Instabilities of scalar fields around oscillating stars

Taishi Ikeda,1 Vitor Cardoso,2,3 and Miguel Zilhão2

1 Dipartimento di Fisica, “Sapienza” Università di Roma, Piazzale Aldo Moro 5, 00185, Roma, Italy
2 CENTRA, Departamento de Física, Instituto Superior Técnico – IST, Universidade de Lisboa – UL, Avenida Rovisco Pais 1, 1049-001 Lisboa, Portugal
3 Niels Bohr International Academy, Niels Bohr Institute, Blegdamsvæj 17, 2100 Copenhagen, Denmark

The behavior of fundamental fields in strong gravity or nontrivial environments is important for our understanding of nature. This problem has interesting applications in the context of dark matter, of dark energy physics or of quantum field theory. The dynamics of fundamental fields has been studied mainly in static or stationary backgrounds, whereas most of our Universe is dynamic. In this paper we investigate “blueshift” and parametric instabilities of scalar fields in dynamical backgrounds, which can be triggered (for instance) by oscillating stars in scalar-tensor theories of gravity. We discuss possible implications of our results, which include constraints on an otherwise hard-to-access parameter space of scalar-tensor theories.

I. INTRODUCTION

General Relativity (GR) is currently the best description of the gravitational interaction, and has been successfully tested on different scales [1]. The recent direct detection of gravitational waves (GWs) [2] indicates that even dynamical, strong-field regions are adequately described by GR (up to the precision probed by current detectors). In spite of its brilliant status, there are a number of conceptual issues with GR, ranging from the large-scale description of the cosmos to the fate of classical singularities in gravitational collapse or the incorporation of quantum effects, see e.g. [3] and references therein. The resolution of some of these challenges most likely requires that GR be superseded by a more sophisticated description.

There is currently no single compelling alternative to GR that solves the above issues without introducing new problems of their own. However, a variety of modified theories have been proposed, mostly with the view to exploring the mathematical and physics content of possible contenders to GR. These frequently include additional degrees of freedom, which might lead to unique observational signatures. The simplest modification of GR are scalar-tensor theories, where a new scalar degree of freedom couples to curvature or matter. Some of these theories arise naturally as possible alternatives, since they have a well-posed initial value problem and simultaneously evade all known constraints. Scalars are also a generic prediction of string theory or of extensions of the standard model of particle physics, and are also natural candidates for dark energy and dark matter [5, 6].

Depending on the coupling to matter, to curvature and on the self-interactions, such theories and new fundamental fields may give rise to wide array of new effects, such as the spontaneous “scalarization” of objects [16–20], screening mechanisms on astrophysical scales [24–26], etc. Potential astrophysical consequences of these theories were mostly based on analysis in stationary settings [16, 20, 23, 25, 27, 28]. Time-dependent setups include spontaneous scalarization during the inspiral and

| C | L0 (km) | ωL0/c | Radial frequency |
|---|---|---|---|
| NS | 0.3 | 10 | 0.6 | 3 kHz |
| WD | 10⁻³ | 10³ | 0.004 | 0.2 Hz |
| Sun | 10⁻⁶ | 7 × 10⁵ | 0.029 | 2 mHz |

merger of neutron stars [29, 32], black holes [33, 34], cosmological evolutions [35, 36] or situations aimed at understanding well-posedness or other fundamental issues [37–39].

Here, we wish to understand possible new phenomena induced by time-periodic motion, in particular by vibrating stars such as the ones summarized in Table I. Theories for which a constant scalar is a ground state typically have the same stationary solutions as GR, making it challenging to tell the two apart. However, when such stars are disturbed (stochastically, like our Sun, or via mergers or accretion for a neutron star), they provide a time-periodic background on which a scalar fluctuation propagates. Indeed, we show that oscillating stars trigger various instabilities of fundamental fields. Such instabilities may facilitate constraints on otherwise hard-to-access parameter space of the theory.

A toy model had been studied in a Minkowski spacetime [40, 41]. Our results confirm and extend the general picture, but in a nontrivial way, by dealing also with general relativistic backgrounds and fluctuations and providing realistic timescales for the mechanism (details on the general relativistic case are discussed in the appended Supplemental material).

We use geometrized units c = G = 1 and parameterize stars by their mass M, radius L0, and compactness

\[ C \equiv \frac{2M}{L_0}. \]
II. SETUP

Our main purpose is to show that oscillating backgrounds can trigger instabilities of fundamental fields nonminimally coupled to matter. We will show this explicitly for the simplest example of a scalar-tensor theory, but our results are valid for more general setups.

We focus on theories in which a scalar field $\Phi$ is coupled to matter, described by the Lagrangian density

$$\mathcal{L} = \frac{R[g]}{16\pi} - \frac{1}{2} g^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi - V(\Phi) + L_m[\Psi_m, \tilde{g}], \quad (2)$$

where $\tilde{g}_{\mu\nu} = A(\Phi)^2 g_{\mu\nu}$ is the physical metric and $L_m$ the Lagrangian describing matter fields $\Psi_m$ (these could be the microscopic, fundamental description leading to notions of density and pressure for example). The equations of motion for the system are

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}, \quad (3a)$$

$$T_{\mu\nu} = \nabla_\mu \Phi \nabla_\nu \Phi - g_{\mu\nu} \left( \frac{1}{2} (\nabla \Phi)^2 + V(\Phi) \right) + T_{\mu\nu}^{(m)},$$

$$\Box \Phi - V'(\Phi) = -A(\Phi)^3 \partial_\Phi A(\Phi) \tilde{T}^{(m)}. \quad (3b)$$

Here, $\tilde{T}^{(m)} = \tilde{g}^{\mu\nu} T_{\mu\nu}^{(m)}$ while $T_{\mu\nu}^{(m)}$ and $\tilde{T}_{\mu\nu}^{(m)}$ are the energy-momentum tensors of the matter fields with respect to metric $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$.

The phenomena we wish to describe are also part of more generic theories: the fundamental ingredient is a position-dependent effective mass function for the scalar $\Phi$. Once linearized around a specific background, the right-hand side of Eq. (3b) is proportional to $\tilde{T}^{(m)} \Phi$. In other words, the coupling of the scalar to matter gives rise to an effective scalar mass which depends on the matter content $\tilde{T}^{(m)} \Phi$. This is one key ingredient of our study. Thus, direct couplings to curvature would also produce similar effects [21, 23].

For concreteness, we focus exclusively on the following scalar potential and coupling function,

$$V(\Phi) = \frac{\mu_0^2}{2} \Phi^2, \quad A(\Phi) = e^{\frac{\beta}{2} \Phi^2}, \quad (4)$$

but our results and methods can be applied to other models. Scalar self-interactions can play an important role in the nonlinear development of the instability; here we focus exclusively on the physics at small $\Phi$. Hence our results describe the early-time dynamics of more general self-interacting theories. Here, $\mu_0$ is the (bare) mass parameter of the scalar field (note that the mass $m_\phi$ of the field is related to the mass parameter via $m_\phi = \mu_0 \hbar$ in these units). With this choice of scalar potential and coupling function, a vanishing scalar field is a solution to the energy-momentum tensor around this solution and can be shown to be effective, time and position-dependent mass. Our analysis can also be generalized to theories with nontrivial, stable scalar profiles, including popular examples such as spontaneous scalarization [10–18, 21–23] and screening mechanisms [24–26, 43, 44].

Let us consider matter fields $\Psi_m$ describing a perfect fluid. Focus on a geometry with vanishing scalar, $\Phi = 0$, describing a static star of (constant, for simplicity) density $\bar{\rho}_0$ and radius $L_0$ [15]. The total mass of this solution can be written as $M = \frac{4\pi}{3} \bar{\rho}_0 L_0^3$.

We ignore the effect of the scalar field on the profile of the fluid. In other words, we deal only with the Klein-Gordon equation (3b) on this fixed geometry. Perturbing around a background solution $\Phi_0(r)$, one finds that the stability of such solutions depend dramatically on the value of the coupling $\beta$ [10–19]. For concreteness, here we take $\beta < 0$ with $\mu_0^2 - \beta \tilde{T}^{(m)} \gg -L_0^2$ and such that $\Phi_0 = 0$ is a stable solution. The assumption of zero background scalar is a conservative assumption, made to highlight the nontrivial effects of the mechanism we discuss below, which provide a unique tool to constrain the theory. When the background scalar is not zero other effects become important. For example, an oscillating body with a nontrivial background scalar will radiate monopolar radiation, a topic explored in the past [38, 46, 47].

III. INSTABILITIES OF OSCILLATING ASTROPHYSICAL OBJECTS

Let us then consider the dynamical behavior of a (nonminimally coupled) scalar field in a geometry describing a radially oscillating star. For simplicity, the results shown below assume a Minkowski background and are thus valid only for Newtonian stars. General Relativity effects change the quantitative but not the qualitative behavior, and are discussed in the appended supplemental material, including an analysis of general-relativistic fluctuations of compact stars [10–18, 49]. Fluctuations around a background value of the scalar field are described by the Klein-Gordon equation

$$\Box \Phi = \mu^2 \Phi, \quad (5)$$

with an effective mass

$$\mu^2 = \mu_0^2 - \beta \tilde{T}^{(m)},$$

which acts as a position-dependent mass term that depends on the energy density of the star. If the star oscillates (with a time-dependent radius $L(t)$), so does the effective mass of the scalar field. For simplicity we here assume the simple model

$$L(t) = L_0 + \delta L \sin \omega t, \quad (6)$$

$$\mu^2(t,r) = \mu_0^2 + \beta (\bar{\rho}_0 + \delta \bar{\rho}(t,r) \sin(\omega t)), \quad (7)$$

where $\delta \bar{\rho}(t,r)$ is the amplitude of the density oscillation, and $\bar{\rho}_0$ is the average of the energy density. We assume $\beta < 0$, in such a way that $\mu_0^2 + \beta \bar{\rho}_0 > 0$ to avoid tachyonic instabilities. The competition between different mechanisms is discussed in detail below and in the Supplemental Material, appended at the end.
Here, we assumed $\delta L \ll L_0$ and $\delta \rho \ll \rho_0$. This dependency has two important aspects:

- Because the effective mass inside the star is smaller than the exterior bare mass $\mu_0$, cf. Eq. (7), long wavelength modes are trapped inside the star. The oscillating star surface therefore provides reflecting boundary conditions at the periodically-varying location $L(t)$. For setups where there is a continuous mass profile, a radially oscillating star corresponds, nevertheless, to periodically varying field profiles at the surface, possibly changing on scales smaller than a wavelength.

- The local density oscillations cause a time-varying effective mass for the scalar, which lends itself to parametric instabilities as we show below.

In astrophysically realistic stars, the time-dependence of the radius and density of the star is quite involved. Normal modes of oscillation of a star will cause a density profile with a sinusoidal-like radial profile as well. We studied other profiles for the perturbation (in particular $\delta \rho \propto \sin ar$) and fully general-relativistic settings and find that they lead to similar results as described below and in the Supplemental material.

We decompose the scalar field into spherical harmonics and evolve each mode, $\Phi_{lm}(t, r)$, using a fourth-order accurate Runge-Kutta scheme for the time integration where spatial derivatives are approximated by fourth-order accurate finite difference stencils. Radiative boundary conditions are imposed at the boundary of the computational domain, which is not in causal contact with the star. Thus, the effective mass inside the star will have a parametric mass profile, which lends itself to parametric instabilities as we show below.

In such cases, we see a transfer of energy from the oscillating star to the scalar field, increasing the scalar frequency. We can see this behavior in Figs. 1-2. Upon each reflection at the surface the scalar drifts to higher frequencies, and after a sufficient amount of reflections it is no longer confined: the field is finally able to leak to outside the star (and has frequency $\omega \sim \mu(1+\epsilon)$, $\epsilon \ll 1$). This mechanism causes the energy to grow as $E_{0} e^{\lambda_{B} t} = E_{0} e^{n_{ref} L_{0} \lambda_{B}}$, with $n_{ref}$ the number of reflections and $\lambda_{B}$ the instability growth rate. Here, $E_{0} \sim \sigma^{-1}$ is the initial dominant spectral content at low energies, the ones that linger long enough to be amplified. Thus, the field is able to leak away from the star and out to spatial infinity when $e^{2n_{ref} L_{0} \lambda_{B}} \sigma^{-1} > \mu_{0}$. This occurs after a number of reflections

$$n_{ref} > n_{*} \sim \frac{\ln[\mu_{0}\sigma]}{2\lambda_{B} L_{0}}.$$  

At the critical number of reflections, the total energy confined inside the star as scalar field is

$$\frac{E}{E_{0\max}} = \sigma \mu_{0},$$

this being also the total amount of energy extracted from the star.

We have thus far focused only on the effect of oscillating boundaries, while the interior is non-dynamical. However, real objects will also have an oscillating density. Thus, the effective mass inside the star will have a periodic time variability, potentially giving rise to “parametric” instabilities. Figure 3 summarizes our results.
for non-vanishing $\beta \delta \tilde{\rho}$. Two mechanisms now compete, a blueshift and a parametric mechanism. For small $L_0^2 \delta \tilde{\rho}$, the evolution of the scalar field is almost identical to that with a vanishing $\delta \tilde{\rho} L_0^2$, and the blueshift mechanism dominates. For large $\delta \mu L_0^2$, we observe instead that the amplitude of the scalar field grows while its frequency is barely changing, a clear sign that we are dealing with a different process. This is in fact a parametric instability. Contrast this with the blueshift instability cases, where the scalar field pulse becomes narrower (due to the oscillating boundary) as time passes, but its amplitude remains constant. Here, the width of the scalar pulse is roughly constant and the instability is driven by a growth in the amplitude of the field (due to the oscillating effective density).

V. APPLICATION TO ASTROPHYSICAL SYSTEMS

We have shown that pulsating stars or other objects may excite important instabilities of nonminimally coupled fields. Compact stars, such as neutron stars, have radial pulsations with frequencies satisfying $\omega L_0 \sim 1$ for the lowest overtones \cite{10}, and are ideal systems where such instabilities might be relevant. These change the local density and distribution of the scalar (which could be one dark matter component) and may even backreact on the star. A precise description of the evolution of the instability requires a precise knowledge of stellar oscillations and careful modeling of the evolution of the scalar in such backgrounds. This is a challenging program that requires further study.

Notice first that one can apply our results for stars as long as other dissipation mechanisms are subdomi-

and on this timescale an energy $E \sim E_0 \sigma \mu_0$ is removed from the star. This result is not very sensitive to the initial conditions. It is, in principle, only weakly affected by backreaction, unless $\sigma \mu_0$ is an extremely large number. Our numerical results indicate that $\lambda_B L_0 \sim 10^{-3}$ is a reasonable estimate for $\delta L > 0.01 L_0$. The instability window for the blueshift mechanism to work is tight, however [cf. Eq. (5)]. Only large overtones are affected by it, unless the star is oscillating nonlinearly with $\delta L \gtrsim 0.1 L_0$.

Consider now the parametric instability. When $\beta \sim -\frac{\mu}{L_0^2}$, with $\bar{\rho}_0$ the temporal average of the density of the star, the relevant dynamics is governed by the Mathieu equation \cite{53,54}. This particular case provides a test on our results, and allows to estimate analytically the timescales involved. For small $\delta \mu^2$, the instability condition amounts to $L_0 \omega = 4\pi/j$ ($j \in \mathbb{Z}$). The instability rate of particular importance are shear viscosity effects, which in neutron stars have a timescale $\tau_0 \approx 100 \rho_17^{5/4} T_5^2 \left(\frac{L_0}{10 \text{km}}\right)^2 \text{ s},$ \hspace{1cm} (12)

where $\rho_17 = \rho/(10^{17} \text{kg/m}^3)$, $T_5 = T/(10^5 \text{K})$ and $T$ is the neutron star temperature.
time scale for \( j = 1 \), for example, is roughly \( t_A \sim 2\omega /\delta \mu^2 \), or

\[
t_A \sim \frac{2\omega}{\delta \mu^2} \sim 1s \left( \frac{10 \text{km}}{L_0} \right) \left( \frac{\omega L_0}{4\pi} \right) \left( \frac{10^{-2} \rho_1}{\delta \rho} \right) \left( \frac{-10}{\beta} \right),
\]

for small \( \delta \mu^2/\omega^2 \). These estimates assume that the field is effectively very light inside the star, which amounts to requiring that

\[
|\beta| \sim 7 \left( \frac{0.3}{C} \right) \left( \frac{\mu}{10^{-10} \text{eV}} \right)^2 \left( \frac{L_0}{10 \text{km}} \right)^2,
\]

but the instabilities discussed here are expected to set in even at small nonzero effective masses. The star oscillations can be induced by accretion, tidal effects or even mergers [55–57]. Note that numerical relativity simulations show that the amplitude of density perturbations during coalescence, for example, can be significant and of the order of the central density itself [58, 59]. Thus, both mechanisms may act on timescales short enough to be relevant. In fact, at large couplings \( \beta \) – not yet ruled out for large bare masses – parametric instabilities will be dominant.

VI. FINAL REMARKS

Nonminimally coupled, massive scalar fields can evade all observational constraints from the observation of nearly stationary configurations, yet produce distinct signatures when evolving around oscillating backgrounds. We have shown that there are at least two possible instability mechanisms: one blueshifts light scalars inside oscillating stars [60]; the second mechanism is of parametric origin, triggered by a periodic oscillation of the star material. Possible excitation mechanisms for the scalar field could include, for instance, coherently or stochastically oscillating dark matter.

Both instabilities act on short timescales when compared to viscous timescales [51], and are expected to play a role in neutron star oscillations. They can backreact on the star – perhaps leading to gravitational collapse, or (in the blueshift mechanism) simply result in a leakage of high frequency, high amplitude scalar. If a fraction, or all of dark matter is made of a scalar component, then these mechanisms can act to produce overdensities close to neutron stars, providing one more route to constraining dark matter. Details on observational signatures of these instabilities require further studies, beyond the scope of this work.

Similar instabilities are expected not only in scalar-tensor theories, but also in other theories with vectors or spinors [61–63]. Scalarized background solutions are not prone to the type of instabilities discussed here, but star oscillations will lead to radiation emission [38]. We expect that the instabilities discussed here could describe and affect other systems where a light degree of freedom is confined to an oscillating background, potentially observable in condensed matter systems.

Finally, other periodic systems include compact binaries; it can be expected that similar blueshift mechanisms act on such binaries. Their high degree of asymmetry makes it more challenging to model, but their immense gravitational potential energy certainly makes them important candidates.

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Supplemental Material

We now give more details and expand on the construction outlined in the main text.

VII. SETUP: NONMINIMAL SCALAR FIELDS

A. The background

We consider matter fields \( \Psi_m \), describing a perfect fluid. Focus on a geometry with vanishing scalar, \( \Phi = 0 \), describing a static star of (constant) density \( \rho_0 \) and radius \( L_0 \). The ADM mass of this solution can be written as

\[
M = \frac{4\pi}{3} \rho_0 L_0^3
\]

and its geometry can be expressed as

\[
\hat{g}_{\mu \nu} dx^\mu dx^\nu = -\hat{\alpha}^2 dt^2 + \hat{\alpha}^2 dr^2 + r^2 d\Omega^2,
\]

where

\[
\hat{\alpha}^2 = \begin{cases} 
\frac{2}{3} \left( \frac{1 - 2M}{L_0} \right)^{1/2} - \frac{1}{3} \left( \frac{1 - 2M}{L_0} \right)^{1/2} & (r < L_0) \\
1 - \frac{2M}{r} & (r > L_0)
\end{cases}
\]
and
\[ \tilde{a}^2 = \begin{cases} \left(1 - \frac{2Mr^2}{L_0^2}\right)^{-1} & (r < L_0) \\ \left(1 - \frac{2M}{r}\right)^{-1} & (r > L_0) \end{cases} . \]

The profile of the pressure inside the star is
\[ \tilde{p}_0(r) = \tilde{p}_0 \left(1 - C \left(1 - \frac{L_0^2}{r^2}\right)^2 / (1 - C)^{1/2} \right) / \left(3(1 - C)^{1/2} - (1 - C)^{1/2} \right) . \] (16)

We can now linearize the equations of motion around this background. Although we focus solely on constant-density stars, previous results indicate that the details of the model are not important [19]. The most relevant feature is the contribution of the star (trace of) energy density stars, previous results indicate that the details of this background. Although we focus solely on constant-time-independent spacetime, since it serves to convey our main message. Let \( \Phi_{\text{eff}} \) be a solution on a fixed geometry. As a concrete situation, we consider relativistic stars of constant density as a solution, parameterized by radius \( L_0 \) and compactness \( C \).

B. The dynamical equations

In Sec. [10] we deal with full dynamical spacetimes on which the scalar evolves. For now, we consider simply a time-independent spacetime, since it serves to convey our main message. Let \( \Phi_0 \) be a static, spherically-symmetric solution of Eq. (15), with the metric (15) and scalar potential and coupling functions given by Eq. (4). \( \Phi_0 \) is then a solution to the equation
\[ \partial_r^2 \Phi_0 + \left(2 - \frac{\partial_r a}{a_0} - \frac{\partial_r a}{a} \right) \partial_r \Phi_0 + a^2 \left(\beta \tilde{T}(m) e^{2\beta \tilde{a}^2 \Phi_0} - \mu_0^2 \Phi_0 \right) = 0 , \] (17)

where \( \alpha \) and \( a \) are trivially related to the functions \( \tilde{a} \) and \( \tilde{\alpha} \) defined above via
\[ (\tilde{a}, \tilde{\alpha}) = (a, \alpha) e^{\tilde{\beta} \tilde{a}^2 \Phi_0} . \]

Let us now consider a general fluctuation around this value,
\[ \Phi(t, r, \theta, \phi) = \Phi_0(r) + \epsilon \sum_{l, m} \frac{x_{lm}(t, r)}{r} Y_{lm}(\theta, \phi) , \] (18)

where \( \epsilon \) is a bookkeeping parameter and \( Y_{lm}(\theta, \phi) \) are the usual scalar spherical harmonics. Then, keeping only first order terms in \( \epsilon \), we find from Eq. (3b) that \( \psi_{lm} \) is governed by
\[ - \partial_r^2 \psi - \frac{\alpha^2}{a^2} \partial_r \left( \log \left( \frac{a}{\alpha} \right) \right) \partial_r \psi + \frac{\alpha^2}{a^2} \partial_r^2 \psi - V_{\text{eff}} \psi = 0 , \] (19)

where we dropped the \( l \) and \( m \) subscripts for clarity and the effective potential \( V_{\text{eff}} \) is given by
\[ V_{\text{eff}} = \frac{a^2}{r \alpha^2} \left( \frac{\partial_r \alpha}{a} - \frac{\partial_r a}{\alpha} \right) + a^2 l(l+1) \frac{1}{r^2} + \alpha^2 \left( \mu_0^2 - e^{2\beta \tilde{a}^2 \Phi_0} (1 + 4\beta \tilde{a}^2 \Phi_0) \right) . \] (20)

The stable background solutions \( \Phi_0(r) \) depend dramatically on the value of the coupling \( \beta \) [16–19]. There is a threshold coupling \( \beta_c \) that divides cases where \( \Phi_0 \) is a stable solution from those where it is not. For \( \beta > \beta_c \), \( \Phi_0 = 0 \) is a stable solution and the geometry of the star and all its properties are identical to those in GR. If \( \beta < \beta_c \), the trivial scalar profile becomes unstable. In such conditions, a GR star quickly develops a nontrivial profile \( \Phi_0 \) and is said, for obvious reasons, to be scalarized. In this case, the geometry and properties of the star change, and it would no longer be described by Eq. (16), for example. All our techniques below can nonetheless be extended to scalarized objects, or directly applied if one’s focus is on configurations where \( |\Phi_0| \ll 1 \) (also known as the decoupling limit).

These scalar-tensor theories are appealing because [16–19, 27]
\[ \beta_c \propto (1 + (\mu_0 R^2)^2) C^{-1} . \] (21)

Thus, they predict scalarization in strong field situations, for example in neutron stars, but not in stars like our Sun. Thus, these theories satisfy solar-system bounds in a natural way. On the other hand, several observational predictions regarding neutron stars can be different from GR, and thus interesting constraints can be imposed. The observation of binary neutron stars can, in principle, be used to constrain scalarization since scalarized stars would radiate more energy (thus leading to a more rapid inspiral, as compared with the standard general relativistic solutions) [16, 19, 64]. However, low energy excitations are unable to propagate if the field is massive. Thus, for fields with
\[ \mu_0 \gg 10^{-16} \text{eV} , \] (22)

it is challenging to impose constraints on this mechanism via neutron star observations [20].

To understand the possible dynamics of the scalar field, it is important to know the structure of the effective potential \( V_{\text{eff}} \). This potential is shown in Fig. 4. The importance of the effective potential is most easily seen from Eq. (19). For fluctuations of energy \( \Omega \),
\[ \psi \sim e^{-i\Omega} \Upsilon(r) , \] (23)
FIG. 4. (Left panel) Effective potential for spherically-symmetric scalar fluctuations around a relativistic uniform density star with $C = 0.3$ and a trivial background scalar $\Phi_0 = 0$ (with a coupling $\beta > \beta_c$). (Right panel) Same as in the left panel, but now for a subcritical coupling $\beta < \beta_c$, leading to scalarization of the star. The behavior of the blue and purple lines is hard to see, but their profile is similar to that of the red and green lines. The corresponding scalar profile is shown in the inset.

it takes the form of a Schrödinger-like equation (notice that $\Omega$ has dimensions of mass$^{-1}$ in geometrical units, but is an energy parameter)

$$\frac{\alpha^2}{a^2} \partial_r^2 \Upsilon - \frac{\alpha^2}{a^2} \partial_r \left( \log \left( \frac{a}{\alpha} \right) \right) \partial_r \Upsilon + (\Omega^2 - V_{\text{eff}}) \Upsilon = 0,$$

Thus for small enough energies $\Omega^2 < V_{\text{eff}}$ the fluctuation is unable to tunnel to infinity if $V_{\text{eff}}$ asymptotes to a constant positive value.

When $\beta < \beta_c$, the object is scalarized with a nontrivial profile $\Phi_0 \neq 0$ (Fig. 4 right panel). Notice from the shape of the effective potential that bound states are hard to achieve inside the object in these cases, since they can easily “tunnel out” (in the language of quantum mechanics associated with the Schrödinger-like equation (24)).

For non-scalarized solutions, with $\Phi_0 = 0$, Eq. (20) shows that fluctuations of the scalar are massive with an effective mass

$$\mu^2 = \mu_0^2 - \beta T^{(m)},$$

which effectively acts as a position-dependent mass term. Consider setups where the effective mass inside the star satisfies $\mu^2 < \mu_0^2$ (cf. Eq. (24) and the left panel of Fig. 4). Then, low-energy fluctuations of the scalar are trapped inside the star. This aspect plays a critical role in our analysis and results. Note that for extremely compact objects the trace of the stress tensor can change sign, and positive couplings may give rise to spontaneous scalarization as well. Our results can in principle be applied to all these different scenarios.

VIII. PERIODIC-MOTION INSTABILITIES IN ONE DIMENSION

As we discussed, some classes of scalar-tensor theories are described by a scalar field with an effective mass that depends on the environment. In particular, the effective mass $\mu$ of the scalar field inside a star, for example, can be lighter than its vacuum counterpart $\mu_0$. The potential experienced by the field looks like those depicted in Fig. 4 and bound states inside the star can appear. Due to this potential barrier, field excitations with wavelength larger than $\mu^{-1}$ are reflected around the surface of the object. Thus, the field propagates within a cavity whose size is varying periodically, and whose density is also varying periodically. We find that these two aspects give rise to two instabilities of different kinds. A varying cavity size allows for the possibility of energy extraction from the cavity via “Doppler-like” exchanges, whereas a varying density gives rise to parametric instabilities.

We describe in details these two types of instability below. Before going into the more complex case of four-dimensional spacetimes, we consider a simple 1+1 setup in this section.

A. The blueshift instability

FIG. 5. A scalar field confined inside a cavity of size $L(t)$. The left boundary is kept fix and the right boundary (periodically) oscillates in time.

We decouple the two effects mentioned above and consider first a scalar field of constant mass, confined to
a one-dimensional oscillating cavity in flat space – see Fig. 5. In particular, one cavity boundary is fixed at $x = 0$ while the other is located at $L(t)$, as described by Eq. (6). The scalar field is then governed by the massive Klein-Gordon equation with Dirichlet boundary conditions imposed at a time-dependent boundary,

$$-\partial^2_t \Phi + \partial^2_x \Phi - \mu^2 \Phi = 0,$$

$$\Phi(t, 0) = \Phi(t, L(t)) = 0,$$  \hspace{1cm} (26)

where $\mu$ is the (effective) mass of the scalar field inside the cavity. This models an oscillating star with a scalar field whose effective mass is small inside the star and large outside. The system has been investigated previously in other contexts, and shown to give rise to blue or redshifted reflections \cite{66, 67}. After a large enough number of reflections, the energy of the field increases without bound for

$$\frac{N\pi}{L_0 + \delta L} < \omega < \frac{N\pi}{L_0 - \delta L},$$  \hspace{1cm} (27)

where $N$ is an integer \cite{67}. We call this the blueshift condition and the associated instability the blueshift instability. In the following we study the mechanism and associated instability in detail.

1. Estimate from Doppler effect

The growth of energy in the field can be understood as a cumulative Doppler effect \cite{20}. To understand this in more depth and from a different perspective, we consider the following toy model. A massless particle is inside a perfectly reflecting cavity whose boundary oscillates, and is emitted at $x = x_{\text{init}}$ moving to the right with initial (angular) frequency $\Omega_0$. After one reflection at the (moving) boundary, the frequency is Doppler-shifted to $\Omega_1$,

$$\Omega_1 = \frac{1 - v(t_1)}{1 + v(t_1)} \Omega_0,$$  \hspace{1cm} (28)

where $v(t) = \dot{L} = \delta L \omega \cos \omega t$ is the velocity of the oscillating boundary, and $t_1$ the coordinate time when the particle hits the boundary determined from the equation $t_1 + x_{\text{init}} = L(t_1)$. Using the same argument, one can obtain a recursion relation for the frequency $\Omega_{n_{\text{ref}}}$, and therefore for the energy $E_{n_{\text{ref}}}$ after $n_{\text{ref}}$ reflections

$$\Omega_n = \frac{1 - v(t_{n_{\text{ref}}})}{1 + v(t_{n_{\text{ref}}})} \Omega_0,$$  \hspace{1cm} (29)

where $t_{n_{\text{ref}}}$ is determined by

$$t_{n_{\text{ref}}} - t_{n_{\text{ref}}-1} - L(t_{n_{\text{ref}}-1}) = L(t_{n_{\text{ref}}}).$$  \hspace{1cm} (30)

The recursion relation \cite{20} can be solved numerically, its solution is shown in Fig. 6 for different choices of parameters fixing $\delta L = 0.1 L_0$. Note that, for this choice of $\delta L$, condition \cite{27} becomes $2.85 < \omega L_0 / N < 3.49$. We thus choose two representative values for the oscillation frequency $\omega$, one outside the instability interval and one inside. The top panel of the figure shows that when the instability criterion is not satisfied the energy of the particle oscillates in time. In some interactions with the boundary it gains energy, in some it loses energy, the average gain is zero. The bottom panel shows that there are conditions that lead to a constant gain of energy by the particle, and where each reflection is accompanied by a transfer of energy from the boundary to the bouncing massless particle. In particular, for certain conditions, the energy grows exponentially,

$$E \propto e^{\lambda_B t},$$  \hspace{1cm} (31)

and the exponent $\lambda_B > 0$ is insensitive (or very mildly dependent only) on the initial position $x_{\text{init}}$.

Figure 7 shows the relation between the instability rate, $\lambda_B$, and the frequency of oscillation of the boundary $\omega$. An approximate relation for the lowest-frequency instability window in Fig. 7 is

$$\lambda_B L_0 \sim a(\omega) \left( \frac{\delta L}{L_0} - \frac{\pi}{\omega L_0} - 1 \right),$$  \hspace{1cm} (32)

valid in the interval $\pi/(L_0 + \delta L) < \omega < \pi/(L_0 - \delta L)$. The coefficient $a(\omega)$ has a nontrivial dependence on the boundary oscillation frequency and is $a \sim 3.5$ for $\omega L_0 = \pi$, and $a \sim 4.6$ for $\omega L_0 = 0.9\pi$.### Fig. 6. Time evolution of energy of a photon inside a box with an oscillating boundary, solution of recursion relation \cite{20}. The amplitude of oscillation of the boundary is $\delta L = 0.1 L_0$, the initial position $x_{\text{init}}$ takes different selected values. For oscillation frequencies $\omega L_0 = 2.8$ (top panel), the energy oscillates, which means that there’s energy transfer between the photon and the boundary, and they reached equilibrium. This also means that the blueshift instability criterion \cite{27} is not satisfied. For oscillation frequencies that do satisfy that condition, the boundary is constantly giving energy to the photon, resulting in an exponential growth of its energy, as seen in the bottom panel.
Here, $k$ which ensures that the massless KG equation is satisfied.

The solution to this recursion relation solves the problem. In one dimension, the solution of the wave equation takes the form

$$
\Phi = e^{-ik\pi R(t+x)} - e^{-i k \pi R(t-x)},
$$

which ensures that the massless KG equation is satisfied. Here, $k$ is an arbitrary constant and $k\pi R$ is the frequency $\Omega$ of the pulse. To satisfy the BCs one takes

$$
R(t + L(t)) = R(t - L(t)) + 2,
$$

The solution to this recursion relation solves the problem. For prescribed motion,

$$
L(t) = L_0 + \frac{L_0}{2\pi} \left( \arcsin \left( \sin \theta \cos \frac{2\pi t}{L_0} \right) - \theta \right),
$$

$$
\theta = \arctan \frac{\epsilon \pi}{L_0},
$$

where $\epsilon$ describes the amplitude of the motion, the solution can be found as

$$
R(2n_{ref} L_0 + \zeta) = 2n_{ref} + \frac{1}{2} - \frac{1}{\pi} \arctan \left( \cot \frac{\zeta \pi}{L_0} - \frac{2n_{ref} \epsilon \pi}{L_0} \right),
$$

where $\zeta$ is a variable in $[-L_0, L_0]$ and $n \geq 1$ is a positive integer. The branch of arctan must be selected carefully in order to avoid a discontinuity. Notice that for small $\epsilon$,

$$
\frac{L(t)}{L_0} = 1 - \frac{\epsilon}{L_0} \sin^2 \frac{\pi t}{L_0} + O(\epsilon/L_0)^3
$$

(39)

It is easy to show that the system is unstable, precisely because the frequency content is increasing [35]. This construction can be generalized to a 3+1 setup.

3. Time evolutions

The results above are either based on a simple particle picture, or arise within a very special and prescribed boundary motion. We now wish to show that this behavior is a generic consequence also when solving the (massive) wave equation (26). To numerically evolve this equation, we first start by introducing new coordinates $(T, X)$ defined by

$$
T(t, x) = t, \quad X(t, x) = \frac{x}{L(t)}.
$$

(40)

In these coordinates, the equation of motion for the scalar field $\phi(T, X)$ take the form

$$
-\partial_T^2 \Phi + \left( \frac{1}{L^2} - \frac{X^2 L^2}{L^2} \right) \partial_X^2 \Phi + X \left( \frac{\dot{L}}{L} - \frac{2L^2}{L^2} \right) \partial_X \Phi
+ 2X \frac{\dot{L}}{L} \partial_T \partial_X \Phi - \mu^2 \Phi = 0,
$$

(41)

Note that we interchangeably use both dots and $\partial_T$ to denote time derivatives with respect to $T$. The boundary conditions (now at constant spatial coordinate $X$) are

$$
\Phi(T, X = 0) = \Phi(T, X = 1) = 0.
$$

(42)

In the new coordinates, the effect of the moving boundary appears as a shift vector term. To numerically solve Eq. (41), we use the characteristic variables $\Phi_\pm$ defined by

$$
\Phi_\pm \equiv \frac{1}{2} \left( \Phi + \lambda_\mp \partial_X \Phi \right), \quad \lambda_\pm = \pm \frac{1}{L} - X \frac{\dot{L}}{L}.
$$

(43)

The equations of motion are then

$$
\dot{\Phi}_\pm = -\lambda_\pm \partial_X \Phi_\pm - \frac{\mu^2}{2} \Phi_\pm,
$$

(44a)

$$
\dot{\Phi} = L (\lambda_+ \Phi_+ - \lambda_- \Phi_-),
$$

(44b)

with the constraint

$$
\frac{\partial_X \Phi_+}{L} = -\Phi_+ + \Phi_-. \quad (45)
$$

In these variables, the boundary conditions become

$$
\Phi_+(T, 0) = \frac{\lambda_-(T)}{\lambda_+(T)} \Phi_-(T, 0), \quad \Phi_-(T, 1) = \frac{\lambda_+(T)}{\lambda_-(T)} \Phi_+(T, 1).
$$

(46a)

(46b)
Finally, the energy of the scalar field inside the cavity is given by
\[
E = \frac{1}{2} \int_0^{L(t)} \left( (\partial_t \Phi)^2 + (\partial_x \Phi)^2 + \mu^2 \Phi^2 \right) dx
= \int_0^1 LdX \left\{ (\Phi_+^2 + \Phi_-^2) + \frac{\mu^2}{2} \Phi^2 \right\}. \quad (47)
\]

FIG. 8. Convergence study of the evolution of the scalar field at fixed radius for the simulation of Fig. 9. The purple line shows the difference between results obtained with low (4$h$) and medium (2$h$) resolutions, while the (dashed) green line shows the difference between results obtained with medium and high ($h$) resolutions multiplied by 16, the expected factor for fourth-order convergence.

We evolve Eqs. (44) with boundary conditions (46) using a finite differencing scheme where spatial derivatives are approximated with 4th-order accurate upwind stencils and a 4th-order accurate Runge Kutta scheme is used for the time integration. We have checked that this code convergences with the expected fourth-order accuracy, as shown in Fig. 8.

In all our simulations we use time-symmetric initial data, where the scalar field is parameterized by
\[
\Phi(0, X) = e^{-\left(\frac{r-r_0}{\sigma}\right)^2}, \quad \dot{\Phi}(0, X) = \frac{\dot{L}}{L} X \partial_X \Phi, \quad (48)
\]
where $\sigma$ and $r_0$ denote the width, and initial position of the scalar field pulse. Note that the wave equation is linear, hence the amplitude can arbitrarily be set to unity. In this work we focus on $\sigma = 0.2L_0$, $r_0 = 0.5L_0$ and $\delta L = 0.1L_0$. Our results are summarized in Figs. 9-12. We surveyed other regions in parameter space and found no qualitative new features with respect to those discussed below.

Consider first massless fields, $\mu = 0$, inside a cavity with an oscillating boundary. One typical evolution is shown in Fig. 9. This figure is very clear: the scalar field evolves in time with a negligible variation in amplitude, but with a significant evolution of frequency content. Note that each reflection is visible in the figure. The scalar field becomes “sharper” upon each reflection, signaling a frequency increase. The inset shows the associated increase in the total energy inside the cavity. The energy increase is exponential and the exponent is consistent with the simple estimate from the blueshift condition (27). Thus, the picture outlined in Sec. VIII A 1 seems to describe evolutions of the wave equation. For these parameters, the energy content inside the cavity grows exponentially, driven by the blueshift conversion. We also verified that the growth timescale $\lambda$ does not strongly depend on the initial position $r_0$ of the pulse, consistent with the particle picture (cf. Fig. 6).

FIG. 9. Evolution of a gaussian pulse of scalar field and its energy (inset). The field is massless, but confined to a cavity whose right boundary oscillates at $\omega L_0 = 3.1$ with an amplitude $\delta L = 0.1L_0$. The gaussian pulse has width $\sigma = 0.2L_0$ and is centered at $r_0 = 0.5L_0$. The green dotted line in the inset is the exponential $\exp(0.31t/L_0)$.

FIG. 10. Time evolution of the total energy inside the cavity for different boundary oscillation frequencies $\omega L_0$. Notice that the purple line, which does not satisfy the instability condition, remains bounded.

This interpretation is made stronger with the anal-
ysis of different parameters. Figure 10 shows us that the timescale is sensitive to the boundary oscillation frequency $\omega L_0$ and that at small frequencies the instability disappears. In fact, our numerical results are very consistent with the toy model of Sec. VIII A 1, including the threshold frequency prediction. For frequencies outside the instability window, our results show a clear oscillating pattern, again consistent with the previous discussion. To summarize, there is a clear parallel between our toy model of a massless particle in Sec. VIII A 1, and the evolution of the massless wave equation. In particular, the result induced by the boundary oscillation.

To summarize, there is a clear parallel between our toy model of a massless particle in Sec. VIII A 1, and the evolution of the massless wave equation. In particular, there are regions in parameter space where the energy in the scalar field inside the cavity grows exponentially with time. For other parameters, the energy oscillates in time, providing further support to the interpretation in terms of blue and redshift processes.

Thus far, we studied only massless fields. Figure 11 shows the evolution of the energy in the cavity for different field mass $\mu$ and different frequencies $\omega$. Notice that one could expect qualitatively new features depending on whether the Compton wavelength of the field is larger or smaller than the cavity size. Surprisingly, our results show clearly that the instability rate and instability window is only weakly dependent on $\mu$, even for large $\mu L_0$.

Because we are interested also in scalarization, for which the effective mass parameter $\mu_{\text{eff}}^2$ can be negative, we studied this case as well, summarized in Fig. 12. Now, another effect can set in, which affects even stationary geometries. Namely, scalarization induced by a tachyonic mode [16, 27]. The tachyonic instability is described by a rate $\delta t_{\text{tachyon}} \sim \sqrt{|\mu L_0|^2 - 1}$, thus at large $|L_0 \mu|$ it can dominate the entire physics.

Thus is indeed apparent in Fig. 12. The upper panel summarizes the evolution outside the blueshift instability window. For small $|L_0 \mu|$ our results indicate indeed just an oscillatory behavior for the total energy. At large values of this coupling however, the tachyonic mode sets in. We observe a similar phenomena inside the blueshift instability window. At large values of $|L_0 \mu|$ the tachyonic mode takes over the entire process, but for small couplings we recover all the $\mu = 0$ results. Our results indicate that there is not relevant change in the scalarization threshold induced by the boundary oscillation.

B. Parametric instabilities

Up to now, we studied the effects of an oscillating boundary alone. However, whenever an astrophysical object vibrates, it also changes its local density in a periodic way. To mimic this effect in a one-dimensional setting let us consider the system

$$-\partial_t^2 \Phi + \partial_x^2 \Phi - \mu^2 \Phi = 0,$$

$$\Phi(t,0) = \Phi(t,L_0) = 0,$$

(49)

where $\mu^2(t) = \tilde{\mu}^2 + \delta \mu^2 \sin(\omega t)$. Thus, $\tilde{\mu}^2$ corresponds to the average effective mass in the system. The oscillating piece $\delta \mu^2$ mimics the time-varying density. Later on, when dealing with a realistic setup, a certain condition between $\delta \mu^2$ and $\delta L$ will be necessary to ensure mass conservation. For now, we proceed with this idealization and without further constraints.

In Fourier space, that is $\Phi(t,x) = \sum_k \phi_k(t)e^{ikx}$,
Eq. (49) can be written as
\[ \partial^2\phi_k + (\delta + 2\epsilon \sin(\tau)) \phi_k = 0, \] (50)
where \( \phi_k \) is the amplitude of the Fourier mode with wavelength \( k \), and
\[ \tau = \omega t, \quad \delta = \frac{k^2 + \mu^2}{\omega^2}, \quad \epsilon = \frac{\delta \mu^2}{2\omega^2}. \] (51)

Equation (50) is the well-known Mathieu equation [53][54], and we refrain from providing further details. For certain combination of the parameters, Mathieu equation admits unstable solutions for \( \delta \sim j^2/4 \) with \( j \in \mathbb{Z} \). In particular, for small \( \epsilon \) and \( j = 1 \),
\[ \delta = \frac{1}{4} + \epsilon \delta_1, \] (52)
one finds the instability rate \( \lambda_A \) (i.e. \( \Phi \sim e^{\lambda_A t} \)) [69][70],
\[ \lambda_A = \omega \sqrt{1 - \delta_1^2}. \] (53)

This instability differs, fundamentally, from the previous “blueshift” or Doppler-like mechanism. It is of parametric origin, and it does not lead to no migration to higher frequencies. Instead, modes in the frequency range given by Eq. (52) are amplified with the rate (53). Thus, this instability can only be quenched via nonlinear effects.

**IX. INSTABILITIES OF OSCILLATING OBJECTS IN FLAT SPACE**

As we saw, fluctuations in some background value of the scalar are described by the Klein-Gordon equation [5] with an effective mass \( \mu^2 = \mu^2(t,r) \) that depends on the energy density of the star. If the star oscillates, so does the effective mass of the scalar field. We model this dependence as in Eq. (7), where \( \delta \rho(t,r) \) is the amplitude of the density oscillation, and \( \bar{\rho}_0 \) is the temporal and spatial average of the energy density. Imposing conservation of the mass \( M \) of the star, we find
\[ \bar{\rho}_0 = \frac{3}{4\pi} ML_0^{-3}, \quad \delta L(t) = -\int_0^{L_0} dr \frac{r^2 \delta \bar{\rho}}{L_0^2 \rho_0}. \] (54)

To summarize, we evolve equation (5) with an effective mass
\[ \mu^2(t,r) = \begin{cases} \mu_{\text{sm}}^2 + \mu^2 \sin(\omega t) & (r < r_-) \\ \mu_{\text{sm}}^2(r) & (r_- < r < r_+) \\ \mu_0^2 & (r > r_+) \end{cases} \] (55)
where \( r_{\pm} = L(t) \pm \frac{1}{2} \Delta L \), and \( \Delta L \) defines a thickness of the surface of the star, introduced to make \( \mu \) a smooth function. Here, \( \mu_{\text{sm}}^2(r) \) is a smooth fifth-order polynomial connecting the effective mass inside the star, \( \mu_{\infty}^2 + \mu^2 \sin(\omega t) \), to its exterior bare value \( \mu_0^2 \). Note that \( \mu_{\infty}^2 = \mu_0^2 + \beta \bar{\rho}_0 \), from Eq. (7).

### A. Numerical setup

Decomposing the scalar field in spherical harmonics, the evolution of the \((lm)\) mode, \( \Phi_{lm}(t,r) \), is governed by
\[ \left( \partial^2_r + \frac{2}{r} \partial_r - \frac{l(l+1)}{r^2} - \mu^2 \right) \Phi_{lm} - \partial^2_t \Phi_{lm} = 0. \] (56)

Regularity of the field at \( r = 0 \) implies that \( \Phi_{lm} \propto r^l \) for \( l \geq 0 \), and we impose radiative boundary conditions at spatial infinity.

As in Sec. VIII A 3, we numerically solve Eq. (56) using a fourth-order accurate Runge-Kutta scheme for the time integration where spatial derivatives are approximated by fourth-order accurate finite difference stencils. We monitor the energy of the scalar field inside the star, defined as
\[ E_{lm} = \frac{1}{2} \int_0^{L_0} \left( |\Phi_{lm}|^2 + |\partial_t \Phi_{lm}|^2 + \frac{l(l+1)}{r^2} |\Phi_{lm}|^2 + \mu^2(t,r) |\Phi_{lm}|^2 \right) r^2 dr, \] (57)

(although we mostly focus on spherically symmetric modes).

We once again use time-symmetric initial data, with a profile parameterized by (8), where \( \sigma \) and \( r_0 \) denoting the width and initial position of the scalar field pulse. Since the equation to be solved is linear we set the initial amplitude of the pulse to unity. As we will see below, we will mostly focus on \( l = 0 \) cases and we fix \( r_0 \) and \( \delta L \) to
\[ r_0 = 0.5L_0, \quad \delta L = 0.1L_0. \] (58)

**FIG. 13.** The purple line shows the difference between results obtained with low (4h) and medium (2h) resolutions, while the (dashed) green line shows the difference between results obtained with medium and high (h) resolutions multiplied by 16, the expected factor for fourth-order convergence.

All our results are numerically convergent. An example is shown in Fig. 13 showing the expected fourth-order convergence.
B. Results

1. Parametric instabilities

![Plot of instability rate of oscillating star](image)

FIG. 14. Instability rate of oscillating star, when density is allowed to oscillate (for initial condition with width $\sigma = 0.1L_0$ but results are insensitive to this choice). Now, two mechanisms compete: a Doppler like or blueshift instability and a parametric instability. In this example, with $\mu_0L_0 = 100$, the latter always dominates. Crosses are data points for $\delta\mu^2L_0^2 = 16$ and different $\delta L/L_0$; blue line is prediction from Mathieu equation [Eq. (53)] with $k = 2\pi/L_0$.

Figure 14 shows that the typical growth rate $\lambda_A$ of the amplification instability as a function of $\omega L_0$ with $\delta\mu^2L_0^2 = 16$ and different $\delta L/L_0$. We used the evolution of the scalar to estimate the rate $\lambda_A$ of the instability. We also numerically evolved Mathieu equation [Eq. (50)] with $k = 2\pi/L_0$, $\tilde{\mu} = 0$, and different $\delta\mu^2$, and obtained the instability timescale, summarized in Fig. 14 (blue line). Note that $k = 2\pi$ is the longest wavelength which can be excited inside the star and the relevant mode governing the instability. Our results show that the numerical evolution is in good agreement with the simple analysis of Sec. VIII B, leading to a Mathieu equation. For small $\delta L$, the timescale in the numerical simulation is in good agreement with the prediction from Eq. (52), and even for $\delta L = 0.1L_0$ – the difference between the growth rate and the “Mathieu” prediction is only about 10%. Here, using the well-known properties [69, 70] of the Mathieu equation, we can derive the resonance band in small $\epsilon$, defined in Eq. (52):

$$\omega \sim \frac{4\pi}{nL_0}, \quad (59)$$

where $n$ is an integer. This expression and Fig. 14 indicate that there are infinite number of instability windows at low frequency. The rate for $n = 1$ at small $\epsilon$ reads

$$L_0\lambda \sim \frac{\beta L_0^2\delta\rho}{8\pi}, \quad (60)$$

where it is assumed that $L_0\lambda \ll 1$. These results were derived assuming a simple constant profile for $\delta\rho$. We have studied nontrivial profiles, including sinusoidal shapes (more appropriate for the description of normal modes) and our results are still compatible with the above description.

X. INSTABILITY OF OSCILLATING STARS IN GENERAL RELATIVITY

We have thus far been considering toy models for instabilities using a flat space approximation. To study the existence of instabilities in astrophysical objects we will now consider radial perturbations around a relativistic constant density star. The corresponding eigenvalue and stability problem was studied previously in classical works [10, 48, 49, 71]. As we will see, the amplification instability discussed in previous sections also manifests itself in this setting. This is a highly nontrivial result: the governing equations, as well and the perturbation density profile are complicated functions, which can hardly be related to any Mathieu-like equation. Nevertheless, our results show that such stars are prone to parametric instabilities.

A. Formulation

![Plot of first four normal mode frequencies](image)

FIG. 15. The first four normal mode frequencies of a constant-density star with compactness $C = 0.3$, for different index $\gamma$. We recover well known results in the literature [10, 48, 49].

Let us consider perturbations around the metric, density, pressure, and 4-velocity profiles of the relativistic constant density star given by Eq. (15). Any of these dynamical quantities $X(t,r) = (\alpha, a, \rho, p)$, is then written as its background, stationary value corresponding to Eq. (15), plus some small linearized fluctuation,

$$X(t,r) = X_0(r) + \epsilon\delta X(t, r), \quad (61)$$

where
where $\epsilon$ is a bookkeeping parameter for the perturbation. We also need to consider fluid displacements and write the radial component of the four-velocity $u_r(t, r) = \epsilon \frac{\partial \xi}{\partial t}(t, r)$, with $\xi(t, r)$ the Lagrangian displacement. From the equation of motion for fluid one obtains the following equation for $\xi$

$$- \frac{\dot{\alpha}_0^2}{\alpha_0} (\dot{\rho}_0 + \ddot{\rho}_0) \ddot{\xi} = \frac{4}{r} \dot{\rho}_0 \dot{\xi} - \frac{1}{\alpha_0 \dot{\alpha}_0} \left( \frac{\alpha_0 \dot{a}_0^3 \gamma \ddot{\rho}_0}{r^2} \left( r^2 \frac{\dot{\alpha}_0}{\alpha_0} \right) \right)' \,,$$

$$+ 8\pi \dot{\alpha}_0^2 \ddot{\rho}_0 (\dot{\rho}_0 + \ddot{\rho}_0) \xi - \frac{1}{\rho_0 + \ddot{\rho}_0} (\ddot{\rho}_0)^2 \xi \,,$$

where primes stand for radial derivatives and $\gamma$ is the adiabatic index defined as

$$\gamma = \frac{\dot{\rho} + \ddot{\rho}}{\dot{\rho}} \left( \frac{d\dot{\rho}}{d\rho} \right) \,.$$  

For a constant density star, $\gamma$ is infinite (incompressible fluid); however, we will assume that it is of (constant) finite value to perform the perturbation 71 72. The density, pressure, and metric perturbation can be obtained from the Lagrangian displacement

$$\delta \rho = \frac{\dot{\alpha}_0 \dot{\rho}_0 + \ddot{\rho}_0}{\gamma} \left( 4 \pi r^2 \dot{\alpha}_0^2 \ddot{\rho}_0 (1 + \gamma) + \frac{\dot{\rho}_0}{2 \rho_0} (a^2 - 1) - 5 \gamma \ddot{\rho}_0 \right) \xi \,,$$

$$+ \frac{\dot{\alpha}_0 \dot{\rho}_0 + \ddot{\rho}_0}{2 \gamma} \left( -1 + 5 \gamma + \frac{\dot{a}_0^2}{\alpha_0} \right) \dot{\gamma} \,,$$

$$+ \frac{\dot{\alpha}_0 \dot{\rho}_0 + \ddot{\rho}_0}{2 \gamma} \left( -1 - \frac{\dot{a}_0^2}{\alpha_0} \right) \delta a \,,$$

$$+ 8 \pi \dot{\alpha}_0^2 \ddot{\rho}_0 (\dot{\rho}_0 + \ddot{\rho}_0) (1 + 8 \pi r^2 \dot{\rho}_0 - \frac{\dot{\alpha}_0^2}{\alpha_0}) \delta \alpha \,.$$  

We focus on the normal mode of the perturbation and assume that $\xi(t, r) = e^{i \omega t} \zeta_\omega (r) = e^{i \omega t} \frac{\partial \zeta_\omega}{\partial t} \zeta_\omega$. The equation for $\zeta_\omega$ is

$$\frac{\zeta''_{\omega}}{\zeta_\omega} = - \frac{\dot{a}_0^2}{\alpha_0^2} \left( 1 + \frac{\dot{\rho}_0}{\rho_0} \right) \omega^2 + 8 \pi \frac{\ddot{\rho}_0}{\rho_0} \frac{\dot{a}_0 \ddot{a}_0}{\alpha_0^2} \left( \frac{2}{r} \partial_r \phi + \partial_r^2 \phi \right) - \frac{\omega^2}{r^2} \alpha_0^2 \phi + \beta T \alpha_0^2 \phi - \mu_0 \delta a^2 \phi \,.$$  

The boundary condition for $\xi_\omega (r)$ is

$$\xi_\omega (0) = 0 \,, \quad \delta p (L_0) \propto \zeta_\omega (L_0) = 0 \,,$$

which corresponds to the following boundary condition for $\zeta_\omega (r)$

$$\zeta_\omega (0) = 0 \,,$$

$$\zeta'_{\omega} (L_0) = - \left( \frac{\partial \zeta_\omega}{\partial L_0} \right) \zeta_\omega (L_0) \,.$$  

We solved the eigenvalue problem for $\omega$ with the above boundary conditions for different adiabatic index $\gamma$ with a shooting method. Our results give a relation between eigenfrequency $\omega$ and index $\gamma$ for different overtones, labeled by $n$. They are summarized in Fig. 15 and are consistent with classical results in the literature 10 49. For the simulation of the scalar field, we introduce a 5th order polynomial smooth transition function $W(r; R_1, R_2)$ satisfying

$$W(r; R_1, R_2) = 1 \quad (r \leq R_1) \,,$$

$$W(r; R_1, R_2) = 0 \quad (r \geq R_2) \,,$$

$$W' (R_1; R_1, R_2) = 0 \quad W'' (R_2; R_1, R_2) = W'' (R_1; R_1, R_2)$$

$$= 0 \,,$$

and replace the Lagrangian displacement with the smoothed profile as follows

$$\xi_\omega (r) \rightarrow \xi_\omega (r) W (r; R_1, R_2) \,.$$  

With this we construct density, pressure, and metric profile from

$$Q(t, r) = Q_0 (r + \xi (t, r)) + \delta Q(t, r) W (r; R_1, R_2)$$

$$Q_0 (r) = Q_{in}(t, r) W (r; R_1, R_2) + Q_{out} (1 - W (r; R_1, R_2)) \,,$$

where $Q(t, r)$ denotes the density, pressure, and the metric, and $Q_0 (r)$ denotes the background profile for each variable.

We are then able to evolve the scalar field $\phi = \frac{\dot{\xi}}{r}$ through the evolution equation

$$\partial_t^2 \phi = \partial_1 \ln \left( \frac{\alpha_0}{a} \right) \partial_1 \phi + \frac{\alpha^2}{a^2} \partial_1 \ln \left( \frac{\alpha_0}{a} \right) \partial_r \phi + \frac{\alpha^2}{a^2} \left( \frac{2}{r} \partial_r \phi + \partial_r^2 \phi \right) - \frac{\omega^2}{r^2} \alpha_0^2 \phi + \beta T \alpha_0^2 \phi - \mu_0 \delta a^2 \phi \,.$$  

For the simulations reported below, we use a momentarily static Gaussian pulse as initial data parameterized by

$$\phi (0, r) = Ae^{-\left( \frac{r^2}{\alpha^2} \right)} \,,$$

$$\dot{\phi} (0, r) = 0 \,,$$
where $A$, $w$, $r_0$ are the initial amplitude, width, and position of the pulse, respectively. We will always set $r_0 = 0.5L_0$.

During the evolution we monitor the scalar field at fixed radius, and the energy of the scalar field inside star, given by

$$E = \frac{1}{2} \int_0^{L_0} dl\rho r^2 \left(\frac{1}{a^2} \phi'^2 + \frac{1}{a^2} \phi'^2 + \frac{l(l + 1)}{r^2} \phi^2 + (\mu_0^2 - \beta \tilde{T}) \phi^2\right)$$

(68)

**B. Results**

**FIG. 16.** Time evolution with $\mu L_0 = 100$, $\beta = -280000$, $\delta \rho_c \approx 0.08\tilde{\rho}_0$, and different initial width of the pulse. The initial width $w/L_0$ is 0.2 (upper panel), and 0.4 (lower panel). The background star is perturbed with the fundamental mode ($n = 0$).

We now take an oscillating compact star. The star oscillations could be caused by accretion or because it was the product of a merger. The amplitude of such oscillations can in any case be significant [55–59]. In our simulations we consider a single mode of oscillation, characterized by the number of nodes $n$. The generalization to accommodate for different modes is straightforward. The control parameters for the background spacetime are the compactness of the star $C$, the adiabatic index $\gamma$, the central value of the density perturbation $\delta \rho_c$, and $n$. Here, the compactness $C$ and adiabatic index $\gamma$ are fixed to 0.3 and 2.0, which are representative of neutron stars [73]. The most drastic astrophysical situations exciting large fluctuations of neutron stars concerns the remnant phase just after a neutron star merger. According to numerical relativity simulations, the amplitude of the density perturbations can be significant and of the order of the central density itself [58–59]. Although such large density perturbations lie beyond the validity range of the linear approximation for the perturbation, we consider $\max(\delta \rho(t, 0)) \approx 0.28\tilde{\rho}_0, 0.08\tilde{\rho}_0$, and use the linear normal mode derived in previous subsection.

Figure [16] shows the time evolution of the scalar field at $r = 0.5L_0$ with $\mu L_0 = 100$, $\beta = -280000$, and $\delta \rho_c \approx 0.08\tilde{\rho}(0)$, and the fundamental density perturbation mode $n = 0$. Note how the scalar has a time-periodic behavior, but is not growing neither in amplitude nor in frequency. With these parameters we could not observe any instabilities.

**FIG. 17.** Time evolution with $\mu L_0 = 100$, $\beta = -280000$, $\delta \rho_c \approx 0.28\tilde{\rho}_0$, $w/L_0 = 0.2$. We plot radial modes of density perturbation with $n = 0, 1, 2$. Notice that the density fluctuation is large, but still smaller than typical values in seen during the coalescence of two neutron stars [58–59]. Also, for smaller $\beta$, the instability sets in at small density fluctuations.

However, an instability sets in with either larger values of density fluctuation or smaller values of $\beta$ (i.e., larger $\beta^2$). We note that for massive fields – our subject here – observational constraints on $\beta$ are poor or non-existing and any of the values discussed here is compatible with current observations. For example, Fig. [17] shows the time evolution of the scalar field at $r = 0.5L_0$, but now with $\delta \rho_c \approx 0.28\tilde{\rho}_0$. We can compare and contrast the results with those of Fig. [16]. For these larger values of density fluctuations, a parametric instability sets in. The frequency of the fundamental density perturbation mode with $\gamma = 2.0$ is about $0.36/L_0$ and the corresponding period is about $17L_0$. This period is clearly evident in Fig. [17] in the form of a plateau in the evolution of the scalar. In other words, the growth proceeds upon interaction with the star with a periodicity dictated by the star eigenfrequency. As is clear from the evolution equations or the background structure of the star itself, it is far from trivial that such oscillating stars in General Relativity would trigger parametric instabilities. Our results confirm it. We note that Fig. [17] refers to a significant density fluctuation. However, our results are mostly sensitive to the combination $\beta \tilde{\rho}_0$, thus much milder fluctuations also result in instability, provided $\beta^2$ is sufficiently large.
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