Open string T-duality in double space *

B. Sazdović †
Institute of Physics,
University of Belgrade,
11001 Belgrade, P.O.Box 57, Serbia

March 6, 2018

Abstract

The role of double space is essential in new interpretation of T-duality and consequently in an attempt to construct M-theory. The case of open string is missing in such approach because until now there have been no appropriate formulation of open string T-duality. In the previous paper [1], we showed how to introduce vector gauge fields $A^N_a$ and $A^D_i$ at the end-points of open string in order to enable open string invariance under local gauge transformations of the Kalb-Ramond field and its T-dual "restricted general coordinate transformations". We demonstrated that gauge fields $A^N_a$ and $A^D_i$ are T-dual to each other. In the present article we prove that all above results can be interpreted as coordinate permutations in double space.

1 Introduction

It is well known that the M-theory unifies all five consistent superstring theories by web of T and S dualities. In order to formulate the M-theory we should construct one theory which contain the initial theory (any of the five consistent) and all corresponding dual ones.

The 2D dimensional double space with the coordinates $Z^M = (x^\mu, y_\mu)$ (which components are the coordinates of initial space $x^\mu$ and its T-dual $y_\mu$) offers many benefits in interpretation of T-duality. In fact in such a space, the T-duality transformations can be realized simply by exchanging places of some coordinates $x^a$, along which we performed T-duality and the corresponding dual coordinates $y_a$ [2, 3]. It contains the initial and all corresponding T-dual theories. Realization of such program for T-duality in the bosonic case has been done: for flat background in Ref.[2] and for the weakly curved background,

---

*Work supported in part by the Serbian Ministry of Education and Science, under contract No. 171031.
†e-mail: sazdovic@ipb.ac.rs
with linear dependence on coordinates, in Ref.[3]. We hoping that S-duality, which can be understood as transformation of dilaton background field, can be successfully incorporated in such procedure.

T-duality for superstrings is non-trivial extension of the bosonic case. In Ref.[4] we extended such approach to the type II theories. In fact, doubling all bosonic coordinates we have unified types IIA and IIB theories. The formulation of M-theory should include T-dualization along fermionic variables, also. T-dualization along all fermionic coordinates in fermionic double space (where we doubled all fermionic variables) has been considered in Ref.[5].

The remaining step is to extended interpretation of T-duality in double space (which we earlier propose for the case of the closed string) to the case of the open string, also. This will be done in the present article.

The difference between open and closed string appears at the open string end points. Until recently, background fields along Neumann and Dirichlet directions \( A^N_a \) and \( A^D_i \) \((N, D \text{ denote components with Neumann and Dirichlet boundary conditions}) are treated in different way [6,7]. The Neumann vector field has been introduced in the Lagrangian through the coupling with \( \dot{x}^a \). On the other hand, Dirichlet vector field has been introduced as a consistency conditions without contributions to the Lagrangian. In order to realize double space formulation in the open string case we should treat both Neumann and Dirichlet vector fields in the same way. This has recently been done in Ref.[1].

In Refs.[8] it has been shown how to introduce vector gauge fields \( A^N_a \) in order that open string retained symmetries of the closed string. Note that according to Ref.[1], beside well known local gauge invariance of the Kalb-Ramond field we used its T-dual "restricted general coordinate transformations", which includes transformations of background fields but not include transformations of the coordinates. So, above interpretation of the T-duality in double space will confirm expressions for T-dual closed string background fields \( G_{\mu\nu} \) and \( B_{\mu\nu} \) (as in Refs.[2,3]) and gives the same expressions for T-dual vector fields \( \ast A^D_i \) and \( \ast A^N_a \) as that obtained in Refs.[1] with Buscher’s procedure.

2 T-duality of the open string

In this section we will introduce some known features of the bosonic string and shortly repeat main results of Ref.[1]. We will adapt T-duality to be in compliance with boundary conditions on the open string end-points.

We will consider vector gauge fields: \( A^N_a \) with Neumann boundary conditions which compensate not implemented gauge symmetry of the Kalb-Ramond field at the open string end-points and \( A^D_i \) with Dirichlet boundary conditions, which compensate not implemented restricted general coordinate transformations at the open string end-points. We
will show that field $A^D$ is T-dual to the $A^N$ one, as well as the general coordinate transformations are T-dual to gauge symmetry.

2.1 Closed and open Bosonic string

Let us start with the action for closed bosonic string \[ S[x] = \kappa \int_{\Sigma} d^2\xi \sqrt{-g} \left[ \frac{1}{2} g^{\alpha\beta} G_{\mu\nu}[x] + \frac{e_{\alpha\beta}}{\sqrt{-g}} B_{\mu\nu}[x] \right] \partial_\alpha x^\mu \partial_\beta x^\nu, \quad (\varepsilon^{01} = -1). \quad (2.1) \]

It propagates in D-dimensional space-time with background defined by the space metric $G_{\mu\nu}$ and the Kalb-Ramond field $B_{\mu\nu}$. We denoted the string coordinates by $x^\mu(\xi)$, $\mu = 0, 1, ..., D - 1$ and the intrinsic world-sheet metric by $g_{\alpha\beta}$. The integration goes over two-dimensional world-sheet $\Sigma$ with coordinates $\xi^\alpha$ ($\xi^0 = \tau, \xi^1 = \sigma$).

In the conformal gauge $g_{\alpha\beta} = e^{2F} \eta_{\alpha\beta}$ this action can be rewritten in terms of light-cone coordinates $\xi^\pm = \frac{1}{2}(\tau \pm \sigma)$, $\partial_\pm = \partial_\tau \pm \partial_\sigma$ as

\[ S = \kappa \int_{\Sigma} d^2\xi \partial_+ x^\mu \Pi_+ \partial_\alpha x^\nu, \quad (2.2) \]

with following combination of background fields

\[ \Pi_+ = B_{\mu\nu} \pm \frac{1}{2} G_{\mu\nu}. \quad (2.3) \]

In the string theory, variation of the action (2.2) with respect to $x^\mu$ produces not only equation of motion

\[ \partial_+ \partial_- x^\mu + \left( \Gamma^\mu_{\nu\rho} - B^\mu_{\nu\rho} \right) \partial_+ x^\nu \partial_- x^\rho = 0, \quad (2.4) \]

but also boundary conditions

\[ \gamma^{(0)}_\mu(x) \delta x^\mu / \sigma = \pm \gamma^{(0)}_\mu(x) \delta x^\mu / \sigma = 0, \quad (2.5) \]

where $\Gamma^\mu_{\nu\rho}$ is Christoffel symbol and we introduce useful variable

\[ \gamma^{(0)}_\mu(x) = \frac{\delta S}{\delta x^\mu} = \kappa \left[ 2B_{\mu\nu} \dot{x}^\nu - G_{\mu\nu} \dot{x}^\nu \right]. \quad (2.6) \]

From now on, we will denote boundary of the open string with $\partial \Sigma$, so that relation (2.5) we can rewrite as

\[ \gamma^{(0)}_\mu(x) \delta x^\mu / \partial \Sigma = 0. \quad (2.7) \]

As a consequence of periodicity, the boundary conditions are trivially satisfied in the closed string case. In the open string case there are two different ways to satisfy boundary conditions. For some coordinates $x^a$ ($a = 0, 1, \cdots, p$) we will chose Neumann boundary
conditions, when variations of string end points $\delta x^a/\partial \Sigma$ are arbitrary and for the rest ones $x^i$ ($i = p + 1, \cdots, D - 1$) we will chose Dirichlet boundary conditions, when edges of the string are fixed $\dot{x}^i/\partial \Sigma = 0$. In order to satisfy Neumann boundary conditions according to (2.5) we should take $\gamma_a^{(0)}(x)/\partial \Sigma = 0$.

It is well known that closed string theory is invariant under following infinitesimal transformations: local gauge transformations of the Kalb-Ramond field

$$\delta \Lambda G_{\mu\nu} = 0, \quad \delta \Lambda B_{\mu\nu} = \partial_{\mu} \Lambda_{\nu} - \partial_{\nu} \Lambda_{\mu},$$

and general coordinate transformations

$$\delta \xi G_{\mu\nu} = -2 (D_\mu \xi_{\nu} + D_\nu \xi_{\mu}), \quad \delta \xi B_{\mu\nu} = -2 \xi^\rho B_{\mu\rho\nu} + 2 \partial_{\mu} (B_{\nu\rho} \xi^\rho) - 2 \partial_{\nu} (B_{\mu\rho} \xi^\rho).$$

These transformations are connected by T-duality [11, 1]. Let us stress that according to Ref.[1] we are not going to add transformations of the coordinates to (2.9). So, we will call this "restricted general coordinate transformations”.

Both above symmetries are failed at the open string end-points. In order to restore these symmetries the gauge fields have to be introduced. To restore local gauge symmetry we introduce the vector fields $A^N_a$ with Neumann boundary conditions (see Ref.[8]), while to restore restricted general coordinate transformations we introduce the vector fields $A^D_i$ with Dirichlet boundary conditions (see Ref.[1]). Note that as a consequence of the boundary conditions only parts of these gauge fields survive.

So, the action for open bosonic string with above boundary conditions is [1]

$$S_{\text{open}}[x] = \kappa \int_{\Sigma} d^2 \xi \partial_+ x^\mu \prod_{\pm \mu} \partial_\pm x^{\nu'} + 2 \kappa \int_{\partial \Sigma} d\tau \left( A^N_a[x] \dot{x}^a - \frac{1}{\kappa} A^D_i[x] G^{-1} i \gamma_j^{(0)}(x) \right)$$

$$= \kappa \int_{\Sigma} d^2 \xi \partial_+ x^\mu \prod_{\pm \mu} \partial_\pm x^{\nu'} + 2 \kappa \gamma^\alpha \beta \int_{\partial \Sigma} d\tau A_{\alpha\beta}[x] \partial_j x^\mu,$$

where following Ref.[1] we introduced effective variables $A_{\pm \mu}(V) = \{ A^{\pm a}(V), A_{\pm i}(V) \}$ defined as

$$A_{\pm a}(V) = A^N_a(V), \quad A_{\pm i}(V) = 2 \Pi_{\pm j} G^{-1} j k A^D_k(V),$$

and for simplicity we assumed that the metric tensor has a form

$$G_{\mu\nu} = \begin{pmatrix} G_{ab} & 0 \\ 0 & G_{ij} \end{pmatrix}.$$

We introduced pair of effective vector fields $A_{\alpha \mu} = \{ A_0, A_1 \}$ instead of initial one $A_{\mu} = \{ A^N_a, A^D_i \}$. So, we doubled the number of vector fields, but there are two constraints on the effective vector fields

$$A_{1a}(V) = 0, \quad A_0i(V) = -(BG^{-1})_i^j A_{1j}(V) = A_{1i} G^{-1} B V.$$

In the literature $A^N_a[x]$ is known as massless vector field on Dp-brane while $A^D_i[x]$ is known as massless scalar oscillations orthogonal to the Dp-brane.
2.2 Choice of background

The space-time equations of motion are consequence of absence of the conformal anomaly. For the closed string case in the lowest order in slope parameter $\alpha'$, it produces \[12\]

$$
\beta^G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{4} B_{\mu\rho\sigma} B^\rho_{\nu}^\sigma + 2 D_\mu \partial_\nu \Phi = 0,
$$

$$
\beta^B_{\mu\nu} \equiv D_\mu B^\rho_{\nu} + 2 \partial_\mu \Phi B^\rho_{\nu} = 0,
$$

$$
\beta^\Phi \equiv 4 (\partial_\Phi)^2 - 4 D_\mu \partial^\mu \Phi + \frac{1}{12} B_{\mu\nu\rho} B^\mu_{\nu\rho} + 4 \pi \kappa (D - 26)/3 - R = 0. \quad (2.14)
$$

Here $B_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$ is the field strength of the Kalb-Ramond field $B_{\mu\nu}$, and $R_{\mu\nu}$ and $D_\mu$ are Ricci tensor and covariant derivative with respect to space-time metric.

With the same reason, for open string case there are additional space-time equations of motion \[13\]. In our notation they take a form

$$
\beta_a \equiv - \frac{1}{2} B_a^\mu \partial_\mu \Phi + G_E^{-1bc} \partial_\mu B_{ba} + G_E^{-1bc} (\frac{1}{2} B_a^d B_{dce} + K^\mu_{ac} B_{\mu\nu} \partial_\nu f^\nu) = 0,
$$

$$
\beta_\mu \equiv \frac{1}{2} \partial_\mu \Phi + G_E^{-1ab} (\frac{1}{2} B_a^c B_{\mu\nu} - K_{\mu ab}) = 0,
$$

where

$$
B_{ab} = B_{ab} - 2 (\partial_i A_{D}^{j} + \partial_j A_{D}^{i}) , \quad G_E^{ab} = G_{ab} - 4 B_{ac} G^{-1cd} B_{db} , \quad (2.15)
$$

and $K^\mu_{ab}$ is extrinsic curvature.

We will consider the simplest solutions of the closed string part

$$
G_{\mu\nu} = \text{const} , \quad B_{\mu\nu} = \text{const} , \quad \Phi = \text{const} , \quad D = 26 , \quad (2.17)
$$

which satisfies equations \[2.14\]. For the open string part \[2.15\], we will assume that vector fields are linear in coordinates \[1\]

$$
A^N_a (x) = A^0_a x^b , \quad A^D_i (x) = A^0_i - \frac{1}{4} F^{(s)}_{ij} x^j , \quad (2.18)
$$

so that corresponding field strengths are constant. The infinitesimal coefficients $F^{(a)}_{ab}$ and $F^{(s)}_{ij}$ are defined as

$$
F^{(a)}_{ab} = \partial_a A^N_b - \partial_b A^N_a , \quad F^{(s)}_{ij} = -2 (\partial_i A^D_j + \partial_j A^D_i) . \quad (2.19)
$$

Note that the $F^{(a)}_{ab}$ is antisymmetric in $a,b$ indices while the $F^{(s)}_{ij}$ is symmetric in $i,j$ indices. Since we are working with plane Dp-brane the extrinsic curvature is zero and because $\Phi, B_{ab}$ and $G_E^{ab}$ are constant, both $\beta_a$ and $\beta_\mu$ vanish.

So, our choice of background fields \[2.17\] and \[2.18\] satisfy all space-time equations of motion.
2.3 Sigma-model T-duality of the open string

The T-dualization procedure of the theory described by the action (2.10) with background fields (2.17) and (2.18) has been performed in Ref.[1]. The T-dualization of the vector background fields $A^N_a$ and $A^D_i$ is nontrivial because these fields are coordinate dependent and it is not possible to apply standard Buscher’s procedure. Instead, T-dualization procedure of the Ref.[22], which work in absence of global symmetry, has been applied.

So, applying T-dualization procedure along all coordinates, the T-dual action has been obtained

$$^*S[y] = \frac{\kappa^2}{2} \int_\Sigma d^2\xi \partial_+ y_\mu \theta_-^{\mu\nu} \partial_- y_\nu + 2\kappa \int_{\partial \Sigma} d\tau \left( A^D_i(V) G^{-1ij} \dot{y}_j - \frac{1}{\kappa} A^N_a(V) * \gamma^a_{(0)} \right)$$

$$= \frac{\kappa^2}{2} \int_\Sigma d^2\xi \partial_+ y_\mu \theta_-^{\mu\nu} \partial_- y_\nu + 2\kappa \eta^{\alpha\beta} \int_{\partial \Sigma} d\tau * A^i_\alpha(V) \partial_\beta y_i , \quad (2.20)$$

where

$$\theta_-^{\mu\nu} = -\frac{2}{\kappa} (G_E^{-1} G^{-1})^{\mu\nu} = \theta^{\mu\nu} + \frac{1}{\kappa} (G_E^{-1})^{\mu\nu} , \quad (2.21)$$

and

$$G^E_{\mu\nu} \equiv G_{\mu\nu} - 4(B G^{-1} B)_{\mu\nu} , \quad \theta^{\mu\nu} \equiv -\frac{2}{\kappa} (G_E^{-1} B G^{-1})^{\mu\nu} , \quad (2.22)$$

are the symmetric and antisymmetric parts of $\theta_-^{\mu\nu}$. In the literature, $G^E_{\mu\nu}$ is known as open string metric and $\theta^{\mu\nu}$ as non-commutative parameter.

Because T-dual action (2.20) should has the same form as the initial one (2.10) but in terms of T-dual fields we can express T-dual background fields in terms of initial ones

$$^*\Pi_+ \equiv \frac{\kappa}{2} \theta_-^{\mu\nu} , \quad ^*A^D_a(V) = G_E^{-1ab} A^N_b(V) , \quad ^*A^N_a(V) = G^{-1ij} A^D_j(V) . \quad (2.23)$$

The first relation can be rewrite as

$$^*G^{\mu\nu} = (G_E^{-1})^{\mu\nu} , \quad ^*D^{\mu\nu} = \frac{\kappa}{2} \theta^{\mu\nu} . \quad (2.24)$$

Note that the T-dual vector background fields depend not on $y_\mu$ but on $V^\mu = -\kappa \theta^{\mu\nu} y_\nu + G_E^{-1\mu\nu} \dot{y}_\nu , \quad (2.25)$

which is function on both $y_\mu$ and its double

$$\ddot{y}_\mu = \int (d\tau \ddot{y}_\mu + d\sigma \dddot{y}_\mu) . \quad (2.26)$$

With the help of (2.23) we can find effective T-dual vector fields in analogy with relation (2.11)

$$^*A^a_\pm(V) = 2 \Pi_\pm^{ab} G^{-1} * A^D_b(V) = \kappa \theta_-^{ab} A^N_b(V) , \quad ^*A^i_\pm(V) = A^i_N(V) = G^{-1ij} A^D_j(V) . \quad (2.27)$$
We introduced two effective T-dual vector fields $\star A_0^\mu = \{ \star A_0^\mu, \star A_1^\mu \}$ instead of initial one $\star A_0^\mu = \{ \star A_D^\mu, \star A_N^\mu \}$, but we have two constraints

\[
\star A_0^\mu (V) = -2(\star B G^{-1})_a^b \ star A_1^b (V) = 2(G^{-1} B)_a^b \ star A_1^b (V), \\
\star A_1^i (V) = 0. 
\]  

(2.28)

For initial and T-dual open strings boundary conditions at the string end-points take a form

\[
\gamma^{(0)}_\mu (x) \equiv \frac{\delta S}{\delta x^\mu} = 0, \quad \gamma^{(0)}_\mu (y) \equiv \frac{\delta S}{\delta y^\mu} = 0. 
\]  

(2.29)

Here $\gamma^{(0)}_\mu$ (defined in (2.6) for closed string) now obtains new infinitesimal term

\[
\gamma^{(0)}_\mu (x) \equiv \frac{\delta S}{\delta x^\mu} = \kappa \left[ 2B^{\mu \nu} \dot{x}_\nu - G^{\mu \nu} x_\nu - 2A_1^i \Delta (\sigma) \right] = \kappa \left[ 2B^{\mu \nu} \dot{x}_\nu - G^{\mu \nu} x_\nu + 2A_D^i \Delta (\sigma) \right], 
\]  

(2.30)

while for T-dual theory we have

\[
\gamma^{(0)}_\mu (y) \equiv \frac{\delta S}{\delta y^\mu} = \kappa \left[ 2\star B^{\mu \nu} \dot{y}_\nu - \star G^{\mu \nu} y_\nu - 2 \star A_1^i \Delta (\sigma) \right] = \kappa \left[ 2\star B^{\mu \nu} \dot{y}_\nu - \star G^{\mu \nu} y_\nu + 2\star G^{-1} a^i_a \Delta (\sigma) \right]. 
\]  

(2.31)

where $\Delta (\sigma) \equiv \delta (\sigma - \pi) - \delta (\sigma)$.

The terms with vector field $A_1^i$ in (2.30) and $A_6^N$ (2.31) are irrelevant in the expressions for actions (2.10) and (2.20), because they appear as infinitesimal of the second order terms.

2.4 T-duality transformations of the open string

T-dual transformation lows for the open string, connecting the initial and corresponding T-dual variables take the form [1]

\[
\partial_\pm x^\mu \equiv -\kappa \theta^\mu_\pm \partial_{\pm y^\mu} \pm 4\kappa \theta^\mu_\pm A_\pm^\nu V (\Delta (\sigma)), \\
\partial_\pm y^\mu \equiv -2\Pi^\mu_\pm \partial_{\pm x^\mu} \pm 4A_\pm^\nu (x) V (\Delta (\sigma)), 
\]  

(2.32)

where the symbol $\cong$ denotes the T-duality relation.

In fact the second transformation (2.32) can be obtained after T-dualization the T-dual action (2.20). The relations (2.32) are inverse to each other. Both transformations differ from the closed string ones by the infinitesimal term which contains vector background fields $A_\pm^\mu$.

In terms of covariant derivatives

\[
D_\pm x^\mu = \partial_\pm x^\mu + 2(G^{-1})^\mu_\nu A_\pm^\nu \Delta (\sigma), \quad D_\pm y^\mu = \partial_\pm y^\mu + 2\star G^{-1} \mu_\nu \star A_\pm^\nu \Delta (\sigma). 
\]  

(2.33)
we can rewrite the transformations (2.32) in a simple form

\[ D_\pm x^\mu \cong -\kappa \theta^{\mu \nu} D_\pm y_\nu, \quad D_\pm y_\mu \cong -2\Pi^{\mu \nu} D_\pm x^\nu. \]  

(2.34)

From first equation (2.32) we can find the T-dual transformation laws for \( \dot{x}^\mu \) and \( x'^\mu \)

\[ \dot{x}^\mu \cong -\kappa \theta^{\mu \nu} [y'_\nu - 4A_{1\nu} \Delta(\sigma)] + G_E^{-1 \mu \nu} [y'_\nu - 4A_{0\nu} \Delta(\sigma)] \]

\[ = -\kappa \theta^{\mu \nu} y'_\nu + G_E^{-1 \mu \nu} y'_\nu + 4* A_1^\mu \Delta(\sigma) \]  

(2.35)

\[ x'^\mu \cong -\kappa \theta^{\mu \nu} [y'_\nu - 4A_{0\nu} \Delta(\sigma)] + G_E^{-1 \mu \nu} [y'_\nu - 4A_{1\nu} \Delta(\sigma)] \]

\[ = -\kappa \theta^{\mu \nu} y'_\nu + G_E^{-1 \mu \nu} y'_\nu + 4* A_0^\mu \Delta(\sigma) \]  

(2.36)

and from the second one the inverse transformations

\[ y'_\mu \cong -2B_{\mu \nu} \dot{x}^{\nu} + G_{\mu \nu} x^{\nu} + 4A_{1\mu} \Delta(\sigma), \]  

(2.37)

\[ y'_\mu \cong G_{\mu \nu} \dot{x}^{\nu} - 2B_{\mu \nu} x^{\nu} + 4A_{0\mu} \Delta(\sigma). \]  

(2.38)

Using the expression for the canonical momentum of the original and of the T-dual theory

\[ \pi_{\mu} \equiv \frac{\delta S}{\delta \dot{x}^\mu} = \kappa \left[ G_{\mu \nu} \dot{x}^{\nu} + 2B_{\mu \nu} x^{\nu} + 2A_{0\mu} \Delta(\sigma) \right], \quad \pi'^{\mu} \equiv \frac{\delta S}{\delta y'_\mu} = \kappa \left[ (G_E^{-1})^{\mu \nu} y'_\nu - \kappa \theta^{\mu \nu} y'_\nu + 2* A_0^\mu \Delta(\sigma) \right], \]  

(2.39)

we can rewrite the transformations (2.36) and (2.38) in the canonical form

\[ \kappa x'^\mu \cong \pi'^{\mu} + 2\kappa A_0^\mu \Delta(\sigma), \quad \pi \mu + 2\kappa A_{0\mu} \Delta(\sigma) \cong \kappa y'_\mu. \]  

(2.40)

This relation connect momenta and winding numbers.

We can rewrite the transformations (2.35) and (2.37) in the form

\[ \kappa \dot{x}^\mu \cong \pi^{\mu} - 2\kappa A_1^\mu \Delta(\sigma), \quad \pi - 2\kappa A_{1\mu} \Delta(\sigma) \cong -\kappa \dot{y}_\mu, \]  

(2.41)

where \( \gamma^{(0)}_\mu \) is defined in (2.30) and \( * \gamma^{(0)}_\mu \) in (2.31).

It was shown in Ref. [1] that \( \pi^{\mu} \) is generator of general coordinate transformations while \( x'^\mu \) is generator of gauge symmetry. In Ref. [1] T-duality relation between \( \dot{x}^\mu \) and \( * \gamma^{(0)}_\mu \) (as well as between \( \dot{y}_\mu \) and \( \gamma^{(0)}_\mu \)) has been established. The relations (2.40) and (2.41) are extension of T-duality to the open string case. The additional \( A_0^\mu \)-dependent terms stem from variations of the arguments of vector background fields.

Note that the momentum \( \pi^{\mu} \) and variable \( \gamma^{(0)}_\mu(x) \), as well as \( \partial_\alpha x^\mu = \{ \dot{x}^\mu, x'^\mu \} \) are components of the same world-sheet vector.

\[ \pi^{\alpha}_\mu \equiv \frac{\delta S}{\delta \partial_\alpha x^\mu} = \{ \pi^{\mu}, \gamma^{(0)}_\mu(x) \}. \]  

(2.42)
From now on we will call $\gamma^{(0)}_\mu(x)$ $\sigma$-momentum. We can rewrite relations (2.40) and (2.41) in the forms

$$
\pi^\alpha_\mu \cong -\kappa \varepsilon^{\alpha\beta} \partial_\beta y_\mu + 2\kappa \eta^{\alpha\beta} A_\beta \Delta(\sigma), \quad \pi^{0\mu} \cong -\kappa \varepsilon^{0\beta} \partial_\beta x^\mu + 2\kappa \eta^{0\beta} A^\mu \Delta(\sigma).
$$

(2.43)

Therefore, T-duality interchange Neumann with Dirichlet gauge fields. It also interchange $\dot{x}^\mu$ and $x'^\mu$ with $*\gamma^{(0)}_\mu$ and $\pi^{0\mu}$ as well as $\dot{y}_\mu$ and $y'_\mu$ with $\gamma^{(0)}_\mu$ and $\pi^\mu$.

### 3 T-dual background fields of open string in double space

Following Refs. [14, 2, 3, 4, 5] we are going to introduce double space in order to offer simple interpretation of T-dualization as coordinates permutation in double space. Let us start with T-dual transformation lows (2.32). We can express them in a useful form, where on the left hand side we put the terms with world-sheet antisymmetric tensor $\varepsilon_{\alpha\beta}$ (note that $\varepsilon_{\pm\pm} = \pm 1$)

$$
\pm \partial_\pm x^\mu \cong G^{E\mu}_{\nu\rho} \partial_\pm x^\nu - 2(BG^{-1})^\mu_{\nu\rho} \partial_\pm x^\nu + 8(\Pi_{\pm\pm} G^{-1})^\nu_{\mu\rho} A_\pm \pm \Delta(\sigma),
$$

$$
\pm \partial_\pm y^\mu \cong 2(G^{-1} B)^\mu_{\nu \rho} \partial_\pm x^\nu + (G^{-1})^\mu_{\nu \rho} \partial_\pm y^\nu - 4G^{-1}_{\mu\nu} A_\pm \pm \Delta(\sigma).
$$

(3.1)

We can rewrite these T-duality relations in the simple form

$$
\partial_\pm Z^M \cong \pm \Omega^{MN} \left[ H_{NK} \partial_\pm Z^K - 2(\mathcal{H} + \sigma_3 \mathcal{H} \sigma_3) A^K_\pm (\dot{Z}_\arg) \Delta(\sigma) \right],
$$

(3.2)

where we introduced the double coordinates $Z^M$ and corresponding arguments of background fields $\dot{Z}_\arg$

$$
Z^M = \left( \begin{array}{c} x^\mu \\ y^\mu \end{array} \right), \quad \dot{Z}_\arg = \left| \begin{array}{c} V^\mu \\ x^\mu \end{array} \right|_D.
$$

(3.3)

Note a different notation for arguments of background fields, introduced in Ref. [3]. The double space coordinate $Z^M$ has 2D rows, $D$ components of initial coordinates $x^\mu$ in the upper $D$ rows and $D$ components of T-dual coordinates $y^\mu$ in the lower $D$ rows. In arguments of background fields $\dot{Z}_\arg$ in each row there is the complete $D$ dimensional vector. Rewritten in form of the one column the arguments of background fields are 2$D^2$ dimensional vector.

Because arguments of all background fields in $A^M_\pm (\dot{Z}_\arg)$ and $^*A^K_\pm (\dot{Z}_\arg)$ (see (3.10)) are the same in the upper $D$ rows as well as in the lower $D$ rows we can write them in two component notation as in (3.3). We indicated this with index $D$.

We also introduced the so called $O(D,D)$ invariant metric $\Omega^{MN}$, the generalized metric $\mathcal{H}_{MN}$ and constant matrix $\sigma_3$.

$$
\Omega^{MN} = \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right), \quad \mathcal{H}_{MN} = \left( \begin{array}{cc} G^{E\mu}_{\nu\rho} & -2 B_{\mu\rho}(G^{-1})^\rho_\nu \\ 2(G^{-1})^\mu_{\rho\nu} B^\rho_\nu & (G^{-1})^\mu_\nu \end{array} \right),
$$

(3.4)
\[(\sigma_3)_M^N = \begin{pmatrix} 1_D & 0 \\ 0 & -1_D \end{pmatrix}, \quad (3.5)\]

and the double gauge fields
\[
\mathcal{A}_\pm^M(\tilde{Z}_{\text{arg}}) = \begin{pmatrix} *\mathcal{A}_\pm^\mu(V) \\ \mathcal{A}_\pm^\mu(x) \end{pmatrix} = \begin{pmatrix} \kappa \theta_\pm^{\mu\nu} \mathcal{A}_{\pm\nu}(V) \\ \mathcal{A}_{\pm\mu}(x) \end{pmatrix}. \quad (3.6)
\]

Note that as well as in Refs. [14, 2, 3, 4, 5] all coordinates are doubled. It is easy to check that
\[
\mathcal{H}^T \Omega \mathcal{H} = \Omega, \quad (3.7)
\]
which shows that manifest $O(D,D)$ symmetry is automatically incorporated into theory.

### 3.1 T-duality in double space along all coordinates

Let us derive expression for T-dual generalized metric and T-dual double gauge fields following approach of Ref. [3]. Then, beside double space coordinate $Z^M$ we should also transform extended coordinates of the arguments of background fields $\tilde{Z}_{\text{arg}}$ (3.3). We will require that the T-duality transformations (3.2) are invariant under transformations of the double space coordinates $Z^M$ and $\tilde{Z}_{\text{arg}}$

\[
*Z^M = *\mathcal{T}_N^M Z^N, \quad *\tilde{Z}_{\text{arg}} = *\tilde{\mathcal{T}} \tilde{Z}_{\text{arg}}. \quad (3.8)
\]

We want to offer interpretation for the case where T-dualization has been performed along all coordinates. So, we are going to exchange all initial with all T-dual coordinates which is described by the matrices $*\mathcal{T}$ and $*\tilde{\mathcal{T}}$ of the form

\[
*\mathcal{T} = \Omega_2 \otimes 1_D = \begin{pmatrix} 0 & 1_D \\ 1_D & 0 \end{pmatrix}, \quad *\tilde{\mathcal{T}} = \Omega_2 \otimes 1_{D^2} = \begin{pmatrix} 0 & 1_{D^2} \\ 1_{D^2} & 0 \end{pmatrix}. \quad (3.9)
\]

The T-dual coordinates $*Z^M$ and $*\tilde{Z}_{\text{arg}}$ should satisfy the same relation as initial one (3.2), but in terms of T-dual background fields

\[
\partial_\pm *Z^M \cong \pm \Omega^{MN} \left[ *\mathcal{H}_{NK} \partial_\pm *Z^K - 2 (**H + \sigma_3^* \mathcal{H} \sigma_3)_{NK} *\mathcal{A}_\pm^K(*\tilde{Z}_{\text{arg}}) \Delta(\sigma) \right]. \quad (3.10)
\]

This produces the expression for the dual generalized metric and dual double gauge fields in terms of the initial ones

\[
*\mathcal{H} \cong *\mathcal{T} \mathcal{H} *\mathcal{T}, \quad *\mathcal{A}_\pm(*\tilde{Z}_{\text{arg}}) \cong *\mathcal{T} \mathcal{A}_\pm(*\tilde{Z}_{\text{arg}}). \quad (3.11)
\]

It is well known [2, 3] that the first relation gives the standard T-dual transformations of the metric and Kalb-Ramond fields (2.24). Rewriting the second relation in components, with the help of (2.24) and (3.6) we have

\[
*\mathcal{A}_\pm^\mu \cong \kappa \theta_\pm^{\mu\nu} \mathcal{A}_{\pm\nu}. \quad (3.12)
\]
Using expressions (2.11) and the first relation (2.23) we obtain
\[ *A_a^\pm \cong \kappa \theta_\pm^{ab} A_{\pm b}, \quad *A_i^\pm \cong \kappa \theta_\pm^{ij} A_{\pm j} = 2 \kappa \theta_\pm^{ij} \Pi_\pm^+_{jk} G^{-1kq} A_q^D = G^{-1ij} A_j^D. \] (3.13)

On the other hand, the T-dual effective fields should have the form (2.27)
\[ *A_a^\pm = 2 * \Pi_{\pm^a}^b G^{-1c} A_c^D, \quad *A_i^\pm = *A_N^i. \] (3.14)

From (3.13) and (3.14) with the help of (2.24) we have
\[ *A_a^D = G_E^{-1ab} A_b^N, \quad *A_i^D = G^{-1ij} A_j^D. \] (3.15)

which is just Buscher T-duality relation for vector fields (2.23).

So, inclusion of gauge fields does not change interpretation of T-duality in double space. It is again replacement of the initial and T-dual coordinates which shows that these transformations are nonphysical.

### 3.2 Double space field strength

If in addition to (3.3) we introduce new double fields
\[ \tilde{Z}^M = \left( \tilde{x}^\mu, \tilde{y}_\mu \right), \quad \partial_M = \left( \frac{\partial x^\mu}{\partial y_\mu} \right), \quad \tilde{\partial}_M = \left( \frac{\partial \tilde{x}^\mu}{\partial \tilde{y}_\mu} \right), \] (3.16)

we can reexpress the field strengths of both initial and T-dual case (see Eqs.(5.11) and (7.31) of Ref.[1]) in the form
\[ F^{MN} = \Omega^{MK} \left( \hat{\partial}_+ K A_+^N (\tilde{Z}_{\text{arg}}) - \hat{\partial}_- K A_\pm^N (\tilde{Z}_{\text{arg}}) \right) = \left( *F^{\mu\nu} 0 \right), \] (3.17)

where we defined
\[ *\tilde{\partial}_{\pm M} = \partial_M \pm \tilde{\partial}_M. \] (3.18)

### 4 Example: Three torus with $D_1$-brane in double space

In this section the example of three-torus with $D_1$-brane, considering in the Ref.[1], we will present in double space. We will show how to perform T-dualization along all coordinates in double space.
4.1 Initial theory in double space

We will start with definition of background fields of the initial theory in double space. Let us denoted the coordinates of the $D = 3$ dimensional torus by $x^0, x^1, x^2$ and introduce nontrivial components of the background fields as

$$G_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad B_{\mu\nu} = \begin{pmatrix} 0 & B & 0 \\ -B & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$  \hspace{1cm} (4.1)

It is easy to find corresponding effective metric and non-commutativity parameter

$$G^E_{\mu\nu} = \begin{pmatrix} G_E & 0 & 0 \\ 0 & -G_E & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \theta^{\mu\nu} = \begin{pmatrix} 0 & \theta & 0 \\ -\theta & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$  \hspace{1cm} (4.2)

where as we defined in \[1\]

$$G_E \equiv 1 - 4B^2, \quad \theta \equiv \frac{2B}{\kappa G_E}.$$  \hspace{1cm} (4.3)

We will also need expression for combination of background fields

$$\theta^{\mu\nu}_\pm = \theta^{\mu\nu} \mp \frac{1}{\kappa} G^{-1\mu\nu}_E = \begin{pmatrix} \mp \frac{1}{\kappa G_E} & \theta & 0 \\ \theta & \pm \frac{1}{\kappa G_E} & 0 \\ 0 & 0 & \pm \frac{1}{\kappa} \end{pmatrix}.$$  \hspace{1cm} (4.4)

According to \[3,4\] it produces

$$\mathcal{H}_{MN} = \begin{pmatrix} G^E_{\mu\nu} & -2(BG^{-1})_{\mu\nu} \\ 2(G^{-1}B)_{\mu\nu} & (G^{-1})^{\mu\nu} \end{pmatrix} = \begin{pmatrix} G_E & 0 & 0 & 2B & 0 \\ 0 & -G_E & 2B & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 2B & 0 & 1 & 0 & 0 \\ 2B & 0 & 0 & -1 & 0 \end{pmatrix}.$$  \hspace{1cm} (4.5)

Similarly, we have

$$\sigma_3 \mathcal{H} \sigma_3 = \begin{pmatrix} G^E_{\mu\nu} & 2(BG^{-1})_{\mu\nu} \\ -2(G^{-1}B)_{\mu\nu} & (G^{-1})^{\mu\nu} \end{pmatrix} = \begin{pmatrix} G_E & 0 & 0 & 0 & -2B & 0 \\ 0 & -G_E & 0 & -2B & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -2B & 0 & 0 & 0 & -1 & 0 \end{pmatrix}.$$  \hspace{1cm} (4.6)
The double space coordinates are

\[
Z^M = \begin{pmatrix} x^\mu \\ y_\mu \end{pmatrix} = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ y_0 \\ y_1 \\ y_2 \end{pmatrix}, \quad \tilde{Z}_{\arg} = \begin{vmatrix} V^\mu \\ x^\mu \end{vmatrix}_{D=3} = \begin{vmatrix} V^\mu \\ V_\mu \\ V_\mu \\ x^\mu \\ x^\mu \\ x^\mu \end{vmatrix}, \quad (4.7)
\]

while the double gauge field according to (3.6) takes a form

\[
A^M (\tilde{Z}_{\arg}) = \begin{pmatrix} *A^0_+ (V) \\ *A^1_+ (V) \\ *A^0_- (V) \\ *A^1_- (V) \end{pmatrix} = \begin{pmatrix} \kappa \theta^{\mu\nu} A_{\pm \nu} (V) \\ A_{\pm 0} (x) \\ A_{\pm 1} (x) \\ A_{\pm 2} (x) \end{pmatrix}, \quad (4.8)
\]

Note that dimension of \( \tilde{Z}_{\arg} \) is \( 2 \times D^2 = 2 \times 3^2 = 18 \).

We will start with \( D_1 \)-brane define with the Dirichlet boundary conditions \( x^2 (\tau, \sigma) / \sigma = 0 = x^2 (\tau, \sigma) / \sigma = \pi = const \). It means that we will work with Neumann background fields \( A^N_0 \) and \( A^N_1 \) and Dirichlet background field \( A^D_2 \) and according to our convention we will have \( p = 1, \ a, b \in \{0, 1\} \) and \( i, j \in \{2\} \).

In terms of initial Neumann and Dirichlet fields we obtain

\[
A^M (\tilde{Z}_{\arg}) = \begin{pmatrix} \mp \frac{1}{g_E} A^N_0 (V) + \kappa \theta A^N_1 (V) \\ -\kappa \theta A^N_0 (V) \pm \frac{1}{g_E} A^N_1 (V) \\ -A^D_2 (V) \\ A^N_0 (x) \\ A^N_1 (x) \end{pmatrix}, \quad (4.9)
\]

where we used the second expression (4.2).
4.2 T-dual theory in double space

On the other hand, for T-dual case we have

\[ *A^M_{\pm}(\tilde{Z}_{\text{arg}}) = \begin{pmatrix} A_{\pm\mu}(V) & *A^\mu_{\pm}(x) \\ *A^\mu_{\pm}(x) & A_{\pm\mu}(x) \end{pmatrix} = \begin{pmatrix} 2\Pi_{\pm\mu\nu} *A^\nu_{\pm}(V) + 2B *A^1_{\pm}(V) + 2B *A^0_{\pm}(V) + *A^1_{\pm}(V) \\ -2B *A^0_{\pm}(V) \end{pmatrix} \]

or with the help of (2.27), in terms of T-dual Neumann and Dirichlet fields

\[ *A^M_{\pm}(\tilde{Z}_{\text{arg}}) = \begin{pmatrix} G_E *A^0_D(V) & *A^0_D(x) \\ -G_E *A^1_D(V) & *A^1_D(x) \end{pmatrix} \]

Using the second equation (3.11), with the help of (4.9) and (4.11) we obtain

\[ *A^M_{\pm}(\tilde{Z}_{\text{arg}}) = \begin{pmatrix} *A^0_D(V) - 2B *A^1_D(x) + \mp A^1_D(x) \\ *A^2_N(V) \end{pmatrix} \]

\[ \equiv *T A_{\pm}(\tilde{Z}_{\text{arg}}) = \begin{pmatrix} A^0_D(V) & A^0_D(x) \\ A^1_D(V) & A^1_D(x) \end{pmatrix} \]

where for this example we have

\[ *T = \begin{pmatrix} 0 & 1_3 \\ 1_3 & 0 \end{pmatrix} \]

Note that transition from \( \tilde{Z}_{\text{arg}} \) to \( \mp \tilde{Z}_{\text{arg}} \) changes \( x^\mu \leftrightarrow V^\mu \) while operator \( *T \) exchanges first three with last three rows from eq.(4.10). Expression (4.12) produces just T-duality relations

\[ *A^0_D = \frac{1}{G_E} A^N_0, \quad *A^1_D = \frac{1}{G_E} A^N_1, \quad *A^2_N = -A^D_2, \]

in accordance with (2.23), (4.11) and (4.12).

The same relation can be obtained with the help of compact notation which produces \( *A^\mu_{\pm} \cong \kappa \theta^\mu_{\pm} A_{\pm\nu} \), (see Eq.(3.12)). According to (2.11) and (2.27) we have respectively

\[ A_{\pm0} = A^N_0, \quad A_{\pm1} = A^N_1, \quad A_{\pm2} = \mp A^D_2 \]
and
\[ A_0^\pm = \mp A_D^0 - 2B A_D^1, \quad A_1^\pm = -2B A_D^0 \mp A_D^1, \quad A_2^\pm = A_N^2. \] (4.16)

Then the equation (3.12) takes the form
\[
\begin{pmatrix}
\mp A_D^0 - 2B A_D^1 \\
-2B A_D^0 \mp A_D^1 \\
A_N^2 
\end{pmatrix} = \begin{pmatrix}
\pm \frac{1}{G_E} \kappa \theta & 0 \\
-\kappa \theta & \pm \frac{1}{G_E} 0 \\
0 & 0 & \pm 1
\end{pmatrix} \begin{pmatrix}
A_0^N \\
A_1^N \\
A_2^D
\end{pmatrix} = \begin{pmatrix}
\mp \frac{1}{G_E} A_0^N + \kappa \theta A_1^N \\
-\kappa \theta A_0^N \pm \frac{1}{G_E} A_1^N \\
-A_2^D
\end{pmatrix},
\] (4.17)

which again produces relation (4.14).

4.3 Double space field strength

The structure of our example produces \( \gamma_2^{(0)} = \kappa x^2 \) and the action (2.10) takes the form
\[
S_{\text{open}}[x] = \kappa \int_{\Sigma} d^2 \xi \delta_{+} x^\mu \Pi_{+\mu\nu} \partial_{-} x^\nu + 2\kappa \int_{\partial \Sigma} d\tau (A_0^N [x] x^0 + A_1^N [x] x^1 + A_2^D [x] x^2). \] (4.18)

Note an unusual coupling of Dirichlet part \( A_D^2 \) with \( x_2' \).

According with (2.18) the non trivial vector background fields are
\[
A_0^N (x) = A_0^0 - \frac{1}{2} F^{(a)} (a)^1, \quad A_1^N (x) = A_1^0 + \frac{1}{2} F^{(a)} x^0, \quad A_2^D (x) = A_2^0 - \frac{1}{4} F^{(s)} x^2 \] (4.19)

where \( F^{(a)} \equiv F_{01}^{(a)} = \partial_0 A_1^N - \partial_1 A_0^N \) and \( F^{(s)} \equiv F_{22}^{(s)} = -4 \partial_2 A_2^D \). Consequently, the field strength of the initial theory is
\[
F_{\mu\nu} = F_{\mu\nu}^{(a)} + \frac{1}{2} F_{\mu\nu}^{(s)} = \begin{pmatrix}
0 & F^{(a)} & 0 \\
-F^{(a)} & 0 & 0 \\
0 & 0 & \frac{1}{2} F^{(s)}
\end{pmatrix} \] . (4.20)

Note an unusual expression and unusual appearance of symmetric field strength \( F^{(s)} \).

5 Conclusions

In the present article we extended interpretation of T-duality in double space to the case of open string. This includes consideration of T-duality for the vector gauge fields.

In string theory the gauge fields appear at boundary of the open string. Their role is to enable complete local gauge symmetries. In fact, there are two important symmetries of the closed string theory: local gauge symmetry of the Kalb-Ramond field and general coordinate transformations. In Ref.[1] we showed that ”restricted general coordinate transformations” (transformations of background fields without transformations of
the coordinates) are T-dual to local gauge symmetry of the Kalb-Ramond field. Both symmetries are failed at the open string end-points. The function of gauge fields is to restore these symmetries at the string end-points.

To each symmetry of the string theory there is appropriate gauge field. As a consequence of the boundary conditions only parts of these gauge fields survive. From gauge field corresponding to local gauge symmetry of the Kalb-Ramond field the components along coordinates with Neumann boundary conditions \( A^N_a \) survive. From gauge field corresponding to restricted general coordinate transformations the components along coordinates with Dirichlet boundary conditions \( A^D_i \) survive. So, the complete vector field is \( A_\mu = \{ A^N_a, A^D_i \} \).

In Ref.\[1\] it was shown that known fact that \( x'^\mu \) is T-dual to \( \pi_\mu \) produces chain of T-dualities between: restricted general coordinate transformation and local gauge transformations; vector fields with Neumann \( A^N_a \) and Dirichlet boundary conditions \( A^D_i \).

In the present article we showed that all the above results have simple interpretation in double space. The double space contains \( 2D \) coordinates, \( D \) initial \( x^\mu \) and corresponding \( D \) T-dual \( y_\mu \). The T-dualization of the present article (along all coordinates) corresponds to the replacement of all initial coordinates \( x^\mu \) with all T-dual coordinates \( y_\mu \) and all initial arguments of the background fields \( x^\mu \) with all T-dual ones \( \tilde{V}^\mu \). Such operation reproduces all results described above. So, in the open string case complete set of T-duality transformations form the same subgroup of the \( 2D \) permutation group as in the closed string case.

Let us stress that there is essential difference between our approach and that of Double field theories (DFT) \[27, 28\]. In DFT there are two coordinates the initial \( x^\mu \) and its double, denoted as \( \tilde{x}_\mu \). The variable \( \tilde{x}_\mu \) corresponds to our \( y_\mu \) but we have additional dual coordinate \( \tilde{y}_\mu \) defined in first relation \[22, 26\].

Consequently, in the double space we are able to represent the backgrounds of all T-dual open string theories in unified manner as well as in the cases of bosonic \[2, 3\] and type II superstring theories \[4\].

This step is important ingredient in better understanding M-theory. We already explained the role of double space in interpretation of T-duality and consequently in an attempt to construct M-theory \[4, 5\]. The present article is extension of these consideration to the case of open string.

References

[1] B. Sazdović, From geometry to non-geometry via T-duality, arXiv: 1606.01938.

[2] B. Sazdović, Chinese Physics C 41 (2017) 053101.
[3] B. Sazdović, *JHEP* **08** (2015) 055.

[4] B. Nikolić, B. Sazdović, *Eur.Phys. J C* (2017) 77:197.

[5] B. Nikolić and B. Sazdović, *Nucl. Phys. B* **917** (2017) 105.

[6] H. Dorn and H.-J. Otto, *Phys. Lett. B* **381** (1996) 81.

[7] E. Alvarez, J.L.F. Barbon and J. Borlaf, *Nucl. Phys. B* **479** (1996) 218.

[8] B. Zwiebach, *A First Course in String Theory*, Cambridge University Press, 2004.

[9] J. Polchinski, *String theory*, Cambridge University Press, 1998.

[10] K. Becker, M. Becker and J. Schwarz, *String Theory and M-Theory: A Modern Introduction*, Cambridge University Press, 2007.

[11] Lj. Davidović and B. Sazdović, in preparation.

[12] C. G. Callan, D. Friedan, E. J. Martinec and M. J. Perry, *Nucl. Phys. B* **262** (1985) 593.

[13] R. G. Leigh, *Mod. Phys. Lett. A* **4** (1989) 2767.

[14] M. Duff, *Nucl. Phys. B* **335** (1990) 610.

[15] S. Hellerman, J. McGreevy and B. Williams, *JHEP* **01** (2004) 024.

[16] A. Dabholkar and C. Hull, *JHEP* **09** (2003) 054.

[17] J. Shelton, W. Taylor and B. Wecht, *JHEP* **10** (2005) 085.

[18] Lj. Davidović, B. Nikolić and B. Sazdović, *EPJ C* **74** (2014) 2734.

[19] Lj. Davidović, B. Nikolić and B. Sazdović, *EPJ C* **75** (2015) 576.

[20] M. Evans and B. A Ovrut, *Phys.Rev. D* **39** (1989) 3016.

[21] M. Evans and B. A Ovrut, *Phys.Rev. D* **41** (1990) 3149.

[22] Lj. Davidović and B. Sazdović, *JHEP* **11** (2015) 119.

[23] R. Blumenhagen, A. Deser, D. Lüst, E. Plauschinn, F. Rennecke, *J. Phys. A* **44**, 385401 (2011).

[24] R. Blumenhagen, E. Plauschinn, *J. Phys. A* **44**, 015401 (2011).

[25] T. Buscher, *Phys. Lett. B* **194** (1987) 51; **201** (1988) 466.
[26] Lj. Davidović and B. Sazdović, *EPJC* **74** (2014) 2683.

[27] C. Hull and B. Zwiebach, *JHEP* **09** (2009) 099; *JHEP* **09** (2009) 090.

[28] O. Hohm, C. Hull and B. Zwiebach, *JHEP* **08** (2010) 008.