Semileptonic $B$ Meson Decays and Interfering Amplitudes

K. Schubert, R. Waldi

Institut für Kern- und Teilchenphysik, Technische Universität Dresden, D-01062 Dresden, Germany

Abstract

Consequences of the interference between external and internal spectator amplitudes for the lifetimes and semileptonic decay fractions of $B^0$ and $B^+$ mesons are discussed. Extrapolating from the constructive interference observed in 11 exclusive hadronic $B$ decays we find an inclusive semileptonic decay fraction of $(11.2 \pm 0.5 \pm 1.7)\%$, significantly closer to the experimental results than previous predictions.

1. Introduction

Although there has been significant progress in the calculation of QCD corrections in the decays of heavy flavour mesons, there are still some unsolved puzzles. One of the most intriguing is the low semileptonic decay fraction of $B$ mesons [1]. Ignoring the small $b \to u$ fraction, the $b$ quark in the $B$ meson decays to a charm quark and emits a virtual $W$ boson. This can transform itself into a lepton neutrino or a quark anti-quark pair. Taking into account the color factors and making some crude assumption about the quark masses, we can determine the relative rate of these processes and find an semileptonic decay fraction of approximately 15%. To obtain a more precise number we have to correct for hadronic effects due to the exchange of gluons between the quark lines. This enhances the hadronic rate with respect to the semileptonic rates resulting in $B_{s,l} \approx 13 - 14\%$. Bigi et al. have recently performed an evaluation of $B(B \to Xe\nu)$ based on the $1/m_Q$ expansion method in QCD [2] and found that the theory cannot accommodate a semileptonic branching fraction of $B$ mesons of less than 12.5%.

Experimentally, the semileptonic decay fraction of $B$ mesons has been determined by the ARGUS and CLEO collaborations and by the four LEP experiments. $B_{s,l}$ is determined by integrating over the measured lepton momentum spectrum. However, models have to be used to remove the background from $b \to c \to s$ cascade decays. The model dependence can be significantly reduced by selecting $\Upsilon(4S)$ decays with two final state leptons. A high momentum lepton tags this reaction as a $B\bar{B}$ event while the other lepton is used to measure the lepton momentum spectrum in semileptonic $B$ decay. Following this procedure, CLEO has contributed a paper to this conference quoting a value of $(10.36 \pm 0.17 \pm 0.40)\%$ for the semileptonic $B$ decay rate [3]. This is significantly below the lower bound allowed by theory and hence we have a problem.

2. Interfering Amplitudes in Hadronic $B$ Decays

A solution to this problem would be a further enhancement of the hadronic decay rate with respect to the semileptonic rate. Hadronic $B$ decays proceed via external or internal spectator diagrams. While the two diagrams lead to different final states in $B^0$ decays, both processes produce the same final state in charged $B$ decays and hence the corresponding amplitudes will interfere. These two amplitudes combined with the factorization hypothesis form the framework of spectator models such as the model by Bauer, Stech, and Wirbel [4]. These models have been surprisingly successful in describing many features of heavy meson...
decay. Destructive interference between the internal and external spectator amplitude for hadronic $D^+$ decays reproduces the observed $D^0 - D^+$ lifetime difference. Bauer, Stech, and Wirbel describe the two amplitudes by phenomenological parameters $a_1$ and $a_2$. The values of these parameters have to be determined by experiments. Destructive interference as observed in $D^+$ decays is described by a relative minus sign between $a_1$ and $a_2$. The theoretical interpretation of these parameters is controversial but it was generally expected that a similar but less pronounced interference pattern would be found in $B$ decay. It came as a surprise when the CLEO collaboration reported constructive interference in all exclusive hadronic $B^+$ decays observed so far. Combining the experimental

| $B^+ \to D^0 \pi^+$ | $0.45 \pm 0.04$ | $0.265(a_1 + 1.230a_2)^2$ |
| $B^+ \to D^0 \rho^+$ | $1.10 \pm 0.18$ | $0.622(a_1 + 0.662a_2)^2$ |
| $B^+ \to D^0 \phi^+$ | $0.51 \pm 0.08$ | $0.253(a_1 + 1.292a_2)^2$ |
| $B^+ \to D^0 \rho^+$ | $1.32 \pm 0.31$ | $0.70(a_1^2 + 1.490a_1a_2 + 64a_2^2)$ |
| $B^+ \to D^0 \rho^+$ | $0.106 \pm 0.015$ | $1.819a_1^2$ |
| $B^+ \to D^0 \pi^+$ | $0.17 \pm 0.05$ | $2.932a_2$ |

Table 1. Experimental averages and theoretically predicted decay fractions for hadronic $B$ decays, assuming $|V_{ub}|^2 = 2.35 \times 10^{-3}$, $f_B = 220$ MeV

decay fractions measured by ARGUS and CLEO results in the averages listed in Table 2. The partial rates are determined under the assumption of equal decay fractions of the $Y(4S)$ to $B^+B^-$ and $B^0\bar{B}^0$ pairs, i.e. $f^+/f^0 = 1$. This quantity is not well measured experimentally; we assume in the following $f^+/f^0 = 1.0 \pm 0.1$. The relative sign between $a_1$ and $a_2$ can be

| $R_1 = \frac{\Gamma(B^+ \to D^0 \pi^+)}{\Gamma(B^+ \to D^0 \rho^+)}$ | $1.71 \pm 0.38$ | $(1 + 1.23a_2/a_1)^2$ |
| $R_2 = \frac{\Gamma(B^+ \to D^0 \rho^+)}{\Gamma(B^+ \to D^0 \phi^+)}$ | $1.60 \pm 0.46$ | $(1 + 0.66a_2/a_1)^2$ |
| $R_3 = \frac{\Gamma(B^+ \to D^0 \phi^+)}{\Gamma(B^+ \to D^0 \rho^+)}$ | $1.79 \pm 0.39$ | $(1 + 1.29a_2/a_1)^2$ |

Table 2. Experimental results and theoretical predictions for ratios of $B^+$ and $B^0$ decay rates, scaled to $f_{D(D^+)} = 220$ MeV



The experimental results and a model prediction for the decay ratios in the modes $D\pi^-$, $D\rho^-$, and $D^\ast\pi^-$ are given in Table 2. They show a clear preference for the positive sign. The theoretical prediction for the decay $B^+ \to D^0 \rho^+$ is too uncertain to include this mode in the determination of $a_1$ and $a_2$. Taking ratios of $B^+$ and $B^0$ decays eliminates the uncertainties due to $|V_{ub}|$ but leaves those originating from $\tau(B^+)/\tau(B^0)$ and $f^+/(f^0)$. The main difference between different models are details of the $B \to \pi$ and $B \to \rho$ form factors. The predictions also depend on the $D$ and $D^\ast$ decay constants $f_D$ and $f_{D^\ast}$. Following Neubert et al. we assume $f_D = f_{D^\ast} = 220$ MeV. On the experimental side, the error due to the $D^0$ decay fractions cancels in the ratios involving $B \to D^\ast$ decays. A least square fit with seven $B \to D^{(*)}$ modes from Table 2, excluding only $B^+ \to D^{(*)}\rho^+$, gives $a_1 = 1.04 \pm 0.05$ and $a_2 = 0.24 \pm 0.06$.

3. Assumptions

The distinction between interfering amplitudes for the $B^+$ decay and non-interfering for the $B^0$ may only be valid for two-body decays. On the other hand, many-body final states will most likely start as two colour singlet quark antiquark pairs, including intermediate massive resonances. Interference between final states via different resonant channels involves strong phases which modify the rate for each individual final state in a random way and disappear in the sum of all states. It seems therefore reasonable to extend the model for exclusive two body decays to the majority of hadronic final states in an inclusive picture at the quark level. We assume that the formation of two colour singlets is the essential step of hadron production, which is taken into account quantitatively by $a_1$ and $a_2$. We neglect modifications by decays into baryon anti-baryon pairs, where our assumption is not valid. Under the

| $B^+(ba)$ | QCD | $B^0(bd)$ | QCD | CKM | PS |
| $\rightarrow$ | | | | | |
| $cu ev$ | 0.86 | $cd ev$ | 0.86 | 1.00 |
| $cu \mu\nu$ | 0.86 | $cd \mu\nu$ | 0.86 | 0.99 |
| $enu$ | 0.86 | $cdr$ | 0.86 | 0.23 |
| $cu du$ | $3(a_1 + a_2)^2$ | $cd du$ | $3a_1^2$ | $|V_{cd}|^2$ | 1.00 |
| $cu su$ | $3(a_1 + a_2)^2$ | $cd su$ | $3a_1^2$ | $|V_{us}|^2$ | 0.98 |
| $ca sc$ | $3a_2^2$ | $cd sc$ | $3a_2^2$ | $|V_{us}|^2$ | 0.98 |
| $ca dc$ | $3a_2^2$ | $cd dc$ | $3a_2^2$ | $|V_{us}|^2$ | 0.49 |
| $ca du$ | $3a_2^2$ | $cd dd$ | $3a_2^2$ | $|V_{us}|^2$ | 0.49 |

Table 3. Contributions from all $b \to c$ spectator diagrams. Partial widths are obtained as $\Gamma = \Gamma_0(b \to c e^- \bar{\nu}) \times CKM \times QCD \times PS$. 

The experimental average and the theoretical prediction for the $B^+$ decay are $0.45 \pm 0.04$ and $0.265(a_1 + 1.230a_2)^2$, respectively.
assumption of duality, the coefficients $a_1$ and $a_2$ can be used to predict the hadronic and semileptonic widths of the $B^+$ and $B^0$ mesons. The individual contributions are listed in Table 3. $PS$ denotes the relative phase space factor and the perturbative QCD correction for the semileptonic width is given by [10].

$$\Gamma(b \rightarrow ce^{-}\bar{\nu}) = \Gamma_0(1 - \frac{2\pi}{3}a_s + \frac{25}{6\pi}(\alpha_s) \approx 0.86\Gamma_0$$

From the factors in Table 3 we obtain the following total widths, normalized to the lowest order semileptonic width $\Gamma_0(b \rightarrow ce^{-}\nu)$

$$\Gamma(B^+)/\Gamma_0 = 1.91 + 4.44(a_1^2 + a_2^2) + 5.99a_1a_2,$$

$$\Gamma(B^0)/\Gamma_0 = 1.91 + 4.44(a_1^2 + a_2^2).$$

Using these widths, we can calculate two important quantities.

- The average semileptonic decay fraction of $B^0$ and $B^+$,

$$B(B \rightarrow e\nu X) = \frac{1}{2.22 + 5.16(a_1^2 + a_2^2) + 3.49a_1a_2},$$

decreases if $a_2$ changes sign from negative to positive.

- The lifetime ratio

$$\tau(B^+)/\tau(B^0) = 1 - \frac{a_1a_2}{0.32 + a_1a_2 + 0.74(a_1^2 + a_2^2)},$$

is larger than 1 for negative and smaller than 1 for positive values of $a_2$.

To give consistent results, we determine $a_1$ and $a_2$ in a fit to the hadronic decay fractions used above, replacing the assumption of equal $B^+$ and $B^0$ lifetimes with the inclusive prediction in eq. 1 to rescale the theoretical expectations for $B^+$ and $B^0$ decays individually. This fit gives $\chi^2 = 11.6$ for 8 degrees of freedom, and

$$a_1 = 1.05 \pm 0.03 \pm 0.10,$$

$$a_2 = 0.227 \pm 0.012 \pm 0.022$$

which implies

$$B(B \rightarrow e\nu X) = (11.2 \pm 0.5 \pm 1.7)\%,$$

$$\tau(B^+)/\tau(B^0) = 0.83 \pm 0.01 \pm 0.01,$$

where the first error is statistical including uncertainties in the $D^0$ and $D^+$ decay fractions, and the second is from the error on $V_{cb}\sqrt{\tau(B)}$. The uncertainty on $f^{+}/f^{0}$ yields a negligible error. The predicted lifetime ratio is low but not inconsistent with the current experimental average of 1.00 $\pm$ 0.07 [11]. The semileptonic decay fraction is further reduced if we assume a small contribution of penguin decays. Assuming this fraction to be 2.5% leads to $\chi^2 = 11.3$ and $B(B \rightarrow e\nu X) = 10.9\%$, while all errors and the values of $a_1$, $a_2$ and $\tau(B^+)/\tau(B^0)$ remain essentially unchanged.

4. Discussion

The discrepancy between the theoretical and the experimental semileptonic decay fraction of $B$ mesons can be considerably reduced by the interpretation of recent results on hadronic $B$ decays in the framework of a spectator model with interfering amplitudes. Our basic assumption is that the constructive interference observed in a few exclusive hadronic $B$ decays is a general feature of $B$ mesons that can be described by two coefficients $a_1$ and $a_2$. There is some experimental evidence that this assumption is correct

- The coefficient $a_2$ extracted from the interference observed in $B^+ \rightarrow D^{(*)+}$ decays agrees well with the $a_2$ value obtained from $B$ to charmonium transitions that can only proceed through the internal spectator diagram.

- A QCD based calculation [12] of inclusive $\psi$ production in $B$ decay falls short of a recent CLEO measurement [13]. However, if the coefficients in the calculation are replaced by the measured values of $a_1$ and $a_2$ we find good agreement.

- The measured $B \rightarrow \chi_c1$ decay fraction has been used to predict the $B \rightarrow \chi_c2$ rate [12, 13]. Again the agreement with recent experimental results can be improved by using $a_1$ and $a_2$ instead of the QCD coefficients in the calculation.

A careful study of inclusive decays as well as a search for color suppressed decays like $B^0 \rightarrow D^0\pi^0$ will allow us in the not too distant future to determine, if $a_1$ and $a_2$ are really universal coefficients in $B$ decays.

References

[1] G. Altarelli and S. Petrarca, PL B261, 303 (1991); I. I. Bigi et al., PL B293, 430 (1992) and erratum ibid. B297, 477 (1993); W. Palmer, B. Stech, PR D48, 4174 (1993).
[2] CLEO Collaboration, International Conference on HEP, Glasgow, CLEO-CONF-94-6 (GLS 0243).
[3] I. I. Bigi, B. Blok, M. A. Shifman, N. G. Uraltsev, A. I. Vainshtein, “B Decays”, ed. S. Stone, World Scientific (1994).
[4] M. S. Alam et al., Phys. Rev. D50, 43 (1994).
[5] T. E. Browder, K. Honscheid, S. Playfer, “B Decays”, ed. S. Stone, World Scientific (1994).
[6] M. Bauer, B. Stech, M. Wirbel, ZP C34,103(1987).
[7] M. Neubert, V. Rieckert, B. Stech in ‘Heavy Flavors’, ed. by A. J. Buras and M. Lindner, World Scientific 1992.
[8] A. Deandrea et al., Preprint UGVA-DPT 1993/07-824.
[9] V. Rieckert, priv. communication.
[10] N. Cabibbo, L. Maiani, Phys. Lett. B79,109 (1978).
[11] V. Sharma, DPF 94, Albuquerque (1994).
[12] G.T. Bodwin, E. Bratten, T.C. Yuan, G.P. Lepage, Phys. Rev. D46, 3703 (1992). Glasgow, CLEO-CONF-94-11.
[13] CLEO Collaboration, International Conference on HEP, Glasgow, CLEO-CONF-94-11 (GLS 0248).