The electric charge at the two-loop level in the Standard Model and the precision measurements of $\Delta \alpha_{\text{had}}$

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The complete calculation of the 2-loop electroweak corrections to the renormalization of the electric charge in the Standard Model allows to discuss in detail the value of the $\overline{\text{MS}}$ effective coupling $\hat{e}^2(m_Z^2)$. We discuss the phenomenological impact of these results in view of the increasing accuracy in the measurements of the cross-section of the reaction $e^+e^- \rightarrow \text{hadrons}$ at low energies.

1. Introduction

The precision tests of the Standard Model (SM) have reached in the past years an incredibly high level of accuracy: several observables are measured at the per mille level and two key quantities (the $W$ boson mass $m_W$ and the sinus of the effective weak mixing angle $\sin^2 \theta_{\text{lep}}^{\text{eff}}$) are even known with an error of few parts in $10^{-4}$. The prospects for the measurements of these two latter observables at future colliders (Tevatron Run IIb, LHC, TESLA) will further improve the present situation [1]. In order to have a sensible comparison between the data and the theoretical prediction, the complete calculation of the 2-loop electroweak (EW) corrections to all relevant observables is a mandatory step.

The input parameters in the gauge sector of the SM lagrangian are chosen in order to minimize the parametric error that they induce on any theoretical prediction and are typically the three best measured quantities ($\alpha, G_{\mu}, m_Z$), the fine structure constant, the Fermi constant derived from the muon decay and the mass of the $Z$ boson; in particular $\alpha$ is known to the impressive accuracy of 3.7 parts per billion [2]. On the other hand, the effective coupling which appears in the expression of any scattering amplitude is the running electromagnetic coupling $\alpha(s)$, but unfortunately this quantity is not so well known as $\alpha(0)$ for two different reasons: i) the presence of non-perturbative hadronic contributions, usually indicated with $\Delta \alpha_{\text{had}}$, which affect the running and ii) the error due to missing higher-order perturbative corrections.

The precise value of $\Delta \alpha_{\text{had}}$ is directly related, via a dispersion relation, to the measurements of the total cross-section of the reaction $e^+e^- \rightarrow \text{hadrons}$ [3]. The latter are reaching a very high level of accuracy and make questionable if the perturbative contributions to the running of $\alpha$ are fully under control.

Recently the 2-loop renormalization of the electric charge in the EW Standard Model has been calculated (we refer to [4] for all technical details) and using these results the running of the $\overline{\text{MS}}$ electric charge has been studied. We review in the present paper the most relevant features of that calculation and discuss its phenomenological consequences, in view of a precision measurement of $\Delta \alpha_{\text{had}}$.

2. The electric charge renormalization

In pure QED the natural definition of an effective QED coupling at the scale $\sqrt{s}$

$$\alpha(s) = \frac{\alpha}{1 - \Delta \alpha(s)}$$

and

$$\Delta \alpha(s) = 4\pi a \text{Re} [\Pi_{\gamma\gamma}(s) - \Pi_{\gamma\gamma}(0)],$$

is given in terms of the photon vacuum polarization function evaluated at different scales.

In the full SM, the bosonic contribution to the photon vacuum polarization at high momentum transfer is, in general, not gauge-invariant. Thus it cannot be included in a sensible way in Eq.(1). Although, Eq.(1) with only the fermionic contri-
bution included is a good effective coupling at the \( m_Z^2 \) scale, it is clear that at the energy scales tested by the future accelerators an effective QED coupling will have to take into account also the bosonic contributions.

A different definition of a QED effective coupling can be obtained by considering the \( \overline{\text{MS}} \) QED coupling constant at the scale \( \mu \) defined by

\[
\hat{\alpha}(\mu) = \frac{\alpha}{1 + 4\pi \alpha \Pi_{\gamma\gamma}(0)}.
\]

Eq. (3) is expressed in terms of the on-shell electric charge counterterm which is a gauge-invariant quantity and includes bosonic and fermionic contributions. In the Background Field Method (BFM), as it will be explained in the rest of the paper, the electric charge counterterm is expressed in terms of the \( q^2 = 0 \) photon vacuum polarization function.

The electric charge renormalization has been discussed at the one-loop level in [5,6]; at the two-loop level it has been studied in several papers in relation to the \( m_W - m_z \) interdependence [7]; in the framework of the BFM explicit results have been presented in [4].

The electric charge is usually defined in terms of the Thomson scattering amplitude, i.e. the process of emission of a real photon off a fermion, with vanishingly small energy. The on-shell electric charge counterterm is defined as to cancel, order by order, all divergences and all finite corrections which arise through virtual corrections. The classes of virtual diagrams which contribute to this amplitude are schematically depicted in Fig. 1. In QED the Ward-Takahashi identity (WI) implies a cancelation between diagrams (a), (b) and (c), and therefore the relation between the bare and the renormalized charge is given, via Dyson summation, by

\[
e^2 = e_0^2/(1 - e_0^2 \Pi_{\gamma\gamma}^{(f)}(0)),
\]

where \( \Pi_{\gamma\gamma}^{(f)}(0) \) is related to the transverse part \( A_{\gamma\gamma} \) of the photon self-energy via \( A_{\gamma\gamma}(q^2) = e_0^2 q^2 \Pi_{\gamma\gamma}(q^2) \) and \( f \) indicates the fermionic contributions.

The QED WI does not hold in the SM quantized in the \( 't \) Hooft-Feynman gauge, because of a non-trivial relation between the bosonic self-energies and the vertex corrections. Both groups of diagrams are separately gauge dependent and only their combination yields a gauge invariant expression, as expected for the electric charge counterterm [5]. Due to the gauge dependence, the Dyson summation of the bosonic self-energies is not manifestly evident in a renormalizable \( R_\xi \) gauge. At the 2-loop level, the calculation of the Thomson scattering amplitude in the \( 't \) Hooft-Feynman gauge is difficult, due to the presence of irreducible vertex corrections and to non-trivial cancellations between the self-energy contributions.

The BFM has been introduced in [8] as an alternative approach to quantize a gauge field theory. The fields of the classical lagrangian are splitted into two component \( \mathcal{L}(\phi) \rightarrow \mathcal{L}(\phi + \phi) \) where the symbol \( \phi \) indicates all classical fields, while \( \phi \) represents the quantum fluctuations, i.e. the integration variables of the path integral. A gauge-fixing term is introduced to break only the gauge invariance of the quantum-fields \( \phi \). The effective action remains invariant under gauge transformations of the background fields and, as a consequence, the Green’s functions with external background fields satisfy simple WIs. A convenient choice of the gauge-fixing can be exploited to derive simple QED-like WI, now in the full electroweak SM.
It has been shown \[9\] that the following relations hold to all orders in perturbation theory, for arbitrary values of the gauge parameter.

\[
q^\mu \Gamma^\gamma_{\mu} (q, \bar{p}, p) = -e Q_f \left[ \Sigma_f (\bar{p}) - \Sigma_f (-p) \right] \tag{4}
\]

\[
B_{\gamma\gamma}(0) = 0 \tag{5}
\]

\[
B_{\gamma Z}(0) = 0 \tag{6}
\]

where \(\Gamma^\gamma_{\mu} \) is the three-point function \(\gamma \bar{f} f\), \(\Sigma_f\) is the two-point function of the fermion, \(q = \bar{p} + p\) the photon momentum and \(Q_f\) is the charge of the fermion \(f\) in units \(e\). Eq. (4) is the usual QED-like Ward identity. \(B_{\gamma\gamma}\) and \(B_{\gamma Z}\) denote the longitudinal parts of the \(\gamma\gamma\) and \(\gamma Z\) self-energies. From the validity of eqs. (4,5) and from the analyticity properties of the two-point functions, it follows that, to all orders,

\[
A_{\gamma\gamma}(0) = 0 \tag{7}
\]

\[
A_{\gamma Z}(0) = 0 . \tag{8}
\]

The validity of the WIs yields a substantial computational simplification: in fact, it is possible to verify that the contributions of diagrams (a), (b) and (c) of Fig. 1 now add up to zero and that diagram (e) is vanishing. In the BFM, the electric charge renormalization is due only to the photon vacuum polarization. From the gauge invariance implied by eq. (7), also for the bosonic contributions, it is possible to Dyson resum also these terms. The relation between bare and renormalized charge is

\[
e^2 = \frac{e_0^2}{1 - e_0^2 \Pi_{\gamma\gamma}(0)} , \tag{9}
\]

\[
\Pi_{\gamma\gamma}(0) = \Pi_{\gamma\gamma}^{(b)}(0) + \Pi_{\gamma\gamma}^{(5)}(0) , \tag{10}
\]

\[
\Pi_{\gamma Z}^{(f)}(0) = \Pi_{\gamma Z}^{(lep)}(0) + \Pi_{\gamma Z}^{(5)}(0) + \Pi_{\gamma Z}^{(pert)}(0) \tag{11}
\]

where \(\Pi_{\gamma\gamma}^{(b)}\) indicated the bosonic contributions. A further distinction is in order for the fermionic part, where \(\Pi_{\gamma\gamma}^{(lep)}\) indicates the diagrams with one leptonic loop, \(\Pi_{\gamma\gamma}^{(5)}\) those with one light quark loop exchanging a photon or a gluon and \(\Pi_{\gamma\gamma}^{(pert)}\) the ones with one top loop or with one light-quark loop, exchanging a \(W\) or a \(Z\) boson. The term \(\Pi_{\gamma\gamma}^{(5)}(0)\) cannot be evaluated perturbatively, due to strong interactions at low energy.

Rewriting \(\Pi_{\gamma\gamma}^{(5)}(0) = \text{Re} \left( \Pi_{\gamma\gamma}^{(5)}(0) - \Pi_{\gamma\gamma}^{(5)}(m_Z^2) \right) + \text{Re} \Pi_{\gamma\gamma}^{(5)}(m_Z^2)\), the round bracket can be evaluated via a dispersion relation from the experimental data for the total cross-section of \(e^+ e^- \to \text{hadrons}\) at low energies \[10\], while the second term can be calculated perturbatively \[11\], due to the large scale \(q^2 = m_Z^2\). In our 2-loop calculation we have separated the diagrams with one light quark loop and the exchange of a photon/gluon, from those with a light quark loop and the exchange of a \(W\) or a \(Z\). The latter, due to the large scales \(m_W^2, m_Z^2\), can be evaluated perturbatively at \(q^2 = 0\). In contrast, the former have to be treated resorting to the experimental low energy data.

The complete analytical expressions for the 2-loop photon vacuum polarization are rather long and are presented in \[4\].

3. The \(\bar{e}^2(m_Z^2)\) \(\overline{\text{MS}}\) parameter

The relation given by Eq. (9) allows to determine one of the fundamental parameter of the \(\overline{\text{MS}}\) renormalization scheme, \(\bar{e}^2(m_Z^2)\), i.e. the \(\overline{\text{MS}}\) electric charge defined at scale \(m_Z\). The \(\overline{\text{MS}}\) renormalization procedure is defined as the subtraction of pole terms of the form \((n-4)^{-1}\), where \(m\) is an integer \(\geq 1\), \(n\) is the number of dimensions and the identification of the `t Hooft parameter \(\mu\) with the relevant mass scale, in this case \(m_Z\).

In order to obtain the relation between \(\bar{e}^2\) and \(e^2\), one writes \(e_0^2 = \bar{e}^2 / \tilde{Z}_e\) in Eq. (9), and uses the counterterms present in \(\tilde{Z}_e\) to cancel the \((n-4)^{-1}\) terms in the regularized but unrenormalized vacuum polarization function \(\Pi_{\gamma\gamma}(0)\) setting \(\mu = m_Z\) in the explicit expressions \[11\]. We define \(\Pi_{\gamma\gamma}^{(i)}\) the self-energy expression subtracted of its divergent part \(I_i / (n-4)\). Without implementing any decoupling we have

\[
\tilde{Z}_e = 1 + \frac{\hat{\alpha}}{4 \pi} (I_1 + I_4 + I_5 + I_6) \frac{1}{n - 4} \tag{12}
\]

so that

\[
e^2 = \frac{\bar{e}^2}{1 + (\hat{\alpha} / \alpha) \Delta \hat{\alpha} (m_Z^2)} , \tag{13}
\]
\[ \Delta \alpha(m_Z^2) = \Delta \alpha^{(5)}_{had} + \frac{\alpha}{\pi} \left( \frac{55}{27} \right) \]

\[ + \left( \frac{11 \alpha_s(m_Z^2)}{9 \pi} + \frac{35 \alpha(m_Z^2)}{108 \pi} \right) \left( \frac{55}{12} - 4 \zeta(3) \right) \]

\[ - 4 \pi \alpha \left[ \hat{\Pi}_{\gamma\gamma}^{(lep)}(0) + \hat{\Pi}_{\gamma\gamma}^{(bos)}(0) + \hat{\Pi}_{\gamma\gamma}^{(pert)}(0) \right] \]

In the first line we use the value 
\[ 4 \pi^2 \text{Re} \left( \Pi^{(5)}_{\gamma\gamma}(0) - \Pi_{\gamma\gamma}^{(0)}(m_Z^2) \right) = 0.027690 \pm 0.000353 \]

for the hadronic non-perturbative contributions. In the first and second line there is the contribution of \( \Pi^{(5)}_{\gamma\gamma}(m_Z^2) \) due to a loop of light quarks interacting with an internal photon or gluon \( [11] \). In the last line we indicate all perturbative contributions, whose explicit expressions can be found in \([4]\). Eq.\( (14) \) can be easily solved for \( \hat{\epsilon}^2 \), obtaining

\[ \hat{\epsilon}^2(m_Z^2) = \frac{\epsilon^2}{1 - \Delta \alpha(m_Z^2)}. \]  (15)  

Using the following values (in GeV) for the fermion masses \( m_e = 0.00511, m_\mu = 0.105658, m_\tau = 1.777, m_t = 174.3 \) and for the gauge bosons \( m_Z = 91.187, m_H = 150 \), we have evaluated \( \hat{\epsilon}^2(m_Z^2) \).

|                | 1NP  | 2 QCD  | 2 QED  | 2 EW   |
|----------------|------|--------|--------|--------|
| leptons        | 3529.2 | 7.66  | 10.18  |
| bosons        | -140.7 | -1.79  |
| top           | -133.7 | 8.66  | 0.19   | 0.08   |
| \( \Pi^{(5)}_{\gamma\gamma}(0) \)_{EW} |        |        |        | 4.56    |
| Re\( \Pi^{(5)}_{\gamma\gamma}(m_Z^2) \) | 473.4  | -2.39  | -0.04  |
| \( \Delta \alpha^{(5)}_{had}(m_Z^2) \) | 2769.0 |        |        |
| total         | 6497.2 | 6.27 | 7.81 | 13.03 |

Table 1  
Numerical results for \( \Delta \alpha(m_Z^2) \), expressed in units \( 10^{-5} \). The input parameters and the different groups of contributions are specified in the text.

In table \( [11] \) we present separately the contributions from the leptons, from the purely bosonic diagrams, from the diagrams involving the top quark, from the diagrams with light quarks exchanging a massive vector boson (indicated with \( \Pi^{(5)}_{\gamma\gamma}(0)_{[EW]} \)) and from the diagrams with light quarks exchanging a photon or a gluon, evaluated at \( q^2 = m_Z^2 \). In the first column we consider the 1-loop and the non-perturbative contributions. In the other columns we distinguish the 2-loop QCD and purely QED corrections \([11]\), and in the last column the full 2-loop EW (QED+weak) corrections. We have checked, in the lepton and in the top case, that the appropriate subset of diagrams from our results reproduces the numbers presented in \([11]\). Concerning the 2-loop EW diagrams involving a top quark, approximate results including all terms of order \( O(\alpha^2 m_t^2/m_W^2) \) were already available \([12]\) and could also be reproduced.

The largest contributions are due to light fermions (leptons and quarks) exchanging massive vector bosons and have both positive sign. In contrast the 2-loop purely bosonic diagrams have negative sign and are smaller in size. The size of the full 2-loop EW results is more than 13 parts in units \( 10^{-5} \) and almost half of it is due to purely electroweak effects.

**MS vs. on-shell**

The use of the BFM makes evident that the electric charge renormalization is due, also in the EW SM, only to the photon vacuum polarization. In the on-shell scheme the Dyson resummation of the self-energy corrections is not possible for the bosonic contributions at an arbitrary energy scale \( q^2 \), because they would break gauge invariance. Only at \( q^2 = 0 \) the WIs guarantee that such terms are a gauge invariant subset. On the other hand, the QED and QCD fermion corrections are a gauge invariant subset which can be included in the definition of the on-shell running coupling and evaluated at any energy scale. The \( \overline{MS} \) renormalization scheme does not suffer of this problem, because the running of the charge is determined only by the self-energy corrections evaluated at \( q^2 = 0 \). The relevance of this approach becomes evident when we want to evaluate the effective charge at high energy scales, like those of LEP2 (\( \sqrt{s} \geq 160 \text{ GeV} \)) or higher,
where the massive gauge bosons are active degrees of freedom which significantly contribute to the running.

Two-loop corrections and $\Delta \alpha_{\text{had}}$

In the past, the error which affects the determination of $\Delta \alpha_{\text{had}}$ has been considered a bottleneck of all precision tests of the SM. The new results from VEPP, DaΦne, BES and BaBar [13] are now improving the present status. There are prospects to lower the error on $\Delta \alpha_{\text{had}}$ from the present $0.027690 \pm 0.000353$ by a factor 5 to $0.00007$ or even to $0.00005$. [14]. 

We remark that the full 2-loop EW corrections shift the central value of $\Delta \alpha$ by $+0.00013$ and are of the same order of magnitude of the error which affects the determination of $\Delta \alpha_{\text{had}}$. The inclusion of the 2-loop EW radiative corrections for the relevant observables is therefore a mandatory step for all precision tests of the SM. The effect of the 2-loop EW corrections is twofold: (i) they shift the central value of $\hat{\alpha}^2(m_Z^2)$ and (ii) reduce the theoretical perturbative uncertainty on its determination, which is now pushed at the 3-loop level. In the $\overline{\text{MS}}$ scheme, the quantity $\hat{\alpha}^2(m_Z^2)$ thus provides a realistic estimate of the effect of the 2-loop EW corrections which modify the running of the effective electric charge.

Perturbative contributions of quarks

The numerical results presented in Table 1 require two further comments. The quasi-vanishing of the diagrams with the top quark, for realistic values of in the input parameters, is due to a fortuitous numerical cancellation. The asymptotic expansion of the top contributions in powers of $m_t^2/m_Z^2$ is not converging very rapidly, because the expansion parameter, the top Yukawa coupling, is of the same order of magnitude as the gauge couplings [16]. One verifies that the so-called leading-terms $O(m_t^2)$ have the same size as the so-called sub-leading ones. Having opposite sign, they almost cancel as shown in fig.2.

The perturbative contributions due to light quarks and leptons, exchanging a $W$ or a $Z$ boson, are numerically relevant: the zero mass of the fermions compensate the suppression due to a heavy mass. They contain a logarithmic term $\log(m_Y/\mu)$ ($V = W, Z$) which becomes relevant at high energy scales $\mu$.

The running of $\hat{\alpha}$ at higher energies

The running of $\hat{\alpha}$ at higher energy scales can be evaluated by changing the value of the renormalization scale $\mu$ in eq.14. The relevant results for $\mu = 1 \text{NP}$, $2 \text{QCD}$, $2 \text{EW}$, $\hat{\alpha}^{-1}(\mu)$ are presented in table 2. We observe again that the full 2-loop EW corrections are not negligible, compared to the present error on $\Delta \alpha_{\text{had}}^{(5)}$.

![Figure 2. Contributions due to the top quark.

The upper line shows only the $O(m_t^2)$ terms, while the lower line is the full result.](image-url)

Table 2

| $\mu$ | 1NP  | 2QCD | 2EW  | $\hat{\alpha}^{-1}(\mu)$ |
|-------|------|------|------|--------------------------|
| $m_Z$ | 6485.42 | 6.27 | 13.03 | 128.11 ± 0.05 |
| 300   | 6991.91 | 40.90 | 21.45 | 127.35 ± 0.05 |
| 500   | 7209.15 | 55.75 | 25.05 | 127.03 ± 0.05 |
| 800   | 7409.01 | 69.42 | 28.37 | 126.73 ± 0.05 |
| 1000  | 7503.90 | 75.91 | 29.94 | 126.59 ± 0.05 |
| 5000  | 8188.22 | 122.72 | 41.24 | 125.57 ± 0.05 |

Numerical results for $\Delta \hat{\alpha}(\mu^2)$, expressed in units $10^{-5}$, for different values of the energy scale $\mu$. (same input parameters as for Table 1).

An $e^+e^-$ linear collider or at the LHC are presented in Table 2. We observe again that the full 2-loop EW corrections are not negligible, if compared to the present error on $\Delta \alpha_{\text{had}}^{(5)}$.
The limits on the Higgs boson mass

Since the Higgs boson has not yet been observed, all theoretical predictions depend parametrically on the value of its mass $m_H$. The results of the global fit known as “blue-band plot” prepared by the EW Working Group \[18\], show the sensitivity of the $\chi^2$ distribution for the Higgs boson mass to the central value and to the error as well of $\alpha(m_Z^2)$. In fig.3 the red and blue lines show the results obtained with two different setups for $\Delta\alpha_{\text{had}}$; in particular the central values differ by $14 \cdot 10^{-5}$ parts. The combined effect of a different choice of the central value and of a reduction of the error, causes a shift of the minimum of the distribution by $\mathcal{O}(15)$ GeV.

A detailed study of the limits on $m_H$ has been presented in \[17\], where it has been shown that the most sensitive observable to the value of $m_H$ is the effective sinus of the weak mixing angle $\sin^2 \theta_{\text{eff}}$ extracted from the leptonic asymmetries, whereas the mass of the $W$ boson has in this respect a weaker constraining power. On the other hand, the effective sinus is also very sensitive to the precise value of $\alpha(m_Z^2)$ and to its error as well.

At present the determination of the $W$ boson mass is known in the EW SM at the two-loop level completely \[7\], whereas only few classes of two-loop contributions are known for the effective sinus. A definite answer about the size of the missing corrections to the effective sinus and about their effect on the limits on the Higgs boson mass can come only from an explicit two-loop calculation.

A global analysis that uses the $\overline{\text{MS}}$ definition of the weak mixing angle, could consistently use the $\overline{\text{MS}}$ effective coupling $\hat{e}^2(m_Z^2)$, to improve the present evaluation. One could include the effect of the new corrections: in particular of the ones due to a loop of light-fermions exchanging a $W$ or $Z$ bosons, of the so called sub-leading corrections in the expansion of the top contributions in powers of $m_t$ and of the bosonic diagrams. Their effect on the indirect limit on the Higgs boson mass can be roughly estimated to be a reduction of $\mathcal{O}(6-8)$ GeV for the 95% C.L. Obviously, this is just an estimate which can not replace, by any means, the result of the global fit to all EW observables. On the other hand, in the $\overline{\text{MS}}$ scheme we can consistently compare the size of the error on $\Delta\alpha_{\text{had}}$ to that of the 2-loop weak corrections and conclude that they are not completely negligible.

A similar statement, either positive or negative, can not be drawn in the on-shell scheme, because of the gauge-invariance problem which forbids the use of a resummed effective coupling including the weak effects. In the on-shell scheme only a complete two-loop calculation can provide a clean answer about the size and the effect of the missing two-loop corrections.

4. Conclusions

We have described the main features of the calculation of the complete 2-loop EW corrections in the SM to the renormalization of the electric charge. We discussed the computational advantages of the BFM, which makes evident that the charge renormalization is due only to the effect of the photon vacuum polarization, whose bosonic contributions can be Dyson resummed in a manifestly gauge invariant way. The $\overline{\text{MS}}$ effective coupling $\hat{e}^2(m_Z^2)$, evaluated at the scale $q^2 = m_Z^2$, is...
shifted by the full 2-loop EW radiative corrections by $+0.00013$, where 5.8 parts are of purely EW origin. The electric charge renormalization receives, at the 2-loop level, QCD, QED and purely EW contributions, which are comparable in size and with the same sign.

In view of an improvement of the measurement of the low-energy cross-section for $e^+e^- \rightarrow \text{hadrons}$, which would imply a reduction of the error of $\Delta \alpha_{\text{had}}$, we observe that the effect of the perturbative 2-loop EW corrections to the effective electric charge cannot be neglected in all precision tests of the SM. In particular the indirect limits on the Higgs boson mass are very sensitive to the precise value of $\alpha(m_Z^2)$.

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REFERENCES
1. U. Baur et al., [arXiv:hep-ph/0111314]
2. K. Hagiwara et al. [Particle Data Group Collaboration], Phys. Rev. D 66 (2002) 010001.
3. see F. Jegerlehner, these proceedings
4. G. Degrassi and A. Vicini, “Two-loop electric charge renormalization in the Standard Model” [arXiv:hep-ph/0307122]
5. A. Sirlin, Phys. Rev. D 22 (1980) 971.
6. W. J. Marciano and A. Sirlin, Phys. Rev. D 22 (1980) 2695 [Erratum-ibid. D 31 (1985) 213].
7. A. Freitas, W. Hollik, W. Walter and G. Weiglein, Phys. Lett. B 495 (2000) 338 [Erratum-ibid. B 570 (2003) 260] [arXiv:hep-ph/0007091].
8. A. Freitas, W. Hollik, W. Walter and G. Weiglein, Nucl. Phys. B 632 (2002) 189 [Erratum-ibid. B 666 (2003) 305] [arXiv:hep-ph/0202131].
9. M. Awramik and M. Czakon, Phys. Rev. Lett. 89, 241801 (2002) [arXiv:hep-ph/0208113].
10. A. Onishchenko and O. Veretin, Phys. Lett. B 551 (2003) 111 [arXiv:hep-ph/0209010].
P. Gambino and A. Vicini, Phys. Lett. B 350, 75 (1995) [arXiv:hep-ph/9412380].
G. Degrassi, P. Gambino and A. Vicini, Phys. Lett. B 383 (1996) 219 [arXiv:hep-ph/9603374].

17. A. Ferroglia, G. Ossola, M. Passera and A. Sirlin, Phys. Rev. D 65 (2002) 113002

18. LEP Electroweak Working Group, http://lepewwg.web.cern.ch/LEPEWWG/