THE GDH SUM RULE FOR THE $\Delta$ ISOBAR:
A POSSIBLE ANOMALY?

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The GDH sum rule is discussed for the $\Delta(1232)$ resonance. It is shown that apart
from ordinary excitations to higher-energy states, the sum rule contains a large
negative contribution due to de-excitation into the nucleon state. Therefore, a
fulfillment of the sum rule assumes a strong coupling of $\Delta^+$ and $\Delta^0$ to resonances
of spin $\geq \frac{5}{2}$. Calculations performed in quark models suggest that $D_{15}(1675)$ may
be such a resonance. However, its strength is found to be not sufficient for bringing
the GDH sum rule to a theoretically expected positive magnitude.

1 Introduction

The Gerasimov–Drell–Hearn sum rule puts a nontrivial constraint on a spin
dependence of the total photoabsorption cross section by hadrons. For a target
$B$ of spin $s$, the electric charge $eZ$ ($e^2 = 1/137$), and the mass $m$ it states:

$$I_{GDH}^\lambda \equiv \int_{\text{thr}}^{\infty} \left( \sigma_{1+s}(\omega) - \sigma_{1-s}(\omega) \right) \frac{d\omega}{\omega} = 4\pi^2 s \left( \frac{\mu}{s} - \frac{eZ}{m} \right)^2,$$

where $\mu$ is the magnetic moment of $B$ and $\mu_e - eZ_s/m$ is the anomalous
magnetic moment (a.m.m.). The cross sections $\sigma_\lambda(\omega)$ refer to absorption of a
circularly polarized photon by the target with spin $s$ parallel or antiparallel
to the photon helicity, so that $\lambda = 1 \pm s$ is the net helicity.

The only strong assumption needed for the validity of the GDH relation
(1), viz. that spin dependence of the Compton forward scattering amplitude
vanishes at high energies, seems to agree with QCD. Recent measurements
of $\sigma_\lambda(\omega)$ off the proton at MAMI confirm that the GDH integral is indeed
equal to its theoretical value given by the a.m.m. of the proton.

We concentrate here on a contribution to the GDH integral from baryon
resonances $B^*$. In the zero-width approximation and in terms of standard
photocouplings,

$$A_{\lambda} = \sqrt{\frac{\pi}{\omega^*}} \langle B^*(k, \lambda) | J_+ | B(-\frac{1}{2}k, \lambda - 1) \rangle, \quad \omega^* = \frac{|m^* - m^2|}{2m^*}$$

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\( J_\pm = J_x \pm iJ_y \) is the electromagnetic current), this contribution reads

\[
I^\text{GDH}_{\text{res}} = \sum_{B^* \neq B} \frac{2\pi}{\omega} \left( |A_{1+s}|^2 - |A_{1-s}|^2 \right), \quad \omega = \frac{|m^2 - m|^2}{2m}.
\] (3)

Both the experimental data \( ^1 \) and theoretical arguments of quark models \( ^2, ^3 \) and large-\( N_c \) QCD \( ^4 \) suggest that the GDH integral for the nucleon is dominated by energies in the \( \Delta \)-resonance region. The magnetic \( \gamma N \Delta \) transition and \( s \)-wave pion photoproduction dominate the integral, whereas a contribution of resonances of \( N \geq 1 \) oscillator bands is relatively small. For other targets situation is, however, different, and a fulfillment of the GDH sum rule requires \( N = 1 \) resonance contributions, as it will be explained below.

2 M1-part of the GDH integral in NQM

It is known that magnetic spin-transitions like \( \gamma N \Delta \) are generally insufficient for saturation of the GDH sum rule. For example, in the case of a weakly-bound system of a few nonrelativistic \( s \)-wave quarks with the operator of the total magnetic moment \( \mathbf{M} = \sum_q (e_q \sigma_q / 2m_q) \), the (unretarded) magnetic M1-contribution to Eq. (3) takes the closure form

\[
I^\text{GDH}_{\text{M1}} = 2\pi \sum_{B^* \neq B} \left\{ (|B^*(s_z+1)|M_+|B(s_z)|)^2 - (|B^*(s_z-1)|M_-|B(s_z)|)^2 \right\}
= 2\pi^2 \langle B(s_z)|[M_-, M_+]|B(s_z)\rangle - 2\pi^2 \langle B(s_z)|[\mu_-, \mu_+]|B(s_z)\rangle.
\] (4)

Here the magnetic moment \( \mu \) of the state \( B \) appears as

\[
\langle B(s'_z)|\mu|B(s_z)\rangle = \langle B(s'_z)|\mathbf{M}|B(s_z)\rangle, \quad \frac{\mu}{s} = \sum_q \frac{e_q}{m_q} P_q,
\] (5)

where quantities \( P_q = \langle s_q \rangle / s_z \), \( \sum_q P_q = 1 \), are fractions of the total spin \( s_z = s \) carried by different quarks \( q \). In this notation Eq. (4) becomes

\[
I^\text{GDH}_{\text{M1}} = 4\pi^2 s \left[ \left( \frac{\mu}{s} \right)^2 - \sum_q \left( \frac{e_q}{m_q} \right)^2 P_q \right].
\] (6)

Generally, \( I^\text{GDH}_{\text{M1}} \) is not equal to the r.h.s. of Eq. (4). For SU(3)-octet states like the proton \( (p = uud) \), for which \( P_1 = P_2 = \frac{4}{3}, P_3 = -\frac{1}{3} \) and \( e_1 = e_2 \), Eqs. (4) and (6) do coincide, provided all masses of quarks are the same and \( m = 3m_q \). These states include also the neutron and the strange baryons \( \Sigma^\pm, \Xi^0, \Xi^- \). However, in the case of \( \Sigma^0 \) and \( \Lambda \) we have \( I^\text{GDH}_{\text{M1}} < I^\text{GDH} \). For SU(3)-decuplet states all \( P_q = \frac{1}{3} \), so that the a.m.m. \( \mu - e Z s / m = 0 \) (in the

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limit of \(m = 3m_q\) and therefore \(I_{GDH}^0 = 0\). Meanwhile \(I_{GDH}^{M1} < 0\) whenever there are quarks of different electric charges in the baryon \(B\). This is the case for those members of the decuplet which contain exactly one or two \(u\)-quarks and thus have the electric charge 0 or +1 (e.g., \(\Delta^+\) and \(\Delta^0\)).

3 M1-photon scattering in NQM

Since the a.m.m. of \(\Delta\) is zero (at least in the NQM), a naive understanding of the GDH integral \(I_{GDH}\) as an integral over positive \(\omega\) may suggest that (spin-dependent part of) the photoabsorption cross section off \(\Delta\) is small. This, however, is not true since we have seen that the GDH M1-contribution \(I_{GDH}^{M1}\) for \(\Delta^+\) or \(\Delta^0\) is not zero and as large as \(-300 \mu b\). This piece of \(I_{GDH}\) emerges owing to \(\Delta \rightarrow N\) transition which is de-excitation and which corresponds to a negative-energy part of the GDH integral. In order to have \(I_{GDH} = 0\), a big positive contribution must also exist which can only come from resonances of spin \(\geq \frac{5}{2}\). Therefore, the GDH sum rule implies that both \(\Delta^+\) and \(\Delta^0\) must have strong electromagnetic transitions into resonances of higher spins.

In view of importance of such a conclusion, we want to argue more that the negative de-excitation contribution is indeed a part of the GDH integral. Let us write the forward scattering amplitude of a photon with the helicity +1 in the above-considered nonrelativistic quark model. Neglecting the retardation and recoil and keeping only M1 terms in the electromagnetic current, we have

\[
T_{M1}(\omega) = \frac{\omega^2}{2} \sum_n \left( \frac{\langle B|M_-|n\rangle \langle n|M_+|B\rangle}{E_n - E_B - \omega - i\epsilon} + \frac{\langle B|M_+|n\rangle \langle n|M_-|B\rangle}{E_n - E_B + \omega - i\epsilon} \right),
\]

where the sum is taken over all possible intermediate states \(n\). At low energies \(\omega\), the amplitude \(T_{M1}(\omega)\) is dominated by intermediate states \(|n\rangle\) of the energy \(E_n = E_B\). These states are just \(|B\rangle\) with perhaps different spin projections. Using closure, we find that

\[
\omega^{-1}T_{M1}(\omega) \rightarrow -\frac{1}{2} \langle B|[\mu_-, \mu_+]|B\rangle = s \left(\frac{\mu}{s}\right)^2 \text{ when } \omega \rightarrow 0.
\]

In the opposite limit of high energies we neglect \(E_n - E_B\) and, using the closure, write

\[
\omega^{-1}T_{M1}(\omega) \rightarrow -\frac{1}{2} \langle B|[M_-, M_+]|B\rangle = s \sum_q \left(\frac{e_q}{m_q}\right)^2 P_q \text{ when } \omega \rightarrow \infty.
\]

The conclusion is that the full GDH integral \(I_{GDH}\), with both excited and de-excited intermediate states included, is what determines a variation of the quantity \(\omega^{-1}T_{M1}(\omega)\) between low and high energies.
In the real (relativistic) world, the spin-dependent part of the full amplitude, \( T_{\text{spin}}(\omega) \), is determined at low energies by the a.m.m. rather than \( \mu \) and \( \omega^{-1} T_{\text{spin}}(\omega) \) vanishes at high energies rather than goes to the constant (9). With these changes, the full GDH integral is still that determines a variation of \( \omega^{-1} T_{\text{spin}}(\omega) \) between \( \omega = 0 \) and \( \infty \) and therefore gives the r.h.s. of Eq. (1).

4 The GDH sum rule for \( \Delta \) in quark models

A challenge with evaluating the GDH sum rule for \( \Delta \) is in finding a source for a big photoabsorption cross section \( \sigma_{5/2}(\omega) \) which must compensate the large negative de-excitation contribution from the nucleon, as well as negative contributions of all resonances of spin \( \leq \frac{3}{2} \). Theoretical evaluations of the GDH sum rule for weakly-bound systems (1–3) suggest that the full electromagnetic spin-orbit interaction, including a relativistic two-body correction, which leads to p-wave excitations of the system, should play an important role here. Using such an interaction in the framework of the Karl–Isgur non-relativistic quark model, we calculated photocouplings \( A_\lambda \) of the \( \Delta \) resonance to all the lowest \( L = 1 \) baryons and found the resonance contribution (3). We have found that the spin-orbit interaction strongly affects photocouplings of \( |N^4 P_M\rangle \) resonances (lying in the 1700 MeV mass range) and makes \( D_{15}(1675) \) a prominent mode of the \( \Delta \) photoexcitation. Still, the found strength of the \( D_{15}(1675) \) contribution is not sufficient to bring the GDH integral to a positive (even small) value (see Table 1).

Recently, Carlson and Carone estimated photocouplings of \( \Delta \) with the \( L = 1 \) baryons using an operator structure of the large-\( N_c \) QCD and determining unknown coefficients through experimental data on photocouplings of the nucleon (3). Depending on whether two-body operators are included or not included into the fits, two solutions were provided (CC-II and CC-I, respectively) which lead to results which we shown in Table 1. Doing so, we add the contribution of the nucleon with the experimentally known photocoupling, just to be in line with the whole ideology of this approach. The CC predictions reveal a strong photocoupling of \( \Delta \) with \( D_{15}(1675) \) which leads to an essential cancellation between the \( N^4(939) \) and \( D_{15}(1675) \) contributions. Uncertainties in photocouplings make it difficult to predict unambiguously the GDH integral. A clear trend, however, is that the \( D_{15}(1675) \) resonance does not yield all the needed cross section \( \sigma_{5/2} \).

It would be desirable to extend the present consideration by including soft-pion photoproduction which is known to visibly contribute to the GDH integral for the nucleon target and which is partly responsible for reducing the otherwise too big contribution of \( \Delta \) if taken with the experimental strength.
Table 1. Contributions of the nucleon and $[70, 1^-]$ resonances to the GDH integral $I_{GDH}$ (in $\mu$b) for the $\Delta^+$ and $\Delta^{++}$ targets. NQM and NQM+so label the Karl–Isgur nonrelativistic quark model respectively without and with one- and two-body spin-orbit electromagnetic interactions. See other notations in the text. In the case of $\Delta^{++}$, results of NQM and NQM+so are identical, and almost so are results of CC-I and CC-II.

| $B^*$        | $B = \Delta^+$ | $B = \Delta^{++}$ |
|--------------|-----------------|--------------------|
|              | NQM            | NQM+so            | CC-I    | CC-II  | NQM    | CC-I   |
| $N(939)$     | -270           | -270              | -468    | -468   |        |        |
| $S_{11}(1535)$ | -56            | -40               | -84     | -149   |        |        |
| $D_{13}(1520)$ | -28            | -3                | -29     | -25    |        |        |
| $S_{11}(1650)$ | -192           | -66               | -20     | -57    |        |        |
| $D_{13}(1700)$ | -35            | -18               | -8      | -35    |        |        |
| $D_{15}(1675)$ | 189            | 332               | 309     | 532    |        |        |
| $S_{31}(1620)$ | -12            | -12               | -10     | -10    | -47    | -38    |
| $D_{33}(1700)$ | -29            | -29               | -13     | -13    | -116   | -51    |
| total ($\mu$b) | -431           | -105              | -323    | -225   | -163   | -90    |

Then effects of the finite width of the $\Delta$ should be taken into account as well. We might anticipate that essentially $s$-wave pion photoproduction off the $\Delta$ does not contribute much to $\sigma_{5/2}$ but does contribute to $\sigma_{-1/2}$ owing to the reaction $\gamma \Delta \rightarrow \pi N$. It thus can further increase the gap between the negative resonance contribution $I_{res}^{GDH}$ and the positive theoretical value of $I^{GDH}$.

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