High Neutron Star Birth Velocities and Gravitational Radiation during Supernova Explosions

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Abstract. Assuming the observed pulsar velocities to originate during asymmetric collapse of stellar cores, we compute the amplitude of gravitational waves emitted during type II and Ib supernova explosions and their detection rate from within a distance of 30 Mpc. At the rms-level of advanced laser interferometers $h \approx 10^{-22}$ at frequencies $300 \sim 1000$ Hz the expected rate is about 1 per year.

Key words: Gravitational waves — Stars: neutron — Supernovae: general

Supernova explosions are among the most violent events known in nature. Supernovae type II and Ib are triggered by the gravitational collapse of evolved degenerate cores of massive (> 10 $M_\odot$) stars and leave behind neutron stars or, possibly, black holes as a remnant. An enormous amount of energy is released during the collapse (roughly, the binding energy of a neutron star $\sim 0.15M_\odot c^2$). Although most of this energy is carried away by neutrinos, a part of it can escape in the form of gravitational radiation provided that the collapse is not purely symmetrical. This collapse anisotropy (expressed conveniently in terms of the fraction of the energy released, $\epsilon M_\odot c^2$) was first introduced by Ozernoy (1965) and Shklovskii (1970).

It required more than 20 years for the idea of the collapse anisotropy to receive serious observational support. First, the revision of the pulsar distance scale led to increasing the observed pulsar velocities by a factor of two up to values of about 400-500 km/s (Lyne & Lorimer 1994). Observations of pulsars associated with young supernova remnants revealed even higher pulsar velocities up to 900 km/s (Frail et al. 1994). Such high velocities of young pulsars could hardly be obtained without additional recoil (“kick velocity”) during the supernova explosion. Next, a direct evidence for a kick velocity of at least 100 km/s has recently been obtained from observations of precessing binary pulsar orbit in PSR J0045-7319 in the SMC (Kaspi et al. 1996). The collapse asymmetry resulting in the kick velocity of a neutron star at birth may be due to different reasons. For example, recent calculations of Burrows & Hayes (1996) has shown the ability of neutrino anisotropic emission to produce kick velocities of 400-500 km/s, as observed (see also Imshennik 1992; Bisnovatyi-Kogan 1993).

One of the consequences of Lyne and Lorimer’s result was a recognition that the kick velocity imparted to a neutron star at birth has a power-law asymptotic form at high velocities (Lipunov, Postnov & Prokhorov 1996a,b). Using direct Monte-Carlo calculations of binary star evolution (the so-called “Scenario Machine”), they found that the Lyne-Lorimer pulsar transverse velocities are best reproduced assuming the space (3-D) kick velocity distributed as

$$ f(x) \propto \frac{x^{0.19}}{(1 + x^{0.72})^{1/2}} $$

where $x = v/v_0$, $v_0 = 400$ km/s. An important thing about having this distribution is that at high velocities ($v \sim 500$ km/s) it goes practically as $v^{-3.17}$, that is much slower than the maxwellian tail $\propto \exp(-v^2)$, which sometimes is assumed for the kick velocity distribution (e.g. Portegies Zwart & Spreeuw 1996).

The anisotropic stellar collapse may be a source of gravitational waves (GW). To be detectable from the distance of Virgo cluster ($\sim 18$ Mpc) by future ground-based laser interferometric and bar detectors, the GW energy emitted during a SN explosion should be about $\Delta E = 10^{-3}M_\odot c^2$ (Thorne 1995; Schutz 1996). However, in numerical calculations this energy has been found to be very tiny, typically $10^{-9} \sim 10^{-7}$ of the total energy output (see Müller 1996 for a review). Unfortunately, all these calculations are still far from being realistic considering enormous numerical difficulties and uncertainties involved.

Nevertheless, the recognition of high additional velocities neutron stars acquire at birth suggests a way to esti-
mate $\epsilon$ from observations, regardless of the unknown collapse anisotropy mechanism(s). Indeed, the kinetic energy of the neutron star motion, $\Delta E = M_{NS}v^2/2$, where $M_{NS}$ is the neutron star mass, may be considered as a lower limit to the energy emitted in GW. Assuming $M_{NS} = 1.4 M_\odot$, we obtain $\epsilon = 0.7 \times (v/c)^2$. With the distribution law [3], the fraction of high velocity pulsars is

$$P(> x) = \int_x^\infty f(\xi)d\xi \approx 0.38 x^{-2.17}$$  \hspace{1cm} (2)

(the power-law asymptotics is valid for $x > 2$). For example, the fraction of pulsars with $v > 1000 \text{ km/s}$ is $\approx 0.05$. Of course, there should exist a maximum cut-off velocity; it must be higher than the maximum pulsar velocities observed, $\sim 2000 \text{ km/s}$ ($x > 5$), but its exact value only slightly changes the normalization coefficient. For $\epsilon = 10^{-4}$ ($v \approx 3200 \text{ km/s}$) we obtain $P(> 8) \approx 0.004$, i.e. every 250-th neutron star may be born with the high velocity required.

The neutron star galactic birth-rate is determined by that of massive ($> 10 M_\odot$) stars, which is $\approx 1/25$ years assuming Salpeter mass function $f(M)dM \propto M^{-2.35}dM$ and the mean galactic star formation rate of 1 $M_\odot$ per year. Using the relations $\epsilon = 0.7(v/c)^2$, $x = v/(400\text{ km/s})$ and Eq. (2) we may write galactic birth rate of high velocity pulsars as

$$R = 0.38/25 x^{-2.17} \text{ yr}^{-1} \approx 1.3 \times 10^{-5} \epsilon^{-1.085} \text{ yr}^{-1}$$  \hspace{1cm} (3)

(here and below subscripts indicate the quantities measured in units of the corresponding power of ten, e.g. $\epsilon_{-3} \equiv \epsilon/10^{-3}$).

The characteristic amplitude of the GW burst from a SN located at a distance $r$ is (Thorne 1987; Schutz 1996)

$$h_c = 5 \times 10^{-22} \epsilon_{-3}^{1/2} \left(\frac{1 \text{ kHz}}{f_c}\right)^{1/2} \left(\frac{18 \text{ Mpc}}{r}\right)$$  \hspace{1cm} (4)

where $f_c$ is the characteristic frequency of the burst. This level should be detectable by the advanced LIGO interferometer. The event rate from the volume $V$ of the Universe is (e.g. Phinney 1991; see also Lipunov, Postnov & Prokhorov 1996b) $R \approx 0.01 \times (\text{galactic rate}) \times (V/\text{Mpc}^3)$. Using Eqs. (3) and (4) we find

$$R \approx (0.08 \text{ yr}^{-1}) \epsilon_{-3}^{0.42} f_{300 \text{ kHz}}^{-3/2} h_{-22}^{3/2} \approx (1 \text{ yr}^{-1}) \epsilon_{-3}^{0.42} f_{300 \text{ kHz}}^{-3/2} h_{-22}^{3/2}.$$  \hspace{1cm} (5)

This means that we have chance to observe 1 GW-burst per year caused by SN II and Ib explosions at a level of $h_{rms} = 10^{-22}$ provided that the energy is carried out at the frequency 300 Hz.

Since $h_c \propto \sqrt{\epsilon} \sim v$, the distribution of the GW-bursts caused by the SN explosions from a given distance $r$ will have the form [3]. This enable us to compute the cumulative distribution of the number of events with the amplitude higher than a given one ($\log N(> h_c) - \log h_c$) using the realistic baryon matter distribution within $\approx 30 \text{ Mpc}$ according to Tully’s Nearby Galaxies Catalog (Tully 1988) (see Lipunov et al. 1995 for more detail). The result is presented in Fig. 1. The hatched region corresponds to GW-frequencies of the signal from 300 Hz (upper boundary) to 1 kHz (lower boundary). For comparison, we reproduce the log $N$ – log $h$ calculated for constant $\epsilon = 10^{-4}$ (Lipunov et al. 1995). Clearly, the more realistic $\epsilon$-distribution yields an order of magnitude smaller event rate because the rate of SN explosions with higher $\epsilon$ strongly decreases. The shape of the curve is smoothed by the $f(v)$ distribution. The mean $\epsilon$ for Lyne-Lorimer velocity distribution [3] is $\langle \epsilon \rangle \propto \int v^2 f(v)dv \approx 4.4 \times 10^{-6}$.

It is seen from the figure that a few events per year are expected at the noise level of the advanced-LIGO rms-sensitivity $\approx 10^{-22}$. At the initial laser interferometers sensitivity level $h_{rms} \approx 10^{-21}$ the expected SN rate is very low, $\approx 0.01$ per year.

![Fig. 1. The log $N(> h_c) = \log h_c$ curve of the number of GW-bursts caused by core collapse supernovae as seen by a GW-detector with an rms-sensitivity $h_c$ at the unitary signal-to-noise ratio in 1-year integration time. The supernova rate are calculated for the baryonic matter distribution within the region of 30 Mpc from Sun according to Tully’s Nearby Galaxies Catalog (Tully 1988) with account for the SN-rate dependence on the morphological type of the galaxies. The distribution of SN-generated GW-amplitudes is taken assuming Lyne-Lorimer kick velocity distribution (see the text). The hatched region encompasses the assumed GW frequencies from 1 kHz (lower boundary) to 300 Hz (upper boundary). The broken curve represents the SN rate assuming constant GW energy fraction $\epsilon = 10^{-4}$ (from Lipunov et al. 1995) with contributions of local group of galaxies indicated.](image_url)
In our analysis, we have not used any particular mechanism for the collapse anisotropy and tried to rely on the observational properties of pulsars which are reliable products of supernova explosions. The only assumption we used is that the pulsar kick velocities are entirely due to the supernova explosion asymmetry. Of course, not every core collapse supernova may give rise to a pulsar. For example, some pulsar studies suggest the galactic pulsar birth-rate \( \frac{1}{125} - \frac{1}{250} \text{yr}^{-1} \) (Lorimer et al. 1993), much smaller than the galactic SN II rate. Additionally, some other mechanisms of the collapse anisotropy may operate. This means that our results can be considered as a lower limit and the actual detection rate may be higher. We conclude that insofar as the observed pulsar velocities is a measure of the core collapse anisotropy, gravitational radiation bursts from supernova explosions should be detectable by the advanced laser interferometers and bar detectors in 1-year integration.

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**Note added in proof**

In a recent paper, Hartman (1996) argued that the observed transverse pulsar velocity distribution is reproduced equally well assuming the Paczyński (1990) distribution for the initial pulsar velocity \( p(x)dx \propto (1 + x^2)^{-2} \) with \( x = v/(600 \text{km/s}) \). The same analysis as for our distribution (1) shows that the resulting log \( N(> h_c) - \log h_c \) curve does not change appreciably giving approximately the same \( \langle \epsilon \rangle \approx 4 \times 10^{-6} \).

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