Cost-Effective Network Disintegration Through Targeted Enumeration

Zhigang Wang ©, Ye Deng, Petter Holme, Zengru Di ©, Linyuan Lü ©, Senior Member, IEEE, and Jun Wu ©

Abstract—Finding an optimal subset of nodes or links to disintegrate harmful networks is a fundamental problem in network science, with potential applications to anti-terrorism, epidemic control, and many other fields of study. The challenge of the network disintegration problem is to balance the effectiveness and efficiency of strategies. In this article, we propose a cost-effective targeted enumeration (TE) method for network disintegration. The proposed approach includes two stages: 1) searching for candidate objects and 2) identifying an optimal solution. In the first stage, we use rank aggregation to generate a comprehensive ranking of node importance, upon which we identify a small-scale candidate set of nodes to remove. In the second stage, we use an enumeration method to find an optimal combination among the candidate nodes. Extensive experimental results on synthetic and real-world networks demonstrate that the proposed method achieves a satisfying tradeoff between effectiveness and efficiency. Our adaptable TE approach can effectively address a range of combinatorial optimization challenges with significant potential applications, including personnel recruitment, portfolio management, and pharmaceutical development.

Manuscript received 30 May 2023; revised 17 October 2023; accepted 30 August 2024. Date of publication 18 September 2024; date of current version 20 November 2024. This work was supported in part by the National Natural Science Foundation of China under Grant 72201035 and Grant 71871217; in part by the Natural Science Foundation of Guangdong Province under Grant 2022A1515010661; and in part by the Japan Society for the Promotion of Science (JSPS) KAKENHI under Grant JP 21H04595. This article was recommended by Associate Editor E. Chen. (Zhigang Wang and Ye Deng contributed equally to this work.) (Corresponding author: Jun Wu.)

Zhigang Wang is with the Department of Systems Science, Faculty of Arts and Sciences, and the International Academic Center of Complex Systems, Beijing Normal University, Zhihua 519087, China, also with the School of Systems Science, Beijing Normal University, Beijing 100875, China, and also with the Research Domain Complexity Science, Potsdam Institute for Climate Impact Research, 14469 Potsdam, Germany (e-mail: wangzg@mail.bnu.edu.cn).

Ye Deng, Zengru Di, and Jun Wu are with the Department of Systems Science, Faculty of Arts and Sciences, and the International Academic Center of Complex Systems, Beijing Normal University, Zhihua 519087, China (e-mail: yedeng@bnu.edu.cn; zdi@bnu.edu.cn; junwu@bnu.edu.cn).

Petter Holme is with the Department of Computer Science, Aalto University, 02150 Espoo, Finland, and also with the Center for Computational Social Science, Kobe University, Kobe 650-0017, Japan (e-mail: ptrholm@gmail.com).

Linyuan Lü is with the School of Cyber Science and Technology, University of Science and Technology of China, Hefei 230026, China, also with the Yangtze Delta Region Institute (Huzhou), University of Electronic Science and Technology of China, Huzhou 313001, China, and also with the Institute of Fundamental and Frontier Sciences, University of Electronic Science and Technology of China, Chengdu 611731, China (e-mail: linyuan.lv@ustc.edu.cn).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TSMC.2024.3454780.

Digital Object Identifier 10.1109/TSMC.2024.3454780.

Index Terms—Complex network, network disintegration, rank aggregation (RA), targeted enumeration (TE).

I. INTRODUCTION

E XPLORING the internal correlation structure of complex networks is an important research paradigm for understanding complex systems [1], [2]. In most cases, we hope to ensure network connectivity, which has promoted research on network robustness in recent decades [3], [4]. However, if a network is harmful, such as a terrorist network, criminal network, epidemic spreading network, financial contagion network, or cancer network, efficiently disrupting the structure and function of the network becomes a meaningful and challenging task. In recent years, COVID-19, Ebola, and avian influenza have caused considerable losses to human society. How to effectively block the spread of diseases among people or animals is a daunting task. Immunization is an effective way of removing some nodes (people or animals) from the epidemic transmission network through vaccination to disintegrate the network and control the spread of disease. This so-called network disintegration through vaccination is a network disintegration solution [12], researchers have also attempted to calculate the centrality measures of the nodes and then remove them individually, starting with the nodes with the highest centrality values, to develop a network disintegration strategy [7], [8], [9], “graph fragmentation” [10], or “network dismantling” [11], involves removing certain nodes or links to destroy the structure of the network and disrupt the function of the network. The core challenge here is to determine specific nodes or links to be removed, aiming to identify critical nodes of the network under various constraints and disintegration objectives. Therefore, network disintegration can be considered as a special case of critical node detection problems (CNPDs). This problem is typically NP-hard and its mathematical essence is a combinatorial optimization problem. In addition to early research based on exact combinatorial optimization methods to find an optimal network disintegration solution [12], researchers have also attempted to calculate the centrality measures of the nodes and then remove them individually, starting with the nodes with the highest centrality values, to develop a network disintegration strategy [7], [8], [13]. However, the set composed of a single important node may not be the most critical set of nodes, and with the increased availability of large-scale networks, novel heuristic or approximate algorithms have been proposed to find vital nodes in complex networks [14], [15]. Furthermore, some studies introduce evolutionary algorithms to the network.
disintegration problem and attempt to find a near-optimal strategy from the considerable solution space [16], [17]. Inspired by advances in artificial intelligence to solve many practical problems, some studies have developed deep reinforcement learning or machine learning to find influential nodes in complex networks [18], [19].

An outstanding challenge in the network disintegration problem is to take into account the computational cost. Although considerable progress has been made in the study of network disintegration, it remains challenging to achieve a good balance between effectiveness and efficiency. Methods with good effectiveness (giving a more accurate estimate), such as mathematical programming, evolutionary algorithms, and deep learning approaches, typically have poor efficiency (effectiveness per running time), limiting their applications in large-scale networks. On the contrary, high-efficiency methods, such as centrality methods and heuristic algorithms, are typically unsatisfactory in terms of effectiveness, yielding nonoptimal solutions. Similar tradeoff problems have been studied for related tasks to estimate the highest degree and optimal individuals to vaccinate [20]. To find a compromise between effectiveness and efficiency, we propose a targeted enumeration (TE) method in this article.

Intuitively, the optimal solution for network disintegration can be obtained by enumerating all combinations, but there will be a combinatorial explosion for medium-sized and large networks. If reducing the scope of enumeration, we can guarantee effectiveness while being more efficient. Therefore, we consider extracting a small-scale candidate set of nodes to reduce the scope of the enumeration, and then find the optimal combination among the candidate nodes through enumeration. The core and difficulty of the method is to efficiently determine the set of candidates. We propose solving this problem using rank aggregation (RA). The main contributions of this article are summarized below.

1) We propose a cost-effective TE method for network disintegration. Especially, we introduce the concept of RA in searching the candidate objects. It provides a satisfying balance between the network disintegration effect and computational complexity.

2) The resulting two-stage TE method has a highly flexible framework that does not require domain-specific knowledge, allowing us to further optimize the framework to disintegrate harmful networks or identify key influencers in the network.

3) We anticipate that the proposed method will find a broader application in solving the problem of selecting \( n \) entities among \( N \) entities in many application scenarios, encompassing the domains of personnel selection, portfolio investment, and drug design.

The remainder of this article is organized as follows. In Section II, we present related works on network disintegration. In Section III, we give the network disintegration model and the details of the proposed method. We then evaluate the strategy on several synthetic and real-world networks of different types in Section IV. Finally, some conclusions are drawn in Section V.

II. RELATED STUDIES

A. Network Disintegration

With the advancement of technology, research on complex networks has experienced a prosperous development in the past two decades. Researchers have been devoted to adopting various methods, such as optimized design, coordinated control, defense, and repair to ensure the continuous and efficient operation of beneficial networks. However, human society also confronts harmful networks, among which the most typical examples are terrorist networks [21] and epidemic spreading networks [22]. Taking terrorist networks as an example, these organizations have gradually transitioned from conventional hierarchical structures to tightly networked relationships. How to efficiently disintegrate such terrorist networks is currently the foremost challenge in the field of counter-terrorism worldwide. In addition, examples, such as cancer networks [23], criminal networks [24], financial crisis networks [25], and all enemy networks during military confrontations, can be perceived as a type of harmful network. Therefore, the main task of the complex network disintegration problem is how to effectively disintegrate these harmful networks through means, such as immunity, blockade, isolation, interference, and attack, and minimize their network performance.

In general, network disintegration methods can be divided into the following categories.

Mathematical Programming Model: Due to network disintegration being a combinatorial optimization problem, the optimal solution can be found by solving a mathematical programming model. Since the 1960s, scholars in the field of operations research have proposed many models and algorithms. For example, Arulselvan et al. [26] proposed an integer linear programming model to solve the problem of network disintegration. Di Summa et al. [27] used a branch-and-bound method to make the above model solvable in polynomial time by changing constraints and adding relaxation variables. Shen et al. [28] developed a mixed-integer programming model with the number of connected components as the objective function. However, the mathematical programming model has high requirements for the mathematical form of the objective function and constraints.

Node Centrality: For large-scale networks, researchers calculate the centrality of nodes and remove them in descending order, including degree centrality (DC) [29], betweenness centrality (BC) [30], eigenvector centrality (EC) [31], closeness centrality [32], PageRank [33], etc. Holme et al. [8] further studied a dynamic way of centralities in contrast to static disintegration, including interactive degree and interactive betweenness (BI), which removes the node with the highest centrality and then recalculates the centrality. Although node centralities are simple and easy to calculate, the collection of a single important node may not be the essential node set.

Heuristic Algorithm: Many methods based on approximate and heuristic algorithms have been proposed. Collective influence (CI) [34] identifies influential spreaders by the number of nodes within a given radius and removes the node with the highest CI value. Articulation Point Targeted Attacks [35] starting from a random node in the network to identify the
articulation point by performing a variant of depth-first search. Ren et al. [36] presented a generalized network dismantling (GND) method to dismantle networks while considering the costs of node removal. Wandel et al. [37] transformed random attacks into targeted attacks based on explosive node percolation. Although the heuristic algorithm is more effective than the node centrality, it is not optimal in most cases.

Evolutionary Algorithm: In recent years, many researchers have introduced evolutionary algorithms into the network disintegration problem to find a near-optimal solution. For example, Wu’s group studies the optimal disintegration strategy for directed, multilayer, and spatial networks based on the tabu search algorithm (TS) [38], [39], [40]. Li et al. [41] designed an elitism-based multiobjective evolutionary algorithm for network disintegration with the assumption that removal costs are related to degrees of nodes. Wu et al. [42] proposed a gene importance-based evolutionary algorithm to find a set of key nodes in cyber–physical power systems. Recently, the memetic algorithm (ME) has attracted attention for its innovative hybrid approach, which combines local search with evolutionary strategies to address network disintegration challenges. This innovative approach encompasses various strategies, such as the critical node identification in sparse graphs [43], and the “reduce–solve–combine” technique [44]. While evolutionary algorithms can achieve near-optimal solutions, it is essential to consider their computational complexity, which remains a significant challenge in network disintegration applications.

Machine Learning: With the rapid development of machine learning, it provides a new solution paradigm for network disintegration. Finding key players in networks through deep reinforcement learning (FINDER) uses deep reinforcement learning, it provides a new solution paradigm for network disintegration. Liu and Wang [47] developed a self-supervised learning-based network dismantling framework to extract essential structures and identify target nodes for network dismantling. There is a growing emphasis on end-to-end machine learning methods that offer comprehensive solutions from data input to final decision making. There is a growing emphasis on end-to-end machine learning methods that offer comprehensive solutions from data input to final decision making. For instance, Yan et al. [48] introduced a deep reinforcement learning framework for hypernetwork dismantling, formulating it as a node sequence decision problem. Grassia and Mangioni [49] introduced CoreGDM, a geometric deep-learning-based algorithm for efficient network dismantling, merging the graph dismantling machine (GDM) framework with the CoreHD algorithm to enhance performance and computational efficiency.

B. Rank Aggregation

RA is a method of integrating and synthesizing multiple ranking lists to generate a comprehensive ranking list [50], [51]. This methodology has garnered extensive attention and application across diverse domains, including education [52], sports [53], recommendation systems [54], and proposal selection [55]. With the objective of creating consensus, many ranking aggregation methods have been developed in both practical application and theoretical research, among which typical methods include Borda’s method [56], the Markov chain method [57], the minimum violation ranking method [58], the Dowdall method [59], and the competition graph method [60]. Among them, Borda’s method is one of the most common methods. Many researchers have modified and improved this method within varying application scenarios [59], [61]. Besides, there are some simple but widely employed methods, such as the average ranking method and Condorcet’s method, which are heuristic methods that assign scores to individual entities based on heuristic indicators and rank the objects accordingly. In particular, the renowned Web search algorithm PageRank [33] can also be classified as a heuristic method.

As the research progressed, a new perspective emerged: the optimization approach, where certain metrics, such as Kendall tau distance [62] or Spearman footrule distance [63], are used between ranking pairs to identify the ranking with the greatest overall agreement with the input ranking. However, in practice, quantifying the distance between partial rankings poses challenges, thereby finding a median ranking that minimizes the overall distance of the input rankings is a proven solution. It is important to note that this optimization problem has been proven to be NP-hard for any distance metric [57]. Consequently, there has been a large amount of research to expedite techniques for addressing the RA problem [64], [65].

III. METHODOLOGY

A. Network Disintegration Model

Consider an undirected and unweighted graph \( G = (V, E) \) with a finite set of nonempty nodes \( V \) and a set of links \( E \). Let \( N = |V| \) and \( W = |E| \) be the number of nodes and links, respectively, and define different nodes as 1, 2, \ldots, \( N \). We denote \( A(G) = (a_{ij})_{N \times N} \) as the adjacency matrix of \( G \), where \( a_{ij} = 1 \) if nodes \( v_i \) and \( v_j \) are adjacent, and \( a_{ij} = 0 \) otherwise. This article focuses on the removal of nodes and considers that all links connected to the node will be deleted after the node is removed. Let \( \hat{V} \subseteq V \) and \( \hat{E} \subseteq E \) denote the set of nodes and links to be removed, respectively; thus, \( \hat{G} = (V \setminus \hat{V}, E \setminus \hat{E}) \) is the network that remains after removing the nodes in \( \hat{V} \), and \( n = |\hat{V}| \) is the strength of disintegration. As a reference, \( \hat{G} \) is the residual network after randomly removing the \( n \) nodes. We denote the network disintegration strategy as \( X = \{x_1, x_2, \ldots, x_N\} \), and its elements are \( x_i = 1 \) if the corresponding \( i \)-th node satisfies \( i \in \hat{V} \); otherwise, \( x_i = 0 \); thus, we can obtain the disintegration strength as \( n = \sum_{i=1}^{N} x_i \). Regardless of the type and scale of attacks to which the network is subject, it will inevitably damage its inherent structure and functions, which will also be reflected in the objective function of the network performance. Thus, we define the following expression as the effectiveness of the
network disintegration strategy:

\[
\Phi(X) = \frac{\Gamma(G) - \Gamma(\hat{G})}{\Gamma(G) - |\Gamma(\hat{G})|}
\]  

(1)

where \(\Gamma\) represents the measurement function of network performance and \(|\Gamma(\hat{G})|\) represents the expectation of \(\Gamma(\hat{G})\), which can be numerically obtained by conducting multiple experiments and taking the average. For simplicity and brevity, \(\Phi(X)\) will be abbreviated to \(\Phi\) in subsequent parts of this article. To ensure that the network performance \(\Gamma\) strictly decreases monotonically with the network disintegration process, we assume that if \(G_1 \subset G_2\), then \(\Gamma(G_1) < \Gamma(G_2)\). It leads to \(\Phi > 0\) if \(n > 0\). Equation (1) reflects the increase in different strategies compared to the random node removal strategy. In this way, different strategies can be compared for the same network.

\(\Phi\) reflects the disintegration effect of different network disintegration strategies. The larger \(\Phi\) suggests a better disintegration effect. There is an important reference value, that is, \(\Phi = 1\). If \(\Phi > 1\), it means that the disintegration strategy is superior to the random removal of nodes. Equation (1) shows that the goal is to design a node removal strategy, that is, a subset of nodes to be removed, which can maximize the disintegration effect \(\Phi\). Thus, the optimization model for the disintegration strategy can be described as the following general mathematical model:

\[
\max \Phi(X = [x_1, x_2, \ldots, x_N]) \\
\text{s.t.} \sum_{i=1}^{N} x_i = n \\
x_i = 0 \text{ or } 1, i = 1, 2, \ldots, N.
\]  

(2)

Usually, the disintegration effect is measured by the size of the largest connected component [7]. However, it has no noticeable numerical changes when removing a small number of nodes. Therefore, in this study, we employ natural connectivity [66], [67] as \(\Gamma\) among a variety of alternative ways. Natural connectivity is a measure function of structural robustness in complex networks. It characterizes the redundancy of alternative links by weighting the total number of closed walks with all lengths in the network. From a mathematical perspective, it can be derived from the graph spectrum as an average eigenvalue

\[
NC = \ln \left( \frac{S}{N} \right) = \ln \left( \frac{1}{N} \sum_{i=1}^{N} e^{\lambda_i} \right)
\]  

(3)

where \(S\) is the total weighted number of closed walks and \(\lambda_i\) is the \(i\)th largest eigenvalue of the adjacency matrix \(A(G)\). Natural connectivity provides a sensitive, reliable, and precise quantitative analysis of robustness and works in connected and disconnected networks and has been proven to change monotonically with the addition or deletion of nodes or links. For networks with a large spectral gap between the highest eigenvalue \(\lambda_1\) and the second highest eigenvalue, we can consider the following approximation of natural connectivity [68]:

\[
NC = \ln \left( \frac{1}{N} \left( e^{\lambda_1} + \sum_{i=2}^{N} e^{\lambda_i} \right) \right) \approx \lambda_1 - \ln N.
\]  

(4)

Next, the efficiency of the network disintegration strategy is defined as the time complexity, and this article refers to the running time of the methods.

B. Searching the Candidate Objects by Rank Aggregation

From a mathematical perspective, network disintegration is a typical combinatorial problem that considers \(n\) nodes from \(N\) nodes without repetition. We can obtain an optimal solution by enumerating all combinations of \(C_N^n\), that is, an exhaustive enumeration. However, there will be a combinatorial explosion for medium-sized and large networks. If we extract a small-scale candidate set of vital nodes \(\hat{V}\) and then enumerate all combinations only among the candidate set, we can guarantee effectiveness while being more efficient. We use \(\hat{N}\) to denote the size of the candidate set \(\hat{V}\), where \(n \leq \hat{N} \leq N\). Then, the enumeration range can be reduced from \(C_N^n\) to \(C_{\hat{N}}^n\), we call it TE. Now, the core problem is to find candidate objects. To construct a heuristic method, the selected \(n\) nodes should be important according to some criterion. There are numerous criteria that characterize the importance of nodes. If we only use a single criterion, then some potential key nodes may be missed. Therefore, we need to simultaneously consider multiple node importance criteria. The RA is just the resultant tool to generate a comprehensive ranking of node importance, upon which we identify a small-scale candidate set. In network science, the centrality of nodes is a common approach to assessing the importance of nodes. Thus, we first generate multiple node rankings based on various centrality measures. We then combine these individual rankings into a consensus ranking using the RA method. Finally, we determine the candidate objects \(\hat{V}\) based on the consensus ranking.

To verify whether there are differences in the candidate objects given by different ranking aggregation algorithms, we selected the graph-based ranking aggregation method and Borda’s method for comparative analysis using the Highschool Network. Table I presents the top five nodes for each scenario. The numbers in the table denote network node identifiers; D, B, E, and C correspond to degree, betweenness, eigenvector, and closeness centrality. The bolded results highlight the differences in aggregated rankings between the two methods. It reveals a remarkable consistency in node rankings, irrespective of the aggregation method or the number of centrality measures aggregated. Notably, nodes 28 and 37 consistently rank highest. These findings suggest that the aggregation method choice has minimal influence on the outcomes. Given its robustness and computational efficiency, especially with high-dimensional rankings [52], [60], we chose the graph-based method to aggregate rankings into a consensus ranking, \(\hat{R}\). It is worth noting that other ranking aggregation methods could also be selected for our method.

Consider \(M\) rankings of \(N\) nodes given by the \(M\) node importance criterion and use \(R_i = [r_{i1}, r_{i2}, \ldots, r_{iN}]\) to denote the node importance ranking given by the criterion \(c_i\), where \(r_{ij}\) represents the rank of the node \(v_j\) based on the criterion \(c_i\). The transition matrix for the ranking \(R_i\) is denoted by \(P_{ij} = (\rho_{ij}^{(M)})_{N \times N}\), where \(\rho_{ij}^{(M)} = 1\) if node \(v_j\) outranks node \(v_i\) under \(c_i\); otherwise, \(\rho_{ij}^{(M)} = 0\). If there is a tie, that is, \(r_{is} = r_{it}\), let
We denote the out-degree and in-degree of node \( G^c \) as \( d_j^+ \) and \( d_j^- \), respectively. Thus, we can define the ratio of out-in degrees (ROID) as follows:

\[
\text{ROID}_j = \frac{d_j^+ + 1}{d_j^- + 1}
\]

which can be used to quantify the strength of the node \( v_j \) and rank all nodes according to their ROID. The higher the ROID value, the higher the rank of the nodes. In the above example, the ROID values of the three nodes are ROID_1 = 0.5, ROID_2 = 1, and ROID_3 = 2. Therefore, we can obtain the aggregated ranking of \( \hat{R} = [3, 2, 1] \).

To better understand the process of searching for candidate objects, an illustration is shown in Fig. 1. Taking into account a sample network that contains 10 nodes and 23 links and has a network topology as shown in Fig. 1(a), we employ three common centrality measures: DC, BC, and EC. The individual ranking of the nodes based on the three centrality measures is shown in Fig. 1(b)–(d). The aggregated ranking \( \hat{R} \) is shown in Fig. 1(e). The details of the classification are provided in Table II. We set the disintegration strength \( n \) as 2 and the size of the candidate set \( \hat{N} \) as 4 and then obtain the candidate set \( [2, 3, 8, 9] \) based on the aggregated ranking, as shown in the orange node in Fig. 1(e). The comparison results of the node ranking with different centrality measures are visualized in Fig. 1(f). Each curve represents a node, and the height of the curve represents the node ranking according to the corresponding criterion. The wavy curves suggest that there are distinct differences between the three individual rankings.

For example, node 2 ranks first with DC but fifth with BC; node 10 ranks first with EC but sixth with DC. On the far right of Fig. 1(f), the aggregated ranks are also presented. The RA method integrates all information from individual rankings and achieves a comprehensive ranking, effectively overcoming the one-sidedness of the individual measure. To some extent, this method takes the “average” of multiple rankings.

Intuitively, the number of criteria for the importance of the node \( M \) and the combination of these criteria will affect the candidate objects and further influence the disintegration effect. To explore the effect of the node importance criterion on the candidate set \( \hat{V} \), Fig. 2 shows the Venn diagram of candidate sets obtained using various combinations of node importance criteria in three real-world networks. As we see in Fig. 2, if we only use a single criterion \( (M = 1) \), the set of candidates with different combinations of criteria varies significantly. However, as \( M \) increases, the intersection of candidate sets based on different criteria also expands observably. For example, in the network shown in Fig. 2(a),
there are only four overlapping nodes when $M = 1$ but nine overlapping nodes when $M = 3$; these results indicate that RA can help us search for a stable and credible candidate set when considering multiple criteria. Moreover, we have verified other criteria through many experiments that this conclusion does not depend on the selection of specific criteria. Without loss of generality, we choose D-B-E as a combination of the node importance criterion in the following experimental analysis.

**C. Identifying the Optimal Solution by Targeted Enumeration**

In the previous section, we propose selecting $\tilde{N}$ candidate nodes by RA. Now, we need to find the optimal combination among the candidate set through enumeration. The size of the candidate set $\tilde{N}$ will directly affect the effectiveness and efficiency of the proposed method. Taking into account that $n \leq \tilde{N} \leq N$, we assume that $\tilde{N} = n + (N-n)\alpha$, where $0 \leq \alpha \leq 1$ is the redundancy coefficient. When $\alpha$ reaches the maximum value of 1, it becomes an exhaustive enumeration. While $\alpha < 1$, it is TE. A higher $\alpha$ will lead to better effectiveness but worse efficiency. Fig. 3(a) shows the disintegration effect $\Phi$ as a function of the redundancy coefficient $\alpha$ in two typical synthetic networks: the Newman–Watts (NW) model of small-world network [69], and the scale-free (SF) network [70]. The curve first increases and then flattens, indicating that a small value of the redundancy coefficient is sufficient for the TE, and increasing $\alpha$ contributes little to the disintegration effect. These results also suggest that the process of selecting candidate objects is effective to some extent. In practical applications, the value of $\alpha$ can be determined based on real needs.

**D. Time Complexity of the Targeted Enumeration Method**

The algorithmic process of the TE method can be summarized below. First, we choose $\tilde{N}$ candidate nodes based on the aggregate ranking of the nodes. Then, we enumerate all possible combinations among the candidate set. Finally, we find the optimal solution that corresponds to the largest disintegration effect $\Phi$. In the example shown in Fig. 1, if the redundancy coefficient is considered to be $\alpha = 0.25$, then there are $C_N^4 = C_2^4 = 6$ combinations, among which the combination $\{2, 8\}$ is the optimal solution.
Next, we briefly analyze the time complexity of the TE method. As described above, the time complexity of the TE method includes three parts: 1) calculating the centrality of the nodes; 2) aggregating multiple rankings; and 3) enumerating among the candidate sets. In the first part, the time complexity for DC is $O(W)$, the time complexity for BC is $O(NW)$ [71], and the time complexity for EC is $O(N + W)$ [72]. In the second part, the time complexity of the RA is $O(\tilde{N}^2)$. Especially, if $n = \log(N)$ and $\alpha = \log(N)/N$, we can obtain $\tilde{N} = n + (N - n)\alpha \approx 2\log(N)$. Thus, the time complexity of the second part is $O(\log^2(N))$. In the third part, with the assumption that $n = \log(N)$ and $\alpha = \log(N)/N$, the number of enumerations can be given as

$$C_{\tilde{N}}^n = \frac{\log^2 N}{(\log N)!^2}.$$ 

At each enumeration, the natural connectivity of the remaining network as the disintegration effect needs to be calculated, and its time complexity is $O(N^3)$. Based on the analysis presented above, it is easy to see that the time complexity of the TE is dominated by the third part, which can be approximatively expressed as $O(N^3 \cdot [(2\log N)!(\log N)!^2])$.

A schematic of the enumeration times $C_{\tilde{N}}^n$ with a varying network size $N$ when assuming $n = \log(N)$ and $\alpha = \log(N)/N$ is shown in Fig. 3(b). We see that the number of enumerations is less than 1000, even with the large network size $N = 10^6$, which is acceptable.

IV. EXPERIMENTAL ANALYSIS ON SYNTHETIC AND REAL-WORLD NETWORKS

A. Experiments in Synthetic Networks

To demonstrate the efficacy of our proposed method, we conduct evaluations on two widely recognized synthetic network models: the NW and the SF networks. These networks are chosen due to their prevalence in network science research and their relevance in depicting typical network topologies for a comprehensive assessment of our method’s performance. We use eight other methods for comparison: DC, BC, EC, BI, CI (with ball size two), GND, TS algorithm, and ME algorithm.
The rationales for selecting them are listed below. Given the prevalent usage of DC, BC, and EC to identify influential network nodes, we have included these three centralities. In particular, BI has been verified to be the best node centrality-based attacking strategy in most cases [73]. The computation time of BI for sparse networks is at least quadratic in the number of nodes, and cubic for dense networks. The removal node set identified by the CI algorithm includes many previously neglected weakly connected nodes, and the high scalability of CI allows us to find key nodes in large-scale networks. In hierarchical networks, the computation of the CI value can be accomplished in $O(N \log N)$ time. Similarly, GND considers realistic removal costs, which is applicable to large-scale networks and has been shown in the literature to outperform many state-of-the-art methods, which has time complexity $O(N \cdot \log^{2+\epsilon} N)$, with $\epsilon > 0$. We set $\epsilon = 3$ in our experiment. As for the TS algorithm, it is an efficient tool for solving global optimization problems, which can be considered the reference baseline in terms of the disintegration effect. For the disintegration strategy based on the TS algorithm, the time complexity involved in generating the neighboring solution during each iteration is approximately $O(N^2)$. Additionally, the time complexity of computing natural connectivity is $O(N^3)$. Consequently, the overall time complexity of the TS algorithm can be expressed as $O(N^{3+\epsilon})$. The ME algorithm is selected because it efficiently combines the explorative power of genetic algorithms with the precision of local search techniques, making it ideal for tackling complex optimization challenges in network structures. Given that the task in each iteration involves the $O(N^3)$ calculation of natural connectivity, the overall time complexity of the ME algorithm can also be approximated as $O(N^{3+4} N)$.

Fig. 4(a) and (b) shows the disintegration effect $\Phi$ as a function of the disintegration strength $n$ with different disintegration methods. We also set $\alpha$ equal to 0.01. As shown in Fig. 4(a) and (b), the proposed method is almost close to the TS and ME algorithms, which can achieve a good disintegration effect. Three methods consistently outperform other methods on all synthetic networks. It is worth pointing out that even for the heterogeneous SF network with $\gamma = 2.5$, in which the vital nodes are apparent and then all methods work well, the TE method still maintains a weak advantage compared to other methods except for the TS and ME algorithms. In addition to improved effectiveness, the TE is also markedly efficient. Fig. 4(c) and (d) shows the computation time of different methods as a function of network size. Due to the scale of the ordinates, the time of DC, BC, EC, BI, CI, and GND are very close to zero but not zero. For example, for the NW networks with $K = 4$ and $p = 0.1$, when $N = 500$, the running times of DC, BC, EC, BI, CI, and GND are 0.01, 0.63, 0.02, 3.24, 0.60, and 1.94 s, respectively. As shown in Fig. 4(c) and (d), with increasing network scale, the growth rate of the TS and ME algorithms is markedly higher than that of the other methods. In contrast, the proposed method is more efficient.

B. Experiments in Real-World Networks

Since synthetic networks cannot completely summarize the typical properties of real-world networks, we extend our application of the TE method to various realistic scenarios. Table III provides details of the real-world networks used in our study. This selection is made to cover a broad range of network sizes and categories, ensuring a thorough and detailed evaluation of the TE method’s efficiency and effectiveness in practical applications. The data sets are publicly accessible and are retrieved from the KONECT Project (http://konect.cc/), the Network Data Repository (https://networkrepository.com/index.php), and the Colorado Index of Complex Networks (https://icon.colorado.edu). Many networks in the real world are directed and weighted networks. For convenience of analysis, this study only considers undirected and unweighted networks.
Fig. 4. Performance of TE in synthetic networks. For the TS algorithm, we assign the tabu list length to 5, the number of candidate solutions to 5, and the maximum number of iterations without improving the optimal solution to 2000. For the ME algorithm, we use a population size of 100 and run it for 150 generations to explore and refine solutions. The numerical results shown are the averages of 20 different network instances under the same parameters. (a) Disintegration effect of the TE method on the NW network with size $N = 1000$ of varying neighbor numbers $K$ and connection probability $p$. (b) Disintegration effect of the TE method on the SF network with size $N = 1000$ of varying degree exponent $\gamma$. (c) Computation time of different methods as a function of network size $N$ on the NW network. The disintegration strength is $n = 5$. All simulation results are obtained on a desktop computer with an Intel Core i7-9700 CPU with 3.00 GHz and 16.0 GB of RAM. (d) Computation time of different methods on the SF network.

networks. We show the disintegration effect $\Phi$ and the running time of the eight methods in Fig. 5. Along with the TS and ME algorithms, the proposed method achieves superior performance compared to the other six methods concerning the disintegration effect. Furthermore, the disintegration effect of these three methods is more stable. For example, for the disintegration strategy based on EC, its effect is behind to TS, ME, and TE methods in 9-11 Hijackers, Infect-Dublin, and Gnutella networks, but not so good in Autobahn and Facebook networks. However, the TS and ME algorithms lead to good effectiveness but poor efficiency. In other words, the proposed method has a lower cost to obtain a disintegration effect that
TABLE III

| Name            | Category                  | Node meaning | Edge meaning | Number of nodes | Number of links | Average Degree | Density | Diameter |
|-----------------|---------------------------|---------------|--------------|-----------------|----------------|----------------|---------|----------|
| 9-11 Hijackers  | Territorial network      | Person        | Association  | 62              | 199            | 2.129          | 0.084   | 5        |
| PDZBase         | Metabolic network        | Protein       | Interaction  | 212             | 214            | 2.283          | 0.011   | 10       |
| Infect-Dublin   | Human contact network    | Person        | Proximity    | 410             | 2765           | 13.488         | 0.033   | 9        |
| Cegans          | Metabolic network        | Substrates    | Metabolic reactions | 453          | 2025           | 8.940          | 0.029   | 7        |
| Autobahn        | Infrastructure network   | Location      | Highway      | 1168            | 2846           | 2.128          | 0.002   | 62       |
| Gnutella        | Computer network         | Host          | Connection   | 10876           | 35994          | 7.355          | 0.001   | 10       |

Fig. 5. Performance of TE in real-world networks. We evaluated the disintegration effect $\Phi$ and the running time of the nine methods on nine real-world networks of different types and set the disintegration strength as $n = \log N$ for different networks. For the TS algorithm, we assign the tabu list length to 5, the number of candidate solutions to 5, and the maximum number of iterations without improving the optimal solution to 2000. For the ME algorithm, we use a population size of 100 and run it for 150 generations to explore and refine solutions. Subplots (a)–(i) show the experimental results for different real-world networks.

is similar to that achieved by the TS and ME algorithms. Compared to centrality-based methods, although the efficiency of the proposed method is lower than that of centrality-based methods, it is acceptable, indicating that the proposed method achieves a satisfying balance between effectiveness and efficiency.

Authorized licensed use limited to the terms of the applicable license agreement with IEEE. Restrictions apply.
Moreover, we perform the first and second stages of the method separately on the nine real-world networks to test their effectiveness. Specifically, we use the graph-based ranking aggregation method to obtain the aggregated ranking of nodes and then directly remove the top $n$ nodes. We call the disintegration effect of this experiment $\Phi_{RA}$. Second, we do not use the ranking aggregation method and directly target enumeration for DC, BC, and EC with the same $\alpha$ as the TE method. We call the disintegration effect of this experiment $\Phi_{TE,DC}$, $\Phi_{TE,BC}$, and $\Phi_{TE,EC}$, respectively. The experimental results are shown in Tables IV and V. It can be seen that the top-ranked methods according to the disintegration effect are TE in most cases, which further verifies the effectiveness of our method. Directly target enumeration for DC and BC only achieves good results for several networks, and its disintegration effect is unstable. Overall, our method achieves a satisfactory balance between solution quality and computational resources.

### V. Conclusion

In summary, we propose a cost-effective network disintegration method called TE. Specifically, the method is divided into two stages. In the first stage, we use RA to transform multiple node ranking into an aggregated ranking. We then select the top $N$ nodes based on the aggregated ranking as the candidate set of nodes to remove. The size of the candidate set is determined by the redundancy coefficient $\alpha$. We show that RA can find a stable and credible candidate set. The second stage is the TE; instead of enumerating all combinations, we enumerate within the scope of the candidate set. The optimal solution is the combination of nodes corresponding to the largest disintegration effect $\Phi$. We show that a small value of $\alpha$ is sufficient for the TE, which is crucial for the feasibility of TE. Numerical experiments on synthetic and real-world networks have shown that the TE outperforms conventional methods and achieves results that are close to those of high-cost evolutionary algorithms. In terms of efficiency, TE is acceptable compared to conventional methods. The critical point of the TE is to determine a set of valid candidates. In this study, the introduction of RA ensures the validity of the candidate set. The aggregated ranking combines multiple node importance criteria and avoids missing potential key nodes. Although it is not the best in terms of effectiveness or efficiency, the proposed method achieves a satisfying tradeoff between effectiveness and efficiency.

The proposed TE method has a highly flexible framework that does not require domain-specific knowledge. Various node importance criteria, RA methods, and different levels of redundancy coefficient $\alpha$ can be used depending on the real situation. As a typical combinatorial optimization problem, selecting $n$ objects from a set of $N$ objects ($n \ll N$) is common in many application scenarios, including personnel selection, portfolio investment, and drug design. An intuitive approach to these problems is to narrow down the search space to find an optimal solution. The proposed method provides a general executable framework for implementation. For instance, when selecting team members from potential candidates, we can aggregate their rankings based on attributes, such as height, agility, and stamina. By conducting a focused round-robin tournament among the highest-ranked candidates, we can effectively determine the most optimal combination of players. Similarly, when tasked with selecting several stocks from the market for portfolio investment, we can aggregate the rankings based on the historical performance, valuation, and corporate management prowess of all available stocks. Then, we can select the most profitable stock portfolio by diversification.

### REFERENCES

[1] G. Bianconi et al., “Complex systems in the spotlight: Next steps after the 2021 noble prize in physics,” *J. Phys. Complex.*, vol. 4, no. 1, 2023, Art. no. 010201.

[2] R. M. D’Souza, M. di Bernardo, and Y. Liu, “Controlling complex networks with complex nodes,” *Nat. Rev. Phys.*, vol. 5, no. 4, pp. 250–262, 2023.

[3] J. Gao, S. V. Buldyrev, S. Havlin, and H. E. Stanley, “Robustness of a network of networks,” *Phys. Rev. Lett.*, vol. 107, no. 19, 2011, Art. no. 195701.

[4] O. Artme et al., “Robustness and resilience of complex networks,” *Nat. Rev. Phys.*, vol. 6, no. 2, pp. 114–131, 2024.

[5] S. Tan, J. Wu, L. Liu, M. Li, and X. Lu, “Efficient network disintegration under incomplete information: The comic effect of link prediction,” *Sci. Rep.*, vol. 6, no. 1, pp. 1–9, 2016.

[6] Z. Wang, Z. Su, Y. Deng, J. Kurths, and J. Wu, “Spatial network disintegration based on kernel density estimation,” *Rel. Eng. Syst. Saf.*, vol. 245, May 2024, Art. no. 110005.

[7] R. Albert, H. Jeong, and A.-L. Barabási, “Error and attack tolerance of complex networks,” *Nature*, vol. 406, no. 6794, pp. 378–382, 2000.

[8] P. Holme, B. J. Kim, C. N. Yoon, and S. K. Han, “Attack vulnerability of complex networks,” *Phys. Rev. E.*, vol. 65, no. 5, 2002, Art. no. 056109.
[9] T. Yi, X. Chen, Y. Zhu, W. Ge, and Z. Han, “Review on the application of deep learning in network attack detection,” J. Netw. Comput. Appl., vol. 212, Mar. 2023, Art. no. 103580.

[10] M. Aprile, N. Castro, G. Ferreira, J. Piccini, F. Robledo, and P. Romero, “Graph fragmentation problem: analysis and synthesis,” Int. Trans. Oper. Res., vol. 26, no. 1, pp. 41–53, 2019.

[11] S. Feng, Z. Cao, and X. Qi, “Generalized network dismantling via a novel spectral pae-algorithm,” Inf. Sci., vol. 632, pp. 285–298, Jun. 2023.

[12] A. Veremeyev, V. Boginski, and E. L. Pasiliao, “Exact identification of critical nodes in sparse networks via new compact formulations,” Optim. Lett., vol. 8, no. 4, pp. 1245–1259, 2014.

[13] L. Lü, D. Chen, X. Ren, Q. Zhang, Y. Zhang, and T. Zhou, “Vital nodes identification in complex networks,” Phys. Rep., vol. 650, pp. 1–63, Sep. 2016.

[14] S. Mugisha and H. Zhou, “Identifying optimal targets of network attack by belief propagation,” Phys. Rev. E, vol. 94, no. 1, 2016, Art. no. 012305.

[15] Z. Qiu, T. Fan, M. Li, and L. Lü, “Identifying vital nodes by Achlioptas process,” New J. Phys., vol. 23, no. 3, 2021, Art. no. 033036.

[16] M. Lozano, C. García-Martínez, F. J. Rodríguez, and H. M. Trujillo, “Optimizing network attacks by artificial bee colony,” Inf. Sci., vol. 377, pp. 30–50, Jan. 2017.

[17] Y. Deng, J. Wu, Y. Xiao, M. Zhang, Y. Yu, and Y. Zhang, “Optimal disintegration strategy with heterogeneous costs in complex networks,” IEEE Trans. Syst., Man, Cybern., Syst., vol. 50, no. 8, pp. 2905–2913, Aug. 2020.

[18] D. Qi and A. J. Majda, “Using machine learning to predict extreme events in complex systems,” Proc. Nat. Acad. Sci. U.S.A., vol. 117, no. 1, pp. 52–59, 2020.

[19] W. Zhang, Z. Jiang, and Q. Yao, “DND: Deep learning-based directed network disintegrator,” IEEE J. Emerg. Sel. Topics Power Electron., vol. 13, no. 3, pp. 841–850, Sep. 2023.

[20] P. Holme and N. Litvak, “Cost-efficient vaccination protocols for network epidemiology,” PLoS Comput. Biol., vol. 13, no. 9, 2017, Art. no. e1005696.

[21] H. A. Eiselt, “Destabilization of terrorist networks,” Chaos Solitons Fractals, vol. 108, pp. 111–118, Mar. 2018.

[22] Y. Liu et al., “Efficient network immunization under limited knowledge,” Nat. Sci. Rev., vol. 8, no. 1, 2021, Art. no. nwaa229.

[23] A. P. Quayle, A. S. Siddiqui, and S. J. M. Jones, “Preferential network perturbation,” Physica A, Statist. Mech. Appl., vol. 371, no. 2, pp. 823–840, 2006.

[24] F. Calderoni, D. Brunetto, and C. Piccardi, “Communities in criminal networks: A case study,” Soc. Netw., vol. 48, pp. 116–125, Jan. 2017.

[25] T. Kobayashi and K. Hasui, “Efficient immunization strategies to prevent financial contagion,” Sci. Rep., vol. 4, no. 1, pp. 1–7, 2014.

[26] A. Arulselvan, C. W. Commander, L. Elefteriadou, and P. M. Pardalos, “A reduce-solve-combine memetic search for identifying critical nodes in sparse graphs,” PLoS Comput. Biol., vol. 9, no. 5, pp. 3302–3315, Sep./Oct. 2023.

[27] M. Grassia, M. D. Domenico, and G. Mangion, “Machine learning dismantling and early-warning signals of disintegration in complex systems,” Nat. Commun., vol. 12, no. 1, 2021, Art. no. 5190.

[28] Y. Zhou, L. Ji, H. Hao, and Y. G. Zhang, “Finding key players in complex networks through deep reinforcement learning,” Nat. Mach. Intell., vol. 2, no. 6, pp. 317–324, 2020.

[29] Y. Zhou, L. Ji, H. Hao, and F. Glover, “Detecting critical nodes in sparse graphs via ‘reduce-solve-combine’ memetic search,” INFORMS J. Comput., vol. 36, no. 1, pp. 39–60, 2024.

[30] C. Fan, L. Zeng, Y. Sun, and Y. Liu, “Finding key players in complex networks through deep reinforcement learning,” Nat. Mach. Intell., vol. 2, no. 6, pp. 317–324, 2020.

[31] M. Grassia and M. Gionis, “CoreGDM: Geometric deep learning network decaying and dismantling,” in Proc. Int. Workshop Complex Netw., pp. 86–94, 2023.

[32] M. Balcanowski and U. Boryczka, “A comparative study of rank aggregation methods in recommendation systems,” Entropy, vol. 25, no. 1, p. 132, 2023.

[33] Y. Xiao, H. Zhu, D. Chen, Y. Deng, and J. Wu, “Measuring robustness in rank aggregation based on the error-effectiveness curve,” Inf. Process. Manage., vol. 60, no. 4, Jul. 2023, Art. no. 103355.

[34] Y. Zhang, Y. Xiao, J. Wu, and X. Li, “Comprehensive world university ranking based on ranking aggregation,” Comput. Stat., vol. 36, no. 2, pp. 1139–1152, 2021.

[35] F. Radicchi, “Who is the best player ever? A complex network analysis of the history of professional tennis,” PLoS One, vol. 6, no. 2, 2011, Art. no. e17249.

[36] M. Balcanowski and U. Boryczka, “Collaborative rank aggregation in recommendation systems,” Procedia Comput. Sci., vol. 207, pp. 2213–2222, 2022.

[37] L. Canós and V. Lien, “Soft computing-based aggregation methods for human resource management,” Eur. J. Oper. Res., vol. 189, no. 3, pp. 669–681, 2008.

[38] J. de Borda, “Mémoire sur les élections au scrutin,” Histoire de l’Académie Royale des Sciences, vol. 102, pp. 657–665, 1781.

[39] C. Dwork, R. Kumar, M. Naor, and D. Sivakumar, “Rank aggregation methods for the Web,” in Proc. 10th Int. Conf. World Wide Web, 2001, pp. 613–622.

[40] K. E. Pedings, A. N. Langville, and Y. Yamamoto, “A minimum viola-tion ranking method,” Optim. Eng., vol. 13, pp. 349–370, Jun. 2012.

[41] B. Reilly, “Social choice in the south seas: Electoral innovation and the bora count in the pacific island countries,” Int. Political Sci. Rev., vol. 23, no. 4, pp. 355–372, 2002.

[42] Y. Xiao, H. Deng, X. Lu, and J. Wu, “Graph-based rank aggregation method for high-dimensional and partial rankings,” J. Oper. Res. Soc., vol. 72, no. 1, pp. 227–236, 2021.

[43] J. A. Aledo, J. A. Gamez, and D. Molina, “Using extension sets to aggregate partial rankings in a flexible setting,” Appl. Math. Comput., vol. 290, pp. 208–223, Nov. 2016.

[44] F. J. Brandenburg, A. Gleißner, and A. Hofmeier, “Comparing and aggregating partial orders with Kendall Tau distances,” Discrete Math. Algorithm. Appl., vol. 5, no. 2, 2013, Art. no. 1360003.

[45] F. J. Brandenburg, A. Gleißner, and A. Hofmeier, “The nearest neighbor Spearman footrule distance for bucket, interval, and partial orders,” J. Comb. Optim., vol. 26, pp. 310–332, Aug. 2013.

Authorized licensed use limited to the terms of the applicable license agreement with IEEE. Restrictions apply.
[64] Y. Yoo, A. R. Escobedo, and J. K. Skolfield, “A new correlation coefficient for comparing and aggregating non-strict and incomplete rankings,” *Eur. J. Oper. Res.*, vol. 285, no. 3, pp. 1025–1041, 2020.

[65] A. R. Escobedo, E. Moreno-Centeno, and R. Yasmin, “An axiomatic distance methodology for aggregating multimodal evaluations,” *Inf. Sci.*, vol. 590, pp. 322–345, Apr. 2022.

[66] J. Wu, M. Barahona, Y. Tan, and H. Deng, “Natural connectivity of complex networks,” *Chin. Phys. Lett.*, vol. 27, no. 7, 2010, Art. no. 078902.

[67] J. Wu, M. Barahona, Y. Tan, and H. Deng, “Spectral measure of structural robustness in complex networks,” *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 41, no. 6, pp. 1244–1252, Nov. 2011.

[68] S. Tan, J. Wu, M. J. Li, and X. Lu, “Approximating natural connectivity of scale-free networks based on largest eigenvalue,” *EPL*, vol. 114, no. 5, 2016, Art. no. 58002.

[69] D. J. Watts and S. H. Strogatz, “Collective dynamics of ‘small-world’ networks,” *Nature*, vol. 393, no. 6684, pp. 440–442, 1998.

[70] A. L. Barabási and R. Albert, “Emergence of scaling in random networks,” *Science*, vol. 286, no. 5439, pp. 509–512, 1999.

[71] U. Brandes, “A faster algorithm for betweenness centrality,” *J. Math. Sociol.*, vol. 25, no. 2, pp. 163–177, 2001.

[72] P. Bonacich, “Factoring and weighting approaches to status scores and clique identification,” *J. Math. Sociol.*, vol. 2, no. 1, pp. 113–120, 1972.

[73] S. Wandelt, X. Sun, D. Feng, M. Zanin, and S. Havlin, “A comparative analysis of approaches to network-dismantling,” *Sci. Rep.*, vol. 8, no. 1, pp. 1–15, 2018.

Zhizhang Wang received the B.S. degree in information and computing science from Wuhan Textile University, Wuhan, China, in 2017, the M.S. degree in statistics from the Wuhan University of Technology, Wuhan, in 2020, and the Ph.D. degree in system theory from Beijing Normal University, Zhuhai, China, in 2024. From 2022 to 2024, he was a visiting Ph.D. student with the Humboldt University of Berlin, Berlin, Germany, and the Potsdam Institute for Climate Impact Research, Potsdam, Germany. His current research interests include complex networks, with a particular focus on the structural robustness of spatial networks.

Ye Deng received the B.S. degree in communication engineering from Sichuan University, Chengdu, China, in 2013, and the Ph.D. degree in management science from the National University of Defense Technology, Changsha, China, in 2019. His current research interests include complex networks, especially the problem of network disintegration.

Peter Holme received the Ph.D. degree in theoretical physics from Umeå University, Umeå, Sweden, in 2004. He is a Professor of Network Science with the Department of Computer Science, Aalto University, Espoo, Finland, and is also affiliated with the Center for Computational Social Science, Kobe University, Kobe, Japan. Before joining Aalto University, he was a Specially Appointed Professor with the Tokyo Institute of Technology, Tokyo, Japan, and a Professor of Energy Science with Sungkyunkwan University, Seoul, South Korea. He has more than 200 research publications, spanning topics from data-driven topics on social, biological, and technological systems to theoretical issues. His current research focuses on fundamental social processes in our times of rapid technology-driven social change.

Jun Wu received the B.S. degree in management science from Sichuan University, Chengdu, China, in 2002, and the Ph.D. degree in management science from the National University of Defense Technology, Changsha, China, in 2008. From 2007 to 2008, he was a visiting Ph.D. student with the Institute for Mathematical Sciences, Imperial College London, London, U.K. He is a Professor with the Department of Systems Science, Faculty of Arts and Sciences, Beijing Normal University, Zhuhai, China. From 2016 to 2017, he was an Academic Visitor with the Department of Computer Science, University of California at Davis, Davis, CA, USA. His research interests are in the interdisciplinary areas of management science, information science, and network science.

Linyuan Lü (Senior Member, IEEE) received the Ph.D. degree in theoretical physics from the Université de Fribourg, Fribourg, Switzerland, in 2012. She is currently a Professor with the University of Science and Technology of China, Hefei, China. She has published over 80 articles in leading journals, including *National Science Review, Physics Reports*, and *Nature Communications*. Her current research interests include complex systems and higher-order network analysis.

Prof. Lü is a Professor with the Department of Systems Science, Faculty of Arts and Sciences, Beijing Normal University, Zhuhai, China. His research interests lie in the interdisciplinary fields of complex networks, self-organization theory, and their applications in socio-economic and biological systems.