FeynHiggs1.2: Hybrid $\overline{\text{MS}}$/on-shell Renormalization for the $\mathcal{CP}$-even Higgs Boson Sector in the MSSM

M. Frank, S. Heinemeyer, W. Hollik and G. Weiglein

1 Institut für Theoretische Physik, Universität Karlsruhe, D–76128 Karlsruhe, Germany
2 HET, Physics Department, Brookhaven Natl. Lab., Upton, NY 11973, USA
3 Max-Planck-Institut für Physik, Föhringer Ring 6, D–80805 München, Germany
4 Institute for Particle Physics Phenomenology, University of Durham, Durham DH1 3LE, UK

Abstract

An updated version is presented of the Fortran code FeynHiggs for the evaluation of the neutral $\mathcal{CP}$-even Higgs sector masses and mixing angles. It differs from the previous version by a modification of the renormalization scheme concerning the treatment of subleading terms at the one-loop level; the two-loop corrections, for which the leading contributions of $\mathcal{O}(\alpha_t\alpha_s)$ and $\mathcal{O}(\alpha_t^2)$ are implemented, are not affected by the modified renormalization prescription. Besides stabilizing the Higgs mass calculations and avoiding unphysically large threshold effects, the mass of the lightest MSSM Higgs boson, $m_h$, is increased by 1-2 GeV for most parts of the MSSM parameter space.
FeynHiggs1.2: Hybrid MS/on-shell Renormalization for the CP-even Higgs Boson Sector in the MSSM

M. Frank, S. Heinemeyer, W. Hollik, G. Weiglein

Abstract
An updated version is presented of the Fortran code FeynHiggs for the evaluation of the neutral CP-even Higgs sector masses and mixing angles. It differs from the previous version by a modification of the renormalization scheme concerning the treatment of subleading terms at the one-loop level; the two-loop corrections, for which the leading contributions of $O(\alpha_t\alpha_s)$ and $O(\alpha_t^2)$ are implemented, are not affected by the modified renormalization prescription. Besides stabilizing the Higgs mass calculations and avoiding unphysically large threshold effects, the mass of the lightest MSSM Higgs boson, $m_h$, is increased by 1-2 GeV for most parts of the MSSM parameter space.

1 Introduction
The search for the lightest Higgs boson in the Minimal Supersymmetric Standard Model (MSSM) is one of the main goals at the present and the next generation of colliders. Therefore the precise knowledge of the dependence of the masses and mixing angles of the Higgs sector of the MSSM on the relevant supersymmetric parameters is of high importance.

In this note we present an updated version of the Fortran code FeynHiggs [1] that evaluates the neutral CP-even Higgs sector masses and mixing angles [2, 3]. It differs from the previous version as presented in Ref. [1] by a modification of the renormalization scheme concerning the treatment of subleading terms at the one-loop level; the two-loop corrections, for which the leading contributions of $O(\alpha_t\alpha_s)$ and $O(\alpha_t^2)$ are implemented, are not affected. In particular, an $\overline{\text{MS}}$ renormalization for $\tan \beta$ and the field renormalization constants has been used (where the $\overline{\text{MS}}$ quantities are evaluated at the scale $m_t$). The renormalization in the new version of FeynHiggs does no longer involve the derivative of the $A$ boson self-energy and the $AZ$ mixing self-energy. This leads to a more stable behavior around thresholds, e.g. at $M_A \approx 2 m_t$, and avoids unphysically large contributions in certain regions of the MSSM parameter space. Thus, the new renormalization scheme stabilizes the prediction of the masses and mixing angles in the CP-even Higgs sector of the MSSM.

2 Renormalization schemes
The Higgs sector of the MSSM [4] consists of two neutral CP-even Higgs bosons, $h$ and $H$ ($m_h < m_H$), the CP-odd $A$ boson, and two charged Higgs bosons, $H^\pm$. At the tree-level, $m_h$ and $m_H$ can be evaluated in terms of the Standard Model (SM) gauge couplings and two additional MSSM parameters, conventionally chosen as $M_A$ and $\tan \beta$, the ratio of the two vacuum expectation values ($\tan \beta = v_2/v_1$). Beyond lowest order, the Feynman-diagrammatic (FD) approach allows to obtain in principle the most precise evaluation of the
neutral CP-even Higgs boson sector, since in this way the effect of different mass scales of the supersymmetric particles and of the external momentum can consistently be included. The masses of the two CP-even Higgs bosons are obtained in this approach by determining the poles of the $h - H$-propagator matrix, which is equivalent to solving the equation

$$ [ q^2 - m_{h,\text{tree}}^2 + \hat{\Sigma}_h(q^2) ] [ q^2 - m_{H,\text{tree}}^2 + \hat{\Sigma}_H(q^2) ] - [ \hat{\Sigma}_{hh}(q^2) ]^2 = 0, \quad (1) $$

where $\hat{\Sigma}_s, s = h, H, hH$, denote the renormalized Higgs boson self-energies. For the renormalization within the FD approach usually the on-shell scheme is applied [3]. This means in particular that all the masses in the FD result are the physical ones, i.e. they correspond to physical observables. Since eq. (1) is solved iteratively, the result for $m_h$ and $m_H$ contains a dependence on the field renormalization constants of $h$ and $H$, which is formally of higher order. Accordingly, there is some freedom in choosing appropriate renormalization conditions for fixing the field renormalization constants (this can also be interpreted as affecting the renormalization of $\tan \beta$). Different renormalization conditions have been considered, e.g. ($\hat{\Sigma}'$ denotes the derivative with respect to the squared momentum):

1. on-shell renormalization for $\hat{\Sigma}_Z, \hat{\Sigma}_A, \hat{\Sigma}'_A, \hat{\Sigma}_{AZ}$, and $\delta v_1/v_1 = \delta v_2/v_2$ [4]
2. on-shell renormalization for $\hat{\Sigma}_Z, \hat{\Sigma}_A, \hat{\Sigma}_{AZ}$, and $\delta v_i = \delta v_{i, \text{div}}, i = 1, 2$ [4]
3. on-shell renormalization for $\hat{\Sigma}_Z, \hat{\Sigma}_A$ [4], $\overline{\text{MS}}$ renormalization for $\delta Z_h, \delta Z_H, \tan \beta$ [4].

The previous version of FeynHiggs is based on renormalization 1, involving the derivative of the $A$ boson self-energy. The new version of FeynHiggs, see www.feynhiggs.de, is based on renormalization 3 (a detailed discussion can be found in Ref. [7]).

3 Numerical comparison

In this section we numerically compare the output of the previous version (based on renormalization 1) and the new version (based on renormalization 3) of FeynHiggs. We also show results for the recently obtained non-logarithmic $O(\alpha_2^2)$ corrections [8, 9] that are also included in the new version of FeynHiggs. The comparison is performed for the parameters of the three LEP benchmark scenarios [10]. In this way, the effect of the new renormalization and the non-logarithmic $O(\alpha_2^2)$ corrections on the analysis of the LEP Higgs-boson searches can easily be read off.

In Figs. 1–3 we show the results in the “$m_h^{\text{max}}$”, “no-mixing” and “large $\mu$” scenario as a function of $M_A$ (left column) and of $\tan \beta$ (right column) for two values of $\tan \beta$ ($\tan \beta = 3, 50$) and $M_A$ ($M_A = 100, 1000$ GeV for the $m_h^{\text{max}}$ and the no-mixing scenario, $M_A = 100, 400$ GeV for the large $\mu$ scenario), respectively. The solid lines correspond to the new result while the dashed lines show the old results. The dotted lines correspond to the new result including the non-logarithmic $O(\alpha_2^2)$ contributions. Concerning the new renormalization scheme, in the $m_h^{\text{max}}$ (Fig. 1) and the no-mixing scenario (Fig. 2) the new result is larger by $\approx 1-2$ GeV for not too small $M_A$ and $\tan \beta$. For small $\tan \beta$ and large $M_A$ the enhancement can be even larger. In the large $\mu$ scenario (Fig. 3) the largest deviations appear
for small $\tan \beta$ for both large and small $M_A$. While the previous prescription for the field renormalization constants leads to unphysically large threshold effects in some regions of the parameter space, which arise from the $A\Z$ mixing self-energy and the derivative of the $A$ boson self-energy, no threshold kinks are visible for the result based on the new renormalization. The shift in $m_h$ of $\approx 1$–2 GeV related to the modification of the renormalization prescription lies in the range of the anticipated theoretical uncertainty from unknown non-leading electroweak two-loop corrections [11]. The new $O(\alpha^2)$ corrections can further increase $m_h$ by up to $\approx 3$ GeV for large $\tilde{t}$ mixing (a detailed analysis will be presented elsewhere [12]).

The new version of *FeynHiggs* can be obtained from www.feynhiggs.de.

**Acknowledgements**

G.W. thanks the organizers of the Les Houches workshop for the invitation and the pleasant and constructive atmosphere. This work was supported in part by the European Community's Human Potential Programme under contract HPRN-CT-2000-00149 Physics at Colliders.

**References**

[1] S. Heinemeyer, W. Hollik and G. Weiglein, *Comp. Phys. Comm.* 124 (2000) 76; [hep-ph/0002213](http://arxiv.org/abs/hep-ph/0002213); see www.feynhiggs.de.

[2] S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Rev.* D 58 (1998) 091701; [hep-ph/9806250](http://arxiv.org/abs/hep-ph/9806250); *Phys. Lett.* B 440 (1998) 296.

[3] S. Heinemeyer, W. Hollik and G. Weiglein, Eur. Phys. Jour. C 9 (1999) 343.

[4] J. Gunion, H. Haber, G. Kane and S. Dawson, *The Higgs Hunter's Guide*, Addison-Wesley, 1990.

[5] A. Dabelstein, *Z. Phys.* C 67 (1995) 495; *Nucl. Phys.* B 456 (1995) 25.

[6] P. Chankowski, S. Pokorski and J. Rosiek, *Nucl. Phys.* B 423 (1994) 437; S. Heinemeyer, W. Hollik, J. Rosiek and G. Weiglein, *Eur. Phys. Jour.* C 19 (2001) 535.

[7] M. Frank, S. Heinemeyer, W. Hollik and G. Weiglein, *in preparation*.

[8] A. Brignole, G. Degrassi, P. Slavich and F. Zwirner, [hep-ph/0112177](http://arxiv.org/abs/hep-ph/0112177).

[9] J. Espinosa and R.-J. Zhang, *Nucl. Phys.* B 586 (2000) 3.

[10] M. Carena, S. Heinemeyer, C. Wagner and G. Weiglein, [hep-ph/9912224](http://arxiv.org/abs/hep-ph/9912224).

[11] S. Heinemeyer and G. Weiglein, [hep-ph/0102117](http://arxiv.org/abs/hep-ph/0102117).

[12] G. Degrassi et al., *in preparation*.
Figure 1: The new renormalization (3, solid) and the old scheme (1, dashed) are compared in the $m_h^{\text{max}}$ scenario. The dotted line shows the inclusion of the non-logarithmic $O(\alpha_t^2)$ corrections. The lower curves are for $\tan\beta = 3$ (left plot) or $M_A = 100$ GeV (right). The upper curves are for $\tan\beta = 50$ (left) or $M_A = 1000$ GeV (right).

Figure 2: The new renormalization (3, solid) and the old scheme (1, dashed) are compared in the no-mixing scenario. The dotted line shows the inclusion of the non-logarithmic $O(\alpha_t^2)$ corrections. The lower curves are for $\tan\beta = 3$ (left plot) or $M_A = 100$ GeV (right). The upper curves are for $\tan\beta = 50$ (left) or $M_A = 1000$ GeV (right).

Figure 3: The new renormalization (3, solid) and the old scheme (1, dashed) are compared in the large $\mu$ scenario. The dotted line shows the inclusion of the non-logarithmic $O(\alpha_t^2)$ corrections. The lower curves are for $\tan\beta = 50$ (left plot) or $M_A = 100$ GeV (right). The upper curves are for $\tan\beta = 3$ (left) or $M_A = 400$ GeV (right).