Three-component Fulde-Ferrell superfluids in a two-dimensional Fermi gas with spin-orbit coupling

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I. INTRODUCTION

Ever since its experimental realization, synthetic spin-orbit coupling (SOC) in ultracold atomic gases have attracted much attention [1–7]. Over the past few years, SOC-induced exotic phases and phase transitions have been extensively studied in both the Bose and Fermi gases [8–15]. Central to the effects of SOC in these systems is the non-perturbative modification of the single-particle dispersion spectra. For Fermi gases in particular, the SOC-modified single-particle dispersion can induce exotic few-body states [16, 17], as well as highly nontrivial many-body correlations [18–19]. The SOC-induced Fulde-Ferrell (FF) pairing state is an interesting example, where the interplay of SOC and the Zeeman-field-induced Fermi surface asymmetry stabilizes unconventional pairing superfluids with finite center-of-mass momenta [15].

Recently, it has been shown that an alternative FF pairing mechanism exists in a Fermi-Fermi mixture, where a two-component non-interacting Fermi gas, when dressed by SOC, interacts spin selectively with a third fermionic species [19]. As the hyperfine-spin distribution in the SOC-induced helicity branches is asymmetric in momentum space, the spin-selective pairing interaction leads to pairing states with finite center-of-mass momenta in both the two-body and the many-body sectors.

In this work, we demonstrate that exotic FF states can also be stabilized in a three-component Fermi gas where all three hyperfine states are coupled by SOC [20, 21]. From the single-particle dispersion, we show that in the three SOC-induced helicity branches, two exhibit asymmetric hyperfine-spin distributions. Based on this asymmetry, we discuss several different configurations of the spin-selective interactions, under which the ground state of the system can either be a conventional Bardeen-Cooper-Schrieffer (BCS) state with zero center-of-mass momentum, or the FF pairing states. Of particular interest here is the existence of a three-component FF pairing state, in which every two out of the three components form FF pairing. We map out the phase diagram of the system, characterize the properties of the three-component FF state such as the order parameters, the gapless contours and the momentum distributions. Based on these results, we discuss possible experimental detection schemes for the interesting pairing states in the system.
work not only reveals the generality of the FF pairing mechanism induced by SOC and spin-selective interactions, but has interesting implications for future experiments on SOC-induced exotic superfluidity as well.

The paper is organized as the following: in Sec. II, we calculate the single-particle dispersion under SOC and analyze the inherent asymmetry in the hyperfine state distribution. In Sec. III, we present the system as well as the mean-field formalism. We then consider different configurations of spin-dependent interactions and investigate their impact on the pairing superfluidity in Sec. IV. In Sec. V, we discuss the novel three-component FF state. The main results of this paper are summarized in Sec. VI.

II. SINGLE-PARTICLE DISPERSION

We consider a two dimensional three-component Fermi gas, where the three atomic hyperfine states are labeled as $|\pm 1\rangle$ and $|0\rangle$. The hyperfine states are coupled by Raman lasers as illustrated in Fig. 1. Under an appropriate rotating frame, the single-particle Hamiltonian can be written in momentum space as $[8, 11, 22]$

$$H_0(\vec{k}) = \begin{pmatrix} \frac{\hbar^2 (\vec{k} + 2\vec{k}_r \vec{\beta})^2}{2m} - h\delta & -\frac{h\epsilon}{2m} & \frac{h}{2m} \\ -\frac{h\epsilon}{2m} & \frac{\hbar^2 k^2}{2m} - h\epsilon & \frac{h}{2m} \\ \frac{h}{2m} & \frac{h}{2m} & \frac{\hbar^2 (\vec{k} - 2\vec{k}_r \vec{\beta})^2}{2m} + h\delta \end{pmatrix} ,$$

(1)

where $\delta$ is the two-photon detuning of the Raman process, $\epsilon$ accounts for a small quadratic Zeeman shift, $h = \hbar\Omega R/2$ denotes the strength of the effective Zeeman field, which is proportional to the Rabi frequency of the Raman process $\Omega R$. We take the recoil energy $E_r = \hbar^2 k_e^2 / (2m)$ and the corresponding wave vector $k_e$ as the units of energy and wave vector, respectively, with $m$ the atomic mass. In the rest of the paper, we will only consider the case where $\delta = 0$. The Hamiltonian $H_0(\vec{k})$ can be diagonalized as

$$H_0(\vec{k}) \begin{bmatrix} a_i(\vec{k}) \\ b_i(\vec{k}) \\ c_i(\vec{k}) \end{bmatrix} = E_i(\vec{k}) \begin{bmatrix} a_i(\vec{k}) \\ b_i(\vec{k}) \\ c_i(\vec{k}) \end{bmatrix} ,$$

(2)

where the eigenvalues $E_i(\vec{k})$ ($i = 1, 2, 3$) are the single-particle dispersion spectra for the helicity branches, and $\psi_i(\vec{k}) = (a_i, b_i, c_i)^T$ are the corresponding eigenvectors. Importantly, the coefficients $a_i(\vec{k}), b_i(\vec{k}),$ and $c_i(\vec{k})$ are related to the weight of hyperfine states in the corresponding helicity branches.

In Fig. 2 we plot the single-particle dispersion spectra
as well as the momentum distributions of the hyperfine states in the helicity branches along the \( k_y = 0 \) axis. Apparently, the hyperfine-state superpositions in the helicity branches are momentum dependent (Fig. 2(b-d)). A critical observation is that while the momentum distribution of the hyperfine state \(|0\rangle\) is symmetric with respect to \( k_z = 0 \) for all three helicity branches, such a symmetry is absent for the cases of states \(|1\rangle\) and \(|-1\rangle\). Instead, the momentum distributions of \(|1\rangle\) and \(|-1\rangle\) are symmetric with respect to each other, such that \( |a_i(k_x)|^2 = |c_i(-k_x)|^2 \). The presence or absence of this inversion symmetry has crucial effects on pairing physics. In fact, in the weak-coupling limit, pairing tends to occur between two fermions residing on their corresponding Fermi surfaces in the absence of SOC. With SOC mixing up the hyperfine spins into the helicity branches, we must allow for the case of states \(|1\rangle\) and \(|-1\rangle\), the pairing state would have zero center-of-mass momentum. On the other hand, if we turn on a small attractive interaction between states \(|1\rangle\) and \(|-1\rangle\), the pairing state would have a finite center-of-mass momentum. This is similar to the FF pairing mechanism in Ref. [19], where the interplay of SOC and spin-selective interactions plays the key role.

### III. MEAN-FIELD TREATMENT OF PAIRING STATES

With the understanding of the single-particle dispersion, we now study the many-body pairing physics using the standard mean-field formalism. Now the complete Hamiltonian can be written as

\[
H = \sum_{\vec{k}} \Phi^\dagger(\vec{k}) [H_0(\vec{k}) - \mu] \Phi(\vec{k}) + H_1,
\]

where \( \Phi^\dagger(\vec{k}) = (c^\dagger_{\vec{k},1}, c^\dagger_{\vec{k},0}, c^\dagger_{\vec{k},-1}) \) is the creation operator for the three hyperfine states, \( c^\dagger_{\vec{k},i} \) is an annihilation operator of a Fermi atom with wave vector \( \vec{k} \) and pseudospin \( i = 1, 0, -1 \) describing three atomic hyperfine states. \( c^\dagger_{\vec{k},i} \) is the corresponding creation operator, and \( \mu \) is the chemical potential. Here, the most general form of the interaction Hamiltonian \( H_1 \) is

\[
H_1 = \frac{g_{1,0}}{V} \sum_{\vec{Q},\vec{k},\vec{k}'} c^\dagger_{\vec{k},1} c^\dagger_{\vec{Q}+\vec{k},0} c_{\vec{k}',0} c_{\vec{Q}-\vec{k}',1} + \frac{g_{1,-1}}{V} \sum_{\vec{Q},\vec{k},\vec{k}'} c^\dagger_{\vec{k},1} c^\dagger_{\vec{Q}+\vec{k},-1} c_{\vec{k}',-1} c_{\vec{Q}-\vec{k}',1} + \frac{g_{0,-1}}{V} \sum_{\vec{Q},\vec{k},\vec{k}'} c^\dagger_{\vec{k},0} c^\dagger_{\vec{Q}+\vec{k},-1} c_{\vec{k}',-1} c_{\vec{Q}-\vec{k}',0}.
\]

Here, \( \vec{Q} \) is the center-of-mass momentum of the pairing fermions, \( V \) is the quantization volume in two dimensions, and \( g_{i,j} \) \((i, j = \pm 1, 0)\) is the bare interaction rate between different hyperfine states, which can be renormalized following the standard procedure in two dimensions [23][25].

\[
\frac{1}{g_{i,j}} = \frac{1}{V} \sum_{\vec{k}} \frac{1}{\hbar^2 k^2/m + E_{\vec{k},i,j}}.
\]

where the superfluid order parameter is taken as

\[
\Delta_{\vec{Q},i,j} = \frac{g_{i,j}}{V} \sum_{\vec{k}} \langle c_{\vec{Q}+\vec{k},j} c_{\vec{k},i} \rangle, \quad (i, j = 1, 0, -1).
\]

Therefore, in the hyperfine-spin basis \( \Psi_{\vec{Q}}(\vec{k}) = (c^\dagger_{\vec{k},1}, c^\dagger_{\vec{Q}+\vec{k},1}, c^\dagger_{\vec{k},0}, c^\dagger_{\vec{Q}+\vec{k},0}, c^\dagger_{\vec{k},-1}, c^\dagger_{\vec{Q}+\vec{k},-1})^T \), the effective mean-field Hamiltonian can be rewritten as

\[
H_m = \frac{1}{2} \sum_{\vec{k}} \Psi^\dagger_{\vec{Q}}(\vec{k}) M_{\vec{k}} \Psi_{\vec{Q}}(\vec{k}) + \sum_{\vec{k}} \left( \frac{3}{2} \xi_{\vec{k}} + 4 E_F - \frac{1}{2} \hbar^2 \right) - V \left( \frac{|\Delta_{\vec{Q},1,0}|^2}{g_{1,0}} + \frac{|\Delta_{\vec{Q},1,-1}|^2}{g_{1,-1}} + \frac{|\Delta_{\vec{Q},0,-1}|^2}{g_{0,-1}} \right).
\]

The matrix takes the form of...
FIG. 3: (Color online) Contour plots of the thermodynamic potential landscape in the $\Delta_{1,0}-Q_x$ plane for: (a) $\mu = 10E_r$ and $E_{b1} = 0.3E_r$; (b) $\mu = 5E_r$ and $E_{b1} = 0.3E_r$; (c) $\mu = 2E_r$ and $E_{b1} = 0.3E_r$. The global minimum are located at: (a) $(\Delta_{1,0}/E_r \approx 2.3410, Q_x/k_r \approx -2.0184)$; (b) $(\Delta_{1,0}/E_r \approx 1.5917, Q_x/k_r \approx -2.0347)$; (c) $(\Delta_{1,0}/E_r \approx 0.5970, Q_x/k_r \approx -2.0666)$. Other parameters are chosen as $\hbar = 0.2E_r$, $\hbar\delta = 0$, and $h = E_r$.

$$M_k = \begin{pmatrix}
\xi_k + 2k_x \varepsilon_x - h\delta & 0 & -h & -h & h \\
0 & \frac{\hbar}{\Delta_{Q,1,0}} & -\frac{\hbar}{\Delta_{Q,1,0}} & -\frac{\hbar}{\Delta_{Q,0,-1}} & -\frac{\hbar}{\Delta_{Q,0,-1}} \\
\frac{\hbar}{\Delta_{Q,1,0}} & -\frac{\hbar}{\Delta_{Q,1,0}} & 0 & -h & h \\
0 & -\frac{\hbar}{\Delta_{Q,0,-1}} & -\frac{\hbar}{\Delta_{Q,0,-1}} & 0 & -h \\
\Delta_{Q,0,-1} & -\frac{\hbar}{\Delta_{Q,0,-1}} & h & -\frac{\hbar}{\Delta_{Q,0,-1}} & 0 \\
\end{pmatrix},$$

with $\xi_k = \hbar^2 k^2/(2m) - \mu$.

The zero-temperature thermodynamic potential can then be derived

$$\Omega = -k_B T \ln \text{Tr} e^{-H_m/(k_B T)} \bigg|_{T \to 0}$$

$$= \frac{1}{2} \sum_{\vec{k},j=1,2,3,4,5,6} E_{\vec{k},j} \Theta(-E_{\vec{k},j})$$

$$+ \sum_{\vec{k}} \left( \frac{3}{2} \xi_{\vec{k}} + 4E_r - \frac{1}{2} \hbar\epsilon_x \right)$$

$$- V \left( \frac{\Delta_{Q,1,0}^2}{g_{1,0}} + \frac{\Delta_{Q,1,-1}^2}{g_{1,-1}} + \frac{\Delta_{Q,0,-1}^2}{g_{0,-1}} \right),$$

where $\Theta(x)$ is the Heaviside step function, $\text{Tr}$ denotes the trace over both the momentum and spin degrees of freedom, $T$ represents the temperature, and $k_B$ is the Boltzmann constant.

In the following, we first consider the simple case where interatomic interactions are limited between two spin species. Due to the symmetry of the setup at $\delta = 0$, we only need to study two different cases among all three possible combinations of interactions: the case with interaction between $|1\rangle$ and $|0\rangle$; and the case with interaction between $|1\rangle$ and $|-1\rangle$.

IV. PAIRING STATES WITH INTERACTION EXISTING BETWEEN TWO SPIN SPECIES

We first study the case where interaction only presents between states $|1\rangle$ and $|0\rangle$, so that $\Delta_{Q,1,-1} = \Delta_{Q,0,-1} = 0$. We also define $\Delta_{Q,1,0} \equiv \Delta_{1,0}$ and $E_{b1} \equiv E_{b1,0}$ to simplify notations. Under these conditions, we calculate the thermodynamic potential in Eq. (10) for any given $Q$, by numerically diagonalizing the matrix Eq. (9). For the parameters that we have studied, the local minimum in the thermodynamic potential always occurs with the $Q = Q_x e_x$, where $e_x$ is the unit vector along the direction of the SOC. In Figs. (a-c), we show the typical contour plots of the thermodynamic potential in the plane of $\Delta_{1,0}-Q_x$ for a given chemical potential $\mu$ and binding energy $E_{b1}$. From these figures, we can see clearly that in general there exists only one local minimum, which corresponds to the ground state of the system. This is consistent with our previous analysis that under SOC-induced asymmetric hyperfine spin distribution and spin-selective interaction between $|1\rangle$ and $|0\rangle$, the pairing state acquires a nonzero center-of-mass momentum, i.e., it is an FF state.

These observations allow us to minimize the thermodynamic potential with respect to the order parameter $\Delta_{1,0}$ and the center-of-mass momentum $Q = Q_x$ to find the ground state of the system. In Fig. (a), we map out the phase diagram in the $\mu-E_{b1}$ plane for $h\epsilon = 0.2E_r$, $h\delta = 0$, and $h = E_r$. Apparently, a continuous phase boundary
exists between the FF state, which is characterized by a finite \( \Delta_{1,0} \) and a finite \( Q_x \), and the normal state (N), which is characterized by a vanishing order parameter. Furthermore, judging from the minimum excitation gap, both a fully gapped FF state (gFF) and a gapless nodal FF (nFF) state exist on the phase diagram, which are separated by a continuous phase boundary. In Fig. 4(b), we show how the ground-state order parameter \( \Delta_{1,0} \) and the center-of-mass momentum \( Q_x \) evolve with the chemical potential \( \mu \) with fixed \( h = E_r \) and \( E_{b1} = E_r \). In Figs. 4(c) and (d), we also show the typical gapless contours of the nFF state in momentum space.

For the case where the only interaction in the system is between the states \([1]\) and \([-1]\), we may set \( \Delta_{Q,1,0} = \Delta_{Q,0,-1} = 0 \). By diagonalizing the effective Hamiltonian and minimize the thermodynamic potential as we have done previously, we find that the ground state of the system is either a BCS pairing state with \( Q = 0 \), or a normal state with a vanishing order parameter. This is also consistent with our previous analysis in the weakly interacting limit that attractive interactions between states \([1]\) and \([-1]\) would lead to BCS pairing, due to the symmetry in the momentum distribution of these states in the helicity branches.

V. PAIRING STATES WITH MULTIPLE INTERACTIONS AMONG VARIOUS SPIN COMBINATIONS

In this section, we consider the case where attractive interactions are not only present between states \([1]\) and \([0]\), but also exist between states \([1]\) and \([-1]\). Hence, we can set \( \Delta_{\vec{Q},0,-1} = 0, \Delta_{\vec{Q},1,-1} = \Delta_{1,-1} \), and \( E_{b2} = E_{b1,-1} \) for convenience.

As before, we numerically minimize the thermodynamic potential to look for the ground state of the system. In Fig. 5(a), we show the typical phase diagram in the \( \mu-E_{b1} \) plane for \( \hbar = 0.2E_r, \hbar \delta = 0, h = E_r \), and \( E_{b2} = E_r \). The red solid line is the first-order boundary, while the blue solid line represents the phase boundary of continuous phase, and the blue dashed curves denote the phase boundaries between the fully gapped FF state and the gapless nodal FF state. The black dashed line denotes \( E_{b1} = E_r \), and it is the asymptote of the first-order boundary. (b) Typical ground state superfluid order parameters \( \Delta_{1,0}, \Delta_{1,-1} \) and center momentum \( Q_x \) of the pairing fermions versus \( \mu \) for \( \hbar = 0.2E_r, \hbar \delta = 0, h = E_r \), and \( E_{b1} = E_r \). The red solid line is \( \Delta_{1,0} \), the blue solid curve represents \( \Delta_{1,-1} \), and the black solid line denotes \( Q_x \).
that there are four distinct FF phases on the phase diagram, including two nodal (or gapless) (labeled as nFF1 and nFF2) and two fully gapped (gFF1 and gFF2) ones. While the gapped FF states are separated from the nodal FF states by continuous phase boundaries (blue dashed lines in Fig. 5(a)), different nodal FF states or different gapped FF states are separated by a first-order phase boundary (red solid line in Fig. 5(a)). On this phase diagram, we also identify a normal state (N) by numerically setting a small threshold of $|\Delta| = 10^{-3}E_r$.

By adjusting the binding energy $E_{b1}$, the system can be tuned across different phases over a wide range of $\mu$. In Fig. 5(b), we show how the ground-state order parameters $\Delta_{1,0}$, $\Delta_{1,-1}$ and the center-of-mass momentum $Q_z$ evolve with the chemical potential $\mu$ with fixed $h = E_r$, $E_{b2} = E_r$, and $E_{b1} = 1.15E_r$. For this typical parameter set, the system can successively go through multiple phase transitions by increasing $\mu$. In the local density approximation where the effect of a global trapping potential is taken into account by the spatial variation of chemical potential, this is the order of phases that one would observe starting from a trap edge to its center.

To further characterize the properties of these FF states, we demonstrate in Fig. 6 the typical gapless contours and dispersion spectra in momentum space. For the gapless contours, we find that there are two closed gapless rings in nFF1, and the two rings are both symmetric about the $k_y = 0$ axis (see Figs. 6(a) and (b)). By decreasing the chemical potential $\mu$, the two rings become smaller in size, and gradually separate from each other. The gapless contours of nFF2 are shown in Figs. 6(c) and (d). Different from the two gapless rings in nFF1, there is only one gapless ring in nFF2 with given parameters. This gapless ring is symmetric with respect to the origin. As the chemical potential decreases, the gapless ring becomes smaller. We also show in Figs. 6(e-h) the quasiparticle and quasihole dispersion spectra of nFF1 and nFF2 along the $k_y = 0$ axis. These are consistent with the corresponding gapless contours in Figs. 6(a-d).

In principle, one may probe these dispersion spectra experimentally using momentum-resolved radio-frequency spectroscopy.
The interesting features of the gapless contours and the dispersion spectra also leave signatures in the particle-number distribution in momentum space, which may be probed more directly via the time-of-flight measurement. We show in Fig. 7 the number density distributions in momentum space along the $k_y = 0$ axis for different nodal FF states with the same parameters as in Fig. 6. For the nFF1 case with $E_{k1} = 2E_r$, it can be clearly seen from Figs. 7(a-b) that abrupt changes are present in the momentum-space density profiles, particularly for the $|−1⟩$ state. Comparing to the corresponding results of gapless contours, one can find that the discontinuous features are consistent with the right half of the gapless contours. For the nFF2 phase as shown in Figs. 7(c-d), similar discontinuities can be found in the momentum distribution of the $|0⟩$ state, which are consistent with the corresponding structure of gapless contours.

VI. SUMMARY

We have studied the properties of the SOC-induced FF pairing states in a two-dimensional three-component Fermi gas with the recently realized synthetic SOC at zero temperature. The FF state here is the result of SOC-induced asymmetric momentum distribution of hyperfine states and the spin-selective interaction, both of which are experimentally achievable. To illustrate this, we investigate in detail the impact of different combinations of spin-selective interactions on the properties of the pairing states of the system. Interestingly, the interplay of SOC and spin-selective interaction can give rise to a novel three-component FF state, in which every two of the three components form an FF pairing state with a common center-of-mass momentum. We study in detail the stability region of the FF states, the dispersion spectra of quasiparticle/quasihole excitations, the gapless contours and the number distributions in momentum space, and discuss possible experimental detection schemes based on our results. As both the synthetic SOC and the spin-selective interactions have been realized experimentally, our work has interesting implications for future experiments on SOC-induced exotic superfluidity.

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