Lorentz Invariance Violation and Modified Hawking Fermions Tunneling from Black Strings

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Abstract: Recently the modified Dirac equation with Lorentz invariance violation has been proposed, which would be helpful to resolve some issues in quantum gravity theory and high energy physics. In this paper, the modified Dirac equation has been generalized in curved spacetime, and then fermion tunneling of black strings is researched under this correctional Dirac field theory. We also use semi-classical approximation method to get correctional Hamilton-Jacobi equation, so that the correctional Hawking temperature and correctional black hole’s entropy are derived.

Keywords: Black holes, Modes of Quantum Gravity.
1. Introduction

In 1974, Hawking proved black holes could radiate Hawking radiation once considering the quantum effect near the horizons of black holes [1]. This theory indicates black hole would be viewed as thermodynamic system, so that black hole physics can be connected closely with gravity, quantum theory and thermodynamics physics. According to the viewpoint of quantum tunneling theory, the virtual particles inside of black hole could cross the horizon due to the quantum tunneling effect, and become real particles, and then could be observed by observers as Hawking radiation. Wilczek and Parikh et al. proposed a semi-classical method to study the quantum tunneling from the horizon of black hole [2, 3, 4]. Along with this method, the Hamilton-Jacobi method was applied to calculate the Hawking tunneling radiation. According to the Hamilton-Jacobi method, the wave function of Klein-Gordon equation can be rewritten as \( \Phi = C \exp \left( \frac{iS}{\hbar} \right) \) (where \( S \) is semi-classical action), and Hamilton-Jacobi equation is obtained via semi-classical approximation. Using the Hamilton-Jacobi equation, the tunneling rate could be calculated by the relationship \( \Gamma \sim \exp \left( -2\text{Im}S \right) \) (where \( \Gamma \) is the tunneling rate at the horizon of black hole), and then the Hawking temperature can be determined. People have applied this method to research Hawking tunneling radiation of several static, stationary and dynamical black holes.

However, since the Hamilton-Jacobi equation is derived from Klein-Gordon equation, original Hamilton-Jacobi method just can be valid for scalar particles in principle. Therefore, Kerner and Mann studied fermions tunneling of black hole by a new method [5, 6], which assume the wave function of Dirac equation \( \Psi \) as spin-up and spin-down, and then calculate the fermions tunneling respectively. Nevertheless, this method is still impossible to apply in arbitrary dimensional spacetime. Our work in 2009 showed that, the Hamilton-Jacobi equation can also be derived from Dirac equation via semi-classical approximation, so that we proved the Hamilton-Jacobi equation can be used to study the fermions tunneling directly [7].
On the other hand, as the basis of general relativity and quantum field theory, Lorentz invariance is proposed to be spontaneously violated at higher energy scales. A possible deformed dispersion relation is given by

\[ p_0^2 = \bar{p}^2 + m^2 - (Lp_0)^\alpha \bar{p}^2, \]  

(1.1)

where \( p_0 \) and \( \bar{p} \) are the energy and momentum of particle and the \( L \) is "minimal length" with the order of the Plank length. The work of spacetime foam Liouville-string models have introduced this relation with \( \alpha = 1 \), and people also proposed quantum equation of spinless particles by using this relation. Recently, Kruglov considers the deformed dispersion relation with \( \alpha = 2 \) and proposes modified Dirac equation

\[ \bar{\gamma}^\mu \partial_\mu + m - iL (\bar{\gamma}^t \partial_t) (\bar{\gamma}^j \partial_j) \psi = 0, \]

(1.2)

where \( \bar{\gamma}^a \) is ordinary gamma matrix, and \( j \) is space coordinate while \( \mu \) is spacetime coordinate. The effect of the correctional term would be observed in higher energy experiment.

In this paper, we try to generalize the modified Dirac equation in curved spacetime, and then study the correction of Hawking tunneling radiation. In section II, the modified Dirac equation in curved spacetime is constructed and then the modified Hamilton-Jacobi equation is derived via semi-classical approximation. We apply the modified Hamilton-Jacobi equation to the fermions tunneling radiation of 2 + 1 dimensional black string and higher dimensional BTZ-like black strings in section III and IV respectively, and the section V includes some conclusion and the discussion about the correction of black hole’s entropy.

2. Modified Dirac Equation and Hamilton-Jacobi Equation in Curved Spacetime

As we all known, the gamma matrix and partial derivative should become gamma matrix in curved spacetime \( \gamma^a \) and covariant \( D_a \) derivative respectively, namely

\[ \bar{\gamma}^a \rightarrow \gamma^a, \quad \partial_a \rightarrow D_a = \partial_a + \Omega_a + \frac{i}{\hbar} eA_a, \]  

(2.1)

where \( \gamma^a \) satisfy the relationship \( \{ \gamma^a, \gamma^b \} = \gamma^a \gamma^b + \gamma^b \gamma^a = 2g^{ab}I \). \( eA_a \) is charged term of Dirac equation and \( \Omega_\mu = \frac{1}{8} (\gamma^a \gamma^b - \gamma^b \gamma^a) e_\nu^\alpha (\partial_\mu e_{\alpha\nu} - \Gamma^c_{\mu\nu} e_{bc}) \) is spin connection. According to this transformation, we can construct the modified Dirac equation in curved spacetime as

\[ \left[ \gamma^\mu D_\mu + \frac{m}{\hbar} - \sigma h (\gamma^t D^t) (\gamma^j D^j) \right] \Psi = 0, \]  

(2.2)

where we choose \( c = 1 \) but \( \hbar \neq 1 \), while \( c = \hbar = 1 \) in Eq.(1.1) and Eq.(1.2). It is assumed that \( \sigma \ll 1 \), so that the correctional term \( \sigma h (\gamma^t D^t) (\gamma^j D^j) \) is very small.

Now let’s use the modified Dirac equation to derive the modified Hamilton-Jacobi equation. Firstly, we rewrite the wave function of Dirac equation as

\[ \Psi = \zeta(t, x^i) \exp \left[ \frac{i}{\hbar} S(t, x^j) \right], \]  

(2.3)
where \( \zeta(t, x^j) \) and \( \Psi \) are \( m \times 1 \) matrices, and \( \partial_t S = -\omega \). In semi-classical approximation, we can consider the \( \hbar \) is very small, so that we can neglect the terms with \( \hbar \) after dividing by the exponential terms and multiplying by \( \hbar \). Therefore, Eq. (2.2) is rewritten as

\[
[i \gamma^\mu (\partial_\mu S + eA_\mu) + m - \sigma \gamma^t (\omega - eA_t) \gamma^j (\partial_j S + eA_j)] \zeta(t, x^j) = 0.
\]

(2.4)

Considering the relationship

\[
\gamma^\mu (\partial_\mu S + eA_\mu) = -\gamma^t (\omega - eA_t) + \gamma^j (\partial_j S + eA_j),
\]

(2.5)

we can get

\[
[i \gamma^\mu (\partial_\mu S + eA_\mu) + M] \zeta(t, x^j) = 0,
\]

(2.6)

where

\[
\Gamma^\mu = \left[ 1 + i \sigma (\omega - eA_t) \right] \gamma^\mu,
\]

\[
M = m - \sigma g^\mu (\omega - eA_t)^2.
\]

(2.7)

Now, multiplying both sides of Eq. (2.7) by the matrix \(-i \gamma^\nu (\partial_\nu S + eA_\nu)\), so that we can obtain

\[
\Gamma^\nu (\partial_\nu S + eA_\nu) \Gamma^\mu (\partial_\mu S + eA_\mu) \zeta - iM \Gamma^\nu (\partial_\nu S + eA_\nu) \zeta = 0
\]

(2.8)

The second term of above equation could be simplified again by Eq. (2.6), so above equation can be rewritten as

\[
\Gamma^\nu \Gamma^\mu (\partial_\nu S + eA_\nu) (\partial_\mu S + eA_\mu) \zeta + M^2 \zeta = 0,
\]

(2.9)

where we can prove the relation

\[
\Gamma^\nu \Gamma^\mu = \gamma^\nu \gamma^\mu + 2i \sigma (\omega - eA_t) g^{\nu \mu} \gamma^\mu + \mathcal{O}(\sigma^2).
\]

(2.10)

We always ignore \( \mathcal{O}(\sigma^2) \) terms because \( \sigma \) is very small. Now, let’s exchange the position of \( \mu \) and \( \nu \) in Eq. (2.9), and consider the relation of gamma matrices \( \{ \gamma^a, \gamma^b \} = 2g^{ab} I \), then we can obtain

\[
\left\{ \frac{\gamma^a \gamma^\beta + \gamma^\alpha \gamma^\beta}{2} (\partial_\alpha S + eA_\alpha) (\partial_\beta S + eA_\beta) + m^2 - 2\sigma mg^{\mu \nu} (\omega - eA_t)^2 \\
+ 2i \sigma (\omega - eA_t) g^{\rho \mu} (\partial_\rho S + eA_\rho) \gamma^\mu (\partial_\mu S + eA_\mu) \right\} \zeta + \mathcal{O}(\sigma^2)
\]

\[
= \left\{ g^{\alpha \beta} (\partial_\alpha S + eA_\alpha) (\partial_\beta S + eA_\beta) + m^2 - 2\sigma mg^{\mu \nu} (\omega - eA_t)^2 \\
+ 2i \sigma (\omega - eA_t) g^{\rho \mu} (\partial_\rho S + eA_\rho) \gamma^\mu (\partial_\mu S + eA_\mu) \right\} \zeta + \mathcal{O}(\sigma^2) = 0.
\]

(2.11)

Namely

\[
[i \sigma \gamma^\mu (\partial_\mu S + eA_\mu) + M] \zeta(t, x^j) = 0,
\]

(2.12)
where

\[ M = g^{\alpha\beta} (\partial_\alpha S + eA_\alpha) (\partial_\beta S + eA_\beta) + m^2 - 2\sigma mg^{tt} (\omega - eA_t)^2 \]  

(2.13)

Using the idea of Eq.(2.8)-(2.9) again, we can multiply both sides of Eq.(2.13) by the matrix 

\[ -i\gamma^\nu (\partial_\nu S + eA_\nu), \]

so that the equation becomes

\[ \sigma \gamma^\nu (\partial_\nu S + eA_\nu) \gamma^\mu (\partial_\mu S + eA_\mu) \zeta - iM \gamma^\nu (\partial_\nu S + eA_\nu) \zeta = 0. \]  

(2.14)

The second term of above equation could be simplified again by Eq.(2.13). Then, exchange \( \mu \) and \( \nu \), and use the relationship \[ \{\gamma^a, \gamma^b\} = 2g^{ab}I, \]

so above equation can be rewritten as

\[ \left[ \frac{\gamma^\nu \gamma^\mu + \gamma^\mu \gamma^\nu}{2} \right] \sigma^2 (\partial_\nu S + eA_\nu) (\partial_\mu S + eA_\mu) + M^2 \right] \zeta (t, x^j) = 0. \]  

(2.15)

The condition that Eq.(2.15) has non-trivial solution required the determinant of coefficient in Eq.(2.15) should vanish, so we can directly get the equation

\[ \sigma^2 g^{\mu\nu} (\partial_\nu S + eA_\nu) (\partial_\mu S + eA_\mu) + M^2 = 0. \]  

(2.16)

Consider the square root for left side of Eq.(2.16) and ignore all the \( O(\sigma^2) \) terms, so we can directly get the modified Hamilton-Jacobi equation:

\[ g^{\mu\nu} (\partial_\nu S + eA_\nu) (\partial_\mu S + eA_\mu) + m^2 - 2\sigma mg^{tt} (\omega - eA_t)^2 = 0. \]  

(2.17)

Therefore, we find the modified Dirac equation from Lorentz invariance violation could lead to the modified Hamilton-Jacobi equation, and the correction of Hamilton-Jacobi equation depends on the energy and mass of radiation fermions. Using the modified Hamilton-Jacobi equation, we then investigate the fermions Hawking tunneling radiation of 2 + 1 dimensional black string and \( n + 1 \) dimensional BTZ-like string in following two sections.

3. Fermions tunneling of 2+1 dimensional black string

The research of gravity in 2+1 dimension can help people further understand the properties of gravity, and it is also important to construct the quantum gravity. Recently, Murata, Soda and Kanno have researched the 2 + 1 dimensional gravity with dilaton field, which action is given by 

\[ I = M_3 \int d^3x \sqrt{-g} \left( BR + \frac{\lambda^2}{B} \right), \]  

(3.1)

where, \( B, M_3 \) and \( \lambda \) are respectively the dilaton field, 3-dimensional Planck mass and the parameter with mass dimension. The static black string solution is given by

\[ ds^2 = -\ln \left( \frac{r}{r_H} \right) dt^2 + \ln \left( \frac{r}{r_H} \right)^{-1} dr^2 + dy^2, \]

\[ B = \lambda r \]  

(3.2)
It is evident that the horizon of this black hole is \( r_H \), but the black string is unstable as \( r_H \lesssim \mathcal{L} \) (where \( \mathcal{L} \) is scale of compactification), so it is assumed that \( r_H \gg \mathcal{L} \).

Now we research the fermions tunneling of this black hole, so the modified Hamilton-Jacobi equation in this spacetime is given by

\[
- (1 - 2\sigma m) \ln \left( \frac{r}{r_H} \right) - \frac{1}{\omega} \omega^2 + \ln \left( \frac{r}{r_H} \right) \left( \frac{dR}{dr} \right)^2 + \left( \frac{dY}{dy} \right)^2 + m^2 = 0,
\]

where we have set \( S = -\omega t + R(r) + Y(y) \), yielding the radial Hamilton-Jacobi equation as

\[
- (1 - 2\sigma m) \ln \left( \frac{r}{r_H} \right) - \frac{1}{\omega} \omega^2 + \ln \left( \frac{r}{r_H} \right) \left( \frac{dR}{dr} \right)^2 + \lambda_0 + m^2 = 0,
\]

where the constant \( \lambda_0 \) in above equation is from separation of variables, and Eq.(3.4) finally can be written as

\[
R_{\pm}(r) = \pm \int \ln \left( \frac{r}{r_H} \right) - \frac{1}{\omega} \omega^2 - \ln \left( \frac{r}{r_H} \right) (\lambda_0 + m^2),
\]

At the horizon \( r_H \) of the black string, above equation is integrated via residue theorem, and we can get

\[
R_{\pm}(r) = \pm i\pi (1 - \sigma m) r_H \omega,
\]

and the fermions tunneling rate

\[
\Gamma = \exp \left( -\frac{2}{\hbar} \text{Im} S \right) = \exp \left[ -\frac{2}{\hbar} (\text{Im} R_+ - \text{Im} R_-) \right]
= e^{-\frac{4\pi}{\hbar} (1 - \sigma m) r_H \omega} = e^{-\frac{\omega}{T_H}}.
\]

The relationship between tunneling rate and Hawking temperature required

\[
T_H = \frac{\hbar}{4\pi r_H} = (1 + \sigma m) T_0.
\]

where \( T_H \) and \( T_0 \) are the modified and non-modified Hawking temperature of the \( 2 + 1 \) dimensional black string respectively, and \( \sigma \) term is the correction.

4. Fermions tunneling of higher dimensional BTZ-like black strings

As we all know that the linear Maxwell action fails to satisfy the conformal symmetry in higher-dimensional spacetime [11], so Hassaine and Martinez proposed gravity theory with non-linear Maxwell field in arbitrary dimensional spacetime

\[
I = -\frac{1}{16\pi} \int_M d^{n+1}x \sqrt{-g} \left[ R + \frac{2}{l^2} - \beta (\alpha F_{\mu\nu} F^{\mu\nu})^2 \right].
\]
where $\Lambda \equiv -l^{-2}$ is cosmological constant. Hendi researched $n+1$ dimensional static black strings solution with $\beta = 1$, $\alpha = -1$ and $s = n/2$, and it is charged BTZ-like solutions \[12\], which metric is

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 \sum_k (dx^k)^2,$$

where

$$f(r) = \frac{r^2}{l^2} - r^{2-n} \left( M + 2^{n/2} Q^{n-1} A_t \right),$$

and the electromagnetic potential

$$A = A_t dt = Q \ln \left( \frac{r}{l} \right) dt$$

As $n = 2$, this solution is no other than the static charged BTZ solution. We will study the Hawking radiation and black hole temperature at the event horizon $r_H$ of this black string. In Eq.(2.17), we can set $S = -\omega t + R(r) + Y(x^k)$, where $x^k$ are the space coordinates excluding the radial coordinate, so that the modified Hamilton-Jacobi equation is given by

$$-(1-2\sigma m)f^{-1}(r)(\omega - eA_t)^2 + f(r) \left( \frac{dR}{dr} \right)^2 + \frac{1}{r^2} \sum_k \left( \frac{dY}{dx^k} \right)^2 + m^2 = 0,$$

and the radial equation with constant $\lambda_0$ is

$$-(1-2\sigma m)f^{-1}(r)(\omega - eA_t)^2 + f(r) \left( \frac{dR}{dr} \right)^2 + \frac{\lambda_0}{r^2} + m^2 = 0,$$

Therefore, at the horizon $r_H$ of the black string, $f(r_H) = 0$ and we finally get

$$R_\pm (r) = \pm \int f(r)^{-1} \sqrt{(1-2\sigma m)(\omega - eA_t)^2 - f(r)(\lambda_0 + m^2)}$$

$$= \pm i\pi (1-\sigma m) \frac{\omega - \omega_0}{f'(r_H)},$$

where $\omega_0 = eA_t(r_H)$. It means that the fermions tunneling rate is

$$\Gamma = \exp \left( -\frac{2}{\hbar} \text{Im} S \right) = \exp \left[ -\frac{2}{\hbar} (\text{Im} R_+ - \text{Im} R_-) \right]$$

$$= e^{-\frac{4\pi}{\hbar}(1-\sigma m) \frac{\omega - \omega_0}{f'(r_H)}} = e^{-\frac{\omega - \omega_0}{\hbar f'(r_H)}}.$$

and the Hawking temperature is

$$T_H = \hbar \frac{1+\sigma m}{4\pi} f'(r_H) = \hbar \frac{1+\sigma m}{4\pi} \left( \frac{nr_H^2}{l^2} - 2\pi Q^n r_H^{1-n} \right) = (1+\sigma m)T_0.$$
5. Conclusions

In this paper, we consider the deformed dispersion relation with Lorentz invariance violation, and generalize the modified Dirac equation in curved spacetime. The fermions tunneling radiation of black strings is researched, and we find the modified Dirac equation could lead to the Hawking temperature's correction, which depend on the correction parameter $\sigma$ and particle mass $m$ in the modified Dirac equation. Next we will discuss the correction of black hole entropy in this theory.

The first law of black hole thermodynamics require

$$dM = TdS + \Xi dJ + UdQ,$$

where $\Xi$ and $U$ are electromagnetic potential and rotating potential, so the non-modified entropy of black hole is \cite{1,13}

$$dS_0 = \frac{dM - \Xi dJ - UdQ}{T_0}.$$  \hspace{1cm} (5.2)

From above results, we know the relationship between modified and non-modified Hawking temperature is $T_H = (1 + \sigma m)T_0$, since the non-modified black hole entropy is given by

$$S_H = \int dS_H = \int \frac{dM - \Xi dJ - UdQ}{(1 + \sigma m)T_0} = S_0 - m \int \sigma dS_0 + O(\sigma^2),$$  \hspace{1cm} (5.3)

where we can ignore the $O(\sigma^2)$ because $\sigma \ll 1$. Eq.(5.3) shows that the correction of black hole entropy depends on $\sigma$, which is independent from time and space coordinates. However, it is possible that $\sigma$ depends on other parameters in curved spacetime, and it is very interesting that $\sigma$ depends on $S_0$. Especially, as $\sigma = \frac{\sigma_0}{S_0} + \cdots$, we can get the logarithmic correction of black hole entropy

$$S_H = S_0 - m\sigma_0 \ln S_0 + \cdots.$$  \hspace{1cm} (5.4)

In quantum gravity theory, the logarithmic correction has been researched in detail \cite{14,15}, and according to Ref.\cite{16}, it is required that the coefficient of logarithmic correction should be $-\frac{n+1}{2(n-1)}$ in $n+1$ dimensional spacetime, so it indicates the $\sigma_0$ could be $\frac{n+1}{2m(n-1)}$.

On the other hand, from the deformed dispersion relation (1.1) with $\alpha = 2$, it implies that the Klein-Gordon equation could be given by

$$(-\partial_t^2 + \partial_j^2 + m^2 - \sigma^2 \hbar^2 \partial_t^2 \partial_j^2) \Phi = 0,$$

so the generalized uncharged Klein-Gordon equation in static curved spacetime is

$$[g^{tt} \nabla_t^2 + g^{jj} \nabla_j^2 + m^2 + \sigma^2 \hbar^2 (g^{tt} \nabla_t^2) (g^{jj} \nabla_j^2)] \Phi = 0,$$

and using the semi-classical approximation with $\Phi = C \exp (iS/h)$, the modified Hamilton-Jacobi equation in scalar field is given by

$$(1 + \sigma^2 g^{tt} \omega^2) g^{\mu \nu} \partial_{\mu} S \partial_{\nu} S + m^2 - \sigma^2 (g^{tt})^2 \omega^4 = 0,$$  \hspace{1cm} (5.7)
namely

\[ g^{\mu\nu} \partial_\mu S \partial_\nu S + m^2 - \sigma^2 g^{tt} \omega^2 (m^2 + g^{tt} \omega^2) + \mathcal{O}(\sigma^4) = 0. \]  

(5.8)

Contrasting Eq. (2.17) and Eq. (5.8) as uncharged case, we find the correctional terms of Dirac field and scalar field are very different. The fact implies that the corrections of Hawking temperature and black hole entropy from Hawking tunneling radiation with different spin particles could be different, and this conclusion could be helpful to suggest a new idea to research the black hole information paradox. Work in these fields is currently in progress.

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References

[1] S.W. Hawking, Nature 248 (1974) 30;  
S.W. Hawking, Commun. Math. Phys. 43 (1975) 199.

[2] S.P. Robinson, F. Wilczek, Phys. Rev. Lett. 95 (2005) 011303, arXiv:gr-qc/0502074;  
T. Damoar, R. Ruffini, Phys. Rev. D 14 (1976) 332;  
S. Sannan, Gen. Relativ. Gravit. 20 (1988) 239.

[3] P. Kraus, F. Wilczek, Nucl. Phys. B 433 (1995) 403, arXiv:gr-qc/9408003;  
M.K. Parikh, F. Wilczek, Phys. Rev. Lett. 85 (2000) 5042, arXiv:hep-th/9907001.

[4] S. Hemming, E. Keski-Vakkuri, Phys. Rev. D 64 (2001) 044006, arXiv:gr-qc/0005115;  
Q.Q. Jiang, S.Q. Wu, X. Cai, Phys. Rev. D 75 (2007) 064029;  
S. Iso, H. Umetsu, F. Wilczek, Phys. Rev. D 74 (2006) 044017, hep-th/0606018;  
A.J.M. Medved, Phys. Rev. D 66 (2002) 124009, arXiv:hep-th/0207247;  
M.K. Parikh, arXiv:hep-th/0402166;  
M.K. Parikh, J.Y. Zhang, Z. Zhao, JHEP 0510 (2005) 055;  
J.Y. Zhang, Z. Zhao, Phys. Lett. B 638 (2006) 110, arXiv:gr-qc/0512153;  
E.T. Akhmedov, V. Akhmedova, D. Singleton, Phys. Lett. B 642 (2006) 124;  
V. Akhmedova, T. Pilling, A. de Gill, D. Singleton, Phys. Lett. B 666 (2008) 269;  
K. Srinivasan, T. Padmanabhan, Phys. Rev. D 60 (1999) 24007;  
S. Shankaranarayanan, T. Padmanabhan, K. Srinivasan, Class. Quantum Grav. 19 (2002) 2671;  
M. Anghelban, M. Nadalini, L. Vanzo, S. Zerbini, J. High Energy Phys. 0505 (2005) 014;  
S.Z. Yang, D.Y. Chen, Int. J. Theor. Phys. 46 (2007) 2923Y;  
S.Z. Yang, D.Y. Chen, Chin. Phys. B 17 (2008) 817.

[5] R. Kerner, R.B. Mann, Class. Quantum Grav. 25 (2008) 095014, arXiv:0710.0612;  
R. Kerner, R.B. Mann, Phys. Lett. B 665 (2008) 277, arXiv:hep-th/0803.2246.

[6] R. Li, J.R. Ren, S.W. Wei, Class. Quantum Grav. 25 (2008) 125016, arXiv:0803.1410;  
R. Li, J.R. Ren, Phys. Lett. B 661 (2008) 370, arXiv:0802.3954;  
D.Y. Chen, Q.Q. Jiang, X.T. Zu, Class. Quantum Grav. 25 (2008) 205022, arXiv:0803.3248;  

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D.Y. Chen, Q.Q. Jiang, X.T. Zu, Phys. Lett. B 665 (2008) 106, arXiv:0804.0131;
R.D. Criscienzo, L. Vanzo, Europhys. Lett. 82 (2008) 60001;
L.H. Li, S.Z. Yang, T.J. Zhou, R. Lin, Europhys. Lett. 84 (2008) 20003;
Q.Q. Jiang, Phys. Lett. B 666 (2008) 517;
Q.Q. Jiang, Phys. Rev. D 78 (2008) 044009;
K. Lin, S.Z. Yang, Int. J. Theor. Phys. 48 (2009) 2061.

[7] K. Lin, S.Z. Yang, Phys. Rev. D 79 (2009) 064035;
K. Lin, S.Z. Yang, Phys. Lett. B 674 (2009) 127;
K. Lin, S.Z. Yang, Chin. Phys. B 20 (2011) 110403.

[8] G. Amelino-Camelia, Int. J. Mod. Phys. D11, (2002) 35, arXiv:gr-qc/0012051;
G. Amelino-Camelia, New J. Phys. 6, (2004) 188, arXiv:gr-qc/0212002;
J. Magueijo and L. Smolin, Phys. Rev. Lett., 88 (2002) 190403, arXiv:hep-th/0112090;
J. Magueijo and L. Smolin, Phys. Rev. D67 (2003) 044017, arXiv:gr-qc/0207085;
J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Lett. B293 (1992) 37,
arXiv:hep-th/9207103;
Chaos, Solitons and Fractals 10(1999) 345, arXiv:hep-th/9805120;
J. Ellis, N. E. Mavromatos and A. S. Sakharov, Astropart. Phys. 20 (2004) 669,
arXiv:astro-ph/0308403;
S. I. Kruglov, Modified wave equation for spinless particles and its solutions in an
external magnetic field, arXiv:1207.6573 [hep-th];
T. Jacobson, S. Liberati and D. Mattingly, Nature 424 (2003) 1019,
arXiv:astro-ph/0212190.

[9] S. I. Kruglov, Phys. Lett. B 718 (2012) 228 arXiv:1210.0509 [gr-qc]

[10] K. Murata, J. Soda and S.Kanno, Evaporating (2+1)-dimensional black strings,
arXiv:gr-qc/0701137.

[11] M Hassaâne and C Martínez, Phys. Rev. D75 (2007) 027502, arXiv:hep-th/0701058;
M Hassaâne and C Martínez, Class. Quantum Grav. 25 (2008) 195023, arXiv:0803.2946
[hep-th]
[12] S.H.Hendi, Eur. Phys. J. C 75 (2007) 027502, arXiv:1007.2704 [gr-qc].
[13] J.D. Bekenstein, Lett. Nuovo Cimento 4 (1972) 737;
J.D. Bekenstein, Phys. Rev. D 7 (1973) 2333.
[14] R. Banerjee, R.B. Majhi, JHEP 0806 (2008) 095, arXiv:0805.2220 [hep-th];
R. Banerjee, R.B. Majhi, Phys. Lett. B 662 (2008) 62;
R. Banerjee, R.B. Majhi, S. Samanta, Phys. Rev. D 77 (2008) 124035;
R. Banerjee, R.B. Majhi, Phys. Rev. D 79 (2009) 064024, arXiv:0812.0497 [hep-th];
S.K. Modak, Phys. Lett. B 671 (2009) 167, arXiv:0807.0959 [hep-th];
R. Banerjee, R.B. Majhi, Phys. Lett. B 674 (2009) 218, arXiv:0808.3688 [hep-th];
R.G. Cai, L.M. Cao, Y.P. Hu, JHEP 0808 (2008) 090, arXiv:0807.1232 [hep-th];
J.Y. Zhang, Phys. Lett. B 668 (2008) 353, arXiv:0806.2441 [hep-th];
R. Banerjee, S.K. Modak, JHEP 0905 (2009) 063, arXiv:0903.3321 [hep-th];
K. Lin, S.Z. Yang, Europhys. Lett. 86 (2009) 20006;
K. Lin, S.Z. Yang, Phys. Lett. B 680 (2009) 506.

[15] S.W. Hawking, Commun. Math. Phys. 55 (1977) 133;
R.K. Kaul, P. Majumdar, Phys. Rev. Lett. 84 (2000) 5255;
D.V. Fursaev, Phys. Rev. D 51 (1995) R5352;
S. Mukherjee, S.S. Pal, JHEP 0205 (2002) 026;
V.P. Frolov, W. Israel, S.N. Solodukhin, Phys. Rev. D 54 (1996) 2732;
M.R. Setare, Phys. Lett. B 573 (2003) 173;
M.R. Setare, Eur. Phys. J. C 38 (2004) 389.

[16] S. Das, P. Majumdar, R.K. Bhaduri, Class. Quantum Grav. 19 (2002) 2355.