Research Article

Identifying the Influential Latent Edges for Promoting the Co-SIR Model

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Received 14 December 2020; Revised 13 January 2021; Accepted 15 March 2021; Published 24 March 2021

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The network-based cooperative information spreading is a widely existing phenomenon in the real world. For instance, the spreading of disease outbreak news and disease prevention information often coexist and interact with each other on the Internet. Promoting the cooperative spreading of information in network-based systems is a subject of great importance in both theoretical and practical perspectives. However, very limited attention has been paid to this specific research area so far. In this study, we propose an effective approach for identifying the influential latent edges (that is, the edges that do not originally exist) which, if added to the original network, can promote the cooperative susceptible-infected-recovered (co-SIR) dynamics. To be specific, we first obtain the probabilities of each node being in different node states by the message-passing approach. Then, based on the state probabilities of nodes obtained, we come up with an indicator, which incorporates both the information of network topology and the co-SIR dynamics, to measure the influence of each latent edge in promoting the co-SIR dynamics. Thus, the most influential latent edges can be located after ranking all the latent edges according to their quantified influence. We verify the rationality and superiority of the proposed indicator in identifying the influential latent edges of both synthetic and real-world networks by extensive numerical simulations. This study provides an effective approach to identify the influential latent edges for promoting the network-based co-SIR information spreading model and offers inspirations for further research on intervening the cooperative spreading dynamics from the perspective of performing network structural perturbations.

1. Introduction

The influence of information spreading on human society is increasingly profound. Promoting the spreading of some typical information, for instance, the vaccination guidance, technological innovation, commercials, and political propaganda can benefit all the aspects of socio-economic systems [1–5]. Therefore, researchers from multiple fronts, including statistical physics and network science, have been paying substantial attention to the specific subject of modeling and promoting the information spreading [6–10].

The complex network theory plays an important role in the study of information spreading. Under the framework of complex networks, a large amount of studies on information spreading dynamics have been gained. For instance, various network-based information spreading models such as the susceptible-infected-susceptible (SIS) model [11, 12], susceptible-infected-recovered (SIR) model [13, 14], threshold model [15–19], and many of their extensions [20–24] have been proposed to describe the various spreading dynamics of a single piece of information. The theoretical study of network-based spreading dynamics with a single-information has a long and successful history. However, cooperative information spreading widely exists in the real world [25, 26]. For example, the transmission of epidemic outbreak news may promote the spreading of epidemic prevention...
information. Recently, considering the fact that the spreading phenomena in the real world are often influenced by cooperativity effects, researchers go further to study on the cooperative spreading dynamics. Many efforts have been made to uncover and understand the interaction mechanisms [27], evolution patterns [28, 29], phase transition [30], and critical dynamics [31] of the cooperative spreading dynamics [32].

In addition to understanding and modeling of the cooperative spreading dynamics, the problem of how to effectively intervene the cooperative dynamics evolution is also of great importance in both theoretical and practical perspectives. In the study of intervening a single dynamics, researchers have developed numerous strategies from the perspective of designing effective transmission strategies [33–38], identifying vital nodes [39–46] and performing network structural perturbations [3, 47–49]. However, the study of intervening cooperative spreading started to attract attention only recently. Min et al. [50] adopted the message-passing approach to find the influential spreaders by predicting the probability that an extensive coinfection outbreak initiated by two singly infected seed nodes.

Given the limited number of studies so far, no attention has been paid in intervening the cooperative information spreading from the perspective of performing network structural perturbations, to the best of our knowledge. To raise the interest of researchers to fill up this research blankness, in this study, we propose an effective approach to identify the influential latent edges which can promote the cooperative susceptible-infected-recovered (co-SIR) dynamics [50] if added to the original networks. The co-SIR model is first proposed to study the cooperative epidemic spreading. In this study, we extend it to describe cooperative information spreading. Specifically, we first employ the message-passing approach to predict the probabilities of each node being in different node states. Next, based on the obtained state probabilities of nodes, we come up with an indicator for measuring the influence of each latent edge in promoting the cooperative dynamics. Finally, the influential latent edges can be identified by ranking all the latent edges according to their quantified influence. Note that, the indicator we proposed in this study incorporates both the network topology information and dynamics information in the network-based co-SIR dynamical system. This study will verify the rationality and superiority of the proposed indicator in identifying the influential latent edges.

This paper is organized as follows. Firstly, Section 2 will give a detailed description of the co-SIR model. Secondly, Section 3 will show how to identify the influential latent edges in the network-based co-SIR dynamical system. Further, Section 4 will present the extensive numerical simulations to verify the effectiveness of our approach in identifying the influential latent edges. Finally, Section 5 will give a conclusion about this study.

2. Model Description

We consider a discrete-time cooperative co-SIR information spreading model [50] in this study. The co-SIR dynamics runs on a complex network $G$ of $N$ nodes and $M$ edges. Denote the adjacency matrix of network $G$ as $H$; thus, when there is an edge between node $i$ and $j$, we have $H_{ij} = 1$; otherwise, $H_{ij} = 0$. To be specific, the co-SIR model merges two classic SIR dynamics, that is, $a$ and $b$. Each node of the network $G$ can be in three distinct states with respect to each one of the two dynamics. For instance, when dynamics $A$ alone is considered, a node $i$ can be in susceptible ($S^a$), informed and able to transmit the information ($I^a$), or removed ($R^a$) state. Therefore, when we take both dynamics $a$ and $b$ into consideration, there should be overall nine possible node states, that is, $S^aS^b, S^aI^b, S^aR^b, I^aS^b, I^aI^b, I^aR^b, R^aS^b, R^aI^b$, and $R^aR^b$.

Besides, the cooperativity between dynamics $a$ and $b$ is defined as follows: for a node informed with both dynamics, the probability to transmit the information by a single contact increases. Recently, Min and Castellano [50] introduced this kind of cooperative mechanism to describe the cooperativity of epidemics, considering the biological rationale that an infection with a pathogen may cause an individual to pass from the latent state to a fully infective one with respect to another pathogen. Similarly, when it comes to the spreading of information, being informed with message $a$ may encourage an individual to be more active in transmitting another message $b$. For instance, when an individual is informed of an epidemic outbreak, he/she may be more willing to transmit the information about epidemic prevention.

To be specific, the dynamic process is as follows. Initially, all the nodes of network $G$ are set to be in the $S^aS^b$ state. Next, we selected a small fraction of nodes to be the spreading seeds with respect to dynamics $a$ and $b$, respectively. At the beginning of each discrete-time step, for each node $i$ and each one of its neighbor $j$, if $i$ is informed with dynamics $a$ (or $b$) alone and susceptible with respect to the other, then $i$ will transmit the information $a$ (or $b$) to $j$ with probability $\lambda_a$ (or $\lambda_b$). That is to say, in this case, the state of node $j$ will be updated according to the following transition rules:

$$
\begin{align*}
I^aS^b + S^aS^b &\xrightarrow{\lambda_a} I^aS^b + I^aS^b, \\
I^aS^b + S^aI^b &\xrightarrow{\lambda_a} I^aS^b + I^aI^b, \\
I^aS^b + S^aR^b &\xrightarrow{\lambda_a} I^aS^b + I^aR^b, \\
S^aI^b + S^aS^b &\xrightarrow{\lambda_b} S^aI^b + S^aI^b, \\
S^aI^b + I^aS^b &\xrightarrow{\lambda_b} S^aI^b + I^aI^b, \\
S^aI^b + R^aS^b &\xrightarrow{\lambda_b} S^aI^b + R^aI^b.
\end{align*}
$$

(1)
Otherwise, if \( i \) is in the recovered state with respect to the other dynamics, the transmission probability will be \( \lambda_c \) (or \( \lambda_b \)). Accordingly, the updating rules of the state of node \( j \) are as follows:

\[
\begin{align*}
I^a R^b + S^a S^b &\xrightarrow{\lambda_c} I^a R^b + I^a S^b, \\
I^a R^b + S^a I^b &\xrightarrow{\lambda_c} I^a R^b + I^a I^b, \\
I^a R^b + S^a R^b &\xrightarrow{\lambda_c} I^a R^b + I^a R^b, \\
R^a I^b + S^a S^b &\xrightarrow{\lambda_c} R^a I^b + S^a I^b, \\
R^a I^b + I^a S^b &\xrightarrow{\lambda_c} R^a I^b + I^a I^b, \\
R^a I^b + R^a S^b &\xrightarrow{\lambda_c} R^a I^b + R^a I^b.
\end{align*}
\]

Similarly, when \( i \) is in the informed state and \( j \) is not susceptible (i.e., \( j \) is in the informed or recovered state) with respect to the other dynamics, the transmission probability should be \( \lambda_c \) (or \( \lambda_b \)) as well. The corresponding updating rules are shown as follows:

\[
\begin{align*}
I^a I^b + S^a I^b &\xrightarrow{\lambda_c} I^a I^b + I^a I^b, \\
I^a I^b + S^a R^b &\xrightarrow{\lambda_c} I^a I^b + I^a R^b, \\
I^a I^b + I^a S^b &\xrightarrow{\lambda_c} I^a I^b + I^a I^b, \\
I^a I^b + R^a S^b &\xrightarrow{\lambda_c} I^a I^b + R^a I^b.
\end{align*}
\]

Note that when the state of node \( i \) and \( j \) is \( I^a I^b \) and \( S^a S^b \), respectively, node \( i \) may transmit information \( a \) (or \( b \)) with probability \( \lambda_a \) (or \( \lambda_b \)), first and then transmit information \( b \) (or \( a \)) with probability \( \lambda_a^\dagger \) (or \( \lambda_b^\dagger \)) during the same time step. Specifically, the state of the susceptible node \( j \) is updated according to the following transition rules:

\[
\begin{align*}
I^a I^b + S^a I^b &\xrightarrow{\frac{1}{2}} (\lambda_a \lambda_b^\dagger + \lambda_a^\dagger \lambda_b) I^a I^b + I^a I^b, \\
I^a I^b + S^a S^b &\xrightarrow{\frac{1}{2}} \lambda_a (1 - \lambda_b^\dagger) I^a I^b + I^a S^b, \\
I^a I^b + S^a S^b &\xrightarrow{\frac{1}{2}} \lambda_b (1 - \lambda_a^\dagger) I^a I^b + S^a I^b.
\end{align*}
\]

After the transmission process, all the informed nodes recover with probability \( \gamma_a \) (or \( \gamma_b \)) with respect to dynamic \( a \) (or \( b \)); and the time step is over. The recovery rules are as follows:

\[
\begin{align*}
I^a \rightarrow I^a \gamma_a, \\
I^a \rightarrow \gamma_b R^a R^b, \\
I^a \rightarrow (1 - \gamma_a) \gamma_b I^a I^b, \\
I^a \rightarrow 1 - \gamma_a \gamma_b I^a I^b, \\
S^a \rightarrow S^a R^a R^b, \\
R^a \rightarrow R^a R^b.
\end{align*}
\]

Finally, the spreading dynamics will be terminated once there is no node that can further transmit information.

In the rest of the paper, we consider only the symmetric case, where \( \lambda_a = \lambda_b = \lambda \) and \( \lambda_a^\dagger = \lambda_b^\dagger = \lambda^\dagger \), and leave the nonsymmetric case open for future work. Besides, for the sake of simplicity, we assume the recovery probability \( \gamma_a = \gamma_b = 1 \).

### 3. Identifying Influential Latent Edges

Now, we turn to the identification of the influential latent edges of network \( G \), which can promote the spreading of the co-SIR model. For convenience, we call the nodes that are informed with both dynamics as the co-informed nodes. Denote the cooperative spreading prevalence of information \( a \) and \( b \) (that is, the fraction of co-informed nodes) as \( \rho_{ab} \). Specifically, we are interested in finding the influential latent edges, which, if added to the original network \( G \), can maximize the cooperative spreading prevalence \( \rho_{ab} \).

To identify the influential latent edges, we need an indicator to measure the influence of each latent edge in promoting the cooperative spreading prevalence \( \rho_{ab} \). A direct indicator for measuring the influence of latent edge \((i, j)\) is the expected incremental number of co-informed nodes observed after adding an edge between node \( i \) and \( j \). However, this indicator expects a large analytical and computational cost. Inspired by the recent references [7, 51], we narrow our focus to the node set \( U_{ij} \), which is composed of node \( i \), \( j \), and their neighbors. It will be much more convenient to calculate the expected incremental number of co-informed nodes in the node set \( U_{ij} \). Denote the probability of node \( i \) being in the state \( X^a Y^b \) state after the termination of dynamics as \( p_i^{XY} \). Note that we should have \( p_i^{XY} = p_j^{XY} \) in the symmetric case. After adding an edge between node \( i \) and \( j \), consider the information is transmitted from node \( i \) to \( j \); then, according to the transmission rules in Section 2, node \( j \) becomes a new co-informed node with probability:

\[
p_{i \rightarrow j} = \lambda \lambda^\dagger \left( p_i^{RR} p_j^{SS} + 2 \lambda p_i^{RR} p_j^{RS} + 2 \lambda p_i^{RS} p_j^{RS} \right).
\]

The first term on the r.h.s accounts for the case when the final states of node \( i \) and \( j \) are \( R^a R^b \) and \( S^a S^b \), respectively; the second term takes into account the case when the final
state of node $i$ is $R^aR^b$ and the final state of node $j$ is $R^aS^b$ or $S^aR^b$; and the third term accounts for the case when $c = S^aR^b$ and $e_i = R^aS^b$ or $c = R^aS^b$ and $e_j = S^aR^b$. Next, the co-informed node $j$ may transmit both information $a$ and $b$ to its neighbors. Therefore, we can approximately get the expected incremental number of co-informed nodes generated through this way of transmission initiated at node $i$ as

$$
\delta_{i\rightarrow j} = \rho_{i\rightarrow j} \left[ 1 + \sum_{r=1}^{N} H_{jr}(\lambda^1 P_{r}^{SS} + 2\lambda P_{r}^{RS}) \right].
$$

Similarly, we can get the expected incremental number $\delta_{j\rightarrow i}$ by symmetry. Take both cases of $\delta_{i\rightarrow j}$ and $\delta_{j\rightarrow i}$ into consideration; therefore, we can define an indicator $\bar{\delta}$ to measure the influence of latent edge $(i, j)$ as

$$
\bar{\delta}_{ij} = \delta_{i\rightarrow j} + \delta_{j\rightarrow i}.
$$

Hereafter, the problem reduces to solving Equation (8), i.e., finding the probabilities of nodes being in different states after the termination of dynamics.

According to Section 2, the steady state behavior of the co-SIR model considered in this study can be predicted analytically by a message-passing approach [50]. Denote $\sigma_{ij}$ as the probability that node $i$ has not been informed by node $j$ with respect to information $a$. In the symmetric case, we have $\sigma_{ij}^a = \sigma_{ij}^b = \sigma_{ij}$, and the message-passing equations for these quantities are

$$
1 - \sigma_{ij} = \frac{1}{2} \left( \lambda + \lambda^{-1} \left( 1 - \prod_{r \in \partial j \setminus i} \sigma_{jr} \right) \left( 1 - \prod_{r \in \partial j} \sigma_{jr} \right) \right.
$$

$$
+ \lambda \left( 1 - \prod_{r \in \partial j \setminus i} \sigma_{jr} \right) \prod_{r \in \partial j} \sigma_{jr},
$$

where $\partial j$ is the set of neighbor of node $j$ and $\partial j \setminus i$ represents the set of neighboring nodes of node $j$ excluding $i$. The first term on the r.h.s. takes into account the case that node $i$ is informed with one piece of the information by node $j$ that is in the co-informed state at the end of the dynamics. Meanwhile, the second term accounts for the case that node $i$ is informed with one piece of the information by node $j$ that is informed with only one piece of information.

Solving Equation (9) by iteration, we can get the probability that node $i$ being informed with both information $a$ and $b$ at the end of the dynamics as follows:

$$
\rho_{i}^{RR} = \left( 1 - \prod_{j \in \partial i} \sigma_{ij} \right)^2.
$$

Similarly, the probability that node $i$ being informed only with one piece of the dynamics at the end is

$$
\rho_{i}^{SR} = \left( 1 - \prod_{j \in \partial i} \sigma_{ij} \right) \prod_{j \in \partial i} \sigma_{ij}.
$$

Besides, the fraction of co-informed nodes after the termination of the dynamics is

$$
\rho_{ab} = \frac{1}{N} \sum_{r=1}^{N} \left( 1 - \prod_{j \in \partial i} \sigma_{ij} \right)^2.
$$

Combining Equations (10) and (11), we can calculate the influence $\delta_{ij}$ of each latent edge $(i, j)$ according to Equation (8) and then identify the influential latent edges for promoting the spreading of the co-SIR model by ranking all the latent edges with respect to the corresponding values of $\bar{\delta}$.

4. Simulation Results

This section will show the results of extensive numerical simulations on both synthetic and real-world networks to verify the effectiveness of our approach in identifying the influential latent edges. The detailed structural information of the networks employed in this section can be found from Table 1. Note that the message-passing approach [50] mentioned in Section 3 can accurately predict the Monte Carlo simulations; thus, to avoid large computational cost, we obtain the numerical value of cooperative spreading prevalence $\rho_{ab}$ by solving Equation (12) instead of using the Monte Carlo simulation approach.

In our approach, we identify the influential latent edges by ranking the values of $\delta$, which incorporates the information of both network structure (i.e., the adjacency matrices $H$) and spreading dynamics (i.e., $\lambda$ and $\lambda^1$). For comparison, we also test two additional approaches that only rely on network topology. On the one hand, we consider the approach to rank all the latent edges by $\theta^d$ which denotes the product of the degree centrality of nodes connected by the given latent edge. On the other hand, we employ the strategy to rank all the latent edges by $\theta^b$, which is the product of betweenness centrality of nodes connected by the given latent edge. With the help of numerical simulations, now we are going to show the effectiveness and advantage of our approach in identifying the influential latent edges step by step.

Firstly, to verify the rationality and superiority of employing indicator $\delta$ to rank the influence of latent edges in promoting the spreading of co-SIR dynamics, we investigate the correlations between $\delta$ and the incremental cooperative spreading prevalence $\rho_{ab}$. The dynamics runs on a scale-free (SF) network $G_s$ with degree distribution $p(k) \sim k^{-\alpha}$, where $\alpha = 2.3$ denotes the degree exponent. More detailed information about the network $G_s$ can be found in Table 1. Figures 1(a)–1(c) show $\delta$ versus $\rho_{ab}$ when the cooperativity parameters $\beta = \lambda^1/\lambda$ are equal to 1, 1.5, and 2, respectively. Note that the transmission probabilities are set to be $\lambda = 0.4$ in all the numerical simulations of Figures 1(a)–1(c). As can be seen, the values of $\delta$ and $\rho_{ab}$ are almost linearly correlated in all the cases studied; thus, it is rational to use the indicator $\delta$ in ranking all the latent edges. Next, we go further to investigate the correlations between $\delta$ and $\rho_{ab}$ for all the parameter regions by calculating the Spearman rank correlation coefficient $[52, 53]$ between them. The Spearman rank correlation coefficient is defined as follows:
where $\phi_l$ and $\delta_l$ are the ranks of latent edge $l$ scored by $\delta$ and $\rho_{ab}$, respectively, and $M_u = (1/2)N(N - 1) - M$ is the total number of latent edges. For convenience, we refer to the latent edge rank scored by $\delta$ as numerical rank $\delta_l$.

Figures 1(d)–1(f) show the correlation $m_s$ versus transmission probability $\lambda$ when (d) $\beta = 1$, (e) $\beta = 1.5$, and (f) $\beta = 2$. The Spearman rank correlation $m_s$ between $\rho_{ab}$ and $\rho_{ab}$ (blue circles), product $\theta^d$ (pink diamonds), or $\theta^b$ (green squares) versus $\lambda$ when (d) $\beta = 1$, (e) $\beta = 1.5$, and (f) $\beta = 2$.

$$m_s = 1 - \delta\sum_{l=1}^{M_u} \frac{(\phi_l - \delta_l)^2}{M_u(M_u - 1)}.$$  \hspace{1cm} (13)

where $\phi_l$ and $\delta_l$ are the ranks of latent edge $l$ scored by $\delta$ and $\rho_{ab}$, respectively, and $M_u = (1/2)N(N - 1) - M$ is the total number of latent edges. For convenience, we refer to the latent edge rank scored by $\delta$ as numerical rank $\delta_l$.

Figures 1(d)–1(f) show the correlation $m_s$ versus transmission probability $\lambda$ when $\beta = 1$, $\beta = 1.5$, and $\beta = 2$, respectively. Note that we only consider the situation when $\lambda > \lambda_c$, where $\lambda_c$ is the threshold value of the dynamics; since in the case when $\lambda < \lambda_c$, the cooperative spreading prevalence $\rho_{ab}$ should be theoretically equal to 0. The theoretical values of $\lambda_c$ are given by the inverse of the principal eigenvalue $\Lambda$ of the nonbacktracking matrix $B$ [50], where $B$ is a $2M \times 2M$ matrix with elements:

$$B_{i,i'} = \delta_{i,i'}(1 - \delta_{a,b}).$$  \hspace{1cm} (14)

The specific values of $\lambda_c$ of the networks can be found in Table 1. We can observe that the values of $m_s$ stay close to 1 (that is to say, the latent edge ranks scored by $\delta$ and $\rho_{ab}$ are strongly correlated) in all the parameter regions. For comparison, we also calculate the Spearman rank correlations between the latent edge ranks scored by degree centrality product $\theta^d$ or betweenness centrality product $\theta^b$ and $\rho_{ab}$. As shown in Figures 1(d)–1(f), the latent edge ranks

| Name          | $N$  | $M$  | $\langle k \rangle$ | $\langle k^2 \rangle$ | $k_{\text{max}}$ | $\lambda_c$ |
|---------------|-----|-----|---------------------|------------------------|------------------|-------------|
| SF            | 200 | 1000| 10                  | 129.5                  | 28               | 0.084       |
| Physicians    | 117 | 465 | 7.95                | 79.162                 | 26               | 0.114       |
| Inf-USAir97   | 332 | 2126| 12.807              | 568.163                | 139              | 0.025       |
| Jazz musicians| 198 | 2742| 27.697              | 1070.242               | 100              | 0.026       |

$N$: the node number; $M$: the edge number; $\langle k \rangle$: the average degree; $\langle k^2 \rangle$: the second moment of the degree distribution; $k_{\text{max}}$: the maximum degree; $\lambda_c$: the threshold value.
scored by $\theta^d$ or $\theta^b$ are positively correlated with the numerical rank $Î£$ only when $Î»$ is small. However, when $Î»$ becomes large, the underlined correlations become negative. To sum up, Figures 1(a)-1(f) demonstrate that using the proposed indicator $ÎÄ$ to identify the influential latent edges is more rational than using the centrality product $\theta^d$ or $\theta^b$.

Secondly, considering the problem of identifying the influential latent edges, we are particularly interested in finding the optimal latent edge which can maximize the cooperative spreading prevalence; thus, we are going to show that the indicator $ÎÄ$ proposed in this study performs well in identifying the optimal latent edge. Denote $L$, $L^d$, and $L^b$ as the optimal latent edges selected according to indicators $ÎÄ$, $\theta^d$, and $\theta^b$, respectively. Let us continue with the co-SIR dynamics on the SF network $G_s$. Figures 2(a)-2(c) present the numerical rank $Î£L$ of the optimal latent edge $L$ versus transmission probability $Î»$ when $Î² = 1$, $Î² = 1.5$, and $Î² = 2$, respectively. It can be seen that $Î£L = 1$ for most values of $Î»$ regardless of the values of cooperative parameter $Î²$. That is to say, the indicator $ÎÄ$ can well predict the optimal edge which can maximize the cooperative spreading prevalence $Î£ab$. Denote $Î£Î» = Î£/M_s$ as the normalized numerical rank, where smaller $Î£Î»$ indicates higher rank. Figures 2(d)-2(f) further show the normalized ranks of optimal latent edges $L$, $L^d$, and $L^b$ versus $Î»$ when $Î² = 1$, $Î² = 1.5$, and $Î² = 2$, respectively. Visually, $L$, $L^d$, and $L^b$ all rank high when $Î»$ is slight above the threshold value, but once $Î»$ becomes large, the ranks of $L^d$ and $L^b$ fall quickly. Therefore, we can draw a conclusion that the strategy of using the indicator $ÎÄ$ which incorporates both information of network topology and dynamics is better than using those structural indicators (i.e., $\theta^d$ and $\theta^b$) in finding the optimal latent edge.

Thirdly, to better understand the specific influence of the optimal latent edges $L$, $L^d$, and $L^b$ in promoting the co-SIR dynamics, we calculated the corresponding incremental cooperative spreading prevalence $Î£\rho_{ab}$ after adding those optimal latent edges to the original networks. For network $G_s$, Figures 3(a)-3(c) show the incremental cooperative spreading prevalence $Î£\rho_{ab}$ versus $Î»$ when $Î² = 1$, $Î² = 1.5$, and $Î² = 2$, respectively. The results demonstrate that adding the optimal latent edge $L$ selected according to the proposed indicator $ÎÄ$ can effectively promote the cooperative spreading prevalence $Î£\rho_{ab}$ for most of the values of $Î». However, adding the optimal $L^d$ or $L^b$ to $G_s$ has little influence in promoting $Î£\rho_{ab}$ for almost all the cases studied. Note that, on the network $G_s$, when $Î»$ is small, the cooperative spreading prevalence $Î£\rho_{ab} \rightarrow 0$; thus, the incremental spreading prevalence $Î£\rho_{ab} \rightarrow 0$ after adding any

![Image of graphs showing numerical ranks and incremental spreading prevalence](image-url)

Figure 2: The numerical ranks of optimal latent edges on the SF network. The numerical rank of the optimal latent edge scored by $ÎÄ$ versus $Î»$ when (a) $Î² = 1$, (b) $Î² = 1.5$, and (c) $Î² = 2$. The normalized numerical rank $Î£$ of the optimal latent edge scored by $ÎÄ$ (blue circles), product $\theta^d$ (pink diamonds), or $\theta^b$ (green squares) versus $Î»$ when (d) $Î² = 1$, (e) $Î² = 1.5$, and (f) $Î² = 2$. 
Figure 3: Incremental cooperative spreading prevalence $\tilde{\rho}_{ab}$ after adding the optimal latent edge to the SF network. The incremental cooperative spreading prevalence $\tilde{\rho}_{ab}$ after adding the optimal latent edge scored by $\bar{\delta}$ (blue circles), product $\theta^d$ (pink diamonds), or $\theta^b$ (green squares) versus $\lambda$ when (a) $\beta = 1$, (b) $\beta = 1.5$, and (c) $\beta = 2$.

Figure 4: Continued.
Figure 4: Incremental cooperative spreading prevalence $\hat{\rho}_{ab}$ after adding the optimal latent edge to the real-world networks. The incremental cooperative spreading prevalence $\hat{\rho}_{ab}$ after adding the optimal latent edge scored by $\delta$ (blue circles), product $\theta^d$ (pink diamonds), or $\theta^b$ (green squares) to the real-world network Jazz musicians [54] when (a) $\beta = 1$, (b) $\beta = 1.5$, and (c) $\beta = 2$. The corresponding results of real-world network physicians [54] and inf-USAir97 [55] are shown in (d) $\beta = 1$, (e) $\beta = 1.5$, and (f) $\beta = 2$ and (g) $\beta = 1$, (h) $\beta = 1.5$, and (i) $\beta = 2$, respectively.

Figure 5: Continued.
single edge to the network. Similarly, when $\lambda$ becomes too large, $\rho_{ab} \rightarrow 1$; thus, $\bar{\rho}_{ab} \rightarrow 0$ as well. We also test the effectiveness of optimal latent edges $L_a$, $L_b$, and $L_c$ in promoting the spreading dynamics on three real-world networks: (a) Jazz musicians [54]; (b) physicians [54]; and (c) inf-USAir97 [55]. Some structural information of these real-world networks can be found in Table 1. Figure 4 shows the incremental cooperative spreading prevalence $\bar{\rho}_{ab}$ versus $\lambda$ on the three real-world networks with different cooperativity parameter $\beta$. The results are consistent with those of the SF network $G_s$, i.e., using the proposed indicator $\bar{\delta}$ in this study can effectively identify the influential latent edge to promote the co-SIR dynamics.

Finally, we investigate how the structural properties (i.e., degree, betweenness, and eigenvector centrality) of optimal latent edge $L$ change with $\lambda$. Denote $\Phi^d$, $\Phi^b$, and $\Phi^e$ as the normalized latent edge rank scored by the degree centrality product $\theta^d$, betweenness centrality product $\theta^b$, and eigenvector centrality product $\theta^e$, respectively. Figures 5(a)–5(c) show the normalized rank $\Phi^d_L$ of the optimal latent edge $L$ versus $\lambda$ on different networks when $\beta = 1$, $\beta = 1.5$, and $\beta = 2$, respectively. Similarly, the corresponding results about normalized rank $\Phi^b_L$ and $\Phi^e_L$ are shown in Figures 5(d)–5(f) and Figures 5(g)–5(i), respectively. As can be demonstrated from Figure 5, when $\lambda$ is small, the centrality ranks of the optimal latent edge $L$ are high for most of the cases. In other words, adding connection between nodes with high centrality can effectively promote the spreading of co-SIR dynamics when the transmission probability is small. This can be understood by the fact that when $\lambda$ is near, the threshold $\rho_{ab}$ should be small, and adding an edge between nodes with high centrality can help to keep the cluster of co-informed nodes, thus, promoting the co-SIR spreading dynamics. However, when $\lambda$ becomes large, the centrality ranks of $L$ fall quickly in most of the cases studied. This is because nodes with high centrality ranks will have a higher probability to be co-informed when $\lambda$ is large; thus, it is unnecessary to add connection between them. The results in Figure 5 give an intuitive explanation for the failure of indicator $\theta^d$ and $\theta^b$ in identifying influential latent edges when $\lambda$ is large.

5. Conclusions

Cooperative information spreading on networked systems is a common phenomenon in the real world. The study of promoting the network-based cooperative spreading dynamics is of both theoretical and practical importance. In this study, we proposed an effective approach to identify the influential latent edges for promoting the spreading of the co-SIR model on complex networks.

To be specific, we first employ the message-passing approach to obtain the probabilities of each node being in different node states. Next, given the obtained state probabilities of nodes, we proposed the indicator $\bar{\delta}$ to measure the influence of each latent edge in promoting the spreading of the co-SIR model. Then, we can rank all the latent edges by the indicator $\bar{\delta}$ to identify the influential latent edges for promoting the spreading of the co-SIR dynamics on the complex networks. Note that the indicator $\bar{\delta}$ incorporates both the information of network structure and the co-SIR dynamics; and the numerical simulations verified that the indicator $\bar{\delta}$ outperforms those structural indicators (i.e., the degree centrality product $\theta^d$ and betweenness centrality product $\theta^b$) in identifying the influential latent edges on both synthetic and real-world networks. Finally, we investigated how the structure properties of the optimal latent edges change with the transmission probability $\lambda$. It is surprising that the centrality ranks of the optimal latent edges fall quickly when $\lambda$ becomes large in most of the cases studied. This finding gives an intuitive explanation for the failure of those structural indicators in identifying the influential latent edges when $\lambda$ is large.
This study provides an effective approach for identifying the influential latent edges for promoting the network-based co-SIR information spreading dynamics. The research findings offer inspirations for further studies on intervening the cooperative spreading dynamics from the perspective of performing network structural perturbations.

Data Availability

The network data used to support the findings of this study have been deposited in the KONECT: the Koblenz Network Collection repository (10.1145/2487788.2488173) and the Network Data Repository with Interactive Graph Analytics and Visualization (http://networkrepository.com).

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant 11975071 and 62006122, in part by the Scientific Research Foundation of Shantou University under Grant NTF19015, and in part by the 2020 Li Ka Shing Foundation Cross-Disciplinary Research under Grant 2020LKSFG09D.

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