Minimal SU(5) Resuscitated by Higgs Coupling Fixed Points

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Abstract

The issue of gauge unification in the (non-supersymmetric) Standard Model is reinvestigated. It is found that with just an additional fourth generation of quarks and leptons, $SU(3) \otimes SU(2) \otimes U(1)$ gauge couplings converge to a common point $\sim 2.65 \times 10^{15}$ GeV ($\tau_p \sim 10^{34\pm1}$ years) provided the Higgs boson has a mass of at least 210 GeV. The presence of ultraviolet fixed points for the Yukawa and Higgs quartic couplings is found to be the origin of such unification.
It is without any doubt that the search for the Higgs boson is one of the most important endeavors in particle physics. Its discovery would certainly reveal a great deal about the mechanism of symmetry breaking and would probably provide a window into new physics beyond the Standard Model (SM).

It is amazing that sometimes a mere knowledge of the Higgs mass can go a long way in giving us a glimpse of such new physics if it exists. For instance, if the Higgs boson were to be found at LEP II (i.e. $m_H \leq m_Z$), vacuum stability arguments suggest that there must be new physics of some sorts in the TeV region \([1]\). On the other hand, if the Higgs boson were heavier than $2m_Z$, one would see the appearance of Landau poles (or possibly large ultraviolet fixed points) in the Higgs quartic coupling below the Planck scale \([2]\). In addition, this range ($> 2m_Z$) would almost certainly rule out a general class of Supersymmetric SM (SSM) since these models predict the existence of a light Higgs scalar of no more than 150 GeV in mass \([3]\). (The upper bound is even more stringent for the Minimal SSM (MSSM) and is approximately 130 GeV \([4]\).) For $m_H$ between 150 and 180 GeV, the SM is perfectly respectable all the way up to the Planck mass despite the fact that there remains many unanswered fundamental questions. (This is probably the only window where “nothing happens”, if one may say so.) In this note, we would like to explore further the issue of the Landau poles and ultraviolet fixed points, and, in particular, their effects on the evolution of the gauge couplings.

Despite the absence of a direct experimental evidence for proton decay, the idea of a Grand Unification of all known interactions is still very attractive for a number of reasons such as the ones espoused in the classic SU(5) paper \([4]\) for example. In the simplest of such schemes, one usually assumes a “desert” beyond the electroweak scale with the unification scale being determined by the point where the three SM couplings meet. However, it is a standard lore that present measurements of the gauge couplings at the Z mass appear to indicate that all three couplings actually do not meet at the same point and this is somewhat problematic for the simple idea of Grand Unification, such as minimal SU(5) \([4]\). It is also a standard lore that with low-energy supersymmetry broken at around 1 TeV or less, such
a unification is possible and occurs at a desirable energy scale $\sim 10^{16}$ GeV \cite{5}. This is, of course, a remarkable result. However, if no light scalar is found below 150 GeV, one would have to reassess the whole issue of gauge coupling unification within the context of low-energy supersymmetry. We would like to point out in this note that, whether or not the non-supersymmetric SM gauge couplings converge to the same point, the question really depends on, or rather is complicated by, the mass of the Higgs boson. In particular, for a sufficiently heavy Higgs boson ($\geq 174$ GeV) \cite{2}, the mere existence of the Landau poles might drastically affect the evolution of the gauge couplings. Simply stated: Can there be Grand Unification with a “desert” if the Higgs boson is found to be heavier than 174 GeV? Put it in another way, a “heavy” Higgs ($\geq 174$ GeV) scenario might be an alternative to MSSM as far as coupling constant unification is concerned.

The issue mentioned above is of critical importance for, if the Higgs boson were not found below $2m_Z$ ruling out supersymmetry (or at least the simplest version of it), it pertains to the whole question of whether or not there will be some form of grand unification at all and, if so, whether or not it is simple (with a “desert”) or more complex with many intermediate scales.

Of particular importance is the feasibility of the search for proton decay, our only direct evidence of the idea of Grand Unification. The current prediction of supersymmetric GUT for the proton lifetime is approximately $10^{37}$ years, which puts it way beyond any foreseeable future search. (It may be a moot point if there is no light Higgs below 150 GeV.) The well-known prediction of minimal SU(5) for the proton lifetime is roughly two orders of magnitudes lower than the current experimental lower bound of $5.5 \times 10^{32}$ years \cite{6}. Is it possible that, if the proton does decay, its lifetime might be within reach of, say, SuperKamiokande which presumably could extend its search up to $10^{34}$ years? We would like to point out in this note that this might be possible.

First let us briefly recall how “unification” is accomplished in MSSM. Basically, the extra degrees of freedom coming from supersymmetry affect the evolution of the gauge couplings in the following way: the slowing-down of the QCD coupling and, because of the need for two
Higgs doublets, the enhanced asymmetric running of the $U(1)_Y$ and $SU(2)_L$ gauge couplings. The net result is the convergence of all three couplings at an energy scale of approximately $10^{16}$ GeV.

For the non-supersymmetric SM, we shall distinguish two cases: the minimal SM with three generations and one Higgs doublet (case I), and the “non-minimal” case with four generations and one Higgs doublet (case II). We shall leave out multiple Higgs scenarios for future studies.

In what follows, we shall use two-loop RG equations for the two cases. For case I, they are well-known and the explicit expressions can be found in the literature [7]. For the second case with four generations, we shall write down explicitly the two-loop RG equations below [8]. To set the notations straight, our definition of the quartic coupling in terms of the Higgs mass is $m_H^2 = \lambda v^2/3$ (corresponding to $\lambda(\phi^\dagger\phi)^2/6$ in the Lagrangian) while the more common definition is $m_H^2 = 2\lambda v^2$. Therefore our $\lambda$ is six times the usual $\lambda$. The RG equations given below reflect our convention on the quartic coupling. There is a reason for paying close attention to this: at tree level, the value of the usual $\lambda$ where the Higgs self-interactions become strong is $8\pi/3$. In our case, this would be $\lambda = 16\pi$. We shall use this value as a guide- and only as a guide- for the evolution of the couplings, noticing that $16\pi$ is the value at tree level. The breakdown of the perturbation expansion might even occur at $16\pi^2$ instead of $16\pi$. In fact, with our definition of $\lambda$, the tree-level 4-point function goes like $\lambda$ while the one-loop 4-point function goes like $(\lambda^2/32\pi^2)(\log)$. Naively one would expect the one-loop term to be of the order of the tree-level one when $\lambda \approx 32\pi^2$. The tree-level unitarity limit of $16\pi$ is a factor of 6 less than this naive expectation. We will keep this in mind when the question arises as regards to when the perturbation expansion breaks down.

We first show that, for the minimal case with three generations and one Higgs doublet, the numerical solutions to the two-loop renormalization group (RG) equations reveal the following features. 1) For $m_H \geq 174$ GeV, the energy scales where $\lambda/16\pi \geq 1$ appear below the Planck scale. This has been extensively studied earlier by various authors (e.g. Ref. [2]) and these scales are usually associated with the so-called “Landau poles”. If there
were no ultraviolet fixed points, the Landau singularity is expected to appear soon after that scale. However the two-loop RG equations are coupled and something very different happens, which brings us to the 2nd point. 2) $\lambda$ does not actually blow up but tends towards some kind of “ultraviolet” fixed point, $\lambda_{fixed} \approx 79$, regardless of its original “low energy” value as long as $m_H \geq 180$ GeV. Now this value of 79 for the fixed point obtained from the two-loop RG equations is not too different from the tree-level unitarity limit of $16\pi$. Taking into account the possibility that one might be able to evolve $\lambda$ up to $32\pi^2 \approx 316$ before perturbation theory breaks down, one might assume that one can still use the two-loop RG equations to evolve the remaining couplings beyond the scale where this fixed point is first reached. Although the existence of this fixed point arises from the two-loop RG equations, we conjecture that it exists at approximately the same value to all orders. In fact, it is well-known that the first two terms of the $\beta$-function is renormalization-scheme independent and, therefore, have physical significance. The coefficients of the higher loop terms can be small or large depending on a particular renormalization scheme. 3) The evolution of the top Yukawa couplings and the gauge couplings is affected very little by the Higgs quartic coupling as it reaches its fixed point and remains there, if the two-loop approximation can be trusted. As a result, the three SM gauge couplings do not seem to converge. This reconfirms the standard result (although previously it was not clear how the Higgs mass might affect this result). We found that the energy scales where $g_2$ and $g_1$ meet and where $g_3$ and $g_2$ meet differ by three orders of magnitude, for the range of Higgs boson masses considered. Explicitly, for $m_H \geq 180$ GeV, $g_2$ and $g_1$ meet at $\sim 6 \times 10^{13}$ GeV, while $g_3$ and $g_1$ meet at $\sim 4 \times 10^{14}$ GeV and $g_3$ and $g_2$ meet at $\sim 1 \times 10^{16}$ GeV.

This little exercise above was meant to show that, in the two-loop approximation, the non-supersymmetric SM model, with three generations and one Higgs doublet, appears to fail to unify the three gauge couplings even if the Higgs boson is heavy enough to exhibit ultraviolet fixed points at energies below the Planck mass. Nevertheless, one should exercise extreme caution when one of the couplings- $\lambda$ in this case- grows large.

We now turn to the second scenario with four generations and one Higgs doublet. These
four generations fit snugly into $5 + 10$ representations of SU(5), except for the right-handed neutrinos which we should need if we were to give a mass to the neutrinos. This could be incorporated into a 16-dimensional representation of SO(10) which splits into $\bar{5} + 10 + 1$ under SU(5). We could then have a pattern of symmetry breaking like $SO(10) \rightarrow SU(5)$ for example. These details are however beyond the scope of this paper.

The appropriate two-loop RG equations are given by:

$$16\pi^2 \frac{d\lambda}{dt} = 4\lambda^2 + 4\lambda(3g_i^2 + 6g_q^2 + 2g_l^2 - 2.25g_2^2 - 0.45g_1^2)$$

$$-12(3g_i^4 + 6g_q^4 + 2g_l^4) + (16\pi^2)^{-1}\lambda(180g_i^6$$

$$+ 288g_q^6 + 96g_i^6 - 3g_i^4 - 6g_q^4 - 2g_i^2 + 80g_3^2(g_i^2$$

$$+ 2g_q^2)) - \lambda^2(24g_i^2 + 48g_q^2 + 16g_l^2) - (52/6)\lambda^3$$

$$-192g_3^2(g_i^4 + 2g_q^4)]$$ \hspace{1cm} (1a)

$$16\pi^2 \frac{dg_i^2}{dt} = g_i^2\{9g_i^2 + 12g_q^2 + 4g_l^2 - 16g_3^2 - 4.5g_2^2 - 1.7g_1^2$$

$$(8\pi^2)^{-1}\lambda(1.5g_i^4 - 2.25g_2^2(6g_q^2 + 3g_l^2 + 2g_i^2)$$

$$-12g_q^4 - (27/4)g_l^4 - 3g_i^4 + (1/6)\lambda^2 + g_i^2$$

$$(-2\lambda + 36g_i^2) - (892/9)g_i^4\})$$ \hspace{1cm} (1b)

$$16\pi^2 \frac{dg_q^2}{dt} = g_q^2\{6g_i^2 + 12g_q^2 + 4g_l^2 - 16g_3^2 - 4.5g_2^2 - 1.7g_1^2$$

$$(8\pi^2)^{-1}\lambda(3g_q^4 - g_i^2(6g_q^2 + 3g_l^2 + 2g_i^2)$$

$$-12g_q^4 - (27/4)g_l^4 - 3g_i^4 + (1/6)\lambda^2 + g_i^2$$

$$(-8/3)\lambda + 40g_i^2 - (892/9)g_i^4\})$$ \hspace{1cm} (1c)

$$16\pi^2 \frac{dg_l^2}{dt} = g_l^2\{6g_i^2 + 12g_q^2 + 4g_l^2 - 4.5(g_i^2 + g_q^2)$$

$$(8\pi^2)^{-1}\lambda(3g_i^4 - g_q^2(6g_q^2 + 3g_l^2 + 2g_i^2) - 12g_q^4$$

$$-(27/4)g_i^4 - 3g_i^4 + (1/6)\lambda^2 - (8/3)\lambda g_i^2\})$$ \hspace{1cm} (1d)

$$16\pi^2 \frac{dg_3^2}{dt} = g_3^2\{(163/15) + (16\pi^2)^{-1}\lambda(787/75)g_1^2 + 6.6g_2^2$$

$$(352/15)g_3^2 - 3.4g_i^2 - 4.4g_q^2 - 3.6g_l^2\})$$ \hspace{1cm} (1e)
\[ 16\pi^2 \frac{dg_2^2}{dt} = g_2^4 \left\{ -(11/3) + (16\pi^2)^{-1} \left[ 2.2g_1^2 + (133/3)g_2^2 
abla^2 - 3g_1^2 - 3g_q^2 - 2g_q^2 \right] \right\} \] (1f)

\[ 16\pi^2 \frac{dg_3^2}{dt} = g_3^4 \left\{ -(34/3) + (16\pi^2)^{-1} \left[ (44/15)g_1^2 + 12g_2^2 
abla^2 - (4/3)g_3^2 - 4g_t^2 - 8g_q^2 \right] \right\} \] (1g)

In the above equations, we have assumed for the fourth family, for simplicity, a Dirac neutrino mass and, in order to satisfy the constraints of electroweak precision measurements, that both quarks and leptons are degenerate SU(2)_L doublets. The respective Yukawa couplings are denoted by \( g_q \) and \( g_l \). Also, in the evolution of \( \lambda \) and the Yukawa couplings, we have neglected, in the two loop terms, contributions involving \( \tau \) and bottom Yukawa couplings as well as as the electroweak gauge couplings, \( g_1 \) and \( g_2 \). For the range of Higgs and heavy quark (including the top quark) masses considered in this paper, these two-loop contributions are not important to the evolution of \( \lambda \) and the Yukawa couplings.

In what follows, we shall assume that, whatever mechanism (a right-handed neutrino in this case) that is responsible for giving a mass to at least the 4th neutrino will not affect the evolution of the three SM gauge couplings. Also there are reasons to believe that this 4th generation might be rather special, distinct from the other three and having very little mixing with them. The physics scenario behind the 4th neutrino mass might be quite unconventional.

We shall assume, in this paper, that the 4th generation quarks are at least as heavy as the top quark and the leptons are heavier than \( m_Z \). The assumption concerning the 4th generation quarks is for mere convenience. (As of now, there does not appear to be any strict limit on the masses of those quarks if the 4th family is non sequential, i.e. possibly having very little mixing with the other three.) For the leptons, we just take LEPII as a guidance for possible masses.

As with the case of three generations, one has two options: 1) Stop evolving \textit{all} couplings as soon as one or more couplings \((\lambda, g_i^2, i = t, q, l)\) reaches \( 4\pi \), or 2) Evolve the couplings
beyond these values to see if there are any ultraviolet fixed points. Option 2 is what we will adopt in this paper for the following reason: if the fixed point (in the two-loop approximation) is located at a not-ridiculously high value then one might try to see what effects it has on the evolution of the gauge couplings, just as we have done above with case I.

We found the following results. First, for a given 4th generation quark mass \( m_Q \) as well as 4th generation lepton mass \( m_L \), there is a minimum value of the Higgs mass which arises because of the requirement that \( \lambda > 0 \) (vacuum stability). As the Higgs mass exceeds that given lower bound, an ultraviolet fixed point appears. We have observed this behaviour for a large range of 4th generation masses. We found the common ultraviolet fixed points to be: \( \lambda \approx 108, \quad g_t^2 \approx 28, \quad g_q^2 \approx 56, \) and \( g_l^2 \approx 55, \) regardless of the initial values as long as there is an appearance of a fixed point. It is found that these fixed points appear rather “early” \((\sim 1 - 5 \times 10^{10} \text{ GeV})\), which means that they have “time” to influence the evolution of the gauge couplings. It is also found that the ultraviolet fixed points are quite “stable” all the way to the unification scale.

The first set of 4th generation masses that we looked at is the following: \( m_Q = m_t \) and \( m_L = m_Z \) (experimentally motivated choices). This set of values is first chosen so we can find the lowest Higgs mass where our scenario would work. We found \( m_{H, \text{min}} \approx 210 \text{ GeV} \). The three gauge couplings are found to converge to a common value \( \alpha_{\text{G}}^{-1} \approx 37.6 \) at the scale \( M_G \approx 2.55 \times 10^{15} \text{ GeV} \). This would translate into a proton lifetime (within SU(5)) of \( \sim 10^{34\pm1} \) years, a value which is quite welcome from an experimental viewpoint.

The above result indicates that convergence of the three gauge couplings is achieved, for \( m_Q \geq m_t \) and \( m_L \geq m_Z \), only when the Higgs boson is heavier than \( \sim 210 \text{ GeV} \). As we vary the 4th generation masses, the lower bound on the Higgs mass will also increase. Notice that the minimum Higgs mass is always larger than the 4th generation masses.

Since it is impossible to list all possible masses for the 4th generation, we shall illustrate our scenario with another two sets of values. The results are listed in Table 1. In practice, one can deduce the lower bound on the Higgs mass once the 4th generation quark and lepton masses are known. If for some reason, the Higgs boson and one set of fermions (quarks or
leptons) are known first, one can set an upper limit on the remaining set of fermions. The RG equations listed above can be easily used for such purposes. It is beyond the scope of this paper to carry out such a task.

We obtain the same results for widely different Higgs masses, for a given set of 4th generation masses. This comes about because the same fixed point is reached for any $\lambda$ above its minimal value. There is an attractive domain lying above the minimal critical value. For convenience only the minimum masses for the Higgs boson are listed in Table 1.

As can be seen from Fig. 1, the three gauge couplings do actually converge at a scale of approximately $2.65 \times 10^{15}$ GeV and at a value $\alpha_G \approx 1/39$. (These numbers correspond to values of 4th generation masses as shown in Table 1.) Not only do they meet but also at a comfortable mass scale. Note that $\alpha_G$ is practically the same as the “old” value with the only difference being the “unification” scale. It is found that $\alpha_G$ is very weakly dependent on the initial Higgs mass, varying between 1/38 and 1/39 for a large range of masses. It is beyond the scope of this paper to illustrate this minor variation.

The proton partial mean lifetime as represented by $\tau_{p \to e^+\pi^0}$ is predicted to be $\tau_{p \to e^+\pi^0}(yr) \approx 10^{31\pm1}(M_G/4.6 \times 10^{14})^4$. In our case, we obtain the following prediction: $\tau_{p \to e^+\pi^0}(yr) \approx 1 \times 10^{34\pm1}$. This is comfortably larger than the current lower limit of $5.5 \times 10^{32}$ years. In addition, the prediction is not too much larger than the current limit which means that it might be experimentally accessible in the not-too-distant future, in contrast with the MSSM predictions.

This scenario made a number of predictions: 1) the proton decays at an accessible rate $\sim 10^{34\pm1}$ years; 2) there is a fourth generation of quarks and leptons; 3) For given 4th generation quark and lepton masses, there is a lower bound on the Higgs boson mass above which the quartic and Yukawa couplings will evolve into an ultraviolet fixed point (with the consequences discussed above). All of these features can be tested in a not-too-distant future. For example, the fourth generation can be non-sequential and can have exceptionally long lifetimes. This could provide a distinct signature [9]. Last but not least, the Higgs boson, if found, is predicted to be at least 210 GeV and possibly heavier, depending on the 4th
generation masses, for unification to occur.

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FIGURES

FIG. 1. The evolution of the SM gauge couplings squared versus ln(E/175 GeV). $g_3$, $g_2$, and $g_1$ are the couplings of SU(3), SU(2) and U(1) respectively. We have used $g_3^2(m_t) = 1.396$, $g_2^2(m_t) = 0.418$, and $g_1^2(m_t) = 0.2115$. The point where the couplings meet corresponds to a unification scale of $\sim 2.65 \times 10^{15}$ GeV. The common unified coupling is $\alpha_G \sim 1/39$. 
TABLES

TABLE I. The minimum value of the Higgs mass for two sets of 4th generation masses. Here $m_t = 175$ GeV. The corresponding couplings run to a fixed point: $\lambda \approx 108$, $g_t^2 \approx 28$, $g_q^2 \approx 56$, and $g_l^2 \approx 55$. Larger values of Higgs masses also lead to the same fixed point, with the same consequences.

| Parameter | Value 1 (GeV) | Value 2 (GeV) |
|-----------|---------------|---------------|
| $m_{H,\text{min}}$ (GeV) | 224 | 233 |
| $m_Q$ (GeV) | 180 | 180 |
| $m_L$ (GeV) | 135 | 156 |
