Stop&Hop: Early Classification of Irregular Time Series

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ABSTRACT
Early classification algorithms help users react faster to their machine learning model’s predictions. Early warning systems in hospitals, for example, let clinicians improve their patients’ outcomes by accurately predicting infections. While early classification systems are advancing rapidly, a major gap remains: existing systems do not consider irregular time series, which have uneven and often-long gaps between their observations. Such series are notoriously pervasive in impactful domains like healthcare. We bridge this gap and study early classification of irregular time series, a new setting for early classifiers that opens doors to more real-world problems. Our solution, Stop&Hop, uses a continuous-time recurrent network to model ongoing irregular time series in real time, while an irregularity-aware halting policy, trained with reinforcement learning, predicts when to stop and classify the streaming series. By taking real-valued step sizes, the halting policy flexibly decides exactly when to stop ongoing series in real time. This way, Stop&Hop seamlessly integrates information contained in the timing of observations, a new and vital source for early classification in this setting, with the time series values to provide early classifications for irregular time series. Using four synthetic and three real-world datasets, we demonstrate that Stop&Hop consistently makes earlier and more-accurate predictions than state-of-the-art alternatives adapted to this new problem. Our code is publicly available at https://github.com/thartvigsen/StopAndHop.

CCS CONCEPTS
• Computing methodologies → Neural networks; Supervised learning by classification.

KEYWORDS
Time Series, Irregularly-Sampled Time Series, Early Classification, Reinforcement Learning, Recurrent Neural Network, Deep Learning

1 INTRODUCTION

Background. Early classification of time series (ECTS) is advancing quickly. Recent approaches overcome the poor scaling and false alarms inherent to classic ECTS [43] via reinforcement learning, predicting whether to Stop or Wait at every step of an ongoing series [13, 14, 25, 47]. This approach counteracts overconfident classifiers, resulting in better predictions. In addition, by using neural networks they classify multivariate time series seamlessly, a setting known to challenge similarity search methods [8, 9, 15, 46, 48]. However, the state-of-the-art methods, along with all prior ECTS algorithms, disregard uneven gaps between observations.

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Knowledge Gap. Despite rapid improvements, there remain ample opportunities to broaden the reach of early classifiers. In particular, existing early classifiers require that their input time series have even spaces between observations: they decide whether to stop early whenever any new observation arrives. Meanwhile, irregular time series (ITS), which have uneven and often-large gaps...
Our work is the first to consider early classification of irregular time series, bridging a major gap between modern early classifiers and real time-sensitive decision making.

Our contributions are as follows:

1. We define the open problem Early Classification of Irregular Time Series, bridging a major gap between modern early classifiers and real time-sensitive decision making.

2. Our method, Stop&Hop, is the first solution to this problem, integrating a continuous-time representation learner with two cooperative reinforcement learning agents.

3. We show that Stop&Hop classifies ITS earlier and more accurately than state-of-the-art alternatives. We also show that Stop&Hop learns to stop exactly when signals arrive using four synthetic datasets, leading to the earliest-possible classifications.

2 RELATED WORK

Early Classification of Time Series. Early Classification of Time Series (ECTS) is a machine learning problem: correctly predict the label of a streaming time series using as few observations as possible [11]. Solutions choose one early timestep per time series at which the whole instance is classified (without cheating and looking at future values). Classifying sequences early is classically targeted at
time series [8, 9, 15, 27, 33, 45, 46]. Most recent approaches [6, 7, 13, 14, 25] have turned to deep learning, extending beyond traditional methods for univariate time series [28, 44–46], which scale poorly by exhaustively searching for discriminative subsequences [15]. The current best solution is to frame this problem as a Partially-Observe Markov Decision Process, where at each regularly-spaced timestep, a policy decides whether or not to stop and predict the label. Some halt RNNs early [13, 14] while others use Deep Q-Networks [25].

A major limitation of existing ECTS methods is their reliance on inputs being regularly-spaced; they decide whether or not to halt at each possible timestep. This does not account for missing values or gaps between observations, features essential to classifying ITS [3]. In ITS, the gaps between consecutive observations may even be large and unpredictable, so waiting until the next value arrives has consequences. Furthermore, many ITS are multivariate, and multiple variables are rarely observed concurrently. This compounds issues with existing ECTS methods. Further, the times at which observations arrive can itself provide valuable knowledge for accuracy [24] and earliness. A successful solution to our problem should take advantage of this extra source of information.

Learning from Irregular Time Series. Standard machine learning techniques often fail for ITS as they assume fixed-length and regularly-spaced inputs [35], which are especially rare in important medical settings [38]. To bridge this gap, myriad recent works learn from ITS directly, developing models that take irregular series as inputs. Some approaches augment RNNs by either including auxiliary information such as a missingness-indicator [24] or time-since-last-observation [3] as extra features to preserve properties found in the irregularity. Others build more complex value estimators by either learning generative models [23], using gaussian kernel adapters [22, 34], set functions [16], or including decay mechanisms in Recurrent Neural Networks (RNN) to encode information-loss when variables go unobserved over long periods of time [3, 30]. Some recent works have begun parameterizing ordinary differential equations to serve as time series models [18, 19, 21, 32]. Some very recent models have also begun to integrate attention mechanisms into this estimation process [4, 36, 39].

However, ITS model considers when to return predictions to end users in the ongoing timeline. A key constraint of the early classification of irregular time series problem is that when classifying a series at a particular point in its timeline, we cannot use any future values. This constraint hinders the use of methods that use all observations during interpolation [34, 36] or ODE models that encode sequences backwards [32]. Such methods are quickly becoming pervasive [19], though creating online models for ITS remains a burgeoning area [5, 29, 41].

### 3 PROBLEM FORMULATION

Assume we are given a set of $N$ labeled irregular time series $D = \{(X^i, y^i)\}_{i=1}^N$. Each series $X$ is a collection of one sequence of $\tau^d$ (timestep, value) pairs per variable $d$: $X = \{(t^d_j, v^d_j)\}_{j=1}^{\tau^d} \in \mathbb{D}_d$ where each sequence of timesteps is strictly increasing ($t^d_1 < t^d_2 < \ldots < t^d_{\tau^d}$) and $v^d_j$ is the corresponding value of variable $d$ for each timestep. $\tau^d$ denotes the number of observations for variable $d$. $X$ is irregular in that typically $t^d_i \neq t^d_k$ and $t^d_{i+1} - t^d_i \neq t^d_{i+2} - t^d_{i+1}$ for all $i, j, k$.

| Table 1: Basic Notation |
|-------------------------|
| Notation | Description |
| $D$ | Variables per time series. |
| $\tau$ | Predicted halting time. |
| $\hat{y}$ | Predicted class label. |
| $t'$ | Candidate halting time. |
| $\pi_{stop}$ | Stopping policy (chooses Stop or Wait). |
| $\pi_{hop}$ | Hopping policy (chooses hop size). |
| $h_{t'}$ | Hidden state computed at time $t'$. |

Each label $y$ indicates to which of $C$ classes $X$ belongs. Our goal of Early Classification of Irregular Time Series is to learn a function $f$ that maps previously-unseen input time series to their accurate class labels $\hat{y}$ based only on values observed prior to some early time $\tau$, which is an unknown function of $X$. The smaller $\tau$ is, the better. However, fully achieving both goals at the same time in practice is usually impossible since early predictions are often made at the expense of accuracy as less of the series has been observed. Thus we seek a tunable solution that balances earliness and accuracy according to the task at hand.

### 4 METHOD

We propose an intuitive first solution to the open Early Classification of Irregular Time Series (ECITS) problem, which we name **Stop&Hop** and illustrate in Figure 3. The ultimate goal of our proposed method is to predict the best halting time $\tau$ for a given series so as to balance the cost of delaying a prediction with the benefits of accuracy, according to the requirements of the task at hand. Thus, one halting timestep $\tau$ is predicted per series $X$ along with a prediction $\hat{y}$ made using only observations made before time $\tau$.

Since no ECITS solution exists, we first describe a general solution, which we then instantiate as an architecture that builds on representation learning for irregular time series and on deep reinforcement learning. A general solution to ECITS iterates three steps: 1) Predict a candidate halting time $t'$ given only variables recorded before $t'$. 2) Construct a vector representation $h_{t'}$ of the ongoing series $X$ that captures patterns in both values and irregularity up to time $t'$. 3) Predict whether or not to halt and classify $X$ at time $t'$. If so, use $h_{t'}$ to classify $X$. If not, predict the next candidate halting time $t'$. Thus, a solution will march through the continuous timeline with a step-size that is predicted by the model according to the observations as they arrive. At each step, the model will decide whether or not to Stop and return a classification.

Each component of this general solution solves one challenge of the ECITS problem. First, learning when to try to stop is essential in the irregular setting. This is in contrast to the standard ECTS setting where, with knowledge that observations arrive on a fixed schedule, methods simply decide whether or not to stop every time a new measurement arrives. Second, standard supervised learning methods struggle to model ITS data as they are not fixed-length. Learning dense representations of these data instead provides feature vectors that are easy to learn from. Step three can then leverage the vast success of deep learning to classify ongoing ITS.
There has been a recent surge in approaches developed for representing all dynamics of observations in series. Most early classifiers can only use previous observations. However, a real-valued Hop Size computed as a function of representation representations of a series ends, a classifier network predicts the class label of the last-observed value for each variable as follows:

\[ \hat{x}_{t'}^d = m_{t'}^d x_{t'}^d \circ (1 - m_{t'}^d) (\gamma_{x'}^d x_{t'}^d_{prev} + (1 - \gamma_{x'}^d x_{t'}^d)) \]

\[ \hat{h}_{t'} = y_{h_t} \circ \hat{h}_{t'} \]

\[ r_{t'} = \sigma(W_{r} x_{t'} + U_r \hat{h}_{t'} + b_r) \]

\[ z_{t'} = \sigma(W_{z} x_{t'} + U_z \hat{h}_{t'} + b_z) \]

\[ \hat{h}_{t'} = \phi(W_{h} x_{t'} + U_r \hat{h}_{t'} + b_z) \]

\[ h_{t'} := (1 - z_{t'}) \circ \hat{h}_{t'} + z_{t'} \circ \hat{h}_{t'}, \]

where \( \hat{x}_{t'}^d \) is the mean of all values of for the given instance’s variable \( d \) before time \( t' \), \( m_{t'}^d \) is a binary value indicating whether or not any new observations have been made since \( t' \), and \( x_{t'}^d_{prev} \) is the value of the most recent observation of the \( d \)-th variable prior to time \( t' \). \( \circ \) is the hadamard product and \( \gamma_{x'}^d \) is a decay factor for variable \( d \) at time \( t' \) computed by a neural network:

\[ y_{x'}^d = e^{-\max(0, W_{x'} m_{t'} + b_{x'})}, \]

where \( y_{x'}^d \) is the difference between \( t' \) and the time of the last observation of variable \( d \). The GRU-D is a natural choice for this problem, as it computes hidden states at any real-valued times, based on previous hidden states. By incorporating the mask \( m \) and time since last observation \( x_{prev} \), the hidden state reflects input irregularity. When decisions—like when to halt—are made based on the resultant hidden states, they depend on the irregularity of the input series. Further, the GRU-D is a unidirectional state space model, computing hidden states without using future information. This is crucial, as early classifiers can only use previous observations.

An encoding \( h_{t'} \) thus represents knowledge contained in the transitions between the values over time and observation density.

This modular setup solves the ECITS problem and so we instantiate this idea with solutions to each of the three sub-problems, integrating the two goals of accuracy and earliness. First, a continuous-time recurrent network \( R(\cdot) \) constructs a representation \( h_{t'} = R(X_{t,t'}) \) where \( X_{t,t'} \) represents all observations made prior to a timestep \( t' \). Next, a Halting Policy Network decides either to Stop, or Hop forward in time to a new timestep \( t' = \Delta t \) where \( \Delta t \) is a real-valued Hop Size computed as a function of representation \( h_{t'} \). This two-policy setup is novel for the ECTS literature. Since \( h_{t'} \) represents all observations made prior to a \( t' \), it encodes the prefixes of ongoing time series. Further, the GRU-D is a natural choice for this problem, as it encodes the prefixes of ongoing time series.

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4.1 Embedding Irregular Time Series Prefixes

\( \text{StorHop} \) learns to encode ongoing irregular time series via continuous-time representation learning, computing vector representations of a series \( X \) at real-valued timesteps. We refer to this as a Prefix Encoder, as it encodes the prefixes of ongoing time series. There has been a recent surge in approaches developed for representing ongoing ITS [3, 32] and most use a recurrent component to encode the series at real-valued timesteps in the continuous timeline: \( h_{t'}^r = R_0(X, t') \), where \( R(\cdot) \) is a continuous-time recurrent neural network and \( t' \) is a real-valued time. \( h_{t'}^r \) is thus a vector representing all dynamics of observations in series \( X \) prior to time \( t' \), including information found in the irregularity of the values. The only constraint on architecture design for \( R_0(\cdot) \) in the Early Classification setting is that \( h_{t'}^r \) must only be computed with respect to values observed earlier than \( t' \). This disables the use of methods that compute bi-directional hidden states or use future values for imputation [34]. Further, we seek to model the irregularity itself, which can inform both the classification accuracy and the earliness. Thus we compute \( h_{t'} \) using the GRU-D [3], denoted for variable step sizes between embeddings. The hidden state and input values are decayed based on the time since the last-observed value for each variable as follows:

\[ \hat{x}_{t'}^d = m_{t'}^d x_{t'}^d \circ (1 - m_{t'}^d) (\gamma_{x'}^d x_{t'}^d_{prev} + (1 - \gamma_{x'}^d x_{t'}^d)) \]

\[ \hat{h}_{t'} = y_{h_t} \circ \hat{h}_{t'} \]

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\[ \hat{h}_{t'} = \phi(W_{h} x_{t'} + U_r \hat{h}_{t'} + b_z) \]

\[ h_{t'} := (1 - z_{t'}) \circ \hat{h}_{t'} + z_{t'} \circ \hat{h}_{t'}, \]

where \( \hat{x}_{t'}^d \) is the mean of all values of for the given instance’s variable \( d \) before time \( t' \), \( m_{t'}^d \) is a binary value indicating whether or not any new observations have been made since \( t' \), and \( x_{t'}^d_{prev} \) is the value of the most recent observation of the \( d \)-th variable prior to time \( t' \). \( \circ \) is the hadamard product and \( \gamma_{x'}^d \) is a decay factor for variable \( d \) at time \( t' \) computed by a neural network:

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An encoding \( h_{t'} \) thus represents knowledge contained in the transitions between the values over time and observation density.
These hidden states can therefore use patterns in values and observations to support both earliness and accuracy. For instance, if observations are arriving rapidly, more may arrive soon, so waiting to receive them will not incur much penalty for waiting [34]. On the other hand, if the hidden states indicate low observation density, it may be better to cut our losses since more observations may be far in the future. Alternatively, the time-since-last-observation can itself be a valuable feature [24], so waiting to see if more observations arrive can itself be a reasonable policy for accurate classification.

4.2 Classifying Prefixes

Given a prefix embedding \( h_{t'} \), we use a Prefix Classifier \( C_{\theta} \) to predict the class label of the entire series \( X \) based on only values observed up to time \( t' \). We use a standard fully connected network that projects \( h_{t'} \) into a \( C \)-dimensional probabilistic space via the softmax function, parameterizing the conditional class probabilities. In our experiments, we use one hidden layer, but this component can be scaled up depending on the task at hand. Once the Halting Policy Network chooses to stop, the final prediction \( \hat{y} \) for series \( X \) is also generated by the Prefix Classifier.

4.3 Irregularity-Aware Halting Policy Network

\textit{Stop} &\textit{Hop} achieves early halting through an irregularity-aware Halting Policy Network \( H_{\theta} \) that chooses whether to \textit{Stop} or \textit{Wait} at a given time \( t' \) based on the history of an ongoing time series \( X \). If it chooses to \textit{Wait}, it also predicts \( \text{for how long} \) in the form of a real-valued \textit{hop-size}, which is added to \( t' \). To account for the irregular nature of the input time series, we model the hop-time as a continuous variable, samples of which we acquire by parameterizing a normal distribution with a neural network. The hop policy \( \pi_{\text{hop}} \) begins by predicting the a mean value:

\begin{equation}
\mu_{t'} = \phi(W_{h} h_{t'} + U_{h} \hat{y}_{t'} + V_{t'} + b),
\end{equation}

where \( \phi \) is the ReLU function [12]. We then sample a hop-size \( \Delta t \) from the normal distribution with mean \( \mu_{t'} \) and standard deviation \( \sigma \). We leave the standard deviation \( \sigma \) as a hyperparameter, though in principle it can also be learned by the model. To ensure \( \Delta t \geq 0 \), we take the absolute value of \( \Delta t \), which is a common approach in similar scenarios [26]. To compute the new candidate halting time, we add the hop-size to the current candidate halting time:

\begin{equation}
t' + \Delta t.
\end{equation}

In practice, early in training the Halting Policy Network may tend to exploit some actions by predicting too-high probabilities, relative to an optimal policy. To encourage exploration early on in training, we employ a simple \( \epsilon \)-greedy approach to action selection and exponentially decay the values \( \epsilon \) from 1 to 0 throughout training:

\begin{equation}
\epsilon_{t'} = \begin{cases} 
\epsilon, & \text{with probability } 1 - \epsilon \\
\text{random action,} & \text{with probability } \epsilon
\end{cases}
\end{equation}

By exploring more, the model tries out different sequences of actions to cover the space of possible episodes more effectively while early on in training. We also increase the probability that the Halting Policy Network chooses to \textit{Wait} before the model has been thoroughly trained so that the prefix embeddings and classifier also get to observe more of the sequences and increase their performance. Otherwise, a model that learns to stop early very quickly will never have seen the later portions of the training sequences. For all of our experiments, we compute \( \epsilon \) as \( e^{-t} \) while reassigning \( t := i + \frac{\text{log2}}{E} \) for \( E \) training epochs after initializing \( t \) to 0. During testing, we set \( \epsilon = 0 \) to avoid exploration for testing series.

Rewards. The final component of the POMDP is the reward for reaching different states. We encourage the halting policy network to cooperate with the prefix embeddings and classifier by setting the reward \( r_{t'} = 1 \) when the \( \hat{y} \) is accurate and setting \( r_{t'} = -1 \) otherwise. This way, accurate classifications are encouraged. To encourage early classifications, we penalize \textit{Wait} probabilities, as discussed in the next section, which describes how we jointly train all of \textit{Stop} &\textit{Hop}’s components.

4.4 Training

The prefix embedding network \( R_{\theta} \) and Classifier \( C_{\theta} \) are fully differentiable, so we train them together using standard back propagation. We encourage them to predict \( \hat{y} \) as close to \( y \) as possible by minimizing their cross entropy, where \( y_{c} = 1 \) if \( y = c \) and is 0 by a stopping policy \( \pi_{\text{stop}} \):

\begin{equation}
p_{t'} = \text{softmax}(W_{h} h_{t'} + U_{h} \hat{y}_{t'} + V_{t'} + b),
\end{equation}

where \( W, U, \) and \( V \) are weight matrices and \( b \) is a bias vector. Finally, we use the probabilities \( p_{t'} \) to sample an action from a multinomial distribution. If \( a_{t'} = \{\text{Stop}\} \), then the corresponding class prediction \( \hat{y}_{t'} \) is returned at time \( t' \) and we set \( \tau \) to \( t' \).

If the model instead chooses to \textit{Hop}, we run a hopping policy \( \pi_{\text{hop}} \). another small neural network, that predicts a positive real-valued hop-size, which is added to \( t' \). To account for the irregular nature of the input time series, we model the hop-time as a continuous variable, samples of which we acquire by parameterizing a normal distribution with a neural network. The hop policy \( \pi_{\text{hop}} \) begins by predicting the a mean value:

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otherwise in Equation 9, \( \hat{y}_c \) is the predicted probability of class \( c \).

\[
L_{\text{acc}} = -\sum_{c=1}^{C} y_c \log(\hat{y}_c) \quad (9)
\]

The Halting Policy Network, on the other hand, samples its actions and so its training is more intensive, though we follow the standard policy gradient method and use the REINFORCE algorithm [42] to estimate the gradient with which we update the network’s parameters. To balance between earliness and accuracy, the parameters of the Halting Policy Network are updated with respect to two goals: Make \( r \) small, and make \( \hat{y} \) accurate. Following the state-of-the-art [13], we achieve smooth optimization by rewarding accurate predictions and penalizing the cumulative probability of \( \text{Waiting} \) over each episode. Thus the loss function for optimizing the halting policy network is computed as:

\[
L_{\text{early}} = -E \sum_{t'} \log \pi_{\text{stop}}(a_{t'}|h_{t'}) \left[ \sum_{j=t'}^T (r_{j'} - b_{j'}) \right] \\
- E \sum_{t'} \log \pi_{\text{stop}}(a_{t'}|h_{t'}) \left[ \sum_{j=t'}^T (r_{j'} - b_{j'}) \right] \\
- \lambda \sum_{t'} \log \pi_{\text{stop}}(a_{t'} = \text{Stop}|h_{t'}),
\]

where the scale of \( \lambda \) determines the emphasis on earliness and \( \pi_{\text{stop}} \) and \( \pi_{\text{hop}} \) are the probabilities predicted by the stopping and hopping networks, respectively. If \( \lambda \) is large, the halting policy network will learn to maximize the probability of stopping always whereas if \( \lambda \) is small or zero, the model will solely maximize accuracy. Interestingly, the most-accurate classification may not always be achieved by observing the entire series, though this is rare in practice. For example, early signals followed by irrelevant values make classification challenging for memory-based models. Our approach deals with this case naturally by learning not to make late predictions when they are less accurate, regardless of any cost of delaying predictions. The final loss \( L \) is thus

\[
L = L_{\text{acc}} + \alpha L_{\text{early}}, \quad (10)
\]

and minimized via gradient descent. \( \alpha \) scales the loss components.

5 EXPERIMENTS

5.1 Datasets

We evaluate \texttt{Stop\&Hop} on four synthetic and three real-world datasets, which are described as follows:

Synthetic datasets: Since the true halting times are not available for real data, we develop four synthetic datasets with known halting times. Intuitively, a good early classifier will stop as soon as a signal is observed. To generate these data, we first uniformly sample \( T - 1 \) timesteps from the range \([0, 1]\) for each of \( N \) time series. We set \( T = 10 \) and \( N = 5000 \). Then, from a chosen distribution of signal times, we sample one more timestep per series at which the class signal occurs and add it to a series’ timesteps.

We experiment with four distributions of true signal times, each creating a unique dataset: \texttt{Uniform}(0, 1), \texttt{Early} \( \mathcal{N}(\mu = 0.25, \sigma = 0.1) \), \texttt{Late} \( \mathcal{N}(\mu = 0.75, \sigma = 0.1) \), and \texttt{BiModal} where half the signals are from \texttt{Early} and the other half are from \texttt{Late}. For each time series, one value is sampled from one of the distributions—each distribution creates one dataset—which serves as the time at which a signal arrives in the timeline. In all cases, we clamp this value to be within range \([0, 1]\). We generate two classes by giving 2500 time series a 1 at their signal occurrence time, and –1 to the remaining 2500 series in the dataset. Thus we know precisely when the signals arrive for each instance. Values for off-signal timesteps are set to 0 and are uniformly sampled from the timeline.

ExtraSensory: We use the publicly-available ExtraSensory [40] human activity recognition dataset, which contains smartphone sensor data collected across one week while participants labeled which actions they performed and when. These data were collected in the wild, so participants were left to their own devices for the duration of the study with no prescribed behavior. Using these data, we simulate a listening probe on a smartphone’s accelerometer data, which consist of three variables corresponding to the X, Y, and Z coordinates. A listening probe saves a phone’s battery by collecting data only when certain measurements are taken, naturally creating irregular time series. For this dataset, we measure the norm of the 3-dimensional accelerometer data, only taking measurements associated with changes in the norm over 0.001. Here we consider the popular tasks of detecting \texttt{Walking} and \texttt{Running}, classifying whether or not a person performs an activity within a window. Since activity records are often incomparable between people, we use the records from the person who walked or ran the most breaking their series into 100-minute long windows. This creates two independent datasets, one per user, which is common for human activity recognition. We balance each dataset, resulting in 2636 time series with an average of 90 observations per series for \texttt{Walking} and 3000 time series with on average 100 observations for \texttt{Running}. Deeper, extended discussion of this dataset is available in a concurrent submission. In general, down-sampling human activity recognition data is common in the irregular time series literature [32, 34], and simulating a listening probe adds a key new ingredient: non-random down-sampling.

PhysioNet: The PhysioNet dataset [37] contains medical records collected from the first 48 hours after 4000 patients were admitted to an intensive care unit and is publicly-available. There are 42 variables recorded at irregular times for each patient along with one label indicating if they perished. This is a common benchmark for multivariate irregular time series classification [3]. On these data, we train our classifiers to perform mortality prediction for previously-unseen patients. 13.8% of patients have positive labels, so we use the Area Under the Receiver-Operator Curve (AUC) as our primary metric on all three real-world datasets.

5.2 Compared Methods

We compare \texttt{Stop\&Hop} to the two key alternatives. Each is a state-of-the-art early classifier which we update to handle ITS.

- \texttt{E-GRU} [6]. \texttt{E-GRU} thresholds a sequential classifier’s output probability in real time. When the predicted probability \( \hat{y} \) surpasses a threshold \( \alpha \), \( \hat{y} \) is used to classify the series, ignoring all future observations. A hidden state represents the streaming series and is updated whenever a new observation arrives. Each time it is updated, the hidden state is passed to a neural network that predicts \( \hat{y} \).
we use the GRU-D update equations [3] described in Section 4.1.

Figure 4: Trade-off between earliness and accuracy on three real-world time-sensitive datasets. For each dataset in (a)-(c), the black line (Stop&Hop) outperforms the compared methods, as it is above them for most halting times.

- **EARLIEST** [13]. Similar to E-GRU, EARLIEST models ongoing time series with an RNN. Whenever a new observation arrives, a new hidden state is computed. Similar to Stop&Hop, a halting policy then decides whether or not to stop and classify the series with a neural network. This baseline ablates the Hopping Policy Network network, as it only chooses between Stop and Wait, ignoring irregularity.

In our synthetic experiments, we also compare Stop&Hop with a baseline: A GRU classifies irregular time series, then stops at chosen timesteps. This is the simplest early classifier as the halting times are always the same, disregarding the input data. For all methods, we use the GRU-D update equations [3] described in Section 4.1.

### 5.3 Implementation Details

For our synthetic datasets and PhysioNet, we repeatedly split the data into 90% training and 10% testing subsets five times, and report the average performance. We learn each model’s parameters on the training set, and report all final evaluation metrics on the testing set.

The ExtraSensory datasets, on the other hand, contain instances taken from different windows along a single timeline and so we select a timestep for each before which is the training/validation data and after which is the testing to ensure the testing set’s sanctity. For the synthetic SimpleSignal datasets, we use 10-dimensional prefix embeddings, which we compute at intervals of 0.1. For ExtraSensory, we use 50-dimensional prefix embeddings and for PhysioNet they are 20-dimensional.

For each method, we use a batch size of 32 and grid search for a learning rate (options: \{1e^{-2}, 1e^{-3}, 1e^{-4}\}) and weight decay for L2 regularization (options: \{1e^{-3}, 1e^{-4}, 1e^{-5}\}) using our validation data. The validation data is a random 10% of the training dataset and we repeat this random splitting five times. In our experiments, since we use a GRU-D [3] to compute prefix embeddings, its hidden state should be updated in between hop sizes. For simplicity, we only update the embeddings when real data are observed per the baselines, though Stop&Hop’s final Stop time may be between observations. Each model was optimized using Adam [20] with learning rates and weight decays that maximize their performance on the validation data. All training was done using Intel Xeon Gold 6148 CPUs. For all experiments, we set the value $\alpha$ in Equation 10 to 1 as the loss terms are balanced. All of our code is public.\(^1\)

### 5.4 Results on real-world datasets

First, we demonstrate that Stop&Hop produces early and accurate classifications using three real time-sensitive datasets, shown in Figure 4. We use the trade-off curves between earliness and AUC to measure each method’s performance, adjusting their parameters to achieve average halting times that span the timeline. We first observe that Stop&Hop outperforms the comparisons: the black line

\(^1\)https://github.com/thartvigsen/StopAndHop
is consistently highest. This improvement indicates that Stop&Hop stops earliest and makes the most-accurate classifications.

For the PhysioNet dataset, Stop&Hop maintains improvement across all average halting times, converging with the comparisons once all time steps are observed—as expected. Further, this AUC (~0.82) is nearly state-of-the-art for this dataset [36]. For the two ExtraSensory datasets, Stop&Hop’s improvements seem to come earlier in the timeline; at some point, AUC appears to saturate and Stop&Hop and EARLIEST accurately classify all testing time series. As we will show on our synthetic results (Figure 8b), this happens when a method predicts good halting points. We are also unsurprised by this saturation in general: there will always be cases where relevant windows of a time series are isolated [50]. For these datasets, the optimal halting times are unknown, yet seem to be around 40 minutes for Walking and 60 minutes for Running on average. Discovering good halting times retrospectively is an added benefit of a successful early classifier; it can even recover reasonable halting times for individual instances, depending on whether or not the classification was accurate. In summary, Stop&Hop consistently outperforms the alternatives on these three real-world datasets.

We also conduct a hyperparameter study for λ for the Running dataset, shown in Figure 5. Our results indicate that λ has strong control over the earliness-accuracy trade-off: As λ increases, Halting Time and Accuracy steadily decrease. Standard deviations are computed across five replications of the same experiment with different seeds. Thus emphasizing earliness or accuracy is intuitive. To further ablate the impacts of hyperparameters on Stop&Hop’s success, we also consider the standard deviation σ of the hop policy πhop, which controls for how long to wait before trying to stop again. As σ increases, so does the variance in chosen hop sizes. Interestingly, we find that Stop&Hop’s performance is largely robust to changing σ, as shown in Figure 6. Still, varying σ seems to have some impact and we recommend tuning it according to the task at hand. σ also determines to what degree πhop directly controls the sampled hop size. As σ grows, πhop has less direct control, which can improve exploration and regularize the chosen actions.

### 5.5 Stop&Hop finds the true halting times

We next verify that Stop&Hop indeed finds the true halting times by using our four synthetic datasets where we know the halting times. Our results are shown in Figures 7 and 8, where we compare Stop&Hop to a Preset Halting baseline with the same prefix embedding approach as Stop&Hop to isolate the effects of learning when to stop. The preset halting method stops at a set of predetermined halting times. For example, a preset halting method that uses 50% of the timeline stops all instances at time 0.5.

Our experiments show that Stop&Hop clearly achieves higher accuracy while using less of the timeline compared to preset halting times, as expected. This is only possible if Stop&Hop appropriately halts when it sees a signal and waits otherwise. As the four synthetic datasets have different halting distributions, we see that Stop&Hop succeeds to wait longer when signals are all later (Figure 7c) and stop earlier when signals are all earlier (Figure 7b). Note that for the Uniform dataset, Stop&Hop steadily increases to Accuracy of 1.0 (indicating 100% accurate predictions), which makes sense because the true signals are distributed uniformly across the timeline. For the Early dataset, where signals happen early in the timeline, AUC saturates early and Accuracy steeply increases early on. For Late, Accuracy increases slowly early on and steeply later. In each case, the Preset Halting baseline’s Accuracy changes exactly as expected.

For the BiModal dataset—highlighted in Figure 8—we first observe that Stop&Hop again makes early and accurate predictions, even when signals are distributed unevenly across the timeline. Its curve is characterized by a steep increase, followed by a plateau, followed by a steep increase, and finally a plateau. We expand this experiment and also show a snapshot of the halting distribution from the BiModal dataset from Stop&Hop trained with λ = 3e−6 in Figure 8b. Each instance’s predicted halting time is plotted against the proportion of the dataset with halting times earlier than a set of possible halting times, showing the cumulative halting distribution. We color-code the early and late signals and find that Stop&Hop matches the cumulative frequencies of the halting timings almost perfectly. As our method captures all positives exactly on time, matching the true cumulative functions without supervision, we
postulate that Stop&Hop can also learn other complex functions. In contrast, the preset halting comparison’s halting distribution is a step function: All instances halt at the same time. This is not flexible enough to match the halting distributions of real datasets.

6 ETHICAL CONSIDERATIONS

Our work facilitates decision making given partial temporal information. This lack of knowledge can naturally lead to misclassifications, which are more or less dangerous depending on the task. For example, incorrectly predicting Cancer early may cause undue stress and financial burden to a patient. Still, the cost of risking a false positive must be balanced with the cost of delaying predictions, which itself may have negative impacts. Early classifiers do not suffer from or introduce this trade-off, and instead embrace reality—the trade-off is real in practice, regardless of the algorithm. Standard classifiers, for instance, ignore prediction timing, so they always pick one side of the trade-off. Algorithms like Stop&Hop crucially allow end-users to balance their own risk aversion.

Figure 7: Earliness vs. Accuracy on three synthetic datasets—Uniform, Early, and Late. The X axis denotes the average percent of the timeline used as Stop&Hop predicts one halting point per time series. The high black line indicates that Stop&Hop succeeds to stop and classify irregular time series at effective times.

Figure 8: Results for the synthetic BiMOMAL dataset. In (a) Stop&Hop outperforms Preset Halting. In (b), the solid and dashed lines match, indicating that Stop&Hop is nearly optimal.

reinforcement learning-based continuous-time recurrent network that leverages irregularity in the inputs to inform classifications that are both early and accurate. By using irregularity to inform when to classify ongoing series, Stop&Hop advances beyond the state-of-the-art for early classification. Our irregularity-aware halting policy network chooses when to stop, which allows more-flexible halting policies than recent alternatives and shortens the reinforcement learning trajectories, leading to more stable training. We find that Stop&Hop indeed halts at the earliest-possible times on all four synthetic data sets and consistently outperforms alternatives on all three real datasets by making earlier and more-accurate predictions.

With this work, we also advocate for broadening the evaluation of machine predictions. Our community often evaluates machine learning models using solely accuracy (or a similar measure). However, in time-sensitive domains, accuracy is irrelevant if a model is used too late. We pose that earliness, along with other highly-impactful directions, like fairness and explainability, should also be considered when developing machine learning systems.

7 CONCLUSION

Our work introduces the open Early Classification of Irregular Time Series problem. This is an important, challenging, and interesting problem that has yet to be considered by the early classification community. We provide a general formulation for solving this problem, which we instantiate as a modular framework that serves as an effective first solution, named Stop&Hop. Stop&Hop is a novel

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