Persistence in the Zero-Temperature Dynamics of the Random Ising Ferromagnet on a Voronoi-Delaunay lattice

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The zero-temperature Glauber dynamic is used to investigate the persistence probability \( P(t) \) in the random two-dimensional ferromagnetic Ising model on a Voronoi-Delaunay tessellation. We consider the coupling factor \( J \) varying with the distance \( r \) between the first neighbors to be \( J(r) \propto e^{-\alpha r} \), with \( \alpha \geq 0 \). The persistence probability of spins flip, that does not depend on time \( t \), is found to decay to a non-zero value \( P(\infty) \) depending on the parameter \( \alpha \). Nevertheless, the quantity \( p(t) = P(t) - P(\infty) \) decays exponentially to zero over long times. Furthermore, the fraction of spins that do not change at a time \( t \) is a monotonically increasing function of the parameter \( \alpha \). Our results are consistent with the ones obtained for the diluted ferromagnetic Ising model on a square lattice.

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I. INTRODUCTION

The Lenz-Ising model is probably the simplest non-trivial model for cooperative behavior that shows spontaneous breaking of symmetry. Because of this simplicity and the fact that each individual element of the model modifies its behavior according to the other individuals in its vicinity, it has a vast number of applications ranging from solid-state physics to biology [1–3]. An important extension to this model is the introduction of disorder such as; random external fields, random exchange parameters, and dilution, where only a fraction \( p \) of the lattice sites are occupied by spins. In particular, the two-dimensional diluted ferromagnetic random Ising model is very important to describe magnetic properties of several condensed matter systems. Moreover, it constitutes a marginal situation of the Harris criterion [4], and for many years there has been several theoretical and numerical studies [5–8] in order to clarify the properties of the dilution model.

On the other hand, only in the last ten years the “persistence” problem in the random two-dimensional ferromagnetic Ising model has attracted considerable interest [9–11]. In this most general form, this problem involves the fraction of space which persists in its initial state until some time later. Hence, in the non-equilibrium dynamics of spin systems we are interested in the fraction of spins \( P(t) \), that persist in the same state as at \( t = 0 \) up to some later time \( t \). For the pure ferromagnetic d-dimensional \((d < 4)\) Ising model, \( P(t) \) has been found to decay algebraically [9,12]

\[
P(t) \propto t^{-\theta(d)},
\]

where \( \theta(d) \) is a non-trivial persistence exponent. For the same model [10] at higher dimensions \((d > 4)\) and for the two-dimensional ferromagnetic \( q \)-state Potts model [13] \((q > 4)\) it has been shown through computer simulations that \( P(t) \) is not equal to zero for \( t \rightarrow \infty \). This characteristic is some time called as “blocking”. Therefore, if \( P(\infty) > 0 \) we can reformulate the problem by restricting our observation to those spins that eventually flip. Hence, we can consider the behavior of

\[
p(t) = P(t) - P(\infty).
\]

By considering the dynamics of the local order parameter the persistence problem can be generalized to nonzero temperatures [14,15] Recently [16,17] the attention has been turned to the persistence problem in systems containing disorder. Numerical simulations of the zero-temperature dynamics of the bond diluted (weak dilution [16] or strong dilution [18]) two-dimensional Ising model also reported “blocking” evidences. Howard [19] has found evidence of an exponential decay of the persistence with blocking for the homogeneous ferromagnetic Ising model on the homogeneous tree of degree three \((T)\) with random spin configuration at time 0. Here we present results of an extensive numerical study of the persistence of the ferromagnetic Ising model on the Voronoi-Delaunay lattice. In this lattice, the coordination number and the distance between the first neighbors sites is random. As the bond length between the first neighbors varies randomly from neighbor to neighbor, we consider that the coupling factor depends on the relative distance \( r_{ij} \) between sites \( i \) and \( j \) according to the following expression:

\[
J_{ij} = J_0 e^{-\alpha r_{ij}}
\]

where \( J_0 \) is a constant and \( 0 \leq \alpha \leq 1 \). The question to be answered here is: does the persistence behavior change with this type of randomness or is its behavior the same as in the diluted ferromagnetic Ising model in a regular lattice? In the present work, we show that, with this type of randomness, \( p(t) \) presents an exponential decay as in the strongly diluted ferromagnetic Ising model in
two-dimension [17] in contrast with the behavior of the pure and weakly diluted models.

II. MODEL AND SIMULATION

The Voronoi construction or tessellation for a given set of points in the plane is defined as follows. For each point, we first determine the polygonal cell consisting of the region of space nearer to that point than any other point. Whenever two such cells share an edge, they are considered to be neighbors. From the Voronoi tessellation, we can obtain the dual lattice by the following procedure: when two cells are neighbors, a link is placed between the two points located in the cells. From the links, one obtains the triangulation of space that is called the Delaunay lattice. The Delaunay lattice is dual to the Voronoy tessellation in the sense that points correspond to cells, links to edges and triangles to the vertices of the Voronoi tessellation.

We consider a two-dimensional Ising ferromagnetic on this Poissonian random lattice which Hamiltonian is given by:

\[ -KH = - \sum_{<i,j>} J_{ij} S_i S_j , \]  

where \( S_i = \pm 1 \) are the Ising spins situated on every site of a Delaunay lattice with \((L \times L = N)\) sites and periodic boundary conditions; \( K = 1/k_B T \), \( T \) is the temperature and \( k_B \) is the Boltzmann constant. The summation in Eq. (4) runs over all nearest-neighbors pairs of sites (points in the Delaunay construction). In this lattice the coordination number varies locally between 3 and \( \infty \) and the coupling factor \( J_{ij} \) depends on the distance between first neighbors according to Eq. 3.

For simplicity, the length of the system is defined here in terms of the size of a regular lattice, \( L = N^{1/2} \). We perform simulations over a lattice with \( L = 500 \). A randomly initial configuration of spins is obtained and \( P(t) \) is calculated over 6000 of Monte Carlo steps and a quenched average is done over 10 different Delaunay lattices for each Monte Carlo step. The temperature zero Glauber dynamics was utilized in order to check the number of spins that never change their state at a time \( t \). In this dynamics we start with a randomly initial spin configuration and allow it to be updated by selecting one spin to be flipped at random or following a given logical sequence. The selected spin, \( S_i \), is flipped or not according to \( \Delta E_i \), where \( \Delta E_i \) is the energy of site \( i \). If \( \Delta E_i < 0 \) the spin \( S_i \) is flipped with probability one. If \( \Delta E_i = 0 \) the spin \( S_i \) is flipped with probability \( 1/2 \) chosen at random. Finally if \( \Delta E_i > 0 \) the spin is not flipped.

One Monte Carlo step corresponds to application of the above rule for all spins of the lattice. The system configuration is left to evolve until a given Monte Carlo step \( t \approx 6,000 \). The number, \( n(t) \), of sites that do not change at this time \( t \) is computed for each Monte Carlo step for the determination of the persistence probability given by [9]:

\[ P(t) = \frac{\langle n(t) \rangle}{N} \]  

The data have been obtained for 10 \( \alpha \) values in the range \( 0 \leq \alpha \leq 1 \).

III. RESULTS AND CONCLUSION

For \( \alpha \) varying from 0 (where the coupling constant is independent of the distance between first neighbors) to 1 the persistence \( P(t) \) seems not to decay algebraically before the “freezing” as we can see in Fig. 1. In this figure we plot \( \ln P(t) \) versus \( t \) for \( \alpha = 0 , 0.5 \) and 1. We
can also verify that, \( P(t) = P(\infty) \) for \( t > t^* \), where \( t^*(\alpha) \) depends on the parameter \( \alpha \), growing with the \( \alpha \) value. The fraction of frozen sites (i.e. the sites that never flip) in function of the parameter \( \alpha \) is shown in Fig. (2). This fraction has a maximum value near \( \alpha = 0.9 \) decreasing for \( \alpha = 1 \). Finally in Fig. (3) we plot \( \ln p(t) \) versus \( \ln t \) for the same values of \( \alpha \). From this figure we can verify that \( p(t) \) decay exponentially for long times. This result agrees with the results obtained by Newman and Stein [17] for the persistence in the strongly diluted Ising ferromagnet. This behavior occurs for \( \alpha = 0 \). This result is contradictory for us once we know that for \( \alpha = 0 \) the ferromagnetic Ising model on a Delaunay lattice has the same critical exponents that the pure ferromagnetic Ising model in a square lattice has [8]. This fact does not agree with the persistence behavior of the pure and weakly diluted ferromagnetic Ising model reported by Jain [16].

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