A Modified Poly-Harmonic Distortion Model Based on the Canonical Piecewise Linear Functions for GaN HEMTs

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This work was supported in part by the National Natural Science Foundation of China (NSFC) under Grant 61701147 and Grant 61971170, and in part by the Foundation of the State Key Laboratory of Millimeter Waves under Grant K202011.

\textbf{ABSTRACT} A novel, large-signal behavioral modeling methodology for Gallium Nitride (GaN) high-electron-mobility transistors (HEMTs), based on the canonical piecewise-linear (CPL) functions, is presented in this paper. The proposed new model employs the poly-harmonic distortion (PHD) model framework, making use of the CPL functions for interpolation of the amplitude of the dominant input signal. The CPL method is also applied to the quadratic PHD (QPHD) model framework, allowing for application to devices operating under high (greater than 3 dB) compression levels. Compared with the standard PHD/QPHD models, which require lengthy tables of parameter values to account for the varying large signal input power(s), the models described in this paper are able to predict transistor behavior at different levels of input power, from the linear region to the strongly nonlinear region (where gain compression exceeds 1 dB), with one single set of model coefficients. The basic theory of the proposed model for both RF and dc responses is provided in the paper. The proposed modeling technique is validated through simulated and experimental data from separate 6 W and 10 W GaN HEMT devices, over a wide range of load conditions and power levels. In addition, a two-dimensional polynomial-based model is used for performance comparison, with the proposed method providing comparable accuracy while requiring significantly fewer model coefficients.

\textbf{INDEX TERMS} Canonical piecewise-linear function, nonlinear behavioral modeling, poly-harmonic distortion model, RF power transistor.

I. INTRODUCTION

Due to the higher power density of GaN devices when compared with traditional compound semiconductors, such as gallium arsenide (GaAs) [1], there are many recent reports of GaN-based circuits and systems showing excellent performance [2]–[4]. As the performance of such systems is pushed to the physical limits, the requirement for highly accurate but compact GaN models is now stronger than ever.

Many different approaches have been developed for GaN device modeling [5]–[7], from physics-based models, equivalent circuit models, to behavioral models. Both physics-based and equivalent circuit-based models, of which the latter can be described as semi-physical/semi-empirical, show significantly improved performance over their behavioral counterparts when the devices are pushed to operating extremes. These models, however, while considered optimal for many existing devices, are often not easily nor quickly adapted for newer device technologies, such as GaN [8]. On the other hand, full black-box behavioral models can be developed very quickly, and can be more accurate in the regions for which they have been extracted.

With the development of microwave technology, circuit nonlinearities have been exploited to deal with the increasingly complex signal formats and power levels required, which creates a demand for behavioral models capable of handling circuits exhibiting significant nonlinearities. Thus, the PHD model for large-signal behavioral prediction,
introduced in [9]–[11], is targeted towards overcoming the limitation of the traditional scattering-wave formalism, which can only work under linear conditions. It is based on the concept of a describing function that relates the pseudowave phasors at the device ports, which, along with the adoption of the harmonic superposition principle [11], admit a first order Taylor series approximation. The approximation remains valid if the perturbation signals are sufficiently small [11]. This approximation became the first implementation of the X-parameter formulation. Since then, the load-dependent X-parameter model [11], [12], as well as the work of Cardiff University (e.g. the Cardiff model) [13], [14], has extended the first implementation of X-parameters to include further incident wave information in order to increase the generality of the model.

However, the large signal behavioral models discussed above all exhibit a model functional dependence on the dominant incident wave(s). In order to allow for the use of power values that are not directly used in the extraction i.e. to facilitate interpolation, the functional power dependence is typically implemented using spline functions. While this simplifies model implementation still requires a large number of measurements across a range of input power levels [15]. In general, for such models, reliable extrapolation is not possible.

Recent research has followed a few different approaches at overcoming the interpolation issue, in particular. In [16]–[18], artificial neural network (ANN) function approximation techniques are used to model directly the key physical parameters of GaN-based transistors e.g. the conduction and displacement currents. However, in all cases, the neural network size needed to develop an accurate model is not known a priori [24]. In [19]–[21], more advanced machine learning (ML) techniques are applied to device modeling, using Bayesian inference [19], [20] and support vector regression [21] in order to capture as wide a modeling space as possible i.e. to ensure the models remain accurate across a wide range of power levels and/or frequencies, using a minimal number of measurements. This approach has led to excellent results. However, a certain amount of parameter optimization is required in such approaches, leading to increased model development times compared with traditional direct-extraction approaches.

During the past decade, modeling methods based on the one-dimensional canonical piecewise-linear (CPL) functions have gained attention from the research community [22]–[26]. This paper combines the CPL technique with the PHD model formalism in order to eliminate the model dependence on the dominant large-signal tones, resulting in a novel model implementation. This new model can capture the device RF/dc behavior across a range of power levels very accurately, using a single set of model parameters. Validation is performed by comparison with both simulated and experimentally obtained data. The remainder of this paper is organized as follows. The theory of the proposed CPL-PHD and CPL-QPHD models are presented in Section II, simulation and experimental results are provided in Section III, and conclusions are drawn in Section IV.

II. MODIFIED PHD MODEL

A. BASIC THEORY OF PHD AND QPHD MODELS

Considering the transistor device as a two-port network, its scattered wave complex amplitude (phasor) at a given port \( p \) and harmonic \( m \), can be written as (1),

\[
B_{pm} = F_{pm} (A_{11}, A_{12}, \ldots A_{21}, A_{22}, \ldots),
\]

where \( F_{pm} \) is a describing function of several complex-valued inputs, namely the incident wave phasors \( A_{qn} \) at each port \( q \) and harmonic \( n \). In [11], the Taylor series is used to linearize (1) around the dominant input wave(s) – often just \( A_{11} \) i.e., the fundamental incident wave at the input – and hence obtain a compact behavioral model with one nonlinear term, while the smaller incident tones are treated as linear perturbations. This gives the standard linearized PHD (X-parameter) model as

\[
B_{pm} = FB_{pm} (|A_{11}|) P^{mn} + \sum_{qn} S_{pm,qn} (|A_{11}|) P^{m-n} A_{qn} + \sum_{qn} T_{pm,qn} (|A_{11}|) P^{m+n} A_{qn}^* \tag{2}
\]

where \( \angle P = e^{+j\alpha_{11}} \).

This model describes the resulting scattered wave \( B_{pm} \) via the nonlinear mapping \( FB_{pm} (|A_{11}|) P^{mn} \) of the dominant incident wave (limited here to just \( A_{11} \) for demonstrative purposes), along with the linear contributions from the remaining incident wave perturbations and their conjugates. The fact that the conjugates of the incident waves are also required is only a relatively recent realization, and indeed led to poor accuracy in earlier attempts at generalizing the S-parameter approach to nonlinear systems e.g. Hot S-parameters. The conjugates arise due to the fact that \( F_{pm} \) is not holomorphic in the complex-variable sense.

We note from (2), that the \( FB, S \) and \( T \) parameters are all dependent on the fundamental large-signal incident wave, \( A_{11} \), thus making any tabulated list of these parameters unmanageably large for a compact model. The standard PHD model described above takes into consideration only to first-order the tones other than the dominant one. However, if any of these perturbations become large – via a moderate to large load-side impedance mismatch, for example – the model performance begins to degrade. In this case, a quadratic formulation is useful, taking perturbations into account up to second-order.

The QPHD model is given as

\[
B_{pm} = FB_{pm} (|A_{11}|) P^{mn} + \sum_{qn} S_{pm,qn} (|A_{11}|) P^{m-n} A_{qn} + \sum_{qn} T_{pm,qn} (|A_{11}|) P^{m+n} A_{qn} + \sum_{qn} U_{pm,qn} (|A_{11}|) P^{m-2n} A_{qn}^2 \tag{3}
\]
\[ + \sum_{qn} V_{pm,qn} (|A_{11}|) P^{m+2n} A_{qn}^2 
+ \sum_{qn} W_{pm,qn} (|A_{11}|) P^m A_{qn}^* \]

(4)

It is clear that the QPHD model includes more information about the DUT than the first-order case: it accounts for the perturbation in the incident waves (in general, at all ports and harmonics) through a second-order polynomial in two variables \( (A_{qn} \text{ and } A_{qn}^*) \). Because of this, the QPHD model can be expected to give superior results, and in particular to give good prediction for a wider range of reflection coefficients presented to the ports.

**B. BASIC THEORY OF CPL FUNCTION**

The CPL functions, originally proposed by Chua in the 1970s [27], [28], allow a nonlinear function to be approximated by a sum of a series of linear functions defined in a piecewise fashion over multiple nonoverlapping partitions of the independent variable, e.g., in a 1D model, the function is defined over distinct intervals of the \( x \)-axis. Recently, several nonlinear behavioral models based on one-dimensional CPL functions, have been reported in the literature [22]–[26]. These a one-dimensional nonlinear CPL functions take the form of \( f(x) \) given below

\[ y = f(x) = a_0 + b_0 x + \sum_{k=1}^{K} c_k |x - \beta_k|, \quad (5) \]

where \( x \) is the DUT input and \( y \) the DUT output, and \( | \cdot | \) denotes the “absolute” value. The quantity \( K \) (the summation upper limit) is the number of partitions, and \( \beta_k \) is the threshold that defines the boundary of the given partition \( k \). The constants \( a_0 \), \( b_0 \) and \( c_k \) are the CPL model parameters [27].

**C. CPL-PHD AND CPL-QPHD MODELS**

We can apply the CPL technique to the \( |A_{11}| \) dependence of the \( FB, S \) and \( T \) terms in the PHD model of (2). The gives the following expansion

\[ FB_{pm} (|A_{11}|) = a_0^{FB_{pm}} + b_0^{FB_{pm}} |A_{11}| 
+ \sum_{k=1}^{K} c_k^{FB_{pm}} |A_{11}| - \beta_k \]  

(6)

\[ S_{pm,qn} (|A_{11}|) = a_0^{S_{pm,qn}} + b_0^{S_{pm,qn}} |A_{11}| 
+ \sum_{k=1}^{K} c_k^{S_{pm,qn}} |A_{11}| - \beta_k \]  

(7)

\[ T_{pm,qn} (|A_{11}|) = a_0^{T_{pm,qn}} + b_0^{T_{pm,qn}} |A_{11}| 
+ \sum_{k=1}^{K} c_k^{T_{pm,qn}} |A_{11}| - \beta_k \]  

(8)

After inserting (6)-(8) into (2), the CPL-PHD behavioral model can be obtained as (9).

\[ B_{pm} = a_0^{FB_{pm}} P^m + b_0^{FB_{pm}} |A_{11}| P^m 
+ \sum_{k=1}^{K} c_k^{FB_{pm}} |A_{11}| - \beta_k |P^m \]

\[ + \sum_{qn} c_k^{FB_{pm}} |A_{11}| - \beta_k |P^{m-n} A_{qn}^* \]

(9)

A similar approach can be used for the QPHD model, allowing the new CPL-QPHD model to be written as (10).

It can be seen that the large signal incident wave dependence inherent in the parameters of the models (2) and (4), is now eliminated. The parameters of the CPL-PHD and CPL-QPHD models i.e., \( a_0^X \), \( b_0^X \), and \( c_k^X \) values (where \( X \) represents the \( FB, S \) or \( T \) \( X \)-parameters), do not depend on the magnitude of the large-signal incident wave. Hence these models can cover a large range of input power levels, across multiple load impedances, with a single set of model parameters.

\[ B_{pm} = a_0^{FB_{pm}} P^m + b_0^{FB_{pm}} |A_{11}| P^m 
+ \sum_{k=1}^{K} c_k^{FB_{pm}} |A_{11}| - \beta_k |P^m \]

\[ + \sum_{qn} c_k^{FB_{pm}} |A_{11}| - \beta_k |P^{m-n} A_{qn}^* \]

(10)
+ \sum_{qn} a_{0}^{\text{spm}} p_{m} A_{qn} A_{qn}^* + \sum_{qn} b_{0}^{\text{spm}} * |A_{11}| P_{m} A_{qn} A_{qn}^* + \sum_{qn} K c_{k}^{\text{spm}} |A_{11}| - \beta_{k} |A_{m} A_{qn} A_{qn}^* (10)

D. CPL-PHD MODEL FOR DC BEHAVIOR

The dc current into the drain of the DUT can be described according to

\[ I_{dc} = \text{Real} \left[ I_{dc} \right] = \left[ A_{11}, A_{12}, \ldots A_{21}, A_{22}, \ldots, A_{mn} \right], \]

where \( I_{dc} \) is a real-valued function of multiple complex-valued variables. Taking the CPL-PHD case as an example, and assuming for demonstrative purposes that the fundamental input is the only dominant signal, we obtain from (5)

\[ I_{dc} = \text{Real} \left[ I_{dc} \right] = \left[ A_{11}, A_{12}, \ldots A_{21}, A_{22}, \ldots, A_{mn} \right], \]

where \( \text{Real} \left[ \right] \) means the real part of the content between the braces. Thus, as can be seen from (12), the model easily can be extended for the CPL-QPHD case.

E. MODEL EXTRACTION METHODOLOGY

Since the model is ‘linear-in-parameters’, it may be extracted via the least squares method. This guarantees the best fit model in a least squares sense – note that no such guarantees are possible for many of the other model approaches mentioned e.g. ANNs. In order to explain the extraction methodology, we take the fundamental CPL-PHD case in (9), choosing \( K \) equal to two as an example. The expression is described as (13). Load pull data covering the entire Smith chart at different input power levels are used to extract the model. The matrix formulation can be represented symbolically as (14), where \( B \) is a matrix of scattered wave data, and \( A \) is a matrix of the input terms on the right hand side of (13). The solution of (14), \( [S] \), is shown as (15), where the superscript ‘\(^H\)’ refers to the Hermitian conjugate operation.

Once the model is extracted, it can be used to predict the behavior of the DUT over the entire Smith chart across a large range of input power levels, which greatly reduces the complexity of the model compared with the existing input power-dependent model i.e. the proposed model can accurately interpolate across \( A_{11} \) and \( A_{21} \).

\[ B_{21} = a_{01}^{FB_{21}} P + b_{01}^{FB_{21}} * |A_{11}| P + \sum_{k=1}^{2} c_{k}^{FB_{21}} |A_{11}| - \beta_{k} |P| + a_{01}^{S_{21,21}} A_{21} + b_{01}^{S_{21,21}} * |A_{11}| A_{21} + \sum_{k=1}^{2} c_{k}^{S_{21,21}} |A_{11}| - \beta_{k} |A_{21} + a_{01}^{T_{21,21}} A_{21} + b_{01}^{T_{21,21}} * |A_{11}| P_{21,21} A_{21}^* + \sum_{k=1}^{2} c_{k}^{T_{21,21}} |A_{11}| - \beta_{k} |P_{21,21} A_{21}^* \]

\[ [B] = [A] [S]. \]

\[ [S] = ([A]^H [A])^{-1} [A]^H [B]. \]

III. MODEL VALIDATION

Model verification is carried out through both simulation and experimental methods.

A. SIMULATION RESULTS

In the simulation-based verification, we use an equivalent-circuit transistor model simulated using the advanced design system (ADS) simulation environment. The device under test (DUT) is a 6 W GaN packaged transistor manufactured by Wolfspeed, and the equivalent circuit model used in this work is supplied by the manufacturer. The proposed behavioral model is implemented in MATLAB.

In the first example, the ability of the proposed model to interpolate across the fundamental incident wave at port two, \( A_{21} \), is obtained via the simulator and compared to that of the proposed model. The \( A_{11} \) value used during validation is the same as that used to extract the model. The proposed model used in this simulation example is extracted with the device biased at -2.8 V for \( V_{GS} \) and 28 V for \( V_{DS} \). The model is extracted using the procedure presented in Section II-E, and takes the form given in (13). Approximately 60 load sample points used for model extraction, at +16 dBm and +33 dBm. The second and third harmonic load impedances are fixed at 50 \( \Omega \), to ensure there are no reflections from the load impedance at these frequencies.

For model verification, the amplitude of the load reflection coefficient again ranges from 0.05 to 0.95, with a step size of 0.1, while the phase of the reflection coefficient ranges from 0 degrees to 355 degrees, with a step size of 5 degrees. This provides a total of 720 load points which are (mostly) different to the load impedance points used during extraction.

Fig. 9 displays the results from the 50 ohm PHD model, the 50 ohm QPHD model, the CPL-PHD and CPL-QPHD models, at both +16 dBm and +33 dBm input power levels. It is shown that both proposed models give very accurate prediction at both weakly nonlinear region (0.9 dB compression) and strongly nonlinear region (4.5 dB compression), while the 50 ohm PHD model gives reasonable prediction for the...
The model accuracy versus the total number of model parameters, and numbers of partitions, is provided in Table 1. From the comparison, we can see that the normalized mean square error (NMSE) of CPL-PHD model is much less than $-30 \text{ dB}$, while, the CPL-QPHD model provide even better prediction, its NMSE is much less than $-40 \text{ dB}$ when the number of partitions is fixed at 1. The 50 ohm PHD model provides the poorest prediction, with its NMSE greater than $-20 \text{ dB}$ for the strongly nonlinear test case. The 50 ohm QPHD model provides superior prediction compared with the 50 ohm PHD model, as expected, however both models show worse performance compared to the proposed model. The QPHD model NMSE is approximately 1 dB worse than that of the CPL-PHD model, despite the fact that they require the same number of model parameters, and over 7 dB worse than the CPL-QPHD model, although in this case the CPL model requires twice the parameter count to achieve this superiority.

### TABLE 1. Performance of CPL-PHD and CPL-QPHD models with different complexity.

| Model          | Number of partitions ($K$) | Total number of model parameters | NMSE ($\text{dB}$) |
|----------------|----------------------------|----------------------------------|--------------------|
| CPL-PHD (16 dBm) | 1                          | 6                                | -49.3              |
| CPL-QPHD (16 dBm) | 1                          | 12                               | -55.3              |
| 50-PHD (16 dBm)   | /                          | 3                                | -39.0              |
| 50-QPHD (16 dBm)   | /                          | 6                                | -48.4              |
| CPL-PHD (33 dBm)   | 1                          | 6                                | -34.1              |
| CPL-QPHD (33 dBm)   | 1                          | 12                               | -41.4              |
| 50-PHD (33 dBm)   | /                          | 3                                | -19.1              |
| 50-QPHD (33 dBm)   | /                          | 6                                | -33.5              |

FIGURE 1. Comparison of load pull results between circuit model, CPL-PHD model, CPL-QPHD model, 50-Ohm PHD model, and 50 ohm QPHD model at both $+16 \text{ dBm}$ (0.9 dB compression) and $+33 \text{ dBm}$ (4.5 dB compression) input, with number of partitions $K$ equal to 1.

FIGURE 2. AM/AM curve, sample points for proposed model extraction, and sample points for proposed model testing/verification.

This shows the excellent interpolation capability of the proposed model across the $A_{21}$ independent variable.

In this second verification case, the ability of the model to interpolate across both the $A_{11}$ and $A_{21}$ incident waves, is examined. Fig. 2 shows the AM/AM curve with the input available power varying from $-20 \text{ dBm}$ to $+35 \text{ dBm}$, at a frequency of 3 GHz, under a load termination of 50 $\Omega$. On this figure are indicated the power levels used for both model extraction (black crosses) and model verification (red circles). The model used in this example is extracted with the same device as in the previous verification setup, this time biased at $-3 \text{ V}$ for $V_{GS}$, and again at 28 V for $V_{DS}$. The load pull sample points used for training are the same as the previous case, while the specific power levels used for model extraction/training, are: $-5 \text{ dBm}$, $+5 \text{ dBm}$, $+10 \text{ dBm}$, $+16 \text{ dBm}$, $+23 \text{ dBm}$, $+27 \text{ dBm}$ and $+33 \text{ dBm}$. This gives 420 ($6 \times 10 \times 7$) points in total.

Once the proposed model is obtained, the interpolation capability of the model is examined, taking data from two
different regions of device operation (see red circles in Fig. 2): the linear region (+8 dBm, less than 0.5 dB compression), and the strongly nonlinear region (+30 dBm, 3.1 dB compression). These data, along with the predicted results from the model, are shown in Fig. 3 and Fig. 4. During model verification, for each input power level, 720 load points are used.

In Fig. 3 and Fig. 4, the performance of the proposed CPL-PHD and CPL-QPHD models are compared with the circuit model. As can be seen, the prediction capability of the CPL-PHD model suffers under simultaneous $A_{11}$ and $A_{21}$ interpolation at these two operating regions i.e., the linear region (Fig. 3) and strongly nonlinear region (Fig. 4), when compared with the first example – this despite the increase in partition number from five to seven. While the CPL-QPHD model, on the other hand, provides decent performance with an NMSE of less than $-38$ dB for both regions, when $K = 7$.

It is important to note that the input power levels used in the verification above are different to the sample points used for model extraction; hence, the prediction accuracy seen in Fig. 3 and Fig. 4 demonstrates the ability of the model to interpolate across the two dominant input waves, $A_{11}$ and $A_{21}$. Model details, including NMSE, are shown in Table 2 and Table 3. The results show that the proposed CPL-QPHD model can cover a wide range of input power levels, across the full Smith chart, with high prediction accuracy, with 48 model parameters. The performance of 50 ohm PHD and QPHD models are also shown in Fig. 3(b) and Fig. 4(b). As can be seen from the results, the traditional 50 ohm PHD and QPHD models provide significant inferior prediction when compared with the proposed model.

In order to make a more rigorous comparison between the 50 ohm PHD, 50 ohm QPHD, CPL-PHD and CPL-QPHD models, the NMSE performances from these two models when they share the same model complexity (i.e. have the same number of model parameters) are also provided in Table II and Table III. As can be seen from the results, the NMSE of the 50 ohm models is quite poor, at greater than $-10$ dB. Although the CPL-PHD model has better
In some situations, the CPL-QPHD model gives more accurate prediction when the complexity increases. In particular, it is important to note that the performance of the CPL-PHD model saturates as the complexity increases, whereas that for the CPL-QPHD continues to improve. This is because the CPL-QPHD model allows for a second order approximation of $A_{21}$, i.e. improved interpolation in $A_{21}$, while, as its number of coefficients (complexity) increases, the CPL-PHD model can only improve the $A_{11}$ interpolation, and the $A_{21}$ model remains a linear approximation (as in the standard X-parameter model).

In Fig. 5, comparison of the proposed model with the equivalent circuit reference model is provided for available input powers (incident wave power) of 0 dBm, +13 dBm, +20 dBm, and +27 dBm, with partition $K = 7$. As is seen, both models provide decent prediction, just as per the results in Fig. 3 and Fig. 4. A more detailed comparison between the proposed model and the 2-D polynomial model is provided in Table V. As can be seen from the results, the 2-D polynomial model provides the best prediction, with an NMSE of $-35.6$ dB, when $S = T = L = 4$, which is 1.3 dB better than the proposed

| Model       | Number of partitions (K) | Total number of model parameters | NMSE (dB) |
|-------------|--------------------------|---------------------------------|-----------|
| 50-PHD      | /                        | 3                               | -6.7      |
| 50-QPHD     | /                        | 6                               | -8.0      |
| CPL-PHD     | 5                        | 18                              | -13.5     |
| CPL-QPHD    | 5                        | 36                              | -14.2     |
| CPL-PHD     | 7                        | 24                              | -22.2     |
| CPL-QPHD    | 7                        | 48                              | -38.8     |
| CPL-PHD     | 5                        | 18                              | -13.5     |
| CPL-QPHD    | 2                        | 18                              | -16.0     |
| CPL-PHD     | 9                        | 30                              | -21.1     |
| CPL-QPHD    | 4                        | 30                              | -13.7     |
| CPL-PHD     | 13                       | 42                              | -6.2      |
| CPL-QPHD    | 6                        | 42                              | -31.1     |
| CPL-PHD     | 17                       | 54                              | -21.8     |
| CPL-QPHD    | 8                        | 54                              | -42.3     |

The model complexity is determined by the maximum Fourier frequency index $L$, and the nonlinear order $S$ and $T$

$$B_{21} = P_1^m \sum_{j=-L}^{L} \sum_{t=1}^{T} \sum_{s=t}^{t} \sum_{l=1}^{L} (|A_{11}|^s(|A_{21}|^t(Q_1/P_1)^l).$$  

where $P_1 = e^{i(LA_{11})}$, $Q_1 = e^{i(LA_{21})}$.

In Fig. 6, the performance of the proposed CPL-QPHD model is compared with the 2-D polynomial model, and the 50 ohm PHD and QPHD models, for an input power of +25 dBm, with 1 dB compression. As can be seen from the results, the proposed model provides similar prediction compared with the polynomial model. In addition, the model complexity and model accuracy, are provided in Table IV.
FIGURE 6. Comparison between circuit model, 50 ohm PHD model, 50 ohm QPHD model, CPL-QPHD model, and 2-D polynomial model, for verifying model interpolation across $A_{11}$ and $A_{21}$. Plot shows results for an available input power (incident wave power) of +25 dBm.

TABLE 5. Performance of proposed model and 2D polynomial model with different complexity.

| Model            | Model Complexity | Total number of model parameters | NMSE (dB) |
|------------------|------------------|----------------------------------|-----------|
| CPL-PHD          | K=5              | 18                               | -23.5     |
| CPL-QPHD         | K=2              | 18                               | -23.4     |
| CPL-QPHD         | K=7              | 24                               | -23.6     |
| CPL-QPHD         | K=9              | 30                               | -24.7     |
| CPL-QPHD         | K=4              | 18                               | -10.3     |
| 2D Polynomial    | S=1, L=1        | 15                               | -22.4     |
|                  | S=2, L=2        | 42                               | -32.6     |
|                  | S=3, L=3        | 90                               | -35.6     |
|                  | S=4, L=4        | 165                              | -25.4     |

CPL-QPHD model. However, the CPL-QPHD model has significantly fewer model parameters than the 2-D polynomial model.

In the third simulation-based verification, the proposed model for dc drain current prediction is validated. The fundamental load pull is taken over Smith chart, with a total of 720 load points. The indices assigned to these points, shown on the horizontal axis of the plots below, increase from the inner part (low gamma) to the outer part (higher gamma) of the Smith chart.

The dc model equation is shown in (12). In this case, only the fundamental information is taken in to consideration. The input power is fixed at +30 dBm, and the quantity of interest is chosen to be the dc drain current. The model ability to predict the dc drain current is shown as separate subplots in Fig. 7, organized according to the number of partitions (K). As can be seen, the model performance improves with increasing numbers of partitions. Further model details are given in Table VI. From the comparison, we can see that the proposed model provides a very accurate prediction for the dc component of the drain current. The NMSE of the CPL-QPHD model can approach $-35$ dB when the number of partitions is set to seven. While the NMSE of the 50 ohm PHD and QPHD models cannot achieve a value lower than $-15$ dB.

B. EXPERIMENTAL MEASUREMENTS RESULTS

This section uses measured experimental data to further validate this new modeling techniques proposed in this paper. The test bench used here is a Keysight PNA-X combined with a Focus Microwaves load pull system.

TABLE 6. Performance of CPL DC current model for $I_{ds}$.

| Model      | Number of partitions (K) | Number of model parameters | NMSE (dB) |
|------------|--------------------------|----------------------------|-----------|
| 50-PHD     | /                        | 3                          | -12.6     |
| 50-QPHD    | /                        | 6                          | -15.5     |
| CPL-PHD    | 3                        | 12                         | -13.9     |
| CPL-QPHD   | 3                        | 24                         | -19.0     |
| CPL-PHD    | 7                        | 24                         | -23.2     |
| CPL-QPHD   | 7                        | 48                         | -34.6     |
FIGURE 8. Measured and modeled results when the number of partitions $K = 3$, for $+10$ dBm, and $+30$ dBm cases, with a 2 GHz input signal.

In this first experimental-based validation, the 10 W GaN transistor from Wolfspeed is used as the target DUT. The gate and drain are biased at $-3$ V and 28 V, respectively. The input port is excited with a 2 GHz input signal ($A_{11}$), at two different input available power levels, specifically: $+10$ dBm, and $+30$ dBm, giving 216 ($9 \times 12 \times 2$) sample points used in total. For the validation, there are 648 load impedance values, distinct from those used during model extraction. Effectively, the model is predicting the response of the device for fundamental incident waves at port two, i.e. $A_{21}$ values, for which it was not directly extracted, while also allowing for different available input power levels ($A_{11}$).

The comparison between the modeled and measured results is shown in Fig. 8. Clearly, the proposed method provides a high level of accuracy over the entire Smith chart, across different excitation power levels, with only one set of model coefficients. The traditional 50 ohm PHD and QPHD models give much worse prediction, although it requires fewer model coefficients. Further model details are given in Table VII. From this table, we can see that the accuracy of the proposed model is extremely good, with the NMSE of the proposed CPL-QPHD model at less than $-40$ dB, when the partition number is fixed at 3.

A detailed comparison between the two proposed models, when they share the same model complexity (same number of model parameters) is also given in Table VIII. Similar to the results in the previous simulation, the CPL-QPHD model provides a more accurate prediction than the CPL-PHD model i.e. it has a significantly higher upper limit of model accuracy.
In Fig. 9, the performance of the proposed CPL-QPHD model is compared with the 2-D polynomial model with measured data, for an input power of +25 dBm. As can be seen from the results in Fig. 9 and Table IX, the proposed model provides more accurate prediction compared with the polynomial model with much less model parameters in this test.

A more detailed comparison between the proposed model and the 2-D polynomial model is provided in Table X. As can be seen from the results, the 2-D polynomial model provide the best prediction, with an NMSE of −27.2 dB, when $S = T = L = 3$, which is 6 dB worse than the proposed CPL-QPHD model. Furthermore, the CPL-QPHD model has significantly fewer model parameters compared with the 2D polynomial model at the same time.

The DUT dc drain current, at the different load impedances and input power levels, is also presented. The model dc performance is given in Fig. 10, with the number of partitions $K$ equal to 3. As can be seen from the results in Fig. 10, the proposed model provides an excellent dc drain current prediction for the two different input power levels, with a single set of model coefficients. The prediction performance of 50 ohm PHD and 50 ohm QPHD models are also given in Fig. 10 (a). From the results, we can see that their performance is significantly worse than the proposed model in this case. The model complexity and accuracy details are provided in Table VII.

IV. CONCLUSION

In this paper, a new modeling technique for power transistors, based on the canonical piecewise-linear functions, is presented and validated. A detailed mathematical demonstration describing the combination of the CPL technique with the PHD formalism, is provided. Several examples are discussed, and corresponding simulation and measurement data are presented. The new modeling technique can predict accurately the RF scattered wave, and can also determine the dc drain current of this power device. Both CPL-PHD and CPL-QPHD...
model forms are considered. Simulation and experimental data show that the new modelling technique has strong ability to interpolate across input powers and load impedances.

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