1D Luttinger liquid & QED

P. Degiovanni and S. Peysson
Laboratoire de Physique (U.M.R. 5672 du CNRS), Ecole Normale Supérieure de Lyon
46, allée d’Italie, 69364 Lyon Cedex 07, France

Using its Conformal Field Theory description, the Luttinger liquid is coupled to the quantum electromagnetic field. We compute the decoherence properties of a superposition of two states obtained by creating the same elementary excitation at different places in the system.

1 Introduction

The Landau-Fermi theory is the simplest theory for interacting electrons in metals. Its basic low energy excitations called quasiparticles are dressed electrons. Despite its success in 3D, the search for more exotic metallic states has been pursued leading to the discovery of the Luttinger liquid. This powerful and simple effective theory plays the role of the Landau-Fermi theory for one dimensional systems and has many applications. As illustrated in the present volume, various systems such as fractional quantum Hall (FQH) edge states, carbon nanotubes, quasi-1D organic conductors and quantum spin chains can be described using this paradigm. Up to now, this is the only known effective theory of a non-Fermi liquid.

However, the coupling to an external quantum reservoir such as the quantum electromagnetic field has not attracted much attention. A notable exception is the work by Loss and Martin which shows that QED’s fluctuations cannot induce a spontaneous permanent current in a Luttinger ring. A great deal of work has been done within the context of weak localization but mainly in the framework of Landau-Fermi theory. In contrast, questions concerning the real time evolution of a Luttinger liquid coupled to QED have not been addressed. Here, recent results on QED-induced decoherence of Schrödinger cat states in a Luttinger liquid are reported. Our main questions are: does QED induce a modification of the effective Luttinger liquid theory? How strong and fast is the decoherence induced by the coupling of a Luttinger liquid to the quantum electromagnetic field?

The CFT description of the Luttinger liquid is recalled in section 2. Our results concerning decoherence in the Luttinger liquid are presented in section 3 and the conclusion summarizes the main results.

2 Effective CFT for the Luttinger Liquid

We consider a system of spinless interacting electrons on a 1D circle of length \( L \). Such a system can be realized for example using a thin annular FQH fluid.

As was shown by Haldane for gapless systems low energy properties can be described using an effective theory with only two parameters: a renormalized Fermi velocity \( v_S \) and a dimensionless interacting constant \( \alpha \). These parameters, which can be related to the microscopic interaction constants (g-ology) are analogous of the \( f_{k,k'} \) parameters of Laudau-Fermi liquids.

This 1D effective description can conveniently be formulated in terms of a 2D conformal field theory (CFT), the Hilbert state of which can be decomposed according to a \( \hat{U}(1)_R \times \hat{U}(1)_L \) symmetry algebra. For each chirality, \( \hat{U}(1) \) is generated by current modes satisfying: \( [J_n, J_m] = n \delta_{n,-m}L \). The electric charge and current densities are related to these modes by:

\[
\rho(\sigma) = \frac{e}{L\sqrt{\alpha}} \sum_{l \in \mathbb{Z}} (J_l e^{2i\pi l \sigma/L} + J_l^* e^{-2i\pi l \sigma/L}) \\
\jmath(\sigma) = \frac{ev_S}{L\sqrt{\alpha}} \sum_{l \in \mathbb{Z}} (J_l e^{2i\pi l \sigma/L} - J_l^* e^{-2i\pi l \sigma/L})
\]  

Irreducible highest weight states of the \( \hat{U}(1)_R \times \hat{U}(1)_L \) symmetry algebra are associated with conformal primary fields called vertex operators. The operator content of the effective Luttinger CFT can be

*It means that backscattering terms which can potentially open a gap in the spectrum are discarded. For an annular FQH fluid, the two edges sufficiently separated and backscattering is strongly suppressed.
determined by listing these highest weight states. They are indexed by \((n, m) \in \mathbb{Z} \times \mathbb{Z}\) such that \(2n \equiv m \pmod{2} : |n, m\) and we have:

\[
J_0 |n, m\rangle = \left(n\sqrt{\alpha} + \frac{m}{2\sqrt{\alpha}}\right) |n, m\rangle \quad \text{and} \quad \tilde{J}_0 |n, m\rangle = \left(n\sqrt{\alpha} - \frac{m}{2\sqrt{\alpha}}\right) |n, m\rangle
\]

The \(l = 0\) modes \(J_0\) and \(\tilde{J}_0\) are quantized: \(2n\) is the total charge and \(m\) is related to the total current circling around the system. The \(l \neq 0\) modes are called \textit{hydrodynamic} in reference to Wen’s pioneering work on the Luttinger liquid as a description for edge states in the FQHE.

A very simple argument for this identification has been proposed\(^\text{3}\). It relies on the recovery of Laughlin’s thought experiment from the CFT point of view\(^\text{4}\). The Luttinger parameter is related to the filling fraction by \(\alpha \nu = 1\). From the 1D point of view, the original fermion operators renormalize (orthogonality catastrophe) to specific vertex operators: \(\psi^\dagger_k\) (resp. \(\psi^\dagger_L\)) corresponds to \(V_{1/2,1}\) (resp. \(V_{1/2,-1}\)). In the case of a FQH fluid, edge fermions carrying unit charge on one of the two edges appear in the spectrum: \(V_{1/2,\nu-1}\) and \(V_{1/2,-\nu-1}\). These operators generate an extended symmetry algebra with respect to which the Luttinger CFT is rational. In contrast to them, the Luttinger fermions carry a fractional charge on each edge \(q_R = (1 + \nu)/2\) and \(q_L = (1 - \nu)/2\) for \(V_{1/2,1}\).

### 3 Coupling to Quantum Electrodynamics

In the Coulomb gauge, the Hamiltonian for a matter system coupled \(S\) to the electromagnetic field \(\mathcal{E}\) is given by:

\[
H = H_S(\rho, j) + H_\mathcal{E}(A_\perp, E_\perp) - \int A_\perp(r) j_\perp(r) \, d^3r + E_{\text{Coulomb}}
\]

The static Coulomb interaction is taken into account by the \textit{effective} Luttinger CFT. In the following, we are interested by the evolution of the reduced density matrix of the matter system. The electromagnetic field defines an \textit{environment}. The Feynman-Vernon-Keldysh method uses a path integral language to compute the evolution of \(S\)’s reduced density matrix. Whereas in quantum mechanics the usual propagator is associated to a simple contour going from the initial time \(t_i\) to the final time \(t_f\), here, the Keldysh contour goes from \(t_i\) to \(t_f\) (+ branch) and back from \(t_f\) to \(t_i\) (− branch). Integration over transverse photons is gaussian and immediately leads to the Feynman-Vernon influence functional:

\[
\mathcal{F}[j_+(x), j_-(y)] = \exp \left(-\frac{i}{2} \sum_{(\epsilon, \epsilon') \in \{1,-1\}^2} \epsilon \epsilon' \int d^3x d^3y j^\dagger_\epsilon(x) D^\epsilon\ell_{\epsilon\ell'}(x, y) j_{\ell'}(y) \right)
\]

where the Keldysh-Green functions for the electromagnetic field \(D^\epsilon\ell_{\epsilon\ell'}(x, y) = -i(T_K A^\dagger_\ell(x) A^\ell_{\epsilon'}(y))\) should be used\(^\text{3}\). At this point, the problem does not seem straightforward: the effect of the electromagnetic field is to introduce a non local quartic interaction for the fermions.

However, such a situation can be handled by the bosonization technique. We replace fermionic degrees of freedom by charge and current densities, which are bosonic and free. In the one dimensional case, this provides an exact reformulation of the Luttinger effective theory. Using bosonic coordinates and keeping only long wavelength components of the current, the coupled problem Luttinger & QED is gaussian and therefore, in principle, exactly solvable. The method could in principle be extended in \(D \geq 2\) but in this case, bosonization only provides an approximate description of the fermion system.

Using a cylindrical mode decomposition for the electromagnetic field and the usual mode expansion for the Luttinger liquid, the problem becomes equivalent to the linear coupling of an infinite family of harmonic oscillators with distinct baths of oscillators. The underlying elementary problem (usually called “QBM”) has been extensively studied\(^\text{5}\). All dynamical properties are encoded into an infinite family of \textit{dimensionless spectral densities} \(\mathcal{J}_l(\omega)\) indexed by Luttinger mode numbers \(l \geq 0\). At low frequencies, its behaviour is ohmic for \(l = 1\) and supraohmic for any other value: \(\mathcal{J}_l(\omega) \sim (L\omega/c)^{2l-1}\) for \(l \neq 0\) and

\(^{3}\text{Effect of the increase of the magnetic flux through the ring on the system’s ground state.}\)

\(^{\text{4}}\text{c and } \epsilon' \text{ label the branches of the Keldysh contour.}\)
\( J_0(\omega) \approx (L\omega/c)^3 \). The interaction between the Luttinger liquid and the quantum electromagnetic field is characterized by a dimensionless coupling constant:

\[
g = 4\pi \frac{\alpha_{\text{QED}}}{\alpha} \left( \frac{v_S}{c} \right)^2 \approx 10^{-8}
\]  

(6)

We are interested by the real time evolution of a state obtained by coherent superposition of two elementary excitations of the Luttinger liquid. As recalled in section 3, localized excitations of the Luttinger liquid are created by vertex operators \( V_{n,m}(\sigma) \). Let us consider Schrödinger cat states defined as superpositions of the same excitation (with possible different chiralities) at different places around the circle. For example:

\[
|\psi_{R/R}\rangle = \frac{1}{\sqrt{2}} \left( \psi_R^\dagger(\sigma_1)|0\rangle + \psi_R^\dagger(\sigma_2)|0\rangle \right) \quad \text{and} \quad |\psi_{R/L}\rangle = \frac{1}{\sqrt{2}} \left( \psi_R^\dagger(\sigma_1)|0\rangle + \psi_L^\dagger(\sigma_2)|0\rangle \right)
\]  

(7)

In an isolated system, such a coherent superposition will remain coherent. Switching on the coupling to the quantum electromagnetic field changes the situation: according to general works on decoherence, such Schrödinger cats should decohere into a statistical mixing of two states: one excitation at one position, or the excitation at the other place. From a solid state physics point of view, decoherence implies that interference effects are suppressed and transport leaves the purely quantum regime.

Our analysis shows that the \( l = 0 \) modes can be treated by exact diagonalization. Hydrodynamic modes are more complicated to study. But since vertex operators create coherent states in the \( l \geq 1 \) modes, we have been able to obtain the following results.

Zero modes Only the total current around the Luttinger ring couples to the transverse electric modes.

First, the Luttinger parameters \( v_S \) and \( \alpha \) get renormalized, leading to a modification of the highest weight state energies. This renormalisation process takes place within the cutoff time, corresponding to the UV cutoff frequency \( \Lambda \). Writing \( v'_S = v_S \eta \) and \( \alpha' = \alpha/\eta \), we have:

\[
\eta^2 = 1 - \frac{c}{\pi v_S} \int_0^{\infty} J_0(\omega) \frac{d\omega}{\omega}
\]  

(8)

which is a correction of order \( gc/v_S \approx 10^{-5} \). Next, off diagonal matrix elements (with respect to \( m \)) are damped within the same time scale by a factor:

\[
\exp \left( -\frac{t^2}{(L/c)\alpha}(m - m')^2 \int_0^{+\infty} d\omega J_0(\omega) \frac{1 - \cos(\omega t)}{(\omega t)^2} \right)
\]  

(9)

The \( t \to +\infty \) value of the decoherence exponent is

\[
-\frac{(m - m')^2}{\alpha} \int_0^{+\infty} d\omega J_0(\omega) \frac{1}{(\omega t)^2}
\]

This is of typical order \( g \). It is proportional to the square of the difference between current intensities, a quantity which measures the “distance” between the two quantum states. Since \( g \approx 10^{-8} \), low energy zero modes do not loose their quantum character because of the coupling with QED.

Hydrodynamic modes Wigner functions are quite appropriate to study the evolution of coherent states. Using the evolution kernel for the evolution of the density matrix, the exact time evolution of the Wigner function corresponding to each hydrodynamic Luttinger mode \( (l \geq 1) \) can be obtained. In principle, this method gives access to the short time regime of the system \( t \approx \Lambda^{-1} \) (when non-markovian effects are still important) and could also be used to study strong coupling situations. However, due to the weakness of QED’s coupling, the main features can be found in a simple way. First of all, let us mention that energy dissipation into the electromagnetic field takes place within a time \( g^{-1} \) longer than the typical Luttinger time \( L/v_S \). The system is strongly underdamped.

In the intermediate regime, \( \Lambda^{-1} \ll t \ll g^{-1}L/v_S \), a perturbative treatment and a secular approximation enables us to extract terms linear in time for the decoherence exponent. Decoherence times for each of the Luttinger modes can then be obtained. The main result of this analysis, which relies on the estimates of low frequency asymptotics of spectral densities \( J_i(\omega) \) is that the \( l = 1 \) mode dominates the

\(^4\alpha_{\text{QED}} \) denotes the fine structure constant.
Decoherence process. At zero temperature, the typical decoherence time constants $\gamma_l$ for the $l$th Luttinger modes are given by:

$$\frac{L}{v_S} \cdot \gamma_l^{(R/R)}(\sigma_1, \sigma_2) = 4g \Delta_{n,m} \left( \frac{2\pi v_s}{c} \right)^{2(l-1)} \cdot \frac{l^2(l+1)}{(2l+1)!} \cdot \sin^2 \left( \frac{\pi l \sigma_{12}}{L} \right)$$

$$\frac{L}{v_S} \cdot \gamma_l^{(R/L)}(\sigma_1, \sigma_2) = 4g \Delta_{n,m} \left( \frac{2\pi v_s}{c} \right)^{2(l-1)} \cdot \frac{l^2(l+1)}{(2l+1)!} \cdot \left( 1 + \frac{m^2 - 4\alpha^2 n^2}{m^2 + 4\alpha^2 n^2} \cos \left( \frac{2\pi l \sigma_{12}}{L} \right) \right)$$

where $\Delta_{n,m}$ denotes the conformal dimension of $V_{n,m}(\sigma)$. Then $\gamma_{l+1}/\gamma_l \simeq (v_S/c)^2$, which shows the dominance of the $l = 1$ mode. Results for the decoherence time of all hydrodynamic modes are presented here for a circle of radius $5 \mu m$. The Luttinger parameter are $\alpha = 3$ and $v_S/c = 10^{-3}$. The temperature dependance can also be obtained: decoherence is faster as temperature increases. Let us notice that, as could be expected, the $R/R$ Schrödinger cat does not decohere for $\sigma_{12} = \sigma_1 - \sigma_2 = 0$.

Frequencies of each oscillator get renormalized. As explained by Mélin, Douçot and Butaud\cite{3}, such an effect leads to a kind of “dynamical orthogonality catastrophe”, that is to say, a quick decrease of the two fermion Green function due to phase mixing.

**Long time behaviour** The previous approximation is not valid anymore at longer times because of dissipation. A Markovian master equation à la Caldeira-Legett\cite{7} can be obtained for the $l = 1$ modes provided the temperature is bigger than dissipation $k_B T \gg \hbar \gamma$, where the damping coefficient is given by $\gamma = \frac{4\pi}{3} \frac{\alpha_{QED}}{\alpha} \left( \frac{v_s}{c} \right)^2 \cdot \frac{\gamma}{L}$. In this framework, retaining only contributions from the $l = 1$ modes, we are able to compute the decoherence coefficient for $t \to +\infty$. The short time asymptotics of this computation coincides with linear terms computed above. For example the long time dependance of decoherence at zero temperature is given by:

$$d_{(R/R)}(t, \sigma_1, \sigma_2) = 4\Delta_{n,m} \cdot \sin^2 \left( \frac{\pi \sigma_{12}}{L} \right) \cdot (1 - e^{-\gamma t})$$

$$d_{(R/L)}(t, \sigma_1, \sigma_2) = 2\Delta_{n,m} \cdot \left( 1 + \frac{m^2 - 4\alpha^2 n^2}{m^2 + 4\alpha^2 n^2} \cos \left( \frac{2\pi \sigma_{12}}{L} \right) \right) \cdot (1 - e^{-\gamma t})$$

Of course, at non-zero temperature, thermalization effects enter the game within the dissipation time scale (damping of hydrodynamic modes) but analytical computations not reported here can also be performed.
4 Conclusion

As a conclusion, let us sketch the decoherence scenario for Schrödinger cats in a Luttinger liquid. First of all, at very short times \( t \simeq \Lambda^{-1} \), the Luttinger parameters \( \alpha \) and \( \nu_S \) get renormalized. Zero modes decohere (only for \( R/L \) and \( L/R \) cats) very weakly. Hydrodynamic modes undergo a gaussian decoherence \( \exp (-t^2/\tau^2) \). After this transitory regime and before saturation, the main decoherence contribution behaves like \( \exp (-t \gamma(\sigma_1, \sigma_2)) \). The main contribution for this decoherence time is the first hydrodynamic mode. This decoherence time is of the order of the dissipation time, \( \tau \), roughly \( 10^9 \) times longer than the Luttinger time \( L/v_S \). After the decoherence time scale, decoherence saturates to its final value. However, interference terms are not completely suppressed. Although our work is done in a many-body framework, and even in a non-Fermi liquid, it would be very interesting to make the connection with Stern et al. more precise.

To conclude, let us stress that acoustic 3D phonons are expected to produce a different scenario. Their coupling to electrons is much stronger, the sound velocity smaller to \( v_S \), and finally the phonon spectrum presents a natural cutoff at the Debye frequency, which is much smaller than the Luttinger frequency \( v_S/L \). Non-markovian and strong coupling effects should therefore play a much more important role than in the Luttinger & QED case. We hope to come back on these issues in a forthcoming publication.

Acknowledgments

We would like to thank the organizers of the conference for giving us the opportunity to present this work in a very stimulating atmosphere. We would also like to thank B. Douçot, Ch. Chaubet and R. Mélin for many interesting discussions. L. Cugliandolo provided us with useful references on the QBM. During the conference, we also benefited from useful discussions with Th. Martin and D. Loss on Luttinger liquid coupled to QED or phonons.

S. Peysson is supported by the Ministère de l’Education Nationale, de la Recherche et de la Technologie (AC contract 98026). This work also benefited of support from CNRS and the European Union (TMR FMRX-CT96-0012).

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