Rapid cosmological simulations in Scalar Field Dark Matter models

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Abstract. The process of structure formation in the Universe can be explained by means of the gravitational instability of the different matter components, in particular that of the so-called dark matter field. In this work, we consider that dark matter can be described by an ultra-light scalar field minimally coupled to gravity, and its gravitational instability in an expanding universe is studied with the help of standard techniques for fluid matter components. Analytical and numerical solutions are shown, for both linear and non-linear perturbations, and compared with those of the standard cold dark matter model. We conclude that non-linear perturbations of the scalar field are indistinguishable from those of cold dark matter as long as the field mass is larger than $10^{-25}$ eV.

1. Introduction

With a contribution of 25.8\% of the total density in the Universe, dark matter (DM) is very important in the context of the structure formation that we observe in our Universe. The most accepted of all is the Λ Cold Dark Matter (ΛCDM) model, which is in very good agreement with several observations at large scales [1]. However, this model presents some problems when compared with observations at small scales, which are summarized as follows:

- The missing satellites problem: Overpopulation of the substructures predicted by the $N$-body simulations of the ΛCDM model [2–4] is at variance with observations of satellite galaxies of the local group (Milky Way and Andromeda).
- The core-cusp problem. Observations show a shallow density profile at the center of many galaxies, however $N$-body numerical simulations predict a divergent behavior of the density profile, the called cusp density profile [5, 6].
- Too Big, too fail. The Milky Way does have some satellites, as do other galaxies, but when these smaller galaxies are examined they seem to have a lot less DM than suggested by the simulations [7–9].

The problems above motivate the study of the structure formation with alternative models of dark matter. In this paper, we show cosmological simulations in which DM is modeled with an ultra-light scalar field [10–17]. A brief description of the contents is the following. In Sec. 2, we present a brief mathematical background of the dark matter model with a scalar field (details of
which can be found in Refs. [16,17]. In Sec. 3, we describe the numerical codes and parameters to perform cosmological simulations, together with a presentation of the general results. Finally, Sec. 4 is devoted to the general conclusions of this work.

2. Mathematical background

DM is modeled by a scalar field $\phi$ endowed with a quadratic scalar field potential, with the Lagrangian density

$$\mathcal{L} = -\frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2,$$

where $m$ is the boson mass. For the background evolution, we consider the line element of a flat, homogeneous and isotropic universe in the form:

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j,$$

where $a(t)$ is the scale factor. The equations of motion are then

$$H^2 = \frac{8\pi G}{3} \left( \sum_j \bar{\rho}_j + \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} m^2 \phi^2 \right),$$

$$\ddot{\phi} = -3H \dot{\phi} - m^2 \phi,$$

with $H = \dot{a}/a$ being the Hubble parameter and $\sum_j \bar{\rho}_j$ is the collective contribution of other matter components. Also, a dot denotes derivative with respect to the cosmic time $t$.

Likewise, we consider linear perturbations to the background evolution, with a line element written in the synchronous gauge as

$$ds^2 = -dt^2 + a^2(t) (\delta_{ij} + h_{ij}) dx^i dx^j,$$

where $h_{ij}$ are the spatial perturbations of the metric (more details in Ref. [18]). For the scalar field, we write it as a background part plus a linear perturbation,

$$\phi(x,t) = \phi(t) + \varphi(x,t),$$

and then the equation of motion of the scalar field perturbation $\varphi$ in Fourier space for a given wavelength $k$ is

$$\ddot{\varphi} = -3H \dot{\varphi} - \left( k^2 a^2 + m^2 \right) \varphi - \frac{1}{2} \dot{\phi} \dot{h}.$$

A full solution of the system of equations (2), (3) and (4) can only be obtained by numerical means, in which the scalar field perturbations can also be coupled to other matter perturbations for a correct calculation of different observables. In the present case, we use the amended version of the CLASS code that was presented in Ref. [16], from which we computed the different instances of the mass power spectrum (MPS) shown in Fig. 1. The curves in color represent the case of the scalar field with different boson masses in the range $m = 10^{-22} - 10^{-25}$ eV.

Also shown is the standard cosmological result of $\Lambda$CDM, and then we can notice that in comparison the scalar field cases present a sharp cut-off, which means the suppression of power at small length scales. The position of the cut-off, and then the scales that are suppressed, depends upon the boson mass, and in general occurs for larger scales for lighter boson masses.

3. Cosmological simulations and general results

In order to get the non-linear evolution of the cosmological perturbations, we require to evolve the MPS shown in Fig. 1 by means of an $N$-body code. $N$-body simulations are simulations of a dynamical system of particles under the influence of forces, like gravity. In general, numerical codes for $N$-body simulations solve the evolution of the particles in terms of the equation of motion [19]

$$\frac{d^2 \Psi}{d\eta^2} + H(\eta) \frac{d \Psi}{d\eta} + \nabla \Phi = 0,$$

where $\eta$ is the conformal time, and the displacement vector $\Psi$ is defined as

$$x(\eta) = q + \Psi(q, \eta),$$
with \( x \) and \( q \) the Eulerian and Lagrangian coordinates, respectively. The gravitational potential \( \Phi \) in Eq. (5) obeys the Poisson equation,

\[
\nabla^2 \Phi = \frac{3}{2} \Omega_m H \delta(x),
\]

(7)

where \( \Omega_m \) is the matter density parameter (obtained from the background evolution) and \( \delta(x) \) is the density contrast\(^1\), which is calculated from the expression

\[
\delta(x) = \frac{1}{\text{Det}(\delta_{ij} + \partial \Psi_i / \partial q_j)}.
\]

(8)

The formalism of Lagrangian Perturbation Theory allows us to find a solution to the divergence of equation (5), expanding the displacement vector \( \Psi \) as follows,

\[
\Psi = \Psi^{(1)} + \Psi^{(2)}.
\]

(9)

\( ^1 \) The density contrast \( \delta \) is also related to the MPS in the Fourier space through \( P(k) \propto |\delta_k|^2 \)

\[ Figure 1. \] MPS with a scalar field as DM considering different values of the boson mass \( m \), see Eq. (1). The most noticeable feature, in comparison with the case of \( \Lambda \)CDM, is the appearance of sharp cut-off for large wavelengths \( k \). This cut-off depends on the mass parameter and could in principle be used to put constraints on the value of \( m \), see for instance Ref. [16,17] and references therein.

We used the L-PICOLA code [20] to perform our simulations, this choice was made because L-PICOLA is considered a fast code, as it uses the Comoving Lagrangian Acceleration (COLA) methods, to solve the equation (5), which tread accuracy in small scales in order to gain computational speed, without sacrificing accuracy at large scales [21].

This code is based on a Particle Mesh scheme which calculates the gravitational force in the particles using a mesh, in general, those types of codes follow the next steps:

(i) Assign the particle charge to the mesh, by the Cloud-in-Cell linear interpolation, an explanation of this method can be found in Ref. [22].
(ii) Solve the Poisson equation (7) by means of the Fast Fourier Transform.
(iii) Calculate the gravitational force \( F = -\nabla \Phi \) in all the mesh points.
(iv) Using the Cloud-in-Cell interpolation get the force in each particle.
(v) Solve the equation of motion (5) in order to get the acceleration of the particles. The Kick-Drift/Leapfrog Method is used to update the particle velocities and positions. The process then starts again in step (i) above until the required evolution time is reached.

In comparison with other popular codes like GADGET [23,24], that uses a hierarchical tree algorithm, and represents fluids by means of smoothed particle hydrodynamics, are necessary \( \sim 2000 \) timesteps in order to reach the same resolution that COLA gets in 10 timesteps.
The output snapshots of the cosmological simulations carry information about the final positions and velocities of the particles. In our case, we analyze them using the two-point correlation function, which can provide us with more information about the galaxies clustering in the Universe. The two-point correlation function $\xi(r)$ is defined as the probability excess, over random and averaged over all space, of finding two galaxies separated by a distance $r$ (in volume elements $\delta V_1$ and $\delta V_2$) \[\delta P = n^2 \delta V_1 \delta V_2 [1 + \xi(r_{12})]. \tag{10}\]

We used the publicly-available CUTE code \[26\] to directly calculate the two-point correlation function from a given simulation snapshot.

We can also relate the two-point correlation function $\xi(r)$ with the mass power spectrum $P(k)$ by

$$\xi(r) = \frac{1}{2\pi^2} \int P(k) \frac{\sin kr}{kr} k^2 dk. \tag{11}$$

Eq. (11) shows the close relation between the two-point correlation function and the MPS, which allows us to use both of them to describe the clustering properties of the particles in a $N$-body simulation.

Firstly, we show in Fig. 2 the linear two-point correlation function $\xi(r)$ of the MPS presented before in Fig. 1; the correlation function was calculated using Eq. (11). Notice that the linear MPS does not take into account the non-linear features of structure formation that appear at late times, and then the obtained $\xi(r)$ should be considered just an approximation to the real thing. It can be seen that the different curves are indistinguishable from those of $\Lambda$CDM as long as the scalar field mass is larger than $10^{-25}$ eV. For the latter and smaller values, there appear some deviations at small scales that can be attributed to the cut-off scale in the MPS.

For the latter, in the case $m = 10^{-25}$ eV, the power is suppressed for scales smaller than $r_c \sim 10 h^{-1}$ Mpc (which corresponds to the wavenumber $k_c \sim 0.1 h$ Mpc$^{-1}$ such that $kr_c \sim 1$), and then the quantity $r^2 \xi$ should show an enhanced amplitude at around such scale. In contrast, the baryonic acoustic oscillations (BAO) scale is located in the range $r_{BAO} \sim 100 - 120 h^{-1}$ Mpc, for which $k_c r_{BAO} \sim 10$, and then the result in the integral (11) is not affected by the cut-off in the MPS (unless we try with masses much lighter than $10^{-25}$ eV). Notice that the argument above is of general applicability, as it is based upon the mathematical properties of the correlation function (11), and then we expect the same results for other dark matter models that present a cut-off in their MPS, like in the case of warm dark matter \[27-29\].

Assuming the cosmological parameters given in the Ref. [1], we evolved the linear MPS shown in Fig. 1 in a box of size $[1024\,\text{Mpc}/h]^3$, with $N = 256^3$ particles and $N_m = 768$ mesh points. Under these parameters, the size of the box is almost ten times larger than the BAO scale, and the number of mesh points is three times that of particles, which is recommended for L-PICOLA to provide optimum accuracy. In Fig. 3 a comparison between the linear and non-linear two-point correlation function is shown, for different scalar field values of boson mass and $\Lambda$CDM. From this comparison we can appreciate a discrepancy in the BAO scale for the linear and non-linear evolution, the BAO scale should not be affected by the non-linear evolution, therefore, it is necessary to do a BAO reconstruction \[30\] at that scale, in order to have a coincidence in both regimes.

4. Conclusions

$N$-body codes are one of the most helpful tools in cosmology to describe the process of structure formation. With the L-PICOLA code, we were able to get the non-linear evolution of a scalar field model for different boson masses, and the CUTE code allowed us to calculate the two-point correlation function from the simulation snapshots. The two-point correlation function shows
Figure 2. Linear two-point correlation function (that was also used as input in the L-PICOLA code) considering different values of the boson mass $m$. Notice that the different curves are indistinguishable from the result of standard cold dark matter model as long as $m > 10^{-25}$ eV, but smaller values of $m$ shows a noticeable discrepancy due to the existence of a sharp cut-off in the MPS, see also Fig. 1.

Figure 3. Comparison between the linear $\Lambda$CDM (yellow line, see also Fig. 2) and the nonlinear two-point correlation functions, for $\Lambda$CDM and different values of the boson mass. The nonlinear cases were obtained from a L-PICOLA simulation with particle number $N = 256^3$, mesh points $N_m = 768$, and a box size $=\left[1024 \text{ Mpc}/h\right]^3$. It can be seen that both cases, linear and nonlinear, suggest that the boson mass should be $> 10^{-25}$ eV.

that the BAO scale is within the range $100 - 120 \text{ Mpc}/h$ for all scalar field models. In addition, at large scales, a coincidence in the correlation function of the $\Lambda$CDM model and scalar field DM is expected, and this is actually the case for boson masses $m > 10^{-25}$ eV in both regimes (linear and non-linear). This allows us to conclude that the scalar field model is a good alternative candidate for DM. However, at this point, we cannot get to distinguish between $\Lambda$CDM and scalar field at small scales, probably because of some error sources linked to the precision of the L-PICOLA code at those scales.

On the other hand, for boson masses smaller than $10^{-25}$ eV we observe an increase on the amplitude in the peaks of the two-point correlation function at small scales. Apart from the direct effect discussed above for Eq. (11), another reason may be that the cut-off in the linear MPS of the scalar field model is not properly interpreted by the codes L-PICOLA and CUTE, as they were originally designed for the $\Lambda$CDM model only. This can be remedied by appropriate changes in the available $N$-body codes [31], or by particular codes dedicated to scalar field models as those in Refs. [32–34]. Special attention should be paid to scales smaller than that of the BAO signal ($r < 140 h^{-1} \text{ Mpc}$) where the differences with respect to CDM are more noticeable. The differences are easily calculated from the integral (11), but not quite so from the point of view of $N$-body simulations, as it is a difficult numerical task to obtain the large resolution required to model small galaxies in large enough cosmological volumes. This is left for future work that we shall present elsewhere.

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