Reconstructing Supersymmetric Contribution to Muon Anomalous Magnetic Dipole Moment at ILC

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Abstract

We study the possibility to determine the supersymmetric (SUSY) contribution to the muon anomalous magnetic dipole moment by using ILC measurements of the properties of superparticles. Assuming that the contribution is as large as the current discrepancy between the result of the Brookhaven E821 experiment and the standard-model prediction, we discuss how and how accurately the SUSY contribution can be reconstructed. We will show that, in a sample point, the reconstruction can be performed with the accuracy of $\sim 13\%$ with the center-of-mass energy 500 GeV and the integrated luminosity $\sim 500–1000 \text{fb}^{-1}$.

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1 Introduction

It has been known that there exists notable discrepancy between the experimentally measured and theoretically predicted values of the muon anomalous magnetic dipole moment \( g-2 \). The Brookhaven E821 experiment [1] reported, for \( a_\mu = (g-2)/2 \),

\[
a_\mu^{(\text{exp})} = (11 659 208.9 \pm 6.3) \times 10^{-10}.
\]

(1)

There are several theoretical estimates of the standard-model (SM) value of the muon \( g-2 \). Based on the analysis of Refs. [2] and [3] for the hadronic vacuum polarization, the predictions are

\[
a_\mu^{(\text{SM})} = \begin{cases} 
(11 659 182.8 \pm 5.0) \times 10^{-10}, & [2] \\
(11 659 180.2 \pm 4.9) \times 10^{-10}, & [3]
\end{cases}
\]

(2)

where we take account of the five-loop QED calculation [4] and the latest update of the electroweak contribution [5]. Thus, the difference is estimated as

\[
\Delta a_\mu \equiv a_\mu^{(\text{exp})} - a_\mu^{(\text{SM})} = \begin{cases} 
(26.1 \pm 8.0) \times 10^{-10}, & [2] \\
(28.7 \pm 8.0) \times 10^{-10}, & [3]
\end{cases}
\]

(3)

Hence, there exists more than 3-\( \sigma \) discrepancy between the experimental and theoretical values. We call this discrepancy as “muon \( g-2 \) anomaly.” The origin of the muon \( g-2 \) anomaly is yet unknown.

If low-energy supersymmetry (SUSY) exists, the SUSY contribution to the muon \( g-2 \), denoted as \( a_\mu^{(\text{SUSY})} \), can be sizable. In particular, when \( \tan \beta \), which is a ratio of the vacuum expectation values of up- and down-type Higgses, is relatively large, \( a_\mu^{(\text{SUSY})} \) can be easily as large as \( \Delta a_\mu \) [6–8]. Thus, it is possible that the muon \( g-2 \) anomaly originates in the SUSY contribution. The primary purpose of this letter is to point out that we may have a chance to test this possibility by reconstructing \( a_\mu^{(\text{SUSY})} \), if superparticles are found in future collider experiments, and if their properties are determined.

At the leading order, the SUSY contribution to the muon \( g-2 \) is composed of smuon–neutralino and sneutrino–chargino loop diagrams. In order to reconstruct \( a_\mu^{(\text{SUSY})} \), it is necessary to understand properties of sleptons, in particular, those of smuons. Unfortunately, they may not be well studied at LHC. On the contrary, once the International Linear Collider (ILC) [9] is built, it is possible to determine them precisely as long as the superparticles are within the kinematical reach.

In this letter, we raise a question how and how accurately the SUSY contribution to the muon \( g-2 \) can be reconstructed by using ILC measurements of the parameters of the minimal supersymmetric standard model (MSSM). We assume that the muon \( g-2 \) anomaly is due to the SUSY contribution. Since the contribution depends on MSSM parameters, we concentrate on a particular case where it is dominated by so-called Bino diagram. Such a setup is especially interesting, because sleptons are expected to be within the kinematical...
reach of ILC [10]. It will be shown that $a_\mu^{(\text{SUSY})}$ can be reconstructed with the accuracy of $\sim 13\%$ for the sample point we adopt, once ILC runs at the center-of-mass energy $\sqrt{s} = 500\text{ GeV}$ and accumulates the integrated luminosity $L \sim 500$–1000 fb$^{-1}$.

2 Framework

Let us first summarize the framework of the analysis. The SUSY contribution to the muon $g - 2$ strongly depends on MSSM parameters. In this letter, we concentrate on the case where it is dominated by so-called Bino diagram. This situation is realized if the Wino and Higgsino mass parameters are much larger than the Bino mass parameter. In this limit, the leading contribution is given by (cf. Ref. [8])

$$a_\mu^{(B)} = -\frac{g_Y^2}{16\pi^2} \frac{m_\mu M_1 m_\tilde{\mu}^{2}_{LR}}{m_\tilde{\mu}_{1}^{2} m_\tilde{\mu}_{2}^{2}} f_N \left(\frac{m_\tilde{\mu}_{1}^{2}}{M_1^{2}}, \frac{m_\tilde{\mu}_{2}^{2}}{M_1^{2}}\right). \tag{4}$$

In the expression, $M_1$ is the Bino mass parameter, $m_\tilde{\mu}^{2}_A$ ($A = 1, 2$) is the $A$-th lightest smuon mass, and $g_Y$ is the gauge coupling constant for $U(1)_Y$, which comes from the Bino–(s)muons interactions. Also, $m_\tilde{\mu}^{2}_{LR}$ is the left-right mixing parameter in the smuon mass matrix. The loop function $f_N$ is defined as

$$f_N(x, y) = xy \left[ -\frac{3 + x + y + xy}{(x - 1)^2(y - 1)^2} + \frac{2x \ln x}{(x - y)(x - 1)^3} - \frac{2y \ln y}{(x - y)(y - 1)^3} \right]. \tag{5}$$

It is notable that $a_\mu^{(B)}$ can be as large as $\Delta a_\mu$ especially when the Higgsinos are heavy, since $m_\tilde{\mu}^{2}_{LR}$ is enhanced when $\mu \tan \beta$ is large, where $\mu$ is the Higgsino mass parameter. In contrast, the other contributions to the muon $g - 2$, including those from the second-lightest or heavier neutralino, are suppressed if the Higgsinos are decoupled.

The contribution $a_\mu^{(B)}$ can be reconstructed if the Bino mass, smuon masses, and the left-right mixing parameter $m_\tilde{\mu}^{2}_{LR}$ are known. As we will see below, they are expected to be determined very accurately at ILC, if the sleptons and the Bino-like neutralino are within the kinematical reach. In fact, since the ILC measurements are very precise, the leading approximation given in Eq. (1) may not be accurate enough to be compared with the ILC analyses. In addition, there is a subtlety in relating the gaugino coupling constants with the gauge coupling constants in particular when some of the superparticles are relatively heavy [10][13]. Thus, we will use more complete formula for $a_\mu^{(\text{SUSY})}$.

The full one-loop level formula for $a_\mu^{(\text{SUSY})}$ consists of the contribution from smuon–neutralino loop diagrams and sneutrino–chargino diagrams. The smuon–neutralino contribution is given by [8]

$$a_\mu^{(\tilde{\nu})} = \frac{1}{16\pi^2} \sum_{A,X} \frac{m_\tilde{\nu}^{2}_A}{m_\tilde{\mu}^{2}_A} \left[ -\frac{1}{12} \left[(N_{AX}^{\mu \nu})^2 + (N_{AX}^{\mu \nu})^2\right] F_1^{N}(x_{AX}) - \frac{m_\tilde{\nu}^{2}_X N_{AX}^{\mu \nu} N_{AX}^{\mu \nu} F_2^{N}(x_{AX}) \right], \tag{6}$$
which includes the leading contribution \( \alpha_{\mu}^{(b)} \). Here, \( m_{\tilde{\chi}_X}^0 \) (\( X = 1–4 \)) is the neutralino mass, \( x_{AX} = m_{\tilde{\chi}_X}^0 / m_{\tilde{\mu}_A} \), and the loop functions are

\[
F_1^N(x) = \frac{2}{(1-x)^4} \left[ 1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x \right],
\]
(7)

\[
F_2^N(x) = \frac{3}{(1-x)^3} \left[ 1 - x^2 + 2x \ln x \right].
\]
(8)

In addition, \( N_{A\tilde{\mu}}^\ell \) and \( N_{A\tilde{\mu}}^\ell \) are neutralino–muon–smuon coupling constants. Parameterizing interactions of neutralinos as

\[
\mathcal{L}_{\text{int}} = \sum_{\ell=e,\mu,\tau} \sum_{A,X} \tilde{\chi}_X^0 (N_{A\tilde{\mu}}^\ell L + N_{A\tilde{\mu}}^\ell R) \ell \tilde{\ell}^\dagger A + \text{h.c.},
\]
(9)

the coefficients are

\[
N_{A\tilde{\mu}}^\ell = \frac{1}{\sqrt{2}} \hat{g}_{Y,L} (U_{\tilde{\chi}_X}^0)_{XB} (U_{\ell})_{AL} + \frac{1}{\sqrt{2}} \hat{g}_2 (U_{\tilde{\chi}_X}^0)_{XW} (U_{\ell})_{AL} - y_{\ell} (U_{\tilde{\chi}_X}^0)_{XH_d} (U_{\ell})_{AR},
\]
(10)

\[
N_{A\tilde{\mu}}^\ell = -\sqrt{2} \hat{g}_{Y,R} (U_{\tilde{\chi}_X}^0)_{XB} (U_{\ell})_{AR} - y_{\ell} (U_{\tilde{\chi}_X}^0)_{XH_d} (U_{\ell})_{AL}.
\]
(11)

Here, \( y_{\ell} \) is the Yukawa coupling constants in the superpotential. The unitary matrices \( U_{\chi}^0 \) and \( U_{\ell} \) diagonalize the mass matrices of neutralinos and sleptons, respectively. It is assumed that soft SUSY breaking parameters of the sleptons are independent of the generation, and all the complex phases of the SUSY parameters are negligibly small, in order to avoid too large lepton-flavor violations and electric dipole moments. Then, the slepton masses are obtained from the slepton mass matrix,

\[
\mathcal{M}_{\tilde{\ell}}^2 = \begin{pmatrix} m_{\tilde{\ell} LL}^2 & m_{\tilde{\ell} LR}^2 \\ m_{\tilde{\ell} LR}^2 & m_{\tilde{\ell} RR}^2 \end{pmatrix},
\]
(12)

which is diagonalized by the following unitary matrix,

\[
U_{\tilde{\ell}} = \begin{pmatrix} \cos \theta_{\tilde{\ell}} & \sin \theta_{\tilde{\ell}} \\ -\sin \theta_{\tilde{\ell}} & \cos \theta_{\tilde{\ell}} \end{pmatrix}.
\]
(13)

The slepton mixing angle satisfies the relation,

\[
m_{\tilde{\ell} LR}^2 = \frac{1}{2} (m_{\tilde{\ell} 1}^2 - m_{\tilde{\ell} 2}^2) \sin 2\theta_{\tilde{\ell}}.
\]
(14)

This relation will play an important role in the following discussion.

It should be noticed that the coupling constants for the gaugino–lepton–slepton vertices, or the gaugino coupling constants, deviate from the ordinary gauge coupling constants [10–14]. In Eqs. (10) and (11), the parameters \( \hat{g}_{Y,L} \), \( \hat{g}_{Y,R} \), and \( \hat{g}_2 \) are introduced to take account of such an effect. In the SUSY limit, \( \hat{g}_{Y,L} = \hat{g}_{Y,R} = g_Y \) and \( \hat{g}_2 = g_2 \) are satisfied (with
Table 1: Parameters and mass spectrum and at our sample point. The masses are in units of GeV, and \( \tilde{\ell} \) denotes selectrons and smuons.

| Parameters | \( m_{\tilde{\ell}_1} \) | \( m_{\tilde{\ell}_2} \) | \( m_{\tilde{\tau}_1} \) | \( m_{\tilde{\tau}_2} \) | \( m_{\tilde{\chi}_0^1} \) | \( \sin \theta_{\tilde{\mu}} \) | \( \sin \theta_{\tilde{\tau}} \) | \( a_{\mu}^{(ILC)} \) |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Values     | 126            | 200            | 108            | 210            | 90             | 0.027          | 0.36           | \( 2.6 \times 10^{-9} \) |

\( g_Y \) and \( g_2 \) being the gauge coupling constants of U(1)\(_Y\) and SU(2)\(_L\), respectively. These relations are violated when some superparticles are (much) heavier than the sleptons. In the case where all the superparticles except for sleptons and the Bino are heavy, we obtain the following approximate formula for \( \tilde{g}_{Y,L} \) and \( \tilde{g}_{Y,R} \) (cf. Ref. [10, 15]):

\[
\tilde{g}_{Y,L}(Q) \simeq g_Y(Q) \left[ 1 + \frac{1}{4\pi} \left( 4\alpha_Y \ln \frac{M_{\text{soft}}}{Q} - \frac{1}{6}\alpha_Y \ln \frac{M_H}{Q} + \frac{9}{4}\alpha_2 \ln \frac{M_W}{Q} \right) \right],
\]

\[
\tilde{g}_{Y,R}(Q) \simeq g_Y(Q) \left[ 1 + \frac{1}{4\pi} \left( 4\alpha_Y \ln \frac{M_{\text{soft}}}{Q} - \frac{1}{6}\alpha_Y \ln \frac{M_H}{Q} \right) \right],
\]

where \( M_{\text{soft}} \) is a mass scale of colored superparticles and heavy Higgses, \( M_H \) is the Higgsino mass, \( M_W \) is the Wino mass, and \( Q \sim m_\tilde{\ell} \) is an energy scale. The differences among \( g_Y, \tilde{g}_{Y,L} \) and \( \tilde{g}_{Y,R} \) can be \( \mathcal{O}(1\text{--}10)\% \) if \( M_{\text{soft}}, M_H \) and \( M_W \) are larger than \( \sim 1 \text{ TeV} \). Note that the leading contribution of (3) is proportional to the product \( \tilde{g}_{Y,L}\tilde{g}_{Y,R} \) (cf. Eq. (4)). Since the corrections to the gaugino couplings can be sizable, both of the couplings should be determined directly at ILC. It is also noted that \( \tilde{g}_{Y,L}, \tilde{g}_{Y,R} \) and \( \tilde{g}_2 \) are universal for (at least) light generations.

In the following discussion, we choose a specific sample point to make our discussion concrete and quantitative. The mass spectrum at the sample point is summarized in Table 1. All the sleptons and the lightest neutralino are within the reach of ILC with \( \sqrt{s} = 500 \text{ GeV} \). Their masses are set to be close to those of the SPS1a' benchmark point [16], so that results of the previous ILC studies can be applied. The lighter sleptons are chosen to be almost left-handed in order to avoid LHC limits (see below). The lightest neutralino mass is 90 GeV, which is the lightest superparticle among the MSSM ones including sneutrinos. Other superparticles such as colored ones as well as Winos and Higgsinos are assumed to be so heavy that they are not observed at LHC nor ILC (so that their masses are different from those for SPS1a').

Trilinear couplings of sleptons, \( A_{\tilde{\ell}} \), are set to be zero. The left-right mixing parameter, \( m_{\tilde{\mu}LR}^2 \) (or equivalently \( \mu \tan \beta \)) is chosen to realize that \( a_{\mu}^{(ILC)} \) defined in Eq. (17) becomes equal to \( 2.6 \times 10^{-9} \), which is close to the central value of the current discrepancies (3); \( \mu \tan \beta = 6.1 \times 10^3 \) GeV.

The mass spectrum is consistent with present collider limits. Light sleptons decaying to the lightest neutralino are searched for by studying the di-lepton signatures at LHC [17,18]. Our sample point is not excluded because masses of the left-handed selectron and smuon are

\# This setup is minimal to reconstruct the SUSY contributions to the muon \( g - 2 \). If some of the heavy superparticles such as Winos would be additionally discovered, the reconstruction could be improved.
close to that of the neutralino. Also, constraints on the right-handed ones are weak, since the production cross sections are small. On the other hand, collider limits on the stau mass is weaker as \( m_{\tilde{\tau}_1} > 81.9 \text{ GeV} \) at 95% CL by LEP [19]. Exclusions from the three-lepton searches at LHC [18, 20] are also negligible, since Winos and Higgsinos are heavy.

### 3 Fun with ILC

In the rest of this letter, we discuss how and how accurately the SUSY contribution to the muon \( g - 2 \) is determined at ILC. At the sample point, only the sleptons and the lightest neutralino are within the reach of ILC. The observed neutralino is identified as Bino-like by absent signals of charginos, since neutral Winos or Higgsinos are associated by charged partners. Let us define the following quantity (cf. Eq. (6)),

\[
a_{\mu}^{(ILC)} \equiv \frac{1}{16\pi^2} \sum_A \frac{m_{\tilde{\mu}}^2}{m_{\tilde{\mu}A}^2} \left[ \frac{1}{12} \left( N_{\tilde{\mu}A}^L \right)^2 + \left( N_{\tilde{\mu}A}^R \right)^2 \right] F_1^N(x_{A1}) - \frac{m_{\tilde{\chi}_0}^2}{3m_{\tilde{\mu}}^2} \hat{N}_{\mu A}^L \hat{N}_{\mu A}^R F_2^N(x_{A1}),
\]

which depends only on ILC observables. The parameters are defined as

\[
\hat{N}_{\mu A}^L \equiv [N_{\mu A1}(U_{\chi 0}^L)_{1\tilde{\mu}_d}^{-0}] = \frac{1}{\sqrt{2}} \tilde{g}_{1,L}^{(\text{eff})} (U_{\tilde{\mu}})_{AL},
\]

\[
\hat{N}_{\mu A}^R \equiv [N_{\mu A1}(U_{\chi 0}^R)_{1\tilde{\mu}_d}^{-0}] = -\sqrt{2} \tilde{g}_{1,R}^{(\text{eff})} (U_{\tilde{\mu}})_{AR},
\]

where

\[
\tilde{g}_{1,L}^{(\text{eff})} \equiv \tilde{g}_{Y,L}(U_{\chi 0}^L)_{1\tilde{B}} + \tilde{g}_2(U_{\chi 0})_{1\tilde{W}},
\]

\[
\tilde{g}_{1,R}^{(\text{eff})} \equiv \tilde{g}_{Y,R}(U_{\chi 0}^R)_{1\tilde{B}}.
\]

The smuon mixing angle can be determined if the left-right mixing parameter of the smuon, \( m_{\tilde{\mu}LR}^2 \), as well as the smuon mass eigenvalues are determined, as noticed from Eq. (14). Thus, \( a_{\mu}^{(ILC)} \) can be reconstructed if the following quantities are known:

\[
m_{\tilde{\mu}1}, m_{\tilde{\mu}2}, m_{\tilde{\mu}LR}^2, m_{\tilde{\chi}_0}^0, \tilde{g}_{1,L}^{(\text{eff})}, \tilde{g}_{1,R}^{(\text{eff})}.
\]

In the following of this section, we consider the reconstruction of \( a_{\mu}^{(ILC)} \) with the determinations of these parameters at ILC.

The full SUSY contribution \( a_{\mu}^{(\text{SUSY})} \) contains the contribution from charginos and heavier neutralinos. Difference between \( a_{\mu}^{(\text{SUSY})} \) and \( a_{\mu}^{(ILC)} \) will be discussed in Sec. 3.4. We show that, at the sample point, future experiments can confirm that \( a_{\mu}^{(\text{SUSY})} \) is dominated by \( a_{\mu}^{(ILC)} \).
3.1 Determination of the left-right mixing

One of the crucial parameters to calculate \( a^{(ILC)}_\mu \) is the left-right mixing parameter \( m_{\tau LR}^2 \). In order to reconstruct the smuon unitary matrix \( U_{\tilde{\mu}} \), it is necessary to determine the mixing angle \( \theta_{\tilde{\mu}} \) or \( m_{\tilde{\mu} LR}^2 \). Although smuons are produced at ILC, it is challenging to determine them from smuon measurements. Importantly, however, \( m_{\tilde{\mu} LR}^2 \) can be obtained from studies of staus. The mixing parameters are scaled by the lepton masses as

\[
m_{\tilde{\mu} LR}^2 = \frac{m_{\mu}}{m_\tau} m_{\tau LR}^2.
\]

This relation is valid in the limit of \( A_{\tilde{\tau}} \ll \mu \tan \beta \), where \( A_{\tilde{\tau}} \) is the trilinear coupling constant of the slepton \( \tilde{\tau} \) normalized by the corresponding Yukawa coupling constant. This is the case at our sample point. Using Eq. \([12]\), \( m_{\tau LR}^2 \) is determined if \( \sin 2\theta_{\tau LR} \) as well as the mass eigenvalues, \( m_{\tau 1} \) and \( m_{\tau 2} \), are measured. Its accuracy is estimated as

\[
(\delta m_{\tau LR}^2)^2 = \left( \frac{\partial m_{\tau LR}^2}{\partial m_{\tau 1}} \right)^2 (\delta m_{\tau 1})^2 + \left( \frac{\partial m_{\tau LR}^2}{\partial m_{\tau 2}} \right)^2 (\delta m_{\tau 2})^2 + \left( \frac{\partial m_{\tau LR}^2}{\partial \sin 2\theta_{\tau LR}} \right)^2 (\delta \sin 2\theta_{\tau LR})^2,
\]

where the derivatives are evaluated at the sample point. In particular, \( \sin 2\theta_{\tau LR} \) can be naturally as large as \( O(0.1) \) in the parameter region where \( a^{(SUSY)}_\mu \simeq \Delta a_\mu \).

First of all, the stau mass eigenvalues, \( m_{\tau 1} \) and \( m_{\tau 2} \), can be determined by measuring the endpoints of the energy distribution of \( \tau \) decay products from the stau decay, \( \tilde{\tau}^\pm \to \tau^\pm \tilde{\chi}_0^1 \). For such an analysis, information about the lightest neutralino mass is also needed; measurement of \( m_{\tilde{\chi}_0^1} \) will be discussed in the next subsection.

The measurement of stau masses at ILC is discussed in detail in Ref. \([21]\). It is claimed that the mass can be determined with the accuracy of \( \sim 0.1\% \) (3\%) for lighter (heavier) stau with \( \sqrt{s} = 500 \text{ GeV} \), \( (P_{\tau^+}, P_{\tau^-}) = (-0.3, +0.8) \) and the integrated luminosity \( \mathcal{L} = 500 \text{ fb}^{-1} \). Here, \( P_{\tau^-} \) (\( P_{\tau^+} \)) is the degree of transverse polarization of the electron (positron) beam. The right- (left-) handed polarization corresponds to \( P_e = +1 \) \((-1) \). The analysis depends on details of the mass spectrum. In Ref. \([21]\), the SPS1a' benchmark point is adopted, and signal regions are optimized for it. In particular, lighter (heavier) stau is almost right-handed (left-handed), and the neutralino mass is 98 GeV. These are different from our sample point and could affect the accuracy. In fact, the energy profile of the decay products of \( \tau \) depends on the helicity of \( \tau \) \([22]\). For instance, jet energy from \( \tau \to \pi \nu \) is likely to be harder for \( \tau_R \) compared to \( \tau_L \) \([23]\). Also, with the polarization used in Ref. \([21]\), the production cross section of the lighter (heavier) stau at our model point is smaller (larger) than those at SPS1a'. On the other hand, the endpoint energies of \( \tau \)-jet increase, as \( m_{\tilde{\chi}_0^1} \) decreases (cf. Ref. \([23]\)). Then, the contamination of the background due to the process \( \gamma\gamma \to \tau^+\tau^- \) is reduced \([21]\). In this letter, we simply adopt the accuracy of 0.1\% and 3\% as our canonical values for the mass measurements of the staus\(^\#2\)

\(^{\#2}\) Dedicated studies of the threshold production of \( \tilde{\tau}_2 \) can improve the accuracy of its mass measurement \([24, 25]\). At the Snowmass SM2 benchmark point, which is close to the SPS1a point, \( \delta m_{\tau 2} \sim 1 \text{ GeV} \) is available for \( m_{\tau 2} = 206 \text{ GeV} \), where \( \sqrt{s} = 500 \text{ GeV} \) and \( \mathcal{L} = 1000 \text{ fb}^{-1} \) with the electron polarization of 80\%, while no polarization for the positron.
Figure 1: Accuracies of the determination of $\sin 2\theta_{\tilde{\tau}}$ from the measurement of the cross section $\sigma(e^+e^- \rightarrow \tilde{\tau}_i\tilde{\tau}_j)$ as a function of the stau mixing angle with the accuracy of the cross section determination of 10%, 5%, 3% and 1% from top to bottom. The mass measurement is assumed to be sufficiently precise.

Next, the mixing angle $\theta_{\tilde{\tau}}$ can be determined from the measurements of the cross sections of stau pair production processes. The cross sections are given by

$$\sigma(e^+e^- \rightarrow \tilde{\tau}_i\tilde{\tau}_j) = \frac{8\pi\alpha^2}{3s}v^3c_{ij}^2\frac{\Delta Z}{\sin^4 2\theta_W}(P_+L^2 + P_+R^2)$$

$$+ \delta_{ij}\frac{1}{16}(P_++P_-) + \delta_{ij}c_{ij}\frac{\Delta Z}{2\sin^2 2\theta_W}(P_+L + P_-R),$$

where the parameters are defined as

$$v^2 = [1 - (m_{\tilde{\tau}_i} + m_{\tilde{\tau}_j})^2/s][1 - (m_{\tilde{\tau}_i} - m_{\tilde{\tau}_j})^2/s],$$

$$\Delta Z = s/(s - m_Z^2),$$

$$c_{11/22} = \frac{1}{2}[L + R \pm (L - R) \cos 2\theta_{\tilde{\tau}}],$$

$$c_{12} = c_{21} = \frac{1}{2}(L - R) \sin 2\theta_{\tilde{\tau}},$$

$$L = \frac{1}{2} + \sin^2 \theta_W,$$

$$R = \sin^2 \theta_W.$$  

#3Alternatively, the stau mixing angle can be determined by measuring the $\tau$ polarization from energy profile of its decay products [12]. However, the cross section measurement provides a better resolution [21].
The beam polarizations are parameterized as $P_{\pm} = (1 \mp P_{e-})(1 \pm P_{e+})$.

We consider productions of lighter staus to determine $\theta_\tau$. The cross section, $\sigma(\tilde{\tau}_1) = \sigma(e^+e^- \to \tilde{\tau}_1^+\tilde{\tau}_1^-)$, depends on $m_{\tilde{\tau}^i}$ and $\theta_\tau$. The accuracy of the measurement of the stau mixing angle is estimated as

$$
(\delta \sin 2\theta_\tau)^2 = \left(\frac{\partial \sin 2\theta_\tau}{\partial \sigma(\tilde{\tau}_1)}\right)^2 (\delta \sigma(\tilde{\tau}_1))^2 + \left(\frac{\partial \sin 2\theta_\tau}{\partial m_{\tilde{\tau}^1}}\right)^2 (\delta m_{\tilde{\tau}^1})^2.
$$

In the sample point, the error is dominated by that of the cross section. The stau mass contributes to the cross section only through $v$, and $m_{\tilde{\tau}^1}$ is (much) smaller than $\sqrt{s}$. Further, mass of $\tilde{\tau}_1$ can be precisely measured, as mentioned above. According to Ref. [21], the cross section for $e^+e^- \to \tilde{\tau}_1^+\tilde{\tau}_1^-$ can be measured with the accuracy of 3.1\% for SPS1a'. Here, the uncertainty dominates the signal statistics and SUSY background, while those of the luminosity and efficiencies are assumed to be negligible. In our sample point, the production cross section is $\sigma(\tilde{\tau}_1) = 54$ fb with $\sqrt{s} = 500$ GeV and $(P_{e+}, P_{e-}) = (-0.3, +0.8)$, which is smaller than $\sigma(\tilde{\tau}_1) = 135$ fb at SPS1a'. By supposing the same acceptance as Ref. [21], the statistical uncertainty increases from $1/\sqrt{N_{\text{sig}}\text{SPS1a}'} = 2.1$\% to 3.4\% with $L = 500$ fb$^{-1}$, where $N_{\text{sig}}$ is the number of the signals accepted by selections. On the other hand, our setup is free from the SUSY background, since only sleptons and the lightest neutralino are produced at our sample point. Thus, the accuracy of the cross section measurement is estimated to be $\delta \sigma(\tilde{\tau}_1)/\sigma(\tilde{\tau}_1) = 3.4$\%.

In Fig. 1, the accuracy of the measurement of $\sin 2\theta_\tau$ is shown. The accuracy is sensitive to the mixing angle. It becomes better when the angle approaches maximal, $\theta_\tau = \pi/4$. This is because $\sigma(\tilde{\tau}_1)$ depends on $\theta_\tau$ via $\cos 2\theta_\tau$. In the sample point, where $\sin 2\theta_\tau = 0.67$, it is expected that $\sin 2\theta_\tau$ can be determined with the accuracy of 9\% by applying $\delta \sigma(\tilde{\tau}_1)/\sigma(\tilde{\tau}_1) = 3.4$\% (and $\delta m_{\tilde{\tau}^1}/m_{\tilde{\tau}^1} \sim 0.1\%$).

Finally, by combining the uncertainties of the determinations of $m_{\tilde{\tau}^1}$, $m_{\tilde{\tau}^2}$ and $\sin 2\theta_\tau$, the accuracy of the $m_{\tilde{\tau}^1}^2$ determination is estimated by Eq. [21]. The uncertainty due to the measurement of the lighter stau mass is negligible, since $m_{\tilde{\tau}^1}$ and $m_{\tilde{\tau}^2}$ contribute to $m_{\tilde{\tau}^1}^2$ in the combination of $(m_{\tilde{\tau}^1}^2 - m_{\tilde{\tau}^2}^2)$, and the uncertainty of the heavier stau mass is larger than that of the lighter one. Also, correlation of the errors, $\delta m_{\tilde{\tau}^1}$ and $\delta \sin 2\theta_\tau$, is negligible, since $\delta m_{\tilde{\tau}^1}$ barely affects $\delta \sin 2\theta_\tau$ when $\delta m_{\tilde{\tau}^1}$ is sufficiently small. As a result, we obtain $\delta m_{\tilde{\tau}^1}^2/m_{\tilde{\tau}^1}^2 = 12\%$ with $\delta m_{\tilde{\tau}^2}/m_{\tilde{\tau}^2} = 3\%$ and $\delta \sin 2\theta_\tau/\sin 2\theta_\tau = 9\%$. From the relation [23], $m_{\tilde{\mu}^1}^2$ is determined with the same accuracy,

$$
\delta m_{\tilde{\mu}^1}^2/m_{\tilde{\mu}^1}^2 = 12\%,
$$

in the sample point, where $m_{\tilde{\mu}^1}^2 = -645$ GeV$^2$.

The sign of $\sin 2\theta_\tau$ is not determined by the cross section measurements. It corresponds to the sign of $\mu \tan \beta$. Consequently, the reconstruction of the SUSY contribution to the

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#4 The signal region can be optimized for our sample point. Since there is no SUSY background, the acceptance could be enhanced and the uncertainty would be reduced.
muon $g - 2$ is possible with two fold ambiguity. We take the sign of $\mu \tan \beta$ so that the muon $g - 2$ anomaly is solved.

There are several comments in order. (i) In the above analysis, we considered the process $e^+ e^- \rightarrow \tildell_1 \tildell_1$. The determination of the slepton mixing angle is possibly improved if the production cross section of a pair of $\tildell_1$ and $\tildell_2$ is measured accurately \cite{21}, since it is proportional to $\sin^2 2\theta_{\ell R}$. In the sample point, the cross section for the process $e^+ e^- \rightarrow \tildell_1 \tildell_2$ becomes 2.7 fb (3.6 fb) with $\sqrt{s} = 500$ GeV and $(P_{e+}, P_{e-}) = (-0.3, +0.8) ((P_{e+}, P_{e-}) = (0.3, -0.8))$. However, we could not find studies about such a process. In particular, the acceptance of the signal events as well as the accuracy of the cross section measurement has not been known. Thus, in the present study, we do not use this process. (ii) The sneutrino mass is measured precisely, for instance, through the decay channel $\tilde{\ell} \rightarrow \ell \tilde{\chi}_i^0$. In our sample point, the cross section for the process $\tilde{\ell} \rightarrow \ell \tilde{\chi}_i^0$ is determined if the sneutrino mass is measured precisely, for instance, through the decay channel $\tilde{\ell} \rightarrow \ell \tilde{\chi}_i^0$. In such a process, we do not use this process. (iii) The (approximate) chiralities of lighter and heavier smuons are fixed by the sign of $\cos 2\theta_{\ell R}$, which can be determined by measuring the smuon production cross sections. (iv) Eq. (23) can be violated if $A_{\ell}$ depends on generations \cite{24}. If they are comparable to the slepton masses, the violation is negligible compared to the accuracy of the measurement of $m_{\ell LR}^2$ at ILC. Let us suppose that $A_{\ell}$ differs by 100 GeV from $A_{\ell}$ in $m_{\ell LR}^2 = -m_\ell (\mu \tan \beta - A_{\ell})$. In the sample point, $m_{\ell LR}^2$ is mis-measured by $\sim 2\%$ if it is determined by Eq. (23). This is smaller than the above ILC uncertainty \cite{25}.

3.2 Mass determinations

Next, let us consider measurements of $m_{\tilde{\mu}_1}$, $m_{\tilde{\mu}_2}$ and $m_{\tilde{\chi}_1^0}$. If smuon masses are within the reach of ILC, they can be obtained from productions of the smuons that decay into neutralinos. The energy spectra of the muons produced by the smuon decay and the production threshold are sensitive to the masses \cite{28}. In Refs. \cite{25,29,30}, the accuracies are estimated to be $\delta m_{\tilde{\mu}_R} = 170$ MeV and $\delta m_{\chi_1^0} = 210$ MeV at the SPS1a benchmark point \cite{31}. Here, the masses are $m_{\tilde{\mu}_R} = 143$ GeV and $m_{\chi_1^0} = 96$ GeV with $\text{Br}(\tilde{\mu}_R \rightarrow \mu^0 \chi_1^0) = 100\%$. The analysis is based on $\sqrt{s} = 400$ GeV, $(P_{e+}, P_{e-}) = (-0.6, +0.8)$ and $L = 200$ fb$^{-1}$. Another study of the threshold scans yields $\delta m_{\tilde{\mu}_R} = 200$ MeV for $m_{\tilde{\mu}_R} = 135$ GeV by assuming 10 fb$^{-1}$ per each data point with $(P_{e+}, P_{e-}) = (+0.3, -0.8)$ \cite{25,32}. The uncertainties are statistically limited. The muon energy spectrum is independent of the smuon chirality, and the mass

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$^#5$ It is difficult to determine $A_{\ell}$ and $\mu \tan \beta$ individually in $m_{\ell LR}$ by the stau decays. In fact, it is possible if the Higgsinos are light \cite{26}. However, they are decoupled in our sample point. Alternatively, $\tan \beta$ is determined if the sneutrino mass is measured precisely, for instance, through the decay channel $\nu \rightarrow \tilde{\chi}_i^0 \tilde{\ell} \overline{\ell}$ (see Ref. \cite{27}). In our sample point, it is difficult to identify the sneutrinos, because they decay only to the lightest neutralino. $^#6$ The relations between the lepton masses and the Yukawa coupling constants are affected by SUSY radiative correction. The correction violates the relation Eq. (23) if the slepton soft masses depend on the generation. The violation is typically small. $^#7$ The neutralino mass can also be measured from the endpoints in the stau productions. However, the resolution is worse \cite{21}.
resolution is less dependent on the smuon-neutralino mass splitting \[33\]. Since the accuracy is limited by signal statistics, we expect $\tilde{\mu}_1$ has a better mass resolution in our sample point. At SPS1a, the production cross section is $\sigma(\tilde{\mu}_1) = 134$ fb with $\sqrt{s} = 400$ GeV, and $(P_{e+}, P_{e-}) = (-0.6, +0.8)$. In our sample point, it becomes $\sigma(\tilde{\mu}_1) = 154$ fb with $\sqrt{s} = 500$ GeV, and $(P_{e+}, P_{e-}) = (+0.3, -0.8)$.

The mass measurement of the heavier smuon is studied in detail at SPS1a by Ref. [34]. Here, the heavier smuon is almost left-handed, and $\sqrt{s} = 500$ GeV, $(P_{e+}, P_{e-}) = (+0.6, -0.8)$ and $\mathcal{L} = 500$ fb$^{-1}$ are used. The resolution can be $\delta m_{\tilde{\mu}_L} = 100$ MeV for $m_{\tilde{\mu}_L} = 190$ GeV and $m_{\tilde{\chi}_0} = 98$ GeV by studying the endpoints. At SPS1a', most of the produced $\tilde{\mu}_L$'s decay into the lightest neutralino and a muon. In our sample point, all $\tilde{\mu}_2$ decay into the lightest neutralino and a muon. The production cross section is $\sigma(\tilde{\mu}_2) = 80$ fb at SPS1a' for $\sqrt{s} = 500$ GeV and $(P_{e+}, P_{e-}) = (+0.6, -0.8)$, while it is $\sigma(\tilde{\mu}_2) = 44$ fb in our sample point with $\sqrt{s} = 500$ GeV and $(P_{e+}, P_{e-}) = (-0.3, +0.8)$. Thus, the statistical uncertainty is degraded by a factor 1.3. On the contrary, the above resolutions could be improved in our sample point, because SUSY background, for instance, from heavier neutralino productions, is suppressed. Finally, the accuracy of the neutralino mass measurement becomes better if studies about the selectron production processes are combined. In Ref. [33], it is claimed that $\delta m_{\tilde{\chi}_0} = 80$ MeV is achieved at SPS1a.

In the present analysis, we simply assume

$$\delta m_{\tilde{\mu}_1} = 200 \text{ MeV}, \quad \delta m_{\tilde{\mu}_2} = 200 \text{ MeV}, \quad \delta m_{\tilde{\chi}_0} = 100 \text{ MeV}, \quad (34)$$

at the sample point. Then, in the reconstruction of $a_{\mu}^{(ILC)}$, the uncertainties in the mass measurements of smuons and neutralino are less important than that of $m_{\tilde{\mu}_L}^2$.

### 3.3 Coupling measurements

The coupling constants $g_{1L}^{(eff)}$ and $g_{1R}^{(eff)}$ are hardly determined directly from the smuon production processes. Instead, they are available from selectron productions [12][13], because they are common in light generations. Since the Yukawa coupling constant of the electron is negligibly small, $(U_{\tilde{e}})_{11} = (U_{\tilde{e}})_{22} = 1$ holds with very high accuracy. (Thus, we call lighter and heavier selectrons as $\tilde{e}_L$ and $\tilde{e}_R$, respectively.) Consequently, we obtain

$$N_{11}^{\tilde{e}L} = \frac{1}{\sqrt{2}} g_{1,L}^{(eff)}, \quad N_{21}^{\tilde{e}R} = -\sqrt{2} g_{1,R}^{(eff)}. \quad (35)$$

Cross sections for the selectron production processes depend on $N_{11}^{\tilde{e}L}$ and $N_{21}^{\tilde{e}R}$ through the $t$-channel neutralino-exchange diagrams. Thus, $g_{1,L}^{(eff)}$ and $g_{1,R}^{(eff)}$ can be measured by studying the selectron production cross sections as long as contributions of heavier neutralinos are known.

In Refs. [35][37], it is claimed that the Bino coupling with the (s)electrons can be determined with the accuracy of 0.18% from the measurements of the production cross section of
Here, the beam configuration is $\sqrt{s} = 500$ GeV with $\mathcal{L} = 500$ fb$^{-1}$ and the polarizations of 80% (electron) and 50% (positron). In the analysis, the SPS1a benchmark point is adopted, in which the selectron mass is $m_{\tilde{e}_R} = 143$ GeV. Here, all the neutralino masses are assumed to be measured by their productions at ILC. The production cross section of $\tilde{e}_R^+\tilde{e}_R^-$ is very sensitive to $\tilde{g}_{1,R}^{(\text{eff})}$. It can be estimated that the accuracy of the measurement of the $\tilde{e}_R^+\tilde{e}_R^-$ cross section should be better than 0.9% to determine the coupling at the 0.18% level. We reinterpret the result of Refs. [35,36] to estimate how accurately $\tilde{g}_{1,R}^{(\text{eff})}$ can be measured in the sample point. Let us assume that the accuracy of the cross section measurement is limited by the signal statistics, and that the acceptance at our sample point is the same as that in SPS1a. We estimate that the precision of Refs. [35,36] is simply scaled by $\sqrt{N_{\text{sig}}}$. At SPS1a, the cross section is $\sigma(\tilde{e}_R^+\tilde{e}_R^-) = 809$ fb for $\sqrt{s} = 500$ GeV and $(P_{e+}, P_{e-}) = (-0.5, +0.8)$, while $\sigma(\tilde{e}_R^+\tilde{e}_R^-) = 316$ fb in our sample point for $\sqrt{s} = 500$ GeV and $(P_{e+}, P_{e-}) = (-0.3, +0.8)$ with assuming that Winos and Higgsinos are decoupled. Then, the experimental uncertainty of the cross section measurement is degraded to be about 1.5%. We emphasize that Winos and Higgsinos are assumed to be undiscovered in our sample point. In addition to the lightest neutralino, heavier neutralinos, which are mostly composed of Winos and Higgsinos, may be exchanged in the $t$-channel diagrams, and contribute to the selectron production cross sections. In the process $e^+e^- \rightarrow \tilde{e}_R^+\tilde{e}_R^-$, their contamination to the gaugino coupling constant measurement is very small, because they appear only through the mixing between the Bino and the Higgsinos. The direct interactions of the Higgsinos to the (s)electron are negligible due to a tiny coupling. In the case when the Higgsinos are heavier than 500 GeV (1 TeV), we estimate that $\tilde{g}_{1,R}^{(\text{eff})}$ involves a theoretical uncertainty of 0.4% (0.1%). As a result, the coupling is expected to be determined with the accuracy of about 0.7% (0.4%) in total. Hereafter, we adopt a slightly conservative value,

$$\delta \tilde{g}_{1,R}^{(\text{eff})}/\tilde{g}_{1,R}^{(\text{eff})} = 1\%.$$

This uncertainty is sub-dominant in the reconstruction of $a_{\mu}^{(\text{ILC})}$ compared to that in $m_{\tilde{\chi}_{LR}}^2$.

The gaugino coupling to the left-handed (s)electron is measured from the production cross section of the left-handed selectrons. In particular, those of the processes, $e^+e^- \rightarrow \tilde{e}_R^+\tilde{e}_R^-$ or $\tilde{e}_L^+\tilde{e}_R^-$, are sensitive to $\tilde{g}_{1,L}^{(\text{eff})}$ (as well as $\tilde{g}_{1,R}^{(\text{eff})}$)\footnote{The process, $e^+e^- \rightarrow \tilde{e}_L^+\tilde{e}_L^-$, also involves $\tilde{g}_{1,L}^{(\text{eff})}$. However, its cross section depends on the Wino coupling $\tilde{g}_2$ as well as $\tilde{g}_{Y,L}$ mainly through the $t$-channel Wino exchange diagram.}. The cross section can be measured precisely at ILC [28]. In Ref. [38], its accuracy is claimed to be $\sim 2\%$ for $m_{\tilde{e}_R} = 143$ GeV and $m_{\tilde{e}_L} = 202$ GeV. Here, the SPS1a point is adopted with $\sqrt{s} = 500$ GeV and $(P_{e+}, P_{e-}) = (-0.6, -0.8)$, though the luminosity is not explicitly shown. The selectron production processes are discriminated from each others by the electron energy and by changing the beam polarization especially of the positron [28,29,39]. In fact, the analysis in Ref. [39] shows that the neutralino coupling can be measured at similar accuracy as those in Ref. [35,36] by
Table 2: Observables necessary for the reconstruction of \( a_\mu^{(ILC)} \), and their uncertainties with \( \sqrt{s} = 500 \text{ GeV} \) and \( \mathcal{L} \sim 500\text{–}1000 \text{ fb}^{-1} \). Processes relevant to determine each observable are also shown. The second and third rows are the information to determine \( m_{\tilde{\mu}}^{2,LR} \). For the determination of \( m_{\tilde{\chi}^0_1} \), analyses of the productions of selectrons and smuons are combined. The uncertainties in \( \tilde{g}_{1,L}^{\text{(eff)}} \) are those from the experiment and theory, respectively.

| \( X \) | \( \delta X \) | \( \delta_X a_\mu^{(ILC)} \) | Process |
|---|---|---|---|
| \( m_{\tilde{\mu}}^{2,LR} \) | 12\% | 13\% | \( e^+e^- \rightarrow \tau^+\tau^- \) (cross section, endpoint) |
| \( (\sin 2\theta_t) \) | (9\%) | – | \( e^+e^- \rightarrow \tilde{\tau}_1^+\tilde{\tau}_1^- \) (cross section) |
| \( (m_{\tilde{\tau}_2}) \) | (3\%) | – | \( e^+e^- \rightarrow \tilde{\tau}_2^+\tilde{\tau}_2^- \) (endpoint) |
| \( m_{\tilde{\chi}^0_1}, m_{\tilde{\mu}^2} \) | 200 MeV | 0.3\% | \( e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^- \) (endpoint) |
| \( m_{\tilde{\chi}^0_1} \) | 100 MeV | < 0.1\% | \( e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^- / \tilde{e}^+\tilde{e}^- \) (endpoint) |
| \( \tilde{g}_{1,L}^{(\text{eff})} \) | a few+1\% | a few+1\% | \( e^+e^- \rightarrow \tilde{e}_L^+\tilde{e}_R^- \) (cross section) |
| \( \tilde{g}_{1,R}^{(\text{eff})} \) | 1\% | 0.9\% | \( e^+e^- \rightarrow \tilde{e}_R^+\tilde{e}^- \) (cross section) |

changing the polarization. Unfortunately, the acceptance as well as the accuracy of the cross section measurement is not found in the literature. In this letter, we assume that \( \sigma(e_L^+e_R^-) \) is measured with the accuracy of a few percents. Numerically, if it is determined at the 2\% (4\%) level, the accuracy of \( \tilde{g}_{1,L}^{(\text{eff})} \) is estimated to be about 1\% (2\%), where \( \delta \tilde{g}_{1,R}^{(\text{eff})} / \tilde{g}_{1,R}^{(\text{eff})} = 1\% \) is applied.

In addition to the experimental uncertainty, the process \( e^+e^- \rightarrow \tilde{e}_L^+\tilde{e}_R^- \) involves the \( t \)-channel exchange diagrams of heavier neutralinos. They contribute to the cross section via the Wino–Higgsino and Bino–Higgsino mixings. Their contamination to the measurement of \( \tilde{g}_{1,L}^{(\text{eff})} \) depends on their masses. Assuming that Wino and Higgsino masses are above 500 GeV, we estimate that \( \tilde{g}_{1,L}^{(\text{eff})} \) involves a theoretical (systematic) uncertainty of 0.9\%, while it is reduced to be 0.2\% for \( M_{\tilde{W},\tilde{H}} > 1 \text{ TeV} \). On the other hand, contaminations from corrections to the Wino coupling with the (s)electrons are smaller than it. As a result, the accuracy of the measurement of the gaugino coupling is estimated to be

\[
\delta \tilde{g}_{1,L}^{(\text{eff})} / \tilde{g}_{1,L}^{(\text{eff})} = \text{a few\% (exp)} + 1\% \text{ (th)},
\]

or better. Here, the first term in the right-hand side comes from the measurement of the cross section for \( e^+e^- \rightarrow \tilde{e}_L^+\tilde{e}_R^- \), and the second term is due to the contamination from the undiscovered Winos and Higgsinos. Then, the uncertainty is sub-dominant in the reconstruction of \( a_\mu^{(ILC)} \) compared to that in \( m_{\tilde{\mu}}^{2,LR} \).
3.4 Reconstruction of the SUSY contribution to muon $g - 2$

Now let us discuss the accuracy of the reconstruction of $a^{(ILC)}_\mu$ with ILC. The accuracy is estimated by summing all the errors induced by these parameters in quadrature as

$$\delta a^{(ILC)}_\mu \equiv \sqrt{\sum_X \left( \delta X a^{(ILC)}_\mu \right)^2}, \quad \delta_X a^{(ILC)}_\mu \equiv \frac{\partial a^{(ILC)}_\mu}{\partial X} \delta X,$$

(38)

where $X = m^2_{\tilde{\mu}_{LR}}, m_{\tilde{\mu}_1}, m_{\tilde{\mu}_2}, m_{\tilde{\chi}^0_1}, \tilde{g}_{1,L}^{(eff)}$, and $\tilde{g}_{1,R}^{(eff)}$. In Table 2 their uncertainties are summarized. Consequently, we estimate $\delta a^{(ILC)}_\mu$ as

$$\delta a^{(ILC)}_\mu / a^{(ILC)}_\mu = 13\%,$$

(39)

taking $\delta \tilde{g}_{1,L}^{(eff)} \leq 3\%$. The dominant error originates in the determination of the left-right mixing parameter $m^2_{\tilde{\mu}_{LR}}$.

The reconstructed SUSY contribution $a^{(ILC)}_\mu$ may not well approximate the full contribution, $a^{(SUSY)}_\mu$. The difference between $a^{(SUSY)}_\mu$ and $a^{(ILC)}_\mu$ comes from the unobserved neutralino and chargino contributions to the muon $g - 2$, and should be understood as a theoretical error in the reconstruction of $a^{(SUSY)}_\mu$ in our procedure. Let us define

$$\delta a^{(SUSY,th)}_\mu \equiv a^{(SUSY)}_\mu - a^{(ILC)}_\mu.$$

(40)

This depends on $M_2$ and $\mu$ (as well as on the parameters listed in Table 2). In Fig. 2 contours of constant $\delta a^{(SUSY,th)}_\mu$ are shown for $M_2 > 0$ with the underlying parameters in Table 2. In particular, $\mu \tan \beta = 6.1 \times 10^3$ GeV is fixed. Here, the uncertainties in the parameters listed in Table 2 are omitted. Obviously, $\delta a^{(SUSY,th)}_\mu$ is suppressed as $M_2$ and $\mu$ become larger. This is because all the diagrams that contain Wino and Higgsino propagators vanish in this limit, so that $a^{(SUSY)}_\mu$ is well approximated by the Bino–smuon diagram. Thus, using lower bounds on the Wino and Higgsino masses provided by collider experiments, a bound on $\delta a^{(SUSY,th)}_\mu$ can be obtained. They will be searched for effectively at LHC with $\sqrt{s} = 13$ or 14 TeV. If the Wino and Higgsino masses are constrained to be larger than 1 TeV (1.5 TeV) in future, $\delta a^{(SUSY,th)}_\mu$ is known to be smaller than 0.9 $\times 10^{-10}$ (0.3 $\times 10^{-10}$) at our model point, which corresponds to 4% (1%) of $a^{(ILC)}_\mu$. This is smaller than the dominant error of the reconstruction of $a^{(ILC)}_\mu$.

Finally we comment on higher order contributions to $a^{(SUSY)}_\mu$. Ref. [40] calculated photonic SUSY two-loop corrections, which change the one-loop result by $\sim 10\%$. They can be determined at ILC by the above procedure, because all the parameters necessary for them are measured simultaneously. In this letter, they are neglected for simplicity, although it is straightforward to include the contributions. Also, corrections to the gaugino couplings

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#10 We have checked that, when $M_2 < 0$, $|\delta a^{(SUSY,th)}_\mu|$ is smaller than that for $M_2 > 0$ with $|M_2|$ fixed.

#11 Wino can be searched for by multi-lepton plus a large missing energy signature, while Higgsino can be by searches for multi-tau and/or standard model bosons together with a large missing energy.
Figure 2: Contours of the difference between the full SUSY contribution to the muon $g - 2$ and the ILC-reconstructed value, $\delta a_\mu^{(\text{SUSY,th})} \equiv a_\mu^{(\text{SUSY})} - a_\mu^{(\text{ILC})}$, on the Wino mass vs. Higgsino mass plane.

and to the lepton Yukawa couplings in the left-right mixing parameters can be as large as $\sim 10\%$ [10, 15, 41]. Importantly, they are already taken into account in the reconstruction of $a_\mu^{(\text{ILC})}$. Most of the other two-loop contributions are considered to be suppressed in our sample point. However, electroweak and SUSY two-loop corrections to the SUSY one-loop diagrams, which have not been calculated, might be $\sim 10\%$ [42]. Since they could be as large as the dominant error of the reconstruction, it is important to calculate these two-loop contributions.

4 Summary and Discussion

In this letter, we have studied how and how accurately we can reconstruct the SUSY contribution to the muon $g - 2$ by using the information available at ILC. If $a_\mu^{(\text{SUSY})}$ is as large as $2.6 \times 10^{-9}$ to solve the muon $g - 2$ anomaly, and also if all the sleptons as well as the lightest neutralino are within the kinematical reach, ILC will be able to measure the MSSM parameters which are necessary to estimate $a_\mu^{(\text{SUSY})}$. We have discussed the procedures and accuracies of their measurements. It has been shown that, in the sample point we choose, the SUSY contribution to the muon $g - 2$ can be reconstructed with the uncertainty of $\sim 13\%$ at ILC with $\sqrt{s} = 500\text{ GeV}$ and an integrated luminosity $\mathcal{L} \sim 500\text{–}1000\text{ fb}^{-1}$. This provides a very crucial test of the SUSY explanation to the muon $g - 2$ anomaly.

We should emphasize that the uncertainty depends on model points. As we have shown, the dominant error in the reconstructed value of $a_\mu^{(\text{ILC})}$ originates in the uncertainty of the left-right mixing parameter $m^2_{\mu LR}$ in the sample point. For instance, if the heavier stau mass increases with the lighter one fixed, it is inferred from Eq. (14) that the reconstruction
would be degraded. On the contrary, if the charged sleptons in the second or third generation are degenerate in masses, the determination of $\sin 2\theta_{\tilde{\mu}}$ could be improved considerably. Unfortunately, slepton productions have not been studied for ILC in such cases.

The present uncertainty of the experimental and SM values of the muon $g - 2$ is about 30% (see Eq. (3)). Thus, the error in the reconstructed value of $a_{\mu}^{\text{ILC}}$ is sub-dominant when we test the idea of solving the muon $g - 2$ anomaly with the SUSY contribution. However, the experimental measurement and theoretical calculation of the SM prediction will be improved in near future. The Fermilab experiment [13] and the J-PARC New $g - 2$/EDM experiment [14] will reduce the experimental error at least by a factor 4–5. The uncertainty of the SM prediction is dominated by those in the hadronic contributions. They will be improved by experiments as well as lattice calculations. The uncertainty is expected to be reduced by a factor 2 [15]. As a result, if the experimental and SM central values would be unchanged, the error in $\Delta a_{\mu}$ could become as small as $\sim 10\%$, which is comparable to that in $a_{\mu}^{\text{ILC}}$. Then, a precise reconstruction of the SUSY contribution to the muon $g - 2$ becomes crucial.

We made several assumptions to evaluate the uncertainty, since we could not find enough information about the slepton production processes. Precise studies of the slepton production process are strongly recommended to deeply understand how useful ILC is to reconstruct the SUSY contribution to the muon $g - 2$.

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