New advancements in AdS/CFT in lower dimensions

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Abstract

We review recent developments in the study of the AdS/CFT correspondence in lower dimensions. We start summarising the classification of AdS$_3 \times$S$^2$ solutions in massive Type IIA supergravity with (0,4) supersymmetries, and the construction of their 2d dual quiver CFTs. These theories are the seed for further developments, that we review next. First, we construct a new class of AdS$_3$ solutions in M-theory that describe M-strings in M5-brane intersections. Second, we generate a new class of AdS$_2 \times$S$^3$ solutions in massive IIA with 4 supercharges that we interpret as describing backreacted baryon vertices within the 5d $\mathcal{N} = 1$ QFT living in D4-D8 branes. Third, we construct two classes of AdS$_2$ solutions in Type IIB. The first are dual to discrete light-cone quantised quantum mechanics living in null cylinders. The second class is interpreted as dual to backreacted baryon vertices within 4d $\mathcal{N} = 2$ QFT living in D3-D7 branes. Explicit dual quiver field theories are given for all classes of solutions. These are used to compute the central charges of the CFTs, that are shown to agree with the holographic expressions.
1 Introduction

Lower dimensional CFTs\(^1\) play a prominent role in the microscopic description of black holes and black strings. Since these exhibit, in the extremal case, AdS\(_2\) and AdS\(_3\) geometries close to their horizons, a deeper understanding of the AdS/CFT correspondence in lower dimensions is of key importance for their study.

The construction of AdS\(_2\) and AdS\(_3\) geometries and the identification of their dual superconformal field theories, has been the focus of many interesting works. In general, as the dimensionality of the internal space increases, the possible geometrical structures, supersymmetries preserved and topologies also increase, giving rise to a plethora of possible solutions (see for instance [1]-[45]). However, even if many classes of solutions with different amounts of supersymmetries have been constructed this has only been paralleled with a detailed understanding of their dual CFTs for the D1-D5 and D1-D5-D5’ systems, and orbifolds thereof [16], [46]-[51].

In this work we will review recent progress on the construction of AdS\(_3\)/CFT\(_2\) and AdS\(_2\)/CFT\(_1\) pairs with four supersymmetries were both sides of the correspondence are reasonably well-understood. These provide controlled settings where the AdS/CFT correspondence can be

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\(^1\)By which we will mean one and two dimensional.
explicitly checked and where the black hole microstate counting programme can be carried out in detail.

Important progress in our understanding of the AdS$_3$/CFT$_2$ correspondence was provided by the recent constructions in [26]. These are solutions to massive Type IIA supergravity with small (i.e. with SU(2) R-symmetry) $\mathcal{N} = (0, 4)$ supersymmetries, realised as AdS$_3 \times S^2 \times M_4$ foliations over an interval, with $M_4$ either a CY$_2$ or a 4d Kähler manifold. These solutions are dual to interesting classes of 2d CFTs admitting a quiver description in the UV, that can be used to compute their degrees of freedom [52]-[54].

Besides providing for explicit AdS$_3$/CFT$_2$ pairs, the constructions in [26, 52, 53, 54] have been the seed of many other interesting developments. In a nutshell, new classes of AdS$_3$ solutions in M-theory with the same number of supersymmetries have been constructed in [34], which provide the holographic duals of the configurations of M-strings described in [55, 56]. From the latter, new classes of solutions in massless Type IIA have been generated [36], which allow for an explicit defect interpretation as surface quivers embedded in a 6d CFT. Perhaps more interestingly, new examples of explicit AdS$_2$/CFT$_1$ duals have been derived from these AdS$_3$/CFT$_2$ pairs in both Type II supergravities [38, 40, 43].

The construction of new AdS$_2$/CFT$_1$ pairs is of special relevance. AdS$_2$ geometries arise as near horizon geometries of extremal black holes, and are thus ubiquitous in their microscopical studies. However, it is well-known that the precise realisation of the AdS$_2$/CFT$_1$ correspondence presents important technical and conceptual problems [57]-[60], that mainly originate from the fact that the boundary of AdS$_2$ is non-connected [61]. As a result, this correspondence is much less understood than any of its higher dimensional counterparts.

A successful approach taken in [38, 40, 43] was to exploit its connection with the much better understood AdS$_3$/CFT$_2$ correspondence. At the geometrical level AdS$_3$ and AdS$_2$ spaces are related by T-duality. This allows one to construct explicit AdS$_2$/CFT$_1$ pairs in which the CFT$_1$ arises as a discrete light-cone compactification of the 2d CFT dual to the AdS$_3$ solution, thus providing an explicit realisation of the constructions in [62]-[65]. Moreover, if two-spheres are present in the internal space of an AdS$_3$ solution, such solutions are amenable to double analytical continuation techniques, which turn AdS$_3 \times S^2$ spaces into AdS$_2 \times S^3$ geometries. These latter class of solutions can then be dual to more general superconformal quantum mechanics (SCQM) than those arising upon discrete light-cone compactification.

The aim of this review article is to summarise the main features of these recent developments. The paper is organised as follows. We start in Section 2 by reviewing the AdS$_3$/CFT$_2$ pairs constructed in [26, 52, 53, 54], seed of the forthcoming constructions. In Section 3 we review their uplift to M-theory, following [34], and briefly address their interpretation as duals to the M-strings in [55, 56]. In Section 4 we turn our attention to the construction of new AdS$_2$/CFT$_1$ pairs in massive Type IIA, following [40]. We describe in some detail the associated dual SCQM, which allows one to interpret the solutions as backreacted D4-D0
baryon vertices in the 5d CFT living in D4'-D8 brane intersections. In Section 5 we discuss two more AdS/CFT pairs, in this case in Type IIB, following [38, 43]. A first class of solutions is constructed from the seed AdS3 solutions reviewed in Section 2 using T-duality. These solutions are dual to explicit 1d CFTs that arise as discrete light-cone compactifications of the 2d CFTs reviewed in Section 2. The second class of solutions is constructed by T-dualising the AdS2 class reviewed in Section 4. These solutions allow for an interesting interpretation as backreacted D5-D1 baryon vertices in the 4d $\mathcal{N} = 2$ QFT living in D3-D7 brane intersections, that we briefly discuss. Finally, in Section 6 we discuss open lines for further investigation.

2 AdS$_3$/CFT$_2$ in massive IIA

In [26] a complete classification of AdS$_3 \times S^2$ solutions to massive IIA supergravity consistent with non-trivial Romans mass, with small (0,4) supersymmetry and SU(2)-structure was obtained. The solutions are warped products of the form AdS$_3 \times S^2 \times M_4 \times I$, where $M_4$ is either a CY$_2$ or a 4d Kähler manifold. In this review we will restrict ourselves to the particular case when $M_4 = $CY$_2$ and the symmetries of the CY$_2$ are respected by the full solution. These solutions provide the “seed” from which all supergravity backgrounds summarised in this work will be derived. It was proposed in [52, 53, 54] that concrete solutions within this class are holographic duals to two dimensional CFTs preserving $\mathcal{N} = (0,4)$ SUSY that we also summarise below.

The Neveu-Schwarz (NS) sector of this class of solutions looks as follows,

\[
\begin{align}
\text{ds}^2 &= \frac{u}{\sqrt{h_4 h_8}} \left( ds_{\text{AdS}_3}^2 + \frac{h_8 h_4}{4 h_8 h_4 + u'^2} ds_{S^2}^2 \right) + \sqrt{\frac{h_4}{h_8}} ds_{\text{CY}_2}^2 + \sqrt{\frac{h_4 h_8}{u}} d\rho^2, \\
\text{e}^{-\Phi} &= \frac{h_3^3}{2 h_4 \sqrt{u}} \sqrt{4 h_8 h_4 + u'^2}, \quad H_3 = \frac{1}{2} d \left( -\rho + \frac{uu'}{4 h_4 h_8 + u'^2} \right) \wedge \text{vol}_{S^2}, 
\end{align}
\]  

(2.1)

where $\Phi$ is the dilaton, $H_3 = dB_2$ is the NS-NS 3-form and the metric is written in string frame. The warping functions $h_4, h_8$ and $u$ have support on $\rho$, which parameterises an interval. We denote $'=\partial_\rho$. The RR fluxes are,

\[
\begin{align}
F_0 &= h_8', \quad F_2 = -\frac{1}{2} \left( h_8 - \frac{h_8' u' u}{4 h_8 h_4 + u'^2} \right) \text{vol}_{S^2}, \\
F_4 &= -\left( d \left( \frac{uu'}{2 h_4} \right) + 2 h_8 d\rho \right) \wedge \text{vol}_{\text{AdS}_3} - h_4' \text{vol}_{\text{CY}_2},
\end{align}
\]  

(2.2)

with the higher fluxes related to them as $F_p = (-1)^{p/2} \star_{10} F_{10-p}$. The background in (2.1)-(2.2) is a SUSY solution of the massive IIA equations of motion if the functions $h_4, h_8, u$
satisfy,
\[ h_4''(\rho) = 0, \quad h_8''(\rho) = 0, \quad u''(\rho) = 0, \]  
away from localised sources, which makes them linear functions of \( \rho \). The first two equations are Bianchi identities, whereas \( u'' = 0 \) is a BPS equation.

The Page fluxes, defined as \( \hat{F} = e^{-B_2} \wedge F \), are given by,
\[ \hat{F}_0 = h_8', \quad \hat{F}_2 = -\frac{1}{2} \left( h_8 - h_8'(\rho - 2\pi k) \right) \text{vol}_{S^2}, \]
\[ \hat{F}_4 = -\left( \partial_\rho \left( \frac{uu'}{2h_4} \right) + 2h_8 \right) d\rho \wedge \text{vol}_{\text{AdS}_3} - h_4' \text{vol}_{\text{CY}_2}. \]  
(2.4)

Here we have included large gauge transformations of \( B_2 \) of parameter \( k \), \( B_2 \rightarrow B_2 + \pi k \text{vol}_{S^2} \), for \( k = 0, 1, \ldots, P \). The transformations are performed every time a \( \rho \)-interval \( \rho \in [2\pi k, 2\pi (k + 1)] \) is crossed. They ensure that \( B_2 \) lies in the fundamental region, i.e.
\[ \frac{1}{4\pi^2} \int_{S^2} B_2 | \in [0, 1]. \]  
(2.5)

The most general solution to (2.3) is that \( h_4 \) and \( h_8 \) are piecewise linear functions. This allows for source branes in the geometry. In turn, \( u \) needs to be continuous for preservation of supersymmetry. Here we will restrict to the simpler case in which \( u = \text{constant} \).

In piecewise linear solutions with the \( \rho \) direction bounded between 0 and \( 2\pi (P + 1) \), where \( h_4 \) and \( h_8 \) vanish, were proposed. These functions read,
\[ h_4(\rho) = \begin{cases} 0 & 0 \leq \rho \leq 2\pi, \\ \sum_{j=0}^{k-1} \beta_j + \frac{\beta_k}{2\pi} (\rho - 2\pi k) & 2\pi k \leq \rho \leq 2\pi (k + 1), \quad k = 1, \ldots, P - 1 \\ \alpha_P - \frac{\alpha_k}{2\pi} (\rho - 2\pi P) & 2\pi P \leq \rho \leq 2\pi (P + 1), \end{cases} \]  
(2.6)
\[ h_8(\rho) = \begin{cases} 0 & 0 \leq \rho \leq 2\pi, \\ \sum_{j=0}^{k-1} \mu_j + \frac{\mu_k}{2\pi} (\rho - 2\pi k) & 2\pi k \leq \rho \leq 2\pi (k + 1), \quad k = 1, \ldots, P - 1 \\ \mu_P - \frac{\mu_k}{2\pi} (\rho - 2\pi P) & 2\pi P \leq \rho \leq 2\pi (P + 1). \end{cases} \]  
(2.7)

The space begins at \( \rho = 0 \) in a smooth fashion. In turn, at \( \rho = 2\pi (P + 1) \) the behaviour of the metric and dilaton corresponds to a superposition of D2-branes wrapped on \( \text{AdS}_3 \) and smeared on the \( \text{CY}_2 \times S^2 \), and D6-branes wrapped on \( \text{AdS}_3 \times \text{CY}_2^3 \).

The quantities \((\alpha_k, \beta_k, \mu_k, \nu_k)\) are integration constants and, by imposing continuity across the different intervals, they must satisfy
\[ \alpha_k = \sum_{j=0}^{k-1} \beta_j, \quad \mu_k = \sum_{j=0}^{k-1} \nu_j. \]  
(2.8)

\(^2\)The \( u \neq \text{constant} \) case is studied in \[54, 37\].  
\(^3\)This is also compactible with a superposition of O2-O6 planes. The string theory interpretation of smeared orientifold fixed planes is however unclear.
Table 1: BPS brane intersection underlying the geometry given by (2.1)-(2.3). \((x^0, x^1)\) are the directions where the 2d CFT lives. The directions \((x^2, \ldots, x^5)\) span the CY\(_2\), on which the D6- and the D8-branes are wrapped. The coordinate \(x^6\) is the direction associated with \(\rho\). Finally \((x^7, x^8, x^9)\) are the transverse directions realising the SO(3) R-symmetry.

|     | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|---|---|---|---|---|---|---|---|---|---|
| D2  | x | x |   |   |   |   |   |   |   |   |
| D4  | x | x |   |   |   | x | x |   |   |   |
| D6  | x | x | x | x | x | x |   |   |   |   |
| D8  | x | x | x | x | x | x | x | x | x | x |
| NS5 | x | x | x | x | x | x | x | x | x | x |

For the solutions defined by (2.6)-(2.7), the quantised charges associated to the Page fluxes given by (2.4) in the different \(2\pi k \leq \rho \leq 2\pi(k + 1)\) intervals are

\[
Q^{(k)}_{D2} = \alpha_k = \sum_{j=0}^{k-1} \beta_j, \quad Q^{(k)}_{D6} = \mu_k = \sum_{j=0}^{k-1} \nu_j
\]
\[
Q^{(k)}_{D4} = \beta_k, \quad Q^{(k)}_{D8} = \nu_k, \quad Q^{(k)}_{NS5} = 1.
\] (2.9)

The field theory duals to this class of solutions were studied in \[52, 53, 54\]. We summarise them in the following subsection.

### 2.1 Two dimensional dual CFTs

The backgrounds defined by equations (2.1)-(2.3) are associated with the brane intersections depicted in Table 1. In these brane set-ups the D2- and D6-branes play the role of colour branes, while the D4- and D8-branes play the role of flavour branes. This is supported by the analysis of the Bianchi identities, which yields

\[
dF_0 = \sum_{k=1}^{P} \left( \frac{\nu_{k-1} - \nu_k}{2\pi} \right) \delta(\rho - 2\pi k) d\rho
\]
\[
d\hat{F}_4 = \sum_{k=1}^{P} \left( \frac{\beta_{k-1} - \beta_k}{2\pi} \right) \delta(\rho - 2\pi k) d\rho \land \text{vol}_{\text{CY}_2},
\] (2.10)
indicating that at the points \(\rho = 2\pi k\) there is the possibility of having localised D8- and D4-branes. Indeed, explicit D8- and D4-branes need to be present at \(\rho = 2\pi k\) if the slopes of \(h_8\) and \(h_4\) are different at both sides. Their numbers are given by

\[
\Delta Q^{(k)}_{D8} = \nu_{k-1} - \nu_k, \quad \Delta Q^{(k)}_{D4} = \beta_{k-1} - \beta_k.
\] (2.11)

\(^4\)See \[66, 67, 68\] for further developments.
The associated Hanany-Witten brane set-up is then the one depicted in Figure 1.

It was shown in [52, 53, 54] that these brane set-ups define 2d CFTs with $N = (0, 4)$ SUSY. These CFTs describe the strongly coupled IR fixed points of the two dimensional quantum field theories living in them. These field theories are encoded in the quivers depicted in Figure 2. Being the extension of the D2 and D6 branes finite in the $\rho$ direction, the field theory living in their intersection is effectively two dimensional at low energies. These quivers are rendered non-anomalous with adequate flavour groups at each node, coming from D4- and D8-branes. Their dynamics is described in terms of $(0, 4)$ vector multiplets and hypermultiplets, arising from the quantisation of the open strings that connect the different types of branes. This analysis will be presented in [69]. It differs slightly from the one originally considered in [52, 53, 54], which also led to anomaly free quivers with the same central charge to leading order, but was not based directly on open string quantisation. We next summarise the main ingredients of the quivers based on open string quantisation, following [69]:

- To each gauge node corresponds a $(0,4)$ vector multiplet, represented by a circle, plus a $(0,4)$ hypermultiplet in the adjoint representation of the gauge group, represented by a grey line starting and ending on the same gauge group. In terms of $(0,2)$ multiplets, the first consists on a vector multiplet and a Fermi multiplet in the adjoint, and the second on two chiral multiplets forming a $(0,4)$ hypermultiplet.

- Between each pair of horizontal nodes there are two $(0,2)$ Fermi multiplets, forming a $(0,4)$ Fermi multiplet, and two $(0,2)$ chiral multiplets, forming a $(0,4)$ twisted hypermultiplet, each in the bifundamental representation of the gauge groups. The $(0,4)$ Fermi multiplet and the $(0,4)$ twisted hypermultiplet combine into a $(4,4)$ twisted hypermultiplet. They are represented by horizontal black solid lines.

- Between each pair of vertical nodes there are two $(0,2)$ chiral multiplets forming a
(0,4) hypermultiplet, in the bifundamental representation of the gauge groups. They are represented by grey lines.

- Between each gauge node and any successive or preceding node there is a one (0,2) Fermi multiplet in the bifundamental representation. They are represented by dashed lines.

- Between each gauge node and its adjacent global symmetry node there is a one (0,2) Fermi multiplet in the fundamental representation of the gauge group. They are again represented by dashed lines.

- Between each gauge node and its opposed global symmetry node there are two (0,2) Fermi multiplets, forming a (0,4) Fermi multiplet, and two (0,2) chiral multiplets, forming a (0,4) twisted hypermultiplet, each in the fundamental representation of the gauge groups. The (0,4) Fermi multiplet and the (0,4) twisted hypermultiplet combine into a (4,4) twisted hypermultiplet. They are represented by curvy black solid lines.

The previous fields contribute to the anomaly of a generic SU($N_i$) gauge group as

- A (0,2) vector multiplet contributes a factor of $-N_i$.
- A (0,2) chiral multiplet in the adjoint representation contributes with a factor of $N_i$.
- A (0,2) chiral multiplet in the bifundamental representation contributes with a factor of $\frac{1}{2}$.
- A (0,2) Fermi multiplet in the adjoint representation contributes with a factor of $-N_i$. 
A (0,2) Fermi multiplet in the fundamental of bifundamental representation contributes with a factor of $-\frac{1}{2}$.

Putting these together, we have that for generic SU($\alpha_k$) and SU($\mu_k$) colour groups the gauge anomaly cancellation condition is, respectively,

$$F_k = \nu_{k-1} - \nu_k, \quad \tilde{F}_k = \beta_{k-1} - \beta_k,$$

(2.12)

for $F_k$, $\tilde{F}_k$ the respective flavour groups. This is precisely the number of D8 and D4 flavour (source) branes in each interval!, as shown by equations (2.11).

In turn, as shown in [70], the right-handed central charge of the IR SCFT is calculated by associating it with the U(1)$_R$ current two-point function,

$$c_{cft} = 3 \text{Tr}[\gamma_3 Q^2_R].$$

(2.13)

Here $Q_R$ is the charge with respect to the U(1)$_R \subset$ SU(2)$_R$, and the trace is over all Weyl fermions in the theory. The two left-handed fermions inside the (0,4) vector multiplet have R-charge equal to 1. For hypermultiplets, we have that for both right handed fermions inside a (0,4) hypermultiplet the R-charge is -1, while those in a (0,4) twisted hypermultiplet have zero R-charge. Finally, the left-handed fermion inside the (0,2) Fermi multiplet has also vanishing R-charge. Putting this together we find that

$$c_{cft} = 6(n_{hyp} - n_{vec}),$$

(2.14)

where $n_{hyp}$ is the number of (untwisted) (0,4) hypermultiplets and $n_{vec}$ is the number of (0,4) vector multiplets.

In [52, 53, 54] a number of dual holographic pairs were presented that provided stringent support for the validity of the proposed duality. In each example the field theory central charge given by expression (2.14) was shown to coincide (for long quivers with large ranks, when the background is a trustable dual description of the CFT) with the holographic central charge. This was computed from the Brown-Henneaux formula, giving

$$c_{hol} = \frac{3\pi}{2G_N} \text{Vol}_{CY_2} \int_0^{2\pi(P+1)} h_4 h_8 d\rho = \frac{3}{\pi} \int_0^{2\pi(P+1)} h_4 h_8 d\rho.$$  

(2.15)

Here we used that $G_N = 8\pi^{6}$, with $g_s = \alpha' = 1$, and that Vol$_{CY_2} = 16\pi^4$. For the functions $h_4$, $h_8$ displayed above this gives

$$c_{hol} = \sum_{k=1}^{P} \left( 6\alpha_k \mu_k + 3(\alpha_k \nu_k + \beta_k \mu_k) + 2\beta_k \nu_k \right),$$

(2.16)

which can be shown to agree in the holographic limit with the expression coming from (2.14).
Table 2: $\frac{1}{8}$-BPS brane intersection underlying the $\text{AdS}_3 \times S^3/\mathbb{Z}_k$ solutions in M-theory. The 2d CFT lives in the $(x^0, x^1)$ directions, $(x^2, \ldots, x^5)$ span the CY$_2$, $x^6$ is the “field theory” direction and $(x^7, x^8, x^9)$ are the transverse directions on which the SO(3)$_R$ symmetry is realised. Finally, $x^{10}$ is the extra eleventh direction, which spans the $S^1/\mathbb{Z}_k \subset S^3/\mathbb{Z}_k$ and plays the role of Taub-NUT direction of the KK-monopoles.

3 AdS$_3$ solutions in M-theory

In this section we consider the uplift to eleven dimensions of the solutions discussed in the previous section, following [34]. In order to carry out this uplift we need to restrict to vanishing Romans’ mass, $F_0 = 0$, which imposes $h'_8 = 0$, and thus both the absence of D8-branes and the presence of a constant number of D6-branes between all pairs of NS5-branes. In the uplift to eleven dimensions this number becomes a modding parameter of the geometry, associated with KK-monopole charge.

The M-theory solutions are of the form $\text{AdS}_3 \times S^3/\mathbb{Z}_k \times \text{CY}_2$ foliated over an interval. They read as follows,

$$
\begin{align*}
\text{ds}_{11}^2 &= \Delta \left( \frac{u}{\sqrt{h_4 h_8}} \text{ds}^2_{\text{AdS}_3} + \sqrt{\frac{h_4}{h_8}} \text{ds}^2_{\text{CY}_2} + \frac{\sqrt{h_4 h_8}}{u} \text{d}\rho^2 \right) + \frac{h'_8}{\Delta^2} \text{ds}^2_{S^3/\mathbb{Z}_k}, \\
G_4 &= -d \left( \frac{uu'}{2h_4} + 2\rho h_8 \right) \wedge \text{vol}_{\text{AdS}_3} + 2h_8 d \left( -\rho + \frac{uu'}{4h_4 h_8 + u^2} \right) \wedge \text{vol}_{S^3/\mathbb{Z}_k} \\
\Delta &= \frac{h_8^{1/2} (4h_4 h_8 + u'^2)^{1/3}}{2^{2/3} h_4^{1/6} u^{1/3}},
\end{align*}
$$

(3.1)

where $k = h_8$. The quotiented 3-sphere is written as a Hopf fibration over an $S^2$,

$$
\text{ds}_{S^3/\mathbb{Z}_k}^2 = \frac{1}{4} \left[ \left( \frac{dz}{k} + \omega \right)^2 + \text{ds}_{S^2}^2 \right]
$$

with $d\omega = \text{vol}_{S^2}$.

(3.2)

In these solutions the symmetries SL(2, $\mathbb{R}$)$\times$SL(2, $\mathbb{R}$) and SU(2) are geometrically realised by the $\text{AdS}_3$ and the quotiented 3-sphere, respectively.

In the new class of solutions given by (3.1), the number of Type IIA D6-branes becomes the orbifold parameter in $S^3/\mathbb{Z}_k$, $k = h_8$, and thus corresponds to KK-monopole charge.
Figure 3: Generic quiver field theory whose IR is dual to the AdS\_3 × S\_3/\mathbb{Z}_k × CY\_2×I M-theory solutions defined by the functions in (2.6)-(2.7).

D2- and D4-branes of the Type IIA solution become M2- and M5-branes, respectively. Their presence is captured by integrating the Page flux $\hat{G}_7 = G_7 - G_4 \wedge C_3$ and a non-trivial flux of $G_4$ through the CY\_2. In turn, the NS5 branes become M5'-branes. The uplifted brane set-up is the one depicted in Table 2. Recently, it was shown that the solutions emerge in the near horizon limit of this intersecting brane system [36]. The KK-monopoles (wrapped on the CY\_2) and the M2 branes are stretched between parallel M5'-branes, with extra M5-branes providing for flavour groups.

In [34], it was argued that this brane intersection describes the M\_A-strings introduced in [55, 56], supplemented with extra M5-branes. The corresponding dual quivers are the ones depicted in Figure 3 with upper row nodes associated to M2-branes and lower row nodes to KK-monopoles. The M5-branes provide for extra flavour groups that render the quivers non-anomalous (and the supergravity equations of motion satisfied). Thus, the new solutions in M-theory provide for explicit AdS\_3 geometries where M\_A-strings can be studied holographically. In particular, the matching between the field theory and holographic computations of the central charge follows directly upon uplift from ten dimensions. The holographic central charge given by equation (2.15) becomes, in the massless case

$$c_{\text{hol}} = \frac{3}{\pi} h_8 \int_0^{2\pi(P+1)} d\rho h_4 = 6h_8 \sum_{k=1}^P \alpha_k = 6h_8 \sum_{k=1}^P Q_{M2}^{(k)} = 6kQ_{M2} = 6Q_{M_A}, \quad (3.3)$$

where $Q_{M_A}$ stands for the total number of M\_A-strings in the configuration, taking into account the orbifolding by $\mathbb{Z}_k$. This result stresses out that the degrees of freedom of the strongly coupled conformal field theory originate from the M\_A-strings.

Further, in [34], a second class of AdS\_3 solutions to M-theory of the form AdS\_3/\mathbb{Z}_k × S\_3× CY\_2×I was obtained through a double analytic continuation from the previous solutions. In
Table 3: $1/8$-BPS brane intersection underlying the AdS$_3$/$\mathbb{Z}_k \times S^3$ solutions in M-theory. $x^1$ is the direction of propagation of the M0-branes. $(x^2, \ldots, x^5)$ span the CY$_2$, $x^6$ is the direction along the $\rho$-interval and $(x^7, x^8, x^9, x^{10})$ are the transverse directions on which the SO(4) R-symmetry is realised.

|      | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|---|---|---|---|---|---|---|---|---|---|----|
| M0   | x | x |   |   |   |   |   |   |   |   |    |
| M2   | x | x |   |   |   |   |   |   | x |   |    |
| M5   | x | x | x |   | x | x | x | x |   |   |    |
| M5'  | x | x | x | x | x | x | x |   |   |   |    |

Notice that the KK-monopoles turn into M0-branes, or waves, with the Taub-NUT direction of the KK-monopoles becoming the direction of propagation. These solutions are associated to the M0-M2-M5-M5' brane intersections depicted in Table 3. They preserve the same number of supersymmetries as the original AdS$_3 \times S^3$/$\mathbb{Z}_k \times$ CY$_2 \times I$ solutions.
4 AdS$_2$/CFT$_1$ in massive IIA

A new class of AdS$_2 \times S^3 \times \text{CY}_2 \times \text{I}$ solutions to massless Type IIA supergravity can be obtained from (3.6) upon reduction along the Hopf fibre of the AdS$_3/\mathbb{Z}_k$ subspace given by (3.7). These backgrounds are associated to D0-F1-D4-D4$'$ brane intersections that preserve $N = 4$ supersymmetries in one dimension. These solutions can also be obtained through a double analytical continuation from the solutions reviewed in Section 2. In fact, this allows one to extend them to the massive case. The corresponding brane set-up is depicted in Table 4. In this manner we find an AdS$_2 \times S^3 \times \text{CY}_2 \times \text{I}$ class of solutions to massive Type IIA supergravity, with NS-NS sector,

\[
\begin{align*}
\text{ds}^2 &= \frac{u}{\sqrt{h_8 h_4}} \left( \frac{h_4 h_8}{4h_4 h_8 - u^2} \text{ds}^2_{\text{AdS}_2} + \text{ds}^2_{S^3} + \sqrt{h_4 h_8 \over u} \text{d}\rho^2 \right), \\
\text{e}^{-2\Phi} &= {h_8^{3/2} \over 4h_4^{1/2} u} (4h_4 h_8 - u^2) \text{vol}_{\text{AdS}_2}, \\
B_2 &= -{1 \over 2} \left( \rho + {uu' \over 4h_4 h_8 - u^2} \right) \text{vol}_{\text{AdS}_2}.
\end{align*}
\]

(4.1)

The background is further supported by the RR fluxes,

\[
\begin{align*}
F_0 &= h_8', \\
F_2 &= -{1 \over 2} \left( h_8 + {h_8'u' \over 4h_8 h_4 - u^2} \right) \text{vol}_{\text{AdS}_2}, \\
F_4 &= \left( -d \left( {u'u \over 2h_4} \right) + 2h_8 d\rho \right) \wedge \text{vol}_{S^3} - h_4' \text{vol}_{\text{CY}_2}.
\end{align*}
\]

(4.2)

The warping functions $h_8$, $h_4$ and $u$ have support on $\rho$, as in the seed solutions. Note that in this case $(4h_8 h_4 - u^2) > 0$, in order to guarantee a real dilaton and a metric with the correct signature. Supersymmetry and the Bianchi identities of the fluxes (away from localised sources) are imposed by the equations (2.3), which make again $h_8$, $h_4$ and $u$ linear functions of $\rho$.

We quote the Page fluxes, $\hat{F} = e^{-B_2} \wedge F$, as follows,

\[
\begin{align*}
\hat{F}_0 &= h_8', \\
\hat{F}_2 &= -{1 \over 2} \left( h_8 - h_8' (\rho - 2\pi k) \right) \text{vol}_{\text{AdS}_2}, \\
\hat{F}_4 &= \left( 2h_8 d\rho - d \left( {u'u \over 2h_4} \right) \right) \wedge \text{vol}_{S^3} - h_4' \text{vol}_{\text{CY}_2},
\end{align*}
\]

(4.3)

where we included large gauge transformations of $B_2$ of parameter $k$, $B_2 \rightarrow B_2 + \pi k \text{vol}_{\text{AdS}_2}$ (see [40]).

An infinite family of backgrounds with $u = \text{constant}$ and $h_4$ and $h_8$ piecewise continuous as in (2.6)-(2.7) were analysed in [40], together with their dual SCQM description\footnote{A concrete example with $u' \neq 0$ was analysed in [45].}. We summarise this description in the next subsection.
Table 4: Brane set-up associated to the solutions (4.1)-(4.2). $x^0$ is the time direction of the ten dimensional spacetime, $x^1, \ldots, x^4$ are the coordinates spanned by the CY$_2$, $x^5$ is the direction where the F1-strings are stretched, and $x^6, x^7, x^8, x^9$ are the coordinates where the SO(4) R-symmetry is realised.

|   | $x^0$ | $x^1$ | $x^2$ | $x^3$ | $x^4$ | $x^5$ | $x^6$ | $x^7$ | $x^8$ | $x^9$ |
|---|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| D0 | x     |       |       |       |       |       |       |       |       |       |
| D4 | x     | x     | x     | x     |       |       |       |       |       |       |
| D4' | x     |       |       |       | x     | x     | x     | x     | x     | x     |
| D8 | x     | x     | x     | x     | x     |       |       |       |       |       |
| F1 | x     |       |       |       |       |       | x     |       |       |       |

Table 4: Brane set-up associated to the solutions (4.1)-(4.2). $x^0$ is the time direction of the ten dimensional spacetime, $x^1, \ldots, x^4$ are the coordinates spanned by the CY$_2$, $x^5$ is the direction where the F1-strings are stretched, and $x^6, x^7, x^8, x^9$ are the coordinates where the SO(4) R-symmetry is realised.

**4.1 Dual superconformal quantum mechanics**

The superconformal quantum mechanics dual to the previous class of solutions was studied in [40]. The proposal therein is that a 1d $\mathcal{N} = 4$ quantum mechanics lives on the D0-D4-D4'-D8-F1 brane set-up depicted in Table 4 that describes the interactions between D0 and D4 brane instantons and F1 Wilson lines in the 5d gauge theory living in the intersection of D4' and D8 branes. This is a generalisation of the ADHM quantum mechanics described in [71], and of the quiver proposals discussed in [72, 73].

In order to describe the quantum mechanics, the D0-D4-D4'-D8-F1 brane system was split into two subsystems, D4-D4'-F1 and D0-D8-F1, that were first studied independently. The first subsystem was interpreted as describing BPS F1 Wilson lines introduced in the 5d theory living on the D4'-branes by D4-branes [74]. Similarly, the D0-D8-F1 subsystem was interpreted as describing F1 Wilson lines introduced in the worldvolume of the D8-branes by D0-branes [75]. Indeed, the branes in the D4-D4'-F1 and D0-D8-F1 subsystems are displayed exactly as in the D3-D5-F1 brane configuration that describes Wilson lines in antisymmetric representations in 4d $\mathcal{N} = 4$ SYM, studied in [76, 77].

For the solutions defined by (2.6)-(2.7), the quantised charges associated to the Page fluxes given by (4.3) in the different $2\pi k \leq \rho \leq 2\pi(k + 1)$ intervals are

$$Q_{D4}^{e(k)} = \alpha_k = \sum_{j=0}^{k-1} \beta_j, \quad Q_{D0}^{e(k)} = \mu_k = \sum_{j=0}^{k-1} \nu_j,$$

$$Q_{D4'}^{m(k)} = \beta_k, \quad Q_{D8}^{m(k)} = \nu_k, \quad Q_{F1}^{e(k)} = 1,$$  (4.4)

where we use electric and magnetic charges as explained in [40]. The Hanany-Witten brane set-up associated to these brane charges is the one depicted in Figure 4. In order to understand the quantum mechanics associated to this brane configuration it is useful to go to Type IIB and S-dualise. Then after performing Hanany-Witten moves one can go back to Type IIA, where one can interpret the resulting brane set-up (depicted in Figure 5) as
describing $U(\alpha_k)$ and $U(\mu_k)$ Wilson lines in the completely antisymmetric representations $(\beta_0, \beta_1, \ldots, \beta_{k-1})$ of $U(\alpha_k)$ and $(\nu_0, \nu_1, \ldots, \nu_{k-1})$ of $U(\mu_k)$, respectively. Given that the Wilson lines are in the completely antisymmetric representations the D4-D4’-F1 and D0-D8-F1 subsystems describe baryon vertices [78].

This is consistent with an interpretation of the AdS$_2$ solutions as describing backreacted baryon vertices within the 5d $\mathcal{N} = 1$ QFT living in D4’-D8 branes. In this interpretation the SCQM arises in the very low energy limit of a system of D4’-D8 branes, dual to a 5d $\mathcal{N} = 1$ gauge theory, in which one-dimensional defects are introduced. The one dimensional defects consist on D4-brane baryon vertices, connected to the D4’ with F1-strings, and D0-brane baryon vertices, connected to the D8 with F1-strings. In the IR the gauge symmetry on both the D4’ and D8 branes becomes global, turning them from colour to flavour branes. In turn, the D4 and D0 defect branes become the new colour branes of the backreacted geometry. This is in agreement with the defect interpretation found for this class of solutions in [36], where the AdS$_2$ geometries were shown to asymptote locally to the AdS$_6$ background of Brandhuber-Oz [79].
The previous SCQMs can be given a quiver-like description, that can be used to compute their central charge. From the brane set-up depicted in Figure 4 one can construct the quiver shown in Figure 6, where the gauge groups are associated to the colour D0- and D4-branes and the flavour groups to the D4'- and D8-branes. The quantised charges are the ones computed in equation (4.4). The dynamics of the quiver is described in terms of (4,4) vector multiplets, (4,4) hypermultiplets in the adjoint representations and (4,4) hypermultiplets in the bifundamental representations. The connection between colour and flavour branes is through twisted (4,4) bifundamental hypermultiplets (bent black lines) and (0,2) bifundamental Fermi multiplets (dashed lines). This follows directly from the analysis of Appendix B in [40].

As before, a check of the validity of the proposed quivers is given by the matching of the field theory and holographic central charges. In the case at hand, however, we are dealing with a one dimensional field theory, for which the definition of central charge is subtle. One way to think about the central charge is as counting the ground states of the conformal quantum mechanics. In [40] the same expression used in Section 2.1 for the computation of the central charge of the 2d dual CFT was proposed to be valid for the 1d quiver mechanics. For these quivers $n_{hyp}$ and $n_{vec}$ are, respectively, the numbers of $\mathcal{N} = 4$ (untwisted) hypermultiplets and vector multiplets. Using this expression perfect agreement was found (in the large number of nodes with large ranks limit) between the quantum mechanics central charge and the holographic central charge, which in this case is obtained through the integration,

\[
c_{\text{hol,1d}} = \frac{3V_{\text{int}}}{4\pi G_N} = \frac{3}{4\pi} \frac{\text{Vol}_{\text{CY}_2}}{(2\pi)^4} \int_0^{2\pi(P+1)} (4h_4h_8 - u'^2) \, d\rho. \tag{4.5}
\]

This is a striking result, since the superconformal quantum mechanics dual to our class of
solutions does not seem to have a direct relation to 2d CFTs. Comparing with results in the literature for the dimension of the Higgs branch of $\mathcal{N} = 4$ quantum mechanics with gauge groups $\prod_v U(N_v)$ connected by bifundamentals [80], one sees that the expression $c_{cft} = 6(n_{hyp} - n_{vec})$ may be interpreted as an extension of the formulas therein to more general $\mathcal{N} = 4$ quivers including flavours. This is an interesting relation that deserves further investigation.

5 AdS$_2$ solutions in Type IIB

In this section we review two classes of AdS$_2$ solutions to Type IIB supergravity with 4 Poincaré supersymmetries. This is based on the works [38, 43]. These solutions are obtained from the backgrounds reviewed in Sections 2 and 4 respectively, upon T-duality. Moreover, both solutions are related to each other through a double analytical continuation.

The two classes of solutions consist on AdS$_2 \times S^2 \times CY_2 \times S^1$ geometries foliated over an interval. Despite this they have substantially different dual field theory descriptions, that are inherited from their respective T-dual origins. Although we will not review these results in this paper, it was shown in [43] that both types of solutions fit in the general class of AdS$_2 \times S^2 \times CY_2 \times \Sigma_2$ solutions of Type IIB supergravity found in [8, 9]. The solutions fit locally in their classification in the absence of D3- and D7-branes sources. In fact, under certain circumstances they extend this class of solutions. The connection between these qualitatively different backgrounds requires of a subtle zoom-in procedure that was explained in [43].

5.1 Type A

Consider the class given by (2.1)-(2.2), where the AdS$_3$ subspace is written as a Hopf fibration over AdS$_2$, as shown by expression (3.7) for $k = 1$ and $z = \psi$. By applying T-duality on the fibre direction $\psi$ new AdS$_2$ solutions preserving $\mathcal{N} = 4$ supersymmetry are obtained. These backgrounds have the structure of an AdS$_2 \times S^2 \times CY_2 \times S^1$ geometry warped over an interval. The NS-NS sector reads,

$$d\tilde{s}^2 = \frac{u}{\sqrt{h_4 h_8}} \left( \frac{1}{4} d\tilde{s}^2_{AdS_2} + \frac{h_4 h_8}{4 h_4 h_8 + u^2} d\tilde{s}_{S^2}^2 \right) + \sqrt{\frac{h_4}{h_8}} d\tilde{s}_{CY_2}^2 + \frac{\sqrt{h_4 h_8}}{u} (d\rho^2 + d\psi^2),$$

$$e^{-2\Phi} = \frac{h_8}{4 h_4} (4 h_4 h_8 + u^2),$$

$$H_3 = \frac{1}{2} d \left( - \rho + \frac{uu'}{4 h_4 h_8 + u^2} \right) \wedge \text{vol}_{S^2} + \frac{1}{2} \text{vol}_{AdS_2} \wedge d\psi,$$

where the $h_4$, $h_8$ and $u$ functions are inherited from the backgrounds (2.1)-(2.2), and thus have support on $\rho$. 16
Table 5: Brane set-up underlying the geometry in (5.1)-(5.2). $x^0$ is the time direction, $(x^1, \ldots, x^4)$ span the CY$_2$, $x^5$ is the direction associated with the $\rho$ coordinate, $(x^6, x^7, x^8)$ are the transverse directions realising the SO(3) R-symmetry and $x^9$ is the $\psi$ direction.

The 10 dimensional RR fluxes are given by,

\begin{align}
F_1 &= h'_8 d\psi, \\
F_3 &= -\frac{1}{2} \left( h_8 - \frac{h'_8 u' u}{4h_8 h_4 + u'^2} \right) \text{vol}_S^2 \wedge d\psi + \frac{1}{4} \left( d \left( \frac{u' u}{2h_4} \right) + 2h_8 d\rho \right) \wedge \text{vol}_{\text{AdS}_2}, \\
F_5 &= -\left( 1 + \ast \right) h'_4 \text{vol}_{\text{CY}_2} \wedge d\psi \\
&= -h'_4 \text{vol}_{\text{CY}_2} \wedge d\psi + \frac{h'_4 h_8 u'^2}{4h_4 (4h_4 h_8 + u'^2)} \text{vol}_{\text{AdS}_2} \wedge \text{vol}_S^2 \wedge d\rho. 
\end{align}  \tag{5.2}

The Type IIB equations of motion are satisfied imposing the BPS equations and Bianchi identities given by (2.3). In turn, the Page forms $\hat{F} = e^{-B_2} \wedge F$ are given by,

\begin{align}
\hat{F}_1 &= h'_8 d\psi, \\
\hat{F}_3 &= \frac{1}{2} \left( h'_4 (\rho - 2\pi k) - h_8 \right) \text{vol}_S^2 \wedge d\psi + \frac{1}{4} \left( u' (h_4 u' - uh'_4) \right) 2h_8 \text{vol}_{\text{AdS}_2} \wedge d\rho, \\
\hat{F}_5 &= \frac{1}{16} \left( (u - (\rho - 2\pi k) u') \frac{u'h'_4 - h_4 u'}{h_4^2} + 4(\rho - 2\pi k) h_8 \right) \text{vol}_{\text{AdS}_2} \wedge \text{vol}_S^2 \wedge d\rho \\
&\quad - h'_4 \text{vol}_{\text{CY}_2} \wedge d\psi. 
\end{align}  \tag{5.3}

The analysis of these fluxes suggests that the brane content that underlies this class of solutions is the one depicted in Table 5. Here the D1- and D3-branes play the role of colour branes and the D7- and D3-branes play the role of flavour branes. As for the AdS$_3$ solutions reviewed in Section 2 an infinite family of AdS$_2$ backgrounds can be defined by the piecewise linear functions $h_4$ and $h_8$ given by the equations (2.6) and (2.7). We turn next to the description of the superconformal quantum mechanics dual to this class of solutions.
5.1.1 Dual superconformal quantum mechanics

The $\mathcal{N} = 4$ superconformal quantum mechanics dual to the previous class of solutions was studied in [38]. At the geometrical level these solutions are related to the AdS$_3$ solutions reviewed in Section 2 through T-duality. At the level of the dual CFTs the superconformal quantum mechanics dual to the AdS$_2$ solutions should then arise from the 2d CFTs dual to the AdS$_3$ backgrounds upon dimensional reduction.

More concretely, in the coordinates used to obtain the AdS$_2$ geometry, defined in equation (3.7), the boundaries of AdS$_3$ are two null cylinders [63]. For this reason the 2d CFT that lives at these boundaries is effectively discrete light-cone quantised (DLCQ), because just one of the SL(2, $\mathbb{R}$)$\times$SL(2, $\mathbb{R}$) sectors of global AdS$_3$ survives the compactification. T-dualisation in these coordinates is then equivalent to starting with a given $\mathcal{N} = (0, 4)$ 2d CFT, as those described in Section 2.1 and DLCQ it, keeping the $\mathcal{N} = 4$ SUSY right sector. This provides an explicit realisation of the constructions in [62, 63, 64, 65]. Field theoretically, we start with the Lagrangian describing the 2d CFT dual to AdS$_3$ and dimensionally reduce it to a matrix model where only the time dependence and the zero modes in the T-dual direction are kept.

The concrete proposal in [38] is that the dynamics of the UV quantum mechanics is described by the dimensional reduction along the space-direction of the 2d QFTs discussed in Section 2.1. The quiver field theories are then the same ones depicted in Figure 2, but now associated to D1 and D5 colour branes and D3 and D7 flavour branes. In turn, the matter fields are $\mathcal{N} = 4$ multiplets, realised as dimensionally reduced 2d $(0, 4)$ multiplets. Note that these quivers inherit the anomaly cancellation condition of the 2d quivers, even if there is no anomaly cancellation condition in 1d.

As in previous AdS/CFT pairs one can check the agreement between the field theory and holographic central charges to test the proposed duality. In this case the usage of expression (2.14) for computing the quantum mechanics central charge is fully justified, since it arises from a 2d CFT upon compactification. As expected, perfect agreement is found with the holographic central charge, which is computed from the same expression (2.15) given its invariance under T-duality.

5.2 Type B

In this subsection we review the AdS$_2$ solutions to Type IIB supergravity that arise from the backgrounds defined in (4.1)-(4.2) upon T-duality along the Hopf-fibre of the three sphere.
The resulting class of solutions have NS-NS sector,

\[
\begin{align*}
\text{ds}^2 &= \frac{u\sqrt{h_4h_8}}{4h_4h_8 - u'^2} \text{d}s^2_{\text{AdS}_2} + \frac{u}{4\sqrt{h_4h_8}} \text{d}s^2_{\text{S}^2} + \sqrt{\frac{h_4}{h_8}} \text{d}s^2_{\text{CY}_2} + \frac{\sqrt{h_4h_8}}{u} (\text{d}\psi^2 + \text{d}\rho^2), \\
e^{-2\Phi} &= \frac{h_8}{4h_4} (4h_4h_8 - u'^2), \\
H_3 &= -\frac{1}{2} \text{d}\left(\rho + \frac{uu'}{4h_4h_8 - u'^2}\right) \wedge \text{vol}_{\text{AdS}_2} + \frac{1}{2} \text{vol}_{\text{S}^2} \wedge \text{d}\psi,
\end{align*}
\] (5.4)

and RR fluxes,

\[
\begin{align*}
F_1 &= h_8' \text{d}\psi, \\
F_3 &= -\frac{1}{2} \left(\frac{h_8' + h_4'u}{4h_4h_8 - u'^2}\right) \text{vol}_{\text{AdS}_2} \wedge \text{d}\psi + \frac{1}{4} \left(-\text{d}\left(\frac{u'u}{2h_4}\right) + 2h_8 \text{d}\rho\right) \wedge \text{vol}_{\text{S}^2}, \\
F_5 &= -(1 + \ast_{10})h_4' \text{vol}_{\text{CY}_2} \wedge \text{d}\psi, \\
&= -h_4' \text{vol}_{\text{CY}_2} \wedge \text{d}\psi - \frac{h_8h_4'u^2}{4h_4(4h_4h_8 - u'^2)} \text{vol}_{\text{AdS}_2} \wedge \text{vol}_{\text{S}^2} \wedge \text{d}\rho.
\end{align*}
\] (5.5)

Here \(\psi \in [0, 2\pi]\) is the T-dual coordinate. Note that \(4h_4h_8 - u'^2 \geq 0\) must be imposed to have well-defined supergravity fields. Supersymmetry holds whenever \(u'' = 0\). In the same way, the Bianchi identities of the fluxes impose \(h''_8 = 0\) and \(h''_4 = 0\), away from localised sources. The \(\rho\) coordinate describes an interval that we will take to be bounded between 0 and \(2\pi(P + 1)\), as in the previous sections.

The Page fluxes read,

\[
\begin{align*}
\hat{F}_1 &= h_8' \text{d}\psi, \\
\hat{F}_3 &= \frac{1}{2} \left(\frac{h_8'(\rho - 2\pi k) - h_8}{2h_4} - h_8\right) \text{vol}_{\text{AdS}_2} \wedge \text{d}\psi + \frac{1}{4} \left(2h_8 + \frac{u'(uh_4' - h_4'u')}{2h_4^2}\right) \text{vol}_{\text{S}^2} \wedge \text{d}\rho, \\
\hat{F}_5 &= \frac{1}{4} \left(h_8(\rho - 2\pi k) - \frac{(u - (\rho - 2\pi k)u') (uh_4' - h_4'u')}{4h_4^2}\right) \text{vol}_{\text{AdS}_2} \wedge \text{vol}_{\text{S}^2} \wedge \text{d}\rho \\
&- h_4' \text{vol}_{\text{CY}_2} \wedge \text{d}\psi.
\end{align*}
\] (5.6)

The analysis of these fluxes yields the brane set-up summarised in Table 6 as underlying this class of solutions. Here the D1- and D5-branes play the role of colour branes and the D3- and D7-branes of flavour branes. Both the brane set-up and the warped AdS_2 × S^2 × CY_2 × S^1 structure of this class of solutions are shared with those of the solutions reviewed in the previous section. The precise relation between the two types of backgrounds is through the double analytic continuation

\[
\begin{align*}
\text{ds}^2_{\text{AdS}_2} &\rightarrow -\text{ds}^2_{\text{S}^2}, \\
\text{ds}^2_{\text{S}^2} &\rightarrow -\text{ds}^2_{\text{AdS}_2}, \\
e^\Phi &\rightarrow i e^\Phi, \\
F_i &\rightarrow -i F_i, \\
u &\rightarrow -iu, \\
h_4 &\rightarrow ih_4, \\
h_8 &\rightarrow ih_8, \\
\rho &\rightarrow i\rho, \\
\psi &\rightarrow -i\psi.
\end{align*}
\] (5.7)
Table 6: Brane set-up underlying the geometry in (5.4)-(5.5). \( x^0 \) is the time direction, \( x^1, \ldots, x^4 \) are the coordinates spanned by the CY\(_2\), \( x^5 \) is the direction associated with the \( \rho \) coordinate, \((x^6, x^7, x^8)\) are the transverse coordinates realising the SO(3) R-symmetry, and \( x^9 \) is the \( \psi \) direction.

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D1 | x |   |   |   |   |   | x |   |   |   |
| D3 | x |   |   | x | x | x | x |   |   |   |
| D5 | x | x | x | x | x |   | x |   |   |   |
| D7 | x | x | x | x | x | x | x |   |   |   |
| NS5 | x | x | x | x | x |   |   |   |   |   |
| F1 | x |   |   |   |   | x |   |   |   |   |

5.2.1 Dual superconformal quantum mechanics

The superconformal quantum mechanics dual to this new class of solutions was discussed in [43]. Given that they are obtained via T-duality from the class of AdS\(_2\) solutions reviewed in Section 4.1, they should be dual to the same superconformal quantum mechanics. In this case the Wilson lines arise from the massive fermionic strings that stretch between D1-branes (and D5-branes) in the \( k \)-th interval and D7-branes (and D3-branes) in all other intervals. In turn, the Wilson lines are in the \((\nu_0, \ldots, \nu_{k-1}), (\beta_0, \ldots, \beta_{k-1})\) completely antisymmetric representation of the U(\( \mu_k \)) and U(\( \alpha_k \)) gauge groups, respectively. As we indicated in Section 4.1 given that the Wilson lines are in the completely antisymmetric representation, the D1-D7-F1 and D5-D3-F1 subsystems describe baryon vertices [78].

This is consistent with an interpretation of our AdS\(_2\) solutions as describing backreacted baryon vertices within the 4d \( \mathcal{N} = 2 \) QFT living in D3-D7 branes. In this interpretation the SCQM arises in the very low energy limit of a system of D3-D7 branes, dual to a 4d \( \mathcal{N} = 2 \) QFT, in which one-dimensional defects are introduced. The one dimensional defects consist on D5-brane baryon vertices, connected to the D3 with F1-strings, and D1-brane baryon vertices, connected to the D7 with F1-strings. In the IR the gauge symmetry on both the D7 and D3 branes becomes global, turning them from colour to flavour branes. In turn, the D5 and D1 defect branes become the new colour branes of the backreacted geometry. It would be very interesting to realise explicitly this defect interpretation geometrically.

6 Discussion

In this review article we have summarised the salient features of the recent developments in [26, 52, 53, 54, 34, 38, 40, 43], in relation to the construction of AdS\(_3\)/CFT\(_2\) and AdS\(_2\)/CFT\(_1\) pairs with 4 supercharges. For clarity, we have summarised the connections between these
Figure 7: Connections between the different classes of solutions reviewed in this paper.

new classes of solutions in Figure 7.

The construction of these new dual pairs extends existing classifications of AdS$_3$ and AdS$_2$ solutions to the case with 4 supersymmetries. Moreover, the analysis in these references complements the construction of the backgrounds with a comprehensive study of the 2d and 1d CFTs dual to them. The proposed CFTs are described in the UV by means of explicit quivers from which observables such as the central charge can be computed, and checked against holographic calculations. The holographic central charge is interpreted as the entanglement entropy of black strings or black holes with AdS$_3$ or AdS$_2$ near horizon geometries. This can then be cross-checked against the field theory computation, within a well controlled setting where the microstate counting programme can be put to work. This line of research remains to be further exploited. See [81, 82, 83]. In particular it would be interesting to apply exact calculational techniques (see [84]) to the new classes of solutions, since this would provide for a deeper understanding of the IR regime of the different theories.

A study that would be interesting to pursue further is the interpretation of the new classes of solutions as surface or line defect CFTs within higher dimensional conformal field theories. Recent progress in this direction has shown that some sub-classes of the AdS$_3$ and AdS$_2$ solutions to massive IIA supergravity (constructed in Sections 2 and 4) asymptote locally to the AdS$_6$ background of Brandhuber-Oz [79]. This means that they can be interpreted as surface or line defect CFTs, respectively, within the 5d Sp(N) fixed point theory dual to the Brandhuber-Oz solution [36]. In the AdS$_2$ case this is in nice agreement with our proposed interpretation of these solutions as backreacted D4-D0 baryon vertices in a system of D4’-D8 branes. In this description the one dimensional defects consist on D4-D0 baryon vertices connected to the D4’-D8 branes with (fermionic) F1-strings. In the IR the gauge symmetry on the D4’-D8 branes becomes global, turning them from colour to flavour branes. In turn,
the D4-D0 defect branes become the new colour branes of the backreacted geometry.

In order to complete the previous picture we should keep in mind that D4-D8 brane set-ups must include O8 orientifold fixed planes in order to flow to a 5d fixed point theory in the UV [35]. It would be interesting to clarify to what extent the behaviour found in [40] at both ends of the space, compatible with the presence of O8 orientifold fixed planes, would provide for a fully consistent global picture. It is expected that in this set-up baryon vertices affected by the orbifold projection will be removed from the spectrum, corresponding to the fact that USp baryons are unstable against their decay into mesons. A similar interpretation for the D2-D6-NS5 brane surface defects within the D4’-D8 brane intersection, put forward in [36] for the AdS3 case, still remains to be elucidated. In both the AdS2 and AdS3 cases an explicit realisation of the quiver CFTs as embedded in the 5d quiver CFT associated to the D4’-D8 brane system remains as well to be found.

This is in contrast with the interpretation of a sub-class of the M-theory pairs reviewed in Section 3 as surface defects within the 6d (1,0) CFT living in M5-branes on ALE singularities, found in [36]. In this case it has been possible to explicitly realise the 2d quiver CFTs as embedded in the 6d quiver CFT associated to M5-branes intersecting with KK-monopoles.

We have argued that the Type B AdS2 solutions reviewed in subsection 5.2 describe backreacted D5-D1 baryon vertices in the 4d \( \mathcal{N} = 2 \) QFT living in D3-D7 intersections. In this case there is no holographic analogue, and it would be interesting to see if these solutions asymptote locally to an AdS5 background. This would provide further support to the proposed defect interpretation.

Finally, we would like to stress that the full class of AdS3 solutions discussed in [26, 52, 53, 54], which constitute the basis for the developments reviewed in this paper, is much broader than the sub-set of solutions that have been the focus of our CFT analysis. In particular, there is evidence that interesting new CFTs arise when there is dependence on the internal structure of the CY2 manifold. Work is in progress in this direction [69].

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