Abstract

We examine the $B \to D^*$ form factor at zero recoil using a continuum QCD approach rooted in the heavy quark sum rules framework. A refined evaluation of the radiative corrections as well as the most recent estimates of higher order power terms together with more careful continuum calculation are included. An upper bound on the form factor of $F(1) \lesssim 0.93$ is derived, based on just the positivity of inelastic contributions. A model-independent estimate of the inelastic contributions shows they are quite significant, lowering the form factor by about 6% or more. This results in an unbiased estimate $F(1) \approx 0.86$ with about three percent uncertainty in the central value.

* On leave of absence from Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188300, Russia
1 Introduction

The determination of the CKM matrix element $V_{cb}$ from exclusive decays has to rely on calculations of the relevant form factors, which are usually defined as

$$ \frac{d\Gamma}{d\omega}(B \to D^* \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 M_{D^*}^3 (\omega^2 - 1)^{3/2} P(\omega)(\mathcal{F}(\omega))^2 $$

$$ \frac{d\Gamma}{d\omega}(B \to D \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 (M_B + M_D)^2 M_D^3 (\omega^2 - 1)^{3/2} (\mathcal{G}(\omega))^2 $$

with $\omega = v \cdot v' = E_{D^*}/M_{D^*}$ (in the $B$ rest frame), and where $P(\omega)$ is a known phase space factor. Based on the normalization of the form factors at $\omega = 1$, $V_{cb}$ is extracted from an extrapolation of the data to the non-recoil point.

In the heavy quark limit, the normalization of the form factors $\mathcal{F}(1) = \mathcal{G}(1) = 1$ is given by heavy quark symmetry, and the main issue in the $V_{cb}$ determination becomes a reliable calculation of the deviation from the heavy quark limit. The published extractions of $V_{cb}$ along this route rely solely on the lattice calculations currently cited as $[1]$

$$ \mathcal{F}(1) = 0.921 \pm 0.024 $$

$$ \mathcal{G}(1) = 1.074 \pm 0.018 \pm 0.016 $$

However, based on the dynamic heavy quark expansion in Minkowski space, it has been argued that larger deviations from the symmetry limit for $\mathcal{F}(1)$ are natural in continuum QCD $[2]$; these have been further supported by the arguments $[3]$ exploiting the relatively small kinetic expectation value $\mu_\pi^2$ extracted from the fits to inclusive $B$ decays. The same line of reasoning led to a rather precise estimate of $\mathcal{G}(1)$ in $B \to D$ $[4]$, showing significantly smaller deviations from unity compared to Eq. (1.2).

Recent years were of primary importance for heavy flavor physics. Along with advances in theory, many nontrivial nonperturbative predictions were verified with high precision, and heavy quark parameters experimentally extracted in accord with prior theoretical expectations; certain predictions were indirectly confirmed in dedicated lattice calculations. All this raised the credibility of OPE-based methods and favored an early onset of the short-distance expansion instrumental for high-precision predictions; confidence rose in the assumptions underlying dynamic treatment of the nonperturbative physics in heavy quarks. This progress warrants a critical re-examination of the form factors. The goal is to incorporate the accumulated knowledge and to shift the focus from merely establishing the scale of the deviations from the heavy quark symmetry limit towards obtaining a refined estimate with a motivated error assessment.

In the present note we discuss the $B \to D^*$ transition at zero recoil using a dynamic QCD approach inspired by the original treatment of the zero-recoil sum rules for heavy flavor transitions. The details of the analysis will be presented in the extended publication $[5]$.

2 Zero Recoil Sum Rule in QCD

We consider the zero-recoil ($q=0$) forward scattering amplitude $T^{mi}(\varepsilon)$ of the flavor-changing axial current $\bar{c} \gamma^i \gamma_5 b$ off a $B$ meson at rest:

$$ T^{mi}(\varepsilon) = \int d^3x \int dx_0 \ e^{-i\varepsilon(M_B - M_{D^*} - \varepsilon)} \frac{1}{2M_B} \langle B|\frac{1}{3} i T \bar{c} \gamma^i \gamma_5 b(x) \bar{b} \gamma_5 c(0)|B\rangle, $$

(2.1)
Figure 1: The analytic structure of $T^{\pi}(\varepsilon)$ and the integration contour yielding the sum rule. Distant cuts are shown along with the physical cut. The radius of the circle is $\varepsilon_M$.

where $\varepsilon$ is the excitation energy above $M_{D^*}$ in the $B \to X_c$ transition (the point $\varepsilon = 0$ corresponds to the elastic $B \to D^*$ transition). The amplitude $T^{\pi}(\varepsilon)$ is an analytic function of $\varepsilon$ and has a physical decay cut at $\varepsilon \geq 0$, and other distant singularities. The analytic structure of $T^{\pi}(\varepsilon)$ is shown in Fig. 1.

The contour integral

$$I_0(\varepsilon_M) = -\frac{1}{2\pi i} \oint_{|\varepsilon| = \varepsilon_M} T^{\pi}(\varepsilon) \, d\varepsilon$$

(2.2)

with the contour running counterclockwise from the upper side of the cut, see Fig. 1, leads to the sum rule involving $\mathcal{F}^2(1)$. Using the analytic properties of $T^{\pi}(\varepsilon)$ the integration contour can be shrunk onto the decay cut; the discontinuity there is related to the weak transition amplitude squared of the axial current into the final charm state with mass $M_X = M_{D^*} + \varepsilon$.

Separating out explicitly the elastic transition contribution $B \to D^*$ at $\varepsilon = 0$ we have

$$I_0(\varepsilon_M) = \mathcal{F}^2(1) + w_{\text{inel}}(\varepsilon_M), \quad w_{\text{inel}}(\varepsilon_M) = \frac{1}{2\pi i} \int_{\varepsilon > 0}^{\varepsilon_M} \text{disc} T^{\pi}(\varepsilon) \, d\varepsilon,$$

(2.3)

where $w_{\text{inel}}(\varepsilon_M)$ is related to the sum of the differential decay probabilities into the excited states with mass up to $M_{D^*} + \varepsilon_M$ in the zero recoil kinematics.

The OPE allows us to calculate the amplitude in (2.1) – and hence $I_0(\varepsilon_M)$ – in the short-distance expansion provided $|\varepsilon|$ is sufficiently large compared to the ordinary hadronic mass scale. It should be noted that strong interaction corrections are driven not only by $|\varepsilon|$, but also by the proximity to distant singularities. Therefore, $\varepsilon_M$ cannot be taken too large either, and the hierarchy $\varepsilon_M \ll 2m_c$ has to be observed.

The sum rule Eq. (2.3) can be cast in the form

$$\mathcal{F}(1) = \sqrt{I_0(\varepsilon_M) - w_{\text{inel}}(\varepsilon_M)}$$

(2.4)

which is the master identity for the considerations to follow. Since $w_{\text{inel}}(\varepsilon_M)$ is strictly positive, we get an upper bound on the form factor

$$\mathcal{F}(1) \leq \sqrt{I_0(\varepsilon_M)}$$

(2.5)

which relies only on the OPE calculation of $I_0$. Note that this bound depends on the parameter $\varepsilon_M$, while (2.4) is independent of $\varepsilon_M$ since the dependence in $I_0$ and $w_{\text{inel}}$ cancel. Furthermore, including an estimate of $w_{\text{inel}}(\varepsilon_M)$ we obtain an evaluation of $\mathcal{F}(1)$.

The correlator in (2.1) can be computed using the OPE, resulting in an expansion of $T^{\pi}(\varepsilon)$ in inverse powers of the masses $m_c$ and $m_b$. This results in the corresponding expansion of
The perturbative renormalization $\xi^\text{pert}_A(\varepsilon_M, \mu)$ can be expanded in power series in $\alpha_s$. We use the Wilsonian OPE and benefit from well-behaved perturbative series for $\xi^\text{pert}_A(\mu)$. The exact form of the perturbative coefficients depends on the definition chosen for higher-dimension operators; we adopt the often used kinetic scheme [6].

In one-loop perturbative calculations there is a simple connection between the normalization point of the heavy quark operators in the kinetic scheme and the hard cut-off on the gluon momentum in the diagram. This allows to obtain the analytic expression for $\xi^\text{pert}_A(\mu)$ to this order even without explicit calculation of the Wilson coefficients $C_k$ in Eq. (2.6). The expression is rather lengthy and will be presented in Ref. [5]. By the same trick one also obtains all higher-order BLM corrections by performing the one-loop calculations with massive gluon [7].

A similar argument does not apply to non-BLM corrections starting $\alpha_s^2$ where $\varepsilon_M$-dependence of $\xi^\text{pert}_A$ has to be determined expanding in $1/m_Q$; for $O(\alpha_s^2)$ corrections this was done in Ref. [8] through order $1/m_Q^2$. The corresponding coefficient was found to be small numerically, which suggests that omitted terms $\propto \alpha_s^2 \varepsilon_M^3/m_Q^3$ and higher should not produce a significant change.

Perturbative corrections to $\xi^\text{pert}_A(\varepsilon_M)$ appear to be small for practical values of $\varepsilon_M$ between 0.6 GeV and 1 GeV. Taking, for instance, $\varepsilon_M = 0.75$ GeV, $m_c = 1.2$ GeV, $m_b = 4.6$ GeV, $\alpha_s(m_b) = 0.22$ we get the numeric estimates at different orders

$$\sqrt{\xi^\text{pert}_A} = 1 - 0.019 + (0.007 \,-\, 0.004) + 0.0045 + ...$$

(2.7)

Here the first term is the tree value, second is $O(\alpha_s)$ evaluated with $\alpha_s = 0.3$, the next pair of values show the shift upon passing to the $O(\alpha_s^2)$ order (positive for the BLM part and negative from the non-BLM contribution); the last term shows $\beta_0^2 \alpha_s^3$ term as an estimate of even higher-order perturbative corrections. Fig. [2] shows the dependence on $\varepsilon_M$ of these predictions for $\sqrt{\xi^\text{pert}_A}$. In particular, taking the full two-loop result as the central estimate we find

$$\sqrt{\xi^\text{pert}_A(0.75 \text{ GeV})} = 0.985 \pm 0.01;$$

(2.8)

we will use $\varepsilon_M = 0.75$ GeV in the subsequent discussion. We emphasize that the numeric stability applies only to the perturbative renormalization factor in the Wilsonian OPE, and the quoted values refer to the specific renormalization scheme (kinetic) adopted in the analysis.
Figure 2: Left: $\sqrt{\xi_A^{\text{pert}}}$ to order $\alpha_s$ (blue), including $\beta_0\alpha_s^2$ (green), full $\mathcal{O}(\alpha_s^2)$ (red) and including $\beta_0^2\alpha_s^3$ (magenta), assuming $\alpha_s^{\overline{\text{MS}}}(m_b) = 0.22$, $m_c = 1.2$ GeV and $m_b = 4.6$ GeV.

Right: Upper bound (2.5) on $F(1)$ depending on $\varepsilon_M$, with or without $\beta_0^2\alpha_s^3$ term. A fixed $\Delta = 0.11$ is used; assuming the perturbative evolution of $\Delta$ with $\varepsilon_M$ would flatten the dependence.

### 2.2 Power corrections

The leading power corrections to $I_0$ were calculated in Refs. [9, 10] to order $1/m_Q^2$ and to order $1/m_Q^3$ in Ref. [11] and read

$$
\Delta_{1/m^2} = \frac{\mu_G^2}{3m_c^2} + \frac{\mu_G^2 - \mu_\pi^2}{4} \left( \frac{1}{m_c^2} + \frac{2}{3m_c m_b} + \frac{1}{m_b^2} \right),
$$

$$
\Delta_{1/m^3} = \frac{\rho_D^3 - \frac{7}{3}\rho_{LS}^3}{4m_c^3} + \frac{1}{12m_b} \left( \frac{1}{m_c^2} + \frac{1}{m_c m_b} + \frac{3}{m_b^2} \right) \left( \rho_D^3 + \rho_{LS}^3 \right). \tag{2.9}
$$

The nonperturbative parameters $\mu_\pi^2$, $\mu_G^2$, $\rho_D^3$ and $\rho_{LS}^3$ all depend on the hard Wilsonian cutoff. The $\varepsilon_M$-dependence of $\xi_A^{\text{pert}}$ is linked to the power-like scale dependence of the nonperturbative matrix elements through their mixing with lower-dimension operators.

In the numerics we use the values $\mu_\pi^2(0.75 \text{ GeV}) = 0.4 \text{ GeV}^2$, $\rho_D^3(0.75 \text{ GeV}) = 0.15 \text{ GeV}^3$, while for the quark masses $m_c = 1.2$ GeV and $m_b = 4.6$ GeV (the scale dependence of the latter plays a role here only at the level formally beyond the accuracy of the calculation). The dependence on $\mu_G^2$ and on $\rho_{LS}^3$ is minimal and their precise values do not matter; we use for them $0.3 \text{ GeV}^2$ and $-0.12 \text{ GeV}^3$, respectively. We then get

$$
\Delta_{1/m^2} = 0.091, \quad \Delta_{1/m^3} = 0.028; \tag{2.10}
$$

If we employ the values of the OPE parameters extracted from a fit to inclusive semileptonic and radiative decay distributions [12, 13], we find a consistent result

$$
\Delta_{1/m^2} + \Delta_{1/m^3} = 0.11 \pm 0.03. \tag{2.11}
$$

An important question is how well the power expansion for the sum rule converges. Recently, the OPE for the semileptonic $B$-meson structure functions has been extended to order $1/m_Q^4$ and $1/m_Q^5$ [14, 15]. Combined with the estimates [16] of the corresponding expectation values discussed in Ref. [15], this leads to

$$
\Delta_{1/m^4} \simeq -0.023, \quad \Delta_{1/m^5} \simeq -0.013. \tag{2.12}
$$

We then observe that the power series for $I_0$ appears well-behaved at the required level of precision. For what concerns the loop corrections to $\Delta$, the $\mathcal{O}(\alpha_s)$ correction to the Wilson
coefficient for the kinetic operator in Eq. (2.9) was calculated in Ref. [8] and turned out numerically insignificant. At $O(1/m_Q^3)$, even if radiative corrections change the coefficient for the Darwin term by 30% the effect on the sum rule would still be small.

Taking into account all the available information, our estimate for the total power correction at $\epsilon_M = 0.75$ GeV is

$$\Delta = 0.105$$ (2.13)

with a 0.015 uncertainty due to higher orders. On theoretical grounds, larger values of $\mu^2_\pi$ and/or $\rho_3^B$ are actually favored; they tend to increase $\Delta$. Combining the above with the perturbative corrections we arrive at an estimate for $I_0$ and, according to Eq. (2.5) at a bound for the form factor, which in terms of central values at $\epsilon_M = 0.75$ GeV is

$$F(1) < 0.93.$$ (2.14)

As stated above, the upper bound in Eq. (2.5) depends on $\epsilon_M$, see Fig. 2, becoming stronger for smaller $\epsilon_M$. It is advantageous to choose the minimal value of $\epsilon_M$ for which the OPE-based short-distance expansion of the integral (2.2) for $I_0(\epsilon_M)$ sets in. This directly depends on how low one can push the renormalization scale $\mu$ while still observing the expectation values actual $\mu$-dependence in the kinetic scheme approximated by the perturbative one. Since in this scheme $\mu^2_\pi(\mu) \geq \mu^2_G(\mu)$ holds for arbitrary $\mu$, in essence this boils down to the question at which scale $\mu_{\text{min}}$ the spin sum rule and the one for $\mu^2_G(\mu_{\text{min}}) \simeq 0.3$ GeV$^2$. The only vital assumption in the analysis is that the onset of the short-distance regime is not unexpectedly delayed in actual QCD and hence does not require $\epsilon_M > 1$ GeV. This principal question can and should be verified on the lattice. This will complement already available evidence from preliminary lattice data [17] as well as from the successful experimental confirmation [18] in nonleptonic $B$ decays of the predicted 3/2-dominance.

### 2.3 Estimate of the inelastic contribution $w_{\text{inel}}$

On general grounds $w_{\text{inel}}$ is expected to be comparable to the power correction $\Delta$ considered above. To actually estimate it we consider another contour integral

$$I_1(\epsilon_M) = -\frac{1}{2\pi i} \oint \frac{T^e(\epsilon)}{\epsilon - \epsilon_M} d\epsilon$$ (2.15)

for which we can write

$$w_{\text{inel}}(\epsilon_M) = \frac{I_1(\epsilon_M)}{\bar{\epsilon}}$$ (2.16)

where $\bar{\epsilon}$ is the average excitation energy (it depends on $\epsilon_M$). The integral is expected to be dominated by the lowest radial excitations of the ground state, with $\bar{\epsilon} \approx \epsilon_{\text{rad}} \approx 700$ MeV.

$I_1(\epsilon_M)$ can also be calculated in the OPE [10]; the result including $1/m_Q^2$ terms reads

$$I_1 = \frac{-(\rho^2_{\pi G} + \rho^2_A)}{3m_c^2} + \frac{-2\rho^2_{\pi \pi} - \rho^2_{\pi G}}{3m_c m_b} + \frac{\rho^2_{\pi \pi} + \rho^2_{\pi G} + \rho^2_S + \rho^2_A}{4} \left( \frac{1}{m_c^2} + \frac{2}{3m_c m_b} + \frac{1}{m_b^2} \right)$$ (2.17)

where the non-local zero momentum transfer correlators $\rho^2_{\pi \pi}$, $\rho^2_{\pi G}$, $\rho^2_S$ and $\rho^2_A$ are defined in [10]. They can be estimated along the lines described in [15], based on saturation by the appropriate intermediate states. We shall defer the details of this estimate to [3]. Here we note that only the first term survives in the BPS limit [4], where it is positive; the second and third terms are
of first and second order in the deviation from the BPS limit, respectively. The last term is positive being the correlator of two identical operators $\bar{b}(\bar{\sigma}\pi)^2b$. Since the middle term comes with a small coefficient $1/3mc_m$, the expression has only a shallow minimum where the first term is decreased by less than 10%; the sum of the last two terms becomes larger than that only if it is positive. On the other hand, the combination

$$- (\rho^{3}_{G} + \rho^{3}_{L}) + \rho^{3}_{LS}$$

(2.18)

determines the hyperfine splitting to order $1/m_Q^2$ and can be constrained from the observed masses of $B(\ast)$ and $D(\ast)$ mesons. We finally obtain

$$I_1(\epsilon_M) \gtrsim \frac{0.48 \text{ GeV}^3}{3m_c^2}$$

(2.19)

with some uncertainty from perturbative corrections, implying for $\tilde{\epsilon} = \epsilon_{\text{rad}} \approx 700$ MeV

$$w_{\text{inel}} \approx \frac{I_1}{\epsilon_{\text{rad}}} \gtrsim 0.13.$$  

(2.20)

This estimate is derived at leading order in $1/m_Q$ and may be corrected by higher-order terms by as much as 30%. We observe that $w_{\text{inel}}$ is similar in size to the power term in $I_0$ and even exceeds it. Using (2.20) at face value we arrive at our estimate for the expected value of the form factor

$$F(1) \lesssim 0.86.$$  

(2.21)

The quasi-resonant states are expected to dominate $w_{\text{inel}}(\mu)$ at intermediate $\mu \approx 1$ GeV; the continuum contribution to $w_{\text{inel}}(\mu)$ is parametrically $1/N_c$-suppressed and usually smaller. The $D(\ast)\pi$ continuum can independently be evaluated in the soft-pion approximation. It turns out that numerically the dominant effect originates from the heavy quark symmetry breaking difference between the $B^\ast B\pi$, $D^\ast D\pi$ and $D^\ast D^\ast\pi$-couplings which until recently has not been accounted for in this context\footnote{The $D^\ast D^\ast\pi$ channel has not been previously considered while generally required by the heavy quark symmetry.}, although is expected to be significant [19]. The result is shown in Fig. 3; it depends on the upper cutoff in the pion momentum $p_{\pi}^{\text{max}}$ marking the end of the soft continuum domain for $D(\ast)\pi$, presumably somewhat below $\epsilon_{\text{rad}}$. We expect about 5% combined yield for $\Gamma_{D^\ast+} = 96$ keV, i.e. about a third of the overall $w_{\text{inel}}$ in Eq. (2.20) in accord with the $1/N_c$ arguments. This contribution alone would lower the upper bound in Eq. (2.14) by 0.025.

### 3 Conclusions

The direct OPE-based $1/m_Q$ expansion of the zero recoil form factor which we have analyzed through the sum rule for the correlator of the zero-recoil axial currents yields the unbiased estimate $F(1) \approx 0.86$, and suggests a lower bound $F(1) < 0.93$. All the values refer to pure QCD form factors and exclude possible electromagnetic effects, in particular the universal short-distance semileptonic enhancement factor of 1.007.

Since a charm mass expansion is involved, the precision of our estimates is limited. Accuracy-wise two aspects should be kept distinct. On the one hand, the heavy quark parameters enter our analysis, and in particular $\mu_\pi^2$, $\rho_0^2$ and $m_c$. Their determination from inclusive measurements will soon be improved thanks to refined theoretical calculations. On the other hand,
Figure 3: Non-resonant $D\pi$ and $D^*\pi$ contribution to $w_{\text{inel}}$ at $g_{D^*D\pi} = 4.9\text{ GeV}^{-1}$ corresponding to $\Gamma_{D^*} = 96\text{ keV}$. The plots show, from bottom to top, $g_{B^*B\pi} = 1, 0.8, 0.6$ and 0.4, and $g_{D^*D^*\pi} = 1, 0.8$ and 0.6, respectively.

even if these QCD parameters were accurately known, sharpening the upper bound and the estimate of $F(1)$ would require additional nonperturbative input. To some extent the uncertainties may be reduced by dedicated measurements in semileptonic decays or by direct lattice calculation of the relevant heavy quark parameters. In our opinion, the latter can be done in a straightforward way, which will be discussed elsewhere.

In our numerical estimates we used the lowest values of $\mu_\pi^2$ and $\rho_D^3$ consistent with the theoretical bounds; larger values would typically further lower both the upper bound and the central expectation for $F(1)$. We estimate the perturbative and non-perturbative uncertainties in our approach to be about $\pm 0.01_{\text{pt}}$ and $\pm 0.02_{\text{np}}$ ($\pm 0.01_{\text{np}}$ for the upper bound), mostly related to the unavoidable truncation of the $1/m_c$ expansions. The uncertainty in the prediction is dominated by $w_{\text{inel}}$, is not symmetric, and allows for larger deviations towards lower values of $F(1)$.

There is an alternative way of determining $|V_{cb}|$, using inclusive semileptonic decays. It rests on the heavy quark expansion in $1/m_b$ and on the OPE which, in this case, should be considered a mature tool. Recent estimates of higher orders in $1/m_Q$ as well as in $\alpha_s$ did not show signs of failure of the heavy quark expansion in this application. Hence one can employ the value of $V_{cb} = (41.54 \pm 0.44 \pm 0.58) \times 10^{-3}$ from the inclusive decays \[12\] and determine the form factors from the data on $B \to D^{(*)}\ell\nu$ decays; this yields

\begin{align}
F(1) &= 0.86 \pm 0.02 \quad (3.1) \\
G(1) &= 1.02 \pm 0.04 \quad (3.2)
\end{align}

The value for $G(1)$ is in a perfect agreement with the HQE prediction \[4\]; $F(1)$ hits the central value of our estimate. Both form factors in Eq. (3.1) are noticeably below the quoted lattice values Eqs. (1.2); this explains the observed tension between the inclusive and exclusive determinations of $|V_{cb}|$.

Our analysis suggests that the lattice determinations of both form factors are systematically high. The central values of the currently quoted lattice $F(1)$ is very close to our upper bound. Lattice calculations – provided they accommodate the experimentally measured $B$-meson expectation values – seem to imply an extremely small inelastic contribution, which is a priori unnatural. Moreover, it appears in contradiction with the large non-local correlators encountered to order $1/m_Q^2$. The estimate of the non-resonant soft-pion $D^{(*)}\pi$ rates also confirms this assessment. We conclude that values for $F(1)$ in excess of 0.9 would be consistent with unitarity and the short-distance expansion of QCD only with rather contrived assumptions. Values of $F(1)$ larger than 0.93 should be viewed as violation of unitarity assuming that usual short-distance expansion in QCD works.
With the unbiased estimate based on the lower-end $\mu_2^2$ and $\rho_D^3$ yielding $\mathcal{F}(1) \approx 0.86$ – or even smaller for larger $\mu_2^2, \rho_D^3$ – we find a surprisingly good agreement (probably, somewhat accidental) between the exclusive and inclusive approaches. In view of the inherent $1/m_c$ expansion for $\mathcal{F}(1)$ and of the approximations used for proliferating hadronic expectation values, matching the theoretical precision already attained for $V_{cb}$ from the inclusive fits does not look probable. Nevertheless, additional experimental and/or lattice input would make the suggested 3% uncertainty interval more robust.

We estimate that the contribution of excited radial states with mass below 3 GeV constitutes about 10% or more of the total yield. This refers only to the zero-recoil kinematics and only to the axial current; it does not include $P$-wave states. This fraction may even be larger when applied to the full phase space. This suggests that the observed ‘broad state’ yield in this mass range routinely attributed to the $\frac{1}{2}$ $P$-wave excitations is actually dominated by states with different quantum numbers, thus resolving the $\left\{ \frac{1}{2} \right\} \sim \frac{3}{2}$ puzzle’. The suppression of the broad $P$-wave yield was predicted based on the spin sum rules and confirmed indirectly in nonleptonic $B$ decays [18]; a recent discussion can be found in Ref. [20].

Acknowledgments

We gratefully acknowledge discussions with Ikaros Bigi and invaluable help from Sascha Turczyk regarding higher-order power corrections. We thank the Galileo Galilei Institute for Theoretical Physics for the hospitality and the INFN for partial support during the completion of this work. The study enjoyed a partial support from the NSF grant PHY-0807959 and from the RSGSS-65751.2010.2 grant. PG is supported in part by a EU’s Marie-Curie Research Training Network under contract MRTN-CT-2006-035505 (HEPTOOLS). TM is partially supported by the German research foundation DFG under contract MA1187/10-1 and by the German Ministry of Research (BMBF), contracts 05H09PSF.

References

[1] C. Bernard et al., Phys. Rev. D 79, 014506 (2009); M. Okamoto et al., Nucl. Phys. Proc. Suppl. 140, 461 (2005); G. M. de Divitiis, R. Petronzio and N. Tantalo, Nucl. Phys. B 807 (2009) 373 [arXiv:0807.2944 [hep-lat]].

[2] See, e.g., N. Uraltsev, arXiv:hep-ph/0010328. Published in the Boris Ioffe Festschrift ‘At the Frontier of Particle Physics / Handbook of QCD’, eds. by M. Shifman (World Scientific, Singapore, 2001), vol. 3 p. 1577, and references therein.

[3] N. Uraltsev, Phys. Lett. B 545 (2002) 337.

[4] N. Uraltsev, Phys. Lett. B 585 (2004) 253.

[5] P. Gambino, T. Mannel and N. Uraltsev, paper in preparation.

[6] I.I. Bigi, M. Shifman, N. Uraltsev and A. Vainshtein, Phys. Rev. D 56 (1997) 4017; A. Czarnecki, K. Melnikov and N. Uraltsev, Phys. Rev. Lett. 80 (1998) 3189.

[7] D. Benson, I.I. Bigi, T. Mannel and N. Uraltsev, Nucl. Phys. B 665 (2003) 367.

[8] A. Czarnecki, K. Melnikov and N. Uraltsev, Phys. Rev. D 57 (1998) 1769.
[9] M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. D 51 (1995) 2217.
[10] I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. D 52 (1995) 196.
[11] I.I. Bigi, M. Shifman and N. Uraltsev, Ann. Rev. Nucl. Part. Sci. 47 (1997) 591.
[12] Heavy Flavour Averaging Group, update for the Winter 2009 conferences, see http://www.slac.stanford.edu/.
[13] C. Schwanda, private communication, August 2009.
[14] I. Bigi, T. Mannel, S. Turczyk and N. Uraltsev, arXiv:0911.3322; to appear in JHEP.
[15] T. Mannel et al., paper in progress.
[16] I.I. Bigi, N. Uraltsev and R. Zwicky, Eur. Phys. J. C 50 (2007) 539.
[17] D. Becirevic et al., Phys. Lett. B 609 (2005) 298.
[18] A. Kuzmin et al. [Belle Collaboration], Phys. Rev. D 76 (2007) 012006; Nucl. Phys. Proc. Suppl. 162 (2006) 228.
[19] V.M. Belyaev, V.M. Braun, A. Khodjamirian and R. Ruckl, Phys. Rev. D 51 (1995) 6177.
[20] I.I. Bigi et al., Eur. Phys. J. C 52 (2007) 975.