Entropy Production and Thermodynamic Arrow of Time in a Recollapsing Universe

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Abstract

We investigate the thermodynamic arrow of time in a time-symmetrically recollapsing universe by calculating quantum mechanically the entropy production of a massive scalar field. It is found that even though the Hamiltonian has a time-reversal symmetry with respect to the maximum expansion of the universe, the entropy production is generic and the total entropy of the scalar field increases monotonically. We conclude that the thermodynamic arrow of time is a universal phenomenon even in the expanding and subsequently recollapsing universe due to the parametric interaction of matter field with gravity.

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I. INTRODUCTION

The thermodynamic arrow of time has always been a puzzling issue in an expanding and subsequently recollapsing universe. When the Universe is to recollapse time-symmetrically with respect to its maximum expansion, can the entropy of matters always increase or time-symmetrically decrease? Historically, Gold [1] gave the first scientific explanation that the entropy of matter should decrease time-symmetrically with respect to the maximum expansion of the universe, but this argument was refuted by Penrose [2]. In the context of quantum cosmology, Hawking [3] argued that the entropy of matter should be time symmetric since the Hartle and Hawking’s no-boundary wave function [4] should be CTP invariant. But Page [5] refuted this argument by pointing out the time-asymmetric wave function obeying CTP theorem, in which the entropy can increase monotonically throughout the expansion and subsequent recollapse of the universe. Later Hawking et al. [6] admitted the thermodynamic arrow of time by considering the growing density perturbation even through the recollapsing universe. Recently, we have argued that the parametric interaction between the gravity and matter fields may break not only the cosmological symmetry of time [7] but also the thermodynamic symmetry of time [8]. Kiefer and Zeh [9] also argued the thermodynamic arrow of time in the recollapsing universe. (See Ref. [10] for comprehensive review and references.)

In this paper we revisit the issue of thermodynamic arrow of time by studying a massive scalar field in a time-symmetrically recollapsing universe. Matter field in such a time-dependent background spacetime should be described by a nonequilibrium quantum field. We apply the recently introduced unified approach [11] based on the Liouville-von Neumann (LvN) equation to the nonequilibrium massive scalar field and find the time-dependent Fock (Hilbert) spaces throughout the expansion and subsequent recollapse of the universe. The particle production is calculated explicitly and the information-theoretic entropy is computed for the nonequilibrium quantum process. From this we conclude that the thermodynamic arrow of time is a universal phenomenon due to the parametric interaction of the matter field with the gravity even in the recollapsing universe.

The organization of this paper is as follows. In Sec. II we review the nonequilibrium quantum evolution of the massive scalar field in the recollapsing Friedmann-Robertson-Walker (FRW) universe. In Sec. III we study an exactly solvable model for the massive scalar field in the time-symmetrically recollapsing universe. Finally, in Sec. IV we study the thermodynamic arrow of time in the recollapsing FRW universe.

II. MASSIVE SCALAR FIELD IN THE FRW UNIVERSE

In order to study the thermodynamic arrow of time in the time-symmetrically recollapsing universe, we consider the simple model of a closed FRW universe minimally coupled to a massive scalar field. The massive scalar field is described by the action

$$I = -\frac{1}{2} \int d^4x \sqrt{-g} \left[ g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + m^2 \Phi^2 \right].$$

The FRW universe has the metric

$$ds^2 = -dt^2 + R^2(t) d\Omega_3^2,$$
where $d\Omega^2$ is the metric on the unit three-sphere. According to Ref. [12] the scalar field can be decomposed as

$$
\Phi(t, x) = \sum_{n,l,m} \phi_{n,m}^l(t)Q_{n,m}^l(x),
$$

(3)

where $Q_{n,m}^l$ denote the eigenfunctions of the Laplace operator on the three-sphere and are explicitly given by $Q_{n,m}^l = \Pi_l^i Y_{l,m}$, where $Y_{l,m}$ are the spherical harmonics on the two-sphere and $\Pi_l^i$ are the Fock harmonics. Then from the action (1) we obtain the mode-decomposed Hamiltonian for the scalar field

$$
H(t) = \sum_k H_k(t) =: \sum_k \frac{\pi_k^2}{2R^3(t)} + \frac{R^3(t)}{2} \left( m^2 + \frac{n^2 - 1}{R^2(t)} \right) \phi_k^2,
$$

(4)

where $k$ denote collectively $(n, l, m)$ and $\pi_k$ are momenta conjugate to $\phi_k$. Thus, the Hamiltonian is a collection of infinitely many time-dependent harmonic oscillators.

Each time-dependent harmonic oscillator in Eq. (4) describes a nonequilibrium quantum process through the parametric interaction between the matter and gravity. Such time-dependent system leads necessarily to the particle production in quantum mechanics and its nonequilibrium process implies the entropy production in quantum statistical mechanics. To treat the nonequilibrium process appropriately, we shall adopt the recently introduced unified approach [11] based on the LvN equation. This LvN approach provides us explicitly not only with the exact quantum states but also with the density operator. In this LvN approach the fundamental laws for the nonequilibrium quantum system are the time-dependent Schrödinger equation (in unit $\hbar = 1$)

$$
i \frac{\partial \Psi(t)}{\partial t} = \hat{H}(t)\Psi(t),
$$

(5)

and the LvN equation for the density operator

$$
i \frac{\partial \hat{\rho}(t)}{\partial t} + [\hat{\rho}(t), \hat{H}(t)] = 0.
$$

(6)

It was Lewis and Riesenfeld [13] who observed that the eigenstate of any operator $O$ satisfying the LvN equation (3) is an exact quantum state of the time-dependent quantum system up to a trivial time-dependent phase factor. (For development and various applications, see Ref. [14].)

The stratagem of the LvN approach is to find the pair of time-dependent operators

$$
\hat{A}_k(t) = i \left[ u_k^*(t)\hat{\pi}_k - R^3(t)\dot{u}_k^*(t)\hat{\phi}_k \right], \quad \hat{A}_k^\dagger(t) = \text{h.c.} \left( \hat{A}_k(t) \right),
$$

(7)

that satisfy the LvN equation

$$
i \frac{\partial \hat{A}_k(t)}{\partial t} + [\hat{A}_k(t), \hat{H}_k(t)] = 0.
$$

(8)

It readily follows that Eq. (8) is satisfied only when $u_k$ is a complex solution to the corresponding classical equation of motion
\[ \ddot{u}_k(t) + 3 \frac{\dot{R}(t)}{R(t)} \dot{u}_k(t) + \left( m^2 + \frac{n^2 - 1}{R^2(t)} \right) u_k(t) = 0. \] (9)

\( \hat{A}_k(t) \) and \( \hat{A}^\dagger_k(t) \) can be made the annihilation and creation operators with the standard commutation relation at each time \( t \)

\[ [\hat{A}_k(t), \hat{A}^\dagger_{k'}(t)] = \delta_{k,k'} \] (10)

by imposing the Wronskian condition

\[ R^3(t) \left[ u_k(t) \dot{u}_k^*(t) - \dot{u}_k^*(t) u_k(t) \right] = i. \] (11)

Then, neglecting trivial time-dependent phase factors, the number states

\[ |n_k, t\rangle = \frac{1}{\sqrt{n_k!}} (\hat{A}_k(t))^{n_k} |0_k, t\rangle, \] (12)

together with the vacuum state

\[ \hat{A}_k(t) |0_k, t\rangle = 0, \] (13)

constitute the time-dependent Fock (Hilbert) space. In fact, there are one-parameter Gaussian states (13) for each oscillator and the vacuum state is the Gaussian state having the minimum uncertainty [14].

Note that the Hamiltonian system (4) may not be solved exactly since the classical equations of motion (9) can not be solved in closed form for the scale factor

\[ R(t) = \sqrt{\frac{3m_P^2}{8\pi\rho}} \text{csch} \left( \sqrt{\frac{8\pi\rho}{3m_P^2}} t \right), \] (14)

where \( m_P = \frac{1}{\sqrt{\hbar}} \) is the Planck mass and \( \rho \) is the density of dust particles in the closed FRW universe. So it would be interesting to study first a model that can be solved exactly.

III. AN EXACTLY SOLVABLE MODEL

We now consider a model Hamiltonian that shares many physically interesting features with Eq. (4). The frequencies of oscillators in Eq. (4) increase indefinitely as the universe (re)collapses \( (R \to 0) \), and approach to constant values at the maximum expansion. To study the particle production in a curved spacetime, the following Hamiltonian has been often used [15]

\[ \hat{H}(t) = \sum_k \hat{H}_k(t) =: \sum_k \frac{\hat{p}_k^2}{2m_0} + \frac{m_0}{2} \left( \omega_0^2 + \frac{\omega_k^2 t^2}{4} \right) \hat{v}_k^2, \] (15)

whose the classical equations of motion can be solved analytically

\[ \ddot{v}_k(t) + \left( \omega_0^2 + \frac{\omega_k^2 t^2}{4} \right) v_k(t) = 0. \] (16)
In terms of the complex solution for each oscillator one finds the annihilation and creation operators

$$\hat{B}_k(t) = i\left[v_k^*(t)\hat{p}_k - m_0\dot{v}_k^*(t)\hat{q}_k\right], \quad \hat{B}_k^\dagger(t) = \text{h.c.}(\hat{B}_k(t)).$$ (17)

Further, one should impose the Wronskian condition

$$m_0\left[v_k(t)\dot{v}_k^*(t) - v_k^*(t)\dot{v}_k(t)\right] = i,$$ (18)

to guarantee the standard commutation relation $[\hat{B}_k(t), \hat{B}_k^\dagger(t)] = \delta_{k,k'}$ for all times.

The complex solution to Eq. (16) satisfying the Wronskian (18) is given by

$$v_{k, in}(t) = \frac{1}{(4m_0^2\omega_k)^{1/4}}\left[i\sqrt{\kappa}W(a_k, \tau) + \frac{1}{\sqrt{\kappa}}W(a_k, -\tau)\right],$$ (19)

where $W(a, \pm \tau)$ are the parabolic cylinder functions [16], and

$$a_k = -\frac{\omega_0^2}{\omega_k}, \quad \tau = \sqrt{\omega_k t}, \quad \kappa = \sqrt{1 + e^{2\pi a_k} - e^{\pi a_k}}.$$ (20)

In the past infinity $(t \to -\infty)$ the solution (19) has the asymptotic form [16]

$$v_{k, in}(t) = \frac{1}{(4m_0^2\omega_k)^{1/4}}\exp\left[i\left(\frac{\tau^2}{4} - a_k\ln(-\tau) + \frac{\pi}{4} + \frac{\varphi_2}{2}\right)\right],$$ (21)

where $\varphi_2 = \arg\Gamma\left(\frac{1}{2} + ia_k\right)$. Note that the asymptotic solution (21) will be the same as the adiabatic (WKB) solution of Sec. IV. Also another useful form of the solution (19) is given by

$$v_{k, in}(t) = \frac{1}{(4m_0^2\omega_k)^{1/4}}\left[i\left(\frac{k}{2} - \frac{1}{k}\right)E(a_k, \tau) + i\left(\frac{k}{2} + \frac{1}{k}\right)E^*(a_k, \tau)\right].$$ (22)

In the future infinity $(t \to \infty)$, $u_{k, out}$ has the asymptotic form

$$v_{k, out} = \frac{1}{(4m_0^2\omega_k)^{1/4}}E^*(a_k, \tau)$$

$$= \frac{1}{(4m_0^2\omega_k\tau^2)^{1/4}}\exp\left[-i\left(\frac{\tau^2}{4} - a_k\ln(\tau) + \frac{\pi}{4} + \frac{\varphi_2}{2}\right)\right].$$ (23)

From Eqs. (21) and (23) we are able to find the transformation of solutions between two asymptotic regimes

$$v_{k, in}(t) = \frac{i}{2}\left(\kappa + \frac{1}{\kappa}\right)v_{k, out}(t) + \frac{i}{2}\left(\kappa - \frac{1}{\kappa}\right)v_{k, out}^*(t),$$ (24)

and the Bogoliubov transformation

$$\hat{B}_{k, out}(t) = \mu_k\hat{B}_{k, in}(t) + v_k\hat{B}_{k, in}^\dagger(t), \quad \text{h.c.}$$ (25)

where
\[ \mu_k = \frac{i}{2} (\kappa + \frac{1}{\kappa}), \quad \nu_k = -\frac{i}{2} (\kappa - \frac{1}{\kappa}). \]  

(26)

Also the Bogoliubov transformation can be rewritten as

\[ \hat{B}_{k,\text{out}}(t) = \hat{S}_k(z_k) \hat{B}_{k,\text{in}}(t) \hat{S}_k^\dagger(z_k), \quad \text{h.c.} \]  

(27)

in terms of the squeeze operator \[17\]

\[ \hat{S}_k(z_k) = \exp \left[ i \frac{\pi}{2} \hat{B}_{k,\text{in}}^\dagger \hat{B}_{k,\text{in}} \right] \exp \left[ \frac{r_k}{2} (\hat{B}_{k,\text{in}}^2 - \hat{B}_{k,\text{in}}^{12}) \right], \]  

(28)

in which

\[ \mu_k = \cosh r_k e^{i \frac{\pi}{2}}, \quad \nu_k = \sinh r_k e^{i \frac{3\pi}{2}}. \]  

(29)

The vacuum states \[13\] for each oscillator depend on time explicitly and those at two different times are related by the Bogoliubov transformation

\[ \hat{B}_k(t) = \mu_k(t, t_0) \hat{B}_k(t_0) + \nu_k(t, t_0) \hat{B}_k^\dagger(t_0), \quad \text{h.c.} \]  

(30)

where

\[ \mu_k(t, t_0) = im_0 [v_k^* (t) \dot{v}_k (t_0) - \dot{v}_k^* (t) v_k (t_0)], \]  

\[ \nu_k(t, t_0) = im_0 [v_k^* (t) \dot{v}_k (t_0) - \dot{v}_k^* (t) v_k^* (t_0)]. \]  

(31)

The quantum state of each oscillator evolves unitarily by the unitary transformation \[27\], and the vacuum states at two different times are not orthogonal to each other

\[ \langle 0_k, t | 0_k, t_0 \rangle = \left( \frac{e^{i \vartheta_k(t, t_0)}}{|\mu_k(t, t_0)|} \right)^{1/2}, \]  

(32)

where \( \vartheta_k = \arg(v_k(t_0)v_k(t)) \). However, since \( \mu_k \geq 1 \) and can be unity at most for a finite number of modes, the vacuum states for all modes of the scalar field

\[ |0, t \rangle = \prod_k |0_k, t \rangle, \]  

(33)

are orthogonal to each other at two different times

\[ \langle 0, t | 0, t_0 \rangle = \prod_k \left( \frac{e^{i \vartheta_k(t, t_0)}}{|\mu_k(t, t_0)|} \right)^{1/2} = 0. \]  

(34)

Therefore, the time-dependent Fock spaces constructed from the vacuum states form unitary inequivalent representations. The physical implication of the unitary inequivalence of representations is that particles are produced in the infinite future \( (t \to \infty) \) from the initial vacuum state of the infinite past \( (t \to -\infty) \) by the amount

\[ \text{in} (0) \sum_k \hat{B}_{k,\text{out}}^\dagger \hat{B}_{k,\text{out}} |0\rangle_{\text{in}} = \sum_k \nu_k^2. \]  

(35)
Similarly, the particle production from the number states is found to be
\[ \prod_k \langle n_k | \sum_k \hat{B}^\dagger_{k,\text{out}} \hat{B}_{k,\text{out}} \prod_k | n_k \rangle_{\text{in}} = \sum_{n_k} n_k^2 (2n_k + 1). \] (36)

The particle production in the expanding universe was first explained by Parker [18].

We can make use of the operators (17) satisfying the LvN equation to construct a density operator
\[ \hat{\rho}_k(t) = \frac{1}{Z_k(t)} e^{-\beta \Omega_k \left( \hat{B}^\dagger_k(t) \hat{B}_k(t) + \frac{1}{2} \right)}, \]
\[ Z_k(t) = \text{Tr} e^{-\beta \Omega_k \left( \hat{B}^\dagger_k(t) \hat{B}_k(t) + \frac{1}{2} \right)}. \] (37)

Here \( \beta \) is a free parameter for nonequilibrium quantum processes, which becomes \( \beta = \frac{1}{k_B T} \) for an equilibrium system, where \( k_B \) and \( T \) are the Boltzmann constant and temperature, respectively. Using the information-theoretic entropy at each time defined by [19]
\[ S_{k,\text{ent.}}(t) = -k_B \text{Tr} [\hat{\rho}_k(t) \ln \hat{\rho}_k(t)], \] (38)
the entropy production during the evolution from the infinite past to the infinite future is given by
\[ \Delta S_{k,\text{ent.}} = -k_B \left\{ \text{Tr} [\hat{\rho}_{k,\text{out}} \ln \hat{\rho}_{k,\text{out}}] - \text{Tr} [\hat{\rho}_{k,\text{in}} \ln \hat{\rho}_{k,\text{in}}] \right\}. \] (39)

It should be pointed out that the entropy cannot be produced for a system of finite oscillators due to the unitary transformation (27).

However, the Fock space of the scalar field in the infinite future is unitarily inequivalent to that of the infinite past. In other words, Eq. (27) should not be regarded as a unitary transformation but rather as a unitarily inequivalent transformation of the scalar field. Therefore, the entropy production for the scalar field should be evaluated with respect to the same Fock space, for instance, that of the infinite past

\[ \Delta S_{k,\text{ent.}} = -k_B \sum_{n_k} \left\{ \text{in} \langle n_k | \hat{\rho}_{\text{out}} \ln \hat{\rho}_{\text{out}} | n_k \rangle_{\text{in}} - \text{in} \langle n_k | \hat{\rho}_{\text{in}} \ln \hat{\rho}_{\text{in}} | n_k \rangle_{\text{in}} \right\} \]
\[ = -k_B \sum_{n_k, m_k} \left\{ \text{in} \langle n_k | \hat{\rho}_{\text{in}} \ln \hat{\rho}_{\text{in}} | n_k \rangle_{\text{in}} \left( |\text{in} \langle m_k | \hat{S} | n_k \rangle_{\text{in}}|^2 - \delta_{m_k, n_k} \right) \right\}. \] (40)

Here we made use of the transformation (27) in the second line. For a small squeeze parameter \( r_k \) we obtain approximately \( \Delta S_{k,\text{ent.}} \simeq 2r_k S_{k,\text{in}} \), which is different from \( \Delta S_{k,\text{ent.}} \simeq 2r_k \) for the Gaussian states in Refs. [20,21]. The total amount of the information-theoretical entropy is given by
\[ \Delta S_{\text{ent.}} \simeq \sum_k 2 \ln \left( \sqrt{1 + e^{-2r_k^2}} - e^{-r_k^2} \right) S_{k,\text{in}}. \] (41)

It is remarkable that the entropy increases monotonically during the symmetric evolution from the infinite past to the future and the amount of the entropy production is proportional to the entropy present in the past.
IV. THERMODYNAMIC ARROW OF TIME IN THE RECOLLAPSING UNIVERSE

We now return to the scalar field in the FRW universe and its Hamiltonian (4). Oftentimes the instantaneous annihilation and creation operators are used that diagonalize the Hamiltonian at every moment:

$$\hat{A}_{k,\text{dig.}}(t) = e^{i\int \Omega_k(t) \frac{\sqrt{R^3(t)} \omega_k(t)}{2} \hat{q}_k + i \int \frac{1}{R^3(t) \omega_k(t)} \hat{p}_k}, \quad \hat{A}^\dagger_{k,\text{dig.}}(t) = \text{h.c.} \left( \hat{A}_{k,\text{dig.}}(t) \right).$$

(42)

However, the Fock spaces of these number states lead to the infinite production of particle and entropy [15]. On the other hand, the LvN approach in Secs. II and III will yield the correct and physically meaningful result, provided that the analytical solution of Eq. (9) is used for each mode. Since it is difficult to solve analytically Eq. (9) for the scale factor (14), we rely on the adiabatic (WKB) solution. This is done by interpreting Eq. (9) as a one-dimensional Schrödinger equation and, for a slowly varying frequency $\omega_k(t)$, by using the adiabatic solution of the form

$$u_{k,\text{ad}}(t) = \frac{1}{\sqrt{2R^3(t) \Omega_k(t)}} \exp \left[ -i \int \Omega_k(t) \right].$$

(43)

The adiabatic annihilation and creation operators for each oscillator are related with the instantaneous ones via the Bogoliubov transformation

$$\hat{A}_{k,\text{ad}}(t) = \left( 1 - i \frac{\Omega_k}{4m_0 \Omega_k^2} \right) \hat{A}_{k,\text{dig.}}(t) - i \frac{\hat{p}_k}{4m_0 \Omega_k^2} e^{2i \int \Omega_k(t) \hat{p}_k}, \quad \text{h.c..}$$

(44)

As for the model in Sec. III, the adiabatic solution of the infinite past can be written in terms of that of the infinite future as

$$u_{k,\text{ad.in}} = \mu_k u_{k,\text{ad.out}} - \nu_k u^*_{k,\text{ad.out}}.$$  

(45)

The explicit forms of $\mu_k$ and $\nu_k$ depend on $R(t)$, $m$, and $k$. Similarly, the adiabatic solution at $t_p$ in the expanding stage can be expressed in terms of that at $t_f$ in the time-symmetric recollapsing stage as

$$u_{k,\text{ad}}(t_p) = \mu_k(t_f, t_p) u_{k,\text{ad}}(t_f) - \nu_k(t_f, t_p) u^*_{k,\text{ad}}(t_f).$$

(46)

In general, one has $\mu_k(t_f, t_p) \neq 0$ even for symmetric times $t_p$ and $t_f$ with respect to the maximum expansion of the universe. One can also understand this fact from the scattering of a particle by a symmetric potential in quantum mechanics. From the study of the model in Sec. III we may find some characteristic features of Eq. (4). It is rather straightforward to obtain the entropy production for each mode of the scalar field

$$\Delta S_{k,\text{ent.}}(t_f, t_p) \simeq 2r_k(t_f, t_p) S_k(t_p),$$

(47)

where

$$\cosh r_k(t_f, t_p) = |\mu_k(t_f, t_p)|.$$  

(48)
The total entropy production of the scalar field is then given by

$$\Delta S_{\text{ent.}}(t_f, t_p) \simeq \sum_k 2r_k(t_f, t_p)S_k(t_p).$$ (49)

Thus we have shown that the entropy of the matter (scalar) field increases even during the time-symmetric recollapse of the universe. This thermodynamic arrow of time is due to the parametric interaction between the matter field and gravity. The particle and entropy production seems to be a generic feature and thereby, the global thermodynamic arrow of time is a consequence of the evolution of the universe.

V. CONCLUSION

In this paper we have studied the particle and entropy production of a massive scalar field in a time-symmetrically recollapsing universe. In the expanding and subsequently recollapsing universe the matter field is described by a nonequilibrium quantum field due to the parametric interaction of the matter field with the gravity. To treat properly we have applied the unified approach based on the Liouville-von Neumann equation to the nonequilibrium massive scalar field and found the time-dependent Fock spaces throughout the evolution of the universe. It is the parametric interaction that leads to the entropy production during the evolution of the universe. The study of an exactly solvable model which has many similar features with the scalar field revealed the monotonic increase of particle and entropy even for the time-symmetric recollapse of the universe. Likewise, the entropy of a massive scalar field in the expanding and recollapsing universe is found to be monotonically increasing using adiabatic (WKB) solution to the classical equation of motion. Therefore, we conclude that the entropy of the scalar field increases monotonically throughout the evolution of the universe including even the symmetrically recollapsing universe.

In this paper we have focused only on the scalar field in the expanding and subsequently recollapsing universe. The other matter of the Universe is the fermionic field. Recently, the Liouville-von Neumann approach has been applied even to the time-dependent fermionic field to describe correctly its nonequilibrium evolution [22]. Since the coupling constants of the fermionic field depend on time as the Universe expands and subsequently recollapses, the result of Ref. [22] can be employed to study the thermodynamic arrow of time due to the fermionic field, which will be presented in a future publication.

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