New near-threshold mesons

Thomas D. Cohen
Department of Physics, University of Maryland, College Park, MD 20742-4111, USA

Boris A. Gelman
Department of Physics, University of Arizona, Tucson, AZ 85721, USA

Shmuel Nussinov
School of Physics and Astronomy,
Tel Sackler Faculty of Exact Sciences,
Tel Aviv University, Tel Aviv, Israel
and
Department of Physics and Astronomy,
University of South Carolina, SC 29208, USA

(Dated: September 2003)

We show that under a number of rather plausible assumptions QCD spectrum may contain a number of mesons which have not been predicted or observed. Such states will have the quantum numbers of two existing mesons and masses very close to the dissociation threshold into the two mesons. Moreover, at least one of the two mesonic constituents itself must be very close to its dissociation threshold. In particular, one might expect the existence of loosely bound systems of $D$ and $D_s^*$ states. This is an argument against the “minimal” interpretation of $f_0(980)$, $a_0(980)$ and $a_0(980)$ can be bound. The mechanism for binding in these cases is the S-wave $s$ quark exchange. The nearness of the constituents to its decay threshold into a kaon plus a remainder, implies that the range of the kaon exchange force becomes abnormally long—significantly longer than $1/m_K$ which greatly aids the binding.

PACS numbers: 12.38.Aw, 12.40.Yx, 14.80.-j, 21.30.Fe, 21.45.+v

INTRODUCTION

The particle data book abounds with hadronic resonances [1]. However, there are comparatively few states which are very close to the threshold for decay into other mesons. Recently a new near-threshold state—one of the $D^*_sJ(2317)$ ($I = 0$ and possibly $J^P = 0^+$) at about $40$ MeV below $KD$ threshold—was found at BABAR [2], CLEO [3] and Belle [4].

Note, that the $D^*_sJ(2317)$ state can be interpreted in a number of ways: (a) as the missing triplet S-wave ($J^P = 0^+$) $|c\bar{s}\rangle$ “quarkonium state”; (b) as a “single bag” $|(c \bar{s} u \bar{u}) + |c \bar{s} d \bar{d})]/\sqrt{2}$ isosinglet state; or (c) as an isosinglet “molecular” bound state $|(K^+ D^0) + |K^0 D^+)$/2 of two separate hadrons. The two hadrons in the last case can be bound—just like the deuteron—by an attractive potential due to the t-channel exchange of various light mesons. The Lagrangian has “off-diagonal” terms such as $q\bar{q}$ pair creation and annihilation and/or ”bag” fissioning and rejoining interconnecting states of type (a) and (b), and (b) and (c) respectively. As a result we expect that $D^*_sJ(2317)$ is a superposition of all three states in (a), (b), and (c). The question is then which one dominates the state $|D^*_sJ(2317)\rangle$.

Regardless of how one chooses to interpret the state there is one key fact about this state which will play a major role in what follows: the state is extremely close to the $KD$ threshold. This situation parallels a case of the pseudoscalar isosinglet and isoscalar mesons—$f_0(980)$ and $a_0(980)$—which are very close to the $K\bar{K}$ threshold. These states can correspond to any one of the three cases above provided the quark pair $c\bar{s}$ is replaced by $s\bar{s}$.

In Ref. [5] one of us argued in favor of interpretations (b) and (c). The argument in [5] was based on the fact that the mass difference of approximately $20$ MeV between $D^*_sJ(2317)$ and the state $D_0^*$ ($J^P = 0^+$) with a mass of about $2300$ MeV (BELLE [6]) is significantly smaller than an approximate $100$ MeV split between any two “strangeness analogue” $X_\bar{s} - X_{\bar{q}}$ ($q = u, d$) mesonic or baryonic states [7]. Likewise, the isotriplet $(P$-wave) $s\bar{s}$ state is $40$ MeV lighter than the isoscalar $S$-wave $\phi(1020)$ rather than being more than $350$ MeV heavier, as is the case for all other nonets. This is an argument against the “minimal” interpretation of $f_0(980)$ and $a_0(980)$ states as $s\bar{s}$ pairs. To the extent that $f_0(980)$, $a_0(980)$ and $D^*_sJ(2317)$ are indeed of type (b) or (c) then the following prediction can be made. A “QCD inequality” [8] implies yet another pseudoscalar $c\bar{c}$ state approximately $100$ MeV below the threshold [8]. This state can be discovered via the $\eta\eta_c$ decay mode in BABAR and Belle. Ordinary $c\bar{c}$ states are accounted for and such a state would have to be interpreted as being exotic.

Of course, one can take a far more agnostic position as far as the interpretation of the $D^*_sJ(2317)$ or the $f_0(980)$
and $a_0(980)$. Since the three interpretations were expressed in terms of model concepts rather than QCD degrees of freedom, one can argue that even in principle there is no way to distinguish between them. However one chooses to interpret these states, we can rely on the fact that they have $J^P = 0^+$ and are only very slightly below the corresponding break-up thresholds: $40 - 50 \text{MeV}$ below $KD$ and $10 - 20 \text{MeV}$ below $KK$ thresholds respectively. This fact greatly facilitates the possibility that these mesons will be bound weakly into “molecular”-like states: $|DD^*_{sJ}(2317)\rangle$ and $|Kf_0\rangle$, $|Ka_0\rangle$, $|Kf_0\rangle$, $|K\bar{a}_0\rangle$. While any of these states would be interesting to observe, the $|DD^*_{sJ}(2317)\rangle$ is of particular interest owing to the fact that by quantum numbers alone ($S = 1$, $C = 2$) it is manifestly exotic.

\[ g_2 = 4 \frac{m_a m_b}{m_{KK}} \]

\[ V(r) = \frac{g_2^2}{16\pi m_a m_b} \exp \left[-r \sqrt{2m_K} \epsilon_i \right] = \alpha_i \exp \left[-\kappa_i r \right], \]

where $g_1$ ($g_1 = g_{Kf_0(n_0)$, $g_2 = g_{KDD^*_{sJ}(2317)}$) is (the mass dimension two) coupling constant of the $S$-wave Yukawa coupling $K\bar{a}_0H_b$. The factor $4 m_a m_b$ in the denominator in Eq. (1) comes from the non-relativistic normalization of the scalar wave functions of $H_a$ and $H_b$. Consequently, the coupling constant $\alpha_i = -g_2^2 / (16\pi m_a m_b)$ is dimensionless.
The potential in Eq. (1) can be interpreted as the (asymptomatic) profile function of the field strength of the virtual $K$ inside the $H_a$ bound state (up to the coupling constants). It has the form of an outgoing spherical wave with a purely imaginary momentum $k_i = i \kappa_i$ with $\kappa_i$ equal to $\sqrt{2m_K \epsilon}$. Note that the $\kappa_i$ in Eq. (1) replaces $m_K$ in the standard Yukawa-like potential yielding much larger range potentials. This effect is huge for possible $DD^*$ loosely state considered by Törnqvist [10]. As in our case, $\epsilon = m_D + m_\pi - m_{D^*}$ is tiny. However, in that case, the interaction is in $P$-wave with a derivative $\pi DD^*$ coupling. As a result, the increase in the range in this case is essentially compensated by the corresponding decrease in the strength of the coupling. This is not the case for the $S$-wave momentum independent couplings relevant in our case.

**ESTIMATED BINDING ENERGIES**

The central result of this paper is that for a wide range of “reasonable” interactions between $H_a$ and $H_b$ binding results.

In the two cases considered here—(1) $|K f_0\rangle$, $|K a_0\rangle$, $|\bar{K} f_0\rangle$, $|\bar{K} a_0\rangle$ and (2) $|DD^*_{s,j}(2317)\rangle$—the values of $\kappa_i$ are:

$$100 \lesssim \kappa_1 \lesssim 140 \text{ MeV}, \quad 200 \lesssim \kappa_2 \lesssim 220 \text{ MeV}.$$  

(2)

The variation in Eq. (2) is due to the differences in binding energies for various $D$ and $K$ charge states.

The binding energies of the $|H_a H_b\rangle$ “molecules” can now be determined from the Schrödinger equation with a potential given in Eq. (1) and reduced masses $\mu_1 = 2 m_K/3 \approx 330 \text{ MeV}$ (case (1)) and $\mu_2 \approx 1030 \text{ MeV}$ (case (2)). The binding energies and the typical sizes of the ground state wave functions (given by $\sqrt{<r^2>}$) for a number of couplings $\alpha_i$ and values of $\kappa_i$ (Eq. (2)) are shown in Tables I, II, III and IV, Tables V, VI, VII and VIII (case (2)) respectively.

In Ref. 12, the value of the coupling constant $\bar{s}_{K\bar{K} f_0}/(4\pi)$ was determined to be $0.6 \text{ GeV}^2$. The corresponding dimensionless coupling is $\alpha_{K\bar{K} f_0} = - s_{K\bar{K} f_0}^2/(16\pi m_K^2) \approx -0.6$. We also assume that the same value is applicable in the case of $K a_0$ and $DD^*_{s,j}(2317)$ systems.

| $\alpha$ | $E_b$, MeV | $\sqrt{<r^2>}$, fm |
|----------|------------|-------------------|
| $-0.4$   | $-2.9$     | $36.1$            |
| $-0.6$   | $-17.3$    | $17.1$            |
| $-0.8$   | $-44.4$    | $11.8$            |

TABLE I: $|K f_0\rangle$, $|K a_0\rangle$: the binding energies and the size of the ground state wave function for the potential in Eq. (1) with $\kappa = 100 \text{ MeV}$.

| $\alpha$ | $E_b$, MeV | $\sqrt{<r^2>}$, fm |
|----------|------------|-------------------|
| $-0.4$   | $-3.0$     | $125$             |
| $-0.6$   | $-8.3$     | $26.7$            |
| $-0.8$   | $-28.4$    | $15.2$            |

TABLE II: $|K f_0\rangle$, $|K a_0\rangle$: the binding energies and the size of the ground state wave function for the potential in Eq. (1) with $\kappa = 140 \text{ MeV}$.

| $\alpha$ | $E_b$, MeV | $\sqrt{<r^2>}$, fm |
|----------|------------|-------------------|
| $-0.2$   | $-0.6$     | $41.1$            |
| $-0.4$   | $-25.5$    | $8.9$             |
| $-0.6$   | $-90.0$    | $5.0$             |

TABLE III: $|D DD^*_{s,j}(2317)\rangle$: the binding energies and the size of the ground state wave function for the potential in Eq. (1) with $\kappa = 200 \text{ MeV}$.

| $\alpha$ | $E_b$, MeV | $\sqrt{<r^2>}$, fm |
|----------|------------|-------------------|
| $-0.2$   | $-0.2$     | $72.1$            |
| $-0.4$   | $-22.0$    | $8.8$             |
| $-0.6$   | $-82.8$    | $5.3$             |

TABLE IV: $|D DD^*_{s,j}(2317)\rangle$: the binding energies and the size of the ground state wave function for the potential in Eq. (1) with $\kappa = 220 \text{ MeV}$.

For this value of the coupling constant the binding energy of $|K f_0\rangle$, $|K a_0\rangle$, $|\bar{K} f_0\rangle$, $|\bar{K} a_0\rangle$ systems ranges from about 8 to 20 MeV (for various values of $\kappa$). Tables I, II In the case of $|DD^*_{s,j}(2317)\rangle$ the binding energy is about $80 - 90 \text{ MeV}$, Tables III, IV.

The potential given in Eq. (1) treats $f_0(0)$, $K$, $D$ and $D^*_{s,j}(2317)$ as though they were point-like particles. The spatial extent of the $K$ and $D$ mesons are approximately 0.4 and 0.3 fm. The $f_0(0)$ and $D^*_{s,j}(2317)$ mesons are presumably even larger (particularly if the interpretations (b) and (c) discussed in the Introduction are correct). Hence, the Yukawa potential in Eq. (1) cannot apply at distances shorter than perhaps 0.5 fm. The large width (short lifetimes), $\Gamma_{f_0/a_0} = 50 - 100 \text{ MeV}$ and $\tau \approx (1.3 - 0.7) \times 10^{-23} \text{ sec}$, makes observations of such states difficult. Roughly speaking, since $\tau < T$, with $T$ being the time for completing one period in the bound state, $T \approx 2 m_{f_0}$, $m_K/\kappa$, the $f_0$ and $a_0$ decay before “realizing” that they are bound. The size of $f_0(0)$ is of order of $1 - 2 \text{ fm}$ and its velocity in traversing the orbit is approximately $200 \text{ MeV}/500 \text{ MeV} \approx 0.4$, so that $T > (2.5 - 5) \times 10^{-23} \text{ seconds}$.
− In the case of the $|DD\sigma_{J}(2317)|$ “molecule” the spatial extent of the stable $D$ and a very long lived $D_{\sigma_{J}}^{*}(2317)$ can be expected to reduce the binding energies. In this case the range of the Yukawa potential, Eq. (1), is approximately 1 fm. The most conservative approach to the unknown short range physics is to cut off the Yukawa potential at distances shorter than, say, $R = 0.5\text{ fm}$ and assume that $V(r) = V(R)$ (the value of the potential in Eq. (1) at $r = R$) for $r < R$. The binding energies and the corresponding sizes of the wave functions are shown in Tables V-VIII and IX. As can be expected, there is a reduction in the binding energies. The $|DD\sigma_{J}(2317)|$ system is still bound by about $20 - 30\text{ MeV}$. The size of the bound state—of order of 8 fm is dominated by the tail of the $K$-exchange potential, Eq. (1). We note in passing that in both cases the systems become unbound if the potential is taken to be zero at $r < R$ (a radical assumption).

| $\alpha$ | $E_{B}$, MeV | $\sqrt{<r^{2}>}$, fm |
|---------|-------------|----------------|
| −0.4    | −8.71       | 13.7           |
| −0.6    | −27.9       | 8.5            |
| −0.8    | −53.3       | 6.4            |

TABLE V: $|D D_{\sigma_{J}}^{*}(2317)|$: the binding energies and the size of the ground state wave function for the potential in Eq. (1) for $r > R$ and $V(r) = V(R)$ for $r < R$; $\kappa = 200\text{ MeV}, R = 0.5\text{ fm}$.

| $\alpha$ | $E_{B}$, MeV | $\sqrt{<r^{2}>}$, fm |
|---------|-------------|----------------|
| −0.4    | −6.5        | 17.6           |
| −0.6    | −23.3       | 8.9            |
| −0.8    | −46.3       | 7.0            |

TABLE VI: $|D D_{\sigma_{J}}^{*}(2317)|$: the binding energies and the size of the ground state wave function for the potential in Eq. (1) for $r > R$ and $V(r) = V(R)$ for $r < R$; $\kappa = 220\text{ MeV}, R = 0.5\text{ fm}$.

| $\alpha$ | $E_{B}$, MeV | $\sqrt{<r^{2}>}$, fm |
|---------|-------------|----------------|
| −0.4    | −4.9        | 17.5           |
| −0.6    | −16.7       | 10.2           |
| −0.8    | −32.3       | 7.9            |

TABLE VII: $|D D_{\sigma_{J}}^{*}(2317)|$: the binding energies and the size of the ground state wave function for the potential in Eq. (1) for $r > R$ and $V(r) = V(R)$ for $r < R$; $\kappa = 200\text{ MeV}, R = 0.3\text{ fm}$.

Note that as the composite states overlap the strong short range hyperfine interactions come into play since we have largely different quarks in $D_{\sigma_{J}}^{*}(2317)$ and $D$ even for the case (b) above with $D_{\sigma_{J}}^{*}(2317)$ viewed as a four-quark construct. The tendency to form these new loosely bound states would imply that at shorter distances we have even stronger attraction that the extrapolation of the relatively smooth Yukawa potential to short distances and the results without any cutoff and a fortiori those in the case (2) may be relevant!

It is interesting to note the drastic consequence of an even small attractive scattering length—with no bound state in $KK$ (rather than $K\bar{K}$ channel). Arbitrary (sufficiently large) number of $K^{0}$ in a common $S$-wave state would then attract forming a condensate carrying macroscopic strangeness ala Lee and Yang or Coleman’s $Q$-balls. The longest range interaction between two kaons (and in fact any two mesons!) due to the two pion exchange—specifically the $S$-wave projection thereof in the $t$-channel is like a $\sigma$ ($J^{PC} = 0^{++}$) or a scalar graviton exchange which is always attractive. The same also holds for $KN$ interactions. However, the scattering length in the Born approximation appropriate here is given by $\int dr r^{2} V(r)$ and the long range attraction is overcome (surely for $K N$ from scattering data analysis and most likely for $K K$) by the strong short range repulsion so that the condensates may not exist.

It is amusing to note in passing the (admittedly weak) connection between the $|D D_{\sigma_{J}}^{*}(2317)|$ bound state and the “Efimov effect”. The latter (which inspired us to look at the present problem) would arise for a zero energy...
|KD⟩ S-wave bound state and infinite scattering length. This in turn leads to an infinite series of three body |KDD⟩ bound states. The ratios the binding energies and the sizes of the Efimov states scale as $E_B(n+1)/E_B(n) = e^{-2\pi} \sim 0.0016$, $< r > (n+1)/ < r > (n) = e^{\pi} \sim 25$ (see also [17, 18]). Clearly in the present case where the range of the actual potential is only about 1 fm this idealized case and the very extended—$< r > \sim 25 n$ (or contracted)—states in the above series are irrelevant. Note, the the Yukawa potential, Eq. (1) goes to $1/r$ in the limit as $\epsilon$ goes to infinity and $\kappa$ goes to zero rather rather than $1/r^2$ as in the Efimov effect. The reason is that the Efimov effect requires exact diagonalization of the degenerate perturbation transcending the perturbative one-meson exchange [17, 18].

EXPERIMENTAL SIGNATURES

Assuming that the $|DD_s^*(2317)⟩$ “molecular” state exists how can it be produced and detected? Since the production requires two pairs of $c\bar{c}$ quarks the discussion in [14] is relevant, providing an upper bound on the expected rate of the new loosely bound extended $|DD_s^*(2317)⟩$ state which we term $\mathcal{M}$ for molecular.

The bound state should manifest as a narrow peak in the mass distribution of associate $D$ and $D_s^*(2317)$ decays. However, the limited experimental resolution at BABAR and Fermi Lab experiments limits the extent that we can utilize this. The different binding of $D^+$ and $D^0$ and different life-time $\tau_{D^0} \sim 0.5 \tau_{D^+}$ may lead to some extra signatures in specific charge dependence of the width and even in the binding energies.

ACKNOWLEDGMENTS

T.C. was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-93ER-40762. B.G. was supported by the U.S. Department of Energy under Grant No. DE-FG03-01ER-41196. S.N. acknowledges a grant of the Israeli Academy of Science. B.G. and S.N. greatly acknowledge the hospitality of the Theory Group for Quarks, Hadrons and Nuclei at the University of Maryland, College Park.

[1] Particle Data Group (K. Hagiwara et al.), Phys. Rev. D 66, 010001 (2002).
[2] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 90, 242001 (2003).
[3] D. Besson et al. [CLEO Collaboration], arXiv: hep-ex/0305100.
[4] P. Krokovny et al. [Belle Collaboration], arXiv: hep-ex/0308019.
[5] R.L. Jaffe, Phys. Rev. D15, 267 (1977); Phys. Rev. D15, 281 (1977).
[6] S. Nussinov, arXiv: hep-ph/0306187.
[7] K. Abe et al. [Belle Collaboration], arXiv: hep-ex/0307021.
[8] S. Nussinov and R. Shrock, unpublished.
[9] S. Nussinov and M.A. Lampert, Phys. Rept. 362, 193 (2002).
[10] N.A. Törnqvist, Phys. Rev. Lett. 67, 556 (1991).
[11] The numerical solutions were obtained using a Mathematica code in W. Lucha and F.F. Schoberl, Int. J. Mod. Phys. C10, 607 (1999).
[12] P.E. Close, N. Isgur and S. Kumano, Nucl. Phys. B389, 513 (1993).
[13] The $D_s^*(2317) \to D_s\pi$ decay mode is suppressed due to the isospin non-conservation.
[14] S.R. Coleman, Nucl. Phys. B262, 263 (1985), Erratum-ibid. B269, 744 (1986).
[15] V. Efimov, Sov. J. Nucl. Phys. 12, 589 (1970); Phys. Lett. B33, 563 (1970).
[16] A.C. Fonseca, E.F. Redish and P.E. Shanley, Nucl. Phys. A320, 273 (1979).
[17] P.F. Bedaque, H.W. Hammer and U. van Kolck, Phys. Rev. Lett. 82, 463 (1999).
[18] P.F. Bedaque, H.W. Hammer and U. van Kolck, Nucl. Phys. A646, 444 (1999).
[19] B.A. Gelman and S. Nussinov, Phys. Lett. B551, 296 (2003).