RECENT DEVELOPMENTS IN PERTURBATIVE QCD:
$Q^2$ EVOLUTION OF CHIRAL-ODD DISTRIBUTIONS
$h_1(x, Q^2)$ and $h_L(x, Q^2)^a$

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After reviewing QCD definitions of the chiral-odd spin-dependent parton distributions $h_1(x, Q^2)$ and $h_L(x, Q^2)$, I will summarize the main feature of the recent two results in perturbative QCD: (i) Next-to-leading order $Q^2$ evolution of $h_1(x, Q^2)$. (ii) Leading order $Q^2$ evolution of the twist-3 distribution $h_L(x, Q^2)$ and the universal simplification of the $Q^2$ evolution of all the twist-3 distributions in the large $N_c$ limit.

1 Introduction

Recent developments in collider experiments have been providing us with rich information on the quark-gluon distributions in the nucleon. Particularly interesting are the spin structure functions which reveal “spin distributions” carried by quarks and gluons inside the nucleon. Besides the importance in their own right, they play an indispensable role to test the spin-dependent part of QCD – fundamental theory of the strong interaction: Deeper test of QCD in the spin dependent level may eventually lead us to search for a new physics beyond the standard model.

The nucleon’s structure functions measured in hard processes can be written as a sum of parton distributions for each quark or anti-quark flavor and a gluon. They are functions of Bjorken’s $x$ which represent parton’s momentum fraction in the nucleon and a scale $Q^2$ at which they are measured. Until now, most data on the nucleon’s structure functions have been obtained through the lepton-nucleon deep inelastic scattering (DIS). The chiral-odd distributions, $h_{1,L}(x, Q^2)$, however, can not be measured by the inclusive DIS, and hence there has been no data up to now. They can be measured by the nucleon-nucleon polarized Drell-Yan process and semi-inclusive DIS which detect particular hadrons in the final state. They will hopefully be measured by planned experiments using polarized accelerators at BNL, DESY, CERN and SLAC etc. In particular, RHIC at BNL is expected to provide first data on these distributions.

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Invited talk presented at “QCD Corrections and the New Physics”, October 27-29, 1997, Hiroshima, Japan. To be published in the proceedings. (ed. by J. Kodaira et al.)

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$^a$
In the study of these structure functions, perturbative QCD plays an indispensable role in predicting their $Q^2$-dependence: Given a structure function, say $h_1(x,Q_0^2)$, at one scale $Q_0^2$, perturbative QCD predicts the shape of $h_1(x,Q^2)$ at an arbitrary scale $Q^2$. This $Q^2$ evolution is necessary not only in extracting low energy hadron properties from high energy experimental data but also in testing the $x$-dependence predicted by a non-perturbative QCD technique or a model with the high energy data. In this talk, I will first give the QCD definition of $h_{1,L}(x,Q^2)$ and discuss their $Q^2$-dependence studied in the recent literature.

![Figure 1](image)

Figure 1: (a) Quark distribution function. (b) Nucleon structure function in DIS. (c) Cross section for the nucleon-nucleon Drell-Yan process.

2 Chiral-Odd Distributions $h_{1,L}(x,Q^2)$

Inclusive hard processes can be generally analyzed in the framework of the QCD factorization theorem $^2$. This theorem generalizes the idea of the Bjorken-Feynman’s “parton model” and allows us to include QCD correction in a systematic way. Here I restrict myself to the hard processes with the nucleon target, such as lepton-nucleon deep inelastic scattering (DIS, $l + p \rightarrow l' + X$), Drell-Yan ($p + p' \rightarrow l'^+l'^- + X$), semi-inclusive DIS ($l + p \rightarrow l' + h + X$). According to the above theorem, the cross section (or the nucleon structure function) for these processes can be factorized into a “soft part” and a “hard part”: The soft part represents the parton (quark or gluon) distribution in the nucleon and the hard part describes the short distance cross section between the parton and the external hard probe which is calculable within perturbation theory. For example, a nucleon structure function in DIS can be written as the imaginary part of the virtual photon-nucleon forward Compton scattering amplitude. (Fig. 1 (b)) According to the above theorem, in the Bjorken limit, i.e. $Q^2, \nu = P \cdot q \rightarrow \infty$ with $x = Q^2/2\nu$ finite, ($Q^2 = -q^2$ is the virtuality of the space-like photon, $P$ is the nucleon’s four momentum), the structure
function can be written as
\[
W(x, Q^2) = \sum_a \int_x^1 \frac{dy}{y} H^a\left(\frac{x}{y}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) \Phi^a(y, \mu^2),
\]
(1)
where \( \Phi^a \) represents a distribution of parton \( a \) in the nucleon and \( H^a \) describes the short distance cross section of the parton \( a \) with the virtual photon. \( \mu^2 \) is the factorization scale. In Fig. 1(b), \( \Phi^a \) is identified by the dotted line. (Fig.1 (a)). Similarly to DIS, the cross section for the nucleon-nucleon Drell-Yan process can also be written in a factorized form at \( s = (P_A + P_B)^2, Q^2 \rightarrow \infty \) with a fixed \( Q^2/s \) (\( P_{A,B} \) are the momenta of the two nucleons, \( Q \) is the momentum of the virtual photon):
\[
d\sigma \sim \sum_{a,b} \int_{x_a}^1 dy_a \int_{x_b}^1 dy_b H^{ab}\left(\frac{x_a}{y_a}, \frac{x_b}{y_b}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) \Phi^a(y_a, \mu^2) \Phi^b(y_b, \mu^2),
\]
(2)
where the two parton distributions, \( \Phi^a \) and \( \Phi^b \), for the beam and the target appear as was shown by dotted lines in Fig. 1(c).

As is seen from Figs. 1(b),(c), the parton distribution can be regarded as a parton-nucleon forward scattering amplitude shown in Fig. 1(a) which appear in several different hard processes. In particular, the quark distribution in the nucleon moving in the \( +\hat{e}_3 \) direction can be written as the light-cone Fourier transform of the quark correlation function in the nucleon:
\[
\Phi^a(x, \mu^2) = P^+ \int_{-\infty}^{\infty} \frac{dz^+}{2\pi} e^{ix P^+ z} \langle PS|\bar{\psi}_a(0)\Gamma\psi^a(z)|PS\rangle,
\]
(3)
where \( |PS\rangle \) denotes the nucleon (mass \( M \)) state with momentum \( P^\mu \) and spin \( S^\mu \), and \( \psi^a \) is the quark field with flavor \( a \). In (3), we have suppressed for simplicity the gauge link operator which ensures the gauge invariance and \( |\mu\rangle \) indicates the operator is renormalized at the scale \( \mu^2 \). A four vector \( a^\mu \) is decomposed into two light-cone components \( a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3) \) and the transverse component \( \vec{a} \). In (3), \( z^+ = 0 \), \( \vec{z} = \vec{0} \), and \( z^2 = 0 \). \( \Gamma \) generically represents \( \gamma \)-matrices, \( \Gamma = \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}, 1 \). \( \Phi^a(x, \mu^2) \) measures the distribution of the parton \( a \) to carry the momentum \( k^+ = xP^+ \) in the nucleon, which is independent from particular hard processes.

If one puts \( \Gamma = \gamma_\mu, \gamma_\mu\gamma_5 \), the chirality of \( \bar{\psi} \) and \( \psi \) becomes the same, namely it defines the chiral-even distributions. Likewise, putting \( \Gamma = \sigma_{\mu\nu}, 1 \) defines the chiral-odd distributions. For the case of the deep-inelastic scattering (Fig. 1(b)), the quark line emanating from the target nucleon comes back to the original nucleon after passing through the hard interactions. Since the
perturbative interaction in the standard model preserves the chirality except a tiny quark mass effect, the chirality of the two quark lines entering the nucleon in Fig. 1(b) is the same. Hence the DIS can probe only the chiral-even quark distributions. On the other hand, in the Drell-Yan process (Fig. 1 (c)), there is no correlation in chirality between two quark lines entering each nucleon. Therefore the Drell-Yan process probes both chiral even and odd distributions.

The chiral-odd distributions \( h_1^a(x, \mu^2), h_3^a(x, \mu^2) \) in our interest are defined by putting \( \Gamma = \sigma_{\mu\nu}i\gamma^5 \) in (3):

\[
P^+ \int \frac{dz^-}{2\pi} e^{ixP^z} \langle PS|\bar{\psi}^a(0)\sigma_{\mu\nu}i\gamma^5\psi^a(z)|\mu|PS\rangle = 2\left[ h_1^a(x, \mu^2)(S_{\perp\mu}p_{\nu} - S_{\perp\nu}p_{\mu})/M \
+ h_3^a(x, \mu^2)M(p_{\mu}n_{\nu} - p_{\nu}n_{\mu})(S \cdot n) \
+ h_3^a(x, \mu^2)M(S_{\perp\mu}n_{\nu} - S_{\perp\nu}n_{\mu}) \right]
\]

where we introduced two light-like vectors \( p, n (p^2 = n^2 = 0) \) by the relation \( P^+ = p^\mu + \frac{M^2}{2}n^\mu, p \cdot n = 1, p^- = n^+ = 0 \). If we write \( P^+ = P, p = \frac{P}{\sqrt{2}}(1, 0, 0, 1), n = \frac{1}{\sqrt{2P}}(1, 0, 0, -1) \), \( P \) is a parameter which specifies the Lorentz frame of the system: \( P \to \infty \) corresponds to the infinite momentum frame, and \( P \to M/\sqrt{2} \) the rest frame of the nucleon. \( S^\mu_0 \) is the transverse component of \( S^\mu \) defined by \( S^\mu = (S \cdot p)p^\mu + (S \cdot n)n^\mu + S^\mu_0 \). One can show that \( \Phi^a \) defined in (3) has a support \(-1 < x < 1\). If one replaces the quark field \( \psi \) in (3) by its charge conjugation field \( C\bar{\psi}^T \), it defines the anti-quark distribution \( \bar{\Phi}^a \). In particular \( h_1^a(x, \mu^2) \) in (4) are related to their anti-quark distribution by \( h_1^a(x, \mu^2) = -\bar{h}_3^{a\ast}(x, \mu^2) \).

\( \Phi^a \) appears in a physical cross section in the form of the convolution with a short distance cross section in a parton level as is shown in (1) and (2). The cross section can be expanded in powers of \( \frac{1}{\sqrt{Q^2}} \) as

\[
\sigma(Q^2) \sim A(\ln Q^2) + \frac{M}{\sqrt{Q^2}}B(\ln Q^2) + \frac{M^2}{Q^2}C(\ln Q^2) + \cdots,
\]

where each coefficient \( A, B, C \) receives logarithmic \( Q^2 \)-dependence due to the QCD radiative correction. In order to see how \( h_{1,3}\ ) can contribute in the expansion (4), it is convenient to move into the infinite momentum frame (\( P \sim Q \to \infty \)). In this limit the coefficient of \( h_{1,3}\ ) in (4) behaves, respectively, as \( O(Q), O(1), O(1/Q) \). Therefore if \( h_1 \) contributes to the \( A \) term in (4), \( h_L \) can contribute at most to the \( B \)-term, and \( h_3 \) can contribute at most to the \( C \)-term. In general, when a distribution function contributes to hard processes at
Table 1: Classification of the quark distributions based on spin, twist and chirality. Underlined distributions are chiral-odd. Others are chiral-even.

| spin | average longitudinal | transverse |
|------|----------------------|------------|
| twist-2 | $f_1$ | $g_1$ | $h_1$ |
| twist-3 | $e$ | $h_L$ | $g_T$ |

most in the order of \( \left( \frac{1}{\sqrt{Q^2}} \right)^{\tau - 2} \), the distribution is called twist-\( \tau \). Therefore $h_1$, $h_L$, $h_3$ in (3) is, respectively, twist-2, -3 and -4.

Twist-2 distribution $h_1$ can be measured through the transversely polarized Drell-Yan, semi-inclusive deep inelastic scatterings which detect pions, polarized baryons, correlated two pions.

From the discussion above, one sees that it is generally difficult to isolate experimentally higher twist ($\tau \geq 3$) distributions in hard processes, since they are hidden by the leading twist-2 contribution ($A$ term in (3)). However, this is not the case for $h_L$ and $g_T$. In particular spin asymmetries, they contribute to the $B$-term in the absence of $A$-term: $g_T$ can be measured in the transversely polarized DIS, and $h_L$ appears in the longitudinal versus transverse spin asymmetry in the polarized nucleon-nucleon Drell-Yan process. Therefore the $Q^2$-evolution of $g_T$ and $h_L$ can be a new test of perturbative QCD beyond the twist-2 level.

Insertion of other $\gamma$-matrices in (3) defines other distributions. In Table 1, we show the classification of the quark distributions up to twist-3. There $f_1$, $g_1$, $h_L$, $e$ is defined, respectively, by $\Gamma = \gamma_\mu, \gamma_\mu \gamma_5, 1$ in (3). A similar classification can also be extended to the gluon distributions. The distribution $f_1$ contributes to the spin averaged structure functions $F_{1,2}(x, Q^2)$ familiar in DIS. The helicity distribution $g_1$ contributes to the $G_1(x, Q^2)$ structure function measured in the longitudinally polarized DIS. By now there has been much accumulation of experimental data on $f_1$ and $g_1$, and the data on $g_1$ triggered lots of theoretical discussion on the “origin of the nucleon spin”\]. The first nonzero data on $g_2 (= g_T - g_1)$ was also reported in Ref.\[.]

### 3 Next-to-leading order (NLO) $Q^2$-evolution of $h_1(x, Q^2)$

As we saw in the previous section, $h_1$ is the third and the final twist-2 quark distribution. It has a simple parton model interpretation as can be seen by the Fourier expansion of $\psi$ in (3). It measures the probability in the transversely polarized nucleon to find a quark polarized parallel to the nucleon spin minus the probability to find it oppositely polarized. Here the transverse polarization
Because of its chiral-odd nature it does not mix with gluon distributions for singlet and nonsinglet distributions. For $f_6$ been known for some time it is described by the usual DGLAP evolution equation. The $Q^2$-evolution of $h_1$ is described by the usual DGLAP evolution equation. Therefore the $Q^2$-dependence of $h_1$ is described by the same equation both for singlet and nonsinglet distributions. For $f_1$ and $g_1$, the NLO $Q^2$ evolution was derived long time ago and has been frequently used for the analysis of experiments. The leading order (LO) $Q^2$-evolution for $h_1$ has been known for some time. In the recent literature, the next-to-leading order (NLO) $Q^2$-evolution has been completed by two papers almost at the same time: Vogelsang presented the light-cone gauge calculation for the two-loop splitting function of $h_1$ in the formalism originally used for $f_1$. We carried out the Feynman gauge calculation of the two-loop anomalous dimension following the method of Ref. 18. The results of these calculations in the \( \overline{\text{MS}} \) scheme agreed completely. This result was subsequently confirmed by Ref 21. In the following, I briefly discuss the characteristic feature of the NLO $Q^2$ evolution of $h_1$ following Ref. 21.

Analysis of (4) gives the connection between the $n$-th moment of $h_1$ and a tower of twist-2 operators:

\[
M_n[h_1(\mu^2)] = \int_{-1}^{1} dx x^n h_1(x, \mu^2) = \frac{-1}{2M} \langle PS_{\perp} | O_n^\nu(\mu^2) S_{\perp\nu} | PS_{\perp} \rangle, \tag{6}
\]

where $S_{\perp}$ stands for the transverse polarization and $O_n^\nu(\mu^2)$ indicates the operator $O_n^\nu$ renormalized at the scale $\mu^2$. The contraction with $n^\mu$ and $S_{\perp}^\mu$ (recall $S_{\perp} \cdot n = 0$, $n^2 = 0$) in (6) projects out the relevant twist-2 contribution from the composite operator. (“Twist” for local composite operators is defined as dimension minus spin.) By solving the renormalization group equation for $O_n^\nu S_{\perp\nu}$, one gets the NLO $Q^2$ dependence of $M_n[h_1(\mu^2)]$ as

\[
\frac{M_n[h_1(Q^2)]}{M_n[h_1(\mu^2)]} = \left( \frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \right)^{\gamma_n(0)/2\beta_0} \left[ 1 + \frac{\alpha_s(Q^2) - \alpha_s(\mu^2)}{4\pi} \frac{\beta_1}{\beta_0} \left( \frac{\gamma_n(1)}{2\beta_1} - \frac{\gamma_n(0)}{2\beta_0} \right) \right], \tag{7}
\]

where $\alpha_s(Q^2)$ is the NLO QCD running coupling constant given by

\[
\frac{\alpha_s(Q^2)}{4\pi} = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2)} \left[ 1 - \beta_1 \ln(Q^2/\Lambda^2) \right], \tag{8}
\]

refers to the eigenstate of the transverse Pauli-Lubański operator $\gamma_5 S_{\perp}$. If one replaces the transverse polarization by the longitudinal one, it becomes the helicity distribution $g_1$. For nonrelativistic quarks, $h_1(x, \mu^2) = g_1(x, \mu^2)$. A model calculation suggests, $h_1$ is the same order as $g_1$.
with the one-loop and two-loop coefficients of the β-function \( \beta_0 = 11 - 2/3N_f \) and \( \beta_1 = 102 - 38/3N_f \) (\( N_f \) is the number of quark flavor) and the QCD scale parameter \( \Lambda \). \( \gamma_n^{(0)} \) and \( \gamma_n^{(1)} \) are the one-loop and two-loop coefficients of the anomalous dimension \( \gamma_n \) for \( O_n^I S_{\perp \nu} \) defined as

\[
\gamma_n = \frac{\alpha_s}{4\pi} \gamma_n^{(0)} + \left( \frac{\alpha_s}{4\pi} \right)^2 \gamma_n^{(1)} + \cdots. \tag{9}
\]

If one sets \( \beta_1 \to 0 \) and \( \gamma_n^{(1)} \to 0 \) in (7), the leading order (LO) \( Q^2 \) evolution is obtained. \( \gamma_n^{(0)} \) and \( \gamma_n^{(1)} \) are obtained, respectively, by calculating the one-loop and two-loop corrections to the two-point Green function which imbeds \( O_n^I S_{\perp \nu} \). To obtain \( \gamma_n^{(1)} \), calculation of 18 two-loop diagrams is required in the Feynman gauge. Since the expression for \( \gamma_n^{(1)} \) is quite complicated, we refer the readers to Refs. 24, 25 for them.

![Figure 2: The NLO anomalous dimension \( \gamma_n^{h(1)} \) in comparison with \( \gamma_n^{f(g)(1)} \). This figure is taken from Ref. 25.](image)

In order to get a rough idea about the NLO \( Q^2 \) dependence of \( h_1 \), we plotted in Fig. 2 \( \gamma_n^{h(1)} \) (\( \gamma_n^{(1)} \) for \( h_1 \)) in comparison with \( \gamma_n^{f(g)(1)} \) (\( \gamma_n^{(1)} \) for the nonsinglet \( f_1 \) and \( g_1 \)) for \( N_f = 3, 5 \). One sees from Fig. 2 \( \gamma_n^{h(1)} > \gamma_n^{f(g)(1)} \) especially at small \( n \). This suggests that the NLO \( Q^2 \) evolution of \( h_1 \) is quite different from that of \( f_1 \) and \( g_1 \) in the small \( x \) region. The relation \( \gamma_n^{h(1)} > \gamma_n^{f(g)(1)} \) is in parallel with and even more conspicuous than the LO anomalous
dimensions which read

\begin{align}
\gamma_{n}^{h(0)} &= 2C_{F} \left( 1 + 4 \sum_{j=2}^{n+1} \frac{1}{j} \right), \\
\gamma_{n}^{f g(0)} &= 2C_{F} \left( 1 - \frac{2}{(n + 1)(n + 2)} + 4 \sum_{j=2}^{n+1} \frac{1}{j} \right). \tag{10}
\end{align}

To illustrate the generic feature of the $Q^{2}$ evolution, we have applied the obtained $Q^{2}$ evolution to a reference distribution for $g_{1}$ and $h_{1}$. As a reference distribution, we take GRSV $g_{1}$ distribution\textsuperscript{23} and assume $h_{1}(x, \mu^{2}) = g_{1}(x, \mu^{2})$ at a low energy input scale ($\mu^{2} = 0.23 \text{ GeV}^{2}$ for LO and $\mu^{2} = 0.34 \text{ GeV}^{2}$ for NLO evolution) as is suggested by a nucleon model\textsuperscript{4,15}. We then evolve them to $Q^{2} = 20 \text{ GeV}^{2}$ and see how much deviation is produced between them. The result is shown in Fig. 3. As is expected from the anomalous dimension, the drastic difference in the $Q^{2}$ evolution between $h_{1}$ and $g_{1}$ is observed in the small $x$ region, and this tendency is more significant for the NLO evolution\textsuperscript{28,29,26} (Although $g_{1}$ for $u$-quark mixes with the gluon distribution, the same tendency in the difference from $h_{1}$ is observed for the nonsinglet distribution.)

![Figure 3](image-url)

Figure 3: (a) The LO $Q^{2}$ evolution of $h_{1}$ (denoted by $\delta u$) and $g_{1}$ (denoted by $\Delta u$) for the $u$-quark. (b) The NLO $Q^{2}$ evolution of $h_{1}$ and $g_{1}$ for the $u$-quark. This figure is taken from Ref.\textsuperscript{26}

As another example, we showed in Fig.4 the $Q^{2}$ evolution of the tensor
charge (Fig. 4(a)) and the first moment of $h_1$ and the nonsinglet $f_1$ ($g_1$) (Fig. 4(b)). Although the NLO effect is sizable for the tensor charge, it is small for the first moment. (For the lattice QCD calculation of the tensor charge, see Ref. 30.)

In Ref. 31, the Regge asymptotics of $h_1$ was studied and the small-$x$ behavior was predicted to be $h_1(x) \sim \text{constant } (x \to 0)$. On the other hand, the rightmost singularity of $\gamma_n^{h(0)}$ and $\gamma_n^{h(1)}$ are, respectively, located at $n = -2$ and $n = -1$ in the complex $n$ plane. Therefore inclusion of the NLO effect in the DGLAP asymptotics gives consistent behavior at $x \to 0$ as the Regge asymptotics. This is in contrast to the (nonsinglet) $f_1$ and $g_1$ distributions, whose LO and NLO DGLAP asymptotics are the same.

One of the interesting applications of the obtained NLO $Q^2$ evolution of $h_1$ is the preservation of the Soffer’s inequality, $|2h_1(x,Q^2)| \leq f_1(x,Q^2) + g_1(x,Q^2)$. Although the validity of this inequality hinges on schemes beyond LO \cite{32}, the NLO $Q^2$ evolution maintains the inequality at $Q^2 > Q_0^2$ if it is satisfied at some (low) scale $Q_0^2$ in suitably defined factorization schemes such as $\overline{\text{MS}}$ and Drell-Yan factorization schemes \cite{33,24}.

As was discussed in Sec. 2, a physical cross section is a convolution of a parton distribution and a short distance cross section. (See (1) and (2)) For the double transverse spin asymmetry ($A_{TT}$) in the Drell-Yan process, the NLO
short distance cross section has been calculated in Ref. 35 in the \( \overline{\text{MS}} \) scheme. The analysis on \( A_{TT} \) combined with the NLO tranversity distribution predicts modest but not negligible NLO effect.

4 \( Q^2 \)-evolution of \( h_L(x,Q^2) \) and its \( N_c \to \infty \) limit

In general, higher twist (\( \tau \geq 3 \)) distributions represent quark-gluon correlation in the nucleon. Using the QCD equation of motion, one obtains from (11) the following relation:

\[
h_L(x,\mu^2) = 2x \int_x^1 \frac{dy}{y^2} h_1(y,\mu^2) + \tilde{h}_L(x,\mu^2),
\]

where \( z^2 = 0, \ z^+ = 0 \) and \( S_\parallel \) stands for the longitudinal polarization for the nucleon (\( S^\mu = S_\mu^\parallel = p^\mu - M^2 n^\mu \)). This equation means that \( h_L \) consists of the twist-2 contribution and \( \tilde{h}_L \) which represents quark-gluon correlation in the nucleon. We call the latter contribution “purely twist-3” contribution.

(Expansion of (12) produces twist-3 local operators. See (15) below.) Equation (11) reminds us of the Wandzura-Wilczek relation

\[
g_T(x,\mu^2) = \int_x^1 \frac{dy}{y} g_1(y,\mu^2) + \tilde{g}_T(x,\mu^2).
\]

For \( e \) and \( \tilde{g}_T \), one can write down relations similar to (12).

The \( Q^2 \)-evolution of the first and second terms in (11) is described separately. The evolution of \( \tilde{h}_L \) is quite complicated. A detailed analysis of (12) leads to the following relation for the \( n \)-th moment of \( \tilde{h}_L \):

\[
M_n[\tilde{h}_L(\mu^2)] = \sum_{k=2}^{[(n+1)/2]} \left( 1 - \frac{2k}{n+2} \right) \frac{1}{2M} \langle PS_\parallel | R_{nk}(\mu) | PS_\parallel \rangle,
\]

where \( S_\parallel \) stands for the longitudinal polarization for the nucleon (\( S^\mu = S_\mu^\parallel = p^\mu - M^2 n^\mu \)). This equation means that \( h_L \) consists of the twist-2 contribution and \( \tilde{h}_L \) which represents quark-gluon correlation in the nucleon. We call the latter contribution “purely twist-3” contribution.

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\]

\[
R_{nk} = \frac{1}{2} \left[ \bar{\psi} \sigma^\lambda n \lambda i \gamma_5 (in \cdot D)^{k-2} i g G_{\nu\alpha} n^\nu (in \cdot D)^{n-k-2} \psi - (k \to n-k) \right].
\]

We note that the number of independent operators \( \{ R_{nk} \} \) (\( k = 2, \ldots, [(n + 1)/2] \)) increases with \( n \). In the \( Q^2 \)-evolution, the mixing among \( \{ R_{nk} \} \) occurs and the renormalization is described by the anomalous dimension matrix.
For \( \{R_{nk}\} \), if we put the LO anomalous dimension matrix for \( \{R_{nk}\} \) as \( [\gamma_n(g)]_{kl} = (\alpha_s/2\pi) X_n_{kl} \) corresponding to (9), the solution to the renormalization group equation for \( \{R_{nk}\} \) takes the following matrix form:

\[
\langle PS_\parallel | R_{nk}(Q^2) | PS_\parallel \rangle = \sum_{l=2}^{[(n+1)/2]} \left[ L X_n_{/\beta_0} \right]_{kl} \langle PS_\parallel | R_{nl}(\mu^2) | PS_\parallel \rangle,
\]

where \( L \equiv \frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \). \( X_n \) for \( \tilde{h}_L \) was derived in Ref. [39]. The \( Q^2 \)-evolution for \( \tilde{g}_T \) and \( e \) is also described by matrix equation similar to (14), and the solution was obtained in Refs. [40, 41] for \( g_T \) and in Ref. [42] for \( e \). As is clear from (14) and (16), \( M_n[\tilde{h}_L(Q^2)] \) and \( M_n[\tilde{h}_L(\mu^2)] \) are not connected by a simple equation as in the case for the twist-2 distribution (see (7)). Although (16) gives complete prediction for the \( Q^2 \) evolution, it is generally difficult to distinguish contribution from many operators in the analysis of experiments.

Figure 5: (Right) Complete spectrum of the eigenvalues of the anomalous dimension matrix for \( \tilde{h}_L \) obtained in Ref. [39]. The symbol \( \diamond \) denotes the one-loop anomalous dimension for \( h_1 \). The solid line is the anomalous dimension (21) at large \( n \). (Left) Spectrum of the eigenvalues of the anomalous dimension matrix for \( \tilde{h}_L \) at large \( \tilde{N}_c \). The solid line denotes the analytic solution given in (18). This figure is taken from Ref. [37].

In order to get some feeling on the \( Q^2 \)-evolution of \( \tilde{h}_L \), we plotted the eigenvalues of \( X_n \) in Fig. 5 (right). For comparison, we also showed in the
same figure the LO anomalous dimension $\gamma_n^{(0)}/2$ for $h_1$. (Note the difference in convention between (14) and (16)). As is clear from this figure, the $Q^2$ evolution of $\tilde{h}_L$ is much faster than that of $h_1$. (See discussion below.)

In the recent literature\textsuperscript{45,37,42}, it has been shown that at large $N_c$ (the number of colors), a great simplification occurs in the $Q^2$-evolution of the twist-3 distributions. Recall $X_n$ in (16) is a function of two Casimir operators $C_G = N_c$ and $C_F = N_c^2 - 1 - N_c$. If one takes $N_c \to \infty$, i.e. $C_F \to N_c/2$, (14) and (16) is reduced to

$$\mathcal{M}_n[\tilde{h}_L(Q^2)] = L^{\tilde{\gamma}_n^{h}/2} \mathcal{M}_n[\tilde{h}_L(\mu^2)], \quad \tilde{\gamma}_n^{h} = 2N_c \left( \sum_{j=1}^{n} \frac{1}{j} - \frac{1}{4} + \frac{3}{2(n+1)} \right). \quad (17)$$

This evolution equation is just like those for the twist-2 distributions (see (7)). In Fig. 5 (left), we showed the distribution of the eigenvalues of $X_n$ obtained numerically at $N_c \to \infty$. The solid line is the analytic solution in (18), which shows (18) corresponds to the lowest eigenvalues at $N_c \to \infty$. Since (17) was obtained by a mere replacement $C_F \to N_c/2$ in (16), the correction to the result is of $O(1/N_c^2) \sim 10\%$ level, which gives enough accuracy for practical applications.

This large-$N_c$ simplification is a consequence of the fact that the coefficients of $R_{nk}$ in (14) constitutes the left eigenvector of $X_n$ corresponding to the eigenvalue $\gamma_n^{h}$ in this limit:

$$\sum_{k=2}^{[n+1)/2} \left( 1 - \frac{2k}{n+2} \right) [X_n]_{kl} = - \left( 1 - \frac{2l}{n+2} \right) \tilde{\gamma}_n^{h}, \quad (19)$$

which implies that all the right eigenvectors of $X_n$ except the one corresponding to $\tilde{\gamma}_n^{h}$ are orthogonal to the vector consisting of $\left( 1 - \frac{2k}{n+2} \right)$. This leads to (17).

This large-$N_c$ simplification of the $Q^2$ evolution was proved for the nonsinglet $\tilde{g}_T$ in Ref.\textsuperscript{45} and for $\tilde{h}_L$ and $e$ in Ref.\textsuperscript{37}. The corresponding anomalous dimensions for $\tilde{g}_T$ and $e$ are, respectively,

$$\tilde{\gamma}_n^{\tilde{g}} = 2N_c \left( \sum_{j=1}^{n} \frac{1}{j} - \frac{1}{4} + \frac{1}{2(n+1)} \right), \quad (20)$$

$$\tilde{\gamma}_n^{e} = 2N_c \left( \sum_{j=1}^{n} \frac{1}{j} - \frac{1}{4} - \frac{1}{2(n+1)} \right).$$
Corresponding to three twist-3 distributions in table 1, there are three
independent twist-3 fragmentation functions. (Their number is doubled to 6 if
one includes final state interactions. See Ref.) It has been shown in Ref. \[17\] that
at large \(N_c\) the \(Q^2\) evolution of all these nonsinglet fragmentation functions
is also described by a simple evolution equation similar to \[17\]. Therefore
the simplification of the twist-3 evolution equation is universal to all twist-3
nonsinglet distribution and fragmentation functions.

To illustrate the actual \(Q^2\) evolution of \(h_L\), we have applied \[17\] to the bag
model calculation of \(h_L\) (Fig. 6) Fig. 6(a) shows the bag calculation of \(h_L\).
At the bag scale, purely twist-3 contribution \(\tilde{h}_L\) is comparable to the twist-2
contribution. After the \(Q^2\) evolution to \(Q^2 = 10 \text{ GeV}^2\), \(h_L\) is dominated by the
twist-2 contribution. This is a consequence of the large anomalous dimension
\[18\] compared with the LO anomalous dimension of \(h_{1g}\) in the right figure
of Fig. 5. A similar calculation was done for \(g_T\) in Ref.\[23\].

Another simplification of the twist-3 evolution occurs at \(n \to \infty\). In
this limit, all the twist-3 distributions obey a simple DGLAP equation \[17\]
with a common anomalous dimension which is slightly shifted from \[18\] and
\[20\]:

\[
\gamma_n = 4C_F \left( \sum_{j=1}^n \frac{1}{j} - \frac{3}{4} \right) + N_c. \tag{21}
\]
This evolution equation satisfies the complete evolution equation to the $O(\ln(n)/n)$ accuracy. In the right figure of Fig. 5, $(21)$ is shown by the solid line. One sees that it is close to the lowest eigenvalues except for small $n$. Combined with this $n \to \infty$ result, the large-$N_c$ evolution equation in $(17)$ with $(18)$ and $(20)$ for each distribution is valid to $O((1/N_n^2)\ln(n)/n)$ accuracy.

5 Summary

In this talk, I discussed the recent progress in perturbative QCD, especially the $Q^2$ evolution of the chiral-odd spin-dependent parton distributions $h_1(x,Q^2)$ and $h_L(x,Q^2)$.

The NLO $Q^2$ evolution for the transversity distribution $h_1(x,Q^2)$ was completed in the MS scheme. This means the $Q^2$ evolution of all the twist-2 distributions has been understood in the NLO level. The resulting $Q^2$ evolution of $h_1(x,Q^2)$ turned out to cause quite different behavior from the helicity distribution $g_1(x,Q^2)$ in the small $x$ region if $h_1$ is assumed to be equal to $g_1$ at a low energy scale.

The LO $Q^2$ evolution for the twist-3 distribution $h_L(x,Q^2)$ (and $e(x,Q^2)$) was completed. Although their $Q^2$ evolution is quite complicated due to the mixing among increasing number of quark-gluon-quark operators, it obeys a simple DGLAP equation similar to the twist-2 distribution in the $N_c \to \infty$ limit, as was the case for the $Q^2$ evolution of the nonsinglet $g_T(x,Q^2)$ distribution. The same simplification at $N_c \to \infty$ was also proved for the twist-3 fragmentation functions. Therefore this large-$N_c$ simplification was proved to be universal for the twist-3 distribution and fragmentation functions.

By these studies, we now have at hand the necessary tools given by perturbative QCD for the analysis of the whole parton distributions. From the point of view of the “spin distributions” in the nucleon, one is more interested in the nonperturbative $x$ dependence of those parton distributions. Armed with the development in the perturbative $Q^2$ dependence and the nonperturbative QCD techniques, we will be forced to challenge this issue when the new spin colliders start producing data on the new spin distributions.

Acknowledgements

I would like to thank I.I. Balitsky, V.M. Braun, A. Hayashigaki, Y. Kanazawa, N. Nishiyama and K. Tanaka for the collaboration on which this talk is based. I’m also grateful to J. Kodaira for inviting me to such an exciting conference.

References
1. See, for example, Spin Structure of the Nucleon (World Scientific, 1996, eds. T.-A. Shibata et al.); SPIN96 (World Scientific, 1997, eds. C.W. de Jager et al.).

2. J.C. Collins, D. Soper and G. Sterman, in Perturbative Quantum Chromodynamics (World Scientific, Singapore, 1989, ed. A.H. Mueller), and references quoted therein.

3. J.C. Collins and D.E. Soper, Nucl. Phys. 194, 445 (1982).

4. R.L. Jaffe and X. Ji, Nucl. Phys. B375, 527 (1992).

5. J.P. Ralston and D.E. Soper, Nucl. Phys. B152, 109 (1979).

6. X. Artru and M. Mekhfi, Z. Phys. C45 669 (1990); Nucl. Phys. A532 (1991) 351c.

7. J.L. Cortes, B. Pire and J.P. Ralston, Z. Phys. C55, 409 (1992).

8. R.L. Jaffe and X. Ji, Phys. Rev. Lett 71, 2547 (1993).

9. X. Ji, Phys. Rev. D49, 114 (1994).

10. R.L. Jaffe, Phys. Rev. D54, 6581 (1996).

11. R.L. Jaffe, X. Jin and J. Tang, hep-ph/9709322.

12. R.L. Jaffe, Comm. Nucl. Part. Phys. 19, 239 (1990).

13. The Spin Muon Collaboration (D. Adams et al.), Phys. Lett. B336, 125 (1994); E143 Collaboration (K. Abe et al.), Phys. Rev. Lett. 76, 587 (1996).

14. A.V. Manohar, Phys. Rev. Lett. 65, 2511 (1990); 66, 289 (1991); X. Ji, Phys. Lett. B289, 137 (1992).

15. P.V. Pobylitsa and M.V. Polyakov, Phys. Lett. B389, 350 (1996); V. Barone, T. Calarco and A. Drago, Phys. Lett. B390, 287 (1997).

16. For a review on $h_1$, see R.L. Jaffe, hep-ph/9710463, to be published in Proc. of 2nd Topical Workshop on Deep Inelastic Scattering off Polarized Targets: Theory Meets Experiment (DESY-Zeuthen, Sep. 1-5, 1997).

17. V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. 15, 438 (1972), G. A. Altarelli and G. Parisi, Nucl. Phys. B126, 298 (1977), Yu. L. Dokshitzer, Sov. Phys. JETP 46, 641 (1977).

18. E.G. Floratos, D.A. Ross and C.T. Sachrajda, Nucl. Phys. B129, 66 (1977); B139, 545 (1978) (E); Nucl. Phys. B152, 493 (1979).

19. A. Gonzalez-Arroyo, C. Lopez and F.J. Yndurain, Nucl. Phys. B153, 161 (1979).

20. G. Curci, W. Furmanski and R. Petronzio, Nucl. Phys. B175, 27 (1980), W. Furmanski and R. Petronzio, Phys. Lett 97B, 437 (1980).

21. R. Mertig, W.L. van Neerven, Z. Phys. C70 637 (1996); W. Vogelsang, Phys. Rev. D54 2023 (1996).

22. M. Glück, E. Reya and A. Vogt, Z. Phys. C53, 127 (1992); A.D. Martin, W.J. Stirling and R.G. Roberts, Phys. Lett. B354, 155 (1995); H.L.
23. M. Glück, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. D53, 4775 (1996); T. Gehrmann and W.J. Stirling, Phys. Rev. D53, 6100 (1996); G. Altarelli, R. Ball, S. Forte and G. Ridolfi, Nucl. Phys. B496, 337 (1997).

24. W. Vogelsang, hep-ph/9706511, to appear in Phys. Rev. D.

25. A. Hayashigaki, Y. Kanazawa and Y. Koike, Phys. Rev. D56, 7350 (1997).

26. A. Hayashigaki, Y. Kanazawa and Y. Koike, hep-ph/9710421, to be published in Proc. of 2nd Topical Workshop on Deep Inelastic Scattering off Polarized Targets: Theory Meets Experiment (DESY-Zeuthen, Sep. 1-5, 1997).

27. S. Kumano and M. Miyama, Phys. Rev. D56, 2504 (1997). Revised after submission. See also comment [10] of Ref. 26.

28. V. Barone, T. Calarco and A. Drago, Phys. Rev. D56, 527 (1997).

29. S. Scopetta and V. Vento, hep-ph/9707250.

30. S. Aoki, M. Doui, T. Hatsuda and Y. Kuramashi, Phys. Rev. D56, 433 (1997).

31. R. Kirschner, L. Mankiewicz, A. Schäfer and L. Szymanowski, Z. Phys. C74, 501 (1997); R. Kirschner, hep-ph/9710253, to be published in Proc. of 2nd Topical Workshop on Deep Inelastic Scattering off Polarized Targets: Theory Meets Experiment (DESY-Zeuthen, Sep. 1-5, 1997).

32. J. Soffer, Phys. Rev. Lett. 74, 1292 (1995).

33. G.R. Goldstein, R.L. Jaffe and X. Ji, Phys. Rev. D52, 5006 (1995).

34. C. Bourrely, J. Soffer and O.V. Teryaev, hep-ph/9710224.

35. A.P. Contogouris, B. Kamal and Z. Merebashvili, Phys. Lett. B337, 169 (1994); B. Kamal, Phys. Rev. D53, 1142 (1996).

36. O. Martin, A. Schäfer, M. Stratmann, and W. Vogelsang, hep-ph/9710300.

37. I.I. Balitsky, V.M. Braun, Y. Koike and K. Tanaka, Phys. Rev. Lett. 77, 3078 (1996).

38. W. Wandzura and F. Wilczek, Phys. Lett. B72, 195 (1977).

39. Y. Koike and K. Tanaka, Phys. Rev. D51, 6125 (1995).

40. E.V. Shuryak and A.I. Vainshtein, Nucl. Phys. B201, 141 (1982); A.P. Bukhvostov, E.A. Kuraev and L.N. Lipatov, Sov. Phys. JETP 60, 22 (1984); P.G. Ratcliffe, Nucl. Phys. B264, 493 (1986); I.I. Balitsky and V.M. Braun, Nucl. Phys. B311, 541 (1988/89); X. Ji and C. Chou, Phys. Rev D42, 3637 (1990); D. Müller, Phys. Lett. B407, 314 (1997).

41. J. Kodaira, Y. Yasui and T. Uenatsu, Phys. Lett. B344, 348 (1995); J. Kodaira, Y. Yasui, K. Tanaka and T. Uenatsu, Phys. Lett. B387,
855 (1996); J. Kodaira, T. Nasuno, H. Tochimura, K. Tanaka, Y. Yasui, hep-ph/9712395, Prog. Theor. Phys. in press.

42. Y. Koike and N. Nishiyama, Phys. Rev. D55, 3068 (1997).

43. In carrying out the renormalization of the twist-3 distributions in the covariant gauge, there arises complication due to the mixing with an equation-of-motion (EOM) operator and a BRS exact operator as well as “alien” operators in the intermediate step. For this point and a method to handle the issue, see Refs. 39, 41.

44. For the evolution equation for $h_L$ and $e$, see A.V. Belitsky and D. Müller, Nucl. Phys. B503, 279 (1997).

45. A. Ali, V.M. Braun and G. Hiller, Phys. Lett. B266, 117 (1991).

46. A.V. Belitsky, Phys. Lett. B312, 312 (1997); A.V. Belitsky and E.A. Kuraev, Nucl. Phys. B499, 301 (1997).

47. Y. Kanazawa and Y. Koike, Phys. Lett. B403, 357 (1997).

48. M. Stratmann, Z. Phys. C60, 763 (1993).