Kineto-Elastodynamic Analysis of Watt’s Mechanism Using ANSYS

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Abstract

Kineto-elastodynamic analysis deals with the study of kinematic and dynamic analysis of a mechanism by considering the elasticity and distribution of mass in link. In present paper finite element method for kineto-elastodynamic analysis of high speed planar four bar mechanism has been presented. Based on the work done by previous researchers, well known displacement finite element method has been used to develop the mass and stiffness properties of an elastic linkage. The mechanism is modeled as beam element and its equation of motion is expressed in a matrix form. An accurate and efficient method to determine the strain in different link of Watt’s mechanism using ANSYS has been presented, considering flexibility of a link at high speed. The results obtained using ANSYS have been validated by comparing them with experimental results available in literature for four bar mechanism, and are found to be in close agreement with experimental results. The same methodology has been further applied for analysis of Watt’s mechanism.

Keywords: Kineto-elastodynamic analysis, Watt’s mechanism, FEM, Elastic links

1. Introduction

The purpose of this investigation is to focus on the simulation model for the kineto-elastodynamic behavior of high-speed planar four bar and Watt’s mechanism. It is well-known that the dynamic analysis of mechanisms operating at high speed cannot overlook the effect of mass distribution and link elastic flexibility, as well as the effect of backlashes and friction in joints. These effects may affect the dynamic response in particular the output link.

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motion to an extent that the mechanism may fail to perform their desired task adequately. The researcher interest is the development of preferably simple mathematical model which is able to simulate the dynamic behavior of the mechanisms. Since this model make it possible to relate the design and operational parameters of mechanisms to their actual dynamic behavior. Most of the researchers have worked on theoretical analysis of four bar mechanism with few link elements i.e. two or three elements per link to avoid mathematical complexity and results are compared with experimental work. The limitation on number of elements has been overcome by using ANSYS software.

It is essential for learners to understand the various terms used in kinematic, static and dynamic analysis of mechanisms. In this concern Erdman and Sandor [1] have explained the basic terminology used for different analysis of the mechanism. In a simple dynamic model for crank and slider mechanism Sadler and Sandor [2] have assumed concentrated lump mass at the rigid end of the coupler. This lumped mass approach was extended by Sadler and Sandor for the planar four bar mechanism with elastic links [3].Midha et al. [4] have presented the dynamic model of high speed four bar planar mechanism using finite element approach. The Link has been considered as a beam element with uniform cross section. In the model, the mass and stiffness matrix have been developed using displacement finite element method. The assumptions made by them are: (i) the elastic motion of links does not affect the rigid body kinematics of the mechanism (ii) each link of the mechanism is one beam element (iii) the crank rotates with uniform speed and (iv) the damping present in the system is negligible. These assumptions had reduced the complexity in mathematical modeling. Moreover, the various steps for finite element analysis of mechanism have been explained by the authors. To represent the coupling between the axial and transverse displacement the geometrical stiffness matrix has been computed based on large strain theory. Analytical result of studied model was presented for basic four bar planar mechanism. It was observed that stresses and strain produced by using quasi-static method found to be much lower than the traditional quasi-static approach. For the validation of the model proposed [5] and Turcic et al. [6] have conducted the experiments for 32 different speed of input crank. In the experimental set-up, all links were constructed from aluminum.

2. Finite element model:

Assumption made in analysis of planar four bar mechanism at a fixed position, states that structure is made up of discrete members. In the structure, each constituent member is considered as a beam and hence theory of bending is applicable. Effects of rotational inertia and shear information have been not taken into consideration.

3. Elastic beam element in plane:

A general beam element, representing a link of a mechanism, is shown in Fig. 1, other links are not shown the Fig. 1. The two axis of reference are; 1. The fixed (OXY) and, 2. The rotated (Oxy) axis and both have a common origin O. The x axis of the rotated frame is always parallel to the rigid body position of the beam element axis throughout its motion. Dotted line in Fig. 1 represents the elastically deformed beam position and full line shows corresponding rigid body position. Generalized nodal displacement co-ordinate u₁ to u₆ are used to describe elastic deformation of beam. These displacements shown in their positive directions with reference to the rigid body position of the beam (link), in Fig. 1 also locate the deformed position P’ and Q’ of the end points P and Q.

The acceleration vector given by equation 1 is obtained considering link as beam element having planar motion.

\[
\{\ddot{u}_a\} = \{\ddot{u}_r\} + \{\ddot{u}\} + \{\ddot{a}_n\} + \{\ddot{a}_c\} + \{\ddot{a}_t\} \tag{1}
\]

where, the vectors from left to right represent the absolute, rigid body, generalized relative to the rigid body position of link, normal, coriolis and tangential accelerations respectively.

In equation (1) the product terms in vectors \{\ddot{a}_n\}, \{\ddot{a}_c\} and \{\ddot{a}_t\} are considered small compared to corresponding terms in \{\ddot{u}_r\} and \{\ddot{u}\}. Hence, these terms are not considered and the resulting equation may be expressed as
Similarly, it can be shown that
\[ \{ \ddot{u}_a \} = \{ \ddot{u}_r \} + \{ \ddot{u} \} \]  
(2)

Use of linear shape function has been made to interpolate the unknown displacement field motion the beam element. The interpolation becomes more accurate with increase in number of node elements within the beam.

4. Mass and stiffness matrices of element:

The equation of motion of the elastic beam element in Fig. 1 is expressed by Lagrange’s equation.
\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{u}_j} \right) - \left( \frac{\partial T}{\partial u_j} \right) + \left( \frac{\partial U}{\partial \dot{u}_j} \right) = \bar{Q}_i, \quad i = 1, 2, \ldots, 6 \]  
(4)

Where, \( \bar{Q}_i \) represents the generalized forces acting in the direction of generalized coordinators and have no potential.

Considering the strains associated with displacement functions and neglecting those due to temperature variations and any strain initially present, the strain energy in matrix form is written as:
\[ U = \frac{1}{2} \{ U_i \}^T \left[ K \right] \{ U_i \}, \quad i = 1, 2, \ldots, 9 \]  
(5)
Then [5],

\[
\mathbf{\bar{m}} = \rho A L \begin{bmatrix}
\frac{1}{3} \\
0 \\
0 \\
-\frac{13}{420} \\
0 \\
\frac{11}{210} L \\
\frac{1}{105} L^2 \\
0 \\
0 \\
\frac{9}{70} \\
\frac{13}{420} L \\
0 \\
-\frac{13}{420} L \\
-\frac{1}{140} L^2 \\
0 \\
0 \\
0 \\
\frac{13}{35} L \\
\frac{35}{105} \\
0 \\
-\frac{11}{210} L \\
\frac{1}{105} L^2 \\
\end{bmatrix}
\]

\[
\mathbf{\bar{k}} = \begin{bmatrix}
\frac{EA}{L} & 0 & \frac{12EI}{L^2} & 4EI \\
0 & \frac{6EI}{L^2} & -\frac{6EI}{L^2} & 0 \\
-\frac{EA}{L} & 0 & \frac{12EI}{L^2} & 0 \\
0 & \frac{6EI}{L^2} & -\frac{2EI}{L^2} & 0 \\
0 & 0 & \frac{4EI}{L^2} & 0 \\
\end{bmatrix}
\]

With the help of Lagrange’s equations, the equation of motion for the beam element may be expressed as

\[
\left[ \mathbf{\bar{m}} \right] \ddot{\mathbf{u}} + \left[ \mathbf{\bar{k}} \right] \mathbf{u} = \{Q\}
\]

(6)

5. Assembly of the element:

Mass and system matrix for elements is their local co-ordinate systems have been formulated in the previous section. Now system matrices for the whole mechanism are found by defining a global coordinate system for a given mechanism.

In the four bar mechanism shown in Fig. 2, for purpose of analysis, joints at all links are considered as nodes and every link is taken to be an element. Mass and stiffness matrices for all the elements have been systematically superposed to developed mass and stiffness matrices of the mechanism, which is further used for finite element analysis.

Afterwards solve the coupled differential equation of motion with help of modal analysis. Solutions of the equation give displacement of each element at a particular instant which are used to calculate strain and stress at desired point on links.

If the system oriented coordinates \{U\} are used as the generalized coordinates to describe the structural deformation of the linkage from its rigid body position, as in Fig. 2. In first, second and the third terms on the left Hand side of Lagrange’s equation is reduce to \( \sum_{j=l}^{n} M_{ij} \ddot{U}_{uj}, 0 \) and \( \sum_{j=l}^{n} K_{ij} U_{j} \) respectively. In matrix form, the equations of motion are

\[
\left[ M \right] \ddot{u} + \left[ K \right] u = \{Q\}
\]

(7)

In the absence of damping forces in the mechanism and external forces on the follower, the equation of motion may be written in matrix form as [5]

\[
\left[ M \right] \ddot{\mathbf{u}} + \left[ C \right] \mathbf{u} + \left[ K \right] \mathbf{u} = -\left[ M \right] \ddot{\mathbf{F}}
\]

(8)
6. Modeling and simulation of mechanism:

A four-bar linkage is a case of multi-body dynamics. In most cases the analysis is based on the assumption that the links are either rigid or elastic (flexible). The schematic mesh model of the mechanism is shown in Fig. 3 and the Geometrical and material properties details of the links are given as bellow.

Fig. 2. Four bar mechanism with displacement and three elements

Fig. 3. Mesh model of four bar mechanism
Table 1: Dimensions of four bar mechanism [6]

| Parameters                  | Fixed link (1) | Crank (2) | Coupler (3) | Follower (4) |
|-----------------------------|----------------|-----------|-------------|--------------|
| Length (mm)                 | 250            | 110       | 280         | 260          |
| C/S Area (mm$^2$)           | -              | 108       | 40          | 40           |
| Area moment of Inertia (mm$^4$) | -              | 160       | 9           | 9            |
| Density = 2770 kg/m$^3$     |                |           | Crank speed = 308.44 rpm |

The links dimension of Watt’s mechanism as mention in Table 2 and meshed model of Watt’s mechanism are as shown in Fig. 4.

Table 2: Dimensions of Watt’s mechanism

| Parameters                  | Link 1 | Link 2 | Link 3 | Link 4 | Link 5 | Link 6 |
|-----------------------------|--------|--------|--------|--------|--------|--------|
| Length (mm)                 | 250    | 110    | 280    | 260    | 270    | 200    |
| C/S Area (mm$^2$)           | -      | 108    | 40     | 40     | 45     | 40     |
| Area moment of Inertia (mm$^4$) | -      | 160    | 9      | 9      | 10     | 9      |
| Modulus of Elasticity, $E = 7.1 \times 10^4$ MPa |        |        |        |        |        |        |
| Density = 2770 kg/m$^3$     |        |        |        |        |        |        |
| Crank speed = 32.3 rad/sec  |        |        |        |        |        |        |

![Fig. 4 Mesh model of Watt’s mechanism](image-url)
7. Results and discussion:

To obtain even moderately accurate result, it is necessary to divide the links into more than one element. This type of analysis has been carried out in ANSYS software by taking more numbers of elements per link. In order to reduce computational time in four bar mechanism, only the coupler link has been considered as flexible and remaining links are assumed to be rigid. Figure 5 and Figure 6 show angular velocity and angular acceleration of coupler and rocker links of four bar mechanism respectively. Strain developed at middle of coupler in four bar mechanism has been shown in Fig. 7.

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**Fig. 5** Angular velocity versus crank angle by ANSYS

**Fig. 6** Angular acceleration versus crank angle by ANSYS

**Fig. 7** Strain in coupler versus crank angle by ANSYS
Table 3: Root Mean Square of strain in coupler link

|                     | Experimental Results | ANSYS Results |
|---------------------|----------------------|---------------|
| RMS Value           | 0.000134             | 0.000138      |

Thereafter, flexible analysis of four bar mechanism was done in ANSYS. By comparing the RMS (Root Mean Square) value (Table 3) of experimental result mention in literatures [6] and result of ANSYS analysis and found to be in close agreement. After these work same methodology applied for analysis of flexible Watt’s mechanism in ANSYS by considered link 3 and link 5 is flexible. Result of this analysis has been presented in Fig. 8 and Fig. 9.

8. Conclusions:

The main goal of this study has been modeling of a flexible four-bar and Watt’s mechanism running at high-speed using ANSYS and its analysis. Although this problem has been addressed for many years, there were, and indeed are still, areas where more research is needed. The first purpose of this study was to develop a set of routines to deliver the steady state solution quickly and efficiently. The first problem addressed was how to obtain an accurate result using the FEM by minimum number of elements per link. Model of four bar mechanism has been developed in ANSYS software and result was compared to experimental result mention in literature and found to be closer. After this same methodology applied to Watt’ mechanism and found out the strain produce in the middle of link 3 and link 5. It has also been observed that the number of element per link dose not effect accuracy of results obtained.

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