$$(\lambda \Phi^4)_4$$ theory on the lattice: evidence for a non-trivial rescaling of the scalar condensate.

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A lattice simulation in the broken phase of $(\lambda \Phi^4)_4$ theory in the Ising limit suggests that, in the continuum limit, the scalar condensate rescales by a factor different from the conventional wavefunction renormalization. Possible effects on the present bounds of the Higgs mass are discussed.

1. INTRODUCTION

It is widely believed that $[1-7]$ $(\lambda \Phi^4)_4$ theories are “trivial”. The conventional interpretation is based on leading-order Renormalization-Group-Improved-Perturbation-Theory (RG1PT). However, a quite different interpretation is advocated in Refs. $[8]$. A key feature of the alternative picture is the presence of a non-trivial rescaling of the “renormalized” vacuum field:

$$v_R \equiv v_B / \sqrt{Z_\varphi}.$$  (1)

The role of $Z_\varphi$ is essential. It provides the key ingredient to get a non-trivial effective potential in a “trivial” theory. In the continuum limit ($\Lambda \to \infty$)

$$Z_\varphi \sim \ln \frac{\Lambda}{M_h} \to \infty,$$  (2)

so that, although $M_h^2/v_B^2 \to 0$, one finds

$$\frac{M_h^2}{v_R^2} = \Lambda - \text{independent}.$$  (3)

On the other hand in the continuum limit $Z_{\text{prop}} \to 1$ consistently with the trivial nature of the shifted field.

In order to directly test the prediction that $Z_\varphi$ differs from $Z_{\text{prop}}$, we present the results of a lattice simulation of the theory (in the Ising limit) where we compute the mass and the residue $Z_{\text{prop}}$ from a 2-parameter fit to the lattice data for the shifted-field propagator. We then compute the zero-momentum susceptibility

$$\frac{1}{\chi} = \left. \frac{d^2 V_{\text{eff}}}{d \varphi_B^2} \right|_{\varphi_B = \pm v_B}$$  (4)

and hence obtain the dimensionless quantity

$$Z_\varphi \equiv M_h^2 \chi.$$  (5)

Finally, we compare $Z_\varphi$ with $Z_{\text{prop}}$.

2. NUMERICAL SIMULATIONS

The one-component $(\lambda \Phi^4)_4$ theory

$$S = \sum_x \left\{ \frac{1}{2} \sum_{\mu} [\Phi(x + \hat{e}_\mu) - \Phi(x)]^2 + \frac{r_0}{2} \Phi^2(x) + \frac{\lambda_0}{4} \Phi^4(x) - J \Phi(x) \right\}$$  (6)

becomes in the Ising limit

$$S_{\text{Ising}} = -\kappa \sum_x \sum_{\mu} [\phi(x + \hat{e}_\mu)\phi(x) + \phi(x - \hat{e}_\mu)\phi(x)]$$  (7)

with $\Phi(x) = \sqrt{2\kappa} \phi(x)$ and $|\phi(x)| = 1$.

The shifted field propagator, defined at $p_\mu \neq 0$, can be computed as

$$G(p) = \sum_x \exp(ipx)h(x)h(0)$$  (8)

for the values $p_\mu = \frac{2\pi}{L} n_\mu$ with $n_\mu \neq 0$. An excellent fit to the lattice data is obtained by using the 2-parameter formula

$$G(p) = \frac{Z_{\text{prop}}}{p^2 + m_{\text{latt}}^2}.$$  (9)
where $m_{\text{lat}}$ is the dimensionless lattice mass and
\[ \hat{p}_\mu = 2 \sin \frac{p_\mu}{2} \] (see Fig. 1).

The susceptibility $\chi$ is measured directly as
\[ \chi_{\text{lat}} = L^4 \left[ \langle \Phi^2 \rangle - \langle \Phi \rangle^2 \right] \] (10)
with $\Phi$ the average field for each lattice configuration. Moreover we define
\[ Z_\varphi \equiv m_{\text{lat}}^2 \chi_{\text{lat}}. \] (11)

To update our field configurations we used the Swendsen-Wang [13] cluster algorithm on $20^4$, $24^4$ and $32^4$ lattices. After discarding 10K sweeps for thermalization, we have performed 50K sweeps, measuring our observables every 5 sweeps. We have computed at different values of the hopping parameter $\kappa$ in order to obtain a correlation length $\xi_{\text{lat}} = 1/m_{\text{lat}}$ in the range 2 to $L/4$. The upper limit of the correlation length is required in order to avoid finite-size effects [14][15].

Our results for $Z_\varphi$ and $Z_{\text{prop}}$, in the broken phase are reported in Fig. 2, and show a sizeable difference for $m_{\text{lat}} < 0.3$.

We have performed a consistency check that no such effect is present in the symmetric phase (Fig. 3).

As an additional check, we have compared with available data in the literature [14][15] both in the symmetric and broken phase and found good agreement.

3. CONCLUSIONS

Our numerical simulation of $(\lambda \Phi^4)_4$, in the Ising limit, shows a clear difference between two measured quantities: the rescaling of the “condensate” $Z_\varphi$ and the more conventional quantity $Z_{\text{prop}}$, associated with the residue of the shifted field propagator. The effect shows up when increasing the correlation length and should become more and more important by approaching the continuum limit of quantum field theory $m_{\text{lat}} \to 0$. Therefore, the relation of the lattice vacuum field $\langle \Phi \rangle$ to the Fermi constant and the same limits on the Higgs mass can sizeably be affected. Indeed, these have been based on the quantity [16]
\[ R_{\text{prop}} = \frac{m_{\text{lat}}}{\langle \Phi \rangle} \sqrt{Z_{\text{prop}}} \] (12)
rather than
\[ R_\varphi = \frac{m_{\text{lat}}}{\langle \Phi \rangle} \sqrt{Z_\varphi}. \] (13)

The discovery of $Z_\varphi$ requires a “second generation” of lattice simulations to re-check the scaling behaviour of the various quantities and compare with all available theoretical descriptions of the continuum limit.
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