Space-Time Evolution of Ultrarelativistic Quantum Dipoles in Quantum Electrodynamics.

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Abstract

We discuss space-time evolution of ultrarelativistic quantum dipole in QED. We show that the space-time evolution can be described, in a certain approximation, by means of a regularized wave function, whose parameters are determined by the process of the dipole creation by a local current. We derive using these wave functions the dipole expansion law, that is found to coincide parametrically in the leading order with the one suggested in ref. [12].

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I. INTRODUCTION

The problem of a space-time time evolution of a electromagnetic field surrounding electrically charged quantum particle has always been under investigation, starting from the classical refs. [1–3] (see also closely related research in classical electrodynamics [4]). In particular, Gribov stressed the connection between the space-time evolution in quantum field theory and the parton model and deep-inelastic scattering. He argued that at high energies the space-time evolution is a well defined physical concept, that directly follows from the study of the relevant Feynman diagrams.

The problem of the space-time evolution of a neutral dipole in gauge quantum field theory, i.e. QED or QCD, recently attracted a lot of attention. This problem is more complex, than the corresponding problem for a single charged particle since one must take into account several effects, such as field regeneration, charge screening and space-time evolution of the dipole wave function, simultaneously.

The studies of the space-time evolution of the electromagnetic dipoles started long time ago in connection to the propagation of the fast $e^+ - e^-$ pairs through matter. The latter research, both experimental [5] and theoretical [6], led to significant progress in the understanding of the properties of the dipoles both in classical and in quantum electrodynamics.

More recently, a lot of interest was attracted to the space-time evolution and screening in the quantum electromagnetic dipoles, leading to the concept of the charge transparency [7]. This research is closely connected to the recent developments in the study of deep inelastic scattering in QCD, where similar ideas lead to the concept of the color transparency and the discovery that a color neutral dipole is a correct degree of freedom for deep-inelastic scattering (for the review and the proper references see refs. [8–10]). For the sufficiently small dipole cross sections some hard diffractive processes at moderately small Bjorken $x_B$ are unambiguously calculable in terms of QCD evolution equation.

In the same time a detailed study of the space-time evolution of both classical and quantum dipole in the center of mass-reference frame of a dipole was undertaken by Gribov in ref. [11], There the space-time evolution of both the current and the electromagnetic field was considered in the leading logarithmic approach for zero mass particles. It was argued that such system is unstable in the sense that the charges in the dipole are screened out, and both the fields, the charges and the currents flow to zero as $1/\log(t/\tau)$ for large times ($\tau$ is the regularization parameter). Although the actual calculations were carried for zero constituents masses it was argued that the results are valid for small times as compared to the Compton waves of the constituents.

More recently, a space-time evolution of dipoles was analyzed in refs. [12], [8]. In particular, it was argued in ref. [12], that the behaviour of the dipole wave function $\Psi(\rho,t) \sim \exp(i\rho^2/t)$, where $\rho$ is the transverse scale of the dipole, leads to a new physical effect of quantum diffusion, i.e. the average transverse area of a dipole increases as $T/E$. where $T$ is the time interval after dipole production.

In ref. [8] a factorization theorem was proved for the hard processes initiated by longitudinal vector current, between the dipole creation amplitude and the dipole scattering amplitude.

Nevertheless the above approaches did not deal with a dipole as an independent quantum object. Rather, a dipole was considered as a product of two plane waves created by a local
current. The total gauge invariant amplitude including the dipole creation by a local current and the corresponding scattering on the target was considered using Feynman diagrams. In such approach dipole arises as an intermediate state in Feynman diagram calculations and one does not need to worry about its properties.

The aim of the present paper is to study whether the dipole in QED can be described self-consistently, on the same basis as the point-like particle.

We shall consider approximately symmetric dipole, i.e. each of its constituents carry approximately half of its energy. We shall see that such a dipole can be indeed described as a gauge invariant way at least in the approximation of quantum mechanics, i.e. if we neglect the radiation (particle production) and consider only scattering. The parameters of the dipole are part of the initial conditions one must specify, and are determined by the dipole production process by a local current. We shall see that for times small with the dipole coherence time, the expansion law of the dipole is

\[ \rho^2 = DT/E + v_t^2T^2 \]  

Here \( D \) is the diffusion coefficient and \( v_t = p_t/E \) is the average relative transverse velocity of the constituents. In the leading order in \( T \) and for \( D=2 \), this equation corresponds to the dipole created by the longitudinal part of the vector current, coinciding with the result of ref. [12].

The paper is organized in the following way. In the second section we shall list the problems that appear when one tries to describe the dipole in quantum field theory, problem of causality and problem of gauge invariance and how they can be solved. In the third chapter we study the time evolution of the dipole wave function and a space-time evolution of a general free dipole in quantum mechanics. In the fourth chapter we explain the need of the regularization and study a space-time evolution in quantum field theory (assuming the dipole is approximately symmetric, and the characteristic transverse momenta of the constituents are much smaller than the dipole center of mass energy). Our results are summarized in conclusion. In Appendix we review for completeness a classical analogue of a dipole creation and space-time evolution.

For the sake of simplicity throughout the paper we shall consider an axially symmetric dipole with spinless constituents. We carry all our calculations for the simpler QED case, although we expect that they will remain true also for QCD.

II. PROBLEMS WITH DIPOLE IN QED.

A. Problem of causality

We shall consider the fast free bare dipole created at some time which we shall choose as \( T = 0 \) by a neutral current. The word "bare" means here that a dipole created in this way has no classical electromagnetic field. The field is regenerated, as in refs. [1,2,11], starting from \( T = 0 \), and approaches a particular limit at \( T \to \infty \) (see Appendix for a review). The latter field will be different from the dipole analogue of the Lienart-Wiehert potential, if we shall take into account the dipole space-time evolution. Indeed, due to causality, being zero
at \( T = 0 \), the classical field for each finite time \( t \) will be inside the sphere \( r \leq t \), and will not be spread to infinity as the field corresponding to the Lienart-Wiehert potential [13].

Consider the dipole created in this way. There are two problems with its description in QFT. The first problem is connected with causality. Any process involving dipole, created at \( T=0 \), must include vacuum fluctuations that existed since \( T = -\infty \). Thus in general all the boundary conditions on the fields must be imposed at \( T = -\infty \), and then of course there is no sense to study separately the space-time evolution of the dipole. However, it is easy to see from the analysis of Feynman diagrams along the lines of ref. [3] that since perturbative QED vacuum is a ground state there exists a natural time ordering in high energy dipole, with the processes involving vacuum pairs, except properly ordered, being suppressed by the powers of the energy (see also ref. [14]). This result permits one to use effectively old time-dependent perturbation theory, and define the boundary conditions at \( T=0 \), and not at \( T = -\infty \), that is the usual case in quantum field theory [15]. The advantage of this approach is that it enables one to take into account directly the space-time evolution of the dipole wave function.

The direct calculation of Feynman diagrams shows that the sum of the terms unsuppressed by the powers of the center of mass energy of the dipole \( E \), exactly coincides with the result of the old time-dependent perturbation theory (see e.g. refs. [16], for a detailed set of rules for this formalism), with only one causal ordering in time left. All other orderings are suppressed due to energy denominators by the orders of energy \( E \).

Note that in this case the old perturbation theory gives the same results as the light-cone perturbation theory (see e.g. refs. [17,18] for the detailed review). Indeed, the light-cone perturbation theory is causal in light-cone time \( x^+ \), i.e. only causal light cone time ordering survives. It is clear that for high energies time \( t \sim z \), so \( x^- \sim 0 \), and \( t \sim x^+ \). While for small energies the calculations are usually quite complicated, for high energies the longitudinal and transverse coordinates factorize, and the calculations of the scattering processes in light-cone perturbation theory become quite simple. However, there are two basic problems connected with the systematic use of the light-cone gauge. First, contributions from other time orderings really do not disappear, e.g. in QED including fermions, but become contact terms that appear in loop calculations, and are often hard to trace. Second, and more important, the evolution of the wave functions that are solutions of the wave equation, goes in time, note in light-cone time. One has to specify boundary condition on the surface \( t = 0 \), not \( x^+ = 0 \). If one specifies initial conditions at \( x^+ = 0 \), they must satisfy self-consistency conditions. This is of course not surprising, since the solution of the wave equation depends on two arbitrary functions, being the second order hyperbolic equation, while the wave equation on light-cone is formally of the first order in light-cone time \( x^+ \).

In other words, not all boundary conditions are allowed on light-cone, and they must be determined from the dynamics of the theory. The same situation exists in quantum field theory, where the boundary conditions at \( x^+ = 0 \) must be determined dynamically, even for high energies. Thus, once the evolution of the wave functions is taken into account, one either has to determine dynamically the evolution in \( x^+ \) from that in \( t \) and then use light-cone perturbation theory, or directly use the time-ordered perturbation theory as in ref. [3]. Consequently, the use of old perturbation theory has its advantage since it permits one to consider boundary problems with boundary conditions specified at \( T=0 \). The results using both of these methods must be of course the same. Moreover, technically the calculations
in both approaches look very similar at high energies. In our paper we shall use the old perturbation theory, discarding the terms suppressed by powers of E.

**B. Problem of gauge invariance**

The problem of casuality described above actually exists also for the evaluation of the e.m. field of a single particle. The problem special for a dipole as an extended object is a problem of gauge invariance of the calculated amplitudes.

It is well known (see ref. [19], and ref. [8] for a more recent work on the subject) that in order for gauge invariance to be preserved a dipole must be created by a local current. It is easy to see that this condition is indeed necessary and sufficient to ensure the gauge invariance if we neglect radiation (i.e. particle creation). Indeed, consider the amplitude of the scattering of the dipole on the external field in the lowest order over e.m. charge e.

\[
M_{if} = \lim_{T \to \infty} \int_{0}^{T} dt d^{3}x J_{(if)\mu}(\vec{x}, t) A_{\mu}(\vec{x}, t). \tag{2.1}
\]

If we make a gauge transformation, the current is gauge invariant, [19], while the field changes as

\[
A_{\mu} \to A_{\mu} + \partial_{\mu} \psi.
\]

Here \( \psi \) is the arbitrary function of \( \vec{x} \) and \( t \). Under gauge transformation the amplitude \( M_{if} \) changes as

\[
\Delta M_{fi} = \int dt \int d^{3}x J_{\mu} \partial^{\mu} \psi \tag{2.2}
\]

The latter integral can be integrated by parts, and if as usual the integration over \( t \) is from \(-\infty\) to \(+\infty\), and \( \psi(t = \pm\infty) = 0 \), the integral is

\[
\Delta M_{if} = \int d^{4}x \psi \partial^{\mu} J_{\mu}. \tag{2.3}
\]

This integral then vanishes due to a current conservation. However because the integration in eq. (2.2) is from 0 to infinity, one obtains additional term

\[
\Delta M_{fi} = \int \rho_{fi}(\vec{x}, 0)f(\vec{x})d^{3}x. \tag{2.4}
\]

Here \( f(\vec{x}) = \psi(\vec{x}, 0) \). Hence we see that the condition for gauge invariance of the matrix elements is that

\[
\rho_{if}(\vec{x}) = 0 \tag{2.5}
\]

I.e. the density matrix element for the transition between the initial state and any arbitrary state \( f \) must be zero.

It is clear that the latter condition will be fulfilled if a dipole is created by a local current. The proof above is not sufficient for the case when we take into account radiation at finite time, since then the operator of interaction may contain derivatives. Taking radiation into account will be important for nonsymmetric dipole and demands additional analysis [20].
C. Gauge invariant initial conditions

In the previous subsection we found the condition eq. (2.5) for the initial dipole to be gauge invariant. It is easy to see that for the initial wave function $\Psi$ this condition is equivalent to

$$\Psi(\vec{p} + \vec{k}_1, \vec{k}_2) = \Psi(\vec{k}_1, \vec{p} + \vec{k}_2)$$

(2.6)

for arbitrary $\vec{p}$. Suppose the initial wave function can be separated into

$$\Psi(\vec{r}_1, \vec{r}_2) = f_1(\vec{r}_1 + \vec{r}_2)f_2(\vec{r}_1 - \vec{r}_2)$$

This separation corresponds to the separation of the center of mass motion. Then it is easy to see that $f_2(\vec{k})$ must be independent of $k$, i.e. a constant.

Suppose for example that the initial wave packet is axially symmetric and the particles have no spin. Then we can use in the $\vec{k}$ space a gaussian:

$$\lim_{a,\epsilon \to 0} (2\epsilon/\pi)^{1/4} (2a/\pi)^{1/2} \exp(-aq_i^2 - \epsilon q_z^2)$$

(2.7)

Such gaussian describes the initial axially symmetric dipole, with the initial internal wave function $f_2$ normalized to 1, and the initial charge density is $\rho(\vec{x}) = \delta(x) - \delta(\vec{x}) = 0$. In general it is clear that one can use as an initial wave function any function whose square is a delta-function $\delta(\vec{r}_1 - \vec{r}_2)$. In order to take into account spin and angular moment one can either use the spherical harmonics or non symmetrical regularization.

III. TIME EVOLUTION OF THE WAVE FUNCTION OF A NEUTRAL DIPOLE.

A. General theory.

The key to the time-dependent description of the dipole in quantum mechanics is its wave function. This wave function is a solution of a Schroedinger equation:

$$\frac{\partial \Psi(\vec{r}_1, \vec{r}_2, t)}{\partial t} = \hat{H}(\vec{r}_1, \vec{r}_2, t)\Psi(\vec{r}_1, \vec{r}_2, t)$$

(3.1)

The Hamiltonian is

$$H = H_1 + H_2 - V(\vec{r}_1, \vec{r}_2)$$

(3.2)

Here

$$H_i^2 = (\vec{p}_i - e\vec{A}_i)^2$$

(3.3)

In eq. (3.3) $p_i$ is the momentum operator for the $i$-th component, $e$ is the charge of the constituent, $V$ is the potential of interaction between components of the dipole (including the possible interaction with the external field) and $A_i$ is the the quantum vector potential in the point $\vec{r}_i$, determined from the current and charge distribution due to the wave function $\Phi$. In our approximation (no radiation) $A$ can be considered as a classical field.

The latter equation can not be solved analytically in general, and here we shall consider the case of the free dipole. In this section we shall consider a free dipole without regularization. However, it is easy to see that the results remain true also for interacting dipole, for times much smaller than the coherence time.
B. Free dipole evolution.

In this case we neglect the e.m. field acting on the components of the dipole. Then the solution of eq. (3.2) can be easily written using momentum representation:

$$
\Phi(\vec{r}_1, \vec{r}_2, t) = \int d^3 q_1 d^3 q_2 \Phi(\vec{q}_1, \vec{q}_2) \exp(i\vec{q}_1 \vec{r}_1 + i\vec{q}_2 \vec{r}_2) \exp(i\epsilon_{q_1} + i\epsilon_{q_2})t
$$

$$
= \int d^3 q_1 d^3 q_2 \Phi(\vec{Q}, \vec{q}) \exp(i(\vec{Q} \vec{R} + \vec{q} \vec{r})) \exp(i(\epsilon_{(Q+q)/2} + i\epsilon_{(Q-q)/2})t)
$$

(3.4)

Here $\epsilon_q = \sqrt{q^2 + M^2}$, $\vec{Q} = \vec{q}_1 + \vec{q}_2$, $\vec{R} = (\vec{r}_1 + \vec{r}_2)/2$, $\vec{q} = \vec{q}_1 - \vec{q}_2$, $\vec{r} = (\vec{r}_1 - \vec{r}_2)/2$. This wave function is fully defined by its value at $t=0$, i.e. the boundary conditions

$$
\Phi(\vec{r}_1, \vec{r}_2, 0) = \int d^3 q_1 d^3 q_2 \Phi(\vec{q}_1, \vec{q}_2) \exp(i\vec{q}_1 \vec{r}_1 + i\vec{q}_2 \vec{r}_2)
$$

(3.5)

Depending on different type of approximation we obtain different types of the space-time evolution of the wave function.

First simplification is the "frozen dipole" approximation. This approximation means that one assumes factorization between the center of mass motion and the internal wave function, and the internal wave function is assumed to be time independent. It is clear from eq. (3.4) that this approximation means that in the time dependent energy multipliers we assume $|\vec{Q} \pm \vec{q}| \sim |\vec{Q}|$, i.e. $\epsilon_q \sim 2\epsilon_Q \equiv E$

$$
\Phi(\vec{Q}, \vec{q}) = \phi(q)/\sqrt{2(\epsilon^2 + M^2)}.
$$

(3.6)

In this approximation we obtain the wave function of the frozen dipole:

$$
\Phi(\vec{R}, \vec{r}, t) = \frac{\exp(i\vec{Q} \vec{R})}{\sqrt{2\epsilon_Q}} f(\vec{r}) \exp(iEt).
$$

(3.7)

The latter wave function describes the evolution of the dipole with the energy $E$, c.m. momentum $\vec{Q}$ and without internal dynamics.

The simplest internal wave function that can be taken is the Gaussian wave packet in the transverse plane and a narrow slap in the $z$-direction that approximately corresponds to the delta-function due to the relativistic effects:

$$
f(q) = (2a/\pi)^{1/2}(2\epsilon/\pi)^{1/4} \exp(-(a + ib)(q_t - k_t)^2) \exp(-\epsilon q_z^2).
$$

(3.8)

Here $\vec{q} = \vec{q}_1 - \vec{q}_2$, $z$ is the direction of the motion of the dipole, $\rho$ is the transverse coordinate, and $k_t$ is the transverse momenta of the constituents in the transverse plane. Note that a general initial wave function is complex. The limit $\epsilon \to 0$ corresponds to the delta function distribution of the matter in the relativistic disc. Index $t$ means the direction transverse to the direction of $\vec{Q}$. In the coordinate space we obtain

$$
f(\vec{q}_1, \vec{q}_2) = \frac{\sqrt{2a\pi}}{a + ib} \exp(-\rho^2/(4(a + ib)))(2\pi/\epsilon)^{1/4} \exp(-(z_1 - z_2)^2/(4\epsilon)) \exp(ik_t \vec{r}).
$$

(3.9)
For the state described by eq. (3.9)
\[ < \vec{p}^i(t = 0) > = k^i_t, < p^2(t = 0) >= 2/a + k^2_t \]
\[ < r^i(t = 0) >= 0, < r^2(t = 0) >= (a + b^2/a)/2. \]

(3.10)

The frozen dipole approximation is useful because we do not have to think about wave function dynamics and can put the boundary conditions at \( t = -\infty \).

We can approximately take into account the evolution of the free dipole explicitly for small times. The simplest way to do it is, neglecting for simplicity the masses of the constituents, to expand
\[ |\vec{Q} \pm \vec{q}| = Q \pm q_z + q^2_t/(2Q). \]

(3.11)

Then each of the time dependent multipliers in the wave function (3.4) can be written as
\[ \exp(i(Et/2 \pm q_z + q^2_t/(E))t) \]

The gaussian wave packet (3.8) in such approximation acquires a time dependence
\[ \Phi(\vec{r}, \vec{R}, t) = (1/\sqrt{2E})\sqrt{2a/\pi} \exp(i\vec{Q}\vec{R}) \exp(iEt) \int d^2q_t \exp(-(a + ib)(\vec{q}_t - \vec{k}_t)^2 + 2(it/(E))q^2_t) \]
\[ \times \exp(i\vec{q}_t\vec{\rho}), \]

(3.12)

(where we neglect the frozen evolution in z direction). In the coordinate space one obtains in this approximation
\[ \Phi(r, R, t) = (1/\sqrt{2E}) \exp(i\vec{Q}\vec{R}) \exp(iEt) \frac{\sqrt{2a\pi}}{a + i(b + 4t/E)} \]
\[ \times \exp\left(-\left(\vec{\rho} - 4\vec{k}_t t/E\right)^2/((a + i(b + 4t/E)) + i\vec{k}_t(\vec{\rho} - 4\vec{k}_t t/E))\right). \]

(3.13)

For this dipole we have time dependent radius:
\[ \rho^2(t) = a/2 + b^2/2a + 8(b/a)\hbar t/E + 8t^2/(E^2a) \]
\[ \rho^i(t) = 4k^i t/E. \]

(3.14)

(3.15)

We wrote \( \hbar \) in the latter equation in order to emphasise that the linear in time term has purely quantum origin. Let us note that this term is nonzero only for complex initial conditions. This term corresponds to the quantum diffusion phenomena [12]. The term
in the expansion $\sim t^2$ corresponds to the usual classical dynamics, this is the only term surviving for the real wave packet. The wave function (3.13) corresponds to the dipole built of the two components of opposite charges that are separated in the transverse plane by the vector $\vec{\rho}(t) = 4\vec{k}_t t/E$. For $\vec{k}_t \neq 0$ such a dipole has a well defined classical limit, describing two constituents, each with the transverse velocity $\vec{v}_t = \vec{k}_t / (E/2)$, and we shall call such a dipole a quasiclassical dipole. However if $\vec{k}_t = 0$, the dipole classical limit is not well defined. The dipole can’t be considered as being built of two separate components, and is locally electrically neutral. We shall call below such object a quantum dipole. I.e. by definition a quantum dipole is a dipole such that by definition

$$< \vec{r}_1(t) - \vec{r}_2(t) >= 0.$$  \hspace{1cm} (3.16)

Below we shall need a limit of a very small dipole:

$$b/a \rightarrow 1, a \rightarrow 0, b \rightarrow 0.$$  \hspace{1cm} (3.17)

It is easy to check that for small times, i.e. times much less than the coherence time $T_c \sim E/p_t^2$, where $p_t$ is the characteristic transverse momenta in the dipole, the interactions do not change qualitatively the dipole dynamics. Moreover, the quantum diffusion term is not influenced by the interaction potential $V$, and remains the same as for the free particle.

**IV. REGULARIZATION OF A FREE NEUTRAL DIPOLE.**

In the previous section we considered the time evolution of the free dipole in quantum mechanics. In order to go to QED, and to consider dipole created by a local current, one needs, as it was explained in section 2, to take the limit $a, b \rightarrow 0$ (in the notations of the previous section). If one takes this limit in the space-time wave function, one obtains the wave function

$$f(\vec{\rho}, t) \sim \exp(i\rho^2 E/t).$$  \hspace{1cm} (4.1)

This wave function however can not be normalized to one, since the preexponent does not have a well defined limit. Note that this wave function naively implies quantum diffusion [12]: Although $<\rho^2>$ is not well defined, the phase of this wave function strongly oscillates at distances $\rho^2 \sim t/E$.

It is clear why there are problems with the $a, b \rightarrow 0$ limit. Indeed, $1/\sqrt{a}$ is, according to the uncertainty principle, average momentum of transverse quantum fluctuations inside a dipole. When $a$ tends to zero, we eventually come to the situation when $1/\sqrt{a}/E \sim 1$, i.e. there is no separation between transverse and longitudinal degrees of freedom for any sensible period of time. The motion of transverse degrees of freedom is relativistic. It is easy to see that the same problem arises if one does not carry the separation of variables as in the previous section, but considers the evolution of the ultrarelativistic dipole from the beginning using the saddle point method.

The only way to solve the problem is to take the limits $E \rightarrow \infty$ and $a \rightarrow 0$ simultaneously. Let us look at the expression for $<\rho^2>$. It is clear that physically the term proportional
to $t^2$ for free dipole is just $v_t^2 t^2 = p_t^2 / E^2 t^2$. This means that for quantum dipole ($k_t$ in the previous section equal to zero), one must take a limit $E \to \infty, a \to 0, E^2 a \to 1/v_t^2$. Thus we see that in order to define the evolution of quantum dipole, in addition to an initial wave function and center of mass energies of the components one must define the way how the wave function and physical quantities are regularised. The regularisation parameters such as $v_t$ are additional parameters for a quantum dipole that must be specified as the initial conditions, in addition to the initial wave function. Physically they are determined by the physical process that creates dipole, and may be different for different local currents. In order to determine them we must consider in detail the process of the dipole creation in QED.

We now turn to quantum diffusion. It is clear that in the $a,b \to 0$ limit we must also demand $b^2/a \to 0$. It is also clear we must demand that $8b/a \to \text{const} \equiv D$. This means that $b$ and $a$ tend to zero with the same speed. The quantum diffusion coefficient $D$ must be specified as an initial condition. As it was explained above, its value depends on the explicit form of the current creating the dipole. For example, for the case of the longitudinal part of the vector current, such analysis gives $D=2$ [12,22].

Above we considered the case of the quantum dipole, i.e. $\vec{k}_t = 0$ in the notations of the previous section. Such dipole is locally charge neutral even for finite times, and does not have well-defined quantum limit. (The average $< p_t^2 > = v_t^2 E^2$ for such a dipole of course can be nonzero). If $k_t \neq 0$, the only change with the previous analysis is that the classical transverse separation speed $v_t$ is now a sum

$$v_t^2 = u_t^2 + p_t^2 / E^2.$$  \hfill (4.2)

For a space-time evolution of the dipole in quantum field theory one obtains

$$\rho^2 = D t / E + v_t^2 t^2$$ \hfill (4.3)

Other physical quantities must be renormalized along the same lines. Note that our approach to regularization looks very similar to the general Schwinger regularization [19].

V. CONCLUSION.

In this paper we considered the possibility of description of a space-time evolution of a quantum dipole in QED. We have considered a dipole created at the time $T=0$ by a local current. The dipole was assumed to be approximately symmetric (i.e. each constituent carries approximately half of its energy), with the center of mass energy $E$ much higher than the characteristic momenta of the transverse degrees of freedom. We have found that such dipole can be described in a gauge invariant way by the time dependent wave function, at least if one neglects radiation at finite time $T > 0$.

We have seen that the condition of gauge invariance of scattering amplitude (excluding radiation) strongly restricts the possible form of the dipole initial wave function that must be localized in a single point. Moreover, to specify the initial conditions it is not enough to specify space coordinate dependence of the initial wave function, as it will seem naively. In addition one must specify the renormalization procedure and the values for the renormalized
physical parameters, the quantum diffusion coefficient $D$ and the average transverse velocity. The values for these parameters are determined by the dipole creation process and depend on the explicit form of the current creating a dipole.

We were able to give the exact definition to the concepts of the space-time evolution of the quantum dipole, quantum diffusion, and an average size of the quantum dipole as a renormalized quantity. This concepts were first discussed on a more intuitive level in ref. [12].

In this paper we considered a symmetric dipole. It is easy to see that along the same lines one can consider asymmetric dipole, such that the dipole constituents have the energies $xE$ and $(1 - x)E$. The only difference will be that in the equations for the dipole size and the wave functions $E$ must be substituted by $Ex(1 - x)$. However, if $x$ is significantly different from 1/2 the radiation becomes important and the additional analysis of the dipole evolution is needed [20].

Another interesting question is the dipole evolution in the external field how to take into account the interaction between the constituents of the dipole. This is a subject for the future studies, although we expect it will not change qualitatively the results obtained using a free dipole. It is clear that in this case we must use the full system of the wave functions in the external field. It will be very interesting also to study the effects of the spin on the dipole evolution.

Next, it will be interesting to study how and for what times bound states influence the dipole space-time evolution.

Finally, it will be very interesting to include the photon radiation in the above formalism.

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**APPENDIX A**

1. Field generation by a scalar particle.

In this appendix we discuss the issue of field generation by a classical dipole, consisting of two distinct components spreading with a definite average velocity. Similar, but more complicated theory of a field generation may be developed for a quantum dipole.

We start by showing what we mean by field generation by bare particle. We shall start from a purely mathematical problem of solving the wave equation with the source appearing at time $t=0$ and moving rapidly. Similar problem of the charged particle field change during sudden stop or acceleration was considered in refs. [2], [4]. The wave equation has the form:

$$\Box \phi(\vec{x}, t) = \rho(\vec{x}, t)$$

$$\rho(\vec{x}, t) = \delta(z - vt)\delta(\vec{p})\theta(t)$$

(A1)
Note here the appearance of theta function meaning that the source appeared at \( t = 0 \). The speed \( v \) is assumed close to the velocity of light \( v \sim 1 \).

The eq. \( \text{(A1)} \) can be easily solved by making Fourier transform both in \( \vec{x} \) and in \( t \):

\[
\phi(\vec{q}, \omega) = \int d^3 \vec{x} dt \phi(\vec{x}, t) \exp(-i \vec{q} \vec{x} - i \omega t)
\] (A2)

Making Fourier transform of both parts of eq. \( \text{(A2)} \) we obtain

\[
\phi(\omega, \vec{q}) = -\frac{1}{\omega^2 - q^2 + i \epsilon \text{sign} \omega} \frac{1}{\omega - \vec{q} \vec{v} - i \epsilon}.
\] (A3)

Note the prescription for pole we made. This prescription of course corresponds to using the retarded Green function while solving eq. \( \text{(A1)} \), so that all the poles are in the upper half-plane. Then the solution is zero for \( t \leq 0 \). In order to understand the form of the solution in the space-time we shall do the inverse Fourier transform. First, let us do the transform in the \( \omega \) space. We have to sum over 3 poles: \( \omega = \pm q + i \epsilon, \omega = \vec{q} \vec{v} + i \epsilon \). Note that the third pole is just the Lienart-Wiehart contribution, while the first two poles correspond to on-shell field. Explicitly, we obtain:

\[
\phi(\vec{q}, t) = -\frac{\cos(qt) - \cos(\vec{q} \vec{v} t)}{(q_t^2 + (1 - v^2) q_z^2)} - i \frac{q \sin(qt) - \vec{q} \vec{v} \sin(\vec{q} \vec{v} t)}{q(q^2 - (\vec{q} \vec{v})^2)}. \] (A4)

It is worthwhile to go further and do Fourier transform in \( q \) \( [21] \). We see that the Fourier transform of the first term in \( \vec{q} \vec{v} \) is different from zero only for \( r \geq t \), and for \( r \geq t \) the first and the second term in eq. \( \text{(A4)} \) exactly cancel each other, while for \( t \geq r \) only the second term contributes and gives exactly the Lienart-Wiehart contribution (or, more explicitly the contribution of the field of rapidly moving scalar charge). Thus we come to a solution

\[
\phi(\vec{x}, t) = \theta(t - r) \phi_0(\vec{x}, t). \] (A5)

Here

\[
\phi_0(\vec{x}, t) = \frac{1}{\sqrt{\rho^2(1 - v^2) + (z - vt)^2}}. \] (A6)

This is the field of the rapidly moving scalar source \([13]\) created at the moment of time \( t=0 \). The theta function imposes causality (The details of the Fourier transform are given in the next subsection).

### 2. Details of the Fourier transform of the potentials.

In order to find explicitly the space structure of the solutions discussed above, the easiest way is to use explicitly the retarded Green function. Using the Fourier transform

\[
G_{\text{ret}}(\omega, \vec{q}) = 1/(\omega^2 - q^2 + i \epsilon \text{sign} \omega), \] (A7)

and \([21]\)
\[ G_{ret} = \delta(r-t)/r \]  \hspace{1cm} (A8)

and taking into account that under Fourie transform the product becomes a convolution, we obtain the field \( \phi \):

\[ \phi(\vec{r}, t) = \int d\tau d^3r' \frac{\delta(t - \tau - |\vec{r} - \vec{r}'|)}{|\vec{r} - \vec{r}'|} \rho(\vec{r}', \tau) \]  \hspace{1cm} (A9)

Substituting the expression for density (A3): \( \delta(z' - v\tau)\delta(\rho') \), we immediately obtain:

\[ \phi(r, t) = \int_0^t d\tau \frac{\delta(t - \tau - \sqrt{\rho^2 + (z - v\tau)^2})}{\sqrt{\rho^2 + (z - v\tau)^2}}. \]  \hspace{1cm} (A10)

It is straightforward to check by solving a quartic equation that the argument of delta function has a zero, such that \( t \geq \tau \geq 0 \) only if \( t \geq r \), and vice versa. Thus we come to a space-time structure for a solution for \( \phi \) in the previous subsection:

\[ \phi(\vec{r}, t) = \theta(r - t) \frac{1}{\sqrt{\rho^2(1 - v^2) + (z - vt)^2}}. \]  \hspace{1cm} (A11)

### 3. Expanding dipole.

The results of the previous subsection can be extended to electrodynamics, the only subtlety is that we have to ensure the conservation of the electric current:

\[ \frac{d\rho}{dt} + \text{div} \vec{j} = 0. \]  \hspace{1cm} (A12)

The case of the field change for sudden stop or acceleration of the charge was considered in ref. [4]. Here we consider a case of a creation of a point-like classical dipole that satisfies eq. (A12). Suppose the two components of the dipole were created at \( T=0 \), in such a way that they both have a speed component \( v \) along z axis and equal but opposite in magnitude velocities in the transverse plane \( \vec{v}_t \) and \( -\vec{v}_t \). Such symmetric dipole is of course over simplification in real laboratory system, where target is at rest, but the general case is not much different. The charge and current density for such classical symmetric dipole are given by

\[ \rho(r, t) = \delta(z - vt) * (\delta(\vec{r} - \vec{v}_t t) - \delta(\vec{r} + \vec{v}_t t)) \theta(t) \equiv \rho_+(\vec{r}, t) - \rho_-(\vec{r}, t) \]

\[ j_z(r, t) = v \rho(r, t) \]

\[ j_t(r, t) = \vec{v}_t (\rho_+(\vec{r}, t) + \rho_-(\vec{r}, t)). \]  \hspace{1cm} (A13)

It is straightforward to check that this ansatz satisfies charge conservation equation (A12). In writing eqs. A13 we neglected the further evolution of the dipole due to coulombic
interaction between the constituents. The Maxwell equations can be easily solved both in Lorentz and in Coulomb gauges.

\[ \Delta A_\mu - \frac{d^2 A_\mu}{dt^2} = -j_\mu \]

\[ \frac{\partial^i A_i}{\partial x^i} + dA_0/dt = 0 \]  

(A14)

Making a Fourier transform as in the previous section we obtain the solution in Lorentz gauge:

\[ A_0 = i(\phi_1 - \phi_2), \quad A_3 = vA_0 \]

\[ \vec{A}_i = iv_i^i(\phi_1 + \phi_2) \]  

(A15)

Here \( \phi_i \) are given by eq. (A3) for \((\omega, \vec{q})\) and (A4) for \((\vec{q}, t)\) representations, with \( v = v_1 \) for \( \phi_1 \) and \( v = v_2 \) for \( \phi_2 \). In our kinematics \( \vec{v}_1 = (v, \vec{v}_1), \vec{v}_2 = (v, -\vec{v}_1) \).

The same solution in Coulomb gauge is

\[ \phi = \phi_1(1 - \omega(v_1 k)/k^2) - \phi_2(1 - \omega(\vec{v}_2 k)/k^2)) \]

\[ \vec{A}_i = (\vec{v}_1 - \vec{k}(v_1 k)/k^2)\phi_1 - (\vec{v}_2 - \vec{k}(\vec{v}_2 k)/k^2)\phi_2. \]  

(A16)

Once again all poles are in the upper half plane to ensure that the solution is zero at \( t < 0 \). It is clear that the space-time structure of the solution will be the same as in the previous chapter: it will be the standard dipole field multiplied by a causality ensuring theta-functions \( \theta(r_i - t) \). It is straightforward to move into mixed \( q - t \) representation by making Fourier transform in \( \omega \).

Let us find the electric and magnetic field corresponding to these vector potentials. We shall do it first in \( \omega, \vec{q} \) and in \( \vec{q}, t \) spaces. In the Fourier components we have:

\[ E_i^i = i q_i^i(\phi_1 - \phi_2) - \omega v_i^i(\phi_1 + \phi_2) \]

\[ E_3 = i(q^3 v - \omega)(\phi_1 - \phi_2) \]

\[ H_{ti} = \epsilon_{3k} i q^3 v_{t2}(\phi_1 + \phi_2) - i \epsilon_{ik} q^k v(\phi_1 - \phi_2) \]

\[ H_3 = i \epsilon_{ij} q^i v_{tj}(\phi_1 - \phi_2). \]  

(A17)
In the space-time representation we have

\[ \vec{E} = \sum_{i=1,2} \vec{E}_1 \theta(r_i - t) + \vec{E}_2 \delta(t - r_i) \quad \vec{H} = \sum_{i=1,2} \vec{H}_1 \theta(t - r_i) + \vec{H}_2 \delta(t - r_i) \]  

(A18)

Here \( \vec{E}_1, \vec{H}_1 \) are the usual Lienart-Wiehart field of the moving charge, however they now exist only inside the light-cone due to causality. The field \( E_2, H_2 \) exists only on light-cone. It is easy to see that the fields \( E_2, H_2 \) are orthogonal in space-time and to light cone, for each of the constituents and thus correspond to the electromagnetic waves that exists on the boundary of light cone-i.e. on the surface \( r = t \). The explicit form of these fields is easily restored using the Fourier transforms of appendices A and B.
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