RADIO AND GAMMA-RAY PULSED EMISSION FROM MILLISECOND PULSARS

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ABSTRACT

Pulsed γ-ray emission from millisecond pulsars (MSPs) has been detected by the sensitive Fermi space telescope, which sheds light on studies of the emission region and its mechanism. In particular, the specific patterns of radio and γ-ray emission from PSR J0101-6422 challenge the popular pulsar models, e.g., outer gap and two-pole caustic models. Using the three-dimensional annular gap model, we have jointly simulated radio and γ-ray light curves for three representative MSPs (PSR J0034-0534, PSR J0101-6422, and PSR J0437-4715) with distinct radio phase lags, and present the best simulated results for these MSPs, particularly for PSR J0101-6422 with complex radio and γ-ray pulse profiles, and for PSR J0437-4715 with a radio interpulse. We have found that both the γ-ray and radio emission originate from the annular gap region located in only one magnetic pole, and the radio emission region is not primarily lower than the γ-ray region in most cases. In addition, the annular gap model with a small magnetic inclination angle instead of an “orthogonal rotator” can account for the MSPs’ radio interpulse with a large phase separation from the main pulse. The annular gap model is a self-consistent model not only for young pulsars but also MSPs, and multi-wavelength light curves can be fundamentally explained using this model.

Key words: acceleration of particles – gamma rays: stars – pulsars: general – pulsars: individual (J0034-0534, J0101-6422, J0437-4715) – radiation mechanisms: non-thermal

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1 INTRODUCTION

Millisecond pulsars (MSPs) are a population of old neutron stars with short spin period, $P_0 \lesssim 20$ ms, and small characteristic magnetic field, $B_0 < 10^{10}$ G (the small period derivative $\dot{P} \lesssim 10^{-17} \text{s s}^{-1}$). They are believed to be formed from the recycling (accretion spin-up) process in binaries (Bhattacharya & van den Heuvel 1991). There is another possible formation channel for MSPs (even for submillisecond pulsars), which is accretion-induced collapse of white dwarfs in a binary (Nomoto 1982; Bulik et al. 2000). Usov (1983) predicted that this collapse white dwarf could emit γ-rays via synchrotron radiation on the order of 100 GeV. Bhattacharya & Srinivasan (1991) subsequently calculated the γ-ray luminosity of MSPs, estimated their contribution to the diffuse γ-ray background of the Milky Way, and finally discussed the detectability of MSPs as point-γ-ray sources. Using data from the Energetic Gamma Ray Experiment, Kuiper et al. (2000) provided circumstantial evidence for the likely detection of pulsed γ-ray emission from an MSP, PSR J0218+4232, which had long been regarded as a γ-ray pulsar candidate. The γ-ray emission from MSPs was not observationally confirmed until the launch of Fermi with the sensitive Large Area Telescope (LAT) in 2008 June. Using eight-month data of Fermi-LAT, eight γ-ray MSPs have already been detected (Abdo et al. 2009). This number has grown to more than 40 to date (Guillemot et al. 2012a) and is still increasing.

From observations, MSPs are analogous to young pulsars, which have multi-wavelength pulsed emission from radio to γ-rays. Do MSPs and young pulsars share a simple model that describes a similar emission region and acceleration mechanism to self-consistently explain their multi-wavelength emission? More and more multi-wavelength data with high precision provide opportunities to obtain remarkable insights on magnetospheric physics. Multi-wavelength study is a key method to discriminate the various pulsar non-thermal emission models for both MSPs and young pulsars.

Initially, with the aim of explaining the high-energy pulsed emission from young pulsars, four traditional magnetospheric gap models have been suggested: the polar cap model (Daugherty & Harding 1994), the outer gap model (Cheng et al. 1986; Romani & Yadigaroglu 1995; Zhang & Cheng 1997), the two-pile caustic (TPC)/slot gap model (Dyks & Rudak 2003; Harding et al. 2008), and the annular gap model (Qiao et al. 2004, 2007; Du et al. 2010). The most important issues distinguishing these models are the indetermination of acceleration electric field region and related mechanisms to emanate high-energy photons (Du et al. 2011, 2012). One of the key discrepancies of these emission models is the one-pole or two-pole emission pattern, which depends on two corresponding geometric parameters: magnetic inclination angle ζ and viewing angle ξ.

Bulik et al. (2000) adopted the polar-cap model to calculate the γ-ray emission from MSPs. They pointed out that curvature radiation from primary particles contributed to the MeV-to-GeV band, while synchrotron radiation arising from pairs dominated only below 1 MeV. Harding et al. (2005) developed the pair-starved polar cap model and obtained a similar spectral conclusion for high-energy emission from MSPs. In this model, the accelerating field is not screened and the entire open volume is available for particle acceleration and emission of gamma rays. Zhang & Cheng (2003) used the outer gap model with a multi-pole magnetic field to model the X-ray and γ-ray spectra for four MSPs, and the predicted results basically agree with the observations (Harding et al. 2005).
Along with radio observations supplying us with excellent timing solutions for Fermi MSPs, the derived γ-ray and radio light curves with high signal-to-noise ratio allow us to perform a joint simulation which can verify the pulsar emission models. Recently, Venter et al. (2009) simulated both radio and γ-ray light curves for MSPs using the pair-starved polar cap, TPC, and outer gap models, and found that most of their simulated light curves are well explained by the TPC and outer gap models. They especially simulated light curves for a small group of MSPs with phase-aligned radio and γ-ray pulse profiles (Venter et al. 2012). In addition, Johnson et al. (2012) used the geometric slot gap and outer gap model to fit γ-ray and radio light curves for three MSPs.

An MSP, PSR J0101-6422, with complex radio and γ-ray light curves, challenges the popular TPC and outer gap models (Kerr et al. 2012). It is found that neither model can faithfully reproduce the observed light curves and phase lags. For such a complex radio profile the simple beam model they used may have been insufficient.

In this paper, we use a three-dimensional annular gap model to study both radio and γ-ray light curves for three MSPs that represent the relevant types of MSPs with different radio lags. In Section 2, the annular gap and core gap is briefly introduced and the acceleration electric field in the annular gap calculated. In Section 3, we jointly simulate radio and γ-ray band light curves for PSR J0034-0534, J0101-6422, and J0437-4715. The radio phase lags are identified and their causes explored. Conclusions and a discussion are in Section 4.

2. THE ANNULAR GAP AND CORE GAP FOR MSPS

2.1. Definition of Annular Gap and Core Gap

In a pulsar magnetosphere (Goldreich & Julian 1969; Ruderman & Sutherland 1975), the critical field lines divide a polar cap into two distinct parts: the annular gap region and the core gap region (see Figure 1 of Du et al. 2012). The annular gap is constrained between the critical and the last open field lines, and the core gap is around the magnetic axis and within the critical field lines (Du et al. 2010). The size of the polar cap decreases with increasing spin period, but can be quite large for MSPs and young pulsars. The annular gap width is correspondingly larger for short spin-period pulsars, and varies with the magnetic azimuth ψ. For an anti-parallel rotator and ψ = 0°, the radii of the annular polar region is \( r_{\text{ann}} = r_{\text{pc}} - r_{\text{core}} = 0.26 R\left(\Omega R/c\right)^{1/2} \) (Du et al. 2010), where \( R \) is the pulsar’s radius, and \( \Omega \) its angular spin frequency.

Combining the advantages of the outer gap and TPC models, Qiao et al. (2004, 2007) initially suggested the annular gap model, which was further developed by Du et al. (2010, 2011, 2012). The site for generation of high-energy photons is mainly located in the vicinity of the null charge surface. Since they are consistent with the physically calculated spectra (Du et al. 2011), the Gaussian emissivities are assumed numerically when simulating light curves. The key emission geometry parameters \( \alpha \) and \( \xi \) have not been confirmed so far; we adopt values of these two parameters from related literature where they exist. If there are no reliable values, we simply use some random values from the theories of MSP magnetic field evolution. By hypothesizing reasonable emissivities and the magnetic inclination angle, we can model wide fan-like emission beams, which well reproduce the relevant light curves cut by a suitable viewing angle.

2.2. Acceleration Electric Field

A pulsar magnetosphere is filled by a corotating charge-separated plasma (Goldreich & Julian 1969; Ruderman & Sutherland 1975). Upon reaching some regions near the light cylinder, the charged particles cannot exceed the speed of light \( c \), and thus escape from the magnetosphere. To compensate for the escaping particles, the pulsar has to supply enough charged particles to its magnetosphere. This dynamic process happens continuously, thus a huge acceleration electric field is generated in the magnetosphere. This is the general mechanism for the acceleration electric field \( E_{\parallel} \), which is applicable both for young pulsars (Du et al. 2011, 2012) and for MSPs. The charged particles with opposite signs are simultaneously emanating from the annular gap and core gap, and satisfy the condition for circuit closure over the whole magnetosphere.

We assume the outflowing particles’ charge density equals the local GJ density (Goldreich & Julian 1969) at a radial distance of \( r \sim R_{\text{LC}} \), where \( R_{\text{LC}} = (cP/2\pi r) \) is the radius of the light cylinder. Details of the calculation method and formulae are presented in Du et al. (2011). The acceleration electric fields of typical MSPs are shown in Figure 1 using four sets of surface magnetic field \( B \) and spin period \( P \). For MSPs with large or small values of \( P \) and \( B \), we found that, in all cases, the electric field in the inner region of the annular gap is sufficiently high \( (E_{\parallel} \geq 10^8 \text{ V cm}^{-1}) \).

The charged particles accelerated in the annular gap or core gap flow out along a field line in a quasi-steady state. Using the derived acceleration electric field, we can obtain the Lorentz factor \( \Gamma_p \) of the primary particles from the balance of...
acceleration and curvature radiation reaction

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\Gamma_p = \left( \frac{3e^2 E_\parallel}{2\rho_c} \right)^{\frac{1}{4}} = 2.36 \times 10^7 \rho_e^{0.5} E_1^{0.25},
\]

where \( e \) is the charge of an electron, \( \rho_c \) the curvature radius in units of \( 10^7 \) cm and \( E_1, \alpha \) the acceleration electric field in units of \( 10^8 \) V cm\(^{-1}\).

The primary particles are accelerated to ultrarelativistic energy with typical Lorentz factors of \( \Gamma_p \sim 10^6\)–\(10^7 \), because of the huge acceleration electric field in the annular gap. Since a large number of \( \gamma \)-ray photons are generated by the primary particles via curvature radiation and inverse Compton scattering processes, abundant \( e^\pm \) pairs are subsequently created through two-photon annihilation and photon-magnetic-absorption (\( \gamma \)-B) processes.

3. SIMULATING RADIO AND \( \gamma \)-RAY LIGHT CURVES FOR MSPs

Thanks to Fermi-LAT, we now know that some MSPs are multi-wavelength emitters which have detectable radio and \( \gamma \)-ray pulsed emission. According to the observations of phase lag between the radio peak and the \( \gamma \)-ray peak (Abdo et al. 2010a; Kerr et al. 2012), MSPs can be divided into four classes. PSR J0034-0534 represents a class of MSPs which has phase-aligned light curves (\( \Delta = 0 \)); PSR J0101-6422 signifies another class which has a moderate radio phase lag (\( \Delta \sim 0.2\)–\(0.3 \)) with quite complex radio or \( \gamma \)-ray light curves; PSR J0437-4715 represents a third class which has a larger radio lag (\( \Delta \sim 0.43 \)) and PSR J1744-1134 corresponds to a fourth class of MSPs whose \( \gamma \)-ray peak precedes the radio peak (Abdo et al. 2010a). We will model this MSP in future when high signal-to-noise \( \gamma \)-ray light curves are available. We process Fermi Pass 7 data to derive the observed light curves for three MSPs according to the radio timing solutions of MSPs from the Fermi Science Support Center (FSSC).6 We select events with energies of \( >0.1 \) GeV within 2\(^\circ\) of each MSP’s position and with zenith angles smaller than 105\(^\circ\). The key filter conditions for a good time interval are rock angle <52\(^\circ\) and angsep(RA\(_{\text{MSP}}\), decl\(_{\text{MSP}}\), RA\(_{\text{SUN}}\), decl\(_{\text{SUN}}\) <5\(^\circ\), where RA and decl are right ascension and declination. Then we use tempo2 (Hobbs et al. 2006) with a Fermi plug-in to obtain the spin phase for each photon. Finally, we obtain the high signal-to-noise \( \gamma \)-ray light curves for the three MSPs (see the red lines in Figures 2–4).

A convincing model should have a simple, clear emission geometric picture with reasonable input parameters, which can reproduce multi-wavelength light curves not only for young pulsars but also for MSPs. In this paper, we will jointly simulate radio and \( \gamma \)-ray light curves for PSR J0034-0534, PSR J0101-6422, and PSR J0437-4715. We briefly introduce the simulation method here. As shown in Table 1, \( \alpha \) and \( \zeta \) are “first-rank” parameters, which are primarily adopted from the literature, where they exist. When there are no convincing values, we use reasonable values of \( \alpha \) from the relevant theory on the magnetic field evolution of pulsars (Ruderman 1991) and \( \zeta \) is adopted randomly according to the simulated emission pattern (photon sky-map). When \( \alpha \) and \( \zeta \) are fixed, several other model parameters are carefully adjusted for the emission regions until the observed light curve of the corresponding band can be reproduced. The model parameters for three MSPs are listed in Table 1.

3.1. PSR J0034-0534

PSR J0034-0534 is the ninth \( \gamma \)-ray MSP detected by Fermi-LAT (Abdo et al. 2010b), and it has strong \( \gamma \)-ray and radio pulsed

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6 http://fermi.gsfc.nasa.gov/ssc/data/access/lat/ephems/
Figure 3. Similar to Figure 2, but for PSR J0101-6422. The radio data are obtained from Kerr et al. (2012). The inclination angle $\alpha = 30^\circ$ and viewing angle $\zeta = 48^\circ$ are used to model the complex radio and $\gamma$-ray light curves with moderate radio lag for this MSP.

(A color version of this figure is available in the online journal.)

Figure 4. Similar to Figure 2, but for PSR J0437-4715. From the right bottom panel, the radio interpulse at phase $\sim -0.4$ and main peak at phase $\sim 0.4$ are reproduced by the annular gap model, and they are generated from a higher and narrower region in the annular gap region of the same magnetic pole as the $\gamma$-ray emission. The radio profile for this MSP is observed from Kunming 40 m radio telescope (Hao et al. 2010). The inclination angle $\alpha = 25^\circ$ and viewing angle $\zeta = 42^\circ$ are used.

(A color version of this figure is available in the online journal.)

emission with phase-aligned light curves. To show the emission region of this MSP, we use the annular gap model to jointly model the radio and $\gamma$-ray light curves. The simulation method is the same as described in Section 3.1 of Du et al. (2011). The key idea is to project all radiation intensities in both the annular gap and core gap to the “non-rotating” sky, considering some physical effects (e.g., aberration and retardation). Here we use numerical emissivities to speed up the calculations, since the assumed emissivities are consistent with the physically calculated spectra, as shown in Figure 8 of Du et al. (2011).

From Figure 2, we find that both radio and $\gamma$-ray emission are mainly generated in the annular gap region colocated at intermediate altitudes $r \sim 0.24-0.56R_{\text{LC}}$, which leads to the phase-aligned light curves. Abdo et al. (2010b) used the TPC and outer gap geometric models with $\alpha = 30^\circ$ and $\zeta = 70^\circ$ to obtain the light curves for PSR J0034-0534, and they derived the
similar conclusion that the radio emission region extends from 0.6\(R_{\text{LC}}\) to 0.8\(R_{\text{LC}}\) and the \(\gamma\)-ray region extends from 0.12\(R_{\text{LC}}\) to 0.9\(R_{\text{LC}}\). It was found that this MSP has a larger transverse emission region for radio emission. Venter et al. (2012) developed the traditional outer gap and TPC model, adopting the similar idea of numerically assumed emissivity of a piecewise-function, and derived the phase-aligned radio and \(\gamma\)-ray light curves for three MSPs including PSR J0034-0534. It seems that PSR J0034-0534 has off-peak pulsed \(\gamma\)-ray emission up to 100\% of the duty cycle (Ackermann et al. 2011), which is not reproduced by the annular gap model. This might be due to lack of knowledge of the emission geometry and magnetic field configuration, and we will further develop our model to study this MSP in detail in the future.

### 3.2. PSR J0101-6422

The observed light curves of PSR J0101-6422 have complex features: the \(\gamma\)-ray profile is likely to have three peaks, while the radio profile contains three peaks that occupy nearly the whole rotation phase (see the left panels of Figure 3). To model both light curves, we consider both cases of single-pole and two-pole emission with many attempts over a large parameter space. The light curves of this MSP favor a single-pole emission picture with \(\alpha = 30\degree\) and \(\zeta = 48\degree\), and the results are shown in Figure 3. For a viewing angle of \(\zeta = 48\degree\), the radio interpeak originates from the core gap at high altitudes of 0.3\(R_{\text{LC}}\)–0.7\(R_{\text{LC}}\); the other two peaks with a bridge (at phases of \(\sim -0.3\) and \(\sim 0.45\)) originate from the annular gap at altitudes of 0.45\(R_{\text{LC}}\)–0.78\(R_{\text{LC}}\). The \(\gamma\)-ray profile is similar; the interpeak (at phase \(\sim 0\)) mainly originates from the core gap at high altitudes of 0.32\(R_{\text{LC}}\)–0.6\(R_{\text{LC}}\), while the other two peaks originate from the annular gap at altitudes of 0.08\(R_{\text{LC}}\)–0.4\(R_{\text{LC}}\).

We note that, according to the annular gap model, the radio emission from PSR J0101-6422 is quite asymmetric in magnetic azimuth. This is possibly due to the special physical coherence condition and propagation effects in the pulsar magnetosphere. We provide some discussion on radio emission in Section 4.

### 3.3. PSR J0437-4715

PSR J0437-4715 is a nearby MSP, at a distance of 0.16 kpc from Earth (Manchester et al. 2005), and has multi-wavelength emission. Chen et al. (1998) suggested that PSR J0437-4715 is an aligned rotator. We therefore adopt a relatively small magnetic inclination angle of \(\alpha = 25\degree\) and a reasonable viewing angle of \(\zeta = 42\degree\) from high-resolution radio timing observations (van Straten et al. 2001; Hotan et al. 2006). Bogdanov et al. (2007) also used the value of \(\zeta = 42\degree\) to successfully model the thermal X-ray pulsation of this MSP. We apply our annular gap model to jointly simulate its radio and \(\gamma\)-ray light curves, and the results are shown in Figure 4. We emphasize that the radio interpulse can be reproduced by our model, although it does not precisely match the observations. The radio emission including the main peak and interpulse originate from a much higher and narrower region in the annular gap region at high altitudes of 0.48\(R_{\text{LC}}\)–0.57\(R_{\text{LC}}\), while the \(\gamma\)-ray emission is generated in the annular gap region at lower altitudes of 0.064\(R_{\text{LC}}–0.15R_{\text{LC}}\) located in the same magnetic pole. This leads to a large radio lag of \(\Delta \sim 0.43\).

### 4. CONCLUSIONS AND DISCUSSION

Pulsed \(\gamma\)-ray emission from MSPs has been observed by the sensitive \textit{Fermi}-Large Area Telescope. The specific pattern of radio and \(\gamma\)-ray emission from PSR J0101-6422 challenges the outer gap and TPC models. A convincing model should apply not only to young pulsars but also to MSPs. In this paper, we used the annular gap model to jointly model the radio and \(\gamma\)-ray light curves for three representative MSPs, PSR J0034-0534, PSR J0101-6422, and PSR J0437-4715 with distinct radio phase lags. For PSR J0034-0534 with phase-aligned radio and \(\gamma\)-ray light curves, both bands are mainly generated in the annular gap region colocated at intermediate altitudes. For PSR J0101-6422 with complex radio and \(\gamma\)-ray pulse profiles, we presented the best simulated results for this type of MSP. The radio interpulse originates from the core gap at higher altitudes, while the other two radio peak with a bridge originate from the annular gap region. The interpulse originates from the core gap region and the other two peaks from the annular gap region. For PSR J0437-4715, which has a large radio lag, the radio emission (including the interpulse) originates from a much higher and narrower region in the annular gap region, and the \(\gamma\)-ray emission is at lower altitudes.

From simulations of these MSPs, the annular gap model favors a single-pole emission pattern with small inclination.

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### Table 1

Model Parameters of Multi-wavelength Light Curves for Three MSPs

| Band     | \(\kappa\) | \(\lambda\) | \(\epsilon\) | \(\sigma_{\alpha}\) | \(\sigma_{\theta,\alpha}\) | \(\sigma_{\delta}\) | \(\sigma_{\theta,\delta}\) | \(\sigma_{\delta,\gamma}\) |
|----------|------------|-------------|--------------|-------------------|------------------------|----------------|------------------------|-------------------------|
| Radio    | 0.75       | 0.85        | 0.32         | 0.007             | 0.15                   | 0.006          | 0.0052                 |
| J0034-0534 (\(\alpha = 35\degree\); \(\zeta = 70\degree\)) |
| > 0.1 GeV | 0.50       | 0.85        | 0.3           | 0.009             | 0.15                   | 0.006          | 0.006                  |
| J0101-6422 (\(\alpha = 30\degree\); \(\zeta = 48\degree\)) |
| > 0.1 GeV | 0.1        | 0.1         | 1.2           | 0.003             | 0.02                   | 0.001          | 0.001                  |
| J0437-4715 (\(\alpha = 25\degree\); \(\zeta = 42\degree\)) |

Notes. \(\alpha\) and \(\zeta\) are the magnetic inclination angles and viewing angles; \(\kappa\) and \(\lambda\) are two geometry parameters to determine the peak altitude in the annular gap; \(\epsilon\) is a parameter for the peak altitude in the core gap; \(\sigma_{\alpha}\) and \(\sigma_{\delta}\) are length scales for the emission region on each open field line in the annular gap and the core gap in units of \(R_{\text{LC}}\), respectively; \(\sigma_{\theta,\alpha}\) and \(\sigma_{\theta,\delta}\) are the transverse bunch scale for field lines in the annular gap and the core gap, respectively. The detailed description of these symbols can be found in Du et al. (2011).
angles ($\alpha \lesssim 35^\circ$) for MSPs. This result is compatible with theories of magnetic field evolution of MSPs in binaries: some recycled pulsars tend to have aligned magnetic field moment, i.e., small magnetic inclination angle $\alpha$ (Ruderman 1991; Chen et al. 1998). Lamb et al. (2009) presented a concrete discussion on the $\alpha$ evolution of MSPs while they were recycling in low-mass X-ray binaries, and they noted that the strong interactions between spinning superfluid neutrons and magnetized superconducting protons in a pulsar’s core force the spin axis to change. An MSP would be an aligned rotator ($\alpha \sim 0^\circ$) if the star’s north and south magnetic poles were forced toward opposite spin poles by the accretion disc, or would be an orthogonal rotator ($\alpha \sim 90^\circ$) if both of the star’s magnetic poles were forced toward the same spin pole. It is certainly unclear what happens when an MSP’s recycling process finishes; Young et al. (2010) analyzed new pulse width data of normal pulsars, and found that the spin and magnetic axes would align when they spin down due to dipole radiation and particle outflowing.

Several MSPs are simply assumed to be nearly orthogonal rotators because they have a radio interpulse separated by a large phase of $\gtrsim 180^\circ$ from their main pulse (Chen et al. 1998). Guillemot et al. (2012b) studied multi-wavelength light curves for two MSPs (PSR B1937+21 and PSR B1957+20), and found that fitting the radio polarization data of PSR B1937+21 favors an orthogonal rotator. As with most rotating vector model fits the confidence area is large but the orthogonal configuration is further supported by the altitude-limited TPC and outer gap models. However, this is not universally true, at least in the case of the annular gap model. The radio light curve of MSPs (e.g., PSR J0437–4715) with an interpulse can be well explained by the annular gap model with a small magnetic inclination angle.

By simulating light curves for MSPs in the annular gap model, we found that the radio emission mainly originates from the high-altitude narrow region in the annular gap region. The radio emission pattern (photon sky-map) is patch-like in our model. The radio emissivities on each field line (in the annular gap or core gap) vary slightly (they are nearly uniform), but the case of $\gamma$-ray light-curve simulation is quite different: they vary greatly. High-energy ($\gamma$-ray and X-ray) emission is generated by non-coherent radiation from relativistic primary particles and pairs, while radio emission is suggested to be generated by coherent radiation due to two-stream instability of outward and inward pairs (Ruderman & Sutherland 1975). Here we focus on studying radio and $\gamma$-ray light curves for MSPs. The concrete emission mechanism including polarization, spectral properties and long-term stabilities of radio lags are, however, needed for further study, taking into account the coherence condition and propagation effects. Han et al. (1998) systematically studied the radio circular polarization for pulsar integrated pulse profiles, and found that sense reversals of circular polarization are observed across the conal emission in some cases, unrestricted to core components. The polarization property of high-altitude radio emission from both annular and core gaps is a valuable subject for future investigation. We will continue to improve our model to present better simulated light curves, especially for the phases of the leading wing of peak 1, trailing wing of peak 2 and off-peak pulses.

In this paper, we simulated radio and gamma-ray light curves for three MSPs. When high signal-to-noise data at other wavelengths are available in the future, we will re-simulate the light curves and fit the multi-wavelength phase-resolved spectra. In summary, the annular gap model is a self-consistent model not only for young pulsars (Du et al. 2011, 2012), but also for MSPs, and multi-wavelength light curves can be well explained by this model.

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