Application of Advance Constrained Simplex Method for MIMO Systems in Quantum Communication Networks

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Abstract: This paper first describes advanced constrained simplex method (advanced complex method), then it shows that this complex procedure has any problems when it’s taken for finding the maximum of a general nonlinear function of several variables within a constrained region is described in wireless communication systems, especially for multiple-input multiple-out (MIMO Configuration). Next advanced constrained simplex method is described how to resolve the problem of the multiple-input multiple-out, and shown how to be efficient compared with the complex method and the simplex method by some simulations. And this wireless network design can be used to MIMO systems in Quantum Communication networks. The feature of technology by which the system told by this paper can get optimum solution by a little search number of times compared with a conventional system in the MIMO environment with more than one optimal value, and is at the place. This system was applied to the Quantum network environment by this paper. This can achieve more compact than the conventional Quantum network environment.

Keywords: Quantum Communication Networks; Application of Artificial Intelligence (AI); MIMO Systems

1. Introduction

In Quantum Communication networks, resolution is several thousand times. Quantum networks form an important element of quantum computing and quantum cryptography systems. Quantum networks allow for the transportation of quantum information between physically separate quantum systems. In distributed quantum computing, network nodes within the network can process information by serving as quantum logic gates. Secure communication can be implemented using quantum networks through quantum key distribution algorithms. All directions type antenna is needed. But a physical problem occurs to make this request satisfied with the Quantum Communication networks environment.

A MIMO channel antenna is given as the means to settle this problem. Estimating mutual information and multi information in large networks describes information theoretic measures correlated with independent, intuitive measures of the underlying structures in a MIMO channel antenna.

This paper has 2 points.

The 1st point is that the design of a high output antenna by a small size. The 2nd is the point that software was utilized to achieve the above. Focuses upon clustering are because a corrugation from more than one direction intervened in the reason that paper was replaced by space coding, and there was a problem that optimal value isn’t obtained.

And even if there is a problem above-mentioned from easiness of dealing with a data, the 2nd point is adopted much now.

MIMO systems are established the use of the direct estimation method. More sophisticated estimation tools are available which allow reliable inference from smaller data sets, but these tools need to be scaled for application to large networks\cite{1,3,20,22-24}. And, an important aspect of the MIMO systems are the estimation of multi–information. Mobile
propagation modeling (MS and BS) is described in Figure 1. And Figure 2 describes maximal Ratio Combining in MIMO configuration. In MIMO Configuration, most important thing is how to determine the optimum weights.

Many method have been proposed to solve the estimation of multi–information problem. But there are many kind of angular distribution function in the Mobile Propagation Modeling (MS and BS)\cite{2,4-10,12,14-17,19}. One angular distribution is normal distribution, and another angular distribution is in the multiplicative process or the additive process. In this case this problem cannot be solved. So the advanced constrained simplex method was developed\cite{21}.

This paper first describes advanced constrained simplex method (advanced complex method), then it shows that this complex procedure has any problems when it’s taken for finding the maximum of a general nonlinear function of several variables within a constrained region is described in wireless communication systems, especially for multiple-input multiple-out (MIMO Configuration).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure1}
\caption{Linear chain of N quantum repeaters \( r_1, \ldots, r_N \) between the two end-users, Alice \( a = r_0 \) and Bob \( b = r_{N+1} \). The chain is connected by \( N+1 \) quantum channels \( \{ \varepsilon_i \} \).
}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure2}
\caption{Optimal performance of lossy chains. Capacity (target bits per chain use) versus total loss of the line (decibels, dB) for \( N = 1,2,10 \) and 100 equidistant repeaters. Compare the repeater-assisted capacities (solid curves) with the point-to-point repeater-less bound (dashed curve).
}
\end{figure}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{Chain (C)} & \textbf{Network (N) - Single-path capacity (C(N))} & \textbf{Network (N) - Multi-path capacity (C''(N))} \\
\hline
Lost channels (transmissivity \( \eta \)) & \(-\log_2 (1 - \eta) \) & \(-\log_2 (1 - \eta) \) \\
Q-limited amplifiers (gain \( g \)) & \(-\log_2 (1 - \max_i Q_i g) \) & \(-\log_2 (1 - \max_i Q_i g) \) \\
Dephasing channels (probability \( p \)) & \(-H_2 (\max_i p) \) & \(-H_2 (\max_i p) \) \\
Ensure channels (probability \( p \)) & \(-\max_i [H_2 (\max_i p)] \) & \(-\max_i [H_2 (\max_i p)] \) \\
\hline
\end{tabular}
\caption{Analytical formulas for the end-to-end capacities of distillable chains and networks}
\end{table}

Next advanced constrained simplex method is described how to resolve the problem of point-to-point protocols of quantum and private communication are their multiple-user architecture.

Then compute the most entropic distribution, i.e., and take the maximization \( \max_i H(P_i) \).

This is the bottleneck that determines the repeater capacity, so that,

\[ C = \log_2 d - \max_i H(P_i) \]

Generalization to dimension \( d \) is also immediate for the two network capacities \( C \) and \( C_m \).

Next advanced constrained simplex method is described how to resolve the problem of the multiple-input multiple-output, and shown how to be efficient compared with the complex method and the simplex method by some simulations. And this wireless network design can be used to radar detection of weak signals for SIMO and MIMO systems.

Then we can take this problem as the problem of nonlinear.
Received signal vector of m'th terminal:
\[ r^{(m)} = A^{(m)} W_m S_m(t) + A^{(m)} \sum_{k=1, k \neq m}^K w_k S_k(t) + n^{(m)} \]

Maximizing SNR of rx (if possible):
\[ \mu^{(m)}(t) = w_m^H A^{(m)H} A^{(m)} w_m S_m(t) + \text{imppairment} = w_m^H B^{(m)} w_m S_m(t) + \text{imppairment} \]

Where
\[ B^{(m)} = A^{(m)H} A^{(m)} \]

Weight vector for transmit MRC:
\[ W_m^* = e^{(m)}_{\text{max}} \]

Where
\[ e^{(m)}_{\text{max}} : \text{Eigenvector corresponding to maximum eigenvalue} \]

2. Reflection

Same as simplex method, complex method can solve the problem without object function’s figure. But, if there is optimized point near the constrained concision, or optimized point does not satisfy the constrained condition, complex method dose not solve the problem depend on the initial point.

So we give new algorithm to advance constrained simplex method. This algorithm is:

If the new point is the worst point, then now research direction is added new constrained condition, and research restart from initial point, and then this initial point is the best point of now research step.

The Figure 3 and 4 show the flow chart of the complex method.

Figure 3. Diamond quantum network \( N^0 \). a This is a quantum network of four points \( P = \{P_0, P_1, P_2, P_3\} \), with end-points \( P_0 = a \) (Alice) and \( P_3 = b \) (Bob). Two points \( P_1 \) and \( P_2 \) are connected by an edge \( (p_1, p_2) \) if there is an associated quantum channel \( e_{ij} \). This channel has corresponding resource state \( \sigma_{ij} \) in a simulation of the network. There are four (simple) routes: 1. \( a \rightarrow P_1 \rightarrow b \); 2. \( a \rightarrow P_2 \rightarrow P_3 \rightarrow b \); and 3: \( a \rightarrow P_2 \rightarrow P_1 \rightarrow b \); and 4: \( a \rightarrow P_1 \rightarrow P_2 \rightarrow b \). As an example, route 4 involves the transmission through the sequence of quantum channels \( \{e_{ij}\} \) which is defined by \( e_{01}^1 = e_{01}^2 = e_{12}^4 \) and \( e_{23}^4 = e_{32}^4 \). b We explicitly show route \( \omega = 4 \). In a sequential protocol, each use of the network corresponds to using a single route \( \omega \) between the two end-points, with some probability \( P_\omega \). c We show an entanglement cut C of the network, with super Alice A and super Bob B made by the points in the two clouds. These are connected by the cut-set \( \overline{C} \) composed by the dotted edges.

Figure 4. Network protocols of quantum and private communication. a In a sequential protocol, systems are routed through a single path probabilistically chosen by the points. Here it is a - p1 - p2 - b. Each transmission occurs between two adaptive LOCCs, where all points of the network perform LOs assisted by two-way CC. b In a flooding protocol, systems are simultaneously routed from Alice to Bob through a sequence of multipoint communications in such a way that each edge of the network is used exactly once in an end-to-end transmission. Here we show a possible sequence a \( \rightarrow \{p_1,p_2\} \), p2\( \rightarrow \{p_1,b\} \), p1\( \rightarrow \{b\} \). Each multipoint communication occurs between two adaptive LOCCs.
Figure 5. Mobile Propagation.

Figure 6. Maximal Ratio Combining in MIMO

(1) Summary of the method

This paper proposes the advanced constrained simplex method, which solves problem within constrained condition, and does not depend on the figure of object function or initial point. The Figure 7 shows the flow chart of the advanced constrained simplex method.

The equality constrained nonlinear programming problem is

Minimize \( f(x) \)

Subject to \( g_i(x) = 1, \ j = 1, m, \) \hspace{1cm} (1)

Where \( x = (x_1, x_n)^T, \ n > m, \) and both \( f(x) \) and \( g_i(x) \) are assumed to have continuous second partial derivatives.

Step 1) A first point is set, which can be satisfied constraint function.

Step 2) Decide \( n \) points which is produced by random function.

Step 3) If \( m \) (\( m \leq n \)) th value is not sufficient constraint function, 1 through to the \( (m-1) \) th point of middle point \( m \) th point move half distance to the middle point of the 1 through to the \( (m-1) \) th point. If this new point is not better than old \( C \) point, and \( n+1 \) points are within 0.000001, then this research has finished. The iterative procedure is continued to estimate the minimized.

Step 4) Above \( n+1 \) points are estimated by object function.

Step 5) The point \( A \) is the middle point of the \( n \) points without the best value point. The point \( B \) is the middle point of the \( n \) points without the worst value point.

Step 6) There is the point \( C \) at the opposite side of point \( A \) with point \( B \). And point \( C \) to point \( B \) is 2 times longer than point \( A \) to point \( B \).

Step 7) If point \( C \) is not satisfied constrains function, next point \( C \) is middle of point \( C \) and \( B \).

If point \( C \) is not satisfied until distance of point \( C \) and point \( B \) is less than 0.000001, point \( B \) is point \( C \).

Step 8) A method that has been the subject of much recent research for the solution of (1) is the sequential quadratic programming (SQP) technique\(^{11,13,18}\). A current estimate \( x^{(k)} \) to the minimized \( x^* \) is known, and a search direction
\( s^{(c)} \) is generated to solve the quadratic programming problem.

\[
\begin{align*}
\text{Minimize} & \quad \frac{1}{2} s^{(k)T} B^{(k)} s^{(k)} + s^{(k)T} \nabla f(x^{(k)}) \\
\text{Subject to} & \quad \nabla g(x^{(k)})^T s^{(k)} + g(x^{(k)}) = 0,
\end{align*}
\tag{2}
\]

Where \( \nabla g(x^{(k)}) = \{ \nabla g_1(x^{(k)}), \ldots, \nabla g_m(x^{(k)}) \} \), the matrix of the constraint normal evaluated at \( x^{(k)} \), \( g^{(k)} \) is the vector \( (g_1(x), \ldots, g_m(x))^T \), and \( B^{(k)} \) is a positive definite approximation to the Hessian of the Lagrange with respect to \( x \). Here the Lagrange is

\[
l(x, \lambda) = f(x) - \lambda^T g(x)
\]

Where \( \lambda = (\lambda_1, \ldots, \lambda_m)^T \) is the vector of Lagrange multipliers, and \( B^{(k)} \) is an estimate to \( I_x (x^{(k)}, \lambda^{(k)}) \) for an estimate \( \lambda^{(k)} \) to the optimal Lagrange multipliers \( \lambda^* \).

Step 9) in step 8, a new point is found. And this point is estimated object function. If this new point is not good more than old C point, and \( n+1 \) points are not within 0.000001, then we add to new constraint function that it is point B to point C line function. And now the best point is decided original point. Go to step 2.

If this new point is better than old C point, then this new point is new C point. Go to step 5.
Figure 7. The flow chart of the complex method 1.
3. Evaluation

Three cases are evaluated on the simulation.

Case 1:
Object function and constraint function are normal. And the existing method can solve that.

Case 2:
The figure of object function is very narrow, and the existing method cannot solve that without the complex method.

Case 3:
There are two object functions in the research field. In this case, the complex method cannot solve this problem.
The Figure 9, 10, 11 show the proceeding of case 1. The Figure 12, 13, 14 show the proceeding of case 2. The Figure 15, 16, 17 show the proceeding of case 3.
Figure 11. The processing of case 1 (3/3).

Figure 12. The processing of case 2 (1/3).

Figure 13. The processing of case 2 (2/3).

Figure 14. The processing of case 2 (3/3).

Figure 15. The processing of case 3 (1/3).

Figure 16. The processing of case 3 (2/3).
Figure 17. The processing of case 3 (3/3).

|                  | Case 1 | Case 2 | Case 3 |
|------------------|--------|--------|--------|
| The classical simplex method | 36     | 60     |        |
| This complex method      | 3      | 5      | 6      |

Table 2. Compare calculation times with the classical simplex method and this complex method

3. Conclusion

This paper proposed MIMO systems in Quantum Communication networks. There are three main contributions. First, the networking factor of Quantum Communication networks for MIMO systems can be conformed. Second, advanced constrained simplex method is described. Finally, advanced constrained simplex method is described how to resolve the problem of the multiple-input multiple-out.

When a MIMO system calculates the weight of each antenna, the strength of the radio wave in the radio reception territory is clustered and next search optimum solution in clustering is searched. Therefore to need much calculated amount, when getting weight of the correct and most suitable antenna, the feature of the MIMO system couldn’t be utilized. This proposed method is searching for the optimum solution while clustering. This can achieve more compact than the conventional Quantum network environment.

Therefore this advanced constrained simplex method can reduce calculated amount, when getting weight of the correct and most suitable antenna, the feature of the MIMO system in Quantum Communication networks could be utilized.

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