QCD: from four to two dimensions

A. Bassetto

Dipartimento di Fisica “G. Galilei”, INFN Sezione di Padova
Via Marzolo 8, I-35131 PADOVA (Italy)
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Abstract

I review some work done in the past four years concerning the transition of Yang-Mills theories from 1+3 to 1+1 dimensions. The problem is considered both in a perturbative context and in exact solutions when available. Several interesting features are discussed, mainly in relation to the phenomenon of confinement, and some controversial issues are clarified.

I would like to report on some work done in the past four years with the aim of clarifying properties and peculiarities of a Yang-Mills theory when the number of space dimensions $d = D - 1$ is lowered to $d = 1$.

The interest in studying two-dimensional theories is mainly due to the possibility of obtaining sometimes exact solutions, which are believed to share important features with the more realistic situation in four dimensions.

Schwinger’s model (massless electrodynamics in two dimensions ($QED_2$)) is the key example, which can be exactly solved, exhibiting very interesting and peculiar properties, like fermion confinement, theta-vacua and the presence of a non-vanishing chiral condensate.

$QCD_2$ is its non-Abelian generalization and has recently received most attention in many investigations. It is widely believed that several phenomena that can be fairly easily understood in two dimensions, can persist when dimensions are increased.

To be definite, in the following I will limit myself to the “pure” Yang-Mills theory ($YM$) with gauge group $SU(N)$ (or sometimes $U(N)$), in spite of the fact that interesting features emerge when dynamical fermions, either in the fundamental or in the adjoint representation, are present.

I will focus my interest on two topics
• relations between $YM_4$ and $YM_2$ with a particular care when considering the limit $D \rightarrow 2$;

• relations between perturbative and non-perturbative solutions.

As a technical tool, I will use the Wilson loop, owing to its gauge invariance and to its reasonable infrared (IR) properties. It is indeed well known that, when approaching $D = 2$, ultraviolet (UV) singularities are no longer a concern, but wild IR behaviours usually show up.

The investigation started from a perturbative test of gauge invariance in $YM_4$ at $\mathcal{O}(g^4)$.

We consider the following Wilson loop

$$W_\gamma = \frac{1}{N} \langle 0 | \text{Tr} \left[ \mathcal{T} \mathcal{P} \exp \left( i g \int_\gamma dx^\mu A_\mu^a(x) T^a \right) \right] | 0 \rangle,$$

(1)

where $\gamma$ is a rectangle with light-like sides parametrized according to the equations

$$C_1 : x^\mu(t) = n^\mu t,$$

$$C_2 : x^\mu(t) = n^\mu + n^\mu t,$$

$$C_3 : x^\mu(t) = n^\mu + n^\mu (1 - t),$$

$$C_4 : x^\mu(t) = n^\mu (1 - t), \quad 0 \leq t \leq 1,$$

(2)

the vectors $n^\mu = \frac{1}{\sqrt{2}}(L, 0, 0, -L)$ and $n^\star^\mu = \frac{1}{\sqrt{2}}(T, 0, 0, T)$ being indeed light-like and normalized in such a way that $n \cdot n^\star = LT$.

This loop exhibits in four dimensions UV as well as IR singularities; they were both regularized dimensionally. A calculation in Feynman gauge was performed in [1], whereas in [2] the same loop was computed in the light-cone gauge $n \cdot A \equiv A_\perp = 0$. The two results coincide, as required by gauge invariance, provided the propagator in light-cone gauge is endowed with a causal prescription for the “spurious” singularity [3]

$$\frac{1}{n \cdot k} \equiv \frac{1}{n \cdot k + i \varepsilon \text{sign}(n^\star \cdot k) = \frac{n^\star \cdot k}{(n \cdot k)(n^\star \cdot k) + i \varepsilon},$$

which naturally follows from canonical equal-time quantization [4].

If instead the Cauchy principal value prescription (CPV) is adopted

$$\frac{1}{n \cdot k} \equiv P\left(\frac{1}{n \cdot k}\right),$$

as suggested by light-front quantization [5], causality and thereby analyticity properties are jeopardized preventing a consistent renormalization of the theory (see e.g. [3]). The result one gets exhibits divergencies that are not controlled by power counting and does not even resemble to the one of Feynman gauge.

It is interesting to investigate what happens to the same Wilson loop calculation when the number $D$ of space-time dimensions approaches 2. This has been done in
ref. [7], both in Feynman and in light-cone gauge. In Feynman gauge the propagator in strictly 2 dimensions is not a tempered distribution, owing to its singular IR behaviour. Individual diagrams, when dimensionally regularized, exhibit poles at $D = 2$; nevertheless these singularities cancel in the sum, leaving a finite result in the limit $D \to 2$. The same result is recovered in the light-cone gauge, provided the propagator has the “spurious” singularity causally prescribed; at $D = 2$ it is a tempered distribution and, indeed, in the coordinate representation it has the expression

$$D_{++}^{ab}(x) = \frac{i\delta^{ab}}{\pi^2} \int d^2k e^{ikx} \frac{k_+^2}{(k^2 + i\epsilon)^2} = \frac{\delta^{ab}}{\pi} \frac{(x^{-})^2}{(-x^2 + i\epsilon)}$$

(3)

to be compared with the expression it gets following the CPV prescription

$$D_{++}^{(P)ab}(x) = -\frac{i\delta^{ab}}{(2\pi)^2} \int d^2k e^{ikx} \frac{\partial}{\partial k_-} P\left(\frac{1}{k_-}\right) = -\frac{i\delta^{ab}}{2} |x^-| \delta(x^+) .$$

(4)

In the light-cone gauge individual diagrams are finite in the limit $D = 2$. Actually the diagram involving the triple vector vertex vanishes as expected, since in light-cone gauge there is no triple vector vertex in two dimensions. Surprisingly, the diagram with a self-energy loop correction to the vector propagator, does not vanish; what happens is that the vanishing of triple vertices is exactly compensated by the loop integration singularity at $D = 2$ leading, eventually, to a finite result. We would like to stress that it is not a pathology of light-cone gauge; precisely this term is needed, together with the contribution coming from graphs with two propagator lines, to get agreement with the Feynman gauge result.

According to a general theorem [8], the maximally non-Abelian terms are the relevant ones in perturbative calculations (as a matter of fact Abelian terms trivially exponentiate); at $O(g^4)$ we find

$$W^{na} = g^4 C_F C_A \frac{\mathcal{A}^2}{16\pi^2} (1 + \frac{\pi^2}{3}),$$

(5)

the first term coming from the graph containing the self-energy and the second one from the graph with crossed vector propagators. The quantities $C_F$ and $C_A$ are the usual quadratic Casimir operators of the fundamental and of the adjoint representation and $\mathcal{A}$ is the area of the loop.

If the same calculation is performed at exactly $D = 2$, the first term is obviously missing. Therefore the theory is discontinuous at $D = 2$, at least in its perturbative formulation.

The occurrence of a term proportional to $C_A$ is troublesome; if indeed the same result is obtained for a rectangular loop with sides of length $2L$ and $2T$, parallel respectively to a space and to the time directions, this dependence would survive in the large-$T$ limit and would be at odd with the expected Abelian-like exponentiation in this limit [6].
This is the motivation for studying the loop $\gamma$
\[\begin{align*}
\gamma_1 & : \gamma_1^\mu(s) = (sT, L) , \\
\gamma_2 & : \gamma_2^\mu(s) = (T, -sL) , \\
\gamma_3 & : \gamma_3^\mu(s) = (-sT, -L) , \\
\gamma_4 & : \gamma_4^\mu(s) = (-T, sL) ,
\end{align*}\]
\[-1 \leq s \leq 1. \quad (6)\]
describing a (counterclockwise-oriented) rectangle centered at the origin of the plane $(x^1, x^0)$, with sides of length $(2L, 2T)$, respectively.

This has been done in two papers [9] and [10]. In the first one, this loop was computed at $O(g^4)$ in light-cone gauge in exactly 2 dimensions; in the second paper the loop was computed in Feynman gauge at $D = 2 + \epsilon$.

Let us start by discussing the results of the second paper. As long as $\epsilon > 0$, the loop depends also on the dimensionless ratio $\beta = \frac{L}{T}$, besides the area. All terms proportional to $C_F C_A$ are subleading in the limit $T \to \infty$ with respect to the “planar” terms which are proportional to $C_F^2$. Indeed they typically behave like
\[T^{4-2D} A^2.\]

To be more precise [10], the contribution due to diagrams with crossed propagators, in the large-$T$ limit and for $\omega \equiv D/2$ near 1, exhibits a double and a single pole, whose Laurent expansion gives
\[
\frac{\mathcal{W}_{na} \pi^{2\omega} e^{2i\pi\omega}}{g^4 C_F C_A (2T)^{4-4\omega} (LT)^2} = \frac{1}{2(\omega - 1)^2} + \frac{1 - \gamma}{(\omega - 1)} - 1 - 2\gamma + \gamma^2 + \frac{\pi^2}{12} + O(\omega - 1),
\]
$\gamma$ being the Euler-Mascheroni constant.

Similarly, the contribution from diagrams involving the self-energy corrected vector propagator, is
\[
\frac{\mathcal{W}^{(2)} \pi^{2\omega} e^{2i\pi\omega}}{g^4 C_F C_A (2T)^{4-4\omega} (LT)^2} = \frac{1}{(\omega - 1)^2} + \frac{9 - 4\gamma}{2(\omega - 1)} + \frac{39}{2} - 9\gamma + 2\gamma^2 + \frac{\pi^2}{6} + O(\omega - 1),
\]
and again exhibits a double and a single pole at $\omega = 1$.

Finally diagrams involving a triple vector vertex lead to
\[
\lim_{\beta \to 0} \frac{\mathcal{W}^{(3)} \pi^{2\omega} e^{2i\pi\omega}}{g^4 C_F C_A (2T)^{4-4\omega} (LT)^2} = -\frac{3}{2(\omega - 1)^2} + \frac{3\gamma - 11/2}{(\omega - 1)} - \frac{35}{2} + 11\gamma - 3\gamma^2 + \frac{\pi^2}{12} + O(\omega - 1). \quad (9)
\]
Therefore agreement with Abelian-like exponentiation holds and the validity of previous perturbative tests of gauge invariance in higher dimensions (see ref.
(3) is fully confirmed. This rather simple and “universal” way of realizing the exponentiation at $D > 2$ might have a deeper justification as well as far-reaching consequences.

However it is clear from the expression above that if we take first the limit $\epsilon \to 0$, no damping occurs when $T \to \infty$. The two limits do not commute.

Summing the three contributions, double and single poles at $\omega = 1$ cancel; when $\epsilon \to 0$ the dependence on $\beta$ disappears and the result of eq.(3) is exactly recovered, in spite of the fact that the two loops are different.

A pure area dependence would be hardly surprising in view of the invariance of the loop in two dimensions under area-preserving diffeomorphisms. Still we remind the reader that the first contribution in eq.(3) was obtained after a limiting procedure from higher dimensions: in exactly two dimensions it does not occur. Then we find amazing that it respects such a symmetry on its own.

Finally, in ref. [9] the same space-time loop is computed in light-cone gauge at exactly $D = 2$. One obtains only the second term of eq.(3), as expected.

Let us summarize what we have achieved so far.

- For $D > 2$ the $O(g^4)$ result we get is in agreement with the expected Abelian-like area exponentiation in the large-$T$ limit. $C_F C_A$ terms are subleading.

- In the limit $D \to 2$ the $O(g^4)$ result is finite, it depends only on the area of the loop and consists of two addenda (see eq.(3)). No simple area exponentiation occurs in the large-$T$ limit, owing to the presence of a leading $C_F C_A$ contribution. Therefore the limits $T \to \infty$ and $D \to 2$ do not commute.

- At exactly $D = 2$ only the second term of eq.(3) survives. The theory exhibits a discontinuity in the limit $D \to 2$ (for any value of $T$!), at least in its perturbative formulation.

All the above conclusions are shared by Feynman and light-cone gauges.

Moreover, working at exactly $D = 2$, Staudacher and Krauth [11] were able to resum in light-cone gauge with causal prescription the perturbative series at all orders in the coupling constant $g$, thereby generalizing our $O(g^4)$ result. They get for $U(N)$ in the Euclidean formulation

$$\mathcal{W} = \frac{1}{N} \exp\left[ - g^2 A_1 \right] L_{N-1}^1 \left( \frac{g^2 A_1}{2} \right),$$

(10)

the function $L_{N-1}^1$ being a generalized Laguerre polynomial.

This result is definitely different from the exact expression which is known in two dimensions

$$\mathcal{W} = \exp\left[ - g^2 \frac{N A_1}{4} \right],$$

(11)

and has been obtained by different authors using different procedures.
Not only a Laguerre polynomial appears as a factor in eq. (11), but also the string tension, namely the constant in the exponential, turns out to be different from the expected one.

More dramatically, eq. (11) in the limit $N \to \infty$ with $\hat{g}^2 = \frac{g^2 N}{2}$ fixed, becomes
\[
\mathcal{W} \to \frac{1}{\sqrt{\hat{g}^2 A}} J_1(2\sqrt{\hat{g}^2 A}),
\]
and confinement is lost.

So what is wrong (if anything) with eq. (11)?

In order to understand this point, it is worthwhile to study the problem on a compact two-dimensional manifold, that, for simplicity, we choose the sphere $S^2$ \[14\]. We shall also consider the slightly simpler case of the group $U(N)$ (the generalization to $SU(N)$ is straightforward). On $S^2$ we consider a smooth non self-intersecting closed contour $\Gamma$. We call $A$ the total area of the sphere, which eventually will be sent to $\infty$, whereas $\bar{A}$ will be the area “inside” the loop we keep finite in this limit. It is well known that, on $S^2$ at large $N$, a phase transition occurs between two regimes, a weak coupling regime which correspond to small values of $g^2A$ and a strong coupling regime for large $g^2A$ \[13\]. This phase transition is driven by instantons. We follow closely the treatment of this problem given in refs. \[12\], \[14\].

Our starting point are the well-known expressions \[13\] of the exact partition function and of a non self-intersecting Wilson loop for a pure $U(N)$ Yang-Mills theory on a sphere $S^2$ with area $A$
\[
\mathcal{Z}(A) = \sum_R (d_R)^2 \exp \left[ -\frac{g^2 A}{4} C_2(R) \right],
\]
\[
\mathcal{W}(A, \bar{A}) = \frac{1}{\mathcal{Z} N} \sum_{R,S} d_R d_S \exp \left[ -\frac{g^2 A}{4} C_2(R) - \frac{g^2 (A - \bar{A})}{4} C_2(S) \right]
\times \int dU \text{Tr}[U] \chi_R(U) \chi_S^\dagger(U),
\]
\[14\]
\[d_R(S)\] being the dimension of the irreducible representation $R(S)$ of $U(N)$; \[C_2(R)\] \([C_2(S)\] is the quadratic Casimir, the integral in \[14\] is over the $U(N)$ group manifold while \[\chi_R(S)\] is the character of the group element $U$ in the $R(S)$ representation. From eq. \[14\] it is possible to derive, in the large-$A$ decompactification limit, the behaviour \[14\].

We write these equations explicitly for $N > 1$ in the form
\[
\mathcal{Z}(A) = \frac{1}{N!} \exp \left[ -\frac{g^2 A}{48} N(N^2 - 1) \right]
\times \sum_{m_i = -\infty}^{+\infty} \Delta^2(m_1, ..., m_N) \exp \left[ -\frac{g^2 A}{4} \sum_{i=1}^{N} (m_i - \frac{N - 1}{2})^2 \right],
\]
[5]
\[ W(A, A) = \frac{1}{ZN} \exp \left[ -\frac{g^2 A}{48} N(N^2 - 1) \right] \frac{1}{N!} \]
\[ \times \sum_{k=1}^{N} \sum_{m_i=-\infty}^{+\infty} \Delta(m_1, ..., m_N) \Delta(m_1 + \delta_{1,k}, ..., m_N + \delta_{N,k}) \]
\[ \times \exp \left[ -\frac{g^2 A}{4} \sum_{i=1}^{N} \left( m_i - \frac{N-1}{2} \right)^2 - \frac{g^2(A - \mathcal{A})}{4} \sum_{i=1}^{N} \left( m_i - \frac{N-1}{2} + \delta_{i,k} \right)^2 \right]. \quad (16) \]

We have described the generic irreducible representation by means of the set of integers \( m_i = (m_1, ..., m_N) \), related to the Young tableaux, in terms of which we get
\[ C_2(R) = \frac{N}{12} (N^2 - 1) + \sum_{i=1}^{N} (m_i - \frac{N-1}{2})^2, \]
\[ d_R = \Delta(m_1, ..., m_N). \quad (17) \]
\( \Delta \) is the Van der Monde determinant and the integration in eq. (14) has been performed explicitly, using the well-known formula for the characters in terms of the set \( m_i \).

Now, as first noted by Witten [17], it is possible to represent \( Z(A) \) (and consequently \( W(A, A) \)) as a sum over instable instantons, where each instanton contribution is associated to a finite, but not trivial, perturbative expansion. The easiest way to see it, is to perform a Poisson resummation in eqs. (15),(16) [18]
\[ \sum_{m_i=-\infty}^{+\infty} F(m_1, ..., m_N) = \sum_{n_i=-\infty}^{+\infty} \tilde{F}(n_1, ..., n_N), \]
\[ \tilde{F}(n_1, ..., n_N) = \int_{-\infty}^{+\infty} dz_1 ... dz_N \exp \left[ 2\pi i (z_1 n_1 + ... + z_N n_N) \right] F(z_1, ..., z_N). \quad (18) \]

We have carefully repeated the original computations of ref. [14], paying particular attention to the numerical factors and to the area dependences; as a matter of fact, at variance with [14], where interest was focussed on the large-\( N \) limit, we are mainly concerned with decompactification (large \( A \)) and with a comparison with the results of ref. [11] for any value of \( N \). We have obtained
\[ Z(A) = C(g^2 A, N) \sum_{n_i=-\infty}^{+\infty} \exp \left[ -S_{\text{inst}}(n_i) \right] Z(n_1, ..., n_N), \]
\[ W(A, A) = \frac{1}{ZN} C(g^2 A, N) \exp \left[ -\frac{g^2 A(A - \mathcal{A})}{4A} \right] \sum_{n_i=-\infty}^{+\infty} \exp \left[ -S_{\text{inst}}(n_i) \right] \]
\[ \times \sum_{k=1}^{N} \exp \left[ -2\pi i n_k \frac{A - \mathcal{A}}{A} \right] W_k(n_1, ..., n_N), \quad (19) \]
where
\[ C(g^2 A, N) = (i)^{N(N-1)} \frac{(\frac{g^2 A}{2})^{N^2/2}}{N!} \exp \left[ -\frac{g^2 A}{48} N(N^2 - 1) \right] \]

\[ S_{\text{inst}}(n_i) = \frac{4\pi^2}{g^2 A} \sum_{i=1}^{N} n_i^2, \]  

and

\[ Z(n_1, ..., n_N) = \exp(i\pi(N - 1) \sum_{i=1}^{N} n_i) \int_{-\infty}^{+\infty} dz_1...dz_N \exp \left[ -\frac{1}{2} \sum_{i=1}^{N} z_i^2 \right] \times \prod_{i<j} \left( \frac{8\pi^2}{g^2 A} (n_i - n_j)^2 - (z_i - z_j)^2 \right), \]

\[ W_k(n_1, ..., n_N) = \exp(i\pi(N - 1) \sum_{i=1}^{N} n_i) \int_{-\infty}^{+\infty} dz_1...dz_N \exp \left[ -\frac{1}{2} \sum_{i=1}^{N} z_i^2 \right] \times \prod_{i<j} \left( \left( \frac{2\sqrt{2\pi}}{g^2 A} (n_i - n_j) + ig^2 A - 2A \delta_{i,k} - \delta_{j,k} \right)^2 - \left( (z_i - z_j) + i \frac{\sqrt{2\pi}}{g^2 A} (\delta_{i,k} - \delta_{j,k}) \right)^2 \right). \]

These formulae have a nice interpretation in terms of instantons. Indeed, on \( S^2 \), there are non trivial solutions of the Yang-Mills equation, labelled by the set of integers \( n_i = (n_1, ..., n_N) \)

\[ A_\mu(x) = \begin{pmatrix} n_1 A_\mu^0(x) & 0 & \ldots & 0 \\ 0 & n_2 A_\mu^0(x) & \ldots & 0 \\ 0 & \ldots & \ldots & 0 \\ 0 & \ldots & \ldots & n_N A_\mu^0(x) \end{pmatrix} \]  

(22)

where \( A_\mu^0(x) = A_\mu^0(\theta, \phi) \) is the Dirac monopole potential,

\[ A_{\theta}^0(\theta, \phi) = 0, \quad A_{\phi}^0(\theta, \phi) = \frac{1 - \cos \theta}{2}, \]

\( \theta \) and \( \phi \) being the polar (spherical) coordinates on \( S^2 \).

From the above representations it is rather clear why the decompactification limit \( A \to \infty \) should not be performed too early. Indeed on the plane it is not easy to distinguish fluctuations around the instanton solutions from Gaussian fluctuations around the trivial field configuration, since \( S_{\text{inst}}(n_i) \) goes to zero for any finite set \( n_i \) when \( A \to \infty \). For finite \( A \) and finite \( n_i \) instead, in the limit \( g \to 0 \), only the zero instanton sector can survive in the Wilson loop expression (notice that the power-like singularity \( (g^2)^{-N(N-1)/2} \) in the coefficient \( C(g^2 A, N) \) exactly cancels in the normalization). In this limit each instanton contribution is \( \mathcal{O}(\exp(-\frac{1}{g^2})) \); therefore instantons become crucial only when they are completely resummed.

On the other hand the zero instanton contribution should be obtainable in principle by means of perturbative calculations.
In the following we compute from eqs. (19) the exact expression on the sphere \(S^2\) of the zero instanton contribution to the Wilson loop, obviously normalized to zero instanton partition function.

We write eq. (19) for the zero instanton sector \(n_i = 0\). Thanks to its symmetry, we can always choose \(k = 1\) and the equation becomes

\[
W_0 = (2\pi)^{-\frac{3}{2}} \prod_{n=0}^{N} \frac{1}{n!} \exp \left[ -g^2 \frac{A(A - A)}{4A} \right] \int_{-\infty}^{+\infty} dz_1 \ldots dz_N \exp \left[ -\frac{1}{2} \sum_{i=1}^{N} z_i^2 \right] 
\times \prod_{j=2}^{N} \left[ (z_1 - z_j)^2 + i \sqrt{\frac{g^2 A}{2}} (z_1 - z_j) - g^2 \frac{A(A - A)}{2A} \right] \Delta^2(z_2, \ldots, z_N). \tag{23}
\]

We introduce the two roots of the quadratic expression in the integrand \(z_\pm = z_1 + i\alpha \pm i\beta\) with \(\alpha = \sqrt{\frac{g^2 A}{2\sqrt{2}}}\) and \(\beta = \sqrt{\frac{g^2 (2A - A)}{2\sqrt{2}A}}\). The previous equation then becomes

\[
W_0 = (2\pi)^{-\frac{3}{2}} \prod_{n=0}^{N} \frac{1}{n!} \exp \left[ -g^2 \frac{A(A - A)}{4A} \right] \int_{-\infty}^{+\infty} dz_1 \ldots dz_N \exp \left[ -\frac{1}{2} \sum_{i=1}^{N} z_i^2 \right] 
\times \Delta(z_+, z_2, \ldots, z_N) \Delta(z_-, z_2, \ldots, z_N). \tag{24}
\]

The two Van der Monde determinants can be expressed in terms of Hermite polynomials \([14]\) and then expanded in the usual way. The integrations over \(z_2, \ldots, z_N\) can be performed, taking the orthogonality condition into account; we get

\[
W_0 = (2\pi)^{-\frac{3}{2}} \prod_{n=0}^{N} \frac{1}{n!} \exp \left[ -g^2 \frac{A(A - A)}{4A} \right] \prod_{k=2}^{N} (jk - 1)! \epsilon_{j_1 \ldots j_k} \epsilon_{j_1 \ldots j_k}
\times \int_{-\infty}^{+\infty} dz_1 \exp \left[ -\frac{z_1^2}{2} \right] H_{j_1-1}(z_+) H_{j_1-1}(z_-). \tag{25}
\]

Thanks to the relation

\[
\int_{-\infty}^{+\infty} dz_1 \exp \left[ -\frac{z_1^2}{2} \right] H_{j_1-1}(z_+) H_{j_1-1}(z_-) = \sqrt{2\pi}(j_1 - 1)!L_{j_1-1}(\alpha^2 - \beta^2), \tag{26}
\]

we finally obtain our main result

\[
W_0 = \frac{1}{N} \exp \left[ -g^2 \frac{A(A - A)}{4A} \right] L_{N-1}^1(g^2 \frac{A(A - A)}{2A}). \tag{27}
\]

At this point we remark that, in the decompactification limit \(A \to \infty, A\) fixed, the quantity in the equation above exactly coincides, for any value of \(N\), with eq. (10), which was derived following completely different considerations. We recall indeed that that result was obtained by a full resummation at all orders of the perturbative expansion of the Wilson loop in terms of Yang-Mills propagators in light-cone gauge, endowed with the causal prescription.
We conclude that, for any value of $N$, the pure area law exponentiation (eq.(11)) follows, after decompactification, only by resumming all instanton sectors, a procedure which changes completely the zero sector behaviour and, in particular, the value of the string tension.

In the light of the considerations above, there is no contradiction between the use of the causal prescription in the light-cone propagator and the pure area law exponentiation of eq.(11); this prescription is correct but the ensuing perturbative calculation can only provide us with the expression for $W_0$. The paradox of ref. [11] is solved by recognizing that they did not take into account the genuine $O(\exp(-\frac{1}{g^2}))$ non perturbative quantities coming, after decompactification, from the instantons on the sphere.

We find quite remarkable that both expressions in eqs.(10) and (11), respectively, are (different) analytic functions of $g^2$. This is hardly surprising for eq.(10), but not for eq.(11), if it is thought as a sum over instanton contributions. This analytic behaviour is at the root of the possibility of obtaining eq.(11) in a quite different way. As a matter of fact, if the Wilson loops we have previously considered, are computed using the instantaneous 't Hooft-CPV potential of eq.(4), which follows from light-front quantization, and just resumming at all orders the related perturbative series, one exactly recovers the correct pure area exponentiation (11), i.e. the same result which requires the essential introduction of non perturbative effects (instantons), when studied in equal-time quantization [9].

This surprising feature is the origin of almost all recent calculations in QCD2, which make essential use of the 't Hooft’s propagator (4).

Confinement in this picture occurs as a trivial generalization of the QED2 situation, namely as a consequence of a two dimensional “instantaneous” increasing potential between a $q\bar{q}$ pair, giving rise to hadronic strings in a natural way (we recall that, with the potential (4), only planar graphs survive and therefore only $C_F$ can appear). But we feel unlikely that a similar mechanism can be at the root of a realistic confinement in higher dimensions.

A deeper conjecture, which deserves further study, may relate it to some peculiar properties of the light-front vacuum (we remind the reader that the light-cone CPV prescription follows from canonical light-front quantization [4]). In equal-time quantization “axial” ghosts are present [4], [7], which, although expunged from the “physical” Hilbert space, contribute to Green functions. These degrees of freedom are canonically suppressed when quantizing on the light-front. In two dimensions this procedure might perhaps be viable in a “continuum” formulation, as renormalization is no longer a concern, but, in higher dimensions, it is certainly illegitimate and perturbatively incorrect. It can neither be extended smoothly beyond the strictly two-dimensional case nor it can be smoothly continued to any Euclidean formulation and compared to different gauge choices.

Why the instantons we have hitherto considered seem to be crucial only in two dimensions in order to obtain the correct area exponentiation, is at present an interesting open problem.
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