Solving of Clock Problems Using An Algebraic Approach
And Developing An Application For Automatic Conversion

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Abstract. The recent trend in learning Mathematics is through android apps like Byju’s. The clock problems asked in aptitude tests could be learnt using such computer applications. The Clock problems are of four categories namely:
1. What is the angle between the hands of a clock at a particular time
2. When the hands of a clock will meet after a particular time
3. When the hands of a clock will be at right angle after a particular time
4. When the hands of a clock will be in a straight line but not together after a particular time

The aim of this article is to convert the clock problems which were solved using the traditional approach to algebraic equations and solve them. Shortcuts are arrived which help in solving the questions in just a few seconds. Any aptitude problem could be converted to an algebraic equation by tracing the way the problem proceeds by applying our analytical skills. Solving of equations would be the easiest part in coming up with the solution. Also a computer application could be developed by using the equations that were arrived at in the analysis part. The computer application aims at solving the four different problems in Clocks. The application helps the learners of aptitude for CAT and other competitive exams to know the approach of the problem. Learning Mathematics with a gaming tool like this would be interesting to the learners. This paper provides a path to creating gaming apps to learn Mathematics.

1. Introduction
Byju is a post graduate from Indian Institute of Management. He in his institution provides mobile applications that help learn aptitude problems with fun. Developing of such a gaming application in solving complex problems like those asked under clocks would help the learner to a greater extent. Algebra is the fundamental of every aptitude problem. Mathematicians in Banking Training Institutes like R.A.C.E. apply the fundamentals and come out with shortcuts for the problems in various areas like Time and Work, Time and Distance, Boats and Streams, Alligations and Mixtures, Trains, Ages, Averages, etc. Conversion of the dependants of the problems to variables and the situations or states of the problems to equations would be the painstaking part of any problem depending upon its difficulty level. The Management Universities like Harvard Business School, Indian Institute of Management and also the recruiters of every field look into the candidate’s approach to new problems. The out-of-box thinking is expected from every candidate.

In the first part of this paper I have presented the traditional way in which the Clocks problem is approached. Following it, I have presented my approach. Then I have described the way the computer application is being developed.
2. Traditional approach of the Clocks problem:
In the traditional method of clocks problem, the concept that “in 60 minutes, the minute hand gains 55 minutes on the hour hand” is used. Simple formulae are derived for each of the above problems using the same traditional approach.

3. Shortcut to find the time after a particular time when the hands of a clock meet:
3.1. Formula
The formula is
Time in minutes = 60*n/11

3.2. Algebraic approach:
Let us consider that x is the angle swiped by the hour hand after n:00 o’ clock.
To find the time after which the hour hand and minute hand meet after 1:00 clock, $n=1$;
The angle to be swept by the minute hand after 1:00 ‘o clock is 30+x.
Let us consider $z$ to be the time in minutes after 1:00 clock when the hands of the clock meet.
Now we create an Angle and Time Table for the hour hand and minute hand.

| Hand         | Angle to be swept after 1:00 | Minutes after 1:00 when the hands meet |
|--------------|-----------------------------|----------------------------------------|
| Minute Hand  | 30+x                        | $z$                                    |
| Hour Hand    | $x$                         | $z$                                    |

The hour hand covers 1/2 degrees in 1 minute.
Angle swept by the hour hand in $z$ minutes = $z/2$
Angle swept by the hour hand in $z$ minutes = $x$

$z/2=x \quad \rightarrow 1$

The minute hand covers 6 degrees in 1 minute.
Angle swept by the minute hand in $z$ minutes = 6*z
Angle swept by the minute hand in $z$ minutes = 30+x

$6z=30+x \quad \rightarrow 2$

Solving equations 1 and 2,
$z=60/11=\text{minutes}$
Similarly the table for 2:00 o’ clock, 3:00 o’ clock, 4:00 o’ clock etc can be drawn and the equations can be solved.
The following are the tables of 2:00 o’ clock, 3:00 o’ clock, etc.

Table for 2:00 o’ clock:

| Hand         | Angle to be swept after 2:00 | Minutes after 2:00 when the hands meet |
|--------------|-----------------------------|----------------------------------------|
| Minute Hand  | 60+x                        | $z$                                    |
| Hour Hand    | $x$                         | $z$                                    |
Table for 3:00 o’clock:

| Hand         | Angle to be swept after 3:00 | Minutes after 3:00 when the hands meet |
|--------------|------------------------------|----------------------------------------|
| Minute Hand  | $90 + x$                     | $z$                                    |
| Hour Hand    | $x$                          | $z$                                    |

Table for 4:00 o’clock:

| Hand         | Angle to be swept after 4:00 | Minutes after 4:00 when the hands meet |
|--------------|------------------------------|----------------------------------------|
| Minute Hand  | $120 + x$                    | $z$                                    |
| Hour Hand    | $x$                          | $z$                                    |

Table for 5:00 o’clock:

| Hand         | Angle to be swept after 5:00 | Minutes after 5:00 when the hands meet |
|--------------|------------------------------|----------------------------------------|
| Minute Hand  | $150 + x$                    | $z$                                    |
| Hour Hand    | $x$                          | $z$                                    |

Table for 6:00 o’clock:

| Hand         | Angle to be swept after 6:00 | Minutes after 6:00 when the hands meet |
|--------------|------------------------------|----------------------------------------|
| Minute Hand  | $180 + x$                    | $z$                                    |
| Hour Hand    | $x$                          | $z$                                    |

Table for 7:00 o’clock:

| Hand         | Angle to be swept after 7:00 | Minutes after 7:00 when the hands meet |
|--------------|------------------------------|----------------------------------------|
| Minute Hand  | $210 + x$                    | $z$                                    |
| Hour Hand    | $x$                          | $z$                                    |

Table for 8:00 o’clock:

| Hand         | Angle to be swept after 8:00 | Minutes after 8:00 when the hands meet |
|--------------|------------------------------|----------------------------------------|
| Minute Hand  | $240 + x$                    | $z$                                    |
| Hour Hand    | $x$                          | $z$                                    |
Table for 9:00 o’ clock:

| Hand      | Angle to be swept after 9:00 | Minutes after 9:00 when the hands meet |
|-----------|------------------------------|---------------------------------------|
| Minute Hand | 270+x                        | z                                     |
| Hour Hand  | x                            | z                                     |

Table for 10:00 o’ clock:

| Hand      | Angle to be swept after 10:00 | Minutes after 10:00 when the hands meet |
|-----------|------------------------------|---------------------------------------|
| Minute Hand | 300+x                        | z                                     |
| Hour Hand  | x                            | z                                     |

Table for 11:00 o’ clock:

| Hand      | Angle to be swept after 11:00 | Minutes after 11:00 when the hands meet |
|-----------|------------------------------|---------------------------------------|
| Minute Hand | 330+x                        | z                                     |
| Hour Hand  | x                            | z                                     |

Table for 12:00 o’ clock:

| Hand      | Angle to be swept after 12:00 | Minutes after 12:00 when the hands meet |
|-----------|------------------------------|---------------------------------------|
| Minute Hand | x                            | z                                     |
| Hour Hand  | x                            | z                                     |

Solving the equations derived from the above tables the minutes after the particular time when the hands of the clock obtained are furnished in the following table:

| Time after which the hands meet | Minutes after the time specified in improper fraction |
|---------------------------------|------------------------------------------------------|
| 1:00                            | 60/11                                                 |
| 2:00                            | 120/11                                                |
| 3:00                            | 180/11                                                |
| 4:00                            | 240/11                                                |
| 5:00                            | 300/11                                                |
| 6:00                            | 360/11                                                |
| 7:00                            | 420/11                                                |
| 8:00                            | 480/11                                                |
| 9:00                            | 540/11                                                |
| 10:00                           | 600/11                                                |
| 11:00                           | 660/11                                                |
| 12:00                           | 0/11                                                  |

The above result follows the formula, 60*n/11 except for 12:00 where n=0.
4. Shortcut to find the time between the given two timings when the hands of the clock are at 90 degrees:

The problem is to find the minutes after the given starting time when the hands are at 90 degrees to each other. For example let us consider that we have to find the minutes between 1:00 o’clock and 2 o’clock when the hands of the clock are at 90 degrees to each other. The hands are at 90 degrees to each other twice within the given time. If we consider the angle in clockwise direction, once the angle between the hands will be at 90 degrees and once the angle between the hands will be at 270 degrees (ie, the hands will be at 90 degrees in the anticlockwise direction). The formula to find the minutes in case of angle in clockwise direction between hour hand and minute hand being 90 degrees is as follows for the following cases

| Case | Formula |
|------|---------|
| 12:00 to 1:00, 1:00 to 2:00, 2:00 to 3:00 | 60*(x+3)/11 |
| 3:00 to 4:00, 4:00 to 5:00, 5:00 to 6:00, 6:00 to 7:00, 7:00 to 8:00, 8:00 to 9:00, 9:00 to 10:00, 10:00 to 11:00, 11:00 to 12:00 | 60*(x-3)/11 |

4.1. Algebraic approach:

If we consider the clockwise degree case,

Let us consider that in y minutes after 1 o’clock, the hands are at 90 degrees. Then the angle to be swiped by the minute hand after 1:00 o’clock is 30+x+90.
The angle to be swiped by the hour hand after 1:00 o’clock is x.

As in the previous case, the table the angle and minute table could be drawn as follows:

| Table for 1:00 o’clock to 2:00 o’clock: |
| Hand | Angle to be swept after 1:00 | Minutes after 1:00 when the hands meet |
|------|-----------------------------|----------------------------------------|
| Minute Hand | 30+x+90            | y                                      |
| Hour Hand   | x                            | y                                      |

The angle swept by hour hand in 1 minute=1/2 degree

The angle swept by the hour hand in y minutes=y/2

x=y/2

The angle swept by the minute hand in 1 minute=6 degrees

The angle swept by the minute hand in y minutes=6y

6y=30+x+270

Solving equations 1 and 2, z=240/11 minutes

Similarly the minutes and angle table drawn for the other timings are as follows:

Between 2:00 and 3:00:

| Hand   | Angle to be swept after 2:00 | Minutes after 2:00 when the hands are at 90 degrees in clockwise direction |
|--------|-------------------------------|-------------------------------------------------------------------------|
| Minute Hand | 60+x+90                      | y                                                                        |
| Hour Hand  | x                             | y                                                                        |
### Between 3:00 and 4:00:

| Hand     | Angle to be swept after 3:00 | Minutes after 3:00 when the hands are at 90 degrees in clockwise direction |
|----------|-----------------------------|----------------------------------------------------------------------------|
| Minute Hand | \( x \)                     | \( y \)                                                                    |
| Hour Hand  | \( x \)                     | \( y \)                                                                    |

### Between 4:00 and 5:00:

| Hand     | Angle to be swept after 4:00 | Minutes after 4:00 when the hands are at 90 degrees in clockwise direction |
|----------|-----------------------------|----------------------------------------------------------------------------|
| Minute Hand | \( x + 30 \)                | \( y \)                                                                    |
| Hour Hand  | \( x \)                     | \( y \)                                                                    |

### Between 5:00 and 6:00:

| Hand     | Angle to be swept after 5:00 | Minutes after 5:00 when the hands are at 90 degrees in clockwise direction |
|----------|-----------------------------|----------------------------------------------------------------------------|
| Minute Hand | \( x + 60 \)                | \( y \)                                                                    |
| Hour Hand  | \( x \)                     | \( y \)                                                                    |

### Between 6:00 and 7:00:

| Hand     | Angle to be swept after 6:00 | Minutes after 6:00 when the hands are at 90 degrees in clockwise direction |
|----------|-----------------------------|----------------------------------------------------------------------------|
| Minute Hand | \( x + 90 \)                | \( y \)                                                                    |
| Hour Hand  | \( x \)                     | \( y \)                                                                    |

### Between 7:00 and 8:00:

| Hand     | Angle to be swept after 7:00 | Minutes after 7:00 when the hands are at 90 degrees in clockwise direction |
|----------|-----------------------------|----------------------------------------------------------------------------|
| Minute Hand | \( x + 120 \)                | \( y \)                                                                    |
| Hour Hand  | \( x \)                     | \( y \)                                                                    |
### Between 8:00 and 9:00:

| Hand      | Angle to be swept after 8:00 | Minutes after 8:00 when the hands are at 90 degrees in clockwise direction |
|-----------|------------------------------|----------------------------------------------------------------------------|
| Minute Hand | $x + 150$                  | $y$                                                                        |
| Hour Hand  | $x$                         | $y$                                                                        |

### Between 9:00 and 10:00:

| Hand      | Angle to be swept after 9:00 | Minutes after 9:00 when the hands are at 90 degrees in clockwise direction |
|-----------|------------------------------|----------------------------------------------------------------------------|
| Minute Hand | $x + 180$                  | $y$                                                                        |
| Hour Hand  | $x$                         | $y$                                                                        |

### Between 10:00 and 11:00:

| Hand      | Angle to be swept after 10:00 | Minutes after 10:00 when the hands are at 90 degrees in clockwise direction |
|-----------|------------------------------|----------------------------------------------------------------------------|
| Minute Hand | $x + 210$                  | $y$                                                                        |
| Hour Hand  | $x$                         | $y$                                                                        |

### Between 11:00 and 12:00:

| Hand      | Angle to be swept after 11:00 | Minutes after 11:00 when the hands are at 90 degrees in clockwise direction |
|-----------|------------------------------|----------------------------------------------------------------------------|
| Minute Hand | $x + 240$                  | $y$                                                                        |
| Hour Hand  | $x$                         | $y$                                                                        |

### Between 12:00 and 1:00:

| Hand      | Angle to be swept after 12:00 | Minutes after 12:00 when the hands are at 90 degrees in clockwise direction |
|-----------|------------------------------|----------------------------------------------------------------------------|
| Minute Hand  | $x + 90$                  | $y$                                                                        |
| Hour Hand    | $x$                         | $y$                                                                        |
Solving the equations derived from the above tables, the minutes after the given starting time at which the hands are at 90 degrees to each other are obtained as in the following table:

| Time after which the hands of the clock are at 90 degrees to each other in the clockwise direction | Minutes after the time specified in improper fraction |
|---|---|
| 1:00 | 240/11 |
| 2:00 | 300/11 |
| 3:00 | 0/11 |
| 4:00 | 60/11 |
| 5:00 | 120/11 |
| 6:00 | 180/11 |
| 7:00 | 240/11 |
| 8:00 | 300/11 |
| 9:00 | 360/11 |
| 10:00 | 420/11 |
| 11:00 | 480/11 |
| 12:00 | 180/11 |

The formula to find the minutes in case of angle in clockwise direction between hour hand and minute hand being 270 degrees is as follows for the following cases:

| Case | Formula |
|---|---|
| 12:00 to 1:00, 1:00 to 2:00, 2:00 to 3:00 | 60*(x+9)/11 |
| 3:00 to 4:00, 4:00 to 5:00, 5:00 to 6:00, 6:00 to 7:00, 7:00 to 8:00, 8:00 to 9:00, 9:00 to 10:00, 10:00 to 11:00, 11:00 to 12:00 | 60*(x+3)/11 |

### 4.2. Algebraic Approach:

Let us consider the time after 1:00 when the hands of the clock are at 270 degrees in the clockwise direction to be $y$.

Let the angle covered by the hour hand from 1:00 in $y$ minutes be $x$.

The angle to be swept by the minute hand from 1:00 o’clock to be at 90 degrees with the hour hand is $30+x+270$.

The minute and angle table for the above scenario is as follows: Table for 1:00 to 2:00:

| Hand         | Angle to be swept after 1:00 | Minutes after 1:00 when the hands are at 270 degrees in clockwise direction |
|--------------|------------------------------|--------------------------------------------------------------------------------|
| Minute Hand  | $30+x+270$                   | $y$                                                                            |
| Hour Hand    | $x$                          | $y$                                                                            |
Hour hand swipes ½ degree in 1 minute.
Angle swiped by the hour hand in y minutes = y/2°
Minute hand swipes 6 degrees in 1 minute. Angle swiped by the minute hand in y minutes = 6y°
6y = 30° + x + 270°

Solving equations 1 and 2:
The minutes after which the hour hand and minute hand are at 270 degrees in the clockwise direction is
y = 600/11 minutes
Similarly, the tables for other timings could be drawn as follows:

| Between 2:00 and 3:00: | Hand | Angle to be swept after 2:00 | Minutes after 2:00 when the hands are at 270 degrees in clockwise direction |
|------------------------|------|-----------------------------|----------------------------------------------------------------------------|
| Minute Hand            | x+60°+270 | y                           |
| Hour Hand              | x     | y                           |

| Between 3:00 and 4:00: | Hand | Angle to be swept after 3:00 | Minutes after 3:00 when the hands are at 270 degrees in clockwise direction |
|------------------------|------|-----------------------------|----------------------------------------------------------------------------|
| Minute Hand            | 90°+90°+x | y                           |
| Hour Hand              | x     | y                           |

| Between 4:00 and 5:00: | Hand | Angle to be swept after 4:00 | Minutes after 4:00 when the hands are at 270 degrees in clockwise direction |
|------------------------|------|-----------------------------|----------------------------------------------------------------------------|
| Minute Hand            | 120°+x+90° | y                           |
| Hour Hand              | x     | y                           |

| Between 5:00 and 6:00: | Hand | Angle to be swept after 5:00 | Minutes after 5:00 when the hands are at 270 degrees in clockwise direction |
|------------------------|------|-----------------------------|----------------------------------------------------------------------------|
| Minute Hand            | 150°+x+90° | y                           |
| Hour Hand              | x     | y                           |
Between 6:00 and 7:00:

| Hand     | Angle to be swept after 6:00 | Minutes after 6:00 when the hands are at 270 degrees in clockwise direction |
|----------|------------------------------|----------------------------------------------------------------------------|
| Minute Hand | 180 + x + 90                | y                                                                          |
| Hour Hand   | x                            | y                                                                          |

Between 7:00 and 8:00:

| Hand     | Angle to be swept after 7:00 | Minutes after 7:00 when the hands are at 270 degrees in clockwise direction |
|----------|------------------------------|----------------------------------------------------------------------------|
| Minute Hand | 210 + 90 + x                | y                                                                          |
| Hour Hand   | x                            | y                                                                          |

Between 8:00 and 9:00:

| Hand     | Angle to be swept after 8:00 | Minutes after 8:00 when the hands are at 270 degrees in clockwise direction |
|----------|------------------------------|----------------------------------------------------------------------------|
| Minute Hand | 240 + x + 90                | y                                                                          |
| Hour Hand   | x                            | y                                                                          |

Between 9:00 and 10:00:

| Hand     | Angle to be swept after 9:00 | Minutes after 9:00 when the hands are at 270 degrees in clockwise direction |
|----------|------------------------------|----------------------------------------------------------------------------|
| Minute Hand | 270 + x + 90                | y                                                                          |
| Hour Hand   | x                            | y                                                                          |

Between 10:00 and 11:00:

| Hand     | Angle to be swept after 10:00 | Minutes after 10:00 when the hands are at 270 degrees in clockwise direction |
|----------|------------------------------|----------------------------------------------------------------------------|
| Minute Hand | 300 + 90 + x                | y                                                                          |
| Hour Hand   | x                            | y                                                                          |
Between 11:00 and 12:00:

| Hand     | Angle to be swept after 11:00 | Minutes after 11:00 when the hands are at 270 degrees in clockwise direction |
|----------|------------------------------|----------------------------------------------------------------------------|
| Minute Hand | 330+90+x                     | y                                                                          |
| Hour Hand  | x                            | y                                                                          |

Between 12:00 and 1:00:

| Hand     | Angle to be swept after 12:00 | Minutes after 12:00 when the hands are at 270 degrees in clockwise direction |
|----------|------------------------------|----------------------------------------------------------------------------|
| Minute Hand | x+270                       | y                                                                          |
| Hour Hand  | x                            | y                                                                          |

Solving the equations derived from the above tables, the minutes after the given starting time at which the hands are at 270 degrees to each other are obtained as in the following table:

| Time after which the hands of the clock are at 270 degrees to each other in the clockwise direction | Minutes after the time specified in improper fraction |
|-------------------------------------------------------------------------------------------------|---------------------------------------------------|
| 1:00                                                                                           | 600/11                                             |
| 2:00                                                                                           | 660/11                                             |
| 3:00                                                                                           | 360/11                                             |
| 4:00                                                                                           | 420/11                                             |
| 5:00                                                                                           | 480/11                                             |
| 6:00                                                                                           | 540/11                                             |
| 7:00                                                                                           | 600/11                                             |
| 8:00                                                                                           | 660/11                                             |
| 9:00                                                                                           | 720/11                                             |
| 10:00                                                                                          | 780/11                                             |
| 11:00                                                                                          | 840/11                                             |
| 12:00                                                                                          | 540/11                                             |
5. Shortcut to find the time after a particular given time after which the hands of the clock are at 180 degrees to each other:

The shortcut for the above problem is as follows:

| Case | Formula |
|------|---------|
| 12:00 to 1:00, 1:00 to 2:00, 2:00 to 3:00, 3:00 to 4:00, 4:00 to 5:00, 5:00 to 6:00 | 60*(x+6)/11 |
| 6:00 to 7:00, 7:00 to 8:00, 8:00 to 9:00, 9:00 to 10:00, 10:00 to 11:00, 11:00 to 12:00 | 60*(x-6)/11 |

5.1. Algebraic approach:
Let us consider the minutes after 12:00 when the hands of a clock are at 180 degrees to each other to be y.

Let the angle to be swiped by the hour hand in y minutes be x. Then the angle to be swiped by the minute hand is 180+x.

The minute and angle tables for the minutes after 12:00 is as follows:

| Hand   | Angle to be swept after 12:00 | Minutes after 12:00 when the hands are at 180 degrees to each other |
|--------|------------------------------|---------------------------------------------------------------------|
| Minute Hand | x+180                        | y                                                                   |
| Hour Hand  | x                            | y                                                                   |

The hour hand swipes ½ degrees in 1 minute.
The angle swept by the hour hand in y minutes = y/2.

\[ x = \frac{y}{2} \]

The minute hand swipes 6 degrees in 1 minute.
The angle swept by the minute hand in y minutes = 6y

\[ 6y = x + 180 \]

Solving equations 1 and 2,
\[ y = \frac{360}{11} \text{ minutes} \]

The minute and angle tables for the other timings are as follows:

From 1:00 to 2:00:

| Hand   | Angle to be swept after 1:00 | Minutes after 1:00 when the hands are at 180 degrees to each other |
|--------|------------------------------|---------------------------------------------------------------------|
| Minute Hand | x+180+30                    | y                                                                   |
| Hour Hand  | x                            | y                                                                   |
From 2:00 to 3:00:

| Hand          | Angle to be swept after 2:00 | Minutes after 2:00 when the hands are at 180 degrees to each other |
|---------------|-----------------------------|------------------------------------------------------------------|
| Minute Hand   | x + 180 + 60                | y                                                                |
| Hour Hand     | x                           | y                                                                |

From 3:00 to 4:00:

| Hand          | Angle to be swept after 3:00 | Minutes after 3:00 when the hands are at 180 degrees to each other |
|---------------|-----------------------------|------------------------------------------------------------------|
| Minute Hand   | x + 180 + 90                | y                                                                |
| Hour Hand     | x                           | y                                                                |

From 4:00 to 5:00:

| Hand          | Angle to be swept after 4:00 | Minutes after 4:00 when the hands are at 180 degrees to each other |
|---------------|-----------------------------|------------------------------------------------------------------|
| Minute Hand   | x + 180 + 120               | y                                                                |
| Hour Hand     | x                           | y                                                                |

From 5:00 to 6:00:

| Hand          | Angle to be swept after 5:00 | Minutes after 5:00 when the hands are at 180 degrees to each other |
|---------------|-----------------------------|------------------------------------------------------------------|
| Minute Hand   | x + 180 + 150               | y                                                                |
| Hour Hand     | x                           | y                                                                |

From 6:00 to 7:00:

| Hand          | Angle to be swept after 6:00 | Minutes after 6:00 when the hands are at 180 degrees to each other |
|---------------|-----------------------------|------------------------------------------------------------------|
| Minute Hand   | x                           | y                                                                |
| Hour Hand     | x                           | y                                                                |
From 7:00 to 8:00:

| Hand      | Angle to be swept after 7:00 | Minutes after 7:00 when the hands are at 180 degrees to each other |
|-----------|------------------------------|---------------------------------------------------------------|
| Minute Hand | x + 30                      | y                                                             |
| Hour Hand  | x                           | y                                                             |

From 8:00 to 9:00:

| Hand      | Angle to be swept after 8:00 | Minutes after 8:00 when the hands are at 180 degrees to each other |
|-----------|------------------------------|---------------------------------------------------------------|
| Minute Hand | x + 60                      | y                                                             |
| Hour Hand  | x                           | y                                                             |

From 9:00 to 10:00:

| Hand      | Angle to be swept after 9:00 | Minutes after 9:00 when the hands are at 180 degrees to each other |
|-----------|------------------------------|---------------------------------------------------------------|
| Minute Hand | x + 90                      | y                                                             |
| Hour Hand  | x                           | y                                                             |

From 10:00 to 11:00:

| Hand      | Angle to be swept after 10:00 | Minutes after 10:00 when the hands are at 180 degrees to each other |
|-----------|------------------------------|---------------------------------------------------------------|
| Minute Hand | x + 120                      | y                                                             |
| Hour Hand  | x                           | y                                                             |

From 11:00 to 12:00:

| Hand      | Angle to be swept after 11:00 | Minutes after 11:00 when the hands are at 180 degrees to each other |
|-----------|------------------------------|---------------------------------------------------------------|
| Minute Hand | x + 150                      | y                                                             |
| Hour Hand  | x                           | y                                                             |
The minutes obtained by solving the equations derived from the above tables is as follows:

| Time after which the hands of the clock are at 180 degrees to each other | Minutes after the time specified in improper fraction |
|------------------------------------------------------------------------|------------------------------------------------------|
| 1:00                                                                   | 420/11                                               |
| 2:00                                                                   | 480/11                                               |
| 3:00                                                                   | 540/11                                               |
| 4:00                                                                   | 600/11                                               |
| 5:00                                                                   | 660/11                                               |
| 6:00                                                                   | 0/11                                                 |
| 7:00                                                                   | 60/11                                                |
| 8:00                                                                   | 120/11                                               |
| 9:00                                                                   | 180/11                                               |
| 10:00                                                                  | 240/11                                               |
| 11:00                                                                  | 300/11                                               |
| 12:00                                                                  | 360/11                                               |

6. Conclusion:
The above shortcut formulae can be implemented in computer programming to create an application that has a clock which shows the time and angle between the two hands of a clock by rotating the hands of the clock. Close analysis of such a time and angle clock could be used by students preparing aptitude problems to understand the way the angle changes with the time.

References
[1] Quantitative Aptitude For Competitive Examinations – Dr.R.S.Aggarwal.