Isomorphic Jacobian matrix of parallel prototype manipulator for micro/nano scale compliant mechanism

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Abstract. To realize the isomorphism of the three-degree-of-freedom (DOF) rotational motion characteristics between the compliant mechanism and the parallel prototype manipulator, a method of constructing the isomorphic mapping Jacobian matrix with micro/nano scale is proposed in this paper. Based on 3-UPC type of spatial parallel manipulator with three rotational DOF, the differential equivalent method is used to simplify and construct the isomorphic Jacobian matrix. The simulation results show that this type of isomorphic mapping Jacobian matrix of parallel prototype manipulator has the similarity motion characteristics which can be used to analyse the topological structure of compliant mechanisms.

1. Introduction

Distributed compliant mechanisms are widely used in the field of micro/nano fabrication because of their low stress concentration and long fatigue life. As early as 1971, Shoup and McLarnan\(^1\) proposed the design of several kinds of spatial mechanism with flexible slender beam, in the assumption of the large deflection and small strain, many researchers have carried out a large deflection beam distributed compliant mechanism configuration synthesis and modeling method based on literatures\(^2\)\(^-\)\(^5\). As for the spatial configuration of compliant mechanism with micro/nano scale features, the large deflection and small deformation assumption is not established. The topology optimization method is adopted for spatial structure analysis\(^6\). Based on mathematics, physics and mechanical configuration synthesis with hierarchical mapping module, Yu Jingjun\(^7\) proposed the comprehensive configuration method of unified visual flexible mechanism by building the mapping relationship between geometry, movement or constraints, and mechanism based on screw theory. In the aspect of modeling micro/nano compliant mechanisms, the researchers tried to model by finite element method. Wu-Le Zhu\(^8\) taken all the connecting rod as a flexible element, adopted the finite element method to obtain the element stiffness matrix and established the motion model of XY platform.

The Jacobian matrix is to describe the relationship between movement and the end of each joint movement. The design concept of the spatial configuration of the parallel manipulator can be used for the synthesis of compliant mechanism, and the Jacobian mapping matrix of parallel prototype manipulator must be established for characterization the micro/nano scale motion characteristic of spatial compliant mechanism. In this paper, 3-UPC type parallel prototype manipulator with three spatial rotational motion characteristics is adopted as the research object. By using the micro/nano scale equivalent method, the Jacobian mapping matrix is constructed. Numerical examples are presented to demonstrate the effectiveness of the proposed method. Finally, conclusions are developed.
2. Isomorphic mapping matrix of 3-UPC type parallel prototype manipulator

3-UPC type parallel prototype manipulator with three rotational degree-of-freedom (DOF) is composed of a moving platform, fixed platform and three symmetrical links. Each link is composed by a U pair (universal joint) which connected with fixed platform, a C pair (cylindrical joint) which connected with moving platform, and C pair connected with U pair through a actuated joint P (prismatic joint). The U pair of each link is equal to two rotational joints with two orthogonal axes, the axes of C pairs and one rotational joints of U pair in three links are all intersected at the point o. The schematic diagram of the mechanism is shown as Fig.1(a), and the schematic diagram of the structure parameters is shown as Fig.1(b), respectively.

Fig.1. Schematic diagram of the 3-UPC type parallel prototype manipulator
(a-schematic diagram of the mechanism; b-structure parameters)

The kinematic properties of the 3-UPC type parallel manipulator have been analyzed by means of screw theory. We assumed that the moving platform around the three axis rotation parameters are $\alpha$, $\beta$, $\gamma$, respectively. Because of the moving platform of micro/nano scale motion characteristics, without loss of generality, we adopted the following approximation such as $\sin(\cdot) = \cdot$, $\cos(\cdot) = 1$, and $\alpha \cdot \beta = \alpha \cdot \gamma = \beta \cdot \gamma = 0$, which called micro/nano scale equivalent method.

Set up the coordinate system as shown in Fig.1(b), the initial coordinates of each connection point can be written as following.

\[
\begin{align*}
A_1 \left(-a, \frac{\sqrt{3}}{3}a, 0\right), & \quad A_2 \left(a, \frac{\sqrt{3}}{3}a, 0\right), & \quad A_3 \left(0, -\frac{2\sqrt{3}}{3}a, 0\right) \\
B_1 \left(-b, \frac{\sqrt{3}}{3}b, h\right), & \quad B_2 \left(b, \frac{\sqrt{3}}{3}b, h\right), & \quad B_3 \left(0, -\frac{2\sqrt{3}}{3}b, h\right)
\end{align*}
\]

(1)

Using the Euler transform, the rotation transformation matrix is given by

\[
R(\alpha, \beta, \gamma) = R_x(\alpha)R_y(\beta)R_z(\gamma) =
\begin{bmatrix}
\cos\gamma & -\sin\gamma & 0 \\
\sin\gamma & \cos\gamma & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos\beta & 0 & \sin\beta \\
0 & 1 & 0 \\
-\sin\beta & 0 & \cos\beta
\end{bmatrix}
\begin{bmatrix}
\cos\alpha & -\sin\alpha & 0 \\
\sin\alpha & \cos\alpha & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(2)

Where $\sin(\cdot) = s \cdot$ and $\cos(\cdot) = c \cdot$.

According to the assumptions, the Eq.(2) can be simplified as.

\[
R(\alpha, \beta, \gamma) =
\begin{bmatrix}
1 & -\gamma & \beta \\
\gamma & 1 & -\alpha \\
-\beta & \alpha & 1
\end{bmatrix}
\]

(3)

With $B'_i = B_i R(\alpha, \beta, \gamma)$, the transformed coordinates of $B_i$ can be derived as

\[
\begin{align*}
B'_1 \left(-b + \frac{\sqrt{3}}{3}b\gamma - \beta h, b\gamma + \frac{\sqrt{3}}{3}b + \alpha h, -b\beta - \frac{\sqrt{3}}{3}b\alpha + h\right) \\
B'_2 \left(b - \frac{\sqrt{3}}{3}b\gamma + \beta h, b\gamma + \frac{\sqrt{3}}{3}b - \alpha h, -b\beta + \frac{\sqrt{3}}{3}b\alpha + h\right)
\end{align*}
\]
\[ B'_i = \left( -\frac{2\sqrt{3}}{3}b\gamma + \beta h, \frac{2\sqrt{3}}{3}b\alpha - \alpha h, \frac{2\sqrt{3}}{3}b\alpha + h \right) \]  

(4)

Assumed that each link provided an actuated force by peristaltic joint, and the value of actuated is \( \Delta l_i \). The vector calculation equation can be denoted as

\[ \vec{A}_i \hat{\alpha} + \vec{A}_i \hat{\alpha} = \vec{B}'_i + \vec{B}'_j \hat{A}_i \]  

(5)

Then, the transformed coordinate can be derived by

\[ \Delta l_i = \frac{\sqrt{3}ah}{3l^2} - \alpha + \frac{2bh}{l^2} \beta + \frac{2\sqrt{3}(a-b)b}{3l^2} \gamma, \Delta l_j = \frac{\sqrt{3}ah}{3l^2} - \alpha + \frac{ah}{l^2} \beta, \Delta l_3 = \frac{2\sqrt{3}ah}{3l^2} - \alpha \]  

(6)

The isomorphic mapping matrix can be written as following.

\[
\begin{bmatrix}
\Delta l_i \\
\Delta l_j \\
\Delta l_3 \\
\end{bmatrix} =
\begin{bmatrix}
\frac{\sqrt{3}ah}{3l^2} & \frac{2bh}{l^2} & \frac{2\sqrt{3}(a-b)b}{3l^2} \\
\frac{3ah}{3l^2} & \frac{ah}{l^2} & 0 \\
2\sqrt{3}ah & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta \\
\gamma \\
\end{bmatrix}
\]

(7)

3. Simulation model of 3-UPC type parallel prototype manipulator via SimMechanical

Assumed that the structure parameters are \( a = 150 \text{mm} \), \( b = 100 \text{mm} \), \( h = 200 \text{mm} \), and the actuated inputs are \( \alpha = 0.03 \sin(3t) \), \( \beta = 0.05 \sin(3t) \), \( \gamma = 0.025 \sin(3t) + 3 \).

The SimMechanics in Matlab is used to build the model of 3-UPC type parallel prototype manipulator, structure of 3-UPC type parallel prototype manipulator is shown as Fig.2, and the visualization structure is shown as Fig.3, respectively.

![Fig 2 Structure of 3-UPC type parallel prototype manipulator](image)

Based on given parameters, the isomorphic mapping matrix can be rewritten by

\[
\begin{bmatrix}
\Delta l_i \\
\Delta l_j \\
\Delta l_3 \\
\end{bmatrix} =
\begin{bmatrix}
0.404 & 0.932 & 0.135 \\
-0.404 & 0.7 & 0 \\
0.81 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta \\
\gamma \\
\end{bmatrix}
\]

(8)

And its inverse matrix can be given by

\[
\begin{bmatrix}
\alpha \\
\beta \\
\gamma \\
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 1.235 \\
-2.3 \times 10^{-16} & 1.429 & 0.713 \\
7.41 & -9.862 & -8.614 \\
\end{bmatrix}
\begin{bmatrix}
\Delta l_i \\
\Delta l_j \\
\Delta l_3 \\
\end{bmatrix}
\]

(9)
Based on Eq.(8), simulation comparison with forward kinematic characteristics between micro/nano scale equivalent method and model of 3-UPC type parallel manipulator build by Matlab software is shown in Fig.4.

The simulation results (shown in Fig.4.) show that the Jacobian isomorphism mapping matrix obtained micro/nano scale equivalent method to maintain the original characteristics of the movement. According to the simulation results can be obtained in small signal inputs under given errors 0.34%, 0.56%, and 1.56% respectively, which can take error compensation method for precise modeling.

Based on Eq.(9), simulation comparison with inverse kinematic characteristics between micro/nano scale equivalent method and model of 3-UPC type parallel manipulator build by Matlab software is shown in Fig.5 and Fig.6.

Fig.3 Visualization structure of 3-UPC type parallel prototype manipulator

Fig.4. Forward kinematic simulation results between model build by micro/nano scale equivalent method and actual model

Fig.5. Inverse kinematic simulation results of the model build by micro/nano scale equivalent method
Fig. 6. Simulation comparison between two models

4. Conclusions
In this paper, we propose a method to construct the isomorphism mapping matrix of 3-UPC type parallel prototype manipulator for structure synthesis of compliant mechanism. Based on 3-UPC type parallel prototype manipulator with three rotational DOF, the micro/nano scale equivalent method is used to simplify and construct the kinematic Jacobian isomorphism mapping matrix. The following conclusions are drawn from the experimental study:

1. The Jacobian isomorphism mapping matrix derived by using micro/nano scale equivalent method keep the same performance with macro motion characteristics;
2. The structural parameters have great influence on the performance of inverse motion mapping which can be modified according to the parameter optimization method.

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