On almost \((m, n)\)-ideals and fuzzy almost \((m, n)\)-ideals in semigroups

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**ABSTRACT**

In this paper, we define almost \((m, n)\)-ideals of semigroups by using the concepts of \((m, n)\)-ideals and almost ideals of semigroups. An almost \((m, n)\)-ideal is a generalization of \((m, n)\)-ideals and a generalization of almost one-sided ideals. We investigate properties of almost \((m, n)\)-ideals of semigroups. Moreover, we define fuzzy almost \((m, n)\)-ideals of semigroups and give relationships between almost \((m, n)\)-ideals and fuzzy almost \((m, n)\)-ideals.

**1. Introduction and preliminaries**

This notion of \((m, n)\)-ideals of semigroups was first introduced and studied by Lajos in [1]. He investigated remarkable properties of \((m, n)\)-ideals of semigroups in [2–6]. Let \(m \) and \(n \) be non-negative integers. A sub-semigroup \(A \) of a semigroup \(S \) is called an \((m, n)\)-ideal of \(S \) if \(A^{m+n} \subseteq A \). Note that a left ideal of a semigroup \(S \) is a \((0, 1)\)-ideal of \(S \) and a right ideal of \(S \) is a \((1,0)\)-ideal of \(S \). An \((m, n)\)-ideal is a one of generalizations of one-sided ideals. Furthermore, the theory of \((m, n)\)-ideals in other structures have also been studied by many authors, for example, \((m, n)\)-ideals in ordered semigroups were studied by Bussaban and Changphas in [7] and in LA-semigroups were studied by Akram et al. in [8], etc. In [9], Omidi and Davaz defined \((m, n)\)-hyperideals and \((m, n)\)-bi-hyperideals in ordered semihyperrings and investigate some of their related properties. Recently, Khan and Mahboob characterized \((m, n)\)-filters of \((m, n)\)-regular ordered semigroups in terms of its prime generalized \((m, n)\)-ideals in [10].

In 1965, Zadeh introduced the fundamental fuzzy set concept in [11]. Since then, fuzzy sets are now applied in various fields. A fuzzy subset of \(S \) is a function from \(S \) into the closed interval \([0, 1]\). For any two fuzzy subsets \(f \) and \(g \) of \(S \),

\[ f \cap g \text{ is a fuzzy subset of } S \text{ defined by } \]

\[ (f \cap g)(x) = \min\{f(x), g(x)\} = f(x) \wedge g(x) \]

for all \(x \in S \),

\[ f \cup g \text{ is a fuzzy subset of } S \text{ defined by } \]

\[ (f \cup g)(x) = \max\{f(x), g(x)\} = f(x) \lor g(x) \]

for all \(x \in S \) and

\[ f \subseteq g \text{ if } f(x) \leq g(x) \text{ for all } x \in S. \]

For a fuzzy subset \(f \) of \(S \), the support of \(f \) is defined by

\[ \text{supp}(f) = \{x \in S \mid f(x) \neq 0\}. \]

The characteristic mapping of a subset \(A \) of \(S \) is a fuzzy subset of \(S \) defined by

\[ C_A(x) = \begin{cases} 1, & x \in A, \\ 0, & x \notin A. \end{cases} \]

The definition of fuzzy points was given by Pu and Liu [12]. For \(x \in S \) and \(\alpha \in (0, 1) \), a fuzzy point \(x_\alpha \) of a set \(S \) is a fuzzy subset of a set \(S \) defined by

\[ x_\alpha(y) = \begin{cases} \alpha, & y = x, \\ 0, & y \neq x. \end{cases} \]

Some interesting topics of fuzzy points were studied in [13–15]. Let \(F(S)\) be the set of all fuzzy subsets in a semigroup \(S\). The semigroup \(S\) itself is a fuzzy subset of \(S\) such that \(S(x) = 1\) for all \(x \in S\), denoted also by \(S\). For each \(f, g \in F(S)\), the product of \(f\) and \(g\) is a fuzzy subset \(f \circ g\).
almost ideals of semigroups were launched in 1980 by Grosek and Satko [16]. They characterized these ideals when a semigroup $S$ contains no proper left, right, two-sided almost ideals in [16], and afterwards they discovered minimal almost ideals and smallest almost ideals of semigroups in [17,18], respectively. A nonempty subset $A$ of a semigroup $S$ is called a left almost ideal of $S$ if $SA \cap A \neq \emptyset$ for any $s \in S$. A right almost ideal of a semigroup $S$ is defined analogously. A nonempty subset $A$ of a semigroup $S$ is called an almost ideal of $S$ if $SA \cap A \neq \emptyset$ and $At \cap A \neq \emptyset$ for all $s,t \in S$. In 1981, Bogdanovic [19] introduced the notion of almost bi-ideals in semigroups by using the concepts of almost ideals and bi-ideals of semigroups. Likewise, Wattanatripop, Chinram and Changphas examined quasi-almost-ideals of bi-ideals of semigroups. Likewise, Wattanatripop, Chinram and Changphas examined quasi-almost-ideals of bi-ideals of semigroups and gave properties of quasi-almost-ideals in [20]. Furthermore, they defined fuzzy almost ideals of semigroups in [20] and fuzzy almost bi-ideals of semigroups in [21] and provided relationship between almost ideals and fuzzy almost ideals of semigroups. Recently, Gaketem generalized results in [21] to study interval-valued fuzzy almost bi-ideals of semigroups in [22]. In [23], Solano, Suebsung and Chinram extended this idea to study almost ideals of n-ary semigroups.

Our purpose of this paper is to define the notion of almost $(m,n)$-ideals of semigroups by using the concepts of $(m,n)$-ideals and almost ideals of semigroups and study them. Moreover, we define the notion of fuzzy almost $(m,n)$-ideals of semigroups and give relationships between almost $(m,n)$-ideals and fuzzy almost $(m,n)$-ideals of semigroups.

2. Almost $(m,n)$-ideals

Let $m$ and $n$ be non-negative integers. Let $A$ be a non-empty subset of a semigroup $S$ and $s \in S$. Note that $A^0 := \{s\}$. For $k \in \mathbb{N}$, let $A^k := A^k s$ and $A^k Sa := s^k A$. Firstly, we define an almost $(m,n)$-ideal of semigroup by using the concepts of $(m,n)$-ideals defined in [1] and almost ideals of semigroups defined in [16].

Definition 2.1: Let $S$ be a semigroup. A non-empty subset $A$ of $S$ is called an almost $(m,n)$-ideal of $S$ if

$$A^m Sa^n \cap A \neq \emptyset$$

for all $s \in S$.

Remark 2.1: The following statements hold.

1. An almost $(1,0)$-ideal of a semigroup $S$ is a right almost ideal of $S$ defined in [15].
2. An almost $(0,1)$-ideal of a semigroup $S$ is a left almost ideal of $S$ defined in [15].
3. Every $(m,n)$-ideal of a semigroup $S$ is an almost $(m,n)$-ideal of $S$.
4. Consider the semigroup $\mathbb{Z}_6$ under the usual addition. We have $A = \{1,4,5\}$ is an $(1,0)$-ideal of $\mathbb{Z}_6$ but $A$ is not a subsemigroup of $\mathbb{Z}_6$. Therefore, an almost $(m,n)$-ideal of a semigroup $S$ need not be a subsemigroup of $S$ and need not be an $(m,n)$-ideal of $S$.

Proposition 2.2: If $A$ is an almost $(m,n)$-ideal of a semigroup $S$, then every subset $H$ of $S$ such that $A \subseteq H$ is an almost $(m,n)$-ideal of $S$.

Proof: Assume that $A$ is an almost $(m,n)$-ideal of $S$ and $H$ is a subset of $S$ with $A \subseteq H$. Then $H \neq A^m Sa^n \cap A \subseteq H^m Sa^n \cap H$ for all $s \in S$. Therefore $H$ is an almost $(m,n)$-ideal of $S$.

Corollary 2.3: The union of two almost $(m,n)$-ideals of a semigroup $S$ is an almost $(m,n)$-ideal of $S$.

Proof: Let $A_1$ and $A_2$ be any two almost $(m,n)$-ideals of $S$. Then $A_1 \subseteq A_1 \cup A_2$. By Proposition 2.2, $A_1 \cup A_2$ is an almost $(m,n)$-ideal of $S$.

Note that in the proof of Corollary 2.3 is true if $A_1$ or $A_2$ is an almost $(m,n)$-ideal of $S$.

Example 2.4: Consider the semigroup $\mathbb{Z}_6$ under the usual addition. We have $A_1 = \{1,4,5\}$ and $A_2 = \{1,2,5\}$ are almost $(1,0)$-ideals of $\mathbb{Z}_6$ but $A_1 \cap A_2 = \{1,5\}$ is not an almost $(1,0)$-ideal of $S$.

Example 2.4 implies that, in general, the intersection of two almost $(m,n)$-ideals of a semigroup $S$ need not be an almost $(m,n)$-ideal of $S$.

Theorem 2.5: Let $S$ be a semigroup such that $|S| > 1$. A semigroup $S$ has no proper almost $(m,n)$-ideal if and only if for any $a \in S$ there exists $s_a \in S$ such that $(S \setminus \{a\})^m s_a (S \setminus \{a\})^n = \{a\}$.

Proof: Assume that $S$ has no proper almost $(m,n)$-ideal and let $a \in S$. Then $S \setminus \{a\}$ is not an almost $(m,n)$-ideal of $S$. Then there exists $s_a \in S$ such that $((S \setminus \{a\})^m s_a (S \setminus \{a\})^n) \cap (S \setminus \{a\}) = \emptyset$. This implies that $(S \setminus \{a\})^m s_a (S \setminus \{a\})^n = \{a\}$. Conversely, let $a \in S$. Then there exists $s_a \in S$ such that

$$(S \setminus \{a\})^m s_a (S \setminus \{a\})^n = \{a\}.$$
Then $B \subseteq S \setminus \{a'\}$ for some $a' \in S$. By Theorem 2.2, $S \setminus \{a'\}$ is also an almost $(m, n)$-ideal of $S$, this is contradiction. Therefore $S$ has no proper almost $(m, n)$-ideal. ■

**Theorem 2.6:** Let $S$ be a semigroup such that $|S| > 1$ and $a \in S$. If $S \setminus \{a\}$ is not an almost $(m, n)$-ideal of $S$, then at least one of them is true.

1. $a = a^{m+n+1}$.
2. $a = a^{(m+n)^3+1}$.
3. $a = a^{(m+n+1)^{m+n}+1}$.

**Proof:** Assume that $S \setminus \{a\}$ is not an almost $(m, n)$-ideal of $S$. Then there exists $s_a \in S$ such that $[(S \setminus \{a\})^m s_a (S \setminus \{a\})^n] \cap (S \setminus \{a\}) = \emptyset$.

Case 1: $s_a \neq a$. Then $s_a \in S \setminus \{a\}$. This implies that $(s_a)^n s_a = s_a = a$. So $a = (s_a)^{m+n+1}$. Suppose that $a \neq a^{m+n+1}$. Then $a^{m+n+1} \in S \setminus \{a\}$, so $(a^{m+n+1})^m s_a (a^{m+n+1})^n = a$. Hence $a = (a^{m+n+1})^{m+n} s_a$.

Case 1.1: If $a^{(m+n)^2} s_a = a$, then

$$a = (a^{m+n+1})^{m+n} s_a = a^{m+n+1} + n s_a = a^{m+n+1}$$

which is a contradiction.

Case 1.2: If $a^{(m+n)^2} s_a \neq a$, then $a^{(m+n)^2} s_a \in S \setminus \{a\}$.

Thus

$$(a^{(m+n)^2} s_a) s_a (a^{(m+n)^2} s_a) = a.$$ 

This implies that

$$a = (s_a)^{m+n+1} a^{(m+n)^3} = a^{m+n+1+3}.$$ 

Case 2: $s_a = a$. Suppose that $a \neq a^{m+n+1}$. Then $a^{m+n+1} \in S \setminus \{a\}$. So $(a^{m+n+1})^m s_a (a^{m+n+1})^n = a$. Therefore,

$$a = (a^{m+n+1})^m a (a^{m+n+1})^n = a^{m+n+1+1}.$$ 

**Proposition 3.1:** Let $f, g$ and $h$ be fuzzy subsets of $S$.

1. If $f \subseteq g$, then $f^n \subseteq g^n$ for all $n \in \mathbb{N} \cup \{0\}$.
2. If $f \subseteq g$, then $f \circ h \subseteq g \circ h$.
3. If $f \subseteq g$, then $f \cap h \subseteq g \cap h$.

**Proof:** The proof is straightforward. ■

**Definition 3.2:** A fuzzy subset $f$ of a semigroup $S$ is called a fuzzy almost $(m, n)$-ideal of $S$ if

$$(f^m \circ (x)_{\alpha} \circ f^n) \cap f \neq 0$$

for all fuzzy point $(x)_{\alpha}$ of $S$.

This implies that $f$ is a fuzzy almost $(m, n)$-ideal of $S$ if for all fuzzy point $(x)_{\alpha}$ of $S$, there exists $y \in S$ such that $[(f^m \circ (x)_{\alpha} \circ f^n) \cap f](y) \neq 0$.

**Proposition 3.3:** Let $f$ be a fuzzy almost $(m, n)$-ideal of $S$ and $g$ be a fuzzy subset of $S$ such that $f \subseteq g$. Then $g$ is a fuzzy almost $(m, n)$-ideal of $S$.

**Proof:** Assume that $f$ is a fuzzy almost $(m, n)$-ideal of $S$ and $g$ is a fuzzy subset of $S$ such that $f \subseteq g$. Let $(x)_{\alpha}$ be a fuzzy point in $S$. We have

$$0 \neq (f^m \circ (x)_{\alpha} \circ f^n) \cap f \subseteq (g^m \circ (x)_{\alpha} \circ g^n) \cap g.$$ 

Therefore, $g$ is a fuzzy almost $(m, n)$-ideal of $S$. ■

**Corollary 3.4:** Let $f$ and $g$ be fuzzy almost $(m, n)$-ideals of $S$. Then $f \cup g$ is a fuzzy almost $(m, n)$-ideal of $S$.

**Proof:** Since $f \subseteq g \cup f$, by Proposition 3.3, $f \cup g$ is a fuzzy almost $(m, n)$-ideal of $S$.

Note that in the proof of Corollary 3.4 it is true if $f$ or $g$ is a fuzzy almost $(m, n)$-ideal of $S$.

**Example 3.5:** Consider $n = 1, m = 0$ and the semigroup $\mathbb{Z}_6$ under the usual addition $f : \mathbb{Z}_6 \rightarrow [0, 1]$ is defined by

$$f(0) = 0, \quad f(1) = 0.2, \quad f(2) = 0, \quad f(3) = 0,$$

$$f(4) = 0.5, \quad f(5) = 0.3$$

and $g : \mathbb{Z}_6 \rightarrow [0, 1]$ defined by

$$g(0) = 0, \quad g(1) = 0.8, \quad g(2) = 0.4, \quad g(3) = 0,$$

$$g(4) = 0, \quad g(5) = 0.3.$$ 

We have $f$ and $g$ are fuzzy almost $(1, 0)$-ideals of $\mathbb{Z}_6$ but $f \cap g$ is not a fuzzy almost $(1, 0)$-ideal of $\mathbb{Z}_6$.

Example 3.5 implies that, in general, the intersection of two fuzzy almost $(m, n)$-ideals of $S$ need not be a fuzzy almost $(m, n)$-ideal of $S$.
Note that for a subset $A$ of $S$, define $A^0 := S$.

**Lemma 3.6:** Let $A$ be a subset of $S$ and $n \in \mathbb{N} \cup \{0\}$. Then $(CA)^n = CA^n$.

**Proof:** The proof is straightforward. ■

**Theorem 3.7:** Let $A$ be a nonempty subset of a semigroup $S$. Then $A$ is an almost $(m, n)$-ideal of $S$ if and only if $CA$ is a fuzzy almost $(m, n)$-ideal of $S$.

**Proof:** Assume that $A$ is an almost $(m, n)$-ideal of $S$. Then $A^n s A^n \cap A \neq \emptyset$ for all $s \in S$. Let $s \in S$ and $\alpha \in (0, 1]$. Thus there exists $x \in A^n s A^n \cap A$. So

$$[(CA)^n \circ (s)_\alpha \circ CA^n] \cap CA(x) \neq 0.$$  By Lemma 3.6, we have

$$[[CA] ] \circ (s)_\alpha \circ CA^n \cap CA(x) \neq 0.$$

Hence, $CA$ is a fuzzy almost $(m, n)$-ideal of $S$.

Conversely, assume that $CA$ is a fuzzy almost $(m, n)$-ideal of $S$. Let $s \in S$ and $\alpha \in (0, 1]$. Thus

$$[(CA)^m \circ (s)_\alpha \circ CA^n] \cap CA(x) \neq 0.$$

Then there exists $x \in S$ such that

$$[[CA] ] \circ (s)_\alpha \circ CA^n \cap CA(x) \neq 0.$$

By Lemma 3.6, we have

$$[(CA)^m \circ (s)_\alpha \circ CA^n] \cap CA(x) \neq 0.$$

Hence, $x \in A^n s A^n \cap A$. Eventually, $A^n s A^n \cap A \neq \emptyset$. ■

**Theorem 3.8:** Let $f$ be a fuzzy subset of $S$. Then $f$ is a fuzzy almost $(m, n)$-ideal of $S$ if and only if $\text{supp}(f)$ is an almost $(m, n)$-ideal of $S$.

**Proof:** Assume that $f$ is a fuzzy almost $(m, n)$-ideal of $S$. Let $x \in S$. Then for any $\alpha \in (0, 1]$, we have

$$(f^n \circ (x)_\alpha \circ f^n) \cap f \neq 0.$$  Thus, there exists $y \in S$ such that

$$[(f^n \circ (x)_\alpha \circ f^n) \cap f] (y) \neq 0.$$  So, $f(y) \neq 0$ and

$$y = a_1 a_2 \ldots a_m b_1 b_2 \ldots b_n$$

for some $a_1, a_2, \ldots, a_m, b_1, b_2, \ldots, b_n \in S$ such that

$$f(a_1) \neq 0, \ f(a_2) \neq 0, \ldots, f(a_m) \neq 0, \ f(b_1) \neq 0, \ f(b_2) \neq 0, \ldots, f(b_n) \neq 0.$$  This implies that $a_1, a_2, \ldots, a_m, b_1, b_2, \ldots, b_n, y \in \text{supp}(f)$.

Thus,

$$[(\text{supp}(f))^n \circ (x)_\alpha \circ (\text{supp}(f))^n] (y) \neq 0$$

and $\text{supp}(f) \neq 0$. Hence,

$$[(\text{supp}(f))^m \circ (x)_\alpha \circ (\text{supp}(f))^n \cap \text{supp}(f)] (y) \neq 0.$$  So, $\text{supp}(f)$ is a fuzzy almost $(m, n)$-ideal of $S$. By Theorem 3.7, $\text{supp}(f)$ is an almost $(m, n)$-ideal of $S$.

Conversely, assume that $\text{supp}(f)$ is an almost $(m, n)$-ideal of $S$. By Theorem 3.7, $\text{supp}(f)$ is a fuzzy almost $(m, n)$-ideal of $S$. Let $(x)_\alpha$ be a fuzzy point in $S$. Then

$$[((\text{supp}(f))^m \circ (x)_\alpha \circ (\text{supp}(f))^n \cap \text{supp}(f)] (y) \neq 0.$$  Then there exists $y \in S$ such that

$$[((\text{supp}(f))^m \circ (x)_\alpha \circ (\text{supp}(f))^n \cap \text{supp}(f)] (y) \neq 0.$$  Hence,

$$(\text{supp}(f))^m \circ (x)_\alpha \circ (\text{supp}(f))^n (y) \neq 0$$

and $\text{supp}(f) (y) \neq 0$. Then there exist $a_1, a_2, \ldots, a_m, b_1, b_2, \ldots, b_n \in \text{supp}(f)$ and $y = a_1 a_2 \ldots a_m b_1 b_2 \ldots b_n$. Thus

$$f(y) \neq 0, \ f(a_1) \neq 0, \ f(a_2) \neq 0, \ldots, f(a_m) \neq 0, \ f(b_1) \neq 0, \ f(b_2) \neq 0, \ldots, f(b_n) \neq 0.$$  Therefore,

$$f^n \circ (x)_\alpha \circ f^n \neq 0.$$  This implies that

$$[(f^n \circ (x)_\alpha \circ f^n) \cap f] (y) \neq 0.$$  Consequently, $f$ is a fuzzy almost $(m, n)$-ideal of $S$. ■

**3.1. Minimal almost $(m, n)$-ideals and minimal fuzzy almost $(m, n)$-ideals**

In this subsection, we give relationship between minimal almost $(m, n)$-ideals and minimal fuzzy almost $(m, n)$-ideals.

**Definition 3.9:** A fuzzy almost $(m, n)$-ideal $f$ is called minimal if for all nonzero fuzzy almost $(m, n)$-ideals $g$ of $S$ such that $g \subseteq f$, we have $\text{supp}(f) = \text{supp}(g)$.

**Theorem 3.10:** Let $S$ be a non-empty subset of a semigroup $S$. Then $A$ is a minimal almost $(m, n)$-ideal of $S$ if and only if $CA$ is a minimal fuzzy almost $(m, n)$-ideal of $S$.

**Proof:** Assume that $A$ is a minimal almost $(m, n)$-ideal of $S$. By Theorem 3.7, $CA$ is a fuzzy almost $(m, n)$-ideal of $S$. Let $f$ be a fuzzy almost $(m, n)$-ideal of $S$ such that $f \subseteq CA$. Then $\text{supp}(f) \subseteq \text{supp}(CA) = A$. By Theorem 3.8, $\text{supp}(f)$ is an almost $(m, n)$-ideal of $S$. Since $A$ is minimal, $\text{supp}(f) = A = \text{supp}(CA)$. Therefore, $CA$ is minimal.

Conversely, assume that $CA$ is a minimal fuzzy almost $(m, n)$-ideal of $S$. Let $B$ be an almost $(m, n)$-ideal of $S$ such that $B \subseteq A$. Then $B$ is a fuzzy almost $(m, n)$-ideal of $S$ such that $B \subseteq CA$. Hence, $B = \text{supp}(CB) = \text{supp}(CA) = A$. Therefore, $A$ is minimal. ■

**Corollary 3.11:** $S$ has no proper almost $(m, n)$-ideal if and only if for all fuzzy almost $(m, n)$-ideals $f$ of $S$, $\text{supp}(f) = S$.

**Proof:** This follows by Theorem 3.10. ■
3.2. Prime almost \((m, n)\)-ideals and prime fuzzy almost \((m, n)\)-ideals

In this subsection, we give relationship between prime almost \((m, n)\)-ideals and prime fuzzy almost \((m, n)\)-ideals.

**Definition 3.12**: Let \(S\) be a semigroup.

(1) An almost \((m, n)\)-ideal \(A\) of \(S\) is called prime if for all \(x, y \in S\), \(xy \in A\) implies \(x \in A\) or \(y \in A\).

(2) A fuzzy almost \((m, n)\)-ideal \(A\) of \(S\) is called prime if for all \(x, y \in S\), \(f(xy) \leq \max(f(x), f(y))\).

**Theorem 3.13**: Let \(A\) be a non-empty subset of \(S\). Then \(A\) is a prime almost \((m, n)\)-ideal of \(S\) if and only if \(CA\) is a prime fuzzy almost \((m, n)\)-ideal of \(S\).

**Proof**: Assume that \(A\) is a prime almost \((m, n)\)-ideal of \(S\). By Theorem 3.7, \(CA\) is a fuzzy almost \((m, n)\)-ideal of \(S\).

Let \(x, y \in S\). We consider two cases:

Case 1: \(xy \in A\). So, \(x \in A\) and \(y \in A\). Then \(\max(C_A(x), C_A(y)) = 1 \geq C_A(xy)\).

Case 2: \(xy \notin A\). Then \(C_A(xy) = 0 \leq \max(C_A(x), C_A(y))\).

Thus, \(CA\) is a prime fuzzy almost \((m, n)\)-ideal of \(S\).

Conversely, assume that \(CA\) is a prime fuzzy almost \((m, n)\)-ideal of \(S\). By Theorem 3.7, \(A\) is an almost \((m, n)\)-ideal of \(S\). Let \(x, y \in S\) such that \(xy \in A\). Then \(C_A(xy) = 1\). By assumption, \(C_A(xy) \leq \max(C_A(x), C_A(y))\). Therefore, \(\max(C_A(x), C_A(y)) = 1\). Hence, \(x \in A\) or \(y \in A\). Thus, \(A\) is a prime almost \((m, n)\)-ideal of \(S\).

3.3. Semiprime almost \((m, n)\)-ideals and semiprime fuzzy almost \((m, n)\)-ideals

In this subsection, we give relationship between semiprime almost \((m, n)\)-ideals and semiprime fuzzy almost \((m, n)\)-ideals.

**Definition 3.14**: Let \(S\) be a semigroup.

(1) An almost \((m, n)\)-ideal \(A\) of \(S\) is called semiprime if for all \(x \in S\), \(x^2 \in A\) implies \(x \in A\).

(2) A fuzzy almost \((m, n)\)-ideal \(f\) is called semiprime if for all \(x \in S\), \(f(x^2) \leq f(x)\).

**Theorem 3.15**: Let \(A\) be a non-empty subset of \(S\). Then \(A\) is a semiprime almost \((m, n)\)-ideal of \(S\) if and only if \(CA\) is a semiprime fuzzy almost \((m, n)\)-ideal of \(S\).

**Proof**: Assume that \(A\) is a semiprime almost \((m, n)\)-ideal of \(S\). By Theorem 3.7, \(CA\) is a semiprime fuzzy almost \((m, n)\)-ideal of \(S\). Let \(x \in S\) such that \(x^2 \in A\). Then \(C_A(x^2) = 1\). By assumption, \(C_A(x^2) \leq C_A(x)\). Since \(C_A(x) = 1\), \(C_A(x^2) = 1\). Hence, \(x \in A\). Thus, \(A\) is a semiprime almost \((m, n)\)-ideal of \(S\).

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