Justification of Landau Hydrodynamic-Tube Model in Central Relativistic Heavy-Ion Collisions

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Received: date / Revised version: date

Abstract. The statistical event-by-event analysis of inelastic interactions of \(^{16}\text{O}\) and \(^{32}\text{S}\) nuclei in emulsion at 60 \(A\)\text{GeV/c} and 200 \(A\)\text{GeV/c} reveals the existence of groups of high multiplicity events belonging to very central nuclear interactions with Gaussian pseudorapidity distributions for produced particles as suggested by the original hydrodynamic-tube model. Characteristics of these events are presented. The experimental observations are interpreted as a result of quark-gluon plasma formation in the course of central nuclear interactions.

PACS. 25.70.-z, 21.30.Fe Physics and Astronomy

1 Introduction

Interest to the study of relativistic heavy ion collisions is caused by many reasons. After the discovery of the quark-gluon plasma (QGP) at CERN and RHIC [1,2,3,4] a particular attention is paid to different issues related with the QGP formation and evolution in the course of nuclear collisions at very high energies. The data reveal that central collisions of heavy ions at these energies result initially in production of hadronic matter in the form of very hot compressed and a nearly frictionless liquid (QGP) evolution of which produces the final state particles. The data were analyzed in the framework of various theoretical approaches, including different, sometimes very sophisticated, versions of the hydrodynamic model (for recent reviews see e.g. [5,6,7,8]). Considerations based on the hydrodynamic-type models were used not only for discussion of more or less general properties of relativistic heavy ion collisions but also for the analysis of more specific features like properties of hot compressed hadronic matter, comparative yields of hadrons or two- and multiparticle correlations [9,10,11,12] between particles produced.

The shape of inclusive rapidity and pseudorapidity distributions of particles produced in relativistic heavy ion collisions was discussed in many papers (see [6] for a review). It was shown [13] that in a wide range of energies these distributions for symmetrical Au-Au and PbPb collisions may be described reasonably well by a Gaussian distribution which follows from the original hydrodynamic model proposed by L.D.Landau [14,15]. At the same time, it was noticed [6] that below \(\sqrt{s_{NN}} < 10\text{GeV}\) the rapidity gap of the reaction for produced pions is small, any created fireball with longitudinal expansion occupies its entire length with pions, so that the agreement with a Gaussian shape may be fortuitous.

Of course, most of the data on multiparticle production in relativistic heavy ion collisions analyzed so far belong to the inclusive and semi-inclusive reactions. At the same time some important features of the production processes may be revealed more clearly if the experimental data will be analyzed on the event-by-event basis. For example, if we analyze the data in the framework of the hydrodynamic model the position of the central fireball on the longitudinal axis may vary because of geometrical reasons and no clear picture could be revealed for the inclusive distributions. In this paper we analyze on the event-by-event basis the experimental data on the shape of pseudorapidity distributions of relativistic singly charged (shower) particles produced in central collisions of relativistic heavy ions in nuclear emulsion at energies of the CERN SPS and the BNL AGS. More specifically we are looking at the possibility that in central relativistic heavy ion collisions the pseudorapidity distributions of relativistic singly charged particles in individual events follow the Gaussian shape as suggested in the original hydrodynamic model [14,15].

2 Experimental Data

The experimental data of the present paper were accumulated in the framework of the EMU-01 collaboration [6,17,18]. Emulsion stacks were irradiated by \(^{16}\text{O}\) nuclei at 60 \(A\)\text{GeV/c} and 200 \(A\)\text{GeV/c}, by \(^{32}\text{S}\) nuclei at 200 \(A\)\text{GeV/c} at the CERN SPS and by \(^{197}\text{Au}\) nuclei at 11.6 \(A\)\text{GeV/c} at the BNL AGS. In all cases the incident beams were parallel to the surface of emulsion plates, beams densities...
were approximately $5 \cdot 10^3$ nuclei/cm$^2$ with an admixture of foreign ions of no more than 2%.

For the analysis the events of inelastic incoherent interactions of incident nuclei in emulsion were collected, the events of electromagnetic nature were excluded from consideration. In accordance with emulsion technique, the secondary charged particles in events were divided into different groups:

- **black** or $b$-particles, mainly consisting of protons from the target nuclei with momenta $p \leq 0.2$ GeV/c and also heavier nuclear fragments;
- **gray** or $g$-particles corresponding to protons with the momenta $0.2 \leq p \leq 1$ GeV/c; they mainly consist of protons - fragments of the target nuclei, contribution of slow pions does not exceed several percent. Black and gray particles may be combined into a group of strongly ionizing $h$-particles with a multiplicity $N_h = n_b + n_g$;
- **shower** or $s$-particles - singly charged particles with speed $\beta \geq 0.7$ and ionization in emulsion $I < 1.14I_0$, where $I_0$ represents the minimal ionization on tracks of singly charged relativistic particles. Shower particles consist mainly of produced particles (pions) and the singly charged spectator fragments of the projectile nucleus. For the emission angles of the latter ones we use the criterion $\sin \theta_0 \leq \frac{p_0}{m_0}$, where $p_0$ is the initial momentum per nucleon, so that particles with $\theta < \theta_0$ were excluded from particles, whose multiplicity is $n_s$ or simply $n$.
- **projectile fragments** - fast particles with charge $Z \geq 2$ and ionization $I/I_0 \approx 4$, not changing at long distances from the point of interaction in emulsion. These particles are not included into the number of $b$- or $g$-particles.

For all the above types of particles their multiplicity and the emission angles were determined. For the analysis of angular distributions of $s$-particles we use pseudorapidity:

$$\eta = -\ln \tan \frac{\theta}{2} \tag{1}$$

where $\theta$ is the emission angle of the $s$-particle. For pions pseudorapidity is related with true rapidity by a simple equation:

$$\sinh \eta = \frac{m_T}{p_T} \sinh \eta, \quad \text{where} \quad m_T^2 = m^2 + p_T^2. \tag{2}$$

Emulsion has complex composition consisting of groups of light (H,C,N,O) and heavy nuclei (Br, Ag). Therefore selection of events in accordance with the conventional criterion $N_b \geq 8$ corresponds to effective selection of interactions central with respect to heavy target nuclei (Br and Ag).

Some general information on the experimental data together with statistics is presented in Table 1. More information is given in [10-12,15,19].

### 3 The model

The hydrodynamic model of multiparticle production was originally developed for the head on nucleon-nucleon collisions at very high ($> 1 TeV$) energies of a projectile [13]. In the c.m. system colliding nucleons undergo a strong Lorentz contraction. At some instant the whole initial energy is concentrated within a thin disk whose size coincides with the Lorentz contracted nucleon size. The disk is in rest in the c.m. system of colliding nucleons. The hadronic matter within the disk has very high density and high temperature $T \gg \mu c^2$, where $\mu$ is the pion mass, so that following modern concepts it consists of point-like quarks and gluons, rather than usual hadrons. It is the quark-gluon plasma expanding according to the laws of relativistic hydrodynamics of ideal fluid. While expanding, it becomes cooler. When the temperature of hadronic matter reaches $T \approx \mu c^2$, the plasma transforms into hadrons, mostly pions. The pseudorapidity distributions of produced particles in different reference systems follow approximately a Gaussian (normal) shape, but only in the case of a very high multiplicity does the pseudorapidity distribution in an individual event become a meaningful concept.

The model was generalized to the case of nucleon-nucleus collisions [15]. In this case the projectile nucleon can cut out in the nucleus a tube whose cross section is equal to the cross section of the nucleon and interacts only with this part of the target nucleus. The length of a tube may vary in dependence of the geometry of an interaction. In contrast to the case of a nucleon-nucleon collision, an intricate mechanism of compression of nuclear matter treated as a continuous medium comes into play at the first stage of collision with a tube. After compression, the one-dimensional (at the first stage) expansion of nuclear matter (the quark-gluon plasma, in modern terms) proceeds according to the laws of relativistic hydrodynamics of ideal fluid. As in the case of a head-on nucleon-nucleon collision, the pseudorapidity distribution of newly produced particles in a high-multiplicity nucleon-nucleon-tube collision may be approximated by a normal Gaussian distribution.

Hydrodynamic considerations were generalized also to the case of relativistic heavy ion collisions (see, e.g. [9,20,21,22,23]). As we understand, completely self-consistent and comprehensive hydrodynamic description of relativistic-nucleus - nucleus collisions is not yet developed, but ideas of relativistic hydrodynamics are used widely for both interpretation of different, sometimes very intriguing, aspects of the existing experimental data from the LHC and RHIC as well as for prediction of the trends of different experimental observables at these energies. Moreover the hydrodynamic approach to multiparticle production was considerably enriched and developed to incorporate new experimental findings in heavy ion collisions.

We are not discussing these issues. We are dealing with a rather simple old-fashioned hydrodynamic approach considering a hot fireball representing by itself a compressed drop of ideal hadronic fluid whose expansion and cooling leads to emission of final state particles with Gaussian pseudorapidity distribution. The goal of the present paper is to study what the experimental data tell us on this possibility.
Table 1. General characteristics of heavy-ion collisions considered in the present study

| Projectile | \( E_0, \text{ GeV} \) | \( N_{\text{ev}} \) | \( n_s \) | \( n_g \) | \( n_b \) | \( N_h \) |
|------------|------------------|----------------|--------|--------|--------|------|
| \(^{16}\text{O}\) | 60               | 884            | 42.5±1.5 | 5.7±0.4 | 4.5±0.2 | 10.2±0.9 |
| \(^{16}\text{O}\) | 200              | 504            | 58.0±2.8 | 4.3±0.2 | 4.1±0.1 | 8.4±0.4  |
| \(^{32}\text{S}\) | 200              | 884            | 80.3±3.3 | 4.7±0.3 | 3.9±0.2 | 8.6±0.7  |
| \(^{197}\text{Au}\) | 16.7             | 1057           | 80.7±2.5 | 5.9±0.2 | 3.6±0.1 | 9.5±0.3  |

Fig. 1. Pseudorapidity distributions for inclusive and semi-inclusive (\( N_h \geq 8 \)) inelastic interaction of \(^{32}\text{S}\) nuclei at 200 A GeV/c. Curves are Gaussian distributions.

4 Analysis of experimental data

In Figure 1 we present the experimental data on pseudorapidity distributions of relativistic s-particles produced in interactions of \(^{32}\text{S}\) nuclei in emulsion. Separately we show also the data for central collisions with respect to the heavy target nuclei (\( N_h \geq 8 \)). The curves here represent the best fits to the data by Gaussian distributions. We see that in general the experimental data differ in shape from the Gaussian distributions, especially at small and high values of \( \eta \). Even if we assume that in individual events the pseudorapidity distribution follows the Gaussian shape, the inclusive or semi-inclusive distributions, like in Figure 1, may decline from it because of different reasons.

For the analysis of experimental data on the shape of pseudorapidity distributions of relativistic particles in individual events we have applied the statistical approach described in details in \cite{24}. We use the coefficient of skewness \( g_1 \), as a measure of asymmetry, and the coefficient of excess \( g_2 \), as a measure of flattering, which represent parametrically invariant quantities defined as (see Sect.15.8 in \cite{24})

\[
\begin{align*}
g_1 &= m_3m_2^{-3/2}, \\
g_2 &= m_4m_2^{-2} - 3, \\
m_k &= \frac{1}{n} \sum_{i=1}^{n} (\eta_i - \bar{\eta})^k, \\
\bar{\eta} &= \frac{1}{n} \sum_{i=1}^{n} \eta_i
\end{align*}
\]

where \( m_k \) are the central moments of \( \eta \)-distributions and \( n = n_s \) stands here for the multiplicity of \( s \)-particles in an event.

It follows from the mathematical statistics that if quantities \( \eta_1, \eta_2, \ldots, \eta_n \) are independent of one another in events of a subensemble and obey Gaussian distributions, the distribution of these parametrically invariant quantities does not depend on the parameters of the Gaussian distributions, and the number \( n \) of particles in the subensemble event uniquely determines the distribution of
parametrically invariant quantities. In this case the mathematical expectation values and variances of \( g_1 \) and \( g_2 \) are as follows (see eq. (29.3.7) in [21]):

\[
\begin{align*}
\nu_{g_1}(n) &= 0, \quad \sigma_{g_1}^2(n) = 6(n-2)(n+1)^{-1}(n+3)^{-1}, \\
\nu_{g_2}(n) &= -6(n+1)^{-1}, \\
\sigma_{g_2}^2(n) &= 24n(n-2)(n-3)(n+1)^{-2}(n+3)^{-1}(n+5)^{-1}.
\end{align*}
\]

We refer to the model described above, where the pseudorapidities obey a Gaussian distribution, as the \( G \) model.

From the mathematical point of view, our goal is to test the hypothesis that pseudorapidities in the events with different and sufficiently large multiplicity \( n \) are finite representative random samples with the volume \( n \) from the single infinite parent population (see Sect.13.3 in [21]), in which pseudorapidities are distributed according to the Gaussian law. To test this hypothesis, we use the central limit theorem (see Sections 17.1-17.4 in [21]), which asserts that the sum of a large number of independent and equally distributed so-called normalized random variables (see Sect.15.6 in [21]) has a normal distribution in the limit. In mathematical statistics, these normalized quantities are constructed from the random variable and the mathematical expectation and variance obtained from these random variables (see Sect.15.6 in [21]). However, our goal is to test the hypothesis of the normality of pseudorapidity distribution in individual experimental events (that is, in the individual finite samples from an infinite parent population). Therefore, we construct a normalized random variable in a different way, namely: when constructing it for each individual event with a multiplicity of \( n \), we calculate the quantities \( g_1 \) and \( g_2 \) (see eq.(3)), using the experimental values of the event pseudorapidities, and the variances and mathematical expectations are determined by theoretical formulas (4) (see eq. (29.3.7) in [21]) for a quantity with normal distribution.

Thus, if our hypothesis of normality is true (if the \( G \)-model is realized), then by our construction, the normalized quantities \( d_1 \) and \( d_2 \) (see Sect.15.6 in [21])

\[
\begin{align*}
d_1 &= [g_1 - \nu_{g_1}(n)] \sigma_{g_1}^{-1}(n), \\
d_2 &= [g_2 - \nu_{g_2}(n)] \sigma_{g_2}^{-1}(n)
\end{align*}
\]

have dispersions equal to 1 and mathematical expectations equal to 0 both in the subensemble of events (with the fixed number of particles \( n \)) and, consequently, in the ensemble of the events (where \( n \) can take any possible values).

Moreover, if the hypothesis of the normality of the pseudorapidity distribution is true, then, according to the central limit theorem of mathematical statistics, for a sufficiently large number \( N \) of independent random samples (that is, the number of interaction events) the sums of these independent and identically distributed normalized quantities

\[
d_1 \sqrt{N} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} d_{1i}, \quad d_2 \sqrt{N} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} d_{2i}
\]

should be less than 2 with the probability of 95% (see Sections 17.1-17.4 in [21]).

If the hypothesis of normality is true, then the \( G \)-model is quite realistic and for a small \( N \) we can use the asymptotic normality (see Sect.17.4 in [21]) of \( g_1 \) and \( g_2 \) in the subensemble of the events described by the \( G \)-model. Then the normalized quantities \( d_1 \) and \( d_2 \) are equally distributed with parameters 0 and 1 in both the subensemble and in the ensemble of events with the large enough \( n_{\text{min}} \) to make the notion of distribution in individual event meaningful. In this case (for the \( G \)-model) the sums (6) have the same restrictions.

In this paper, the sums (6) were calculated for interaction events with the multiplicity of relativistic shower particles \( n_s \) in the interval from \( n_s^{\text{min}} \) to \( n_s^{\text{max}} \). Calculations were repeated for different intervals \( (n_s^{\text{min}}, n_s^{\text{max}}) \) with fixed \( n_s^{\text{max}} \), whereas the value of \( n_s^{\text{min}} \) was changing from some minimum value of \( n_s \) to the maximum value of \( n_s = n_s^{\text{max}} \), which was defined from the experiment.

The procedures described were applied to all samples of our experimental data.

In Figure 2 we present the experimental data on values of the parameters \( d_1 \sqrt{N} \) and \( d_2 \sqrt{N} \) in dependence on the multiplicity \( n_{\text{min}} \) in interactions of \( ^{107}\text{Au} \) nuclei in emulsion at 11.6 \( A \) GeV/c. The shaded area is the area where \( |d_1 \sqrt{N}| \) & \( |d_2 \sqrt{N}| < 2 \).
Fig. 3. Dependence of parameters $d_1\sqrt{N}$ and $d_2\sqrt{N}$ on $n_{\text{min}}$ in interactions of oxygen nuclei in emulsion at 60 $A\text{GeV}/c$ and 200 $A\text{GeV}/c$ and in interactions of sulfur nuclei at 200 $A\text{GeV}/c$. The shaded area is the area where $|d_1\sqrt{N}|$ & $|d_2\sqrt{N}| < 2$.

Table 2. The numbers and characteristics of events with the Gaussian pseudorapidity distributions

| Projectile | $E_0$, GeV | $N_{\text{ev}}$ | $n_s$ | $n_g$ | $n_b$ | $N_h$ |
|------------|------------|----------------|------|------|------|------|
| $^{16}\text{O}$ | 60 | 4 | 185.8 | 21.8 | 8.5 | 30.3 |
| $^{16}\text{O}$ | 200 | 9 | 247.0 | 18.8 | 8.8 | 27.6 |
| $^{32}\text{S}$ | 200 | 6 | 440.0 | 15.0 | 5.2 | 20.2 |

have also observed that the overwhelming majority of inelastic incoherent events have pseudorapidity distributions of relativistic particles which do not follow the Gaussian shape. Only in very small groups of very high multiplicity events we have found that both $d_1\sqrt{N}$ and $d_2\sqrt{N}$ have values which are less than 2 in their absolute magnitudes and for these very events pseudorapidity distributions of $s$-particles represent by themselves statistical samplings from the Gaussian distributions. This fact is illustrated in Figure 3. Obviously pseudorapidity distributions in these events obey the Gaussian shape.

The experimental data on the numbers of events in which pseudorapidity distributions of $s$-particles are Gaussian distributions together with average multiplicities in them are shown in Table 2 for relativistic heavy-ion collisions analyzed in this paper. In Table 3 we show the average values and dispersions of pseudorapidity distributions in these 9 and 6 individual events found in interactions of $^{16}\text{O}$ and $^{32}\text{S}$ nuclei in emulsion at 200 $A\text{GeV}/c$. We see that average multiplicities of produced $s$-particles in these events are extremely high, exceeding (4-5) times average multiplicities in considered heavy ion collisions, so we are dealing with the central heavy ion collisions. Comparing multiplicities of $h$-particles from Tables 1 and 2 we see that these events belong to interactions of incident ions with heavy emulsion nuclei.

We see from the data of Table 3 that the centers and dispersions of pseudorapidity distributions in events selected by our analysis do not fluctuate much, so that in Figure 4 we show pseudorapidity distributions for all these events. Gaussian distributions describe them well. In order to estimate the energy density for these events we can use approach suggested by D.Bjorken [25]. Of course, the exact values depend on some parameters, in particular on the radius of a volume, where the QCD transition could take a place. Having in mind our experimental data on $\frac{dN}{d\eta}$ from Figure 4, our estimates may vary in the limits from 1 GeV/fm$^3$ to 16 GeV/fm$^3$, if the radius of the volume changes from 4 to 1 fm.

We have verified empirically the results of application of the statistical approach based on parametrically invariant quantities $g_1$ and $g_2$ to the conditions of our experiments. In order to do so we have used ensembles of Monte Carlo events generated following the phenomenological model of independent emission of $s$-particles (IEM) [26,27]. In the framework of this model we assume that: (i) multiplicity ($n_s$) distributions of simulated events reproduce the experimental distributions for the interactions considered; (ii) one-particle pseudorapidity distributions
of s-particles in each one of simulated subensembles of events (within, for instance, the fixed range of \( n_s \)) reproduce the experimental distribution for the same range of \( n_s \); (iii) emission angles of s-particles in each one of simulated events are statistically independent.

As an example in Figure 5 we show the values of parameters \( d_1 \sqrt{N} \) and \( d_2 \sqrt{N} \) in dependence on the multiplicity \( n_{\text{min}} \) in Monte Carlo events generated in accordance with multiplicity and pseudorapidity distributions of s-particles for interactions of \( ^{32}\text{S} \) nuclei in emulsion at 200 \( A \) GeV/c. We see that even at much higher statistics (5000 events) than the experimental ones Monte Carlo events do not reveal existence of any group of events with the Gaussian shape of pseudorapidity distributions. We conclude from this figure that the probability of accidental formation of the Gaussian pseudorapidity distributions in groups of individual events, not recognizable by the present statistical approach, is negligibly small for the conditions of our experimental data.

### 5 Conclusions

In the present paper we have analyzed on the event-by-event basis the shape of pseudorapidity distributions of produced particles in relativistic heavy ion collisions at CERN SPS and BNL AGS energies. The goal was to search for events in which pseudorapidity distributions of produced s-particles are Gaussian distributions. We have used for this purpose the statistical method of parametrically invariant quantities [24]. Utilizing this approach to the experimental data on interactions of \( ^{16}\text{O} \) nuclei at 60 \( A \) GeV/c and 200 \( A \) GeV/c and interactions of \( ^{32}\text{S} \) nuclei at 200 \( A \) GeV/c in emulsion we have discovered the

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**Table 3. Characteristics of pseudorapidity distributions in events selected by the analysis in interactions of \( ^{16}\text{O} \) and \( ^{32}\text{S} \) nuclei in emulsion at 200 \( A \) GeV/c**

| No | \( n_s \) | \( \langle \eta \rangle \) | \( \sigma (\eta) \) | \( d_1 \sqrt{N} \) | \( d_2 \sqrt{N} \) |
|----|----------|----------------|---------------|----------------|----------------|
| 1  | 232      | 2.91±0.09      | 1.41          | 0.68           | -0.80          |
| 2  | 245      | 2.98±0.09      | 1.44          | 0.55           | -0.10          |
| 3  | 225      | 2.79±0.09      | 1.37          | -0.68          | 3.72           |
| 4  | 249      | 3.02±0.09      | 1.36          | 1.36           | -1.84          |
| 5  | 232      | 2.97±0.10      | 1.48          | 0.27           | -1.19          |
| 6  | 275      | 3.02±0.07      | 1.17          | -1.15          | 0.10           |
| 7  | 261      | 3.10±0.08      | 1.35          | 0.91           | 0.09           |
| 8  | 268      | 2.74±0.08      | 1.39          | 1.20           | 0.16           |
| 9  | 236      | 2.82±0.09      | 1.35          | -0.06          | -1.14          |

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**Fig. 4.** Pseudorapidity distributions in selected events for interactions of \( ^{16}\text{O} \) and \( ^{32}\text{S} \) nuclei at 200 \( A \) GeV/c. Curves are Gaussian distributions.

**Fig. 5.** Dependence of parameters \( \overline{d_1 \sqrt{N}} \) and \( \overline{d_2 \sqrt{N}} \) on \( n_{\text{min}} \) in Monte Carlo events generated for interactions of sulfur nuclei in emulsion at 200 \( A \) GeV/c. The shaded area is the area where \( |\overline{d_1 \sqrt{N}}| \leq |\overline{d_2 \sqrt{N}}| < 2 \).
existence of small groups of incoherent inelastic events with the Gaussian pseudorapidity distributions of produced particles. At the same time no events of this type were observed in interactions of gold $^{107}$Au nuclei at lower incident energy at 11.6 A GeV/c. Also we have found from results of Monte Carlo simulations that the probability of accidental formation of the Gaussian pseudorapidity distributions in high-multiplicity individual events is negligibly small for our experimental conditions.

The experimental data show that the multiplicity of produced particles in these events is much higher than the average multiplicity in corresponding interactions and belongs to the high end of multiplicity distribution, i.e. to the central heavy-ion collisions. For these events the probability of formation is small enough (at the level of 1% or less of the total statistics of inelastic events) and most probably increases with the energy of an interaction.

The original hydrodynamic model is, to our best knowledge, the only model suggesting some certain shape for pseudorapidity distributions of produced particles - the Gaussian distribution. Simplicity of the model probably is one of the reasons why the original hydrodynamic model is considered to be a wildly extremal proposal \[\text{[6].}\] One can note, of course, that the model was introduced to describe only few general characteristics of multiparticle production processes, pseudorapidity distribution of charged particles being one of the simplest characteristics of the production process. So, the scope of the model was rather narrow. To describe more complex features and characteristics of the production process, like observed in modern experiments, it is necessary to go beyond and to exploit more advanced versions of the hydrodynamic model. From this point of view modern versions probably are more plausible but less certain in predictions.

The experimental observations of the present paper encourage us to interpret the existence of events with the Gaussian pseudorapidity distributions of produced particles in central relativistic heavy-ion collisions as a result of formation in the course of an interaction of a droplet of hadronic matter - the quark-gluon plasma, i.e. the primordial high density state, whose expansion and cooling leads to its decay with production of final state particles. This interpretation may be supported by following considerations.

Calculations in the framework of lattice QCD show \[\text{[23, 29].}\] that at the energy densities exceeding a critical value of about 1 to 1.5 GeV per fm$^3$ achievable at incident energies of about $\sqrt{s_{NN}} \gtrsim 5$ GeV, the hadronic phase of matter disappears, giving rise to the primordial high density state (QGP) whose evolution is governed by the elementary interactions of quarks and gluons. One can note that top SPS energies exceed by far these incident energies and realization of such a phase transition in heavy-ion collisions at these energies was confirmed experimentally by many characteristic signals (see, e.g. [31]).

Of course, realization of the phase transition cannot be taken for granted in all relativistic heavy ion collisions at these energies, even central ones. It is a rather rare and random phenomenon and at the energies considered fluctuations in the energy density could play an important role so that the produced primordial QCD objects may vary in some initial characteristics, in the volume, for example. Therefore it was recommended at the beginning of the QGP age to search for these objects experimentally on the event-by-event basis \[\text{[31].}\] Evolution of these objects is probably reflected by the original hydrodynamic model and may lead to the Gaussian distributions of final state particles in pseudorapidity. Therefore we believe that it is important to study and to confirm this possibility in other experiments as well.

We are grateful to all members of the EMU-01 collaboration, and especially to professor Ingvar Otterlund, for the joy to work together and for the excellent quality of the data.

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