Study of spatial meson correlators at finite temperature in quenched anisotropic lattice QCD*

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We analyze the meson correlator in the spatial direction at finite temperature. To achieve fine resolution in the spatial direction, we use an anisotropic lattice with the standard Wilson plaquette gauge action and the $O(a)$ improved Wilson quark action. Below and above $T_c$, properties of correlators are investigated by two methods: fits with ansatz for the spectral function, and direct reconstruction of the spectral function using the maximum entropy method.

1. Introduction

The extraction of the spectral functions from the correlators measured in lattice QCD simulation is of great importance to understand the QCD phase transition at finite temperature, both in theoretical and phenomenological points of view. Our goal is to investigate the both of spatial and temporal structure of hadron correlators near the deconfining transition \[1\]. Particularly for the latter, one needs to develop techniques to extract detailed and reliable information from the correlators. On the other hand, the correlators in the spatial direction are rather easy to measure precisely, and therefore are expected to serve as a good test ground for the method. In this report, we focus on the spatial meson correlators at finite temperature, and develop procedures to analyze them. These analyses will give us insights also for the static screening structure of QCD at $T > 0$, which still remains not well understood \[2\].

We introduce a spectral function for the correlator propagating in the spatial direction in analogous to the ordinary spectral function obtained from the correlator propagating in the temporal direction. In order to reconstruct this spectral function from the lattice data, we apply two kinds of analysis procedures: (a) Fit with ansatz on the shape of spectral function. At present stage, we assume that the spectral function is represented by a sum of several strong peaks, i.e. delta functions, and fit the data to this form to obtain the screening masses corresponding to the peaks. We adopt single, two, and three pole(s) fittings. (b) Reconstruction of spectral function from the correlator by the maximum entropy method (MEM) \[3\]. Comparing the results of these two methods, we discuss their efficiency and reliability.

For these analysis, it is better to have as many points of correlators as possible. We use the anisotropic lattice which has small lattice spacing in $z$-direction to measure the spatial hadron correlators in high resolution. Although one needs additional effort for the calibration of anisotropy parameters, once it has been done, the systematic uncertainties due to the anisotropy can be controlled within reasonable accuracy \[4\]. In the following, we show only the results for the pseudoscalar meson correlators, while the procedures are applicable for other channel in the same manner.

2. Numerical Simulations and Results

Our numerical simulations are carried out on quenched lattices of sizes $12^2 \times 96 \times N_t$ where $N_t = 12$, 5, 3 and 2 which roughly correspond to the temperatures $\approx 0$, $0.8T_c$, $1.3T_c$ and $2T_c$, respectively.
respectively. The gauge configurations are generated with the Wilson plaquette action at \( \beta = 5.75 \) with the anisotropy \( \xi = a/a_z = 4 \), where \( a_z \) and \( a \) are the lattice spacings in \( z \)-direction and other three directions. With these parameters, the lattice cutoff set by the string tension is \( a^{-1}_z \simeq 4 \) GeV. Configuration numbers are 400 at \( T \simeq 0 \) and 510 at other \( T \)'s. The quark action is the \( O(a) \) improved Wilson action [4], at three values of hopping parameter, \( \kappa = 0.124, 0.122 \) and 0.120, which cover the range of quark mass \( m_q = (0.5 \sim 1.5) m_s \). The bare quark anisotropy is set as \( \gamma_F = \), which is determined using the meson dispersion relation from the lattice Klein-Gordon type action.

In the following, we show the results for the pseudoscalar meson channel at \( \kappa = 0.120 \) \( (m_q \simeq 1.5m_s) \). The fit is performed with the fitting region \([L_{\min}, L_{\max}]\), where \( L_{\max} \) is fixed to 48 and \( L_{\min} \) is varied. The screening masses as the results of single, double and triple pole fits are displayed in the figures for each value of \( L_{\min} \). Existence of a region where the masses are stable indicates that the assumed form of fit is a good representation of the spectral function. On the other hand, absence of such a region signals that the spectral function is not approximated by such a form. In both cases, we can extract the spectral function reconstructed by MEM as an alternative procedure. In MEM analysis we use the default model function \( m_0(\omega) = 16\omega^2 \), frequency region \( 0 \leq \omega \leq 2.0 \) and the fitting region of correlators \( 1 \leq t \leq 40 \).

Let us start with the zero temperature. The upper panel of Figure 1 shows the results fitting analysis. For each fit (i.e. number of poles), there is a region where the masses do not change within errors. For the ground and first excited states, three fits give consistent masses. This shows that the assumed form of spectral function (the sum of poles) well represent the correlator, in accord with the physical expectation. The spectral function reconstructed by MEM also exhibits consistent position of peaks, as shown in the lower panel of Fig. 1. Since the masses from the fits shows clear plateaus, and observing the errors associated to the regions of spectral function, the observed widths of peaks may not be practical ones but artifacts through the statistical and systematic errors MEM.

Now we apply the same procedures to the correlators at finite temperature. The result of fit method at \( 0.8T_c \) is almost the same as at \( T = 0 \). Therefore there seems no significant change in the spectral functions at \( T = 0.8T_c \) from that at \( T = 0 \). On the other hand, as shown in Figure 2, the result of MEM shows a deference: there is two peaks at the position of the first excited state obtained by the fits. This is unphysical structure can be understood by the failure of MEM in reconstructing the spectral function, probably due
to lack of statistics. In order to detect the structure of spectral function in high frequency region from MEM, one would probably need larger statistics that the present one which of $O(500)$.

Figure 3 shows the results at $1.3T_c$. We again find the regions of constant masses for each fit and a consistency in the masses of the ground and first excited states. Result of spectral function also have peaks at consistent positions. Thus we find that the sum of poles are a good approximation of the spectral function. The ground state mass is slightly less than the twice lowest Matsubara frequency. For the quantitative analysis, however, present lattice is so coarse that the number of temporal degrees of freedom is just three. We also observe similar result at $T = 2T_c$.

We conclude that these two procedures are promising as useful ways to extract the structure of the spectral function, although one need careful treatment particularly for MEM. Fits assuming various forms for the spectral function serve complementary way of analysis, and supply the reliability for the result of MEM. Such a study is particularly important for the analysis of temporal correlators, since the degrees of freedom is restricted even with anisotropic lattices and the data are inevitably in the high frequency region. In this case, since the spectral function have physical width, one also need to try other fitting forms, such as the peaks with widths. To obtain more quantitative result on the screening mass and the spectral function for the spatial correlators, we should use smaller lattice spacing as well as higher statistics.

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