Non-coherent contributions in charge-exchange reactions and $\eta-\eta'$ mixing

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Abstract

We analyse $K^-p \rightarrow (\eta, \eta', \pi^0)\Lambda$ on the basis of the fit of data in a wide region of energies, and $\pi^-p \rightarrow (\eta, \eta')n$ at the energies of GAMS-4$\pi$. We show that disagreements between the data and the predictions of Regge theory may be explained by the mode change of summation of intermediate contributions at increasing energy, from coherent to non-coherent. A method of experimental measurement of the non-coherent contributions is proposed. On the basis of available data on the charge-exchange reactions the $\eta-\eta'$ mixing is estimated.

1 Introduction

It has been observed [1] that binary charge-exchange reactions of hadrons at high energies go through the charge-exchange scattering of fast quarks located in the beginning of quantum fluctuations, the splitting and recombination of partons inside hadrons. This fact has far-reaching effects. First, the mentioned fast quark scattering is accompanied by high virtualities [1]. Therefore the appropriate subprocesses are hard and may be described in the parton model. Second, the mentioned scattering interrupts the fluctuations, which results in the formation of a cloud of uncorrelated partons. In the exclusive reaction they are to be captured by the flying away clusters, and this induces destruction of the coherence. So one can expect destruction of the coherence of intermediate contributions. Due to increasing duration of the interaction with increasing energy of the collisions [2, 3, 4, 5], the latter effect should be increasing with increasing the energy.

Based on the above considerations [1] proposed a model for the description of $\pi^-p \rightarrow M^0n$ and $K^-p \rightarrow M^0\Lambda$, $M^0 = \eta, \eta', \pi^0$. The model also used the idea that contributions of soft interactions that follow the hard scattering, obey the Regge behavior [6, 7]. However, the mode of summations of elementary contributions may be coherent or non-coherent. In the former case the model reproduced conventional Regge approach. In the case of non-coherent summation the model gave non-trivial predictions. Since the mode of summation should change with increasing the energy, one of non-trivial predictions was the emergence of the energy dependence in the
vertex functions, prohibited in the Regge approach. Such a dependence is really observed at comparing at different energies the dependence on the transfer of the ratio of yields of $\eta'$ and $\eta$ in the $\pi^-$ beams [1, 8]. However, the most striking effect was disappearance of a dip near $|t| = 0.4 \text{ (GeV/c)}^2$ with increasing the energy in the differential cross-section $K^-p \rightarrow \eta\Lambda$. In reality, the dip is observed at the $K^-$ momentum 3–8 GeV/c in the laboratory frame [9, 10, 11, 12] and disappears at the momentum 32.5 GeV/c [13]. In the Regge approach the dip is explained as a consequence of a dominance of the vector-exchange trajectory and simultaneously its zeroing by the signature factor in the region $\alpha_V(t) \approx 0$, where $\alpha_V(t)$ is the vector trajectory [14]. However, this mechanism is independent of the energy, and it is not clear why it ceases to operate with the increasing energy. Ref. [1] explained the effect by the mode change of summation of intermediate contributions. But the data fit was not carried out in [1] and therefore the explanation was a qualitative.

In this paper we carry out a full fit of available data at the relatively low and high energies, and on this basis we confirm the above explanation. In addition, we solve the problem of the description at intermediate energies where contributions of both modes are possible. In effect, we propose an algorithm for measuring a relative value of the coherent and non-coherent contributions. As a by-product we get an independent estimate of the $\eta$–$\eta'$ mixing. A possible admixture of the glueball is considered, as well.

We carry out the analysis in a modified version of the model [1]. The modification concerns mainly the version of the Regge phenomenology. First of all, we defreeze the parameters responsible for the quark symmetry breaking. This modification is particularly important for determining the $\eta$–$\eta'$ mixing. Further, following [7] we change a parameterization of the vertex functions in the spirit of Veneziano model. At last, we consider the trajectories non-degenerate and we take into account the effect of interference between their contributions.

The paper is organized as follows. In the next section we formulate modifications in the model. The data fit is carried out in sect. 3. A generalization of the model to a simultaneous consideration of the coherent and non-coherent contributions is discussed in sect. 4. In sect. 5 we discuss the results and make conclusions. In appendix A we transform data [13] from the form of numbers of pairs of gamma-quanta to the form of differential cross-sections.

2 The model

As noted in the introduction, the high-energy charge-exchange reactions occur via the inelastic scattering of fast quarks. The appropriate subprocesses are hard and can be described in the QCD perturbation theory. The calculations in the cases of $\pi^-p \rightarrow (\eta, \eta', \pi^0)\, n$ and $K^-p \rightarrow (\eta, \eta', \pi^0)\Lambda$ [1] show that at low transfers the relevant contributions to the amplitude are equal in absolute value and independent of the flavors and energies of the colliding quarks. On the background of hard subprocesses the contributions of soft processes are further formed. We assume that they are independent of the flavors in the hard subprocesses. In this case the contributions of hard
subprocesses actually are factorized, and their non-trivial result is a certain mode of summation of soft contributions. For a quantitative description of the soft contributions we apply the Regge phenomenology. In doing so, we consider contributions associated with the valence quarks in the final state as elementary ones. They are formed in different ways and they are summed differently in the different modes. As a result, in the mode of coherent summation with a certain choice of the signs the formulas of the conventional Regge approach are reproduced. In the non-coherent mode the elementary contributions are summed in the cross-section. In general, this leads to non-trivial consequences.

Basically, the above discussion defines the model up to the definition of Regge amplitudes. Turning to the latter issue, we recall that binary processes in the leading approximation at large \( s \) and small \( t \) are described as a sum of contributions of the leading trajectories [6, 7],

\[
A_{ab}(s, t) = \sum_i \beta_{aib}^\pm(t) \frac{1 \pm e^{-i\pi \alpha_i^\pm(t)}}{\sin(\pi \alpha_i^\pm(t))} \left(\frac{s}{s_0}\right)^{\alpha_i^\pm(t)}.
\]  

(1)

Here \( a \) and \( b \) mean initial and final states, \( \alpha_i^\pm(t) \) are the trajectories of particular parity, \( \beta_{aib}^\pm(t) \) are the vertex functions, \( s_0 \) is a scale parameter. The numerator in (1) represents the signature factor. The zeros in the denominator give Regge poles in the region of bound states \( (t > 0) \). In the scattering region \( (t < 0) \) the poles must be compensated by zeros in \( \beta_{aib}^\pm(t) \). The latter property is explicitly realized in the parameterization in the spirit of the Veneziano model, which includes the gamma function instead of the sine in the denominator [7]:

\[
A_{ab}(s, t) = \sum_i \beta_{aib}^\pm(t) \Gamma(1 - \alpha_i(t)) \left[1 \pm e^{-i\pi \alpha_i^\pm(t)}\right] \left(\frac{s}{s_0}\right)^{\alpha_i^\pm(t)}.
\]  

(2)

Here vertex functions \( \beta_{aib}^\pm(t) \) differ from those in (1) and have smoother behavior. Phenomenologically they are well described by an exponential function, possibly with a modification by polynomial factors describing spin-flip contributions.

In fact, the presence of exponential factors in the vertex functions is set by formula (2) itself. Really, the r.h.s in (2) does not change with simultaneous substitutions \( s_0 \rightarrow \tilde{s}_0, \beta_{aib}(t) \rightarrow \tilde{\beta}_{aib}(t) \), where

\[
\tilde{\beta}_{aib}(t) = \beta_{aib}(t) \left(\frac{s_0}{\tilde{s}_0}\right)^{\alpha_i(t)}.
\]  

(3)

Hereinafter we omit the sign of the signature \( (\pm) \) assuming it is included in the index \( i \). In linear approximation for the trajectory, \( \alpha_i(t) = \alpha_i(0) + \alpha'_i(0) t \), we have

\[
\tilde{\beta}_{aib}(t) = \left(\frac{s_0}{\tilde{s}_0}\right)^{\alpha_i(0)} \beta_{aib}(t) \times e^{\alpha'_i(0) \ln(\tilde{s}_0/s_0) t}.
\]  

(4)

So, if at some \( s_0 \) the vertex functions do not include an exponential factor, then this factor does appear at the transition to another scale parameter. For example, in the Veneziano model the vertex factor initially appears as a constant at the scale parameter \( s_0 = 1/\alpha' \), where \( \alpha' \) is a slope of the trajectory. However, without changing
the amplitude, we can choose another scale parameter with simultaneous change of the vertex function in accordance with (4). In particular, at the transition to the scale $\tilde{s}_0$ the vertex factor in the Veneziano model is converted to an exponential function with the slope $\alpha' \ln(\tilde{s}_0 \alpha')$.

Below we assume that the vertex functions have purely exponential behavior (up to the spin-flip factors) and we consider their normalizations and slopes as free parameters. Simultaneously we put $s_0 = s$ in formula (2). This condition unifies the normalizations and collects exponential $t$-dependence completely in the vertex functions. At the same time, at the transition to different energies the results can be recalculated by means of (4).

Further, we assume that the slopes in the vertex functions are determined by the trajectories, not by real particles in the final state. So at small $t$, we put

$$\beta_{aib}(t) = \beta_{aib}(0) \exp(c_i t). \tag{5}$$

The dependence on real particles is included in the normalization $\beta_{aib}(0)$, which is manifested in the overall strength of the coupling, in the group factors and the mixing parameters. Violation of the flavor symmetry is described by additional exponent-like factors,

$$\xi_i = \xi_{0i} \exp(\xi_i' t). \tag{6}$$

Here $\xi_{0i}$ and $\xi_i'$ are determined by the trajectory and the valence quarks in the final state. (Indices for the valence quarks are omitted to avoid bulkiness.) Actually (6) defines the splitting of the contributions of trajectories due to violation of the flavor symmetry.$^1$

Let us consider for definiteness the charge-exchange processes in the $\pi^-$ beams with yields of $\eta$ and $\eta'$. In the leading approximation they are determined by the $a_2$-trajectory $^6$ and the corresponding formulas are independent of the mode of summation of intermediate contributions $^1$. Taking into account above comments, we proceed directly to the differential cross-sections,

$$\frac{d\sigma}{dt}(\pi^- p \to \eta n) = g^2_{a_2}(t) (1 - rt) \cos^2 \frac{\pi \alpha_{a_2}}{2} \left( \cos \theta - \xi_{a_2} \sqrt{2} \sin \theta \right)^2, \tag{7}$$

$$\frac{d\sigma}{dt}(\pi^- p \to \eta' n) = g^2_{a_2}(t) (1 - r't) \cos^2 \frac{\pi \alpha_{a_2}}{2} \left( \sin \theta + \xi_{a_2} \sqrt{2} \cos \theta \right)^2. \tag{8}$$

Here in both formulas the first factor is a flavor-independent contribution to the vertex function. In the case of arbitrary trajectory "$i$", in accordance with (2) and (5), it is defined as

$$g_i(t) = g_{0i} \Gamma (1 - \alpha_i(t)) \exp(c_i t), \tag{9}$$

$^1$Here we proceed from the provision, that the splitting accumulates during formation of the trajectories and is weakly related to the formation of real particles in the final state. In support we note that the characteristic time of the former process is much greater than that of the latter one. Really, the formation time of a relativistic particle is of order $E/\mu^2$, where $E$ and $\mu$ are its energy and mass. In the case of trajectories the characteristic time is determined by the formation time of fast partons (at the recombination of the fluctuations). Since the masses of the partons are much smaller than the hadron masses and their momenta are comparable, the above ratio occurs.
where \( g_{0i} \) and \( c_i \) are phenomenological parameters. (Remember, in (7) and (8) \( i = a_2 \)). The second factor in (7), (8) is the spin-flip contribution. The third is the signature factor. The last factors stand for flavor-dependent contributions to the vertex functions. They are determined by the angle of \( \eta - \eta' \) mixing, by the group factors, and by the nonet-symmetry-breaking factor \( \xi_{a_2} \) introduced in (6). Here, we consider the mixing in the simplest scheme [15],

\[
|\eta\rangle = \cos \theta |\eta^8\rangle - \sin \theta |\eta^0\rangle, \\
|\eta'\rangle = \sin \theta |\eta^8\rangle + \cos \theta |\eta^0\rangle.
\]  

(10)

It is worth mentioning that in the case of exact nonet symmetry (\( \xi_{a_2} = 1 \)) the sine and cosine in (7), (8) define the non-strange component in the wave functions of \( \eta, \eta' \).

By this means formulas (7), (8) describe the reactions with the aid of seven parameters. Six of them, \( g_{0a_2}, c_{a_2}, r', r, \xi_{0a_2}, \xi'_{a_2} \), are specific. Parameter \( \theta \) is universal.

An important characteristic in the \( \pi^- \) beams is the ratio of the differential cross-sections. After simple transformations, we get

\[
R_{\eta'\eta}(\tau) \equiv \frac{d\sigma/dt(\pi^-p \rightarrow \eta'n)}{d\sigma/dt(\pi^-p \rightarrow \eta n)} = \frac{1-r't}{1-rt} \tan^2(\theta + \theta_{id} - \delta).
\]  

(11)

Here \( \theta_{id} = \arctan \sqrt{2} \) (\( \theta_{id} \approx 54.7^0 \)) and

\[
\delta = \arctan \frac{\sqrt{2}(1 - \xi_{a_2})}{1 + 2\xi_{a_2}}.
\]  

(12)

It is seen from (11) that at zero transfer \( R_{\eta'\eta}(0) \) is determined by the difference \( \theta - \delta_0 \), where \( \delta_0 = \delta(0) \). Traditionally \( R_{\eta'\eta}(0) \) is used for determining the mixing angle \( \theta \). However, we see that on this basis only the difference \( \theta - \delta_0 \) can be determined, not \( \theta \) alone.

In the case of charge-exchange reactions in the \( K^- \) beam with yields of \( \eta \) and \( \eta' \), there are two leading trajectories, \( K^* \) and \( K^*_2 \) [6]. They have different signatures and different symmetry properties, which complicates the description. Moreover, the trajectories can be non-degenerate. In this case an interference term should appear.

In the coherent mode the model leads to the following formulas for differential cross-sections (the same formulas arise in the Regge approach):

\[
\frac{d\sigma}{dt}(K^-p \rightarrow \eta\Lambda) = 3 g_{\varphi}^2(t) \sin^2 \frac{\pi \alpha_{\varphi}}{2} \cos^2 \theta + \frac{1}{3} g_{\tau}^2(t) \cos^2 \frac{\pi \alpha_{\tau}}{2} \left( \cos \theta + 2\sqrt{2} \xi \sin \theta \right)^2
\]

\[
- 2 g_{\varphi}(t) g_{\tau}(t) \cos \frac{\pi \alpha_{\tau}}{2} \sin \frac{\pi \alpha_{\varphi}}{2} \sin \frac{\pi (\alpha_{\varphi} - \alpha_{\tau})}{2} \cos \theta \left( \cos \theta + 2\sqrt{2} \xi \sin \theta \right),
\]  

(13)
\[
\frac{d\sigma}{dt}(K^- p \to \eta' \Lambda) =
\]
\[
3 g_v^2(t) \sin^2 \frac{2\pi\alpha_v}{2} \sin^2 \theta + \frac{1}{3} g_r^2(t) \cos^2 \frac{2\pi\alpha_r}{2} \left( \sin \theta - 2\sqrt{2} \xi \cos \theta \right)^2
\]
\[
- 2 g_v(t) g_r(t) \cos \frac{\pi\alpha_r}{2} \sin \frac{\pi\alpha_r}{2} \sin \frac{\pi(\alpha_v - \alpha_r)}{2} \sin \theta \left( \sin \theta - 2\sqrt{2} \xi \cos \theta \right).
\]

(14)

Hereinafter we introduce indices \( V \) and \( T \) instead of \( K^* \) and \( K_2^* \), respectively, and we omit \( T \) in \( \xi' \). The spin-flip factors are not included since the data do not need this [9, 10, 11, 12]. So, above formulas include seven parameters: \( \theta, g_v, c_v, g_r, c_r, \xi_0, \xi' \), where \( \xi_0 \) and \( \xi' \) are involved in \( \xi \), cf. (6). Notice that the vertex functions with purely singlet final states, in conformity with antisymmetric properties, are zero in the case of vector trajectories. Accordingly, \( \xi \) does not appear in the vector channel.

It is helpful noting that expressions in the large round brackets in (13), (14) are reduced to cosine and sine of \( \theta + \tilde{\theta}_{id} - \tilde{\delta} \), where \( \tilde{\theta}_{id} = -\arctan(2\sqrt{2}) \) and

\[
\tilde{\delta} = -\arctan \frac{2\sqrt{2}(1 - \xi)}{1 + 8\xi}.
\]

(15)

With the aid of this property it is easy to understand the reason of the appearance of a dip in \( K^- p \to \eta \Lambda \). The point is that \( (\cos \theta + 2\sqrt{2} \xi \sin \theta) \) in the tensor contributions in (13) is proportional to \( \cos(\theta + \tilde{\theta}_{id} - \tilde{\delta}) \), which in view of \( \tilde{\theta}_{id} \approx -70.5^0 \) is approximately zero at \( \theta \approx -20^0, \tilde{\delta} \approx 0 \). Consequently the contributions of the tensor trajectory in (13) are strongly suppressed in the region where \( \alpha_{K^*}(t) \approx 0 \), i.e. at \( t \approx -0.4 \) (GeV/c)² [14].

In the non-coherent mode the formulas for the differential cross-sections are significantly different. Recall that in this case the elementary contributions, associated with the valence quark, are summed in the cross-section. In doing so, the strange and non-strange quark-antiquark valence pairs appear in the mesonic final states in the equal parts (as in the coherent mode, as well) [1]. This gives

\[
\frac{d\sigma}{dt}(K^- p \to \eta \Lambda) = \frac{5}{3} g_v^2(t) \sin^2 \frac{2\pi\alpha_v}{2} \sin^2 \theta
\]
\[
+ \frac{1}{3} g_r^2(t) \cos^2 \frac{2\pi\alpha_r}{2} \left[ (\cos \theta - \sqrt{2} \xi \sin \theta)^2 + (\cos \theta + \sqrt{2} \xi \sin \theta)^2 \right]
\]
\[
+ \frac{2}{3} g_v(t) g_r(t) \cos \frac{\pi\alpha_r}{2} \sin \frac{\pi\alpha_r}{2} \sin \frac{\pi(\alpha_v - \alpha_r)}{2} \cos \theta \left( 5 \cos \theta + \sqrt{2} \xi \sin \theta \right).
\]

(16)

\[
\frac{d\sigma}{dt}(K^- p \to \eta' \Lambda) = \frac{5}{3} g_v^2(t) \sin^2 \frac{2\pi\alpha_v}{2} \sin^2 \theta
\]
\[
+ \frac{1}{3} g_r^2(t) \cos^2 \frac{2\pi\alpha_r}{2} \left[ (\sin \theta + \sqrt{2} \xi \cos \theta)^2 + (\sin \theta - \sqrt{2} \xi \cos \theta)^2 \right]
\]
\[
+ \frac{2}{3} g_v(t) g_r(t) \cos \frac{\pi\alpha_r}{2} \sin \frac{\pi\alpha_r}{2} \sin \frac{\pi(\alpha_v - \alpha_r)}{2} \sin \theta \left( 5 \sin \theta - \sqrt{2} \xi \cos \theta \right).
\]

(17)
These formulas include the same parameters as (13), (14), but the parameters values may be different.

Finally, we note that there is a third charge-exchange reaction in the $K^-$ beams, the $K^-p \rightarrow \pi^0\Lambda$, which is determined by the same trajectories. In this case the formula for the differential cross-section is independent of mode of summation of elementary contributions [1]. Owing to the absence of mesonic singlets in the final state, it does not contain the mixing parameter $\theta$ and the singlet-channel splitting $\xi$. However, a similar splitting can occur because of another isotopic spin in the final state. So we have

$$\frac{d\sigma}{dt}(K^-p \rightarrow \pi^0\Lambda) = \zeta^2 \left\{ g_V^2(t) \sin^2 \frac{\pi \alpha_V}{2} + g_T^2(t) \cos^2 \frac{\pi \alpha_T}{2} + 2 g_V(t) g_T(t) \sin \frac{\pi \alpha_V}{2} \cos \frac{\pi \alpha_T}{2} \sin \frac{\pi (\alpha_V - \alpha_T)}{2} \right\}. \quad (18)$$

Here

$$\zeta = \zeta_0 \exp(\zeta' t), \quad (19)$$

and are $\zeta_0$, $\zeta'$ are parameters. So, the process with $\eta, \eta', \pi^0$ are described in total by nine parameters.

3 The fit

To determine the mode of summation of intermediate contributions we proceed to the fit of data. Recall that in accordance with previous qualitative analysis [1] at relatively low and high energies the coherent and non-coherent mode is realized, respectively. Our task is to confirm or deny this result on the basis of the fit of data. Correspondingly, we do the fit of data at relatively low and high energies independently. In order to eliminate false solutions, we introduce restrictions on the parameters. Namely, we assume that a solution is physical if $\theta, \xi_0, \zeta_0$ belong to intervals $-35^\circ < \theta < -5^\circ$, $0.5 < \xi_0 < 1.5$, $0.5 < \zeta_0 < 1.5$. The first condition cuts off solutions clearly inconsistent with the results of other studies [15]. The second and third conditions mean that the violation of the flavor symmetry should not be too large. A similar condition for the slope parameters implies that $\xi'$ and $\zeta'$ in absolute value should not exceed $c_T$ and $c_V$, and within each trajectory the slopes should not split significantly. We demand also a positivity of the resulting slopes in the vertex functions, which means decreasing of contributions of the trajectories with increasing $-t$. Lastly, we consider linear trajectories and we define them based on the spectroscopy data.

3.1 $\pi^-$ beams

In the first place we consider data in the $\pi^-$ beams. Recall that appropriate cross-sections are given in (7), (8) and they are independent of the mode of summation of intermediate contributions.
Figure 1: Differential cross-sections $\pi^-p \rightarrow (\eta, \eta')n$ [mb/(GeV/c)] at 32.5 GeV/c [13]. Theoretical curves correspond to (7), (8) with parameter values specified in the text.

Based on the spectroscopy data [15] the $a_2$-trajectory in the linear approximation is

$$\alpha_{a_2}(t) = 0.45 + 0.89t,$$

where $t$ is given in (GeV/c)². The errors in (20) are of order of percents, which is insignificant for subsequent results. Notice that earlier determinations of $\alpha_{a_2}(t)$ proceeding directly from the scattering data, gave $\alpha_{a_2}(t) = 0.4 + 0.7t$ [16] and $\alpha_{a_2}(t) = 0.37 + 0.79t + 0.03t^2$ [17]. The discrepancy with our result (20) is explained by the differences in the formulas for the differential cross-sections used in the mentioned references.

The charge-exchange reactions in the $\pi^-$ beams with yields of $\eta$, $\eta'$ have been studied in detail at 8.45 GeV/c [8] and at higher momenta [13, 18, 16]. Unfortunately, data [8] and [18] are not available, and data [16] without [18] are not complete. For this reason we to consider only data [13] at 32.5 GeV/c. (Of course, this narrows our capabilities, but we will be able to work out important details and verify the accuracy of the model itself.) The fit with these data reveals a series of solutions with close $\chi^2$ distributed in the ($\theta$, $\xi_0$)-plane near a curve defined by condition $\theta - \delta_0 = -18.5^\circ$. When doing the fit with different fixed ($\theta$, $\xi_0$), the surface of minima of $\chi^2$, which appears over the ($\theta$, $\xi_0$)-plane, has a trough of almost constant depth located along the mentioned curve. The boundaries of the curve are determined by the boundaries of the physical region for the parameters: ($-5^\circ$, 0.62) and ($-28.6^\circ$, 1.5). In these points $\chi^2$/d.o.f. takes values 30.4/35 and 30.0/35, respectively, and along the curve $\chi^2$ remains within the mentioned limits. (The other parameters vary typically within 10–20%.) So, all the points of the curve determine practically equivalent solutions.

For an illustration we point out one of the solutions that appear when all the parameters are free: $\theta = -21^\circ \pm 8^\circ$, $g_{9a_2} = 1.7 \pm 0.4$, $c_{a_2} = 3.37 \pm 0.08$, $\xi_{0a_2} = 1.1 \pm 0.4$, $\xi'_{a_2} = 0.40 \pm 0.08$, $r = 19.7 \pm 0.3$, $r' = 18.4 \pm 1.7$, $\chi^2$/d.o.f. = 31.0/37. Hereinafter the normalization constant(s) are determined in $\mu b^{1/2}/(\text{GeV/c})$ and the slopes in (GeV/c)⁻² (see footnote²). In fig. 1 the corresponding differential cross-sections are

²We have recalculated data [13], obtained initially in the form of numbers of pairs of gamma-quanta arising from $\eta$ and $\eta'$, into the units of differential cross-sections, see appendix A. The error
presented together with the data used in the fit.

It is worth noting that in all the solutions the errors of \( \theta \) and \( \xi_{0a2} \) are significant. However, both these parameters are highly correlated and their difference is determined quite accurately. In particular, for the solution given above \( \theta - \delta_0 = (-18.5 \pm 0.6)^\circ \), and this result is kept for all the solutions along the curve. From here and (11) we restore the ratio of the cross sections \( R_{\pi}^{\eta/\eta}(0) = 0.53 \pm 0.02 \) with \( R_{\pi}^{\eta/\pi}(0) = 0.54 \pm 0.04 \) in [13]. In fact, \( R_{\pi}^{\eta/\eta}(0) \) is practically independent of energy. Really, [8] obtained \( R_{\pi}^{\eta/\eta}(0) = 0.500 \pm 0.092 \) at 8.45 GeV/c, and [18] obtained \( R_{\pi}^{\eta/\eta}(0) = 0.55 \pm 0.06 \) at 25 GeV/c and 40 GeV/c. So, regardless \( \theta \), the value of \( \xi_{0a2} \) remains constant (within errors) with changing the energy. This means that \( \xi_{0a2} \) is independent of (weakly dependent on) the mode of summation of intermediate contributions. However, \( \xi_{a2} \) depends significantly which follows from the energy dependence of the slope of \( R_{\pi}^{\eta/\eta}(t) \), see (11), (12) and discussion in [1].

Concluding, we note that high reliability of the solutions along the curve means that our model describes data well and it is quite suitable for the analysis of the charge-exchange reactions. In particular, our ansatz about the presence of the gamma function in the amplitude and the parameterization of the vertex functions is confirmed. If one removes the gamma function but preserves the signature factor, the quality of the description falls catastrophically: in the physical region the minimum of \( \chi^2/\text{d.o.f.} \) constitutes 126/35.

3.2 \( K^- \) beams

A priori, we do not know whether the leading \( K^* \) and \( K_2^- \) trajectories are non-degenerate. An independent fit of spectroscopy data [15] clearly indicates a preference of their non-degeneracy.\(^3\) Assuming linearity of the trajectories, we obtain

\[
\alpha_{K^*} = 0.33 + 0.84 t ,
\]

\[
\alpha_{K_2^-} = 0.11 + 0.93 t .
\]

The errors in the coefficients in (21), (22) do not exceed 2%, which is insignificant for our purposes.

Further, at relatively low energies we use formulas (13), (14) and (18). The differential cross-sections in this energy range were measured by several groups [9, 10, 11], but only data [11] at 4.2 GeV/c are suitable for the fit. Unfortunately, we can not use the totality of these data as they cover a large area of the transfer while our formulas with one-reggeon exchanges are valid at small \( t \) only. Since we do not a priori know how small \( t \) should be, we do a series of the fits gradually expanding the area of \( t \). We continue this procedure until the quality of the description worsens sharply or physical solutions disappear. In this manner we define the limiting values \(-t = 1.4, 1.0, 0.35 \) (GeV/c)\(^2\) with the numbers of experimental points 8, 10, 10 in the cases with

\(^3\)The hypothesis of the degeneracy leads to \( \chi^2/\text{d.o.f.} = 26.6 \) against \( \chi^2/\text{d.o.f.} = 1.7 \) in the case of non-degeneracy.
Table 1: Solutions of the fit of data [11] (Fit1, Fit2) and [13] (Fit3–Fit6). Fit1–Fit3 and Fit4–Fit6 match the coherent and non-coherent mode, respectively. Parameters $g_T$, $g_v$ are given in $\mu b^{1/2}/(\text{GeV/c})$, the slopes in $(\text{GeV/c})^{-2}$.

|       | Fit1  | Fit2  | Fit3  | Fit4  | Fit5  | Fit6  |
|-------|-------|-------|-------|-------|-------|-------|
| $\chi^2$/d.o.f. | 11.4/19 | 12.1/20 | 36.4/37 | 37.4/37 | 35.0/37 | 37.5/39 |
| $\theta$ [$^\circ$] | $-12 \pm 12$ | $-20.8$ [input] | $-25.9 \pm 2.32$ | $-21.8 \pm 5.0$ | $-22.2 \pm 4.3$ | $-20.8 \pm 4.9$ |
| $g_T$ | 8.1 $\pm$ 5.3 | 12.2 $\pm$ 4.4 | 1.7 $\pm$ 1.3 | 2.0 $\pm$ 0.4 | 2.1 $\pm$ 0.1 | 2.07 $\pm$ 0.09 |
| $c_T$ | 1.2 $\pm$ 0.8 | 1.5 $\pm$ 0.8 | 12.1 $\pm$ 6.4 | 4.1 $\pm$ 0.4 | 4.3 $\pm$ 0.3 | 4.1 $\pm$ 0.2 |
| $g_v$ | 12.2 $\pm$ 0.6 | 12.8 $\pm$ 0.6 | 2.2 $\pm$ 0.4 | 0.2 $\pm$ 1.4 | $-2.2 \pm 1.2$ | 0 [input] |
| $c_v$ | 1.7 $\pm$ 0.1 | 1.7 $\pm$ 0.1 | 3.0 $\pm$ 0.9 | 0.7 $\pm$ 13.7 | 110 $\pm$ 70 | — |
| $\xi_0$ | 1.13 $\pm$ 0.77 | 0.72 $\pm$ 0.31 | 1.02 $\pm$ 0.86 | 0.96 $\pm$ 0.18 | 0.91 $\pm$ 0.08 | 0.95 $\pm$ 0.08 |
| $\xi'$ | 1.0 $\pm$ 0.8 | 0.8 $\pm$ 0.9 | $-10.0 \pm 6.5$ | $-2.3 \pm 0.6$ | $-2.4 \pm 0.4$ | $-2.2 \pm 0.4$ |
| $\zeta_0$ | 1.19 $\pm$ 0.07 | 0.91 $\pm$ 0.24 | — | — | — | — |
| $\zeta'$ | $-0.6 \pm 0.5$ | $-0.5 \pm 0.4$ | — | — | — | — |

$\eta$, $\eta'$, $\pi^0$, respectively. In the mentioned areas there is only one solution. We present it in table 1 as Fit1. (The contributions to $\chi^2$ associated with $\eta$, $\eta'$, $\pi^0$ are 4.5, 3.8, 3.2, respectively.) In fig. 2 we show this solution by solid (green) thick curves. The solid thin curves continue the solution beyond the region of the fit. The discrepancy with the data in this region means that some unaccounted contributions dominate. Typically, they are reggeon-pomeron or multi-reggeon contributions characterized by a lower slope and hence prevailing at large $-t$ [6].

As we can see, the mentioned solution describes data well, but most parameters are determined with large errors. In particular, the mixing angle $\theta$ in essence is undetermined. So we can assign a certain value to $\theta$ and then with this value do the fit again. We take $\theta = -20.8^\circ$, the value obtained below. In this case again there is only one solution. We present it in table 1 as Fit2. (The contributions to $\chi^2$ associated with $\eta$, $\eta'$, $\pi^0$ are 4.4, 4.0, 3.7, respectively.) In fig. 2 we show it by dashed (red) curves. We see that Fit1 and Fit2 coincide within the errors, and the curves almost coincide in the region of the fit. This means that in terms of conformity to the data both solutions are equivalent.

We can clarify the reason why angle $\theta$ in Fit1 is poorly defined. The point is that formula (18) for the cross-section with $\pi^0$ does not contain $\theta$, while dependence on $\theta$ in formulas (13), (14) in the tensor channel appears in the combination $\theta - \tilde{\delta}$. So when considering the two latter reactions, we get a situation which is similar to that in the case in the $\pi^-$ beams, although with a correction for the presence of two trajectories. Namely, two series of solutions with close $\chi^2$ appear, which differ by a sign of the interference between contributions of the trajectories. All these solutions are distributed in the $(\theta, \xi_0)$-plane along the curve $\theta - \tilde{\delta}_0 = \text{constant}$, where $\tilde{\delta}_0 = \tilde{\delta}(0)$.
Figure 2: Differential cross-sections $K^-p \rightarrow (\eta, \eta', \pi^0)\Lambda$ [\(\mu b/(GeV/c)^2\)] at 4.2 GeV/c [11]. Theoretical curves in (a), (b), (c) correspond to (13), (14), (18), respectively. Solid (green) and dashed (red) curves represent solutions Fit1 and Fit2, respectively. In the regions of the fit the data and curves are shown by thick lines.

Figure 3: Differential cross-sections $K^-p \rightarrow (\eta, \eta', \pi^0)\Lambda$ [\(\mu b/(GeV/c)^2\)] at 32.5 GeV/c. The data in (a) and (b) are taken from [13]. Solid (blue) curves represent Fit6. Solution Fit2 in recalculation to the energy [13] is shown by (red) dashed curves. Solution Fit3 is shown by (orange) points. In the frame (c) the curves for Fit3 and Fit6 are shown at $\zeta = 1$ in (18).

Simultaneously, they all describe the dip in $K^-p \rightarrow \eta\Lambda$ and monotonic behavior of $K^-p \rightarrow \eta'\Lambda$. The inclusion of data with $\pi^0$ excludes one of the series with “wrong” interference, and fixes $\theta, \xi_0$ via the correlation with other parameters. However, fixing via the correlation implies poor definition, as it happens.

Now we turn to relatively high energies. The available data in this range are [13], obtained at the $K^-$ momentum 32.5 GeV/c. Unfortunately, these data cover reactions with $\eta$ and $\eta'$ only. However, the fit in this case leads to more precise outcomes.

First of all, we check whether there is a solution in the coherent mode. It turns out that such a solution exists, see Fit3 in table 1. Its essential features are the negative sign of $\xi'$ and the approximate equality in absolute value of $\xi'$ and $c_T$. The negative sign of $\xi'$ means exponential growth of $\xi(t)$ with increasing $-t$. So at $t = -0.4 (GeV/c)^2$ we have $\xi \approx 55$. Such a great value destroys the mechanism of appearance of a dip in $K^-p \rightarrow \eta\Lambda$. However, both features are unacceptable to a physical solution.
since they mean large splitting between the slope parameters in the vertex function. Really, the slope in the pure singlet component in this case is \( c_T + \xi' = 2.1 \pm 0.3 \) (strong correlation is taken into account). This is almost 6 times smaller than the slope \( c_v \) in the octet component. The difference is too great for the flavor symmetry violation. So Fit3 must be considered as unphysical solution. However that is not all. Besides the slopes in Fit3 are incompatible with Regge behavior. Really, \( \xi' \) should be unchanged with changing the energy while \( c_T \) and \( c_v \) should evolve in accordance with (4). However, \( \xi' \) in Fit3 changes the sign and in absolute value exceeds tenfold \( \xi' \) in Fit1. Moreover, proceeding from Fit2, \( c_T \) and \( c_v \) both must equal 3.3 at the energy of Fit3. However, \( c_T \) in Fit3 is significantly larger. This means violation of Regge behavior. The mentioned inconsistencies are clearly visible in fig. 3(a,b), where solution Fit2 recalculated to the energy of Fit3 is shown by dashed (red) curves, and solution Fit3 is shown by (orange) points. In fig. 3(c) the analogous curves have meaning of predictions for \( K^-p \to \pi^0\Lambda \) at 32.5 GeV/c.

On the basis of the above discussion, we conclude that the high-energy data are incompatible with predictions of the Regge approach. So in this energy range a modified approach is required. For this purpose we take advantage of our model in the non-coherent mode.

The corresponding fit of data [13] with formulas (16), (17) leads to two solutions, shown in table 1 as Fit4 and Fit5. They almost coincide, except \( g_v \) and \( c_v \). Namely, \( g_v \) in Fit4 is compatible with zero, whereas \( g_v \) in Fit5 is not. Besides, \( c_v \) in Fit5 is anomalously large. However, on close examination the mentioned differences are illusory. Really, due to the large \( c_v \) the vector contribution in Fit5 is nonzero at \( t \approx 0 \) only. So, both solutions are compatible with zero contribution of the vector trajectory. Assuming initially \( g_v = 0 \), we come to solution Fit6. In view of above discussion, Fit6 is practically equivalent to Fit4 and Fit5. In fig. 3 we show Fit6 by solid (blue) curves.

An essential feature of the solutions Fit4-Fit6 as compared to Fit1, are essentially smaller errors in \( \theta \) and \( \xi_0 \). This is a consequence of the fact that both these parameters are determined based on the data in the reactions with \( \eta \) and \( \eta' \) only, without involving the data with \( \pi^0 \) (see above discussion). As the final outcome for \( \theta \) we take its value in Fit6,

\[
\theta = -(20.8 \pm 4.9)^\circ. \tag{23}
\]

Our main conclusion in this subsection is that in the framework of conventional Regge approach the data in the \( K^- \) beams are well described at relatively low energies, but are no longer described at the transition to higher energies. At the same time, at the higher energies the data are well described in the mode with non-coherent summation of intermediate contributions. The data fit at low energies does not determine the \( \eta-\eta' \) mixing angle. The fit at higher energies leads to estimate (23).

### 3.3 Gluonium admixture

The above analysis was carried out in the particular scheme (10) of the \( \eta-\eta' \) mixing. However analysis can easily be generalized to any scheme. Here we consider a
generalization which includes a gluonium admixture in the \( \eta' \). In this scheme

\[
|\eta\rangle = \cos \theta |\eta^8\rangle - \sin \theta |\eta^0\rangle, \\
|\eta'\rangle = \cos \theta_G \left( \sin \theta |\eta^8\rangle + \cos \theta |\eta^0\rangle \right) + \sin \theta_G |\eta^G\rangle.
\]

(24)

where \( |\eta^G\rangle \) is a gluonium state and \( \theta_G \) is an additional mixing angle. This scheme was first proposed as a solution to the axial Ward identities for the relevant composite interpolating fields on the condition of the renormalization-group invariance of the pattern of the mixing [19]. Afterwards this scheme was repeatedly considered on purely phenomenological basis, see bibliography in [1]. In scheme (24) the above formulas do not change in the case of \( \eta \) and only slightly change in the case of \( \eta' \). Namely, a factor \( \cos^2 \theta_G \) appears in the r.h.s. in (8), (11), (14), (17). We complement the allowable range of parameters by condition \( 0.5 < \cos^2 \theta_G < 1 \), where the lower bound means that \( \eta' \) is predominantly a quark state, not a glueball.

The analysis of data [13] in the \( \pi^- \) beams, similar to that in sect. 3.1, leads to a series of solutions with close \( \chi^2 \) grouped in the \((\theta, \zeta_0, \theta_G)\)-space near a surface specified by condition

\[
R^\eta/\eta(0) = \cos^2 \theta_G \tan^2(\theta + \theta_{id} - \delta_0).
\]

(25)

Among these solutions \( \cos^2 \theta_G \) varies from \( 0.68 \pm 0.14 \) to \( 0.92 \pm 0.20 \), in all cases with \( \chi^2 / \text{d.o.f.} \approx 30/34 \). Thereby, the data in the \( \pi^- \) beams do not provide solid information about the gluonium admixture in \( \eta' \).

The fit of data [11] and [13] in the \( K^- \) beams with all the parameters free, do not reveal any physical solutions. Nevertheless, at various fixed \( \cos^2 \theta_G \) the solutions appear, and we can trace their evolution. Recall that at \( \cos^2 \theta_G = 1 \) all the solutions are presented in table 1. With decreasing \( \cos^2 \theta_G \) from 1, the \( \chi^2 \) is typically increasing in the appropriate branches of the solutions with the parameters are smoothly evolving. In so doing, the solutions in the branches Fit1 and Fit3 continue to be incompatible with each other, and the solutions in Fit3 are characterized by abnormal splitting of the slope parameters as was discussed in sect. 3.2.

Within \( 0.6 \leq \cos^2 \theta_G \leq 1 \) the solutions exist in all cases with free \( \theta \). In particular, at \( \cos^2 \theta_G = 0.6 \) the \( \chi^2 / \text{d.o.f.} \) in branches Fit1 and Fit6 takes values 11.7/19 and 38.0/39, respectively, and in both cases \( \theta \) remains within the errors of table 1. In addition, in the branch Fit2 with \( \theta = -20.8^\circ \) and free \( \cos^2 \theta_G \), there is only one solution: \( \cos^2 \theta_G = 0.68 \pm 0.67, g_\tau = 14.3 \pm 7.9, c_\tau = 1.6 \pm 0.8, g_\nu = 12.8 \pm 0.6, \ c_\nu = 1.7 \pm 0.1, \ z_0 = 0.74 \pm 0.30, \ x' = 0.7 \pm 1.0, \ z_0 = 0.82 \pm 0.33, \ x' = -0.5 \pm 0.5, \ \chi^2 / \text{d.o.f.} = 12.0 / 19 \). Notice that \( \cos^2 \theta_G \) is compatible with 1.

So the data on the charge-exchange reactions are rather indifferent to a possible gluonium admixture in the \( \eta' \), and on this basis we can not draw a conclusion about the presence or absence of this admixture. However the conclusion about the mode change of summation of intermediate contributions remains in force in the presence of the gluonium admixture. In the case of small gluonium admixture the estimate for the angle \( \theta \) undergoes insignificant changes.
4 Solution at intermediate energies

We have seen above that the charge-exchange reactions in the $K^-$ beams at relatively low and high energies are well described by the formulas obtained in the coherent and non-coherent modes, respectively. Now we consider the problem of the description in the intermediate energy region where contributions of both modes are possible.

We offer a solution in the spirit of the density matrix. Specifically, we assume that each mode can be implemented with a certain probability. Since there is no third, the sum of the two probabilities must be equal to one. So, we consider the differential cross-section in the form

$$\frac{d\sigma}{dt}(s, t) = w \frac{d\sigma_c}{dt}(s, t) + (1 - w) \frac{d\sigma_{nc}}{dt}(s, t).$$

(26)

Here $w$ is the probability to find the system in the coherent mode, indices “c” and “nc” mean the modes of coherent and non-coherent summation. In the most general case $w$ is a function of $s$ and $t$. Asymptotically at low and high $s$, we expect $w \to 1$ and $w \to 0$, respectively. Our task is to find an algorithm of experimental measurement of $w$.

Below, we do this under the assumption that at small transfer one can neglect in $w$ the dependence on $t$, so that $w = w(s)$. In fact, reliable measurements are needed at least at two energies. At $s = s_1$ we have

$$\frac{d\sigma}{dt}(s_1, t) = \left[ w_1 \frac{d\sigma_c}{dt}(s_1, t) \right] + \left[ (1 - w_1) \frac{d\sigma_{nc}}{dt}(s_1, t) \right].$$

(27)

Here $w_1 = w(s_1)$, $d\sigma_c/dt$ and $d\sigma_{nc}/dt$ are described by appropriate formulas in the coherent and non-coherent modes. The large square brackets mean that factors $w_1$ and $(1 - w_1)$ are not considered as independent parameters, but at the fit are considered absorbed by normalization constants in the cross-sections. At $s = s_2$ an analogous formula includes $w_2 = w(s_2)$ as a hidden parameter. However, we write this formula in a modified form. Namely, we factor out numbers $x$ and $y$, defined as

$$x = \frac{w_2}{w_1}, \quad y = \frac{1 - w_2}{1 - w_1}.$$

(28)

So at $s = s_2$, we have

$$\frac{d\sigma}{dt}(s_2, t) = x \left[ w_1 \frac{d\sigma_c}{dt}(s_2, t) \right] + y \left[ (1 - w_1) \frac{d\sigma_{nc}}{dt}(s_2, t) \right].$$

(29)

Further, we notice that all the parameters in the square brackets in (29) are actually determined by the fit at $s = s_1$ with (27). Really, some parameters are unchanged at changing the energy, and the remaining ones are changed by means of (4) with $s_0 = s_2, s_0 = s_1$. So in (29) only numerical factors $x$ and $y$ are unknown. They may be determined based on the fit at $s = s_2$. Then, solving system (28), we obtain the probabilities:

$$w_1 = \frac{y - 1}{y - x}, \quad w_2 = x \frac{y - 1}{y - x}.$$

(30)
Figure 4: Differential cross-sections $K^- p \rightarrow (\eta, \eta')\Lambda$ [$\mu b/(GeV/c)^2$] at 8.25 GeV/c [12]. Solid (blue) and dashed (red) curves represent solutions Fit6 and Fit2, respectively, recalculated to energy [12]. Dash-dot (violet) curves represent solution built by (26) at $w = 0.95$.

In this way, we have presented an algorithm to measure $w(s)$. Of course, in practice it is expedient to do a joint fit of data simultaneously at different energies. On this basis we can determine $w(s)$ in a wide range of energy. We expect that $w(s)$ is monotonically decreasing with increasing the energy from asymptotic value 1 at relatively low energies up to asymptotic value 0 at higher energies.

Unfortunately, the currently available data are not sufficient to carry out such an analysis because data [11] contain too few measured points and [13] cover too small region of $t$. Besides, in the case of the common use the data [13] from the very beginning must be recalculated to units of the differential cross-sections, which leads to an additional 12%-error, see appendix A. This further reduces the quality of the fit. As for the data [12] at 8.25 GeV/c, which fall into the intermediate region, their quality is not sufficient to do the fit with them because, in particular, they contain too few measured points.

However, all the mentioned data may be used for illustrative purposes. Namely, let us suppose that at the energies of [11] and [13] the probability $w$ takes values 1 and 0, respectively. Then, recalculating solutions Fit2 and Fit6 to the intermediate energy [12] and substituting the result into (26), we obtain a prediction for the differential cross-section at 8.25 GeV/c. In fig. 4 we show data [12] and we present by dashed (red) and solid (blue) curves solutions Fit2 and Fit6, respectively, recalculated to this energy. The dash-dot curve gives the result defined by (26) at $w = 0.95$, where 0.95 is chosen arbitrarily. We see that the latter curve lies between the former two and on the whole better matches the data. One should bear in mind, however, that the positions of the curves in fig. 4 are shown without taking into account the appropriate errors.

5 Discussion and conclusion

The analysis in this work confirms the earlier qualitative conclusion [1] about the mode change with increasing the energy of summation of intermediate contributions
in the charge-exchange reactions. In support of this conclusion, we put forward the following results obtained based on the fit of data. First, we found a solution which describes in the Regge approach the charge-exchange reactions $K^- p \rightarrow (\eta, \eta', \pi^0)\Lambda$ at relatively low energies. In our model this solution implies the mode of coherent summation of intermediate contributions. Then, we showed the absence of a similar solution at relatively high energies. Instead, a solution appears that corresponds to the mode of non-coherent summation. Finally, we compared the obtained solutions by recalculating to a common energy and established fundamental difference between them.

We emphasize that the above solutions are statistically well-founded. Moreover, the data on the charge-exchange reactions in the $\pi^-$ beams are fitted very well. This indicates that our model describes the charge-exchange reactions well, and results obtained on its basis are reliable.

At the same time, the range of applicability of the model is limited by small transfers. This is a consequence of the fact that the model takes into account the contributions of leading trajectories only, and ignores the contributions of the Regge cuts that manifest themselves at higher $-t$. In our approach the region where the contributions of the cuts may be ignored is determined empirically by gradually increasing the upper limit of $|t|$ until the data are well described. So, in principle, the range of applicability of the model can be expanded by including in analysis the reggeon-pomeron or multi-reggeon contributions. We do not consider such an extension since this implies a substantial increasing in the number of parameters in conditions of limited data.

In our analysis we proceeded from the assumption that the angle of the $\eta-\eta'$ mixing is unknown, and we considered it as a parameter which is to be determined by the fit. This approach is valuable in itself in view of the lack of well-established estimate of the mixing on the basis of the charge-exchange reactions \cite{20, 21}. Furthermore, unlike \cite{1} we considered the parameter of violation of the nonet symmetry as unknown, which must be determined by the fit, as well. Unfortunately, this leads to the impossibility of determining the mixing angle on the basis of data in the $\pi^-$ beams, since corresponding cross-sections are determined by the difference of the mixing angle and the parameter of the nonet symmetry breaking. For this reason, we determined the mixing by basing on the data in the $K^-$ beams. Unluckily, the appropriate estimate includes large error and therefore in practical terms is little informative. However, we show that the gluonium admixture in the $\eta'$ has little effect on the $\eta-\eta'$ mixing. Moreover, the gluonium admixture does not affect the conclusion about the mode change with increasing the energy.

Finally, we found a solution to the problem of description of the charge-exchange reactions at the intermediate energy range where both modes are possible, the coherent and non-coherent. The solution is based on an idea that each mode occurs with a certain probability, and the probability is dependent on the energy. By this means the

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\footnote{In the case of $\pi^-$ beams the mode change is independently confirmed by effect of the energy dependence of the slope of $R_{\eta'/\eta}(t)$, see discussion in sect. 3.1 and in \cite{1}.}

\footnote{We do not take into consideration the daughter trajectories as they have close slope but lower intercept. Therefore their contributions are suppressed.}
differential cross-section is determined as a sum of contributions with certain modes, weighted with the probabilities of finding the system in the particular mode. We propose an algorithm for measuring the probabilities. A study of the charge-exchange reactions with taking into account this solution might be a subject of future research provided that sufficiently detailed data become available.

In summary, with high reliability we confirmed the effect of the mode change with increasing the energy of summation of intermediate contributions in the differential cross-sections of charge-exchange reactions. The analysis in this paper more thoroughly specifies the model for further systematic investigation of this phenomenon. In particular, one can study the energy dependence of the coherent and non-coherent contributions to the cross-sections of the mentioned reactions.

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Appendix A

The data [13] are presented in the form of numbers of pairs of gamma-quanta arising due to the decays of \( \eta \) and \( \eta' \). Our task is to determine the proportionality factor between the numbers of the registered pairs of gamma-quanta and the corresponding differential cross-sections.

In the case of \( \pi^- \) beams we take advantage of formula (12) in [16],

\[
\frac{d\sigma}{dt}(\pi^- + p \rightarrow \eta \rightarrow \gamma \gamma n)\bigg|_{t=0} = (37.1 \pm 3.6) \left( \frac{s}{s_0} \right)^{-1.26 \pm 0.04} \times \mu b (\text{GeV/c})^{-2}. \tag{31}
\]

Here \( s_0 = 10 (\text{GeV/c})^2 \). At \( s = 62.1 (\text{GeV/c})^2 \), which means \( p_{\text{LAB}} = 32.5 \text{ GeV/c} \), the r.h.s. in (31) is equal to \((3.71 \pm 0.45) \times \mu b (\text{GeV/c})^{-2}\). On the other hand, at \( p_{\text{LAB}} = 32.5 \text{ GeV/c} \) [13] obtained

\[
\frac{dN}{dt}(\pi^- + p \rightarrow \eta \rightarrow \gamma \gamma n)\bigg|_{t=0} = (2.21 \pm 0.02) \times 10^6 \times (\text{GeV/c})^{-2}, \tag{32}
\]

where \( N \) is the number of pairs of gamma-quanta. The sought-for factor is a ratio of (31) to (32). Numerically it is \( r_{\pi^-} = (1.68 \pm 0.20) \times 10^{-6} \mu b \). In the case of yield of \( \eta' \) the appropriate factor numerically is the same, because the gamma-quanta in [13] were registered in the same experiment. Moreover, this factor determines also the ratio of \( d\sigma(\pi^- + p \rightarrow \eta'^\eta n)/dt \) to \( \Delta N(\pi^-, \eta'^\eta)/\Delta t \), where \( \Delta N(\pi^-, \eta'^\eta) \) is the number of \( \eta'^\eta \) formed in the \( \pi^- \) beam in the course of experiment [13] in the bin \( t \) of the width \( \Delta t \).

In the case of \( K^- \) beams, we take advantage of the fact that the beam of negative particles in [13] consisted of 98% \( \pi^- \) and 2% \( K^- \), and the admixture of other particles was negligible. On this basis, we get \( r_{K^-} = (0.98/0.02)r_{\pi^-} = (8.23 \pm 0.98) \times 10^{-5} \mu b \). Here \( r_{K^-} \) has the meaning of the ratio of \( d\sigma(K^- + p \rightarrow \eta'^\eta n)/dt \) to \( \Delta N(K^-, \eta'^\eta)/\Delta t \),
where $\Delta N(K^-, \eta^{[d]})$ is the number of $\eta^{[d]}$ formed in the $K^-$ beam in the corresponding bin $t$ of the width $\Delta t$.

Given the branchings of $\eta^{[d]} \rightarrow \gamma \gamma$, the obtained factors determine different conversion factors depending on the choice of the final state.

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