Vector Meson Electroproduction at small Bjorken-\(x\) and Generalized Parton Distributions

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Abstract

We analyze electroproduction of light vector mesons (\(V = \rho, \omega\)) at small Bjorken-\(x\) in an approach that includes the gluonic generalized parton distributions and a partonic subprocess, \(g \rightarrow (q\bar{q})g, q\bar{q} \rightarrow V\). The subprocess is calculated to lowest order of perturbative QCD taking into account the transverse momenta of the quark and antiquark as well as Sudakov suppressions. Our approach allows to investigate the transition amplitudes for all kind of polarized virtual photons and polarized vector mesons. Modelling the generalized parton distributions through double distributions and using simple Gaussian wavefunctions for the vector mesons, we compute the longitudinal and transverse cross sections at large photon virtualities as well as the spin density matrix elements for the vector mesons. Our results are in fair agreement with the findings of recent experiments performed at HERA.

(Revised version)
1 Introduction

Vector meson electroproduction at large photon virtuality, $Q^2$, has always attracted a lot of theoretical interest. Its diffractive nature as well as the interesting correlation between the $Q^2$ and the energy dependence are challenging issues. At most traditional concepts like vector meson dominance (see e.g. [1]) or the Regge model with its prominent Pomeron exchange (see e.g. [2]) have been exploited to analyze the electroproduction data. In 1987 Donnachie and Landshoff viewed the Pomeron as the exchange of two gluons between the proton and the quark-antiquark pair created by the virtual photon and which subsequently form the outgoing meson. Brodsky et al. [4] treated the two-gluon exchange contribution to electroproduction at large $Q^2$ and small $B$-pomeron-$x$, $x_{Bj}$, in the framework of QCD factorization. They showed that in their approach, known as the $\ln (1-x_{Bj})$ approximation, the emission and reabsorption of the gluons by the proton can be related to the usual gluon distribution. Many variants of the leading $\ln (1-x_{Bj})$ approximation can be found in the literature which differ mainly by the treatment of the subprocess $g \to Vg$, see Ref. [5] to name a few. These approaches describe many features of vector meson electroproduction quite well.

In 1996 vector meson electroproduction has been taken up by theory again. Exploiting the new concept of generalized parton distributions (GPD) [9,10] it has been shown [9,11] that, at large $Q^2$, the process factorizes into a hard parton-level subprocess - meson electroproduction off partons - and soft proton matrix elements which represent generalized parton distributions. The process is depicted in Fig. 1, where also the momenta of the involved particles are specified. It has also been shown in Ref. [8] that the process is dominated by transitions from longitudinally polarized photons to longitudinally polarized vector mesons ($L \to L$) at large $Q^2$; the amplitudes for other transitions are suppressed by inverse powers of $Q$. The production of vector mesons at small $x_{Bj}$ ($< 10^{-2}$) is controlled by gluonic GPDs where quasi-on-shell gluons are emitted and reabsorbed by the proton. These GPDs which represent the soft physics embodied in the proton matrix elements, are unknown as yet and have to be modelled.

Detailed experimental information on electroproduction of light vector mesons in the region of small $x_{Bj}$ is available from HERA. Cross sections and spin density matrix elements have been measured by H1 and ZEUS. Despite the sound theoretical basis of the handbag approach not much has been done as yet in analyzing these data within this framework. There is only the explorative study of the longitudinal cross section for production performed by Mankiewicz et al. [12]. The normalization of the cross section was however not understood in this work. Martin et al. [13] on the other hand, started from the $\ln (1-x_{Bj})$ approximation and estimated effects due to the replacement of the gluon distribution by the corresponding GPD. Here, in this work we attempt a complete and systematic analysis of the available electroproduction data at small $x_{Bj}$. In order to analyze the spin density matrix elements of the vector mesons we also calculate the amplitudes for transitions from transversely polarized photons to transversely and longitudinally polarized vector mesons ($T \to T$ and $T \to L$). We allow for quark transverse momentum and take into account Sudakov suppressions. As it will turn out this approach leads to the correct
Figure 1: The handbag-type diagram for meson electroproduction off protons. The large blob represents a GPD while the small one stands for meson electroproduction off partons. The momenta of the involved particles are specified.

normalization of the cross sections at finite but large $Q^2$. Infrared singularities which occur for the $T ! T$ transition amplitude in collinear approximation are also regularized in our approach although in an admittedly model-dependent way.

The plan of the paper is the following: A kinematical prelude and the handbag amplitude are presented in Sect. 2. The amplitudes for the subprocess $g!Vg$ are discussed in Sect. 3 to leading order of perturbative QCD and including transverse momenta of the quarks and antiquarks making up the meson. The impact parameter representations of the full handbag amplitudes for electroproduction of vector mesons are presented in Sect. 4. The following section, Sect. 5, is devoted to the construction of the GPDs. Numerical results, obtained from the handbag approach, for the cross sections of vector meson electroproduction and for the vector meson’s spin density matrix elements are compared to recent experimental results in the small $x_{Bj}$ region in Sects. 6 and 7, respectively. In the next section, Sect. 8, we discuss the helicity correlation $A_{LL}$ and the role of the GPD $\hat{H}$ and summarize in Sect. 9.

2 The handbag factorization

We will work in a photon-proton center of mass (cm) frame, see Fig. 1, in a kinematical situation where

$$W^2 = (p + q)^2;$$

and the virtuality of the incoming photon, $q^2 = Q^2$, are large while Bjorken’s variable,

$$x_{Bj} = Q^2 = (2p \cdot q);$$

is small ($x_{Bj} < 10^{-2}$). We also assume the square of the momentum transfer, $\Delta^2 = p^0 \cdot p$, to be much smaller than $Q^2$. 

3
In light-cone components, defined by \( a = (a^0, a^3) = \frac{p}{2} \) and \( b = a b + b a \), the momenta of the protons and the photon read

\[
\begin{align*}
\mathbf{p} &= (1 + \gamma) \gamma \frac{m^2 + \frac{2}{2} \gamma = 4}{2 (1 + \gamma) \gamma^2} \\
\mathbf{p}^0 &= (1 + \gamma) \gamma \frac{m^2 + \frac{2}{2} \gamma = 4}{2 (1 + \gamma) \gamma^2} \\
\mathbf{q} &= (1 + \gamma) \gamma \frac{Q^2 + \frac{2}{2} \gamma = 4}{2 (1 + \gamma) \gamma^2}
\end{align*}
\]

(3)

where \( \gamma \) equals \( x_B \) up to corrections of order \( m^2/Q^2 \) and \( \gamma \). Here, \( m \) denotes the mass of the proton. The average proton momentum is defined by

\[
\mathbf{p} = \frac{1}{2} (\mathbf{p} + \mathbf{p}^0) ;
\]

(4)

and the skewness parameter by

\[
\frac{\mathbf{q} \cdot \mathbf{p}^*}{(\mathbf{p} + \mathbf{p}^0)^*} ;
\]

(5)

In the photon-proton c.m. frame and for small \( x_B \), the skewness parameter is related to Bjorken-\( x \) by

\[
\frac{\mathbf{q} \cdot \mathbf{p}^*}{(\mathbf{p} + \mathbf{p}^0)^*} = \frac{x_{B,j}}{x} , \quad x_{B,j} = 2 ;
\]

(6)

For Mandelstam \( t \), given by

\[
t = 2 = \frac{4}{1} \frac{m^2 + \frac{2}{2}}{2} ;
\]

(7)

a minimum value is implied by the positivity of \( \frac{2}{2} \)

\[
t_{min} = \frac{4m^2}{1} \frac{2}{2} ;
\]

(8)

Since we are interested in the region of small Bjorken-\( x \) and, hence, small skewness we will use \( t_{min} = 0 \) in the following. We also will neglect the proton and meson (\( m_V \)) masses in the kinematics.

Let us now consider the dynamics of vector meson electroproduction in the kinematical regime specified above. The dominant contribution in this kinematical region comes from the emission and reabsorption of collinear gluons from the protons accompanied by \( g \) ! \( g \) scattering. The neglect of an analogous quark contribution is justified by the fact that, at small \( x_B \), partons with small momentum fractions dominantly participate in hard meson electroproduction. Since, at small \( t \), the GPDs are expected to reflect the magnitudes of the usual parton distributions the gluon contribution should outweigh the
quark one. This is in particular the case for electroproduction of mesons where only the small strange quark content of the proton is probed. Even for the production of mesons the gluonic contribution seems to be still sizeable for $x_{Bj}$ as large as 0.1 as is indicated by the ratio of and electroproduction cross sections.

The momenta of the gluons which, approximately, are collinearly emitted or absorbed by the protons, are parameterized as

$$\begin{align*}
k_g &= (x + p^\perp) \hat{p}^\parallel; \\
k_g^0 &= (x - p^\perp) \hat{p}^\parallel;
\end{align*}$$

(9)

In general the partons may have small virtualities of the order of $\frac{2}{\Lambda}$. As usual we have introduced an average parton momentum

$$k = \frac{1}{2} (k_g + k_g^0);$$

and an average momentum fraction

$$x = k^+ p^+.$$  

(10)

In order to facilitate comparison with other work we also provide the relations between the variables $x$ and and the usual Mandelstam variables for the hard subprocess. They read ($\ell'$ = 0)

$$\begin{align*}
s &= (q + k_g)^2, \quad \frac{x}{2} Q^2; \\
d &= (q - k_g^0)^2, \quad \frac{x^+}{2} Q^2;
\end{align*}$$

and are valid at large $Q^2$ and small $\Lambda$.

Radyushkin has calculated the asymptotically leading handbag contribution to meson electroproduction at small $x_{Bj}$. As he showed this contribution involves $L \cdot L$ transitions. Leaving aside for the time being a potential breakdown of factorization, Radyushkin’s result can straightforwardly be generalized to other transitions. The crucial point in the derivation of the handbag amplitude is the use of light-cone gauge for the gluon field, $\Lambda = 0$, where

$$n = [0; 1; 0,?];$$

(13)

and $a$ is a color label. This gauge allows to express the gluon field by an integral over the gluon field strength tensor $G^a$ (the limit $u \rightarrow 0$ is to be understood)

$$A^a(z) = \int_0^1 dz \, e(z + n): G^a(z + n):$$

(14)
With the help of this expression one can replace the products of fields appearing in the perturbatively calculated amplitude for $p! Vp$ by

$$A^a(0) A^{a_0}(z) = \frac{gg}{N_c^2} \frac{X}{1}\left(k_g; \right) \left(k_{g_0}; \right)^d d^0 e^0 e^{00}$$

$$n^n G^a \left(0 \right) G^{a_0} \left(z + n \right) \left(k_g; \right) \left(k_{g_0}; \right)^0; \quad (15)$$

where we have also made a helicity projection for the gluons. The use of the approximation for the gluon momenta forces the relative distance of the fields on the light cone $z! z = [0; z; 0]$. The vectors $(k_g;)$ and $(k_{g_0};)$ specify the polarization of the on-shell gluons, the corresponding momenta, $k_g$ and $k_{g_0}$ are defined in Eq. 16. The first set of polarization vectors in Fig. is to be used to contract the hard scattering kernel leading to gauge invariant parton-level helicity amplitudes $H_{g,t}$ for $g! Vg$ (and $0$ denote the helicities of $g$ and $V$, respectively). The contraction of the field strength tensors with the second set of polarization vectors leads to

$$n^n G \left(0 \right) G_{a_0} \left(z + n \right) \left(k_g; \right) \left(k_{g_0}; \right)^0 =$$

$$\frac{1}{2} n^n G \left(0 \right) G_{a_0} \left(z + n \right) \left(g_0 + i \gamma^0 \right) \left(t_0^1 \right)^i; \quad (16)$$

where

$$g_{11} = g_{22} = g_{12} = g_{21} = 0, \quad g_{12} = \frac{i}{2}$$

$$t_{12} = t_{21} = 1. \quad (17)$$

while all other components of these tensors are zero. That only the transverse components in the contraction are is a consequence of the chosen light-cone gauge and of the fact that the polarization vectors have zero plus components in the c.m. frame we are working.

Proton matrix elements of the gluon helicity non-$p\bar{p}$ contributions $g_0$ and $i \gamma^0$ in define the unpolarized, $H^g(\mathbf{x}; \mathbf{t})$ and $E^g(\mathbf{x}; \mathbf{t})$, and the polarized, $\bar{H}^g(\mathbf{x}; \mathbf{t})$ and $\bar{E}^g(\mathbf{x}; \mathbf{t})$, gluon GPD's, respectively. The proton matrix elements of these gluon field operators are related to the GPD's by

$$H^g(\mathbf{x}; \mathbf{t}) = \frac{1}{2} \left[ e^{i\mathbf{x} \cdot \hat{\mathbf{p}}} \mathbf{u}(p) \mathbf{u}(p) \right] \mathbf{H}^g(\mathbf{x}; \mathbf{t}) + \frac{1}{2} \left[ e^{i\mathbf{x} \cdot \hat{\mathbf{p}}} \mathbf{u}(p) \mathbf{u}(p) \right] \mathbf{E}^g(\mathbf{x}; \mathbf{t})$$

$$+ \frac{1}{2} \left[ e^{i\mathbf{x} \cdot \hat{\mathbf{p}}} \mathbf{u}(p) \mathbf{u}(p) \right] \mathbf{H}^g(\mathbf{x}; \mathbf{t}) + \frac{1}{2} \left[ e^{i\mathbf{x} \cdot \hat{\mathbf{p}}} \mathbf{u}(p) \mathbf{u}(p) \right] \mathbf{E}^g(\mathbf{x}; \mathbf{t})$$
and

$$\hat{H}^g(x; t) \frac{2}{1} E^g(x; t)$$  \hspace{1cm} (20)

occur. Since we are interested in small the \(E^g\) and \(E^g\) terms can safely be neglected in the expressions. For proton helicity \(\hat{p}\), on the other hand, \(H^g\) and \(\hat{H}^g\) do not contribute but only

$$\frac{P \cdot t}{2m} \frac{1}{1} E^g(2E^g)$$  \hspace{1cm} (21)

where is a phase factor reading

$$= \frac{1 + \frac{1}{2j}}{j^2}$$  \hspace{1cm} (22)

for proton momenta of the form.

The gluon helicity \(\hat{p}\) contribution in which defines four more GPDs will be neglected in the following since it is strongly suppressed at small \(t\). The mismatch between the proton and gluon helicities in the proton matrix elements has to be compensated by orbital angular momentum. For each unit of \(t\) a factor \(\frac{P \cdot t}{m} \) is picked up.

Further suppression comes from the subprocess amplitudes which behave as

$$H_{00}^V \sqrt{p} \frac{1}{t=0} \frac{1}{j} \hat{g}_{0j}$$  \hspace{1cm} (23)

at small \(t\) and from the fact that the amplitude \(H_{00}^V\) vanishes for \(=1\).

Combining all this, we finally obtain the helicity amplitudes for electroproduction of vector mesons \(^3\):

$$M_{0+} = \frac{e}{C_V} n_h \frac{\partial}{2} \int_0^1 \frac{dx}{(x + 1)(x + i^n)}$$

$$= \frac{H_{0+}^V + H_{0+}^V}{h} i \quad E^g(x; t)$$

$$+ \frac{H_{0+}^V + H_{0+}^V}{h} i \quad \hat{H}^g(x; t)$$  \hspace{1cm} (24)

for proton helicity non- \(\hat{p}\) (explicit helicities are labelled by their signs) and for helicity \(\hat{p}\)

$$M_{0+} = \frac{e}{C_V} n_h \frac{\partial}{2} \int_0^1 \frac{dx}{(x + 1)(x + i^n)}$$

$$= \frac{H_{0+}^V + H_{0+}^V}{h} i \quad E^g(x; t)$$

$$+ \frac{H_{0+}^V + H_{0+}^V}{h} i \quad \hat{H}^g(x; t)$$  \hspace{1cm} (25)

\(^3\)We note in passing that our helicities are light-cone helicities which naturally occur in the handbag approach. The difference to the usual c.m. frame helicities is of order \(\frac{P \cdot t}{m} \approx 2m^2\) and can be ignored in the kinematical region of interest in this work.
The subprocess amplitudes, \( H^V \), are functions of \( Q^2 \), \( x \), and \( t \). The flavor weight factors, \( C_V \), read for and mesons

\[
C = \frac{1}{2} \left( e_u, e_d \right) = 1 = \frac{P}{2}; \quad C = e_s = 1=3; \quad (26)
\]

where \( e_i \) denotes the quark charge in units of the positron charge \( e \). The remaining helicity amplitudes are obtained with the help of parity invariance

\[
M_{00} = (1)_{0}^{0} \quad M_{00}: \quad (27)
\]

An analogous relation holds for the subprocess amplitudes.

3. The partonic subprocess \( g ! V g \)

The parton-level amplitudes for the subprocess \( g ! V g \) are calculated from the Feynman graphs shown in Fig. 2; the outgoing quark pair is to be combined into the vector meson regarding its quantum numbers. This is conveniently done by means of a covariant spin wavefunction. As is well-known from analyses of hadron form factors at large momentum transfer, leading-twist perturbative calculations are unstable in the end-point regions since the contributions from large transverse separations, \( b \), of quark and antiquark forming the meson are not sufficiently suppressed. In order to eliminate that defect Li and Sterman retained the quark transverse degrees of freedom and took into account Sudakov suppressions. Including, in addition, meson wavefunctions with their intrinsic transverse momentum dependence instead of distribution amplitudes, the perturbative contributions to form factors can reliably and self-consistently be calculated, the end-point regions are strongly damped.

Since the subprocess \( g ! V g \) bears resemblance to the meson form factors it is tempting to apply this so-called modified perturbative approach also here in order to
suppress the contributions from the soft end-point regions and, simultaneously, to regularize this way infrared divergencies that may occur in the T ! L and T ! T amplitudes.

The modified perturbative approach applied to the subprocess, is, to some extent, similar to the mechanism proposed in Ref. for the suppression of the leading-twist gluon contribution to hard meson electroproduction. It is however to be stressed that in Ref. the leading ln(1=xB) approximation of Ref. has been utilized.

Let us now turn to the description of the soft qq ! V transition matrix element. We start from a frame where the hadron rapidly moves along the 3-direction (q0 = [0;0;0,]) with q0 = Q = (2^3P). This frame is termed the hadron-out one. The momenta of the quark and the antiquark which form the valence Fock state of the meson, are parameterized as

\[ q_1 = q^0 + k_1; \quad q_2 = q^0 + k_2; \]

where

\[ k_1 = [k_1^+; 0; k_1^-]; \quad k_2 = [k_2^+; 0; k_2^-]; \]

The variables \( q \) and \( k \) are the usual fractions of the light-cone minus-component of the meson's momentum the constituents carry. Momentum conservation provides the constraints

\[ q_0 = 1; \quad k_0 = k_0; \quad k; \]

It can be shown that the variables \( q \) and \( k \) are invariant under all kinematical Poincare transformations, i.e. under boosts along and rotations around the 3-direction as well as under transverse boosts. Moreover - and this is an important point - the light-cone wavefunction associated with the valence Fock state \( \psi = \psi(\kappa, k) \), is independent of the hadron's momentum and is invariant under these kinematical transformations too. The light-cone wavefunction may diverge for longitudinally and transversally polarized vector mesons.

As is customary in the parton approach we neglect the binding energy. That possibility crude approximation can be achieved by putting the individual k^+ -component to zero. In fact starting from a parameterization of the various momenta in the meson's rest frame and boosting to the hadron-out frame, one sees that the k^+ -component are of order \( m^2 = q^0 \). The plus-component of the difference of the momenta

\[ K = \frac{1}{2}(k_1 - k_2); \]

is zero with this choice of k^+ -components and, hence,

\[ K = [0; 0; k_0]; \]

and \( q^0 K = 0 \). The quarks are treated as massless in the hadron-out frame; they are not strictly on-shell.

It is convenient to couple the spinors representing quark and antiquark in a covariant spin wave function for the vector meson. The Dirac indices of it (omitted for convenience)
are to be contracted with the corresponding ones of the hard scattering kernel (see below). For the construction of the spin wave function we adapt the method presented in Ref. (27) straightforwardly to vector mesons. The product of spinors \( \psi(q_1) \psi(q_2) \) is boosted to the hadron’s rest frame, coupled there into the quantum number basis of the vector meson and boosted back to hadron-out frame. Separating terms with and without \( K \) and neglecting term \( s / K^2 \), one arrives at

\[
\psi = \psi_0 + \psi K ;
\]

where

\[
\psi_0 = \frac{1}{2} (q_0 + m_v) \psi ;\quad \psi = \frac{1}{M_v} f \psi_0 \quad g ;
\]

The polarization state of the meson is described by the vector \( \psi \). The soft physics parameter \( M_v \) is of order \( m_v \); its model dependence results from the specific treatment of the quarks in the meson’s rest frame. In the following we will use \( M_v = m_v \) for simplicity but we will comment on other choices of it.

Since the anticommutator \( f \psi_0, \psi q_0 \) is zero the 4-vector \( K \) is only determined up to a multiple of the meson moment. This property can be used to identify \( K \), given in Eq. (33), with the quark-antiquark relative momentum

\[
K = \frac{1}{2} (q_1 + q_2) ;
\]

where the parton momenta, \( q_i \), are defined in Eq. (28). This choice, although not forced, is very convenient. Its main advantage is that \( K \) now represents one unit of orbital angular momentum in a covariant manner. As discussed in this article, the relative momentum \( K \) is a 4-transverse vector which are defined by \( K = K \bigl( m_q = q_0 \bigr) \). In the hadron-out frame and up to corrections of order \( m^2_v = q_0^2, K = K \). In the meson’s rest frame on the other hand, clearly \( K = (0; \omega) \), and one has an object transforming as a 3-vector under the three-dimensional rotation group \( O(3) \).

One of the basis ingredients of the hard scattering picture is the collinear approximation which says that all constituents move along the same direction as their parent hadron up to a scale of the order of the Fermi motion \( h \omega^2 \), which typically amounts to a few 100 MeV. The (nearly) collinear kinematics justifies an expansion of the spin wave function upon a power series in \( k \) or, in order to retain a covariant formulation, in \( K \). Up to terms linear in \( K \) this expansion is given above for vector mesons.

The transformation from the hadron-out frame to our c.m. frame where the meson momentum has a transverse component \( \gamma = 2 \), is executed by a transverse boost (cf. e.g., Ref. (27)) that leaves the minus component of any momentum vector unchanged, and which involves a parameter \( d = a \) and a transverse vector \( d_\perp \), is defined as

\[
[a^i ; a \ ; a^\perp ] \quad [a \quad a^2 \quad d + \frac{\kappa a}{2} \frac{d_\perp^2}{(d_\perp)^2} ; a \ ; a^\perp \quad \frac{a}{d} d_\perp ] ;
\]

The transverse boost is one of the kinematical Poincaré transformation that leaves the hadron wave function invariant. Taking for the parameter \( d = q_0^2 \) and \( d_\perp = \gamma = 2 \), we readily
nd from the expressions for the quark momenta and the relative momentum in the c.m. frame.

Provided quark transverse momenta are taken into account, the general structure of the $g!Vg$ amplitude is

$$H^V = \frac{Z}{16^3} \int \frac{d^2k_\perp}{\pi} \, v(\sqrt{\kappa^2}) \text{Tr}[v^T \, T_H] : \quad (37)$$

The hard scattering kernel, $T_H = T_H (\,;x;Q^2;k^2; t)$, can be written as follows

$$T_H = T_0 (\,;x;Q^2;k^2; t) + T (\,;x;Q^2;k^2; t) K + \cdots : \quad (38)$$

Terms $/K K$ (and higher) in the nominator are neglected while, in the spirit of the modified perturbative approach, the $k^2$ terms in the denominators are kept. Inserting Eq. $\square$ as well as the spin wave function $\square$ into Eq. $\square$, one obtains

$$H^V = \frac{Z}{16^3} \int \frac{d^2k_\perp}{\pi} \, v(\sqrt{\kappa^2}) \text{Tr}[v^T_0 T_0 + v T_0 K + v T K K + \cdots :] \quad (39)$$

Obviously, the terms $/K$ integrate to zero while the $K K$ term survives the $k^2_\perp$-integration. Hence,

$$H^V = \int \frac{d^2k_\perp}{16^2} \, v(\sqrt{\kappa^2}) \text{Tr}[v^T_0 T_0 + \frac{1}{2} k^2_\perp g_\perp \, v \, T + \cdots :] \quad (40)$$

where $g_\perp$ is the transverse metric tensor defined in Eq. $\square$. In order to simplify matters we only take into account the first non-zero term in this expansion for each amplitude, i.e., we neglect any correction of order $mV$ or $\kappa^2$ to its leading term $^4$. As we said above we however retain the $k^2_\perp$ terms in the denominators of the propagators. Moreover, any $t$ dependence of the subprocess amplitudes is ignored except the factors of required by angular momentum conservation. This is justified in the small $t$ region we are interested in.

For longitudinally polarized vector mesons the first term in Eq. $\square$ contributes, the other term represents a $k^2_\perp$ correction to it which we, according to our strategy, neglect as well as all other terms indicated by the ellipses. For transversely polarized mesons, on the other hand, the first term in Eq. $\square$ disappears since the number of matrices in the trace is odd $^5$. The second term in Eq. $\square$, $/k^2_\perp$, contributes in this case; it scales as $/k^2_\perp = (mV Q)$, see Eq. $\square$. Combining this property with the behavior of the subprocess amplitudes near the forward direction $\square$ and utilizing the fact that $\sqrt{k^2_\perp} = \sqrt{mV + q_0}$ is of order 1, the various photon-meson transitions respect the following hierarchy

$^4$Note that the hard scattering kernel $T_H$ does not depend on the vector meson mass; it occurs through the spin wave function.

$^5$We remind the reader that for longitudinally polarized vector mesons, $0 = q_0^2 = mV$ up to corrections of order $mV = q_0^2$.  

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The following form the relevant sums and differences of positive and negative gluon helicities can be cast into the following form

\[ \mathcal{H}_0^v + \mathcal{H}_0^v = \frac{8}{2N_c} N_R \frac{Z_1}{Z} \frac{d^2 k_T}{16} V_D (\mu^2) (\sigma^2) \mathcal{F}_0 \mathcal{D} ; \] (42)

where the product of propagator denominators reads

\[ D = k_T^2 + Q^2 k_T^2 + Q^2 . \] (43)

This hierarchy propagates to the proton non-\(p\) amplitudes for the full process and justifies the neglect of \(L! T\) and \(T! T\) transitions in the analysis. The amplitudes for proton helicity \(p\) have an extra factor \(\frac{t}{m} \), see (24). Our interest in this work is focused on unpolarized protons. In the corresponding cross sections there is no interference between \(p\) and non-\(p\) amplitudes. Hence, proton \(p\) is suppressed by a factor of \(t\) and since there is no theoretical or phenomenological indication that \(F^g\) is much larger than \(F^g\), neglected by us. Information on the proton \(p\) amplitudes may be extracted from data on meson electroproduction with polarized protons. As a last simplification we neglect contributions from \(H^g\) in the evaluation of the amplitudes. Since in the forward limit \(t \to 0\) \(H^g\) and \(H^g\) reduce to \(x g(x)\) and \(x g(x)\), respectively, it is plausible to expect that the relative size of \(g\) and \(g\) is reflected in that of \(g\) and \(g\) at small and \(t\). Since \(j g(x)\) is much smaller than \(g(x)\), the contribution from \(g\) can safely be neglected.

The model GPDs we are going to construct in Sect. 5 do indeed respect this assertion. As a consequence of parity invariance, see (22), and (42), there is anyway no contribution from the GPD \(H^g\) to the most important amplitude, \(L! L\). Care is required for observables for which the contribution from \(H^g\) partially if not totally cancels. An example of such an observable is the correlation of the electron and proton helicities. We will comment on this observable in Sect. 8.
Here, $N_c$ denotes the number of colors. The functions $f_0^+$ read

$$f_{0+}^+ = Q^{11} \left( \frac{x^2}{q^2} \right)^{2/3} \; ;$$
$$f_{00}^+ = Q^{10} \left( \frac{x^2}{q^2} \right)^{1/2} \; ;$$
$$f_{1+}^+ = \frac{k^2}{m_v} Q^{10} \left( \frac{x^2}{q^2} \right)^{2/3} \; ;$$
$$f_{10}^+ = \frac{k^2}{m_v} Q^{10} \left( \frac{x^2}{q^2} \right)^{2/3} \; ;$$

Following Ref. \[23\], we retain $k^2$ terms in the denominators of the propagators. These terms play an important role since they compete with terms / $(Q^2)$ which become small in the end-point regions where either or tends to zero. They lead to the suppression of contributions with large quark-antiquark separations as we mentioned above.

In collinear approximation and utilizing distribution amplitudes up to twist-3 accuracy the subprocess amplitudes for T ! T transitions are infrared divergent, signaling the breakdown of factorization. Neglecting transverse momenta in Eq. \[24\], one finds

$$H_{+\gamma}^+ + H_{+\gamma}^+ \propto \frac{d}{2} \frac{x^2}{x^2} \frac{2}{2} \left( \frac{\delta x^2}{x^2} \right) \frac{Z}{2} \frac{k^2}{k^2_{\gamma}} \left( \frac{1}{k^2_{\gamma}} \right) ;$$

Assuming for instance a Gaussian wavefunction \[\gamma \propto \exp \left( \frac{1}{2} k^2_{\gamma} \right) \], an ansatz that has been shown to work successfully in many cases (see for instance \[24\]) and will be used by us in the numerical analysis of meson electroproduction, we find that in fact the integral is regular. As pointed out in \[24\] the $x$ integral in Eq. \[24\] may not exist due to the double pole $(x^2 + i\nu^2)$ occurring. Whether or not this happens depends on properties of the GPDs. In Sect. 5 we will take up this problem again.

One may also consider a transverse momentum dependence of the GPDs. That issue has been investigated in Ref. \[20\] for meson electroproduction at intermediate values of $x_{Bj}$. In this kinematical region the emission and reabsorption of quarks from the proton dominates. We however think that the $k_\gamma$ dependence of the GPDs is of minor importance. In contrast to the meson where the hard process enforces the dominance of the compact Fock state of the meson, all proton Fock states contribute to the GPDs at small $x_{Bj}$. If the gluons are distributed in the proton like the quarks, an assumption that is supported by the slope of the di hard peak in elastic proton-proton scattering, the $k_\gamma$ dependence of the GPD $H^g$ should roughly re ect the change radius of the proton $(hk_{2,1}^{1/2} \sim 200 \text{ MeV})$. Consequently, we expect $H^g$ to be only mildly dependent on the transverse momentum, a potential e ect we neglect.
4 The impact parameter space

Transverse momenta in the subprocess amplitudes, see Section 1 imply finite quark-antiquark separations in the configuration space which are accompanied by gluon radiation. On the grounds of previous work by Collins and Soper, Stem an and collaborators calculated this radiation to next-to-leading-log approximation using resummation techniques and having recourse to the renormalization group. The result is a Sudakov factor which suppresses large quark-antiquark separations and which we also have to take into account in our analysis in order to have consistency with the retention of the transverse degrees of freedom. Since the Sudakov factor is given in the transverse separation or impact parameter space - only in this space the gluonic radiative corrections exponentiate - we have to work in this space.

The two-dimensional Fourier transformation between the canonical conjugated $b$ and $k_{T}$ spaces is defined by

$$f(k) = \frac{1}{(2\pi)^2} \int d^2 k_{T} \exp\left[i k_{T} \cdot b\right] f(k) :$$

(46)

For the meson wavefunctions we adopt the same Gaussian parameterization as is used for the pion

$$\psi_i(k^2_T) = 8^{-\frac{q}{2}N} \psi_i^2 \psi_k a_{\psi_i L}^2 \exp\left[-2a_{\psi_i L}^2 k_{T}^2\right] ;$$

(47)

$i = L; T$ which strictly speaking represent full wavefunctions with their perturbative tails removed. Transverse momentum integration of these wavefunctions lead to the associated distribution amplitudes which represent the soft hadronic matrix elements entering calculations within the collinear factorization approach. Actually, the wavefunction lead to the so-called asymptotic meson distribution amplitude

$$a_S = 6 :$$

(48)

For the decay constants, $f_{VL}$ of longitudinally polarized vector mesons we take the values

$$f_L = 0.216 \text{ GeV} ; \quad f_{L} = 0.237 \text{ GeV} :$$

(49)

The decay constants for transversely polarized vector mesons are almost unknown. The only available information comes from QCD sum rules. In Reference $f_T$ has been estimated to $1.60 \pm 10$ MeV. We actually these decay constants to experiment. Identifying for instance the soft parameter $M_V$ in the spin wave function with the meson mass, we obtain

$$f_T = 0.250 \text{ GeV} ; \quad f_T = 0.275 \text{ GeV} :$$

(50)

Choosing $M_V$ to be smaller than the meson mass results in smaller values of the decay constants $f_{VT}$. The transverse size parameters $a_{VL}$ are fixed by the requirement of equal probabilities for the vector meson and pion valence Fock states, namely 0.25. This leads to

$$a_L = 0.52 \text{ GeV}^{-1} ; \quad a_L = 0.45 \text{ GeV}^{-1} :$$

(51)
The transverse size parameters for transversely polarized vector mesons are adjusted to experiment. The numerical results we are going to present below are obtained with
\begin{align}
a_T &= 0.65 \text{ GeV}^{-1}; \\
a_T &= 0.60 \text{ GeV}^{-1};
\end{align}

With the parameter values quoted in and the rms transverse momenta, evaluated from and , as just quoted in GeV and GeV for the longitudinally polarized and mesons, respectively. These values are much larger than the one for the proton rms transverse momentum.

The Fourier transform of the meson wavefunction reads
\begin{equation}
\hat{V}_1(\beta; \delta) = 2 \frac{f_{V_1}}{2N_c} \Lambda_s(\beta) \exp \frac{b^2}{4a_{V_1}^2};
\end{equation}
The product of the propagator denominators can be decomposed into single-pole terms which are either of the form
\begin{equation}
T_1 = \frac{1}{k_0^2 + d_1 Q^2};
\end{equation}
or
\begin{equation}
T_2 = \frac{1}{k_0^2 + q (x) Q^2} i^*;
\end{equation}
where \( d_1 \neq 0 \). The Fourier transform of these pole terms can readily be obtained
\begin{align}
\hat{T}_1 &= \frac{1}{2} K_0(q \delta_1 bQ) \\
\hat{T}_2 &= \frac{1}{2} K_0(q \delta_2 (x) bQ) \\
&+ \frac{i}{4} H_0^{(1)}(q \delta_2 (x) bQ)
\end{align}
where \( K_0 \) and \( H_0^{(1)} \) are the zeroth order modified Bessel function of second kind and Hankel function, respectively.

Putting all this together and including the Sudakov factor, \( \exp[ -S(\beta; \delta Q) ] \), the gluonic contributions to the helicity amplitudes for vector-meson electroproduction read
\begin{align}
M_{\sigma^+;+} &= 0 + M_{\sigma^+;+} + M_{\sigma^+;+}; \\
M_{\sigma^+;+} &= \frac{e}{2N_c} C_V \int dx \int d^2 \beta \hat{V}_1(\beta; \delta) \hat{D}^\ell(\beta; \delta) \hat{D}^\ell(\beta; Q) \exp[ -S(\beta; \delta Q) ]; \\
M_{\sigma^+;+} &= \frac{e}{2N_c} C_V \int dx \int d^2 \beta \hat{V}_1(\beta; \delta) \hat{D}^\ell(\beta; \delta) \hat{D}^\ell(\beta; Q) \exp[ -S(\beta; \delta Q) ];
\end{align}
which is the b-space version of the amplitude. The functions $D$ and $f_0$ are given in (24) and (25). Since the Fourier transformed wavefunctions, the product of propagator denominators as well as the Sudakov factor only depend on $b$ the angle integration in the last integral is trivial and a three-dimensional integral $(dx dB dB)$ remains to be evaluated numerically. Parity invariance lead to the following relations among the amplitudes:

$$
M_{H^+;+} = M_{H^+;+}^H + i M_{H^+;+}^H,
M_{H^0;+} = M_{H^0;+}^H + i M_{H^0;+}^H \tag{58}
$$

The Sudakov exponent $S$ in (57) is given by:

$$
S(\;;b;Q) = s(\;;b;Q) + s(\;;b;Q) \frac{4}{3} \ln \left( \frac{R_{QCD}^2}{4\pi} \right) \tag{59}
$$

where a Sudakov function $s$ occurs for each quark line entering the meson and the abbreviation

$$
\hat{b} = \ln (\frac{Q}{b}) \tag{60}
$$

is used. The last term in (59) arises from the application of the renormalization group equation ($\gamma = \frac{11}{2} n_f$) where $n_f$ is the number of active flavors taken to be 3. A value of 220 MeV for $Q_{QCD}$ is used here and in the evaluation of $s$ from the one-loop expression. The renormalization scale $R$ is taken to be the largest mass scale appearing in the hard scattering amplitude, i.e. $R = \max (Q; Q; 1=\mu)$. For small $b$ there is no suppression from the Sudakov factor; as $b$ increases the Sudakov factor decreases, reaching zero at $b = 1 = Q_{QCD}$. For even larger $b$ the Sudakov is set to zero. The Sudakov function $s$ reads

$$
s(\;;b;Q) = \frac{8}{3} \ln \left( \frac{Q}{b} \right) + NLL \text{ terms } \tag{61}
$$

where

$$
q = \ln \left( \frac{Q}{2 \cdot Q_{QCD}} \right) \tag{62}
$$

Actually we do not use the explicit form of the next-to-leading-log corrections quoted in (59) but those given in Ref. [23]. The latter ones contain some minor corrections which are hardly relevant numerically. Due to the properties of the Sudakov factor any contribution to the amplitudes is damped asymptotically, i.e. for $\ln (Q^2; Q_{QCD}) = 1$, except those from configurations with small quark-antiquark separations. $b$ plays the role of an infrared cut-off; it sets up the interface between non-perturbative soft gluon contributions - still contained in the hadronic wavefunction - and perturbative soft gluon contributions accounted for by the Sudakov factor.

---

6 The same relations as for the $H^0$ term also hold for the t-channel exchange of a particle with natural parity, $P = (1)$. The relations for the $H^0$ term in (58) correspond to those obtained for an unnatural parity exchange.

7 The definition of the Sudakov factor is completed by the following rules: $\exp [S] = 1$ if $s(\;;b;Q) = 0$ if $b = Q_{QCD}$ and $s(\;;b;Q) = 0$ if $b = Q$. 

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5 Modelling the GPDs

In order to calculate the electroproduction amplitudes, we still need the gluon GPDs. A model for them can be constructed with the help of double distributions which guarantee polynomiality of the GPDs. The gluonic double distribution $f(\cdot, t' \cdot 0)$ is customarily parameterized as

$$f(\cdot, t' \cdot 0) = g(\cdot) \frac{(2n+2)}{2^{2n+1} (n+1)} \frac{j \int_{1}^{2} \frac{2j}{j}}{j \int_{1}^{2n+1}};$$

(63)

where $g(x)$ is the usual gluon distribution. Its definition is extended to negative $x$ by

$$g(-x) = g(x).$$

(64)

A popular choice of $n$ is 1 for quarks and 2 for gluons. This is motivated by the interpretation of the dependence like a meson distribution amplitude for hard exclusive processes. The cases $n = 1$ and 2 correspond to the asymptotic behavior of a quark distribution amplitude / $(1 - x)^2$ and for gluons / $(1 - x)^2$, respectively. This correspondence is not demanded by theory. Therefore, we will consider both the cases, $n = 1$ and 2, for the construction of the gluon GPD. A parameterization of the $t$ dependence of $f$ is difficult. The multiplication of $f$ as given in (63) by a $t$-dependent form factor, although frequently used in default of a better idea, is unsatisfactory. Paramaterizations of the GPDs as well as results from lattice QCD revealed that a factorization of $f$ in $t'$ is on the one hand and in $t$ on the other hand is most likely incorrect. Fortunately, the knowledge of the GPDs at $t' = 0$ suffices for our purposes as will become clear in Sect.

According to [36], the GPD $H^g$ is related to the double distribution by (since we will only work with GPDs at $t' = 0$ we omit the variable $t$ in the GPDs in the following)

$$H^g(x; ) = \int_{0}^{x} \int_{0}^{1} \frac{Z_{x_1}^{x_3}}{x_3} d + \int_{0}^{x} \int_{0}^{1} \frac{Z_{x_1}^{x_3}}{x_3} d - f(\cdot, = x)$$

(65)

The definition of $H^g$ is completed by noting that it is an even function of $x$

$$H^g(x; ) = H^g(x; ).$$

(66)

The integration limits in Eq. (65) are given by

$$x_1 = \frac{x + 1}{1 + x}; \ x_2 = \frac{x}{1}; \ x_3 = \frac{x}{1}.$$ 

(67)

The limit $x_1 (x_2)$ is the momentum fraction the emitted (reabsorbed) gluon carries with respect to the incoming (outgoing) proton. The last term in the definition is the so-called D-term. Its support is the region $x$ and it ensures the correct
polynomiality property of the GPD. Since the D term is and our interest lies in small skewness, we neglect it.

We take the gluon distribution from the NLO CTEQ5M results and use an interpolation of it which has been proposed in Ref. and which is valid in the range \( Q_0^2 = 4 \text{ GeV}^2 \quad Q^2 \quad 40 \text{ GeV}^2 \)

\[
g(x) = \left( \frac{Q^2}{Q_0^2} \right)^\frac{5}{2} \prod_{i=0}^{3} c_i x_i^{2i-2}; \quad (68)
\]

where

\[
c_0 = 1.94; \quad c_1 = 3.78 + 0.24 \ln \left( \frac{Q^2}{Q_0^2} \right); \quad c_2 = 6.79 - 2.13 \ln \left( \frac{Q^2}{Q_0^2} \right); \quad (69)
\]

This is a very good approximation to the CTEQ gluon distribution for \( Q^2 < 0.5 \). At the largest value of \( Q^2 \) we are going to use the interpolation, namely 40 GeV^2, and for \( Q^2 > 0.2 \) it deviates less than 5% from the CTEQ gluon distribution. For values of \( Q^2 \) in the range \( 10^4 \quad 10^1 \), the interpolation agrees with the CTEQ gluon distribution within 1%. In this region the \( Q^2 \)-dependence of \( g(x) \) is approximatively given by

\[
(\frac{Q^2}{Q_0^2}) = 0.17 + 0.07 \ln \left( \frac{Q^2}{Q_0^2} \right) - 0.05 \ln^2 \left( \frac{Q^2}{Q_0^2} \right); \quad (70)
\]

The parameterization and effectively take into account the evolution of the gluon distribution in a large but finite range of \( Q^2 \) as calculated in Ref. At small the gluon distribution has a typical error of about 15%. Within this error there is agreement with the analysis presented in Ref. An error assessment of the power provides an uncertainty of about 10–15% for \( Q^2 < 0.5 \).

For the various terms in the ansatz, the integrations occurring in Eq. can be performed analytically, see Ref. One finds

\[
H_{11}(x; ) = \frac{3}{2^3} \left( \frac{1 + i=2}{4 + i=2} \right) \frac{1}{n} \left( \frac{2 + i=2}{x} \right)^{2 + i=2} x_{i=0}^{2 + i=2} \left( \frac{1}{x} \right) x_i^{2i-2} \left( \frac{1}{x} \right); \quad (71)
\]

for the case \( n = 1 \). A similar but somewhat more complicated result is obtained for the case \( n = 2 \). This way we obtain an expansion of \( H^g \)

\[
H^g(x; ) = \sum_{i=1}^{X} c_i H_{n1}(x; ); \quad (72)
\]

with coefficients following from . The evolution of the GPD is here approximated by that of the gluon distribution. The dominant contribution to vector meson electroproduction comes from the imaginary part of the \( L \! L \) amplitude (see Sect. which is
Figure 3: Model results for the GPD $H^g$ in the small $x$ range at $t' = 0$ and for the case $n = 1$. The solid (dashed, dash-dotted) line represents the GPD at $n = 5$ (1; 0.5) and at the scale 2 GeV.

/ $H^g(\,; \, )$. Since, to a good approximation, $H^g(\,; \, )$ equals $x g(x_B)$ at small $x$, we have approximately taken into account the evolution. The full evolution of the gluonic GPD is complicated because of mixing with the flavor-singlet quark GPD. Its modelling would counteract any possible gain of accuracy obtained by the inclusion of the full evolution behavior.

The GPD $H^g$ and its derivatives up to order $n$ are continuous at $x = 0$. For $x$ one can convince oneself that $H^g(x; \, ) \approx x g(x)$ up to corrections of order $x^2$. In the forward limit, $|t| \to 0$, the GPD $H^g$ reduces to the ordinary parton distribution $x g(x)$. Results for $H^g$ in the case $n = 1$ are shown in Fig. 3. For $x$ larger than 1 there is practically no dependence on the skewness in contrast to the region $x \ll 1$ in accord with the general behavior of the model GPD just mentioned. The GPDs for $n = 1$ and 2 agree with each other on the percent level at small $x$. As we checked the numerical results for the cross sections obtained with both these GPDs are very similar; the differences in the imaginary (real) parts of the amplitudes are typically smaller than 1(7)%. In the following we will therefore show only numerical results for the case $n = 1$.

Considering the collinear limit of the subprocess amplitude, one notices a double pole $(x + i\nu)^2$ occurring in the $T^1$ amplitude. Partial integration leads to the integral

$$ \int_0^1 \frac{dx}{x} \frac{d}{d\nu} H^g(x; \, ) f(x; \, ) ; $$

(73)

where $f$ arises from the subprocess amplitude. Since the derivatives of $H^g$ and $f$ are continuous at $x = 0$, the integral exist. The transverse quark momenta are not needed for the regularization of the $T^1$ amplitude.

A model for the GPD $H^g$ can be constructed analogously to the parton dis-
The GPD \( \tilde{g} \) is only to be replaced by its polarized counterpart \( g(\cdot) \). The continuation to negative \( x \) is defined by
\[
g(\cdot) = g(\cdot)
\]
(74)
The GPD \( \tilde{g} \) is antisymmetric in \( x \)
\[
\tilde{g}(x; \cdot) = \tilde{g}(\cdot; x)
\]
(75)
We take \( g \) from Ref. and parameterize it analogously to
\[
g(\cdot) = \sim \left(1 - \sum_{i=0}^{\infty} e_i x^2\right)
\]
(76)
where
\[
e_0 = 3.39 \quad 0.864 \ln(Q^2=Q^2_0) \quad e_1 = 1.73 + 0.24 \ln(Q^2=Q^2_0) \quad 0.17 \ln(Q^2=Q^2_0)
\]
e_2 = 0.42 \quad 0.15 \ln(Q^2=Q^2_0) \quad 0.069 \ln(Q^2=Q^2_0)
(77)
and
\[
\sim (Q^2) = 0.78 \quad 0.173 \ln(Q^2=Q^2_0)
\]
(78)
The GPD \( \tilde{g} \) can than calculated analytically for either case, \( n = 1 \) and \( 2 \), with, for instance, the help of \( \tilde{g} \). It is then represented by a sum analogously to \( \tilde{g} \). We finally remark that the polarized gluon distribution and hence \( \tilde{g} \) is subject to much larger uncertainties than \( H \).

6 Cross sections

Vector meson electroproduction in the di-rective region has been extensively investigated at HERA for large \( W \) and \( Q^2 \) but small \( x_\perp \). Preliminary data from H1 and ZEUS extend the range of \( Q^2 \) for which electroproduction data are available. In order to confront the data with the theory developed in the preceding sections, one has either to extrapolate the data to \( t' \to 0 \) or to take into account the \( t \) dependencies of the GPD and the subprocess amplitudes. The latter recipe is not straightforward. As we mentioned in Sect. It is not easy to find a plausible parameterization for the \( t \) dependence of the GPD because factorization in \( t \) and \( x_\perp \) most likely does not hold. We therefore use a variant of the first recipe and multiply the \( t' \to 0 \) amplitudes by exponentials
\[
\exp \left[tB_{\perp}^V\right]
\]
(79)
with slope parameters, \( B_{\perp}^V (i = LL;LT;TT \text{ for } L^+; L^+; T^+ \; \text{transitions, respectively}) \), adjusted to experiment. The ansatz is in accord with the expected exponential behavior of the GPD structure. Differences in the slope parameters arise from the \( t \) dependence of the subprocess amplitudes.
In the one-photon exchange approximation the $ep!epV$ cross section integrated over the azimuthal angle, reads

$$\frac{d^3\langle ep!epV\rangle}{dydQ^2dt} = \frac{\sin \frac{1}{2} y}{2} y Q^2 \int dT \frac{dL}{dt} ;$$

(80)

where high-energy, small-$x_{Bj}$ approximations have been applied to the phase space factor. Under the same kinematical conditions the ratio of longitudinal to transversal polarization of the virtual photon is given by

$$\int \frac{1}{1+ (1-y)^2} ;$$

(81)

where $y$ is the fraction of longitudinal electron momentum carried by the photon

$$y = \frac{q}{p} = \frac{W^2 + Q^2}{s} :$$

(82)

Here, $k_e$ is the momentum of the incident electron and $s = (k_e + p)^2$. The $p!Vp$ partial cross sections in ity for transversally and longitudinally polarized virtual photons are related to the amplitudes by

$$\frac{dT}{dt} = \frac{1}{16 W^2 (W^2 + Q^2)} \int M_{H^{0+}, \rho^+}^H i + M_{H^{0+}, \rho^+}^H i ;$$

$$\frac{dL}{dt} = \frac{1}{16 W^2 (W^2 + Q^2)} \int M_{H^{0+}, \rho^+}^H i ;$$

(83)

where we made use of Eqs. and . Terms of order $\hbar^8 q^2$ have been neglected in the cross sections since there is no interference between the $H^0$ and $H^0$ contributions.

The di erential cross section data for $ep!epV$ exhibits a characteristic di action peak at small $t$. The slope of the di action peak is found to be nearly independent of $W$ but is mildly varying with $Q^2$. Most of the di erential cross section data for and production are compatible with a single exponential within errors. The combined H1 and ZEUS data on the slopes in the range $4 \lesssim Q^2 < 40 \text{ GeV}^2$ can be condensed into

$$B_{LL}^V = B_{LT}^V \left( 2B_{TT}^V = B_{LL}^V \right) ;$$

(84)

This parameterization is in rather good agreement with experiment, keeping in mind that the experimental slopes are not always extracted from cross section data in the same range of $t$. Possible deviations from a single exponential behavior of the cross sections then lead to di erent slopes. We naturally assign the slope to the dominant $L!L$ transition amplitude. The slopes of the other amplitudes are not well determined as yet. A detailed analysis of the spin density matrix elements presented in Section 7 seem to favor the choice $B_{TT}^V = 2B_{LT}^V = B_{LL}^V$ slightly. These slope values lead to results from our GPD based
Figure 4: The differential cross section $d\sigma/df|nb/GeV^2|$ for $ep \rightarrow ep$ versus $-t$ integrated over the kinematical region available to the ZEUS experiment from which the data are taken. The solid line is our fit to the data at $W = 75$ GeV and $Q^2 = 6$ GeV$^2$ (see text). The dashed (dot-dashed, short-dashed) line represents the individual contributions from the $L \times L$ ($T \times T, T \times L$) amplitudes.

approach is in fair agreement with the HERA data. It is to be stressed that the magnitude of the transverse cross section is controlled by the product of parameters $f_{T T} = m_V^2 = B_{TT}$, leaving aside the mild $Q^2$ dependence of the slope. The just described fit is based on the choice $M_V = m_V$ (see the remark subsequent to Eq. (34)). Taking a smaller value for $M_V$ and a corresponding value for the decay constant, the slope $B_{TT}$ can be closer to that one for the $L \times L$ amplitude. For instance, choosing $M_V = m_V = 2$, one may use $B_{TT} = B_{LL}$ (for $f_T = 170$ MeV and $f_T = 190$ MeV) and obtains almost identical results for the cross sections.

As a check of our choice of the slopes, we show the ZEUS data for the differential cross section of production in Fig. 4. These data indicate deviations from a single exponential behavior. They are integrated over the $W$ and $Q^2$ region accessible to ZEUS; $W$ varies between 32 and 167 GeV in dependence on $Q^2$ which varies between 3 and 50 GeV$^2$. The associated normalization uncertainty is of no bearing to us since we are interested in the process $p + V \rightarrow p$. The forward amplitudes are evaluated from the model GPD $H^9$ shown in Fig. 3 multiplied with the exponentials, leading to the results for the $ep \rightarrow ep$ differential cross section shown in Fig. 4. The agreement between our result and experiment is not too good. Obviously, the value of the slope taken from Eq. (84) at $Q^2 = 6$ GeV$^2$, is a bit too small. However, the data shown in Fig. 4 need confirmation. We can also see from the figure that our result, although obtained with different slopes, do not deviate from a single exponential behavior substantially. Also shown in Fig. 4 are the three individual contributions $L \times L$, $T \times T$ and $T \times L$ separately. As expected the $L \times L$ contribution dominates. The $T \times T$ contributions amounts to about 25% of the
Figure 5: The integrated cross section for \( p!p \) (left) and \( p!p \) (right) versus \( Q^2 \) at \( W' = 75 \text{ GeV} \). Data taken from \( \text{H1} \) (filled squares) and \( \text{ZEUS} \) (open squares) for and production, respectively. The solid lines represent our results.

Let us now turn to the discussion of the process \( p!Vp \). The integrated cross section for this process is related to the integrated partial cross sections \( \sigma \) by

\[
( p!Vp) = T( p!Vp) + L( p!Vp) ;
\]

The \( \text{H1} \) and \( \text{ZEUS} \) data on the cross sections for \( p!Vp \ (V = \gamma, \phi) \), integrated over the di-raction peak, are compared to our results in Figure 5. We repeat our results are evaluated from the handbag amplitude multiplied by the exponentials and using the GPD \( H^g \) shown in Figure 4 and agreed well between model and experiment is achieved for both processes provided \( Q^2 \) is larger than about \( 4 \text{ GeV}^2 \).

The HERA experiments also measured the decay angular distributions of the \( V = \gamma, \phi \) and determined their spin density matrix elements. This information allows for a determination of the cross section ratio

\[
R(V) = \frac{L(p!Vp)}{T(p!Vp)} ;
\]

from which, in combination with \( L \), the longitudinal cross section, \( L \), can be isolated as well. The HERA data for \( L \) and \( R \) are compared to our results in Figures 6 and respectively. Again, reasonable agreement is to be observed for \( Q^2 \) larger than \( 4 \text{ GeV}^2 \). The ratio \( R \) increases with \( Q^2 \) since the transverse cross section is suppressed by \( 1 = Q^2 \) as compared to the longitudinal one, see the hierarchy.

The experimental results on cross section ratio are derived from data on the spin density matrix element \( r_{00}^{V} \). The extracted ratio is therefore a ratio of the differential cross sections.
Figure 6: The integrated cross section for longitudinally polarized photons versus $Q^2$ at $W' = 75$ GeV. Left: $Lp \rightarrow p$; right: $Lp \rightarrow p$. The data are taken from H1 (filled squares) and ZEUS (open squares), respectively. The solid lines represent our results.

Figure 7: The ratio of longitudinal and transverse cross sections for $(Lp \rightarrow p)$ production versus $Q^2$ at $W' = 75$ GeV. Data taken from H1 (filled squares) and ZEUS (open squares) for $Lp \rightarrow p$ production, respectively. The open triangles represent preliminary ZEUS data for $Lp \rightarrow pp$ and electroproduction. The solid (dashed) lines are our results for the ratio of differential (integrated) cross sections, $R'(R)$. The ratio $R'$ is evaluated at $t = 0 \pm 5$ GeV$^2$. 

24
\[ R(V) = \frac{d_{L}(p_1 V p)}{d_{T}(p_1 V p)}; \]

which equals the ratio of integrated cross sections, \( R \), only if both the differential cross sections show the same dependence \( 8 \). This is however not the case if the slopes differ. Therefore, \( R \), measured at \( t' = 0.15 \text{ GeV}^2 \), is about 10-20% larger than \( R \). In Fig. we also display our prediction for \( R \). Very good agreement with experiment is to be seen now. It is to be stressed that the uncertainties of the gluon distribution entail a typical error of about 30% for our predictions for the cross sections. In the ratios these errors cancel to some extent. As we remarked above \( B_{V}^{0+} \), is also in agreement with the present data provided the value of the product \( (f_{V,T} = M_{V})^2 = B_{V}^{0+} \) is kept constant. The ratio \( R \) for this practically falls together with \( R \) in the presented above.

In Fig. we display an Argand diagram of the three forward amplitudes for electron production at \( Q^2 = 4 \text{ GeV}^2 \), \( t = 0.15 \text{ GeV}^2 \) and \( W = 75 \text{ GeV} \). Both, \( M_{0+}^{H} \) and \( M_{0+}^{L+} \) are dominantly in agreement while the \( T \) \( L \) one is nearly real. The latter phase is a consequence of the branch point of \( x^2 \) in Eq. The hierarchy is here seen again. The phase of the production amplitude \( M_{0+}^{H} \) at \( t' = 0 \) is shown in more detail on the right hand side of Fig. The real part in agreement with \( Q^2 \) and takes values between 0.2 and 0.4 in the \( Q^2 \) region of interest. The real part of the \( L \) \( L \) amplitude therefore contributes only about 10% to the cross sections.

A number of comments concerning the leading-twist contribution are in order. As we mentioned above it is given by the collinear approximation of the dominant amplitude \( M_{0+}^{H} \). The salient features of the leading-twist contribution are passed on to the full \( L \) \( L \) amplitude, the quark transverse momenta and Sudakov suppressions essentially act its absolute value. The exam ination of the leading-twist contribution therefore elucidates many properties of our results in a simple way. Neglecting the \( k \) \( t \) terms in and using the standard definition of a meson distribution amplitude

\[ \frac{f_{V}^{L}}{2N_c} \frac{d^3k_7}{16} = v_{L}(z); \]

we obtain the subprocess amplitude \( H_{0+}^{V} \) in collinear approximation from Eq. and, inserting it into, the leading-twist contribution to the \( L \) \( L \) amplitude

\[ M_{0+}^{coll} = \frac{8}{N_c Q} f_{V}^{L} \left( 1 - \frac{H(\kappa)}{\kappa + 1} \right); \]

The moment of the meson's distribution amplitude \( v_{L} \) occurring now, is denoted by \( h_{L} = i_{L} \). For the wavefunction the associated distribution amplitude is the asymptotic form which leads to a value of 3 for the \( l = \) moment.

\( ^{8} \) For single exponentials the relation between \( R \) and \( R \) is given by

\[ R = B_{TT} = B_{LL} \exp \left[ \frac{B_{LL}}{B_{TT}} \right] R \].
Figure 8: Left: The production amplitudes for the three transitions L ! L, T ! T and T ! L at Q^2 = 4 GeV^2, t = 0.15 GeV^2 and W = 75 GeV. Right: Real over imaginary part of the amplitude \( M^H_{0+,0+} \) for production versus \( Q^2 \) at \( t' = 0 \) and \( W = 75 \) GeV.

We can now easily understand the growth of the real over imaginary part ratio with \( Q^2 \). Applying the derivative analyticity relation, frequently but unjustifiably termed the local dispersion relation, to the imaginary part of the leading-twist amplitude

\[
\mathcal{M}_{0+,0+}^{\text{coll}} \frac{\partial}{\partial \ln x_B} \frac{i}{2} \text{Im} \, \mathcal{M}_{0+,0+}^{\text{coll}}
\]

(90)

and using the low-\( x_B \) behavior of the model GPD \( H^g(p, x_B) \) (see Eq. 71), we find

\[
\text{Re} \, \mathcal{M}_{0+,0+}^{\text{coll}} = \frac{1}{2} \text{Im} \, \mathcal{M}_{0+,0+}^{\text{coll}} \left( \frac{1}{2} Q^2 \right)
\]

(91)

The increase of \( \text{Re} \, \mathcal{M}_{0+,0+}^{\text{coll}} \) with \( Q^2 \) (see Eq. 72) which has been calculated by the CTEQ group with the help of QCD evolution, is what we see at the right-hand side of Fig. 8.

Up to corrections from the real part the integrated longitudinal cross section reads

\[
\sigma_{\text{coll}}^{L} = \frac{16}{N_c^2} \frac{4}{B_{LL}} \frac{e_m}{Q^2} \left[ s f_{V,L} C_{V} h_{L} = i_{V,L} \right]^2 H^g(\; ; )^2
\]

(92)

in collinear approximation. The ratio of the and cross sections is given by \( (f_{L,C} = f_{L,C})^2 \). Our results shown in Figs. 5 and 6 approach this value with increasing \( Q^2 \). Due to the behavior of \( H^g(\; ; ) \) at small \( x \) the cross section behaves as

\[
\frac{\sigma_{\text{coll}}^{L}}{W^4 Q^2};
\]

(93)

at \( x_{B_L} \). The power behavior comes about as a consequence of the behavior of the GPD and the underlying gluon distribution. We note in passing that in the
Figure 9: Left: The integrated cross section for $p \rightarrow p$ versus $W$ for five values of $Q^2$. Data taken from Ref. The solid lines represent our result. Right: The power versus $Q^2$ for electroproduction. The solid curve represents the power as determined in Ref. with an error estimate given by the shaded band. Data are taken from Ref. ( ) and Ref. ( ).

Regge picture: the exponent $(Q^2)$ is associated with Pomeron exchange. The intercept of the Pomeron trajectory is related to by $p_0 = 1 + (Q^2)$. In the Regge model is a free parameter.

In Fig. we display the cross section for $p \rightarrow p$ as a function of $W$ for sample values of $Q^2$. Fair agreement between experiment and our predictions is to be seen. The $W$ dependence of the predictions from the full approach is very close to that given in . Deviations from the power law at lower values of $W$, to be observed in Fig. arise from various corrections to the leading-twist contribution we take into account, such as the quark transverse momenta, the $T$ amplitude and the real parts of the $L$ amplitude. This interpretation of the power behavior of is supported by a comparison of as taken from the analysis presented in Ref. with the powers obtained from fits to the cross section data. Rough agreement between both results is to be seen in Fig. although the errors of the HERA data do not permit a definite conclusion as yet. Preliminary HERA data seem to improve the agreement.

For very small one can estimate the size of the collinear contribution using the leading terms in the model GPD (94). One obtains

$$\sigma_{\text{coll}}(p \rightarrow p) = 57.2 \ \text{b GeV}^6 \ \text{GeV}^6 \ \text{GeV}^6 \text{GeV}^6 \ \text{GeV}^6 \text{GeV}^6 \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \ .$$
where $c_0 = c_0 = [(1 = -3)(1 = -2)]$ is the coefficient of the first term in the power series of $H^g$ associated with Eq. $\xi$. This cross section is rather large, well above experiment. The quark transverse momentum and the Sudakov factor suppress it such that agreement with experiment is found, see Fig. 6. 

Exploiting the leading ln($1 - x_{Bj}$) approximation, Brodsky et al. [4] found a result $^9$ that equals any except that $H^g(\xi; x_B)$ is replaced by the usual gluon distribution $x_{Bj} q(x_{Bj})$ (see also [5]). At very small $x_B$, i.e., if $x_B$ is so small that the first terms in Eq. [4] are the only ones that contribute, the usual gluon distribution and the GPD only differ by about 20% resulting from the difference between $c_0$ and $c_0$. For larger $x_B$, however, the difference between both the functions becomes substantial, growing up to about a factor of 1.6 - 2 at $x_B = 0.1$, see Fig. 10. 

The use of the ln($1 - x_{Bj}$) approximation at values of around 0.1 may therefore lead to an underestimate of the gluonic contribution to cross sections by a factor 3 to 4. We repeat that, in contrast to the ln($1 - x_{Bj}$) approximation, we do not require small $x_B$ in principal. We only restrict ourselves to small $x_B$ in order to avoid complications with potential contributions from quark emissions and reabsorptions by the proton. The enhancement effect apparent in Fig. 10 is known as the skewing effect, and has been discussed by several authors [8,15,54,55]. The size of the skewing effect estimated in these papers, is compatible with our model result for $\xi > 1$. 

In any case the leading-twist as well as the ln($1 - x_{Bj}$) result provide cross sections that are too large. In order to settle this problem for the ln($1 - x_{Bj}$) approximation Frankfurt et al. [6] estimated a correction factor by allowing for quark transverse momentum in the loop. This mechanism bears resemblance to our approach as we mentioned in Sect. 3. The correction

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure10.png}
\caption{Ratios of the GPDs and the parton distribution for the $n = 1$ and 2 models at a scale of 4 GeV$^2$.}
\end{figure}

\textsuperscript{9}Note, in the decay constant includes theavor weight factor $C_v$.}
factor obtained in is large enough to achieve agreement with experiment. This factor has also been used by Mankiewicz et al. in an exploratory study of $L_1$ in an otherwise collinear GPD approach. Martin et al. exploited the ln($l=x_0$) approximation in their analysis of vector meson electroproduction by including parton transverse momenta and an unintegrated gluon distribution.

7 Spin density matrix elements

With the help of Eqs. (27), (41) and (58) the spin density matrix elements extracted from the decay angular distributions measured with unpolarized leptons and protons simplify to ($\mathcal{R}$ is defined in (90))

\[
N_L = 2 \Re M_{0^+,0^+}^{H}; \\
N_T = 2 \Im M_{0^+,0^+}^{H} + \Re M_{0^+,0^+}^{H} ; \\
r_{00}^{04} = \frac{1}{2} \frac{\Re}{\mathcal{R}^2_N} M_{0^+,0^+}^{H} + \frac{\Re}{\mathcal{R}^2_N}; \\
\Re r_{10}^{04} = \Re \frac{\Im}{\mathcal{R}^2} \frac{\Re}{\mathcal{R}^2} M_{0^+,0^+}^{H}; \\
r_{10}^{10} = \frac{1}{\mathcal{R}} \frac{\Re}{\mathcal{R}^2} M_{0^+,0^+}^{H} ; \\
r_{11}^{10} = \frac{1}{\mathcal{R}} \frac{\Re}{\mathcal{R}^2} M_{0^+,0^+}^{H}; \\
r_{50}^{00} = 4 \frac{\Re}{2N_L N_T} \frac{\Re}{\mathcal{R}^2} M_{0^+,0^+}^{H} ; \\
\Re r_{10}^{05} = \Re \frac{\Im}{\mathcal{R}^2} \frac{\Re}{\mathcal{R}^2} M_{0^+,0^+}^{H}; \\
\Im r_{11}^{05} = \frac{1}{\mathcal{R}} \frac{\Re}{\mathcal{R}^2} M_{0^+,0^+}^{H}; \\
\Im r_{50}^{01} = 4 \frac{\Re}{2N_L N_T} \frac{\Re}{\mathcal{R}^2} M_{0^+,0^+}^{H} ;
\]

(95)

while

\[
r_{04}^{04} = r_{11}^{11} = r_{11}^{05} = r_{11}^{05} = \Im r_{11}^{05} = 0 ;
\]

(96)

because of the neglect of $L^! \rightarrow T$ and $T^! \rightarrow T$ transitions. The relations obtained in the GPD approach under the assumption of the dominance of the $H^q$ terms, coincide with those found assuming dominance of natural parity t-channel exchanges and the neglect of proton helicity $H^q$. The contributions from $H^q$ enter the spin density matrix elements only as bilinears, there are no interferences between $H^q$ and $H^q$ terms.

The data for the spin density matrix elements from H1 and ZEUS are shown in Figs. 11 and 12 and compared to the results from the GPD based approach. The general pattern of the data is reproduced. The dominance of the $L^! \rightarrow L$ transition
amplitude is clearly visible in the angular distribution of the production and decay of the vector mesons, in particular in the value of $r_{00}^2$ which tends towards 1 with increasing $Q^2$. This behavior is well reproduced by our approach as we already discussed in connection with the cross section ratio $R$.

The $T!L$ amplitude is probed by the matrix elements $r_{00}^1$ and $r_{00}^5$. While the rest matrix element is approximately $/M_0^+r_+^0f=\hat{M}_0^+r_+^0f$, the ratio $\text{Im} M_0^+r_+^0=\text{Im} M_0^+r_+^0$ essentially controls the second. Both $r_{00}^1$ and $r_{00}^5$ are found to be rather small. The ratio of the $T!T$ and $L!L$ amplitudes is approximately measured by $r_{11}^1$ and $\text{Re} r_{10}^5$, quadratically in the first case, linearly in the second one since both the amplitudes have about the same phase as is shown in Fig.8. The fair agreement between theory and experiment for these spin density matrix elements tells us that our approach provides the correct sizes and relative phases of the $T!T$ and $L!L$ amplitudes. The matrix elements $\text{Re} r_{10}^0 = \text{Re} r_{10}^5 = \text{Im} r_{10}^5$ measure an interference term between the $T!T$ and $T!L$ amplitudes which is very small. Also this prediction is in acceptable agreement with experiment.

The $t$-dependence of the spin density matrix elements confirm the above observations, see Fig.10. The $T!L$ sensitive matrix elements behave proportional to $t$ while those controlled by ratios of the $T!T$ and $L!L$ amplitudes exhibit an $t$ dependence according to the different slopes chosen for them. As we mentioned in Sec.6 the freedom in choosing a suitable value of $M_V$ also allows to test with $B_V^+$ which, while the transverse cross section is nearly insensitive to this choice provided the product $(f_V M_V)^2=b_{TT}$ is approximately kept fixed, does the $t$ dependence of some of the spin density matrix elements (e.g. $r_{00}^0, r_{11}^1$) change; they become very small at $t$. Given the accuracy of the present data such a behavior is not in conflict with experiment.

Finally, in Fig.11 we show our predictions for electroproduction at $W = 10 \text{ GeV}$ characteristic of the COMPASS experiment.

Other theoretical analyses of the spin density matrix elements base on variants of the $\Lambda$ approximation. The variants differ from each other in the detailed treatment of the subprocess $g+g$ and $ggg$. The same hierarchy of the amplitudes is obtained as we mention and, in general, rather similar results are obtained for the spin density matrix elements. What differentiates the $t$ dependence of the $T!L$ amplitude and a somewhat different $t$ dependence of the $T!T$ matrix.

8 The helicity correlation

Last we want discuss the role of the GPD $\hat{H}^g$. For this purpose we consider the initial state helicity correlation $A_{LL}$ which can be measured with longitudinally polarized beam and target. After integration over the azimuthal angle this correlation reads

$$A_{LL} = \frac{p_1 \cdot n}{32 W^2 (W^2 + Q^2)} \int d^2 T \frac{d^2 T}{d^2 L} \frac{1}{M^{++} + \frac{\hat{f}}{2} + \frac{\hat{M}^{0+}}{2} + \frac{\hat{f}}{2}}$$

(97)
Figure 11: The spin density matrix elements of electroproduced mesons versus $Q^2$ at $W' = 75 \text{ GeV}$ and $t' = 0.15 \text{ GeV}^2$. Data, taken from [12] (illed circles) and [14] (open circles), are compared to our results (solid line). Preliminary data on $r_{00}^{04}$ from ZEUS (open triangles) are also shown.
Figure 12: The spin density matrix elements for electroproduction versus $Q^2$ at $W' = 75$ GeV and $t' = 0$ GeV². The H1 data are compared to our results (solid line).
Figure 13: The spin density matrix elements of electroproduced mesons versus $t$ at $Q^2 = 5 \text{ GeV}^2$ and $W' = 75 \text{ GeV}$. Data taken from [12] (filled circles). The solid (dashed) lines represent our results for the choice $B_{TT} = B_{LL} = 2$ ($B_{LL}$).
Figure 14: The spin density matrix elements of electroproduced mesons versus $Q^2$ at $W' = 10$ GeV, $y' = 0.6$ and $t' = 0.15$ GeV$^2$. The solid lines represent our results.
where the amplitudes and cross sections refer to the process $p!Vp$ and are given in Eqs. (24) and (83). As can easily be seen from Eq. (58), $A_{LL} = 0$ if the $\bar{g}^g$ terms are neglected as we did in the preceding sections. Yet in contrast to the cross sections and spin density matrix elements where the correction are bilinear in the $\bar{g}^g$ terms and, hence, extremely small, the leading term in $A_{LL}$ is an interference between the $H^g$ and the $\bar{H}^g$ terms. In fact, with the help of Eqs. (27) and (58), one obtains from Eq. (98)

$$A_{LL}[ep!e'pV] = 2\frac{p}{1} \mathrm{Re} \left\{ \frac{\hat{M}_{H^{++},+} M_{H^p}^{g_i}}{\hat{M}_{H^{0^+},+}^{g_i} + \hat{M}_{H^{++},+}^{g_i}} \right\}. \quad (98)$$

Obviously, this ratio is of order $\hbar k_i^2 = Q^2 H_i^g = H_i^{g}g_i$ and, therefore, very small values for $A_{LL}$ are to be expected. Indeed exploiting the model $GPD$s presented in Section 5 we confirm this assertion as can be seen from Fig. 15 where results for $A_{LL}$ for $e\bar{p}$ and electroproduction at $t' = 0$ are displayed. The results for production, only shown at $W = 15 \text{ GeV}$, is compared to the SM C data. At this energy and in the range of $Q^2$ shown in the plot, the contribution from the quark $GPD$ is expected to be small. Our results for $A_{LL}$ are not in disagreement with experiment given the admitted large experimental errors and the rather large value of the skewness. Results for electroproduction are shown at energies typical for the HERMES and COMPASS experiments. The dominance of the gluon over the sea quarks permit this. At $W = 5 \text{ GeV}$ $A_{LL}$ is not very small since the major contribution to it comes from the region $0.1 < x < 0.2$ where $g-g$ is not small.

The proton helicity $ip$ contribution, related to the $GPD E^g$, may change these results but likely not substantially. The helicity correlation will increase with growing momentum.
transfer if the slope of the $T^T$ amplitude is smaller than that of the $L^L$ one. Besides allowing predictions for $A_{LL}$, this calculation also supports our assumption of negligible contributions from $A_{T^T}$ to cross sections and spin density matrix elements.

9 Summary

We analyzed electroproduction of light vector mesons at small $x_{Bj}$ within a GPD-based approach. In this kinematical domain the gluonic GPD $H^g$, parameterizing the response of the proton to the emission and reabsorption of gluons, controls the process. The gluonic GPD, not calculable at present, is constructed from an ansatz for the double distributions currently in use. In order to examine the influence of the model GPD on the numerical results for vector meson electroproduction we used two different versions for it ($n = 1$ and 2). The differences in the numerical results obtained from these two models, are on the percent level. The subprocess amplitudes for $g^V g$ are calculated by us to lowest order of perturbative QCD but transverse momenta of the quark and antiquark that form the vector meson, are taken into account as well as Sudakov suppression which sum up gluonic radiative corrections.

The GPD approach reproduces all main features of vector meson electroproduction at small $x_{Bj}$ known from phenomenology. The dominance of the contributions from the GPD $H^g$ over those from $F^g$ and $E^g$, leads to the relations and, hence, to results equivalent to those obtained assuming the dominance of natural parity exchange. Approximate $s$-channel helicity conservation holds due to the hierarchy of the amplitudes respect in our GPD-based approach. The behavior of the longitudinal cross section as a $Q^2$ dependent power of $W$ at fixed $Q^2$ is a consequence of low-$x$ properties of the GPD and QCD evolution. The numerical results we obtain from our approach are in reasonable agreement with the small $x_{Bj}$ data on cross sections and spin density matrix elements for electroproduction of vector mesons measured by H1 and ZEUS. The $t$ dependence of vector meson electroproduction is not yet satisfactorily settled. In principle it is generated by a combination of the $t$ dependence of the GPD and, with lesser importance, that of the subprocess amplitudes. Due to the lack of a plausible parameterization of the $t$ dependence of the GPD we have evaluated the electroproduction amplitudes at $t' = 0$ and multiplied them by exponentials in $t$. In proven cases this recipe is demanded and will be unavoidable as soon as detailed differential cross section data are at hand.

We also compared in some detail our approach to the leading-twist contribution and to the leading $\ln(1-x_{Bj})$ approximation. The latter is rather close to the GPD approach at low $x_{Bj}$ and small $t$ but not identical. For $x_{Bj}$ larger than about 0.01 the replacement of $H^g(\xi)$ by $F^g(\xi)$ becomes inappropriate. The GPD has, in contrast to the leading $\ln(1-x_{Bj})$ approximation, the potential to investigate the $t$ dependence of electroproduction. The lack of understanding of the GPD's $t$ dependence prevents this at present.
Acknowledgements

We thank A. Borissov, A. Efremov, O. Nachtmann, H. Meyer, E. Paul, A. Sandacz and L. Szymanski for discussions and A. Buni for providing us with preliminary ZEUS data. This work has been supported in part by the Russian Foundation for Basic Research, Grant 03-02-16816, the Integrated Infrastructure Initiative "Hadron Physics" of the European Union, contract No. 506078 and by the Heisenberg-Landau program.

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