Accelerating the 2-point and 3-point galaxy correlation functions using Fourier transforms

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1 INTRODUCTION

In studying the large-scale clustering of galaxies, we commonly use the two and three-point correlation functions $\xi$ (2PCF) and $\zeta$ (3PCF) (Peebles 1980; Bernardeau et al. 2002; Szapudi 2005 for reviews; recent observational work on the 2PCF is Anderson et al. 2014; on the 3PCF, Kayo et al. 2004; McBride et al. 2011a, b; Guo et al. 2014). These are correlations of the fractional overdensity field $\delta(x) = \rho(x)/\bar{\rho} - 1$, where $\rho(x)$ is the density field and $\bar{\rho}$ is the mean density. Also commonly used is the anisotropic 2PCF $\xi_{\text{aniso}}$, often written as multipole moments of the 2PCF with respect to the line of sight (Cabré & Gaztañaga 2009; Okumura & Jing 2011; Reid et al. 2012; Samushia, Percival & Raccanelli 2012; Chuang & Wang 2012; Chuang & Wang 2013a, b; Chuang et al. 2013; Sánchez et al. 2013; Xu et al. 2013; Ross, Percival & Manera 2015; White et al. 2015 for recent modeling and observational work). Typically, direct counting is used to compute $\xi$, $\xi_{\text{aniso}}$, and $\zeta$. The calculation of $\xi$ and $\xi_{\text{aniso}}$ scales as $N n V_{\text{max}}$, while that of $\zeta$ scales as $N (n V_{\text{max}})^2$, where $N$ is the number of galaxies in the survey, $n$ is the number density, and $V_{\text{max}}$ is the volume of a sphere of the maximum radius out to which the correlation is measured. For surveys such as the Sloan Digital Sky Survey (SDSS) (Alam et al. 2015 for latest release), with order one million galaxies, and upcoming efforts like Euclid (Laureijs et al. 2011), Large Synoptic Survey Telescope (LSST) (LSST Dark Energy Science Collaboration 2012), Dark Energy Spectroscopic Instrument (DESI) (Levi et al. 2013), and WFIRST (Spergel et al. 2013) (see also Jain et al. 2015), with tens of millions to billions of galaxies, these scalings render the 2PCF computationally expensive and the 3PCF computationally prohibitive for large $V_{\text{max}}$. Therefore, techniques that scale more favorably with number and number density of objects merit consideration. In this work, we argue for the utility of Fourier methods in measuring the two and three-point correlation functions, as their computational cost depends primarily on the number of grid cells $N_g$ into which the survey volume is discretized (though gridding the density field depends linearly on the number of galaxies). While the methods will introduce artifacts due to the grid resolution, the balance of performance versus accuracy may be desirable for some applications, for example the analysis of large-scale correlations in large numbers of mock catalogs.

It has long been known that the 2PCF can be computed using FTs, but, for reasons we detail further below (Section 2), this has not been the favored approach. It has been less appreciated that the anisotropic 2PCF and 3PCF can also be computed using FTs, though they have been used for the projected 3PCF (Zheng 2004) and the 3PCF of the Cosmic Microwave Background (Chen & Szapudi 2005), as well as multipole moments of the power spectrum (Bianchi et al. 2013; Scoccimarro 2015) and bispectrum (Scoccimarro 2015). Though this will not be our focus here, FTs are standardly used to compute the power spectrum, typically with
FKP weighting (Feldman, Kaiser & Peacock 1994). A recent implementation is discussed in Anderson et al. (2014) and Percival et al. (2014).

In this paper, we begin by showing that several perceived disadvantages of using FTs for the standard 2PCF can be avoided with a particular way of setting up the Landy-Szalay estimator (Section 2). We then show that the anisotropic 2PCF can also be straightforwardly handled with FTs (Section 3). In Section 3, we show that the 3PCF algorithm recently presented in Slepian & Eisenstein (2015; hereafter SE15), which already offers roughly a factor of 500 speed improvement relative to naive triplet counting, can in many circumstances be further accelerated using FTs. Section 5 concludes.

2 PAIR COUNTING WITH FOURIER TRANSFORMS

2.1 Estimator and passage to the FT

It is well-known that the 2PCF of a field can be computed quickly using the FT: it is simply the convolution of the density field with itself, and hence by the Convolution Theorem reduces to multiplication in Fourier space:

$$\xi(r) \equiv \int dx \delta(x)\delta(x+r) = \int \frac{dk}{(2\pi)^3} \hat{\delta}(k)^2 e^{-ikr} \quad (1)$$

where here and throughout $\hat{\delta}(k) = \int dx \delta(x)e^{ikx}$ is the FT of the field. Numerically, the integrals become sums over wavenumbers quantized to fit in the finite box on which the integral is evaluated, and for periodic boundary conditions the largest wavelength is simply the periodicity. For a non-periodic survey, we can still use this method by expanding the grid to be well larger than the survey and setting $\delta = 0$ outside the survey volume.

However, this is not usually the method employed in cosmology, particularly in wide-field surveys. Instead, one typically uses an explicit counting of pairs, such as the Landy-Szalay (1993) estimator,

$$\hat{\xi}(s) = \frac{NN}{RR} \int \frac{dr_1 dr_2 \theta(r_1-r_2;S)N(r_1)N(r_2)}{dr_1 dr_2 \theta(r_1-r_2;S)R(r_1)R(r_2)}, \quad (2)$$

with $N \equiv (D-R)$, $D$ the data, $R$ the random counts, and $\theta$ a binning function that is non-zero where its first argument is within a three-dimensional volume labeled by $S$. In this Section we focus on bins of fixed Cartesian separation. The random catalog is chosen to be a Monte Carlo realization of the mean density field, with weights such that the mean densities of the randoms and the data match. Changing variables to the separation $s \equiv r_2 - r_1$, we have

$$\hat{\xi}(s) = \frac{\int ds \theta(s;S) \int dr_1 N(r_1)N(r_1+s)}{\int ds \theta(s;S) \int dr_1 R(r_1)R(r_1+s)} \frac{\int dr_1 N(r_1)N(r_1+s)}{\int dr_1 R(r_1)R(r_1+s)} \quad (3)$$

Thus the binned estimator is simply a binned convolution, and the convolution can be evaluated with FTs in $O(N_c \log N_p)$ time (Cooley & Tukey 1965; Press et al. 2007). Forming the gridded density field the FTs require is linear in the number of particles (e.g., using triangular cloud-in-cell interpolation (Hockney & Eastwood 1981; Jing 2005)), so we avoid any quadratic scaling with the galaxy density. Therefore, in principle the Landy-Szalay estimator can be straightforwardly evaluated using FTs.

2.2 Comparing FTs and pair counting

We now discuss the practical disadvantages and advantages of pair counting relative to a Fourier approach. Ultimately we will show that several perceived issues with the FT method that seemed to make pair counting more favorable can be resolved by the approach of this paper.

Relative to the Fourier method, pair counting has the disadvantage that the work scales as the square of the number of points. It is a simple optimization, using trees or grids, to avoid working with pairs that are much further separated than the maximum scale of interest. However for work on large scales, the large number of pairs can be computationally burdensome. This is particularly true because the number of randoms typically should be much larger than (of order 100 times) the number of data points. The computational expense can be somewhat reduced by using tree methods that aggregate many points into cells that are then added to the count of a given separation bin as a unit. However, this only helps if the binning is substantially coarser than the interparticle spacing, which often is not the case in practical galaxy surveys.

Pair counting does have some important advantages. First, it avoids any gridding of the data. Gridding results in small displacements of the effective particle positions, which in turn produce correlation results that are smoothed versions of the true answer (see Jing 2005 for discussion of the effect in the power spectrum and how it can be ameliorated). Cartesian gridding can also cause artifacts when summing over separation bins in spherical coordinates. Decreasing the grid spacing can decrease these biases, but at an increased computational cost. Second, pair counting allows easy computation of the anisotropy of the correlations relative to the line of sight, since in real space the orientation of each pair to the line of sight is clear. Fourier methods, on the other hand, use a Cartesian basis that treats positions in the survey without preference as to orientation to the line of sight, appearing to destroy this information. Third, pair counting produces an unbiased estimate of the correlation function regardless of the survey geometry. In contrast, an FT-based convolution of the $\delta$ field yields a misnormalized result due to the zero-padding outside of the survey volume.

In this work, we show that the last two of these problems can be easily avoided when using FTs. Here, we discuss the zero-padding issue, deferring the anisotropic correlations to Section 3. Equation (2) shows that there is no need to form the $\delta$ field. The zero padding of the grid beyond the survey boundary is of no consequence because it will enter both the numerator and denominator of equation (2) and hence cancel out. Thus the value of $\hat{\xi}$ should be the same, up to grid smoothing, whether one has used pair counting or the FT. Consequently any advantage of the Landy-Szalay computation of the monopole of the correlation function, e.g., as regards the integral constraint (Coil 2012 for definition), will remain.

Finally, we note that there is a common misunderstanding that for non-periodic volumes one must embed in a periodic domain that is twice the size of the original survey.
This is not true if one is only interested in a limited range of separations \( s \). One need only use a periodic embedding large enough that the separation between any point in the survey and any point in the periodic duplicate is larger than \( |s| \).

3 ANISOTROPIC CORRELATIONS

We now turn to the anisotropic 2PCF, described in terms of its multipole moments \( \xi \). The anisotropies of the correlation functions, and most importantly the quadrupole \( \xi_2 \), have important cosmological purpose for the measurement of the Alcock-Paczynski effect (Alcock & Paczynski 1979), redshift-space distortions (Kaiser 1987; Hamilton 1998 for a review), and anisotropic baryon acoustic oscillation (BAO) signature (Gaztañaga, Cabrè & Hui 2009).

Here the multipole moments are with respect to the angle between the pair separation \( \mathbf{s} \) and the line of sight \( \mathbf{n} = (r_1 + r_2)/2 \), and we define \( \mu = \mathbf{s} \cdot \mathbf{n} \). We first write \( \xi_{\text{aniso}} \) as a function of bin \( S \) in separation magnitude \( s = |s| \) and \( \mu \):

\[
\xi_{\text{aniso}}(S, \mu) = \frac{N(S, \mu)}{R(S, \mu)},
\]

where \( N \) and \( R \) respectively denote \( NN \) and \( RR \). We expand \( \xi_{\text{aniso}} \), \( N \), and \( R \) as multipole series:

\[
\xi_{\text{aniso}}(S, \mu) = \sum_{\ell=0} \xi_{\ell}(S) P_{\ell}(\mu), \quad N(S, \mu) = \sum_{k=0} \mathcal{N}_k(S) P_{k}(\mu),
\]

\[
R(S, \mu) = \sum_{j=0} \mathcal{R}_j(\tau) P_{j}(\mu).
\]

\( P_{\ell} \) is a Legendre polynomial, and as usual, these relations imply \( \xi_{\ell}(S) = [(2\ell+1)/2] \int d\mu P_{\ell}(\mu) \xi_{\text{aniso}}(S, \mu) \) and similarly for \( \mathcal{N}_k(S) \) and \( \mathcal{R}_j(S) \). Multiplying equation (1) through by \( R \) and then inserting equations (5), we find

\[
\sum_{ij} \xi_{\ell}(S) \mathcal{R}_j(2+1) \left( \begin{array}{ccc} \ell & j & q \\ 0 & 0 & 0 \end{array} \right) P_{1} = \sum_{k} \mathcal{N}_k P_{k}, \quad (7)
\]

The Wigner 3j-symbol above describes angular momentum coupling. Using orthogonality, separating out the \( j = 0 \) term, dividing through by \( \mathcal{R}_0 \), and defining \( \mathcal{F}_j = \mathcal{R}_j/\mathcal{R}_0 \), we obtain

\[
\mathcal{N}_j = \xi_{\ell} + \sum_{j>0} \xi_{\ell}(2k+1) \left( \begin{array}{ccc} \ell & j & k \\ 0 & 0 & 0 \end{array} \right)^2 \mathcal{F}_j = \sum_{k} \mathcal{N}_k P_{k}, \quad (8)
\]

Defining a multipole coupling matrix \( \mathcal{M} \) with elements

\[
M_{kj} = (2k+1) \sum_{j>0} \left( \begin{array}{ccc} \ell & j & k \\ 0 & 0 & 0 \end{array} \right)^2 \mathcal{F}_j \quad (9)
\]

we see that equation (8) can be written as

\[
\mathcal{N}/\mathcal{R}_0 = (\mathbf{I} + \mathcal{M}) \mathcal{E}_{\text{aniso}} \equiv \mathbf{A} \mathcal{E}_{\text{aniso}}, \quad (10)
\]

where \( \mathcal{N} = \{ \mathcal{N}_0, \mathcal{N}_1, \cdots, \mathcal{N}_{\ell_{\text{max}}} \} \) and analogously for \( \mathcal{E}_{\text{aniso}} \). The edge-correction equation (10) can then be solved by matrix inversion. Formally we need all multipoles of the randoms \( \mathcal{R}_j \) to obtain the solution, but in practice the \( f_j \) should fall off so quickly that measuring only out to some \( \ell_{\text{max}} \) is sufficient. For more detailed discussion of similar issues arising in the 3PCF, see SE15 Section 4.2. Note also that if we are computing an auto-correlation, then by parity all odd-order coefficients in equation (5) vanish.

With equation (10) for the vector of multipole coefficients \( \mathcal{E}_{\text{aniso}} \), our task now becomes measuring the \( \mathcal{N}_k \) and \( \mathcal{R}_j \); it requires. We restrict to the case where the pair project to only a small angle on the sky; for discussion of wide-angle effects, see Samushia, Percival & Raccanelli (2012) and Raccanelli et al. (2013). In this limit, we then approximate \( \mu = \mathbf{s} \cdot \mathbf{n} \) by \( \mathbf{s} \cdot \tilde{r}_1 \), i.e., approximating that the line of sight to the pair is very nearly the line of sight to one member (Yamamoto et al. 2006). We write

\[
\mathcal{N}_k(S) = (2k+1) \int s^2 ds \theta(s; S) \int d\tau_1 \mathcal{P}_k(\mathbf{s} \cdot \tilde{r}_1) N(r_1) N(r_1 + s) \quad (11)
\]

and analogously for \( \mathcal{R}_j \). The integral over \( d\tau \) averages over translations. Consolidating integrals, we find

\[
\mathcal{N}_k(s) = (2k+1) \int d\theta(s; r) \int d\tau_1 P_k(\mathbf{s} \cdot \tilde{r}_1) N(r_1) N(r_1 + s) \quad (12)
\]

We now show how to cast the inner integral as a convolution, which will then permit its evaluation via FTs. Using the spherical harmonic addition theorem (Arfken, Weber & Harris 2013, hereafter AWH13, equation 16.57) the integral becomes

\[
\mathcal{N}_k(s) = 4\pi \sum_{k=-k}^{k} \int ds \theta(s; S) \int d\tau_1 Y_{km}(\mathbf{s}) Y_{km}^* (\tilde{r}_1) N(r_1) N(r_1 + s) \quad (13)
\]

The approach here generalizes to any separable kernel inserted in place of the \( P_{\ell} \) above.

With the problem thus set up as a convolution, to compute a particular multipole \( \mathcal{N}_k \) we need \( 2\ell + 2 \) real forward FTs, one for the data minus randoms, \( N \), and then \( 2\ell + 1 \) for the independent components of the \( \mathcal{N}_k \). We then need \( 2\ell + 2 \) real inverse transforms after taking the products in Fourier space. Note that once the convolution is computed, we can perform all of the integrals over the binning as needed. Further, we can separate the real and imaginary components of the spherical harmonics and compute all of these \( 2\ell + 1 \) terms sequentially, which allows us to store only 3 copies of the full grid at a time (\( N \) and its FT, plus the working space for the convolution), while accumulating the resulting contributions to \( \mathcal{N}_k \).

The computation of \( \mathcal{R}_j \) proceeds identically. However, we note that because the \( \mathcal{R} \) pair count does not involve a near-cancellation as \( \mathcal{N} = (D - R)^2 \) does, one can use a substantially smaller random catalog when computing \( \mathcal{R}_j \).
Inspecting equation (11) shows that \( \mathcal{R}_0 \) appears as a normalization of the correlation function, while the \( f_s \) ratios only slightly mix terms. Moreover, when computing repeatedly on large numbers of mock catalogs, one would generally not need to repeat the computation of \( \mathcal{R} \) for each.

\section{3PCF with Fourier Transforms}

We now show how to use FTs to accelerate the algorithm for measuring multipole moments of the 3PCF presented in SE15. Note that here we consider only the isotropic 3PCF and do not track orientation to the line of sight. We first recall that this algorithm measures the binned radial coefficients in an expansion of the 3PCF as

\[ \zeta(S_1, S_2; \mathbf{r}_1 \cdot \mathbf{r}_2) = \sum \zeta(S_1, S_2) P_{\ell}(\hat{s}_1 \cdot \hat{s}_2). \]  

(SE15 equation 14), and finally averaged over all possible origins \( \mathbf{x} \) to find

\[ \zeta(S_1, S_2) = \frac{1}{V} \int d\mathbf{x} \zeta(S_1, S_2; \mathbf{x}) \]  

(SE15 equation 12).

Here we show that computing the local \( a_{\ell m}(S; \mathbf{x}) \) is simply a convolution and so can be accelerated with FTs. We begin with SE15 equation 14 for the \( a_{\ell m} \) about a particular origin of an arbitrary density field \( \delta \):

\[ a_{\ell m}(S; \mathbf{x}) = \int d\mathbf{r}' Y_{\ell m}^{*}(\mathbf{r}') \int r'^2 dr' \theta(|\mathbf{r}'|; S) \delta(\mathbf{r}' + \mathbf{x}). \]  

(17)

Consolidating integrals, this becomes

\[ a_{\ell m}(S; \mathbf{x}) = \int d\mathbf{r}' Y_{\ell m}^{*}(\mathbf{r}') \theta(|\mathbf{r}'|; S) \delta(\mathbf{r}' + \mathbf{x}), \]  

(18)

which clearly has the form of a convolution. Hence by the Convolution Theorem

\[ a_{\ell m}(S; \mathbf{x}) = \text{FT}^{-1} \left\{ \tilde{K}_{\ell m}(k; S) \tilde{\delta}(k) \right\} (\mathbf{x}), \]  

(19)

where again \( \tilde{\delta}(k) \) is the FT of the density field \( \delta(\mathbf{r}') \) and \( \tilde{K}_{\ell m}(k; S) \) is the FT of the kernel

\[ K_{\ell m}(r'; S) \equiv Y_{\ell m}^{*}(r') \theta(|r'|; S). \]  

(20)

Thus where in SE15 we needed an \( O(nV_{\text{max}}) \) operation about each possible origin \( \mathbf{x} \) to compute each \( a_{\ell m} \) for a given radial bin, and hence \( O(nN_{\text{max}}) \) operations total for each radial bin, we now simply need \( N_{\ell m} \log N_{\ell m} \) operations total to compute each \( a_{\ell m} \) in a given radial bin and for \textit{all} origins.

In detail, in SE15 the \( \zeta \) are expressed in terms of multipole moments of the \( NNN \) and \( RRR \) fields (see SE15 Section 4), which can be obtained using equations \[17\] - \[20\] with \( \delta = NNN \) and then \( \delta = RRR \) successively. If we want the multipole coefficients of, e.g., \( NNN \), up to \( \ell_{\text{max}} \) in \( N_{\ell m} \) radial bins, we need one real forward FT of the density field, \( N_{\ell m}(\ell_{\text{max}}+1)^2 \) real forward transforms for the kernels \( K_{\ell m} \), and finally this same number of real inverse transforms after taking products in Fourier space. The same holds for \( RRR \).

The kernel \( K_{\ell m} \) is simple and so its forward FTs can be done analytically, essentially halving the total number of FTs required. We have

\[ \tilde{K}_{\ell m}(k; S) = \int dr' e^{i \mathbf{k} \cdot \mathbf{r}'} \theta(|\mathbf{r}'|; S) Y_{\ell m}^{*}(\mathbf{r}') \]

\[ = (4\pi)^{\delta} Y_{\ell m}^{*}(\mathbf{k}) \tilde{\gamma}_{\ell m}(k; S). \]  

(21)

We expanded the plane wave using AWH13 equation 16.61, performed the angular integral by orthogonality, and defined

\[ \tilde{\gamma}_{\ell m}(k; S) = \int a^2 du j_{\ell}(ku) \theta(u; S). \]  

(22)

Analytically evaluating the kernel’s FTs is not always favorable. Doing the FTs all numerically treats data and kernel on the same footing: both will be gridded using, e.g., cloud-in-cell and then transformed. If one computes \( \tilde{K}_{\ell m} \) analytically, however, one must then grid in Fourier space so as to match the gridded, transformed data. Transforming and then gridding only reduces to gridding and then transforming in the limit of a very fine grid. Otherwise we expect this reordering of operations might introduce additional artifacts. However it is precisely in applications where the grid is extremely fine that the FT will take longest and so imply the greatest need to reduce the number of transforms by analytic evaluation of \( \tilde{K}_{\ell m} \).

\section{Discussion and Conclusions}

We have presented Fourier Transform methods for the computation of the 2PCF, anisotropic 2PCF, and 3PCF. For the 2PCF, we have shown that the familiar Landy-Szalay estimator can be immediately translated into an FT computation. We then show that the multipoles of the anisotropic 2PCF can be computed by FTs, despite the curvature of the sky relative to the Cartesian grid. After computing the multipoles of the \( NN \) numerator and \( RR \) denominator, one can easily transform to the multipoles of \( \xi_{\text{aniso}} \). For the 3PCF, we have shown that the SE15 estimator for the Legendre decomposition of the 3PCF can be computed with FTs.

In all cases, the resulting algorithm scales only linearly with the number of survey objects (or random samples) and only \( O(N_s ln N_s) \) with the grid size. For some important applications, this is faster than the \( O(N_s N_{\ell_{\text{max}}}) \) scaling of the pair-counting methods (and the SE15 3PCF method). The speed advantages are maximized when one considers larger separations, higher number densities, and coarser grids.

However, Fourier methods do introduce artifacts due to grid resolution, the level of which will depend on the ratio of the grid spacing to the radial bin width being used. For example, in the BAO analysis of Anderson et al. (2014), 2PCF separation bins of 8h^{-1} Mpc were used. One would then want the FT grid to be comfortably smaller than this. It is worth noting that the FT artifacts would be reduced if one used radial separation bins with smoother edges instead of the traditional tophats. Smoother bins are acceptable for science applications such as BAO, which is a smooth...
feature itself, and indeed are numerically more stable for
the spherical Bessel transforms needed to form model 2PCF
from theoretical power spectra. One might proceed by tun-
ing the bins to have negligible support beyond wavenum-
ber $k \approx 0.4 h \text{Mpc}^{-1}$, where the acoustic oscillations have
been damped away, while choosing an FT grid scale of $3-4h^{-1} \text{Mpc}$ so as to place the Nyquist frequency comfortably
above that smoothing scale.

We expect that an important application of these meth-
ods is to the calculation of correlation functions from mock
catalogs. The computation of covariance matrices now is
commonly performed by repeating the calculation on hun-
dreds or thousands of mock catalogs. This dominates the
computational effort of the cosmology analysis. But it is a
place where making a mild sacrifice in the accuracy of the
pair count may be acceptable to gain the speed FTs offer. An
interesting aspect of these FT approximations is that they
should converge to the pair-counting answer as the grid size
increases. One might opt to compute mocks with a reason-
ablely efficient grid, accepting an error that is small compared
to the survey variations, while still processing the actual
data with a finer grid or an explicit pair-counting code.

While we were preparing this work for submission,
Bianchi et al. (2015) and Scoccimarro (2015) posted pre-
prints suggesting use of FTs to compute the anisotropies of
the large-scale power spectrum about the line-of-sight, us-
ing respectively the Yamamoto et al. (2006) estimator and
a newly constructed estimator. Scoccimarro also uses FTs
to estimate the redshift-space bispectrum. The spherical harmonics can be directly translated to
Legendre polynomials, whereas we expand in spherical harmon-
ics. The mathematical approach of these works is the same as what we
present in Section 2 for the anisotropic 2PCF. In detail,
they present their results as polynomial expansions of Legendre polynomials, whereas we expand in spherical harmonics.
The spherical harmonics can be directly translated to
polynomials when computing (SE15 Section 2), and extensions to higher multipoles are likely more convenient to track with $Y_{lm}$. We note that the very large computational advantage reported in Bianchi et al. (2015) is specific to the power spectrum, which otherwise required summing over all pairs of survey objects. For the 2PCF, common methods only need
to count pairs within $V_{\text{max}}$, so the FT advantage over pair counting is more modest for realistic grid sizes.

The coming generation of large galaxy surveys will
stress our computational resources not simply because of
the survey size but also because of the drive for increasing
analysis accuracy, which manifests itself in larger numbers of mock catalogs and analysis variations. We believe that Fourier methods such as those presented here offer an important means of enhancing the computational speed of future cosmological analyses.

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