Energy separation in supersonic boundary layer: joint effect of suction and longitudinal pressure gradient

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Abstract. In the framework of study of gas-dynamic temperature stratification method a numerical investigation of the temperature stratification is carried out for gas suction from a turbulent boundary layer in a supersonic flow on a permeable plate. The joint effect of gas suction through the plate and longitudinal pressure gradient in outer flow on the temperature stratification value is investigated for both accelerating and braking flows. Stratification is most pronounced for gases with small Prandtl numbers and significantly depends on the total suction consumption. The effect of stepped suction of varying intensity on the value of the temperature stratification is studied. It is shown that for a number of combinations of permeable inserts of different lengths with variable suction intensity, a significant difference can be obtained between the average temperature of the gas in the boundary layer and the average temperature of the suctioned gas.

At the same time, the presence of a negative pressure gradient reduces the stratification value in comparison with the gradientless and braking flow.

1. Introduction
The process of redistribution of the total heat content of the gas flow is considered to be a method of gas-dynamic machine-free energy separation, which is carried out without the performance of external work by the gas and in the absence of heat transfer with the external environment. A review of the currently known methods of machine-free energy separation is presented in [1]. The method of gas-dynamic temperature stratification, proposed in [2], is considered most thoroughly in this review. The essence of the method consists in the following: when the supersonic and subsonic flows with the same braking parameters are separated by the flat plate, the recovery wall temperature in supersonic flow differs from the total gas temperature, whereas on the subsonic flow side these values are practically identical. There is a temperature difference on the wall and, consequently, there is a heat flux, the direction of which depends on the Prandtl number of the gas. In paper [1], a one-dimensional technique for calculating the parameters of a temperature stratification device was proposed, and the results of experiments on the study of limit values of energy separation for air (Pr=0.71) are presented. Some technical applications of the method and its calculations are proposed in [3-5].

For supersonic and subsonic gas flows with the same braking parameters separated by the flat plate, a numerical study was carried out in the range of values of the Prandtl number Pr=0.1..4.0 for the both cases of impermeable and permeable heat-conducting plate in the presence of gas injection and suction through it. For the impermeable plate, it was shown, as in [6], that the stratification increased with decreasing of the Pr number. The injection of gas into the supersonic flow reduced the stratification compared with the impermeable plate, and the suction - increased it. This effect was due to the fact that a
"cold" gas was sucked from the boundary layer of the supersonic flow, the temperature of which is lower than the temperature of the braking gas of the oncoming flow (see figure 1). It was shown in [6] that when the gas was suctioned from a turbulent boundary layer through a permeable insert of sufficient length, a significant temperature difference between the gas in the boundary layer and the suctioned gas can be obtained. The effect of the Prandtl and Mach numbers of the oncoming flow on the value of the temperature stratification, which is largely determined by the value of the suction intensity and, consequently, the total consumption of the suctioned gas, was investigated. Stratification is most pronounced for gases with small Prandtl numbers. This effect was experimentally confirmed in [7].

The purpose of the study is to investigate numerically the temperature stratification that means the difference between the average temperature of compressible boundary layer on a heat insulated plate (boundary layer gas in figure 1) and the temperature of suctioned gas for a number of combinations of porous inserts of different lengths with a variable suction intensity under longitudinal pressure gradient.

![Figure 1. Temperature stratification under suction in the supersonic flow.](image)

2. Formulation of the task
To calculate the flow and heat transfer in a compressible turbulent boundary layer, a system of equations of continuity, motion, and energy were set as in the previous studies of this task [6, 8]. To determine the value of the turbulent friction in the equation of motion, a three-parameter turbulence model is used [9], where the transfer equations are taken for the shear stress $\tau = -\langle u'v' \rangle$, turbulence energy $E$ and the parameter $\omega = E / L^2$ containing the transverse integrated scale of turbulence $L$. The equations of the turbulence model [9] are also given in [10, 11].

To determine the turbulent heat flux $\rho q_t = -\rho \langle v' h' \rangle$ included in the energy equation, a hypothesis of the turbulent Prandtl number $Pr_t$ constancy over the thickness boundary layer ($Pr_t = 0.9$) is used:

$$\rho q_t = \mu_t \frac{\partial h}{\partial y}, \quad \mu_t = \rho \tau \frac{\partial u}{\partial y}$$

The boundary conditions for their solution were also described in [6, 8].

The variation of the temperature stratification value $\Delta T$, the difference between the braking temperature $T_*$ and the average temperature of the suctioned gas $<T_j>$, was determined as follows:

$$<T_j> = \frac{1}{G_j} \int_{x_0}^{x} j_u T_j \, dx, \quad \text{where} \quad G_j = \int_{x_0}^{x} j_u \, dx$$

The results of the study of the effect of the stepped suction and the longitudinal pressure gradient on the value of the temperature stratification are presented below.
3. Calculation results

3.1. Effect of stepped suction
The calculations were carried out in the following formulation (see figure 2). The plate on the one side was flowed by a supersonic gas flow with a constant velocity $u_e$ corresponding to a given Mach number at a pressure of 0.1 MPa and a braking temperature $T_e^* = 400 K$. On the other side of the plate was a cavity, where the gas suctioned from the boundary layer of the supersonic flow was collected. The inlet section of the plate $x_0$ was assumed to be impermeable and heat insulated. Its length $x_0 = 10 \text{ mm}$ (Re$_x \approx 10^6$) was chosen so that the first permeable insert of the plate 1 was beyond the laminar-turbulent transition region in the boundary layer. Further, through the porous inserts, gas was suctioned with a constant rate along the length of the plate $j_w = \frac{(\rho v)_w}{(\rho u)_e}$. Behind each permeable insert there was a section of an impermeable heat insulated plate with the same length. This scheme of organization of gas suction is called as stepped suction. Calculations were made for a helium-xenon mixture at $M = 3$. 

![Figure 2. Calculation scheme of stepped suction regime.](image)

Figure 3(a) shows the $\Delta T$ value for one porous insert ($L_p = 100 \text{ mm}$), two porous inserts of the same length ($L_p = 50 \text{ mm}$) and four porous inserts of the same length ($L_p = 25 \text{ mm}$). For all three variants, the specific consumption of the suctioned gas was the same and equal to $G_j = 1 \text{ kg/s·m}$. The rate of gas suction in this case was $j_w = 0.0033$. As one can see, the sectioning of the permeable plate into two or four inserts with the same suction rate $j_w$ at a sufficiently high gas consumption $G_j$ gives a slight increase in the temperature stratification (lines 2, 3, figure 3(a)).

An increase in the temperature stratification can be realized with smaller total gas consumption $G_j$ and an unequal suction rate $j_w$ for each of the porous inserts. Figure 3(b) shows the variation of the temperature stratification $\Delta T$ along the length for one (line 1), two (line 2) and three (line 3) porous inserts. The porous inserts were of different length, but the total consumption of the suctioned gas was the same for all three variants and was equal to $G_j = 0.5 \text{ kg/s·m}$. 

![Figure 3. The variation of the temperature stratification value along the plate length for several permeable inserts with the corresponding length $L_p$ and different rate of gas suction $j_w$ for two consumptions of the suctioned gas: (a) – $G_j = 1 \text{ kg/s·m}$; (b) – $G_j = 0.5 \text{ kg/s·m}$.](image)

For one and two porous inserts, the rate $j_w$ of the suctioned gas was the same, and for three inserts it was variable along the length with growth along the last insert. This allowed to increase the temperature stratification to a value $\Delta T$ of 58 K, which is 7 K more than for one porous insert with the same insert suctioned gas rate. Further increase in the temperature stratification is associated with a
decrease in the total gas consumption, the sectioning of the porous inserts and the choice of the varying rate of the suctioned gas for each porous insert. These measures depend on the design features of the heat exchanger and should be determined by setting the value of the temperature stratification and the required rate of the suctioned gas.

3.2. Effect of longitudinal pressure gradient

The calculation scheme is presented on the figure 4(a). As the parameter of acceleration (braking) regimes was taken the following value:

$$K = \frac{v}{u_e} \frac{du_e}{dx}$$

(3)

In the case of negative pressure gradient $dp/dx<0$ parameter $K$ is positive $K>0$ and on the contrary for $dp/dx>0$ – $K<0$ (see figure 4(b)). For the purpose of subsequent comparison of the results, the calculations were carried out with a constant total flow rate of the sucked gas $G_j$. In this case calculations were carried out for the constant suction rate $j_w$ for several values of the parameter $K$ (for example line $j_w=0.0044$, $K=5 \times 10^{-3}$, figure 5(a)). Then the total consumption $G_j$ was found and the $j_w$ for the case of $K=0$ was calculated respectively (see line $K=0$, $j_w=0.00387$). In this formulation of the task the total consumption for both cases – with or without pressure gradient – is equal anywhere along the plate length.

![Figure 4](image4.jpg)

**Figure 4.** Calculation scheme of the regime with joint effect of the suction and pressure gradient (a); the variation of the pressure gradient $dp/dx$ along the plate length $x$ for several values of parameter $K$ (b).

![Figure 5](image5.jpg)

**Figure 5.** Calculation of the total consumption of the suctioned gas for the cases $K=0$ and $K \neq 0$ (a); the variation of the temperature stratification value $\Delta T$ for several values of suction rate for both regimes – braking and gradientless flow (b).
On the figure 5(b) the variation of the temperature stratification value $\Delta T$ along the plate length $x$ for several values of suction rate for both regimes – braking and gradientless flow is shown. Instead of parameter $K$ the x-axis was presented by the total consumption of the suctioned gas $G_j$. As one can see the $\Delta T$ value can be increased under the braking of the outer flow in comparison with the same value in gradientless flow. At the same time, the presence of a negative pressure gradient reduces the amount of stratification. The limiting of stratification values in the braking flow exceed those of the temperature stratification for a gradientless flow for the air by 10 K, and the He-Xe mixture by 70 K.

4. Conclusion

In the framework of study of gas-dynamic temperature stratification method a numerical investigation of the temperature stratification is carried out for gas suction from a turbulent boundary layer in a supersonic flow on a permeable plate. The joint effect of gas suction and longitudinal pressure gradient in outer flow on the temperature stratification value is investigated for both accelerating and braking flows. Stratification is most pronounced for gases with small Prandtl numbers and significantly depends on the intensity of the total suction consumption. The effect of stepped suction of varying intensity on the value of the temperature stratification is studied. It is shown that for a number of combinations of permeable inserts of different lengths with variable suction intensity, a significant difference can be obtained between the average temperature of the gas in the boundary layer and the average temperature of the suctioned gas. At the same time, the presence of a negative pressure gradient reduces the stratification value in comparison with the gradientless and braking flow.

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