Comment on the $S$-wave masses of singly heavy mesons

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Based on the study of the string model methods of singly heavy mesons and singly heavy baryons, we calculate the mass spectrum of $1S$- and $2S$-wave for both charm and bottom mesons ($D/D_s, B/B_s$).

Experimentally, there are most masses spectra of $1S$-wave have been found, while the masses part of the $2S$-state is not determined. In this paper, we will use singly light quark or diquark model images and Regge trajectory models, combined with perturbation processing methods, to analyze and study the observed singly heavy mesons, further predict the unobserved mesons masses and their corresponding spin-parity quantum numbers.

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I. INTRODUCTION

The $D/D_s$ mesons with charm quark and the $B/B_s$ mesons with bottom quark are typical heavy-light mesons, which is structurally analogous to a the hydrogen-like atoms (the singly light antiquark and the heavy quark resemble the extranuclear electron and the proton, respectively). In recent decades, more and more singly heavy (SH) mesons states $Qar{q}$ ($Q = c, b$; $\bar{q} = \bar{n}, \bar{s}$) have been discovered by BaBar, Belleale, CLEO, and the LHCb experiment [1], so the study of heavy-light mesons system has been attracting great attention. It can be seen from PDG [2] that the experimental values of some low energy $D, D_s$ mesons [3] have been basically determined. Since the experimental observation of the singly charm mesons states $D(2550), D^*(2600), D(2750)$ and $D^*(2760)$, there have been different research methods [4–10] for the calculation and analysis of these charm mesons states. For example, $^3P_0$ model [11–13], Chiral quark model

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[14], lattice QCD model [4, 5, 15], other models [16–20], etc. In addition, the high energy $B, B_s$ mesons [21–25] have been extensively studied, there are different interpretations for the bottom meson states, such as $B_f(5840), B'_f(5970)$ [26–31]. So far, these quantum states are still controversial and need to be further confirmed by experiments.

In this paper, the purpose of this work is to calculate the masses of $1S$- and $2S$-wave $D/D_s$ and $B/B_s$ mesons states from the linear Regge trajectory (spin independent mass) formula and the spin dependent potential (spin dependent mass) with the corresponding spin-parity quantum numbers $J^P = 0^-$ or $1^-$, respectively. In Table I, we list the heavy quark and singly light quark masses of charm and bottom mesons, and the string tensions $\alpha(Q\bar{q})$.

### Table I: The effective masses(GeV) of quarks determined by Regge trajectory and the relativistic quark model are compared, with $\alpha$ in GeV$^{-2}$.

| Parameters | $M_c$ | $M_b$ | $m_n$ | $m_s$ | $\alpha(c\bar{n})$ | $\alpha(c\bar{s})$ | $\alpha(b\bar{n})$ | $\alpha(b\bar{s})$ |
|------------|-------|-------|-------|-------|----------------------|----------------------|----------------------|----------------------|
| Ref.[32]   | 1.44  | 4.48  | 0.23  | 0.328 | 0.223                | 0.249                | 0.275                | 0.313                |
| EFG [33]   | 1.55  | 4.88  | 0.33  | 0.5   | 0.64/0.58            | 0.68/0.64            | 1.25/1.21            | 1.28/1.23            |

This paper is organized as follows. We analyze the Regge trajectory formula to give $S$-wave spin-average masses in Sec. II. In Sec. III, we review about the spin-dependent Hamiltonian. In Sec. IV, we talk about the scaling relations. In Sec. V, we mainly employ the Breit-Fermi spin interaction to calculate their spin coupling parameters and wave functions. We present conclusions in Sec. VI

### II. THE REGGE TRAJECTORY AND THE SPIN AVERAGE MASSES

Considering the color interaction between quarks and quarks in the heavy-light mesons system, in order to estimate the masses splitting in orbitally excited charm and bottom mesons, we use Regge-like masses relation [34] to comprehensively analyze the whole singly heavy system of mesons,

$$ (\tilde{M}_L - M_Q)^2 = \alpha \pi L + a_0, \quad (1) $$

where, $\tilde{M}_L, M_Q$ are the spin-average mass and the heavy quark mass of the singly heavy meson, respectively. $L$ is the orbital angular momentum of the mesons system($L = 0, 1, 2, \cdots$). $\alpha$ is the QCD string tension coefficient between heavy quark and light antiquark. The intercept factor $a_0$ depends on the light antiquark mass $m_{\bar{q}}$ and the non-relativistic kinematic energy $P_Q^2/M_Q$ of the heavy quark,

$$ a_0 = (m_{\bar{q}} + \frac{P_Q^2}{M_Q})^2, \quad (2) $$
note that non-relativistic kinematic 3–momentum of heavy quark in Eq. (2) has been associated with both 
\( M_Q \) and \( v_Q \),

\[
P_Q \equiv M_Q v_Q, \quad v_Q = \left(1 - \frac{m_{bareQ}^2}{M_Q^2}\right)^{1/2},
\]

(3)

here, \( v_Q \) is the velocity of the heavy quark, and the 3–momentum \( P_Q \) is conserved in the heavy quark limit of \( M_Q \to \infty \).

Using Eqs. (1), (2) and (3), one can obtain the spin-averaged masses \([32, 35]\) by

\[
\bar{M}_L = M_Q + \sqrt{\alpha \pi (L + \frac{\pi}{2} n) + \left(m_{\bar{q}} + M_Q \left(1 - \frac{m_{bareQ}^2}{M_Q^2}\right)\right)^2},
\]

(4)

where, \( m_{bareQ} \) and \( m_{\bar{q}} \) are the heavy quark bare mass of singly heavy meson and the singly light anti-
quark mass, respectively. \( n \) is a radial quantum number \((n = 0, 1, 2, \cdots)\). The selection of these parameters is listed in Table I. We get the 1S-wave spin-average mass \((L = 0, n = 0)\) in Eq. (4) is

\[
\bar{M}_L = M_Q + \left(m_{\bar{q}} + M_Q \left(1 - \frac{m_{bareQ}^2}{M_Q^2}\right)\right). \quad \text{The 2S-wave spin-average mass} \quad (L = 0, n = 1) \quad \text{in Eq. (4) is}
\]

\[
\bar{M}_L = M_Q + \sqrt{\alpha \pi \times \frac{\pi}{2} + \left(m_{\bar{q}} + M_Q \left(1 - \frac{m_{bareQ}^2}{M_Q^2}\right)\right)^2}. \quad \text{Given trajectory parameters in Table I, one can pre-
dict Regge trajectories of the charm excited mesons} \ D/D_s \text{ in FIG. 1–2 and the bottom excited mesons} \ B/B_s \text{ in FIG. 3–4 with radial quantum number} \ n = 0, 1, 2, 3 \text{ and 4}.
\]

![FIG. 1: D meson spin-average mass](image1)

![FIG. 2: D_s meson spin-average mass](image2)

Regge trajectories of the heavy-light hadron systems relating the shifted spin-averaged mass squared to
the orbital angular momentum \( L \) of the systems, with the parameters in Table I corresponding to Eq. (4).
The red solid circles correspond to the observed (mean) masses, and the empty circles indicate the predicted value in FIG. 1 – 4.
III. THE SPIN-DEPENDENT POTENTIAL

Due to the spin-spin interaction between singly light antiquark and heavy quark of singly heavy mesons, in order to estimate the mass splitting, we consider the spin-dependent Hamiltonian $H^{SD}$ [36, 37],

$$H^{SD} = a_1 \mathbf{L} \cdot \mathbf{S}_q + a_2 \mathbf{L} \cdot \mathbf{S}_Q + b S_{12} + c S_q \cdot S_Q,$$

where, the first two terms are spin-orbit interactions, the third is the tensor energy, and the last is the contact interaction between the heavy quark spin $\mathbf{S}_Q$ and the antiquark spin $\mathbf{S}_q$. Here, $a_1, a_2, b, c$ are spin coupling parameters.

For the singly heavy mesons, the singly light antiquark spin and the heavy quark spin are $S_q = \frac{1}{2}$, $S_Q = \frac{1}{2}$, respectively. Therefore, there are two kinds of the total spin $S$, one is 0 and the other is 1. In the scheme of $LS$ coupling, note that the total angular momentum $J = S + L$. Coupling of $L = 0$ with the spin $S = 0$ gives states with the total angular momentum $J = 0$, while coupling with $S = 1$ leads to states the angular momentum $J = 1$.

We consider the $S$-wave ($L = 0$) states in $Q\bar{q}$ mesons case. Then, the first three terms of in Eq. (5) are eliminated, only the last term survives,

$$H^{SD} = c S_q \cdot S_Q.$$  

It is very convenient to analyze spin-spin interaction into the non-trivial terms for the mass splitting, the eigenvalues (two diagonal elements) of $<\mathbf{S}_q \cdot \mathbf{S}_Q>$ can be obtained,

$$<\mathbf{S}_q \cdot \mathbf{S}_Q> = [S(S + 1) + S_Q(S_Q + 1) + S_q(S_q + 1)]/2,$$
\[
< S_q \cdot S_Q > = \begin{bmatrix}
\frac{3}{4} & 0 \\
0 & \frac{1}{4}
\end{bmatrix}.
\]  
(8)

combining with Eqs. (4) and (8), the S-wave masses are,

\[
M(S) = \bar{M}_L + c \begin{bmatrix}
\frac{3}{4} & 0 \\
0 & \frac{1}{4}
\end{bmatrix}.
\]  
(9)

Using the above Eq.(9), we can calculate the mass spectrum of the charm and the bottom mesons states \((D/D_s, B/B_s)\). There is much discussion about the hyperfine splitting terms, and here we mainly determine the parameter values and then derive the effective mass of the mesons.

**IV. THE HYPERFINE SPLITTING PARAMETERS**

It is possible to apply the charm and bottom mesons masses of \(M(D, 1S) = 1869.59\text{MeV} \), \(M(D^*, 1S) = 2010.26\text{MeV} \) and \(M(D_s, 1S) = 1968.3\text{MeV}, M(D_s^*, 1S) = 2112.2\text{MeV} \) [1]. A linear relationship between them is their spin-weighted sum:

\[
\sum_{J} (2J+1) \Delta \bar{M} = 0,
\]

\[
\bar{M}_{D,1S} = \frac{1869.59\text{MeV} + 3 \times 2010.26\text{MeV}}{4} = 1975.09\text{MeV},
\]

\[
\bar{M}_{D_s,1S} = \frac{1968.3\text{MeV} + 3 \times 2112.2\text{MeV}}{4} = 2076.23\text{MeV},
\]

the spin-averaged masses calculated by Eq. (4) are nearly equal to the results of the experimental value, namely, Eq. (10) and Eq. (11), respectively. In addition, the mass splitting are

\[
c(D, 1S) = M(1^3S_1, 1^-) - M(1^1S_0, 0^-) = 140.6\text{MeV},
\]

\[
c(D_s, 1S) = M(1^3S_1, 1^-) - M(1^1S_0, 0^-) = 143.9\text{MeV}.
\]

From Eqs. (12) and (13), it is assumed that \(c(D, 1S) \approx c(D_s, 1S)\) is about 140MeV in the charm meson of the 1S-wave. However, one can use a similar approach in the 1S-wave bottom mesons \((B/B_s)\), corresponding to the scaling relations from the charm meson \((c\bar{n})\) and then to the bottom meson \((b\bar{s})\), as well as the following rough estimate of the parameter \(c\),

\[
c(b\bar{s}) \approx c(b\bar{n}).
\]

To estimate the mass of the bottom flavor meson with vector singly light quarks \((\text{spin } S_q = 1/2)\), we use the spin coupling parameters obtained from experimental measurements \(D\) and \(D_s\) mesons values, which should theoretically have an approximate mapping with the following scaling relationship,

\[
c(b\bar{q}) = \frac{M_c}{M_b} c(c\bar{q}),
\]

(15)
with the heavy quarks relation \( M_c/M_b \) for the parameter of \( c(B, 1S), c(B_s, 1S) \) are about 45 MeV. We will show the results of our calculations, as compared with the experimental values, with reasonable values of the parameters achieved.

V. SPIN-SPIN COUPLINGS IN RELATIVIZED QUARK MODEL

In this section, we mainly analyze the mass splitting of the singly heavy meson in a relativistic quark model, in which the Breit-Fermi spin interaction is used to calculate the spin coupling parameter \( c \) and compare it with the experimentally matched ones.

For this, we consider the quasi-static potential of the Brett-Fermi spin interaction [6, 38] is

\[
V_{\text{quasi-static}}^L = V(r) + S(r) + \left( \frac{V' S}{r} \right) L \cdot \left( \frac{S}{2m_q} + \frac{S}{2M_Q} \right) \\
+ \left( \frac{V'}{r} \right) L \cdot \left( \frac{S}{m_qM_Q} \right) + \frac{1}{3m_qM_Q} \left( \frac{V'}{r} - V'' \right) S_{12} \\
+ \frac{1}{3m_qM_Q} \left( \nabla^2 V(r) \right) S_1 \cdot S_2,
\]

(16)

here, \( V \) and \( S \) are the respective vector and scalar potentials, and \( V', S' \) and \( V'' \) their derivatives. In \( S \)-wave \((L = 0)\), the \( L \cdot S \) terms will be eliminated, the spin-spin interaction of the last term contains a delta function in configuration space and is treated nonperturbatively in Eq. (16).

\[
V_{\text{L}=0}^{\text{quasi-static}} = V(r) + S(r) + \frac{2k_s \times 4\pi}{3m_qM_Q} \langle \tilde{\delta}(r) \rangle S_1 \cdot S_2,
\]

(17)

\[
V(r) = \frac{k_s}{r},
\]

(18)

where \( k_s = -\frac{3\alpha_s}{4} \), and \( \alpha_s \) is the strong coupling. One finds

\[
c = \frac{2k_s \times 4\pi}{3m_qM_Q} \langle \tilde{\delta}(r) \rangle.
\]

(19)

In Ref. [39], \( \tilde{\delta}(r) = \left( \frac{3\sigma}{\sqrt{\pi}} \right)^3 e^{-\sigma^2r^2} \), which is a Gaussian-smeared term with a parameter \( \sigma \) and the quantum average \( \langle \rangle \) is made over the S-wave wavefunction \( R_{nL}(r) \) of the meson \((Q\bar{q})\) system. Then, the S-wave wavefunction \( R_{nL}(r) \) with \( n = 0, 1, 2, 3, 4 \) and \( L = 0 \) of the mesons are shown in Figure 5–8.

In order to get the masses of the S-wave meson system, there are the second parameters \((\alpha_s, \sigma)\) are determined by fitting the mass spectrum, and the values of the parameters \( c_1, c_2, c_3, c_4 \) and \( c_5 \) obtained by fitting with Eq. (19) correspond to \( 1S, 2S, 3S, 4S \) and \( 5S \) waves in Table II, respectively. The fitting result of parameters \( c \) is consistent with the value calculated by Eqs. (14) and (15).

With the help of the relation Eq. (19) for fitting the \( S \)-wave masses, the results are shown in the Table III. One can find that our results for excited charm mesons \( D_0(2550) \) and \( D^*(2640) \) are considered as the
TABLE II: Model Parameters (in GeV).

| State | \( \alpha_s \) | \( \sigma \) | \( c_1 \) | \( c_2 \) | \( c_3 \) | \( c_4 \) | \( c_5 \) |
|-------|----------------|----------------|--------|--------|--------|--------|--------|
| \( D \) | 0.564 | 0.230 | 0.143 | 0.089 | 0.049 | 0.022 | 0.005 |
| \( D_s \) | 0.537 | 0.260 | 0.144 | 0.100 | 0.048 | 0.012 | 0.003 |
| \( B \) | 0.532 | 0.216 | 0.041 | 0.027 | 0.016 | 0.007 | 0.002 |
| \( B_s \) | 0.526 | 0.200 | 0.048 | 0.038 | 0.022 | 0.006 | 0.001 |
TABLE III: The S-wave mass spectrum (MeV) of the charmed and bottomed mesons are given and compared with the different quark models.

| State $J^P$ | Meson | Mass   | Ours | GI [18, 40] | EFG [33] | Mass   | Ours | GI [18, 40] | EFG [33] |
|-------------|-------|--------|------|-------------|----------|--------|------|-------------|----------|
| $1^1S_0$ 0° | $D^\pm$ | 1869.59 | 1870.86 | 1877 | 1871 | $D_s$ | 1968.3 | 1968.18 | 1979 | 1969 |
| $1^3S_1$ 1° | $D^*$ | 2010.26 | 2014.13 | 2041 | 2010 | $D_s^*$ | 2112.2 | 2112.35 | 2129 | 2111 |
| $2^1S_0$ 0° | $D_0$ | 2549 ± 16 | 2551.58 | 2581 | 2581 |   | 2642.42 |   | 2673 | 2688 |
| $2^3S_1$ 1° | $D^*$ | 2637 ± 2 | 2640.86 | 2643 | 2632 | $D_{s1}^*$ | 2708.3 | 2742.72 | 2732 | 2731 |
| $3^1S_0$ 0° |   | 2981.16 | 3068 | 3062 | 3094.93 |   | 3154 | 3219 |
| $3^3S_1$ 1° |   | 3029.66 | 3110 | 3096 | 3143.35 |   | 3193 | 3242 |
| $4^1S_0$ 0° |   | 3318.03 | 3468 | 3452 | 3453.16 |   | 3547 | 3652 |
| $4^3S_1$ 1° |   | 3339.75 | 3497 | 3482 | 3464.92 |   | 3575 | 3669 |
| $5^1S_0$ 0° |   | 3601.57 | 3814 | 3793 | 3743.86 |   | 3894 | 4033 |
| $5^3S_1$ 1° |   | 3606.49 | 3837 | 3822 | 3746.42 |   | 3912 | 4048 |
| $1^1S_0$ 0° | $B^0$ | 5279.6 | 5279.17 | 5312 | 5280 | $B_s$ | 5366.9 | 5366.58 | 5394 | 5372 |
| $1^3S_1$ 1° | $B^*$ | 5324.7 | 5320.27 | 5371 | 5326 | $B_s^*$ | 5414.4 | 5414.15 | 5450 | 5414 |
| $2^1S_0$ 0° | $B_J$ | 5851 ± 19 | 5892.94 | 5904 | 5890 |   | 6000.83 | 5984 | 5976 |
| $2^3S_1$ 1° | $B_J$ | 5964 ± 5 | 5920.12 | 5933 | 5906 |   | 6039.26 | 6012 | 5992 |
| $3^1S_0$ 0° |   | 6316.81 | 6335 | 6379 | 6450.78 |   | 6410 | 6467 |
| $3^3S_1$ 1° |   | 6332.52 | 6355 | 6387 | 6473.15 |   | 6429 | 6475 |
| $4^1S_0$ 0° |   | 6661.00 | 6689 | 6781 | 6820.55 |   | 6759 | 6874 |
| $4^3S_1$ 1° |   | 6667.82 | 6703 | 6786 | 6826.35 |   | 6773 | 6879 |
| $5^1S_0$ 0° |   | 6956.72 | 6997 | 7129 | 7133.48 |   | 7063 | 7231 |
| $5^3S_1$ 1° |   | 6958.36 | 7008 | 7133 | 7134.81 |   | 7076 | 7235 |

Candidate of the 2S-wave, while our model calculations for the experimentally determined $M(D^*_{s1}(2700)) = 2708.3MeV$ meson is slightly 40MeV higher. On the other hand, according to the LHCb Collaboration’s suggestion [27], the $B_J(5840)$ and $B_J(5970)$ can be $2^1S_0$ and $2^3S_1$ states, respectively, the mass splitting is about 110$MeV$, which appears relatively large compared to our value of 27$MeV$ in Table II.

VI. CONCLUSIONS

In this paper, one can based on studying the singly heavy mesons ($D/D_s$) and ($B/B_s$) of the quark and antiquark bound state system. We mainly combine the linear Regge trajectories and the relativistic quark model, discuss the similarity scaling relationship of the spin coupling parameters, and then revisit the 2S-wave and 3S-wave mass spectrums of the spin parity $J^P = 0^-$ or $1^-$, and predict the higher energy $S$-wave
states mass spectrum of mesons.

Therefore, comparing the experimental data of heavy-light mesons with the predictions of existing theoretical models will provide favorable evidence for the internal interaction and the internal structure of the mesons.

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