Applying Isotropic Fractional-Rational Curves Toward Surface Modeling: New Insights from the Applied Mathematics Perspective

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Abstract-- Given curves or points, one of the problems that have been documented in the recent past entails building smooth surfaces. The problem is pronounced due to computer technology and industry developments. Initially, zero Gaussian curvature shells and minimal surfaces were used. These surfaces relied on isotropic analytic curves. However, consumer properties restrict these curves. Hence, there is a growing need to steer expansions in surface shaping. In response to this trend, this paper proposed a technique for surface construction based on the role of isotropic fractional-rational curves. In the framework, the building of surfaces involved the use of flat orthogonal and isothermal grids, a process governed by the Weierstrass technique. Indeed, the latter approach was adopted due to the associated trend of demanding minimal surfaces.

1. Introduction

In engineering, intensive developments have been documented. Other areas that have felt this trend include computer technology and the construction industry [1, 2]. In the process, the problem of smooth surface development relative to the given curves or points has been documented [3-5]. Indeed, one of the approaches that have been adopted relative to solving this dilemma has been the case of Gaussian curvature shells of zero [6, 7]. The use of the latter approach is associated with simple manufacture and design [8]. However, the outcome has not been the best when the procedure is applied to complex structures [9], especially because the shells’ carrying capacity relies on the overall contour’s small deviations from the actual shapes [9, 10]. Hence, there has been growing demand for the elimination of such limitations naturally. One of the approaches that have been adopted involves minimal surfaces.

It is also worth indicating that isotropic curves have been applied successfully relative to minimal surface simulation [2, 8]. In situations involving surface shape interactive controls, it has also been documented that parametric curves are worth adopting, especially when they use characteristic polygons [7-9]. A specific example is the use of Bezler curves [10, 11]. Furthermore, some studies affirm that expansions of possibilities about surface shaping demand that the operators stretch beyond the control of point coordinates and also focus on point weights [1-4]. The eventuality is that fractional-rational curve representations ought to be utilized. In this study, the central purpose was to establish mathematical models of surfaces containing differential properties relative to the leading role of fractional-rational curves. Hence, isotropic fractional-rational curves were constructed in the selected space and plane. In turn, the curves aided in developing a network, with grids and curves also used in surface construction.

2. Materials and Methods

Given the nth order’s fractional-rational curve in the form:
Isotropic fractional-rational curves were built to fulfill the condition:

$$x(t)^2 + y(t)^2 + z(t)^2 = 0$$

Separately, 3D and 2D curves were modeled. Initially, 2D curves were modeled. The basic elements applied were in the form of isotropic segments. In turn, the elements were used for developing isotropic chords and isotropic characteristic polygons. Conditions that governed the construction of isotropic polygons were defined in the form:

$$(x_1 - x_0)^2 + (y_1 - y_0)^2 = 0,$$

$$\cdots,$$

$$(x_n - x_{n-1})^2 + (y_n - y_{n-1})^2 = 0,$$

$$(x_0 - x_n)^2 + (y_0 - y_n)^2 = 0,$$

Given the developed isotropic fractional-rational curves, they were used to model 3D surfaces. The basis on which the surfaces were modeled was a conformal change of the equation of the curve’s parameter, as well as the real part selection. In so doing, minimal surfaces were obtained. It is also worth indicating that several options for third order isotropic curve creation were considered based on the conditions:

$$\sum_{r=x,y,z} (r_1 - r_0)^2 = 0,$$

$$\sum_{r=x,y,z} (r_1 - r_0) (r_2 - r_1) = 0,$$

$$\omega_3 = \frac{-2\omega_2^2 \sum_{r=x,y,z} (r_2 - r_1)^2}{\omega_1 \sum_{r=x,y,z} (r_1 - r_0) (r_3 - r_2)},$$

$$2 \sum_{r=x,y,z} (r_2 - r_1)^2 + \sum_{r=x,y,z} (r_1 - r_0) (r_3 - r_2) = 0,$$

$$\omega_0 = \frac{-2\omega_1^2 \sum_{r=x,y,z} (r_2 - r_1)^2}{\omega_2 \sum_{r=x,y,z} (r_1 - r_0) (r_3 - r_2)},$$

$$\sum_{r=x,y,z} (r_1 - r_0) (r_2 - r_1) = 0,$$

$$\sum_{r=x,y,z} (r_3 - r_2)^2 = 0.$$
From the consideration of the conditions above, aspects that were developed and represented for further analysis included the isotropic guide curve arising from the analytical function, the guide curve arising from the deployment of a flat curve deformation, isotropic fractional-rational curve of the third order simulation (arising from a flat curve’s deformation), and the modeling of isothermal and orthogonal grids (governed by isotropic planar fractional-rational curves). The resultant cubic fractional-rational curves that were obtained were in the form:

![Figure 1: The resultant cubic fractional-rational curves](image)

3. Results and Discussion

Indeed, the study focused on surface construction based on isotropic fractional-rational curves. This procedure was followed due to the need to determine the degree to which the proposed model would lead to informed decision-making in industrial settings, especially in relation to the trend of controlling the shape of grids and curves in user modes without necessarily engaging in the process of expression recalculation. Previously, the recalculation of surface parameters had been employed by many scholars. In this study, the motivation was to discern the potentially beneficial effect of the proposed model’s capacity to avoid surface parameter recalculation, as well as the perceived secondary and other beneficial effect of the speedy processing of images. The research context involved cases where computer graphics are used.

It is also notable that the proposed framework sought to foster the perceived benefit of speedy processing but caution was taken to ensure that crucial surface properties were not compromised or overlooked. Some of the properties that proved crucial and worth considering to ensure that the proposed model would not compromise them included issues such as the grid lines’ orthogonality and isotropism states. In the study, it is further notable that close formulas through which the selected surfaces’ parameters would be calculated were proposed. In so doing, the proposed model culminated into the establishment of the reference curves’ isotropy conditions, hence discerning the level at which the model would yield optimal outcomes. In the findings, the proposed framework exhibited superior outcomes compared to previous models that had been simulated and applied to similar settings. This inference arose from the outcomes that were obtained after validating the feasibility of the proposed model and comparing its performance outcomes with those that had been documented in most of the previous literature.
4. Conclusion and Future Directions

In summary, one of the problems that have been documented in the recent past entails building smooth surfaces. The problem is pronounced due to computer technology and industry developments. Initially, zero Gaussian curvature shells and minimal surfaces were used. These surfaces relied on isotropic analytic curves. However, consumer properties restrict these curves. Hence, there is a growing need to steer expansions in surface shaping. In response to this trend, this paper proposed a technique for surface construction based on the role of isotropic fractional-rational curves. In the framework, the building of surfaces involved the use of flat orthogonal and isothermal grids, a process governed by the Weierstrass technique. Indeed, the latter approach was adopted due to the associated trend of demanding minimal surfaces.

Hence, this study proved informative and the results were of practical importance to industrial and other real-world contexts in various ways. For instance, the results demonstrated that several parameters determine the design and outcome validity in relation to the performance of smooth surfaces. Some of these factors include the nature of the shapes, their texture, and materials that might have been used to make them. Of importance to note further is that the study yielded new insights into some of the ways in which mathematical models could be used to design and determine the performance of smooth surfaces, as well as predict some of the problems that might be encountered while using the designed surfaces, especially in industrial and other real-world scenarios. Furthermore, the results aided in understanding some of the challenges that previous scholarly investigations targeting similar curves might have faced, eventually developing a framework through which solutions to the perceived problems could be addressed.

For decision-makers in industries and engineering fields, it becomes important to analyze current problems facing the design of smooth surfaces, especially based on some of the information documented in the previous literature. In so doing, points of departure of the expected shape design might be identified and optimization or mathematical models aimed at responding to the perceived problems developed accordingly. With industrial processes supported in terms of improved material functionality and minimal challenges, the long-term impact of mathematical models that would have been developed is that there might be significant improvements in productivity at the firm levels, hence increased profitability. With the promising nature of the optimization models, it remains notable that this study’s proposed framework is unexceptional. Given its promising nature and confirmed feasibility relative to the validation outcomes that were obtained, this study recommends that the framework is employed in industrial scenarios involving smooth surfaces, especially due to the projected improvements in firm operations. In future, there is a need for scholarly investigations to examine further some of the drawbacks that might accrue from the implementation of the proposed model. Also, there is a need to study the feasibility of the proposed framework when applied to different industrial scenarios with varying experimental parameters different from those that were considered in this study, as well as situations with assumptions that are different from the ones that were defined in this study.

5. References

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