Probability theory in construction logistics

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Abstract. The article deals with the modeling of logistic processes of construction with the help of probability theory. Random deviations accompany any natural process, and even more so logistic processes in construction. Practice sets such tasks in which various factors play a significant role in the processes under consideration, but the number of these factors is so large that it is not always possible to trace cause-and-effect relationships between them. Elements of uncertainty, complexity, multiple causes inherent in random phenomena and processes in logistics, and therefore require special methods for their investigation, study and management. Such methods are developed by probability theory.

1. Introduction

In General, the term modeling is interpreted as the study of phenomena and processes occurring on certain objects, by building and studying models of these objects. In economic systems, as a set of principles and rules that determine the form and content of economic relations arising in material production and services, as a rule, mathematical modeling is used as a description of economic processes by mathematical dependencies. While modeling is a tool by which the analysis of the occurring phenomena and processes, and its result is carried out planning, forecasting and management of objects of production and economic activity [1-4].

Logistics, as a control system of flow processes, including material flows, in the economy of the national economy is one of the leading areas. This position of logistics is determined by the fact that all manufactured products (industrial products and consumer goods) are subject to standard procedures, which include packaging and repacking, warehousing and storage of products, sorting, picking orders, distribution, loading and unloading, transportation and delivery to end users.

In turn, there are global problems in logistics, in which modeling plays a significant role, and the developed mathematical models today and in the future allow us to solve problems from the standpoint of rationalization and optimization of logistics procedures and operations. Global problems
of logistics include: rationalization of procurement activities; minimization of costs in the transport and warehouse sector; optimization of objects of stored orders of products; rationalization of product distribution processes; rationalization of product distribution processes; coordination of the movement of material and technical resources in production; timely renovation of technical devices.

Each of these problems includes a number of problems solved with the help of mathematical models. So in procurement activity process of purchases is regulated by the Federal law 44-FZ "About contract system in the sphere of purchases of goods, works, services for ensuring the state and municipal needs" of 05.04.2013 N 44-FZ. At the same time, purchases are carried out on a competitive basis. However, there is a certain range of material resources that business entities acquire independently from suppliers. And here there is a question of an assessment and a choice of suppliers on various criteria to which we concern the following: production capacity; progressive technologies; the range of the delivered products; remoteness of the supplier; conditions and terms of delivery of products; constancy of packing; financial position of the supplier [5-10].

Probability theory in logistics considers random variables due to logistic processes and operations. In logistics, the following stochastic random variables occur:

1. Demand (capacity to pay).
2. Sales volume (sales volume).
3. Duration (implementation period).
4. Revenue from product sales.
5. Costs (General, logistics, transaction).
6. Time of loading-unloading of vehicles.
7. Delivery time (product travel).
8. The level of use of load capacity and cargo capacity of vehicles.
9. Time of service of buyers (consumers).
10. Trade turnover of the trade enterprise
11. The turnover of material resources.
12. The flow of consumers (flow of requests for servicing).
13. Time of employment of means of service.
14. The movement of the material stock.
15. The volume of shipment of sold products.
16. Distribution of products by ABC groups.
17. Delivery process-reliability of delivery and others.

2. Materials and Methods

If the studied phenomenon is represented as a complete group of events that are incompatible and equally possible, then the probability of this event is equal to the ratio of the number \( m \) of favorable cases to the total number of \( n \) possible cases, i.e. the probability is:

\[
P(A) = \frac{m}{n}. \tag{1}
\]

In practice, statistical probability is usually considered, as a result of the accumulated statistics on the favorable events \( m \) and the total number of events \( n \).

In construction logistics, the most common are the following probability distribution laws: normal, exponential, binomial, Poisson [11-14].

2.1 Normal law of probability distribution

The density of the normal distribution is as follows:

\[
y = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}}, \tag{2}
\]

where \( a \) - is the center of the probability distribution or the expectation of a given random variable, i.e. \( a = M(x) \);

\( \sigma \) - is the standard deviation of a given random variable.

In practice, the corresponding statistical estimates are calculated. So, the estimate for the expectation
is the average value:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n},$$  \hspace{1cm} (3)

where $n$ is the amount of data in the considered statistical array.

Expectation is the theoretical value of a given random variable, which tends to the average value with an unlimited increase in the number of data.

Standard deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}},$$  \hspace{1cm} (4)

In construction logistics, a value of a quantity is estimated by a value of:

$$x = \bar{x} \pm \sigma,$$  \hspace{1cm} (5)

the coefficient of variation is estimated:

$$k = \frac{\sigma}{\bar{x}}.$$  \hspace{1cm} (6)

Figure 1 shows a graph of the normal probability distribution of logistics processes.

![Graph of the normal probability distribution](image)

**Figure 1.** The normal law of probability distribution.

### 2.2 Exponential law of probability distribution

The density of the exponential probability distribution law is as follows:

$$f(x) = \lambda e^{-\lambda x},$$  \hspace{1cm} (7)

where $e$ is the base of the natural logarithm.

The exponential law describes the time parameters of random logistic processes. The following random variables fall under the exponential law:

1) service time;
2) loading-unloading time of vehicles;
3) time spent on other logistics operations;
4) the interval between requests coming to the service.

The peculiarity of the exponential law is that it is defined by one parameter $\lambda$. At the same time:

$$\lambda = \frac{1}{\bar{T}},$$  \hspace{1cm} (8)

where $\bar{T}$ is the average value of the time parameter under study.

For quantities that obey the exponential law, the expectation $M$ and the mean square value are equal to each other:

$$M = \frac{1}{\lambda}, \quad \sigma = \frac{1}{\bar{T}}.$$  \hspace{1cm} (9)
Figure 2 shows a graph of the exponential law.

![Exponential Law Graph](image)

**Figure 2.** Exponential law of probability distribution.

### 2.3 Binomial probability distribution law

The binomial law of probability distribution is expressed by the formula:

\[ P_{n,m} = C_n^m \cdot p^m \cdot q^{n-m}. \]  \(10\)

This law determines the probability of occurrence of \( m \) events out of the total number of events \( n \), where \( p \) is the probability of occurrence of one event from this group of events; \( q \) - probability of occurrence of the specified event,

\[ q = 1 - p. \]  \(11\)

The value \( C_n^m \) - is the number of combinations of \( n \) by \( m \), determined by the formula:

\[ C_n^m = \frac{n!}{m!(n-m)!}. \]  \(12\)

Equality is used to calculate the number of combinations:

\[ C_n^m = C_n^{m-n}. \]  \(13\)

In binomial distribution the most probable number of events is \( N \):

\[ np-q \leq N \leq np+q \]  \(14\)

or

\[ N = np. \]  \(15\)

### 3. Results and Discussion

In probability theory, there are methods to assess the degree to which actual probability distributions correspond to their theoretical values. For this purpose, the so-called consent criteria are used, the most famous of which is the criterion \( X^2 \). This criterion allows to compare among themselves the empirical laws of distribution received on the same actual data.

The smaller the value of \( X^2 \), the better this empirical law agrees with the theoretical one. To compare the empirical laws of probability distribution, the values of \( X^2 \) are calculated using the following formula:

\[ X^2 = \sum \frac{(n_f-n_t)^2}{n_t}, \]  \(16\)

Where \( n_f \), \( n_t \) - are, respectively, the actual and theoretical values of the frequencies of the studied distribution laws.

The value \( X^2 \) is also random, and therefore obeys its law of distribution. The approach to the comparison of empirical distribution laws can be shown by an example.
Let us determine which law of probability distribution - normal or exponential-better reflects the
distribution of this value, i.e. we will test hypotheses. As the studied value we take the volume of sales
of building materials. The initial data are presented in table 1:

| Date | Sales (thousand rubles) | Date | Sales (thousand rubles) |
|------|-------------------------|------|-------------------------|
| 1    | 3.5                     | 16   | 27.0                    |
| 2    | 3.8                     | 17   | 29.5                    |
| 3    | 2.7                     | 18   | 22.1                    |
| 4    | 14.5                    | 19   | 48.3                    |
| 5    | 18.3                    | 20   | 64.5                    |
| 6    | 13.4                    | 21   | 18.5                    |
| 7    | 7.5                     | 22   | 19.5                    |
| 8    | 24.8                    | 23   | 27.4                    |
| 9    | 16.5                    | 24   | 35.0                    |
| 10   | 12.4                    | 25   | 42.0                    |
| 11   | 34.5                    | 26   | 54.0                    |
| 12   | 41.2                    | 27   | 32.1                    |
| 13   | 27.4                    | 28   | 14.5                    |
| 14   | 24.5                    | 29   | 9.4                     |
| 15   | 25.5                    | 30   | 10.0                    |

The task is formulated as follows: to build the probability distribution of the demand for building
material, if the results of the study obtained the results of the implementation, in thousand rubles, per
day.

When calculating the value of $X^2$ for different distributions, it was obtained:

1) $X^2 = 7.86$ – with exponential distribution.
2) $X^2 = 5.46$ – under normal distribution.

5.46 is less than 7.86, hence the theoretical normal distribution is more consistent with the actual
one than the exponential one.

In General, a number of logistic processes in construction, namely: supply, shipment, movement of
materials, consumption of materials, etc. is described by the normal law of probability distribution. A
distinctive feature of this distribution is the presence of a pronounced symmetry of random variables
relative to their average value. For these processes, the normal law applies to all building materials,
certain assortment groups or individual names of resources.

In ABC-analysis of the structure of logistic processes in construction, the obtained characteristics
in value or natural terms are subject to an exponential distribution.

The fact that the sale of building materials complies with the normal law is important for the
logistics of construction, because it allows you to determine the amount of material stock, for which
the following formula is recommended:

$$V = \overline{G} + 3\sigma, \quad (17)$$

where $V$ – is the required amount of inventory for an indefinite period, $\overline{G}$ – is the average sales per
unit time (day, week, month), $\sigma$ - is the standard deviation.

This model shows that any requirement of the buyer for this or that quality of goods has to be
satisfied with probability close to 1. This model uses the "three Sigma" rule. In the normal law, $3\sigma$
corresponds to a probability of 0.99.

In modern conditions, computer technology allows you to track in the current time mode, the
average implementation and standard deviations, as well as adjust the value of the material stock in
warehouses.
4. Conclusions
Random deviations accompany any natural process, and even more so logistic processes in construction. Practice sets such tasks in which various factors play a significant role in the processes under consideration, but the number of these factors is so large that it is not always possible to trace cause-and-effect relationships between them. Elements of uncertainty, complexity, multiple causes inherent in random phenomena and processes in logistics, and therefore require special methods for their investigation, study and management. The provided model of determination of material stock in warehouses of the construction organizations can be used both for civil, and for industrial construction [15-23].

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