A New Method of Reconstructing Galactic 3D Structures Using Ultralong-wavelength Radio Observations

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Abstract

The free–free absorption of low-frequency radio waves by thermal electrons in the warm ionized medium of our Galaxy becomes very significant at \( \lesssim 10 \text{MHz} \) (ultralong wavelength), and the absorption strength depends on the radio frequency. Upcoming space experiments such as the Discovering Sky at the Longest Wavelength and Farside Array for Radio Science Investigations of the Dark Ages and Exoplanets will produce high-resolution multifrequency sky maps at the ultralong wavelength, providing a new window to observe the universe. In this paper we propose that from these ultralong-wavelength multifrequency maps, the 3D distribution of the Galactic electrons can be reconstructed. This novel and robust reconstruction of the Galactic electron distribution will be a key science case of those space missions. Ultralong-wavelength observations will be a powerful tool for studying the astrophysics relevant to the Galactic electron distribution, for example, the impacts of supernova explosions on electron distribution, and the interaction between interstellar atoms and ionizing photons escaped from the H II regions around massive stars.

Unified Astronomy Thesaurus concepts: Interstellar medium (847); Interstellar plasma (851); Interstellar absorption (831); Milky Way Galaxy (1054); Radio interferometers (1345)

Supporting material: animation

1. Introduction

Understanding our neighborhood and environment has long been a human pursuit, and the vicinity of the solar system is of great interest, as it has direct impacts on the solar system. Besides the well-observed nearby stars (Gaia Collaboration et al. 2022) the space is permeated with the interstellar medium (ISM), which serves as the reservoir of the material from which stars formed. Our Sun resides in a low-density cavity named the “Local Bubble” (LB) or “Local Hot Bubble” (Cox & Reynolds 1987), which is filled with X-ray-emitting hot gas (Snowden et al. 1998). The LB has a typical electron number density of \( \sim 5 \times 10^{-3} \text{ cm}^{-3} \), which is only \( \sim 20\% – 50\% \) of the average value at that radius on the Galactic disk (Ocker et al. 2020), and it extends to \( \sim 100 \) pc (Frisch et al. 2011). This low-density cavity in the ISM is thought to be created by a series of supernova explosions in prehistoric time (Zucker et al. 2022). The spatial distribution and morphology of the ISM reflects the complex interactions between the gas and stars in the Galactic “ecosystem,” but our knowledge of this neighborhood is far from complete. For example, the Loop I bubble is another low-density cavity adjacent to the LB, and the well-known giant North Polar Spur (NPS) feature in low-frequency radio sky maps is believed by many to be the brightest part of its bubble shell (Wolleben 2007). However, there are also arguments that the NPS is located at the much more distant Galactic center and is related to the Fermi Bubble (Sarkar 2019).

The ISM has a number of different components or phases, which are observed or probed by various methods (Draine 2011). The atomic and molecular hydrogen, accounting for \( \sim 80\% \) of the ISM hydrogen in mass, are traced by the 21 cm and CO lines, respectively. The dust (\( \sim 1\% \) of ISM in mass) is measured using interstellar extinction (Lallement et al. 2014; Capitanio et al. 2017). The hot gaseous halo (\( \lesssim 5\% \) of ISM mass within a Galactocentric distance of \( \sim 20 \) kpc) is measured from the X-ray absorption or emission lines of highly ionized metals (Miller & Bregman 2013). The diffuse warm ionized medium (WIM), which occupies \( \sim 20\% – 30\% \) of the volume near the Galactic plane and accounts for \( \sim 30\% \) of the total ISM mass (Kulkarni & Heiles 1988; Reynolds 1991a), has been observed via the diffuse H\( \alpha \) or other faint nebular emission lines (Reynolds 1977; Reynolds & Haffner 2000; Dickinson et al. 2003; Finkbeiner 2003; Haffner et al. 2003), or by the pulsar dispersion measures (DM), which could be used to reconstruct the 3D distribution of the electrons (Cordes et al. 1991; Taylor & Cordes 1993; Ferrière 2001; Gómez et al. 2001; Schnitzeler 2012; Greiner et al. 2016; Ocker et al. 2020), though the spatial resolution is limited by the available pulsars. Moreover, the scattering measure can provide information on the intrinsic fluctuations of the WIM (Williamson 1972).

The ISM can be probed using low-frequency radio observations, based on the fact that the ionized gas has a stronger absorption of radio waves at lower frequencies. The radiative transfer equation of the radiation is

\[
\frac{dI_v}{ds} = -\alpha_v I_v + j_v, \tag{1}
\]
where $I_\nu$ is the intensity at frequency $\nu$, and the absorption coefficient $\alpha_\nu$ of the ISM at low radio frequency is dominated by the electron free–free process$^5$, given by Condon & Ransom (2016)

$$
\alpha_\nu \approx 3.28 \times 10^{-3} \left( \frac{T_e}{10^4 \, \text{K}} \right)^{-1.35} \left( \frac{\nu}{\text{GHz}} \right)^{-2.1} \left( \frac{n_e}{\text{cm}^{-3}} \right)^2 \text{pc}^{-1},
$$

where $T_e$ and $n_e$ are the free electron temperature and number density, respectively. For diffuse Galactic electrons the absorption effect becomes significant at $\nu \lesssim 10 \, \text{MHz}$, while for dense H II regions the absorption becomes significant at even higher frequencies. The absorption strength contains information of electron densities at different distances. By using the multi-frequency data, the density distribution of the electron can be reconstructed, if the radiation emissivity $j_\nu$ (or in terms of brightness temperature $T = \frac{c^2 J_\nu}{2 \pi k}$, $\epsilon = \frac{2 c^2 J_\nu}{3 k}$) is known. As this method is sensitive to free electron density, it probes primarily the WIM component of the ISM. Given the large volume and mass fraction of the WIM, it is obviously highly interesting and useful to reconstruct its 3D structures.

The data at frequencies below 30 MHz, which we shall refer to as the ultralong-wavelength band, and especially below 10 MHz, are very scarce, as ground-based observation is hindered by ionosphere and radio frequency interference (Jester & Falcke 2009). There were some ground-based observations in Tasmania of Australia and Canada (e.g., Reber & Ellis 1956; Ellis et al. 1962; Ellis & Hamilton 1966a; Can & Whitham 1977; Can 1979; Reber 1994, for its history see also George et al. 2015a, 2015b, 2015c, 2016; Orchardon et al. 2015a, 2015b) and some early space observations (e.g., Hertz 1964; Alexander & Stone 1965; Smith 1965; Alexander et al. 1969; Brown 1973; Alexander & Novaco 1974; Novaco & Brown 1978; Manning & Dulk 2001). From these observations, it is noted that there is a downturn in the global brightness below $\sim 3 \, \text{MHz}$, which is believed to be caused by the free–free absorption mechanism of the ISM (Ellis et al. 1962; Ellis & Hamilton 1964; Alexander & Stone 1965; Novaco & Brown 1978). It has also been noted that at frequencies below a few MHz, in contrast to higher frequencies, the Galactic poles appear to be brighter than the Galactic plane, which is attributed to the stronger absorption on the Galactic plane (Alexander et al. 1970; Brown 1973; Novaco & Brown 1978; Ellis 1982; Manning & Dulk 2001).

Information on the Galactic electron distribution had been inferred from the ultralong-wavelength observations, even with the crude data obtained in the early observations. From the measured ultralong-wavelength global spectrum, by assuming an absorption length, electron density was derived (Hoyle & Ellis 1963; Smith 1965; Ellis & Hamilton 1966b; Alexander et al. 1970; Brown 1973; Can 1979; Kassim 1989; Fleishman & Tokarev 1995), which is found to be $\sim 0.03-0.1 \, \text{cm}^{-3}$ near the Galactic plane. These early works assumed a simple constant electron density. A slightly more sophisticated model assumes that the electron density depends on the vertical distance to the Galactic plane, which is constrained by the pulsar DM (e.g., Reynolds 1991b; Nordgren et al. 1992; Taylor & Cordes 1993; Gómez et al. 2001). Peterson & Webber (2002) found however that in this model the electron density derived from the pulsar observations is too low to raise the observed downturn of the ultralong-wavelength global spectrum. To fit both observations, they suggested that the electrons in the WIM are clumpy, which would induce stronger free–free absorption.

Jones et al. (2016) proposed that the free–free absorption can change the spectral index of the extragalactic background sources, and this can be used to construct the 2D distribution of electrons (the column density along the line of sight). However, by using discrete point sources as the background, this approach would be limited by the number of available point sources, just as in the pulsar observations.

The existing ultralong-wavelength observations have relatively low resolution. However, reconstruction of the full 3D Galactic electron structures will be feasible when high-resolution ultralong-wavelength sky maps are available. Recently, a number of ultralong-wavelength space missions have been proposed (Chen et al. 2019), such as the Discovering the Sky at the Longest Wavelength Lunar Orbit Array (Chen et al. 2021; Shi et al. 2022a, 2022b), and the Farside Array for Radio Science Investigations of the Dark Ages and Exoplanets (Burns et al. 2019), which have the capability of producing high-resolution maps, and will enable the full 3D reconstruction. Anticipating high-resolution multifrequency sky maps in the near future, we propose here to reconstruct the full 3D distribution of electrons using the ultralong-wavelength observation. The spatial distribution of the electrons is encoded in the spectrum: at the lower frequency, there is a higher contribution of absorption from more nearby absorption. If the emissivity along a line of sight (LOS) is known, then the electron density profile along it can be uniquely determined from a high-resolution spectrum, without requiring any prior information about the density profile. Combining the observations of many LOS toward different directions one can obtain the full 3D distribution of the electrons. To our knowledge, such a multifrequency tomographic method has not been applied to the full reconstruction of the ISM before.

The sky radio radiation at different frequencies is produced by various physical processes. Besides the cosmic microwave background, the thermal emission of the Galactic dust dominates the radiation above $\sim 60 \, \text{GHz}$. The sky between $\sim 10$ and $60 \, \text{GHz}$ is dominated by the bremsstrahlung radiation from Galactic thermal electrons. In this frequency range, there is also the spinning dust emission that peaks at $\sim 20 \, \text{GHz}$. Below $\sim 10 \, \text{GHz}$, the synchrotron radiation produced by the Galactic cosmic-ray electrons increases rapidly and becomes dominant (de Oliveira-Costa et al. 2008). In the synchrotron background, roughly one-third (Seiffert et al. 2011) are from extragalactic sources such as star-forming galaxies, active galactic nuclei, galaxy clusters, and so on (Singal et al. 2018). Our proposed reconstruction depends on the observation of the ultralong-wavelength radiation, and the understanding of the unabsorbed synchrotron emissivity. The unabsorbed synchrotron radiation has a nearly power-law spectrum, and the variation of the spectral index is relatively small, which simplifies the reconstruction.

The outline of this Paper is as follows: In Section 2 we introduce our methods. In Section 3 we present the statistics of our results, and show the reconstructed 1D, 2D (Galactic plane), and 3D structures in simulations and compare them

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$^5$The synchrotron self-absorption is unimportant for the typical Galactic magnetic field ($\sim 1 \, \mu \text{G}$; Sun et al. 2008) and cosmic-ray density ($\sim 10^{-13} \, \text{GeV}^{-1} \, \text{cm}^{-3}$; Peron et al. 2021) above $\sim 1 \, \text{kHz}$ (Ghisellini 2013). The radio wave cannot propagate below the ISM plasma frequency $\sim 2 \, \text{kHz}$ (Jester & Falcke 2009), so in the frequency range considered in this Paper, free–free absorption is the dominant absorption mechanism.
with the input electron model. In Section 4 we summarize the results and discuss some potential uncertainties.

2. Methods

At the frequency of interest, the emissivity is dominated by the synchrotron radiation from cosmic-ray electrons, which are believed to be produced primarily by supernova remnants and other accelerators and propagate over the Galaxy (Orlando & Strong 2013a). Due to the diffusive propagation, its intensity is relatively smooth, though it can be enhanced near the injecting source or at regions with stronger magnetic fields, which can trap the particles. Here, we model the emissivity as (Cong et al. 2021)

$$\epsilon(\nu, R, Z) = A\left(\frac{R + \tau_i}{R_0}\right)^{\alpha} e^{-R/R_0\epsilon[Z/Z_0]}\left(\frac{\nu}{\nu_0}\right)^{\gamma}, \quad (3)$$

where $R, Z$ are the Galactic cylindrical coordinates, $\tau_i = 0.1$ kpc is a small cutoff radius, $\nu_0 = 408$ MHz, $A$, $R_0$, $\alpha$, $Z_0$, and $\gamma$ are model parameters; they are obtained by fitting the Haslam 408 MHz map (Haslam et al. 1982; Remazeilles et al. 2015). That is, we integrate this emissivity along each LOS to obtain a sky map at 408 MHz, then find the best-fit emissivity parameters for which the sky map is closest (measured by the Euclidean distance) to the observed map. Details can be found in Cong et al. (2021). We use the constant $\beta_2 = -2.51$ that is fitted from multifrequency observations between 45 and 408 MHz.

The purpose of this paper is to demonstrate the feasibility of the algorithm; therefore the simple emissivity model of Equation (3) is used. One may also use more sophisticated models, for example the emissivity constructed by cosmic-ray propagation and magnetic distribution model, and constrained by observations like gamma-ray sky maps, e.g., GALPROP (Porter et al. 2021). Moreover, there have been some attempts to construct the synchrotron emissivity from observations. The opaque H II regions can fully absorb the synchrotron radiation behind them. Therefore they can be used to separate the synchrotron emission from regions in front of and behind the H II regions (Nord et al. 2006; Hindson et al. 2016; Su et al. 2017, 2018; Polderman et al. 2019).

We make the mock multifrequency sky maps $T_{\text{sky}}(\nu_j, l, b)$ using the ULSA sky model (Cong et al. 2021). The ULSA generates the sky maps at ultralong-wavelengths including the free–free absorption effect of the Galactic electrons. There are a number of Galactic electron distribution models by synthesizing different observations (Cordes et al. 1991; Gaensler et al. 2008; Schnitzeler 2012; Yao et al. 2017; here the NE2001 model (Cordes & Lazio 2002, 2003) is adopted. The basic distribution is described by a few components with analytical formulae, for example the thin and thick Galactic disks, and the spiral arms. Known structures such as voids and dense H II regions are also added. Fluctuations caused by unresolved small-scale (< 1 pc) electron structures are described by fluctuation parameters. When simulating the mock sky maps, we adopt the same fluctuation parameter for all the electron components in NE2001 for simplicity. Cong et al. (2021) found that using the default fluctuation parameters in NE2001 would overpredict the sky brightness below $\sim 3$ MHz. Instead, if increasing the fluctuation parameter for the thick disk from 0.2 pc$^{-2/3}$ to 3.0 pc$^{-2/3}$, the predicted global spectrum is consistent with the current observations. Since the thick disk dominates the all-sky absorption, we adopt the fluctuation parameter 3.0 pc$^{-2/3}$ for all components. The produced sky maps are similar to those in Figure 8 of Cong et al. (2021). Meanwhile, as our reconstruction relies on the free–free absorption, adopting a larger fluctuation parameter, and hence a more efficient absorption effect, is helpful for reconstructing the electron content. Moreover, we adopt a constant electron temperature of 8000 K. In Figure 1 we plot the mock sky maps $T_{\text{sky}}(\nu_j, l, b)$ at $\nu = 0.1$ and 0.5 MHz, respectively, as examples. For sky maps at $\geq 1$ MHz, we refer interested readers to Figure 8 of Cong et al. (2021).

We demonstrate the reconstruction of electron density distribution from the mock maps. The sky is pixelized with the HealPix scheme, with NSIDE = 32, so there are a total of 12,288 pixels for the full sky. Better performance is expected if observations have higher resolutions. Along each pixel direction, the LOS is divided into $N$ bins, each with an emissivity and electron density. The emissivity that is dominated by the synchrotron is assumed to be known. The sky brightness temperature of this LOS is modeled as

$$T_{\text{sky}}(\nu_j, l, b) \approx \sum_{i=1}^{N} \tilde{\epsilon}_i(\nu, R, Z) \Delta \xi_i \times 1 - \exp(-\Delta \tau_{i-1}) \exp(-\Delta \tau_i) + T_E^{\text{iso}}(\nu) \exp(-\tau_N), \quad (4)$$

where $\tilde{\epsilon}_i$, $\Delta \xi_i$, and $\Delta \tau_i$ are the mean emissivity, length, and optical depth of the $i$th bin, and

$$\Delta \tau_{i-1} = \sum_{i'=1}^{i-1} \Delta \tau_{i'}; \quad (5)$$

$T_E^{\text{iso}}(\nu)$ is the isotropic extragalactic background used in ULSA model, given by $T_E^{\text{iso}} = 1.2(\nu/$GHz$)^{-2.58}$ K (Seiffert et al. 2011).

We make log-spaced bins along the LOS, starting from $\nu_{\text{min}} = 0.005$ kpc with a step interval of $\Delta \nu = 0.2$, as the brightness temperature is related to the exponential of optical depth. We have limited the LOS to a distance at most 20 kpc from the Galactic center. The free–free optical depth of each bin is

$$\Delta \tau_i = \alpha_{\nu_i} \Delta \xi_i = 3.28 \times 10^{-7} \frac{T_e}{(10^4 \text{ K})}^{-1.35} \left( \frac{\nu}{\text{GHz}} \right)^{2.1} \left( \frac{n_e^2}{\text{cm}^{-6}} \right) \Delta \xi_i \text{ cm pc}^{-1}. \quad (6)$$

If $T_e$ is known, then the reconstructed quantity is $\langle n_e^2 \rangle_i$. Here the symbol $\langle \cdot \rangle$ denotes the mean inside each bin. From $\langle n_e^2 \rangle_i = F_{\text{flux}} / \langle n_e \rangle_i^2$, and if the fluctuation parameter$^6$ $F_{\text{flux}}$ is further known, it is translated into $\langle n_e \rangle_i$. For demonstration purposes we assume $T_e = 8000$ K (Reynolds 1990) and

$^6$ When converting the $\langle n_e^2 \rangle_i$ into $\langle n_e \rangle_i$, we define the fluctuation parameter $F_{\text{flux}} = \langle n_e^2 \rangle_i / \langle n_e \rangle_i^2$. It is a dimensionless and phenomenological parameter that describes the connection between the electron density and the free–free absorption strength, without specifying a physical picture of the electron distribution. In NE2001, the physically motivated fluctuation parameter is defined as $F_{\text{NE2001}} = \zeta \nu^{\beta} \nu^\omega \nu^{\beta'}/\nu_0^{\beta'}/\nu$, where $\nu$ (to avoid confusing the variable emissivity, in this paper we replace the $\nu$ in NE2001 paper with $\nu$) describes the electron fluctuations inside electron clouds, $\zeta$ describes the cloud-to-cloud fluctuations, $\nu$ is the filling factor of electron clouds, and $\nu_0$ is their outer scale. Therefore $F_{\text{flux}}$ performs as $\nu^{\beta'/2} F_{\text{NE2001}}^{\nu'/(\gamma'/2 - 1 + \omega)}$ in the formula. We have checked that when $\nu_0 = 1$ pc is adopted as in NE2001, the same $F_{\text{flux}}$ and $F_{\text{NE2001}}$ values give the same conversion factors. For this reason, we keep the dimensionless definition of the fluctuation parameter as it is only used to display the reconstruction results more intuitively.
$F_{\text{fluc}} \equiv 3.0$ is known, to show the reconstructed electron density as result. Otherwise, our method only reconstructs a combined quantity $T_e^{-1.35} F_{\text{fluc}} \langle n_e, l \rangle^2$. In observations $T_e$ varies between 5500 and 20000 K (Reynolds 1990) or 6000–10000 K (Haffner et al. 2009 and references therein), and $F_{\text{fluc}}$ can be $\geq 3$ (Gaensler et al. 2008). We assume that the emissivity $\epsilon(\nu, R, Z)$ is known and takes the form as given by Equation (3).

For the multifrequency maps, we assume the frequency interval is 0.1 MHz for the frequency range of 0.1–1.0 MHz, and 0.2 MHz for the frequency range of 1.0–10.0 MHz, so there are a total of 55 mock maps at 55 frequencies. We treat $\langle n_e, l \rangle$ of each bin as an unknown parameter. By comparing the multifrequency sky brightness temperature of Equation (4) that is a function of $\langle n_e, l \rangle$ with observations (mock maps), we can simultaneously find $\langle n_e, l \rangle$ of all bins that minimize

$$
\chi^2(l, b) = \sum_{j=1}^{N_{\text{freq}}} \left[ \frac{T_{\text{sky}(\nu_j, l, b)} - T_{\text{sky} \text{obs}(\nu_j, l, b)}}{\sigma_{\text{noise}}} \right]^2,
$$

using the Monte Carlo Markov Chain (MCMC) procedure (Foreman-Mackey et al. 2013), where $N_{\text{freq}}$ is the number of frequency bins.

In this work we use fixed grids for solving the electron densities, which may not accurately capture some small and dense clumps. The absorption by a single dense clump may obscure the whole LOS, but if we already know some information about these clumps, i.e., their location, size, and density from other observations, we can use such information to improve the fitting. If a bin contains a dense clump, then the density contribution from this clump to the bin is known, leaving only the diffuse electron density as an unknown quantity.

Figure 1. The mock sky maps at 0.1 MHz (top) and 0.5 MHz (bottom) used in this Paper.

$$
\sigma_{\text{noise}} \sim \frac{D^2 T_{\text{sky}}}{A_{\text{eff}} \sqrt{N_d(N_d - 1) \tau_{\text{obs}} \Delta \nu}} = \frac{4\pi T_{\text{sky}}}{\theta_{\text{res}}^2 \sqrt{N_d(N_d - 1) \tau_{\text{obs}} \Delta \nu}},
$$

Regarding the noise level of the observations, at a low frequency the system temperature is dominated by the sky temperature (Shi et al. 2022b).
where \( A_{\text{eff}} = \frac{\lambda^2}{4} \) is the effective area of the antenna with physical size \( \ll \lambda \), \( N_a \) is the number of antennas, \( D \) is the baseline length, and \( \theta_{\text{res}} \sim \frac{\lambda}{D} \) is the angular resolution; \( t_{\text{obs}} \) is the integration time and \( \Delta \nu \) is the width of the frequency point. For \( \theta_{\text{res}} = 1.0^\circ \), \( N_a = 8 \) and \( \Delta \nu = 0.1 \) MHz, we have \( \sigma_{\text{true}}/T_{\text{sky}} \sim 1\% \) if \( t_{\text{obs}} = 0.1 \) yr and \( \sigma_{\text{true}}/T_{\text{sky}} \sim 10\% \) if \( t_{\text{obs}} = 0.5 \) day. We adopt a 1\% noise level for all simulations in this Paper.

### 3. Results

In Figure 2 we show the minimum \( \chi^2 \) normalized by the number of frequency bins for each LOS as the HealPix sky map (top), and the probability density of their distribution (bottom). About 90\% of the LOS have \( \chi^2_{\text{min}}/N_{\text{freq}} < 1.5 \), so the MCMC fitting does indeed have a good performance. About 2\% of the LOS have \( \chi^2_{\text{min}}/N_{\text{freq}} > 3.0 \), for displaying purposes they are not shown in Figure 2. The distribution of \( \chi^2_{\text{min}}/N_{\text{freq}} \) is well fitted by a Gaussian function with the peak 1.72, the center 1.12, and the radius 0.23. We check that most LOS with \( \chi^2_{\text{min}}/N_{\text{freq}} > 2.0 \) have dense clumps. As described in Section 2, if there is a dense clump in a LOS, we assume that its contribution to the density in the relevant bin is known, and only fit the density of diffuse electrons in that bin. This can improve the reconstructed density profile; see Figure 4 later on. But it cannot solve the problem completely. Because usually the size of the dense clump is much smaller than the bin width, its absorption to the emissivity in the bin also depends on its exact location inside the bin, but this information is not used in the fix grid computation. In the northern hemisphere between \(-30^\circ \leq l \leq 30^\circ \) there is the Loop I bubble, it is a low-density cavity with huge angular size; the absorption within the bubble is weak; see the sky maps in Figure 1. Moreover, such a bubble has an edge that is modeled as a sharp break in the NE2001. The sharp break feature is hard to capture because of the bin resolution limit. Because of the above reasons, the \( \chi^2_{\text{min}}/N_{\text{freq}} \) in this region is biased to higher values.

We then compare our reconstructed electron density with NE2001.\(^7\) The same fluctuation parameter of 3.0 is assumed for all gas components in order to translate the reconstructed \( \langle n_e^2 \rangle \) into the mean electron density that is more intuitive for displaying. We plot the mean absolute value of relative errors between the reconstructed electron density and that of NE2001 \( |\epsilon_{\text{rel}}| \) for all LOS as a map in the top panel of Figure 3. This is natural because in the Galactic plane the absorption is stronger, making it possible to reconstruct the electron density profile more accurately. High Galactic latitude regions have larger errors, not only because the absorption is weaker, but also because the extragalactic background has a larger fraction in the total received flux. Around \( l \approx 330^\circ \); there is a patch where the relative error is much higher than the surrounding regions. We check that in NE2001 there is a low-density void with density as low as \( 0.001 \) cm\(^{-3} \). In our reconstruction usually the low-density regions have larger relative errors. Nevertheless, our reconstruction has good performance, considering the fact that in our method not only each LOS, but also the electron density in each bin of a LOS, are all independent of the others.

We show the reconstructed 1D density profiles for some LOS in Figure 4 as examples. In the Galactic plane our algorithm correctly reproduces the electron density profiles up to \(~10\) kpc, and in high Galactic latitude regions up to \(~3\) kpc, although there are deviations on small scales. For all of these LOS, within \(~0.03–0.2\) kpc, the electron density is relatively low, indicating that our Sun is located in a bubble. For the LOS pointing to \( l \approx 90^\circ \) however, the low-density environment extends to \(~2\) kpc; this is the low-density region simply called “LDR” in the NE2001 model (Toscano et al. 1999; Cordes & Lazio 2003). A LOS penetrating the Gum Nebula is shown in panels (g) and (h). Panel (g) is the result directly reconstructed, and panel (h) is the result when the location, size, and density of the Gum Nebula are known and its density is added to the relevant bin. Indeed, the density profile in panel (h) is much improved than in panel (g). Nevertheless, except for the Gum Nebula, other dense clumps have much smaller angular sizes on the sky; the large-scale morphology of our reconstructed maps almost does not change, if we simply remove those LOS penetrating dense clumps.

With the mock full-sky map, we can reconstruct the 3D distribution of Galactic electrons by synthesizing all LOS. The tomographic reconstruction result of our Sun’s vicinity is shown in the top panel of Figure 5, and for comparison the original NE2001 model distribution is shown in the bottom panel. This figure uses 12,288 LOS, and the density volume is a heliocentric cuboid of \( 6 \times 6 \times 4 \) kpc\(^3\), with pixel size \( 0.05 \) kpc. However, the poorest resolution at the edge is \(~0.5\) kpc, limited by the bin width. The reconstructed volume map is also shown in a video linked to Figure 5. Both the reconstructed density and the NE2001 density are smoothed by a Gaussian kernel with radius \( 0.05 \) kpc. Our method reconstructs major features from NE2001, including the LB, the LSB, the Gum Nebula, and so on.

In our reconstructed electron density field, there are some streaks as shown in the top panel of Figure 5. This is a consequence of our solving the density along the radial direction from our position as the center. Since we adopt a grid along the radial direction, the grid cells will have shapes that either appear elongated if their size along the radial direction is larger than the tangential direction, or obl ate if the reverse is true. In our case we adopted a log-spaced grid, which is good for the numerical problem, but at the far end the streak is more apparent. Increasing the number of LOS and reducing the bin width can relieve the streak problem. However, if the bin width...
is too small then the noise of each bin is large since the contribution of a single bin to the total absorption is small.

In Figure 6 we show the probability density as a function of \(|f_{\text{err}}|\) and \(n_e\), where \(|f_{\text{err}}|\) is the absolute value of the relative error between the top and bottom panels of Figure 5 for each pixel, and \(n_e\) is the density of the relevant pixel in the bottom panel of Figure 5. About 70% pixels have \(|f_{\text{err}}| < 30\%\), and the median of \(|f_{\text{err}}|\) is 17%. About 8% pixels have \(|f_{\text{err}}| > 100\%\), most of them are low-density regions; see the top left part of Figure 6. This is not surprising because the absorption in low-density regions is weaker, therefore hard to constrain the density accurately. Considering the above, any structures with size comparable to the resolution limit and/or with density contrast comparable to the uncertainty level are hard to see from the figure.

Moving further from our vicinity on to larger scales, we meet two spiral arms that bracket our Sun, as shown in the reconstructed electron density on the Galactic plane in Figure 7. The reconstructed electron map clearly shows the Carina-Sagittarius arm in the direction toward the Galactic center, and the Perseus arm in the anticenter direction. Previously the spiral arms have been mapped via neutral hydrogen, CO (trace the molecular hydrogen, Englmaier et al. 2011; Nakanishi & Sofue 2016), a dust extinction survey (Rezaei et al. 2018), and other tracers for high-mass star formation (Hou & Han 2014). The ultralong-wavelength observations will allow reconstruction of the diffuse free electrons in the spiral arms.

4. Summary and Discussions

The ultralong-wavelength radio spectrum below \(\sim 10\) MHz has largely been constrained from the ground and space at poor spatial resolution. Some space missions have been proposed to obtain high-resolution sky maps in this band. In this Paper we
proposed that, thanks to the frequency-dependent absorption to the electromagnetic wave, from the upcoming ultralong-wavelength observations the 3D structures of the Galactic electrons can be reconstructed. Using mock sky maps we performed simulations to prove the feasibility of this method.

There are dense clumps in the sky, and they often have active star formation and also emit synchrotron radiation. If such emission is not accurately known, then it may cause errors in reconstruction. To assess this uncertainty, we consider the nearby Gum Nebula. According to Woermann (1998), at 408 MHz the Gum Nebula has a typical surface brightness temperature of roughly 10 K, and the typical spectrum index derived from 408 and 2326 MHz is roughly $-2.5$. Taking the Gum Nebula as a sphere with radius 0.14 kpc and electron density $0.43 \text{ cm}^{-3}$, at a distance 0.5 kpc from us (Cordes & Lazio 2002), and the emissivity is uniformly distributed in the sphere, we expect $\sim 5 \times 10^4$ K surface brightness at 1 MHz if $F_{\text{flux}} = 3.0$. This is actually smaller than the mean brightness of the Galactic plane in our sky model at the same frequency. According to this simple estimate, a large fraction of the clump emission might be absorbed by itself, since it has density much higher than the ambient medium. So at least in this case, the influence of the clump emission is modest.

Our synchrotron emissivity model is cylindrical. However the real emissivity could have noncylindrical large-scale structures, induced by the spiral structures in the regular magnetic field (Orlando & Strong 2013b). We have tested that for such a spiral emissivity model, the electron density can be still reconstructed and all conclusions do not change. The reconstruction errors could come from the unknown random small-scale fluctuations in the synchrotron emissivity. To estimate its impact, we add fluctuations to the emissivity model of Equation (3), and then reconstruct the electron density profile assuming the small-scale fluctuations are not present. Motivated by the measured angular power spectrum of Galactic synchrotron radiation $C_l \propto l^{-2}$ (Iacobelli et al. 2013), we model the small-scale fluctuations of emissivity to
be Gaussian with the power spectrum
\[ P_\nu(k) \propto k^{-3}, \quad k > k_{\text{min}}, \]  
(9)
where \( k \) is the spatial wavenumber. We assume that fluctuations only exist on scales smaller than \( 2\pi/k_{\text{min}} \) (\( k > k_{\text{min}} \)). On larger scales, we assume the Equation (3) correctly describes the cosmic-ray emissivity. We normalize \( P_\nu \) so that the emissivity has a mean relative fluctuations level \( \sim 50\% \) at the smoothing scale 0.1 kpc. That is, when the fluctuations field following the power spectrum of Equation (9) is added to the cylindrical emissivity field, and the new emissivity field is smoothed with a Gaussian kernel with radius = 0.1 kpc, then \( \sqrt{\langle \delta \epsilon \rangle^2}/\epsilon \sim 50\% \). We found that, despite this perturbation, which induces errors in the reconstruction at small scales, the reconstruction results remain largely unchanged at large scales. The results of this reconstruction are shown in Figure 8, for \( k_{\text{min}} = 5.0 \text{ kpc}^{-1} \) and \( k_{\text{min}} = 0.5 \text{ kpc}^{-1} \), respectively.

Our method directly reconstructs \( \langle n_e^2 \rangle \) of each bin, a quantity that produces absorption equivalent to real electron distribution inside this bin. Since \( \langle n_e^2 \rangle = F_{\text{fluc}} \langle n_e \rangle^2 \), we actually cannot distinguish the cases if a region has a smaller electron density but larger fluctuation parameter, or the reverse, unless the \( F_{\text{fluc}} \) is derived from other methods. Just for convenience of translating the results into a more intuitive quantity \( \langle n_e \rangle \) we assume a constant \( F_{\text{fluc}} \). The \( F_{\text{fluc}} \) may vary bin by bin since the fluctuations of the WIM can depend on locations (Reynolds 1991a; Gaensler et al. 2008); however we have checked that it will not change our reconstruction of \( \langle n_e^2 \rangle \). Just, if the correct \( F_{\text{fluc}} \) is not known, the translated \( \langle n_e \rangle \) would deviate from the real mean electron density. For example, in Gaensler et al. (2008) they investigated the volume filling factor of electron clouds. If the electron fluctuations inside clouds are negligible, then the fluctuation parameter can be well approximated as the reciprocal of the filling factor. If this case the fluctuation parameter is \( \sim 25 \) in the Galactic plane and \( \sim 3 \) at \( |Z| = 1.4 \text{ kpc} \). For such a fluctuation parameter then our \( \langle n_e \rangle \) at the Galactic plane is overestimated by a factor \( \sim 3 \). In Ocker et al. (2021) they considered a complicated electron cloud model rather than uniform density. According to the observations of scattering and dispersion measures of the Galactic pulsars, they found that the fluctuation parameters at the low Galactic latitudes and inner Galaxy are \( \gtrsim 10\text{--}1000 \) times larger than the typical thick disk at high latitudes. Considering this, the error on the reconstructed electron density by assuming a constant fluctuation parameter can even be much larger, particularly at low Galactic latitudes and for the LOS toward the inner Galaxy. Future studies trying to disentangle the electron density would merit a more sophisticated model for the spatial variation in the fluctuation parameter. Perhaps values independently inferred from pulsar observations along similar LOS could be employed, although this would be limited by the sparseness of pulsar spatial distribution. In some biased regions, the WIM has larger fluctuations. The synchrotron
emissivity from such regions will be also biased, because usually such regions are more close to the sources of the cosmic ray—supernova remnants. We tested that, if the biased emissivity is correctly involved in the emissivity model, then no matter if the fluctuation parameter is biased or not in the same regions, the \( \langle n_e^2 \rangle \) can still be correctly reconstructed. However if the biased emissivity is missed in the model, then like what we have discussed in the last paragraph, it would induce small-scale uncertainties on the reconstructed \( \langle n_e^2 \rangle \) and the final \( \langle n_e \rangle \).

Despite some modeling uncertainties, the reconstruction from ultralong-wavelength observation is fairly robust. Once the ultralong-wavelength sky is systematically surveyed, we will gain new knowledge about our residence in the Milky Way, and pin down the long-lasting debate on the distance of...
the NPS. It will be an important step in obtaining a synthetic picture of our Galaxy.

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![Figure 8](image-url)

**Figure 8.** The emissivity with small-scale fluctuations and its known smooth component (top panels), and the reconstructed electron density profiles for the LOS with $l = 2\degree$ on the Galactic plane (bottom panels). For left panels we adopt $k_{\text{min}} = 5.0$ kpc$^{-1}$ and for right panels we adopt $k_{\text{min}} = 0.5$ kpc$^{-1}$. All density profiles, including the NE2001, have been smoothed by a Gaussian kernel with radius equal to 1 bin.
