Quarks, diquarks and QCD mixing in the $N^*$ resonance spectrum

Qiang Zhao\textsuperscript{1,2}† and Frank E. Close\textsuperscript{3}‡
1) Institute of High Energy Physics, Chinese Academy of Sciences, Beijing, 100049, P.R. China
2) Department of Physics, University of Surrey, Guildford, GU2 7XH, United Kingdom
3) Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Keble Rd., Oxford, OX1 3NP, United Kingdom

(Dated: October 21, 2018)

PACS numbers: 12.39.-x, 13.60.-r, 13.60.Rj

I. INTRODUCTION

It is remarkable that forty years after the quark model was first applied to the problem of baryon resonances\textsuperscript{1} it is still not well established whether three constituent quarks are the minimal effective degrees of freedom or whether a quark-diquark dynamics, where a pair of quarks is “frozen” into a ground state, suffices. Indeed, the vast literature inspired by the apparent discovery of a metastable “pentaquark” baryon in 2003\textsuperscript{2} showed both how little the strong dynamics of quarks is understood and raised renewed speculation about the role and existence of highly correlated diquarks\textsuperscript{3,4}. Furthermore, an acceptable description of baryon resonance spectroscopy has been proposed based on a quark-diquark picture\textsuperscript{5}.

The relative coordinate between the two quarks forming the diquark is constrained to be in the $l_\rho = 0$ state. (We use the symbols $\rho$, $\lambda$ as in Ref\textsuperscript{6} to denote the antisymmetric and symmetric two-body substates within a three-body wavefunction.) In such a case, the familiar and established $[SU(6), L_\rho]$ multiplets\textsuperscript{[56, $L_\rho$]} and $[70, L_\rho]$ occur, but it is impossible to form $[SU(6), l_\rho]$ correlations, $[20, L_\rho]\textsuperscript{[6]}$. Within a $qqq$ dynamics, where both $\lambda$ and $\rho$ spatial oscillators can be excited, the spectrum is richer and such $20$ states can occur.

Whereas $[56, L_\rho]$ and $[70, L_\rho]$ excitations are well established, the search for $[20, L_\rho]$ has been largely ignored, primarily because they cannot be excited by mesons or photons from a $56$ nucleon. This is because photons and $qq$ beams transform under SU(6) as $35$, which with the SU(6) forbidden transition $56\otimes 35 \rightarrow 20$ causes them to decouple from nucleons in naive SU(6)\textsuperscript{[7,8]}. The purpose of this paper is to re-examine the assumptions underlying resonance production in the quark model. This will lead us to a selection rule, that appears to have been overlooked in the literature, and also to identify circumstances where $20$ states can be excited.

A standard and phenomenologically successful assumption common to a large number of papers in the quark model is that photon transitions are additive in the constituent quarks\textsuperscript{[9, 10, 11]}. This assumption also underlies models of hadronic production and decay in the sense that when $q_1q_2q_3 \rightarrow |q_1q_2q_1| + |q_3q_3|$, the quark pair $q_1q_2$ are effectively spectators and only $q_3$ is involved in driving the transition. Such approximations lead to well known selection rules, which have proved useful in classifying resonances\textsuperscript{[11]}. We adopt this approximation as a first step and show that within it there is a further selection rule that appears to have been overlooked in the literature. We shall refer to this as the “$\Lambda$ selection rule” and show how it may help classify $N^*$ resonances.

The above “spectator-hypothesis” for transition amplitudes will be violated by the spin-dependent forces that act between pairs of quarks and break SU(6), such as those generating the $N-\Delta$ mass gap. For example, when the nucleon is in an electromagnetic field, gauge invariance and the presence of such two-body forces imply that there occur diagrams where the photon interacts with quark number 3, say, and the exchange force acts between quarks 3 and 1 or 2. Thus electromagnetic interactions can transfer momentum to both $\lambda$ and $\rho$ oscillators, which both spoil the spectator-hypothesis and opens the possibility of exciting $20$-plets.

* Dedicated to the memory of R.H. Dalitz.
† e-mail: Qiang.Zhao@surrey.ac.uk
‡ e-mail: F.Close@physics.ox.ac.uk
As a specific and quantifiable example we shall assume these spin-dependent forces arise from gluon-exchange in QCD. This has considerable quantitative support \[12\] and also has been shown to induce mixings, including $^{70}_{35}$-plets configurations, into the nucleon wavefunction \[13\]. Taking into account that the $N$-$\Delta$ mass gap of 300 MeV is on the scale of $\Lambda_{QCD}$, the resulting mixing effects can be sizeable. Whereas $^{56}_{35}$ $\rightarrow$ $^{20}_{15}$, the coupling $^{70}_{35}$ $\rightarrow$ $^{20}_{15}$ is allowed. Consequently the SU(6) breaking that induces $^{70}_{35}$ correlations into the nucleon enables the excitation of $^{20}_{15}$-plets by photons and mesons. Interestingly, we shall see that the $\Lambda$ selection rule still manifests itself in $N^* \rightarrow K\Lambda$ transitions even though the SU(6) symmetry is broken. This phenomenon will be useful for clarifying the excitation of $^{20}_{15}$-plets in the $N^*$ spectrum.

In the next section we first present the “$\Lambda$ selection rule”. We then show how QCD-generated wavefunction mixing allows production of $^{20}_{15}$-plets. We formulate these ideas in a QCD quark model \[13\] though the qualitative results should be more generally true. In the final section we quantify these effects and discuss their application to $N^*$ classification.

II. THE MODEL

For reference, we specify our nomenclature. The standard SU(6)$\otimes$O(3) wavefunction can be constructed from three fundamental representations of group $S_3$:

$$SU(6): \quad 6 \otimes 6 \otimes 6 = 56_s + 70_\rho + 70_\lambda + 20_a,$$

where the subscripts denote the corresponding $S_3$ basis for each representation, and the bold numbers denote the dimension of the corresponding representation. The spin-flavor wavefunctions can be expressed as $|N_6, {2S+1}N_3\rangle$, where $N_6$ ($=56$, $70$ or $20$) and $N_3$ ($=8$, $10$, or $1$) denote the SU(6) and SU(3) representation and $S$ stands for the total spin. The SU(6)$\otimes$O(3) (symmetric) wavefunction is

$$|\text{SU}(6) \otimes \text{O}(3)\rangle = |N_6, {2S+1}N_3, N, L, J\rangle,$$

where explicit expressions follow the convention of Isgur and Karl \[14, 15, 16\].

The basic rules follow from application of the Pauli exclusion principle to baryon wavefunctions together with an empirically well tested assumption that electroweak and strong decays are dominated by single quark transitions where explicit expressions follow the convention of Isgur and Karl \[14, 15, 16\]. For example, the Moorhouse selection rule \[17\] states that transition amplitudes for $\gamma p$ to all resonances of representation \[70, 4^8\], such as $D_{15}(1675)$, must be zero due to the vanishing transition matrix element for the charge operator.

Such correlations also lead to a “$\Lambda$ selection rule”, which appears to have been overlooked in the literature. It states that $N^*$ in \[70, 4^8\] decouple from $\Lambda K$ and $\Lambda K^*$ channels. This follows because the $[ud]$ in the $\Lambda$ has $S = 0$ and in the spectator approximation, the strangeness emissions in $N^* \rightarrow \Lambda K$ or $\Lambda K^*$, the spectator $[ud]$ in the $N^*$ must also be in $S_{[ud]} = 0$, whereby such transitions for the $N^*$ of \[70, 4^8\] with $S_{[ud]} = 1$ are forbidden.

Note that the $\Lambda$ selection rule applies to both proton and neutron resonances of \[70, 4^8\], in contrast to the Moorhouse selection rule, which applies only to the proton. The nearest that we can find to this in the literature is that $\Lambda^*(70, 4^8) \rightarrow KN$ \[18\]. While the associated zero in $N^*(70, 4^8) \rightarrow K\Lambda$ is implicit in work that has calculated the couplings of baryon resonances \[19\], the source and generality of the rule does not seem to have been noted \[20\].

III. APPLICATION TO $N^*$ SPECTRUM

An immediate application of the rules is to the $D_{15}(1675)$, which is in \[70, 4^8\]. According to the Moorhouse selection rule, the amplitudes for $\gamma p \rightarrow D_{15}$ should vanish. However, the experimental values are not zero, though they are small. Non-zero amplitudes arise from QCD mixings induced by single gluon exchange in the physical nucleon \[13\].

The effective interaction

$$H_{FB} = \frac{2\alpha_s}{3m_cm_j} \left[ \frac{8\pi}{3} S_i \cdot S_j \delta^3(r_{ij}) + \frac{1}{r^3_{ij}} \left( \frac{3(S_i \cdot r_{ij})(S_j \cdot r_{ij})}{r^2_{ij}} - S_i \cdot S_j \right) \right]$$

induces significant mixings between the $^28$ and $^48$ in the $^{56}_{35}$ and $^{70}_{35}$ \[10\] and the nucleon wavefunction becomes \[13\]

$$|N\rangle = 0.90|S_S\rangle - 0.34|S_{S'}\rangle - 0.27|S_M\rangle - 0.06|D_M\rangle,$$
where subscripts, $S$ and $M$, refer to the spatial symmetry in the $S$ and $D$-wave states for the nucleon internal wavefunction. Thus, the $O(\alpha_s)$ admixtures at $N = 2$ comprise a 34% in amplitude excited $56$ and $27\%$ $70$ each with $L = 0$ and $6\%$ $70$ with $L = 2$. The $70$ admixture quantitatively agrees with the most recent data [21] for the $\gamma p \to D_{15}$ amplitudes, neutron charge radius and $D_{05} \to KN$ [13]. The results assume that mixing effects in the $D_{15}$ are negligible relative to those for the nucleon [13]: this is because there is no $[70,2; L^p = 1^-]$ state available for mixing with the $D_{15}$, and the nearest $J = 5/2$ state with negative parity is over 500 MeV more massive at $N = 3$. Within this $O(\alpha_s)$ analysis, such mixing is negligible: transitions from the large components of the nucleon to small in $D_{15}$ have $\Delta N = 3$. The leading $O(\alpha_s)$ amplitude for $\gamma p \to D_{15}$ is dominantly driven by the small components in the nucleon and the large component in the $D_{15}$ [15], for which $\Delta N = 1$.

This violation of the Moorhouse selection rule supports the hypothesis of QCD mixing in the wavefunction of the $N$. The $\Lambda$ selection rule also remains robust, in the context of the diquark model, as admixtures of $[ud]$ with spin one, which would violate it, are only expected at most to be $20\%$ in amplitude [22], to be compared with $27\%$ for the nucleon in Eq. (4). Therefore, we expect the $\Lambda$ selection rule to be at least as good as the Moorhouse rule even at $O(\alpha_s)$. Thus decays such as $D_{15} \to K\Lambda$ will effectively still vanish relative to $K\Sigma$; for the $D_{15}(1675)$ the phase space inhibits a clean test but the ratio of branching ratios for the analogouse state at $\gamma p \to N$ make an effective bosonic “diquark” with mass comparable to that of an isolated quark. Comparison of masses of $N^*(u[ud])$ and mesons $ud$ with $L \geq 1$ support this hypothesis of a tight correlation, at least for excited states [3,22]. If the quark-diquark dynamics is absolute, then $SU(6) \otimes O(3)$ multiplets such as $[20,1^+]$ cannot occur. The spatial wavefunction for $20$ involves both $\rho$ and $\lambda$ degrees of freedom; but for an unexcited diquark, the $\rho$ oscillator is frozen. Therefore, experimental evidence for the excitations of the $20$ plts can distinguish between these prescriptions.

Pauli symmetry requires an antisymmetric spatial wavefunction for a $20$ e.g. for the lowest state $[20,1^+]$

$$\psi_{211}(\rho, \lambda) = \sqrt{2}(\rho + \lambda_2) - \lambda_2 + \rho_2) \frac{\alpha_5}{\pi^{5/2}} e^{-\alpha_5^2 (\rho^2 + \lambda^2)^{1/2}},$$

where $\rho_\pm \equiv \pm(\rho_x \pm i \rho_y)/\sqrt{2}$ and $\lambda_\pm \equiv \pm(\lambda_x \pm i \lambda_y)/\sqrt{2}$, and hence both $\rho$ and $\lambda$ degrees of freedom have to be excited. Furthermore $20$ decouples from $35 \otimes 56$ but is allowed to $35 \otimes 70$. A $27\%$ $70$ admixture in the nucleon has potential implications for resonance excitation that may be used to look for $20$-plets. For transitions between representation $[70,2; 8, 2, 0, 1/2]$ and $[20,2; 8, 2, 1, J]$ the matrix element $\langle \psi_{210} | e^{ikr_3} | \psi_{200} \rangle \equiv 0$. Non-zero photon transitions can occur by the orbital flip “electric” term. Since $L = 1$, the nucleon component $[70,2; 8, 2, 0, 1/2]$ can be excited to $J = 1/2$ and $3/2$ corresponding to $P_{11}$ and $P_{13}$ in $[20,2; 8, 2, 1, J]$. The helicity amplitudes (not including the mixing angle) are presented in Table I with the spatial integral

$$A = 6 \sqrt{\frac{\pi}{k_0}} \mu_0 \frac{1}{g} \langle \psi_{110} | e^{ikr_3} p_3 \psi_{200} \rangle$$

$$= -6 \sqrt{\frac{\pi}{k_0}} \mu_0 \frac{1}{g} \times \frac{2k}{3\sqrt{3}} e^{-k^2/6\alpha_5^2}.$$ 

These amplitudes may be compared with those for $\gamma p \to D_{15}$ as listed in Table I for which the spatial integral is

$$B = 6 \sqrt{\frac{\pi}{k_0}} \mu_0 k \langle \psi_{110} | e^{ikr_3} \psi_{200} \rangle$$

$$= 6 \sqrt{\frac{\pi}{k_0}} \mu_0 k \frac{k}{3\alpha_5} e^{-k^2/6\alpha_5^2}.$$ 

Additional $P_{11}$ and $P_{13}$ from representation $20$ automatically raise questions about the quark model assignments of the observed $P_{11}$ and $P_{13}$ states, among which $P_{11}(1440)$, $P_{11}(1710)$, and $P_{13}(1720)$ are well-established resonances, while signals for $P_{13}(1900)$ and $P_{11}(2100)$ are quite poor [21].
IV. POSITIVE PARITY $N^*$ UP TO 2 GEV

At $N = 2$ in the quark model a quark-diquark spectrum allows $[56, 0^+]$, $[56, 2^+]$, and $[70, 0^+][70, 2^+]$. If all $qqq$ degrees of freedom can be excited, correlations corresponding to $[20, 1^+]$ are also possible. This implies the following $N^*$ and $\Delta$ states (the superscripts denoting the $qqq$ net spin state as $2S + 1$):

$$
[56, 0^+] = P_{11}(2N); P_{33}(4\Delta)
$$

$$
[56, 2^+] = P_{13}, F_{15}(2N); P_{31}, P_{33}, F_{35}, F_{37}(4\Delta)
$$

$$
[70, 0^+] = P_{11}(2N); P_{33}(4N); P_{31}(4\Delta)
$$

$$
[70, 2^+] = P_{11}, P_{13}, F_{15}, F_{17}(2N); P_{31}, F_{15}(2N); P_{33}, F_{35}(2\Delta).
$$

(8)

Without the $20$-plets, a commonly accepted scheme is as follows $[6, 10]$. The $P_{11}(1440)$ is assigned to $[56, 0^+]$ at $N = 2$ with $P_{33}(1660)$ as its isospin $3/2$ partner $[24, 25]$. The photoproduction amplitudes off proton and neutron, $A_{1/2}^p = -0.065 \pm 0.004$ and $A_{1/2}^n = +0.040 \pm 0.010 (\text{GeV})^{-1/2}$, are consistent with M1 transitions in ratio $\sim 3 : 2$ as for the nucleons $[9, 10, 21]$. The mass splitting of the $N(1440) - \Delta(1660)$ is consistent with the hyperfine splitting.

For the $[56, 2^+]$ the $F_{17}(1950)$ is uniquely assigned. The hyperfine mass splitting naturally associates the $F_{15}(1680)$ as a partner; this is further confirmed by its photoproduction amplitudes, which satisfy selection rules from both proton and neutron targets $[9, 10]$. The $P_{13}(1720)$ is prima facie associated with the $F_{15}(1680)$ but the photoproduction amplitudes do not easily fit; the helicity $3/2$ amplitude of the $P_{13}$ is predicted to be one-half that of the $F_{15}$, which does not fit well with the data $[21]$. The overall message from the $56$ is that the constituent quark degrees of freedom are manifested at $N = 2$ even though $N\pi$ couplings are strong as evidenced by $\Gamma_{\gamma} \sim 350$ MeV for $P_{11}(1440)$. This sets the challenge of assessing the situation for the rest of the $N = 2$ levels.

Only three $P_{11}$ states are expected at $N = 2$ if the $20$-plets cannot be excited. As to assigning these states: (i) The $P_{11}(1440)$ has already been assigned to $56$; (ii) the $P_{11}$ of $[70, 4^+; 2^+]$ is expected to be heavier $[10]$ and the signals for $F_{17}(1900), F_{15}(2000), P_{33}(1900)$ and $P_{13}(2100)$ $[21]$ lead to a plausible assignment of these four in $[70, 4^+; 2^+]$; (iii) this leaves $P_{11}(1710)$ a natural candidate for $[70, 2^+; 2^+]$. Nonetheless, there are still states of $[70, 2^+; 2^+]$ missing, and another $P_{13}$ and $F_{15}$ are needed.

Within the approximations that exist and decays are dominated by single quark transition, and that there is no mixing among the $56$ and $70$ basis states, the assignments of $P_{13}$ and $F_{15}$ can be determined. The Moorhouse and $\Lambda$ selection rules give the following filters for these states: $P_{33}(2N)$ in $[70, 0^+]$ decouples from $K\Lambda$ and $\gamma\gamma$ but couples to $\gamma\gamma$. This contrasts with $P_{13}(2N)$ in either of $[70, 2^+]$ or $[56, 2^+]$ which couple to all three channels. The helicity $3/2$ amplitude of $F_{15}$ in either of the $[70, 2^+]$ or $[56, 2^+]$ couples to photons with amplitude that is twice the size of its $P_{13}$ counterparts.

Table [11] shows the photoproduction amplitudes expected. The $P_{11}(1710)$ fits with $[70, 2^+; 2^+, 0, 1/2]$ due to the large error bars, however neither of the $P_{13}(1710/1900)$ fit easily with being pure $56$ or $70$ states. When the QCD mixing effects are included the agreement improves, in that small couplings of $4N$ states to $\gamma\gamma$ are predicted, in accord with data. However, the implication is the added complexity that an additional $P_{11}$ and $P_{13}$ correlation in $[20, 1^+]$ is allowed. Most immediately this prevents associating the $P_{11}(1710)$ as $[70, 0^+]$ simply on the grounds of elimination of alternative possibilities. Thus we now consider what are the theoretical signals and what does experiment currently say.

Qualitatively one anticipates $P_{13}(4N)$ having a small but non-zero coupling to $\gamma\gamma$, the $\gamma\gamma$ being larger while the $\Lambda\Delta$ decay is still forbidden. For the $20$ states $P_{11, 13}(4N)$ both $\gamma\gamma$ and $\gamma\gamma$ amplitudes will be small and of similar magnitude. However, mixing with their counterparts in $56$ and $70$ may be expected. In Table [11] we list the helicity amplitudes for the $P_{13}(1710), P_{13}(1720)$ and $P_{13}(1900)$ with all the possible quark model assignments and the mixing angles from Eq. (3). The amplitudes for the $P_{11}$ and $P_{13}$ of $[20, 2^+; 2, 1, J]$ are the same order of magnitude as the Moorhouse-violating $\gamma\gamma \rightarrow D_{15}(1675)$ in Table [11].

For the $P_{11}(1710)$ all three possible configurations have amplitudes compatible with experimental data. For $P_{13}(1720)$, assignment in either $[56, 2^+; 2^+]$ or $[70, 2^+; 2^+]$ significantly overestimates the data $[27, 28]$ for $A_{1/2}^p$ if it is a pure state. Table [11] shows that the presence of $20$ cannot be ignored, should be included in searches for so-called “missing resonances”, and that a possible mixture of the $20$-plets may lead to significant corrections to the results based on the conventional $56$ and $70$.

As a benchmark for advancing understanding we propose the following scenario, as a challenge for experiment: can one eliminate the extreme possibility that $P_{13}(1710)$ and $P_{13}(1720)$ are consistent with being in $20$ configurations? There are already qualitative indications that they are not simply $56$ or $70$. Their hadronic decays differ noticeably from their sibling $P_{11}(1440)$; compared with the $P_{11}(1440)$ in $56$ for which $\Gamma_{\gamma} \sim 350$ MeV with a strong coupling to $N\pi$, their total widths are $\sim 100$ MeV, with $N\pi$ forming only a small part of this.

Consider now the phenomenology were these states in $20$. As $20 \rightarrow 56 \otimes 35$, whereas $20 \rightarrow 70 \otimes 35$ is allowed, decay to $N\pi$ will be allowed only through the $70$ admixtures in the nucleon. Using the wavefunction in Eq. (4), this
implies that $N\pi$ widths will be suppressed by an order of magnitude relative to allowed widths such as from 56 or 70 initial states. Decays $20 \rightarrow N^*(70) + \pi \rightarrow N\pi\pi$ or $N\eta\pi$ can occur in leading order when phase space allows. These results make the $P_{11}(1710)$ and $P_{13}(1720)$ extremely interesting as not only are their total widths significantly less than the $P_{11}(1440)$ but the dominant modes for $P_{11}(1710)$ and $P_{13}(1720)$ are to $N\pi\pi$, which allows a possible cascade decay of $20 \rightarrow N^*(70)\pi \rightarrow N\pi\pi$. In particular, $B.R. P_{13}(1720) \rightarrow N\pi\pi \sim 70\%$ could contain significant $N^*(70)\pi$; present partial wave analysis has not included this possibility [24]. For $P_{11}(20)$ the S-wave decay $P_{11} \rightarrow S_{11}\pi$ is expected to be a dominant two-body decay; hence we urge a search for $P_{11} \rightarrow N\eta\pi$. Decays to 56 + $M$ (where $M$ denotes a meson) will occur through mixing, as for photoproduction, and so $K\Lambda$ may be expected at a few percent in branching ratio. In general we advocate that partial wave analyses of $N\pi\pi$ allow for the possible presence of $N^*(70)\pi$.

In order to classify these states, we advocate that partial wave analysis quantifies the couplings to $N^*(70)\pi$ and also includes data on $K\Lambda$ and/or $K^*\Lambda$ relative to $K\Sigma$ as the $\Lambda$ selection rule can then be brought to bear. The $\Lambda$ selection rule is useful for classifying the $P_{11}$ and $P_{13}$ in either [56,2] and [70,2] or [70,4] and [20,2], by looking at their decays into $K\Lambda$ and/or $K^*\Lambda$. The Moorhouse selection rule can distinguish $[70,4]$ and $[20,2]$ since the [70,4] will be suppressed in $\gamma p$ but sizeable in $\gamma n$, while the [20,2] will be suppressed in both. The [70,4] decays to $K\Lambda$ will be suppressed relative to $K\Sigma$ for both charged and neutral $N^\ast$. $J/\psi \rightarrow \bar{p} + N^\ast$ is a further probe of $N^\ast$ assignments, which accesses 56 in leading order and 70 via mixing while 20 is forbidden. Hence for example $J/\psi \rightarrow \bar{p} + (P_{11} : P_{13})$ probes the 56 and 70 content of these states. Combined with our selection rule this identifies $J/\psi \rightarrow \bar{p} + K\Lambda$ as a channel that selects the 56 content of the $P_{11}$ and $P_{13}$.

In summary, with interest in $N^\ast$ with masses above 2 GeV coming into focus at Jefferson Laboratory and accessible at BEPC with high statistic $J/\psi \rightarrow \bar{p} + N^\ast$, this new selection rule should be useful for classifying baryon resonances and interpreting $\gamma N \rightarrow K\Lambda$, $K^*\Lambda$, $K\Sigma$ and $K^*\Sigma$ [30, 31]. A coherent study of these channels may provide evidence on the dynamics of diquark correlations and the presence of 20-plets, which have hitherto been largely ignored.

Acknowledgement

We are grateful to G. Karl for helpful comments. This work is supported, in part, by grants from the U.K. Engineering and Physical Sciences Research Council (Grant No. GR/S99433/01), and the Particle Physics and Astronomy Research Council, and the EU-TMR program “Eurodice”, HPRN-CT-2002-00311, and the Institute of High Energy Physics, Chinese Academy of Sciences.

[1] R.H. Dalitz, p251, Les Houches 1965, eds. C. DeWitt and M. Jacob (Gordon and Breach, New York, 1965); p215, Proc. XIII International Conf., Berkeley 1966 (Univ. of California press).
[2] T. Nakano et al., Phys. Rev. Lett. 91, 012002 (2003).
[3] R. Jaffe and F. Wilczek, Phys. Rev. Lett. 91, 232003 (2003).
[4] M. Karliner and H.J. Lipkin, Phys. Lett. B 575, 249 (2003).
[5] F. Wilczek, arXiv:hep-ph/0409168.
[6] See S. Capstick and W. Roberts, Prog. Part. Nucl. Phys. 45, S241 (2000), and references therein.
[7] M. Anselmino, E. Predazzi, S. Ekelin, S. Fredriksson and D. B. Lichtenberg, Rev. Mod. Phys. 65, 1199 (1993).
[8] D. Faiman and A. W. Hendry, Phys. Rev. 173 (1968) 1720.
[9] L. A. Copley, G. Karl and E. Obryk, Nucl. Phys. B 13, 303 (1969).
[10] R. P. Feynman, M. Kislinger and F. Ravndal, Phys. Rev. D 3, 2706 (1971).
[11] F.E. Close, A. Donnachie and G. Shaw Electromagnetic Interactions and Hadronic Structure (Cambridge Univ. Press, 2006); F.E. Close, Introduction to quarks and partons (Academic Press 1978).
[12] A. De Rujula, H. Georgi and S. L. Glashow, Phys. Rev. D 12, 147 (1975).
[13] N. Isgur, G. Karl and R. Koniuk, Phys. Rev. Lett. 41, 1269(1978); Erratum 45, 1738(1980).
[14] N. Isgur and G. Karl, Phys. Lett. 72B, 109 (1977).
[15] N. Isgur and G. Karl, Phys. Rev. D 18, 4187 (1978).
[16] N. Isgur and G. Karl, Phys. Rev. D 19, 2653 (1979); Erratum 23, 817 (1981).
[17] R.G. Moorhouse, Phys. Rev. Lett. 16, 772 (1966).
[18] R. Cashmore, A.J.G. Hey and P.J. Litchfield, Nucl. Phys. B95, 516 (1975).
[19] S. Capstick and W. Roberts, Phys. Rev. D58, 074011 (1998).
[20] W.P. Petersen and J.L. Rosner, Phys. Rev. D 6, 820 (1972); D. Faiman, Phys. Rev. D 15, 854 (1977).
[21] S. Eidelman et al. (Particle Data Group), Phys. Lett. B 592, 1 (2004).
[22] N. Isgur and G. Karl, Phys. Rev. D 21, 3175 (1980).
[23] F.E. Close, Proc. of the 58th Scottish Summer School in Physics, St. Andrews (2004).
[24] L.D. Roper, Phys. Rev. Lett. 12, 340 (1964).
TABLE I: Helicity amplitudes for \([70, 2, 8, 0; \frac{1}{2}] \rightarrow [20, 2, 8, 2, 1, J]\) for \(P_{11}\) and \(P_{13}\), and \([70, 2, 8, 0; \frac{1}{2}] \rightarrow [70, 4, 8, 1, 1, J]\) for \(D_{15}\). \(A\) and \(B\) are the corresponding spatial integrals. The theory calculations with the mixing angle and experimental data \([21]\) for \(\gamma p \rightarrow D_{15}(1675)\) are also listed with unit \(10^3 \times \text{GeV}^{1/2}\).

TABLE II: Helicity amplitudes for the \(P_{11}(1710)\) and \(P_{13}(1720)\) with all the possible quark model assignments for them. The data are from PDG \([21]\), and numbers have a unit of \(10^3 \times \text{GeV}^{1/2}\). The numbers with ** are from multichannel studies \([26]\).