DOMINANT THREE-BODY DECAYS
OF A HEAVY HIGGS AND TOP QUARK

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ABSTRACT

We calculate the dominant three body Higgs decays, $H \rightarrow W^+W^-(Z^0, \gamma)$ and $H \rightarrow t\bar{t}(Z^0, \gamma, g)$, in the Standard Model. We find that the branching ratios of these decays are of the order of few percent for large Higgs masses. We comment on the behaviour of the partial decay width $\Gamma(H \rightarrow t\bar{t}W^-)$ below the $t\bar{t}$ threshold. Numerical results of the following three body top decays, $t \rightarrow W^+b(\gamma, g, Z^0)$ and $t \rightarrow W^+bH$, are also given. We discuss the feasibility of observing these Higgs and top decays at future high energy colliders.
1. Introduction

In spite of the beautiful confirmation of the Standard Model (SM) by LEP data [1] the Higgs sector [2] still remains quite unconstrained. The only ‘real’ upper bound on the Higgs mass is $M_H \leq 1 TeV$ given to us by unitarity arguments [3]. On the other hand if the Higgs particle is indeed heavy, $M_H \geq 500 GeV$, then three body Higgs decays like $H \to W^+W^-(\gamma, Z^0)$ and $H \to t\bar{t}(Z^0, \gamma, g)$ have appreciable branching ratios too. In this case in order to confirm that an observed scalar (should one find it) is indeed the Higgs particle predicted by SM one will have to study the subdominant partial decays such as those mentioned above, as well.

As is well known two body decays of a heavy Higgs decays are dominated by $H \to W^+W^-, Z^0Z^0, t\bar{t}$. Some of the more important three body decays have already been discussed in the literature. The pure bosonic decay modes $H \to W^+W^- Z^0$ and $H \to W^+W^- \gamma$ have been calculated in refs. [4] and [5], [6], respectively. Ref. [6] corrects the results of [5]. We have redone the calculation for both these channels. For the latter we agree with [6], however we disagree with ref. [4] for the former by two orders of magnitude. Furthermore a heavy top means a substantial Yukawa coupling $g_{tH}$. Due to this fact the decays $H \to t\bar{t}(\gamma, g, Z^0)$ are also not negligible. Only one of them ($H \to t\bar{t}Z^0$) has been discussed so far [7]. Here we take the opportunity to correct an omission of factor 3 in [7] and present the full results for all the three decays.

By now it is clear that the top quark is likely to be heavy ($90 GeV \leq m_t \leq 200 GeV$). Therefore its three body decays become interesting, too. Here we calculate $t \to W^+b(\gamma, g), t \to W^+bZ^0$ and $t \to W^+bH$. The first two have been discussed by two theoretical groups, which reported similar results [8,9]. Our numerical results agree with ref. [9] and the revised results of [8].

In view of the discrepancies that existed between different calculations for the three body decays of $H$ and $t$ we take the opportunity to consolidate and to present some new results for all these decays.

Our paper is organized as follows. In section 2 we discuss three body Higgs decays. Section 3 deals with the corresponding top decays. Whenever the result for the matrix element squared is not too unwieldy we present the analytical expression as well. We summarize our conclusions in the end.
2. Dominant three body Higgs decays

We consider here the following partial decays of a heavy Higgs

\[ H \rightarrow W^+W^- (\gamma, Z^0) \]
\[ H \rightarrow t\bar{t}(Z^0, g, \gamma) \]
\[ H \rightarrow t\bar{t}W^- \] (1)

The corresponding diagrams are depicted in figs. 1 and 2, respectively. All these decays listed above gain from the fact that the Higgs couples predominantly to heavy particles e.g.

\[ g_{HWW} = g M_W \]
\[ g_{HZZ} = g_{HWW} \frac{M_Z^2}{M_W^2} \]
\[ g_{HHt} = \frac{m_t}{2M_W} \] (2)

The interesting feature of the first decay channel in (1) are the triple gauge boson couplings. The trilinear vertices for \( W^+(p) - W^-(q) - V^0(r) \) (\( V^0 = \gamma, Z^0 \)) are given by

\[ V_{\rho\mu\nu}^{WVV}(p, q, r) = g_{\rho\mu\nu} \left[ (r - q)_\rho g_{\mu\nu} + (q - p)_\nu g_{\rho\mu} + (p - r)_\mu g_{\nu\rho} \right] \]
\[ g_{\gamma,WW} = e, \quad g_{ZWV} = g \cos \theta_W \] (3)

which have been defined for all momenta incoming. Additionally, non-standard couplings like the anomalous magnetic moment vertex

\[ ie(\kappa_V - 1) \left[ g_{\rho\nu\tau \mu} - g_{\mu\nu\rho\tau} \right] \] (4)

and others [10] will be not taken into account here. We think that a rare decay mode of Higgs is not the best place to look for such couplings.

Since we consider here a heavy Higgs we have to a very good approximation

\[ \Gamma_{tot}^{Higgs} = \Gamma(H \rightarrow W^+W^-) + \Gamma(H \rightarrow Z^0Z^0) + \Gamma(H \rightarrow t\bar{t}) \]
\[ \Gamma(H \rightarrow VV) = \frac{\alpha_w}{16n_V} M_H x_V^{-2} (1 - 4x_V^2)^{1/2} (1 - 4x_V^2 + 12x_V^4), \quad V = Z^0, W \] (5)
\[ \Gamma(H \rightarrow t\bar{t}) = \frac{3\alpha_w}{8} M_H \left( \frac{m_t}{M_W} \right)^2 (1 - 4x_t^2)^{3/2} \]

where \( \alpha_w = g^2/4\pi \), \( x_i = M_i/M_H \) and \( n_V \) equals 1(2) for the \( W^+W^- \) (\( Z^0Z^0 \)) boson pairs.
The partial width of the three body Higgs decay can generally be put into the form

\[ \Gamma(H \to 3\text{body}) = \frac{1}{256\pi^3 M_H^4} \int_{s_2^-}^{s_2^+} ds_2 \int_{s_1^-}^{s_1^+} ds_1 |T(H \to 3\text{body})|^2 \]  

(6)

where \( s_1 \) and \( s_2 \) are invariants (and \( s_{1,2}^\pm \) the corresponding phase space boundaries) which we define below for each case under consideration.

Throughout the paper we will use the following set of parameters

\[ \alpha_{em}(M_W^2) = \frac{1}{128} \]
\[ \alpha_s(M_Z^2) = 0.12 \]
\[ M_W = 80.6 \text{ GeV} \]
\[ M_Z = 91.161 \text{ GeV} \]  

(7)

### 2.1 \( H \to W^+W^-(Z^0,\gamma) \)

The sum of the three diagrams for the decay \( H(p) \to W^+(k_+)W^-(k_-)Z^0(k) \) (see fig. 1) can be conveniently written as

\[ T(H \to W^+W^-Z^0) = -ig_{zw}g_{hw} (T^WZW_A + T^WZW_B + T^WZW_C) \]

(8a)

\[ T^WZW_A + T^WZW_B + T^WZW_C = \frac{\epsilon_\rho^*(k_-)\epsilon_\mu^*(k)\epsilon_\alpha^*(k_+)}{\Delta_W(s_1)\Delta_W(s_2)\Delta_Z(s_3)^*} \]

\[ \left\{ -k^\rho g^{\alpha\mu} \Delta_Z(s_3) \left[ 2(\Delta_W(s_1) + \Delta_W(s_2)) - \xi_Z \Delta_W(s_2) \right] \\
+ k_+^\rho g^{\alpha\mu} \Delta_W(s_1) \left[ 2 \Delta_Z(s_3) + \xi_Z \Delta_W(s_2) \right] \\
+ k_+^\mu g^{\rho\mu} \Delta_Z(s_3) \left[ 2(\Delta_W(s_1) + \Delta_W(s_2)) - \xi_Z \Delta_W(s_1) \right] \\
- k_-^\mu g^{\rho\mu} \Delta_W(s_2) \left[ 2 \Delta_Z(s_3) + \xi_Z \Delta_W(s_1) \right] \\
- k_-^\rho g^{\mu\rho} \xi_Z \Delta_W(s_1) \left[ \Delta_Z(s_3) + \Delta_W(s_2) \right] \\
+ k_+^\rho g^{\mu\rho} \xi_Z \Delta_W(s_2) \left[ \Delta_Z(s_3) + \Delta_W(s_1) \right] \right\} \]

(8b)

where we have used eq.(2) and defined

\[ \Delta_i(s) = s - M_i^2 \]
\[ \xi_Z = \frac{M_Z^2}{M_W^2} \]
\[ s_1 = (k_+ + k)^2 \]
\[ s_2 = (k_- + k)^2 \]
\[ s_3 = M_H^2 + M_Z^2 + 2M_W^2 - s_1 - s_2 \]  

(9)
The phase space boundaries which correspond to these variables are as follows

\[ s_2^+ = (M_H - M_W)^2, \quad s_2^- = (M_W + M_Z)^2 \]
\[ s_1^+ = M_W^2 + M_Z^2 - \frac{1}{2s_2} \left[ (s_2 - M_H^2 + M_W^2)(s_2 + M_Z^2 - M_W^2) \right. \]
\[ \left. \pm \lambda^{1/2}(s_2, M_H^2, M_W^2) \lambda^{1/2}(s_2, M_W^2, M_Z^2) \right] \]

\[ \lambda(x, y, z) = (x - y - z)^2 - 4xy \]

It seems unreasonable to give here the lengthy expression for \(|T(H \rightarrow W^+W^-Z^0)|^2\) in full detail. We used the algebraic manipulation program REDUCE to evaluate this expression and performed the integration numerically. The results are presented in fig.3. One can see that at about \(M_H \geq 600\) GeV the branching ratio is

\[ Br(H \rightarrow W^+W^-Z^0) \simeq O(1\%) \] (11)

which is indeed sizeable.

We will discuss the visibility of each of the three body Higgs decays at the end of section 2.

Our result disagrees with ref.[4] where only the mode \(H \rightarrow W_LW_LZ_L\) has been evaluated with claim that this is the biggest contribution. The estimate given in [4] is

\[ \frac{\Gamma(H \rightarrow W_LW_LZ_L)}{\Gamma(H \rightarrow WW) + \Gamma(H \rightarrow ZZ)} = 3.07 \times 10^{-4}(M_H^2/1\text{TeV}) \] (12)

Our results e.g. for \(M_H = 1\text{TeV}\) is about 2 orders of magnitude larger which indicates that \(W_LW_LZ_L\) is not yet the dominant contribution to \(H \rightarrow WWZ\) (in contrast what one would naively expect) \(^1\).

In case of \(H(p) \rightarrow W^+(k_+)W^-(k_-)\gamma(k)\) we have only two amplitudes \(T_{A}^{WW\gamma}\) and \(T_{B}^{WW\gamma}\) (fig.1). This simplifies the form of the matrix element considerably and it is given by

\[ T(H \rightarrow W^+W^-\gamma) = -iegM_W\epsilon^+_\mu\epsilon^-\nu^* \left\{ \frac{p \cdot k}{(k_+ \cdot k)(k_- \cdot k)} (k_\rho g_{\mu\nu} - k_\mu g_{\rho\nu}) \right. \]
\[ \left. + g_{\mu\rho} \left[ \frac{k_\nu}{k_+ \cdot k} - \frac{k_-}{k_- \cdot k} \right] \right\} \] (13)

\(^1\) After our manuscript has been completed we become aware of ref. [11] where it is nicely explained why the decay \(H \rightarrow W_LW_LZ_L\) is forbidden due to parity conservation on the tree level. Our results agree with the ones obtained in [11].
Written in this way, the amplitude is trivially transverse \((k_\nu T^{\mu\nu} = 0)\). This simple form enables us to express \(|T(H \to W^+ W^- \gamma)|^2\) in a relatively compact form given in the appendix (eq.(A2)). The variables \(s_1\) and \(s_2\) as well as the phase space limits are obtained from eqs.(10) by putting formally \(M_Z = 0\). To avoid infrared singularities we impose a cut on the photon energy which in terms of our integration variables is
\[
E_\gamma = \frac{k \cdot p}{M_H} = \frac{s_1 + s_2 - 2M_W^2}{2M_H} > E_{\text{cut}}^\gamma
\]  
(14)

In the numerical calculation we have chosen \(E_{\text{cut}}^\gamma = 10, 20, 50 \text{ GeV}\) corresponding to the full, dashed and dotted lines in fig.4, respectively. As in the case \(H \to W^+ W^- Z^0\) the branching ratio \(Br(H \to W^+ W^- \gamma)\) is also of the order \(O(1\%)\) and hence not negligible. Our calculation confirms the results of ref.[6] which in turn corrected the mistake in [5]. We also mention that a detailed treatment of \(H \to WW\gamma\) for the case of soft bremsstrahlung (as a part of radiative corrections) as well as for hard photon emission is given in ref. [12]. There a partial analytical integration over the phase space (in the range \(E_{\text{cut}}^\gamma << E_\gamma < E_{\text{max}}^\gamma\) can also be found. We have checked that our numerical results agree with fig. 6 of ref. [12].

2.2 \(H \to t\bar{t}(Z^0, \gamma, g)\)

In this section we will mainly concentrate on the decay channels \(H \to t\bar{t}(\gamma, g)\). The partial decay \(H \to t\bar{t}Z^0\) has been calculated by us in [7]. Note the in [7] an overall colour factor 3 is missing which we have now included in the numerical results for this channel presented in fig.5. For \(M_H \geq 600 \text{ GeV}\) the branching ratio for this particular decay mode is of the order \(10^{-3}\) and higher.

It is also evident that once the process \(H \to t\bar{t}\gamma\) has been calculated the corresponding gluonic mode \(H \to t\bar{t}g\) can be obtained from the former by including the Casimir factor of \(SU(3)_C\) (and reducing it by \(1/N_C\))
\[
\frac{1}{N_C} \text{Tr} \left( \frac{\lambda^a \lambda^a}{2} \right) = \frac{4}{3}
\]  
(15)
and by replacing
\[
\alpha_{em} \to \frac{9}{4} \alpha_s
\]  
(16)

This leads to
\[
\Gamma(H \to t\bar{t}g) = 3\frac{\alpha_s(M^2_H)}{\alpha_{em}}\Gamma(H \to t\bar{t}\gamma)
\]  
\[\simeq 38.4\Gamma(H \to t\bar{t}\gamma)
\]  
(17)
The total matrix element \( T(H \rightarrow t\bar{t}\gamma) \) is given by the sum of the two amplitudes

\[
T^A_{t\bar{t}\gamma} = -\frac{i}{2} \epsilon^\mu(k_{\gamma}) \frac{g e}{s_1 - m_t^2} m_t \bar{u}_t (j + m_t)\gamma_\mu v_{\bar{t}}
\]

\[
T^B_{t\bar{t}\gamma} = -\frac{i}{2} \epsilon^\mu(k_{\gamma}) \frac{g e}{s_3 - m_t^2} m_t \bar{u}_t \gamma_\mu (j' + m_t) v_{\bar{t}}
\]

where

\[
l = -k_{\gamma} - k_{\bar{t}}, \quad l^2 = s_1
\]

\[
r = k_{\gamma} + k_{t}, \quad r^2 = s_3
\]

\[
s_3 = M_H^2 + 2m_t^2 - s_1 - s_2
\]

The phase space boundaries to be used in the numerical evaluation of the eq.(6) are given by

\[
s_2^+ = M_H^2, \quad s_2^- = 4m_t^2
\]

\[
s_1^\pm = \frac{1}{2} (M_H^2 + 2m_t^2 - s_2) \pm \frac{1}{2} (M_H^2 - s_2) \left[ 1 - \frac{4m_t^2}{s_2} \right]^{1/2}
\]

After factorizing common constants we put the squared matrix element into the following form

\[
|T(H \rightarrow t\bar{t}\gamma)|^2 = \frac{64}{3} \pi^2 \alpha_w \alpha_em \frac{m_t^2}{M_W^2} \left( |\tilde{T}^A_{t\bar{t}\gamma}|^2 + |\tilde{T}^B_{t\bar{t}\gamma}|^2 + 2\text{Re}\tilde{T}^A_{t\bar{t}\gamma} \cdot (\tilde{T}^B_{t\bar{t}\gamma})^* \right)
\]

and refer the reader to the appendix where the analytical form is given (eq.(A3)).

Fig.6a and 6b show the numerical results for two different values of \( E_{cut}^\gamma \) imposed on the photon energy. The branching ratio is of the order \( 10^{-3} \). From this using eq.(17) we conclude that

\[
Br(H \rightarrow t\bar{t}g) \simeq O(1\%)
\]

which is comparable to \( Br(H \rightarrow W^+W^-Z^0) \).

Analytical expressions in term of Spence functions for hard bremsstrahlung in \( H \rightarrow f\bar{f}\gamma \) and \( H \rightarrow q\bar{q}g \) are presented in [13] and [14], respectively.

### 2.3 \( H \rightarrow t\bar{b}W^- \)

This decay relevant for the kinematical range \( M_H \leq 2m_t \) has been discussed in detail in [7]. We supplement here this discussion by giving in the appendix (eqs.(A4)-(A6)) the analytical expression for \( |T(H \rightarrow W^-t\bar{b})|^2 \) which we write in the form

\[
|T(H \rightarrow W^-t\bar{b})|^2 = 6\alpha_w \pi^2 |V_{tb}|^2 \left( |\tilde{T}^W_{A,Wtb}|^2 + |\tilde{T}^W_{C,Wtb}|^2 + 2\text{Re}(\tilde{T}^W_{A,Wtb} \cdot (\tilde{T}^W_{C,Wtb})^*) \right)
\]
Note that a third amplitude in which the Higgs couples directly to bottom-quarks can be safely neglected since

\[ |T_B(H \rightarrow b\bar{b}^* \rightarrow bW^+t)|^2 \simeq \left( \frac{m_b}{m_t} \right)^2 |T_A^{Wtb}|^2 < 2 \times 10^{-3} |T_A^{Wtb}|^2 \] (24)

The tilded quantities in eq.(23) depend on the two Mandelstam variables

\[ s_1 = (k_W + k_b)^2 \]
\[ s_2 = (k_t + k_b)^2 \] (25)

Fig.7 displays the threshold behaviour of \( H \rightarrow W^+\bar{t}t \). The branching ratio can be as large as \( 10^{-3} \) for \( M_H < 2m_t \). If the decay channel \( H \rightarrow t\bar{t} \) is kinematically open one of course has

\[ \Gamma(H \rightarrow W^-\bar{t}t) \simeq \Gamma(H \rightarrow t\bar{t}) \] (26)

to a very high accuracy.

Before closing the section on Higgs decays let us give the branching ratio of all three body decay modes considered above. We define

\[ Br_{3\text{body}}(M_H) = \frac{\sum_{i=3\text{body}} \Gamma(H \rightarrow i)}{\Gamma_{Higgs\text{tot}}} \] (27)

Putting \( m_t = 150 \text{ GeV} \) and \( E_{\text{cut}}^{\gamma,q} = 20 \text{ GeV} \) we obtain

\[ Br_{3\text{body}}(M_H = 400, 600, 800, 1000 \text{ GeV}) = (0.9, 3.5, 7.1, 9.4) \cdot 10^{-2} \] (28)

Thus we see that the three body decay modes of a heavy Higgs can have branching ratios as high as 10%.

Some comments on the feasibility to observe the discussed three body Higgs decays are in order. We note that, out of the few three body channels, \( H \rightarrow W^+W^-(Z^0, \gamma) \) are the best candidates to be seen at SSC energies (\( \sqrt{s} = 40\text{TeV} \)) provided \( M_H \geq 600 \text{ GeV} \). The reason is that the background from the ‘direct’ production is [15] (for SSC)

\[ \sigma(pp \rightarrow W^+W^-Z^0 + X) \simeq 0.4 \text{ pb} \]
\[ \sigma(pp \rightarrow W^+W^-\gamma + X)|_{p_{\gamma} \geq 40 \text{ GeV}} \simeq 0.2 \text{ pb} \] (29)
whereas in the mass range $M_H = (600 - 1000) \text{ GeV}$ one has [16]

$$\sigma(pp \to H + X) \simeq (10 - 1) \text{ pb} \quad (30)$$

Together with $Br(H \to W^+W^-(Z^0, \gamma))$ from figs.3 and 4 we find that the three gauge bosons production through Higgs decay is ‘only’ 10 times smaller than the ‘direct’ one ($q\bar{q} \to W^+W^-Z^0$). Therefore suitably chosen cuts should, in principle, make it possible to observe such decays.

The situation for $H \to t\bar{t}(g, \gamma, Z^0)$ is a little more involved. It is well known that there is an overwhelming background from $gg$ fusion [17]

$$\sigma(pp \to t\bar{t} + X) \simeq (10^4 - 10^5) \text{ pb} \quad (31)$$

which makes it quite hard to observe hadronic Higgs decays in general. The situation might be more promising for $V^0 = \gamma, Z^0$ once some suitable cuts are applied, but this would require more detailed Monte Carlo simulations.

Let us also investigate the potential of the ambitious project of a linear $e^+e^-$ super-collider operating at $\sqrt{s} = 2\text{ TeV}$ [18]. Here the heavy Higgs production is dominated by $W^*W^*$ and $Z^*Z^*$ fusion. Again for the range $M_H = (600 - 1000) \text{ GeV}$ one has [18]

$$\sigma(e^+e^- \to W^*W^* \to \nu\bar{\nu}H) \simeq \mathcal{O}(10^{-1}) \text{ pb}$$

$$\sigma(e^+e^- \to Z^*Z^* \to e^+e^-H) \simeq \mathcal{O}(10^{-2} - 10^{-3}) \text{ pb} \quad (32)$$

whereas the top production yields ($m_t = (100 - 200) \text{ GeV}$) [19]

$$\sigma(e^+e^- \to W^*W^* \to \nu\bar{\nu}t\bar{t}) \simeq \mathcal{O}(10^{-2} - 10^{-3}) \text{ pb} \quad (33)$$

$$\sigma_{beamstr.}(e^+e^- \to e^+e^-t\bar{t}) \simeq \mathcal{O}(10 - 1) \text{ pb}$$

where the last cross section refers to beamstrahlung of electron in the field of $e^+e^-$-bunches (and vice versa) [20]. However, designs of $e^+e^-$-machines which give rise to high large amount of beamstrahlung seem to be disfavoured since such designs would make the, usually ‘clean’, $e^+e^-$-collider ‘messier’ due to possible underlying events caused by $\gamma\gamma$ interactions [21]. Keeping this in mind it is clear that a high energy $e^+e^-$-collider has a better potential to observe hadronic Higgs decays as compared to a $pp$- machine. We also mention here that using more realistic beamstrahlung spectra the cross section $\sigma_{beamstr.}$ is smaller [21] as compared to the value we have quoted in (33).
3. Dominant three body top decays

The relevant diagrams for the decays \( t \rightarrow W^+ b (\gamma, g, Z^0) \) as well as \( t \rightarrow W^+ b H \) are given in fig.8. As in the case of three body Higgs decays (eq.(6)) the generic form of the partial width for the top decays under consideration is

\[
\Gamma(t \to 3\text{body}) = \frac{1}{256\pi^3 m_t^3} \int_{s_1^2}^{s_2^2} ds_1 \int_{s_1^1}^{s_1} ds_2 \int_{s_1^1}^{s_1} ds_3 |T(t \to 3\text{body})|^2
\]

(34)

Below we will give the new result on \( t \rightarrow W^+ b Z^0 \). Our calculations of \( t \rightarrow W^+ b (\gamma, g) \) confirms the results of ref.[9] and the revised ones of ref.[8]. For completeness we quote the numerical values for \( \Gamma(t \rightarrow W^+ b H) \) which has been discussed in [7].

3.1 \( t \rightarrow W^+ b (g, \gamma, Z^0) \)

The general form of the three amplitudes contributing to \( t \rightarrow W^+ b V^0 \) \((V^0 = \gamma, g, Z^0)\) can be cast into the following expressions

\[
T_A^{WbV} = -i \frac{gQ_T^V e^{\mu\nu}(k_V) \epsilon^{\mu\nu}(k_W)}{2\sqrt{2}} \overline{u}_b \gamma_\mu \gamma_\nu (p + m_t) \gamma_\nu \left[g_V^t \gamma_5 + g_A^t \gamma_5 \right] T^a u_t
\]

(35)

\[
T_B^{WbV} = -i \frac{gQ_b^V e^{\mu\nu}(k_V) \epsilon^{\mu\nu}(k_W)}{2\sqrt{2}} \overline{u}_b \gamma_\mu \gamma_\nu (p + m_b) \gamma_\nu \left[g_V^b \gamma_5 + g_A^b \gamma_5 \right] T^a (p' + m_b) \gamma_\mu \gamma_\nu - u_t
\]

\[
T_C^{WbV} = -i \frac{g g_{VVW} e^{\mu\nu}(k_V) \epsilon^{\mu\nu}(k_W)}{2\sqrt{2}} \frac{1}{s_3 - M_W^2} \left\{-g_{VVW}^\rho + q^\rho q^\mu / M_W^2 \right\} f_{\rho\mu\nu} \overline{u}_b \gamma_\lambda \gamma_\nu - u_t
\]

where

\[
\gamma_\pm = 1 \pm \gamma_5
\]

\[
f_{\rho\mu\nu} = (k_W - k_V) \rho g_{\mu\nu} - (2k_W + k_V) \nu g_{\rho\mu} + (2k_V + k_W) \mu g_{\rho\nu}
\]

The coupling \( g_{VVW} \) has been defined in eq.(3) \((g_{VVW} = 0)\) and \( Q_T^V \) are overall coupling constants of \( V^0 \) to quarks (charges in case of the photon). Obviously for \( V^0 = \gamma, g \) one has

\[
g_V^t = 1, \quad g_V^b = 0, \quad q = t, b
\]

(37)

whereas for \( V^0 = Z^0 \) we employ the common definitions

\[
g_V^t = \frac{1}{4 \cos \theta_W} \left[ 1 - \frac{8}{3} \sin^2 \theta_W \right], \quad g_V^b = -\frac{1}{4 \cos \theta_W}
\]

\[
g_A^t = \frac{1}{4 \cos \theta_W} \left[ -1 + \frac{4}{3} \sin^2 \theta_W \right], \quad g_A^b = \frac{1}{4 \cos \theta_W}
\]

(38)
The kinematical variables entering (35) are defined as follows

\[ l = k_W + k_b, \quad l^2 = s_2 \]
\[ r = k_V + k_b, \quad r^2 = s_1 \]
\[ q = k_W + k_V, \quad q^2 = s_3 = M_{WV}^2 + (M_V^2 + m_t^2 + m_b^2 - s_1 - s_2) \]

in which the phase space boundaries are readily found to be

\[ s_2^- = (M_W + m_b)^2, \quad s_2^+ = (m_t - M_V)^2 \]
\[ s_1^\pm = M_{WV}^2 + M_V^2 - \frac{1}{2s_2^2} \left( (s_2 - m_t^2 + M_V^2) (s_2 + M_{WV}^2 - m_b^2) \right) \]
\[ \mp \lambda^{1/2}(s_2, M_{WV}^2, m_b^2) \lambda^{1/2}(s_2, m_t^2, M_V^2) \]  

(40)

In case of \( M_V = 0 \) we introduce a cut in the phase space on the photon (gluon) energy

\[ E_{\gamma, g} = \frac{m_t^2 - s_2}{2m_t} > E_{\text{cut}}^{\gamma, g} \]  

(41)

Since we keep \( m_b \neq 0 \) there are no collinear singularities.

We will not spell out here the expressions for the squared matrix elements since for \( V^0 = \gamma, g \) they can be found in [9] and the corresponding formula for \( V^0 = Z^0 \) is too lengthy. Instead we will briefly comment on the factorization of the amplitude \( T(t \rightarrow W^+ b\gamma) \). Splitting this amplitude into an abelian part which does not contain gauge bosons couplings and a remainder \( \tilde{T} \)

\[ T(t \rightarrow W^+ b\gamma) = -\frac{1}{3} T_{\text{abelian}} + \tilde{T} \]
\[ T_{\text{abelian}} = -i \frac{g e}{2\sqrt{2}} \epsilon_{\gamma*}^{\nu*} \epsilon_W^{\mu*} \gamma^\mu u^\nu \]
\[ \gamma_\mu \gamma_- \left( \frac{1 + m_t}{-2k_{\gamma} \cdot k_t} \right) \gamma_\nu \frac{(y + m_b)}{2k_{\gamma} \cdot k_b} \gamma_\nu \gamma_- \]
\[ u = \left( \frac{-2k_{\gamma} \cdot k_b}{2k_{\gamma} \cdot k_w} \right) T_{\text{abelian}} \]  

(42)

one can show after some algebraic manipulation that the following factorization holds

\[ \tilde{T} = \left( \frac{-2k_{\gamma} \cdot k_b}{2k_{\gamma} \cdot k_w} \right) T_{\text{abelian}} \]  

(43)

This fact simplifies the calculation to a large extent.

The numerical results for \( Br(t \rightarrow W^+ b(Z^0, \gamma, g)) \) are plotted versus the top mass in figs.9-11. Our results for photon and gluon channel confirm the ones obtained in [9] and [23]. The decay \( t \rightarrow Wb(\gamma, g) \) as part of radiative corrections to semi-leptonic
top decays is also discussed in [24]. The order of magnitude of these decays can be summarized by \((E_{cut}^{\gamma,g} < 20 \text{ GeV}, m_t = 150 \text{ GeV})\)

\[
Br(t \to W^+bg) \simeq \mathcal{O}(10^{-1}) \\
Br(t \to W^+b\gamma) \simeq \mathcal{O}(10^{-3})
\]  

(44)

In SM with \(m_t = 200 \text{ GeV}\) we get

\[
Br(t \to W^+bZ^0) \simeq \mathcal{O}(10^{-5})
\]

(45)

where the suppression is essentially due to phase space. There are, however, indeed very minor extensions of SM which allow values up to \(m_t = 300 \text{ GeV}\) [22] (here the branching ratio reaches the \(10^{-3}\) mark).

### 3.2 \(t \to W^+bH\)

The decay \(t \to W^+bH\) has been discussed by us in [7]. Here we will just write down the relevant formulas.

Neglecting, as in the case \(H \to W^-t\bar{b}\) (see eq.(24)), a third possible diagram proportional to \(m_b\) we are left with the two amplitudes (see fig.8)

\[
T_{WbH}^A = -i\epsilon^{\mu*}(k_W) \frac{g^2}{4\sqrt{2}} \frac{m_t}{M_W s_1 - m_t^2} \frac{V_{tb}^*}{u_b} \bar{u}_b \gamma_\mu \gamma_- (l' + m_t) u_t \\
T_{WbH}^B = -i\epsilon^{\mu*}(k_W) \frac{g^2}{2\sqrt{2}} \frac{M_W V_{tb}^*}{s_2 - M_W^2} \bar{u}_b \gamma_+ \left[ \gamma_\mu - \frac{m_t q_\mu}{M_W^2} \right] u_t
\]

(46)

with

\[
l^2 = (k_W + k_b)^2 = s_1 \\
q^2 = (k_H + k_W)^2 = s_2
\]

(47)

The reader will find the squared matrix element in the appendix (eqs.(A4)-(A6)). The latter we write in the form

\[
|T(t \to W^+bH)|^2 = c_{w,\pi}^2 |V_{tb}|^2 \sum_{\text{pol.}} |\tilde{T}_{WbH}^A + \tilde{T}_{WbH}^B|^2
\]

(48)

The numerical results for this decay channel are displayed in fig.12. Since the LEP data already restrict the Higgs mass \(M_H \geq 50 \text{ GeV}\) [25] the branching ratio is expected to be small (like in the case \(t \to W^+bZ^0\)). Indeed for \(M_H = 60 \text{ GeV}\) and \(m_t = 200 \text{ GeV}\) we obtain

\[
Br(t \to W^+bH) \simeq \mathcal{O}(10^{-4})
\]

(49)
We conclude by mentioning that the large number of $10^8 t\bar{t}$ pairs produced at SSC per year (which in case of hadronic Higgs decays gave rise to an enormous background) is now of course of great advantage to observe rare top decays (two body rare top decays like $t \rightarrow c(g, \gamma, H)$ have been investigated in [26]). Hence such decays would be observable. In particular, if b-quark identification will be accessible at high energies with high efficiency, then such three-body decays may be probed by tagging three b-jet events in the five well-isolated jet signals of the top decay.

We also note that with minor modifications (e.g. mixing angles) three body decays of heavy fermions could become important in some extensions of SM like two Higgs doublet models ($t \rightarrow W^+ b H^0$) or fourth generations extensions ($t' \rightarrow W^+ b Z^0$). The latter is still not excluded provided $m_{\nu_4} > M_Z/2$ (some models even favour such an extension to explain the $\tau$-decay puzzle). The three body decays with $Z^0$ bremsstrahlung of new particles have been discussed in [27].

4. Conclusions

The future high energy colliders like SSC or even a $2TeV e^+e^-$ machine will produce enough Higgs particles to look also into subdominant decays of this yet missing ingredient of the Standard Model. Here we have concentrated on the most important three body decays of a heavy Higgs. We have shown that the branching ratio $Br(H \rightarrow 3\text{body})$ can be as high as 10% and cannot therefore be neglected, should indeed the Higgs turn out to be heavy. For the channel $H \rightarrow W^+W^-Z^0$ we disagree with previous calculations. This decay mode contributes substantially to the subclass of three body decays of $H$.

In addition we have presented the dominant three body decays of a heavy top quark. Some of these decays, like $t \rightarrow W^+ b Z^0$, have small (phase space suppressed) branching ratios. However, the enormous number of $t\bar{t}$ pairs expected at SSC should make it possible to observe even such rare decays provided the top quark mass is around $200 GeV$. Others, like $t \rightarrow W^+ b(g, \gamma)$, with a hard photon (gluon) have non-negligible branching ratios.

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REFERENCES

[1] For a summary see F. Dydak in *Proceedings of the 25th International Conference on High Energy Physics*, eds. K. K. Phua, Y. Yamaguchi

[2] For a review see J. F. Gunion, H. E. Haber, G. Kane and S. Dawson, *The Higgs Hunter’s Guide* Addison Wesley 1990

[3] B. W. Lee, C. Quigg and G. B. Thacker, Phys. Rev. **D16** (1977) 1519 and references therein

[4] T. G. Rizzo, Phys. Lett. **B217** (1989) 191

[5] T. G. Rizzo, Phys. Rev. **D31** (1985) 2366

[6] D. A. Dicus et al., Phys. Rev. **D34** (1986) 2157

[7] R. Decker, M. Nowakowski and A. Pilaftsis, Mod. Phys. Lett. **A6** (1991) 3491; **E A7** (1992) 819

[8] G. Couture, Phys. Rev. **D40** (1989) 2927; **E42** (1990) 1855.

[9] G. Tupper et al., Phys. Rev. **D43** (1991) 274

[10] K. Hagiwara, K. Hikasa, R. D. Peccei and D. Zeppenfeld, Nucl. Phys. **B282** (1987) 253

[11] P. Langacker and J. Liu, Philadelphia preprint UPR-0497T (1992)

[12] B. Kniehl, Nucl. Phys. **B357** (1991) 439

[13] B. Kniehl, Nucl. Phys. **B376** (1992) 3

[14] M. Drees and K. Hikasa, Phys. Lett. **B240** (1990) 455

[15] V. Barger and T. Han, Phys. Lett. **B212** (1988) 117

[16] H. M. Georgi et al., Phys. Rev. Lett. **40** (1978) 692

G. Altarelli, B. Mele and F. Pittolli, Nucl. Phys. **B287** (1987) 205

[17] D. Denegri in *Large Hadron Collider Workshop*, CERN 90-10, Vol.1, 1990

[18] G. Altarelli in *Proceedings of the Workshop on Physics of Future Accelerators* CERN 87-07, Vol.1, 1987

[19] R. P. Kauffman, Phys. Rev. **D41** (1990) 3343

[20] R. Blankenbecler and S. D. Drell, Phys. Rev. **D36** (1987) 277
[21] M. Drees and R. M. Godbole, Phys. Rev. Lett. 67 (1991) 1189; M. Drees, R. Godbole DESY-preprint 92-044

[22] S. Bertolini, A. Sirlin, Phys. Lett. B237 (1991) 179.

[23] V. Barger, A. Stange and W.-Y. Keung, Phys. Rev. D42 (1990) 1835

[24] M. Jezabek and J. H. Kühn, Phys. Lett. B207 (1988) 91

[25] D. Decamp et al., ALEPH collab., Phys. Lett. B236 (1990) 233; B. Adeva et al., L3 collab., Phys. Lett. B248 (1990) 203; P. Aarnis et al., DELPHI collab., Phys. Lett. B245 (1990) 276; M. Z. Akrawy et al., OPAL collab., Phys. Lett. B236 (1990) 224; L3 collab., CERN-preprint CERN-PPE/92-40

[26] G. Eilam, J. L. Hewett and A. Soni, Phys. Rev. D44 (1991) 1473

[27] V. Barger, W.-Y. Keung and T. G. Rizzo, Phys. Rev. D40 (1989) 2274
Figure captions

**Fig.1** Feynman diagrams contributing to $H \rightarrow W^+W^-V^0$, $V^0 \in \{Z^0, \gamma\}$. In case of $V^0 = \gamma$ clearly the amplitude $T_C = 0$

**Fig.2** Feynman diagrams relevant for the decay $H \rightarrow t\bar{t}V$, $q \in \{t, b\}$ and $V \in \{Z^0, \gamma, g, W\}$. In case $H \rightarrow t\bar{t}(\gamma \text{ or } g)$ diagram $C$ does not contribute. For $H \rightarrow W^-\bar{t}b$ diagram $B$ is negligible.

**Fig.3** Branching ratio $Br(H \rightarrow W^+W^-Z^0)$ versus the Higgs mass.

**Fig.4** Branching ratio $Br(H \rightarrow W^+W^-\gamma)$ with three different energy cuts: $E_{cut}^\gamma = 10 \text{ GeV}$ (full line), 20 GeV (dashed), 50 GeV (dotted).

**Fig.5** Branching ratio $Br(H \rightarrow t\bar{t}Z^0)$ versus the Higgs mass with three different values of $m_t$: $m_t = 90 \text{ GeV}$ (full line), 150 GeV (dashed), 200 GeV (dotted).

**Fig.6a** Branching ratio $Br(H \rightarrow t\bar{t}\gamma)$ with $E_{cut}^\gamma = 20 \text{ GeV}$. The full line corresponds to $m_t = 100 \text{ GeV}$, the dashed one to 150 GeV and the dotted one to 200 GeV.

**Fig.6b** The same as Fig.6a but with $E_{cut}^\gamma = 50 \text{ GeV}$.

**Fig.7** Branching ratio $Br(H \rightarrow t\bar{t}W^-)$ versus the Higgs mass with three different values of the top mass indicated in the figure. The dashed line corresponds to $Br(H \rightarrow t\bar{t})$.

**Fig.8** Feynman diagrams relevant for $t \rightarrow W^+bB$, $B \in \{\gamma, g, Z^0, H\}$. In case $B = g$ the diagram $C$ does not contribute. In case of $t \rightarrow W^+bH$ diagram $B$ is negligible.

**Fig.9** Branching ratio $Br(t \rightarrow W^+bZ^0)$ versus the top mass.

**Fig.10** Branching ratio $Br(t \rightarrow W^+b\gamma)$ with three different energy cuts: $E_{cut}^\gamma = 10 \text{ GeV}$ (full line), 20 GeV (dashed), 50 GeV (dotted).

**Fig.11** Branching ratio $Br(t \rightarrow W^+bg)$ with the same values of the energy cuts as in fig.10.

**Fig.12** Branching ratio $Br(t \rightarrow W^+bH)$ versus the top mass for $M_H = 50 \text{ GeV}$ (full line), 60 GeV (dashed), 70 GeV (dotted).
Appendix

Below we give the expressions for different squared matrix elements. It is convenient to define the following mass ratios

\[ \xi_H = \frac{M_H^2}{M_W^2} \]
\[ \xi_t = \frac{m_t^2}{M_W^2} \]

We start with the decay \( H \to W^+W^-\gamma \) (eq.(12)).

\[
|T(H \to W^+W^-\gamma)|^2 = \frac{e^2 g^2 M_W^2}{4} \left\{ 4M_W^2 - M_H^2 (2\xi_H - \xi_H^2 - 14) - 2(s_1 + s_2)(9 + \xi_H^2) + \frac{s_1^2 + s_2^2}{M_W^2} (\xi_H + 4) + \frac{2s_1 s_2}{M_W^2} \left( 2 + 2\xi_H - \frac{s_1 + s_2}{M_W^2} \right) \right. \\
+ \left( \frac{M_W^2 - s_1}{s_2 - M_W^2} \right) \left[ 6M_W^2 - M_H^2 (3 - 2\xi_H) + s_1 \left( 5 - \xi_H - \frac{2s_1 + \xi_H s_2}{M_W^2} \right) \right. \\
+ s_2 (5 - 3\xi_H - \xi_H^2) + \frac{s_2^2}{M_W^4} (2M_H^2 - s_1 - s_2) \right] \\
+ \left( \frac{M_W^2 - s_2}{s_1 - M_W^2} \right) \left[ 6M_W^2 - M_H^2 (3 - 2\xi_H) + s_2 \left( 5 - \xi_H - \frac{2s_2 + \xi_H s_1}{M_W^2} \right) \right. \\
+ s_1 (5 - 3\xi_H - \xi_H^2) + \frac{s_1^2}{M_W^4} (2M_H^2 - s_1 - s_2) \right\} \]

(A2)

The three contributions (see eq.(21)) to \( H \to t\bar{t}\gamma \) are

\[
|\tilde{T}^{t\bar{t}\gamma}_A|^2 = \frac{1}{(s_1 - m_t^2)^2} \left[ m_t^2 (3m_t^2 - 3M_H^2 + 6s_1 + s_2) + s_1(M_H^2 - s_1 - s_2) \right] \\
|\tilde{T}^{t\bar{t}\gamma}_B|^2 = \frac{1}{(s_3 - m_t^2)^2} \left[ m_t^2 (11m_t^2 + M_H^2 - 2s_1 - 3s_2) + s_1(M_H^2 - s_1 - s_2) \right] \\
2Re\tilde{T}^{t\bar{t}\gamma}_A(\tilde{T}^{t\bar{t}\gamma}_B)^* = \frac{2}{(s_1 - m_t^2)(s_3 - m_t^2)} \left[ m_t^2 (7m_t^2 - 5M_H^2 + 2s_1 - s_2) + s_1(M_H^2 - s_1 - s_2) + M_H^2 s_2 \right] \\
(A3)

It is clear that the expressions for the squared matrix elements of \( H \to t\bar{b}W^- \) and \( t \to W^+bH \) are related to each other by a relative minus sign, which is a result of crossing symmetry and an additional check for us. With obvious notation we will therefore write
\[
\left( \left| \tilde{T}_{Wtb} \right|^2 \right) = \pm \frac{\xi_t}{(s_1 - m_t^2)^2} \left[ m_t^2 (m_t^2 - 2s_2 + s_1 + M_H^2 - 6M_W^2) + 2M_H^2 M_W^2 \right.
\]
\[
+ s_1 (\xi_t s_2 + 4\xi_t s_1 - \xi_t M_H^2 - 2M_H^2 - 2M_W^2 + 2s_1 + 2s_2 - s_1 s_2/M_W^2) \right].
\] (A4)

\[
\left( \left| \tilde{T}_{Wtb}^C \right|^2 \right) = \pm \frac{1}{(s_2 - M_W^2)^2} \left[ m_t^2 (9m_t^2 - 2\xi_t m_t^2 M_H^2 - 4s_1 - 4M_H^2 \right.
\]
\[
+ 8M_W^2 - 5s_2) + M_H^2 (\xi_t^2 M_H^2 + 4M_W^2) + 4s_1 (s_1 - M_H^2 - M_W^2 + \xi_t M_H^2)
\]
\[
+ 2s_2 (-\xi_t^2 M_H^2 - \xi_t m_t^2 + \xi_t s_2 - 2\xi_t s_1 + \xi_t M_H^2 + 2s_1 - 4M_W^2 + \xi_t \xi_H s_2)
\]
\[
+ \xi_t s_2 (\xi_t s_2 - \xi_H M_H^2 - s_2^2/M_W^2) \right].
\] (A5)

\[
\left( \frac{2Re\tilde{T}_{Wtb}^*(\tilde{T}_{Wtb}^{C})^*}{2Re\tilde{T}_{Wtb}^C(\tilde{T}_{Wtb}^{C})^*} \right) = \pm \frac{2\xi_t}{(s_1 - m_t^2)(s_2 - M_W^2)} \left[ m_t^2 (4m_t^2 - 3M_H^2 + 5M_W^2 \right.
\]
\[
- 3s_2 - 2s_1) - 2M_W^2 (2s_2 - M_H^2 + 2M_W^2) + s_1 s_2 (-2\xi_t - \xi_H + 1 + s_2/M_W^2)
\]
\[
+ 2s_1 (\xi_t M_H^2 + 2s_1 - M_H^2) + 2M_H^2 s_2 \right].
\] (A6)

where the \( \tilde{T} \)'s enter eq. (23) for the Higgs decay and (48) for the top decay.