Numerical Solution for Linear Fredholm Integro-Differential Equation Using Touchard Polynomials

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Abstract:
A new method based on the Touchard polynomials (TPs) was presented for the numerical solution of the linear Fredholm integro-differential equation (FIDE) of the first order and second kind with condition. The derivative and integration of the (TPs) were simply obtained. The convergence analysis of the presented method was given and the applicability was proved by some numerical examples. The results obtained in this method are compared with other known results.

Key words: Fredholm equation, Integro-differential equation, Integral equation, Touchard polynomial, Numerical solution

Introduction:
Integral and integro-differential equations are originated in many scientific and engineering applications. In particular Fredholm integral equation and (FIDE) can be derived from boundary value problem. The (IDE) contains both differential and integral signs and the derivative of the unknown variable may appear to any order. (FIDE) is an equation derived from the boundary value problem with given initial boundary condition, where both the differential and integral signs appeared together in the same equation. In addition, limits of the integration are constants. (FIDE) of the first order and second kind contains the unknown variable and its derivative inside and outside the integral sign respectively. It is noted that initial condition should be given for (FIDE) to find the particular solution (1). (FIDEs) often come in applications being the mathematical models of processes in biological problems, physics, economy and chemistry, etc. (2). (FIDEs) are difficult to be solved analytically, so it requires effective numerical methods (3). For these reasons, many scientists have been encouraged to study many numerical methods to solve (FIDEs). All methods have pros or cons but that hasn't stopped scientists from developing various methods such as Bernstein collocation matrix method (4), the well-posedness method (5), reproducing kernel method (6), exponential spline method (7), improved reproducing kernel method (8), priori Nystrom method (9), and Fibonacci polynomials method (10).

The general form of the linear (FIDE) of 1st order and 2nd kind is given by (1, 11):

\[
X'(\gamma) + a_2 X(\gamma) + \int_{a_1}^{a_2} T(\gamma, u) X(u) du = \eta, \quad \gamma \\
X_{a_1}, \quad \text{with initial boundary condition } X(a_1) = X_0,
\]

where \( a_1, a_2 \) and \( \eta \) are constants, \( T(\gamma, u) \) is a known function of the variables \( \gamma \) and \( u \), called the nucleus (kernel) of the Integral equation. The unknown function \( X(\gamma) \) will be calculated, which exists inside and outside the integral sign. \( h(\gamma) \) is a given function, \( X'(\gamma) = \frac{d}{d\gamma} X(\gamma) \) and \( X(a_1) = X_0 \) is a constant initial boundary condition.

This paper is ordered as follows: Touchard polynomials, approximation function, solution the (FIDE), convergence analysis, test examples with tables and graphs are presented, brief of conclusions and recommendations, and finally, the references are listed.
Touchar Polynomials:
Let’s begin with the definition of the (TPs) that was studied by the French mathematician Jacques Touchar. The (TPs) consist of a polynomial sequence of binomial type, it’s defined on [0, 1] as follows (12, 13, 14, and 15):
\[
l_n(y) = \sum_{m=0}^{n} \frac{n!}{m!} E(n,m) y^m = \sum_{m=0}^{n} \binom{n}{m} y^m , \quad \binom{n}{m}
\]
where n and m are the degree and index of the (TPs) respectively.
The 1st five polynomials of the (TPs) are given below:
\[
l_0(y) = 1 \quad l_1(y) = 1 + y \\
l_2(y) = 1 + 2y + y^2 \\
l_3(y) = 1 + 3y + 3y^2 + y^3 \\
l_4(y) = 1 + 4y + 6y^2 + 4y^3 + y^4
\]
Approximation Function:
Suppose that the function \(X_n(y)\) is approximated using the (TPs) as in the following:
\[
X_n(y) = \alpha_0 l_0(y) + \alpha_1 l_1(y) + \ldots + \alpha_n l_n(y) \\
= \sum_{m=0}^{n} \alpha_m l_m(y)
\]
for m\geq0, the function \(l_m(y)\) denotes the Touchar basis polynomials of nth degree, as defined in Eq. (2). \(\alpha_m\) (m = 0,1,...,n) are the unknown Touchar coefficients that will be calculated later.
Now Eq. (3) can be written as dot product:
\[
X_n(y) = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} \cdot \begin{bmatrix} l_0(y) \\ l_1(y) \\ \ldots \\ l_n(y) \end{bmatrix}, \quad \ldots (4)
\]
Eq. (4) can be converted into:
\[
X_n(y) = [1 \ y \ y^2 \ldots y^n] \begin{bmatrix} \beta_{00} & \beta_{01} & \beta_{02} & \ldots & \beta_{0n} \\ 0 & \beta_{11} & \beta_{12} & \ldots & \beta_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \beta_{nn} \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix}, \quad \ldots (5)
\]
where \(\beta_{rr}\) (r = 0,1,2,...,n) are the coefficients of the power basis that are used to obtain the (TPs) coefficients and the matrix is invertible. For example n=1, 2, and 3, the operational matrices are shown in Eqs. (6), (7), and (8) respectively:
\[
X_1(y) = [1 \ y] \begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix}, \quad \ldots (6)
\]
\[
X_2(y) = [1 \ y \ y^2] \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix}, \quad \ldots (7)
\]
\[
X_3(y) = [1 \ y \ y^2 \ y^3] \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}, \quad \ldots (8)
\]
Since the derivative of Eq. (2) is:
\[
l_n'(y) = \frac{d}{dy} \sum_{m=0}^{n} \binom{n}{m} y^m \\
= \sum_{m=1}^{n} \binom{n}{m} m y^{m-1} . \quad \ldots (9)
\]
Then, the derivative of Eqs. (5), (6), (7) and (8) respectively is:
\[
X_1'(y) = [0 \ 1 \ 1 \ 1] \begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix}, \quad \ldots (10)
\]
\[
X_2'(y) = [0 \ 1 \ 2 \ y \ 3 y^2] \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix}, \quad \ldots (11)
\]
\[
X_3'(y) = [0 \ 1 \ 2 \ y \ 3 y^2 \ y^3] \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}, \quad \ldots (12)
\]
Solutions (the FIDE) of 1st order and 2nd Kind:
Since Eq. (1) has the following form:
\[
X'(y) = h(y) + \eta \int T(y,u) X(u)du , \quad \gamma \\
\in [a_1, a_2] \quad \text{and} \quad X(a_1) = X_0 \ldots (14)
\]
By using Eqs. (5) and (10), suppose that:
\[
X(y) \equiv X_n(y) \begin{bmatrix} \beta_{00} & \beta_{01} & \beta_{02} & \ldots & \beta_{0n} \\ 0 & \beta_{11} & \beta_{12} & \ldots & \beta_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \beta_{nn} \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix}, \quad \ldots (15)
\]
and
\[
X(y) \equiv x_0(y) = \begin{bmatrix}
\gamma_0 \\
\gamma_1 \\
\vdots \\
\gamma_n
\end{bmatrix} = \begin{bmatrix}
\alpha_0 \\
\alpha_1 \\
\vdots \\
\alpha_n
\end{bmatrix} \begin{bmatrix}
\alpha_0 \\
\alpha_1 \\
\vdots \\
\alpha_n
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

now, by substituting Eqs. (15) and (16) into Eq. (14) yields:

\[
0 \leq y \leq 0 \to \gamma_{n+1} = \begin{bmatrix}
\beta_0 \\
\beta_1 \\
\vdots \\
\beta_n
\end{bmatrix} \begin{bmatrix}
\alpha_0 \\
\alpha_1 \\
\vdots \\
\alpha_n
\end{bmatrix} = \begin{bmatrix}
\beta_0 \\
\beta_1 \\
\vdots \\
\beta_n
\end{bmatrix} \begin{bmatrix}
\alpha_0 \\
\alpha_1 \\
\vdots \\
\alpha_n
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
\beta_0 \\
\beta_1 \\
\vdots \\
\beta_n
\end{bmatrix} \begin{bmatrix}
\alpha_0 \\
\alpha_1 \\
\vdots \\
\alpha_n
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
\beta_0 \\
\beta_1 \\
\vdots \\
\beta_n
\end{bmatrix} \begin{bmatrix}
\alpha_0 \\
\alpha_1 \\
\vdots \\
\alpha_n
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

Now, in order to determine \( \alpha_0, \alpha_1, ..., \alpha_n \) the integrations in Eq. (17) must be computed by selecting \( y_r \in [0,1], \ r = 0,1, ..., n \), and applying the given initial condition, to get a system of \((n+1)\) linear algebraic equations with the \((n+1)\) of the unknown values. Solving this system by using Gauss elimination method, the unknown coefficients \( \alpha_0, \alpha_1, ..., \alpha_n \) are obtained and have a unique solution, hence, by substituting Touchard coefficients into Eq. (3) to get the approximate numerical solution for Eq. (1)

**Convergence Analysis:**

In this section, the convergence for the suggested method is proved. The unknown Touchard coefficients \( \alpha_0, \alpha_1, ..., \alpha_n \) are uniquely determined by Eq. (17). Therefore Eq. (1) with the boundary condition has a unique solution and this solution is given by truncated Touchard series in Eq. (3). Now, when the approximate numerical solution \( X_n(y) \) and its derivatives are substituted in Eq. (1), the following equation should be satisfied approximately, then for \( y = y_0 \in [0,1] \), \( \lambda = 0, 1, 2, ..., n \),

\[
ER_n(y) = \left| \sum_{m=0}^{n} \alpha_m l_m(y) \right| \leq h(y) - \eta \int_{a_1}^{a_2} T(y, u) \sum_{m=0}^{n} \alpha_m l_m(u) du \to 0,
\]

and \( ER_n(y) \leq 10^{-\gamma} \).

If \( \gamma = 10^{-\gamma} \) is specified, then the truncation limit \( n \) is increased until the difference \( ER_n(y) \) between each of the points \( y_0 \) becomes less than or equal \( 10^{-\gamma} \). In other words, the error function \( ER_n(y) \) can be estimated by the relation:

\[
ER_n(y) = \left( \sum_{m=0}^{n} \alpha_m l_m(y) \right) \to h(y) - \eta \int_{a_1}^{a_2} T(y, u) \sum_{m=0}^{n} \alpha_m l_m(u) du,
\]

then, \( ER_n(y) \to 0 \) when \( n \) is a very large, then the error function decreases (3 and 10)

**Numerical Examples:**

This section checks the computational accuracy of the (TPs) method, by testing three examples of linear (FIDE) and one example of a nonlinear (FIDE). The accuracy of the solution method was measured by the absolute error for the first three examples, while the fourth example was measured by the maximum absolute error, convergence rate and time of CPU, also all calculations and charts were accomplished on my PC using the matlab2018 program.

The general formulas of the testes were defined as follows:

**Absolute error:**

\[|ER| = |X(y_0) - X_n(y_0)|, \quad y_0 \in [0,1] \text{ and } \lambda = 0, 1, ..., n\]

**Maximum error:**

\[\|ER\| = \max_{y_0 \in [0,1]} |X(y_0) - X_n(y_0)|, \quad X(y_0) \text{ and } X_n(y_0)\]

**Example 1:** Solve the linear (FIDE) given in (1):

\[X'(y) = h(y) + \int_{0}^{1} xu X(u) du, \quad 0 \leq y \leq 1\]

where \( h(y) = 3 + 6y, \quad \eta = 1, T(y, u) = xu, \quad X(0) = 0, \quad \text{and the exact solution } X(y) = 3y + 4y^2 \]

Now, by applying the (TPs), the approximate solutions of this example for \( n = 1, 2 \) and 3 are respectively:

\[X_1(y) = \sum_{m=0}^{1} \alpha_m l_m(y) = \alpha_0 l_0(y) + \alpha_1 l_1(y) = (-3.01) l_0(y) + (3.0) l_1(y)\]

\[X_2(y) = \sum_{m=0}^{2} \alpha_m l_m(y) = \alpha_0 l_0(y) + \alpha_1 l_1(y) + \alpha_2 l_2(y) = (0.76) l_0(y) + (-4.5556) l_1(y) + (3.7778) l_2(y)\]
The approximate solutions and absolute error were compared in Tables 1 and 2, respectively, showing that the accuracy of the results increases as $n$ increases. In Fig. 1, the exact solution was compared with Touchard solution for $n = 3$.

### Table 1. Approximate Numerical and Exact Solutions of Example 1.

| $\gamma$ | Exact Solution | Approximate Solutions |
|----------|----------------|-----------------------|
| 0.0      | 0.0            | -1.0000e-02           |
| 0.1      | 3.4000e-01     | 2.9000e-01            |
| 0.2      | 7.6000e-01     | 5.9000e-01            |
| 0.3      | 1.2600e+00     | 8.9000e-01            |
| 0.4      | 1.8400e+00     | 1.1900e+00            |
| 0.5      | 2.5000e+00     | 1.4900e+00            |
| 0.6      | 3.2400e+00     | 1.7900e+00            |
| 0.7      | 4.0600e+00     | 2.0900e+00            |
| 0.8      | 4.9600e+00     | 2.3900e+00            |
| 0.9      | 5.9400e+00     | 2.6900e+00            |
| 1.0      | 7.0000e+00     | 2.9900e+00            |

### Table 2. Comparison of the Absolute Error of Example 1.

| $\gamma$ | Absolute Errors |
|----------|-----------------|
|          | $n = 1$         | $n = 2$         | $n = 3$         |
| 0.0      | 1.0000e-02      | 1.7778e-02      | 2.2222e-02      |
| 0.1      | 5.0000e-02      | 1.9999e-02      | 2.0001e-02      |
| 0.2      | 1.7000e-01      | 2.6666e-02      | 1.3334e-02      |
| 0.3      | 3.7000e-01      | 3.7778e-02      | 2.2223e-03      |
| 0.4      | 6.5000e-01      | 5.3333e-02      | 1.3333e-02      |
| 0.5      | 1.0100e+00      | 7.3333e-02      | 3.3333e-02      |
| 0.6      | 1.4500e+00      | 9.7777e-02      | 5.7777e-02      |
| 0.7      | 1.9700e+00      | 1.2667e-01      | 8.6667e-02      |
| 0.8      | 2.5700e+00      | 1.6000e-01      | 1.2000e-01      |
| 0.9      | 3.2500e+00      | 1.9778e-01      | 1.5778e-01      |
| 1.0      | 4.0100e+00      | 2.4000e-01      | 2.0000e-01      |

### Figure 1. Numerical Result and Exact Solution of Example 1 for $n = 3$.

#### Example 2: Solve the linear (FIDE) given in (16, and 17)

$$X'(\gamma) = h(\gamma) + \int_{0}^{1} \gamma X(u) \, du, \quad 0 \leq \gamma \leq 1$$
where \( h(\gamma) = \gamma e^\gamma + e^\gamma - \gamma, \eta = 1, \ T(\gamma, u) = \gamma \), \( X(0) = 0 \) and the exact solution is \( X(\gamma) = \gamma e^\gamma \).

The approximate numerical results are obtained for \( n = 1, 3 \) and 4, respectively:

\[
\begin{align*}
X_1(\gamma) &= (-0.97)I_0(\gamma) + (0.9657)I_1(\gamma), \\
X_3(\gamma) &= (-0.45)I_0(\gamma) + (0.5351)I_1(\gamma) \\
&\quad + (-0.7436)I_2(\gamma) \\
&\quad + (0.6523)I_3(\gamma).
\end{align*}
\]

Table 3 shows the absolute errors for \( n = 4 \), and compares with methods included in (16 and 17). In Fig. 2, the exact solution is compared with Touchard solution for \( n = 4 \).

\[
X_5(\gamma) = (-0.15)I_0(\gamma) + (-0.6898)I_1(\gamma) + \\
(1.0507)I_2(\gamma) + (-0.4411)I_3(\gamma) \\
+ (0.2278)I_4(\gamma).
\]

Example 3: Solve the linear (FIDE) given in (11, 16, and 17)

\[
X'(\gamma) = h(\gamma) + \int_0^1 \gamma u X(u) \, du, \quad 0 \leq \gamma \leq 1
\]

where \( h(\gamma) = 1 - \frac{1}{\gamma}, \eta = 1, \ T(\gamma, u) = \gamma u \), \( X(0) = 0 \) and the exact solution is \( X(\gamma) = \gamma \).

By applying suggested method for this example, for \( n = 5 \), the Touchard solution is:

\[
X_5(\gamma) = (\gamma).
\]

In Table 4, the absolute error in the current method is compared with those in (11, 16 and 17), and it is found that the absolute error in the current method is the highest accuracy. In Fig. 3, for \( n = 5 \), the Touchard solution is compared with the exact solution.
Table 4. Comparison of the Absolute Error of Example 3.

| γ  | Current Method n=5 | Method in (11) | Method in (16) | Method in (17) |
|----|-------------------|----------------|----------------|----------------|
| 0.1| 0.0               | 3.7900e−06     | 2.1794e−04     | 1.6667e−03     |
| 0.2| 0.0               | 1.5160e−05     | 6.3855e−04     | 6.0939e−03     |
| 0.3| 0.0               | 3.4110e−05     | 7.9137e−04     | 1.3202e−02     |
| 0.4| 0.0               | 6.0640e−05     | 2.1559e−02     | 2.2914e−03     |
| 0.5| 0.0               | 9.4750e−05     | 4.9936e−03     | 3.5158e−02     |
| 0.6| 0.0               | 1.3644e−04     | 2.2173e−02     | 6.6965e−02     |
| 0.7| 0.0               | 1.8571e−04     | 1.0565e−04     | 7.1243e−02     |
| 0.8| 0.0               | 2.4256e−04     | 1.4323e−03     | 8.6398e−02     |
| 0.9| 0.0               | 3.0699e−04     | 2.0775e−02     | 1.0810e−01     |

Figure 3. Numerical Result and Exact Solution of Example 3 for n = 5.

Example 4: Finally, solve the nonlinear (FIDE) given in (18)

\[
X'(\gamma) = h(\gamma) + \int_{0}^{1} \gamma^{3} (X(u))^2 \, du, \quad 0 \leq \gamma \leq 1
\]

where \( h(\gamma) = 1 - \frac{1}{3} \gamma^{3} \), \( \eta = 1 \), \( T(\gamma, u) = \gamma^{3} \), \( X(0) = 0 \) and the exact solution is \( X(\gamma) = \gamma \).

By applying suggested method for \( n = 1, 2 \) and 3, the Touchard solutions are obtained respectively:

\[
X_1(\gamma) = (-1)I_0(\gamma) + (1)I_1(\gamma)
\]

\[
X_2(\gamma) = (-1)I_0(\gamma) + (1)I_1(\gamma) + (4.4522e-16)I_2(\gamma).
\]

\[
X_3(\gamma) = (-1)I_0(\gamma) + (1)I_1(\gamma) + (-7.1067e-15)I_2(\gamma) + (1.8000e-15)I_3(\gamma).
\]

The comparison of the maximum error, ratio of error and CPU times of the current method with those in (18) is shown in Table 5, and shows that the current method for \( n = 1, 2 \) and 3 has a much higher accuracy than those in (18) for \( n = 5, 9 \) and 17. In Fig. 4, for \( n = 2 \) and 3, the Touchard solutions were compared with the exact solution.

Table 5. Comparison of the Maximum Absolute Error, Error Ratio and CPU Time of Example 4.

| n  | \( \|e\|_{\infty} \) | Ratio | Time | n  | \( \|e\|_{\infty} \) | Ratio | Time |
|----|------------------|-------|------|----|------------------|-------|------|
| 1  | 1.3989 e−14      | −     | 0.036| 5  | 3.26 e−03        | −     | 0.37 |
| 2  | 6.2172 e−15      | 2.25  | 0.039| 9  | 8.44 e−04        | 3.87  | 0.41 |
| 3  | 3.3307 e−15      | 1.87  | 0.092| 17 | 2.16 e−04        | 3.90  | 0.56 |
Conclusions and Recommendations:
In this study, numerical solutions are obtained for linear (FIDEs) of the first order and second kind under condition, using Touchard polynomials, and different degrees for purpose of comparing. This method reduces the (FIDEs) into a set of algebraic equations. It's worth noting that one of the important features of this method is that the Touchard coefficients of the solutions are found easily by using PC programs. Also, another advantage is the obtaining solution is polynomials of the degree equal or less than selected n. However, the solution converges rapidly to the exact solution when n increases. The comparison between the absolute errors for four test examples and those methods included in (11, 16, 17 and 18), shows that the accuracy of the current method is almost similar or better than those of the existing methods. As a future work, the current method can also applied to the system of linear (FIDEs), because it is effective and applicable for the linear and nonlinear for these kinds of equations and the results obtained support this claim. Because the solutions obtained here are approximate solutions, it is expected in some examples that the absolute error increases when γ approaches 1 in the interval [0, 1] as in examples 1 and 2.

All the methods referred to in the introduction to this study are approximate numerical methods that have been used to solve the Fredholm integro-differential equations that are difficult to solve analytically. These methods have been used to solve them numerically. The pros of these methods are to obtain approximate solutions and the possibility of writing algorithms for solutions in these methods. Programming these algorithms on personal computers by writing computer programs to identify unknown values and then the possibility of comparing the results obtained in these methods by graphs. The cons of these methods are the existence of errors in the accuracy of the results in reaching the approximate solutions.

Author's declaration:
- Conflicts of Interest: None.
- I hereby confirm that all the Figures and Tables in the manuscript are mine. Besides, the Figures and images, which are not mine, have been given the permission for re-publication attached with the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in Wasit University.

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حل عددي لمعادلة فريدهولم التفاضلية التكاملية الخطية باستخدام متعددة حدود تشارد.

جليل طلبة عبدالله
قسم الإحصاء، كلية الإدارة والاقتصاد، جامعة واسط، واسط، العراق

الخلاصة:
تم تقديم طريقة جديدة تستند إلى متعددة حدود تشارد لحل العددي لمعادلات فريدهولم التفاضلية التكاملية من المرتبة الأولى والثاني مع الشرط. تم الحصول ببساطة على متعددة حدود تشارد وتكاملها. وعُدت في أن الطرق الهمتية وتبني تقنيات تطبيقها ببعض الأمثلية العددية. تم مقارنة النتائج التي تم الحصول عليها مع النتائج المعروفة الأخرى.

الكلمات المفتاحية: معادلة فريدهولم، معادلة تفاضلية تكاملية، متعددة تشارد، حل العددي.