Study of excitation energies and transmission potential of the reduced quadruple B (E2) for the $^{70}$Zn isotope by applying the nuclear shell model

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Abstract. In this paper, the excited energy levels and reduced electric quadrupole transition probabilities (BE2 ↓) in both pure and mixed configurations of $^{70}$Zn nucleus have been calculated by applying the nuclear shell model. In the present study, the surface delta and modified surface delta interactions within the model space ($1p_{3/2}, 0f_{5/2}, 1p_{1/2}$, and $0g_{9/2}$) have been used for two protons outside the closed nuclear core represented by the $^{68}$Ni nucleus. The equations are programmed for the above two interactions and the reduced electric quadrupole transition probabilities (BE2 ↓) equations using the Fortran version 95. Through the used Fortran version 95 program code, the elements of the matrix have been calculated for two particles which are necessary to calculate the eigenvalues, the eigenvectors and then the final energies of the nucleus that has been used in the current study. Through the calculations, theoretical results that have well agreement with the available practical values have been obtained. The total angular momentum and parity of some uncertain and unspecified energy levels have been confirmed and determined by the total angular momentum and the practical parity. Also, new values have been identified for both the excited energy levels and the reduced electric quadrupole transition probabilities (BE2 ↓), these values are considered as a suggestion that increases theoretical knowledge of each of the energy levels and the expected transition probabilities during this study.

Key words: excitation energies, electric quadrupole transition probability B(E2).

1. Introduction

The structure of the nuclear shell is the basis for the theory of multiparticles in the nuclei, as the nucleus is divided into a closed core and equivalent nucleons (particles and gaps) that occupy the outer orbits. One of the important advantages of the nuclear shell model is the periodic change of nuclear properties continuously [1,2]. According to the nuclear shell model, nucleons are distributed in separate shells in the nucleus, and the capacity of each specific shell is determined by the greatest number of nucleons, in line with the rules of quantum mechanics and the Pauli Exclusion Principle, as each nucleon moves in its own orbit independently of any other nucleons. This is under the influence of an output interaction of all nucleons in the nucleus, due to the fact that the shell model is called the (Independent Particle Model) [2]. The scientist Bartlett)) had proposed the model of the independent
particle in 1932 [3] and assumed that the nuclei that possessed a stable structure such as the nuclei (4He and 16O) were caused by the arrangement of nucleons in similar shells to those observed for electrons in atomic physics. And during the year 1934 AD That idea expanded to include the heavy nuclei by the scientist Elsasser when he had presented evidence that the shells are occupied with neutrons or protons in numbers represented by {20, 28, 50, 82 and 126} [4] but at that time the available experimental data was very limited to support the nuclear theory which led to the rejection of a shell model heavily by many scientists for several years. During 1948 Mayer presented convincing evidences using the measurements of nuclear binding energies and an abundance of isotopes filled with neutrons or protons in numbers {20, 28, 50, 82 and 126} which made the nuclear shell model back in the forefront of nuclear physics research [5]. Despite the success of the shell model in low orbits, it was unable to achieve this in the upper orbits when using the well potential as a nuclear interaction, and after proving the Scattering studies of neutrons and protons that nucleons are arranged in sequential energy levels according to the quantitative numbers [6,7]. According to the nuclear shell model, the angular momentum and parties of the nuclear energy levels were expected as each level was named by the quantitative number of radials (n) and the value of the orbital angular momentum (l) and the value of the angular momentum of a single particle (j), this type of level is called the level of one particle in the nuclear shell model. The first steps in improving the shell model are choosing more realistic potentials, as it is the basic premise shell model and the effect of interactions between nucleons can be represented by a single particle potential, it may be believed that with very high density and strong forces the nucleons will collide all the time and thus the orbit of the single particle cannot be maintained. But according to Pauli's exclusion principle, in which nucleons are limited to only a limited number of allowed orbits, the typical shell model potential can be expressed:

\[ V(r) = \frac{-V_0}{1 + e^{(r-R)/a}} \]  

(1)

Whereas, R, a and V_0 are the mean radius, the thickness of the nucleus and the depth of the voltage respectively since; the typical values for the coefficients are V_0 ≈ 57 MeV, R≈1.25A, R ≈ 1.25A^{1/3} fm and a ≈ 0.65 fm. In addition, it can add corrections to the depth of the well arising from the symmetry energy of an unequal number of neutrons and protons with the ability of a neutron to interact with a proton in more ways than one: a neutron with a neutron (n-n) and a proton with a proton (p-p) that represents coulomb repulsion and the last interaction of a proton and a neutron (n-p) (hence the strength of n-p is stronger than (n-n) and (p-p). For a spherically symmetrical potential \( V(r) \), it can consider the boundary conditions for energy levels that can be calculated from the Schrödinger equation for the central effort \( V(r) \) starting with the following mathematical formula:

\[ -\frac{\hbar^2}{2m} \nabla^2 \Psi(r) + \left\{\frac{(l+\frac{1}{2})\hbar^2}{2mr^2}V(r)\right\} \Psi(r) = E\Psi(r) \]  

(2)

Where (E) is the eigenvalue of energy.

2. Theory

The remain residual effect between two nucleons is the interaction part that does not include the central interaction rate and represents the force resulting from the collision between nucleons [8], and allowing the residual effect between nucleons to remove the disintegration in the levels which represents a characteristic of the shell model, so the effect will cancel out the simple independent particle model [9,10], and the residual effect causes a disturbance in the Hamilton effect, which is represented by the energy potential of the two nucleons outside the closed core \( V(F_1, F_2) \). According to this, Hamilton is extracted for the perturbation state by the following equation [11,12]:

\[ H = H_{\text{core}} + \sum_{i=1}^{2} H_i + V(F_1, F_2) \]  

(3)

On the basis of this equation the energy equation will be:

\[ E = E_{\text{core}} + \sum_{i=1}^{2} \varepsilon_i + \langle j_1 j_2 | V(F_1, F_2) | j_1 j_2 \rangle \]  

(4)
Whereas, \( E_{\text{core}} \) represents the closed core binding energy and \( \varepsilon_i \) represents the single particle energy (SPE) of the orbit \( i \), whose value can be found from the vicinity of the closed shell with a mass number that is more than one nucleon than the closed core as in the following equation [12]:

\[
\varepsilon_i = BE(\text{core} + 1) - BE(\text{core})
\]

\( \langle j_1|2|V(r_1^2, r_2^2)|j_2 \rangle \) is known as Two-Body Matrix Elements (TBME) for the influence interaction between nucleon-nucleon for the orbits outside of the closed core and can be calculated from the remaining of surface delta interaction(SDI) as follows [12,13]:

\[
V_{\text{SDI}}(r_1^2, r_2^2) = -4\pi A_T \delta(r_1 - r_2) \delta(r_1 - R_0)
\]

Since \( AT \) represents the coefficient of interaction and \( \delta(r_1 - r_2) \) \( \delta(r_1 - R_0) \) is the delta function that describes the residual interaction between two nucleons on the surface of the nuclei whose coordinates are \( r_1 \) \& \( r_2 \) while \( R_0 \) represents the radius of the nucleus, the amount that is produced from this interaction is called the matrix of two-dimensional elements of two different states of two nucleons in the model space orbits [12-15]:

\[
\langle j_1|2|V_{\text{SDI}}(r_1^2, r_2^2)|j_3 \rangle = \frac{A_T}{2(2J + 1)} \left\{ \left( \frac{2J_1 + 1}{2J_2 + 1} \right) \left( \frac{J_3 + 1}{2J_4 + 1} \right) \right\} \delta(r_1 - r_2) \delta(r_1 - R_0)
\]
In this equation, $T_{1/2}$ is the half-life of the initial state and $\alpha$ is the total conversion coefficient of the emitted gamma rays. From equation (11) and equation (12), $B(\sigma \lambda ; J_i \rightarrow J_f)$ can be calculated from the following equation[12-15]

$$B(\sigma \lambda ; J_i \rightarrow J_f) = \frac{\lambda(2\lambda + 1)!!}{8\pi(\lambda + 1)} \frac{h \ln 2}{T_{1/2} (1 + \alpha)} \left( \frac{hc}{E_\gamma} \right)^{2\lambda + 1}$$  \hspace{1cm} (13)

The probability of reduced transmission is given in $e^2 fm^2$ units for electrical transitions and $(\mu_N = e\hbar 2Mpc^2fm^2$) for magnetic transitions. The experimental values of the reduced transition probability are often measured in Weisskopf units, these estimates are based on a model with the following assumptions[19]:

- The nucleus consists of an inert core plus one active particle.
- Transition occurs between the states $J_i = \ell \pm 1/2 \rightarrow J_f = 1/2$.
- The radiation part of the wave functions of the initial and final state inside the nucleus and ends on the outside.

The reduced transition probabilities in Weisskopf units can be transformed by the following conversions[12]:

$$B(E\lambda) = \left(\frac{1.2}{4\pi}\right)^{2\lambda} \left(\frac{3}{\lambda + 3}\right)^2 A^{2\lambda + 3} e^2 f m^{2\lambda}.$$  \hspace{1cm} (14)

$$B(M\lambda) = \frac{10(1.2)^{2\lambda - 2}}{\pi} \left(\frac{3}{\lambda + 3}\right)^2 A^{(2\lambda - 2)/3} \left(\frac{e\hbar}{2Mpc}\right)^2 f m^{2\lambda - 2}$$  \hspace{1cm} (15)

The reduced transition probability is related to the mono-particle electromagnetic effect[19]:

$$B(\sigma \lambda ; J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \left| \langle J_i \alpha_f | O^{\alpha \lambda} | J_i \alpha_i \rangle \right|^2$$  \hspace{1cm} (16)

As $O^{\alpha \lambda}$ is the multipolar electromagnetic operation of the configuration $\lambda$ and is similar to the residual interaction matrix between the two particles, the elements of the reduced matrix of a single-particle operation can be decayed into the elements of the single particle transition matrix[18]:

$$\langle J_i \alpha_i | O^{\alpha \lambda} | J_i \alpha_i \rangle = \sum_{J_1, J_2} c_1(\ell_3, l_4, l_3, l_1, l_2, l_1) \langle J_3, J_4 | O^{\alpha \lambda} | J_1, J_2 \rangle$$  \hspace{1cm} (17)

The probability of transition for a reduced quadrupole electrode can be written based on equation (16) and as follows:

$$B(E2 ; J_i \rightarrow J_f) = \frac{1}{2J_f + 1} \left| \langle J_f \alpha_f | O^{E2} | J_i \alpha_i \rangle \right|^2$$  \hspace{1cm} (18)

3. Results and discussion

Theoretical calculations of the energy spectrum, eigenvectors and the reduced electric quadrupole transition in the $^{70}$Zn nucleus have been performed in both pure and mixed configurations using the nuclear shell model. In the present study, the Fortran 95 language code has been used to program the interactions of the surface delta and the modified surface delta as well as the reduced electric quadrupole transition probability equations. The final energy levels of the above-mentioned nucleus have been calculated in each of the allowable total angular momentum states that depend on the energies of the proton particle through the energy of the singular particle $^{69}$Cu relative to the closed core $^{68}$Ni of the levels$(\epsilon_{01p^3/2} = -9.0456$ MeV, $\epsilon_{00f^5/2} = -7.8321$ MeV, $\epsilon_{00g^7/2} = -6.4938$ MeV) and $(\epsilon_{01p^3/2} = -6.9438$ MeV), the calculations include two nucleons outside the closed nuclear core represented by the $^{68}$Ni nucleus within the possible configurations represented in table (1) for each total angular momentum state and symmetry, the calculations for the nuclei that have been used in the current study will be discussed as follows:
Table 1. Possible configurations for the $^{70}$Zn nucleus.

| $I^n$ | Configurations |
|-------|-----------------|
| $0^+$ | $(1p_{3/2})^2, (0f_{5/2})^2, (1p_{1/2})^2, (0g_{9/2})^2$ |
| $2^+$ | $(1p_{3/2})^2, (1p_{3/2} - 0f_{5/2}), (1p_{3/2} - 1p_{1/2}), (0f_{5/2})^2, (0f_{5/2} - 1p_{1/2}), (0g_{9/2})^2$ |
| $3^+$ | $(1p_{3/2} - 0f_{5/2}), (0f_{5/2} - 1p_{1/2})$ |
| $4^+$ | $(1p_{3/2} - 0f_{5/2}), (0f_{5/2})^2, (0g_{9/2})^2$ |
| $1^+$ | $(1p_{3/2} - 0f_{5/2}), (1p_{3/2} - 1p_{1/2})$ |
| $3^-$ | $(1p_{3/2} - 0g_{9/2}), (0f_{5/2} - 0g_{9/2})$ |
| $4^-$ | $(1p_{3/2} - 0g_{9/2}), (0f_{5/2} - 0g_{9/2}), (1p_{1/2} - 0g_{9/2})$ |
| $5^-$ | $(1p_{3/2} - 0g_{9/2}), (0f_{5/2} - 0g_{9/2}), (1p_{1/2} - 0g_{9/2})$ |
| $6^-$ | $(1p_{3/2} - 0g_{9/2}), (0f_{5/2} - 0g_{9/2})$ |
| $7$  | $(1p_{3/2} - 0g_{9/2})$ |
| $6^+$ | $(0g_{9/2})^2$ |
| $8^+$ | $(0g_{9/2})^2$ |

3.1 $^{70}$Zn isotope

3.1.1 calculation of surface delta interaction (SDI)

The calculation of the binding energies and matrix elements has been performed using equations (5 and 7) based on the value of the reaction force ($A_T = 0.3270$ MeV) for both the pure and mixed configuration s, the results of calculating the excitation energies have been obtained and listed in Table (2):
Table 2. Excitation energy calculations for the $^{70}$Zn nucleus in the pure and mixed configurations of the surface delta interaction [20].

| J$^\pi$ | SDI (Pure) $E_x$ (MeV) | J$^\pi$ | SDI (Mixing) $E_x$ (MeV) | J$^\pi$ |
|---------|------------------------|---------|--------------------------|---------|
| $0^+_1$ | 0                      | $0^+_1$ | 0                        | $0^+_1$ |
| $2^+_1$ | 0.5232                 | $2^+_1$ | 1.2524                  | $2^+_1$ |
| $2^+_2$ | 1.4884                 | $2^+_2$ | 2.1877                  | $2^+_2$ |
| $4^+_1$ | 1.4938                 | $4^+_1$ | 2.3503                  |        |
| $1^+_1$ | 1.75                   | $1^+_1$ | 2.6504                  |        |
| $2^+_1$ | 1.7554                 | $2^+_1$ | 2.7361                  | $2^+_1$ |
| $1^+_1$ | 1.8675                 | $1^+_1$ | 2.7524                  |        |
| $3^+_1$ | 1.8675                 | $3^+_1$ | 2.7679                  | $1^+_2,2^+,3^+$ |
| $0^+_0$ | 2.1                    | $0^+_0$ | 2.7679                  | $2^+,3^+$ |
| $0^+_1$ | 2.519                  | $0^+_1$ | 3.4536                  | $3^+$   |
| $2^+_1$ | 2.5711                 | $2^+_1$ | 3.4671                  | $2^+$   |
| $3^+_1$ | 2.5829                 | $3^+_1$ | 3.8085                  | $3^+$   |
| $2^+_1$ | 2.8568                 | $2^+_1$ | 3.8111                  | $0^+$   |
| $3^+_1$ | 2.9635                 | $3^+_1$ | 3.8639                  | $3^+$   |
| $4^+_1$ | 2.9876                 | $4^+_1$ | 3.9068                  | $4^+$   |
| $5^+_1$ | 3.0076                 | $5^+_1$ | 3.9445                  | $2^+$   |
| $4^+_1$ | 3.2058                 | $4^+_1$ | 4.1062                  |        |
| $6^+_1$ | 3.2058                 | $6^+_1$ | 4.1062                  |        |
| $7^+_1$ | 3.9391                 | $7^+_1$ | 4.8395                  | $5^+,6^+$ |
| $5^+_1$ | 4.0045                 | $5^+_1$ | 4.859                   | $5^+,6^+$ |
| $0^+_0$ | 4.1226                 | $0^+_0$ | 5.2022                  |        |
| $5^+_1$ | 4.2364                 | $5^+_1$ | 5.2644                  |        |
| $4^+_1$ | 4.3018                 | $4^+_1$ | 5.2822                  |        |
| $3^+_1$ | 4.3344                 | $3^+_1$ | 5.3197                  |        |
| $2^+_1$ | 4.4193                 | $2^+_1$ | 5.3197                  |        |
| $4^+_1$ | 4.4193                 | $4^+_1$ | 5.3197                  |        |
| $6^+_1$ | 4.4193                 | $6^+_1$ | 5.7795                  |        |
It is expected a good match between the practical level (0.8849 MeV; 2\(^+\)) and theoretical level (0.8116 MeV; 2\(^+\)) in the mixed configuration. 

- It is expected to determine the total angular momentum and parity (1\(_1^+\)) for the practical level (1.5540 MeV) through its approaching with theoretical level (1.5936 MeV) in the pure configuration. 

- It is predicted that the theoretical level in mixed configuration (1.7499 MeV; 2\(^+\)) has a good match with the practical level (1.7591 MeV; 2\(^+\)) for the same total angular momentum and parity. 

- It is predicted a good match between the practical level (1.7867 MeV; 4\(^+\)) and theoretical level (1.8985 MeV; 4\(_1^-\)) in the mixed configuration. 

- The appearance of good match between theoretical level in mixed configuration (2.1792 MeV; 2\(_1^-\)) and practical level (1.9572 MeV; 2\(^-\)). 

- It is expected a good match between the practical level (2.1406 MeV; 0\(^+\)) and theoretical values (2.1782 MeV; 0\(_1^-\)) and (2.3805 MeV; 0\(_2^-\)) in the pure and mixed configurations respectively. 

- Theoretical plane in mixed configuration (3.3\(_1^-\) 1\(_2^-\); 2.2089 MeV) has appeared matched with the practical level (2,1,3 \(_1^-\);2.3750 MeV) so it is expected to confirm its total angular momentum with values (1,3). 

- It is expected a good match between the theoretical level in the pure configuration (2.5085 MeV; 2\(_4^-\)) and the practical level (2.5383 MeV; 2\(^-\)). 

- It is expected a good match between the theoretical level in the pure configuration (2.7540 MeV; 2\(_5^-\)) and the practical level (2.6650 MeV; 2\(^-\)). 

- It is expected to determine the total angular momentum and parity (3\(_2^-\)) for the practical level (2.8050 MeV) through its approaching with theoretical level (3\(_2^-\); 2.8071 MeV) in the pure configuration. 

- It is expected a good match between the practical level (3.0382 MeV; -5) theoretical level (2.8986 MeV; 5\(_1^-\)) in the mixed configuration. 

- Theoretical level in mixed configuration (2.9760 MeV; 2\(_4^-\)) has a good match with the practical level (1\(^+\), 2\(_+\), 3\(_+\); 2.9497 MeV). 

- Theoretical level in the pure configuration (3.0494 MeV; 4\(_1^-\), 6\(_1^-\)) appeared in a good match with the practical level (3.0220 MeV) so it is expected to determine the total angular momentum and its parity with the values (4\(_1^-\), 6\(_1^-\)). 

- It is expected to confirm the total angular momentum and parity (0\(_1^-\)) for the practical level (3.3280 MeV; 3\(_1^-\)) compared to the theoretical level (3.2551 MeV) in mixed configuration by showing a good match between them. 

- It is expected a good match between the theoretical level in the mixed configuration (3.3049 MeV; 3\(_2^-\)) and the practical level (3.2350 MeV; 3\(_+\) 4\(_+\) 5\(_+\)). 

- The theoretical level in the mixed configuration (3.5472 MeV; 4\(_1^-\), 6\(_1^-\)) appeared in a good match with the practical level (3.4767 MeV) so it is expected to determine the total angular momentum and its parity with the values (4\(_1^-\), 6\(_1^-\)). 

- It is expected to determine the total angular momentum and parity (7\(_i^-\)) for the practical level (3.9140 MeV) compared to the theoretical level (3.8975 MeV; 7\(_i^-\)) in the pure configuration. 

- Theoretical level in the pure configuration (3.9192 MeV; 5\(_2^-\)) appeared in a good agreement with the practical level (4.0015 MeV; (5,6,7)) so it is expected to confirm the total angular momentum with the value (5) and determine its parity (-).
It is predicted a good match between theoretical level (4.1237 MeV; \(5\frac{3}{2}^-\)) in the pure configuration and practical level (4.1720 MeV; \(5\)).

It is expected to confirm the total angular momentum (6\(\frac{3}{2}\)) and determine the parity (-) of the practical level (4.2645 MeV; (5,6,7)) compared to the theoretical level (4.2629 MeV; 2\(\frac{1}{2}^+\), 4\(\frac{3}{2}^-\), 6\(\frac{3}{2}^-\)) in the pure configuration by good match appears between them.

It is expected to confirm the total angular momentum and parity (7\(\frac{1}{2}\)) of the practical level ((5,6,7) ;4.4648 MeV) compared to the theoretical level (4.3945 MeV; 7\(\frac{1}{2}^-\)) in mixed configuration.

It is expected to confirm the total angular momentum (5) and determine the parity (-) of the practical level (4.7101 MeV; (5,6,7)) by appearing a match with the theoretical level (4.7250 MeV; 5\(\frac{3}{2}^-\)) in the mixed configuration.

It is expected to confirm the total angular momentum (6\(\frac{3}{2}\)) and determine the parity (-) of the practical level (4.7917 MeV; (5,6,7)) compared to the theoretical level (4.7607 MeV; 2\(\frac{1}{2}^+\), 4\(\frac{3}{2}^-\), 6\(\frac{3}{2}^-\)) in mixed configuration by appearing a good match between them.

Figure 1.a. shows the excitation pattern diagram of the \(^{70}\)Zn nucleus using the SDI surface delta interaction in the pure and mixed configurations in comparison with the practical [20].
Figure 1.b. shows the excitation pattern diagram of the $^{70}\text{Zn}$ nucleus using the SDI surface delta interaction in the pure and mixed configurations in comparison with the practical [20].
Figure 1.c. shows the excitation pattern diagram of the $^{70}$Zn nucleus using the SDI surface delta interaction in the pure and mixed configurations in comparison with the practical [20].

3.1.2 Calculation of modified surface delta interaction (MSDI)
Using equations (2 and 8), the calculations of the binding energies and the matrix elements have been carried out. The force strength values ($AT = 0.2488$ MeV, $B = 0.3889$ MeV, $C = -0.9482$ MeV) have been performed for both the pure and mixed configurations for calculating the excitation energies listed in the table (3).
Table 3. Calculations of the excitation energy of the $^{70}$Zn nucleus in the pure and mixed configurations of the modified surface delta interaction [20].

| $J^\pi$ | MSDI (Pure) | $E_x$ (MeV) | $J^\pi$ | $E_x$ (MeV) | $E_{Exp.}$ (MeV) | $J^\pi$ | $E_{Exp.}$ (MeV) |
|---------|-------------|-------------|---------|-------------|-----------------|---------|-----------------|
| 0$^+$   | 0           | 0           | 0$^+$   | 0           | 0               | 0$^+$   | 0               |
| 2$^+$   | 0.3981      | -----       | 2$^+$   | 0.8116      | 0.8849          | 2$^+$   | 0.8849          |
| 2$^+$   | 1.3946      | -----       | 2$^+$   | 1.7499      | 1.7591          | 2$^+$   | 1.7591          |
| 4$^+$   | 1.4268      | -----       | 4$^+$   | 1.8985      | 1.7867          | 4$^+$   | 1.8985          |
| 1$^+$   | 1.5936      | 1.5540      | 1$^+$   | 2.0914      | -----           | 1$^+$   | 2.0914          |
| 2$^+$   | 1.6258      | -----       | 2$^+$   | 2.1792      | 1.9572          | 2$^+$   | 1.9572          |
| 1$^+$   | 1.7111      | -----       | 1$^+$   | 2.2089      | 2.3750          | (2,1,3)$^+$ |
| 3$^+$   | 1.7111      | -----       | 3$^+$   | 2.2089      | -----           | 3$^+$   | 2.2089          |
| 0$^+$   | 2.1782      | 2.1406      | 0$^+$   | 2.3805      | 2.1406          | 0$^+$   | 2.1406          |
| 0$^+$   | 2.4408      | -----       | 2$^+$   | 2.976       | 2.9496          | 1$^+,2^+,3^+$ |
| 2$^+$   | 2.5085      | 2.5383      | 2$^+$   | 3.0547      | -----           | 2$^+$   | 3.0547          |
| 3$^+$   | 2.5755      | -----       | 3$^+$   | 3.2551      | 3.3280          | (3)$^+$ |
| 2$^+$   | 2.754       | 2.6650      | 2$^+$   | 3.3049      | 3.2350          | 3$^+,4^+,5^+$ |
| 3$^+$   | 2.8071      | 2.8050      | 5$^+$   | 3.3402      | -----           | 3$^+$   | 3.3402          |
| 4$^+$   | 2.8535      | -----       | 4$^+$   | 3.3626      | -----           | 4$^+$   | 3.3626          |
| 5$^+$   | 2.8986      | 3.0381      | 5$^+$   | 3.3869      | -----           | 5$^+$   | 3.3869          |
| 4$^+$   | 3.0494      | 3.0220      | 4$^+$   | 3.5472      | 3.4767          | 4$^+$   | 3.5472          |
| 6$^+$   | 3.0494      | 3.0220      | 6$^+$   | 3.5472      | 3.4767          | 6$^+$   | 3.5472          |
| 7$^+$   | 3.8975      | 3.9140      | 5$^+$   | 4.3698      | -----           | 7$^+$   | 4.3698          |
| 5$^+$   | 3.9192      | 4.0014      | (5,6,7)$^-$ | 7$^+$   | 4.3954      | 4.4647        | (5,6,7)$^+$ |
| 5$^+$   | 4.1237      | 4.1720      | 5$^+$   | 4.6432      | -----           | 5$^+$   | 4.6432          |
| 4$^+$   | 4.1454      | -----       | 3$^+$   | 4.7148      | -----           | 4$^+$   | 4.7148          |
| 3$^+$   | 4.1983      | -----       | 5$^+$   | 4.7356      | 4.7101          | (5,6,7)$^+$ |
| 2$^+$   | 4.2629      | -----       | 4$^+$   | 4.7607      | -----           | 2$^+$   | 4.7607          |
| 4$^+$   | 4.2629      | 4.2645      | (5,6,7)$^-$ | 2$^+$   | 4.7607      | 4.7917        | (5,6,7)$^+$ |
| 6$^+$   | 4.2629      | 4.2645      | (5,6,7)$^-$ | 2$^+$   | 4.7607      | 4.7917        | (5,6,7)$^+$ |
| 0$^+$   | 4.3572      | -----       | 0$^+$   | 5.3319      | -----           | 0$^+$   | 5.3319          |
| 2$^+$   | 5.2996      | -----       | 2$^+$   | 5.864       | -----           | 2$^+$   | 5.864           |
| 4$^+$   | 5.4446      | -----       | 4$^+$   | 5.9572      | -----           | 4$^+$   | 5.9572          |
- It is expected a good match between the practical level (0.8849 MeV; $2^+$) and theoretical level (0.8116 MeV; $2^+$) in the mixed configuration.
- It is expected to determine the total angular momentum and parity (1$^+_1$) for the practical level (1.5540 MeV) through its approaching with theoretical level (1.5936 MeV) in the pure configuration.
- It is predicted that the theoretical level in mixed configuration (1.7499 MeV; $2^+_2$) has a good match with the practical level (1.7591 MeV; $2^+$) for the same total angular momentum and parity.
- It is predicted a good match between the practical level (1.7867 MeV; $4^+_1$) and theoretical level (1.8985 MeV; $4^+_1$) in the mixed configuration.
- The appearance of good match between theoretical level in mixed configuration (2.1792 MeV; $2^+_2$) and practical level (1.9572 MeV; $2^+$).
- It is expected a good match between the practical level (2.1406 MeV; 0$^-$) and theoretical values (2.1782 MeV; 0$^-$) and (2.3805 MeV; 0$^-$) in the pure and mixed configurations respectively.
- Theoretical plane in mixed configuration ($3^+_11^-_2 ;2.2089$ MeV) has appeared matched with the practical level (2,1,3) $^+$;2.3750 MeV) ,so it is expected to confirm its total angular momentum with values (1,3).
- It is expected a good match between the theoretical level in the pure configuration (2.5085 MeV; $2^+_3$) and the practical level (2.5383 MeV; $2^+$).
- It is expected a good match between the theoretical level in the pure configuration (2.7540 MeV; $2^+_3$) and the practical level (2.6650 MeV; $2^+$).
- It is expected to determine the total angular momentum and parity (3$^+_1$) for the practical level (2.8050 MeV) through its approaching with theoretical level (3$^+_1$; 2.8071 MeV) in the pure configuration.
- It is expected a good match between the practical level (3.0382 MeV; 5$^-$) theoretical level (2.896 MeV; 5$^+_1$) in the pure configuration.
- Theoretical level in mixed configuration (2.9760 MeV; $2^+_2$) has appeared matched with the practical level (1, 2, 3 $^+;2.9497$ MeV).
- Theoretical level in the pure configuration (3.0494 MeV; 4$^-;6^-;6^-;6^-)$ appeared in a good match with the practical level (3.0220 MeV) so it is expected to determine the total angular momentum and its parity with the values (4$^-;6^-;6^-)$.
- It is expected to confirm the total angular momentum and parity (0$^-$) for the practical level (3.2820 MeV; 3$^+$) compared to the theoretical level (3.2551 MeV) in mixed configuration by showing a good match between them.
- It is expected a good match between the theoretical level in the mixed configuration (3.3049 MeV; $3^+_2$) and the practical level (3.2350 MeV; 3$^+4^-5^-$).
- The theoretical level in the mixed configuration (3.5472 MeV; 4$^-;6^-;6^-)$ appeared in a good match with the practical level (3.4767 MeV) ,so it is expected to determine the total angular momentum and its parity with the values (4$^-;6^-;6^-)$.
- It is expected to determine the total angular momentum and parity (5$^-;5^-$) for the practical level (3.9140 MeV) compared to the theoretical level (3.8975 MeV; 7$^-;7^+$) in the pure configuration.
- Theoretical level in the pure configuration (3.9192 MeV; 5$^+;5^-$) appeared in a good agreement with the practical level (4.0015 MeV; 5$^+;5^-$) so it is expected to confirm the total angular momentum with the value (5) and determine its parity (-).
- It is predicted a good match between theoretical level (4.1237 MeV; 5$^+_3$) in the pure configuration and practical level (4.1720 MeV; 5$^-;5^-$).
It is expected to confirm the total angular momentum (6) and determine the parity (-) of the practical level (4.2645 MeV; (5,6,7)) compared to the theoretical level (4.2629 MeV; 2\(^{-}\); 4\(^{-}\), 6\(^{-}\)) in the pure configuration by good match appears between them.

- It is expected to confirm the total angular momentum and parity (7\(^{-}\)) of the practical level (4.4648 MeV) compared to the theoretical level (4.3945 MeV; 7\(^{-}\)) in the mixed configuration.

- It is expected to confirm the total angular momentum (5) and determine the parity (-) of the practical level (4.7101 MeV; (5,6,7)) by appearing a match with the theoretical level (4.7250 MeV; 5\(^{-}\)) in the mixed configuration.

- It is expected to confirm the total angular momentum (6) and determine the parity (-) of the practical level (4.7917 MeV; (5,6,7)) compared to the theoretical level (4.7607 MeV; 2\(^{-}\); 4\(^{-}\), 6\(^{-}\)) in mixed configuration by appearing a good match between them.

**Figure 2.a.** excitation energy diagram of the \(^{70}\)Zn nucleus of the modified surface delta interaction MSDI in the pure and mixed configurations in comparison with the practical values [20].
Figure 2.b. excitation energy diagram of the $^{70}$Zn nucleus of the modified surface delta interaction MSDI in the pure and mixed configurations in comparison with the practical values [20].
Figure 2.c. Excitation energy diagram of the $^{70}$Zn nucleus of the modified surface delta interaction MSDI in the pure and mixed configurations in comparison with the practical values [20].

3.2 reduced electric quadrupole transition probabilities ($BE_2$)

3.2.1 Transition probability calculations for SDI and modified surface delta interactions MSDI

The reduced transition probabilities of the electric quadrupole can be calculated depending on equation (18) through the levels of excitation energies in addition to the eigenvectors as well as determining the value of the necessary coefficients ($e = 3.935$, $\alpha = 0.4901$ fm$^{-1}$) for both the surface delta and the modified surface delta. The results of the transition calculations have been obtained and shown in Table 4:
Table 4. Theoretical calculations of the reduced transition probabilities of electric quadrupole B (E2) for both the surface delta and the modified surface delta interactions compared to the practical [20].

| J₁ → Jₖ       | B(E2) e² fm⁴ | SDI         | MSDI         | Exp. [19]        |
|---------------|--------------|-------------|-------------|------------------|
| 2⁺ₙ → 0⁺ₙ      | 250.7503     | 286.4241    | 286.1968    |                  |
| 2⁺ₙ → 1⁺ₙ      | 1140.054     | 1473.272    | ----        |                  |
| 3⁺ₙ → 2⁺ₙ      | 78.7265      | 94.3559     | 188.5129    |                  |
| 3⁺ₙ → 1⁺ₙ      | 0            | 0           | ----        |                  |
| 4⁺ₙ → 3⁺ₙ      | 1.9555       | 1.2487      | ----        |                  |
| 4⁺ₙ → 2⁺ₙ      | 175.0001     | 187.1429    | 325.6131    |                  |
| 6⁺ₙ → 4⁺ₙ      | 2.5142       | 1.4779      | ----        |                  |
| 6⁺ₙ → 5⁺ₙ      | 199.3322     | 199.3322    | ----        |                  |

Figure 3. Diagram of theoretical transition probabilities of electric quadrupole B (E2) for the 70Zn nucleus of the surface delta and modified surface delta interactions in the pure and mixed configurations in comparison with the practical results [20].

Table (4) and Figure (3) show the following:
- There is an excellent transmission compatibility of 2⁺ → 0⁺₁ for the modified delta surface interaction at the value (86.4241 e² fm⁴).
- New values were found for the quadrupole electric transitions in e² fm⁴ for both the surface delta and the modified surface delta interactions {1140.054 ;2⁻₁ → 1⁻₁ and {1140.054 ;4⁻₁
\[ \rightarrow 3^+; 1.9555 \text{ and } \{1.2487; 6^+ \rightarrow 4^+; 2.5142 \text{ and } \{1.4779; 8^+ \rightarrow 6^+; 199.3322 \text{ and } 199.3322 \} \text{ respectively.} \]

4. Conclusions:

Through the results obtained for the excited energy levels of the \(^{70}\text{Zn}\) nucleus using the interaction of the surface delta and the modified surface delta and for the mixed and pure configurations in addition to reduced electric quadrupole transition probabilities (BE2 ↓) it turns out that there is an acceptable match for the expected theoretical nuclear energy levels with the chosen practical values, some of which have been confirmed and identified some of the values of total angular momentum and parity of the practical energy levels of the nucleus.

The mixed configuration has given values for energy levels more accurate and closer to the practical values than the pure configuration and this is due to the lack of neglecting the effect of the levels that represent the configuration space on each other in the mixed configuration and this is very clear in the higher levels of energy.

By calculating the probability of reduced electric quadrupole transition probabilities (BE2 ↓), there has been an excellent approaching of the transitions values with the available process, especially this approaching has been very clear to the modified surface delta interaction.

An appearing of new values for both energy levels and the reduced electric quadrupole transition probabilities (BE2 ↓) and this has led to a suggestion that these values increase the theoretical knowledge of both energy levels and the probability of transitions. The nuclear shell model is a successful model for calculating energy levels and reduced electric quadrupole transition probabilities for the selected nucleus and within the studied model space.

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