Robust Transmission Network Expansion Planning in Energy Systems: Improving Computational Performance

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Abstract

Recent advances about the problem of transmission network expansion planning propose the use of robust optimization techniques, as an alternative to stochastic mathematical programming methods, to make the problem tractable in realistic systems. They consider different sources of uncertainty, mainly related to the capacity and availability of generation facilities and demands, and make use of adaptive robust optimization models. The resulting formulations materialize on three-level mixed-integer optimization problems, which are solved using different strategies. Although it is true that these robust methods are more efficient than their stochastic counterparts, it is also true that solution times for mixed-integer linear programming problems grow exponentially with respect to the size of the problem. This fact encourages researchers, practitioners and system operators to use computational efficient methods when solving this type of problems. This paper addresses the issue of improving computational performance by taking different features from existing algorithms. In particular, we replace the lower-level problem by its dual, and solve the resulting bi-level problem using a primal cutting plane algorithm within a decomposition scheme. Using this alternative and simple approach, the computing time for solving transmission expansion planning problems reduces drastically. Numerical results on an illustrative example, the IEEE-24 and IEEE 118-bus test systems demonstrate that the algorithm

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is superior in terms of computational performance with respect to existing methods.

1. Introduction

The problem of Transmission Network Expansion Planning (TNEP) aims to resolve the issue of how to expand or reinforce an existing electricity transmission network so that adequately serves system loads over a given horizon. The main difficulty of this problem is to take decisions under the great amount of uncertainty associated with i) demands, ii) renewable generation, such as wind and solar power plants, and iii) equipment failure. Two main frameworks have been used to tackle these uncertainties, stochastic programming [1] and robust optimization [2, 3, 4, 5, 6, 7, 8].

In stochastic programming problems, uncertain data is assumed to follow a given probability distribution and are usually dealt with by using scenarios or finite sampling from the joint probability density function. Examples of the successful application of stochastic programming in TNEP problems are given by [9, 10, 11, 12] among others. However, the number of scenarios needed to represent the actual stochastic processes can be very large, which may result in intractable problems. In contrast, robust optimization can avoid the intractability issue related to stochastic programming approaches. Note that tractability is not the only difficulty arising from stochastic programming, but it is the most important aspect from this paper perspective. Examples of the successful application of robust optimization in TNEP problems are given by [13, 14]. However, in this paper we focus on the seminal works proposed by [15], [16] and [17].

The three works on which our proposal is based formulate the problem using an adaptive robust optimization (ARO) framework [18, 19]. In consequence, all of them use three-level formulations: i) first level minimizes the cost of expansion ([16] also minimizes the maximum regret), the decision variables for this level are those associated with construction or expansion of lines, ii) second level selects the realization of the uncertain parameters that maximizes the system’s operating costs within the uncertainty set, the variables related to this level are the uncertain parameters, i.e., generation capacity and demand, and iii) in the third level the system operator selects the optimal decision variables to minimize operating costs for given values of first and second level variables. Note that in this work we do not consider
contingencies explicitly, however, they could be taken into account within the robust formulation following the guidelines proposed by [20].

[15] transforms the second and third levels into one single-level maximization problem (so called subproblem in this work) by using the dual of the third level. It takes advantage of the fact that all uncertain parameters need to be equal to their upper or lower limits in the worst case scenario, which allows using binary variables to deal with these uncertain parameters. This simplification imposes the limitation that the uncertainty budget, typically used in robust optimization, must be integer. However, in our opinion, it does not dwarf the benefits of robust optimization while simplifying the resolution of the problem substantially. In contrast, [16, 17] transforms the second and third levels into one single level maximization problem by using the Karush-Kuhn-Tucker (KKT) conditions of the third level. In this particular case, the number of constraints, continuous and binary variables of the subproblem increases with respect to the other approach.

Regarding the resolution of the resulting bi-level problem, [15] proposes a Benders [21] approach where the dual information from subproblems is used to construct additional Benders cuts. The main concern of this approach is the slow convergence typical from this type of decomposition algorithms, which forced the author to include additional linear constraints in order to improve convergence. On the other hand, [16, 17] apply a constraint-and-column [22] generation method solely based on primal cuts. This method is computationally advantageous with respect to the Benders decomposition and converges in a small number of iterations (authors reported solution results with only 3-4 iterations required). Note that the methodologies proposed by [16] and [17] are basically the same from the mathematical point of view, however, they analyse different situations and [17] proposed a different way to construct the uncertainty sets.

This paper proposes an alternative solution approach to the three-level mixed-integer optimization problem associated with TNEP. The idea is to combine the way [15] solves the inner level problems (subproblems) and the constraint-and-column generation method used by [16] and [17] for the first level problem (master problem). Note that although the master problem solved using primal cuts is very efficient, computing times associated with [16] and [17] approaches increase exponentially with respect the size of the problem. The reason is the subproblem, which takes most of the resolution time because of the use of the KKT optimality conditions of the third level into the second level problem. The resulting model calculates simultaneously
the primal and dual variables of the third level problem, and the primal variables are not required to construct the primal cuts of the master. Therefore, this alternative of using the KKT conditions is inefficient.

The remainder of the paper is structured as follows. Section 2 describes the adaptive robust formulation of the TNEP problem in compact matrix form, and discusses the solution approaches proposed by [15], [16] and [17]. The detailed formulation is only given in the Appendix A. In Section 3 the definition of the uncertainty set and the description of the solution approach are presented. Numerical results for different networks are given in Section 4 and compared with those obtained using the method proposed by [16] and [17]. Finally, the paper is concluded in Section 5.

2. Transmission network expansion planning problem: ARO compact formulation

According to [15], [16] and [17], the robust TNEP problem can be written in compact matrix form as the following three-level mathematical programming problem:

\[
\begin{align*}
\text{Minimize} & \quad (c^T x + \max_{d \in D} \min_{y \in \Omega(x,d)} b^T y) \\
\text{subject to} & \quad c^T x \leq \Pi \\
& \quad x \in \{0,1\},
\end{align*}
\]

(1)

subject to

\[
\begin{align*}
c^T x & \leq \Pi \\
x & \in \{0,1\},
\end{align*}
\]

(2)

(3)

where \(x\) is the vector of first stage binary variables representing the investment vs no investment in reinforcing or building new lines, \(c\) is the investment cost vector, \(d\) is the vector of second stage continuous variables representing the random or uncertain parameters, i.e. generation capacities and demands, \(D\) is the uncertainty set, \(b\) is the vector including operating costs, and \(y\) is the vector of third stage continuous variables referring to the operating variables. These operating variables include powers consumed, power flows, power produced by generating units, load shed by demands, and voltage angles at buses (see Appendix A for a detail description of the formulation). \(\Pi\) represents the maximum budget for investment in transmission expansion. Finally, \(\Omega(x,d)\) defines the feasibility region for the operating variables \(y\), as
a function of investment decisions $x$ and given realizations of the uncertain parameters $d$, as follows:

$$
\Omega(x, d) = \begin{cases} 
Ax + By = E : \lambda \\
Fx + Gy \leq K : \mu \\
I_{eq}y = d : \alpha \\
I_{ineq}y \leq d : \varphi,
\end{cases}
$$

(4)

where $A, B, E, F, G$ and $K$ are matrices of constant parameters dependent on the network configuration and element characteristics, $I_{eq}$ selects the components of $y$ that are equal to the uncertain parameters (demands), and $I_{ineq}$ selects the components of $y$ that are limited by the uncertain parameters (i.e. maximum power generation and maximum load shedding). The first set of equality constraints correspond to constraints enforcing the power balance at every bus, the power flow through each line, and fixing the voltage angle of the reference bus. The second set of inequality constraints are associated with line flow limits, and limits on the voltage angles at every bus. Note that $\lambda, \mu, \alpha$ and $\varphi$ are the dual variable vectors associated with each set of constraints, respectively.

For a detailed physical interpretation of the mathematical formulation (1)-(3) we recommend the paper by [17].

2.1. Bi-level approach proposed by [15]

[15] proposes dealing with problem (1)-(3) by decomposing and iteratively solving a subproblem and a master problem. The master variables correspond to $x$, i.e. the vector of first stage binary variables. Thus, for given values of these master variables, the subproblem corresponds to:

$$
\text{Maximum } d \in D \quad \text{Minimum } y \in \Omega(x, d) \quad b^T y.
$$

(5)

Considering that the dual problem associated with the third-level is equal to:

$$
\text{Maximize } (E - Ax)^T \lambda - (K - Fx)^T \mu + d^T (\alpha - \varphi)
$$

subject to

$$
B^T \lambda - G^T \mu + I_{eq}^T \alpha - I_{ineq}^T \varphi = b
$$

(7)

$$
\mu \geq 0
$$

(8)

$$
\varphi \geq 0,
$$

(9)
it can be substituted into (5), resulting in the following single level maximization problem:

$$\text{Maximize } f_{\text{dual}} = (E - Ax)^T \lambda - (K - Fx)^T \mu + d^T (\alpha - \varphi)$$  \hspace{1cm} (10)$$

subject to (7), (8), (9), and

$$d \in D.$$  \hspace{1cm} (11)$$

Subproblem (10)-(11) is a bilinear mathematical programming problem, which can be linearized and transformed into a mixed-integer linear mathematical programming problem at the expense of introducing binary variables associated with the uncertainty set (see [15] for more details). This is a simplification consisting in the assumption that the uncertain parameters are either at the nominal, or upper or lower limits of their uncertainty range, as shown by [13] and [15].

The optimal solution of subproblem (10)-(11) provides the dual variable information and the uncertain parameter values to construct Benders cuts for the master problem, which corresponds to the following optimization problem at iteration $k$:

$$\text{Minimize } c^T x + \gamma$$  \hspace{1cm} (12)$$

subject to

$$\gamma \geq (E - Ax)^T \lambda^{(i)} - (K - Fx)^T \mu^{(i)} + d_{(i)}^T (\alpha^{(i)} - \varphi^{(i)}); \forall i < k$$  \hspace{1cm} (13)$$

$$c^T x \leq \Pi$$  \hspace{1cm} (14)$$

$$x \in \{0, 1\}.$$  \hspace{1cm} (15)$$

Since this type of decomposition algorithms are known to have slow convergence properties, [15] proposes adding additional linear constraints obtained from a number of previously computed realizations of the uncertainty vector. These additional cuts accelerate convergence.

2.2. Bi-level approach proposed by [16] and [17]

[16] and [17] also propose dealing with problem (1)-(3) by decomposing and iteratively solving a subproblem and a master problem. The master variables correspond to $x$, i.e. the vector of first stage binary variables, however, they propose a different solution strategy. In particular, they substitute the
third-level minimization problem within subproblem (5) by its KKT conditions, resulting in the following single level maximization problem:

$$\text{Maximize } b^T y$$

subject to

$$Ax + By = E$$  \hspace{1cm} (17)

$$I_{eq} y = d$$  \hspace{1cm} (18)

$$0 = b + B^T \lambda - G^T \mu + I_{eq}^T \alpha - I_{ineq}^T \varphi$$  \hspace{1cm} (19)

$$0 \leq K - Fx - Gy \perp \mu \geq 0$$  \hspace{1cm} (20)

$$0 \leq d - I_{ineq} y \perp \varphi \geq 0$$  \hspace{1cm} (21)

$$d \in D,$$  \hspace{1cm} (22)

where constraint (19) results from differentiating the Lagrangian of the third-level problem with respect to third-level variables $y$, and constraints (20) and (21) represent the complementarity conditions associated with inequality constraints from (4).

Subproblem (16)-(22) is a nonlinear mathematical programming problem, which can be linearized and transformed into a mixed-integer linear mathematical programming problem at the expense of using the Fortuny-Amat [23] transformation, which requires including a binary variable for each inequality constraint. For more details about Fortuny-Amat transformation related to this specific application check reference [17].

The optimal solution of subproblem (16)-(22) provides the uncertain parameter values $d_{(i)}$ to construct primal cuts for the master problem, which corresponds to the following optimization problem at iteration $k$:

$$\text{Minimize } c^T x + \gamma$$

$$\forall i = 1, \ldots, k - 1$$

$$\text{Minimize } x, y_{(i)};$$

$$7$$
subject to

\[
\begin{align*}
\gamma & \geq b^T y(i); \quad \forall i = 1, \ldots, k - 1 \quad (24) \\
\gamma & \geq 0 \quad (25) \\
Ax + By(i) & = E; \quad \forall i = 1, \ldots, k - 1 \quad (26) \\
Fx + Gy(i) & \leq K; \quad \forall i = 1, \ldots, k - 1 \quad (27) \\
I_{eqy(i)} & = d(i); \quad \forall i = 1, \ldots, k - 1 \quad (28) \\
I_{ineqy(i)} & \leq d(i); \quad \forall i = 1, \ldots, k - 1 \quad (29) \\
c^T x & \leq \Pi \quad (30) \\
x & \in \{0, 1\}. \quad (31)
\end{align*}
\]

Note that the master problem, besides variable \(\gamma\), includes one operating variable vector \(y(i)\) for each realization of the uncertain parameters obtained from subproblem (16)-(22). In contrast with respect to the Benders master problem proposed by [15], this algorithm converges in a few iterations without the need to include additional primal cuts besides those included by the iterative process itself.

3. Proposed solution technique

This section justifies and explain in detail the proposed solution technique. However, before focusing on the mathematical description of the solution method, we justify the type of uncertainty set used in this paper.

3.1. Uncertainty set

To deal with uncertainty we use cardinality constrained uncertainty sets [8]. In particular, we use the same definition of uncertainty set than that given by [15] but distributed by type of random parameter (generation capacity \(G\) or demand \(D\)) and per region \(r\):

\[
D = \begin{cases} 
\tilde{d}^G_i = \bar{d}^G_i + \tilde{d}^G_i z^G_i; & \forall i \in \Omega^G \\
\tilde{d}^D_i = \bar{d}^D_i + \tilde{d}^D_i z^D_i; & \forall i \in \Omega^D 
\end{cases}
\]

where \(\tilde{d}^G_i\) is the uncertain generation limit related to the \(i\)th variable within vector \(d\), \(\bar{d}^G_i\) is the corresponding nominal value, \(\tilde{d}^G_i\) is the maximum positive distance from the nominal value that can take the random parameter, and \(\Omega^G\) is the set of indices of the generating units. Analogously, \(\tilde{d}^D_i\), \(\bar{d}^D_i\), \(\tilde{d}^D_i\)
and $\Omega^D$ correspond to the same values but for demands. In addition, the following constraints associated with budget uncertainty must hold:

\begin{align}
\sum_{i \in (\Omega^G \cap \Omega^r)} |z_i^G| & \leq \Gamma_r^G; \quad \forall r, \tag{33} \\
\sum_{i \in (\Omega^D \cap \Omega^r)} |z_i^D| & \leq \Gamma_r^D; \quad \forall r, \tag{34}
\end{align}

where $\Omega^r$ is the set of indices for region $r$, $z_i^G$ and $z_i^D$ are auxiliary continuous variables with the following characteristics $|z_i^G| \leq 1; \forall i \in \Omega^G$ and $|z_i^D| \leq 1; \forall i \in \Omega^D$, and $\Gamma_r^G$ and $\Gamma_r^D$ are the maximum number of random parameters of generation capacity and demand, respectively, which are allowed to reach their lower or upper limits within region $r$. We include the discrimination by region according to the proposal by [17].

It is important to remark that the auxiliary variables $z_i^G$ and $z_i^D$ are initially assumed to be continuous, however, in order to solve the robust TNEP such as in [15], these variables will be considered as binary. The only limitation introduced by this simplification is that uncertainty budgets $\Gamma_r^G$ and $\Gamma_r^D$ must be integer values, which in our opinion does not dwarf the benefits of robust optimization from the practical perspective.

In addition to the simplification proposed by [15], we also consider the fact noticed by [17] that the worst realization of “nature” try to make generation capacities to be as lower as possible and demand loads as higher as possible. According to this fact, our uncertainty sets are finally defined as follows:

\begin{align}
d_i^G &= \bar{d}_i^G - \hat{d}_i^G z_i^G; \quad \forall i \in \Omega^G, \tag{35} \\
d_i^D &= \bar{d}_i^D + \hat{d}_i^D z_i^D; \quad \forall i \in \Omega^D, \tag{36} \\
\sum_{i \in (\Omega^G \cap \Omega^r)} z_i^G & \leq \Gamma_r^G; \quad \forall r, \tag{37} \\
\sum_{i \in (\Omega^D \cap \Omega^r)} z_i^D & \leq \Gamma_r^D; \quad \forall r, \tag{38} \\
z_i^G & \in \{0, 1\}; \forall i \in \Omega^G, \tag{39} \\
z_i^D & \in \{0, 1\}; \forall i \in \Omega^D. \tag{40}
\end{align}

Alternatively, instead of uncertainty budgets associated with generation, demand and regions $\Gamma_r^G$ and $\Gamma_r^D$, a unique uncertainty for each region $\Gamma_r$, or for the system $\Gamma$ could be used instead.
Note that we do not use the uncertainty set defined by [17] because in its actual form does not provide an appropriate interpretation with respect robustness. The definition of the uncertainty set given by [17] is:

\[
\frac{\sum_{i \in \Omega_r} (d_{G-i}^{G_{\text{max}} - d_i^G})}{\sum_{i \in \Omega_r} (d_{G-i}^{G_{\text{max}} - d_{G-i}^{G_{\text{min}}}})} \leq \Gamma_r^G; \forall r \\
\frac{\sum_{i \in \Omega_r} (d_{D-i}^D - d_{D-i}^{D_{\text{min}}})}{\sum_{i \in \Omega_r} (d_{D-i}^{D_{\text{max}} - d_{D-i}^{D_{\text{min}}}})} \leq \Gamma_r^D; \forall r.
\]

This formulation presents the following problem of interpretation. If \( \Gamma_r^G = 1 \) or \( \Gamma_r^D = 1 \), the uncertainty variables associated with generation take the minimum possible values \( d_{G-i}^{G_{\text{min}}} \) and those random variables related to load demand take the maximum possible values \( d_{D-i}^{D_{\text{max}}} \), this case corresponds to the maximum level of uncertainty [2] as pointed out by [17]. However, if \( \Gamma_r^G = 0 \) or \( \Gamma_r^D = 0 \), the uncertainty variables associated with generation take the maximum values possible \( d_{G-i}^{G_{\text{max}}} \) and those random variables related to load demand take the minimum possible values \( d_{D-i}^{D_{\text{min}}} \), this case does not correspond to the no uncertainty case as pointed out by [17], in fact it corresponds to the most favourable case from the system operation perspective, and capacity expansion plannings using this uncertainty budget would probably result in null investments and the resulting network could be congested for all possible values of the random parameters within the uncertainty set. This fact makes difficult to interpret intermediate situations \( 0 < \Gamma_r^G < 1 \) and \( 0 < \Gamma_r^D < 1 \).

### 3.2. Proposed decomposition method

Once the uncertainty set is properly defined, we focus on our methodological proposal for solving problem (1)-(3). It is based on the following observations:

1. The subproblem defined by [15] is much more efficient than that proposed by [16] and [17]. Note that the number of binary variables is equal to the number of uncertain parameters (cardinality of \( \Omega^G \) plus cardinality of \( \Omega^D \)), which are the only variables involved including the dual variables of the third-level problem and the uncertain parameters themselves. In contrast, the subproblem defined by [16] and [17] includes the uncertain parameters, the primal and dual variables of the third-level problem, and the number of binary variables is equal to the
number of inequality constraints. The number of binary variables associated with the maximum generation capacities and upper bounds of load-shedding are just equal to the number of uncertain parameters, and we have to include additional binary variables for line flow limits and voltage angle limits. This results in a bigger and complex subproblem to solve.

2. Besides, the only variables required for the master problem proposed by [16] and [17] are the uncertain parameters. Therefore, the calculation of the primal variables within subproblem is useless and inefficient.

3. The master problem proposed by [16] and [17] based on primal cuts is computationally advantageous with respect to the Benders master problem based on dual cuts proposed by [15]. It converges in a few iterations and does not require including additional cuts artificially.

For the reasons given above, we use as subproblem the one proposed by [15] and as master problem the one proposed by [16] and [17], as follows:

**Subproblem:** It consist of the maximization of the objective function (10) subject to constraints (7), (8), (9) and the uncertainty set definition (35)-(40).

**Master problem:** It consist of the minimization of the objective function (23) subject to constraints (24)-(31).

The only additional detail required for the proper definition of the method is the linealization of the bilinear term included in the objective function of the subproblem, i.e. $d^T(\alpha - \varphi)$. Considering equations (35)-(36), this bilinear term becomes:

$$
    d^T(\alpha - \varphi) = \sum_{\forall i \in \Omega^G} d^G_i \varphi^G_i + \sum_{\forall i \in \Omega^D} e_i d^D_i \varphi^D_i + \sum_{\forall i \in \Omega^D} d^D_i \alpha^D_i \\
    = \sum_{\forall i \in \Omega^G} (\bar{d}^G_i - \hat{d}^G_i z_i^G) \varphi^G_i + \sum_{\forall i \in \Omega^D} e_i (\bar{d}^D_i + \hat{d}^D_i z_i^D) \varphi^D_i \\
    + \sum_{\forall i \in \Omega^D} (\bar{d}^D_i + \hat{d}^D_i z_i^D) \alpha^D_i = \sum_{\forall i \in \Omega^G} (\bar{d}^G_i \varphi^G_i - \hat{d}^G_i z_i^G \varphi^G_i) \\
    + \sum_{\forall i \in \Omega^D} e_i (\bar{d}^D_i \varphi^D_i + \hat{d}^D_i z_i^D \varphi^D_i) + \sum_{\forall i \in \Omega^D} (\bar{d}^D_i \alpha^D_i + \hat{d}^D_i z_i^D \alpha^D_i).
$$

(43)

Note that for consistency, the negative sign on the compact expression for $\varphi$ is implicitly included in its components $\varphi^G_i$ and $\varphi^D_i$, which are defined
as negative in the detailed formulation given in the Appendix A. $e_i$ is the percentage of load shed by demand $i$.

The terms to be linearized are $z_i^G \varphi_i^G, \forall i \in \Omega^G$, $z_i^D \alpha_i^D, \forall i \in \Omega^D$, which can be replaced by the following continuous variables $z_i^G$, $z_i^D$, and $z_i^D$, respectively, iff the following set of constraints is included:

$$\varphi_i^G \leq z_i^G \leq \varphi_i^G, \forall i \in \Omega^G$$

$$\varphi_i^D \leq z_i^D \leq \varphi_i^D, \forall i \in \Omega^D$$

$$\alpha_i^D \leq z_i^D \leq \alpha_i^D, \forall i \in \Omega^D$$

where parameters $\varphi_i^G, \varphi_i^G$, $\varphi_i^D, \varphi_i^D$, $\alpha_i^D, \alpha_i^D$ are lower and upper limits of the dual variables, which can be replaced by $-M$ and $M$, respectively, being $M$ a positive and large enough constant. Note that in comparison with the subproblem proposed by [15], we take advantage of the fact that we know in advance what uncertain variables tend, respectively, to the upper and lower limit of the uncertainty set, which allows reducing the number of variables by half.

Finally, the proposed iterative scheme is described step by step on the following algorithm:

**Algorithm 3.1. (Iterative method).**

**Input:** Selection of uncertainty budgets $\Gamma^G_r$ and $\Gamma^D_r$ for each region and the tolerance of the process $\varepsilon$. These data are selected by the decision maker.

**Step 1: Initialization.** Initialize the iteration counter to $\nu = 1$, and upper and lower bounds of the objective function $z^{(up)} = \infty$ and $z^{(lo)} = -\infty$.

**Step 2: Solving the master problem at iteration $\nu$.** Solve the master problem (23) subject to constraints (24)-(31). The result provides values of the decision variables $x^{(\nu)}$ and $\gamma^{(\nu)}$. Update the optimal objective function lower bound $z^{(lo)} = c^T x^{(\nu)} + \gamma^{(\nu)}$. Note that at the first iteration the optimal solution corresponds to the no investment case, alternatively, we could start with any other vector of decision variables.
Step 3: Solving subproblem at iteration $\nu$. For given values of the decision variables $x(\nu)$, we calculate the worst operating costs within the uncertainty set $f_{\text{dual}}^{(\nu)}$, obtaining also the corresponding uncertain parameters $d(\nu)$. This is achieved by solving problem (10) subject to constraints (7), (8), (9) and the uncertainty set definition (35)-(40). Update the optimal objective function upper bound $z^{(\text{up})} = c^T x_{\nu} + f_{\text{dual}}^{(\nu)}$.

Step 4: Convergence checking. If $\left( z^{(\text{up})} - z^{(\text{lo})} \right) / z^{(\text{up})} \leq \varepsilon$ go to Step 5, else update the iteration counter $\nu \rightarrow \nu + 1$ and continue in Step 2.

Step 5: Output. The solution for a given tolerance corresponds to $x^* = x(\nu)$.

4. Numerical case studies

In this section, we present numerical experiments of our model proposal and its comparison with the method proposed by [16] and [17]. We use an illustrative example i.e. the Garver system [24], and two realistic case studies: the IEEE 24-bus system [25] and the IEEE 118-bus test system [26]. It is important to point out that in this paper we just focus on numerical performance. The interpretation of robust solutions is out of the scope of this work since it is studied in detail in references [15], [16] and [17].

All examples have been implemented and solved using GAMS [27, 28] and CPLEX 12, on a PC with four processors clocking at 2.39 GHz and 3.2 GB of RAM memory. Note that computing times reported correspond to the sum of running times used by the solvers in order to solve the masters and subproblems until the final solution is attained. We use the same tolerance for all problems and equal to $\varepsilon = 10^{-6}$.

4.1. Illustrative example. Garver system

The proposed model is illustrated with the Garver 6-bus system, depicted in Figure 1. This system comprises 6 buses, 3 generators, 5 inelastic demands and 6 lines. Nominal values of generation capacities and demands and their offering and nominal costs can be found in Table 1. The load-shedding cost is equal to the nominal cost of each demand. It is considered that a maximum of three lines can be installed between each pair of buses. Line data are
obtained from Table I of [11] including construction costs, and the maximum available investment budget is €40 million.

The investment return period of each line is considered to be 25 years, and the discount rate is 10%, resulting in an annual amortization rate of 10%. Since investment cost is annualized, the weighted factor $\sigma$ is equal to the number of hours of a year, i.e. 8760, obtaining an annualized load-shedding and power generation cost.

Regarding the uncertainty set, power generation capacity can increase or decrease a maximum of 50% of their nominal values, while demand are allowed to change a maximum of 20%.

We have solved the robust TNEP problem for three different combinations of uncertainty budgets associated with generation capacities and demands. We have used both the method proposed by [16] and [17] and the method proposed in this paper. In order to obtain statistically sound conclusions about computing times, we have solved the problem 100 times and obtain mean and standard deviation executions times. The solution times from this numerical experiments are given in Table 2.

From Table 2 the following observations are pertinent:

1. The proposed method is computationally faster in average than the
method proposed by [16] and [17] for all cases, from the first and worst possible case corresponding to Soyster’s solution [2] up to the deterministic case when uncertainty budgets are null. Regarding the standard deviation of computing times the values are comparable. Computational time is reduced in about 30%.

2. Both methods provide the same solution and converge in the same number of iterations. Note that if the uncertainty budgets are integer both approaches are equivalent because the master problem is the same and the subproblems are equivalents.

3. The maximum number of iterations required is four.

The difference in computing time is associated with the solution of the subproblem. The one proposed by [16] and [17] has 517 equations, 460 continuous and 161 binary variables, while the subproblem proposed in this paper contains 119 equations, 195 continuous and 8 binary variables. Even though we are dealing with a small example, the difference in complexity between both subproblems is considerable.

4.2. IEEE 24-bus Reliability Test System

The following case study is based on the IEEE 24-bus Reliability Test System (RTS) [25], depicted in Fig. 2. The system comprises 24 buses, 34 existing corridors which can accept a maximum of three equal lines and 7 new corridors, 10 generating units and 17 loads. Data for lines in existing corridors are taken from [25], while line data for new corridors are obtained.
Table 2: Computational results for Garver’s 6-bus example

| Γ^G | Γ^D | Optimal sol. (M€) | δ iter. | Mean (s) | Std. (s) | Mean (s) | Std. (s) |
|-----|-----|------------------|---------|----------|----------|----------|----------|
| 3   | 5   | 35505.31         | 3       | 0.973    | 0.048    | 0.644    | 0.054    |
| 2   | 3   | 25832.22         | 4       | 1.339    | 0.106    | 0.936    | 0.094    |
| 1   | 2   | 5861.92          | 4       | 1.605    | 0.095    | 1.011    | 0.072    |
| 0   | 0   | 440.07           | 2       | 0.539    | 0.071    | 0.428    | 0.065    |

from Table I in [29]. Investment costs are €20 million, and analogously to the Garver illustrative example, the investment return period of each line is considered to be 25 years, and the discount rate is 10%, resulting in an annual amortization rate of 10%. Table 3 provides the location of generators and demands in the network, the maximum power and the corresponding costs for generating units, and the load and the nominal load-shedding cost for demands. The load-shedding cost is equal to 100 times the nominal value of each demand.

Regarding the uncertainty set, power generation capacity can increase or decrease a maximum of 50% of their nominal values, while demand are allowed to change a maximum of 20%.

We have solved the same computational experiment than in the previous illustrative example, also comparing with the method proposed by [16] and [17]. The solution times from these numerical experiments is given in Table 4.

From Table 4 we can extract the same conclusions than in the previous illustrative example:

1. The proposed method is considerably faster in average computing time than the method proposed by [16] and [17] for all cases, from the worst possible case corresponding to Soyster’s solution [2] up to the deterministic case, being the latter the one with lowest computing times. The reason is that the values of the binary variables of the subproblems are known in advance and only 2 iterations are required. Average computational time is reduced in about 235 times.
2. The maximum number of iterations required is three.

The difference in computing time is associated with the solution of the subproblem. The one proposed by [16] and [17] has 1604 equations, 1507 continuous and 553 binary variables, while the subproblem proposed in this paper contains 370 equations, 583 continuous and 27 binary variables. The difference in complexity between both subproblems is considerable.

4.3. IEEE 118-bus test system

Finally, we run additional computational tests using the IEEE 118-bus test system [26]. The system comprises 118 buses, 186 existing lines, 54 generating units and 91 loads. In addition, it is possible to construct up to 61 additional lines to duplicate each one of the following existing lines: 8, 12, 23, 32, 38, 41, 51, 68, 78, 96, 104, 118, 119, 121, 125, 129, 134, 159, 7,
### Table 3: Generator and demand data for IEEE 24 RTS case study

| Bus | Power size (MW) | Cost (€/MWh) | Load (MW) | NLS cost (€/MWh) |
|-----|----------------|--------------|-----------|------------------|
| 1   | 230            | 95           | 259       | 99               |
| 2   | 230            | 96           | 233       | 98               |
| 3   | –              | –            | 432       | 100              |
| 4   | –              | –            | 178       | 99               |
| 5   | –              | –            | 171       | 100              |
| 6   | –              | –            | 326       | 99               |
| 7   | 360            | 96           | 300       | 100              |
| 8   | –              | –            | 411       | 93               |
| 9   | –              | –            | 420       | 99               |
| 10  | –              | –            | 468       | 100              |
| 13  | 709            | 80           | 636       | 92               |
| 14  | –              | –            | 466       | 90               |
| 15  | 258            | 82           | 761       | 87               |
| 16  | 186            | 77           | 240       | 84               |
| 18  | 480            | 73           | 799       | 91               |
| 19  | –              | –            | 435       | 94               |
| 20  | –              | –            | 307       | 95               |
| 21  | 480            | 74           | –         | –                |
| 22  | 360            | 79           | –         | –                |
| 23  | 792            | 78           | –         | –                |
| 24  | –              | –            | –         | –                |

NLS: Nominal load-shedding

9, 36, 117, 71, 131, 133, 147, 103, 65, 144, 168, 4, 13, 132, 69, 66, 67, 5, 89, 29, 167, 145, 70, 42, 90, 16, 174, 98, 99, 185, 93, 94, 128, 164, 97, 153, 146, 116, 163, 31, 92, 130. Data for lines in existing corridors are taken from [26]. Investment costs are €100 million, and analogously to both previous examples, the investment return period of each line is considered to be 25 years, and the discount rate is 10%, resulting in an annual amortization rate of 10%.

Table B.6 in Appendix B provides the location of generators and demands in the network, the maximum power and the corresponding cost for generating units, and the load and nominal load-shedding cost for demands. The load-shedding cost is equal to 1.2 times the nominal load-shedding cost.
of each demand.

Regarding the uncertainty set, power generation capacity can increase or decrease a maximum of 50% of their nominal values, while demand are allowed to change a maximum of 20%.

We have solved the same computational experiment than in the previous illustrative example, also comparing with the method proposed by [16] and [17]. The solution times from this numerical experiments is given in Table 5.

| Table 4: Computational results for IEEE 24 RTS case study | [16] and [17] method | Proposed method |
|----------------------------------------------------------|-----------------------|-----------------|
| [173x649] | | |
| | Optimal sol. (M€) | [16] and [17] method | Proposed method |
| | # iter. | Mean (s) | Std. (s) | Mean (s) | Std. (s) |
| Γ^G = 10 | 500752.14 | 3 | 165.64 | 46.63 | 1.51 | 0.30 |
| Γ^D = 17 | | | | | | |
| Γ^G = 7 | 454917.47 | 3 | 308.20 | 75.97 | 1.80 | 0.39 |
| Γ^D = 12 | | | | | | |
| Γ^G = 3 | 348941.40 | 3 | 518.75 | 1.15 | 1.12 | 0.11 |
| Γ^D = 5 | | | | | | |
| Γ^G = 0 | 219161.52 | 2 | 144.29 | 0.17 | 0.414 | 0.08 |
| Γ^D = 0 | | | | | | |

From Table 5 the following observations are pertinent:
1. The proposed method is extremely faster in average computing time than the method proposed by [16] and [17] for all cases, in fact the latter method does not reach the optimal solution within the time limit of 36000 s (10 hours) imposed. Note that if any of the solvers (for master or subproblems) does not find an optimal solution within that time the process is stopped. In contrast, the proposed method reached the optimal solution in less than one minute for all instances of budget uncertainties.

2. The maximum number of iterations required is 5.

The difference in computing time is associated with the solution of the subproblem. The one proposed by [16] and [17] has 5168 equations, 5491 continuous and 2071 binary variables, while the subproblem proposed in this paper contains 1548 equations, 1857 continuous and 145 binary variables. The difference in complexity between both subproblems is apparent and explains the large difference in computing time between both methods.

4.4. Discussion

Numerical experiments demonstrate that the proposed method is computationally more efficient than that proposed by [16] and [17]. Nevertheless, this result is evident considering the difference in complexity of the subproblem. Note that [17] already claimed that the most time consuming stage of their process was the subproblem resolution, and our subproblems make a difference in terms of saving computational time while master problems are exactly the same.

We have not reported results with respect to [15] method. Nevertheless, it is also evident that our method is more efficient because we use the same subproblem but reducing binary variables to a half. This difference might imply large computing time differences for large problems. In fact, the larger the problem, the larger the difference according to the functioning of standard branch-and-cut solvers. The only doubt about if our method is faster might come from the master problem, however, the number of iterations reported in this paper and in the works by [16] and [17] are always lower than or equal to 5, and the number of iterations reported by [15] varies considerably depending if additional cuts are included or not but they are usually above 5 or more iterations. Besides, this strategy of including additional cuts could be reproduced using the primal master problem of our algorithm by just considering additional realizations of random parameters like [15] does in his work. Nevertheless, the convergence behavior does not require it.
5. Conclusions

This paper proposes a new decomposition algorithm to attain the exact solution of the TNEP problem derived from using a two-stage adaptive robust strategy. Although there is nothing novel with respect the formulation of the problem, we manage to combine formulations and findings from different authors in the proper manner so that our method beats in computational efficiency, same cases by large, prior methodologies.

We have run several computational experiments to show the functioning of the proposed algorithm with respect to the approach proposed by [16] and [17]. Nevertheless, it has been demonstrated that the computational time required by this algorithm is always lower than those proposed by [15], [16] and [17]. The subproblem proposed in this paper has lower complexity and number of binary variables than the others, while the primal master problem is equal to the one proposed by [16] and [17], which is computationally advantageous with respect to the Benders master problem used by [15], and besides, it is not required to include additional cuts from given realizations of the random parameters.

Bearing in mind that the proposed method keeps all features about robust optimization design with respect to their predecessors, the proposed algorithm turns out to be the most efficient method to solve the robust transmission network expansion planning to date, and takes advantage of state-of-the-art mixed-integer mathematical programming solvers. The only limitation is the integrality constraint of budget uncertainty, which simplifies the subproblem considerably and does not dwarf the benefit of using robust optimization.

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Appendix A. Detailed formulation of the TNEP problem

Before describing the detailed formulation, the main notation used and not defined previously is stated below for quick reference.
Constants:

- $b_k$ Susceptance of line $k$ (S).
- $c_i^G$ Generation cost for generator $i$ (€/MWh).
- $c_j^U$ Load-shedding cost for consumer $j$ (€/MWh).
- $c_k$ Investment cost of building line $k$ (€).
- $e_j$ Percentage of load shed by the $j$-th demand.
- $f_k^{\text{max}}$ Capacity of line $k$ (MW).
- $o(k)$ Sending-end bus of line $k$.
- $r(k)$ Receiving-end bus of line $k$.

Primal variables:

- $d_j$ Power consumed by the $j$-th demand (MW).
- $f_k$ Power flow through line $k$ (MW).
- $g_i$ Power produced by the $i$-th generating unit (MW).
- $r_j$ Load shed by the $j$-th demand (MW).
- $x_k$ Binary variable that is equal to 1 if line $k$ is built and 0 otherwise.
- $\theta_s$ Voltage angle at bus $s$ (radians).

Dual variables: Note that dual variables are provided after the corresponding equalities or inequalities separated by a colon.

Indices and Sets:

- $s(i)$ Bus index where the $i$-th generating unit is located.
- $s(j)$ Bus index where the $j$-th demand is located.
- $\Psi^D_s$ Set of indices of the demands located at bus $s$.
- $\Psi^G_s$ Set of indices of the generating units located at bus $s$. 

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$\Omega^D$ Set of indices of the demands.

$\Omega^G$ Set of indices of the generating units.

$\Omega^L$ Set of all transmission lines, prospective and existing.

$\Omega^{L+}$ Set of all prospective transmission lines.

$\Omega^N$ Set of all networks buses.

**TNEP detailed problem:**

The decision making problem pertaining to a transmission planner minimizes simultaneously network investment, generation costs and cost of load-shedding. Problem (1)-(3) for given values of the random parameters $d$ is as follows:

Minimize $x, y$ $R \left( \sum_{k \in \Omega^{L+}} c_k x_k \right) + \sigma \left( \sum_{i \in \Omega^G} c_i^G g_i + \sum_{j \in \Omega^D} c_j^U r_j \right)$ \hspace{1cm} (A.1)

subject to

\begin{align*}
x_k &= 1; \forall k \in \Omega^L \setminus \Omega^{L+} \hspace{1cm} (A.2) \\
x_k &\in \{0, 1\}; \forall k \in \Omega^L \hspace{1cm} (A.3) \\
\sum_{k \in \Omega^{L+}} c_k x_k &\leq \Pi \hspace{1cm} (A.4) \\
\sum_{i \in \Psi^G} g_i - \sum_{k | o(k) = s} f_k + \sum_{k | r(k) = s} f_k + \sum_{j \in \Psi^D} r_j &= \sum_{j \in \Psi^D} d_j : \lambda_s; \forall s \in \Omega^N \hspace{1cm} (A.5) \\
f_k &= b_k x_k (\theta_{o(k)} - \theta_{r(k)}) : \phi_k; \forall k \in \Omega^L \hspace{1cm} (A.6) \\
\theta_s &= 0 : \chi_s; s : \text{slack} \hspace{1cm} (A.7) \end{align*}
\begin{align*}
  f_k & \leq f_k^{\max} : \phi_k^{\max} ; \quad \forall k \in \Omega^L \quad (A.9) \\
  f_k & \geq -f_k^{\max} : \phi_k^{\min} ; \quad \forall k \in \Omega^L \quad (A.10) \\
  \theta_s & \leq \pi : \xi_s^{\max} ; \quad \forall s \in \Omega^N \setminus \text{slack} \quad (A.11) \\
  \theta_s & \geq -\pi : \xi_s^{\min} ; \quad \forall s \in \Omega^N \setminus \text{slack} \quad (A.12) \\
  g_i & \geq 0 ; \quad \forall i \in \Omega^G \quad (A.13) \\
  r_j & \geq 0 ; \quad \forall j \in \Omega^D \quad (A.14) \\
  d_j & = d_j^D : \alpha_j^D ; \quad \forall j \in \Omega^D \quad (A.15) \\
  g_i & \leq d_i^G : \varphi_i^G ; \quad \forall i \in \Omega^G \quad (A.16) \\
  r_j & \leq e_j d_j^D : \varphi_j^D ; \quad \forall j \in \Omega^D . \quad (A.17)
\end{align*}

The generation and load-shedding costs are multiplied by the weighting factor \( \sigma \) to make the annual investment cost and the generation and load-shedding cost comparable quantities. Investment costs in (A.1) are multiplied by the capital recovery factor \( R \), calculated as \( \frac{r(1+r)^n}{(1+r)^n - 1} \), where \( r \) is the interest rate and \( n \) is the number of considered years. Note that dual variables are provided after the corresponding equalities or inequalities separated by a colon. Constraint (A.4) enforces an upper bound on the investment cost and corresponds to (2). Constraints (A.5) enforce the power balance at every bus. Constraints (A.6) represent the power flow through each line. Each of these constraints is multiplied by a binary variable, thus, if the corresponding line is not physically connected to the network the power flow through it is zero. Equation (A.7) fixes the voltage angle of the reference bus. Note that (A.5)-(A.7) corresponds to the linear constraint set associated with \( \Omega(x, d) \) in the first row of (4).

Constraints (A.9)-(A.10) enforce the line flow limits. Constraints (A.11)-(A.12) impose limits on the voltage angles at every bus, and (A.13)-(A.14) ensures positiveness of power generation and load-shedding. All these inequality constraints corresponds to the inequality set related to \( \Omega(x, d) \) in the second row of (4).

Constraints (A.15) force demands to be equal to the uncertain demand variable, and corresponds to the equality constraint set related to \( \Omega(x, d) \) in the third row of (4). And finally, (A.16) and (A.17) limit power generation and load-shedding to be lower than the uncertain generation capacity and a percentage of uncertain demand, respectively. These last two sets of constraints correspond to the inequality constraint set related to \( \Omega(x, d) \) in the
fourth row of (4).

**Detailed subproblem:**

For the sake of completeness, we also provide the detailed formulation of the subproblem constituted by the objective function (10) subject to constraints (7), (8), (9) and the uncertainty set definition (35)-(40). It is as follows:

Maximize

\[
\begin{array}{l}
\sum_{k \in \Omega^L} \left( \phi_k^{\max} - \phi_k^{\min} \right) f_k^{\max} + \sum_{s \in \Omega^N \setminus s: \text{slack}} \pi \left( \xi_s^{\max} - \xi_s^{\min} \right) \\
+ \sum_{i \in \Omega^G} \varphi_i G_i^{D} + \sum_{j \in \Omega^D} \alpha_j D_j^{D} + \sum_{j \in \Omega^D} \left( e_j D_j^{D} \right)
\end{array}
\]

subject to:

\[
\begin{align*}
\lambda_{s(i)} + \varphi_i^G & \leq c_i^G \sigma; \quad \forall i \in \Omega^G & (A.19) \\
-\lambda_{s(j)} + \alpha_j^D & \leq 0; \quad \forall j \in \Omega^D & (A.20) \\
\lambda_{s(j)} + \varphi_i^D & \leq c_j^D \sigma; \quad \forall j \in \Omega^D & (A.21) \\
-\lambda_{o(k)} + \lambda_{r(k)} + \phi_k + \phi_k^{\max} + \phi_k^{\min} & = 0; \quad \forall k \in \Omega^L & (A.22) \\
- \sum_{k | o(k)=s} b_k x_k \phi_k + \sum_{k | r(k)=s} b_k x_k \phi_k & + \xi_s^{\max} + \xi_s^{\min} = 0 \quad \forall s \in \Omega^N \setminus s: \text{slack} & (A.23) \\
- \sum_{k | o(k)=s} b_k x_k \phi_k + \sum_{k | r(k)=s} b_k x_k \phi_k & + \chi_s = 0 \quad s: \text{slack} & (A.24) \\
-\infty & \leq \lambda_s \leq \infty; \quad \forall s \in \Omega^N & (A.25) \\
-\infty & \leq \phi_k \leq \infty; \quad \forall k \in \Omega^L & (A.26) \\
-\infty & \leq \chi_s \leq \infty; \quad s: \text{slack} & (A.27) \\
\phi_k^{\max} & \leq 0; \quad \forall k \in \Omega^L & (A.28) \\
\phi_k^{\min} & \geq 0; \quad \forall k \in \Omega^L & (A.29) \\
\xi_s^{\max} & \leq 0; \quad \forall s \in \Omega^N \setminus s: \text{slack} & (A.30) \\
\xi_s^{\min} & \geq 0; \quad \forall s \in \Omega^N \setminus s: \text{slack} & (A.31) \\
-\infty & \leq \alpha_j^D \leq \infty; \quad \forall j \in \Omega^D & (A.32) \\
\varphi_i^G & \leq 0; \quad \forall i \in \Omega^G & (A.33) \\
\varphi_i^D & \leq 0; \quad \forall i \in \Omega^D. & (A.34)
\end{align*}
\]
Note that in order to transform the non-linear problem into a mixed-integer linear mathematical programming problem, bilinear terms present in the second row of objective function (A.18) must be replaced by (43) and constraints (44)-(49) included in the formulation above.

The correspondence of dual variables between the compact and detailed formulation is as follows:

\[
\begin{align*}
\lambda &= (\lambda_s; \forall s, \phi_k; \forall k, \chi_s)^T \quad (A.35) \\
\mu &= (\phi_k^{\text{max}}; \forall k, \phi_k^{\text{min}}; \forall k, \xi_s^{\text{max}}; \forall s, \xi_s^{\text{min}}; \forall s)^T \quad (A.36) \\
\alpha &= (\alpha_j^D; \forall j)^T \quad (A.37) \\
\phi &= -(\phi_i^G; \forall i, \phi_j^D; \forall j)^T. \quad (A.38)
\end{align*}
\]

Appendix B. Generation and load data for the IEEE-118 test system

Table B.6 provides the location of generators and demands in the network, the maximum power and the corresponding cost for generating units, and the load and nominal load-shedding cost for demands.

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Table B.6: Generator and demand data for IEEE 118 case study

| Bus | Generators | Demands | Generators | Demands | Generators | Demands | Generators | Demands |
|-----|------------|---------|------------|---------|------------|---------|------------|---------|
|     | Pow       | Cost    | Load       | NLSC    | Pow       | Cost    | Load       | NLSC    | Pow       | Cost    | Load       | NLSC    |
| 1   | 689        | 50      | 31         | 150     | 27        | 581     | 50         | 61      | 1000      | 14      | 23         | 91      |
| 2   | 270        | 50      | 32         | 500     | 18        | 797     | 50         | 62      | 500       | 18      | 1040       | 50      | 92         | 1500    |
| 3   | 327        | 50      | 33         | –       | 311       | 50      | 64         | –       | –         | 93      | –          | 162     |
| 4   | 150        | 27      | 405        | 50      | 34        | 150     | 27        | 797     | 50      | 64         | –       | 94         | 405     |
| 5   | –          | –       | –          | –       | 35        | –       | 446        | 50      | 65         | 2100    | 10        | –       | 95      |
| 6   | 150        | 27      | 702        | 50      | 36        | 500     | 18        | 419     | 50      | 66         | 2100    | 10        | 527     | 50      | 96      |
| 7   | –          | –       | 257        | 50      | 37        | –       | –          | –       | 67        | –       | 378        | 50      | 97      |
| 8   | 150        | 27      | –          | –       | 38        | –       | –          | –       | 68        | –       | –          | 98      |
| 9   | 1500       | 14      | 39         | –       | 365       | 50      | 69         | 1500    | 14      | –         | –       | 459      |
| 10  | 1500       | 14      | 40         | 150     | 27        | 270     | 50         | 70      | 400       | 17      | 891        | 50      | 199       |
| 11  | –          | 445     | 50         | 41      | –         | 696     | 50         | 77      | –         | 104     |
| 12  | 1500       | 14      | 42         | 150     | 27        | 500     | 50         | 72      | 150       | 27      | –         | 102     |
| 13  | –          | 459      | 50         | 43      | –         | 243     | 50         | 73      | 150       | 27      | –         | 103     |
| 14  | –          | 189      | 50         | 44      | –         | 216     | 50         | 74      | 100       | 38      | 918       |
| 15  | 150        | 27      | 1216       | 50      | 45        | –       | 716        | 50      | 75        | –       | 635       |
| 16  | –          | 338      | 50         | 46500    | 18        | 378     | 50         | 76      | 500       | 18      | 918       |
| 17  | –          | 129      | 50         | 47      | –         | 459     | 50         | 77      | 500       | 18      | 824       |
| 18  | 500        | 18      | 810        | 50      | 48        | –       | 270        | 50      | 78        | –       | 959       |
| 19  | 150        | 27      | 608        | 50      | 49        | 1250    | 13        | 1175    | 50      | 79        | –       | 527       |
| 20  | –          | 233      | 50         | 50      | –         | 230     | 50         | 80      | 1500      | 14      | 1755      |
| 21  | –          | 189      | 50         | 51      | –         | 230     | 50         | 81      | –         | –       | 114       |
| 22  | –          | 135      | 50         | 52      | –         | 243     | 50         | 82      | 500       | 18      | 729       |
| 23  | 35         | 50      | 35         | –       | 311       | 50      | 84         | –       | –         | 279     |
| 24  | 150        | 27      | –          | –       | 54        | 1250    | 13        | 1526    | 50      | 84        | –       | 149       |
| 25  | 1500       | 14      | –          | –       | 55        | 500     | 18        | 851     | 50      | 85        |
| 26  | 1500       | 14      | –          | –       | 150       | 27        | 324       | 50      | 115       | –       | 297       |
| 27  | 1500       | 14      | –          | –       | 150       | 27        | –         | –       | 284       | 50      | 110       |
| 28  | –          | 230      | 50         | 58      | –         | 162     | 50         | 88      | –         | 648     |
| 29  | –          | 324      | 50         | 591000   | 14        | 3739    | 50         | 89      | 1000      | 14      | –         |
| 30  | 60         | 459000   | 1053      | 50      | –         | –       | –          | –       | –         | –       | –         |

Offer (MW), bid (MW), price (€/MWh), NLSC: Nominal Load-Shedding Cost

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