Enhance primordial black hole abundance through the non-linear processes around bounce point

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Abstract. The non-singular bouncing cosmology is an alternative paradigm to inflation, wherein the background energy density vanishes at the bounce point, in the context of Einstein gravity. Therefore, the non-linear effects in the evolution of density fluctuations ($\delta \rho$) may be strong in the bounce phase, which potentially provides a mechanism to enhance the
abundance of primordial black holes (PBHs). This article presents a comprehensive illustration for PBH enhancement due to the bounce phase. To calculate the non-linear evolution of $\delta \rho$, the Raychaudhuri equation is numerically solved here. Since the non-linear processes may lead to a non-Gaussian probability distribution function for $\delta \rho$ after the bounce point, the PBH abundance is calculated in a modified Press-Schechter formalism. In this case, the criterion of PBH formation is complicated, due to complicated non-linear evolutionary behavior of $\delta \rho$ during the bounce phase. Our results indicate that the bounce phase indeed has potential to enhance the PBH abundance sufficiently. Furthermore, the PBH abundance is applied to constrain the parameters of bounce phase, providing a complementary to the surveys of cosmic microwave background and large scale structure.

**Keywords:** alternatives to inflation, physics of the early universe, primordial black holes

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1 Introduction

Primordial black holes (PBHs) are believed to originate from extremely over-dense regions in the early Universe [1–3]. Unlike astrophysical black holes, which evolve from massive stars and contain masses \(\gtrsim 5 M_\odot\) [4], PBHs can be formed much earlier than the birth of first-generation stars, and their masses are allowed to distribute in a very wide range — in principle, from Planck mass \((\sim 10^{-5} \text{ g})\) to the mass of observable Universe \((\sim 10^{55} \text{ g})\) [5–8]. In view of this, there are various motivations to introduce PBHs in cosmology and astrophysics. For example, the massive PBHs \((> 10^{15} \text{ g})\) can serve as a candidate for cold dark matter [9, 10] and seeds of super massive black holes at high redshifts [11, 12]. Moreover, the light PBHs \((< 10^{15} \text{ g})\) have strong Hawking radiations [13, 14], so they may be responsible for some electromagnetic emission phenomena [5, 15–18].

The attentions on PBHs keep stupendously increasing in recent years, especially since LIGO and Virgo collaborations achieve fruitful successes on gravitational wave (GW) detection. LIGO and Virgo so far have announced tens of GW events, and have also brought several potential evidences for PBHs [19–23]. In particular, for the event GW190521 [24], one black hole’s mass \(85^{+21}_{-14} M_\odot\) is believed to reside in the \((60–130) M_\odot\) mass gap, which is forbidden in stellar evolution theories [25]. The literature [26] shows that this black hole may have a primordial origin, if the PBHs can accrete efficiently before reionization epoch. In addition, although the events GW200105 and GW200115 are officially announced as GW signals from neutron star-black hole binaries [27], they are also compatible with the scenario of PBH binaries [28]. Moreover, other GW detection programs, including NANOGrav [29], LISA [30], Taiji [31], TianQin [32], Gaia and THEIA [33] etc., have all treated PBHs as a potential target. Therefore, one can expect that the requirement for PBH researches will become more and more emergent in the epoch of GW astronomy.

The abundance and mass function of PBHs rely on power spectrum and probability distribution function (PDF) of primordial density fluctuations [34]. In the literature, a Gaussian
PDF is generally assumed, which is named Press-Schechter formalism [35]. In this case, a power spectrum with extra enhancement on certain length scales (much smaller than the scales of cosmic microwave background (CMB) surveys $1-10^4$ Mpc [36]) is required to generate PBHs with certain masses [37–44]. To achieve the enhancement, particular mechanisms in the early Universe are introduced, such as hybrid inflation [37], inflection-point inflation [38], pre-big bang inflation [44], and sound speed resonance of the fluctuations [40–43] etc.. Moreover, recent literature [45–57] has pointed out that the extremely compact regions should inevitably have highly non-linear evolution, i.e. obeying a non-Gaussian PDF. Therefore, the non-Gaussianity (NG) can have a significant impact on the PBH abundance.

Since inflation is currently the most widely accepted paradigm of early Universe, most PBH researches are based on inflationary scenario. Meanwhile, the bouncing cosmology [58–61] can satisfy the CMB constraints [36] as well, hence it is deemed as an alternative scenario to inflation. In the non-singular bouncing scenario, the Universe typically starts with a contracting phase, then turns to expand when the scale factor is small enough (but still larger than 0), and finally evolves to the expanding Big Bang phase. Usually, the moment of the minimum scale factor is called bounce point [60]. So far, only few works [62, 63] have investigated the PBH formation in bouncing cosmology [see 64, as well], and the analyses therein concentrate on the contracting phase, not including the bounce point. The results of PBH abundance given by these researches seem pessimistic. For example, our previous work [63] shows that the density fluctuation given by matter-contracting phase does not enhance the PBH abundance significantly, unless the phase is physically disfavored, e.g. the Hubble parameter is close to or larger than Planck scale.

However, it is inspiring to note that, the background density ($\bar{\rho}$) around the bounce point approaches 0, in context of Einstein gravity [63, 65, 66]. This implies that the density fluctuation ($\delta \rho/\bar{\rho}$) may diverge, and the evolution of $\delta \rho$ can be highly non-linear and the PDF may have strong NGs. These non-linear effects/NGs have been confirmed at least for some specific cases [66]. Therefore, the non-linear processes around bounce point naturally provide a mechanism to increase PBH abundance, and we will illustrate it in this article.

The rest of this article is organized as follows. In section 2, we briefly review the non-singular bouncing scenario and present a model-agnostic parametrization of the bounce phase. In section 3, we introduce the Rychaudhuri equation to numerically calculate the non-linear evolution of density fluctuation in the bounce phase. In section 4, we evaluate the PBH abundance in the bouncing scenario, and then use the PBH abundance to constrain the parameters of bounce phase. Finally, we summarize in section 5. Throughout this paper, the Planck unit with $c = h = 8\pi G = 1$ is adopted, unless otherwise specified. Under this convention, the Planck mass is $M_p = (8\pi G)^{-1/2} = 4.6 \times 10^{-6} \text{g}$, and the present Hubble parameter is $H_0 = 67.4 \text{km s}^{-1} \text{Mpc}^{-1} = 5.9 \times 10^{-61}$ [36].

2 A brief description of non-singular bounce phase

The non-singular bouncing scenario can be realized by multiple mechanisms [60, 61], including those modifying the General Relativity (GR) [75–80] and not [81–89]. In this article, we only consider the context of GR or alternative gravity theories which are equivalent to GR with a dynamical fluid. In this case, the background dynamics can be described by the Friedmann-

\footnote{It is an unphysical divergent as long as $\delta \rho$ is limited. The realization of bounce phase without pathology can be found in the literature [67–74].}
Figure 1. An illustration for the evolution of $\dot{H}(t)$ throughout a bouncing cosmology.

Lemaître-Robertson-Walker (FLRW) equation

$$\begin{cases} \dot{H}^2 = \frac{1}{3}\dot{\rho} \\ \dot{H} + \dot{H}^2 = -\frac{1}{6}(\dot{\rho} + 3\dot{P}) \end{cases}.$$  \hspace{1cm} (2.1)

Here $\dot{H} = \dot{a}/\bar{a}$, $\dot{\rho}$ and $\dot{P}$ denote background Hubble parameter, density and pressure, respectively. The $\bar{a}$ means the background scale factor. The “$\bar{}$” denotes a variable for the background fluid, and the “$\dot{}$” means derivative with respect to background time $t$. Furthermore, the background pressure is usually parameterized as $\bar{P} = w\bar{\rho}$, with $w$ named equation-of-state (EOS) parameter.

As mentioned in section 1, the Universe starts with a contracting phase, during which $\dot{H}$ is negative and $w$ is generally larger than $-1/3$ [60]. Subsequently, the bounce phase takes place when the Universe is small enough to avoid the cosmological singularity, hence the null energy condition (NEC) $\dot{\rho} + P \geq 0$ should be violated, according to the singularity theorems [90]. This indicates that the constraints $w < -1$ and $\dot{H} > 0$ are required during the bounce phase. Additionally, the condition $\dot{H} = 0$ holds at the bounce point [60]. Given above, the evolutionary behavior of $\dot{H}$ during the bounce phase can be sketched as follows — $\dot{H}$ is negative before bounce point, vanishes at the point, and becomes positive after it (see figure 1 as well). The final stage of the Universe is the standard Big Bang phase, during which the background dynamics can be simply described by the spatially flat $\Lambda$CDM model:

$$\dot{H}(z) = H_0\sqrt{\Omega_\Lambda + \Omega_m(1+z)^3 + \Omega_m(1+z_{eq})^{-1}(1+z)^4},$$  \hspace{1cm} (2.2)

where $z \equiv 1/\bar{a} - 1$ is the cosmological redshift, $H_0 \simeq 67.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ denotes the present Hubble parameter, $\Omega_m \simeq 0.315$ stands for density fraction for matter component today, $\Omega_\Lambda \simeq 1 - \Omega_m$ represents density fraction for dark energy today, and $z_{eq} \simeq 3400$ is the redshift when matter and radiation have equal densities [36].

Since the PBH formation in contracting phase have been studied in [62, 63], we concentrate our consideration on the bounce phase in this article, i.e. from the end of contracting
phase ($t_C$) to the beginning of Big Bang phase ($t_{BB}$). Hereafter, the Hubble parameters at $t_C$ and $t_{BB}$ are taken as two free parameters $H_C$ and $H_{BB}$. For not losing generality, we adopt a model-agnostic parametrization of $\ddot{H}$ during bounce phase, i.e. Taylor expanding it as

$$
\ddot{H}(t) = \sum_{\gamma=1}^{\infty} \Upsilon_{\gamma} t^{\gamma}, \quad t_C \leq t \leq t_{BB},
$$

(2.3)

where the coefficients $\Upsilon_{\gamma}$ are parameters to be determined, and $t = 0$ is set as the bounce point (see figure 1). It is seen that to ensure the condition $\dot{H} > 0$ during the bounce phase, $\ddot{H}$ should be dominated by one or several terms with odd $\gamma$. For simplicity, we only consider one odd term in this work, and equation (2.3) reduces to

$$
\ddot{H}(t) \simeq \Upsilon_{\gamma} t^{\gamma}, \quad t_C \leq t \leq t_{BB},
$$

(2.4)

with $\gamma$ being an odd number (e.g. 1, 3, 5…). It is important to note that, although the parametrization of equation (2.4) is not as generic as equation (2.3), it still applies for a variety of bouncing models [65, 66, 81, 91–93]. As a result, the e-folding number with respect to the bounce point can be calculated as

$$
n \equiv \int_0^t \frac{\dot{H}(t')dt'}{t'} = \frac{\Upsilon_{\gamma} t^{\gamma+1}}{\gamma+1} = \frac{1}{\gamma+1} \left( \frac{H^{\gamma+1}}{\Upsilon_{\gamma}} \right)^{\frac{1}{\gamma}},
$$

(2.5)

and the scale factor is

$$
\bar{a}(t) \equiv a_b e^n = a_b \exp \left[ \frac{1}{\gamma+1} \left( \frac{H^{\gamma+1}}{\Upsilon_{\gamma}} \right)^{\frac{1}{\gamma}} \right],
$$

(2.6)

where $a_b$ denotes the scale factor at the bounce point. Since $\gamma$ is odd, it is seen that $n$ is positive before and after the bounce point.

At the end of this part, we give a short guidance on how to practically calculate the background dynamics during the bounce phase, using our parametrization in equation (2.4). In our treatment, we set $H_C$, $H_{BB}$, $\gamma$, $n_{BB}$ as four independent parameters, with the last one denoting the e-folding number at $t_{BB}$. Once the four parameters are given, the evolution of $\bar{a}(t)$ and $\bar{H}(t)$ will be determined as follows. Firstly, taking these parameters into equation (2.4) and equation (2.5), one obtains the values of $t_C$, $t_{BB}$ and $\Upsilon_{\gamma}$. Furthermore, taking $H_{BB}$ into equation (2.2) and numerically solve the equation, one can get the scale factor at the beginning of the Big Bang phase (denoted as $a_{BB}$, hereafter). Hence, the scale factor at the bounce point is known as $a_b = a_{BB} \exp(-n_{BB})$. Finally, $\bar{a}$ and $\bar{H}$ can be calculated from equation (2.6) and equation (2.4), respectively. We notice that the our parametrization may lead to discontinuities of $\dot{H}$ and $w$ at the joint points $t_C$ and $t_{BB}$, but the discontinuities can be allowed in this work. Furthermore, the values of the above parameters may be constrained in some specific bouncing models, we do not consider those constraints here.

3 Density fluctuations during bounce phase

3.1 Raychaudhuri equation

To investigate the non-linear evolution of density fluctuations during the bounce phase, we introduce the Raychaudhuri equation [94–96], which describes fluctuations with isotropic
stress in a comoving gauge:

\[
\frac{dH}{dt_c} + H^2 = -\frac{1}{6} (\rho + 3P) - \frac{1}{3} \nabla \cdot \nabla \frac{P}{\rho + P},
\]

(3.1)

where \( H, \rho \) and \( P \) denote the local Hubble parameter, energy density and pressure, respectively. The density and pressure are usually parameterized as

\[
\begin{align*}
\rho &= \bar{\rho} + \delta \rho = \bar{\rho}(1 + \delta) \\
P &= \bar{P} + \delta P = \bar{\rho}(w + c_s^2 \delta),
\end{align*}
\]

(3.2)

with \( \delta \rho, \delta, \delta P \) and \( c_s \) being density fluctuation, density contrast, pressure fluctuation and adiabatic sound speed, respectively. The \( t_c \) is the local time (proper time along a comoving worldline), which can be achieved by

\[
\nabla^2 \left( \frac{dt}{dt_c} \right) = \nabla^2 \left( \frac{\delta P}{\rho + P} \right).
\]

(3.3)

Since \( t_c \) denotes time along a comoving worldline, all the fluctuations, e.g. \( \delta \rho \) and \( \delta P \), are analyzed under the comoving gauge in this article. Additionally, to calculate the evolution of fluctuations, one also requires a continuity equation

\[
\frac{d\rho}{dt_c} = -3H(\rho + P).
\]

(3.4)

Note that the variables \( H \) and \( t_c \) are complicated functions of \( \delta \rho \). Additionally, the pressure-gradient term \( \nabla \cdot \nabla \frac{P}{\rho + P} \) in equation (3.1) yields an non-linear term of \( \delta \rho \). These make equation (3.1) an non-linear equation in \( \delta \rho \).

In the following, we will calculate the evolution of the fluctuated regions around bounce point using equation (3.1)–equation (3.4). Let us start with making some simplifications. Firstly, all the fluctuated regions are considered as isolated and spherical clouds or voids embedded in the background fluid (with density \( \bar{\rho} \) and pressure \( \bar{P} \)). This assumption has been widely adopted in the Press-Schechter formalism [35]. Hereafter, the \( \rho \) and \( P \) denote the mean density and pressure inside a fluctuated region, so \( \delta \rho \) means the density difference inside and outside the region. Hence, equation (3.3) reduces to

\[
\frac{dt}{dt_c} \equiv \xi = 1 + \frac{\delta P}{\rho + P} = 1 + \frac{c_s^2 \delta \rho}{\rho + P}.
\]

(3.5)

Note that \( dt_c \) is a function of \( \delta \rho \) and leads to inconvenience in the analyses of the evolution of \( \delta \rho \). Therefore, one should take the background time \( t \) as the time variable in calculating equation (3.1). In the replacement of \( dt_c \rightarrow dt \), the coefficient \( \xi \) (and its time derivative \( \dot{\xi} \)) will appear into the equation of motion. Secondly, we assume that the gradient operator \( \nabla \) is related to the physical radius of the fluctuated region \( R \). Technically, the gradient operator can be simply replaced by \( \nabla \rightarrow i/R = ik/a \) when it acts linearly on \( \delta \rho \), e.g. \( |\nabla \delta \rho| \simeq |\delta \rho/R| \) and \( |\nabla^2 \delta \rho| \simeq |\delta \rho/R^2| \). Here \( a \) is the scale factor inside the fluctuated region \( d\ln a/dt_c \equiv H \), and \( k = a/R \) the comoving wavenumber of the region. According to equation (3.4), the local scale factor can be calculated as

\[
a = a_C \exp \left( -\frac{1}{3} \int \frac{d\rho}{\rho + P} \right),
\]

(3.6)
with $a_C$ being the background scale factor at the initial moment $t_C$. Thirdly, we set $c_s$ as a constant for both time and spatial coordinates, during the bounce phase.

As a result, equation (3.1) reduces to

$$-\frac{\xi \dot{\rho} + \xi^2 \ddot{\rho}}{\rho + P} + \frac{4\xi^2 \ddot{\rho}_P + 3\xi \dot{\rho} \dot{P}}{3(\rho + P)^2} = -\frac{1}{2} \left( 1 + 3P \right) + \frac{c_s^2 k^2}{a^2} \left[ \frac{\delta \rho}{\rho + P} - \frac{(1 + c_s^2) \delta \rho^2}{(\rho + P)^2} \right],$$  \tag{3.7}

with $\xi$ given by equation (3.5), $a$ given by equation (3.6), and the background variables $\dot{\rho}$ and $\ddot{P}$ given by equation (2.1), respectively. Clearly, equation (3.7) is non-linear in $\delta \rho$, showing in three aspects: (1) the coefficients $\xi$ and $\dot{\xi}$, from the transformation $dt_c \rightarrow dt$; (2) the $\frac{k^2 \delta \rho^2}{(\rho + P)^2}$ term, from the non-linear contribution of the $\nabla \times \frac{\Sigma P}{\rho + P}$ term in equation (3.1); (3) other non-linear terms, e.g. $\frac{\dot{\rho}}{\rho + P}$ and $\frac{\ddot{P}}{\rho + P \gamma}$, from $H$. By numerically solving equation (3.7), the non-linear evolution of $\delta \rho$ can be calculated, which will be discussed in the following parts.

### 3.2 Linear approximations

Since the non-linear evolution of $\delta \rho$ is significant only around the bounce point, a linear dynamics remains valid during the rest epochs. Therefore, the linear approximation of equation (3.7) will be studied firstly, which can greatly simplify the analysis and provide initial conditions for the non-linear computations.

Up to the linear order of $\delta \rho$, equation (3.5) reduces to $\xi = 1 + c_s^2 \delta \rho / (\dot{\rho} + \ddot{P})$ and the scale factor can be treated as unperturbed ($a = \bar{a}$). Therefore, equation (3.7) reduced to

$$\dot{\delta \rho} + \left( 5 \dot{H} - \frac{\ddot{H}}{\bar{H}} \right) \delta \rho + \left( \frac{c_s^2 k^2}{a^2} + 6 \bar{H}^2 + 4 \ddot{H} - \frac{3 \bar{H} \ddot{H}}{\bar{H}} \right) \delta \rho = 0.$$  \tag{3.8}

Inserting the background dynamics of the bounce phase equation (2.4), one obtains

$$\dot{\delta \rho} + \left( 5 \bar{Y}_r \gamma - \frac{\gamma - 1}{t} \right) \delta \rho + \left[ \frac{c_s^2 k^2}{a^2} + (3 + \gamma) \bar{Y}_r \gamma^{-1} + 6 \gamma \gamma^2 \right] \delta \rho = 0,$$  \tag{3.9}

with $\gamma$ being an odd number. It is seen that $\delta \rho$ has an oscillatory behavior during bounce phase, even for the long-wavelength modes $k \rightarrow 0$. For example, in the case $\gamma = 1$ and $t \rightarrow 0$, equation (3.9) reduces to an equation of harmonic oscillator $\delta \rho + 4 \bar{Y}_1 \delta \rho = 0$. This means that an initial void may become a cloud during the bounce phase, and vice versa. Therefore, the probability of PBHs originated from both initial clouds and voids should be considered in the following numerical computations.

Moreover, taking the background dynamics of the contracting phase $\dot{H} = 2/[3(1 + w)t]$ (with $w$ being a constant) into equation (3.8), one obtains

$$\dot{\delta \rho} + \frac{2(8 + 3w)}{3(1 + w)t} \delta \rho + \frac{4(3 + w)}{3(1 + w)^2 t^2} \delta \rho = 0,$$  \tag{3.10}

$k \rightarrow 0$ & $t \leq t_C$.  \tag{3.10}

The leading-order solution of equation (3.10) is $\delta \rho \propto t^{-(3+w)/(1+w)}$. Hence, at the initial moment of the bounce phase ($t_C$), one obtains

$$\delta \rho \approx -\frac{3(3 + w)}{2} H_C \delta \rho.$$  \tag{3.11}

This will be used as an initial condition\(^2\) in the subsequent numerical computations of equation (3.7) during the bounce phase.

\(^2\)The modes of interest are super-Hubble at initial moment $c_s k \ll |a_C H_C|$. Hence, equation (3.11) based on the limit $k \rightarrow 0$ applies for them approximately.
3.3 Numerical realization

In this part, we perform numerical computations of equation (3.7) to obtain non-linear evolution of $\delta \rho$ during the bounce phase.

First of all, the results depend on the initial values of $\delta \rho$, $\dot{\delta} \rho$ and $k$. The initial $\delta \rho$ can be expressed as $3H_0^2 \delta_i$, and the initial $\dot{\delta} \rho$ can be fixed by equation (3.11). Hence, $\delta_i$ and $k$ will be two input parameters in the following computations.

Furthermore, combing equation (3.6) and equation (3.7), we have to handle a set of differential-integral equations, which are very complicated. Hence, we choose the method of iteration as a practical approach, as shown in figure 2. It is seen that the initial $a(t)$ of the iteration is taken as the background one $\bar{a}(t)$ given by equation (2.6). The iteration stops at the $(i+1)$-th ($i \geq 0$) step, if the resulting $a^{(i)}(t)$ and $a^{(i+1)}(t)$ nearly overlap.

A specific result of the evolution for $\delta \rho$ during a bounce phase is shown in figure 3. Firstly, it is seen that the difference between linear and non-linear results becomes significant when $|\delta \rho|$ exceeds $\bar{\rho}$, as expected in section 1. Secondly, the evolution of $\delta \rho$ shows oscillating behaviors as mentioned in subsection 3.2 — the initial cloud/void becomes a void/cloud during the span $t \in (7, 8) \times 10^{12}$, and then returns cloud/void later. Furthermore, at the late stages ($t > 3 \times 10^{12}$), $|\delta \rho|$ becomes much smaller than $\bar{\rho}$, and the evolutionary behaviors of $\delta \rho$ from both non-linear and linear theories are nearly the same, except overall amplifications. It indicates that the non-linear effects become negligible in the late stage of bounce phase and the subsequent Big Bang phase. In the shown case by figure 3, the non-linear effects amplify $|\delta \rho|$ by factors of 1.64 (for the case $\delta_i > 0$) and 2.65 (for the case $\delta_i < 0$), respectively, compared with the result given by the linear theory. However, it is important to note that the non-linear effects may also reduce $|\delta \rho|$ in some cases, hence the PBH abundance is not always enhanced. This brings requirements on the parameters of bounce phase to enhance PBH abundance, which will be discussed in section 4.

3.4 Power spectrum $P_\zeta$ and PDF for $\delta$

The power spectrum of comoving curvature fluctuation is typically parameterized as $P_\zeta(k) = A_s (k/k_p)^{n_s-1}$. This power-law parametrization has been constrained well at the scales of
CMB and large scale structure (LSS) surveys, with \( k_p = 0.05 \text{ Mpc}^{-1} \), \( A_s = 2.1 \times 10^{-9} \) and \( n_s = 0.965 \) \cite{36}. However, considering the evolution of \( \delta \rho \) during bounce phase, \( \zeta \) may deviate the power-law form at smaller scales. This will be shown in the following.

Firstly, \( \zeta \) is related to \( \delta \rho \). For a cluster with radius \( R \), one yields \( \sigma_{re}^2(R) = \int (dk/k) W(kR/a)(16/81)(kR/a)^4 P_\zeta(k) \) \cite{40}. Here \( \sigma_{re}^2 \) denotes the variance of density contrast \( \langle \sigma^2 \rangle = \langle \delta^2 \rangle \) at the horizon re-entry moment \((c_s k = \bar{a} H)\) in the radiation dominated stage), and \( W(kR/a) \) is a Window function. As mentioned in subsection 3.1, for a fluctuation with physical scale \( R \), we only consider the contribution from the mode \( k = a/R \). Hence the corresponding Window function is \( W(kR/a) = \delta(kR/a - 1) \), with \( \delta(x) \) being the Dirac function. As a result, the power spectrum can be obtained by \( P_\zeta(k) = (81/16) \sigma_{re}^2(k) \), with \( k = a/R \).

Secondly, a proper initial condition of \( \delta_i \) should be taken to calculate \( \sigma_{re}^2 \). For example, the Bunch-Davies vacuum is usually adopted for the matter-bounce scenario, in which \( \sigma_i^2(k) \propto k^4 \), with \( \sigma_i^2 \) being the variance of \( \delta_i \) \cite{63}. In this article, the initial condition is set as a power-law function \( \sigma_i^2(k) \propto k^{n_s+3} \), so that the parametrization \( P_\zeta(k) = A_s(k/k_p)^{n_s-1} \) is satisfied at the CMB scales. Furthermore, the PDF of initial density contrast is assumed to be Gaussian \( \delta_i \in N(0, \sigma_i^2) \). Note that the non-linear processes may make the PDF \( p(\delta) \) non-Gaussian after bounce phase. In this case, the variance is obtained as \( \sigma^2 = \int \delta^2 p(\delta) d\delta \).

The resulting power spectrum \( P_\zeta \) and PDF of \( \delta \) after the bounce phase are illustrated in figure 4. One can see that the profile of \( P_\zeta \) has a fluctuation around the power-law form at small scales. It is inferred that the fluctuated scales are related to the e-folding number of the bounce phase \( n_{BB} \) — a larger \( n_{BB} \) corresponds to larger fluctuated scales. For example, for the \( n_{BB} = 0.8 \) case, the fluctuated scales are large \( (k > 10^5 \text{ Mpc}^{-1}) \), while for the \( n_{BB} = 0.3 \) case, the fluctuated scales are small \( (k > 10^{13} \text{ Mpc}^{-1}) \). Additionally, for the

**Figure 3.** An illustration for the evolution of \( \delta \rho \) during the bounce phase with \( H_{BB} = -H_C = 6.23 \times 10^{-12}, n_{BB} = 5, \gamma = 11, c_s = 1. \) The orange and blue thick curves sketch the fluctuations with initial values \( \delta_i = 10^{-17} \) and \( \delta_i = -10^{-17} \) respectively, and the wavenumber \( k \) for both cases is taken as 0. The black thin curve represents \( \delta \rho \) given by the linear theory (3.9), and the green filled area represents the regime \( |\delta \rho| \leq \bar{\rho} \).
Figure 4. (a) Power spectra $P_\zeta(k)$ in bounce scenarios. The black curve represents the result with $n_{BB} = 0.3$ and $\gamma = 3$, and the magenta one represents the one with $n_{BB} = 0.8$ and $\gamma = 1$. The green dashed line stands for the power-law parametrization $P_\zeta(k) = A_s (k/k_P)^{n_s - 1}$. (b) The red thick curve represents the PDF of $\delta$ after bounce phase, with $k = 10^{12} \text{Mpc}^{-1}$, $n_{BB} = 0.6$ and $\gamma = 3$. The magenta thin curve stands for the standard Gaussian PDF. (c) The cyan thick curve represents the PDF of $\delta$ for $k = 10^{12} \text{Mpc}^{-1}$, $n_{BB} = 0.8$ and $\gamma = 1$. The other parameters in the calculation are taken as $H_{BB} = -H_C = 5.68 \times 10^{-19}$ and $c_s = 1$.

extremely small scales (close to the cut-off $k \simeq |a CH_C|/c_s$), $P_\zeta$ decays significantly, because the pressure is important at small scales. Moreover, panels (b) and (c) show that the PDFs of $\delta$ indeed become non-Gaussian after bounce phase. The change of the PDF profiles indicates that the non-linear processes around bounce point are physical, not due to a gauge issue. Furthermore, the probability of $\delta > \sigma$ increases significantly for the shown PDFs, which is helpful to enhance the PBH abundance.

4 PBH enhancement in bouncing cosmology

4.1 Probability of PBH formation

It is known that the physical scales of fluctuations re-enter Hubble lengths during the Big Bang phase.\(^3\) If a fluctuated region is compact enough at the Hubble re-entry moment, i.e. with $\delta_{re}$ being larger than a threshold $\delta_c$, it will finally collapse to a PBH. Here $\delta_{re}$

\(^3\)For simplicity, we only consider the PBH formation in the radiation dominated stage $z > z_{eq}$ in this article.
denotes the \( \delta \) at Hubble re-entry moment. The probability of PBH formation depends on the PDF of \( \delta_{\text{re}} \). For a Gaussian PDF \( \delta_{\text{re}} \in N(0, \sigma^2_{\text{re}}) \), the probability is the well-known \( \beta = \frac{1}{2} \text{Erfc}(\delta_{\text{c}}/\sqrt{2}\sigma_{\text{re}}) \) [5, 7, 34] given by the Press-Schechter theory [35]. Here the \text{Erfc} denotes the complementary error function. However, in the bouncing scenario, the non-linear evolution of \( \delta \) may be significant and the PDF may have strong NGs after the bounce point, as shown in figure 4. Hence the probability of PBH formation should be beyond the Press-Schechter formalism, and will be evaluated in the following.

In this case, the probability of PBH formation can be expressed as \( \beta = \int_{\delta_{\text{re}} > \xi} p(\delta_{\text{re}}) d\delta_{\text{re}} \), with \( p(\delta_{\text{re}}) \) being a non-Gaussian PDF. Since the profile of \( p(\delta_{\text{re}}) \) may be extremely complicated (e.g. see panel (c) in figure 4) and its calculation is computationally expensive, we express \( \beta \) in terms of \( \delta \) at another moment instead. As mentioned in subsection 3.4, because the non-Gaussian PDF of \( \delta \) arises from the bounce phase, it is reasonable to assume \( \delta_i \in N(0, \sigma^2_i) \). Furthermore, the PBH formation probability at a fixed \( k \) can be calculated as

\[
\beta = \int_{\mathcal{F}(\delta_i)} \frac{1}{\sqrt{2\pi}\sigma_i} \exp \left( -\frac{\delta_i^2}{2\sigma_i^2} \right) d\delta_i,
\]

(4.1)

where \( \mathcal{F}(\delta_i) \) is a criterion equivalent to \( \delta_{\text{re}} > \delta_c \). Clearly, equation (4.1) has a similar form to the result in the Press-Schechter formalism [7].

As mentioned in subsection 3.4, the initial variance is parameterized as a power-law form \( \sigma^2_i(k) \propto k^{n_\delta+3} \). Furthermore, to fix \( \mathcal{F}(\delta_i) \), one should know the evolution of \( \delta \) — in details, from \( \delta_i \) to \( \delta_{\text{BB}} \), and then to \( \delta_{\text{re}} \). As mentioned in subsection 3.3, the evolution from \( \delta_{\text{BB}} \) to \( \delta_{\text{re}} \) is linear, which can be evaluated as \( \delta \propto k^2/(\bar{a}\dot{H}) \) [97]. Therefore, the criterion \( \delta_{\text{re}} \geq \delta_i \) is equivalent to \( \delta_{\text{BB}} > k^2\delta_i/(3\bar{a}_{\text{BB}}^2\dot{H}_{\text{BB}}^2) \). Additionally, the evolution from \( \delta_i \) to \( \delta_{\text{BB}} \) is given by equation (3.7), i.e. \( \delta_{\text{BB}} \) can be written as a function of \( \delta_i \) in principle. Therefore, the criterion is

\[
\mathcal{F}(\delta_i) : \delta_{\text{BB}}(\delta_i) \geq \frac{k^2}{3\bar{a}_{\text{BB}}^2\dot{H}_{\text{BB}}^2}\delta_c.
\]

(4.2)

Here, the value of \( \delta_c \) is set as 0.37 [6]. Given above, the probability of PBH formation in the bounce scenario can be calculated.

Figure 5 illustrates the \((k, \delta_i)\) parameter space which can form PBHs, as well as the probability \( \beta(k) \). Firstly, it is clear that the non-linear effects during the bounce phase can significantly improve the probability of PBH formation. In details, the linear theory requires \( |\delta_i|/\sigma_i \gtrsim 10^4 \) for each \( k \) to form PBHs, corresponding to negligibly small probabilities \( \beta(k) \lesssim 10^{-2\times10^7} \). Meanwhile, the non-linear theory enlarges the probabilities to \( \beta(k) > 10^{-20} \), for some wavelengths. Secondly, it is clear that the non-linear effects are significant only at small scales \( k \gtrsim 10^{10} \text{Mpc}^{-1} \) for the illustrated case), which will not affect the power spectrum and PDF of \( \delta \) at CMB scales. Moreover, the profile of the parameter space forming PBHs is not continuous, consisting of discrete points, in the non-linear theory. This is because the non-linear evolutionary behavior of \( \delta\rho \) is complicated. For example, an initially less denser region may finally evolve to a PBH, while an initial denser one may not.

### 4.2 PBH abundance

Since \( \beta(k) \) has been obtained, the PBH abundance and mass function will be evaluated in the following.
typically used to represent the PBH abundance \[5, 9, 10\], which is expressed as

\[ \frac{\sigma_i}{\sigma_t} \]

for the result in linear theory (equation (3.7)), and the blue dots represent the result in non-linear theory (equation (3.7)). The numerical result is achieved under the precision \( \Delta[\log_{10}(|\delta_i|/\sigma_i)] = 2^{-6} \) and \( \Delta(\log_{10} k) = 2^{-7} \).

First of all, the mass of PBH originating from a region with wavenumber \( k \) is usually estimated as the mass inside the Hubble horizon at the re-entry moment \[5, 7\]

\[ m(k) \simeq H^{-1}|_{k=\sqrt{3}aH} \simeq 10^{15}g \left( \frac{k}{k_{15}} \right)^{-2}, \]  

with \( k_{15} \simeq 10^{15}\text{Mpc}^{-1} \). Furthermore, the comoving number density of PBHs formed by fluctuated regions with wavenumber \( k \) is \( n(k) \simeq k^3\beta(k) \) \[7\]. As a result, the total number of PBHs inside the observable Universe (with comoving wavenumber \( k_0 \simeq H_0 \)) is

\[ N = \sum_{k=ik_0} n(k) \left( \frac{k}{k_0} \right)^3 \simeq \int \frac{k^3\beta(k)dk}{k_0^3}, \]

with \( i = 1, 2, 3, \ldots \). Here the technique of box normalization \( (k = ik_0) \) is used.

Moreover, since the heavy PBHs \( (m > 10^{15}g) \) contribute to part of dark matter components, the density fraction of PBHs against the total dark matter \( f \equiv \Omega_{\text{PBH}}/\Omega_{\text{dm}} \) is typically used to represent the PBH abundance \[5, 9, 10\], which is expressed as

\[ f \simeq \int_{m(k')>10^{15}g} \frac{m(k')n(k')dk'}{3H_0^2\Omega_{\text{dm}}} \approx 1.55 \times 10^{36} \int_{k'<k_{15}} \frac{k'\beta(k')dk'}{k_{15}^2}, \]

with \( \Omega_{\text{dm}} \simeq 0.264 \) being the density fraction for the dark matter \[36\]. Furthermore, the PBH abundance around a specific mass (actually in the PBH mass range \( (m, m + \Delta m) \), with
Figure 6. The PBH abundances $f(m)$ in bouncing cosmology. The green dots represent the result given by the bounce phase with $\gamma = 5$ and $n_{\text{BB}} = 0.31$, and the blue dots given by $\gamma = 5$ and $n_{\text{BB}} = 0.59$ — the other parameters are the same as those in figure 5. The results are achieved under the numerical precision $\Delta[\log_{10}(|\delta_i|/\sigma_i)] = 0.1$ and $\Delta(\log_{10}k) = 0.04$. The red curve denotes the upper limit of $f(m)$ by observations of $\gamma$-ray background, Voyager positron flux and annihilation line radiation; the cyan curve denotes the upper limit of $f(m)$ by microlensing observations [7, 8, 18, 98, 99].

$\Delta m \simeq m$, or equivalently $\Delta k \simeq -k/2$ according to equation (4.3)) is also used [7], which is

$$f(m) \simeq 1.55 \times 10^{36} \int \frac{k'\beta(k')dk'}{k_{15}^2} W(k';k), \quad (4.6)$$

with the window function

$$W(k';k) = \begin{cases} 1, & k' \in \left(\frac{k}{2}, k\right) \text{ & } k' < k_{15} \\ 0, & \text{else} \end{cases} \quad (4.7)$$

Figure 6 illustrates the PBH abundances $f(m)$ given by specific bounce phases. It is seen that the bounce phase, at lest for the specific cases, can sufficiently improve the PBH abundance and do not break the observational constraints. Hence it can serve as an potential mechanism of PBH formation.

4.3 Constraints of bounce phase through PBH abundance

Since the PBH abundance depends on the background dynamics of bouncing scenario, as shown in figure 6, the PBHs can conversely provide a probe to the parameters of bounce phase, which includes $H_C$, $H_{\text{BB}}$, $n_{\text{BB}}$, $\gamma$ and $c_s$. Note that, the parameters $H_C$, $H_{\text{BB}}$ and $c_s$ can be effectively constrained combining CMB observations [65], but $n_{\text{BB}}$ and $\gamma$ are generally poorly measured. Therefore, the constraints of $n_{\text{BB}}$ and $\gamma$ by PBHs are worthwhile.

The PBH abundance $f(m)$ in a wide mass range ($10^{15}$–$10^{55}$)g has been constrained [7, 18, 98]. For a bounce model with parameters $n_{\text{BB}}$ and $\gamma$, it will be ruled out if too many PBHs are generated, i.e. in the following two cases: (1) the abundance $f(m)$ calculated
Figure 7. Constraints of bounce phase parameters $n_{BB}$ and $\gamma$, with $H_{BB} = -H_C = 6.23 \times 10^{-12}$ and $c_s = 1$. The red dots represent the excluded bounce phases, which generate too many PBHs. The gray dots denote the disfavored bounce phases, which cannot generate PBHs effectively ($N < 1$). The green dots stand for the favored bounce phases. The results are achieved under the numerical precision $\Delta[\log_{10}(|\delta|/\sigma)] = 0.1$ and $\Delta(\log_{10} k) = 0.04$.

by equation (4.6) breaks an observational constraint; (2) the total abundance $f$ given by equation (4.5) is larger than 1. Furthermore, if the PBH number $N$ given by equation (4.4) is smaller than 1, the corresponding bounce phase cannot generate PBHs effectively. In this case, although the bounce phase is safe in the observational constraint, it is disfavored in the motivation of PBH formations. Given above, only the bounce phases neither generating too many PBHs nor too few PBHs are favored.

The constraints of $n_{BB}$ and $\gamma$ are illustrated in figure 7. Firstly, it is clear that the bounce phase can be constrained by PBHs. Furthermore, the favored bounce phases (green dots) distribute discretely in the parameter space, and the rule of the distribution is still unknown. This is probably because the numerical precision in our calculations is not high enough. Note that an increasing precision may improve the results significantly. Although the constraints in figure 7 look ruleless in current precision, they are still conducive to the model construction of bounce phase. For example, the models with $\gamma = 3, 5$ are generally more dangerous to generate too many PBHs than the other ones.

5 Conclusions and outlook

In this article, we apply the non-linear evolution of density fluctuation around bounce point to enhance PBH abundance. Note that the non-linear effects naturally exist in the relativistic bouncing scenario, hence the PBH formation does not require an extra physical mechanism. Our results in section 4 indicate that it is plausible to produce PBHs sufficiently through bounce phase, which improve the pessimistic conclusions in the earlier researches [62, 63]. We express the PBH abundance through the parameters of bounce phase, and the PBH abundance can also provide a probe to the parameters $\gamma$ and $n_{BB}$, which are usually difficult
to be measured by the CMB or LSS observations. Therefore, the PBH observations may provide a complementary to the future surveys of CMB and LSS.

The current work can be extended in the following three aspects.

Firstly, the non-linear equation (3.7) is so complicated that the numerical errors may have considerable impacts on the results. For example, the linear equation (3.9) and the non-linear equation (3.7) actually lead to different transfer functions at $\delta_i \to 0$ due to the numerical errors, with the deviation being several orders of magnitude. To reduce the impacts from numerical errors, the linear transfer function in this work is actually achieved by solving the non-linear equation (3.7) at $\delta_i = 10^{-40}$. This algorithm should be further tested or improved in the future. Moreover, the precision in this article is not high enough. For example, the parameter $\epsilon = 0.1$ in the iteration (see figure 2) seems large in usual numerical works. Additionally, the constrained parameters in figure 7 distributes randomly, due to the low precision in parameters $\delta_i$ and $k$. As mentioned in subsection 4.3, the results will change significantly as the numerical precision increases, so will be the time of the computations. Therefore, our results can only serve as a rough estimation, and it is worthwhile to spend more time improving the precision in the follow-up researches.

Secondly, the subsequent evolution of the formed PBHs, including evaporation [13], accretion and merger [100], are not considered in this article, which will change the abundance and mass function for the PBHs of interest. Hence we also plan to include these effects and update the constraints of bounce phase in the future.

Moreover, since the PBH enhancement in this work has a close connection to the NGs of density fluctuation, the PBH abundance can be expressed in terms of the parameters of NGs, such as $f_{NL}$ and $g_{NL}$ etc. [45, 47, 48]. This is another point worth being investigated in the follow-up researches. Additionally, the NG parameters $f_{NL}$ and $g_{NL}$ have been constrained by CMB surveys [101]. Hence the bounce phase can be constrained though the joint of the PBH abundance and observational results of $f_{NL}$ and $g_{NL}$.

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As we have found, for $|\delta_i| \sim 10^{-40}$, the linear and non-linear equations approximately yield the same results; if $|\delta_i| < 10^{-40}$, the results will be dominated by numerical errors.
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