Wormhole inspired by non-commutative geometry

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Abstract In the present work we search for a new wormhole solution inspired by noncommutative geometry with the additional condition of allowing conformal Killing vectors (CKV). A special aspect of noncommutative geometry is that it replaces point-like structures of gravitational sources with smeared objects under Gaussian distribution. However, the purpose of this paper is to obtain wormhole solutions with noncommutative geometry as a background where we consider a point-like structure of gravitational object without smearing effect. It is found through this investigation that wormhole solutions exist in this Lorentzian distribution with viable physical properties.

Keywords General Relativity; Non-commutative geometry; wormholes

1 Introduction

A wormhole, which is similar to a tunnel with two ends each in separate points in spacetime or two connecting black holes, was conjectured first by Weyl [1] and later on by [2]. In a more concrete physical definition it is essentially some kind of hypothetical topological feature of spacetime which may acts as shortcut through spacetime topology.

It is argued by Morris et al. [3,4] and others [5,6,7] that in principle a wormhole would allow travel in time as well as in space and can be shown explicitly how to convert a wormhole traversing space into one traversing time. However, there are other types of wormholes available in the literature where the traversing path does not pass through a region of exotic matter [8,9]. Following the work of [8], a new type of thin-shell wormhole, which was constructed by applying the cut-and-paste technique to two copies of a charged black hole [10], is of special mention in this regard.

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Thus a traversable wormhole, tunnel-like structure connecting different regions of our Universe or of different universes altogether, has been an issue of special investigation under Einstein’s general theory of relativity [11]. It is argued by Rahaman et al. [12] that although just as good a prediction of Einstein’s theory as black holes, wormholes have so far eluded detection. As one of the peculiar features a wormhole requires the violation of the null energy condition (NEC) [4]. One can note that phantom dark energy also violates the NEC and hence could have deep connection to in formation of wormholes [13,14].

It is believed that some perspective of quantum gravity can be explored mathematically in a better way with the help of non-commutative geometry. This is based on the non-commutativity of the coordinates encoded in the commutator, \([x_\mu, x_\nu] = \theta_{\mu\nu}\), where \(\theta_{\mu\nu}\) is an anti-symmetric and real second-ordered matrix which determines the fundamental cell discretization of spacetime [15,16,17,18,19]. We also invoke the inheritance symmetry of the spacetime under conformal Killing vectors (CKV). Basically CKVs are motions along which the metric tensor of a spacetime remains invariant up to a certain scale factor. In a given spacetime manifold \(M\), one can define a globally smooth conformal vector field \(\xi\), such that for the metric \(g_{ab}\) it can be written as

\[
\xi_{a,b} = \psi g_{ab} + F_{ab},
\]

where \(\psi : M \rightarrow R\) is the smooth conformal function of \(\xi\) and \(F_{ab}\) is the conformal bivector of \(\xi\). This is equivalent to the following form:

\[
L_\xi g_{ik} = \xi_{i;k} + \xi_{k;i} = \psi g_{ik},
\]

where \(L\) signifies the Lie derivatives along the CKV \(\xi^a\).

In favour of the prescription of this mathematical technique CKV we find out the following features: (1) it provides a deeper insight into the spacetime geometry and facilitates the generation of exact solutions to the Einstein field equations in a more comprehensive forms, (2) the study of this particular symmetry in spacetime is physically very important as it plays a crucial role of discovering conservation laws and to devise spacetime classification schemes, and (3) because of the highly non-linearity of the Einstein field equations one can reduce easily the partial differential equations to ordinary differential equations by using CKV. Interested readers may look at the recent works on CKV technique available in the literature [20,21,22].

In a more precise description the idea of noncommutative geometry is that one can average coordinate operators on noncommutative coherent states to obtain a mean value of a function defined on a noncommutative manifold. As one physically expects the effect of noncommutativity on classical Dirac delta distributions is simply a spread of the profile into a Gaussian. Obviously, as is expected, there might be additional effects, but they are subleading with respect to the Gaussian term. Furthermore, one can re-interpret the whole business of noncommutative geometry inspired solutions in terms of nonlocal gravity actions. All in all such an approach overcomes the limitations of the star product approach concerning the unitarity of the theory and the lack of short scale regularization of black holes [23,24,25,26,27].

In the present work therefore we search for some new solutions of wormhole admitting conformal motion of Killing Vectors. It is a formal practice to consider
the inheritance symmetry to establish a natural relationship between spacetime geometry and matter-energy distribution for a astrophysical system. Thus our main goal here is to examine the solutions of Einstein field equations by admitting CKV under non-commutative geometry. The scheme of the investigation is as follows: in the Sec. 2 we provide the mathematical formalism and Einstein’s field equations under the framework of this technique. A specific matter-energy density profile has been employed in Sec. 3 to obtain various physical features of the wormhole under consideration by addressing the issues like the conservation equation, stability of the system, active gravitational mass and gravitational energy. Sec. 4 is devoted for some concluding remarks.

2 Conformal Killing vector and basic equations

We take the static spherically symmetric metric in the following form

\[ ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]

where \( r \) is the radial coordinate. Here \( \nu \) and \( \lambda \) are the metric potentials which have functional dependence on \( r \) only.

Thus, the only survived Einstein’s field equations in their explicit forms (rendering \( G = c = 1 \)) are

\[ e^{-\lambda} \left( \frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = 8\pi \rho, \]

\[ e^{-\lambda} \left( \frac{1}{r^2} + \frac{\nu'}{r} \right) - \frac{1}{r^2} = 8\pi p_r, \]

\[ \frac{1}{2} e^{-\lambda} \left[ \frac{1}{2} (\nu')^2 + \nu'' - \frac{1}{2} \lambda' \nu' + \frac{1}{r} (\nu' - \lambda') \right] = 8\pi p_t, \]

where \( \rho, p_r \) and \( p_t \) are matter-energy density, radial pressure and transverse pressure respectively for the fluid distribution. Here \( t \) over \( \nu \) and \( \lambda \) denotes partial derivative w.r.t. radial coordinate \( r \).

The conformal Killing equations, as mentioned in Eqs. (2), then yield as follows:

\[ \xi^1 \nu' = \psi, \]

\[ \xi^4 = C_1 = \text{constant}, \]

\[ \xi^3 = \psi r, \]

\[ \xi^1 \lambda' + 2 \xi_3 = \psi, \]

where \( \xi^\alpha \) are the conformal 4-vectors and \( \psi \) is the conformal function as mentioned earlier.

This set of equations, in a straight forward way, imply the following simple forms:

\[ e^{\nu} = C_2^2 r^2, \]

\[ e^{\lambda} = \frac{C_2^2}{\psi^2}, \]

\[ \xi^i = C_1 \delta^i_4 + \left( \frac{\psi r}{2} \right) \delta^i_1, \]
where $C_2$ and $C_3$ are integration constants. Here the non-zero components of the conformal Killing vector $\xi^a$ are $\xi^0$ and $\xi^1$.

Now using solutions (7) and (8), the equations (3)-(5) take the following form as

$$\frac{1}{r^2} \left[ 1 - \psi^2 \frac{2C_2}{C_3^2} \right] - 2\psi \psi' \frac{C_2}{C_3^2} = 8\pi \rho, \quad (10)$$

$$\frac{1}{r^2} \left[ 1 - \frac{3\psi^2}{C_3} \right] = -8\pi p_t, \quad (11)$$

$$\left[ \frac{\psi^2}{C_3^2 r^2} \right] + 2\psi \psi' \frac{C_2}{C_3^2} = 8\pi p_t. \quad (12)$$

These are the equations forming master set which has all the information of the fluid distribution under the framework of Einstein’s general theory of relativity with the associated non-commutative geometry and conformal Killing vectors.

### 3 Matter-energy density profile and physical features of the wormhole

As stated by Rahaman et al. [11], the necessary ingredients that supply fuel to construct wormholes remain an elusive goal for theoretical physicists and there are several proposals that have been put forward by different authors [26, 29, 30, 31, 32, 33]. However, in our present work we consider cosmic fluid as source and thus have provided a new class of wormhole solutions. Keeping the essential aspects of the noncommutativity approach which are specifically sensitive to the Gaussian nature of the smearing as employed in Ref. [18], we rather get inspired by the work of Rizzo-Mehdipour-Nicolini [34, 35, 36] to search for a new fluid model admitting conformal motion. Therefore, we assume a Lorentzian distribution of particle-like gravitational source and hence the energy density profile as given in Ref. [34, 35, 36] as follows:

$$\rho(r) = \frac{M \sqrt{\phi}}{\pi^2 (r^2 + \phi)^2}, \quad (13)$$

where $\phi$ is the noncommutativity parameter and $M$ is the smeared mass distribution given by

$$M = 4\pi \int_0^r \rho r^2 dr = \frac{2M}{\pi} \left[ \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) - \frac{r \sqrt{\phi}}{r^2 + \phi} \right]. \quad (14)$$

Now, solving equation (10) we get

$$\psi^2 = C_3^2 - \left( \frac{4C_2^2 M}{\pi r} \right) \left[ \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) - \frac{r \sqrt{\phi}}{r^2 + \phi} \right] + \frac{D_1}{r}, \quad (15)$$

where $D_1$ is an integration constant and can be taken as zero.

The parameters, like the tangential pressure and metric potentials, are found as

$$p_t = \frac{1}{8\pi} \left[ \frac{1}{r^2} - 8\pi \left( \frac{M \sqrt{\phi}}{\pi^2 (r^2 + \phi)^2} \right) \right]. \quad (16)$$
Fig. 1 Diagram of the shape function of the wormhole for the specific values of the parameters as $\phi = 0.01$, $M = 1$.

\[ e^\nu = C_2^2 r^2, \]  
(17)

\[ e^\lambda = \frac{1}{1 - \left(\frac{4M}{\pi r}\right) \left(\tan^{-1}\left(\frac{r}{\sqrt{\phi}}\right) - \frac{r\sqrt{\phi}}{r^2 + \phi}\right)}. \] 
(18)

Let us now write down the metric potential conveniently in terms of the shape function $b(r)$ as follows:

\[ e^\lambda = \frac{1}{1 - b(r)}. \]  
(19)

where $b(r)$ is given by

\[ b(r) = \left(\frac{4M}{\pi}\right) \left[\tan^{-1}\left(\frac{r}{\sqrt{\phi}}\right) - \frac{r\sqrt{\phi}}{r^2 + \phi}\right]. \]  
(20)

The behavioural effects of different aspects of the above shape function $b(r)$ and its derivative are shown in Figs. 1 - 4. Here from Fig. 1 we observe that the shape function is exponentially increasing, so $b'(r) > 0$. The throat of the wormhole is located at $r = r_0$, where $b(r) - r$ cuts the $r$-axis, which is numerically closer to 2 km [Fig. 2]. One can also observe that for $r > r_0$, $b(r) - r < 0$. This immediately implies that $b(r)/r < 1$ [Fig. 3] which is an essential requirement for a shape function. We have now the flare-out condition $b'(r_0) < 1$ for $r > r_0$. We also observe the asymptotic behaviour from Fig. 4 such that $b(r)/r \to 0$ as $r \to 0$. Unfortunately, this has the similar explanation as done in Ref. 20 that the redshift function does not approach zero as $r \to 0$ due to the conformal symmetry. This means the wormhole spacetime is not asymptotically flat and hence will have to be cut off at some radial distance which smoothly joins to an exterior vacuum solution.

One can now define the redshift function $f(r)$ as follows:

\[ f(r) = \ln(C_2 r). \]  
(21)
It can be observed from the above expression that the wormhole presented here is traversable one as redshift function remains finite.

The above solution should be matched with the exterior vacuum spacetime of the Schwarzschild type at some junction interface with radius $R$. Using this matching condition, one can easily find the value of unknown constant $C_2$ as

$$C_2 = \frac{e^{f(R)}}{R},$$

so that the redshift function now explicitly becomes

$$f(r) = \ln \left[ \frac{r e^{f(R)}}{R} \right].$$

The redshift function is therefore finite in the region $r_0 < r < R$, as required because this will prevent an event horizon.
Fig. 4 Diagram of the derivative of the shape function of the wormhole for the specific values of the parameters as $\phi = 0.01, M = 1$.

3.1 Volume integral quantifier

The total amount of averaged null energy condition (ANEC) violating matter in the spacetime with a cut-off of the stress energy at $R$ is given by the integral

$$I = \int_{r_0}^{R} (p_r + \rho) dV. \quad (24)$$

Using field Eqs. (4) and (5) one can get

$$p_r + \rho = \frac{1}{8\pi r} \left( 1 - \frac{b}{r} \right) \left[ \ln \left( \frac{re^2f}{r - b} \right) \right]', \quad (25)$$

which are graphically shown in Figs. 5 and 6. Here $\rho$ being positive, as is expected from Eq. (13), these are therefore in accordance with the physical properties of the fluid distribution.

Hence, by substituting (24) in (23), we get

$$I = \left[ (r - b) \ln \left( \frac{re^2f}{r - b} \right) \right]_{r_0}^{R} - \int_{r_0}^{R} \left( 1 - b' \right) \ln \left( \frac{re^2f}{r - b} \right) dr. \quad (26)$$

Since at the throat $r_0$, one has $b(r_0) = r_0$, therefore the above integral takes the following form as

$$I = -\int_{r_0}^{R} \left( 1 - b' \right) \ln \left( \frac{re^2f}{r - b} \right) dr, \quad (27)$$

which we have solved numerically as can not be done the integration in a straight way.

In Fig. 4 we have considered $M = 1, \phi = 0.01, r_0 =$ throat radius $= 1.865$, the upper limit $R$ is varying from 2 to 3. Regarding this integration some observations are as follows: (1) When we are taking $r_0$ less than 1.865, the value of the
Fig. 5 The variation of energy density is shown against $r$.

Fig. 6 The violation of weak energy condition is shown against $r$. 
The variation of $I$ is shown against $r$.

**Table 1** Data for plotting FIG. 7

| Upper limit $R$ | Value of $I$       |
|-----------------|--------------------|
| 2.0             | -0.718869          |
| 2.1             | -1.140190          |
| 2.2             | -1.526710          |
| 2.3             | -1.890460          |
| 2.4             | -2.237580          |
| 2.5             | -2.571790          |
| 2.6             | -2.895580          |
| 2.7             | -3.210700          |
| 2.8             | -3.518450          |
| 2.9             | -3.819850          |
| 3.0             | -4.115690          |

Integration become complex; (2) The imaginary part of that complex value is not changing with the change of the upper limit for any particular value of lower limit less than the throat radius $r_0$; (3) The imaginary part of that complex value is not changing with the change of the value of $f$ for a particular value of upper limit and a particular value of lower limit less than the throat radius $r_0$; (4) The numerical value of the integration becomes real when the value of the throat radius $r_0$ is 1.865 or greater than 1.865.
3.2 Stability of the model

Following the suggestion of Ponce de Leon [37], we write the Tolman-Oppenheimer-Volkov (TOV) equation in the following form

\[-\frac{M_G}{r^2} (\rho + p_r) \frac{\lambda''}{r^2} - \frac{dp_r}{dr} + \frac{2}{r} (p_t - p_r) = 0, \tag{28}\]

where \(M_G = M_G(r)\) is the effective gravitational mass within the region from \(r_0\) up to the radius \(r\) and is given by

\[M_G(r) = \frac{1}{2} r^2 e^{\frac{r^2}{2} \nu'} \tag{29}\]

For equilibrium the above Eq. (29) can be easily written as

\[F_g + F_h + F_a = 0, \tag{30}\]

where

\[F_g = -\frac{\nu'}{2} \rho \frac{\lambda''}{r^2} - \frac{M_G}{\pi^2 r^3 (r^2 + \phi)} \tag{31}\]

\[F_h = -\frac{dp_r}{dr} = -\frac{\pi^2 r^3}{2\pi^2 r^3 (r^2 + \phi)} + \frac{3M\sqrt{\phi}}{2\pi^2 r^3} \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) \tag{32}\]

\[F_a = \frac{2}{r} (p_t - p_r) = -\frac{1}{2\pi^2 r^3} - \frac{2M\sqrt{\phi}}{\pi^2 r^3 (r^2 + \phi)} \tag{33}\]

From the Fig. 8 it can be observed that stability of the system has been attained by gravitational and anisotropic forces against hydrostatic force. However, initially instability prevails to some extent and then gradually they balance each other at around 4 km radius of the throat.

3.3 Active gravitational mass

The active gravitational mass within the region from the throat \(r_0\) up to the radius \(R\) can be found as

\[M_{active} = 4\pi \int_{r_0}^{R} \rho r^2 \frac{dr}{2} = \frac{2M}{\pi} \left[ \tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) - \frac{r\sqrt{\phi}}{r^2 + \phi} \right]_{r_0}^{R} \tag{34}\]

We observe here that the active gravitational mass \(M_{active}\) of the wormhole is positive under the constraint \(\tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) > \frac{r\sqrt{\phi}}{r^2 + \phi}\) and also the nature of variation is physically acceptable as can be seen from Fig. 9.
Fig. 8 The variation of forces are shown against $r$.

Fig. 9 The variation of $M_{\text{active}}$ is shown against $r$. 
3.4 Total gravitational energy

Using the prescription given by Lyndell-Bell et al. [38] and Nandi et al. [39], we calculate the total gravitational energy of the wormhole as

\[ E_g = \frac{1}{2} \int_{r_0}^{R} [1 - \sqrt{g_{rr}}] \rho_{eff} r^2 dr + \frac{r_0}{2}, \]  

(35)

where the second part is the contribution from the effective gravitational mass. The range of the integration is considered here from the throat \( r_0 \) to the embedded radial space of the wormhole geometry. We have solved the above Eq. (35) numerically.

In Fig. 10 we have considered \( M = 1, \phi = 0.01, r_0 = \) throat radius = 2, the upper limit \( R \) is varying from 2 to 3. Here our observations are as follows: (1) When we are taking \( r_0 \) less than 1.865, the value of the integration become complex; (2) The numerical value of the integration becomes real when the value of the throat radius \( r_0 \) is 1.865 or greater than 1.865. This real and positive value, \( E_g > 0 \), at once indicates that which means that there is a repulsion around the throat. Obviously this result is expected for construction of a physically valid wormhole to maintain stability of the fluid distribution.

4 Concluding remarks

In the present paper we have considered anisotropic real matter source for constructing new wormhole solutions. The background geometry is inspired by non-commutativity along with conformal Killing vectors. Speciality of this noncommutative geometry is to replace point-like structure of gravitational source by
Table 2 Data for plotting FIG. 10

| Upper limit R | Value of $E_g$ |
|---------------|---------------|
| 2.0           | 0.931300      |
| 2.1           | 0.931002      |
| 2.2           | 0.930800      |
| 2.3           | 0.930651      |
| 2.4           | 0.930534      |
| 2.5           | 0.930441      |
| 2.6           | 0.930364      |
| 2.7           | 0.930300      |
| 2.8           | 0.930246      |
| 2.9           | 0.930200      |
| 3.0           | 0.930160      |

The main observations of the present study therefore are as follows:

(1) The stability of the wormhole has been attained by gravitational and anisotropic forces against hydrostatic force. However, initially instability prevails to some extent and then gradually they balance each other at around 4 km radius of the throat.

(2) The active gravitational mass $M_{\text{active}}$ of the wormhole is positive under the constraint $\tan^{-1} \left( \frac{r}{\sqrt{\phi}} \right) > \frac{r\sqrt{\phi}}{r^2 + \phi}$ as is expected from the physical point of view.

(3) Since the total gravitational energy, $E_g > 0$, there is a repulsion around the throat which is expected usually for stable configuration of a wormhole.

As stated earlier in the text, in the present work we employ energy density given by Rizzo-Mehdipour-Nicolini [34,35,36] instead of Nicolini-Smailagic-Spallucci type [18]. However, our overall observation is that in our present approach the solutions and properties of the model are physically valid and interesting as much as in the former approach. As a special mention we would like to look at the Fig. 1 where we observe that the shape function is exponentially increasing instead of monotone increase as in the former case (Fig. 2 of Ref. [20]) whereas the redshift function does not approach zero as $r \to 0$ due to the conformal symmetry in both the approaches. So, exploration can be done with some other rigorous studies between the two approaches, i.e. Refs. [18] and [35], which can be sought for in a future project.
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