Interactions from SSB of scale symmetry: applications to problems of quintessence, galaxy dark matter and fermion family

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We study a scale invariant two measures theory where a dilaton field \( \phi \) has no explicit potentials. The scale transformations include a translation of a dilaton \( \phi \rightarrow \phi + \text{const} \). The theory demonstrates a new mechanism for generation of the exponential potential: in the conformal Einstein frame (CEF), after SSB of scale invariance, the theory develops the exponential potential and, in general, non-linear kinetic term is generated as well. The scale symmetry does not allow the appearance of terms breaking the exponential shape of the potential that solves the problem of the flatness of the scalar field potential in the context of quintessential scenarios. As examples, three different sets of the magnitudes of the parameters and integration constants (SPaIC) are presented where the theory permits to get interesting cosmological and astrophysical results in the analytic form. For one SPaIC, the theory has standard scaling solutions for \( \phi \) usually used in the context of the quintessential scenario. For the second SPaIC, the theory has scaling solutions with equation of state \( p_\phi = w \rho_\phi \) where \( w \) is predicted to be restricted by \(-1 < w < -3/8\). For the third SPaIC, the theory allows a static spherically symmetric particles in CEF that appears to suggest a new approach to the family problem of particle physics. It is automatically achieved that for two of them, fermion masses are constants, gravitational equations are canonical and the "fifth force" is absent. For the third type of particles, four fermionic interaction appears from SSB of scale invariance.

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I. INTRODUCTION

Recent observations imply that the Universe now is undergoing era of acceleration [1]. This is most naturally explained by the existence of a vacuum energy which can be of the form of an explicit cosmological constant. Alternatively, there may be a slow rolling scalar field, whose potential (assumed to have zero asymptotic value) provides the negative pressure required for accelerating the Universe. This is the basic idea of the quintessence [2]. Some of the problems of the quintessence scenario connected to the field theoretic grounds of this idea, are: i) what is the origin of the quintessence potential; ii) why the asymptotic value of the potential vanishes (this is actually the "old" cosmological constant problem [1]) iii) the needed flatness of the potential [3]. iv) without the symmetry φ → φ + const it is very hard to explain the absence of the long-range force if no fine tuning is made [4]. But such a translation-like symmetry is usually incompatible with a nontrivial potential.

One of the main aims of this paper is to show how the above problems can be solved in the context of the two measures theories (TMT) [9–12]. These kind of models are based on the observation that in a generally covariant formulation of the action principle one has to integrate using an invariant volume element, which is not obliged to be dependent of the metric. In GR, the volume element $\sqrt{-g}d^4x$ is indeed generally coordinate invariant, but nothing forbids us from considering the invariant volume element $\Phi d^4x$ where $\Phi$ is a scalar density that could be independent of the metric [5].

If the measure $\Phi$ is allowed, we have seen in a number of models [1] that, in the conformal Einstein frame (CEF), the equations of motion have the canonical GR structure, but the scalar field potential produced in the CEF is such that zero vacuum energy for the ground state of the theory is obtained without fine tuning, that is the "old" cosmological constant problem can be solved [11].

If two measures are allowed, this opens new possibilities concerning scale invariance [12–13]. In this context we study here a theory which is invariant under scale transformations including also a translation-like symmetry for a dilaton field of the form thought by Carroll [6]. For the case when the original action does not contain dilaton potentials at all, it is found that the integration of the equation of motion corresponding to the measure $\Phi$ degrees of freedom, spontaneously breaks the scale symmetry and the generation of a dilaton potential is a consequence of this spontaneous symmetry breaking (SSB). When studying the theory in the CEF, it is demonstrated in Sec. III that the spontaneously induced dilaton potential has the form of the exponential one and in addition, also non-linear kinetic terms appear.

In Secs. IV and V, we discuss possible applications of the theory to cosmological and astrophysical questions when the dilaton field is the dominant fraction of the matter: it is found that quintessential solutions are possible and, for a different region of parameter space, halo dark matter solutions are also possible.

In Sec. VI we show that in the presence of fermions, the theory displays a successful fermionic mass generation after the SSB, and this is actually the second main aim of this paper. In the regime when the fermionic density is of the order typical for the normal particle physics (which in the laboratory conditions is always much higher than the dilaton density ), there are constant fermion masses, gravitational equations are canonical and the "fifth force" is absent, - all this without any additional restrictions on the parameters of the theory. A possible explanation to the "family puzzle" of particle physics also appears naturally in the context of this model. For one of the families, a quartic fermion interaction appears as a result of the SSB of scale symmetry.

II. TWO MEASURES THEORY (TMT)

The main idea of these kind of theories [9] is to reconsider the basic structure of generally relativistic actions, which are usually taken to be of the form

$$ S = \int d^4x \sqrt{-g} L $$

where $L$ is a scalar and $g = \det(g_{\mu\nu})$. The volume element $d^4x \sqrt{-g}$ is an invariant entity. It is however possible to build a different invariant volume element if another density, that is an object having the same transformation properties as $\sqrt{-g}$, is introduced. For example, given four scalar fields $\varphi_a$, $a = 1, 2, 3, 4$ we can build the density

$$ \Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d $$

and then $\Phi d^4x$ is also an invariant object. Notice also that $\Phi$ is a total derivative since

$$ \Phi = \partial_\mu (\varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d) $$

\[2\]
Therefore if we consider possible actions which use both $\Phi$ and $\sqrt{-g}$ we are lead to TMT

$$S = \int L_1 \Phi d^4 x + \int L_2 \sqrt{-g} d^4 x$$  \hspace{1cm} (4)$$

Since $\Phi$ is a total derivative, we see that a shift of $L_1$ by a constant, $L_1 \rightarrow L_1 + \text{const}$, has the effect of adding to $S$ the integral of a total derivative, which does not change equations of motion. Such a feature is not showed by the second piece of Eq. (4) since $\sqrt{-g}$ is not a total derivative. It is clear then that the introduction of a new volume element has consequences on the way we think about the cosmological constant problem, since the vacuum energy is related to the coupling of the volume element with the Lagrangian. How this relation is modified when a new volume element is introduced, was discussed in [12,13].

It has been shown that a wide class of TMT models [11], containing among others a scalar field, can be formulated which are free of the "old" cosmological constant problem. An important feature of those models consists in the use of the "first order formalism" where the connection coefficients $\Gamma^\lambda_{\mu\nu}$, metric $g_{\mu\nu}$, and in our case also $\varphi_a$ and any matter fields that may exist are treated as independent dynamical variables. Any relations that they satisfy are a result of the equations of motion. The models allow the use of the so called conformal Einstein frame (CEF) where the equations of motion have canonical GR form and the effective potential has an absolute minimum at zero value of the effective energy density without fine tuning. This was verified to be the case in all examples studied in Ref. [11], provided the action form [1] is preserved, where $L_1$ and $L_2$ are $\varphi_a$-independent. If this is so, an infinite symmetry appears [11]: $\varphi_a \rightarrow \varphi_a + f_a(L_1)$, where $f_a(L_1)$ is an arbitrary function of $L_1$.

### III. SCALE INVARIANT MODEL WITH SPONTANEOUS SYMMETRY BREAKING

**GIVING RISE TO A POTENTIAL**

If we believe that there are no fundamental scales in physics, we are lead to the notion of scale invariance. In the context of TMT, to implement global scale invariance one has to introduce a "dilaton" field [12,13]. In this case the measure $\Phi$ is used, provided $\Phi$ degrees of freedom also can participate in the scale transformation [12,13]. In [12,13], explicit potentials (of exponential form) which respect the symmetry were introduced. Fundamental theories however, like string theories, etc. give most naturally only massless particles, which means that on ly kinetic terms and no explicit potentials appear [12,13]. In [12,13], explicit potentials (of exponential form) which respect the symmetry were introduced. Fundamental theories however, like string theories, etc. give most naturally only massless particles, which means that only kinetic terms and no explicit potentials appear from the beginning naturally. Let us therefore explore a similar situation in the context of a scale invariant TMT model. We postulate then the form of the action

$$S = \int d^4 x \Phi e^{\alpha\phi/M_p} \left[ -\frac{1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} \right] + \int d^4 x \sqrt{-g} e^{\alpha\phi/M_p} \left[ -\frac{b_0}{\kappa} R(\Gamma, g) + \frac{b_k}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} \right]$$  \hspace{1cm} (5)$$

where we proceed in the first order formalism and $R(\Gamma, g) = g^{\mu\nu} R_{\mu\nu}(\Gamma)$, $R_{\mu\nu}(\Gamma) = R_{\mu\nu\alpha}(\Gamma)$ and $R_{\mu\nu\sigma}(\Gamma) \equiv \Gamma^\lambda_{\mu\sigma} \Gamma^\mu_{\lambda\nu} - (\nu \leftrightarrow \sigma)$. By means of a redefinition of factors of $\phi$ and of $\Phi$ one can always normalize the kinetic term of $\phi$ and the $R$-term that go together with $\Phi$ as done in [3]. Once this is done, this freedom however is not present any more concerning the second part of the action going together with $\sqrt{-g}$. The appearance of the constants $b_0$ and $b_k$ is a result of this. Concerning the possible magnitudes of $b_0$ and $b_k$ we will here assume only that they are positive.

The action (5) is invariant under the scale transformations:

$$g_{\mu\nu} \rightarrow e^\theta g_{\mu\nu}, \quad \phi \rightarrow \phi \frac{M_p}{\alpha} \theta, \quad \Gamma^\nu_{\mu\sigma} \rightarrow \Gamma^\nu_{\mu\sigma}, \quad \text{and} \quad \varphi_a \rightarrow \lambda_a \varphi_a \quad \text{where} \quad \Pi \lambda_a = e^{2\theta}.$$  \hspace{1cm} (6)$$

Notice that (5) is the most general action of TMT invariant under the scale transformations (3) where the Lagrangian densities $L_1$ and $L_2$ are linear in the scalar curvature and quadratic in the space-time derivatives of the dilaton but without explicit potentials. In Refs. [12,13], actions of such type were discussed, but with explicit potentials and without kinetic term going with $\sqrt{-g}$. A different definition of the metric have been used also in Ref. [12,13] ($g^{\mu\nu}$ in Ref. [12,13] instead of the combination $e^{\alpha\phi/M_p} g^{\mu\nu}$ here) so that no factor $e^{\alpha\phi/M_p}$ appeared multiplying $\Phi$ in Ref. [12,13]. Also it is possible to formulate a consistent scale invariant model keeping only the simplest structure (namely, only the measure $\Phi$ is used), provided $L_1$ contains 4-index field strengths and an exponential potential for the dilaton [4]. Then SSB of the scale invariance can lead to a quintessential potential [4]. Another type of the field theory models with explicitly broken scale symmetry have been studied in Ref. [13] where it is shown that the quintessential inflation [10] type models can be obtained without fine tuning.
We examine now the equations of motion that arise from \( S_c \). Varying the measure fields \( \varphi_a \), we get
\[
A^\mu_\nu \partial_\mu \left[ -\frac{1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\alpha \beta} \partial_\alpha \varphi_{\beta \varphi} \right] = 0 \tag{7}
\]
\[
A^\mu_\nu = \varepsilon^{\mu_\nu \alpha \beta} \varepsilon_{\alpha \beta \varphi_\lambda} \partial_\varphi \varphi_\lambda \partial_\varphi \varphi_\varphi \partial_\varphi \varphi_\varphi. \tag{8}
\]
Since \( \text{Det}(A^\mu_\nu) = \frac{4\pi^2}{\kappa^3} \Phi^3 \) it follows that if \( \Phi \neq 0 \),
\[
-\frac{1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu \nu} \partial_\mu \partial_\nu = sM^4 = \text{const},
\]
where \( s = \pm 1 \) and \( M \) is a constant of the dimension of mass. It can be noticed that the appearance of a nonzero integration constant \( sM^4 \) spontaneously breaks the scale invariance \( \Phi \).

The variation of \( S \) with respect to \( g^{\mu \nu} \) yields
\[
-\frac{1}{\kappa} R_{\mu \nu}(\Gamma)(\Phi + b_g \sqrt{-g}) + \frac{1}{2} \phi_{\mu \nu}(\Phi + b_k \sqrt{-g}) - \frac{1}{2} \sqrt{-gg_{\mu \nu}} \left[ -\frac{b_g}{\kappa} R(\Gamma, g) + \frac{b_k}{2} g^{\alpha \beta} \partial_\alpha \varphi_{\beta \varphi} \right] = 0 \tag{10}
\]
Contracting Eq. (10) with \( g_{\mu \nu} \), solving for \( R(\Gamma, g) \) and inserting into Eq. (9) we obtain the constraint
\[
M^4(\zeta - b_g) e^{-\alpha \phi/M_p} + \frac{\Delta}{2} g^{\alpha \beta} \partial_\alpha \varphi_{\beta \varphi} = 0, \tag{11}
\]
where the scalar \( \zeta \) is defined as
\[
\zeta \equiv \frac{\Phi}{\sqrt{-g}} \tag{12}
\]
and \( \Delta = b_g - b_k \).

Varying the action with respect to \( \phi \) and using Eq. (9) we get
\[
(-g)^{-1/2} \partial_\mu \left[ (\zeta + b_k) e^{\alpha \phi/M_p} \sqrt{-gg^{\mu \nu}} \partial_\nu \phi \right] - \frac{\alpha}{M_p} \left[ M^4(\zeta + b_g) - \frac{\Delta}{2} g^{\alpha \beta} \partial_\alpha \varphi_{\beta \varphi} e^{\alpha \phi/M_p} \right] = 0 \tag{13}
\]
Considering the term containing connection \( \Gamma^\lambda_{\mu \nu} \), that is \( R(\Gamma, g) \), we see that it can be written as
\[
S_\Gamma = -\frac{1}{\kappa} \int \sqrt{-g} e^{\alpha \phi/M_p} (\zeta + b_g) g^{\mu \nu} R_{\mu \nu}(\Gamma) = -\frac{1}{\kappa} \int \sqrt{-gg^{\mu \nu}} R_{\mu \nu}(\Gamma), \tag{14}
\]
where \( \tilde{g}_{\mu \nu} \) is determined by the conformal transformation
\[
\tilde{g}_{\mu \nu} = e^{\alpha \phi/M_p} (\zeta + b_g) g_{\mu \nu} \tag{15}
\]
It is clear then that the variation of \( S_\Gamma \) with respect to \( \Gamma \) will give the same result expressed in terms of \( \tilde{g}_{\mu \nu} \) as in the similar GR problem in Palatini formulation. Therefore, if \( \Gamma^\lambda_{\mu \nu} \) is taken to be symmetric in \( \mu, \nu \), then in terms of the metric \( \tilde{g}_{\mu \nu} \), the connection coefficients \( \Gamma^\lambda_{\mu \nu} \) are Christoffel’s connection coefficients of the Riemannian space-time with the metric \( \tilde{g}_{\mu \nu} \):
\[
\Gamma^\lambda_{\mu \nu} = \{ \lambda_{\mu \nu} \}_{\tilde{g}_{\mu \nu}} = \frac{1}{2} \tilde{g}^{\lambda \alpha} (\partial_\alpha \tilde{g}_{\mu \nu} + \partial_\mu \tilde{g}_{\alpha \nu} - \partial_\alpha \tilde{g}_{\mu \nu}). \tag{16}
\]
So, it appears that working with \( \tilde{g}_{\mu \nu} \), we recover a Riemannian structure for space-time. We will refer to this as the conformal Einstein frame (CEF). Notice that \( \tilde{g}_{\mu \nu} \) is invariant under the scale transformations \( \tilde{g}_{\mu \nu} \) and therefore the spontaneous breaking of the global scale symmetry (see Eq. (9) and discussion after it) is reduced, in CEF, to the spontaneous breaking of the shift symmetry \( \phi \rightarrow \phi + \text{const} \) for the dilaton field. In this context, it is interesting to notice that Carroll [4] pointed to the possible role of the shift symmetry for a scalar field in the resolution of the long range force problem of the quintessential scenario.

Equations (11) and (12) in CEF take the following form:
\[ G_{\mu
u}(\bar{g}_{\alpha\beta}) = \frac{\kappa}{2} T_{\mu
u}^{\text{eff}} \quad (17) \]

\[ T_{\mu
u}^{\text{eff}} = \frac{1}{2} \left( 1 + \frac{b_k}{b_g} \right) (\phi_{,\mu}\phi_{,\nu} - K\bar{g}_{\mu\nu}) - \frac{\Delta^2 K e^{2\alpha\phi/M_p}}{2b_g M^4} \left( \phi_{,\mu}\phi_{,\nu} - \frac{1}{2} K\bar{g}_{\mu\nu} \right) + \bar{g}_{\mu\nu} \frac{s M^4}{4 b_g} e^{-2\alpha\phi/M_p} \quad (18) \]

\[ \left( b_g + b_k - \frac{\Delta^2}{M^4} K e^{2\alpha\phi/M_p} \right) \left[ \left( -\bar{g} \right)^{-1/2} \partial_{\mu} (\sqrt{-g} g^{\mu\nu} \partial_{\nu} \phi) + \tilde{g}^{\alpha\beta} \partial_{\alpha} \phi \partial_{\beta} \ln \left( \frac{1}{2} \left( 1 + \frac{b_k}{b_g} \right) - \frac{\Delta^2}{2b_g M^4} K e^{2\alpha\phi/M_p} \right) \right] + \frac{\alpha \Delta^2}{M_p M^4} K e^{2\alpha\phi/M_p} - \frac{\alpha M^4}{M_p} e^{-2\alpha\phi/M_p} = 0 \quad (19) \]

Here \( K \equiv \frac{1}{2} \delta^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} \), \( G_{\mu\nu}(\bar{g}_{\alpha\beta}) \) is the Einstein tensor in the Riemannian space-time with metric \( \bar{g}_{\mu\nu} \) and the constraint (11) have been used which in CEF takes the form

\[ \zeta = b_g \frac{M^4 - \Delta K e^{2\alpha\phi/M_p}}{M^4 + \Delta K e^{2\alpha\phi/M_p}} \quad (20) \]

Notice that in \( T_{\mu\nu}^{\text{eff}} \) we can recognize an effective potential

\[ V_{\text{eff}} = \frac{s M^4}{4 b_g} e^{-2\alpha\phi/M_p} \quad (21) \]

which appears in spite of the fact that no explicit potential term was introduced in the original action (5). As we see, the existence of \( V_{\text{eff}} \) is associated with the constant \( s M^4 \), appearance of which spontaneously breaks the scale invariance. This is actually a new mechanism for generating the exponential potential [5].

Notice also that if \( b_g \neq b_k \), the effective energy-momentum \( T_{\mu\nu}^{\text{eff}} \) as well as the dilaton equation of motion contain the non-canonical terms nonlinear in gradients of the dilaton \( \phi \). It will be very important that the non-canonical in \( \phi_{,\alpha} \) terms are multiplied by a very specific exponential of \( \phi \). As we will see, these non-canonical terms may be responsible for the most interesting scaling solutions.

In the context of FRW cosmology, this structure provides conditions for quintessential solutions if \( s = 1 \). In the case of static solutions, it garantees the existence of solutions which could play the role of the halo dark matter of galaxies provided that \( s = -1 \).

**IV. SCALING SOLUTIONS**

In the context of a spatially flat FRW cosmology with a metric \( ds_{\text{eff}}^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \), the equations (17)-(19), with the choice \( s = +1 \), become:

\[ H^2 = \frac{1}{3 M_p^2} \rho_{\text{eff}}(\phi) \quad (22) \]

\[ \left( b_g + b_k - \frac{\Delta^2}{2M^4} \phi^2 e^{2\alpha\phi/M_p} \right) \left[ \tilde{g} + 3H \phi + \phi \partial_t \right] \left( \frac{1}{2} \left( 1 + \frac{b_k}{b_g} \right) - \frac{\Delta^2}{4b_g M^4} \phi^2 e^{2\alpha\phi/M_p} \right) + \frac{\alpha \Delta^2}{4M^4 M_p} \phi^2 e^{2\alpha\phi/M_p} - \frac{\alpha M^4}{M_p} e^{-2\alpha\phi/M_p} = 0 \quad (23) \]

where the energy density of the dilaton field is

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1See for comparison Refs. [3] [4] and a general discussion in Ref. [3].

2Another origins for non-linear kinetic terms, known in the literature [24], are higher order gravitational corrections in string and supergravity theories.
\[ \rho_{\text{eff}}(\phi) = \frac{1}{4} \left( 1 + \frac{b_k}{b_g} \right) \phi^2 - \frac{3 \Delta^2}{16 b_g M^4} \phi^4 e^{2 \alpha \phi/M_p} + \frac{M^4}{4 b_g} e^{-2 \alpha \phi/M_p} \]  
(25)

and the pressure

\[ p_{\text{eff}}(\phi) = \frac{1}{4} \left( 1 + \frac{b_k}{b_g} \right) \phi^2 - \frac{\Delta^2}{16 b_g M^4} \phi^4 e^{2 \alpha \phi/M_p} - \frac{M^4}{4 b_g} e^{-2 \alpha \phi/M_p} \]  
(26)

One can see that Eqs. (22)-(25) allow solutions of a familiar quintessential form [2,3]

\[ \phi(t) = \frac{M_p}{2 \alpha} \phi_0 + \frac{M_p}{\alpha} \ln(M_p t) \]  
(27)

\[ a(t) = t^\gamma \]  
(28)

which provides scaling behaviors of the dilaton energy density

\[ \rho_{\phi} \propto 1/a^n. \]  
(29)

The important role for possibility of such solutions belongs to the remarkable feature of the nonlinear terms in Eqs. (22)-(25) that appear only in the combination \( \dot{\phi}^2 e^{2 \alpha \phi/M_p} \) which remains constant for the solutions (27) and (28):

\[ \dot{\phi}^2 e^{2 \alpha \phi/M_p} = \text{const} \]  
(30)

Eqs. (27) and (28) describe solutions of Eqs. (22)-(25) if

\[ \gamma = \frac{b_g + b_k - y}{4 b_g \alpha^2} \]  
(31)

where

\[ y = \frac{\Delta^2 M_p^4 e^{\phi_0}}{2 M^4 \alpha^2} \]  
(32)

is a solution of the cubic equation

\[ y^3 - 2(b_g + b_k - b_g \alpha^2) y^2 + (b_g + b_k)(b_g + b_k - \frac{4}{3} b_g \alpha^2) y - \frac{2}{3} b_g \alpha^2 \Delta^2 = 0. \]  
(33)

Up to now we did not make any assumptions about parameters of the theory. We will suppose that \( b_g \) and \( b_k \) are positive. One can notice immediately that if \( b_k = b_g \) then Eqs. (22)-(25) describe the FRW cosmological model in the context of the standard GR when the minimally coupled scalar field \( \phi \) with the potential \( \frac{M^4}{4 b_g} e^{-2 \alpha \phi/M_p} \) is the only source of gravity.

Another interesting possibility consists of the assumption that

\[ b_k \ll b_g \]  
(34)

Then ignoring corrections of the order of \( b_k/b_g \), the solutions of Eq. (33) are

\[ y_1 = b_g \]  
(35)

\[ y_2 = \frac{b_g}{2} \left[ 1 - 2 \alpha^2 + \sqrt{4 \alpha^4 - \frac{20}{3} \alpha^2 + 1} \right] \]  
(36)

\[ y_3 = \frac{b_g}{2} \left[ 1 - 2 \alpha^2 - \sqrt{4 \alpha^4 - \frac{20}{3} \alpha^2 + 1} \right] \]  
(37)
The solution \( y_1 \) corresponds to the static universe \((\gamma = 0 \text{ and } a(t) = \text{const})\) supported by the slow rolling scalar field \( \phi \), Eq. (27). However, taking into account corrections of the order \( b_k/b_g \) to \( y_1 \) we will get \( \gamma \propto O(b_k/b_g) \).

Solutions \( y_2 \) and \( y_3 \) exist and are positive (see the definition (32)) only if

\[
\alpha^2 \leq \frac{1}{6}
\]

The solution \( y_2 \) corresponds to the values of the parameter \( \gamma \) monotonically varying from \( \gamma_{\text{min}} = 2/3 \) up to \( \gamma = 1 \) as \( \alpha^2 \) changes from 0 up to 1/6.

The most interesting solution is given by \( y_3 \) that provides the values of the parameter \( \gamma \) monotonically varying from \( \gamma_{\text{min}} = 1 \) up to \( \gamma = \infty \) as \( \alpha^2 \) changes from 1/6 up to zero. For the dilatonic matter equation-of-state \( p = w\rho \) we get

\[
-1 \leq w \leq -\frac{32}{39} \approx -0.82
\]

In the conclusion of this section let us revert to one of the problems of the quintessence discussed in Introduction, namely to the flatness problem [5]. This is a question of the field theoretic basis for the choice of the flat enough potential. In fact, Kolda and Lyth noted [5] that an extreme fine tuning is needed in order to prevent the contribution from another possible terms breaking the flatness of the potential (see also for a review by Binetruy in Ref. [4]). In the theory we study here, there is a symmetry (scale symmetry (6)) which forbids the appearance of such dangerous contributions into \( V_{\text{eff}} \), at least on the classical level. One can hope that the soft breaking of the scale symmetry guarantees that the symmetry breaking quantum corrections to the classical effective potential (21) will be small.

V. POSSIBILITY FOR HALO-DARK-MATTER-LIKE SOLUTIONS FROM SPONTANEOUS BREAKING OF SCALE SYMMETRY

The idea that scalar field(s) configuration can give a "halo dark matter" has been explored in the literature. For example, using a variation of the Barriola and Vilenkin topologically nontrivial global monopole [21], which provides with an energy density behaving as \( 1/r^2 \), Nucamendi, Selgado and Sudarsky [22] were able to find a solution of the halo dark matter problem.

Another interesting and more simple model for dilatonic dark matter have been studied by Matos, Guzman and Nunez [23]. They showed that a single spherically symmetric scalar field with exponential potential of the form (21) could serve as a dark matter in galaxies provided that the overall sign of the potential (21) is opposite to that used in quintessence cosmology [23]. The physical origin of this opposite sign is a serious problem of the model [23].

We will now see that the scale invariant model with a single scalar field discussed in this paper can, after SSB of scale symmetry, give rise to the halo dark matter type solutions similar to those studied in Ref. [23]. The appearance of the overall negative sign in the potential is here a result of the choice of the negative integration constant in Eq.(11), i.e. \( s = -1 \) in Eq.(21).

Let us consider Eqs. (17)-(19) for the static spherically case

\[
ds^2 = B(r)dt^2 - A^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)
\]

Motivated from the cosmological solution where condition (10) was satisfied, we now look for solutions when

\[
\phi' e^{\alpha \phi/M_p} = \text{const}
\]

where \( \phi' \equiv d\phi/dr \). Then similar to the solutions of Ref. [23], we get

\[
\phi = \frac{M_p}{\alpha} \ln(r/r_0) + \frac{M_p}{2\alpha} \phi_1, \quad B(r) = r^{2l}, \quad A = \text{const}
\]

with the following equations for parameters \( l \) and \( \frac{b_k}{b_g} \) where \( x = \frac{M_p^2 e^{\phi_1}}{\alpha^2 M_p^4} \):

\[
l = \frac{\pi}{b_g \alpha^2} \left[ 2(b_g + b_k) + s\Delta^2 \frac{x A}{r_0} \right]
\]
(1 + 2l)Δ^2 \left( \frac{x A}{r_0^3} \right)^2 + 4s(1 + l)(b_g + b_k) \frac{x A}{r_0^3} + 4 = 0 \tag{44}

In contrast to the cosmological applications of the theory studied in the previous section, a positive solution for $\frac{x A}{r_0^3}$ of Eq. (44) exists only when $s = -1$.

Solution (42) can be used for description of the halo dark matter if $l \ll 1$ (see Ref. [22,23]). Having this in mind and taking into account that $|\Delta|/(b_g + b_k) < 1$, we get from Eqs. (43) and (44)

$$l \approx \frac{\pi}{\alpha^2} (1 + \frac{b_k}{b_g}) \left[ 2 - \left( \frac{b_g - b_k}{b_g + b_k} \right)^2 \right]. \tag{45}$$

Notice that for the particular case when $b_g = b_k$, the solution (45) gives $l = \frac{8\pi}{\alpha^2}$ which coincides with the appropriate relation between $l$ and exponent in the potential of Ref. [22].

This means that the halo dark matter type solution can be achieved if $\alpha^2$ is large enough

$$\alpha^2 \gg 4\pi \tag{46}$$

in contrast to the condition (38) needed for the existence of the cosmological scaling solutions.

VI. A NOTE ON QUANTIZATION

If $\Delta \neq 0$ then one can see from Eq. (25) that there is a possibility of negative energy contribution from the space-time derivatives of the dilaton. This raises of course the suspicion that the quantum theory may contain ghosts. Let us check this question when considering small perturbations around the backgrounds studied in Secs. IV and V.

To see this, let us calculate the canonically conjugate momenta to $\phi$, starting from the original action (5) and expressing it in terms of the variables defined in CEF, Eq. (15):

$$\pi_\phi = \frac{1}{2b_g} \left( b_g + b_k - \frac{\Delta^2}{sM^2}Ke^{2\alpha\phi/M_p} \right) \sqrt{-\tilde{g}^{00}} \dot{\phi}. \tag{47}$$

As we have seen in Secs. IV and V, both the cosmological scaling solutions and the halo-like solutions provide backgrounds where $Ke^{2\alpha\phi/M_p} = const$. Moreover, it is easy to see that for the scaling solutions

$$\pi_\phi = \frac{1}{2b_g} (b_g + b_k - y) a^3 \dot{\phi} = 2\alpha^2 \gamma a^3 \dot{\phi}, \tag{48}$$

where $\gamma$ and $y$ are defined by Eqs. (11) and (12). We have seen that for scaling solutions studied in Se. IV, $\gamma$ gets positive values. Therefore we conclude that in such backgrounds $\pi_\phi$ and $\dot{\phi}$ have the same sign, that guaranties a ghost-free quantization. The only exclusion is the particular case when $b_k = 0, y = b_g$. As we have seen, such solution describes a static universe. In this case the canonically conjugate momenta $\pi_\phi = 0$ and therefore it appears that in this vacuum there are no particles associated with the scalar field $\phi$.

For the background constituted by the halo dark matter solution, Sec. V, we obtain

$$\pi_\phi = \frac{1}{8b_g} \left[ 4(b_g + b_k) - \frac{\Delta^2 x A}{r_0^3} \right] A^{-1} r^{2 - l} |\sin \theta| \dot{\phi}, \tag{49}$$

where $x$ is defined in the text before Eq. (43). The coincidence of the signs of $\pi_\phi$ and $\dot{\phi}$ follows then from Eq. (43) as $s = -1$ and positivity of $l$.

Thus, at least for the the physically interesting cases studied in Secs. IV and V, the problem of ghosts does not appear.

VII. INCLUSION OF FERMIONIC MATTER CONSISTENT WITH SCALE INVARIANCE AND THE "FAMILY BIRTH EFFECT"

In general scalar-tensor theories, particle masses depend on time, when the theory is studied in the frame where Newton’s constant is really a constant. However, for all the fermionic matter observed in the universe, the cosmological
variation of particle masses (including those of electrons) is highly constrained. We want to show now how the theory presented in this paper avoids this problem and also the so called fifth force problem, in spite of the need to include exponential couplings of the dilaton field to fermionic matter in order to ensure global scale invariance.

To describe fermions, normally one uses the vierbein \( e^a_{\mu} \) and spin-connection \( \omega^{ab}_\mu \) formalism where the metric is given by \( g^{\mu\nu} = e^a_{\mu} e^b_{\nu} \eta^{ab} \) and the scalar curvature is \( R(\omega, e) = e^{\mu\nu} e^{\rho\sigma} R_{\mu\nu\rho\sigma} \) where

\[
R_{\mu\nu\rho\sigma}(\omega) = \partial_{\mu} \omega_{\nu\rho} + \omega_{\mu\sigma} \omega_{\nu\rho} - (\mu \leftrightarrow \nu).
\]  

(50)

Following the general idea of the model, we now treat the geometrical objects \( e^a_{\mu}, \omega^{ab}_\mu \), the measure fields \( \varphi_a \), as well as the dilaton \( \phi \) and the fermionic fields as independent variables. In this formalism, the natural generalization of the action \( L \) keeping the general structure \( \bar{L} \), when a fermion field \( \Psi \) is also present and which also respect scale invariance is the following:

\[
L_1 = e^{\alpha\phi/M_p} - \frac{1}{\kappa} R(\omega, e) + \frac{1}{2} g^{\mu\nu} \phi_{\mu} \phi_{\nu} + \frac{i}{2} \sqrt{\gamma} e^{\mu} \left[ \gamma^\alpha e^a_{\mu} \left( \partial_\mu + \frac{1}{2} \omega^c_{\mu} \sigma_{cd} \right) - \left( \partial_\mu - \frac{1}{2} \omega^c_{\mu} \sigma_{cd} \right) \gamma^a e^\mu \right] \Psi - m \Psi \Psi e^{\frac{2\alpha\phi}{M_p}}
\]  

(51)

\[
L_2 = e^{\alpha\phi/M_p} \left[ -\frac{b_g}{\kappa} R(\omega, e) + \frac{b_k}{2} g^{\mu\nu} \phi_{\mu} \phi_{\nu} - h_m \Psi \Psi e^{\frac{2\alpha\phi}{M_p}} \right]
\]  

(52)

The action \( \bar{L} \) with such \( L_1 \) and \( L_2 \) is invariant under the scale transformations

\[
e^a_{\mu} \rightarrow e^{\theta/2} e^a_{\mu}, \quad \omega^{ab}_\mu \rightarrow \omega^{ab}_\mu, \quad \varphi_a \rightarrow \lambda_a \varphi_a \quad \text{where} \quad \Pi \lambda_a = e^{2\theta}
\]

\[
\phi \rightarrow \phi - M_p \theta, \quad \Psi \rightarrow e^{-\theta/4} \Psi, \quad \bar{\Psi} \rightarrow e^{-\theta/4} \bar{\Psi}
\]  

(53)

Notice that two types of fermionic "mass-like terms" which respect scale invariance have been introduced. In contrast, for simplicity, we have restricted the coupling of the fermionic kinetic term to the measure \( \Phi \) only.

We can immediately obtain the equations of motion. From these going through similar steps to those performed in Sec. III, a constraint follows again which replaces \( \bar{L} \) and which contains now a contribution from the fermions. The spin-connection can be found by the variation of \( \omega^{ab}_\mu \).

Similar to what we learned from the treatment of Sec. III, we can consider the theory in the CEF which in this case involves also a transformation of the fermionic fields:

\[
\tilde{g}_{\mu\nu} = e^{\alpha\phi/M_p} (\zeta + b_g) g_{\mu\nu}, \quad \tilde{e}^{a}_{\mu} = e^{\frac{1}{2} \alpha\phi/M_p} (\zeta + b_g)^{1/2} e^{a}_{\mu},
\]

\[
\Psi' = e^{\frac{1}{4} \alpha\phi/M_p} (\zeta + b_g)^{-1/4} \Psi.
\]  

(54)

Notice that variables \( \tilde{g}_{\mu\nu}, \tilde{e}^{a}_{\mu}, \Psi' \) and \( \bar{\Psi} \) are in fact invariant under the scale transformations \( \bar{L} \). In the CEF the only field which still has a non trivial transformation property is the dilaton \( \phi \) which gets shifted (according to \( \bar{L} \)). Thus, the presence of fermions does not change a conclusion made in Sec. III after Eq. (6): the spontaneous breaking of the scale symmetry is reduced, in the CEF, to the spontaneous breaking of the shift symmetry \( \phi \rightarrow \phi + const \) for the dilaton field.

In terms of \( \tilde{e}^{a}_{\mu}, \Psi', \bar{\Psi} \) and \( \phi \), the constraint which now replaces \( \bar{L} \) and which contains now a contribution from the fermions is

\[
(\zeta + b_g) M^4 e^{-\frac{2\alpha\phi}{M_p}} + \Delta (\zeta + b_g) K + F(\zeta) (\zeta + b_g)^{1/2} m \bar{\Psi} \Psi' = 0.
\]  

(55)

where we have chosen \( s = +1 \) for definiteness and the function \( F(\zeta) \) is defined by

\[
F(\zeta) \equiv \frac{1}{2} \left( \zeta + \frac{2b_g h}{\zeta} + 3h \right)
\]  

(56)

The dilaton and the fermion field equations are respectively

\[
\begin{align*}
\left( \zeta + b_k \right) \left[ (-\tilde{g})^{-1/2} \partial_\mu (\sqrt{-\tilde{g}} g^{\mu\nu} \partial_\nu \phi) + \tilde{g}^{\alpha\beta} \partial_\alpha \phi \partial_\beta \ln \frac{\zeta + b_k}{\zeta + b_g} \right] \\
+ \frac{\alpha \Delta}{M_p} K - \frac{\alpha M^4}{M_p} e^{-2\alpha\phi/M_p} - \frac{\alpha m M}{M_p \sqrt{\zeta + b_g}} F(\zeta) \bar{\Psi} \Psi' = 0.
\end{align*}
\]  

(57)
where

$$\omega_{cd}^{\mu}(\tilde{e}) = \omega_{cd}^{\mu}(e) + \frac{1}{4M_{P}^{2}}\eta_{cd}e_{abde}\gamma^{5}\gamma^{5}\Psi'\gamma^{i}\Psi',$$

(59)

$$\omega_{cd}^{\mu}(\tilde{e})$$ is the Riemannian part of the connection and $$C_{ab}^{p}$$ is the trace of the Ricci rotation coefficients \([24]\) in the new variables. Eq. (59) coincides with the well-known solution for the spin connection in the context of the first order formalism approach to the Einstein-Cartan theory \([24]\) where a Dirac spinor field is the only source of a non-Riemannian part of the connection. For details see also \([11]\).

The gravitational equations are of the standard form \([17]\) with

$$T_{\mu\nu}^{\text{eff}} = \frac{\zeta}{\zeta + b_{y}}\varphi_{,\mu}\varphi_{,\nu} - K\tilde{g}_{\mu\nu} + \frac{b_{y}M_{P}^{4}}{(\zeta + b_{y})^{2}}e^{-2\alpha\phi/M_{P}}\tilde{g}_{\mu\nu} + \frac{\zeta}{\zeta + b_{y}}T_{\mu\nu}^{(f,\text{canonical})} - \frac{mF(\zeta)}{\sqrt{\zeta + b_{y}}}\nabla'_{\mu\nu}\Psi'_{\nu},$$

(60)

where

$$T_{\mu\nu}^{(f,\text{canonical})} = \frac{i}{2}\tilde{\nabla}_{\mu}\gamma_{a}(\nabla'_{\nu})\Psi' - (\nabla'_{(\mu})\gamma_{a}\nabla'_{\nu)}\psi'_{a}$$

(61)

is the canonical energy-momentum tensor for the fermionic field in the curved space-time \([25]\) and $$\nabla'_{\nu}\Psi' = (\partial_{\nu} + \frac{1}{2}\omega_{\nu cd}\sigma_{cd})\Psi'$$ and $$\nabla'_{\nu} = \partial_{\nu} - \frac{1}{2}\omega_{\nu cd}\sigma_{cd}.$$ The scalar field $$\zeta$$ is defined by the constraint \([58]\) in terms of the dilaton and fermion fields as a solution of the fifth degree algebraic equation that makes finding $$\zeta$$ in general a very complicate question. However there are two physically most interesting limiting cases when solving \([58]\) is simple enough. The only assumption we will make about dimensionless parameters of the theory in what follows will be that $$b_{y}, b_{k}$$ and $$h$$ are not too large.

Let us first analyze the constraint \([58]\) when the fermionic density (proportional to $$\tilde{\nabla}'\Psi'$$) is very low as compared to the contributions of the dilaton potential ($$\propto M_{P}^{4}e^{-2\alpha\phi/M_{P}}$$) and kinetic term $$K$$. In this limiting case, the constraint gives again the expression \([24]\) for $$\zeta$$. If we assume then the quintessential cosmological solution of Sec.IV or the halo dark matter solution of Sec.V where $$K_{e^{2\alpha\phi/M_{P}}} = \text{const}.$$ we get a constant value of $$\zeta$$. Inserting this value of $$\zeta$$ into \([58]\) we see that the mass of a "test" fermion (that is when we ignore the effect of the fermion itself on the quintessential or halo dark matter background) is constant. Notice that the constant mass of fermions can be different in the cosmological and in the halo solutions (remind that parameters of the theory needed for these solutions are also different). Notice, however, that if $$\Delta = 0$$ (that is $$b_{y} = b_{k}$$), then $$\zeta = b_{y}$$ and the mass of a "test" fermion is constant for any dilatonic background.

An opposite regime is realized when the contribution of the fermionic density to the constraint \([58]\) is very high as compared to the contributions of the dilaton potential and kinetic term. In the context of the quintessence model of the present day universe, this regime corresponds in particular to the normal laboratory conditions in particle physics. Then according to the constraint \([58]\), one of the possibilities for this to be realized consists in the condition

$$F(\zeta) \equiv \frac{1}{2}\left(\zeta + \frac{2b_{y}h}{\zeta} + 3h\right) \approx 0$$

(62)

from which we find two possible constant values for $$\zeta$$

$$\zeta \approx -\frac{3}{2}h \left(1 + \sqrt{1 - \frac{8b_{y}}{9h}}\right) = \text{const.}$$

(63)

Of course, these solutions have sense only if $$b_{y}/h < 9/8$$. We see from \([58]\) that two different constants $$\zeta$$ given by \([58]\) define in general two specific masses for the fermion. Notice that in the special case when $$b_{y} = 0$$, Eq. \([58]\) is linear in $$\zeta$$ and we obtain therefore only one effective fermion mass $$m_{\text{term}}^{\text{eff}} \approx \frac{2m}{3\sqrt{3}h}$$.

Surprisingly that the same factor $$F(\zeta)$$ appears in the last terms of Eqs. \([17]\) and \([60]\). Therefore, in the same regime of fermion dominance, the last terms of Eqs. \([17]\) and \([60]\) automatically vanish. In Eq. \([58]\), this means that the fermion density $$\tilde{\nabla}'\Psi'$$ is not a source for the dilaton and thus the long-range force disappears automatically. Notice that there is no need to require no interactions of the dilaton with barionic matter at all to have agreement
with observations but it is rather enough that these interactions vanish in the appropriate regime where barionic matter dominates over other matter fields. In Eq. (60), the condition (62) means that in the region where the fermionic matter dominates, the fermion energy-momentum tensor, up to a constant, becomes equal to the canonical energy-momentum tensor of a fermion field in GR.

The separate possibility relevant to the very high fermionic density (again, as compared to the contributions of the dilaton potential and kinetic term) is the case when

\[ \zeta + b_g \approx 0 \] (64)

is a solution. Then \( F(\zeta) \approx F(-b_g) = h - b_g, \quad \zeta - b_g \approx -2b_g \) and it follows from the constraint (65)

\[ \frac{1}{\sqrt{\zeta + b_g}} \approx \frac{m}{4M^4} \left( \frac{h}{b_g} - 1 \right) \Psi \Psi^e 2a\phi/M_p. \]

In this case, Eq. (68) describes fermion with an effective quartic self-interaction like in NJL model [26]. The coupling constant of this self-interaction depends on the dilaton \( \phi \). For example, the condition (64) is realized as \( \phi \to \infty \) that corresponds to the late universe in the quintessence scenario.

Here, as opposed to the solutions (63), we get quartic interaction instead of mass generation. We expect however that after \( \Psi \Psi^e \) develops an expectation value, mass generation will be possible as in NJL model [26] (for recent progress in this subject see e. g. Ref. [27]). It is interesting to note that appearance of the quartic self-interaction here is related to the SSB of the scale invariance. In fact, Eq. (65) tells us that without SSB of scale invariance such quartic interaction is not defined.

Concluding this analysis of equations when the fermionic density is of the order typical for the normal particle physics (which in the laboratory conditions is always much higher than the dilaton density ) we see that starting from a single fermionic field we obtain (if \( b_g \neq 0 \)) exactly three different types of spin 1/2 particles in CEF. This appears to be a new approach to the family problem in particle physics. This is why we will refer to the described effect as the "family birth effect".

All what has been done here concerning fermions is in the context of a toy model without Higgs fields, gauge bosons and the associated \( SU(2) \times U(1) \times SU(3) \) gauge symmetry of the standard model. As we have seen in other models (see [11], the second reference of [13] and [15]), it is possible to incorporate the two measure ideas with the gauge symmetry and Higgs mechanism. Now the differences consist of: i) the presence of global scale symmetry, ii) the most general TMT structure for gravitation and dilaton sector but including only kinetic terms. The complete discussion of the standard model in the context of such TMT structure will be presented in a separate publication [28]. Here we want only to explain shortly the main ideas that provides us the possibility to implement this program.

It is important that in a simple way gauge fields can be incorporated so that they will not appear in fundamental constraint\(^3\) in contrast to the fermions (see for comparison Eq. (24)). As it is easy to see, the different constant values of \( \zeta \) corresponding to the solutions of the constraint do not change the expectation value of the Higgs field. We can also work without significant changes in the discussion of the fermionic sector if instead of explicit mass-like terms we will work with similar terms where the coupling constants with the dimensionality of the mass are replaced by gauge invariant Yukawa couplings to the Higgs field. Once again we find three fermion families, as was done above in the toy model. Generating mass of two of them is automatic as in the previous discussion. For the third we need again some quantum effect that gives rise to expectation value of the gauge invariant Yukawa coupling terms. Since the object that gets expectation value is gauge invariant, we don’t expect further breaking of gauge symmetry (as opposed to usual analysis on top quark condensates [27] [1]).

\(^3\)The decoupling of the dilaton in the CEF in the case of high fermion density was discussed also in a simpler scale invariant model (with \( b_g = b_k = 0 \) and explicit exponential potentials) in Ref. [13].

\(^4\)This may be done by making the gauge field kinetic terms coupled to \( \sqrt{-g} \) and the Higgs field kinetic term coupled to measure \( \Phi \). Both of these things are dictated also by local scale invariance of that part of the action.

\(^5\) Notice that in our case there is an explicit Higgs field as opposed to the top quark condensate models, and in spite of this we need the condensates of the Yukawa coupling terms so as to get a normal mass term for the third family.
VIII. DISCUSSION AND CONCLUSIONS

In this paper the possibility of a spontaneously generating exponential potential for the dilaton field in the context of TMT with spontaneously broken global scale symmetry was studied. The symmetry transformations formulated in terms of the original variables (\( \xi \)) (or (\( \mathcal{E} \)) in the presence of fermions) include the global scale transformations of the metric, of the scalar fields \( \varphi_a \) related to the measure \( \Phi \) (and of the fermion fields) and in addition the dilaton field \( \phi \) undergoes a global shift. In the CEF (see Eqs. (15) or (54) where the theory is formulated in the Riemannian (or Einstein-Cartan) space-time), all dynamical variables are invariant under the transformations (\( \xi \)) (or (\( \mathcal{E} \))) except for the dilaton field which still gets shifted by a constant. Thus, SSB of the scale symmetry that appears firstly in (\( \xi \)) when solving Eq. (7), is reduced, in the CEF, to SSB of the shift symmetry \( \phi \rightarrow \phi + const. \)

The original action does not includes potentials but in the CEF, the exponential potential appears as a result of SSB of the scale symmetry. In the generic case \( \Delta = b_g - b_k \neq 0 \), the process of SSB also produces terms with higher powers in derivatives of the dilaton field.

Cosmological scaling solutions of the theory were studied. The flatness of the potential \( V_{eff} \) which is associated here with the exponential form, is protected by the scale symmetry. Quintessence solutions (corresponding to accelerating universe) were found possible for a range of parameters if the integration constant \( sM^4 \) in Eq. (9) is chosen to be positive.

Also in the same model, but for a negative integration constant \( sM^4 \) and for a different range of the parameter \( \alpha \) it is found that halo-like solutions exist. They give rise to a constant velocity for test particles moving at large distances in circular orbits.

Finally, the behavior of fermions in such type of models was investigated. Scale invariant fermion mass-like terms can be introduced in two different ways since they can appear coupled to each of the two different measures of the theory. Although an exponential of the dilaton field \( \phi \) couples to the fermion in both of these terms, it is found that when the fermions are treated as a test particles in the scaling background, their masses in the CEF are constant.

Even more surprising is the behavior of the fermions in the limit of high fermion density as compared to the dilaton density. This approximation is regarded as more realistic if we are interested in the regular particle physics behavior of these fermions under normal laboratory conditions. It is found then that in the CEF, a given fermion can behave in three different ways according to the three different solutions of the fundamental constraint (55). Two of the solutions correspond to fermions with constant masses and the other - to a NJL model [26], which is known can generate mass on the quantum level. From one fermion three are obtained for free. This suggests a new approach to the "family problem" in particle physics.

In addition to this, for the two mentioned above solutions (\( \xi \)) corresponding to constant fermion masses, the fermion-dilaton coupling in the CEF (proportional to \( F(\zeta) \), Eq. (56)) disappears automatically. If one of these types of fermions is associated to the first family (regular matter, i.e., \( u \) and \( d \) quarks, \( e^- \) and \( \nu_e \)), we obtain that normal matter decouples from the dilaton.

The analysis of the constraint (55) in the case where the fermionic density is of the same order as the dilaton energy density will provide in general five solutions for \( \zeta \). It could be that those "low energy families" may be a good candidate for dark matter.

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