Design sensitivity analysis in the kinematics of the 4SS-axle guiding mechanism with Panhard bar

C Alexandru
Transilvania University of Braşov, Romania
E-mail: calex@unitbv.ro

Abstract. This work deals with the design sensitivity analysis in the kinematics of the vehicle rear axle guided by four points - on four spheres (so called 4SS), with Panhard bar. The geometric parameters that define the kinematic scheme of the axle guiding mechanism are considered in this study, which aims to identify the influence of these parameters on the specific kinematic functions. The purpose is to achieve a separation of geometric parameters depending on their influences on the objective functions to be optimized from a kinematic point of view, so as to simplify the process of effective optimization, by taking into account (as design variables) only the main parameters.

1. Introduction

Relative to car body, the vehicle wheels can be guided independently - by means of a guiding mechanism for each wheel, or dependently - by a guiding mechanism of the rigid axle. The first solution is frequently used for the front & rear wheels of the passenger cars, while the second one is mainly used for the rear axles of larger gauge cars. For the rear axle guidance, spatial mechanisms formed by a number of binary links (bars) are interposed between axle and car body. The bars’ connections to axle and car body are made by using compliant joints (i.e. bushings) [1-3]. Usually, for the kinematic study (where the car body is fixed), the bushings are modelled as spherical joints, the corresponding models having a low number of degree of mobility (DOM=1 or DOM=2) [4, 5].

The guidance of the rear axle is made by driving a number of its points on suitably chosen surfaces and curves (sphere, circle or coupler curve). By the guidance of four axle points on four spheres with centres on car body (4SS), bi-mobile (DOM=2) guiding mechanisms are obtained. According to the study carried out in [6], for this structural group/class of axle guiding mechanisms, the spherical joint model assumption can be accepted because the behaviour of these mechanisms is closer to that of the real model with compliant joints (bushings). The structural variants of 4SS axle guiding mechanism are presented in Figure 1. For the scheme shown in Figure 1.a, all the guiding bars of the mechanism are arranged in the longitudinal direction, while for the mechanism shown in Figure 1.b one of the bars (4) is arranged transversely (so called Panhard bar), which ensures a better takeover of the transversal forces from the wheels, the latest solution being addressed in this work.

The kinematic optimization of the axle guiding mechanisms is a rather complex problem, considering both the variety of kinematic parameters that need to be optimized and the multitude of geometric parameters that define the mechanism. For this reason, it would be preferable to achieve a separation of geometric parameters depending on the influence they have on the objective functions to be optimized. Such a study will allow that in the effective optimization only the main parameters, which significantly influence the behaviour of the axle guiding mechanism, to be taken into account.
2. Geometric and kinematic parameters

Its position and orientation, in relation to the global reference frame OXYZ that is fixed in car body, describe the spatial movement of the rear axle. The global coordinates $X_p^0$, $Z_p^0$ determine the initial position of the axle in OXYZ reference frame. Due to the symmetry of the guiding linkage relative to the longitudinal plane of the car, there are the following relationships between the geometrical parameters: $X_{M01}=X_{M02}$, $|Y_{M01}|=Y_{M02}$, $Z_{M01}=Z_{M02}$, $Y_{M03}=0$, $X_{M1(P)}=X_{M2(P)}$, $|Y_{M1(P)}|=Y_{M2(P)}$, $Z_{M1(P)}=Z_{M2(P)}$, $Y_{M3(P)}=0$. Therefore, the geometrical model of the 4SS guiding mechanisms is defined by 19 geometrical parameters, as follows (Figure 2): $X_{M01,2}$, $X_{M01,2}$, $Z_{M01,2}$, $X_{M03}$, $Z_{M03}$, $X_{M04}$, $Y_{M04}$, $Z_{M04}$, $X_{M1,2(P)}$, $Y_{M1,2(P)}$, $Z_{M1,2(P)}$, $X_{M3(P)}$, $Z_{M3(P)}$, $X_{M4(P)}$, $Z_{M4(P)}$, $X_{M1,2(P)}$, $Y_{M1,2(P)}$, $Z_{M1,2(P)}$, $X_{M3(P)}$, $Z_{M3(P)}$, $X_{M4(P)}$, $Z_{M4(P)}$, $l_{1,2}$, $l_{3}$, $l_{4}$.

Relative to the car body, the rear axle must have the possibility of vertical displacement and rotation around the longitudinal axis of the vehicle (X). When the vehicle is moving, besides the above mentioned necessary motions, secondary undesirable motions can occur, as follows: displacements of the axle centre P along the longitudinal and transversal axes, axle pivoting movement, and axle rotation around its own axis. The minimization of the undesirable motions represents the goal of the kinematic optimization [4, 5].

The spatial position & orientation of the axle are defined by the following kinematic parameters:

- the linear displacements of the axle’s centre:
  \[ \Delta X_p = X_p - X_p^0, \Delta Y_p = Y_p - Y_p^0, \Delta Z_p = Z_p - Z_p^0; \]  

- the rotations of the rear axle in the global coordinate system’s plans:
The global coordinates of the characteristic points (Gx, Gy, G) and of the axle’s center (P) are determined in accordance with the numerical method depicted in [**]. The vertical coordinates of the wheels’ centers / axle’s ends (ZGx, ZGay) are independent kinematic parameters.

Usually, the coordinates of the guiding points on axle (X, Y, Z)Mopi are established by constructive criteria. Afterwards, it will be analyzed the influence of the other parameters on the kinematic behavior of the guiding mechanisms (in fact, on the undesirable motions). The spatial configuration of the mechanism is defined by taking into account the disposing of the guiding bars, in accordance with the schemes shown in Figure 3 (the longitudinal bars - 1, 2, 3) and Figure 4 (the transversal bar - 4).

The unitary position vectors u1,3 (for the longitudinal bars) and u4 (for the transversal bar) are defined by the following equations:

\[
\begin{align*}
    u_i = u_i^x & = \begin{bmatrix} 
        \cos \varphi_{iy} \\
        \sin \varphi_{iz} \\
        -\sin \varphi_{iy}\cos \varphi_{iz}
    \end{bmatrix}, \\
    u_i^y & = \begin{bmatrix} 
        \sin \varphi_{iz} \\
        \cos \varphi_{iy} \\
        \cos \varphi_{iz}\sin \varphi_{iy}
    \end{bmatrix}, \\
    u_i^z & = \begin{bmatrix} 
        \cos \varphi_{iz} \\
        -\sin \varphi_{iz}\cos \varphi_{iy}
    \end{bmatrix}, \\
    i = 1...3, \\
    u_4^x & = \begin{bmatrix} 
        \sin \varphi_{4z} \\
        \cos \varphi_{4z} \\
        \cos \varphi_{4z}\sin \varphi_{4z}
    \end{bmatrix}, \\
    u_4^y & = \begin{bmatrix} 
        \cos \varphi_{4z} \\
        -\sin \varphi_{4z}\cos \varphi_{4z}
    \end{bmatrix}, \\
    u_4^z & = \begin{bmatrix} 
        \sin \varphi_{4z}
    \end{bmatrix}, \\
    i = 1...3, \\
\end{align*}
\]

where:

\[
\varphi_{iy} = \arctg(tg\varphi_{iy} \cdot \cos \varphi_{iz}), \\
\varphi_{4z} = \arctg(tg\varphi_{4z} \cdot \cos \varphi_{4z}).
\]

The global coordinates of points/joints on car body (XMi, YMi, ZMi) will be:

\[
\begin{align*}
    X_{Mi} = X_{Mc} - l_i \cos \varphi_{iz} \cdot \cos \varphi_{iy}, \\
    Y_{Mi} = Y_{Mc} - l_i \sin \varphi_{iz} \cdot \cos \varphi_{iy}, \\
    Z_{Mi} = Z_{Mc} + l_i \cdot \sin \varphi_{iy}, \\
    i = 1...3, \\
    X_{M0i} = X_{Mc} - l_i \cdot \sin \varphi_{iz} \cdot \cos \varphi_{4z}, \\
    Y_{M0i} = Y_{Mc} + l_i \cdot \cos \varphi_{iz} \cdot \cos \varphi_{4z}, \\
    Z_{M0i} = Z_{Mc} + l_i \cdot \sin \varphi_{4z},
\end{align*}
\]

where the global coordinates of the guiding points on axle (XM, YM, ZM) correspond to the known initial position of the mechanism.

By noting k = l1/l1(1) (the ratio between the lengths of the upper and lower longitudinal bars), there are obtained the following geometrical parameters (whose influence on the kinematic behaviour of the axle guiding mechanism will be analysed): k, l1, l4, \varphi_{iz}, \varphi_{4z}, \varphi_{4y}, \varphi_{4z}. 

\[
\eta_{iy} = \arctg\frac{X_{Giy} - X_{Gzi}}{Y_{Giy} - Y_{Gzi}}, \\
\eta_{ix} = \arctg\frac{Z_{Giy} - Z_{Gzi}}{X_{Giy} - X_{Gzi}}, \\
\eta_{iz} = \arcsin\frac{\left|Z_{Giy} - Z_{Gzi}\right| - \left|Z_{P} - Z_{Gzi}\right|}{\left|X_{Giy} - X_{Gzi}\right|}.
\]

(2)

The positioning of the longitudinal bars.

Figure 3.

The positioning of the transversal bar.

Figure 4.
3. Results and conclusions

The results of the design sensitivity study are presented in the diagrams shown in Figures 5-9, corresponding to the functional case $Z_{Gz}=Z_{Gd}$, considering the vertical travel of the wheels in the interval $\Delta Z \in [-80, 80]$mm relative to the initial position (vehicle in rest). The influence of parameters $l_1, \varphi_{1z}$ and $\varphi_{4z}$ on the undesirable motions $\Delta X_P, \Delta Y_P, \eta_{xy}$ and $\eta_{xz}$ is insignificant; on the other hand, all geometrical parameters have a small influence on the axle pivoting motion $\eta_{xy}$ (for this reason, the corresponding variation diagrams are no longer presented). The following values of the geometrical parameters were considered: $\varphi_{1y} = \{-6, 0, +6\}^\circ$, $\varphi_{3y} = \{-6, 0, +6\}^\circ$, $l_1 = \{400, 500, 600\}$ mm, $k = \{0.4, 0.6, 0.8\}$, $\varphi_{4x} = \{-6, 0, +6\}^\circ$.

![Figure 5. The influence of parameter $\varphi_{1y}$.](image1)

![Figure 6. The influence of parameter $\varphi_{3y}$.](image2)

![Figure 7. The influence of parameter $l_1$.](image3)
According to these results, a separation of the geometrical parameters was obtained, as follows:
main parameters, with great influence on kinematic behaviour of the axle guiding mechanism: \(k, \varphi_{1y}, \varphi_{3y}\); secondary parameters, with small influence: \(l_{1(2)}, l_{4}, \varphi_{1(2)x}, \varphi_{4x}, \varphi_{4z}\). There can be considered that the undesirable motions are given by the following functions, in pairs [kinematic parameters; (geometric parameters)]: \(\Delta X_P \equiv F_1[Z_{Gr.d}; (k, \varphi_{1y}, \varphi_{3y})], \Delta Y_P \equiv F_2[Z_{Gr.d}; (\varphi_{4x})], \eta_{xy} \equiv F_3[Z_{Gr.d}], \eta_{xz} \equiv F_d[Z_{Gr.d}; (k, \varphi_{1y}, \varphi_{3y})]\). Afterwards, the kinematic synthesis of the axle guiding mechanism can be carried out on the basis of the main geometrical parameters, by neglecting the secondary parameters, which simplifies the optimal design process, with beneficial effects on the allotted time.

References
[1] Knapczyk J and Maniowski M 2002 Selected effects of bushings characteristics on five-link suspension elastokinematics *Mobility and Vehicle Mechanics* 3(2) pp 107-121
[2] Knapczyk J and Maniowski M 2006 Elastokinematic modeling and study of five-rod suspension with subframe *Mechanism and Machine Theory* 41(9) pp 1031-1047
[3] Tică M, Dobre G and Mateescu V 2014 Influence of compliance for an elastokinematic model of a proposed rear suspension *International Journal of Automotive Technology* 15(6) pp 885-891
[4] Alexandru C 2009 The kinematic optimization of the multi-link suspension mechanism used for rear axle of the motor vehicle *Proceedings of the Romanian Academy - A* 10(3) pp 244-253
[5] Simionescu PA and Beale D 2002 Synthesis and analysis of the five-link rear suspension system used in automobile *Mechanism and Machine Theory* 37(9) pp 815-832
[6] Țoțu V and Alexandru C (2013) Study concerning the effect of the bushings’ deformability on the static behavior of the rear axle guiding linkages *Applied Mechanics and Materials* 245 pp 132-137
[7] Alexandru C 2019 Method for the quasi-static analysis of beam axle suspension systems used for road vehicles *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering* 233(7) pp 1818-1833