Short distance signatures in Cosmology: Why not in Black Holes?

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Current theoretical investigations seem to indicate the possibility of observing signatures of short distance physics in the Cosmic Microwave Background spectrum. We try to gain a deeper understanding on why all information about this regime is lost in the case of Black Hole radiation but not necessarily so in a cosmological setting by using the moving mirror as a toy model for both backgrounds. The different responses of the Hawking and Cosmic Microwave Background spectra to short distance physics are derived in the appropriate limit when the moving mirror mimics a Black Hole background or an expanding universe. The different sensitivities to new physics, displayed by both backgrounds, are clarified through an averaging prescription that accounts for the intrinsic uncertainty in their quantum fluctuations. We then proceed to interpret the physical significance of our findings for time-dependent backgrounds in the light of nonlocal string theory.

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I. INTRODUCTION

Although short distance physics belongs in the realm of quantum gravity, it is possible that some traces may have survived in the low energy observables. Many efforts have been devoted to the exploration of this issue and the claims fall in two categories: Hawking radiation is robust against nonlinear modifications of short distance physics; but Cosmic Microwave Background (CMB) spectrum may generically be sensitive to short distance modes (see also for a partial list of other relevant References), except for special classes of modification that ensure adiabatic time evolution of the modes at late times.

These claims should be taken as suggestive rather than conclusive for at least the two following reasons:
i) we do not, as yet, have a fundamental theory to describe physics at energies higher than the Planck mass. This means that even for specific short distance models introduced in literature, we do not have a set of equations by which to study these models. Einstein equations and in particular the equation of energy conservation almost always break down in the presence of nonlinear trans-planckian physics. We can hope that quantum field theory and modifications to Einstein equations which satisfy Bianchi identity may remain a good description, as approximate tools, while taking the low energy limit;
ii) the spectrum is very sensitive to the initial conditions. We do not know what the initial conditions are in Cosmology but they can contaminate the result of the investigation of the role of nonlinear physics in the CMB spectrum.

Despite the difficulties of carrying out such an investigation, it is very exciting to consider the possibility that we may find low energy signatures from very high energy processes in Cosmology. If trans-planckian physics is described within the framework of String Theory (e.g. see Ref. [1]) with our present state of knowledge, then its first evidence would be found in cosmological grounds.

Let us take the above claims as true. In these notes we want to gain a deeper understanding about probes of Planck scale physics, by posing the following question: Why do the CMB spectrum and Hawking radiation respond differently to nonlinear short distance physics? In order to achieve a comparison between Black Holes and Cosmology, the setup here is the following: Consider a moving mirror which follows a certain trajectory along a cartesian direction (see [1] and References therein). In Section II we discuss the response function of a detector in the space to the right of the moving mirror for two limiting cases: a) when the mirror’s motion “mimics” a Black Hole background (Section II A); b) and when the mirror’s motion follows the evolution of the Hubble horizon (Section II B). We then compare the spectra and argue that the difference between Black Holes and Cosmology with respect to their sensitivity to short distance physics, arises from the fact that since the stationary background of Black Holes is characterized by a length scale much larger than the Planck length, there are lesser degrees of freedom in the first instance (surface degrees of freedom “confined” to the Black Hole horizon), as compared to the dynamic background of Cosmology which, due to non-locality, results in volume degrees of freedom.

This leads us on to introducing a spatial averaging prescription to account for the uncertainty in the origin of Hawking radiation that indicates a “wash-out” of trans-planckian modes, while in Cosmology there is a natural time averaging to account for the uncertainty in the initial conditions which (at best) enhances the role played...
by short scale physics at later times. We conclude in Section 11 that any traces of Planck scale physics in the spectra, as measured by their departure from thermality, may be larger and thus observable in the cosmological case. We then speculate that our results are related and point very much in the same direction as recent works on cosmological background solutions in String Theory (see [12] and References therein). An observable effect becomes possible when quantum fluctuations are comparable to thermal fluctuations. The competition between quantum and thermal fluctuations is larger for volume degrees of freedom than for surface degrees of freedom, thus the possibility that they may be within reach of observation in Cosmology.

Recent work in String Theory [14] indicates that non-locality may be required for all non-stationary, cosmologically relevant backgrounds (i.e. Lorentzian vacua). We relate the findings of [12] to our backgrounds in order to obtain a physical interpretation for the sensitivity of the CMB spectrum to Planck scale physics as originating from non-locality of time-dependent solutions in String Theory.

We use units with $c = \hbar = 1$ and denote by $m_p$ the Planck mass and by $\ell_p = m_p^{-1}$ the Planck length. Finally, $k_B$ is the Boltzmann constant.

II. MOVING MIRRORS AND THE DETECTOR

There are several instances of non-trivial vacua in flat space-time, such as the time dependent Casimir effect and the moving mirror, which can be dealt with as time-dependent boundary conditions for the (Klein-Gordon) wave equation for a massless scalar field $\phi = \phi(t, x)$:

$$\Box \phi \equiv \ddot{\phi} - \phi'' = 0$$

$$\phi(t, z(t)) = 0 ,$$

with $(t, x)$ an (inertial) reference frame and $x = z(t)$ is the trajectory of the moving mirror in such a frame. We shall assume $z(t < 0) = 0$, so that the mirror starts moving at time $t = 0$. For the Casimir effect one also demands

$$\phi(t, L) = 0 ,$$

where $x = L \gg z(t)$ is the location of the second (fixed) wall of the box.

We note that the case corresponding to the standard moving mirror [11] is then recovered as $L \to \infty$ by assuming that the space-time extends from $t = -\infty$ (the infinite null past denoted by $\mathcal{I}^-$) to $t = +\infty$ (the infinite null future $\mathcal{I}^+$). One then chooses appropriate positive frequency modes for the vacuum state on $\mathcal{I}^-$ and evolves the system to $\mathcal{I}^+$, where negative frequencies and the out-vacuum are well-defined. This approach is much in the same spirit as the usual in-out formalism of quantum field theory and is therefore consistent, provided that the relevant part of the process occurs in a relatively small portion of the space-time outside of which the fields propagate freely [13].

The same approach is taken for a Black Hole background and in fact, one finds out that for a particular choice of the mirror’s trajectory, the moving mirror case reproduces Hawking radiation, when backreaction is ignored [29]. Both processes can be viewed as a projection from the surface $\mathcal{I}^-$ to the mirror (horizon) and then back to the surface $\mathcal{I}^+$, which points to the holographic nature of the effect.

A more complex approach which fully incorporates the backreaction of particles on the background geometry appears more appropriate for describing cosmological particle creation. In this case one can assume the space-time begins at $t = 0$ (corresponding, e.g. to the onset of inflation). Positive frequency modes can correspondingly be defined only with respect to the initial time surface at $t = 0$ and the initial vacuum evolved according to the in-in, or closed time path formalism of quantum field theory [14]. The notion of particles is not unique in this fully time-dependent context [13], although one may be able to define it if a “preferred” frame exists. However, no projection of the sort described in the previous paragraph is naturally conceivable because of the arbitrary time dependence of the mirror (cosmological scale factor).

A. Casimir effect and Response function for the “Black Hole” mirror

The analogy between Hawking radiation from a Black Hole and a moving mirror is well known. Here we shall simply recall the main points reported in Ref. [11], chap. 4.4. Let us introduce the usual advanced and retarded coordinates, $u = t - x$ and $v = t + x$, which cover all of Minkowski space-time to the right of the mirror. The mirror trajectory is described by $x = z(t)$. A convenient choice of modes solving Eqs. (1a) and (1b) is then given by

$$\phi_{\omega}^{in}(u, v) = \frac{i}{\sqrt{4 \pi \omega}} e^{i \omega v - e^{-i \omega (2 t_u - u)}} ,$$

where $t_u = u + z(t_u)$. Such modes are positive frequency on the infinite null past $(\mathcal{I}^-)$ or $u = -\infty$ and define the in-vacuum $|0_{in}\rangle$ which, for $t < 0$, remains devoid of particles (since $t_u = u$ for $u < 0$). Modes of negative frequency can analogously be introduced on $\mathcal{I}^+$ (i.e. $v = +\infty$).

We are interested in finding the squeezing of the final state at $\mathcal{I}^+$ or $u = +\infty$ as compared to the initial vacua, which results from the mirror’s motion. When the mirror starts moving at $t = 0$, the in-vacuum is modified by the Doppler factor appearing in the right-moving part of the modes. The case which mimics Hawking radiation is given by a mirror’s trajectory that approaches the speed...
of light,
\[ z(t) = -t + v_0 \left( 1 - e^{-2\kappa t} \right) . \]  

The last term above represents a transient between the steady state and the asymptotic null curve \( v = v_0 \). Left-moving parts of modes (\[3\]) get reflected off the mirror for \( v \leq v_0 \), while they pass unaffected for \( v > v_0 \). The piling up of (equispaced) lines of constant \( x \) along the “causal horizon” \( v = v_0 \) occurs also for a collapsing body and is at the origin of the effect (see Refs. [17] for a neat exposition).

A detector can now be introduced as a quantum system with (discrete) internal energy spectrum \(| E_n \rangle\) and (sharply) localized on the trajectory \( x = y(t) \). It interacts with the scalar field via a dipole term,
\[ V_{\text{int}} = q \delta (x - y(t)) \hat{Q} \phi(t, x) , \]  

where \( q \) is a (small) coupling constant and \( \hat{Q} \) the operator which causes transitions between inner energy levels. The response function of such a detector is just sensitive to positive frequencies of the scalar field (to lowest order in \( q \)), and its probability of excitation from the ground state \(| E_0 \rangle\) to the state with energy \( E_n \) is given by [11]

\[ P_{0 \to n} = q^2 \left| \langle E_n | \hat{Q} | E_0 \rangle \right|^2 \frac{1}{e^{(E_n - E_0)/k_B T} - 1} , \]  

with Doppler shifted temperature
\[ k_B T = \frac{\kappa}{2\pi} \sqrt{1 - w} \equiv \frac{\kappa}{2\pi} \Gamma(w) , \]

where \( w \) is the (instantaneous) velocity of the detector in the \((t, x)\) frame, \( y(t) \approx y(0) + w t \).

The above simple result follows from the spatial \( \delta \)-function in Eq. [4] forcing the Wightman function to be strictly evaluated on the trajectory’s points \( y(t) \) and \( y(t') \) without smearing [3]. A realistic detector would however be better described in terms of a wave-packet \( \Psi(t, x) \) peaked along the trajectory \( x = y(t) \) with a “width” \( \Delta \). Hence, the \( \delta \)-function in Eqs. [4] and [3] must be replaced by \( |\Psi(t, x)|^2 \) and one can show that scalar field modes with short wavelengths \( \lambda \ll \Delta \) are averaged over and do not contribute appreciably to the transition probability [3] (for the details see, e.g. [18] and References therein). One can understand the origin of this suppression by considering a region of size \( \Delta \sim N \lambda \) (with \( N \gg 1 \)) and estimating the order of magnitude in fluctuations therein by using the uncertainty principle for the location of the \( \Delta \) region. Assuming a Poisson distribution, one finds
\[ \frac{\delta \Delta}{\Delta} \sim \frac{\delta N}{N} \sim \sqrt{\frac{N}{N}} \sim \sqrt{\frac{\lambda}{\Delta}} \ll 1 . \]  

For a realistic detector \( \Delta \gg \lambda = \ell_p \), thus it would never respond to trans-planckian modes and the result [3] holds with very good approximation for sufficiently low (detected) temperature \( k_B T \lesssim \Delta^{-1} \).

It is futile to ask where those thermal particles come from since the very notion of a particle is likely lost near the source region [3]. One could say that particle production roughly occurs in the strip \( 0 < v < v_0 \) where the mirror changes its motion between the two asymptotic regimes. However, detection is performed well outside the strip \( 0 < v < v_0 \), and most commonly at very late times (near \( 3^+ \)), where a particle interpretation is recovered. In any case, since the typical wavelengths of the produced quanta are of order \( \kappa^{-1} \), a stationary detector (with \( w = 0 \)) cannot resolve the position of the source with accuracy better then \( \kappa^{-1} \). There is thus one fundamental and intrinsic (i.e. detector independent) length scale in the effect, namely \( \ell \sim \kappa^{-1} \), which determines the statistical mechanics of the emitted flux as well as the source fuzziness. The other length \( v_0 \) reflects an irrelevant detail of the mirror motion, while \( w \) accounts for the relative state of the detector with respect to the mirror.

Let us now see how this picture translates into Black Hole language. We recall that for a Schwarzschild Black Hole of Arnowitt Deser Misner (ADM) mass parameter \( M \) the Hawking temperature is
\[ T_H = \frac{1}{8\pi M k_B} , \]

and the Tolman factor for a stationary detector placed at \( r \gg 2M \) which receives signals from a source located at radial position \( r = r_s \) is
\[ \Gamma_T = \sqrt{1 - \frac{2M}{r_s}} . \]
Hence, if the particles came precisely from \( r_s = 2M \), they had to be produced with a local energy \( k_B T \to \infty \) (achieved by taking \( w \to -1 \) in Eq. (3)). One can then say that \( k_B T \) in Eq. (3) is to be viewed as the energy of Hawking quanta at the point of emission with \( \kappa = 2\pi k_B T_H = 1/4M \) the surface gravity of the Black Hole, and that the Doppler factor \( \Gamma(w) \) plays the role of the Tolman factor, with \( w \) a function of the radius \( r_s \), the origin of the radiation. Since \( \kappa \ll m_p \) (as \( M \gg \ell_p \) for a Black Hole), the Hawking temperature \( T_H \) is far less than the Planck mass and trans-Planckian energies will not reach a stationary detector (which could anyways not detect them according to the uncertainty principle argument above since its width \( \Delta \gg \ell_p \)).

The problem remains whether modes of trans-Planckian energies are excited at some small radius \( r_s \). Since Hawking quanta have typical wavelengths of order \( M \) besides the gravity scale (the Planck length \( \ell_p \)), one can therefore roughly consider the Black Hole as an emitter of size \( \Delta_{BH} \sim M \sim N\ell_p \) and repeat the Poisson counting which led to Eq. (8). One obtains an order of magnitude estimate for fluctuations inside the region of size \( \Delta_{BH} \).

The above argument can be refined by introducing an effective Tolman factor (for the detector far away from the Black Hole) which takes into account the uncertainty in the radius of the source \( r_s \) and consequently of the temperature \( T \) of the emitted Hawking radiation. This is obtained by averaging the Doppler factor \( \Gamma \) in Eq. (6) over a corresponding range of equivalent source positions to yield the average energy at the point of emission corresponding to a given detected \( T_H \).

\[
\rho \equiv \frac{\langle k_B T \rangle}{k_B T_H} = \frac{1}{\ell} \int_0^\ell dx \sqrt{1 - \frac{1}{1 + w(x)}}.
\]

Let us then replace the Doppler factor with the Tolman factor (11) and assume \( \ell = \alpha/\langle k_B T \rangle \), with \( \alpha \approx 1 \). Upon substituting in Eq. (12), one obtains

\[
\rho = \frac{\langle k_B T \rangle}{\alpha} \int_{2M}^{2M+\alpha} \frac{dr_s}{\sqrt{1 - 2M/r_s}} = \frac{\rho}{4\pi\alpha} \int_0^{\pi/\ell} dx \sqrt{1 + x^{-1}}.
\]

The above expression requires the self-consistency condition for \( \rho \), independent of \( M \),

\[
\frac{1}{4\pi\alpha} \int_0^{\pi/\ell} dx \sqrt{1 + x^{-1}} = 1.
\]

The (numerical) solution for \( \alpha = 1 \) is

\[
\langle k_B T \rangle \approx 1.23 k_B T_H,
\]

which is much smaller than \( m_p \) for \( M \gg \ell_p \). This result clarifies the fact that high energy modes “wash-out due to averaging”.

Let us note that, since the Tolman factor \( \Gamma_T \) does not depend on the energy \( k_B T_H \), neither does the ratio \( \rho \), and the result (15) holds for each frequency of the Hawking spectrum separately. Finally, for \( \ell \ll M \) (\( \alpha \ll 1 \)) one would recover the usual result \( k_B T \to \infty \).

### B. Casimir effect and Response function for the “cosmological” mirror.

The CMB spectrum in Cosmology is formally calculated by the same methods as particle creation in curved space (9), since the nonlinear time-dependent frequency of short distance physics can be attributed to a curved space time geometry (10). In our case this role is played by the “cosmological mirror’s trajectory”. In this part we will follow a different approach from the conventional one in calculating the Bogolubov coefficients. Instead of finding the S-matrix element for the asymptotic vacua at past and future null infinity, the time-dependent Bogolubov coefficients are calculated iteratively at each time slice that is taken consecutively after small time intervals. The purpose for following this procedure is in order to enable us to trace step by step the stage at which the effects of trans-Planckian signatures may arise. Recall that unlike the ADM mass parameter \( M \) in Black Holes there is no characteristic time scale in Cosmology besides the gravity scale (the Planck length \( \ell_p \)), thus the time step can be taken as small as \( \ell_p \).

Let us define the initial Fock space from positive frequency modes at \( t = 0 \) and then evolve to larger values of \( t \). In doing so, we are assuming that the initial conditions are known and well defined. In particular, we note that solutions to Eq. (14) are given by superposition of plane waves in the coordinates \( x \) and \( t \). Then imposing Eqs. (14) and (13) yields the (discrete) spectrum

\[
\phi_n(t, x) = N e^{-i\omega_n t} \sin \left( n \pi \frac{x - z}{L - z} \right),
\]

where \( n \) is a positive integer and \( N \) is a normalization factor. The frequency \( \omega \) equals the spatial momentum \( k \) and is given by

\[
\omega_n = k_n = \frac{n \pi}{L - z}.
\]
The modes (17) indeed solve Eq. (14) only if one can neglect the time dependence of \( z \) (and, consequently, in \( \omega \)). This is basically the lowest order adiabatic approximation, which helps in defining the concept of particles [13]. We shall provide later a way of measuring its validity.

In order to build up an Hilbert space one introduces the usual (fixed time) scalar product

\[
(\phi, \psi) \equiv -i \int_z^L dx \left( \phi \partial_t \psi^* - \psi^* \partial_t \phi \right),
\]

which yields the (time independent) result \( (\phi_n, \phi_m) = \delta_{n,m} \), provided the normalization is \( N = 1/\sqrt{n \pi} \) and the time dependence of \( z \) is neglected. A general instantaneous solution can therefore be written as

\[
\phi(t, x) = \sum_{n=1}^{\infty} \left[ a_n \phi_n(t, x) + a_n^* \phi_n^*(t, x) \right],
\]

where \( a_n \) and \( a_n^* \) become the annihilation and creation operators of the quantized theory [1].

To take fully into account the time dependence of \( z \) one should consider time-dependent coefficients \( a_n = a_n(t) \) and substitute the expression (19) into Eq. (14). One would then find that \( a \sim \delta t/L \equiv \epsilon \), where \( \delta t \ll L \) is the typical time step considered (see below). Therefore \( \epsilon \) is the parameter which measures the departure from adiabaticity and the time dependence of \( a \) yields second order effects in \( \epsilon \). A systematic treatment consisting in a perturbative expansion in \( \epsilon \) can be carried out [20], but does not seem necessary for the treatment of the conceptual issue investigated here.

The instantaneous vacuum at \( t = t_1 \) is defined as usual by \( |\phi_n(t_1, t_1)\rangle = 0 \). At a different time \( t = t_2 > t_1 \) one has a different set of modes and consequently a different expansion of the form (19). This also means a different vacuum \( |0(t_2), t_2\rangle \) or, equivalently, that the state \( |0(t_1), t_2\rangle \) will generically contain “particles”. Due to the time dependence of the background, the calculation of the spectrum in this scenario reduces to the familiar problem of particle creation in curved space [2, 11, 13, 21].

Below we denote by unbarred symbols quantities evaluated at \( t = t_1 \) and with a bar the same quantities at the slightly later time \( t_2 = t_1 + \delta t \) (with \( \delta t \ll t_2 \)). Further, the “cosmological mirror position” at \( t = t_2 \) is given by \( \bar{z} = z + \delta z \) (\( |\delta z| \ll |z| \delta t \ll \delta t \ll L \)). The increase \( \delta N_n \) in the “number of particles” in the mode \( n \) stored in the state \( |0(t_1), t_1 + \delta t\rangle \) is obtained from the Bogolubov coefficients

\[
\beta_{nm} \equiv \langle \bar{\phi}_n, \phi_m^* \rangle = e^{-i(\omega_n + \bar{\omega}_n) \delta t} \frac{\omega_m - \bar{\omega}_n}{\pi \sqrt{n m}} \int_{z + \delta z}^L dx \sin \left[ k_n (x - z - \delta z) \right] \sin \left[ k_m (x - z) \right]
\]

\[
= e^{-i(\omega_n + \bar{\omega}_n) \delta t} \frac{\omega_m - \bar{\omega}_n}{\pi \sqrt{n m}} \frac{(L - z) n}{(L - z) n + (L - z - \delta z) m} \sin \left( \frac{m \pi \delta z}{L - z} \right),
\]

and is given by

\[
\delta N_n = \sum_{m=1}^{\infty} |\beta_{nm}|^2 = \sum_{m=1}^{\infty} \frac{n}{\pi^2 m} \sin^2 \left( \frac{m \pi \gamma}{n + (1 - \gamma) m} \right),
\]

with \( \gamma \equiv \delta z/(L - z) \sim \epsilon \ll 1 \). An example of \( \delta N_n \) for \( \gamma \sim 10^{-3} \) is plotted in Fig. 4. It is interesting to note that in the adiabatic approximation of a slowly varying \( z \), the shape of the curve resembles a planckian distribution. Decreasing \( \gamma \) lowers the peak and shifts it to larger values of \( n \).

For \( \delta z \to 0 \) (stationary mirror) and/or \( L - z \to \infty \), \( \gamma \to 0 \), each term in the sum vanishes, hence \( \delta N_n \to 0 \). However, for \( L \) finite, the total number of particles \( N_n \) in the mode \( n \) after a (large) lapse of time \( T \) is obtained from the expression in Eq. (21) summing over a large number \( (T/\delta t) \) of small steps \( \delta t \). This would yield \( N_n \sim (T/\delta t) \delta N_n \). Eventually \( T \to \infty \) and it is therefore clear that \( 0 = \lim_{T \to \infty} (\lim_{L \to \infty} N_n) \neq \lim_{L \to \infty} (\lim_{T \to \infty} N_n) = \infty \).

The wall at \( x = L \) can also be removed by relaxing the condition (14) from the very beginning which then results in a continuous spectrum of states (as distributions)

\[
\phi_k(t, x) = \frac{N}{\sqrt{\omega}} e^{-i \omega t} \sin \left[ k (x - z) \right],
\]

where \( \omega \) is a positive real and \( N \) the normalization factor in the scalar product (13). It is now possible to repeat the same steps as before and, on observing that

\[
\int_{z + \delta z}^L dx \sin \left[ k (x - z - \delta z) \right] \sin \left[ q (x - z) \right] = \frac{k}{k^2 - q^2} \sin \left( q \delta z \right),
\]

one obtains (modulo numerical normalization factors) a spatial density of produced particles

\[
\delta N_k \approx \frac{4 \pi}{\sqrt{q}} \frac{k^2}{(k^2 + q^2)^2} \sin^2 \left( q \delta z \right).
\]

The above expression has the same features as that in Eq. (23) for the discrete case: the peak in the distribution...
lows and shifts to larger values of \( k \) for decreasing \( \delta z \) (see Fig. 2 for an example). The total density of particles produced in the mode \( k \) would then be given by

\[
N_k = \sum_{\delta z} \delta N_k ,
\]

where the sum is over the (discretized) displacements \( \delta z(t) \) of the moving mirror. For a mirror which approaches the speed of light \( \delta z \sim 1 \) and one finds

\[
N_k(T) \sim T \delta N_k ,
\]

assuming the motion has started at \( t = 0 \) and ended at \( t = T \), and \( \delta N_k \) is that plotted in Fig. 2.

It is now straightforward to study the consequences of non-linear dispersion relations

\[
\omega_n = \omega(k_n) \neq k_n .
\]

In fact, again neglecting the time dependence of \( z \), the only modification that “registers” parameters of short distance non-linearity occurs in the factor in front of the integral in Eq. (20a). For example, one can simply place a sharp cut-off at \( \omega = m_p \) or consider the smooth Epstein dispersion relations of Ref. 23.

\[
\omega_n \sim k_n \operatorname{sech} \left( \frac{k_n}{\mu} \right) ,
\]

(28)

(where \( \mu \approx 1 \) determines the location of the peak in the spectrum), to obtain the (generally large) damping of particle production displayed in Fig. 1. Although the high wave-number effect can be directly appreciated at the time of production only for modes with wavelengths sufficiently large (i.e. \( n \) sufficiently small), the cosmological red-shift will make the entire region in Figs. 1 and 2 (and possibly a larger one) visible to present detection, as we now proceed to discuss. Therefore this effect may become more important at present than during inflation.

In order to translate the Casimir results into Cosmology we first need to put down the dictionary. One can consider the size of the box \( L - z \) as the scale factor of the Universe \( A(t) \) whose evolution is a priori not known. Having this in mind the mirror’s trajectory \( z(t) \) was kept arbitrary, the evolution of the system followed continuously (and not just looking at the asymptotic behavior of the mirror) and particle production was seen to occur homogeneously. For sufficiently smooth trajectories (i.e. cosmological evolution), a stationary \( (w = 0) \) detector would measure a portion \( 0 < k \lesssim \Delta^{-1} \) of the instantaneous spectrum shown in Fig. 1 and 2 at the time \( t_s \) at which particle production occurs. Energies are successively red-shifted by the expansion of the Universe and the same detector probes the portion \( 0 < k \lesssim \Gamma(w) \Delta^{-1} \) at the time \( t \gg t_s \), where \( \Gamma \) is the Doppler factor of Eq. (7) with \( w \) a function of the time of creation \( t_s \).

If the cosmological evolution is such that \( w \rightarrow -1 \) for \( t_s \rightarrow 0 \) (considered, e.g. as the onset of inflation), trans-Planckian modes certainly appear in the CMB spectrum at the late time \( t \).

For the Black Hole case the uncertainty in the location of the origin of Hawking quanta led us to introduce a spatial average over the region of size \( \ell \sim \kappa^{-1} \gg \ell_p \) outside the horizon [Eq. (12)] by which we have shown that short distance physics does not in any way affect the statistical mechanics of the detected Hawking spectrum (see also Refs. 3, 4, 5). In the cosmological case, the significant uncertainty is in the creation time \( t_s \) of cosmological fluctuations for a given detected energy at time \( t \). One could thus envisage a temporal averaging over such a time uncertainty. However, no such procedure could prevent the high frequency branch of the spectrum to become visible in an inflationary scenario 6. In fact, suppose the duration of the de Sitter phase, with scale factor

\[
A \sim e^{H t} ,
\]

(29)

is \( T \pm \Delta T \) with uncertainty \( \Delta T > 0 \), then an effective
cosmological blue-shift would be given by
\[
\langle \Gamma \rangle \sim \frac{1}{2 |\Delta T|} \int_{-\Delta T}^{\Delta T} dt_s \left( e^{H T} - 1 \right)
\]
\[
= \frac{e^{H T}}{H |\Delta T|} \sinh(\Delta T) - 1 > e^{H T}, \tag{30}
\]
which, for all uncertainties $\Delta T > 0$, results in a larger blue-shift than one would have for $\Delta T = 0$.

We can conclude that the most important difference between the Hawking effect and the CMB spectrum in Cosmology is that, due to their stationarity, Black Holes have a fixed characteristic length scale $\ell \sim M \gg \ell_p$ which sets the size of relevant fluctuations well above the Planck scale. In Cosmology the analog scale is the inverse of the *time-dependent* Hubble constant $H = \dot{A}/A$ whose value varies in time and, at best, is just bound from below by the natural quantum gravity scale $\ell_p$. Quantum fluctuations in a cosmological setting would therefore be expected to correspond to Planck scales. This is to say that according to the uncertainty principle, the Hubble radius $H^{-1}$ is uncertain within (roughly) one Planck length. As discussed in the next Section, the interpretation of this statement in terms of a realistic cosmological scenario of Lorentzian vacua given by String Theory in time-dependent backgrounds, is that non-locality seems to be required for at least Planck length scales $\ell_p$.

The authors in Refs. [22, 23] used similar quantum statistical mechanics arguments to get an order of magnitude estimate of the contributions of Planck scale fluctuations in Cosmology and found it to be of the order $(H \ell_p)^2$. Other works estimate the imprints of Planck scale physics to be of the order $(H \ell_p)^{1/2}$. This issue is still under debate due to the ambiguity in the definition of initial conditions. However the important point for our investigation is that if, besides the quantum gravity scale $\ell_p$, the only other characteristic scale of the cosmological system is the inverse Hubble constant $H^{-1}$ which, due to its time dependence, has a lower bound of order $\ell_p$, then observational signatures of new physics at Planck scale are not far out of reach.

Let us illustrate with a concrete example, by considering the response function of a detector for a mirror's trajectory $z(t)$, whose motion in the intermediate regime resembles the nonlinear short distance physics of a Bianchi type I Cosmology [11] (with the anisotropic parameters $\delta_1 = \beta = 0$). Below we have to take into account that the time dependence in $z(t)$ given by the parameter $\alpha_k$ and Eq. (31), when combined with the Bogolubov coefficients [20a], becomes a non-vanishing complicated function of the initial vacuum $|0(t_s), t\rangle$ that registers parameters of short distance physics contained in $z(t)$ and does not reduce to the thermal distribution of Eq. (4). In this example the generalized frequency term that enters Eq. (12), for modes with wavenumber $k$, is given by
\[
\omega^2 = k^2 \left( 1 - e^{-\alpha_k \eta^2} \right), \tag{31}
\]
with $\eta$ the conformal time. The (diagonal) Bogolubov coefficients are [11]
\[
\beta_{kk} = -i \frac{\pi^{1/2}}{2 \omega \sqrt{\alpha_k}} \exp \left( \frac{2 \omega}{\alpha_k} \right). \tag{32}
\]

The expression for $\beta_{kk}$, which enters in the response function and registers the short distance parameter $\alpha_k$, illustrates that unlike the Black Hole case where degrees of freedom are “projected” into its surface and thus any traces of Planck scale physics are lost due to this confinement to the horizon, in Cosmology no such confinement occurs. The detector registers details of the mirror’s motion ($\alpha_k$) and the response function is generally time dependent. In this case the detector is registering volume degrees of freedom due to non-locality of the time-dependent space-time.

III. THE REAL RACE: BLACK HOLES VERSUS COSMOLOGY

Let us try below to interpret our findings of Section II and search for a better conceptual understanding for the display of different sensitivities of Black Holes and Cosmology to Planck scale physics. In doing so, we rely on the assumption that String Theory [23] is the viable candidate for describing very high energy physics for both cases. We also speculate on the relation of the sensitivities of these backgrounds to a Holographic interpretation [20] where possible.

From a string theoretical point of view, non-locality seems to be required for time-dependent backgrounds (see [12] and References therein), while locality is sufficient for describing stationary solutions. Despite much of the current efforts in literature, the Holographic Principle is not yet well understood or known for most of the realistic cases of time-dependent boundaries. It is possible that the difficulty of a holographic interpretation for a realistic cosmological scenario may be originating from the requirement of non-locality of String Theory actions, arising in time-dependent boundaries. For this reason, the projection of the degrees of freedom on the surface might not be possible due to long-range correlations produced by non-local terms on the world-sheet action. This might be why in Section II the sensitivity of the radiation spectrum to short scale physics, for a stationary versus a dynamic scenario turned out equivalent to having surface projection versus volume degrees of freedom in the latter. Non-locality introduces long-range correlations and therefore it does not allow a projection of the degrees of freedom to the boundary.

Time-dependent string backgrounds solutions, relevant for Cosmology, were studied in [12] for a special class of geometries of particular interest because, although not an exact CFT, they are still solutions to Einstein equations. The results of Ref. [12] seem to indicate that within the framework of perturbative String Theory, a squeezed state on the boundary (equivalent to having particle creation there) is generically derived from nonlocal actions
on the boundary which are obtained from vertex interactions that deform the string world-sheet. However, conformal invariance on the world-sheet is preserved when string perturbation theory is valid but, *Nonlocal String Theory seems to be required to describe Lorentzian vacua* since in dynamic backgrounds the Bogolubov transformation is nontrivial. Here, our understanding of the discrepancy between the Planck physics sensitivity of the Hawking radiation and CMB spectrum due to the volume degrees of freedom for non-stationary backgrounds, is very much in accord with the findings of Ref. [12] for the necessity of non-locality for time-dependent backgrounds.

In Section II we showed through the moving mirror that for a “constant” Black Hole mass we have a projection and confinement of the relevant information from the surface $3^{-}$ to the (smeared) horizon of the Black Hole. Although the results for the moving mirror that reproduces Hawking radiation are well known, our interpretation of the results by comparison to Cosmology is new. In order to inquire about the possible departure of the spectrum from thermality due to the new physics at short distances, we needed to compare quantum fluctuations to thermal fluctuations. Assuming that the uncertainty principle is valid at least up to Planck scale, the Planck scale physics becomes irrelevant while averaging over such a “large region”. Much in the same spirit the Holographic Principle provides an explanation for the Black Hole’s degrees of freedom being projected to its fixed boundary, the horizon [26]. Mathematically this projection of the degrees of freedom to the surface becomes clear from the $\delta$-function that appears in the response function of the detector (see Section [IA]). It therefore seems that the memory loss that the spectrum suffers in terms of the Planck scale physics comes from the reduction of the degrees of freedom to the surface of the Black Hole, which results from stationary backgrounds with a fixed characteristic scale $M \gg \ell_p$. In this case, one can understand the reduction of the Black Hole’s degrees of freedom as arising from two principles: holography and uncertainty, which are both valid for stationary boundaries of size larger than the Planck length.

We would naively expect a similar situation to occur in Cosmology. In Cosmology the number of e-foldings, $N = \ln(A)$, is given by the Hubble constant $H$ (where $H = \dot{A}$ and $A$ is the scale factor). Thus $H$ would be the parameter equivalent to the ADM mass of the Black Hole since they both determine the red-shift (of radiation respectively propagating forward in time or approaching the Black Hole horizon). From Section [IB] we see that the detector will register, besides its non-inertial motion with respect to the comoving frame with velocity $w = w(H)$, also the nonlinear deviations of short scale physics (e.g. Bianchi type I in [11] can be our imperfect fluid) and the nonlinear deviations from the Hubble linear law. As we know, in our universe $H$ has changed tremendously with time, say between $z = 10^{60}$ inflation era, to $z = 10^3$ the red-shift of the last scattering surface. The only instance when we can obtain an analogous situation with the Black Hole case for the detector, is if we have a perfectly linear Hubble law $z = H s$, with $H$ constant, $s$ the proper distance and the temperature is rescaled as $T = AT$ in the comoving frame. Then a similar situation to the Black Hole should arise, provided that our detector is moving non-inertially with some velocity $w = H/\sqrt{H^2 + T}$. By transforming to the boundary comoving frame with constant speed, one recovers the stationary case and a holographic interpretation similar to the Black Hole case may be possible. In this case the detector simply registers its “own motion” $w$ [11].

However, when the Hubble constant is time-dependent or when there is any departure from the linear Hubble law (e.g. Brane-World Cosmologies), the response function of the detector becomes very complicated due to long range correlations arising from non-locality (see Section [IB]). For the same reason, the characteristic fixed scale $M$ of a Black Hole which introduces a lower bound on the range of quantum fluctuations and correlation lengths probed by the detector, is lost in Cosmology. In fact, $H(t)^{-1}$ can be as small as $\ell_p$ at early times. In this case, the response function does not pick a $\delta$-function projection to the surface and it thus contains memory of the parameters of short distance physics due to the nonlocal physics at Planck scale. These arguments lead us to speculate that the detector registers volume degrees of freedom whenever a “nonconstant mass” $H(t)$ is introduced for the background. If $H$ is time dependent from near the Planck scale then the detector will register those features. The motion of a “cosmological mirror” (the time-dependent boundary) can be traced back in time as far as we want, up to the string scale where short distance imprints are found. There is no known physical reason why one should choose to stop or start the motion of the “cosmological mirror” at a given (possibly large) time rather than imposing that condition ad hoc. Therefore the uncertainty principle does not prevent the theory from being sensitive to Planck scale physics and it allows probing of these string scale distances. It is not clear how or if one can apply holography for a time-dependent boundary $H(t)$, since String Theory for time-dependent backgrounds is not yet well understood. The recent progress for the specific dynamic backgrounds considered in [12] indicates that nonlocal interactions in space-time for the boundary action are required for time-dependent backgrounds. Accordingly, due to the correlations for the squeezed state on the boundary, the response function (i.e. the spectrum) will contain information of the parameters of Nonlocal String Theory (in space and time) giving rise to the time-dependent boundary, i.e. details of the boundary’s past trajectory/history. The validity of the uncertainty principle introduces a lower bound on the range of correlations of order $\ell_p$. When the uncertainty principle is applied to volume degrees of freedom,
it results in larger quantum fluctuations as compared to surface fluctuations. Therefore the chance of observing deviations from thermality in the spectrum in Cosmology increases.

The entropy change expected to occur from string interactions on the world-sheet for time-dependent backgrounds, would indicate that there is an energy flow between the world-sheet and the moving boundary. This issue may become more severe in the case of non-perturbative String Theory and would have consequences for the validity of the second law of thermodynamics in the early universe. Such physical arguments support the same conclusion: time-dependent backgrounds generically would contain volume degrees of freedom, thus they register imprints of short scale physics. Perhaps there may be a special class of time-dependent string solutions which may be described by local String Theory, however we are not aware whether they would correspond to realistic cosmologies.

In the absence of an Holographic Principle in Cosmology, perhaps stochastic processes provide a relevant description of physics at Planck scale (as discussed in Refs. [13]). We do not elaborate along this direction here, although it is interesting to explore whether a stochastic approach would change our results. Although Planck scale physics remains beyond our grasp of understanding, the considerations presented here are exciting and a step forward because they indicate that the possibility of finding, in cosmological observables, signatures of Planck scale physics and indirect evidence for String Theory, are not far out of reach.

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[27] We restrict our considerations to 1+1 Minkowski space-time for the sake of simplicity.

[28] With a forethought to modified short distance physics, one might also trade such sharp boundary conditions for fuzzier constraints.

[29] For an attempt to extend the analogy when the backreaction is included see Ref. [14].

[30] Let us note that such a δ-function implies a sum over all normal modes of the detector’s position wave-function which would also be affected by modifications in the trans-planckian regime. We gloss over this fact here precisely for the reason presently explained.

[31] Also notice that due to the spherical symmetry and the stationary Black Hole background this case becomes one dimensional in terms of fluctuations.

[32] However, the validity of these statements and the uncertainty principle near the Planck scale should be treated with caution since we have no knowledge of the physics at these energies. Besides space-time non-locality raises important doubts for the unitarity of the theory at these scales, perhaps stochastic processes might best describe physics in this regime and they may smear out any relevant information of new physics [13].