Dense Polarized Positrons from Laser-Irradiated Foil Targets in the QED Regime

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Dense positrons are shown to be effectively generated from laser-solid interactions in the strong-field quantum electrodynamics (QED) regime. Whether these positrons are polarized has not yet been reported, limiting their potential applications. Here, by QED particle-in-cell simulations including electron-positron spin and photon polarization effects, we investigate a typical laser-solid setup that an ultraintense linearly polarized laser irradiates a foil target with μm-scale-length pre-plasma. We find that once the positron yield becomes appreciable with the laser intensity exceeding $10^{24}$ W/cm$^2$, the positrons are obviously polarized. The polarized positrons can acquire >30% polarization degree and >30 nC charge with a flux of $10^{12}$ sr$^{-1}$. The polarization relies on the deflected angles and can reach 60% at some angles and energies. The angularly-dependent polarization is attributed to the asymmetrical laser fields positrons undergo in the skin layer of overdense plasma, where the radiative spin-flip and radiation reaction play significant roles. The positron polarization is robust and could generally appear in future 100-PW-class laser-solid experiments for various applications.

Polarized positrons with a preferential orientation of spins can exhibit unique features in many areas, such as searching new physics beyond the Standard Model in International Linear Collider (ILC) [1, 2] and probing spin phenomena at material surfaces [3, 4]. Besides, polarized electron-positron ($e^- e^+$) plasmas are believed to be ubiquitous in pulsar magnetospheres [5]. There are a few methods to generate high-energy polarized positrons. Ultrarelativistic positrons in tesla-level magnetic fields of storage rings can be polarized via radiative spin-flip [6, 7] but rather slowly. Alternatively, polarized positrons are usually produced via Bethe-Heitler (BH) process by hitting high-Z targets with circularly polarized $\gamma$ photons [8, 9] or prepolarized electrons [10]. These BH methods suffer low conversion efficiency of $\sim 10^4$ positrons ($10^{-6}$ nC) per shot, and thus high repetitions are necessary to meet the high-charge or -density requirements of ILC (3.2 nC) and laboratory astrophysics.

Dense positrons can be efficiently generated from single-shot laser-matter interactions in the strong-field quantum electrodynamics (QED) regime [11–15]. This approach is becoming experimentally feasible with advances in high-intensity laser technologies [16]. Recently, the intensity of $1.1 \times 10^{23}$ W/cm$^2$ has been realized by a 4-PW laser system [17], and higher-power laser facilities of 10-PW [18] to 100-PW classes will be available [19–22]. In such strong laser fields, $\gamma$ photons can be radiated by electrons and in turn annihilate into $e^- e^+$ pairs via Breit-Wheeler (BW) process [23]. For all-optical configurations of lasers colliding with unpolarized multi-GeV electrons [24, 25], polarized positrons can be obtained if asymmetric laser fields are employed, such as elliptically polarized [26] or two-color linearly polarized laser pulses [27]. Limited by the charge of electron beams from laser wakefield accelerators, the corresponding positron yield is at the $10^{-4}$ nC level. Furthermore, constructing such asymmetric strong laser fields is challenging due to the low damage threshold of optical devices [28]. Recent QED particle-in-cell (PIC) simulations have shown that impinging on a stationary target by two counter-propagating 10-PW-class lasers [29–31] or one 100-PW-class laser [32–34] is capable of generating much denser positrons over 100 nC via self-sustained QED cascades. However, it remains unclear whether such positrons are polarized or not because the QED model being widely adopted in the existing QED-PIC codes [34–37] overlooks the positron spin dynamics.

In this Letter, we employ a recently-developed QED-PIC code including pair spin and photon polarization effects to clarify the above problem. By QED-PIC simulations, we investigate a linearly polarized laser interaction with a solid foil target with μm-scale-length pre-plasma, as depicted in Fig. 1(a). When the laser intensity exceeds $10^{24}$ W/cm$^2$, substantial positrons are created primarily in the skin layer of solid-density plasma, where the dominance of laser magnetic components is favorable for $e^- e^+$ pair creation. The positrons are then quickly pushed into deeper plasma and escape from the laser fields, causing them only experience subcycle laser fields. In such asymmetric fields, the created positrons are split into two populations of opposite spin polarization at the positive and negative deflected angles, respectively, due to radiative spin-flip and radiation reaction. Above 30% polarization of a 30 nC positrons can be achieved and the polariza-
I. Introduction
tion can even reach 60% at some angles and energies. Note that in future 100-PW laser-solid experiments even aiming at other applications [32, 34, 38], a skin layer can be certainly formed, where polarized positron generation could be ubiquitous, therefore, pair spin and photon polarization effects should be considered.

Simulation setups.—We perform QED-PIC simulations to investigate the positron polarization sketched above with the code YUNIC [39, 40]. Multiphoton Compton scattering and multiphoton BW process can be characterized by quantum invariant parameters \( \chi_e = (|e|h/m_e^3c^4)[F_{\mu\nu}p^\mu p^\nu] \) and \( \chi_\gamma = (|e|h^2/m_e^3c^4)[F_{\mu\nu}k^\mu k^\nu] \), respectively, where \( F_{\mu\nu} \) is the field tensor, \( p^\mu (\hbar k^\mu) \) is the pair (photon) four-momentum, \( h \) is the reduced Planck constant, \( c \) is the speed of light, and \( e \) and \( m_e \) are the electron charge and mass. These two leading QED processes are implemented through the standard Monte-Carlo algorithm [35–37], but including \( e^-e^+ \) spin and photon polarization effects [12, 41–43]. Since we select the mean axis as quantization axis, the spin vector \( \mathbf{S} \) of non-radiating electrons/positrons and Stokes parameter \( \xi \) of non-decaying photons also need to be updated [43]. More implementation details can be found from the Supplemental Material [44].

We first adopt one-dimensional (1D) PIC simulations to better get insight into the positron polarization mechanism with higher numerical resolutions, where the geometry is 1D while \( \mathbf{S}, \xi \) and the particle momentum \( \mathbf{p} \) still remain fully three-dimensional (3D). A laser pulse lin-
of high energies are weakly linearly polarized in the $x$-$y$ plane [44]. Here, $\mathbf{B}' = [E + \beta \times \mathbf{B} - \beta(\mathbf{\beta} \cdot \mathbf{E})] \times \mathbf{\beta}$ and $\beta$ denotes unit vector along the positron velocity. Considering $\mathbf{\beta}$ is directed in the $x$-$y$ plane, one can obtain $\zeta \approx (0, 0, B_z/B_e)$ in the MFDR, hence $\mathbf{S}_z$ of newly created positrons has the same sign with $B_z$. From Fig. 3(a), both positron number and polarization at birth are essentially symmetrical with respect to $\theta_y = 0$. Similarly, positrons born in adjacent positive half-cycles also exhibit similar distributions, but with $S_z > 0$ due to $B_z > 0$ (see Fig. S5 in [44]). Thus, the polarization of positrons born at positive and negative half-cycles could be counteracted each other and the angularly-dependent polarization would not be achieved if their spins or deflection angles were not changed later.

The positron polarization in Fig. 1(c) is attributed to the asymmetric laser fields that positrons experience later, where radiative spin-flip and radiation reaction mainly in the second MFDR play significant roles. The marked positrons are pushed forward after being created, gradually divide into bunch-I and bunch-II [Fig. 2(b)]. Two bunches successively escape the laser fields from two adjacent half-cycles, with the relative number $N^1_+ : N^1_- \approx 3 : 5$. Only experiencing the first MFDR where they are born, bunch-I positrons are quickly pushed forward into the deeper plasma, because their initial negative momenta $p_y < 0$ [see the red-dotted line in Fig. 3(c)] lead to strong forward Lorentz forces, i.e., $\beta_y B_z$ along the $+x$ direction. Without undergoing an EFDR for acceleration, bunch-I is generally less energetic and weakly radiating [Fig. 3(c)], hence almost retaining the initial negative polarization [Fig. 3(d)]. By contrast, bunch-II positrons travel through an EFDR to obtain higher energies and then through the second MFDR to radiate strongly [Fig. 3(e) and also $P_{\text{rad}}$ in Fig. 2(b)]. Due to quantum stochasticity, spins of only a fraction of bunch-II positrons can flip parallel to $\zeta$ and achieve an opposite polarization of $S_z > 0$ [Figs. 3(f)] as $B_z > 0$ in the second MFDR.

Figures 3(c) and 3(f) show that part bunch-II positrons with the final $S_z > 0$ mainly appear at $\theta_y < 0$, because they undergo strong radiation reaction. This can be explained by tracking a typical bunch-II positron [Figs. 2(c)-2(e)]. Under the Lorentz force and radiation reaction, its transverse momentum $p_y$ can be approximated as $p_y \approx p_{y0} + \int dt \left[ e(E_y - \beta_z B_z) - \frac{p_{y0}}{\gamma m_e c^2} P_{\text{rad}} \right]$, where $p_{y0}$ is the initial $y$-momentum, $\gamma$ is the relativistic
tic factor, and the last term is radiation reaction whose direction is opposite to the velocity. Figure 2(d) shows that the tracked positron first undergoes the gyration motion in the first MFDR due to its low initial energy and then enters the EFDR for a significant acceleration. As $E_y > B_z$ and small $P_{\text{rad}}$ in these two regimes, the positron gains $p_y > 0$. After entering the second MFDR, the positron emits a high-energy photon and simultaneously its $p_y$ is sharply decreased by radiation reaction (i.e., $P_{\text{rad}}$ is large enough). As $E_y < B_z$ and $\beta_x > 0$ in the second MFDR, $p_y$ gradually decreases and changes the sign to achieve $\theta_y < 0$. Here, the strong radiation of $\varepsilon_{\text{rad}} > 0.2$ GeV is necessary for bunch-II to flip their spins [46] and sharply decrease $p_y$. This is supported by statistical results in Fig. 3(e) that $\theta_y$ tends to change from positive to negative values as $\varepsilon_{\text{rad}}$ increases. Bunch-I finally obtains a small positive deflection angle of $\theta_y \approx 10^\circ$ in average, making less contribution to the overall polarization.

Therefore, positrons born in negative half-cycles are generally polarized with $\mathcal{S}_z > 0$ at $\theta_y < 0$ and $\mathcal{S}_z < 0$ at $\theta_y > 0$ [Fig. 3(b)] due to the joint action of spin-flip and radiation reaction. It also holds for positrons born in positive half-cycles [Fig. S5(b) in [44]], hence leading to the overall polarization as displayed in Fig. 1(c). Note that the obtained electrons are also polarized like positrons (Fig. S4 in [44]), but with weaker polarizations due to a mixing of unpolarized target electrons.

**Parameter influences.**—The dependence of positron yield $N_+$ and positron polarization $\mathcal{S}_z$ on the preplasma scale-length $L$ and laser amplitude $a_0$ are presented in Figs. 4(a) and 4(b), respectively. Preplasmas due to prepulses are unavoidable and also adjustable in real experiments. Preplasmas of relatively low densities favor both $N_+$ and $\mathcal{S}_z$ by enhancing the laser absorption and generating more ultrarelativistic electrons to trigger QED cascades [34]. A small scale-length preplasma of $L = 0.4$ µm leads to significant laser reflection, where positrons are mainly created in the strong standing wave away from the target surface (Fig. S6 in [44]). It implies that they would experience quasi-symmetrically multicy-
polarization above 30% for a 30° incidence angle (Fig. S8 in [44]).

In conclusion, we have investigated the generation of dense polarized positrons in a conventional setup of laser-solid interaction in the QED regime. Over 30 nC transversely polarized positrons with the polarization degree above 30% can be generated at the laser intensity $3 \times 10^{24} \text{ W/cm}^2$. The high density and charge of such polarized positrons can meet the requirements of future electron-positron colliders and exploration of polarized plasma collective behavior [47]. The positron polarization mechanism is robust since the laser fields experienced by positrons are naturally asymmetric in the skin layer. Therefore, the positron polarization could be ubiquitous in future 100-PW laser-solid experiments.

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I. QED PROBABILITIES AND MONTE-CARLO METHODS

In this work, the spin- and polarization-resolved probabilities of photon emission and \( e^- e^+ \) pair production derived by Baier and Katkov are employed, which are written as [1–3]

\[
\frac{d^2W_{\gamma}}{dudt} = \frac{C_{\text{rad}}}{4} \left\{ \frac{u^2 - 2u + 2}{1-u}K_{2/3}(y_1) - uK_{1/3}(y_1) - uK_{1/3}(y_1)(S_i \cdot e_2) - \frac{u}{1-u}K_{1/3}(y_1)(S_f \cdot e_2) + \left( 2K_{2/3}(y_1) - \text{Int}K_{1/3}(y_1) \right)(S_i \cdot S_f) + \frac{u^2}{1-u}K_{2/3}(y_1)(S_i \cdot e_1)\right\} \xi_1 + \left( 2u - \frac{u^2}{1-u}K_{2/3}(y_1) - u\text{Int}K_{1/3}(y_1) \right)(S_i \cdot e_2)\xi_2
\]

\[
\frac{d^2W_{\pm}}{d\epsilon_{\pm}dt} = \frac{C_{\text{pairs}}}{2} \left\{ \frac{\epsilon_+^2 + \epsilon_-^2}{\epsilon_+ \epsilon_-}K_{2/3}(y_2) + \text{Int}K_{1/3}(y_2) - \xi_1K_{2/3}(y_2) + \frac{\epsilon_+^2}{\epsilon_+ \epsilon_-}K_{1/3}(y_2)\right\} \xi_1 \xi_2 + \left( \frac{\epsilon_+^2}{\epsilon_+ \epsilon_-}K_{1/3}(y_2) + \frac{\epsilon_-^2}{\epsilon_+ \epsilon_-}K_{2/3}(y_2) \right)(S_{\pm} \cdot e_0)
\]

where \( K_r(y) \) is the modified Bessel function of the second kind with a noninteger factor \( r \), \( \text{Int}K_{1/3}(y) \equiv \int_y^\infty K_{1/3}(x)dx \), \( y_1 = 2u/[3(1-u)\chi_{\epsilon}] \), \( y_2 = \epsilon_+^2/(3\chi_{\epsilon,\epsilon_+}) \), \( u = \epsilon_+ / \epsilon_\epsilon \), \( C_{\text{rad}} = (\alpha m_i^2 c^4)/(\sqrt{3}\pi \epsilon_\epsilon) \), \( C_{\text{pairs}} = (\alpha m_i^2 c^4)/(\sqrt{3}\pi \epsilon_\epsilon) \), and \( \alpha \approx 1/137 \) is the fine structure constant. Particle energies \( \epsilon_\epsilon \), \( \epsilon_\gamma \), \( \epsilon_+ \), and \( \epsilon_- \) are those of emitting leptons (electrons or positrons), emitted \( \gamma \) photons, and the created positrons and electrons, respectively. \( S_i \) and \( S_f \) are the spin vectors of leptons before and after the radiation. \( S_\pm \) are the spin vectors of the newly created electrons and positrons. When Eqs. (S1) and (S2) are applied to the electron (positron), \( e_0 \) represents the unit vector along the electron (positron) velocity, \( e_1 \) and \( e_1' \) represent the unit vectors along the transverse electron (positron) acceleration, \( e_2 = e_\epsilon \times e_1 \) and \( e_2' = e_\epsilon \times e_1' \), respectively. The Stokes parameter \( \xi \) is defined with the orthonormal basis \( (e_1, e_2, e_\epsilon) \), while \( \xi' \) is defined with \( (e_1', e_2', e_\epsilon) \). Based on the locally constant-field approximation, the QED model above holds for the ultraintense laser field of the normalized intensity \( a_0 \gg 1 \) [4].

The spin and polarization relevant Monte-Carlo algorithms in our QED-PIC code YUNIC [5, 6] are basically based on the single-particle code CAIN [7], in which the mean axes of leptons and photons are selected to be their quantization axes. After a photon emission, the lepton spin flips with respect to \( S_R = \pm S_R^*/|S_R^*| \), where
\[ S^*_R = \left(2K_{2/3}(y_1) - \text{Int}K_{1/3}(y_1)\right) \mathbf{S}_i + \frac{u}{1-u} K_{2/3}(y_1) \mathbf{e}_2 + \frac{u^2}{y_1} \left[ K_{2/3}(y_1) - \text{Int}K_{1/3}(y_1) \right] (\mathbf{S}_i \cdot \mathbf{e}_r) \mathbf{e}_r. \]

The Stokes parameters of the emitted photon at birth are chosen as \( \xi = \pm \xi^*/|\xi^*| \), where \( \xi^* = (\xi^i_1, \xi^i_2, \xi^i_3) \), \( \xi^i_1 = \frac{u}{1-u} K_{1/3}(y_1)(\mathbf{S}_i \cdot \mathbf{e}_i) \), \( \xi^i_2 = \frac{2u\gamma^2}{y_1} K_{2/3}(y_1) - u \text{Int}K_{1/3}(y_1) \) \( (\mathbf{S}_i \cdot \mathbf{e}_r) \) \( (\mathbf{S}_i \cdot \mathbf{e}_z) \). The spin vector of created pairs at birth is parallel or antiparallel to \( \mathbf{S}^*_R/|\mathbf{S}^*_R| \), where \( \mathbf{S}^*_R = -\xi_1^i (\varepsilon_\gamma/\varepsilon_\pm)\text{Int}K_{1/3}(y_2) \mathbf{e}_1 - (\varepsilon_\gamma/\varepsilon_\pm - \xi_3^i \varepsilon_\gamma/\varepsilon_\pm)K_{1/3}(y_2)\mathbf{e}_2 + \xi_2^i [(-\varepsilon_\gamma/\varepsilon_\pm)\text{Int}K_{1/3}(y_2) + (\varepsilon^*_3 - \varepsilon^*_2)/|\varepsilon_\pm|]K_{2/3}(y_2) \mathbf{e}_r .\)

The spin vector of leptons \( \mathbf{S}_{NR} \) without photon emissions and the Stokes parameters of photons \( \xi_{ND} \) before decaying into \( e^- e^+ \) pairs must be treated carefully due to selection effects as pointed in CAIN \([7]\). This is because we have chosen the mean axes as the quantization axes, indicating that every particle in the simulation actually represents an ensemble of particles. Taking the radiative polarization process as an example. The average polarization of non-emitting leptons would change since the photon emission probability \( d^2W_\gamma/(dudt) \) relies on the lepton spin vector \( \mathbf{S}_i \) \( \left[ \text{see the third term} -uK_{1/3}(y_1)(\mathbf{S}_i \cdot \mathbf{e}_r) \right] \) in Eq. (S1). It means that the non-emitting leptons are more prone to be \( \mathbf{S}_i \cdot \mathbf{e}_2 > 0 \) due to the smaller emission probability \( d^2W_\gamma/(dudt) \) compared with those of \( \mathbf{S}_i \cdot \mathbf{e}_2 < 0 \), which causes their mean spin vectors to deviate from the initial ones. Therefore, for non-emitting leptons, their spin vectors should also be updated according to another quantization axis \( \mathbf{S}^*_{NR} = \pm \mathbf{S}^*_{NR}/|\mathbf{S}^*_{NR}| \), where \( \mathbf{S}^*_{NR} = \mathbf{S}_i \left\{ 1 - C_{rad}\Delta t \int J \left[ \frac{u^2 - 2u + 2}{1 - u} K_{2/3}(y_1) - \text{Int}K_{1/3}(y_1) \right] du \right\} e_2 C_{rad}\Delta t \int J uK_{1/3}(y_1)du [3, 7] \). The similar selection effect should also be applied to non-decaying photons because the third term \( -\xi_1^i K_{2/3}(y_2) \) in Eq. (S2) also plays a selection role in the photon polarization. The quantization axes of non-decaying photons are \( \xi_{ND} = \pm \xi_{ND}/|\xi_{ND}| \), where \( \xi_{ND} = \xi \left\{ 1 - C_{pairs}\Delta t \int \varepsilon_\pm K_{2/3}(y_2) + \text{Int}K_{1/3}(y_2) d\varepsilon_\pm \right\} + e^* C_{pairs}\Delta t \int \varepsilon_\pm K_{2/3}(y_2) d\varepsilon_+ \) and \( e^* = (0, 0, 1) \).

Note that the selection effect of non-emitting leptons is extensively considered recently, while that of non-decaying photons has not attracted much attention.

To illustrate the importance of the selection effect, we consider 10 GeV electrons or photons are injected into the perpendicular static magnetic field of a strength \( B_0 = 1.07 \times 10^6 \) T. Three Monte-Carlo methods based on different quantization axes are employed: (I) the mean axis, as described above; (II) the fixed axis, i.e., \( \pm e_2 \) axis for leptons and \( \left(0, 0, \pm 1 \right) \) axis for photons, which does not require additional calculations for non-emitting leptons or non-decaying photons; (III) the mean axis but excluding the selection effects. The time evolution of the average transverse polarization \( \overline{S}_1 \) of primary electrons and the average linear polarization \( \overline{S}_3 \) of primary photons are showed in Figs. S2(a) and S2(b), respectively. We can see that methods I and II are in good agreement, while method III cannot give the correct results.

Although method II can give the same results as method I in the case of a static magnetic field above, it cannot preserve the completely 3D polarization dynamics. Therefore, our employed method I is more suitable for the complex electromagnetic environments in laser-plasma interactions.

**II. POLARIZATIONS OF LEPTONS AND PHOTONS**

Here, we utilize the simplified models specific to this work to analyze the involved polarization processes. In our considered configuration, the linear laser polarization along the \( y \) direction leads to that leptons move almost in the laser polarization plane (\( x-y \) plane). Note that \( \mathbf{S} \) is defined in the lepton rest frame, its average spin component \( \overline{S}_z \) can always characterize the transverse polarization since positrons move completely (mainly) in the \( x-y \) plane for 1D
(later 3D) geometry. The electromagnetic fields in the rest frame of an ultrarelativistic lepton is
\[
E' = \gamma(E + \beta \times B) - \frac{\gamma^2}{\gamma + 1} \beta (\beta \cdot E) \approx \gamma F_0 (\beta_y, -\beta_x, 0),
\]
\[
B' = \gamma(B - \beta \times E) - \frac{\gamma^2}{\gamma + 1} \beta (\beta \cdot B) \approx \gamma F_0 (0, 0, 1),
\]
where \(\gamma\) is the lepton relativistic factor, \(\beta \approx (\beta_x, \beta_y, 0)\) is the unit vector along the lepton velocity, and \(F_0 = B_z - \beta_z E_y\).

As expected, the electric field \(E'\) and magnetic field \(B'\) of the same magnitude are orthogonal to \(\beta\). In the magnetic field dominated regime where the photon emission and the pair creation mainly happen, the rest-frame magnetic field direction \(\mathbf{\zeta} \equiv B'/|B'|\) is approximated along the laser magnetic field direction of the laboratory frame, i.e. \(\mathbf{\zeta} \approx (0, 0, B_z/|B_z|)\). For convenience of expression, the spin vector of electrons is defined with respect to \(-\mathbf{\zeta}\), while that of positrons is with respect to \(\mathbf{\zeta}\); in other words, the same polarization values for electrons and positrons means that their spins are directed oppositely here. In addition, electrons and positrons cannot acquire the longitudinal polarization due to an initially unpolarized target that we take, and the emitted photons are only linearly polarized with \(\xi_3 \neq 0\). We no longer distinguish between \(\xi_3\) and \(\xi_3^*\) due to \(\xi_3 \approx \xi_3^*\) in the linearly polarized laser fields [6]. Equations (S1) and (S2) can therefore be simplified to
\[
\frac{d^2 W_\pm}{d \varpi dt} = \frac{C_{\text{pairs}}}{2} \left\{ \frac{\varepsilon_+^2 + \varepsilon_-^2}{\varepsilon_+ \varepsilon_-} K_{2/3}(y_2) + \text{Int} K_{1/3}(y_1) - \xi_3 K_{2/3}(y_2) \right\} S_f S_f\]
\[
+ \left\{ \frac{u}{1-u} K_{1/3}(y_1) S_f + \frac{u}{1-u} K_{1/3}(y_1) S_i \right\} \xi_3,
\]
\[
\frac{d^2 W_\pm}{d \varepsilon_\pm dt} = \frac{C_{\text{pairs}}}{2} \left\{ \frac{\varepsilon_+^2 + \varepsilon_-^2}{\varepsilon_+ \varepsilon_-} K_{2/3}(y_2) + \text{Int} K_{1/3}(y_1) - \xi_3 K_{2/3}(y_2) \right\} S_{\pm}\]
\[
+ \left\{ \frac{\varepsilon_+}{\varepsilon_\pm} - \frac{\xi_3}{\varepsilon_\pm} \right\} K_{1/3}(y_2) S_{\pm}.
\]

Accordingly, we can obtain the averaged polarization \(\overline{S}_f\), \(\overline{\xi}_3\) and \(\overline{S}_{\pm}\), respectively.

1. The average polarization of the positron after a photon emission:
\[
\overline{S}_f = \frac{u}{1-u} K_{1/3}(y_1) + \frac{u}{1-u} K_{1/3}(y_1) S_i
\]
\[
\overline{S}_f = \frac{u}{1-u} K_{1/3}(y_1) + \frac{u}{1-u} K_{1/3}(y_1) S_i.
\]
\[
\overline{\xi}_3 = \frac{K_{2/3}(y_1) + u K_{1/3}(y_1)}{u K_{1/3}(y_1) + u K_{1/3}(y_1)} S_i.
\]

2. The average polarization of the emitted photon:
\[
\overline{S}_{\pm} = \frac{\left( \frac{\varepsilon_+^2}{\varepsilon_\pm^2} - \frac{\xi_3}{\varepsilon_\pm} \right) K_{1/3}(y_2)}{(\frac{\varepsilon_+^2}{\varepsilon_\pm^2} - \frac{\xi_3}{\varepsilon_\pm} K_{2/3}(y_2) + \text{Int} K_{1/3}(y_1)}.\]

These polarizations exhibit distinct characteristics between low- and high-energy regimes. For the photon emission as shown in Figs. S2(a) and S2(b), when emitting a low-energy photon of \(\varepsilon_+/\varepsilon_e \ll 1\), the lepton nearly keeps its polarization unchanged, and the average photon polarization always has a positive value of around 0.5, insensitive to the lepton polarization. While for a high-energy emitted photon \(\varepsilon_+/\varepsilon_e \to 1\), the lepton spin tends to flip along \(\mathbf{\zeta}\), which is parallel (antiparallel) to \(\mathbf{B}\) for positrons (electrons); meanwhile, the polarization of emitted photon is highly dependent of the lepton polarization, with \(\xi_3 \approx S_i\). For the \(e^-e^+\) pair production as shown in Fig. S2(c), the spin of low-energy lepton is more likely to be parallel to \(\mathbf{\zeta}\), but the high-energy case is determined by the photon polarization. Above asymptotic relationships under low- and high-energy limits can be summarized as
\[
\overline{S}_f = \begin{cases} 1, & \varepsilon_+/\varepsilon_e \to 1 \\ 0.5, & \varepsilon_+/\varepsilon_e \to 0 \end{cases} , \quad \overline{\xi}_3 = \begin{cases} \xi_3, & \varepsilon_+/\varepsilon_e \to 0 \\ 1, & \varepsilon_+/\varepsilon_e \to 1 \end{cases} , \quad \overline{S}_{\pm} = \begin{cases} 1, & \varepsilon_+/\varepsilon_\pm \to 0 \\ 0, & \varepsilon_+/\varepsilon_\pm \to 0 \end{cases}.
FIG. S2. (a) The average lepton polarization $S_f$ after radiation and (b) the average emitted photon polarization $\xi_3$ versus the energy fraction $\varepsilon_\gamma/\varepsilon_e$ for various lepton polarizations $S_i$ before radiation with the QED parameter $\chi_e = 2$. The average polarization $S_\pm$ of generated leptons via the $e^- e^+$ pair production versus the energy fraction $\varepsilon_\pm/\varepsilon_\gamma$ for various photon polarizations $\xi_3$ with the QED parameter $\chi_\gamma = 2$.

III. ADDITIONAL SIMULATION RESULTS

A. Photon polarization

For an initially unpolarized foil target, the emitted $\gamma$ photons have a positive polarization $\xi_3 > 0$, which is decreased with the photon energy $\varepsilon_\gamma$ as shown in Fig. S3. This is consistent with the theoretical plot of Fig. S2(b). It means that for high-energy photons that are responsible for the $e^- e^+$ pair production, they are weakly polarized. Therefore, according to Fig. S2(c), the generated positrons have a positive polarization $S_z > 0$ in the positive laser cycle $B_z > 0$, while a negative value $S_z < 0$ in the negative laser cycle $B_z < 0$, i.e., their spin vectors are dominantly parallel to laser magnetic field $B$. According to the third term $-\xi_3 K_{2/3}(y_2)$ of Eq. (S6), the positive $\xi_3$ would decrease the $e^- e^+$ pair production probability, leading to a 7% positron yield decrease observed in our simulations compared to that excluding the spin and polarization effects.

FIG. S3. The photon polarization $\xi_3$ versus the photon energy $\varepsilon_\gamma$ and photon energy spectrum $f_\gamma(\varepsilon_\gamma)$. All parameters are the same as Fig. (1).

B. Electron polarization

Figures S4(a) and S4(b) show the electron polarization (including initial target electrons and later generated electrons). For the generated electrons of $e^- e^+$ pairs, they can also be polarized like positrons, with $S_z > 0$ at $\theta_y < 0$ and
\( \overline{S}_z < 0 \) at \( \theta_y > 0 \). However, due to the existence of substantial unpolarized target electrons, the global polarization degree is weakened to some extent. From the blue line of Fig. S4(b), the electron polarization can arrive \( \sim 20\% \) at \( |\theta_y| > 20^\circ \), smaller that 30\% of positrons. With some certain deflection angles and energies, the electron polarization can also reach around 60\%.

FIG. S4. (a) The number distribution \( f_-(\theta_y, \varepsilon_-) \) and (b) polarization \( \overline{S}_z \) of electrons versus the deflection angle \( \theta_y \) and electron energy \( \varepsilon_- \) at the end of the interaction \( t = 28T_0 \). All parameters are the same as those in Fig. (1).

C. Polarization of positrons born in the positive half-cycle

For comparison, we also present the polarization properties of positrons born in a positive laser cycle in Figs. S5(a) and S5(b) [next to the elliptical zone of Fig. 2(a) we previously focused]. As born in the positive laser cycle of \( B_z > 0 \), positrons at birth would be polarized along the \( B \) direction to achieve \( \overline{S}_z > 0 \), as shown in Fig. S5(a), which is opposite to the negative cycle case of \( \overline{S}_z < 0 \) in Fig. 3(a). Therefore, the initial positron polarization between positive and negative cycles vanishes. Under the radiation reaction and radiative polarization, positrons deflected along the \( \pm z \) directions are polarized along the \( \mp z \) directions, which is consistent with the negative cycle case in Fig. 3(b), thus leading to the global positron polarization.

FIG. S5. For positrons born in a positive laser cycle. The number distribution \( f_+(\theta_y) \) and polarization \( \overline{S}_z \) versus the deflection angle \( \theta_y \) (a) at birth from \( t = 21.4T_0 \) to \( 22T_0 \) and (b) at the end of the interaction \( t = 28T_0 \).

D. Small-scale preplasma

In the case of a small scale-length of preplasmas, the laser pulse cannot quickly trigger the QED cascades near the target surface. We show in Fig. S6 that positrons are generated away from the target surface for \( L = 0.4 \) \( \mu \)m, rather than near the target surface in the case of \( L = 1.5 \) \( \mu \)m in Fig. 2(a). Therefore, after being generated, positrons would experience multicycle laser fields before escaping, consequently the polarization mechanism breaks, leading to a weaker polarization.
E. Influence of target density

Here, we investigate the influence of target density (i.e., $n_0 = 300n_c$, $530n_c$, and $700n_c$) on the positron yield $N_+$ and the positron polarization $S_z$, as shown in Fig. S7(a) and S7(b), respectively. For the relatively lower-density target of $n_0 = 300n_c$, the positron yield is much less than the other two higher-density cases. Meanwhile, its optimal laser strength for the positron polarization is around $a_0 = 1500$, less than $a_0 = 1700$ of $n_0 = 530n_c$. This is because the light pressure is more likely to destroy the lower-density target with the laser strength increasing.

F. 3D simulation parameters and obliquely incident laser pulse at 30°

In Fig. 5, the 3D simulation domain is $L_x \times L_y \times L_z = 20\lambda_0 \times 12\lambda_0 \times 12\lambda_0$, resolved by $640 \times 384 \times 384$ cells, and filled with 25 electrons and 16 C$^{6+}$ ions per cell. Other laser and target parameters are the same as Fig. 1.

We have also performed an additional 3D simulation to investigate the positron polarization in the case of the obliquely p-polarized incident laser pulse at 30°, since in real experiments the oblique incidence is often employed to avoid the damage of reflected lights to optical devices. From Fig. S8, we can see that the obvious deflection-angle-dependent polarization is still observed. There are still 22 nC positrons of a polarization degree above 30%, only slightly smaller than 30 nC of normal incidence case. Thus, the proposed scheme is easily conducted in experiments...
FIG. S8. Angular distributions of (a) positron number, as well as two positron polarization components (b) $S_z$ and (c) $S_y$ versus the polar angle $\theta$ and azimuthal angle $\phi$, where the laser pulse polarized along $\phi = 0, 180^\circ$ propagates along $\theta = 0$. Other parameters are the same as those in Fig. 5 expect that the laser incident angle is $30^\circ$.

with future 100-PW-class laser facilities.

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