Article

Sufficient Criteria for the Absence of Global Solutions for an Inhomogeneous System of Fractional Differential Equations

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Abstract: A nonlinear inhomogeneous system of fractional differential equations is investigated. Namely, sufficient criteria are obtained so that the considered system has no global solutions. Furthermore, an example is provided to show the effect of the inhomogeneous terms on the blow-up of solutions. Our results are extensions of those obtained by Furati and Kirane (2008) in the homogeneous case.

Keywords: absence of global solutions; inhomogeneous system; fractional differential equations

MSC: 26A33; 34A08

1. Introduction

The theory of fractional calculus provides useful mathematical tools for modeling several phenomena from Science and Engineering (see e.g., [1–7]). This fact has motivated researchers to investigate fractional differential equations in various directions including theory (existence and uniqueness of solutions [8–10], comparison principles [11,12], blow-up profile of solutions [13,14]), and numerical methods (see e.g., [15–19]).

In this paper, we study the inhomogeneous system of fractional differential equations

\[
\begin{aligned}
\frac{dx}{dt}(t) + \frac{dx}{dt^\alpha}(t) &= |y(t)|^q + z_1(t), \\
\frac{dy}{dt}(t) + \frac{dy}{dt^\beta}(t) &= |x(t)|^p + z_2(t), \\
(x(0), y(0)) &= (x_0, y_0),
\end{aligned}
\]

(1)

where \( t > 0, \ p, q > 1, \ z_1, z_2 \in C([0, \infty)), \ z_1, z_2 \geq 0, \ 0 < a, \beta < 1, \) and \( \frac{d}{dt^\tau}, \ \tau \in \{a, \beta\}, \) is the derivative of order \( \tau \) in the sense of Caputo. Namely, we obtain sufficient criteria for which global solutions for Equation (1) do not exist, i.e., a finite time blow-up occurs.

The study of the absence of global solutions for differential equations (or fractional differential equations) furnishes important indications on limiting behaviors of many physical systems. In industry, the knowledge of the finite time blow-up can prevent accidents and malfunctions. It can also be useful for the improvement of the performance of machines and the extension of their life-span.
In [20], Furati and Kirane investigated the system

\[
\begin{aligned}
    dx(t) + \frac{dx}{dt}(t) &= |y(t)|^q, \\
    dy(t) + \frac{dy}{dt}(t) &= |x(t)|^p, \quad (x(0), y(0)) = (x_0, y_0),
\end{aligned}
\]

which is a special case of Equation (1) with \( z_i \equiv 0, i = 1, 2 \). They proved that, if \( x_0, y_0 > 0 \) and

\[
1 - \frac{1}{pq} \leq \max \left\{ \frac{\alpha}{p}, \frac{\beta}{q} \right\},
\]

then Equation (2) has no global solutions. Next, Kirane and Malik [14] studied the profile of the blowing up solutions of Equation (2).

Let us mention that several results related to the finite time blow-up of solutions of fractional differential equations were obtained in previous contributions (see e.g., [13,21–25] and references therein). However, systems of the type seen in Equation (1) were not studied previously. Here, our aim is to study the effect of the inhomogenous terms \( z_i(t), i = 1, 2 \), on the blow-up of solutions of Equation (2).

Before presenting the main results, we first recall briefly certain standard notions on fractional calculus that will be used throughout this paper. For more details, see e.g., [9].

Let \( \xi > 0 \). The fractional integrals of order \( \xi \) of a function \( \eta \in C([0, \mu]), \mu > 0 \), are given by

\[
I_{0}^{\xi} \eta(t) = \frac{1}{\Gamma(\xi)} \int_{0}^{t} (t - \tau)^{\xi - 1} \eta(\tau) \, d\tau \quad \text{(left-sided fractional integral)}
\]

and

\[
I_{\mu}^{\xi} \eta(t) = \frac{1}{\Gamma(\xi)} \int_{\mu}^{t} (\tau - t)^{\xi - 1} \eta(\tau) \, d\tau \quad \text{(right-sided fractional integral)},
\]

for all \( t \in [0, \mu] \).

For all \( \eta, \lambda \in C([0, \mu]) \), one has

\[
\int_{0}^{\mu} \lambda(t) I_{0}^{\xi} \eta(t) \, dt = \int_{0}^{\mu} \eta(t) I_{0}^{\xi} \lambda(t) \, dt.
\]

Let \( \psi(t) = \left( 1 - \frac{t}{\mu} \right)^{\kappa}, \) for all \( t \in [0, \mu] \),

where \( \kappa \geq 0 \). Then

\[
I_{\mu}^{\xi} \psi(t) = \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa + \xi + 1)} \mu^{-\kappa} (\mu - t)^{\xi + \kappa}, \quad \text{for all } t \in [0, \mu].
\]

Suppose now that \( 0 < \xi < 1 \). The derivative of order \( \xi \) of a function \( \eta \in C^{1}([0, \mu]) \) in the Caputo sense, is given by

\[
\frac{d\eta}{dx^{\xi}}(t) = I_{0}^{1-\xi} \left( \frac{d\eta}{dt} \right)(t), \quad \text{for all } t \in [0, \mu].
\]

The pair of functions \( (x, y) \), where \( x, y \in C^{1}([0, \infty)) \), is a global solution of Equation (1), if it satisfies Equation (1) for all \( t > 0 \). Let us recall that the system of Equation (1) is investigated under the assumptions:

\[\text{(A1) } p, q > 1 \text{ and } 0 < \alpha, \beta < 1.\]

\[\text{(A2) } z_i \in C([0, \infty)), z_i \geq 0, i = 1, 2.\]
Now, we present our results.

**Theorem 1.** Let \( x_0, y_0 \geq 0 \). If

\[
\limsup_{T \to +\infty} T^{(\alpha-1)pq + \beta q + 1 \over pq - 1} \int_0^T z_1(s) \, ds = +\infty \quad \text{or} \quad \limsup_{T \to +\infty} T^{(\beta-1)pq + \alpha p + 1 \over pq - 1} \int_0^T z_2(s) \, ds = +\infty,
\]

then Equation (1) has no global solutions.

Consider now the case of a single equation

\[
\begin{cases} 
  dx(t) + dx(t)^{\alpha}(t) = |x(t)|^p + z(t), \\
  x(0) = x_0,
\end{cases}
\]

where \( 0 < \alpha < 1 \), \( p > 1 \) and \( z \in C([0, \infty)), z \geq 0 \). From Theorem 1, we deduce

**Corollary 1.** Let \( x_0 \geq 0 \). If

\[
\limsup_{T \to +\infty} T^{\rho q - 1} \int_0^T z(s) \, ds = +\infty,
\]

then Equation (7) has no global solutions.

The following example shows the effect of the inhomogeneous terms \( z_i(t), i = 1, 2 \), on the blow-up of solutions of Equation (2)

**Example 1.** Consider the system

\[
\begin{cases} 
  dx(t) + dx(t)^{\alpha}(t) = |y(t)|^q + (t + 1)^\rho, \\
  dy(t) + dy(t)^{\beta}(t) = |x(t)|^p, \\
  (x(0), y(0)) = (x_0, y_0),
\end{cases}
\]

where \( t > 0, x_0 \geq 0, y_0 \geq 0, p > 1, q > 1 \) and

\[
\rho > - \min \left\{ 1, \frac{q(\alpha p + \beta)}{pq - 1} \right\}.
\]

System Equation (8) is a special case of Equation (1) with

\[
z_1(t) = (t + 1)^\rho \quad \text{and} \quad z_2(t) = 0,
\]

for all \( t \geq 0 \). One observes easily that

\[
T^{(\alpha-1)pq + \beta q + 1 \over pq - 1} \int_0^T z_1(s) \, ds \sim T^{(\alpha-1)pq + \beta q + 1 \over pq - 1} + \rho + 1, \quad \text{as } T \to +\infty.
\]

Furthermore, it follows from Equation (9) that

\[
\frac{(\alpha - 1)pq + \beta q + 1}{pq - 1} + \rho + 1 > 0,
\]

which yields

\[
\lim_{T \to +\infty} T^{(\alpha-1)pq + \beta q + 1 \over pq - 1} \int_0^T z_1(s) \, ds = +\infty.
\]
Hence by Theorem 1, one deduces that under condition Equation (9), for all \( p, q > 1 \), Equation (8) has no global solutions.

2. Proof of the Main Result

The proof is based on the nonlinear capacity method (see e.g., [26]). More precisely, we first suppose that \((x, y)\) is a global solution of Equation (1). Next, we multiply both equations in Equation (1) by an adequate test function that depends on a parameter \( T \gg 1 \), and we integrate by parts over the interval \((0, T)\). Using standard integral inequalities, the condition in Equation (6) and passing to the limit as \( T \to \infty \), a contradiction follows.

The detailed proof of Theorem 1 is given below.

Proof. We follow the steps mentioned previously.

Step 1 (Multiplication of both equations in Equation (1) by an adequate test function that depends of a parameter \( T \gg 1 \)):

Suppose \((x, y)\) is solution of (1) which is global. For \( \kappa, T \gg 1 \), let

\[
\nu(t) = T - \kappa(T - t), \quad \text{for all } 0 \leq t \leq T. \tag{10}
\]

After multiplication of the first equation in Equation (1) by \( \nu(t) \) and integration over \((0, T)\), we obtain

\[
\int_0^T \frac{dx}{dt} \nu \, dt + \int_0^T \frac{dx}{dt} \nu \, dt = \int_0^T |y|^q \nu \, dt + \int_0^T z_1 \nu \, dt. \tag{11}
\]

Integrating by parts, we have

\[
\int_0^T \frac{dx}{dt} \nu \, dt = x(T)\nu(T) - x(0)\nu(0) - \int_0^T x \frac{d\nu}{dt} \, dt.
\]

Since \( \nu(T) = 0 \) and \( \nu(0) = 1 \), we get

\[
\int_0^T \frac{dx}{dt} \nu \, dt = -x(0) - \int_0^T x \frac{d\nu}{dt} \, dt. \tag{12}
\]

Using Equations (5) and (3), we have

\[
\int_0^T \frac{dx}{dt} \nu \, dt = \int_0^T \mathcal{I}_T^{1-a} \left( \frac{dx}{dt} \right) \nu \, dt = \int_0^T \frac{dx}{dt} \mathcal{I}_T^{1-a} \nu \, dt.
\]

Again, we integrate by parts, we obtain

\[
\int_0^T \frac{dx}{dt^a} \nu \, dt = x(T) \left( \mathcal{I}_T^{1-a} \nu \right)(T) - x(0) \left( \mathcal{I}_T^{1-a} \nu \right)(0) - \int_0^T x \frac{d\mathcal{I}_T^{1-a} \nu}{dt} \, dt.
\]

On the other hand, by Equation (4), one has \( \left( \mathcal{I}_T^{1-a} \nu \right)(T) = 0 \). Therefore, it holds that

\[
\int_0^T \frac{dx}{dt^a} \nu \, dt = -x(0) \left( \mathcal{I}_T^{1-a} \nu \right)(0) - \int_0^T x \frac{d\mathcal{I}_T^{1-a} \nu}{dt} \, dt. \tag{13}
\]

Further, using Equation (11)—(13), we deduce that

\[
x(0) \left( 1 + \left( \mathcal{I}_T^{1-a} \nu \right)(0) \right) + \int_0^T |y|^q \nu \, dt + \int_0^T z_1 \nu \, dt \leq \int_0^T \left| \frac{d\nu}{dt} \right| \, dt + \int_0^T \left| x \frac{d\mathcal{I}_T^{1-a} \nu}{dt} \right| \, dt.
\]
Since \( x(0) \geq 0 \) and \( \left( T_0^{1-\alpha} v \right) (0) \geq 0 \), by Equation (4), we obtain
\[
\int_0^T |y|^q v \ dt + \int_0^T z_1 v \ dt \leq \int_0^T |x| \left| \frac{dv}{dt} \right| \ dt + \int_0^T |x| \left| \frac{dI_{T_0}^{1-\alpha} v}{dt} \right| \ dt. \tag{14}
\]

Similarly, after multiplication of the second equation in Equation (1) by \( v(t) \) and integration over \((0,T)\), we have that \( y(0) \geq 0 \) and \( \left( T_0^{1-\beta} v \right) (0) \geq 0 \), we obtain
\[
\int_0^T |x|^p v \ dt + \int_0^T z_2 v \ dt \leq \int_0^T |y| \left| \frac{dv}{dt} \right| \ dt + \int_0^T |y| \left| \frac{dI_{T_0}^{1-\beta} v}{dt} \right| \ dt. \tag{15}
\]

Next, we set \( \mathbb{I} = \int_0^T |x|^p v \ dt \) and \( \mathbb{J} = \int_0^T |y|^q v \ dt \).

Using Hölder’s inequality to get
\[
\int_0^T |x| \left| \frac{dv}{dt} \right| \ dt = \int_0^T |x| \left( \frac{1}{q} \right)^{\frac{1}{q}} \left( \frac{1}{p} \right)^{\frac{1}{p}} \left| \frac{dv}{dt} \right|^{\frac{1}{p}} \ dt \leq \|x\|_p \left( \int_0^T \left| \frac{dv}{dt} \right|^{\frac{1}{p}} \ dt \right)^{\frac{1}{p}} \tag{16}
\]
and
\[
\int_0^T |x| \left| \frac{dI_{T_0}^{1-\alpha} v}{dt} \right| \ dt \leq \|x\|_p \left( \int_0^T \left| \frac{dI_{T_0}^{1-\alpha} v}{dt} \right|^{\frac{1}{p}} \ dt \right)^{\frac{1}{p}} \tag{17}
\]

Similarly, one has
\[
\int_0^T |y| \left| \frac{dv}{dt} \right| \ dt \leq \|y\|_q \left( \int_0^T \left| \frac{dv}{dt} \right|^{\frac{1}{q}} \ dt \right)^{\frac{1}{q}} \tag{18}
\]
and
\[
\int_0^T |y| \left| \frac{dI_{T_0}^{1-\beta} v}{dt} \right| \ dt \leq \|y\|_q \left( \int_0^T \left| \frac{dI_{T_0}^{1-\beta} v}{dt} \right|^{\frac{1}{q}} \ dt \right)^{\frac{1}{q}}. \tag{19}
\]

Next, setting
\[
\mathbb{A}_1 = \int_0^T v^{\frac{1}{q}} \left| \frac{dv}{dt} \right|^{\frac{p}{q}} \ dt \quad \text{and} \quad \mathbb{A}_2 = \int_0^T v^{\frac{1}{q}} \left| \frac{dI_{T_0}^{1-\alpha} v}{dt} \right|^{\frac{p}{q}} \ dt,
\]
using Equations (14), (16), and (17), one deduces that
\[
\mathbb{J} + \int_0^T z_1 v \ dt \leq \|x\|_p \left( \mathbb{A}_1 + \mathbb{A}_2 \right). \tag{20}
\]

Similarly, setting
\[
\mathbb{B}_1 = \int_0^T v^{\frac{1}{q}} \left| \frac{dv}{dt} \right|^{\frac{q-1}{q}} \ dt \quad \text{and} \quad \mathbb{B}_2 = \int_0^T v^{\frac{1}{q}} \left| \frac{dI_{T_0}^{1-\beta} v}{dt} \right|^{\frac{q-1}{q}} \ dt,
\]
using Equations (15), (18), and (19), one deduces that

\[ I + \int_0^T z_2 v \, dt \leq J_1 \left( \frac{q-1}{B_1^T + B_2^T} \right). \tag{21} \]

Consider now the case

\[ \lim \sup_{T \to +\infty} T^{\frac{(\alpha - 1)p + \beta + 1}{p-1}} \int_0^T z_1(s) \, ds = +\infty. \tag{22} \]

Since

\[ \int_0^T z_2 v \, dt \geq 0, \]

it follows from Equation (21) that

\[ I \leq J_1 \left( \frac{q-1}{B_1^T + B_2^T} \right). \]

The above inequality with Equation (20) yields

\[ J + \int_0^T z_1 v \, dt \leq J_1 \left( \frac{q-1}{B_1^T + B_2^T} \right)^{\frac{1}{p}} \left( \frac{\nu_1}{A_1^T} + \frac{\nu_1}{A_2^T} \right). \tag{23} \]

Step 2 (Estimates and conclusion):

Further, we shall estimate the term in right-hand side of the above inequality. Using the inequality

\[ (a_1 + a_2)^\gamma \leq 2^\gamma (a_1^\gamma + a_2^\gamma), \quad 0 < \gamma < 1, \ a_1, a_2 > 0, \]

one obtains

\[ \left( \frac{q-1}{B_1^T + B_2^T} \right)^{\frac{1}{p}} \left( \frac{\nu_1}{A_1^T} + \frac{\nu_1}{A_2^T} \right) \leq 2^{\frac{1}{p}} \left( \frac{\nu_1}{A_1^T} B_1^T + A_2^T + A_1^T B_2^T + A_2^T B_1^T \right). \tag{24} \]

On the other hand, using Equation (10), elementary calculations yield

\[ \mathcal{A}_1 = \frac{1}{\kappa - \frac{1}{\gamma - 1}} T^{\frac{1}{\gamma - 1}} \quad \text{and} \quad \mathcal{B}_1 = \frac{1}{\kappa - \frac{1}{\gamma - 1}} T^{\frac{1}{\gamma - 1}}. \tag{25} \]

Similarly, using Equations (4) and (10), we obtain

\[ \mathcal{A}_2 = \left[ \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa + 1 - a)} \right] \frac{T^{\frac{1}{\gamma - 1}}}{\kappa - \frac{\gamma}{\gamma - 1} + 1} \quad \text{and} \quad \mathcal{B}_2 = \left[ \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa + 1 - \beta)} \right] \frac{T^{\frac{1}{\gamma - 1}}}{\kappa - \frac{\gamma}{\gamma - 1} + 1}. \tag{26} \]

Hence, using Equations (24)–(26), one deduces easily that

\[ \left( \frac{q-1}{B_1^T + B_2^T} \right)^{\frac{1}{p}} \left( \frac{\nu_1}{A_1^T} + \frac{\nu_1}{A_2^T} \right) \leq C T^{1-a-\frac{p}{\gamma - 1}}, \]

where \( C > 0 \) is a constant. The above inequality with Equation (23) yield

\[ J + \int_0^T z_1 v \, dt \leq C \mathcal{J}_1 T^{1-a-\frac{p}{\gamma - 1}}. \tag{27} \]
Next, using Young’s inequality
\[ ab \leq \frac{1}{pq} + \frac{pq - 1}{pq}, \quad a, b > 0, \]
we get
\[ C \frac{1}{pq} T^{1-\alpha - \frac{\beta}{p} - \frac{1}{q}} \leq \frac{1}{pq} + \left( \frac{pq - 1}{pq} \right) C \frac{1}{pq} T^{1-\alpha - \frac{\beta}{p} - \frac{1}{q}}, \]

\[ \text{together with Equation (27) implies} \]
\[ \int_0^T z_1 v \, dt \leq \tilde{C} T^{\frac{1}{pq}} \left( 1-\alpha - \frac{\beta}{p} - \frac{1}{q} \right), \quad (28) \]

\[ \text{where } \tilde{C} = \left( \frac{pq - 1}{pq} \right) C \frac{1}{pq} . \]

On the other hand, by Equation (10), one has
\[ \int_0^T z_1 v \, dt = T^{-\xi} \int_0^T z_1(t) (T - t)^{\xi} \, dt \geq \frac{1}{2^{\xi}} \int_0^T z_1(t) \, dt. \]

The above inequality with (28) yields
\[ \frac{1}{2^{\xi}} \frac{(\alpha - 1) + \beta + 1}{\alpha + 1} \int_0^T z_1(t) \, dt \leq \tilde{C}, \]

which contradicts Equation (22).

Next, consider the case
\[ \limsup_{T \to +\infty} T^{\frac{(\beta - 1) + \alpha + 1}{\alpha + 1}} \int_0^T z_2(s) \, ds = +\infty. \quad (29) \]

Using similar argument as above, one obtains
\[ \frac{1}{2^{\xi}} \frac{(\beta - 1) + \alpha + 1}{\alpha + 1} \int_0^T z_2(t) \, dt \leq \tilde{C}', \]

for some constant \( \tilde{C}' > 0 \), which contradicts (29). \( \Box \)

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