Equation of state dependencies and universal relations studies

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1 INTRODUCTION

Asteroseismology that enables us to understand the interior structure of

neutron stars is coming to the forefront, with added tools of grav-

itational wave astronomy applied in detecting gravitational wave

signal from two binary neutron star mergers (GW170817 Abbott

et al. (2017) and GW190425 Abbott et al. (2020)), electromagnetic

observations like NICER data; this assists in resolving degenerate

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et al. (2020).

Some of the earlier works on gravitational asteroseismology includes

Andersson & Kokkotas (1998); Allen et al. (1998); Pons

et al. (2002); Benhar et al. (2004) (see Kokkotas (1996) for the pul-

sations of relativistic stars and references therein). Through multiple

studies it has been shown that the fundamental oscillation mode (f-

mode) of the star has a strong resonance with the orbital frequencies
closer to merger of a coalescing binary neutron star. Resonant tidal
excitation of oscillation modes in merging binary neutron stars has
been carried out by Lai (1994); Lai & Wu (2006); Xu & Lai (2017).
Other oscillation modes of the star, like p and g modes, may also
become relevant throughout the inspiral, due to nonlinear coupling
to the tide, as discussed by Weinberg et al. (2013); Weinberg (2016);
Zhou & Zhang (2017); Nouri et al. (2021).

Schmidt & Hinderer (2019) describes an f-mode tidal model
(fm tidal) in the frequency domain under stationary phase approxi-
mation (SPA), discussed in Finn & Chernoff (1993). They com-
pute tidal effects in the phase as described in Flanagan & Hinderer
(2008). The Love numbers are measured by estimating the best-fit
parameters using Poisson & Will (1995). Based on the “fmtidal”
model, Pratten et al. (2020) quotes f-mode frequency estimates us-
ing GW170817 data in the range of 1.37 – 1.47 and 1.47 – 1.59
kHz, (for the larger and the smaller NS mass companions, respec-
tively) while taking into account the various PN order corrections
effects of adiabatic and dynamical tides. They also provide up-
per bounds by including the universal relations which are around
\( \approx 2.8 - 2.9 \) kHz and \( \approx 3.1 - 3.2 \) kHz for the two NS’s. Phenomenological models connecting tidal deformability with frequency have been also discussed by Andersson & Pinhó (2019, 2020).

Contribution due to spin during the binary neutron star merger on \( f \)-modes and dynamical tides have been recently carried out by Ma et al. (2020) and Steinhoff et al. (2021) based on some of the earlier works by Hinderer et al. (2010, 2016) and Steinhoff et al. (2016). Gravitational-wave asteroseismology with \( f \)-modes from neutron star binaries at the merger phase has been also carried out recently by Ng et al. (2020) where they use NR BNS simulations carried out in Rezzolla & Takami (2016); Dietrich et al. (2017a,b) and find less than one percent difference between BNS merger frequency (based on the merger peak amplitude) and \( f \)-modes computed for isolated neutron stars. They use the universal relations between \( f \)-mode and tidal deformability as given in Chan et al. (2014) to discuss UR between merger frequency and the rescaled tidal coupling parameter defined in terms of the dimensionless tidal Love numbers of the component neutron stars. Frequency deviations in the universal relations of isolated neutron stars and postmerger remnants have been discussed by Liou et al. (2021), whereas UR for damping time have been carried out in Liou & Stergioulas (2018).

Universal relations among compactness \( (C = M/R) \), moment of inertia \((I)\) and tidal deformability, and their validity and deviations have been studied in Maselli et al. (2013); Chan et al. (2014); Chirenti et al. (2015); Yagi & Yunes (2017a) for the various choices of equations of state (see Yagi & Yunes (2017b) for review).

Jiang & Yagi (2020) performs an analytical study showing about choices of equation of state (see Yagi & Yunes (2017b) for review). These studies are based on the standard approach laid out by Thorne & Campolattaro (1967) to solve the stellar perturbation equations in curved spacetime by applying the appropriate boundary conditions. The \( f \)-mode oscillations have also been studied using numerical simulations, employing the cowling approximation i.e. by evolving hydrodynamic equations in the fixed background of general relativistic spacetime for a single perturbed neutron star Font et al. (2000); Font et al. (2001); Shibata & Karino (2004); Kastaun et al. (2010); Chirenti et al. (2015), also in full GR to study bar mode instability De Pietri et al. (2014), as well as under conformally flat condition (CFC) Dimmelmeier et al. (2006); Bucciantini & Del Zanna (2011); Pili et al. (2014) where the 3-metric is assumed to be conformally flat and the spacetime dynamics is coupled with fluid dynamics. Ng et al. (2020) employ CFC approach as discussed in Dimmelmeier et al. (2006) and use a publicly available code XNS described in Bucciantini & Del Zanna (2011); Pili et al. (2014).

Recently, Rosofsky et al. (2019) carried out studies in the fully dynamical spacetime and extracted the fundamental modes for the polytropic equations of state. There have been studies using piece-wise poly-tropic EoS to impose constraints on the neutron star structure Bauswein et al. (2020); Miller et al. (2020) or using the parametrised EoS with continuous sound speed O’Boyle et al. (2020).

In the current work, we extract \( f \)-mode frequencies by evolving non-rotating neutron star in the dynamical spacetime while considering tabulated realistic EoS to describe its internal composition.

We also probe the possibility of using these single star simulations to study the tidal effects expected during a binary inspiral by perturbing the stars in the Cowling regime and then evolving them using full General Relativity. Single star simulations are computationally less expensive to carry out, providing the possibility to reach higher resolutions in the future which may be important for studying higher order harmonics. We, further, study the validity of our results using the URs associated with tidal deformability and stellar parameters.

This is a step towards extending our study incorporating spin effects which we shall report in the follow up work. Our work provides leeway for future studies of tidal effects of rotating stars in an inspiral and also post-merger remnants with differential rotation.

In section 2 we briefly outline mathematical and numerical framework for general relativistic hydro-dynamical system, the initial setup and matter configuration as described by a set of equations of state under consideration. We analyse our simulation data and describe the result findings in section 3. We compute fundamental mode (\( f \)-mode frequency), its relations as a function of compactness and mass, then compare our fits with some of the recently carried out works. Our findings match well with the universal relations described with a deviation less than of a few percent. Section 4 discusses our results and conclusions. Appendix B briefly shows convergence test for our simulations.

## 2 Basic Numerical Framework

We solve the Einstein equations using Numerical Relativity methods of 3+1 decomposition of the spacetime.

\[
G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}
\]

\[
d\sigma^2 = (-\alpha^2 + \beta_i \beta^i) dt^2 + 2\beta_i dt dx^i + \gamma_{ij} dx^i dx^j
\]

Alcubierre (2008); Rezzolla & Zanotti (2013); Baumgarte & Shapiro (2010); Shibata (2015) here, \( \alpha \) is the lapse function, \( \beta^i \) is the shift vector and \( \gamma_{ij} \) is the spatial metric. The units used are \( G = c = 1 \) unless mentioned otherwise.

### 2.1 Dynamical Evolution

The sapcetime evolution is achieved using the Baumgarte-Shapiro-Shibata-Nakamura-Oohara-Kojima (BSSNOK) formalism (Nakamura et al. 1987; Shibata & Nakamura 1995; Baumgarte & Shapiro 1998; Alcubierre et al. 2000) which is a conformal formulation of the ADM equations (Arnowitt et al. 1959, 2008). The \( 1+log \) and Gamma-driver gauge conditions are adopted for the evolution of lapse and shift (Alcubierre 2008; Baiotti & Rezzolla 2017). We use the MacLACHLAN code (Brown et al. 2009) for evolution of spacetime variables which is a publicly available code in the EINSTEINTOOLKIT (Babiuc-Hamilton et al. 2019; Goodale et al. 2003; Schnetter et al. 2006, 2004) suite. A Kreiss-Oliger dissipation (Rezzolla & Zanotti 2013; Alcubierre 2008; Shibata 2015) is added to spacetime variables for removing high frequency noise.

To model Neutron stars, a relativistic perfect fluid is assumed. The conservation equations for the energy-momentum tensor \( T_{\mu\nu} \) and the matter current density \( J_{\mu} \) are solved numerically after being re-cast into a flux-conservative formulation (Rezzolla & Zanotti 2013; Font 2008; Baiotti & Rezzolla 2017)

\[
\nabla_{\mu} J^\mu = 0, \nabla_{\mu} T^{\mu\nu} = 0
\]

The system of equations is complete with an Equation of State
of the type $p = p(\rho, Y_e, T)$ which for our models is described in the sec. 2.3. (For more details readers can refer to (Rezzolla & Zanotti 2013; Font 2008; Baiotti & Rezzolla 2017; Shibata 2015).)

We carry out the hydrodynamics using the publicly available WhiskyTHC code (Radice et al. 2013, 2014; Radice, D. & Rezzolla, L. 2012) which works within the EinsteinToolKit framework and uses high-resolution shock capturing methods. For the time integration, method of lines is used with fourth-order Runge-Kutta methods. The fifth-order MPS flux-reconstruction method is used along with Harten-Lax-van Leer-Einfeldt (HLL) Riemann solver.

2.2 Initial data

We use a perturbed Tolman–Oppenheimer–Volkoff (TOV) star for the initial data. The initial data is generated using the PizzaTOV thorn for both polytropic and tabulated equations of state. A perturbation is added for the density as:

$$\delta \rho = A \rho (r/R) Y_{22}$$

where, $\delta \rho$ is the perturbed density, $\rho$ is the density, $A$ is the perturbation amplitude which we set to 0.01 to introduce a small perturbation, $r$ is the radial distance from centre of the star and $R$ is the radius of the star. We use the (2,2) eigenfunction which is expected to be similar to tidal interactions of binary neutron stars (Rosofsky et al. 2019; Pratten et al. 2020). For all the models we choose the artificial background atmosphere density as $\rho_{\text{atm}} = 10^{-14} M \approx 6.17 \times 10^3 g \text{ cm}^{-3}$. The simulations are performed at minimum grid spacing of 0.105$M \approx 155$m. For the tabulated EoS 1.4$M_\odot$ cases we also perform simulations at 0.07$M$ and for polytropic at 0.06$M$. The convergence test for the DD2 equation of state for three different resolutions is given in appendix B.

2.3 Equations of State

We use several finite-temperature, composition dependent nuclear-theory based equations of state (fig. 1). Three of them are based on relativistic mean field (RMF) models. These equations of state are publicly available in tabulated form at https://stellarcollapse.org.

(i) LS220 Lattimer & Swesty (1991) (Lattimer & Swesty EoS with the incompressibility K = 220 MeV); contains neutrons, protons, alpha particles and heavy nuclei. It is based on the single nucleus approximation for heavy nuclei. LS220 has been widely used in many supernova simulations.

(ii) DD2 Hempel et al. (2012): contains neutrons, protons, light nuclei such as deuterons, helions, tritons and alpha particles and heavy nuclei. DD2 is an RMF with a density-dependent nucleon-meson coupling for treating high density nuclear matter.

(iii) SFHo Steiner et al. (2013): Another RMF, and similar to DD2 it contains neutrons, protons, light nuclei such as deuterons, helions, tritons and alpha particles and heavy nuclei. However, the RMF parameters are tuned to fit the NS mass-radius observation.

(iv) BHB Banik et al. (2014): Another RMF similar to DD2 and SFHo with the same particle composition, but BHB EoS additionally includes $\Lambda$ hyperons and hyperon-hyperon interactions allowed by $\phi$ mesons.

(v) SLY Chabanat et al. (1998): contains only protons, neutrons and electrons. It is developed out of a refined Skyrme-like effective potential, originating from the shell-model description of the nuclei.

3 ANALYSIS AND RESULTS

We evolve a set of non-rotating configurations in the mass range of $1.2 - 2.0 M_\odot$ for each of the EoS, described in section 2.3. The mass, radius, central density for these have been listed in the first four columns of table 2. The frequency of the mode is computed by taking the Fourier transform of the time series data for the gravitational waveform generated during the evolution of each simulation. For

\begin{equation}
\Gamma = 2
\end{equation}

\begin{equation}
P(\rho, \epsilon) = P(\rho) + \rho (\epsilon - \epsilon_{\text{cold}}(\rho))(\Gamma_{th} - 1)
\end{equation}
smoothing the peaks in the FFTs we use cubic spline. We consider the waveform that is extracted at the radius \( r = 10M \) for the \( l = 2, m = 2 \) mode of the \( \Psi_4 \) data using the Newman-Penrose formalism (Newman & Penrose 1963; Bishop & Rezzolla 2016). We choose this radius since it has the least amount of noise among all the considered extraction radii while the computed fundamental mode stays the same as could be seen in Fig. B.4.

We also notice that change in resolution does not affect the extracted \( f \)-mode frequencies Fig. B.3 (appendix B). We run the simulations for \( 2100M \approx 10.35ms \).

### 3.1 fundamental \((f)\)-modes

We present our models and results of our simulations in table 2. In Fig. 2 (top panels) we plot the \( f \)-mode frequency value in terms of mass and compactness for the considered set of equations of state.

We notice that the \( f \)-mode frequency is much smaller for the stiff EoS such as DD2 and BHB. It increases for the higher masses and softer EoS. Based on the frequency band of the observed gravitational signal and the inferred mass, it is possible to put bounds on the soft/stiffness of the EoS that characterises the interior of the neutron star. For example, if the detected \( f \)-mode frequency is below 1.8 kHz for a star having mass above \( 1.4M_\odot \), then all the considered soft EoS would be ruled out. On the other hand, frequency above 1.8 kHz would permit only some stiff EoS if the object is more massive i.e. \( \approx 1.8M_\odot \) or more and compactness above 0.20. Similar trends we see in terms of effective compactness \( \eta \) and tidal deformability \( \lambda_2 \) (see bottom panels of Fig. 2). We also observe that while few of the considered EoS show linear trend, some of the other, such as BHB, LS220, SLy and SFHo deviate and show a faster rise in \( f \)-mode values for larger parameter values (as could be seen in all the four panels of Fig 2). This doesn’t seem to be dependent only on the softness or stiffness of the matter, but could be due to the finer micro-physics involved. To understand this better, in our follow-up studies, we plan to consider a larger set of EoS, including the ones having hyperons, strange quark matter, etc.

In Fig. 3 we present the fits of our models with the relation as given in ref. (Chirenti et al. 2015):

\[
f = a + b \sqrt{\frac{M}{R^3}}
\]

We find a good linear fit for our data across the EoS used for our study and list the values of \( a \) and \( b \) in table 1. LS220 EoS shows the steepest change (the red line), followed by BHB (the Beige color line) in Fig. 3. We compare our fit obtained in table 1 to the Table II in ref. (Chirenti et al. 2015) for the LS220 and APR4 equation of states, and find our results to be in agreement (within 10%).

### 3.2 Universal Relations

In order to study, whether the equations of state specific trends that we notice above, contribute to the deviation from Universal relations or not, we verify some of the URs. First, we compute the relation between \( f \)-mode frequency and the tidal deformability as given in (Pratten et al. 2020; Chan et al. 2014)

\[
M\omega = \sum_i a_i \left(\frac{\xi}{\lambda_2}\right)^i
\]

where \( a_i \) are the numerical coefficients presented in table 3. \( M \) is the mass of the star and \( \omega \) is the angular \( f \)-mode frequency, \( \xi = \log(\lambda_2) \) where \( \lambda_2 \) is the dimensionless electric tidal deformability calculated from the tidal Love number \( k_2 \) as (Chan et al. 2014):

\[
\lambda_2 = \frac{2}{3} \frac{k_2}{(M/R)^5}
\]

The tidal Love number \( k_2 \) is calculated by solving the metric perturbation equation as defined in ref. (Hinderer 2008). For this computation we integrate Eq. (15) from Hinderer (2008) for the metric perturbation function \( H \), from center to surface using fourth order Runge-Kutta method. We use the Runge-Kutta ODE solver with adaptive step size routine from Numerical recipe (Press et al. 2007). Finally, the tidal Love number \( k_2 \) is computed from Eq.(23) from Hinderer (2008). Figure 2 (bottom-right panel) shows the relation of tidal deformability \( \lambda_2 \) with the \( f \)-mode.

The comparison and deviation for \( f \)-Love UR, Eq. 8 is carried out in two ways: first, using these equations we compute the coefficients for our data. Second, using the same values of coefficients as given in ref. (Chan et al. 2014) but, with the \( f, \lambda_2 \) and \( \eta \) that we calculate for each of our simulation. The last three columns of table 2 show comparison and percent deviations with \( f \)-Love UR’s. Table 3 lists the coefficients for ref. (Chan et al. 2014) and the ones computed for our data. In fig. 4 we compare the our results with that of ref. (Chan et al. 2014).

It has been observed that universal behaviour also exist between the \( f \)-mode and the effective compactness \( \eta \) (Lau et al. 2010; Chirenti et al. 2015). We also test these URs described by the model

\[
M\omega = c_1 + c_2 \eta + c_3 \eta^2
\]

where \( \eta = \sqrt{M^2/I} \) and \( I \) is the moment of inertia. We obtain the fit as \( c_1 = -0.00747, c_2 = 0.1471 \) and \( c_3 = 0.55328 \). We show this universality in the fig. 5. Our results agree well with the results of the earlier works (Lau et al. 2010; Chirenti et al. 2015). The fundamental mode \( f \)-mode and effective compactness \( \eta \) that we compute for our configurations are also plotted in Fig 2 (bottom left panel).

Universal relation between the compactness and tidal Love numbers have been discussed in appendix A.

### 3.3 Calculation of damping times

We compute fundamental frequency from \( \Psi_4 \) data of our simulations (see discussion in the beginning of section 3) by evolving each initial configuration for about \( 2100M \approx 10 \) ms. This duration is insufficient, and it is required to have a longer simulation and higher resolution to extract reliable damping times. Thus, we choose the recently established relations, which use compactness and effective

| EoS    | a   | b   |
|--------|-----|-----|
| DD2    | 0.657| 32.061|
| BHB    | 0.429| 39.801|
| LS220  | 0.286| 44.653|
| SFHo   | 0.663| 35.101|
| SRO-APR| 0.881| 29.958|
| SLy    | 0.557| 36.544|
| APR4   | 0.795| 30.791|

Table 1. Data fitted for the various compact star models with Eq. 7. We find that our results are in agreement with the ref. (Chirenti et al. 2015).
\[ f \text{-mode oscillations in dynamical spacetimes} \]

Figure 2. \( f \)-mode frequency for different EoS is shown in different panels with respect to mass \( M \) (top left); compactness \( C = M/R \), \( R \) being the radius of the star (top right); effective compactness \( \eta = \sqrt{M^3/\ell} \) where \( I \) is the moment of inertia (bottom left) and tidal deformability \( \lambda_2 \) as in Eq. 9 (bottom right). The softer EoS has larger \( f \)-mode frequencies as compared to the intermediate (LS220) and stiffer (DD2 and BHB) EoS.

Figure 3. Fit from Eq. 7. The \( a \) and \( b \) values are listed on the figure above and also table 1.

Figure 4. The universality observed between tidal deformability and \( f \)-modes. Here, we compare our fit with that of ref. (Chan et al. 2014).

\[
\frac{M}{\tau_1} = 0.112 \left( \frac{M}{R} \right)^4 - 0.53 \left( \frac{M}{R} \right)^5 + 0.628 \left( \frac{M}{R} \right)^6
\]  

where \( \tau_1 \) and \( \tau_2 \) are the damping times in terms of compactness \( C = M/R \) and effective compactness \( \eta \), respectively. Figure 6 shows

\[
\frac{I^2}{M^3 \tau_2} = 0.0068 - 0.025 \eta^2
\]  

(12)
Table 2. $f$ modes for the different EoS for different masses. A clear trend is visible. Frequency increases with mass and with softness of the EoS. Agreement of $f$-Love URs (Chan et al. 2014; Fraternali et al. 2020) is also presented for the different models for $f = 2$ by calculating the % difference between LHS and RHS of Eq. 8. Second to last two columns are obtained by fitting the UR with our data. And the last column lists the difference between the UR provided in (Chan et al. 2014) and our work.

Table 3. Fitting parameters $a_i$ from Eq. 8 for the $f$ -Love universal relations. Comparing this work and (Chan et al. 2014). The $a_i$ values have been calculated without the polytropic models.

damping time vs frequency plot, where the damping time is taken to be the average of $\tau_1$ and $\tau_2$. We find that for the polytrope case one of the relations overestimates the damping time obtained from linear perturbations methods (Baiotti et al. 2009; Rososky et al. 2019) while the other underestimates, taking an average of $\tau_1$ and $\tau_2$ gives us a value close to the expected value for the polytrope, hence we report the average values. In the subsequent study, we intend to extract damping times by evolving our systems for a much longer time and at a much higher resolution as one needs to be sure that the damping obtained is not due to numerical errors.

4 CONCLUSIONS

In this work, we evolve isolated non-spinning Neutron star in fully dynamical spacetime. We consider a set of realistic EoS as listed in section 2.3 to describe internal composition of the star. For each of the considered EoS, we evolve 5-6 configurations having mass in
The range of between approach. We compute tidal deformability for each of the case using perturbative of the case and its dependence on the equation of state. We also
time here is calculated as

\[ \tau = \frac{\tau_1 + \tau_2}{2} \]  

from Eq. 11 and Eq. 12. We compute \( \tau \) for each \( M \) from DD2 (stiff EoS) and LS220 (having

\[ f\text{-mode oscillations in dynamical spacetimes} \]

the black dashed line is the fit that we obtained for Eq. 10. The brown dashed line is the one presented in the ref. (Chirenti et al. 2015).

![Figure 5](image)

**Figure 5.** The universality observed between \( \eta \) and \( f\)-mode. The black dashed line is the fit we obtained for Eq. 10. The brown dashed line is the one presented in the ref. (Chirenti et al. 2015).

The data underlying this article will be shared on reasonable request to the corresponding authors.

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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding authors.

Figure 6. \( f\)-mode vs the damping time for all the models. The damping time here is calculated as \( (\tau_1 + \tau_2)/2 \) from Eq. 11 and Eq. 12.

![Figure 6](image)

Data for this study will be shared on reasonable request to the corresponding authors.
APPENDIX A: COMPACTNESS - TIDAL DEFORMABILITY UNIVERSAL RELATIONS

For our compact star models, we also study another universal relation in addition to the discussion in sec.3.2 between the compactness and tidal Love number using the initial data that we use in our simulations. We also use this as a check for our initial data against already established results (Maselli et al. 2013; Godzieba et al. 2021). The universal relations are given as,

\[ C = \sum_b b_i (\log(\lambda_2))^i \]  

(A1)

The universal relation between compactness and tidal deformability provides us the constraint on the radius of the star as a less compact star will be deformed more by a tidal potential for a given mass (Godzieba et al. 2021), providing a relation between radius and \( \lambda_2 \). We present our obtained fit for \( b_i \) in table A1. In fig. A1, we compare our results to a recent study by (Godzieba et al. 2021). Our results are consistent with the results of (Maselli et al. 2013).

APPENDIX B: NUMERICAL TESTS

B1 \( f \)-mode frequency at different resolutions

The performance of \( \text{Ψ} \) at different resolutions. Convergence can be seen.

The FFT of \( \text{Ψ}_4 \) at different resolutions shows the \( f \)-mode value does not change. The black dashed line represents the value of \( f \)-mode frequency.

The performance of \( \text{Ψ} \) at different resolutions. Convergence can be seen.
Table A1. Fitting parameters $b_i$ from Eq. A1 for the C-Love universal relations. Comparing this work with the coefficient values in (Maselli et al. 2013) and (Godzieba et al. 2021). The values of $b_i$’s have been calculated without using the polytrope EoS cases.

|       | $b_0$       | $b_1$       | $b_2$       | $b_3$       | $b_4$       | $b_5$       | $b_6$       |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| This work | $3.71 \times 10^{-1}$ | $3.770 \times 10^{-1}$ | $1.056 \times 10^{-3}$ | $1.149 \times 10^{-3}$ | $5.424 \times 10^{-3}$ | $-3.188 \times 10^{-6}$ | $6.181 \times 10^{-5}$ |
| (Maselli et al. 2013) | $3.71 \times 10^{-1}$ | $-3.91 \times 10^{-2}$ | $-2.636 \times 10^{-4}$ | $-9.058 \times 10^{-2}$ | $-9.628 \times 10^{-4}$ | $3.155 \times 10^{-5}$ |
| (Godzieba et al. 2021) | $3.768 \times 10^{-1}$ | $-2.30 \times 10^{-3}$ | $2.877 \times 10^{-3}$ | $1.205 \times 10^{-2}$ | $-9.628 \times 10^{-4}$ | $3.155 \times 10^{-5}$ |

Figure B4. The FFT of $\Psi_4$ at 5 different radii with a refinement layer between each.

resolutions test for a single case, i.e. DD2 EoS with $M = 1.4M_\odot$. We choose our standard resolution with grid spacing equals to 0.105$M$ as the intermediate level, and 0.07$M$ and 0.14$M$ as the higher and lower resolutions respectively.

The results of this test are presented in Fig. B1 for the lapse function, and Fig. B2 for $\Psi_4$ function. The Fourier transform over $\Psi_4$, which gives the oscillation frequency spectrum is shown in Fig B3. These results indicate that the observed quantities converge to one solution as we move from low to high resolutions, and the value of $f$-mode frequencies do not change significantly across different resolutions. This convergence study confirms that the intermediate resolution used in our numerical simulations, to report the $f$-mode frequencies, is in the convergence regime.

B2 $f$-mode frequency at different $\Psi_4$ extraction radii

As the second numerical test, we investigate the accuracy of the $\Psi_4$ functions extracted from different radii. For this test we choose our polytropic case with $\Gamma = 2$, $\kappa = 100$ and $M = 1.4M_\odot$, with minimum grid spacing equals to 0.06$M$ for the resolution. The $\Psi_4$ outputs are extracted at $r = 10, 40, 70, 100, 130M$ for the Fourier transform.

In fig. B4, we show that the $f$-mode frequency does not change with different radii of extraction of $\Psi_4$ for Polytropic equation of state. We also notice that $f$-mode is most prominent and has less noise for $r = 10M$, and hence we chose it for comparison in with other equations of state.

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