Renormalization of nonequilibrium dynamics in FRW cosmology

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We derive the renormalized nonequilibrium equations of motion for a scalar field and its quantum back reaction in a conformally flat Friedmann-Robertson-Walker universe. We use a fully covariant formalism proposed by us recently for handling numerically and analytically nonequilibrium dynamics in one-loop approximation. The system is assumed to be in a conformal vacuum state initially. We use dimensional regularization; we find that the counter terms can be chosen independent of the initial conditions though the divergent leading order graphs do depend on them.

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1. INTRODUCTION

Nonequilibrium processes in cosmology have been considered recently by various authors. The main interest has been centered around the possible inflationary period of the universe (see e.g. [1–3]) and the subsequent reheating [4,5]. It has already been found by considering the parametric resonance [6] associated with oscillations of the inflaton field and by exact computations including back reaction both in Minkowski space [7] and in an expanding universe [8] that the time-dependent inflaton field produces particles or classical fluctuations preferentially at low momenta and not in a distribution corresponding to thermal equilibrium. This process of particle production has therefore been termed preheating [6,10].

The equations of motion for nonequilibrium systems have been presented by various authors [11–13] using the CTP formalism introduced by Schwinger [14] and Keldysh [15]. Their application to inflation within a conformally flat FRW universe has been initiated by Ringwald [16–18]; they have been recently implemented numerically in [19] and [20]. Similar computations have been performed recently in configuration space for the case that the fluctuations are treated as classical ones [21,22]. Again fluctuations with rather low momenta are strongly excited, thus justifying the classical approximation. Apart from such exact numerical computations there exist also various analyses based on analytical approximations to the solution of the mode equations [9,23–27].

We have proposed recently a computation scheme for nonequilibrium dynamics which has two aspects: on the one hand it separates cleanly the divergent and the finite contributions of the quantum fluctuations in one-loop approximation, and so is attractive for numerical computations. On the other hand it leads, by the analysis of the divergent leading order contributions, to a simple formulation of the renormalized equations of motion. The fact that the divergent contributions are removed from the numerical computation allows a free choice of regularization. We have chosen dimensional regularization since it is easier to handle in the presence of quartic and quadratic divergencies than e.g. Pauli-Villars regularization. This scheme has been applied and implemented numerically for scalar fields [28], fermion fields and for the SU(2) Higgs model [29]. We will present here the renormalized equations of motion for a scalar field in flat Friedmann-Robertson-Walker (FRW) cosmology.

We will not attempt here a numerical implementation. There exist already several numerical analyses based on various assumptions on the initial conditions and on the parameters of the theory. We do not expect that our formulation will lead to major differences in the qualitative features of the results. Nevertheless, we think that in the presence of quartic and quadratic divergencies it is important to formulate the renormalization scheme in a way that is fully covariant and independent of the initial conditions. It turns out that the resulting formalism represents at the same time a rather attractive numerical computation scheme.

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II. FRW COSMOLOGY

We consider the Friedmann-Robertson-Walker metric with curvature parameter $k = 0$, i.e. a spatially isotropic and flat space-time. The line-element is given in this case by

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2.$$  \hspace{1cm} (1)

The time evolution of the $a(t)$ is governed by Einstein’s field equation

$$G_{\mu\nu} + \alpha H^{(1)}_{\mu\nu} + \beta H^{(2)}_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa \langle T_{\mu\nu}\rangle,$$  \hspace{1cm} (2)

with $\kappa = 8\pi G$. The Einstein curvature tensor $G_{\mu\nu}$ is given by

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R.$$  \hspace{1cm} (3)

The Ricci tensor and the Ricci scalar are defined as

$$R_{\mu\nu} = R^\lambda_{\mu\lambda\nu},$$  \hspace{1cm} (4)

$$R = g^{\mu\nu}R_{\mu\nu}. $$  \hspace{1cm} (5)

where

$$R^\lambda_{\alpha\beta\gamma} = \partial_\gamma \Gamma^\lambda_{\alpha\beta} - \partial_\alpha \Gamma^\lambda_{\gamma\beta} \Gamma^\gamma_{\alpha\beta} - \Gamma^\lambda_{\alpha\sigma} \Gamma^\gamma_{\sigma\beta}.$$  \hspace{1cm} (6)

The terms $H^{(1)}_{\mu\nu}$ and $H^{(2)}_{\mu\nu}$ arise if terms proportional to $R^2$ and $R^\mu\nu R_{\mu\nu}$ are included into the Hilbert-Einstein action. If space-time is conformally flat, these terms are related by

$$H^{(2)}_{\mu\nu} = \frac{1}{3} H^{(1)}_{\mu\nu},$$  \hspace{1cm} (7)

so that we can set $\beta = 0$ in (2) without loss of generality \[30\]. We also replace $H^{(1)}_{\mu\nu}$ by $H_{\mu\nu}$ in the following.

These terms are usually not considered in standard cosmology. They are included here, as well as the cosmological constant, only for the purpose of renormalization; they will absorb divergencies of the energy momentum tensor. So in principle they should appear on the right hand side as counter terms for the energy momentum tensor; they are related to the coefficients of these counter terms by $\Lambda = \kappa \delta \tilde{\Lambda}$ and $\alpha = \kappa \delta \tilde{\alpha}$. We will need a further counter term proportional to $G_{\mu\nu}$, which can be considered as a wave function renormalization for the gravitational field; either we have to replace $G_{\mu\nu}$ by $ZG_{\mu\nu}$ on the left hand side of the Einstein equations or to introduce a counter term $\delta \tilde{Z}G_{\mu\nu}$ for the energy momentum tensor. Again both alternatives are related by $Z = 1 + \kappa \delta \tilde{Z}$. Writing the renormalized equation (2) in the form

$$\left(\frac{1}{\kappa} + \delta \tilde{Z}\right) G_{\mu\nu} + (\tilde{\alpha} + \delta \tilde{\alpha}) H_{\mu\nu} + (\tilde{\Lambda} + \delta \tilde{\Lambda}) g_{\mu\nu} = T^{\text{ren}}_{\mu\nu},$$  \hspace{1cm} (8)

we can identify $\delta \tilde{Z}$ as a correction to the gravitational coupling via $\kappa^{-1} \rightarrow \kappa^{-1} + \delta \tilde{Z}$. As usual, we can reduce the Einstein field equations to an equation for the time-time component and one for the trace of $G_{\mu\nu}$, the Friedmann equations

$$G_{tt} + \alpha H_{tt} + \Lambda = -\kappa T_{tt},$$  \hspace{1cm} (9)

$$G^\mu_\mu + \alpha H^\mu_\mu + 4\Lambda = -\kappa T^\mu_\mu. $$  \hspace{1cm} (10)

For the line element (1) the various terms take the form \[30\]

$$G_{tt}(t) = -3H^2(t),$$  \hspace{1cm} (11)

$$G^\mu_\mu(t) = -R(t), $$  \hspace{1cm} (12)

$$H_{tt}(t) = -6 \left[ H(t)\dot{H}(t) + H^2(t)R(t) - \frac{1}{12}R^2(t) \right],$$  \hspace{1cm} (13)

$$H^\mu_\mu(t) = -6 \left[ \dot{R}(t) + 3H(t)\dot{H}(t) \right]. $$  \hspace{1cm} (14)

\[1\]See \[37\] for their precise definitions.
with the curvature scalar
\[ R(t) = 6 \left[ \dot{H}(t) + 2H^2(t) \right], \]
and the Hubble expansion rate
\[ H(t) = \frac{\dot{a}(t)}{a(t)}. \]

### III. NONEQUILIBRIUM EQUATIONS FOR THE SCALAR FIELD

The Lagrangian density of a $\phi^4$-theory in curved space-time is given by
\[ \mathcal{L} = \sqrt{-g} \left\{ \frac{1}{2} \partial_{\mu} \Phi \partial^{\mu} \Phi - \frac{1}{2} m^2 \Phi^2 - \frac{\xi}{2} R \Phi^2 - \frac{\lambda}{4!} \Phi^4 \right\}, \]
where $R(x)$ is the curvature scalar and $\xi$ the bare dimensionless parameter describing the coupling of the bare scalar field to the gravitational background. We split the field $\Phi$ into its expectation value $\phi$ and the quantum fluctuations $\psi$:
\[ \Phi(\vec{x}, t) = \phi(t) + \psi(\vec{x}, t), \]
with
\[ \phi(t) = \langle \Phi(\vec{x}, t) \rangle = \frac{\text{Tr} \Phi \rho(t)}{\text{Tr} \rho(t)}, \]
where $\rho(t)$ is the density matrix of the system which satisfies the Liouville equation
\[ i \frac{d \rho(t)}{dt} = [\mathcal{H}(t), \rho(t)]. \]

The one-loop equation of motion of a scalar field with $\lambda \phi^4$ interaction has been obtained in the FRW universe by Ringwald [16]; we follow closely his formulation. The equation of motion for the classical field is
\[ \ddot{\phi} + 3H \dot{\phi} + (m^2 + \xi R)\phi + \frac{\lambda}{6} \phi^3 + \frac{\lambda}{2} \langle \psi^2 \rangle \phi = 0. \]
The expectation value of the quantum fluctuations $\langle \psi^2 \rangle$ can be expressed as
\[ \langle \psi^2 \rangle = -i G(t, \vec{x}; t, \vec{x}) \]
in terms of the non-equilibrium Green function $G(t, \vec{x}; t', \vec{x}')$ which satisfies
\[ \left[ \frac{\partial^2}{\partial t^2} + 3H \frac{\partial}{\partial t} + a^{-2}(t) \nabla^2 + m^2 + \xi R(t) + \frac{\lambda}{2} \phi^2(t) \right] G(t, \vec{x}; t', \vec{x}') = \frac{i}{a^3(t)} \delta(t, \vec{x}; t', \vec{x}') \].

The boundary conditions for this Green function will be given below. Due to the presence of the term $H(t) \partial / \partial t$ the differential operator on the left hand side of this equation is non-hermitian. It is made hermitian by introducing conformal time and appropriate scale factors. Conformal time is defined as
\[ \tau = \int_0^t dt' \frac{1}{a(t')} . \]
In conformal time the line-element (11) reads
\[ ds^2 = C(\tau)(d\tau^2 - d\vec{x}^2) , \]
where the conformal factor $C(\tau)$ is given by
The Green function is rescaled accordingly via

\[ \tilde{G}(\vec{x}, \tau; \vec{x}', \tau') = a(t)a(t')G(\vec{x}, t; \vec{x}, t'). \]  

We further impose the initial conditions

\[ U_k(0) = 1 ; U_k'(0) = -i\Omega_k(0), \]  

where the primes denote derivatives with respect to conformal time. The two-point-function \( \tilde{G} \) now satisfies

\[ \tilde{G}(\vec{x}, \tau; \vec{x}, \tau') = -\delta(\vec{x}, \tau; \vec{x}', \tau'). \]  

The expectation value of the fluctuation fields is given, therefore, by the fluctuation integral

\[ F(\tau) = \langle \tilde{\psi}^2(\tau) \rangle = -i\tilde{G}(\vec{x}, \tau; \vec{x}, \bar{\tau}) = \int \frac{d^3k}{(2\pi)^3} \frac{|U_k(\tau)|^2}{2\Omega_k}. \]
The unrenormalized equation of motion of the inflaton field reads

\[ \varphi'' + a^2(\tau) \left[ m^2 + \left( \xi - \frac{1}{6} \right) R(\tau) \right] \varphi + \frac{\lambda}{6} \varphi^3 + \frac{\lambda}{2} \varphi^2 \mathcal{F}(\tau) = 0 . \]  

(40)

The regularization of the fluctuation integral and the renormalized form of this equation will be discussed below.

The boundary conditions for the mode functions and the corresponding definition of the fluctuation integral are related to the definition of the initial quantum state and/or density matrix of our nonequilibrium expansion. It would, e.g., look different if we had used the time variable related to the definition of the initial quantum state and/or density matrix of our nonequilibrium expansion. It would, in curved space time. For a classical field it reads [30]

\[ \text{to study.} \]

that the precise initial conditions do not influence too strongly that period of the cosmological expansion one wants artificial. Since there is no solution of principle to this problem one has to hope (or try out by numerical experiments) that the quantum fluctuations. So starting with the conformal vacuum state [30], have already been chosen in [17], they are discussed to some extent in [8]. The initial conditions for the cosmological expansion are even more subtle since even in principle we cannot start from an equilibrium situation, except if the classical field is at a minimum of the effective potential and the energy momentum tensor vanishes there - obviously not an interesting situation. When considering nonequilibrium processes in Minkowski space-time one can imagine that an initial classical field is maintained away from the minimum of the effective potential by a source (like capacitor plates for an electric field) which is switched off at time zero. In such a situation it makes sense to assume that the initial state is the vacuum state corresponding to that initial classical field. In the cosmological context the presence of a nonvanishing classical energy leads already to a cosmological expansion which in turn influences the quantum fluctuations. So starting with the conformal vacuum state or a thermal state constructed on it seems a priori artificial. Since there is no solution of principle to this problem one has to hope (or try out by numerical experiments) that the precise initial conditions do not influence too strongly that period of the cosmological expansion one wants to study.

### IV. THE ENERGY MOMENTUM TENSOR

In order to formulate Einstein’s field equation we have to discuss the energy momentum tensor of the scalar field in curved space time. For a classical field it reads [31]

\[ T_{\mu\nu} = (1 - 2\xi)\phi_{,\mu}\phi_{,\nu} + (2\xi - \frac{1}{2}) g_{\mu\nu} g^{\rho\sigma} \phi_{,\rho}\phi_{,\sigma} - 2\xi \phi_{,\mu}\phi_{,\nu} \]

\[ + 2\xi g_{\mu\nu} \Box \phi - \xi G_{\mu\nu} \phi^2 + \frac{1}{2} m^2 g_{\mu\nu} \phi^2 + \frac{\lambda}{24} g_{\mu\nu} \phi^4 . \]  

(41)

In the conformally flat FRW metric the energy momentum tensor is diagonal. One obtains for its time-time component and its trace

\[ T_{\mu}^{\mu cl} = \frac{\lambda}{3} \phi^2 + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{24} \phi^4 - \xi G_{\mu\nu} \phi^2 + 6\xi H \dot{\phi} \]

\[ T_{\mu}^{\mu} = -\dot{\phi}^2 + 2m^2 \phi^2 + \frac{\lambda}{6} \phi^4 - \xi G_{\mu\nu} \phi^2 + 6\xi (\phi'' + \phi' + 3H \dot{\phi}) . \]  

(42)

We introduce again conformal time and the conformal rescaling of the fields. Furthermore, we include the quantum fluctuations of the field \( \varphi \). We obtain [3]

\[ T_{tt} = \frac{1}{2a^4} \varphi'^2 + \frac{1}{2a^2} m^2 \varphi^2 + \frac{\lambda}{4a^4} \varphi^4 + (1 - 6\xi) \left( \frac{H^2}{2a^2} \varphi^2 - \frac{H}{a^2} \varphi' \right) \]

\[ + \frac{1}{a^2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\Omega k_0} \left\{ \frac{1}{2a^2} |U'_k|^2 + \frac{1}{2a^2} \Omega(\tau) |U_k|^2 - \frac{1}{2} (\xi - \frac{1}{6}) R |U_k|^2 \right. \]

\[ \left. - \frac{1}{2} (6\xi - 1) H^2 |U_k|^2 + \frac{1}{2} (6\xi - 1) \frac{H}{a} \frac{d}{d\tau} |U_k|^2 \right\} , \]  

(43)

\[ ^2 \text{We continue to consider } T_{tt} \text{ instead of } T_{\tau\tau} = a^2 T_{tt} \text{ for convenience.} \]
reaction to the potential by making the ansatz

\[ T^\mu = (1 - 6\xi) \left( -\frac{\varphi'^2}{a^4} + \frac{H^2}{a^2} \varphi^2 + 2 \frac{H}{a^3} \varphi \varphi' \right) + 6 \frac{\xi}{a^4} \varphi \varphi'' + 2 m^2 \varphi^2 + \frac{\lambda}{6a^4} \varphi^4 \]

\[ + \frac{1}{a^2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\Omega_{k0}} \left\{ (1 - 6\xi) \left[ \left( \frac{|U'_k|^2}{a^2} + \Omega^2(\tau) \frac{|U_k|^2}{a^2} \right) \right. \right. \]

\[ \left. \left. - \left( \frac{2|U'_k|^2}{a^2} + H^2|U_k|^2 - \frac{H}{a} \frac{d}{d\tau} |U_k|^2 \right) \right] \right. \]

\[ + \left[ m^2 + \frac{\lambda}{2a^2} - (\xi - \frac{1}{6}) R \right] |U_k|^2 \} \right) . \]  

(44)

Energy density and pressure are related to the energy momentum tensor via

\[ T_{\mu\nu} = \mathcal{E} , \]
\[ T^\mu_\mu = \mathcal{E} - 3p . \]  

(45)

It is straightforward to show, using the equations of motion for the classical field and for the mode functions (23), that the energy is covariantly conserved:

\[ \mathcal{E}'(\tau)/a(\tau) + 3H(\tau)(\rho(\tau) + \mathcal{E}(\tau)) = 0 . \]  

(46)

V. PERTURBATIVE EXPANSION

In order to prepare the renormalized version of the equations given in the previous section we introduce a suitable expansion of the mode functions, which was successfully used in [28, 29] for the inflaton field coupled to itself, to fermions, and gauge bosons in Minkowski-space. Adding the term \( M^2(0) \) on both sides of the mode function equation it takes the form

\[ \left[ \frac{d^2}{d\tau^2} + \Omega_{k0}^2 \right] U_k(\tau) = -V(\tau) U_k(\tau) , \]  

(47)

with

\[ V(\tau) = M^2(\tau) - M^2(0) , \]
\[ \Omega_{k0} = \left[ \tilde{k}^2 + M^2(0) \right]^{1/2} \]  

(48)

(for the definition of \( M^2(\tau) \) see eq.(42)). The first and second derivatives of this potential will be needed in the perturbative expansion and are given by

\[ V'(\tau) = 2aHM^2(\tau) + a^2 \left( \xi - \frac{1}{6} \right) R' + \lambda \varphi \varphi' - \lambda aH \varphi^2 , \]  

(49)

\[ V'' = \frac{1}{a} a^2 R M^2 - 2a^2 H^2 M^2 + 2aHM'(\tau) \]

\[ + 2a^3 H \left( \xi - \frac{1}{6} \right) R' - 2\lambda aH \varphi \varphi' + a^2 \left( \xi - \frac{1}{6} \right) R'' \]

\[ + \lambda (\varphi \varphi'' + \varphi'^2) - \frac{\lambda a^2}{6} R \varphi^2 + \lambda a^2 H^2 \varphi^2 . \]  

(50)

Including the initial conditions (36) the mode functions satisfy the equivalent integral equation

\[ U_k(\tau) = e^{-i\Omega_{k0} \tau} + \int_0^\infty d\tau' \Delta_{k,ret}(\tau - \tau') V(\tau') U_k(\tau') , \]  

(51)

with

\[ \Delta_{k,ret}(\tau - \tau') = -\frac{1}{\Omega_{k0}} \Theta(\tau - \tau') \sin \left( \Omega_{k0}(\tau - \tau') \right) . \]  

(52)

We separate \( U_k(\tau) \) into the trivial part corresponding to the case \( V(\tau) = 0 \) and a function \( h_k(\tau) \) which represents the reaction to the potential by making the ansatz
\[ U_k(\tau) = e^{-\Omega_{k0}\tau}(1 + h_k(\tau)) . \]  

\( h_k(\tau) \) satisfies then the integral equation
\[ h_k(\tau) = \int_0^\tau d\tau' \Delta_{k,\text{ret}}(\tau - \tau')V(\tau')(1 + h_k(\tau'))e^{i\Omega_{k0}(\tau - \tau')} , \]  

and an equivalent differential equation
\[ h_k''(\tau) - 2i\Omega_{k0}h_k'(\tau) = -V(\tau)(1 + h_k(\tau)) , \]  

with the initial conditions \( h_k(0) = \dot{h}_k(0) = 0 \). We expand now \( h_k(\tau) \) with respect to orders in \( V(\tau) \) by writing
\[ h_k(\tau) = h_k^{(1)}(\tau) + h_k^{(2)}(\tau) + h_k^{(3)}(\tau) + \cdots , \]  

where \( h_k^{(n)}(\tau) \) is of \( n \)'th order in \( V(\tau) \) and \( h_k^{(n)}(\tau) \) is the sum over all orders beginning with the \( n \)'th one:
\[ h_k^{(n)}(\tau) = \sum_{i=n}^\infty h_k^{(n)}(\tau) . \]  

The \( h_k^{(n)} \) are obtained by iterating the integral equation (54) or the differential equation (53). The function \( h_k^{(1)}(\tau) \) is identical to the function \( h_k(\tau) \) itself which is obtained by solving (55). The function \( h_k^{(2)}(\tau) \) can again be obtained by iteration via
\[ h_k^{(2)}(\tau) = \int_0^\tau d\tau' \Delta_{k,\text{ret}}(\tau - \tau')V(\tau')h_k^{(1)}(\tau')e^{i\Omega_{k0}(\tau - \tau')} , \]  

or, using the differential equation, via
\[ h_k''(\tau) - 2i\Omega_{k0}h_k'(\tau) = -V(\tau)h_k^{(1)}(\tau) . \]  

This iteration has the numerical aspect that it avoids computing \( h_k^{(2)} \) via the small difference \( h_k^{(2)} - h_k^{(1)} \). However, the integral equations are used as well in order to derive the asymptotic behaviour as \( \Omega_{k0} \to \infty \) and to separate divergent and finite contributions. We will give here the relevant leading terms for \( h_k^{(1)}(\tau) \) and \( h_k^{(2)}(\tau) \). We have
\[ h_k^{(1)}(\tau) = \frac{i}{2\Omega_{k0}} \int_0^\tau d\tau' \left( \exp(2i\Omega_{k0}(\tau - \tau')) - 1 \right)V(\tau') . \]  

Integrating by parts we obtain
\[ h_k^{(1)}(\tau) = -\frac{i}{2\Omega_{k0}} \int_0^\tau d\tau' V(\tau') - \frac{1}{4\Omega_{k0}^2} V(\tau) + \frac{1}{4\Omega_{k0}^2} \int_0^\tau d\tau' \exp(2i\Omega_{k0}(\tau - \tau'))V'(\tau') , \]  

or, by another integration by parts,
\[ h_k^{(1)}(\tau) = -\frac{i}{2\Omega_{k0}} \int_0^\tau d\tau' V(\tau') - \frac{1}{4\Omega_{k0}^2} V(\tau) + \frac{i}{8\Omega_{k0}^3} V'(\tau) \]
\[ -\frac{i}{8\Omega_{k0}^3} \int_0^\tau d\tau' \exp(2i\Omega_{k0}(\tau - \tau'))V''(\tau') . \]
We will need often the real part of $h_k^{(1)}$ for which we find
\[
\text{Re } h_k^{(1)}(\tau) = -\frac{1}{4\Omega_{k0}^2}V(\tau) + \frac{1}{4\Omega_{k0}^2} \int_0^\tau d\tau' \cos(2\Omega_{k0}(\tau - \tau'))V'(\tau')
\]
\[= -\frac{1}{4\Omega_{k0}^2}V(\tau) + \frac{1}{4\Omega_{k0}^2}C(V', \tau) . \tag{65}\]

Since various Fourier integrals will appear in the finite parts of the fluctuation integrals we introduce for later convenience the notations
\[
S(f, \tau) = \int_0^\tau d\tau' \sin(2\Omega_{k0}(\tau - \tau'))f(\tau') , \tag{66}
\]
\[
C(f, \tau) = \int_0^\tau d\tau' \cos(2\Omega_{k0}(\tau - \tau'))f(\tau') . \tag{67}
\]

For the leading behaviour of $h_k^{(2)}(\tau)$ we find
\[
h_k^{(2)}(\tau) = -\frac{1}{4\Omega_{k0}^2} \int_0^\tau \int_0^{\tau'} d\tau'' V(\tau')V(\tau'') + O(\Omega_{k0}^{-3}) . \tag{68}\]

In terms of this perturbative expansion we can write the mode functions appearing in the fluctuation integrals in the equation of motion and in the energy momentum tensor as
\[
|U_k|^2 = 1 + 2\text{Re } h_k^{(\text{T})} + |h_k^{(\text{T})}|^2 , \tag{69}
\]
and
\[
|U_k'|^2 = \Omega_{k0}^2 \left( 1 + 2\text{Re } h_k^{(\text{T})} + |h_k^{(\text{T})}|^2 + |h_k^{(\text{T})}|^2 \right.
\]
\[\left. - i\Omega_{k0} \left( -2i\text{Im } h_k^{(\text{T})} - 2i\text{Im } h_k^{(\text{T})} + h_k^{(\text{T})} \right) \right) . \tag{70}\]

As the potential is real, the leading behaviour of the sums is
\[
1 + 2\text{Re } h_k^{(\text{T})} + |h_k^{(\text{T})}|^2 = 1 - \frac{1}{2\Omega_{k0}^2}V(\tau) + \frac{1}{4\Omega_{k0}^3}\sin(2\Omega_{k0}\tau)V'(0) + \frac{1}{8\Omega_{k0}^4}V''(\tau)
\]
\[\left. - \frac{1}{8\Omega_{k0}^4}(2\Omega_{k0}\tau)V''(0) + \frac{3}{8\Omega_{k0}^4}V^2(\tau) + O(\Omega_{k0}^{-5}) \right) , \tag{71}\]
and
\[
-2i\text{Im } h_k^{(\text{T})} - 2i\text{Im } h_k^{(\text{T})} h_k^{(\text{T})} = \frac{i}{\Omega_{k0}}V(\tau) - \frac{i}{2\Omega_{k0}^2}\sin(2\Omega_{k0}\tau)V'(0) - \frac{i}{4\Omega_{k0}^3}V''(\tau)
\]
\[\left. + \frac{i}{4\Omega_{k0}}\cos(2\Omega_{k0}\tau)V''(0) - \frac{3i}{4\Omega_{k0}^4}V^2(\tau) + O(\Omega_{k0}^{-4}) \right) . \tag{72}\]

From the Wronskian relation
\[
U_k U_k' - U_k^* U_k^* = 2i\Omega_{k0} \tag{73}
\]
we obtain the relation
\[
2i\Omega_{k0} \left( 2\text{Re } h_k^{(\text{T})} + |h_k^{(\text{T})}|^2 \right) - 2i\text{Im } h_k^{(\text{T})} - 2i\text{Im } h_k^{(\text{T})} h_k^{(\text{T})} = 0 , \tag{74}\]
which proves to be useful in simplifying the mode integrals occurring in the energy momentum tensor.
VI. RENORMALIZATION OF THE EQUATION OF MOTION

The expansion of the mode functions allows a free choice of the regularization scheme. We will use dimensional regularization here since in the presence of quadratic and quartic divergencies it is much easier to handle than in Pauli-Villars regularization. We will show that in this scheme the divergence structure of the equation of motion and of the energy momentum tensor have the correct form for a consistent renormalization. Furthermore, we will present Pauli-Villars regularization. We will show that in this scheme the divergence structure of the equation of motion and of the energy momentum tensor have the correct form for a consistent renormalization. Furthermore, we will present the explicit form of the finite parts of the fluctuation integrals occurring in the equation of motion and the energy momentum tensor.

The fluctuation integral of the equation of motion can be split into a divergent and a convergent part. Using (63) we obtain

$$F(\tau) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\Omega k_0} \left( 1 + 2\Re h_k^T(\tau) + |h_k^T(\tau)|^2 \right)$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\Omega k_0} \left( 1 - \frac{1}{2\Omega k_0} V(\tau) + \frac{1}{2\Omega k_0} C(V', \tau) + 2\Re h_k^T(\tau) + |h_k^T(\tau)|^2 \right).$$

(75)

The first two terms in the integrand have to be regularized. We first rewrite the basic equation of motion, including appropriate counter terms, as

$$\varphi'' + a^2 \left[ m^2 + \delta m + (\xi - \frac{1}{6} + \delta \xi) R \right] \varphi + \frac{\lambda + \delta \lambda}{6} \varphi^3 + \frac{\lambda}{2} \varphi F = 0.$$  (76)

Next we separate from the term $\lambda \varphi F/2$ the dimensionally regularized divergent parts

$$\left\{ \frac{\lambda}{2} \varphi(\tau) \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\Omega k_0} \right\}_{\text{reg}} = \frac{\mu^2}{2} \varphi(\tau) \int \frac{d^4k}{(2\pi)^4} \frac{1}{2 \left( k^2 + M^2(0) \right)^{1/2}}$$

$$= -\frac{\lambda M^2(0) \varphi(\tau)}{32\pi^2} \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{M^2(0)} - \gamma + 1 \right\},$$  (77)

and

$$\left\{ -\frac{\lambda}{2} \varphi(\tau) V(\tau) \int \frac{d^3k}{(2\pi)^3} \frac{1}{4\Omega k_0} \right\}_{\text{reg}} = -\frac{1}{8} (\mu^2)^{\epsilon} \lambda \varphi(\tau) V(\tau) \int \frac{d^4k}{(2\pi)^4} \frac{1}{2 \left( k^2 + M^2(0) \right)^{3/2}}$$

$$= \frac{\lambda \varphi(\tau) V(\tau)}{32\pi^2} \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{M^2(0)} - \gamma \right\}.$$  (78)

Recalling that $V(\tau) = M^2(\tau) - M^2(0)$ and $M^2(\tau) = a^2(\tau) \left[ m^2 + (\xi - 1/6) R \right] + \lambda \varphi^2/2$ one sees that $M^2(0)$ cancels for the divergent terms and that therefore the counter terms can be chosen independent of the initial conditions as

$$\delta m^2 = \frac{\lambda m^2}{32\pi^2} \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{m^2} - \gamma \right\},$$  (79)

$$\delta \lambda = \frac{3\lambda^2}{32\pi^2} \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{m^2} - \gamma \right\},$$  (80)

$$\delta \xi = \frac{\lambda (\xi - \frac{1}{6})}{32\pi^2} \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{m^2} - \gamma \right\}.$$  (81)

The renormalized equation of motion now reads

$$\varphi'' + a^2 \left[ m^2 + \Delta m + (\xi + \Delta \xi - \frac{1}{6}) R \right] \varphi + \frac{\lambda + \Delta \lambda}{6} \varphi^3 + \frac{\lambda}{2} \varphi F_{\text{fin}} = 0,$$  (82)

with
\[ F_{\text{fin}} = \frac{M^2(0)}{16\pi^2} + \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\Omega_{k0}} \left( \frac{1}{2\Omega_{k0}} C(V', \tau) + 2\text{Re} h_k^{(\mathbb{T})} + |h_k^{(\mathbb{T})}|^2 \right), \] (83)

and with the finite corrections

\[ \Delta m^2 = -\frac{\lambda m^2}{32\pi^2} \ln \frac{m^2}{M^2(0)}, \] (84)
\[ \Delta \lambda = -\frac{3\lambda^2}{32\pi^2} \ln \frac{m^2}{M^2(0)}, \] (85)
\[ \Delta \xi = -\frac{\lambda(\xi - \frac{1}{d})}{32\pi^2} \ln \frac{m^2}{M^2(0)}. \] (86)

**VII. RENORMALIZATION OF THE ENERGY MOMENTUM TENSOR**

In order to derive a renormalized form of the Friedmann equations we have to renormalize the energy momentum tensor as well. In principle this has been discussed long ago and it will not be a surprise that the divergent parts are in one-to-one correspondence to those given e.g. in [30]. However, we have to discuss this subject in the framework of nonequilibrium quantum field theory, and we are interested in particular in the precise form of the finite parts which will be the subject of a numerical computation. In order to renormalize the energy we introduce the available counter terms into the unrenormalized expression so that it reads now

\[ \mathcal{E} = \frac{1}{2a^2} \varphi^2 + \frac{1}{2a^2} (m^2 + \delta m^2) \varphi^2 + \frac{\lambda + \delta \lambda}{4a^4} \varphi^4 \]

\[ -6 \left( \xi - \frac{d}{6} + \delta \xi \right) \left( \frac{H^2}{2a^2} \varphi^2 - \frac{H}{a^2} \varphi \varphi' \right) + \delta \lambda + \delta \alpha H_{tt} + \delta \tilde{Z} G_{tt} \]

\[ + \frac{1}{a^2} \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{\Omega_{k0}}{2a^2} \left( 1 + 2\text{Re} h_k^{(\mathbb{T})} + |h_k^{(\mathbb{T})}|^2 \right) + \frac{1}{4\Omega_{k0}a^2} |h_k^{(\mathbb{T})}|^2 \right\} \]
\[ - \frac{i}{4a^2} \left( -2i \text{Im} h_k^{(\mathbb{T})} - 2i \text{Im} h_k^{(\mathbb{T})} \ast h_k^{(\mathbb{T})} \right) \]
\[ + \frac{1}{4\Omega_{k0}a^2} V(\tau) \left( 1 + 2\text{Re} h_k^{(\mathbb{T})} + |h_k^{(\mathbb{T})}|^2 \right) \]
\[ - \frac{1}{4\Omega_{k0}} (6\xi - 1) \left( \frac{R}{6} + H^2 \right) \left( 1 + 2\text{Re} h_k^{(\mathbb{T})} + |h_k^{(\mathbb{T})}|^2 \right) \]
\[ + \frac{1}{4\Omega_{k0}} (6\xi - 1) \frac{H}{d} \left( 1 + 2\text{Re} h_k^{(\mathbb{T})} + |h_k^{(\mathbb{T})}|^2 \right). \] (87)

The divergent parts of the fluctuation integral are

\[ \mathcal{E}_{\text{div. fluc}} = \frac{1}{a^2} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{\Omega_{k0}}{2a^2} + \frac{1}{4\Omega_{k0}a^2} V(\tau) - \frac{1}{16\Omega_{k0}^2} V^2(\tau) \right] \]
\[ - \frac{1}{2} (6\xi - 1) \left( \frac{R}{6} + H^2 \right) \left( \frac{1}{2\Omega_{k0}} - \frac{1}{4\Omega_{k0}} V(\tau) \right) \]
\[ - \frac{1}{2} (6\xi - 1) \frac{H}{d} \frac{1}{4\Omega_{k0}} V(\tau). \] (88)

Dimensional regularisation of the divergent integrals yields

\[ \int \frac{d^3k}{(2\pi)^3} \frac{1}{a^2} \left[ \frac{\Omega_{k0}}{2} + \frac{1}{4\Omega_{k0}} V(\tau) - \frac{1}{16\Omega_{k0}^2} V^2(\tau) \right] \]
\[ = -\frac{M^4(\tau)}{64\pi^2a^2} \left\{ 2 + \ln \frac{4\pi \mu^2}{M^2(0)} - \gamma \right\} + \frac{M^4(0)}{128\pi^2a^2} \frac{M^2(0)M^2(\tau)}{32\pi^2a^2} \] (89)
\[ = -\frac{a^2 \left[ m^2 + \left( \xi - \frac{1}{d} \right) R + \frac{\lambda}{d} \varphi^2 \right]^2}{64\pi^2} \left\{ 2 + \ln \frac{4\pi \mu^2}{M^2(0)} - \gamma \right\} + \frac{M^4(0)}{128\pi^2a^2} - \frac{M^2(0)M^2(\tau)}{32\pi^2a^2}. \]
The terms proportional $\lambda m^2 \varphi^2$ and $\lambda^2/4\varphi^4$ in (89) are cancelled by the mass and coupling constant counter terms. The divergent term which depends on $m^4$ determines the cosmological constant counter term, that is

$$
\delta \Lambda = \frac{m^4}{64\pi^2} \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{m^2} - \gamma \right\} .
$$

(90)

The remaining terms in (89) combine with the corresponding expressions of the following two dimensionally regularized integrals

$$
-\frac{1}{2} (6\xi - 1) \left( \frac{R}{6} + H^2 \right) \int \frac{d^3 k}{(2\pi)^3} \left[ \frac{1}{2\Omega_{k0}} - \frac{1}{4\Omega_{k0}^3} V(\tau) \right]
$$

$$
= (6\xi - 1) \left( \frac{R}{6} + H^2 \right) \frac{M^2(\tau)}{32\pi^2} \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{M^2(0)} - \gamma \right\}
$$

$$
+ \frac{1}{32\pi^2} (6\xi - 1) \left( \frac{R}{6} + H^2 \right) M^2(0) ,
$$

(91)

and

$$
-\frac{1}{2} (6\xi - 1) \frac{H}{a} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{4\Omega_{k0}^3} V'(\tau)
$$

$$
= -\frac{1}{16\pi} (6\xi - 1) H^2 M^2(\tau) \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{M^2(0)} - \gamma \right\}
$$

$$
- \frac{1}{32\pi^2} (6\xi - 1) \frac{H}{a} \left[ (6\xi - 1) R' a^2 + \lambda \varphi' \lambda - \lambda a H \varphi^2 \right] \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{M^2(0)} - \gamma \right\} .
$$

(92)

The term $\lambda(\xi - 1/6)R\varphi^2$ in (89) is cancelled by the same term with opposite sign in (91). The $H^2\varphi^2$-terms in (91) and (92) are absorbed by the counter term proportional to $\delta \xi$, and the divergence proportional to $\varphi^2$ is compensated by this counter term as well. The remaining $\varphi$-independent but still time-dependent divergent terms are absorbed into the counter terms $\delta \alpha H_{tt}$ and $\delta \tilde{Z} G_{tt}$. We choose

$$
\delta \alpha = -\frac{(6\xi - 1)}{32\pi^2} \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{m^2} - \gamma \right\} ,
$$

(93)

$$
\delta \tilde{Z} = -\frac{(6\xi - 1)}{16\pi^2} \frac{m^2}{\epsilon} \left\{ \frac{2}{\epsilon} + \ln \frac{4\pi\mu^2}{m^2} - \gamma \right\} .
$$

(94)

That is, all divergent integrals appearing in the unrenormalized divergent fluctuation integral of the energy are removed by the corresponding counter terms and we finally arrive at the renormalized expression for the energy

$$
\mathcal{E}_{ren} = \frac{1}{2a^2} \varphi^2 + \frac{1}{2a^2} (m^2 + \Delta m^2) \varphi^2 + \frac{\lambda + \Delta \lambda}{4a^4} \varphi^4 - 6 (\xi - \frac{1}{6}) \left( \frac{H^2}{2a^2} \varphi^2 - \frac{H}{2a^2} \varphi' \right)
$$

$$
+ \Delta \Lambda + \Delta \alpha H_{tt} + \Delta \tilde{Z} G_{tt} + \frac{M^2(0)}{32\pi^2 a^4} (6\xi - 1) \left( \frac{R}{6} + H^2 \right) + \frac{M^2(0)}{128\pi^2 a^4} - \frac{M^2(0) M^2(\tau)}{32\pi^2 a^4}
$$

$$
+ \frac{1}{a^2} \int \frac{d^3 k}{(2\pi)^3} \left\{ \frac{V(\tau)}{4\Omega_{k0} a^2} \left( 2Re h_k^T + |h_k^T|^2 \right) + \frac{1}{4\Omega_{k0} a^2} |h_k^T|^2 \right\}
$$

$$
+ \frac{1}{2\Omega_{k0} a^2} \Re \left( h_k^{(1)} h_k^{(T)} \right) + \frac{1}{16\Omega_{k0} a^2} C^2(V', \tau) + \frac{1}{16\Omega_{k0} a^2} S^2(V', \tau)
$$

$$
- \frac{1}{4\Omega_{k0}} (6\xi - 1) \left( \frac{R}{6} + H^2 \right) \left\{ \frac{1}{2\Omega_{k0}^2} C(V', \tau) + 2Re h_k^T + |h_k^T|^2 \right\}
$$

$$
+ \frac{1}{4\Omega_{k0}} (6\xi - 1) \frac{H}{a} \left\{ \frac{1}{2\Omega_{k0}^2} C(V'', \tau') + \frac{1}{2\Omega_{k0}^2} \cos 2\Omega_{k0} \tau V'(0)
$$

$$
+ 2Re h_k^T + 2Re h_k^T h_k^T \right\} .
$$

(95)

The finite corrections are
\begin{align}
\Delta \bar{\alpha} &= \frac{(\xi - \frac{7}{2})^2}{32\pi^2} \ln \frac{m^2}{M^2(0)}, \\
\Delta \bar{\lambda} &= -\frac{m^4}{64\pi^2} \ln \frac{m^2}{M^2(0)}, \\
\Delta \bar{Z} &= \frac{(\xi - \frac{7}{2}) m^2}{16\pi^2} \ln \frac{m^2}{M^2(0)}.
\end{align}

Next we have to consider the renormalization of the trace of the energy momentum tensor. We introduce the available counter terms into the unrenormalized expression for \( T_\mu^\mu \) so that it reads now

\begin{equation}
T_\mu^\mu = -[1 - 6(\xi + \delta \xi)] \left( \frac{\varphi'^2}{a^4} + \frac{H^2}{a^2} \varphi^2 - \frac{2}{a^4} \varphi \varphi' \right) + \frac{6(\xi + \delta \xi)}{a^4} \varphi \varphi'' + 2(m^2 + \delta m^2) \frac{\varphi^2}{a^2} + \frac{\lambda + \delta \lambda}{6a^4} \varphi^4 + 4\delta \bar{\lambda} + \delta \bar{\lambda} H^2 + \delta \bar{Z} G^\mu_\mu
\end{equation}

\begin{equation}
+ \frac{1}{a^2} \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{a^2} (1 - 6\xi) \left[ -\frac{1}{2(\Omega k_0)} |h_1^\tau|^2 + \frac{i}{2} \left( -2i \text{Im} h_1^\tau h_k^\tau \right) + \frac{1}{2(\Omega k_0)} V(\tau) \left( 1 + 2 \text{Re} h_k^\tau \right) \right] + \frac{H}{2(\Omega k_0)} (1 - 6\xi) \frac{d}{dr} \left( 1 + 2 \text{Re} h_k^\tau \right) \right\}.
\end{equation}

We can split the trace of the stress tensor into a divergent and into a convergent part. The divergent part reads

\begin{equation}
T_\mu^\mu_{\text{div}} = \frac{1}{a^2} \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{a^2} (1 - 6\xi) \frac{1}{8(\Omega k_0)^3} V''(\tau) - (1 - 6\xi) \frac{H}{a} \frac{1}{4(\Omega k_0)^3} V'(\tau)
\end{equation}

\begin{equation}
+ \left[ m^2 + \frac{\lambda}{2a^2} \varphi^2 - (1 - 6\xi) \left( H^2 - \frac{R}{6} \right) \right] \left[ \frac{1}{2(\Omega k_0)} - \frac{1}{4(\Omega k_0)^3} V(\tau) \right] \right\}.
\end{equation}

The first derivative of the potential cancels with the corresponding term from (50) in (100). The remaining divergent parts are

\begin{equation}
T_\mu^\mu_{\text{div}} = \frac{1}{a^2} \int \frac{d^3k}{(2\pi)^3} \left\{ (1 - 6\xi) \frac{1}{8(\Omega k_0)^3} \left[ \frac{1}{4} RM^2(\tau) - 2H^2 M^2(\tau)
\end{equation}

\begin{equation}
+ 2aH (\xi - \frac{1}{6}) R' - 2\lambda \frac{H}{a} \varphi \varphi' + (\xi - \frac{1}{6}) R''
\end{equation}

\begin{equation}
+ \frac{1}{a^2} \lambda (\varphi \varphi'' + \varphi') - \frac{\lambda}{6} R \varphi^2 + \lambda H^2 \varphi^2
\end{equation}

\begin{equation}
+ \left[ m^2 + \frac{\lambda}{2a^2} \varphi^2 - (1 - 6\xi) \left( H^2 - \frac{R}{6} \right) \right] \left[ \frac{1}{2(\Omega k_0)} - \frac{1}{4(\Omega k_0)^3} V(\tau) \right] \right\}.
\end{equation}

After dimensional regularization again the counter terms absorb all divergent terms in the fluctuation integral of \( T_\mu^\mu \). The renormalized trace of the stress tensor takes the final form

\begin{equation}
T_\mu^\mu = -[1 - 6(\xi + \Delta \xi)] \left( \frac{\varphi'^2}{a^4} + \frac{H^2}{a^2} \varphi^2 - \frac{2}{a^4} \varphi \varphi' \right) + \frac{6(\xi + \Delta \xi)}{a^4} \varphi \varphi''
\end{equation}
\[+2(m^2 + \Delta m^2)\frac{\varphi^2}{a^2} + \frac{\lambda + \Delta \lambda}{6a^4} \varphi^4 + 4\Delta \tilde{\lambda} + \Delta \tilde{G}^\mu_\mu
\]
\[-\frac{M^2(0)}{16\pi^2a^2} \left[ m^2 + \frac{\lambda}{2a^2} \varphi^2 - (1 - 6\xi) \left( H^2 - \frac{R}{6} \right) \right]
\]
\[+ \frac{1}{a^2} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1}{a^2} (1 - 6\xi) \left( -2\text{Im} h_k^{(\overline{\tau})} - 2i \text{Im} h_k^{(\overline{\tau})} h_k^{(\tau)} \right) \right.
\]
\[+ \frac{1}{2\Omega k_0} V(\tau) \left( 2\Re h_k^{(\overline{\tau})} + |h_k^{(\overline{\tau})}|^2 \right) + \frac{1}{4\Omega k_0} V'(0) \sin 2\Omega k_0 \tau
\]
\[+ \frac{1}{8\Omega^3 k_0} V''(0) \cos 2\Omega k_0 \tau - \frac{1}{8\Omega k_0} C(V'', \tau) \right]
\[+ \frac{H}{2\Omega k_0} (1 - 6\xi) \left[ \frac{1}{2\Omega^2 k_0} C(V'', \tau) + 2\Re h_k^{(\overline{\tau})} \right.
\]
\[+ \frac{1}{2\Omega^2 k_0} \cos 2\Omega k_0 \tau V'(0) + 2\Re h_k^{(\overline{\tau})} h_k^{(\tau)} \left. \right]
\[+ \frac{1}{2\Omega k_0} \left[ m^2 + \frac{\lambda}{2a^2} \varphi^2 - (\xi - \frac{1}{6}) \left( H^2 - \frac{R}{6} \right) \right]
\]
\[\times \left[ \frac{1}{2\Omega^2 k_0} C(V', \tau) + 2\Re h_k^{(\overline{\tau})} + |h_k^{(\overline{\tau})}|^2 \right]. \tag{102} \]

**VIII. CONCLUSIONS**

We have presented here the renormalized equations of motion for a scalar field in a conformally flat FRW universe including the quantum back reaction in one-loop approximation. We have used dimensional regularization and an $\overline{MS}$ renormalization. However, within this formalism other renormalization conditions can be employed, as well as other regularizations. The formalism is fully covariant and the counter terms can be chosen independent of the initial conditions. In contrast to the adiabatic regularization it is not based on the WKB expansion and can therefore be generalized to coupled systems \[29\]. Furthermore, we do not have to perform delicate subtractions in the divergent integrals since in our formulation they are finite from the outset. The method can be adapted easily to finite temperature computations as well, since no new ultraviolet divergencies are introduced.

The intention of this work is of course to supply at the same time a useful scheme for numerical computations. We have shown previously \[28\] that the numerical implementation is straightforward. A realistic application to one of the inflationary scenarios needs, however, a judicious choice of initial conditions and of the parameters of the theory, as well as extensive numerical experiments. While such computations are in progress we prefer to present here the formalism as such; we think that is by itself of general theoretical as well as of practical interest.

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