Quarkonium Production in an Improved Color Evaporation Model

Yan-Qing Ma\textsuperscript{1,2,*} and Ramona Vogt\textsuperscript{4,†}

\textsuperscript{1}School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
\textsuperscript{2}Center for High Energy Physics, Peking University, Beijing 100871, China
\textsuperscript{3}Collaborative Innovation Center of Quantum Matter, Beijing 100871, China
\textsuperscript{4}Nuclear and Chemical Sciences Division, Lawrence Livermore National Laboratory, Livermore, CA 94551, USA
\textsuperscript{5}Physics Department, University of California at Davis, Davis, CA 95616, USA

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I. INTRODUCTION

The study of heavy quarkonium production is one of the best ways to understand hadronization in QCD. Currently, the most widely used theory for heavy quarkonium production is the nonrelativistic QCD (NRQCD) approach\textsuperscript{28} proposed in 1994. By introducing a systematic velocity expansion, this theory can naturally solve the infrared divergence problem encountered in the color singlet model (CSM)\textsuperscript{29–31}. In this sense, NRQCD factorization can be thought of as a generalized version of CSM. Furthermore, it also successfully explained the $\psi'$ surplus found at Tevatron\textsuperscript{32} by including color octet contributions.

Nevertheless, recent studies have shown that NRQCD factorization encounters serious difficulties\textsuperscript{33}. First, naive power counting implies that $\psi(nS)$ and $\Upsilon(nS)$ productions at hadron colliders are dominated by the $^3S_1$ color octet channel which results in transverse polarization at high transverse momentum, $p_T$. However, experimental measurements found these states to be almost unpolarized. Current explanations of $J/\psi$ production include $^1S_0$ color octet dominance\textsuperscript{34,35} and cancelation of transverse polarization between the $^3S_1$ and $^3P_J$ color octet channels\textsuperscript{34,37,38}. Whether these explanations can be generalized to other quarkonium states is still in question. Second, the nonperturbative color octet long-distance matrix elements (LDMEs) extracted from hadron colliders\textsuperscript{39–41} are inconsistent with the upper bound set by $e^+e^-$ collisions\textsuperscript{32}. Thus the LDMEs are not universal. Finally, there is still no convincing proof of NRQCD factorization to all orders in $\alpha_s$. The state-of-art proof is only to next-to-next-to-leading order for special cases\textsuperscript{43}.

Considering the above difficulties, one should definitely study NRQCD factorization in more detail, but, at the same time, one may need to turn to other theories of quarkonium production. A theory which is known to satisfy all-order factorization is the color evaporation model (CEM)\textsuperscript{44,45}. In this model, to produce a charmonium states $\psi$, one first produces a charm quark-antiquark pair $c\bar{c}$ with invariant mass smaller than the $D$-meson threshold. The pair then hadronizes to the $\psi$ by randomly emitting soft particle\textsuperscript{4}. The production cross section is expressed as

\[
\frac{d\sigma_{\psi}(P)}{d^3P} = F_\psi \int_{2m_c}^{2MD} dM \frac{d\sigma_{c\bar{c}}(M,P)}{dM d^3P},
\]

(1)

where $m_c$ ($M_D$) is the mass of charm quark ($D$ meson) and $M$ is the invariant mass of the $c\bar{c}$ pair. In this model, it is assumed that the $\psi$ momentum, $P$, is approximately the same as the momentum of the $c\bar{c}$ pair. The predictive power of the CEM is based on the assumption that the hadronization factor $F_\psi$ is universal and thus independent of the kinematics and spin of the $\psi$, as well as the production process.

Although CEM is intuitive, simple, and successful to explain $J/\psi$ production data, it has very fatal flaw. A straightforward conclusion from the CEM is that the ratio of differential cross sections of two charmonia states is independent of the kinematics and independent of the colliding species. However, it has long known that experimental results of ratio of production cross section of $\psi'$ over that of $J/\psi$ depend on their transverse momentum (recent experimental data see Refs.\textsuperscript{46,47}). This disagreement is regarded as the main evidence that CEM is a wrong.

Considered the advantages of CEM mentioned above, we may need to study whether a modification of CEM can provide a correct theory for quarkonium production. In this paper, by taking into account physical effects overlooked in the original CEM, we propose an improved

\footnote{\textsuperscript{1}We refer to them as soft gluons here.}
color-evaporation model (ICEM). On the one hand, the nice features of CEM are retained in the ICEM, including having only one parameter for each quarkonium state and satisfying all-order factorization. On the other hand, the ICEM can correctly describe charmonium production cross section ratios.

II. THE IMPROVED COLOR-EVAPORATION MODEL

Our picture of heavy quarkonium (say charmonium) production is as follows. To produce a charmonium state $\psi$, it is necessary to produce a $c\bar{c}$ pair in the hard collision, because the mass of the $c\bar{c}$ pair is much larger than the QCD nonperturbative scale $\Lambda_{\text{QCD}}$. Before the $c\bar{c}$ pair hadronizes to charmonium, it will exchange many soft gluons between various color sources, as well as emit soft gluons. An illustration of this picture is given in Fig. 1. In this figure, the blob marked by ‘$H$’ denotes the hard collision kernel, the blob marked by ‘$S$’ denotes soft interactions, and the thick double lines denotes the $c\bar{c}$ pair with momentum $P$. To separate the hard part from the other parts, we introduce a scale $\lambda$ with $m_c \gg \lambda \gg \Lambda_{\text{QCD}}$, and define the hard part as all particles that are off shell by more than $\lambda^2$.

We emphasize that we distinguish soft gluons exchanged between the $c\bar{c}$ pair and other color sources (with momentum denoted by $P_S$) from soft gluons emitted by the $c\bar{c}$ pair (with momentum denoted by $P_X$). Indeed, these two kinds of soft gluons are significantly different. The total energy of exchanged gluons can be either positive or negative. However, the emitted gluons will eventually evolve to experimentally observable particles. Thus their total momentum must be time-like and their total energy must be positive.

In our model, we construct a relationship between $P$ and $\langle P_\psi \rangle$, the average momentum of $\psi$ that has hadronized from a $c\bar{c}$ pair with fixed momentum $P$. The relationship is easy to obtain in the rest frame of $P$, with $P = (M, 0, 0, 0)$. For each event, we have

$$P = P_\psi + P_S + P_X.$$  \hspace{1cm} (2)

In the spirit of the traditional CEM, we assume the distributions of $P_S$ and $P_X$ are rotation invariant in this frame, which implies $\langle P_S \rangle = (m_S, 0, 0, 0)$ and $\langle P_X \rangle = (m_X, 0, 0, 0)$. Because exchanged gluons can flow in either direction, we may expect $m_S \approx 0$. Thus $\langle P_\psi \rangle = (M - m_X, 0, 0, 0)$ with $m_X > 0$. Therefore,

$$M_\psi < M - m_X < M,$$  \hspace{1cm} (3)

where we use the fact that $\langle P_\psi^0 \rangle$ must be larger than $M_\psi$. Equation (3) sets a lower limit on $M$ that is significantly different from the lower limit $2m_c$ of the traditional CEM.

As both $P_S$ and $P_X$ are order of $\lambda$, power counting of $P_c$ gives $(O(m_c), O(\lambda), O(\lambda), O(\lambda))$. Combining with the on-shell condition $P_\psi^2 = M_\psi^2$, we arrive at $P_\psi^0 = M_\psi + O(\lambda^2/m_c)$. Thus we have

$$\langle P_\psi \rangle = \frac{M_\psi}{M} P + O(\lambda^2/m_c),$$  \hspace{1cm} (4)

which again differs from the relation used in the traditional CEM where $P_c$ is identified with $P$. Note that the proportionality between the momenta of the mother and daughter particles in Eq. (4) was first proposed in Ref. [18] to relate the momentum of the $\chi_{cJ}$ and the $J/\psi$ produced by its decay. It has since been used in many calculations of quarkonium production in the NRQCD framework. In this paper, we prove the relation rigorously with clear assumptions. By combining Eqs. (3) and (4), we arrive at the improved color evaporation model (ICEM):

$$\frac{d\sigma_\psi(P)}{d^3P} = F_\psi \int_{M_\psi}^{2M_\psi} d^3P' dM \frac{d\sigma_{c\bar{c}}(M, P')}{dMd^3P'} \delta^3(P - M_\psi P')$$  

$$= F_\psi \int_{M_\psi}^{2M_\psi} dM \frac{d\sigma_{c\bar{c}}(M, P')}{dMd^3P} \frac{(M/M_\psi)P}{M},$$  \hspace{1cm} (5)

with correction at $O(\lambda^2/m_c^2)$. If one is only interested in the transverse momentum distribution, we have

$$\frac{d\sigma_\psi(P)}{dp_T} = F_\psi \int_{M_\psi}^{2M_\psi} dM \frac{d\sigma_{c\bar{c}}(M, P')}{dMdp_T} |_{p_T=(M/M_\psi)p_T}.$$  \hspace{1cm} (6)

Before performing any numerical calculations, we can already expect some advantages of the ICEM. First, because there is an explicit charmonium mass dependence in Eq. (6), the ratio of differential cross sections of two charmonia is no longer $p_T$-independent in the ICEM. Thus it is possible to explain data such as $d\sigma_{\psi(2S)}/d\sigma_{J/\psi}$. 

FIG. 1. An illustration of charmonium production in a high energy collision. See text for details.
Second, by making a distinction between the momentum of the $c\bar{c}$ pair and that of charmonium, the predicted $p_T$ spectra will be softer and thus may explain the high $p_T$ data better.

We emphasize that the ICEM Eq. [3] does not mean that $c\bar{c}$ pair with invariant mass smaller than $M_{c\bar{c}}$ has no possibility to hadronize to $\psi$. In fact, this kind of $c\bar{c}$ pair can absorb energy by interacting with other color source, and thus can have larger invariant mass and hadronize to $\psi$. At the same time, even if the invariant mass of $c\bar{c}$ pair is larger than $M_{c\bar{c}}$, it may loss energy by interacting with other color source, and eventually cannot hadronize to $\psi$ because of invariant mass being too small. By assuming $m_S \approx 0$, we effectively approximate that the two effects cancel each other. As a result, the Eq. [3] should be only interpreted at the integration level.

An exception for the above argument is for the ground state particle production, say $\eta_c$ for charmonium. Based on the quark-hadron duality, $c\bar{c}$ pair with invariant mass smaller than $D$ meson threshold must hadronize to charmonium, therefore it is not possible for a $c\bar{c}$ pair with $M_{c\bar{c}} > M_{\eta_c}$ to emit too much energy so that its invariant mass becomes smaller than $M_{\eta_c}$. This means that the approximation $m_S \approx 0$ is not reasonable here and thus ICEM is not good for $\eta_c$ production. However, for $\eta_c$ production, as the condition Eq. [4] is not needed, the original CEM should be good.

### III. NUMERICAL RESULTS

To confront our model with experimental data, we updated the CEM parameters determined in Ref. [50]. In that work, in an attempt to reduce the uncertainty on the total charm cross section, the charm mass was fixed at $1.27 \pm 0.09$ GeV while the factorization and renormalization scales were fit to a subset of the measured total charm cross section data. The values found were $\mu_F/m = 2.1^{+2.55}_{-0.85}$ and $\mu_R/m = 1.6^{+0.11}_{-0.12}$ employing the CT10 proton parton densities [51].

The central open charm parameter set $(m, \mu_F/m, \mu_R/m) = (1.27, 2.1, 1.6)$ was used to calculate the energy dependence of the forward $J/\psi$ cross section, $\sigma(x_F > 0)$, in the CEM using the exclusive $c\bar{c}$ production code described in Ref. [52]. Because the NLO $c\bar{c}$ code is an exclusive calculation, the mass cut is on the invariant average over kinematic variables of the $c$ and $\bar{c}$. Thus, in this calculation $\mu_F$ and $\mu_R$ are defined relative to the transverse mass of the charm quark, $\mu_{F,R} \propto m_T = \sqrt{m^2 + p_T^2}$, where $p_T^2 = 0.5(p_T^2 + p_{\bar{T}}^2)$. The normalization $F_\psi$ is the scale factor that adjusted the fraction of the total charm cross section in the mass range $2m < M < 2m_{D}$ to the forward cross section data.

To determine the uncertainty on the $J/\psi$ calculation, the charm mass was varied between the upper and lower limits, 1.36 and 1.18 GeV respectively, for the central values of $\mu_F/m$ and $\mu_R/m$, and the scales were varied around their central values while the charm mass was held fixed at its central value of 1.27 GeV: $(\mu_F/m, \mu_R/m) = (C, L), (L, C), (L, L), (C, H), (H, C), (H, H)$ where $H(L)$ is the upper (lower) limit of the factorization and renormalization scales determined from the charm fits. Using the same value of $F_\psi$ in all cases, the uncertainty band on the $J/\psi$ cross section was calculated by finding the upper and lower limits of the mass and scale variations and adding them in quadrature, as discussed in Refs. [53, 54].

To calculate the charmonium $p_T$ dependence, a Gaussian transverse momentum broadening is added to the final state. The value of the average $k_T$ kick applied was taken to be $\langle k_T^2 \rangle = 1 + (1/12) \ln(\sqrt{3}/20) \text{GeV}^2$ [50], giving 1.19 GeV$^2$ at RHIC and 1.49 GeV$^2$ at 7 TeV.

Since the ICEM calculation discussed here reduces the cross section relative to the calculation in Ref. [50], the value of $F_\psi$ had to be increased by 40% to retain agreement with the data. The $\psi'$ cross section and its uncertainty was calculated with the same parameters but with a value of $F_{\psi'}$ scaled to the $\psi'$ data.

To obtain the uncertainty on the $\psi'/\psi$ ratio, the mass and scale uncertainties were assumed to be correlated. The resulting uncertainty band is dominated by the scale uncertainty, the mass uncertainty is small.

Our results for $J/\psi$ production cross section as a function of $p_T$ are shown in Fig. 2, where we compare with data at hadron colliders for center of mass energies of 0.2 TeV and 7 TeV. The 0.2 TeV RHIC data are measured by the PHENIX Collaboration [46] at central rapidities, $|y| < 0.35$, and the 7 TeV LHC data are measured by the LHCb Collaboration [49] at forward rapidity, $2.5 < y < 4$. The largest discrepancy between the model and the data is in the RHIC data at intermediate $p_T$, $4 < p_T < 7$ GeV. However, since the experimental uncertainty is rather large in this region, our results are in general agreement with the data.

We now turn to the $\psi'$ production cross section as a function of $p_T$ in Fig. 3. We again compare with the midrapidity PHENIX data [46] at 0.2 TeV and the forward LHCb data [47] at 7 TeV. Since the $\psi'$ rates are generally lower, the measured uncertainty is larger. Given this, the agreement of the calculation with the data is also good.

The ratio of the production cross sections of $\psi'$ to that of $J/\psi$ as a function of $p_T$ is given in Fig. 4. The 0.2 TeV RHIC data and 7 TeV LHC data are taken from Ref. [46] and Ref. [47], respectively. Although the original CEM predicts a constant for this ratio, in contradiction with the data, our ICEM calculations are in good agreement with all data.

### IV. SUMMARY AND DISCUSSION

By distinguishing between exchanged and emitted soft gluons and considering some physical constraints, we propose an improved color evaporation model for charmonium production. Comparison with data shows that the
ICEM can nicely reproduce the $p_T$ dependence of the ratio of the $\psi'$ to $J/\psi$ production cross sections. Thus, this improved model overcomes one of the main obstacles of the original CEM. The success of the ICEM calculation confirms our picture of charmonium production.

We note that the question of polarization in the ICEM as well as the original CEM has not yet been addressed. As seen in the NRQCD approach, the polarization is an important test of models. The prediction of the final-state charmonium polarization depends on whether soft gluons change spin and angular momentum of the $c\bar{c}$ pair. A preliminary study of charmonium polarization in the CEM will be presented elsewhere.

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FIG. 4. Results for ratio of the $\psi'$ production cross section to that of $J/\psi$. The 0.2 TeV PHENIX data and 7 TeV LHCb data are taken from Ref. [16] and Ref. [17], respectively.

[1] G. T. Bodwin, E. Braaten, and G. P. Lepage, Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium, *Phys. Rev. D* **51** (1995) 1125, hep-ph/9407393 [InSPIRE].

[2] S. Ellis, M. B. Einhorn, and C. Quigg, Comment on Hadronic Production of Psions, *Phys. Rev. Lett.* **36** (1976) 1263 [InSPIRE].

[3] C. Carlson and R. Suaya, Hadronic Production of $\psi/J/\psi$ Mesons, *Phys. Rev. D* **14** (1976) 3115 [InSPIRE].

[4] C.-H. Chang, Hadronic Production of $J/\psi$ Associated With a Gluon, *Nucl. Phys. B* **172** (1980) 425–434 [InSPIRE].

[5] E. Braaten and S. Fleming, Color octet fragmentation and the $\psi'$ surplus at the Tevatron, *Phys. Rev. Lett.* **74** (1995) 3327, hep-ph/9411365 [InSPIRE].

[6] N. Brambilla, S. Eidelman, B. Heltsley, R. Vogt, G. Bodwin, et al., Heavy quarkonium: progress, puzzles, and opportunities, *Eur. Phys. J. C* **71** (2011) 1534, arXiv:1010.5827 [InSPIRE].

[7] K.-T. Chao, Y.-Q. Ma, K.-T. Chao, Y.-Q. Ma, K. Wang, and K.-T. Chao, $J/\psi$ Polarization at Hadron Colliders in Nonrelativistic QCD, *Phys. Rev. Lett.* **108** (2012) 242004, arXiv:1201.2675 [InSPIRE].

[8] G. T. Bodwin, H. S. Chung, U.-R. Kim, and J. Lee, Fragmentation contributions to $J/\psi$ production at the Tevatron and the LHC, *Phys. Rev. Lett.* **113** (2014) 022001, arXiv:1403.3612 [InSPIRE].

[9] P. Faccioli, V. Knunz, C. Lourenco, J. Seixas, and H. K. Wohri, Quarkonium production in the LHC era: a polarized perspective, *Phys. Lett. B* **736** (2014) 98–109, arXiv:1403.3970 [InSPIRE].

[10] H. Han, Y.-Q. Ma, C. Meng, H.-S. Shao, and K.-T. Chao, $\eta$ Production at LHC and Implications for the Understanding of $J/\psi$ Production, *Phys. Rev. Lett.* **114** (2015) 092005, arXiv:1411.7350 [InSPIRE].

[11] H.-F. Zhang, Z. Sun, W.-L. Sang, and R. Li, Impact of $\eta$ hadroproduction data on charmonium production and polarization within NRQCD framework, *Phys. Rev. Lett.* **114** (2015) 092006, arXiv:1412.0508 [InSPIRE].

[12] Y.-Q. Ma, K. Wang, and K.-T. Chao, $J/\psi(\psi')$ production at the Tevatron and LHC at $O(\alpha_s^4v^4)$ in nonrelativistic QCD, *Phys. Rev. Lett.* **106** (2011) 042002, arXiv:1009.3655 [InSPIRE].

[13] M. Butenschoen and B. A. Kniehl, Reconciling $J/\psi$ production at HERA, RHIC, Tevatron, and LHC with NRQCD factorization at next-to-leading order, *Phys. Rev. Lett.* **106** (2011) 022003, arXiv:1009.5662 [InSPIRE].

[14] B. Gong, L.-P. Wan, J.-X. Wang, and H.-F. Zhang, Polarization for Prompt $J/\psi, \psi(2S)$ production at the Tevatron and LHC, *Phys. Rev. Lett.* **110** (2013) 042002, arXiv:1205.6682 [InSPIRE].

[15] Y.-J. Zhang, Y.-Q. Ma, K. Wang, and K.-T. Chao, QCD radiative correction to color-octet $J/\psi$ inclusive production at $B$ Factories, *Phys. Rev. D* **81** (2010) 034015, arXiv:0911.2166 [InSPIRE].

[16] G. C. Nayak, J.-W. Qiu, and G. Sterman, NRQCD Factorization and Velocity-dependence of NNLO Poles in Heavy Quarkonium Production, *Phys. Rev. D* **74** (2006) 074007, hep-ph/0608066 [InSPIRE].

[17] H. Fritzsch, Producing Heavy Quark Flavors in Hadronic Collisions: A Test of Quantum Chromodynamics, *Phys. Lett. B* **67** (1977) 217 [InSPIRE].

[18] F. Halzen, Cvc for Gluons and Hadroproduction of Quark Flavors, *Phys. Lett. B* **69** (1977) 105 [InSPIRE].

[19] PHENIX Collaboration, A. Adare et al., Ground and excited charmonium state production in $p+p$ collisions at $\sqrt{s} = 200$ GeV, *Phys. Rev. D* **85** (2012) 092004, arXiv:1105.1966 [InSPIRE].

[20] LHCb Collaboration, R. Aaij et al., Measurement of $\psi(2S)$ meson production in pp collisions at $\sqrt{s}=7$ TeV, *Eur. Phys. J. C* **72** (2012) 3100, arXiv:1204.1258 [InSPIRE].

[21] Y.-Q. Ma, K. Wang, and K.-T. Chao, QCD radiative corrections to $\chi_{cJ}$ production at hadron colliders.
[2] LHCb Collaboration, R. Aaij et al., Measurement of J/ψ production in pp collisions at √s = 7 TeV, *Eur. Phys. J. C* 71 (2011) 1645 [arXiv:1103.0423].

[3] R. Nelson, R. Vogt, and A. Frawley, Narrowing the uncertainty on the total charm cross section and its effect on the J/ψ cross section, *Phys. Rev. C* 87 (2013) 014908 [arXiv:1210.4610].

[4] M. L. Mangano, P. Nason, and G. Ridolfi, Heavy quark correlations in hadron collisions at next-to-leading order, Nucl. Phys. B 373 (1992) 295.

[5] M. Cacciari, P. Nason, and R. Vogt, QCD predictions for charm and bottom production at RHIC, *Phys. Rev. Lett.* 95 (2005) 122001 [hep-ph/0502203].

[6] H. S. Cheung and R. Vogt in progress (2016).

[7] G. T. Bodwin, E. Braaten, and G. P. Lepage, Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium, *Phys. Rev. D* 51 (1995) 1125 [hep-ph/9407339].

[8] C. Carlson and R. Suaya, Hadronic Production of ψ/J Mesons, *Phys. Rev. D* 14 (1976) 3115 [InSPIRE].

[9] C.-H. Chang, Hadronic Production of J/ψ Associated With a Gluon, *Nucl. Phys.* B172 (1980) 425–434 [InSPIRE].

[10] E. Braaten and S. Fleming, Color octet fragmentation and the ψ’ surplus at the Tevatron, *Phys. Rev. Lett.* 74 (1995) 3327 [hep-ph/9411365].

[11] N. Brambilla, S. Eidelman, B. Heltsley, R. Vogt, G. Bodwin, et al., Heavy quarkonium: progress, puzzles, and opportunities, *Eur. Phys. J. C* 71 (2011) 1534 [arXiv:1010.5827].

[12] K.-T. Chao, Y.-Q. Ma, H.-S. Shao, K. Wang, and Y.-J. Zhang, J/ψ Polarization at Hadron Colliders in Nonrelativistic QCD, *Phys. Rev. Lett.* 108 (2012) 242004 [arXiv:1201.2675].

[13] G. T. Bodwin, H. S. Chung, U.-R. Kim, and J. Lee, Fragmentation contributions to J/ψ production at the Tevatron and the LHC, *Phys. Rev. Lett.* 113 (2014) 022001 [arXiv:1403.3612].

[14] P. Faccioli, V. Kunnz, C. Lourenco, J. Seixas, and H. K. Wolfr, Quarkonium production in the LHC era: a polarized perspective, *Phys. Lett. B* 736 (2014) 98–100 [arXiv:1403.3970].

[15] H. Han, Y.-Q. Ma, C. Meng, H.-S. Shao, and K.-T. Chao, ηc Production at LHC and Implications for the Understanding of J/ψ Production, *Phys. Rev. Lett.* 114 (2015) 092005 [arXiv:1411.7350].

[16] H.-F. Zhang, Z. Sun, W.-L. Sang, and R. Li, Impact of ηc hadroproduction data on charmonium production and polarization within NRQCD framework, *Phys. Rev. Lett.* 114 (2015) 092006 [arXiv:1412.0508].

[17] Y.-Q. Ma, K. Wang, and K.-T. Chao, J/ψ(ψ′) production at the Tevatron and LHC at O(αs^2) in nonrelativistic QCD, *Phys. Rev. Lett.* 106 (2011) 042002 [arXiv:1009.3655].

[18] M. Butenschoen and B. A. Kniehl, Reconciling J/ψ production at HERA, RHIC, Tevatron, and LHC with NRQCD factorization at next-to-leading order, *Phys. Rev. Lett.* 106 (2011) 022003 [arXiv:1009.5662].

[19] B. Gong, L.-P. Wan, J.-X. Wang, and H.-F. Zhang, Polarization for Prompt J/ψ, ψ(2S) production at the Tevatron and LHC, *Phys. Rev. Lett.* 110 (2013) 042002 [arXiv:1205.6882].

[20] Y.-J. Zhang, Y.-Q. Ma, K. Wang, and K.-T. Chao, QCD radiative correction to color-octet J/ψ inclusive production at B Factories, *Phys. Rev. D* 81 (2010) 034015 [arXiv:0911.2166].

[21] G. C. Nayak, J.-W. Qu, and G. Sterman, NRQCD Factorization and Velocity-dependence of NNLO Poles in Heavy Quarkonium Production, *Phys. Rev. D* 74 (2006) 074007 [hep-ph/0608066].

[22] H. Fritzsch, Producing Heavy Quark Flavors in Hadronic Collisions: A Test of Quantum Chromodynamics, *Phys. Lett.* B67 (1977) 217 [InSPIRE].

[23] F. Halzen, Csc for Gluons and Hadroproduction of Quark Flavors, *Phys. Lett.* B69 (1977) 105 [InSPIRE].

[24] PHENIX Collaboration, A. Adare et al., Ground and excited charmonium state production in p + p collisions at √s = 200 GeV, *Phys. Rev. D* 85 (2012) 092004 [arXiv:1105.1966].

[25] LHCb Collaboration, R. Aaij et al., Measurement of ψ(2S) meson production in pp collisions at √s=7 TeV, *Eur. Phys. J. C* 72 (2012) 2100 [arXiv:1204.1258].

[26] Y.-Q. Ma, K. Wang, and K.-T. Chao, QCD radiative correction to χcJ production at hadron colliders, *Phys. Rev. D* 83 (2011) 111503 [arXiv:1002.3987].

[27] LHCb Collaboration, R. Aaij et al., Measurement of J/ψ production in pp collisions at √s = 7 TeV, *Eur. Phys. J. C* 71 (2011) 1645 [arXiv:1103.0423].

[28] R. Nelson, R. Vogt, and A. Frawley, Narrowing the uncertainty on the total charm cross section and its effect on the J/ψ cross section, *Phys. Rev. C* 87 (2013) 014908 [arXiv:1210.4610].

[29] H.-L. Lai, M. Guzzi, J. Huston, Z. Li, P. M. Nadolsky, J. Pumplin, and C. P. Yuan, New parton distributions for charm and bottom production at RHIC, *Phys. Rev. Lett.* 110 (2013) 014908 [arXiv:1205.6882].

[30] M. Cacciari, P. Nason, and R. Vogt, QCD predictions for charm and bottom production at RHIC, *Phys. Rev. Lett.* 95 (2005) 122001 [hep-ph/0502203].