News on Penguins

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Abstract. We summarize recent theoretical developments in the field of radiative and semileptonic penguin decays.

PACS: 13.20.He, 12.38.Bx, 12.39.St. MZ-TH/11-32

AMBIGUITY OF THE NEW PHYSICS SCALE

Within the indirect search for New Physics (NP) there is an ambiguity of the new physics scale. In the model-independent approach using the effective electroweak hamiltonian, the contribution to one specific operator $O_i$ can be parametrized via

$$\left(\frac{C_{i,SM}}{M_W} + \frac{C_{i,NP}}{\Lambda_{NP}}\right) \times \mathcal{O}_i$$

where the first term represents the SM contribution at the electroweak scale $M_W$ and the second one the NP contribution with an unknown coupling $C_{i,SM}$ and an unknown NP scale $\Lambda_{NP}$.

The radiative and semi-leptonic penguin modes, $b \to s\gamma$ and $b \to s\ell^+\ell^-$, are flavour changing neutral current (FCNC) processes and, thus, are highly sensitive for new degrees of freedom via virtual effects (for reviews, see [1, 2, 3]). The non-existence of large NP effects in flavour observables in general [4, 5] implies the famous flavour problem, namely why FCNC are suppressed. Either the mass scale of the new degrees of freedom, $\Lambda_{NP}$, is very high or the new flavour-violating couplings, $C_{i,NP}$, are small for (symmetry?) reasons that remain to be found. For example, assuming generic new flavour-violating couplings of $O(1)$, the present data on $K-\bar{K}$ mixing implies a very high NP scale of order $10^3-10^4$ TeV depending on whether the new contributions enter at loop- or at tree-level. In contrast, theoretical considerations on the Higgs sector, which is responsible for the mass generation of the fundamental particles in the SM, call for NP at order 1 TeV. As a consequence, any NP below the 1 TeV scale must have a non-generic flavour structure. In addition, also the present electroweak data indicate a slightly higher NP scale, a data-driven problem known as little hierarchy problem.

The present measurements of $B$ decays, especially of FCNC processes, already significantly restrict the parameter space of NP models. In general such bounds from flavour physics are model-dependent, but often much stronger than the ones derived from other measurements. In any case, the indirect flavour information will be most valuable when the general nature and the mass scale of NP will be known.

THE INCLUSIVE DECAY $\bar{B} \to X_s\gamma$

Perturbative contributions: Among the rare decay modes, the inclusive decay $\bar{B} \to X_s\gamma$ is the most important one, because it is theoretically well-understood and at the same time it has been measured extensively at the $B$ factories. While non-perturbative corrections to this decay mode are subleading and recently estimated to be well below 10% [6], perturbative QCD corrections are the most important corrections. Within a global effort, a perturbative QCD calculation to the next-to-next-to-leading-logarithmic order level (NNLL) has quite recently been performed and has led to the first NNLL prediction of the $\bar{B} \to X_s\gamma$ branching fraction [7] with a photon cut at $E_\gamma = 1.6$ GeV (including the error due to nonperturbative corrections):

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\text{NNLL}} = (3.15 \pm 0.23) \times 10^{-4}.$$
This result is based on various highly-nontrivial perturbative calculations \[8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\]. The combined experimental data leads to (Heavy Flavor Averaging Group (HFAG) \[19\])

\[
\mathcal{B}(\bar{B} \to X_s\gamma) = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4},
\]

(3)

where the first error is combined statistical and systematic, and the second is due to the extrapolation in the photon energy. Thus, the SM prediction and the experimental average are consistent at the 1.2\sigma level. This is one important example that the CKM theory is not only confirmed by the data entering into the CKM unitarity fit, but also by many additional flavour mixing phenomena.

**Nonperturbative contributions:** It was noted long ago \[20\], that there is no local OPE for the inclusive decay $\bar{B} \to X_s\gamma$ if one considers operators beyond the leading electromagnetic dipole operator $\mathcal{O}_7$. Then, there are so-called resolved photon contributions which contain subprocesses in which the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex \[6\]. Only recently, a systematic analysis \[6\] of all resolved photon contributions related to other operators in the weak Hamiltonian has established this breakdown of the local OPE within the hadronic power corrections as a generic result. Clearly, estimating such nonlocal matrix elements is very difficult, and an irreducible theoretical uncertainty of $\pm(4 - 5)\%$ for the total $CP$ averaged decay rate, defined with a photon-energy cut of $E_\gamma = 1.6$ GeV, remains \[6\]. This result strongly indicates that the theoretical efforts for the $B \to X_s\gamma$ mode have reached the nonperturbative boundaries, but it also reconfirms the dominance of the perturbative contributions.

There is another positive result induced by this new analysis: Until recently, it was believed that the long-distance contributions from the intermediate $u$-quark in the penguin loops are critical. They are suppressed in the $B \to X_s\gamma$ mode by the CKM matrix elements. In $\bar{B} \to X_d\gamma$, there is no CKM suppression, and one must account for the nonperturbative contributions that arise from the $u$-quark loops. A simple dimensional estimate leads to an uncertainty of at least 10%. However, this contribution vanishes in the total $CP$-averaged rate of $\bar{B} \to X_s\gamma$ at order $\Lambda/m_b$ \[3\]. This result applies to the total rate of $\bar{B} \to X_d\gamma$ as well. Thus, there is no power correction due to the $u$-quark loops in the total rate of $\bar{B} \to X_d\gamma$ at order $\Lambda/m_b$, which implies that the $CP$-averaged decay rate of $\bar{B} \to X_d\gamma$ is as theoretically clean as the decay rate of $\bar{B} \to X_s\gamma$ \[1\]. The present NLL-prediction reads \[21, 22\] (for $E_\gamma > 1.6$ GeV):

\[
\mathcal{B}(\bar{B} \to X_d\gamma) = \left(1.38 \pm 0.14 \pm 0.02_{ CKM} \pm 0.09_{ param} \pm 0.05_{ scale}\right) \times 10^{-5}.
\]

(4)

The large CKM-uncertainty is due to $V_{td}$. The uncertainty due to the charm-scheme dependence can be reduced significantly by a NNLL analysis in analogy to $\bar{B} \to X_s\gamma$. Already the NLL prediction in combination with the first real measurement of this inclusive mode \[23\] leads to interesting bounds on NP \[24\].

But there is also a negative consequence: Without CP-averaging the non-perturbative the $u$-quark contribution survives and dominates the direct $CP$ asymmetry in $\bar{B} \to X_d\gamma$. Perturbative contributions exhibit a triple suppression; one has an $\alpha_s$ suppression in order to have a strong phase, a Cabibbo suppression of $\lambda^2$, and a GIM suppression of $(m_c/m_b)^2 \approx \Lambda/m_b$ reflecting the fact that in the limit $m_c \to 0$ the direct CP violation vanishes. Thus, the non-perturbative (resolved) $u$-quark contribution is enhanced by a factor $1/\alpha_s$ compared to the perturbative contributions. Model estimates of the resolved contribution lead to \[25\]

\[-0.6\% < \mathcal{A}(\bar{B} \to X_s\gamma)^{SM} < 2.8\% ,
\]

(5)

which covers most of the experimentally allowed range \[19\]: $\mathcal{A}(\bar{B} \to X_s\gamma) = -(1.2 \pm 2.8)\%$. But the untagged direct $CP$ asymmetry survives. All resolved contributions cancel at order $\Lambda/m_b$, so we still have the zero-prediction for the untagged $CP$ asymmetry for $\bar{B} \to X_s+d\gamma$ \[26, 27, 21\]. This prediction is directly based on the CKM-unitarity and on the smallness of the U-spin breaking parameters $(m_c/m_b)^2$. Thus, it represents a clean test whether new $CP$ phases are active or not.

There are still some open issues in the decay $\bar{B} \to X_s\gamma$ which ask for further study. First, there are three-loop matrix elements of the leading four-quark operators, which have first been calculated within the so-called large-$\beta_0$ approximation \[17, 28, 29\]. A calculation that goes beyond this approximation by employing an interpolation in the charm quark mass $m_c$ from $m_c > m_b$ to the physical $m_c$ value has been presented in Refs. \[18, 30\]. In this interpolation the $\alpha_s^2\beta_0$ result \[17\] is assumed to be a good approximation for the complete $\alpha_s^2$ result for vanishing charm mass. It is this part of the NNLL calculation which is still open for improvement. Indeed, there are several collaborations presently working on this issue. A complete calculation of the three-loop matrix elements of the four-quark operators $\mathcal{O}_{1,2}$ for vanishing charm mass is in progress and will cross-check the error estimate due to the interpolation \[31, 32\].
Partial results are already available \cite{33}. In addition, there is an effort \cite{34} to calculate the matrix elements for arbitrary $m_c$ directly. All these efforts will finally eliminate the 3\% uncertainty due to the interpolation within the present NNLL prediction \cite{7}.

Second, in the measurement of the inclusive mode $\bar{B} \rightarrow X_s \gamma$ one needs cuts in the photon energy spectrum to suppress the background from other $B$ decays which induces additional sensitivities to non-perturbative physics. These shape-function effects were taken into account in the experimental analysis. The corresponding theoretical uncertainties due to this model dependence are reflected in the extrapolation error of the experimental results. The extrapolation is done from the experimental energy cut values down to 1.6 GeV. But a cut around 1.6 GeV might not guarantee that a theoretical description in terms of a local OPE is sufficient because of the sensitivity to the additional scale $\Delta = m_b - 2E_\gamma$ \cite{35}. A multiscale OPE with three short-distance scales $m_b$, $\sqrt{m_b} \Lambda$, and $\Delta$ has been proposed to connect the shape function and the local OPE region. Recently, such additional perturbative cutoff-related effects have been calculated to NNLL precision by the use of SCET methods \cite{36,37,38}. Such perturbative effects due to the additional scale are negligible at 1.0 GeV but of order 3\% at 1.6 GeV \cite{36}. The size of these effects at 1.6 GeV is similar to the 3\% higher-order uncertainty in the present NNLL prediction. However, the numerical consistency of the SCET analysis has recently been questioned \cite{39}. Far away from the endpoint ($E_\gamma = 1.6$ GeV), the logarithmic and nonlogarithmic terms cancel; this kind of cancellation was already observed in Ref.\cite{40}. Within the resummation of the cutoff-enhanced logarithms this feature leads to an overestimate of the $O(\alpha_s^2)$ terms \cite{39}. Further work is needed to clarify this issue.

Third, the $\bar{B} \rightarrow X_s \gamma$ decay rate is normalized to the charmless semileptonic rate in order to separate the charm dependence. Then the quantity $C = |V_{ub}|^2/|V_{cb}|^2 \times \Gamma(B \rightarrow X_s e \bar{\nu})/\Gamma(B \rightarrow X_s e \bar{\nu})$ enters the theoretical prediction. Recently, a scheme dependence in the determination of the prefactor $C$ was noticed \cite{41}, which is around 3\%, thus within the perturbative uncertainty \cite{39}. The two determinations in the $1S$ scheme \cite{42} and in the kinetic scheme \cite{41} differ through renormalization schemes, methodology, and experimental input. An update of the analysis within the $1S$ scheme might resolve part of the corresponding uncertainty.

**THE INCLUSIVE DECAY $B \rightarrow X_s \ell^+ \ell^-$**

**Perturbative contributions:** This inclusive mode is also dominated by perturbative contributions, if one eliminates $c\bar{c}$ resonances with the help of kinematic cuts. In the so-called ‘perturbative $q^2$-windows’ below and above the resonances, namely in the low-dilepton-mass region $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$, and also in the high-dilepton-mass region with $q^2 > 14.4\text{GeV}^2$, theoretical predictions for the invariant mass spectrum are dominated by the perturbative contributions. QCD corrections are calculated to NNLL precision \cite{43,44,45,46,47,48,49,50,51,52}. More recently electromagnetic corrections were calculated: NLL quantum electrodynamics (QED) two-loop corrections to the Wilson coefficients are of $O(2\%)$ \cite{49}. Also, in the QED one-loop corrections to matrix elements, large collinear logarithms of the form $\log(m_{\gamma\gamma}^2/m_{\ell\ell}^2)$ survive integration if only a restricted part of the dilepton mass spectrum is considered. These collinear logarithms add another contribution of order $+2\%$ in the low-$q^2$ region for $B \rightarrow X_s \mu^+ \mu^-$. For the high-$q^2$ region, one finds $-8\%$ \cite{52}.

**Quark-Hadron-Duality:** The integrated branching fraction of $\bar{B} \rightarrow X_s \ell^+ \ell^-$ is dominated by the resonance background which exceeds the nonresonant charm-loop contribution by two orders of magnitude. As has been recently noticed \cite{54}, this feature should not be misinterpreted as a striking failure of global parton-hadron duality which postulates that the sum over the hadronic final states, including resonances, should be well approximated by a quark-level calculation \cite{55}. Crucially, the charm-resonance contributions to the decay $\bar{B} \rightarrow X_s \ell^+ \ell^-$ are expressed in terms of a phase-space integral over the absolute square of a correlator. For such a quantity global quark-hadron duality is not expected to hold. Nevertheless, local quark-hadron duality (which, of course, also implies global duality) may be reestablished by resumming Coulomb-like interactions \cite{54}.

**THE EXCLUSIVE DECAY $B \rightarrow K^{(*)} \ell^+ \ell^-$**

**QCD factorization:** The method of QCD factorization (QCDF) \cite{56} and its field-theoretical formulation SCET \cite{57,58} form the basis of the up-to-date predictions of exclusive $B$ decays. There is also a factorization formula for the exclusive semileptonic $B$ decays, such as $B \rightarrow K^* \ell^+ \ell^-$. The hadronic form factors can be expanded in the small ratios $\Lambda/m_b$ and $\Lambda/E$, where $E$ is the energy of the light meson. If we neglect corrections of order $1/m_b$ and $\alpha_s$, the seven a priori independent $B \rightarrow K^*$ form factors reduce to two universal form factors $\xi_{\perp}$ and $\xi_{\parallel}$ \cite{61}. These theoretical
simplifications of the QCDF/SCET approach are valid to the kinematic region in which the energy of the $K^*$ is of the order of the heavy quark mass, that is, $q^2 \ll m_b^2$. Thus, factorization formula applies well in the dilepton mass range 1 GeV$^2 < q^2 < 6$ GeV$^2$.

**Full angular analysis:** In the near future, a full angular analysis will become possible. This rich information allows for the design of observables with specific NP sensitivity and reduced hadronic uncertainties [62, 63]. These observables are constructed in such a way that the soft form factor dependence cancels out at leading order in $\alpha_s$ and in $\Lambda/m_b$ for all low dilepton masses, and they have much higher sensitivity to new right-handed currents than observables that are already accessible via the projection fits [64, 62, 63]. In these optimized observables, the unknown $\Lambda/m_b$ corrections are the source of the largest uncertainty. Further detailed NP analyses of such angular observables have been presented in Refs. [65, 66]. A full angular analysis provides high sensitivity to various Wilson coefficients, but the sensitivity to new weak phases is restricted, mainly due to large experimental uncertainties [67, 63]. Observables defined at high-$q^2$ also represent very interesting observables due to the fact that $\Lambda/m_b$ corrections can be estimated with the help of heavy-quark effective theory (HQET), but formfactors at high-$q^2$ have to be presently extrapolated from the low-$q^2$ region [68, 69, 70].

**ACKNOWLEDGMENTS**

TH thanks the organizers of the conference for the interesting and valuable meeting and the CERN theory group for its hospitality during his regular visits to CERN where part of this work was written. TH is also thankful to Leonardo Vernazza for a careful reading of the manuscript.

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