Ballistic Injection and Acceleration of Positrons in the Laser-Plasma Bubble Regime

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Abstract

A novel approach for positron injection and acceleration in laser driven plasma wakefield is proposed. A theoretical model is developed and confirmed through simulations. The proposal is based on employing two co-axially propagating beams ring-shaped and Gaussian beams to drive wakefields in a preformed plasma volume filled with both electrons and positrons. The laser’s ponderomotive force is utilized to provide the transverse momenta for positron injection and those positrons can be trapped by the focusing field and then accelerated by the wake wave. The simulation shows that a relatively high-charge, quasi-monoenergetic positrons beams can be achieved. The positrons are accelerated to more than 200 MeV within 2mm, which is similar to the acceleration of electrons in the same scenario, with the same normalized peak laser intensities of $a = 2$ for both Gaussian and ring-shaped lasers.

Keywords: positron injection, positron acceleration, laser wakefield acceleration, laser plasma interaction
Introduction

Intense relativistic positron beams are crucial for basic plasma physics studies and pair-production in the field of fundamental physics[1]. They are also believed to exist in violent high-energy astrophysical phenomena[2]. A stable method to generate intense, mono-energetic and fully tunable positron beams enables experimental study of gamma-ray bursts and black holes [3,4]. Bremsstrahlung-based high-energy positrons are usually produced in linear accelerators (LINACs) and synchrotron facilities via propagating the relativistic electron beams through a thick, high Z targets. However, a positron beam generated by this method is of a continuous energy spectrum and a large transverse geometrical emittance, thus having limited applications.

Since the laser wakefield accelerator(LWFA) concept was proposed in 1979[5], a lot of work has been done on electron acceleration using ultrafast terawatt laser systems[6-19]. Researchers have made great progress in producing high quality electron beams [6-8,10-12] as well as in boosting the electron energy [12-14,19-22]. Until recently, experiments demonstrated the generation of sub-hundred MeV positrons by interacting electrons beams from laser-plasma accelerators with high-Z solid targets [23]. However, the resulting positron beams, in addition to the above-mentioned drawbacks, are limited both in positron yield and energy. Hybrid schemes have also been proposed and conducted experimentally to generate low energy ($E < 20$ MeV) and broad divergence ($\sim 1$ rad) positron jets with a high positron yield (up to $10^{11}$ per shot) [24-26]. However, a scheme for positron injection into laser-plasma positron accelerators has yet been absent.

The difficulty of positron acceleration in the “bubble” regime[9] is that positrons, due to their positive charge, are easily expelled away from the bubble in the transverse direction[27,28]. A short and intense ring-shaped laser with azimuthal symmetric intensity profile, such as Laguerre-Gaussian mode($l$, $p$) = (1,0), where p is radial index and l is azimuthal index, can excite a donut-shaped bubble wakefield in plasma, which is thought to be a good candidate for positron acceleration [29-33]. Positrons are severely affected in the first half bucket of the donut bubble by the presence of the laser pulse, thus the injection dynamics is completely different from that of electrons.

Here, in this paper a proposal is presented for trapping and injection of positrons utilizing the front half of the second bubbles driven by a Gaussian laser beam and a co-axial
propagating ring-shaped laser beam. The Gaussian beam focus spot size is smaller than that of the ring-shaped beam. In the simulations, the positrons fill the whole plasma region with much lower density. Simulations show that the positrons can be scattered either by the center bubble front or by the donut bubble front, depending on their initial location. The time delay between the two laser beams plays an important role regarding the charge of injected positrons into the center and donut bubbles. The time delay between the two laser beams is examined carefully to investigate the dynamics of the injection and acceleration. The energy spectrum of the accelerated positron beams is quasi-monoenergetic and having a maximum kinetic energy up to 200 MeV in this scenario.

**Results**

**Model for Laser-Plasma Injection and Acceleration of Positrons**

In this scheme, laser profiles of \( a^G(t, r) = a_0^G \exp\left(\frac{-r^2}{r_0^2} - \frac{(t-t_0)^2}{t_0^2}\right) \) for the Gaussian beam and \( a^R = a_0^R \exp\left(\frac{-(r-r_0)^2}{r_R^2}\right) \exp\left(\frac{-(t-t_0)^2}{t_0^2}\right) \) for the ring-shaped beam are used. These two laser beams have the same temporal profile, but with a time delay \( \tau \). A positive \( \tau \) means the ring-shaped beam is ahead of the center Gaussian beam. The spatial profile of the ring-shaped beam can be characterized by inner radius \( r_{in} \) and outer radius \( r_{out} \), or ring radius \( r_0 = (r_{out} + r_{in})/2 \) and ring width \( r_d = (r_{out} - r_{in})/2 \). The ring-shaped beam has the same normalized peak strength \( a_0^L \) at \( r = r_0 \). It can excite a series of donut bubbles with width of \( r_d = 2\sqrt{a_0^R \frac{c}{\omega_p}} \) when the matching condition is met, as predicted by nonlinear theory for bubble[9]. Here, \( c \) is the speed of light in vacuum, \( \omega_p = \sqrt{\frac{4\pi n_0 e^2}{m_e}} \) is the plasma frequency, \( n_0 \) is the ambient electron density, \( m_e \) is the electron or positron mass, and \( e \) is the unit charge of electrons (negative \(-e\)) or positrons (positive \(+e\)). To clarify the dynamic processes for the scattering and injection, it is better to let the two laser beams interact with their surrounding plasma independently. Thus, the relation \( 2\left(\sqrt{a_0^G} + \sqrt{a_0^R}\right) \frac{c}{\omega_p} < r_0 \) should be satisfied. In this case, the excited plasma wakefields by Gaussian and ring-shaped laser beams can be analyzed independently.
In the moving frame of the laser, the positrons in a plasma wakefield structure can experience the process of scattering, trapping and acceleration before they are emitted or decelerated. In the beginning, the positrons are repelled by the electromagnetic force from the laser tail [27,28] like ions scattered by nuclei. The positrons will be injected into the bubbles if the transverse momentum is high enough to let the positrons penetrate the bubble and stay inside. The transverse motion of the positrons is restricted by the focusing electromagnetic field inside the bubble. The longitudinal electric field can accelerate these positrons to relativistic velocities if they stay long enough inside the bubble.

It is key for this injection scheme to complete the transfer of positrons between the center and the donut bubbles. “Elastic” scattering provides the positrons with transverse momentum, which makes them possible to move from the center bubble to the donut bubble or from the donut bubble to the center bubble for trapping. Otherwise, the positrons will be pushed away by the first half of bubble which is usually overlapped with the rear of the laser pulse. The scattering process happens solely in the first half of the bubble, where the bubble radius $r_b$ maps out a circle.

For laser propagating along the $x$-axis, under the quasi-static approximation, assume all variables depend on $\xi = x - v_g t$ instead of $x$ and $t$, where $v_g = c \sqrt{1 - \frac{n_0}{n_c}}$ is the group velocity of light in plasma, $n_0$ is the ambient electron density, $n_c$ is the critical density. In the moving frame of the bubble, the currents and densities are time independent, as well as the bubble boundary.

The total force on a positron inside bubble given in Ref. [34] can be written as

$$F(r) = \frac{r}{2}, r \leq r_b$$  \hspace{1cm} \text{Eq. 1}$$

The total force of electromagnetic field on a positron acts as a conservative repulsive force, pointing from the center of bubble to the position of the positron, with a strength proportional to the distance between them.

The form of the repulsive force keeps the trajectory of a positron in the same plane. The Hamiltonian in the polar coordinates of the plane can be expressed as:

$$h = \sqrt{1 + p_r^2 + p_\theta^2} - \frac{r^2}{4}$$  \hspace{1cm} \text{Eq. 2}$$

Assuming the scattering procedure of positrons is non-relativistic, two constants of
motion can be derived, namely the conservation of angular momentum:

\[ rp_\theta = L_0 \quad \text{Eq. 3} \]

and as the Hamiltonian does not depend on time, the conservation of energy:

\[ \sqrt{1 + p_r^2 + p_\theta^2} - \frac{r^2}{4} = h_0 \quad \text{Eq. 4} \]

The equations of motion (EOM) in polar coordinate are

\[
\begin{align*}
\frac{d^2 r}{dt^2} - r \left( \frac{d \theta}{dt} \right)^2 &= F(r) = \frac{r}{2} \\
\frac{r}{dt^2} + 2 \frac{d r}{dt} \frac{d \theta}{dt} &= 0
\end{align*} \quad \text{Eq. 5}
\]

Defining \( u = 1/r \), Eq. 3 can be rewritten as

\[ \frac{d \theta}{dt} = L_0 u^2 \quad \text{Eq. 6} \]

Thus, we have

\[ \frac{d}{dt} = L_0 u^2 \frac{d}{d \theta} \quad \text{Eq. 7} \]

Therefore

\[ \frac{dr}{dt} = \frac{d}{dt} \left( \frac{1}{u} \right) = -\frac{1}{u^2} \frac{du}{dt} = -L_0 \frac{du}{d \theta} \quad \text{Eq. 8} \]

And

\[ \frac{d^2 r}{dt^2} = -L_0^2 u^2 \frac{d^2 u}{d \theta^2} \quad \text{Eq. 9} \]

Substituting Eq. 6 and Eq. 9 into Eq. 5 gives:
If the scattering angle $\theta$ is concerned, a convenient solution can be given as follows. The equations of motion in Cartesian coordinate are:

$$\begin{align*}
\frac{d^2x}{dt^2} &= \frac{x}{2} \\
\frac{d^2y}{dt^2} &= \frac{y}{2}
\end{align*} \quad \text{Eq. 11}$$

For initial conditions, assume $x(t = 0) = x_0; \ y(t = 0) = y_0; \ v_x(t = 0) = v_0$; and $v_y(t = 0) = 0$. The solutions for Eq. 11 can be written as:
\begin{align*}
\begin{cases}
x = \frac{v_0}{\omega} \sinh \omega t + x_0 \cosh \omega t \\
y = y_0 \cosh \omega t \\
\omega = \sqrt{\frac{1}{2}}
\end{cases} & \text{Eq. 12} \\
\text{So that,} \\
\frac{x^2}{x_0^2 \omega^2 - v_0^2} + \frac{y^2}{y_0^2 \omega^2} - \frac{2x_0 xy}{y_0(x_0^2 \omega^2 - v_0^2)} + \frac{v_0^2}{\omega_0(x_0^2 \omega^2 - v_0^2)} = 0 & \text{Eq. 13}
\end{align*}

The trajectory of positrons inside a bubble is a hyperbola and the scattering angle can be determined from Eq. 13.

Fig. 2 (color online) The scattering angle of positrons by the bubble as a function of initial impact parameter $b$ for case $n_0 = 3 \times 10^{18}/\text{cm}^3$ and $\alpha_0 = 2$. Theoretical calculation (blue dotted line) is based on Eq. 13 and simulation results (solid black line) is from PIC simulation. It should be noted that the theoretical scattering angle cuts off at $b = 2\sqrt{2}$ due to the radius of bubble is $2\sqrt{2}$.

In the accelerating stage, consider the spherical bubble excited by a Gaussian laser
\( a^G(t, r) = a_0^G \exp\left(\frac{-r^2}{r_0^2} - \frac{(t-t_0)^2}{t_0^2}\right) \). For paraxial electrons and positrons, the one-dimensional fluid model is used: the scalar potential \( \phi(\xi) \) satisfies the Poisson-like equation:

\[
\frac{\partial^2 \phi}{\partial \xi^2} = k_p^2 v_p^2 \left[ v_p \left(1 - \frac{1+\alpha^G}{\gamma_p^2(1+\gamma^2)}\right)^{-\frac{1}{2}} - 1 \right]
\]  \hspace{1cm} \text{Eq. 14}

\( \gamma_p = (1 - v_p^2/c^2)^{-1/2} \) is the Lorentz factor corresponding to the plasma wave phase velocity, \( k_p = \omega_p/v_p \) is the plasma wave vector. The Hamiltonian can be expressed as

\[
H = \sqrt{1 + p_{\parallel}^2 + p_{\perp}^2 \pm \phi(\xi)}
\]  \hspace{1cm} \text{Eq. 15}

The minus sign is for electrons and plus sign is for positrons, respectively.

Under the canonical transformation \((x, p_{\parallel}) \rightarrow (\xi, p_{\parallel})\), the Hamiltonian takes the form of

\[
H = \sqrt{1 + p_{\parallel}^2 + p_{\perp}^2 \pm \phi(\xi) - v_p p_{\parallel}}
\]  \hspace{1cm} \text{Eq. 16}

This gives several constants of motion. The first is the conservation of transverse canonical momentum.

\[
p_{\perp} \pm a^G = \text{const.}
\]  \hspace{1cm} \text{Eq. 17}

For positrons initially at rest and far from a sufficiently short laser pulse, this is effectively \( p_{\perp} = -a^G \).

The other constant of motion is the energy. So for a positron with an initial energy \( H_0 \), the solution for longitudinal momentum is

\[
p_{\parallel} = v_p \gamma_p (H_0 - \phi) \pm \gamma_p \sqrt{\gamma_p^2 (H_0 + \phi)^2 - p_{\perp}^2} - 1
\]  \hspace{1cm} \text{Eq. 18}

This equation gives the positron trajectory in \((\xi, p_{\parallel})\) phase space similar to Fig. 1 in Ref. [27]. In the first half-cycle of the wakefield, the wakefield electrostatic force and laser ponderomotive force both act as repulsive force, effectively reflecting back positrons coming from the front like a mirror. While the one dimensional model shows the possibility that these positrons could be accelerated to high energy, in higher dimensions due to the lack of a focusing mechanics, the transverse electrostatic force will scatter these positrons transversely. Positrons will leave the bubble area and can't be accelerated further. The trapped orbits in the second and third front half-cycles show the potential of actual acceleration of positrons, as the transverse electrostatic force act as a focusing force.
However, unlike electron self-injection, positrons initially at rest cannot be self-injected into these areas [27], thus an injection scheme is required. An injection scheme for positrons can either change the structure of wakefield during the course to reconnect fluid orbits to trapped orbits, or let positrons enter the trapped orbits from higher dimensions. The proposed “ballistic” injection scheme employs the latter method and let positrons scattered by other bubbles enter transversely. Figure 3 illustrates the configuration of this ballistic injection scheme with a time delay of $\tau=0.62$ as an example.

Fig. 3 (color online) The configuration of this ballistic injection scheme. The blue and green colors are contour surfaces of electron densities of donut and center bubbles, respectively. The red color represents injected positrons. The $x$-$y$ and $x$-$z$ planes are transverse slices of the density distribution and the longitudinal electric field $E_x$. The red curve in the $x$-$y$ plane is the trajectory of an injected positron (corresponding to the red spheres in the 3D model). The leading oscillating colors (amber and grey) denotes the laser
beams in the $x$-$z$ plane. The $y$-$z$ plane is the projection of electron density (blue) and injected positron density (red).

In the ballistic injection scheme, the positrons can gain enough transverse momenta, through scattering by the front of other bubbles or the rear of the laser pulse, to penetrate the bubble and enter the focusing field. In the focusing field, the positrons will experience both longitudinal acceleration and transverse oscillation before they exit the region. The initial injection phase of positrons makes a great difference on the acceleration stage as the dynamics of scattering process is completely different. The delay of the two laser beams can be used as an optimizing tool to “adjust” the injection of positrons favoring in the bubbles driven either by the Gaussian beam or by the ring-shaped beam.

Wide transverse distribution of positrons will lead to the injection of positrons in different bubbles due to different scattering paths, which will result in multi-bunches. Positrons with wide longitudinal distribution, even each of them can experience the same acceleration field in the injected bubble, will lead to the broadening of energy spectra of positrons in each bubble, similar with the case of continuous injection in electron acceleration.

**Discussion**

As discussed previously, there are possibilities for injection and trapping with respect to the original distribution of positrons. The injection rates of positrons are different in different initial positions. The injection rate is different with different $\tau$ even in the same initial position, as the structure of scattering field is different.

The ponderomotive force of driving laser and the space charge force in the front of the first plasma wave period behave as defocusing forces on the positrons. The process of positron injection can be controlled through a delay time $\tau$ between the two laser pulses. The volume of donut bubbles is much larger than the center bubbles geometrically, thus they can capture and hold more positrons in principle. In the case of positive delay $\tau$, which means the center laser is delayed relative to the ring-shaped laser, the center laser can possibly scatter positrons into the second period of the donuts. Thus, majority of the injected positrons are in the donut bubbles. A negative delay $\tau$ enhances the scattering of positrons by donuts into the center bubbles, so both the total number and the ratio of
positrons in the center bubbles are increased. This can be used as an effective tool for a different scenario of acceleration as we will discuss later. Simulations show that the initial transverse phase of injection of the positron bunch has an essential influence on the energy spread while the energy spread is not very affected by the witness positron threshold momentum.

For simplicity, two cases with $\tau = 0.62$ and $\tau = -0.16$ were presented out of a wide range of simulations as they have relatively high injection rates. In the case of $\tau = 0.62$, roughly 0.46% of total positrons are injected in the donut bubbles and 98% of injected positrons can be accelerated to more than 80 MeV. In the case of $\tau = -0.16$, significantly more positrons are injected in the center bubbles. The total injection rate is roughly 0.54% and 37% of injected positrons can be accelerated to higher than 80 MeV.

For a better comparison, simulation results of electron density distribution, accelerating and focusing fields in different time and delays are shown in Fig. 4. There are areas with both positive $E_x$ and negative focusing field gradient near the axis in the front of each bubble from the second period, which are shown as black rectangles in Fig. 4 (a), (d) and (g). The positrons in this area will be focused during acceleration. The focusing field can also possibly trap positrons passing through this area, which is the process of injection. This is an ideal region for positron acceleration in the bubble. The positrons can be accelerated in the front of the 2nd and 3rd of donut and center bubbles with different $\tau$. The positrons can be trapped and accelerated and remain in the bubbles for 7 ps, and then most of them start deceleration. The donuts hold most positrons, but the number of positrons in the 2nd center bubble greatly increases with $\tau = -0.16$. The typical energy spectrum of the accelerated positrons at $t = 7$ ps is given in Fig 5. The maximum energy of accelerated positrons is similar in both kinds of bubbles, but the charges and spectra are different.
Fig. 4 (color online) Simulation results of (a)-(c): electron density distribution $n_e$ (positrons are colored in red), (d)-(f): accelerating field $E_x$ and (g)-(i): focusing field ($E_y - B_z$) in $t = 1$ ps (near the entry of plasma region) with $\tau = 0.62$ ps ((a), (d), (g)) and $t = 7$ ps (near the exit of plasma region) with $\tau = 0.62$ ps ((b), (e), (h)) and $\tau = -0.16$ ps ((c), (f), (i)).
Fig. 5 (color online) Typical energy spectrum of positrons of (a) $\tau = 0.62$ and (b) $\tau = -0.16$ at $t = 7$ ps (corresponds to Fig. 4 (b) and (c), respectively). The injected positrons are counted separately in each center bubble or donut. The positron number injected later than the 3rd period can be neglected compared to previous periods. Only positrons with energy higher than 80 MeV are counted in the spectrum. The total charge of positrons with energy greater than 80 MeV is 2.06 pC for $\tau = 0.62$ ps and 0.89 pC for $\tau = -0.16$ ps.

In the case of $\tau = 0.62$ ps (shown as Fig. 4 (a)), about 99.8% of positrons (2.06 pC in
total) are injected in the donut bubbles, and only 0.2% of positrons are in the center bubbles. The relative energy spread \( \delta = \left( \frac{\Delta E}{E_{\text{peak}}} \right) \), \( \Delta E \) and \( E_{\text{peak}} \) are the energy FWHM (full width at half maximum) and the peak energy of the concerned positron bunch) of the accelerated positrons in the 2\textsuperscript{nd} and 3\textsuperscript{rd} donut bubbles are roughly 6\% and 20\% with a peak energy of about 170 MeV and 120 MeV, respectively. The relative energy spread of the positrons are roughly 5\% and 10\% with a peak energy of about 200 MeV and 150 MeV in the 2\textsuperscript{nd} and 3\textsuperscript{rd} center bubbles, respectively. Even the charges are comparatively low in the center bubbles, the quality of the accelerated positrons is higher in terms of peak energy and relative energy spread.

In the case of \( \tau = -0.16 \) ps (shown as Fig. 4), the trapped and accelerated positrons increased greatly in both center bubbles and donut bubbles. It is worth noting that the number of positrons in the center bubbles is almost 12 times higher as compared with the case of \( \tau = 0.62 \) ps, contributing up to 4.6\% of the total injected positrons. Both of the relative energy spread and the peak energy degrade as compared with the case of \( \tau = 0.62 \) ps.

There may be an operational way to control the accelerated positron beam shape, which will be considered in a following work. The results also suggest that the relative time delay \( \tau \) between the two laser pulses can also be used for different purposes of positron acceleration such as achieving a better relative energy spread or a higher total charge. Interestingly, a kind of ring shaped dual quasi-monoenergetic high-energy positron beams can be achieved in this scenario. As expected, the magnitude of the accelerating field for positrons is close to that of electrons through estimation. More energetic positrons can also be achieved through increasing the distance of acceleration field, which needs optimization of the matching parameters of both laser and plasma densities as those can be done for electrons [9,10,12,22].

In this paper a ballistic injection method for positrons in laser wakefield acceleration is proposed with the requirement of co-axial co-propagating Gaussian and ring-shaped laser pulses. A theoretical model is presented through the description of the dynamical processes experienced by the positrons: scattering, injection and acceleration. The injection method is confirmed through PIC simulations. The simulation shows that a relatively high-charged, quasi-monoenergetic positrons beams (around 200 MeV) can be achieved, the high-energy
collimated positron beams are appropriate for applications and further experiments.

**Methods**

**Approximation and normalization**

Normalization was considered for simplicity, where we use \( t/\omega_p \) instead of \( t \) for time, \( c/\omega_p l \) instead of \( l \) for length, \( \mathbf{A} = \frac{eA}{mc} \) instead of \( \mathbf{A} \) for vector potential, \( \phi = \frac{e\phi}{mc^2} \) instead of \( \phi \) for scalar potential, \( \frac{eE}{mc\omega_p} \) instead of \( \mathbf{E} \) for electric field, \( \frac{eB}{m\omega_p} \) instead of \( \mathbf{B} \) for magnetic field, \( n_e/n_0 \) for plasma density, \( \mathbf{P}/mc \) instead of \( \mathbf{P} \) for momentum, and \( v/c \) instead of \( v \) for velocity, respectively. The Lorentz gauge \( \nabla \cdot \mathbf{A} + \frac{1}{c} \frac{\partial \phi}{\partial t} = 0 \) is used.

For easy understanding, delay \( \tau \) is normalized to \( \lambda_p/c \), where \( \lambda_p \) is the spatial period of a plasma wake, and the value means a relative delay of Gaussian laser with respect to the ring-shaped laser.

The positron density is assumed to be \( n_p \), which is assumed to be much lower than \( n_e \), e.g. \( n_p = 0.01n_e \), to guarantee there are no apparent affection to the plasma field and the models for laser interacting with cold plasma could be applied. The contribution of positrons can also be neglected when analyzing the shape and fields of the plasma bubble.

**Particle-in-cell simulations**

The simulations are conducted by the Particle-in-cell code EPOCH[35]. In this method, collections of physical particles are represented using a smaller number of pseudoparticles, and the fields generated by the motion of these pseudoparticles are calculated using a finite difference time domain technique on an underlying grid of fixed spatial resolution. The forces on the pseudoparticles due to the calculated fields are then used to update the pseudoparticle velocities, and these velocities are then used to update the pseudoparticle positions. This leads to a scheme which can reproduce the full range of classical micro-scale behavior of a collection of charged particles.

**Simulation parameters**

The incident laser propagates along the \( x \)-axis and is linearly polarized in the \( x-y \) plane. Two beams enter the simulation region from the left boundary with spatial and temporal
Gaussian profiles, with \( a_0^G = a_0^R = 2. \) The pulse duration is fixed to 20 fs in FWHM for both beams, and the wavelength is set to 0.8 um. The center laser beam has a focused spot size of 10um (FWHM). The ring-shaped beam has a ring radius \( r_0 = 30\text{um} \) and ring width \( r_d = 10\text{um} \). The plasma region is placed 10um away from the left boundary of the simulation box. The ambient plasma electron density is set to \( n_0 = 3 \times 10^{18}\text{cm}^{-3} \), and the first 100um of plasma region is filled with positrons. The positron density is assumed to be \( n_p = 1 \times 10^{16}\text{cm}^{-3} \) in the simulation, which is already demonstrated experimentally[24]. The initial temperature of positrons is assumed to be 2 MeV which is considered to be higher enough to prevent annihilation with electrons before they can be accelerated.

The so-called “moving window” technique is used in simulation. The simulation box corresponds to a physical volume of 120um * 160um, and is sampled by 20 cells per laser wavelength in the laser propagation direction and 8 cells per wavelength in each transverse direction. It travels in laser direction with a speed of \( v_g \). 2 macro-electrons, 2 macro-protons and 64 macro-positrons are placed in each cell.

**Data availability**

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

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**Author Contributions**

H. Lu proposed the scheme. Z. Xu, C. Xiao, R. Hu and J. Yu conducted the work. Z. Xu, Z. Gong, Y. Shou, C. Chen, and X. Yan developed the basic theory. Z. Xu, C. Xiao and H. Lu write the manuscript. Z. Najmudin, N. Hafz, J. Liu, S. Chen, C. Xie, R. Li, P. Rajeev and D. Neely helped in revision of the manuscript. All authors reviewed the manuscript. Z.
Xu and C. Xiao contributed equally to this work.

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