Optimal Design for PID Controller Based on DE Algorithm in Omnidirectional Mobile Robot

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Abstract. This paper introduces an omnidirectional mobile robot based on Mecanum wheel, which is used for conveying heavy load in a small space of the automatic warehousing logistics center. Then analyzes and establishes the omnidirectional chassis inverse and forward kinematic model. In order to improve the performance of motion, the paper proposes the optimal PID controller based on differential evolution algorithm. Finally, through MATLAB simulation, the results show that the kinematic model of mobile robot chassis is correct, further more the controller optimized by the DE algorithm working better than the traditional Z-N PID tuned. So the optimal scheme is reasonable and feasible, which has a value for engineering applications.

1 Introduction

Mobile robots are increasingly used in automated industrial environments [1]. The mobile robot technology includes the latest research results of many subjects, such as machinery, sensors, electronic, computer, information processing, automatic control and the artificial intelligence, it represents the highest achievement of electromechanical integration [2].

The Mecanum wheel shown in Fig. 1 which is widely used in omnidirectional mobile mechanism is a kind of the mature technology. It first proposed in 1973 by a Swedish engineer called Bengt Ilon [3]. The Mecanum wheel bearing capacity is strong, the relative position between wheel and suspension on the omni-directional mobile mechanism is fixed, it turns without independent steering mechanism, just depends on coordinating 4 wheels velocity and direction that can realize the omnidirectional movement, and mechanical structure of the system is simple, reliable and easy to control [4].

2 Kinematics analysis

In order to establish and do the research on the kinematics model, we construct the reference coordinate system shown in Fig. 3. The coordinate system origin $O_XY$ is based on the center of the mechanical structure of the mobile platform, and the coordinate system origin $O_{XY'}$ is based on the center of the mechanical structure of each Mecanum wheel, any of the coordinate system conforms to the right-hand rule.

Figure 1. Structure of the Mecanum wheel.

Figure 2. Structure of the mobile robot platform.

In particular, the installation of the wheel body has deferent spin around (the left forward and right rear wheel is left-spin, the right forward and left rear wheel is right-spin, specifically, the angle between wheel axis and roller axis is 45°). it’s worth noting that the figure 1-a is observed from top to down of the omnidirectional mobile robot platform, as well as the oblique dashed lines drawn in each wheel is the contact line between roller surface and road surface. In Fig. 3, L is the distance that along Y-axis between the center of wheel and chassis, 1 is the distance that along X-axis between the center of wheel and chassis, and we assume that this mobile platform is running with the velocity $(V_x, V_y, W_z)$, at the same time,
the wheel i is running with the line velocity $V_i$ (angular velocity is $W_i$), $\alpha_i$ is the offset angular of wheel i, $R$ is the radius of each Mecanum wheel.

According to the reference paper [5], it deduced the general kinematic mathematical model of the 4 Mecanum wheels omnidirectional mobile platform.

$$[\alpha_1, \alpha_2, \alpha_3, \alpha_4] = K_i [V_x, V_y, V_z]$$  \hspace{1cm} (1)

in formula (1), the inverse kinematic matrix $K_i$ is:

$$K_i = \begin{bmatrix}
cota  & 1 & -(Lcota + 1) \\
-cota & 1 & -(Lcota + l) \\
-cota & 1 & Leota + l \\
cota & 1 & Leota + l
\end{bmatrix}$$  \hspace{1cm} (2)

Thus, we transform the matrix formula (1), then get the forward kinematic model in formula (3).

$$[V_x, V_y, V_z] = K_i [\alpha_1, \alpha_2, \alpha_3, \alpha_4]$$  \hspace{1cm} (3)

in formula (3), the forward kinematic matrix $K_i$ is:

$$K_i = \frac{R}{4} \begin{bmatrix}
tana & -tana & -tana & tana \\
1 & 1 & 1 & 1 \\
-1 & -1 & 1 & 1 \\
Leota + l & Leota + l & Leota + l & Leota + l
\end{bmatrix}$$  \hspace{1cm} (4)

From the above formulas, it shows that the velocity of omnidirectional mobile robot is decided by the cooperation from revolving velocity of the 4 Mecanum wheels. The detailed moving direction by different cooperation wheel speed of the chassis is shown in Fig. 4.

3 Principle of DE

Differential Evolution (DE) Algorithm is a simple and efficient method to solve the problem of global optimization, it is also a kind of stochastic search algorithm based on population [6]. The DE Algorithm first proposed by Rain Storn et al. who came from California State University at Berkeley[7]. In view of the advantage of efficient, DE has drawn great attention of the relative scholars all over the world, which quickly became a hot research topic of optimizing computation.

3.1 Initialization

The initial population is generated a size of $N_p$ randomly which include D dimension vector in each population, for example:

$$X_{iG} = \{X_{iG}^1, X_{iG}^2, ..., X_{iG}^D\}, i = 1, 2, ..., N_p$$  \hspace{1cm} (5)

In formula (5), the $N_p$ is a constant, every individual need make mutation, recombination and selection in turn, generations of population to be made evolution until computing over.

3.2 Mutation

The mutant vector individual is computed by the follow formula.

$$V_{iG} = X_{iG} + F(X_{r1G} + X_{r2G} + X_{r3G}), r_1, r_2, r_3 \epsilon [1, 2, ..., N_p]$$  \hspace{1cm} (6)

In formula (6), $i \neq r_1 \neq r_2 \neq r_3$, F is a real constant factor to control the differential variation and $F > 0$. The $V_{iG}$ is prepared for the next generation mutation vector, $X_{iG}$ is the current generation vector from different population.

3.3 Recombination

The recombination operation cross over each pair of the target vector $X_{iG}$ and its corresponding mutant vector $V_{iG}$ for generating a trial vector $U_{iG}$
In the basic version, DE employs the crossover defined as follows:

\[ U_{i,t}^{j} = \begin{cases} V_{i,t}^{j}, & \text{if } (\text{rand}[0,1] \leq CR) \text{ or } j \neq \text{rand}j \, \forall j, j = 1, 2, \ldots, D \end{cases} \]

In formula (8), CR controls the fraction of parameter values copied from the mutant vector, CR is a user-defined real constant and rand j is a random real constant, they are both between the range \([0,1]\).

### 3.4 Selection

The DE algorithm uses greedy method that the best individuals are selected to replace parent individuals to generate new individual in general evolutionary strategies. The selection operation can be expressed as follows:

\[ X_{i,t}^{j} = \begin{cases} U_{i,t}^{j}, & \text{if } f(U_{i,t}^{j}) \leq f(X_{i,t}^{j}) \, \forall j, j = 1, 2, \ldots, D \end{cases} \]

In formula (9), the fitness function \( f(x) \) is to evaluate the best result (individual).

### 4 Optimal controller design

As we know, PID controller has been widely used in most industry control system, optimizing the PID parameters has been a considerable topic for the people [8]-[10]. In this paper, we use DE algorithm for optimizing and tuning parameters of PID controller in omnidirectional mobile robot. The principle of optimal PID controller designing based on DE algorithm is shown in Fig. 6.

#### 4.1 Encoding

In the PID controller, there are 3 parameters \( K_p, K_i, K_d \) to be optimized. DE algorithm using floating-point encoding method so that the 3 parameters could be directly served as a vector of individual. Then DE algorithm searching space is worked from the results of Ziegler-Nichols (Z-N) tuning method, thus making full use of the reasonable PID value of Z-N method. Assuming that \( K_p^*, K_i^*, K_d^* \) are the tuned value by Z-N method, \( \sigma, \beta \) are the scaling factors, we can define the boundry of searching by formula (10):

\[
\begin{align*}
\alpha K_p &\leq K_p \leq \beta K_p^* \\
\alpha K_i &\leq K_i \leq \beta K_i^* \\
\alpha K_d &\leq K_d \leq \beta K_d^*
\end{align*}
\]

### 4.2 Fitness function

The target of optimizing PID controller is to improve the key performance indicators, such as stability, precision and highly response, so we introduce overshoot (\( \delta \)), rise time (\( t_r \)) and accumulative error (\( \Sigma |e_k| \)) into the fitness function \( F \) as follows:

\[
F = w_1 \delta + w_2 t_r + w_3 \sum_{k=1}^{\infty} |e_k|
\]

in formula (11), the constant weight coefficient \( w_1, w_2, w_3 \) are used for adjusting the influence of corresponding variables in system.

### 4.3 Optimizing flow

Step 1: Define searching boundaries for PID parameters \( (K_p, K_i, K_d) \) by formula (10).

Step 2: Define the population size \( N_p \), crossover probability \( CR \), maximum number of iteration \( G_{max} \), then randomly generating initial population in searching boundaries.

Step 3: According to the formula (6) make mutation for individuals in population.

Step 4: According to the formula (8) make recombination for mutation individuals.

Step 5: According to the formula (9) Compute the fitness values of every individual vector.

Step 6: According to the formula (9) make selection for the next generation individuals.

Step 7: If the number of iterations less than \( G_{max} \), repeat Step 2-7, otherwise end.

### 5 Simulation and analysis

#### 5.1 Simulation model

In order to verify the correctness of the forward and inverse kinematics model, as well as the effectiveness of the optimized PID controller by DE algorithm, based on the actual conditions we set up an omnidirectional robot...
motion control platform by MATLAB simulink, please see Fig. 7.

In Fig. 7, the ‘JoyInput’ model is a 3*1 column vector with sine signal, each element respectively corresponds to the input of direction X, Y, Wz; the ‘Inverse Kinematic’ model represents the matrix \( K_i \) in formula(2); ‘Random Load’ model generates the 3Nm torque random disturbance to the wheels; ‘Servo system’ model is the servo driver with space-vector PWM technique, each unit has a double-loop control system, furtherly the PID controller of speed loop is optimized by DE algorithm; the ‘Forward Kinematic’ model represents the matrix \( K_f \) in formula(4).

5.2 Simulation parameters

a) Mobile robot chassis parameters: \( L=1.0m, l=0.5m, \) transmission ratio \( i=100; \)

b) Mecanum wheel parameters: roller offset angle \( \alpha =45^\circ, \) wheel radius \( R=0.25m; \)

c) PMSM parameters: rated speed \( w_n=3000, \) torque \( T_n=20Nm; \)

d) The fitness function weight coefficient: \( w_1=10, w_2=2, w_3=5; \)

e) The DE algorithm population size \( N_p=30, \) crossover probability \( CR=0.8, \) iterations max \( G_{max}=50. \)

Figure 8. Simulink model for omnidirectional mobile robot

Figure 9. Sine response for omnidirectional mobile robot

In Fig. 8, the convergence curve is the processing of DE algorithm. The Z-N PID parameters are \( K_p=31.2, K_i=5.3, K_d=1.2, \) after optimized by DE algorithm, the PID parameters are \( K_p=42.4, K_i=6.3, K_d=0.92. \)

When we set the target sine signal with angular frequency \( \omega=40rad/s, \) amplitude of three DOF \( V_x=0.2m/s, V_y=0.2m/s, W_z=0.2rad/s, \) the simulation results are shown as following Fig. 9.

5.3 Simulation results

Given that any velocity loop PID controller of the 4 wheels is working in the same way, so we show the optimized results by an example here.

From the simulation results of sine response, at the three DOF direction, the optimized PID controller by DE algorithm works with following error much less than the PID controller by Z-N tuned. It indicates the optimal controller design method is effective and control system has strong robustness, the results of simulation also show that the proposed inverse and forward kinematic are both correct and feasible.

6 Conclusion

In this paper, we analyzed the inverse and forward kinematics of omnidirectional mobile robot based on Mecanum wheel, then put forward the DE algorithm for optimizing PID controller of speed loop in servo system. Through MATLAB simulation platform, we bulit the simulation model of system, the results showed that the inverse and forward kinematics model are reasonable, DE algorithm optimization for the PID controller is better than traditional Z-N tuned. The DE algorithm can be implemented with ease and few parameters need to be tuned, which could be widely used in engineering practice.

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