Sensitivity analysis of a new model to predict the survival probability of artificial rock blocks upon dynamic impact

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Abstract. Rockfall fragmentation is a very complex phenomenon that is still poorly understood and modelled. Being able to adequately model fragmentation of impacting blocks, including change of shape, size, and energy after breakage, is essential to be able to predict realistic trajectories and design effective mitigation measures. In order to develop an accurate predictive model for rockfall fragmentation, it is necessary to better understand the fragmentation process and its likely outcomes. A novel model was recently proposed by the authors which can predict the survival probability (SP) of brittle spheres upon impact from the statistical distribution of material parameters, obtained by standard quasi-static tests (Brazilian tests and unconfined compression tests). The survival probability is described as a Weibull function whose two parameters (shape parameter -m- and scale parameter - critical kinetic energy) are predicted by the model. The model is based on theoretically-derived (from Hertzian contact theory) conversion factors used to transform the critical work required to fail disc samples in quasi-static indirect tension into the critical kinetic energy to cause failure of spheres at impact in vertical drop tests. This paper presents a sensitivity analysis on the parameters which influence the prediction of the critical kinetic energy.

1. Introduction

Rockfall research has made considerable progress since the early 1980s, particularly in terms of design of rockfall protection measurements and of rockfall trajectory modelling. The latter is a key step in the design of rockfall protection measurements and hazard assessment, as it provides information on possible block trajectories and impact energy. After a rockfall event, fragmentation of rock blocks is often observed [1]. Rock fragmentation upon impact is a still poorly understood aspect of rockfall and it is not accurately modeled because the current state of knowledge on the topic is deficient. Undoubtedly, it is a complex phenomenon which can be influenced by many factors such as rock strength, block shape, presence and properties of discontinuities, stiffness of the ground and impact conditions [5]. In recent years, Corominas and co-workers significantly contributed to modelling fragmentation in rockfall engineering with two fragmentation models. The first one is a “Rockfall Fractal Fragmentation Method” (RFFM) [8] to obtain the rockfall block size distribution (RBSD) from in situ block size distribution (IBSD). The second model is a GIS-based software, called RockGIS, that can stochastically simulate fragmentation [11]-[12]. Although this model represents a significant progress in fragmentation modeling, it relies on several assumptions and, in particular, a survival probability of the rock block upon impact must be assumed between 0 and 1 allowing a percentage of blocks to survive despite having reached a certain breakage energy threshold. As far as the authors know, there is scant experimental data about survival...
probability at impact, its relation to impact energy and to material properties. The authors [13] have recently proposed a model to predict the survival probability (SP) of brittle spheres upon impact from the statistical distribution of material parameters obtained by quasi-static Brazilian tests. This paper briefly recalls the model and presents a sensitivity analysis, in order to assess the effect that parameter uncertainty may have on the prediction of the survival probability.

2. The concept of survival probability
The concept of survival probability is commonly used in comminution [14]. It describes the likelihood that grains or particles sustain a certain load (typically, in terms of force or stress) without breaking. The concept can be extended to any mechanical test performed on a series of specimens. For example, after a series of compression tests, if the cumulative distribution of stress at failure indicates that 30% of specimens require 20 MPa of stress to reach failure, it means that 70% of specimens have survived a stress of 20 MPa. It is hence possible to turn a cumulative distribution of resistance into a survival probability. Survival probabilities are commonly fitted using a Weibull distribution [15], which is a sigmoidal cumulative curve fully characterised by a shape parameter and a scale parameter. In rockfall, the load imparted onto particles or rocks is a kinetic energy at impact and experimental evidence obtained by the authors suggest that the survival probability between 0% and 100% is approximately linear [13].

3. A model to predict the survival probability of brittle spheres upon dynamic impact
The model is described in its entirety in Guccione et al. [13] so, for conciseness, only its key elements are recalled here. Although the objective is to propose a model that is generally applicable to real rockfall problems, the model was initially developed for a simple case of mortar spheres, in free fall, without rotation, impacting normal to a strong concrete slab. It constitutes a first step towards the prediction of survival probability for more complex and realistic scenarios. The model relies on the analogy (in terms of failure pattern) between fragmentation of spheres at impact and failure of discs in indirect tension (Brazilian test) in the range of kinetic energy relevant for the survival probability [16]. The two key ideas of the model are that:

- the work required to fail discs in indirect tension can be converted, via a series of conversion factors (recalled in later sections and fully derived in Guccione et al. [13]), into a kinetic energy required for fragmentation. The process is illustrated in figure 1. This amount of kinetic energy reflects the relative position of the survival probability (i.e. Weibull scale parameter).
- the shape or flatness of the survival probability for the drop tests is directly related to the shape parameter of the Weibull distribution of work required to fail rock specimens under indirect tension.

To apply the model, it is first necessary to conduct a series of Brazilian tests on small discs (minimum of 20 tests recommended, but the more the better) and interpret the results in terms of work required to fail each specimen. Then, the distribution of work values is converted into a survival probability for impacting spheres (as per explanations in Section 2). After plotting the entire survival probability curve for work and fitting it with a Weibull distribution, it is possible to identify the shape and scale Weibull parameters for work: \(m_{BT-W}\) and \(W_{BT}^{ct}(d)\), respectively. These two parameters become the input of the model for fragmentation survival probability.

The position of the fragmentation survival probability (Weibull scale parameter) is the critical kinetic energy \(E_{K(D)}^{ct}\) obtained by:

\[
E_{K(D)}^{ct} = C_{Size} \cdot C_{shape} \cdot C_{Rate} \cdot W_{BT}^{ct}(d)
\]

where \(C_{Size}, C_{shape}, C_{Rate}\) are conversion factors to infer dynamic breakage survival of spheres from static tests on discs, as indicated in figure 1 and explained in the next sections. The shape Weibull parameter of the fragmentation survival probability \(m_{K}^{ct}\) is assumed to be equal to that of the survival probability for work \(m_{BT-W}\):

2
$$m_E = m_{BT} - W$$

(2)

Figure 1. Key steps of the model to predict the critical kinetic energy for failure of sphere of diameter $D$ upon dynamic impact from distribution of work required to fail a disc of diameter $d$ under indirect tension.

Following experimental observations by the authors [13], the model describes the fragmentation survival probability for the brittle spheres in drop test as a linear function of kinetic energy at impact ($E_K(D)$):

$$SP(E_K(D)) = 37 + \frac{100 \cdot m_E}{e} \cdot \left(1 - \frac{E_K(D)}{E_{cr}(D)}\right)$$

(3)

Derivations of equation (3) are elaborated in detail in Guccione et al. [13].

3.1. Shape conversion factor $C_{shape}$

The shape conversion factor $C_{shape}$ converts the work required to fail a disc of diameter $d$ during a quasi-static Brazilian test ($W_{BT}(d)$) into the work required to fail a sphere of the same diameter $d$ under compressive loading at the same rate ($W_{SC}(d)$). The conversion factor is a combination of separate conversion factors for the maximum force and the maximum displacement and it was derived assuming that the load-displacement response of discs and spheres is linear. The force factor was obtained by considering the equations that yield the tensile strength of a sphere [20] and disc [21] under compression. The displacement factor was derived by using Hertzian contact theory [22] which provides the total
deformation of a sphere or a disc compressed against a flat surface. The shape conversion factor $C_{\text{shape}}$ is given by equation 4:

$$
C_{\text{shape}} = \frac{0.92 \cdot \pi \cdot Y_m \cdot h}{F_{BT(d)}^{1/3} \cdot Y_{ms}^{2/3} \cdot (1 - \nu_m^2) \cdot d^{1/3} \cdot \left[0.41 + \ln(2d) - 0.5 \cdot \ln\left(\frac{2 \cdot d \cdot F_{BT(d)}}{\pi \cdot h \cdot Y_{ms}}\right)\right]}
$$

(4)

where $d$ and $h$ are diameter and thickness of a disc tested under indirect tensile test; $F_{BT(d)}$ is the force required to fail a disc under indirect tension with a quasi-static loading rate; $Y_m$ and $\nu_m$ are the elastic modulus and Poisson’s ratio of the mortar (sphere); $Y_{ms}$ is the equivalent modulus for the mortar-steel platen system defined as:

$$
\frac{1}{Y_{ms}} = \left(\frac{1 - \nu_m^2}{Y_m} + \frac{1 - \nu_s^2}{Y_s}\right)
$$

(5)

where $Y_s = 210$ GPa and $\nu_s = 0.3$ are the elastic modulus and Poisson’s ratio of the steel platen, respectively.

3.2. Size conversion factor $C_{\text{size}}$

The size conversion factor $C_{\text{size}}$ converts the work done to fail a sphere of diameter $d$ in compression ($W_{SC(d)}$) into the work required to fail a sphere of different (typically larger) diameter $D$ in compression ($W_{SC(D)}$). Frossard et al. [23] proposed a relationship to account for the size effect for the force required to crush mineral particles. This scaling relationship was extended to the work required to fail spheres by the authors [13] using size-dependent Hertzian contact theory. The size conversion factor $C_{\text{shape}}$ is given by equation 6:

$$
C_{\text{size}} = \left(\frac{D}{d}\right)^{3 - \frac{5}{m_{BT-F}}}
$$

(6)

where $m_{BT-F}$ is the Weibull shape parameter for the distribution of failure forces for the Brazilian tests conducted on discs of mortar to characterise the material.

3.3. Rate conversion factor $C_{\text{rate}}$

The rate conversion factor $C_{\text{rate}}$ accounts for strain rate difference between quasi-static tests and drop tests. The factor converts the work required to fail a sphere of diameter $D$ in quasi-static compression ($W_{SC(D)}$) into the kinetic energy requested to fail a sphere of same diameter $D$ under dynamic loading ($E_{K(D)}$). As for the shape conversion factor, the rate conversion factor is a combination of a conversion factor for force and one for displacement. The factor for force is based on the work by Wu et al. [24] who studied the dynamic strength of concrete. The displacement factor is obtained from Hertzian contact theory [22] and reflects the fact that under a higher load (i.e. dynamic as opposed to static), the sphere will deform more. The rate conversion factor $C_{\text{rate}}$ is given by equation (7):

$$
C_{\text{rate}} = \frac{\text{ISR}^{0.092}}{2} \cdot \alpha \cdot \left(\frac{Y_{ms}}{Y_{mc}}\right)^{2/3}
$$

(7)

where ISR is the increase in strain rate between quasi-static testing and dynamic testing (established from time to failure in quasi-static tests and impact duration in drop tests); $Y_{ms}$ is the equivalent modulus for the mortar-steel platen system (see equation (5)); $Y_{mc}$ is the equivalent modulus for the mortar-concrete slab system and $\alpha$ is a factor used to estimate the part of deformation that occurs in the sphere
(assumed to be 0.8, see [13]). The equivalent modulus for the mortar-concrete slab system \( Y_{mc} \) is defined as:

\[
\frac{1}{Y_{mc}} = \left( \frac{1 - \nu_m^2}{Y_m} + \frac{1 - \nu_c^2}{Y_c} \right)
\]

(8)

where \( Y_m \) and \( Y_c \) are the elastic moduli of mortar and concrete slab, respectively; \( \nu_m \) and \( \nu_c \) are the Poisson’s ratios of mortar and concrete slab, respectively.

4. Sensitivity analysis

The three different conversion factors (equations 4 to 8) rely on the elastic modulus and Poisson’s ratio of both impacting material and impacted material \((Y_m, Y_c, \nu_m \text{ and } \nu_c)\), as well as on an estimate of strain rate difference (ISR) and a representative force that describe the response of a series of specimens tested in indirect tension \( (B_{RT(d)}) \). Because the response in compression of mortar or concrete is not perfectly linear, measuring the Poisson’s ratio is time consuming and estimating a dynamic strain rate is not trivial, there is some uncertainty associated with the determination of the moduli, Poisson’s ratios and ISR. \( B_{RT(d)} \) is one value meant to capture the whole distribution of forces of Brazilian tests at failure and so it is affected by uncertainty in deciding which value should be used in the distribution and what is the effect of number of tests on the distribution. The fact that equations 4 to 8 are nonlinear means that assessing the effect of variation of the selected parameters on the model predictions is not trivial.

Consequently, a sensitivity analysis was conducted using the minimum and maximum values reported in table 1. A constant value of critical Brazilian work \( (W_{BT}^{cr}(d) = 0.663 \text{ J}) \) was converted to a critical kinetic energy \( (E_k^{cr}(D)) \) using the conversion factors (equations 4 to 8 with \( d = 54 \text{ mm} \) and \( D = 100 \text{ mm} \), as per [13]). When assessing the sensitivity of a specific parameter, the range given in table 1 was considered and all other parameters were assigned a base value (see table 1). In the first part of the sensitivity analysis, parameters were changed one at a time. Note that the sensitivity to \( W_{BT}^{cr}(d) \) was not tested as \( W_{BT}^{cr}(d) \) and \( E_k^{cr}(D) \) are linearly related (equation 1). Then, minimum and maximum values of parameters were combined in order to obtain lower and upper bounds of survival probability, predicted using equation 3 with \( m_E = 4.2 \) (for the range of parameters given in table 1).

Table 1. Range of parameters used for the sensitivity analysis. Base values are those used in Guccione et al. [13]. Other input parameters are: \( m_{BT-f} = 8.9; W_{BT}^{cr}(d) = 0.663 \text{ J}; m_E = 4.2 \).

| Parameter | Base Value | Min Value | Max Value |
|-----------|------------|-----------|-----------|
| \( Y_m \) [GPa] | 4.4 | 2.2 | 6.6 |
| \( Y_c \) [GPa] | 11.7 | 5.9 | 17.6 |
| \( \nu_m \) | 0.20 | 0.10 | 0.30 |
| \( \nu_c \) | 0.15 | 0.10 | 0.30 |
| \( B_{RT} \) [N] | 4849 | 2400 | 7300 |
| ISR | 115,385 | 57,500 | 173,000 |

5. Results

Figures 2, 3, 4 and 5 illustrate the effects of uncertainty on the moduli (mortar and concrete), Poisson’s ratio (mortar and concrete), force and strain rate increase, respectively. Figure 2 shows that the calculation of the critical kinetic energy is highly influenced by the elastic modulus of both materials. The first observation is that the stiffer the sphere (mortar), the higher the critical kinetic energy but the stiffer the slab (concrete), the lower the critical kinetic energy. This is due to the fact that the stiffer the mortar, the higher \( C_{\text{Shape}} \) (equation 4) while the stiffer the slab, the lower \( C_{\text{Rate}} \) (equation 7). The prediction is more sensitive to the modulus of the sphere than that of the slab; increasing \( Y_m \) and \( Y_c \) by a factor 3 changes \( E_k^{cr}(D) \) by a factor 1.55 and 1.20, respectively.
Figure 2. Influence of elastic modulus (mortar and concrete) variations on the critical kinetic energy predicted by the model (equations 4 to 8).

Figure 3. Influence of Poisson’s ratio (mortar and concrete) variations on the critical kinetic energy predicted by the model (equations 4 to 8).

Figure 4. Influence of varying the value of critical force required to fail a disc under indirect tension ($F_{BT}$) on the critical kinetic energy predicted by the model (equations 4 to 8).

Figure 5. Influence of ISR variations on the critical kinetic energy predicted by the model (equations 4 to 8).

Figure 3 shows that the critical kinetic energy is relatively insensitive to the Poisson’s ratio of the sphere (mortar) and the slab (concrete). Indeed, increasing Poisson’s ratio by a factor 3 leads to less than 5% change in critical kinetic energy.

Figure 4 shows that increasing the representative force required to fail a disc under quasi-static indirect tension reduces the critical kinetic energy, which is consistent with equation 4. The prediction is quite sensitive to the value of $F_{BT}$: a threefold increase (from 2400 to 7300 N) leads to diminution of $E_k^c(D)$ by a factor 1.25.

Finally, figure 5 shows the sensitivity of $E_k^c(D)$ to the increase in strain rate. The higher ISR, the higher the critical kinetic energy. However, like for the Poisson’s ratio, the effect is marginal: a fourfold increase of ISR only leads to 10% increase of critical kinetic energy. So, the three most influential factors are the sphere modulus (mortar), the slab modulus (concrete) and the representative force required to fail a disc under quasi-static indirect tension $F_{BT}$. 
To complete the sensitivity analysis, all six factors were combined in order to obtain the upper and lower bounds of predictions (for the minimum and maximum values of table 1). Minimum values of $Y_c$, $\nu_c$ and $F_{BT}$ and maximum values of $Y_m$, $\nu_m$ and ISR were used to get the maximum value of critical kinetic energy, and vice versa. Figure 6 reports the experimental values of survival probability for 100 mm diameter spheres measured by Guccione et al. [13] and presents the extreme predicted survival probability envelope. It can be seen that the upper and lower bounds are only 2 to 3 J either side of the prediction using the base value. This shows that the model is quite robust and that some inaccuracy in the input parameters is possible without significantly affecting the prediction of survival probability.

![Figure 6. Experimental values (dots) and predicted values (lines) of fragmentation survival probability as a function of kinetic energy.](image)

**6. Conclusions**

A sensitivity analysis on a novel model proposed by the authors [13] to predict the survival probability of brittle spheres upon impact from the statistical distribution of material parameters obtained by quasi-static Brazilian tests is presented. The influence of six parameters was investigated: elastic modulus and Poisson’s ratio of mortar (used for the spheres) and concrete (used for the slab), a representative force required to fail a disc under indirect tension and the increase in strain rate, from quasi-static loading to dynamic impact. The predicted critical kinetic energy is shown to be most sensitive to elastic modulus values of both materials (impacting block and impacted surface), as well as the value of force required to fail a disc under indirect tension. In contrast, predicted critical kinetic energy is only moderately sensitive to the estimated value of the strain rate parameter and the values of Poisson’s ratio have a marginal influence. The six parameters were then combined to obtain lower and upper bounds of survival probability prediction and it was concluded that the model is robust and that it is possible to have some uncertainty in the input parameters without observing changes in the predictions that would render them worthless.

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