Research on unified analytical solution and corresponding time-effect deformation characteristics of rock mass viscoelastic deformation under rigid and flexible bearing plate tests

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Abstract. Describing the phenomena and laws of rock mass aging deformation is an important research area on bearing plate tests. Presently, no unified analytical solution exists for characterizing time-dependent displacements of flexible and rigid circular bearing plate tests. Because of this deficiency, based on the rigid and flexible bearing plate rock mass rheology tests and according to the viscoelastic theory, the unified time-dependent deformation analysis solution of the half-space viscoelastic body under a normal uniform load on the surface is derived. Based on this derivation, the viscoelastic analytical solution of rock mass aging deformation and the elastic aftereffect displacement equation for the Maxwell, Kelvin–Voigt, and Burgers models, respectively, were determined. The viscoelastic characteristics of rock mass aging deformation under bearing plate tests and the influence law of viscoelastic parameters on rock mass aging deformation are discussed. The results show that the viscoelastic analytical solution of the rock mass aging deformation at the center of the rigid circular bearing plate is π/4 that of the flexible circular bearing plate test. Also, the viscoelastic analytical solution of the time-dependent deformation of the rock mass at the edge of the flexible circular bearing plate is 2/π that at the center of the bearing plate. However, the evolution trend and law of the time-dependent deformation of the rock mass at the center of both circular bearing plate tests are consistent. Furthermore, the viscoelastic analytical solutions of the rock mass deformation at the center of the circular bearing plate obtained by the above three rheological models have elastic aftereffect characteristics. Precisely, although the Maxwell rheological model does not have elastic aftereffect characteristics, the viscoelastic analytical solution equation for rock mass deformation at the center of the circular bearing plate test obtained from this model has an exponential term related to time, which results in the gradual decay of the rock mass deformation with time after unloading, i.e., showing elastic aftereffect characteristics. When the Maxwell and the Burgers models are used, the rock mass under the bearing plate test condition has the same final residual deformation value \( \frac{q}{2\eta_0} \frac{\tau_1}{\tau_2} \) after unloading. However, when the Kelvin–Voigt model is used, no residual deformation of the rock mass exists after unloading under the bearing plate test conditions, which is consistent with the law of the elastic aftereffect phenomenon of the Kelvin–Voigt model.

Keywords: On-site rheological test, rigid and flexible bearing plate, viscoelastic, unified analytical solution, aging deformation characteristics.
1. Introduction

A field test is an important method to obtain rock mass deformation parameters. The field bearing plate method is the most widely used test method for measuring rock mass deformation parameters, and it is also a commonly used method for on-site rock mass creep tests [1, 2]. Due to the convenient installation and operation of the pressure-bearing plate test method, high accuracy of measurement parameters, and wide application range, it is widely used in various projects [3–7].

Commonly used test methods for bearing plates in field tests are divided based on rigidity, including the flexible and rigid bearing plate test methods. The main difference between these two bearing plate test methods lies in the difference in the pressure-loading equipment acting on the surface of the rock mass during the test. For the flexible bearing plate test method, pressure steel pillows are used as the bearing plate, which is flexibly in contact with the rock mass surface. Within the range of the bearing plate, the rock mass surface bears uniform pressure, but the vertical displacement produced is nonuniform. For the rigid bearing plate test method, it is assumed that the bearing plate has absolute rigidity. The equipment for applying the pressure load is the jack and the steel plate, which are in rigid contact with the rock mass surface and produce uniform vertical displacement within the range of the bearing plate. But the pressure on the rock mass surface is nonuniform. The on-site rock mass flexible and rigid circular bearing plate tests are the most widely used methods to obtain the time-dependent rock mass deformation characteristics and parameters. Describing the phenomena and laws of the rock mass aging deformation is an important research area on bearing plate tests. Presently, no unified analytical solution exists for characterizing time-dependent displacements of flexible and rigid circular bearing plate tests, which is not conducive for understanding and promoting the application of rheological test results of circular bearing plate tests. Because of this deficiency, based on the rigid and flexible bearing plate rock mass rheology tests and according to the viscoelastic theory, the unified time-dependent deformation analysis solution of the half-space viscoelastic body under a normal uniform load on the surface is derived. Based on this derivation, the viscoelastic analytical solution of rock mass aging deformation and the elastic aftereffect displacement equation for the Maxwell, Kelvin–Voigt, and Burgers models, respectively, were developed. Furthermore, the viscoelastic characteristics of rock mass aging deformation under the bearing plate tests and the influence law of viscoelastic parameters on rock mass aging deformation are discussed.

2. Unified solution of rheological model of circular bearing plate

Suppose a rock mass is a homogeneous and isotropic medium and that the impacts of other interference factors in the test are ignored. A circular bearing plate test is close to the situation of the half-space body experiencing the action of a circular normal distribution force on the boundary. The vertical displacement (subsidence) of the load action center point caused by a normal uniform force on the semi-infinite elastic body boundary (rock mass surface) can be obtained through Boussinesq solutions that are based on elastic mechanics [8].

For a flexible circular bearing plate, according to the theory of elasticity, the vertical displacement (subsidence) of the load action center point caused by a normal uniform force is as follows

\[ w = \frac{2(1-\mu^2)qR_0}{E}. \]  

(1-1)

Here, \( q \) is the normal uniform distribution load, and \( R_0 \) is the radius of the circular bearing plate. Likewise, the vertical elastic displacement load of the edge-measuring point of a flexible circular bearing plate is given as
\[ w = \frac{4(1-\mu^2)qR_0}{\pi E}. \quad (1-2) \]

For a rigid circular bearing plate, according to the theory of elasticity, the vertical displacement (subsidence) of the load action center point caused by a normal uniform force is given as

\[ w = \frac{(1-\mu^2)\pi qR_0}{2E}. \quad (2-1) \]

Because the contact between the rigid circular bearing plate and the bedrock is rigid, a uniform vertical displacement is produced on the rock mass surface. Thus, the vertical elastic displacement solution of the edge-measuring point of a rigid circular bearing plate is given as

\[ w = \frac{(1-\mu^2)\pi qR_0}{2E}. \quad (2-2) \]

The unified equation of the load action rock mass vertical displacement (subsidence) caused by a normal uniform force of a circular bearing plate can be obtained by comparing equations (1) and (2), and is given as:

\[ w = \omega \frac{2(1-\mu^2)qR_0}{E}. \quad (3) \]

In Equation (3), \( \omega \) is the coefficient which is related to the bearing plate test method, where, when a flexible circular bearing plate is used in the test and the test displacement measuring point is in the center point of the bearing plate, \( \omega = 1 \); when a flexible circular bearing plate is used in the test and the test displacement measuring point is at the edge of the bearing plate, \( \omega = \frac{2}{\pi} \); and when a rigid circular bearing plate is used in the test, \( \omega = \frac{\pi}{4} \).

In three-dimensional (3D) space, the relationships between the elastic modulus \( E \), Poisson’s ratio \( \mu \), shear elastic modulus \( G_0 \), and bulk elastic modulus \( K_0 \) are given as

\[
\begin{align*}
E &= \frac{9G_0K_0}{3K_0+G_0} \\
\mu &= \frac{2K_0-2G_0}{2(3K_0+G_0)}.
\end{align*}
\]

(4)

The unified equation of the load action vertical displacement (subsidence) caused by a normal uniform force on the boundary of a semi-infinite elastic body (rock mass surface) can be obtained by substituting Equation (4) into Equation (3) to give Equation 5.

\[ w = \omega \frac{(3K_0 + 4G_0)qR_0}{2G_0(3K_0 + G_0)}. \quad (5) \]

To consider the versatility of the viscoelastic solution and obtain the same solution as the instantaneous elastic solution when \( t = 0 \), the expression of J.C. Jaeger and N.G.W. Cook was used for a 3D viscoelastic constitutive relation [9].

\[
\begin{align*}
S_{ij} &= 2 \frac{Q'}{P'} \epsilon_{ij} \\
\sigma_m &= 3 \frac{Q'}{P'} \epsilon_m
\end{align*}
\]

(6)

In Equation (6), \( S_{ij} \), \( \sigma_m \), \( \epsilon_{ij} \), and \( \epsilon_m \) are the component and volume stress of the deviator stress tensor and the component and volume strain of the deviator strain tensor, respectively. \( Q' \) and \( P' \) are differential operators, namely,
\[ P' = \sum_{k=0}^{m} p^k \frac{d^k}{d\tau^k} \]
\[ Q' = \sum_{k=0}^{m} q^k \frac{d^k}{d\tau^k} \]  \hspace{1cm} (7)

Similarly, all elastic and viscoelastic moduli and coefficients in \( Q \) and \( P \) must be replaced with the shear elastic modulus, viscoelastic modulus, and viscoelastic coefficient, whereas \( Q'' \) and \( P'' \) are the operators that reflect the material viscoelastic volume deformation. If the material volume deformation is elastic, \( Q'' = K \) and \( P'' = 1 \).

The Laplace transformation of Equation (5) was obtained, and the 3D elastic constitutive relationship \( S_{ij} = 2G_0 \epsilon_{ij} \), \( \sigma_{ii} = 3K \epsilon_{ii} \), and Equation (6) were focused on to obtain a viscoelastic unified analytical solution of rock mass-aging deformation for a bearing plate test condition. In the Laplace space, the derivation can be expressed in the following general formula
\[ \hat{w}(s)_{z=0} = \omega \frac{q R_0}{2} \left[ \frac{p(s)}{sQ'(s)} + \frac{3p(s)p'(s)}{s(3p(s)Q'(s) + p'(s)Q'(s))} \right]. \]  \hspace{1cm} (8)

3. Viscoelastic analytical solutions under circular bearing plate

According to rock rheological theory, commonly used viscoelastic combination models include the Maxwell, Kelvin–Voigt, and Burgers models. Assuming that the volume deformation of the rock mass shows elastic characteristics, the viscoelastic analytical expressions of the rock mass-aging deformation under a normal uniform load in these several different viscoelastic combination models can be derived.

3.1. Maxwell model

The Maxwell model is composed of independent elastic and viscous components (Fig. 1). For the Maxwell model of the 3D stress state, the components of the deviator strain tensor \( e_{ij} \) and deviator stress tensor \( s_{ij} \) are used; then, the volume stress \( \sigma_m \) and strain \( \epsilon_m \) are combined, and the rheological model equation is given as
\[ \eta_0 \dot{\epsilon}_{ij} = \eta_0 \ddot{s}_{ij} + s_{ij} \]
\[ 3K_0 \epsilon_m = \sigma_m \]  \hspace{1cm} (9)

Equations (6) and (7) are combined; therefore, the differential operator of the Maxwell rheological model in the 3D stress state is as follows
\[ P'(s) = 1 + p_1 s \] \hspace{1cm} and \hspace{1cm} \[ P''(s) = 1 \]
\[ Q'(s) = q_0 s \] \hspace{1cm} and \hspace{1cm} \[ Q''(s) = K_0 \]  \hspace{1cm} (10)

where, \( p_1 = \frac{\eta_0}{G_0} \) and \( q_0 = \eta_0 \). These equations are substituted into Equation (8) for simplification, and then a viscoelastic analytical solution of rock mass-aging deformation for the condition of a bearing plate test in the Laplace space during loading is given as
\[ \hat{w}(s) = \omega \frac{q R_0}{2} \left[ \frac{1 + p_1 s}{q_0 s^2} + \frac{3(1 + p_1 s)}{s(A + B s)} \right], \]  \hspace{1cm} (11)

where, \( A = 3K_0 \) and \( B = 3K_0 p_1 + q_0 \). The Laplace inverse transformation is performed on Equation (11). After simplification, the viscoelastic analytical expression of rock mass-aging deformation using the Maxwell rheological model is given as
$$w(t) = \omega \frac{qR_0}{2} \left[ \frac{t}{q_0} + \frac{3q_0 + Ap_1}{Aq_0} - \frac{3(B - Ap_1)}{AB} e^{-\frac{A}{B}t} \right]. \quad (12)$$

When $t = 0$,

$$w(0) = \omega \frac{qR_0}{2} \frac{3K_0 + 4G_0}{G_0(3K_0 + G_0)}. \quad (13)$$

Suppose that when the rock mass flows to time $t = t_1$ under the action of the bearing plate constant load $q$ that it is then suddenly unloaded to zero (which is achieved by applying a counteracting constant load $-q$); then, the elastic aftereffect expression $w'(t)$ of the rock mass-aging displacement in the Maxwell rheological model can be derived.

At $t_1$ and afterward, the contrary load $-q$ is applied, and then, Equation (14) can be obtained according to Equation (12) as follows

$$w(t) = \omega \frac{qR_0}{2} \left[ \frac{t - t_1}{q_0} + \frac{3q_0 + Ap_1}{Aq_0} - \frac{3(B - Ap_1)}{AB} e^{-\frac{A}{B}(t - t_1)} \right]. \quad (14)$$

When $t \geq t_1$, Equation (15) can be obtained by adding Equations (12) and (14) as follows

$$w'(t) = \omega \frac{qR_0}{2} \left[ \frac{t_1}{q_0} + \frac{3(B - Ap_1)}{AB} (e^\frac{A}{B(t_1)} - 1)e^{-\frac{A}{B}t} \right], \quad (t \geq t_1). \quad (15)$$

When $t = \infty$,

$$w'(\infty) = \omega \frac{qR_0}{2} \frac{t_1}{q_0} = \omega \frac{qR_0}{2} \frac{t_1}{\eta_0} \quad (16)$$

**Figure 1.** Schematic of 1D component of Maxwell model.

### 3.2. Kelvin–Voigt model

The Kelvin–Voigt model is composed of an independent elastic component and a Kelvin body (Fig. 2). For the Kelvin–Voigt model in the 3D stress state, the components of the deviator strain tensor $e_{ij}$ and deviator stress tensor $s_{ij}$ are used; then, volume stress $\sigma_m$ and strain $\varepsilon_m$ are combined, and the rheological model equation is given as

$$\frac{\eta_1}{G_1} \dot{e}_{ij} + e_{ij} = \frac{\eta_1}{G_0 G_1} \dot{s}_{ij} + \frac{G_0 + G_1}{G_0 G_1} s_{ij}, \quad (17)$$

Equations (6) and (7) are combined, and then, the differential operator of the Kelvin–Voigt model in the 3D stress state is given as

$$P'(s) = p_0 + p_1 s \quad \text{and} \quad P''(s) = 1 \quad (18)$$

where $p_0 = \frac{G_0 + G_1}{G_0 G_1}$, $p_1 = \frac{\eta_1}{G_0 G_1}$, and $q_1 = \frac{\eta_1}{G_1}$. For simplification, Equation (18) is substituted into Equation (8), and then a viscoelastic analytical solution of rock mass-aging deformation for the condition of a bearing plate test in the Laplace space during loading is obtained as follows:
where, \( A = 3K_0 p_0 + 1 \) and \( B = 3K_0 p_1 + q_1 \). The Laplace inverse transformation of Equation (19) is obtained; after simplification, the viscoelastic analytical expression of rock mass-aging deformation in the Kelvin–Voigt model is obtained as

\[
\hat{w}(s) = \omega q R_0 \frac{p_0 + p_1 s + 3(p_0 + p_2 s)}{s(1 + q_1 s)} + \frac{3(p_0 + p_2 s)}{s(A + Bs + Cs^2)},
\]  

(19)

When \( t = 0 \), the instantaneous displacement \( w(0) \) is the same as in Equation (1). The elastic aftereffect expression \( w'(t) \) of the rock mass-aging displacement in the Kelvin–Voigt model can be obtained through the same derivation process as that of the elastic aftereffect expression of the bearing plate rock mass-aging deformation in the Maxwell rheological model. When \( t \geq t_1 \),

\[
w'(t) = \omega q R_0 \frac{p_1 - p_0 q_1}{q_1} \left( 1 - e^{-\frac{t}{q_1}} \right) e^{-\frac{t}{q_1}} + \frac{3(Ap_1 - Bp_0)}{AB} e^{-\frac{A}{B}t}, \quad (t \geq t_1).
\]  

(20)

When \( t = \infty \),

\[
w'(\infty) = 0
\]  

(22)

3.3. Burgers model

The Burgers model comprises an independent elastic component, an independent viscous component, and a Kelvin body (Fig. 3). For the Burgers model in the 3D stress state, the components of the deviator strain tensor \( e_{ij} \) and deviator stress tensor \( s_{ij} \) are used; then, the volume stress \( \sigma_m \) and strain \( \varepsilon_m \) are combined, and the rheological model equation is given as

\[
\ddot{e}_{ij} + \frac{G_i}{\eta_1} \dot{e}_{ij} = \frac{1}{G_0} \ddot{s}_{ij} + \left( \frac{\eta_0 G_i + G_0 \eta_1 + G_0 \eta_2}{G_0 \eta_0 \eta_1} \right) \dot{c}_{ij} + \frac{G_i}{\eta_0 \eta_1} s_{ij},
\]  

(23)

Equations (6) and (7) are combined; therefore, the differential operator of the Burgers rheological model in the 3D stress state is given as

\[
P'(s) = p_0 + p_1 s + p_2 s^2 \quad \text{and} \quad P''(s) = 1, \\
Q'(s) = q_1 s + s^2 \quad \text{and} \quad Q''(s) = K_0.
\]  

(24)

where, \( p_0 = \frac{G_i}{\eta_0 \eta_1} \), \( p_1 = \frac{\eta_0 G_i + G_0 \eta_1 + G_0 \eta_2}{G_0 \eta_0 \eta_1} \), \( p_2 = \frac{1}{G_0} \), and \( q_1 = \frac{G_i}{\eta_1} \). For simplification, Equation (24) is substituted into Equation (8), and then the viscoelastic analytical solution of rock mass-aging deformation for the condition of a bearing plate test in the Laplace space during loading is obtained as

\[
\tilde{w}(s) = \omega q R_0 \frac{p_0 + p_1 s + 3(p_0 + p_2 s)}{s(1 + q_1 s)} + \frac{3(p_0 + p_2 s)}{s(A + Bs + Cs^2)},
\]  

(25)
where \( A = 3K_0p_0 \), \( B = 3K_0p_1 + q_1 \), and \( C = 3K_0p_2 + 1 \). The Laplace inverse transformation of Equation (25) is obtained; after simplification, the viscoelastic analytical expression of rock mass-aging deformation in the Burgers rheological model is given as

\[
\begin{align*}
\omega(t) &= \frac{qR_0}{2} \frac{p_0}{q_1} t + \frac{3p_0q_1^2 + A_1q_1 - Ap_0}{Aq_1^2} + \frac{p_2q_1^2 - p_1q_1 + p_0}{q_1^2} e^{-q_1t} + \\
&\quad \frac{3(Ap_2 - Cp_0)}{AC \sqrt{\frac{b^2}{4} - AC}} \left\{ \sqrt{\frac{b^2}{4} - AC} \cos h \left( \frac{b^2}{4} - AC \right) e^{-\frac{b^2}{4}t} + \frac{t}{\sqrt{\frac{b^2}{4} - AC}} - e^{-\frac{b^2}{4}t} \right\}.
\end{align*}
\]

When \( t = 0 \), the instantaneous displacement \( \omega(0) \) is the same as in Equation (13). The elastic aftereffect expression \( \omega'(t) \) of the bearing plate rock mass-aging displacement in the Burgers rheological model can be obtained through the same derivation process as that of the elastic aftereffect expression of the bearing plate rock mass-aging deformation in the Maxwell rheological model; therefore, when \( t \geq t_1 \),

\[
\begin{align*}
\omega'(t) &= \omega \frac{qR_0}{2} \frac{p_0}{q_1} t_1 + \frac{p_2q_1^2 - p_1q_1 + p_0}{q_1^2} (1 - e^{-q_1t_1}) e^{-q_1t} + \\
&\quad \frac{3(Ap_2 - Cp_0)}{AC \alpha_1} \left\{ \alpha_1 - \alpha_1 e^{\alpha_1t} \cos h (\alpha_2t) - \alpha_3 e^{\alpha_3t} \sin h (\alpha_2t) \right\} \cos h (\alpha_2t) \sin h (\alpha_2t) \sin h (\alpha_2t) \left\{ \alpha_2 - \alpha_2 e^{\alpha_2t} \sin h (\alpha_2t) - \alpha_3 e^{\alpha_3t} \cos h (\alpha_2t) \sin h (\alpha_2t) \right\} e^{-\alpha_4t}, \quad (t \geq t_1),
\end{align*}
\]

where, \( \alpha_1 = \sqrt{\frac{b^2}{4} - AC} \), \( \alpha_2 = \sqrt{\frac{b^2}{4} - AC} \), \( \alpha_3 = \frac{ABp_2 + BCp_0 - 2ACP_1}{2(Ap_2 - Cp_0)} \), and \( \alpha_4 = \frac{B}{2C} \). When \( t = \infty \),

\[
\omega'(\infty) = \omega \frac{qR_0}{2} \frac{p_0}{q_1} t_1 = \omega \frac{qR_0}{2} \frac{t_1}{\eta_0}.
\]

4. **Viscoelastic characteristics of rock mass**

Equations (12), (20), and (26) are the viscoelastic analytical equations of rock mass-aging deformation for the condition of a bearing plate test when the rock mass rheological models are the Maxwell, Kelvin–Voigt, and Burgers models, respectively. Equations (14), (21), and (27) are the elastic aftereffect equations of rock mass-aging deformation when the rock mass rheological models are the Maxwell, Kelvin–Voigt, and Burgers models, respectively. Based on these viscoelastic analytical equations and elastic aftereffect equations of rock mass-aging deformation for the condition of a bearing plate test, the parameters of the rheological mechanics of the rock mass (Table 1) were used to obtain the viscoelastic analytical solutions and elastic aftereffect characteristics of rock mass-aging deformation at the centers of flexible and rigid circular bearing plates when the rock mass rheological models are the Maxwell, Kelvin–Voigt, and Burgers models. Fig. 4 shows that when the rheological model is established, under the same load action, the viscoelastic analytical solution of the rock mass-aging deformation at the center of a rigid circular bearing plate is smaller than that at the center of a flexible circular bearing plate and is \( \frac{\pi}{4} \) times the viscoelastic analytical solution of the rock mass-aging deformation at the center of a flexible circular bearing plate. However, the evolution trends and
laws of the rock mass-aging deformation at the centers of these two circular bearing plates are consistent with each other. The viscoelastic analytical solutions of the rock mass-aging deformation at the center of a circular bearing plate obtained from these three rheological models all have elastic aftereffect characteristics. For the Kelvin–Voigt and Burgers rheological models, the elastic aftereffect characteristics of the viscoelastic analytical solutions of the rock mass-aging deformation at the center of a circular bearing plate, namely, the evolution law of the displacement decaying with time, have similarities with the elastic aftereffect characteristics of the Kelvin–Voigt and Burgers rheological models. The Maxwell rheological model does not have elastic aftereffect characteristics, but the viscoelastic analytical solution of the rock mass-aging deformation at the center of a circular bearing plate obtained from the Maxwell rheological model has elastic aftereffect characteristics. This characteristic is related to the time exponential term in the viscoelastic analytical solution (Equation (12)) of the rock mass-aging deformation. Besides, the elastic aftereffect expression of the rock mass-aging deformation derived from this solution has an exponential term related to time; therefore, the rock mass deformation decays gradually as time increases, i.e., the elastic aftereffect phenomenon of aging deformation. This kind of phenomenon is also shown in Fig. 4 (a).

### Table 1. Parameters of viscoelastic analytical expression of rock mass-aging deformation for different viscoelastic combination models ($q = 1.5 \text{ MPa}$ and $R_0 = 0.5 \text{ m}$).

| Viscoelastic combination model | Bulk modulus $K_0$ (GPa) | Shear modulus $G_0$ (GPa) | Shear viscosity coefficient $\eta_0$ (GPa.h) | Shear modulus $G_1$ (GPa) | Shear viscosity coefficient $\eta_1$ (GPa.h) |
|-------------------------------|-------------------------|---------------------------|---------------------------------------------|-------------------------|---------------------------------------------|
| Maxwell model                 | 2.0                     | 1.2                       | 200                                        | -                       | -                                           |
| Kelvin–Voigt model            | 2.0                     | 1.2                       | -                                           | 2                       | 15                                          |
| Burgers model                 | 2.0                     | 1.2                       | 200                                        | 2                       | 15                                          |

Furthermore, as time tends to infinity, in the Maxwell and Burgers models, the viscoelastic analytical solutions of the rock mass-aging deformation at the center of a circular bearing plate both have residual deformations, and the residual deformations have the same expressions, $\omega \frac{qR_0^2 t_1}{2 \eta_0}$, whereas for the elastic aftereffect behavior of the viscoelastic analytical solutions of the rock mass-aging deformation at the center of a circular bearing plate in the Kelvin–Voigt model, the displacement equals zero when time approaches infinity, that is, there is no residual deformation, which is consistent with the law of the elastic aftereffect phenomenon of the Kelvin–Voigt model.
Fig. 4. Viscoelastic analytical solutions and elastic aftereffect characteristics of rock mass-aging deformation at the centers of flexible and rigid circular bearing plates. (a) Maxwell, (b) Kelvin–Voigt, and (c) Burgers rheological models.

Fig. 5 shows the viscoelastic analytical solutions of the rock mass-aging deformation at the center and edge of a flexible circular bearing plate for the Maxwell, Kelvin–Voigt, and Burgers models. When the rheological model is established, under the same load action, the viscoelastic analytical solution of the rock mass-aging deformation at the edge of a flexible circular bearing plate is smaller than the viscoelastic analytical solution at the center of a flexible circular bearing plate. Also, it is \( \frac{2}{\pi} \) times the viscoelastic analytical solution at the center of flexible circular bearing plate. Nonetheless, the evolution trends and laws of rock mass-aging deformation at the center and edge of a flexible circular bearing plate are consistent with each other as time increases.

Fig. 6 compares the analytical solutions of the rock mass-aging deformation at the center of a bearing plate for different rheological models. Whether a flexible circular bearing plate or rigid circular bearing plate is used, the evolution law of rock mass-aging deformation mainly depends on the adopted rheological model. When the Maxwell model is used, the rock mass-aging deformation shows elastic-viscous rheological characteristics and has a transient displacement. The displacement has a constant creep as time increases; in the constant creep stage, the creep rate is constant, and the displacement grows as time increases. This law is consistent with the deformation law of the Maxwell model. Likewise, when the Kelvin–Voigt model is used, the rock mass-aging deformation shows elastic-viscoelastic rheological characteristics and has a transient displacement. The displacement has an attenuation creep; therefore, the creep rate is zero in the stable stage, and the displacement stabilizes as time increases. This law is consistent with the deformation law of the Kelvin–Voigt model. When the Burgers model is used, the rock mass-aging deformation shows elastic-viscous-viscoelastic rheological characteristics and has a transient displacement. The displacement experiences attenuation and constant creep as time increases. In the constant creep stage, the creep rate is constant, and the displacement grows as time increases. This law is consistent with the deformation law of the Burgers model. Furthermore, when the same transient parameter is used, the transient vertical displacements of the bearing plate rock mass surface obtained from three rheological models are equal, and all displacements are equal to the results of Equation (13). The abovementioned results also show that the derived viscoelastic analytical solutions of rock mass-aging deformation for the condition of a bearing plate test are correct.
Figure 5. Comparison of viscoelastic analytical solutions of rock mass-aging deformation at the center and edge of a flexible circular bearing plate. (a) Maxwell, (b) Kelvin–Voigt, and (c) Burgers rheological models.

Figure 6. Comparison of viscoelastic analytical solutions of rock mass-aging deformation at the center of a bearing plate for different rheological models for (a) flexible and (b) rigid circular bearing plates.

From the above analyses, the description of the evolution law of rock mass-aging deformation for the condition of a bearing plate test is closely related to the rheological model. It is essential to identify the rheological model and its parameters after the test curve of a bearing plate is obtained to obtain a suitable rheological model and its parameters and reasonably express the test curve of the bearing plate simultaneously.

5. Conclusions

Based on rigid and flexible bearing plates’ rock mass rheological tests and viscoelastic theory, an
aging deformation analytical solution of a half-space viscoelastic body was derived for a surface normal uniform load. Furthermore, the rock mass-aging deformation viscoelastic analytical expression and elastic aftereffect displacement equation were established for several rheological models, such as the Maxwell, Kelvin–Voigt, and Burgers models. Next, the viscoelastic characteristics of rock mass deformation in a bearing plate test were discussed. Theoretically, the viscoelastic analytical solution of rock mass-aging deformation at the center of a rigid round bearing plate was $\frac{\pi}{4}$ times that at the center of a round flexible bearing plate. The viscoelastic analytical solution of the rock mass-aging deformation at the edge of a round flexible bearing plate was $\frac{2}{\pi}$ times of that at the center of the bearing plate. In the Maxwell and Burgers models, the elastic aftereffects of the viscoelastic analytical solution of rock mass-aging deformation for bearing plate test conditions had the same residual deformation expression.

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