EM-RBR: a reinforced framework for knowledge graph completion from reasoning perspective

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Abstract

Knowledge graph completion aims to predict the new links in given entities among the knowledge graph (KG). Most mainstream embedding methods focus on fact triplets contained in the given KG, however, ignoring the rich background information provided by logic rules driven from knowledge base implicitly. To solve this problem, in this paper, we propose a general framework, named EM-RBR(embedding and rule-based reasoning), capable of combining the advantages of reasoning based on rules and the state-of-the-art models of embedding. EM-RBR aims to utilize relational background knowledge contained in rules to conduct multi-relation reasoning link prediction rather than superficial vector triangle linkage in embedding models. By this way, we can explore relation between two entities in deeper context to achieve higher accuracy. In experiments, we demonstrate that EM-RBR achieves better performance compared with previous models on FB15k, WN18 and our new dataset FB15k-R. We make the implementation of EM-RBR available at https://github.com/1173710224/link-prediction-with-rule-based-reasoning.

1 Introduction

Knowledge graph (KG) has the ability to convey knowledge about the world and express the knowledge in a structured representation. The rich structured information provided by knowledge graphs has become extremely useful resources for many Artificial Intelligence related applications like query expansion [9], word sense disambiguation [32], information extraction [11], etc. A typical knowledge representation in KG is multi-relational data, stored in triplets of the form (head entity, relation, tail entity), e.g., (Paris, Capital-Of, France). However, due to the discrete nature of the logic facts [30], the knowledge contained in the KG is meant to be incomplete [25]. Consequently, knowledge graph completion(KGC) has received more and more attention, which attempts to predict whether a new triplet is likely to belong to the knowledge base (KB) by leveraging existing triplets of the KB.

Currently, the popular embedding-based KGC methods aim at embedding entities and relations in knowledge base to a low-dimensional latent feature space. The implicit relationships between entities can be inferred by comparing their distributed representations in this vector space. These researchers [2, 19, 31, 12, 18, 20] make their own contributions for more reasonable and competent embedding. But the overall effect is highly correlated with the density of the knowledge base. Because embedding method always fails to predict weak and hidden relations which are observed in the KG with a low frequency. The embedding will converge to a solution that is not suitable for triplets owned weak relations, since the training set for embedding cannot contain all factual triplets. However, reasoning over the hidden relations can covert the testing target from weak triplet to other more frequent triplets, i.e. conducting multi-relation reasoning link prediction according to the frequent triplets rather than the single weak relation examination. For example, there is an existing triplet (Paul, LeaderOf, SoccerTeam) and a rule LeaderOf(x, y) → MemberOf(x, y)
which indicates the leader of a soccer team is also a member of a sport team. Then we can apply the rule on the triplet to obtain a new triplet \((Paul, Memberof, SportTeam)\) even if the relation \(Memberof\) is weak in knowledge base.

Besides, some innovative models try to harness rules for better prediction. Joint models \([24, 29, 10]\) utilize the rules in loss functions of translation models and get a better embedding representation of entities and relations. An optimization based on ProPPR \([30]\) embeds rules and then uses those embedding results to calculate the hyper-parameters of ProPPR. These efforts all end up on getting better embedding from rules and triplets, rather than solving completion through real rule-based reasoning, which is necessary to address weak relation prediction as mentioned before. Compared with them, EM-RBR can perform completion from the reasoning perspective.

**Challenge:** In the development of the joint framework EM-RBR, we meet two challenges. On the one hand, it is not feasible and tractable to mine rules manually because it is time-consuming and laborious. A suitable method is to extract rules by some rules-mining algorithms such as AMIE \([7]\). However, these rules automatically mined sometimes are not completely credible. Therefore, it is necessary to propose a reasonable way to measure rules to pick proper rules for the given triplets. On the other hand, it is known that traditional reasoning-based methods will give only 0 or 1 to one triplet to indicate accept or refuse for the given knowledge base. This conventional qualitative analysis lacks the quantitative information as the embedding models. So the result of our reasoning-based framework need to reflect the probability that one triplet belongs to the knowledge base more accurately.

We propose a novel framework EM-RBR combing embedding and rule-based reasoning. Given a new triplet, we apply rules to conduct reasoning. With the help of the structured information of logic rules in the the same semantic space with embedding, a score can be output by EM-RBR to measure the quality of the result.

**Contribution:** Three main contributions in EM-RBR are summarized as follows. Firstly, EM-RBR is flexible and general enough to optimise any type of embedding models. Secondly, we give rule a sensible measurement. The evaluation strategy referred as rule embedding not only takes embedding results into consideration but also unifies the magnitude of embedding and rules’ credibility. Thirdly, we propose novel rating mechanism combined with reasoning process, which can distinguish a given triplet with other wrong triplets better.

In the remaining of this paper, we will explain how our model works in Section 2, experiments in Section 3 and related work in Section 4.

### 2 Method

The core idea of our framework is to conduct multi-relation path prediction in deeper context from reasoning perspective. As discussed above, two core problems are to be solved. One is the evaluation of rules, and another is the measurement of the reasoning results. We solve the first one in Section 2.2 and solve the second by constructing a heuristic dissimilarity score function in Section 2.3. Before this, the overview of our framework is in Section 2.1.

#### 2.1 Overview

**Definition 2.1. Rule:** A rule in our framework is in the form of \(B_1(x, z) \land B_2(z, y) \implies H(x, y)\), where the entities order in one triplet is random, i.e. \(B_3(z, x) \land B_4(z, y) \implies R(x, y)\) is also a valid rule.

We model a knowledge graph as a collection of facts \(G = \{(h, r, t)\mid h, t \in \mathcal{E}, r \in \mathcal{R}\}\), where \(\mathcal{E}\) and \(\mathcal{R}\) represent the set of entities and relations in the knowledge graph, respectively. The steps of our framework are as follows corresponding to Figure 1.

**Step 1.** We invoke an embedding model to get a set \(\Xi \in \mathbb{R}^{|\mathcal{E}|+|\mathcal{R}| \times k}\) containing the \(k\)-dimensional embedding of entities and relations in \(G\). Then a triplet will be embedded as \((h, r, t)\), where \(h, r, t \in \Xi\). **Step 2.** We apply AMIE \([7]\) on \(G\) to get a reasoning rules set \(\Omega\), where each rule meets Definition 2.1. **Step 3.** The reasoning rules are measured based on \(\Xi\) to select appropriate rules in the next step. More detailed description is in Section 2.2. **Step 4.** Then reasoning is conducted for a given triplet \((h, r, t)\), which will be described in Section 2.3.
2.2 Measurement of Reasoning Rule Based on Embedding

The measurements of rules provided by AMIE, such as PCA confidence [7], cannot fit in our systems, since they do not take embedding results into consideration, and the different magnitude of embedding and rules’ credibility brings some difficulties to the design of the compatible evaluation strategy. Thus, we need a new method to measure the rules. We score the rules based on the embedding results. Specifically, we develop a scalable matrix approach that puts the structural information of logic rules into the same semantic space with the help of embedding models in EM-RBR.

For example, to measure a rule $B_1(x, z) \land B_2(z, y) \Rightarrow H(x, y)$, we can draw the three triplets in space as shown in Figure 2. In EM-RBR, if the rule has a high credibility, $\|x + H - y\| \approx \|x + B_1 - z + z + B_2 - y\|$. Then we can get $H \approx B_1 + B_2$, which can be projected as a rule triangle $\Delta \in \mathbb{R}^{3 \times k}$ at the back plane to be scored with $\|B_1 + B_2 - H\|$. The basic idea of such measurement is that the functional relation induced by the $H$-labeled edges corresponds to a translation of the other two relations in the rule triangle. In other words, we want that $\|B_1 + B_2 - H\|$ is closer to 0 when $\Delta$ has a higher credibility ($H$ should be a nearest neighbor of $B_1 + B_2$), while $B_1 + B_2$ should be far away from $H$ otherwise. In this way, the rule is represented as a matrix of $\mathbb{R}^{3 \times k}$. The rule evaluation has nothing to do with the embedding of entity, which matches the real-life scenarios and intuition.
2.3 Rule-based Reasoning

2.3.1 Reasoning

Definition 2.2. score: The score $s$ of a triplet is a real number that meets $s \geq 1$. The object of the score is that the smaller $s$ is, the triplet belongs to knowledge graph with greater probability. A triplet $(h, r, t)$’s score is denoted as $s(h, r, t)$. For a given triplet, EM-RBR will give out a score as Definition 2.2. Therefore, the goal of link prediction is converted to make the score of the correct triplet as close to 1 as possible, and the opposite of the wrong triplet.

We use reasoning (implemented as breadth first search) to calculate a score $s_{\sim}(h, r, t)$ for each triplet $(h, r, t)$. Next, we analyze the search process. Firstly, a initial state of the search is the triplet itself, which will be transformed into a new state by matching the rules. The transformation and matching methods are presented in the part of Example in Section 2.3.2 Secondly, a score function $\mathcal{L}(h, r, t)$, which is described in Section 2.3.2 is used to measure the score of each state. Lastly, a state is extensible if and only if the state can be transformed into a new state with a smaller score. When a state’s score can’t get smaller, we call the state as a termination state. After traversing all the termination states in the search process, the minimum value of all ending states’ scores is the score of triplets.

2.3.2 Score Function

Definition 2.3. Final triplets: The given original triplet $(h, r, t)$ is extended to a set of triplets recursively. When each of extended triplets has already contained in $\mathcal{O}$ or has no rules to match according to $r$, the extension ends. We call all the triplets in the set as final triplets. The final triplet set is denoted as $\mathcal{O}$ in general or $\mathcal{O}_{\sim}(h, r, t)$ to denote a specific original triplet $(h, r, t)$.

Example: As seen in Figure 3 reasoning starts with $(h, r, t)$, we match it with a rule $p(x, z) \land q(z, y) \Rightarrow r(x, y)$ according to $r$. Then the new state is $\{(x, p, z), (z, q, y)\}$, each of which is not in $\mathcal{O}$ and can find rule to match. So we continue to extend them to match $p$ with $B_1(x, z) \land B_2(z, y) \Rightarrow p(x, y)$ and $B_3(x, z) \land B_4(z, y) \Rightarrow q(x, y)$, respectively. Then current state is $\{(h, B_1, e_1), (e_1, B_2, e_0), (e_0, B_3, e_2), (e_2, B_4, t)\}$. Supposing $(h, B_1, e_1), (e_1, B_2, e_0)$ are in $\mathcal{O}$ and $(e_0, B_3, e_2), (e_2, B_4, t)$ have no rules to match, they all meet the requirements in Definition 2.3, then current state is a termination state. There may also be multiple rules matching one triplet, so there may exist many termination states.

A triplet will be extended to a group of final triplets as Definition 2.3 by rules in $\Omega$. And $\mathcal{L}(h, r, t)$ is designed to evaluate these final triplets. Because the probability of $(h, r, t)$ belonging to the knowledge graph is closely related to those triplets in $\mathcal{O}$, the $s_{\sim}(h, r, t)$ will rise with increasing of $\mathbf{s}_{\sim \text{trans}X}(h, r, t)$ for any $(h, r, t) \in \mathcal{O}$, where $\mathbf{s}_{\sim \text{trans}X}$ represents a triplet’s translation score in transX model. If we use transE in EM-RBR, then $\mathbf{s}_{\sim \text{trans}E}(h, r, t) = \mathbf{s}_{\sim \text{trans}E}(h, r, t) = \frac{|h| + |r - t|}{k}$.

Considering the above relationship and the necessity to integrate reasoning rules with uncertainty, we can calculate $\mathcal{L}(h, r, t)$ as Equation 1 where the factor $\alpha_{\sim}(\mathcal{O}_h, \mathcal{O}_r, \mathcal{O}_t)$ is only related to rules that used in computing $\mathcal{O}_{\sim}(h, r, t)$. We put these related rules in a set $\Delta_{\text{path}}$, then the factor can be calculated as Equation 2. As Equation 1 shows, smaller $s_{\sim}(h, r, t)$ indicates higher credibility of the rules and final triplets used in the reasoning process and greater probability the original triplet belongs to the KG.

\[
\mathcal{L}(h, r, t) = \prod_{(\mathcal{O}_h, \mathcal{O}_r, \mathcal{O}_t) \in \mathcal{O}} \left( \frac{(s_{\sim \text{trans}X}(\mathcal{O}_h, \mathcal{O}_r, \mathcal{O}_t)/k + 1) * \alpha_{\sim}(\mathcal{O}_h, \mathcal{O}_r, \mathcal{O}_t)}{} \right) \quad (1)
\]

\[
\alpha_{\sim}(\mathcal{O}_h, \mathcal{O}_r, \mathcal{O}_t) = \prod_{(B_1 \land B_2 \Rightarrow H) \in \Delta_{\text{path}}} \exp((||B_1 + B_2 - H||)/k) \quad (2)
\]
For TransE, TransH, and TransR, we set the same parameters, i.e., the dimensions. We only adopt above three models in the experiment to present EM-RBR’s performance, for EM-RBR has the competence to make sure the improvement of embedding models used in our framework. An open-source implementation of EM-RBR(X) denotes the embedding model in this experiment group is TransX. [‡]: test set available at https://github.com/1173710224/link-prediction-with-rule-based-reasoning/blob/master/FB15k-R-test.txt

Table 1: Experimental results on FB15k and FB15k-R test set. [⋆]: We used at code https://github.com/thunlp/Fast-TransX, configured new parameters, and re-run the program to get the experimental results. [‡]: We used at code https://github.com/thunlp/Fast-TransX, configured new parameters, and re-run the program to get the experimental results. [†]: EM-RBR(X) denotes the embedding model in this experiment group is TransX.

| Method | FB15k | FB15k-R+ |
|--------|-------|---------|
|        | MR    | MRR     | H@10 | H@5 | H@3 | H@1 | MR    | MRR     | H@10 | H@5 | H@3 | H@1 |
| TransE | 70.30 | 45.77  | 74.27 | 64.44 | 55.79 | 29.98 | 71.33 | 26.11  | 48.10 | 35.80 | 28.65 | 14.90 |
| EM-RBR (E) | 68.36 | 50.01  | 76.23 | 67.84 | 60.62 | 34.44 | 3.12  | 79.88  | 96.40 | 95.60 | 94.45 | 65.10 |
| (up 0.94) | (+4.24) | (+1.96) | (+3.40) | (+4.83) | (+4.46) | (up 68.21) | (up 53.77) | (up 48.3) | (up 59.8) | (up 65.8) | (up 50.2) |
| TransH | 72.56 | 45.81  | 74.01 | 64.09 | 55.53 | 30.37 | 50.65 | 30.43  | 54.95 | 42.55 | 34.45 | 18.25 |
| EM-RBR (H) | 70.72 | 52.39  | 76.52 | 68.28 | 61.13 | 38.82 | 3.52  | 85.61  | 97.80 | 96.50 | 95.60 | 75.45 |
| (up 0.84) | (+6.58) | (+2.51) | (+4.19) | (+5.60) | (+8.45) | (up 47.13) | (up 55.18) | (up 42.85) | (up 53.95) | (up 61.15) | (up 57.20) |
| TransR | 55.98 | 47.88  | 77.04 | 68.01 | 59.47 | 31.10 | 29.64 | 18.51  | 37.60 | 26.65 | 19.65 | 7.65 |
| EM-RBR (R) | 55.47 | 51.93  | 78.35 | 70.54 | 63.42 | 35.86 | 1.73  | 86.01  | 99.20 | 98.60 | 97.85 | 74.30 |
| (up 0.51) | (+4.05) | (+1.31) | (+2.53) | (+3.95) | (+4.76) | (up 27.91) | (up 67.50) | (up 61.60) | (up 71.95) | (up 78.20) | (up 66.65) |

3 Experiment

3.1 Experiment Setup

Dataset: We evaluate EM-RBR on FB15k [2] and FB15k-R. FB15k has 14,951 entities, 1,345 relations and 59,071 test triplets in total. We create FB15k-R, a subset of FB15k, which contains 1000 tested triplets that have weak and hidden relations between head entity and tail entity.

Metrics: We use a number of commonly used metrics, including Mean Rank (MR), Mean Reciprocal Rank (MRR), and Hit ratio with cut-off values n = 1, 3, 5, 10. MR measures the average rank of all correct entities and MRR is the average inverse rank for correct entities. Hits@n measures the proportion of correct entities in the top n entities. MR is always greater or equal to 1 and the lower MR indicates better performance, while MRR and Hits@n scores always range from 0.0 to 1.0 and higher score reflects better prediction results. We use filtered setting protocol [2], i.e., filtering out any corrupted triplets that appear in the KB to avoid possibly flawed evaluation.

Baseline: To demonstrate the effectiveness of EM-RBR, we compare with a number of competitive baselines—TransE [2], TransH [31], TransR [18]. We only adopt above three models in the experiment to present EM-RBR’s performance, for EM-RBR has the competence to make sure the improvement of embedding models used in our framework. An open-source implementation of TransE, TransH and TransR is available from the project website https://github.com/thunlp/Fast-TransX

Implementation: For TransE, TransH, and TransR, we set the same parameters, i.e., the dimensions of embedding $k = 100$, learning rate $\lambda = 0.001$, the margin $\gamma = 1$. We traverse all the training triplets for 1000 rounds. Other parameters of models are set as the same with the parameters in the published works [2][31][18].

3.2 Experimental Results

EM-RBR integrated with TransE, TransH and TransR greatly improve the performance of the original methods on all metrics according to the results in Table 1. This implies that our reasoning-enhanced framework can efficiently help the knowledge graph embedding models to perform link prediction more accurately.

Our rule-enhanced method significantly reaches some promising absolute performance scores on Hits@1, Hits@3, and Hits@5 metrics, e.g., in FB15k, EM-RBR(H)’s improvements of 38.82 – 30.37 = 8.45 in Hits@1, 61.13 – 55.53 = 5.60 in Hits@3 and also obtains 68.28 – 64.09 = 4.19 absolute improvement in Hits@5. These metrics paying attention on front ranks are the most important in real knowledge inference task. The higer Hits@n score indicates the greater possibilities to rank the correct triplets in the front.

More impressively in FB15k-R, EM-RBR(E)’s, EM-RBR(H)’s and EM-RBR(R)’s advance in Hits@1 is 65.1 – 14.9 = 50.2, 75.45 – 18.25 = 57.20 and 74.30 – 7.65 = 66.65 respectively.
Thus, EM-RBR can make knowledge graph embedding method capable of being used in the real and large scale knowledge inference tasks.

### 3.3 Model Analysis

#### Hyper-Parameter Analysis

**Consecutive times \( t \):** When the score of a solution keeps the same for continuous \( t \) popped nodes, we will stop reasoning and return current score to accelerate the conduction of reasoning. We select times \( t \) among \{1, 3, 5, 15, 20, 40, 60, 80\} and conduct the experiment by sampling 1000 test triplets from FB15k. If we set \( t \) too large, the time overhead of reasoning will be unbearable. The relations between \( t \) and the performance of EM-RBR are shown in Figure 4. We see that, when \( t \) vary in the interval of 0-20, the effect on the capability of EM-RBR is obvious because the value closer to 0 will lead to the less reasoning to be applied and our model to degenerate to the embedding models. Figure 4 shows \( t = 20 \) yields the best overall performance.

**Rule number:** Naturally, the limited number of rules will result in the lack of proper rules to reason for a correct triplets while too many rules will bring the rules of bad quality to improve the ranks of wrong triplets. Both conditions can interfere the performance of EM-RBR. Thus we select rules’ number among \{6500, 6000, … 1000, 500, 0\} and conduct the experiment on EM-RBR(E) for 1000 test triplets that we sample from FB15k. Every time we remove the rules that rank in the lowest 500 rules by PCA-credibility [7]. The result is presented in Figure 5.

#### Case Study

To explore how much EM-RBR can optimize on each triplet on earth, we sort all the tested triplets according to \( \text{trans_rank} - \text{reasoning_rank} \) from largest to smallest. The analysis of result and top 10 cases of each experimental group are shown in Table 2. Excellent results demonstrate the powerful capabilities of our model.

Comparing the statistics in two tables, we can find that although EM-RBR has achieved very successful results in most test cases, there are still some triplets whose rank has not been optimized to the top, and even some triplets’ ranks have fallen after optimization. Next, we will take a few cases to analyze these phenomena briefly.

**Case#1** The 23339-th test case on EM-RBR(H) is the 19372-th case in Table 2. The initial score of this triplet is 1.0669, after reasoning, it’s optimized to 1.03551 with the help of 3645-th rule. However, five other triplets are processed to a smaller score with the same rule. So there is something wrong with the measurement of rules. This is also a point where our model can be improved.

**Case#2** The 0-th test triplet’s rank is the same to that of translation model. We check the rules that can be matched by the relation in this test triplet, and find that no rule is matched. The number of cases with the same situation is about 40000, nearly two-thirds of all the test triplets. So an idea to further optimize EM-RBR is to extract more rules that can be adapted to more relations.

### 4 Related work

For the path-based methods, the Path Ranking Algorithm (PRA) [16] uses random walks to perform multiple bounded depth-first search processes. Ref. [17] uses PRA to estimate the probability...
Table 2: Optimized case analysis. [+] the line number of test case in test file starting from 0. [∗] the rank of the test case in EM-RBR. [‡] the rank of the test case in the embedding model used in EM-RBR. [⋄] L corresponds to replacing the head entity and R the tail entity.

| Rank | id | EM-RBR(E) | EM-RBR(H) | EM-RBR(R) |
|------|----|-----------|-----------|-----------|
| 1    | 47722 | 2 | 14141 | R | 18355 | 2 | 12689 | L | 15105 | 2 | 966 | L |
| 2    | 47722 | 2 | 13900 | L | 32966 | 3 | 8551 | R | 42675 | 1 | 868 | R |
| 3    | 18355 | 2 | 7525 | L | 18355 | 2 | 7231 | R | 34891 | 2 | 733 | R |
| 4    | 36133 | 2 | 6884 | L | 47722 | 2 | 4589 | L | 24314 | 2 | 714 | R |
| 5    | 33004 | 1 | 6253 | L | 24243 | 1 | 4547 | L | 32849 | 1 | 701 | L |
| 6    | 33243 | 2 | 5883 | R | 33004 | 1 | 4490 | L | 55951 | 2 | 673 | L |
| 7    | 30883 | 2 | 5674 | L | 47722 | 2 | 3977 | R | 38773 | 1 | 640 | L |
| 8    | 14035 | 2 | 4862 | L | 13558 | 5 | 3741 | R | 54283 | 5 | 674 | L |
| 9    | 18355 | 2 | 4525 | R | 55951 | 2 | 3699 | L | 25500 | 1 | 585 | R |
| 10   | 24243 | 1 | 3655 | L | 50019 | 1 | 3386 | R | 34891 | 2 | 555 | L |
| . . . | 19372 | 52886 | 4 | 2 | R | 23339 | 6 | 1 | L | 44273 | 2 | 1 | R |
| 19373 | 52707 | 13 | 11 | L | 23288 | 7 | 2 | R | 43969 | 2 | 1 | R |
| 19374 | 52529 | 9 | 7 | R | 23218 | 7 | 2 | R | 43664 | 2 | 1 | L |
| 19375 | 51447 | 3 | 1 | R | 21906 | 7 | 2 | R | 43483 | 2 | 1 | L |
| 19376 | 50932 | 4 | 2 | R | 20794 | 7 | 2 | R | 42380 | 2 | 1 | R |
| . . . | . . . | . . . | . . . | . . . | . . . | . . . | . . . | . . . | . . . | . . . | . . . | . . . |

of an unseen triplet as a combination of weighted random walks. Ref. [27] and [23] are both the combination of Markov logic network and embedding. Ref. [14] is mainly a clustering algorithm, clustering entity sets under multiple relationship categories. Ref. [8] makes use of an external text corpus to increase the connectivity of KB. The Neural LP model [25] compiles inferential tasks into differentiable numerical matrix sequences. Besides, many studies have modeled the path-finding problem as a Markov decision-making process, such as the DeepPath model [33] and MINERVÁ [4]. For the embedding methods, ref. [21] has organized the existing work. Our paper divides all embedding methods into four categories, which are: translation, Bilinear & Tensor, neural network and complex vector. Firstly, for translation, the Unstructured model [1] assumes that the head and tail entity vectors are similar without distinguishing relation types. The Structured Embedding (SE) model [3] assumes that the head and tail entities are similar only in a relation-dependent subspace. Later, there are transE, transR, transH [18, 31, 2], etc. Ref. [25] mines first-order logical rules from knowledge graphs and uses those rules to solve KBC. Additionally, other work [35, 5, 7] can extract some high-quality rules from knowledge base. For the second type, DISTMULT [34] is based on the Bilinear model [22] where each relation is represented by a diagonal matrix rather than a full matrix. SimplE [13] extends DISTMULT to allow two embeddings of each entity to be learned dependently. The third method is to implement embedding with a neural network. Apart from the models mentioned in Section 1, NTN [26] and ER-MLP [6] also belong to this method. Fourthly, instead of embedding entities and relations in real-valued vector space, ComplEx [28] is an extension of DISTMULT in the complex vector space. ComplEx-N3 [15] extends ComplEx with weighted nuclear 3-norm. Also in the complex vector space, RotatE [27] defines each relation as a rotation from the head entity to the tail entity. QuatE [36] represents entities by quaternion embeddings (i.e., hypercomplex-valued embeddings) and models relations as rotations in the quaternion space.

5 Conclusion & Future work

This paper introduces an innovative framework called EM-RBR combining embedding and rule-based reasoning, which can be easily integrated with any translation based embedding model. Besides, we propose a scalable matrix approach that puts the structured information of logic rules into the same semantic space with embedding models. Unlike previous joint models trying to get better embedding results from rules and triplets, our model allows solving completion from the reasoning perspective by conducting multi-relation path prediction. We also demonstrate that EM-RBR can efficiently improve the performance of embedding methods for KGC. Surprisingly, we show a significant improvement by using EM-RBR in the filtered Hits@1, Hits@3 and Hits@5, i.e., the evaluations focusing on front ranks. This makes the existing translation based embedding methods more suitable and reliable to be used in the real and large scale knowledge inference tasks. Potential paths for future work include extending the integrated model to reason over more various tasks related to knowledge graph and enlarging the types of rules contained in our framework.
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