Floyd-Warshall Reinforcement Learning: Learning from Past Experiences to Reach New Goals

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Abstract

Consider multi-goal tasks that involve static environments and dynamic goals. Examples of such tasks, such as goal-directed navigation and pick-and-place in robotics, abound. Two types of Reinforcement Learning (RL) algorithms are used for such tasks: model-free or model-based. Each of these approaches has limitations. Model-free RL struggles to transfer learned information when the goal location changes, but achieves high asymptotic accuracy in single goal tasks. Model-based RL can transfer learned information to new goal locations by retaining the explicitly learned state-dynamics, but is limited by the fact that small errors in modelling these dynamics accumulate over long-term planning. In this work, we improve upon the limitations of model-free RL in multi-goal domains. We do this by adapting the Floyd-Warshall algorithm for RL and call the adaptation Floyd-Warshall RL (FWRL). The proposed algorithm learns a goal-conditioned action-value function by constraining the value of the optimal path between any two states to be greater than or equal to the value of paths via intermediary states. Experimentally, we show that FWRL is more sample-efficient and learns higher reward strategies in multi-goal tasks as compared to Q-learning, model-based RL and other relevant baselines in a tabular domain.

1 Introduction

Reinforcement learning (RL) allows for agents to learn complex behaviors in a multitude of environments while requiring supervision only in the form of reward signals. RL has had success in various domains ranging from playing Atari games (Mnih et al., 2015) from purely visual input and defeating world GO champions (Gibney, 2016), to applications in robotic navigation (Mirowski et al., 2018) and manipulation (Levine et al., 2018). In the realm of multi-goal tasks, this work introduces Floyd-Warshall Reinforcement Learning (FWRL), a new algorithm that facilitates the transfer of learned behaviours in environments with dynamic goal locations.

There are two types of RL algorithms, model-based and model-free, which differ in whether the state-transition function is learned explicitly or implicitly (Sutton and Barto, 1998). In model-based RL, the dynamics that govern an environment’s transitions is explicitly modelled. At any point in an episode, agents use this model to predict future states and utilize this information to maximize possible reward. This formulation is known to be sample-efficient while normally not achieving high asymptotic performance (Pong et al., 2018). In contrast, in model-free RL, algorithms such as policy gradients, actor-critic and Q-learning directly learn the expected “value” for each state without explicitly learning the environment-dynamics. This paradigm has been behind most of the successes in such diverse applications like Atari games, Go championships etc.

In multi-goal tasks that involve static environments and dynamic goals, such as goal-directed navigation and pick-and-place in robotics, model-free RL struggles to transfer learned behavior from one goal location to another within the same environment (Dhiman et al., 2018; Quillen et al., 2018). This occurs because model-free RL represents the

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environment as value functions, which confute the state-dynamics and reward distributions into a single representation. On the other hand, model-based RL allows for the separation of environment dynamics and reward, but can have lower asymptotic performance due to the accumulation of small errors in the modelling function.

In this work we introduce Floyd-Warshall Reinforcement Learning, an adaptation of the Floyd-Warshall shortest path algorithm ([Floyd, 1962]), for multi-goal tasks. The Floyd-Warshall shortest path algorithm is itself a generalization of Dijkstra’s algorithm for multi-goal domains on graphs. The algorithm works by learning a goal conditioned value function ([Schaul et al., 2015]), called the Floyd-Warshall (FW) function, that is defined to be the expected reward in going from a start state-action pair \((s, a)\) to a given goal state \(s’\):

\[
F_\pi (s’|s, a) = \mathbb{E}_\pi \left[ \sum_{i=0}^{t=k} r_i \mid s_0 = s, a_0 = a, s_k = s’ \right].
\]  

(1)

In order to learn the FW-function, we employ the following triangular-inequality constraint for shortest paths, which is our main contribution,

\[
F_{\pi, \gamma}^* (s_j \mid s_i, a_i) \geq F_{\pi, \gamma}^* (s_k \mid s_i, a_i) + \max_{a_k} F_{\pi, \gamma}^* (s_j \mid s_k, a_k) \quad \forall s_k \in S. \tag{2}
\]

Section 5 describes the terminology, equations and the method in more detail.

This constraint allows FWRL to remember paths even if they do not lead to the goal location during particular episodes. The motivation is similar to the Hindsight Experience Replay (HER) ([Anderson et al., 2017]), where agents learn by re-imagining the final states of past failed experiences as successful. Our method allows us to utilize hindsight experience in more a fine-grained manner because we consider all states as potential goals. Other works using a goal-conditioned value function for multi-goal tasks include Universal Value Function Approximators (UVFA) ([Schaul et al., 2015]) and Temporal Difference Models (TDM) ([Pong et al., 2018]). UVFA introduces the use of goal-conditioned value functions and a factorization approach for faster learning of neuronal networks that depend upon goals from the state space. TDM combines model-based and model-free algorithms by modeling the learning as a constrained objective function using a horizon dependent goal-conditioned value function. In contrast to these works, we present an alternative mechanism for learning these functions that is horizon independent.

Experimentally, FWRL is shown to be more sample-efficient and achieve higher reward standard Q-Learning and model-based methods in a tabular setting. FWRL is found to achieve two times median rewards than the next best baseline.

2 Related work

Goal-conditioned value functions Multi-goal reinforcement learning has gained attention lately with important work like UVFAs ([Schaul et al., 2015], HER ([Andrychowicz et al., 2016]) and TDM ([Pong et al., 2018]) making progress towards learning goal-conditioned value functions for multi-goal tasks. Introduced by ([Schaul et al., 2015]), universal goal-conditioned value functions (UVFs), \(V(s, g)\), measures the utility function of achieving any goal from any state in an environment. In this work, UVFs are learned using traditional Q-learning based approaches coupled with matrix factorization based methods for faster learning. In Hindsight Experience Replay (HER), ([Andrychowicz et al., 2016]) learn UVFs from previous episodes accounting for when the goal location has not been achieved. Their method works by utilizing past failed experiences and re-imagining the last states of these episodes as goal states. ([Pong et al., 2018]) propose Temporal Difference Models (TDM) that estimate goal directed horizon dependent value functions, \(Q(s, a, g, \tau)\). There work is limited because they assume that rewards are available densely in the form of some distance like measure from the goal. In contrast to all the above works our contribution is a novel algorithm for learning UVFs via leveraging a triangular-inequality like constraint within the space of these functions.

Goal directed visual navigation There has been considerable interest in using Deep Reinforcement Learning (DRL) algorithms for the goal-driven visual navigation of robots ([Mirowski et al., 2016, 2017], [Dhiman et al., 2018], [Gupta et al., 2017], [Savinov, Dosovitskiy, and Koltun, 2018]). Mirowski et al. (2016) demonstrate that a DRL algorithm called Asynchronous Advantage Actor Critic (A3C) can learn to find a goal in 3D navigation simulators, using only a front facing first person view as input, while Mirowski et al. (2017) demonstrate goal directed navigation in Google’s street view graph. Gupta et al. (2017) addresses goal directed mapping but their method depends upon navigation-specific occupancy grids which limit the method’s generalizability to mutli-goal tasks. Moving the successes of these works from simulations to the real world is an active area of research because of the high sample complexity of model-free RL algorithms ([Zhu et al., 2017], [Anderson et al., 2018], [Dhiman et al., 2018]) empirically evaluate Mirowski et al. (2016)’s approach and show that when goal locations are dynamic, the path chosen to reach the goal are often far from optimal. In contrast to our method, these works focus on the navigation domain and employ domain specific auxiliary rewards and data-structures making them less generalizable to other multi-goal tasks.

Binary goal tasks In Oh et al. (2016), agent’s DRL-driven navigation is complimented with external memory modules indexed in time. In a similar vein, Parisotto and Salakhutdinov (2017) experiment with indexing memory modules with the agent’s spatial coordinates. Both works consider goal specification as a binary signal hidden somewhere in the observation space. In such problems the challenge is to learn the goal specification protocol while avoiding the wrong goal and exploring for the right one. These problems domains do not evaluate the ability of agents to find the shortest path to the goal.
3 Background

We present a short review of the background material that our work depends upon.

3.1 Dijkstra

Dijkstra (Dijkstra, 1959) is a shortest path finding algorithm from a given vertex in the graph. Consider a weighted graph \( G = (S, E) \), with \( S \) as the vertices and \( E \) as the edges. Dijkstra algorithms works by maintaining a data-structure \( D : S \to \mathbb{R} \), that represents the shortest path length from the source. The data structure \( D \) is initialized with zero at start location \( D[s_0] \leftarrow 0 \) and a high value everywhere else \( D[i] \leftarrow \infty \forall i \in S \). The algorithm then sequentially updates \( D \) by

\[
D[j] \leftarrow \min\{D[j], r_{i,j} + D[i]\} \forall (i,j) \in E, \tag{3}
\]

where \( r_{i,j} \) is the edge-weight for directed edge \((i,j)\in E\). The shortest path \( \langle s_0, s_1, \ldots \rangle \) starting from vertex \( s_0 \) can be read from \( D \) via \( s_{t+1} = \arg\min_{i \in Nbr(s_t)} D[i] \) where \( Nbr(s_t) = \{i | (i, s_t) \in E\} \) denotes the neighborhood of \( s_t \).

With a carefully chosen data-structure and traversal order, the Dijkstra Algorithm can be made to run in \( O(|S| \log |S|) \).

3.2 Q-Learning

Q-learning (Watkins and Dayan, 1992) is a reinforcement learning (RL) algorithm that allows agent to explore enviornment and simultaneously compute maximum reward paths.

An RL problem is formalized as an Markov Decision Process (MDP). A MDP is defined by a four tuple \( \langle S, A, T, R \rangle \), where \( S \) is the state space, \( A \) is the action space, \( T : S \times A \to S \) is the system dynamics and \( R : S \to \mathbb{R} \) is the reward yielded on a execution of an action. The objective of a typical RL problem is to maximize the expected cumulative reward over time, called the returns \( R_t = \sum_{t'=t}^{T} r_{t'} \).

Q-learning works by maintaining an action-value function \( Q : S \times A \to \mathbb{R} \) which is defined as the expected return

\[
Q_t(s_t, a_t) = \mathbb{E}_\pi[R_t] \text{ from a given state-action pair. The Q-learning algorithm works by updating the Q-function using the Bellman equation for every transition from } s \text{ to } s' \text{ on action } a \text{ yielding reward } r, \\
Q_t(s, a) = \mathbb{E}_\pi \left[ r + \max_{a'} Q_t'(s', a') | s, a \right]. \tag{4}
\]

3.3 Floyd-Warshall

The Floyd-Warshall algorithm (Floyd, 1962) is a shortest path finding algorithm from any vertex to any other vertex in a graph. Similar to Dijkstra’s algorithm, the Floyd-Warshall algorithm finds the shortest path by keeping maintaining a shortest distance data-structure \( D : S \times S \to \mathbb{R} \) between any two pair of vertices \( i, j \in S \). The data-structure \( D \) is initialized with edges \( D[i, j] \leftarrow r_{i,j} \forall (i,j) \in E \) and the uninitialized edges are assigned a high value, \( D[i, j] \leftarrow \infty \forall i, j \in S \). The algorithm works by sequentially observing all the nodes in the graph and updating \( D \) with the shortest explored path known so far:

\[
D[i, j] \leftarrow \min\{D[i, j], D[i, k] + D[k, j]\} \forall i, j, k \in S. \tag{5}
\]

The update equation in the algorithm depends upon the triangular inequality for shortest paths distances \( D[i, j] \leq D[i, k] + D[k, j] \) and hence works only in the absence of negative cycles in the graph. Fortunately, many practical problems can be formulated such that negative cycles do not occur. The Floyd-Warshall algorithm runs in \( O(|S|^3) \) and is suitable for dense graphs. There also exists extensions of the algorithm like Johnson’s algorithm (Johnson, 1977) that run in \( O(|S|^2 \log |S| + |S||E|) \) while working on the same principle.

From the parallels between Eq. (3) and Eq. (4), Q-learning can be seen as a generalization Dijkstra’s algorithm. Both the algorithms work by taking one step minimum (or maximum) over the neighboring state. Unlike Dijkstra, in Q-learning one has to compute an additional maximum over actions. This is because in an MDP, the agent cannot directly choose the next state to be in. Instead, it chooses an action that leads it to the next state based on transition probabilities. Moreover, Q-learning has to explore the state space before it can exploit the learned information to find most-rewarding path. With these parallels in mind, we generalize the Floyd-Warshall algorithm to work on an MDP and call it Floyd-Warshall Reinforcement Learning.

4 Problem definition

Consider an agent interacting with an environment, \( \varepsilon \). At every time step, \( t \), the agent takes an action \( a_t \in A \), observes a state, \( s_t \in S \) and a reward \( r_t \in [-R_{\text{goal}}, R_{\text{goal}}] \). A goal state, \( g \in S \), is provided to the agent and it receives \( R_{\text{goal}} \) the highest reward in the environment—on reaching it with respect to some threshold \( ||s - g|| < \delta_{\text{goal-thresh}} \). An episode is defined as of a fixed number of time steps, \( T \). For every episode, a new goal state is provided to the agent as input. If the agent reaches the goal state before the episode ends, the agent is re-spawned at a new random location within the environment while the goal state remains unchanged for the
episode. The agent’s objective is to find the sequence of actions to take that maximizes the total reward per episode.

Model-free Reinforcement learning methods assume that the rewards are being sampled from the a static reward function. In a problem where the goal location changes it becomes hard to transfer the learned value-function or action-value function to the changed location. One alternative is to concatenate the goal location to the state, making the new state space \( [s_t, g]^T \in \mathcal{S}^2 \) larger. This method is wasteful in computation and more importantly in sample complexity [Schaul et al. (2015)].

5 Method

We present a model-free reinforcement learning method that transfers learned behavior when goal locations are dynamic. We call this algorithm Floyd-Warshall Reinforcement Learning, because of its similarity to Floyd-Warshall algorithm [Floyd (1962)] a shortest-path planning algorithm on graphs. Similar to universal value function [Schaul et al. (2015)], we define the Floyd-Warshall (FW) function as the expected cumulative reward on going from a start state to an end state within an episode:

\[
F_\pi(s'|s, a) = \mathbb{E}_\pi \left[ \sum_{i=0}^{t=k} r_t \mid s_0 = s, a_0 = a, s_k = s' \right], \quad (6)
\]

where \( \pi \) is the stochastic policy being followed. Note that we do not use discounted rewards, instead assuming that episodes are of finite time length. In keeping with the Floyd-Warshall shortest path algorithm, we assume that there are no positive reward cycles in the environment.

Note that the FW-function is closely related to the Q-function,

\[
Q_\pi(s, a) = \sum_{s'} P_\pi(s'|s, a) F_\pi(s'|s, a), \quad (7)
\]

where \( P_\pi(s'|s, a) \) is the probability of the agent arriving at \( s' \) within the episode. We define the optimal FW-function as the maximum expected value that a path between a start state and a goal state can yield,

\[
F^*_\pi(s'|s, a) = \max_{\pi_{\pi'}} F_{\pi'}(s'|s, a) \quad (8)
\]

where \( \pi_{\pi'} \) is the optimal policy towards the goal state \( s' \). Once the FW-function approximation is learned, the optimal policy can be computed from FW-function similar to the Q-learning algorithm, \( \pi^*_\pi(s) = \arg \max_a F^*_\pi(s'|s, a) \)

When the policy is optimal, the Floyd-Warshall function must satisfy the constraint

\[
F^*_\pi(s_j|s_i, a_i) \geq F^*_\pi(s_k|s_i, a_i) + \max_{a_k} F^*_\pi(s_j|s_k, a_k) \quad \forall s_k \in \mathcal{S}. \quad (9)
\]

In other words, the value for the optimal path from given start state to a given goal state should be greater than or equal to value of path via any intermediate state. This triangular-inequality like constraint is the main contribution of our work. To the best of our knowledge it has not been employed in any previous works utilizing goal-conditioned value functions.

Aside from the dynamic goal locations, we assume environments underlying reward distributions to be static. We also assume that the goal reward is the highest reward in the reward space. The pseudo-code for the algorithm is shown in Algorithm 1.

**Algorithm 1:** Floyd-Warshall Reinforcement Learning (Tabular setting)

```
/* By default all states are unreachable */
Initialize \( F(s_j|s_i, a; \theta_F) \leftarrow -\infty \);
Define \( \pi^*(s_t, s_g, F) = \arg \max_a F(s_g|s_t, a; \theta_F) \);
Input \( s_g \);
Set \( t \leftarrow 0 \);
Observe \( s_t \);
for \( t \leftarrow 1 \) to \( T \) do
    Take action \( a_t \sim \epsilon\text{-greedy}(\pi^*(s_t, s_g, F)) \);
    Observe \( s_{t+1}, r_t \);
    if \( r_t \geq R_{goal} \) then
        /* Do not update the value function with goal reward */
        continue;
    \( F(s_{t+1}|s_t, a_t) \leftarrow r_t \);
    for \( s_k \in \mathcal{S}, a_k \in \mathcal{A}, s_t \in \mathcal{S} \) do
        \( F(s_t|s_k, a_k) \leftarrow \max \{ F(s_l|s_k, a_k), F(s_t|s_k, a_k) + \max_{a_p \in \mathcal{A}} F(s_l|s_t, a_p) \} \); Result: \( \pi^*(s_k, s_g, F) \)
```

6 Experiments

Experiments are conducted in a grid-world like setup as displayed in figure [2]. The agent can occupy any one of the white blank squares. The agent’s observations is the numbered location of each square i.e. each square \( x,y \) coordinate with the origin being the top left corner of the environment, \( s_t = (x, y) \). Agents can act by moving in four directions, \{up, down, left, right\}. Experiments are conducted in a tabular domain showcasing results that are intuitive to understand while still displaying large performance gaps between FWRL and standard baselines methods.

7 Environment

We use two types of grid world environments in the experiments, a four room grid world and a windy version of the four room grid world.

**Four room grid world** Four room grid world is a grid world with four rooms connected to each other as shown in Figure [2]. This example is chosen due to it’s intentional difficulty for random exploration based agents. Since the exit points, are narrow, random agents tend to get stuck in individual rooms.
Four room windy world  In four room windy world, the previous setup is augmented in some cells with wind. This wind, indicated by arrows, increases the probability of the agent randomly going in the direction of the arrow by 0.25. Conceived in [Sutton and Barto (1998)], this setup increases the dependence of the dynamics model upon environment conditions.

7.1 Metrics
The metrics used to quantify and compare agent performance across FWRL to baseline methods are described here.

Reward  As in typical in reinforcement learning, the reward earned per episode by the agent is treated as a metric of success.

Distance-Inefficiency  The distance-inefficiency (Dhiman et al., 2018) is the ratio of the distances travelled by the agent during an episode to the sum of the shortest path to the goal at every point of initialization. Mathematically it is defined as:

\[
\text{Dist-ineff.} = \frac{\sum_{i=1}^{N-1} \sum_{k=\tau_{i}+1}^{\tau_{i}+1} \|x_{k+1} - x_{j}\|}{\sum_{i=1}^{N-1} \delta(x_{\tau_{i}+1}, x_{g})},
\]

where \(\delta(x_{\tau_{i}+1}, x_{g})\) denotes the shortest path distance between spawn location \(x_{\tau_{i}+1}\) and goal location \(x_{g}\). The numerator is the total distance covered by the agent while skipping the jumps where the agent gets re-spawned after reaching the goal location. The denominator is the total shortest distance during the episodes.

7.2 Baselines
We compare our FWRL against three baselines: two versions of Q-Learning [Watkins and Dayan (1992)] and Model based RL (MBRL).

Q-Learning: QL and QLCAT  We implement two versions of Q-learning. In the first version, called QL, we reset the Q-function after every episode. This version of Q-Learning does not use the prior knowledge of the goal state, but depends upon the goal reward to build a new Q-function in every episode. In the second version, called QLCAT, we concatenate the state with the goal location and retain the learned Q-function function across episodes.

7.3 Model-based RL
We implement a simple version of tabular model-based RL, where we maintain a transition count data-structure \(T(s'|s,a)\). This allows to compute a frequentist estimate of dynamics model. We also keep a tabular record of the reward from each state-action pair \(R(s,a)\). The dynamics model is then used to find the maximum-reward-path to the goal state.

8 Results

Quantitative Results  We evaluate Q-learning Concatenated (QLCAT), Q-learning (QL), model-based RL (MBRL) and Floyd-Warshall Reinforcement Learning (FWRL) on two metrics in two different environments. The two metrics we use are Distance inefficiency and average reward per episode. The baselines and metrics are explained in the experiment section (6). The results are shown in Figure 3. We find that FWRL consistently achieves higher rewards, and lower distance inefficiency than the baselines. The reward curves also show that FWRL learns to find the higher reward much faster than the baselines, demonstrating higher sample efficiency.

Qualitative Results  To demonstrate the claim made in Fig[1] we train QLCAT and FWRL for two episodes each with start and goal locations such that the path meets in the center. Unlike the quantitative section, in this experiment the episode ends when the agent reaches the goal. After two episodes of training, in which both FWRL (Floyd-Warshall RL) and QLCAT (Q-Learning with goal concatenated to the state) reach the desired goals via exploration, we put the algorithms to test. For the test episode, the goal is chosen from first training episode but start location is chosen from second training episode. During the test episode, we turn off \(\epsilon\)-greedy exploration and follow the learned policy greedily.

We find that QLCAT decides to repeat the action that pushes it into the wall, therefore is unable to move. However, FWRL reaches the goal using the shortest path. The trajectories and value functions are visualized in Figure[7].
Figure 4: Reward curves on grid world. FWRL reward climbs much faster than all other baselines showcasing the improved sample efficiency of the algorithm.

Figure 5: Results on windy world. FWRL beats other baselines consistently. Lower is better for Distance-Inefficiency. Higher is better for reward per episode.

Figure 6: Reward curves on windy world. FWRL reward climbs much faster than all other baselines showcasing the improved sample efficiency of the algorithm.

Figure 7: Qualitative results: Visualization of value-function in H-Maze. Green box with G represents goal location, Blue box with S represents start location. The obstacles are shown black. The trajectories are shown by arrows. The color of remaining boxes show the expected value of each state computed by the corresponding algorithm. Each row different algorithm, while columns show different episodes. We find that QLCAT is unable to learn a new trajectory when given an unseen combination of start and end location. FWRL (ours) finds the shortest path easily in such a test case.

9 Conclusion
Floyd-Warshall Reinforcement Learning (FWRL) allows us to learn a goal conditioned action-value function which is invariant to the change in goal location as compared to the action-value functions typically used in typical Q-learning. This allows FWRL to transfer learned behaviors about the environment when the goal location changes. Many goal-conditioned tasks like navigation and robotic pick and place can benefit from this framework.

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