Measurement of the Berry Phase in a Single Solid-State Spin Qubit

Kai Zhang, Naufar M. Nusran, Bradley R. Slezak, and M. V. Gurudev Dutt*
Department of Physics and Astronomy, 3941 O’Hara Street, Pittsburgh PA 15260
(Dated: October 13, 2014)

We present measurements of the Berry Phase in a single solid-state spin qubit associated with the nitrogen-vacancy center in diamond. Our results demonstrate the remarkable degree of coherent control achievable in the presence of a highly complex solid-state environment. We manipulate the spin qubit geometrically by careful application of microwave radiation that creates an effective rotating magnetic field, and observe the resulting phase via spin-echo interferometry. We find good agreement with Berry’s predictions within experimental errors. We also investigated the role of the environment on the geometric phase, and observed that unlike other solid-state qubit systems, the dephasing was primarily dominated by fast radial fluctuations in the path.

PACS numbers: 76.30.Mi,03.65.Vf, 03.67.Lx

In quantum mechanics, the Hamiltonian for a spin-1/2 qubit interacting with an external magnetic field is given by

$$H = \hbar \gamma_B \mathbf{B} \cdot \mathbf{\sigma}/2,$$

where $\mathbf{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli operators, $\hbar$ is Planck’s constant, $\gamma_B$ is the appropriate gyromagnetic ratio for the particle, and $\mathbf{B}$ is the magnetic field vector. In the Bloch sphere picture (Fig. 1(b)), the qubit state $\mathbf{S} = \hbar \mathbf{\sigma}/2$ continually precesses about the vector $\mathbf{B}$ acquiring dynamical phase $\beta_B(t) = \gamma_B B t$ where $B = |\mathbf{B}|$. When the direction of $\mathbf{B}$ is now changed adiabatically in a time, i.e. at a rate slower than $\gamma_B B$, the qubit additionally acquires Berry phase while remaining in the same superposition with respect to the quantization axis $\mathbf{B}$. When $\mathbf{B}$ completes the closed circular path $C$ shown in Fig. 1(a) the geometric phase acquired by an eigenstate is $\pm \Theta_C/2$ where $\Theta_C$ is the solid angle of the cone subtended by $C$ at the origin. The $\pm$ sign refers to opposite phases acquired by the ground or excited state of the qubit, respectively, giving rise to a relative geometric phase $\beta_B = \Theta_C$. For the circular path shown in

$$\Theta_C = \int_{C} \mathbf{A} \cdot d\mathbf{S} = \int_{C} \mathbf{A} \cdot \mathbf{B} \sin \phi \, ds,$$

where $\mathbf{A}$ is the vector potential of the magnetic field, $\phi$ is the angle between the magnetic field vector and the vector $\mathbf{S}$, and $ds$ is an infinitesimal path length along the closed path $C$. In the case of the circular path $C$ in Fig. 1(a), the geometric phase is given by

$$\Theta_C = \int_{C} \mathbf{A} \cdot d\mathbf{S} = \int_{C} (\mathbf{A} \cdot \mathbf{B}) \sin \phi \, ds.$$
the solid angle is given by \( \Theta_C = 2\pi(1 - \cos \theta) \), depending only on the cone angle \( \theta \).

The NV center (Fig. 2(a)) is a spin-1 system in the ground state, quantized along the C_3v symmetry axis between the nitrogen and vacancy sites, with the \( |m_s = 0 \rangle \) and \( |m_s = \pm 1 \rangle \) levels split by 2.87 GHz at zero magnetic field. The spin state can be initialized by optical pumping with 532 nm laser excitation, and the spin polarization can be detected by measuring the spin-dependent fluorescence signal. We use a single NV center in a type-IIa bulk diamond sample, and apply a static magnetic field \( B_0 \) oriented along the NV centers \( z \)-axis, allowing us to form a pseudo-spin \( \sigma = 1/2 \) qubit system with the \( |m_s = 0 \rangle \leftrightarrow |m_s = -1 \rangle \) spin states. The magnetic field is chosen to bias the system near the excited-state level anti-crossing, resulting in complete polarization of the associated \(^{14}\text{N}\) nuclear spin of the NV center[28] and realizing a nearly ideal two-level system for our experiments. Microwave pulses are applied to the NV center using an impedance-matched microstrip line coupled to a thin copper wire on the diamond surface, allowing us to attain a \( \pi/2 \) rotation in \( \sim 20 \) ns.

In the rotating frame of the microwave drive, and under the rotating wave approximation (RWA), the Hamiltonian for the NV center can be written as,

\[
H = \hbar \Delta \sigma_z + \hbar \Omega (\sigma_x \cos \Phi + \sigma_y \sin \Phi)
\]

Here \( \Delta \) is the detuning between the microwave and the transition frequency, \( \Omega \) is the Rabi frequency of the on-resonance drive field and \( \Phi \) is an adjustable control phase of the microwave. In the rotating frame, we can immediately identify the effective magnetic field as given by

\[
B = (\Omega \cos \Phi , \Omega \sin \Phi , \Delta).
\]

In our experiments, we typically keep \( \Delta \) fixed and trace circular paths with different radii \( \Omega \). The cone angle in our experiment would therefore by given by \( \cos \theta = \Delta/(\Omega^2 + \Delta^2)^{1/2} \).

The corresponding adiabatic circuit for the magnetic field can be achieved by fast amplitude and phase modulation of the magnetic field, as shown in Fig. 2(b). We apply a single microwave to the system as shown in Fig. 2(c): one at a frequency \( f_{\text{det}} \) that carries out the adiabatic circuit for \( B \), and another tuned to the resonance transition frequency \( f_{\text{res}} \) that is used for the spin-echo interference, and subsequent state tomography to extract the spin vector. The spin-echo sequence initializes the qubit into an eigen-state of \( \sigma_x \) with a resonant \( \pi/2 \) pulse. The off-resonant drive is then carefully ramped up, the effective field \( B \) traces the contour \( C \) in Fig. 1(a) in the counter-clockwise direction, and is then ramped down back to zero, generating a relative phase \( \phi_+ = (\beta_g - \beta_d) \) where \( \beta_g \) is the geometric phase and \( \beta_d \) is the dynamic phase. To cancel out dynamic phase and decoherence effects, we apply a \( \pi \) pulse at this point, and reverse the direction of the circuit by setting \( \Phi \rightarrow -\Phi \). During this part of the evolution, the Hamiltonian has effectively been flipped and so the dynamic phase changes sign, but the geometric phase is unaffected, and the phase becomes \( \phi_- = (\beta_g + \beta_d) \).

After the full sequence, the qubit state has acquired a cumulative relative phase \( \phi = \phi_+ + \phi_- = 2\beta_g \) which is purely geometric. By increasing the number of times \( (N) \) we trace the contour, the relative phase becomes \( \phi = (N/2)\phi_+ + (N/2)\phi_- = N\beta_g \). If we do not reverse the direction of the circuit, then the geometric phase should cancel out which we use as a control experiment to verify that there is no quantum phase accumulated during the spin-echo.

At the end of the sequence, we can extract the values of the spin vector through quantum state tomography. The value of \( \langle S_z \rangle \) can be obtained using either Rabi oscillations or rapid adiabatic passage experiment to calibrate our fluorescence levels [27]. The \( \langle S_x \rangle \) and \( \langle S_y \rangle \) values can be obtained by applying a \( \pi/2 \) pulse around the different axes \( x \) or \( y \), serving as a tomography pulse. The experimental phase can then be extracted as \( \phi = \arctan(\langle S_y \rangle/\langle S_x \rangle) \). Even in the presence of decoherence, which is a minor effect in our experiments [27], the coherence in either \( x \) or \( y \) direction would be equally affected and thereby will not alter the geometric phase we
measure from taking the ratio. The total pulse sequence time $T = 4 \mu$s and the shape of the waveforms was chosen to both preserve adiabaticity and to allow for the local spin environment of our qubit to return to its original state [27].

\[ \langle S_x \rangle = A \cos \left( 2\pi N \left( 1 - \frac{\Delta}{(\Omega^2 + \Delta^2)^{1/2}} \right) \right) \]
\[ \langle S_y \rangle = -A \sin \left( 2\pi N \left( 1 - \frac{\Delta}{(\Omega^2 + \Delta^2)^{1/2}} \right) \right) \]  

The theoretical prediction shown in Fig. 3(a) differs from the measured values in a systematic fashion that is at first puzzling. However, we have carried out a number of control experiments, and other checks on our microwave parameters [27], and developed a model that is in good agreement with these experiments. Due to imperfections in our microwave circuitry, such as phase imbalance, amplitude imbalance and channel nonlinearities, distortions of the effective field path from the ideal circuit occur. As the value of the Rabi frequency is increased by increasing the amplitude of the I and Q channels in our modulators, the corresponding shape is modified, and results in deviations from the theoretical geometric phase.

We can model all of these effects in numerical simulation, using independently measured values of the imperfections in the microwave circuitry [27]. As shown in Fig. 3(b), our simulations taking into account all these various errors, as well as any small deviations from adiabaticity, show good agreement with the data. In particular, it is clear that the distortions seem to cause the measured phase to lead the theoretical phase. To compare our data further to the theory, we decided to phenomenologically assume that the effective Rabi frequency in Berry phase experiments is a fit parameter (labeled $\Omega_{eff}$ hereon) rather than being completely determined by the value of the amplitude we sent to the IQ modulators. Our data is well fit to this model as shown by the thick lines Fig. 3(a).

When we correct the solid angle uniformly by this one fit parameter ($\Omega_{eff}$ at a fixed value of $\Delta$), we obtain nearly perfect agreement between theory and our measurements. In Fig. 3(c) we plotted the values of the quantum phase versus the solid angle for different $N$. The data points were uncorrected, and the solid lines show the theory which has now been corrected by replacing $\Omega \rightarrow \Omega_{eff}$ which was extracted from the fit parameters. The data for $N = 0$ is also a confirmation that the spin-echo is highly successful in canceling any quantum phase accumulated in either half of the sequence. We have verified that the adiabaticity parameter $a = \Phi \sin \theta / |B| \ll 0.1$ in the regime of evolution times that were used in our experiments [27]. We should also note that a similar deviation from theory can be seen in another solid-state qubit system (Ref.[12]) although it was not explicitly identified as due to these imperfections in that work.

Our final set of measurements explores the question of how the geometric phase decays in the presence of the complex solid-state environment that interacts with
FIG. 4. (a) (left) Path followed by effective field \( B \) for fractional circuits executed during \( N \) scans. (right) Microwave sequence for the \( N \) scans. (b) Measured values of \( \langle S_x \rangle \) and \( \langle S_y \rangle \) from state tomography as a function of \( N \) for fixed values of the solid angle \( \Theta_C \sim 0.8 \) radians. Inset shows predicted decay from theory taking into account adiabatic dephasing. (b) Measured values of \( \langle S_x \rangle \) and \( \langle S_y \rangle \) from state tomography for the pulse sequence shown in the inset. See main text for explanation and interpretation.

the central NV spin. Decay of the geometric phase is an important parameter to be measured for feasibility of geometric quantum information processing [8, 9] and for NV spin gyroscopes and mechanical rotation sensors [24–26]. As shown in Fig. 3(a), we were unable to detect any geometric dephasing over the limited range of solid angles obtainable by scanning the Rabi frequency \( \Omega \). To increase the amount of geometric phase accumulated, we could keep increasing \( \Omega \), but this may have undesirable effects such as sample heating or greater distortion in the circuital trajectory caused by the phase modulators. Instead, we chose a different route by using the fact that the total quantum phase \( \phi = N \phi_g \). Although \( N \) was previously restricted to be an integer, there is no reason that this should be so, indeed as long as the effective field returns to its original point the above equation holds even for fractional circuits of the parameter space (Fig. 4(a)). Hence, we carried out an experiment with \( N \) continuously varying while keeping the solid angle \( \Theta_C \) fixed, using the pulse sequence in Fig. 4(a).

Our experimental data in Fig. 4(b) clearly showed decay as the amount of geometric phase is increased, which could be potentially explained as due to slow (adiabatic) fluctuations in either \( \Delta \) or \( \Omega \). Indeed this was the model used in Refs. [12, 19, 20] to study geometric dephasing. We can estimate the strength of these fluctuations for \( \Delta \) and \( \Omega \) independently for our NV center from Ramsey and Rabi measurements [27]. Assuming a gaussian model for the noise spectrum in \( \Delta \), we obtain the following expression for the decay in the Berry phase as a function of the circuital number \( N \):

\[
\langle S_x \rangle = A \cos(N \Theta_C) \exp(-\frac{(N/N^*)^2}{2})
\]  

where \( N^* = \frac{\sqrt{2}}{\sigma_\Delta} = \frac{2\pi T_2^*}{\sigma} \), \( \sigma_\Delta \) is the standard deviation of the noise, \( \sigma = |d\Theta_C/d\Delta| = \frac{\Omega^2}{(1+\beta \Delta)^{3/2}} \). Given the numerical values in our experiments, we obtained \( N^* \sim 400 \), and generated the anticipated prediction (see inset to Fig. 4(b)). Clearly, the noise in \( \Delta \) from this environment is insufficient to explain the decay in our data. Similarly, our long-time Rabi oscillation data shows that the noise \( \sigma_\Omega \) is even smaller and cannot explain this decay [27].

To understand the decay, we carried out an experiment where no geometric phase is present but only dynamic phase. To do this, we dragged the magnetic field out to the value used in our experiment, but did not carry out the adiabatic closed circuit. We merely allowed the spin vector to precess dynamically around this value of the magnetic field, as shown in Fig. 4(c). Our expectation was that because the value of the field remains the same in both halves of the spin echo sequence, we should obtain constant projection of the spin vector along \( x \) and \( y \) axis. As we scan the time of the sequence, and thereby the number of cycles of geometric phase that we could have accumulated during this time, we instead see a clear decay of the probability. This measurement indicates that our Rabi frequency \( \Omega \) is not the same from one half of the sequence to the other, but instead has fast noise fluctuations. Our numerical simulations using the density matrix show that such fluctuations would indeed result in the observed decay of the spin coherence [27]. We thereby attribute both the decay in the \( N \) scan and in this dynamic phase experiment to the fast radial fluctuations in the geometric path caused by the technical limitations of our AWG and microwave circuitry.

Our work reports on detailed measurements of the Berry phase in a single solid-state spin qubit. Although the NV spin can be treated as an effective spin-1/2 sys-
NV centers in our type-IIa single crystal diamond sample (sumitomo with [1 1 1] orientation) are located with our home-built confocal microscope. A high NA dry microscope objective (Olympus 0.95 NA) is used in this confocal setup for NV excitation and collection of fluorescence emission. Phonon-mediated fluorescent emission (650-750 nm) for the single NV center is detected under coherent optical excitation (LASERGLOW IIIB 532nm laser) using a single photon counting module (PerkinElmer SPCM-AQR-14-FC). Green excitation of the NV center polarizes the electron spin into $|m_S = 0\rangle$ sublevel of the $^3A_2$ ground state due to optical pumping. The rate of fluorescence signal counts varies for the $|m_S = 0\rangle$ and $|m_S = \pm 1\rangle$ states, which enables the optical detection of the electron spin. The photon counting takes place at the both ends of the optical excitation pulse (Fig. ??(a)). The counts at the beginning known as "signal" ($S_i$) is highly dependant on the NV state while the counts at the end of optical excitation known as "reference" ($R_i$) is not. By taking the ratio of $S_i$ and $R_i$ as our fluorescence level (a.u.), we minimize the effect of laser fluctuations on our experiments. A dc bias magnetic field $B_0 \approx 450$ Gauss is applied along the NV axis by a permanent magnet. The transition frequency between $|m_S = 0\rangle$ and $|m_S = -1\rangle$ states is measured by Optically Detected Magnetic Resonance (ODMR) experiment and Pulsed ESR frequency scan experiment (Fig). Detuned microwave for Berry phase experiment is generated by Rohde&Schwarz (SMIQ03B) signal generator with built-in IQ modulator. On resonance microwave is generated by PTS3200 synthesizer. Both microwaves are delivered via a 20-micron-diameter copper wire placed on the diamond sample. The input waveforms for IQ channels are generated by Tektronics AWG520 Arbitrary waveform generator (1G sample/s). The value of $\Delta$ is extremely well controlled in our experiments as we frequently track the resonance frequency of the NV center, and both synthesizers in our experiment are synchronized by an atomic clock and measured to drift less than $\sim 1 \times 10^{-3}$ Hz.

SPIN ECHO SEQUENCE: LARMOR REVIVALS

The $^{13}$C nuclear spin bath that has a natural abundance of $\approx 1.1\%$ effectively produces a random field with frequency set by the nuclear gyromagnetic ratio $\gamma_n = 2\pi(10.75) \text{ MHz/T}$ and the dc bias field $B_0$. This random field causes collapses and revivals in the CP signals. For Berry phase experiment, it is required to operate on a revival point of spin echo sequence. In our Berry phase amplitude scan, the spin echo sequence time is set to the first revival. In N scan, the sequence time is set to the fifth revival.

SUPPLEMENTARY MATERIAL

**NV CENTER ESR AND EXPERIMENTAL SETUP**

This work was supported by the DOE Office of Basic Energy Sciences (DE-SC 0006638), NSF CAREER (DMR-0847195), NSF PHY-1005341, and the Alfred P. Sloan Research Fellowship.
CALIBRATION TEST EXPERIMENTS

Fluorescence Level of $|0\rangle$ and $|1\rangle$ State

In our experiments, the direct signal we were measuring is fluorescence counts. The fluorescence counts signal was mapped to probability in $|0\rangle$ or $|1\rangle$ state by calibration of fluorescence levels at $|0\rangle$ and $|1\rangle$ states with Rabi oscillation. However, imperfections in $\pi$ pulses might result in imperfect rotations and thereby cause errors in the fluorescence calibration.

Adiabatic passage is another way of calibrating fluorescence levels, and is used to check our Rabi oscillation calibration. By modulating both the amplitude and frequency of the driving microwave, an effective magnetic field adiabatically varying from $+z$ direction to $-z$ direction was created, as shown in Fig. S3(a). The detuned microwave starts ramping up at $t = t_0$ while modulating its frequency, and then ramps back down to 0 with an opposite detuning at $t = t_f$. The magnitude of the effective $B$ field is 12.5 MHz. Fluorescence level at different time $t$ was measured and shown in Fig(b).

The data points before $t_0$ and after $t_f$ is fitted to fluorescence levels in $|0\rangle$ and $|1\rangle$ state respectively. The fitted fluorescence levels fall in the confidence interval from our Rabi calibration, meaning our fluorescence level calibration is accurate.

Microwave Power at Different Frequencies

To test the frequency response of our microwave circuit, Rabi oscillation were measured under detuned microwave driving field with different detuning frequencies. Oscillations at 0MHz, 10MHz and 20MHz detuning are shown as examples in Fig (a),(b) and (c). The fitted parameters, oscillation frequencies $\Omega(\Delta)$, amplitudes $amp(\Delta)$ and equilibrium position $bkg(\Delta)$ are plotted as functions of detuning frequency in Fig (d),(e) and (f).

The theoretical expectations are as following:

$$\Omega(\Delta) = \sqrt{\Omega(0)^2 + \Delta^2}$$  \hspace{1cm} (4)

$$amp(\Delta) = amp(0) \frac{\Omega(0)^2}{\Omega(0)^2 + \Delta^2}$$  \hspace{1cm} (5)

$$bkg(\Delta) = bkg(0) + amp(0) \frac{\Delta^2}{\Omega(0)^2 + \Delta^2}$$  \hspace{1cm} (6)

![Figure S4](image_url)

FIG. S4. a), b) and c) Rabi oscillation at 0 MHz, 10 MHz, and 20 MHz detuning. d), e) and f) Fitted frequencies, amplitudes and equilibrium positions at different detuning. Error bars come from standard error of fitted parameters. Red curve is a fit to the theory and green curve is the theoretical prediction based on our on-resonance Rabi data.

The data demonstrates that our knowledge of MW parameters are accurate as a function of the microwave frequency.

Single Channel Nonlinearity of IQ Modulator

One usual systematic error in our system is single channel nonlinearity of the IQ modulator. It means the microwave output amplitude is not always proportional to voltage input in I/Q channel. A Rabi experiment with
ADIABATICITY

FIG. S7. a) Schematic Microwave Sequence. b) Probability in |0⟩ state vs period of cyclic motion T and ramping time of microwave power Ta. Red curves are calculated adiabaticity parameter.

The Berry phase theory is based on adiabatic approximation. The Berry phase is not robust to transitions between |0⟩ and |1⟩ state, and there could be a discrepancy from the theory if the transition probability is not negligible.

In order to check our adiabaticity, almost the same waveform as Berry phase experiment (N=2) was used. The only difference is that the first π/2 pulse and the last π/2 pulse were removed Fig. S7. If the adiabatic condition is good, the spin should stay in |0⟩ state before the π pulse and stay in |1⟩ state after the π pulse. The probability in state |0⟩ at the end of the sequence should be close to 0. This probability in state |0⟩ for both scanning ramping up/down time Ta and the scanning cyclic period T is shown in Fig. S7.

The data is taken at 5MHz detuning, 12.5MHz Rabi frequency. Adiabaticity parameter calculated according to T(Ta) are shown as red curves. In our Berry phase experiments (10MHz detuning, 12.5MHz Rabi frequency for amplitude scan), we chose T=800ns, Ta=350ns. We believe we were well within the adiabatic regime.

From the fit parameter we get the time scale of decay $T_2^\ast = 2.2\mu s$, larger than the $T_2$ (about 0.8μs for this NV). This means that we do have slow fluctuation in microwave amplitude, but the overwhelming fluctuation is still the fluctuation in resonance frequency.

The calibrated actual value at different input value $P_v$ is $0.2, 0.4, 0.6, 0.8, 1.0$. The only difference is that the first $T_a=350\mu s$. We chose $T=800\mu s$, $T_a=350\mu s$. The data is taken at the same microwave power as the Berry phase experiment (12.5MHz), and the total experiment time is comparable to Berry phase experiment. From the fit parameter we get the time scale of decay $T_2^\ast = 2.2\mu s$, larger than the $T_2$ (about 0.8μs for this NV). This means that we do have slow fluctuation in microwave amplitude, but the overwhelming fluctuation is still the fluctuation in resonance frequency.

A simple long time Rabi experiment was done to test the microwave power stability. If the microwave amplitude has a probability distribution around the expected value, the Rabi signal will decay as pulse length increases, which is shown in Fig. S6.

In the experiment, the microwave frequency is on resonance. The microwave pulse length is fixed to be 100 ns and different DC voltages are sent to I channel of the modulator. If there is no single channel nonlinearity, the microwave output amplitude should be proportional to the DC voltage on I channel, and so also the Rabi frequency. A sinusoidal signal in fluorescence level is expected.

As shown in Fig. S5, the green curve is the perfect sinusoidal function we expect, but the data points are clearly off by a certain amount. So the data is fitted to nonlinear function $ax + bi$ instead. The fit parameters are $a = 1.17, b = -0.17$.

FIG. S5. a) Rabi experiment with fixed microwave pulse length of 100ns and scanning DC voltage input to I channel. Green curve is expectation values without single channel nonlinearity, and red curve is the fit using 1st order nonlinearity. b) The calibrated actual value at different input value according to fitted parameters $a = 1.17, b = -0.17$. Clearly, The actual value is larger than expectation, resulting in larger solid angle.

FIG. S6. Rabi Oscillation up to long times. The fitted time scale of decay is $T_2^\ast = 2.2\mu s$. fixed pulse length and scanning microwave amplitude is used to calibrate this single channel nonlinearity.

Microwave Power Stability
CANCELLATION OF BERRY PHASE

By flipping the direction of phase modulation at the 2nd half of the Berry phase sequence, the accumulated Berry phase survives while the dynamic phase gets cancelled by the spin echo. This implies that the Berry phase will be cancelled if the direction of phase modulation is not flipped. The cancellation of Berry phase is proved experimentally by applying the same sequence as amplitude scan (N=2), but without flipping the direction of phase modulation in the 2nd half.

The data shows no oscillation at all, meaning a perfect cancellation of Berry phase by spin echo. And this is the raw data from which the $N = 0$ phase data in the main paper is extracted.

SIMULATION OF DYNAMIC DEPHASING

* Correspondence to: gdutt@pitt.edu

[1] M. V. Berry, Proc. R. Soc. Lond. A 392, 45 (1984).
[2] Y. Aharonov and D. Bohm, Phys. Rev. 115, 485 (1959).
[3] R. Y. Chiao and Y.-S. Wu, Phys. Rev. Lett. 57, 933 (1986).
[4] R. Y. Chiao, A. Antaramian, K. M. Ganga, H. Jiao, S. R. Wilkinson, and H. Nathel, Phys. Rev. Lett. 82, 1959 (2010).
[5] F. Wilczek and A. Zee, Phys. Rev. Lett. 52, 2111 (1984).
[6] Y. Zhang, Y.-W. Tan, H. L. Stormer, and P. Kim, Nature 438, 201 (2005).
[7] D. Xiao, M.-C. Chang, and Q. Niu, Rev. Mod. Phys. 82, 1959 (2010).
[8] P. Zanardi and M. Rosetti, Phys. Lett. A 264, 94 (1999).
[9] J. Pachos, P. Zanardi, and M. Rosetti, Phys. Rev. A 61, 010305(R) (2000).
[10] E. Sjoqvist, Physics 1, 35 (2008).
[11] J. A. Jones, V. Vedral, A. Ekert, and G. Castagnoli, Nature 403, 869 (2000).
[12] P. J. Leek, J. M. Fink, A. Blais, R. Bianchetti, M. Goppl, J. M. Gambetta, D. I. Schuster, L. Frunzio, R. J. Schoelkopf, and A. Wallraff, Science 318, 1889 (2007).
[13] D. Leibfried, B. DeMarco, V. Meyer, D. Lucas, M. Barrett, J. Britton, W. M. Itano, B. Jelenkovic, C. Langer, T. Rosenband, and D. J. Wineland, Nature 422, 412 (2003).
[14] B. Kane, Nature 393, 133 (1998).
[15] D. Loss and D. DiVincenzo, Phys. Rev. A 57, 120 (1998).
[16] M. W. Doherty, N. B. Manson, P. Delaney, F. Jelezko, J. Wrachtrup, and L. C. Hollenberg, Physics Reports 528, 1 (2013), the nitrogen-vacancy colour centre in diamond.
[17] M. V. G. Dutt, L. Childress, L. Jiang, E. Togan, J. Maze, F. Jelezko, A. S. Zibrov, P. R. Hemmer, and M. D. Lukin, Science 316, 1312 (2007).
[18] T. van der Sar, Z. H. Wang, M. S. Blok, H. Bernien, T. H. Taminiau, D. M. Toyli, D. A. Lidar, D. D. Awschalom, R. Hanson, and V. V. Dobrovitski, Nature 484, 82 (2012).
[19] G. De Chiara and G. M. Palma, Phys. Rev. Lett. 91, 090404 (2003).
[20] A. Carollo, I. Fuentes-Guridi, M. F. Santos, and V. Vedral, Phys. Rev. Lett. 90, 160402 (2003).
[21] S.-L. Zhu and Z. D. Wang, Phys. Rev. Lett. 89, 097902 (2002).
[22] P. San-Jose, G. Zarand, A. Shnirman, and G. Schön, Phys. Rev. Lett. 97, 076803 (2006).
[23] R. S. Whitney, Y. Makhlin, A. Shnirman, and Y. Gefen, Phys. Rev. Lett. 94, 070407 (2005).
[24] D. Maclaurin, M. W. Doherty, L. C. L. Hollenberg, and A. M. Martin, Phys. Rev. Lett. 108, 240403 (2012).
[25] A. Ajoy and P. Cappellaro, Phys. Rev. A 86, 062104 (2012).
[26] M. P. Ledbetter, K. Jensen, R. Fischer, A. Jarmola, and...
D. Budker, Phys. Rev. A 86, 052116 (2012).

[27] “See supplementary material at [url to be inserted by publisher] for more information on materials and methods.”.

[28] V. Jacques, P. Neumann, J. Beck, M. Markham, D. Twitchen, J. Meijer, F. Kaiser, G. Balasubramanian, F. Jelezko, and J. Wrachtrup, Phys. Rev. Lett. 102, 057403 (2009).

[29] E. Sjoqvist, D. M. Tong, L. M. Andersson, B. Hessmo, M. Johansson, and K. Singh, New J. Phys. 14, 103035 (2012).

[30] C. Zu, W. B. Wang, L. He, W. G. Zhang, C. Y. Dai, F. Wang, and L. M. Duan, Nature 514, 72 (2014).