Almost Interior Gamma-ideals and Fuzzy Almost Interior Gamma-ideals in Gamma-semigroups

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Abstract Ideal theory plays an important role in studying in many algebraic structures, for example, rings, semigroups, semirings, etc. The algebraic structure Γ-semigroup is a generalization of the classical semigroup. Many results in semigroups were extended to results in Γ-semigroups. Many results in ideal theory of Γ-semigroups were widely investigated. In this paper, we first focus to study some novel ideals of Γ-semigroups. In Section 2, we define almost interior Γ-ideals and weakly almost interior Γ-ideals of Γ-semigroups by using the concept ideas of interior Γ-ideals and almost Γ-ideals of Γ-semigroups. Every almost interior Γ-ideal of a Γ-semigroup S is clearly a weakly almost interior Γ-ideal of S but the converse is not true in general. The notions of both almost interior Γ-ideals and weakly almost interior Γ-ideals of Γ-semigroups are generalizations of the notion of interior Γ-ideal of a Γ-semigroup S. We investigate basic properties of both almost interior Γ-ideals and weakly almost interior Γ-ideals of Γ-semigroups. The notion of fuzzy sets was introduced by Zadeh in 1965. Fuzzy set is an extension of the classical notion of sets. Fuzzy sets are somewhat like sets whose elements have degrees of membership. In the remainder of this paper, we focus on studying some novelities of fuzzy ideals in Γ-semigroups. In Section 3, we introduce fuzzy almost interior Γ-ideals and fuzzy weakly almost interior Γ-ideals of Γ-semigroups. We investigate their properties. Finally, we give some relationship between almost interior Γ-ideals [weakly almost interior Γ-ideals] and fuzzy almost interior Γ-ideals [fuzzy weakly almost interior Γ-ideals] of Γ-semigroups.

Keywords Almost Interior Γ-ideals, Weakly Almost Interior Γ-ideals, Fuzzy Almost Interior Ideals, Fuzzy Weakly Interior Γ-ideals.

1 Introduction and preliminaries

In 1965, Zadeh [25] first introduced the notion of fuzzy subsets. Applications of fuzzy subsets have been developed in many fields. Rosenfeld applied the fuzzy subsets to define fuzzy subgroups of groups in [14]. Applications of fuzzy subsets in semigroups were first considered by Kuroki [11, 12]. Now, fuzzy subsets were studied in many algebraic structures (for example, in ternary semigroups [21]), in non-associative ordered semigroups [11], etc. The definition of almost ideals of semigroups (or A-ideals) was first studied by Grosek and Satko [5, 6, 7] in 1980. Recently, Wattanatripop, Chinram and Changphas [23, 24] introduced the notion of quasi almost ideals (or quasi-A-ideals) of semigroups and gave their basic properties. Moreover, they applied fuzzy subsets to define fuzzy almost ideals, fuzzy almost bi-ideals and fuzzy quasi almost ideals of semigroups and showed relationship between almost ideals [almost bi-ideals, quasi almost ideals] and their fuzzifications of semigroups. Furthermore, Kaopusek, Kaewnoi and Chinram [9] using the concepts of interior ideals and almost ideals of semigroups, defined the notions of almost interior ideals and weakly almost interior ideals of semigroups. Moreover, they investigated their basic properties. Now, the notion of almost ideals in semigroups were extend to some generalizations of semigroups, for example, almost ideals in ternary semigroup [21], almost hyperideals in semihypergroups [20], etc. The notion of Γ-semigroups were first introduced by Sen [16]. This algebraic structure generalized from the classi-
cal semigroup. Many definitions and results in semigroups were extended to that in $\Gamma$-semigroups. Ideal theory in $\Gamma$-semigroups have been widely studied, for example, Chinram [4] studied quasi-$\Gamma$-ideals (or quasi-ideals) of $\Gamma$-semigroups in 2006 and he joined with Jirokul [3] to studied bi-$\Gamma$-ideals (or bi-ideals) of $\Gamma$-semigroups in 2007, Hila [8] focused on regular $\Gamma$-semigroups, semiprime $\Gamma$-semigroups and quasi-reflexive $\Gamma$-semigroups, and focused to study minimal quasi-ideals of $\Gamma$-semigroups, Ansari and Khan [2] focused on $(m, n)$ bi-$\Gamma$-ideal in $\Gamma$-semigroups, etc.

Firstly, we recall the definitions and some notations of fuzzy sets. A fuzzy subset of a set $S$ is a membership function from $S$ into the closed unit interval $[0, 1]$. Let $S$ be any set and $f$ and $g$ be any two fuzzy subsets of $S$.

1. The intersection of $f$ and $g$ ($f \cap g$) is a fuzzy subset of $S$ defined as follows:

$$(f \cap g)(x) = \min\{f(x), g(x)\} \text{ for all } x \in S.$$  

2. The union of $f$ and $g$ ($f \cup g$) is a fuzzy subset of $S$ defined as follows:

$$(f \cup g)(x) = \max\{f(x), g(x)\} \text{ for all } x \in S.$$  

3. $f \subseteq g$ if $f(x) \leq g(x)$ for all $x \in S$, and we say that $f$ is a fuzzy subset of $g$.

For any fuzzy subset $f$ of any set $S$, the support of $f$ ($supp(f)$) is a subset of $S$ defined by

$$supp(f) = \{x \in S \mid f(x) \neq 0\}.$$  

For any subset $A$ of any set $S$, the characteristic mapping $C_A$ of $A$ is a fuzzy subset of $S$ defined by

$$C_A(x) = \begin{cases} 1 & x \in A, \\ 0 & x \notin A. \end{cases}$$  

For any element $x$ of any set $S$ and a real number $t \in (0, 1]$, a fuzzy point $x_t (\{13\})$ of $S$ is a fuzzy subset of $S$ defined by

$$x_t(y) = \begin{cases} t & y = x, \\ 0 & y \neq x. \end{cases}$$  

Now, we recall the definition of $\Gamma$-semigroups defined by Sen and Saha in [17].

**Definition 1.1.** [17] Let $\Gamma$ be a nonempty set. A nonempty set $S$ is called a $\Gamma$-semigroup if it satisfies:

1. for all $a, b \in S$ and for all $\alpha \in \Gamma$, $a \alpha b \in S$ and

2. for all $a, b, c \in S$ and for all $\alpha, \beta \in \Gamma$,

$$(a \alpha b) \beta c = a \alpha (b \beta c).$$  

For nonempty subsets $A, B$ of a $\Gamma$-semigroup $S$, let

$$A \Gamma B = \{a \alpha b \mid a \in A, b \in B, \alpha \in \Gamma\}.$$  

If $x \in S$, we let $A \Gamma x := A \Gamma \{x\}$ and $x \Gamma B := \{x\} \Gamma B$. For $\alpha \in \Gamma$, we let

$$A \alpha B = \{a \alpha b \mid a \in A, b \in B\}.$$  

Let $S$ be a $\Gamma$-semigroup and $F(S)$ be the set of all fuzzy subsets of $S$. For each $\alpha$ in $\Gamma$, let $\circ_{\alpha}$ be a binary operation on $F(S)$ defined by

$$(f \circ_{\alpha} g)(x) = \begin{cases} \sup \{\min\{f(a), g(b)\} \} & \text{if } x \in S \alpha S, \\ 0 & \text{otherwise}. \end{cases}$$  

We have that $(F(S), \Gamma^*)$ is also a $\Gamma$-semigroup where $\Gamma^* := \{\circ_{\alpha} \mid \alpha \in \Gamma\}$.

**Definition 1.2.** Let $S$ be a $\Gamma$-semigroup and $T$ be a nonempty subset of $S$.

1. $T$ is called a sub $\Gamma$-semigroup of $S$ if $TT \subseteq T$.

2. $T$ is called a left $\Gamma$-ideal of $S$ if $ST \subseteq T$.

3. $T$ is called a right $\Gamma$-ideal of $S$ if $TS \subseteq T$.

Next, we recall the definitions of interior $\Gamma$-ideals of $\Gamma$-semigroups that we will use later.

**Definition 1.3.** A sub $\Gamma$-semigroup $I$ of a $\Gamma$-semigroup $S$ is called an interior $\Gamma$-ideal of $S$ if $S \Gamma S \subseteq I$.

In 2010, Sardar, Davvaz and Majumder studied fuzzy interior $\Gamma$-ideals (or interior ideals) of $\Gamma$-semigroups and investigated some of their basic properties in [15]. They obtained the characterization of simple $\Gamma$-semigroups in terms of fuzzy interior $\Gamma$-ideals.

**Definition 1.4.** [15] A fuzzy subsemigroup $f$ of a $\Gamma$-semigroup $S$ is called a fuzzy interior $\Gamma$-ideal of $S$ if it satisfies,

$$f(xa \alpha by) \geq f(a)$$  

for all $a, x, y \in S$ and $\alpha, \beta \in \Gamma$.

Wattanatripop and Changphas first defined and studied almost $\Gamma$-ideals in $\Gamma$-semigroups [22] by using the concept of almost ideal in semigroups defined by Grosek and Satko. They defined left (right) almost $\Gamma$-ideals in $\Gamma$-semigroups as the following.

**Definition 1.5.** [22] A nonempty subset $I$ of a $\Gamma$-semigroup $S$ is called a left almost $\Gamma$-ideal [right almost $\Gamma$-ideal] of $S$ if $S \Gamma I \cap I \neq \emptyset$ [I $\Gamma S \cap I \neq \emptyset$].

Simuen, Wattanatripop and Chinram [19] using the concepts of quasi $\Gamma$-ideals in $\Gamma$-semigroups and almost $\Gamma$-ideals in $\Gamma$-semigroups, studied the basic properties of both almost quasi $\Gamma$-ideals and fuzzy almost quasi $\Gamma$-ideals of $\Gamma$-semigroups. Moreover, they gave the remarkable relationship between almost quasi-$\Gamma$-ideals and their fuzzification. In addition, Simuen, Abdullah, Yonthanthum and Chinram [18] introduced the concepts of almost bi-$\Gamma$-ideals and fuzzy almost bi-$\Gamma$-ideals of $\Gamma$-semigroups. Also, they gave some properties
and investigated relationship between almost bi $\Gamma$-ideals of $\Gamma$-semigroups and their fuzzification.

Similar to quasi-$\Gamma$-ideals and bi-$\Gamma$-ideals, interior $\Gamma$-ideals are elementary ideals in $\Gamma$-semigroups. This is motivation to study almost interior $\Gamma$-ideals and their fuzzification. The purpose of this paper is to introduce and study almost interior $\Gamma$-ideals and weakly almost interior $\Gamma$-ideals of $\Gamma$-semigroups. Moreover, we investigate their properties. Furthermore, we define the fuzzifications of almost interior $\Gamma$-ideals [weakly almost interior $\Gamma$-ideals] of $\Gamma$-semigroups and give some relationship between almost interior $\Gamma$-ideals [weakly almost interior $\Gamma$-ideals] and their fuzzification.

## 2 Almost interior $\Gamma$-ideals in $\Gamma$-semigroups

In this section, we give the definitions of almost interior $\Gamma$-ideals and weakly almost interior $\Gamma$-ideals of $\Gamma$-semigroups. Moreover, we investigate their properties.

**Definition 2.1.** Let $S$ be a $\Gamma$-semigroup. A nonempty subset $I$ of $S$ is called an almost interior $\Gamma$-ideal of $S$ if $a\Gamma I \subseteq I$ for all $a, b \in S$.

**Theorem 2.1.** Let $S$ be a $\Gamma$-semigroup. Every interior $\Gamma$-ideal of $S$ is an almost interior $\Gamma$-ideal of $S$.

**Proof.** Assume that $I$ is any interior $\Gamma$-ideal of a $\Gamma$-semigroup $S$ and let $a, b \in S$. Then $a\Gamma I \subseteq I$. This implies that $a\Gamma I \subseteq I$. It is conclude that $I$ is an almost interior $\Gamma$-ideal of $S$.

Example 2.1 shows that the converse of Theorem 2.1 is not generally true.

**Example 2.1.** Consider the $\Gamma$-semigroup $Z_5$ with $\Gamma = \{0, 1\}$ such that $\pi \Gamma b := \pi + \gamma + \delta$. We have that $I = \{0, 2\}$ is an almost interior $\Gamma$-ideal of $Z_5$. It is easy to see that $I$ is not an interior $\Gamma$-ideal of $Z_5$.

**Theorem 2.2.** Let $I$ and $H$ be any two nonempty subsets of a $\Gamma$-semigroup $S$ such that $I \subseteq H$. If $I$ is an almost interior $\Gamma$-ideal of $S$, then $H$ is also an almost interior $\Gamma$-ideal of $S$.

**Proof.** Let $H$ be a subset of $S$ containing $I$ and let $a, b \in S$. Then $a\Gamma I \subseteq I$. Thus $a\Gamma H \subseteq H$. Hence, $H$ is an almost interior $\Gamma$-ideal of $S$.

From Theorem 2.2, we have the following corollary.

**Corollary 2.3.** Let $S$ be a $\Gamma$-semigroup and $I_1, I_2$ be any two almost interior $\Gamma$-ideals of a $\Gamma$-semigroup $S$. Then $I_1 \cup I_2$ is also an almost interior $\Gamma$-ideal of $S$.

**Proof.** Since $I_1 \subseteq I_1 \cup I_2$, by Theorem 2.2, $I_1 \cup I_2$ is an almost interior $\Gamma$-ideal of $S$.

**Example 2.2.** Consider the $\Gamma$-semigroup $Z_5$ with $\Gamma = \{0, 2\}$ such that $\pi \Gamma b := \pi + \gamma + \delta$. Let $I_1 = \{0, 2\}$ and $I_2 = \{1\}$. It is easy to show that $I_1 \cap I_2 = \emptyset$, so it is not an almost interior $\Gamma$-ideal of $Z_5$.

Remark 2.1 follows from Example 2.2.

**Remark 2.1.** If $I_1$ and $I_2$ are almost interior $\Gamma$-ideals of $\Gamma$-semigroups, then $I_1 \cap I_2$ need not be an almost interior $\Gamma$-ideal of $\Gamma$-semigroups.

**Definition 2.2.** Let $S$ be a $\Gamma$-semigroup. A nonempty subset $I$ of $S$ is called a weakly almost interior $\Gamma$-ideal of $S$ if $a\Gamma I \cap I \neq \emptyset$ for all $a \in S$.

**Example 2.3.** Consider the $\Gamma$-semigroup $Z_4$ with $\Gamma = \{\emptyset, 1\}$ such that $\pi \Gamma b := \pi + \gamma + \delta$. We have that the set $I = \{0, 2\}$ is a weakly almost interior $\Gamma$-ideal of $Z_4$.

From Example 2.3, we have the following remark.

**Remark 2.2.** Every almost interior $\Gamma$-ideal of a $\Gamma$-semigroup $S$ is a weakly almost interior $\Gamma$-ideal of $S$.

However, the converse of Remark 2.2 is not true by the following example.

**Example 2.4.** Consider the $\Gamma$-semigroup $Z_6$ with $\Gamma = \{\emptyset\}$ such that $\pi \Gamma b := \pi + \gamma + \delta$. We see that $I = \{0, 3\}$ is a weakly almost interior $\Gamma$-ideal but not an almost interior $\Gamma$-ideal of $Z_6$.

**Theorem 2.4.** Let $I$ and $H$ be any two nonempty subsets of a $\Gamma$-semigroup $S$ such that $I \subseteq H$. If $I$ is a weakly almost interior $\Gamma$-ideal of $S$, then $H$ is also a weakly almost interior $\Gamma$-ideal of $S$.

**Proof.** Let $a$ be any element of $S$. Since $I \subseteq H$, then $a\Gamma I \cap I \subseteq a\Gamma H \cap H$. Hence, $H$ is a weakly almost interior $\Gamma$-ideal of $S$.

From Theorem 2.4, The following corollary holds.

**Corollary 2.5.** Let $I_1$ and $I_2$ be weakly almost interior $\Gamma$-ideals of a $\Gamma$-semigroup $S$. Then $I_1 \cup I_2$ is a weakly almost interior $\Gamma$-ideal of $S$.

**Proof.** Since $I_1 \subseteq I_1 \cup I_2$, it follows from Theorem 2.4 that $I_1 \cup I_2$ is a weakly almost interior $\Gamma$-ideal of $S$.

**Example 2.5.** Consider the $\Gamma$-semigroup $Z_4$ with $\Gamma = \{\emptyset, 1\}$ such that $\pi \Gamma b := \pi + \gamma + \delta$. Let $I_1 = \{0, 2\}$ and $I_2 = \{\emptyset, 3\}$. We see that $I_1$ and $I_2$ are weakly almost interior $\Gamma$-ideals of $Z_4$ but $I_1 \cap I_2 = \emptyset$, so it is not a weakly almost interior $\Gamma$-ideal of $Z_4$.

The following remark holds by Example 2.5.

**Remark 2.3.** If $I_1$ and $I_2$ are any two weakly almost interior $\Gamma$-ideals of $\Gamma$-semigroups, then $I_1 \cap I_2$ need not be a weakly almost interior $\Gamma$-ideal of $\Gamma$-semigroups.

**Theorem 2.6.** A $\Gamma$-semigroup $S$ has no proper weakly almost interior $\Gamma$-ideals if and only if for each element $a$ of $S$, there exists $s_a$ such that $s_a \Gamma (S \setminus \{a\}) \Gamma s_a = \{a\}$.

**Proof.** Assume that a $\Gamma$-semigroup $S$ has no proper weakly almost interior $\Gamma$-ideals. Let $a$ be a fixed element of $S$. Thus $S \setminus \{a\}$ is not a weakly almost interior $\Gamma$-ideal of $\Gamma$-semigroup $S$. Thus there exists $s_a \in S$ such that $s_a \Gamma (S \setminus \{a\}) \Gamma s_a \cap (S \setminus \{a\}) = \emptyset$. Therefore, $s_a \Gamma (S \setminus \{a\}) \Gamma s_a = \{a\}$. 

Remark 2.6 follows from Example 2.6.
3 Fuzzy almost interior $\Gamma$-ideals in $\Gamma$-semigroups

In this section, we give the definitions of fuzzy almost interior $\Gamma$-ideals and fuzzy weakly almost interior $\Gamma$-ideals of $\Gamma$-semigroups. Moreover, we investigate their properties.

The definition of fuzzy almost interior $\Gamma$-ideals of $\Gamma$-semigroups is defined as follows:

**Definition 3.1.** A fuzzy subset $f$ of a $\Gamma$-semigroup $S$ is said to be a fuzzy almost interior $\Gamma$-ideal of $S$ if for all fuzzy points $x_t, y_t$ of $S$, there exist $\alpha, \beta \in \Gamma$ such that $x_t, \alpha f \cap y_t \cap f \neq \emptyset$.

**Example 3.1.** Consider the $\Gamma$-semigroup $Z_6$ with $\Gamma = \{0, \emptyset\}$ such that $\pi \gamma b := \pi + \gamma + b$. Define a fuzzy subset $f$ of $Z_6$ by

$$f(0) = 0, f(\emptyset) = 0.9, f(\emptyset) = 0.8, f(\emptyset) = 0.5.$$ 

It is easy to see that $f$ is a fuzzy almost interior $\Gamma$-ideal of $Z_6$.

Moreover, fuzzy weakly almost interior $\Gamma$-ideals of $\Gamma$-semigroups is defined as follows:

**Definition 3.2.** A fuzzy subset $f$ of a $\Gamma$-semigroup $S$ is said to be a fuzzy weakly almost interior $\Gamma$-ideal of $S$ if for all fuzzy points $x_t, x_t$ of $S$, there exist $\alpha, \beta \in \Gamma$ such that $x_t, \alpha f \cap y_t \cap f \neq \emptyset$.

**Example 3.2.** Consider the $\Gamma$-semigroup $Z_6$ with $\Gamma = \{0\}$ such that $\pi \gamma b := \pi + \gamma + b$. Define a fuzzy subset $f$ of $Z_6$ by $f(0) = 0.6, f(\emptyset) = 0, f(\emptyset) = 0, f(\emptyset) = 1$ and $f(\emptyset) = 0$. We have that $f$ is a fuzzy weakly almost interior $\Gamma$-ideal of $Z_6$.

Furthermore, we have the following remark.

**Remark 3.1.** Every fuzzy almost interior $\Gamma$-ideal of a $\Gamma$-semigroup $S$ is a fuzzy weakly almost interior $\Gamma$-ideal of $S$.

**Example 3.3.** Consider the $\Gamma$-semigroup $Z_6$ with $\Gamma = \{0\}$ such that $\pi \gamma b := \pi + \gamma + b$. Define a fuzzy subset $f$ of $Z_6$ by $f(0) = 0.1, f(\emptyset) = 0, f(\emptyset) = 0, f(\emptyset) = 0.5, f(\emptyset) = 0$. We have that $f$ is a fuzzy weakly almost interior $\Gamma$-ideal but not a fuzzy almost interior $\Gamma$-ideal of $Z_6$.

**Theorem 3.1.** Let $f$ and $g$ be any two fuzzy subsets of a $\Gamma$-semigroup $S$ such that $f \subseteq g$. Suppose that $f$ is a fuzzy almost interior $\Gamma$-ideal of $S$. Then $g$ is also a fuzzy almost interior $\Gamma$-ideal of $S$.

**Proof.** Let $x_t, y_t$ be fuzzy points of $S$. Since $f$ is a fuzzy almost interior $\Gamma$-ideal of $S$, there exist $\alpha, \beta \in \Gamma$ such that $x_t, \alpha f \cap y_t \cap f \neq \emptyset$. Since $f \subseteq g$, we have that $x_t, \alpha f \cap y_t \cap g \neq \emptyset$. This implies that $x_t, \alpha g \cap y_t \cap g \neq \emptyset$. Therefore, $g$ is a fuzzy almost interior $\Gamma$-ideal of $S$.

From Theorem 3.1, we have the following corollary.

**Corollary 3.2.** Let $f$ and $g$ be any two fuzzy almost interior $\Gamma$-ideals of a $\Gamma$-semigroup $S$. Thus $f \cup g$ is also a fuzzy almost interior $\Gamma$-ideal of $S$.

**Proof.** Since $f \subseteq f \cup g$, by Theorem 3.1, $f \cup g$ is a fuzzy almost interior $\Gamma$-ideal of $S$.

**Theorem 3.3.** Let $f$ and $g$ be any two fuzzy subsets of a $\Gamma$-semigroup $S$ such that $f \subseteq g$. Assume that $f$ is a fuzzy weakly almost interior $\Gamma$-ideal of $S$. Then $g$ is also a fuzzy weakly almost interior $\Gamma$-ideal of $S$.

**Proof.** The proof of this theorem is similar to the proof of Theorem 3.1.

From Theorem 3.3, we have the following corollary.

**Corollary 3.4.** Let $f$ and $g$ be any two fuzzy weakly almost interior $\Gamma$-ideals of a $\Gamma$-semigroup $S$. Thus $f \cup g$ is also a fuzzy weakly almost interior $\Gamma$-ideal of $S$.

**Proof.** The proof of this corollary is similar to the proof of Corollary 3.2.

**Example 3.4.** Consider the $\Gamma$-semigroup $Z_7$ with $\Gamma = \{0\}$ such that $\pi \gamma b := \pi + \gamma + b$. Define fuzzy subsets $f$ and $g$ of $Z_7$ by $f(0) = 0.1, f(\emptyset) = 0.3, f(\emptyset) = 0, f(\emptyset) = 0.1, f(\emptyset) = 0, f(\emptyset) = 0, f(\emptyset) = 0$ and $g(0) = 1, g(\emptyset) = 0, g(\emptyset) = 0.6, g(\emptyset) = 0.1, g(\emptyset) = 0.5, g(\emptyset) = 0$. We have that $f$ and $g$ are fuzzy almost interior $\Gamma$-ideals of $Z_7$ but $f \cap g$ is not a fuzzy almost interior $\Gamma$-ideal of $Z_7$.

The Remark 3.2 follows from Example 3.4.

**Remark 3.2.** If $f$ and $g$ are any two fuzzy almost interior $\Gamma$-ideals of $\Gamma$-semigroups, then $f \cap g$ need not be a fuzzy almost interior $\Gamma$-ideal of $\Gamma$-semigroups.

**Example 3.5.** Consider the $\Gamma$-semigroup $Z_6$ with $\Gamma = \{0\}$ such that $\pi \gamma b := \pi + \gamma + b$. Define fuzzy subsets $f$ and $g$ of $Z_7$ by $f(0) = 1, f(\emptyset) = 0, f(\emptyset) = 0, f(\emptyset) = 0, f(\emptyset) = 0, f(\emptyset) = 0$ and $g(0) = 0.5, g(\emptyset) = 0, g(\emptyset) = 0.3, g(\emptyset) = 0, g(\emptyset) = 1, g(\emptyset) = 0$. We have that $f$ and $g$ are fuzzy weakly almost interior $\Gamma$-ideals of $Z_6$ but $f \cap g$ is not a fuzzy weakly almost interior $\Gamma$-ideal of $Z_6$.

Likewise, we have the following remark.

**Remark 3.3.** If $f$ and $g$ are any two fuzzy weakly almost interior $\Gamma$-ideals of $\Gamma$-semigroups, then $f \cap g$ need not be a fuzzy weakly almost interior $\Gamma$-ideal of $\Gamma$-semigroups.

**Theorem 3.5.** Let $I$ be a nonempty subset of a $\Gamma$-semigroup $S$. We have that $I$ is an almost interior $\Gamma$-ideal of $S$ if and only if $C_I$ is a fuzzy almost interior $\Gamma$-ideal of $S$.

**Proof.** Suppose that $I$ is an almost interior $\Gamma$-ideal of $S$ and let $x_t, y_t$ be any two fuzzy points of $S$. Then $x_Iy_I \cap I \neq \emptyset$. Thus there exists an element $z \in S$ such that $z \in x_Iy_I$ and $z \in I$. Thus $z \in x_Iy_I$ for some $\alpha, \beta \in \Gamma$. So $(x_t, \alpha g, y_t, \beta g)(z) \neq 0$ and $C_I(z) = 1$. Hence,
Theorem 3.6. Let $I$ be a nonempty subset of a $\Gamma$-semigroup $S$. We have that $I$ is a weakly almost interior $\Gamma$-ideal of $S$ if and only if $C_I$ is a fuzzy weakly almost interior $\Gamma$-ideal of $S$.

Proof. The proof of this theorem is similar to the proof of Theorem 3.5.

Theorem 3.7. Let $f$ be a fuzzy subset of a $\Gamma$-semigroup $S$. Then $f$ is a fuzzy almost interior $\Gamma$-ideal of $S$ if and only if $\text{supp}(f)$ is an almost interior $\Gamma$-ideal of $S$.

Proof. Let $f$ be a fuzzy almost interior $\Gamma$-ideal of $S$. Let $x_t$ and $y_t$ be any two fuzzy points of $S$. Then $x_t \circ f \circ y_t \cap f \neq 0$ for some $\alpha, \beta \in \Gamma$. Hence there exists $a \in S$ such that $[(x_t \circ f \circ y_t) \cap f](a) \neq 0$. So $f(a) \neq 0$ and there exists $b \in S$ such that $a = xab3y$ and $f(b) \neq 0$. That is $a, b \in \text{supp}(f)$. Thus $(x_t \circ f \circ y_t)(a) \neq 0$ and $C_{\text{supp}(f)}(a) \neq 0$. Therefore, $(x_t \circ f \circ y_t) \cap C_{\text{supp}(f)} = 0$. Hence $C_{\text{supp}(f)}$ is a fuzzy almost interior $\Gamma$-ideal of $S$. By Theorem 3.5, $\text{supp}(f)$ is an almost interior $\Gamma$-ideal of $S$.

To prove the converse, suppose that $\text{supp}(f)$ is an almost interior $\Gamma$-ideal of $S$. By Theorem 3.5, we have that $C_{\text{supp}(f)}$ is a fuzzy almost interior $\Gamma$-ideal of $S$. Let $x_t$ and $y_t$ be any two fuzzy points of $S$. Then $(x_t \circ f \circ y_t) \cap C_{\text{supp}(f)} = 0$ for some $\alpha, \beta \in \Gamma$. Then there exists $a \in S$ such that $(x_t \circ f \circ y_t) \cap C_{\text{supp}(f)}(a) \neq 0$. Hence $(x_t \circ f \circ y_t)(a) \neq 0$ and $C_{\text{supp}(f)}(a) \neq 0$. Then $f(a) \neq 0$ and there exists $b \in S$ such that $a = xab3y$ and $f(b) \neq 0$. This means that $(x_t \circ f \circ y_t) \cap f \neq 0$. Therefore, $f$ is a fuzzy almost interior $\Gamma$-ideal of $S$.

Theorem 3.8. Let $f$ be a fuzzy subset of a $\Gamma$-semigroup $S$. Then $f$ is a fuzzy weakly almost interior $\Gamma$-ideal of $S$ if and only if $\text{supp}(f)$ is a weakly almost interior $\Gamma$-ideal of $S$.

Proof. The proof of this theorem is similar to the proof of Theorem 3.7.

Next, we define minimal fuzzy almost interior $\Gamma$-ideals in $\Gamma$-semigroups and give some relationship between minimal almost interior $\Gamma$-ideals and minimal fuzzy almost interior $\Gamma$-ideals of $\Gamma$-semigroups.

Definition 3.3. A fuzzy almost interior $\Gamma$-ideal $f$ is called minimal if $\text{supp}(g) = \text{supp}(f)$ for every fuzzy almost interior $\Gamma$-ideal $g$ of $S$ such that $g \subseteq f$.

Theorem 3.9. A nonempty subset $I$ of a $\Gamma$-semigroup $S$ is a minimal almost interior $\Gamma$-ideal of $S$ if and only if $C_I$ is a minimal fuzzy almost interior $\Gamma$-ideal of $S$.

Proof. Let $I$ be a minimal almost interior $\Gamma$-ideal of $S$. By Theorem 3.5, we have that $C_I$ is a fuzzy almost interior $\Gamma$-ideal of $S$. Let $g$ be any fuzzy almost interior $\Gamma$-ideal of $S$ such that $g \subseteq C_I$. By Theorem 3.7, we have that $\text{supp}(g)$ is an almost interior $\Gamma$-ideal of $S$. Moreover, we have that $\text{supp}(g) \subseteq \text{supp}(C_I) = I$. Since $I$ is minimal, $\text{supp}(g) = I = \text{supp}(C_I)$. This implies that $C_I$ is minimal.

To prove the converse, assume that $C_I$ is a minimal fuzzy almost interior $\Gamma$-ideal of $S$. Let $I'$ be any almost interior $\Gamma$-ideal of $S$ such that $I' \subseteq I$. Thus $C_I$ is a fuzzy almost interior $\Gamma$-ideal of $S$ such that $C_{I'} \subseteq C_I$. Hence, $I' = \text{supp}(C_{I'}) = \text{supp}(C_I) = I$. It is conclude that $I$ is minimal.

From Theorem 3.9, we have the following corollary.

Corollary 3.10. Let $S$ be a $\Gamma$-semigroup. Then $S$ has no proper almost interior $\Gamma$-ideal if and only if $\text{supp}(f) = S$ for every fuzzy almost interior $\Gamma$-ideal $f$ of $S$.

Proof. Let $f$ be a fuzzy almost interior $\Gamma$-ideal of $S$. By Theorem 3.7, we have that $\text{supp}(f)$ is an almost interior $\Gamma$-ideal of $S$. Since $S$ has no proper almost interior $\Gamma$-ideal, $\text{supp}(f) = S$.

Conversely, suppose that $I'$ is an proper almost interior $\Gamma$-ideal of $S$. By Theorem 3.9, we have that $C_{I'}'$ is a fuzzy almost interior $\Gamma$-ideal of $S$. Thus $\text{supp}(C_{I'})' = I' \neq S$, a contradiction. Hence, $S$ has no proper almost interior $\Gamma$-ideal.

Next, we will study the minimality of fuzzy weakly almost interior $\Gamma$-ideals of $\Gamma$-semigroups.

Definition 3.4. A fuzzy weakly almost interior $\Gamma$-ideal $f$ is called minimal if for each fuzzy weakly almost interior $\Gamma$-ideal $g$ of $S$ such that $g \subseteq f$, we have $\text{supp}(g) = \text{supp}(f)$.

Theorem 3.11. A nonempty subset $I$ of a $\Gamma$-semigroup $A$ is a minimal weakly almost interior $\Gamma$-ideal of $S$ if and only if $C_I$ is a minimal weakly almost interior $\Gamma$-ideal of $S$.

Proof. The proof of this theorem is similar to the proof of Theorem 3.9.

From Theorem 3.11, we have the following corollary.

Corollary 3.12. Let $S$ be a $\Gamma$-semigroup. Then $S$ has no proper weakly almost interior $\Gamma$-ideals if and only if for all fuzzy weakly almost interior $\Gamma$-ideals $f$ of $S$, $\text{supp}(f) = S$.

Proof. The proof of this corollary is similar to the proof of Corollary 3.10.

4 Conclusions

In this paper, we define some novel of ideals and fuzzy ideals of $\Gamma$-semigroups and investigate their remarkable properties. In Section 2, we define almost interior $\Gamma$-ideals and weakly almost interior $\Gamma$-ideals of $\Gamma$-semigroups by using ideas of almost interior $\Gamma$-ideals and weakly almost interior $\Gamma$-ideals of $\Gamma$-semigroups in [9]. Moreover, The union of two (weakly)
almost interior $\Gamma$-ideals is also a (weakly) almost interior $\Gamma$-ideals but the intersection of them need not be a (weakly) almost interior $\Gamma$-ideals in $\Gamma$-semigroups. Moreover, we investigate the necessary and sufficient condition of $\Gamma$-semigroups having no proper weakly almost interior $\Gamma$-ideals in Theorem 2.6. In Section 3, we introduce fuzzy almost interior $\Gamma$-ideals and fuzzy weakly almost interior $\Gamma$-ideals of $\Gamma$-semigroups. Moreover, The union of two fuzzy (weakly) almost interior $\Gamma$-ideals is also a fuzzy (weakly) almost interior $\Gamma$-ideals but the intersection of them need not be a fuzzy (weakly) almost interior $\Gamma$-ideals in $\Gamma$-semigroups. We give some relationship between almost interior $\Gamma$-ideals and their fuzzification were shown in Theorem 3.5-3.9 and Theorem 3.11. We give the necessary and sufficient condition of $\Gamma$-semigroups having no proper (weakly) almost interior $\Gamma$-ideals by checking all (weakly) almost interior $\Gamma$-ideals in Corollary 3.10 and 3.12. Moreover, the results in this paper generalized the results in [9] and [10].

In the future work, we can study other kinds of almost ideals and fuzzifications in $\Gamma$-semigroups or almost ideals and fuzzifications in other algebraic structures.

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