Loschmidt echo of local dynamical processes in integrable and non integrable spin chains

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Received 22 October 2018, revised 11 June 2019
Accepted for publication 28 June 2019
Published 1 August 2019

Abstract

We consider a local instantaneous quantum dynamical process (QDP) that disturbs the background unitary Hamiltonian dynamics of a spin chain. We consider both a non-unitary incoherent QDP and a coherent unitary QDP intervention of the dynamical evolution of the spin chain. To track the effect of QDP on the dynamics, we investigate the Loschmidt echo, which is quite sensitive to whether the background dynamics is integrable or not. For the integrable case, namely the Heisenberg model, the Loschmidt echo depends on the parameters corresponding to the QDP as well as the time of occurrence of QDP. The probability of reviving the system to its initial state is higher for non-unitary/incoherent QDPs occurring at large time intervals. In the case of a unitary/coherent QDP, some amount of the probability of reviving the state is lost. For the non-integrable background dynamics we consider a kicked Harper model. It exhibits a decaying behaviour when contrasted with integrable dynamics. The decay rate is slower when the corresponding classical Hamiltonian is non-chaotic.

Keywords: Loschmidt echo, quantum dynamical process, chaos, spin chain, entanglement

(Some figures may appear in colour only in the online journal)

1. Introduction

An isolated system evolves unitarily, which allows a pure state to remain pure throughout the evolution. So, such systems can be brought back to their initial state through a time reversal operation. If the system interacts with the environment, which can lead to decoherence, this is no longer true and the system cannot be brought back to their initial state by time reversal operation. Moreover, it has been proposed that the probability of restoring the system back
to the initial state is a decreasing function of time. In addition, the revival probability shows a contrasting behaviour for integrable and non-integrable background dynamics. Peres [1] first showed that classically chaotic and integrable system behave differently under imperfect time reversal. The Loschmidt echo is a measure of the revival of the state when an imperfect time reversal procedure is applied during the evolution of a quantum system. It quantifies the sensitivity of the quantum state to the local decohering process when the system interacts with the environment.

The Loschmidt echo in many body systems has been studied extensively over the last decade [2]. It has been studied in contexts of a quantum quench [3–5], the sensitivity to perturbations in many body systems [6], the many body localised phase [7], the Lyapunov exponents of classical systems [8], and a many body system in a decohering environment [9–12]. It has been observed that the Loschmidt echo decays exponentially in ergodic systems [5, 7, 13]. The time scale of the decay can be related to the Lyapunov exponent of the classical system [7, 8, 14]. In a recent study, there is an exact computation of the Loschmidt echo for a quantum quench in interacting systems [15].

We consider a system that is undergoing a unitary evolution due to background Hamiltonian dynamics. The dynamical evolution is interrupted by a quantum process operating locally and instantaneously, and the system is further evolved through the background Hamiltonian dynamics. The interruption can be thought of as interaction of the system with the environment, which can lead to decoherence. We will consider single-qubit operations occurring once for the most part, and consider many single-qubit processes occurring at different times and location operations also. The local operation that interrupts the background evolution can occur from local quantum decohering processes or local coherent operations. A local quantum dynamical process (QDP) occurring on a many qubit state during unitary evolution, can change the distribution of correlations and entanglement structure. The speed of the signal propagating due to the QDP occurrence has been studied [16], and the interference of the signal and the state propagation in the spin chain dynamics has been investigated recently [17]. Here, we study the effect of a local QDP, occurring for a given qubit at a given epoch of time during the unitary evolution of an initial state, using the Loschmidt echo, calculated from the overlap of the time-evolved multi-qubit states with and without the QDP intervention.

In this paper, we will consider the disturbance of the unitary background dynamics from both coherent and incoherent QDPs. An incoherent QDP will cause decoherence in the system, and a unitarily evolving pure state will become a mixed state as a result. Multiple incoherent QDPs intervening the dynamics at regular intervals can be thought of as if the system is interacting with an external decohering environment. The action of a multiple number of coherent QDPs does not cause decoherence but, nevertheless, can generate non-integrability in the system. Certain non-integrable systems described by time-dependent Hamiltonians can be thought of as a multiple coherent QDPs intervening the background integrable dynamics at different qubits with different strengths. We will discuss how non-integrable dynamics can be obtained by introducing multiple coherent operations at a regular interval on integrable dynamics. We will try to show below that the Loschmidt echo can show a contrasting behaviour between the integrable and non-integrable dynamics.

The paper is arranged as follows. In section 2, we will discuss a general approach to compute the Loschmidt echo for anisotropic Heisenberg dynamics for both incoherent and coherent QDPs using Green’s functions. We will also consider the case of a multiple number of QDPs occurring on the spin chain. In section 3, we will study the kicked Harper model dynamics, that allows us to go from an integrable to a non-integrable dynamics by turning the kicking time. We will investigate the contrast between the background integrable XY dynamics and Harper dynamics, i.e. chaotic and non-chaotic Harper dynamics, and the effect of an
incoherent QDP intervening the dynamics by computing the Loschmidt echo. In the last section, we give a summary and conclusions.

2. Local QDP interrupting the dynamics of the Heisenberg model

We consider a spin chain in an initial pure state evolving through the unitary Hamiltonian dynamics. We will first discuss below the Heisenberg dynamics, an exactly solvable and integrable model for spin chains. A non-integrable dynamics will be discussed in the later section. This background dynamics is intervened with an instantaneous local QDP, which disturbs a given spin at a given time. The state of the system will be a mixed state, leading to decoherence, if the QDP is non-unitary, represented as a noisy quantum channel. If the QDP is also a unitary process, however, the state remains a pure state. Further, the state evolves through the background Hamiltonian dynamics. In previous studies [16, 17], the signal propagation of the QDP in the spin chain, and the interference of this signal with the quantum state transport have been investigated, for various initial states using different model Hamiltonians. For simple initial states, i.e. one-magnon and two-magnon states, analytical calculations can be done in great detail. We take a similar approach here, and investigate the Loschmidt echo of the QDP in spin chains, for simple states for both integrable and non-integrable background dynamics for representative unitary or non-unitary QDP.

We will first discuss the effect of local operations (both coherent and incoherent) in the Heisenberg model, an exactly solvable and integrable model. Let us consider a one-dimensional chain of \( N \) spins interacting through anisotropic Heisenberg exchange interaction. The Hamiltonian is given by,

\[
H = -\frac{1}{2} \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z). \tag{1}
\]

Using the \( \sigma^z \) basis states (up-spin and down-spin states for each spin), the last term signifies the interaction between spins, and the first two terms signify the hopping of down spins or up spins to neighbouring sites. The total number of up(down) spins or \( z \) component of the total spin is a constant of motion in the dynamics. Thus, the eigenstates have a definite number of up(down) spins. The ground state is a ferromagnetic state with all the spins up or down, with the energy \( E_g = -N\Delta/2 \) for a chain of \( N \) spins. One-magnon excitations can be created by turning any one of the spins, giving \( N \) localised one-magnon states, which can be labelled by the location of the down spin. The state for one down spin at the site \( x \) can be represented by \( |x\rangle = \sigma_x^- |F\rangle \), using the ferromagnetic ground state \( |F\rangle \) with all spins up. One-magnon eigenstates are labelled by the momentum of the down spin, with plane-wave eigenfunctions. The one-magnon eigenvalue is given by \( \epsilon_1(p) = \epsilon_0 - 2 \cos p \), where the momentum \( p = 2\pi I/N \) is determined by an integer \( I = 1, 2, \ldots, N \). The one-magnon eigenstates are not affected by the interaction term in the Hamiltonian. The two-magnon state \( |x_1, x_2\rangle \), denoted by the locations of the two down spins \( x_1, x_2 \). The eigenvalue is given by \( \epsilon_2(p_1, p_2) = \epsilon_0 + 2(\Delta - \cos p_1) + 2(\Delta - \cos p_2) \). The two-magnon eigenstates involve both scattering states and bound states. The two-magnon eigenstates are labelled by two momenta \( p_1, p_2 \) that can be computed [19] from Bethe ansatz equations, \( p_1N = 2\pi I_1 + \theta(p_1, p_2), \ p_2N = 2\pi I_2 - \theta(p_1, p_2) \). The phase angle depends on the interaction strength \( \Delta \).

For our discussion below, we will consider three different types of initial states. Consider an unentangled initial state \( \alpha |F\rangle + \beta |1\rangle \), which is a linear combination of the ferromagnetic ground state and a one-magnon state with the down spin localised on the first site. This is an unentangled state, being a direct product of up-spin states for all sites and a linear combination
of up- and down-spin states for the first spin. For later times, the one-magnon component evolves through the spreading of the down spin wave function. We also consider an entangled one-magnon state, a one-magnon initial state $\alpha|1\rangle + \beta|r\rangle$, where the first and the $r$th qubits are entangled initially, the rest of the spins are in a direct product state. For later times, the down-spin wave function will evolve by spreading out, generating entanglement for other pairs of spins. Another type of initial state we consider is a combination of zero and two-magnon states, $\alpha|F\rangle + \beta|1, r\rangle$. Here the first and $r$th spins are entangled, but all other spins in a direct product state. The dynamics of this entangled state is different from the one-magnon entangled state, as the two-magnon component evolves through the two-magnon propagator that depends on the interaction strength. Since the initial state is a pure state, the time evolution leads to a pure state. There are more complicated states with three or more magnons [15], which are quite difficult to track analytically for the time evolution. The time-evolved state $|\psi(t)\rangle$, in general, can have various types of entanglement, i.e. bipartite entanglement, pairwise entanglement distribution, three-body entanglement etc.

For the unentangled initial state $|\Psi(0)\rangle = \alpha|F\rangle + \beta|1\rangle$, the time-evolved state can be written using the one-magnon propagator as

$$|\Psi(t)\rangle = e^{-iE_g t} \left[ \alpha|F\rangle + \beta \sum_x G_x^f(t)|x\rangle \right],$$

where $E_g$ is the ground state energy, and the one-magnon Green’s function is given in terms of the eigenfunctions and eigenvalues given above, defined by

$$G_x^f(t) = e^{i\epsilon_x t} \sum_p \psi_p^\dagger(t) e^{-i\epsilon_x r}.$$  

Using the one-magnon eigenfunctions we can express the Green’s function [16] for a large system as,

$$G_x^f(t) = (-i)^{(s-x')} J_{s-x'}(2t),$$

where $J_n(2t)$ is the $n$’th order Bessel function. The real part of the Green function $G_x^f(t)$ is plotted, as a density plot, as a function of time $t$ and site index $l$ in figure 1(a). It quantifies the propagation of the down spin from the first site of the chain to $l$th site at time $t$. The time taken for the down spin to propagate is linear in $l$, implying a finite speed of propagation, as expected for integrable dynamics. The maximum value of the function is obtained at time $l/2$.

We now turn to evaluating operator expectation values in this state, that we will need later on. We can evaluate the expectation values of the single-spin operators at later times using the one-magnon Green’s function directly, we get after straightforward calculations,

$$\langle \sigma_m^a(t) \rangle = \langle \Psi(t) | \sigma_m^a | \Psi(t) \rangle = 1 - 2|\beta|^2 |G_m^f(t)|^2,$$

$$\langle \sigma_m^a(t) \rangle = \langle \Psi(t) | \sigma_m^a | \Psi(t) \rangle = 2\alpha^* \beta \text{Re}(e^{-i\epsilon_d G_m^f(t)}).$$

Let us consider a one-magnon entangled initial state $|\Psi(0)\rangle = \alpha|1\rangle + \beta|r\rangle$, with the first spin in an entangled state with the spin at $r$. For this entangled single-magnon initial state, the evolution for both the components is similar to the previous case, the single-magnon Green’s function propagator will determine the location of the down spin in the time-evolved state. The state for later times is given by, $|\Psi(t)\rangle = \sum_x \alpha G_x^f(t) + \beta G_x^r(t)|x\rangle \equiv \sum_x K_x |x\rangle$, where we have defined a new composite Green’s function $K_m$ in terms of the single-magnon Green’s function evaluated above. Now the expectation values of the spin operators can be similarly calculated, we get
\[ \langle \sigma_x^m(t_0) \rangle = 0. \] (6)

The expectation value of \( \sigma_x^m \) is zero because, the spin operator acts on a one-magnon state and generates a superposition of zero-magnon and two-magnon states, which have no overlap with the one-magnon state.

The time evolution of the initial state \( |\Psi(0)\rangle = \alpha |F\rangle + \beta |1, r\rangle \), is more complicated as we have a superposition of zero-magnon and two-magnon states here. The two-magnon component evolves through a two-magnon Green’s function \( G_{x,y}^{1,2}(t) \), the amplitude for the two down spins at locations \( l \) and \( m \) at \( t = 0 \) moving to the locations \( x \) and \( y \) at a later time \( t \). Now, the time-evolved state is given by,

\[ |\Psi(t)\rangle = e^{-i\epsilon g t} \alpha |F\rangle + \beta \sum_{x,y} G_{x,y}^{1,2}(t)|x,y\rangle. \] (7)

The computation of the two-magnon Green’s function, shown above, is complicated as both the scattering states (where the state has two independent one-magnon excitations) and two-magnon bound states contribute (see [17] for the details of these contributions). For comparison, we have shown the density plots of the single-magnon and two-magnon Green’s functions in figure 1. The real part of the scattering state Green functions \( G_{l,l+1}^{1,2+1}(t) \) and \( G_{l,l+10}^{1,2+10}(t) \) for anisotropy constant \( \Delta = 1.0 \) has been plotted in figures 1(b) and (c). Unlike the case of one-magnon Green function the two-magnon Green function decays fast with both time and site index, which can be intuitively understood from the normalisation condition \( \sum_l \sum_r |G_{l,m}^{1,2}(t)|^2 = 1 \). The expectation values of the spin operators can be straightforwardly calculated; we have

\[ \langle \sigma_z^m(t) \rangle = 1 - 2|\beta|^2 \left( \sum_{x < 0} |G_{l,x}^{1,m}(t)|^2 + \sum_{x > 0} |G_{l,x}^{m,x}(t)|^2 \right), \]
\[ \langle \sigma_x^m(t) \rangle = 0. \] (8)

The expectation value of \( \sigma_x^m \) is zero in this case also, as the state is an even magnon state, the operation of the spin operator will result in an odd magnon state.
Now, let us turn our attention to the disturbance of the unitary background dynamics, which we have considered until now, by an instantaneous local QDP. Since, the QDP can lead to decoherence, we will use the density matrix notation for representing the state, which is a natural approach for describing both pure and mixed quantum states. We consider here the time evolution of a Heisenberg spin chain starting with an initial state, represented by the density operator \( \rho(0) = \ket{\Psi(0)}\bra{\Psi(0)} \) at time \( t = 0 \). The background Hamiltonian dynamics is interrupted due to QDP occurring at time \( t = t_0 \) during the evolution of the state from time \( t = 0 \) to time \( t \). Thus, the time evolution here occurs in three stages: the first stage is a unitary evolution from \( t = 0 \) to the epoch of QDP at time \( t = t_0 \), represented by a unitary operator \( U_{t_0,0} \). The time evolution is given above for various initial states, the state is given by \( \rho(t_0^-) = \ket{\Psi(t_0)}\bra{\Psi(t_0)} \). The second stage is the action of an instantaneous QDP at \( t = t_0 \), which will be represented by Kraus operators for an incoherent QDP, and a unitary operation for a coherent QDP. Just after the instantaneous action of the QDP, we can represent the state using the Kraus operators \( \{ E_i \} \), which represent the dynamical process. After the QDP occurs at \( t = t_0 \) at a site \( m \) the state can be written using the Kraus operators as,

\[
\tilde{\rho}(t_0^+) = E_0\rho(t_0)E_0^† + E_1\rho(t_0)E_1^†. \tag{9}
\]

We have used \( \tilde{\rho}(t_0) \) for a state evolved with a QDP intervention, to distinguish it from \( \rho(t_0) \) for a state with no QDP occurrence. We consider here single-qubit operations, i.e. the QDP occurs at given time \( t = t_0 \) on a spin at the location \( m \). Some examples of single-qubit operations are a phase flip gate quantum channel action, a bit flip gate channel action, a projective measurement in the eigen basis of \( \sigma^z \). The Kraus operators for a QDP are given by, for a phase flip gate we have \( E_0 = \sqrt{p}\mathbf{1}, E_1 = \sqrt{1-p}\sigma^z \). The state just after the QDP operation at the site \( m \) is an incoherent superposition of two pure states, we have

\[
\tilde{\rho}(t_0^+) = p\ket{\Psi(t_0)}\bra{\Psi(t_0)} + (1-p)\sigma^z_m\ket{\Psi(t_0)}\bra{\Psi(t_0)}\sigma^z_m. \tag{10}
\]

Here, the two pure states occurring above have the same character as the state before the quantum process occurring. For instance, a one-magnon state will still remain a one-magnon state as \( \sigma^z_m \) operation preserves that character. However, for a bit-flip gate QDP the character is not preserved. The Kraus operators for a bit-flip gate QDP are given by \( S = E_0 = \sqrt{p}\mathbf{1}, E_1 = \sqrt{1-p}\sigma^z \). Now, \( \sigma^z_m \) operating on a one-magnon state leads to a zero-magnon and a two-magnon state, due to flipping a spin. For a projective measurement along \( z \)-axis, the Kraus operators are given by \( E_0 = (1 + \sigma^z)/2, E_1 = (1 - \sigma^z)/2 \). In all these case, the occurrence of a non-unitary QDP results in a state \( \rho(t_0^-) \) that is an incoherent superposition of two pure states as shown above, as there are two Kraus operators in each case. For a unitary or coherent QDP intervening the background dynamics, the resultant state is a pure state. But the QDP can change the character of the state, a one-magnon pure state can become a coherent superposition of one-magnon and two-magnon states, which we will discuss below. The third stage of the evolution is the time evolution from \( t = t_0 \) to a later time \( t \) through the background Hamiltonian dynamics, we have

\[
\tilde{\rho}(t) = U(t,t_0)\tilde{\rho}(t_0^+)U^†(t,t_0). \tag{11}
\]

The evolution in this stage proceeds similar to the first stage, different components of \( \rho(t_0^+) \), shown in equation (10), evolving through the corresponding Green’s function propagators.

The Loschmidt echo is quantified by the overlap of the two states evolved under two different Hamiltonians. It measures the probability of the state to revive to the initial state by an imperfect time reversal operation, as the Hamiltonian in the reverse time direction is different from the forward time evolution of the initial state. The Loschmidt echo is defined as
\[ L(t) = \left| \langle \tilde{\Psi}(t)|\tilde{\Psi}(i) \rangle \right|^2, \]
where the two states \(|\tilde{\Psi}(i)\rangle\) and \(|\tilde{\Psi}(i)\rangle\) are obtained by two different Hamiltonian evolutions. It is sensitive to the difference of the Hamiltonians, and also the behaviour is different for integrable and non-integrable dynamics. In our context, after the local instantaneous local operation, the system will not be able to revive to the initial state if the time reversal process is performed using the background dynamics, which means the Loschmidt echo may not be equal to unity.

In this case, the evolution of both the states \(\tilde{\rho}(t_0)\) with QDP occurrence and \(\rho(t_0)\) without the QDP, evolve unitarily from \(t = t_0\) to time \(t\), which implies that the Loschmidt echo will not be a function of time \(t\), but a function of only \(t_0\), the time of QDP occurrence and in general the location index \(m\) where it occurs.

In this section, and in the next section, we will describe the Loschmidt echo dynamics of many qubit systems (both integrable and non-integrable) under local instantaneous operations.

The Loschmidt echo, for unitarily evolved states, is just the overlap of the two states evolved under two different Hamiltonians. In our case, the evolution with a QDP can lead to a mixed state, represented by \(\tilde{\rho}(t)\). The state evolved without a QDP remains a pure state, represented by \(\rho(t) = |\Psi(i)\rangle\langle \Psi(i)|\). Thus, the Loschmidt echo \(L(m,t_0)\) can be defined as,

\[
L(m,t_0) = \text{Tr}(\tilde{\rho}(t_0)\rho(t)) = \langle \tilde{\Psi}(t)|\tilde{\rho}(t)|\tilde{\Psi}(i) \rangle = \langle \Psi(t_0)|\tilde{\rho}(t_0)|\Psi(t_0) \rangle, \tag{12}
\]
where \(m\) keeps track of the location of QDP, and \(t_0\) the time of QDP occurrence. In the final step, we have used the fact that the evolution is same for \(\rho(t)\) and \(\tilde{\rho}(t)\), from the time \(t_0\) to the time \(t\). Thus, the Loschmidt echo in general depends on the initial state, the Kraus operators of the QDP and the parameters, the time of the QDP, and the location of the QDP occurrence.

Now, the expression for the Loschmidt echo can be rewritten in terms of the Kraus operators, as

\[
L(m,t_0) = \sum_i \left| \langle \Psi(0)|U_{t_0,i}^{\dagger}E_iU_{t_0,0}|\Psi(0) \rangle \right|^2 = \sum_i \left| \langle E_i(t_0) \rangle \right|^2. \tag{13}
\]

For a phase-flip gate QDP acting on the spin at site \(m\) at time \(t_0\), the Loschmidt echo can be calculated as

\[
L(m,t_0) = p + (1-p)|\langle \sigma_{m}^z(t_0) \rangle|^2, \tag{14}
\]
and for the bit-flip QDP (here the Kraus operator involves \(\sigma_{m}^x\)),

\[
L(m,t_0) = p + (1-p)|\langle \sigma_{m}^x(t_0) \rangle|^2. \tag{15}
\]

The Loschmidt echo for a projective measurement along either \(z\) axis or \(x\)-axis can be obtained from a phase-flip QDP or a bit-flip QDP correspondingly by taking \(p = 1/2\). The dependence of the initial state is implicit through the expectation value of the corresponding spin operator at site \(m\) where the QDP occurs. We can use the expectation values we have calculated for various types of initial states (given in equation (5) for the unentangled initial state, in equation (6) for the entangled one-magnon state, and in equation (8) for the two-magnon initial state), and get a corresponding expression for the Loschmidt echo.

For the unentangled initial state, \(|\Psi(0)\rangle = \alpha|F\rangle + \beta|1\rangle\), using equation (5), we get an expression for the Loschmidt echo for the phase-flip QDP as,

\[
L(m,t_0) = p + (1-p)(1-2|\beta|^2|G_{1}^{\sigma_{m}^z}(t_0)|^2)^2, \tag{16}
\]
and correspondingly for a bit-flip gate QDP, we have

\[
L(m,t_0) = p + 4|\alpha|^2|\beta|^2(1-p)(\text{Re}(e^{-i\alpha \theta}G_{1}^{\sigma_{m}^x}(t_0)))^2. \tag{17}
\]
Now, for the entangled initial state $|\Psi(0)\rangle = \alpha|1\rangle + \beta|r\rangle$, using equation (6) for the expectation values for the spin operators for later times, are given by, thus, the expression for the Loschmidt echo for the phase-flip QDP and the bit-flip QDP are given correspondingly as

$$L(m, t_0) = p + (1 - p)(1 - 2|K_m(t_0)|^2)^2, \quad L(m, t_0) = p. \tag{18}$$

There is no time dependence in the second case above; as for the bit-flip QDP the Kraus operator involving $\sigma^x_m$ gives zero-magnon and two-magnon components that are orthogonal to the one-magnon state, thus the expectation value is zero as given in equation (8). Similarly, for an initial state with a two-magnon component, $|\Psi(0)\rangle = \beta|1\rangle + \beta|1, r\rangle$, using equation (8), we get an expression for the Loschmidt echo. For the phase-flip and bit-flip QDPs correspondingly, we have

$$L(m, t_0) = p + (1 - p)(1 - 2|\beta|^2(\sum_{x < m} |G^{\sigma_m}_{1, x}(t_0)|^2 + \sum_{x > m} |G^{\sigma_m}_{1, x}(t_0)|^2))^2, \tag{19}$$

$$L(m, t_0) = p.$$

For a projective measurement along the $z$-axis at site $m$, the expression for the Loschmidt echo is similar to that of the phase-flip gate QDP, as the Kraus operators involve $\sigma^z_m$. Similarly, for a projective measurement along the $x$-basis, the expression for the Loschmidt echo is similar to that of the bit-flip QDP.

The Green function appearing in the above equations for the Loschmidt echo falls off inversely with the QDP occurrence time $t_0$, since the Green’s function is a Bessel function with time $t_0$ as the argument, it is easy to see that for large values of $t_0$ the $\sigma^z$ expectation value for both entangled and unentangled initial states approaches unity. This implies that for a phase-flip QDP or a projective measurement QDP in $\sigma^z$ basis, all the states are fully reversible for large value of $t_0$. Now, the $\sigma^z$ expectation value is zero for the entangled state $\alpha|1\rangle + \beta|r\rangle$ for any value of $t_0$, as the operation changes the parity of the state. Similarly, it is zero for the two-magnon state also. For the unentangled state $\alpha|F\rangle + \beta|1\rangle$, the expectation value goes to zero for a large value of $t_0$. This would imply that, for the bit-flip gate QDP, and a projective measurement QDP in $\sigma^x$ basis, the Loschmidt echo tends to $p$ and $1/2$ respectively for the three states for large value of $t_0$; the states are not time reversible here. Figure 2(a) shows the Loschmidt echo for three different initial states as a function of $t_0$. As $t_0$ increases, $L(m, t_0)$ tends to unity. As shown in figure 1, the encoded initial state propagates through the chain with a finite speed, the encoded state from the first site will reach at the site $m$ after a time $m/2$. The minimum value of the Loschmidt echo at $t_0 = m/2$ indicates the location of the state at site $m$.

Now, we will consider the case where the quantum system undergoes multiple sequential phase-flip QDPs during the background unitary evolution. Let QDPs occur at site $m_1$ at time $t_0$, at site $m_2$ at time $2t_0$ and so on. After two such QDPs at times $t_0$ and $2t_0$, the expression for $L(m_1, m_2; 2t_0)$ can be given by extending equation (14) as,

$$L(m_1, m_2; 2t_0) = p^2 + p(1 - p)|\langle \sigma^z_{m_1}(t_0) \rangle|^2 + p(1 - p)|\langle \sigma^z_{m_2}(2t_0) \rangle|^2 + (1 - p)^2|\langle \sigma^z_{m_1}(2t_0)\sigma^z_{m_2}(t_0) \rangle|^2. \tag{20}$$

Similarly, the expression for the Loschmidt echo after $n$ sequential QDPs at sites $(m_1, m_2, ..., m_n)$ at a time interval of $t_0$ is given by,

$$L(m_1, m_2, ..., m_n; nt_0) = p^n + p^{n-1}(1 - p)\sum_j |\langle \sigma^z_{m_1}(jt_0) \rangle|^2 + p^{n-2}(1 - p)^2\sum_{jk} |\langle \sigma^z_{m_2}(jt_0)\sigma^z_{m_1}(jt_0) \rangle|^2$$

$$\cdots + (1 - p)^n|\langle \sigma^z_{m_1}(nt_0)\cdots\sigma^z_{m_1}(nt_0) \rangle|^2. \tag{21}$$
By setting \( p = \frac{1}{2} \), we get the expression for the Loschmidt echo for \( n \) sequential projective measurements. The expectation values of operator product of multiple \( \sigma^z \) operators above sum can be written in generally as
\[
\langle \sigma^z_{m_1}(T_1) \sigma^z_{m_2}(T_2) \cdots \sigma^z_{m_n}(T_n) \rangle = 1 + |\beta|^2 \left[ -2 \sum_{j=1}^{n} (G_{m_j}^j(T_j))^2 \right.
\]
\[+ 4 \sum_{k=2}^{n} \sum_{j<k}^{n} G_{m_k}^k(T_j) G_{m_k}^{m_j}(T_k - T_j) G_{m_j}^{m_k}(T_k) + \ldots \]
\[+ (-2)^n G_{m_1}^{m_1}(T_1) \prod_{j=2}^{n} G_{m_{j-1}}^{m_j}(T_j - T_{j-1}) G_{m_j}^{m_{j-1}}(T_n) \].

The above expression is obtained by using the identity for the addition of the Green functions, given by
\[
\sum_{\nu'} G_{\nu_1}^{\nu_1}(t_1 + t_2) G_{\nu_2}^{\nu_2}(t_1) = G_{\nu_2}^{\nu_2}(t_2).
\]
Now, for the entangled initial state $\alpha|1\rangle + \beta|r\rangle$ the same expression is valid with the quantity $G^n_{\alpha r}$ replaced by $K_m$. Similar to the above expression, the different terms that arise are: a sum of product of two Green functions, a sum of product of three Green functions, and so on. The contribution becomes smaller as the number of Green function increases, as each of them is less than unity. So the right-hand side in equation (22) tends to unity for large value of $t_0$. That means that even if the system is interrupted multiple times at large time intervals $t_0$, the Loschmidt echo takes a large value. For example, figure 2(b) shows the expectation value $(\sigma^z_{m_3=3}(3t_0)\sigma^z_{m_2=2}(2t_0)\sigma^z_{m_1=1}(t_0))$, calculated for the initial state $\alpha|F\rangle + \beta|1\rangle$ using the second-order, the third-order and the fourth-order terms of the Green functions. The contribution becomes much smaller as the number of Green functions increases. The Loschmidt echo $L(\{m_i\};n)$ after the occurrence of multiple projective measurements, along $\sigma^z$ basis, is shown as a function of the number of QDPs $n$ in figure 2(c) for some representative values of $t_0$. The locations of QDPs $m_i$ are some some random number between 1 and 10. Thus, the Loschmidt echo is a function of the locations of QDPs, and it can be seen that it takes larger values for large $t_0$ in general. This means that the system gets enough time to revive between two QDPs.

Now, let us consider QDPs involving $\sigma^x$ or $\sigma^y$ operators. Similar to the above case of QDPs with $\sigma^z$ operators, it can be argued that the $\sigma^x$ or $\sigma^y$ expectation values can be written as a sum of individual Green functions, which tend to zero for large value of $t_0$. Because the operator $\sigma^x$ or $\sigma^y$ changes the number of magnon in the state, in the case of conserving dynamics this does not contribute to the expectation value. Let us consider the initial state $\alpha|F\rangle + \beta|1\rangle$. The QDP operator $\sigma^z$ acts on this state at time $t$, generating a state with a linear combination of a zero-magnon state, a one-magnon state and a two-magnon state. The zero-magnon state is generated from one-magnon component of the initial state, carrying a one-magnon Green’s function. Thus, its overlap with the original state at time $t_0$ tends to zero in the large $t_0$ limit. This argument can be extended to multiple operations also. Since only zero-magnon and one-magnon sector states contribute to the Loschmidt echo, these states can come from, at least, one-magnon and two-magnon states through the $\sigma^z$ operation. Thus, the expression for the Loschmidt echo contains the terms of the form $\sum G^{z*}(n_{t_0})G^{x'*y'*z'*}(n_{t_0})$. These terms will eventually go to zero in thermodynamic limit and for $t_0 \rightarrow \infty$, as the $n$-magnon Green function varies with system size as $1/N^{n/2}$. Thus, the Loschmidt echo saturates to $p^n$, which is the minimum value for large $t_0$ as shown in equation (21), by replacing the expectation value of $\sigma^z$ by that of $\sigma^x$.

The calculation of the Loschmidt echo is more challenging for initial states with a large number of magnons as it requires calculating Green functions for multiple magnon states. As magnon increases in the dynamics each magnon sector contributes to the Loschmidt echo and hence it decreases for phase flip gate operations. Figures 2(d) and (e) show numerical calculations of the Loschmidt echo $L(m,t_0)$ as a function of $t_0$ for a selective set of multi-magnon initial states with QDPs using projective measurement operators of $\sigma^x$ and $\sigma^y$. We can see from figure 2(d) that the Loschmidt echo decreases substantially for states with more number magnons. For a state with a three-magnon component, it quickly reaches 0.5 for a QDP using $\sigma^z$ operators. For a QDP using $\sigma^y$ operators, the Loschmidt echo reaches the value 0.5, for all the initial states considered, as expected from our analytical calculations for simple initial states. We now discuss the computation of the Loschmidt echo for a coherent QDP, which is easier than the incoherent QDP discussed above. Since the QDP occurs at $t_0$ during the evolution the state $|\tilde{\Psi}(t)\rangle$ with QDP interruption should match with the state $|\Psi(t)\rangle$ without QDP interruption except at time $t_0$. $L(m,t)$ should depend on $t_0$ like the previous case. Denoting the instantaneous coherent operation at the site $m$ by a unitary operator $V_m$ the quantity $L(m,t_0)$ in terms of the state at $t_0$ and the instantaneous local gate operator can be written as,
In the last step, we have written it as the overlap of the state $|\tilde{\Psi}(t_0^+)\rangle = V_m U_{t_0,0} |\Psi(0)\rangle$, and the state $|\Psi(t_0)\rangle = U_{t_0,0} |\Psi(0)\rangle$ obtained from the time-reversed unitary process from $t_0^+$ to 0. The dependence of $L(m,t_0)$ on the $m$ and $t_0$ is more transparent, in the above, when it is written as the squared expectation value of $V_m$ in the state $|\Psi(t_0)\rangle$; The operation of the unitary operator $V_m$ on the basis states of $m$ th spin is given by

$V_m[0] = \gamma |0\rangle + \delta |1\rangle$, $V_m[1] = -\delta^* |0\rangle + \gamma^* |1\rangle$. (25)

Here also we take three states, namely one magnon unentangled state $\alpha|F\rangle + \beta|1\rangle$, a one-magnon entangled state $\alpha|1\rangle + \beta|r\rangle$ and a two-magnon entangled state $\alpha|F\rangle + \beta|1, r\rangle$. The state $\alpha|F\rangle + \beta|1\rangle$ after interrupted by the coherent QDP at time $t_0$ is given by,

$V_m|\Psi(t_0)\rangle = \gamma|\Psi(t_0)\rangle - 2i\beta\gamma|G^m_{1r}(t_0)|m\rangle + \alpha \delta e^{-i\epsilon_0} |m\rangle + \beta \delta \sum_i G^m_i(t_0)|m, x\rangle - \beta \delta^* G^m_{1r}(t_0)|F\rangle$. (26)

Similarly, for the entangled initial state $\alpha|1\rangle + \beta|r\rangle$ is given by

$V_m|\Psi(t_0)\rangle = \gamma|\Psi(t_0)\rangle + i\gamma\left(|\Psi(t_0)\rangle - 2K_m(t_0)|m\rangle\right)$

$+ \delta \sum_x K_m(t_0)|m, x\rangle - \delta^* K_m(t_0)|F\rangle$. (27)

Similarly, for the two-magnon entangled initial state $\alpha|F\rangle + \beta|1, r\rangle$ is given by

$V_m|\Psi(t_0)\rangle = \gamma|\Psi(t_0)\rangle - 2i\beta\gamma(\sum_{x<m} G^m_{1r}(t_0)|x, m\rangle + \sum_{x>m} G^m_{1r}(t_0)|m, x\rangle)$

$+ (\text{one magnon sector } + \text{three magnon sector})$. (28)

It can be seen from the above that when the operator $V_m$ acts on a state it gives a part $\gamma$ of the original state, this results in the following forms of the Loschmidt echo. The Loschmidt echo can calculated from the quantity $\langle V_m(t_0) \rangle$ for the state $|\Psi(t_0)\rangle$ as

$L(m, t_0) = |\langle V_m(t_0) \rangle|^2 = |\gamma - 2i\gamma|\beta G^m_{1r}(t_0)|^2 + 2i\text{Im}(\alpha \beta^* \delta e^{-i\epsilon_0} G^m_{1r}(t_0))|^2$. (29)

The corresponding expression for the initial state $\alpha|1\rangle + \beta|r\rangle$ is given by

$L(m, t_0) = |\langle V_m(t_0) \rangle|^2 = |\gamma - 2i\gamma|K_m(t_0)|^2|^2$. (30)

Now, for an initial state with a two-magnon component, $\alpha|F\rangle + \beta|1, r\rangle$, we can write the Loschmidt echo as, for a coherent QDP,

$L(m, t_0) = |\gamma - 2i\gamma|\beta|^2 (\sum_{x<m} |G^m_{1r}(t_0)|^2 + \sum_{x>m} |G^m_{1r}(t_0)|^2)^2$, (31)

using two-magnon Green’s functions, analogous to equation (19).

It can be seen from the above that the probability of reversal is less than unity for large time. This is because, in this case of coherent QDP, the state obtained after the operation is a combination of zero-, one- and two-magnon states and each sector has its own conserved dynamics. Since the initial state is a combination of zero- and one-magnon states, the two-magnon sector does not contribute to the Loschmidt echo. Thus, the probability of reviving the system to the initial state is reduced.
For a large value of $t_0$, since the contribution from the Green’s function term drops off, the value of $L(m; t_0)$ saturates to $|\gamma|^2$, which is less than unity. For different coherent operations, this value is different depending on the parameters. For the X gate and the Y gate QDPs, the value is zero; for the Hadamard gate QDP, it takes the value 1/2, and for the Z gate QDP the value is 1 which is maximum. The Loschmidt echo $L(m, t_0)$ as a function of $t_0$ is shown in figure 2(f) for the three initial states. The value of $\gamma$ and $\delta$ are set as $(1+i)/\sqrt{3}$ and $\frac{1}{\sqrt{3}}$ respectively.

$L(t_0)$ fluctuates around $|\gamma|^2$ which is 2/3 in this case, for large time $t_0$. In the case of multiple coherent QDP interruptions, each time the operator $V_m$ operates on the state, the resultant amplitude for the state will get a coefficient factor $\gamma$. Thus, we can argue that after $n$ operations, the Loschmidt echo saturates to a value $|\gamma|^{2n}$ for large $t_0$ limit.

3. Kicked Harper model

We now turn our attention to the non integrable dynamics of the background Hamiltonian evolution. There are sharp differences in the eigenvalue spacing distribution and the structure of the eigenfunctions of the integrable and non-integrable dynamics that have been widely investigated [18, 20, 21]. The traditional model for studying the effects of non-integrable dynamics is the Ising chain with a kicked magnetic field in both transverse and tangential directions. This model, though simple, is not exactly solvable, making it impossible for tracking the Loschmidt echo analytically. Instead, we turn to a Harper model with a kicked magnetic field, which has been studied in the context of the dynamics of pair entanglement [20]. The interference of the QDP signal and the state transfer has been studied in detail [17], exhibiting distinct features in the non-integrable regime. Following that approach in the present context of the Loschmidt echo, using this model we can study the effects of QDP in non-integrable systems, in a simple model Hamiltonian with a tuneable parameter, to go continuously from completely integrable to completely non-integrable regimes. We use a one-dimensional periodically kicked Harper model, a simple model of fermions hopping on a chain with an inhomogeneous site potential, appearing as a kick at regular intervals. The spin operator version of the Hamiltonian is given by

$$H(t) = \sum_{j=1}^{N} \left[ -\frac{1}{2}(\sigma^x_j \sigma^x_{j+1} + \sigma^y_j \sigma^y_{j+1}) + g \sum_{n=-\infty}^{\infty} \delta(t - n) \cos\left(\frac{2\pi j}{N}\right)\sigma^z_j \right].$$

The first term is the XY term of the Heisenberg model considered above, which causes hopping of up or down spins. The last term is an inhomogeneous magnetic field in the $z$ direction that comes into play through kicks at an interval of $\tau$. The coupling strength $g$ and the kicking time $\tau$ can be independently varied, which can affect the nature of the dynamics as we will see below.

The classical version of the kicked Harper Hamiltonian is regular for $\tau \to 0$ and completely chaotic for a large value of $\tau$, and similarly the eigenvalue and eigenfunctions of the quantum version display correspondingly regular or chaotic characteristics [21]. The Harper model dynamics conserves the magnon number through the evolution. So the dynamics can be thought of as site-dependent kicks interrupting the background XY dynamics at a regular interval. Through the time evolution, the down spins can hop around to other sites. We will consider evolution at discrete times, namely $t = \tau^+, 2\tau^+$ etc, which is at instants just after a kick. The unitary operator for the evolution between two kicks is straightforwardly given by

$$U(g, \tau) = e^{-i\tau \sum_j \frac{1}{2}(\sigma^z_j \sigma^z_{j+1} + \sigma^z_j \sigma^z_{j+1})} e^{-i\tau g \sum_j \cos\left(\frac{2\pi j}{N}\right)\sigma^z_j}.$$  

$$e^{-i\tau \sum_j \frac{1}{2}(\sigma^z_j \sigma^z_{j+1} + \sigma^z_j \sigma^z_{j+1})} e^{-i\tau g \sum_j \cos\left(\frac{2\pi j}{N}\right)\sigma^z_j},$$
where the two operator factors appearing above do not commute. The time evolved state at time \( n \tau \) just after \( n \) kicks is \(| \Psi(t) \rangle = U^n(g, \tau) | \Psi(0) \rangle \). The system evolves between the two kicks, from \( t = n \tau \) to \( t = (n + 1) \tau \), through the XY dynamics which introduces a location-dependent phase factor to the Green function. This is different from the case we have discussed earlier where the background dynamics was interrupted by a coherent QDP at one site; where each kick is a coherent operation that occurs at all sites.

For the initial state with a one-magnon component, \(| \Psi(0) \rangle = \alpha | F \rangle + \beta | 1 \rangle \), the time-evolved state will be given by

\[
| \tilde{\Psi}(t = n \tau) \rangle = \alpha e^{-i \epsilon_0 t} | F \rangle + \beta \sum_x \tilde{G}^n_x(t = n \tau) | x \rangle.
\] (34)

Here we have introduced a different composite Green function, related to the Green’s function studied in the last section, given by

\[
\tilde{G}^n_x(t = n \tau) = \sum_{x_1, x_2, \ldots, x_n} \prod_{j=1}^{n-1} G^{n}_{x_j}(\tau) e^{2i g \cos(\frac{2\pi \eta \tau}{n} + 1)}.
\] (35)

It can be seen that after each kick, a site-dependent new phase is introduced in the Green function, which indicates the qualitative change in the dynamics from the previous section. By setting \( g \tau = 0 \) in the above, the Green function \( \tilde{G}^n_x(t) \) is reduced to the Green function \( G^n_x(t) \), the one-magnon propagator function of the Heisenberg model, as a result of the identity in equation (21). The real part of the Green function \( \tilde{G}^n_x(t = n \tau) \) is plotted for some representative values of \( \tau \) and \( g \) as a function of site index \( l \) and \( t \) in figures 3(a)–(f). For comparison, the real part of \( G^1_x(t) \), which determines the Heisenberg dynamics, is plotted in figure 1(a). The qualitative nature depends on the value \( g \tau \). \( \tilde{G}^n_x(t = n \tau) \) resembles the \( G^1_x(t) \) for small value of \( g \tau \).
this can be seen from figures 3(a) and (d) with the values 0.01 and 0.09 respectively, whereas, for larger values the light cone structure changes and becomes non-linear for larger time; this can be seen from figures 3(b) and (e). For \( g \tau \) greater than 0.5 the Green function becomes localised in space and time which can be seen from figures 3(c) and (f). In this regime, the dynamics are quite different from the Heisenberg dynamics.

Since the Loschmidt echo is also sensitive to the dynamics being integrable or non-integrable we investigate the variation of the Loschmidt echo with time we take a one magnon initial state and evolve the system unitarily with a Hamiltonian and reverse the system unitarily with a different Hamiltonian. In this context, we want to compute the variation of the Loschmidt echo as a function of number of kicks, kicking interval \( \tau \) and potential strength parameter \( g \), where in one side evolution there is no kicks, i.e. for simple XY dynamics on the other side there is Harper dynamics. The Loschmidt echo in this case is given by

\[
L(t) = |\langle \Psi(t) | \tilde{\Psi}(n\tau) \rangle|^2 = |\alpha|^2 + |\beta|^2 \sum_x \tilde{G}_x(t) G_x(t) \]

In figure 4, we show the time dependence of the Loschmidt echo \( L(t) \) for different values of \( g \) and \( \tau \) for both small and large systems. \( L(t) \) saturates to the value 1/3 for large time. This shows
that the products of integrable and non-integrable Green functions, shown in equation (36), go to zero for large value of \( t \). In case of large systems there is a local revival of the value of \( L(t) \) for small values of \( g\tau \) and then decays slowly, which can be seen from figure 4(a). This can be thought of as a signature for non chaotic behaviour. For larger values of \( g\tau \) the value of \( L(t) \) decays monotonically and fast to the value 1/3 shown in figure 4(b). It should be pointed out here that the Loschmidt echo fluctuates around 1/3 as shown in figure 3(b), after averaging over all possible input states from equation (28), and it would fluctuate around 1/4 for using a particular set of parameters as shown in figures 4(c) and (d). The Loschmidt echo is shown for small systems in figures 4(c) and (d), for different values of \( g \). It decays very quickly and then fluctuates around 1/4, as we are not doing any averaging over all possible input states. The fluctuations are larger for small values of \( g\tau \). For smaller systems, the value will continue to fluctuate because of repetitive reflections from the boundaries, which is explained before.

Now, let us turn to the case of a QDP intervening the background dynamics of the Harper model given above. If an incoherent QDP (e.g. local projective measurement along \( \sigma^z \) or \( \sigma^x \))
equation (7) can still be used, with \( \sigma \) replaced by \( \sigma^2 \) for the case of QDP along the \( \sigma^2 \) basis and \( \sigma^2 \) respectively; this can be seen from figures 5(a) and (b) respectively. This is similar to the Heisenberg model except for smaller size systems, the behaviour is similar but half for large values of \( \sigma \), but it often touches unity. But, for larger values of \( \sigma \), the fluctuations are less, and the Loschmidt echo is always less than unity. In figure 5(c) the value of \( L(t) \) shows more fluctuations for small values of \( \sigma \) but mostly bounded between 0.5 and 0.7.

A contrast between the XY dynamics and the Harper model can be drawn from the value of the Loschmidt echo when the QDP occurs at a farther site. The equations (5) and (6) are valid for Harper dynamics with the composite Green function \( \tilde{G}(t) \). In figure 3, it can be seen that the composite Green function \( \tilde{G}(t) \) becomes non-zero in a very small region in the \( l - t \) diagram as the value of \( \sigma \) increases, or in other words, if we start with a one-magnon state with the down spin at the first site, the further sites are not connected via the Green function for any later time. In that case the values of the Loschmidt echo become exactly unity, zero and \( |\gamma|^2 \) respectively from equations (16), (17) and (29).

We now turn to the case where the system evolves unitarity forward in time with one value of kicking interval \( \tau_1 \), and in reverse direction evolves unitarily with different value \( \tau_2 \). Here, we can expect different behaviour depending on whether one or both the evolutions are in a non-integrable regime. The Loschmidt echo as a function of time in this case in given by

\[
L(t = n_1\tau_1 + n_2\tau_2) = \langle \tilde{\psi}(n_1\tau_1)\tilde{\psi}(n_2\tau_2) \rangle^2 = ||\alpha||^2 + ||\beta||^2 \sum_x \tilde{G}_1^x(n_1\tau_1)\tilde{G}_1^x(n_2\tau_2). \tag{37}
\]

We have plotted the Loschmidt echo \( L(t) \) as a function of time for two different values of \( \tau_1 \) and \( \tau_2 \) corresponding to forward and backward evolutions. In smaller size systems, since we evolve the system numerically, \( L(t) \) is plotted as a continuous function of time \( t \) in figures 6(a) and (b). It is seen that the peaks are obtained at time \( t = n_1\tau_1 + n_2\tau_2 \) where \( n_1 \) and \( n_2 \) are
integer. For a larger-size system, we have calculated the values of $L(t)$ only at the values of $t = n_1 \tau_1 = n_2 \tau_2$ and plotted in figure 6(c) where $n_1$ and $n_2$ are integer, which corresponds to the peaks in a small system. However the qualitative nature is the same for both cases. For smaller values of $\tau_1$ and $\tau_2$, e.g. $\tau_2 = 0.1, 0.3$ and $\tau_1 = 0.2, 0.4$, the peak value is almost unity and remains constant for large $t$. But for larger values of $\tau_1$ and $\tau_2$ e.g. $\tau_1 = 0.1$ and $\tau_2 = 0.9$ the peak value is slightly less around 0.95 and slowly vary with time. For $\tau_1 = 0.1$ and $\tau_2 = 0.9$, the value of $L(t)$ is much less than unity and slowly varies around 0.7 for large time.

4. Summary and conclusions

In this paper, we have investigated the reversibility of a many body quantum state when local quantum dynamical processes interrupt the background dynamics for both integrable and non-integrable dynamics. For the Heisenberg model we study a set of simple initial states, for which the dynamics is interrupted by a local instantaneous operation at a certain epoch of time during evolution. The quantity Loschmidt echo captures the effect of local operation interrupting the background dynamics. The Loschmidt echo depends on the time of operation and the Kraus operators corresponding to the operation. It has been shown that for a phase-flip operation, and a projective measurement in $\sigma_z$ basis, the Loschmidt echo tends to unity for long time limit $t_0 \to \infty$, for initial states with only one-magnon and two-magnon components. This means that the system loses the memory of the occurrence of QDP, when the QDP occurs after a long time during the evolution. For example, for a one-magnon initial state, in the time-evolved quantum state after a long time, the down spin is spread throughout the system. The QDP operation does not change the system considerably, thus the Loschmidt echo is close to unity. In contrast, for a bit-flip operation or a projective measurement in the $\sigma^x$ basis, the Loschmidt echo tends to $p$ or $1/2$ respectively for large time limit $t_0 \to \infty$, as the operation changes the number of magnons in the quantum state.

We have also considered multiple sequential QDPs interrupting the background dynamics at different sites at regular intervals of time $t_0$. The expression for the Loschmidt echo in cases can be written a series of sums, where each of the terms can be expanded in terms of the product of one-particle Green functions, but the leading terms only contribute. There we have shown for the Kraus operators functions of $\sigma^z$ operator the quantum state does not revive to its initial state for finite value of $t_0$ and the value of Loschmidt echo increases with $t_0$ but decreases with the number of operations $n$. For the operations where the Kraus operators functions of $\sigma^x$ or $\sigma^y$ number of magnon either increases or decreases after each operation. After some number of operations, the probability of revival to the initial state will become small and in the limit $t_0 \to 0$.

We have discussed the effects of local coherent operations on the dynamics. Since such operations also change the number of magnons in the state, the probability of revival is always less than unity. In case of multiple numbers of coherent operations a fraction of the initial state is possible to revive. Even for large time interval $t_0$ of operations the Loschmidt echo tends to a value that depends on the QDP operation parameters. The asymptotic value, for a large $t$, deceases with the number of operations exponentially.

The effect of non-integrable background dynamics has been studied by considering a simple non-integrable model, where an inhomogeneous kicked site-dependent potential is added to the integrable XY model. Since the potential is periodic and kicked at regular intervals, this can be thought of as a QDP interrupting the XY dynamics periodically. Each such operation introduces a site-dependent phase factor in the Green’s function. It is seen that after the action of a few kicks, depending on the potential strength $g$ and kicking interval $\tau$, the qualitative
nature of the Green’s function changes. Above a certain value of $g\tau$ the Green’s function starts to get localised in space and time. This regime corresponds to the chaotic behaviour in the classical Harper maps.

In this context of multiple number of coherent QDPs interrupting the background dynamics, we have considered the reversibility of a one-magnon initial state where forward evolution is governed by the kicked Harper Hamiltonian, and the reverse evolution is governed by integrable XY Hamiltonian. In such cases, the Loschmidt echo shows two different behaviours depending on the value of $g\tau$. For smaller values of $g\tau$, for which the corresponding classical dynamics is non chaotic, the Loschmidt echo decays slowly from unity and shows a temporary revival for a certain time interval for large systems. However, for small-size systems, though it decays quickly, the Loschmidt echo shows large fluctuations as a function of time. For larger values of $g\tau$, the Loschmidt echo always quickly decays with time.

We have considered the case where a local incoherent QDP interrupts the background Harper dynamics. In this case, the Loschmidt echo, though it shows a behaviour similar to that of the integrable models, decays very quickly. This is due to the fact that the composite Green’s function that arises in Harper dynamics does not show oscillatory behaviour, whereas the Green’s function for the Heisenberg dynamics oscillates with time, as it is basically a Bessel function with time as the argument. We also considered the case where both the forward and the backward evolutions of the dynamics are governed by the kicked Harper Hamiltonian, but with two different values of kicking periods $\tau_1$ and $\tau_2$ respectively for the forward and backward time evolutions. In this case we see that the Loschmidt echo decays very slowly and oscillates around a certain value depending on the values of $\tau_1$ and $\tau_2$ instead of going to zero. This value is almost unity for small values of $\tau_1$ and $\tau_2$ small, corresponding to the integrable behaviour.

Acknowledgment

SS acknowledges the financial support from CSIR, India.

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