THE PROFIT-SHARING RULE THAT MAXIMIZES SUSTAINABILITY OF CARTEL AGREEMENTS

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Abstract. We study the profit-sharing rule that maximizes the sustainability of cartel agreements when firms can make side-payments. This rule is such that the critical discount factor is the same for all firms (“balanced temptation”). If a cartel applies this rule, contrarily to the typical finding in the literature, asymmetries among firms may increase the sustainability of the cartel. In an illustrating example of a Cournot duopoly with asymmetric production costs, the sustainability of collusion is maximal when firms are extremely asymmetric.

1. Introduction. The literature on the sustainability of cartels typically considers an infinitely repeated game where each firm may either: act in accordance with a collusive agreement that specifies each firm’s output (or the price charged to consumers); or break the agreement by increasing output (or decreasing price), taking advantage of the fact that the other firms are restricting output (or charging the cartel price). If one of the firms deviates from the agreement, the industry becomes non-cooperative and profits in the subsequent periods become lower.

The choice of a firm between these two options depends on the relative importance that it gives to present and future profits. If the discount factor is sufficiently close to unity (meaning that similar importance is given to present and future profits), the firm will prefer to abide by the cartel rules; if it is sufficiently close to zero (meaning that almost no importance is given to future profits), the firm will prefer to break the cartel to enjoy a higher profit in the present, in spite of suffering the consequent decrease of future profits. The critical value of the discount factor, above which no firm finds it profitable to break the agreement, is an indicator of the sustainability of the cartel.

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When firms are heterogeneous, sharing the profit of the cartel is not a trivial matter.\(^1\) If they avoid side-payments in order to decrease the probability of detection, a natural profit-sharing rule is to allocate production efficiently and let each firm keep its individual profit.\(^2\) However, some firms may be displeased with this rule, particularly if they are relatively inefficient and anticipate that they would get a small share of the gains from collusion. In that case, firms may use side-payments to redistribute the cartel profit among them. Side-payments provide an additional degree of freedom: the cartel can allocate production efficiently and, independently, choose how to share profits. In this context, a natural profit-sharing rule is based on the solution of the Nash bargaining game: each firm gets its non-cooperative profit plus an equal share of the increase of the industry profit generated by the cartel agreement.\(^3\)

Depending on the source of firm heterogeneity and on the profit-sharing rule that is adopted by the cartel, some firms will be more prone to break the cartel than others. Since the sustainability of the cartel only depends on the behavior of the firm that has the strongest propensity to deviate, it seems to be straightforward that firm heterogeneity decreases the sustainability of cartels. This conclusion is supported by the theoretical contributions of Bae [2], Harrington [10], Compte et al. [6], Kühn [12], Vasconcelos [18], Miklós-Thal [14], and Brandão et al. [4].

To address the issue of sustaining collusion in the presence of asymmetries among firms when side-payments are possible, we study the profit-sharing rule that maximizes the sustainability of the cartel. With this rule, all firms have the same critical discount factor, i.e., the balanced-temptation property proposed by Friedman [8] is satisfied. If one of the firms had a higher critical discount factor (i.e., a stronger propensity to break the cartel), the sustainability of the cartel agreement could be increased by marginally increasing this firm’s share of the cartel profits (e.g., by slightly increasing the side-payment from the other firms to this firm).\(^4\)

For illustrative purposes, we apply this profit-sharing rule in a simple example of a Cournot duopoly in which firms have different (quadratic) production costs. We find a U-shaped relationship between the degree of firms’ asymmetry and the sustainability of collusion.\(^5\) Collusion is least sustainable if firms are moderately asymmetric and is most sustainable if firms are extremely asymmetric.\(^6\)

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\(^1\) According to Ganslandt et al. [9]: “in the 43 European cartel cases between 2002 and 2007, the size of the second largest firm was on average 70% of the size of the largest firm, and the size of the third largest firm was on average 32% of the size of the largest firm” (p. 768).

\(^2\) This profit-sharing rule is perhaps the most commonly used in the literature. See the discussion by Bos and Harrington [3].

\(^3\) This sharing rule was firstly proposed by Osborne and Pitchik [16].

\(^4\) As a solution concept, balanced temptation was criticized by Harrington [10], who advocated the application of a bargaining solution to select one among the set of sustainable collusive agreements. On the other hand, Verboven [19] showed that when perfect collusion is not sustainable, the outcome of bargaining is likely to satisfy the balanced-temptation property.

\(^5\) According to Levenstein and Suslow [13], the effect of cost asymmetries on the propensity to collude is ambiguous (p. 47).

\(^6\) Ganslandt et al. [9] found that collusion is easiest to sustain when firms are moderately asymmetric. In their model, however, this results from the existence of an indivisible cost of collusion that the largest firm bears for running the cartel. The higher is this cost, the more asymmetric must firms be for the sustainability of collusion to be maximal. In our model, there is no cost of running the cartel. Another major distinction between the two contributions is that Ganslandt et al. [9] rule out side-payments between firms.
Previous contributions have also considered the balanced-temptation solution to study the effect of cost asymmetries on the sustainability of a cartel, but in environments without side-payments. In a price-setting supergame where firms have constant unit costs, Bae [2] concluded that cost asymmetries hinder the sustainability of collusion. In a contribution that is the closest to ours, Collie [5] provided numerical evidence suggesting that cost asymmetry enhances the sustainability of collusion in a quantity-setting supergame with linear demand and quadratic costs. We find a similar result in an environment in which side-payments are possible.

2. Model. Consider an industry composed by heterogeneous firms in a stationary infinite-horizon setting. The objective of each firm is to maximize the discounted value of its flow of profits: \( \sum_{t=0}^{\infty} \delta^t \pi_{it} \), where \( \pi_{it} \) denotes the profit of firm \( i \) in period \( t \), and \( \delta \in (0, 1) \) is the common discount factor.

Before period \( t = 0 \), firms \( i \in \{1, ..., N\} \) establish a collusive agreement, which may stipulate the output produced by each firm, the price set by each firm, the geographical division of the market, or the advertising expenditure by each firm. The agreement may include all firms in the industry or not, and may be conducive to cartel profit maximization or not.\(^7\) It lasts forever, as long as there are no defections. Firms use grim trigger strategies (Friedman [8]): following a deviation, they permanently revert to a Nash equilibrium of the stage game in which they have lower joint profits than under collusion.\(^8\) There is no renegotiation.

Let \( \pi^s_i \) denote the single-period profit of firm \( i \) in each of the possible competitive scenarios: \( s = m \) if firms collude; \( s = d \) if firm \( i \) unilaterally deviates from the collusive agreement; and \( s = p \) along the punishment path.

Firms prefer to abide by the collusive agreement if the discounted value of the flow of collusive profits exceeds the sum of the deviation profit with the discounted value of the flow of the subsequent punishment profits. More precisely, collusion is sustainable (i.e., is a subgame perfect Nash equilibrium) if and only if the following incentive compatibility constraint is satisfied for all firms \( i \in \{1, ..., N\} \):

\[
\sum_{t=0}^{\infty} \delta^t \pi^m_i \geq \sum_{r=1}^{\infty} \delta^r \pi^p_i \iff \delta \geq \frac{\pi^d_i - \pi^m_i}{\pi^d_i - \pi^p_i} \equiv \delta^*_i.
\]

(1)

Therefore, the critical discount factor for collusion sustainability is: \( \delta^* = \max_{i \in \{1, ..., N\}} \delta^*_i \).

The magnitude of each firm’s critical discount factor depends on how the profit of the cartel is shared among firms. Denote by \( \lambda_i \in (0, 1) \) the profit-share of firm \( i \): \( \pi^m_i = \lambda_i M \), where \( M \) denotes the industry profit under collusion.

We consider the profit-sharing rule that maximizes sustainability of collusion, i.e., firms are assumed to choose the allocation, \( \lambda \equiv (\lambda_1, ..., \lambda_N) \), that minimizes \( \delta^* \).

\(^7\)We abstract from the issue of participation by assuming that unanimity is required for cartel formation. As a result, firms prefer to accept to participate and then deviate rather than reject to participate. Readers interested in the topic of endogenous cartel formation can see, for example, d’Aspremont et al. [7], and Bos and Harrington [3].

\(^8\)Punishment strategies that lead to non-stationary profit streams along the punishment path, such as “stick-and-carrot” punishments (Abreu [1]), can also be considered. It is only necessary to calculate the continuation value after a deviation, \( V_i \), and replace the single-stage punishment profit, \( \pi^p_i \), by \( (1 - \delta)V_i \).
Proposition. The allocation of the cartel profit, $\lambda^*$, that maximizes sustainability of collusion is such that:

$$
\delta^* = \frac{D - M}{D - P} \quad \text{and} \quad \lambda^*_i = \frac{M - P}{M(D - P)} \pi^d_i + \frac{D - M}{M(D - P)} \pi^p_i,
$$

where $M = \sum_{i=1}^{N} \pi^m_i$ is the cartel profit under collusion, $D = \sum_{i=1}^{N} \pi^d_i$ is the sum of the individual deviation profits, and $P = \sum_{i=1}^{N} \pi^p_i$ is the joint profit along the punishment path.

Proof. It is rather trivial to show that $\delta^*_i = \delta^*_j$, $\forall i, j$. Let $A = \{i: \delta^*_i = \delta^*\}$ and $B = \{i: \delta^*_i < \delta^*\}$. If $B$ is not empty, a marginal increase of the profit shares of the firms in $A$ at the cost of the profit shares of the firms in $B$ decreases $\delta^*$.

Thus, the $N$ incentive compatibility constraints (1) are satisfied with the equality sign. We have, therefore, a linear system of $N + 1$ equations with $N + 1$ unknowns:

$$
\begin{align*}
\delta^* &= \frac{\pi^d_i - \lambda^*_i M \pi^d_i}{\pi^d_i - \pi^p_i}, \quad \forall i \\
\sum_{i=1}^{N} \lambda^*_i &= 1,
\end{align*}
$$

whose solution is (2).

The Proposition states that the profit-sharing rule that maximizes the sustainability of the collusive agreement is the one according to which all cartel members have the same incentives to deviate, i.e., have the same critical discount factor. The intuition for this result is the following. A collusive agreement is only sustainable if no firm has incentives to defect. As a result, the sustainability of the agreement only depends on the critical discount factor of the firms that are the most tempted to defect. The critical discount factors of the remaining firms are not relevant. If side-payments are allowed, the cartel’s critical discount factor, $\delta^*$, can be lowered by asking firms with lower critical discount factors to make a small transfer to the firms whose critical discount factor is the highest, i.e., to the most disruptive firms.

Observe that the resulting critical discount factor is always lower than unity, which means that, if firms can make side-payments, the balanced-temptation collusive agreement is sustainable if firms are sufficiently patient.

3. Cournot duopoly with asymmetric production costs. Consider the case of an infinitely-repeated Cournot duopoly with homogeneous goods and asymmetric production costs. In each period $t \in \{0, 1, \ldots\}$, firms 1 and 2 simultaneously choose their output levels, $q_{1t}$ and $q_{2t}$. The price charged to consumers is the same for both firms and given by:

$$
P_t = 1 - (q_{1t} + q_{2t}).
$$

Firms need capital to produce. A firm with more capital can produce the same units of output at a lower cost. The cost function of firm $i \in \{1, 2\}$ is given by:

$$
C_i(q_i) = \frac{q_i^2}{2k_i},
$$

where $k_i$ is the capital stock of firm $i$. For simplicity, we normalize the joint capacity to 1 and denote by $k \in (0, 1)$ the capacity of firm 1, i.e., we set $k_1 = k$ and $k_2 = 1 - k$. When $k = 0.5$, the firms are symmetric; if $k > 0.5$, firm 1 is more efficient; if $k < 0.5$, it is firm 2 that is more efficient.

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9This is a simplified version of the cost function proposed by Perry and Porter [17].
Under Cournot competition (punishment path), firm $i$ produces the quantity that maximizes its individual profit, given the quantity produced by firm $j \neq i$:

$$\max_{q_i} \left\{ (1 - q_i - q_j) q_i - \frac{q_i^2}{2k_i} \right\}.$$ 

The competitive output and profit of each firm $i \in \{1, 2\}$ is given by:

$$q^p_i = \frac{k_i}{(1 + k_i)(1 + S)} \quad \text{and} \quad \pi^p_i = \frac{k_i(1 + 2k_i)}{2(1 + k_i)^2(1 + S)^2}, \quad (3)$$

where $S = \sum_{i=1}^{2} \frac{k_i}{1 + k_i}$.

We follow most theoretical contributions that study collusion sustainability in assuming that, along the collusive path, firms maximize their joint profit:

$$\max_{(q_1, q_2)} \left\{ (1 - q_1 - q_2) (q_1 + q_2) - \frac{q_1^2}{2k} - \frac{q_2^2}{2(1 - k)} \right\}.$$ 

Under perfect collusion, the output of each firm is proportional to its capital stock, $q^m_i = \frac{k_i}{3}$, and the cartel profit is $M = \frac{1}{5}$.

If firm $i$ disrupts the collusive agreement, it produces the quantity that maximizes its individual profit, assuming that the other firm produces the collusive output:

$$\max_{q_i} \left\{ (1 - q_i - \frac{k_j}{3}) q_i - \frac{q_i^2}{2k_i} \right\}.$$ 

The deviation output and the corresponding single-period profit are:

$$q^d_i = \frac{k_i(2 + k_i)}{3(1 + 2k_i)} \quad \text{and} \quad \pi^d_i = \frac{k_i(2 + k_i)^2}{18(1 + 2k_i)}.$$ 

Replacing the expressions for profits in (2), we obtain the critical discount factor that is associated with the profit-sharing rule that maximizes collusion sustainability (illustrated in Figure 1):\(^{10}\)

$$\delta^* = \frac{(3 - 4k + 4k^2)(1 + k - k^2)^2}{6 + 3k - 12k^2 + 14k^3 + 3k^4 - 12k^5 + 4k^6}. \quad (5)$$

Notice, however, that for this to be the critical discount factor, the most efficient firm (i.e., the firm that has more capital) must make a side-payment to the other firm, in order to increase its collusive gains and thus decrease its incentives to defect.

As illustrated in Figure 1, collusion is easiest to sustain when one firm owns almost all the industry capital (i.e., when $k \to 0$ or $k \to 1$). Collusion is hardest to sustain when one firm owns approximately 33% of the industry capital. Symmetry between firms does not necessarily facilitate collusion. This is in contrast with the existing literature, which defends symmetry across firms as facilitating collusion (Motta [15]).\(^{11}\)

Let us now study the sustainability of collusion with two alternative profit-sharing rules, which are usually considered in the literature on collusion among heterogeneous firms: (i) Nash bargaining; and (ii) perfect collusion without side-payments.\(^{12}\)

\(^{10}\)As defined in the Proposition, the profit-sharing rule that maximizes the sustainability of collusion is such that the critical discount factor is the same for the two firms, despite their cost asymmetry. Thus, the expression (5) can be obtained by replacing in (2) the expression for the profits either of firm 1 or firm 2.

\(^{11}\)An exception is the contribution of Collie [5], which is close to ours.

\(^{12}\)For a deeper discussion on these two sharing rules see, for instance, Brandão et al. [4] and the references therein.
If firms divide their joint profit through Nash bargaining, the profit share of firm $i \in \{1, 2\}$ is given by:

$$\lambda_i^N = \arg\max_{\lambda_i} \left\{ \left( \lambda_i M - \pi_i^m \right) \left[ (1 - \lambda_i) M - \pi_j^m \right] \right\},$$

where $j \neq i$. Replacing $\pi_i^m = \lambda_i^N M$ and expressions (3) and (4) in (1), we obtain the critical discount factors above which firms abide by the collusive agreement:

$$\delta_1^N = \frac{-4 - 8k + 45k^2 + 62k^3 - 68k^4 + 6k^5 + 51k^6 + 57k^7 + 6k^8 + 6k^9}{3k^2(1 + k - k^2)(3 - 2k + 2k^2)(4 + 6k - 3k^2 - k^3)}$$

and

$$\delta_2^N = \frac{39 - 35k - 165k^2 + 285k^3 - 92k^4 - 207k^5 + 324k^6 - 207k^7 + 60k^8 - 6k^9}{3(1 - k)^2(1 + k - k^2)(3 - 2k + 2k^2)(6 + 3k - 6k^2 + k^3)}.$$

Thus, when firms divide the cartel profit through Nash bargaining, collusion is sustainable if $\delta > \delta^N = \max\{\delta_1^N, \delta_2^N\}$. It is straightforward to prove that, with this profit-sharing rule, the biggest firm is the most tempted to deviate (as illustrated in Figure 2).

Suppose now that side-payments between firms are not feasible. Assuming that firms continue to maximize cartel profits and allocate production efficiently, as we have seen before, the collusive output of firm $i \in \{1, 2\}$ is $q_i^m = \frac{k_i}{s}$, and, therefore, the collusive profit is $\pi_i^m = \frac{k_i^2}{s}$. Replacing this expression and expressions (3) and (4) in (1), we obtain the critical discount factor for each firm in the absence of side-payments:

$$\delta_1^P = \frac{(1 - k) \left( 1 + k - k^2 \right)^2}{k^2(4 + 6k - 3k^2 - k^3)} \quad \text{and} \quad \delta_2^P = \frac{k (1 + k - k^2)^2}{(1 - k)^2(6 + 3k - 6k^2 + k^3)}.$$

Again, collusion is sustainable in the absence of side-payments if and only if $\delta > \delta^P = \max\{\delta_1^P, \delta_2^P\}$. As illustrated in Figure 3, in this case, it is the smallest firm that has the strongest incentives to deviate. This occurs because, due to its inefficiency, the smallest firm is asked to produce a very small share of the
cartel output. Since the firm is not compensated (through a side-payment) from restricting output so much, it may even profit more in a non-cooperative scenario (i.e., $\pi_i^c < \pi_i^p$ and, therefore, $\delta^{P^*} > 1$). This is why, if firms are very asymmetric, $k \in (0, 0.39) \cup (0.61, 1)$, perfect collusion is not sustainable without side-payments.

Figure 4 illustrates the fact that the profit-sharing rule that maximizes the sustainability of collusion by balancing temptation significantly decreases the critical discount factor.
4. Conclusions. When asymmetries between firms are very pronounced, perfect collusion without side-payments may not be possible. However, if firms can make side-payments to redistribute their collusive profits in an appropriate way, they may be able to sustain perfect collusion (as long as they are sufficiently patient). With the profit-sharing rule that maximizes collusion sustainability, which is the one that satisfies the “balanced-temptation” requirement of Friedman [8], asymmetries between firms may increase the sustainability of collusion. This is in line with the results of Collie [5], but contrasts with the typical message in the literature.

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