On the Possibility of Creating a UCN Source at a Periodic Pulsed Reactor

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Abstract—The possibility of creating a UCN source at a periodic pulsed reactor is considered. It is shown that the implementation of the principle of time focusing based on nonstationary neutron diffraction and the idea of pulse filling of the UCN trap allow one to create a sufficiently intense UCN source at a periodic pulsed reactor.

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INTRODUCTION

As is known, ultracold neutrons (UCNs) were discovered by F.L. Shapiro and coworkers [1]. In an experiment at a reactor with a mean power of 6 kW, they observed neutrons capable of being in a closed volume (a curved tube) for several seconds. For the history of this discovery, see [2]. In world literature, Dubna (Russian Federation) is rightfully considered the birthplace of UCN.

To date, there are a number of UCN sources in the world [3, 4], and several more of them are under construction. Unfortunately, there is no UCN source in Dubna, although attempts have been made to construct one [5]. The reasons for this are largely due to the features of the pulsed reactor at the JINR Laboratory of Neutron Physics [6, 7]. Its mean power of 2 MW is relatively low for creating a continuous UCN source, and the repetition rate of 5 Hz is too high to accumulate neutrons produced in each individual pulse. At the same time, the pulsed flux of thermal neutrons is very large, since the interval between pulses significantly exceeds their duration, which is about 350 μs. It is obvious that the pulsed UCN flux from a thin moderator can also be quite significant under certain conditions. Consequently, the problem is how to take advantage of this circumstance.

A possible solution to the problem was proposed by Shapiro [8, 9]. It consists in filling the trap with UCNs only during the pulse and efficiently isolating it the rest of the time. In the ideal case of no losses, the UCN density in the trap corresponds to the peak neutron density, which may exceed the time-average density by several orders of magnitude.

This idea cannot be feasible due to the fact that in practice the trap cannot be close to the moderator due to biological shielding. In this case, it is necessary to provide a transport neutron guide several meters long feeding the trap. Placing an isolating valve next to the moderator, the UCN source, results in the neutron guide becoming part of the trap. Because of the small transverse size of the neutron guide, the frequency of collisions of neutrons with its walls is quite high. This greatly reduces the storage time of UCNs in the system of trap and neutron guide and, accordingly, noticeably decreases the gain factor. Placing a valve at the entrance to the trap at a distance of several meters from the source is useful only if the sources have a low repetition rate [10–12]. For sources with a repetition rate of a few hertz, the spread of the UCN transit times exceeds the intervals between pulses, and a valve at the entrance to the trap is useless.

This problem can be solved by using a device that acts as a time lens and forms a time image of the source in the immediate vicinity of the trap. In this case, the duration of the pulse of the time image can be on the same order of magnitude as the duration of the true pulsed UCN flux in the source. A time lens can play here about the same role as an illuminator in a microscope, which allows one to combine the object under study with the image of the light source, while it turns out to be impossible to combine it with the true source.

This proposal was formulated in [13, 14], where the principles of forming a time image of a point source were discussed. Note that in the literature, the term of "time lens" is used in relation to devices with significantly different functions. While Frank and Gähler [13, 14] dealt with a lens that formed a given time distribution...
of the neutron flux at a known point in space, Rauch and coworkers [15–17] discussed the possibility of compressing the velocity interval at the observation point, that is, an additional monochromatization of the neutron beam.

The purpose of this work is to show that the principle of time focusing can serve as a methodological basis for constructing intense UCN sources at periodic pulsed reactors.

THE OPERATING PRINCIPLE
OF A TIME FOCUSING SOURCE

Time Lens and Methods of Changing the Neutron Energy

The main element of the UCN source is a time lens, the action of which is based on controlling the velocity and, accordingly, the neutron energy. Apparently, there is a wide range of possibilities for this.

In [15–17], it was proposed to provide the effect on the motion of the neutron with the help of the appropriately formed configuration of a stationary or alternating magnetic field. The change in the velocity of neutrons is achieved due to the action of a force on them,

\[ \mathbf{F} = \mathbf{V} (\mu \mathbf{B}) \]

where \( \mu \) is the magnetic moment of the neutron and \( \mathbf{B} \) is the magnetic induction. In this case, a magnetic time lens that forms a magnetic field is a device extended along the direction of the neutron beam.

In contrast to these proposals, it was assumed in [13, 14] that the lens is a local device. We explain the principle of its operation (see Fig. 1a).

Let neutrons be emitted from a point \( x = 0 \) in the positive direction of the \( X \) axis at the time instant \( t = 0 \); their velocities \( v \) are distributed in a certain range of values. The time \( t_f \) of their arrival at the observation point \( x = L \) is distributed in the interval \( t_1 < t_L < t_f \). Suppose that a time lens is located at the point \( x = a \); it changes the neutron energy by an amount of \( \Delta E (t) \) according to a given time law in the time interval \( t \) in \( t_1 < t < t_f \). Then, the dependence \( \Delta E (t) \)

\[ \Delta E (t) = \frac{1}{2} \left( \frac{b}{t_0 - t} \right)^2 \left( \frac{a}{V_a} \right)^2, \quad t = \frac{a}{V_a}, \quad t_1 < t < t_2 \]

\[ \Delta E (t) = \frac{m}{2} \left( \frac{b}{t_0 - t} \right)^2 \left( \frac{a}{V_a} \right)^2, \quad t = \frac{a}{V_a}, \quad t_1 < t < t_2 \]

where \( m \) is the neutron mass.

Time focusing is accompanied by a change in the velocity distribution of neutrons just as in optics the imaging is associated with the transformation of the angular distribution of beams. At the same time, the duration of a time pulse is transformed, which allows one to introduce the concept of time magnification \( M \).

Suppose that the lens action period \( T = t_f - t_1 \) coincides with the repetition period of the source pulses. Then, the repetition rate is \( f_s = T^{-1} \); the distance \( a \) from the source to the lens and the range of received neutron velocities \( V_{a\min} \leq V \leq V_{a\max} \) are related

\[ T = \frac{a}{V_{a\min}} - \frac{a}{V_{a\max}}. \]

The distance \( a \) and the maximum velocity \( V_{a\max} \) captured by the lens determine the minimum time of flight from the source to the lens

\[ t_{a\min} = t_1 = \frac{a}{V_{a\max}}. \]

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Fig. 1. Scheme of action of a time lens. (a) A lens that does not change the mean neutron energy and (b) a lens that moderates neutrons.
The minimum velocity captured by the lens and the maximum time of flight of the distance \( a \) are determined as follows

\[
V_{\text{min}} = \frac{aV_{\text{max}}}{TV_{\text{max}} + a}, \quad t_2 = \frac{a}{V_{\text{min}}}. \tag{5}
\]

The velocities \( V_{\text{max}} \) and \( V_{\text{min}} \) determine the magnitude of the neutron flux transformed by the lens. The range of captured velocities can be called the velocity aperture of the lens. The magnitude of the maximum neutron velocity after passing through the lens \( V_{\text{max}} \) is limited by the properties of the trap that accumulates the neutrons. By setting this velocity, the minimum time of flight of the second section of the neutron guide \( t_{\text{bmin}} = b/V_{\text{max}} \) is also determined. Obviously, the neutrons that spent the most time in the first section of the flight should have the highest velocities after passing through the lens. Therefore, the total time of flight \( t_0 \) is

\[
t_0 = t_{\text{max}} + t_{\text{bmin}} = \frac{a}{V_{\text{min}}} + \frac{b}{V_{\text{max}}}. \tag{6}
\]

The maximum energy transfer is determined by the velocities of \( V_{\text{max}} \) and \( V_{\text{min}} \)

\[
\Delta E = \frac{m}{2} \left[ V_{\text{max}}^2 - V_{\text{min}}^2 \right]. \tag{7}
\]

The most important problem is the problem of how the neutron energy changes according to a given time law (2). The authors of [13, 14] proposed to turn to quantum nonstationary phenomena. At that time it seemed an attractive but somewhat extravagant possibility. The phase modulation of the neutron wave (the phase diffraction grating moves across the direction of its propagation) and the resonant spin-flip of the neutron in a magnetic field were considered. Both of these proposals were quite bold, since the phenomenon of nonstationary diffraction of neutrons by a moving grating was only an object of theoretical consideration [18], and the change in the neutron velocity during resonant spin-flip, which was observed earlier at the limit of experimental possibilities [19], was more convincingly demonstrated in [20] only after the publication of [13].

A short time later, the nonstationary UCN diffraction by a moving grating was observed experimentally [21]. Sometime later, the time focusing was also demonstrated in experiments with a moving grating [22, 23]. The possibility of time focusing based on resonant spin-flip of the neutron was experimentally confirmed [24, 25].

**Moderation of Neutrons by a Lens**

Initially, the action of a time lens was considered as the time of arrival of the neutron bunch at the point of the image, and the problem of the ratio of the neutron velocities in the region of neutron generation and at the point of the image was not discussed. In particular, it is clearly seen from Fig. 1a, as in [14], that the mean velocity (central line in Fig. 1a) is unchanged, but the velocity distribution changes. However, there may be a case when the lens can change the mean velocity and mean energy of neutrons (Fig. 1b). This possibility was mentioned in [14]. It should be noted that a time lens based on quantum energy transformation consistently affects the neutrons arriving at it. Apparently, Liouville’s theorem on the invariability of the phase density of particles is inapplicable to neutron moderation due to its nonpotential nature. This problem needs to be analyzed more carefully.

**Neutron Trap and Accumulation Effect**

Now, we explain the idea of Shapiro [9] about the accumulation of neutrons from a pulsed source. Suppose we have a trap of ultracold neutrons, and a periodic pulsed flux with pulse duration \( \tau \) enters the entrance hole of area \( s \). Then, the number of neutrons entering the trap in a pulse is

\[
N_m = F_s \tau, \tag{8}
\]

where \( F_s \) is the neutron flux averaged over the pulse duration. Assume that the entrance window of the trap is open only during the pulse, and the rest of the time is blocked by some ideal shutter. If the number of neutrons absorbed in the trap walls during the time between pulses is relatively small, then the neutron flux in the trap increases from pulse to pulse until it reaches a certain equilibrium value \( \Phi \). Neglecting the probability of neutron decay during the time of the equilibrium setting, we assume that the channels of UCNs escape from the trap are only absorbed in its walls and escape through the entrance hole during the time when it is open. Equating the number of neutrons entering the trap in one pulse (8) to the number of neutrons leaving it during the duration of one cycle \( T \), we obtain

\[
F_s \tau = \Phi (s \tau + \Sigma T \mu), \tag{9}
\]

where \( \Sigma \) is the area of the trap surface, \( \mu \) is the UCN absorption coefficient upon collision with the trap wall, and \( T \) is the pulse repetition period. For the equilibrium flux density, we have

\[
\Phi = \frac{F_s}{1 + (\Sigma \mu T / s \tau)}. \tag{10}
\]

From formula (10), it is easy to obtain expressions for the ratio of the flux density in the trap to the mean input flux of the trap \( \Phi / (\langle F \rangle) = F_s \tau / T \), that is, the gain factor.

\[
G = \frac{\Phi}{\langle F \rangle}, \quad G = \frac{s T}{s \tau + \Sigma \mu T}. \tag{11}
\]
Formula (11) somewhat differs from formula (8) in [9], which is associated with two circumstances. First, the possibility of neutron outflow from the trap to the user is not considered in (11). Second, the authors of [9] compare the neutron flux density in the trap in the case of the pulsed and stationary filling methods, rather than the ratio of the fluxes in the source and the trap.

The absorption coefficient of neutrons reflected from the wall $\mu$ is determined by the ratio $\eta = W/E_b$ of the imaginary and real parts of the effective potential of interaction between neutrons and the trap material

$$U = E_b - iW, \ E_b = \frac{2\pi h^2}{m} \rho b, \ W = \frac{\hbar}{2} \rho \sigma_{\text{loss}} v.$$  \hfill (12)

Here, $b$ is the neutron scattering length on the nuclei of the trap material, $\rho$ is the number of nuclei per unit volume, $\sigma_{\text{loss}}$ is the cross section of all processes in the trap material that lead to the loss of UCNs from it, and $v$ is the neutron velocity. The real part of the potential $E_b$ is usually called the boundary energy of the substance.

For an isotropic flux, the angular-averaged absorption coefficient $\mu$ [9, 26] is

$$\mu = \frac{2\eta}{y^2} \left( \arcsin y - y \sqrt{1 - y^2} \right), \hfill (13)$$

where $y = \sqrt{E/E_b}$ and $E$ is the neutron energy. If the energy $E$ is not too close to the boundary energy, then the values $\mu$ and $\eta$ in a wide range of variation of the parameter $y$ differ only slightly. For a number of good materials, the value $\eta$ can be $\eta = (3-5) \times 10^{-5}$ [27–29] and the parameter $\Sigma_k T/3\pi$ in the denominator of Eq. (11) can be of the order of unity even for a trap of a large volume. In this case, the gain factor $G$ turns out to be of the order of $T/\tau$.

**Transformation of the Neutron Pulse Time: Time Magnification**

Since the gain factor $G$ significantly depends on how long the trap valve is open, the problem of the duration of the pulse generated by the time lens turns out to be very important. The authors of [14] assumed that the formula for a thin lens is used with a relatively small energy transfer by the lens $|\Delta E| \ll E$

$$M = \tau_{\text{fin}}/\tau_{\text{ini}}, \quad M = b/a,$$  \hfill (14)

where $\tau_{\text{ini}}$ and $\tau_{\text{fin}}$ are the durations of the initial time pulse and its image formed by the lens. However, this is true only when the focusing conditions are symmetric (Fig. 1a), that is, when some of the neutrons are accelerated, and the same number decelerate. This is not the case for the moderating lens. The problem of the relationship between the duration of a neutron flux pulse generated by a neutron source and its time image formed by a lens based on a nonstationary energy transformation is considered in more detail in [30]. It was shown that neutrons emitted not exactly at the instant of time $t = 0$, but at a moment $\delta$ close to it, arrive at the observation point $L = a + b$ not at the calculated time $t_0$, but at the instant

$$t = t_0 + \delta b \left( \frac{V_a}{V_b} \right)^3.$$  \hfill (15)

Assuming that the lens action period (time aperture) $t_1 - t_0 \ll t_0$ is small, which is similar to the paraxial approximation in optics, it is easy to obtain the following relation for the time magnification

$$M = b \left( \frac{V_a}{V_b} \right)^3,$$  \hfill (16)

where $V_a^0$ and $V_b^0$ are the mean velocities in sections $a$ and $b$, respectively.

Thus, the duration of the image formed by the time lens depends not only on the geometric factor $b/a$, but it also significantly depends on the ratio of the initial and final velocities, and the shape of the time pulse can also be determined by the velocity spectra before and after passing through the lens. A significant time magnification for a moderating lens significantly limits the gain factor (11) due to the accumulation mode of the trap.

**Nonstationary Neutron Diffraction as a Physical Basis for Energy Transformation by a Time Lens**

The action of a local time lens is based on the possibility of a strictly defined and time-variable energy transfer to a neutron. The two energy transfer methods based on the use of quantum nonstationary phenomena are mentioned above. They are a spin-flip of the neutron by a spin flipper (a prerequisite is an alternating magnetic field with a circular frequency $\Omega$) and the flux modulation by some device that provides amplitude or phase modulation of the flux with frequency of $f = \Omega/2\pi$. In both cases, energy $\Delta E = h\Omega$ equal to or a multiple of an energy quantum is transferred to the neutron. It could be assumed that the flipper is more efficient than the modulator. Indeed, all neutrons, whose spin turned out to be reversed under the action of an alternating field with a frequency of $\Omega(t)$ in the presence of a relatively slowly varying magnetic field $B(t)$, change their energy by an amount of $\Delta E(t) = 2\mu B(t) = h\Omega(t)$.

For periodic modulation of the flux, the resulting state is a superposition of a large number of waves with different amplitudes $A_s < 1$ and discrete frequency values of $\omega_n = \omega_0 + n \Omega$, where $n$ is integer, regardless of whether such modulation is carried out using a fast
periodic chopper [31, 32] or a periodic structure moving across the beam [18]. Of the complete set of waves at each given moment of time, only one has an energy corresponding to the necessary condition for time focusing, and its amplitude depends both on the parameters of the modulator and, generally speaking, on the time-varying velocity of the incident neutrons. It would seem that the problem of choosing between two approaches to controlling the neutron energy should be solved in favor of the electromagnetic method.

However, there are two practically important factors; they are the maximum possible amount of transferred energy and the ease of quickly changing the required frequency. The latter should change from its maximum to its minimum value during the period of action of the source and lens and, at the same time, almost instantaneously reset again to the initial value at the end of each cycle. It turns out that both of these conditions are currently better met by a device based on the phenomenon of nonstationary diffraction by a moving structure.

It is also important that the phenomenon of nonstationary diffraction of neutrons by a moving grating has been sufficiently well studied theoretically and experimentally. We give the basic information about this phenomenon, which is necessary for what follows. There are several theoretical approaches to solving the problem of neutron diffraction by a moving grating. Here, we present a simplified version of the solution to this problem as in [18, 21, 33].

Let a plane neutron wave be incident on a thin grating, the grooves of which are oriented along the $y$ axis,

$$
\Psi_0(x, z, t) = \exp[i(k_{0x}x + k_{0z}z - \omega_0 t)],
$$

where $k_{0x} = mv_{0x}/h$, $k_{0z} = mv_{0z}/h$, $v_{0x}$, and $v_{0z}$ are the tangential and normal components of the velocity, respectively; $\hbar$ is the Planck constant; and $\omega_0 = \hbar k_0^2/2m$ and $k_0 = (k_{0x}^2 + k_{0z}^2)^{1/2}$ are the frequency and wave number.

Assume that the grating moves with velocity $V_g$ in the positive direction of the $x$ axis. Solving the problem of diffraction in a moving system, in which the grating is at rest, one can find the projections of the wave vectors of all diffraction orders and their amplitudes $\alpha_n$,

$$
\alpha_n = \frac{1}{d} \int_0^d H(x) \exp(-in\omega_0 x) dx,
$$

where $g_0 = 2\pi/d$, and $H(x) = H(x + d)$ is the periodic transmission function of the grating. The wave function of diffracted neutrons is found by transform-

the found solution back into the laboratory coordinate system. It is

$$
\Psi(x, y, z) = \sum_{n=\pm \infty} A_n \exp[i(k_{nx}x + k_{nz}z - \omega_n t)],
$$

where

$$
k_{nx} = k_{0x} + g_n,
$$

$$
k_{nz} = \left[k_{0z}^2 + 2(k_{V} - k_{0x}) g_n - g_n^2\right]^{1/2},
$$

$$
g_n = n g_0
$$

is the magnitude of the reciprocal lattice vector; $d$ is the spatial period of the grating; $n = 0, \pm 1, \pm 2, ...$ are integers; and the frequencies of the diffracted waves $\omega_n = \omega_0 + n\Omega$ are characterized by spectral splitting

$$
\Omega = 2\pi f, \quad \text{where} \quad f = V_g / d.
$$

The amplitudes of the diffracted waves in the laboratory system are determined from the flux conservation condition

$$
A_n = \alpha_n k_0^2 [(k_0^2 + 2k_V g_n)]^{1/4}.
$$

A very satisfactory agreement with the predictions of such a theory was obtained in [21, 32]. As suggested in [18], the experiments were performed using a grating of a rectangular profile, for which the phase of the transmitted wave changed abruptly by $\pi$ every half period

$$
\Delta \varphi = k_{0z} (1 - n_{gr}) h = \pi.
$$

Here, $h$ is the height of the grating tooth and $n_{gr}$ is the neutron refractive index in the grating material. For such a $\pi$ phase grating, the amplitudes of even orders, including zero, are zero, and the amplitudes of odd orders are $\alpha_n = 2/\pi \alpha_n$, and they decrease with increasing order number $n$. However, this is true only when neutrons fall normally on the grating, which is possible only for a grating at rest since neutrons fall on the grating at a certain angle in the coordinate system of a moving grating. Consequently, the phase change is of a trapezoidal form due to different path lengths in the substance near the edges of the U-shaped teeth of the grating.

This circumstance was considered in [34], where the trapezoidal dependence of the phase on the coordinate is used to calculate the amplitudes of diffraction orders in accordance with (14) (see Fig. 2). The phase dependence profile is characterized by a geometric parameter

$$
C = (2h/d)(V_g - v_{0x})/V_{0z},
$$

which increases with increasing grating velocity and profile depth and decreasing period. When $C \ll 1$, the phase profile is close to rectangular, and when $C = 1$, it is of a triangular shape.

In a more rigorous approach, it is necessary to consider that waves propagating in a grating material of finite thickness can interact. To take this circumstance
into account, it is necessary to apply a dynamic approach to diffraction formulated in [35]. The authors of [35] concluded that for a given grating velocity and neutron energy, the ratio between the intensities of order substantially depends on the profile depth $h$. Therefore, if one disregards Eq. (22), it is possible to set the profile depth $h$ and the parameter $C$ according to the specific values of the grating velocity and the value of the transferred energy. A number of predictions in [35] found their confirmation in experiments [36, 37].

We also note that, although the dynamical theory predicts somewhat different values for the amplitudes of diffraction orders than the modified kinematic approach, both theories give qualitatively not too different results [35].

**UCN SOURCE FOR PERIODIC PULSED REACTOR**

A schematic diagram of a UCN source based on the principle of time focusing is shown in Fig. 3. The main moderator 1 is a source of a pulsed flux of cold neutrons. Thin converter moderator 2 serves as a pulsed UCN source. Its thickness is limited by the product of the minimum velocity captured by the lens by the required duration of the pulsed UCN flux. It can be separated from the volume of the mirror neutron guide 3 by a membrane transparent to UCN. Probably, the need to suppress the background excludes the possibility of using a straight neutron guide, but for now we ignore this circumstance.

Neutron lens 4 is a set of diffraction gratings located at the periphery of the rotating disk. The rotation period of the lens disk is equal to or a multiple of the repetition period of the pulsed neutron source. The rotation phase is synchronized with the source. The grating parameters are optimized to provide a focusing condition.

Omitting for now the practically important problem of the arrangement of the pulse valve at the entrance to the trap, we only analyze the transfer and transformation properties of the combination of neutron guides and a lens, the principle of which is based on nonstationary neutron diffraction by a moving grating. We give the results of some calculations that make it possible to estimate the possible parameters of such a source. To be specific, we proceeded from the parameters, which were typical of the IBR-2 reactor [6, 38]. In calculations, the neutron guide length is $L = 10$ m, and the distance from the source to the lens is $a = 6$ m.
ON THE POSSIBILITY OF CREATING A UCN SOURCE

UCN Flux Density in the Converter

The pulsed flux of thermal and cold neutrons is formed by the main moderator $I$ of the source. The energy distribution of neutrons is assumed to be Maxwellian with an effective temperature $T_n$. Then, the mean UCN flux with energies less than certain boundary energy $E_b$ [39] is

$$\langle F \rangle = \frac{\langle \Phi_b \rangle}{8} \left( \frac{E_b}{T_n} \right)^2,$$

where $\langle \Phi_b \rangle$ is the total neutron flux in the moderator. In certain cases, the UCN flux in the converter can exceed the flux in the moderator, which is considered by introducing a certain amplification factor into consideration (for example, see [39]).

The mean flux of thermal neutrons in the IBR2-M reactor is $\Phi_0 = 2 \times 10^{12}$ cm$^{-2}$ s$^{-1}$. Assuming that the temperature of the Maxwellian spectrum of neutrons is 400 K and the boundary energy is $E_b = 190$ meV, we obtain the UCN flux density, $\Phi = 8$ cm$^{-2}$ s$^{-1}$. The choice of the value of $E_b$ is determined by setting the maximum neutron velocity at the entrance to the trap.

Probably, it is possible to use a converter, in which the UCN flux density exceeds the flux in the external moderator by an order of magnitude. For example, such a converter is ice at low temperature [5]. Consequently, the UCN flux density is $\Phi = 80$ cm$^{-2}$ s$^{-1}$.

Lens Parameters, Transport Time, and Transmission of Neutron Guides

In the calculations, we assume that the maximum longitudinal velocity after passing through the lens should not exceed 3 m s$^{-1}$ so that the total velocity at the entrance to the trap was less than or of the order of the boundary velocity for beryllium, $v_{b,Be} = 6.9$ m s$^{-1}$, which we chose as the trap material. The total neutron transport time depends on the position of the lens and the maximum velocity it captures. If a neutron guide length is $L = 10$ m, the total neutron transport time is 2–3 seconds.

The calculation was carried out by the Monte Carlo method. We assume that the flux in the moderator is isotropic, the neutron velocities are limited to a certain value $V_{\text{max}}$, and the probability of finding a neutron in the velocity range from $V$ to $V + dV$ is proportional to $V^3$. The choice of the value $V_{\text{max}}$ is rather arbitrary, since the final results were normalized to the standard phase space determined by the boundary velocity of the trap (see below). It is only necessary to fulfill the condition $V_{\text{max}} > V_{\text{ng}}$, where $V_{\text{ng}}$ is the maximum longitudinal velocity captured by the lens. In the calculations, it was assumed that $V_{\text{max}} = \sqrt{3}V_{\text{ng}}$.

The velocity $V_{\text{ng}}$ and the distance to the lens determine the time of neutron transport from the moderator to the trap. Since the neutron guide length is much larger than its transverse diameter, then nearly all neutrons, for which the velocity component $v_\perp$ normal to the axis of the neutron guide is greater than its boundary velocity $v_{bg}$, are not captured by the neutron guide and are lost. Neutrons with a lower transverse velocity experience multiple collisions with the walls, are absorbed in them with some probability, and propagate in the direction of the lens and the trap. Since the probability of absorption of a neutron with a velocity normal to the surface of the substance and not too close to the boundary velocity [26] is

$$\mu = 2\eta \frac{v_\perp}{\sqrt{v_{bg}^2 - v_\perp^2}},$$

then the probability of successful neutron transport is determined by the expression

$$\vartheta = \left( 1 - 2\eta_{\text{ng}} \frac{v_\perp}{\sqrt{v_{bg}^2 - v_\perp^2}} \right)^n,$$

where $n$ is the number of collisions of neutrons with the neutron-guide walls and $\eta_{\text{ng}}$ is the parameter of neutron absorption by the substance of the neutron guide. According to the conditions of time focusing, the lens provides isochronism of neutron transport, and the number of collisions of neutrons that reach the trap with the neutron guide walls is

$$n = \frac{v_\perp}{D} t_0,$$

where $D$ is the transverse size of the neutron guide and $t_0$ is the total transport time through both sections of the neutron guide. In the calculations, we assume that the diameter of the neutron guide is $D = 8$ cm, the boundary velocity is $v_{bg} = 6.5$ m s$^{-1}$, and the absorption parameter is $\eta_{\text{bg}} = 10^{-4}$. These values correspond to the parameters of the nonmagnetic NiMo alloy, which is widely used in UCN physics.

Calculations have shown that, under the assumption of specular neutron guides, the losses of neutrons due to absorption in the walls do not exceed a few percent, and the transport efficiency is almost completely determined by the fraction of the phase space captured by the system. The latter depends on the choice of magnitude $V_{\text{max}}$, boundary velocity $v_{bg}$, and velocity aperture of the lens.

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1 Hereinafter, as is customary in neutron physics, we use the terms flux and flux density in a single sense implying in both cases the flux density with the dimension cm$^{-2}$ s$^{-1}$.
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The diffraction efficiency of the lens is calculated in a modified kinematic approximation. It is assumed that neutrons are incident normally onto the grating in the laboratory system. The depth of the grating profile and the distance between the grooves varied continuously depending on the current value of the velocity of the incident neutrons. Neglecting the tangential velocity with respect to the grating, Eq. (23) is written as

\[ (28) \]

where \( \phi \) is the modulation frequency specified by the current value of the required energy transfer (2) and the relation \( \Delta E (t) = 2nf (t) \). It is assumed that the time dependence of the frequency is provided by a continuous change in the slowly varying parameter \( d \) with a constant grating velocity \( V_g \). Knowing the value of \( f \) at each time instant, it is possible from Eq. (30) to obtain the maximum height of the grating tooth \( h_{\text{max}} \), for which the boundary condition \( C = 1 \) is satisfied.

In this case, the phase modulation function is trapezoidal [34]. In contrast to the case shown in Fig. 2, the phase changes from zero to a certain value \( \phi \), which was determined by the neutron velocity, the height of the tooth, and the grating material with the boundary velocity \( V_{\text{b,grat}} \)

\[ \phi = k(1 - n)h = \frac{2\pi}{\lambda} \left[ 1 - \sqrt{1 - \left( \frac{V_{\text{b,grat}}}{V_1^2} \right)^2} \right] h. \]  

(29)

It can be shown that the wave amplitude of the minus first order is described by the formula

\[ \alpha_{-1} = \frac{i\phi}{2\pi} \times \left( e^{i\phi} e^{-iC} - 1 \right) \left( \frac{\phi}{\pi} - C \right) \left( e^{i\pi C} - e^{i\phi} \right) \left( \frac{\phi}{\pi} + C \right) \]  

(30)

The diffraction efficiency (the intensity of the minus first order wave \( |A_{-1}|^2 \)) is calculated using Eqs. (30) and (21) for a large set of values \( 0 < h < h_{\text{max}} \). The value of \( h \) providing the maximum diffraction efficiency is chosen based on the results. For different ranges of neutron velocity, one of four values \( V_{\text{b,grat}} \) was chosen from 3.2 m s\(^{-1}\) (Si) to 7.8 m s\(^{-1}\) (\(^{58}\)NiMo alloy).

The results of calculating the diffraction efficiency of the lens are shown in Fig. 4. The efficiency jumps seen in Fig. 4 are due to transitions to a new grating material with a different value \( V_{\text{b,grat}} \). Note that in accordance with Eq. (21) the intensity of diffraction orders corresponding to neutron moderation can be higher than for a grating at rest, which does not change the velocity spectrum. This circumstance is used in presenting the results shown in Fig. 4.

The diffraction efficiency of the lens and the efficiency of capture and transport of neutrons determine the value of the efficiency of the system as a whole \( T \).

It was taken into account that the calculation of the probability of transport was carried out for the phase space of velocities inside a sphere with a radius \( V_{\text{max}} = \sqrt{2}V_{\text{a,grat}} \), and the neutron flux density in such a phase space exceeds the UCN flux density with velocities from zero to \( V_{\text{b,grat}} \) in the ratio \( K = (V_{\text{max}}/V_{\text{b,grat}})^4 \), where \( V_{\text{b,grat}} = \sqrt{2E/\sqrt{m}} \) (see Eq. (26)). Thus, the transport efficiency of a system consisting of two neutron guides and a lens is the ratio of the flux at the exit from the neutron guide, that is, at the entrance to the trap, to the flux of neutrons with velocities less than \( V_{\text{b,grat}} = 6 \) m s\(^{-1}\) in the converter. The dependence of the efficiency \( T \) determined in this way on the maximum longitudinal velocity \( V_{\text{max}} \) is shown in Fig. 5. The minimum velocity \( V_{\text{a,grat}} \) captured by the lens is calculated using Eq. (5).

**Pulse Duration, Accumulation Effect, and Intensity of the UCN Source**

The ratio of the velocities before and after passing through the lens changes depending on the selected value of the maximum longitudinal velocity. In accordance with (15), this means that each value of the longitudinal velocity corresponds to its own value of the
deviation of the arrival time to the trap $t$ from the calculated time $t_0$. Therefore, the shape of the resulting pulse is calculated with consideration of the distribution of longitudinal velocities at the exit of the neutron guide. The initial pulse shape was specified as a Gaussian with a half-width of $350\,\mu s$.

The results of such calculations are shown in Figs. 6 and 7. For the pulse duration in Fig. 7, the width at half maximum was taken. The pulse duration and the specified trap parameters determine the value of the gain factor from the pulse accumulation mode. Therefore, the latter depends on the maximum longitudinal velocity.

Figure 8 shows the dependence of the gain factor $G$ due to the pulsed filling of the trap depending on its radius for several values of the maximum longitudinal velocity. The calculation considers the increase in the duration of the pulse generated by the lens. For the absorption parameter of the trap material, a value $\eta = 3 \times 10^{-5}$ is taken that corresponds to the experimental data for a beryllium trap at low temperatures [27, 40]. For this value of the absorption parameter, the accumulation time in the trap becomes so long that neglecting the finite neutron lifetime with respect to beta decay becomes inadmissible. Therefore, the gain factor is calculated according to the formula

$$G = \frac{sT}{(s + \Sigma T\mu + 4W\gamma T/v)},$$

which differs from Eq. (11) by an additional factor in the denominator. Here, $W$ is the trap volume; $\gamma = 1.13 \times 10^{-3}\,s^{-1}$ is the decay constant and $v$ is the neutron velocity.

However, the total efficiency of the source, that is, the ratio of the UCN flux density in the trap to the UCN flux density in the converter, is characterized not by the $G$ factor, but by its product by the value of the transport efficiency of the channel $T$, which can be greater than unity for a moderating lens. The dependence of the total gain factor $\Theta = GT$ on the trap radius is shown in Fig. 9. As can be seen from Fig. 9, the gain factor increases with an increase in the longitudinal neutron velocity; this is due to the compression of the flux by the moderating lens. However, the gain factor does not increase with further increase in

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**Fig. 5.** Transport efficiency of a channel with the lens as a function of the maximum longitudinal neutron velocity (see text).

**Fig. 6.** The calculated shape of the pulse formed by the lens for three values of the maximum longitudinal velocity.

**Fig. 7.** Width of the pulse formed by the lens as a function of the maximum longitudinal velocity.
velocity, which is due to the drop in the diffraction efficiency of the lens (Fig. 4).

Multiplying the gain factor $\Theta$ by the value of the mean flux in the moderator, we obtain an estimate of the UCN flux in the trap, which is the main result of the calculation.

RESULTS AND DISCUSSION

It can be seen from Fig. 10 that the principle of time focusing accompanied by neutron moderation makes it possible to create a sufficiently intense UCN source at a periodic pulsed reactor even with a small value of the mean neutron flux in the moderator.

The above results demonstrate the potential of the concept discussed, but at the same time they are rather evaluative. We summarize the main parameters of the calculation and the approximations.

1. The mean flux of thermal neutrons is $\Phi_0 = 2 \times 10^{12}$ n (cm$^2$ s)$^{-1}$.

2. The effective temperature of the Maxwellian spectrum of neutrons is $T = 400$ K.

3. The gain factor of the converter is 10.
(4) The length of the neutron guide is 10 m, and the lens is located at a distance of 6 m from the converter.

(5) The neutron guide is absolutely specular.

(6) Calculation of the diffraction efficiency of gratings is based on a modified kinematic approximation.

(7) A pulse valve at the entrance to the trap is ideal.

(8) The boundary velocity and absorption parameter of the trap material are 6 m s⁻¹ and η = 3 × 10⁻⁵, respectively.

These approximations lead to multidirectional factors that distort the result. Thus, overly optimistic approximations that probably overestimate the values by one and a half to two times are noted in items 5–7. On the other hand, the UCN flux density in the moderator is inversely proportional to the square of the neutron temperature, and the use of a cryogenic moderator can not only compensate for the errors, but also increase the total estimate at least several times.

It seems that the concept of an intense UCN source based on a pulsed reactor presented here should be the object of more careful calculations, the results of which, in turn, should be confirmed by experiments.

This seems to be especially relevant in planning the construction of a new intense neutron source IBR3 Neptun at JINR.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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