b\rangle \langle a| + d_{ab} |a\rangle \langle b|). \]  

Here \( d_{ba} = \langle b|d|a\rangle \) and \( d_{ab} = \langle a|d|b\rangle = d_{ba}^* \) are the matrix elements of the operator \( d = d_{ba} |b\rangle \langle a| + d_{ab} |a\rangle \langle b| \) for the electric dipole of the atom. The evolution of the density matrix \( \rho \) of the atom is governed by the equations

\[ \dot{\rho}_{aa} = -\frac{i}{\hbar} (\mathcal{E} u e^{-i\omega t} + \mathcal{E}^* u^* e^{i\omega t}) \cdot (d_{ba} \rho_{ab} - d_{ab} \rho_{ba}) + \gamma_b \rho_{bb}, \]  

\[ \dot{\rho}_{ba} = -i(\omega_b - \omega_a - i\gamma_b/2) \rho_{ba} - i\frac{1}{\hbar} (\mathcal{E} u e^{-i\omega t} + \mathcal{E}^* u^* e^{i\omega t}) \cdot d_{ba} (\rho_{bb} - \rho_{aa}), \]  

where \( \gamma_b \) is the lifetime of the atomic upper level \( |b\rangle \).

We assume that the atom is initially in the lower level \( |a\rangle \). We consider the case where the magnitude of the detuning \( \omega - (\omega_b - \omega_a) \) is large as compared to the atomic decay rate \( \gamma_b \) and to the magnitude of the Rabi frequency \( \Omega = d_{ba} \mathcal{E}/\hbar \). In this case, we have \( \rho_{aa} \simeq 1 \) and \( \rho_{bb} \simeq 0 \). In the adiabatic approximation, we have \( \dot{\rho}_{aa} \simeq 0 \) and \( \dot{\rho}_{ba} \simeq 0 \). With these assumptions and approximations, we solve equations (A.3) in the quasistationary regime. We use the formula \( \Gamma_{sca} = \gamma_b \rho_{bb} \) to calculate the scattering rate. Then, we find

\[ \Gamma_{sca} = \frac{|\mathcal{E}|^2}{2\hbar^2} \text{Im} \left( \frac{|\langle b|u \cdot d|a\rangle|^2}{\omega_{ba} - \omega - i\gamma_b/2} + \frac{|\langle a|u \cdot d|b\rangle|^2}{\omega_{ba} + \omega - i\gamma_b/2} \right). \]  

Another method to estimate the scattering rate is given in the following: the time-averaged power absorbed by the atom is the work that the field does on the induced dipole and is given by

\[ W = \langle \mathbf{d}(t) \cdot \mathbf{E}(t) \rangle. \]  

In the classical picture, the energy of the driving field is dissipated continuously. However, in the quantum mechanical picture the photons in the driving field are scattered by the atom. When we use the formula \( \Gamma_{sca} = W/\hbar \omega \) to estimate the scattering rate, we obtain

\[ \Gamma_{sca} = \frac{|\mathcal{E}|^2}{2\hbar^2} \text{Im} \left( \frac{|\langle b|u \cdot d|a\rangle|^2}{\omega_{ba} - \omega - i\gamma_b/2} + \frac{|\langle a|u \cdot d|b\rangle|^2}{\omega_{ba} + \omega + i\gamma_b/2} \right). \]  

10
We note that the second terms in equations (A.4) and (A.6) are slightly different from each other. Indeed, the signs in front of \(i\gamma_b/2\) in these terms are opposite to each other. This difference is a consequence of the fact that the derivations of equations (A.4) and (A.6) are based on the population attenuation and the energy attenuation, respectively. Meanwhile, the second terms in equations (A.4) and (A.6) describe the contributions from the counter-rotating terms of the interaction Hamiltonian to the scattering rate. Due to the counter-rotating interaction terms, the total excitation number of the combined atom–field system is not conserved. This is the reason why the two methods give different results. Note that, since \(\gamma_b \ll \omega, \omega_{ba}\), the second terms in equations (A.4) and (A.6) and, consequently, the difference between them are very small. We neglect these terms, that is, we adopt the expression

\[
\Gamma_{sca} = \frac{|\mathcal{E}|^2}{2\hbar} \text{Im} \left( \frac{|\langle b|\mathbf{u} \cdot \mathbf{d}|a\rangle|^2}{\omega_{ba} - \omega - i\gamma_b/2} \right)
\]

(A.7)

for the scattering rate of light from the two-level atom.

A.2. Scattering of light from a multilevel atom

Consider now a multilevel atom interacting with a far-off-resonance light field of arbitrary polarization. Assume that the atom is initially in a ground-state level \(a\). We generalize expression (A.7) to this case. Then, the scattering rate is given by

\[
\Gamma_{sca} = \frac{|\mathcal{E}|^2}{2\hbar} \text{Im} \sum_b \frac{|\langle b|\mathbf{u} \cdot \mathbf{d}|a\rangle|^2}{\omega_{ba} - \omega - i\gamma_b/2},
\]

(A.8)

where the summation is performed over all upper levels \(b\). We do not take into account the ac Stark shift and the hyperfine structure (hfs) splitting in frequency. However, we take into account the hfs by using the hfs basis.

We assume that initial level \(a\) is a magnetic sublevel \(|nJ F_m\rangle\) of the ground state \(|nJ\rangle\). We can write

\[
\Gamma_{sca} = \langle a|\mathcal{G}|a\rangle,
\]

(A.9)

where

\[
\mathcal{G} = -\frac{|\mathcal{E}|^2}{2\hbar} (\mathbf{u}^* \cdot \mathbf{d}) \mathcal{I} (\mathbf{u} \cdot \mathbf{d})
\]

(A.10)

with

\[
\mathcal{I} = -\frac{1}{\hbar} \sum_b \text{Im} \left( \frac{1}{\omega_{ba} - \omega - i\gamma_b/2} \right) |b\rangle \langle b|.
\]

(A.11)

When we use the Wigner–Eckart theorem [45] and perform some spherical-tensor calculations, we find

\[
\Gamma_{sca} = -\frac{|\mathcal{E}|^2}{2\hbar} \sum_{K = 0,1,2} (-1)^K \langle \mathbf{u}^* \otimes \mathbf{u} \rangle_{K,0} (-1)^{F-m_F} \begin{pmatrix} F & K & F \\ -m_F & 0 & m_F \end{pmatrix} \tilde{\rho}^{(K)}_F,
\]

(A.12)

where

\[
\tilde{\rho}^{(K)}_F = (-1)^{J+l+F+K} (2F+1) \left\{ \begin{array}{ccc} F & K & F \\ J & I & J \end{array} \right\} \rho^{(K)}_J
\]

(A.13)
with
\[ \beta_{J}^{(k)} = (-1)^{J+k+1} \sqrt{2K+1} \sum_{n'J} (-1)^{J'} \left[ \begin{array}{cc} K & 1 \\ J & J' \end{array} \right] |\langle n'J'|d|nJ \rangle|^2 \frac{1}{\hbar} \text{Im} \left( \frac{1}{\omega_{n'J'} - \omega_{nJ} - \omega - i \gamma_{n'J'}/2} \right). \]

(A.14)

Here, \( |\langle nJ|d|n'J' \rangle| = (-1)^{J-J'} |\langle n'J'|d|nJ \rangle| \) are the reduced matrix elements of the dipole moment of the atom in the \( J \) basis.

We consider the case of alkali-metal atoms, where \( J = 1/2 \) and consequently \( \beta_{J}^{(2)} = 0 \). In this case, we can simplify equation (A.12) to the form
\[ \Gamma_{\text{sca}} = I_L [\eta_s + (-1)^{F-l+1/2} m_F C \eta_{vc}]. \]

(A.15)

This quantity determines the off-resonant excitation rate of the atom and is therefore denoted as \( \Gamma_{\text{exc}} \) in the main manuscript. Here we have introduced the notation \( I_L = c_0 |E|^2/2 \) for the conventional intensity of the light field and the notation \( C = 2 \text{Im}(u^*_x u_y) = -i[u^* \times u]_y \) for the ellipticity of the component of the field in the \( (x, y) \) plane. The coefficients \( \eta_s \) and \( \eta_{vc} \) in equation (A.15) are given by the expressions
\[ \eta_s = \frac{1}{6h^2 c_0} \sum_{n'J} |\langle n'J'|d|nJ \rangle|^2 \text{Im} \left( \frac{1}{\omega_{n'J'nJ} - \omega - i \gamma_{n'J'}/2} \right), \]
\[ \eta_{vc} = \frac{1}{6h^2 c_0} \sum_{n'J} \left[ \frac{11}{4} - J'(J' + 1) \right] |\langle n'J'|d|nJ \rangle|^2 \text{Im} \left( \frac{1}{\omega_{n'J'nJ} - \omega - i \gamma_{n'J'}/2} \right). \]

(A.16)

It is clear that \( \eta_s \) and \( \eta_{vc} \) do not depend on \( F \).

In equation (A.15), the terms associated with the coefficients \( \eta_s \) and \( \eta_{vc} \) have the scalar and vector nature, respectively. Therefore, we call \( \eta_s \) and \( \eta_{vc} \) the scalar and vector scattering coefficients, respectively.

**Appendix B. Transformation of trapping potentials**

Here we present a preliminary study on the possibility of transforming a two-color nanofiber-based trap \([1, 2]\) into a nanofiber-based magnetic trap. We take a simplified approach where we model the two-color trap by taking into account solely the scalar polarizability of the atom. For the description of the nanofiber-based magnetic trap we use the formalism outlined in the main text, i.e. we consider the vector polarizability of the atom. In order to compute the intermediate potential that is present when transforming one trap into the other, we simply sum up the contribution of the individual traps. We note that a rigorous treatment would require one to compute the spatially varying eigenenergies of the full interaction Hamiltonian that comprises the Stark interaction of the three trapping light fields, the hyperfine interaction and the magnetic interaction of the atom with the external light field.

Within the framework of the simplified model, we observe that a two-color nanofiber-based trap \([1, 2]\) can be continuously transformed into the nanofiber-based magnetic trap discussed in the main text. Given that the atoms stored in the two-color trap have been prepared in a low-field seeking Zeeman state this might allow one to load atoms into the nanofiber-based magnetic trap. In the following, we show the transformation of the trapping potentials starting...
Figure B.1. Transformation of a two-color nanofiber-based trap into a nanofiber-based magnetic trap. The panels show contour plots of trapping potential in the fiber transverse plane. Contour lines are show between $-400$ and $0 \, \mu K$ in steps of $15 \, \mu K$. Panel (A) shows a two-color trap, (B)–(E) are intermediate trapping potentials, and (F) shows the nanofiber-based magnetic trap with configuration (b) from table 1. The cross section of the nanofiber is marked as a solid gray disc at the bottom of the figure.

with a cesium atom in the state $|6S_{1/2}, F = 4, m_F = 4\rangle$ confined in a compensated two-color trap [2]. In order to reduce the chance for spin flips and to maintain the Zeeman state, a homogeneous external magnetic offset field of $1 \, \text{G}$ is applied. The orientation of this field is chosen to be in the $-z$ direction. The initial compensated two-color potential is created by a combination of a blue-detuned traveling standing-wave [2] and a conventional red-detuned standing-wave guided light field. All two-color trapping fields are quasi-linearly polarized, and the blue-detuned fields are orthogonally polarized with respect to the red-detuned fields. For our calculations, we further assume the wavelengths $\lambda_{\text{blue}} = 686 \, \text{nm}$ and $\lambda_{\text{red}} = 937 \, \text{nm}$ as well as the powers $P_{\text{blue}} = 2 \times 26.45$ and $2 \times 0.575 \, \text{mW}$ for the blue-detuned and red-detuned field, respectively. The radius of the fiber is $a = 230 \, \text{nm}$.

We provide contour plots of the trapping potential in the fiber transverse plane in figure B.1. Panel (A) shows the compensated two-color nanofiber trap with parameters as specified and a trap depth of about $310 \, \mu K$. While going from (A) to (F), the power of the blue- and the red-detuned light fields as well as the externally applied magnetic offset field in the $-z$ direction are linearly ramped to zero. Meanwhile, the power of the guided trapping laser field at the tune-out wavelength is linearly increased from zero to $P = 1.2 \, \text{mW}$. The homogeneous bias field along the $-x$ direction as required for the nanofiber-based magnetic trap is also, in parallel, switched
on linearly, finally reaching 22 G. In the end of the transformation sequence, the nanofiber-based magnetic trap with configuration (b) as specified in table 1 is formed and shown in panel (F) of figure B.1. Note that the suggested sequence is only an example and by no means optimized.

It is apparent from the figure that the depth of the trapping potential remains large during the entire transformation. In order to keeping heating as small as possible, the ramping sequence (A)–(F) should be performed slowly enough such that the atoms in the trap can adiabatically adapt to the evolving potential. Assuming no further loss mechanisms to be present during the transformation, all atoms should be transferred from the two-color trap to the nanofiber-based magnetic trap.

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