Analysis of Data on Polarized Lepton-Nucleon Scattering

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Abstract

We re-analyze data on deep inelastic polarized lepton-nucleon scattering, with particular attention to testing the Bjorken sum rule and estimating the quark contributions to the nucleon spin. Since only structure function data at fixed $Q^2$ can be used to test sum rules, we use E142 asymmetry measurements and unpolarized structure function data to extract $g_1^n$ at fixed $Q^2 = 2$ GeV$^2$. When higher-twist effects, which are important at low $Q^2$, are included, both the E142 and SMC data are compatible with the Bjorken sum rule within one standard deviation. Assuming validity of the Bjorken sum rule, we estimate the quark contributions to the nucleon spin, finding that their total net contribution is small, with the strange quark contribution non-zero and negative. The quark spin content of the nucleon spin is in agreement with Skyrme model.
1 Introduction

It was the Bjorken sum rule \[1\],\[2\] for polarized lepton-nucleon scattering, derived initially \[1\] from quark current algebra at short distances, that led to the first proposal that deep inelastic structure functions should scale. However, although approximate scaling has been observed in deep inelastic scattering and quark model relations for structure functions hold in general, with deviations which are understood quantitatively in terms of perturbative QCD and asymptotic freedom at large momentum transfers, the Bjorken sum rule was never actually tested during the 25 years following its discovery. Data on polarized lepton-proton scattering have been available for about 15 years \[3\]-\[7\], but data on polarized lepton-neutron scattering have become available only very recently \[8\],\[9\]. The stimulus for performing these experiments, finally, was provided by the results of the EMC experiment \[6\],\[7\] on polarized muon-proton scattering, which differed from predictions based on naive constituent quark model ideas \[10\], but were consistent with perturbative QCD. Polarized lepton-neutron scattering would also have to differ from naive constituent quark model ideas if the Bjorken sum rule were to be valid. If it were not valid, the whole edifice of perturbative QCD and asymptotic freedom would collapse \[11\].

One of the two results on polarized lepton-neutron scattering announced recently, the one by the SMC \[8\], does indeed find a deviation from naive constituent quark model ideas, and, when combined with the EMC proton data, verifies the Bjorken sum rule within admittedly large errors. However, the other polarized neutron experiment by the E142 collaboration \[9\], which has much smaller statistical errors, is claimed, when combined with the EMC data, to disagree with the Bjorken sum rule by two standard deviations. This result therefore presents a puzzle whose resolution apparently needs affirmative answers to one or more of the following questions: are the EMC data wrong? are the E142 data wrong? is the Bjorken sum rule wrong?

We take the point of view in this paper that one must be prudent before jumping to any such dramatic conclusion, especially before concluding that the Bjorken sum rule is wrong. We recall that in every polarization experiment the data have been taken in different ranges of $Q^2$, which varied from bin to bin in the Bjorken scaling variable $x$, with average $Q^2$ values that are known in the case of the SMC and EMC experiments, but have not been published by the E142 collaboration. The EMC and the SMC have used the fact, observed also in other experiments, that the asymmetry $A_1(x)$ has no detectable dependence on $Q^2$ in the range studied, to estimate the polarized structure functions at a fixed value of $Q^2$. Whereas the EMC used their own measurements of the unpolarized structure function $F_2(x, Q^2)$ and a perturbative QCD model for the ratio $R(x, Q^2)$ of longitudinal to transverse virtual photon cross-sections to calculate $g_1(x, Q^2)$ for $Q^2$ fixed at 10.7 GeV$^2$, the SMC used the more recent NMC parametrization \[12\] of the unpolarized structure function $F_2(x, Q^2)$ and a SLAC parametrization \[13\] of the ratio $R(x, Q^2)$ to calculate $g_1(x, Q^2)$ for $Q^2$ fixed at 4.6 GeV$^2$. This procedure is a priori better suited for testing
sum rule predictions, which are formulated for structure functions integrated over all \( x \) values at fixed \( Q^2 \), than would be an integral over \( x \) at variable values of \( Q^2 \).

In this paper, we first check whether there is indeed any evidence against the validity of the Bjorken sum rule. To do this, we start by rescaling the EMC polarized proton data \(^1\) to \( Q^2 \) at 4.6 GeV\(^2\), for comparison with the SMC \(^2\), and at 2 GeV\(^2\), for comparison with E142 \(^3\) using the latest NMC parametrization \(^4\) of the unpolarized structure function and the latest SLAC parametrization of \( R(x, Q^2) \) \(^5\). For the latter comparison, we use the measurements of \( A_n^1 \) reported by E142 and the same NMC and SLAC parametrizations to extract \( g_n^0(x, Q^2) \) for \( Q^2 \) fixed at 2 GeV\(^2\). Next, we discuss the uncertainties in the extrapolations of the E142 data to \( x = 0 \) and 1. Since the appropriate power behaviour at small \( x \) is not tightly constrained by other data or by theory, this introduces some extra uncertainty beyond that discussed by E142. \( A \) priori, the error in the SMC extrapolation to \( x = 0 \) is 5 times smaller, since they have measurements down to \( x = 0.006 \), as opposed to the 0.03 of E142. Contrary to what is stated in ref. \(^6\), the behaviour of the polarization asymmetry at large \( x \) is not determined by perturbative QCD without extra assumptions about the non-perturbative structure of the nucleon, which also tends to increase the uncertainty beyond that assumed by E142. Next we show that, when proper attention is paid to the \( Q^2 \) dependence of the structure functions at large \( x \), the SMC data are largely compatible with a recently derived bound \(^7\). We then extract the integrals of \( g_1^p - g_1^n \) at fixed \( Q^2 = 4.6 \text{ GeV}^2 \) by combining the rescaled EMC and SMC data, and at 2 GeV\(^2\) by combining the rescaled EMC and E142 data, including realistic allowances for the errors. Particularly in the low-\( Q^2 \) case of the comparison of E142 and the EMC, higher-twist and mass corrections must be taken into account. According to the indicative estimate in ref. \(^8\), these corrections are more than 10\% at \( Q^2 = 2 \text{ GeV}^2 \) for the Bjorken sum rule. \(^9\) The estimates of ref. \(^8\) should therefore be included in the theoretical prediction. After including these subasymptotic effects, both the E142 as well as the SMC data are in good agreement with the Bjorken prediction. Encouraged by this agreement, we assume that the Bjorken sum rule is indeed correct, and go on to extract the contributions of the various quark flavours to the proton spin using all the available polarized structure function data. We find

\[ \Delta u + \Delta d + \Delta s = 0.22 \pm 0.10 \]  

(1)

which is far from the naïve quark model. It is close to the prediction of the Skyrme model for light quarks \(^10\). We conclude that a quantitative understanding of axial current matrix elements is now emerging.

\(^1\)See however ref. \(^11\) for a recent discussion of higher-twist effects in this context.
2 Data at Fixed $Q^2$

We first discuss the rescaling of the available polarized structure function data to a fixed value of $Q^2$, which is a crucial step for testing QCD sum rules, that are formulated at fixed $Q^2$. Experiments measure directly the polarization asymmetry $A_1(x, Q^2)$, in terms of which the polarized structure function $g_1(x, Q^2)$ is given by

$$g_1(x, Q^2) = \frac{A_1(x, Q^2) F_2(x, Q^2)}{2 x [1 + R(x, Q^2)]}$$

(2)

where $F_2(x, Q^2)$ is the conventional unpolarized structure function, and $R(x, Q^2)$ is the ratio of virtual longitudinal to transverse virtual photon cross sections. As already mentioned, each experiment takes data over a range of $Q^2$ that varies with $x$, and differs from experiment to experiment: $\langle Q^2 \rangle = 2, 4.6, 10.7 \text{ GeV}^2$ for E142, the SMC and the EMC respectively. Each experiment reports that the asymmetry that it measures exhibits no detectable variation with $Q^2$ in any $x$ bin. It is therefore reasonable to use this $Q^2$-independent value, together with the unpolarized $F_2(x, Q^2)$ and $R(x, Q^2)$ measured in other experiments, to estimate $g_1(x, Q^2)$ at some fixed $Q^2$, taken ideally to be the average $\langle Q^2 \rangle$ for each experiment. However, to combine the experiments that have different $\langle Q^2 \rangle$ to test the Bjorken sum rule, it is necessary to estimate $g_1(x, Q^2)$ for at least one non-ideal value of $Q^2$: the $Q^2$ variation of $g_1(x, Q^2)$ is substantial, even if that of $A_1(x, Q^2)$ is negligible.

Both the EMC [7] and the SMC [8] have presented their data at fixed $Q^2$ as advocated above. However, the EMC used a parametrization of the unpolarized data that has now been superseded by more recent results from the NMC [12] and SLAC [13]. The latter have been used by the SMC to estimate their $g_1^p(x, Q^2)$, and we use them here to re-estimate $g_1^p(x, Q^2)$. Figure 1 shows the rescaled EMC $g_1^p(x, Q^2)$ at fixed $Q^2 = 2, 4.6$ and $10.7 \text{ GeV}^2$, using a smooth parametrization [7] of the $A_1^p$ EMC data. The two former values of $Q^2$ are needed for combining with the E142 and SMC data respectively: the $Q^2$-dependence of $g_1^p(x, Q^2)$ is seen clearly, and the corresponding integrals $\Gamma_1^p(Q^2) \equiv \int_0^1 g_1^p(x, Q^2)$ are shown in Table 1.

| $Q^2 (\text{GeV}^2)$ | $\Gamma_1^p(Q^2)$ |
|----------------------|-------------------|
| 2.0                  | 0.124 ± 0.013 ± 0.019 |
| 4.6                  | 0.125 ± 0.013 ± 0.019 |
| 10.7                 | 0.128 ± 0.013 ± 0.019 |

The integrals $\Gamma_1^p(Q^2) \equiv \int_0^1 g_1^p(x, Q^2)$ obtained from the EMC [7] data on $A_1^p(x, Q^2)$, at $Q^2 = 2 \text{ GeV}^2$, combined with the NMC [12] data for $F_2^p(x, Q^2)$ and SLAC parametrization [13] of $R(x, Q^2) \equiv \sigma_L/\sigma_T$. The first is the statistical and the second is the systematic error.
We assume here that the systematic error on $\Gamma_1^p$ is as quoted by the EMC $^3$, even though the systematic error on the NMC $F_2^p$ is smaller than the systematic error on the EMC $F_2^p$. If there were no error at all on $F_2^p$ the systematic error in $\Gamma_1^p$ would be reduced from $\pm 0.019$ down to $\pm 0.018$. The difference from the value quoted by the EMC for $Q^2 = 10.7 \text{ GeV}^2$ is well within their quoted errors: we will comment later on the $Q^2$-dependence of $\Gamma_1^p(Q^2)$.

We have also applied the same procedure to the E142 data $^4$, using their values of $A_1^n(x)$ and the NMC and SLAC parametrizations of $F_2(x,Q^2)$ and $R(x,Q^2)$ respectively to estimate $g_1^n(x,Q^2)$ at $Q^2 = 2 \text{ GeV}^2$, their average value. This procedure yields

$$\int_{0.03}^{0.6} dx g_1^n(x,Q^2=2 \text{ GeV}^2) = -0.022 \pm 0.006 \text{ (stat.)} \pm 0.006 \text{ (syst.)} \quad (3)$$

to be compared with the estimate of $-0.019 \pm 0.006 \text{ (stat.)} \pm 0.006 \text{ (syst.)}$ given by E142.

Before estimating $\Gamma_1^p(Q^2 = \text{GeV}^2)$ on the basis of equation (3), we first comment on the extrapolations of their data beyond $x = 0.6$ and below $x = 0.03$. Models for the polarization asymmetry that combine a non-perturbative Ansatz for the neutron wave function with perturbative QCD at large $Q^2$ can be used to estimate the limiting value of $A_1^n(x)$ as $x \to 1$, but perturbative QCD alone does not predict a limiting value. Therefore, we prefer to estimate the error due to the high-$x$ extrapolation by allowing $|A_1^n(x)| \leq 1$, rather than by specifying a limiting value. Thus we estimate a contribution $0.000 \pm 0.003$ from the $x > 0.6$ region. The low-$x$ extrapolation of the E142 data is a priori more uncertain than that of the SMC, because the latter measure down to lower $x = 0.006$. E142 have chosen to extrapolate using a power form $g_1^n(x) = Ax^\alpha$, where $\alpha$ was fixed at the value $0.18$ predicted in ref. $^5$ and adopted in ref. $^6$. The model used in ref. $^6$ is however incompatible with the new data, and hence cannot be used $^7$ to support the value of $\alpha$ adopted by E142. Other data do not fix the power $\alpha$: for example, a fit to the EMC data at low $x$ gave $^8 \alpha = 0.07^{+0.32}_{-0.42}$, whereas we consider $^9$ a plausible theoretical range to be $0 \leq \alpha \leq 0.5$, as assumed by the SMC. Using this latter range of $\alpha$ to estimate the contribution of the range $x < 0.03$ as $-0.006 \pm 0.006$ $^3$, we estimate the full integral to be

$$\Gamma_1^n(Q^2=2 \text{ GeV}^2) = -0.028 \pm 0.006 \text{ (stat.)} \pm 0.009 \text{ (syst.)} \quad (4)$$

to be compared with the E142 estimate of $-0.022 \pm 0.011$.

The value of $\Gamma_1^n(Q^2)$ at $Q^2 = 4.6 \text{ GeV}^2$ can be extracted from the SMC data $^8$ together with the appropriate value of $\Gamma_1^p(Q^2)$ in Table 1. This results in

$$\Gamma_1^n(Q^2=4.6 \text{ GeV}^2) = -0.076 \pm 0.046 \text{ (stat.)} \pm 0.037 \text{ (syst.)} \quad (5)$$

$^2$We note that one would obtain a lower value if one combined E142 and SMC data, which are lower at small $x$. 

4
to be compared with the SMC estimate of $-0.08 \pm 0.04 \pm 0.04$.

There has been some question [14] whether the SMC data at large $x$ are compatible with general positivity bounds derived within the framework of the quark-parton model. An interesting bound is derived in ref. [14] by eliminating the $u$ quark-parton contributions to $g_1^n(x, Q^2)$ and $g_1^p(x, Q^2)$:

$$\left|4g_1^n(x, Q^2) - g_1^p(x, Q^2)\right| = \left|\frac{15}{18} \Delta d(x, Q^2) + \frac{3}{18} \Delta s(x, Q^2)\right| \leq \frac{15}{18} d(x, Q^2) + \frac{3}{18} s(x, Q^2),$$

(6)

It should be noted that this bound could be violated by higher-twist effects, that are expected to grow at smaller $Q^2$ and larger $x$. An additional possible source of error is the assumption, made in ref. [14] that $g_1^n(x, Q^2) + g_1^p(x, Q^2) = g_1^d(x, Q^2)$ locally in $x$. The superficially similar relation among the first moments used by the SMC [8] $\Gamma_1^n(Q^2) + \Gamma_1^p(Q^2) \simeq \Gamma_1^d(Q^2)/(1 - 1.5 \omega_D)$, where $\omega_D = 0.058$ is the probability of the deuteron to be in a $D$-state, is well justified, but the local relation is violated by smearing of the deuteron at large $x$.

We re-express the bound (6) in terms of the directly-measured quantities $A_1^p(x)$ and $A_1^n(x)$ and the $Q^2$-dependent quantities $R(x, Q^2)$ and $\xi(x, Q^2) \equiv F_2^n(x, Q^2)/F_2^p(x, Q^2)$:

$$\left|4A_1^n(x, Q^2)\xi(x, Q^2) - A_1^p(x, Q^2)\right| \leq \left[1 + R(x, Q^2)\right] \left[4\xi(x, Q^2) - 1\right]$$

(7)

We have evaluated both sides of this version of the bound, using the same fixed values $Q^2 = 2, 4.6 \text{ GeV}^2$ on each side to compare EMC data with the E142 and SMC data respectively.

The E142 data satisfy the bound comfortably, whereas the SMC data are in marginal disagreement, as shown in figure 2. We do not take seriously the latter disagreement, in view of the large errors and the possible contributions of higher twist effects. Shifting the central values of the SMC data at $x \geq 0.4$ so as to be consistent with the bound (6) would not in any case change significantly our estimate of $\Gamma_1^n(Q^2)$ based on the SMC data.

3 Testing the Bjorken Sum Rule

We are now in a position to evaluate the difference between $\Gamma_1^n(Q^2)$ and $\Gamma_1^p(Q^2)$ at $Q^2 = 2 \text{ GeV}^2$ using the EMC and E142 data, and at $Q^2 = 4.6 \text{ GeV}^2$ using the EMC and SMC data:

$$\Gamma_1^{p-n}(Q^2=2.0 \text{ GeV}^2) = 0.152 \pm 0.014 \pm 0.021$$

$$\Gamma_1^{p-n}(Q^2=4.6 \text{ GeV}^2) = 0.201 \pm 0.048 \pm 0.042$$
These are to be compared with the Bjorken sum rule, which takes the following form when the leading-order perturbative QCD corrections are included [2]:

$$\Gamma^{p-n}(Q^2) \equiv \int_0^1 dx \left[ g_1^p(x, Q^2) - g_1^n(x, Q^2) \right] = \frac{1}{6} g_A \left[ 1 - \alpha_s(Q^2)/\pi \right] \quad (9)$$

To estimate the appropriate values of $\alpha_s(Q^2)$ at $Q^2 = 2, 4.6$ GeV$^2$, we take the latest estimate at $\alpha_s(m^2_{\tau}) = 0.330 \pm 0.046 [22]$, which is consistent with determinations at higher $Q^2$, and use the leading order corrections to calculate $\alpha_s(2, 4.6$ GeV$^2) = 0.371 \pm 0.039$, together with $g_A = 1.2573 \pm 0.0028 [24]$, leading to the following values of the right-hand-side of equation (9):

$$\Gamma^{p-n}(Q^2=2.0 \text{ GeV}^2) = 0.185 \pm 0.004 \quad (10)$$

$$\Gamma^{p-n}(Q^2=4.6 \text{ GeV}^2) = 0.189 \pm 0.003$$

However, these values cannot to be compared with the evaluations (8) before including subasymptotic $O(1/Q^2)$ effects [15] due to mass corrections and higher-twist operators, which are particularly important at the low average $Q^2$ of E142.

The mass corrections [15] are proportional to moments of $g_1^{p,n}(x)$, which can be estimated using the data themselves:

$$\langle x^2 \rangle_p \equiv \int dx \, x^2 g_1^p(x, Q^2) \approx \begin{cases} 0.0168, & (Q^2 = 2.0 \text{ GeV}^2) \\ 0.0130, & (Q^2 = 4.6 \text{ GeV}^2) \end{cases}$$

$$|\langle x^2 \rangle_n| \equiv \left| \int dx \, x^2 g_1^n(x, Q^2) \right| < 7 \times 10^{-5}. \quad (11)$$

Errors in $\langle x^2 \rangle_p$ can be neglected, since the mass corrections include an additional small factor $(4/9)(m_N^2/Q^2)$ where $m_N$ is the nucleon mass. Estimating the higher-twist corrections requires calculating the reduced matrix elements \( \langle \langle U^{NS} \rangle \rangle \) and \( \langle \langle U^S \rangle \rangle \) of the local operators of spin one and twist four [23], [15]:

$$U^S_\mu = \bar{u} g \tilde{G}_{\mu\nu} \gamma^\nu u + \bar{d} g \tilde{G}_{\mu\nu} \gamma^\nu d + \frac{18}{5} \bar{s} g \tilde{G}_{\mu\nu} \gamma^\nu s,$$

$$U^{NS}_\mu = \bar{u} g \tilde{G}_{\mu\nu} \gamma^\nu u - \bar{d} g \tilde{G}_{\mu\nu} \gamma^\nu d,$$

$$\langle N | U_\mu | N \rangle = s_\mu \langle \langle U \rangle \rangle, \quad s_\mu \equiv \bar{N} \gamma_\mu \gamma_5 N. \quad (13)$$

where $\tilde{G}_{\mu\nu} \equiv \epsilon_{\mu\nu\alpha\beta} G_a^{\alpha\beta} \lambda_\alpha/2$ and $\bar{N}, N$ are nucleon spinors. The matrix elements of the non-strange quark operators have been estimated using sum rule techniques in [15]: the possible error quoted is estimated at 50% for $\Gamma^{p-n}$ and 100% for $\Gamma^{p+n}$, and
it has been suggested that the matrix element of the strange quark operator should not be larger than this uncertainty. Ref. [15] found the following magnitudes for the higher-twist correction:

\[
\begin{align*}
\delta \Gamma_1^{p-n}(Q^2=2.0 \text{ GeV}^2) &= -0.022 \pm 0.011 \\
\delta \Gamma_1^{p-n}(Q^2=4.6 \text{ GeV}^2) &= -0.010 \pm 0.005
\end{align*}
\]

(14)

\[
\begin{align*}
\delta \Gamma_1^{p+n}(Q^2=2.0 \text{ GeV}^2) &= +0.017 \pm 0.017 \\
\delta \Gamma_1^{p+n}(Q^2=4.6 \text{ GeV}^2) &= +0.007 \pm 0.007
\end{align*}
\]

The net contribution of the higher-twist and mass corrections to \(\Gamma_1^p(Q^2)\) is quite small, becoming negligible at the \(Q^2\) of the EMC experiment, and has the effect of decreasing the integral slightly at smaller \(Q^2\). This is consistent with the trend found in Table 1 above on the hypothesis that \(A_1^p\) is independent of \(Q^2\), though the inferred variation cannot be considered significant. The net contribution of the \(1/Q^2\) corrections to \(\Gamma_1^n(Q^2)\) are much larger, positive, and particularly significant for the low \(Q^2\) of E142. Thus it is not surprising that their value of \(\Gamma_1^n(Q^2)\) happens to be larger than that of the SMC. The net correction to the Bjorken sum rule is important, exceeding 10\% at \(Q^2 = 2 \text{ GeV}^2\):

\[
\begin{align*}
\Gamma_1^{p-n}(Q^2=2.0 \text{ GeV}^2) &= 0.163 \pm 0.012 \\
\Gamma_1^{p-n}(Q^2=4.6 \text{ GeV}^2) &= 0.180 \pm 0.006
\end{align*}
\]

(15)

Comparing these theoretical estimates with the experimental evaluations (8), we conclude that the E142 and EMC data are consistent with the Bjorken sum rule within less than one standard deviation. This point is shown graphically in figure 3. One way of phrasing this consistency is to quote the effective values of \(g_A\) extracted from the two combinations of experiments by subtracting the subasymptotic corrections (14) and removing the \(\alpha_s(Q^2)\) correction appearing in (9):

\[
\begin{align*}
\text{EMC & SMC:} & \quad g_A^{\text{eff}} = 1.39 \pm 0.38 \\
\text{EMC & E142:} & \quad g_A^{\text{eff}} = 1.19 \pm 0.17
\end{align*}
\]

(16) (17)

Combining all three experiments, we find

\[g_A^{\text{eff}} = 1.22 \pm 0.15\]

(18)

The Bjorken sum rule is consistent with all the available data on polarized structure functions, and is now verified at the 12\% level.
4 Evaluation of the Quark Contributions to the Nucleon Spin

Having reassured ourselves that the available data on polarized nucleon structure functions are consistent with the Bjorken sum rule, we now extract the contributions of the $u, d$ and $s$ quark flavours to the total nucleon spin, and compare them with theoretical estimates. We treat independently the EMC data on polarized protons, the SMC data on polarized deuterons, and the E142 data on polarized $^3$He. In the case of the SMC result, we fit the data after applying the D-wave correction. In the case of the E142 result, we assume the same model of $^3$He as in ref. [9] (see also ref. [25] for a detailed discussion of $^3$He wavefunction in this context). Before fitting, we apply to each value of the integral of $g_1(x, Q^2)$ the appropriate higher-twist, mass and perturbative QCD corrections. We denote the resulting corrected first moments $\tilde{\Gamma}_1^p(Q^2)$ and $\tilde{\Gamma}_1^n(Q^2)$, which correspond to the net charge-weighted sums of quark helicities. Thus we have

$$\frac{1}{2} \left( \frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s \right) = \tilde{\Gamma}_1^p \quad (Q^2=10.7\text{ GeV}^2) = +0.140 \pm 0.023$$
$$\frac{1}{2} \left( \frac{1}{9}\Delta u + \frac{4}{9}\Delta d + \frac{1}{9}\Delta s \right) = \tilde{\Gamma}_1^n \quad (Q^2= 2.0\text{ GeV}^2) = -0.056 \pm 0.015$$
$$\frac{1}{2} \left( \frac{5}{9}\Delta u + \frac{5}{9}\Delta d + \frac{2}{9}\Delta s \right) = \tilde{\Gamma}_1^p + \tilde{\Gamma}_1^n \quad (Q^2= 4.6\text{ GeV}^2) = +0.048 \pm 0.055$$

We combine each of these inputs with [24]

$$\Delta u - \Delta d = g_A = 1.2573 \pm 0.0028$$

and take [24]

$$F = 0.46 \pm 0.01; \quad D = 0.79 \pm 0.01; \quad F/D = 0.58 \pm 0.02$$

Each of eqs. (13) and (21) can be rewritten as linear constraint relating the values of $\Delta \Sigma$ and $\Delta s$. In fig. 4 we plot the allowed regions in the $\Delta \Sigma - \Delta s$ plane, corresponding to these constraints. The three data sets above yield the following compatible estimates of the total quark contribution to the proton spin:

- EMC ($Q^2 = 10.7$ GeV$^2$): $\Delta u + \Delta d + \Delta s = +0.16 \pm 0.21$
- SMC ($Q^2 = 4.6$ GeV$^2$): $\Delta u + \Delta d + \Delta s = +0.06 \pm 0.25$
- E142 ($Q^2 = 2.0$ GeV$^2$): $\Delta u + \Delta d + \Delta s = +0.29 \pm 0.14$

Ignoring any possible $Q^2$ dependence in $\Delta u + \Delta d + \Delta s$, we combine these in quadrature to obtain

$$\Delta \Sigma \equiv \Delta u + \Delta d + \Delta s = 0.22 \pm 0.10$$

where the individual quark contributions are

$$\Delta u = +0.80 \pm 0.04$$
$$\Delta d = -0.46 \pm 0.04$$
$$\Delta s = -0.13 \pm 0.04$$
The global estimate of $\Delta s$ is approximately three standard deviations away from zero, providing good evidence that the naive assumption of ref. [10], namely that strange quarks should not contribute to the nucleon spin, is false. This does not strike us as implausible, since we have much more sophisticated models of nucleon structure then when [10] was written almost 20 years ago.

In particular, we note that the value (23) of $\Sigma \Delta q$ is quite close to zero, which is the value expected in the class of chiral soliton models pioneered by the computation [17] in the Skyrme model in the limit of massless quarks and a large number of colours. It has been suggested that corrections to this limiting value might amount to about 30% in the case of realistic quark masses and three colours $^3$, which would be consistent with the experimental value (23). We are not aware of any other dynamical model of nucleon structure which predicts a value of $\Sigma \Delta q$ close to zero, though some models are able to accommodate it. For example, it has been suggested [23] that gluons inside the nucleon might be highly polarized, and that this polarization would communicate itself to the quarks via radiative corrections and the axial anomaly. Unfortunately, this interesting suggestion does not give insight why $\Sigma \Delta q$ should vanish in any limit $^4$. There have been some initial calculations of matrix elements “from first principles” using lattice techniques [31]–[33], which are consistent within errors with our determinations.

It is also worthwhile noticing that $\Delta s$ being nonzero is a part of an interesting pattern $^5$: experiment indicates that certain strange-quark bilinear operators, such as $\bar{s}\gamma_\mu\gamma_5 s$ have relatively large matrix elements in the proton, while others are very small. The presence of a substantial non-valence component of $\bar{s}s$ pairs in the proton has some striking consequences. One of these is the evasion of the OZI rule in the couplings of $\bar{s}s$ mesons to baryons $^3$, leading to surprisingly large branching ratios for $\phi$ production in $\bar{p}p$ annihilation at rest $^3$.

## 5 Interpretation

We conclude that experiment and theory are converging on consistent and quite quantitative decompositions of the spin of the nucleon. The chiral soliton models pioneered by the Skyrme model are perhaps unique in providing understanding of these results, which disagree with naive quark model ideas. The basic idea of these chiral models is that a nucleon contains a very large number of very light, relativistic quarks, that are best described by a topological lump in an effective bosonic field. The data seem to indicate that this is a better starting-point for understanding the

$^3$See ref. [27] for discussion of $1/N_c$ corrections and ref. [28] for a current assessment of the situation in Skyrme type models.

$^4$For proposals in this direction, see ref. [30].
spin of the nucleon than the naïve picture of three non-relativistic valence constituent quarks. However, the constituent quark model does provide good understanding of many other aspects of nucleon structure. We are therefore confronted with the challenge \[37\] of relating constituent quarks to the chiral picture that works so well for the polarized structure function data re-analyzed in this paper. There is already some indication \([38]-[41]\) that this program has good chances of success.

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Figure Captions:

Fig. 1. (a) The polarized structure function $g_p(x, Q^2)$ at $Q^2 = 2.0$, 4.6 and 10.7 GeV$^2$, obtained from the EMC $A_p$ data [7], smoothed and combined with the NMC [12] data for $F_2^p(x, Q^2)$ and SLAC parametrization [13] of $R(x, Q^2) \equiv \sigma_L/\sigma_T$. Continuous curve: $Q^2 = 2$ GeV$^2$, dots: $Q^2 = 4.6$ GeV$^2$, dot-dash: $Q^2 = 10.7$ GeV$^2$. Compensation between the decrease at large $x$ and the increase at small $x$ results in the relatively small $Q^2$-variation of $\Gamma_p^p(Q^2)$ found in Table 1. (b) The polarized structure function $g_n(x, Q^2)$ at $Q^2 = 2.0$, 4.6 and 10.7 GeV$^2$, obtained from the E142 $A_n$ data [9], smoothed and combined with the NMC [12] data for $F_2^n(x, Q^2)$ and $F_2^n(x, Q^2)$, and SLAC parametrization [13] of $R(x, Q^2) \equiv \sigma_L/\sigma_T$. Continuous curve: $Q^2 = 2$ GeV$^2$, dots: $Q^2 = 4.6$ GeV$^2$, dot-dash: $Q^2 = 10.7$ GeV$^2$. Unlike in the case of $g_p(x, Q^2)$, there is no compensation between the large and small $x$, and this results in the relatively large $Q^2$-variation of $\Gamma_n^p(Q^2)$.

Fig. 2. The difference between the right-hand and left-hand sides of the bound [2]. The actual errors are slightly larger than those indicated by the error bars, as the latter refer to the error in the left-hand side only. (a) E142 data [4], combined with EMC data [7], rescaled to $Q^2 = 2$ GeV$^2$; (b) SMC data [8], combined with EMC data [7], rescaled to $Q^2 = 4.6$ GeV$^2$.

Fig. 3. Experimental tests at $Q^2 = 2$ GeV$^2$: E142 and EMC, $Q^2 = 4.6$ GeV$^2$: SMC and EMC of the Bjorken sum rule, including perturbative QCD corrections (dot-dashed lines) and higher-twist corrections (solid lines). The asymptotic value $g_A/6$ is denoted by a dotted line.

Fig. 4. The allowed regions in the $\Delta \Sigma - \Delta s$ plane, corresponding to the linear constraints [14] and [21]. Continuous lines: $\Delta \Sigma - 3\Delta s = 3F - D$; dots: $\Gamma_1^p(Q^2)$ constraint; dot-dash: $\Gamma_1^n(Q^2)$; dashes: $\Gamma_1^p(Q^2) + \Gamma_1^n(Q^2)$. 

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Fig. 1(a)

\[ g_1p(x, Q^2) \]
Fig. 1(b)

\[ g_1n(x, Q^2) \]

\[ g_1n(x, Q^2); Q^2 = 2, 4.6, 10.7 \text{ GeV}^2 \]
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rhs - lhs

Fig. 2(a)

E142 & EMC, $Q^2 = 2$ GeV$^2$
Fig. 2(b)

\[ \text{rhs - lhs} \]

SMC & EMC, \( Q^2 = 4.6 \text{ GeV}^2 \)
