THE MINIMUM WIDTH OF THE ARRIVAL DIRECTION DISTRIBUTION OF ULTRA-HIGH-ENERGY COSMIC RAYS DETECTED WITH THE YAKUTSK ARRAY

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ABSTRACT
This paper presents the results of searches for anisotropy in the arrival directions of ultra-high-energy cosmic rays (CRs) detected with the Yakutsk Array during the 1974–2008 observational period as well as searches in available data from other giant extensive air shower arrays working at present. A method of analysis based on a comparison of the minimum width of distributions in equatorial coordinates is used. As a result, a hypothesis of isotropy in arrival directions is rejected at the 99.5% significance level. The observed decrease in the minimum width of the distribution can be explained by the presence of CR sources in energy intervals and sky regions according to recent indications inferred from data of the Yakutsk Array and Telescope Array experiments.

Key words: astroparticle physics – cosmic rays

1. INTRODUCTION

The origin of ultra-high-energy cosmic rays (UHECRs) is a long-standing challenge in cosmic ray (CR) physics. Extensive air shower (EAS) arrays detecting CRs at energies above 1 EeV (= 10^{18} eV) mainly observe isotropic arrival directions with no sign of fluxes from sources significantly exceeding instrumental errors (e.g., Aab et al. 2014a; Kampert & Tinyakov 2014).

At the same time, there are indications of small anisotropies in the arrival directions, which are revealed by comparing of CR intensities in adjacent sky regions (e.g., Ivanov et al. 2003; Sommers & Westerhoff 2009; Abbasi et al. 2014). To confirm or refute the indications, it is useful to diversify the approaches and methods used in the analysis of CR arrival directions aside from just independent experimental data.

Recently, the minimum width of arrival direction distribution (MWADD) method was used to search for sources of UHECRs in a circle of right ascensions (Ivanov et al. 2015a). The method is a specific variant of testing for the equality of population variances as an alternative to the equality of means (Ivanov 2013).

In this paper, a two-dimensional generalized method is developed for application in equatorial coordinates to test a null hypothesis, \( H_0 \), of isotropy in the arrival directions of CRs and an alternative hypothesis of a possible source of CRs with fitted position and luminosity. Our aim is to reject or confirm and to refine the characteristics of possible CR sources indicated in previous papers.

The present enhancement of the method considers the non-uniformity of the array acceptance area in CR arrival angles due to the unequal time-integrated flux from different directions of the sky.

2. THE YAKUTSK ARRAY EXPERIMENT AND SAMPLING OF THE DATA SET

The main purpose of the Yakutsk Array (Y) is to investigate CRs by measuring EAS in the energy range of \( 10^{15} \sim 10^{20} \) EeV. Construction of the array near Yakutsk, Russia, at geographical coordinates 61°7N, 129°4E, 105 m above sea level (1020 g cm^{-2}), was completed in 1973 (Dyakonov et al. 1991). In the years following construction, the array has been reconfigured several times. Before 1990, the total area covered by the detectors was \( \sim 17 \) km²; now, it is 8.2 km². At present, it consists of 58 ground-based and four underground scintillation counters to measure charged particles (electrons and muons), and 48 detectors of air Cherenkov light (Egorova et al. 2001; Ivanov et al. 2010).

EAS event selection from the background is realized with a two-level trigger of detector signals: the first level is the coincidence of signals from two scintillation counters in a station within 2 \( \mu \)s; the second level is the coincidence of signals from at least three nearby stations within 40 \( \mu \)s. The functioning procedures and types of array detectors are described in Dyakonov et al. (1991), Ivanov (2009), and Ivanov et al. (2013).

The location of the shower core is based on the fitting of the lateral distribution of particles by a Greisen-type trial function. Core location errors are \( \sim 30 \) to \( \sim 50 \) m, depending mainly on the number of triggered stations in the EAS event (Dyakonov et al. 1991).

The arrival angles of EAS primary particles are calculated with the plane shower front approximation using the trigger times of stations. A clock pulse transmitter at the center of the array provides pulse timing with \( \sim 100 \) ns accuracy. Errors in the arrival angles depend on the primary energy, decreasing from \( \sim 7^\circ \) at \( E = 1 \) EeV to \( \sim 3^\circ \) above \( E = 10 \) EeV. For more detailed information, see Ivanov et al. (2009, 2010).

In contrast to two other giant arrays functioning at present, the Pierre Auger Observatory (PAO, Abraham et al. 2004) and the Telescope Array (TA, Kawai et al. 2009) with fluorescent detectors, atmospheric Cherenkov light is used here to estimate the energy of primary particles initiating EAS (Ivanov et al. 2007, 2009, 2015b).

In this work, the same sample of the Yakutsk Array data is used as in the previous paper (Ivanov et al. 2015a); this consists of EAS events detected in the period 1974 January–2008 June, with shower axes within the array area with zenith angles \( \theta < 60^\circ \). The unified energy estimation procedure is applied to all showers throughout the period, following Egorova et al. (2001). An additional cut is caused by the

\[ \text{For this method in the context of directional statistics see the Appendix.} \]

\[ \text{Website: http://eas.ysn.ru} \]
increased threshold energy, which will be set out in the next subsection.

2.1. Exposure of the Ground EAS Array for Celestial Regions

Because the array aperture is bounded in the zenith angle interval \( \theta \in (0, \theta_{\text{thr}}) \). CRs can be detected only from a part of the sky in a horizontal system. The diurnal cycle of the array functioning provides nearly uniform exposure in right ascensions and apparent non-uniformity in declinations.

A simple and convenient way to calculate the non-uniform exposure of the array is to use a Monte Carlo (MC) method (for basics, see, e.g., Metropolis & Ulam 1949; Hammersley & Handscomb 1975; an application to the array exposure is demonstrated by Ivanov et al. 1997, 2003). An algorithm is based on the angular distribution of isotropic rays in a horizontal system attached to the flat array on the ground. Within the infinitesimal time interval \((t, t + dt)\), with stationary Earth, the azimuthal distribution is uniform and the zenith angle distribution is as follows: if \( \theta < \theta_{\text{thr}} \), then \( f_\theta = \sin(2\theta)/(1 - \cos^2 \theta_{\text{thr}}) \), else \( f_\theta = 0 \). If we assume the inverse time \( f \rightarrow -f \), so that all the exposed rays move from the array to the sky, then the rays are integrated over the diurnal cycle due to the Earth’s reverse rotation.

In this algorithm, random directions, \((\phi_i, \theta_i)\), \(i = 1, ..., N\), are sampled in the horizontal system from a uniform distribution in the azimuth and \( f_\theta \) in zenith angles. A uniform distribution of the sidereal time is used as well. The directions are converted, for instance, to equatorial angles \((\phi_i, \theta_i) \rightarrow (\alpha_i, \delta_i)\) (e.g., Green 1985; Collins 2004) to form the expected-for-isotropy distribution of CR arrival directions on the celestial sphere.

Resultant declination distributions for the ground arrays working at present are shown in Figure 1, while the right ascension distribution is uniform. Actually, the distributions obtained are formed by the directional exposures of the EAS arrays, that is, the effective time-integrated collecting area for flux from each direction of the sky.

Minor deviations from the uniform distribution in right ascension are caused by the diurnal and seasonal variations of the array exposure. The efficiency of array detection is affected by weather, shutdowns of detectors, the geomagnetic modulation of the EAS event rate, and so on (Ivanov et al. 1999, 2007; Pravdin et al. 2001). Attenuation of showers in the atmosphere results in a deviation from the “isotropic” zenith angle distribution, \( f_\theta \), at low energies. However, above some threshold energy, \( E_{\text{thr}} \), all these effects can be neglected in comparison with statistical errors arising with the energy. In this paper, the following values are chosen: \( E_{\text{thr}} = 3.2 \text{ EeV}, \theta_{\text{thr}} = 60^\circ \) for the Yakutsk Array (Ivanov et al. 2015a), \( E_{\text{thr}} = 52 \text{ EeV}, \theta_{\text{thr}} = 80^\circ \) for PAO (Aab et al. 2014b), and \( E_{\text{thr}} = 57 \text{ EeV}, \theta_{\text{thr}} = 55^\circ \) for TA data (Abbasi et al. 2014).

3. ANALYSIS OF CR ARRIVAL DIRECTIONS IN EQUATORIAL COORDINATES

3.1. The Method of Minimum Width of Distribution to Analyze Arrival Directions of CRs

The isotropic distribution of arrival directions has no mean value because the limits of the region on the sphere that should be integrated over are undefined. Meanwhile, if we assume an arbitrary direction, \((\alpha, \delta)\), as a trial mean, then we can find a dispersion of the isotropic distribution, namely, the width \(2\omega_i\), which is independent of the trial mean:

\[
\omega_i = \frac{1}{4\pi} \int_0^{2\pi} \sin(\psi) \, d\phi \int_0^\pi \sin(\psi) \, dyd\psi = \frac{\pi}{2},
\]

where \(\psi\) is the angular distance to \((\alpha, \delta)\). In the case of \(N\) data points on the celestial sphere, a sum of the angular distances is applicable instead:

\[
\omega_i = \frac{1}{N} \sum_{i=0}^{N} \psi_i,
\]

where the asymptotic limit is equal to \(\pi/2\) as \(N\) approaches \(\infty\).

So, the width of the isotropic distribution on the sphere is \(2\omega_i = 180^\circ\). On the other hand, if there is a source of CRs with angular size \(S \ll \omega_i\), lurking in the isotropic background, then the aggregate width of the distribution (i) reaches a minimum when the trial mean points to the source; and (ii) has a minimum that is distinctly less than \(2\omega_i\), depending on the fraction of the source luminosity in the overall flux of CRs.

For instance, if the flux from the source is half of the total, then \(\omega_{\text{min}} = 45^\circ\), while for a fraction of 0.1, half of the minimum width is \(\omega_{\text{min}} = 81^\circ\). It seems that by measuring the width of the distribution of arrival directions, one is able to reject the null hypothesis and to find the coordinates and the fraction of CR flux from a source, if there is any.

3.2. Application of the Method to MC data

In this section, simulation results of the MWADD method applied to \(N\) points on the equatorial sphere, taking into account the array exposure, are given. In this case, the width of the distribution is strongly influenced by the exposure, so the directional dependence \(\omega_i(\alpha, \delta)\) should be calculated for the particular array.

It is straightforward to use the MC algorithm described above (Section 2.1) to compute the width with the trial mean scanning the whole \(\alpha \in (0, 360^\circ), \delta \in (-90^\circ, 90^\circ)\) equatorial area. The results for three arrays are shown in Figure 2. At \(E > E_{\text{thr}}\), the distribution width is the same in right ascensions, so the width variation is shown for the trial mean scanning declinations.

The method is applicable only in searching for a single source, SS, of CRs. Indeed, in the case of two sources located at the angular distance \(L\) from each other, with the fractions of CR fluxes \(f \) and \((1 - f)\), MWADD is \(\omega_2 = 2f(1 - f)L\). For opposite sources with equal fractions, a half width is \(\omega_2 = \pi/2\), just as in the isotropic alternative. In what follows, we will...
explicitly suppose an SS of CRs within a particular energy interval.

The statistical power of the MWADD method is the efficiency depending on the sample size $N$. We have to find a lower bound of $N$ needed to reject $H_0$ at a confidence level of 99% when an alternative hypothesis, $H_1$, is true. To estimate $N_{\text{min}}$, we used $H_1$ consisting of an SS as a $\delta$-function located in $(\alpha_{\text{SS}}, \varphi_{\text{SS}})$, within the field of view of the Yakutsk Array, with the fraction of the total CR flux $\varphi$ and an isotropic background that provides $(1 - \varphi)$ of the flux. The distribution width for each trial mean is normalized using the “exposed” isotropic distribution width as a measure.

The result of the MC simulation is given in Figure 3 in comparison with the power of harmonic analysis in right ascension. The first harmonic amplitude, $A_1$, under $H_1$ is a weighted vector sum of 2 and $2/\sqrt{N}$ (Ivanov et al. 2015a). Using the probability $P(> A_1) = \exp(-NA_1^2/4) = 0.01$, one can find $N_{\text{min}}$ for $H_0$ as a function of $f$. A conclusion to be drawn is that the minimum width method is more powerful than harmonic analysis in R.A..

3.3. Application of the Method to Experimental Data
3.3.1. Testing the Null Hypothesis with the Yakutsk Array Data

The Yakutsk Array data at energies above 3.2 EeV are divided into four intervals with widths $4 \log E = 0.25, 0.5$. Below, energy scaling factors for EAS arrays derived from comparison of the observed energy spectra (Ivanov 2010, 2014; Dawson et al. 2013) are used (PAO: 1.04; TA: 0.96; Yakutsk: 0.561). The scaled energy is marked $E_{\text{WG}}$.

The width of the observed distribution of the arrival directions is normalized using the expected isotropic distribution width for a given trial mean. Only in the energy interval $E_{\text{WG}} \in (5.6, 10)$ EeV is there a definite minimum, $\omega_{\text{obs}}/\omega_0 = 0.89$, of the distribution width of CRs detected with the Yakutsk array; this is shown in Figure 4, the central map. Another minimum in the width of the arrival directions at energies above 55 EeV is revealed in data provided by the TA Collaboration (Abbasi et al. 2014, mapped on the right).

To estimate the probability of MWADD under $H_0$ being less than or equal to the observed value, the MC algorithm is used with the number of isotropic events in a set equal to the number

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To estimate the probability of MWADD under $H_0$ being less than or equal to the observed value, the MC algorithm is used with the number of isotropic events in a set equal to the number

of observed EAS events, $N_{\text{obs}}$, in the particular energy bin. The exact algorithm of the MWADD calculation that was used for the data is then performed on the MC event set.

The procedure is repeated $M$ times to find the fraction of MC event sets where the minimum distribution width is equal to or less than the experimental value. This fraction is interpreted as a probability for quoting the significance of the anisotropy signal. The number of MC event sets used in the simulation and the number of events in the energy bins detected in the experiments are presented in Table 1.

The resultant probability for the Yakutsk array data in the energy bin $E_{\text{WG}} \in (5.6, 10)$ EeV is $P_1 = 1.15 \times 10^{-3}$, which is equivalent to a $\sim 3.1\sigma$ deviation in normal distribution terms. However, a penalty factor should be applied to the probability, which is calculated a posteriori. Assuming equally possible anisotropies in any of the four energy bins, with comparable deviations, one obtains a final probability $P = 4.6 \times 10^{-3}$, equivalent to $\sim 2.6\sigma$. Consequently, the null hypothesis can be rejected based on the Yakutsk Array data at the significance level of 99.5%.

Observed values of MWADD calculated using available data from PAO (Aab et al. 2014b) and TA (Abbasi et al. 2014) are shown in comparison with the Yakutsk Array data in Figure 5. There is no deviation from isotropic expectation in the data from PAO, while TA data exhibit a pronounced deviation of MWADD in the energy range where a “hotspot” was indicated (Abbasi et al. 2014).

The probability of 72 isotropic EAS events above $E_{\text{WG}} > 55$ EeV having an MWADD less than that observed by the TA is $P_0 = 1.3 \times 10^{-3}$ ($\sim 3\sigma$). A penalty factor can only be calculated by the TA Collaboration using all the data observed.

3.3.2. Searching for the Coordinates of Possible CR Sources and Fraction of the Flux

Our alternative hypothesis, $H_1$, has free parameters to adjust to the observed MWADD: the position of the SS in an equatorial system and the fraction of the total CR flux that has arrived from the source. The same iterative procedure is used to calculate the “most probable” parameters yielding the observed distribution width. For implementation, the MC program mentioned above is adapted to calculate MWADD under $H_1$ with free parameters as input.

Fitted parameters are then used to calculate the random dispersion of MWADD for a fixed $N$ equal to the number of detected CRs in a particular energy bin. By varying a parameter, its confidence interval is determined where the

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3 In other words, the Rayleigh test with the first harmonic amplitude as a measure of dispersion (Jupp 2001).
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Figure 4. Ratio of the observed to expected-for-isotropy width of the distribution of arrival directions in arbitrary colors (a scale bar is given on the left) mapped using Hammer–Aitoff projection of equatorial coordinates. The FOV border lines of the arrays are shown by dotted curves.

Table 1

| Experiment, Energy Bin, EeV | M   | P(%) |
|-----------------------------|-----|------|
| PAO, E_WG > 54              | 57.3|
| TA, E_WG > 55               | 0.1 |      |
| Yakutsk, (3.1, 5.6)         | 0.1 |      |
| Yakutsk, (5.6, 10.0)        | 0.1 |      |
| Yakutsk, (10.0, 17.7)       | 0.1 |      |
| Yakutsk, (17.7, 56.1)       | 0.1 |      |

Note. The number of EAS events observed in energy bins, N_obs, and sample size, M, used in the simulation are given for arrays.

Figure 5. Minimum width of the distribution of arrival directions as a function of CR energy. A normalized difference in the widths of observed and isotropic distributions is derived using the data from EAS arrays.

resultant deviation of MWADD is within random dispersion limits.

Hypothesis H₁ is applied to the Yakutsk Array data in the energy interval E_WG ∈ (5.6, 10) EeV and to TA data at E_WG > 55 EeV. The most probable parameters for the Yakutsk Array data are α_Y = 36°±33, δ_Y = 48°±25, f_Y = 0.11 ± 0.08. The results for TA are α_TA = 144°±29, δ_TA = 42°±23, f_TA = 0.2 ± 0.1.

A hint of the possible source of CRs in the interval E_WG ∈ (5.6, 10) EeV, derived from the Yakutsk Array data using the MWADD method in the right ascension circle (Ivanov et al. 2015a), is confirmed; the resulting α_Y intervals are within experimental errors.

The coordinates of a hypothesized TA source are in agreement with that of a hotspot revealed by the TA Collaboration (Abbasi et al. 2014). An additional bonus in our case is an estimate of the probable fraction of CRs attributed to this source.

4. CONCLUSION

The Yakutsk Array data on the arrival directions of CRs above 3.2 EeV in equatorial coordinates are analyzed using the minimum width of distribution method. A previous hint of large-scale anisotropy in the energy range 5.6 < E_WG < 10 EeV is confirmed by this enhanced method. The null hypothesis is rejected at the 99.5% significance level.

For comparison, our method of analysis is applied to available data from other giant EAS arrays. PAO data demonstrated no deviation of the MWADD from the isotropic distribution width at energies above 54 EeV. However, arrival directions of UHECRs detected with TA have a decreased minimum width at E_WG > 55 EeV with a statistical significance of ~3σ.

The MWADD method is applied with the alternative hypothesis of a single CR source in a weighed combination with the uniform background. Free parameters of a source are fitted using experimental data in the energy intervals where H₀ is ruled out: α_Y = 36°±33, δ_Y = 48°±25, f_Y = 0.11 ± 0.08 for the Yakutsk Array data in the energy interval E_WG ∈ (5.6, 10) EeV, and α_TA = 144°±29, δ_TA = 42°±23, f_TA = 0.2 ± 0.1 for TA data at E_WG > 55 EeV.

Although our alternative hypothesis is not a unique explanation of the observed decrease in MWADD, the indicated coordinates and fitted CR fraction of the possible sources may be useful in a future search with enhanced statistics for anisotropy in arrival directions of UHECRs.

The equatorial coordinates of the possible source derived from the TA data are in agreement with a hotspot position found by comparing CR events summed within sky regions (Abbasi et al. 2014). Our approach is different, as it is based on the overall distribution width of arrival directions rather than on the excess flux of CRs in a particular angular region.

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APPENDIX

Our objective in this paper is to compare a circular uniform distribution on the unit sphere $S^2$ in $\mathbb{R}^3$ and, as antithesis, a point source of CRs. For simplicity, all considerations in this appendix will be illustrated on a circle, where $\psi_0$ is the point source position.

The most common way in directional statistics is to use $n$th moments of a distribution defined as $m_n = \frac{1}{N} \sum_{i=1}^{N} \exp(in\psi_i)$, where $N$ is the number of data points at $\psi_i$, with mean angle $\overline{\psi} = \text{Arg}(m_1)$ and the circular variance $S = 1 - |m_1|$. However, there are other measures of the distribution moments in use. For instance, we used in this paper the measure of dispersion of angles $\frac{1}{\pi} \sum_{i=1}^{N} (|\psi - \overline{\psi}| + |\psi_i - \psi_0||)$ considered by Mardia (1972). In this approach the mean angle is where the dispersion is minimum (for distributions under consideration, the median coincides with the mean).

The MWADD method is used in this paper to locate the mean and to test for uniformity of directions against the alternative hypothesis with a point source. In Section 3.2, the method is compared to the Rayleigh test known as one of the most powerful tests for unimodal data (Jupp 2001).

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