An algorithm of Saxena-Easo on fuzzy time series forecasting

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Abstract. This paper presents a forecast model of Saxena-Easo fuzzy time series prediction to study the prediction of Indonesia inflation rate in 1970-2016. We use MATLAB software to compute this method. The algorithm of Saxena-Easo fuzzy time series doesn’t need stationarity like conventional forecasting method, capable of dealing with the value of time series which are linguistic and has the advantage of reducing the calculation, time and simplifying the calculation process. Generally it’s focus on percentage change as the universe discourse, interval partition and defuzzification. The result indicate that between the actual data and the forecast data are close enough with Root Mean Square Error (RMSE)= 1.5289.

1. Introduction

One of soft computing methods that presented by Song and Chissom based on fuzzy set theory by Zadeh is known as Fuzzy time series. In recent years, it has been implemented in time series data such as enrollment data for the University of Alabama [1], [2], [3], [4], daily temperature [5], and car fatalities [6].

In 1998, Indonesia has experienced big economic crisis where inflation rate reach 77.63%. Various approaches have been developed by researchers to forecast Indonesia inflation rate so that government can anticipate impact from fluctuation of Indonesia inflation rate in future [7]. Now we try to implement fuzzy time series from Saxen and Easo on Indonesia inflation rate data. The Indonesia inflation rate is the change in the price increase of goods and services in Indonesia generally and continuously over a period of time which expressed as a percentage. This algorithm doesn’t need stationarity like conventional forecasting method, capable of dealing with the value of time series which are linguistic, and has the advantage of reducing the calculation, time and simplifying the calculation process. It’s also points out the forecast data of Alabama Universities enrollments in 1978-1992 get the better result than the other algorithm which proposed before and produce better accuracy. The research is continued by calculating the accuracy from algorithm of Saxena-Easo which has been implemented on Indonesia inflation rate data. Here we used Root Mean Square Error (RMSE) and we compared the result of forecasting with actual data of Indonesia inflation rate.

The rest of this paper is organize as follow. In section 2, we give basic concept of fuzzy time series. In section 3, we give explanation about algorithm of Saxena-Easo Fuzzy Time Series. The precision value of a forecasting method is given in section 4. The conclusions are discussed in section 5 where the result given value close to actual data.

2. The Basic Concept of Fuzzy Time Series

Definition 1 (Time Series): The sequence of data which is measured at successive time spaced at uniform time interval is called time series.
Definition 2 (Fuzzy set): The elements of sets with degrees of membership is called fuzzy set. Let \( \mathcal{U} = \{u_1, u_2, \ldots, u_n\} \) and let fuzzy set in \( \mathcal{U} \) is \( \mathcal{A} \). \( \mathcal{A} \) is defined below:

\[
A = \frac{f_A(u_1)}{u_1} + \frac{f_A(u_2)}{u_2} + \ldots + \frac{f_A(u_n)}{u_n}
\]

Where the membership function of the fuzzy set \( \mathcal{A} \) is called \( f_A \). \( f_A : \mathcal{U} \rightarrow [0,1] \). \( f_A(u_i) \) denote the degree of membership of \( u_i \) in fuzzy set \( \mathcal{A} \), \( f_A(u_i) \in [0,1] \) and \( 1 \leq i \leq n \).

Definition 3 (Fuzzy Time Series): Time series with fuzzy data. Let \( Y(t) (t = \ldots, 0, 1, 2, \ldots) \) is a subset of \( \mathbb{R} \) and the universe discourse for \( f_i(t) (i = 1, 2, \ldots) \). Then the collection of \( f_i(t) \) is known as \( F(t) \). \( F(t) \) is fuzzy time series on \( Y(t) (t = \ldots, 0, 1, 2, \ldots) \).

Definition 4 (Forecast Error): The difference between actual data and forecasted value of time series.

3. Algorithm of Saxena Easo Fuzzy Time Series

In this section, we will give explain about algorithm of Saxena-Easo Fuzzy Time Series that we use.

The procedure for this method is shown below:

Step 1. Change the actual data of Indonesia inflation rate into percentage of change using formula:

\[
percChange = \left( \frac{x_t - x_{t-1}}{x_{t-1}} \right) \times 100 \tag{1}
\]

rate for \( t-1 \). The percentage change of Indonesia inflation rate year to year from 1970 until 2016 is shown in Table 1. For example percentage change for 1970-1971:

\[
percChange_{1970-1971} = \left( \frac{x_{1971} - x_{1970}}{x_{1970}} \right) \times 100
= \left( \frac{8.94}{2.62-8.94} \right) \times 100
= -70.69
\]

Table 1. The year to year percentage change of Indonesia inflation rate.

| Years     | Percentage Change of Indonesia Inflation Rate |
|-----------|-----------------------------------------------|
| 1970-1971 | -70.69                                        |
| 1971-1972 | 885.11                                        |
| 1972-1973 | 5.27                                          |
| 1973-1974 | :                                             |
| 2013-2014 | -0.24                                         |
| 2014-2015 | -59.93                                        |
| 2015-2016 | -9.85                                         |
Step 2. Define the universe of discourse $U$ in $U=[D_{\text{min}},D_{\text{max}}]$ and partition it into $n$ equal intervals. $D_{\text{min}}$ and $D_{\text{max}}$ are the minimum and maximum from actual data of Indonesia inflation rate. For example, in this paper $U=[-98, 886]$ is partitioned into seven equal intervals. Shown in Table 2.

Table 2. Intervals and frequency.

| Interval       | Frequency |
|----------------|-----------|
| [-98; 42.5714] | 34        |
| [42.5714; 183.1429] | 8        |
| [183.1429; 323.7143] | 1        |
| [323.7143; 464.2857] | 1        |
| [464.2857; 604.8571] | 1        |
| [604.8571; 745.4286] | 0        |
| [745.4286; 886] | 1        |

Step 3. Divide the intervals before into several equal length based on the number of frequency from each interval in Table 2 and compute mid points from each subinterval. Next, give linguistic values for each subinterval. The result from this step is called fuzzy interval. For example, first interval in Table 2 has 34 frequency so that we divide first interval into 34 subinterval then we give linguistic value $(X_1, X_2, \ldots, X_{34})$ and calculate mid points from each subinterval. The result shown in Table 3 where there’s 46 subinterval from each interval in Table 2 then give linguistic value from $X_1$ until $X_{46}$.

Table 3. Linguistic value, interval and midpoint.

| Linguistic | Interval       | Mid Points |
|------------|----------------|------------|
| $X_1$      | [-98; -93.8655] | -95.9328   |
| $X_2$      | [-93.8655; -89.7311] | -91.7983  |
| $X_3$      | [-89.7311; -85.5966] | -87.6639  |
| $\vdots$   | $\vdots$       | $\vdots$  |
| $X_{44}$   | [323.7143; 464.2857] | 394       |
| $X_{45}$   | [464.2857; 604.8571] | 534.5714  |
| $X_{46}$   | [745.4286; 886] | 815.7143  |

Step 4. Compute prediction of percentage change with this formula (2):
Where $a_{j-1}$, $a_j$, $a_{j+1}$ are mid points of fuzzy intervals $X_{j-1}, X_j, X_{j+1}$. For example prediction of percentage change for 1971, we have percentage change about -70.69 and it is in interval $X_7$. The result shown in Table 4.

$$t_{1971} = \frac{0.5 - 1 + 0.5}{0.5 + 1 + 0.5} \left( \frac{a_{j-1}}{a_j} + \frac{a_j}{a_{j+1}} \right)$$

$$= \frac{0.5 - 75.26 + 0.5}{0.5 - 71.13 + 0.5}$$

$$= -71.0057$$

Step 5. Defuzzify the fuzzy data with this formula (3).

$$F(t) = \left( \frac{t_j}{100} \times x_{t-1} \right) + x_{t-1}$$

Where $x_{t-1}$ is actual data of Indonesia inflation rate for $t - 1$, $t_j$ is predicted percentage change. The result shown in Table 4 and for example below:

$$F(1971) = \left( \frac{-71.0057}{100} \times 8.94 \right) + 8.94$$

$$F(1971) = 2.5921$$

Step 6. Count RMSE. The result from this step is shown in Table 4.

4. The Precision Value of A Forecasting Method

We use Root Mean Square Error (RMSE) to measure the accuracy of Saxena-Easo fuzzy time series for forecasting Indonesia inflation rate with formula (4):

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n}(x_t - F_t)^2}{n}}$$

Where $x_t$ is actual data of Indonesia inflation rate, $n$ is number of data, $F_t$ is forecasting data of Indonesia inflation rate.

| Years | Actual Inflation rate | Percentage Change | Fuzzy Set | Predicted Percentage | Forecast | SE   |
|-------|-----------------------|-------------------|-----------|----------------------|----------|------|
| 1970  | 8.94                  | -                 | -         | -                    | -        | -    |
| 1971  | 2.62                  | -70.6935          | $X_7$     | -71.0057             | 2.5921   | 0.0008|
| 1972  | 25.81                 | 885.1145          | $X_{46}$  | 694.0434             | 20.8039  | 25.0607|
We implemented Saxena-Easo fuzzy time series using MATLAB software. Table 4 summarizes the result of Saxena Easo FTS method for Indonesia inflation rate in 1970 to 2016, where the universe of discourse is divided into 7 intervals and each intervals is divided based on frequency of each intervals with equal length until we get 46 of fuzzy set which is denoted \((X_1, X_2, \ldots, X_{46})\). Every mid point of fuzzy set is used to compute predicted percentage. The predicted percentage change and actual data for \(t-1\) are used to forecast Indonesia inflation rate for \(t\) period. The result shows between actual data and forecast data aren’t too different. We can see in 1971, between actual data and forecast data have small square error (SE) about 0.0008. In the following, we compute the accuracy of this method with root mean square error (RMSE). The RMSE of the forecasting of this method is small enough about 1.5289. Figure 1 shows the black line for actual data and the grey line for forecast data with Saxena Easo fuzzy time series. We can see from this graph that the grey line is close enough to the black line, means that result of Saxena Easo fuzzy time series is quite accurate if it’s compared with actual data.

![Figure 1. The result of method for forecasting of Indonesia inflation rate](image)

5. Conclusion

The study present algorithm of Saxena-Easo Fuzzy Time Series to be applied on Indonesia inflation rate data from 1970 – 2016. From table 4, we can see that this method provide RMSE = 1.5289 which is small enough so that this method gets a higher forecasting accuracy rate for forecasting. The smaller the RMSE is the more accurate are the forecast method. In future work, this method need to be developed to do forecasting for \(t+1\) period if there’s no actual data for that period, provide smaller interval to make higher accuracy rate, and compare this method with conventional time series analysis.

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