Angle Tracking Robust Learning Control for Pneumatic Artificial Muscle Systems

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ABSTRACT Pneumatic artificial muscle systems have been widely used in the applications of biomimetic robots and medical auxiliary devices. The existence of high nonlinearities, uncertainties and time-varying characteristics in pneumatic artificial muscle systems brings much challenge for accurate system modeling and controller design. In this paper, a robust adaptive iterative learning control scheme is proposed to solve the angle tracking problem for a kind of pneumatic artificial muscle-actuated mechanism. After deriving the system model according to the feature of mechanism, Lyapunov synthesis method is used to design the control law and adaptive learning laws. Robust strategy and full saturation learning strategy are jointly used to compensate parametric/ nonparametric uncertainties and reject external disturbances. Alignment condition is applied to solve the initial position problem of iterative learning control. As the iteration number increases, the system state can accurately track the reference trajectory over the whole interval. In the end, a simulation example is presented to demonstrate the effectiveness of the designed control scheme.

INDEX TERMS Pneumatic artificial muscle systems; adaptive iterative learning control; initial position problem; alignment condition; Lyapunov approach

I. INTRODUCTION

Robotics and the related technologies have developed rapidly in recent years. For pursuing higher performances, it has been a hot issue to design robotic systems with new materials and technologies [1], [2]. Pneumatic artificial muscle (PAM) systems are a kind of novel tube-like biomimetic actuators. They can contract or extend by inflating and deflating pressurized air through servo valves, which are similar to human skeletal muscles in size and power output capability [2]- [5]. Thanks to no cumbersome mechanisms used, the devices actuated by PAMs have many advantages, such as cheapness, light weight, compliance, high power/weight ratio and high power/volume ratio [6], [7]. In view of these above advantages, PAM systems has earned increasing attentions and have been widely applied in rehabilitation medicine, bionic robot and human-computer integration over the past ten years.

Due to the complex nonlinearities caused by the pressurized air and geometric features of PAM systems, there exist many difficulties in system modeling and controller design for PAM systems. The past two decades witnessed the venturous exploration in this field. In [3], sliding mode control technique was applied to solve the tracking problem for PAM systems. In [8], an advanced nonlinear PID control scheme was proposed for the position control of a PAM-actuated setup. In [9], a neural network based predictive control algorithms was developed for the trajectory tracking control of PAM sytems. In [10] and [11], a backstepping adaptive control controller was designed for PAM systems, respectively. In [12], the fuzzy control for pneumatic muscle driven rehabilitation robot was studied, and a modified genetic algorithm was developed to identify the optimal set of parameters for the fuzzy controller. In [13], a novel CDESO-based nonlinear active disturbance rejection control scheme was designed to control the PAM system. To the best of authors’ knowledge, there has been few reports on adaptive iterative learning control (ILC) application to PAM systems, so far.
ILC is one of the most effective control strategies in dealing with repeated tracking control or periodic disturbance rejection for nonlinear systems [14]-[18]. After the advent of ILC in the early 1980s, ILC gained more and more attentions because of its advantages such as high control precision and less knowledge on system model. While an ILC system operates, the controller can gradually update the control input by using the system information in the previous trial, till excellent control performance is obtained. Up to now, ILC has been widely applied in many practical applications, e.g., robotic manipulators, power electronic circuits, hard disk drives, and chemical plants [19]-[24]. Adaptive ILC can be regarded as a combination of ILC and adaptive control [14], [25]. Owing to its convenience in controller design for parametric and nonparametric uncertain systems, adaptive ILC has received increasing attention over the past 20 years. However, while putting an adaptive ILC theoretical algorithm into solving the real control tasks including angle tracking for PAM systems, one has to face the so-called initial position problem of ILC. This problem means a contradiction to be resolved between theory and practice as follows. On the one hand, many traditional ILC theoretical algorithms require the initial error should be zero at each learning cycle; otherwise, a slight nonzero initial error may lead to the divergence of tracking error. On the other hand, as far as real industrial processes are concerned, due to the limitation of physical resetting, it is impossible to achieve zero initial error at each learning cycle. Therefore, for the need of applying ILC theory into practice, researchers have to investigate the adaptive ILC schemes under nonzero error initial condition. Up to now, a few solutions have been reported in recent years, e.g., time-varying boundary layer approach [26], error-tracking approach [27], initial rectifying action [28]-[29] and alignment condition strategy [30]-[33]. Among them, alignment condition strategy is suitable for the cases where the reference trajectory is smoothly closed. While adopting this strategy, initial position resetting can be replaced by setting the value of initial state at current cycle equal to the value of final state at previous cycle at current cycle. In [30], ILC for nonlinear MIMO nonparametric systems under alignment condition was investigated. In [31], a barrier composite energy function approach was proposed to solve the tracking problem for position-constrained robot manipulators under alignment condition. In [32], a suboptimal learning control algorithm was studied for improving the convergence speed of nonlinearly parametric time-delay ILC systems under alignment condition. In [33], the tracking problem for tank gun control systems under alignment condition was investigated, and good performance was achieved by using the proposed robust adaptive ILC design. However, how to develop an effective adaptive ILC scheme for PAM systems under alignment condition is still unclear.

Motivated by the above discussion, this work focuses on the adaptive ILC algorithm design for PAM systems with nonzero initial errors. The main results and contributions are given as follows.

(1) By constructing a novel Lyapunov–Krasovskii functional, an adaptive ILC law is developed to make the state of PAM systems track the reference trajectory over the whole time interval.

(2) By using parameter separation technique and signal replacement mechanism, robust strategy and adaptive ILC strategy are jointly to deal with nonparametric uncertainties and parametric uncertainties.

(3) In ILC design, the alignment condition is used to remove the zero initial error condition, which is used to solve the initial position problem of ILC.

The paper is organized as follows. The system model and problem formulation is introduced in Section II. The detailed procedure of controller design is addressed in Section III. The convergence analysis of closed-loop PAM systems is given in Section IV. In Section V, the simulation results are illustrated to verify the effectiveness of the proposed control scheme. Finally, Section VI concludes this work.

II. PROBLEM FORMULATION

Let us consider a kind of PAM-actuated mechanism [34] operating over a fixed finite time interval repeatedly, whose control structure is shown in Fig. 1. The two control variables of this mechanism may be described as follows:

$$\begin{align*}
   u_a(t) &= u_p + k_a u(t), \\
   u_b(t) &= u_p - k_a u(t),
\end{align*}$$

where $t \in [0, T]$, $k_a$ is the coefficient of voltage distribution, $u_p$ is the preloaded voltage, $u(t)$ is the control law, and $u_a(t)$ and $u_b(t)$ are two input control voltages of pressure proportional valves, respectively. Here, $T$ represents the operation time at each iteration.

![Fig. 1. Control structure of the PAM-actuated mechanism](image)

The internal pressures of two PAM actuators are given as

$$\begin{align*}
   P_1(t) &= P_0 + \Delta P(t) = k_0 u_a(t), \\
   P_2(t) &= P_0 - \Delta P(t) = k_0 u_b(t),
\end{align*}$$

where $k_0$ is the proportional coefficient of the input control voltages and the output pressures for pressure proportional valves, $P_0$ denotes the preload internal pressure of actuators, $\Delta P(t)$ represents the variation of pressure, and $P_1(t)$ and...


\[
P_2(t) \text{ are two internal pressures of PAM actuators. The pulling forces of PAM actuators are derived according to (3).}
\]

\[
\begin{align*}
F_1(t) &= P_1(t)(k_1\epsilon_1^2(t) + k_2\epsilon_1(t) + k_3) + k_4, \\
F_2(t) &= P_2(t)(k_1\epsilon_2^2(t) + k_2\epsilon_1(t) + k_3) + k_4,
\end{align*}
\]

(3)

where \(F_1(t)\) and \(F_2(t)\) are two pulling forces of PAM actuators, \(k_1, k_2, k_3\) and \(k_4\) are four parameters, and \(\epsilon_1(t)\) and \(\epsilon_2(t)\) are derived according to (4).

\[
\begin{align*}
\epsilon_1(t) &= \epsilon_0 + r_0^{-1}\theta(t), \\
\epsilon_2(t) &= \epsilon_0 - r_0^{-1}\theta(t),
\end{align*}
\]

(4)

where \(\theta(t)\) is the deflection angle of the mechanism, \(\epsilon_0\) and \(l_0\) are the initial shrinking rate and initial length of PAM actuators, respectively. The driving moment of this mechanism is expressed as:

\[
T_p(t) = J\dot{\theta}(t) + b_v\dot{\theta}(t) = F_1(t)r - F_2(t)r + \nu(\theta(t), \dot{\theta}(t), t),
\]

(5)

in which \(J\) and \(b_v\) represent a moment of inertia and a damping coefficient, respectively, and \(\nu(\theta(t), \dot{\theta}(t), t)\) denotes the sum of unknown external disturbances and unmodeled dynamics. Substituting (1)-(4) into (5) yields

\[
J\dot{\theta}(t) + b_v\dot{\theta}(t) = k_0u_r(4k_1\epsilon_0r_0^{-1} + 2k_2r_0^{-1})\theta(t) + k_0k_ur(2k_1\epsilon_0^2 + 2k_1r(\theta(t)r_0^{-1})^2 + 2k_2\epsilon_0 + 2k_3)u(t) + \nu(\theta(t), \dot{\theta}(t), t)
\]

(6)

Due to the deflection angle \(\theta(t)\) is very small, \(2k_1r(\theta(t)r_0^{-1})^2 \approx 0\) [34]. Then, from (6), we have

\[
\dot{\theta}(t) = -\frac{b_v}{J}\dot{\theta}(t) + \frac{2k_0u_r(2k_1\epsilon_0 + k_2)r_0^{-1}}{J}\theta(t) + \frac{2k_0k_ur(k_1\epsilon_0^2 + k_2\epsilon_0 + k_3)}{J}u(t) + \nu(\theta(t), \dot{\theta}(t), t)
\]

(7)

where \(\nu(\theta(t), \dot{\theta}(t), t) = \nu(\theta(t), \dot{\theta}(t), t)/J\). By setting \(x_1(t) = \theta(t)\) and \(x_2(t) = \dot{\theta}(t), y(t) = x_1(t)\) from (7), we get the state-space model of PAM systems at the \(j\)th iteration as follows:

\[
\begin{align*}
\dot{x}_{1,j}(t) &= x_{2,j}(t), \\
\dot{x}_{2,j}(t) &= u_{p,j}\beta_1 x_{1,j}(t) + \beta_2 x_{2,j}(t) + \omega u_{j}(t) + \nu(x_j(t), t) \\
y_k(t) &= x_{1,k}(t)
\end{align*}
\]

(8)

where \(\beta_1 = \frac{2k_0u_r(2k_1\epsilon_0 + k_2)r_0^{-1}}{J}\), \(\beta_2 = -\frac{b_v}{J}\), and \(\omega = \frac{2k_0k_ur(k_1\epsilon_0^2 + k_2\epsilon_0 + k_3)}{J}\).

Define \(x_1(t) = [x_1(t), x_2(t)]^T, x_2(t) = [x_{1d}(t), x_{2d}(t)] = [x_{1d}(t), \dot{x}_d(t)]^T\) and \(e_j(t) = [e_{1,j}, e_{2,j}]^T = x_j(t) - x_d(t)\). Without loss of generality, we make the assumptions as follows:

Assumption 1:

\[
\nu(x_j(t), t) = \nu_1(x_j(t), t) + \nu_2(x_j(t), t)
\]

where \(\nu_1(x_j(t), t) - \nu_1,k(x_d(t)) \leq l_1\|e_j(t)\|\) and \(\omega_2(x_j, t) \leq l_2, l_1\) and \(l_2\) are unknown constants.

Assumption 2: \(x_d(t) \in C^1[0, T], x_d(T) = x_d(0)\).

The control task of this work is to make \(x_{1,k}(t)\) accurately track its reference signal \(x_{1d}(t)\) over \([0, T]\), as the iteration index \(j\) increases. For the sake of brevity, the arguments in this paper are sometimes omitted when no confusion is likely to arise. We denote \(\nu_1(x_j(t), t)\) and \(\nu_2(x_j(t), t)\) briefly by \(\nu_{1,j}\) and \(\nu_{2,j}\), respectively.

Remark 1: In this work, we consider the situation that the reference trajectory meets \(x_d(T) = x_d(0)\). The solution to initial position problem of ILC is to combine \(x_{1,j}(0) = x_{1,j}(-1)\) with \(x_d(T) = x_d(0)\). This kind of strategy is called alignment condition in the literature [30].

III. CONTROLLER DESIGN

From (8), we have

\[
\begin{align*}
\dot{e}_{1,j}(t) &= e_{2,j}(t), \\
\dot{e}_{2,j}(t) &= u_{p,j}\beta_1 x_{1,j}(t) + \beta_2 x_{2,j}(t) + \omega u_{j}(t) + \nu_j(x_j(t), t) - \dot{x}_d.
\end{align*}
\]

(9)

Define \(s_j = 2e_{1,j} + e_{2,j}\) and \(s_{\phi,j} = s_j - \phi\text{ sat}(\frac{s_j}{\phi})\), where \(\phi\) is a positive constant and \(\text{sat}_{\phi}(\cdot)\) is defined as follows: for a scalar \(\bar{a}\),

\[
\text{sat}_{\phi,\bar{a}}(\bar{a}) = \begin{cases} 
\bar{a} & \bar{a} \geq \phi \bar{a} \\
\phi \bar{a} & \phi \bar{a} < \phi \bar{a}
\end{cases}
\]

for a vector \(\hat{a} = [\hat{a}_1, \hat{a}_2, \cdots, \hat{a}_m] \in \mathbb{R}^m, \text{sat}_{\phi,\bar{a}}(\hat{a}) = [\text{sat}_{\phi,\bar{a}}(\hat{a}_1), \text{sat}_{\phi,\bar{a}}(\hat{a}_2), \cdots, \text{sat}_{\phi,\bar{a}}(\hat{a}_m)]^T\).

Taking the derivative of \(s_k\) with respect to \(t\), we have

\[
s_j = 2e_{2,j} + u_{p,j}\beta_1 x_{1,j} + \beta_2 x_{2,j} + \omega u_{j} + \nu_{1,j} + \nu_{2,j} - \dot{x}_d
\]

(10)

While \(|s_j| > \phi\), taking time derivative of \(V_j = \frac{1}{2}\|s_{\phi,j}\|\) yields

\[
\dot{V}_j = \frac{1}{\omega} s_{\phi,j} \dot{s}_{\phi,j}
\]

(11)

By Assumption 2, we obtain

\[
\frac{1}{\omega} s_{\phi,j} \nu_{1,j} \leq \frac{1}{\omega} \|e_j\| + \frac{1}{\omega} s_{\phi,j} \nu_1(x_d, t)
\]

(12)

and

\[
\frac{1}{\omega} s_{\phi,j} \nu_{2,j} \leq \frac{1}{\omega} \|l_2\| s_{\phi,j}.
\]

(13)
Substituting (12) and (13) into (11), we have
\[
V_j \leq s_{\phi_j} \left( \frac{1}{\omega} 2e_{2,j} + \frac{1}{\omega} u_{p,j} \beta x_{1,j} + \frac{1}{\omega} \beta_2 x_{2,j} + u_j \right) \\
- \frac{1}{\omega} \dot{e}_j + \frac{l_1}{\omega} s_{\phi_j} \| e_j \| s_{\phi_j} \| e_j \| \frac{1}{\phi} + \frac{1}{\omega} s_{\phi_j} \nu_1 (x_d, t) \\
+ \frac{1}{\omega} \int_{t-1}^{t} s_{\phi_j} \| e_j \| s_{\phi_j} \| e_j \| \frac{1}{\phi} \, ds_{\phi_j}(t) - u_j \\
= s_{\phi_j} (\eta_{1,j}^T \varphi_j + \eta_{2,j}^T \psi_j + u_j)
\]
(14)
where \( \eta_1 \triangleq \left[ \frac{\beta}{\omega}, \frac{\beta_2}{\omega}, \frac{\nu_1 (x_d, t)}{\omega} \right]^T \), \( \eta_2 \triangleq \left[ \frac{2e_{2,j} - \dot{x}_d, u_{p,j} x_{1,j}, x_{2,j}}{\omega} \right]^T \), \( \psi_j \triangleq \left[ \| e_j \| s_{\phi_j} \| e_j \| \frac{1}{\phi} \right]^T \). Now, on the basis of (14), we design control law and learning laws as follows:
\[
\eta_{1,j} = - \mu_1 s_{\phi_j} - \eta_{1,j}^T \varphi_j - \eta_{2,j}^T \psi_j,
\]
\( \eta_{1,j} \text{sat}_{\frac{1}{\lambda_1}}(\eta_{1,j}) + 3 \mu_3 s_{\phi_j} \varphi_j, \eta_{1,j} = 0 \),
(15)
\[
\eta_{2,j} = \text{sat}_{\frac{1}{\lambda_2}}(\eta_{2,j} - 1) + 3 \mu_3 s_{\phi_j} \varphi_j, \eta_{2,j} = 0 \),
(16)
where \( u_{p,j} \) is set as
\[
u_{p,j} = \begin{cases} 
\mu_i, & \text{for } j = 0 \\
\mu_{i-j}(T), & \text{for } j \geq 1
\end{cases}
\]
and \( \mu_i > 0 \) for \( i = 1, 2, 3, 4 \).

Remark 2: Each element in \( \eta_2 \) is positive. \( \eta_{2,j} \) is used to estimate the vector \( \eta_2 \). By the property of ILC system, the system error may converge whether the elements in \( \eta_{2,j} \) are positive, negative, or zeroes. In order to get higher convergence speed, the elements in \( \eta_{2,j} \) should be set to small positive numbers or zeroes, rather than negative numbers. In this work, the elements in \( \eta_{2,j} \) are set to zeroes. It is also reasonable to let the elements in \( \eta_{2,j} \) be small positive numbers, for example, \( \eta_{2,j} = [0.02, 0.02]^T \).

IV. CONVERGENCE ANALYSIS

Theorem 1:
For the PAM system (8) satisfying Assumption 1 and Assumption 2, by means of the proposed robust learning control law (15) and learning laws (16)-(17), the angle tracking error can converge in the sense that \( |e_{1,j}(t)| \leq \frac{\phi}{2}, \forall t \in [0, T] \), as the iteration index \( j \) increases, and all variables of the closed-loop PAM system are ensured to be bounded.

Proof 1:
Firstly, let us define a Lyapunov functional at the \( j \)th learning cycle as follows:
\[
L_j = V_j + \frac{1}{2 \mu_2} \int_{0}^{t} \tilde{\eta}_{1,j}^T \tilde{\eta}_{1,j} \, dt + \frac{1}{2 \mu_3} \int_{0}^{t} \tilde{\eta}_{2,j}^T \tilde{\eta}_{2,j} \, dt,
\]
(18)
where \( \tilde{\eta}_{1,j} = \eta_{1,j} - \eta_{1,j}^* \) and \( \tilde{\eta}_{2,j} = \eta_{2,j} - \eta_{2,j}^* \).
According to the definition of \( L_k \), we obtain
\[
L_j = L_{j-1} - V_{j-1} + \frac{1}{2 \mu_2} \left( \int_{0}^{t} \tilde{\eta}_{1,j}^T \tilde{\eta}_{1,j} \, dt - V_{j-1} \right) + \frac{1}{2 \mu_3} \left( \int_{0}^{t} \tilde{\eta}_{2,j}^T \tilde{\eta}_{2,j} \, dt \right)
\]
(19)
for \( j > 0 \).
Combining (14) with (15) leads to
\[
V_j = V_j(0) - \mu_1 \int_{0}^{t} s_{\phi_j}^2 \, dt + \int_{0}^{t} s_{\phi_j} \tilde{\eta}_{1,j}^T \varphi_j + \tilde{\eta}_{2,j}^T \psi_j \, dt,
\]
(20)
By substituting (20) into (19), we obtain
\[
L_j = L_{j-1} - V_{j-1} + \frac{1}{2 \mu_2} \left( \int_{0}^{t} \tilde{\eta}_{1,j}^T \tilde{\eta}_{1,j} \, dt - V_{j-1} \right) + \frac{1}{2 \mu_3} \left( \int_{0}^{t} \tilde{\eta}_{2,j}^T \tilde{\eta}_{2,j} \, dt \right)
\]
(21)
For \( a, b, c \in R, (a-b)^2-(a-c)^2 = -2(a-b)(b-c)-(b-c)^2 \) holds. By using this property and (16), we deduce
\[
\frac{1}{2 \mu_2} \left( \tilde{\eta}_{1,j}^T \tilde{\eta}_{1,j} - \tilde{\eta}_{1,j-1}^T \tilde{\eta}_{1,j-1} \right) + s_{\phi_j} \tilde{\eta}_{1,j}^T \varphi_j
\]
\[
= \frac{1}{2 \mu_2} (\eta_{1,j} - \eta_{1,j-1})^T (\eta_{1,j} - \eta_{1,j-1}) - \mu_2 s_{\phi_j} \varphi_j
\]
\[
- \frac{1}{2 \mu_2} (\eta_{1,j} - \eta_{1,j-1})^T (\eta_{1,j} - \eta_{1,j-1})
\]
\[
= \frac{1}{2 \mu_2} (\eta_{1,j} - \eta_{1,j-1})^T (\eta_{1,j} - \eta_{1,j-1}) \leq 0.
\]
(22)
In a similar way, from (17), we have
\[
\frac{1}{2 \mu_3} \left( \tilde{\eta}_{2,j}^T \tilde{\eta}_{2,j} - \tilde{\eta}_{2,j-1}^T \tilde{\eta}_{2,j-1} \right) + s_{\phi_j} \tilde{\eta}_{2,j}^T \psi_j
\]
\[
= \frac{1}{2 \mu_3} (\eta_{2,j} - \eta_{2,j-1})^T (\eta_{2,j} - \eta_{2,j-1}) - \mu_3 s_{\phi_j} \psi_j
\]
\[
- \frac{1}{2 \mu_3} (\eta_{2,j} - \eta_{2,j-1})^T (\eta_{2,j} - \eta_{2,j-1})
\]
\[
= \frac{1}{2 \mu_3} (\eta_{2,j} - \eta_{2,j-1})^T (\eta_{2,j} - \eta_{2,j-1}) \leq 0.
\]
(23)
Substituting (22) and into (21) yields
\[
L_j = L_{j-1} - V_{j-1} + \mu_1 \int_{0}^{t} s_{\phi_j}^2 \, dt
\]
(24)
In view of the alignment condition, \( x_{j-1}(T) = x_j(0) \) and \( x_d(T) = x_d(0) \) hold. Then, from (24), we have
\[
L_j(T) = L_{j-1}(T) = - \mu_1 \int_{0}^{T} s_{\phi_j}^2 \, dt.
\]
(25)
Further, we have
\[
L_j(T) \leq L_0(T) - \mu_1 \sum_{i=1}^{j} \int_{0}^{T} s_{\phi_i}^2 \, dt.
\]
(26)
According to the continuity of Lyapunov functional, we can see that $L_0(t)$ is bounded for $t \in [0, T]$. Thus, from (26), we have

$$\lim_{j \to +\infty} \int_0^T s_{j\alpha}^2 d\tau = 0. \quad (27)$$

On the other hand, it follows from (24) that

$$L_j(t) \leq L_{j-1}(T) - \mu_1 \int_0^t s_{j\alpha}^2 d\tau. \quad (28)$$

From (25), we can see that $L_{j-1}(T)$ is bounded. Thus, from (28), we conclude that $L_j(t)$ is bounded. According to this conclusion, we can easily get the boundedness of $s_j$, $e_j$, and $a_j$. By the property of saturation functions, we can further deduce that $s_j$ is bounded. Now, we can deduce that $|s_{j\alpha}(t)|$ is bounded, which means $s_{j\alpha}(t)$ is equicontinuous. On account of the above conclusion, it follows from (28) that there must exist a positive integer $\beta$,

$$|s_{j\alpha}(t)| = 0 \quad \forall t \in [0, T], \forall j \geq \beta. \quad (29)$$

According to the relationship $\dot{e}_{1,i} + 2e_{1,i} = s_j$, we have

$$e_{1,j}(t) = e^{-2t}e_{1,j}(0) + e^{-2t} \int_0^t e^{2\tau} s_{j}(\tau) d\tau, \quad (30)$$

$$e_{1,j-1}(T) = e^{-2T}e_{1,j-1}(0) + e^{-2T} \int_0^T e^{2\tau} s_{j-1}(\tau) d\tau, \quad (31)$$

$$e_{1,1}(T) = e^{-2T}e_{1,1}(0) + e^{-2T} \int_0^T e^{2\tau} s_{1}(\tau) d\tau, \quad (32)$$

$$e_{1,0}(T) = e^{-2T}e_{1,0}(0) + e^{-2T} \int_0^T e^{2\tau} s_{0}(\tau) d\tau. \quad (33)$$

By using the fact $e_{1,i}(0) = e_{1,i-1}(T)$ for $i = 1, 2, \cdots, j$, from (30)-(33), we have

$$e_{1,j}(t)$$

$$= e^{-2(t+iT)}e_{1,0}(0) + \sum_{i=j-p+1}^j e^{-2(t+iT)} \int_0^T e^{2\tau} s_{j-i}(\tau) d\tau$$

$$+ \sum_{i=0}^{j-p} e^{-2(t+iT)} \int_0^T e^{2\tau} s_{j-i}(\tau) d\tau$$

$$\leq e^{-2(t+jT)}e_{1,0}(0) + \sum_{i=j-p+1}^j e^{-2(t+iT)} \int_0^T e^{2\tau} s_{j-i}(\tau) d\tau$$

$$+ \sum_{i=0}^{j-p} e^{-2(t+iT)} \int_0^T e^{2\tau} s_{j-i}(\tau) d\tau + e^{-2t} \int_0^t e^{2\tau} s_{j}(\tau) d\tau. \quad (34)$$

From (29), we obtain that $|s_i(t)| \leq \phi$ while $i \geq \beta$. Therefore,

$$e^{-2t} \int_0^t e^{2\tau} s_j(\tau) d\tau + \sum_{i=1}^{j-\beta} e^{-2(t+iT)} \int_0^T e^{2\tau} s_{j-i}(\tau) d\tau$$

$$\leq \frac{\phi}{2} e^{-2t} (e^{2t} - 1) + \frac{\phi}{2} e^{-2T} e^{2T} (e^{2T} - 1)$$

$$+ \frac{\phi}{2} e^{-2T} e^{2T} (e^{2T} - 1) + \cdots$$

$$+ \frac{\phi}{2} e^{-2T} e^{2T} (e^{2T} - 1)$$

$$\leq \frac{\phi}{2} (1 - e^{-2t}) + \frac{\phi}{2} e^{-2T} (1 - e^{-2t}) + \frac{\phi}{2} e^{-2T} (e^{2T} - e^{-4T})$$

$$+ \cdots + \frac{\phi}{2} e^{-2T} (e^{2T} - e^{2T} - e^{-4T}) \quad (35)$$

Combining (34) with (35), we have

$$e_{1,j}(t)$$

$$\leq e^{-2(t+jT)}e_{1,0}(0) + \sum_{i=j-\beta+1}^j e^{-2(t+iT)} \int_0^T e^{2\tau} s_{j-i}(\tau) d\tau$$

$$+ \frac{\phi}{2} - \frac{\phi}{2} e^{-2t} e^{2T} (j-\beta)T \quad (36)$$

Since $e_{1,0}(0)$ and $\int_0^T e^{2\tau} s_j(\tau) d\tau$ are bounded, $e^{-2(t+jT)}e_{1,0}(0) + \sum_{i=j-\beta+1}^j e^{-2(t+iT)} \int_0^T e^{2\tau} s_{j-i}(\tau) d\tau - \frac{\phi}{2} e^{-2t} e^{2T} (j-\beta)T$ converges to zero as the iteration index $j$ increases. Combining this with (36), we can draw a conclusion that

$$|e_{1,j}(t)| \leq \frac{\phi}{2}, \quad \forall t \in [0, T] \quad (37)$$

holds as the iteration index $j$ increases.

In this work, alignment condition is used to deal with the initial position problem in the design of PAM ILC system. Full saturation learning method is applied to estimate unknown time-invariant constants and unknown time-varying but iteration-independent uncertainties, which is of significance to ensure the boundedness of all signals in the closed-loop PAM system.

V. NUMERICAL SIMULATION

In this section, we carry out a numerical simulation for the PAM system (8), where $\mathbf{x}_d = [-2 \cos(0.4t), 0.8 \sin(0.4t)]^T$, $v(\mathbf{x}_j(t), t) = 0.6 \sin(x_{1,j})x_{2,j} + 0.1 \sin(x_{1,j}x_{2,j}) + 0.2 \text{rand}(j)$ with rand($j$) being a random number between 0 and 1. The model parameters are listed in TABLE 1.

| Parameters |
|------------|
| $k_0 = 0.09$ | $k_1 = 0.1$ | $k_2 = 0.15$ | $k_3 = 0.4$ | $\epsilon_0 = 0.5$ |
| $k_u = 1$ | $r = 40mm$ | $l_0 = 200mm$ | $u_p = 2.5V$ | $J = 1kg \cdot m^2$ |

The control parameters and gains in control law (15) and learning laws (16)-(17) are set as follows: $\phi = 0.002, \mu_1 = \ldots$
5, $\mu_2 = 0.5$ and $\mu_3 = 0.05$. The of system initial states are set as $x_0(0) = [0.7, 0]^T$ and $x_j(0) = x_{j-1}(T)$ for $j \geq 1$.

After 30 cycles, the simulation results are shown in Figs. 2-7. Figs. 2-3 show the profiles of angle and angular velocity over $[0, T]$ at the 30th learning cycle, and the corresponding tracking error profiles presented in Figs. 4-5. From Figs. 2-5, we conclude that $x(t)$ can precisely track $x_d(t)$. The control input at the 30th learning cycle is shown in Fig. 6. Fig. 7 gives the convergence history of $s_{\phi_j}$, where $J_k \triangleq \max_{t \in [0, T]} |s_{\phi_j}|$. The above simulation results verify the effectiveness of theoretical analysis in this work.

VI. CONCLUSION
The angle tracking problem of PAM systems is studied in this work. A novel robust adaptive iterative learning control scheme is proposed to solve the angle tracking problem for PAM systems. By using parameter separation technique and signal replacement mechanism, robust strategy and adaptive ILC strategy are jointly to deal with nonparametric uncertainties and parametric uncertainties. The alignment condition is used to remove the zero initial error condition. Simulation results show the effectiveness of our proposed robust adaptive ILC scheme.

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