Characterization of dynamical regimes and entanglement sudden death in a microcavity quantum dot system

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Abstract
The relation between the dynamical regimes (weak and strong coupling) and entanglement for a dissipative quantum dot microcavity system is studied. In the framework of a phenomenological temperature model an analysis in both temporal (population dynamics) and frequency domain (photoluminescence) is carried out in order to identify the associated dynamical behavior. The Wigner function and concurrence are employed to quantify the entanglement in each regime. We find that sudden death of entanglement is a typical characteristic of the strong coupling regime.

1. Introduction
In the last few years, the study of microsystems that involve the interaction between an active medium and confined light has made possible the observation of interesting phenomena in two dynamical regimes: weak and strong coupling [1–3]. In the first regime, spontaneous emission control was successfully realized (the Purcell effect) [4, 5]; in the second one, several groups are now searching for coherent polaritonic phenomena such as lasing [6] or condensation [7, 8], which can open applications in quantum information and quantum optics [9–12]. In addition, some recent theoretical works have shown that the dynamical properties for a coupled quantum dot–cavity system may be described using a simple dissipative model [13, 14].

Our purpose in this work is to study the relations between weak and strong coupling regimes with the dynamical exciton–photon entanglement, by using the concurrence measure and the Wigner quasiprobability function. Despite the fact that the Wigner function depends only on the photonic state, it has been shown that it can be used as a qualitative criterion for determining the separability of the quantum exciton–photon state [15]. In [16] a similar study has been carried out by studying two quantum dots and only in the stationary limit.

2. Theoretical background
We are interested in the evolution of a quantum dot interacting with a confined mode of the electromagnetic field inside a
coupling between the system (exciton + external reservoir degrees of freedom and assuming the validity this model can be found in [13]. After tracing over the The explicit form of the system–reservoir interaction for Hamiltonian of the reservoirs, which is made of electron–hole photons of the cavity and the exciton. The second one is the The whole system–reservoir Hamiltonian can be split into three 2.1. Dynamics We will employ a Fock state basis \( |n\rangle \) with the non-conservative processes included are given by: The master equation we shall consider is \([13, 22, 23]\)

\[
\frac{d\rho}{dt} = i[\rho, H] + \frac{\kappa}{2}(2\sigma a^\dagger a - a^\dagger a^\dagger a^\dagger a) + \frac{\gamma}{2}(2\sigma \rho \sigma^\dagger - \sigma^\dagger \sigma \rho - \rho \sigma^\dagger \sigma) + \frac{P}{2}(2\sigma^\dagger \rho \sigma - \rho \sigma \sigma^\dagger - \sigma \sigma^\dagger \rho).
\]

By changing the values of the free parameters in this model, two different dynamical regimes are reached: weak and strong coupling. Possible transitions between these two regimes can be achieved as loss and pump rates are modified. The dynamical behavior of the system as well as the size of the basis employed are governed by the competition of the timescales involved in the model. The timescales associated with the non-conservative processes included are given by: \(\tau_P = 1/P, \tau_\kappa = 1/\kappa\) and \(\tau_\gamma = 1/\gamma\), whereas the interaction timescale is \(\tau_g = 1/g\). We address now the interpretation of these timescales. In figure 1 three possible cases are shown whose dynamics can be described using a basis including up to one photon (cases (A) and (B)) and more than one photon (case (C)).

In case (A), for instance, the relation \(\kappa \gtrsim P \gg g\) holds, and the system is operating in the weak coupling regime. When an exciton is pumped during the typical timescale \(\tau_P\), the elapsed time until it recombines into a photon is given by (the largest) scale \(\tau_g\). Bear in mind that throughout this period no further excitation can be done over the quantum dot, because the Pauli exclusion principle does not allow an additional excitation. The photon generated in this way quickly leaves the cavity due to the small timescale \(\tau_\kappa\) in which it can inhabit the cavity. The latter mechanism applies for any photon number is never expected to be greater than one. If the initial condition for the electromagnetic field is the vacuum state, the photon mean number is never expected to be greater than one.

A similar reasoning can be done in case (B), where \(g \gtrsim \kappa \gg P\). In this case each photon created by the interaction

\[\text{Figure 1. Schematic diagram of some relations among the typical timescales associated with the effective rates involved in the master equation. Cases (A) and (B) show situations in which the basis containing up to one photon is enough to describe the dynamics. On the other hand, case (C) shows a set of parameters where this description fails and the requirement for a larger basis arises. See the text for additional details.}\]
can be re-absorbed and produce Rabi oscillations (i.e. strong coupling) before it leaves the cavity. A basis including up to one photon is enough to describe the system because the characteristic time related to losses is much smaller than the pump one. The important issue here is that the dynamical situation is clearly different to the previous one.

Finally, in case (C), where $g \gtrsim P \gg \kappa$, we have a situation in which the photons can be efficiently stored within the cavity. Indeed, since the coherent emission rate is small, its characteristic timescale is large enough compared with the associated excitonic pump and interaction rates. Photons created by exciton recombination remain in the cavity a long time before they leak through the cavity mirrors. This case is a clear example where the multiphoton model must be implemented.

2.2. Photoluminescence

The Fourier transform of the first-order correlation function directly gives the photoluminescence spectrum of the system [20]:

$$S(\omega, t) \propto \int_{-\infty}^{\infty} \langle a^\dagger(t + \tau) a(t) \rangle e^{i\omega \tau} d\tau.$$  

(3)

Note that in this expression a knowledge of the time-dependent expected value for the product of creation and annihilation operators is needed in order to compute the photoluminescence spectrum. However, a non-analytical expression for such an expectation value is available for our system. This problem can be solved by representing the operators $a$ ($a^\dagger$) in the interaction picture, and then using the quantum regression theorem [22]. This theorem states that, given a set of operators $O_j$ satisfying

$$\frac{d}{d\tau} O_j(t + \tau) = \sum_k L_{jk} O_k(t + \tau),$$  

(4)

then

$$\frac{d}{d\tau} \langle O_j(t + \tau) O(t) \rangle = \sum_k L_{jk} \langle O_k(t + \tau) O(t) \rangle,$$  

(5)

for any operator $O$. The validity of this theorem holds whenever a closed set of operators are involved in the dynamics in the Markovian approximation. Unfortunately, representing creation and annihilation operators in the interaction picture does not lead to a complete set. It is necessary to add two new operators in order to close the system. The final set of equations for the operators are

$$\begin{align*}
a^\dagger_{G0}(t) &= |Gn + 1\rangle\langle Gn| e^{i(\omega - \Delta)\tau} \\
a^\dagger_{N}(t) &= |Xn + 1\rangle\langle Xn| e^{i(\omega - \Delta)\tau} \\
\sigma^+_n(t) &= |Xn\rangle\langle Gn| e^{i\omega\tau} \\
\zeta_n(t) &= |Gn + 1\rangle\langle Xn - 1| e^{i(\omega - 2\Delta)\tau}.
\end{align*}$$  

(6)

In the framework of the previous discussion regarding the dynamical timescales, we can consider the simplest model involving all dissipative and interaction processes, that is, we shall study a problem having just three levels: $|G0\rangle$, $|X0\rangle$ and $|G1\rangle$. Taking into account this consideration we only need to define two operators whose dynamical equations are

$$\begin{align*}
\frac{d}{d\tau} \langle a^\dagger_{G0}(t) \rangle &= -\left(\frac{\kappa}{2} + P - i(\omega - \Delta)\right) \langle a^\dagger_{G0}(t) \rangle + i g \langle \sigma^+_n(t) \rangle \\
\frac{d}{d\tau} \langle \sigma^+_n(t) \rangle &= i g \langle a^\dagger_{G0}(t) \rangle - \left(\frac{P + \gamma}{2} - i\omega\right) \langle \sigma^+_n(t) \rangle.
\end{align*}$$  

(7)

Thus by using the quantum regression theorem we can write the dynamics for the delayed operators as

$$\begin{align*}
\dot{X}(t + \tau) &= \frac{d}{d\tau} \left(\begin{array}{c} \langle a^\dagger_{G0}(t + \tau) \rangle \\ \langle \sigma^+_n(t + \tau) \rangle \end{array}\right) \\
&= \left(-\left(\frac{\kappa}{2} + P\right) + i(\omega - \Delta) \begin{array}{c} ig \\ \frac{P + \gamma}{2} + i\omega \end{array}\right) \times \left(\begin{array}{c} \langle a^\dagger_{G0}(t + \tau) \rangle \\ \langle \sigma^+_n(t + \tau) \rangle \end{array}\right) = AX.
\end{align*}$$  

The last linear system has the following formal solution:

$$X(t + \tau) = e^{\Delta t} X(t) = Be^{\Lambda t} B^{-1} X(t),$$  

(8)

where $A = B\Lambda B^{-1}$ and $\Lambda = \text{diag}(\lambda_+, \lambda_-)$ is a diagonal matrix containing the eigenvalues of $A$.

The eigenvalues $\lambda_\pm$ are directly related to the peaks $\omega_{\pm}$ and widths $\Gamma_\pm$ of the spectrum:

$$\omega_{\pm} + i\Gamma_\pm = i\lambda_\pm.$$  

(9)

2.3. Temperature effects

In order to make a more realistic description of the experimental data we also include a temperature dependence in the model. It is important to explicitly point out that the developed master equation describes the system at zero temperature, that is, the states of the reservoirs considered in the derivation of the master equation (2) are zero-temperature states. This assumption can be justified as follows.

First, notice that when a finite temperature reservoir of photons is considered in the derivation of the master equation one obtains two Lindblad terms [24]. One accounts for thermally induced absorption and is proportional to the average number of photons of the reservoir and the other term is proportional to the average number of photons of the reservoir plus one and accounts for spontaneous emission.

The average number of photons of a thermal reservoir at temperature $T$ is $1/(e^{\hbar\omega_0/k_BT} - 1)$. The standard experimental values for the photon energy in a micropillar are $\omega_0 \sim 1$ eV, whereas for temperatures of the order of $10^2$ K the thermal energy $k_B T \sim 1$ meV and thus the average number of photons is $N(\omega) \sim e^{-10^2} \sim 10^{-400}$. We see that effects due to finite temperature in the master equation for the system considered here are quite small.

Summarizing, the above discussion, we can add thermal effects on the system through its effective parameters, such as the quantum dot energy gap and the cavity refractive index, without considering the fundamental issues introduced into the model by the dephasing processes. First, let us consider the temperature photon energy dependence. The usual
microwavers are built from GaAs/AlGaAs layers. For these cavities the resonant wavelength depends on the refractive index as \( \lambda = \lambda_{\text{air}}/n \), where \( \lambda_{\text{air}} \) is the light wavelength in vacuum. We can then modify the resonant frequency by changing the refractive index \( n(T) \) [27]. Experiments have shown that this index has an almost linear temperature dependence within the range from 0 K up to a few hundred Kelvin. In [25] it has been shown that the refractive index can be modeled with the simple formulae

\[
n(T) = n_0(1 + aT),
\]

(10)

where \( a \approx 10^{-5} \text{K}^{-1} \). It is seen that the corrections to the wavelength due to temperature are rather small, and because of this will not be considered in this work. On the other hand, the temperature effects, related to the active medium, can be included through the energy gap in the quantum dot. We shall use here the Varshini model [26], which fits the bandgap thermal dependence in the low temperature region. A more detailed discussion about the validity of this model can be found in [27]

\[
\omega(T) = E_G(0) - \frac{\alpha T^2}{\beta + T},
\]

(11)

where \( \alpha \) and \( \beta \) are the square roots of the decreasingly ordered eigenvalues of the positive-definite non-Hermitian matrix \( \rho \rho^* \) with

\[
\rho = \sum_i \rho_i^{(p)} \otimes \rho_i^{(X)},
\]

and \( \rho^* \) is the complex conjugated matrix representation of \( \rho \) in some suitable basis.

For instance, note that if our system is described by using the basis \( \{|0\}, \{|1\}\} \otimes \{\{G\}, \{X\}\} \) (where \( |0\rangle, |1\rangle \) are photon Fock states), the exciton–photonic field system can be thought of as two interacting qubits. Once a (numerical) solution of the master equation is obtained, it is straightforward to compute the concurrence following the previous recipe. It is to be noticed that the validity of this basis is constrained to the dynamical regime where this cutoff holds. When the basis is larger than the one considered earlier, the concurrence measure is not applicable and hence a new criterion must be established. Despite the Wigner function only depending on the photon state, it has been demonstrated that it yields qualitative information about the separability between the exciton and photon parts of the global quantum state [15]. The Wigner function for the photonic field can be easily computed as [29]

\[
W(\alpha) = 2Tr_P[D(-\alpha)\rho^{(P)}D(\alpha)P_f],
\]

(12)

where \( P_f = e^{i\alpha a^\dagger a} \) is the field parity operator and \( D(\alpha) = e^{i\alpha \sigma^\dagger a} \) is the displacement operator, \( \rho^{(P)} \) is the density operator of the photons and \( Tr_P \) is the trace operation in Fock space. Our system involves both excitonic and photonic states; hence this definition is not directly applicable and a previous differentiation among the possible combinations has to be done, that is, we have to separately consider the Wigner matrix elements \( W_{ij} [15] \):

\[
W_{ij}(\alpha) = 2Tr_P[D(-\alpha)|i\rangle\langle j|D(\alpha)P_f],
\]

where indexes \( i, j \) run over excitonic states \( \{|X\}, \{|G\}\} \) and \( \rho \) is the density operator of the whole system. Notice that \( \langle i|\rho|j \rangle \) is an operator that acts only in the state space of the photons so that the operations in the above equation are well defined.

Note that if the system is separable at a given time then the matrix elements \( W_{XX} \) and \( W_{GG} \) have the same shape in phase space, leading to a separable system, which is itself related to a vanishing concurrence situation. Indeed, recognizing that each Fock state has a well-defined signature within the phase space, it is possible to identify the predominant photonic state by observing the Wigner function. A more detailed analysis shows that if the quantum state is separable

\[
\rho = \sum_I \rho_i^{(p)} \otimes \rho_i^{(X)},
\]

then the corresponding Wigner function of the system can be written as [15]

\[
W_{ij}(\alpha) = \rho_i^{(X)} W(\alpha),
\]

(13)

where \( W(\alpha) = \sum_I W_{ij}(\alpha) \). Therefore, if the system is separable at a given time all the matrix elements \( W_{ij} \) of the Wigner matrix have the same shape for every \( i, j \). In opposition to the quantitative concurrence measure, the Wigner function only shows qualitative information about the entanglement.

3. Results

3.1. Weak coupling regime

In the weak coupling (WC) regime the relation [23]

\[
16g^4 < (\kappa - \gamma)^2,
\]

(14)

holds. Hence by selecting a strong enough emission rate WC is obtained. We choose \( g = 15 \mu eV, \gamma = 1 \mu eV, \kappa = 85 \mu eV \) and \( P = 20 \mu eV \) corresponding to case (A) of figure 1. As we stated before, this situation can be described with the reduced basis including up to one photon.

We stress that relation (14) is derived under the assumption that there is no incoherent pumping over the system. The effects of the pumping have been studied in detail in [30].
Figure 2. (Color online) (Left panel) Crossing emission peaks in the WC regime computed with a simplified model (solid line), the whole model using a 40 photonic level basis (squares). (Right panel) Time evolution of the populations and the average photon number in the WC regime, computed using the simplified model, $g = 15 \, \mu eV$, $\gamma = 1 \, \mu eV$, $\kappa = 85 \, \mu eV$ and $P = 20 \, \mu eV$. The average photon number was computed using 40 Fock states.

Figure 3. (Color online) (Left panel) Emission peaks in the SC regime. Anticrossing can be observed at $T = 12 \, K$. The solid line was computed by using the simple three-level model. Meanwhile dots are the peaks obtained with a basis involving 40 Fock states. (Right panel) Time evolution of populations and the average photon number for the strong coupling regime; again only the average photon number was computed with 40 Fock states. Differences with the weak coupling regime are clear.

We have taken a photon decay rate of $\kappa = 85 \, \mu eV$ and a cavity mode of frequency $\omega = 1296.11 \, meV$, so that the quality factor of the cavity we are considering is $Q \approx 15 \, 000$. The spectra for this set of parameters is shown in figure 2. These values were chosen following the results of [11] where strong coupling in a single quantum dot microcavity system is reported.

In figure 2 it is seen that at approximately $T = 12 \, K$ the peaks of the cavity and exciton coincide. The parameters used for the temperature dependence were $E_G(0) = 1299.6 \, meV$, $\alpha = 0.81 \, meV \, K^{-1}$ and $\beta = 457.6 \, K$.

We pointed out that the weak coupling regime could be reached by setting an emission rate higher than the coupling constant. Due to the small time the photons spend in the cavity the chance of them to interact with the exciton is quite small and because of this no oscillations appear in the populations as seen in figure 2.

3.2. Strong coupling

The strong coupling (SC) regime is even more interesting since it enables the existence of polaritons. From an experimental point of view it is more complicated to obtain due to the fine scales of the variables involved. This regime is also studied with the same three-level model described above and with a basis of 40 Fock states but now taking $g = 35 \, \mu eV$, $\kappa = 25 \, \mu eV$, $\gamma = 1 \, \mu eV$, $P = 1 \, \mu eV$, $E_G(0) = 1299.6 \, meV$, $\alpha = 0.81meV \, K^{-1}$, $\beta = 457.6 \, K$ and a cavity resonant frequency of $\nu = 1299.35 \, meV$.

It is clear that in this regime oscillations in the populations are observed since the time a photon can live in the cavity is long enough for it to interact with the exciton, i.e. $1/g = \tau_e > \tau_c = 1/\kappa$. Peak positions are shown in figure 3.

Note that, for detunings far from the resonance, a small shift of the cavity peak appears. This effect is not observed
Figure 4. (Color online) Concurrence time evolution as a function of the dissipative parameters: (a) changing $\kappa$ and fixing $P = 3 \text{ \mu eV}, \Delta = 0$, (b) changing $P$ and fixing $\kappa = 3 \text{ \mu eV}, \Delta = 0$ and (c) changing $\Delta$ and keeping $P = 2 \text{ \mu eV}, \kappa = 3 \text{ \mu eV}$. In (d) time evolution of linear entropy ($M(\rho) = 1 - \text{Tr}(\rho^2)$) for the last set of parameters. In (e) and (f) we plot the concurrence ($C(\rho)$), linear entropy ($M(\rho)$), $\sqrt{\rho_{X0}G_0X_0}$ and $|\rho_{X0}G_1|$ for $\Delta = 0 \text{ eV}$ and $\Delta = 0.1 \text{ eV}$, respectively, with $\kappa = 3 \text{ \mu eV}, P = 2 \text{ \mu eV}$. Dynamics was solved with the initial condition $\rho_{X0}X_0 = 1$ and the coupling constant $g = 25 \text{ \mu eV}$ in all cases.

in the WC regime. This shift is due to the strong interaction between matter and light.

The closest approximation of the two emission peaks (cavity and excitonic) is reached when $T = 12 \text{ K}$, where the separation (Rabi splitting) is

$$R = 68.89 \text{ \mu eV}.$$ 

Another important fact which enables us to conclude that we are dealing with a system in the SC regime is that the following condition holds:

$$R \approx 2g.$$ 

That is, the Rabi splitting is approximately two times the coupling constant [23]. Finally, to determine if the reduced basis was enough to describe the system we computed the average photon number $\langle n_{40} \rangle$ using a basis of 40 Fock states. This information is in the right panels of figures 2 and 3 and can be compared with the average photon number using the
elapsed time in the concurrence revivals becomes longer.

described in 3.1; however, since we are mainly interested in the strong coupling regime with a set of parameters such as those described in section 2.4. First we study the system in the to do so we now turn our attention to the simplified system of strong and weak coupling regimes. Now we want to study the behavior of the entanglement in these regimes. In order

3.3. Concurrence

In section 2 a study of the relation between dynamics and photoluminescence was made, leading to a direct identification of strong and weak coupling regimes. Now we want to study the behavior of the entanglement in these regimes. In order to do so we now turn our attention to the simplified system described in section 2.4. First we study the system in the strong coupling regime with a set of parameters such as those described in 3.1; however, since we are mainly interested in the effects that dissipation has on the entanglement we set $\Delta = 0$. Since spontaneous emission processes are small compared with the other effects, we also set $\gamma = 0$. Taking $g = 25 \, \mu$eV we obtain the results in the upper panels of figure 4. In the middle left panel the evolution of the concurrence as a function of detuning is shown. Notice that when the system moves from strong coupling to weak coupling the entanglement becomes larger, leading to a non-vanishing value of the concurrence for all times.

The range of parameters for this situation was carefully chosen since large variations of them lead to unphysical results. Indeed, if $\kappa$ is increased the cavity eventually will get empty, while if $P$ is increased the basis is not large enough to describe the system, since the mean number of photons becomes larger than 1.

Results on the evolution of concurrence show that it does not distinguish between the two dissipative processes (parameters) considered ($\kappa$ and $P$). The reason for this behavior is that the concurrence of our system depends essentially on the difference between the absolute value of the coherence term $\rho_{G1X0}$ and the square root of the product of the populations $\rho_{G0G0}$ and $\rho_{X1X1}$. Notice, for example, that in figure 4 panel (e) the concurrence vanishes for precisely the times when $|\rho_{G1X0}| < \sqrt{\rho_{G0G0}\rho_{X1X1}}$ and that the difference between the concurrence and $|\rho_{G1X0}|$ in their minima grows as the coherence $\sqrt{\rho_{G0G0}\rho_{X1X1}}$ grows as seen in panel (f). In the dynamics of the density matrix we are considering it is seen that the coherences decay exponentially with rates proportional to $\kappa + P$ and that, on the other hand, $P$ tends to increase the population of $\rho_{X1X1}$ and $\kappa$ tends to increase the population of $\rho_{G0G0}$ so that the difference $|\rho_{G1X0}| - \sqrt{\rho_{G0G0}\rho_{X1X1}}$ does not distinguish between the two processes.

Even more interesting is the fact that, as the dissipation increases, the zones where concurrence vanishes (the so-called entanglement sudden death [31]) become wider and revivals in the concurrence become more separated as dissipation increases. Note that when dissipation increases the maxima of the concurrence are less defined and eventually disappear. In order to quantify these two effects we compute the temporal length of the first two collapses of the concurrence ($\delta t_1$ is the time interval of the first collapse and $\delta t_2$ the time interval of the second one). The results obtained are plotted in figure 5. It is clearly seen that, as the non-Hamiltonian effects become important, the length of both intervals gets longer and that for any $\kappa$ and $P$ the second interval $\delta t_2$ is wider than the first one $\delta t_1$. This behavior can be understood as follows: as explained in the last paragraph the degree of entanglement between the subsystems is the difference $|\rho_{G1X0}| - \sqrt{\rho_{G0G0}\rho_{X1X1}}$ and the dynamical behavior of the coherence term is an oscillatory function times an exponential decaying function with decay rate proportional to the sum of $\kappa$ and $P$. In order to have entanglement the absolute value of $|\rho_{G1X0}|$ must be greater than $\sqrt{\rho_{G0G0}\rho_{X1X1}}$. Now notice that, as $P$ or $\kappa$ are increased, the oscillations gets more damped. Because of the damping the amplitude of the oscillations is smaller and the absolute value of the coherence has to be in a time $t$ closer to a maximum in order to be greater than $\sqrt{\rho_{G0G0}\rho_{X1X1}}$, that is, the finite time where there is no entanglement approaches the time interval between two successive maxima of $|\rho_{G1X0}|$ as $\kappa$ or $P$ is increased. To understand the fact that the first interval of sudden death $\delta t_1$ is smaller than the second one $\delta t_2$, simply notice that for the times in the second interval the amplitude of $|\rho_{G1X0}|$ will be smaller than in the first one, so that

Figure 5. (Color online) Revivals time $\delta t_1$ (left panel) and $\delta t_2$ (right panel) in ps as functions of $\kappa$ and $P$. As dissipative factors increase the elapsed time in the concurrence revivals becomes longer.
Figure 6. (Color online) Time behavior for Wigner function matrix elements at different concurrence regimes. Notice that in the times of non-vanishing concurrence ($t_1$ and $t_3$) the shape of the matrix elements is quite different, in particular the non-diagonal elements cannot be obtained by multiplying the trace (which is a real-valued function) since they have more than one lobe. On the other hand, in the vanishing concurrence case ($t_2$) the shape is the same. To further quantify the separability of the quantum state of the whole system we can define the following quantity:

$$\delta_{W_{ij}}(\alpha) = W_{ij}(\alpha) - W_{ij}(0)$$

if the state is separable then $\delta_{W_{ij}}(\alpha) = 0$ for all $\alpha$. Actually when one calculates $\delta_{W_{ij}}(\alpha)$ in $t_1$ and $t_3$ it is bounded as follows: $0 \leq |\delta_{W_{ij}}(\alpha)| \leq 3$. On the other hand, in $t_2$ it has the following bound: $0 \leq |\delta_{W_{ij}}(\alpha)| \leq 0.04$. The times $t_1$, $t_2$ and $t_3$ are indicated in figure 4.

Now we compute the elements of the Wigner function $W_{GG}$ and $W_{XX}$ (see figure 6) in representative zones of the concurrence function for typical values of the parameters: $\kappa = 3 \, \mu eV$ and $P = 2 \, \mu eV$, at the times $t_1$, $t_2$ and $t_3$ indicated in figure 4. Note that, in the regions where concurrence goes to zero, phase space is similar (up to a multiplicative constant) as suggested by $W_{XX}$. $W_{GG}$, $\text{Re}(W_{GX})$ and $\text{Im}(W_{GX})$, so that the photonic states are similar for both ground and excited excitonic states and hence the photonic states are separable from the excitonic part leading to a vanishing entanglement.
On the other hand, for non-vanishing concurrence it is clear that photonic states cannot be separated.

4. Conclusions

We have built a phenomenological model that is able to describe both weak and strong coupling and that accounts for temperature effects in a microcavity quantum dot system. We have used a concurrence criterion and the Wigner function to carry out the entanglement analysis related to the dynamical regimes in a simple dissipative model. The strong coupling regime shows a periodic disentanglement that depends on the dissipation rates. On the other hand, the weak coupling regime shows no complete dynamical losing of entanglement. This relation between dynamical regimes and entanglement sudden death has been investigated and we have shown that, on the one hand, both the coherent emission (κ) and the incoherent pumping (P) affect the time windows where there is no entanglement. In the same way, as time goes by this window becomes wider due to the damping in the dynamics of the non-diagonal terms of the density matrix caused by κ and P.

Finally, we would like to point out that our results suggest that one can control the entanglement between the subsystems by manipulating external parameters such as the pumping rate P and the cavity quality factor (which is related to the emission rate κ).

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