Are Gamma-Ray Bursts a Standard Energy Reservoir?

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Abstract. One of the most important discoveries in the observation of gamma-ray bursts (GRBs) is that the total energy emitted by a GRB in $\gamma$-rays has a very narrow distribution around $10^{51}$ erg, which has led people to claim that GRBs are standard energy explosions. As people made the claim they have ignored the selection biases which must be important since GRB observations are strongly fluence or flux-limited. In this paper we show that, when the selection effects are considered, the intrinsic distribution of the GRB energy can be very broad. The number of faint GRBs has been significantly underestimated because of the fluence or flux limit. The bright part of the distribution has been affected by another important selection effect arising from the beaming of GRB jets, which is instrument-independent and caused by the fact that brighter GRBs tend to have smaller jet angles and hence smaller probabilities to be detected. Our finding indicates that GRBs are not a standard energy reservoir, and challenges the proposal that GRBs can be used as standard candles to probe cosmology.

Key words: cosmology: theory – gamma-rays: bursts – gamma-rays: observations

1 Introduction

A characteristic observed feature of cosmological gamma-ray bursts (GRBs) is that they emitted a huge amount of energy in $\gamma$-rays in a very short time and their isotropic-equivalent $\gamma$-ray energy (i.e., the total $\gamma$-ray energy emitted by a GRB if the GRB radiates isotropically) spans a very large range—more than five orders of magnitude. The measured isotropic-equivalent energy of GRBs, $E_{iso}$, appears to have a log-normal distribution with a mean $\sim 10^{53}$ erg, and a dispersion $\sim 0.9$ in $\log E_{iso}$ (Amati 2006, 2007; Li 2007). However, there is evidence that GRBs are beamed (Harrison et al. 1999; Kulkarni et al. 1999; Stanek et al. 1999). Assuming that a GRB radiates its energy into two oppositely directed jets, each having a half-opening angle $\theta_{jet}$. The total solid angle spanned by the jets is then $4\pi\omega$, where $\omega \equiv 1 - \cos \theta_{jet} < 1$. If the emission of a jet is distributed more or less uniformly on its cross-section, the total $\gamma$-ray energy emitted by the GRB is approximately $E_{\gamma} = \omega E_{iso}$, smaller than $E_{iso}$ by a beaming factor $\omega$.

One of the most important discoveries in GRB observations has been that the value of $E_{\gamma}$ has a very narrow distribution with a mean $\sim 10^{51}$ erg comparable to ordinary supernovae, which has led people to claim that GRBs are a standard energy reservoir involving an approximately constant explosion energy (Frail et al. 2001; Piran et al. 2001; Berger, Kulkarni & Frail 2003; Bloom, Frail & Kulkarni 2003; Friedman & Bloom 2005). Theoretical models for interpreting the clustering GRB energy have also been proposed (see, e.g., Zhang & Mészáros 2002). It is well-known that observations of GRBs are strongly fluence or flux-limited, hence GRB samples seriously suffer from Malmquist-type selection biases (Malmquist 1920; Teerikorpi 1997). That is, an observer will see an increase in the averaged luminosity or the total energy of GRBs with the distance, caused by the fact that less luminous or sub-energetic bursts at large distances will not be detected. Although this Malmquist bias for a flux-limited sample of astronomical objects looks obvious, sometimes people made serious mistakes in interpreting data by neglecting it. For instance,
with a study of nearby galaxies it had been incorrectly claimed that the Hubble constant increases with the distance (de Vaucouleurs 1972; Teerikorpi 1975; Sandage 1994).

Unfortunately, as people drew the conclusion on the distribution of the GRB energy and claimed that GRBs are standard energy explosions, they have treated the observed distribution as the intrinsic distribution and have neglected the selection biases that are very important for GRBs at cosmological distances. As a result, the number of faint GRBs has been significantly underestimated, since a GRB will not be detected if its flux or fluence falls below the detection limit. The bright part of the GRB energy distribution suffers from another important selection bias, which arises from the fact that brighter GRBs tend to have smaller jet opening angles and hence smaller probabilities to be detected. This beaming bias is independent of instruments and thus cannot be reduced by improving the sensitivity of detectors.

The aim of this paper is to show that the influence of the selection biases from the fluence limit and the jet beaming is strong enough that the observed distribution of the GRB energy does not represent the intrinsic distribution at all. We present a simple model that explains nicely the observed distribution of the GRB energy, yet the burst energy reservoir in the model is not standard. Hence, the collimation-corrected energy of GRBs can have a very broad intrinsic distribution despite the fact that it is observed to cluster to a narrow distribution. Our results lead to the suggestion that GRBs are not a standard energy reservoir, contrary to the previous claim.

2 Intrinsic versus Observed Energy Functions of GRBs

To estimate the influence of selection biases on the observed distribution of the GRB energy, we assume that a GRB will be detected if one of its jets points toward the observer, and its fluence exceeds the limit \( F_{\text{bol,lim}} = 1.2 \times 10^{-6} \text{ erg cm}^{-2} \). Although this is an over-simplified approximation for the selection effect for GRBs, we will see that this simple selection effect already affects the observed distribution of the GRB energy strongly enough. In addition, it appears that the above fluence limit can reasonably represent the selection effect for GRBs with measured peak spectral energy and isotropic-equivalent energy (Li 2007).

The limit in fluence corresponds to a lower limit in the isotropic-equivalent energy of a detectable GRB at redshift \( z : E_{\text{iso,lim}} = 4\pi D_{\text{com}}^2(1 + z)F_{\text{bol,lim}} \), where \( D_{\text{com}} \) is the comoving distance to the burst. Here we assume a cosmology with \( \Omega_m = 0.3, \Omega_\Lambda = 0.7 \), and a Hubble constant \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \).

For the GRB rate, as people often do we adopt the simplest assumption that GRBs follow the cosmic star formation history (Totani 1997; Natarajan et al. 2005). Then, up to a normalization factor, the intrinsic distribution of GRB redshifts is given by

\[
 f(z) = \frac{\Sigma_{\text{SFR}}(z)}{1 + z} \frac{dV_{\text{com}}}{dz},
\]

where \( \Sigma_{\text{SFR}}(z) \) is the comoving star formation rate, and \( V_{\text{com}} \) is the comoving volume.

We adopt a star formation rate (Hopkins & Beacom 2006; Le & Dermer 2007)

\[
 \Sigma_{\text{SFR}}(z) = \frac{1 + az}{1 + (z/b)^c}.
\]

The parameters \( a, b \) and \( c \) are not well constrained. However, a model with \( a = 8, b = 3 \) and \( c = 1.3 \) fits the observed distribution of GRB redshifts reasonably well (Le & Dermer 2007). Hence, we fix \( a, b \) and \( c \) to these values.

For simplicity, we assume that except the number density, the property of GRBs does not evolve with the cosmic redshift, although this might not be true in reality (Li 2007). Then, the intrinsic
distribution function of $z$, $E_{\text{iso}}$, and $y \equiv \log \left( \tan \theta_{\text{jet}} \right)$ must have a form

$$P(z, E_{\text{iso}}, y) = f(z) \phi_{\text{iso}} (E_{\text{iso}}) \psi (E_{\text{iso}}, y),$$  \hspace{2cm} (3)$$

where we assume that $\psi (E_{\text{iso}}, y)$ is normalized with respect to $y$: $\int_{y=0}^{\infty} \psi (E_{\text{iso}}, y) \, dy = 1$. We choose $E_{\text{iso}}$ rather than $E_\gamma$ as an independent variable, since in practice $E_{\text{iso}}$ is easier to measure than $E_\gamma$ although the latter might be more fundamental.

For a GRB with a beaming factor $\omega$, the probability for it to be detected by an observer is $\omega$, without consideration of the fluence limit. The observed distribution of $E_{\text{iso}}$ is then

$$\hat{\phi}_{\text{iso}} (E_{\text{iso}}) = \phi_{\text{iso}} (E_{\text{iso}}) \langle \omega \rangle (E_{\text{iso}}) \Xi (E_{\text{iso}}),$$ \hspace{2cm} (4)$$

where

$$\langle \omega \rangle (E_{\text{iso}}) \equiv \int_{y=0}^{\infty} \omega \psi (E_{\text{iso}}, y) \, dy$$ \hspace{2cm} (5)$$

is the averaged beaming factor, and the function

$$\Xi (E_{\text{iso}}) \equiv \int_{0}^{\infty} f(z) \, dz$$ \hspace{2cm} (6)$$

reflects the selection effect from the fluence limit.

In equation (6), for a given $E_{\text{iso}}$, the value of $z_{\text{limit}} = z_{\text{limit}} (E_{\text{iso}})$ is solved from the equation $E_{\text{iso}} = E_{\text{iso,limit}}$, or just given by the maximum redshift of GRBs if $E_{\text{iso}} > E_{\text{iso,limit}}$ at $z = z_{\text{max}}$ (assuming that the distribution of GRB redshifts is cut off at $z = z_{\text{max}}$).

The intrinsic distribution function of the collimation-corrected energy $E_\gamma = \omega E_{\text{iso}}$, derived from the distribution in equation (5), is

$$\phi_\gamma (E_\gamma) = \int_{0}^{\infty} \omega^{-1} \phi_{\text{iso}} \left( \omega^{-1} E_\gamma \right) \psi \left( \omega^{-1} E_\gamma, y \right) \, dy.$$ \hspace{2cm} (7)$$

The observed distribution of $E_\gamma$ is then

$$\hat{\phi}_\gamma (E_\gamma) = \int_{0}^{\infty} \phi_{\text{iso}} \left( \omega^{-1} E_\gamma \right) \psi \left( \omega^{-1} E_\gamma, y \right) \Xi \left( \omega^{-1} E_\gamma \right) \, dy.$$ \hspace{2cm} (8)$$

It is observed that the jet opening angle of GRBs is anti-correlated to the isotropic-equivalent energy [Frail et al. 2001; Bloom et al. 2003; Friedman & Bloom 2005]. We define $x \equiv \log E_{\text{iso}}$ and assume that $\psi(x, y)$ has a Gaussian form

$$\psi(x, y) = \frac{1}{\sqrt{2\pi}\sigma_y} \exp \left[ -\frac{(y - mx - p)^2}{2\sigma_y^2} \right],$$ \hspace{2cm} (9)$$

where $m$, $p$, and $\sigma_y$ are constants.

For a given $x$, the normalized observed distribution of $y$ is

$$\hat{\psi}(y) = \int_{y=0}^{\infty} \omega \psi(x, y) \, dy = \frac{\omega(y) \psi(x, y)}{\langle \omega \rangle (x)}.$$ \hspace{2cm} (10)$$

Because of normalization, the selection effect from the fluence limit [the function $\Xi (E_{\text{iso}})$ defined by eq. (5)] is canceled out in equation (10) so it does not influence the observed distribution of the jet opening angle for a given $E_{\text{iso}}$. However, the selection effect from beaming [i.e., the function $\omega(y)$] is retained.
Figure 1: The observed distribution of the GRB energy is shaped by the fluence limit of the detector and the distribution of the jet opening angle. The solid curve shows the fluence-selection function defined by equation (6) normalized by $\Xi_0 \equiv \int_0^{z_{\text{max}}} f(z)dz$, where we have set $z_{\text{max}} = 10$. The dashed curve shows the averaged jet beaming factor, defined by equation (5). The detection criteria are defined as follows: a GRB is detected if (1) one of its jets points toward the observer; and (2) its observed fluence exceeds the limit $F_{\text{bol,lim}} = 1.2 \times 10^{-6}$ erg cm$^{-2}$. The fluence-selection function leads to a quasi-exponential cut-off to the GRB energy function at the low-energy end. The jet beaming factor reduces the number of detected GRBs of high energy, caused by the fact that brighter GRBs tend to have smaller jet angles.

A maximum-likelihood fit of equation (10) [with $\psi(x,y)$ given by eq. 9] to the 23 GRBs with available $E_{\text{iso}}$ and $\theta_{\text{jet}}$ (Friedman & Bloom 2005) leads to $m = -0.216$, $p = -0.825$ and $\sigma_y = 0.148$ ($E_{\text{iso}}$ in 10$^{52}$ erg).

In Fig. 1 we show the fluence-selection function defined by equation (6) (with $z_{\text{max}} = 10$) and the averaged $\omega$ defined by equation (5). The fluence-selection effect affects the observed distribution of faint GRBs, while the beaming effect affects the observed distribution of bright GRBs dramatically. For a given intrinsic distribution of the GRB energy, the combination of these two effects determines the shape of the distribution observed by an observer (if other selection effects are neglected).

Finally, we assume that the intrinsic function of the isotropic-equivalent energy of GRBs is a power law with an exponential cut-off

$$\phi_{\text{iso}} (E_{\text{iso}}) = E_{\text{iso}}^{\alpha} \exp \left( -E_{\text{iso}} / E_\ast \right) ,$$

where $\alpha$ and $E_\ast$ are constant parameters to be determined from observational data.
Figure 2: The dotted line histogram is the observed distribution of $E_{\text{iso}}$ for 48 long-duration GRBs, with the number of GRBs in each bin indicated by a dark point with Poisson error bars. The dashed curve shows the intrinsic distribution of log $E_{\text{iso}}$ defined by a power law with an exponential cut-off in equation (11), with $\alpha = -0.733$, and $E_\star = 3.21 \times 10^{54}$ erg. The solid curve is the observed distribution of log $E_{\text{iso}}$ derived from equation (4), which well fits the observed data, with $\chi^2_r = 0.49$. The fluence limit of the detector is $1.2 \times 10^{-6}$ erg cm$^{-2}$. The maximum GRB redshift is set to be $z_{\text{max}} = 6$, in accordance with the redshift distribution of the 48 GRBs. The intrinsic distribution is normalized so that the area under the dashed curve is the same as that under the solid curve.

3 Results

Fitting the observed distribution of $E_{\text{iso}}$ for 48 long-duration GRBs (Amati 2006, 2007; Li 2007) by the function in equation (4), we get $\alpha = -0.733$ and $E_\star = 3.21 \times 10^{54}$ erg (Fig. 2). The reduced chi-square of the fit is $\chi^2_r = 0.49$, indicating a very good fit. This result clearly shows the fact that the intrinsic distribution of GRB energy is very different from the observed distribution, because of the strong selection effects from beaming and fluence-limit. The observed distribution has a Gaussian shape, but the intrinsic distribution is consistent with a power-law with an exponential cut-off \cite{Amati2006, Amati2007, Li2007}. The derived intrinsic distribution indicates the existence of a large amount of faint GRBs, which have not been detected because of the fluence-limit. In addition, the intrinsic distribution $E_{\text{iso}} \phi(E_{\text{iso}})$ peaks at $E_{\text{iso}} \sim 10^{54}$ erg, an order of magnitude larger than the value $\sim 10^{53}$ erg directly inferred from the observed distribution.

With $\alpha$ and $E_\star$ fixed at the above values, we then fit the observed distribution of $E_{\gamma}$ for 23 GRBs

\footnote{We tried to fit the observed distribution of $E_{\text{iso}}$ with a Gaussian intrinsic distribution of log $E_{\text{iso}}$, but we obtained an unrealistically large mean of $E_{\text{iso}} \sim 10^{56}$ erg.}
Figure 3: Distribution of the collimation-corrected energy of GRBs, $E_\gamma$, derived from the intrinsic distribution function of the isotropic-equivalent energy in equation (11), with $\alpha = -0.733$ and $E_* = 3.21 \times 10^{54}$ erg (the dashed curve in Fig. 2). The solid curve is the observed distribution of $\log E_\gamma$ (eq. 8). The dashed curve is the intrinsic distribution of $\log E_\gamma$ (eq. 7). The dotted line histogram is the distribution of the measured $E_\gamma$ for 23 long-duration GRBs, with the number of GRBs in each bin indicated by a dark point with Poisson error bars. By varying the normalization, the derived distribution (the solid curve) fits the observation perfectly, with $\chi^2_r = 0.26$. The intrinsic distribution is normalized so that the area under the dashed curve is the same as that under the solid curve.

(Friedman & Bloom 2005) by equation (6), varying only the normalization. The best fit is shown in Fig. 3 with $\chi^2_r = 0.26$. Although this is not an independent fit given the fact that $E_{iso}$ and $\theta_{jet}$ are anti-correlated which has been adopted by our model, the goodness of the fit is still impressive, confirming that the relation between $E_{iso}$ and $\theta_{jet}$ assumed in equation (9) is a good approximation.

Fig. 3 shows the dramatic difference between the intrinsic distribution and the observed distribution of $\log E_\gamma$. The observed distribution (the solid curve) has an exponential decay toward the faint burst end, but the intrinsic distribution (the dashed curve) decays toward the faint end by a power law: $E_\gamma \phi_\gamma (E_\gamma) \propto E_\gamma^{0.45}$ [slightly faster than the decay of the intrinsic distribution of $\log E_{iso}$, $E_{iso} \phi_{iso} (E_{iso}) \propto E_{iso}^{0.27}$]. The existence of a large amount of undetected faint GRBs broadens the intrinsic distribution of $\log E_\gamma$ significantly.

Here, by ‘faint GRBs’ we refer to those bursts with an $E_{iso}$ or $E_\gamma$ that is smaller than the $E_{iso}$ or $E_\gamma$ at the maximum of the distribution of $\log E_{iso}$ and $\log E_\gamma$. Although the detection of highly sub-luminous and sub-energetic nearby GRBs 980425, 031203 and 060218 has led people to propose that there exists a unique population of faint GRBs (Cobb et al. 2006; Pian et al. 2006; Soderberg et al. 2006; Guetta & Della Valle 2007; Liang et al. 2007), the results in this paper do not rely on the...
Figure 4: This figure shows how the observed distribution of GRB energy sensitively depends on the selection biases from the detector fluence limit and the beaming of GRB jets. Upper panel: The intrinsic (solid curve) and the observed (dashed and dotted curves) distribution function of the GRB isotropic-equivalent energy, \( E_{\text{iso}} \). The solid curve is a plot of \( \Phi(E_{\text{iso}}) \), defined by equation (11) with \( \alpha = -0.733 \) and \( E_* = 3.21 \times 10^{54} \) erg. The dashed curves are plots of \( \Phi(E_{\text{iso}}) \), calculated by equation (4) with \( z_{\text{max}} = 10 \) and \( F_{\text{bol}, \lim} = 10^{-6}, 10^{-7} \) and \( 10^{-8} \) erg cm\(^{-2}\) respectively (from right to left, counted by the peak). The dotted curve shows \( \Phi(E_{\text{iso}}) \) in the limiting case of \( F_{\text{bol}, \lim} = 0 \) (i.e., GRBs can be detected down to any small value of fluence). All distributions are normalized so that the integral over \( E_{\text{iso}} \) is unity. Lower panel: Similar to the upper panel but for the intrinsic (solid curve) and observed (dashed and dotted curves) distribution of the collimation-corrected GRB energy, \( E_{\gamma} = \omega E_{\text{iso}} \), calculated by equations (7) and (8) with different values of \( F_{\text{bol}, \lim} \) as in the upper panel. All distributions are normalized so that the integral over \( E_{\gamma} \) is unity.
existence of this unique population of faint GRBs. The extension of the derived energy function of normal GRBs to the low-energy end already significantly broadens the GRB energy function.

The effect of the selection biases is more clearly illustrated in Fig. 3 which shows the dependence of the shape of $E_{\text{iso}} \phi_{\gamma} (E_{\text{iso}})$ (upper panel) and $E_{\gamma} \phi_{\gamma} (E_{\gamma})$ (lower panel) on the beaming effect, and how the shape changes with the fluence limit of the detector. If the fluence limit decreases by a factor 10 from $10^{-6}$ erg cm$^{-2}$ (or from $10^{-7}$ erg cm$^{-2}$), the width of $E_{\text{iso}} \phi_{\gamma} (E_{\text{iso}})$ and $E_{\gamma} \phi_{\gamma} (E_{\gamma})$ increases by $\sim 0.8$ in log $E_{\text{iso}}$ and $\sim 0.5$ in log $E_{\gamma}$, respectively. Correspondingly, the value of $E_{\text{iso}}$ at the maximum of $E_{\text{iso}} \phi_{\gamma} (E_{\text{iso}})$ decreases by a factor $\sim 10^{0.8}$, and the value of $E_{\gamma}$ at the maximum of $E_{\gamma} \phi_{\gamma} (E_{\gamma})$ decreases by a factor $\sim 10^{0.5}$.

If we had an ideal detector that can detect an arbitrarily faint GRB, we would see a distribution of energy given by the dotted lines in Fig. 3 which is dramatically different from the intrinsic distribution. For example, the observed distribution of $E_{\gamma} (E_{\text{iso}})$ would peak at $\sim 10^{48}$ erg, rather than $\sim 10^{51}$ erg ($\sim 10^{44}$ erg) as indicated by the intrinsic distribution.

4 Conclusions and Discussion

The observation that the total energy emitted in $\gamma$-rays by long-duration GRBs clusters around $10^{51}$ erg (Frail et al. [2001]), which has been considered as the most intriguing finding in GRB research (Zhang & Meszéros [2004]), is only a superficial result since the strong selection biases from the detector selection effect and the beaming of GRBs have been ignored. The previous claim that the energy output of the central engine of long-duration GRBs has a universal value (Frail et al. [2001]; Piran et al. [2001]), which was derived from the above superficial result, is likely to be incorrect since the observed narrow distribution of $E_{\gamma}$ is consistent with a broad intrinsic distribution of $E_{\gamma}$.

In fact, our results show that for both log $E_{\text{iso}}$ (Fig. 2) and log $E_{\gamma}$ (Fig. 3), the distribution on the left-hand side to the maximum is well modeled by the cut-off from the fluence limit (the solid curve in Fig. 1). It would be surprising that the intrinsic distribution happens to have a low-energy cut-off that is coincident with the fluence limit cut-off.

The influence of the flux or fluence limit of detectors on the observation of GRBs is well-known and has been taken into account either thoroughly or partly in many GRB works, e.g. in deriving the luminosity function of GRBs (Schmidt 1999, 2001; Firmani et al. 2004; Guetta, Piran & Waxman 2005; Liang et al. 2007). However, the influence has sometimes been ignored or seriously underestimated. The claim that the collimation-corrected energy of GRBs has a narrow distribution and hence GRBs are a standard energy reservoir is an example where the selection effects have been ignored and wrong physical conclusions have been drawn.

Although it is generally conceived that the jet opening angle is anti-correlated to the GRB energy, in the study on the luminosity function of GRBs the effect of jet beaming has often not been properly taken into account. For example, in Guetta et al. [2005] and Liang et al. [2007], an isotropic luminosity function was derived by comparing the model prediction with the observed flux or the luminosity distribution without a consideration of beaming, then the derived luminosity function was used to calculate a weighted and averaged beaming factor. As we can see from equation (4), the isotropic luminosity function derived by them should be the product of the intrinsic isotropic luminosity function and the averaged beaming factor as a function luminosity, not the intrinsic isotropic luminosity itself.

Our results also challenge the proposal that GRBs can be used as standard candles to probe cosmology (Bloom et al. 2003; Friedman & Bloom 2005; Schaefer 2007 and references therein). Although the constancy of the GRB energy is not a necessary condition for GRBs to be standard candles because of the identification of several good correlations among GRB observables, the existence of a large amount of faint bursts that have not been observed might significantly increase the
scatter in those correlations. In addition, the recent work of Butler et al. (2007) indicates that some of those relations arise from partial correlation with the detector threshold and hence are unrelated to the physical properties of GRBs.

Finally, a prediction of this work that can be tested with future observations is that as the sensitivity of GRB detectors increases the observed distribution of $E_\gamma$ broadens towards the low-energy end.

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