B decays into two light pseudoscalar mesons
and possible effects of enhanced $b \rightarrow sg$

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Abstract

We consider the branching ratios and CP asymmetry in the $B$ meson decays into two pseudoscalar mesons in the generalized factorization approximation. We also investigate the possible effects of the enhanced chromomagnetic interaction $b \rightarrow sg$ on these exclusive $B$ meson decays that was suggested as a possible solution to the semileptonic branching ratio of $B$ mesons, the missing charm puzzle and the large $B \rightarrow \eta' + X_s$. We finds that such enhanced $b \rightarrow sg$ interaction degrades the agreement between the data and the model predictions for $B$ meson decays into two light mesons in the generalized factorization approximation.

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I. INTRODUCTION

Recently, the CLEO collaboration reported the observation of some nonleptonic $B$ meson decays into two light pseudoscalar mesons with the following branching ratios \([1,2]\):

\[
\begin{align*}
\mathcal{B}(B^\pm \to \pi^\pm K^0) &= (2.3^{+1.1}_{-1.0} \pm 0.3 \pm 0.2) \times 10^{-5} \\
\mathcal{B}(B_d \to \pi^\pm K^\mp) &= (1.5^{+0.5}_{-0.4} \pm 0.1 \pm 0.1) \times 10^{-5} \\
\mathcal{B}(B^\pm \to K^\pm \eta') &= (6.5^{+1.5}_{-1.4} \pm 0.9) \times 10^{-5} \\
\mathcal{B}(B_d \to K^0 \eta') &= (4.7^{+2.7}_{-2.0} \pm 0.9) \times 10^{-5},
\end{align*}
\]  

(1)

And the upper limits of the branching ratios for other decay modes were of the order of $10^{-5}$. These measurements at CLEO and the future experiments at B-factories motivate a great deal of theoretical interests in nonleptonic $B$ decays into two light mesons \([3\text{–}10]\).

The standard theoretical framework to study non-leptonic $B$ decays is based on the effective Hamiltonian approach. The short and long distance QCD effects in the non-leptonic weak decays are separated by means of the operator product expansion \([11]\). The resulting effective Hamiltonian consists of products of scale-dependent Wilson coefficients $C_i(\mu)$ and the local four-quark operators at the renormalization scale $\mu$. The short distance contributions of the coefficients $C_i(\mu)$ has been evaluated up to the next-to-leading logarithmic (NLL) order \([12,13]\). In the NLL precision, the Wilson coefficients depend on the renormalization scheme as well as renormalization scale. These dependences should be canceled by corresponding scheme/scale dependence of the matrix element of the operators. However, hadronic matrix elements are usually calculated under the factorization approximation in which they are replaced by the scale/scheme independent form factors and decay constants. In order to achieve the scale/scheme independence of the matrix elements of the effective Hamiltonian, Ali and Greub \([4]\) included the corrections of the one-loop penguin-like diagrams and some process independent part of the vertex corrections associated with the four-fermi operators to the partonic matrix elements before doing factorization step. They also have taken into account the effects of the $O(\alpha_s)$ tree-level matrix element associated with the chromomagnetic dipole operator. Such corrections compensate the scale/scheme dependences of the Wilson coefficients and one finally get the effective Hamiltonian involving the scale/scheme independent Wilson coefficients and four-quark operator.

The factorization method is successful to describe the heavy-to-heavy nonleptonic decays \([14]\). The intuitive argument for the factorization is given by Bjorken \([15]\) based on the idea of color transparency. The quark pairs with high energy coming from the heavy meson decay hadronize after they have traveled some distance from each other. Hence the decay process is expected to factorize into the color singlet current pairs since soft gluon effects are small after the hadronization occurs. There is more theoretical argument about the factorization based on QCD. In the specific kinematic region in which the two light quarks are highly collinear and all quarks are almost on-shell, the leading term of the Green function in expansion of inverse powers of the heavy quark masses and the large energy transferred to the light quark pairs exhibits factorization \([16]\). The authors in Ref. \([4]\) extended this framework to the heavy-to-light transitions. They also introduce a new free parameter $\xi$ that describes nonfactorization effects such as color octet contributions and concluded that the range $0 \leq \xi \leq 0.5$ is consistent with data \([4]\). However one should keep in mind that the
factorization for the heavy-to-light transition is still only a model to describe the complex hadronic matrix elements since there is no theoretical justification as the heavy-to-heavy transition case [10]. In principle there is no reason that only one single value of the parameter $\xi$ can explain the branching ratios of all kind of different modes. For example, the authors of Ref. [16] introduce two different $\xi$’s corresponding to the different currents structures which give different nonfactorization corrections. However, we will use the factorization method including one nonfactorization parameter $\xi$ in this work.

On the other hand, there are some possible anomalies in the inclusive $B$ decays. The persistent discrepancy of the measurements of the semileptonic branching ratio and charm multiplicity with the theoretical predictions have been known quite for a while [17]. One can argue that these problems arise because of the breakdown of local quark-hadron duality that was invoked when one estimates the nonleptonic $B$ decay rate. However, several authors have noticed that these puzzles can be solved if one assumes that the Wilson coefficient of the chromomagnetic operator is enhanced by some new physics contributions [18]. Incidentally, this may help to understand the recently measured branching ratio for $B(B \to \eta/\eta')X_s$, which is surprisingly larger than previously expected. Such an enhanced $b \to s \gamma$ should affect inevitably the exclusive decay rates of $B$ mesons, and it is one of the main themes of our present work to study such effects.

In this work, we consider the nonleptonic $B$ decays into two light pseudoscalar mesons and investigate the possible effects through the enhanced chromomagnetic dipole contribution. The branching ratios are calculated by the generalized factorization method proposed in Ref. [4]. The QCD and EW penguin contributions are included in this work. The two-angle mixing formalisms are used in the calculation of the decay rates involving $\eta, \eta'$ mesons, and the amplitudes $b \to s(gg) \to s(\eta, \eta')$ are included using the QCD anomaly, instead of the intrinsic charm contents of $\eta(1')$. We also study the CP asymmetry in $B$ meson decays, both the direct $CP$ asymmetries of charged $B$ meson decays and the time integrated $CP$ asymmetries of neutral $B$ meson decays. In this calculation, we assume that the strong phases are given by the penguin-type diagrams with internal light quarks which could have on-shell momentum [19].

II. CALCULATIONAL FRAMEWORK

The $\Delta B = 1$ effective Hamiltonian is given by

$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} \left[ V_{ub} V_{uq}^* (C_1 O_1^u + C_2 O_2^u) + V_{cb} V_{cq}^* (C_1 O_1^c + C_2 O_2^c) - V_{tb} V_{tq}^* \left( \sum_{i=3}^{10} C_i O_i + C_g O_g \right) \right]$$

(2)

where the operators are
Here the $O_1$ and $O_2$ are the current-current operators. The $O_3 \sim O_6$ and $O_7 \sim O_{10}$ are called the gluonic and electroweak penguin operators respectively, and $O_g$ is the gluonic dipole moment operator. The subscripts $V \pm A$ represent the projection operators $1 \pm \gamma_5$ onto left- and right-handed spinor and $e_{\gamma'}$ indicates the electromagnetic charge of the corresponding quarks. The subindex $\alpha$ and $\beta$ are the $SU(3)$ color indices and $\lambda_{\alpha\beta}^A$ ($A = 1 \sim 8$) are the Gell-Mann matrices.

If we take $m_{top}^{pole} = 175$ GeV, $\alpha_s(M_Z) = 0.118, \alpha(M_Z) = 1/128$, we have the following numerical values of the Wilson coefficients at the renormalization scale $\mu = 2.5$ GeV in the naive dimensional renormalization scheme:\[4\]:

\[
\begin{align*}
C_1 &= 1.117 & C_2 &= -0.257 & C_3 &= 0.017 & C_4 &= -0.044 \\
C_5 &= 0.011 & C_6 &= -0.056 & C_7 &= -1 \times 10^{-5} & C_8 &= 5 \times 10^{-4} \\
C_9 &= -0.010 & C_{10} &= 0.002 & C_g &= -0.158
\end{align*}
\]

where the Wilson coefficients $C_1 \sim C_6$ are taken as the NLL values with respect to QCD. The values of the remaining coefficients are given at the leading logarithmic precision.

In the NLL precision, the matrix elements of local 4-fermion operators $O_i$'s are to be treated at the one-loop level. In Ref.\[4\], the contributions arising from the penguin-type diagram of the operators $O_1 \sim O_6$ and the tree-level diagram of the dipole operator $O_g$ have been calculated and absorbed into the effective Wilson coefficients $C_i^{eff}$. The process-independent contributions from the vertex-type diagrams are also considered. These full NLL considerations are sufficient to compensate the scale and scheme dependence arising from the factorization approximation and the resulting effective Wilson coefficients are given in Ref.\[4\].

In this paper, we use the factorization approximation in order to calculate the hadronic matrix elements of the type $\langle h_1 h_2 | O_i | B \rangle$, where $h_{1,2}$ is a light pseudoscalar meson. In this approximation, the hadronic matrix elements are factorized into a product of two matrix elements of quark bilinear operators. As an example, let us consider the decay $\bar{B}^0 \rightarrow \pi^+ \pi^-$, whose matrix element is given by

\[
M = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* \left[ a_4 + a_{10} + \frac{2m^2_Z(a_6 + a_8)}{(m_u + m_d)(m_b - m_u)} \right] \right\} \\
\times \langle \pi^- | d\bar{u} | 0 > < \pi^+ | \bar{u} d | \bar{B}^0 >
\]

where

\[
a_i = C_i^{eff} + \xi C_i^{eff} (i = odd), \quad a_i = C_i^{eff} + \xi C_i^{eff} (i = even).
\]
Here, instead of $1/N_c$, we introduce a free parameter $\xi$ which is supposed to describe the nonfactorization contribution. (The amplitudes for other decays are explicitly listed in Appendix.)

When we express the matrix elements of quark bilinear operators in terms of meson decay constants and form factors, it is important to remember which convention we use for the meson wave function in terms of quark flavor contents. We adopt the following convention, for which the meson state is the same as the isospin eigenstate without extra sign.

\[ |\pi^+ > = |ud >, \quad |\pi^0 > = |\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u}) >, \quad |\pi^- > = |d\bar{u} >, \quad |K^- > = |s\bar{u} >, \quad |K^0 > = |s\bar{d} >, \quad |B^- > = |b\bar{u} >, \quad |\bar{B}^0 > = |b\bar{d} >, \quad |\eta_8 > = |\frac{1}{\sqrt{6}}(2s\bar{s} - u\bar{u} - d\bar{d}) >, \quad |\eta_0 > = |\frac{1}{\sqrt{3}}(-s\bar{s} - u\bar{u} - d\bar{d}) > \quad (7) \]

With this convention, we define the decay constants and form factors as follows:

\[ <\pi^-(p)|\bar{d}u_0|0 >= if_\pi p_\mu, \quad <K^-(p)|\bar{s}u_0|0 >= if_K p_\mu \]
\[ <\eta_8(p)|\frac{1}{\sqrt{6}}(\bar{u}u_0 + \bar{d}d_0 - 2s\bar{s}_0)|0 >= if_8 p_\mu \]
\[ <\eta_0(p)|\frac{1}{\sqrt{3}}(\bar{u}u_0 + \bar{d}d_0 + s\bar{s}_0)|0 >= if_0 p_\mu \]
\[ <\eta^{(0)}(p)|\bar{u}u_0|0 >= if_{\eta^{(0)}} u_\mu, \quad <\eta^{(0)}(p)|\bar{s}u_0|0 >= if_{\eta^{(0)}} s_\mu, \quad <\eta^{(0)}(p)|\bar{c}c_0|0 >= if_{\eta^{(0)}} c_\mu \quad (8) \]

and

\[ <\pi^-(p')|\bar{d}b|B^-(p) >= [(p + p')_\mu - \frac{m_B^2 - m_\pi^2}{q^2}q_\mu]F_{B \rightarrow \pi}^{(1)}(q^2) + \frac{m_B^2 - m_\pi^2}{q^2}q_\mu F_{0 \rightarrow \pi}^{(0)}(q^2) \]
\[ <K^-(p')|\bar{s}b|B^-(p) >= [(p + p')_\mu - \frac{m_B^2 - m_K^2}{q^2}q_\mu]F_{B \rightarrow K}^{(1)}(q^2) + \frac{m_B^2 - m_K^2}{q^2}q_\mu F_{0 \rightarrow K}^{(0)}(q^2) \]
\[ <\eta^{(0)}(p')|\bar{u}b|B^-(p) >= [(p + p')_\mu - \frac{m_B^2 - m_{\eta^{(0)}}^2}{q^2}q_\mu]F_{B \rightarrow \eta^{(0)}}^{(1)}(q^2) + \frac{m_B^2 - m_{\eta^{(0)}}^2}{q^2}q_\mu F_{0 \rightarrow \eta^{(0)}}^{(0)}(q^2) \quad (9) \]

Other matrix elements are related to the above matrix elements using the following isospin relations.

\[ <\pi^-(p)|\bar{d}u_0|0 >= \sqrt{2} <\pi^0|\bar{u}u_0|0 >= -\sqrt{2} <\pi^0|\bar{d}d_0|0 > \]
\[ <K^-(p)|\bar{s}u_0|0 >= - <K^0|\bar{s}d_0|0 >= - <K^0|\bar{d}s_0|0 >, \quad <\eta^{(0)}|\bar{u}u_0|0 >= <\eta^{(0)}|\bar{d}d_0|0 > \]
\[ <\pi^-(p')|\bar{d}b|B^-(p_B) >= <\pi^0|\bar{u}b|B^0 >= \sqrt{2} <\pi^0|\bar{d}b|B^0 >= \sqrt{2} <\pi^0|\bar{u}b|B^- > \]
\[ <K^-(p')|\bar{s}b|B^-(p_B) >= <K^0|\bar{s}b|B^0 >, \quad <\eta^{(0)}|\bar{u}b|B^- >= - <\eta^{(0)}|\bar{d}b|B^0 > \quad (10) \]

We choose the numerical values of the relevant decay constants, $f_\pi, f_K$ as follows:

\[ f_\pi = 131 \text{ MeV}, \quad f_K = 160 \text{ MeV}. \quad (11) \]

The values of form factors at $q^2 = m_H^2$ is required for the calculation of decay rate. As the final states involve only light hadrons we can safely neglect the $q^2$ dependence of form
factors. Hence we assume that $F_{0,1}(m_{b}^{2}) = F_{0,1}(0)$. The values used for the form factors are obtained using the BSW model \[20\],

$$F_{0,1}^{B \to K} = 0.33, \quad \text{and} \quad F_{0,1}^{B \to \pi} = 0.33 \quad (12)$$

In the case with nonleptonic $B$ decays into the final state involving $\eta$ or $\eta'$, we use two-angle mixing formalism developed by Leutwyler \[21\]. This formalism is phenomenologically adequate to explain the various experimental data, the decay width $\Gamma(\eta \to 2\gamma)$, $\Gamma(\eta' \to 2\gamma)$ and the ratio $\Gamma(J/\psi \to \eta')/\Gamma(J/\psi \to \eta\gamma)$, as recently discussed in Refs. \[21\]-\[22\]. In this formalism, the $\eta$ and $\eta'$ states are defined by the mixture of the octet and singlet states with different mixing angles such as

$$\eta > = \cos \theta_{8}|\eta_{8} > - \sin \theta_{0}|\eta_{0} >, \quad \eta' > = \sin \theta_{8}|\eta_{8} > + \cos \theta_{0}|\eta_{0} > \quad (13)$$

Then, the decay constants $f^{u}_{\eta'}, f^{s}_{\eta'}$ are obtained through the two mixing angles and $f_{8}, f_{0}$:

$$f^{u}_{\eta'} = \frac{f_{u}}{\sqrt{3}} \cos \theta_{8} - \frac{f_{0}}{\sqrt{3}} \sin \theta_{0}, \quad f^{v}_{\eta'} = \frac{f_{u}}{\sqrt{3}} \sin \theta_{8} + \frac{f_{0}}{\sqrt{3}} \cos \theta_{0}.$$

And the relevant form factors for $B \to \eta$ and $B \to \eta'$ are:

$$F_{0,1}^{B \to \eta} = F_{0,1}^{B \to \pi} \left[ \frac{\cos \theta_{8}}{\sqrt{3}} - \frac{\sin \theta_{0}}{\sqrt{3}} \right], \quad F_{0,1}^{B \to \eta'} = F_{0,1}^{B \to \pi} \left[ \frac{\sin \theta_{8}}{\sqrt{3}} + \frac{\cos \theta_{0}}{\sqrt{3}} \right] \quad (14)$$

For numerical analysis, we choose the following values:

$$\theta_{8} = -22.2^{\circ}, \quad \theta_{0} = -9.1^{\circ}, \quad \frac{f_{8}}{f_{\pi}} = 1.28, \quad \frac{f_{0}}{f_{\pi}} = 1.20 \quad (16)$$

which are given by fitting of data on the decay width $\Gamma(\eta \to 2\gamma)$, $\Gamma(\eta' \to 2\gamma)$ and the ratio $\Gamma(J/\psi \to \eta')/\Gamma(J/\psi \to \eta\gamma)$ \[22\]. The parameters $f^{u}_{\eta}, f^{v}_{\eta}$ quantify the contributions from the decay $b \to s(\bar{c}c) \to s(\eta, \eta')$. Their magnitude and the sign were estimated using the QCD anomaly method in Ref. \[3\], and we use their values in this article: $f^{u}_{\eta} = -0.9 \text{ MeV}$ and $f^{v}_{\eta} = -2.3 \text{ MeV}$ for $m_{c} = 1.5 \text{ GeV}$.

For numerical calculation of the transition matrix elements, we also need numerical values of CKM elements and quark masses. We use the Wolfenstein parameterization \[23\] of CKM matrix element. Two parameters $A, \lambda$ are well determined using $|V_{cb}|$ through the fitting of the $B \to D^{*}l\nu$ decay spectrum and $|V_{us}|$ through the $K \to \pi e\nu$ and hyperon decays : $A = 0.81 \pm 0.06$ and $\lambda = \sin \theta_{c} = 0.0025 \pm 0.0018$. In this paper, we choose the central value of these parameters. Other two parameters in CKM matrix elements are constrained by CKM unitarity fitting \[24\] as $0.025 \leq \eta \leq 0.52$ and $-0.25 \leq \rho \leq 0.35$ (95% C.L.). However the lower bound on the mass mixing ratio $\Delta M_{s} / \Delta M_{d}$ and the experimental value of $R_{1} \equiv B(B^{0}(\bar{B}^{0}) \to \pi^{0}K^{\pm})/B(B^{0} \to \pi^{\pm}K^{0}) = 0.65 \pm 0.40$ disfavor negative $\rho$ region. Using the relation, $\sqrt{\rho^{2} + \eta^{2}} = |V_{ub}|/\lambda |V_{cb}|$, we choose the three typical values of $\rho$ and $\eta$ satisfying the above constraint following ref. \[5\],

$$(\rho = 0.05, \quad \eta = 0.36), \quad (\rho = 0.30, \quad \eta = 0.42), \quad (\rho = 0.00, \quad \eta = 0.22).$$

These correspond $|V_{ub}|/|V_{cb}| = 0.08, 0.11$, and 0.05 representing the center value and upper and lower limit of the experimental value (90% C.L.) of the ratio.
In the calculation of Wilson coefficients, we use the internal quark masses as constituent quark masses: \( m_b = 4.88 \text{ GeV}, \ m_c = 1.5 \text{ GeV}, \ m_s = 0.5 \text{ GeV}, \ m_d = m_u = 0.2 \text{ GeV}. \) The mass terms in the matrix elements of several decay modes come form the equation of motion. Hence they are current quark masses rather than the constituent quark masses and the values of them are given by \( m_b = 4.88 \text{ GeV}, \ m_c = 1.3 \text{ GeV}, \ m_s = 122 \text{ MeV}, \ m_d = 7.6 \text{ MeV}, \ m_u = 4.2 \text{ MeV} \) at the renormalization scale \( \mu = 2.5 \text{ GeV}. \)

### III. BRANCHING RATIOS

Armed with the strategies described in the previous section, it is straightforward to calculate the amplitudes and the branching ratios of \( B \) meson decays into two light pseudoscalar mesons. The full amplitudes are given in the Appendix, and we will consider the numerical results only in this section. We present the results for the averaged branching ratio, since CLEO has measured the averaged one : for example,

\[
B(B^\pm \to \eta \pi^\pm) = \frac{1}{2} \left[ B(B^- \to \eta \pi^-) + B(B^+ \to \eta \pi^+) \right].
\]  

(17)

In Table 1, we present the numerical results using the typical parameters : \( \xi = 1/3, \) \( f_\eta^{(c)} = -0.9 \text{ MeV}, \ f_\rho^{(c)} = -2.3 \text{ MeV} \) ( \( m_c = 1.5 \text{ GeV} \) ), \( p = 0.05, \) \( \eta = 0.36 \). In Figs. 1, 2 and 3, we plot the branching ratio as a function of nonfactorization parameter \( \xi \) and using three different set of CKM angle \( \rho, \eta. \)

The additional effects of EW penguin diagrams in \( B \to \pi \pi, \pi K, KK \) modes, except \( B_d \to \pi^0 \pi^0, \pi^0 K^0 \) and \( B^\pm \to \pi^0 K^\pm \) modes, are generally negligible and our predictions on the decay rates including EW penguin on the several modes are similar to that of Ref. [1], and consistent on the recent CLEO data [1]. In \( B^\pm \to K^\pm \eta' \), the theoretical prediction has about 2\( \sigma \) deviation from the CLEO data. In \( B_d \to \pi^0 \pi^0, \pi^0 K^0 \) modes, the EW penguin effects decrease the decay rate about 33\% and 25\% for \( \xi = 1/3 \), respectively. In \( B^\pm \to \pi^0 K^\pm \) mode, the decay rate is increased by the effects of EW penguin diagrams about 29\% for \( \xi = 1/3 \). The decay rate of \( B^\pm \to K^0 K^\pm \) is same as that of \( B_d \to K^0 K^0 \) because of the isospin symmetry.

In \( B \to \pi \eta' \), \( K \eta' \), \( \eta' \eta' \) modes except \( B^\pm \to K^\pm \eta, \) \( B_d \to K^0 \eta \), \( \eta \eta \) modes, the EW penguin effects are also negligible and give at most \( \mathcal{O}(\text{few \%) corrections in decay rates. For } B_d \to \eta \eta \) modes, the effects increase the decay rate about 20\%. In the \( B^\pm \to K^\pm \eta \), \( B_d \to K^0 \eta \), the decay rates are decreased by the effects about 39\% and 36\% respectively. In \( B_d \to \pi^0 \pi^0, \eta' \eta' \) modes, the branching ratio plot has minimum value between \( \xi = 0.2 \) and \( \xi = 0.3 \) and its values become very small in this region and very sensitive to the actual values of \( \xi \), so that we cannot trust our predictions too much.

We also compare the decay rates with and without the \( b \to s(c\bar{c}) \to s(\eta, \eta') \) contributions. These corrections do not change the decay rates significantly. Our results are quite different from those obtained in Ref. [23], where \( b \to s(c\bar{c}) \to s(\eta, \eta') \) effect was interpreted as the intrinsic charm contents in \( \eta, \eta' \) mesons. The authors of Ref. [23] estimated the numerical value of \( f_\eta^{(c)}/f_\eta^{(u)} \) to be \( \mathcal{O}(1) \), and predicted that the branching ratios are about \( 0.194 \times 10^{-5}(5.83 \times 10^{-5}) \) for the decay modes \( B^\pm \to K^\pm \eta(\eta') \), and \( 0.027 \times 10^{-5}(5.73 \times 10^{-5}) \) for the decays modes \( B_d \to K^0 \eta(\eta') \), which are substantially larger than our predictions. However,
such large $f_{\eta(c)}^{(c)}$ are not compatible with other theoretical/experimental considerations [3,22], and cannot be taken too seriously.

IV. CP ASYMMETRIES

We also consider the direct CP violating rate asymmetry in the charged $B$ meson decay and the time integrated $CP$ asymmetry in neutral $B$ meson decay. In the inclusive charmless $B$ decays, the necessary strong phase differences are obtained from the absorptive part of the penguin diagram [15,20,27]. Such method is applied to exclusive nonleptonic decays of $B$ meson decays into two light pseudoscalar mesons [28]. It had been usually assumed that the final state interactions in nonleptonic $B$ decays may be negligibly small, because the mass of $B$ meson is far above the usual resonance region. However such effects become important in some cases. For example, the soft rescattering effects [29–31] might affect the method to constrain and determine the CKM angle $\gamma$ using $B \to \pi K$ modes [22].

In this article, we neglect the final state interactions and calculate the typical values and various parameter dependences of $CP$-asymmetries.

The $CP$-violating rate asymmetry is defined as

$$a_{CP} = \frac{\Gamma(B^- \to f) - \Gamma(B^+ \to \bar{f})}{\Gamma(B^- \to f) + \Gamma(B^+ \to \bar{f})} \tag{18}$$

where $\bar{f}$ is the charge conjugated state of the final state $f$.

The time integrated $CP$-asymmetry of neutral $B_d$ meson [33] is

$$a_{CP}(B_d \to f) = \frac{1}{1 + x_d^2} \left[ A_{CP}^{\text{dir}}(B_d \to f) + A_{CP}^{\text{mix-ind}}(B_d \to f) \right], \tag{19}$$

with

$$A_{CP}^{\text{dir}}(B_d \to f) = \frac{1 - |\xi_f|^2}{1 + |\xi_f|^2}, \quad A_{CP}^{\text{mix-ind}}(B_d \to f) = \frac{-2x_d m \xi_f}{1 + |\xi_f|^2}. \tag{20}$$

and

$$x_d = \frac{\Delta m_{B_d}}{\Gamma_{B_d}} \sim 0.71, \quad \xi_f = \frac{q}{p} \frac{A(B^0 \to f)}{A(B^0 \to \bar{f})}. \tag{21}$$

where $f$ is $CP$ eigenstates and $\Delta m$ denotes the mass difference of $B^0$ and $\bar{B}^0$ mesons. The ratio $q/p$ represents the CKM matrix elements ratio contributing to the $B^0 - \bar{B}^0$ mixing. If we consider $B$ meson decay into the final states with $K^0(\bar{K}^0)$ meson, the parameter $\xi_f$ has additional factor $(q/p)_K$ for the $K^0 - \bar{K}^0$ mixing effects.

We present the typical values of $CP$ asymmetry of several modes and estimate the EW penguin effects and $f_{\eta(c)}^{(c)}$ effects in Table 2. We also consider the effects of different CKM angles $\rho, \eta$. In the Figures 4,5 and 6, the $CP$ asymmetry is given by the function of nonfactorization parameter $\xi$ with three different CKM angles $\rho, \eta$. In Ref [28], the authors present the results of $CP$ asymmetry as a function of $q^2$. We will fix this value as $m_b^2/2$ in this analysis. The EW penguin effects and the $b \to d(s)[c\bar{c} \to gg \to \eta(c)]$ type corrections
on CP violation are generally small. The two-mixing scheme also gives generally small corrections to the values of the CP asymmetry compared with the one-mixing formalism for \( \eta - \eta' \) system.

In the CP asymmetry in charged B decays, the nonfactorization parameter \( \xi \) dependence is generally mild. The CP asymmetry in \( B^\pm \to \pi^\pm \pi^0, K^\pm \pi^0 \) is very small, at most 0.2\% in magnitude. For \( B^\pm \to K^\pm \pi^0, K^\pm \eta \) mode, the magnitude of CP asymmetry is less than about 10\% in the most parameter sets. In \( B^\pm \to K^\pm K^0, \pi^\pm \eta(\prime) \), and \( K^\pm \eta \) modes, the range of the magnitude of CP asymmetry is 10 \sim 30\%.

In the CP asymmetry in neutral B meson decays, \( B_d \to \pi^0 \pi^0 \) and \( B_d \to \eta(\prime) \eta(\prime) \) modes give very steep \( \xi \) dependence in 0.2 \sim 0.4 region of \( \xi \) region. The sign of CP asymmetry is changed in the region near \( \xi \sim 0.3 \). Hence the typical values for such modes in table 2 (which values are given in \( \xi = 1/3 \)) should be largely changed in small shift of \( \xi \). In other mode, the CP asymmetry change slowly in varying \( \xi \).

V. POSSIBLE EFFECTS THROUGH ENHANCEMENTS IN \( B \to SG \)

For the last several years, there have been some speculations about the enhanced \( b \to sg \) by several authors [18] in order to resolve some discrepancies in the data and theoretical expectations in inclusive B decays. The CLEO and ARGUS collaboration [34,35] have measured the semileptonic branching ratio:

\[
B_{SL}^{exp} = 10.23 \pm 0.39\%
\]  

(22)

And the CLEO 1.5, CLEO II and ARGUS data [34] give

\[
n_c^{exp} = 1.15 \pm 0.05
\]  

(23)

for the average number of charm(anti-) quarks per \( B^+/B^0 \)-decay. The result (22) is considerably smaller than the theoretical prediction in the parton model. It has been found that charm mass corrections to \( \Gamma(b \to c\bar{c}s) \) are large and can reduce the theoretical prediction for \( B_{SL} [36] \):

\[
B_{SL} = (11.7 \pm 1.4 \pm 1.0)\%
\]  

(24)

Then, there is no spectacular discrepancy between (22) and (24). However, as we lower the theoretical prediction for \( B_{SL} \) by increasing \( \Gamma(b \to c\bar{c}s) \), we simultaneously increase the prediction for \( n_c \). The theoretical prediction for \( n_c \) obtained from eq.(24) reads

\[
n_c = 1.34 \mp 0.06
\]  

(25)

The discrepancy between (23) and (25) constitutes the ”missing charm puzzle”.

Furthermore CLEO collaboration reported rather large branching ratio of \( B \to \eta' X_s \) decay mode [37]:

\[
B(B \to \eta' X_s) = (6.2 \pm 1.6 \pm 1.3) \times 10^{-4} \text{ for } 2.0 < P_{\eta'} < 2.7GeV
\]  

(26)

These anomalies may be solved through an enhancement of the chromomagnetic dipole coefficient \( C_g \) by new physics. For example, Lenz et.al [38] showed that \( |C_g(M_W)| \) must be
enhanced by a factor of 9 to 16 in order to explain the observed charm deficit if the CKM structure of new physics contribution is the same as in the Standard Model.

It is straightforward to include such effects of the enhanced chromomagnetic moment $b \to sg$ on the nonleptonic two body decays of $B$ mesons. This is accomplished by a simple replacement of

$$C_g \to \tilde{C}_g \equiv C_{g,SM} + C_{g,new}. \quad (27)$$

Here, to see the possible effects on the nonleptonic two body decays of $B$ meson, we take two possibilities:

$$\tilde{C}_g(2.5\text{GeV}) = +5 C_{g,SM}(2.5\text{GeV}), \quad \text{and} \quad -5 C_{g,SM}(2.5\text{GeV}) \quad (28)$$

In Fig. 7 and Fig. 8, we show the predictions for $B \to \pi K$ and $B \to \eta' K$ with $C_{g,new} = \pm 5 C_{g,SM}$ at $\mu = 2.5$ GeV. The agreement between the data and the factorization predictions is generally degraded when one includes the enhanced $b \to sg$. For the positive $C_{g,new} = +5 C_{g,SM}$, all the predictions become worse because of destructive interference between the SM amplitude and the enhanced $b \to sg$. For the negative $C_{g,new} = -5 C_{g,SM}$, one can improve the branching ratio for $B \to \eta K$, only by paying a price to the worse predictions for $B \to \pi K$. All these observations are based on the generalized factorization assumption. In such an approximation, there is a tendency that the enhanced $b \to sg$ makes worse the agreements between the data and predictions. It has to be kept in mind that the enhanced $b \to sg$ scenario should be also tested in the exclusive $B$ decays in a better way than considered in this work, if possible.

**VI. CONCLUSIONS AND DISCUSSIONS**

In this work, we considered the branching ratios and $CP$ asymmetries in $B$ meson decays into two light pseudoscalar mesons, and the possible effects of the enhanced $b \to sg$ vertex suggested as a solution to the semileptonic branching ratio problem and the missing charm puzzle in $B$ meson decays. The typical branching ratios in various parameter set are given in Table 1. Their $\xi$ parameter dependences are given in Figs. 1, 2 and 3. In $B \to \pi\pi, \pi K, KK$ modes, we included the EW penguin effects as well as gluonic penguin effects. Especially in $B_d \to \pi^0\pi^0, \pi^0K^0, K^0\eta$ modes and $B^\pm \to \pi^0K^\pm, K^\pm\eta$ modes, EW penguin effects give important contributions to the branching ratio: about $+33\%, +25\%, -36\%, +29\%, -38\%$ variation respectively. In $B_d \to \pi^0\pi^0, \eta^{(s)}\eta^{(s)}$ modes, the branching ratio plot has minimum value between $\xi = 0.2$ and $\xi = 0.3$ and its values become very small in this region and very different from the values outside such the region.

The EW penguin effects and the $b \to d(s)[c\bar{c} \to gg \to \eta^{(s)}]$ type corrections on $CP$ violation generally give small contributions to $CP$ asymmetry. The typical values of the $CP$ asymmetry of $B$ meson decays are given in Table 2. In Figs. 4, 5 and 6, the $\xi$ dependences of $CP$ asymmetry in several modes are given in three different set of CKM angle $\rho, \eta$. For a fixed $\xi$, the important corrections of $CP$ asymmetry are given by varying the CKM angle itself. The $\xi$ dependence of $CP$ asymmetry in charged $B$ decays are rather mild. In the neutral $B$ meson decay modes, $CP$ asymmetry of $B_d \to \pi^0\pi^0$ and $B_d \to \eta^{(s)}\eta^{(s)}$ modes
change very sharply in the region $0.2 < \xi < 0.4$. In other modes, the $CP$ asymmetry changes slowly when one varies $\xi$.

Lastly, we made an observation that the enhanced $b \to s g$ tends to degrade the agreement between the data and the factorization predictions for $B$ meson decays into two light mesons, which seems to disfavor the enhanced $b \to s g$ scenario. The enhanced $b \to s g$ idea was put forward within the inclusive $B$ decays [18], which can be studied with much less theoretical uncertainties compared to the exclusive cases. But it is certainly true that the enhanced $b \to s g$ will affect the individual exclusive $B$ decay modes as well. It would be very welcome to study the effect of this enhanced $b \to s g$ on the individual exclusive $B$ decays in a way more reliable than the generalized factorization method employed in the present work.

VII. APPENDIX

In this appendix, we present the complete matrix elements including the EW penguin effects and $f_{n'(c)}^{(c)}$ effects.

- $B^- \to \pi^- \pi^0$

$$M = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{ud}^*(a_1 + a_2) - V_{tb}V_{td}^* \left[ \frac{3}{2}(a_9 - a_7 + a_{10}) + \frac{2m_d^2 a_8}{(m_b + m_d)(m_b - m_u)} \right] \right\} < \pi^-|\bar{d}u_-|0 > < \pi^0|\bar{u}b_-|B^- >$$

where

$$< \pi^-|\bar{d}u_-|0 > < \pi^0|\bar{u}b_-|B^- > = i \frac{f_\pi}{\sqrt{2}} (m_B^2 - m_\pi^2) F_{B^0 \to \pi}^{\pi} (m_\pi^2).$$

- $\bar{B}^0 \to \pi^+ \pi^-$

$$M = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{ud}^*a_1 - V_{tb}V_{td}^* \left[ a_4 + a_{10} + \frac{2m_d^2 (a_6 + a_8)}{(m_u + m_d)(m_b - m_u)} \right] \right\} \times < \pi^-|\bar{d}u_-|0 > < \pi^+|\bar{u}b_-|\bar{B}^0 >$$

where

$$< \pi^-|\bar{d}u_-|0 > < \pi^+|\bar{u}b_-|\bar{B}^0 > = i f_\pi (m_B^2 - m_\pi^2) F_{B^0 \to \pi}^{\pi} (m_\pi^2).$$

- $\bar{B}^0 \to \pi^0 \pi^0$

$$M = \frac{2G_F}{\sqrt{2}} \left\{ V_{ub}V_{ud}^*a_2 + V_{tb}V_{td}^* \left[ a_4 + \frac{3}{2}(a_7 - a_9) - \frac{1}{2}a_{10} + \frac{m_d^2 (2a_6 - a_8)}{2m_d(m_b - m_d)} \right] \right\} \times < \pi^0|\bar{u}u_-|0 > < \pi^0|\bar{d}b_-|\bar{B}^0 >$$

where

$$< \pi^0|\bar{u}u_-|0 > < \pi^0|\bar{d}b_-|\bar{B}^0 > = i \frac{f_\pi}{2} (m_B^2 - m_\pi^2) F_{B^0 \to \pi^0}^{\pi} (m_\pi^2).$$

11
• $B^- \to K^0\pi^-$

$$M = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ a_4 - \frac{1}{2} a_{10} + \frac{m_K^2 (2a_6 - a_8)}{(m_d + m_s)(m_b - m_d)} \right] < \bar{K}^0|\bar{s}d_>| 0 > < \pi^-|\bar{d}b_| B^- >$$

(35)

where

$$< \bar{K}^0|\bar{s}d_>| 0 > < \pi^-|\bar{d}b_| B^- > = -i f_K (m_B^2 - m_\pi^2) F_0^{B\to\pi}(m_K^2).$$

(36)

• $B^- \to K^-\pi^0$

$$M = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{us}^* (a_1 + a_2 R_{K^-\pi^0}) - V_{tb} V_{ts}^* \left[ a_4 + a_{10} + \frac{2m_K^2 (a_6 + a_8)}{(m_u + m_s)(m_b - m_u)} + \frac{3}{2} (a_9 - a_7 R_{K^-\pi^0}) \right] \right\} < K^-|\bar{s}u_>| 0 > < \pi^0|\bar{u}b_| B^- >$$

(37)

where

$$R_{K^-\pi^0} \equiv < \pi^0|\bar{u}u_>| 0 > < K^-|\bar{s}b_| B^- > = \frac{f_\pi m_B^2 - m_K^2 F_0^{B\to\pi}(m_\pi^2)}{f_K m_B^2 - m_K^2 F_0^{B\to\pi}(m_\pi^2)},$$

$$< K^-|\bar{s}u_>| 0 > < \pi^0|\bar{u}b_| B^- > = i \frac{f_K}{\sqrt{2}} (m_B^2 - m_\pi^2) F_0^{B\to\pi}(m_K^2).$$

(38)

• $\bar{B}^0 \to K^-\pi^+$

$$M = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* \left[ a_4 + a_{10} + \frac{2m_K^2 (a_6 + a_8)}{(m_u + m_s)(m_b - m_u)} \right] \right\} 
\times < K^-|\bar{s}u_>| 0 > < \pi^+|\bar{u}b_| \bar{B}^0 >$$

(39)

where

$$< K^-|\bar{s}u_>| 0 > < \pi^+|\bar{u}b_| \bar{B}^0 > = i f_K (m_B^2 - m_\pi^2) F_0^{B\to\pi}(m_K^2).$$

(40)

• $\bar{B}^0 \to \bar{K}^0\pi^0$

$$M = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{us}^* a_2 R_{K^0\pi^0} - V_{tb} V_{ts}^* \left[ a_4 - \frac{1}{2} a_{10} + \frac{m_K^2 (2a_6 - a_8)}{(m_d + m_s)(m_b - m_d)} + \frac{3}{2} (a_9 - a_7 R_{K^0\pi^0}) \right] \right\} < \bar{K}^0|\bar{s}d_>| 0 > < \pi^0|\bar{d}b_| \bar{B}^0 >$$

(41)

where

$$R_{K^0\pi^0} \equiv < \pi^0|\bar{u}u_>| 0 > < \bar{K}^0|\bar{s}b_| \bar{B}^0 > = \frac{f_\pi m_B^2 - m_K^2 F_0^{B\to\pi}(m_\pi^2)}{f_K m_B^2 - m_K^2 F_0^{B\to\pi}(m_\pi^2)},$$

$$< \bar{K}^0|\bar{s}d_>| 0 > < \pi^0|\bar{d}b_| \bar{B}^0 > = -i \frac{f_K}{\sqrt{2}} (m_B^2 - m_K^2) F_0^{B\to\pi}(m_K^2).$$

(42)
\[ B^{-} \rightarrow K^{0} K^{-} \]

\[
M = -\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^{*} \left\{ a_4 - \frac{1}{2} a_{10} + \frac{m_{K}^2 (2a_6 - a_8)}{(m_d + m_s)(m_b - m_s)} \right\} \\
\times < K^0 | \bar{d}s_\rightarrow |0 > < K^- | \bar{s}b_\rightarrow |B^- >
\]

where

\[
< K^0 | \bar{d}s_\rightarrow |0 > < K^- | \bar{s}b_\rightarrow |B^- > = -i \ f_K (m_B^2 - m_K^2) F_{0}^{B \rightarrow K} (m_K^2).
\]

\[
\bar{B}^0 \rightarrow K^{0} \bar{K}^0
\]

\[
M = -\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^{*} \left\{ a_4 - \frac{1}{2} a_{10} + \frac{m_{K}^2 (2a_6 - a_8)}{(m_d + m_s)(m_b - m_s)} \right\} \\
\times < K^0 | \bar{d}s_\rightarrow |0 > < \bar{K}^0 | \bar{s}b_\rightarrow |\bar{B}^0 >
\]

where

\[
< K^0 | \bar{d}s_\rightarrow |0 > < \bar{K}^0 | \bar{s}b_\rightarrow |\bar{B}^0 > = -i \ f_K (m_B^2 - m_K^2) F_{0}^{\bar{B} \rightarrow \bar{K}} (m_K^2).
\]

\[
B^{-} \rightarrow \pi^{-} \eta
\]

\[
M = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{ud}^{*} ( a_2 + a_1 R_{\pi^- \eta} ) + V_{cb} V_{cd}^{*} a_2 \frac{f_{\eta}^{(c)}}{f_{\eta}^{u}} \right\} \\
- V_{tb} V_{td}^{*} \left[ 2(a_3 - a_5) + a_4 + \frac{m_{\pi}^2 (2a_6 - a_8)}{2m_s (m_b - m_s)} \left( 1 - \frac{f_{\eta}^{u}}{f_{\eta}^{s}} \right) - \frac{1}{2} (a_7 - a_9 + a_{10}) \right] \\
+ \left\{ a_4 + a_{10} + \frac{2m_{\pi}^2 (a_6 + a_8)}{(m_u + m_d)(m_b - m_u)} \right\} R_{\pi^- \eta} \\
+ \left\{ a_3 - a_5 + \frac{1}{2} (a_7 - a_9) \right\} \frac{f_{\eta}^{s}}{f_{\eta}^{u}} + \left\{ a_3 - a_5 - a_7 + a_9 \right\} \frac{f_{\eta}^{(c)}}{f_{\eta}^{u}} \right\} \\
\times < \pi^- | \bar{d}b_\rightarrow | B^- > < \eta | \bar{u}u_\rightarrow |0 >
\]

where

\[
R_{\pi^- \eta} = \frac{< \pi^- | \bar{u}u_\rightarrow |0 > < \eta | \bar{d}b_\rightarrow |B^- >}{< \eta | \bar{u}u_\rightarrow |0 > < \pi^- | \bar{d}b_\rightarrow |B^- >} = \frac{f_{\pi} m_B^2 - m_{\pi}^2}{f_{\eta}^{u} m_B^2 - m_{\eta}^2} \frac{F_{0}^{B \rightarrow \pi} (m_{\pi}^2)}{F_{0}^{B \rightarrow \eta} (m_{\eta}^2)},
\]

\[
< \pi^- | \bar{d}b_\rightarrow | B^- > < \eta | \bar{u}u_\rightarrow |0 > = i \ f_{\eta}^{u} (m_B^2 - m_{\pi}^2) F_{0}^{B \rightarrow \pi} (m_{\pi}^2).
\]

\[
\bar{B}^0 \rightarrow \pi^0 \eta
\]
\[ M = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{us}^* a_2 \left( 1 + R_{\pi^0\eta} \right) + V_{cb} V_{cs}^* a_2 \frac{f_{s}^{(c)}}{f_{u}^{(c)}} \right. \\
- V_{tb} V_{td}^* \left[ 2(a_3 - a_5) + a_4 + \frac{m_{d}^2(2a_6 - a_8)}{2m_s(m_b - m_s)} \left( 1 - \frac{f_{u}^{(c)}}{f_{d}^{(c)}} \right) - \frac{1}{2}(a_7 - a_9 + a_{10}) \right] \\
- \left\{ a_4 + (a_6 - \frac{1}{2}a_8) \frac{m_{d}^2}{m_d(m_b - m_d)} + 3 \frac{2(a_7 - a_9)}{2} - \frac{1}{2}a_{10} \right\} R_{\pi^0\eta} \\
+ \left\{ a_3 - a_5 + \frac{1}{2}(a_7 - a_9) \right\} \frac{f_{u}^{(c)}}{f_{d}^{(c)}} + \left\{ a_3 - a_5 - a_7 + a_9 \right\} \right\} \right\} \\
\times < \eta | \bar{\eta} u u | 0 > \langle \pi^0 | d b | B^0 > \right] (49) \]

where
\[ R_{\pi^0\eta} \equiv < \pi^0 | \bar{\eta} u u | 0 > \langle \eta | d b | B^0 > \right] = - \frac{f_{s}^{(c)}}{f_{u}^{(c)}} \frac{m_{d}^2}{m_d(m_b - m_d)} \frac{1}{2} \left( m_{s}^2(m_b - m_s) \right) \frac{F_{0}^{B \rightarrow \eta}(m_{K}^2)}{F_{0}^{B \rightarrow K}(m_{\eta}^2)} \right], \]
\[ < \eta | \bar{\eta} u u | 0 > \langle \pi^0 | d b | B^0 > = i \frac{f_{u}^{(c)}}{\sqrt{2}} \left( m_{s}^2(m_b - m_s) \right) \frac{F_{0}^{B \rightarrow \pi}(m_{\eta}^2)}{F_{0}^{B \rightarrow K}(m_{\eta}^2)} \right], (50) \]

- The matrix elements of the $B^- \rightarrow \pi^- \eta', \bar{B}^0 \rightarrow \pi^0 \eta'$ modes might be obtained by replacing $\eta$ with $\eta'$ in $B^- \rightarrow \pi^- \eta, \bar{B}^0 \rightarrow \pi^0 \eta$ modes.

- $B^- \rightarrow K^- \eta$

\[ M = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{us}^* a_2 + V_{cb} V_{cs}^* a_2 \frac{f_{s}^{(c)}}{f_{u}^{(c)}} - V_{tb} V_{td}^* \left[ 2(a_3 - a_5) - \frac{m_{d}^2(2a_6 - a_8)}{2m_s(m_b - m_s)} \right] \right. \]
\[- \frac{1}{2}(a_7 - a_9) + \left\{ a_4 + \frac{2m_{K}^2(a_6 + a_8)}{(m_s + m_u)(m_b - m_u)} + a_{10} \right\} R_{K^- \eta} \]
\[+ \left\{ a_3 + a_4 - a_5 + \frac{m_{d}^2(2a_6 - a_8)}{2m_s(m_b - m_s)} + \frac{1}{2}(a_7 - a_9 - a_{10}) \right\} \frac{f_{u}^{(c)}}{f_{d}^{(c)}} \]
\[+ (a_3 - a_5 - a_7 + a_9) \frac{f_{s}^{(c)}}{f_{u}^{(c)}} \right\} \left\langle \eta | \bar{\eta} u u | 0 > \langle K^- | s b | B^- > \right] (51) \]

where
\[ R_{K^- \eta} \equiv \left\langle K^- | \bar{s} b | 0 > \langle \eta | \bar{\eta} b | B^- > \right\rangle = - \frac{f_{s}^{(c)}}{f_{u}^{(c)}} \frac{m_{s}^2(m_b - m_s)}{m_d(m_b - m_d)} \frac{1}{2} \left( m_{s}^2(m_b - m_s) \right) \frac{F_{0}^{K \rightarrow \eta}(m_{K}^2)}{F_{0}^{K \rightarrow \pi}(m_{\eta}^2)} \right], \]
\[ < \eta | \bar{\eta} u u | 0 > \langle K^- | \bar{d} b | B^- > = i \frac{f_{u}^{(c)}}{m_{s}^2(m_b - m_s)} \frac{F_{0}^{K \rightarrow \pi}(m_{\eta}^2)}{F_{0}^{K \rightarrow \pi}(m_{\eta}^2)} \right], (52) \]

- $\bar{B}^0 \rightarrow \bar{K}^0 \eta$

\[ M = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{us}^* a_2 + V_{cb} V_{cs}^* a_2 \frac{f_{s}^{(c)}}{f_{u}^{(c)}} - V_{tb} V_{td}^* \left[ 2(a_3 - a_5) - \frac{m_{d}^2(2a_6 - a_8)}{2m_s(m_b - m_s)} - \frac{1}{2}(a_7 - a_9) \right] \right. \]
\[ + \left\{ a_4 + \frac{m_K^2 (2a_6 - a_8)}{(m_s + m_d)(m_b - m_d)} - \frac{1}{2} a_{10} \right\} R_{K^0}\eta \]
\[ + \left\{ a_3 + 4 - a_5 + \frac{m_Q^2 (2a_6 - a_8)}{2m_s (m_b - m_s)} + \frac{1}{2} (a_7 - a_9 - a_{10}) \right\} \frac{f_{q}^*}{f_q} \]
\[ + (a_3 - a_5 - a_7 + a_9) \frac{f_{q}^{(c)}}{f_q} \} \right\} < \eta |\bar{u}u_- | 0 > < K^0 |\bar{s}b_- | B^0 > \]

where

\[ R_{K^0}\eta \equiv \frac{< K^0 |\bar{s}d_- | 0 > < \eta |\bar{d}b_- | B^0 >}{< \eta |\bar{u}u_- | 0 > < K^0 |\bar{s}b_- | B^0 >} = \frac{f_K m_B^2 - m_\eta^2}{f_q^*} \frac{F_0^{B^0 \rightarrow \eta}(m_K^2)}{F_0^{B^0 \rightarrow \eta}(m_\eta^2)}, \]
\[ < \eta |\bar{u}u_- | 0 > < K^0 |\bar{s}b_- | B^0 > = i \frac{f_{q}^*}{f_q} (m_B^2 - m_\eta^2) F_0^{B^0 \rightarrow \eta}(m_\eta^2). \]  

- The matrix elements of the $B^- \rightarrow K^- \eta', \bar{B}^0 \rightarrow \bar{K}^0 \eta'$ modes might be obtained by replacing $\eta$ with $\eta'$ in $B^- \rightarrow K^- \eta, \bar{B}^0 \rightarrow \bar{K}^0 \eta$ modes.

- $\bar{B}^0 \rightarrow \eta \eta$

\[ M = \frac{2G_F}{\sqrt{2}} \left\{ V_{ub} V_{ud}^* a_2 + V_{cb} V_{cd}^* a_2 \frac{f_{q}^{(c)}}{f_q} - V_{tb} V_{td}^* \left[ 2(a_3 - a_5) + a_4 + \frac{m_n^2 (2a_6 - a_8)}{2m_s (m_b - m_s)} \left( 1 - \frac{f_{q}^*}{f_q} \right) - \frac{1}{2} (a_7 - a_9 + a_{10}) \right] + \left\{ a_3 - a_5 + \frac{1}{2} (a_7 - a_9) \right\} \frac{f_{q}^*}{f_q} + (a_3 - a_5 - a_7 + a_9) \frac{f_{q}^{(c)}}{f_q} \} \right\} \times < \eta |\bar{u}u_- | 0 > < \eta |\bar{d}b_- | B^0 > \]

where

\[ < \eta |\bar{u}u_- | 0 > < \eta |\bar{d}b_- | B^0 > = -i \frac{f_{q}^*}{f_q} (m_B^2 - m_\eta^2) F_0^{B^0 \rightarrow \eta}(m_\eta^2). \]  

- The matrix elements of the $B^0 \rightarrow \eta' \eta'$ might be obtained by replacing $\eta$ with $\eta'$ in $B^0 \rightarrow \eta \eta$ modes.

- $B^0 \rightarrow \eta \eta'$

\[ M = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{ud}^* a_2 + V_{cb} V_{cd}^* a_2 \frac{f_{q}^{(c)}}{f_q} - V_{tb} V_{td}^* \left[ 2(a_3 - a_5) + a_4 + \frac{m_n^2 (2a_6 - a_8)}{2m_s (m_b - m_s)} \left( 1 - \frac{f_{q}^*}{f_q} \right) - \frac{1}{2} (a_7 - a_9 + a_{10}) \right] + \left\{ a_3 - a_5 + \frac{1}{2} (a_7 - a_9) \right\} \frac{f_{q}^*}{f_q} + (a_3 - a_5 - a_7 + a_9) \frac{f_{q}^{(c)}}{f_q} \} \right\} < \eta |\bar{u}u_- | 0 > < \eta' |\bar{d}b_- | B^0 > + (\eta \rightarrow \eta') \]
where
\[
\begin{align*}
\langle \eta|\bar{u}u|0 \rangle &< \eta'|\bar{d}b|B^0 \rangle = -i f^{u}_{\eta} (m_B^2 - m_{\eta'}^2) F_{0}^{B \rightarrow \eta'}(m_{\eta}^2), \\
\langle \eta'|\bar{u}u|0 \rangle &< \eta|\bar{d}b|B^0 \rangle = -i f^{u}_{\eta'} (m_B^2 - m_{\eta}^2) F_{0}^{B \rightarrow \eta}(m_{\eta'}^2).
\end{align*}
\]

(60)

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TABLE I. Combined branching ratios in unit of $10^{-5}$. 'QCD' and 'EW' present the QCD penguin and EW penguin effects respectively. 'Full' includes the $b \to s[c\bar{c} \to gg \to \eta\eta']$ effects: 

$\xi = 1/3$, $f^{(c)}_{\eta} = -0.9 MeV$, $f^{(c)}_{\eta'} = -2.3 MeV$ ( $m_c = 1.5 GeV$ ), $\rho = 0.05$, $\eta = 0.36$

| Decay Mode | Tree   | Tree+QCD | Tree+QCD+EW | Exp.  |
|------------|--------|----------|-------------|-------|
| $B^\pm \to \pi^\pm\pi^0$ | 0.51   | 0.51     | 0.51        | $<2.0$|
| $B^\pm \to \pi^\pm K^0$ | 0      | 1.63     | 1.61        | $2.3^{+1.1}_{-1.0} \pm 0.3 \pm 0.2$ |
| $B^\pm \to \pi^0 K^\pm$ | 0.038  | 0.84     | 1.08        | $<1.6$|
| $B^\pm \to K^\pm K^0$  | 0      | 0.082    | 0.081       | $<2.1$|
| $B_d \to \pi^\pm\pi^+$ | 0.94   | 0.93     | 0.93        | $<1.5$|
| $B_d \to \pi^0\pi^0$   | $0.11 \times 10^{-2}$ | 0.021 | 0.014 | $<0.93$|
| $B_d \to \pi^\pm K^\mp$ | 0.071  | 1.68     | 1.71        | $1.5^{+0.5}_{-0.4} \pm 0.1 \pm 0.1$ |
| $B_d \to \pi^0 K^0$     | $0.53 \times 10^{-4}$ | 0.83  | 0.62        | $<4.1$|
| $B_d \to K^0\bar{K}^0$  | 0      | 0.082    | 0.081       | $<1.7$|

| Decay Mode | Tree   | Tree+QCD | Tree+QCD+EW | Full  | Exp.  |
|------------|--------|----------|-------------|-------|-------|
| $B^\pm \to \pi^\pm\eta$ | 0.23   | 0.25     | 0.26        | 0.26  | $<1.5$|
| $B^\pm \to \pi^\pm\eta'$ | 0.16   | 0.18     | 0.17        | 0.18  | $<3.1$|
| $B_d \to \pi^0\eta$     | $0.18 \times 10^{-4}$ | 0.026 | 0.026 | 0.026 | $<0.8$|
| $B_d \to \pi^0\eta'$    | $0.71 \times 10^{-5}$ | 0.011 | 0.79 $\times 10^{-2}$ | 0.81 $\times 10^{-2}$ | $<1.1$|
| $B^\pm \to K^\pm\eta$   | 0.017  | 0.18     | 0.11        | 0.11  | $<1.4$|
| $B^\pm \to K^\pm\eta'$  | 0.012  | 2.15     | 2.05        | 2.08  | $6.5^{+1.5}_{-1.4} \pm 0.9$ |
| $B_d \to K^0\eta$       | $0.37 \times 10^{-4}$ | 0.15  | 0.096 | 0.094 | $<3.3$|
| $B_d \to K^0\eta'$      | $0.24 \times 10^{-4}$ | 2.16  | 2.04 | 2.07 | $4.7^{+2.7}_{-2.0} \pm 0.9$ |
| $B_d \to \eta\eta$      | $0.32 \times 10^{-3}$ | $0.83 \times 10^{-2}$ | 0.010 | 0.010 | $<1.8$|
| $B_d \to \eta\eta'$     | $0.44 \times 10^{-3}$ | $0.78 \times 10^{-2}$ | $0.83 \times 10^{-2}$ | $0.86 \times 10^{-2}$ | $<2.7$|
| $B_d \to \eta'\eta'$    | $0.15 \times 10^{-3}$ | $0.17 \times 10^{-3}$ | $0.14 \times 10^{-3}$ | $0.15 \times 10^{-3}$ | $<4.7$|
TABLE II. $CP$ asymmetries in %. 'QCD' and 'EW' present the QCD penguin and EW penguin effects respectively. 'Full' includes the $b \to s[c\bar{c} \to gg \to \eta'\eta]$ effects. 'Full(one-mixing)' presents the one-mixing scheme for $\eta - \eta'$ mixing: CKM phases are $\rho = 0.05, \eta = 0.36(\rho = 0.30, \eta = 0.42; \rho = 0, \eta = 0.22), \xi = 1/3$.

| Decay Mode      | QCD           | QCD+EW       |
|-----------------|---------------|--------------|
| $B^\pm \to \pi^\pm \pi^0$ | 0 (0, 0)      | -0.05(-0.03, -0.09) |
| $B^\pm \to \pi^\pm K^0$ | 1.6 (1.8, 1.0) | 1.6 (1.8, 1.0) |
| $B^\pm \to \pi^0 K^\pm$ | 8.4 (12.9, 5.0) | 6.7 (10.0, 4.0) |
| $B^\pm \to K^0 K^\pm$ | -12.2(-20.5, -7.3) | -12.3(-20.6, -7.3) |
| $B_d \to \pi^\pm \pi^\mp$ | 5.1 (21.4, 22.8) | 5.4 (21.5, 23.3) |
| $B_d \to \pi^0 \pi^0$ | -10.5(-18.8, -6.5) | -13.9(-24.4, -8.6) |
| $B_d \to \pi^0 K_S$ | 31.1 (41.1 19.8) | 31.1 (41.1, 19.8) |
| $B_d \to K_S K_S$ | 15.2 (26.0, 9.1) | 15.2 (26.1, 9.1) |

| Decay Mode      | QCD           | QCD+EW       | Full | Full(one-mixing) |
|-----------------|---------------|--------------|------|------------------|
| $B^\pm \to \pi^\pm \eta$ | -18.2(-9.6, -23.5) | -18.0(-9.6, -22.4) | -18.0(-9.6, -22.4) | -18.6(-9.9, -23.1) |
| $B^\pm \to \pi^\pm \eta'$ | -17.9(-9.4, -25.9) | -18.0(-9.4, -26.3) | -17.9(-9.4, -26.1) | -17.7(-9.3, -25.9) |
| $B^\pm \to K^\pm \eta$ | -6.7(-5.3, -4.8) | -10.9(-7.9, -8.2) | -11.1(-8.0, -8.4) | -13.4(-7.5, -14.5) |
| $B^\pm \to K^\pm \eta'$ | 4.5 (5.8, 2.7) | 4.7 (6.1, 2.8) | 4.6 (6.0, 2.8) | 4.8 (6.3, 2.9) |
| $B_d \to \pi^0 \eta$ | 18.8(31.4, 11.3) | 18.8(31.4, 11.3) | 18.9(31.6, 11.3) | 19.2(32.0, 11.5) |
| $B_d \to \pi^0 \eta'$ | 23.5(38.4, 14.2) | 27.9(44.5, 16.8) | 28.1(44.8, 17.0) | 28.1(44.7, 17.0) |
| $B_d \to K_S \eta$ | 31.8 (41.7, 20.3) | 32.0 (41.9, 20.5) | 32.1 (41.9, 20.5) | 34.3 (43.6, 22.1) |
| $B_d \to K_S \eta'$ | 29.9 (40.1, 19.0) | 29.9 (40.0, 18.9) | 29.9 (40.1, 18.9) | 29.9 (40.0, 18.9) |
| $B_d \to \eta \eta$ | 37.3 (52.4, 22.9) | 33.1 (48.1, 20.2) | 33.1 (48.1, 20.2) | 33.5 (48.6, 20.4) |
| $B_d \to \eta \eta'$ | 45.5 (59.6, 28.5) | 43.9 (58.3, 27.4) | 43.8 (58.1, 27.4) | 43.9 (58.3, 27.4) |
| $B_d \to \eta' \eta'$ | 57.7 (67.8, 37.8) | 63.1 (70.6, 42.5) | 62.4 (69.9, 41.9) | 62.5 (69.8, 42.0) |
FIG. 1. Combined branching ratio of the charged $B$ meson decays as a function of nonfactorization parameter $\xi$: $\rho = 0.05, \eta = 0.36$ for real line, $\rho = 0.30, \eta = 0.42$ for dashed line and $\rho = 0, \eta = 0.22$ for dotted line.
FIG. 2. Combined branching ratio of the neutral $B$ meson decays as a function of nonfactorization parameter $\xi$: $\rho = 0.05, \eta = 0.36$ for real line, $\rho = 0.30, \eta = 0.42$ for dashed line and $\rho = 0, \eta = 0.22$ for dotted line.
FIG. 3. Combined branching ratio of the neutral $B$ meson decays as a function of nonfactorization parameter $\xi$: $\rho = 0.05, \eta = 0.36$ for real line, $\rho = 0.30, \eta = 0.42$ for dashed line and $\rho = 0, \eta = 0.22$ for dotted line.
FIG. 4. The CP asymmetry of the charged B meson decays as a function of nonfactorization parameter $\xi$: $\rho = 0.05, \eta = 0.36$ for real line, $\rho = 0.30, \eta = 0.42$ for dashed line and $\rho = 0, \eta = 0.22$ for dotted line.
FIG. 5. The $CP$ asymmetry of the neutral $B$ meson decays as a function of nonfactorization parameter $\xi$: $\rho = 0.05, \eta = 0.36$ for real line, $\rho = 0.30, \eta = 0.42$ for dashed line and $\rho = 0, \eta = 0.22$ for dotted line.
FIG. 6. The $CP$ asymmetry of the neutral $B$ meson decays as a function of nonfactorization parameter $\xi$: $\rho = 0.05, \eta = 0.36$ for real line, $\rho = 0.30, \eta = 0.42$ for dashed line and $\rho = 0, \eta = 0.22$ for dotted line.
FIG. 7. Combined branching ratios of the $B$ meson decays as functions of nonfactorization parameter $\xi$ in the presence of the enhanced $b \to s g$ with $C_{g,new} = +5C_{g,SM}$ at $\mu = 2.5$ GeV: $\rho = 0.05, \eta = 0.36$ for real line, $\rho = 0.30, \eta = 0.42$ for dashed line and $\rho = 0, \eta = 0.22$ for dotted line.
FIG. 8. Combined branching ratios of the $B$ meson decays as functions of nonfactorization parameter $\xi$ in the presence of the enhanced $b \to sg$ with $C_{g,\text{new}} = -5C_{g,\text{SM}}$ at $\mu = 2.5$ GeV: $\rho = 0.05, \eta = 0.36$ for real line, $\rho = 0.30, \eta = 0.42$ for dashed line and $\rho = 0, \eta = 0.22$ for dotted line.