Gluino Production in Electron-Positron Annihilation

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Abstract
We discuss the pair production of gluinos in electron-positron annihilation at LEP, in a model with soft supersymmetry breaking, allowing for mixing between the squarks. In much of the parameter space of the Minimal Supersymmetric Model (MSSM) the cross section corresponds to a $Z$ branching ratio above $10^{-5}$, even up to $10^{-4}$. A non-observation of gluinos at this level restricts the allowed MSSM parameter space. In particular, it leads to lower bounds on the soft mass parameters in the squark sector.

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1 Introduction

Recent searches for gluinos by the CDF Collaboration have established a lower mass bound of the order of 140 GeV/c² \[1\]. This bound depends on the assumed decay mode of the gluino, it is valid for the case of direct decay to the lightest supersymmetric particle, \(\tilde{g} \to q\bar{q}\tilde{\chi}\). The analysis is insensitive to light gluinos, \(m_{\tilde{g}} \leq \mathcal{O}(40 \text{ GeV})\). However, various other experiments, in particular those at the CERN SPS \[2, 3\] exclude most of the region below 40 GeV, except for a narrow range around 3–5 GeV/c² \[4\].

The existence of this low-mass gluino window has recently been pointed out \[4\], and it is even argued that data on \(\alpha_s(m_Z)\) favour a light gluino \[5, 6, 7\]. (See also ref. \[8\], however.) Some of its further consequences are explored in ref. \[9\].

The importance of searching for light gluinos has long been stressed \[10\]. Clearly, if the gluino is very light, it should be produced at LEP, either by radiation in pairs off a quark \[11, 12\], or in pairs via the triangle diagram \[13, 14, 15\]. In the former case, the final four-jet state would be rather hard to isolate, because of the QCD background \[16\]. For the latter mechanism, the cross section was at low energies (ref. \[13\], photon exchange) found to depend very much on the mass splitting between the squarks, being in general rather small. A similar analysis has been performed for the \(Z\) decay \[14, 15\], and the cross section was found to depend sensitively on the mass splitting between the top and bottom quarks. Because the previous analyses are limited to low top-quark masses, and in order to also study the effects of chiral mixing, we find it important to present a new analysis of the gluino pair production cross section.

The notation to be used is in part given by the MSSM Lagrangian density

\[
\mathcal{L} = \mathcal{L}_{\text{SU}(3)} + \mathcal{L}_{\text{SU}(2)\times U(1)} + \mathcal{L}_{\text{Soft}},
\]

with the SU(3) part given by (subscripts “s” for “strong”)

\[
\mathcal{L}_{\text{SU}(3)} = \left\{ \frac{1}{8g_s^2} \text{Tr} [W_s^a W_s^a]_{\theta\bar{\theta}} + \text{h.c.} \right\} + \left[ Q_{\text{SU}(3)}^L e^{2g_s V_s} Q_{\text{SU}(3)}^L + Q_{\text{SU}(3)}^R e^{-2g_s V_s} Q_{\text{SU}(3)}^R \right]_{\theta\bar{\theta}\bar{\theta}},
\]

and the SU(2)×U(1) part by

\[
\mathcal{L}_{\text{SU}(2)\times U(1)} = \left\{ \left[ \frac{1}{8g^2} \text{Tr} [W^a W_a] + \frac{1}{4} w^a w_a - \mu \hat{H}_1^T \epsilon \hat{H}_2 \right] \right\},
\]
Here, “hats” refer to superfields. The gauge-invariant (soft) supersymmetry breaking part is given in terms of component fields as

\[
L_{\text{Soft}} = \left\{ \beta^{H \bar{H}_1^T} e H_2 + \frac{g m_e A_e}{\sqrt{2} m_W \cos \beta} L^T e H_1 \bar{e} R + \frac{g m_d A_d}{\sqrt{2} m_W \cos \beta} Q^T e H_1 \bar{d} R - \frac{g m_u A_u}{\sqrt{2} m_W \sin \beta} Q^T e H_1 \bar{u} R + \text{h.c.} \right\}
\]

Subscripts \(u\) (or \(U\)) and \(d\) (or \(D\)) refer generically to up and down-type quarks. We shall mostly focus on the contributions from the third generation. Thus, these symbols will actually often refer to top and bottom quarks. Spinors are here expressed in two-component Weyl notation, since the chiral mixing acts at this level. The notation is further explained in ref. \[17\] and references quoted there.

The gluino mass is given explicitly by \(m_\tilde{g}\), whereas squark masses depend not only on the explicit mass parameters \(\tilde{M}_U, \tilde{M}_U\) and \(\tilde{M}_D\), but also on \(m_u, m_d, m_Z, m_W, A_u, A_d, \mu\) and \(\beta\). For each flavour, there are two squarks, whose masses are given in terms of a similar parameterization in ref. \[13\]. (See also ref. \[17\].) In the limit of no mixing, i.e., with \(\mu = 0\), and \(A_d = A_u = 0\), the masses of the squarks associated with left- (L) and right- (R) chiral quarks are given by

\[
m_{u_L}^2 = m_u^2 + \tilde{M}_U^2 - \left( \frac{1}{6} m_Z^2 - \frac{2}{3} m_W^2 \right) \cos(2\beta),
\]

\[
m_{u_R}^2 = m_u^2 + \tilde{M}_U^2 + \left( \frac{2}{3} m_Z^2 - \frac{2}{3} m_W^2 \right) \cos(2\beta),
\]
\[ m_{dL}^2 = m_d^2 + \tilde{M}_U^2 - \left( \frac{1}{6} m_Z^2 + \frac{1}{3} m_W^2 \right) \cos(2\beta), \]
\[ m_{dR}^2 = m_d^2 + \tilde{M}_U^2 - \left( \frac{1}{3} m_Z^2 - \frac{1}{3} m_W^2 \right) \cos(2\beta). \] (1.5)

We shall however consider the case of mixing, for which the mass formulas are more complicated [17, 18].

It should be noted that the above Lagrangian represents a model which is different from the recently considered “constrained” models based on Grand Unification and supergravity [19, 20]. In particular, the gluino mass is here not tied to the other gaugino masses. The model is “minimal” in the sense that it has only two Higgs doublets, the soft mass terms are however “non-minimal”.

2 The $Z\tilde{g}\tilde{g}$ amplitudes

In the decay of the $Z$, or more generally in electron-positron annihilation, the pair production of gluinos can proceed via the two generic diagrams $(a)$ and $(b)$ of figure 1, where the internal lines of the triangles are quarks and squarks. Allowing for mixing between the squarks associated with the left- and right-chiral quark superfields, we find the Feynman rules for the vertices as given in figure 2.

We shall write the amplitude for

\[ e^+ e^- \rightarrow \tilde{g}\tilde{g} \] (2.1)

as

\[ \mathcal{M} = L^\mu i D_{F\mu\nu} \tilde{G}^\nu \delta_{ab}, \] (2.2)

where the lepton current is given as

\[ L^\mu = \tau(p_2) \left\{ \frac{-ig\gamma^\mu}{2\cos\theta_W} (g_V - g_A\gamma_5) \right\} u(p_1), \] (2.3)

and the gluino current $\tilde{G}^\mu$ will consist of a sum over contributions from different diagrams to be discussed presently. Furthermore, $iD_{F\mu\nu}$ is the $Z$ propagator, and $\delta_{ab}$ is a Kronecker delta in the gluino colour indices. For each quark flavour, there are two uncrossed and two
crossed diagrams of type (a). If we label them by the quark and squark propagators of the triangle, then we can write the terms involving $u$-quarks as

$$G_{uu1}^\mu = -N_u \bar{u}(k_2)(C_u^+ - C_\tilde{u} \gamma_5) T_{uu1}^\mu(k_1, k_2)(C_\tilde{u}^+ + C_u^+ \gamma_5) C^{-1} \tilde{u}^T(k_1),$$

$$G_{uu2}^\mu = -N_u \bar{u}(k_2)(C_\tilde{u}^- + C_u^+ \gamma_5) T_{uu2}^\mu(k_1, k_2)(C_u^- - C_\tilde{u}^+ \gamma_5) C^{-1} \tilde{u}^T(k_1).$$

(2.4)

Here, $C$ denotes the charge conjugation matrix and $T$ transposition. Since the gluino is a Majorana fermion, the currents contain the factor $C^{-1} \tilde{u}^T(k_1)$ rather than the $v(k_1)$ associated with Dirac fermions, but one could alternatively have used antiparticle spinors of opposite spins [24]. However, this is less convenient in dealing with the interference terms between uncrossed and crossed diagrams. Furthermore, the subscripts 1 and 2 refer to the mass eigenstates of the squarks. The quark-squark-gluino couplings depend on the chiral mixing (see figure 2), and are proportional to the coefficients

$$C_{\tilde{u}}^\pm = \cos \theta_{\tilde{u}} \pm \sin \theta_{\tilde{u}}.$$

(2.5)

Furthermore,

$$N_u = \frac{g g_s^2}{16(2\pi)^4 \cos \theta_W}.$$  

(2.6)

Here, $g$ and $g_s$ are the $SU(2)$ and $QCD$ coupling constants. For photon exchange, the corresponding factor is

$$N_u = \frac{e g_s^2}{6(2\pi)^4}.$$  

(2.7)

The triangle integral associated with this diagram (a) is given by

$$T_{uu1}^\mu(k_1, k_2) = \int d^4q \left[ \frac{q + k_1 + m_u}{(q + k_1)^2 - m_u^2 + i\epsilon} \gamma^\mu(g_V^u - g_A^u \gamma_5) \right] \times \frac{1}{(q - k_2)^2 - m_u^2 + i\epsilon} \frac{1}{q^2 - m_\ell^2 + i\epsilon},$$

(2.8)

with

$$g_V^u = 1 - \frac{8}{3} \sin^2 \theta_W, \quad g_A^u = 1.$$  

(2.9)

For each quark flavour, there are also four uncrossed and four crossed diagrams of type (b). The gluino currents corresponding to the uncrossed diagrams involving the
For photon exchange, the corresponding factors are matrices. It is convenient to expand the first one, eq. (2.8), in terms of "even" (u) and the other one, eq. (2.13) as

\[ T^\mu_{iju}(k_1, k_2) = \int dq q \left( \frac{4}{3} \sin^2 \theta_w - \cos^2 \theta_W \right) \left( \frac{g g_s^2}{8(2\pi)^4} \sin(2\theta_w) \right) \]

with over-all factors

\[ N_{11u} = -N_{22u} = \frac{g g_s^2}{8(2\pi)^4} \left( \frac{4}{3} \sin^2 \theta_w - \cos^2 \theta_W \right), \]
\[ N_{12u} = N_{21u} = \frac{g g_s^2}{16(2\pi)^4} \sin(2\theta_w). \]

For photon exchange, the corresponding factors are

\[ N_{11u} = N_{22u} = -\frac{e g_s^2}{6(2\pi)^4}, \]
\[ N_{12u} = N_{21u} = 0. \]

The triangle integral associated with this diagram (b) is

\[ T^\mu_{iju}(k_1, k_2) = \int dq q \left( \frac{4}{3} \sin^2 \theta_w - \cos^2 \theta_W \right) \left( \frac{g g_s^2}{8(2\pi)^4} \sin(2\theta_w) \right) \]

\[ \times \left( \frac{1}{m_1^2 + i\epsilon} \right) \left( \frac{1}{m_2^2 + i\epsilon} \right) \]

\[ \times \left( \frac{1}{(q + k_1)^2 - m_1^2 + i\epsilon} \right) \left( \frac{1}{(q - k_2)^2 - m_2^2 + i\epsilon} \right). \]

We need to also discuss the structure of the triangle integrals in terms of Dirac matrices. It is convenient to expand the first one, eq. (2.8), in terms of "even" (E) and "odd" (O) scalar integrals as

\[ T^\mu_{ui}(k_1, k_2) = E^\mu_{ui} a \gamma^\alpha (g^{a}_{V} - g^{a}_{A} \gamma_5) + E^b_{ui} \gamma^\mu a \gamma^\alpha (g^{b}_{V} + g^{b}_{A} \gamma_5) \]

\[ + O^a_{ui} \gamma^\mu (g^{a}_{V} - g^{a}_{A} \gamma_5) + O^b_{ui} \gamma^\mu \gamma^\alpha (g^{b}_{V} + g^{b}_{A} \gamma_5), \]

and the other one, eq. (2.13) as

\[ T^\mu_{iju}(k_1, k_2) = E^\mu_{ij} + O^\mu_{ij} \gamma^\alpha. \]
These integrals are discussed in Appendix A.

The gluino current of eq. (2.2) can now be written as

$$\tilde{G}^\mu = \sum_{\text{flavours}} \tilde{G}^\mu_q,$$

(2.16)

with the $u$-quark contribution

$$\tilde{G}^\mu_u = \left( \tilde{G}^{\mu}_{uu1} + \tilde{G}^{\mu}_{uu2} \right) + \left( \tilde{G}^{\mu}_{11u} + \tilde{G}^{\mu}_{22u} + \tilde{G}^{\mu}_{12u} + \tilde{G}^{\mu}_{21u} \right)$$

+ crossed terms

(2.17)

For each diagram there is a crossed diagram, whose amplitude is obtained by interchanging the gluino momenta, $k_1 \leftrightarrow k_2$, and reversing the over-all sign. Thus, the first terms of the amplitudes corresponding to the crossed diagrams are obtained from eqs. (2.4) and (2.10) as

$$\tilde{G}^{\mu(\text{cr})}_{uu1} = N_u \bar{u}(k_1)(C_u^+ - C_u^- \gamma_5) T^{\mu}_{uu1}(k_2, k_1)(C_u^+ + C_u^- \gamma_5) C^{-1} \bar{u}^T(k_2),$$

$$\tilde{G}^{\mu(\text{cr})}_{11u} = N_{11u} \bar{u}(k_1)(C_1^+ - C_1^- \gamma_5) T^{\mu}_{11u}(k_2, k_1)(C_1^+ + C_1^- \gamma_5) C^{-1} \bar{u}^T(k_2).$$

(2.18)

Furthermore, there are $4 + 8$ amplitudes involving the $d$-quark, with chiral mixing given by

$$C_d^\pm = \cos \theta_d \pm \sin \theta_d,$$

(2.19)

over-all factors,

$$N_d = N_u,$$

$$N_{11d} = -\frac{g g_s^2}{8(2\pi)^4 \cos \theta_W} \left( \frac{2}{3} \sin^2 \theta_W - \cos^2 \theta_d \right),$$

$$N_{22d} = -\frac{g g_s^2}{8(2\pi)^4 \cos \theta_W} \left( \frac{2}{3} \sin^2 \theta_W - \sin^2 \theta_d \right),$$

$$N_{12d} = N_{21d} = -\frac{g g_s^2}{16(2\pi)^4 \cos \theta_W} \sin(2\theta_d),$$

(2.20)

and

$$g_V^d = -1 + \frac{4}{3} \sin^2 \theta_W, \quad g_A^d = -1.$$  

(2.21)
The no-mixing limit

For comparison, we quote also the simple forms obtained for the amplitudes (2.4) and (2.10) in the limit of no mixing (nm) between the squarks,

\[ \tilde{G}_{uu1}^{\mu(nm)} = -N_u \bar{u}(k_2)(1 - \gamma_5)T_{uu1}^{\mu}(k_1, k_2)(1 + \gamma_5)C^{-1}\bar{u}^T(k_1), \]
\[ \tilde{G}_{uu2}^{\mu(nm)} = -N_u \bar{u}(k_2)(1 + \gamma_5)T_{uu2}^{\mu}(k_1, k_2)(1 - \gamma_5)C^{-1}\bar{u}^T(k_1), \]

and

\[ \tilde{G}_{11u}^{\mu(nm)} = -N_{11u}^{(nm)} \bar{u}(k_2)(1 - \gamma_5)T_{11u}^{\mu}(k_1, k_2)(1 + \gamma_5)C^{-1}\bar{u}^T(k_1), \]
\[ \tilde{G}_{22u}^{\mu(nm)} = -N_{22u}^{(nm)} \bar{u}(k_2)(1 + \gamma_5)T_{22u}^{\mu}(k_1, k_2)(1 - \gamma_5)C^{-1}\bar{u}^T(k_1), \]
\[ \tilde{G}_{12u}^{\mu(nm)} = \tilde{G}_{21u}^{\mu(nm)} = 0, \]

with

\[ N_{11u}^{(nm)} = -\frac{g g_s^2}{8(2\pi)^4\cos\theta_W} \left(1 - \frac{4}{3}\sin^2\theta_W\right), \]
\[ N_{22u}^{(nm)} = \frac{g g_s^2}{8(2\pi)^4\cos\theta_W} \frac{4}{3}\sin^2\theta_W. \]

Indices 1 and 2 will then refer to the squarks associated with the left- and right-chiral quarks. Their masses are given by eq. (1.13). In the presence of mixing, however, indices 1 and 2 will refer to the heavier and lighter of the two squarks, respectively.

3 The Gluino Current

The gluino current (2.17) can be written as a sum of pairs of terms, corresponding to the uncrossed and crossed diagrams. Furthermore, there are 8 amplitudes with two, and 4 with one internal squark line, a total of twelve diagrams for each quark flavour. For the \( u \)-quark loops we have

\[ \tilde{G}_u^{\mu} = \sum_i [\bar{u}(k_2)M_{uu}^{\mu}(k_1, k_2)C^{-1}\bar{u}^T(k_1) + \bar{u}(k_1)M_{uu}^{\mu(cr)}(k_1, k_2)C^{-1}\bar{u}^T(k_2)] \\
+ \sum_{i,j} [\bar{u}(k_2)M_{ijuu}^{\mu}(k_1, k_2)C^{-1}\bar{u}^T(k_1) + \bar{u}(k_1)M_{ijuu}^{\mu(cr)}(k_1, k_2)C^{-1}\bar{u}^T(k_2)] \]
\[
\begin{align*}
\bar{u}(k_2) [M^\mu_{uu}(k_1, k_2) - C^{-1} M^{\mu(\text{cr})T}_{uu}(k_1, k_2) C] C^{-1} \bar{u}^T(k_1) ] \\
+ \sum_{i,j} \{ \bar{u}(k_2) [M^\mu_{ij}(k_1, k_2) - C^{-1} M^{\mu(\text{cr})T}_{ij}(k_1, k_2) C] C^{-1} \bar{u}^T(k_1) \},
\end{align*}
\]

(3.1)

where in the last step we have transposed the crossed terms, using \( C^{-1T} = -C^{-1} \). It follows from eqs. (2.4) and (2.10) that

\[
M^\mu_{uu1} = -N_{uu} (C^+_\bar{u} - C^-_{\bar{u}} \gamma_5) T^\mu_{uu1}(k_1, k_2)(C^+_\bar{u} + C^-_{\bar{u}} \gamma_5),
\]

\[
M^\mu_{uu2} = -N_{uu} (C^-_{\bar{u}} + C^+_\bar{u} \gamma_5) T^\mu_{uu2}(k_1, k_2)(C^-_{\bar{u}} - C^+_\bar{u} \gamma_5),
\]

(3.2)

and

\[
M^\mu_{11u} = -N_{11u}(C^+_\bar{u} - C^-_{\bar{u}} \gamma_5) T^\mu_{11u}(k_1, k_2)(C^+_\bar{u} + C^-_{\bar{u}} \gamma_5),
\]

\[
M^\mu_{22u} = -N_{22u}(C^-_{\bar{u}} + C^+_\bar{u} \gamma_5) T^\mu_{22u}(k_1, k_2)(C^-_{\bar{u}} - C^+_\bar{u} \gamma_5),
\]

\[
M^\mu_{12u} = -N_{12u}(C^+_\bar{u} - C^-_{\bar{u}} \gamma_5) T^\mu_{12u}(k_1, k_2)(C^+_\bar{u} - C^-_{\bar{u}} \gamma_5),
\]

\[
M^\mu_{21u} = -N_{21u}(C^-_{\bar{u}} + C^+_\bar{u} \gamma_5) T^\mu_{21u}(k_1, k_2)(C^-_{\bar{u}} + C^+_\bar{u} \gamma_5).
\]

(3.3)

The crossed amplitudes are related by a change of sign, and interchange of \( k_1 \) and \( k_2 \),

\[
M^{\mu(\text{cr})}_{uu1} = N_{uu} (C^+_\bar{u} - C^-_{\bar{u}} \gamma_5) T^\mu_{uu1}(k_1, k_2)(C^+_\bar{u} + C^-_{\bar{u}} \gamma_5),
\]

\[
M^{\mu(\text{cr})}_{11u} = N_{11u}(C^+_\bar{u} - C^-_{\bar{u}} \gamma_5) T^\mu_{11u}(k_2, k_1)(C^+_\bar{u} + C^-_{\bar{u}} \gamma_5),
\]

(3.4)

etc. If we introduce a sign factor,

\[
S_1 = -, \quad S_2 = +,
\]

(3.5)

then these results (3.2)–(3.4) can be expressed more compactly as

\[
\begin{align*}
M^\mu_{uu} &= -S_i N_{iu} \left( C^-_{\bar{u}} - S_i C^S_{\bar{u}} \gamma_5 \right) T^\mu_{uu}(k_1, k_2) \left( S_i C^-_{\bar{u}} - C^S_{\bar{u}} \gamma_5 \right), \\
M^{\mu(\text{cr})}_{uu} &= S_i N_{iu} \left( C^-_{\bar{u}} + S_i C^S_{\bar{u}} \gamma_5 \right) T^\mu_{uu}(k_1, k_2) \left( S_i C^-_{\bar{u}} - C^S_{\bar{u}} \gamma_5 \right), \\
M^\mu_{ij} &= -S_j N_{iju} \left( C^-_{\bar{u}} - S_i C^S_{\bar{u}} \gamma_5 \right) T^\mu_{iju}(k_1, k_2) \left( S_j C^-_{\bar{u}} - C^S_{\bar{u}} \gamma_5 \right), \\
M^{\mu(\text{cr})}_{ij} &= S_j N_{iju} \left( C^-_{\bar{u}} + S_i C^S_{\bar{u}} \gamma_5 \right) T^\mu_{iju}(k_1, k_2) \left( S_j C^-_{\bar{u}} - C^S_{\bar{u}} \gamma_5 \right).
\end{align*}
\]

(3.6)
Exploiting now the fact that

\[ (C_+^u)^2 - (C_-^u)^2 = 2 \sin(2\theta_u), \]
\[ (C_+^u)^2 + (C_-^u)^2 = 2, \]
\[ 2C_+^u C_-^u = 2 \cos(2\theta_u), \] (3.7)

and the expansion (2.14) in terms of Dirac matrices, we find the structure of \( M_{uui}^\mu \) to be given by

\[ M_{uui}^\mu = 2S_i N_u E_{uui,\alpha}^a (k_1, k_2) \sin(2\theta_u) \gamma^\alpha \gamma^\mu (g^u_V - g^u_A \gamma_5) \]
\[ + 2S_i N_u E_{uui,\alpha}^b (k_1, k_2) \sin(2\theta_u) \gamma^\mu \gamma^\alpha (g^u_V + g^u_A \gamma_5) \]
\[ - 2N_u O_{uui}^a (k_1, k_2) \gamma^\mu \{ g^u_V + S_i g^u_A \cos(2\theta_u) - \gamma_5 [g^u_A + S_i g^u_V \cos(2\theta_u)] \} \]
\[ - 2N_u O_{uui,\alpha\beta}^b (k_1, k_2) \gamma^\alpha \gamma^\mu \gamma^\beta \{ g^u_V - S_i g^u_A \cos(2\theta_u) + \gamma_5 [g^u_A - S_i g^u_V \cos(2\theta_u)] \} . \] (3.8)

Similarly, we find [cf. eq. (2.13)]

\[ M_{iju}^\mu = S_j N_{iju} \left\{ -S_j C_{-i}^u S_i C_{-j}^u + S_i C_{-j}^u C_{-i}^u \right\} \]
\[ + \gamma_5 \left\{ C_{-i}^u C_{-j}^u - S_i S_j C_{-i}^u C_{-j}^u \right\} E_{iju}^\mu (k_1, k_2) \]
\[ - S_j N_{iju} \gamma^\alpha \left\{ S_j C_{-i}^u S_i C_{-j}^u + S_i C_{-j}^u C_{-i}^u \right\} \]
\[ - \gamma_5 \left\{ C_{-i}^u C_{-j}^u + S_i S_j C_{-i}^u C_{-j}^u \right\} O_{iju,\alpha}^\mu (k_1, k_2) \] (3.9)

From eq. (3.1), we define \( M^\mu \) by

\[ \tilde{G}^\mu = \sum_{\text{generations}} (\tilde{G}^\mu_u + \tilde{G}^\mu_d) \]
\[ = \bar{u}(k_2) [M^\mu(k_1, k_2) - C^{-1} M^{\mu(\text{cr})T}(k_1, k_2) C] C^{-1} \bar{u}^T(k_1). \] (3.10)

Thus, when summed over flavours [cf. eq. (2.17)], we have

\[ M^\mu = \sum_{\text{generations}} \left\{ (M_{u11}^\mu + M_{u12}^\mu) + (M_{u11}^\mu + M_{u22}^\mu + M_{12}^\mu + M_{21}^\mu) \right\} \]
\[ + (M_{d1}^\mu + M_{d2}^\mu) + (M_{11}^\mu + M_{22}^\mu + M_{12}^\mu + M_{21}^\mu) \] (3.11)
and a similar expression $M^{\mu (cr)}$ for the crossed amplitudes.

Using eqs. (3.8) and (3.9), we get the following structure in terms of Dirac matrices

$$M^\mu - C^{-1} M^{\mu (cr)} T C = (\mathcal{V}_a^a + \gamma_5 A_\alpha^a) \gamma^\alpha \gamma^\mu + \left( \mathcal{V}_b^b + \gamma_5 A_\alpha^b \right) \gamma^\mu \gamma^\alpha + (\mathcal{V}_c^c + \gamma_5 A_\alpha^c) \gamma^\alpha \gamma^\beta + \mathcal{V}_e^e \gamma^\mu + \gamma_5 A_\alpha^e \gamma^\alpha \right).$$

(3.12)

The $\mathcal{V}^c$ and $\mathcal{V}_a^f \gamma^\mu$ contributions vanish since two Majorana fermions cannot form a vector current:

$$\overline{\Psi}_g \gamma^\mu \Psi_g = \left( \overline{\psi}_g \overline{\sigma}^\mu \psi_g \right) \left( \begin{array}{c} 0 \\ \sigma^\mu \\ 0 \end{array} \right) = \overline{\psi}_g \overline{\sigma}^\mu \psi_g + \psi_g \sigma^\mu \overline{\psi}_g = 0.$$  

(3.13)

The other $\mathcal{V}$ and $\mathcal{A}$ terms are given in Appendix A. All the remaining $\mathcal{V}$ and also the pseudoscalar $A_\alpha^e$ vanish, and eq. (3.12) takes the simple form

$$M^\mu - C^{-1} M^{\mu (cr)} T C = \mathcal{A}_\alpha^a \gamma^\alpha \gamma^\mu \gamma_5^a + \mathcal{A}_\alpha^b \gamma^\mu \gamma^\alpha \gamma_5^b + \mathcal{A}_\alpha^d \gamma^\alpha \gamma^\mu \gamma_5^d - \mathcal{A}_\alpha^e \gamma^\mu \gamma_5^e.$$  

(3.14)

4 The Cross Section

Evaluating the spin sum, we get [cf. eq. (2.2)]

$$X = \frac{1}{4} \sum_{\text{spin}} \mathcal{M}^\dagger \mathcal{M}$$

$$= \frac{1}{4} \sum_{\text{spin}} \left( L_\mu D_{F \mu \nu} \tilde{G}^\nu \right) \left( \tilde{G}^{\alpha \dagger} D_{F \alpha \beta}^\dagger L^{\beta \dagger} \right)$$

$$= \frac{g_\psi^2 + g_A^2}{4 \cos^2 \theta_W} \left( p_{1 \nu} p_{2 \nu} + p_{1 \nu} p_{2 \mu} - (p_1 \cdot p_2) g_{\mu \nu} \right) \mathcal{T}^{\mu \nu},$$

(4.1)

where [cf. eq. (3.10)]

$$\mathcal{T}^{\mu \nu} = \sum_{\text{spin}} \bar{u}(k_2) \left[ M^\mu (k_1, k_2) - C^{-1} M^{\mu (cr)} (k_1, k_2) C \right] C^{-1} \bar{u}^T(k_1)$$
\begin{align*}
&\times u^T(k_1)\gamma^{0T}(-C^{-1}) \left[ M^{\mu(k_1,k_2)} - CM^{\nu(k_1,k_2)}C^{-1} \right] \gamma^0 u(k_2) \\
&= \text{Tr} \left[ \left\{ M^{\mu(k_1,k_2)} - CM^{\nu(k_1,k_2)}C \right\} (k_1 - m_\tilde{g}) \right. \\
&\quad \left. \times \gamma^0 \left\{ M^{\nu(k_1,k_2)} - CM^{\mu(k_1,k_2)}C^{-1} \right\} \gamma^0 (k_2 + m_\tilde{g}) \right].
\end{align*}
\tag{4.2}

We have here used $\gamma^{\mu T} = -C\gamma^{\mu}C^{-1}$.

Invoking eq. (3.14), we obtain the structure of the tensor $T^{\mu\nu}$ in terms of Dirac matrices as
\begin{align*}
T^{\mu\nu} &= -\text{Tr} \left[ \left\{ A^{a}_\alpha \gamma^{\alpha} \gamma^{\mu} \gamma_5 + A^{b}_\alpha \gamma^{\mu} \gamma^{\alpha} \gamma_5 - A^{c}_\alpha \gamma^{\mu} \gamma_5 - A^{f}_\alpha \gamma^{\alpha} \gamma_5 \\
&\quad - A^{d}_{\alpha\beta} \gamma^{\alpha} \gamma^{\mu} \gamma^{\beta} \gamma_5 \right\} (k_1 - m_\tilde{g}) \\
&\quad \times \left\{ A^{a\dagger}_\rho \gamma^{\rho} \gamma_5 + A^{b\dagger}_\rho \gamma^{\rho} \gamma_5 + A^{c\dagger} \gamma^{\rho} \gamma_5 + A^{f\dagger} \gamma^{\rho} \gamma_5 \\
&\quad + A^{d\dagger}_{\rho\sigma} \gamma^{\rho} \gamma^{\sigma} \gamma_5 \right\} (k_2 + m_\tilde{g}) \right],
\end{align*}
\tag{4.3}

and evaluate the trace using computer algebra [25, 26].

By summing over the eight gluino colours, and integrating over the solid angle, we find that the cross section is proportional to the square of the sum of two partial amplitudes, corresponding to the contributions of the two diagrams (a) and (b). This is possible, since by general arguments [13], there is essentially only one invariant amplitude. The integrated cross section thus takes the form
\begin{equation}
\sigma = \frac{g^2\pi^3 (g_V^2 + g_A^2) \left( \sqrt{E^2 - m_\tilde{g}^2} \right)^3}{12E \cos^2 \theta_W \left[ (s - m^2_Z)^2 + \Gamma^2_Z m^2_Z \right]} \left| \sum (A_a + A_b) \right|^2,
\tag{4.4}
\end{equation}

with $E$ the beam energy and the sum running over quark flavours $q$. The two partial amplitudes correspond to diagrams (a) and (b) and are given as
\begin{align*}
A_a &= 4 \sum_i S_i N_q \left\{ F^{00}_{q\bar{q}} \left( \hat{b}_q m_\tilde{g} m_q + f_{q\bar{q}} m_\tilde{g}^2 - v_{q\bar{q}} m_\tilde{g}^2 \right) - 4 F^{01}_{q\bar{q}} m_\tilde{g} (\hat{b}_q m_q + f_{q\bar{q}} m_\tilde{g}) \\
&\quad + 2 F^{02}_{q\bar{q}} f_{q\bar{q}} m_\tilde{g}^2 - 2 F^{11}_{q\bar{q}} f_{q\bar{q}} \left( \frac{1}{2} s - m_\tilde{g}^2 \right) + G_{q\bar{q}} f_{q\bar{q}} \right\}, \\
A_b &= 4 \sum_{ij} S_j N_{ij} b_{ij} G_{ij},
\tag{4.5}
\end{align*}
with $S_i$ the sign factor of eq. (3.3) and the dependence on the electroweak and chiral mixing angles given by the coefficients

\[
\hat{b}_q = -2g_A^q \sin(2\theta_q),
\]

\[
b_{ijq} = C^{-S_i}_{\bar{q}} C^{S_i}_{\bar{q}} + S_i S_j C^{S_i}_{\bar{q}} C^{-S_j}_{\bar{q}},
\]

\[
f_{qqi} = 2S_i \{g_A^q - S_i g_V^q \cos(2\theta_q)\},
\]

\[
v_{qqi} = -2S_i \{g_A^q + S_i g_V^q \cos(2\theta_q)\},
\]

which are read off from the contributing $A$ terms of eq. (4.3). We note that the amplitudes (4.3) contain terms that apparently are odd in the masses, i.e., proportional to $m_{\bar{g}} m_q$. These arise from the chiral mixing, i.e., they are multiplied by factors $\hat{b}_q$ which also are odd in these masses, and vanish in the limit of no mixing. The integrals $F^{ab}_{qqi}$, $G_{qqi}$ and $G_{ijq}$ are given in Appendix A.

The above result, eq. (4.4), is given as an integrated cross section. Actually, since there is only one invariant amplitude, whose structure is determined by the fact that it describes the annihilation of two massless fermions to a pair of self-conjugate fermions [13], the angular distribution is given by the familiar expression

\[
\frac{d\sigma}{d\Omega} = \frac{3}{16\pi} \sigma (1 + \cos^2 \theta).
\]

5 Results

In order to better understand what is required for the cross section to be large, let us first state the conditions that must be satisfied in order for it to vanish.

Conditions for vanishing cross section

The gluino pair production cross section would vanish if the following two conditions were both satisfied. These conditions are [13]

1. mass degeneracy in each quark isospin doublet, $m_d = m_u$ (this is violated),

2. mass degeneracy in each squark isospin doublet, i.e., $m_{\tilde{d}_1} = m_{\tilde{d}_2} = m_{\tilde{u}_1} = m_{\tilde{u}_2}$, for each generation.
Kane and Rolnick [14] state that in the case of $Z$ decay, the cross section vanishes when $m_\tilde{q} = m_q$. We do not reproduce this requirement, but instead the conditions (1) and (2) above.

For comparison, in the case of no axial coupling to the $Z$, i.e., in the QED limit, the cross section would vanish if there is

- mass degeneracy in each squark chiral doublet, i.e., $m_{\tilde{u}_1} = m_{\tilde{u}_2}$, and $m_{\tilde{d}_1} = m_{\tilde{d}_2}$ for each generation. This condition is less strong than item (2) above.

The magnitude of the cross section will depend on how strongly these conditions (1) and (2) are violated. Especially for the third generation, item (1) is violated. This is generally believed to imply that the squark isospin doublets are not degenerated either. However, in a consistent MSSM, the squark masses cannot be specified as free parameters, they emerge as dependent on the more fundamental parameters of the Lagrangian. Furthermore, there are four squark masses for each generation. It is therefore not possible to make simple (and correct) statements about the magnitude of the cross section.

For the purpose of developing some intuition for how large the gluino pair production cross section would be at LEP, we show in figure 3 the ratio

$$R = \frac{\sigma(e^+e^- \rightarrow \tilde{g}\tilde{g})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

vs. maximal squark mass splitting $\delta m_\tilde{q}$. The plot is based on a scan of the MSSM parameter space, at grid points given by

$$\begin{align*}
\tan \beta & \in \{1.1, 5, 15, 30\}, \\
\mu & \in \{0, \pm 20, \pm 40, \pm 70, \pm 100, \pm 200, \pm 300, \pm 500\} \text{ GeV,} \\
A_t & \in \{0, 10, 20, 40, 70, 100, 200, 300, 500, 800, 1000\} \text{ GeV,} \\
A_b & \in \{0, 10, 20, 40, 70, 100, 200, 300, 500, 800, 1000\} \text{ GeV,} \\
\tilde{M}_T & \in \{0, 10, 20, 40, 70, 100, 200, 300, 500, 800, 1000\} \text{ GeV,} \\
\tilde{m}_T & \in \{0, 10, 20, 40, 70, 100, 200, 300, 500, 800, 1000\} \text{ GeV,} \\
\tilde{m}_B & \in \{0, 10, 20, 40, 70, 100, 200, 300, 500, 800, 1000\} \text{ GeV,}
\end{align*}$$

(5.2)
for gluino, bottom and top quark masses given by the “standard values”,

\[ m_{\tilde{g}} = 3.5 \text{ GeV}, \quad m_b = 4.8 \text{ GeV}, \quad m_t = 170 \text{ GeV}. \]  

(5.3)

We here consider only the contributions from the third generation, so \( t \) (or \( T \)) and \( b \) (or \( B \)) refer to \( u \) (or \( U \)) and \( d \) (or \( D \)) in the Lagrangian (1.1)–(1.4). All encountered cross section ratios lie in the light shaded region, where the horizontal axis gives the largest resulting squark mass difference, \( \delta m_{\tilde{q}} = \max_{i,j} |m_{\tilde{q}_i} - m_{\tilde{q}_j}| \). No values are found within the dark shaded or the white regions. The cross section ratios are thus typically between \( 10^{-5} \) and \( 10^{-2} \). (The \( Z \) branching ratio is obtained upon multiplying by 3.3\%). The jagged borders are ascribed to the discreteness of the sampling, as well as the rather complex dependence the cross section has on the many parameters. Parameter sets that lead to any one of the squarks being light, \( m_{\tilde{q}} < 45 \text{ GeV} \), are left out, since such light squarks would have been detected at LEP [22].

The value for the gluino mass, \( m_{\tilde{g}} = 3.5 \text{ GeV} \), has been chosen as representative of the “light-gluino window”. Actually, the cross section has only a very weak dependence on the gluino mass, as long as it is well below the kinematical threshold [23].

**Dependence on squark and top masses**

As noted previously [14], the gluino cross section tends to increase with increasing top mass, but the way it increases depends on the other parameters. This is illustrated in figure 4, where we show the ratio \( R \) as a function of stop mass (denoted \( m_{\tilde{u}} \)), for different values of top mass (denoted \( m_t \)). However, this figure is somewhat idealized in the sense that the squark masses are set by hand, they do not result naturally from some set of fundamental parameters of the Lagrangian. Two sets of parameters are considered, each set is for \( m_{\tilde{u}_1} = m_{\tilde{u}_2} \equiv m_{\tilde{u}} \) and \( m_{\tilde{b}_1} = m_{\tilde{b}_2} \equiv m_{\tilde{b}} \). The three steep curves are for \( m_{\tilde{d}} = 50 \text{ GeV} \), whereas the other three are for \( m_{\tilde{d}} = 200 \text{ GeV} \). For each set, three values of the top quark mass are considered, \( m_t = 0, 50, \) and \( 170 \text{ GeV} \). We note that if \( m_u = m_d \) (= 0 GeV) and \( m_{\tilde{u}} = m_{\tilde{d}} \), then the cross section vanishes, in accordance with items (1) and (2) above.

**Dependence on \( \tan \beta \) and \( \mu \)**

We can now start to address the question of which parameters would be restricted by an experimental limit on the cross section. With the parameters \( A_t, A_b, \bar{M}_T, \bar{m}_T, \) and \( \bar{m}_B \)
allowed to take on values in the set

\[ A_t \in \{0, 10, 20, 50, 100, 300, 500, 800\} \text{ GeV}, \]
\[ A_b \in \{0, 10, 20, 50, 100, 300, 500, 800\} \text{ GeV}, \]
\[ \tilde{M}_T \in \{0, 10, 20, 50, 100, 300, 500, 800\} \text{ GeV}, \]
\[ \tilde{m}_T \in \{0, 10, 20, 50, 100, 300, 500, 800\} \text{ GeV}, \]
\[ \tilde{m}_B \in \{0, 10, 20, 50, 100, 300, 500, 800\} \text{ GeV}, \]

we have scanned for extrema of the gluino cross section as a function of \( \tan \beta \) and \( \mu \). It turns out that there is little dependence on the latter parameters. In fact, the minimal values found are \( R_{\text{min}} \simeq 10^{-6} \), whereas the maximal values are \( R_{\text{max}} \simeq 0.01 - 0.02 \), with a rather weak dependence on \( \tan \beta \) and \( \mu \), for \( 1.1 \leq \tan \beta \leq 50 \) and \( |\mu| \leq 500 \text{ GeV} \). Thus, \emph{an upper limit on the gluino pair production cross section does not significantly restrict neither \( \tan \beta \) nor \( \mu \)}. The lightest squark, which is the lightest stop, \( \tilde{t}_2 \), will exceed about 350 GeV for the values of \( A_t, A_b, \tilde{M}_T, \tilde{m}_T \) and \( \tilde{m}_B \) which minimize \( R \), in the given range of \( \tan \beta \) and \( \mu \).

\textbf{Dependence on \( \tan \beta \) and \( \tilde{M}_T \)}

In figure 5 we indicate the dependence of the cross section on \( \tan \beta \) and \( \tilde{M}_T \), for the following choice of the other parameters,

\[ \tilde{M}_T = \tilde{m}_T = \tilde{m}_B, \]
\[ A = A_t = A_b, \]

with

\[ \mu \in \{0, \pm 20, \pm 40, \pm 70, \pm 100, \pm 200, \pm 300, \pm 500\} \text{ GeV}, \]
\[ A \in \{0, 10, 20, 40, 70, 100, 200, 300, 500, 800, 1000\} \text{ GeV} \]

and for the “standard values” for gluino, \( b \) and \( t \) quark masses given by eq. (5.3). Clearly, an upper bound on the cross section ratio of e.g. \( 10^{-3} \), would rule out values of \( \tilde{M}_T \) below about 350 GeV.
A lower bound on $\tilde{M}_T$ would also lead to a lower bound on the heaviest squark (for this set of parameters, always the heaviest stop, $\tilde{t}_1$) about similar in magnitude to $\tilde{M}_T$ \cite{23}.

**Dependence on $\tilde{m}_B$ and $\tilde{m}_T$**

The correlation between the cross section ratio $R$ and $\tilde{M}_T$ is however not quite as simple as that shown in figure 5 if we relax the condition (5.5). It turns out that $R$ can become larger than $10^{-3}$ even for rather low values $\tilde{m}_T \leq 100$ GeV, *provided $\tilde{m}_B$ is high*. This is illustrated in figure 6, where we show regions in the $\tilde{m}_B$--$\tilde{m}_T$ plane where $R$ exceeds $10^{-3}$ for given upper bounds on $\tilde{M}_T$. Two cases are considered, $R_{\text{min}}$ in (a), and $R_{\text{max}}$ in (b), where “min” and “max” refer to scans over the parameter values

$$\tan \beta \in \{1.1, 5, 15, 30\},$$

$$\mu \in \{0, \pm 50, \pm 100, \pm 200, \pm 500\} \text{ GeV},$$

$$A_t \in \{0, 10, 20, 50, 100, 300, 800\} \text{ GeV},$$

$$A_b \in \{0, 10, 20, 50, 100, 300, 800\} \text{ GeV}. \quad (5.7)$$

The gluino and quark masses considered are the “standard values”, and for $\tilde{M}_T$ we have taken

$$\tilde{M}_T \in \{50, 100, 300, 800\} \text{ GeV}. \quad (5.8)$$

**6 Discussion**

The present study does not address the question of decay or fragmentation. In order to consider a “worst case” scenario, we basically assume the gluinos are stable and form gluinoballs. If they are unstable and decay, detection would be easier. These gluinoballs must be colour singlets, but could be electrically charged, in which case they would show up in the detectors, or neutral, in which case they would presumably escape undetected. *However, in the latter case, since they are produced far above threshold, one would expect a few ordinary hadrons (e.g., pions) to also emerge from the fragmentation process*. These would be detected, and give standard SUSY-triggers of considerable missing energy.
For the sake of definiteness, suppose one can rule out the production of gluino pairs at a level of at most 10 events per 1 million $Z$ decays. This would imply $R < 10^{-5}/3.3\%$, or $R < 3 \cdot 10^{-4}$. It follows from figure 3 that this condition would exclude much of the “Physical Region”. From figures 5 and 6 we see that lower limits on the soft-supersymmetry-breaking parameters would be obtained, but that the precise limits would depend on whether these parameters are related.

In summary, the pair production of gluinos, without accompanying quark jets, is in $Z$ decay large enough to be measurable in much of the MSSM parameter space, and should therefore be searched for vigorously.

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Appendix A

This appendix provides some information on the triangle integrals.

The integrals of eqs. (2.14) and (2.15):

The quantities appearing in eqs. (2.14) and (2.15) can be expressed in terms of more basic integrals as

\[ F_{uuu}^{\mu}(k_1, k_2) = -i\pi^2 \left[ k_1^\mu F_{uuu}^{00} + k_2^\mu \left( F_{uuu}^{01} - F_{uuu}^{10} \right) \right], \]

\[ F_{uuu}^{\nu}(k_1, k_2) = i\pi^2 \left[ k_1^\nu \left( F_{uuu}^{01} - F_{uuu}^{00} \right) + k_2^\nu F_{uuu}^{10} \right], \]

\[ O_{uuu}^{\mu\nu}(k_1, k_2) = \frac{1}{2} g^{\mu\nu} \left( \frac{1}{\epsilon} - \gamma + 2 - 2G_{uuu} \right) + k_1^\mu k_1^\nu \left( F_{uuu}^{01} - F_{uuu}^{02} \right) \]

\[ + k_1^\mu k_2^\nu \left( F_{uuu}^{10} - F_{uuu}^{11} + F_{uuu}^{10} - F_{uuu}^{01} \right) + k_2^\mu k_2^\nu \left( F_{uuu}^{10} - F_{uuu}^{20} \right), \]

with 1/\(\epsilon\) representing the UV-divergent part, \(\gamma\) the Euler constant, and the integrals over Feynman parameters defined by

\[ F_{qqi}^{ab} = \int_0^1 dx \int_0^{1-x} dz \frac{z^a x^b}{h_{qqi}}, \]

\[ F_{ijq}^{ab} = \int_0^1 dx \int_0^{1-x} dz \frac{z^a x^b}{h_{ijq}}, \]

\[ G_{qqi} = \int_0^1 dx \int_0^{1-x} dz \log \frac{h_{qqi}}{\mu^2}, \]

\[ G_{ijq} = \int_0^1 dx \int_0^{1-x} dz \log \frac{h_{ijq}}{\mu^2}. \]

(A.2)

with

\[ h_{qqi} = m_\tilde{g}^2 (x + z)(x + z - 1) - sxz - (m_\tilde{q_i}^2 - m_\tilde{q}^2)(x + z) + m_\tilde{q_i}^2 - i\epsilon, \]

\[ h_{ijq} = m_\tilde{g}^2 (x + z)(x + z - 1) - sxz + (m_\tilde{q_j}^2 - m_\tilde{q}^2)x \]

\[ + (m_\tilde{q_i}^2 - m_\tilde{q}^2)z + m_\tilde{q}^2 - i\epsilon. \]

(A.3)
The parameter $\mu$ is a renormalization mass. When the amplitude is summed over both members of an isospin doublet, the $\mu$-dependence cancels. The integrals $F_{qqi}^{ab}$, $G_{qqi}$ and $G_{ijq}$ can be evaluated in terms of dilogarithms ($F_{ijq}^{ab}$ does not contribute). Performing the integration over $z$, we find that the $F_{qqi}^{ab}$, $G_{qqi}$ and $G_{ijq}$ can be expressed in terms of the one-dimensional integrals

$$I^m = \int_0^1 dx x^m \log \left[ \frac{ax + b \pm \sqrt{c(x^2 + 2dx + e)}}{c(x^2 + 2dx + e)} + \pm ie \right]$$

$$J = \int_0^1 dx \sqrt{c(x^2 + 2dx + e)} \log \left[ \frac{ax + b \pm \sqrt{c(x^2 + 2dx + e)}}{c(x^2 + 2dx + e)} + \pm ie \right]$$

$$K^n = \int_0^1 dx x^n \log \left[ (ax + b)^2 - c(x^2 + 2dx + e) - ie \right].$$

Here $m = 0, 1, 2$, and $n = 0, 1$. The $K^n$ integral is straightforward. The arguments of the square roots in $I^m$ and $J$ may change sign within the domain of integration. The $I^m$ and $J$ integrals are evaluated using the following substitutions

$$x = y - d \rightarrow y = u\sqrt{d^2 - e} \rightarrow \begin{cases} u = \cosh \alpha \rightarrow v = \tanh(\alpha/2) & \text{when } u \geq 1, \\ u = -\cosh \alpha \rightarrow v = \tanh(\alpha/2) & \text{when } u \leq 1, \\ u = \sin \alpha \rightarrow v = \tan(\alpha/2) & \text{when } |u| < 1. \end{cases}$$

The integrals $F_{uui}^{ab}$ and $F_{ijq}^{ab}$ satisfy the symmetry relations

$$F_{uui}^{ab} = F_{uui}^{ba} \quad \text{and} \quad F_{ijq}^{ab} = F_{ijq}^{ba},$$

This is easily checked by interchanging the parametric integrations.

The integrals of eq. (3.12):

The $V$ integrals are defined by

$$V^a_{\alpha} = 2 \sum_{iq} S_i N_q g^q_V \sin(2\theta_q) \left[ E_{qqi}^{a\alpha}(k_1, k_2) + E_{qqi}^{b\alpha}(k_2, k_1) \right],$$

$$V^b_{\alpha} = 2 \sum_{iq} S_i N_q g^q_V \sin(2\theta_q) \left[ E_{qqi}^{a\alpha}(k_2, k_1) + E_{qqi}^{b\alpha}(k_1, k_2) \right],$$

$$V^d_{\alpha\beta} = -2 \sum_{iq} N_q \left[ O_{qqi}^{b\alpha\beta}(k_1, k_2) - O_{qqi}^{b\alpha\beta}(k_2, k_1) \right] \left[ g^q_V - S_i g^q_A \cos(2\theta_q) \right],$$

$$V^{e\mu} = -\sum_{ijq} S_j N_{ijq} \left[ E_{ijq}^{\mu}(k_1, k_2) + E_{ijq}^{\mu}(k_2, k_1) \right] \left( S_i C_{ijq}^{S_i} C_{ijq}^{S_i} - S_i C_{ijq}^{S_i} C_{ijq}^{S_i} \right).$$

(A.7)
Using eqs. (A.1) and (A.6), all these terms can be shown to vanish, i.e., we are left with only the $A$ type terms,

\[
\mathcal{A}_a = -2 \sum_{iq} S_i N_q g_A^a \sin(2\theta_{\tilde{q}}) \left[ E_{qqi\alpha}^a(k_1, k_2) - E_{qqi\alpha}^b(k_2, k_1) \right], \\
\mathcal{A}_b = -2 \sum_{iq} S_i N_q g_A^a \sin(2\theta_{\tilde{q}}) \left[ E_{qqi\alpha}^a(k_2, k_1) - E_{qqi\alpha}^b(k_1, k_2) \right], \\
\mathcal{A}_c = -2 \sum_{iq} N_q \left[ O_{qqi\alpha}^a(k_1, k_2) + O_{qqi\alpha}^a(k_2, k_1) \right] \left[ g_A^q + S_i g_{\nu}^q \cos(2\theta_{\tilde{q}}) \right], \\
\mathcal{A}_d = 2 \sum_{iq} N_q \left[ O_{qqi\alpha\beta}^b(k_1, k_2) + O_{qqi\beta\alpha}^b(k_2, k_1) \right] \left[ g_A^q - S_i g_{\nu}^q \cos(2\theta_{\tilde{q}}) \right], \\
\mathcal{A}_e = \sum_{ijq} S_j N_{ijq} \left[ E_{ijq\mu}^\mu(k_1, k_2) + E_{ijq\mu}^\mu(k_2, k_1) \right] \left( C_{\tilde{q}j}^S C_{ijq}^S - S_i S_j C_{ijq}^S C_{\tilde{q}j}^S \right), \\
\mathcal{A}_f = -\sum_{ijq} S_j N_{ijq} \left[ O_{ijq\mu}^\mu(k_1, k_2) + O_{ijq\mu}^\mu(k_2, k_1) \right] \left( C_{\tilde{q}j}^S C_{\tilde{q}j}^S - S_i S_j C_{\tilde{q}j}^S C_{\tilde{q}j}^S \right). \\
\]  

(A.8)

Using eqs. (A.1) and (A.6), we find that $\mathcal{A}_e^\mu$ vanishes, and eq. (3.12) reduces to eq. (3.14).
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Figure captions

Fig. 1. The two classes of Feynman diagrams for $e^+e^- \rightarrow \tilde{g}\tilde{g}$.

Fig. 2. The couplings involved in the process $e^+e^- \rightarrow \tilde{g}\tilde{g}$.

Fig. 3. Cross section ratios $R = \sigma(e^+e^- \rightarrow \tilde{g}\tilde{g})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ at the $Z$ resonance. The figure shows the result of a scan of parameter space, eq. (5.2), against the largest resulting squark mass difference.

Fig. 4. Cross section ratios $R$ vs. stop mass $m_{\tilde{t}_1} = m_{\tilde{t}_2} \equiv m_{\tilde{t}}$. Two values of sbottom mass are considered, $m_{\tilde{b}_1} = m_{\tilde{b}_2} \equiv m_{\tilde{b}} = 50$ GeV and 200 GeV, together with three values of $(u$ or) top quark mass.

Fig. 5. Regions of lower bounds on $R$ in the plane spanned by the soft squark mass parameter $\tilde{M}_T$ [cf. eqs. (1.4) and (1.5)] and $\tan \beta$. A somewhat special case is considered, cf. eq. (5.5). We here consider the values of $m_{\tilde{g}}$, $m_{\tilde{b}}$, and $m_t$ given by eq. (5.3).

Fig. 6. Regions where $R \geq 10^{-3}$ are for different values of $\tilde{M}_T$ outlined in the plane spanned by $\tilde{m}_T$ and $\tilde{m}_B$. In (a), we show the regions where the minimum values of $R$, obtained when scanning the other parameters, fulfill $R \geq 10^{-3}$. In (b), we show the regions where the maximum values of $R$, obtained when scanning the other parameters, fulfill $R \geq 10^{-3}$. The region where $\tilde{m}_B < 50$ GeV is not allowed.