Abstract—We consider the compound capacity of polar codes under successive cancellation decoding for a collection of binary-input memoryless output-symmetric channels. By deriving a sequence of upper and lower bounds, we show that in general the compound capacity under successive decoding is strictly smaller than the unrestricted compound capacity.

I. HISTORY AND MOTIVATION

Polar codes, recently introduced by Arikan [1], are a family of codes that achieve the capacity of a large class of channels using low-complexity encoding and decoding algorithms. The complexity of these algorithms scales as $O(N \log N)$, where $N$ is the blocklength of the code. Recently, it has been shown that, in addition to being capacity-achieving for channel coding, polar-like codes are also optimal for lossy source coding as well as multi-terminal problems like the Wyner-Ziv and the Gelfand-Pinsker problem [2].

Polar codes are closely related to Reed-Muller (RM) codes. The rows of the generator matrix of a polar code of length $N = 2^n$ are chosen from the rows of the matrix $G^\otimes n = [1 0] \otimes n$, where $\otimes$ denotes the Kronecker product. The crucial difference of polar codes to RM codes is in the choice of the rows. For RM codes the rows of largest weight are chosen, whereas for polar codes the choice is dependent on the channel. We refer the reader to [1] for a detailed discussion on the construction of polar codes. The decoding is done using a successive cancellation (SC) decoder. This algorithm decodes the bits one-by-one in a pre-chosen order.

Consider a communication scenario where the transmitter and the receiver do not know the channel. The only knowledge they have is the set of channels to which the channel belongs. This is known as the compound channel scenario. Let $\mathcal{W}$ denote the set of channels. The compound capacity of $\mathcal{W}$ is defined as the rate at which we can reliably transmit irrespective of the particular channel (out of $\mathcal{W}$) that is chosen. The compound capacity is given by [3]

$$C(\mathcal{W}) = \max_P \inf_{W \in \mathcal{W}} I_P(W),$$

where $I_P(W)$ denotes the mutual information between the input and the output of $W$, with the input distribution being $P$.

Note that the compound capacity of $\mathcal{W}$ can be strictly smaller than the infimum of the individual capacities. This happens if the capacity-achieving input distribution for the individual channels are different. On the other hand, if the capacity-achieving input distribution is the same for all channels in $\mathcal{W}$, then the compound capacity is equal to the infimum of the individual capacities. This is indeed the case since we restrict our attention to the class of binary-input memoryless output-symmetric (BMS) channels.

We are interested in the maximum achievable rate using polar codes and SC decoding. We refer to this as the compound capacity using polar codes and denote it as $C_{P,SC}(\mathcal{W})$. More precisely, given a collection $\mathcal{W}$ of BMS channels we are interested in constructing a polar code of rate $R$ which works well (under SC decoding) for every channel in this collection. This means, given a target block error probability, call it $P_B$, we ask whether there exists a polar code of rate $R$ such that its block error probability is at most $P_B$ for any channel in $\mathcal{W}$. In particular, how large can we make $R$ so that a construction exists for any $P_B > 0$?

We consider the compound capacity with respect to ignorance at the transmitter but we allow the decoder to have knowledge of the actual channel.

II. BASIC POLAR CODE CONSTRUCTIONS

Rather than describing the standard construction of polar codes, let us give here an alternative but entirely equivalent formulation. For the standard view we refer the reader to [1].

Binary polar codes have length $N = 2^n$, where $n$ is an integer. Under successive decoding, there is a BMS channel associated to each bit $U_i$ given the observation vector $Y_0^{n-1}$ as well as the values of the previous bits $U_0^{i-1}$. This channel has a fairly simple description in terms of the underlying BMS channel $W$

Definition 1 (Tree Channels of Height $n$): Consider the following $N = 2^n$ tree channels of height $n$. Let $\sigma_1 \ldots \sigma_n$ be the $n$-bit binary expansion of $i$. E.g., we have for $n = 3$, $0 = 000$, $1 = 001$, $\ldots$, $7 = 111$. Let $\sigma = \sigma_1 \sigma_2 \ldots \sigma_n$. Note that for our purpose it is slightly more convenient to denote the least (most) significant bit as $\sigma_n (\sigma_1)$. Each tree channel consists of $n + 1$ levels, namely $0, \ldots, n$. It is a complete binary tree. The root is at level $n$. At level $j$ we have $2^{n-j}$ nodes. For $1 \leq j \leq n$, if $\sigma_j = 0$ then all nodes on level $j$ are check nodes; if $\sigma_j = 1$ then all nodes on level $j$ are variable nodes. All nodes at level $0$ correspond to independent observations of the output of the channel $W$, assuming that the input is 0.

An example for $W_{011}$ (that is $n = 3$ and $\sigma = 011$) is shown in Figure 1.

Let us call $\sigma = \sigma_1 \ldots \sigma_n$ the type of the tree. We have $\sigma \in \{0, 1\}^n$. Let $W^\sigma$ be the channel associated to the tree of type $\sigma$. Then $I(W^\sigma)$ denotes the corresponding capacity. Further, by $Z(W^\sigma)$ we mean the corresponding Bhattacharyya functional (see [4, Chapter 4]).

1We note that in order to arrive at this description we crucially use the fact that $W$ is symmetric. This allows us to assume that $U_0^{n-1}$ is the all-zero vector.
Consider the channels $W^{(i)}_N$ introduced by Arıkan in [1]. The channel $W^{(i)}_N$ has input $U_i$ and output $(Y_i^{N-1}, U_0^{-1})$. Without proof we note that $W^{(i)}_N$ is equivalent to the channel $W^\sigma$ introduced above if we let $\sigma$ be the $n$-bit binary expansion of $i$.

Given the description of $W^\sigma$ in terms of a tree channel, it is clear that we can use density evolution [4] to compute the channel law of $W^\sigma$. Indeed, assuming that infinite-precision density evolution has unit cost, it was shown in [5] that the total cost of computing all channel laws is linear in $N$.

When using density evolution it is convenient to represent the channel in the log-likelihood domain. We refer the reader to [4] for a detailed description of density evolution. The BMS $W$ is represented as a probability distribution over $\mathbb{R} \cup \{\pm \infty\}$. The probability distribution is the distribution of the variable $\log(W(Y_{10}))$, where $Y \sim W(y|0)$.

Density evolution starts at the leaf nodes which are the observation nodes and proceeds up the tree. We have two types of convolutions, namely the variable convolution (denoted by $\oplus$) and the check convolution (denoted by $\otimes$). All the densities corresponding to nodes which are at the same level are identical. Each node in the $j$-th level is connected to two nodes in the $(j-1)$-th level. Hence the convolution (depending on $\sigma_j$) of two identical densities in the $(j-1)$-th level yields the density in the $j$-th level. If $\sigma_j = 0$, then we use a variable convolution ($\oplus$), and if $\sigma_j = 1$, then we use a check convolution ($\otimes$).

**Example 2 (Density Evolution):** Consider the channel shown in Figure 1. By some abuse of notation, let $W$ also denote the initial density corresponding to the channel $W$. Recall that $\sigma = 011$. Then the density corresponding to $W^{011}$ (the root node) is given by

$$
(W^{02})^{\otimes2} = (W^{02})^{\otimes4}.
$$

### III. Main Results

Consider two BMS channels $P$ and $Q$. We are interested in constructing a common polar code of rate $R$ (of arbitrarily large block length) which allows reliable transmission over both channels.

Trivially,

$$
C_{P, SC}(P, Q) \leq \min\{I(P), I(Q)\}.
$$

We will see shortly that, properly applied, this simple fact can be used to give tight bounds.

For the lower bound we claim that

$$
C_{P, SC}(P, Q) \geq C_{P, SC}(BEC(Z(P)), BEC(Z(Q))) = 1 - \max\{Z(P), Z(Q)\}.
$$

To see this claim, we proceed as follows. Consider a particular computation tree of height $n$ with observations at its leaf nodes from a BMS channel with Bhattacharyya constant $Z$. What is the largest value that the Bhattacharyya constant of the root node can take on? From the extreme values of information combining framework ([4, Chapter 4]) we can deduce that we get the largest value if we take the BMS channel to be the $BEC(Z)$. This is true, since at variable nodes the Bhattacharyya constant acts multiplicatively for any channel, and at check nodes the worst input distribution is known to be the one from the family of $BEC$ channels. Further, $BEC$ densities stay preserved within the computation graph.

The above considerations give rise to the following transmission scheme. We signal on those channels $W^\sigma$ which are reliable for the $BEC(\max\{Z(P), Z(Q)\})$. A fortiori these channels are also reliable for the actual input distribution. In this way we can achieve a reliable transmission at rate

$$
1 - \max\{Z(P), Z(Q)\}.
$$

**Example 3 (BSC and BEC):** Let us apply the above mentioned bounds to $C_{P, SC}(P, Q)$, where $P = BEC(0.5)$ and $Q = BSC(0.11002)$. We have

$$
I(P) = I(Q) = 0.5,
$$

$$
Z(BEC(0.5)) = 0.5,
$$

$$
Z(BSC(0.11002)) = 2\sqrt{0.1102(1 - 0.11002)} \approx 0.6258.
$$

The upper bound [1] and the lower bound [2] then translate to

$$
C_{P, SC}(P, Q) \leq \min\{0.5, 0.5\} = 0.5,
$$

$$
C_{P, SC}(P, Q) \geq 1 - \max\{0.6258, 0.5\} = 0.3742.
$$

Note that the upper bound is trivial, but the lower bound is not.

In some special cases the best achievable rate is easy to determine. This happens in particular if the two channels are ordered by degradation.

**Example 4 (BSC and BEC Ordered by Degradation):** Let $P = BEC(0.22004)$ and $Q = BSC(0.11002)$. We have $I(P) = 0.770098$ and $I(Q) = 0.5$. Further, one can check that the $BSC(0.11002)$ is degraded with respect to the $BEC(0.22004)$. This implies that any sub-channel of type $\sigma$ which is good for the $BSC(0.11002)$, is also good for the $BEC(0.22004)$. Hence, $C_{P, SC}(BEC(0.22004), BSC(0.11002)) = I(Q) = 0.5$.

More generally, if the channels $W$ are such that there is a channel $W \in W$ which is degraded with respect to every channel in $W$, then $C_{P, SC}(W) = C(W) = I(W)$. Moreover, the sub-channels $\sigma$ that are good for $W$ are good also for all channels in $W$.

So far we have looked at seemingly trivial upper and lower bounds on the compound capacity of two channels. As we...
will see now, it is quite simple to considerably tighten the result by considering individual branches of the computation tree separately.

Theorem 5 (Bounds on Pairwise Compound Rate): Let $P$ and $Q$ be two BMS channels. Then for any $n \in \mathbb{N}$

$$C_{P, SC}(P, Q) \leq \frac{1}{2^n} \sum_{\sigma \in \{0, 1\}^n} \min \{I(P^\sigma), I(Q^\sigma)\},$$

and

$$C_{P, SC}(P, Q) \geq 1 - \frac{1}{2^n} \sum_{\sigma \in \{0, 1\}^n} \max \{Z(P^\sigma), Z(Q^\sigma)\}.$$ 

Further, the upper as well as the lower bounds converge to the compound capacity as $n$ tends to infinity and the bounds are monotone with respect to $n$.

Proof: Consider all $N = 2^n$ tree channels. Note that there are $2^{n-1}$ such channels that have $\sigma_1 = 0$ and $2^{n-1}$ such channels that have $\sigma_1 = 1$. Recall that $\sigma_1$ corresponds to the type of node at level $n$.

This level transforms the original channel $P$ into $P^0$ and $P^1$, respectively. Consider first the $2^{n-1}$ tree channels that correspond to $\sigma_1 = 1$. Instead of thinking of each tree as a height $n$ with observations from the channel $P$, think of each of them as a tree of height $n-1$ with observations coming from the channel $P^1$. By applying our previous argument, we see that if we let $n$ tend to infinity then the common capacity for this half of channels is at most $0.5 \min \{I(P^1), I(Q^1)\}$. Clearly the same argument can be made for the second half of channels. This improves the trivial upper bound (1) to

$$C_{P, SC}(P, Q) \leq 0.5 \min \{I(P^1), I(Q^1)\} + 0.5 \min \{I(P^0), I(Q^0)\}.$$ 

Clearly the same argument can be applied to trees of any height $n$. This explains the upper bound on the compound capacity of the form $\min \{I(P^\sigma), I(Q^\sigma)\}$.

In the same way we can apply this argument to the lower bound (2). From the basic polarization phenomenon we know that for every $\delta > 0$ there exists an $n \in \mathbb{N}$ so that

$$\frac{1}{2^n} |\{\sigma \in \{0, 1\}^n : I(P^\sigma) \in [\delta, 1 - \delta]\}| \leq \delta/4.$$ 

Equivalent statements hold for $I(Q^\sigma)$, $Z(P^\sigma)$, and $Z(Q^\sigma)$.

In words, except for at most a fraction $\delta$, all channel pairs $(P^\sigma, Q^\sigma)$ have “polarized.” For each polarized pair both the upper as well as the lower bound are loose by at most $\delta$. Therefore, the gap between the upper and lower bound is at most $(1 - \delta)2\delta + \delta$.

To see that the bounds are monotone consider a particular type $\sigma$ of length $n$. Then we have

$$\min \{I(P^\sigma), I(Q^\sigma)\}$$

$$= \min \left\{ \frac{1}{2}(I(P^\sigma_0) + I(P^\sigma_1)), \frac{1}{2}(I(Q^\sigma_0) + I(Q^\sigma_1)) \right\}$$

$$\geq \frac{1}{2} \min \{I(P^\sigma_0), I(Q^\sigma_0)\} + \frac{1}{2} \min \{I(P^\sigma_1), I(Q^\sigma_1)\}.$$ 

A similar argument applies to the lower bound.

Example 7 (Bounds on Compound Rate of BMS Channels): In the previous example we considered the compound capacity of two BMS channels. How does the result change if we consider a whole family of BMS channels? E.g., what is $C_{P, SC}(\{\text{BMS}(I = 0.5)\})$?

We currently do not know of a procedure (even numerical) to compute this rate. But it is easy to give some upper and lower bounds.

In particular we have

$$C_{P, SC}(\{\text{BMS}(I = 0.5)\}) \leq C(\text{BSC}(0.11002), \text{BEC}(0.5))$$

$$\leq 0.4817,$$

$$C_{P, SC}(\{\text{BMS}(I = 0.5)\}) \geq 1 - Z(\text{BSC}(I = 0.5)) \approx 0.374.$$ 

The upper bound is trivial. The compound rate of a whole class cannot be larger than the compound rate of two of its members. For the lower bound note that from Theorem 5 we know that the achievable rate is at least as large as $1 - \max \{Z\}$, where the maximum is over all channels in the class. Since the BSC has the largest Bhattacharyya parameter of all channels in the class of channels with a fixed capacity, the result follows.

IV. A BETTER UNIVERSAL LOWER BOUND

The universal lower bound expressed in (3) is rather weak. Let us therefore show how to strengthen it.

Let $W$ denote a class of BMS channels. From Theorem 5 we know that in order to evaluate the lower bound we have to optimize the terms $Z(P^\sigma)$ over the class $W$.

To be specific, let $W$ be BMS$(I)$, i.e., the space of BMS channels that have capacity $I$. Expressed in an alternative way, this is the space of distributions that have entropy equal to $1 - I$.

The above optimization is in general a difficult problem. The first difficulty is that the space $\{\text{BMS}(I)\}$ is infinite dimensional. Thus, in order to use numerical procedures we have to approximate this space by a finite dimensional space. Fortunately, as the space is compact, this task can be accomplished. E.g., look at the densities corresponding to the class $\{\text{BMS}(I)\}$ in the $|D|$-domain. In this domain, each BMS channel $W$ is represented by the density corresponding to the probability distribution of $|W(Y | 0) - W(Y | 1)|$, where $Y \sim W(y | 0)$. For example, the $|D|$-density corresponding to $\text{BSC}(\epsilon)$ is $\Delta_{1-2\epsilon}$.

We quantize the interval $[0, 1]$ using real values $0 = p_1 < p_2 < \cdots < p_m = 1$, $m \in \mathbb{N}$. The $m$-dimensional polytope
approximation of \{BMS(I)\}, denoted by \(W_m\), is the space of all the densities which are of the form \(\sum_{i=1}^{m} \alpha_i \Delta_{p_i}\). Let \(\alpha = [\alpha_1, \cdots, \alpha_m]^T\). Then \(\alpha\) must satisfy the following linear constraints:

\[
\alpha^T 1_{m \times 1} = 1, \quad \alpha^T H_{m \times 1} = 1 - I, \quad \alpha_i \geq 0, \tag{4}
\]

where \(H_{m \times 1} = [b_2(\frac{1-p_2}{2})]_{m \times 1}\) and \(1_{m \times 1}\) is the all-one vector.

Due to quantization, there is in general an approximation error.

**Lemma 8 (m versus \(\delta\)):** Let \(a \in BMS(I)\). Assume a uniform quantization of the interval \([0, 1]\) with \(m\) points \(0 = p_1 < p_2 < \cdots < p_m = 1\). If \(m \geq 1 + \frac{1}{\sqrt{1-\delta^2}}\), then there exists a density \(b \in W_m\) such that \(|Z(a) - Z(b)| \leq \delta\).

**Proof:** For a given density \(a\), let \(Q_u(a)(Q_d(a))\) denote the quantized density obtained by mapping the mass in the interval \((p_i, p_{i+1})\) to \(p_{i+1}\). Let \(\mathcal{Q}_u(a)\) be upgraded (degraded) with respect to \(a\). Thus, \(H(Q_u(a)) \leq H(a) \leq H(Q_d(a))\). The Bhattacharyya parameter \(Z(a)\) is given by

\[
Z(a) = \int_0^1 \int_0^1 \sqrt{1 - x^2 y^2} dx dy.
\]

Since \(\sqrt{1 - x^2} = \sqrt{1 - x^2 y^2}\) is decreasing on \([0, 1]\), we have

\[
Z(Q_d(a)) - Z(a) \leq \sum_{i,j} \int_{p_i}^{p_{i+1}} \int_{p_j}^{p_{j+1}} \left(1 - p_i^2 p_j^2 - \sqrt{1 - x^2 y^2}\right) dx dy,
\]

\[
Z(a) - Z(Q_u(a)) \leq \sum_{i,j} \int_{p_i}^{p_{i+1}} \int_{p_j}^{p_{j+1}} \left(1 - p_i^2 p_j^2 - \sqrt{1 - x^2 y^2}\right) dx dy.
\]

Now note that the maximum approximation error, call it \(\delta\), happens when \(xy\) is close to 1. This maximum error is equal to \(\sqrt{1 - 1 - \left(1 - \left(\frac{1}{m-1}\right)\right)^2} - \sqrt{1 - 1^2}\)

Solving for \(m\) we see that the quantization error can be made smaller than \(\delta\) by choosing \(m\) such that

\[
m \geq 1 + \frac{1}{\sqrt{1-\delta^2}}. \tag{5}
\]

Note that if \(a \in W\) then in general neither \(Q_d(a)\) nor \(Q_u(a)\) are elements of \(W_m\), since their entropies do not match. In fact, as discussed above, the entropy of \(Q_d(a)\) is too high, and the entropy of \(Q_u(a)\) is too low. But by taking a suitable convex combination we can find an element \(b \in W_m\) for which \(Z(b)\) differs from \(Z(a)\) by at most \(\delta\).

In more detail, consider the function \(f(t) = H(tQ_u(a) + (1-t)Q_d(a))\), \(0 \leq t \leq 1\). Clearly, \(f\) is a continuous function on its domain. Since every density of the form of \(tQ_u(a)+(1-t)Q_d(a)\) is upgraded with respect to \(Q_u(a)\) and degraded with respect to \(Q_d(a)\), we have \(Z((Q_u(a))^{\otimes 2}) \leq Z((tQ_u(a)+(1-t)Q_d(a))^{\otimes 2}) \leq Z((Q_d(a))^{\otimes 2})\). As a result: \(|Z((tQ_u(a)+(1-t)Q_d(a))^{\otimes 2}) - Z(a^{\otimes 2})| \leq \delta\). We further have \(f(0) = H(Q_d(a)) \leq H(a) \leq H(Q_u(a)) = f(1)\). Thus there exists a \(0 \leq t_0 \leq 1\) such that \(f(t_0) = H(a)\). Hence, \(t_0Q_u(a) + (1-t_0)Q_d(a) \in BMS(I)\) and \(t_0Q_u(a) + (1-t_0)Q_d(a) \in W_m\).

Therefore \(t_0Q_u(a) + (1-t_0)Q_d(a)\) is the desired density.

**Example 9** (Improved Bound for \(BMS(I = \frac{1}{2})\)): Let us derive an improved bound for the class \(W = BMS(I = \frac{1}{2})\). We pick \(n = 1\), i.e., we consider tree channels of height 1 in Theorem 5.

For \(\sigma = 0\) the implied operation is \(\otimes\). It is well known that in this case the maximum of \(Z(a \otimes a)\) over all \(a \in W\) is achieved for \(a = BSC(0.11002)\). The corresponding maximum \(Z\) value is 0.3916.

Next consider \(\sigma = 1\). This corresponds to the convolution \(\boxast\). Motivated by Lemma 8 consider at first the maximization of \(Z\) within the class \(W_m\):

\[
\begin{align*}
\text{maximize : } & \sum_{i,j} \alpha_i \alpha_j Z(\Delta_{p_i} \boxast \Delta_{p_j}) = \sum_{i,j} \alpha_i \alpha_j \sqrt{1 - (p_i p_j)^2} \\
\text{subject to : } & \alpha^T 1_{m \times 1} = 1, \quad \alpha^T H_{m \times 1} = \frac{1}{2}, \quad \alpha_i \geq 0.
\end{align*}
\]

In the above, since the \(p_i\)s are fixed, the terms \(\sqrt{1 - (p_i p_j)^2}\) are also fixed. The task is to optimize the quadratic form \(\alpha^T P \alpha\) over the corresponding \(\alpha\) polytope, where the \(m \times m\) matrix \(P\) is defined as \(P_{ij} = \sqrt{1 - (p_i p_j)^2}\). We claim that this is a convex optimization problem.

To see this, expand \(\sqrt{1 - x^2}\) as a Taylor series in the form

\[
\sqrt{1 - x^2} = 1 - \sum_{l \geq 0} t_l x^{2l},
\]

where the \(t_l \geq 0\). We further have

\[
\alpha^T P \alpha = \sum_{i,j} \alpha_i \alpha_j \left(1 - (p_i p_j)^2\right) = 1 - \sum_{l \geq 0} t_l \left(\alpha_i p_i^{2l}\right)^2.
\]

Thus, since \(t_l \geq 0\) and the \(p_i\)s are fixed, each of the terms -\(t_l(\sum_i \alpha_i p_i^{2l})^2\) in the above sum represents a concave function. As a result the whole function is concave.

To find a bound, let us relax the condition \(0 \leq \alpha_i \leq 1\) and admit \(\alpha \in \mathbb{R}\). We are thus faced with solving the convex optimization problem

\[
\begin{align*}
\text{maximize : } & \alpha^T P \alpha \\
\text{subject to : } & \alpha^T 1_{m \times 1} = 1, \quad \alpha^T H_{m \times 1} = \frac{1}{2}.
\end{align*}
\]

The Kuhn-Tucker conditions for this problem yield

\[
\begin{bmatrix}
2P & 1 & \alpha_1 \\
1^T & 0 & \alpha_2 \\
H^T & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_n
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}.
\]

As \(P\) is non-singular, the answer to the above set of linear equations is unique.

We can now numerically compute this upper bound and from Lemma 8 we have an upper bound on the estimation.
error due to quantization. We get an approximate value of 0.799. We conclude that
\[
C_{SC}(\{BMS(I = 0.5)\}) \geq 1 - \frac{1}{2}(0.392 + 0.799)
= 0.404.
\]
This slightly improves on the value 0.374 in (3). In principle even better bounds can be derived by considering values of \( n \) beyond 1. But the implied optimization problems that need to be solved are non-trivial.

\[\diamond\]

V. CONCLUSION AND OPEN PROBLEMS

We proved that the compound capacity of polar codes under SC decoding is in general strictly less than the compound capacity itself. It is natural to inquire why polar codes combined with SC decoding fail to achieve the compound capacity. Is this due to the codes themselves or is it a result of the sub-optimality of the decoding algorithm? We pose this as an interesting open question.

In [6] polar codes based on general \( \ell \times \ell \) matrices \( G \) were considered. It was shown that suitably chosen such codes have an improved error exponent. Perhaps this generalization is also useful in order to increase the compound capacity of polar codes.

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REFERENCES

[1] E. Arıkan, “Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels,” submitted to IEEE Trans. Inform. Theory, 2008.
[2] S. B. Korada and R. Urbanke, “Polar codes are optimal for lossy source coding,” submitted to IEEE Trans. Inform. Theory, 2009.
[3] D. Blackwell, L. Breiman, and A. J. Thomasian, “The capacity of a class of channels,” The Annals of Mathematical Statistics, vol. 3, no. 4, pp. 1229–1241, 1959.
[4] T. Richardson and R. Urbanke, Modern Coding Theory. Cambridge University Press, 2008.
[5] R. Mori and T. Tanaka, “Performance and Construction of Polar Codes on Symmetric Binary-Input Memoryless Channels,” Jan. 2009, available from http://arxiv.org/pdf/0901.2207.
[6] S. B. Korada, E. Şaşoğlu, and R. Urbanke, “Polar codes: Characterization of exponent, bounds, and constructions,” submitted to IEEE Trans. Inform. Theory, 2009.