Radiative and Semileptonic B Meson Decay spectra

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Abstract

We review semi-inclusive charmless B decays, focussing on threshold logarithmic resumming and on universality of QCD dynamics in radiative and semileptonic decays.
1 Semileptonic B decays

The precise determination of the magnitude of the Cabibbo-Kobayashi-Maskawa (CKM) element $V_{ub}$ with a clear uncertainty remains one of the key goals of the heavy flavor physics programs, both experimentally and theoretically. The smallest element in the CKM mixing matrix $|V_{ub}|$ plays a crucial role in the examination of the unitarity constraints and of the related fundamental questions.

The charmless semileptonic $B \rightarrow X_u \ell \bar{\nu}$ decay channel provides a possible path for the determination of $|V_{ub}|$. Semileptonic $B$ decays present several advantages, such as the possibility of using the systematic framework provided by the Heavy Quark Effective Theory (HQET), with the additional assumption of quark-hadron duality. HQET is implemented through the operator product expansion (OPE) in the form of a heavy quark expansion \[1\]. It allows to evaluate inclusive transition rates as an expansion in inverse powers of the heavy quark mass. Note that quark hadron duality is not derived from first principles, but it is a necessary assumptions for many applications of QCD. For semi-leptonic decays the property of duality is often referred as global. For non-leptonic decays, where the total hadronic mass is fixed, it is only the the fact that one sums over many hadronic states that provides an averaging (so called local duality). The success of the QCD predictions for the inclusive semi-hadronic $\tau \rightarrow \nu_{\tau} + \text{hadrons}$ decays widths is a strong test of global duality \[2\].

Theoretically, issues regarding the calculation of the total semileptonic partial width $\Gamma(B \rightarrow X_u \ell \bar{\nu})$ via OPE are well understood \[3, 4\]. The semi-leptonic decay rate can be calculated as

$$\Gamma = \Gamma_0 + \frac{1}{m_b^2} \Gamma_2 + \frac{1}{m_b^4} \Gamma_3 + \cdots$$  

(1)

Here the OPE is both a nonperturbative power series in $1/m_b$ and a perturbative expansion in $\alpha_s$. The leading term $\Gamma_0$ is the decay rate of an on-shell $b$ quark treated within renormalization group improved perturbative QCD. The perturbative corrections are known to order $\alpha_s^2$ in the strong interactions \[5\]. The most remarkable feature of Eq. \[1\] is the absence of a contribution of order $1/m_b$. That means that non-perturbative corrections are suppressed by at least two powers of the heavy quark mass. They can be expressed as matrix elements of higher dimension operators in HQET and parameterized by non perturbative parameters. The absence of contribution of order $1/m_b$ was observed in \[3\] and it is often referred as Luke’s theorem.

Theoretically, the total inclusive rate would allow determination of $|V_{ub}|$ to better than 10%, the main sources of uncertainties being the uncertainty in the $b$ quark mass and uncertainty on potential violation of the underlying assumption of global quark–hadron duality. However, experimentally, the much more copious $B \rightarrow X_s \ell \bar{\nu}$ process, which has a rate about 60 times higher, does not makes feasible a measurement over the full phase space.

To overcome this background, inclusive $B \rightarrow X_u \ell \bar{\nu}$ measurements utilize restricted regions of phase space in which the $B \rightarrow X_u \ell \bar{\nu}$ process is kinematically highly suppressed. The background is forbidden in the regions of large charged lepton energy $E_l > (M_B^2 - M_D^2)/2M_B$ (the endpoint), low hadronic mass $M_X < M_D$ and large dilepton mass $q^2 > (M_B - M_D)^2$. Extraction of $|V_{ub}|$ from such a measurement requires knowledge of the fraction of the total $B \rightarrow X_u \ell \bar{\nu}$ that lies within the utilized region of phase space, which complicates the theoretical issues considerably.

Let us consider the first two kinematic regions for which the charm background is absent, that is the large lepton energy region, $E_l > (M_B^2 - M_D^2)/2M_B$, and the small hadronic invariant mass region $M_X < M_D$. In both cases one needs to consider the following kinematic region

$$E_X \sim m_b, \quad m_X^2 \sim \Lambda_{QCD} m_b \ll m_b.$$  

(2)

This kinematic region has sufficient phase space for many different resonances to be produced in the final state, so an inclusive description of the decays is still appropriate. However, in this region, the differential rate is very sensitive to the details of the wave function of the $b$ quark in the $B$ meson. The parton level differential distribution at the end-point region has its own problems, as well, related to the presence of large logarithms which spoil the perturbative expansion.

A third way to isolate the charmless semileptonic signal is to use a selection based on the $q^2$ of the leptonic system. Restriction of phase space to regions of large $q^2$ also restores the validity of the OPE \[4\] and suppresses effects due to the details of the wave function of the $b$ quark in the $B$ meson. Taking only the region kinematically forbidden to $b \rightarrow c \ell \bar{\nu}$, $q^2 > (m_B - m_D)^2$ unfortunately introduces a low mass scale \[5\] into the OPE and the
uncertainties blow up to be of order \((\Lambda_{QCD}/m_b^2)\). Another drawback of this method is the elimination of higher energy hadronic final states, which may exacerbate duality concerns.

### 1.1 Fermi motion and shape function

Let us begin by discussing Fermi motion. This phenomenon, originally discovered in nuclear physics, is classically described as a small oscillatory motion of the heavy quark inside the hadron, due to the interaction with the valence quark; in the quantum theory it is also the virtuality of the heavy flavor that matters. Generally, as the mass of the heavy flavor becomes large, we expect that the heavy particle decouples from the light degrees of freedom and becomes “frozen” with respect to strong interactions. That is the basic assumption of HQET. However, even if Fermi motion can be neglected in the “bulk” of the phase space of the decay products, it still plays a role close to kinematical boundaries, such as the region of interest. Kinematically, that is easy to show, since a small virtuality of the heavy flavor in the initial state produces relatively large variations of the fragmentation mass in the final state. A \(b\) quark in a \(B\) meson has momentum

\[ p_b^\mu = m_b v^\mu + k^\mu \]  

where \(v^\mu\) is the four–velocity of the quark, which we can take at rest without any loss of generality: \(v^\mu = (1; 0, 0, 0)\). \(k^\mu = p_b^\mu - m_b v^\mu\) is the residual \(b\) quark momentum after the “mechanical” portion of momentum is subtracted off and it is of order \(\Lambda_{QCD}\). If the momentum transfer to the final state leptons is \(q\), the momentum and the invariant mass of the final state hadrons are

\[ p_X^\mu = m_b v^\mu + k^\mu - q^\mu \quad m_X^2 = p_X^2 \]  

The boundary kinematic region is characterized by relations \(\mu\); being \(k^\mu\) of order \(\Lambda_{QCD}\), large values of \(E_X\) can originate only from \(m_b v^\mu - q^\mu\), inducing an almost light–like behavior

\[ m_b v^\mu - q^\mu = (E_X, 0, 0, E_X) + O(\Lambda_{QCD}) \]

\[ (m_b v - q)^2 = O(E_X \Lambda_{QCD}). \]  

The invariant mass of the final state hadrons is

\[ m_X^2 = (m_b v + k - q)^2 = (m_b v - q)^2 + 2k \cdot (m_b v - q) + O(\Lambda_{QCD}^2) \]

\[ \simeq (m_b v - q)^2 + 2E_X k_+ \]  

where \(n^\mu \equiv (1, 0, 0, 1)\) and \(k_+ = k \cdot n\). Over most of phase space, the second term is suppressed relative to the first by one power of \(\Lambda_{QCD}/m_b\), and so it may be treated as a perturbation. This corresponds to the usual OPE. However, in the region of interest the first two terms are of the same order.

This can be also seen in a more compact way, imposing directly

\[ m_b v^\mu - q^\mu \equiv E_X n^\mu + k^\mu, \]  

where \(k^\mu\) is of order \(\Lambda_{QCD}\). Then Eq. (9) can be written as

\[ m_X^2 \simeq (m_b v + k - q)^2 = (m_b v - q)^2 + 2k \cdot (m_b v - q) + O(\Lambda_{QCD}^2) \]

\[ \simeq E_X k' \cdot n + 2E_X k \cdot n = E_X (k'_+ + k_+). \]  

A fluctuation in the heavy quark momentum of order \(\Lambda_{QCD}\) in the initial state produces a variation of the final invariant mass of the hard subprocess of order

\[ \delta m_X^2 \sim O(\Lambda_{QCD} E_X). \]  

An amplification by a factor \(E_X\) has occurred, as anticipated.

The differential rate in this region is therefore sensitive to the wave function \(f(k_+)\) which describes the distribution of the light cone component of the residual momentum of the \(b\) quark. The shape function is a non-perturbative function and cannot be calculated analytically, so the rate in that region is model dependent.
even at leading order in $\Lambda_{QCD}/m_b$. If we consider a heavy quark with the given off-shell momentum and a final state consisting of a massless on-shell quark at the tree level, we find for the shape function

$$f(k_+)^{\text{part}} = \delta \left( k_+ + \frac{(m_b v - q)^2}{2E_X} \right) + O(\alpha_S),$$

(10)
as it should be, since $m_X^2 = 0$. Selecting the hadronic final state, i.e. $k_+$, we select the light-cone virtuality of the heavy flavor which participates in the decay.

We note that even with the amplification effect, Fermi motion effects are irrelevant in most of the phase space, where typical values for the final hadron mass are

$$m_X^2 \sim O(E_X^2),$$

(11)
in agreement with physical intuition.

The shape function $f(k_+)$ is written as a function of $k_+ = k^0 + k_\parallel$; spatial components $k_\parallel$ and $k_\perp$ have been defined with respect to the $m_b v - q^\mu$ direction (roughly the recoiling u quark). At this order, possible contributions due to $k_\perp$ are ignored. The shape function can also be seen as a resummation of the OPE to all orders in $E_X \Lambda_{QCD}/m_b^2$ [4, 9, 10].

Because the shape function depends only on parameters of the B meson, it is reasonable to expect that this leading order description holds for any B decay to a light quark. In general, we aspect QCD to factorize long distance effects into structure functions, with universal characteristics. In particular, one would like to get an estimation of structure functions for the semileptonic decays through the differential rate of $B \rightarrow Xs\gamma$ decay. However, due to the different kinematics between two body and three body processes, and to the presence of more than one energy scale ($E_X$, $m_X$ and $m_b$), universality of structure functions is not trivially applicable [11, 12, 13].

1.2 Perturbative resumming of large logarithms

In general, the differential partial width is given by the convolution of a non-perturbative structure function with the perturbative calculable parton level differential distribution. Large remnants of the long distance dynamics occur also at the perturbative level by the presence of large logarithms near the threshold regions of phase space. Threshold resummation is a well known calculation technique which organizes the logarithmic enhancements to all orders in perturbation theory, thereby extending the QCD predictive power. In perturbative QCD, the hadronic subprocess in $\bar{B} \rightarrow Xs\bar{l}l\nu$ consists of a heavy quark decaying into a light quark which evolves later into a jet of soft and collinear partons. The series of large infrared logarithms is due to an only partial cancellation of infrared divergencies in real and virtual diagrams. Let us consider f.i. the light quark produced in a process with a hard scale $Q$, evolving into a jet whose invariant mass is kinematically limited to a value $m$ well below $Q$; the smaller integration region of the real diagrams induces a left-over double logarithm in the ratio $Q/m$. Multiple gluon emission occurs at high orders of perturbation theory, originating a double logarithmic series [14, 15].

2 Radiative B decays

Let us consider the radiative decay with a real photon in the final state,

$$B \rightarrow X_s + \gamma$$

(12)

A systematic resummation is best done in the $N$-moment space or Mellin space, because it leads to the exponentiation of the logarithmic corrections. In the $N$-moment space the threshold region corresponds to the limit $N \rightarrow \infty$ [16]. Let us consider the Mellin transform of the normalized spectrum

$$\Gamma_{R,N} \equiv \frac{1}{\Gamma_{R}} \int_0^1 (1 - t_s)^{N-1} \frac{d\Gamma_R}{dt_s} dt_s$$

(13)
In Mellin space threshold logarithms manifest themselves as a series in \( \log N \). The order by order perturbative expansion contains double logarithmic contributions and it has the following schematic form

\[
\Gamma_{R,N} = 1 + \alpha (c_{12} L^2 + c_{11} L + c_{10}) + \alpha^2 (c_{24} L^4 + c_{23} L^3 + c_{22} L^2 + c_{21} L + c_{20}) + \cdots
\]  

where \( L \equiv \log N \) and \( \alpha \) is the running coupling evaluated at the hard scale \( m_b \).

The logarithmic terms have an exponential structure and we can write the following factorized form

\[
\Gamma_{R,N} = C_R(\alpha) \sigma_N(\alpha) + d_{R,N}(\alpha),
\]

where the form factor \( \sigma_N(\alpha) \) describes hard and collinear partons, while \( \sigma_N(\alpha) \) approximates at next-to-leading order (NLO), \( \alpha \ll \alpha \).

\( d_{R,N}(\alpha) \) is the hard scale of the process: in radiative decays, \( \alpha \equiv \beta_0 \alpha L \) and \( \beta_0 \) is the first coefficient of the \( \beta \) function expansion in \( \alpha \). By maintaining in the exponent of \( \sigma_N(\alpha) \) only the function \( g_1 \) one approximates at leading order (LO); similarly keeping \( g_1 \) and \( g_2 \) at next-to-leading order (NLO), \( g_1 \), \( g_2 \) and \( g_3 \) at next-to-next-to leading order (NNLO) and so on. This is equivalent to resumming series up to \( L(\alpha L)^n \), \( (\alpha L)^n \) and \( \alpha(\alpha L)^n \), and so on. The explicit expressions of \( g_i \) are known up to NNLO \[17\] \[18\] \[19\] \[20\].

The exponent in \[16\] can be obtained by means of the following resummation formula \[14\] \[15\] \[20\]

\[
G_N(\alpha) = \int_0^1 d z \frac{z^{N-1} - 1}{1 - z} \left\{ \int \frac{Q^2(1-z)}{Q^2(1-z)^2} \frac{d k_t^2}{k_t^2} A(\alpha(k_t^2)) + B(\alpha(Q^2(1-z))) + D(\alpha(Q^2(1-z)^2)) \right\},
\]

where \( Q = 2E_X \) is the hard scale of the process: in radiative decays, \( Q = m_b \). The functions \( A(\alpha) \), \( B(\alpha) \) and \( D(\alpha) \) have a standard fixed order expansion in \( \alpha \), with numerical coefficients

\[
A(\alpha) = A_1 \alpha + A_2 \alpha^2 + \cdots, \quad B(\alpha) = B_1 \alpha + B_2 \alpha^2 + \cdots, \quad D(\alpha) = D_1 \alpha + D_2 \alpha^2 + \cdots.
\]

\( A(\alpha) \) describes the emission of partons which are both soft and collinear, \( B(\alpha) \) describes hard and collinear partons, while \( D(\alpha) \) describe partons which are soft and at large angles. The values of the known first coefficients of the functions \( A(\alpha) \), \( B(\alpha) \) and \( D(\alpha) \) are reported in \[11\].

By truncating \( \alpha \) expansions for the functions \( A(\alpha) \), \( B(\alpha) \) and \( D(\alpha) \) in eq. \[21\] one is implicitly assuming \( \alpha \ll 1 \). That is not always correct since the running coupling \( \alpha(k_t^2) \) is integrated over all gluon transverse...
momenta \( k_t \) from the hard scale \( Q \) down to zero: inside the integration region, the Landau pole is hit and the running coupling diverges. A prescription has to be assigned to give a meaning to the formal expression \( \sigma(t_s; \alpha) \). Even after integration, an effect of the presence of the Landau pole persists: the series in eq. \( (19) \) is divergent as the higher order functions have factorially growing coefficients \( [19, 21] \). By truncating it to its first few terms, we stay within the so-called fixed logarithmic accuracy. Moreover, the functions \( g_i(\lambda) \) have branch cuts starting at \( \lambda = 1/2 \) and going up to infinity. When \( \lambda \to \frac{1}{2} - \), by definition of \( \lambda \), a singularity in \( N \)-space occurs at \( N \to \exp \left[ 1/2 \beta_0 \alpha_S \left( Q^2 \right) \right] \), that is at \( N_{\text{crit}} \sim Q/\Lambda \) (\( \Lambda \) is the QCD scale)\(^2\). The form factors are then formally well defined up to a critical value \( N_{\text{crit}} \), above which they acquire a (completely unphysical) imaginary part.

To return from \( N \) space to the physical space one uses the Mellin inverse transform:

\[
\sigma(t_s; \alpha) = \int_{C-i\infty}^{C+i\infty} \frac{dN}{2\pi i} (1 - t_s)^{-N} \sigma_N(\alpha) .
\] (23)

The evaluation of the inverse transform requires a prescription, in order to overcome the problems just mentioned and to obtain a form factor formally well defined in the whole \( t_s \)-space, which resums all the logarithmic terms at the requested order. Several strategies can be pursued towards this result. The simplest possibility is to restrict oneself to a fiducial region in \( N \)-space below \( N_{\text{crit}} \). Another possibility is to use the so-called minimal-prescription \( [22] \), which regularizes the form factor by means of an additional prescription for the contour integration in \( N \)-space in \( (23) \). The problem of the presence of an integration over the Landau pole in resummation formulas, and of the ambiguities associated, has been also examined in the contest of the occurrence of infrared renormalons, to get information about non-perturbative effects, in the form of power-suppressed corrections to the cross sections (see f.i. \( [21] \)).

Another approach is to give a prescription for the infrared pole directly in \( N \)-space, in such a way that the form factors are well-defined for any \( N \). Then it is not necessary to give a prescription for the Mellin inverse transform. A recent analysis \( [23] \) substitutes an effective coupling to the standard running coupling in resumming formula \( [21] \). The effective coupling is built in a way to maintain the high energy behaviour of the standard running coupling, without reaching the Landau pole at low energies. Therefore the effective coupling never straddles far outside the perturbative domain. The resummation formula is free of Landau pole pathologies, and no prescription is needed because \( \sigma_N(\alpha) \) is analytic on the integration contour \( [24] \).

The effective form factor may also include absorptive effects related to the coupling constant in order to derive an improved expression for the resummation formula. Such absorptive effects are related to the decay of time-like gluons in the jet evolution. As well known from perturbation theory, at higher orders, one has to consider multiple emissions off the heavy and the light quark in \( B \) and secondary emissions off the radiated gluons. Because of the presence of these higher-order terms, the coupling in the resumming formula is evaluated at the transverse momentum of the primary emitted gluon \( [24] \). By including the \(-i\pi\) terms in the integral over the discontinuity, i.e. the absorptive effects, usually neglected, the coupling in the resumming formulas is replaced by an effective coupling, evaluated at the transverse momentum of the primary emitted gluon \( [23, 25] \).

\[
\alpha \to \tilde{\alpha}(k_1^2).
\] (24)

There is another advantage by using a perturbative approach with an effective coupling. In the standard approach, after resumming the large perturbative logarithmic contributions, one has to postulate a physically motivated non-perturbative model. Generally, an \( ad-hoc \) non-perturbative form factor is convoluted with the perturbative distributions. Universal aspects of QCD dynamics, common to different processes, are not easily discovered this way. On the other hand, by describing different processes with the appropriate perturbative formulas, and the same effective coupling by assumption, one can deal simultaneously with perturbative and non-perturbative effects (without introducing model-depending parameters), avoid mistakes related to double counting, and underline universal effects by comparing with the data \( [23, 24, 26] \).

\(^2\)Let us also observe that the degree of singularity of the functions \( g_i \) for \( \lambda \to 1/2 \), and therefore also of the form factor, increases with the order of the function, i.e. with \( i \) \( [21] \).
3 Universal aspects of QCD dynamics

Let us now consider the decay

\[ B \to X_u + l + \nu. \]  

(25)

It is possible to obtain a factorized form for the triple differential distribution, the most general distribution in process (25) (its integration leads to all other spectra):

\[ \frac{1}{\Gamma} \frac{d^3 \Gamma}{dx du dw} = C[x, w; \alpha(w m_b)] \sigma[u; \alpha(w m_b)] + d[x, u, w; \alpha(w m_b)], \]  

(26)

where:

\[ w \equiv \frac{2 E_X}{m_b} \quad (0 \leq w \leq 2), \quad x \equiv \frac{2 E_l}{m_b} \quad (0 \leq x \leq 1) \]  

(27)

and

\[ u \equiv \frac{E_X - \sqrt{E_X^2 - m_X^2}}{E_X + \sqrt{E_X^2 - m_X^2}} = \frac{1 - \sqrt{1 - (2 m_X/Q)^2}}{1 + \sqrt{1 - (2 m_X/Q)^2}} \simeq \left( \frac{m_X}{Q} \right)^2 \quad (0 \leq u \leq 1). \]  

(28)

We have called \( Q \) the hard scale of the process and, at the threshold, set \( Q = 2 E_X \). Both the logarithms and the argument of the running coupling depend on the kinematics of the problem, by means of the hard scale \( Q \). There are two important kinematical differences with respect to the radiative case. First, in the three-body semileptonic decay, the distribution also depends on the charged lepton energy \( E_l \). Second, while in the radiative decay it is always \( Q = 2 E_X = m_b \), in process (25) there are regions of phase space where \( E_X \) is substantially reduced \(^3\).

In analogy with what done in the previous section, we can study the distribution in the Mellin space, defining

\[ \sigma_N(\alpha) \equiv \int_0^1 du \ (1 - u)^{N-1} \sigma(u; \alpha) \]  

(29)

At this level, there is universality among radiative and semi-leptonic decays, meaning that the same QCD form factor \( \sigma_N \) appears in both distributions \(^{15}\) and \(^{24}\). Consequently, the form factor in the physical space is the same \( \sigma(u; \alpha) \). It is evaluated at the argument \( u \simeq m_X^2/(4 E_X^2) \) in the semileptonic case. In the radiative case, by imposing the kinematical relation between hadronic energy \( E_X \) and hadronic mass \( m_{X_s} \), we have

\[ u|_{E_X = m_b/2(1 + m_{X_s}^2/m_b^2)} = t_s. \]  

(30)

where \( t_s \equiv m_{X_s}^2/m_b^2 \).

The coupling constant argument is set at the hard scale \( Q = 2 E_X \) in both processes; in the radiative decay, that implies it is fixed to \( m_b \).

This simple connection between radiative and semileptonic processes is sometimes lost when one passes to the double or single differential distributions for the processes \(^{24}\) \(^{15}\) \(^{11}\) \(^{12}\) \(^{13}\). All double and single distributions are obtained by integrating the triple differential distribution \(^{24}\). As seen before, in semileptonic decays the hadronic energy \( E_X \) is not fixed and it can be integrated over; this affects the infrared structure of the distribution, since the form factor \( \sigma \) depends on \( E_X \) \(^{15}\) \(^{11}\) \(^{12}\) \(^{13}\). This class of distributions, f.i. the single differential distribution in \( m_X \), cannot be directly compared with the radiative spectra. The structure of the threshold logarithms is not the same and there is no universality of long distance effects. On the other side, distributions as the single differential one in \( E_X \), where the energy \( E_X \) is not integrated over, keep the infrared structure of the radiative spectrum. They can be directly related via short-distance coefficients to the radiative spectrum.

\(^3\)One can think, f.i., to the kinematical configuration with a large invariant mass for the lepton pair.
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