The characteristics of the $b$-value anomalies preceding the 2004 $M_w$9.0 Sumatra earthquake

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ABSTRACT

For earthquakes ($M \geq 4.9$) that occurred in the region limited by latitudes $2^\circ$S–$16^\circ$N and longitudes $90^\circ$E–$101^\circ$E from 1 January 1973 to 25 December 2004, we investigated temporal variations of the $b$ value in the aftershock zone to determine its abnormal characteristics prior to the 2004 $M_w$9.0 Sumatra earthquake. The $b$ value was estimated via the maximum likelihood method. We found that the $b$ value significantly decreased by $\sim 36\%$ in the aftershock zone from January 1990 to December 1999. From 2000 onwards until the Sumatra earthquake, the $b$ value was persistently low (between 1.0 and 1.2). Furthermore, we calculated the spatial distribution of the relative changes in $b$ value from 1990 to 1999. Only one area of larger relative declines in $b$ value ($>25\%$) was located very near to the epicentre of the Sumatra event. We also calculated the $b$ values as a function of time for earthquakes in this area and its immediate vicinity. The result shows that the changes of $b$ value with time around the epicentre is similar to that in the aftershock zone, but the relative decline amount is $\sim 40\%$ around the epicentre, greater than that in the aftershock zone. Therefore, if our finding is confirmed, it will help to better constrain the location of future great earthquakes in seismic hazard analysis.

1. Introduction

The relation between earthquake magnitudes and frequencies can be formulated by (Gutenberg and Richter 1944)

$$\log_{10}(N_M) = a - bM$$

which is called the Gutenberg–Richter (GR) law. The constant $b$ describes the relative occurrence of large and small events. A low $b$ value indicates a larger proportion of large earthquakes. The results of laboratory experiments indicate a reverse relation...
between $b$ value and stress (Scholz 1968a; Wyss 1973). Fluctuations of $b$ are also related to the degree of material heterogeneity (Mogi 1962), even geothermal gradients (Warren and Latham 1970). On the basis of an idea that the tectonic stress along faults in the crust may increase during the loading cycle for great earthquakes, the $b$-value has been applied to reflect the stress state in the Earth’s crust (Suyehiro et al. 1964; Smith 1981; Schorlemmer et al. 2005; Narteau et al. 2009). Some investigators have sought spatial-temporal $b$ value variations preceding earthquakes. Lower $b$ values have been found in the foreshock activity for moderate-size to great earthquakes (Suyehiro et al. 1964; Suyehiro 1966; Gibowicz 1973; Papazachos 1975; Wu et al. 1976; Papadopoulos and Minadakis 2016; Dascher-Cousineau et al. 2020; Gulia et al. 2020). Observations showed the $b$-value decreasing prior to some earthquakes (Wyss 1973; Imoto 1991; Nakaya 2006; Xie et al. 2010; Nanjo et al. 2012; Schurr et al. 2014; Shi et al. 2018; Chen and Zhu 2020). The decrease in the $b$ value preceding an earthquake is considered in the form of stress increase (Scholz 1968b; Main et al. 1989; Urbancic et al. 1992; Hainzl et al. 1999). The $b$ value is often applied to seismic hazard analysis (Li et al. 1978; Huang et al. 1989; Wang et al. 2012; Yi et al. 2008, 2013).

A giant earthquake ($M_w = 9.0$) occurred off the coast of NW Sumatra on 26 December 2004. One previous study (Nuannin et al. 2005) of spatial and temporal changes in $b$ value leading up to the Sumatra earthquake reported a significant drop in the $b$ value towards the end of 2004. Instead, Nanjo et al. (2012) found that the $b$ value decreased by 14.3% from 1.26 in 1984 to 1.08 in 1994 but remained almost unchanged from 1995 to immediately before the Sumatra earthquake (Nanjo et al. 2012). We investigated long-term spatio-temporal changes of $b$-value prior to the 2004 great Sumatra earthquake within its aftershock area, and found the area with the largest relative decline in $b$ value using the grid searching method.

2. Data and methods

The catalogue used in this study was obtained from USGS (https://earthquake.usgs.gov/earthquakes/search/) for the period of 1 January 1973–25 December 2004. The epicentres of aftershocks following the Sumatra mainshock from 26 December 2004 to 31 January 2005 were plotted in Figure 1(a). The region, covered mostly by the aftershock cloud (the polygon in Figure 1(a)), was selected for the calculation of $b(t)$ curves, i.e. the time variation of $b$ value. The region (latitude 2°S–16°N and longitude 90°E–101°E), covering the polygon and the area to the south of it, was selected to calculate the spatial distribution of relative changes of $b$. In this region there were 2719 earthquakes ($M \geq 4.0$), and the spatial distribution of their epicentres is shown in Figure 1(b). We computed the magnitude of completeness $M_C$ as a function of time [$M_C(t)$] using the maximum curvature technique (Wiener and Wyss 2000), applying sample sizes of 300 events and a step size of 10 events. Note that $M_C \leq 4.7$ during the whole time period, and $M_C \leq 4.5$ since 1996 (Figure 1(c)). Due to underestimation of $M_C$ by the maximum curvature technique, an increment of 0.2 should be added to $M_C$ (Huang et al. 2016). Therefore, in analysis, we took $M_C$ to be 4.9. There are 798 $M \geq 4.9$ shocks in total. Most of them are shallow events ($h \leq 70$ km),
138 events occurred at depths of $70 < h \leq 257$ km. In calculation events with depths of $h \leq 257$ km were used. We selected 462 earthquakes of $M \geq 4.9$ earthquakes that occurred in the aftershock zone (the polygon in Figure 1(a)) for calculation of $b(t)$ curve.

The maximum likelihood method was applied to estimate the $b$ value (Aki 1965):

$$b = \frac{\log e}{M - M_{\text{min}}}$$

(2)

The 95% confidence standard deviation of $b$ is

$$\sigma(b) = 1.96 \frac{b}{\sqrt{N - 1}}$$

(3)
where $\bar{M}$ represents the average magnitude, $M_{\text{min}}$ is the minimum magnitude of a given sample, and $N$ is the sample size.

For calculation of the $b$ value as a function of time, we applied a moving time window containing a constant number of events to ensure that the analysis was not influenced by a change in sample size. First, we calculated the $b$ value as a function of time for the aftershock area, applying a time window comprising 160 events and moving by ten events. Then spatial distribution of relative changes in the $b$ value was also mapped. To map the spatial distribution of the relative changes in $b$ value, the region limited by latitudes $2^\circ \text{S}–16^\circ \text{N}$ and longitudes $90^\circ \text{E}–101^\circ \text{E}$ was divided into 4949 grids of $0.1^\circ \times 0.1^\circ$. A square space window of $3^\circ \times 3^\circ$ (with sixty or more events) centred at each node on each grid were considered. We calculated the $b(t)$ curve for earthquakes that occurred in each space window, using a time window comprising 50 events and moving by one event. From each $b(t)$ curve, we calculated the relative change in $b$ value during the period from 1990 to 1999, taking the result as the value at the corresponding grid node. After calculating the relative changes in $b$ value at all grid nodes, the spatial distribution of relative changes in $b$ value was estimated.

For a $b(t)$ curve, the relative change in $b$ value is calculated by

$$\frac{\Delta b}{b} = \frac{b_2 - b_1}{b_1} \times 100$$

where $b_1$ is the value of $b$ at time $t_1$, and $b_2$ at time $t_2$. After we have calculated the relative changes in $b$ value at all nodes, the spatial distribution of the relative changes in $b$ value can be created.

3. Results

Figure 2 shows the $b$ values (heavy line) as a function of time for the aftershock zone, where the grey area indicates the 95% confidence limit. Here was a decreasing...
trend in $b$ value prior to the Sumatra event. The $b$ value decreased $\sim 36\%$ from 1.76 to $\sim 1.12$. The $b$ value declined 0.56 (a relative decline of $\sim 31\%$) towards the end of 1999, and $\sim 0.08$ from the beginning of 2000 until immediately before the Sumatra mainshock.

The spatial distribution of the relative changes in $b$ value is exhibited in Figure 3. The relative declines in $b$ value were larger in the area around the epicentre than those in other areas before the Sumatra $M_w9.0$ event. The spatial extent of this area is between $2^\circ$N and $4.5^\circ$N, where the larger relative declines greater than 25% distributed, at the western edge of it the epicentre of the 2004 Sumatra $M_w9.0$ main shock took place (Figure 3(b)). It reflects that the high relative declines in $b$ value appeared around the epicentre area before the Sumatra $M_w9.0$ event occurred. We also calculated the $b$ value versus time using 202 earthquakes of $M \geq 4.9$ in this area, applying a time window comprising 70 events and moving by five events. Figure 4 shows the result (heavy line). In this area, the change pattern of $b$ value with time is similar to that in the aftershock zone, the $b$ value dropped from 1.65 to $\sim 1.0$, (a relative decline of $\sim 40\%$), before the end of 1999. In contrast, the $b$ value increased only 0.08 from the beginning of 2000 towards immediately before the occurrence of the Sumatra event. Evidently, the relative decline of the $b$ value around the epicentre is greater than that in the aftershock zone.

4. Discussions and conclusion

Based on our investigation of $b$ value versus time for the aftershock area, and the spatial distribution of the relative changes of the $b$ value, we conclude the following:

In the aftershock area of the Sumatra event, there was a distinct decreasing trend in the $b$ value from 1990 to 1999, but no significant changes in $b$ value occurred in the five years immediately before the Sumatra earthquake.

In the area concentrated around the epicentre of the Sumatra event, the decreases of the $b$-value from 1990 to 1999 were larger than those in the aftershock area, and $b$ value increased by $\sim 0.08$ in the five years immediately before the Sumatra earthquake.
We performed a similar analysis for other three strong earthquakes of $M \geq 7.0$ and achieved similar results (Figure 5) despite their contrasting geological tectonic background. These three strong earthquakes are the 11 March 2011 Mw9.0 Tohoku-Oki, Japan earthquake, the 6 July 2019 Ridgecrest, USA earthquake and the 12 May 2008 Wenchuan, China earthquake. The Sumatra and the Tohoku-Oki events were located around the globe-scale plate boundaries where the oceanic crust and lithosphere begin their descent into the mantle. The convergence associated with this subduction process is responsible for their preparatory process. The Wenchuan earthquake was a thrust event in the intraplate, while the Ridgecrest mainshock was a strike-slip event nearby the boundary between the Pacific plate and the North America plate.

Despite the different geological tectonic settings of these events, the spacial and temporal changes in $b$ value were very similar (Figure 5). First, around the epicentre there was a 3.5 to 10-year decreasing trend in $b$ value (10 years for the Sumatra event, 8.75 years for the Tohoku-Oki, 5 years for the Ridgecrest and 3.5 years for the Wenchuan) and low $b$ values (between 1.0 and 1.2) several years before each strong earthquake. Secondly, during the period of decreasing $b$ value, the only area of high relative declines in $b$ was located very near to the epicentres. Nanjo et al. (2012) reported a 10-year decrease in $b$ value prior to the Tohoku-Oki mainshock, agreeing with our result. For the Wenchuan event (Nanjo et al. 2012), Chen and Zhu (2020) observed a steep drop in $b$ value several months before the main shock (Chen and Zhu 2020), but Shi et al. (2018) recorded similar observations to ours. As for the 2019 $M_w$7.1 Ridgecrest event, the $b$ value was lower during the fore shock sequence (Dascher-Cousineau et al. 2020; Gulia et al. 2020). For earthquakes around the $M_w$6.4 epicentre the $b$ values were mostly above 1 before 2010, and then gradually decreases to values around 0.7 (Nanjo 2020).

Given the different geological tectonic settings, the similarity of the spatial and temporal change trends in $b$ value of these four events suggest a common underlying mechanism. We observed a 10-year decline in $b$ value both around the epicentre and in the whole aftershock zone, and low $b$ values several years before the occurrence of
Figure 5. Map of high relative decreases in $b$ value (diagrams in the left column) and $b$ values (heavy lines in right diagrams) as a function of time for the regions enclosed by the black heavy lines in left diagrams. The grey area indicates the 95% confidence limit. The vertical arrows "↓" mark the times of occurrence of three events respectively. A1 and A2: the 2011 $M_w$9.0 Tohoku-Oki, Japan earthquake. The dataset from USGS catalog ($M ≥ 4.9$) over the period of January 1, 1973 to March 10, 2011 was used. A time window of 380 events moved by 1 events for $b(t)$ curve. B1 and B2: the 2019 $M_w$7.1 Ridgecrest, USA earthquake. The dataset from USGS catalog ($M ≥ 1.8$) over the period of January 2000 to June 2019 was used. A time window of 1500 events moved by 20 events for $b(t)$ curve. C1 and C2: the 2008 $M_s$8.0 Wenchuan, China earthquake. The dataset from the China Earthquake Networks Center, China Earthquake Administration ($M_s ≥ 1.5$) over the period of January 2000 to April 2008 was used. A time window of 600 events moved by 20 events for $b(t)$ curve.
the Sumatra earthquake. Larger relative declines in $b$ value clumped together around the epicentral area during the decline in $b$ value. Based on the inverse correlation between $b$ value and stress, we suggest that stress around the epicentre started to increase in the early stages of earthquake development, and was maintained at a high level for several years before the Sumatra earthquake. The stress enhancement was greatest around the epicentre. Observations usually show that the stress is usually high not only around the epicentre but also in other places preceding a strong earthquake. It has even been said that some large earthquakes do not occur in the area of high stress or no large earthquake took place in some areas of high stress. However, based on our results, we conclude that highest relative enhancement of stress is near the epicentral area during the period of increasing stress, and it could be the magnitude of relative increase of stress rather than the absolute level of stress that is associated large earthquakes. Therefore, we hypothesize that the analysis of spatio distribution of relative decrease of $b$ values, such as that applied in our study, could be used to constrain the location of future earthquakes when seismic hazard analysis is carried out.

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