Quantum memory as a perpetuum mobile of the second kind

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Abstract

It is argued that a scalable quantum memory could be used as a perpetuum mobile of the second kind and hence cannot be realized in Nature. The reasoning is based on the assumption that the Landauer’s principle for measurements is a consequence of the second law of thermodynamics and not an independent postulate. This implies a modification of the Landauer’s principle when applied for discrimination of equilibrium states. In Appendices I,II the entropy, heat and work balance for open systems is discussed. In Appendix III a model of measurement violating the standard formulation of Landauer’s principle is presented.

1 Introduction

This note concerns the fundamental question in quantum information: Is fault-tolerant Quantum Information Processing (QIP) feasible? The extensively studied and well-developed theory of fault-tolerant quantum computation [1, 2, 3, 4] provides a positive answer, however its phenomenological assumptions are doubtful and often criticized [5, 6, 7, 8]. On the other hand first principle models are very difficult to analyze and do not give a complete and general answer yet. Therefore, one can ask an easier question: Is a scalable quantum memory feasible? In the recent years several models of quantum memory based on self-correcting spin systems have been proposed [9, 10, 11] and few of them rigorously analyzed [12, 13, 14, 15, 16]. Despite certain stability properties, proved or expected for some of those models, there is no proof that any of the proposals satisfies all the needed conditions for scalable quantum memory. A natural question arises: Can phenomenological thermodynamics provide restrictions or even no-go theorems for Quantum Memory, or generally for QIP?
An attempt of [17] based on the KMS theory was not convincing for many experts because it was based on the mathematical structures related to thermodynamical limit in terms of quasi-local algebras. Here another, more heuristic, approach is presented involving the modified Landauer’s Principle for quantum measurements and two examples of Gedankenexperiment on a system implementing quantum memory.

The main problem with thermodynamical arguments is that the laws of thermodynamics are usually formulated in a natural language and have a common sense character. To apply them to some subtle problems one needs more rigorous formulations, than those found in the most textbooks. This is particularly important in the quantum theory, which often seems to be far from a "common sense".

Another problem is the question of applicability of thermodynamics to QIP. There exist two points of view:

1) Physical systems used for QIP are different from those considered in thermodynamics and therefore thermodynamical restrictions do not apply.

2) Thermodynamics applies.

The author of the present note shares the second opinion following the famous statements:

_But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation._ - Sir Arthur Stanley Eddington.

_(Thermodynamics)...is the only physical theory of universal content which I am convinced that within the framework of applicability of its basic concepts, it will never be overthrown._ - Albert Einstein.

## 2 Applicability of thermodynamics to information processing

The laws of thermodynamics possess a phenomenological and common sense character. In particular the limits of their applicability lie beyond the scope of phenomenological thermodynamics and need serious considerations based on first principle microscopic models. For example one can take a possible formulation of the Zero-th Law:

_Any system coupled to a thermal bath relaxes to the thermodynamical equilibrium state at the bath’s temperature,_
and the Second Law:

*It is impossible to obtain a process such that the unique effect is the subtraction of a positive heat from a reservoir and the production of a positive work.*

In both cases one can ask the questions: How long does thermal relaxation or subtraction of heat can take? Are there any relations between the size of the systems and the time scale of those processes? Similarly, what should be a scale of produced work and how should it depend on the size of the system?

As a simple example consider a ferromagnet consisting of \( N \) microscopic constituents ("spins") below the critical temperature. In principle, any polarized macroscopic state ultimately relax to the unique "equilibrium state" for which the direction of magnetisation \( \vec{M} \) is completely unpredictable. The states with fixed direction of \( \vec{M} \) are metastable with relaxation times increasing *exponentially* with \( N \). For large \( N \) such metastable states possess all expected features of equilibrium states and moreover the rigorous approach involving thermodynamical limit treats them as true equilibrium states corresponding to pure thermodynamical phases. To find an additional relation between admissible time scales of thermodynamic processes one can be guided by the analogous problems in computer science. In the theory of complexity the problem can be solved efficiently if the time needed for the solution grows polynomially with the input size. This suggests the following reformulation of the Zero-th Law:

*Any \( N \)-particle system coupled to a thermal bath relaxes to the (possibly nonunique) thermodynamical equilibrium state at the bath’s temperature with relaxation time growing at most polynomially in \( N \),* and the Second Law:

*It is impossible to obtain an effective process such that the unique effect is the subtraction of a positive heat from a reservoir and the production of a positive work of the order of at least \( kT \). By effective process we mean a process which takes at most polynomial time in the number of particles \( N \).*

The study of applicability of thermodynamics to classical and quantum systems reached a mature status quite recently with the development of fluctuation theorems [18]. The rough formulation of the fluctuation theorem is the following:

*For a system consisting of \( N \) particles the probability of observing during time \( t \) an entropy production opposite to that dictated by the second law of thermodynamics decreases exponentially with \( Nt \).*

Such a formulation has an immediate consequence for classical and quantum
computing. Namely, one cannot hope that the efficiency of any computing scheme can follow from the hypothetical deviations from phenomenological thermodynamics.

3 Measurements and Landauer’s Principle

Landauer’s Principle, first argued in 1961 by Rolf Landauer of IBM [19], holds that any logically irreversible manipulation of information, such as the erasure of a bit or the merging of two computation paths, must be accompanied by a corresponding entropy increase in non-information bearing degrees of freedom of the information processing apparatus or its environment [20].

Specifically, each bit of lost information will lead to the release of an amount $kT \ln 2$ of heat, where $k$ is the Boltzmann constant and $T$ is the absolute temperature of the circuit. On the other hand, if no information is erased, computation may in principle be achieved which is thermodynamically reversible, and require no release of heat. This has led to a considerable interest in the study of reversible computing.

The Landauer’s Principle has a direct relation to thermodynamics of quantum measurement processes. Here, a quantum measurement is a projective von Neumann measurement of an observable $A = \sum_j a_j P_j$ such that for an initial state $\rho$ of the system the final state after measurement is given by:

$$\rho_k = \frac{P_k \rho P_k}{\text{Tr}(P_k \rho P_k)}, \quad \text{when the outcome } a_k \text{ is recorded}$$

(1)

or

$$\rho' = \sum_j P_j \rho P_j, \quad \text{when the outcome is not recorded.}$$

(2)

In order to use the measurement’s outcome for the system’s control one has to assume that after a measurement the system remains in the corresponding state for the time at least of the order $O(1)$.

Consider a 2-level quantum system with a trivial initial Hamiltonian $H(t_0) = 0$ coupled to a heat bath at the temperature $T$. One can design a cyclic procedure of extracting work from a bath consisting of the following steps [22]:

i) Measurement on the system at the initial state $\rho(t_0) = I/2$ in the basis of $\sigma^z$ which yields the outcome $s = \pm 1$ with a collapsed state $\rho(t_1) = |s\rangle\langle s|$. 

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ii) Fast (in comparison to the thermal relaxation time) switching on an external field producing the Hamiltonian $H_s(t_1) = (sE/2)(sI-\sigma^z)$ which increases the energy of the state $|-s\rangle$ by $E >> kT$ and does not change the energy of the state $|s\rangle$.

iii) Slow (again in comparison to the thermal relaxation time) switching off the external fields such that $H(t_2) = 0$. Reseting of the measuring device.

One can compute the balance of work $W(t)$, heat $Q(t)$ and internal energy $E(t)$ during the full cycle $t_0 \rightarrow t_1 \rightarrow t_2$ using the basic definitions discussed in [23, 22] and recalled in the Appendix I:

$$E = \text{Tr}(\rho H), \quad dW = \text{Tr}(\rho dH), \quad dQ = \text{Tr}(d\rho H).$$

In the step i) the state of the system evolves from the complete mixture $\rho(t_0) = I/2$ to the pure state $|s\rangle\langle s|$. No work is performed on the system. In the step ii) the state remains the same and again no work is performed. During the step iii) the system equilibrates at any moment and the work $W \leq kT \ln 2$ is adiabatically extracted from the bath. The system ends the cycle again in the state $\rho(t_0) = I/2$. To avoid the conflict with the Second Law we have to conclude that:

A completion of a binary measurement, including reseting of a measuring device needs at least $kT \ln 2$ of work.

One can call this statement standard Landauer’s Principle for Measurement (sLPM).

The argument of above does not apply if both states $|\pm 1\rangle$ are equilibrium ones in the sense of the definition of above, i.e. their relaxation times to the unique Gibbs state are exponentially long$^1$.

Therefore, one faces the following alternative:

1) The sLPM is valid for all measurements, and therefore is not a consequence of the Second Law, but must be added as an independent additional postulate.

2) The sLPM is a consequence of the Second Law and therefore does not need to hold for equilibrium states.

$^1$A multitude of metastable states for a glassy system must be also treated as an equilibrium state, because the determination of an energy landscape is a computationally hard problem what makes impossible to design a cyclic process of work extraction from a heat bath.
The second possibility is much more likely as the laws of thermodynamics seem to provide ultimate, model independent limitations on physical processes. Assuming now that the task which does not violate the laws of thermodynamics can be realized in principle by a certain physical process one can formulate the following:

**Landauer’s Principle for Measurement (LPM)** A measurement which allows to distinguish between two different states of a system coupled to a heat bath:

a) needs at least $kT \ln 2$ of work if those states relax to their uniform statistical mixture,

b) does not need a net amount of work if those states are equilibrium ones.

**Remarks** It is often claimed that the work (at least $kT \ln 2$) needed to perform a binary measurement is actually used to erase a bit of information in a "memory" of a measuring device [20]. To support this picture microscopic models of erasure have been proposed involving certain entropy-energy balance [24]. However, as shown in the Appendix II this argument is generally not convincing. Therefore, we are left with phenomenological arguments as presented above which do not depend on the detailed model of the measurement procedure. To develop a microscopic model of measurement we need a detailed dynamical description of involved physical processes similar to that presented in the Appendix I. Such an example of a measurement’s model which violates the sLPM is presented in the Appendix III.

4 Perpetuum mobile based on quantum memory

A scalable quantum memory for a single qubit is a system which consists of $N$ microscopic constituents (e.g. atoms, spins...) interacting with a heat bath at the temperature $T > 0$. The system is designed in such a way that the information bearing degrees of freedom form a quantum subsystem (encoded qubit) described by 2-dimensional Hilbert space spanned by the vectors of the form $|\psi\rangle \equiv |\psi\rangle \otimes |\omega_R\rangle$, with $|\omega_R\rangle$ being a purification of the fixed thermal equilibrium state for all other degrees of freedom. The minimal needed assumptions concerning operations on the single-qubit quantum memory are the following:

I) One can perform effectively von Neumann measurements of the observ-
ables $\sigma^z$ and $\sigma^x$.

II) The external Hamiltonian control of the form $H_{\text{control}}(t) = h(t)\sigma^z + g(t)I$
can be effectively executed,

III) The eigenstates of $\sigma^z$ and $\sigma^x$ are equilibrium ones (i.e. metastable with
life-times exponentially long in $N$).

Here again ”effectively” means that one needs time at most polynomial in $N$.

Alternatively, instead of I) and II) one can assume that:
I') One can perform effectively von Neumann measurements of the observable $\sigma^z$.
II') The observables $\sigma^x, \sigma^y$ and $\sigma^z$ can be implemented effectively to construct
an interaction Hamiltonian with another quantum system of the form

$$H_{\text{int}} = \sigma^x \otimes X + \sigma^y \otimes Y + \sigma^z \otimes Z. \quad (4)$$

One can design now two examples of cyclic processes which effectively
extract work from a heat bath using such quantum memory.

**Process A**

In the first example one assumes I), II) , III) and the process consists of
the following steps:

A1) Measurement on the system at the initial state $\rho(t_0) = I/2$ in the basis
of $\sigma^z$ which yields the outcome $s = \pm 1$ with a collapsed state $\rho(t_1) = |s\rangle\langle s|.$

A2) Switching on the external field producing the Hamiltonian $H_s(t_1) = (sE/2)(sI - \sigma^z)$ which increases the energy of the state $|-s\rangle$ by $E \simeq kT$ and
does not change the energy of the state $|s\rangle$.

A3) Measurement on the system at the state $\rho(t_1) = |s\rangle\langle s|$ in the basis of $\sigma^x$,
without recording the result, which yields the outcome state $\rho(t_2) = I/2$.

A4) Switching off the external field such that $H(t_2) = 0$. Reseting of the
measuring device.

One can again compute the balance of work $W(t)$, heat $Q(t)$ and internal
energy $E(t)$ during the full cycle $t_0 \rightarrow t_1 \rightarrow t_2$ using (3). During A1) the
state of the system evolves from the complete mixture $\rho(t_0) = I/2$ to the
pure state $|s\rangle\langle s|$. According to the LPM no work is needed to measure
metastable states. During the step A2) no work is needed due to (3). In
the step A3) again due to the LPM no work is performed but the state is
changed to a complete mixture what demands that heat $Q = E/2 \simeq kT/2$ is
absorbed from the heat bath. In the last step this heat is transformed into work $W \simeq kT/2$. As the measuring device is reseted the information about the state is lost and the system is again in the completely mixed state.

**Remark** If the 2-dimensional subspace of qubit states is not an eigenspace of the temporal qubit Hamiltonian the qubit evolution may interfere with the measurement process. However, it is reasonably to assume that if the level splitting is $E \leq kT$ the heat bath fluctuations always provide the energy amount necessary to satisfy the energy balance during measurement. This is the reason to choose $E \simeq kT$ in the process of above.

**Process B**

In the second process one assumes I’), II’) and III)\(^2\)

B1) Measurement of the memory’s observable $\sigma^z$,

B2) Switching on the coupling Hamiltonian (4), where $X,Y,Z$ are Pauli operators for the relaxing qubit, one performs a SWAP operation [21], transferring the post-measurement memory’s state to the relaxing qubit.

B3) Knowing the state of the relaxing qubit one applies the procedure described in Section 3 to extract $kT\ln 2$ work from the bath.

In this process we use the memory to measure indirectly the relaxing qubit without spending work.

Therefore, the net effect of both cyclic processes is a subtraction of heat from the bath and the production of work what violates the Second Law of Thermodynamics.

### 5 Conclusions

The discussion of *Gedankenexperiment* shows that a scalable quantum memory could be used as a perpetuum mobile of the second kind and hence cannot be realized in Nature. The fundamental assumptions behind the analysis of this model is that the properly formulated laws of thermodynamics are valid and the physical processes which are not forbidden by them can be realized. The arguments of this type can be generally accepted only if there are supported by a large body of independent theoretical and experimental evidence. Therefore, the analysis of microscopic models of the candidates for quantum memory is still important and will be for sure continued in the near future.

\(^2\)The idea of this Gedankenexperiment is based on the discussions with Hector Bombin
Appendix I. Markovian model reproducing the laws of thermodynamics

The laws of thermodynamics can be derived from the following model of the open system coupled to several heat baths and controlled by external forces \[23\]. The density matrix of the system \(\rho(t)\) satisfies the following Markovian Master Equation

\[
\frac{d}{dt}\rho(t) = -i[H(t),\rho(t)] + \sum_j L_j(t)\rho(t),
\]

where for any \(0 \leq t \leq \infty\) \(L_j(t)\) is a generator of a completely positive dynamical semigroup which satisfies

\[
L_j(t)\rho^eq_j(t) = 0, \quad \rho^eq_j(t) = \frac{e^{-\beta_j H(t)}}{\text{Tr}e^{-\beta_j H(t)}}.
\]

Here \(\beta_j = 1/kT_j\) is the inverse temperature of the \(j\)-th heat bath. The equation of motion \((5),(6)\) can be derived from a microscopic Hamiltonian model using the weak coupling assumption and for slowly varying external fields \[25\].

The First Law of thermodynamics becomes now the definition of work \(W\) performed on a system and heat \(Q\) absorbed by the system with the obvious definition of the internal energy \(E\)

\[
E(t) = \text{Tr}(\rho(t)H(t)), \quad \frac{d}{dt}W(t) = \text{Tr}(\rho(t)\frac{dH(t)}{dt}),
\]

\[
\frac{d}{dt}Q(t) = \text{Tr}\left(\frac{d\rho(t)}{dt}H(t)\right) = \sum_j \text{Tr}(H(t)L_j(t)\rho(t)) \equiv \sum_j \frac{d}{dt}Q_j(t).
\]

where \(Q_j\) is the heat absorbed by the system from the \(j\)-th bath. Defining the entropy as \(S(t) = -k\text{Tr}(\rho(t)\ln\rho(t))\) one obtains the Second Law

\[
\frac{d}{dt}S(t) - \sum_j \frac{1}{T_j} \frac{d}{dt}Q_j(t) = \sum_j \sigma_j(t) \geq 0
\]

where the entropy production caused by the \(j\)-baths is given by

\[
\sigma_j(t) = k\text{Tr}\left(L_j(t)\rho(t)[\ln\rho(t) - \ln\rho^eq_j(t)]\right) \geq 0
\]

and its positivity follows from \[6\] and complete positivity of the map \(\exp\{sL_j(t)\}, s \geq 0\).
7 Appendix II. Argument based on energy and entropy balance

Quite often the microscopic derivations of Landauer’s principle are based on the following picture. We consider a process with an initial and final product states for a system coupled to a heat bath

\[ \rho_{\text{in}} \otimes \omega(\beta) \rightarrow \rho_{\text{fin}} \otimes \omega' \]  

(11)

where \( \omega(\beta) \) is a Gibbs state of a bath at the inverse temperature \( \beta \) and \( \omega' \) is a final state of a bath, not necessarily given by another Gibbs state.

Using the definitions and notation

\[ \omega(\beta) = \frac{e^{-\beta H_{\text{bath}}}}{Z(\beta)} , \quad E(\beta) = \text{Tr}(\omega(\beta)H_{\text{bath}}) , \quad S(\beta) = -k\text{Tr}(\omega(\beta)\ln(\omega)) \]  

(12)

one can easily compute

\[ \frac{d}{d\beta} S(\beta) = k\beta \frac{d}{d\beta} E(\beta) . \]  

(13)

From (11) and the fact that the total system is an isolated Hamiltonian one, the entropy balance follows

\[ S(\rho_{\text{fin}}) - S(\rho_{\text{in}}) = S(\beta) - S(\beta') \simeq k\beta(E(\beta) - E(\beta')) \]  

(14)

where \( \beta' \) is the inverse temperature of the Gibbs state such that \( S(\beta') = -k\text{Tr}(\omega'\ln(\omega')) \). In the last equality in (14) we use the fact that a heat bath is a large physical system and its interaction with a small open system can only infinitesimally change bath’s intensive parameters what implies \( \beta' \simeq \beta \).

As the Gibbs state minimizes internal energy under the condition of a fixed entropy the energy gain of a bath satisfies

\[ \Delta E = \text{Tr}(\omega'\rho_{\text{bath}}) - E(\beta) \geq E(\beta') - E(\beta) . \]  

(15)

Defining

\[ \Delta S = S(\rho_{\text{fin}}) - S(\rho_{\text{in}}) = S(\beta) - S(\beta') \]  

(16)

and using (14), (15) one obtains the inequality

\[ \Delta E + T\Delta S \geq 0 . \]  

(17)
One can apply now the scheme of above to a model of resetting a single bit of information in a memory of measuring device. The bit is supported by two degenerated eigenstates of the memory \( |0\rangle, |1\rangle \). The initial state encodes an unknown bit what corresponds to the initial state \( \rho_{in} = 1/2(|0\rangle\langle 0| + |1\rangle\langle 1|) \) with the entropy \( k \ln 2 \), and the final state is a fixed reference state, say \( |0\rangle \), with the entropy equal to 0. Therefore, using (17) one obtains the lower bound for the increase of the bath’s internal energy

\[
\Delta E \geq kT \ln 2.
\]  

(18)

As the process is cyclic in the sense that the external time-dependent control fields switched on at the beginning of the process are switched off at its end, and the energy of the 2-level system is not changed one can attribute \( \Delta E \) to the amount of work performed by the external forces and dissipated into the bath’s degrees of freedom. Hence, the Landauer’s principle for bit’s erasure seems to be justified on the microscopic basis. Moreover, as an actual measurement which transforms the initial reference state \( |0\rangle \) into \( |0\rangle \) or \( |1\rangle \) does not change the entropy this part of a cyclic measurement process does not need work and hence the sLPM seems to be valid as well.

Unfortunately, the arguments of above are not convincing. The main problem is the entropy balance based on the assumption of the exact product structure for initial and final states (11). Indeed, a weak coupling to a heat bath suggests an approximative product state structure but due to the discontinuity of entropy in the limit of large systems we cannot use it for the estimation of entropy. This fact is expressed in terms of Fannes inequality [26] for two close density matrices of a system with \( D \)-dimensional Hilbert space and \( \| \cdot \|_1 \) denoting the trace norm

\[
|S(\rho) - S(\rho')| \leq \| \rho - \rho' \|_1 \ln D - \| \rho - \rho' \|_1 \ln(\| \rho - \rho' \|_1) .
\]  

(19)

To illustrate this problem one can consider the model discussed in the Appendix I. The validity of the Markovian approximation means that the state of the total system is well-approximated by the product \( \rho(t) \otimes \omega_\beta \) where \( \rho(t) \) is a solution of the Master equation \([5]\). Obviously, this product form is not consistent with the constant entropy of the total Hamiltonian system. The missing entropy is hidden in the small correction terms describing the residual system - bath correlations and local perturbations of the bath’s state which practically do not influence the values of measured observables. Therefore, to obtain a proper entropy, heat and work balance one has to use their
definitions as presented in Appendix I and needs an equation of motion for a system like that obtained in the Markovian limit [5].

8 Appendix III. A measurement model violating standard Landauer’s Principle

We present a classical model of a complete measurement process, including reseting of the measuring device, which can discriminate between two equilibrium states of the system and does not need work (see Fig.1). The system is a macroscopic ferromagnetic bar placed along the \( x \)-direction which can be found in two states of polarization. The measuring device is a compass needle with the magnetic moment \( \mu \) rotating in the \( xy \)-plane which can be transported along the \( z \)-axis. The magnetic field \( B \) acting on the needle is directed along the \( x \)-axis and depends on \( z \). Therefore, the Hamiltonian of the needle is given by

\[
H(\phi, z) = -\mu B(z) \cos \phi
\]

where \( \phi \) is an angle between the compass needle and the \( x \)-axis.

![Fig.1. Measurement of magnet’s polarization](image)

The work performed on the needle by slowly moving it from the position \( z_1 \) at time \( t_1 \) to \( z_2 \) at time \( t_2 \) can be computed using the classical version of
the formula (7)

\[ W(t_2, t_1) = \int_{t_1}^{t_2} dt \int_0^{2\pi} d\phi p_{eq}(\phi, z(t)) \frac{d}{dt} H(\phi, z(t)) = \int_{z_1}^{z_2} F(z) dz \]  

(21)

where the thermodynamical force \( F(z) = \int_0^{2\pi} d\phi p_{eq}(\phi, z) \frac{\partial}{\partial z} H(\phi, z) \) and \( p_{eq} = Z^{-1} \exp\{-H/kT\} \) is a canonical distribution. In the derivation of (21) we use the assumption that the process is slow and therefore can be considered as isothermal.

The cyclic measurement process begins with the compass needle placed in the position \( z_0 \) far away from the magnet (\( |\mu B(z_0)| \ll kT \)) and hence the initial state of the needle is a uniform distribution with respect to \( \phi \). Then the needle is slowly moved to the position \( z_m \) such that \( |\mu B(z_m)| \gg kT \) and hence the direction of the needle shows the polarization state of the magnet - this is the actual measurement. To reset the needle’s state one moves it slowly back to \( z_0 \). According to the formula (21) the net work performed on the whole system is equal to zero.

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