Entanglement through qubit motion and the dynamical Casimir effect

Andrés Agustí,1 Enrique Solano,2,3,4 and Carlos Sabin1

1 Instituto de Física Fundamental, CSIC, Serrano, 113-bis, 28006 Madrid (Spain)
2 Department of Physical Chemistry, University of the Basque Country UPV/EHU, Apartado 644, 48080 Bilbao, Spain
3 IKERBASQUE, Basque Foundation for Science, Maria Diaz de Haro 3, 48013 Bilbao, Spain
4 Department of Physics, Shanghai University, 200444 Shanghai, China

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We explore the interplay between acceleration radiation and the dynamical Casimir effect (DCE) in the field of superconducting quantum technologies, analyzing the generation of entanglement between two qubits by means of the DCE in several states of qubit motion. We show that the correlated absorption and emission of photons is crucial for entanglement, which in some cases can be linked to the notion of simultaneity in special relativity.

I. INTRODUCTION

Superconducting quantum technologies together with circuit quantum electrodynamics (cQED)1 form one of the most promising candidates for processing quantum information as well as the experimentation on the foundations of quantum mechanics. The advantages of this technology are, on the one hand, the strong coupling between the superconducting qubits and the resonant cavities, as well as the ability to widely tune this coupling and all the parameters of the system, allowing the investigation of new phenomena2-3 such as, among many others, the Dynamical Casimir Effect (DCE)4 or the Unruh effect5.

The DCE is a member of a large family of effects linked to the quantum fluctuations of the vacuum, among which are the Lamb displacement6, the magnetic moment of the electron or the Casimir (static) effect7-9. The latter is produced by a reduction in the density of modes imposed by certain boundary conditions, which leads to a pressure of radiation exerted by the vacuum. Its dynamical counterpart shares a similar origin, except for the fact that boundary conditions must be time-dependent, which can be achieved by means of SQUIDs4. This effect has also been measured by modulating the effective speed of light10. On the other hand, the DCE can be considered a resource to generate quantum correlations, including quantum entanglement11-15. In this paper, we will focus on this feature.

The Unruh effect is another member of the family of aforementioned phenomena, since it consists in the measurement of thermal radiation by a detector moving with constant proper acceleration through the quantum vacuum16-17. Unfortunately, this has never been observed since it requires unachievable accelerations to generate detectable signals, although it can be increased by several orders of magnitude when two-level systems are accelerated through resonant cavities by means of non-adiabatic boundary conditions18. However, this modification of the original effect is still difficult to reach experimentally and has not been observed. In this report we will analyse another scenario: it is possible to simulate the cavity-enhanced Unruh effect by modulating the qubit-cavity coupling19. In the same way as with the DCE, this simulation of the Unruh effect can be used as a mechanism for the generation of entanglement20.

In this paper we consider an scenario where both the DCE and the simulated enhanced Unruh radiation can take place (see Fig. 1). It consists of two superconducting resonators sharing a common SQUID and each of them coupled to a superconducting qubit. DCE radiation can be generated by means of the modulation of the magnetic flux threading the SQUID, while the coupling of the qubits can be tuned to simulate their motion. We show that the correlated absorption and re-emission of the DCE radiation by the qubits is crucial for the generation of entanglement, as in the case where the qubits are static13. If the motion of the qubit preserves the correlated nature of the absorptions and emissions, entanglement is preserved as well. However, in general, uncorrelated motion of the qubits will result in the vanishing of entanglement, even for low simulated velocities. In the case of equal-length cavities, this physics can be linked to the notion of simultaneity in special relativity: breaking down simultaneity in the absorption would make entanglement vanish.

The rest of the paper has been structured as follows. First, we present the superconducting setup, its Hamiltonian and some relevant features. Then in the next section, we discuss the results of, on the one hand, applying perturbation theory to the calculation of the concurrence, and on the other hand, calculating this same magnitude solving the master equation of the system numerically. Finally, we summarize our conclusions.

II. SETUP: DCE AND SIMULATED QUBIT MOTION

We consider a system composed of two superconducting qubits -in particular, modifications of the usual design of a transmon qubit21- whose coupling to the electromagnetic field can be controlled by the magnetic flux threading the SQUIDs that compose them, which offers an opportunity to simulate the generation of acceleration radiation in the cavity-enhanced Unruh effect. In addi-
The first line contains the static Hamiltonian, where the sum runs over both cavities and both qubits. Experimentally, the product \( \phi \) being indicated the transmission line resonators or cavities, of which their characteristic frequencies \( \omega_{1,2} \) as well as the coupling between them. The capacitive coupling between cavities and qubits mentioned above is also indicated in red.

The Hamiltonian is

\[
H = \hbar \sum_{i=1}^{2} \left[ \omega_{c_i} \left(a_i^\dagger a_i + \frac{1}{2}\right) + \frac{\omega_{g_i}}{2} \sigma_i^z \right] + \\
+ \hbar \sum_{i=1}^{2} g_i \cos(k_i x_{q_i}(t)) \sigma_i^x (a_i^\dagger + a_i) + \\
+ \hbar g_{1,2}(t)(a_1^\dagger + a_1)(a_2^\dagger + a_2),
\]

where the sum runs over both cavities and both qubits. The first line contains the static Hamiltonian, where \( \omega_{c_i} \) is the frequency of cavity 1 or 2, depending on the subscript, as well as \( \omega_{g_i} \), that is of the qubits. On the other hand, \( a_i^\dagger \) and \( a_i \) are the creation and annihilation operators of the corresponding cavity and \( \sigma_i^z \) the third Pauli operator of each qubit. The second line of the equation contains the interaction of the qubits with their cavities, being \( k_i \) the wave vector of the cavity, \( \omega_q \) the maximum intensity of its coupling, \( \sigma_i^x \) the first Pauli operator of the qubit and \( x_{q_i}(t) \) simulated trajectory of qubit motion. Experimentally, the product \( k_i x_{q_i}(t) \) is actually \( f = \phi(t)/\phi_0 \) with \( \phi_0 \) the quantum of magnetic flux and \( \phi(t) \) the flux through the SQUID [19, 20]. The third line contains the interaction between the cavities, being \( g_{1,2}(t) \) the time-dependent coupling assuming that the boundary conditions produced by the SQUID do not destroy the structure of normal field modes or make resonances of new modes with the qubits. This is the case when \( g_{1,2}(t) = g_0 \cos(\omega_{d} t) \) with \( \omega_{d} = \omega_{c_1} + \omega_{c_2} \). Moreover, in this case the interaction Hamiltonian can be approximated by a two-mode squeezing Hamiltonian [13]:

\[
g_{1,2}(t)(a_1^\dagger + a_1)(a_2^\dagger + a_2) \rightarrow g_0/2(a_1^\dagger a_2^\dagger + a_1 a_2). \tag{2}
\]

So, finally:

\[
H = \hbar \sum_{i=1}^{2} \left[ \omega_{c_i} \left(a_i^\dagger a_i + \frac{1}{2}\right) + \frac{\omega_{g_i}}{2} \sigma_i^z \right] + \\
+ \hbar \sum_{i=1}^{2} g_i \cos(k_i x_{q_i}(t)) \sigma_i^x (a_i^\dagger + a_i) + \\
+ \hbar g_{1,2}(t)(a_1^\dagger e^{i\omega_{d} t} + a_1)(a_2^\dagger e^{-i\omega_{d} t}), \tag{3}
\]

Previous work [19] has shown that, even for moderate values of the coupling, the modulation of the coupling strength might resonate with the counterrotating terms of the Hamiltonian. Therefore, we will not perform the rotating wave approximation (RWA).

Finally, notice that if either or both of the trajectories \( x_i(t) \) are changed by \( L_i - x_i(t) \) and the relevant coupling constant \( g_i \) is inverted, then the Hamiltonian does not undergo any change, which is quickly deduced using the expression of the cosine of the sum and substituting \( k_i = \pi/L_i \). This symmetry can be interpreted as a mechanism by which a path \( x_{q_i}(t) \) that passes between the two ends of the cavity in a finite time can be extended beyond that time reflecting it with respect to the center of the cavity. That is, if for a time \( \tau \) we have \( x_i(\tau) = L_i \) and at a later time \( x(\tau + \delta t) \) exceeds \( L_i \), then the path can be modified as follows \( x_i(\tau + \delta t) = L_i - x_i(\delta t) \), producing the same Hamiltonian which governed the evolution up to \( \tau \), except for the sign of \( g_i \). In other words, this symmetry offers a natural bounce condition to continue trajectories that reach the ends of the cavities, so natural that it will prove useful throughout the work.

### III. RESULTS

We will use two different methods to analyze entanglement generation in the setup discussed in the previous section, namely perturbation theory to obtain an approximate expression of the concurrence and then numerical simulations to integrate the master equation governing the system. These two methods make up the following two subsections.
A. Perturbative results

Expanding the evolution for the global state $|\Psi(t)\rangle$ under hamiltonian [3] up to third order in the couplings $g_1$, $g_2$, $g_0$ gives non-zero projections onto the states: \{00gg, 11gg, 00eg, 01eg, 00ee, 22gg, 21ge, 12eg, 33gg\}, where \{0, 1\} are Fock number states of each cavity and \{g, e\} are the ground and excited states of each qubit. We aim to compute the entanglement dynamics of the qubits, so it is necessary to trace over the cavity field states. We find:

$$\rho_{\text{qubits}} = \begin{pmatrix} \rho_{11}^{(6)} & 0 & 0 & \rho_{14}^{(3)} \\ 0 & \rho_{22}^{(4)} & 0 & 0 \\ 0 & 0 & \rho_{33}^{(4)} & 0 \\ \rho_{41}^{(3)} & 0 & 0 & \rho_{44}^{(6)} \end{pmatrix}$$

where $\rho_{11}$ is the matrix element $\langle ee | \rho | ee \rangle$, $\rho_{14}$ the matrix element $\langle ee | \rho | gg \rangle$ and so on. The superscript indicates its perturbative order with respect to the coupling constants and if we restrict ourselves to the third order, only the non-diagonal elements survive. Then, using the concurrence $C(\rho)$ [22], as entanglement measurement, we find:

$$C(\rho) = 2|\rho_{14}| = 2|\langle ee | \rho | gg \rangle| = 2|\langle 00ee | \rho_{\text{total}} | 00gg \rangle| = g_1 g_2 g_0 \times$$

\[ \int_0^t dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \cos(f_1(t_1)) \cos(f_2(t_2)) e^{i \omega_d t_3} + \int_0^t dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \cos(f_1(t_1)) \cos(f_2(t_2)) e^{i \omega_d t_3}, \]

where $f_i = k_i x_i(t)$. It should be noted that in this system the concurrence can be interpreted in a very intuitive fashion: it is proportional to the probability that two photons are emitted, one in each cavity, and that each qubit absorbs one. At first sight, Eq. (5) seems to conclude that the qubit motion will only reduce concurrence, since integrating cosines will not provide any further contribution. However, these can cause a resonance, either with each other or with the term due to the emission. In the following paragraphs, we will study these resonances produced by different trajectories, paying attention to the conditions that must be fulfilled for their existence.

1. Static qubits

If the qubits are static, Eq. (5) further simplifies, since the cosine functions are all constant. By performing the first integration, we get

$$\int_0^{t_2} dt_3 e^{i \omega_d t_3} = e^{i \omega_d t_2} - 1/i \omega_d$$

and the constant term will eventually give rise to:

$$C_{\text{rest}} = \frac{g_0 g_1 g_2}{\omega_d} t^2 + O(t). \quad (6)$$

This quadratic behaviour of the concurrence seems to be in agreement with [13] for the moderate values of time where the perturbative approach is valid -the perturbative approximation will eventually break down, as we will see in detail below.

2. Constant velocity

Particularizing Eq. (5) for the case of qubits moving with constant velocities $v_i$ with initial positions at $x = 0$, we get:

$$C_{\text{const}} = \frac{g_1 g_2 g_0}{4 \omega_d} \times$$

\[ \int_0^t dt_1 \int_0^{t_1} dt_2 (e^{ik_1 v_1 t_1} + e^{-ik_1 v_1 t_1}) \times \\ (e^{ik_2 v_2 t_2} + e^{-ik_2 v_2 t_2}) (e^{i \omega_d t_2} - 1) + \\ + \int_0^t dt_1 \int_0^{t_1} dt_2 (e^{ik_1 v_1 t_2} + e^{-ik_1 v_1 t_2}) \times \\ (e^{ik_2 v_2 t_1} + e^{-ik_2 v_2 t_1}) (e^{i \omega_d t_2} - 1) \quad (7) \]

By inspection of Eq. (7), we find that resonances will appear if either or both following conditions are verified:

$$\omega_d = k_1 v_1 \\ k_1 v_1 = k_2 v_2$$

Note that $\omega_d = \omega_1 + \omega_2$, so the above conditions turn into:

$$c/L_1 + c/L_2 = |v_1|/L_i \\ |v_1|/L_1 = |v_2|/L_2, \quad (9)$$

The first one implies a superluminal velocity of at least one of the qubits, and was already found in [23]. It is related with the emission of Ginzburg radiation at superluminal constant velocities. The second one -since both qubits start in the same position- means that the distance of the qubits to $x = 0$ -in units of the corresponding cavity length- is always the same for both qubits $x_1/L_1 = -x_2/L_2$. Then the hamiltonians of both qubits are equivalent at any time, entailing that the absorption and re-emission of DCE photons is perfectly correlated. In particular, if the lengths of the cavities are equal $L_1 = L_2$ then the distances are exactly the same, which means that absorptions and emissions occur always simultaneously. This suggests an interesting link between the generation of entanglement in a quantum setup and a key notion of special relativity, such as simultaneity.

When both conditions in Eq. (9) are met at the same time, namely $|v_1|/L_1 = |v_2|/L_2 = c/L_1 + c/L_2$, then we find:

$$C_{\text{const}, 1} = \frac{g_0 g_1 g_2}{\omega_d^2} | \sin(\omega_d t)| t + O(t^0), \quad (10)$$
while if only the first condition is met \( |v|/L = c/L + c/L_2 = kv/\pi \), then
\[
C_v \text{ const, } = \frac{g_0 g_1 g_2}{2\omega_d k_1} \sin(kvt) t + O(t^3).
\]
In the latter case, it is the velocity of the non-resonant qubit and not the mirror frequency what modulates the generation of concurrence. Even if we assume \( kv \approx \omega_d \), then the concurrence in Eq. (11) is reduced by a factor 1/2 with respect to the concurrence in Eq. (10), which highlights again the importance of the correlations among the absorptions and emissions of photons.

Interestingly, we can use the symmetry of the Hamiltonian described at the end of Section III that in this type of trajectory translates into an inversion of the speed of the qubits when they try to leave the cavities. The full symmetry would change the sign of the relevant coupling, but these signs can be canceled if both trajectories arrive at the ends of the cavities simultaneously and their speeds are then inverted. With these bounces, trajectories of constant velocity can be extended in time. The concurrence (7) inherits this Hamiltonian symmetry: after \( n \) bounces the generated entanglement is \( n \) times the entanglement after the first bounce. We see that in this case the simultaneity in the bounce plays a crucial role by simplifying the calculation of the concurrence and extending their relevance. Finally, if the second condition in Eq. (9) is met but not the first it is not possible to obtain a closed analytical expression of the concurrence. However, we will present numerical results in the next section.

3. Other trajectories

A convenient family of qubit trajectories would be given by:
\[
x_1(t) = \frac{L_1}{\pi} \arccos \left(2 \left(\frac{t}{\tau}\right)^n - 1\right)
\]
\[
x_2(t) = -\frac{L_2}{\pi} \arccos \left(2 \left(\frac{t}{\tau}\right)^n - 1\right),
\]
\( \tau \) being the flight time of the qubits, namely the time that it takes for each qubit to traverse its cavity. These trajectories can be seen in Fig. (2). They exhibit divergencies in the velocity and the acceleration when time gets close to the flight time, and also at \( t = 0 \) for the particular case \( n = 1 \).

We find the following concurrence:
\[
C_{\text{arccos}} = \frac{4g_0 g_1 g_2}{\omega_d(n+1)^2} \frac{v^{2n}}{2^n} t^{2n+2} + O(t^{2n}).
\]
Therefore, with these trajectories we are able to produce resonances with arbitrary powers of time. Of course, all the above is restricted by the perturbative approximations adopted, which will eventually break down. We now proceed to show results obtained by a numerical integration of the master equation of the system, which enables the exploration of the long-time dynamics.

B. Numerical results

1. Static qubits and constant velocities

We consider four trajectories (see Fig. (3)). The first one is the case where the qubits are static, already discussed in [13]. Then, we consider the trajectory analyzed perturbatively in the previous section, where the qubits start at \( x = 0 \) and move with opposite velocities, giving rise to correlated -simultaneous for equal cavities- absorption and emission of photons. The third case is related to the second by the symmetry relation discussed throughout this work, since one qubit starts at the other end of its cavity. Finally, we consider a trajectory which breaks the correlations among the absorptions and emissions of photons, since one qubit starts out at the center of its cavity while the other starts at 0.

In Fig. 4 we show the numerical results for the concurrences at long times. We reproduce the results for the static case in [13] up to the point of maximum concurrence, where they propose to switch off the coupling in order to optimize the entanglement generation. We show that in the two trajectories which preserve the correlated absorptions and emissions the high concurrence is indeed preserved, even achieving larger maximum values. Finally, in the asymmetric case, the concurrence is significantly reduced, as expected. Interestingly, this effect occurs already at low non-relativistic velocities, which highlights the role of the correlation/simultaneity in the generation of entanglement in this setup. In order to
achieve the regime of large velocities, it is convenient to use trajectories similar to the ones in Section III A 3. We will explore them in the next subsection.

2. Other trajectories

In this case, we have considered the following trajectories (see Fig. 5). We first use similar trajectories as in Eq. (12), but extended by means of the bounce symmetry:

\[ f_n(x) = \frac{1}{\pi} \arccos(2x^n - 1) \]

\[ x_1(t) = -L_1 f_n \left( \frac{t}{\tau} - \left\lfloor \frac{t}{\tau} \right\rfloor \right) \quad \text{if} \quad \left\lfloor \frac{t}{\tau} \right\rfloor \text{even} \]

\[ = -L_1 + L_1 f_n \left( \frac{t}{\tau} - \left\lfloor \frac{t}{\tau} \right\rfloor \right) \quad \text{if} \quad \left\lfloor \frac{t}{\tau} \right\rfloor \text{odd} \]

\[ x_2(t) = L_2 f_n \left( \frac{t}{\tau} - \left\lfloor \frac{t}{\tau} \right\rfloor \right) \quad \text{if} \quad \left\lfloor \frac{t}{\tau} \right\rfloor \text{even} \]

\[ = L_2 - L_2 f_n \left( \frac{t}{\tau} - \left\lfloor \frac{t}{\tau} \right\rfloor \right) \quad \text{if} \quad \left\lfloor \frac{t}{\tau} \right\rfloor \text{odd}, \quad (14) \]

\[ \tau = v_1/L_1 = v_2/L_2 \] being again the time of flight which takes the qubits to traverse their cavities and \(|x|\) being the floor function. In this case, as can be seen in Fig. 5, the trajectories are synchronized in such a way that the absorption and emission of photons is correlated.

Another case that we consider is obtained by replacing in Eq. (14) \(x_2(t)\) by \(x_2(t+\tau/2)\), which preserves the symmetry between the qubits. Finally, making instead the replacement with, for instance, \(x_2(t+0.1\tau)\), then the qubits are out of phase. We show the concurrences for these trajectories in Fig. 6, which again show that the correlations/simultaneity between the qubits are crucial to understand the magnitude of entanglement generation. In order to further illuminate this point, it is interesting to discuss the population in the Bell basis. In the static case and all the cases where the qubit positions are correlated...
FIG. 6. Concurrences for the trajectories in Fig. 5, using the same color code, but with down-facing triangle markers for green, right-facing for red and up-facing for cyan. As a reference, we plot in dark blue with left-facing triangles the static case.

in the way discussed above, photons are emitted and absorbed in pairs, and therefore it is expected that all the population is in the Bell states $|\phi_{\pm}\rangle = 1/\sqrt{2}(|gg\rangle \pm |ee\rangle)$. However, if the positions of the qubits are uncorrelated it is possible that one of the qubits emits or absorbs a photon while the other does not. This enables the population of the other Bell states $|\psi_{\pm}\rangle = 1/\sqrt{2}(|ge\rangle \pm |eg\rangle)$. In Fig. 7 we confirm that this is indeed the case: only in the low-concurrence case a significant population eventually appears in $|\psi_{\pm}\rangle$. Comparing Fig. 6 with 7 we see that jumps in the population of $|\psi_{\pm}\rangle$ are correlated with falls in the value of the concurrence, as expected.

IV. CONCLUSIONS

We have analyzed the entanglement dynamics between two qubits in a system where each one interacts with a resonant cavity with tunable coupling, which allows to simulate their motion. The cavities interact in turn with each other through a SQUID, which implements a boundary condition that can be modulated by the magnetic flow threading it. This results in a two-mode squeezing hamiltonian which is the source of entanglement. We show that a high degree of entanglement can be generated both in the case where the qubits are static -previously discussed in [1] - and in the cases where the motion preserves the fact that photons are absorbed and re-emitted in pairs -one by each qubit- populating only the Bell states $|\phi_{\pm}\rangle$. Otherwise, if the motion of the qubits is such that photons can be emitted or absorbed by only one qubit, we find that the states $|\psi_{\pm}\rangle$ are also populated and the concurrence is dramatically reduced. If the cavities are equal in length, this means that high-concurrence trajectories are characterized by simultaneous absorption and emission of photons, which suggests an interesting link with the notion of simultaneity in special relativity.

These results pave the way for the exploration of special relativistic effects in a quantum setup. For instance, we envision the quantum simulation of the gedanken textbook experiments where trains moving at relativistic speeds are used to illustrate the relativity of simultaneity. In our setup, the magnitude of entanglement could be used, in principle, as a witness of simultaneity, and viceversa. Our results are fully within reach of current technology.

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