Higgs boson decays in the littlest Higgs model

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Abstract

We calculate the two body Higgs boson decays in the framework of the littlest Higgs model. The decay $H \rightarrow \gamma Z$ is computed at one loop level and, using previous results, we evaluate the branching fractions in the framework of the littlest Higgs model. A wide range of the space parameter of the model is considered and possible deviations from the standard model are explored.

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Although the standard model (SM) has passed tests up to the highest energies accessible today, it remains unanswered the mechanism responsible for the generation of mass and behind the spontaneous breaking of the $SU(2)_L \times U(1)_Y$ symmetry. The precision electroweak measurements indicate a possible light Higgs boson \cite{1}, but the SM suffers of the so-called hierarchy or fine-tuning problem which is due to the presence of quadratic divergences in the loop processes for the scalar Higgs boson self-energy. This lead to the open question of how to prevent the Higgs boson mass from getting grand unification theory (GUT) scale contributions, in order to remain it light. An answer has come from supersymmetry (SUSY), where the introduction of supersymmetric partners with opposite statistics of the existing particle cancel out the divergences \cite{2}. Strong dynamics at the TeV-scale has also been invoked by theories, such as topcolor models \cite{3} where the electroweak symmetry breaking is done dynamically. Recently has appeared a new proposal, called Little Higgs models, which could solve the problem in a natural way by making the Higgs boson a pseudo-Goldstone boson, whose mass is protected to get quadratic divergences from the gauge sector by a global symmetry \cite{4}.

In the following we use the littlest Higgs model which is based on a global $SU(5)$ symmetry \cite{5}. It is a non-linear sigma model which is broken when the symmetric tensor gets an expectation value, down the symmetry to global $SO(5)$. At the same time, the subgroup $[SU(2) \times U(1)]^2$ of $SU(5)$ is gauged and broken to its diagonal subgroup $SU(2)_L \times U(1)_Y$, the SM symmetry. The key point is in the restoration of part of the global symmetry by ungauging some part of the gauge symmetry. In this model the restored symmetry is $SU(3)$. Therefore the Higgs boson remains light because it becomes a Goldstone boson of spontaneously broken $SU(3)$ and acquires a mass at the electroweak scale by radiative corrections. Hence, the Higgs boson mass does not acquire contributions from gauge loops because it is protected by an approximate global symmetry and the quadratic divergences are cancel off because the gauge sector of the two $SU(2) \times U(1)$ are coupling with the Higgs boson with opposite sign.

On the other hand, the contributions to the Higgs boson mass from interactions with fermions and from quartic couplings, should obey the approximate global symmetries. Thus, to control the quadratic divergences coming from the top quark coupling, the model adds a new like-vector T-quark and then allows the top quark to get a mass from a collective breaking. These cancellations allow that the little Higgs theory be valid up to a scale of
the order of 10 TeV without any fine-tuning. At this scale the theory becomes a strongly
interacting one \[4, 5\]. At the end, in the littlest Higgs model there are four new gauge
bosons $W_H^\pm$ and $B_H$, a vector-like T-quark and a new triplet scalar field of $SU(2)_L$, all of
them playing together in order to successfully cancel off the quadratic divergences of the
Higgs boson self-energy \[4, 5, 6\].

The effects of the new states at low energies have been studied and several constraints
on the model parameters have been imposed in the literature. A very exhaustive study
of this model was presented in reference \[6\]. Different studies using precision electroweak
measurements \[7\] and possible signatures at future colliders have also been done \[8\].

One important job for the present and future experiments is to search for the Higgs boson
and investigate its properties. But, even in the framework of the SM the Higgs boson has
several possibilities to be produced. Depending on the Higgs boson mass, different decay
modes open up \[9\]. In the framework of the SM for instance, the decay to $b\bar{b}$ is dominant but
the decay $h \rightarrow \tau^+\tau^-$ is also sizable. With increasing mass, the Higgs boson decays heavier
particles become dominant, mostly into the channels with $ZZ$ and $W^+W^-$ pairs. However,
the ratio of $h \rightarrow \gamma\gamma$ decay, although it is small one because it is at one-loop level, can be
important below the $W^+W^-$ pair threshold because of its well-defined final state. Thus, the
production processes at colliders in combination with all opened decay channels result in a
large number of possible final states. Our aim is to study the Higgs boson decays in the
framework of the littlest Higgs model using as a paradigm the SM framework. In this way
we can compare the different modes and we can use different constraints on the parameter
space taken from the literature in the framework of the littlest Higgs model.

The littlest Higgs (LH) model is based on an $SU(5)/SO(5)$ non-linear sigma model.
A vacuum expectation value (VEV) breaks the $SU(5)$ global symmetry into its subgroup
$SO(5)$ and breaks the local gauge symmetry $[SU(2) \otimes U(1)]^2$ into its diagonal subgroup
$SU(2)_L \otimes U(1)_Y$ at the same time, which is identified as the SM electroweak gauge group.
At the scale $\Lambda_s \sim 4\pi f$, the VEV associated with the spontaneous symmetry breaking,
proportional to the scale $f$, is parameterized by

$$
\Sigma_0 = \begin{pmatrix}
1_{2\times 2} \\
1 & 1 \\
1_{2\times 2}
\end{pmatrix}.
$$

(1)
The scalar fields of the non-linear $\sigma$-model can be written as

$$\Sigma(x) = e^{i\Pi(x)/f} \Sigma_0 e^{i\Pi(x)^T/f},$$  \hspace{1cm} (2)$$

where $\Pi(x) = \pi^a(x)X^a$ is the Goldstone boson matrix. $X^a$ are the broken generators of $SU(5)$. The Goldstone boson matrix $\Pi(x)$ can be expressed as

$$\Pi = \begin{pmatrix} h^t/\sqrt{2} & \phi^t \\ h/\sqrt{2} & h^*/\sqrt{2} \\ \phi & h^T/\sqrt{2} \end{pmatrix},$$  \hspace{1cm} (3)$$

where

$$h = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}, \quad \phi = \begin{pmatrix} \phi^{++} & \phi^+/\sqrt{2} \\ \phi^+/\sqrt{2} & \phi^0 \end{pmatrix},$$  \hspace{1cm} (4)$$

are doublet and triplet under the unbroken SM gauge group $SU(2)_L \otimes U(1)_Y$, respectively.

The leading order dimension-two term for the scalar fields $\Sigma(x)$ in the littlest Higgs model can be written as

$$\mathcal{L} = \frac{1}{4} f^2 \frac{1}{4} \text{Tr} [D_\mu \Sigma]^2,$$  \hspace{1cm} (5)$$

where $D_\mu$ is the covariant derivative for the gauge group $[SU(2) \otimes U(1)]^2 = [SU(2)_1 \otimes U(1)_1] \otimes [SU(2)_2 \otimes U(1)_2]$ and can be written in the following way

$$D_\mu \Sigma = \partial_\mu \Sigma - i \sum_{j=1}^2 \left[ g_j (W_j \Sigma + \Sigma W_j^T) + g'_j (B_j \Sigma + \Sigma B_j^T) \right],$$  \hspace{1cm} (6)$$

where $W_{\mu j} = \sum_{a=1}^3 W_{\mu j}^a Q_j^a$ and $B_j = B_{\mu j} Y_j$ are the $SU(2)_j$ and $U(1)_j$ gauge fields, respectively. The generators of two $SU(2)$’s ($Q_j^a$) and two $U(1)$’s generators ($Y_j$) are as follows

$$Q_1^a = \left( \begin{array}{c} \sigma^a_2 \\ 0_{3 \times 3} \end{array} \right), \quad Q_2^a = \left( \begin{array}{c} 0_{3 \times 3} \\ -\sigma^a_2 \end{array} \right),$$  \hspace{1cm} (7)$$

$$Y_1 = \text{diag}\{-3, -3, 2, 2, 2\}/10, \quad Y_2 = \text{diag}\{-2, -2, -2, 3, 3\}/10,$$

where $\sigma^a (a = 1, 2, 3)$ are the Pauli matrices. As we expect, the breaking of the gauge symmetry $[SU(2) \times U(1)]^2$ into its diagonal subgroup $SU(2)_L \times U(1)_Y$ gives rise to heavy gauge bosons $W'$ and $B'$, and the unbroken subgroup $SU(2)_L \times U(1)_Y$ introduces the massless gauge bosons $W$ and $B$. 
In the littlest Higgs model there is no Higgs potential at tree-level. Instead, the Higgs potential is generated at one-loop level and higher orders due to the interactions with gauge bosons and fermions. The Higgs potential can be presented in the standard form of a Coleman-Weinberg potential as

$$V = \lambda_\phi f^2 \text{Tr}(\phi^\dagger \phi) + i \lambda_{h\phi h} f (h \phi^\dagger h^T - h^* \phi h^\dagger) - \mu^2 hh^\dagger + \lambda_{h} (hh^\dagger)^2$$

$$+ \lambda_{h\phi h} h^\dagger \phi h^\dagger + \lambda_{s} (h^2 + \phi^2) \lambda_{s} (h^2 + \phi^2) + \lambda_{s} \lambda_{s} (h^2 + \phi^2)$$

where $\lambda_\phi$, $\lambda_{h\phi h}$, $\lambda_{h}$, $\lambda_{s} (h^2 + \phi^2)$ and $\lambda_{s} \lambda_{s} (h^2 + \phi^2)$ are the coefficients of the original Higgs potential. By minimizing the Coleman-Weinberg potential, we obtain the vacuum expectation values $\langle h^0 \rangle = v/\sqrt{2}, \langle i\phi^0 \rangle = v'$ of the Higgs boson doublet $h$ and triplet $\phi$, which give rise to the electroweak symmetry breaking (EWSB). In order to get the correct vacuum for the electroweak symmetry breaking with $m_H^2 > 0$, it is possible to express all four parameters in the Higgs potential to leading order in terms of the physical parameters $f$, $m_H^2$, $\nu$ and $\nu'$. After EWSB, the gauge sector gets additional mass and mixing term due to the VEVs of $h$ and $\phi$. By diagonalizing the quadratic term of the gauge sector, we may get the mass eigenstates for the light bosons $A_L, Z_L, W_L$, and for the heavy ones, $A_H, Z_H$ and $W_H$, with masses of the order of $\nu^2/f^2$. To leading order they are

$$M_{WL}^2 = m_w^2 \left[ 1 - \frac{\nu^2}{f^2} \left( \frac{1}{6} + \frac{1}{4}(c^2 - s^2)^2 - \frac{x^2}{4} \right) \right],$$

$$M_{WH}^2 = m_w^2 \frac{f^2}{s^2 c^2 \nu'^2},$$

$$M_{\Phi}^2 = 2m_H^2 \frac{f^2}{(1 - x^2)\nu^2},$$

where $m_w = g\nu/2$; the mixing between the two gauge groups $SU(2)_i$ is parametrized by $c$ and in the Higgs sector the parameter $x = 4f\nu'/\nu^2$ is defined.

Large quadratic divergence in the Higgs boson mass due to the heavy top Yukawa interaction is a problem in the SM. The littlest Higgs model solve this problem by introducing a pair of new fermions $\tilde{t}$ and $\tilde{t}'$ which are a vector-like pair. Their interactions are included in the following Lagrangian:

$$\mathcal{L}_Y = \frac{1}{2} \lambda_1 f \epsilon_{ijk} \epsilon_{xy} \chi_i \Sigma_{j, x, \Sigma_{k, y}} u^c_3 + \lambda_2 f \tilde{t}\tilde{t}' + h.c.$$
the Lagrangian and diagonalizing the mass matrix, we get the physical states of the top quark and a new heavy vector-like quark \( T \). The masses of the two physical states are

\[
m_t = \frac{v\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} \left( 1 + \frac{v^2}{f^2} \left[ -\frac{1}{3} + \frac{f
u'}{v^2} + \frac{1}{2} \frac{\lambda_1^2}{\lambda_1^2 + \lambda_2^2} \left( 1 - \frac{\lambda_2^2}{\lambda_1^2 + \lambda_2^2} \right) \right] \right),
\]

\[
m_T = \frac{m_t f}{s_t c_t \nu},
\]

respectively, and the mixing between the top quark and the heavy vector-like quark \( T \) is parametrized by \( c_t \). Since the top quark mass is already obtained in the SM, we can then get the parameter relation from Eq. (11)

\[
\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} \approx \frac{v^2}{m_t^2} \approx 2.
\]

In general, all the couplings to the Higgs boson in the Littlest Higgs model are modified at order \( \nu^2/f^2 \), so we use the following parameterization,

\[
\mathcal{L} = -\frac{m_t}{\nu} y_t \bar{t}H - \frac{m_t}{\nu} y_T \bar{T}TH + 2 \frac{m_W^2}{\nu} y_{wL} W^+_L W^-_L H + 2 \frac{m_W^2}{\nu} y_{wH} W^+_H W^-_H
\]

\[
- 2 \frac{m_{\Phi^+}}{\nu} y_{\Phi^+} \Phi^+ \Phi^- H - \frac{m_b}{\nu} y_{bbH} + \frac{m_L^2}{\nu} y_{LZL} Z_L Z_H
\]

(14)

where the \( y_i \) factors are derived from the Higgs couplings given in ref \[6\] and they can be written as

\[
y_t = 1 + \frac{\nu^2}{f^2} \left( -\frac{2}{3} + \frac{1}{2} x - \frac{1}{4} x^2 + c_t^2(1 + c_t^2) \right)
\]

\[
y_b = 1 + \frac{\nu^2}{f^2} \left( -\frac{2}{3} + \frac{1}{2} x - \frac{1}{4} x^2 \right)
\]

\[
y_T = \frac{\nu}{f} c_t (1 + c_t^2)
\]

\[
y_{wL} = 1 + \frac{\nu^2}{f^2} \left( -\frac{1}{6} + \frac{3}{4}(c^2 - s^2)^2 - x^2 \right)
\]

\[
y_{ZL} = 1 + \frac{\nu^2}{f^2} \left( -\frac{1}{6} - \frac{1}{4}(c^2 - s^2)^2 - \frac{5}{4}(c^2 - s^2)^2 + \frac{1}{4} x^2 \right)
\]

(15)

The partial widths will be proportional to the above factors \( y_i \) or combinations of them.

The width of the tree level decay \( H \rightarrow \bar{f}f \) is

\[
\Gamma(H \rightarrow \bar{f}f)^{\text{LH}} = \frac{N_c}{8\pi} \sqrt{2} G_F m_f^3 \beta^3 m_H y_f^2 y_{G_F}^2
\]

(16)

where

\[
y_{G_F}^2 = 1 + \frac{\nu^2}{f^2} \left( -\frac{5}{12} + \frac{1}{4} x^2 \right)
\]

(17)
We use only the $H \to \bar{b}b$ decay coming from LH model, while the other options like the decay into leptons are taken as in the SM model. Here we should note that the factors $y_i$ for the standard particles go to one when new physics is turn off, but they become zero for the new particles of the model. This means that the SM expressions can be obtained asymptotically when the new physics is decoupled in the limit $f \to \infty$. The decays into WW or ZZ bosons are like the SM ones, but they get a new factor $y_{WW}$ or $y_{ZZ}$ respectively,

$$
y_{WW} = 1 + \frac{\nu^2}{4f^2}(-3 + 6(c^2 - s^2) - 7x^2)
$$

$$
y_{ZZ} = 1 + \frac{\nu^2}{4f^2}(-3 - 2(c^2 - s^2) - 10(c^2 - s^2) + 3x^2)
$$

These factors are coming from products of $y_i$ related with the corrections of new physics.

We use the W boson mass as an input, not the Z boson mass; note that the factor $y_{WW}$ is different of the factor used in reference [11].

The one loop-level decays $H \to \gamma\gamma$ and $H \to gg$ are taken from reference [6] and the decay $H \to \gamma Z$ is

$$
\Gamma(H \to \gamma Z) = \frac{\sqrt{2}G_F \alpha^2 m_h^2 y_{GF}^2}{128\pi^3} (1 - \frac{m_Z^2}{m_h^2})^3 |A_F + A_W|^2
$$

where

$$
A_F = \frac{-2}{s_w c_w} y_{top}(I_1(\tau_t, \lambda_t) - I_2(\tau_t, \lambda_t)) - \frac{2}{3} t_w y_T(I_1(\tau_T, \lambda_T) - I_2(\tau_T, \lambda_T))
$$

$$
A_W = \cot_w \left\{ y_{WL} \left[ I_1(\tau_w, \lambda_w)(-4 + 2t_w^2 - \frac{m_h^2}{m_{W_L}^2} \frac{c_{2w}}{2c_w} - \frac{1}{c_w^2}) + 4I_2(\tau_w, \lambda_w)(4 - \frac{1}{c_w^2}) \right] + y_{WH} \left[ I_1(\tau_W, \lambda_W)(-4 + 2t_w^2(1 + \frac{m_{W_L}}{m_{W_H}}) \frac{m_{W_L}}{m_{W_H}^2} - \frac{m_W^2}{m_{W_H}^2} \frac{c_{2w}}{2c_w^2}) + 4I_2(\tau_{W}, \lambda_{W})(4 - \frac{2m_{W_L}^2 t_w^2}{m_{W_H}^2}) \right] + y_{\phi}(2I_2(\tau_{\phi^+}, \lambda_{\phi^+}) - I_1(\tau_{\phi^+}, \lambda_{\phi^+})) \right\}
$$

the integrals $I_1$ and $I_2$ can be found in [9]. The arguments depend on the masses where the integrals are to be evaluated, and the new factor

$$
y_{top} = \frac{1}{2} - \frac{4}{3} s_w^2 + \frac{\nu^2}{f^2}(-\frac{2}{3} + c_t^2(2 + c_t^2) + \frac{1}{2} x - \frac{1}{4} x^2 + \frac{1}{2} c^2(c^2 - s^2) - \frac{5}{2}(c^2 - s^2)(-\frac{4}{5} + c^2 - c_t^2(\frac{20}{15} - \frac{2}{3} c^2)))
$$

\( (21) \)
is the result of the product of $y_t$ and the correction of the $\bar{t}tZ_L$ coupling.

The results are shown in figures 1-5. The regions between lines correspond to accessible values on some parameters. The parameters should be taken in the intervals $0 \leq c_t \leq 1$, $0 \leq x \leq 1$ and $0.4 \leq c' \leq 1$. Figure 1 shows the ratio $BR(H \to \bar{t}t)_{LH}/BR(H \to \bar{t}t)_{SM}$ versus the scale $f$; this ratio depends on different parameters including the Higgs boson mass because it is the ratio between the branching fraction. For $f = 1$ TeV, $m_H = 120$ GeV, the branching fraction of the littlest Higgs model is modified from $+2\%$ to $-8\%$. As we expect, the region is converging to one as $f$ increase, where the new physics is decoupling. If we consider the ratio between the widths of $H \to \bar{t}t$, the Higgs boson mass dependence cancels out, as shown in figure 3. Figure 2 presents the dependence of the ratio between the branching fractions with the Higgs boson mass, where we have fixed the scale at $f = 1$ TeV. In that plot we note that for $m_h = 180$ GeV the branching can be deviated $\pm 9\%$ from the SM value. Figure 3 shows the case of ratio of widths for $H \to \bar{t}t$ and, as already mentioned, it does not depend on the Higgs mass. For $f = 1$ TeV the decay in the framework of the littlest Higgs model is suppressed respect to the SM between $11\%$ and $6\%$; this is because of the factor $y_b^2 y_{G_F}^2\nu^2/f^2$, that has a negative term of the order of $\nu^2/f^2$. When $f$ grows up the rate is getting closer to one, as expected.

Figure 4 shows the ratio between the widths for the one-loop level decay $H \to \gamma Z$ against the scale $f$; we expect a deviation about $+9\%--15\%$ for $f = 1$ TeV. Finally, figure 5 shows the branching fractions for a wide range of parameters, fixed $f = 1$ TeV versus the Higgs boson mass. The curves are showing zones where the parameters of the model are valid. In fact the branching fraction of the SM would be inside the region.

In conclusion, we estimate the branching fraction of the different channels for the Higgs boson in the framework of the littlest Higgs model in particular we show the one-loop level $H \to \gamma Z$ and in different figures we show the behavior respect to the SM and in general deviations about $10\%$ are expected.

Note added: During the elaboration of this work another paper [11] on a similar subject appeared.

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FIG. 1: The ratio $BR(H \to \bar{b}b)^{LH}/BR(H \to \bar{b}b)^{SM}$ versus the scale $f$, $m_H = 120$ GeV.

FIG. 2: The ratio between the branching fractions $BR(H \to \bar{b}b)^{LH}/BR(H \to \bar{b}b)^{SM}$ versus the Higgs boson mass, where we have fixed the scale at $f = 1$ TeV.
FIG. 3: The ratio of widths for $H \to b \bar{b}$ versus the scale $f$, it does not depend on the Higgs mass.

FIG. 4: The ratio between the widths for the one-loop level decay $H \to \gamma Z$ against the scale $f$. 

$\Gamma_{HH}^{LH}/\Gamma_{HH}^{SM}$
FIG. 5: The branching fractions for a wide range of parameters, fixed $f = 1$ TeV versus the Higgs boson mass.