Approach to Express The Second Moment with A Class of Stochastic Completion Time

(Running head: Second Moment with A Class of Stochastic Completion Time)

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1 Introduction

Glazebrook [1] (1984) investigated a class of scheduling problem with machine subject to breakdowns. In 1990, Birge et al. [2] provided a description of the processing environment with machine subject to a sequence stochastic breakdowns in the sense as follows.

For \( k = 1, 2, \cdots \), the \( k \)-th period of machine uptime and downtime is characterized by two nonnegative random variables \( U_k \) and \( D_k \). \( U_k \) represents the \( k \)-th machine uptime, namely the length of the period between the \((k - 1)\)-th and \( k \)-th breakdown. \( D_k \) represents the \( k \)-th machine downtime, namely the length of the \( k \)-th breakdown. Further, \( \{U_k\} \) and \( \{D_k\} \) are the sequences of independent and identically distributed (i.i.d.) nonnegative random variables.
Moreover, the uptimes are independent of the downtimes.

We call the process environment as BP Environment next.

Note that the machine can be interpreted as a dynamic system, such as a person, a market or a workshop; and the machine uptimes and downtimes can be interpreted as two kinds of different states, such as healthful and diseased, prosperous and desolate or robust state and non-robust. The BP Environment can be interpreted as an alternative system of two different states. Therefore, we can easily know that a lot of real situation can be modeled as the BP Environment. For example, working on line can be interpreted as a BP Environment for it is inevitably disturbed by some unexpected events, such as viruses, damages of component, power cuts, etc., randomly. Due to the health and mood of a person being change, we can divide the state of a person into two states, healthful state and diseased state, in terms of a threshold determined by theory and experience. That is, a person can be regarded as health (uptime) if his state satisfies the threshold, otherwise as disease (downtime). According to the same way, a economic system can also be interpreted as a BP Environment. Moreover, the traffic of cities, which is randomly blocked up, is either an important instance of BP Environment. In a word, the BP Environment is very important in practice, with which the stochastic scheduling problems have been extensively studied for recent twenty years, e.g. see [2-9].

For BP Environments, a job to be processed under a BP Environment is called as a preempt-repeat model job if the work done on the job is lost when a breakdown happen before it is completed, that is it must be processed from beginning after a breakdown except it has been completed before the breakdown happen, see [2]. The situation to process a repeat model job under a BP Environment can be found in many industrial application. It arise commonly in process industries, where the product requires continuous being processed without interruption. One example is metal refinery, where the raw materials are purified by melting them in very high temperature. If a breakdown (such as power outage) occurs before the metal is purified to
the required level, it will quickly cool down and the heating process has to start again after the breakdown is over. Other examples include running a program on a computer, downloading a file from the internet, implementing a reliability test on a facility, etc. Generally, if a job must be continuously processed with no interruption until it is completed totally, then the processing pattern of the job should be modeled by the preempt-repeat formulation in the presence of machine breakdowns.

Given a BP Environment, let $j$ be a repeat version job to process under the BP Environment and with processing time $p_k$ for $k$-th uptime, and $\{p_k\}$ be an i.i.d. sequence of random variables. Then, it is obvious that the real processing time $R$ of the job is a random variable. For the random variable $R$, one of important topic is to express its first and second moments $E[R]$ and $E[R^2]$. Under the conditions that $U_k$ follows an exponential distribution, $D_k$ is a continuous random variable with first moment and $p_k = t$ for any $k$, Birge et al.[2] presented an expression of $E[R]$ with some distribution characters of the uptime and downtime. In the present work, we address the second moment of $R$. We will propose an approach to express $E[R^2]$ with some distribution characters of the uptime, downtime and processing time under the conditions that $U_k, D_k$ and $p_k$ are general nonnegative continuous random variables.

The rest of the paper is arranged as follows. Section 2 presents the preliminary formulations. Section 3 is devoted to express the second moment. Section 4 provides a few of remarks. Section 5 make a simple conclusion.

2 Formulation

This section provides a further description of the problem we will attack and establishes necessary formulations for the sequel researchers.

Throughout the paper, all variables and constants are real-valued. And variables denoted by large letters or black body letters represent random variables. For example, $U_k$ expresses that the $k$-th uptime is a random variable; $p_j$ expresses that the processing time of job $j$ is a
We call the problem that we will attack to be SMCT (second moment of the completion time), its meaning is as follows.

**SMCT.** For a given BP Environment, process a preempt-repeat model job $j$ on the machine. Assume: (a) $\{U_k\}, \{D_k\}$ and $\{p_k\}$ are mutually independent, and all the sequences of i.i.d. and nonnegative continuous random variables; (b) $D_k$ is with finite first and second moment $\mu$ resp. $\nu$; (c) the machine is available at the beginning of process, that is, the uptimes and downtimes are arranged as: $U_1, D_1, U_2, D_2, \ldots, U_k, D_k, \ldots$; (d) The starting time of the job (the time when $j$ begin to process ) is zero. Let $R$ be the real completion time. Express $E[R]$ and $E[R^2]$ with some distribution characters of the uptime, downtime and processing time.

For convenience afterwards, we now construct the following formule expressions.

Let $I_A$ denote the indicator of an event $A$, which takes value 1 if $A$ occurs and otherwise 0. In terms of the meaning of $R$, if $R < +\infty$, then we have

$$R = p_1 \cdot I_{\{U_1 \geq p_1\}} + \left[ p_2 + (U_1 + D_1) \right] \cdot I_{\{U_1 < p_1, U_2 \geq p_2\}} + \cdots + \left[ p_n + \sum_{k=1}^{n-1} (U_k + D_k) \right] \cdot I_{\{U_1 < p_1, U_n \geq p_n; 1 \leq i \leq n-1\}} + \cdots$$

$$= p_1 \cdot I_{\{U_1 \geq p_1\}} + \sum_{n=2}^{\infty} \left[ p_n + \sum_{k=1}^{n-1} (U_k + D_k) \right] \cdot I_{\{U_1 < p_1, U_n \geq p_n; 1 \leq i \leq n-1\}}. \quad (1)$$

Note that

$$I_{\{U_1 < p_1, U_n \geq p_n; 1 \leq i \leq n-1\}} \cdot I_{\{U_1 < p_1, U_m \geq p_m; 1 \leq i \leq m-1\}} = \begin{cases} 0, & m \neq n \\ I_{\{U_1 < p_1, U_n \geq p_n; 1 \leq i \leq n-1\}}, & m = n. \end{cases}$$

When $R < +\infty$, (1) leads to

$$R^2 = p_1^2 \cdot I_{\{U_1 \geq p_1\}} + \left[ p_2 + (U_1 + D_1) \right]^2 \cdot I_{\{U_1 < p_1, U_2 \geq p_2\}} + \sum_{n=3}^{\infty} \left[ p_n + \sum_{k=1}^{n-1} (U_k + D_k) \right]^2 \cdot I_{\{U_1 < p_1, U_n \geq p_n; 1 \leq i \leq n-1\}}. \quad (2)$$

Finally, set $a = E[p_k | U_k \geq p_k], b = E[U_k | U_k < p_k], c = E[p_k^2 | U_k \geq p_k], d = E[U_k^2 | U_k < p_k]$
and \( q = P\{U_k < p_k\} \) for every \( k \geq 1 \).

3 Approach

This section focuses on expressing \( E[R] \) and \( E[R^2] \).

Theorem 1. For a given problem SMCT, if \( 0 < q < 1 \), then

\[
E[R] = a + (b + \mu) \cdot \frac{q}{1 - q}.
\]  

(3)

Proof. It is obvious that \( \{R < +\infty\} = \{U_1 \geq p_1\} \cup (\bigcup_{n=2}^{\infty} \{U_i < p_i, U_n \geq p_n : 1 \leq i \leq n - 1\}) \].

Note that \( \{U_i < p_i, U_n \geq p_n : 1 \leq i \leq n - 1\} \cap \{U_i < p_i, U_m \geq p_m : 1 \leq i \leq m - 1\} = \emptyset \) when \( n \neq m \). We have

\[
P\{R < +\infty\} = P\{U_1 \geq p_1\} + \sum_{n=2}^{\infty} P\{U_i < p_i, U_n \geq p_n : 1 \leq i \leq n - 1\}
\]

\[
= \sum_{n=1}^{\infty} q^{n-1}(1 - q) = 1.
\]

This shows (1) holds almost surely. Hence, we obtain

\[
E[R] = E[p_1 \cdot I\{U_1 \geq p_1\}] + \sum_{n=2}^{\infty} E[p_n \cdot I\{U_i < p_i, U_n \geq p_n : 1 \leq i \leq n - 1\}]
\]

\[
+ \sum_{n=2}^{\infty} E\left[\sum_{k=1}^{n-1} D_k \cdot I\{U_i < p_i, U_n \geq p_n : 1 \leq i \leq n - 1\}\right]
\]

\[
+ \sum_{n=2}^{\infty} \sum_{k=1}^{n-1} E[U_k \cdot I\{U_i < p_i, U_n \geq p_n : 1 \leq i \leq n - 1\}].
\]  

(4)
Note that \( \{p_k\}, \{U_k\} \) and \( \{D_k\} \) are mutually independent. From (4), we get

\[
E[R] = E[p_1|U_1 \geq p_1] \cdot p_1 \cdot \mathbb{P}(U_1 \geq p_1) + \sum_{n=2}^{\infty} E[p_n \cdot I(U_n \geq p_n)] \\
    \left( \prod_{i=1}^{n-1} \mathbb{P}(U_i < p_i) \right) + \sum_{n=2}^{\infty} (n-1)\mu \left( \prod_{i=1}^{n-1} \mathbb{P}(U_i < p_i) \right) \cdot \mathbb{P}(U_n \geq p_n) \\
    + \sum_{n=2}^{\infty} \sum_{k=1}^{n-1} E[U_k \cdot I(U_k < p_k)] \cdot \left( \prod_{i=1, i \neq k}^{n-1} \mathbb{P}(U_i < p_i) \right) \cdot \mathbb{P}(U_n \geq p_n)
\]

\[
= a(1-q) + \sum_{n=2}^{\infty} a(1-q) q^{n-1} + \sum_{n=2}^{\infty} (n-1)\mu (1-q) q^{n-1} \\
    + \sum_{n=2}^{\infty} (n-1)b(1-q) q^{n-1} \\
= a(1-q) + a(1-q) \left( \sum_{n=1}^{\infty} q^n \right) + (b+\mu)(1-q) q \left( \sum_{n=1}^{\infty} nq^{n-1} \right) \\
= a(1-q) + a(1-q) \cdot \frac{q}{1-q} + (b+\mu)(1-q) q \cdot \left( \sum_{n=1}^{\infty} q^n \right) \\
= a + (b+\mu)(1-q) q \cdot \frac{q}{1-q} = a + (b+\mu) \cdot \frac{q}{1-q} .
\]

This completes the proof.

**Theorem 2.** For a given problem SMCT, if \( 0 < q < 1 \), then

\[
E[R^2] = c + (2ab + 2\mu a + 2\mu b + d + \nu) \cdot \frac{q}{1-q} + 2(\mu + b)^2 \cdot \left( \frac{q}{1-q} \right)^2 .
\]
Proof. Note $P\{R < +\infty\} = 1$. By (2), we obtain
\[
E[R^2] = E[p_1^2 \cdot I_{(U_1 \geq p_1)}] + E[(p_2 + (U_1 + D_1))^2 \cdot I_{(U_1 < p_1, U_2 \geq p_2)}]
+ \sum_{n=3}^{\infty} E[p_n^2 \cdot I_{(U_n < p_n, U_{n+1} \geq p_{n+1} \leq i \leq n-1)}]
+ 2(\sum_{n=3}^{\infty} E[p_n((\sum_{k=1}^{n-1} (U_k + D_k)) \cdot I_{(U_n < p_n, U_{n+1} \geq p_{n+1} \leq i \leq n-1)})]
+ \sum_{n=3}^{\infty} E[\sum_{k=1}^{n-1} U_k \cdot U_{i+1} \cdot I_{(U_n < p_n, U_{n+1} \geq p_{n+1} \leq i \leq n-1)}]
+ 2(\sum_{n=3}^{\infty} E[(\sum_{k=1}^{n-1} U_k \cdot D_k) \cdot I_{(U_n < p_n, U_{n+1} \geq p_{n+1} \leq i \leq n-1)})]
+ \sum_{n=3}^{\infty} E[\sum_{k=1}^{n-1} U_k \cdot U_{i+1} \cdot D_k \cdot D_1 \cdot I_{(U_n < p_n, U_{n+1} \geq p_{n+1} \leq i \leq n-1)}].
\]

On the other hand, we have
\[
E[(p_2 + (U_1 + D_1))^2 \cdot I_{(U_1 < p_1, U_2 \geq p_2)}]
= E[p_2^2 + 2p_2(U_1 + D_1) + U_1^2 + 2U_1D_1 + D_1^2 \cdot I_{(U_1 < p_1, U_2 \geq p_2)}]
= E[(p_2^2 \cdot I_{(U_2 \geq p_2)}) \cdot I_{(U_1 < p_1)}]
+ 2E[(U_1 \cdot I_{(U_1 < p_1)}) \cdot (p_2 \cdot I_{(U_2 \geq p_2)})]
+ 2E[(D_1 \cdot I_{(U_1 < p_1)}) \cdot (p_2 \cdot I_{(U_2 \geq p_2)})]
+ E[(U_1^2 \cdot I_{(U_1 < p_1)}) \cdot I_{(U_2 \geq p_2)}]
+ 2E[D_1 \cdot (U_1 \cdot I_{(U_1 < p_1)}) \cdot I_{(U_2 \geq p_2)}]
+ E[D_1^2 \cdot I_{(U_1 < p_1, U_2 \geq p_2)}]
= c(1 - q)q + 2ab(1 - q)q + 2\mu a(1 - q)q +
\]
\[
d(1 - q)q + 2\mu b(1 - q)q + \nu(1 - q)q,
\]
\[
E[(p_n^2 \cdot I_{(U_n < p_n, U_{n+1} \geq p_{n+1} \leq i \leq n-1)}]
= E[(p_n^2 \cdot I_{(U_n \geq p_n)}) \cdot I_{(U_n < p_n, U_{n+1} \geq p_{n+1} \leq i \leq n-1)}]
= E[(p_n^2 | U_n \geq p_n) \cdot P(U_n \geq p_n) \cdot (\prod_{i=1}^{n-1} P(U_i < p_i))] = c(1 - q)q^{n-1};
\]
\[
\sum_{n=3}^{\infty} c(1-q)q^{n-1} = c(1-q)(\sum_{n=3}^{\infty} q^{n-1}) = c(1-q) \cdot \frac{q^2}{(1-q)} = cq^2,
\]

(8)'

\[
E[p_n(\sum_{k=1}^{n-1} (U_k + D_k)) \cdot I\{U_i < p_i, U_n \geq p_n : 1 \leq i \leq n-1\}] \\
= \sum_{k=1}^{n-1} E[p_n \cdot I\{U_n \geq p_n\} \cdot ((U_k \cdot I\{U_k < p_k\}) \cdot I\{U_i < p_i : 1 \leq i \leq n-1, i \neq k\}] \\
\quad + D_k \cdot I\{U_i < p_i : 1 \leq i \leq n-1\}] \\
= \sum_{k=1}^{n-1} [a(1-q) \cdot bq \cdot q^{n-2} + a(1-q) \cdot \mu \cdot q^{n-1}] \\
= (n-1)ab(1-q)q^{n-1} + (n-1)a\mu(1-q)q^{n-1};
\]

(9)'

\[
\sum_{n=3}^{\infty} [(n-1)ab(1-q)q^{n-1} + (n-1)a(1-q) \cdot \mu \cdot q^{n-1}] \\
= a(b + \mu)(1-q)q(\sum_{n=2}^{\infty} nq^{n-1}) = a(b + \mu)(1-q)q(\sum_{n=2}^{\infty} q^n)'
\]

(9)''

\[
E[(\sum_{k=1}^{n-1} \sum_{l=1}^{n-1} U_k U_l) \cdot I\{U_i < p_i, U_n \geq p_n : 1 \leq i \leq n-1\}] \\
= \sum_{k=1}^{n-1} E[U_k^2 \cdot I\{U_i < p_i, U_n \geq p_n : 1 \leq i \leq n-1\}] + \\
\sum_{1 \leq k, l \leq n-1, k \neq l} E[U_k U_l \cdot I\{U_i < p_i, U_n \geq p_n : 1 \leq i \leq n-1\}] \\
= \sum_{k=1}^{n-1} E[(U_k \cdot I\{U_k < p_k\}) \cdot I\{U_i < p_i, U_n \geq p_n : 1 \leq i \leq n-1, i \neq k\}] + \\
\sum_{1 \leq k, l \leq n-1, k \neq l} E[(U_k \cdot I\{U_k < p_k\}) \cdot (U_l \cdot I\{U_l < p_l\}) \cdot I\{U_i < p_i, U_n \geq p_n : 1 \leq i \leq n-1, i \neq k, l\}] \\
= (n-1)d(1-q)q^{n-1} + (n-2)(n-1)b^2(1-q)q^{n-1};
\]

(10)'''

\[
\sum_{n=3}^{\infty} [(n-1)d(1-q)q^{n-1} + (n-2)(n-1)b^2(1-q)q^{n-1}] \\
= d(1-q)q(\sum_{n=2}^{\infty} nq^{n-1}) + b^2(1-q)q^2(\sum_{n=2}^{\infty} n(n-1)q^{n-2}) \\
= d(1-q)q(\sum_{n=2}^{\infty} q^n)' + b^2(1-q)q^2(\sum_{n=2}^{\infty} q^n)''
\]

(10)"
\[ E[(\sum_{k=1}^{n-1} \sum_{l=1}^{n-1} U_k D_l) \cdot I\{U_i < p, U_n \geq p_n : 1 \leq i \leq n-1\}] \]

\[ = \sum_{k=1}^{n-1} \sum_{l=1}^{n-1} E[D_k \cdot (U_k \cdot I\{U_k < p\}) \cdot I\{U_i < p : 1 \leq i \leq n-1, i \neq k\} \cdot I\{U_n \geq p_n\}] \]

\[ = \sum_{k=1}^{n-1} (n-1) bq\mu q^{n-2}(1-q) = (n-1)^2 \mu b(1-q)q^{n-1}; \quad (11) \]

\[ \sum_{n=3}^{\infty} (n-1)^2 \mu b(1-q)q^{n-1} \]

\[ = \mu b(1-q)[\sum_{n=3}^{\infty} (n-1)(n-2)q^{n-1} + \sum_{n=3}^{\infty} (n-1)q^{n-1}] \]

\[ = \mu b(1-q)[(\sum_{n=2}^{\infty} n(n-1)q^{n-2})q^2 + (\sum_{n=2}^{\infty} nq^{n-1})q] \]

\[ = \mu b(1-q)[(\sum_{n=2}^{\infty} q^n)''q^2 + (\sum_{n=2}^{\infty} q^n)'q] \]

\[ = \frac{\mu b(2-q)q^2}{1-q} + 2\mu bq^2 \frac{q}{(1-q)^2}, \quad (11)' \]

\[ E[\sum_{k=1}^{n-1} \sum_{l=1}^{n-1} D_k D_l \cdot I\{U_i < p, U_n \geq p_n : 1 \leq i \leq n-1\}] \]

\[ = (\sum_{k=1}^{n-1} E[D_k^2] + \sum_{1 \leq k, l \leq n-1, k \neq l} E[D_k D_l]) \cdot q^{n-1}(1-q) \]

\[ = [(n-1)\nu + (n-1)(n-2)\mu^2](1-q)q^{n-1}; \quad (12) \]

\[ \sum_{n=3}^{\infty} [(n-1)\nu + (n-1)(n-2)\mu^2](1-q)q^{n-1} \]

\[ = \nu(1-q)[\sum_{n=3}^{\infty} (n-1)q^{n-1}] + \mu^2(1-q)[\sum_{n=3}^{\infty} (n-1)(n-2)q^{n-1}] \]

\[ = \nu(1-q)q(\sum_{n=2}^{\infty} q^n)' + \mu^2(1-q)q^2(\sum_{n=2}^{\infty} q^n)' \]

\[ = \nu(2-q)q^2 + 2\nu^2 q^2 \frac{(1-q)^2}{(1-q)^2}, \quad (12)' \]
Combining (6)-(12), we obtain

\[ E[R^2] = c(1 - q) + (c + 2ab + 2\mu a + 2\mu b + d + \nu)(1 - q)q + \\
cq^2 + \frac{2a(b+\mu)a^2(2-q)}{1-q} + \frac{[\mu(2-q)a^2]}{1-q} + \\
\frac{2\mu b^2}{1-q} + \frac{2\mu q^2}{1-q} + \frac{2\mu^2 q^2}{1-q}, \]

\[ = c[(1 - q) + (1 - q)q + q^2] + 2a[(b + \mu)(1 - q)q + \frac{(b+\mu)(2-q)q^2}{1-q}] + \\
d[(1 - q)q + \frac{(2-q)q^2}{1-q}] + \nu[(1 - q)q + \frac{(2-q)q^2}{1-q}] + \\
2\mu b[(1 - q)q + \frac{(2-q)q^2}{1-q}] + \frac{2\mu^2 q^2}{1-q} + 2\mu q^2, \]

\[ = c + [2ab + 2\mu(a + b) + d + \nu] \cdot \frac{q}{1-q} + 2(\mu + b)^2 \cdot \left(\frac{q}{1-q}\right)^2. \]

The Proof completes.

4 Remarks

(i) For the Theorem 1, actually \(D_k\) needs only to possess first moment.

(ii) When \(q = 0\), we have \(P\{U_1 \geq p_1\} = 1\). Thus \(R = p_1\) almost surely. Furthermore, \(E[R] = E[p_1]\) and \(E[R^2] = E[p_1^2]\). When \(q = 1\), \(P\{U_k < p_k\} = 1\) for all \(k \geq 1\). So \(j\) cannot be completed almost surely in the case.

(iii) Suppose that \(U_k\) follows nonnegative exponential distribution with parameter \(\lambda\) and \(p_k = t\) for all \(k \geq 1\). Then

\[ a = E[t|U_1 \geq t] = t, \]

\[ b = E[U_1|U_1 < t] = \frac{\int_0^t \lambda e^{-\lambda s} ds}{P(U_1 < t)} = \frac{1}{\lambda} - \frac{t}{e^{\lambda t} - 1}, \text{ and} \]

\[ q = P\{U_1 < t\} = \int_0^t \lambda e^{-\lambda s} ds = 1 - e^{-\lambda t}. \]

Substituting \(a, b\) and \(q\) with these results in (3) respectively, we obtain

\[ E[R] = \left(\frac{1}{\lambda} + \mu\right)(e^\lambda - 1). \]

This is the same with the result of Birge et al. [2].

(iv) When \(\mu = o(1)\) and \(\nu = o(1)\), we call the Birge Processing Environment as Instan-
taneous Birge Processing Environment (i.e., Birge Processing Environments with stochastic instantaneous breakdowns). Under this situation, we have

\[
E[R] = a + \frac{bq}{1-q} + o(1),
\]

\[
E[R^2] = c + (2ab + d) \cdot \frac{q}{1-q} + 2b^2 \left( \frac{q}{1-q} \right)^2 + o(1).
\]

In practice, for large part conditions, \( \mu \) and \( \nu \) are actually small enough to processing time \( p \) and uptime \( U_k \). This is often the realistic case. That is, the Instantaneous Birge Processing Environment is very important in practice.

5 Conclusion

We have successfully express the second moment of the completion time about a preempt-repeat model job processed on a machine subject to stochastic breakdowns, by some distribution characters of the uptime, the downtime and the processing time. The result and the method we use are going to largely stimulate the development of the research on the problems with machines subject to stochastic breakdowns.

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