The earthquakes aftershock processes of the Tien Shan and its surrounding area

V N Sychev and N A Sycheva
Research station of Russian Academy of Science in Bishkek city, Research Station RAS, Bishkek-49, Kyrgyzstan
E-mail: sychev@gdirc.ru

Abstract. The earthquakes and aftershock sequences catalogue of 13 moderate earthquakes occurred in the Tian Shan and the Pamir Mountains from 1983 till 1997 have been examined from the point of view of non-equilibrium thermodynamics using Tsallis statistics and its compliance with Omori-Utsu power-law. It has been shown that the earthquake sequence is the system with memory and long-range correlations. The Tsallis parameter \( q \) for the aftershock sequences is larger than the \( q \), calculated for all the earthquake catalogue, which indicates the preservation of increased long-range correlations during the aftershock sequence. Dynamic definition of the parameter \( q \) of the four events has demonstrated the sharp increase of the long-range correlations in the target region before the main event and the sharp decrease straight after it followed by return to the average value observed before the mainshock. It has been illustrated that the parameters of the researched aftershock sequences are described by the power dependences and follow the Omori-Utsu law.

1. Introduction
The Tian Shan region aftershock sequences have been examined before in works [1-2]. The first work addresses the aftershock sequences following moderate and weak earthquakes occurred on the territory of Bishkek geodynamic ground (The Northern Tian Shan) and its surrounding area. It has been indicated that the epicenters of the mainshocks are located in the areas with the big regional faults of various types. Different aftershock characteristics have been studied, its mainshock class dependency and tendencies have been defined. Aftershock sequence of the latest powerful earthquake on the territory of the Tian Shan – Suusamyr Valley (19.08.1992, \( M = 7.3 \)) has been studied in the second work. Earthquake and aftershock sequences catalogue of Suusamyr earthquake has been examined from the point of view of their non-equilibrium thermodynamics using Tsallis statistics and its long-range correlations have been analysed. It has been illustrated that the parameters of the researched aftershock sequence are described by the power dependences and follow the Omori-Utsu law. Dynamic definition of the parameter \( q \) before and after the Suusamyr sequence has allowed to indicate the sharp increase of the long-range correlations in the researched region before the main event and the sharp decrease straight after it followed by the return to the average value, observed before the mainshock.

The present work continues the research described above and addresses the moderate earthquakes aftershock sequences analysis occurred in the Tian Shan and the Pamir Mountains during the 1983-1997 period. The work [3] marks that the aftershock sequence energy and time distribution is described by the power-laws and it might indicate considerably non-linear, deterministic and chaotic fault system behavior where aftershocks occur, and their different characteristics can represent the current state of
the system. Therefore, the main objective of the present work is to evaluate the aftershock process from the point of view of their non-equilibrium thermodynamics using the Tsallis statistics and validate its compliance with the Omori-Utsu power-law in order to confirm the conclusion made for the sole powerful earthquake in the work [2]. Allotment of the aftershock sequences is provided by the catalogue of Seismology Institute of the National Science Academy of the Kyrgyz Republic (KSI) including the earthquakes during the 1980-2001 period.

2. Data and method

The approach proposed in the work [4] has been taken to extract the aftershock sequence from the KSI catalogue. Some statistical characteristics of the earthquake catalogue are illustrated by figure 1A: an unbiased sample is the events of 7 – 16 classes, maximum of events is in 1992 (Suusamyr earthquake, 19.08.1992, M=7.3), depth of the major part of the earthquakes ranges from 0 to 5 km. The events which depths are not defined are classified to 0-5 km depth range.

The figure 1B illustrates the seismicity distribution, the epicentral location of 13 earthquakes (hereinafter mainshock) and their focal mechanisms: few earthquakes occurred in the area of the Southern Tian Shan and the Pamir Mountains junction with Tarim, one – in the area of Talas-Fergana fault, three – along the Fergana Valley borders, one – in the eastern part of Kyungei Alatoo and one in the eastern part of the southern Kokshal Too foothills. The Mainshock focal mechanisms have the upthrusting and upthrust-shifting nature. Compression axes have submeridional direction which matches with the regional direction defined in many seismological works and GPS-data [5-7].

Some characteristics of the mainshocks and aftershock sequences are given in the table.

![Figure 1](image_url)
Table. The main parameters of the mainshock and aftershock sequences.

| №  | Date       | Time      | φ,°  | λ,°  | H, км | K   | M   | N   | T   | K_aft | p    | q   |
|----|------------|-----------|------|------|-------|-----|-----|-----|-----|-------|------|-----|
| 1  | 13.02.1983 | 01:40:09  | 40.23| 75.23| 20    | 16.1| 6.72| 624 | 1879| 13.6  | 1.11 | 1.610|
| 2  | 16.12.1983 | 13:15:53  | 39.38| 72.92| 15    | 14.5| 5.83| 321 | 1351| 12.8  | 0.67 | 1.586|
| 3  | 17.02.1984 | 23:26:52  | 40.85| 71.02| 25    | 14.0| 5.56| 355 | 1021| 12.4  | 1.0  | 1.518|
| 4  | 26.10.1984 | 20:22:17  | 39.20| 71.23| 15    | 14.5| 5.83| 287 | 1332| 12.1  | 0.97 | 1.577|
| 5  | 23.08.1985 | 12:41:55  | 39.43| 75.48| 20    | 16.5| 6.94| 551 | 1021| 12.4  | 1.73 | 1.586|
| 6  | 24.01.1987 | 08:09:17  | 41.43| 79.27| 13    | 14.7| 5.94| 620 | 1353| 13.6  | 1.05 | 1.566|
| 7  | 17.04.1990 | 01:59:19  | 39.45| 74.55| 29    | 15.0| 6.11| 426 | 672 | 10.0  | 0.97 | 1.595|
| 8  | 12.11.1990 | 12:28:51  | 42.98| 77.92| 15    | 15.0| 6.11| 137 | 1312| 10.8  | 1.04 | 1.674|
| 9  | 01.12.1990 | 18:09:27  | 40.88| 73.65| 5     | 12.6| 4.78| 104 | 601 | 9.1   | 0.77 | 1.583|
| 10 | 15.05.1992 | 08:07:59  | 41.10| 72.42| 10    | 15.0| 6.11| 1380| 1752| 11.4  | 0.80 | 1.527|
| 11 | 19.03.1996 | 15:00:26  | 40.08| 76.65| 0     | 14.6| 5.89| 180 | 1086| 11.7  | 0.65 | 1.634|
| 12 | 21.01.1997 | 01:47:12  | 39.43| 76.98| 0     | 14.4| 5.78| 479 | 84  | 10.8  | 1.52 | 1.601|
| 13 | 15.04.1997 | 18:19:21  | 39.67| 76.97| 0     | 15.0| 6.11| 1154| 1715| 12.0  | 0.49 | 1.549|

According to the Omori law [8] the aftershock frequency after the powerful earthquake decreases hyperbolically within the time. In practice, the more generalized formula, known as the Omori-Utsu law [9], is usually used:

\[ n(t) = \frac{K}{(c + t)^p} \]  \hspace{1cm} (1)

Here \( k > 0, \ c > 0, \ t \geq 0 \) are constants, \( p \) depends on the location and changes from case to case [10]. The Omori law describes the monotonically decreasing aftershock sequence. Calculation of the Omori parameter depends on the chosen time interval, thus, cumulative number of events is preferred to calculate \( p \) [11], resulting from the generalized Omori law via the integration:

\[ N(t) = \frac{K(c + t)^{1-p}}{1-p} \]  \hspace{1cm} (2)

The latter expression is used to calculate included in the modified Omori law coefficients \( k, c \) and parameter \( p \).

Worth noting that the laws describing earthquake sequences are mainly empiric and cannot be derived from the equilibrium thermodynamics. But lately the situation has changed and in order to describe an earthquake sequence the non-equilibrium thermodynamics laws are used more often [12] – the non-extensive analysis of the seismic event sequence [13-15].

Tsallis has proposed to switch from logarithmic to the power distribution [12], thus he generalized Boltzmann statistics, introducing the parameter \( q \) into the entropy expression representing the power of non-additivity:

\[ S_q = k \frac{1}{l - q} \left( 1 - \sum_{i=1}^{N} p_i^q \right) \]
where \( p_i = \frac{N_i}{N(\varepsilon)} \), \( N_i \) is the number of system elements of \( \varepsilon \) - division, \( N(\varepsilon) \) is complete number of elements of \( \varepsilon \) coverage, \( k \) is size constant. Boltzmann statistics complies with the \( q \rightarrow 1 \) limit. The \( q < 1 \) case implies the higher events limit. \( q > 1 \) indicates that there are long-range correlations in the non-equilibrium system. For example, corresponding expression for the earthquakes magnitude will be:

\[
\log \left( \frac{N(M > M_{th})}{N} \right) = \left( \frac{2-q}{1-q} \right) \log \left[ 1 - \left( \frac{1-q}{2-q} \right) \left( \frac{10^{M_{th}}}{a} \right) \right]
\]

where \( N(M > M_{th}) \) is the number of earthquakes with the energy value higher than the threshold \( M_{th} \), \( N \) is complete number of earthquakes. Unknown \( a \) and \( q \) parameters are calculated in the right part of the derived expression. The calculated value \( q \) indicates whether there are long-range correlations in the system and how they change.

The calculated parameters also allow to define how the obtained results comply with the ones given in the studies of other seismically active regions.

3. Result
The Omori law parameter \( p \), Tsallis parameter \( q \) and distributions within the energy, time and space scale have been calculated for all the researched aftershock sequences. However, within the present article context only some of the events are presented (Figure 2).

**Figure 2.** Distribution of aftershocks within the space (top row) and the time (second row) scale and illustrations necessary for calculation of Omori-Utsu parameters and their value (third row) and Tsallis parameter \( q \) and its value (bottom row) for events 1, 3, 5 and 10 from the table.
For the majority of the researched events the main event is located on the board of seismic cloud indicating the point of the Earth’s crust fault. As a rule, the fault expands along the one side of the mainshock.

The class of events decrease within the time is observed only for the main part of the researched sequences, however, there are earthquakes (11, 12, 13, tabl.) with the events occurring during the aftershock process whose energy characteristic is close or higher than the mainshock’s one. This may be due to the imperfection of the aftershock sequence determination algorithm or the specific features of the seismic process. For each of the aftershock sequence the cumulative number is constructed and using the expression (2), the Omori-Utsu law’s parameters $K$, $c$ and $p$ are calculated. Energy distribution of event number in the left part of the expression (3) is calculated likewise. From the right part of the expression (3) the Tsallis parameter $q$ and coefficient $a$ are calculated.

The calculated value of the parameter $p$ (Omori-Utsu law) for the aftershock sequence ranges from 0.49 to 1.73 (table). The work [16] indicates that the value $p$ usually ranges from 0.5 to 1.8 and typical value according to the empiric data of the works [17-21] is considered to be $\sim 1.0$ which means that the researched regions are seismically active. The higher value $p$ signifies that the event sequence velocity is decreasing faster when the $p$ is lower. In our case for the 12th event $p$ is 1.52 and for the 13th $p$ is 0.49. Duration of the aftershock process for the first one is 84 days, the second one is 1715 days, however both events have the same magnitude.

Before the observation of the Tsallis parameter $q$ for the aftershock process all the necessary calculations have been made as well as the Tsallis parameter $q$ for all the catalogue (KIS) has been calculated. The calculated value of $q=1.525$ is close to the $q=1.569$ calculated in the work [2] which corresponds well to the given values of the other seismically active regions [22, 14, 11].

The Tsallis aftershock sequence parameter $q$ (tabl.) ranges from 1.518 to 1.674, which in comparison to $q$ differs on average in $\sim 6\%$ (from 13 to 2.4 \%). In all the cases, excluding the first one (event №3) the value $q$ is higher than the specific value of all the catalogue. This event has occurred in the Fergana Valley with $M=5.5$ followed by 355 aftershocks during 1021 days. The one fourth of the aftershocks (95) have occurred during the first days of that event whereas the others occurred during the 3 years distinguishing that event from the others.

The four most powerful earthquakes whose aftershock number is $N > 400$ (tabl., indicated grey) were studied to examine the Tsallis parameter behavior in the dynamics. Not all the events (the big area, Figure 1) were researched to calculate the Tsallis parameter in the dynamics, but the ones located within the 200 km radius of the mainshock. 200 events with the step of 20 events were used in this specific sequence. The results of the calculation are illustrated in the Figure 3. The arrow in the figure marks the time of the mainshock.

The increase of the Tsallis parameter is observed in all the close to the mainshock date researched cases and its decrease during some time after. In other words, any catastrophe is followed by the increase of the long-range correlations [23] till some threshold value along with the accumulated intensities decrease of these long-range correlations

4. Conclusion

The result of the 13 moderate earthquakes aftershock sequences analysis of the Tian Shan and the Pamir Mountains is as follows. The calculated value $p$ (Omori-Utsu) for majority of these events is close or higher than unity, and this indicates that the researched region is seismically active.

The Tsallis parameter $q$ calculated for all the catalogue is $q=1.525$ which complies with the values obtained from the other seismically active regions. The Tsallis parameter calculated for the aftershock sequences ranges from 1.518 to 1.674 which is higher than the value obtained for all the catalogue meaning that before the main event in the researched system the long-range correlations are increasing and preserving during the aftershock activity. The behavior of the Tsallis parameter $q$ has been examined in the dynamics of the four events of the highest magnitudes which is indicated by the sharp increase of the long-range correlations straight before the main event and then, after the main event, the decrease to the previous level.
Figure 3. The behavior of the Tsallis parameter q in the dynamics near the date of the mainshock within the 200 km radius for events (table.): 13.02.1983, M=6.72; 17.02.1984, M=5.56; 23.08.1985, M=6.94; 15.05.1992 M=6.11 and 19.08.1992, B=7.3. The dashed vertical line marks the time of the mainshock.

Acknowledgements
The research was performed within the framework of the state assignment for Federal State Budgetary Institution of Science Research Station of Russian Academy of Sciences in Bishkek (topics No. AAAA-A19-119020190064-9 and No. AAAA-A19-119020190066-3).

Reference
[1] Muhamadeeva V A and Sycheva N A 2018 About the aftershock processes accompanying moderate and weak earthquakes in the territory of the Bishkek geodynamic proving ground and in its vicinities J. Geosystems of transition zones 3, 2165–80
[2] Sychev V N, Sycheva N A and Imashev S A 2019 Study of the aftershock sequence of the Suusamyr earthquake J. Geosystems of transition zones 1, 3, 35–43
[3] Shebalin P N 2004 Aftershocks as indicators of the stress state in the fault system Doklady Earth Sciences 2, 398 p 249–54
[4] Molchan G M and Dmitrieva O E 1991 Identification of aftershocks: a review and new approaches J. Computational seismology. 24 pp 19–50
[5] Yunga S L 1990 Method and result the study of seismotectonic deformation (Moscow: Nauka) p 191
[6] Sycheva N A, Yunga S L, Bogomolov L M et al 2008 Seismotectonic deformations and recent tectonics of the Tien Shan J. Izvestiya Physics of the Solid Earth 5, 44, p 351–63
[7] Kostuk A D, Sycheva N A, Yunga S L et al 2010 Deformation of the earth crust in the Northern Tien Shan according to the earthquake focal data and satellite geodesy J. Izvestiya Physics of the Solid Earth 3, 230–43
[8] Omori F 1894 On after-shocks of earthquakes J. of the College of Science Imperial Univ. of Tokyo 7, 111–200
[9] Utsu T 1961 A statistical study on the occurrence of aftershocks Geophysical Magazine 30, 521–605
[10] Gulielmi A V 2016 Interpretation Omori's law J. Izvestiya Physics of the Solid Earth 5, 165–66. https://link.springer.com/article/10.1134/S1069351316050165
[11] Telesca L 2011 Tsallis-based nonextensive analysis of the Southern California seismicity

   Entropy 7. 13 127–1280

[12] Tsallis C 1988 Possible generalization of Boltzmann-Gibbs statistics J. of Statistical Physics 1–2.

   52 479–87

[13] Telesca L Cuomo V, Lapenna V, Vallianatos F et al 2001 Analysis of the temporal properties of

   Greek aftershock sequences J. Tectonophysics 1–4. 341 163–78

   www.gein.noa.gr/Greek/Staff/GD/WEB_PAPERS/B21_Tectonophysics_2001.pdf

[14] Telesca L and Chen C C 2010 Nonextensive analysis of crustal seismicity in Taiwan J. Nat.

   Hazards Earth Syst. Sci. 10 1293–97

[15] Chochlaki K, Michas G and Vallianatos F 2018 Complexity of the Yellowstone park volcanic

   field seismicity in Terms of Tsallis entropy Entropy 20 721

[16] Olsson R 1999 An estimation of the maximum b-values in the Gutenberg–Richter relation J. of

   Geodynamics 4–5. 27 547–52

[17] Utsu T, Ogata Y and Matsuura R S 1995 The centenary of the Omori formula for a decay law of

   aftershock activity J. of Physics of the Earth 43 1–33

[18] Kagan Y Y 2004 Short-term properties of earthquake catalogs and models of earthquake source

   J Bulletin of the Seismological Society of America 94 1207–28

[19] Mandal P, Chadha R K, Raju I P et al 2007 Are the 7 March 2006 Mw 5.6 event and the 3 February

   2006 Mw 4.58 event triggered by the five years continued occurrence of aftershocks of the

   2001 Mw 7.7 Bhuj event? J. Current Science 92 1114–24

[20] Ommi S, Zafarani H and Smirnov V B 2016 Bayesian estimation of the Modified Omori Law

   parameters for the Iranian Plateau J. Seismol 20 953–70

[21] Gulielmi A V 2017 Omori’s law: A note on the history of geophysics Advances in physical

   Sciences 3. 187 pp 343–48

[22] Silva R, Franca G S, Vilar C S et al 2006 Nonextensive models for earthquakes J. Phys. Rev. E.

   73 026102

[23] Prigoghin I and Stengers I 1986 Order out of chaos Man's new dialogue with nature English

   translation (Moscow: Progress) p 432