Abstract: We consider the downlink of an orthogonal frequency division multiplexing (OFDM)-based cell that services calls from many service-classes. The call arrival process is random (Poisson) or quasi-random, i.e., calls are generated by an infinite or a finite number of sources, respectively. In order to determine congestion probabilities and resource utilization, we model the cell as a multirate loss model. Regarding the call admission, we consider the restricted accessibility, the bandwidth reservation (BR), and the complete sharing (CS) policies. In a system of restricted accessibility, a new call may be blocked even if resources do exist. In a BR system, subcarriers can be reserved in favor of calls of high subcarrier requirements. Finally, in a CS system, a new call is blocked due to resource unavailability. In all three policies, we show that there exist recursive formulas for the determination of the various performance measures. Based on simulation, the accuracy of the proposed formulas is found to be quite satisfactory.

Keywords: OFDM; congestion; random; quasi-random; recursive; restricted; reservation; complete sharing

1. Introduction

The determination and evaluation of the main quality of service (QoS) parameters, such as call blocking probabilities (CBP) and network resources utilization, is a complex task in contemporary networks, due to the growth of network traffic and the high traffic stream diversity [1]. The latter necessitates research on teletraffic loss or queueing models, either at call-level or at packet-level [2–7]. Such models not only assist in network optimization and dimensioning procedures but they may also be used in combination with machine learning techniques [8,9] or as an input to computational intelligent techniques, such as the fuzzy analytical hierarchy process techniques [10,11]. In this paper, we concentrate on call-level teletraffic loss models.

The simplest call-level loss models adopt as a call arrival process the classical Poisson process, which leads to analytically tractable formulas for the determination of performance measures, such as CBP and resources utilization. The origination of calls in the Poisson process results from an infinite number of users (or traffic sources). This means that the Poisson process cannot capture the case of calls generated via a finite number of users. The latter can be well described by the quasi-random
arrival process. This call arrival process depends on the number of idle users (that are capable of generating traffic) and is smoother than the Poisson process. Because of this, the CBP in a system that carries quasi-random traffic are much lower compared to the corresponding CBP of a system that carries random (Poisson) traffic. For recent applications of the quasi-random process in loss systems, the interested reader may resort to [12–16].

We consider the cases of random and quasi-random traffic in the downlink of an orthogonal frequency division multiplexing (OFDM)-based cell which provides service to calls from many service-classes. OFDM is a dominant technology in 4th generation (4G) networks and can also be considered as a candidate technology in 5th generation (5G) networks [17–21] and in cognitive radio networks [22]. The analysis of this OFDM-based cell relies on the loss models of [23–26]. The case of a batch arrival process and two types of calls, narrow-band and wide-band, has been studied in [27] under two different batch blocking disciplines: (a) the complete and (b) the partial batch blocking discipline. A possible extension of [27] that may result in efficient formulas for the CBP determination can be based on the works of [28–30] (for the partial batch blocking discipline) and [31] (for the complete batch blocking discipline).

In [23], Paik and Suh (P-S) consider an OFDM-based cell that accommodates different service-classes whose calls follow a Poisson process. The P-S system is described as a loss system, i.e., calls are blocked and lost in the case of resource unavailability. This means that the adopted policy for resource sharing in [23] is the complete sharing (CS) policy. The CS policy is the default policy in teletraffic loss models, but it may result in an unfair resource allocation among calls [1,2].

Contrary to [24,25], where a new call is accepted in the OFDM-based cell only if the required subcarriers are available, in the P-S model the admission of a new call is based on the availability of both subcarriers and power. Apart from this significant modification in the call admission, it is important to mention that the model of [23] has a product form solution (PFS) for the steady-state probabilities. The existence of a PFS is significant in loss/queueing models, since it results in efficient algorithms (of recursive or convolutional form) for the performance measures calculation [32–37]. However, in the P-S model, the calculation of CBP and resource utilization is based on complicated algorithms which are unattractive for network planning engineers. To overcome this problem of complexity that appears in [23], a recursive formula is proposed in [26] for the calculation of CBP and resource utilization.

In this paper, firstly, we extend the model of [23] by incorporating restricted accessibility. We name this model P-S/res. In such a system, each state is related to a pre-specified blocking probability and therefore a call may be blocked and lost even if subcarriers do exist at the time of the call’s arrival [38]. For the proposed model, we show that the calculation of CBP and resource utilization can be based on recursive formulas. In addition, we show the relationship of the P-S/res model with the P-S model under the bandwidth reservation (BR) policy (P-S/BR model). This policy permits the subcarriers’ reservation so as to favor those calls that have high subcarrier requirements. In that sense, the BR policy provides QoS to certain service-classes. Secondly, we propose the quasi-random P-S model with restricted accessibility (qr-P-S/res model) and provide recursive formulas for the determination of time and call congestion (TC and CC, respectively) probabilities as well as resource utilization which are the main performance measures. TC probabilities, for a particular service-class, can be calculated via the proportion of time the system has no available resources for this service-class. On the other hand, CC probabilities can be determined via the proportion of arriving calls that find no available resources in the system. Note that in the P-S model, TC and CC probabilities coincide (and named CBP) due to the Poisson process. Thirdly, we show the relationship between the qr-P-S/res model and the qr-P-S/BR, qr-P-S models, where the arrival process remains quasi-random but the adopted policy is the BR and the CS policy, respectively. The calculation of all performance measures in the qr-P-S/BR and the qr-P-S models can also be based on recursive formulas. Based on simulation results, the accuracy of all proposed formulas is quite satisfactory.

This paper is organized as follows: In Section 2, we review the P-S model and present the formulas for the CBP determination and resource utilization. In Section 3, we propose the P-S/res model
We conclude in Section 6.

with mean \( \alpha \) (i.e., a blocked call cannot retry to be accepted in the cell with the same or less subcarrier requirements).

\[ K \]

\[ \pi \]

\[ L \]

\[ R \]

\[ \gamma \]

\[ \lambda \]

\[ \mu \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]

\[ \gamma \]

\[ \lambda \]
and are impractical compared to the formula of Kaufman–Roberts (K–R) \[44,45\]. The K–R formula is recursive and therefore leads to an efficient way for the CBP calculation in a loss system that services multirate Poisson traffic. Because of this, the interested reader can find many applications of the K–R formula in PFS and non-PFS models \[46–58\].

The complexity problem of (1) can be circumvented via a recursive formula, proposed in \[26\], that resembles the K–R formula. The presentation of this formula, requires the following notation: 

\[ j_1 = \sum_{k=1}^{K} \sum_{l=1}^{L} n_{kl}b_l \] refers to the occupied subcarriers, i.e., \( j_1 = 0, \ldots, M \) and 

\[ j_2 = \sum_{k=1}^{K} \sum_{l=1}^{L} p_k n_{kl}b_l \] refers to the occupied cell’s power, i.e., \( j_2 = 0, \ldots, P \). Furthermore, let \( q(j) = q(j_1, j_2) \) be the occupancy distribution, denoted as:

\[ q(j) = q(j_1, j_2) = \sum_{n \in \Omega_j} \pi(n), \]

where \( \Omega_j \) refers to those states where the occupied subcarriers and power is \( j_1 \) and \( j_2 \), respectively.

The recursive calculation of \( q(j_1, j_2) \)'s is based on (4), whose complexity is \( O(MPKL) \):

\[ q(j_1, j_2) = \begin{cases} 
1, & \text{for } j_1 = j_2 = 0 \\
\frac{1}{\pi} \sum_{k=1}^{K} \sum_{l=1}^{L} a_{kl}b_l q(j_1 - b_l, j_2 - p_k b_l) & \text{for } j_1 = 1, \ldots, M \text{ and } j_2 = 1, \ldots, P
\end{cases} \]

Having obtained the unnormalized values of \( q(j_1, j_2) \), we can calculate the CBP \( B(k,l) \) of service-class \((k,l)\) via:

\[ B(k,l) = \sum_{(j_1 + b_l > M) \cup (j_2 + p_k b_l > P)} G^{-1} q(j_1, j_2), \]

and the mean number of service-class \((k,l)\) in-service calls, \( E(k,l) \), via:

\[ E(k,l) = a_{kl}(1 - B(k,l)), \]

where \( G \) is the normalization constant, given by 

\[ G = \sum_{j_1 = 0}^{M} \sum_{j_2 = 0}^{P} q(j_1, j_2). \]

Based on \( E(k,l) \), we can determine the blocking probability (BP) of the entire system, the subcarrier and the power utilization (SU and PU, respectively) via:

\[ \text{BP} = \sum_{k=1}^{K} \sum_{l=1}^{L} B(k,l) \lambda_{k,l} / \Lambda, \quad \Lambda = \sum_{k=1}^{K} \sum_{l=1}^{L} \lambda_{k,l}, \]

\[ \text{SU} = \sum_{k=1}^{K} \sum_{l=1}^{L} E(k,l) b_l / M, \]

\[ \text{PU} = \sum_{k=1}^{K} \sum_{l=1}^{L} p_k E(k,l) b_l / P. \]

### 3. The P-S Multirate Loss Model under Restricted Accessibility

#### 3.1. The Analytical Model

We consider again the P-S model and apply the notion of restricted accessibility. To this end, let each state \( j_1 > 0 \) be associated with a blocking probability factor, \( pb_{k,l}(j_1) \). Note that if the available subcarriers for service-class \((k,l)\) calls are not enough (i.e., when \( j_1 \geq M - b_l +1 \)), then \( pb_{k,l}(j_1) = 1 \). Similarly, if the system is empty, then \( pb_{k,l}(0) = 0 \).
The admission mechanism for a call of service-class \((k,l)\) in the P-S\(\text{/res}\) model is the following: (a) if \((M - j_1 \geq b_l) \cap (j_2 + p_kb_l \leq P)\) then the service-class \((k,l)\) call is accepted in the cell with probability \(1 - pb_{k,l}(j_1)\) and remains for a service-time which is generally distributed with mean \(\mu^{-1}\), (b) if \((M - j_1 < b_l) \cup (j_2 + p_kb_l > P)\) then the call is blocked due to subcarriers’ unavailability.

The proof of a recursive formula for the unnormalized values of \(q(j_1, j_2)\)'s requires the existence of local balance between states \((j_1 - b_l, j_2 - p_kb_l)\) and \((j_1, j_2)\). These two states differ only by one in-service call of service-class \((k,l)\). The form of local balance equation is as follows:

\[
a_{kl}(1 - pb_{k,l}(j_1 - b_l))q(j_1 - b_l, j_2 - p_kb_l) = y_{kl}(j_1, j_2)q(j_1, j_2),
\]

where \(y_{kl}(j_1, j_2)\) refers to the mean number of service-class \((k,l)\) calls in state \((j_1, j_2)\).

Multiplying both sides of (10) by \(b_l\) and summing over \(k\) and \(l\) we obtain the following formula for the recursive calculation of \(q(j_1, j_2)\)'s in the P-S\(\text{/res}\) model:

\[
\sum_{k=1}^{K} \sum_{l=1}^{L} a_{kl}(1 - pb_{k,l}(j_1 - b_l))q(j_1 - b_l, j_2 - p_kb_l) = j_1q(j_1, j_2),
\]

or

\[
q(j_1, j_2) = \begin{cases} 1, & \text{for } j_1 = j_2 = 0 \\ \frac{1}{b_l} \sum_{k=1}^{K} \sum_{l=1}^{L} a_{kl}(1 - pb_{k,l}(j_1 - b_l))q(j_1 - b_l, j_2 - p_kb_l) & \
\text{for } j_1 = 1, \ldots, M \text{ and } j_2 = 1, \ldots, P \end{cases}
\]

Based on the unnormalized values of \(q(j_1, j_2)\), we can determine the CBP \(B(k,l)\) of service-class \((k,l)\) via:

\[
B(k,l) = \sum_{(j_1+b_l>M)\cup(j_2+p_kb_l>P)} G^{-1}q(j_1, j_2)pb_{k,l}(j_1),
\]

while \(E(k,l), \text{BP}, \text{SU}, \text{and PU}\) are calculated via (6)–(9), respectively.

### 3.2. The Case of the BR Policy (P-S/BR Model)

A proper selection of the values of \(pb_{k,l}(j_1)\) results in the classical BR policy. In that policy, a call of service-class \((k,l)\) has a reservation parameter \(t_l\) and a requirement of \(b_l\) subcarriers. The parameter \(t_l\) denotes the number of subcarriers reserved to benefit calls of all service-classes except for \((k,l)\). Since the BR policy is used to favor calls of high subcarrier requirements, it is obvious that it provides QoS to certain service-classes.

The call admission mechanism in the case of the BR policy (P-S/BR model) consists of the following two cases: (a) if \((M - j_1 - t_l \geq b_l) \cap (j_2 + p_kb_l \leq P)\) then the service-class \((k,l)\) call is accepted for service in the system, (b) if \((M - j_1 - t_l < b_l) \cup (j_2 + p_kb_l > P)\) then call blocking occurs.

By assuming that \(pb_{k,l}(j_1) = 0\) when \(j_1 \leq M - b_l - t_l\) and \(pb_{k,l}(j_1) = 1\) when \(j_1 > M - b_l - t_l\), then the BR policy is incorporated in the model.

In the P-S/BR model, the determination of \(q(j_1, j_2)\)'s is based on (14), whose complexity is \(O(MPKL)\) [56]:

\[
q(j_1, j_2) = \begin{cases} 1, & \text{for } j_1 = j_2 = 0 \\ \frac{1}{b_l} \sum_{k=1}^{K} \sum_{l=1}^{L} a_{kl}(j_1 - b_l)b q(j_1 - b_l, j_2 - p_kb_l) & \\
\text{for } j_1 = 1, \ldots, M \text{ and } j_2 = 1, \ldots, P \end{cases}
\]

where \(a_{kl}(j_1 - b_l) = \begin{cases} a_{kl}, & \text{for } j_1 \leq M - t_l \\ 0, & \text{for } j_1 > M - t_l \end{cases}\).
Having obtained the unnormalized values of \( q(j_1, j_2) \), the calculation of the CBP \( B(k, l) \) of service-class \((k, l)\) can be based on:

\[
B(k, l) = \sum_{(|j_1+b_t+i|>M)\cup(j_2+p_kb_l>P)} G^{-1}q(j_1, j_2),
\]

(15)

while \( E(k, l) \), BP, SU, and PU are calculated via (6)–(9), respectively.

4. The Quasi-Random P-S Multirate Loss Model with Restricted Accessibility

4.1. The Analytical Model

Consider again the model of [23] that provides service to \( KL \) service-classes. In the proposed qr-P-S/res model, new calls of service-class \((k, l)\) are generated from a finite number of sources \( N_{kl} \). The mean arrival rate of idle service-class \((k, l)\) sources is given by \( \lambda_{kl, idle} = (N_{kl} - n_{kl})v_{kl} \), where \( n_{kl} \) is the number of in-service calls of service-class \((k, l)\) and \( v_{kl} \) is the per idle source arrival rate. Based on the above, the per idle source offered traffic-load of service-class \((k, l)\) is determined by \( \alpha_{kl, idle} = v_{kl} / \mu \) (in erlang). If \( N_{kl} \to \infty \) for all service-classes and the total offered traffic-load is constant, then we have the P-S/res model (since the arrival process becomes Poisson).

Upon its arrival, a service-class \((k, l)\) call \((k = 1, \ldots, K \text{ and } l = 1, \ldots, L)\) requires \( b_l \) subcarriers. Let the occupied subcarriers and power in the cell be \( j_1 \) and \( j_2 \), respectively, when the new call arrives. Then, the admission mechanism for the new call is as follows: (a) if \((M - j_1 \geq b_l) \cap (j_2 + p_kb_l \leq P)\) then the service-class \((k, l)\) call is accepted in the cell with probability \( 1 - pb_{k,l}(j_1) \). In that case, the service-time is generally distributed with mean \( \mu^{-1} \), (b) if \((M - j_1 < b_l) \cup (j_2 + p_kb_l > P)\) then the call is blocked due to subcarriers’ unavailability.

Let \( q_{fin}(j) \) be the occupancy distribution in the proposed qr-P-S/res model. To prove a formula for the unnormalized values of \( q_{fin}(j_1, j_2) \)'s, we assume that local balance exists between the states \((j_1 - b_l, j_2 - p_kb_l)\) and \((j_1, j_2)\). The form of local balance equation is as follows:

\[
(N_{kl} - y_{kl, fin}(j_1 - b_l, j_2 - p_kb_l))y_{kl, idle}(1 - pb_{k,l}(j_1 - b_l))q_{fin}(j_1 - b_l, j_2 - p_kb_l) = y_{kl, fin}(j_1, j_2)q_{fin}(j_1, j_2),
\]

(16)

where \( y_{kl, fin}(j_1 - b_l, j_2 - p_kb_l) \) and \( y_{kl, fin}(j_1, j_2) \) refer to the mean number of service-class \((k, l)\) calls in states \((j_1 - b_l, j_2 - p_kb_l)\) and \((j_1, j_2)\), respectively.

Multiplying both sides of (16) by \( b_l \) and summing over \( k \) and \( l \) we obtain:

\[
\sum_{k=1}^{K} \sum_{l=1}^{L} (N_{kl} - y_{kl, fin}(j_1 - b_l, j_2 - p_kb_l))y_{kl, idle}b_l(1 - pb_{k,l}(j_1 - b_l))q_{fin}(j_1 - b_l, j_2 - p_kb_l) = j_1q_{fin}(j_1, j_2).
\]

(17)

The value of \( y_{kl, fin}(j_1 - b_l, j_2 - p_kb_l) \) in (17) is unknown. To determine it, the following lemma is necessary [59]: Two stochastic systems will be equivalent and result in the same congestion probabilities, if they have (a) the same traffic parameters \((K, L, N_{kl}, \alpha_{kl, idle})\), where \( k = 1, \ldots, K \text{ and } l = 1, \ldots, L \) and (b) are the same states.

Therefore, the purpose is to find a stochastic system, whereby the values of \( y_{kl, fin}(j_1 - b_l, j_2 - p_kb_l) \) can be determined. The subcarriers’ requirements of calls of all service-classes and the values of \( M \) and \( P \) in the new system are chosen so that both conditions (a) and (b) are valid and the occupancy \((j_1, j_2)\) of each state \( j \) is unique.

In that case, state \((j) = (j_1, j_2)\) is reached only via state \((j_1 - b_l, j_2 - p_kb_l)\). Thus, \( y_{kl, fin}(j_1 - b_l, j_2 - p_kb_l) = n_{kl} - 1 \). Based on the above, (17) can be written as:

\[
q_{fin}(j_1, j_2) = \begin{cases} 
1, & \text{for } j_1 = j_2 = 0 \\
\frac{1}{M} \sum_{k=1}^{K} \sum_{l=1}^{L} (N_{kl} - n_{kl} + 1)\alpha_{kl, idle}b_l(1 - pb_{k,l}(j_1 - b_l))q_{fin}(j_1 - b_l, j_2 - p_kb_l) & \text{for } j_1 = 1, \ldots, M \text{ and } j_2 = 1, \ldots, P
\end{cases}
\]

(18)
Note that if $N_{kl} \to \infty$ for all service-classes and the total offered traffic-load is constant, then we obtain (12) of the proposed P-S/res model.

### 4.2. Performance Measures Calculation

Having obtained the unnormalized values of $q_{\text{fin}}(j_1, j_2)$, we can calculate the TC probabilities of service-class $(k,l)$ calls, $B_{TC}(k,l)$, via:

$$B_{TC}(k,l) = \sum_{(j_1 + b_I > M) \cup (j_2 + p_k b_I > P)} G^{-1}q_{\text{fin}}(j_1, j_2)p b_k, j_1(j_1),$$

and the CC probabilities of service-class $(k,l)$ calls via (19) but for a cell with $N_{kl} - 1$ sources.

Furthermore, we can determine the average number of service-class $(k,l)$ in-service calls, $E_{\text{fin}}(k,l)$, via:

$$E_{\text{fin}}(k,l) = \sum_{j=1}^{M} \sum_{j_2=1}^{P} G^{-1}y_{k,l, fin}(j_1, j_2)q_{\text{fin}}(j_1, j_2),$$

where $G_{\text{fin}} = \frac{M}{j=1} \sum_{j_2=0}^{P} q_{\text{fin}}(j_1, j_2)$ and $y_{k,l, fin}(j_1 - b_I, j_2 - p_k b_I)$ is the mean number of service-class $(k,l)$ calls in state $(j_1 - b_I, j_2 - p_k b_I)$ calculated via:

$$y_{k,l, fin}(j_1, j_2) = \left(\frac{N_{kl} - n_{kl}}{N_{kl} + 1}\right) \alpha_{k,l, idle} \left(1 - p b_k, j_1(j_1 - b_I)\right) q_{\text{fin}}(j_1 - b_I, j_2 - p_k b_I).$$

In addition, we can determine the entire system BP based on the TC probabilities of all service-classes, $B_{PTC}$, the SU$_{fin}$, and the PU$_{fin}$, via:

$$B_{PTC} = \sum_{k=1}^{K} \sum_{l=1}^{L} B_{TC}(k,l) N_{kl} v_{kl} / \Lambda_{\text{fin}}, \quad \Lambda_{\text{fin}} = \sum_{k=1}^{K} \sum_{l=1}^{L} N_{kl} v_{kl},$$

$$SU_{\text{fin}} = \sum_{k=1}^{K} \sum_{l=1}^{L} E_{\text{fin}}(k,l) b_I / M,$$

$$PU_{\text{fin}} = \sum_{k=1}^{K} \sum_{l=1}^{L} p_k E_{\text{fin}}(k,l) b_I / P.$$ 

In order to determine the values of $q_{\text{fin}}(j_1, j_2)$ according to (18), the unknown values of $n_{kl}$ are required. These values can be obtained via a stochastic system, with the same parameters and the same states as already described for the proof of (18). However, the state space determination of this system becomes complex due to the large number of service-classes. To this end, we propose an algorithm which is simpler and easy to implement:

(a) Determine the values of $q(j_1, j_2)$ according to (12) (i.e., via the P-S/res model).

(b) Determine the values of $y_{k,l}(j_1, j_2)$ via the formula:

$$y_{k,l}(j_1, j_2) = \alpha_{k,l} \left(1 - p b_k, j_1(j_1 - b_I)\right) q(j_1 - b_I, j_2 - p_k b_I).$$

(c) Modify (18) to the following formula, where $y_{k,l}(j_1, j_2)$ is given by (25):

$$q_{\text{fin}}(j_1, j_2) = \begin{cases} 1, & \text{for } j_1 = j_2 = 0 \\ \frac{1}{L} \sum_{l=1}^{L} (N_{kl} - n_{kl}(j_1 - b_I, j_2 - p_k b_I)) \alpha_{k,l, idle} b_I \left(1 - p b_k, j_1(j_1 - b_I)\right) q_{\text{fin}}(j_1 - b_I, j_2 - p_k b_I) & \text{for } j_1 = 1, \ldots, Mand \ j_2 = 1, \ldots, P \end{cases}$$

$$q_{\text{fin}}(j_1, j_2) = \begin{cases} 1, & \text{for } j_1 = j_2 = 0 \\ \frac{1}{L} \sum_{l=1}^{L} (N_{kl} - n_{kl}(j_1 - b_I, j_2 - p_k b_I)) \alpha_{k,l, idle} b_I \left(1 - p b_k, j_1(j_1 - b_I)\right) q_{\text{fin}}(j_1 - b_I, j_2 - p_k b_I) & \text{for } j_1 = 1, \ldots, Mand \ j_2 = 1, \ldots, P \end{cases}$$
(d) Determine $E_{\text{fin}}(k, l)$ via (20), where the values of $y_{kl, \text{fin}}(j_1, j_2)$ are given by:

$$
y_{kl, \text{fin}}(j_1, j_2) = \frac{(N_{kl} - y_{kl}(j_1 - b_l, j_2 - p_k b_l))\alpha_{kl, \text{idle}}(1 - p b_{kj}(j_1 - b_l))q_{\text{fin}}(j_1 - b_l, j_2 - p_k b_l)}{q_{\text{fin}}(j_1, j_2)}.
$$

(e) Determine: (1) the TC probabilities of service-class $(k, l)$ calls, $B_{TC}(k, l)$, via (19), and (2) the BP$_{TC}$, the SU$_{\text{fin}}$, and the PU$_{\text{fin}}$, via (22)–(24), respectively.

4.3. The Case of the BR Policy (qr-P-S/BR Model)

The admission mechanism in the qr-P-S/BR model is the same with that of the P-S/BR model. Since the local balance is destroyed (due to the BR policy), the recursive formulas presented in this subsection are approximate. Following the previous analysis of Section 4, we propose an algorithm for the calculation of performance measures in the qr-P-S/BR model:

(a) Determine the values of $q(j_1, j_2)$ according to (14) (i.e., via the P-S/BR model).
(b) Determine the values of $y_{kl}(j_1, j_2)$, for $j_1 \leq M - t_l$, via the formula:

$$
y_{kl}(j_1, j_2) = \frac{\alpha_{kl}q(j_1 - b_l, j_2 - p_k b_l)}{q(j_1, j_2)}.
$$

(c) Modify (18) to the following formula where $y_{kl}(j_1, j_2)$ has been calculated via (28):

$$
q_{\text{fin}}(j_1, j_2) = \left\{\begin{array}{ll}
1, & \text{for } j_1 = j_2 = 0 \\
\frac{1}{L_k} \sum_{k, j_1 = 1}^L (N_{kl} - y_{kl}(j_1 - b_l, j_2 - p_k b_l))\alpha_{kl, \text{idle}}b_{q_{\text{fin}}}(j_1 - b_l, j_2 - p_k b_l), & \text{for } j_1 = 1, \ldots, M \text{ and } j_2 = 1, \ldots, P
\end{array}\right.
$$

where $\alpha'_{kl, \text{idle}} = \alpha_{kl, \text{idle}}^\prime(j_1 - b_l) = \alpha_{kl, \text{idle}}$ for $j_1 \leq M - t_l$.

(d) Determine the average number of in-service calls of service-class $(k, l)$, $E_{\text{fin}}(k, l)$ via (20), where $y_{kl, \text{fin}}(j_1, j_2)$ is given by, for $j_1 \leq M - t_l$:

$$
y_{kl, \text{fin}}(j_1, j_2) = \frac{(N_{kl} - y_{kl}(j_1 - b_l, j_2 - p_k b_l))\alpha_{kl, \text{idle}}q_{\text{fin}}(j_1 - b_l, j_2 - p_k b_l)}{q_{\text{fin}}(j_1, j_2)}.
$$

(e) Determine the TC probabilities of service-class $(k, l)$ calls, $B_{TC}(k, l)$, via:

$$
B_{TC}(k, l) = \sum_{(j_1 + b_l + t_l > M) \cup (j_2 + p_k b_l > P)} G^{-1}q_{\text{fin}}(j_1, j_2),
$$

and the BP$_{TC}$, the SU$_{\text{fin}}$, and the PU$_{\text{fin}}$, via (22)–(24), respectively.

4.4. The Case of the CS Policy (qr-P-S Model)

In the qr-P-S multirate loss model, a new service-class $(k, l)$ call requires $b_l$ subcarriers. Assuming that the occupied subcarriers and power in the cell are $j_1$ and $j_2$, respectively, then, the new call: (a) is accepted for a generally distributed service-time with mean $\mu^{-1}$, if $(M - j_1 \geq b_l) \cap (j_2 + p_k b_l \leq P)$ and (b) is blocked and lost, if $(M - j_1 < b_l) \cup (j_2 + p_k b_l > P)$.

Following Section 4, it can be proved that the (unnormalized) values of $q_{\text{fin}}(j_1, j_2)$ in the qr-P-S model are given by:

$$
q_{\text{fin}}(j_1, j_2) = \left\{\begin{array}{ll}
1, & \text{for } j_1 = j_2 = 0 \\
\frac{1}{L_k} \sum_{k, j_1 = 1}^L (N_{kl} - y_{kl}(j_1 - b_l, j_2 - p_k b_l))\alpha_{kl, \text{idle}}b_{q_{\text{fin}}}(j_1 - b_l, j_2 - p_k b_l), & \text{for } j_1 = 1, \ldots, M \text{ and } j_2 = 1, \ldots, P
\end{array}\right.
$$
Note that if $N_{d} \rightarrow \infty$ for all service-classes and the total offered traffic-load is constant, then we have (4) of the P-S model.

In order to overcome the equivalent stochastic system determination required for the calculation of $q_{\text{fin}}(j_{1}, j_{2})$ via (32), we present the following algorithm for the calculation of the various performance measures in the qr-P-S model:

(a) Determine the values of $q(j_{1}, j_{2})$ according to (4) (i.e., via the P-S model).
(b) Determine the values of $y_{kl}(j_{1}, j_{2})$ via the formula:

$$y_{kl}(j_{1}, j_{2}) = \frac{\alpha_{kl}q(j_{1} - b_{l}j_{2} - p_{k}b_{l})}{q(j_{1}, j_{2})}. \quad (33)$$

(c) Modify (32) to the following formula, where $y_{kl}(j_{1}, j_{2})$ has been determined via (33):

$$q_{\text{fin}}(j_{1}, j_{2}) = \left\{ \begin{array}{ll} 1, & \text{for } j_{1} = j_{2} = 0 \\
\frac{1}{N} \sum_{j_{1}=1}^{K} \sum_{j_{2}=1}^{L} (N_{d} - y_{kl}(j_{1} - b_{l}j_{2} - p_{k}b_{l})) \alpha_{kl, \text{idle}}b_{l}q_{\text{fin}}(j_{1} - b_{l}j_{2} - p_{k}b_{l}) & \text{for } j_{1} = 1, \ldots, M \text{ and } j_{2} = 1, \ldots, P.
\end{array} \right. \quad (34)$$

(d) Determine $E_{\text{fin}}(k, l)$ via (20), where $y_{kl, \text{fin}}(j_{1}, j_{2})$ is given by:

$$y_{kl, \text{fin}}(j_{1}, j_{2}) = \frac{(N_{d} - y_{kl}(j_{1} - b_{l}j_{2} - p_{k}b_{l})) \alpha_{kl, \text{idle}}q_{\text{fin}}(j_{1} - b_{l}j_{2} - p_{k}b_{l})}{q_{\text{fin}}(j_{1}, j_{2})}. \quad (35)$$

(e) Determine the TC probabilities of service-class $(k, l)$ calls, $B_{\text{TC}}(k, l)$, via:

$$B_{\text{TC}}(k, l) = \sum_{\{(j_{1}, j_{2}) : (j_{1} + b_{l}j_{2} + p_{k}b_{l}) > M\}} G^{-1}q_{\text{fin}}(j_{1}, j_{2}), \quad (36)$$

and the BP$_{\text{TC}}$, the SU$_{\text{fin}}$, and the PU$_{\text{fin}}$, via (22)–(24), respectively.

5. Performance Evaluation

In this section, we consider a cell that accommodates KL service-classes and provide simulation and analytical congestion probabilities results for the P-S, the P-S/res, and the qr-P-S models. In addition, we provide simulation and analytical SU and PU results for the P-S and the P-S/res models. The required input for these models is: $B = 20$ MHz, $M = 256$, $P = 25$ Watt, $R = 329.6$ kbps, $L = 64$, $b_{l} = l$, $l = 1, \ldots, 64$, while we assume that $b_{l}$ is uniformly distributed. Due to this assumption, a new call has an average subcarrier requirement $\hat{\chi} = 32.5$. In addition, let $K = 3$, which means that the cell accommodates $KL = 192$ service-classes. In the case of the qr-P-S model, we assume that $N_{d} = 20$ sources for all service-classes. Let the integer representations of $p_{k}$ ($k = 1, 2, 3$) and $P$ be: $p_{1} = 6$, $p_{2} = 10$, $p_{3} = 16$, $P_{r} = 2500$. The values of $p_{k}^{*}$ require that: $p_{1} \approx 0.06$, $p_{2} \approx 0.01$, $p_{3} \approx 0.16$ achieved via the following values of the average channel gain $\gamma_{k}$ ($k = 1, 2, 3$): $\gamma_{1} = 24.679$ dB, $\gamma_{2} = 22.460$ dB, $\gamma_{3} = 20.419$ dB. In addition, an arriving call has an average channel gain $\gamma_{k}$ with a probability that is determined via: (1) set 1: $r_{k} = 1/3$ ($k = 1, 2, 3$) and (2) set 2: $r_{1} = 1/4$, $r_{2} = 1/4$, $r_{3} = 1/2$. Furthermore, let $\lambda_{kl} = \lambda r_{k} / L$, where $\lambda$ is the total arrival rate given by $\Lambda = \alpha M \mu / \hat{\chi}$, $\alpha$ is the cell’s traffic intensity and $\mu$ is the service rate of calls with $\mu = 0.00625$. In the case of the P-S/res model, we assume that $pb_{j_{1}}(j_{1}) = 0$ when $j_{1} \leq M - b_{l} - t_{l}$ and $pb_{j_{2}}(j_{1}) = 1$ when $j_{1} > M - b_{l} - t_{l}$. Due to this assumption, we have the P-S/BR model. The values of $t_{l}$ are $t_{l} = 64 - l$, $l = 1, \ldots, 64$, and they are chosen in such a way so that $b_{l} + t_{l} = \ldots = b_{64} + t_{64}$.

Regarding the simulation results, they are based on Simscript III [60]. Each simulation run is based on 10 million generated calls while the results presented herein are mean values of 7 runs. Furthermore, the blocking events of the first 3% of these calls are not taken into account in the results, in order to account for a warm-up period. In all figures of this section, the analytical and simulation results are
quite close. Note that in the x-axis of Figures 1–5 the value of $\alpha$ increases from 0.2 to 1.0 in steps of 0.1, while in the x-axis of Figures 6 and 7 the value of $\alpha$ increases from 0.05 to 0.2 in steps of 0.025.

In Figures 1 and 2, we consider the P-S and the P-S/BR models and present the simulation and analytical CBP of service-classes $(3, 64), (2, 64), (1, 64)$ (Figure 1) and service-classes $(3, 48), (2, 48), (1, 48)$ (Figure 2). Note that service-classes $(3, 64), (2, 64)$ and $(1, 64)$ have the highest requirement in terms of subcarriers ($l = 64$). Regarding the average channel gain we consider set 1 ($\eta_k = 1/3$ ($k = 1, 2, 3$)). In Figure 1, it is obvious that the BR policy decreases the CBP values of service-classes $(3, 64), (2, 64)$ and $(1, 64)$ compared to the corresponding CBP values of the P-S model. In Figure 2, the BR policy increases (in most of the cases) the CBP values of service-classes $(3, 48), (2, 48), (1, 48)$ compared to the corresponding CBP values of the P-S model. A similar behavior appears in most of the service-classes whose calls have a requirement of less than 64 subcarriers. In addition, the same behavior (in terms of CBP) appears when we consider set 2 for the average channel gain.

![Figure 1. CBP—Service-classes (3, 64), (2, 64), and (1, 64).](image1)

![Figure 2. CBP—Service-classes (3, 48), (2, 48), and (1, 48).](image2)
In Figure 3, we present the entire system BP for both sets of $r_k$. We observe that the BP increases for both sets of $r_k$ when the BR policy is considered. This is because the values of $t_l$ parameters are chosen to increase the CBP of service-classes with low subcarrier requirements and benefit service-classes with high subcarrier requirements. The increase of BP in the case of the P-S/BR model results in a slight decrease of the SU and PU (for both sets of $r_k$) compared to the P-S model, as we show in Figures 4 and 5, respectively. A similar behavior appears in the case of the corresponding quasi-random models.
In Figures 6 and 7, we consider the PS and the qr-P-S models for both sets of $r_k$. Figures 6 and 7 show the simulation and analytical TC probabilities of service-classes (3, 16) and (3, 64), respectively. We observe that: (1) in the qr-P-S model the TC probabilities are lower compared to those obtained in the P-S model, which is due to the quasi-random process which is smoother than the Poisson process, and (2) the selection of set 2 for the values of $r_k$ increases the TC probabilities since the power assigned to calls in the case of set 2 is larger compared to set 1.
6. Conclusions

We propose loss models for the analysis of the downlink of an OFDM cell that accommodates random or quasi-random generated calls from different service-classes under the restricted accessibility, the BR and the CS policies. The cell is analysed as a loss system, i.e., calls are blocked in case of resource unavailability. To determine the main performance measures, such as congestion probabilities and resource utilization, we propose approximate but recursive formulas. All formulas are quite accurate, compared to simulation, and can be applied to network planning and dimensioning procedures.

Author Contributions: Conceptualization, all authors; methodology, all authors; software, P.I.P., I.D.M.; validation, P.I.P., I.D.M.; writing—original draft preparation, all authors; writing—review and editing, all authors.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References
1. Moscholios, I.D.; Logothetis, M.D. Efficient Multirate Teletraffic Loss Models beyond Erlang; Wiley IEEE Press: Hoboken, NJ, USA, 2019. [CrossRef]
2. Stasiak, M.; Głąbowski, M.; Wisniewski, A.; Zwierzykowski, P. Modeling and Dimensioning of Mobile Networks: From GSM to LTE; Wiley: Hoboken, NJ, USA, 2011. [CrossRef]
3. Shioda, S. Fundamental trade-offs between resource separation and resource share for quality of service guarantees. IET Netw. 2014, 3, 4–15. [CrossRef]
4. Bouloukakis, G.; Moscholios, I.D.; Georgantas, N.; Issarny, V. Performance modeling of the middleware overlay infrastructure of mobile things. In Proceedings of the 2017 IEEE International Conference on Communications (ICC), Paris, France, 21–25 May 2017. [CrossRef]
5. Tadayon, N.; Kaddoum, G. Packet-level modeling of cooperative diversity: A queueing network approach. IEEE Access 2018, 6, 35223–35242. [CrossRef]
6. Benson, K.; Boulouchakis, G.; Issarny, V.; Mehrotra, S.; Moscholios, I.; Venkatasubramanian, N. FIREDEX: A prioritized IoT data exchange middleware for emergency response. In Proceedings of the ACM/IFIP/USENIX 19th International Middleware Conference, Rennes, France, 10–14 December 2018. [CrossRef]

7. Hanczewski, S.; Stasiak, M.; Weißenberg, J. Non-full-available queueing model of an EON node. Opt. Switch. Netw. 2019, 33, 131–142. [CrossRef]

8. Xue, J.; Yan, F.; Riska, A.; Smirni, E. Scheduling data analytics work with performance guarantees: Queuing and machine learning models in synergy. Clust. Comput. 2016, 19, 849–864. [CrossRef]

9. Ataie, E.; Gianniti, E.; Ardagna, D.; Movaghar, A. A combined analytical modeling machine learning approach for performance prediction in mapreduce jobs in cloud environment. In Proceedings of the 18th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing (IEEE SYNASC), Timisoara, Romania, 24–27 September 2016. [CrossRef]

10. Abbasi, A.; Jin, H.; Wu, S. A software-defined cloud resource management framework. In Proceedings of the Asia-Pacific Services Computing Conference, Bangkok, Thailand, 7–9 December 2015. [CrossRef]

11. Abbasi, A.; Jin, H. v-Mapper: An application-aware resource consolidation scheme for cloud data centers. Future Internet 2018, 10, 90. [CrossRef]

12. Vardakas, J.; Zorba, N.; Verikoukis, C. Power demand control scenarios for smart grid applications with finite number of appliances. Appl. Energy 2016, 162, 83–98. [CrossRef]

13. Moscholios, I.; Vassilakis, V.; Logothetis, M.; Boucouvalas, A. State-dependent bandwidth sharing policies for wireless multirate loss networks. IEEE Trans. Wirel. Commun. 2017, 16, 5481–5497. [CrossRef]

14. Głąbowski, M.; Kmiecik, D.; Stasiak, M. Modelling of multiservice networks with separated resources and overflow of adaptive traffic. Wirel. Commun. Mob. Comput. 2018, 2018, 17. [CrossRef]

15. Abd El-atty, S.; Gharsseldien, Z.; Lizos, K. Predictive reservation for handover optimization in two-tier heterogeneous cellular networks. Wirel. Pers. Commun. 2018, 98, 1637–1661. [CrossRef]

16. Efstratiou, P.; Moscholios, I. User mobility in a 5G cell with quasi-random traffic under the complete sharing and bandwidth reservation policies. Autom. Control Comp. Sci. 2019, 53, 376–386. [CrossRef]

17. Vahidi, V.; Saberinia, E. OFDM high speed train communication systems in 5G cellular networks. In Proceedings of the 15th IEEE Annual Consumer Communications Networking Conference (IEEE CCNC), Las Vegas, NV, USA, 12–15 January 2018. [CrossRef]

18. Zhang, L.; Xiao, A.; Molu, M.; Tafazolli, R. Filtered OFDM systems, algorithms, and performance analysis for 5G and beyond. IEEE Trans. Commun. 2018, 66, 1205–1218. [CrossRef]

19. Yang, G.; Liang, Y.; Zhang, R.; Pei, Y. Modulation in the air: Backscatter communication over ambient OFDM carrier. IEEE Trans. Commun. 2018, 66, 1219–1233. [CrossRef]

20. Na, Z.; Wang, Y.; Xiong, M.; Liu, X.; Xia, J. Modeling and throughput analysis of an ADO-OFDM based relay-assisted VLC system for 5G networks. IEEE Access 2018, 6, 17586–17594. [CrossRef]

21. Lu, W.; Fang, S.; Hu, S.; Liu, X.; Li, B.; Na, Z.; Gong, Y. Energy efficiency optimization for OFDM based 5G wireless networks with simultaneous wireless information and power transfer. IEEE Access 2018, 6, 75937–75946. [CrossRef]

22. Sultana, A.; Fernando, X.; Zhao, L. Power allocation using geometric water filling for OFDM-based cognitive radio networks. In Proceedings of the 2016 IEEE 84th Vehicular Technology Conference (IEEE VTC-Fall), Montreal, QC, Canada, 18–21 September 2016. [CrossRef]

23. Paik, C.; Suh, Y. Generalized queueing model for call blocking probability and resource utilization in OFDM wireless networks. IEEE Commun. Lett. 2011, 15, 767–769. [CrossRef]

24. Pla, V.; Martinez-Bauzet, J.; Casares-Giner, V. Comments on call blocking probability and bandwidth utilization of OFDM subcarrier allocation in next-generation wireless networks. IEEE Commun. Lett. 2008, 12, 349. [CrossRef]

25. Chen, J.; Chen, W. Call blocking probability and bandwidth utilization of OFDM subcarrier allocation in next-generation wireless networks. IEEE Commun. Lett. 2006, 10, 82–84. [CrossRef]

26. Moscholios, I.; Vassilakis, V.; Panagoulias, P.; Logothetis, M. On call blocking probabilities and resource utilization in OFDM wireless networks. In Proceedings of the 2018 11th International Symposium on Communication Systems, Networks Digital Signal Processing (CSNDSP), Budapest, Hungary, 18–20 July 2018. [CrossRef]

27. Zhang, Y.; Xiao, Y.; Chen, H. Queueing analysis for OFDM subcarrier allocation in broadband wireless multiservice networks. IEEE Trans. Wirel. Commun. 2008, 7, 3951–3961. [CrossRef]
28. Kaufman, J.; Rege, K. Blocking in a shared resource environment with batched Poisson arrival processes. *Perf. Eval.* 1996, 24, 249–263. [CrossRef]
29. Moscholios, I.; Logothetis, M. The Erlang multirate loss model with batched Poisson arrival processes under the bandwidth reservation policy. *Comput. Commun.* 2010, 33, S167–S179. [CrossRef]
30. Moscholios, I.; Vassilakis, V.; Sarigiannidis, P. Performance modelling of a multirate loss system with batched Poisson arrivals under a probabilistic threshold policy. *IET Netw.* 2018, 7, 242–247. [CrossRef]
31. Ezhilchelvan, P.; Mitran, I. Multi-class resource sharing with batch arrivals and complete blocking. In Proceedings of the International Conference on Quantitative Evaluation of Systems (QEST), Berlin, Germany, 5–7 September 2017; Lecture Notes in Computer Science. Springer: Cham, Switzerland, 2017; Volume 10503. [CrossRef]
32. Iversen, V. The exact evaluation of multi-service loss system with access control. *Teleteknik* 1987, 31, 56–61.
33. Moscholios, I.; Logothetis, M.; Koukias, M. An ON-OFF multirate loss model of finite sources. *IEICE Trans. Commun.* 2007, 90, 1608–1619. [CrossRef]
34. Głąbowski, M.; Kalisz, A. Convolution algorithm for overflow calculation in integrated services networks. In Proceedings of the 17th Asia Pacific Conference on Communications (APCC), Sabah, Malaysia, 2–5 October 2011. [CrossRef]
35. Stasiak, M.; Parniewicz, D.; Zvierzykowski, P. Traffic engineering for multicast connections in multiservice cellular network. *IEEE Trans. Ind. Inform.* 2013, 9, 262–270. [CrossRef]
36. Moscholios, I.; Vassilakis, V.; Logothetis, M.; Boucouvalas, A. A probabilistic threshold-based bandwidth sharing policy for wireless multirate loss networks. *IEEE Wirel. Commun. Lett.* 2016, 5, 304–307. [CrossRef]
37. Głąbowski, M.; Sobieraj, M. Analytical modelling of multiservice switching networks with multiservice sources and resource management mechanisms. *Telecom. Syst.* 2017, 66, 559–578. [CrossRef]
38. Iversen, V. Modelling restricted accessibility for wireless multi-service systems. *LNCS* 2006, 3883, 93–102. [CrossRef]
39. Pinsky, E.; Conway, A. Computational algorithms for blocking probabilities in circuit-switched networks. *Ann. Oper. Res.* 1992, 35, 31–41. [CrossRef]
40. Choudhury, G.; Leung, K.; Whitt, W. An algorithm to compute blocking probabilities in multi-rate multi-class multi-resource loss models. *Adv. Appl. Prob.* 1995, 27, 1104–1143. [CrossRef]
41. Caro, F.; Simchi-Levi, D. Optimal static pricing for a tree network. *Ann. Oper. Res.* 2012, 196, 137–152. [CrossRef]
42. Wang, M.; Li, S.; Won, E.; Zukerman, M. Performance analysis of circuit-switched multi-service multi-rate networks with alternative routing. *J. Light. Technol.* 2014, 32, 179–200. [CrossRef]
43. Beard, C.; Frost, V. Prioritized resource allocation for stressed networks. *IEEE/ACM Trans. Netw.* 2001, 9, 618–633. [CrossRef]
44. Kaufman, J. Blocking in a shared resource environment. *IEEE Trans. Commun.* 1981, 29, 1474–1481. [CrossRef]
45. Roberts, J. A service system with heterogeneous user requirements. *Performance of Data Communications Systems and Their Applications;* North Holland: Amsterdam, The Netherlands, 1981; pp. 423–431.
46. Vardakas, J.; Moscholios, I.; Limnios, N.; Stylianakis, V. Performance analysis of OCDMA PON configuration supporting multirate bursty traffic with retrials and QoS differentiation. *Opt. Switch. Netw.* 2014, 13, 112–123. [CrossRef]
47. Huang, Y.; Rosberg, Z.; Ko, K.; Zukerman, M. Blocking probability approximations and bounds for best-effort calls in an integrated service system. *IEEE Trans. Commun.* 2015, 63, 5014–5026. [CrossRef]
48. Vassilakis, V.; Moscholios, I.; Logothetis, M. Uplink blocking probabilities in priority-based cellular CDMA networks with finite source population. *IEICE Trans. Commun.* 2016, 99, 1302–1309. [CrossRef]
49. Casares-Giner, V. Some teletraffic issues in optical burst switching with burst segmentation. *Electr. Lett.* 2016, 52, 941–943. [CrossRef]
50. Głąbowski, M.; Kalisz, A.; Stasiak, M. Modelling overflow systems with distributed secondary resources. *Comp. Netw.* 2016, 108, 171–183. [CrossRef]
51. Vassilakis, V.; Moscholios, I.; Logothetis, M. Efficient radio resource allocation in SDN/NFV based mobile cellular networks under the complete sharing policy. *IET Netw.* 2018, 7, 103–108. [CrossRef]
52. Sagkriotis, S.; Pantelis, S.; Moscholios, I.; Vassilakis, V. Call blocking probabilities in a two-link multi rate loss system for Poisson traffic. *IET Netw.* 2018, 7, 233–241. [CrossRef]
53. Vakilinia, S.; Cheriet, M. Preemptive cloud resource allocation modeling of processing jobs. *J. Supercomput.* 2018, 74, 2116–2150. [CrossRef]

54. Panagoulias, P.; Moscholios, I. Congestion probabilities in the X2 link of LTE Networks. *Telecommun. Syst.* 2019, 71, 585–599. [CrossRef]

55. Francisco, C.; Martins, L.; Medhi, D. Dynamic multicriteria alternative routing for single-and multi-service reservation-oriented networks and its performance. *Ann. Telecommun.* 2019, 74, 697–715. [CrossRef]

56. Panagoulias, P.; Moscholios, I.; Logothetis, M. Performance metrics in OFDM wireless networks under the bandwidth reservation policy. In Proceedings of the International Conference on Image Processing and Communications, Bydgoszcz, Poland, 11–13 September 2019. [CrossRef]

57. Hanczewski, S.; Horiushkina, A.; Stasiak, M.; Weissenberg, J. The analytical model of 5G networks. In Proceedings of the International Conference on Image Processing and Communications, Bydgoszcz, Poland, 11–13 September 2019. [CrossRef]

58. Chousainov, I.-A.; Moscholios, I.; Kaloxyllos, A.; Logothetis, M. Performance evaluation of a C-RAN supporting quasi-random traffic. In Proceedings of the 2019 International Conference on Software, Telecommunications and Computer Networks (Softcom), Split, Croatia, 19–21 September 2019. [CrossRef]

59. Stamatelos, G.; Hayes, J. Admission control techniques with application to broadband networks. *Comp. Commun.* 1994, 17, 663–673. [CrossRef]

60. Simscript III. Available online: http://www.simscript.com (accessed on 8 December 2019).

© 2019 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).