Analysis of rotation capacity of a novel 2R1T mechanism based on origami of thick panels

Boyan Chang1, 2, Jifu Zhang1, Dong Liang1, 2, Yang Zhou1
1Department of Mechanical Engineering, Tiangong University, Tianjin, 300387
2Tianjin Key Laboratory of Advanced Mechatronics Equipment Technology, Tiangong University, Tianjin, 300387
mmts_tjpu@126.com

Abstract: A foldable and symmetrical lower-mobility parallel mechanism was proposed based on Waterbomb origami of thick panels. It consists of a moving platform, a base plate and three deployable foldable legs between moving platform and base plate. Firstly, constraint wrenches of each leg were formulated based on screw theory and the results illustrated that the moving platform is in possession of two degrees of orientation freedom and one translational degree of freedom. Secondly, it was approved that base and moving platform are always symmetrical about a middle plane and the moving platform can rotate continuously about any axis chosen freely on this plane. Solving models including forward and inverse position problems were established to determine the maximum rotational angle and workspace. Finally, performance index of maximum rotational angle of the PM was analyzed, and effects of two structural variables to the performance were summarized. Conclusions obtained can provide a theoretical basis for the structural design and engineering application of this 2T1R parallel mechanism.

1. Introduction
Origami mechanism[1] is a branch of mechanism and belongs to the category of folding mechanism. It is widely used in the design of robots, such as skeleton origami robot, driving origami robot, shell origami robot, etc.[2-3] because of its advantages of light weight, convenient folding and small occupied space after folding. According to the different deformation resistance of origami mechanism during folding and unfolding, it can be divided into rigid origami mechanism and flexible origami mechanism[4]. During folding and unfolding, other parts of rigid origami mechanism except creases remain rigid, ensuring the stability of mechanism motion performance [5]. The design of rigid origami mechanism is usually based on the zero thickness characteristics of paper [6-8]. However, in practical engineering applications, components have a certain thickness. If the traditional origami method is directly applied to thick plate folding, it will cause physical interference between components. Therefore, rigid origami cannot be directly used for the design of folding mechanism. Considering the influence of component thickness, Feng et al. [9] took the generalized waterbomb origami tube as the research object, analyzed the influence of geometric design parameters on the rigid folding of origami tube, as well as the bifurcation behavior and possible physical interference in the folding process, so as to provide a theoretical basis for the kinematic analysis of complex origami structure. Dai et al.[10-11] proposed a new type of continuous manipulator, which uses the Waterbomb origami mechanism as the external wrapping frame to realize the bending and expansion of the manipulator through wire transmission. Parallel mechanism has the characteristics of high speed, high strength, large bearing
capacity and good dynamic response \cite{12}, but that most 2R1T spatial parallel mechanisms cannot be folded or fully folded due to the constraints of branch chain structure and rotation angle of kinematic pair, resulting in large overall space occupation of the mechanism, difficult storage or transportation, and unfavorable for the mechanism to work in a closed and narrow space. Based on the above research work, this paper introduces the thick plate origami theory into the design of parallel mechanism, and obtains a 3-DOF spatial parallel mechanism. By constructing the forward and inverse solution models of the mechanism, the working space and maximum rotation range of the mechanism are determined by using the limit search method. By constructing the velocity constraint equation of the mechanism, the Jacobian matrix of the mechanism is solved. Aiming at the optimization of dexterity, stiffness, bearing capacity and maximum rotation angle in the workspace, the structural parameters of the mechanism are optimized, which can realize the complete folding of the parallel mechanism and reduce the actual storage space.

2. Structure composition and degree of freedom analysis of mechanism

Based on the thick plate origami theory, a new parallel mechanism is proposed, as shown in Figure 1. The mechanism is composed of static platform 7, dynamic platform 8 and three branched chains leg1, leg2 and leg3 connecting the two platforms. Each branch chain has the same structural composition and is a waterbomb thick plate folding unit \cite{9}, as shown in Figure 2(a). Each branch chain is connected with the static platform and the moving platform through the rotating pairs \( R_{7i} \) and \( R_{8i} \). The centers of the rotating pairs are correspondingly marked \( A_i \) and \( C_i \). The axes of the six rotating pairs \( R_{1i}, R_{2i}, R_{3i}, R_{4i}, R_{5i}, \) and \( R_{6i} \) inside the branch chain intersect at point \( B_i \). When the folding unit is fully unfolded and fully folded, they are shown in Figure 2(b) respectively. As shown in, in order to ensure that the branch chain can be completely folded without interference, the inner angle \( \alpha_i (i=1, 2, \ldots, 6) \) satisfies the following relationship: \( \alpha_1=\alpha_6, \alpha_2=\alpha_3=\alpha_4=\alpha_5, \alpha_2+\alpha_4\leq\alpha_1 \). And the thickness of the branch chain after complete folding is \( 2\ h \).

![Figure 1. Foldable PM of thick panels with 3 legs](image1)

(a) Waterbomb thick plate folding unit

![Figure 2. Schematic diagram of legs](image2)

(b) Unfold unit fully

According to the principle that the correlation and inversion of helix are independent of the selection of coordinate system, taking a single waterbomb thick plate folding unit shown in Figure 2 as the research object, the \( B_rX_rY_rZ_r \) coordinate system is established. \( X_r \) axis coincides with \( R_3 \) axis, \( Y_r \) axis is in the plane composed of \( R_5 \) and \( R_2 \) and perpendicular to \( X_r \) axis, and \( Z_r \) conforms to the right-hand rule, as shown in Figure 1. The motion spiral system is

Where \( a_j, b_j \) and \( c_j \) \((j=1, 2, \ldots, 6; i=1, 2, 3)\) are the directions corresponding to each rotating pair. The kinematic screw system and constrained screw system of leg \( i \) can be expressed as

\[
\begin{align*}
S_j &= (a_j, b_j, c_j; 0,0,0) \\
S_0 &= (a_2, b_2, 0; 0,0,0) \\
S_1 &= (a_1, 0; c_1; 0,0,0) \\
S_2 &= (a_1, b_1, c_1; 0,0,0) \\
S_3 &= (a_1, 0, 0; 0,0,0) \\
S_4 &= (a_1, 0; 0,0,0) \\
S_5 &= (a_1, b_1, 0; 0,0,0)
\end{align*}
\]
It can be seen that folding units in each branch chain are equivalent to the generalized spherical pair, and its center is at point $B_i$, and the kinematic screw system of the generalized spherical joint is 

$$
S^{(i)}_0 = (0,0;0,0,1), \quad S^{(i)}_1 = (0,0;0,1,0), \quad S^{(i)}_2 = (0,0;1,0,0)
$$

(2)

In the initial position and attitude of the mechanism, when the dynamic and static platforms are parallel to each other and the plane symmetry about point $B_i$ ($i=1,2,3$) is formed, i.e. when $R_8^i$ and $R_7^i$ are parallel to each other, the kinematic screw system of each branch chain can be obtained as follows

$$
S_0 = (l,m;0,mz,ml), \quad S_1 = (0,0;0,0,0), \quad S_2 = (0,0;0,1,0), \quad S_3 = (l,m;0,mz,ml)
$$

(3)

The corresponding constrained screw is

$$
S^*_0 = (l,m;0,0,0)
$$

(4)

From equation (5), it can be seen that each branch chain imposes a force constraint on the platform, which is located in the plane $B_1B_2B_3$ and passes through the spherical center $B_i$ of the broad spherical pair. Therefore, a total of 3 branches impose 3 coplanar but non-intersecting constraints on the platform, as shown in Figure 3.

Three coplanar constraint forces limit two degrees of freedom of movement of the moving platform in plane $B_1B_2B_3$ and one degree of freedom of rotation along the normal direction of the plane. In conclusion, the moving platform has degrees of freedom of rotation along any two orthogonal axes in plane $B_1B_2B_3$ and one translational degree along the normal direction of the plane.

3. Position Analysis

3.1. Inverse Position Solution

Inner tangential circle radius and edge length of dynamic and static platform are set as $r$ and $a$ and $a = 2\sqrt{3}r/3$. According to the principle of equivalent replacement of kinematic chain, the branch chain can be equivalent to RSR chain. The equivalent length of rod $AB_i$ and $BC_i$ in RSR branch chain are both $l$. Because each branch chain needs to be folded completely without interference, the restriction condition of length $l \leq r$ is provided. Establishing the fixed coordinate system $O-XYZ$ on the stationary platform as shown in Figure 4 and the $OXY$ plane coincides with the surface of the stationary platform.
In $O$-XYZ, the coordinate of the center point of the moving platform is

$$O_p = R_{\psi}(\sin \psi \cos \phi, \sin \psi \sin \phi, \cos \psi)^T$$

As $O_p$ and $O$ are symmetrical with respect to plane $B_1B_2B_3$, it can be seen that the coordinates of intersection $M$ is

$$M = \frac{R_{\psi}}{2}(\sin \psi \cos \phi, \sin \psi \sin \phi, \cos \psi)^T$$

Thus the equation for plane $B_1B_2B_3$ is

$$x \sin \psi \cos \phi + y \sin \psi \sin \phi + z \cos \psi = \frac{R_{\psi}}{2}$$

(6)

The expression of point $B_i$ in the coordinate system $O$-XYZ can be written as

$$B_i = T_w \cdot B_i \quad (i = 1, 2, 3)$$

(7)

In equation (7)', $T_w$ is coordinate transformation matrix from $A_1-x'y'z'$ coordinate system to basic coordinate system $O$-XYZ, $B_i$ are coordinate point of $B_i$ in $A_1-x'y'z$.

$$T_w = \begin{bmatrix}
\cos \phi_i & -\sin \phi_i & 0 & r \cos \phi_i \\
\sin \phi_i & \cos \phi_i & 0 & r \sin \phi_i \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad xB_i = \begin{bmatrix}
-l \cos \beta_i \\
0 \\
l \sin \beta_i \\
1
\end{bmatrix}$$

where $\phi_i$ is expressed as

$$\phi_i = (i - 1) \frac{2\pi}{3} \quad (i = 1, 2, 3)$$

(8)

equation (8)' is substituted to equation (6)' to obtain

$$\begin{bmatrix}
(r-lc\beta_i)s\psi c\phi + ls\beta_i c\psi = \frac{R_{\psi}}{2} \\
\frac{\sqrt{3}}{2}(r-lc\beta_i)s\psi s\phi + ls\beta_i s\psi - \frac{1}{2}(r-lc\beta_i)s\psi c\phi - \frac{R_{\psi}}{2} \\
l s\beta_i c\psi - \frac{1}{2}(r-lc\beta_i)s\psi c\phi - \frac{\sqrt{3}}{2}(r-lc\beta_i)s\psi s\phi = \frac{R_{\psi}}{2}
\end{bmatrix}$$

(9)

As $t_i = \tan(\frac{\beta_i}{2})$ equation (9)' can be expressed as

$$C_i t_i^2 + D_i t_i + E_i = 0$$

(10)

In equation,

$$C_1 = (l + r) \sin \psi \cos \phi - \frac{R_{\psi}}{2} \quad C_2 = \frac{1}{2}(r+l) \sin \psi \cos \phi + \frac{\sqrt{3}}{2}(r+l) \sin \psi \sin \phi - \frac{R_{\psi}}{2} \quad C_3 = -\frac{1}{2}(l+r) \sin \psi \cos \phi + \frac{\sqrt{3}}{2}(l+r) \sin \psi \sin \phi - \frac{R_{\psi}}{2}$$

$$E_1 = (r-l) \sin \psi \cos \phi - \frac{R_{\psi}}{2} \quad E_2 = \frac{1}{2}(l-r) \sin \psi \cos \phi + \frac{\sqrt{3}}{2}(l-r) \sin \psi \sin \phi - \frac{R_{\psi}}{2} \quad E_3 = \frac{\sqrt{3}}{2}(l-r) \sin \psi \sin \phi + \frac{1}{2}(l-r) \sin \psi \cos \phi - \frac{R_{\psi}}{2}$$

$$D_i = 2l \cos \psi \quad (i = 1, 2, 3)$$
The solution of equation (10) can be expressed as
\[ t_i = \frac{-D_i \pm \sqrt{D_i^2 - 4C_i E_i}}{2C_i} \]  \hspace{1cm} (11)
where \( \beta_i = 2 \arctan(t_i) \)

3.2. Positive Position Solution

From equation (6)', the matrix expression of plane equation can be obtained as follows
\[ \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \begin{pmatrix} r - l \cos \beta_i \\ \frac{\sqrt{3}}{2} (r - l \cos \beta_i) \\ \frac{\sqrt{3}}{2} (l \cos \beta_i - r) \end{pmatrix} + \begin{pmatrix} l \sin \beta_i \\ l \sin \beta_i \\ l \sin \beta_i \end{pmatrix} = 0 \]  \hspace{1cm} (12)

In equation (13)', the equation of plane B1B2B3 can be further obtained
\[ Ax + By + Cz + D = 0 \]  \hspace{1cm} (13)

In equation,
\[ A = \frac{1}{2} [\sqrt{3} l \cos \beta_2 \sin \beta_1 + \sqrt{3} l \cos \beta_2 \sin \beta_3 - \sqrt{3} l \cos \beta_3 \sin \beta_1 - \sqrt{3} l \cos \beta_3 \sin \beta_2 - 2 \sqrt{3} l \sin \beta_1 + \sqrt{3} l \sin \beta_2 + \sqrt{3} l \sin \beta_3]; \]
\[ B = l^2 \cos \beta_1 \sin \beta_2 - l^2 \cos \beta_2 \sin \beta_1 + \frac{1}{2} [l^2 \cos \beta_1 \sin \beta_3 - l^2 \cos \beta_3 \sin \beta_1 - l^2 \cos \beta_2 \sin \beta_3 + l^2 \cos \beta_3 \sin \beta_2 + 3 l \sin \beta_1 - 3 l \sin \beta_2]; \]
\[ C = \frac{\sqrt{3}}{2} (3 r^2 + l^2 \cos \beta_1 \cos \beta_2 + l^2 \cos \beta_1 \cos \beta_3 + l^2 \cos \beta_2 \cos \beta_3 - 2 l r \cos \beta_1 - 2 l r \cos \beta_2 - 2 l r \cos \beta_3); \]
\[ D = -\frac{\sqrt{3}}{2} (2 r \sin \beta_1 + 2 r \sin \beta_2 + 2 r \sin \beta_3 - l^2 \cos \beta_1 \sin \beta_2 - l^2 \cos \beta_2 \sin \beta_1 - l^2 \cos \beta_3 \sin \beta_1 - l^2 \cos \beta_3 \sin \beta_2 - l^2 \cos \beta_2 \sin \beta_3 + l^2 \cos \beta_1 \sin \beta_2 + l^2 \cos \beta_2 \sin \beta_3 + l^2 \cos \beta_3 \sin \beta_1); \]

Due to the symmetry of dynamic and static platforms with respect to the middle plane, the expression of coordinate and direction angle of center point of dynamic platforms can be obtained as
\[ \begin{align*}
x_p &= -\frac{AD}{A^2 + B^2 + C^2} \\
y_p &= -\frac{DB}{A^2 + B^2 + C^2} \\
z_p &= -\frac{CD}{A^2 + B^2 + C^2} \\
\phi &= \arctan \frac{y_p}{x_p} \\
\Omega &= 2 \arccos \frac{z_p}{\sqrt{x_p^2 + y_p^2 + z_p^2}}
\end{align*} \]  \hspace{1cm} (14)

3.3. Example

Position and attitude parameters of the dynamic platforms can be obtained according to equation (12)' to equation (13)', as shown in Table 1.

| Coefficient | \(x_p\) | \(y_p\) | \(z_p\) | \(R_\phi\) | \(\psi/(^\circ)\) | \(\phi/(^\circ)\) |
|-------------|--------|--------|--------|----------|----------------|----------------|
| Value 1     | 4.961  | -0.538 | 61.429 | 61.622   | 4.645          | -6.184         |
| Value 2     | 4.4095 | 2.068  | 47.614 | 47.863   | 5.840          | 25.126         |

In Table 1, input angle adopts two groups of arbitrary data satisfying the constraints of the mechanism, i.e. the first group \(\beta_1 = 43.232^\circ\), \(\beta_2 = 63.7326^\circ\), \(\beta_3 = 60.1426^\circ\) and the second group \(\beta_1 = 30^\circ\), \(\beta_2 = 40^\circ\) and \(\beta_3 = 50^\circ\).
Table 2. Verification of \( \beta_i \) Angular Posture Inverse Solution

| Drive angle | \( \beta_1 \) | \( \beta_2 \) | \( \beta_3 \) |
|-------------|-------------|-------------|-------------|
| Value 1     | 43.2328°    | 63.7326°    | 60.1426°    |
| Value 3     | 30.0344°    | 39.9008°    | 50.0822°    |

4. Analysis of mechanism workspace and rotational performance

Workspace shown in Figure 5 is an important index in parallel mechanism. It can intuitively show the working range of mechanism. Limit coordinate search method can be used to determine the working space of 2R1T thick plate folding parallel mechanism. The constraints of \( \beta_1, \beta_2, \beta_3 \) is \([0, \pi], \phi \in [0, 2\pi]\), and rod length is \( l \leq r \). The searching range of \( R_0 \) is \( 2h \leq R_0 \leq \sqrt{(0.5a^2 + 2l^2)(1 - \cos(2\alpha_i))} \) when \( \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 < 360^\circ \), and \( 2h \leq R_0 \leq 2l \) when \( \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 \geq 360^\circ \).

From Figure 5, it can be seen that the workspace of the mechanism is regular and symmetrical with respect to \( Z \)-axis. With the increase of \( Z \)-axis coordinates, the cross-sectional area of the workspace first increases and then decreases. When the \( R_0 \) is 50.93mm, the moving platform is the largest. In order to study the influence rule of structure parameters on rotating ability of moving platform, the radius of tangential circle \( r \) and equivalent rod length \( l \) of dynamic and static platforms are selected as two independent design variables, the plate thickness is the design constant, and other parameters, \( a = 2r\sqrt{5}/3, \alpha_4 = \alpha_5 = \alpha_6 = 0.5\alpha_1, \) and

\[
\alpha_i = 2\arctan \left( \frac{\sqrt{3}r}{3\cos(\arcsin(h/l))} \right)
\]

The turning performance index can be obtained by programming as shown in Figure 6.

![Figure 5. workspace of 2R1T PM](image-url)
It can be seen from Figure 6 that the turning performance index of the mechanism can reach 59.01° at the maximum and 21.54° at the minimum in the design space, which shows that the influence of design parameters on the turning performance of the mechanism is obvious. In addition, the rotational performance index increases with the increase of $l$ and decreases with the increase of $r$. The better range of rotational performance index is concentrated in the area of $42\text{mm} \leq l \leq 60\text{mm}$ and $60\text{mm} \leq r \leq 75\text{mm}$.

5. Conclusion
A completely foldable 2R1T parallel mechanism is designed by combining the theory of thick plate folding with the parallel mechanism. The degree of freedom of the mechanism is verified based on the screw theory. The positive and inverse position equation of the mechanism are established, and the correctness of the equation is verified by an example. According to the inverse model of the position, the working space of the mechanism is determined by the limit coordinate search method. The working space of the mechanism is regular in shape and symmetrical in relation to the $Z$-axis. With the increase of the $Z$-axis coordinates, the cross-sectional area of the working space increases first and then decreases. Rotational performance of the mechanism is analyzed and performance map of the maximum rotation angle $\Omega_{\text{max}}$ is offered which can provide a theoretical basis for the structural design and engineering application of this 2T1R parallel mechanism.

Reference
[1] Zhang X, Chen Y. The diamond thick-panel origami and the corresponding mobile assemblies of plane-symmetric Bricard linkages[J]. Mechanism and Machine Theory, 2018, 130.
[2] Beech R. The practical illustrated encyclopedia of origami: The complete guide to the art of paperfolding. Amness, 2009.
[3] Feng H J, Yao G Q, Chen Y, et al. Origami robots [J]. Chinese Science: technical science, 2018, 48(12): 1259-1274.
[4] Tachi T. Rigid origami mechanisms[J]. Journal of the Robotics Society of Japan, 2016, 34(3):184-191.
[5] Guo Z, Yu H Y, Hua Z X, Zhao D. Kinematic analysis and simulation of folding process for rigid origami mechanisms [J]. Journal of Jilin University (Engineering Edition), 2020, 50(01): 66-76.
[6] Yang M Y, Ma J Y, Li J M, et al. Minimally invasive surgical forceps based on thick plate origami theory [J]. Journal of Mechanical engineering, 2018, 54(17): 36-45.
[7] Chen Y, Peng R, Z You. Origami of thick panels[J]. Science, 2015, 349(6246): 396-400.
[8] Zhang X, Chen Y. Mobile assemblies of bennett linkages from four-crease origami patterns.[J]. Proceedings. Mathematical, physical, and engineering sciences, 2018, 474(2210).
[9] Feng H J, Ma J Y, Chen Y. Rigid folding of waterbomb origami tubes [J]. Journal of Mechanical engineering.2020, 56(19): 143-159.
[10] Zhang K T, Qiu C, Dai J S.An origami paraller structure integrated deployable continuum robot. In: ASME 2015 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference. American Society of Mechanical Engineers 2015. V05BT08A032-V05BT08A032.
[11] Zhang K T, Qiu C, Dai J S. An extensible continuum robot with integrated origami parallel modules. J Mech Robotics, 2016, 8: 0301010.
[12] Li Q C, Chai Q H. Review on 2R1T 3-DOF parallel mechanisms [J].Scientific Bulletin,2017, 62(14): 1507-1519.
[13] Chen Z M, Zhang Y, Huang K. Symmetrical 2R1T parallel mechanism without parasitic motion[J].Journal of Mechanical Engineering,2016, 52(03): 9-17.