Neutrino elastic scattering on polarized electrons as tool for probing neutrino nature

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Abstract Possibility of using the polarized electron target (PET) for testing the neutrino nature is considered. One assumes that the incoming electron neutrino ($\nu_e$) beam is the superposition of left chiral states with right chiral ones. Consequently the non-vanishing transversal components of $\nu_e$ spin polarization may appear, both T-even and T-odd. $\nu_e$s are produced by the low energy monochromatic (un)polarized emitter located at a near distance from the hypothetical detector which is able to measure both the azimuthal angle and polar angle of the recoils, and/or also the energy of the outgoing electrons with a high resolution. A detection process is the elastic scattering of $\nu_e$s (Dirac or Majorana) on the polarized electrons. Left chiral (LC) $\nu_e$s interact mainly by the standard $V-A$ interaction, while right chiral (RC) ones participate only in the non-standard $V+A$, scalar $S_R$, pseudoscalar $P_R$ and tensor $T_R$ interactions. We show that a distinction between the Dirac and Majorana $\nu_e$s is possible both for the purely left chiral states and the left-right superposition. In the first case a departure from the standard prediction of the azimuthal asymmetry of recoil electrons is caused by the interferences between the non-standard complex S and T couplings, proportional to the angular correlations (T-even and T-odd) among the polarization of the electron target, the incoming neutrino momentum and the outgoing electron momentum. Such a deviation would indicate the Dirac nature of $\nu_e$s. In the second variant the azimuthal asymmetry, polar distribution and energy spectrum of scattered electrons are sensitive to the interference terms between the standard and exotic interactions, proportional to the various angular correlations (T-even and T-odd) among the transversal $\nu_e$ spin polarization (related to the $\nu_e$ source), the electron target polarization, the incoming $\nu_e$ momentum and the outgoing electron momentum. The basic difference between the Dirac and Majorana $\nu_e$s arises from the absence of T and V interactions in the Majorana scenario. Moreover, in the Majorana case the above observables contain the non-vanishing interference between $V-A$ and $V+A$ interactions, proportional to the T-even longitudinal $\nu_e$ polarization. Our model-independent study is carried out for the flavor $\nu_e$ eigenstates in the relativistic $\nu_e$ limit.

1 Introduction

One of the basic questions in the neutrino physics is whether the $\nu$s are the Dirac or Majorana fermions. At present the neutrinoless double beta decay is viewed as the main tool to investigate $\nu$ nature [1–3], however the purely leptonic processes (e.g. the neutrino-electron elastic scattering (NEES)) may also shed some light on this problem [4, 5]. Kayser and Langacker have analyzed the $\nu$ nature problem in the context of non-zero $\nu$ mass and of the standard model (SM) $V$-$A$ interaction [6–10] of only the LC vs. There is an alternative opportunity of distinguishing between the Majorana and Dirac $\nu$s by admitting the exotic $V+A$, scalar $S$, pseudoscalar $P$ and tensor $T$ interactions coupling to the LC and RC vs in the leptonic processes within the relativistic $\nu$ limit. The appropriate tests have been considered by Rosen [11] and Dass [12]. It is also worthwhile noticing the other interesting papers devoted to the $\nu$ nature [13–18]. The above ideas involve the unpolarized detection target. When the target-electrons are polarized by an external magnetic field, one has possibility of changing the rate of weak interaction by inverting the direction of magnetic field. This feature is very important in the detection of low energy $\nu$s because the background level would be precisely controlled [19]. PET seems to be a more sensitive laboratory for probing the $\nu$ nature and time reversal symmetry violation in the leptonic processes (TRSV) than the unpolarized...
target due to the mentioned control of contribution of the interaction to the cross section. It is worth reminding that the PET has been proposed to test the flavor composition of (anti)neutrino beam [20] and various effects of non-standard physics. We mean the neutrino magnetic moments, TRSV in the leptonic processes [21], axions, spin–spin interaction in gravitation [22–28] The possibility of using polarized targets of nucleons and of electrons for the fermionic, scalar and vector dark matter detection is also worth noticing [29–31]. The methods of producing the spin-polarized gases such as helium, argon and xenon are described in [32, 33].

It is also essential to mention the measurements confirming the possibility of realizing the polarized target crystal of Gd$_2$SiO$_4$ (GSO) doped with Cerium (GSO:Ce) [34].

Let us recall that there is no difference between the Dirac and Majorana vs in the case of NEES with the standard V-A interaction in the relativistic limit, when the target is unpolarized. In addition the standard couplings have to be the real numbers as a consequence of the hermiticity condition of interaction lagrangian for the NEES.

The SM does not allow to clarify the origin of parity violation, observed barion asymmetry of universe [35] through a single CP-violating phase of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix (CKM) [36] and other fundamental problems. This situation led to the appearance of many non-standard models: the left-right symmetric models (LRSM) [37–41], composite models [42–44], models with extra dimensions (MED) [45], the unparticle models (UP) [46–58] and other schemes outside the SM [59–82]. It is also noteworthy that the current experimental results still leave some space for the scenarios with the exotic interactions.

Recently the study of the $\nu$ nature with a use of PET in the case of standard V-A interaction, when the evolution of $\nu$ spin polarization in the astrophysical environments is admitted, has been carried out in [83].

In this paper we consider the elastic scattering of low energy $\nu_e$ (\~{}1 MeV) on the polarized electrons of target in the presence of non-standard complex scalar, pseudoscalar, tensor couplings and $V + A$ interaction as a useful tool for testing the $\nu$ nature. We show how the various types of azimuthal asymmetry, the polar distribution and the energy spectrum of scattered electrons enable to distinguish between the Dirac and Majorana $\nu_e$s both for the purely left chiral states (only longitudinal $\nu_e$ polarization ($\vec{\eta}_L^{\nu}$)\(^{\perp}\)) and the left–right superposition (non-zero transversal $\nu_e$ polarization ($\vec{\eta}_L^{\nu}$)\(^\perp\), taking into account TRSV. Our study is model–independent and carried out for the flavor–eigenstate (Dirac and Majorana) $\nu_e$s in the relativistic limit. One assumes that the monochromatic low energy and (un)polarized $\nu_e$ emitter with a high activity is placed at a near distance from the detector (or at the detector centre). The hypothetical detector is assumed to be able to measure both the azimuthal angle $\phi_e$ and polar angle $\theta_e$ of the recoil electrons, and/or also the energy of the outgoing electrons with a high resolution, Fig.1.

We utilize the experimental values of standard couplings: $c_{V}^{L} = 1 + (\pm 0.04 \pm 0.015)$, $c_{A}^{L} = 1 + (\pm 0.507 \pm 0.014)$ to evaluate the predicted effects [84]. The laboratory differential cross sections (see Appendix I for the Majorana $\nu_\chi$s and [21] for the Dirac case) are calculated with the use of the covariant projectors for the incoming $\nu_e$s (including both the longitudinal and transversal components of the spin polarization) in the relativistic limit and for the polarized target-electrons, respectively [85].

### 2 Elastic scattering of Dirac electron neutrinos on polarized electrons

We analyze a scenario in which the incoming Dirac $\nu_e$ beam is assumed to be the superposition of LC states with RC ones. The detection process is the elastic scattering of Dirac $\nu_e$s on the polarized target-electrons. LC $\nu_e$s interact mainly by the standard $V - A$ interaction and small admixture of

![Fig. 1 Production plane of the $\nu_e$ beam is spanned by the polarization unit vector $\hat{S}$ of source and the $\nu_e$ LAB momentum unit vector $\hat{q}$. Reaction plane is spanned by $\hat{q}$ and the transverse electron polarization vector of target ($\hat{\eta}$)\(^{\perp}\) for $\nu_e + e^- \rightarrow \nu_e + e^\rightarrow$. $\theta_1 = \pi/2$ is the angle between the orientation of polarization of the electron target $\hat{\eta}$ and $\hat{q}$, so $\hat{\eta} \cdot \hat{q} = 0$ and $\hat{\eta} = (\hat{\eta})^{\perp}$. $\theta_2$ is the polar angle between $\hat{q}$ and the unit vector $\hat{p}_{\nu}$ of recoil electron momentum. $\phi_3$ is the angle between $(\hat{\eta})^{\perp}$ and the transversal component of outgoing electron momentum $(\hat{q})^{\perp} \cdot \hat{\eta} = (\sin \theta_2 \cos \phi_3, \sin \theta_2 \sin \phi_3, \cos \theta_2)$.](attachment:image.png)
non-standard scalar $S_L$, pseudoscalar $P_L$, tensor $T_L$ interactions, while RC ones take part only in the exotic $V + A$ and $S_R, P_R, T_R$ interactions. As a result of the superposition of the two chiralities the spin polarization vector have the nonvanishing transversal polarization components, which may give rise to both T-even and T-odd effects. As an example of process in which the transversal $\nu$ polarization may be produced, we refer to the ref. [86], where the muon capture by proton has been considered. The amplitude for the $\nu_e e^-$ scattering in low energy region is of the form:

$$M^{D}_{\nu_e e^-} = \frac{G_F}{\sqrt{2}} \left\{ (\bar{\nu}_e \gamma^\mu (c_L^e - c_A^e) u_e)(\bar{\nu}_\nu, \gamma^\mu (1 - \gamma_5) u_{\nu_e}) + e^e_{LR} (\bar{\nu}_e, \gamma^\mu (1 + \gamma_5) u_e)(\bar{\nu}_\nu, \gamma^\mu (1 + \gamma_5) u_{\nu_e}) + e^e_{RR} (\bar{\nu}_e, \gamma^\mu (1 - \gamma_5) u_e)(\bar{\nu}_\nu, \gamma^\mu (1 - \gamma_5) u_{\nu_e}) + 1/2 (\bar{\nu}_e \gamma^\mu (1 - \gamma_5) u_e)(\bar{\nu}_\nu, \gamma^\mu (1 - \gamma_5) u_{\nu_e}) + 1/2 (\bar{\nu}_e \gamma^\mu (1 + \gamma_5) u_e)(\bar{\nu}_\nu, \gamma^\mu (1 + \gamma_5) u_{\nu_e}) \right\},$$

where $G_F = 1.1663788(7) \times 10^{-5} \text{GeV}^{-2} (0.6 \text{ ppm})$ [87] is the Fermi constant. The coupling constants are denoted as $c^e_L, c^e_A, c^e_S, c^e_P, c^e_T$ respectively to the incoming $\nu_e$ of left- and right-handed chirality. All the non-standard couplings $c^e_S, c^e_P, c^e_T$ are the complex numbers denoted as $c^e_S = |c^e_S| e^{i\theta_5}, c^e_P = |c^e_P| e^{i\theta_5}, \text{etc.}$, $c^e_T, c^e_L$ coupling constants are the real numbers as a consequence of hermitian interaction lagrangian. Moreover, we take into account the relations between the non-standard complex couplings with left- and right-handed chirality appearing at the level of interaction lagrangian: $c^e_{S,T,P} = c^e_{S,T,P}$. 

3 Elastic scattering of Majorana electron neutrinos on polarized electrons

The fundamental difference between the Majorana and Dirac $\nu_e$ arises from a fact that the Majorana $\nu_e$s do not participate in the vector $V$ and tensor $T$ interactions. This is a direct consequence of the $(u, l)$-mode decomposition of the Majorana field. The amplitude for the NEES on the PET for the Majorana low energy $\nu_e$s is as follows:

$$M^{D}_{\nu_e e^-} = \frac{2G_F}{\sqrt{2}} \left\{ (\bar{\nu}_e \gamma^\mu (c_v - c_A^e) u_e)(\bar{\nu}_\nu, \gamma^\mu (1 - \gamma_5) u_{\nu_e}) + (\bar{\nu}_e \gamma^\mu (c_v + c_A^e) u_e)(\bar{\nu}_\nu, \gamma^\mu (1 + \gamma_5) u_{\nu_e}) + (\bar{\nu}_e \gamma^\mu (1 - \gamma_5) u_e)(\bar{\nu}_\nu, \gamma^\mu (1 - \gamma_5) u_{\nu_e}) + (\bar{\nu}_e \gamma^\mu (1 + \gamma_5) u_e)(\bar{\nu}_\nu, \gamma^\mu (1 + \gamma_5) u_{\nu_e}) \right\}.$$

We see that the $\nu_e$ contributions from $A, S, P$ are multiplied by the factor of 2 as a result of the Majorana condition. The indexes $L, (R)$ for the standard $V - A$ and non-standard $V + A$ interactions are omitted. It means that both LC and RC $\nu_e$s may take part in the above interactions. All the other assumptions are the same as for the Dirac case.

4 Distinguishing between Dirac and Majorana neutrinos through azimuthal asymmetries of recoil electrons

In this section we analyze the possibility of distinguishing the Dirac from Majorana $\nu_e$s through probing the azimuthal
Fig. 3 Dirac (or Majorana) $\nu_e$ with $V-A$ interaction: dependence of $d^2\sigma/d\theta_e d\theta$ on $\phi_e$ for $\hat{n}_V \cdot \hat{q} = -1$, $E_V = 1\,\text{MeV}$, $\theta_1 = \pi/2$; $\theta_2 = \pi/12$ (dotted line); $\theta_3 = \pi/6$ (solid line); $\theta_4 = \pi/3$ (dashed line).

Fig. 4 Presence of non-standard couplings with $\hat{n}_V \cdot \hat{q} = -1$: dependence of $d^2\sigma/d\phi_e$ on $\phi_e$ for different values of $\theta_1$ and $E_V = 1\,\text{MeV}$; upper plot for the case of Dirac $\nu_e$ with $V-A$ and $S_R$ when $|\epsilon^d_1| = 0.4$, $\theta_3 = 0$; lower plot for Majorana $\nu_e$ with $V-A$ and $V+A$ when $|\epsilon^m_3| = |\epsilon^m_4| = 0.4$; dotted line for $\theta_1 = \pi/4$ solid line for $\theta_1 = \pi/2$; dashed line for $\theta_1 = 3\pi/4$.

Fig. 5 Dirac $\nu_e$, presence of non-standard couplings with $\hat{n}_e \cdot \hat{q} = -1$: upper plot; dependence of $\Phi_{\text{max}}$ on $\Delta \theta_{\text{PT}}$ with $\theta_1 = \pi/2$, $E_V = 1\,\text{MeV}$ in case of $V-A$ with $S_R$ and $T_R$ when $|\epsilon^d_1| = |\epsilon^d_2| = 0.2$ (dashed line); plot of $\Phi_{\text{max}}$ on $\Delta \theta_{\text{PT}}$ for $\theta_1 = \pi/2$, $E_V = 1\,\text{MeV}$ in case of $V-A$ with $P_R$ and $T_R$ when $|\epsilon^d_1| = |\epsilon^d_2| = 0.2$ (dotted line); lower plot; dependence of $A(\Phi_{\text{max}})$ on $\Delta \theta_{\text{PT}}$ (dashed line) and on $\Delta \theta_{\text{PT}}$ (dotted line), respectively, with same assumptions as for $\Phi_{\text{max}}$.

Fig. 6 Presence of non-standard couplings with $\hat{n}_e \cdot \hat{q} = -1$ for Majorana $\nu_e$: plot of $A_{\Phi}(\Phi_{\text{max}})$ as a function of $\theta_1$ for the case of $V-A$ with $V+A$ when $E_V = 1\,\text{MeV}$; solid line for $\theta_1 = \pi/18$; dotted line for $\theta_1 = \pi/2$; dashed line for $\theta_1 = 17\pi/18$. 

asymmetries of recoil electrons (defined in the Appendix 2).

Let us remind that the Dirac and Majorana $\nu_s$ cannot be discriminated in the case of V-A interaction with $\hat{n}_e \cdot \hat{q} = -1$ in the relativistic $v$ limit, even if the target-electrons are polarized. The illustration of this regularity are the Fig. 2 and Fig.3. The Fig. 2 shows how the asymmetries $A_{\nu_s}(\Phi_{\max})$, $A_{\theta_s}(\Phi_{\max})$ depend on the angle $\theta_s$ between $\hat{n}_e$ and $\hat{q}$. For simplicity, Fig. 1 is made for $\theta_s = \pi/2$. We see that the maximum values of $A_{\nu_s}(\Phi_{\max})$ and $A_{\theta_s}(\Phi_{\max})$ grow from 0.008 at $\theta_s = \pi/6$ for $\theta_s = \pi/18$ (upper plot) to 0.42 at $\theta_s = \pi/12$ for $\theta_s = 17\pi/18$ (lower plot). Although the magnitude of the asymmetries may change, orientation of the asymmetry axis is fixed at $\Phi_{\max} = \pi/2$. This is also illustrated on the plot of $d^2\sigma/d\Phi_d d\theta_e$ in Fig.3.

When one departs from the pure V-A interaction and, still assuming fully longitudinal polarization of incoming $\nu_s$ (â€œâ€œ $\hat{n}_e \cdot \hat{q} = -1$), one introduces the non–standard couplings in the detection process, the asymmetries $A(\Phi_{\max})$, $A_{\nu_s}(\Phi_{\max})$ and $A_{\theta_s}(\Phi_{\max})$ can distinguish between the Dirac and Majorana $\nu_s$ in the vanishing $\nu_s$ mass limit. The Fig. 4 displays that the azimuthal distribution of recoil electrons for the Dirac $\nu_s$ has the maximum at $\phi_s = \pi$ for $\theta_s = \pi/4, \pi/2, 3\pi/4$ (upper plot), while for the Majorana ones the maximum is shifted to $\phi_s = 0 = 2\pi$ (lower plot). The presence of non–standard S,T,P complex couplings of Dirac $\nu_s$ produces, among other terms, the non–vanishing triple angular correlations composed of $\hat{q}, \hat{p}_e, (\hat{n}_e)^{-1}$ vectors. It allows to search
for the effects of TRSV in the NEES. In the Majorana case the interference between $V - A$ and $V + A$ interactions proportional to $T$-even correlations only survives. The Fig. 5 shows how the asymmetry axis location $\Phi_{\text{max}}$ (upper plot) and the magnitude of $A(\Phi_{\text{max}})$ (lower plot) depend on the phase differences $\Delta \theta_{ST,R} = \theta_{c,R} - \theta_{T,R}$ (dashed lines) and $\Delta \theta_{SD,R} = \theta_{P,R} - \theta_{TR}$ (dotted lines) for $\theta_1 = \pi/2$. For illustrative purposes, we present the formula on $A(\Phi)$ with $\theta_1$ dependence for the Dirac scenario with $V - A$, $SR$ and $TR$ interactions when $\theta_v = \pi$, assuming the experimental values of standard couplings, $E_v = 1\, MeV$, $|c_\beta|^2 = |c_\gamma|^2 = 0.2$:

\[
A_{\Phi_{\text{max}}} = \left\{ \begin{array}{l}
1.699 \sin \theta_1 (0.362 \sin(\Delta \theta_{ST,R} - \Phi))
+ 0.04 \sin(\Delta \theta_{ST,R} + \Phi) - 3.07 \sin(\Phi) \\
- 2(1.338 \cos(\Delta \theta_{ST,R} - 30.531)) \cos \theta_1 \\
- 0.885 \cos \Delta \theta_{ST,R} + 82.191 \end{array} \right.
\]

(3)

We see that for $\theta_1 = 0 = \pi$ the asymmetry vanishes. The case of $V - A$ with $P_R$ and $T_R$ interactions when $\theta_v = \pi$ has been added to show the differences between both scenarios. The observation of departure of the asymmetry axis location from $\Phi = \pi/2$ would indicate the Dirac $\nu_e$ and signalize the possibility of TRSV.

The other asymmetry $A_{\nu}(\Phi_{\text{max}})$ shown in Fig. 6 may attain the extreme values close to 1 at $\theta_w = 5\pi/18$ (dashed line) for the Majorana scenario with $V - A$ and $V + A$ interactions when $\theta_1 = \pi$ and $\theta_1 = 17\pi/18$. The standard value is expected to be 0.42 at $\theta_1 = \pi/12$.

When one assumes that the incoming $\nu_e$ beam is the superposition of LC vs with RC ones and there is the experimental control of angle $\phi_v$ connected with $(\hat{\eta}_v)_{\perp}$ shown in the Fig.1, we have a new possibility of testing the $\nu_e$ nature and the TRSV by probing the dependence of $A(\Phi_{\text{max}})$ and the asymmetry axes location $\Phi_{\text{max}}$ on $\phi_v$. The Fig. 7 illustrates the effects for the scenario with $V - A$ and $SR$ interactions. The detection of such regularity would indicate the existence of exotic scalar couplings of RC vs. The precise measurement of magnitude of $A(\Phi_{\text{max}})$ would help to distinguish between the Dirac and Majorana $\nu_e$, and detect the TRSV.

The Fig. 8 displays the impact of $\theta_1$ on the possibility of distinction between the Dirac and Majorana $\nu_e$ by measuring the asymmetry $A_{\theta_1}(\Phi_{\text{max}})$ at fixed location of $(\hat{\eta}_v)_{\perp}$ ($\phi_v = 0$) for the variant of $V - A$ with $SR$ interactions. The maximum values of $A_{\theta_1}(\Phi_{\text{max}})$ for $\theta_1 = 0$ increase to 0.02 in the Dirac case (solid line in upper plot), and to 0.04 in the Majorana case (right upper plot) in comparison to the standard expectation of 0.008, when $\theta_1 = \pi/2$ the magnitude of $A_{\theta_1}(\Phi_{\text{max}})$ may decrease to around 0.01 for the Majorana $\nu_e$ (solid line in middle right plot), while the standard prediction gives 0.08 at $\theta_1 = \pi/6$. When the TRSV takes place the maximum value of $A_{\theta_1}(\Phi_{\text{max}})$ for $\theta_1 = \pi$ decreases to around 0.2 for the Majorana $\nu_e$ (solid line in lower right plot), and to around 0.05 in the Dirac case (dashed line in lower left plot) compared to the standard expectation of 0.42 at $\theta_1 = \pi/12$. The change of configuration $\phi_v$ causes the change of values of $A_{\theta_1}(\Phi_{\text{max}})$, Fig. 9.

It is necessary to point out that from an experimental point of view a searching for the differences between the Dirac and Majorana $\nu_e$ by the measurement of observables dependent on $(\hat{\eta}_v)_{\perp}$ related to the production process would be extremely difficult. In order to measure $A_{\theta_1}(\Phi_{\text{max}})$ one should determine the location of $\Phi_{\text{max}}$ by counting the events along the azimuthal angle (at fixed $\theta_1$ for any configuration of $\phi_v$) from $\Phi$ to $\Phi + \pi$ and from $\Phi + \pi$ to $\Phi + 2\pi$ for various $\Phi$; in this way $\Phi_{\text{max}}$ and $A_{\theta_1}(\Phi_{\text{max}})$ would be found according to its definition. These measurements have to be repeated for different $\theta_1$.s. The drawn curve with respect to $\theta_1$ should fit to one of the curves on the Fig. 6. The measurement of $A(\Phi_{\text{max}})$ would proceed in the similar way as above, but now $\theta_1$ is not fixed (azimuthal orientation of $(\hat{\eta}_v)_{\perp}$ of the incoming $\nu_e$ described by $\phi_v$ is fixed instead). One counts events along azimuthal angle from $\Phi$ to $\Phi + \pi$ and from $\Phi + \pi$ to $\Phi + 2\pi$ for all $\theta_1$.s. The repetition of the measurements for different $\phi_v$ would give the curve with respect to $\phi_v$ which should fit to a one of the curves on the Fig. 7.

5 Distinguishing between Dirac and Majorana neutrinos via spectrum and polar angle distribution of scattered electrons

In this section we explore the $\nu_e$ nature problem by using the electron energy spectrum and polar angle distribution of scattered electrons. To begin with, it is worth recalling that
Fig. 11 Dirac (or Majorana) $\nu_e$ with $V-A$ interaction $\hat{n}_V \cdot \hat{q} = -1$, $E_\nu = 1\text{MeV}$: plot of $d\sigma/dy$ as a function of $y$ for different values of $\theta_1$; solid line for $\theta_1 = 0$; dashed line for $\theta_1 = \pi/2$; dotted line for $\theta_1 = \pi$.

Fig. 12 Superposition of LC $\nu_e$s with RC ones in presence of nonstandard couplings with $\hat{n}_V \cdot \hat{q} = -0.95$: dependence of $d\sigma/dy$ on $y$ for $E_\nu = 1\text{MeV}$, $\phi_V = 0$, $\theta_1 = \pi/2$. Upper plot for TRSC: standard $V - A$ interaction (solid line); Dirac $\nu_e$ with $V - A$ and $T_R$ when $|e|^2 = 0.3$, $\theta_{SR} = 0$ (dashed-dotted line); Dirac case of $V - A$ and $S_R$ when $|e|^2 = 0.3$, $\theta_{SR} = 0$ (dotted line); Majorana $\nu_e$ for $V - A$ with $S_R$ when $|e|^2 = 0.3$, $\theta_{SR} = \pi/2$ (dashed line); Dirac case of $V - A$ and $S_R$ when $|e|^2 = 0.3$, $\theta_{SR} = \pi/2$ (dotted line); Majorana $\nu_e$ for $V - A$ with $S_R$ when $|e|^2 = 0.3$, $\theta_{SR} = \pi/2$ (dashed line).

Fig. 13 Superposition of LC $\nu_e$s with RC ones in presence of nonstandard couplings with $\hat{n}_V \cdot \hat{q} = -0.95$: dependence of $d\sigma/d\theta_2$ as a function of $\theta_2$ for $E_\nu = 1\text{MeV}$, $\phi_V = 0$, $\theta_1 = \pi/2$. Upper plot for TRSC: standard $V - A$ interaction (solid line); Dirac $\nu$ with $V - A$ and $T_R$ when $|e|^2 = 0.3$, $\theta_{SR} = 0$ (dashed-dotted line); Dirac case of $V - A$ and $S_R$ when $|e|^2 = 0.3$, $\theta_{SR} = 0$ (dotted line); Majorana $\nu_e$ for $V - A$ with $S_R$ when $|e|^2 = 0.3$, $\theta_{SR} = 0$ (dashed line). Lower plot for TRSV: standard $V - A$ interaction (solid line); Dirac $\nu$ with $V - A$ and $T_R$ when $|e|^2 = 0.3$, $\theta_{SR} = \pi/2$ (dashed-dotted line); Dirac case of $V - A$ and $S_R$ when $|e|^2 = 0.3$, $\theta_{SR} = \pi/2$ (dotted line); Majorana $\nu_e$ for $V - A$ with $S_R$ when $|e|^2 = 0.3$, $\theta_{SR} = \pi/2$ (dashed line).

The above observables do not allow to differentiate between the Dirac and Majorana $\nu_e$s in the case of standard $V-A$ interaction in the relativistic limit; see Figs. (10-11) which are made for $\theta_1 = 0, \pi/2, \pi$.

If one assumes that the $\nu_e$ source produces the superposition of LC with RC and one has the fixed location of $(\hat{n}_V)^{-1}$ with respect to the production plane, the cross sections $d\sigma/d\theta_e$, $d\sigma/dy$ for the detection of Dirac and Ma-
for different values of \( T - \text{odd} \) among proportional to the various angular correlations \( T - \text{even} \) and \( \theta \).

Fig. 14 Superposition of LC \( \nu_S \) with RC ones in presence of nonstandard couplings with \( \tilde{\eta}_V \cdot \hat{q} = -0.95 \): dependence of \( d\sigma/d\theta \), as a function of \( \theta \), for different values of \( \theta_1 \) when \( E_{\nu} = 1\,\text{MeV} \), \( \phi_\nu = 0 \). Upper plot for \( \theta_1 = 0 \); middle plot for \( \theta_1 = \pi/2 \); lower plot for \( \theta_1 = \pi \); dashed line for Dirac \( \nu_L \) with \( V - A \) and \( T_R \), \( |\epsilon_\phi^R| = 0.3 \), \( \theta_{S,R} = 0 \); dotted line for Majorana \( \nu_L \) with \( V - A \) and \( S_R \), \( |\epsilon_\phi^R| = 0.3 \), \( \theta_{S,R} = 0 \); solid line for Dirac \( \nu_L \) with \( V - A \) and \( S_R \) when \( |\epsilon_\phi^R| = 0.3 \), \( \theta_{S,R} = 0 \).

Fig. 15 Superposition of LC \( \nu_S \) with RC ones in presence of nonstandard couplings with \( \tilde{\eta}_V \cdot \hat{q} = -0.95 \): dependence of \( d\sigma/d\theta \), as a function of \( \theta \), for different values of \( \theta_1 \) when \( E_{\nu} = 1\,\text{MeV} \), \( \phi_\nu = 0 \). Upper plot for Dirac \( \nu_L \) with \( V - A \) and \( S_R \) when \( |\epsilon_\phi^R| = 0.4 \), \( \theta_{S,R} = 0 \); solid line for \( \theta_1 = 0 \); dotted line for \( \theta_1 = \pi/2 \); dashed line for \( \theta_1 = \pi \). Lower plot for Majorana \( \nu_L \) with \( V - A \) and \( S_R \) when \( |\epsilon_\phi^R| = 0.4 \), \( \theta_{S,R} = 0 \); solid line for \( \theta_1 = 0 \); dotted line for \( \theta_1 = \pi/2 \); dashed line for \( \theta_1 = \pi \).

Majorana \( \nu_S \) contain the interferences between the standard couplings of LC \( \nu_S \) and non–standard couplings of RC ones proportional to the various angular correlations \( T \)-even and \( T \)-odd) among \( (\tilde{\eta}_V)^\pm \), \( \hat{q}, \hat{p}_e, (\tilde{\eta}_V)^\pm \) vectors. Consequently the linear contributions from the non–standard interactions allow to distinguish between the Dirac and Majorana \( \nu_S \), and search for the TRSV, Figs. (12–16). The significant differences in the low energy region of recoil electrons spectrum for the Dirac \( \nu_S \) with \( V - A \) and \( T_R \) interactions, both for

\[ \text{TRSC (dashed-dotted line in upper plot) and TRSV (dashed-dotted line in lower plot) when } \theta_1 = \pi/2 \text{ can be noticed, Fig. 12. The polar distribution of scattered electrons seen in the Fig. 13 displays a similar departure for the same scenario. The Fig. 14 shows how the change of } \theta_1 \text{ affects the energy spectrum of recoil electrons in the presence of interferences related to } (\tilde{\eta}_V)^\pm, \text{ both for the Dirac and Majorana } \nu_S. \text{ The noticeable deviations for the low energy recoil electrons in the case of Dirac scenario with } V - A \text{ and } T_R \text{ interactions when } \theta_1 = 0, \pi/2 \text{ are seen (dashed line in upper and middle plots). The departure from the standard prediction in the Majorana case with } V - A \text{ and } S_R \text{ interactions when } \theta_1 = \pi \text{ is visible in the high energy scattered electrons region (dotted line in lower plot). The Figs. (15-16) show the impact of } \theta_1 \text{ on the magnitude of effects caused by the contributions.} \]
Fig. 16 Superposition of LC νs with RC ones for Dirac νc with V − A and T_R. [\textbf{c}] = 0.4, \theta_{\gamma R} = 0 and \hat{\eta}_\nu \cdot \hat{q} = -0.95; plot of d\sigma/d\theta_e as a function of \theta_e for different values of \theta_1 when \E_v = 1\,\text{MeV}, \phi_v = 0; solid line for \theta_1 = 0; dotted line for \theta_1 = \pi/2; dashed line for \theta_1 = \pi.

Fig. 17 Presence of non–standard couplings with \hat{\eta}_\nu \cdot \hat{q} = -1; plot of d\sigma/d\theta_e as a function of \theta_e for different values of \theta_1 and \E_v = 1\,\text{MeV}; upper plot for the case of Dirac νc with V − A and S_R when |\textbf{c}^\eta| = 0.4, \theta_{\gamma R} = 0; lower plot for Majorana νc with V − A and V + A when |\textbf{c}_V| = |\textbf{c}_A| = 0.4; dotted line for \theta_1 = \pi/4 solid line for \theta_1 = \pi/2; dashed line for \theta_1 = 3\pi/4.

Fig. 18 Presence of non–standard couplings with \hat{\eta}_\nu \cdot \hat{q} = -1; plot of d\sigma/d\theta_e as a function of \theta_e for different values of \theta_1 and \E_v = 1\,\text{MeV}; upper plot for the case of Dirac νc with V − A and S_R when |\textbf{c}^\eta| = 0.4, \theta_{\gamma R} = 0; lower plot for Majorana νc with V − A and V + A when |\textbf{c}_V| = |\textbf{c}_A| = 0.4; dotted line for \theta_1 = \pi/4 solid line for \theta_1 = \pi/2; dashed line for \theta_1 = 3\pi/4.

connected with (\hat{\eta}_\nu)^\perp in the case of polar distribution of recoil electrons. The shape of distributions is similar for the Dirac and Majorana νc s with V − A and S_R interactions, but the maximum values are slightly larger in the Majorana case Fig. 15. The Dirac scenario with V − A and T_R interactions illustrated in the Fig. 16 would mean that the maximum values of distribution should be observed for different values of \theta_1 than the standard location, Fig. 10. It is worth noting that all the Figs. (12-16) have been made at fixed configuration of azimuthal angle \phi_v = 0. Each change of \phi_v would affect the shape and maximum values of polar distributions. The similar changes would manifest in the case of spectrum.

When the incoming νc beam has only longitudinal component of polarization, i.e. \hat{\eta}_\nu \cdot \hat{q} = -1, there is still the opportunity of distinguishing between the Dirac and Majorana νc s, but the effects of TRSV can not be observed due to the annihilation of interferences between the standard and exotic couplings proportional to the T-odd correlations, Fig. (17-18). Both plots illustrate the influence of \theta_1 on the possibility of distinguishing between the Dirac and Majorana νc s for the various scenarios. The polar distribution of recoil electrons d\sigma/d\theta_e for the Dirac νc s with V − A and S_R interactions has maximum at \theta_e \simeq 0.7 for \theta_1 = \pi/4, while for
the Majorana $\nu_e$ with $V - A$ and $V + A$ interactions $d\sigma / d\theta_c$ achieves maximum at $\theta_c \simeq 0.5$ for the same $\theta_1$. Moreover, the maximum values of both distributions are different for a given $\theta_1$, Fig. 17. The differences on the recoil electrons spectrum are enough visible for the low energy recoil electrons and in the high energy region for $y \in [0.5, 0.8]$, Fig. 18.

6 Conclusions

We have shown that the various types of the azimuthal asymmetries of recoil electrons, the energy spectrum and the polar angle distribution of scattered electrons are sensitive to the differences between the effects caused by the Dirac and Majorana $\nu_e$ sources interacting with PET in the presence of exotic interactions, both for $\theta_1 = \pi$ and $\theta_1 \neq \pi$ with $|\hat{\eta}_e|^\dagger \neq 0$. The high-precision measurements of these quantities can shed some light on the fundamental problems of $
u$ nature and TRS in the leptonic processes. It is necessary to stress that new tests require the strong low-energy (monochromatic) $\nu_e$ sources, the large PET, and the detectors sensitive to the measurement of the azimuthal angle and polar angle of recoil electrons with the high angular resolution. The proposals of this type of detectors have been discussed in the literature [88–93]. The high-resolution measurements of the spectrum of low energy outgoing electrons need the detectors with the ultra low threshold and background. The interesting concepts of various (monochromatic) $\nu_e$ sources are also worth noticing [94–99]. A preliminary study on the feasibility of electron polarized scintillating GSO target has been carried out by [34]. In order to make the detection of $|\hat{\eta}_e|^\dagger$–dependent effects feasible, further studies on the appropriate choice of $\nu_e$ source, in which the exotic couplings of RC $\nu_e$s in addition to the LC ones take part, are needed to explain the basic role of production process in generating $\nu_e$ beam with non-zero $(\hat{\eta}_e)^\dagger$ and in controlling the angle $\phi_e$, Fig. 1. Today the controlled production of $\nu_e$ beam with the fixed direction of $(\hat{\eta}_e)^\dagger$ with respect to the production plane is still impossible, so the variant with the use of unpolarized $\nu_e$ source generating only the longitudinally polarized $\nu_e$s seems to be more available.

7 Appendix 1- General formula on laboratory differential cross section for elastic scattering of Majorana $\nu_e$s on PET

The laboratory differential cross section for the Majorana $\nu_e$s, when $\hat{\eta}_e \perp \hat{q}$ ($\theta_1 = \pi/2$), is of the form:

$$\frac{d^2\sigma}{dyd\phi_c} = \left(\frac{d^2\sigma}{dyd\phi_c}\right)_{V-A} + \left(\frac{d^2\sigma}{dyd\phi_c}\right)_{V+A}$$  \hspace{1cm} (4)
\[
(\frac{d^2 \sigma}{d\eta d\Phi})_{\text{low energy}} = B \left\{ \frac{y}{2} \left[ \eta \cdot \hat{\Phi} \eta \cdot \hat{\Phi} + \eta \cdot \hat{\Phi} \eta \cdot \hat{\Phi} \right] + \eta \cdot \hat{\Phi} \eta \cdot \hat{\Phi} \right\},
\]

\[
\left( \frac{d^2 \sigma}{d\eta d\Phi} \right)_{\text{V-A}} = 2B \left\{ 2e \left( \frac{2y}{2} + y \right) \left[ \eta \cdot \hat{\Phi} \eta \cdot \hat{\Phi} + \eta \cdot \hat{\Phi} \eta \cdot \hat{\Phi} \right] + \eta \cdot \hat{\Phi} \eta \cdot \hat{\Phi} \right\},
\]

where \( T_e \) is the kinetic energy of the recoil electron; \( E_\eta \) is the incoming \( \eta \) energy; \( m_e \) is the electron mass; \( B \equiv (E_\eta m_e/2\pi^2) (Q^2_\eta/2) \). \( \eta \) is the unit 3-vector of \( \eta \) spin polarization in its rest frame, \( \hat{\Phi} \). \( \eta \cdot \hat{\Phi} \) is the longitudinal component of \( \eta \) spin polarization. \( |\eta \cdot \hat{\Phi}| = |1 - 2Q^2_\eta| \), where \( Q^2_\eta \) is the probability of producing the LC \( \eta \). We see that the interference terms between standard \( V - A \) and exotic \( S_P, P_\eta \) couplings depend on the transversal \( \eta \) spin polarization related to the production process (similar regularity as in the Dirac case [21]).

8 Appendix 2 - Definitions of asymmetry functions

The asymmetry functions \( A(\Phi), A_y(\Phi), A_{0,\eta}(\Phi) \) are defined as

\[
A(\Phi) := \frac{1}{2} \int \frac{d\Phi}{d\Phi} d\Phi - \frac{1}{2} \int \frac{d\Phi}{d\Phi} d\Phi,
\]

\[
A_{0,\eta}(\Phi) := \frac{1}{2} \int \frac{d\Phi}{d\Phi} d\Phi - \frac{1}{2} \int \frac{d\Phi}{d\Phi} d\Phi,
\]

\[
A_y(\Phi) := \frac{1}{2} \int \frac{d\Phi}{d\Phi} d\Phi + \frac{1}{2} \int \frac{d\Phi}{d\Phi} d\Phi.
\]
References

1. M. Doi et al., Phys. Lett. B 103, 219 (1981)
2. W. C. Haxton et al., Phys. Rev. Lett. 47, 153 (1981)
3. H. Ejiri, J. Phys. Soc. Jpn. 74, 2101 (2005)
4. B. Kayser, R. E. Shrock, Phys. Lett. B 112, 137 (1982)
5. P. Langacker, D. London, Phys. Rev. D 39, 266 (1989)
6. S. L. Glashow, Nucl. Phys. 22, 579 (1961)
7. S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967)
8. A. Salam, A. Salam, in Elementary Particle Theory (Almquist and Wiksells, Stockholm, 1969)
9. R. P. Feynman, M. Gell-Mann, Phys. Rev. 109, 193 (1958)
10. E. C. G. Sudarshan, R. E. Marshak, Phys. Rev. 109, 1860 (1958)
11. S. P. Rosen, Phys. Rev. Lett. 48, 842 (1982)
12. G. V. Dass, Phys. Rev. D 32, 1239 (1985)
13. M. Zrałek, Acta Phys. Polon. B 28, 2225 (1997)
14. M. Doi, T. Kotani, H. Nishiura, K. Okuda, E. Takasugi, Prog. Theor. Phys. 67, 281 (1982)
15. V. B. Semikoz, Nucl. Phys. B 498, 39 (1997)
16. S. Pastor, J. Segura, V. B. Semikoz, J. W. F. Valle, Phys. Rev. D 59, 013004 (1999)
17. D. Singh, N. Mobed, G. Papini, Phys. Rev. Lett. 97, 041101 (2006)
18. T. D. Gutierrez, Phys. Rev. Lett. 96, 121802 (2006)
19. M. Misiaszek et al., Nucl. Phys. B 734, 203 (2006)
20. P. Minkowski, M. Passera, Phys. Lett. B 541, 151 (2002)
21. W. Sobkó, A. Blaut, Eur. Phys. J. C 713, 258 (2018)
22. J. Bernabeu et al., Phys. Lett. B 613, 162 (2005)
23. V. A. Guseinov et al., Phys. Rev. D 75, 073021 (2007)
24. S. Ciechanowicz et al., Phys. Rev. D 71, 093006 (2005)
25. T. I. Rashba, V. B. Semikoz, Phys. Lett. B 479, 218 (2000)
26. W.-T. Ni et al., Phys. Rev. Lett. 82, 2439 (1999)
27. W. Białek et al., Phys. Rev. Lett. 56, 1623 (1986)
28. P. V. Vorobyov, Y. I. Gitarts, Phys. Rev. D 76, 053001 (2007)
29. C.-T. Chiang, M. Kamionkowski, G. Z. Knijn, Phys. Dark Univ. 1, 109 (2012)
30. T. Franarin, M. Fairbairn, Phys. Rev. D 94, 053004 (2016)
31. G. D. Starkman, D. N. Spergel, Phys. Rev. Lett. 74, 2623 (1995)
32. M. A. Bouchiat, T. R. Carver, C. M. Varnum, Phys. Rev. Lett. 5, 373 (1960)
33. T. G. Walker, W. Happer, Rev. Mod. Phys. 69, 629 (1997)
34. B. Babussinov et al., Nucl. Instrum. and Meth. A 694, 335 (2012)
35. A. Riotto, M. Trodden, Annu. Rev. Nucl. Part. Sci. 49, 35 (1999)
36. M. Kobayashi, T. Maskawa, Prog. Theor. Phys. 49, 652 (1973)
37. J.C. Pati, A. Salam, Phys. Rev. D 10, 275 (1974)
38. R. Mohapatra, J.C. Pati, Phys. Rev. D 11, 566 (1975); Phys. Rev. D 11, 558 (1975)
39. R.N. Mohapatra, G. Senjanovic, Phys. Rev. D 12, 1502 (1975); Phys Rev D 23, 165 (1981)
40. M. A. B. Beg et al., Phys. Rev. Lett. 38, 1252 (1977)
41. P. Herczeg, Phys. Rev. D 34, 3449 (1986)
42. A. Jodidio et al., Phys. Rev. D 34, 1967 (1986)
43. E. J. Eichten, K. D. Lane, M.E. Peskin, Phys. Rev. Lett. 50, 811 (1983)
44. P. Herczeg, Prog. Part. Nucl. Phys. 46, 413 (2001)
45. N. Arkani-Hamed, S. Dimopoulos, G. Dvali, J. March-Russell, Phys. Lett. B 429, 263 (1998)
46. T. Banks, A. Zaks, Nucl. Phys. B 196, 189 (1982)
47. H. Georgi, Phys. Rev. Lett. 98, 221601 (2007)
48. H. Georgi, Phys. Lett. B 650, 275 (2007)
49. K. Cheung, W.Y. Keung, T.C. Yuan, Phys. Rev. Lett. 99, 051803 (2007)
50. K. Cheung, W.Y. Keung, T.C. Yuan, Phys. Rev. D 76, 055003 (2007)
51. S. Zhou, S. He, Phys. Rev. D 76, 093011 (2007)
52. A. B. Balantekin, K. O. Ozansoy, Phys. Rev. D 76, 095014 (2007)
53. J. Barranco et al., Phys. Rev. D 79, 073011 (2009)
54. D. Montanino, M. Picariello, J. Pulido, Phys. Rev. D 77, 093011 (2008)
55. S. Zhou, Phys. Lett. B 659, 336 (2008)
56. B. Grinstein, K. A. Intriligator, I. Z. Rothstein, Phys. Lett. B 662, 367 (2008)
57. M. Deniz et al., Phys. Rev. D 82, 033004 (2010)
58. J. Barranco et al., Int. J. Mod. Phys. A 27, 1250147 (2012)
59. L. Wolfenstein, Phys. Rev. D 17, 2369 (1978)
60. J. W. F. Valle, Phys. Lett. B 199, 432 (1987)
61. E. Roulet, Phys. Rev. D 44, 935 (1991)
62. M. M. Guzzo, A. Masiero, S. T. Petcov, Phys. Lett. B 260, 154 (1991)
63. J. Schechter, J. W. F. Valle, Phys. Rev. D 22, 2227 (1980)
64. A. Zee, Phys. Lett. B 93, 389 (1980)
65. L. J. Hall, V. A. Kostelecky, S. Raby, Nucl. Phys. B 267, 415 (1986)
66. K. S. Babu, Phys. Lett. B 203, 132 (1988)
67. M. Hirsch, J. W. F. Valle, New J. Phys. 6, 76 (2004)
68. N. Fornengo et al., Phys. Rev. D 65, 013010 (2001)
69. P. S. Amanik, G. M. Fuller, B. Grinstein, Astropart. Phys. 24, 160 (2005)
70. G. L. Fogli, E. Lisi, A. Mirizzi, D. Montanino, Phys. Rev. D 76, 053001 (2007)
72. O. G. Miranda, M. Maya, R. Huerta, Phys. Rev. D 53, 1719 (1996)
73. O. G. Miranda, V. Semikoz, J. W. F. Valle, Nucl. Phys. Proc. Suppl. 66, 261 (1998)
74. J. Barranco, O. G. Miranda, T. I. Rashba, Phys. Rev. D 76, 073008 (2007)
75. A. Bolanos et al., Phys. Rev. D 79, 113012 (2009)
76. S. Davidson et al., JHEP 0303, 011 (2003)
77. J. Barranco et al., Phys. Rev. D 73, 113001 (2006)
78. J. Barranco et al., Phys. Rev. D 77, 093014 (2008)
79. C. Biggio, M. Blennow, E. Fernandez-Martinez, JHEP 0903, 139 (2009)
80. C. Biggio, M. Blennow, E. Fernandez-Martinez, JHEP 0908, 090 (2009)
81. J. Barranco, O. G. Miranda, T. I. Rashba, JHEP 0512, 021 (2005)
82. K. Scholberg, Phys. Rev. D 73, 033005 (2006)
83. J. Barranco et al., Phys. Lett. B 739, 343 (2014)
84. M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)
85. L. Michel, A. S. Wightman, Phys. Rev. 98, 1190 (1955)
86. S. Ciechanowicz, M. Misiaszek, S. Sobkow, Eur. Phys. J. C 32, s01, s151 (2003)
87. D. M. Webber et al., Phys. Rev. Lett. 106, 041803 (2011)
88. F. Arzarello et al., Report No. CERN-LAA/94-19, College de France LPC/94-28, 1994
89. J. Seguinot et al., Report No. LPC 95 08, College de France, Laboratoire de Physique Corpusculaire, 1995
90. A. Sarrat, Nucl. Phys. Proc. Suppl. 95, 177 (2001)
91. R. E. Lanou et al., The Heron project, Abstracts of Papers of the American Chemical Society 2(217), 021-NUCL 1999
92. Y.H. Huang, R. E. Lanou, H. J. Maris, G. M. Seidel, B. Sethumadhavan, W. Yao, Astropart. Phys. 30, 1 (2008)
93. J. S. Adams, Y. H. Huang, Y. H. Kim, R. E. Lanou, H. J. Maris, G. M. Seidel, The HERON project, chapter 8, pages 70-80, 2002
94. P. Zucchelli, Phys. Lett. B 532, 166 (2002)
95. J. Sato, Phys. Rev. Lett. 95, 131804 (2005)
96. J. Bernabeu, J. Burguet-Castell, C. Espinoza, M. Lindroos, JHEP, issue 12, 14 (2005)
97. A. L. Barabanov, O. A. Titov, Eur. Phys. J. A 51, 96 (2015)
98. J. Bernabeu, C. Espinoza, C. Orme, S. Palomares-Ruiz, S. Pascoli, JHEP, issue 06, 040 (2009)
99. M. E. Estevez Aguado et al., Phys. Rev. C 84, 034304 (2011)