Revisiting the Phase Structure of the Polyakov-quark-meson Model in the presence of Vacuum Fermion Fluctuation.

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We have considered the contribution of fermionic vacuum loop in the effective potential of Polyakov loop extended Quark Meson Model (PQM) for the two quark flavour case and explored the phase structure and thermodynamics of the resulting PQMVT model (Polyakov Quark Meson Model with Vacuum Term) in detail at non zero as well as zero chemical potential. The temperature variations of order parameters and their derivatives have been calculated and the phase diagram together with the location of critical end point (CEP) has been obtained in $\mu$, and $T$ plane in both the models PQMVT and PQM. The PQMVT model analysis has been compared with the calculations in PQM model in order to bring out the effect of fermionic vacuum term on the physical observables. We notice that the critical end point (CEP) which is located near the temperature axis at $(\mu = 81.0, T = 167 \text{ MeV})$ in the PQM model gets shifted close to the chemical potential axis at $(\mu_{\text{CEP}} = 294.7, T_{\text{CEP}} = 84.0 \text{ MeV})$ in the PQMVT model calculations of the phase diagram. Since it emerges from a background of second order transition in the chiral limit of massless quarks, the crossover occurring at $\mu = 0$ in PQMVT model for the realistic case of explicitly broken chiral symmetry, has been identified as quite a soft and smooth transition. We have presented and compared the results for temperature variations of thermodynamic observables at zero and different non-zero quark chemical potentials. It is noticed that the presence of fermionic vacuum term in the effective potential leads to a smoother and slower temperature variation of thermodynamic quantities.

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I. INTRODUCTION

Quantum Chromo-dynamics (QCD), the commonly accepted theory of strong interaction predicts that normal hadronic matter undergoes a phase transition, where the individual hadrons dissolve into their constituents and produce a collective form of matter known as the Quark Gluon Plasma (QGP) under the extreme conditions of high temperature and/or density [1–4]. Relativistic heavy ion collision experiments at RHIC (BNL), LHC (CERN) and the future CBM experiments at the FAIR facility (GSI-Darmstadt) aim to create and study such a collective state of matter. Study of the different aspects of this phase transition, is a tough and challenging task because Quantum Chromodynamics (QCD) becomes nonperturbative in the low energy limit.

It is well known that the basic QCD Lagrangian has the global $SU_{L+R}(N_f) \times SU_{L-R}(N_f)$ symmetry for $N_f$ flavours of massless quarks. The axial (A=L+R) part of this symmetry known as the chiral symmetry is spontaneously broken by the formation of a chiral condensate in the low energy hadronic vacuum of QCD and one gets $(N_f^2 - 1)$ massless Goldstone bosons according to the Goldstone’s theorem. Since quarks are not massless in real life, chiral symmetry of the QCD lagrangian gets explicitly broken and massless modes become pseudo-Goldstone bosons after acquiring mass. Nevertheless, the observed lightness of pions in nature suggests that we have an approximate chiral symmetry for QCD with two flavours of light u and d quarks. In the opposite limit of infinitely heavy quarks, QCD becomes a pure $SU(N_c)$ gauge theory which remains invariant under the global $Z(N_c)$ center symmetry of the color gauge group. The Center symmetry which is a symmetry of hadronic vacuum, gets spontaneously broken in the high temperature/density regime of QGP. The expectation value of the Wilson line (Polyakov loop) is related to the free energy of a static color charge. It vanishes in the confining phase as the quark has infinite free energy and becomes finite in the deconfined phase. Hence the Polyakov loop serves as the order parameter of the confinement-deconfinement phase transition [5]. Even though the center symmetry is always broken with the inclusion of dynamical quarks in the system, one can regard the Polyakov loop as an approximate order parameter because it is a good indicator of a rapid crossover in the confinement-deconfinement transition [6, 7].

The first principle lattice QCD Monte Carlo simulations (see e.g. [8–17]) give us important information and insights regarding various aspects of the QGP transition, like the restoration of chiral symmetry in QCD, order of the

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confinement-deconfinement phase transition, richness of the QCD phase structure and mapping of the phase diagram. Unfortunately progress in lattice QCD calculations has got severely hampered due to the QCD action becoming complex on account of the fermion sign problem [8] when baryon density/chemical potential is non zero. Though several methods have been developed to circumvent the sign problem at small baryon chemical potentials, a general solution to the sign problem for all chemical potentials is yet to be devised. Further since lattice calculations are technically involved and various issues are not conclusively settled within the lattice community, one resorts to the calculations within the ambit of phenomenological models developed in terms of effective degrees of freedom. These models serve to complement the lattice simulations and give much needed insight about the regions of phase diagram inaccessible to lattice simulations.

In recent years, effective chiral models, having the pattern of chiral symmetry breaking as that of QCD like the linear sigma models(LSM) [18–24], the quark-meson (QM) models (see e.g., [25–35]), Nambu-Jona-Lasinio (NJL) type models [25, 36–39], have led to the investigation of the properties and structure of chiral symmetry restoring phase transition at sufficiently high temperature and density. Further these models were extended to incorporate the features of confinement-deconfinement transition where chiral condensate and Polyakov loop got simultaneously coupled to the quark degrees of freedom. Thus Polyakov loop augmented PNJL models [40–56], PLSM models and PQM models [57–65] have facilitated the investigation of the full QCD thermodynamics and phase structure at zero and finite quark chemical potential and it has been shown that bulk thermodynamics of the effective models agrees well with the lattice QCD data.

In most of the QM/PQM model calculations, the fermion vacuum contributions to the free energy is frequently neglected [25, 26, 31, 33, 38] because here, the spontaneous breaking of chiral symmetry is generated by the mesonic potential itself. While in the NJL/PNJL model investigations, fermion vacuum term leads to the dynamical breaking of the chiral symmetry, hence it gets explicitly included up to a momentum cutoff Λ. Very recently, it has been shown by Skokov et. al. in Ref. [64] that in a mean field approximation, where the fermion vacuum contribution to the free energy is neglected, the order of the phase transition for two flavour QM model in the massless chiral limit becomes first order at zero baryon chemical potential. They have further shown that the quark-meson model, with appropriately renormalized fermionic vacuum fluctuations in the thermodynamic potential, becomes an effective QCD-like model because now it can reproduce the second order chiral phase transition at μ = 0 as expected from the universality arguments [66] for the two massless flavours of QCD. It has also been shown that in the presence of an external magnetic field, the structure of the phase diagram in the PQM model is considerably affected by the fermionic vacuum contribution [67]. In this paper, we will investigate the effect of fermionic vacuum fluctuations on the phase structure and thermodynamics of PQM/QM models in detail at non zero as well as zero chemical potential. In order to bring out the effect of fermionic vacuum term on the physical observables, we will compare the results of our calculation with the corresponding PQM model calculations without vacuum term.

The arrangement of this paper is as follows. In Sec. II, we have given the formulation of PQM model for the two quark flavour. The Polyakov loop potential and the thermodynamic grand potential has been given in subsection II.A. After giving a brief description of the appropriate renormalization of fermionic vacuum loop contribution, the subsection II.B describes how the new model parameters are obtained in vacuum when renormalized vacuum term is added to the effective potential. The section III investigates the effect of fermionic vacuum term on the phase structure and thermodynamics. The subsection III.A explores how, the temperature variation of order parameters and their derivatives at different chemical potentials, structure of the phase diagram in the μ and T plane and the location of critical end point, gets affected in the presence of vacuum term. The effect on the temperature variation of thermodynamic observables namely pressure, entropy, energy density and interaction measure has been discussed in the subsection III.B while the discussion of specific heat, speed of sound and \( \frac{\partial \mu}{\partial T} \) has been presented in subsection III.C and finally the subsection III.D describes the results for quark number density and quark number susceptibility. Summary together with the conclusion has been presented in Sec. IV. The first and second partial derivatives of \( \mathcal{U}_{\text{log}} \) and \( \Omega_{90}^T \) with respect to temperature and chemical potential has been evaluated in appendix A.

II. MODEL FORMULATION

We will be working in the two flavor quark meson linear sigma model which has been combined with the Polyakov loop potential [57] in this model, quarks coming in two flavor are coupled to the \( SU_L(2) \times SU_R(2) \) symmetric four mesonic fields \( \sigma \) and \( \pi \) together with spatially constant temporal gauge field represented by Polyakov loop potential. Polyakov loop field \( \Phi(\vec{x}) \) is defined as the thermal expectation value of color trace of Wilson loop in temporal direction

\[
\Phi = \frac{1}{N_c} \text{Tr}_c L, \quad \Phi^* = \frac{1}{N_c} \text{Tr}_c L^\dagger
\]  

(1)
where $L(x)$ is a matrix in the fundamental representation of the $SU_c(3)$ color gauge group.

$$L(\vec{x}) = \mathcal{P} \exp \left[ i \int_0^\beta d\tau A_0(\vec{x}, \tau) \right]$$ \hspace{1cm} (2)

Here $\mathcal{P}$ is path ordering, $A_0$ is the temporal component of Euclidean vector field and $\beta = T^{-1}$ [5].

The model Lagrangian is written in terms of quarks, mesons, couplings and Polyakov loop potential $\mathcal{U}(\Phi, \Phi^*, T)$.

$$\mathcal{L}_{PQM} = \mathcal{L}_{QM} - \mathcal{U}(\Phi, \Phi^*, T)$$ \hspace{1cm} (3)

where the Lagrangian in quark meson linear sigma model

$$\mathcal{L}_{QM} = \bar{q} f \left[ i\gamma^\mu D_\mu - g(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) \right] q + \mathcal{L}_m$$ \hspace{1cm} (4)

The coupling of quarks with the uniform temporal background gauge field is effected by the following replacement

$$D_\mu = \partial_\mu - i A_\mu$$

and

$$A_\mu = \delta_{\mu0} A_0 \text{ (Polyakov gauge)}$$

where

$$A_\mu = g_s A_a^\mu \lambda^a / 2.$$ $g_s$ is the $SU_c(3)$ gauge coupling. $\lambda^a$ are Gell-Mann matrices in the color space, $a$ runs from 1 · · · 8.

$q_f = (u, d)^T$ denotes the quarks coming in two flavors and three colors. $g$ is the flavor blind Yukawa coupling that couples the two flavor of quarks with four mesons; one scalar ($\sigma, J^P = 0^+$) and three pseudoscalars ($\vec{\pi}, J^P = 0^-$).

The quarks have no intrinsic mass but become massive after spontaneous chiral symmetry breaking because of nonvanishing vacuum expectation value of the chiral condensate. The mesonic part of the Lagrangian has the following form

$$\mathcal{L}_m = \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \vec{\pi})^2 - U(\sigma, \vec{\pi})$$ \hspace{1cm} (5)

The pure mesonic potential is given by the expression

$$U(\sigma, \vec{\pi}) = \frac{\lambda}{4} \left( \sigma^2 + \vec{\pi}^2 - v^2 \right)^2 - h\sigma,$$ \hspace{1cm} (6)

Here $\lambda$ is quartic coupling of the mesonic fields, $v$ is the vacuum expectation value of scalar field when chiral symmetry is explicitly broken and $h = f_m m^2$. 

A. Polyakov loop potential and thermodynamic grand potential

The effective potential $\mathcal{U}(\Phi, \Phi^*, T)$ is constructed such that it reproduces thermodynamics of pure glue theory on the lattice for temperatures up to twice the deconfinement phase transition temperature. In this work, we are using the logarithmic form of Polyakov loop effective potential Ref. [41]. The results produced by this potential are known to be fitted well to lattice results.

$$\frac{\mathcal{U}_{\log} (\Phi, \Phi^*, T)}{T^4} = - \frac{a(T)}{2} \Phi^* \Phi + b(T) \ln(1 - 6\Phi^* \Phi$$

$$+ 4(\Phi^3 + \Phi^3) - 3(\Phi^4)^2)$$ \hspace{1cm} (7)

where the temperature dependent coefficients are as follow

$$a(T) = a_0 + a_1 \left( \frac{T_0}{T} \right) + a_2 \left( \frac{T_0}{T} \right)^2$$

and

$$b(T) = b_3 \left( \frac{T_0}{T} \right)^3.$$ 

The critical temperature for deconfinement phase transition $T_0 = 270$ MeV is fixed for pure gauge Yang Mills theory. In the presence of dynamical quarks $T_0$ is directly linked to the mass-scale $\Lambda_{QCD}$, the parameter which has a flavor and chemical potential dependence in full dynamical QCD and $T_0 \to T_0(N_f, \mu)$ [57, 65]. For our numerical calculations in this paper, we have taken a fixed $T_0 = 208$ for two flavours of quarks.

The parameters of Eq.(7) are

$$a_0 = 3.51, \quad a_1 = -2.47,$$

$$a_2 = 15.2, \quad b_3 = -1.75.$$
In the mean-field approximation, the thermodynamic grand potential for the PQM model is given as [57]

\[ \Omega_{MF}(T, \mu; \sigma, \Phi, \Phi^*) = U(T; \Phi, \Phi^*) + U(\sigma) + \Omega_{q\bar{q}}(T, \mu; \sigma, \Phi, \Phi^*). \]  (8)

Here, we have written the vacuum expectation values \( \langle \sigma \rangle = \sigma \) and \( \langle \vec{\pi} \rangle = 0 \).

The quark/antiquark contribution in the presence of Polyakov loop reads

\[ \Omega_{q\bar{q}}(T, \mu; \sigma, \Phi, \Phi^*) = \Omega_{vac}^q + \Omega_{T}^q = -2N_f \int \frac{d^3 p}{(2\pi)^3} \left\{ N_c E_q \theta(\Lambda^2 - \vec{p}^2) + T \left[ \ln g_q^+ + \ln g_q^- \right] \right\} \]  (9)

The first term of the Eq. (9) denotes the fermion vacuum contribution, regularized by the ultraviolet cutoff \( \Lambda \). In the second term \( g_q^+ \) and \( g_q^- \) have been defined after taking trace over color space.

\[ g_q^+ = \left[ 1 + 3 \Phi e^{-E_q^+ / T} + 3 \Phi^* e^{-2E_q^+ / T} + e^{-3E_q^+ / T} \right] \]  (10)

\[ g_q^- = \left[ 1 + 3 \Phi^* e^{-E_q^- / T} + 3 \Phi e^{-2E_q^- / T} + e^{-3E_q^- / T} \right] \]  (11)

Here we use the notation \( E_q^\pm = E_q \mp \mu \) and \( E_q \) is the single particle energy of quark/antiquark.

\[ E_q = \sqrt{\vec{p}^2 + m_q^2} \]  (12)

where the constituent quark mass \( m_q = g\sigma \) is a function of chiral condensate. In vacuum \( \sigma(0,0) = \sigma_0 = f_\pi = 93.0 \text{MeV} \)

**B. The renormalized vacuum term and model parameters**

The fermion vacuum loop contribution can be obtained by appropriately renormalizing the first term of Eq. (9) using the dimensional regularization scheme, as done in Ref.[64]. A brief description of essential steps is given below.

Fermion vacuum term is just the one-loop zero temperature effective potential at lowest order [68]

\[ \Omega_{q\bar{q}}^\text{vac} = -2N_f N_c \int \frac{d^4 p}{(2\pi)^4} \ln (p_0^2 + E_q^2) + K, \]  (13)

the infinite constant \( K \) is independent of the fermion mass, hence it is dropped.

The dimensional regularization of Eq. (13) near three dimensions, \( d = 3 - 2\epsilon \) leads to the potential up to zeroth order in \( \epsilon \) as given by

\[ \Omega_{q\bar{q}}^\text{vac} = \frac{N_c N_f}{16\pi^2} m_q^4 \left\{ \frac{1}{\epsilon} - \frac{1}{2} \left[ -3 + 2\gamma_E + 4 \ln \left( \frac{m_q}{2\sqrt{\pi} M} \right) \right] \right\}, \]  (14)

here \( M \) denotes the arbitrary renormalization scale.

The addition of a counter term \( \delta \mathcal{L} \) in the Lagrangian of the QM or PQM model

\[ \delta \mathcal{L} = \frac{N_c N_f}{16\pi^2} g^4 \sigma^4 \left\{ \frac{1}{\epsilon} - \frac{1}{2} \left[ -3 + 2\gamma_E - 4 \ln (2\sqrt{\pi}) \right] \right\}, \]  (15)

gives the renormalized fermion vacuum loop contribution as

\[ \Omega_{q\bar{q}}^\text{reg} = -\frac{N_c N_f}{8\pi^2} m_q^4 \ln \left( \frac{m_q}{M} \right). \]  (16)

Now the first term of Eq. (9) which is vacuum contribution will be replaced by the appropriately renormalized fermion vacuum loop contribution as given in Eq. (16).
The relevant part of the effective potential in Eq. (8) which will fix the value of the parameters \( \lambda \) and \( v \) in the vacuum at \( T = 0 \) and \( \mu = 0 \) is the purely \( \sigma \) dependent mesonic potential \( U(\sigma) \) plus the renormalized vacuum term given by Eq. (16).

\[
\Omega(\sigma) = \Omega_{\text{reg}}^{\text{eq}} + U(\sigma) = -\frac{N_c N_f}{8\pi^2} g^4 \sigma^4 \ln \left( \frac{g\sigma}{M} \right) - \frac{\lambda v^2}{2} \sigma^2 + \frac{\lambda}{4} \sigma^4 - h\sigma,
\]

(17)

The first derivative of \( \Omega(\sigma) \) with respect to \( \sigma \) at \( \sigma = f_\pi \) in the vacuum is put to zero

\[
\frac{\partial \Omega_{\text{MF}}(0; 0; \sigma, \Phi, \Phi^*)}{\partial \sigma} = \frac{\partial \Omega(\sigma)}{\partial \sigma} = 0
\]

(18)

The second derivative of \( \Omega(\sigma) \) with respect to \( \sigma \) at \( \sigma = f_\pi \) in the vacuum gives the mass of \( \sigma \)

\[
m^2_\sigma = \frac{\partial^2 \Omega_{\text{MF}}(0; 0; f_\pi, \Phi, \Phi^*)}{\partial \sigma^2} = \frac{\partial^2 \Omega(\sigma)}{\partial \sigma^2}
\]

(19)

Solving the equations (18) and (19), we obtain

\[
\lambda = \lambda_s + \frac{N_c N_f}{8\pi^2} g^4 \left[ 3 + 4 \ln \left( \frac{g f_\pi}{M} \right) \right]
\]

and

\[
\lambda v^2 = (\lambda v^2)_s + \frac{N_c N_f}{4\pi^2} g^4 f_\pi^2
\]

(20)

where \( \lambda_s \) and \( (\lambda v^2)_s \) are the values of the parameters in the pure sigma model

\[
\lambda_s = \frac{m^2_\sigma - m^2_\pi}{2f_\pi^2}
\]

(22)

\[
(\lambda v^2)_s = \frac{m^2_\sigma - 3m^2_\pi}{2}
\]

(23)

It is evident from the equations (20) and (21) that the value of the parameters \( \lambda \) and \( v^2 \) have a logarithmic dependence on the arbitrary renormalization scale \( M \). However, when we put the value of \( \lambda \) and \( \lambda v^2 \) in Eq.(17), the \( M \) dependence cancels out neatly after the rearrangement of terms. Finally we obtain

\[
\Omega(\sigma) = -\frac{N_c N_f}{8\pi^2} g^4 \sigma^4 \ln \left( \frac{\sigma}{f_\pi} \right) - \frac{\lambda v^2}{2} \sigma^2 + \frac{\lambda}{4} \sigma^4 - h\sigma,
\]

(24)

Here, we define \( \lambda_r \) and \( \lambda_r v^2_r \) as the values of the parameters after proper accounting of the renormalized fermion vacuum contribution.

\[
\lambda_r = \lambda_s + \frac{3N_c N_f}{8\pi^2} g^4
\]

and

\[
\lambda_r v^2_r = (\lambda v^2)_s + \frac{N_c N_f}{4\pi^2} g^4 f_\pi^2
\]

(25)

(26)

Now the thermodynamic grand potential for the PQM model in the presence of appropriately renormalized fermionic vacuum contribution (PQMVT model) will be written as

\[
\Omega_{\text{MF}}(T, \mu; \sigma, \Phi, \Phi^*) = U(T; \Phi, \Phi^*) + \Omega(\sigma) + \Omega_{\text{reg}}^{\text{eq}}(T, \mu; \sigma, \Phi, \Phi^*).
\]

(27)

Thus in the PQMVT model, One can get the chiral condensate \( \sigma \), and the Polyakov loop expectation values \( \Phi, \Phi^* \) by searching the global minima of the grand potential in Eq.(27) for a given value of temperature \( T \) and chemical potential

\[
\frac{\partial \Omega_{\text{MF}}}{\partial \sigma} = \frac{\partial \Omega_{\text{MF}}}{\partial \Phi} = \frac{\partial \Omega_{\text{MF}}}{\partial \Phi^*} = 0
\]

(28)

We will take the values \( m_\pi = 138 \text{ MeV}, m_\sigma = 500 \text{ MeV}, \) and \( f_\pi = 93 \text{ MeV} \) in our numerical computation. The constituent quark mass in vacuum \( m^0_q = 335 \text{ MeV} \) fixes the value of Yukawa coupling \( g = 3.3 \).
III. EFFECT OF THE VACUUM TERM ON THE PHASE STRUCTURE AND THERMODYNAMICS

We are presenting the results of our calculation for studying the temperature variation of the order parameters $\sigma$, $\Phi$, $\Phi^*$, their temperature derivatives and various thermodynamic observables at zero and non-zero quark chemical potentials in the presence of the renormalized fermionic vacuum term in the effective potential of the PQM model. These results have been termed as PQMVT model calculations and we have investigated the interplay of chiral symmetry restoration and confinement-deconfinement transition in the influence of fermionic vacuum term. The phase diagram together with the location of critical end point (CEP) has been obtained in $\mu$, and $T$ plane for both the cases with and without fermionic vacuum contribution in the effective potential. In order to have a comparison, we have also shown the temperature variations of order parameters and their derivatives in the PQM model calculation with the same parameter set. The temperature variations of thermodynamic observables namely pressure, energy density and entropy density at three different chemical potentials ($\sigma$, $\mu_{CEP}$ and $\mu > \mu_{CEP}$) have been shown in PQMVT model calculations. In order to study the effect of fermionic vacuum term at zero chemical potential, the temperature variation of the interaction measure, speed of sound, $p/\epsilon$ ratio and specific heat, has been calculated in PQMVT model and QMVT model (Quark Meson model with vacuum term) and these results have been compared with the corresponding results in the PQM and QM model calculations. Finally we will be presenting the results of the temperature variation of baryon number density and quark number susceptibility at different chemical potentials in PQMVT model calculation.

A. Phase structure

The solutions of the coupled gap equations, Eq.(28) determine the nature of chiral and deconfinement phase transition through the temperature and chemical potential dependence of chiral condensate $\sigma$, the expectation value of the Polyakov loop $\Phi$ and $\Phi^*$. Fig.1(a) shows the temperature variation of the chiral condensate $\sigma$ normalized with the vacuum value on the left while the right end of the plot shows the Polyakov loop $\Phi$ and $\Phi^*$ temperature variation for the PQMVT model calculations, the corresponding temperature variation of the chiral and Polyakov loop order parameters in PQM model calculations has been shown in Fig.2(a). In Fig.1(a), the continuous dots, thin dash and thick solid lines represent the variation of $\sigma_0$ on the left and $\Phi$ on the right at $\mu = 0$, 294.7 (CEP) and 300 MeV respectively. Thick dash and thick solid lines in the right end of the plot represent the $\Phi^*$ variations at $\mu = 294.7$ and 300 MeV respectively. Fig.1(b), 1(c) and 1(d), show the temperature derivatives of $\sigma$, $\Phi$ and $\Phi^*$ fields as a function of temperature respectively at three different chemical potentials $\mu = 0, 100$ and 294.73 MeV in PQMVT model calculations while the temperature variations of the same derivatives in the PQM model at $\mu = 0$ MeV has been shown in Fig.2(b). The characteristic temperatures (pseudocritical temperature) for the chiral transition $T_\chi^\sigma$ and the confinement-deconfinement transition $T_\chi^\Phi$, are defined by the peak positions (infection point) in the temperature derivatives of $\sigma$ and $\Phi$, $\Phi^*$ fields.

The chiral crossover transition for the realistic case of explicitly broken chiral symmetry, becomes quite soft and smooth at $\mu = 0$ because the corresponding chiral phase transition for massless quarks turns second order in the chiral limit after having a proper accounting of the fermionic vacuum contribution in the PQMVT model. The smoothness of crossover at $\mu = 0$ is evident from the temperature variation of the chiral order parameter in Fig.1(a) while the Polyakov loop order parameter variation at the same chemical potential, is sharp in comparison. The chiral crossover at $\mu = 0$ becomes less smooth as we increase the chemical potential. We find a large range ($\mu = 0$ at $T_\chi^\sigma = 186.5$ MeV to $\mu_{CEP} = 294.7$ MeV at $T_\chi^\sigma = 84.0$ MeV) in the values of chemical potential that makes the temperature variation of chiral order parameter, sharp and sharper such that eventually the crossover turns into a second order transition at CEP. The narrow width of the coincident variation of $\Phi$ and $\Phi^*$ temperature derivative at zero chemical potential in Fig.1(b), signifies a sharp crossover for the confinement-deconfinement transition at $T_\chi^\Phi = 169.0$ MeV The $\sigma$ derivative shows a broad double peak structure at $\mu = 0$ similar to the findings of NJL model calculation in Ref[48], we have identified the chiral crossover temperature $T_\chi^\sigma = 186.5$ MeV as the second peak position at higher pseudocritical temperature in Fig.1(b). The first peak in the $\sigma$ derivative is driven by the sharp peak of the Polyakov loop variation. As the chemical potential is increased, the variation of Polyakov loop $\Phi$ derivative becomes smoother and broader with increasing width, while the $\sigma$ derivative variation shows a decreasing width and double peak structure starts getting smeared after $\mu = 100$ MeV as shown in Fig.1(c). For the chiral crossover transition in the chemical potential range $\mu = 100$ to 160 MeV, the identification of pseudocritical temperature $T_\chi^\sigma$ becomes ambiguous with an ambiguity of about 5 MeV. For $\mu > 160$ MeV in the PQMVT model, double peak structure disappears from the temperature variation of the chiral order parameter temperature derivative as shown in Fig.1(d) and it's width decreases becoming narrow, narrower and narrowmost till the CEP at $\mu = 294.7$ MeV and $T = 84.0$ MeV is reached where the chiral transition turns second order.

For the realistic case of explicitly broken symmetry, the temperature variation of chiral order parameter at $\mu = 0$,
FIG. 1: Temperature variations in the PMQVT model. (a) The continuous dots, dash and solid lines represent the variation of $\sigma$ on the left end and $\Phi$ on the right end of the plot at $\mu = 0$, 294.7 (CEP) and 300 MeV respectively. Thick dash and thick solid lines in the right end of the plot represent the $\Phi^*$ variations at $\mu = 294.7$ and 300 MeV respectively. (b), (c) and (d), show the temperature derivatives of $\sigma$, $\Phi$ and $\Phi^*$ fields as a function of temperature respectively at three different chemical potentials $\mu = 0$, 100 and 200 MeV.

turns out to be quite sharp and rapid in Fig.2(a) in comparison to the corresponding PQMVT model variation in Fig.1(a) because the chiral transition in the massless quark limit, is first order in the PQM model. Further the chiral transition remains a crossover in quite a small range from $\mu = 0$ at $T^c = 171.5$ MeV to $\mu = 81$ MeV at $T^c = 167$ MeV in the PQM model results of Fig.2(a). Since the chiral crossover is sharper than the confinement-deconfinement crossover in the PQM model calculations, the single peak of the $\sigma$ field temperature derivative in Fig.2(b) at $\mu = 0$ is narrower and a lot higher than the peak in the variation of temperature derivatives of $\Phi$ and $\Phi^*$. We have scaled the variation of $\Phi$ and $\Phi^*$ temperature derivatives in Fig.2(b) by a multiple of 5 which shows a very small double peak kind of structure. We consider the chiral and confinement-deconfinement crossovers nearly coincident at $\mu = 0$, $T^c = 171.5$ and we get exact coincidence as we move towards the CEP ($T = 167.0$ MeV and $\mu = 81.0$ MeV) of the model where on account of the transition turning second order, we get highest and narrowmost peak.

In order to probe the issue of double peak structures emerging in Fig.1(b), 1(c), the temperature derivatives of $\sigma$, $\Phi$ and $\Phi^*$ fields have been evaluated as function of temperature by taking the Polynomial form [40] for Polyakov loop potential instead of the logarithmic form in PQMVT model. AT $\mu = 0$, none of the field derivatives have double peak structure. It starts appearing at $\mu = 200$ MeV separately in $\Phi$ and $\Phi^*$ derivatives as shown in Fig.3(a) and we find robust noncoincident second peaks respectively at $T^c = 176$ MeV, $T^c = 156$ MeV for $\mu = 280$ MeV as shown in Fig.3(b). The first and highest peak in $\sigma$ field temperature derivative gives the location of CEP at $T_{CEP} = 78.2$ MeV and $\mu_{CEP} = 293$ MeV. Though we have not evaluated the whole phase diagram for this case, we find that the confinement-deconfinement transition line (obtained from nearly coincident peaks in $\Phi$ and $\Phi^*$)
The scenario is ruled out in our PQMVT model calculation with constant symmetry restoration occurs at a higher temperature than the deconfinement transition. Thus the quarkyonic phase MeV as discussed in the second paragraph. Here we find support for the standard scenario [57, 65] where chiral shown the chiral crossover transition by a dash line which starts from the Polyakov loop potential while the calculation with polynomial Polyakov loop potential supports its occurrence in different lines which get located above the chiral crossover phase boundary from µ = 200 to µ = µ_{CEP} = 293 MeV . These findings being similar to the results of Ref.[38, 65] support the quarkyonic phase[70] like scenario, having a certain range of µ the Polyakov loop derivative while the Polyakov field Φ derivative shows a single peak in a chemical potential range µ > 0 to µ = 0 while thick and thin dash lines represent the Φ∗ and Φ variations at µ = 81 MeV respectively. Thus the meaning of the peaks in Φ, Φ∗ and σ field temperature derivatives and their relation to the nature of crossover transition in confinement-deconfinement and chiral sector is debatable. Similar to the findings of NJL model calculation in Ref[48], our calculations in PQMVT model with phenomenologically improved logarithmic Polyakov loop potential (which describes the gluon dynamics more appropriately with a better correlation to the effect of dynamical quarks), show double peak in σ field temperature derivative while the Polyakov field Φ derivative shows a single peak in a chemical potential range µ = 0 to µ = 200 MeV as discussed in the second paragraph. Here we find support for the standard scenario [57, 65] where chiral symmetry restoration occurs at a higher temperature than the deconfinement transition. Thus the quarkyonic phase scenario is ruled out in our PQMVT model calculation with constant T_{0} when we have taken a logarithmic form for the Polyakov loop potential while the calculation with polynomial Polyakov loop potential supports its occurrence in certain range of µ and T.

In Fig.4, we have obtained the phase diagram in our calculation with logarithmic Polyakov loop potential and located the critical end point (CEP) in the PQMVT as well as PQM model calculations for m_{σ} = 500 MeV. The structure of the phase diagram is very sensitive to the chosen value of the sigma meson mass. For the value m_{σ} = 600 MeV, the transition becomes a crossover in the entire µ and T plane for the PQMVT model calculation. We have shown the chiral crossover transition by a dash line which starts from T_{χ} = 186.5 MeV at µ = 0 axis and ends at CEP; T_{CEP} = 84 MeV and µ_{CEP} = 294.7 MeV in PQMVT model. Due to the smearing of double peak structure in the temperature derivative of chiral order parameter in the range µ = 100 to 160 MeV, the chiral crossover transition temperature T_{χ} is identified with an ambiguity of about 5 MeV. We get a unique T_{χ} for µ > 160 MeV in the phase diagram because of a single peak structure which gets narrow and narrower for higher chemical potentials till we reach the CEP. The dash dotted line which starts at T_{χ} = 169 MeV and ends at CEP of the PQMVT model, signifies the confinement- deconfinement crossover transition. The chiral and confinement-deconfinement crossover transition lines merge at µ = 250 and T_{χ} = T_{χ}^{∗} = 132. The thin solid line for µ > µ_{CEP} represents the first order phase transition corresponding to the jump in all the order parameters at the same critical temperature. The chiral crossover transition line lies above the crossover line for the confinement-deconfinement transition. Thus our results of the PQMVT model calculation are in tune with the standard scenario [65] where chiral symmetry restoration occurs at a higher critical temperature T_{χ} = 186.5 MeV than the confinement- deconfinement transition temperature T_{χ}^{∗} = 169 MeV at µ = 0 axis. Further the crossover transition temperature at µ = 0 compare well with the lattice [12, 15] results and QCD.

![Graph](image-url)
FIG. 3: Temperature variation of order parameter derivatives with polynomial Polyakov loop potential in PQMVT model calculation.

FIG. 4: Phase Diagram.

Based computations [65, 69] in two flavour model. The chiral and confinement-deconfinement crossover transition lines are coincident (as shown by the continuous dots) and start from $T_{\chi} = T_{\Phi} = 171.5$ MeV at $\mu = 0$ MeV to end at the CEP of the PQM model. The first order transition for $\mu > \mu_{\text{CEP}}$ in the PQM model, has been shown by the thick solid line. The CEP of the PQM model gets located near the temperature axis at $\mu_{\text{CEP}} = 81$ MeV and $T_{\text{CEP}} = 167$ MeV because the chiral crossover at $\mu = 0$, having the background of a first order phase transition in the chiral limit, is rapid and sharp and soon it gets converted to a first order phase transition as we increase the chemical potential. While the critical end point (CEP) gets shifted close to the chemical potential axis in PQMVT model because the chiral crossover transition at $\mu = 0$ is quite soft and smooth as it emerges from a phase transition which turns second order in the chiral limit due to the effect of renormalized fermionic vacuum contribution in the effective potential and further it remains a crossover for large values of the chemical potential.

B. Thermodynamic Observables: Pressure, Entropy and Energy Density

The negative of grand potential gives the thermodynamic pressure

\[ p(T, \mu) = -\Omega_{\text{MF}}(T, \mu) \] (29)
Thermodynamic pressure divided by the QCD Stefan-Boltzmann (SB) limit has been shown for three chemical potentials \( \mu = 0, 294.7 \) (CEP) and 300 MeV in Fig.5 for PQMVT model. It has been normalized to vanish at \( T = \mu = 0 \). We have shown the pressure calculated in PQM model also for comparison at \( \mu = 0 \). For \( N_f \) massless quarks and \( N_c^2 - 1 \) massless gluons in the deconfined phase, the QCD pressure in the SB limit is given by

\[
\frac{p_{SB}}{T^4} = (N_c^2 - 1) \frac{\pi^2}{45} + N_c N_f \left[ \frac{7 \pi^2}{180} + \frac{1}{6} \left( \frac{\mu}{T} \right)^2 + \frac{1}{12 \pi^2} \left( \frac{\mu}{T} \right)^4 \right].
\]  
(30)

The fermionic vacuum contribution makes the pressure variation in PQMVT model smooth at \( \mu = 0 \) and this curve (thin solid line) lies slightly below the curve (line with continuous dots) obtained in PQM model. The pressure variations at \( \mu_{CEP} = 294.7 \) and \( \mu = 300 \) MeV of PQMVT model are represented by the thick solid and dash line respectively. The pressure increases near the chiral transition due to the melting of the constituent quark masses and saturates at about eighty percent of the SB limit.

The entropy density is defined as negative of the temperature derivative of the grand potential.

\[
s = -\frac{\partial \Omega_{MF}}{\partial T}
\]  
(31)

The implicit variation of \( \sigma, \Phi \) and \( \Phi^* \) fields with respect to temperature has been accounted for, in the temperature derivative of \( \Omega(\sigma) \), \( U_{\log} \) and \( \Omega_{qg}^T \) as evaluated in the appendix A. The temperature variation of entropy density normalized by its QCD SB limit has been shown in Fig.6. It is continuous for crossover transition and attains about 40-45 percent of its SB value at pseudocritical transition temperature. Again due to the fermionic vacuum fluctuations, the entropy density variation (thin solid line) at \( \mu = 0 \) turns out to be a smoother function of temperature in PQMVT model when it is compared with corresponding curve (line with continuous dots) of PQM model calculation. At \( \mu_{CEP} = 294.7 \) MeV, the entropy density curve (thick solid line) shows a steep rise at \( T_{CEP} = 84 \) MeV in PQMVT model, then it takes a bend to reach its saturation. The PQM model entropy density curve (dah dotted line) at \( \mu = 294.7 \) MeV shows a large jump because chiral transition is strong first order at this chemical potential. The first order chiral transition of PQMVT model at \( \mu = 300.0 \) MeV, generates another jump in the entropy density curve (line with dash), though this jump is smaller than the first order jump seen in PQM model entropy curve at a lower chemical potential \( \mu = 294.7 \) MeV.

The energy density in the presence of chemical potential is given as

\[
\epsilon = -p + Ts + \mu n
\]  
(32)

where \( n \) is the number density. The temperature variation of energy density normalized by its QCD SB limit value has been shown in Fig.7 for \( \mu = 0, 280, 294.7 \) (CEP) and 300 MeV in PQMVT model. The energy density variation (thin
solid line) at \( \mu = 0 \) similar to entropy density variation, is smoother in comparison to the corresponding variation in PQM model calculation (line with continuous dots), this again is due to the influence of fermionic vacuum fluctuations. Similar to the entropy density variation at \( \mu_{\text{CEP}} = 294.7 \text{ MeV} \), the energy density (thick solid line) also shows a very steep and large rise at \( T_{\text{CEP}} = 84.0 \text{ MeV} \), then it curves to attain the saturation. At \( \mu = 300.0 \text{ MeV} \), we get a large jump in the energy density curve (dash-dotted line) which of course is a signature of the first order chiral transition. Since quark degrees of freedom get liberated and become light, the energy density registers a rapid increase near the crossover/phase transition point and reaches almost to the value of SB limit.

The trace anomaly of energy momentum tensor is also known as interaction measure. The temperature variation of the interaction measure \( \Delta = (E - 3p)/T^4 \) has been shown in Fig.8 at \( \mu = 0 \text{ MeV} \) in PQMVT, QMVT and PQM, QM model calculations. The QM model variation of the interaction measure (line with continuous dots) shows a sharp and narrow peak near the pseudocritical transition temperature which becomes very broad and smooth in the corresponding variation (thick dash line) of QMVT model calculation due to the effect of inclusion of fermion vacuum term contribution in the effective potential of QM model. The peak of interaction measure temperature variation (solid line) in PQMVT model shifts to a slightly higher temperature value in comparison to the corresponding peak in the variation (line with thin dash) of PQM model calculation.
The expression of specific heat at constant volume is given by

\[ C_V = \left. \frac{\partial \epsilon}{\partial T} \right|_V = -T \left. \frac{\partial^2 \Omega_{\text{MF}}}{\partial T^2} \right|_V \] (33)

The second partial temperature derivatives of \( \sigma \), \( \Phi \) and \( \Phi^* \) fields contribute in the double derivatives of \( \Omega(\sigma) \), \( \Omega_{\text{log}} \) and \( \Omega_{\text{eq}}^T \) with respect to temperature as given in the appendix A. Fig. 9 shows the temperature variation of the specific heat \( C_V \) normalized by \( T^3 \) in QM, QMVT and in PQM, PQMVT model calculations at \( \mu = 0 \). The specific heat variation while growing with the temperature, peaks at the crossover transition temperature and then saturates at the corresponding SB limit at the higher temperature. The QM model specific heat variation shows a large and sharp peak which becomes quite smooth and broad in the corresponding variation of QMVT model calculation due to the presence of fermionic vacuum term, further the peak position gets shifted to a higher transition temperature. The qualitative difference of structures in the curves of QM and QMVT model gets reduced due to the influence of Polyakov loop potential and we notice that the PQM model specific heat variation has quite a high and sharp
peak which becomes small and a little less sharp in the PQMVT model variation and the peaks occur at the same transition temperature. Further, we remark that the peak positions of the temperature variation of order parameter derivatives in Fig.1(b) and Fig.2(b) give different transition temperatures for chiral crossover in PQM and PQMVT model calculations while for confinement-deconfinement crossover, the transition temperature is almost same in both the models.

The speed of sound is an important quantity for hydrodynamical investigations of relativistic heavy-ion collisions. It is given by

\[ C_s^2 = \left. \frac{\partial p}{\partial \epsilon} \right|_{S} = \left. \frac{\partial p}{\partial T} \right|_{V} \left/ \left. \frac{\partial \epsilon}{\partial T} \right|_{V} \right. = \frac{s}{C_V}, \]

The equation of state parameter \( p(T)/\epsilon(T) \) also represents the information contained in trace anomaly. The velocity of sound \( C_s^2 \) and the equation of state parameter \( p(T)/\epsilon(T) \) ratio has been shown as a function of temperature in QM, QMVT and PQM, PQMVT model calculations at \( \mu = 0 \) in Fig.10. Thick lines denote the result for the sound velocity \( C_s^2 \) and thin lines show the variation of \( p(T)/\epsilon(T) \) ratio. The presence of fermion vacuum term in QMVT model leads to a very smooth temperature variation for \( C_s^2 \) (line with thick long dash) and \( p(T)/\epsilon(T) \) ratio (line with thin long dash). The \( C_s^2 \) temperature variation (line with thick continuous dots) in the QM model calculation, shows a very sharp drop followed by a rapid rise while the EOS parameter \( p(T)/\epsilon(T) \) ratio (line with thin continuous dots) shows a cusp at crossover transition temperature. The temperature variation of \( C_s^2 \) (thick solid line) and \( p(T)/\epsilon(T) \) ratio (thin solid line) in the PQMVT model turns out to be smoother than the corresponding variation of \( C_s^2 \) (line with thick,short and dark dash) and \( p(T)/\epsilon(T) \) ratio (thin dash line) in the PQM model calculation. At higher temperatures \( C_s^2 \) and \( p(T)/\epsilon(T) \) ratio approach the ideal gas value \( 1/3 \) in all the cases of model calculation. In PQM and PQMVT models, the value of \( C_s^2 \) almost matches with the \( p(T)/\epsilon(T) \) ratio for lower temperatures and \( C_s^2 \) value is always larger than the \( p(T)/\epsilon(T) \) ratio accept near the transition temperature, similar as in Ref.[43, 60]. The minimum value of \( p(T)/\epsilon(T) \) ratio is about .033 in PQMVT model which being slightly larger than the PQM model value .026, is less than the lattice result .075 [16, 71]. Similar to the findings of Ref.[60], interestingly the \( C_s^2 \) value is found to be less than 0.1 around half the crossover transition temperature in our PQM and PQMVT model results. In contrast, using a model of confinement, the results of Ref. [72] find values of about \( C_s^2 \sim 0.2 \) around half the transition temperature and \( C_s^2 = 0.15 \) around the transition temperature.

![FIG. 10: The variation of \( p(T)/\epsilon(T) \) has been shown by thin lines while thick lines show the variation of \( C_s^2 \).](image)

D. Quark number density and Susceptibility

The first derivative of grand potential with respect to chemical potential gives the quark number density

\[ n = -\frac{\partial \Omega_{\text{MF}}}{\partial \mu}. \]

The implicit variation of \( \sigma, \Phi \) and \( \Phi^* \) fields with respect to chemical potential has been accounted for, in the evaluation of first derivative of \( \Omega(\sigma), U_{\text{log}} \) and \( \Omega^X_{q\bar{q}} \) with respect to chemical potential as given in the appendix A.
The temperature variation of the quark number density normalized by $T^3$ in the PQMVT model calculation has been shown in Fig.11 for three quark chemical potentials $\mu = 280, 294.7$ (CEP) and 300 MeV. The dash dotted line shows the number density variation for a crossover transition at $\mu = 280$ MeV, here we see a small peak structure. The dotted line number density variation shows a sharper rise with a narrow peak at $\mu_{\text{CEP}} = 294.7$ MeV and it approaches the SB value of number density variation (shown by thick dots) for higher temperatures. At $\mu = 300$ MeV, the Phase transition becomes first order, hence the quark number density being a first derivative of the grand potential with respect to the chemical potential, shows a jump in the solid line temperature variation.

The expression of quark number susceptibility is obtained as

$$\chi_q = \frac{\partial^2 \Omega_{\text{MF}}}{\partial \mu^2} \quad (36)$$

The second partial derivatives of $\sigma$, $\Phi$ and $\Phi^*$ fields with respect to chemical potential contribute in the double derivatives of $\Omega(\sigma)$, $U_{\log}$ and $Q_{q\bar{q}}^T$ with respect to chemical potential as given in the appendix A. Fig.12 shows the variation of quark number susceptibility normalized by $T^2$ as a function of temperature in the PQMVT model calculation for chemical potentials $\mu = 280, 294.7$ (CEP) and 300 MeV. The dash dotted line susceptibility variation at $\mu = 280$ MeV shows a continuous peak structure at the crossover transition temperature. Since at $\mu_{\text{CEP}} = 294.7$ MeV, the phase transition turns second order, the dotted line of quark number susceptibility variation shows a very large and strongly divergent peak at $T_{\text{CEP}}$. The solid line shows the quark number susceptibility at $\mu = 300$ MeV for the first order transition case, we get a discontinuous variation because order parameter registers a jump in the first order transition.

IV. SUMMARY AND DISCUSSION

We have investigated the temperature variation of the order parameters $\sigma$, $\Phi$, $\Phi^*$, their temperature derivatives and various thermodynamic physical observables at non zero and zero quark chemical potentials in the presence of renormalized fermionic vacuum term in the effective potential of the PQM model. The results termed as the PQMVT model calculations have been compared with the results of PQM model without vacuum term. We have used logarithmic Polyakov loop potential for our calculation.

The chiral crossover transition for the realistic case of explicit chiral symmetry breaking, becomes quite soft and smooth at $\mu = 0$ in PQMVT model due to the proper accounting of the fermionic vacuum term contribution in the PQM model because the corresponding phase transition at $\mu = 0$ turns second order in the chiral limit of massless quarks. The $\sigma$ derivative shows a broad double peak structure at $\mu = 0$. The second peak position at higher transition temperature $T_{\chi} = 186.5$ MeV identifies the chiral crossover because the first peak results due to a sharp peak in the Polyakov loop temperature variation which signals a rapid confinement-deconfinement crossover transition at $\mu = 0$. In a large range of $\mu$, $T$ values (from $\mu = 0$ and $T = 186.5$ MeV to $\mu = 294$ MeV and $T = 84$ MeV), the chiral transition remains a crossover and it keeps on becoming sharper with the increase in chemical potential till the point of second order transition at $\mu_{\text{CEP}}$ is reached in the PQMVT model. Instead of logarithmic form, if we take polynomial
form for Polyakov loop potential in our PQMVT model calculation, the temperature derivatives of Polyakov loop field $\Phi$ and its conjugate $\Phi^*$ show distinct non coincident double peak structure in the chemical potential range $\mu = 200$ MeV to $\mu_{CEP} = 293$ MeV and we do not find any double peak structure near $\mu = 0$ in the temperature derivative of $\sigma$ field. Hence confinement-deconfinement crossover transition lines for $\Phi^*$ and $\Phi$ fields will be located above the chiral crossover phase boundary from $\mu = 200$ to $\mu = \mu_{CEP} = 293$ MeV. This finding support the quarkyonic phase like scenario, having a region of confinement with restored chiral symmetry. Our calculation with logarithmic form for the Polyakov loop potential does not support this quarkyonic phase like scenario. Since the chiral transition in the massless quark limit is first order at zero chemical potential, the corresponding crossover transition for the realistic case has been found to be quite sharp and rapid in the PQM model without any vacuum term. Further the chiral transition remains a crossover in quite a small range only from $\mu = 0$ and $T^c_\chi = 171.5$ MeV to $\mu = 81$ MeV and $T^c_\chi = 167$ MeV in the PQM model calculations.

The phase diagram together with the location of critical end point (CEP) has been obtained in $\mu$, and $T$ plane for $m_\sigma = 500$ MeV in both the models PQMVT and PQM. The structure of the phase diagram is very sensitive to the value of sigma meson mass. For the value $m_\sigma = 600$ MeV, the transition becomes a crossover in the entire $\mu$ and $T$ plane for the PQMVT model calculation. We do not have a coincident chiral and confinement-deconfinement crossover transitions in the PQMVT model as the chiral crossover transition line lies above the crossover line for the confinement-deconfinement transition. Our results of the PQMVT model calculation, are in tune with the standard scenario where chiral symmetry restoration occurs at a higher critical temperature than the confinement-deconfinement transition temperature. The quarkyonic phase scenario is ruled out in our PQMVT model calculation with constant $T_0$ when we have taken a logarithmic form for the Polyakov loop potential while the calculation with polynomial form of Polyakov loop potential supports its occurrence in certain range of $\mu$ and $T$. The critical end point (CEP) gets shifted close to the chemical potential axis ($\mu_{CEP} = 294.7$ MeV, $T_{CEP} = 84.0$ MeV ) in PQMVT model because the chiral crossover transition at $\mu = 0$ emerging from a second order phase transition in the chiral limit, becomes quite soft and smooth due to the effect of fermionic vacuum contribution in the effective potential and further it remains a crossover for large values of the chemical potential. The chiral and confinement-deconfinement crossover transition lines are coincident and the CEP of the PQM model gets located near the temperature axis at $\mu_{CEP} = 81$ and $T_{CEP} = 167$ because the chiral crossover at $\mu = 0$, having the background of a first order phase transition in the chiral limit, is quite rapid and sharp and soon it gets converted to a first order phase transition as we increase the chemical potential.

The temperature variation of thermodynamic observables namely pressure, energy density, entropy density at three different chemical potentials (zero, $\mu_{CEP}$ and $\mu > \mu_{CEP}$) has been shown in PQMVT model. Due to the proper accounting of appropriately renormalized fermionic vacuum fluctuations, the pressure ,entropy density and energy density variations at $\mu = 0$ turn out to be a smoother function of temperature in PQMVT model when it is compared with corresponding curves in PQM model calculation. The temperature variations of the interaction measure, speed of sound, $p(T)/\epsilon(T)$ and specific heat, have been calculated in PQMVT model and QMVT (Quark Meson model with vacuum term) model and these results have been compared with the corresponding results in the PQM and QM model calculations. Again we find that the presence of fermionic vacuum contribution in effective potential leads to the smoother variation of the thermodynamic quantities.

Finally we have shown the results of the temperature variations of baryon number density and quark number susceptibility at different chemical potentials in PQMVT.
model calculations.

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Appendix A: First and second partial derivatives of grand potential

First partial derivative of logarithmic Polyakov loop potential with respect to chemical potential and temperature

\[
\frac{\partial \mathcal{U}_{\text{log}}}{\partial \mu} = T^4 \left[ -\frac{a(T)}{2} \left\{ \frac{\partial \Phi}{\partial \mu} \Phi^* + \Phi \frac{\partial \Phi^*}{\partial \mu} \right\} - 6b(T) \ X_\mu \right]
\]  
(A1)

\[
\frac{\partial \mathcal{U}_{\text{log}}}{\partial T} = 4T^3 \left[ -\frac{a(T)}{2} \Phi^* \Phi + b(T) \ln|W| \right] + T^4 \left[ -\frac{1}{2} \left\{ \frac{\partial a(T)}{\partial T} \Phi \Phi^* + a(T) \frac{\partial \Phi}{\partial T} \Phi^* + a(T) \Phi \frac{\partial \Phi^*}{\partial T} \right\} 
\right.

\left. + \frac{d b(T)}{dT} \ln|W| - 6b(T) \ X_T \right]
\]  
(A2)

where

\[
X_y = \frac{(1 + \Phi \Phi^*) \left( \Phi \frac{\partial \Phi^*}{\partial y} + \Phi^* \frac{\partial \Phi}{\partial y} \right) - 2 \left( \Phi^2 \frac{\partial \Phi}{\partial y} + \Phi \Phi^* \frac{\partial \Phi}{\partial y} \right)}{W}
\]  
(A3)

\[
W = 1 - 6\Phi \Phi^* + 4(\Phi^3 + \Phi^3) - 3(\Phi^* \Phi)^2
\]  
(A4)

Second partial derivative of logarithmic Polyakov loop potential with respect to chemical potential and temperatures

\[
\frac{\partial^2 \mathcal{U}_{\text{log}}}{\partial \mu^2} = T^4 \left[ -\frac{a(T)}{2} \left( \frac{\partial^2 \Phi}{\partial \mu^2} \Phi^* + 2 \frac{\partial \Phi}{\partial \mu} \frac{\partial \Phi^*}{\partial \mu} + \Phi \frac{\partial^2 \Phi^*}{\partial \mu^2} \right) - 36b(T) X_\mu^2 
\right.

\left. - \frac{6b(T)}{W} \left\{ \left( \frac{\partial \Phi^*}{\partial \mu} + \Phi^* \frac{\partial \Phi}{\partial \mu} \right)^2 + (1 + \Phi \Phi^*) \left( \Phi \frac{\partial^2 \Phi^*}{\partial \mu^2} + 2 \frac{\partial \Phi}{\partial \mu} \frac{\partial \Phi^*}{\partial \mu} + \Phi^* \frac{\partial^2 \Phi}{\partial \mu^2} \right) 
\right. 

\left. - 2 \left( \Phi^2 \frac{\partial^2 \Phi}{\partial \mu^2} + 2 \Phi \left( \frac{\partial \Phi}{\partial \mu} \right)^2 + \Phi^* \left( \frac{\partial^2 \Phi^*}{\partial \mu^2} \right) + 2 \Phi^* \left( \frac{\partial \Phi}{\partial \mu} \right)^2 \right) \right\} \right]
\]  
(A5)
\[ \frac{\partial^2 \mathcal{H}_0}{\partial T^2} = 12T^2 \left[ -\frac{a(T)}{2} \Phi^* \Phi + b(T) \ln[W] \right] \]

\[ + 8T^3 \left[ -\frac{1}{2} \left( \frac{da(T)}{dT} \Phi^* + a(T) \Phi \frac{\partial \Phi^*}{\partial T} + a(T) \Phi \frac{\partial \Phi^*}{\partial T} \right) + \frac{db(T)}{dT} \ln[W] - 6b(T)X_T \right] \]

\[ + 4 \left[ -\frac{1}{2} \left( \frac{d^2a(T)}{dT^2} \Phi^* + a(T) \frac{\partial^2 \Phi^*}{\partial T^2} \right) - \left( \frac{da(T)}{dT} \frac{\partial \Phi^*}{\partial T} + a(T) \Phi \frac{\partial^2 \Phi^*}{\partial T} \right) \right] + \frac{db(T)}{dT} \ln[W] \]

\[ - 12 \frac{db(T)}{dT} X_T - 36b(T)X_T^2 \]

\[ - 6b(T) \left[ \left( \frac{\Phi^*}{\partial \Phi} + \frac{\Phi}{\partial T} \right)^2 + (1 + \Phi \Phi^*) \left( \frac{\partial^2 \Phi^*}{\partial T^2} + 2 \frac{\partial \Phi^*}{\partial T} \frac{\partial \Phi^*}{\partial T} + \Phi \frac{\partial^2 \Phi^*}{\partial T^2} \right) \right] \left( \frac{1}{W} \right) \] (A6)

First partial derivative of \( \Omega_{q\bar{q}}^T \) with respect to chemical potential and temperature:

\[ \frac{\partial \Omega_{q\bar{q}}^T}{\partial \mu} = -12 \int \frac{d^3p}{(2\pi)^3} \left[ T \left( B_{q,\mu}^+ + B_{q,\mu}^- \right) \right] \] (A7)

\[ \frac{\partial \Omega_{q\bar{q}}^T}{\partial T} = -4 \int \frac{d^3p}{(2\pi)^3} \left[ \ln g_q^+ + \ln g_q^- + 3T \left( B_{q,T}^+ + B_{q,T}^- \right) \right] \] (A8)

where

\[ A_q^+ = \Phi e^{-\beta E_q^+} + 2\Phi^* e^{-2\beta E_q^+} + e^{-3\beta E_q^+} \] (A9)

\[ A_q^- = \Phi^* e^{-\beta E_q^-} + 2\Phi e^{-2\beta E_q^-} + e^{-3\beta E_q^-} \] (A10)

\[ B_{q,x}^+ = \frac{1}{g_q^+} \left\{ A_q^+ \frac{\partial}{\partial x} (-\beta E_q^+) + \frac{\partial \Phi}{\partial x} e^{-\beta E_q^+} + \frac{\partial \Phi^*}{\partial x} e^{-2\beta E_q^+} \right\} \] (A11)

\[ B_{q,x}^- = \frac{1}{g_q^-} \left\{ A_q^- \frac{\partial}{\partial x} (-\beta E_q^-) + \frac{\partial \Phi^*}{\partial x} e^{-\beta E_q^-} + \frac{\partial \Phi}{\partial x} e^{-2\beta E_q^-} \right\} \] (A12)

Second partial derivative of \( \Omega_{q\bar{q}}^T \) with respect to chemical potential and temperature:

\[ \frac{\partial^2 \Omega_{q\bar{q}}^T}{\partial \mu^2} = -12T \int \frac{d^3p}{(2\pi)^3} D_{q,\mu} \] (A13)

\[ \frac{\partial^2 \Omega_{q\bar{q}}^T}{\partial T^2} = -12 \int \frac{d^3p}{(2\pi)^3} \left[ 2 \left( B_q^+ + B_q^- \right) + T D_{q,T} \right] \] (A14)
where

\[
D_{q,x} = \left[ -3 \left( B_q^+ + B_q^- \right) + \frac{1}{g_q} \left\{ C_q^+ \left[ \frac{\partial}{\partial x} (-\beta E_q^+) \right]^2 + \left( 2 \frac{\partial \Phi}{\partial x} e^{-\beta E_q^+} + 4 \frac{\partial \Phi^*}{\partial x} e^{-2\beta E_q^+} \right) \frac{\partial}{\partial x} (-\beta E_q^+) + A_q^+ \frac{\partial^2}{\partial^2 x} (-\beta E_q^+) \right] \\
+ \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi^*}{\partial x^2} \right) e^{-\beta E_q^+} \right\} \\
+ \frac{1}{g_q} \left\{ C_q^- \left[ \frac{\partial}{\partial x} (-\beta E_q^-) \right]^2 + \left( \frac{\partial \Phi^*}{\partial x} e^{-\beta E_q^-} + 4 \frac{\partial \Phi}{\partial x} e^{-2\beta E_q^-} \right) \frac{\partial}{\partial x} (-\beta E_q^-) + A_q^- \frac{\partial^2}{\partial^2 x} (-\beta E_q^-) \right] \\
+ \left( \frac{\partial^2 \Phi^*}{\partial x^2} + \frac{\partial^2 \Phi}{\partial x^2} \right) e^{-\beta E_q^-} \right\} \right]
\]

\begin{align}
C_q^+ &= \Phi e^{-\beta E_q^+} + 4\Phi^* e^{-2\beta E_q^+} + 3e^{-3\beta E_q^+} \\
C_q^- &= \Phi^* e^{-\beta E_q^-} + 4\Phi e^{-2\beta E_q^-} + 3e^{-3\beta E_q^-}
\end{align}

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