A DISC FORMULA FOR PLURISUBHARMONIC SUBEXTENSIONS IN MANIFOLDS

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Abstract. We provide a sufficient condition for open sets $W$ and $X$ such that a disc formula for the largest plurisubharmonic subextension of an upper semicontinuous function on a domain $W$ to a complex manifold $X$ holds.

1. Introduction and the main result

The theory of disc functionals was initiated by E. A. Poletsky in the late 1980s [Po1] and it offers a different approach to certain extremal functions of pluripotential theory (see Klimek [Kl]). Extremal functions are usually defined as suprema of classes of plurisubharmonic functions with certain properties, and many of them are envelopes of appropriate disc functionals: the largest plurisubharmonic minorant of an upper semicontinuous function, the largest nonpositive plurisubharmonic function whose Levi form is bounded below by the Levi form of a given plurisubharmonic function, pluricomplex Green functions and Siciak-Zaharyuta extremal function [Po2, BS, LS1, Ro1, Ro2, LS2, RS, LS3, LS4, MS, LS5, Ma, DF2, DF3, Ku].

Recently, F. Larusson and E. A. Poletsky [LP] proved a disc formula for the largest plurisubharmonic subextension of an upper semicontinuous function on a domain $W$ to a larger domain $X$ in a Stein manifold under suitable conditions on $W$ and $X$. Their conditions depend on the topological properties of the space $A^W_X$ of analytic discs in $X$ with boundaries in $W$. They also provide an example which shows that the disc formula can fail in general. The aim of this note is to provide a more geometric condition on $X$ and $W$ which also implies the disc formula.

Theorem 1.1. Let $X$ be a $(n-1)$-complete complex manifold of dimension $n > 1$, with smooth $(n-1)$-complete exhaustion function $\rho: X \to \mathbb{R}$. Let $W = \{ x \in X : \rho(x) > c \}$ be a superlevel set of $\rho$ for some $c \in \mathbb{R}$. If $\phi: W \to [-\infty, \infty)$ is upper semicontinuous, then

$$
\sup\{ u \in PSH(X) : u|W \leq \phi \} = \inf \left\{ \int_T \phi \circ f \, d\lambda : f \in A^W_X, \ f(0) = x \right\}.
$$

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First we set some notations. Let $\mathcal{D} = \{ \zeta \in \mathbb{C} : |\zeta| < 1 \}$ and $\mathcal{T} = b\mathcal{D} = \{ \zeta \in \mathbb{C} : |\zeta| = 1 \}$. Given a domain $W$ in a complex manifold $X$ we denote by $\mathcal{A}_X$ the set of all analytic discs in $X$, that is, continuous maps $\overline{\mathcal{D}} \to X$ that are holomorphic in $\mathcal{D}$, and by $\mathcal{A}_W^X$ the set of all analytic discs in $X$ with boundaries in $W$. We denote by $PSH(X)$ the class of all plurisubharmonic functions on $X$ (we take the constant $-\infty$ to be plurisubharmonic).

If $\varphi : W \to [-\infty, \infty)$ is an upper semicontinuous function, then we denote by $S\varphi$ the supremum of all plurisubharmonic subextensions of $u$, that is, plurisubharmonic functions $u \in PSH(X)$ satisfying $u|W \leq \varphi$. As noted in [LP], $S\varphi$ is plurisubharmonic if, for example, $X$ is covered by analytic discs with boundaries in $W$. In this case, $S\varphi$ is the largest plurisubharmonic subextension of $\varphi$.

2. **Proof**

In the proof we use the method of gluing sprays, which was developed in [DF1], and it was used in the Poletsky theory in [DF2, DF3] (For an exposition of the method we refer to [Fu1, §5.8–§5.9]; a brief exposition can also be found in [DF2]).

The following lemma is essentially [DF1] Lemma 6.2:

**Lemma 2.1.** Let $X$ be a complex manifold of dimension $n > 1$, and let $d$ be a complete distance function on $X$. Let $A \Subset X$ be relatively compact open subset of $X$ and let $B$ be a 2-convex bump on $A$ [DF1, Definition 6.1]. Let $P$ be a domain in $\mathbb{C}^N$ containing 0 and assume that $f : P \times \mathbb{D} \to X$ is a spray of discs with the exceptional set containing 0 such that $f_0(b\mathcal{D}) \cap \overline{A} = \emptyset$. (Here $f_0 = f(0, \cdot)$ is the core map of the spray.) Let $E \subset \mathcal{T}$ be a union of finitely many closed circular arcs such that $f_0(\mathcal{T} \setminus E) \cap \overline{A \cup B} = \emptyset$.

Given $\epsilon > 0$ there are a domain $P' \subset P$ containing $0 \in \mathbb{C}^N$ and a spray of maps $g : P' \times \mathbb{D} \to X$ with the exceptional set containing 0 such that
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(i) \( g(t, z) \cap A \cup B = \emptyset \) for \( t \in P' \) and \( z \in T \),
(ii) \( d(g(t, z), f(t, z)) < \epsilon \) for \( t \in P' \) and \( z \in T \setminus E \), and
(iii) \( f_0(0) = g_0(0) \).

Let us give the idea of the proof of the above lemma. A 2-convex bump gives a continuously varying family of small analytic discs in \( X \) with boundaries outside the bump along the part of the boundary of the initial disc that is mapped into the bump. By solving a certain Riemann-Hilbert boundary value problem we obtain a continuous map defined on a neighborhood of a boundary arc in the closed unit disc, holomorphic in the interior, such that the boundary arc is mapped close to the boundary of the above chosen family of analytic discs, and hence the boundary arc is mapped outside the bump. The globalization of the construction is obtained by the method of patching holomorphic sprays. The local modification method gives a new spray near a part of the boundary \( T \); by insuring that the two sprays are sufficiently close to each other on the intersection of their domains, we patch them into a new spray. We obtain property (ii) by possibly shrinking the parameter set \( P \) at the start.

We use sprays that are continuous up to the boundary and as it is remarked in \[DF1\], the assumption that sprays are of class \( C^r, r \geq 2 \), is needed only for the existence of a Stein neighborhood. Using \[Fo2\ Corollary 1.3\] instead of \[DF1\ Theorem 2.6\] we obtain the same result for all \( r \in \mathbb{Z}_+ \). Here we shall use these results with \( r = 0 \).

The following lemma is essentially obtained by the same proof as \[DF1\ Lemma 6.3\]: property (ii) below is a consequence of Lemma 2.1 (ii) which we use in the proof instead of using \[DF1\ Lemma 6.2\].

**Lemma 2.2.** Let \( X \) be a \((n - 1)\)-complete complex manifold of dimension \( n > 1 \), with smooth \((n - 1)\)-complete exhaustion function \( \rho: X \to \mathbb{R} \) and let \( d \) be a complete distance function on \( X \). Let \( f \in \mathcal{A}_X \) be an analytic disc, let \( c_0 < c_1 \) and assume that \( \rho(f(z)) < c_1 \) for all \( z \in T \) and furthermore assume that \( E \subset T \) is a union of finitely many closed circular arcs such that \( \rho(f(z)) \in (c_0, c_1) \) for all \( z \in T \setminus E \). Then for each \( \eta > 0 \) there exists an analytic disc \( g \in \mathcal{A}_X \) satisfying the following properties:

(i) \( \rho(g(z)) \in (c_0, c_1) \) for \( z \in T \),
(ii) \( d(g(z), f(z)) < \eta \) for \( z \in T \setminus E \), and
(iii) \( f(0) = g(0) \).

Let us review the idea of the proof. Since the Levi form \( \mathcal{L}_\rho \) is assumed to have at least two positive eigenvalues at every point \( x_0 \in X \), we get at least one positive eigenvalue in a direction tangential to the regular level set of \( \rho \) at each point \( x_0 \). So we can choose local coordinates near \( x_0 \) such that the sublevel set \( \{ x: \rho(x) \leq \rho(x_0) \} \) near \( x_0 \) is convex in one direction and we can fill the space between regular sublevel sets by adding finitely many 2-convex bumps.

By Lemma 2.1 we can push the boundary of the analytic disc outside a 2-convex bump while we make only a small perturbation on the part of the boundary that already lies outside some neighborhood of the bump. In finitely many steps we can push the boundary to a given higher super-level set of $\rho$. To cross each critical level set of $\rho$ we use a different function constructed especially for this purpose: we make a small perturbation if necessary and we proceed by the noncritical procedure with perturbed $(n-1)$-convex function. Once the boundary is above the critical level set we proceed with the noncritical construction for the original exhaustion function.

Proof of Theorem 1.1. Note that $E_{A_X^W} \phi \leq \phi$ on $W$ since constant discs in $W$ belong to $A_X^W$. Once we prove that $E_{A_X^W} \phi$ is plurisubharmonic the equality in the theorem holds.

The proof is reduced to the case when the function $\phi: W \to \mathbb{R}$ is continuous and bounded from below exactly as in the case of Poisson envelope [DF2].

Next we show that $E_{A_X^W} \phi$ is upper semicontinuous on $X$. The proof goes along the same lines as in [DF2]; we include it for the convenience of the reader since it applies sprays. Pick a point $x \in X$ and a number $\epsilon > 0$. Assume first that $E_{A_X^W} \phi(x) > -\infty$. By the definition of $E_{A_X^W} \phi$ there exists a disc $f_0 \in A_X^W$ with $f_0(0) = x$ such that $E_{A_X^W} \phi(x) \leq H_\phi(f_0) < E_{A_X^W} \phi(x) + \epsilon$. We embed $f_0$ as the central map $f_0 = f(0, \cdot)$ in a spray of holomorphic discs $f: P \times \overline{B} \to X$, where $P$ is an open set in $\mathbb{C}^m$ containing the origin. If $P'$, $0 \in P' \subset P$, is small enough then $f(t, \cdot) \in A_X^W$, $H_\phi(f(t, \cdot)) < H_\phi(f_0) + \epsilon$ for each $t \in P'$, hence

$$E_{A_X^W} \phi(f(t, 0)) \leq H_\phi(f(t, \cdot)) \leq H_\phi(f_0) + \epsilon < E_{A_X^W} \phi(x) + 2\epsilon.$$  

By the domination property the set $\{f(t, 0): t \in P'\}$ fills a neighborhood of the point $x = f_0(0)$ in $X$, so we see that $E_{A_X^W} \phi$ is upper semicontinuous at $x$. A similar argument works at points where $E_{A_X^W} \phi(x) = -\infty$.

The main part is to prove that $E_{A_X^W} \phi$ is plurisubharmonic on $X$: we need to show that for every analytic disc $h \in A_X$ we have the submeanvalue property

$$E_{A_X^W} \phi(h(0)) \leq \int_T (E_{A_X^W} \phi)(h(w)) d\lambda(w).$$

Therefore, it is enough to construct for each $\epsilon > 0$ and for each continuous function $v \geq E_{A_X^W} \phi$ on $X$, an analytic disc $g \in A_X^W$ such that $g(0) = h(0)$, and

$$H_\phi(g) \leq H_v(h) + \epsilon. \quad (2.1)$$

We proceed in two steps. First we construct an analytic disc $f$ with large part of the boundary lying in $W$ and satisfying an integral estimate similar to (2.1) on the part of the boundary lying in $W$. Then we correct the disc around small pieces of the boundary while we do not perturb the rest of the boundary too much and we obtain $g$. 

More precisely, for any given $\epsilon > 0$ we shall construct $f \in \mathcal{A}_X$, a union of finitely many closed circular arcs $E \subset \mathbb{T}$, and real numbers $c < c_0 < c_1$ with the following properties:

(a) $\rho(f(w)) < c_1$ for every $w \in \mathbb{T}$,
(b) $\rho(f(w)) \in (c_0, c_1)$ for every $w \in \mathbb{T} \setminus E$,
(c) $\int_{\mathbb{T} \setminus E} \phi \circ f \, d\lambda \leq H_v(h) + 2\epsilon/3$,
(d) $\max\{\phi(x): \rho(x) \in [c_0, c_1]\} \cdot \lambda(E) \leq \epsilon/6$,
(e) $f(0) = h(0)$.

Fix a point $w \in \mathbb{T}$. By the definition of $v$ there exists an analytic disc $g_w \in \mathcal{A}_X^w$ with center $g_w(0) = h(w)$ such that

$$\int_{\mathbb{T}} \phi(g_w(z)) \, d\lambda(z) < v(h(w)) + \epsilon/6. \tag{2.2}$$

We embed the disc $g_w$ into a dominating spray of discs $G(t, \cdot) \in \mathcal{A}_X$ depending holomorphically on the parameter $t$ in some ball $P$ in Euclidean space $\mathbb{C}^m$ with the central map $G(0, \cdot) = g_w$. Since the spray $G$ is dominating, the set $G(P, 0) = \{G(t, \cdot): t \in P\}$ covers a neighborhood of the point $f(w)$ in $X$. By shrinking the parameter set $P$ if necessary we obtain, in addition, that $G(t, \cdot) \in \mathcal{A}_X^w$ for every $t \in P$. By the implicit mapping theorem there is a disc $D \subset \{z \mid \rho(z) < c\}$ centered at the point $w \in \mathbb{T}$ and a holomorphic map $\varphi: D \to P$ such that

$$\varphi(w) = 0 \quad \text{and} \quad G(\varphi(\zeta), 0) = h(\zeta), \quad \zeta \in D.$$ 

The map $g: D \times \mathbb{D} \to X$ defined by

$$g(\zeta, z) = G(\varphi(\zeta), z), \quad \zeta \in D, \quad z \in \mathbb{D}$$

is continuous, holomorphic in $D \times \mathbb{D}$, and

$$g(w, \cdot) = G(0, \cdot) = g_w; \quad g(\zeta, 0) = h(\zeta), \quad \zeta \in D.$$ 

Since $g(\zeta, \cdot)$ is uniformly close to $g_w$ when $\zeta$ is close to $w$, it follows from (2.2) that there is a small arc $I \subset \mathbb{T} \cap D$ around the point $w$ such that

$$\int_I d\lambda(w) \int_{\mathbb{T}} \phi(g(w, z)) \, d\lambda(z) < \int_I v(h(w)) \, d\lambda(w) + \frac{\lambda(I) \epsilon}{6}.$$ 

By repeating this construction at other points of $\mathbb{T}$ we find finitely many open circular arcs $I''_j$ and discs $D_j$ contained in $r_0 \mathbb{D}$ such that $I''_j \subset \mathbb{T} \cap D_j$ ($j = 1, \ldots, l$), $\cup I''_j = \mathbb{T}$, and holomorphic families of discs $g_j(\zeta, z)$ for $\zeta \in D_j$ and $z \in \mathbb{D}$ such that

$$\int_{I_j} d\lambda(w) \int_{\mathbb{T}} \phi(g_j(w, z)) \, d\lambda(z) < \int_{I_j} v(h(w)) \, d\lambda(w) + \frac{\lambda(I_j) \epsilon}{6}. \tag{2.3}$$ 

for any $I_j \subset I''_j$.

There are real numbers $c < c_0 < c_1$ such that $\rho(x) \in (c_0, c_1)$ for each $x \in \bigcup g_j(\overline{D_j}, \mathbb{D})$. Choose open circular arcs $I_j$ and $I_j'$ such that $I_j \subset I_j' \subset I_j''$, and
Indeed, the integral over $\cup_i \Delta_i$ estimate and for every $e_i$ (2.6)
\[
\xi(\zeta) = \begin{cases} 
g_j(\zeta, \chi(\zeta)z), & \zeta \in \Delta_j, \ z \in \mathbb{T}, \ j = 1, \ldots, l; 
\ h(\zeta), & \chi(\zeta) = 0, \ z \in \mathbb{T}.
\end{cases}
\]
Note that $\xi$ is continuous and is holomorphic in the second variable. Then
\[
\int_{\mathbb{T} \setminus \overline{E}} d\lambda(w) \int_{\mathbb{T}} \phi(\xi(w, z)) \ d\lambda(z) < \int_{\mathbb{T}} v(h(w)) \ d\lambda(w) + \epsilon/3.
\]
Indeed, the integral over $\cup_j I_j$ is estimated by adding up the inequalities (2.3) and the rest by (2.4).

Fix an index $j \in \{1, \ldots, l\}$. We shall apply [DF2] Lemma 3.1 over $\Delta_j$ to find an analytic disc $f_j': \Delta_j \to X$ that approximates $h$ uniformly as close as desired outside of a small neighborhood of the arc $T_j$ in $\Delta_j$ and satisfies the estimate
\[
\int_{I_j} \phi'(f_j'(w)) \ d\lambda(w) < \int_{I_j} d\lambda(w) \int_{\mathbb{T}} \phi(\xi(w, z)) \ d\lambda(z) + \frac{\lambda(I_j) \epsilon}{6}.
\]
Consider the function
\[
\phi_j'(\zeta, z) = \phi(g_j(\zeta, z)), \quad \zeta \in \Delta_j, \ z \in \mathbb{T}
\]
and the smooth family of analytic discs in $\mathbb{C}^2(\zeta, z)$ given by
\[
g_j'(\zeta, z) = (\zeta, \chi(\zeta) z), \quad \zeta \in \partial \Delta_j, \ z \in \mathbb{T}.
\]
Recall from (2.5) that $\xi(\zeta, z) = g_j(\zeta, \chi(\zeta)z)$ for $\zeta \in \Delta_j$ and $z \in \mathbb{T}$ thus
\[
\xi = g_j \circ g_j' \text{ and } \phi_j' = \phi \circ g_j \text{ on } \partial \Delta_j \times \mathbb{T}.
\]
Applying [DF2] Lemma 3.1 with $\mathbb{D}$ replaced by $\Delta_j$, $u$ replaced by $\phi_j'$ and $g$ replaced by $g_j'$ furnishes an analytic disc $h_j': \Delta_j \to \mathbb{C}^2$ which approximates the disc $\zeta \mapsto (\zeta, 0)$ outside of a small neighborhood of the arc $T_j$,
\[
\int_{I_j} \phi_j'(h_j'(w)) \ d\lambda(w) < \int_{I_j} d\lambda(w) \int_{\mathbb{T}} \phi_j'(g_j'(w, z)) \ d\lambda(z) + \frac{\lambda(I_j) \epsilon}{6},
\]
and for every $e^{it} \in I_j$ the point $h_j'(e^{it})$ is so close to the set $\{e^{it}\} \times \mathbb{T}$ that $\rho(g_j(h_j'(e^{it}))) \in (e_0, c_1)$. Since $g_j: \Delta_j \times \mathbb{T} \to X$ is holomorphic in the interior
Δ_j × D, the map
\[ f_j' := g_j \circ h_j' : \Delta_j \to X \]
is an analytic disc in X such that \( f_j' (\zeta) \approx g_j (\zeta, 0) = f (\zeta) \) for \( \zeta \) outside a small neighborhood of \( T_j \) in \( \Delta_j \). From (2.8) and (2.10) it follows that
\[ \phi \circ f_j' = \phi \circ g_j \circ h_j' = \phi_j \circ h_j' , \quad \phi \circ \xi = \phi \circ g_j \circ g_j' = \phi_j \circ g_j' \]
hold on \( b\Delta_j \times \overline{\Delta} \). Hence the integrals in (2.7) equal the corresponding integrals in (2.9), and so the disc \( f_j' \) satisfies the desired properties.

If the approximation of \( h \) by \( f_j' \) is close enough for each \( j = 1, \ldots, l \), we can glue this collection of discs into a single analytic disc \( f : \overline{\Delta} \to X \) with the same center as \( h \), which approximates \( h \) away from the union of arcs \( \cup_{j=1}^{l} I_j \), and which approximates the disc \( f_j' \) over a neighborhood of \( I_j \) for each \( j \) exactly as in the proof of [DF2, Theorem 1.1]. If the approximation is good enough the properties (a) and (b) hold. In particular, we can insure that for each \( j = 1, \ldots, l \) we have
\[ \int_{I_j} \phi (f (w)) \, d\lambda (w) < \int_{I_j} d\lambda (w) \int_{T} \phi (g (w, z)) \, d\lambda (z) + \frac{\lambda (I_j) \epsilon}{3}. \]

By adding up these terms and by the inequality (2.6) we get
\[ \int_{T \setminus E} \phi (f (w)) \, d\lambda (w) < \int_{T \setminus E} d\lambda (w) \int_{T} \phi (\xi (w, z)) \, d\lambda (z) + \epsilon / 3 \]
\[ < \int_{T} v (h (w)) \, d\lambda (w) + 2 \epsilon / 3. \]
(2.11)

This implies that \( f \) satisfies (a)-(e).

Next we use Lemma [2.2] for \( f, E \) to get analytic disc \( g \in A_X \). Property (i) implies that \( g (z) \in W \) for every \( z \in T \). Since \( \phi \) is continuous we may choose \( \eta > 0 \) so small that property (ii) implies that
\[ \int_{T \setminus E} \phi \circ g \, d\lambda \leq \int_{T \setminus E} \phi \circ f \, d\lambda + \epsilon / 3, \]
and by (iii) we get \( g (0) = f (0) \).

Combining the inequalities (2.11), (2.12) and (d) we obtain (2.1). This completes the proof of Theorem 1.1. □

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