Initial Scales, Supersymmetric Dark Matter and Variations of Neutralino-Nucleon Cross Sections

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Abstract

The neutralino-nucleon cross section in the context of the MSSM with universal soft supersymmetry-breaking terms is compared with the limits from dark matter detectors. Our analysis is focussed on the stability of the corresponding cross sections with respect to variations of the initial scale for the running of the soft terms, finding that the smaller the scale is, the larger the cross sections become. For example, by taking $10^{10−12}$ GeV rather than $M_{GUT}$, which is a more sensible choice, in particular in the context of some superstring models, we find extensive regions in the parameter space with cross sections in the range of $10^{-6}−10^{-5}$ pb, i.e. where current dark matter experiments are sensitive. For instance, this can be obtained for $\tan\beta > \sim 3$.

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1 Introduction

Recently there has been some theoretical activity analyzing the compatibility of regions in the parameter space of the minimal supersymmetric standard model (MSSM) with the sensitivity of current dark matter detectors, DAMA \[11\] and CDMS \[12\]. These detectors are sensitive to a neutralino-nucleon cross section \(\sigma_{\tilde{\chi}^0_1-p}\) in the range of \(10^{-6} - 10^{-5}\) pb. Working in the supergravity framework for the MSSM with universal soft terms, it was pointed out in \[1, 2, 4\] that the large tan \(\beta\) regime allows regions where the above mentioned range of \(\sigma_{\tilde{\chi}^0_1-p}\) is reached. Besides, working with non-universal soft scalar masses, they also found \(\sigma_{\tilde{\chi}^0_1-p} \approx 10^{-6}\) pb for small values of tan \(\beta\). In particular, this was obtained for \(\tan \beta \gtrsim 25\) (\(\tan \beta \gtrsim 4\)) working with universal (non-universal) soft terms in \[4\]. The case of non-universal gaugino masses was also analyzed in \[5\] with interesting results.

The above analyses were performed assuming universality (and non-universality) of the soft breaking terms at the unification scale, \(M_{GUT} \approx 10^{16}\) GeV, as it is usually done in the MSSM literature. Such a scale can be obtained in a natural manner within superstring theories. This is e.g. the case of type I superstring theory \[14, 15\] and heterotic M-theory \[14, 16\]. However, recently, going away from perturbative vacua, it was realized that the string scale may be anywhere between the weak scale and the Planck scale. For instance D-brane configurations where the standard model lives, allow these possibilities in type I strings \[17, 18, 19, 20\]. Similar results can also be obtained in type II strings \[21\] and weakly and strongly coupled heterotic strings \[22, 23\]. Hence, it is natural to wonder how much the standard neutralino-nucleon cross section analysis will get modified by taking a scale \(M_I\) smaller than \(M_{GUT}\) for the initial scale of the soft terms \[3\].

The content of the article is as follows. In Section 2 we will briefly review several scenarios suggested by superstring theory where the initial scale for the running of the soft terms is \(M_I\) instead of \(M_{GUT}\). In particular, we will see that \(M_I \approx 10^{10-14}\) GeV is an attractive possibility. The issue of gauge coupling unification, which is important for our calculation, will also be discussed. Then, in Section 3, we will study in detail the

\footnote{1See also \[6, 8, 9, 10\] for other works.}

\footnote{2Let us recall that the lightest neutralino \(\tilde{\chi}^0_1\) is the natural candidate for dark matter in supersymmetric theories with conserved R parity, since it is usually the lightest supersymmetric particle (LSP) and therefore stable \[13\].}

\footnote{3This question was recently pointed out in \[24\] for a different type of analysis. In particular, the authors studied low-energy implications, like sparticle spectra and charge and colour breaking constraints, of a string theory with a scale of order \(10^{11}\) GeV. Similar phenomenological analyses were carried out in the past \[25\] for \(M_{Planck}\) rather than \(M_{GUT}\).}
stability of the neutralino-nucleon cross section with respect to variations of $M_I$. For the sake of generality, we will allow $M_I$ to vary between $10^{16}$ GeV, which corresponds to $M_{GUT}$, and $10^{10}$ GeV. Of course the results will be valid not only for low-scale string scenarios but also for any scenario with an unification scale smaller than $M_{GUT}$. Let us finally remark that the analysis will be carried out for the case of universal soft terms. This is the most simple situation in the framework of the MSSM and can be obtained e.g. in superstring models with dilaton-dominated supersymmetry breaking \[^26\] or in weakly and strongly coupled heterotic models with one Kähler modulus \[^27\]. Finally, the conclusions are left for Section 4.

2 Initial scales

As mentioned in the Introduction, it was recently realized that the string scale is not necessarily close to the Planck scale but can be as low as the electroweak scale. In this context, two scenarios are specially attractive in order to attack the hierarchy problem of unified theories: a non-supersymmetric scenario with the string scale of order a few TeV \[^18\], \[^19\], and a supersymmetric scenario with the string scale of order $10^{10–12}$ GeV \[^20\]. Since we are interested in the analysis of supersymmetric dark matter, we will concentrate on the latter.

In supergravity models supersymmetry can be spontaneously broken in a hidden sector of the theory and the gravitino mass, which sets the overall scale of the soft terms, is given by:

$$m_{3/2} \approx \frac{F}{M_{Planck}},$$

(1)

where $F$ is the auxiliary field whose vacuum expectation value breaks supersymmetry. Since in supergravity one would expect $F \approx M_{Planck}^2$, one obtains $m_{3/2} \approx M_{Planck}$ and therefore the hierarchy problem solved in principle by supersymmetry would be re-introduced, unless non-perturbative effects such as gaugino condensation produce $F \approx M_WM_{Planck}$. However, if the scale of the fundamental theory is $M_I \approx 10^{10–12}$ GeV instead of $M_{Planck}$, then $F \approx M_I^2$ and one gets $m_{3/2} \approx M_W$ in a natural way, without invoking any hierarchically suppressed non-perturbative effect \[^20\].

For example, embedding the standard model inside D3-branes in type I strings, the string scale is given by:

$$M_I^4 = \frac{\alpha M_{Planck}^3}{\sqrt{2}} M_c^3,$$

(2)

where $\alpha$ is the gauge coupling and $M_c$ is the compactification scale. Thus one gets $M_I \approx 10^{10–12}$ GeV with $M_c \approx 10^{8–10}$ GeV.
There are other arguments in favour of scenarios with initial scales $M_I$ smaller than $M_{GUT}$. For example in [22] scales $M_I \approx 10^{10-14}$ GeV were suggested to explain many experimental observations as neutrino masses or the scale for axion physics. These scales might also explain the observed ultra-high energy ($\approx 10^{20}$ eV) cosmic rays as products of long-lived massive string mode decays. Besides, several models of chaotic inflation favour also these initial scales [28].

Inspired by these scenarios we will allow the initial scale $M_I$ for the running of the soft terms to vary between $10^{16}$ GeV and $10^{10}$ GeV, when computing the neutralino-nucleon cross section below. As we will see, the values of the gauge coupling constants at those scales will be crucial in the computation. This leads us for a brief discussion of gauge coupling unification in models with low initial scale:

(a) Non universality of gauge couplings
An interesting proposal in the context of type I string models was studied in [15, 29]: if the standard model comes from the same collection of D-branes, stringy corrections might change the boundary conditions at the string scale $M_I$ to mimic the effect of field theoretical logarithmic running. Thus the gauge couplings will be non universal and their values will depend on the initial scale $M_I$ chosen. This is schematically shown in Fig. 1a for the scale $M_I = 10^{11}$ GeV, where $g_3 \approx 0.8$, $g_2 \approx 0.6$ and $g_1 \approx 0.5$. Clearly, another possibility giving rise to a similar result might arise when the gauge groups came from different types of D-branes. Since different D-branes have associated different couplings, this would imply the non universality of the gauge couplings.

(b) Universality of gauge couplings
On the other hand, if gauge coupling unification at $M_I$, $\alpha_i = \alpha$, is what we want to obtain, then the addition of extra fields in the massless spectrum can achieve this task [20]. An example of additional particles which can produce the beta functions, $b_3 = -3$, $b_2 = 3$, $b_1 = 19$, yielding unification at around $M_I = 10^{11}$ GeV was given in [24]

\[ 2 \times [(1, 2, 1/2) + (1, 2, -1/2)] + 3 \times [(1, 1, 1) + (1, 1, -1)] , \tag{3} \]

where the fields transform under $SU(3)_c \times SU(2)_L \times U(1)_Y$. In this example one obtains $g(M_I) \approx 0.8$. This is schematically shown in Fig. 1b.

As a matter of fact, once the Pandora’s box containing extra matter fields is open, other possibilities arise. Note that the example above does not contain extra triplets, and therefore $\alpha_3$ runs as in the MSSM

\[ \frac{1}{\alpha_3(Q)} = \frac{1}{\alpha_3(M_{susy})} - \frac{b_3}{2\pi} \ln \frac{Q}{M_{susy}} , \tag{4} \]
where \( b_3 = -3 \) and \( M_{suss} \) indicates the supersymmetric threshold. However, introducing \( n_3 \) extra triplets, \( b_3 = -3 + \frac{1}{2} n_3 \) will increase and therefore \( \alpha_3 \) will also increase. Likewise, extra doublets and/or singlets will allow to increase the value of \( \alpha_2 \) and \( \alpha_1 \) as in the example above and therefore we will be able to obtain unification at \( M_I \), but for bigger values of \( \alpha_i = \alpha \). For example, the additional particles

\[
3 \times [(3, 1, 2/3) + (3, 1, -2/3)] + 6 \times [(1, 2, 1/2) + (1, 2, -1/2)] ,
\]

produce \( b_3 = 0, b_2 = 7, b_1 = 27 \), yielding unification again at around \( M_I = 10^{11} \) GeV but for \( g(M_I) \approx 1.3 \).

We will see in the next section that due to the different values of the gauge couplings at \( M_I \), scenarios (a) and (b) give rise to qualitatively different results for cross sections.

### 3 Neutralino-nucleon cross sections versus initial scales

In this section we will consider the whole parameter space of the MSSM with the only assumption of universality. In particular, the requirement of correct electroweak breaking leave us with four independent parameters (modulo the sign of the Higgs mixing parameter \( \mu \) which appears in the superpotential \( W = \mu H_1 H_2 \)). These may be chosen as follows: \( m, M, A \) and \( \tan \beta \), i.e. the scalar and gaugino masses, the coefficient of the trilinear terms, and the ratio of Higgs vacuum expectation values \( \frac{\langle H_2 \rangle}{\langle H_1 \rangle} \).

On the other hand, we will work with the usual formulas for the elastic scattering of relic LSPs on protons and neutrons that can be found in the literature [7, 30, 9, 3]. In particular, we will follow the re-evaluation of the rates carried out in [3], using their central values for the hadronic matrix elements.

As mentioned in the introduction, the initial boundary conditions for the running MSSM soft terms are usually understood at a scale \( M_{GUT} \). Smaller initial scales, as for example \( M_I \approx 10^{11} \) GeV, will imply larger neutralino-nucleon cross sections. Although we will enter in more details later on, basically this can be understood from the variation in the value of \( \mu^2 \) with \( M_I \), since the cross sections are very sensitive to this value.

Let us discuss then first the variation of \( \mu^2 \) with \( M_I \). Recalling that this value is determined by the electroweak breaking conditions as

\[
\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} M_Z^2 ,
\]
we observe that, for $\tan \beta$ fixed, the smaller the initial scale for the running is, the smaller the numerator in the first piece of (6) becomes. To understand this qualitatively, let us consider e.g. the evolution of $m_{H_1}^2$ (neglecting for simplicity the bottom and tau Yukawa couplings)

$$m_{H_1}^2(t) = m^2 + M^2 G(t), \quad (7)$$

where $t = \ln(M_I^2/Q^2)$,

$$G(t) = \frac{3}{2} f_2(t) + \frac{1}{2} f_1(t), \quad (8)$$

and the functions $f_i(t)$ are given by:

$$f_i(t) = \frac{1}{b_i} \left( 1 - \frac{1}{(1 + \frac{\alpha_i(0)}{4\pi} b_i t)^2} \right). \quad (9)$$

For $M_I = M_{\text{GUT}}$ we recover the usual case of the MSSM: $t \approx 61$ for $Q \approx 1$ TeV. However, for $M_I$ smaller than $M_{\text{GUT}}$, the value of $t$ will decrease and therefore $f_i(t)$ will also decrease, producing a smaller value of $G(t)$. As a consequence $m_{H_1}^2$ at $Q \approx 1$ TeV will also be smaller. For $m_{H_2}^2(t)$ the argument is similar: the smaller the initial scale for the running is, the less important the negative contribution $m_{H_2}^2$ to $\mu^2$ in (6) becomes.

In fact, when the initial scale is decreased, it is worth noticing that the values of the gauge couplings are modified. Therefore, this effect will also contribute to modify the soft Higgs mass-squared (see e.g. (3)). Let us consider first the case (a) with non-universal gauge couplings at $M_I$ discussed in the previous section (see also Fig. 1a), where $\alpha_2(M_I)$ and $\alpha_1(M_I)$ are smaller than $\alpha(M_{\text{GUT}})$. For instance, for $m_{H_1}^2$ this has obvious implications: $f_i(t)$ decrease for $M_I$ smaller than $M_{\text{GUT}}$, not only because $t$ decreases but also because $\alpha_i(0)$ are smaller.

The above conclusions have important consequences for cross sections. As is well known, when $|\mu|$ decreases the Higgsino components, $N_{13}$ and $N_{14}$, of the lightest neutralino

$$\tilde{\chi}_1^0 = N_{11} \tilde{B}^0 + N_{12} \tilde{W}^0 + N_{13} \tilde{H}_1^0 + N_{14} \tilde{H}_2^0 \quad (10)$$

increase and therefore the spin independent cross section also increases. We show both facts in Figs. 2 and 3.

These figures correspond to $\mu < 0$. Opposite values of $\mu$ imply smaller cross sections and, moreover, well known experimental constraints as those coming from the $b \to s\gamma$...
process highly reduce the $\mu > 0$ parameter space. The other parameters are chosen as follows. For a given $\tan \beta$ and neutralino mass, the common gaugino mass $M$ is essentially fixed. For the common scalar mass $m$ we have taken $m = 150$ GeV. Finally, for the common coefficient of the trilinear terms $A$ we have taken $A = -M$. This relation is particularly interesting since it arises naturally in several string models \cite{26, 15}. In any case we have checked that the cross sections and our main conclusions are not very sensitive to the specific values of $A$ and $m$ in a wide range. In particular this is so for $|A/M| \lesssim 5$ and $50 \text{ GeV} \lesssim m \lesssim 250 \text{ GeV}$.

We have checked that our results are consistent with present bounds coming from accelerators and astrophysics. The former are LEP and Tevatron bounds on supersymmetric masses and CLEO $b \rightarrow s\gamma$ branching ratio measurements. The latter are relic neutralino density bounds and will be discussed in some detail later on.

In Fig. 2, for $\tan \beta = 10$, we exhibit the gaugino-Higgsino components-squared $N_{11}^2$ of the LSP as a function of its mass $\tilde{\chi}_0^1$ for two different values of the initial scale, $M_I = 10^{16}$ GeV $\approx M_{\text{GUT}}$ and $M_I = 10^{11}$ GeV. Clearly, the smaller the scale is, the larger the Higgsino components become. In particular, for $M_I = 10^{16}$ GeV the LSP is mainly Bino since $N_{11}$ is extremely large. The scattering channels through Higgs exchange are suppressed (recall that the Higgs-neutralino-neutralino couplings are proportional to $N_{13}$ and $N_{14}$) and therefore the cross sections are small as we will see explicitly below. As a matter of fact, the scattering channels through squark exchange are also suppressed by the mass of the first-family squarks. Indeed in this limit the cross section can be approximated as

$$\sigma_{\tilde{\chi}_0^1-p} \propto \frac{m_r^2}{4\pi} \left( \frac{g^2 \sin \theta}{m_\tilde{q}^2 - m_{\tilde{\chi}_0^1}^2} \right)^2 |N_{11}|^4,$$

where $m_r$ is the reduced mass and $m_\tilde{q}$, $\theta$ are the mass and the mixing angle of the first-family squarks respectively. However for $M_I = 10^{11}$ GeV the Higgsino contributions $N_{13}$ and $N_{14}$ become important and even dominant for $m_{\tilde{\chi}_0^1} \lesssim 130$ GeV (e.g. with $\tan \beta = 3$ this is obtained for $m_{\tilde{\chi}_0^1} \lesssim 65$ GeV). Following the above arguments this will imply larger cross sections. Indeed scattering channels through Higgs exchange are now important and their contributions to cross sections can be schematically approximated as

$$\sigma_{\tilde{\chi}_0^1-p} \propto \frac{m_r^2 \lambda_4^2}{4\pi m_h^4} |N_{1i} (g' N_{11} - g_2 N_{12})|^2,$$

\footnote{Let us remark however that for $m$ in the range $50 - 100$ GeV the neutralino is not the LSP in the whole parameter space. In some regions the stau is the LSP.}
where \( i = 3, 4, \lambda_q \) are the quark Yukawa couplings and \( m_h \) represent the Higgs masses. It is also worth noticing that, for any fixed value of \( M_I \), the larger \( \tan \beta \) is, the larger the Higgsino contributions become. The reason being that the top(bottom) Yukawa coupling decreases(increases) since it is proportional to \( \frac{1}{\sin \beta} \left( \frac{1}{\cos \beta} \right) \). This implies that the negative(positive) contribution \( m_{H_2}^2 (m_{H_1}^2) \) to \( \mu^2 \) is less important. The discussion of the cases with \( \tan \beta > 10 \) is more subtle and will be carried out below.

The consequence of these results on the cross section is shown in Fig. 3, where the cross section as a function of the LSP mass \( m_{\tilde{\chi}_0^1} \) is plotted for five different values of the initial scale \( M_I \). For instance, when \( m_{\tilde{\chi}_0^1} = 100 \text{ GeV} \), \( \sigma_{\tilde{\chi}_0^1-p} \) for \( M_I = 10^{11} \text{ GeV} \) is two orders of magnitude larger than for \( M_{GUT} \). In particular, for \( \tan \beta = 3 \), one finds \( \sigma_{\tilde{\chi}_0^1-p} < 10^{-6} \text{ pb} \) if the initial scale is \( M_I = 10^{16} \text{ GeV} \). However \( \sigma_{\tilde{\chi}_0^1-p} \approx 10^{-6} \text{ GeV} \) is possible if \( M_I \) decreases. For \( M_I \lesssim 10^{12} \text{ GeV} \), taking into account the experimental lower bound on the lightest chargino mass \( m_{\tilde{\chi}_1^\pm} = 90 \), the range \( 70 \text{ GeV} \lesssim m_{\tilde{\chi}_0^1} \lesssim 100 \text{ GeV} \) is even consistent with the DAMA limits. As discussed above, the larger \( \tan \beta \) is, the larger the Higgsino contributions become, and therefore the cross section increases.

For \( \tan \beta = 10 \) we see in Fig. 3 that the range \( 60 \text{ GeV} \lesssim m_{\tilde{\chi}_0^1} \lesssim 130 \text{ GeV} \) is now consistent with DAMA limits. This corresponds to \( M_I \lesssim 10^{14} \text{ GeV} \).

Finally, we show in Fig. 3 the case \( \tan \beta = 20 \). Then the above range increases \( 50 \text{ GeV} \lesssim m_{\tilde{\chi}_0^1} \lesssim 170 \text{ GeV} \), corresponding now to \( M_I \lesssim 10^{16} \text{ GeV} \). It is worth noticing here that the value of \( \mu^2 \) is very stable with respect to variations of \( \tan \beta \) when this is large (\( \tan \beta \gtrsim 10 \)). This is due to the fact that \( \mu^2 \approx -m_{H_2}^2 - \frac{1}{2}M_Z^2 \) (see (6)). Since \( \sin \beta \approx 1 \), the top Yukawa coupling is stable and therefore the same conclusion is obtained for \( m_{H_2}^2 \) and \( \mu^2 \). Thus, for a given \( M_I \), the reason for the cross section to increase when \( \tan \beta \) increases cannot be now the increment of the Higgsino components of the LSP. Nevertheless there is a second effect in the cross section which is now the dominant one: the contribution of the down-type quark Yukawa couplings (see (12)) which are proportional to \( \frac{1}{\cos \beta} \).

In Fig. 4 we plot the neutralino-proton cross section as a function of \( \tan \beta \). Whereas large values of \( \tan \beta \) are needed in the case \( M_I = M_{GUT} \) to obtain cross sections in the relevant region of DAMA experiment, the opposite situation occurs in the case \( M_I = 10^{11} \text{ GeV} \) since smaller values are favoured.

Let us consider now the case (b) with gauge coupling unification at \( M_I \) discussed in Section 2 (see also Fig. 1b). The result for the cross section as a function of \( m_{\tilde{\chi}_0^1} \) is plotted in Fig. 5 for \( \tan \beta = 20 \). Clearly, the cross section increases when \( M_I \) decreases. However, this increment is less important than in the previous case. The reason being that now \( \alpha_2(M_I) \) and \( \alpha_1(M_I) \) are bigger than \( \alpha(M_{GUT}) \) instead of smaller. For example this counteracts the increment of \( f_i(t) \) in (9) due to the smaller value of \( t \) when \( M_I \).
is smaller. Due to this effect, only with \( \tan \beta \gtrsim 20 \) we obtain regions consistent with DAMA limits.

The results for the case with extra triplets (see e.g. (3)) are worst since the gauge coupling at the unification point \( M_I \) is bigger than above. The value of \( \sigma_{\tilde{\chi}_1^0 - p} \) at \( M_I \) may be even smaller than its usual value at \( M_{GUT} \).

Before concluding let us discuss briefly the effect of relic neutralino density bounds on cross sections. The most robust evidence for the existence of dark matter comes from relatively small scales. Lower limits inferred from the flat rotation curves of spiral galaxies \([13, 31]\) are \( \Omega_{\text{halo}} \gtrsim 10 \Omega_{\text{vis}} \) or \( \Omega_{\text{halo}} h^2 \gtrsim 0.01 - 0.05 \), where \( h \) is the reduced Hubble constant. On the opposite side, observations at large scales, \((6 - 20) h^{-1} \text{ Mpc}\), have provided estimates of \( \Omega_{\text{CDM}} h^2 \approx 0.1 - 0.6 \) \([32]\), but values as low as \( \Omega_{\text{CDM}} h^2 \approx 0.02 \) have also been quoted \([33]\). Taking up-to-date limits on \( h \), the baryon density from nucleosynthesis and overall matter-balance analysis one is able to obtain a favoured range, \( 0.01 \lesssim \Omega_{\text{CDM}} h^2 \lesssim 0.3 \) (at \( \sim 2\sigma \) CL) \([34, 35]\). Note that conservative lower limits in the small and large scales are of the same order of magnitude.

In this work, the expected neutralino cosmological relic density has been computed according to well known techniques (see \([13]\)). In principle, from its general behaviour \( \Omega_{\chi} h^2 \propto 1/\langle \sigma_{\text{ann}} \rangle \), where \( \sigma_{\text{ann}} \) is the cross section for annihilation of neutralinos, it is expected that such high neutralino-proton cross sections as those presented above will then correspond to relatively low relic neutralino densities. However, our results show that for some of the largest cross-sections, with e.g. \( M_I \approx 10^{12-11} \text{ GeV} \), the value of the relic density is still inside the conservative ranges we considered above. As a function of the intermediate scale our results show a steady increase in the value of the relic density when we move from \( M_I \approx 10^{11} \text{ GeV} \) down to \( M_I \approx 10^{16} \text{ GeV} \). In this respect, the main conclusion to be drawn from our results is that it should be always feasible, for large areas of supersymmetric parameters, to find an “intermediate” initial scale \( M_I \) which represents a compromise between a high neutralino cross section and an adequate relic neutralino density.

We expect that neutralino coannihilations\([5]\) do not play an important role here since the mass differences among the LSP and the NLSP are not too small in general. For example, for the relative mass difference among the two lightest neutralinos, we have found \( \Delta m_{\text{NLSP}}/m_{\text{LSP}} \gtrsim 0.2 \) in most of the interesting regions, being typically much higher.

\footnote{The computation of the relic density carried out here, following \([13]\), contains only a partial treatment of neutralino coannihilation channels.}
4 Conclusions

In this paper we have analyzed the relevant implications for dark matter analyses of the possible existence of an initial scale $M_I$ smaller than $M_{GUT}$ to implement the boundary conditions.

We have noted that the neutralino-nucleon cross section in the MSSM is quite sensitive to the value of the initial scale for the running of the soft breaking terms. The smaller the scale is, the larger the cross section becomes. In particular, by taking $M_I \approx 10^{10-12}$ GeV rather than $M_{GUT} \approx 10^{16}$ GeV for the initial scale, which is a more sensible choice e.g. in the context of some superstring models, we find that the cross section increases substantially being compatible with the sensitivity of current dark matter experiments $\sigma_{\tilde{\chi}_0^0-p} \approx 10^{-6} - 10^{-5}$ pb, for $\tan \beta \gtrsim 3$. For larger values of the initial scale, as e.g. $M_I = 10^{14}$ GeV, the compatibility is obtained for $\tan \beta \gtrsim 10$. Let us remark that these results have been obtained assuming non-universal gauge couplings at $M_I$, as discussed in Section 2. They should be compared with those of the MSSM with initial scale $M_{GUT}$, where $\tan \beta \gtrsim 20$ is needed.

We have also discussed the corresponding relic neutralino densities and checked that they are of the right order of magnitude in large areas of the parameter space for the neutralino being a CDM candidate.

The above computations have been carried out for the case of universal soft terms. This is not only the most simple possibility in the framework of the MSSM, but also is allowed in the context of superstring models. This is e.g. the case of the dilaton-dominated supersymmetry breaking scenarios or weakly and strongly coupled heterotic models with one Kähler modulus. In this sense the analysis of neutralino-nucleon cross sections of those models is included in our computation.

Obviously, non-universality of the soft terms will introduce more flexibility in the computation, in particular in the value of $\mu^2$, in order to obtain regions in the parameter space giving rise to cross sections compatible with the sensitivity of current detectors.

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Figure 1: Running of the gauge couplings with energy assuming non-universality (a) and universality (b) of couplings at the initial scale $M_I = 10^{11}$ GeV. Dashed lines indicate the usual running of the MSSM couplings.
Figure 2: Gaugino-Higgsino components-squared $N_{li}^2$ of the lightest neutralino as a function of its mass, for the case $\tan \beta = 10$, and for two different values of the initial scale, $M_I = 10^{16}$ GeV and $M_I = 10^{11}$ GeV.
Figure 3: Neutralino-proton cross section as a function of the neutralino mass for several possible values of the initial scale $M_I$, and for different values of $\tan \beta$. Current experimental limits, DAMA and CDMS, are shown. The region on the left of the line denoted by $m_{\chi^+} = 90$ GeV is excluded because of the experimental lower bound on the lightest chargino mass.
Figure 4: Neutralino-proton cross section as a function of $\tan \beta$ for two possible values of the initial scale, $M_I = 10^{16}$ GeV, $M_I = 10^{11}$ GeV, and for different values of the neutralino mass, namely 50, 75 and 100 GeV. DAMA limits are also shown.
Figure 5: The same as in Fig. 3 but only for \( \tan \beta = 20 \) and with gauge coupling unification at \( M_1 \).