Quantum Superposition of Two Temperatures

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In the classical world, temperature is a measure of how hot or cold a physical object is. We never find a physical system which can be both hot and cold at the same time. Here, we show that for a quantum system, it is possible to have superposition of two temperatures which can lead to a situation that it can be found both in hot and cold state. We propose a physical mechanism for how to create a quantum state which is superposition of two temperatures. Furthermore, we define an operator for the inverse temperature and show that the thermal state is, in fact, an eigenstate of this operator. The quantum state which represents superposition of two temperatures is not an eigenstate of the inverse temperature operator. Our findings can have new applications in quantum thermodynamics, quantum nano scale devices and quantum statistical mechanics.
I. INTRODUCTION

Quantum superposition, which has been puzzling since its inception, lies at the heart of quantum theory. This superposition leads to quantum coherence [1], and exhibits non-classical correlations [2–5] among distant parties that offer technological advances in communication [6–8], information processing [9–11], computation [12] and sensing [13, 14].

While quantum theory is about a century old, thermodynamics [15] which stems from the quest of utilizing an energetic resource for various work efficiently has been well established over the centuries. This pragmatic branch of physics has witnessed several conceptual breakthroughs without violation of its principles [16].

Only few decades ago people have started exploring thermodynamic behavior of objects at the atomic and nano-scales, in particular to explore what happens to thermodynamic quantities at these scales where fluctuations cannot be ignored [17, 18]. Thus, stochastic thermodynamics [19, 20] and quantum thermodynamics [21–24] have emerged as active fields of research of late. Several intriguing phenomena starting from heat flowing from cold to hot body [25], to several second laws [26, 27] have been established without contradicting the traditional thermodynamics. Besides these foundational issues, designing quantum thermal machines [28, 29] and exploring advantages of quantum resources like coherence and entanglement in functioning of heat machines [30–37] have also become active fields of study.

Classically the concept of temperature was originated as the average kinetic energy of an ideal gas [15]. Though the notion of temperature of a quantum particle is much more intrigued, it can be visualized as the Lagrange multipliers that arises while maximizing the von Neumann entropy of a quantum particle with fixed average energy [38, 39]. Thus, the temperature of the quantum particle can well be explained by the Gibbs distribution of it at some inverse temperature $\beta$. This lies at the heart of quantum thermometry [40] where a quantum probe interacts with a heat bath and reaches the thermal distribution corresponding to the bath temperature. From this distribution of the probe, specifically some properties of the probe state that depends monotonically on this distribution, one can estimate the temperature of the probe.

In the quantum world, physical objects can exist in a superposition of two or more distinct states. Here, we ask can a quantum system be in a superposition state characterized by two different temperatures? In classical systems, this is not possible. But in quantum mechanics the possibility of quantum superposition of two temperatures is possible. There is also experimental evidence that electrons carrying heat propagate as two-dimensional quantum waves and the ripples in those waves are predicted to lead to hot and cold spots [41]. This is in sharp contrast to our everyday experience of temperature and heat flow. Even though we do not intend to model the experiment, it is possible that there are quantum superposition of two temperatures and the hot and the cold spot could be due to quantum interference of these states.

While locally thermal states become important to inspect the role of quantum entanglement in storing work [42], we inquire a fundamental question: Suppose we have two physical systems at thermal distributions at two different temperatures. Is it possible to create a coherent quantum superposition of these two systems at two temperatures? We find that the answer to this question is yes. We show that if we take the purifications of the thermal states and perform Bell-state measurement on the ancillary systems, then that can create a quantum state which is superposition of two temperatures. Our protocol works in a probabilistic manner, but nevertheless brings out the main idea of our paper, i.e., ‘making temperature quantum’. We also show how the amplitudes of quantum superposition of two temperatures can display interference pattern which is a key feature of quantum world. The interference pattern depends on the two different temperatures which may be hot and cold. Furthermore, we define an inverse temperature operator and show that the purified state of any thermal state is actually an eigenstate of this operator. That is consistent with our understanding that thermal states are in a definite temperature. We can also check that the superposition of two temperatures is not an eigenstate of the inverse temperature operator, thus hinting towards superposition of two temperatures. Our findings could have a deep impact on the foundations of quantum statistical physics and could have implications for quantum nano technology, thermal machines at small scale and ultimately in quantum technology.

Before we present our main results, we would like to mention that if we have two thermal states at two different temperatures, then we can imagine that there are two pure entangled states corresponding to these two thermal states. Then, using the coherent quantum control or indefinite causal order [43–45] one can create a superposition of these two pure entangled states. However, that will not corresponding to superposition of two temperatures in quantum world [46].
II. SUPERPOSITION OF TEMPERATURES

In this section, we provide a protocol where we can create a coherent superposition of quantum states with two distinct temperatures. We present our main idea for the case of qubits, but it can be generalized for higher dimensions.

First, consider a simple case of two qubits with density operators $\rho_A = e^{-\beta_A H_A}/Z_A$ and $\rho_B = e^{-\beta_B H_B}/Z_B$ which are in thermal states at inverse temperatures $\beta_A = 1/kT_A$ and $\beta_B = 1/kT_B$, respectively. Here $k$ is the Boltzmann constant and $T_A$, $T_B$ are temperatures of the thermal states $A$ and $B$. Let the Hamiltonian of the two qubit systems be

$$H_A = E_0|00\rangle\langle00| + E_1|11\rangle\langle11|,$$
$$H_B = E'_0|00\rangle\langle00| + E'_1|11\rangle\langle11|,$$  
where $E_0$, $E_1$ are the eigenvalues of the Hamiltonian $H_A$ and $E'_0$, $E'_1$ are the eigenvalues of the Hamiltonian $H_B$. The density matrix for the first qubit can be expressed as

$$\rho_A = p_0|00\rangle\langle00| + p_1|11\rangle\langle11|,$$  
where $p_0 = e^{-\beta_A E_0}/Z_A$, $p_1 = e^{-\beta_A E_1}/Z_A$ and $Z_A = e^{-\beta_A E_0} + e^{-\beta_A E_1}$ being the partition function. Similarly, the density matrix for the second qubit can be expressed as

$$\rho_B = f_0|00\rangle\langle00| + f_1|11\rangle\langle11|,$$  
where $f_0 = e^{-\beta_B E'_0}/Z_B$, $f_1 = e^{-\beta_B E'_1}/Z_B$ and $Z_B = e^{-\beta_B E'_0} + e^{-\beta_B E'_1}$ being the partition function.

Now consider two purification of $A$ and $B$ such that $\rho_A = Tr_A(\rho_{A'}\Psi(|\Phi\rangle))$ and $\rho_B = Tr_B(\rho_{B'\Phi}|\Phi\rangle)$, where

$$|\Psi\rangle = \sqrt{p_0}|00\rangle + \sqrt{f_0}|00\rangle,$$
$$|\Phi\rangle = \sqrt{p_1}|11\rangle + \sqrt{f_1}|11\rangle,$$
Now consider the joint state of the composite system $A'A$ and $BB'$ as given by

$$|\Psi\rangle_{A'A} = \frac{1}{\sqrt{2}}(|\Phi^+\rangle_{A'B'}\langle\Phi^+\rangle + |\Phi^-\rangle_{A'B'}\langle\Phi^-\rangle),$$

where $|\Phi^\pm\rangle$ and $|\Phi^\mp\rangle$ are the well known Bell states. Now if we perform a Bell basis measurement on the system $A'B'$.

If we find that $|\Phi^+\rangle_{A'B'}$ clicks (for example), which has the probability $P_{\Phi^+} = \frac{1}{2}p_0f_0 + p_1f_1$ of occurrence, then the system $AB$ can be found in the following state

$$|\Psi(\Phi^+)\rangle_{AB} = \frac{1}{\sqrt{2}N}(\sqrt{p_0f_0}|00\rangle + \sqrt{p_1f_1}|11\rangle)$$

where $N = \sqrt{p_0f_0 + p_1f_1}$ is the normalization constant. Now, we can write the above state as

$$|\Psi(\Phi^+)\rangle_{AB} = \frac{1}{\sqrt{2Z_AZ_B}}(e^{-(\beta_A E_0 + \beta_B E_0')/2}|00\rangle_{AB} + e^{-(\beta_A E_1 + \beta_B E_1')/2}\sqrt{2Z_AZ_B}|11\rangle_{AB}).$$

In the above state, if we chose $E_0' = 0$ and $E_1' = 0$, then we have a state

$$|\Psi(\Phi^+)\rangle_{AB} = \frac{1}{\sqrt{2Z_AZ_B}}(e^{-\beta_A E_0/2}|00\rangle_{AB} + e^{-\beta_B E_1'/2}\sqrt{2Z_AZ_B}|11\rangle_{AB}).$$

The above state is a quantum superposition of two temperatures $\beta_A$ and $\beta_B$. The coherent oscillation between two amplitudes actually depends not on one temperature but on temperatures of two different baths which could be at $T_A$ and $T_B$. Also, this is not a superposition of two purified thermal states. This is genuine quantum superposition of two temperature state. If we view temperature as arising from the entanglement between the system and the
FIG. 1. We plot the interference pattern of this superposition of two temperature states with $E_0 = 5$, $E'_1 = 1$ and $\beta_A = 1$.

bath, then the Bell measurement creates the entanglement between two independent qubits which are at two different temperatures.

To see the coherent oscillation between two branches of the amplitudes, let us imagine that we perform a controlled NOT gate on $AB$ and discard the system $B$. Then, we apply a Hadamard gate on the qubit $A$ and measure in the basis energy basis $\{|0\rangle, |1\rangle\}$. The probability of finding the system in these energy eigenbasis will reveal interference effect of two branches which are at two different temperatures. For example, the probability of finding the qubit in the state $|0\rangle$ is given by

$$
\frac{1}{2} \left[ 1 + \frac{1}{N^2 Z_A Z_B} e^{-\beta_A E_0^2/2} \cos \phi \right].
$$

The above expression displays interference which is a result of coherent superposition of two amplitudes at temperatures $T_A$ and $T_B$, respectively. There is well defined phase relationship between the amplitudes in the superposition. We plot this interference pattern in Fig. 1.

Let us explore what kind of superposition of two temperatures can be created if we get other outcomes. If $|\Phi^+\rangle_{A'B'}$ clicks, which also has the probability $P_{\Phi^+} = \frac{1}{2}[p_0 f_0 + p_1 f_1]$ of occurrence, then following the same procedure we get the state of the two qubits $AB$ as

$$
|\Psi(\Phi^+)\rangle_{AB} = \frac{1}{N' \sqrt{2 Z_A Z_B}} (e^{-\beta_A E_0^2/2} |00\rangle_{AB} - e^{-\beta_B E'_1^2/2} e^{i \phi} |11\rangle_{AB}),
$$

which differs from Eq. (9) by a local unitary. For example, by applying $(I \otimes \sigma_z)$, we can obtain the same state as given in (9), i.e., a quantum state with superposition of two temperatures.

If $|\Psi^-\rangle_{A'B'}$ clicks with probability $P_{\Psi^-} = \frac{1}{2}[p_0 f_1 + p_1 f_0]$, then the system $AB$ can be found in the following state

$$
|\Psi(\Psi^-)\rangle_{AB} = \frac{1}{N' \sqrt{2 Z_A Z_B}} (e^{-\beta_A E_0^2/2} |01\rangle_{AB} + e^{-\beta_B E'_1^2/2} e^{i \phi} |10\rangle_{AB}),
$$

where $N' = \sqrt{p_0 f_1 + p_1 f_0}$. Similarly, for $|\Psi^-\rangle_{A'B'}$ with probability $P_{\Psi^-} = \frac{1}{2}[p_0 f_1 + p_1 f_0]$, we get

$$
|\Psi(\Psi^-)\rangle_{AB} = \frac{1}{N' \sqrt{2 Z_A Z_B}} (e^{-\beta_A E_0^2/2} |01\rangle_{AB} - e^{-\beta_A E_1^2/2} e^{i \phi} |10\rangle_{AB}).
$$

Clearly in the last two cases the choice $E'_0 = 0$ and $E'_1 = 0$ does not work. Therefore, our protocol for creating superposition of two temperatures is probabilistic with success probability given by

$$
P_{\text{suc}} = p_0 f_0 + p_1 f_1.
$$
It may be noted that if we chose \( E_1 = 0 \) and \( E'_1 = 0 \), then we can create similar quantum superposition of two temperatures with total success probability \( p_{\text{succ}} = [p_0 f_1 + p_1 f_0] \). The quantum state which is a superposition of two temperatures can have testable consequences. One can carry out an experiment to observe such temperature superpositions. For example, if we create such a state and send through a Mach-Zehnder interferometer then we should be able to see an interference pattern on the screen which will depend on the temperature of hot and cold bath.

### III. INVERSE TEMPERATURE OPERATOR

If we think of making temperature as a quantum variable, then the natural question that comes to mind: Can there be a Hermitian operator for temperature? When a quantum state is in a thermal state, we know that it is in a thermal equilibrium and remains in a fixed temperature. Intuitively, one can then imagine that if such an operator exists for temperature, then the thermal state should be an eigenstate of this operator.

Here, we would like to mention that after we defined the inverse temperature operator in 2017, we noticed that there is a proposal to define temperature as an operator [47]. However, our approach is different than what is given in the above paper.

In this section, we define an inverse temperature operator and show that the purification of the thermal state is indeed an eigenstate of this operator. Let us consider a physical system \( S \) which is in a thermal state at inverse temperature \( \beta \) as given by

\[
\rho_S = \frac{e^{-\beta H}}{Z} = \sum_n \frac{e^{-\beta E_n}}{Z} |n\rangle \langle n|,
\]

where \( H \) is the Hamiltonian of the system, \( |n\rangle \) is the \( n \)th energy eigenstate of the Hamiltonian with energy \( E_n \) and \( Z = \sum_n e^{-\beta E_n} \) is the partition function. Let us now consider a purification of \( \rho_S \) as

\[
|\Psi\rangle_{SR} = \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} |n\rangle_S \otimes |n\rangle_R
\]

such that \( \rho_S = T_{FR}(|\Psi\rangle_{SR}\langle \Psi|) \) and \( R \) denotes the bath degrees of freedom.

We define the (squared) inverse temperature operator \( \hat{K} = \sum_n \left( \frac{i\partial}{\partial E_n} \otimes -\frac{i\partial}{\partial E_n} \right) \). This is a Hermitian operator that acts on \( \mathcal{H}_S \otimes \mathcal{H}_R \). Moreover, the inverse temperature operator is a non-local operator on the purified Hilbert space. We will prove that the purification of any thermal state is an eigenstate of the (squared) inverse temperature operator with eigenvalue \( c\beta^2 \), where \( c = 16 \), i.e., we have

\[
\hat{K} |\Psi\rangle_{SR} = \frac{\beta^2}{16} |\Psi\rangle_{SR}.
\]

The action of the inverse temperature operator \( \hat{K} \) on the purified thermal state is given by

\[
\hat{K} |\Psi\rangle_{SR} = \sum_n \left( \frac{i\partial}{\partial E_n} \otimes -\frac{i\partial}{\partial E_n} \right) |\Psi\rangle_{SR}
= \frac{1}{\sqrt{Z}} \sum_n \sum_m \left( \frac{i\partial}{\partial E_n} \otimes -\frac{i\partial}{\partial E_n} \right) e^{-\beta E_m/4} |m\rangle \otimes e^{-\beta E_m/4} |m\rangle
= \frac{1}{\sqrt{Z}} \sum_n \sum_m \left( -\frac{i\beta}{4} \right) \delta_{nm} e^{-\beta E_n/4} |m\rangle \otimes \left( \frac{i\beta}{4} \right) \delta_{nm} e^{-\beta E_n/4} |m\rangle
= \frac{\beta^2}{16} \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} |n\rangle \otimes |n\rangle = \frac{\beta^2}{16} |\Psi\rangle_{SR}.
\]

Therefore, \( |\Psi\rangle_{SR} \) is an eigenstate of \( \sum_n \left( \frac{i\partial}{\partial E_n} \otimes -\frac{i\partial}{\partial E_n} \right) \) with eigenvalue \( \frac{\beta^2}{16} \). Thus, the thermal state is, indeed, in a definite temperature state. However, the quantum superposition of two temperatures as given in (9) and (11) are not eigenstates of the (squared inverse) temperature operator. Thus, we cannot associate a definite temperature to the quantum superposition. Also, the individual qubits are not in a thermal state with definite temperature. More details of the inverse temperature operator and its applications to Thermodynamics uncertainty relations will be reported in future.
IV. DISCUSSIONS AND CONCLUSIONS

The notion of temperature, even though commonly used, it still raises deep questions when we apply thermodynamical concepts for quantum systems. Recent progresses suggest that there can be thermalisation in isolated quantum systems which hints that a temperature can be assigned even to individual, pure quantum states. In classical physics, temperature of a physical system in thermal equilibrium can be found at a fixed temperature. In this paper, we have shown that physical system can not only exists in a definite temperature state, but also in a superposition of two temperatures. In particular, we have given a protocol how to create superposition of qubits where the spin-up and spin-down states can be at two different temperatures. This coherent superposition of pure state at two different temperatures can lead to quantum interference where interference pattern will depend on temperature of two baths. We have also defined a (squared inverse) temperature operator and shown that the purification of the thermal state is an eigenstate of this operator. This is in concurrence with our understanding that thermal state is in a definite temperature. The quantum superposition of two temperatures state is not an eigenstate of the temperature operator. We hope that our proposal for the superposition of two temperatures can be tested in experiment involving quantum interferometry.

Finally, it may be noted that if one uses two thermal channels and applies the coherent control of channels for the system along with a control qubit, one can create superposition of two purified thermal states. But we believe that it is not a superposition of two temperatures. We hope that our protocol will find useful applications in quantum thermodynamics, quantum nano scale devices and quantum statistical mechanics.

Note added:- After completion of our work in 2017, we came to know about the preprint [48] which has addressed similar question, but the results are completely different.

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