Gravitational waves as a probe of SUSY scale

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We investigate the sources of the Hubble-induced mass for a flat direction in supersymmetric theories and show that the sign of the Hubble-induced mass generally changes just after the end of inflation. This implies that global cosmic strings generally form after the end of inflation in most supersymmetric models, including the Minimal Supersymmetric Standard Model. The cosmic strings emit gravitational waves whose frequency corresponds to the Hubble scale, until they disappear when the Hubble parameter decreases down to the soft mass of the flat direction. As a result, the peak frequency of gravitational waves is related to the supersymmetric scale. The observation of this gravitational wave signal will give us information of supersymmetric scale and reheating temperature.

Introduction. The observation of gravitational waves (GWs) will open a new window onto the early Universe and provide information on physics at correspondingly high energy scales [1]. Stochastic GW signals are generated by non-equilibrium phenomena during the post-inflationary period, such as preheating [2], first order phase transition [3,17], turbulent motions [18–21], topological defects [22–29], and self-ordering scalar fields [30–34]. Quantum fluctuations during inflation are another source of GWs, called the inflationary GW background [35,36]. Each source may predict characteristic GW signals measurable by future GW detectors; such as LISA [37], DECIGO [38], Advanced LIGO [39], and ET [40]. The observation of these GW signals will improve our understanding on particle physics beyond the Standard Model.

Supersymmetric (SUSY) theories are well-motivated in particle physics, because it addresses the hierarchy problem and also achieves gauge coupling unification. In SUSY theories, there usually exist scalar fields called flat directions, whose potentials are flat as long as they maintain SUSY and renormalizability (see Ref. [41]). In this letter, we investigate the dynamics of a flat direction and show that cosmic string network generically forms after the end of inflation. They eventually disappear when the Hubble parameter decreases down to the mass of the flat direction. Since the cosmic strings generate a stochastic GW background with a peak frequency corresponding to the Hubble scale, the information of the mass of the flat direction is imprinted on the GW spectrum. This mechanism will give us information of SUSY scale through GW detection even if its energy scale is beyond the reach of future accelerators.

Dynamics of flat direction. Let us focus on one flat direction, which we denote as $\phi$. During inflation, the flat direction obtains Hubble-induced mass through higher-dimensional Kähler potentials like

$$\int d^2 \theta d^2 \bar{\phi} c_H^I H_{\text{int}}^I |X|^2 |\phi|^2,$$

where $M_*$ is a cut-off scale, and $c_H^I$ is an $O(1)$ constant. We expect that $M_*$ is less than or equal to the Planck scale $M_{\text{Pl}}$ ($\simeq 2.4 \times 10^{18}$ GeV). The $F$-term of $X$ drives inflation and satisfies the relation of $F_X^2 = 3H_{\text{int}}^I M_{\text{Pl}}^2$. This implies that the flat direction obtains a Hubble-induced mass of $c_H^I H_{\text{int}}^I |\phi|^2$ during inflation, where

$$c_H^I = 3 c_H^I M_{\text{Pl}}^2 M_*^2.$$  

While the coefficient $c_H^I$ is assumed to be negative in the context of the Affleck-Dine baryogenesis [42], we assume $c_H^I > 0$ in this paper. In this case, the flat direction stays at the origin, i.e., $\phi = 0$, during inflation.

After inflation ends and before reheating completes, the energy density of the Universe is dominated by the oscillation of a scalar field (denoted by $I$) and the Hubble parameter decreases with time as $H(t) \propto a^{-3/2}(t)$, where $a(t)$ is the scale factor. During this oscillation era, the flat direction obtains a Hubble-induced mass through

$$\int d^2 \theta d^2 \bar{\phi} c_H^I H_{\text{int}}^I |I|^2 |\phi|^2 \geq c_H^I |I|^2 |\phi|^2.$$  

Since the oscillation time scale of $I$ is much smaller than $H^{-1}$, we can average $|I|^2$ over the oscillation time scale like $|I|^2 \approx \rho_I(t)/2 \approx (3/2)H^2(t)M_{\text{Pl}}^2$. Thus, we obtain the Hubble induced mass of $c_H^I H^2(t) |\phi|^2$ with

$$c_H = 3 c_H^I M_{\text{Pl}}^2$$

during the oscillation era. The coefficient of Hubble-induced mass during inflation, $c_H^I$, is generally different from the one during the oscillation era, $c_H$, because the field $I$ is generally different from the field $X$. For example, in the chaotic inflation model proposed in Ref. [43].
the field $X$ (in this letter) is identified with the field $X$ (in Ref. [14]) while $I$ is identified with the inflaton $\varphi$. In the simplest hybrid inflation model proposed in Ref. [15] (Ref. [16]), the field $X$ is identified with the field $\Phi$ ($S$) while $I$ is a water-fall field $\Psi_1$ and $\Psi_2$ ($\phi$ and $\bar{\phi}$). While the non-renormalizable terms are heavily dependent on the high-energy physics beyond the cut-off scale, we assume that among many flat directions in SUSY theories (at least) one flat direction has $c_{H_{\text{int}}} > 0$ and $c_{H} < 0$. Let us investigate the dynamics of such a flat direction. The flat direction stays at $\phi = 0$ during inflation, and then, it obtains a large VEV after the end of inflation.

If the flat direction has a non-renormalizable superpotential, the resultant GW background is generally too small to be detected [17]. Thus, we consider the case that the superpotential of the flat direction is absent due to discrete R-symmetries. In this case, the potential of the flat direction obtains higher-dimensional terms coming from non-renormalizable Kähler potential and is written as

$$V(\phi) = c_{H}H^2(t) |\phi|^2 + m_{\phi}^2 |\phi|^2 + a_{H}H^2(t) |\phi|^{2n-2} M_{Pl}^{-2n-1} + \ldots.$$  

where $n \geq 3$ is a certain integer, $m_{\phi}$ is the soft mass of the flat direction, and $a_{H}$ is given by

$$a_{H} = a_{H}' \left( \frac{M_{Pl}}{M_{*}} \right)^{2n-2} ,$$

with $a_{H}'$ being an $O(1)$ constant. The dots ... represent the other irrelevant higher-dimensional terms. Here we assume that higher-dimensional terms breaking $U(1)$ symmetry like $\phi^n + \text{c.c.}$ are absent due to R-symmetries. The potential minimum is determined as

$$\langle \phi \rangle = \left( \frac{|c_{H}|}{a_{H}(n-1)} \right)^{1/(2n-4)} M_{Pl} \sim M_{*} ,$$

as long as $c_{H}H^2(t) \gg m_{\phi}^2$.

Here we investigate the components of the flat direction in detail. Let us consider $L_i H_u$ flat direction as an illustration, where $i$ is a flavour index. Since the above non-renormalizable Kähler potentials generally breaks flavour symmetry, $L_i H_u$ flat direction has the local $SU(2)_L \times U(1)_Y$ symmetry and the global $U(1)_L$ symmetry. The non-zero VEV of the flat direction breaks the local $SU(2)_L \times U(1)_Y$ symmetry to the $U(1)_{\text{EM}}$ symmetry and breaks the global $U(1)_L$ symmetry completely. As a result, global cosmic string network forms after the end of inflation.

By using numerical simulations (see below), we confirm that cosmic string network reaches a scaling regime within a certain time. In this regime, the number of cosmic strings in the Hubble volume is $O(1)$.

Since the Hubble-induced mass decreases with time as $\propto H(t) \propto a^{-3/2}(t)$, the soft mass $m_{\phi}$ eventually dominates the potential of the flat direction and the flat direction starts to oscillate around $\phi = 0$. Let us denote that time as $t_{\text{decay}}$, which is estimated by

$$c_{H}H^2(t_{\text{decay}}) \simeq m_{\phi}^2 .$$

Note that for low-scale SUSY models we should require $T_{RH} \lesssim 10^9 \text{ GeV}$ to avoid the gravitino problem [48][49]. Even for high-scale SUSY models such as pure gravity mediation, $T_{RH} \lesssim 10^{10} \text{ GeV}$ is required to avoid an over-production of LSP ($= O(100) \text{ GeV}$) [50][52]. In such well-motivated cases, the cosmic strings dissipate after reheating completes, that is, $t_{\text{decay}} < t_{RH}$, where $t_{RH}$ is the time reheating completes. Hereafter we consider such a case.

**Calculation of GWs.** GWs are emitted by the dynamics of the cosmic strings [22][25][28][32][33]. We calculate the spectrum of GWs using the method proposed in Ref. [7][27], which is suitable to our situation compared with the method to calculate GW amplitudes from localized sources derived in Ref. [53][6]. Hereafter, we change the time variable from $t$ to the conformal time $\tau$ which is defined by $dt = a d\tau$.

The energy density of GWs can be calculated from [7]

$$\Omega_{gw}(\tau) \equiv \frac{1}{\rho_{total}(\tau)} \frac{1}{d \log \bar{k}} \frac{1}{24V a^4 H^2(\tau)} \int d\Omega_k \sum_{ij} (|A_{ij}|^2 + |B_{ij}|^2) ,$$

where $\rho_{total}(\tau) (= 3M_{Pl}^2 H^2(\tau))$ is the total energy density of the Universe. The frequency-dependent coefficients $A_{ij}$ and $B_{ij}$ are given as [27]

$$A_{ij}(k) = -16\pi G \int_{\tau_i}^{\tau_f} d\tau' \, \tau' a(\tau') f_A(k\tau') T_{ij}^{TT}(\tau',k) ,$$

$$B_{ij}(k) = 16\pi G \int_{\tau_i}^{\tau_f} d\tau' \, \tau' a(\tau') f_B(k\tau') T_{ij}^{TT}(\tau',k) .$$

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1. The absence of the non-renormalizable superpotential for $L_i H_u$ is disfavored from the viewpoint of the observed neutrino oscillations. In this letter, however, we take $L_i H_u$ flat direction as an example to take advantage of its simple flavour structure.

2. If the flat direction has a larger flavour symmetry like $SU(3)_{\text{flavour}}$, the non-zero VEV of the flat direction breaks the global $SU(3)_{\text{flavour}} \times U(1)_L$ symmetry to the $SU(2)_L \times U(1)_{\text{EM}}$ symmetry. Also in this case, GWs are emitted by the dynamics of randomly distributed Nambu-Goldstone modes of the flat direction [39][44][47], though cosmic strings are absent.

3. Since $d^2 V(\phi)/d\phi^2 \sim H^2(t)$ at $\phi = \langle \phi \rangle$, a typical width of cosmic strings is of the order of the Hubble radius. This implies that the Nambu-Goto approximation, which was used in Ref. [44] for example, is inappropriate to describe these cosmic strings.
where \( T_{ij}^{TT} \) is the Fourier transformed transverse-traceless part of the anisotropic stress. Here we have implicitly assumed that the source term \( T_{ij}^{TT} \) lasts during the interval of \([\tau_i, \tau_f]\). The functions \( f_A \) and \( f_B \) are calculated by matching solutions of the Einstein equation in the oscillation era with that in the radiation dominated era at \( \tau = \tau_{RH} \) as\(^4\):

\[
\begin{align*}
 f_A(k\tau') &= [a_1 n_1(k\tau') - a_2 j_1(k\tau')], \\
 f_B(k\tau') &= [-b_1 n_1(k\tau') + b_2 j_1(k\tau')],
\end{align*}
\]

where \( j_i \) and \( n_i \) are the spherical Bessel and Neumann functions of order \( l \), and the coefficients are given as
\[
\begin{align*}
 a_1 &= x^2 [j_1(x) \partial_x n_0(x) - n_0(x) \partial_x j_1(x)] |_{x \rightarrow k\tau_{RH}}, \\
 a_2 &= x^2 [n_1(x) \partial_x n_0(x) - n_0(x) \partial_x n_1(x)] |_{x \rightarrow k\tau_{RH}}, \\
 b_1 &= -x^2 [j_1(x) \partial_x j_0(x) - j_0(x) \partial_x j_1(x)] |_{x \rightarrow k\tau_{RH}}, \\
 b_2 &= -x^2 [n_1(x) \partial_x j_0(x) - j_0(x) \partial_x n_1(x)] |_{x \rightarrow k\tau_{RH}}.
\end{align*}
\]

Here let us estimate the spectrum of GWs from the cosmic strings before we show our simulation results. GWs are most efficiently emitted for the frequency corresponding to the Hubble radius, which is given as \( \tau \sim \tau^{-1} \).

Since the emission of GWs proceeds through the Planck suppressed interaction (see Eqs. (9), (10), and (11)), the produced energy density of GWs can be estimated as\(^4\):

\[
\frac{\Delta \Omega_{gw}}{\Delta \log \tau} \sim \left( \frac{\phi}{M_{Pl}} \right)^4 \sim \left( \frac{M_*}{M_{Pl}} \right)^4,
\]

where we use Eq. (7). The GW energy density decreases with time as \( \propto a^{-1(\tau)} \) in the oscillation era and the peak frequency of the emitted GWs decreases with time as \( \propto \tau^{-1} \). Therefore, the GW spectrum for the frequency \( k \tau^{-1} \) is proportional to \( k^{-2} \).

For large-scale modes \( k \lesssim \tau^{-1} \), the Fourier transformed transverse-traceless part of the anisotropic stress \( T_{ij}^{TT} \) is independent of \( k \) due to the loss of causality at the large scale\(^4\). Thus, the spectrum of GWs bends at the wavenumber around \( k \approx \tau_{RH}^{-1} \). The GW emission terminates at the time of \( \tau_{\text{decay}} \), when the cosmic strings disappear. The GW peak energy density and frequency at that time is roughly estimated as Eq. (14) and \( k_{\text{peak}} \sim \alpha H(\tau_{\text{decay}}) \approx a(t_{\text{decay}}) c_H^{-1/2} m_\phi \), respectively. Then the GW amplitude decreases with time as \( \propto a^{-1(\tau)} \) until reheating completes.

### Results of numerical simulations.

To calculate the GW spectrum from Eqs. (7), (10), and (11), we have performed lattice simulations with \( N^3 = 256^3 \) grid points in the oscillation era, in which \( H(\tau) \propto a^{-3/2} \). We use a numerical method similar to the one used in Ref. [29]. While details are given in Ref. [17], we stress that our results are independent of the choices of the time step, simulation size, and grid size by changing their values by a factor of 50%.

Initial fluctuations of the field value are seeded by the vacuum fluctuations though we have checked that our results are qualitatively insensitive to the detailed form of the initial conditions. Numerical simulations are performed in a unit with \( a(\tau_i) \equiv a_i = 1 \) and \( H(\tau_i) \equiv H_i = 1 \), though we explicitly write \( H_i \) below.

Figure 1 shows evolution of GW spectra obtained from our numerical simulations. We confirm that cosmic string network reaches scaling regime at the time around \( \tau H_i \approx 15 \). Then the produced energy density of GWs \( \Delta \Omega_{gw}/\Delta \log \tau \) becomes constant while its peak frequency \( k_{\text{peak}} \) decreases with time as \( k_{\text{peak}} \propto \tau^{-1} \). When the flat direction starts to oscillate at the time around \( \tau H_i = 35 (\approx \tau_{\text{decay}} H_i) \), the peak frequency becomes constant and the GW spectrum begins to redshift adiabatically as \( \propto a^{-1} \). We find that GW spectra for \( k \lesssim \tau_{\text{decay}}^{-1} \) are consistent with analytical estimation with \( T_{ij}^{TT} \) independent of \( k \), which we expect from the viewpoint of causality. From Fig. 1 we obtain numerical factors such that

\[
\Omega_{gw}(t_{\text{decay}}) \approx 2 \left( \frac{\phi}{M_{Pl}} \right)^4,
\]

\[
\frac{k_{\text{peak}}}{a(t_{\text{decay}})} \approx \frac{3 m_\phi}{\sqrt{H_i}},
\]

where \( \langle \phi \rangle \) and \( t_{\text{decay}} \) are given by Eqs. (7) and (8), respectively.
FIG. 1. Evolution of GW spectra obtained by numerical calculations. We show the obtained spectra at $\tau_{\text{RH}} = 15$ (red line), $\tau_{\text{RH}} = 35$ (green line), and $\tau_{\text{RH}} = 100$ (magenta line). We take $n = 4$, $c_H = 15$, and $m_{\phi}/H_i = 5 \times 10^{-4}$. The blue (dotted) curve represents the contour of the peak frequency and the peak GW energy density for the case of $m_{\phi}/H_i = 5 \times 10^{-4}$ (0). The red dashed curve represents an analytic estimation given by Eqs. (10) and (11) with $k$-independent $T_{ij}^{\phi \phi}$.

(entropy) density. We have used Eq. (13) and assumed $c_H^2 = a_H^2 = 1$ and $n = 4$ in the last line (see Eqs. (4), (6), and (7)). Taking redshift into account, we obtain the present value of peak frequency $f_0$ like

$$f_0 \approx \left( \frac{g_*(t_0)}{g_*(t_{\text{RH}})} \right)^{1/3} \left( \frac{T_0}{T_{\text{RH}}} \right) \left( \frac{H_{\text{RH}}}{H_{\text{decay}}} \right)^{2/3} \frac{k_{\text{peak}}}{2\pi a(t_{\text{decay}})} \approx 7 \times 10^2 \text{ Hz} \left( \frac{m_{\phi}}{10^3 \text{ GeV}} \right)^{1/3} \left( \frac{T_{\text{RH}}}{10^9 \text{ GeV}} \right)^{1/3} \left( \frac{M_*}{M_{\text{Pl}}} \right)^{1/3},$$

(18)

where we use Eq. (16) in the last line. As explained above, the spectrum of GWs bends at the wavenumber corresponding to the Hubble scale at the time of reheating ($k = k_{\text{bend}} \approx a(t_{\text{RH}})H_{\text{RH}}$). The present value of bending frequency $f_{\text{bend}}$ is given as

$$f_{\text{bend}} = \left( \frac{g_*(t_0)}{g_*(t_{\text{RH}})} \right)^{1/3} \left( \frac{T_0}{T_{\text{RH}}} \right) \frac{k_{\text{bend}}}{2\pi a(t_{\text{RH}})} \approx 30 \text{ Hz} \left( \frac{T_{\text{RH}}}{10^9 \text{ GeV}} \right).$$

(19)

We can obtain the reheating temperature $T_{\text{RH}}$ from observation of the bend in the GW spectrum at the large scale as observation of the inflationary GW background [53]. In addition, we can obtain the mass of the flat direction $m_{\phi}$ and the cut-off scale $M_*$ from observations of energy density and peak frequency of GWs (see Eqs. (17) and (18)).

Figure 2 shows examples of GW spectra predicted by the present mechanism. We also plot single detector sensitivities for LISA [57] and Ultimate DECIGO [58] by using the online sensitivity curve generator in [56] with the parameters in Table 7 of Ref. [57]. We plot cross-correlation sensitivities for Advanced LIGO [39] and ET (ET-B configuration) [40], assuming two detectors are co-aligned and coincident. In the figure, we take the signal to noise ratio $\text{SNR} = 5$, the angular efficiency factor $F = 2/5$, the total observation time $T = 1 \text{ yr}$, and the frequency resolution $\Delta f/f = 0.1$. CMB constraints (horizontal lines) are put on the integrated energy density of GWs $\int d\log f \Omega_{gw} h^2(t_0)$ [58]. We find that GW signals can be observed by ET and Ultimate DECIGO.

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