Fourth order gravity and experimental constraints on Eddington parameters

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PPN-limit of alternative theories of gravity represents a still controversial matter of debate and no definitive answer has been provided, up to now, about this issue. By using the definition of the PPN-parameters $\gamma$ and $\beta$ in term of $f(R)$ theories of gravity, we show that a family of third-order polynomial theories, in the Ricci scalar $R$, turns out to be compatible with the PPN-limit and the deviation from General Relativity, theoretically predicted, can agree with experimental data.

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1. General Relativity (GR) is the cornerstone theory among the several attempts proposed to describe gravity. It represents an elegant approach furnishing several phenomenological predictions and its validity, in the Newtonian limit regime, is experimentally probed up to the Solar System scales. However, also at these scales, some conundrums come out as the indications of an apparent, anomalous, long–range acceleration revealed from the data analysis of Pioneer 10/11, Galileo, and Ulysses space crafts which are difficult to be framed in the standard scheme of GR and its low energy limit \cite{1,2}. Furthermore, at galactic distances, huge bulks of dark matter are needed to provide realistic models matching with observations. In this case, retaining GR and its low energy limit implies the introduction of an actually unknown ingredient. We face a similar situation even at larger scales: clusters of galaxies are gravitationally stable and bound only if large amounts of dark matter are supposed in their potential wells. Finally, an unknown form of dark energy is required to explain the observed accelerated expansion of cosmic fluid. Summarizing, almost $95\%$ of matter-energy content of the universe is unknown in the framework of Standard Cosmological Model while we can experimentally probe only gravity and ordinary (baryonic and radiation) matter. Considering another point of view, anomalous acceleration (Solar System), dark matter (galaxies and galaxy clusters), dark energy (cosmology) could be nothing else but the indications that shortcomings are present in GR and gravity is an interaction depending on the scale. The assumption of a linear Lagrangian density in the Ricci scalar $R$ for the Hilbert-Einstein action could be too simple to describe gravity at any scale and more general approaches should be pursued to match observations. Among these schemes, several motivations suggest to generalize GR by considering gravitational actions where generic functions of curvature invariants are present. Specifically, actions of the form

$$A = \int d^4 x \sqrt{-g} \left[ f(R) + L_m \right], \quad (1)$$

where $f(R)$ is an analytic function of $R$ and $L_m$ is the standard matter Lagrangian density, result particularly interesting. The variation of \cite{1} gives rise to fourth-order field equations

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = T^\text{curv}_{\alpha\beta} + T^m_{\alpha\beta}/f'(R) \quad (2)$$

where the curvature stress-energy tensor is defined as

$$T^\text{curv}_{\alpha\beta} = \frac{1}{f'(R)} \left\{ \frac{1}{2} g_{\alpha\beta} \left[ f(R) - R f'(R) \right] + f'(R) g^\mu\nu(g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\beta} g_{\mu\nu}) \right\}, \quad (3)$$

with prime denoting derivative with respect to $R$ and $T^m_{\alpha\beta}$ the standard matter contribution. For $f(R) = R$, the curvature stress-energy tensor identically vanishes and Eqs. \cite{2} reduce to the standard second-order Einstein field equations. From Eq. \cite{2}, it is clear that the curvature stress-energy tensor plays the role of a further source term in the field equations so that it can be considered as an effective fluid of purely geometric origin.

This approach is physically motivated by several unification theories of fundamental interactions and by the field quantization on curved space-times \cite{3}. At cosmological level, it is well known that further curvature contributions can solve shortcomings at early epochs (giving rise to inflationary solutions \cite{4}) and explaining the today observed
accelerated behavior by a sort of curvature quintessence, i.e. curvature can act as a fluid implementing acceleration. This result can be achieved in metric and affine (Palatini) approaches. In addition, reversing the problem, one can reconstruct the form of the gravity Lagrangian by observational data of cosmological relevance through a "back scattering" procedure. All these facts suggest that the function $f(R)$ should be more general than the linear Hilbert-Einstein one implying that higher order gravity could be a suitable approach to solve GR shortcomings without introducing mysterious ingredients as dark energy and dark matter (see e.g. 21, 22).

In recent papers, some authors have confronted this kind of theories even with the Post Parameterized Newtonian prescriptions in metric and Palatini approaches. However, the results seem controversial since in some cases it is argued that GR is always valid at Solar System scales and there is no room for other theories; nevertheless, some other studies find that recent experiments as Cassini and Lunar Laser Ranging allow the possibility that extended theories of gravity could be seriously taken into account. In particular, it is possible to define PPN-parameters in term of $f(R)$ functions and several classes of fourth order theories result compatible with experiments in Solar System 23.

In this letter, we follow a different approach. Starting from the definitions of PPN-parameters in term of a generic analytic function $f(R)$ and its derivatives, we deduce a class of fourth order theories, compatible with data, by means of an inverse procedure which allows to compare PPN-conditions with data. As a matter of fact, it is possible to show that a third order polynomial, in the Ricci scalar, is compatible with observational constraints on PPN-parameters. The degree of deviation from GR depends on the experimental estimate of PPN-parameters.

A useful method to take into account deviation with respect to GR is to develop expansions about the GR solutions up to some perturbation orders. A standard approach is the Parameterized-Post-Newtonian (PPN) expansion of the Schwarzschild metric. In isotropic coordinates, it is

$$ds^2 = \left(1 - \frac{\tilde{r}_g}{r}\right)^2 dt^2 - \left(1 + \frac{\tilde{r}_g}{4r}\right)^4 \left(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2\right)$$

(4)

where $\tilde{r}_g = 2GM/c^2$ is the Schwarzschild radius. Eddington parameterized deviations with respect to GR, considering a Taylor series in term of $r_g/\tilde{r}$ assuming that in Solar System, the limit $r_g/\tilde{r} \ll 1$ holds 27. The resulting metric is

$$ds^2 \simeq \left[1 - \alpha \frac{\tilde{r}_g}{r} + \frac{\beta}{2} \left(\frac{\tilde{r}_g}{r}\right)^2 + \ldots\right] dt^2 - \left[1 + \gamma \frac{\tilde{r}_g}{r} + \ldots\right] \left(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2\right)$$

(5)

where $\alpha, \beta, \gamma$ are unknown dimensionless parameters (Eddington parameters) which parameterize deviations with respect to GR. The reason to carry out this expansion up to the order $(r_g/\tilde{r})^2$ in $g_{00}$ and only to the order $(r_g/\tilde{r})$ in $g_{ij}$ is that, in applications to celestial mechanics, $g_{ij}$ always appears multiplied by an extra factor $v^2 \sim (M/\tilde{r})$. It is evident that the standard GR solution for a spherically symmetric gravitational system in vacuum is obtained for $\alpha = \beta = \gamma = 1$ giving again the Schwarzschild solution. Actually, the parameter $\alpha$ can be settled to the unity due to the mass definition of the system itself 27. As a consequence, the expanded metric (5) can be recast in the form:

$$ds^2 \simeq \left[1 - \frac{\tilde{r}_g}{r} + \frac{\beta - \gamma}{2} \left(\frac{\tilde{r}_g}{r}\right)^2 + \ldots\right] dt^2 - \left[1 + \gamma \frac{\tilde{r}_g}{r} + \ldots\right] dr^2 - r^2 d\Omega^2,$$

(6)

where we have restored the standard spherical coordinates by means of the transformation $r = \tilde{r} (1 + \frac{\tilde{r}_g}{\tilde{r}})^2$. The two parameters $\beta, \gamma$ have a physical interpretation. The parameter $\gamma$ measures the amount of curvature of space generated by a body of mass $M$ at radius $r$. In fact, the spatial components of the Riemann curvature tensor are, at post-Newtonian order,

$$R_{ijkl} = \frac{3}{2} \gamma \frac{\tilde{r}_g}{r^3} N_{ijkl}$$

(7)

independently of the gauge choice, where $N_{ijkl}$ represents the geometric tensor properties (e.g. symmetries of the Riemann tensor and so on). On the other side, the parameter $\beta$ measures the amount of non-linearity ($\sim (r_g/\tilde{r})^2$) in the $g_{00}$ component of the metric. However, this statement is valid only in the standard post-Newtonian gauge.

If one takes into account a more general theory of gravity, the calculation of the PPN-limit can be performed following a well defined pipeline which straightforwardly generalizes the standard GR case 27. A significant development in this sense has been pursued by Damour and Esposito-Farèse which have approached to the calculation of the PP-limit of scalar-tensor gravity by means of a conformal transformation $\tilde{g}_{\mu\nu} = F(\phi)g_{\mu\nu}$ to the standard Einstein frame. In fact, a general scalar-tensor theory

$$A = \int d^4x \sqrt{-g} \left[F(\phi)R + \frac{1}{2} \phi_{,\mu} \phi^{;\mu} - V(\phi) + L_m\right],$$

(8)
so that (11) is usually referred as a Brans-Dicke description of \( f \)-O’Hanlon Lagrangian [33]. However, the typical Brans-Dicke action is

\[
\mathcal{L}_{\text{ST}} \rightarrow E + \phi \quad \leftrightarrow \quad \mathcal{L}_{f(R)}
\]

where, by analogy,

\[
\psi \quad \rightarrow \quad \text{fifth order gravity}.
\]

Recasting fourth-order gravity as a scalar-tensor theory, often the following steps, in terms of a generic scalar-tensor gravity and fourth order gravity, although mathematically straightforward, requires a careful physical analysis. Recasting fourth-order gravity as a scalar-tensor theory, often the following steps, in terms of a generic scalar field \( \psi \), are considered

\[
f(R) + \mathcal{L}_m \rightarrow F'(\psi)R + F(\psi) - F'(\psi)\psi + \mathcal{L}_m \rightarrow F'(\psi)R - V(\psi) + \mathcal{L}_m,
\]

where, by analogy, \( \psi \rightarrow R \) and the "potential" is \( V(\psi) = F(\psi) - F'(\psi)\psi \). Clearly the kinetic term is not present so that (11) is usually referred as a Brans-Dicke description of \( f(R) \) gravity where \( \omega_{BD} = 0 \). This is the so-called O’Hanlon Lagrangian [38]. However, the typical Brans-Dicke action is

\[
\mathcal{A} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \ddot{R} + \frac{1}{2} \ddot{\phi} + \psi \right] - V(\psi) + \mathcal{L}_m,
\]

where no scalar field potential is present and \( \omega_{BD} \) is a constant. In summary, O’Hanlon Lagrangian has a potential but has no kinetic term, while Brans-Dicke Lagrangian has a kinetic term without potential. The most general situation is in [38, 39] where we have non-minimal coupling, kinetic term, and scalar field potential. This means that fourth-order gravity and scalar tensor gravity can be "compared" only by means of conformal transformations where kinetic and potential terms are preserved. In particular, it is misleading to state that PPN-limit of fourth order gravity is bad defined since these models provide \( \omega_{BD} = 0 \) and this is in contrast with observations [23, 24].

Scalar-tensor theories and \( f(R) \) theories can be rigorously compared, after conformal transformations, in the Einstein frame where both kinetic and potential terms are present. With this consideration in mind, \( F(\phi) \) and \( f(R) \) can be considered analogous quantities in Jordan frame and then the PPN limit can be developed [1].

Starting from this analogy, the PPN results for scalar-tensor gravity can be extended to fourth order gravity [25]. In fact, identifying \( \phi \rightarrow R \) [35], it is possible to extend the definition of the scalar-tensor PPN-parameters [28, 36] to the case of fourth order gravity:

\[
\gamma - 1 = -\frac{f''(R)^2}{f'(R) + 2f''(R)^2}, \quad \beta - 1 = \frac{1}{4} \frac{f''(R) \cdot f''(R) - 2f'(R) + 3f''(R)^2}{2f'(R) + 3f''(R)^2} \quad \frac{d\gamma}{dR},
\]

*1 To be precise, conformal transformations should be operated "before" performing PPN-limit and results discussed in the same frame. A back conformal transformation, after PPN limit, could be misleading due to gauge troubles.*
In [25], these definitions have been confronted with the observational upper limits on $\gamma$ and $\beta$ coming from Mercury Perihelion Shift [37] and Very Long Baseline Interferometry [38]. Actually, it is possible to show that data and theoretical predictions from Eqs. [13] agree in the limits of experimental measures for several classes of fourth order theories. Such a result tells us that extended theories of gravity are not ruled out from Solar System experiments but a more careful analysis of theories against experimental limits has to be performed. A possible procedure could be to link the analytic form of a generic fourth order theory with experimental data. In fact, the matching between data and theoretical predictions, found in [25], holds provided some restrictions for the model parameters but gives no general constraints on the theory. In general, the function $f(R)$ could contain an infinite number of parameters (i.e. it can be conceived as an infinite power series [24]) while, on the contrary, the number of useful relations is finite (in our case we have only two relations). An attempt to deduce the form of the gravity Lagrangian can be to consider the relations [13] as differential equations for $f(R)$, so that, taking into account the experimental results, one could constrain, in principle, the model parameters by the measured values of $\gamma$ and $\beta$.

3. The idea is supposing the relations for $\gamma$ and $\beta$ as differential equations. This hypothesis is reasonable if the derivatives of $f(R)$ function are smoothly evolving with the Ricci scalar. Formally, one can consider the r.h.s. of the definitions [13] as differential relations which have to be matched with values of PPN-parameters. In other words, one has to solve the equations [13] where $\gamma$ and $\beta$ are two parameters. Based on such an assumption, on can try to derive the largest class of $f(R)$ theories compatible with experimental data. In fact, by the integration of Eqs. [13], one obtains a solution parameterized by $\beta$ and $\gamma$ which have to be confronted with the experimental quantities $\beta_{exp}$ and $\gamma_{exp}$.

Assuming $f'(R) + 2f''(R)^2 \neq 0$ and defining $A = \left| \frac{1 - \gamma}{2\gamma - 1} \right|$, we obtain from [13] a differential equation for $f(R)$:

$$[f''(R)]^2 = Af'(R).$$

(14)

The general solution of such an equation is a third order polynomial $f(R) = aR^3 + bR^2 + cR + d$ whose coefficients have to satisfy the conditions: $a = b = c = 0$ and $d \neq 0$ (trivial solution) or $a = \pm \frac{A}{3}, b = \pm \sqrt{\frac{2\gamma}{3}A},$ with $c, d \neq 0$. Thus, the general solution for the non-trivial case, in natural units, reads

$$f(R) = \frac{1}{12} \left| \frac{1 - \gamma}{2\gamma - 1} \right| R^3 \pm \frac{\sqrt{\gamma}}{2} \sqrt{\frac{1 - \gamma}{2\gamma - 1}} R^2 + cR + d.$$  

(15)

It is evident that the integration constants $c$ and $d$ have to be compatible with GR prescriptions and, eventually, with the presence of a cosmological constant. Indeed, when $\gamma \to 1$, which implies $f(R) \to cR + d$, the GR-limit is recovered. As a consequence the values of these constants remain fixed ($c = 1$ and $d = \Lambda$, where $\Lambda$ is the cosmological constant). Therefore, the fourth order theory provided by Eq. [15] becomes

$$f_{\pm}(R) = \frac{1}{12} \left| \frac{1 - \gamma}{2\gamma - 1} \right| R^3 \pm \frac{\sqrt{\gamma}}{2} \sqrt{\frac{1 - \gamma}{2\gamma - 1}} R^2 + R + \Lambda,$$  

(16)

where we have formally displayed the two branch form of the solution depending on the sign of the coefficient entering the second order term. Since the constants $a, b, c, d$ of the general solution satisfy the relation $3ac - b^2 = 0$, one can easily verify that it gives:

$$\left. \frac{d\gamma}{dR} \right|_{f_{\pm}(R)} = -\frac{d}{dR}\left[ \frac{f''(R)^2}{f'(R) + 2f''(R)^2} \right]_{f_{\pm}(R)} = 0,$$  

(17)

where the subscript $f_{\pm}(R)$ refers to the calculation to the solution [16]. This result, compared with the second differential equation Eq. [14], implies $4(\beta - 1) = 0$, which means the compatibility of the solution even with this second relation.

4. Up to now we have discussed a family of fourth order theories [16] parameterized by the PPN-quantity $\gamma$; on the other hand, for this class of Lagrangians, the parameter $\beta$ is compatible with GR value being unity.

Now, the further step directly characterizes such a class of theories by means of the experimental estimates of $\gamma$. In particular, by fixing $\gamma$ to its observational estimate $\gamma_{exp}$, we will obtain the weight of the coefficients relative to each of the non-linear terms in the Ricci scalar of the Lagrangian [16]. In such a way, since GR predictions require exactly $\gamma_{exp} = \beta_{exp} = 1$, in the case of fourth order gravity, one could take into account small deviations from this values as inferred from experiments. Some plots can contribute to the discussion of this argument. In Fig [16] the Lagrangian [16] is plotted. It is parameterized for several values of $\gamma$ compatible with the experimental bounds coming from the Mercury perihelion shift (see Table.1 and [55]). The function is plotted in the range $R \geq 0$. Since the property $f_+(R) = -f_-(R)$ holds for the function [16], one can easily recover the shape of the plot in the negative region. As it is reasonable, the deviation from GR becomes remarkable when scalar curvature islarge.
Mercury Perihelion Shift | $|2\gamma - \beta - 1| < 3 \times 10^{-3}$
Lunar Laser Ranging | $43 - \gamma - 3 = -(0.7 \pm 1) \times 10^{-3}$
Very Long Baseline Interf. | $|\gamma - 1| = 4 \times 10^{-4}$
Cassini Spacecraft | $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$

TABLE II: A schematic resume of recent experimental constraints on the PPN-parameters. They are the perihelion shift of Mercury [37], the Lunar Laser Ranging [38], the upper limit coming from the Very Long Baseline Interferometry [38], and the results obtained by the estimate of the Cassini spacecraft delay into the radio waves transmission near the Solar conjunction [40].

FIG. 1: Plot of the two branch solution provided in Eq. (16). The $f_{+}(R)$ (dotted line) branch family is up to GR solution (straight line), while the one indicated with $f_{-}(R)$ (dotted-dashed line) remains below this line. The different plots for each family refer to different values of $\gamma$ fulfilling the condition $|\gamma - 1| \leq 10^{-4}$ and increased by step of $10^{-5}$.

In order to display the differences between the theory [10] and Hilbert-Einstein one, the ratio $f(R)/R$ is plotted in Fig. 2. Again it is evident that the two Lagrangians differ significantly for great values of the curvature scalar. It is worth noting that the formal difference between the PPN-inspired Lagrangian and the GR expression can be related to the physical meaning of the parameter $\gamma$ which is the deviation from the Schwarzschild-like solution. It measures the spatial curvature of the region which one is investigating, then the deviation from the local flatness can be due to the influence of higher order contributions in Ricci scalar. On the other hand, one can reverse the argument and notice that if such a deviation is measured, it can be recast in the framework of fourth order gravity, and in particular its “amount” indicates the deviation from GR. Furthermore, it is worth considering that, in the expression [10], the modulus of the coefficients in $\gamma$ (i.e. the strength of the term) decreases by increasing the degree of $R$. In particular, the highest values of cubic and squared terms in $R$ are, respectively, of order $10^{-4}$ and $10^{-2}$ (see Fig. 3) then GR remains a viable theory at short distances (i.e. Solar System) and low curvature regimes.

FIG. 2: The ratio $f(R)/R$. It is shown the deviation of the fourth order gravity from GR considering the PPN-limit. Dotted and dotted-dashed lines refer to the $f_{+}(R)$ and $f_{-}(R)$ branches plotted with respect to several values of $\gamma$ (the step in this case is $2.5 \times 10^{-5}$).
A remark is in order at this point. The class of theories which we have discussed is a third order function of the Ricci scalar $R$ parameterized by the experimental values of the PPN parameter $\gamma$. In principle, any analytic $f(R)$ can be compared with the Lagrangian (16) provided suitable values of the coefficients. However, more general results can be achieved relaxing the condition $\beta = 1$ which is an intrinsic feature for (16) (see for example [24]). These considerations suggest to take into account, as physical theories, functions of the Ricci scalar which slightly deviates from GR, i.e. $f(R) = f_0 R^{1+\epsilon}$ with $\epsilon$ a small parameter which indicates how much the theory deviates from GR [41]. In fact, supposing $\epsilon$ sufficiently small, it is possible to approximate this expression

$$f_0 |R|^{(1+\epsilon)} \simeq f_0 |R| \left(1 + \epsilon \ln |R| + \frac{\epsilon^2 \ln^2 |R|}{2} + \ldots \right).$$

This relation can be easily confronted with the solution (16) since, also in this case, the corrections have very small "strength" [42].

5. We have shown how a polynomial Lagrangian in the Ricci scalar $R$, compatible with the PPN-limit, can be recovered in the framework of fourth order gravity. The approach is based on the formulation of the PPN-limit of such gravity models developed in analogy with scalar-tensor gravity [25]. In particular, considering the local relations defining the PPN fourth order parameters as differential expressions, one obtains a third-order polynomial in the Ricci scalar which is parameterized by the PPN-quantity $\gamma$ and compatible with the limit $\beta = 1$. The order of deviation from the linearity in $R$ is induced by the deviations of $\gamma$ from the GR expectation value $\gamma = 1$. Actually, the PPN parameter $\gamma$ may represent the key parameter to discriminate among relativistic theories of gravity. In particular, this quantity should be significatively tested at Solar System scales by forthcoming experiments like LATOR [43]. From a physical point of view, any analytic function of $R$, by means of its Taylor expansion, can be compared with (16). Therefore, a theory like $f(R) = f_0 R^{1+\epsilon}$, indicating small deviations from standard GR, is in agreement with the proposed approach, so, in principle, the experimental $\gamma$ could indicate the value of the parameter $\epsilon$. In conclusion, one can reasonably state that generic fourth-order gravity models could be viable candidate theories even in the PPN-limit. In other words, due to the presented results, they cannot be a priori excluded at Solar System scales.

[1] J.D. Anderson et al., Phys. Rev. Lett. 81 (1998) 2858.
[2] J.D. Anderson, et al., Phys. Rev. D 65 (2002) 082004.
[3] N.D. Birrell, P.C.W. Davies, Quantum Fields in Curved Space, Cambridge Univ. Press, Cambridge (UK), 1982.
[4] A.A. Starobinsky, Phys. Lett. B, 91, 99, 1980.
[5] Capozziello S., Int. J. Mod. Phys. D 11, 483, (2002).
[6] Capozziello S., Cardone V.F., Carloni S., Troisi A., Int. J. Mod. Phys. D12, 1969 (2003).
[7] Carroll S.M., Duvvuri V., Trodden M., Turner M.S., Phys. Rev. D 70, 043528 (2004).
[8] Nojiri S., Odintsov S.D., Phys. Lett. B 576, 5 (2003).
[9] Nojiri S., Odintsov S.D., Mod. Phys. Lett. A 19, 627 (2003).
[10] Nojiri S., Odintsov S.D., Gen. Rel. Grav. 36, 1765 (2004).
[11] Capozziello, S., Carloni, S. and Troisi, A., Rec. Res. Devel. Astronomy. & Astrophys. 1, 625 (2003).
[12] Allemandi G., Borowiec A., Francaviglia M., Phys. Rev. D 70, 043524 (2004).
[13] Allemandi G., Borowiec A., Francaviglia M., Phys. Rev. D 70, 103503 (2004).
[14] Vollick D. N., Phys. Rev. D 68, 063510 (2003).
[15] Meng X. H., Wang P., Class. Quant. Grav. 20, 4949 (2003).
[16] Flanagan E. E., Phys. Rev. Lett. 92, 071101 (2004).
[17] Flanagan E. E., Class. Quant. Grav. 21, 417 (2004).
[18] Meng X. H., Wang P., Class. Quant. Grav. 21, 951 (2004).
[19] G. M. Kremer and D. S. M. Alves, Phys. Rev. D 70, 023503 (2004).
[20] T. Multamaki and I. Vilja, Phys. Rev. D, 73, 024018, 2006.
[21] Capozziello S., Cardone V.F., Troisi A., Phys. Rev. D 71, 043503 (2005).
[22] Capozziello S., Cardone V.F., Carloni S., Troisi A., Phys. Lett. A 326, 292 (2004).
[23] Olmo G.J., Phys. Rev. Lett. 95, 261102 (2005).
[24] Olmo G.J., Phys. Rev. D 72, 083505 (2005).
[25] Capozziello S., Troisi A., Phys. Rev. D 72 (2005).
[26] Allemandi G., Francaviglia M., Ruggiero M., Tartaglia A., Gen. Rel. Grav. 37, 1891 (2005).
[27] Will C. M., Theory and Experiments in Gravitational Physics, Cambridge Univ. Press, Cambridge (1993).
[28] Danour T., Esposito-Farese G., Class. Quant. Grav. 9, 2093 (1992).
[29] Danour T., Esposito-Farese G., Phys. Rev. Lett. 70, 2220 (1993).
[30] Danour T., Esposito-Farese G. Phys. Rev. D 54, 1474 (1996).
[31] Danour T., Esposito-Farese G. Phys. Rev. D 58, 042001 (1998).
[32] Faraoni V., Cosmology in Scalar-tensor Gravity, Kluwer, Dordrecht, 2004.
[33] Teyssandier P., Tourrenc P., J. Math. Phys. 24, 2793 (1983).
[34] Schmidt H.J., Class. Quant. Grav. 7, 1023 (1990).
[35] Wands D., Class. Quant. Grav. 11, 269 (1994).
[36] Schimd C., Uzan J. P., and Riazuelo A., Phys. Rev. D 71, 083512 (2005).
[37] Shapiro I.I., in General Relativity and Gravitation 12, Ashby N., et al., Eds. Cambridge University Press (1993).
[38] Shapiro S.S., et al., Phys. Rev. Lett. D 92, 121101 (2004).
[39] Williams J.G., et al., Phys. Rev. D 53, 6730 (1996).
[40] Bertotti B., Iess L., Tortora P., Nature 425, 374 (2003).
[41] T. Clifton, J.D. Barrow, Phys. Rev. D 72, 103505 (2005).
[42] Capozziello S., Francaviglia M. in preparation.
[43] Turyshev S.G., Shao M., K.L. Nordtvedt K.L., pre-print: gr-qc/0601035.