Nonlinear Realizations in Tensorial Superspaces
and Higher Spins

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Abstract

In the first part of the talk I report on surprising relations between higher spin (HS) theory and nonlinear realizations of the supergroup $OSp(1|8)$, a minimal superconformal extension of $N = 1, 4D$ supersymmetry with tensorial charges. The second part is a review of the “master” model of HS particle which makes manifest the classical equivalence of two previously known HS particle formulations and gives rise to new massless HS multiplets.

Talk given at the XXII Max Born Symposium “Quantum, Super and Twistors” in honor of the 70th birthday of Prof. Jerzy Lukierski, Wroclaw, 26 - 29 September 2006
1. **Introduction.** Since the seminal papers by Fradkin and Vasiliev [1], the theory of higher spin (HS) fields attracts vast interest (see e.g. [2, 3] and refs. therein), especially due to its profound relations to string theory. A concise and suggestive way of dealing with higher spins is to treat them in spaces with additional coordinates, e.g. the tensorial ones [4–7].

M. Vasiliev has shown in [6] that the free 4D HS field theory can be described by the pair of bosonic and fermionic fields \( b(Y), f_\dot{a}(Y) \) \((\dot{a} = 1, 2, 3, 4)\) defined on ten-dimensional real tensorial space

\[
Y^{\dot{a}\dot{b}} = Y^{\beta\dot{a}} = \frac{1}{2} x^m (\gamma_m)^{\dot{a}\dot{b}} + \frac{1}{4} y^{[mn]} (\gamma_{[mn]})^{\dot{a}\dot{b}},
\]

where \( x^m \) are Minkowski space coordinates. A nice superfield form of these equations, in the tensorial superspace \( R^{(10|4)} = (Y^{\dot{a}\dot{b}}, \theta^{\dot{a}}) \), was given in [7].

One of the goals of my talk is to argue [8] that an adequate setting for the HS equations in extended (super)spaces is provided by nonlinear realizations of the supergroup \( OSp(1|8) \), a minimal superconformal extension of \( N = 1, 4D \) supersymmetry with tensorial charges [9–11]. Besides reproducing the HS equations of refs. [6, 7] in a nice geometric way, such framework opens new avenues in the HS theory. In particular, it suggests an important role of the \( OSp(1|8) \) generalization of the \( N = 1, 4D \) chirality concept which underlies gauge \( N = 1 \), 4D theories including supergravity [12].

The simple and, at the same time, powerful device for the analysis of the geometric structure of extended (super)spaces is provided by (super)particles propagating in them. As the second subject of my talk I describe the new “master” model of the bosonic HS particle [13] which encompasses two previously known HS particle models and gives rise to new massless free HS multiplets.

2. **\( OSp(1|8) \) as a generalized superconformal group.** The even (bosonic) sector of the superalgebra \( osp(1|8) \) is the generalized 4D conformal algebra \( sp(8) \) which is a closure of the standard conformal algebra \( so(2, 4) \) and the algebra \( sl(4, R) \). The algebra \( so(2, 4) \approx su(2, 2) \) is spanned by the generators \( (L_{\dot{a}\dot{b}}, T_{\dot{a}\dot{b}}, P_{\dot{a}\dot{b}}, K_{\dot{a}\dot{b}}, D) \). The algebra \( sl(4, R) \) is spanned by the generators \( (L_{\alpha\beta}, T_{\alpha\beta}, A, F_{\alpha\dot{b}}, \overline{F}_{\alpha\dot{b}}) \). The extra generators \( A, F_{\rho\dot{\tau}}, \overline{F}_{\rho\dot{\tau}} \equiv (F_{\tau\rho})^* \) satisfy the following non-zero commutation relations

\[
[F_{\alpha\dot{b}}, \overline{F}_{\beta\dot{\nu}}] = 2\epsilon_{\alpha\beta}\epsilon_{\dot{b}\dot{\nu}}A + 2(\epsilon_{\alpha\beta}L_{\beta\dot{b}} - \epsilon_{\dot{b}\dot{\nu}}L_{\alpha\beta}), \quad [A, F_{\alpha\dot{b}}] = 2F_{\alpha\dot{b}}
\]

(and c.c.). The algebra \( sp(8) \) includes the following additional 12 Abelian generators:

\( (Z_{\alpha\beta}, \overline{Z}_{\dot{a}\dot{b}}) \) describing six standard tensorial translations and \( (\overline{Z}_{\alpha\beta}, \overline{Z}_{\dot{a}\dot{b}}) \) describing six conformal tensorial translations.

The odd (fermionic) sector of \( osp(1|8) \) involves \( N = 1 \) super Poincaré generators \( Q_{\alpha}, \dot{Q}_{\dot{a}} \) and the generators \( S_{\alpha}, \dot{S}_{\dot{a}} \) of conformal supersymmetry:

\[
\{Q_{\alpha}, \dot{Q}_{\dot{a}}\} = 2P_{\alpha\dot{a}}, \quad \{Q_{\alpha}, Q_{\beta}\} = 2Z_{\alpha\beta}, \\
\{S_{\alpha}, \dot{S}_{\dot{a}}\} = 2K_{\alpha\dot{a}}, \quad \{S_{\alpha}, S_{\beta}\} = 2\overline{Z}_{\alpha\beta}, \\
\{Q_{\alpha}, \dot{S}_{\dot{b}}\} = F_{\alpha\dot{b}}, \quad \{Q_{\alpha}, S_{\beta}\} = \epsilon_{\alpha\beta} \left( iA - \frac{1}{2} A \right) + L_{\alpha\beta}
\]

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(and c.c.). The remaining $OSp(1|8)$ (anti)commutators can be found in [8].

3. **Nonlinear realizations of $OSp(1|8)$**. We construct nonlinear realization of $OSp(1|8)$ in the supercoset $OSp(1|8)/SL(4, R)$. It contains generators $\{Q, \tilde{Q}, P, Z, Z, S, \bar{S}, K, \bar{Z}, \bar{K}, \bar{Z}, D\}$ (where, for brevity, we suppressed Lorentz indices) and is parametrized by the coordinates

$$Y^M \equiv (Y^{\hat{\alpha}\hat{\beta}}, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}), \quad \bar{Y}^{\hat{\alpha}\hat{\beta}} \equiv (x^{\alpha\beta}, z^{\alpha\beta}, \bar{z}^{\dot{\alpha}\dot{\beta}}),$$  \hspace{1cm} (4)

the fermionic coordinates $\theta, \bar{\theta}$ being associated with the generators $Q, \tilde{Q}$ and the remaining bosonic coordinates with $P, Z, \bar{Z}$. In addition, it involves the Goldstone superfields $\{\psi^\alpha(\bar{Y}), \bar{\psi}^{\dot{\alpha}}(\bar{Y}), k_{a\dot{a}}(\bar{Y}), t_{a\beta}(\bar{Y}), \bar{t}_{\dot{a}\dot{\beta}}(\bar{Y}), \phi(\bar{Y})\}$ associated with the generators $S, \bar{S}, K, \bar{Z}$ and $D$, respectively.

For the supercoset elements we use the exponential parametrization

$$G = e^{i(\theta Q + \bar{\theta} \tilde{Q})} e^{i(x \cdot P + z \cdot Z)} e^{i(k \cdot K + t \cdot \bar{Z})} e^{i(\psi S + \bar{\psi} \bar{S})}.$$  \hspace{1cm} (5)

The left-covariant Cartan forms are defined by:

$$G^{-1}dG = i(\Omega_Q \cdot Q + \Omega_S \cdot S + \Omega_P \cdot P + \Omega_Z \cdot Z + \Omega_D D + \Omega_K \cdot K + \Omega_{\bar{Z}} \cdot \bar{Z} + \Omega_L \cdot L + \Omega_A A) = i\Omega,$$  \hspace{1cm} (6)

All the Goldstone superfields can be covariantly expressed through the dilatonic one $\phi(\bar{Y})$ by the inverse Higgs [14] constraint

$$\Omega_D = 0 \quad \Rightarrow \quad k_{a\dot{a}} = -e^{-\phi} \partial_{a\dot{a}} \phi, \quad t_{a\beta} = \frac{1}{2}e^{-\phi} \partial_{a\beta} \phi, \quad \bar{t}_{\dot{a}\dot{\beta}} = \frac{1}{2}e^{-\phi} \partial_{\dot{a}\dot{\beta}} \phi,$$  \hspace{1cm} (7)

$$\psi_a = -e^{-\frac{1}{2} \phi} D_a \phi, \quad \bar{\psi}_{\dot{a}} = -e^{-\frac{1}{2} \phi} D_{\dot{a}} \phi,$$  \hspace{1cm} (8)

$$D_a = \frac{\partial}{\partial \theta^a} - i\bar{\theta}^\dot{\beta} \partial_{\dot{a}\dot{\beta}} + i\theta^\beta \partial_{a\beta}, \quad \bar{D}_{\dot{a}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{a}}} + i\theta^\beta \partial_{\dot{a}\beta} - i\bar{\theta}^\dot{\beta} \partial_{\dot{a}\dot{\beta}}.$$  \hspace{1cm} (9)

The HS field dynamics in this setting also arises as a sort of dynamical inverse Higgs effect. Namely, it amounts to the vanishing of the full set of the $OSp(1|8)$ covariant spinor derivatives of the Goldstone superfields $\psi^\alpha, \bar{\psi}^{\dot{\alpha}}$ (defined as the projections of the Cartan forms $\Omega^\alpha, \Omega_{\dot{\alpha}}, \Omega_D$ on the $\theta, \bar{\theta}$ covariant differentials $\Omega^{\alpha}, \Omega_{\dot{\alpha}}, \Omega_D$):

$$\nabla_\beta \psi^\alpha = 0, \quad \nabla_{\dot{\beta}} \bar{\psi}^{\dot{\alpha}} = 0, \quad \nabla_\beta \phi = 0 \quad \text{and c.c.}.$$  \hspace{1cm} (10)

These equations yield both the expressions [8], [9] and the equations

$$(D)^2 e^\phi = (\bar{D})^2 e^\phi = 0, \quad [D^\alpha, \bar{D}^{\dot{\alpha}}] e^\phi = 0.$$  \hspace{1cm} (11)

Eqs. (12) are recognized as the two-component spinor form of the equation suggested in [7] (for $\Phi = e^\phi$).

4. **Tensorial chiral superspace**. The underlying $N = 1$ supergravity gauge group is a group of general diffeomorphisms of chiral $N = 1$ superspace $C(4|2) = (x^m_L, \theta^a_L)$ [12].
This manifests the fundamental role of the principle of preserving chiral representations in $N = 1$ supergravity. The question arises whether an analog of this principle exists for higher-spin generalization of $N = 1$ supergravity. From the analysis of the full set of the (anti)commutation relations of the superalgebra $osp(1|8)$ it follows that the minimal analog of $C^{(4|2)}$ is the coset spanned by the following generators $(P_{\alpha\dot{\alpha}}, Z_{\alpha\beta}, F_{\dot{\beta}\dot{\beta}}, Q_{\alpha})$, i.e. it contains only one holomorphic half of the tensorial central charges and, in addition, the complex generator $F_{\dot{\beta}\dot{\beta}}$. Thus the set of the relevant coset parameters, i.e. $C^{(11|2)} = (x^{\alpha}_{L}, z_{L}^{\alpha\beta}, f^{\alpha\dot{\alpha}}_{L}, \theta^{\alpha}_{L}) \equiv (Y_{L})$ is closed under the left action of the supergroup $OSp(1|8)$ and provides a natural generalization of $C^{(4|2)}$. It is interesting to inquire whether some higher-spin dynamics can be associated with superfields given on $C^{(11|2)}$ as an alternative to eqs. (12) and what is the theory enjoying invariance under general diffeomorphisms of $C^{(11|2)}$ (the HS analog of $N = 1, 4D$ conformal supergravity?) Some examples of the $OSp(1|8)$ invariant actions of the tensorial chiral superfields were presented in [8]. It still remains to reveal their component field contents and their relation to the dynamics of HS fields.

5. HS particles. The unfolded formulation of the HS theory [6] is reproduced by quantizing the tensorial particle [5], [7], or the equivalent HS particle [6], in which tensorial coordinates were gauged away. There also exists a different formulation of the HS particle exhibiting invariance under the even counterpart of supersymmetry [15].

In the unfolded formulation without tensorial coordinates [6] the basic equation for the real HS field $\Phi(x, y, \bar{y})$ reads

$$\left(\partial_{\alpha\dot{\alpha}} + i \frac{\partial}{\partial y^\alpha} \frac{\partial}{\partial \bar{y}^\alpha}\right) \Phi = 0, \quad (13)$$

where $y^\alpha$ is a commuting Weyl spinor, $\bar{y}^\dot{\alpha} = (y^\alpha)$. Solution of eq. (13) can be found, assuming the polynomial dependence of $\Phi$ on $y^\alpha$, $\bar{y}^\dot{\alpha}$

$$\Phi(x, y, \bar{y}) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \bar{y}^\alpha_{m} \cdots \bar{y}^\alpha_{m} \bar{y}^\dot{\alpha}_{1} \cdots \bar{y}^\dot{\alpha}_{n} \varphi_{\alpha_{1} \cdots \alpha_{m} \dot{\alpha}_{1} \cdots \dot{\alpha}_{n}}(x). \quad (14)$$

The independent space-time fields in this expansion are self–dual $\varphi_{\alpha_{1} \cdots \alpha_{m}}$ and anti–self–dual $\bar{\varphi}_{\dot{\alpha}_{1} \cdots \dot{\alpha}_{n}}$ field strengths of all helicities. Eq. (13) leads to Klein–Gordon and Dirac equations for them.

A classical counterpart of this unfolded formulation is the particle action

$$S_{1} = \int d\tau \left(\lambda_{\alpha} \bar{\lambda}_{\dot{\alpha}} \dot{x}^{\alpha} + \lambda_{\dot{\alpha}} \bar{y}^\alpha + \bar{\lambda}_{\dot{\alpha}} \dot{\bar{y}}^{\dot{\alpha}}\right). \quad (15)$$

The spinors $\lambda_{\alpha}, \bar{\lambda}_{\dot{\alpha}}$ are canonical momenta for $y^\alpha$, $\bar{y}^\dot{\alpha}$. The constraints

$$P_{\alpha\dot{\alpha}} - \lambda_{\alpha} \bar{\lambda}_{\dot{\alpha}} \approx 0 \quad (16)$$

are first class and after quantization reproduce the unfolded equation (13).
A different model of the massless HS particle was proposed in [15]:

\[ S_2 = \int d\tau \left( P_{\alpha\dot{\alpha}} \dot{\omega}^{\alpha\dot{\alpha}} - eP_{a\dot{a}} P^{a\dot{a}} \right), \quad \omega^{\alpha\dot{\alpha}} \equiv \dot{x}^{\alpha} - i\dot{\zeta}^{\alpha} \dot{\zeta}^{\dot{\alpha}} + i\dot{\bar{z}}^{\dot{\alpha}} \zeta^{\alpha}. \tag{17} \]

The crucial difference of (17) from the \( N = 1 \) superparticle action is that the Weyl spinor \( \zeta^\alpha, \bar{\zeta}^{\dot{\alpha}} = (\bar{\zeta}^{\dot{\alpha}}) \), is commuting. This model is manifestly invariant under the even counterpart of 4D supersymmetry [16, 17, 15, 18]

\[ \delta x^{\alpha\dot{\alpha}} = i(e^{\alpha\dot{\alpha}} - \bar{\zeta}^{\dot{\alpha}} \zeta^\alpha), \quad \delta \zeta^\alpha = e^\alpha, \quad \delta \bar{\zeta}^{\dot{\alpha}} = \bar{e}^{\dot{\alpha}}, \tag{18} \]

where \( e^\alpha \) is a commuting Weyl spinor.

The wave function of the HS particle model (17) is

\[ \Psi(x_L, \zeta) = \sum_{n=0}^{\infty} \zeta^{\alpha_1} \cdots \zeta^{\alpha_n} \psi_{\alpha_1 \cdots \alpha_n}(x_L), \quad D_{\dot{a}} \Psi = 0, \tag{20} \]

which are quantum counterparts of the first class constraints. Due to eqs. (21) the fields in the expansion (20) are complex self–dual field strengths of the massless particles of all helicities. As a result, the spectrum contains all helicities, every non-zero helicity appearing only once. In this picture the scalar field is complex, as opposed to the unfolded formulation.

6. Master HS particle model. The master HS system action [13] involves the variables of both systems (15) and (17) plus a complex scalar \( \eta \):

\[ S = \int d\tau \left[ \lambda_\alpha \bar{\lambda}_{\dot{\alpha}} \dot{\omega}^{\alpha\dot{\alpha}} + \lambda_\alpha \dot{y}^\alpha + \bar{\lambda}_{\dot{\alpha}} \dot{\bar{y}}^{\dot{\alpha}} + i(\eta \dot{\eta} - \dot{\eta} \bar{\eta}) + 2i(\dot{\eta} \bar{\lambda}_{\dot{\alpha}} \dot{\zeta}^{\dot{\alpha}} - \dot{\eta} \dot{\zeta}^{\alpha} \lambda_\alpha) - l(N - c) \right]. \tag{22} \]

The field \( l \) acts as a Lagrange multiplier for the constraint

\[ N - c \equiv i(y^\alpha \lambda_\alpha - \bar{\lambda}_{\dot{\alpha}} \bar{y}^{\dot{\alpha}}) - 2\eta \bar{\eta} - c \approx 0, \tag{23} \]

which generates, in the Hamiltonian formalism, local \( U(1) \) transformations of the involved complex fields (except for \( \zeta, \dot{\zeta} \)).

The action (22) produces the following primary constraints

\[ T_{a\dot{a}} \equiv P_{a\dot{a}} - \lambda_\alpha \bar{\lambda}_{\dot{\alpha}} \approx 0, \tag{24} \]
\[ D_\alpha \equiv D_\alpha + 2i\bar{\eta}\lambda_\alpha \approx 0, \quad \bar{D}_{\dot{\alpha}} \equiv \bar{D}_{\dot{\alpha}} - 2i\bar{\eta}\lambda_{\dot{\alpha}} \approx 0 \] (25)

and

\[ g \equiv p_\eta + i\bar{\eta} \approx 0, \quad \bar{g} \equiv \bar{p}_\eta - i\eta \approx 0, \] (26)

where \( \lambda_\alpha \) and \( \bar{\lambda}_{\dot{\alpha}} \) are treated as conjugate momenta for \( y^\alpha \) and \( \bar{y}^\dot{\alpha} \). The constraints (26) are second class and so can be treated in the strong sense by introducing Dirac brackets. Then \( \eta, \bar{\eta} \) form the canonical pair: \([\eta, \bar{\eta}]_D = \frac{i}{2}\).

The systems (17) and (22) are (classically) equivalent to each other in the common sector of their phase space which is singled out by choosing the definite sign of the energy \( P_0 \). Two second class constraints contained in the spinor constraints (19) can be converted into the first class ones by adding two degrees of freedom carried by the complex scalar field \( \eta \) and introducing a commuting spinor \( \lambda_\alpha \) to ensure the Lorentz covariance of the new spinor constraints (25). The closure of the algebra of the new spinor constraints, \([D_\alpha, \bar{D}_{\dot{\alpha}}]_D = 2iT_{\alpha\dot{\alpha}}\), (27)
gives just the constraint (24) resolving four-momentum in terms of the spinor product. This resolution is defined up to an arbitrary phase transformation of \( \lambda_\alpha \), \( \bar{\lambda}_{\dot{\alpha}} \). To ensure this \( U(1) \) gauge invariance in the full modified action, we just add the constraint (23).

The world-line particle model (15) also follows from the master model (22) under a particular gauge choice. The spinor constraints (25) and the gauge-fixing condition \( \zeta^\alpha \approx 0 \) together with its complex conjugate can be used to eliminate the variables \( \zeta^\alpha, \pi_\alpha \) and their complex conjugates. Then the constraint (23), together with the gauge fixing condition \( \chi \equiv \varphi - \text{const} \approx 0 \), can be used to gauge away the variable \( \eta \equiv \sqrt{\rho e^{i\varphi}} \).

7. First-quantized theory. The equations for the wave function \( F^{(q)}(x, \zeta, \bar{\zeta}, y, \bar{y}, \eta) \) read

\[
\left( \partial_{\alpha\dot{\beta}} + i \frac{\partial}{\partial y^\alpha} \frac{\partial}{\partial \bar{y}^\dot{\beta}} \right) F^{(q)} = 0 , \quad (a) \quad \left( D_\alpha + \frac{\partial}{\partial \eta} \frac{\partial}{\partial y^\alpha} \right) F^{(q)} = 0 , \quad (b) \quad \left( \bar{D}_{\dot{\alpha}} - 2\eta \frac{\partial}{\partial \bar{y}^\dot{\alpha}} \right) F^{(q)} = 0 ,
\]

\[
\left( y^\alpha \frac{\partial}{\partial y^\alpha} - \bar{y}^\dot{\alpha} \frac{\partial}{\partial \bar{y}^\dot{\alpha}} - \eta \frac{\partial}{\partial \eta} \right) F^{(q)} = q F^{(q)} .
\]

(30)

Here, the operators \( D_\alpha = -i(\partial_\alpha + i\partial_\alpha \bar{\zeta}^\dot{\alpha}) \) and \( \bar{D}_{\dot{\alpha}} = -i(\bar{D}_{\dot{\alpha}} - i\zeta^\alpha \partial_\alpha) \) are quantum counterparts of the “covariant momenta” (19). The external \( U(1) \) charge \( q \) defined by (30) is the quantum counterpart of the constant \( c \) in (23). Eq. (30) implies \( U(1) \) covariance of the wave function

\[ F^{(q)}(x, \zeta, \bar{\zeta}, e^{i\varphi}y, e^{-i\varphi}\bar{y}, e^{-i\varphi}\eta) = e^{q\varphi} F^{(q)}(x, \zeta, \bar{\zeta}, y, \bar{y}, \eta) . \]

(31)

Requiring \( F^{(q)} \) to be single-valued restricts \( q \) to the integer values.

The set (28)–(30) can be solved in two different ways. One can first fix the dependence on the variables \( \zeta, \bar{\zeta}, \eta \) and end up with some complex function of \( (x, y, \bar{y}) \) having a fixed \( U(1) \) charge \( q \) and subjected to some holomorphic version of the unfolded equation (13).
Another way is to start by fixing the dependence on \( y, \bar{y}, \eta \), that gives rise to a set of equations like (21) for some chiral functions of \( (x, \zeta, \bar{\zeta}) \). Thus two alternative formulations of the massless HS equations are naturally realized as two alternative ways of solving the master set (28)–(30). The eventual sets of fields are the same in both cases. Since the wave function \( F(q) \) carries an external \( U(1) \) charge \( q \) and bears a dependence on the extra coordinate \( \eta \), the set (28)–(30) yields a richer set of massless HS multiplets as compared to (13) or (21). Without entering into details, let us list these multiplets, following ref. [13].

\[ q = 0 \]. The space of physical states is spanned by the complex self–dual field strengths \( \phi_{\alpha_1...\alpha_k} \), \( k = 0, 1, 2, \ldots \), of the massless particles of all integer and half-integer helicities. Thus the case of \( q = 0 \) basically amounts to the standard HS multiplet of ref. [6] (modulo the complexity of the scalar field).

\[ q > 0 \]. The space of physical states is spanned by the self–dual field strengths of the massless particles with helicities \( \frac{q}{2}, \frac{q}{2} + \frac{1}{2}, \frac{q}{2} + 1, \ldots \). Thus the scalar field is absent in the spectrum for non-zero positive \( q \).

\[ q < 0 \]. The physical fields describe massless particles with all positive helicities including the zero one, and a finite number of massless states with negative helicities \( -\frac{1}{2}, -1, \ldots, -\left\lfloor \frac{|q|}{2} \right\rfloor \). Taken with its conjugate, this multiplet reveals a partial doubling of fields with a given helicity.

8. Summary. To summarize, the new HS particle gives rise to an extended set of the massless HS equations with novel HS multiplets as their solutions. Its distinguishing features are the external \( U(1) \) charge \( q \) which basically coincides with the minimal helicity of given HS multiplet and the presence of complex scalar coordinate \( \eta \) on which the HS fields depend holomorphically. It would be interesting to elaborate on a possible role of this new coordinate in the geometry of HS (super)gravity (e.g. in the approach of ref. [7]). Also, it is tempting to incorporate supersymmetry into this picture (see [18] for the first steps) and to extend the master HS model to include the tensorial coordinates, thus establishing links with the first half of this talk.

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