Conversions between Electrical Network Representations

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Abstract—The behaviour of electrical networks can be described with many different representations, each with their distinct benefits. In this paper, we consider Z, Y, G, H, ABCD, S and T parameters. Formulas exist to go from one representation to another, but implementing them is an error-prone procedure. In this paper, we present a more elegant way to implement the transformations based on matrix calculations.

Let us start by defining the different circuit parameters we will consider in this paper.

**Impedance and Admittance parameters:** The behaviour of a circuit with \(N\) ports is described by the following relations:

\[
\begin{bmatrix}
V_1 \\
\vdots \\
V_N
\end{bmatrix}
= Z
\begin{bmatrix}
I_1 \\
\vdots \\
I_N
\end{bmatrix}
\quad\text{and}\quad
\begin{bmatrix}
I_1 \\
\vdots \\
I_N
\end{bmatrix}
= Y
\begin{bmatrix}
V_1 \\
\vdots \\
V_N
\end{bmatrix}
\]  

where the \(Y\) and \(Z\) matrices are \(N \times N\) matrices that contain the impedance parameters or admittance parameters of the circuit.

**Mixed parameters:** When two-port circuits are considered, some specialised representations exist that mix voltages and currents at different ports of the circuit.

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix}
= G
\begin{bmatrix}
V_2 \\
I_2
\end{bmatrix}
\quad\text{and}\quad
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix}
= H
\begin{bmatrix}
V_2 \\
I_2
\end{bmatrix}
\]  

When the voltage and current at one of the ports is considered as input signals, the ABCD parameters are obtained. Two different sets of ABCD parameters are used here:

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix}
= A
\begin{bmatrix}
V_2 \\
I_2
\end{bmatrix}
\quad\text{and}\quad
\begin{bmatrix}
V_2 \\
I_2
\end{bmatrix}
= B
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix}
\]  

The \(B\) matrix is the inverse of the \(A\)-matrix if it exists.

**Wave-based parameters:** Instead of working with voltages and currents, incident and reflected waves are used as inputs and outputs of the circuit representation. The incident and reflected waves at a port are related to the voltages and currents measured at the same port in the following way:

\[
A_i = k(V_i + Z_0 I_i) \quad B_i = k(V_i - Z_0 I_i)
\]  

where \(Z_0\) is the normalisation impedance. The parameter \(k\) depends on the preferred definition of the waves. Two different options are available in literature [1], [2]:

\[
k = \frac{1}{2 \sqrt{\text{Re}\{Z_0\}}} \quad k = \alpha \frac{\sqrt{\text{Re}\{Z_0\}}}{2 |Z_0|}
\]

Figure 1. The currents are measured flowing into the circuit and all voltages are referenced to a common ground node.

where \(\text{Re}\{Z_0\}\) indicates the real part of \(Z_0\) and \(\alpha\) is a free parameter of modulus 1.

The S-paramters are now defined as:

\[
\begin{bmatrix}
B_1 \\
\vdots \\
B_N
\end{bmatrix}
= S
\begin{bmatrix}
A_1 \\
\vdots \\
A_N
\end{bmatrix}
\]  

where \(S\) is an \(N \times N\) matrix that contains the S-parameters of the circuit.

The second wave-based representation we consider are the T-parameters:

\[
\begin{bmatrix}
A_1 \\
A_2
\end{bmatrix}
= T
\begin{bmatrix}
B_1 \\
B_2
\end{bmatrix}
\]  

I. TRANSFORMING BETWEEN THE REPRESENTATIONS

All these different parameters describe the behaviour of the circuit. Usually, the designer chooses the representation that best suits the task at hand. It is therefore common to have to transform the circuit representations from one into the other. There are some references available that provide formulas to carry out these transformations, but copying them is an error-prone task and many of the lists of conversion formulas contain errors [3].

In general, the circuit parameters are in a certain representation \(R\) which links the input signals \(U\) to output signals \(O\):

\[
O = RU
\]

They have to be transformed into another representation \(R^N\) with its inputs \(U^N\) and outputs \(O^N\):

\[
O^N = R^NU^N
\]

the transformation from one representation to another can be described by looking at the transformation on the stacked input-output vectors:

\[
\begin{bmatrix}
O^N \\
U^N
\end{bmatrix}
= \begin{bmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{bmatrix}
\begin{bmatrix}
O \\
U
\end{bmatrix}
\]  

where \(\text{Re}\{Z_0\}\) indicates the real part of \(Z_0\) and \(\alpha\) is a free parameter of modulus 1. The S-parameters are now defined as:

\[
\begin{bmatrix}
B_1 \\
\vdots \\
B_N
\end{bmatrix}
= S
\begin{bmatrix}
A_1 \\
\vdots \\
A_N
\end{bmatrix}
\]  

where \(S\) is an \(N \times N\) matrix that contains the S-parameters of the circuit.
The goal of the transformation is to write $R^N$ as a function of the original representation $R$ and the transformation matrix $P$. Solving (7) for $O^N$ gives

$$O^N = (P_{11}R + P_{12})(P_{21}R + P_{22})^{-1} U^N$$

This gives us the expression for $R^N$ in function of $R$:

$$R^N = (P_{11}R + P_{12})(P_{21}R + P_{22})^{-1}$$

This expression allows us to transform any representation into another when we know the transformation matrix $P$. The different transformation matrices for most of the representations are listed at the end of the paper. This approach is based on the excellent paper on mixed-mode S-parameters where a similar transformation is constructed to transform from single-ended to mixed-mode [4].

**Example: Transforming from $Z$ to $G$**

As an example, we show how we obtained the transformation matrix to go from Z-parameters to G-parameters. For this specific transformation, the following two stacked input-output vectors are obtained

$$\begin{bmatrix} O \\ U \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix}, \quad \begin{bmatrix} O^N \\ U^N \end{bmatrix} = \begin{bmatrix} I_1 \\ V_2 \\ V_1 \\ I_2 \end{bmatrix}$$

The transformation matrix $P$ can now be found by finding the permutation matrix that links the two vectors:

$$\begin{bmatrix} I_1 \\ V_2 \\ V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix}$$

Which results in the following conversion formula

$$G = \left( \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} Z + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right) \left( \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} Z + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right)^{-1}$$

**II. CONCLUSION**

A very simple-to-implement method is obtained to go from one circuit representation to another. The method uses a transformation matrix $P$, which is listed for most of the transformations at the end of the paper. The actual transform boils down to splitting $P$ in four parts

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

and then calculating

$$R^N = (P_{11}R + P_{12})(P_{21}R + P_{22})^{-1}$$

where $R$ is the matrix describing the circuit in its original representation and $R^N$ is the circuit matrix in the new representation.

**REFERENCES**

[1] K. Kurokawa, “Power waves and the scattering matrix,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 13, no. 2, pp. 194–202, Mar 1965.

[2] R. B. Marks and D. F. Williams, “A general waveguide circuit theory,” *Journal of Research-National Institute of Standards and Technology*, vol. 97, pp. 533–533, 1992.

[3] D. A. Frickey, “Conversions between s, z, y, h, abcd, and t parameters which are valid for complex source and load impedances,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 42, no. 2, pp. 205–211, Feb 1994.

[4] A. Ferrero and M. Pirola, “Generalized mixed-mode s-parameters,” *IEEE Trans. on Microwave Theory and Techniques*, vol. 54, no. 1, pp. 458–463, Jan 2006.
| From (σ) | Y | Z | G | H | A | S | T |
|----------|---|---|---|---|---|---|---|
| Y (1)    | \[ \begin{bmatrix} 0 & Z & 0 & 1 \\ Z & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \] | \[ \begin{bmatrix} 0 & Z & 0 & 1 \\ Z & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \] | \[ \begin{bmatrix} 0 & Z & 0 & 1 \\ Z & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \] | \[ \begin{bmatrix} 0 & Z & 0 & 1 \\ Z & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \] | \[ \begin{bmatrix} 0 & Z & 0 & 1 \\ Z & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \] | \[ \begin{bmatrix} 0 & Z & 0 & 1 \\ Z & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \] | \[ \begin{bmatrix} 0 & Z & 0 & 1 \\ Z & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \] |