Remarks on Black Hole Evolution
a la Firewalls and Fuzzballs

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ABSTRACT

Assume that there exists a fundamental theory of gravity which has a fundamental length scale and which is capable of explaining black hole evolution process fully in terms of fundamental microscopic degrees of freedom. The resultant evolution will clearly be unitary and will have no information loss. Carrying this assumption to its logical conclusion and, in particular, by considering the size of the region obtained by resolving the black hole singularity in such a theory, we arrive at a scenario very similar to that in Mathur’s fuzz ball proposal where there is no horizon. We also comment on the consistency of such a picture with the recent results of Almheiri et al.
1. We start by assuming that there exists a fundamental theory of gravity, such as string theory or loop quantum gravity or spin foams or spin networks, which has a fundamental length scale and which is capable of explaining black hole evolution process fully in terms of fundamental microscopic degrees of freedom. The resultant evolution will clearly be unitary and will have no information loss.

We carry this assumption to its logical conclusion. Thus, black hole singularity will be resolved in such a theory. Furthermore, it is physically reasonable to expect that such a resolution in terms of microscopic degrees of freedom will contain information about various constituents that went into the black hole and, in the semiclassical approximation, disappeared down its singularity. We then consider the possible size of the region obtained by resolving the black hole singularity, and argue that it is parametrically larger than Planck length. Analysing it further, we arrive at a scenario very similar to that in Mathur’s fuzz ball proposal where there is no horizon [1].

We then comment on the consistency of such a scenario with the recent results of Almheiri et al [2]. Essentially, statements (ii) and (iii) given in the abstract of their paper are not true in our scenario. We also point out one challenge (among many others) that must be faced in our scenario or in fuzzball proposal. This challenge is about the construction of a Oppenheimer – Volkoff type static solution for a star of arbitrary mass, its stability against collapse, and its entropic content.

In section 2, we present our main arguments and arrive at a scenario similar to that in fuzz ball proposal. In section 3, we comment on the consistency of such a scenario with the results of Almheiri et al. We then conclude in section 4, by pointing out a challenge for our scenario and for fuzzball proposal.

2. Consider the formation of a black hole by the collapse of, for example, a massive star in a pure state. As per the standard picture, such a black hole has a central singularity, has a horizon which covers it, emits Hawking radiation from its horizon, becomes smaller, and evaporates away completely in a finite time, leaving only the Hawking radiation. Such an evolution is non unitary since a pure state evolves to a mixed state, and leads to information loss.

Consider a fundamental theory of gravity, such as string theory or loop
quantum gravity or spin foams or spin networks, which has a fundamental length scale and which is assumed to be capable of explaining black hole evolution process fully in terms of fundamental microscopic degrees of freedom. The resultant evolution will clearly be unitary and will have no information loss. Such a theory should, beside other things, also be able to explain Hawking radiation and resolve black hole singularity.

- Indeed, string theory explains black hole entropy and Hawking radiation as arising from low energy excitations living on intersecting branes. Developing these ideas further, Mathur has proposed a fuzz ball picture for black hole evolution [1]. Its salient feature, for our purposes here, is that there is no horizon in the fuzz ball description of black holes.

- Ashtekar and Bojowald, and also Hayward, consider the resolution of black hole singularity by loop quantum gravity, and propose a paradigm for black hole evolution [3]. Its salient feature, again, is that there is no event horizon but only an apparent horizon. Thus, collapsing matter and other things that go into black hole can eventually escape to outside regions. See the corresponding Penrose diagrams in these works. Assuming that one understands the singularity resolution in sufficient detail, the black hole evolution may be described unitarily. According to these works, information escapes near the end of evaporation. If this is true then one encounters ‘remnant problems’, but see below.

- Stephens, ’t Hooft, and Whiting have also studied black evolution assuming that it is unitary, and argue that there will be no information loss [4]. They then describe the Penrose diagram for the evolution process. The salient feature here, again, is the absence of an event horizon.

- Several years ago, Hawking himself has argued for the unitary evolution of black hole and for no information loss [5]. According to Hawking’s quote given in the first paper of Hayward in [3], this seems to come about because ‘... a true event horizon never forms, just an apparent horizon.’

- Wald has always argued that the presence of an event horizon will lead to information loss and non unitary evolution [6]. This is because the
causal structure inside the event horizon, by definition, prevents the inside information from reaching outside.

The above points all arise in the course of attempting to describe black hole evolution process in terms of fundamental microscopic degrees of freedom. They strongly suggest that if an event horizon forms in the process of black hole formation then the subsequent evolution of that black hole will be non unitary and information will be lost. A straightforward corollary then is that if the black hole formation and evolution process is to be unitary then no event horizon may form. An apparent horizon may or may not form, its formation is not a necessary part of the corollary.

One often assumes that the distinction between event and apparent horizons becomes important only near the end of evaporation process when the radius of spacetime curvature and the black hole size are of order of a few Planck lengths. Until this end stage, the standard description given by the semiclassical action is expected to be valid. In particular, just as for event horizons, nothing from inside the horizon is expected to reach outside until near the end. It then follows that information about matter that formed the black hole, and about the Hawking photons which went inside the horizon during evaporation, must all escape from an end-stage object of Planckian size. The amount of such information depends on the initial size of the black hole and may be arbitrarily large. Hence, its storage in, and escape from, Planckian sized objects lead to various conundrums which are referred to as ‘remnant problems’.

Let there be a theory which resolves, or is capable of resolving, the black hole singularities. Let \( l_f \) be the fundamental length scale for this theory, far above which one obtains the standard semiclassical action. For example, \( l_f \sim l_{pl} \), the Planck length, in loop quantum gravity, spin foam, or spin network theories; whereas \( l_f \sim l_s \) in string theory. If string theory coupling constant \( g_s \sim O(1) \), as may perhaps be required for singularity resolution, then \( l_s \sim l_{pl} \). Here, we take \( g_s \sim O(1) \) and set \( l_f = l_{pl} \).

Let \( l_{cld} \) be the size of the region obtained by resolving the singularity. Then, \( l_{cld} \) is the length scale where the semiclassical action describing the black hole evolution process should be corrected and where the distinction between event and apparent horizons becomes important. Also, if the size of black hole horizon becomes comparable to \( l_{cld} \) then information from inside the horizon may be expected to reach outside. One may therefore say that
there is no horizon when horizon size and $l_{\text{cld}}$ are comparable.

It is generally assumed that $l_{\text{cld}} \sim l_{\text{pl}} = l_f$. Now, in a spacetime obtained from a semiclassical action, one expects the action to be corrected when the spacetime curvature approaches Planckian strengths. Within the semiclassical calculations, this typically happens a few Planck lengths away from the singularity. One should then use the fundamental theory and resolve the singularity. Such a resolution, given by the fundamental theory in terms of its microscopic degrees of freedom, must contain information about what all had gone into the singularity. Logically, it does not then follow that the size of the region obtained upon resolving the singularity will also be of order Planck length. Hence, we denote the size of the resolved region by $l_{\text{cld}}$. Also, for the sake of convenience, we refer to the resolved region as singularity cloud.

This singularity cloud resides at the center of a black hole and is made up of objects from fundamental theory - such as stringy excitations, or Ashtekar loops, or spins of spin foams or spin networks. Although more things may fall into the cloud, interact with its constituents, and get transformed to fundamental objects, nothing from the cloud may escape and reach outside the black hole horizon until the black hole size and $l_{\text{cld}}$ are comparable.

We now argue that $l_{\text{cld}}$ must be parametrically larger than the fundamental length scale $l_f$, must depend on the initial black hole mass $M_{\text{init}}$, and must be of the form

$$l_{\text{cld}} \gtrsim (l_f M_{\text{init}})^\alpha l_f$$

where it is physically reasonable to expect that $\alpha \geq \frac{1}{d-1}$ for a $d$-dimensional black hole.\footnote{However, even if $\alpha < \frac{1}{d-1}$, the arguments given below and the conclusions they lead to are all valid as long as $\alpha$ is strictly positive. The validity of our arguments requires only that $l_{\text{cld}}$ be dependent on $M_{\text{init}}$ and be parametrically larger than $l_f$.} Consider the initial constituents which went in to form the black hole of mass $M_{\text{init}}$. Upon black hole formation, these constituents are taken to have disappeared down the singularity. But they must show up when the singularity is resolved using the fundamental theory. After this resolution, the initial constituents may be thought of as being transformed mostly to $N \sim l_f M_{\text{init}}$ number of fundamental objects each of whose mass is $\sim l_f^{-1}$. It is then reasonable to expect that the closest that $N$ number of fundamental objects, each of whose mass is $\sim l_f^{-1}$, can be packed is in a spatial region of size $\sim N^{\frac{1}{d-1}} l_f$. Considering that the actual singularity resolution may not
correspond to the closest packing, one obtains the estimate in equation (1). This estimate is analogous to saying that the smallest region where one mole of hydrogen can be packed is given by \( N_a^3 \lambda_{pr} \) where \( N_a \) is the Avagadro’s number and \( \lambda_{pr} \) is proton’s Compton wavelength. In the case of hydrogen, of course, we know that a more realistic estimate is vastly larger than this, and is obtained by replacing \( \lambda_{pr} \) by Bohr radius.

Once the black hole forms, it starts to emit Hawking radiation. A pair of photons tunnel out of vacuum near the horizon; one of them, the \( \text{out} \)—photon, goes outside to infinity; and the other, the \( \text{in} \)—photon, falls inside onto the singularity. Now, this \( \text{in} \)—photon is supposed to carry negative energy and is supposed to reduce the black hole mass. Then, should \( M_{\text{init}} \) in equation (1) be perhaps replaced by \( M_{\text{now}} \), the net black hole mass \( M_{\text{now}} \) at the time of observation?

It is not clear how such an ‘energy annihilation’ is to take place; and, whether and how it would result in the shrinking of the singularity cloud. By contrast, a ‘charge annihilation’ is simple: Negative charges fall onto a positively charged cloud. In a typical process, a negative and a positive charge meet and cancel each other’s charges and emit a photon. Such a photon may escape to infinity, or may be confined within a finite region. Note that the negative and positive charges need not necessarily disappear by annihilating each other. For example, they may form an ‘atomic’ bound state; or, negative charges may swarm around positive charges anchored at lattice sites. Thus, although the net charge is reduced, the charges themselves may not necessarily disappear.

Going by this analogy, the Hawking \( \text{in} \)—photons will fall onto the singularity cloud and will interact there with the matter quanta that initially formed the black hole. The resultant product, if any and whose nature is not clear to us, must stay within the singularity cloud. Until the black hole size and \( l_{\text{cld}} \) become comparable, the matter quanta, the \( \text{in} \)—photons, and the resultant interaction and decay products must all stay within the cloud. At the face of it, it seems that all such processes can only increase the size of the cloud. We may well be biased in thinking so, but it is not clear to us what processes may decrease the size of the cloud, except the process whereby black hole and cloud sizes become comparable to each other and objects from the cloud may escape to outside. Therefore, we assume that the inflow of Hawking \( \text{in} \)—photons, as well as anything else that falls into
the black hole, will only increase the size of the cloud or, at best, keep it unchanged. 2

We are now led to an interesting situation. Imagine that we observe a black hole now, which was formed earlier. Its mass is $M_{\text{now}}$, and was $M_{\text{init}}$ at the time of formation. In the description of this black hole by a fundamental theory, there are three length scales now: The two expected ones, namely $l_f = l_{pl}$ and the horizon size $\sim (l_{pl} M_{\text{now}})^{\frac{1}{d-3}} l_{pl}$, which already appear in the semiclassical description; and a new third scale $l_{cld}$ corresponding to the size of the singularity cloud which appears only after the singularity is resolved in terms of fundamental microscopic degrees of freedom. Note that $l_{cld}$ depends on $M_{\text{init}}$ as given in equation (1), and that $M_{\text{init}}$ could be arbitrarily larger than $M_{\text{now}}$. It may be that $M_{\text{init}} = 10 M_{\text{now}}$, or it may equally well be that $M_{\text{init}} = 10^{100} M_{\text{now}}$. So, it seems that one cannot predict this third scale $l_{cld}$ from the given data, namely the mass of the black hole $M_{\text{now}}$.

One can then envision some strange possibilities. For example, $M_{\text{init}}$ is so large that $l_{cld} \gg (l_{pl} M_{\text{now}})^{\frac{1}{d-3}} l_{pl}$, i.e. the singularity cloud is larger than the horizon size now. Therefore, this black hole must be emitting information about the initial matter in the form of fundamental objects from the singularity cloud; hence, effectively, it has no horizon. Or, $l_{cld}$ may be one half or one tenth of the present horizon size. This would imply that the present horizon will be ‘covered’ by the singularity cloud, and will disappear at a time which can not be predicted until the actual time of disappearance.

Note that our estimate of the size $l_{cld}$ of the singularity cloud is physically reasonable, conservative, and rests on the existence of a fundamental theory which is capable of explaining black hole evolution process fully in terms of microscopic degrees of freedom. It is also reasonable that the singularity cloud stores the information about the initial matter. It then seems that the above strange features arise because of the assumption that nothing can escape from the cloud to outside until the horizon size becomes comparable to $l_{cld}$.

The simplest way to eliminate such strange features and regain predictability is to assume that the estimate for $l_{cld}$ given in equation (1) is a vast underestimate if $\alpha \leq \frac{1}{d-1}$. It can further be seen that taking $\frac{1}{d-1} < \alpha < \frac{1}{d-3}$ will not suffice, but that taking $\alpha = \frac{1}{d-3}$ will suffice to eliminate the strange

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2See [7, 8] for some recent work on bit models of Hawking evaporation where similar issues arise.
features and regain predictability. This is because $\alpha = \frac{1}{d-3}$ implies that the scale $l_{dd}$ is same as the initial horizon scale. Hence, objects from the cloud can escape to outside right from the moment of black hole formation.\footnote{Avery, Chowdhury, and Puhm in [9] arrive at a similar conclusion from a very different point of view.} This effectively means that there is no horizon. Also, since objects from the cloud can escape to outside, it follows that the size of the singularity cloud will keep decreasing and remain comparable to black hole size throughout its evolution.

Such a description of black hole evolution is very similar to Mathur’s fuzz ball proposal. Here we have argued that this must be the case in any fundamental theory which has a fundamental length scale and which is capable of explaining black hole evolution process fully in terms of fundamental microscopic degrees of freedom. The unitarity of evolution and the absence of information loss will now follow ‘trivially’ just as for any lump of burning coal.

3. We now comment on the consistency of our scenario with the results of Almheiri et al [2]. As expressed concisely in their abstract, Almheiri et al “argue that the following three statements cannot all be true: (i) Hawking radiation is in a pure state, (ii) the information carried by the radiation is emitted from the region near the horizon, with low energy effective field theory valid beyond some microscopic distance from the horizon, and (iii) the infalling observer encounters nothing unusual at the horizon.”

Our scenario is similar in spirit to Mathur’s fuzz ball proposal, but is argued to be applicable for any fundamental theory which can explain black hole evolution in terms of microscopic degrees of freedom. In our scenario, statements (ii) and (iii) of Almheiri et al are not true. The information is stored in, and emitted to outside from, the singularity cloud whose size is argued to be comparable to black hole size; and, the dynamics of objects in the cloud are to be described by the fundamental theory in terms of microscopic degrees of freedom. Hence, statement (ii) is not true. Also, an infalling observer will be falling through the singularity cloud and, depending on the details such as the density of the cloud and the energy and the nature of the probes used, the infalling observer may see things unusual while falling through the black hole. Hence, statement (iii) is not true.
The singularity cloud here may perhaps be thought of as similar to the ‘firewall’ of Almheiri et al, but only in as much as infalling objects interact with the cloud and get ‘burnt’ down to fundamental objects. But these are really two different things. The ‘firewall’ stays behind or coincides with the horizon, which is a well defined null surface, and the time of its formation is still under debate. The singularity cloud is a lot more similar to fuzz ball. Just like fuzz ball, it is argued to be of same size as black hole. Furthermore, there strictly is no horizon since objects from the cloud can escape to outside. Also, the cloud is argued to have been formed at the time of black hole formation.

Our scenario will, probably, be deemed to be far from conservative. But the physical motivations behind it are the standard ones: requiring the evolution to be unitary, information to be not lost, and assuming that a fundamental theory with a fundamental length scale exists which is capable of explaining black hole evolution process fully in terms of microscopic degrees of freedom. It is this last motivation that is not often invoked, nor carried to its logical conclusion. String theory is one such fundamental theory and, using it, Mathur indeed has been led to his fuzzball proposal.

Also, note that if an event horizon is assumed to have been formed then, by definition and as Wald has always argued, information will be lost and evolution will be non unitary. Of course, as has been pointed out often, a simple way out is that an event horizon does not form, only an apparent horizon forms. This, together with a few standard assumptions about singular regions, then leads to ‘remnant problems’. One is then led to issues related to firewalls, complementarity, et cetera as can be seen in the works of Almheiri et al and several other works based on it [9]. With the physically motivated present assumptions about singular regions, these issues are all bypassed.

Use of holographic principle to understand black hole evolution is an interesting approach, particularly with Maldacena’s conjecture relating gravity in $AdS_5$ to a four dimensional Yang-Mills (YM) theory. It is known that a black hole in $AdS_5$ corresponds to YM theory at finite temperature [10], and that absorption by black hole corresponds to thermalisation in YM theory [11]. However, to the best of our knowledge, it is not yet known as to e.g. what an infalling observer sees; or, whether Mathur’s fuzz ball proposal is correct, and whether horizon is absent; or, how the black hole singularity gets resolved.

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4. We conclude by pointing out one challenge (among many others) that must be faced in our scenario, or in fuzzball proposal, where it is argued that there is no horizon when a black hole is described by a fundamental theory in terms of fundamental microscopic degrees of freedom. The horizon arises only in the description by a semiclassical theory.

Consider a star. Following Oppenheimer and Volkoff, one can construct static star solutions assuming an equation of state for the constituent matter. Typically, the radius of the star increases with its mass and the mass-radius relation can be made linear for some parameter values. However, in all such solutions known so far, an instability sets in for sufficiently high mass and/or sufficiently high central density. This instability causes the star to collapse. Hence, it is believed that any sufficiently massive star will collapse, and that the final collapse product will be a standard black hole with horizon [12].

Our present scenario, or fuzzball proposal, argues that there is no horizon. Then, using the microscopic degrees of freedom of the fundamental theory, (1) it must be possible to describe the singularity cloud or fuzz ball by an effective equation of state; to construct an Oppenheimer-Volkoff type static star solution; to show that the size of such a star is of the order of its Schwarzschild radius; and, most importantly, to show that there is no instability for any value of mass and/or central density. Moreover, (2) the relevant equation of state for the constituent fundamental objects must be such that the entropy of such a star is comparable, if not exactly equal, to the entropy of a corresponding standard black hole.

Obtaining these two results will perhaps make the present scenario or fuzzball proposal more compelling. At present, we do not know how to prove or disprove these results but we believe that it is a goal worth pursuing.

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