Forecasting the Urban Skyline with Extreme Value Theory

Jonathan Auerbach and Phyllis Wan

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Abstract

The world’s urban population is expected to grow fifty percent by the year 2050 and exceed six billion. The major challenges confronting cities, such as sustainability, safety, and equality, will depend on the infrastructure developed to accommodate the increase. Urban planners have long debated the consequences of vertical expansion—the concentration of residents by constructing tall buildings—over horizontal expansion—the dispersal of residents by extending urban boundaries. Yet relatively little work has predicted the vertical expansion of cities and quantified the likelihood and therefore urgency of these consequences.

We regard tall buildings as random exceedances over a threshold and use extreme value theory to forecast the skyscrapers that will dominate the urban skyline in 2050 if present trends continue. We predict forty-one thousand skyscrapers will surpass 150 meters and 40 floors, an increase of eight percent a year, far outpacing the expected urban population growth of two percent a year. The typical tall skyscraper will not be noticeably taller, and the tallest will likely exceed one thousand meters but not one mile. If a mile-high skyscraper is constructed, it will hold fewer occupants than many of the mile-highs currently designed. We predict roughly three-quarters the number of floors of the Mile-High Tower, two-thirds of Next Tokyo’s Sky Mile Tower, and half the floors of Frank Lloyd Wright’s The Illinois—three prominent plans for a mile-high skyscraper. However, the relationship between floor and height will vary across cities.
1. Introduction

The world is urbanizing at an astonishing rate. Four billion people live in urban areas, up from two billion in 1985. By 2050, the United Nations predicts urban areas will encompass more than six billion residents. These increases are due to growth in both the world population and the proportion of the population that resides in urban areas. Roughly half the world’s population is urban, up from forty-percent in 1985 and projected to rise above two-thirds in 2050 (UN (2018)). The future preponderance of cities suggests the major challenges confronting civilization will be urban challenges. Moreover, the particular nature of these challenges will depend on how cities choose to accommodate urbanization (Rose (2016), page 15).

Cities change in response to population growth by either increasing density—the population per land area—or extending boundaries—the horizontal distance between city limits. The prevailing paradigm among urban planners is to preserve city boundaries and encourage density (Angel et al. (2011)). It argues that density affords certain economies of scale, such as reducing the cost of infrastructure and social services like roads, water, safety, and health care. Density is also advocated to promote sustainability by preserving the city periphery for agriculture or wildlife (Swilling (2016)). Yet density, if not properly accommodated, can lead to overcrowding and impede quality of life. Nearly one third of urban residents in developing regions live in overcrowded slums that concentrate poverty (UN (2015), page 2).

Some urban planners have argued that density requires vertical expansion, through the construction of skyscrapers, to prevent overcrowding and maintain quality of life (Gottmann (1966)), (Al-Kodmany (2012)), (Barr (2017)). This three-dimensional solution to a two-dimensional problem was stated as early as 1925 by architect Le Corbusier: “We must decongest the centers of our cities by increasing their density” (Kashef (2008)). In this spirit, economist Glaeser (2011) recommends policies that ease height restrictions and increase financial incentives for skyscraper development.

Other urban planners warn that urbanization is too rapid to be adequately addressed by vertical expansion (James (2001), page 484), (Cohen (2006), page 73), (Canepari (2014)). Angel et al. (2011) argue cities must “make room” for urbanization by moving boundaries and recommend policies that extend the radius of public services like the transit system.

It stands to reason that cities will utilize multiple strategies to accommodate urbanization. For example, cities will incentivize some vertical expansion and extend the radius of some public services. The challenges facing cities in 2050 will depend on which policies are implemented. Anticipating these challenges requires an answer to questions such as: if present trends continue, how much vertical growth will the typical city experience by 2050? How far will the typical city boundary extend?

This paper demonstrates that extreme value theory provides a principled basis for forecasting vertical growth. It regards tall buildings as random exceedances over a threshold and uses the statistical laws governing extreme values to extrapolate the characteristics of skyscrapers that will dominate the urban skyline in 2050. Similar arguments have produced successful forecasts in a wide variety of fields, most notably those concerned with risk management: finance (e.g. Gencay and Selcuk (2004), Bao et al. (2006), Chan and Gray (2006), Herrera and González (2014)) and climate (e.g. Garreau, RD. (2004), Ghil (2011), D’Amico et al. (2015), Thompson et al. (2017)). However, we know of no work that applies these arguments to forecast how cities will respond to rapid urbanization.

The findings are arranged into five sections. Section 2 describes the dataset for this study: a database maintained by the Council on Tall Buildings and Urban Habitat (2017) of nearly all skyscrapers worldwide taller than 150 meters and completed as of December 2017. A brief review of the skyscraper literature follows, motivating the consistency of skyscrapers with extreme value theory. Section 3 outlines the log-linear model used to conclude that forty-one thousand skyscrapers will be completed by 2050. Section 4 outlines the generalized Pareto distribution (GPD) used to conclude that there is a seventy-five percent chance a skyscraper will exceed one thousand meters by 2050 and a nine percent chance it will exceed a mile. Section 5 outlines the censored asymmetric bivariate logistic distribution used to conclude that a mile-high skyscraper, if built, will have around 250 floors. The paper concludes in the final two sections by discussing some methodological and policy consequences of these predictions and identifying areas for future research.
2. Extreme Value Theory is Consistent with Skyscraper Theory

This paper predicts the prevalence and nature of skyscrapers in the year 2050 if present trends continue. The present section reviews the definition of a skyscraper and motivates the use of extreme value theory to characterize tall buildings.

Strictly speaking, the term skyscraper refers not to height but to the mode of construction. A skyscraper is defined as any multi-story building supported by a steel or concrete frame instead of traditional load-bearing walls (Curl and Wilson (2015), page 710). The tall buildings capable of sustaining the dense cities of the future will almost certainly be skyscrapers. Over the past century, building beyond a few floors has required a supporting frame to be economical.

In contrast to the precise definition of a skyscraper, the definition of a tall building depends on context. From a safety perspective, high-rises—multi-story buildings beyond 23 meters (75 feet)—are harder to evacuate than low-rises. For characterizing the urban skyline, however, a twenty story skyscraper in Sydney, Australia might go unnoticed while the same building in Sidney, New York would dominate the skyline. The Council on Tall Buildings and Urban Habitat (CTBUH) sets international standards for the purposes of research and arbitrating titles, such as the world’s tallest building. They designate buildings higher than fifty meters as tall, three-hundred meters as supertall, and six-hundred meters as megatall. Height is defined as the distance from “the level of the lowest, significant, open-air, pedestrian entrance to the architectural top of the building, including spires, but not including antennae, signage, flag poles or other functional-technical equipment. This measurement is the most widely utilized ...” (Council on Tall Buildings and Urban Habitat (2017)). The New York City Fire Department relies on all of these definitions when inspecting buildings, dispatching firefighters, and conducting investigations.

CTBUH also maintains The Skyscraper Center, a database of every tall building worldwide. The data is recognized as “the premier source for accurate, reliable information on tall buildings around the world” (ibid.) However, the data relies on CTBUH members and the public to add entries, resulting in the partial or complete omission of some smaller buildings. For this reason, this paper refers to buildings exceeding 150 meters and 40 floors as tall, roughly the size of the United Nations Headquarters in New York City. CTBUH catalogs 3,251 tall skyscrapers in 258 cities as of December 2017. In later sections, it will also be convenient to consider skyscrapers exceeding 225 meters or 59 floors, roughly the size of One Penn Plaza in New York City. Due to the nature of our analysis, we refer to these buildings as extremely tall. CTBUH catalogs 325 extremely tall skyscrapers in 81 cities as of December 2017.

The height and year each skyscraper was completed is displayed in the top-left panel of Figure 1. From the panel, it appears as though a simple statistical relationship might govern skyscraper heights, ignoring the hiatus between the Great Depression and the Second World War, which is assumed to be anomalous. But it cannot be taken for granted that skyscrapers can be modeled statistically. These buildings are modern marvels, requiring enormous cooperation across teams of architects, engineers, financers, and multiple levels of government. Extrapolation from the data is only meaningful if the determinants underlying skyscraper construction are varied and independent enough to be characterized statistically. The remainder of this section outlines an argument supporting this view.

Skyscrapers appeared in the nineteenth century only after technological innovations, such as the elevator brake and the mass production of steel, made building beyond a few floors economical. By the turn of the twentieth century, a handful of twenty floor buildings had been completed in major cities across the United States. By the mid-twentieth century, engineering advances allowed for mile-high buildings (1609.34 meters), and various architects have proposed designs, such as the Houston Pinnacle (1 mile, 500 floors), the Ultima Tower (2 miles, 500 floors), and the Sky Mile Tower (1 mile, 400 floors). Perhaps most famously, architect Frank Lloyd Wright proposed a mile-high skyscraper, The Illinois, in 1957, planning for 528 floors (Council on Tall Buildings and Urban Habitat (2017)).

But none of these designs have been realized. Despite the technical ability to build tall, mile-high designs are considered impractical because they are unlikely to turn a profit. Indeed, highly ambitious projects often fail for financial reasons. (Lepik (2004), pages 21-2) For example, the Jeddah Tower was originally planned
for one mile (330 floors). After the Great Recession, the height was reduced by more than a third to one thousand meters (167 floors).

Skyscrapers are a commercial response to an economic phenomenon. They arise when land values produce rents that exceed the enormous cost of construction and maintenance. Aesthetics are a secondary concern (Clark and Kingston (1930)), (Willis (1995)), (Ascher and Vroman (2011), pages 12, 22). Indeed, it was in the aftermath of the 1871 Chicago fire that the urgent need for new office space produced the First Chicago School of skyscrapers (Lepik (2004), page 6). Skyscraper construction has since followed the rise of oil-rich Middle Eastern countries in the 1980s, the former Soviet-bloc countries in the 1990s, and the Pacific Rim countries in the 2000s (Sennott (2004), page 1217). Barr and Luo (2017) find that half of the variation in China’s skyscraper construction can be explained by population and gross city product alone.

The enormous cost of a skyscraper is not only the result of additional construction materials. Taller buildings require concrete to be pumped at higher pressures (Ascher and Vroman (2011), page 83). Nor is the cost entirely in raising the building itself. Taller buildings require more elevators, which reduces the floor area available for occupancy and thus the revenue potential of the building (ibid., page 33). Human comfort is also a factor. Excessive elevator speed (ibid., page 103) and building sway (ibid., page 61) can produce motion sickness even if safe. Additional considerations include government policy (permits and zoning, financial incentives, and public infrastructure), culture (aesthetic, equity, and sustainability), and environment (foundation quality, prevailing winds, and natural disaster frequency).

In short, a litany of factors must align favorably to produce a skyscraper. One could hypothetically account for every factor in every city property and predict the future of skyscraper development. Yet it is convenient, and perhaps more accurate, to regard unobserved heterogeneity among these factors as a source of randomness that obeys the laws of probability. Statistical models can then be used to quantify the probability a property possesses the factors that will produce a particular skyscraper where the parameters of the models are determined from data.

Extreme value theory provides a principled strategy for choosing a statistical model. The theory is simplified by assuming skyscraper development is independent. This assumption is certainly violated for contemporaneously completed skyscrapers within a city since many of the aforementioned factors are correlated within a city at a specific period in time. But a series of investigations by economist and skyscraper expert Barr suggest that the conditions underlying skyscraper development are largely independent across cities and time periods. For example, Barr (2012) finds competition within cities is limited to periods close in time and space. Barr, Mizrach, and Mundra (2015) find skyscraper height is not a useful indicator of economic bubbles or turning points. Barr and Luo (2017) find little evidence that cities in China compete for the tallest building.

Considerable economic pressure produces the factors that drive skyscraper development, and the catalyst for this pressure has varied idiosyncratically by city and time period. It is perhaps because of this idiosyncratic variation—and the fact that no city possesses more than ten percent of all skyscrapers—that the prevalence and nature of skyscrapers so closely follows the distributions predicted by extreme value theory, as demonstrated in the following sections, despite periods of economic and political turmoil within nearly every city since the Second World War.

3. Predicting the Quantity of Skyscrapers Completed by 2050

Skyscraper construction has increased at a remarkably steady rate. The number of tall skyscrapers—skyscrapers exceeding 150 meters and 40 floors—has risen eight percent each year since 1950. If this trend continues, the eight percent annual growth rate of skyscrapers will far outpace the two percent annual growth rate of urban populations. Forty-one thousand tall skyscrapers will be completed by 2050, and if the UN’s population predictions hold, there will be roughly 6,800 per billion city residents in 2050 compared to the roughly 800 per billion city residents today.

The eight percent rate was determined by fitting the following log-linear model. Let $N_t$ be the number of skyscrapers completed in year $t$, $t = 1950, \ldots, 2017$. The $N_t$’s are assumed to be independent and follow a Poisson distribution with mean
\[ E[N_t] = \exp(\alpha + \beta t) \]

The Poisson distribution can be justified theoretically by regarding skyscraper development as the sum of independent Bernoulli trials. Suppose \( D(t) \) is a dataset containing the height of every building in the world completed in year \( t \). Define \( D_u(t) \) to be the subset of all buildings in \( D(t) \) exceeding the height threshold \( u \). For sufficiently large \( u \), the number of skyscrapers in \( D_u(t) \), \( N_t \), is well approximated by a Poisson distribution. See Coles et al. (2001) (page 124) for a more detailed discussion of the Poisson Process limit for extremes.

We use the \texttt{glm} function in the R Core Team (2018) package \texttt{stats} to choose parameter estimates \( \hat{\alpha} \) and \( \hat{\beta} \) that maximize the likelihood and to calculate their standard errors. The plug-in estimate, \( \hat{E}[N_t] = \exp(\hat{\alpha} + \hat{\beta} t) \), is plotted against the data in the top-right panel of Figure 1. An inner 95 percent predictive distribution for the years 2020 to 2050 is added in the bottom-left panel. A cumulative thirty-eight thousand skyscrapers is estimated for completion between 2018 and 2050 if present trends continue (forty-one thousand total, with a standard error of seven thousand), about twelve times the current number.

The top-right panel of Figure 1 exhibits short-term serial correlation in the residuals. Most noticeably, the historic trend was not maintained in the late 1990s. Nevertheless, the log-linear relationship remains predictive. To demonstrate the accuracy of the model for predicting 2050, thirty-three years after the data was collected in 2017, we predicted the last thirty-three years using data up until 1984. The bottom-right panel shows that if such predictions had been made in 1984 (dotted blue line), they would align closely with the actual number of skyscrapers built each year. The log linear model anticipates 3,082 skyscrapers by the end of 2017 when in fact 2,988 were built between 1950 and 2017, a difference of three percent.

4. Predicting the Height of Skyscrapers Completed by 2050

The tallest skyscraper has doubled in height since 1950, yet the height increase of the typical extremely tall skyscraper is not statistically significant. The same distribution describes the height of skyscrapers exceeding 225 meters since 1950. We conclude the tallest skyscraper is increasing not because skyscrapers are getting taller but because more buildings are being constructed and thus more buildings are eligible to be the tallest. Assuming this distribution continues to describe the height of skyscrapers completed by 2050, the probability a new building will exceed the current tallest building—the Burj Khalifa (828 meters)—is estimated to be nearly 100 percent. The probability that a new building will exceed the Jeddah Tower (1,000 meters)—scheduled to open in 2020—is 77 percent. The probability that a new building will exceed one mile is 9 percent.

The distribution of skyscraper heights was approximated by the family of generalized Pareto distributions (GPD). These distributions can be justified both theoretically—by regarding tall buildings as random exceedances over a threshold—and numerically—by comparing the observed skyscraper heights to random draws from a GPD. We begin with the theoretical justification. Let \( X \) be a random variable with unknown distribution function \( F \). The exceedance conditional distribution describes the behavior of \( X \) given that it exceeds a large threshold \( u \),

\[ F_u(y) = \Pr(X - u \leq y | X > u) \]

for \( u \) large. The Pickands-Balkema-de Haan Theorem (Balkema and De Haan (1974), Pickands III (1975)) states that for a sufficiently large class of distributions \( F \),

\[ F_u(y) \to H(y) = 1 - \left( 1 + \frac{\xi y}{\sigma} \right)^{-1/\xi}, \quad \text{as } u \to \infty, \]

where \( \sigma > 0, y > 0 \) when \( \xi > 0 \) and \( 0 \leq y \leq -\frac{\sigma}{\xi} \) when \( \xi < 0 \). For \( \xi = 0 \), the value of the function is taken as its limit \( H(y) = 1 - \exp(-y/\sigma) \). The set of distribution functions \( H(\cdot) \) is the GPD family. See Coles et al. (2001) (page 74) for a more detailed discussion of the GPD for modeling threshold exceedances.
The number of skyscrapers has grown 8 percent each year since 1950.

A log-linear model predicts a cumulative 41,000 skyscrapers (±7,000) will be built by 2050.

Figure 1: We estimate the quantity of skyscrapers in 2050, thirty-three years after the data was collected at the end of 2017. The top-left panel shows the height (meters) and year all 3,251 skyscrapers exceeding 150 meters and 40 floors were completed. The top-right panel shows the number of skyscrapers completed each year (points) and the expected number after fitting a log-linear model (blue line). The bottom-left panel extrapolates the number of new skyscrapers to be completed each year until 2050. The bottom-right panel shows that, had the same prediction been performed thirty-three years ago at the end of 1984, an estimated 3,082 new skyscrapers would have been completed by 2018. This prediction would have been three percent more than the actual number.
Suppose $D$ is a dataset of observations, and $D_u \subset D$ is the subset of all observations whose measurement, $X$, is greater than threshold $u \gg 0$. The Pickands-Balkema-de Haan Theorem justifies approximating the distribution of each $X$ with GPD

$$\Pr(X \leq x | X > u) = 1 - \left( 1 + \frac{\xi(x - \mu)}{\sigma} \right)^{-1/\xi},$$

where the parameters $(\mu, \xi, \sigma)$ can be estimated from dataset $D_u$ by maximizing the likelihood.

As in the previous section, $D$ is taken to be the dataset containing the height of every building in the world, and $D_u$ is the subset of all buildings in $D$ exceeding the height threshold $u$. However, unlike the previous section, the default threshold of $u = 150$ meters may not be sufficiently high to reliably extrapolate the tail of $D$. We note that, were $u$ sufficiently high so that $X - u | X > u$ followed a GPD, $X - u' | X > u'$ would also follow a GPD with the same $(\sigma, \xi)$ for any $u' > u$. This suggests the following strategy for choosing $u$. Produce a sequence of candidate thresholds $u_1 < \ldots < u_d$ and estimate the GPD parameters $(\mu_{u_i}, \sigma_{u_i}, \xi_{u_i})$, yielding $(\hat{\mu}_{u}, \hat{\sigma}_{u_i}, \hat{\xi}_{u})$. Then select any threshold $u$ for which the estimates $\hat{\xi}_{u'}$ and $\hat{\sigma}_{u'}$ remain stable for all $u' > u$. See Gencay and Selcuk (2004) for a general discussion of GPD threshold choice and prediction.

The top-left panel of Figure 2 shows maximum likelihood estimates for the shape parameter, $\hat{\xi}_u$, when fit to skyscraper heights exceeding a sequence of thresholds from 150 to 350 meters. Point estimates are colored red, and 50 (95) percent confidence intervals are depicted with thick (thin) lines. The point estimates increase as the threshold increases from 0 at 150 meters and stabilize around .2 after 225 meters. Hill plots (not shown) also indicate a stable $\hat{\xi}$ of .2. In contrast, the estimate for the scale parameter, $\hat{\sigma}$, changes little as the threshold is increased. Since the extreme value assumptions appear satisfied at the 225 meter threshold, we refer to skyscrapers exceeding this height as “extremely tall”. Computation is discussed further in the Appendix.

A numeric justification for the GPD can be made by comparing simulations of skyscraper heights to the observed data. In fact, at the threshold of 225, simulations from the same GPD describe the heights of skyscrapers completed at different time periods. To demonstrate this, we first obtain the maximum likelihood estimates for the GPD parameters, $\hat{\mu}_{225}$, $\hat{\sigma}_{225}$, and $\hat{\xi}_{225}$. We then partition the skyscrapers built after 1950 into sextiles according to the year in which they were completed. The top-right panel of Figure 2 shows q-q plots for the skyscrapers in each time period. However, instead of plotting skyscraper heights against the cdf,$\Pr(X > u)$, we transform both axes by the GPD $1 - \left( 1 + \frac{\hat{\xi}_{225}(X - \hat{\mu}_{225})}{\hat{\sigma}_{225}} + 1 \right)^{-1/\hat{\xi}_{225}}$. After transformation, the theoretical distribution is a standard uniform. We find these q-q plots more stable than the unadjusted plot, and it is easier to assess the fit.

The close fit in each time period suggests the distribution of extremely tall skyscraper heights does not change over time. This is investigated further in the bottom-right panel of Figure 2, which shows the median height of extremely tall skyscrapers each year since 1950. A median regression line is estimated using the R Core Team (2018) package quantreg (Koenker (2018)). The height of the typical extremely tall skyscraper increases less than half a meter each year, a relatively small 3.6 percent increase over sixty-eight years. The standard errors of the regression parameters are also retained, and the increase is not statistically significant (p-value .41).

We conclude that the urban skyline is driven primarily by the exponential increase in the number of buildings completed each year. Years with more construction are more likely to build extremely tall skyscrapers, and the increase of the typical skyscraper is negligible by comparison. For the purpose of predicting the height of the tallest skyscraper in 2050, we assume this trend will continue. We use the gpdSim function in the R Core Team (2018) package fExtremes (Wuertz, Setz, and Chalabi (2017)) to draw from the GPD with parameters $\hat{\mu}_{225}$, $\hat{\sigma}_{225}$, and $\hat{\xi}_{225}$, approximately 8,400 times, where 8,400 is the number of new skyscrapers estimated to be completed by 2050 in the previous section multiplied by the empirical probability a skyscraper above 150 meters will exceed 225 meters. The maximum height is retained, and this process is repeated one thousand times. Figure 3 shows the resulting distribution of maximum heights (blue). A right-sided 95 percent predictive interval ends at 1,800 meters.

To demonstrate the accuracy of this approach for predicting 2050, thirty-three years after the data was collected in 2017, we conduct a second simulation using only data that would have been available before 1984.
The GPD is fit using all skyscrapers above the 225 threshold, and we repeatedly simulate the maximum skyscraper one thousand times. These simulations assume approximately three thousand tall skyscrapers will be built between 1984 and 2017, around fifteen percent of which will exceed 225 meters. The distribution of the maximum height predicted in 1984 is shown in Figure 3 (red). We find that the current tallest skyscraper, at 828 meters, would have been considered likely. The median simulation is 924 meters, roughly ten percent above this value.

5. Predicting the Number of Floors in Skyscrapers Completed by 2050

The marginal number of floors in the typical skyscraper decreases as height increases. Height alone overstates the ability of skyscrapers to accommodate a growing population. Assuming the marginal number of floors continues to decrease as skyscraper height increases, the one thousand meter building is estimated to have seventy percent the floors of the mile-high building—despite being sixty-two percent of the height. While diminishing marginal floors is reflected in most architectural designs, we find the estimated number of floors will diminish faster with height than most designs anticipate. However, the exact relationship between height and number of floors will vary by city.

The relationship between height and number of floors is extrapolated to extremely tall skyscrapers using the following bivariate extreme value model. Let $(X,Y)$ be a bivariate random variable with GPD margins as justified in Section 4. Denote the respective parameter sets indexing the GPDs as $(\mu_x, \sigma_x, \xi_x)$ and $(\mu_y, \sigma_y, \xi_y)$, and apply the monotone transformations

\begin{align*}
\tilde{X} &= -\log \left( \frac{1 + \xi_x (X - \mu_x)}{\sigma_x} \right)^{-1/\xi_x} \\
\tilde{Y} &= -\log \left( \frac{1 + \xi_y (Y - \mu_y)}{\sigma_y} \right)^{-1/\xi_y}
\end{align*}

such that $(\tilde{X}, \tilde{Y})$ has standard exponential margins. The joint distribution $(\tilde{X}, \tilde{Y})$ is modeled using the asymmetric logistic distribution, where given thresholds $u, v \gg 0$, for $X > u, Y > v$,

\[ \Pr(\tilde{X} > x, \tilde{Y} > y) \propto \exp \left( - (1 - \theta_x) x - (1 - \theta_y) y - (x^r \theta_x^r + y^r \theta_y^r)^{1/r} \right), \quad \theta_x, \theta_y \in [0, 1], \quad r \geq 1. \]

This model has the advantage of being simple and flexible, and it is suitable for larger sample problems, as in our case. Tawn (1988) and Coles et al. (2001) (page 142) provide a detailed discussion of bivariate models for threshold exceedances.

The thresholds $u = 225$ and $v = 59$ are selected based on the stability of the marginal distribution as described in Section 4. The nine parameters $\mu_x, \sigma_x, \xi_x, \mu_y, \sigma_y, \xi_y, \theta_x, \theta_y,$ and $r$ are estimated using the heights and floors of all skyscrapers exceeding 225 meters or 59 floors, maximizing the censored likelihood:

\[ L_c(\mu_x, \sigma_x, \xi_x, \mu_y, \sigma_y, \xi_y, \theta_x, \theta_y, r) = \prod_{x_i > u, y_i > v} f(x_i, y_i) P(X > u, Y > v) \prod_{x_i \leq u, y_i > v} f_Y(y_i) P(x \leq u) \prod_{x_i > u, y_i \leq v} f_X(x_i) P(y \leq v), \]

where $f, f_X, f_Y$ are the joint and marginal density function of $(X,Y)$ from the transformations $^1$ and $^2$ and the distribution function $^3$.

The top left panel of Figure 4 displays the height and number of floors of every tall skyscraper, colored by its contribution to the censored likelihood, $L_c$. Skyscrapers below 225 meters and 59 floors (blue) do not
The maximum likelihood estimate of the shape rises from 0 at 150 to stabilize at .2 after 225 meters.

The same generalized Pareto distribution fits skyscraper heights over different time periods:

- 1950–1992
- 1993–2003
- 2004–2008
- 2009–2011
- 2012–2014
- 2015–2017

These plots indicate the same generalized Pareto distribution accurately describes the distribution of skyscraper heights exceeding 225 meters at different time periods.

The median height of the tallest skyscrapers grows 0.19 meters per year since 1950.

We conclude the typical extremely tall skyscraper will not increase noticeably by 2050.
The tallest skyscraper is predicted to be 1134 meters in 2050. There is a 9 percent chance it will exceed one mile. There is a 0 percent chance the current tallest skyscraper will remain the tallest.

Figure 3: We simulate the height of the tallest skyscraper in 2050. The simulated density (red) suggests the tallest building in the world is unlikely to exceed one mile (dashed line on right side). However, it will almost certainly be taller than the current tallest building, the Burj Khalifa (828 meters, dashed line on left side) and likely taller than the Jeddah Tower (one thousand meters), expected for completion in 2020. Had the same simulation been conducted in 1984, the density (blue) would have found the tallest skyscraper in 2018 to be between six hundred and one thousand meters.
contribute to the likelihood and are not used to estimate the parameters. Skyscrapers exceeding 225 meters and 59 floors (red) make up the first factor. The remaining skyscrapers (green) make up the second two factors. For example, a 250 meter skyscraper with 50 floors is treated like a 250 meter skyscraper whose floors are only known to be below 59. This approach is similar to the censored likelihood in Huser et al. (2016), except that skyscrapers at or below 225 meters and 59 floors are excluded from the analysis. Computation is discussed further in the Appendix.

The maximum likelihood parameters are retained to estimate the conditional density of the number of floors for a one-thousand-meter skyscraper (Figure 4, top-right panel) and a one-mile tall skyscraper (Figure 4, bottom-left panel). Dark blue regions represent the right-sided 50 percent region, and light blue regions represent the right-sided 95 percent region. These densities are compared with actual skyscraper plans (dotted line). The median one-thousand-meter skyscraper is estimated to have 107 percent the number of floors of the Jeddah Tower (to be completed in 2020). The median one-mile skyscraper is estimated to have roughly three-quarters the number of floors of the Mile-High Tower, two-thirds of Next Tokyo’s Sky Mile Tower, and half the floors of Frank Lloyd Wright’s The Illinois.

As in the previous two sections, the same analysis is performed with only data available in 1984. The bottom-right panel shows the conditional density of the 828 meter skyscraper as it would have been estimated using the threshold of 225 meters or 59 floors. In 1984, an 828 meter skyscraper would have been nearly twice the height of the current tallest building, the Willis Tower (then Sears Tower, 442 meters and 108 floors). The conditional median predicts the typical 828 meter skyscraper would have 179 floors, ten percent more than the Burj Khalifa completed twenty-four years later. Simply scaling the Willis Tower to the height of the Burj Khalifa would yield 202 floors, overestimating the actual number by twenty-four percent. A linear regression model fit with extremely tall skyscrapers overestimates by fifteen percent.

The estimated relationship between floor and heights varies considerably across cities. The top panel of Figure 5 shows the empirical median height (meters) and number of floors of extremely tall skyscrapers (exceeding 225 meters or 59 floors) in select cities. The medians are each based on roughly ten observations, and thus sampling variation overstates the likely differences between the typical extremely tall skyscrapers of cities in the year 2050.

We augment the bivariate model to estimate city-level medians. We allow the marginal GPD parameters to vary by city, according to a normal distribution with an unknown mean and variance. Such hierarchical models are often used to produce city-level estimates that have smaller errors on average than the corresponding stratified estimates. The use of a parameter hierarchy also has a Bayesian interpretation. See Coles et al. (2001) (page 169) and Vehtari (2017) for two discussions of Bayesian inference and extremes.

The bottom panel of Figure 5 shows the estimated median for the select cities in the hierarchical model with parameters selected by maximum likelihood. The median of the non-hierarchical model in the previous Figure is represented by a black dot. (Note that Hong Kong and New York City contain a disproportionately large number of extremely tall skyscrapers.) These city-level estimates can be seen as a compromise between the noisy empirical medians in the top panel and the more accurate but global median estimated by the non-hierarchical model. However, despite partial pooling across cities, the typical height per floor ratio still spans a considerably large range: 3.6 (Hong Kong) to 4 (Moscow).

6. Discussion

This paper applied extreme value analysis to predict the prevalence and nature of skyscrapers if present trends continue until the year 2050. The findings of these sections have both methodological and policy consequences. This section discusses the methodological consequences, while the following section considers policy.

Section 4 found that skyscraper heights are well described by a generalized Pareto distribution with a positive shape, $\xi \approx .2$. This means that the distribution of skyscraper heights has a heavy tail, and, theoretically speaking, the maximum does not exist. While this is obviously false—skyscrapers as defined in Section 1 are certainly bounded above—it suggests sample averages and sums may be unreliable for inference and
The marginal number of floors diminishes as height increases. Blue observations are excluded from model, and green censored.

The typical one-thousand-meter skyscraper will have 178 floors.

The typical mile-high skyscraper will have 255 floors.

The typical 828-meter skyscraper, if estimated 33 years ago, would overestimate the Burj Khalifa by ten percent.

Figure 4: We estimate the number of floors in the tallest skyscrapers in 2050. The top-left panel shows the height (meters) and number of floors for each skyscraper exceeding 150 meters and 40 floors. The black line represents the floor to height ratio of the median skyscraper. This ratio is not preserved as height increases. A bivariate asymmetric logistic distribution is used to approximate the joint distribution, and colors depict the use of each observation for estimating the parameters. Skyscrapers below 225 meters and 59 floors (blue) are not used to estimate the parameters. Skyscrapers above 225 meters and 59 floors (red) are modeled using the complete likelihood. The remaining skyscrapers (green) are modeled using the censored likelihood. The joint distribution indexed by the maximum likelihood parameters is then used to make probabilistic statements of future skyscrapers. The bottom-left (top-right) panel shows the conditional density of the number of floors for a one-mile (one-thousand-meter) tall skyscraper. Right-side 50 (95) percent intervals are shaded dark (light) blue. This distribution is compared with actual skyscraper plans (dotted line). For example, the one-mile skyscraper will have roughly three-quarters the number of floors in the Mile-High Tower, two-thirds in Next Tokyo’s Sky Mile Tower, and half the floors in Frank Lloyd Wright’s The Illinois. The bottom-right panel shows that, had the same analysis been conducted in 1984, it would have estimated the Burj Khalifa to have 179 floors, ten percent more than it actually had when completed twenty-four years later.
The height and number of floors of the typical tall building varies considerably across cities.

A hierarchical Asymmetric Logistic model partially pools city−level estimates to improve accuracy. Depicted cities are estimated to have more similar buildings than the top−panel would suggest.

Figure 5: We estimate the height and number of floors of the tallest buildings of major cities in 2050. The top panel shows the empirical median height (m) and number of floors for extremely tall skyscrapers (exceeding 225 meters or 59 floors) in select cities. The medians are based on roughly ten observations each and are likely unreliable estimates of skyscrapers in 2050. More accurate city−level predictions can be made using a hierarchical bivariate asymmetric logistic distribution, where we allow the marginal GPD parameters to vary by city according to a normal distribution with unknown mean and variance. All parameters are then estimated by maximizing the likelihood. The bottom panel shows the estimated median for the select cities in the hierarchical model. The median of the non-hierarchical model in the previous Figure is represented by a black dot. These city−level estimates can be seen as a compromise between the noisy empirical medians in the top panel and the accurate, but global median estimated by the non-hierarchical model. The line reflects the 3.8 meter per floor of the typical tall skyscraper.
extrapolation. Researchers must be careful interpreting these quantities as evidence for their theories, especially with small sample sizes. For example, the fact that the average height does not increase with specific economic conditions may not indicate that the skyscraper construction is unrelated to those conditions. Furthermore, increasing average height in recent years may provide a poor basis for anticipating heights in future years. Percentiles, such as the median, can be more stable representatives of their theoretical analogs, and may prove better alternatives for conducting inference and extrapolating as demonstrated in the bottom-right panel of Figure 2. Cirillo and Taleb (2016) have made similar points for researchers using the total number of war casualties to determine whether humans are less violent than in the past and whether wide-scale war will return in the future.

Section 4 also found that the GPD parameters change little over time after choosing an appropriate threshold. This suggests that the height of the tallest skyscrapers is driven by the exponential increase in the number of new buildings constructed each year and not a desire to build the typical building taller. The distinction may be important for researchers who use skyscraper heights as evidence of a cause-and-effect relationship and then attempt to predict skyscraper heights from its cause. For example, our findings are consistent with the theory that the demand for extremely tall buildings led to the use of innovative technologies, such as faster elevators. Had the reverse been true—had the development of innovative technologies prompted extremely tall skyscrapers—an increase in skyscrapers would have been observed across the board as the technology became available, and the GPD parameters would have changed substantially over time. While it is not the intent of this paper to draw any causal conclusions, we point out that researchers need to be careful of “reverse-causality”, attributing taller buildings as a consequence of a given factor instead of its cause, without additional evidence. Spurious correlations provide a poor basis for prediction.

Section 5 demonstrated how city-level effects might be estimated with a hierarchical model. The dataset contains 258 cities, although the typical city has only one skyscraper exceeding 150 meters and 40 floors. Predictions are still possible for these cities because the model borrows information across cities. Future researchers might benefit from augmenting the hierarchy to include country and region effects. Or, alternatively, covariate information, such as population and gross city product at the time each skyscraper was completed, could be used instead. These covariates would make the modeling assumptions more plausible and may give insight into how cities might change policies to increase or decrease skyscraper activity, provided covariates are chosen judiciously and not based on spurious correlations. Spatiotemporal dependence could even be modeled directly as discussed by Bao et al. (2006), Chan and Gray (2006), and Ghil et al. (2011).

7. Conclusion

The major challenges confronting cities, such as sustainability, safety, and equality, will depend on the infrastructure developed to accommodate urbanization. Some urban planners have suggested that vertical growth—the concentration of residents by constructing tall buildings—be used to accommodate density. Others have argued that urbanization will be too rapid to be accommodated by vertical growth alone.

This paper finds that skyscraper construction will outpace urbanization if present trends continue. Cities currently have around 800 skyscrapers per billion people. By 2050, it is estimated that cities will have 6,800 skyscrapers per billion people. It also finds that the tallest among these will be around fifty percent higher than those today and therefore able to accommodate more people. However, these skyscrapers will not have fifty percent more floors since the marginal capacity will diminish as heights increase. For example, the one thousand meter building will have seventy percent the floors of the mile-high building, despite being sixty-two percent of the height.

This paper has not investigated whether skyscrapers will be constructed in the cities with the most rapid urbanization. Nor has it investigated whether skyscraper development should be used to accommodate density in the first place. Instead, extreme value analysis provided a principled basis to forecast future trends and quantify uncertainty. It relies on the assumption that present trends continue, and there are a variety of reasons why future trends may deviate from the past sixty-eight years. For example, unprecedented technological changes may result in new materials or methods that substantially reduce the cost of construction. Other technological changes could cause a cultural shift in how residents live or work, perhaps freeing up
commercial space for residential purposes. There is also the possibility of a hiatus due to global upheaval, not unlike the period spanning the Great Depression and Second World War. That period was ignored in this analysis, but future work may choose to investigate this frightening prospect.

We conclude by stressing that extreme value analysis is one of many principled strategies that could be used to predict skyscraper development and the effects of urbanization more broadly. The previous sections could be augmented by integrating theories from architecture, engineering, policy, and social science. The incorporation of expert knowledge is always useful, but it is particularly desirable with extreme value analysis, as heavy tailed distributions are sensitive to outliers and benefit from the context afforded by the theory of other disciplines. More broadly, all disciplines will be necessary to anticipate how cities will respond to the greatest migration in human history, and solve perhaps the principal challenge of our time.

8. Appendix

All models in Sections 4 and 5 are written in the Stan probabilistic programming language (Carpenter et al. (2017)), and maximum likelihood estimates are computed using the LBFGS algorithm in the R Core Team (2018) package rstan (Stan Development Team (2018)). Standard errors are computed from the observed Fisher information. Constrained parameters such as scales and shapes are transformed so that they are unconstrained, the Fisher information is calculated, and the constrained standard error is computed using the delta method. Vehtari (2017) provides an introduction to univariate Extreme Value Analysis with Stan. Section 2.6 of Carpenter et al. discusses the LBFGS algorithm for maximum likelihood estimation.

A 225 meter threshold for height (59 for floors) is utilized when fitting a GPD model. Note that in the limit, $\mu$ should be equal to the threshold $u$. We allow $\mu$ to vary ($0 < \mu < \min(x_i)$) in order to add flexibility to the model. This choice does not impact the conclusions in Section 4. Our point estimates match the output from the univariate gpdFit function in the package fExtremes (Wuertz, Setz, and Chalabi (2017)), which sets $\mu = u$.

Before fitting the bivariate models, the number of floors was scaled by 3.8. The median skyscraper rises roughly 3.8 meters per floor, and scaling by this number aligns the marginal distributions.

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