Diffraction driven steep rise of spin structure function
\( g_{LT} = g_1 + g_2 \) at small \( x \) and DIS sum rules

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Abstract

We derive a unitarity relationship between the spin structure function \( g_{LT}(x, Q^2) = g_1(x, Q^2) + g_2(x, Q^2) \), the LT interference diffractive structure function and the spin-flip coupling of the pomeron to nucleons. Our diffractive mechanism gives rise to a dramatic small-\( x \) rise
\( g_{LT}(x, Q^2) \sim g_2(x, \vec{Q}^2) \sim \left( \frac{1}{x} \right)^{2(1+\delta_g)} \), where \( \delta_g \) is an exponent of small-\( x \) rise of the unpolarized gluon density in the proton \( g(x, \vec{Q}^2) \) at a moderate hard scale \( \vec{Q}^2 \) for light flavour contribution and large hard scale \( \vec{Q}^2 \sim m_f^2 \) for heavy flavour contribution. It invalidates the Burkhardt-Cottingham sum rule. The found small-\( x \) rise of diffraction driven \( g_{LT}(x, Q^2) \) is steeper than given by the Wandzura-Wilczek relation under conventional assumptions on small-\( x \) behaviour of \( g_1(x, Q^2) \).

1 Introduction and motivation

The combination \( g_{LT}(x, Q^2) = g_1(x, Q^2) + g_2(x, Q^2) \) of familiar spin structure functions \( g_1 \) and \( g_2 \) of deep inelastic scattering (DIS) is related to the absorptive part of amplitude \( A_{\mu\rho,\nu\lambda}(\Delta = 0) \) of forward (\( T \)) transverse to (\( L \)) longitudinal photon scattering accompanied by the target nucleon spin-flip,

\[
\sigma_{LT} = \frac{1}{(Q^2 + W^2)} \text{Im} A_{-1, \frac{1}{2}, L, \frac{1}{2}}(\Delta = 0) = \frac{4\pi^2 \alpha_{em}}{Q^2} \cdot \frac{4m_p}{\sqrt{Q^2}} \cdot x^2 g_{LT}(x, Q^2),
\] (1)
where $\Delta$ is the momentum transfer and $\mu, \nu = \pm 1, L$ and $\rho, \lambda = \pm \frac{1}{2}$ are helicities of particles in $\gamma^* p_\lambda \rightarrow \gamma^* p'_\rho$ scattering, $Q^2, W^2$ and $x = Q^2/(Q^2 + W^2)$ are standard DIS variable. The motto of high energy QCD – the quark helicity conservation, the common wisdom that high energy scattering is spin-independent, some model considerations including \cite{1} the vanishing one-pomeron exchange contribution to $A_{-1-\frac{1}{2},L+\frac{1}{2}}(\Delta = 0)$, all suggest that the corresponding spin asymmetry $A_2 = \sigma_{LT}/\sigma_T$ vanishes in small-$x$ limit of DIS.

In this communication we demonstrate that this is not the case. We find about $x$-independent spin asymmetry $A_2$ and scaling and steeply rising $g_{LT}(x, Q^2)$ at small $x$,

$$g_{LT}(x, Q^2) \sim \frac{G^2(x, \overline{Q}^2)}{x^2},$$

(2)

where $G(x, Q^2) = x g(x, Q^2) \sim \left(\frac{x}{Q^2}\right)^{\delta_g}$ is the conventional unpolarized gluon structure function of the target nucleon and $\overline{Q}^2$ is flavour dependent scale to be specified below.

The case of the helicity amplitude $A_{-1-\frac{1}{2},L+\frac{1}{2}}(\Delta)$ is quite tricky. On the one hand, QCD motivated considerations strongly suggest a nonvanishing pomeron spin-flip in diffractive nucleon-nucleon scattering \cite{2}. On the other hand, recent studies have shown that the s-channel helicity nonconserving (SCHNC) LT interference cross section $\sigma_{LT}^D$ of diffractive DIS \cite{3} and related SCHNC spin-flip amplitudes of diffractive vector meson production do not vanish \cite{4, 5} at small $x$. As Zakharov emphasized \cite{2} such spin-flip does not conflict the quark helicity conservation because in scattering of composite objects helicity of composite states is not equal to the sum of helicities of quarks, which arguably holds way beyond the perturbative QCD (pQCD) domain. The recent work on SCHNC vector meson production illustrates this point nicely \cite{3, 4, 5}.

Consequently, pomeron exchange well contributes to this helicity amplitude but the Procrustean bed of Regge factorization enforces the forward zero, $A_{-1-\frac{1}{2},L+\frac{1}{2}}(\Delta) \propto \Delta^2$, and vanishing $\sigma_{LT}$ in one-pomeron exchange approximation. The principal point behind our result (2) is Gribov’s observation \cite{6} that such kinematical zeros can be lifted by two-pomeron exchange (two-pomeron cut) which can contribute to helicity amplitudes vanishing in one-pomeron exchange approximation. A good example is a recent derivation \cite{7} of a rising tensor structure function $b_2(x, Q^2)$ for DIS off spin-1 deuterons. In defiance of common wisdom, it gives rise to dependence of total cross section on the deuteron tensor polarization which persists at small $x$. Such a rise of $b_2(x, Q^2)$ invalidates the Close-Kumano sum rule \cite{8}. Incidentally, it derives from the most part from diffractive mechanism which we pursue in this paper. Another example due to Karnakov \cite{9} is a difference of $\gamma\gamma$ total cross sections for parallel and perpendicular linear polarizations of colliding photons - the quantity which vanishes in one-pomeron exchange approximation. The keyword behind these new effects is unitarity \cite{10}, two-pomeron cut is simply a first approximation to imposition of unitarity constraints.

## 2 Regge theory expectations and sum rules

We recall that our expectations for small-$x$ behaviour of different structure functions, $\sim \left(\frac{x}{Q^2}\right)^{\delta}$, have been habitually driven by the Regge picture of soft interactions, in which the exponent (intercept) $\delta = \alpha - 1$ is controlled by quantum numbers of the relevant $t$-channel exchange (a good summary is found in textbook \cite{11}). For instance, the Regge theory suggests $\alpha_{IP} \sim 1$
for helicity-diagonal pomeron ( vacuum) exchange dominated \( F_2(x, Q^2) \) and \( F_L(x, Q^2) \) and \( \alpha_R \sim \frac{1}{2} \) for the secondary reggeon \((\rho, A_2)\)-exchange quantities like \( F_{2p}(x, Q^2) - F_{2n}(x, Q^2) \) and \( \omega\)-exchange \( F_3(x, Q^2) \). The dominant \( A_1 \) and \( f_1 \) reggeon exchange in the axial vector channel suggests \( \alpha_1 \sim 0 \) for \( xg_1(x, Q^2) \). These Regge theory intercepts are not stable against QCD evolution, but extensive studies of small-\( x \) asymptotics of generalized two-gluon and quark-antiquark ladder diagrams have revealed only marginal modifications of the above hierarchy of intercepts (for the BFKL pomeron exchange see \[12\], for reggeon exchange and/or non-singlet structure function see \[13\], for different spin structure functions see: \( g_1(x, Q^2) \) in \[14\], \( g_2(x, Q^2) \) in \[13\], \( F_{3\gamma}(x, Q^2) \) in \[10\]). The corollary of these studies is that \( g_1(x, Q^2) \) and \( g_2(x, Q^2) \) of two-parton ladder approximation have the \( x \)-dependence typical of the reggeon exchange and their contributions to spin asymmetries \( A_1 \) and \( A_2 \) do indeed vanish in the small-\( x \) limit. We recall that works \[14\] focused on exactly forward, \( \Delta = 0 \), Compton scattering amplitudes.

Burkhardt and Cottingham \[1\] argued that because neither pomeron nor high lying reggeon exchanges contribute to \( A_{-1-\frac{1}{2}L+\frac{1}{4}}(\Delta = 0) \), then unsubtracted (superconvergent) dispersion relation holds for this Compton scattering amplitudes. Precisely superconvergence has been the principal assumption behind the much discussed BC sum rule \[1\]

\[
\int dxg_2(x, Q^2) \propto \int_{Q^2/2}^\infty d\nu \text{Im} A_2(Q^2, \nu, \Delta = 0) = 0 , \tag{3}
\]

for thorough reviews see \[11, 17, 18, 19\]. The tricky point is that the BC amplitude \( A_2(Q^2, \nu, \Delta) \) (which differs from our \( A_{-1-\frac{1}{2}L+\frac{1}{4}}(\Delta) \) only insignificantly) receives a contribution from pomeron exchange, and the integral

\[
\int_{Q^2/2}^\infty d\nu \text{Im} A_2(Q^2, \nu, \Delta)
\]

would diverge at any finite \( \Delta \neq 0 \), which makes the BC sum rule quite a singular one. As we emphasized above, \( A_{-1-\frac{1}{2}L+\frac{1}{4}}(\Delta) \propto \Delta^2 \) and vanishes at \( \Delta = 0 \) only because of rigours of Regge factorization, Gribov’s two-pomeron exchange breaks Regge factorization and gives \( A_{-1-\frac{1}{2}L+\frac{1}{4}}(\Delta = 0) \neq 0 \), see also \[20\]. The specter of resulting dramatic small-\( x \) rise of \( g_2(x, Q^2) \) and of divergence of the BC integral permeates the modern literature on spin structure functions (see textbook \[13\] and recent reviews \[17, 18, 19\]). The aforementioned breaking of the Close-Kumano some rule is of the same origin; if cast in the Regge language, the scaling and rising tensor structure function found in \[7\] falls into the pomeron-cut category. Numerical estimates show that tensor asymmetry is quite large, the related evaluations of \( g_{LT}(x, Q^2) \) are as yet lacking.

## 3 Diffractive DIS and unitarity driven \( g_{LT}^U(x, Q^2) \)

In this communication we fill this gap and report the first ever evaluation of unitarity or diffractive driven contribution to \( g_{LT}(x, Q^2) \) in terms of the two other experimentally accessible spin observables: the SCHNC LT interference diffractive DIS structure function \[3\] and the pomeron spin-flip amplitude in nucleon-nucleon and pion-nucleon scattering \[2\]. By unitarity relation, the opening of diffractive DIS channel \( \gamma^*p \rightarrow p'X \) affects the elastic scattering amplitude \((10)\) and references therein). The best known unitarity effect is Gribov’s absorption or shadowing correction \[21\] to one-pomeron exchange. Besides simple shadowing, for spinning
particles unitarity corrections can give rise to new spin amplitudes absent in one-pomeron exchange, which was precisely the case with tensor structure function for DIS off spin-1 deuteron [7].

In the related evaluation of unitarity driven $\sigma_{LT}^{(U)}$ we start with the eikonal unitarity diagram in fig. 1. Here the eikonal refers to the ‘elastic’ $pX$ intermediate state, the effect of so-called ‘inelastic’ intermediate states $p^*X$ when the proton excites into resonances or low-mass continuum states will be commented on below. Hereafter all unitarity corrections will be supplied by a superscript $(U)$. As an input we need amplitudes $A_{\mu\nu,\rho,\lambda}^D$ of diffractive DIS $\gamma^*\nu p \rightarrow X\mu p'$, where $\mu$ stands for spin states of the diffractive state $X$. Applying the optical theorem to this unitarity contribution to forward scattering amplitude, we find [10]

$$\sigma_{LT}^{(U)} = \text{Re} \frac{1}{16\pi^2(W^2 + Q^2)^2} \int d^2\Delta dM^2 \sum_{\mu,\rho} A_{\mu-1,\rho,\lambda}^D A_{\mu\rho,L+\frac{1}{2}}^D,$$

where $M$ is the invariant mass of the intermediate state. In order for this unitarity diagram to contribute to $\sigma_{LT}$, the r.h.s. of (4) must have a structure which in the convenient polarization vector-spinor representation has the form

$$\sigma_{LT} \propto \langle f|\sigma[n e^i(-)]|in \rangle ,$$

where $\sigma$ is the nucleon spin operator, $n$ is the unit vector along the $\gamma^*p$ collision axis, and

$$e(\nu) = -\frac{1}{\sqrt{2}}(\nu, i)$$

is the photon polarization vector for helicity $\nu$.

In the polarization-vector representation the factorized one-pomeron amplitude for diffractive DIS reads [4, 5]

$$A^D = (i + \frac{\pi}{2} \delta_{IP}) \left\{ T_{0L} V_0^\dagger e_L + T_{\pm\pm}(V^\dagger e) + T_{\pm0} V_0^\dagger(\Delta e) + T_{\pm L}(\Delta V^\dagger) e_L + ... \right\}$$

$$\times \{ 1 + i \frac{r_5}{m_p}(\sigma[n \Delta]) \} ,$$

where $T_{\mu\nu}$ is the imaginary part of diffractive amplitude for an unpolarized target, $V$ stands for the transverse polarization vector of diffractive state $X$ and $r_5 = r_5(0) \exp(-B_5 \Delta^2)$ is the ratio of the spin-flip to non-flip pomeron-nucleon couplings. The signature factor

$$\eta = i + \frac{\pi}{2} \delta_{IP} .$$

is defined through the $Q^2$- and $x$-dependent effective intercept

$$\delta_{IP} = \frac{d \log \text{Im}T(x, Q^2)}{d \log \frac{1}{x}} ,$$

which is the same as $\delta_g$ taken at a relevant average hard scale. The sight difference of this scale and of $\delta_{IP}$ thereof for different helicity amplitudes can be neglected for the purposes of our discussion and to a good approximation $r_5$ can be considered a real valued quantity.
The Regge factorization (1) is equally applicable to one-pomeron exchange elastic scattering, in which case it dictates (hereafter we suppress the helicity (−))

\[ A_{-1-\frac{1}{2}L+\frac{1}{2}}(\Delta) \propto T_{-L}(e^\dagger\Delta)(\sigma[n\Delta]), \tag{9} \]

which, as we mentioned in the Introduction, vanishes in the forward case \( \Delta = 0 \).

We need the LT transition in either of the diffractive \( \gamma^* \leftrightarrow X \) vertices and spin-flip transition in either of the pomeron-nucleon vertices in unitarity diagram of fig. 1, the other two vertices are spin non-flip ones. The both spin-flip transitions are off-forward with finite momentum transfer \( \Delta \) to the intermediate state and the integrand of eq. (4) will be \( \propto (e^\dagger\Delta)(\sigma[n\Delta]) \). Summing over the phase space of the intermediate state \( X \) includes an integration over azimuthal angle \( \phi \) of \( \Delta \) of the form

\[ \int \frac{d\phi}{2\pi} (e^\dagger\Delta)(\sigma[n\Delta]) = \frac{1}{2}\Delta^2(\sigma[ne^\dagger]), \tag{10} \]

which has precisely the desired spin structure (5).

Now we notice that the LT interference term in differential cross section of diffractive DIS on unpolarized nucleons \( \gamma^* p_{\lambda} \rightarrow X_{\mu}p'_\rho \) equals

\[ \frac{d\sigma_{LT}^D}{dM^2d^2\Delta} = \frac{1}{16\pi^2(W^2 + Q^2)^2} \sum_{\mu,\rho,\lambda} A_{D-1,\mu\rho}^* A_{D,\mu_\rho,L\lambda} \tag{11} \]

and differs from the r.h.s. of (4) only by complex conjugation of one of diffractive amplitudes. In principle \( \sigma_{LT}^D \) can be measured experimentally. The scaling properties of \( d\sigma_{LT}^D \) have been established in [3]. The conventionally defined LT interference diffractive structure function \( F_{LT}^{(4)} \) is twist-3 [3], for the purposes of the present discussion it is convenient to factor out the kinematical factor \( \Delta/Q \) and define the scaling and dimensionless LT diffractive structure function \( g_{LT}^D(x_{\text{IP}}, \beta, Q^2) \) such that

\[ (M^2 + Q^2) \frac{d\sigma_{LT}^D}{dM^2d^2\Delta} = \frac{4\pi^2\alpha_{em}^2}{Q^2} \cdot \frac{(\Delta e)}{Q} \cdot \left(1 + |r_5|^2 \frac{\Delta^2}{m_p^2}\right) \cdot g_{LT}^D(x_{\text{IP}}, \beta, Q^2) B_{LT} \exp(-B_{LT} \Delta^2), \tag{12} \]

where \( \beta = Q^2/(Q^2 + M^2) \) and \( x_{\text{IP}} = x/\beta \) are diffractive DIS variables. In what follows we shall neglect corrections \( \propto |r_5|^2 \), because nucleon spin-flip effects are numerically very small within the diffraction cone. Then, making use of (4),(12) and of the factorization property of one-pomeron amplitude (1), we obtain

\[ g_{LT}^{(U)}(x, Q^2) = \frac{1}{x^2} \cdot r_5(0) \sin(\pi x_{\text{IP}}) \cdot \int_x^1 \frac{d\beta}{\beta} \cdot \frac{B_{LT}}{4m_p^2(B_{LT} + B_5)} \cdot g_{LT}^D(x_{\text{IP}}, \beta, Q^2). \tag{13} \]

4 The model evaluations of \( g_{LT}^{(U)} \)

Eq. (13) is our central result and up to now we have been completely model independent. In principle, the \( g_{LT}^D \) can be measured experimentally, in the lack of such direct data in our numerical estimates of \( g_{LT} \) we resort to QCD model for diffractive DIS developed in [3]. We refer to this paper for details, here we only recall the salient results.
The driving term of diffractive DIS is excitation of $q\bar{q}$ Fock states of the photon (fig. 2). We notice that only $q\bar{q}$ pairs with the sum of helicities zero contribute to $\sigma_{LT}$. Consider first the contribution from intermediate heavy flavour excitation, in which case the mass $m_f$ of a heavy quark provides the large pQCD hard scale [22, 23]

$$Q^2 \approx \frac{m_f^2}{1 - \beta}.$$  

(14)

The lower blobs in the diagram of fig. 2a are related to skewed unintegrated gluon structure function of the proton which can be approximated by the conventional diagonal unintegrated gluon structure function taken at $x_{eff} = \frac{1}{2}x_p$. To a log $Q^2$ accuracy, gross features of $g_{LT}^{D}(x_p, \beta, Q^2)$ are described by [3]

$$g_{LT}^{D} \approx \frac{e_f^2}{3B_{LT}m_f^2} \cdot \beta^4 (1 - \beta)(2 - 3\beta)\alpha_s^2(Q^2)G^2\left(\frac{1}{2}x_p, Q^2\right),$$

(15)

where $e_f$ is the quark charge in units of the electron charge, $\alpha_s$ is the strong coupling, and we assumed $Q^2 \gg 4m_f^2$. Notice that the hard scale (14) rises as $\beta \to 1$. The QCD scaling violations in the gluon structure function are strong and at moderate values of $Q^2$ the crude approximation is

$$G^2\left(\frac{1}{2}x_p, Q^2\right) \propto Q^2 \gamma \left(\frac{1}{x_p}\right)^{2\delta_g} \propto \frac{\beta^{2\delta_g}}{(1 - \beta)^\gamma} \left(\frac{1}{x}\right)^{2\delta_g},$$

(16)

where $\gamma \sim 1$ and $2\delta_g \sim 0.4$-0.5 for moderate $Q^2$ and $x \lesssim 10^3$, see below. Both the scaling violations and, to a lesser extent the small-$x$ rise of gluon densities, enhance the contribution from $\beta \sim 1$. Then the contribution from intermediate open heavy flavor to the l.h.s. of (13) can be evaluated as

$$g_{LT}^{U}(x, Q^2) \approx -\frac{1}{30x^2} \frac{r_5(0)e_f^2}{(B_{LT} + B_5)^2m_p^2m_f^2} \cdot \alpha_s^2(Q^2)G^2\left(\frac{1}{2}x, Q^2 \approx m_f^2\right).$$

(17)

The numerical factor in the r.h.s. of (17) is only a crude estimate for $\gamma \approx 1$, it depends strongly on the pattern of scaling violations which for heavy flavours is under the control of pQCD.

The detailed discussion of pQCD hard scale for light flavour contribution to $g_{LT}^{D}$ is found in [3]. We only emphasize that because the dominant contribution to $\sigma_{LT}$ comes from $\beta \sim 1$ where the hard scale (14) is enhanced by the factor $\propto 1/1 - \beta$, even for light flavours $\sigma_{LT}$ receives a dominant contribution from hard to semihard gluons. The final result for $g_{LT}^{D}$ is similar to (15) with the substitution of $m_f^2$ by the semihard scale $Q^2 \approx (0.5$-1) GeV$^2$. Still it is not under the full control of pQCD because of substantial contribution of soft momenta in the quark loop integration. Also, because $g_{LT}^{D}$ changes the sign, the numerical results for the moment (13) depend on the pattern of scaling violations at the semihard scale $Q^2 \approx (0.5$-1) GeV$^2$ which can be affected by soft dynamics. Furthermore, the overall result for unitarity driven $g_{LT}$ is proportional to $\langle \Delta^2 \rangle$, which is the soft quantity controlled by the size of the target proton and large transverse size of diffractive $q\bar{q}$ states [24]. All this sensitivity to soft input notwithstanding, our unitarity driven $g_{LT}^{U}$ has precisely the same QCD status as standard
diffractive structure functions: it exists, it is a scaling phenomenon, its QCD evolution is reasonably well understood [25], but its numerical magnitude is not calculable from first principles of pQCD, although QCD motivated models do correctly reproduce all features of diffractive DIS [23, 26]. To this end we recall that no one has ever requested pQCD to provide the input for standard DGLAP evolution of the proton structure function.

The small-\(x\) dependence of \(x^2 g^{LT}_{LT}(x, Q^2)\) is the same as of the unpolarized diffractive structure function [22, 23] and, in principle, could be borrowed from experiment. The effective exponent \(\delta_g\) depends on \(x_g\), at \(x_g = 10^{-3}\) for light flavour contribution we find \(2\delta_g \approx 0.4\), for the numerically smaller charm contribution \(2\delta_g \approx 0.6\).

5 Numerical estimates: unitarity driven \(g^{LT}_{LT}(U)\) vs. the Wandzura-Wilczek relationship

The nucleon spin-flip defines a brand new skewed gluon distribution [27, 28], without going into details we only state that anomalous dimensions which control the small-\(x\) dependence of this skewed structure function are identical to those for unpolarized gluon distribution and the spin-flip parameter \(r_5\) would depend on neither \(x\) nor \(Q^2\). Zakharov’s sound arguments [2] in favor of non-vanishing \(r_5\) do not require pQCD and the existence of our unitarity driven \(g^{LT}_{LT}(x, Q^2)\) is beyond doubts. However, as a soft parameter \(r_5\) is quite sensitive to models of soft wave function of the nucleon [4]. Incidentally, it is of great interest for the polarimetry of stored proton beams and the whole spin physics program at RHIC [29]. Different model estimates of \(r_5\) and the experimental situation are summarized in recent review [29]. The experimental data on pion-nucleon scattering give \(|r_5| = 0.2 \pm 0.3\), the experimental data on proton-proton scattering leave a room for quite a strong spin-flip, \(r_5 = -0.6\) with about 100 per cent uncertainty. The theoretical models give \(|r_5| \lesssim 0.1-0.2\), the sign of \(r_5\) remains open.

Even if \(r_5\) were known, there will be corrections to our eikonal estimate (13) from proton excitations \(p^*\) (resonances and continuum) in the intermediate state (fig. 1b). Although SCHC diffraction excitation amplitudes are smaller than elastic ones (for suppression of diffraction excitation by the node effect see [30]), the inelastic \((p, p^*)\) spin-flip transitions can be enhanced compared to elastic \((p, p')\) ones [1]. Consequently, one can not exclude that the contribution of \(p^*\) excitations would enhance the effective \(r_5\) by a large factor. Here for the sake of definiteness we evaluate \(x^2 g^{LT}_{LT}(x, Q^2)\) assuming the conservative value \(r_5 = -0.1\). We take \(B_{LT} = 10\) GeV\(^2\) for light flavours and \(B_{LT} = 5\) GeV\(^2\) for charm as evaluated in [3], the slope \(B_5\) remains unknown and we put \(B_5 = 0\). This conservative estimate is shown in fig. 3, at the moment we can not exclude even one order in magnitude larger effect. At \(Q^2 \lesssim 4m_c^2\) the charm contribution to diffraction is small, the difference between small-\(Q^2\) and large-\(Q^2\) curves in fig. 3 illustrates the significance of charm contribution to \(g^{LT}_{LT}\). A crude parameterization of our numerical results for \(x \lesssim 10^{-3}\) and \(Q^2 = 5\) GeV\(^2\) is

\[
x^2 g^{LT}_{LT}(U)(x, Q^2) \approx r_5(0) \cdot 10^{-4} \left(\frac{0.001}{x}\right)^{0.4}
\]

It corresponds to spin symmetry

\[
A_2 \approx 6 \cdot 10^{-4} \cdot r_5(0) \frac{m_p}{\sqrt{Q^2}}
\]
which is approximately flat in the region of \( x = (10^{-3} - 10^{-4}) \), cf. the \( x \)-dependence of tensor asymmetry in [4].

The steep rise (18) of \( g^{(U)}_{LT} \) can not go forever as it would conflict the unitarity bounds. The same is true of the experimentally observed rise of unpolarized structure functions. The scale of unitarity effects is set by the ratio of diffractive to nondiffractive DIS [25, 31]. One could evaluate higher order unitarity effects consistently by the technique developed in [32], here we only notice that higher order unitarity corrections are known to play marginal role in related nuclear shadowing in DIS on even heaviest nuclei [33].

Above we focused on the two-pomeron cut contribution which dominates at very small \( x \). The related contribution from secondary reggeon-pomeron cut will be of the form

\[
x^2 g^{(RP)}_{LT} \propto \left( \frac{1}{x} \right)^{\delta_R + \alpha_R - 1}.
\]

Because of a large spin-flip coupling of secondary reggeons (for the review see [29]), this subleading term can well dominate the unitarity correction at moderately small \( x \). It will definitely dominate the small-\( x \) behaviour of proton-neutron difference \( g^p_{LT} - g^n_{LT} \). It could eventually be evaluated with the further progress in QCD modeling of reggeon effects in diffractive DIS [34]. With the reference to reggeon studies in [34], here we only emphasize that \( g^{(RP)}_{LT}(x, Q^2) \) is a scaling function of \( Q^2 \).

As a reference value for the comparison with our small-\( x \) result, we show in fig. 3 the so-called Wandzura-Wilczek (WW) result [35]

\[
x^2 g^{WW}_{LT}(x, Q^2) = x^2 \int_x^1 \frac{dy}{y} g_1(y, Q^2).
\]

The standard parameterizations and QCD ladder estimate [14] give \( g_1(x, Q^2) \sim \left( \frac{1}{x} \right)^{\delta_1} \) with the exponent \( \delta_1 \sim 0.5 \). Then \( x^2 g^{WW}_{LT}(x, Q^2) \) would vanish at small \( x \) as \( x^{2-\delta} \). To the extent that they are fitted to the same experimental data, all available parameterizations of \( g_1 \) give approximately the same WW integral, our WW curve shown in fig. 3 is for the parameterization [36]. Our diffractive mechanism takes over at \( x \lesssim 10^{-3} \).

The uncertain status of the WW relation has been much discussed in the literature [17, 18, 19]. The WW relation has never been supposed to, and evidently does not, hold in presence of such a diffractive component of \( g_{LT} \). However, in the spirit of duality sum rules one may still hope that full \( g_{LT} \) minus our diffractive component minus the \( R\overline{P} \) cut contribution has the required superconvergence properties and one can hypothesize that the WW relation is applicable to \( g_{LT} - g^{(U)}_{LT} \). In other words, it is tempting to identify our unitarity effect \( g^{(U)}_{LT} \) with the long sought deviation from the WW relation and to hypothesize that WW relation would hold approximately at large and moderate \( x \) where the diffractive component is numerically small. This seems to be the case in the experimentally studied range \( x \gtrsim 0.01 \) [37].

As well known, \( g_{LT}(x, Q^2) \) does not admit any obvious parton model interpretation. The OPE content of unitarity corrections to structure functions deserves a dedicated study. We only notice that in close similarity to leading twist unpolarized diffractive structure function [23], the upper blob in diagrams of fig. 2 receives a substantial contribution from large and moderate transverse distances. So to say, in the quark loop we are way along the light cone, but finite, not \( 1/Q \), transverse distance from the light cone.
6 Unitarity driven $g_{LT}^{(U)}$ from vector meson production?

We wish to point out that diffractive vector mesons offer a direct experimental window at effective $r_5(0)$ which sums up the elastic and inelastic rescattering contributions. Evidently, unitarity diagrams of figs. 1,2 do contribute to $\gamma^* p_\lambda \rightarrow V p'_\rho$, too. Our diffractive mechanism would give rise to finite $A_{0-\frac{1}{2},+1+\frac{1}{2}}^V(\Delta = 0)$, whereas in two-parton ladder (= one pomeron exchange) $A_{0-\frac{1}{2},+1+\frac{1}{2}}^V(\Delta) \propto \Delta^2$. The marginal difference from the above evaluation of diffractive $g_{LT}$ is emergence of $q\bar{q}V$ vertex instead of one of the pointlike QED $q\bar{q}\gamma$ vertices and that the two gluon distributions in vector meson production are skewed differently than in the calculation of $g_{LT}^{(U)}$, but the ratio of diffraction driven SCHNC LT and SCHC conserving dominant one-pomeron amplitudes can be worked out. What counts is that the lower blob of the diagram is identical to that in the case of $g_{LT}^{(U)}$. For the experimental determination of $A_{0-\frac{1}{2},+1+\frac{1}{2}}^V(\Delta = 0)$ one needs to isolate the polarization dependence of production of longitudinally polarized vector mesons by circular polarized photons on transverse polarized targets, which is doable experimentally because decays of vector mesons are self analyzing. Notice, that unitarity considerations in section 3 were quite general and did not require an applicability of pQCD, finite $Q^2$ was only needed to have longitudinal photons. In the case of vector mesons, nonvanishing $A_{0-\frac{1}{2},+1+\frac{1}{2}}^V(\Delta = 0)$ is possible not only with virtual photons in polarized DIS but also for circular polarized real photons. Although real photoproduction of $\rho$ and $\phi$ mesons will be utterly nonperturbative process, the cross sections are large, high luminosity external real photon beams can readily be produced either by laser backscattering or coherent bremsstrahlung in crystals, and vector meson production seems to be an ideal testing ground for existence of diffraction driven $g_{LT}^{(U)}$. Because much of the dependence on the model of the vector meson wave function would cancel in the ratio of SCHNC spin-flip and SCHC non-flip amplitudes, even nonperturbative real photoproduction of vector mesons would provide useful constraints on $r_5(0)$.

7 Can the Burkhardt-Cottingham sum rule be salvaged?

Our diffraction driven contribution to $g_{LT}(x,Q^2)$ rises at small $x$ faster than $g_1(x,Q^2)$. Incidentally, the diffractive mechanism does not contribute to the difference helicity of amplitudes which gives rise to the spin asymmetry $A_1$. Consequently, at small $x$ the diffraction driven $g_{LT}^{(U)}$ is dominated by $g_2$. The resulting small-$x$ rise of $g_2$ invalidates the superconvergence assumption behind the derivation of the BC sum rule [1]. There were suggestions reviewed in [17, 19] that the BC sum rule might be salvaged because the residues of pomeron cuts might vanish at large $Q^2$. This is not the case with our diffractive $g_{LT}^{(U)}(x,Q^2)$ which is a manifestly scaling function of $Q^2$. To this end we recall that unpolarized diffractive DIS is a well established scaling phenomenon (22, 23, 26), for the corresponding phenomenology and review of the HERA data on diffractive DIS see [23, 88].
8 Impact of diffraction driven $g_{LT}^{(U)}$ to extraction of $g_1$
from longitudinal asymmetry

As well known there is a $\propto \frac{1}{Q^2}$ correction from $g_2$ in the extraction of the small-$x$ spin structure function $g_1$ from asymmetry $A_1$:

$$A_1 = \frac{1}{F_1(x, Q^2)} \left\{ g_1(x, Q^2) - \frac{4m_p^2x^2}{Q^2} g_2(x, Q^2) \right\}$$

$$= \frac{1}{F_1(x, Q^2)} \left\{ g_1(x, Q^2)(1 + \frac{4m_p^2x^2}{Q^2}) - \frac{4m_p^2x^2 g_{LT}(x, Q^2)}{Q^2} \right\}$$  (21)

The small-$x$ growth of $x^2 g_{LT}^{(U)}(x, Q^2)$ invalidates the common lore assumption that this correction can be neglected at low $x$. Quite to the contrary, it is not dissimilar to, or even somewhat faster than, the usually discussed small-$x$ rise of $g_1(x, Q^2)$ and this term can not be neglected off hand. Our conservative estimate \[18\] suggests that it is small, though.

9 Summary and conclusions

We have shown how $s$-channel helicity nonconserving LT interference in diffractive DIS in conjunction with the pomeron spin-flip in diffractive nucleon-nucleon scattering gives rise to a steep rise eq. (1) of the spin structure function $g_{LT}(x, Q^2)$ at small $x$. The transverse spin asymmetry considered in this paper and tensor spin asymmetry discussed earlier in \[7\] fall into a broad family of unitarity (diffraction) driven spin effects which, in the opposite to the common wisdom, persist in high energy and/or small-$x$ limit (the work on straightforward extension of the above to related spin structure functions in DIS off photons is in progress). The rate of rise of diffraction driven $g_{LT}^{(U)}(x, Q^2)$ is related to that of other experimental observable - the unpolarized gluon structure function of the target proton or, still better, to the experimentally measurable $x_F$ dependence of unpolarized diffractive structure function. Whether the diffraction driven $x^2 g_{LT}^{(U)}(x, Q^2)$ is numerically large or small is not an issue, the crucial point is that the found rate of the small-$x$ rise of $g_{LT}(x, Q^2)$ invalidates the superconvergence assumptions behind the Burkhardt-Cottingham sum rule and behind the Wandzura-Wilczek relation. There exists an interesting possibility of testing the existence of diffractive mechanism for $g_{LT}$ in vector meson production by circular polarized real photons on transverse polarized proton targets, which deserves dedicated study.

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Figure captions.

Fig. 1. The unitarity diagram with (a) ‘elastic’ $pX$ intermediate state and (b) ‘inelastic’ $p^*X$ intermediate state with excitation of the proton to resonances or low-mass continuum $p^*$.

Fig. 2. (a) The QCD model for unitarity diagram of Fig. 1. The lower blobs are related to unintegrated gluon structure function of the proton and contain the pomeron spin-flip amplitude. (b) The pQCD Feynman diagram content of the shaded upper blobs.

Fig. 3. The conservative estimate assuming $r_5(0) = -0.1$ for the unitarity driven $x^2 g_{LT}^{(U)}(x, Q^2)$ vs. the expectation from WW relation (the dashed curve) for GS parameterization for $g_1(x, Q^2)$. The difference between the curves for $Q^2 = 5$ GeV$^2$ (diamonds) and $Q^2 = 100$ GeV$^2$ (triangles) is due to the charm contribution at large $Q^2$. 
\[ \gamma_L^* \rightarrow X \rightarrow \gamma_T^* \]

\[ \begin{array}{c}
\text{IP} \\
\Delta \\
\Delta \\
\text{IP}
\end{array} \]

\[ p \quad p \quad p \]

\[ \begin{array}{c}
\text{IP} \\
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\end{array} \]

\[ p \quad p^* \quad p \]

\[ \gamma_L^* \rightarrow X \rightarrow \gamma_T^* \]

\[ \begin{array}{c}
\text{IP} \\
\Delta \\
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\end{array} \]

\[ p \quad p \quad p \]

\[ \begin{array}{c}
\text{IP} \\
\Delta \\
\Delta \\
\text{IP}
\end{array} \]

\[ p \quad p^* \quad p \]

a) b)
\[ \gamma T = \frac{1}{2} \left( \frac{q(q+p) - m^2}{(p+q)^2 - m^2} \right) \]

\[ = \gamma + \gamma + \gamma + \gamma \]
Unitarity effect:

- $Q^2 = 5 \text{ GeV}^2$
- $Q^2 = 100 \text{ GeV}^2$

$\chi^2 g_{\text{LT}}(x, Q^2)$