Constraints on the R-parity violating couplings using the newest measurement of the decay $B^0_s \rightarrow \mu^+\mu^-$

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**Abstract**

Recently, the LHCb collaboration reported the first evidence for the decay $B^0_s \rightarrow \mu^+\mu^-$. A branching ratio of $\mathcal{B}(B^0_s \rightarrow \mu^+\mu^-) = (3.2^{+1.5}_{-1.2}) \times 10^{-9}$ is given. Using the newest data, and together with the most precise predictions of the Standard Model contributions to the decay, we derive the constraints on the combinations of the R-parity violating parameters. Our results are several orders of magnitudes stronger than the constraints in the previous literature. We also update the constraints on the relevant parameters using the upper limit of $\mathcal{B}(B^0_d \rightarrow \mu^+\mu^-)$.

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1. INTRODUCTION

The helicity suppressed rare decay process $B_s^0 \rightarrow \mu^+\mu^-$ is induced by $Z$ boson mediated penguin diagram and box diagram in the Standard Model (SM). This double suppression mechanism makes the SM prediction for the process very small \cite{1}

$$B(B_s^0 \rightarrow \mu^+\mu^-) = (3.23 \pm 0.27) \times 10^{-9}. \quad (1)$$

The fact that there are only leptons in the final states makes it a golden channel for the discovery and/or constraining the new physics model parameter space, since the new physics contributions can be larger than the SM effects and there is the least hadronic uncertainty.

The minimal supersymmetric standard model with R-parity violation (MSSM-RPV) is an extension of the minimal supersymmetric standard model (MSSM) by abandoning the discrete symmetry, the R-parity, which is defined by $R_p = (-1)^{3B+L+2S}$, where $B$ is the baryon number, $L$ is the lepton number, and $S$ is the spin of the particle. The most general R-parity violating term can be included in the MSSM by introducing the following superpotential \cite{2, 3}:

$$W_{R_p} = \mu_i H_u L_i + \frac{1}{2} \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \frac{1}{2} \lambda''_{ijk} U^c_i D^c_j D^c_k, \quad (2)$$

where additional factor of 1/2 is added because of the fact that the first two indices of the couplings $\lambda_{ijk}$ and $\lambda''_{ijk}$ are antisymmetric. It is easy to see from this superpotential that the $B_s^0 \rightarrow \mu^+\mu^-$ decay can be induced at the tree level from the lepton number violating terms $\lambda$ and $\lambda'$.

Study of the MSSM-RPV have been performed in many rare decay processes. The bounds on the relevant parameters in the MSSM-RPV obtained from the decay $B_s^0 \rightarrow \mu^+\mu^-$ were derived in Ref. \cite{4} and revised in the literature \cite{5, 6}. However, there were only upper bounds from experiments at that time.

Recently, the LHCb collaboration reported the first measurement of the branching ratio of $B_s^0 \rightarrow \mu^+\mu^-$ \cite{7}

$$B(B_s^0 \rightarrow \mu^+\mu^-) = (3.23^{+1.5}_{-1.2}) \times 10^{-9}. \quad (3)$$

This just lies on the central regions of the SM prediction in Eq. (1), which will put severe constraints on every new physics models. In this brief report, we will use the newest data in Eq. (3) to constrain the relevant parameters in the framework of the minimal supersymmetric standard model with R-parity violation. Since the experimental measurement
is quite close to the SM prediction, we have to include the contributions of the standard model together with the new physics contribution. Using the first time measurement of the branching ratio of the decay process, we give the most stringent constraints on the relevant parameters in the MSSM-RPV. We also update the constraint on the sneutrino exchange term and the squark exchange term from the newest experimental upper limits [7]:

\[ \mathcal{B}(B^0_d \rightarrow \mu^+\mu^-) < 9.4 \times 10^{-10}. \]  

This brief report is organized as follows, in Sec. II we present the analytical expressions; and then we use these equations to give the numerical results and discussions in Sec. III. We close this paper with a conclusion in Sec. IV.

II. ANALYTICAL EXPRESSIONS

In this section, we present the formalism for the calculation of the branching ratio of the \( B^0_s \rightarrow \mu^+\mu^- \) in both the SM and the MSSM-RPV. The same formalism can also be applied to the process \( B^0_d \rightarrow \mu^+\mu^- \) with changing the corresponding Cabibbo-Kobayashi-Maskawa matrix elements. In the SM, the effective Hamiltonian governing \( B^0_s \rightarrow \mu^+\mu^- \) can be written as [8]:

\[ \mathcal{H}^{SM}_{eff} = -\frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} V_{tb}^* V_{ts} \eta_Y Y_0(x_t)(\bar{b}\gamma_\mu P_L s)(\bar{\mu}\gamma_\mu P_L \mu) + h.c., \]

with \( P_L = (1 - \gamma_5)/2 \). The function \( Y_0 \) is the famous Inami-Lim function calculated from the electroweak penguin and box diagrams [9]:

\[ Y_0(x) = \frac{x}{8} \left( \frac{4 - x}{1 - x} + \frac{3x}{(1 - x)^2} \ln x \right). \]

While \( \eta_Y = 1.026 \pm 0.006 \) includes higher order QCD corrections [10].

The branching fraction of \( B^0_s \rightarrow \mu^+\mu^- \) can be given by

\[ \mathcal{B}(B^0_s \rightarrow \mu^+\mu^-) = \frac{\tau(B^0_s)}{16 \pi m_{B^0_s}} |\mathcal{H}^{SM}_{eff}|^2 \sqrt{1 - \frac{4m_{\mu^\pm}^2}{m_{B^0_s}^2}}, \]

where \( \tau(B^0_s) \) is the lifetime of the \( B^0_s \) meson.

In the MSSM-RPV, the relevant effective Hamiltonian can be obtained by matching the amplitudes in full theory as shown in Fig. I onto the effective four fermion operators. For
FIG. 1: Feynman diagrams contributing to $B_s^0 \rightarrow \mu^+\mu^-$ in the minimal supersymmetric standard model with R-parity violation.

For the up-squark contribution shown in the right diagram of Fig. 1, the effective Hamiltonian is

$$H_{eff}^{RPV} = \sum_i \frac{B_s^{RPV}}{i}^2 \frac{m_{\tilde{u}_i}^2}{m_{\tilde{u}_i}^2} \lambda_{2i2}^* \lambda_{2i3}^*.$$

where $B = \sum_i \frac{B_{2i2}^* B_{2i3}}{m_{\tilde{u}_i}^2}$. For this scalar quark contribution has the same structure of current as the standard model case, and the SM prediction lies in the central values of the experimental data, we can reasonably assume that the interference term of the SM and the MSSM-RPV is greatly

$$B(B_s^0 \rightarrow \mu^+\mu^-) = B^{SM} + B_{A}^{RPV},$$

where we will use Eq. 11 as input value of $B^{SM}$ in our numerical calculations, while

$$B_{A}^{RPV} = |A|^2 \frac{\tau(B_s^0)}{16\pi} \frac{1}{f_{B_s^0}^2 m_{B_s^0}^3} \left( 1 - \frac{2 m_{\tilde{u}_i}^2}{m_{\tilde{u}_i}^2} \right) \sqrt{1 - \frac{4 m_{\tilde{u}_i}^2}{m_{B_s^0}^2}}.$$

For this new kind of contribution, the interference of the $(\bar{\mu} P_R \mu)$ density with the standard model $(\bar{\mu} \gamma^\mu P_L \mu)$ current leads to zero, so we can directly separate the SM and the MSSM-RPV contributions to the branching ratio as

$$B(B_s^0 \rightarrow \mu^+\mu^-) = B_{SM} + B_{A}^{RPV},$$

where we will use Eq. 11 as input value of $B^{SM}$ in our numerical calculations, while

$$B_{A}^{RPV} = |A|^2 \frac{\tau(B_s^0)}{16\pi} \frac{1}{f_{B_s^0}^2 m_{B_s^0}^3} \left( 1 - \frac{2 m_{\tilde{u}_i}^2}{m_{\tilde{u}_i}^2} \right) \sqrt{1 - \frac{4 m_{\tilde{u}_i}^2}{m_{B_s^0}^2}}.$$

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$$B(B_s^0 \rightarrow \mu^+\mu^-) = B_{SM} + B_{A}^{RPV},$$

where $P_R = (1 + \gamma_5)/2$ and

$$A = \sum_i \frac{\lambda_{2i2}^* \lambda_{2i3}^*}{m_{\tilde{u}_i}^2}.$$
TABLE I: Input parameters for \( B_0^s \) and \( B_0^d \) mesons used in numerical calculations. Uncertainties of these parameters are not considered for the reason that the uncertainties induced by these parameters on the relevant constraints on the couplings of the MSSM-RPV are far beyond the scope of the experimental data.

| \( B_0^s \) | \( \tau_B (\text{ps}) \) | \( f_B (\text{MeV}) \) | \( m_B (\text{GeV}) \) |
|---------|----------------|----------------|----------------|
| \( B_0^s \) | 1.466 | 227 | 5.36677 |
| \( B_0^d \) | 1.519 | 190 | 5.27958 |

larger than the pure MSSM-RPV contributions, so we will approximately write the total contributions as

\[
B(B_0^s \to \mu^+ \mu^-) = B^{SM} + B^\text{int}_B,
\]

where we also use the numerical value for \( B^{SM} \) from Eq. (1); while the interference term can be written as:

\[
B^\text{int}_B = B \frac{\tau(B_0^s)G_F \alpha}{16\sqrt{2}\pi^2 \sin^2 \Theta_W} V_{tb}^\ast V_{ts} f^2_B m_{B_0^s} m_{\mu^2} \eta Y(x_t) \sqrt{1 - \frac{4m_{\mu^2}}{m_{B_0^s}^2}}.
\]

The \( B_0^s \) meson decay constant \( f_{B_0^s} \) shown in the above equations arises from the calculation of the hadronic matrix element, which is defined as

\[
\langle 0 | \bar{\ell} \gamma^\mu \gamma_5 s | B_0^s \rangle = i f_{B_0^s} p_{B_0^s}^\mu.
\]

The hadronic matrix elements for the pseudo-scalar density can be derived from the equation of motion under the assumption that \( m_b \simeq m_{B_0^s} \).

III. NUMERICAL RESULTS

In this section we present our numerical results. Following Ref. [1], the SM parameters are taken as \( G_F = 1.16638 \times 10^{-5} \text{GeV}^{-2} \), \( \alpha = 1/127.937 \), \( m_W = 80.385 \text{GeV} \) [11], \( m_t = 173.2 \text{GeV} \) [12], \( m_\mu = 105.6584 \text{MeV} \), \( |V_{tb}^\ast V_{ts}| = 0.0405 \) and \( |V_{tb}^\ast V_{td}| = 0.0087 \). The relevant parameters of neutral \( B \) mesons are collected in table II. Uncertainties of these parameters are not considered in the numerical calculation, since the uncertainties induced by these parameters on the relevant constraints on the couplings of the MSSM-RPV are far beyond the scope of the experimental data.
As described in the last section, the contributions of the SM are taken as input, and then
we calculate the total effects from the sum of the SM and the scalar neutrino contribution
or the sum of SM and scalar quark contribution of the MSSM-RPV. By demanding the
total contributions do not exceed the experimental upper and lower bounds, we obtain the
following constraints on the relevant combinations of the parameters in the MSSM-RPV,
respectively,

$$\left| \sum_i \frac{\lambda_{i22}^* \lambda_{i23}}{m_{\tilde{\nu}_i}^2} \right| < 6.52 \times 10^{-11},$$
$$-2.29 \times 10^{-9} < \sum_i \frac{\lambda_{i11}^* \lambda_{i23}}{m_{\tilde{u}_i}^2} < 2.87 \times 10^{-9}.$$  \quad (17)

The scalar neutrino coupling suffers roughly 2 orders of stronger constraints than the scalar
quark coupling since there is no helicity suppression in the sneutrino contributions. Due to
the more stringent experimental limit, our results are several orders of magnitude stronger
than previous results in the literature [5, 6]. If we further assume the mass of the sparticles
to be several hundreds GeV, a roughly estimate shows that the above combinations of the
R-parity violating couplings are around $10^{-6}$ and $10^{-4}$, respectively, which means that the
magnitudes of the couplings are not too far away from unity.

We also give the constraints from the newest experimental upper limits on the $B(B_d \rightarrow \mu^+\mu^-)$ shown in Eq. (4). The corresponding constraints on the sneutrino exchange term
and the squark exchange term are given below

$$\left| \sum_i \frac{\lambda_{i22}^* \lambda_{i13}}{m_{\tilde{\nu}_i}^2} \right| < 7.85 \times 10^{-11},$$
$$\sum_i \frac{\lambda_{i21}^* \lambda_{i23}}{m_{\tilde{u}_i}^2} < 1.17 \times 10^{-8}.$$  \quad (18)

IV. SUMMARY

In conclusion, using the newest experimental data, we have calculated the contributions
to the $B_s(d) \rightarrow \mu^+\mu^-$ in the framework of the MSSM-RPV. We gave the constraints on the
relevant combinations of the parameters in the MSSM-RPV as

$$\left| \sum_i \frac{\lambda_{i22}^* \lambda_{i23}}{m_{\tilde{\nu}_i}^2} \right| < 6.52 \times 10^{-11},$$
$$-2.29 \times 10^{-9} < \sum_i \frac{\lambda_{i11}^* \lambda_{i23}}{m_{\tilde{u}_i}^2} < 2.87 \times 10^{-9};$$  \quad (19)
\[ |\sum_i \lambda_{i13} \lambda_{1i3}^*| \frac{1}{m_{\tilde{\nu}_i}^2} < 7.85 \times 10^{-11}, \]
\[ \sum_i \frac{\lambda_{i1} \lambda_{1i3}^*}{m_{\tilde{u}_i}^2} < 1.17 \times 10^{-8}. \] \tag{20}

Our results are several orders of magnitude stronger than the previous results in the literature.

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