Spin-polarized transport and Andreev reflection in semiconductor/superconductor hybrid structures

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We show that spin-polarized electron transmission across semiconductor/superconductor (Sm/S) hybrid structures depends sensitively on the degree of spin polarization as well as the strengths of potential and spin-flip scattering at the interface. We demonstrate that increasing the Fermi velocity mismatch in the Sm and S regions can lead to enhanced junction transparency in the presence of spin polarization. We find that the Andreev reflection amplitude at the superconducting gap energy is a robust measure of the spin polarization magnitude, being independent of the strengths of potential and spin-flip scattering and the Fermi velocity of the superconductor.

We study theoretically spin-polarized carrier transport across semiconductor/superconductor (Sm/S) hybrid structures. Our motivation arises from the increasing current interest in spin transport in electronic materials and the possibility of using spin degrees of freedom in nanoelectronic devices. The new terminology “spintronics,” coined as an alternative to the charge based conventional electronic technology, has emerged encompassing diversified efforts to understand and exploit various aspects of spin dynamics which could lead to the design of potentially novel devices. Many of the recent advances have focused on studying Sm and hybrid Sm structures as the likely candidates for spintronic devices. Although the well established growth and materials fabrication techniques for semiconductors coupled with their tunable electronic properties, such as carrier density and Fermi velocity, provide important advantages over other materials, it is nevertheless essential to demonstrate the feasibility of creating and controlling spin-polarized carriers in semiconductors, if they are going to be utilized in proposed spintronic devices. In addition to the optically created spin-polarized electrons, it is now possible to employ several other methods to produce spin-polarized carriers in semiconductors, such as, the direct injection from a ferromagnet, using a novel class of ferromagnetic semiconductors based on Mn-doped GaAs, and utilizing the Bychkov-Rashba effect.

For the operation of hybrid Sm structures it is often desirable to achieve high junction transparency (i.e. a large transmission coefficient) at the interface of a semiconductor with other materials. The junction transparency, at the semiconductor interface with a metal, is typically limited by a native Schottky barrier. Recently, in the context of unpolarized spin transport, novel fabrication techniques were developed to suppress the Schottky barrier, providing, as a consequence, ohmic contacts and ballistic transport, across Sm/metal interfaces.

Our aim in this paper, motivated by the recent advances in hybrid Sm structures, is to investigate and quantify their degree of carrier spin polarization and the junction transparency. For this purpose we study Sm/S structures and the process of Andreev reflection (governing the low bias transport) which is very sensitive to both the degree of the spin polarization and the junction transparency. Such sensitivity can be explained by noticing that the Andreev reflection is a two-particle process. An incident electron of spin $\sigma = \uparrow, \downarrow$ on a Sm/S interface is reflected as a hole belonging to the opposite spin subband $\overline{\sigma}$, back to the Sm region while a Cooper pair is transferred to the superconductor. The probability for Andreev reflection at low bias voltage is thus related to the square of the normal state transmission coefficient and has a stronger dependence on the junction transparency than the ordinary single particle tunneling. For spin-polarized carriers, with different populations in two spin subbands, only a fraction of the incident electrons from a majority subband will have a minority subband partner in order to be Andreev reflected. Specifically, in the limit of full spin polarization Andreev reflection is completely suppressed. In the superconducting state, for an applied voltage smaller than the superconducting gap, single particle tunneling is not allowed in the S region and the modification of the Andreev reflection amplitude by spin polarization or junction transparency will be manifested in transport quantities such as the differential conductance or current-voltage characteristics of Sm/S junction.

In previous work on Sm/S structures, only spin-unpolarized transport was considered, and the spin-dependent Andreev reflection has only been addressed in the context of ferromagnet/S junctions. However, in the latter case, calculations were performed assuming the equality of the effective masses in the two regions across the interface. For Sm/S structures, which have a typical ratio of the effective masses $\sim 10$, such an assumption is clearly incorrect, and therefore we generalize to a situation with both unequal effective masses and Fermi velocities.

We consider transport of injected spin-polarized carriers by choosing a geometry where the Sm occupies $x < 0$ region and is separated by the flat interface, at $x = 0$, from the S region, at $x > 0$. We solve the Bogoliubov-de Gennes (BdG) equations for a Sm/S junction assuming specular reflection and translational invariance parallel to the interface ($y-z$ plane), implying the conservation of the parallel component of the wavevector $k_\parallel$ in the scattering process.
processes. At the interface we consider both potential (non-spin-flip) and spin-flip scattering. For an electron with spin \( \sigma \), incident from the Sm region, the allowed scattering processes and the corresponding amplitudes include the Andreev reflection, \( a_{\sigma} \), ordinary (normal) reflection, \( b_{\sigma} \), transmission as an electronlike quasiparticle, \( c_{\sigma} \), the transmission with the branch crossing as a holelike quasiparticle, \( d_{\sigma} \), together with their spin-flip counterparts, denoted by \( a_{\sigma} \), \( b_{\sigma} \), \( c_{\sigma} \), \( d_{\sigma} \). For example, the amplitude \( b_{\sigma} \) will correspond to the reflection of the incident electron accompanied by a spin-flip. The appropriate angle for each scattering trajectory is determined by an analogue of Snell’s law. In the presence of spin-flip scattering BdG equations can no longer be decoupled into spin up/down sectors. Instead, we look for four dimensional solutions of the form \( \Psi_{\sigma} = (\phi_{\sigma}, \chi_{\sigma}, \chi_{\sigma}, \lambda_{\sigma})^T \), where \( \phi_{\sigma}, \chi_{\sigma} \) are the electronlike quasiparticle (ELQ) and the holelike quasiparticle (HLQ) amplitudes, respectively. We consider ballistic transport in Sm and S regions and model the interfacial scattering by \( U(\mathbf{r}) = \hat{U} \delta(\mathbf{r}) \), where \( \delta(\mathbf{r}) \) is the Dirac delta function implying the problem. For our purposes, it is sufficient to focus on the low temperature regime where the Fermi function valid for \( \epsilon \), \( \rho \) vanishes, and for the remaining amplitudes we obtain

\[
\psi_{1\sigma}(0) = \psi_{2\sigma}(0) = \frac{\hbar}{m_1} \partial_x \psi_{1\sigma}(0) = \frac{\hbar}{m_2} \partial_x \psi_{2\sigma}(0) - \frac{1}{\hbar} \hat{U} \psi_{1\sigma}(0),
\]

where subscripts 1, 2 corresponds to the quantities in the Sm and S regions, respectively. We use a step function form of the pair potential \( \Delta(\mathbf{r}) = \Delta e^{i\theta(\mathbf{r})} \), \( \Delta \) is the superconducting gap. We employ the Andreev approximation valid for \( \epsilon, \Delta \ll E_F \), where \( \epsilon \) and \( E_F \) are the excitation and the Fermi energy, respectively. The solution in the Sm region, for spin \( \sigma \) electrons incident at an angle \( \theta \) with the normal to the interface is

\[
\psi_{1\sigma}(x) = e^{ik_{1\sigma}x} \left[ \begin{array}{c}
\delta_{\sigma} \\
0 \\
0 \\
0
\end{array} \right] + a_{\sigma} e^{ik_{1\sigma}x} \left[ \begin{array}{c}
\delta_{\sigma} \\
0 \\
0 \\
0
\end{array} \right] + a_{f\sigma} e^{ik_{1\sigma}x} \left[ \begin{array}{c}
\delta_{\sigma} \\
0 \\
0 \\
0
\end{array} \right] + b_{\sigma} e^{-ik_{1\sigma}x} \left[ \begin{array}{c}
\delta_{\sigma} \\
0 \\
0 \\
0
\end{array} \right] + b_{f\sigma} e^{-ik_{1\sigma}x} \left[ \begin{array}{c}
\delta_{\sigma} \\
0 \\
0 \\
0
\end{array} \right],
\]

where by \( k_{1\sigma}, k_{f\sigma} \), we denote, generally different \( x \)-components of \( k_1 \), and \( \delta_{\sigma} \) is the Kronecker symbol. Analogously, solutions in the S region can be explicitly written, and consequently all the scattering amplitudes analytically determined from Eq. (6) using the explicit forms of the wavefunctions. The conservation of the parallel component implies \( k_{1\sigma} = k_{f\sigma}, \sigma = \parallel, \perp, i = 1, 2 \), and within the Andreev approximation, conservation of energy requires

\[
k_{1\sigma}^2 = k_{f\sigma}^2, \quad k_{2\sigma}^2 = k_{2\sigma}^2, \quad k_{1\sigma}^2 = k_{2\sigma}^2 + k_{2\sigma}^2, \quad \lambda = \parallel, \perp,
\]

where \( k_{1\sigma}, k_{2\sigma}, k_{2\sigma} \) are the Fermi wavevectors in the Sm and S regions, respectively. We assume that in the presence of spin polarization, \( k_{f\sigma} \geq k_{f\sigma} \) holds, as a consequence of the splitting, \( \Delta \), of the spin subbands with corresponding bandwidths \( E_{F\sigma} = E_F + \rho \sigma \hbar \), where \( \rho \sigma = \pm 1 \). Consequently, there can exist a range for \( \theta \) (less than the angle of total reflection) where \( k_{f\perp} \) is purely imaginary while there is still nonvanishing transmission across the junction. For this purpose, we extend the above notation to the \( x \)-components of velocity and define \( v'_{\parallel\sigma} \equiv Re(v_{\parallel\sigma}) \). In contrast, from \( k_{f\perp} \geq k_{f\perp} \), it follows that \( v'_{\perp\sigma} \) and \( v'_{\perp\sigma} \) can be interchanged since the imaginary part would only arise at \( \theta \) greater than the critical angle for total reflection.

To obtain the differential conductance and related quantities of interest in spin-polarized transport studies, we modify the BTK method along the line of the Landauer-Büttiker scattering formalism in the normal state transport. Employing the conservation of probability current it is necessary to calculate only the amplitudes from Sm region, giving the differential conductance

\[
G_{\text{Sm/S}} = \frac{e^2}{h} \sum_{k_{1\sigma}, \sigma} \int_{-\infty}^{\infty} \left| 1 - \frac{v'_{\parallel\sigma}}{v_{\parallel\sigma}} (|a_{\sigma}|^2 - |b_{f\sigma}|^2) + |a_{f\sigma}|^2 - |b_{\sigma}|^2 \right| f(\epsilon - eV) - f(\epsilon) d\epsilon,
\]

where \( V \) represents the bias voltage, and \( f(\epsilon) \) is the Fermi function, which introduces temperature dependence in the problem. For our purposes, it is sufficient to focus on the low temperature regime where the Fermi function difference in Eq. (6) can be replaced by the Dirac delta function implying \( \epsilon = eV \). The substitution \( 1 + \frac{v'_{\parallel\sigma}}{v_{\parallel\sigma}} (|a_{\sigma}|^2 - |b_{f\sigma}|^2) + |a_{f\sigma}|^2 - |b_{\sigma}|^2 \rightarrow 1 - \frac{v'_{\parallel\sigma}}{v_{\parallel\sigma}} (|a_{\sigma}|^2 + |b_{f\sigma}|^2) - |a_{f\sigma}|^2 - |b_{\sigma}|^2 \) in Eq. (6), gives an explicit expression for the spin conductance. Noticing that the latter term is proportional to the quasiparticle current, we can immediately infer that the subgap spin conductance will vanish as there is no quasiparticle tunneling below the superconducting gap.

In the absence of spin-flip scattering of \( a_{f\sigma} \) and \( b_{f\sigma} \) vanish, and for the remaining amplitudes we obtain

\[
a_{\sigma} = \frac{4v'_{1\sigma}v'_{2\sigma} \Gamma}{v_{1\sigma}v_{2\sigma} + v^2_{2\sigma} + 4H^2_{2\sigma}/\hbar^2 + 2i(v_{1\sigma} - v_{1\sigma})H_{2\sigma}/\hbar(1 - \Gamma^2) + v_{2\sigma}(v_{1\sigma} + v_{1\sigma})(1 + \Gamma^2)},
\]

(4)
where $\Gamma$ is related to the BCS coherence factors and is given by $\Gamma = [\epsilon - i \sqrt{\Delta^2 - \varepsilon^2}/\Delta$ for $|\varepsilon| \leq \Delta$, and $\Gamma = [\epsilon - \text{sign}(\varepsilon)\sqrt{\Delta^2 - \varepsilon^2}/\Delta$ for $|\varepsilon| > \Delta$. Using the Eq. (4) and (3) it is interesting to compare the effective transmission coefficients for Sm/normal metal (Sm/N) and Sm/S junctions at zero bias

$$T_{\sigma}^{\text{Sm/N}}(\theta) = \frac{4v_{\sigma}v_{\sigma'}}{(v_{\sigma} + v_{\sigma'})^2 + 4H_0^2/h^2}, \quad T_{\sigma}^{\text{Sm/S}}(\theta) = \frac{4v_{\sigma}v_{\sigma'}v_{\sigma''}^2}{(v_{\sigma}v_{\sigma'} + v_{\sigma''} + 4H_0^2/h^2)^2 + 4(v_{\sigma} - v_{\sigma''})H_0^2/h^2}. \quad (6)$$

For normal incidence, $\theta = 0$ ($v_{\sigma}v_{\sigma'} = v_{\uparrow}\uparrow v_{\downarrow}\downarrow$, $v_{\sigma''} = v_{\uparrow}\downarrow$), and in the absence of potential scattering $H_0 = 0$: therefore in order to attain perfect transparency ($T = 1$), it is necessary in both Sm/N and unpolarized Sm/S junctions for the Fermi velocities $v_{\uparrow}\uparrow$, $v_{\downarrow}\downarrow$ to be equal. However, in the presence of finite spin polarization in Sm/S junctions it is possible to have perfect transparency even when all the Fermi velocities differ, satisfying $v_{\uparrow}\downarrow = \sqrt{m_F v_{\uparrow}\downarrow}$. There is also a broader regime, $H_0, \theta \neq 0$, which can be determined using Eq. (5), (6), where finite spin polarization ($v_{\uparrow}\downarrow \neq v_{\downarrow}\downarrow$) enhances junction transparency as compared to the unpolarized case.

To investigate the effect of the variable degree of spin polarization we first consider a geometry which models the in-plane transport of a two-dimensional electron gas (2DEG) which has a quasi one-dimensional interface, of length $L$ with a superconductor. For illustration, we choose parameters $m_{2D}/m_1 = 15$, and $k_{2D}/k_1 = 20$, corresponding approximately to Al/GaAs interface with a Sm 2DEG carrier density $n_s \sim 10^{10} \text{cm}^{-2}$, and thickness $\lesssim 70 \text{nm}$ with occupation only of the first quantized level. We focus on the normalized conductance $G_0 = G^{\text{Sm/S}}/G_{2DN}$, where $G_{2DN} = e^2L(k_{F\uparrow} + k_{F\downarrow})/\hbar \pi$ is the normal state conductance, and $G^{\text{Sm/S}}$ is given in Eq. (4). The summation in Eq. (3), can be replaced by $\int d\theta \cos\theta$, favoring the forward scattering channel. We parameterize potential scattering by $Z_{\sigma} = H_0/(v_{\uparrow}\downarrow v_{\uparrow}\downarrow)^{1/2}$ and spin-flip scattering by $F = H_F/(v_{\uparrow}\downarrow v_{\downarrow}\downarrow)^{1/2}$, where $H_F$ is the phenomenological parameter in matrix $\tilde{U}$, which determines the strength of $\phi_{\sigma} \rightarrow \phi_{\sigma'}$ and $\chi_{\sigma} \rightarrow \chi_{\sigma'}$ scattering. In this approach, our intention is to investigate some of the qualitative features of spin-flip scattering, without specifying its precise origin. To model the subband spin-splitting and the related degree of spin polarization, we define the parameter $X = h_0/E_F$. In Fig. 4, we give the calculated $G_0(eV/\Delta)$ for various degrees of spin polarization (including strong polarization, currently not attainable in the hybrid semiconductor structures), in the absence of interfacial scattering, $Z_\sigma = F = 0$. In the inset we show the effect of interfacial scattering at a fixed spin polarization. While individually potential and spin-flip scattering each tends to suppress the conductance below and above the gap, their combined effect may surprisingly lead to the enhancement of subgap conductance. Conductance amplitude for fixed $X$, at the bias voltage of the superconducting gap, depends (using the conservation of probability current) only on amplitudes $a_\sigma$ and $a_{\sigma'}$. From Eq. (3) and by noticing that $\Gamma = 1$, at $eV = \Delta$, it follows that $a_\sigma$ depends on the ratio of $v_{\uparrow}\sigma$ and $v_{\uparrow}\sigma'$, i.e., on the subband spin-splitting only, while it can be shown that $a_{\sigma'}$ vanishes identically in this situation.

We next consider the geometry for a bulk Sm/S or a 2DEG/S junction with a two-dimensional interface, parallel to the 2DEG. Calculating the conductance from Eq. (3) entails summation over the two $k_1$ components which could be replaced by $\int d\theta \cos\theta \sin\theta$. We define $G_3 = G^{\text{Sm/S}}/G_{3N}$ where $G_{3N} = e^2A(k_{F\uparrow}^2 + k_{F\downarrow}^2)/\hbar \pi = k_F^2/\hbar \pi$ is the Sharvin conductance in the normal state and $A$ is the contact area of the Sm/S interface. In Fig. 5 we use the same parameters as in the previous figure, omitting results for $X > 0.6$. For a bulk Sm region, the chosen ratio of $k_{2D}/k_1$ can be achieved with a large surface doping. The angular dependence, of the combination of scattering amplitudes in $G_2$ and $G_3$ is identical. There are, however, differences between the figures, including the smaller subgap conductance in the latter case, which can be explained by the angular integration, weighting more contributions near $\theta = \pi/4$, rather than $\theta = 0$ region (as in the former case). The inset in Fig. 5 shows that the conductance peak is no longer exactly at the superconducting gap, as compared to the one in the previous figure. The maximum value depends only weakly on the presence of potential and spin-flip scattering and together with other features in $G_2$ can be used to estimate the degree of spin polarization from the conductance data. The effect of combined spin-flip and potential scattering is more pronounced in Fig. 5 and is qualitatively different from Fig. 4 in particular the double conductance peak in Fig. 5 has no analog in Fig. 4.

In summary, we have shown that the low temperature transport properties in Sm/S structures may serve as a sensitive and quantitative probe for determining the degree of spin polarization and the strength of interfacial scattering. The high Sm/S junction transparency achieved in recent fabrication techniques, can be further enhanced with spin-polarized carriers. Experimental studies of Sm/S junctions should provide an important testing ground for studying the feasibility of spintronic devices based on hybrid semiconductor structures. Because of the various simplifying approximations (e.g. specular scattering with conserved parallel component of wavevectors, delta function model for interface scattering, step function approximation for the superconducting gap function, etc.) our results should be taken as of qualitative rather than of quantitative validity. However, all of these approximations are
nonessential, made for the sake of analytical simplicity and may be improved upon in future numerical treatments of the problem when experimental data on spin-polarized transport in Sm/S hybrid structures becomes available.

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FIG. 1. Normalized conductance, $G_2(eV/\Delta)$, curves from top to bottom represent $X=0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,0.95$ at $Z_s = F = 0$. The inset shows $X = 0.4$ results, the upper four curves (from top to bottom at zero bias) have $Z_s$, $F$ values given by $(0,0)$, $(0.5,0.5)$, $(0.5,0.25)$, $(0.5,0.25)$, and $(0,0)$ for the bottom curve, corresponding to $k_{2F}/k_{1F} = 70$ and $n_s \sim 10^{-2}$ cm$^{-2}$.

FIG. 2. Normalized conductance, $G_3(eV/\Delta)$. Curves from top to bottom correspond to $X = 0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,0.95$, all the other parameters and ordering are the same as in Fig. 1 and its inset.
$Z_{\sigma} = F = 0$

$k_{2F} / k_{1F} = 20$

$m_2 / m_1 = 15$