Static Properties of Hydrostatic Thrust Bearing Considering Couple Stress

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Abstract. In this paper, it studied the static properties of hydrostatic thrust bearings combined the effects of recess depth, fluid inertia and couple stress. It is found that couple stress and centrifugal inertia has notable effects on the static properties of hydrostatic thrust bearing.

Introduction

In recent years, the hydrostatic thrust bearing becomes more and more important in mechanical industry for its remarkable advantages of high accuracy, great rigidity and little vibration. Plenty of works have been done to improve the bearing’s working performance in the past decades. Many researchers have realized that, at high rotating velocity, the inertia effects play an important role on the bearing properties. Dowson [1] first studied the hydrostatic thrust bearings considering the inertia effects and found that loading capacity decreased for the parallel bearing, while increased for the step bearing. For different structures of bearings, the pressure distributions were quite different. Soon after, Coombs and Dowson [2] conducted an experiment to investigate the properties of a hydrostatic thrust bearing with inertia effects. Without considering the thermal effect, the theoretical analysis agreed well with the experiment results.

Although the results of Dowson’s work were excellent, the lubricant in his study was identified as Newtonian fluid. Most of the lubricants used in the modern industry are not the tradition Newtonian lubricants any more. To get better property of lubrication, different additives are put into the lubricants. And a lot of micro-continuum models [3, 4, 5] have been developed to study the properties of these new fluids. Among these theories, the Stokes theory [5] is the simplest one which includes polar effects such as non-symmetric tensors, body forces and couple stresses.

In the present study, the centrifugal effects and couple stress are both considered.

Analysis

The schematic diagram of the hydrostatic thrust bearing is showed in Figure 1. Define the dimensionless quantities:

\begin{align*}
P &= \frac{\overline{P}}{P_0}, \quad Q = \frac{\mu \overline{Q}}{\pi R^2 P_0}, \quad r = \frac{\overline{r}}{R}, \quad r_i = \frac{\overline{r_i}}{R}, \quad S_0 = \frac{\pi \Omega^2 R^2}{P_0}, \quad S = \frac{3}{20} S_0, \quad u = \frac{\overline{v}}{R \Omega}, \quad v = \frac{\overline{v}}{R \Omega}, \quad w = \frac{\overline{w}}{R \Omega}.
\end{align*}

(1)

Using the couple stress theory and considering the axial symmetry of the thrust bearing, the simplified dimensionless Navier-Stokes equations of motion in cylindrical coordinates can be expressed as follows:
\[
\frac{S_0 v^2}{r} = \frac{\partial P}{\partial r} + \frac{1}{\delta} \frac{\partial^2 u}{\partial z^2} - \frac{I^2}{\delta} \frac{\partial^2 u}{\partial \tilde{z}^2} 
\]

(2)

\[
0 = \frac{\partial^2 v}{\partial \tilde{z}^2} - I^2 \frac{\partial^4 v}{\partial \tilde{z}^4} 
\]

(3)

\[
0 = \frac{\partial P}{\partial \tilde{z}} 
\]

(4)

In the equation (3), with the boundary conditions: \(v|_{\tilde{z} = 0 \rightarrow \beta h} = 0\), \(v|_{\tilde{z} = \beta h} = \frac{r\beta}{\beta h}\), the velocity component in the circumferential direction could be derived as: \(v = \frac{r\beta}{\beta h}\)

Substituting the expression of \(v\) into the equation (2), using the boundary condition:

\[
0 = \frac{\partial v}{\partial \tilde{z}} 
\]

(5)

We could obtain:

\[
\frac{\partial^3 u}{\partial \tilde{z}^3} \bigg|_{\tilde{z} = 0, \tilde{z} = \beta h} = 0, \quad \frac{\partial^3 u}{\partial \tilde{z}^3} \bigg|_{\tilde{z} = 0, \tilde{z} = \beta h} = 0 
\]

(5)

Multiply equation (6) with \(2r\) and integrate with respect to \(z\) from 0 to \(\beta h\), then set \(S = \frac{3S_0}{20}\). The pressure gradient in the radial direction can be reached as:

\[
\frac{\partial P}{\partial r} = 2\left(\frac{Sf_r(\beta h, l)}{f_r(\beta h, l)} + \frac{20}{3} \frac{S^2}{\beta^2 h^2}\right) - \frac{6Q}{f_r(\beta h, l)} 
\]

(7)

where \(f_r(\beta h, l) = 24l^3 \tan \frac{\beta h}{2l} + \beta^3 h - 12 \beta^2 h l^2\), \(f_s(\beta h, l) = 40l^3 \tan \frac{\beta h}{2l} + \beta^3 h^3 - 20 \beta^2 h l^2\), \(Q = \frac{\mu Q}{\pi R P_o} = 2r \int_0^{\beta h} \frac{\partial u}{\partial \tilde{z}} d\tilde{z}\)

According to the bearing structure in the Figure 1, the film thicknesses in and outside the recess are different, and the pressure distributions are also different. Integrate the equation (7) with respect to \(r\) with the pressure boundary conditions, \(P|_{r=\infty} = 1, P|_{r=0} = 0\), the pressure distribution along the radial direction with can be represented as:

\[
P = 1 + \left(\frac{Sf_s(\beta h, l)}{f_s(\beta h, l)} + \frac{20}{3} \frac{S^2}{\beta^2 h^2}\right) \left(r^3 - r_0^3\right) - \frac{6Q}{f_s(\beta h, l)} \ln r 
\]

(8)

\[
P = \left(\frac{Sf_s(\beta h, l)}{f_s(\beta h, l)} + \frac{20}{3} \frac{S^2}{\beta^2 h^2}\right) \left(r^3 - 1\right) - \frac{6Q}{f_s(\beta h, l)} \ln r 
\]

(9)

At \(r = r_1\) (the step location), assuming the pressure is continuous, we obtain the dimensionless flow rate from equations (8) and (9):

\[
Q = \frac{\left(r_1^3 - 1\right) g(h, l) - \left(r_1^3 - r_0^3\right) g(\beta h, l) - 1}{6 \left(f_s(\beta h, l) \ln r_1 - f_s(\beta h, l) \ln r_0\right)} f_s(\beta h, l) f_r(\beta h, l) 
\]

(10)

where

\[
g(\beta h, l) = \frac{Sf_s(\beta h, l)}{f_s(\beta h, l)} + \frac{20}{3} \frac{S^2}{\beta^2 h^2} 
\]

(11)

And the expression of dimensionless load-carrying capacity can be calculated as
W = r_0^2 + 2 \int_{r_0}^{r_1} r P dr + 2 \int_{r_1}^{r_0} r P dr = \frac{1}{2} \left( \frac{S f_S (\beta h, l)}{f_I (\beta h, l)} + \frac{20 S l^2}{3} \right) \left( r_1^2 - r_0^2 \right)^2 - \frac{6Q}{f_I (\beta h, l)} \left( \frac{r_1^2 \ln \frac{r_1}{r_0} - r_1^2 - r_0^2}{2} \right) \left( \frac{r_1^2 - 1}{h^2} \right) + \frac{6Q}{f_I (\beta h, l)} \left( r_1^2 \ln r_1 + \frac{1-r_1^2}{2} \right) \right)

\begin{align*}
W &= r_0^2 + 2 \int_{r_0}^{r_1} r P dr + 2 \int_{r_1}^{r_0} r P dr = \frac{1}{2} \left( \frac{S f_S (\beta h, l)}{f_I (\beta h, l)} + \frac{20 S l^2}{3} \right) \left( r_1^2 - r_0^2 \right)^2 - \frac{6Q}{f_I (\beta h, l)} \left( \frac{r_1^2 \ln \frac{r_1}{r_0} - r_1^2 - r_0^2}{2} \right) \left( \frac{r_1^2 - 1}{h^2} \right) + \frac{6Q}{f_I (\beta h, l)} \left( r_1^2 \ln r_1 + \frac{1-r_1^2}{2} \right) \right)
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\end{align*}

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W &= r_0^2 + 2 \int_{r_0}^{r_1} r P dr + 2 \int_{r_0}^{r_1} r P dr = \frac{1}{2} \left( \frac{S f_S (\beta h, l)}{f_I (\beta h, l)} + \frac{20 S l^2}{3} \right) \left( r_1^2 - r_0^2 \right)^2 - \frac{6Q}{f_I (\beta h, l)} \left( \frac{r_1^2 \ln \frac{r_1}{r_0} - r_1^2 - r_0^2}{2} \right) \left( \frac{r_1^2 - 1}{h^2} \right) + \frac{6Q}{f_I (\beta h, l)} \left( r_1^2 \ln r_1 + \frac{1-r_1^2}{2} \right) \right)
\end{align*}

Figure 1. Schematic diagram of thrust bearing.

Results and Discussion

To discuss the combined effects of recess depth, centrifugal inertia and couple stress on the static properties of hydrostatic thrust bearing, the results of pressure, radial flow rate and load capacity are calculated for different recess depths $\beta$, couple stress coefficients $l$ and inertia effect coefficients $S$.

Dimensionless Pressure Distribution

Figure 2 illustrates the dimensionless pressure distribution. It is found that both in and outside the recess, the pressure decreases at first and then increases. With the couple stress increasing, the location of minimum pressure in the recess moves toward to oil supply hole, while outside the recess, the location of minimum pressure moves away from the supply hole. The variation trend of the pressure would reverse at the location of step. And the effect of the couple stress on the rotating bearing is much more notable than that on the static situation.

Figure 2. Variation of dimensionless pressure distribution with different couple stress parameter $l$.

\begin{align*}
S = 5, \beta = 2.18, r_0 = 0.05, r_1 = 0.5.
\end{align*}

Load Capacity

Figure 3 and Figure 4 show the relationship between dimensionless load capacity and step radius. Figure 3 depicts the load capacity at different step radius, for $r_0 = 0.05, \beta = 5$, and different couple stress coefficients: (a) $l = 0$; (b) $l = 0.1h$; (c) $l = 0.3h$; (d) $l = 0.5h$. The results show that, load capacity becomes larger as the increasing of couple stress, and for larger value of $S$, the change is more apparent. At different couple stress and inertia effect, the change of load capacity with step radius is analogous. As the step radius increases, load capacity first becomes larger, arrives at a peak and then decreases. If the radius of the step is 0.465 of the bearing radius, load capacity is
independent of both inertia effect and couple stress, and this conclusion is in accordance with the results of Dowson [1]. Figure 4 shows the comparison of load capacity at $S = 2$ (considering inertia effects) and $S = 0$ (the situation of no inertia). It can be seen that load capacity becomes larger with the increase of recess depth, and also with the increasing of couple stress. For a larger value of $\beta$, the effect of couple stress is becoming less remarkable. It means that the rising of the recess depth would impair the effect of couple stress on the load capacity. To get maximum load capacity, as the recess depth becomes larger, the step radius should be designed larger correspondingly.

![Diagram](image1)

Figure 3. Variation of load capacity as step position $r_1$. $r_0 = 0.05$, $\beta = 5$, (a) $l = 0$; (b) $l = 0.1h$; (c) $l = 0.3h$; (d) $l = 0.5h$.

![Diagram](image2)

Figure 4. Variation of load capacity as step position $r_1$. $l=0$, $0.1h$, $0.2h$, $0.3h$, $r_0=0.05$. (a) $S = 0$, (b) $S = 2$.

**Lubricant Flow Rate**

Figure 5 and Figure 6 describe the effects of fluid inertia and couple stress on the lubricant flow rate at different depth of recess. It is obviously that flow rate becomes larger with the increasing inertia effect but smaller with increasing of couple stress. In Figure 5, the relation between flow rate and inertia parameter is linear, and the rate of change grows up as the recess depth becomes larger. The effect of the recess depth on flow rate is smaller for a larger recess depth, and when $\beta > 4$, the effect is negligible. In Figure 6, with larger recess depth, the lubricant flow rate is larger. And the effect is also not obviously when $\beta > 4$. As couple stress becomes larger, the effect of recess depth on the lubricant flow rate becomes smaller. It means that the increasing of couple stress would impair the effect of recess depth on the lubricant flow rate.
Figure 5. Variation of flow rate as inertia parameter $S$, $\beta = 1$, 2, 4, $\infty$, $l = 0$, $r_0 = 0.05$, $r_1 = 0.5$.

Figure 6. Variation of flow rate as couple stress parameter $l/h$, $\beta = 1$, 2, 4, $\infty$, $S = 5$, $r_0 = 0.05$, $r_1 = 0.5$.

Conclusions
In this paper, the combined effects of recess depth, fluid inertia and couple stress on static properties (pressure, flow rate and load capacity) of hydrostatic thrust bearing are studied. Based on the discussion above, following conclusions could be obtained:

1. Couple stress and inertia has notable effects on the static properties of hydrostatic thrust bearing.
2. The recess depth also affects the properties of hydrostatic thrust bearing obviously.
3. For a specific recess structure, a constant load capacity is obtained. It means that for this structure, both the inertia and couple stress have no effect on the loading capacity.

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