Thermal ground state and nonthermal probes

Thierry Grandou† and Ralf Hofmann*

† Institut Nonlinéaire de Nice
1361 route des Lucioles, Sophia Antipolis
F-06560 Valbonne, France

* Institut für Theoretische Physik
Universität Heidelberg
Philosophenweg 16
69120 Heidelberg, Germany

Abstract

The Euclidean formulation of SU(2) Yang-Mills thermodynamics admits periodic, (anti)selfdual solutions to the fundamental, classical equation of motion which possess one unit of topological charge: (anti)calorons. A spatial coarse graining over the central region in a pair of such localised field configurations with trivial holonomy generates an inert adjoint scalar field \( \phi \), effectively describing the pure quantum part of the thermal ground state in the induced quantum field theory. The latter’s local vertices are mediated by just-not-resolved (anti)caloron centers of action \( \hbar \). This is the basic reason for a rapid convergence of the loop expansion of thermodynamical quantities, polarization tensors, etc., their effective loop momenta being severely constrained in entirely fixed and physical unitary-Coulomb gauge. Here we show for the limit of zero holonomy how (anti)calorons associate a temperature independent electric permittivity and magnetic permeability to the thermal ground state of SU(2)\(_{\text{CMB}}\), the Yang-Mills theory conjectured to underlie photon propagation.
1 Introduction

Quantum Mechanics is a highly efficient framework to describe the subatomic world \cite{1,2,3}, including coherence phenomena that extend to macroscopic length and time scales \cite{4,5,6}. The key quantity to describe deviations from classical behavior is Planck’s quantum of action $\hbar = \frac{\lambda}{2\pi} = 6.58 \times 10^{16}$ eVs which determines the fundamental interaction between charged matter and the electromagnetic field and thus also the shape of blackbody spectra by relating frequency $\omega$ and wave vector $k$ to particle-like energy $E = \hbar \omega$ and momentum $p = \hbar k$ and by appeal to Bose-Einstein statistics. In Quantum Mechanics, $\hbar$ sets the strength of multiplicative noncommutativity for a pair of canonically conjugate variables such as position and momentum, implying the respective uncertainty relations.

Although generally accepted as a universal constant of nature and in spite of the fact that we are able to efficiently compute quantum mechanical amplitudes and quantum statistical averages for a vast variety of processes in particle collisions, atoms and molecules, extended condensed-matter systems, and astrophysical objects to match experiment and observation very well, one should remain curious concerning the principle mechanism that causes the emergence of a universal quantum of action. In \cite{7,8} it was argued that the irreconcilability of classical Euclidean and Minkowskian time evolution as expressed by a time-periodic SU(2) (anti)selfdual gauge field configuration – a (anti)caloron –, whose action $\hbar$ is associated with one unit of winding about a central spacetime point, gives rise to indeterminism in the process it mediates. That each unit of action assigned to (anti)calorons of radius $\rho = |\phi|^{-1}$, which dominate the emergence of the thermal ground state, equals $\hbar$ follows from the value of the coupling $e$ in the induced, effective, thermal quantum field theory \cite{11} of the deconfining phase in SU(2) Yang-Mills thermodynamics. The coupling $e$, in turn, obeys an evolution in temperature (flat almost everywhere) which represents the validity of Legendre transformations in the effective ensemble where the thermal ground state co-exists with massive (adjoint Higgs mechanism) and massless (intact U(1)) thermal fluctuations. The thermal ground state thus is a spatially homogeneous ensemble of quantum fluctuations carried by (anti)caloron centers. At the same time, as we shall see, this state provides electric and magnetic dipole densities supporting the propagation of electromagnetic waves in an SU(2) Yang-Mills of scale $\Lambda \sim 10^{-4}$ eV, SU(2)$_{\text{CMB}}$ \cite{9}.

In the present work, we establish this link between quantised action, represented by $\phi$, and classical wave propagation enabled by the vacuum parameters $e_0$ and $\mu_0$ in terms of the central and peripheral structure of a trivial-holonomy (anti)caloron, respectively. That is, by allowing a fictitious temperature $T$ to represent the energy density of an electromagnetic wave (nonthermal, external probe) via the thermal ground state through which it propagates we ask what this implies for $e_0$ and $\mu_0$. As a result, both $e_0$ and $\mu_0$ neither depend on $T$ nor, as we shall argue, on any singled-out inertial frame. But this means no more and no less than the rivival of the luminiferous aether, albeit now in a Poincaré invariant way.
This paper is organised as follows. In the next section we shortly discuss key features of the effective theory for the deconfining phase of SU(2) Yang-Mills thermodynamics. Sec. 3 contains a reminder to principles in interpreting a Euclidean field configuration in terms of Minkowskian observables. In a next step, general facts are reviewed on Euclidean, periodic, (anti)selfdual field configurations of charge modulus unity concerning the central locus of action, their holonomy, and their behaviour under semiclassical deformation. Finally, we review the anatomy of zero-holonomy Harrington-Shepard (HS) caloron in detail, pointing out its staticity for spatial distances from the center that exceed the inverse of temperature, and discuss which static charge configuration it resembles depending on two distinct distance regimes. In Sec. 4 we briefly review the postulate that an SU(2) Yang-Mills theory of scale \( \Lambda \sim 10^{-4} \text{eV} \), SU(2)\text{CMB}, describes photon propagation [9]. Subsequently, the large-distance regime in a HS (anti)caloron is considered in order to deduce an expression for \( \varepsilon_0 \) based on knowledge about the electric dipole moment provided by a (anti)caloron of radius radius \( \rho = |\phi|^{-1} \), the size of the spatial coarse-graining volume \( V_{cg} \), and the fact that the energy density of the probe must match that of the thermal ground state. As a result, \( \varepsilon_0 \) and \( \mu_0 \) turn out to be \( T \) independent, the former representing an electric charge, large on the scale of the electron charge, of the fictitious constituent monopoles giving rise to the associated dipole density. Zooming in to smaller spatial distances to the center, the HS (anti)caloron exhibits isolated (anti)selfdual monopoles. For them to turn into dipoles shaking by the probe fields is required. We then show that the definitions of \( \varepsilon_0 \) and \( \mu_0 \), which were successfully applied to the large-distance regime, become meaningless. Finally, our results are discussed. Sec. 5 summarises the paper and discusses the universality of \( \varepsilon_0 \) and \( \mu_0 \) for the entire electromagnetic spectrum.

2 Sketch of deconfining SU(2) Yang-Mills thermodynamics

For deconfining SU(2) Yang-Mills thermodynamics, a spatial coarse graining over the (anti)selfdual, that is, the nonpropagating [11], topological sector with charge modulus \( |Q| = 1 \) can be performed, see [9] and references therein, to yield an inert adjoint scalar field \( \phi \). Its modulus \( |\phi| \) sets the maximal possible resolution in the effective theory whose ground state energy density essentially is given as \( \text{tr} \frac{\Lambda^6}{\phi^2} = 4\pi\Lambda^3 T \) (\( \Lambda \) a constant of integration of dimension mass) and whose propagating sector is, in a totally fixed, physical gauge (unitary-Coulomb) characterised by a massless mode (\( \gamma \), unbroken U(1) subgroup of SU(2)) and two thermal quasiparticle modes of equal mass \( m = 2e |\phi| \) (\( V^\pm \), mass induced by adjoint Higgs mechanism) which propagate thermally, that is, on-shell only. Interactions within this propagating sector are mediated by isolated (anti)calorons whose action is argued to be \( \hbar \) [7, 8]. Judged in terms of inclusive quantities such as radiative corrections to the one-loop pressure or the energy density of blackbody radiation, these interactions are
feeble [9], and their expansion into 1-PI irreducible bubble diagrams is conjectured to terminate at a finite number of loops [12]. However, spectrally seen, the effects of $V^\pm$ interacting with $\gamma$ lead to severe consequences at low frequencies and temperatures comparable to the critical temperature $T_c$ where screened (anti)monopoles, released by (anti)caloron dissociation upon large-holonomy deformations [13], rapidly become massless and thus start to condense.

3 Caloron structure

3.1 Euclidean field theory and interpretable quantities

Nontrivial solutions to an elliptic differential equation, such as the Euclidean Yang-Mills equation $D_\mu F_{\mu\nu} = 0$, no longer are solutions of the corresponding hyperbolic equation upon analytic continuation $x_4 \equiv \tau \to i\tau_0$ (Wick rotation). To endow meaning to quantities computed on classical field configurations on a 4D Euclidean spacetime in SU(2) Yang-Mills thermodynamics in terms of observables in a Minkowskian spacetime we thus must insist that these quantities are not affected by the Wick rotation. That is, to assign a real-world interpretation to a Euclidean quantity it needs to be (i) either stationary (not depend on $\tau$) or (ii) associated with an instant in Euclidean spacetime because, by exploiting time translational invariance of the Yang-Mills action, this instant can be picked as $(\tau = 0, x)$ in Euclidean spacetime.

3.2 Review of general facts

If not stated otherwise we work in supernatural units, $\hbar = c = k_B = 1$, where $\hbar$ is the reduced quantum of action, $c$ the speed of light in vacuum, and $k_B$ Boltzmann’s constant. A trivial-holonomy caloron of topological charge unity on the cylinder $S_1 \times \mathbb{R}^3$, where $S_1$ is the circle of circumference $\beta \equiv 1/T$ ($T$ temperature) describing the compactified Euclidean time dimension ($0 \leq \tau \leq \beta$), is constructed by an appropriate superposition of charge-one singular-gauge instanton prepotentials [14] with the temporal coordinate of their instanton centers equidistantly stacked along the infinitely extended Euclidean time dimension [15] to enforce temporal periodicity, $A_\mu(\tau = 0, x) = A_\mu(\tau = \beta, x)$. For gauge group SU(2) this Harrington-Shepard (HS) caloron is given as (antihermitian generators $t_a$ ($a = 1, 2, 3$) with $\text{tr} t_a t_b = -\frac{1}{2} \delta_{ab}$):

$$A_\mu = \bar{\eta}_{\mu\nu}^a t_a \partial_\nu \log \Pi(\tau, r),$$

where $r \equiv |x|$, $\bar{\eta}_{\mu\nu}^a$ denotes the antiselfdual 't Hooft symbol, $\bar{\eta}_{\mu\nu}^a = \epsilon_{\mu\nu}^a - \delta_{a\mu} \delta_{\nu4} + \delta_{a\nu} \delta_{\mu4}$, and

$$\Pi(\tau, r) = 1 + \frac{\pi \rho^2}{\beta r} \frac{\sinh \left( \frac{2\pi r}{\beta} \right)}{\cosh \left( \frac{2\pi r}{\beta} \right) - \cos \left( \frac{2\pi r}{\beta} \right)}.$$
Here $\rho$ is the scale parameter of the singular-gauge instanton to seed the "mirror sum" within $S^1 \times \mathbb{R}^3$, leading to Eq. (2). The associated antiselfdual field configuration is obtained in replacing $\bar{\eta}^a_{\mu\nu}$ by $\eta^{a\mu\nu}$ (selfdual 't Hooft symbol) in Eq. (1).

Configuration (1) is singular at $\tau = r = 0$. This point is the locus of the configuration's topological charge $Q = 1$ in the sense that the integral of the Chern-Simons current $K_\mu = \frac{1}{16\pi^2} \epsilon_{\mu\alpha\beta\gamma} \left( A_\alpha^a \partial_\beta A_\gamma^a + \frac{1}{3} \epsilon^{abc} A_\alpha^a A_\beta^b A_\gamma^c \right)$ over a three-sphere $S^3_\delta$ of radius $\delta$, which is centered there, yields unity independently of $\delta \geq 0$. Selfduality implies that the action of the HS caloron is given as

$$S_C = \frac{8\pi^2}{g^2} \int_{S^3_\delta} d\Sigma_\mu K_\mu = \frac{8\pi^2}{g^2}, \quad (3)$$

where $g$ is the coupling constant in Euclidean (classical) theory. Eq. (3) holds in the limit $\delta \to 0$, meaning that $S_C$ can be attributed to the singularity of the HS solution at $\tau = r = 0$ and thus has a Minkowskian interpretation, see Sec. 3.1. Based on [10] and on the fact that the thermal ground state emerges from $|Q| = 1$ caloron/anticalorons, whose scale parameter $\rho$ essentially coincides with the inverse of maximal resolution, $|\phi|^{-1}$, in the effective theory for deconfining SU(2) Yang-Mills thermodynamics, it was argued in [7], see also [8], that $S_C$ (as well as the action of a HS anticaloron $S_A$ with $\rho \sim |\phi|^{-1}$) equals $\hbar$ if the effective theory is to be interpreted as a local quantum field theory.

The HS caloron is the trivial-holonomy limit of the selfdual Lee-Lu-Kraan-van-Baal (LLKvB) configuration with $Q = 1$ and total magnetic charge zero [16, 17] which is constructed via the Nahm transformation of selfdual fields on the Euclidean four-torus [18]. For nontrivial holonomy ($A_4(r \to \infty) = iu \mathbb{T}^3$ with $0 < u < \frac{2\pi}{\beta}$) the LLKvB solution exhibits a pair of a magnetic monopole (m) and its antimonopole (a) w.r.t. the Abelian subgroup $U(1) \subset SU(2)$ left unbroken by $A_4(r \to \infty) \neq 0$. Their masses are $m_m = 4\pi u$ and $m_a = 4\pi \left( \frac{2\pi}{\beta} - u \right)$ such that in the trivial-holonomy limits $u \to 0, 2\pi/\beta$, one of these magnetic constituents becomes massless and thus completely spatially delocalised. For nontrivial holonomy, where both monopole and antimonopole are of finite mass, localised, and separated by a spatial distance

$$s = \pi \frac{\rho^2}{\beta}, \quad (4)$$

they can be considered static by an exact cancellation of attraction, mediated by their $U(1)$ magnetic fields, and repulsion due to the field $A_4$. As was shown in [13] by investigating the effective action of a LLKvB caloron (integrating out Gaussian fluctuations), this balance is distorted, leading to monopole-antimonopole attraction for

$$0 < u \leq \frac{\pi}{\beta} \left( 1 - \frac{1}{\sqrt{3}} \right) \quad \text{or} \quad \frac{\pi}{\beta} \left( 1 + \frac{1}{\sqrt{3}} \right) < u \leq \frac{2\pi}{\beta}, \quad (5)$$

and to repulsion in the complementary range of (large) holonomy. Because there is no localised counter part to a monopole or antimonopole in the trivial-holonomy
limit, HS calorons must be considered stable under Gaussian fluctuations, in contrast to the case of nontrivial holonomy which is unstable. The latter statement is also mirrored by the fact that a nontrivial, static holonomy leads to zero quantum weight in the infinite-volume limit (which is realistic at high temperatures \[^9\] where the radius of the spatial coarse-graining volume for a single caloron diverges as \(|\chi|^{-1} = \sqrt{2x^T / \Lambda^3} \), \(\Lambda\) the Yang-Mills scale). As a consequence, nontrivial holonomy can only occur transiently in configurations which do not saturate (anti)selfduality bounds to the Yang-Mills action. Again, this is equivalent to stating the instability of the LLKvB solution. It can be shown \[^9\] that the small-holonomy case of monopole-antimonople attraction by far dominates the situation of monopole-antimonople repulsion when a caloron dissociates into its constituents.

The spatial coarse graining over (anti)selfdual calorons of charge modulus \(|Q| = 1\), which do not propagate (due to (anti)selfduality their energy-momentum tensor vanishes identically \[^11\] ), yielding a highly accurate a priori estimate of the deconfining thermal ground state in terms of an inert, adjoint scalar field \(\phi\) and a pure-gauge configuration \(a_{\mu}^{gs}\), is performed over isolated and stable HS solutions \[^9\]. The coarse-grained field \(a_{\mu}^{gs}\) represents a posteriori the effects of small holonomy changes due to (anti)caloron overlap and interaction.

### 3.3 Anatomy of a relevant Harrington-Shepard caloron

Let us now review \[^19\] how the field strength of a HS caloron depends on the distance from its center at \(\tau = r = 0\). For \(|x| \ll \beta\) \((|x| \equiv \sqrt{x^2} \equiv \sqrt{x_\mu x_\mu}, x_4 \equiv \tau)\) one has

\[
\Pi(x) = (1 + \frac{\pi}{3} s + \frac{\rho^2}{x^2} + O(x^2/\beta^2)),
\]

where \(s\) is defined in Eq. \(4\). From Eqs. \(6\) and \(1\) one obtains with \(|x| \ll \beta\) the following expression for \(F_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\kappa\lambda} F_{\kappa\lambda} \equiv \tilde{F}_{\mu\nu}\)

\[
F_{\mu\nu}^{a} = -4 \rho^2 \frac{\hat{\eta}_{\alpha\beta}^{a}}{(x^2 + \rho^2)^2} I_{\alpha\mu} I_{\beta\nu} + O(x^2/\beta^4),
\]

where \(I_{\alpha\mu} \equiv \delta_{\alpha\mu} - 2 \frac{x_\alpha x_\mu}{x^2}\). At small four-dimensional distances from the caloron center the field strength thus behaves like the one of a singular-gauge instanton with a renormalised scale parameter \(\rho^2 = \frac{\rho^2}{\sqrt{3+3}}\). Therefore, the field strength of the HS solution exhibits a dependence on \(\tau\) and as such has no Minkowskian interpretation, see Sec.\[3.1\]. What can be inferred for a Minkowskian spacetime though is that the action of the configuration is attributable to winding of the caloron around the group manifold \(S_3\) as induced by a spacetime point, the instanton center. This is because, in the sense of Eq. \(3\), an instant has no analytic continuation or Wick rotation. (The 4D action or topological-charge density of the caloron is regular at \(\tau = r = 0\), does depend on Euclidean spacetime in the vicinity of this point, and thus has no Minkowskian interpretation.)
For $r \gg \beta$ the selfdual electric and magnetic fields $E^a_i$ and $B^a_i$ are static and can be written as

$$E^a_i = B^a_i \sim -\frac{\hat{x}^a_i}{r^2} - \frac{1}{rs}(\delta^a_i - 3\hat{x}^a\hat{x}_i)\left(1 + \frac{s}{r}\right)^2,$$

where $\hat{x}_i \equiv \frac{x_i}{r}$ and $\hat{x}^a \equiv \frac{x^a}{r}$. For $\beta \ll r \ll s$ Eq. (8) simplifies as

$$E^a_i = B^a_i \sim -\frac{\hat{x}^a_i}{r^2},$$

and thus describes a static non-Abelian monopole of unit electric and magnetic charges (dyon). For $r \gg s \gg \beta$ Eq. (8) reduces to

$$E^a_i = B^a_i \sim \frac{s\delta^a_i - 3\hat{x}^a\hat{x}_i}{r^3}.$$  

This is the field strength of a static, selfdual non-Abelian dipole field, its dipole moment $p^a_i$ given as

$$p^a_i = s\delta^a_i.$$  

Interestingly, the same distance $s$, which sets the separation between the charge centers of an Abelian magnetic monopole and its antimonopole in a nontrivial-holonomy caloron, prescribes here for the case of trivial holonomy how small $r$ needs to be in order to reduce the non-Abelian dipole of Eq. (10) to the non-Abelian monopole constituent, see Eq. (9). For a HS anticaloron one simply replaces $E^a_i = B^a_i$ by $E^a_i = -B^a_i$ in Eqs. (8), (9), and (10).

Finally, let us remark that the condition $s \gg \beta$, which is required for Eqs. (9) and (10) to be valid, is always satisfied for the caloron scale $\rho \sim |\phi|^{-1}$ which is relevant for the building of the thermal ground state in the deconfining phase of SU(2) Yang-Mills thermodynamics [9]. Namely, one has

$$\frac{s}{\beta} = \pi \left(\frac{\rho}{\beta}\right)^2 = \pi \left(\frac{\lambda^{3/2}}{2\pi}\right)^2 = \frac{\lambda^3}{4\pi} \geq 212.3,$$

where $\lambda \equiv \frac{2\pi T}{\Lambda} \geq \lambda_c = 13.87$.

4 Thermal ground state as induced by a probe

The postulate that photon propagation should be described by an SU(2) rather than a U(1) gauge principle was put forward in [20] and has undergone various levels of investigation ever since, see [9, 21, 22]. As a result, the associated Yang-Mills scale $\Lambda \sim 1.0638 \times 10^{-4}$ eV is fixed by low-frequency observation of the Cosmic Microwave Background (CMB) [23] to correspond to the critical temperature for the deconfining-preconfining phase transition being the CMB’s present baseline temperature $T_0 = 2.725$ K [24]. This prompted the name SU(2)$_\text{CMB}$. In the following we
would like to investigate in what sense the vacuum parameters of classical electrodynamics, namely the electric permittivity \( \varepsilon_0 \) and the magnetic permeability \( \mu_0 \), can be reduced to the physics of the static, non-Abelian, and (anti)selfdual monopole and dipole configurations represented by HS (anti)calorons in the regimes \( \beta \ll r \ll s \) and \( r \gg s \gg \beta \), respectively, see Sec.3.3. To do this, the concept of a thermal ground state together with information on how it is obtained [9] as well as the results of Sec.3.3 [19] are invoked.

### 4.1 Preexisting dipole densities

Let us discuss the case \( r \gg s \). In order to not affect spatial homogeneity on scales comparable to or smaller than \( s \) the electromagnetic field, which propagates through the deconfining thermal ground state in the absence of any explicit electric charges, is considered a plane wave of wave length \( l \) much larger than \( s \). Such a field effectively sees a density of selfdual dipoles, see Eq. (11). Because they are given by \( p_i^a = s \delta_i^a \) their dipole moments align along along direction of the exciting electric or magnetic field both in space and in the SU(2) algebra su(2). Note that at this stage the definition of what is to be viewed as an Abelian direction in su(2) is a global gauge convention such that all spatial directions of the dipole moment \( p_i^a \) are a priori thinkable. That is, dynamical Abelian projection of the non-Abelian situation of Eq. (10) is owed to the Abelian and dipole aligning nature of the exciting, massless field [9]. Modulo global gauge rotations, this field is exists because of the adjoint Higgs mechanism invoked by the inert field \( \phi \).

Per spatial coarse-graining volume \( V_{cs} \) of radius \( |\phi|^{-1} = \rho = \sqrt{\frac{\Lambda^2}{2\pi T}} \) with

\[
V_{cs} = \frac{4}{3} \pi |\phi|^{-3},
\]

the center of a selfdual HS caloron and the center of an antiselfdual HS anticaloron [9] reside. Note the large hierarchy between \( s \) (the minimal spatial distance to the center of a (anti)caloron, which allows to identify the static, (anti)selfdual dipole) and the radius of the sphere \( |\phi|^{-1} \) defining \( V_{cs} \),

\[
\frac{s}{|\phi|^{-1}} = \frac{1}{2} \Lambda^{3/2} \geq 25.83 \left( \frac{\lambda}{\lambda_c} \right)^{3/2}.
\]

(14)

If the exciting field is electric then it sees twice the electric dipole \( p_i^a \) (cancellation of magnetic dipole between caloron and anticaloron), if it is magnetic it sees twice the magnetic dipole \( p_i^a \) (cancellation of electric dipole between caloron and anticaloron, \( E = -B \iff -E = B \)). To be definite, let us discuss the electric case in detail, characterised by an exciting Abelian field \( E_e \). The modulus of the according dipole density \( D_e || E_e \) is given as

\[
|D_e| = \frac{2s}{V_{cs}} = \frac{3}{4\pi} \Lambda^2 \lambda_c^{1/2} \left( \frac{\lambda}{\lambda_c} \right)^{1/2}.
\]

(15)
In classical electromagnetism the relation between the fields $E_e$ and $D_e$ is

$$D_e = \epsilon_0 E_e,$$  \hspace{1cm} (16)

where

$$\epsilon_0 = 5.52703 \times 10^7 \frac{Q}{\text{V m}}$$  \hspace{1cm} (17)

is the electric permittivity of the vacuum, and $Q = 1.602 \times 10^{-19}$ A·s denotes the electron charge (unit of elementary charge), now both in SI units.

According to electromagnetism the energy density $\rho_{\text{EM}}$ carried by an external electromagnetic wave with $|E_e| = |B_e|$ is

$$\rho_{\text{EM}} = \frac{1}{2} \left( \epsilon_0 E_e^2 + \frac{1}{\mu_0} B_e^2 \right) = \frac{1}{2} \left( \epsilon_0 + \frac{1}{\mu_0} \right) E_e^2.$$  \hspace{1cm} (18)

In natural units we have $\epsilon_0 \mu_0 = 1/c^2 = 1$, and therefore $\mu_0 = 1/\epsilon_0$. Thus

$$\rho_{\text{EM}} = \epsilon_0 E_e^2.$$  \hspace{1cm} (19)

The $E_e$-field dependence of $\rho_{\text{EM}}$ is converted into a fictitious temperature dependence by demanding that the temperature of the thermal ground state of SU(2) CMB adjusts itself such as to accomodate $\rho_{\text{EM}}$,

$$\rho_{\text{EM}} = 4\pi \Lambda^3 T \quad \Leftrightarrow \quad |E_e| = \Lambda^2 \sqrt{2 \frac{\Lambda_e}{\epsilon_0}} \left( \frac{\lambda}{\lambda_c} \right)^{1/2}. \hspace{1cm} (20)$$

Eq. (20) generalises the thermal situation of ground-state energy density of Sec. 3.2 where ground-state thermalisation is induced by a thermal ensemble of excitations, to the case where the thermal ensemble is missing but the probe field induces a fictitious temperature and energy density to the ground state. Combining Eqs. (15), (16), and (20), and introducing the ratio $\xi$ between the non-Abelian monopole charge $Q'$ in the dipole and the (Abelian) electron charge $Q$, we obtain

$$\epsilon_0 [Q (\text{V m})^{-1}] = \frac{3}{\sqrt{32\pi}} \left( \frac{\Lambda[\text{m}^{-1}]}{\Lambda[\text{eV}]} \right)^{1/2} \xi \sqrt{\epsilon_0 [Q (\text{V m})^{-1}]} \Leftrightarrow$$

$$\epsilon_0 [Q (\text{V m})^{-1}] = \frac{9}{32\pi^2} \frac{\Lambda[\text{m}^{-1}]}{\Lambda[\text{eV}]} \xi^2. \hspace{1cm} (21)$$

Notice that $\epsilon_0$ does not exhibit any temperature dependence and thus no dependence on the field strength $E_e$. It is a universal constant. In particular, $\epsilon_0$ does not relate to the state of fictitious ground-state thermalisation which would associate to the rest frame of a local heat bath.

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1To assume $\epsilon_0 \mu_0 = 1$ just represents a short cut, it would have come out automatically if we had treated the magnetic case explicitly.

2In natural units, the actual charge of the monopole constituents within the (anti)selfdual dipole is $1/g$ where $g$ is the undetermined fundamental gauge coupling. This is absorbed into $\xi$. 

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To produce the measured value for $\epsilon_0$ as in Eq. (17) the ratio $\xi$ in Eq. (21) is required to be

$$\xi \equiv \frac{Q'}{Q} = 19.56.$$  

(22)

Thus, compared to the electron charge, the charge unit associated with a (anti)self-dual non-Abelian dipole, residing in the thermal ground state, is gigantic.

Discussing $\mu_0$, we could have been proceeded in complete analogy to the case of $\epsilon_0$. (It would be $\mu_0^{-1}$ defining the ratio between the modulus of the magnetic dipole density and the magnetic flux density $|\mathbf{B}|$.) Here, however, the comparison between non-Abelian magnetic charge and an elementary, magnetic, and Abelian charge is not facilitated since the latter does not exist in electrodynamics.

Finally, let us see what the condition that the wavelength $l$ of the electromagnetic disturbance considered in this section is much larger than $s$ means in units of meters when invoking SU(2)$_{\text{CMB}}$. One has

$$l \gg \frac{\lambda^2}{2\Lambda} \left( \frac{\lambda}{\lambda_c} \right)^2 = 1.1254 \text{ m} \left( \frac{T}{2.725 \text{ K}} \right)^2.$$  

(23)

Setting $T = T_c = 2.725 \text{ K}$ in Eq. (23), we obtain a lower bound on the wavelength of $l_{\text{min}} = 1.1254 \text{ m}$.

### 4.2 Explicitly induced dipole densities

Let us now discuss the case $\beta \ll |\phi|^{-1} \ll r \ll s$. To rely on the presence of the inert adjoint scalar field $\phi$ of the effective theory, $r$ needs to be larger than the spatial coarse-graining scale $|\phi|^{-1} = \frac{1}{2\pi} \lambda_c^{3/2} \left( \frac{\lambda}{\lambda_c} \right)^{3/2} \beta \geq 8.22 \beta$. Within the according regime $|\phi|^{-1} \leq r \ll s$ of spatial distances from the caloron center at $(\tau = 0, \mathbf{x} = 0)$ an electromagnetic wave of wave length $l$ sees the self-dual field of a static, non-Abelian monopole of electric and magnetic charge as in Eq. (9) which is centered at $\mathbf{x} = 0$. A self-dual Abelian field strength $E_i = B_i$ of this monopole is obtained \cite{25} as

$$E_i = B_i = \frac{\phi^a}{|\phi|} \tilde{E}_i^a = \frac{\phi^a}{|\phi|} B_i^a$$  

(24)

with the field $\phi$ gauged from unitary gauge $\phi^a = 2|\phi|\delta^{a3}$ into "hedgehog" gauge $\phi^a = 2|\phi|\delta^{a3}$. The according gauge transformation is give in terms of the group element $\Omega \equiv \cos \frac{1}{2} \psi - ik \cdot \sigma \sin \frac{1}{2} \psi$ where $\sigma_i, (i = 1, 2, 3)$, are the Pauli matrices, $\hat{k} \equiv \hat{e}_3 \times \frac{\hat{e}_\phi}{\sin \theta}$, $\hat{e}_3$ is the third vector of an orthonormal basis of space, $\theta \equiv \angle(\hat{e}_3, \hat{x})$, and $\psi = \theta$ for $0 \leq \theta < \pi - \epsilon$, which smoothly drops to zero at $\theta = \pi$, and the limit $\epsilon \to 0$ is understood \cite{25}. For the monopole field $E_i$ to be normalized to charge $-2Q'$ one \footnote{The factor two in front of the monopole charge $Q'$ is due to a contribution to the monopole field strength of the anticaloron identical to that of the caloron.}
Thus has
\[ E_i = B_i = -\frac{2Q'}{4\pi\epsilon_0} \hat{x}_i = -\frac{2Q'\mu_0}{4\pi r^2} \hat{x}_i. \]  
(25)
The electric or magnetic poles of Eq. (25) should independently react by harmonic and linear acceleration to the presence of an external electric or magnetic field \( E_e \) or \( B_e \), respectively, forming a monochromatic electromagnetic wave of frequency \( \omega = \frac{2\pi}{\lambda} \). At \( x = 0 \) one has
\[ E_e = E_0 \sin(\omega t), \]  
(26)
and readily derives (as in Thomson scattering) that the induced dipole moment \( p \), say, for the electric case, is given as
\[ p = -\frac{E_e(2Q')^2}{m\omega^2}. \]  
(27)
Interestingly, by virtue of Eq. (25) the squared charge of the pole, \((2Q')^2\), cancels out in \( p \) because its mass \( m \) carries an identical factor (only the electric (magnetic) monopole is linearly and harmonically accelerated by the external electric (magnetic) field \( E_e \) (\( B_e \)) and hence \( m \) carries electric (magnetic) field energy only):
\[ m = \frac{1}{2} \epsilon_0 4\pi \int_0^{\infty} \frac{E_i^2}{r^2} dr \Rightarrow \]
\[ p = -\frac{8\pi\epsilon_0 E_e |\phi|}{|\phi|\omega^2}. \]  
(28)
Again, the volume \( V_{cg} \), which underlies the dipole moment \( p \) by containing a caloron and an anticaloron center, is given by Eq. (13), and we have
\[ |D| = \frac{|p|}{V_{cg}} = 6 \epsilon_0 |E_e||\phi|^2/\omega^2, \]  
(29)
and therefore
\[ \epsilon_0 \equiv \frac{|D|}{|E_e|} = 6 \epsilon_0 |\phi|^2/\omega^2. \]  
(30)
In Eq. (30) also the vacuum permittivity \( \epsilon_0 \) cancels out, and we are left with the condition
\[ \omega = \sqrt{6} |\phi| \Leftrightarrow l = \sqrt{\frac{2}{3}\pi} |\phi|^{-1} = \sqrt{\frac{2}{3}\pi\Lambda^{-1}\lambda_c^{1/2}} \left( \frac{\lambda}{\Lambda} \right)^{1/2}, \]  
(31)
where temperature \( T \) (or \( \lambda \)), again, is set by the local field strengths of the electromagnetic probe according to Eqs. (18) and (20). Let us see whether the second of
Eqs. (31) is consistent with $|\phi|^{-1} \leq r = l \ll s$. The former inequality is self-evident, and the latter follows from

$$s = \sqrt{\frac{3 \lambda_c^{3/2}}{8 \pi}} \left(\frac{\lambda}{\lambda_c}\right)^{3/2} = 10.069 \left(\frac{\lambda}{\lambda_c}\right)^{3/2}.$$  \hspace{1cm} (32)

By setting $\lambda = \lambda_c$ we obtain from Eqs. (31) a minimal wavelength

$$l_{\text{min}} = \sqrt{\frac{2}{3}} \pi \Lambda^{-1} \lambda_c^{1/2} = 0.112 \text{ m}.$$  \hspace{1cm} (33)

This wavelength is about a factor of ten smaller than the lowest possible value as expressed by Eq. (23).

### 4.3 Discussion

In Secs. 4.1 and 4.2 an analysis was performed to clarify to what extent the thermal ground state of SU(2)$_{\text{CMB}}$ can be regarded as the luminiferous aether, supporting the propagation of an external electromagnetic wave (probe) of field strengths $|E_e| = |B_e|$ and wavelength $l$ which, by itself, is not thermal.

Sec. 4.1 has focussed on wave lengths that are large compared to the distance $s = \frac{\pi|\phi|^{-2}}{\beta}$, very large compared to the resolution limit $|\phi|^{-1}$ of the effective theory for deconfining SU(2)$_{\text{CMB}}$ and even more so on the scale of inverse temperature $\beta$, see Eq. (12), when (anti)calorons of SU(2)$_{\text{CMB}}$ manifest themselves as static (anti)selfdual dipoles whose dipole moment is set by a fictitious temperature representing the intensity of the probe via Eq. (20). And indeed, in this case vacuum permittivity $\epsilon_0$ and permeability $\mu_0$ turn out to be universal constants, see Eq. (21). When confronted with their experimental values the charges of the “constituent” non-Abelian monopoles in a dipole follow in units of electron charge, see Eq. (22).

Eqs. (23) and (20) indicate that an uncertainty-like relation between field $|E_e|$ strength and wave length $l$ takes place as follows

$$|E_e|^4 l^{-1} \ll \frac{8 \Lambda^9}{\epsilon_0^2}.$$  \hspace{1cm} (34)

Therefore, the larger the probe intensity the longer its wave length is required to be in order to be supported by thermal ground-state physics. In any case, in SU(2)$_{\text{CMB}}$ wave lengths need to be larger than the meter scale, see Eq. (23).

Things are different for wave lengths that are large on the scale $|\phi|^{-1}$ but short on the scale $s = \frac{\pi|\phi|^{-2}}{\beta}$. This case is investigated in Sec. 4.2. Then a (anti)caloron can no longer be viewed as a static, (anti)selfdual dipole but rather is represented by a static, (anti)selfdual monopole. However, an attempt to consider dipole moments as induced dynamically by monopole shaking through the probe fields renders the definitions of vacuum parameters $\epsilon_0$ and $\mu_0$ meaningless, see Eq. (30). It does yield
a fixation of the probe’s wave length $l$ in terms of $|\phi|^{-1}$ though, see Eq. (31). While the former situation is not surprising because single magnetic charges violate the Bianchi identities for the electromagnetic field strength tensor $F_{\mu\nu}$ it is nontrivial that $l$ turns out to selfconsistently satisfy the constraint that $s \gg l > |\phi|^{-1}$. Note that the minimal wave lengths $l_{\text{min}} = 1.1254 \text{ m}$ and $l_{\text{min}} = \sqrt{\frac{2}{3}} \pi \Lambda^{-1} \lambda_{c}^{1/2} = 0.112 \text{ m}$ as obtained in Secs. 4.1 and 4.2 respectively, are off by a factor of ten only.

5 Summary and Conclusions

We have addressed the question how the concept of a thermal ground state of SU(2)$_{\text{CMB}}$, which in a fully thermalised situation coexists with a spectrum of partially massive (adjoint Higgs mechanism) thermal excitations of the same temperature, can be employed to understand the propagation of a nonthermal probe (monochromatic electromagnetic wave) in vacuum, characterised by electric permittivity $\epsilon_{0}$ and magnetic permeability $\mu_{0}$. To do this, we have appealed to the fact that the thermal ground state emerges by a spatial coarse graining over (anti)selfdual fundamental Yang-Mills fields of topological charge modulus unity at finite temperature: Harrington-Shepard (anti)calorons of trivial holonomy. Knowing how large the coarse-graining volume is, which contains one caloron and one anticaloron center, where the unit of action $\hbar$ is localised (Sec. 3.2), and by exploiting the structure of these field configurations spatially far away (Sec. 3.3) from their centers, we were able to deduce densities of electric and magnetic dipoles in Sec. 4.1. Dividing these dipole densities by the respective field strengths of the probe, selfconsistently adjusted to the energy-density of the thermal ground state (small, transient (anti) caloron holonomies), yields definitions of $\epsilon_{0}$ and $\mu_{0}$. In the electric case a match with the experimental value predicts the charge of one of the monopoles, which constitutes the dipole, in terms of electron charge. The former charge turns out to be substantially larger than the latter.

As shown in Sec. 4.2 this way of reasoning, which is valid for large wave lengths ($l \gg s$) only, cannot be extended to smaller wave lengths $l$. Namely, in a region of spatial distances to the (anti)caloron center, where the configuration resembles (anti)selfdual, static monopoles, the definition of $\epsilon_{0}$ and $\mu_{0}$ in terms of dipole densities that are explicitly induced by the probes oscillating field strengths becomes meaningless. This is expected since the existence of resolved magnetic monopoles would violate the Bianchi identities for the field strength tensor $F_{\mu\nu}$ of electromagnetism.

We conclude that the thermal ground state of SU(2)$_{\text{CMB}}$ supports the propagation of a nonthermal probe purely in terms of Harrington-Shepard (anti)calorons (trivial holonomy) if the probe’s wave length $l$ is sufficiently large (the regime $l \gg s = \frac{\pi |\phi|^{-2}}{\beta} \geq 1.1254 \text{ m}$) and that there is an uncertainty-like relation between $l$ and the square of the probe’s intensity, see (31).

Let us now address the question how the propagation of shorter wave lengths
and/or larger intensities of nonthermal probes can be understood in terms of non-trivial ground-state structure. To do this, one could argue as follows. Since the result of Eq. (21) does not exhibit a $T$ dependence it "forgets" about the assumptions $l \gg s = s(T)$ and that field strength and temperature are related as in Eq. (20), and therefore should be considered valid for any probe and thus an invariant under the full Poincaré group. Conversely seen, the assumption of Poincaré covariance allows to transform any probe into a frame where the uncertainty-like relation (34) is obeyed (redshift by relativistic Doppler effect). In such a frame, the selfconsistency of Poincaré covariance is demonstrated by the invariance of the vacuum parameters $\epsilon_0$ and $\mu_0$ under further redshifting.

**Acknowledgments**

One of us (RH) would like to thank Stan Brodsky for a stimulating conversation.

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