A reconnection switch to trigger gamma-ray burst jet dissipation

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ABSTRACT
Prompt gamma-ray burst (GRB) emission requires some mechanism to dissipate an ultrarelativistic jet. Internal shocks or some form of electromagnetic dissipation are candidate mechanisms. Any mechanism needs to answer basic questions, such as what is the origin of variability, what radius does dissipation occur at, and how does efficient prompt emission occur. These mechanisms also need to be consistent with how ultrarelativistic jets form and stay baryon pure despite turbulence and electromagnetic reconnection near the compact object and despite stellar entrainment within the collapsar model. We use the latest magnetohydrodynamical models of ultrarelativistic jets to explore some of these questions in the context of electromagnetic dissipation due to the slow collisional and fast collisionless reconnection mechanisms, as often associated with Sweet–Parker and Petschek reconnection, respectively.

For a highly magnetized ultrarelativistic jet and typical collapsar parameters, we find that significant electromagnetic dissipation may be avoided until it proceeds catastrophically near the jet photosphere at large radii ($r \sim 10^{13} - 10^{14}$ cm), by which the jet obtains a high Lorentz factor ($\gamma \sim 100 - 1000$), has a luminosity of $L_j \sim 10^{50} - 10^{51}$ erg s$^{-1}$, has observer variability time-scales of the order of 1 s (ranging from 0.001 to 10 s), achieves $\gamma \theta_j \sim 10 - 20$ (for opening half-angle $\theta_j$) and so is able to produce jet-breaks, and has comparable energy available for both prompt and afterglow emission. A range of model parameters are investigated and simplified scaling laws are derived. This reconnection switch mechanism allows for highly efficient conversion of electromagnetic energy into prompt emission and associates the observed prompt GRB pulse temporal structure with dissipation time-scales of some number of reconnecting current sheets embedded in the jet. We hope this work helps to motivate the development of self-consistent radiative compressible relativistic reconnection models.

Key words: accretion, accretion discs – black hole physics – magnetic reconnection – MHD – methods: numerical – gamma-ray burst: general – galaxies: jets.
energy would dominate the prompt energy as opposite to observed (Willingale et al. 2007); particle-in-cell (PIC) simulations show that shocks are dominated in energy by Maxwellian electrons whose emission may be inconsistent with the observed double power-law Band function (Giannios & Spitkovsky 2009); and if GRB jets contain strong toroidal fields, then the shocks should inefficiently dissipate the kinetic energy (Kennel & Coroniti 1984; Zhang & Kobayashi 2005; Narayan, Kumar & Tchekhovskoy 2011) and inefficiently accelerate particles to high energies (Sironi & Spitkovsky 2009). One alternative to the internal shock model includes dissipation from nuclear and Coulomb collisions (Beloborodov 2010).

In light of these issues with shock models and since GRB jets are expected to be highly magnetized, it is interesting to explore alternatives for which electromagnetic dissipation directly leads to acceleration and emission (Romanova & Lovelace 1992; Thompson 1994; Usos 1994; Levinson & van Putten 1997; Lyutikov & Blackman 2001; Lyutikov 2006b; Thompson 2006). Such investigations are motivated by studies of pulsar winds (Kennel & Coroniti 1984; Coroniti 1990; Lyubarsky & Kirk 2001; Kirk & Skjeraasen 2003a; Nagata et al. 2008), field reversals in active galactic nucleus (AGN) jets (Lovelace, Romanova & Newman 1994; Lovelace, Newman & Romanova 1997), and magnetars (Thompson & Duncan 1995; Lyutikov 2003a; Lyutikov 2006a). Electromagnetic dissipation mechanisms are appealing because they do not require the generation of highly relativistic relative motion between shells of matter or highly relativistic turbulent motion. Dissipation can instead proceed in situ. Indeed, high-resolution three-dimensional magnetohydrodynamic (MHD) simulations of jet propagation show that toroidal magnetic fields shield jets against many of the shear instabilities noted in purely hydrodynamical simulations (Keppens et al. 2009; Migone et al. 2010), which suggests that it is difficult for a magnetized jet with a strong toroidal field to produce numerous shells moving at varying relativistic speeds as required by the internal shock model to generate temporal variability and efficient dissipation. Further, modern simulations of magnetized GRB jets suggest that even if relatively relativistic shells were generated, internal shocks would be unable to explain the high efficiency of prompt GRB emission (Narayan et al. 2011). This implies that local electromagnetic dissipation may be the only efficient dissipation mechanism possible in highly magnetized jets.

Further, several observations point to a requirement of strong electromagnetic fields in GRB jets. The typical absence of a thermal photospheric emission component requires the jet to contain a significant low-entropy (e.g. magnetized) component, and the presence of non-pair-producing GeV photons requires emission to be at larger radii than expected in the internal shock model (Zhang & Pe’er 2009). The typical non-detection by Fermi of a GeV spectrum excess due to synchrotron self-Comptonization (SSC) in prompt GRBs can be achieved by a highly magnetized jet since fewer electrons are required to support a strong electromagnetic field, as compared to the number of electrons required by the internal shock model that should generate an SSC component (Fan 2009). Possible measurements of highly polarized gamma-rays from GRBs could require an ordered magnetic field in the emitting region (Lyutikov, Pariev & Blandford 2003). The absence of bright optical flashes in the very early afterglow may require the reverse shock region to be somewhat magnetized (Mimica, Giannios & Aloy 2009; Mizuno et al. 2009).

While electromagnetic fields are commonly understood to be dynamically important in the jets produced by GRB engines (Narayan, Paczynski & Piran 1992; Thompson 1994; Usos 1994; Vlahakis & Königl 2003; Lyutikov 2006b; Uzdensky & MacFadyen 2006, 2007), only recently have self-consistent MHD simulations been able to model the formation and large-scale axisymmetric structure of such jets (Komissarov et al. 2009; Lyubarsky et al. 2009a, including what happens beyond a stellar envelope in the collapsar model (Tchekhovskoy, Narayan & McKinney 2010b). Such large-scale idealized MHD jet simulations are complemented by small-scale general relativistic MHD (GRMHD) axisymmetric and three-dimensional simulations that consider the role of magnetorotational instability (MRI) driven turbulence (Balbus & Hawley 1991) and magnetic field geometries close to the BH (McKinney 2006b; Beckwith, Hawley & Krolik 2008; McKinney & Blandford 2009) and by realistic GRMHD simulations of the engine in the collapsar model (Mizuno et al. 2004; Barkov & Komissarov 2008; Nagataki 2009). Of relevance to GRBs is that a dipolar field appears to be required to launch a jet from the BH (Beckwith et al. 2008; McKinney & Blandford 2009), yet magnetic field advection from a presupernova core collapsing on to a BH leads to small a magnetic flux to generate the power of a cosmological GRB (Komissarov & Barkov 2009). The MRI must then be invoked (Akiyama et al. 2003), but that only generates small-scale (not dipolar) field. One solution is for a magnetic dynamo to generate quasi-periodic large-scale dipolar fields over tens of dynamical times as it occurs for toroidal fields in accretion discs (Davis, Stone & Pessah 2010), but this necessarily implies one must understand electromagnetic dissipation at the interface between each successive flip of the jet’s dipolar field.

However, it remains difficult to understand how the electromagnetic field dissipates in GRB jets. The most advanced simulations of relativistic jets from collapsing stars forming BH accretion flows use the MHD approximation (Komissarov & Barkov 2009), so they cannot incorporate all the plasma effects observed in PIC simulations that are required to self-consistently study magnetic reconnection. Even the most advanced PIC simulations are inapplicable to GRB jets that require relativistic reconnection in the presence of pair creation and annihilation, neutrinos, photons, radiative cooling in both optically-thick and optically-thin regimes, relativistic compression, etc. There is still no coherent physical theory of relativistic reconnection (see e.g. Blackman & Field 1994; Lyutikov & Uzdensky 2003; Lyubarsky 2005; Jaroschek & Hoshino 2009; see also section 5.5 of Uzdensky & MacFadyen 2006). Therefore, quasi-analytical approaches must be used to investigate the role of electromagnetic dissipation in GRB jets.

In prior quasi-analytical work, electromagnetic dissipation through magnetic reconnection in GRB jets has been suggested as a possible source of acceleration and emission at large radii from the central compact object (Thompson 1994; Spruit, Daigne & Drenkhahn 2001; Drenkhahn & Spruit 2002a; Giannios & Spruit 2006; Lyutikov 2006b). It was considered problematic for ideal MHD processes to efficiently accelerate and collimate jets, so non-ideal electromagnetic dissipation was favoured to accelerate GRB jets (Thompson 1994; Drenkhahn & Spruit 2002a) or be a source of energy for emission in GRB jets (Thompson 1994). Prior works also assume that the flow is already relativistic ($\gamma > 1$) and continues to speed up with radius as based upon one-dimensional jet models. The dissipation rate was assumed to be Alfvenic or some fraction of it, which assumes a fast reconnection mode.

However, it is now known that a relativistic quasi-conical outflow (i.e. split-monopole near-compact object and otherwise unconfined) accelerates efficiently near the polar-axis in ideal MHD (Tchekhovskoy, McKinney & Narayan 2009), and any sufficiently globally collimated outflow is efficiently accelerated (Komissarov et al. 2009; Tchekhovskoy et al. 2010b). This dramatically changes...
the behaviour of the Lorentz factor as a function of radius compared to prior one-dimensional models (for a review, see Spruit 2010). In addition, settings following ideal MHD acceleration near the compact object start with $\gamma \approx 1$ until reaching the Alfvén surface. Thus, if dissipation were always nearly Alfvénic, then (unabated by time-dilation) fast reconnection could occur near the jet base. Reconnection near the jet base can severely quench the Blandford–Znajek effect (Beckwith et al. 2008; McKinney & Blandford 2009), which may be required to drive the relativistic electromagnetic jet in the first place. Also, if reconnection were quite close to Alfvénic, then the jet could dissipate deep inside the photosphere, leading to a hot fireball instead of an electromagnetic jet. This would lead to a dominant thermal spectrum, which is inconsistent with observations.

Prior quasi-analytical works have also investigated the potential role of the current-driven magnetic kink instability, which is assumed to operate rather efficiently on Alfvénic time-scales (Spruit et al. 2001; Giannios & Spruit 2006). However, modern stability analyses have shown that the kink instability operates somewhat slower than comoving Alfvén time-scales for collimating flows (Narayan, Li & Tchekhovskoy 2009), and much of a relativistic quasi-conical flow can be out of causal contact with itself to eliminate the instability except within narrow regions right around the polar-axis that contain little electromagnetic energy flux (Spruit et al. 2001; McKinney & Blandford 2009).

Prior work also considered other aspects of electromagnetic dissipation. For example, a violation of ideal MHD occurs when the plasma density drops below the Goldreich–Julian density, and this leads to dissipation (Lyutikov & Blackman 2001). However, for reasonable lower limits on the baryon loading (Levinson & Eichler 2003), such a violation of ideal MHD only occurs at $r \gg 10^{19}$ cm (Spruit et al. 2001). Reconnecting layers in accelerated jets have been found to be potentially unstable to magnetic interchange instabilities leading to enhanced reconnection (Lyubarsky 2010b), although they considered a sharp pressure boundary instead of a distributed radiative photosphere as present in GRB jets. Turbulence-driven electromagnetic dissipation from magnetic instabilities induced by internal shocks has also been recently considered (Zhang & Yan 2011), although the conditions for turbulence in highly magnetized jets remain undetermined.

One aspect of electromagnetic dissipation not investigated so far is the role of critical transitions between collisional and collisionless reconnection and its association with, respectively, rather slow resistive-MHD reconnection (perhaps as slow as Sweet–Parker reconnection: Sweet 1958; Parker 1957) and fast Petschek or Petschek-like reconnection (Petschek 1964). [For a review, see Priest & Forbes (2000).]

Prior simulations and laboratory experiments show that collisional plasmas avoid the fast Petschek-type reconnection in favour of the slow Sweet–Parker type reconnection. For collisional plasmas, the Petschek reconnection configuration promptly collapses to a thin Sweet–Parker layer (Uzdensky & Kulsrud 1998, 2000). No fast (inflow velocity of the order of the Alfvén velocity) Petschek reconnection is seen to occur in collisional plasmas (Biskamp 1986a; Ma & Bhattacharjee 1996; Ji et al. 1998; Uzdensky & Kulsrud 2000; Kulsrud 2001; Breslau & Jardin 2003; Malyshekin, Linde & Kulsrud 2005)

On the other hand, a collisionless plasma allows for non-ideal MHD effects that can force a plasma into a fast Petschek-like reconnection regime occurring on the ion skin depth scale and operating independently from the classical resistivity. [For a review, see Kirk, Melrose & Priest (1994), Kulsrud (2005) and Zweibel & Yamada (2009).] Simulations of pure pair plasmas also show fast reconnection but on the electron skin depth (instead of the ion skin depth) as perhaps due to the electron pressure effect (Bessho & Bhattacharjee 2007). Petschek reconnection plays an important role in the Sun, where nanoflares remain the most plausible source of coronal heating (Parker 1988a; Klimchuk et al. 2009). In fact, the solar corona may even exist in a balanced marginally-collisionless state governed by the density-controlled transitions between these two modes of reconnection (Uzdensky 2007a,c). There is also observational evidence that the transitions between the collisional and collisionless reconnection regimes may also control coronal heating in BH accretion discs (Goodman & Uzdensky 2008). Solar and stellar flares could also be triggered by these critical transitions (Cassak, Drake & Shay 2006; Cassak, Mullan & Shay 2008). Therefore, it is important to determine whether a reconnecting system is collisional or collisionless in order to determine whether current sheets operate in the fast reconnection mode (Cassak et al. 2005; Malyshekin et al. 2005; Yamada et al. 2006; Bessho & Bhattacharjee 2007; Uzdensky 2007a,c).

There do exist purely-resistive-MHD alternatives to fast collisionless Petschek reconnection, including due to externally driven MHD turbulence (Lazarian & Vishniac 1999; Kowal et al. 2009b; Loureiro et al. 2009) and plasmoid-dominated reconnection (Loureyo, Schekochihin & Cowley 2007; Cassak, Shay & Drake 2009; Kowal et al. 2009a; Samtaney et al. 2009; Huang & Bhattacharjee 2010; Uzdensky, Loureyo & Schekochihin 2010). However, relativistic Poynting-flux dominated jets are strongly electromagnetically dominated and relativistically expanding flows, and hence they may only be susceptible to turbulence or similar cascades in regions that remain in causal contact across the jet – such as very close to the polar-axis where there is little total electromagnetic energy and so little possible emission. These alternatives require further consideration in such relativistic regimes.

This paper builds upon three novel key elements. The first key element of this work is that we consider the role of transitions between slow collisional and fast collisionless reconnection. The fact that the reconnection rate depends upon the collisionality of the plasma is particularly important for GRB jets that are collisional at the jet base and out to large radii. We compute the properties of the collisional and collisionless reconnecting layers to determine which reconnection mode dominates at each radius and angle within the GRB jet. Once collisionless reconnection is initiated at some ‘transition radius’, reconnection proceeds at a rate much faster than the collisional rate and can initiate prompt GRB dissipation and emission.

The second key element of this work is that the collisional layer’s properties are determined as due to collisions among species of ions, photons, electrons, positrons and neutrinos. The current layer structure is treated similarly to radiative accretion discs (see e.g. Kohri, Narayan & Piran 2005). We also make use of relativistic reconnection work by Lyubarsky (2005), which shows that the relativistic Sweet–Parker layer has a reconnection rate as expected from non-relativistic theory. We incorporate radiative effects into the reconnection switch model by following Uzdensky & McKinney (2011) who showed that the current layer compresses in the strong cooling limit of non-relativistic radiative reconnection.

The third key element of this work is that the large-scale jet is modelled using the latest ideal MHD models of ultra-relativistic jets rather than assuming inefficiently accelerated one-dimensional flows. We also extend the study of Spruit et al. (2001) and determine a range of possible ways for field-polarity reversals to occur that lead to current sheets in the substructure of the jet.
2 BASIC ARGUMENT

In this section, a basic argument is presented to demonstrate the existence of a reconnection switch mechanism that leads to dissipation near the GRB jet photosphere.

Fig. 1 shows a basic picture of the reconnection switch model for a jet containing multiple current sheets each corresponding to a layer wherein oppositely directed magnetic field lines are dissipated. Near the central engine, dissipation proceeds via slow collisional reconnection due to the high collisional rate. This allows the electromagnetic field to avoid significant dissipation despite the presence of current sheets, and this allows the generation of a baryon-pure strong electromagnetic field for launching an ultrarelativistic jet. At large radii, pairs annihilate and the densities go down, leading to infrequent collisions. Collisionless plasma effects (e.g. due to electron–proton or electron–positron decoupling) can then initiate the much faster collisionless reconnection mode (operating in a Petschek-like geometry) that disrupts the slow collisional mode (operating in a Sweet–Parker like geometry) (Cassak et al. 2005).

This leads to a reconnection switch that triggers jet dissipation, which (as shown below) initiates prompt emission near the GRB jet photosphere. As mentioned in the introduction, the concept of a reconnection switch has been applied to many astrophysical phenomena, and in this paper we simply apply the same concept to GRB jets.

The relevant length-scale for fast collisionless reconnection in baryon-dominated plasmas is the proton skin depth

\[ d_p = \frac{c}{\omega_{pp}}. \]  

with proton plasma frequency \( \omega_{pp} = \sqrt{4\pi n_p e^2/m_p} \), proton number density \( n_p \), charge \( e \), proton mass \( m_p \) and speed of light \( c \). The relevant scale for pair-dominated plasmas is the pair skin depth

\[ d_e = \frac{c}{\omega_{pe}}. \]  

with pair plasma frequency \( \omega_{pe} = \sqrt{4\pi n_{e,\text{tot}} e^2/m_e} \), electron+pair number density \( n_{e,\text{tot}} = n_e + n_{\text{ion}} \) and electron mass \( m_e \).

Collisions ensure that resistive MHD applies, which forces the current layer to avoid the fast Petschek-like geometry in favour of a Sweet–Parker like geometry (see e.g. Uzdensky & Kulsrud 2000). The Sweet–Parker solution requires pressure equilibrium across the current sheet. Assume, as valid for most of this paper, that the connecting field is not significantly weaker than the guide field. Then, the Sweet–Parker solution without a guide field can be used to estimate any quantities to within factors of order unity. The electromagnetic pressure \( p_{\text{EM}} = \rho_{\text{EM}} c^2 \), where \( \rho_{\text{EM}} \) is the electromagnetic energy density) and thermal gas pressure (which, as borne out in this paper, is dominated by photon pressure \( p_{\gamma} = u_{\gamma}/3 \), where \( u_{\gamma} \) is the photon energy density) balance via

\[ p_{\text{EM}} \sim p_{\gamma}. \]  

Next, assume, as also borne out in this paper, that the collisional resistivity against current-carrying electrons and positrons is dominated by Compton scattering, then the corresponding magnetic diffusivity is

\[ \eta \approx (4/3) d_e^2 (u_{\gamma} \sigma_T c^2) / (m_e c^2) \]  

(Goodman & Uzdensky 2008), for Thomson scattering cross-section \( \sigma_T \). Then, for electromagnetically dominated jets with relativistic Alfven speeds, the collisional Sweet–Parker current sheet thickness is

\[ \delta_{\text{SP}} \sim \sqrt{\frac{L_0 \eta}{c}}, \]  

where \( L_0 \) is the comoving length of the current layer as estimated by the scale for variations in the electromagnetic field.

Figure 1. Reconnection switch concept: collapsar model or some other system produces a jet (with opening half-angle \( \theta \)) corresponding to a generalized striped wind containing many field reversals that develop into dissipative current sheets. The jet collimates due to confinement by the stellar envelope out to a radius \( r_{\text{mono}} \), after which the jet (having become ultrarelativistic) cannot expand laterally and becomes a nearly radial (monopolar) flow. The figure shows toroidal field polarity reversals produced by dynamo processes near the central engine or by turbulent entrainment at boundary layers. In the reconnection switch model, these current sheets avoid significant dissipation while in the collisional regime until they reach sufficiently large radii where the plasma becomes collisionless and fast collisionless reconnection is triggered. The curved dotted lines denote that the striped wind continues down to the central engine. This paper focuses on the dissipation processes within current sheets that ultimately lead to prompt emission.

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The fast Petschek-like collisionless reconnection mode takes over when \( d_p \gtrsim \delta_{\text{SP}} \) for baryon-dominated plasmas, as discussed above. Using the above equations, the condition for fast collisionless reconnection is then given by

\[
1 \lesssim \left( \frac{d_p}{\delta_{\text{SP}}} \right)^2 \sim \frac{(1 + n_{\text{pairs}}/n_e)^2}{8\gamma_{\text{sync}}},
\]

where \( \mu = u_{\text{EM}}/(p_{\text{e}}c^2) \) is the electromagnetic energy per baryon rest-mass energy in the jet, \( \rho_k \approx 2n_p/c^2 \) is the baryon rest-mass density for a proton number density \( n_p \), assuming protons and neutrons are at equal number densities, \( n_e = n_p \) is the baryonic-associated electron density from charge neutrality, \( \tau_{\gamma\text{e}} \approx n_s c \tau_{\gamma} L_0 \) is the scattering optical depth and \( L_0 \) corresponds to the density scaleheight for photons, electrons and positrons for an emitting slab of length \( L_0 \) (Popham & Narayan 1995). For a pair-dominated plasma in the limit that baryons play no role, the condition for fast reconnection becomes

\[
1 \lesssim \left( \frac{d_p}{\delta_{\text{SP}}} \right)^2 \sim \frac{\left( m_\gamma/m_p \right)^2 (1 + n_{\text{pairs}}/n_e)}{8\gamma_{\text{sync}}}. \tag{7}
\]

Future studies can clarify how fast reconnection operates in a plasma with baryons that typically dominate the rest-mass energy even if pairs have a non-negligible number density.

Assume \( n_{\text{pairs}} \lesssim n_e \), then the condition for transition to fast reconnection simply becomes

\[
\tau_{\gamma\text{e}} \ll (8\mu)^{-1}. \tag{8}
\]

Because MHD jets for GRBs have \( \mu \sim 1 \) at large radii (Tchekhovskoy et al. 2010b), \textit{fast reconnection in GRB jets is predicted to occur near the photosphere}. Such a dissipation is required by dissipative photosphere models (see e.g. Rees & Mészáros 2005). Further, once fast dissipation starts, the value of \( \mu \) drops, which forces the fast collisionless reconnection condition to be maintained. Even if fast reconnection were relativistic with speed of the order of \( c \), dissipation is still suspended until large radii so that a baryon-pure ultrarelativistic jet can form.

The rest of this paper computes the radius where the reconnection switch occurs, the photon optical depth at this radius to determine whether quasi-thermal photospheric emission is possible, and some other observables. Realistic MHD GRB jet models are used since the polar jet’s ideal MHD acceleration is substantially different from that considered in prior reconnection jet models. The important pair contribution is non-trivial to compute because near the photosphere the photons and pairs are only marginally optically thick. Also, the plasma temperature near the transition to fast reconnection ends up low enough that pairs are deep in the suppressed regime. A simple exponential suppression factor would lead to more than an order of magnitude error in the transition radius, so pairs must be treated more accurately. Despite pairs being in the suppressed regime, the pair number density can greatly exceed the baryonic-associated electron number density and so crucially affect the transition radius as seen from equation (6). We must also consider neutrino cooling near the compact object, because strong cooling can lead to an effective drop in \( \delta_{\text{SP}} \) (Uzdensky & McKinney 2011) and so trigger fast reconnection causing the jet to dissipate before it is launched. Overall, this requires us to compute the properties of a jet filled with a complex of multiple reconnecting slabs, each consisting of baryons with electrons, pairs and neutrinos at arbitrary optical depths. This is achieved by treating the current layer structure similarly to that done for radiative accretion discs (see e.g. Kohri et al. 2005).

### 3 LARGE-SCALE JET STRUCTURE

In this section, the large-scale axisymmetric structure of relativistic GRB jets from BHs or magnetars is presented. The large-scale jet structure acts as a key constraint on the reconnection physics by fixing the radial dependence of magnetic pressure. During reconnection, the magnetic pressure balances the thermal pressure within current layers, which determines the contributions of baryons, photons, pairs and neutrinos.

Fig. 2 shows the basic elements of the central engine and the production of an axisymmetric jet. The jet could be confined laterally (up to a radius \( r_{\text{mono}} \)) by an accretion disc, disc corona, disc wind, stellar envelope, or some ambient medium. A generalized jet solution that applies for both small and large radii is obtained in Appendix A, which collects together results from recent analytical works and simulations (Tchekhovskoy, McKinney & Narayan 2008; Tchekhovskoy et al. 2009, 2010b). However, the full solution can be cumbersome for obtaining simple scaling laws for how the results depend upon model parameters.

Therefore, this section outlines a simplified jet structure that approximates the full jet structure model and helps to highlight the basic elements of the full solution. The radius is assumed to be much larger than the size of the compact object (i.e. \( r \gg r_{\text{fp}} \)), at which a field line attaches at a foot-point radius \( r_{\text{fp}} \) and at a foot-point angle \( \theta_{\text{fp}} \) across the surface of the compact object (see Fig. 2). The radius is also assumed to be larger than the deconfinement radius \( r_{\text{mono}} \). The following expressions apply for \( 0 \leq \theta \leq \pi/2 \) with an assumed equatorial symmetry.

The degree of collimation inside \( r_{\text{mono}} \) is given by a parameter \( \nu \), such that for angles \( \theta \ll 1 \) (i.e. cylindrical radius \( R \ll r \)) one can show that the field lines obey

\[
\frac{r}{r_{\text{fp}}} \approx \theta_{\text{fp}}^{-2/\nu} \left( 2 \sin \frac{\theta_{\text{fp}}}{2} \right)^{2/\nu}, \tag{9}
\]

![Figure 2](https://example.com/figure2.png)

**Figure 2.** GRB jet structure near the central engine: a relativistic MHD jet is driven by a compact object with rotation frequency \( \Omega \) that causes the field line at the foot-point radius \( r_{\text{fp}} \) and opening half-angle \( \theta_{\text{fp}} \) to have a field-line rotation frequency \( \Omega_{\text{2p}} \) (offset from the axis for clarity). At larger radii, each field line follows a collimating trajectory with opening half-angle \( \theta_{\text{fp}} \), while the entire jet has an opening half-angle \( \theta_{\text{j}} \) corresponding to the largest \( \theta_{\text{fp}} \) allowed by the central engine’s accretion disc, corona or wind.
where the field collimates with $2 > v > 0$ out to $r_{\text{mono}}$, after which it follows the $v = 0$ type radial geometry on a particular field line with fixed opening half-angle of $\theta_i$ giving

$$\theta_i \approx \left( \frac{r_{\text{mono}}}{r_{fp}} \right)^{-1/2} \left( \frac{\sin \theta_{fp}}{2} \right),$$

(10)

where the total jet opening angle $\theta_i$ corresponds to the value of $\theta_{fp}$ for the largest value of $\theta_{fp}$ that is allowed by the presence of a confining medium near the compact object.

For $r \geq r_{\text{mono}} \gg r_{fp}$, the radial field strength is

$$B_r \approx B_{r,fp} \left( \frac{r_{\text{mono}}}{r_{fp}} \right)^{-2} \left( \frac{r}{r_{\text{mono}}} \right)^{-2} \tan(\theta_i/2).$$

(11)

The $\theta$ component is small and can be neglected. The toroidal $\phi$ component is

$$B_\phi \approx B_{\phi,fp} \left( \frac{r_{\text{mono}}}{r_{fp}} \right)^{-2} \left( \frac{r}{r_{\text{mono}}} \right)^{-1},$$

(12)

where for rapidly rotating BHs or neutron stars (NSs) $r_{fp}\Omega_{fp} \lesssim 0.25c$ and $\Omega_{fp}$ is the field rotation frequency one can set at each foot point. For a BH angular rotation rate of $\Omega = \Omega_{H} = (jc)/(2r_{H})$ with horizon radius $r_{H}$ and dimensionless BH spin $j$, the value of the field line angular rotation rate is $\Omega_{fp} \approx \Omega_{H}/2$. For a Lorentz factor of $\gamma \gg 1$, one can show that $b^2 \approx B_\phi^2/\gamma^2$, which allows one to obtain the electromagnetic pressure and energy density via $\rho_{EM} = \rho_{EM}^0 = b^2/(8\pi c^2)$, where $b$ is the comoving field strength. The baryonic rest-mass density is

$$\rho_b \approx \rho_{b,fp} \left( \frac{B_{\phi,fp}}{B_{\phi,fp}} \right) \left( \frac{1}{\gamma} \right),$$

(13)

where $\gamma$ is some estimate of the Lorentz factor for $r \geq r_{\text{mono}}$. We define a magnetization parameter as

$$\zeta = \frac{B_\phi^2}{8\pi \rho_{b,fp} c^2},$$

(14)

which is used to define $\rho_{b,fp}$ from $B_{\phi,fp}$. Near the compact object, the value of $\zeta$ is similar to the electromagnetic energy per particle.

Overall, for a given $\zeta$, $B_{\phi,fp}, \gamma_{\text{min}}, \theta_{fp}$ and an estimate of $\gamma$, one can determine the radial and angular dependence of $b^2, \rho_b, \theta_i$ and $\theta_{fp}$, and other quantities defined in Appendix A, such as the electromagnetic energy per unit rest-mass energy ($\mu$), the electromagnetic energy flux per unit rest-mass flux ($\mu$), the electromagnetic energy flux per unit mass-energy flux ($\sigma$) and the jet power ($P_j$). This MHD jet solution only has to be applicable up to the radius where significant dissipation occurs, because determining the dissipation radius is the primary goal of this work.

For all models, we choose a rapidly rotating BH foot-point radius of $r_{fp} \approx 4.4 \text{ km}(M_{BH}/M_\odot)$ ($M_{BH}$ is the BH mass) with a rapid rotation rate such that $r_{fp}\Omega_{fp} = 0.25c$. The collapsar case with a BH of mass $M_{BH} = 3M_\odot$ is chosen. For the collapsar model, $r_{\text{mono}} \approx 3 \times 10^{10} \text{ cm}$ is typically chosen (i.e. radius of the progenitor star), while for the short-duration GRB compact object merger model, $r_{\text{mono}} \approx 1.2 \times 10^{7} \text{ cm} = 120 \text{ km}$ is chosen (i.e. extent of the newly formed disc–corona–wind that helps collimate the jet); Tchekhovskoy et al. (2008, 2010b).

A BH is chosen instead of a NS because it is more likely to be a generator of a powerful baryon-pure jet. First, the BH cleans magnetic field lines of mass (MacDonald & Thorne 1982; Levinson & Eichler 1993), so the magnetization can be quite high in the BH case. The magnetization $\mu$ could be limited by neutron diffusion (Levinson & Eichler 2003), which still leads to quite high magnetizations of $\mu \approx 10^3–10^4$ (McKinney 2005a) (corresponding to $\zeta \approx 10^3–10^4$). Secondly, a BH can produce a more powerful jet since its Kerr parameter is $j \sim 1$. For the magnetar case, the outflow only becomes highly magnetized at late time when the power has significantly diminished, and NSs have a Kerr parameter only up to $j \sim 0.6$ before break-up. These issues make it potentially more difficult for the magnetar to operate as both an efficient and a powerful engine of a highly magnetized ultrarelativistic jet (Metzger, Thompson & Quataert 2007; Bucciantini et al. 2008b; Metzger et al. 2011).

One must use Appendix A to obtain an accurate dependence for all MHD quantities (i.e. including $\gamma$) within the jet as a function of radius and angle. Consider typical collapsar model parameters that would lead to an ultrarelativistic jet with $\gamma \sim 1000$. If $\zeta \approx 10^4$, $B_{\phi,fp} = 3 \times 10^{20} \text{ G}$, $r_{\text{mono}} = 3 \times 10^{10} \text{ cm}$ and $\theta_{fp} = \pi/2$, then $\mu \approx 5400$, $\gamma (r = 10^{15} \text{ cm}) \approx 800$, $P_j (r = r_{fp}) \approx 2.2 \times 10^{51} \text{ erg s}^{-1}$, $(\gamma \theta)(r = 10^{14} \text{ cm}) \approx 18$ and $\sigma (r = 10^{14} \text{ cm}) \approx 6$. At $r = 10^{14} \text{ cm}$, there is about approximately six times less kinetic energy than electromagnetic energy, which can be tapped for prompt GRB emission if there exists some mechanism to dissipate the energy.

### 4 JET FIELD SUBSTRUCTURE: GENERALIZED STRIPED WIND

In this section, we consider the process whereby small-scale field reversals and current sheets become embedded within the large-scale jet structure. The comoving length-scale, $\Delta_\phi$, of these jet field substructures plays a prominent role in later calculations because the dissipation rate due to collisional reconnection is dominated by the smallest value of $\Delta_\phi$.

Fig. 3 shows the different types of field geometries that are considered. All of these substructure types probably coexist to some degree. The jet structure presented in Section 3 was formally constructed with a single polarity for the electromagnetic field. However, after some time, there can be many different field polarities within the jet, as in the case of the jet currently emerging from black hole XTE J1550-564. The different field polarities can be produced by field reversals that are set by the substructure in the jet. This can be represented as a pattern of alternating magnetic field polarities, where the alternating field polarities are displaced either in the $\phi$ direction (type B) or the $\theta$ direction (type C). Type D corresponds to a dipolar field that is unstable at large radii. All types are expected to some degree.
field where the return field is presumed to connect back to the region slightly beyond the compact object. However, the polarity of the field can have arbitrary reversals without changing the large-scale force balance of the jet as long as the current sheets are inserted so that only the field direction changes. For an assumed field perturbation of approximately \(\exp[i(k \cdot R_j - \Omega_1 t)]\) with cylindrical radius \(R_j\) for the entire jet, Maxwell’s equations give an overall smallest characteristic electromagnetic field length-scale of \(\Delta \sim 1/[l/(\pi R_j) + \Omega_1/(2\pi c)]\). The comoving length-scale \(\Delta_0\) generally depends upon the substructure type.

Substructure type A corresponds to a time-dependent alternating polarity for a dipolar field near the compact object. The injection of field reversals occurs at an angular rate of \(\sim m\Omega_0\) near the compact object for some spherical harmonic quantum number \(m \geq 0\). Such an injection can occur when the compact object’s rotation-axis is misaligned with the magnetic field axis as in the standard striped wind (Coroniti 1990; Thompson 1994; Spruit et al. 2001; Palenzuela et al. 2010), a NS’s internal large-scale poloidal field dynamo flips the sign of the field, the compact object is fed alternating polarity from the accretion disc (Lovelace et al. 1997), or a large-scale poloidal field accretion disc dynamo determines the field near the BH. Values of \(0 < m < 1\) are allowed since a magnetic dynamo can operate on time-scales much longer than the dynamical time-scale. For example, an accretion disc could have \(m = 0.1\) (Davis et al. 2010).

Substructure type B corresponds to a time-independent non-axisymmetric \((m \geq 1)\) multipolar field threading the compact object or accretion disc (Akiyama et al. 2003). The length-scale of the electromagnetic field is determined by reversals at different foot-point angles in the \(\phi\) direction near the compact object, and so type B is also related to the angular rate of \(\sim m\Omega_0\).

At large distances, substructure types A and B both have field reversals that are radially stacked such that the laboratory-frame \(\Delta = 2\pi c/(m\Omega_0)\). In the laboratory frame, the pattern of reversals in \(\Delta\) is advected to large radii at roughly the speed of light, so that the comoving scale is \(\Delta_0 \sim \Delta \gamma\) (Spruit et al. 2001). There may be many “stripes” separated radially by \(\Delta_0\).

Substructure type C corresponds to a time-independent axisymmetric (e.g. spherical harmonic \(l = 1, 2, 3, \ldots\) corresponding to dipolar, quadrupolar, etc.) multipolar field threading the compact object or disc. Note that \(l\) cannot be zero because jet formation requires a non-zero axial field at the jet base. The scale of the electromagnetic field is determined by reversals at different foot-point angles in the \(\theta\) direction near the compact object. The field-reversal scale is \(\Delta \sim \pi R_j/l\). The scale of changes in the field is perpendicular to the flow direction so that \(\Delta = \Delta_0 \sim \pi R_j/l\). If \(\gamma \theta \geq 1\), then the jet becomes causally disconnected across \(\theta\) due to finite wave speeds for any finite magnetization. The causally connected region has a size \(\Delta_0 \sim \pi R_j/(l\gamma \theta)\), so physical models effectively have \(l \geq \gamma \theta\) when using the original \(\Delta_0\) formula. The model described in fig. 2 of Romanova & Lovelace (1992) corresponds to this substructure type C with \(l = 2\) and with a prescribed structure in \(\phi\). Another example of type C is a magnetic tower model, where there is a single large current sheet that separates two regions (Lynden-Bell 1996; Uzdensky & MacFadyen 2006, 2007). This substructure is qualitatively different from substructure types A and B because \(\Delta_0 \propto r\), while the other substructures do not depend upon \(r\) once \(\gamma\) is roughly constant.

Substructure type D corresponds to an ordered dipolar field at the jet base, but the jet becomes unstable at large radii. Such instabilities could be due to intrinsic jet properties leading to the, for example, current-driven magnetic kink instability or due to turbulence at boundary layers such as the jet–wind or jet–envelope boundary. For example, in the relativistically magnetized limit of MHD jets, if the comoving toroidal field exceeds the comoving poloidal field by factors of a few to tens, then the growth time-scale for the kink instability is \(T \sim R_j/c\) generating structure from \(\Delta_0 \sim 2\pi R_j\) to \(\Delta_0 \sim r\) (Narayan et al. 2009). Such kink modes would be stabilized by jet expansion or finite mass-loading leading to causal disconnection across much of the jet except within \(\theta \lesssim 1/l\gamma\) (Spruit et al. 2001). In general, the scale of the perturbations could be randomly oriented across \(\sim \pi R_j\) and generated on a time-scale of \(\sim \gamma 2\pi c\) in the laboratory frame. Hence, this substructure corresponds to some mixture of substructures A–C for \(m \geq 0\) and \(l > 0\) within \(\Delta_0 \sim 1/[l/(\pi R_j) + m\Omega_0/(\gamma 2\pi c)]\).

Fig. 4 shows the jet substructures at large radii. Types A and B are similar, but in detail they differ. For types A–C, at all radii (including near the central compact object), the comoving poloidal field is at most to within factors of order unity comparable in magnitude to the comoving toroidal field. For types A–C, the toroidal field dominates in the laboratory and comoving frames for \(r \gg r_{\text{magn}}\). Generally, substructure D might correspond to having a strong poloidal field.

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Figure 4. Magnetic field reversals at large radii: shows substructure types A–D at large radii where the toroidal field dominates and the flow carries many field reversals harbouring current layers separated by length-scale \(\Delta_0\). Type A corresponds to time-dependent reversals with \(\Delta_0 \gtrsim \gamma(2\pi c/\Omega_0)\). Type B corresponds to a ‘barber pole’ of reversals with \(\Delta_0 \sim \gamma(2\pi c/(m\Omega_0))\) \((m \geq 0\) is the toroidal spherical harmonic quantum number). Type C corresponds to reversals in \(\theta\) with \(\Delta_0 \sim \gamma R_j/l\) \((l > 0\) is the poloidal spherical harmonic quantum number). Type D corresponds to reversals due to instabilities or turbulence at boundary layers and is some mixture of types A–C. In general, types A–D can be described by some particular (more generally a power spectrum of) \(m \geq 0\) and \(l > 0\) to obtain \(\Delta_0 \sim 1/[l/(\pi R_j) + m\Omega_0/(\gamma 2\pi c)]\).
However, magnetic kink driven modes are expected to typically leave the jet’s comoving toroidal and poloidal fields as comparable (i.e. when the so-called ‘safety factor’ is of the order of unity in the comoving frame), because that is when the instability is initiated. Overall, one concludes that any guide field (which does not reconnect) in the current sheet is not much stronger than the reconnecting field. This means the analyses in Section 2 and in later sections that assume a ‘weak’ guide field are accurate to within factors of order unity or better, because the Sweet–Parker analysis is only significantly modified if the guide field is significantly stronger than the reconnecting field.

In summary, the jet can develop various substructures that have comoving scale \( \Delta_0 \) satisfying

\[
\frac{1}{\Delta_0(l, m)} \sim \frac{l}{\pi R_j} + \frac{m \Omega \rho}{\gamma^2 \pi c},
\]

for some real numbers \( l > \max(0, \gamma \theta) \) and \( m \geq 0 \). Only dynamo and jet-medium interaction computations can determine (the power spectrum of) \( l \) and \( m \), but many dynamos and instabilities saturate at low \( l, m \sim 1 \). The length-scale \( (\Delta_0) \) over which the comoving electromagnetic field changes is a natural scale over which any dissipation term enters the comoving induction equation, and so \( \Delta_0 \) plays a central role in determining the comoving physics of reconnection discussed in the next section. At large radii, these modes correspond to the jet having current sheets with a guide field that is not significantly stronger than the reconnecting field, which allows one to estimate (to within factors of order unity) the properties of the collisional layer using the Sweet–Parker analysis without a guide field.

5 RECONNECTION PHYSICS

In this section, the conditions for a plasma to undergo collisional or collisionless reconnection in the comoving frame of the jet are computed. As described in Section 4, the characteristic length-scale of the electromagnetic field (before reconnection occurs) is given by \( \Delta_0 \). As discussed below, such current distributions can be unstable to collapse to a much thinner size \( \delta \). The jet is assumed to be filled with a complex of multiple current layers each of thickness \( \delta \ll \Delta_0 \). Instead of seeking a solution that is in global force balance within the jet, all fluid quantities (\( \rho, u, v, B \)) in the background jet solution) are assumed to be roughly homogeneous within the current layer at any given radius.

In Section 5.1, the formation and length of current sheets is discussed, because the length enters models of slow and fast reconnection. In Section 5.2, the EoS and emission rates for the current layer are summarized (see Appendix B for details). In Section 5.3, the transition from slow to fast reconnection is discussed (see Appendix C for more details). In Section 5.4, how the electromagnetic field is modified by fast dissipation is computed. Finally, in Section 5.5, the time-scales for observed dissipation/emission variability are estimated.

5.1 Layer formation and length

First, we must establish that current sheet formation is possible on time-scales shorter than the jet-propagation time-scale to distances relevant for prompt GRB emission. Current layer formation starts with a distributed current of size \( \Delta_0 \), that spans a null point in the magnetic field strength. Such current distributions can collapse to current sheets on fast Alfvén crossing time-scales through a variety of mechanisms: (1) exponential collapse of the distributed current until the thermalized gas pressure within a thickness \( \delta < \Delta_0 \) can support the external electromagnetic pressure (Kulsrud 2005); (2) collapse as part of the evolution of the global magnetic field as occurs for the equatorial current sheet in a pulsar wind; or (3) intrinsically unstable collapse at an X-point (Dungey 1953, 1958; Imshennik & Syrovatskii 1967; Waeltbroeck 1993; Loureiro et al. 2005). For example, X-type collapse forms current sheets on a time-scale

\[
t_t \sim \frac{\Delta_0}{2 \nu a} \ln S
\]

(Priest & Forbes 1986), where \( v_A = c |\mathbf{\beta}_A/\sqrt{\rho_A} | \sim c \) is the Alfvén speed outside the newly forming current layer, \( \xi = \rho_a c^2 + u_\perp^2 + p_a + b_\perp^2/(4\gamma) \), where \( |\mathbf{\beta}_a| \) is the upstream comoving electromagnetic field strength, \( u_\perp \) is the upstream gas internal energy and \( p_a \) is the upstream gas pressure. The relevant controlling parameter is the global Lundquist number, \( S \equiv \Delta_0 \nu_\perp/\eta \), describing the ratio of the resistive time \( (T_{\text{res}} = \Delta_0^2 \eta/\nu_\perp \) for resistivity \( \eta \) to the typical electromagnetic advection time \( (T_A = \Delta_0/\nu_A) \). Typically \( S \gg 1 \) and so ideal MHD is valid on scales of the order of \( \Delta_0 \). The above time-scale shows that current sheet formation only depends logarithmically on the resistivity. For rapidly rotating BHs generating jets with \( \gamma \sim 100 \), half-opening angle \( \theta_j \sim 2^\circ \), substructure types A or B with \( m = 1 \), and \( S \sim 10^5 \), current sheet formation occurs by \( r > 10^{13} \) cm as comparable with the expected radius of prompt GRB emission. Thus, current sheet formation is a relatively fast process that can occur during jet propagation.

Secondly, the length of the current layer is needed because this is what enters the slow and fast reconnection models. The length of the current layer in the comoving frame \( (L_0) \) can be estimated from the characteristic reversal length-scale \( (\Delta_0) \) of the electromagnetic field, because this is the natural length-scale that enters the MHD equations. The X-type collapse described above can feed off perturbations introduced initially in the jet, producing structures with \( L_0 \sim \Delta_0 \) (Priest & Forbes 1986). In addition, the presence of a complex of multiple current layers can itself induce structures with \( L_0 \sim \Delta_0 \) (Otto & Birk 1992; Yan et al. 1994). Ideal MHD-type processes such as magnetic Rayleigh–Taylor instability can also produce structure along the layer with \( L_0 \sim \Delta_0 \) (Lyubarsky 2010a). Even in the absence of any current layers (i.e. \( m = 0 \) and \( l = 1 \)), the jet still has a fundamental characteristic scale of \( \Delta_0(1, 0) = \pi R_j \).

These arguments suggest that the length of the current layer should be within the range \( \Delta_0 \lesssim L_0 \lesssim \pi R_j \). Because \( \Delta_0 \sim R_j \) is represented by \( l \sim 1 \) and for most of the paper \( \Delta_0 \) never appears explicitly independent of \( L_0 \), for simplicity, the allowed range for \( L_0 \) is subsumed as an allowed variation in \( l \) and \( m \). With the length of the layer determined, one determines the thickness of the slow collisional reconnection Sweet–Parker layer in Section 5.3.

5.2 Current layer pressure, temperature, density and emission rates

The reconnection physics is determined not only by the externally imposed magnetic field, rest-mass density, internal energy density, field-reversal scale and length of the current layer as described in the previous sections, but also by the form of energy that the magnetic energy is dissipated into.

The solution for the multi-species EoS and emission rates are described in Appendix B, which follows similar physics to that used for radiative accretion discs. In summary, the gas EoS consisting of baryons, photons, non-pair-produced electrons, pair-produced
electrons–positrons, and neutrinos is given by the total internal energy density

\[ u_e = u_\gamma + u_v + u_n + u_{\text{pairs}} + u_e \]  

and total thermal gas pressure

\[ p_g = p_\gamma + p_v + p_n + p_{\text{pairs}} + p_e, \]  

where each species is listed, respectively, and is a function of temperature (T) and baryon density (\( n_b \)).

Given the proton number density (\( n_p \)), the baryon number density (\( n_b = n_p/m_p \)) and charge neutrality such that the electron number density \( n_e = n_b \), the electron fraction is \( Y_e = n_e/n_b \). A value of \( Y_e \approx 0.1 \) is assumed (see Appendix B). Also, \( n_e \equiv n_{e-} + n_{e+} \), where \( n_{e-} \) and \( n_{e+} \) are the total electron and total positron number densities, respectively. Let the total electron+positron number density be \( n_{e,\text{tot}} \equiv n_{e-} + n_{e+} \), then the pair-only number density is \( n_{\text{pairs}} = n_{e,\text{tot}} - n_e \).

The reconnecting layer is treated as a radiating slab, with a total cooling rate \( \dot{Q}_c \) for photons, pairs and neutrinos of

\[ \dot{Q}_c = \dot{Q}_\gamma + \dot{Q}_{\text{pairs}} + \dot{Q}_n, \]  

for each species, respectively. The respective optical depths in the jet are given by \( \tau_\gamma, \tau_{\text{pairs}} \) and \( \tau_v \), which are separated into scattering and absorption optical depths (e.g. \( \tau_{\gamma,\text{scat}} \) and \( \tau_{\gamma,\text{abs}} \), respectively). A non-negligible \( \dot{Q}_{\text{pairs}} \) occurs when \( \tau_{\text{pairs}} \ll 1 \) as allowed when pairs leave the current layer by travelling along field lines.

### 5.3 Transition from slow collisional to fast collisionless reconnection

In this section, we discuss how the current layers within the GRB jet make a transition from collisional to collisionless reconnection, which triggers the Petschek-like reconnection geometry leading to fast reconnection. Then, dissipation and emission from fast reconnection can contribute significantly to prompt GRB emission.

When the collisionless reconnection layer thickness is larger than the Sweet–Parker thickness (Sweet 1958; Parker 1957), the layer enters a Petschek-like reconnection geometry (Petschek 1964) allowing for fast reconnection (see e.g. Ma & Bhattacharjee 1996; Uzdensky & Kulsrud 2000; Kulsrud 2001; Cassak et al. 2005; Yamada et al. 2006). For current layers where plasma \( \beta = p_l/p_b \sim 1 \), the condition that the plasma no longer obeys the resistive MHD equations on the scale of the current layer can be shown to be equivalent to

\[ \delta_{\text{SP}} < d_i = c/\omega_{pi}, \]  

where \( \omega_{pi} \) is the ion plasma frequency (Kulsrud 2005). Here, \( \delta_{\text{SP}} = \delta_{\text{SP}} A^{-1/2}, \) where \( \delta_{\text{SP}} \sim L_0 S^{-1/2} \) is the nominal Sweet–Parker thickness that is determined using a balance of gas pressure (\( p_g \)) and electromagnetic pressure (\( p_{\text{EM}} \)) across the layer by following the Sweet–Parker analysis. The compression ratio \( A \sim \max(1, Q_c/\delta_{\text{SP}}) \) (where \( Q_{\text{SP}} \) is the dissipation rate in the Sweet–Parker layer) accounts for radiative cooling in the layer (Uzdensky & McKinney 2011). Typically, \( A \sim 1 \) is found in this paper. The value of the Lundquist number, \( S \), is obtained from Spitzer and Compton drag resistivities. For more details, see Appendix C.

When the above condition is satisfied, the reconnection operates in a fast collisionless mode. The transition radius to fast collisionless reconnection is then given by

\[ r_{\text{pairs}} = r[\delta_{\text{SP}} = d_i]. \]  

This transition radius is used to identify where fast magnetic reconnection starts and so leads to dissipation and emission.

One interesting caveat is that once collisionless reconnection is initiated, the higher dissipation rate may lead to different plasma parameters such as the temperature. The loss of magnetic energy may raise the local temperature in the optically-thick limit, but it can lower the temperature at slightly larger radii if the flow is already optically thin because a loss of magnetic energy leads to lower gas pressures. If resistivity increases due to dissipation, then the Sweet–Parker thickness increases and may lead to recovery of the collisional mode of reconnection. In such a situation, the plasma may enter a ‘marginally collisionless’ reconnection mode in which \( \delta_{\text{SP}} = d_i \) is maintained as the plasma dissipates and continues to enter and exit the fast and slow reconnection modes. It is difficult to estimate the average rate of reconnection in such a scenario (Uzdensky 2007b; Goodman & Uzdensky 2008). However, in some studies, the transition back to slow reconnection was shown to be avoided by a hysteresis effect (Cassak et al. 2005).

Slow collisional reconnection can be as slow as Sweet–Parker, giving an inflow velocity of \( v_i \sim v_A S^{-1/2} \ll v_A \sim c \), or it may be as fast as \( v_i \sim 0.01v_A \) (Uzdensky et al. 2010). Various Studies have found that the fast collisionless reconnection rate is of the order of \( v_i \sim 0.1v_A \) or even \( v_i \sim v_A \) (for details see Appendix C4). For simplicity and definiteness, the reconnection rate is chosen to be \( v_i \sim 0.1c \) in all cases. The exact number used for \( v_i \) only moves the expected dissipation radius and does not significantly affect the transition radius or any other calculations. Once significant dissipation has occurred, the value of \( v_A \) drops below \( c \), but in all cases, we will find it only drops to \( v_A \sim 0.01c \) by the time the slow collisional mode would recover. Also, since \( v_A \) dropping below \( c \) indicates most of the electromagnetic energy flux has been dissipated (i.e. the goal of this work), we simply avoid detailed discussions about the solution at large radii where this occurs.

### 5.4 Effect of magnetic reconnection on the jet structure

In this section, we determine how the jet electromagnetic energy density, \( b^2/(8\pi t) \), versus \( r \) is modified by fast dissipation once the transition to fast reconnection has been triggered. The resulting function for \( b^2 \) versus radius modifies the original large-scale jet structure dependence given in Section 3 and Appendix A. The radial extent over which significant dissipation occurs also determines the photon/pair/neutrino opacity integrals used to determine the EoS and emission rates for the radiative current layer in Appendix B3.

The electromagnetic energy dissipated in the jet is given by the electromagnetic enthalpy flux into both sides of a current layer where each side harbours a region of size \( \Delta_0/2 \) from which magnetic flux can be accumulated for each sheet. The comoving electromagnetic dissipation energy density rate is then

\[ \dot{Q}_{\text{EM}} = \frac{d}{d_\text{co}} \left( \frac{b^2}{8\pi t} \right) = -2 \frac{b^2}{4\pi t} v_i \Delta_0, \]  

where \( v_i \) is the comoving rate of reconnection, corresponding to the inflow of magnetic flux into the current layer. Note that the relevant length-scale is \( \Delta_0 \), the distance between layers instead of the length of the layer, because magnetic flux must be brought in from the space between layers.

For the jet structure, the relativistic MHD energy–momentum equations of motion are dominated by the radial electromagnetic...
field advection term either for an electromagnetically dominated jet with \( \gamma \gg 1 \) or for \( \gamma \) and \( \theta_1 \) roughly constant as occurs for \( r \gtrsim r_{\text{trans}} \) (as applicable to all astrophysically relevant models studied in this paper). Then, equation (22) is a sink term to the quasi-steady relativistic MHD energy equation given by

\[
0 = -\left( \frac{1}{r^2 \sin \theta} \right) \partial_r (r^2 \sin \theta b^2 \gamma^2 c - 4\pi Q_{\text{EM}} \gamma). \tag{23}
\]

For \( Q_{\text{EM}} \to 0 \) (i.e. \( v_t \to 0 \)) corresponding to the ideal MHD jet solution, the comoving electromagnetic energy density is a power-law dependence with \( b^2 \propto r^{-3} \). In general, equation (23) can be solved for \( b^2(r) \) to give

\[
b^2(r) = \exp \left[ -\int r_0 \, dr \left( -\frac{2}{r} - \frac{2\gamma}{c^2 \gamma \Delta_0} \right) \right], \tag{24}
\]

which assumes \( v_t \ll v_A \sim c \) as relevant for electromagnetically dominated jets. Once \( v_t \ll c \), dissipation (i.e. the goal of this work) has already occurred, and then we simply avoid detailed discussions about the overall solution for such radii rather than seeking the correct \( b^2 \) versus \( r \) in the weakly magnetized regime.

For \( m \neq 0 \) modes, the value of \( \Delta_0 \) is fixed (i.e. does not scale with \( r \) for \( r \gtrsim r_{\text{mono}} \)), and then the jet solution has an exponential term such that

\[
br^2(r = r_{\text{trans}}) = \left( \frac{r}{r_{\text{trans}}} \right)^{-2} \exp \left[ -\int r_0 \, dr \left( -\frac{2\gamma}{c^2 \gamma \Delta_0} \right) \right], \tag{25}
\]

for a radius of transition to fast reconnection \( r_{\text{trans}} \) and \( r \gtrsim r_{\text{trans}} \).

On the other hand, for \( l > 0 \) modes \( \Delta_0 \propto r \), and then the dissipation does not lead to exponential decay and instead leads to a modified power law given by

\[
br^2(r = r_{\text{trans}}) = \left( \frac{r}{r_{\text{trans}}} \right)^{-2(1 + \frac{\ln \frac{v_t}{c}}{\ln \frac{v_A}{c}})}, \tag{26}
\]

for \( r \gtrsim r_{\text{trans}} \). In general, one obtains \( r_{\text{obs}} / r_{\text{trans}} \sim f^{-(\pi \gamma c \theta_1 / 24 \pi)} \) for a fraction \( f \) of magnetic energy dissipated (e.g. \( r_{\text{obs}} \sim 10^6 r_{\text{trans}} \)) for \( l = 5 \), \( v_t/c = 1 \), \( \gamma \theta_1 = 10 \) and \( f = 1/2 \). A simple conclusion would be that negligible change in \( b^2 \) occurs for large enough \( x_t \equiv (\gamma \theta_1 / \pi c) \) as due to a given \( l \) mode becoming causally disconnected across the jet. However, the \( l \) modes generate plasmoids with radial size \( L_0 \) that may remain fixed in size once significant dissipation occurs. This alternative assumes, perhaps optimistically, that for \( l \) modes the magnetic flux can be accumulated and dissipated from all directions instead of only in the \( \theta \) direction. Thus, instead of assuming some \( l \) modes fail to dissipate, an alternative is also considered that the \( l \) modes produce plasmoids similar to substructures A and B except with an effective \( \Delta_0 \to L_0 \) whose value is fixed with radius as evaluated at the transition radius.

Once fast reconnection starts at \( r_{\text{trans}} \), then equation (25) shows that reconnection completes a single e-folding of dissipation/emission by

\[
r_{\text{obs}} \approx r_{\text{trans}} + \frac{\gamma c \Delta_0}{2v_t}, \tag{27}
\]

assuming \( m \) modes or the optimistic case for \( l \) modes. For example, consider a model with \( m = 0.1 \), \( \gamma \sim 300 \), a rapidly rotating BH such that \( \Omega_r \sim 0.25c/r_{\text{trans}}, \Delta_0 \sim 2\pi c \gamma / (m \Omega_r) \approx 3 \pi \times 10^6 \text{ cm}, \) and \( v_t \sim 0.1c \). This model gives \( r_{\text{obs}} \approx r_{\text{trans}} + 10^4 \text{ cm}, \) which results in a reasonable radius for prompt GRB emission for small enough \( r_{\text{trans}} \). For \( r \gtrsim r_{\text{trans}} \), most of the dissipation has completed and leaves relatively few photons or pairs. Thus, this radial extent can be used to determine the opacity integral for photons, pairs and neutrinos within a relativistic jet as done in Appendix B3.

Between the transition and dissipation radii, most of the jet’s electromagnetic power, \( P_j^{\text{EM}} \), is lost as radiation from plasma blobs moving at Lorentz factor \( \gamma \), being observed at some angle away from the jet-axis. For a laboratory-frame radial dissipation range of \( dr \sim \gamma R_{\text{trans}} / v_t \), laboratory-frame cross-sectional area \( \sim \pi R_j^2 \) and laboratory-frame dissipation rate \( \sim \gamma Q_{\text{EM}} \), the total isotropic jet luminosity is

\[
L_j \sim (\gamma Q_{\text{EM}}) (\gamma R_{\text{trans}} / v_t) (\pi R_j^2) \sim \gamma^2 b^2 c R_j^2 \sim P_j^{\text{EM}} \sim P_j, \tag{28}
\]

which shows that the laboratory-frame loss rate of electromagnetic energy equals the laboratory-frame electromagnetic jet power. This also shows that the reconnection rate, \( v_t \), does not determine the luminosity from the region \( dr \). Instead, a larger number of layers within the range \( dr \) dissipate simultaneously for smaller \( v_t / c \), so a fixed jet power leads to a fixed total isotropic luminosity.

### 5.5 Variability from fast reconnecting layers

In this section, we compute various time-scales from reconnecting layers that may be imprinted in observations of the prompt GRB emission. The jet substructure type determines the temporal structure of the prompt GRB emission via the number of dissipating layers. Also, emission time-scales can be linked to the total event time-scale, the event time-scale per unit number of reconnecting current layers imbedded in the jet, the dissipation time-scale of a single current layer, and the thermal photon radiation emission time-scale.

Consider a reconnecting layer (plasma blob) that begins to dissipate at a transition radius \( r_{\text{trans}} \) with a comoving dissipation time-scale of \( d t_{\text{obs}} \). This is related to the laboratory-frame time-scale via

\[
d t_{\text{lab}} \approx \gamma d t_{\text{obs}}. \tag{29}
\]

Photons emitted at the start of fast reconnection travel outwards as the plasma blob also travels outwards. The observed photon time difference from the beginning to the end of current sheet dissipation is

\[
d t_{\text{obs}} \approx d t_{\text{lab}} / (2\gamma^2), \tag{29}
\]

such that the observed photon time-scale accounting for light-travel effects is

\[
d t_{\text{obs}} \approx d t_{\text{obs}} / (2\gamma^2). \tag{29}
\]

For the \( m \) modes and for the optimistic case for \( l \) modes, the comoving dissipation time-scale is

\[
d t_{\text{obs},\text{m}} \sim \Delta_0 / (4v_t) \sim \Delta_0 / (4v_t), \tag{29}
\]

and the observed dissipation time-scale is

\[
d t_{\text{lab},\text{m}} \sim \Delta_0 / (4v_t), \tag{30}
\]

Each layer present in the jet over the duration of the event gives one expected emission time-scale. For substructure types A and B, the laboratory-frame cycle time to produce a single slab is

\[
d t_{\text{lab},\text{m}} \sim 2\pi / (m \Omega_r) \approx 2\pi / (m \Omega_r). \tag{30}
\]

These time-scales may be used to determine the number of layers that contribute to the prompt GRB emission.
associated with each shell that emerges from $r = r_{\text{trans}}$. For substructure type C, there are $f$ simultaneous emitting layers that in principle could dissipate stochastically in time rather than going off simultaneously. In that case, for an event duration $T$, one expects a variability time-scale of

$$dr_{\text{obs}} \sim \frac{T}{l}. \quad (31)$$

Reconnection is expected to proceed stochastically, but it will also have a short time-scale associated with the transit time of fluid through a single reconnection layer of exhaust length $L_0$. The comoving transit time is of the order of $dr_{\text{trans}} = L_0/v_{\lambda}$, such that each reconnection event has an observed time-scale of

$$dr_{\text{obs},\tau} = \frac{L_0}{2\gamma v_{\lambda}} \quad (32)$$

The thermal photon radiation emission time-scale is

$$dr_{\text{obs},\gamma} \sim \frac{u_{\gamma}}{2\gamma q_{\gamma}}, \quad (33)$$

corresponding to the time-scale over which the thermal radiation contributes fully to its component of force balance against the magnetic pressure in the current layer complex.

For scale-height $H$ and mean free path of $\lambda = 1/(dr/ds)$, the photon diffusion time-scale is

$$dr_{\text{obs},\lambda} = \frac{1}{2\gamma} \max \left[ \frac{3H^2}{\lambda c^2}, \frac{H}{c} \right], \quad (34)$$

where for $H = H_{\text{abs}}$ one has $H_{\lambda} \equiv r$, the optical depth. The observer absorption diffusion time-scale is denoted by $dr_{\text{obs},\lambda}$ when setting $H = H_{\text{abs}}$, while the observer scattering diffusion time-scale is denoted by $dr_{\text{obs},\lambda}$ when setting $H = H_{\text{sc}}$.

6 RESULTS: JET DISSIPATION

This section computes several results obtained with the procedure:

(i) Given the independent variables $\xi$, $B_{\gamma}$, $v$, $r_{\text{mono}}$, and $\theta_{\gamma}$, and the jet structure from Section 3 (specifically, using Appendix A), one obtains $\rho_0(r, \theta)$, $b^2(r, \theta)$, $\gamma(r, \theta)$, $Q_0(r, \theta)$ and $\theta_{\gamma}(r)$.

(ii) Given $l$ and $m$ and the jet substructure from Section 4, one obtains the single current layer spacing of $\Delta_0(l, m)$ and layer length of $L_0(l, m)$ at any point in the jet.

(iii) The multiple current layer complex equations determined in Section 5, with the non-ideal MHD jet solution for $b^2(r)$ (equations 25 and 26) used after $r = r_{\text{trans}}$, are iteratively solved as a set of simultaneous equations for the temperature $T$, pair number density $n_{\text{pairs}}$ and the compression ratio $A$. A single (unique) solution is always found.

(iv) Given $T$, $n_{\text{pairs}}$, and $A$, the remaining quantities are iteratively computed.

The solution at each radius is obtained independently. The transit time through the current layer is roughly the equilibrium time-scale, which is sufficiently small compared with the jet flow time for this to be a valid approximation except for radii much larger than the transition or dissipation radii.

In what follows, a fiducial model is discussed in detail in Section 6.1, a model parameter exploration is presented in Section 6.2, the simplified jet structure given in Section 3 is used to write down scaling laws for results as functions of independent variables, a comparison is made to prior work in Section 6.4, and finally in Section 6.5 a discussion is provided about how the reconnection switch model applies to other (non-GRB) jet systems in order to identify whether GRB jets are special.

6.1 Results: fiducial model with full jet structure

Consider a fiducial model of a rapidly rotating BH with a magnetization $\zeta = 10^4$ that leads to $10^4 \lesssim \gamma \lesssim 1000$; a radial magnetic field at the compact object of $B_{\gamma}$ that leads to $P_j \sim 10^{49} \sim 10^{52}$ erg s$^{-1}$; a collimating field geometry with $v = \gamma/3$ as roughly consistent with a shocked stellar envelope (see discussion in Tchekhovskoy et al. 2008, 2010b); a deconfinement radius of $r_{\text{mono}} = 3 \times 10^{19}$ cm as roughly the radius of a Wolf–Rayet star in the pre-supernova phase; and a foot-point opening half-angle of $\theta_{\gamma} \approx \pi/2$ as applicable for a neutrino-dominated accretion disc that takes on a quite thin geometry near a rotating BH where the jet is launched (see e.g. Kohri et al. (2005) and Chen & Beloborodov (2007), and for simulations of such magnetized thin discs, see e.g. Shafee et al. (2008) and Penna et al. (2010)). The chosen $\zeta = 10^4$ corresponds to an electromagnetic energy flux per unit rest-mass flux of $\mu \approx 5400$ and an electromagnetic energy per baryon rest energy at the field-line foot point of $\mu \approx 3 \times 10^{10}$ cm as roughly the radius of a Wolf–Rayet star in the pre-supernova phase; and a foot-point opening half-angle of $\theta_{\gamma} \approx \pi/2$ as applicable for a neutrino-dominated accretion disc that takes on a quite thin geometry near a rotating BH where the jet is launched (see e.g. Kohri et al. (2005) and Chen & Beloborodov (2007), and for simulations of such magnetized thin discs, see e.g. Shafee et al. (2008) and Penna et al. (2010)). The chosen $\zeta = 10^4$ corresponds to an electromagnetic energy flux per unit rest-mass flux of $\mu \approx 5400$ and an electromagnetic energy per baryon rest energy at the field-line foot point of $\mu \approx 3 \times 10^{10}$ cm as roughly the radius of a Wolf–Rayet star in the pre-supernova phase; and a foot-point opening half-angle of $\theta_{\gamma} \approx \pi/2$ as applicable for a neutrino-dominated accretion disc that takes on a quite thin geometry near a rotating BH where the jet is launched (see e.g. Kohri et al. (2005) and Chen & Beloborodov (2007), and for simulations of such magnetized thin discs, see e.g. Shafee et al. (2008) and Penna et al. (2010)). The chosen $\zeta = 10^4$ corresponds to an electromagnetic energy flux per unit rest-mass flux of $\mu \approx 5400$ and an electromagnetic energy per baryon rest energy at the field-line foot point of $\mu \approx 3 \times 10^{10}$ cm as roughly the radius of a Wolf–Rayet star in the pre-supernova phase; and a foot-point opening half-angle of $\theta_{\gamma} \approx \pi/2$ as applicable for a neutrino-dominated accretion disc that takes on a quite thin geometry near a rotating BH where the jet is launched (see e.g. Kohri et al. (2005) and Chen & Beloborodov (2007), and for simulations of such magnetized thin discs, see e.g. Shafee et al. (2008) and Penna et al. (2010)). The chosen $\zeta = 10^4$ corresponds to an electromagnetic energy flux per unit rest-mass flux of $\mu \approx 5400$ and an electromagnetic energy per baryon rest energy at the field-line foot point of $\mu \approx 3 \times 10^{10}$ cm as roughly the radius of a Wolf–Rayet star in the pre-supernova phase; and a foot-point opening half-angle of $\theta_{\gamma} \approx \pi/2$ as applicable for a neutrino-dominated accretion disc that takes on a quite thin geometry near a rotating BH where the jet is launched (see e.g. Kohri et al. (2005) and Chen & Beloborodov (2007), and for simulations of such magnetized thin discs, see e.g. Shafee et al. (2008) and Penna et al. (2010)).
Figure 5. Jet structure for a fiducial rapidly rotating BH model with $\xi = 10^4$, $B_{r,fp} = 3.2 \times 10^{15}$ G, $v = 3/4$, $r_{mono} = 3 \times 10^{10}$ cm, $\theta_{fp} = \pi/2$ and modes $m = 0.1$, $l = 1$, for quantities along the outermost field line. Top left-hand panel: opening half-angle ($\theta_j$) as the thick solid line. The location of $r = r_{mono}$ is shown by the thin vertical line. Top right-hand panel: Lorentz factor ($\gamma$) as the thick solid line. The total energy flux per unit rest-mass flux ($\mu \geq \gamma$) as the mostly horizontal dotted line. In this and other panels, the label 'Id' marks the same quantity for that line type, but corresponding to the ideal MHD jet solution without electromagnetic dissipation. The ideal MHD jet solution has constant $\mu$, and $\gamma$ continues to rise beyond the dissipation radius. Bottom left-hand panel: magnetic field components ($B_r$ as the solid line and $-B_\phi$ as the dotted line). The ideal MHD jet solution at large radii follows roughly $-B_\phi \propto 1/r$. Bottom right-hand panel: baryon rest-mass energy density ($\rho_b c^2$) as the solid line and comoving electromagnetic energy density ($b^2/(8\pi)$) as the short-dashed line. The thin horizontal solid line shows $|b| = b_{QED}$, the QED critical field. As with the toroidal field strength, the comoving electromagnetic energy is dissipated away in the non-ideal MHD jet solution beyond the transition radius. This is in contrast to the ideal MHD jet solution that would continue with roughly $b^2 \propto r^{-2}$, which is a similar power-law scaling to rest-mass density at large radii. Overall, by $r \approx 10^{14}$ cm, the jet obtains an opening half-angle of $1^\circ$, a Lorentz factor of $\gamma \approx 800$ with $\gamma \theta_j \approx 18$, and a jet power of $P_j \approx 8 \times 10^{51}$ erg s$^{-1}$.

Fig. 5 also shows how the ideal MHD jet solution is modified by the current sheet dissipation. Beyond the transition radius at $r_{trans} \approx 3 \times 10^{13}$ cm, dissipation proceeds due to fast collisionless reconnection. This causes $|b|$ (and so $|B_\phi|$) to drop exponentially for several e-foldings. Fast dissipation finally ceases once the transition back to slow collisional reconnection occurs due to low temperatures leading to a dominant Spitzer resistivity. Despite the significant drop in the comoving toroidal field, only by $r \approx 6 \times 10^{15}$ cm does it become smaller than the comoving poloidal field. Beyond this radius, any current sheet analysis requires consideration of a guide field, but this is beyond the dissipation radius so that all of our work remains valid.

Fig. 6 shows the temperature, collisional and collisionless current layer thicknesses, number densities, and optical depths for neutrinos, photons and pairs as functions of $r$. The sequence of events from small to large radii is as follows: (1) the optically-thick current layer’s temperature decreases until pairs drop out at $r \approx 10^{10}$ cm; (2) photons dominate the gas pressure. However, after the photon absorption photosphere at $r \approx 2 \times 10^{11}$ cm, the photon densities decrease faster; (3) the loss of photons causes the temperature to begin to increase at $r \approx 2 \times 10^{12}$ cm, which eventually leads to a re-emergence of pairs at $r \approx 4 \times 10^{12}$ cm (with $n_{pairs} = n_e$ at $r \approx 8 \times 10^{12}$ cm) and then to a slight quenching of the rise in temperature because the pairs suspend the decrease in the photon scattering opacity; (4) the transition to fast collisionless reconnection occurs at $r_{trans} \approx 3 \times 10^{13}$ cm, where $n_{pairs} \approx 7n_e$; (5) the transition radius is nearly coincident with where photons begin to free stream leading to $\tau_{\gamma,sc} \sim 1$ at $r \approx 5 \times 10^{13}$; (6) eventually, the pairs drop out again at $r \approx 10^{15}$ cm due to the pair absorption and scattering optical depths approaching unity, after which the temperature rises rapidly.
Figure 6. Reconnection layer solution using the jet model shown in Fig. 5. Top left-hand panel: current layer temperature ($T$). The vertical line shows the location of $r_{\text{mono}}$, where the jet becomes deconfined. The jumps in temperature are resolved by several points and explained in the text. Top right-hand panel: fast collisionless (Petschek-like) ion skin depth ($d_i$) as the solid line and slow collisional (Sweet–Parker like) layer thickness ($\delta_{\text{SP}}'$) as the short-dashed line. Note that $d_i = \delta_{\text{SP}}'$ at $r \approx 3 \times 10^{13}$ cm, where fast collisionless reconnection is triggered. As in Fig. 5, the label 'Id' marks the case when the ideal MHD jet solution is used to determine the properties of the collisional layer. Bottom left-hand panel: electron number density ($n_e$) as the solid line, pair number density ($n_{\text{pairs}}$) as the short-dashed line and neutrino number density ($n_\nu$) as the long-dashed line. Jet expansion leads to cooling and a loss of pairs at $r \approx 10^{10}$ cm until they re-emerge with $n_{\text{pairs}} \approx n_e$ at $r \approx 8 \times 10^{12}$ cm. Bottom right-hand panel: neutrino scattering optical depth ($\tau_\nu$, sca) as the solid line, photon scattering optical depth ($\tau_\gamma$, sca) as the short-dashed line, and pair scattering optical depth ($\tau_{\text{pairs},\text{sca}}$) as the long-dashed line. The thin horizontal line shows an optical depth of unity, where $\tau_\gamma$, sca $\approx 1$ at $r \approx 5 \times 10^{13}$ cm. Overall, the key result is that the fast collisionless reconnection mode is triggered at $r \approx 3 \times 10^{13}$ cm where $\tau_\gamma$, sca $\approx 1$, so the transition roughly coincides with where the jet becomes optically thin for photons.

The key point from Fig. 6 is that $d_i = \delta_{\text{SP}}'$ at $r_{\text{trans}} \approx 3 \times 10^{13}$ cm, marking the transition radius to fast collisionless reconnection. Beyond $r_{\text{trans}}$, the collisional resistivity is no longer effective and dissipation proceeds due to collisionless processes that operate on the larger scale of the ion skin depth. This disrupts the current layer into a Petschek-like geometry, which allows a relatively fast reconnection rate of $v_r \sim 0.1$–1.0c rather than a slower Sweet–Parker like rate. This is why we identify this as a catastrophic reconnection switch.

Note that the radial range where electrons are degenerate is far inside the transition radius, so degeneracy does not affect the resistivity nor the value of $r_{\text{trans}}$. Also, note that even with the loss of pairs for $r \sim 10^{10}$–$4 \times 10^{12}$ cm, the proton–electron plasma remains highly collisional, which forces the transition to fast collisionless reconnection to be at large radii.

The transition to fast collisionless reconnection might lead to a marginally collisionless state that forces $d_i \sim \delta_{\text{SP}}'$. Using the full non-ideal MHD jet solution shows that the collisional layer thickness rises once dissipation occurs, and fast dissipation ceases when the two thicknesses become roughly equal because the collisional mode recovers. However, by this radius, significant electromagnetic dissipation has already occurred – and in fact this is what allows the collisional mode to recover. In addition, the possibility for a return to collisional reconnection was tested by using different $v_r$ up to $v_r \sim c$, which leads to a different radial dependence of $b^2$. We find that marginal collisionality was unable to be established for any $v_r$ until there has already been significant electromagnetic dissipation. Also, note that the dissipation included in the model does not itself control the transition to fast reconnection. This was tested by using...
the ideal MHD jet solution for $b^2$ versus $r$, which shows that the thickness for the collisional layer remains much smaller than for the collisionless layer once fast collisionless reconnection would have started.

The condition for fast reconnection in electron–positron plasmas is the pair-dominated switch condition $d_e = \delta_{SP}$. This condition occurs at $r \approx 6 \times 10^{15}$ cm in the ideal MHD jet solution that assumes no dissipation up to this radius. If the baryon-dominated switch condition applies and dissipation did occur, then the pair-dominated condition is reached at $r \approx 2 \times 10^{13}$ cm after significant dissipation has already occurred. However, the pair-dominated switch condition is not expected to be relevant for the fiducial model or for a broad range of variations in model parameters. Pairs only re-emerge after the photon scattering opacity becomes of the order of unity, and the transition to fast collisionless reconnection has occurred to within factors of order unity. Also, the total mass energy of the plasma is found to be carried by protons by the radius where $d_e = \delta_{SP}$ or $d_e = \delta_{SP}$. Thus, the condition for fast reconnection may still be dominated by the protons, although future studies should consider how fast reconnection proceeds when pairs dominate in number density but ions dominate in energy density.

Fig. 6 also shows the scattering optical depth for neutrinos, photons, and pairs. The neutrinos are never optically thick in this model. The quantity $\tau_{\gamma}$ corresponds to the opacity towards the observer, accounting for locally generated pairs and downstream baryonic electrons as computed via equation (B22). The photons become optically thin to scattering at $r \sim r_{\text{trans}}$, which is consistent with our basic argument presented in Section 2. This means that the reconnection switch mechanism occurs once non-thermal and quasi-thermal photons can be produced. Beyond $r \sim r_{\text{trans}}$, the non-ideal MHD jet solution has a photon scattering opacity that deviates only slightly from that of the ideal MHD jet solution. Thus, dissipation is not crucial for lowering the photon scattering optical depth at larger radii. The pair scattering opacity is dominated by Coulomb interactions when the temperature is low, but then the pair density is negligible (e.g. beyond $r \sim r_{\text{trans}}$ leading to a jump in the pair opacity). For the ideal MHD jet solution, the pair optical depth roughly follows that of the photons and continues to drop at all larger radii.

Perhaps the most distinct non-intuitive feature of the solution is the two-step rise and final drop in temperature. Let us consider a simplified calculation to explain why the temperature is forced to rise once the jet becomes optically thin to scattering. Cyclo-synchrotron radiation dominates over free–free emission at all radii, so the optically-thin energy density loss rate is

$$Q \approx n_e m_e^4 \sigma_{TC} \left( \gamma_e^2 - 1 \right) u_{\text{EM}},$$  

(35)

where $\gamma_e \approx 1 + \Theta_e (\Gamma - 1)$ (recall $\Theta_e \equiv k_{B} T (m_e c^2)$) is the electron thermal Lorentz factor. Then, $\tau_{\gamma, \text{abs}} \approx Q \sigma_{TC} (\epsilon_{\gamma, \text{abs}})$ is the absorption optical depth from Kirchhoff’s law, and the scattering optical depth is $\tau_{\gamma, \text{scat}} \sim \sigma_{\text{SP}} n_{\text{pairs}} \Delta_0$, since pairs (when present) typically dominate the photon scattering opacity. The energy density of photons is obtained from the two-stream approximation using equation (B14), which in the limit $\tau_{\gamma, \text{abs}} \ll 1$ and $\tau_{\gamma, \text{scat}} \sim 1$ gives

$$u_{\gamma} \approx 3.2 \tau_{\gamma, \text{abs}} u_{\text{EM}},$$  

(36)

where $u_{\text{EM}} = c \sigma_{TC} T^4$ is the optically-thick radiation energy density, and $p_{\gamma} = u_{\gamma}/3$ is the radiation pressure that dominates over other pressures. Then, the pressure equilibrium condition across the current layer of $p_{\text{EM}} = u_{\text{EM}} = p_{\gamma}$ determines the temperature implicitly via

$$\tau_{\gamma, \text{abs}}^{-1} \approx 2.9 \left( \frac{\Theta_e}{\Gamma - 1} \right) + 1.4 \left( \frac{\Theta_e}{\Gamma - 1} \right)^2 \tau_{\gamma, \text{scat}}$$  

(37)

then $\tau_{\gamma, \text{abs}} \ll 1$ (otherwise the temperature is set by $p_{0, \gamma} = p_{\text{EM}}$). Thus, a drop in the scattering optical depth leads to a rise in the temperature in this limit. One also obtains

$$\tau_{\gamma, \text{abs}} \approx \frac{u_{\text{EM}}}{u_{0, \gamma}},$$  

(38)

which indicates that the absorption opacity drops significantly when the electromagnetic energy density is not high enough to produce optically-thick photons. Equation (37) shows that once the flow becomes optically thin, the temperature is forced to rise up to $T \sim 10^9$ K regardless of the temperature in the optically-thick regime. The full solution has a slightly lower temperature of $T \sim 10^8$ K because the cyclo-synchrotron emission is slightly dominated by higher energy electrons than those with the mean temperature. This gives a solution closer to using $\Gamma = 4/3$ than $\Gamma = 5/3$ in the above simplified equations. Overall, the behaviour of the temperature versus radius can be explained as follows: (1) the temperature decreases with radius due to adiabatic expansion in the optically-thick regime; (2) the temperature rises when the photons become optically thin due to the need to maintain pressure equilibrium with electromagnetic field; (3) the slight flattening in the temperature during its rise (two-step rise) is because pairs re-emerge and increase the scattering opacity, which suppresses the rapid temperature rise; (4) eventually, the pair opacity also decreases and the pairs drop out, which leads to a continued rise in the temperature as the scattering opacity decreases; and (5) finally, the temperature decreases independently of the photon scattering opacity once a significant loss of electromagnetic energy has occurred.

The scattering and absorption photospheres lead to thermal emission and might correspond to the observed peak energy. In addition, low-energy electrons’ cyclo-synchrotron emission is self-absorbed and then Comptonization can dominate their emission leading to a Comptonized thermal component as the peak energy corresponding to $E_{\text{peak}} \sim 10$ keV–2 MeV (Rees & Mészáros 2005; Thompson, Mészáros & Rees 2007). Electromagnetic dissipation at the transition radius (coincident with the photon scattering photosphere) leads to a thermal peak of $E_{\text{peak}} \approx \gamma T \sim 20$ MeV for the fiducial model value of $\gamma \approx 800$. The thermal peak obtained at the photon absorption photosphere is smaller at $T \approx 10^7$ K, giving $E_{\text{peak}} \sim 600$ keV at this relatively high $\gamma \approx 800$. This peak energy is roughly consistent with the observed peak energy for long-duration GRBs.

Fig. 7 shows the Lundquist number, observer time-scales, energy-density loss rates and pressures within the collisional current layer at each radius. For the ideal MHD jet solution, one would estimate that the resistivity is dominated by Compton drag at all radii, with the Spitzer (due to proton–electron Coulomb collisions) becoming nearly as important at $r \approx 7 \times 10^{6}$ cm and at $r \approx 2 \times 10^{15}$ cm. For the full dissipative MHD jet solution, Spitzer resistivity begins to play a role only after significant electromagnetic dissipation occurs once there is a significant decrease in temperature within the layer at $r \approx 6 \times 10^{15}$ cm. The associated Lundquist number is of the order of $S \sim 10^{20}$ and eventually drops at large radii due to MHD dissipation. Fig. 7 also shows various observed time-scales for photon emission. Once fast collisionless reconnection kicks in at $r_{\text{trans}}$, these time-scales give an estimate for observed variability. The rise in the time-scales at large radii is due to the jet being optically thin.
Figure 7. Continued reconnection layer solution (otherwise identical to Fig. 6). Top left-hand panel: Lundquist number ($S$) as the short-dashed line. As in Fig. 5, the label 'Id' marks the ideal MHD jet solution. Using the ideal MHD jet solution for the current layer shows that the Lundquist number stays roughly constant, while the full non-ideal MHD jet solution shows the Lundquist number drops as associated with a high Spitzer resistivity that dominates over the Compton drag resistivity. Top right-hand panel: observer time for transit through the current layer ($t_{\text{obs,tra}}$) as the solid line, observer time for single e-folding for fast reconnection dissipation ($t_{\text{obs,diss}}$) as the short-dashed line, observer time for $m = 0.1$ radial structure induced variability ($t_{\text{obs,m}}$) as the dotted line, and observer time for photon emission from the current layer ($t_{\text{obs,\gamma}}$) as the long-dashed line. Overall, the observer time-scales during significant electromagnetic dissipation are of the order of 1 s and span 0.001–10 s. Bottom left-hand panel: photon energy density cooling rate ($Q_\gamma$) as the solid line, neutrino energy density cooling rate ($Q_\nu$) as the long-dashed line, and upper limit for cooling rate in the strong-cooling regime ($Q_{SP}$) as the dotted line. The layer is not in the strong-cooling regime because $Q_\gamma \leq Q_{SP}$, which means the layer thickness can be estimated by the standard Sweet–Parker type solution. Bottom right-hand panel: photon pressure ($p_\gamma$) as the solid line, pair pressure ($p_{\text{pairs}}$) as the short-dashed line, and neutrino pressure ($p_\nu$) as the long-dashed line. Baryonic-associated pressures are always negligible until significant dissipation has already occurred. The photon pressure is comparable to or dominates all other sources of thermal pressure.

and so the diffusion time-scale reaches the light-crossing time that increases with radius. The transit time for fluid to pass through the current layer would lead to variable emission directly due to reconnection, while some of the other time-scales give variability due to photon diffusion and emission rates. The $m$ mode produces many pulses due to new reconnection layers passing beyond $r_{\text{trans}}$. Emission would decay over the time-scale for completing dissipation. The observer time for photon absorption diffusion from the current layer photosphere ($t_{\text{obs,ad}}$) and the observer time for photon scattering diffusion from the current layer photosphere ($t_{\text{obs,sc}}$) roughly follow $t_{\text{obs,\gamma}}$. Between $r_{\text{trans}}$ and $r_{\text{diss}}$, these time-scales are $t_{\text{obs,tra}} \sim 0.002$–0.007 s, $t_{\text{obs,m}} \sim 0.004$ s, $t_{\text{obs,sc}} \sim 0.001$–0.04 s, $t_{\text{obs,\gamma}} \sim 0.002$–0.04 s, $t_{\text{obs,\gamma}} \sim 0.004$–0.06 s and $t_{\text{obs,diss}} \sim 0.005$ s. After a few e-foldings of dissipation, the time-scales are within the range of 1–10 s. These time-scales are roughly consistent the observed GRB pulse durations that typically are $\sim 0.5$ s, but range from 0.01 to 10 s (Norris et al. 1996). Because the time-scale is always associated with a fixed radius, pulse time-scales do not evolve over the event duration as happens in the internal shock model.

Fig. 7 shows that neutrinos dominate the energy density loss rate up to $r \sim 10^{10}$ cm, after which photons dominate. Near the jet base, neutrino energy density loss rates do not exceed $Q_{SP}$, which means that the current layer does not compress due to radiative cooling and the non-radiative Sweet–Parker analysis is approximately valid. The gas energy cooling rate $Q_{\text{g}} \sim Q_{SP}$ within the dissipation region after fast collisionless reconnection is triggered, so radiative cooling has become marginally dynamically important to the current layer structure.
Fig. 7 also shows that the photon pressure is comparable to or dominates other pressures at all radii including in the optically-thin regime. Electron and baryon pressures are negligible at all radii.

Lastly, consider some checks on the composition of the jet. The electrons and positrons are found to be non-degenerate at all radii except for a region between \( r_{\text{mono}} \) and about 100\( r_{\text{mono}} \). Nucleons are computed to everywhere be non-degenerate and \( \beta \) equilibrium implies \( Y_e = 1/2 \) everywhere as consistent with this work’s assumptions. However, the nuclear statistical equilibrium (NSE) time-scale can be compared to the jet flow time such that complete NSE occurs at \( r_{\text{NSE}} \sim \sqrt{\gamma} d_{\text{NSE}} \). For the fiducial model, the value of \( r_{\text{NSE}} \) is always found to be much greater than the local radius. NSE only completes to about a tenth of a single e-folding NSE time-scale. Thus, the composition of the jet would primarily be determined by the accretion disc (either directly by advection and reconnection or by neutron diffusion) rather than by NSE or \( \beta \) equilibrium.

6.2 Results: parameter dependence for full jet structure

In this section, several model parameters are explored to determine how they affect the results. All fiducial parameters are held fixed except a single parameter that is allowed to vary. For each model parameter and across all radii, a total of 40 \( \times \) 40 solutions (i.e. radius \( \times \) model parameter) are sought.

Fig. 8 shows results for \( B_1(r_{fp}) \) versus \( r \) at \( \zeta = 10^4 \) for which \( \mu \approx 5400 \). We find that a stronger electromagnetic field near the compact object leads to a transition to fast reconnection at larger radii. For expected collapsar parameters with \( B_1 \sim 10^{15} \) G, the transition to fast collisionless reconnection occurs at \( r \sim 10^{13} - 10^{14} \) cm in all cases. At \( r \gtrsim 10^{14} \) cm, the Lorentz factor is \( \gamma \approx 800 \) for the ideal MHD jet solution. The electromagnetic energy flux per unit mass-energy flux is \( \alpha \approx 6 \), indicating a large reservoir of energy exists to be electromagnetically dissipated. This solution has \( \gamma \theta_j \approx 18 \) for all field strengths at a radius of \( r \sim 10^{14} \) cm, while asymptotically \( \gamma \theta_j \approx 20 \). Jet-breaks can occur because \( \gamma \theta_j \gtrsim 1 \). For all of parameter space, \( A \sim 1 \), so the current layer is not in the strong-cooling limit. The break in the behaviour of \( r_{\text{diss}} \) at \( B_1 \sim 10^{13} \) G is due to the Lorentz factor saturating at large radii. When \( r_\tau \gtrsim 1 \), NSE is found to hold and free nucleons are generated in the jet independently of the accretion disc composition. Above \( B_1 \sim 10^{15} \) G, near the compact object, NSE partially holds and the jet contains a non-negligible fraction of free nucleons that typically freeze out before they might convert to \( \alpha \) particles at large radii. Below \( B_1 \sim 10^{13} \) G, the jet is found to never be in NSE on jet flow time-scales, and the composition would be set by the accretion disc that feeds the jet with baryons. The \( \beta \) equilibrium value of \( Y_e \) is \( 1/2 \) holds for all field strengths. However, NSE time-scales are typically long compared to the jet flow time, so reaction rates need not be considered. Thus, for the field strengths required to explain typical cosmological GRBs, the composition in the accretion disc would determine the jet composition rather than equilibrium conditions or reaction rates.

Plots similar to Fig. 8, but with different \( \zeta \), show similar results. For all \( \zeta \gtrsim 10^2 \), the flow is optically thin when fast collisionless reconnection initiates. Consider the case with \( \zeta = 10^2 \) for which \( \mu = 55 \). The transition radius to fast collisionless reconnection is also roughly linear in log-log and varies from \( r_{\text{trans}} = 10^{17} \) cm to \( r_{\tau} \) to \( 10^{17} \) cm for a variation of \( B_1 \) to \( 10^9 \) G, respectively. Also, \( r_{\text{diss}} \approx 10^{13} \) cm for \( B_1 \approx 10^{13} \) G, while otherwise \( r_{\text{diss}} \approx r_{\text{trans}} \). In addition, the region where pairs drop out shifts down along with this \( r_{\text{trans}} \) line. Asymptotically, \( \gamma \sim 50 \) and \( \sigma \sim 1 \). However, the solution always has \( \gamma \theta_j \lesssim 2 \), which implies jet-breaks may not be discernible for such solutions even if they produce efficient non-thermal or quasi-thermal emission. For collapsar parameters and this value of \( \zeta \sim 10^2 \), the transition to fast collisionless reconnection occurs at \( r \sim 10^9 \) cm. For collapsar parameters and this value of \( \zeta \sim 10^2 \), the transition to fast collisionless reconnection occurs at \( r \sim 10^9 \) cm.

Fig. 9 shows how changing the effective magnetization parameter, \( \zeta \), determines the solution for otherwise typical collapsar parameters that we adopted for our fiducial model in Section 6.1. This shows that dissipation starts at \( r \sim 10^{14} \) cm if \( \zeta \gtrsim 10^4 \), which gives \( \gamma \sim 800 \) with \( \sigma \sim 6 \) and \( \sigma \theta_j \sim 18 \). We consider this a fiducial model that is fairly insensitive to other free parameters. This type of progenitor system generates a jet fast enough to avoid the compactness problem, harbours about six times more energy available for prompt emission than for afterglow emission (assuming the energy is mostly dissipated and radiated away instead of accelerating the jet), and can produce jet-breaks. For \( \zeta < 5000 \), the dissipation completes within the region where photon scattering opacity is unity, which forces a non-negligible fraction of photons to thermalize. Such events should have a thermal emission component in their spectra. In this parameter space for the ideal MHD jet solution, the comoving toroidal field has increased to a few times the comoving poloidal field, triggering the kink instability (Narayan et al. 2009), at \( r \sim 10^{13} \) cm for \( \zeta \sim 10^4 \) and at \( r \sim 10^{10} \) cm for \( \zeta = 50 \). However, for all \( \zeta \), the transition to vigorous kink instability still occurs beyond \( r_{\text{mono}} \), where \( \gamma \theta_j \gg 1 \), so dissipation takes place only
collisionless reconnection varies only from $10^{13.7}$ to $10^{13.8}$ cm. There is little electromagnetic energy flux for field lines with $\theta_{fp} \lesssim 0.1$, so dissipation there is not important compared to the total jet dissipation.

Figs 10 and 11 show that the variations in $l$ and $m$ lead to significant changes in the behaviour of the solution. A significant portion of the dissipated electromagnetic energy is thermalized and produces a hot magnetic fireball when $l \gtrsim 10^7$ and $m \gtrsim 0.8$ rather than direct non-thermal emission. This effect is caused by the larger $l$, $m$ leading to a smaller current sheet length and hence a smaller Sweet–Parker scale $\delta_{SP}$, which then becomes equal to the ion skin depth near the rotational axis within $\theta < 1/\gamma$ due to larger angles being causally disconnected. This angle is typically small compared to the jet opening angle, and so this region contains very little electromagnetic energy flux due to both the electromagnetic power dependence on angle and the efficient ideal MHD acceleration at small angles (Tchekhovskoy et al. 2009, 2010b).

All other free model parameters were similarly explored for otherwise fixed fiducial parameters. The free parameter $v$ controls the field geometry ranging from monopolar ($v = 0$) to parabolic ($v = 1$) with the most likely value for the collapsar model being $v \sim 3/4$. The solution dependence on $v = \{10^{-4}, 2\}$ is weak, where $r_{\text{trans}}$ varies from $10^{15}$ to $10^{16}$ cm for $v = 10^{-4}$ to 2 and where $r_{\text{diss}}$ varies from $10^{13.1}$ to $10^{15}$ cm for $v = 10^{-4}$ to 2.

The free parameter $r_{\text{mono}}$ controls when the flow transitions from a collimating solution (e.g. as caused by a stellar envelope or accretion disc corona/wind) to a monopolar solution as occurs when the jet becomes unconfined (i.e. no collimating agent is present). We find that for $r_{\text{mono}} = \{GM/c^2, 10^{16}\}$ cm, the transition radius to fast collisionless reconnection increases slowly/monotonically from $\{r_{\text{trans}}, r_{\text{mono}}\}$ of $\{10^{15}, 10^{5.7}\}$ cm to $\{10^{15}, 10^{18}\}$ cm. For $r_{\text{mono}} > 10^{15.3}$ cm, jet-breaks are not expected because $\gamma \theta_j \sim 1$ once $r \sim 10^{15.3}$ cm.

Consider short-duration GRBs formed by BH–NS or NS–NS mergers that involve only an accretion disc instead of both an accretion disc and a stellar envelope. Then, $r_{\text{mono}} \sim 10^7$ cm is applicable, because this corresponds to the cylindrical radial extent of the accretion disc that collimates the jet. Such a system is found to have a jet opening half-angle of the order of $\theta_j \sim 0.4 \approx 23^\circ$, and the transition radius to fast collisionless reconnection is at $r_{\text{trans}} \approx 10^{15}$ cm.

The dependence on $\theta_{fp} = [0.16, \pi/2]$, over which much of the jet power resides, is very weak. The transition radius to fast

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Figure 9. Contour plot for the magnetization ($\zeta$) versus $r$ showing $\gamma \theta_j = 1$ (lower mostly horizontal dashed line) above which jet-breaks can occur, and otherwise like in Fig. 8. Neutrinos are never optically thick for this case. By $r \gtrsim 10^{14}$ cm and for $\zeta \sim 10^4$, the Lorentz factor is $\gamma \sim \mu/7 \sim \zeta/13$. Also, $\sigma \sim 1$ for $\zeta = 10^2$ and increases for increasing $\zeta$, while $\delta \delta_j \sim 1$ for $\zeta = 10^4$ and increases for increasing $\zeta$. This implies jet-breaks are predicted for a broad range in $\zeta$. For $\zeta \gtrsim 5000$, the flow becomes optically thin towards the observer near the transition radius to fast collisionless reconnection. This shows for collapsar parameters with $B_j \sim 10^{15}$ G that the transition to fast collisionless reconnection occurs at $r \sim 10^{14}$ cm if $\zeta \sim 10^4$ corresponding to $\mu \sim 5400$ and $\gamma \sim 800$. 

Figure 10. Contour plot for the substructure $l$ mode versus $r$ and otherwise like in Fig. 8. Unlike small $l$, large $l$ causes the transition radius to fast collisionless reconnection to occur within the optically-thick region. The line labelled by $r_{\text{diss}}$ corresponds to equation (26) for $f = 0.5$ corresponding to dissipating half of the electromagnetic energy flux. This is a stronger constraint on the dissipation radius, which shows that for $l \lesssim 100$ the dissipation is very slow even for fast reconnection due to the (possible) continued radial and lateral expansion of the jet on dissipating structures.

Figure 11. Contour plot for the substructure $m$ mode versus $r$ and otherwise like in Fig. 8. Unlike small $m$, large $m$ causes the transition radius to fast collisionless reconnection to occur within the optically-thick region.
at smaller radii. One may generally have a spectrum of \(l, m\) modes, and then dissipated electromagnetic energy contributes both thermal and non-thermal components to the photon spectrum. A stronger constraint on the \(l\) modes is also shown using equation (26) with \(f = 0.5\) corresponding to dissipating half of the electromagnetic energy flux. This would be the dissipation radius if the collisional structures (that formed before or at the transition radius) continue to expand with radius as naively predicted by \( \Delta \theta \). It seems more likely that once the collisionless mode is triggered, the dissipation decouples from any collisionally induced structures, in which case the weaker constraint can be used.

Overall, for collapsar-type parameters, the parameter space study shows that the transition to fast reconnection often occurs beyond or near the photosphere unless the magnetization is too low (\(\zeta \lesssim 10^4\)). The transition occurs near, but below, the photosphere when the \(l\) mode is too high (\(l \gtrsim 10\)) or the \(m\) mode is too high (\(m \gtrsim 1\)). For collapsar-type parameters, the transition radius is roughly linear in a log–log plot of the foot-point field strength \([B_l(r_\text{fp})]\) versus radius, such that, by coincidence, the transition radius happens to have a value in centimetres roughly the same as the value of the foot-point field in Gaussian units. Interestingly, the natural \(m \sim 1\) spiral mode ( McKinney & Blandford 2009) demarcates a boundary where the transition to fast reconnection occurs at the jet photosphere. Thus, small parameter variations can lead to events with thermal or non-thermal components in the prompt GRB spectrum.

### 6.3 Results: parameter dependence for simplified jet structure and current layer physics

In this section, the simplified jet structure presented in Section 3 is used to obtain formulae that describe the essence of the above results. First, one must obtain an estimate of the bulk jet Lorentz factor, \(\gamma\), such as can be obtained from the full jet structure solution in Appendix A. Secondly, one must approximate the current layer physics, which is possible for some portions of the relevant parameter space. The results are not generally accurate to within an order of magnitude, but the calculation at least highlights the basic ideas and principles.

For typical collapsar parameters and at the transition radius to fast collisionless reconnection, the photon pressure dominates, and the absorption opacity (dominated by the cyclo-synchrotron process) is very small while the scattering opacity is not small. Cyclo-synchrotron radiation can be approximated with non-relativistic electrons having a delta function distribution at energy \(3k_B T/2\). Radiative cooling within the layer is often weak, so that \(A = 1\) can be chosen. The Coulomb logarithm is of the order of 30.

Photon drag resistivity generally dominates over the Spitzer resistivity up to and far beyond the transition radius. A non-relativistic temperature approximation is accurate, although pairs do provide non-negligible cyclo-synchrotron radiation and opacity. The pair and photon opacities are such that \(g_{\text{pairs}} \) and \(g_{\gamma}\) (see equation B14) are roughly comparable. These approximations greatly simplify the force balance condition given by equation (C7), leaving a couple of equations for \(T\) and \(n_{\text{pairs}}\) to be solved numerically.

Even with these simplifications, a difficulty in obtaining a closed-form approximation is that the temperature suppression of pairs is transcendental. However, one can leave \(n_{\text{pairs}}\) as a free parameter, which can be estimated from the full solution or the approximate solution in the preceding paragraph.

We find that the value of \(\xi\) is implied from the value of \(\gamma\), because at large radii \(\gamma \approx \xi/14\) for models that end up with \(\sigma\) of the order of several. For models near the collapsar-type parameters and \(\xi < 10^6\), the expressions in the rest of this section tend to be accurate to a couple orders of magnitude. For very large \(\zeta \gtrsim 10^6\), the dependence of \(\gamma\) on free parameters can become difficult to estimate without the full jet structure, and so the below expressions should be used with caution when arbitrarily varying \(\gamma\).

For compactness of the below formulae, we rescale the free model parameters by collapsar values: \(r_\text{fp} = r_\text{fp}/r_g\) with \(r_g \equiv GM_\odot/c^2 = 3GM_\odot/c^2, \tilde{\tau}_\text{mono} = \tau_\text{mono}/(3 \times 10^3\text{ cm}), B_l = B_l/(3.2 \times 10^{15}\text{ G}), \tilde{\gamma} = \gamma/700, \tilde{\theta}_e = \sin(\theta_e)/2\sin(\pi/4)\) and \(\zeta = \xi/10^4\). Both \(l\) and \(m\) modes are considered but in general have different properties. For the case considered, however, \(n_{\text{pairs}} = n_{\text{pairs}}/(\Omega_\text{f})\) is reasonable for both mode types. Such a choice for \(n_{\text{pairs}}\) is a reasonable estimate for only a limited range of model parameters such as chosen for the fiducial model near the transition radius as estimated by this simplified model. Generally, both \(\gamma\) and \(n_{\text{pairs}}\) need to be chosen based upon the results from the full solution. Otherwise, for \(l\) modes, free model parameters are rescaled as follows: \(\tilde{r}_l = r/l (1 \times 10^{14}\text{ cm})\), and \(L_{\text{o},l} = L_{\text{o}}/(\pi R_l/l)\) such that \(l = \gamma D_l\), where \(L_{\text{o},l}\) assumes that typically \(L_\odot\) scales with \(R_l\) as when \(m = 0\). For \(m\) modes, free model parameters are rescaled as follows: \(\tilde{r}_m = r/l (1 \times 10^{14}\text{ cm})\), and \(L_{\text{o},m} = L_o/[\gamma c/(m\Omega_\text{f})]\) with \(m = 0.1\), which assumes that \(L_\odot\) does not scale with \(R_l\) for \(l = 1\) and \(m = 0\). As applicable to most of parameter space, we assume the optical depth integrals use \(\Delta \theta \gtrsim 10^4 \zeta \lesssim \gamma/HR\) for \(m\) modes and \(L_\odot \sim \Delta \theta \gtrsim \zeta \gammaHR\) for \(l\) modes.

Then, the transition radius is

\[
r_{\text{trans,}l} \sim 10^{14}\text{ cm } B_l/r_\text{fp}^{1/3} \tilde{r}_l^{1/3} \tilde{\tau}_l^{1/3} \tilde{\nu}_l^{2/3} \tilde{\gamma}^{-4/3} n_{\text{pairs}}^{-1/3} \tilde{\xi}^{-1/3}
\]

for \(l\) modes, and

\[
r_{\text{trans,}m} \sim 10^{14}\text{ cm } B_l/r_\text{fp}^{1/3} \tilde{r}_m^{1/3} \tilde{\tau}_m^{1/3} \tilde{\nu}_m^{2/3} \tilde{\gamma}^{-1/2} n_{\text{pairs}}^{-1/2} \tilde{\xi}^{-1/2}
\]

for \(m\) modes. This transition radius is consistent with our full solution from the previous sections. These estimates are valid for both non-relativistic or relativistic temperatures, whereas other scaling laws obtained later tend to require a non-relativistic approximation for reliable scalings. Note that the transition radius is only weakly sensitive to \(n_{\text{pairs}}\). In the case where \(n_{\text{pairs}} \gtrsim n_e\), one can roughly take \(n_{\text{pairs}} \rightarrow n_e\) in the above expressions. For remaining estimates, \(n_{\text{pairs}} \gg n_e\) within the current layer is assumed, although one can readily solve the system of equations that emerge in the other limit.

The dissipation radius is

\[
r_{\text{diss,}l} \sim n_{\text{trans,}l} + 10^{14}\text{ cm } (B_l/r_\text{fp} \tilde{L}_{\text{o},l}^{1/3})^{1/4} \tilde{r}_l^{4/3-2/3} \tilde{\tau}_l^{2/3} \tilde{\nu}_l^{2/3} \tilde{\gamma}^{-1/3} n_{\text{pairs}}^{1/3} \tilde{\xi}^{-1/3}
\]

for \(l\) modes, and

\[
r_{\text{diss,}m} \sim n_{\text{trans,}m} + 10^{14}\text{ cm } L_{\text{o},m} \tilde{r}_m \tilde{\gamma}^{2/3}
\]

for \(m\) modes.

The optical depth towards the observer for thermal photons is

\[
\tau_{\gamma, l} \sim 2 \tilde{L}_{\text{o},l} \tilde{r}_l n_{\text{pairs}} \tilde{\tau}_l \tilde{\gamma}^{-1} + 0.04 \tilde{r}_l^{2/3} \tilde{\tau}_l^{2/3} \tilde{\nu}_l^{2/3} \tilde{\gamma}^{-2/3} n_{\text{pairs}}^{2/3} \tilde{\xi}^{-2/3}
\]

for \(l\) modes, and

\[
\tau_{\gamma, m} \sim 1 \tilde{L}_{\text{o},m} \tilde{r}_m n_{\text{pairs}} + 0.1 \tilde{r}_m^{2/3} \tilde{\tau}_m^{2/3} \tilde{\nu}_m^{2/3} \tilde{\gamma}^{-2/3} n_{\text{pairs}}^{2/3} \tilde{\xi}^{-2/3}
\]

for \(m\) modes. The left-hand term corresponds to the optical depth due to the current layer, and the right-hand term corresponds to the optical depth due to electrons associated with baryons downstream in the jet. The local pair contribution tends to dominate the downstream electron contribution to \(\tau_{\gamma}\). However, as \(L_\odot\) decreases, the right-hand term dominates.

Optically-thin synchrotron could be the source of the observed high-energy photons near the peak of the energy spectrum for...
prompt GRB emission. Internal shocks are a fast heating process that could accelerate particles to high non-thermal energies. However, reconnection is a slow heating process such that heating and optically-thin synchrotron cooling must be in balance, which leads to low-energy particles and too low synchrotron photon energies to explain the observed peak energies (unless e.g. high-energy electrons cool downstream where field is much weaker) (Ghisellini & Celotti 1999).

Quasi-thermal emission could instead explain the observed prompt spectrum (Rees & Mészáros 2005; Thompson et al. 2007; Giannios 2008). There is necessarily incomplete photon thermalization because $\tau_{\gamma} \lesssim 1$ for $r \gtrsim r_{\text{trans}}$. Equation (37) already established that at a unity scattering optical depth there is a stable temperature of $T \approx 4 \times 10^{5}$ K independent of model parameters. Because the transition to fast reconnection occurs once $\tau_{\gamma, \text{csa}} = 1$, this provides a naturally stable temperature. The temperature is high and would only explain the observed peak energy for $\gamma \lesssim 50$. However, several scatters may be required to thermalize the photons, and incomplete thermalization might lead to lower temperatures allowing up to $\gamma \sim 1000$ while still obtaining consistency with the observed peak energy (Thompson 1994; Giannios & Spruit 2005; Giannios 2006; Giannios & Spruit 2007; Giannios 2008).

Incomplete thermalization at the scattering photosphere may lead to a temperature closer to that at the absorption photosphere. Let $\Theta_{e} = \Theta_{e}/0.0004$, and assume $\eta_{\text{pairs}} < n_{e}$ as applicable for many model parameters, then the temperature at $\tau_{\gamma, \text{abs}} = 1$ satisfies

$$\Theta_{e}^{-1} \approx \left( \frac{B_{l}}{\xi} \right)^{4/3} \Theta_{e}^{e} + 0.05 \left( \frac{B_{l}}{\xi} \right)^{2/3} \Theta_{e},$$

for both $l$ and $m$ modes, that for fiducial parameters $\Theta_{e} = 1$ or $\Theta_{e} = 0.0004$. Typically, the left-hand term dominates, then $\Theta_{e} \approx \left[ \xi/(B_{l}) \right]^{4/9}$. Then, the observed thermal peak energy is at

$$E_{\text{peak obs}} \approx \tilde{\gamma} \Theta_{e} 140 \text{ keV},$$

which is close to the observed peak energy. However, a more detailed study is required to determine whether $E_{\text{peak}} \propto \gamma^{13/9}$ is too much variation to be consistent with, for example, the Ghirlanda relation (Ghirlanda, Ghisellini & Lazzati 2004).

One of the shortest time-scales for observed prompt variability can be estimated from the transit time (as seen by an observer) for a plasma to pass through the reconnection layer as given by

$$d_{\text{obs, tr, l}} \sim 0.02 \, s \, L_{l} \tilde{f}_{l} \tilde{r}^{-2}$$

for $l$ modes, and

$$d_{\text{obs, tr, m}} \sim 0.004 \, s \, L_{m} \tilde{f}_{0} \tilde{r}_{l}$$

for $m$ modes. This and the other time-scales in Section 5.5 give a range of values from 0.001 s up to about 10 s for both $m$ and $l$ modes, which is roughly consistent with observations. For field substructure types A–C, this suggests that the prompt GRB emission probes the turbulent field generated in the accretion disc near the BH. For field substructure type D, the prompt emission probes the turbulent layer that develops between the jet and stellar envelope or accretion disc corona/wind. While one expects reconnection to proceed stochastically, the saturated dissipation of a collection of current sheets would lead to a delay in the dissipation of current sheets just passing the fast transition radius. Thus, one expects a correlation between pulse widths and intervals, and no spreading of pulses at later times because the transition radius is roughly fixed. More modelling is required to validate these suggestive agreements with observations (Piran 2004).

### 6.4 Comparison with other works

Pioneering studies of reconnection in GRB jets by Spruit et al. (2001) determined the radius (their equation 61) at which dissipation proceeds. In this paper’s language, they assumed substructure type A (and type B with $m = 1$) to obtain a dissipation radius of $r_{d} \approx (\pi c)/(\epsilon \Omega_{c})v_{\gamma}^{2}$, where $\epsilon \sim 0.1$ controls the reconnection velocity normalized to Alfvén velocity. They assumed that fast reconnection occurs over the entire length-scale $\Delta_{r}$, giving a laboratory-frame time-scale for dissipation of $T \sim \gamma/(\Delta_{r} v_{\gamma})$. They conclude that $r_{d} \sim 2 \times 10^{13}$ cm for $\Omega_{c} \approx 10^{7} \text{ s}^{-1}$ and a fixed value of $\gamma \sim 300$.

One should check that this condition holds at smaller radii where $\gamma \sim 1$ in order to ensure the jet can form in the first place without dissipation disrupting the jet formation process. At small radii, the jet has $\gamma \sim 1 + r/2c$ and $\theta \sim (r/r_{d})^{\gamma/2} \sin(\theta r_{d}/2)$. For any $\nu$, one can solve for the dissipation radius accounting for the changes in the Lorentz factor. Their conclusions appear accurate as long as the jet contains substructure type A (or B with $m = 1$) and dissipation occurs over $\Delta_{r}$. Some problems may arise in their model because the dissipation radius decreases inversely with $m$.

More serious is that if the jet contains field reversals in $\theta$ as in substructure type C, then their equation (57) shows that dissipation would occur near the jet base because $\gamma/\theta \sim 1$ at small radii for a magnetized jet. GRMHD simulations of accreting BHs show that substructure type C, which is generated by MHD turbulence, is quite common. This suggests that only by having a slow reconnection rate at small radii and a fast reconnection rate at large radii can one ensure magnetized jet formation in the first place. The reconnection switch model presented in this paper naturally provides such a mechanism.

### 6.5 Applications to other jet systems

AGNs and X-ray binaries may also exhibit a transition to fast collimationless reconnection. Here we consider only a couple of example systems. The ranges $r_{d} \lesssim r_{\text{mono}} \lesssim 10^{10} r_{g}$ and $1 \lesssim \xi \lesssim 10^{3}$ are investigated. One expects $\xi \sim \gamma \lesssim 100$ (Fender 2003).

First, consider the case of the jet in M87 (similar arguments apply to blazars), which has a BH with mass $M \approx 6 \times 10^{9} M_{\odot}$ accreting at $M \sim 10^{-5} M_{\odot} \text{yr}^{-1}$ with a bolometric luminosity of $L_{\text{bol}} \sim 10^{42} \text{ erg s}^{-1}$ (Gebhardt & Thomas 2009), and so operates at a radiative efficiency of the order of $\eta \lesssim 0.01$ corresponding to a low-luminosity radiatively inefficient accretion flow. Equiapitation arguments combined with GRMHD simulations of accretion discs and the Blandford–Znajek funnel region (McKinney 2004) then imply that the magnetic field strength at the base of the jet is of the order of $B_{z0} \sim 10^{4} \text{ G}$.

Our solutions show that the M87 jet is already in the fast collimationless regime at the base of the jet (see also Giannios, Uzdensky & Begelman 2010). This suggests that an electromagnetically dominated jet from M87 would not survive unless an ordered dipolar field is present at the jet base (McKinney & Blandford 2009). Because the magnetic field should be well ordered near the BH to produce a jet in the first place, this suggests that pairs may dominate the mass energy because baryons would only enter the jet base via reconnection or turbulent diffusion. This may have important implications for polarized disc and jet emission and composition (see e.g. Broderick & McKinney 2010; Dexter et al. 2010; Shcherbakov, Penna & McKinney 2010).

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3 A hot MHD jet can be pushed to slightly larger $\gamma \theta_{j}$ at small radii.

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Secondly, consider the case of the jet in the BH X-ray binary 
GRS1915+105, which has a BH with mass $M \sim 14 M_\odot$ 
(Greiner, Cuby & McCaughrean 2001) accreting at near-Eddington 
rates during outburst and production of a fast transient jet (Fender 
et al. 1999). The system has $M \sim 2.4 \times 10^{-8} M_\odot \, yr^{-1}$ (Körding, 
Fender & Migliari 2006) and a bolometric luminosity of $L_{\text{bol}} \sim 
10^{38}$ erg s$^{-1}$, so operates at a radiative efficiency of the order of 
$\eta \sim 0.1$ corresponding to a radiatively efficient accretion flow. 
Equation and arguments combined with GRMHD simulations of accretion 
discs and the Blandford–Znajek funnel region (McKinney 2004) then 
implies that the magnetic field at the base of the polar jet is $B_{\text{fp}} \sim 10^9$ G. 

Interestingly, for GRS1915+105 with $\zeta \lesssim 10^4$ and only within 
20 gravitational radii, the jet is in the slow-reconnection regime (see 
also Goodman & Uzdensky 2008). Such a confined region where 
reconnection is essentially marginally collisionless could promote 
the production of the fast transient jet as triggered by a transition 
from slow to fast collisionless reconnection. More modelling is 
required to investigate this effect.

7 DISCUSSION

For GRB jets, why must reconnection be slow enough near the 
central engine and yet fast at large radii?

Fast relativistic reconnection at speeds up to the order of 
$v_r \sim c$ might occur (Lyutikov & Uzdensky 2003; Lyubarsky 2005). 
If this occurred near the central engine and the jet contained current 
sheets, then dissipation of the jet would occur in situ and no jet 
might emerge or would be heavily baryon-loaded (Beckwith et al. 
2008; McKinney & Blandford 2009). If somehow a rate of $v_r \sim c$ 
was suspended until $\gamma \gg 1$, at which point relativistic time 
dilation would stall dissipation, then still the dissipation radius would 
be inside the photon’s photosphere (i.e. $r_{\text{diss}} \approx r_{\text{trans}}$, where $r_{\gamma} \approx 1$ 
at $r = r_{\text{trans}}$). For example, for $m = 0.1$ and otherwise fiducial 
dimensions, having $v_r \sim c$ leads to $r_{\text{diss}} \sim 10^{18}$ cm while $r_{\gamma} = 1$ 
at $r = 10^{18}$ cm. All dissipation would complete inside the photosphere 
and non-thermal emission would be unlikely. Hence, in general, fast 
reconnection should be delayed somehow.

On the other hand, slow collisional reconnection leads to a de-
layed dissipation rate relative to fast reconnection by a large factor. 
For example, in the extreme case of Sweet–Parker reconnection, 
the slowdown is by a factor of $\sim \sqrt{3}$, where $S \gg 1$ is the 
Lundquist number (see equation 29). At the transition radius, fast 
reconnection has an observed time-scale of the order of a second 
for the fiducial model. A typical Lundquist number at the transition 
radius is $S \sim 10^{20}$. This indicates that a time-delay of about $10^{10}$ s 
(i.e. hundreds of years) would occur before the slow reconnection 
can dissipate a sizeable fraction of the available energy, which is 
an unacceptable delay. Thus, slow collisional Sweet–Parker reconnec-
tion (with any jet substructure type) cannot be responsible for 
powering the prompt GRB emission. Therefore, fast reconnection is 
required, but under general considerations, it must be delayed until 
large radii to avoid complete thermalization of the photons. The 
reconnection switch mechanism satisfies both these requirements.

An alternative to fast collisionless reconnection is fast collision-
reational reconnection. Collisional reconnection may proceed faster 
than Sweet–Parker due to secondary tearing instability (plasmoid-
dominated reconnection) or MHD turbulence. Preliminary sim-
ulations show that, for otherwise fixed parameters, non-relativistic 
Petschek may only be 10 times faster than non-relativistic collision-
reational reconnection (Loureiro et al. 2007; Cassak et al. 2009; 
Kowal et al. 2009a; Loureiro et al. 2009; Samtaney et al. 2009; Huang 
& Bhattacharjee 2010; Uzdensky et al. 2010). While fast collision-
less reconnection would dominate this collisional mechanism once 
the plasma becomes collisionless, one should consider the effect 
at smaller radii. The jet starts with velocity $v \sim c$ on scales of the 
order of the BH horizon and the Lorentz factor grows rapidly with 
$\gamma \propto R$. Unless the reconnection rate were $v_r \to c$, the relativistic 
time-dilation stalls reconnection until $\gamma$ flattens out for $r > r_{\text{mono}}$. 
Assuming fast collisional (e.g. plasmoid-dominated) reconnection is 
10 times slower than the typical speeds of fast collisionless recon-
nection, then the dissipation radius is at 10 times the distance given 
by equations (41) and (42). Thus, fast collisionless dissipation al-
ready completes before fast collisional reconnection even becomes 
important. Also, for the fast collisional plasmoid-dominated mode, 
it remains unclear whether causal disconnection across the jet would 
allow for plasmoid chains to grow. As with suppression of the 
magnetic kink instability, turbulence may be avoided except within the 
narrow region of $\theta \lesssim 1/\gamma$ where the jet is still causally connected. 

Because this region contains very little power, this would lead to 
negligible dissipation compared to the total jet dissipation.

Interestingly, GRB systems are quite unique compared to AGNs 
and X-ray binaries. GRBs exhibit a transition from slow collisional 
to fast collisionless reconnection at roughly billions of gravitational 
radii from the central compact object. On the other hand, X-ray 
binary systems may exhibit a transition to fast reconnection within 
tens of gravitational radii, while AGN jets tend to always be in the 
fast collisionless regime. This means that while GRB systems might 
sustain a disordered field at the jet base, it appears unlikely that 
a disordered field can be sustained at the jet base in most AGNs and 
some X-ray binary systems. Significant dissipation and emission 
can then occur when the jet harbours current sheets. This suggests 
that ultrarelativistic jets may be more difficult to obtain from AGNs 
and X-ray binaries unless the magnetic field geometry consists of an 
organized dipolar field (McKinney & Blandford 2009). For AGNs, 
such a requirement is tolerable if the observed coherence length of 
magnetic fields in the interstellar medium, which is high enough to 
trap a significant flux near a supermassive BH, is assumed to be typi-
cal of galaxies with AGNs (Narayan, Igumenshchev & Abramowicz 
2003).

This suggests that current sheets within jets in AGN and X-ray 
binary systems may limit how efficiently electromagnetic energy 
flux can be converted into kinetic energy flux through ideal MHD 
acceleration. Normally, the electromagnetic energy flux per unit 
mass flux ($\mu$, measuring the degree of mass-loading) is understood 
to limit the terminal Lorentz factor. However, dissipation of electro-
magnetic energy in current sheets can lead to significant decreases 
in $\mu$. This may explain why GRB jets tend to have higher Lorentz 
factors than jets from AGN and X-ray binary systems.

Another interesting result is that the current-driven kink mode 
is an unlikely source of current sheets. As discussed in Section 4, 
the $|m| = 1$ kink instability operates when the magnitude of the 
comoving toroidal field $(|b_\phi|)$ is about $3–10$ times the magnitude 
of the comoving poloidal field $(|b_p|)$ (Narayan et al. 2009). The 
comoving poloidal field’s relative growth only occurs for $r \gg r_{\text{mono}}$.

In the fiducial model for the ideal MHD jet solution 
with $\theta_\phi = \pi/2$, 
these comoving fields become comparable by $r \sim 10^{18}$ cm 
and the comoving toroidal field is about 10 times larger by $r \sim 8 \times 
10^{18}$ cm. Across all modes, $|b_p|$ only grows to $3–10$ times $|b_\phi|$ 
for $r \gg r_{\text{mono}}$. Thus, only after this radius would one expect 
vigorous comoving kink instabilities. However, beyond $r = r_{\text{mono}}$, 
the jet loses significant causal contact across in angle due to the 
typical value of $\gamma \theta_j \sim 10–20$. Thus, one only expects the region 
within $\theta < 1/\gamma$ around the rotational axis to undergo the instability.
For typical parameters of $\gamma \theta_j \sim 15$, this region contains negligible electromagnetic power. Further, the structure of the jet has $|b_\perp| \ll |b_j|$ at such small angles due to the even more efficient acceleration that occurs closer to the rotational axis (Tchekhovskoy et al. 2009, 2010b). Overall, kink instabilities appear ineffective at leading to significant dissipation of electromagnetic energy.

In addition, the interaction between the jet and a disc wind or stellar envelope is also an unlikely source of current sheets (or numerous shells moving at varying relativistic speeds) due to toroidal-field-dominated relativistic jets tending to suppress boundary layer instabilities as discovered via three-dimensional relativistic MHD simulations of AGN jets (Keppens et al. 2009; Mignone et al. 2010), although specific studies for GRB jets should be performed.

8 CONCLUSIONS

We set out to explore a jet dissipation mechanism, denoted by ‘reconnection switch’, that relies on the growing evidence that current sheets dissipate at different rates in collisional and collisionless plasmas. GRB jets naturally transition from being collisional to collisionless at large radii where dissipation is initiated near the jet photosphere due to a transition to fast collisionless reconnection.

Our picture corresponds to a highly magnetized jet that is presumed to be launched by a rotating NS or BH. Current sheets are assumed to be generated by being injected into the jet by some dynamo action near the jet base or by instabilities at large radii. These reversals form a complex of numerous current sheets that dissipate slowly at small radii when the plasma is collisional and rapidly at large radii when the plasma becomes collisionless. The fast Petschek-like collisionless mode occurs once the ion skin depth is larger than the collisional current layer thickness, which allows for the disruption of the thinner collisional layer geometry and enables the fast Petschek-like reconnection geometry.

For typical long-duration collapsar GRB parameters, reconnection stays collisional until the jet transitions to fast collisionless reconnection at $r \sim 10^{13}$–$10^{14}$ cm. Even if the reconnection switch mechanism fails to be valid and dissipation simply proceeds at the somewhat fast reconnection rate of $v_r \lesssim 0.01c$, then, regardless, all of our calculations remain intact because the dissipation radius (where significant electromagnetic dissipation occurs) is beyond the expected transition radius for many models. However, if $v_r \gtrsim 0.01c$, then the reconnection switch is required to avoid complete photon thermalization in many models.

Between the transition and dissipation radii, the Lorentz factor is $\gamma \sim 100$–1000 and the electromagnetic energy flux in the jet exceeds the kinetic energy flux by factors of 5–10. Assuming this energy is mostly dissipated and radiated away instead of accelerating the jet, it allows the electromagnetic dissipation to produce a prompt GRB luminosity that is equal to (or even exceeds) the associated afterglow emission, which could be what is observed but is difficult to obtain in the internal shock model (Willingale et al. 2007). Because $\gamma \theta_j \sim 10$–20, the afterglow can exhibit jet-breaks. The jet is electromagnetically dominated up to the transition radius. However, a reverse shock can be present because dissipation decreases the electromagnetic energy flux to roughly the kinetic energy flux by the radius where an external shock forms.

In our reconnection model, the number of prompt GRB pulses is suggested to be related to the number of current sheets embedded in the jet by the time the jet enters the dissipation regime and also related to the time-scale for reconnection and radiative emission to occur. For typical collapsar parameters, pulse durations are of the order of 1 s and range from 0.001 to 10 s. Pulses have a rise time associated with the transit time through a current layer as associated with heating, while the decay time is the time-scale for dissipation of magnetic flux in the jet. These time-scales are associated with a fixed transition/dissipation radius during the entire event so that pulses do not spread in time for fixed engine parameters.

Significant electromagnetic dissipation only occurs once the flow has become optically thin for photons that reach the observer, and hence the compactness problem is naturally avoided. However, some portion of the dissipated energy could lead to some photospheric emission for certain model parameters. This may help identify why some fraction of events seem to have thermal peaks (Ryde 2005). More work is required to obtain quasi-thermal emission spectra (see e.g. Giannios 2008).

One goal of this work was to motivate future studies of reconnection in the presence of (non-traditional) effects, including relativistic flow, pairs, radiation, supercritical electromagnetic fields, a complex of multiple current sheets, etc. This paper identifies important new directions in future reconnection research, and motivates them by identifying a new area of application of such studies to real astrophysical systems.

ACKNOWLEDGMENTS

JCM thanks Alexander Tchekhovskoy and Ramesh Narayan for discussions regarding the jet structure; Lukasz Stawarz for discussions regarding jet beaming effects; Dimitrios Giannios, Chris Thompson, Bing Zhang, Maxim Lyutikov, Tsvi Piran and Gabriele Ghisellini for useful comments; and the anonymous referee for helpful suggestions. This work was supported by NASA Chandra Fellowship PF7-80048 (JCM), the National Science Foundation (NSF) Grant PHY-0903851 (DAU) and the NSF through TeraGrid resources provided by the NCSA (Abe), LONI (QueenBee) and NICS (Kraken) under grant number TG-AST080025N.

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In this section, a generalized axisymmetric steady-state jet solution is presented as based upon approximate analytical solutions that have been confirmed numerically (Tchekhovskoy et al. 2008, 2009, 2010b) using the \texttt{HARM} code (Gammie, McKinney & T"{o}rh 2003; Noble et al. 2006; Migone & McKinney 2007; Tchekhovskoy, McKinney & Narayan 2007).

In essence, highly magnetized jets are accelerated by the toroidal field gradient. The key to efficient acceleration is that field lines around the jet-axis must redistribute so that an electromagnetic ‘exhaust nozzle’ forms. At very small angles near the polar-axis, the redistribution is highly non-self-similar, while slightly offset from...
the polar-axis (where most of the energy flux is) the redistribution is approximately self-similar. In this approximately self-similar region, the toroidal field rapidly unwind due to a drop in $R^2 B_r$ along field lines (Tchekhovskoy et al. 2009, 2010b), where $B_r$ is the poloidal field strength. One can obtain quite accurate yet simple analytical models of such jet solutions that apply to most of the jet undergoing acceleration containing most of the electromagnetic energy flux. The narrow non-self-similar region very close to the polar-axis contains only a small fraction of the total energy flux, so only its effects on the rest of the mostly self-similar flow (not its own internal structure) need to be considered.

Aspects of magnetized jets are well modelled by a force-free approach (infinitely magnetized limit of ideal MHD) solution. The vector potential is roughly independent of rotation and given by

$$P_{\text{coll}} \equiv RA_{\phi, \text{coll}} \propto \left( \frac{r}{r_{\text{fp}}} \right) \gamma (1 - \cos \theta),$$

where $A_{\phi, \text{coll}}$ is the vector potential for a collimating jet, $R = \sin \theta$ is the cylindrical radius, $r_{\text{fp}}$ is the foot-point radius, $0 \leq \theta \leq \pi$ is a free parameter, and $\theta$ is the position angle in the domain. $P_{\text{coll}}$ is normalized by assuming that the radial field has the value $B_r(r_{\text{fp}}) \propto \pi/2$. The MHD invariants are constant on field lines (constant $P_{\phi, \text{coll}}$), such that, for example, the invariant field-line rotation frequency can be expressed as $\Omega_{\phi} \propto (2r_{\text{fp}} \phi_0)$. The force-free Lorentz factor ($\gamma_{\text{coll}}$) approaches the local poloidal radius of curvature of field lines and $C \approx 3$ (to within factors of order unity, $C$ depends upon the details of the solution). For large $r$,

$$\gamma_{\text{coll}} \approx \frac{2}{\delta} \sqrt{\frac{C}{(2 - v)v}},$$

where $R_c$ is the local poloidal radius of curvature of field lines and $C \approx 3$ (to within factors of order unity, $C$ depends upon the details of the solution). For large $r$,

$$\gamma_{\text{coll}} \approx \frac{2}{\delta} \sqrt{\frac{C}{(2 - v)v}}.$$  

(A8)

The second asymptotic acceleration regime is dominated by the poloidal field curvature. The force-free Lorentz factor can be used together with MHD energy conservation constraints to obtain an accurate full MHD Lorentz factor. Given the total energy flux per unit mass flux ($\mu$, which is the upper limit of $\gamma$) and the electromagnetic energy flux per unit rest-mass flux ($\gamma$, then one can show that $\mu = \gamma - \Phi RQ_e B_\phi / (c^2 \sqrt{\gamma}) = \gamma (1 + \sigma)$ with the conserved quantity $\sqrt{\gamma} R \Phi \equiv B_\phi / (\gamma R^2 u_\phi) = B_\phi / (\gamma R^2 u_\phi) = B_\phi / (\gamma R^2 u_\phi)$. With the general estimate that $\gamma_{\text{coll}} \approx \mu \sigma$ with $\sigma \to 0$ as $\gamma \to \mu$, one has that

$$\frac{1}{\gamma_{\text{coll}}} \approx \frac{1}{\gamma_{\text{coll}}} + \frac{1}{\mu}.$$  

(A9)

which accurately predicts the full Lorentz factor (i.e. $\gamma = \gamma_{\text{coll}}$) for collimating field geometries for MHD jets with $v > 0$ (Tchekhovskoy et al. 2008, 2009). The value of $\gamma_{\text{coll}}$ is chosen such that $\gamma = \gamma_{\text{coll}}$ at $r = r_{\text{fp}}$.

If the MHD jet was never confined or becomes unconfined, then the jet has $v \sim 0$ and behaves qualitatively differently from a collimating MHD jet (Tchekhovskoy et al. 2010b). The first asymptotic remains the same, while the monopolar solution has a modified second asymptotic solution given by the cubic equation

$$\gamma_{\text{2.m}} = C_1 \left[ \left( \frac{\mu - \gamma_{\text{2.m}}}{\sin \theta} \right) \ln \left( 1 + C_2 \frac{r_{\text{fp}} \sin \theta}{c \gamma} \right) \right]^{1/3},$$

where $C_1 \approx 2$, $C_2 \approx 0.4$, $\theta \sim \theta$ is the angle at which the jet passes the causality surface and $\gamma_{\text{coll}} \approx (\mu/\sin^2 \theta)^{1/3}$ is the Lorentz factor at the causality surface (Tchekhovskoy et al. 2009). The maximum value of $\gamma_{\text{2.m}} \to \mu$ occurs as $\theta \to 0$, which leads to a maximally efficient conversion of electromagnetic energy to kinetic energy. This behaviour of the Lorentz factor near $\gamma \sim 0$ captures the crucial effects of the fast critical surface and the related causality surface.

A sharp transition from the collimating to the monopole solution is assumed to occur at radius $r = r_{\text{monopolar}}$, which has been shown to be a quite accurate treatment of the time-dependent ideal MHD solution (Tchekhovskoy et al. 2010b). The final Lorentz factor is then $\gamma_{\text{coll}}$ inside $r_{\text{monopolar}}$ and otherwise follows the $\mu = 0$ monopole solution with the second asymptotic given by $\gamma_{\text{2.m}}$. At very small $r_{\text{fp}} \lesssim 10^{-3}$, these approximations can break down giving $\gamma < 1$, in which case $\gamma = 1$ is enforced because it is accurate for such regions.

Now that the full Lorentz factor and magnetic field structure are defined, the ideal MHD invariants can be used to constrain the rest of the jet structure. The $\gamma$ component of the 4-velocity can be obtained from the conserved angular momentum flux per unit rest-mass flux ($\lambda = \lambda_0 = (R u_\phi - \Phi R B_\phi) / (c \sqrt{\gamma})$) to obtain the conserved quantity $\psi = \psi_{\text{coll}} = \lambda - \Omega_0 \mu = \gamma - R \Omega_0 u_\phi / c^2$, where $\Omega_0$ is the angular velocity parameter and $u_\phi$ is the azimuthal component of the 4-velocity.
\[ u_\theta = c \left( \frac{\gamma - \psi}{R^2 \gamma} \right), \]  
\[ (A11) \]

\( \Phi \) is used to obtain \( u_{\phi,fp} = \Omega_0 \theta_p R_{fp} + B_{\phi,fp} v_{p,fp} / B_{\phi,fp}. \) Foot-point velocities are used to obtain \( \gamma' (\theta_{fp}, \theta_p) \).

The poloidal field-aligned component of the 4-velocity \( (u_\phi = \sqrt{u_\phi^2 + u_p^2}) \) is given by the definition of the Lorentz factor using \( \gamma' = \sqrt{1 - v^2/c^2} = 1 + (u_\phi^2 + u_p^2) \gamma^2 \), where \( v^2 = v_\phi^2 + v_p^2 \) and \( u_i = \gamma' \psi_i \) such that \( u_\phi^2 = c^2 (\gamma'^2 - 1) - u_p^2 \).

The rest-mass density can be obtained from the conservation of mass and magnetic flux such that
\[ \rho_0 = \frac{B_{\phi,fp}^2}{8 \pi R_{fp} u_p^2 c^2}. \]  
\[ (A12) \]

A useful measure of the magnetization is the parameter
\[ \zeta = \frac{B_{\phi,fp}^2}{2 \pi \rho_0 u_p c^2}. \]  
\[ (A13) \]

From the definition of \( \mu \), one can show that \( \mu \approx \gamma_{fp} + 4 c^2 (\Omega_0 f_{fp, fb} / \gamma_{fp})^2 \sin (\theta_{fp}/2) / (\gamma_{fp} v_{p,fp}) \). For example, with \( \zeta \gg \gamma_{fp} \approx 1 \), one has that \( \zeta \approx 2 \mu \) when \( \theta_{fp} = \pi/2, \Omega_0 f_{fp} = 0.25 \tau_{fb} \), and \( \gamma_{fp} \approx 1.15 \). In general, a foot-point launching velocity of \( v_{p,fp} \approx c/2 \) is chosen (Tchekhovskoy et al. 2008), but it is unimportant as long as \( 1 \ll \gamma_{fp} \ll \mu \).

To enforce consistency with the ideal MHD invariants, \( B_{\phi,fp} \) is recovered from the definition of \( \mu \), given the computed \( \gamma_{fp} \) and \( u_\phi \).

The true solution’s deviation of \( \int d \phi \left[ \frac{\rho_{b,fp}(\theta) - \rho_{b,fp}(\theta)}{\rho_{b,fp}(\theta)} \right] \) for other fixed parameters. How-ever, because \( \rho_{b,fp}(\theta) \) and \( c/\Omega_0 \) are assumed to be non-degenerate within the jet. Assuming NSE, the fraction of free nucleons is
\[ X_{\text{nuc}} \approx \min \left[ 1, 0.296 \rho_{b,fp}^{-3/4} T_{11}^{9/8} e^{-0.6209 \mu} \right], \]  
\[ (B1) \]

(Woosley & Baron 1992; Kohri et al. 2005). The comoving timescale for establishing NSE is \( \tau_{\text{NSE}} \sim \rho_{b,fp}^{0.5} \exp (1.8 \times 10^{-4}/T - 39) \) s in the electron degenerate regime and \( \tau_{\text{NSE}} \sim T/(3.15 \times 10^5) \) s in the hot non-degenerate regime (Kohiklov 1989; Qian & Woosley 1996). These timescales are compared with the jet flow time given in Section 6.1.

Given \( X_{\text{nuc}} \), the baryon internal energy density is
\[ u_b = \rho_b \left( \frac{3 k_b T}{8 m_N} \frac{(3 - E_{\text{ratio}}) X_{\text{nuc}} + 1}{1 - E_{\text{ratio}}} \right), \]  
\[ (B2) \]

where \( E_{\text{ratio}} = E_{\text{nuc}}/(4 m_N c^2) \), the \( \alpha \)-particle binding energy is \( E_{\text{bin}} = 28.3 \text{ MeV}, m_{\text{nuc}} c^2 \approx (m_n + m_p c^2)/2 \approx 938.919 \text{ MeV} \) and \( k_b \) is Boltzmann’s constant. This directly accounts for the nuclear binding energy of \( \alpha \) particles and the effect of photodisintegration directly.

These jet-structure equations do not account for the thermal or general relativistic effects. First, if the thermal energy is subdominant to the electromagnetic and rest-mass energy, then the above jet-structure equations are valid and remain accurate because only weak shocks occur in a highly magnetized flow. Secondly, GR effects accumulate magnetic flux towards the BH spin-axis leading to a non-constant \( B_{\phi,fp}(\theta) \) (McKinney 2005b; Komissarov & McKinney 2007; McKinney & Narayan 2007a,b; Tchekhovskoy, Narayan & McKinney 2010a). The spin enhancement of the magnetic field would also change how one would estimate \( B_{\phi,fp} \) from the mass-accretion rate (Gammie, Shapiro & McKinney 2004; McKinney 2005b). However, these effects occur only within a few gravitational radii, and beyond this radius the flux deconcentrates without much difference to the non-rotating BH case. This suggests a smaller true \( \theta_{fp} \) for a rapidly rotating BH with an accretion disc of height-to-radius ratio \( \approx 0.1 \) (as for a neutrino-dominated accretion disc) should give comparable results to generally choosing \( \theta_{fp} \approx \pi/2 \).

APPENDIX B: DENSITIES, PRESSURES AND EMISSION RATES

In this section, the EOS and emission rates for baryonic and radiative species are presented. For radiative species, the optically-thick densities/pressures and optically-thin energy/number loss rates are used with the two-stream radiative transfer slab approximation presented in Section B2. This approximation determines the pressure and energy rates for general optical depths. These are then used in energy and force balance conditions that set the reconnection layer’s physical geometry and reconnection rate in Section C3.

The distribution function for all species is assumed to be thermal except as modified by a radiative transport approximation to account for the range of optically-thin and optically-thick behaviours for radiative species. As tested using equation (B8), the thermal assumption is often valid for the collisional layer.

B1 Baryons

A mixture of free nucleons (protons + neutrons) and \( \alpha \) particles can exist at the densities and temperatures of interest. Baryons are assumed to be at a single temperature \( T \) in thermal equilibrium. The baryons have a number density of \( n_b = \rho_b/m_b \), particle mass \( m_b \), and are assumed to be non-degenerate within the jet. Assuming NSE,
within the EoS. The baryonic pressure is then
\[ p_b = (\Gamma - 1)n_b, \]  
with \( \Gamma = 5/3 \), corresponding to a sum over all non-degenerate and non-relativistic baryons. The non-degeneracy assumption for baryons is ensured by checking that \( n_b \lesssim (m_b k_B T/2\pi \hbar^2)^{3/2} \) (Kohri & Mineshige 2002). Tests for the assumption that \( Y_e = n_e/n_b \sim 1/2 \) are discussed in Section B5.

### B2 Two-stream radiation approximation

The energy densities and cooling rates for radiative species are obtained via the two-stream approximation limit of the Boltzmann equation (Rybicki & Lightman 1979; Hubeny 1990; Popham & Narayan 1995; Sawyer 2003). The theory of radiative reconnection of current layers is quite analogous to the radiative transfer problem in accretion disc theory (Uzdensky & McKinney 2011), for which this approximation has been used extensively for both photons (Popham & Narayan 1995) and neutrinos (Popham, Woosley & Fryer 1999; Di Matteo, Perna & Narayan 2002; Kawanaka & Mineshige 2007). In this one-zone approximation, radiation is either lost from the slab or remains and is not redistributed within the slab.

The two-stream approximation requires optical depths for absorption (\( \tau_{abs} \)), scattering (\( \tau_{sca} \)) and their total
\[ \tau_{tot} = \tau_{abs} + \tau_{sca}. \]  

For scattering species interacting with a bath of scatterer species \( B \) with number density \( n_B \), scatterer density scale-height \( H_B \) and scatter cross-section \( \sigma_B \), the scattering optical depth is
\[ \tau_{sca} = \sum_B n_{A,B} \sigma_B H_B. \]  

Changes of the order of unity in the momentum of species \( A \) are required to achieve an effective collision corresponding to large-angle scattering and an effective exchange of energy. The actual number density of scatterer species \( B \) given by \( n_B \) is effectively reduced by diffuse (rather than direct) scattering, such that
\[ n_{A,B,eff} = n_B X[p_A] \]  
and
\[ X[p_A] \sim \min \left[ \frac{\Delta p_A}{p_A}, \left( \frac{\Delta p_A}{p_A} \right)^2 \right]. \]

where \( p_A \) is the momentum of species \( A \). The centre-of-momentum frame change in momenta is approximated as \( \Delta p_A \sim \min(p_A, p_B) \), where \( p_B \) is the momentum of species \( B \). The minimum (min [ ]) conditional in \( X[p_A] \) accounts for the fact that effective collisions occur when \( (\Delta p_A/p_A)^{\sim 1} \), while the momentum undergoes a random walk for \( (\Delta p_A/p_A) \lesssim 1 \). This correction is not applied to Coulomb scattering as computed later because this effect is already included directly in the cross-section.

Thermal equilibrium between a pair of particles is achieved if the transit time of plasma through the current layer is longer than their collisional energy-exchange time. This condition is written as
\[ C_{\text{thermalization,}\ A\ B} = \frac{L_0}{v_A} v_{\text{cc,}\ A\ B} \gg 1, \]  
where the effective collision energy-exchange rate due to factors of order unity changes in the momentum of species \( A \) is
\[ v_{\text{cc,}\ A\ B} \approx v_{\text{cc,}\ A\ B} X[p_A], \]  
where \( X[p_A] \) is given by equation (B7) and \( v_{\text{cc,}\ A\ B} \) is the actual collisional frequency for particle type \( A \) to collide with particle type \( B \). In the non-relativistic limit, \( v_{\text{cc,}\ e-e} \sim n_e T_e^{-3/2} m_e^{-1/2} \) and \( v_{\text{cc,}\ e-i} \sim (m_i/m_e)^{1/2} v_{\text{cc,}\ e-e} \sim (m_i/m_e) v_{\text{cc,}\ i-i}, \) where ions are scattered diffusively by electrons. The value of \( C_{\text{thermalization,}\ A\ B} \) checked for all models in order to ensure sufficient thermalization, although even in the collisionless regime one expects a dominant thermal component (Giannios & Spitkovsky 2009).

The absorption opacities are obtained using Kirchhoff’s law, such that for each absorber with number density \( n_B \) and some absorption scale-height \( H_B \) one obtains an energy and number density loss rate based absorption optical depth of
\[ \tau_{abs} = \sum_B n_B H_B \sigma_{abs} = \sum_B \frac{Q_{B-A,0} H_B}{v_B n_B} \]  
and
\[ \tau_{n_B,abs} = \sum_B n_B H_B \sigma_{n_B-\text{abs}} = \sum_B \frac{R_{B-A,0} H_B}{v_B n_B}, \]  
respectively, where \( n_B \) and \( n_B \) are the optically-thick (i.e. those corresponding to the thermodynamic equilibrium at a given temperature) energy and number densities, respectively, and \( v_B \) is the free-stream velocity (e.g. \( v_B = c \) for photons and neutrinos). (Note that the subscript \( \text{script} \) only indicates the quantity is some known quantity used to construct new quantities that are valid at general optical depths.) The cross-section \( \sigma_{abs} \) is for the inverse reaction to that operating at the optically-thin rate \( Q_{B-A,0} R_{B-A,0} \).

Now that the absorption and scattering optical depths have been obtained, the two-stream approximation can be used to obtain densities and rates valid for general optical depths. The internal energy and number density are given by
\[ u = u_0 g[\tau_B], \]  
and
\[ n = n_0 g[\tau_B], \]  
respectively, with
\[ g[\tau] = \frac{\tau_{	ext{tot}}/2 + 1/\sqrt{3}}{\tau_{	ext{tot}}/2 + 1/\sqrt{3} + 1/(3\tau_{abs})}, \]  
and energy and number surface fluxes lost from the slab of
\[ F_u = v_B u_0 h[\tau_B], \]  
and
\[ F_n = v_B n_0 h[\tau_B], \]  
respectively, with
\[ h[\tau] = \frac{1/3}{\tau_{	ext{tot}}/2 + 1/\sqrt{3} + 1/(3\tau_{abs})}. \]

The volumetric energy and number density rates are then given by
\[ Q = \frac{F_u}{H_{n,\text{abs}}}, \]  
and
\[ R = \frac{F_n}{H_{n,\text{abs}}}, \]  
respectively, where the absorption depth is
\[ H_{n,\text{abs}}(\tau_{abs}) = \frac{\tau_{abs}}{d\tau_{abs}/d\sigma}. \]

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where from Kirchhoff’s law one obtains \(d\tau_{abs}/d\nu = \sum_{\nu} n_{\nu}\sigma_{\nu,abs} = \sum_{\nu} Q_{\nu,abs}/(\nu\Delta_{\nu,abs}) = Q_0/(\nu\Delta_{\nu,abs})\) and \(d\tau_{abs}/d\nu = \sum_{\nu} R_{\nu,abs}/(\nu\Delta_{\nu,abs}) = R_0/(\nu\Delta_{\nu,abs})\). Note that \(Q\) and \(R\) become the optically-thin energy and density rates \(Q_0\) and \(R_0\), respectively, when \(\tau \ll 1\).

B3 Optical depth in a relativistic jet

In order to compute the optical depth in the prior section, one requires some estimate of the density scale-height for scatterers, given by \(H\), that is applicable for a relativistic jet.

In the laboratory frame, the optical depth is

\[
\tau = \int c dt' (\sigma n) = \int d\nu [\sigma n \gamma (1 - \beta \cos \theta)],
\]

where \(c dt' = d\nu'\) is the comoving time-difference, \(d\nu' = \gamma (1 - \beta \cos \theta) d\nu\) is the laboratory-frame radial difference, \(n\) is the comoving number density of particles, \(\gamma\) is the Lorentz factor, \(c\beta\) is the 3-velocity and \(\theta\) is the angle in the laboratory frame. An angle-averaged invariant cross-section \((\sigma)\) is assumed to be used. For a fluid with \(\gamma > 1\) in the laboratory frame, a typical photon emitted isotropically in the comoving frame is emitted parallel to the jet in the laboratory frame giving

\[
\tau \approx \int_{r}^{r + \Delta r} d\nu [\sigma n/(2\gamma)],
\]

(B22)

where \(\Delta r\) is the laboratory-frame distance the photon traverses through the medium of density \(n\). For photons to escape a radiating slab of comoving size \(L_0\), \(\Delta r \sim 2\gamma L_0\) per layer would be chosen.

In our case, the dissipating complex of current sheets consists of numerous narrow dissipation regions covered in a photon–neutrino photosphere (when a species is optically thick). As shown in Section 5.4, the range over which the electromagnetic energy is dissipated into these species is given by \(\Delta r \sim c \gamma \Delta_{\nu}/(2\nu)\) for all \(m\) modes and some \(l\) modes. Assume that radiation densities drop as roughly \(r^{-2}\) (accurate for photons up to relevant radii), and assume that the Lorentz factor and \(c\rho, T\) vary as some power of \(r\) (including constant with radius). The opacity integral then gives

\[
\left(\frac{\tau_{\text{rad}}}{\sigma_{\text{rad}}n_{\text{rad}}}\right)^{-1} \sim \frac{2\nu}{c\Delta_{\nu}} \gamma \sin(\theta) \equiv (\Delta_{\nu}')^{-1},
\]

(B23)

where \(n_{\text{rad}}\) is the number density of radiation at radius \(r\). For downstream baryonic-associated electrons, a similar calculation gives

\[
\left(\frac{\tau_{\text{baryonic}}}{\sigma_{\text{baryonic}}n_{\text{baryonic}}}\right)^{-1} \sim \frac{2\nu}{c\Delta_{\nu}} \gamma \sin(\theta) \equiv (\Delta_{\nu}')^{-1},
\]

(B24)

where baryons are assumed to extend to large radii such that \(\Delta r \to \infty\).

Thus, we have determined that \(H \sim \Delta_{\nu}'\) for scatterers of radiative species generated by the current layers, and \(H \sim \Delta_{\nu}'\) for scatterers consisting of baryonic-associated electrons downstream in the jet. A more detailed relativistic radiative transfer calculation is left for future work (see e.g. Meszaros 1992; Beloborodov 2011).

The two-stream approximation can now be applied to photons, pairs and neutrinos in order to determine their behaviour in the optically-thick and marginally-optically-thin regime when the single-temperature approximation is still good to within factors of order unity. This two-stream approximation determines the densities \((n)\), pressures \((\rho)\), energy rates \((Q)\) and number rates \((R)\) for photons, pairs and neutrinos as obtained from the base optically-thin rates \((Q_0, R_0)\), base optically-thick densities \((n_0, n_0)\) and scattering-cross sections \((\sigma_{\text{sca}})\) written down for each species in the next section.

B4 Photons

Photons are created within the dissipative current layer by processes such as bremsstrahlung (e.g. free–free emission, free–bound emission), cyclo-synchrotron, pair annihilation, radiative pair annihilation and double Compton scattering. Most photons are created inside the dissipative current layers, and photons can travel across field lines and interact with other layers. For photons, the two-stream approximation requires knowing the scattering and absorption opacities as computed below, and one also needs the optically-thick limit for the energy and number densities. The optically-thick photon internal energy density is

\[
u_0,T = a_T T^4,
\]

(B25)

where \(a_T = \pi^2 k_B^4/[15(\hbar c)^3]\). The photon pressure is then given by \(p_0,T = \nu_0,T/3\) as for a \(\Gamma\) is \(4/3\) ideal gas for adiabatic constant \(\Gamma\). The optically-thick number density of photons is

\[
\nu_{0,T} = (2\zeta[3/(\pi^2)]a_T/k_B)T^3 \approx (1/4)[\nu_{0,T}/(k_B T)]^3.
\]

(B26)

B4.1 Photon scattering

The scattering opacity for photons is \(\sigma_{\text{sca}} = n_{\text{e,eff}} \Delta_{\nu}' + n_{\gamma-e,e,eff} \Delta_{\nu}'\),

(B27)

for an electron scattering opacity \(\sigma_{\text{sca}}\), where \(n_{\gamma-e,e,eff}\) is the effective number density of pairs and \(n_{\gamma-e,e,eff}\) the effective number density of electrons as computed from equation (B6), and the scale-heights for scatterers \(\Delta_{\nu}'\) are given by equations (B23) and (B24), respectively. Radiative effects discussed later can lead to an enhancement of the baryon and associated electron density, but the opacity is an integral of density that is roughly fixed for a conserved amount of mass across the jet.

In the subcritical QED regime, the thermal spectrally averaged cross-section for the Klein–Nishina effect is roughly

\[
\left(\frac{\sigma_{\text{KN}}}{\sigma_T}\right)^{-1} \sim 1 + \left(\frac{3m_e c^2}{8k_B T}\right)^{-1},
\]

(B28)

where \(\sigma_T\) is the Thomson cross-section.

In the supercritical QED regime, the extraordinary mode (E mode; electric polarization perpendicular to the magnetic field), electron scattering cross-section depends upon whether the particle energy is above/below the rest-mass of electrons and above/below the energy of the first Landau level given by \(h\omega_{le}(1) = [(m_e c^2)^2 + 2\hbar c q B]/2 - m_e c^2\) (Silantev & Iakobov 1980; Meszaros 1992; Thompson & Duncan 1995; Lai 2001). For the E mode, the thermal spectrally averaged cross-section is

\[
\left(\frac{\sigma_{\gamma-e}}{\sigma_T}\right)^{-1} \sim 1 + \left(\frac{\sqrt{3} m_e c^2 k_B T}{\hbar q B}\right)^{-2},
\]

(B29)

which only applies for supercritical-field strengths. The high-field suppression of the scattering cross-section for the E mode and the efficient conversion of the ordinary mode (O mode) to E mode (Meszaros 1992; Thompson & Duncan 1995) means that radiative emission is dominated by the E mode with this scattering cross-section when above the critical field strength. Photon splitting and merging are assumed to be in detailed balance in the optically-thick regime (Thompson & Duncan 1995), and often the thermal photons...
are optically thick for supercritical fields. Defining an interpolation parameter \( e_B = \exp \left\{ \left( -m_e c^2 \right)/(h\omega_{\text{abs}}(1)) \right\} \), the total electron scattering cross-section for any \( T \) and \( b \) is

\[
\sigma_{e\text{s}} \sim \sigma_B e_B + \sigma_{\text{Kn}}(1 - e_B).
\]

(B30)

### B4.2 Photon absorption and emission

The absorption opacity is determined from the optically-thin emission rate and Kirchhoff’s law. Because the true number density of photons is not required to be accurate in this work, \( \tau_{\nu,\text{abs}} \approx \tau_{\nu,y,\text{abs}} \approx \tau_{\nu,y,\text{abs}} \) is set and only energy density loss rates are considered. Thermal free–free, thermal synchrotron and thermal pair annihilation are considered. The total photon absorption optical depth is

\[
\tau_{\nu,\text{abs}} \sim (\sigma_{\text{ff}} + \sigma_{\text{synch}}) n_{\text{pairs}} \Delta \gamma + \sigma_{\text{e}\rightarrow\gamma\gamma} n_{\text{pairs}} \Delta \gamma,
\]

(B31) as due to free–free, synchrotron and pair annihilation given below. While the non-pair-producing electrons (with number density \( n_e \)) outside the current layer’s absorption photosphere may be initially cold in the jet, those electrons are assumed to be heated by a sufficient number of photons, bringing that portion of the jet into thermal equilibrium if the region is optically thick. The local differential absorption optical depth is \( d\tau_{\nu,\text{abs}}/ds = (\sigma_{\text{ff}} + \sigma_{\text{synch}})(n_{\text{pairs}} + n_e) + \sigma_{\text{e}\rightarrow\gamma\gamma} n_{\text{pairs}} \).

### B4.3 Free–free

The optically-thin free–free emission rate that includes relativistic effects, electron/positron–ion collisions and electron/positron–electron/positron collisions is

\[
Q_{1,\text{ff}} \sim 1.4 \times 10^{-27} T^{1/2} \left\{ n_{\text{e,tot}} \left( 1 + \frac{\text{KE}_e}{m_e c^2} \right)^2 \right\},
\]

(B32) where \( \text{KE}_e \approx 3k_B T \) is the average kinetic energy of relativistic electrons/pairs, \( n_e \sim n_p \sim m \) is assumed, and protons are assumed to be non-relativistic (Böttcher, Pohl & Schlickeiser 1999). Kirchhoff’s law gives

\[
\sigma_{0,\text{ff}} = \frac{Q_{1,\text{ff}}}{n_{\text{e,tot}} c u_\gamma y},
\]

(B33) for the cross-section per emitting/absorbing particle. In the supercritical QED regime, the thermal spectrally averaged cross-section is suppressed such that

\[
\left( \frac{\sigma_{\text{ff}}}{\sigma_{0,\text{ff}}} \right)^{1/2} \sim 1 + \left( \frac{\sqrt{\pi} m_e c^2 k_B T}{h c q B} \right)^{-2} \]

(B34)\]

\[\text{(Thompson \\& Duncan 1995; Lai 2001). Note that one can define the QED optically-thin emission rate, } Q_{0,\text{ff}}, \text{ from Kirchhoff’s law: } \sigma_{0,\text{ff}} = Q_{0,\text{ff}}/(n_{\text{e,tot}} c u_\gamma y). \text{ These equations give, as required, a free–free contribution to } Q_{\theta} \text{ of } Q_{0,\text{ff}} \text{ in the optically-thin limit.}

### B4.4 Cyclo-synchrotron

The synchrotron (and approximate cyclotron) energy density loss rate integrated over all angles, frequencies, and over an isotropic distribution of thermal particles with Lorentz factor \( \gamma_e \) and pitch angle \( \theta \) is

\[
Q_{1,\text{synch}} \approx n_{\text{e,tot}} \int_{\theta=0}^{\pi} \sin \theta \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} d\gamma_e \left\{ f(\gamma_e) j(\gamma_e, \theta) \right\},
\]

(B35) where the frequency-integrated emissivity (erg s\(^{-1}\)) for each pitch angle is

\[
j(\gamma_e, \theta) = \frac{2q e^2 (\gamma_e^2 - 1) \sin^2 \theta}{3m_e c^3},
\]

(B36) and for simplicity the non-degenerate relativistic electron thermal distribution,

\[
f(\gamma_e) = \frac{1}{\Theta K_{\gamma}(1/\Theta)} \gamma_e^2 \beta_e \exp(-\gamma_e/\Theta),
\]

(B37) is used, where \( \Theta = k_B T/(m_e c^2) \), \( \beta_e = \sqrt{1 - 1/\gamma_e^2} \) and \( K \) is the Bessel \( K \) function (Rybicki \\& Lightman 1979; Mahadevan, Narayan \\& Yi 1996; Özel, Psaltis \\& Narayan 2000). In cases when the gas is degenerate, synchrotron is found not to be crucial for this study. The electron and photon azimuthal angles have both already been defined unity for \( \gamma_e \rightarrow 0 \).

### B4.5 Pair annihilation into photons

Electron–positron pair annihilation into photons for thermal pairs has a number density rate of

\[
R_{0,\text{e}\rightarrow\gamma\gamma} \approx 3 \times 10^{-6} \left( \begin{array}{c} \sigma_{\text{e}\rightarrow\gamma\gamma} n_e \left[ 1 + \frac{2\Theta^2}{\ln(1.12\Theta + 1.3)} \right] \end{array} \right)^{-1} \]

(B40) for each thermal pair unit (one electron and one positron), where \( \Theta = k_B T/(m_e c^2) \), \( n_e = (n_{\text{e,tot}} - n_p)/2 \approx n_{\text{pairs}}/2 \) and \( n_p = (n_{\text{e,tot}} + n_p)/2 = n_{\text{pairs}}/2 + n_e \) (Svensson 1982; Sikora \\& Zbyszewska 1986). The corresponding energy density loss rate is

\[
Q_{0,\text{e}\rightarrow\gamma\gamma} \approx E_{\text{pairs}} R_{0,\text{e}\rightarrow\gamma\gamma},
\]

(B41)
where $E_{\text{pairs}} \approx 2(m_e c^2 + u_e/n_e)$, and $u_e$ is the electron/positron internal energy density given later. By Kirchhoff’s law,

$$\sigma e^+ e^- \rightarrow \gamma \gamma = \frac{Q_{e^+ e^- \rightarrow \gamma \gamma}}{n_{\text{pairs}} c u_{0,0}} \quad (B42)$$

QED effects on the optically-thin pair annihilation rates are neglected, but only tend to be important at very low baryon loading of the jet.

### B5 Electrons and positrons

Electron–positron pairs can be created by processes such as photon annihilation, one-photon pair production in supercritical fields, ion–electron collisions, electron–electron collisions, ion–photon collisions and electron–photon collisions. The pairs do not readily cross field lines and are stuck within the dissipation region until they can annihilate into photons and spread throughout the photon photosphere or travel along field lines within the layer.

In thermal equilibrium, the asymmetry between electrons and positrons is represented by the electron chemical potential $\mu_e \equiv \mu_{e^-} - \mu_{e^+}$, which is determined by the condition of charge neutrality among protons, electrons and positrons given by $n_e \equiv n_{e^-} + n_{e^+} = n_0$, where $n_0$ is the total number density of protons (free or bound). The degeneracy parameter of electrons is given by $\eta_e = (n_e/m_e)T$, where $n_e \gtrsim 1$ implies electrons are degenerate. A value of $Y_e \equiv n_e/n_0 = 1/2$ is assumed, but the $\beta$-equilibrium value of $Y_e$ is computed from $1/(1+Y_e) = n_e/n_0 = \exp(Q - n_0)$ with $Q = (m_e - m_0)c^2/(k_B T)$, which assumes only free nucleons are present (Kohri & Mineshige 2002). This gives a test of whether the assumption of $Y_e = 1/2$ is violated.

The Fermi–Dirac distribution function of particles in thermal equilibrium is

$$f_e(E) = \frac{1}{\exp(E/k_B T) - 1} + 1. \quad (B43)$$

The number densities of electrons and positrons are

$$n_{e^-} = \frac{1}{\pi^2 (mc^2)^2} \int_0^{\infty} d\tilde{p} \left\{ \tilde{p}^2 [f_e(\tilde{p}E)] \right\}, \quad (B44)$$

where $\tilde{p} \equiv p c$ and $E_e = \sqrt{\tilde{p}^2 + (mc^2)^2}$. For the total number density of (free and bound) protons ($n_p = Y_e \rho_0/m_0$) and $T$, the value of $\mu_e$ is iteratively solved for from the definition $n_e(\mu_e)/n_0 = Y_e$. Then, $n_{e^-}$ and $n_{e^+}$ and any other quantities where $\mu_e$ appears can be computed. The electron–positron pressure is

$$p_{e^-} = \frac{1}{3\pi^2 (mc^2)^2} \int_0^{\infty} d\tilde{p} \left\{ \tilde{p}^4 E_e [f_e(\tilde{p}E_e)] \right\}, \quad (B45)$$

The electron rest-mass plus internal energy gives an internal energy density of

$$u_{e^-} = \frac{1}{\pi^2 (mc^2)^2} \int_0^{\infty} d\tilde{p} \left\{ \tilde{p}^2 E_e [f_e(\tilde{p}E_e)] \right\}. \quad (B46)$$

The non-degenerate limit gives

$$p_e = n_e k_B T \quad (B47)$$

and

$$u_e \approx \frac{p_e}{\Gamma - 1}. \quad (B48)$$

where the parameter $\Theta \equiv k_B T/(mc^2)$ is used to linearly interpolate from $\Gamma = 5/3$ to $4/3$ such that at and beyond a value of unity this gives $\Gamma = 4/3$.

Then, the pair pressure, internal energy density and number density are

$$p_{0,\text{pairs}} = p_e^- + p_{e^+} - p_e, \quad (B49)$$

$$u_{0,\text{pairs}} = u_e^- + u_{e^+} - u_e \quad (B50)$$

and

$$n_{0,\text{pairs}} = n_{e^-} + n_{e^+} - n_e = 2n_{e^+}, \quad (B51)$$

respectively. For more details, see Kohri et al. (2005). Note that the sum of, for example, $p_e + p_{0,\text{pairs}}$ that enters the total gas pressure is unaffected by this decomposition.

#### B5.1 Electron and positron $EoS$ with QED corrections

In the supercritical-field regime, the pair internal energy density depends upon whether the particle energy is above/below the rest-mass of electrons and above/below the energy of the first Landau level. Let $e_1 = \exp(-m_e c^2/k_B T)$ and introduce an interpolation factor $e_2 \equiv \exp\{-(k_B T)(\ln\Theta(1))\}$ (Meszaros 1992; Thompson & Duncan 1995). Then, the following approximate QED expressions are used if $e_2 > 0.1$, and otherwise the more accurate non-QED expressions given previously are used.

When $\ln\Theta(1) \ll k_B T$ and $k_B T \gg m_e c^2$, the pairs behave like classical radiation with an internal energy density

$$u_{0,A,\text{pairs}} \approx (7/4)u_{0,0}(1 - e_2), \quad (B52)$$

which gives a number density of pairs roughly equal to the number density of $m_e c^2$ photons. When $\ln\Theta(1) \ll k_B T$ and $k_B T \ll m_e c^2$, then the pairs are classical and non-relativistic with an internal energy density

$$u_{0,B,\text{pairs}} \approx 2^{3/2} \frac{(m_e c^2)^4}{\pi^2 (k_B T)^2} \left( \frac{k_B T}{m_e c^2} \right)^{3/2} e_1 (1 - e_2). \quad (B53)$$

When $\ln\Theta(1) \gg k_B T$ and $k_B T \gg m_e c^2$, then photons and quantized relativistic pairs are in detailed balance with an internal energy density

$$u_{0,C,\text{pairs}} \approx \frac{1}{12} \frac{h c q_b}{(k_B T)^2} \gamma_1 e_2. \quad (B54)$$

When $\ln\Theta(1) \gg k_B T$ and $k_B T \ll m_e c^2$, then the pairs are quantized and non-relativistic with an internal energy density

$$u_{0,D,\text{pairs}} \approx \frac{(h c q_b)}{2\pi^2 (k_B T)^2} \left( \frac{k_B T}{m_e c^2} \right)^{1/2} \gamma_1 e_2. \quad (B55)$$

Across these four regimes, a sufficient interpolation procedure is to simply sum all terms together to obtain an internal energy density

$$u_{0,\text{pairs}} \sim u_{0,A,\text{pairs}} + u_{0,B,\text{pairs}} + u_{0,C,\text{pairs}} + u_{0,D,\text{pairs}}. \quad (B56)$$

pressure

$$p_{0,\text{pairs}} \sim u_{0,A,\text{pairs}}/3 + 2u_{0,B,\text{pairs}}/3 + u_{0,C,\text{pairs}}/3 + 2u_{0,D,\text{pairs}}/3 \quad (B57)$$

and pair number density

$$n_{0,\text{pairs}} \sim \frac{p_{0,\text{pairs}}}{k_B T}. \quad (B58)$$

These expressions are only accurate to within factors of order unity for $k_B T \gtrsim m_e c^2$ and otherwise less accurate, but the QED effects tend to only be relevant at high temperatures.
B5.2 Opacity effects on electrons–positrons

The pair-production processes $\gamma\gamma \rightarrow e^+e^-$, $\gamma e \rightarrow ee^+e^-$ and $ee \rightarrow eee^+e^-$ are considered, where $ep \rightarrow epe^+e^-$ and $\gamma p \rightarrow epe^+$ have been shown to be less efficient for thermal plasmas considered in this paper (Zdziarski 1982; Sikora & Zbyszewska 1986).

Pairs act as radiation when annihilation into photons dominates pair creation and pairs fill in the region above the current layer. In the limit that Kirchhoff’s laws apply, the creation of photons via the inverse reactions to $\gamma\gamma \rightarrow e^+e^-$ and $\gamma e \rightarrow ee^+e^-$ as seen below, dominate these inverse processes of $ee \rightarrow eee^+e^-$ except for $k_B T \gg mc^2$, such that any absorption opacity leads to photons that can readily cross field lines and come into equilibrium throughout the photon photosphere of size $L_0$. Pairs also act as radiation (with the addition of Coulomb interactions) when pairs flow down the current layer along straight field lines that open up after a length $L_0$, then, both across and along the layer, the photon and pair density scale-heights are of the order of $L_0$. In the collisionless limit, the pairs are determined by their prior history instead of equilibrium, but this work only seeks to find the collisional layer structure.

The two-stream approximation is applied to pairs whether they annihilate and travel above the layer or they travel down field lines along the layer. Factors of the order of unity in equations (B14) and (B17) would slightly change depending upon the allowed trajectories, but such minor changes are ignored. Kirchhoff’s law is used as usual with angle and energy averaged rates given below, where recall that $(\sigma v) / (c) = (1 + \delta/3)R_2/(cn_1n_2)$ for species 1 and 2 and number density rate $R_2$ (see e.g. Weaver 1976). For simplicity, the underlying photon, electron and pair distributions are assumed to be thermal, although the photon densities are modified by the two-stream approximation to account for optical depth effects. Because the (fully or marginally) optically-thin energy density and pressure of pairs are not required to be accurate in this work, while the number density of pairs in the marginally-optically-thin limit should be somewhat accurate, the number rates are treated directly while the energy rates are approximated.

Consider the $\gamma\gamma \rightarrow e^+e^-$ process. Following the notation and assumptions of Weaver (1976), let $c = \exp[- \mu / (k_B T)]$ with photon chemical potential $\mu_p$, $\Phi = mc^2 E_p$ for the centre-of-momentum photon energy $E_p$, $\Theta = k_B T / mc^2$, and $x = 1/(\Theta \Phi)$ such that $xk_B T = E_p$, then

$$
\sigma_{\gamma\gamma \rightarrow e^+e^-} = \frac{3\pi}{8} \Phi^2 \left[ (2 + 2\Phi^2 - \Phi^4) \cosh^{-1} \left( \frac{1}{\Phi} \right) - \Phi^2 \right].
$$

(B59)

where the unit of pairs has thermal energy of roughly $E_{\gamma\gamma} \approx 2E_\gamma$, where $E_\gamma \sim u_p / n_\gamma$.

The $\gamma e \rightarrow ee^+e^-$ process has a fitted cross-section

$$
\sigma_{\gamma e \rightarrow ee^+e^-} = \frac{10^{-3} (y - 4)^2}{(5.6 + 20.4(y - 4) - 10.9(y - 4)^2)} \times 3.6(y - 4)^3 + 7.4(y - 4)^4.
$$

(B62)

Pairs also act as radiation (see e.g. Weaver 1976). For simplicity, the number density rate for each pair unit is then

$$
R_{\gamma\gamma \rightarrow e^+e^-} = \frac{2(k_B T)^3}{(\hbar c)^6} \sum_{n=1}^{\infty} \left( \frac{n!}{C^{\nu+1}} \right)_{-1}^{n} \left( \frac{\mu_p}{c} \right)^{2n+2} \int_{0}^{\infty} dy \left[ y^{2} \sigma_{\gamma\gamma \rightarrow e^+e^-} \left[ y \right] e^{-y/\Theta} - \frac{y^{2}}{2} \sigma_{\gamma\gamma \rightarrow e^+e^-} \left[ y \right] e^{-y/\Theta} \right],
$$

(B60)

where $Q_{\gamma\gamma \rightarrow e^+e^-} \approx E_{\gamma\gamma}R_{\gamma\gamma \rightarrow e^+e^-}$, (B70)

where $E_{\gamma\gamma} \sim E_\gamma + KE_e$ and $KE_e \sim u_e / n_e$.

The $ee \rightarrow eee^+e^-$ process has a number density rate for each pair unit of

$$
R_{ee \rightarrow eee^+e^-} = 8.4 \times 10^{-6} n_e^2 \sigma_{ee \rightarrow eee^+e^-} \left[ \ln(\Theta) \right]^3,
$$

(B71)

for $\Theta > 1$ and $R_{ee \rightarrow eee^+e^-} \rightarrow 0$ for $\Theta < 1$ (Svensson 1984, 1987; Sikora & Zbyszewska 1986). The energy density loss rate is

$$
Q_{ee \rightarrow eee^+e^-} \approx E_{ee}R_{ee \rightarrow eee^+e^-},
$$

(B72)

where $E_{ee} \approx 2(KE_e)$.

The total optically-thin energy and number density rates are

$$
R_{0, pairs} = R_{\gamma\gamma \rightarrow e^+e^-} + R_{\gamma e \rightarrow ee^+e^-} + R_{ee \rightarrow eee^+e^-},
$$

(B73)

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The scattering optical depth is

\[ \tau_{\text{pairs, sca}} \approx \sigma_{e^+ e^-} n_{\text{pairs}} \Delta_0' + \sigma_C (n_{\text{pairs}} \Delta_0 + n_e R) , \]  

(B75)

where \( n_{\text{pairs}} \) is the effective number density of photons computed from equation (B6) using \( \rho_p \sim (1/c)(u_\text{L}/h_p) \), and \( \sigma_C \) is the Coulomb scattering cross-section given later by equation (C2). The absorption optical depths are given by Kirchhoff’s law as

\[ \tau_{\text{abs, \ pairs}} \approx \frac{2 R_0 \text{pairs} \Delta_0}{v_{\text{L}, 0} n_{\text{pairs}}} \]  

(B76)

and

\[ \tau_{\text{abs, \ pairs}} \approx \frac{Q_{\text{pairs}} \Delta_0'}{v_{\text{L}, 0} n_{\text{pairs}}} \]  

(B77)

where the factor of 2\( R_0 \text{pairs} \) appears because \( R_0 \text{pairs} \) corresponds to the rate of producing a pair unit, while \( n_{\text{pairs}} \) is the total number of pairs separately counting electrons and positrons. \( H_{\text{abs}} = \Delta_0' \) as happens because there is only one depth involved for pairs and photons involved in the absorption depths. The effective free-stream velocity (\( v_{\text{S}} \)) for pairs traversing across field lines over the scale-height \( \Delta_0' \) is determined by the distance per unit time travelled as photons and pairs. The fractional distance travelled as photons is \( L_{\text{pairs}} = c/((R_0 + r_{\text{pairs}}) + R_0 + r_{\text{pairs}}) \) and the fractional distance travelled as pairs is \( L_{\text{pairs}} = v_{\text{L}, \text{pairs}} / (L_{\text{pairs}} + L_{\text{pairs}}) \). The total fractional travel-time is \( T = 1/(R_0 + r_{\text{pairs}}) + 1/((R_0 + r_{\text{pairs}}) + R_0 + r_{\text{pairs}}) \), such that the effective average free-stream velocity is \( v_{\text{L}, \text{pairs}} \sim (L_{\text{pairs}} + L_{\text{pairs}})T \).

Across the field lines \( v_{\text{L}, \text{pairs}} \approx 0 \) due to particle gyrations around magnetic field lines, while parallel to the field lines, \( v_{\text{L}, \text{pairs}} \sim c \sqrt{1 - \gamma_{\text{e}}^2} \) is the electron–positron thermal speed. In addition, along the length of the layer, the pairs can be advected at the speed \( v_{\text{L}} \sim c \). For the temperatures and densities considered, a good approximation is found to be \( v_{\text{L}, \text{pairs}} \approx v_{e} \sim c \), where lower \( c \) corresponds to low temperatures where pairs are not dynamically important. The total opacity is \( \tau_{\text{pairs, tot}} = \tau_{\text{pairs, sca}} + \tau_{\text{pairs, abs}} \). The two-stream approximation gives [equations (B81) and (B82) corrected after online publication on 2011 December 14].

\[ p_{\text{pairs}} = \rho_{\text{pairs}} g(\tau_{\text{pairs}}) \]  

(B78)

\[ u_{\text{pairs}} = h_{\text{pairs}} g(\tau_{\text{pairs}}) \]  

(B79)

\[ n_{\text{pairs}} = n_{\text{pairs}} g(\tau_{\text{pairs}}) \]  

(B80)

\[ F_{\text{pairs}} = v_{\text{L}, \text{pairs}} n_{\text{pairs}} g(\tau_{\text{pairs}}) \]  

(B81)

\[ F_{\text{pairs}} = v_{\text{L}, \text{pairs}} n_{\text{pairs}} g(\tau_{\text{pairs}}) \]  

(B82)

QED effects (e.g. one-photon and modifications to two-photon annihilation) are only important when the pairs are optically thick to both absorption and scattering, so the QED effects (see e.g. Baring 1988; Baring & Harding 1992) on the optically-thin rates need not be considered for the density of pairs.

### B6 Neutrinos

At the highest densities (\( \rho_\nu \gtrsim 10^{10} \text{g cm}^{-3} \)) and temperatures (\( T \gtrsim 10^{10} \text{K} \)) considered, neutrino emission is the dominant source of cooling and pressure. Kirchhoff’s law is used as usual, except because the true number density of neutrinos is not required to be accurate in this work, \( \tau_{\text{nu, abs}} \approx \tau_{\text{nu, abs}} \approx \tau_{\text{nu, abs}} \) is set and only energy density loss rates are considered.

We follow Kohri & Mineshige (2002) (see also Kohri et al. 2005), except their energy density loss rates assume that there exists an optically-thick thermalized photon and pair bath, while in this work the photon and pair densities are reduced by a factor of \( g[\tau_{\gamma}] \) and \( g[\tau_{\text{pairs}}] \), respectively. When photons and pairs are involved in the reaction, a \( g[\tau] \) factor is applied for that given number density (\( \propto T^3 \)) as it enters the original integral, so that these rates are consistent with the number densities of photons and pairs. Without this correction, for example, the pair annihilation rate would be erroneously large at high temperatures in the regime where photons and pairs are optically thin. This assumes, as accurate in this work, that pairs dominate electrons in number density when neutrinos are being produced. The neutrino energy density loss rate for capture of non-degenerate pairs on nucleons is then

\[ Q_{\nu, \text{Ne} \rightarrow} \approx 9.2 \times 10^{53} T_{\text{abs}}^9 (\rho_{\text{p}} X_{\text{nu}}) g[\tau_{\text{pairs}}] \]  

(B83)

and on degenerate pairs is

\[ Q_{\nu, \text{Ne} \rightarrow} \approx 1.1 \times 10^{34} n_e^9 T_{\text{abs}}^9 g[\tau_{\text{pairs}}] \]  

(B84)

where \( Ne \rightarrow \) denotes a sum of processes \( e^+ + n \rightarrow p + \nu_e \) and \( e^- + p \rightarrow n + \nu_e \) and \( \nu_e \rightarrow Ne \) is used to denote the sum of their inverse reactions. The value of \( n_e \) is used to linearly interpolate between regimes, such that \( n_e = 1 \) only the degeneracy expression is used. Neutrino pair production by annihilating pairs gives

\[ Q_{\nu, e^- e^- \rightarrow \gamma \gamma} \approx 4.8 \times 10^{53} T_{\text{abs}}^9 \sigma_{e^- e^-} (1 + \gamma_{\gamma}) \]  

(B85)

where

\[ \gamma_{\gamma} = \frac{2 + \gamma_{\gamma}^2}{1 + \gamma_{\gamma}} \]  

(B87)

The plasmon process gives

\[ Q_{\nu, \text{p} \rightarrow \nu e} \approx 1.5 \times 10^{53} T_{\text{abs}}^9 \sigma_{\text{p} e} (1 + \gamma_{\gamma}) \]  

(B86)

where \( \gamma_{\gamma} = \frac{2 + \gamma_{\gamma}^2}{1 + \gamma_{\gamma}} \) is the total optically-thin neutrino energy density loss rate is

\[ Q_{\nu, \text{v} \rightarrow} = Q_{\nu, \text{Ne} \rightarrow} + Q_{\nu, e^- e^- \rightarrow \gamma \gamma} + Q_{\nu, \text{p} \rightarrow \nu e} + Q_{\nu, \nu e \rightarrow} \]  

(B88)

The neutrinos can be optically thick at sufficiently high densities/temperatures, which is important to include because an artificially high neutrino cooling rate would spuriously lead to compressible solutions for the collisional layer (see Section C3). The electron-type neutrino is treated most accurately among all the neutrino species because it generally dominates the energy density loss rates. Both anti-neutrinos and neutrinos are treated using a single opacity and all neutrino chemical potentials are assumed to be zero. The scattering optical depth is

\[ \tau_{\nu, \text{sc}a} = \frac{1}{\sigma_{\nu, \text{b}} (n_{\nu, \text{b}} \sigma_{\text{b}})} R_1 + n_{\nu, \text{e}} \sigma_C (n_{\nu, \text{e}} \sigma_C') \]  

(B89)

where \( n_{\nu, \text{b}} \sigma_{\text{b}} \) is the effective number density of free baryons, \( n_{\nu, \text{e}} \sigma_{\text{b}} \) is the number density of free baryons, and \( n_{\nu, \text{b}} \sigma_{\text{b}} \) is the effective number density of electrons as computed from equation (B6). Free nucleon scattering has

\[ \sigma_{\nu, \text{b}} = 7.7 \times 10^{-17} \mu_\text{b} (C_{\nu, \text{b}} Y_\text{b} + C_{\nu, \text{c}} Y_\text{c}) T_{11}^2 \]  

(B90)

where \( Y_\text{b} = Y_\text{e} \) and \( Y_\text{c} = 1 - Y_\gamma \); \( C_{\nu, \text{b}} = [4(C_{\nu, \text{b}} - 1)^2 + 5a_\text{b}^2]/24 \) and \( C_{\nu, \text{c}} = (1 + 5a_\text{c}^2)/24 \), with vector coupling \( C_{\nu, \text{c}} = 1/2 + 2 \sin^2 \theta_W \).
\( \alpha_s \approx 1.25 \), and Weinberg angle is \( \sin^2 \theta_W = 0.23 \). For electron–positron scattering

\[
\sigma_{\text{el+e-}} = \sigma_0' \left[ 1 + \frac{n_e}{4} \left( (C_V + C_A)^2 + \frac{1}{3} (C_V - C_A)^2 \right) \frac{k_B T}{m_e c^2} \right]^2,
\]

(B91)

where \( C_A = 1/2 \) for electron neutrinos and \( C_A = -1/2 \) for electron anti-neutrinos, \( \sigma_0' = (3\pi/8)(2700\xi[5]/(7\pi^2) \approx (3/2)\sigma_0 \) and \( \sigma_0 \approx 1.7 \times 10^{-14} \). Because both electron neutrinos are treated using a single opacity, \( C_A = 1/2 \) is set as applicable for the electron neutrino because their emission rate is generally larger or equal to the electron anti-neutrino emission rate when the neutrino chemical potential is zero. The electron–neutrino scattering expression above assumes the neutrino energy is thermal, which is inaccurate in the optically-thin regime. However, this only leads to order unity corrections in our results for the marginally-optically-thick regime.

The absorption optical depth is

\[
\tau_{\nu, \text{abs}} = (\sigma_{\nu-N_e} + \sigma_{\nu-N_p-v-N_n}) n_B R^2_{\nu} + (\sigma_{\nu-N_p+e^++\gamma-N_n}) n_p \Delta s^
u,
\]

(B92)

where baryon reactions and photon-pair reactions have been collected together, and from Kirchoff’s law \( \sigma_{\nu-N_e} = Q_{\nu-N_e}/(n_{\nu} c \mu_{0} v_0) \), \( \sigma_{\nu-N_p-v-N_n} = Q_{\nu-N_p-v-N_n}/(n_{\nu} c \mu_{0} v_0) \), \( \sigma_{\nu-N_p+e^++\gamma-N_n} = Q_{\nu-N_p+e^++\gamma-N_n}/(n_{\nu} c \mu_{0} v_0) \), and \( \sigma_{\nu-N_p+e^-+\gamma-N_n} = Q_{\nu-N_p+e^-+\gamma-N_n}/(n_{\nu} c \mu_{0} v_0) \), where \( n_{\nu} = (7/8)a_\nu T^4 \) includes both neutrinos and anti-neutrinos. Because the emission rate that is used merges both electron and pair-capture processes, a single \( n_{\nu} \) is used instead of each \( n_{\nu} \) and \( n_{\bar{\nu}} \) (for \( Y_e = 1/2 \), this is accurate). Note that \( n_p \) and \( n_T \) cancel out when obtaining the optical depth.

The total neutrino optical depth is \( \tau_{\nu, \text{tot}} = \tau_{\nu, \text{scat}} + \tau_{\nu, \text{abs}} \). As for photons, the two-stream approximation is used to obtain the properties of the neutrinos for general optical depths. The \( \mu \) and \( \tau \) neutrinos have been neglected up to this point because their optically-thin emission rates are smaller than the electron types. However, in the optically-thick regime, all neutrino species have the same densities and pressure when assuming zero neutrino chemical potentials. To approximately capture this simple thermalization effect, note that the absorption and scattering opacities in the optically-thick regime are similar to within an order of magnitude among neutrino species. Thus, the optical depth factors \( g(\tau_\nu) \) and \( h(\tau_\nu) \) are used to interpolate to the optically-thick regime in order to include the \( \mu \) and \( \tau \) neutrinos. The neutrino energy density loss rate is

\[
Q_\nu = \frac{F_\nu}{H_{\nu, \text{abs}}},
\]

(B93)

where \( F_\nu = c \mu_0 h(\tau_\nu)(1 + 2h(\tau_\nu)) \), \( H_{\nu, \text{abs}} \equiv \tau_{\nu, \text{abs}}/(d\tau_{\nu, \text{abs}}/ds) \) and \( d\tau_{\nu, \text{abs}}/ds = n_B (\sigma_{\nu-N_e} + \sigma_{\nu-N_p+e^+v-N_n}) + n_p (\sigma_{\nu-N_p+e^-v-N_n} + \sigma_{\nu-N_p+e^++\gamma-N_n}) Q_{\nu-N_e}/(c_\mu_0 v_0) \). The \( (1 + 2h(\tau_\nu)) \) factor in \( F_\nu \) approximately accounts for \( \mu \) and \( \tau \) neutrinos in the optically-thick regime by modifying the value of \( n_{\nu, \text{scat}} \) to be three times larger as required. The number density, internal energy density and pressure of neutrinos are, respectively,

\[
n_\nu = n_{\nu, \text{scat}} g(\tau_\nu)(1 + 2g(\tau_\nu)),
\]

(B94)

\[
u = n_{\nu, \text{scat}} g(\tau_\nu)(1 + 2g(\tau_\nu)),
\]

(B95)

and

\[
p_\nu = (1/3)\nu c^2,
\]

(B96)

where \( n_{\nu, c} = [45\xi(3/[2\pi^2])/(a_{\nu}/k_B) T^3 \approx (1/3)(a_{\nu}/k_B) T^3 \). The \( (1 + 2g(\tau_\nu)) \) factor approximately accounts for \( \mu \) and \( \tau \) neutrinos in the optically-thick regime. Future work can consider all neutrino species in detail and consider QED effects on the neutrinos that may be important when \( \tau_\nu \gtrsim 1 \).

**APPENDIX C: COLLISIONAL AND COLLISIONLESS RECONNECTION**

The goal of the following sections is to determine the thickness of the collisional current layer. The key point is that collisional reconnection dominates unless dissipation (occurring on the scale of the layer thickness) is dominated by collisionless effects that occur on the scale of the plasma skin depth.

The resistivity \( \eta \) is computed (Section C1) in order to determine the thickness of the collisional current layer (Sections C2 and C3). These calculations assume the background jet’s values of \( b^2 \) and \( \rho_0 \), the current sheet length \( L_0 \), and the sheet-separation length \( \Delta_0 \). Then, collisionless reconnection is discussed (Section C4).

**C1 Classical resistivity**

Consider the collisional resistivity on current-carrying electrons and positrons (with number density \( n_e' \approx n_{e, \text{scat}} \)) as due to photon drag, proton collisions or electron–positron collisions.

The resistivity is presumed to be associated with comoving 4-current \( j = e_{\nu} \gamma_{\nu} q_{\nu} \) with comoving relative electron drift 4-speed \( \gamma_{\nu} q_{\nu} \) and electron charge \( e \). Then, the resistivity \( \eta' = (v_{\nu} m_{\nu})/(n_{\nu, \text{scat}}^2 \gamma_{\nu}) \) can be determined by using the electron drift momentum \( p_x = \gamma_{\nu} q_{\nu} m_{\nu} \), comoving electric field strength \( |E_{\text{col}}| \), effective collisional frequency \( \nu_{\text{eff}} \), interaction force balance \( e|E_{\text{col}}| \approx p_x v_{\nu} \), and Ohm’s law \( \epsilon j = e|E_{\text{col}}| \eta' \). The magnetic diffusivity is then given by \( \eta = \eta' c^2/(4\pi n_e) \), which can also be written as \( \eta = d^2 v_{\nu} \); where \( d_x = c/\omega_{\nu} \) is the skin depth of current-carrying electrons and positrons, \( \omega_{\nu, \text{rel}} = \sqrt{4\pi n_e e^2/m_e} \) is the current-carrying electron–positron non-relativistic plasma frequency, \( \omega_{\nu, \text{col}} = \omega_{\nu, \text{rel}} G_{\text{coll}}^{(0)}(\mu^2/4\pi^2/1,1,3/2)/[2K_3(\mu)] \) is the associated relativistic plasma frequency, \( \mu = m_e c^2/(k_B T_e) \), \( G(\cdot) \) is the Meijer G function and \( K_3(\cdot) \) is the modified Bessel function of the second kind (Bergman & Elasson 2001).

In addition to collisions between charged particles, current-carrying electrons and positrons with drift momentum \( p_x = \gamma_{\nu} q_{\nu} m_{\nu} \) experience a photon radiative drag force \( F_x = -4/(3\pi)(\gamma_{\nu} q_{\nu} c^2 n_{e, \text{scat}} v_{\nu}) \), where \( F_x = -|E_{\text{col}}| \). This gives an effective collisional frequency of \( v_{\nu} \approx F_x/p_x \). From the magnetic diffusivity of \( \eta = d^2 v_{\nu} \), one obtains the resistivity due to photon drag of \( \eta_{\nu} = (4/3)m_e^2 (u_\nu, \mathcal{C} / (m_e c^2) \)

(Goodman & Udzenisky 2008).

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8 For the regimes of interest in this paper, generalized Ohm’s law shows that the current rise time for pairs (\( \tau_{\text{rise}} \approx \sqrt{\eta' / \epsilon} \)) is not longer than the current pair annihilation time (\( \tau_{\text{pair}} \approx (n_{\nu, \text{pair}} / m_e c^2)^{-1} \)) such that pairs always contribute to the current. Protons are assumed to contribute negligibly to the current density, which is accurate for the regimes considered.

9 The collisionless resistivity computed here assumes all particles are non-degenerate, but the degeneracy of nucleons and electron–positrons is computed and the fully degenerate electron–proton resistivity code by Poteckin et al. (1999) was used to check how electron–proton collisions are affected. Our work’s results end up not depending upon electron–proton or pair–proton collisions, so the degeneracy effects (such as discussed in Rossi, Armitage & Menou 2008 for electron–proton collisions) can be neglected in this study. Computing radiative drag and pair drag on the degenerate current-carrying electrons and positrons is left for future work.
The Coulomb resistivity for current-carrying electrons and positrons interacting with protons, electrons or positrons is determined by the mean free path given by \( \lambda_{\text{Coul}} \approx 1/(n_{\text{pairs}} + \eta_\gamma) \), where in general \( n_{\text{pairs}} + n_\gamma \geq n_\gamma \), so that pairs contribute an extra opacity that sets a lower limit on the mean free path. The use of \( n_{\text{pairs}} + n_\gamma \) estimates the fact that the electron–positron pair bath and ion bath contribute (to within factors of order unity) the same to the per-particle Coulomb collision on the current-carrying electrons and positrons. For an effective collisional rate as given above \( (\nu_v) \), the magnetic diffusivity is then \( \eta = (n_{\text{pairs}} + n_\gamma)/n_e (\gamma \sigma m_e c^2)/(e^2 4\pi \tau_\gamma) \) if each scatter is effective. If \( n_{\text{pairs}} \to 0 \) or \( n_\gamma \sim n_{\text{pairs}} \gg n_\gamma \), then \( \eta = (\gamma \sigma m_e c^2)/(e^2 4\pi \tau_\gamma) \), which is independent of the density of current-carrying electrons and positrons. In order to determine the resistivity, the cross-section (\( \sigma \)) must be determined.

Coulomb collisions have naive cross-section of \( \sigma_0 = \pi \lambda_C^2 \), where \( \lambda_C \approx \sqrt{\gamma/(16\pi)} e^2/KE \) is the length-scale over which Coulomb forces are important, where the kinetic energy of electrons in the centre-of-momentum frame is given by \( KE \approx (m_e \gamma_0 v_\text{th,e})^2 + m_e c^2 - m_e c^2 \), where \( v_\text{th,e} \approx \gamma_0 \approx \gamma_0 \approx \gamma_{\text{th}} \approx c \gamma_0 \left( 2 + \gamma_0 \right)/(1 + \gamma_0) \) is the thermal speed of electrons, \( \gamma_0 = 1/\sqrt{1 -(v_\text{th,e}/c)^2} \) and \( \gamma_0 \approx (v_\text{th,e}/m_e c^2). \) There are additional relativistic corrections to \( \lambda_C \) of the order of unity not considered here (see e.g. McKinley & Feshbach 1948). Also note that there are QED corrections that suppress the electron–electron cross-section, but there is only weak suppression of electron–proton interactions that then dominate the Coulomb cross-section (Storey & Melrose 1987; Sadooghi & Jalili 2007). For weakly coupled plasmas, the Debye screening gives a corrected Coulomb cross-section of

\[
\sigma_C \approx \sigma_0 \ln \Lambda, \tag{C2}
\]

where \( \ln \Lambda \) is the Coulomb logarithm and \( \lambda_D \approx v_\text{th,e}/m_e c \) is the Debye length.

The classical resistivity for current-carrying electrons and positrons interacting with protons, electrons and positrons, which accounts for Debye screening, gives the Spitzer resistivity of \( \eta_s \approx d^2_\gamma v_\gamma \). While the relativistic expression is used, in the non-relativistic limit, this becomes simply

\[
\eta_s \approx \left( \frac{5 \sqrt{3}}{32 \pi} \right) c \sigma_0 (\theta_\gamma)^{-3/2} \ln \Lambda, \tag{C3}
\]

where \( \theta_\gamma = (k_B T)/\left(m_e c^2 \right) \) is the dimensionless electron temperature and \( r_\gamma = e^2/(m_e c^2) \) is the classical electron radius.

The total resistivity is then taken to be

\[
\eta = \eta_s + \eta_\gamma, \tag{C4}
\]

which is a simple sum because the resistivities act on the same current-carrying electrons and positrons. In most of parameter space for the GRB jets considered, photon drag dominates the Spitzer resistivity. Note that neutrino drag is generally negligible.

**C2 Reconnection models**

Given the length \( L_\text{C} \) of the current layer (as determined in Section 5.1), one can derive two classes of solutions for magnetic dissipation: the slow Sweet–Parker type reconnection (Sweet 1958; Parker 1957, 1963a) and the fast Petschek-type reconnection (Petschek 1964). Which of these two modes of reconnection is ultimately realized will be determined by the plasma collisionality as discussed in Section 5.3.

As shown in Fig. C1, the Sweet–Parker regime corresponds to an elongated reconnection layer. Plasma is forced into a confined space before being ejected out along a geometrically narrow channel, which causes the reconnection rate to be slow.\(^{10}\)

The Petschek model uses the fact that topological changes in the electromagnetic field do not have to take place in the same region where most of dissipation occurs. The Petschek model has a very small inner Sweet–Parker like layer (called the central diffusion region) at the X-point where topological changes occur, but the current layer generates slow-mode shocks that dissipate most of the electromagnetic energy and deflect the incoming plasma into two relatively wide exhaust channels. The ejected plasma therefore has a large geometrical exit channel allowing faster ingoing plasma and so faster reconnection. Petschek reconnection occurs with an asymptotic reconnection speed that typically relaxes to \( v_\gamma/v_\Lambda \sim 1/\ln S \) (including in the relativistic regime; Lyubarsky 2005). For a Lundquist number \( S \gg 1 \), the Petschek reconnection rate is \( v_\gamma/v_\Lambda \approx \pi/[4 \ln(5)] \), giving \( v_\gamma \sim 0.018c \) for \( S = 10^{22} \) for a highly magnetized jet (Lyubarsky 2005).

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10 Many effects are neglected, such as secondary instabilities (Dahlburg, Klimchuk & Antiochos 2003, 2005), two-dimensional tearing instability (Loureiro et al. 2007; Jaroschek & Hoshino 2009) (but a temperature-dependent resistivity, non-uniform flow along the layer, and pressure anisotropy can stabilize the tearing mode; Biskamp 1986a; Priest & Forbes 2000), interchange instability (Uchida & Sakurai 1977), resistive kink mode (Schnack & Killeen 1979), an anisotropic Spitzer resistivity, realistic thermal conduction, and viscosity.
C3 Collisional reconnection

Collisional reconnection is assumed to be approximately like the Sweet–Parker model of a stationary dissipating current sheet. Lyubarsky (2005) found that the relativistic Sweet–Parker reconnection rate, layer thickness and ejection velocity all behave as expected from non-relativistic Sweet–Parker theory. For Sweet–Parker layer extent $L_0$, the Sweet–Parker layer has thickness

$$\delta_{SP} \sim \sqrt{\frac{L_0 \eta}{v_A}} = L_0 S^{-1/2},$$

(C5)

which is associated with a reconnection velocity of

$$v_{r,SP} \sim \frac{\eta}{\delta_{SP}} = v_A S^{-1/2},$$

(C6)

and an ejection velocity of $u \sim v_A$, where we recall that $S = L_0 v_A/\eta$ is the Lundquist number. The reconnection time-scale is $\tau_{SP} \sim \tau_A \sqrt{S}$, where $\tau_A = L_0/v_A$ is the fluid transit flow time along the length of the layer. For very large $S$, the Sweet–Parker solution may be unstable to plasmoid formation (see e.g. Uzdensky et al. 2010).

Uzdensky & McKinney (2011) found that one can readily extend the non-relativistic Sweet–Parker theory to include radiative cooling. The results in their Section 2 apply for arbitrary optical depths. They obtained such results by using the energy equation rather than the incompressible assumption as done in Lyubarsky (2005). An interesting point is that the effects of cooling decouple from the rest of equations, which allows one to obtain the Sweet–Parker type result from pressure equilibrium as usual and then independently obtain the constraint from radiative cooling conditions that determine a baryon compression ratio, $A = n_b/n_{b,0}$, that is, the ratio of plasma density in the centre of the layer, $n_b$, to the plasma density in the upstream region, $n_{b,0}$.

The solution that Uzdensky & McKinney (2011) obtained for general optical depths ultimately involves two equations: pressure equilibrium across the layer and radiative energy balance across the layer. While a global jet-layers structure is not sought in this paper, a global solution would show that radiation pressure, like the plasma density, would have its own structure across the jet. This justifies balancing the total gas radiation pressure of the complex against the electromagnetic pressure of the jet.

To obtain the radiative Sweet–Parker solution, first, one imposes the pressure balance condition

$$p_{EM} \sim \rho_g(T),$$

(C7)

where $p_{EM} = b^2/(8\pi\gamma)$ is the electromagnetic pressure. This equation corresponds to a loss of magnetic pressure within the layer that is recovered by a balanced thermal gas pressure.

Secondly, the energy equation determines the effects of radiative transport (Uzdensky & McKinney 2011). The result is that one sets the compression ratio of the rest-mass density within the current layer of size $\delta_{SP}$ (given below) to be

$$A = \frac{Q_g(T)}{Q_{SP}}.$$

(C8)

where recall that $Q_g(T)$ is the gas energy volume loss rate and

$$Q_{SP} = \frac{b^2}{4\pi L_0} \frac{v_A}{c},$$

(C9)

where $v_A$ is the usual upstream Alfvén velocity (Uzdensky & McKinney 2011). The quantity $Q_{SP}$ is the energy density dissipation rate if the layer corresponds to the standard thin Sweet–Parker solution with thickness $\delta_{SP}$. Note that $A$ does not depend upon the resistivity. Equation (C8) applies only in the strong cooling regime, that is, if $A \gg 1$. If $A < 1$, then one sets $A = 1$ corresponding to the weak cooling limit, giving back the original non-radiative incompressible Sweet–Parker solution. This $A$ factor is applied to all baryon densities ($n_b$) and baryon-associated electron densities ($n_e$), except when used in opacity integrals because the total baryonic mass across the jet is conserved regardless of localized compression in each thin current sheet. For example, there are no changes to the opacity integrals over baryonic-associated mass when performed for equations (B27), (B31), (B89) and (B92).\footnote{Because the free nucleon fraction depends upon density and not column density, for a pre-collapse baryon density $n_b$, one should integrate to get $r \approx A X_{nuc} [A_{nuc} b + X_{nuc} (n_b) R_j]$. However, because generally $R_j \gg A$ is found, this is a negligible effect and $r \approx X_{nuc} [A_{nuc} R_j]$ is set as usual.}

Once the energy density loss rate ($Q_g$) is comparable to (or larger than) the energy dissipation rate ($Q_{SP}$), the layer must compress and undergo more rapid reconnection in order to balance the radiative losses. The reconnection velocity and current layer thickness are, respectively, given by

$$v_{r,SP} \sim v_{r,SP} A^{1/2},$$

(C10)

$$\delta_{SP} \sim \delta_{SP} A^{-1/2},$$

(C11)

(Uzdensky & McKinney 2011). This is only an approximation of a so-far undeveloped fully relativistic radiation reconnection theory. However, if $A = 1$, then the radiation only contributes to the pressure and internal energy, and the original non-relativistic Sweet–Parker solution should be accurate even in the relativistic regime (Lyubarsky 2005). In most of the parameter space considered in this study, $A \sim 1$ is found to hold. This implies that the usual Sweet–Parker solution is generally valid to within factors of order unity.

The time-rate of change of connected magnetic flux is

$$\frac{\partial \Phi}{\partial t} = -E_z = |v_B/c|,$$

(C12)

where $B_\| \sim |b|$ (the comoving electromagnetic field strength in the jet). In the non-relativistic incompressible limit, this gives a time rate of change of magnetic flux of $\partial \Phi/(\partial z\partial t) \sim |v_b S^{-1/2}| c |b|/c \ll v_A \delta_{SP}/c$, which implies a slow reconnection rate compared to Alfvén time-scales. The reconnection time-scale is given by the time needed to reconnect a finite amount of flux of the order of $|b| \delta_{SP}$ present in the volume between multiple current sheets. This gives $\tau_c \sim (|b| \delta_{SP})/(\partial \Phi/\partial t) \sim (\delta_{SP} v_A L_0 S)$, which is much longer than it takes an Alfvén wave to cross $\delta_{SP}$.

In the relativistic regime, the inflow speed may reach $v \sim v_A \sim c$ and significant Lorentz contraction might occur when there is a significant loss of energy through radiation or if the Lundquist number $S$ is smaller than the comoving magnetization $b^2/(8\pi \gamma n_e c^2)$ (Lyutikov & Uzdensky 2003). However, when including both the energy and momentum equations (Lyubarsky 2005), this is found not to occur in the Sweet–Parker regime. This validates the assumptions used in this work. Relativistic reconnection is an active area of research (Watanabe & Yokoyama 2006; Hesse & Zenitani 2007; Komissarov, Barkov & Lyutikov 2007; Zenitani & Hoshino 2007; Zenitani & Hesse 2008; Zenitani, Hesse & Klimas 2009; Tenbarge et al. 2010), so aspects of this work may require modification.

C4 Collisionless reconnection

Sweet–Parker reconnection is slow because plasma has to flow through a narrow channel. Microphysics (e.g. anomalous resistivity
or the Hall effect) alone cannot enhance the reconnection rate if the current layer is preserved in the Sweet–Parker configuration. For example, as the current layer thickness $\delta \rightarrow d_i$, Hall effects dominate the resistivity. The Sweet–Parker analysis yields a reconnection speed of $v_r \sim (d_i/L_0) v_A$, which still gives $v_r \ll v_A$ simply because of the assumed Sweet–Parker geometry. However, if a transition to collisionless reconnection occurs as discussed in Section 5.3, then the Sweet–Parker geometry can be disrupted into a Petschek-like geometry leading to fast reconnection.

One candidate for fast collisionless reconnection is the Hall effect. In generalized Ohm’s law, the Hall term is $j \times B / (n_e e c)$ operating on the ion skin depth where electrons and ions decouple. The Hall effect involves a two-fluid laminar reconnection configuration with $v_i \leq 0.1 v_A$ (Mandt, Denton & Drake 1994; Shay et al. 1998; Bhattacharjee, Ma & Wang 2001; Birn et al. 2001; Cassak et al. 2005; Daughton et al. 2006; Yamada et al. 2006).

Another candidate is an anomalous resistivity, which can be due to plasma microinstabilities (Parker 1963b; Hasegawa 1975; Coroniti & Eviatar 1977; Tsuda & Ugai 1977; Parker 1979; Syrovatskii 1981; Biskamp 1986b; Melrose 1986; Taylor 1986; Begelman & Chiueh 1988; Parker 1988b; Scholer 1989; Masuda et al. 1994; Kulsrud 1998; Biskamp & Schwarz 2001; Erkaev et al. 2001; Kulsrud 2001; Malyshkin et al. 2005). Anomalous resistivity can be triggered, for example, when the drift velocity ($v_d$) exceeds some critical velocity ($v_c$), such that $v_d = j/(e n_e c) > v_c \sim v_{th}$. This process can drive plasma instabilities developing microturbulence where scattering of electrons by waves enhances resistivity. As the layer’s thickness $\delta$ decreases down to the critical thickness $\delta_c = c B / (4 \pi j_e c)$, where $j_e = e n_e \gamma_{th} v_{th}$, anomalous resistivity turns on (this scale is usually comparable to $d_i$). Anomalous resistivity allows for higher resistivity than Spitzer, but it also enables the Petschek-like geometry (see e.g. Biskamp & Schwarz 2001; Kulsrud 2001; Yamada et al. 2006; Zweibel & Yamada 2009).

Overall, collisionless Petschek-type reconnection operates on the ion skin depth

$$d_i = c / \omega_{pi}, \quad (C13)$$

where $\omega_{pi}$ is the ion plasma frequency. The ejection velocity is the same as in the Sweet–Parker case with velocity of the order of $v_A$.

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