Frequency mixing in a ferrimagnetic sphere resonator

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Abstract – Frequency mixing in ferrimagnetic resonators based on yttrium iron garnet (YIG) and calcium vanadium bismuth iron garnet (CVBIG) is employed respectively for studying their nonlinear interactions. The ferrimagnetic Kittel mode is driven by applying a pump tone at a frequency close to resonance. We explore two nonlinear frequency mixing configurations. In the first one, mixing between a transverse pump tone and an added longitudinal weak signal is explored, and the experimental results are compared with the predictions of the Landau-Zener-Stuckelberg model. In the second one, intermodulation measurements are employed by mixing pump and signal tones both in the transverse direction for studying a bifurcation between a stable spiral and a stable node attractors. Our results are applicable for developing sensitive signal receivers with high gain for both the radio frequency and the microwave bands.
of magnetic anisotropy is disregarded, and the sphere is treated as a macrospin. The polarization vector \( \mathbf{P} \) evolves in time \( t \) according to the Bloch-Landau-Lifshitz equation
\[
\frac{d \mathbf{P}}{dt} = \mathbf{P} \times \mathbf{\Omega} + \Gamma,
\]
where \( \mathbf{\Omega} = \gamma_e \mathbf{B} \) is the rotation vector, with \( \mathbf{B} \) being the externally applied magnetic induction and \( \gamma_e = 28 \text{GHz T}^{-1} \) being the gyromagnetic ratio, and the vector \( \Gamma = -\Gamma_1 \mathbf{P}_z \mathbf{\hat{x}} - \Gamma_2 \mathbf{P}_y \mathbf{\hat{y}} - \Gamma_2 \mathbf{P}_x \mathbf{\hat{z}} \) represents the contribution of damping, with \( \Gamma_1 = 1/\tau_1 \) and \( \Gamma_2 = 1/\tau_2 \) being the longitudinal and transverse relaxation rates, respectively, and \( P_{z,x} \) being the steady-state polarization. Consider the case where \( \mathbf{\Omega}(t) = \omega_1 \cos(\omega t) \mathbf{\hat{x}} + \sin(\omega t) \mathbf{\hat{y}} + \omega_0 \mathbf{\hat{z}} \). Here \( \omega_1 \) and \( \omega \) are both real constants, and \( \omega_0 \) (Kittel mode angular frequency) oscillates in time according to \( \omega_0 = \omega_c + \omega_h \sin(\omega_m t) \), where \( \omega_c, \omega_h \), and \( \omega_m \) are all real constants. Nonlinearity of the Bloch-Landau-Lifshitz equation gives rise to frequency mixing between the transverse driving at angular frequency \( \omega \) and the longitudinal driving at angular frequency \( \omega_m \). The resonance condition of the \( l \)-th order frequency mixing process reads \( \omega + l \omega_m = \omega_c \), where \( l \) is an integer (see appendix D of ref. [35]). The complex amplitude \( P_+ \) (in a rotating frame) of the corresponding \( l \)-th side band is given by (see appendix D of ref. [35])
\[
P_+ = \frac{\frac{\omega_1 \omega_c}{\tau_2} \left( 1 + \frac{\omega_1^2}{\tau_1^2} \right) P_{z,a} / \left( 1 + \frac{\omega_1^2}{\tau_1^2} \right)^2}{\left( \frac{\omega_1 \omega_c}{\tau_2} \right)^2 / \left( \frac{\omega_1^2}{\tau_1^2} \right)^2 + \left( \frac{\omega_1 \omega_c}{\tau_2} \right)^2}.
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Landau-Zener-Stuckelberg interferometry. – Landau-Zener-Stuckelberg interferometry is based on a mixing process between transverse and longitudinal driving frequencies that are simultaneously applied to a resonator [30,31]. In this section, for simplicity, the effect of magnetic anisotropy is disregarded, and the sphere is treated as a macrospin. The polarization vector \( \mathbf{P} \) evolves in time \( t \) according to the Bloch-Landau-Lifshitz equation
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\]

Anisotropy-induced Kerr nonlinearity. – The experimental setup used for intermodulation measurements is shown in fig. 3(a). Here the device under test (DUT2 with the ferrimagnetic resonator made of YIG) is the same

MW (RF) band. All measurements are performed at room temperature.

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In this approach, the Hamiltonian $\mathcal{H}_M$ is expressed in the form $\hbar^{-1}\mathcal{H}_M = \omega_c N_M + K_M N_M^2 + Q_M N_M^4 + \cdots$, where $\omega_c = \mu_0 \gamma_c H$ is the angular frequency of the Kittel mode [6,37], $\mu_0 = 4\pi \times 10^{-7} \text{ N} \cdot \text{A}^{-2}$ is the permeability of free space, $H$ is the externally applied uniform magnetic field (which is assumed to be parallel to the $z$-axis), $N_M$ is a number operator, $K_M$ is the so-called Kerr frequency, and $Q_M$ is the coefficient of quartic nonlinearity. When nonlinearity is taken into account to lowest nonvanishing order only, i.e., when the quartic and all higher-order terms are disregarded, the response can be described using the Duffing-Kerr model. This model predicts that the response of the system to an externally applied monochromatic driving can become bistable.

In general, the number of magnons $\langle N_M \rangle$ in a resonantly driven sphere having total linear damping rate $\gamma_c$ with pump power $P_p$ is given for the case of critical coupling by $\langle N_M \rangle \simeq P_p / (\hbar \omega_c \gamma_c)$. On the other hand, the expected number of magnons $\langle N_M \rangle$ at the onset of Duffing-Kerr bistability is $\simeq \gamma_c / K_M$ (see eq. (42) of ref. [38] and note that, for simplicity, cubic nonlinear damping is disregarded). Thus, from the measured values of the linear damping rate $\gamma_c/(2\pi) \simeq 1\text{ MHz}$ and $P_p \simeq 1\text{ mW}$, at the bistability onset point one obtains $K_M/(2\pi) \simeq -2 \times 10^{-9} \text{ Hz}$ (the minus signs indicates that the Kerr nonlinearity gives rise to softening). Note, however, that the above estimate, which is based on the Duffing-Kerr model, is valid provided that the quartic and all higher-order terms can be disregarded near (and below) the bistability onset. For the quartic term this condition can be expressed as $|Q_M| \ll |K_M|^3 / \gamma_c^2$.

The values of $K_M$ and $Q_M$ are estimated below for the case where nonlinearity originates from magnetic anisotropy. The Stoner-Wohlfarth energy $E_{SW}$ is expressed as a function of the magnetization vector $\mathbf{M} = M \mathbf{\hat{u}}_M$, and the first-order $K_{c1}$ and second-order $K_{c2}$ anisotropy constants as [39]

$$E_M = -\mu_0 \mathbf{M} \cdot \mathbf{H} + K_{c1} \sin^2 \phi + K_{c2} \sin^4 \phi,$$

(2)

where $V_s = 4\pi R_s^3 / 3$ is the volume of the sphere having radius $R_s$, and $\phi$ is the angle between $\mathbf{u}_M$ and the unit vector $\mathbf{\hat{u}}_s$ parallel to the easy axis. It is assumed that the sphere is fully magnetized, i.e., $|\mathbf{M}| \simeq M_s$, where $M_s$ is the saturation magnetization. In terms of the dimensionless angular momentum vector $\mathbf{\Sigma} = -2M V_s / (\hbar \gamma_c) \equiv (\Sigma_x, \Sigma_y, \Sigma_z)$ eq. (2) is rewritten as $E_M = h \omega_{K1} (1 + K_{c2} / K_{c1}) = \mathcal{H}_M$, where

$$\hbar^{-1}\mathcal{H}_M = \frac{\omega_c \Sigma_z}{2} + \left(1 + \frac{2K_{c2}}{K_{c1}}\right) \frac{K_M (\mathbf{\Sigma} \cdot \mathbf{\hat{u}}_A)^2}{4} + \frac{K_{c2} K_M^2 (\mathbf{\Sigma} \cdot \mathbf{\hat{u}}_A)^4}{16\omega_{K1}},$$

(3)

$\omega_{K1} = h^{-1} V_s K_{c1}$ and $K_M = h \gamma_c^2 K_{c1}/(V_s M_s^2)$ is the Kerr frequency [20].

Fig. 3: Intermodulation. (a) An intense pump and a relatively weak signal are simultaneously injected into the MW loop antenna, and the reflected signal is measured using a spectrum analyzer. (b) Experimentally obtained hysteretic resonance curve (with no signal) showing bistability corresponding to the forward (red arrow) and backward (blue arrow) microwave frequency sweep directions (see fig. 3(b)). The measured response curve, which is obtained via the forward and backward microwave frequency sweep directions. (c) Spectrum analyzer measurement of the reflected signal intensity as a function of the detuning frequency with respect to the pump frequency. Frequency mixing between the input pump tone at angular frequency $\omega_p$ and the input signal tone at angular frequency $\omega_s$ gives rise to output idler tones. The angular frequency of the $l$-th order idler tone is given by $\omega_p + l\omega_{id}$, where $l \neq 1$ is an integer ($l = -1$ for the primary idler tone).

as that shown in fig. 1, where the RF antenna (RFA) is removed from the setup. The nonlinearity gives rise to bistability, which, in turn, yields a hysteretic resonance curve, which is obtained via the forward and backward sweeping directions (see fig. 3(b)). The measured response becomes bistable when the input pump power $P_p$ is of the order of mW. The subsequent idler tones generated due to the nonlinear frequency mixing of pump and signal tones in the ferrimagnetic resonator are shown in fig. 3(c).

Nonlinearity in the response of ferrimagnetic sphere resonators is reviewed below. Both the Kerr coefficient and the coefficient of quartic nonlinearity are estimated for the case where nonlinearity originates from magnetic anisotropy. The Kerr nonlinearity is expected to give rise to bistability. The values of the nonlinear coefficients allow estimating the effect of quartic nonlinearity near the bistability onset. As is explained below this effect is expected to be small, though, non-negligible.

The technique of bosonization can be applied to model the nonlinearity in ferrimagnetic sphere resonators [36].
In the Holstein-Primakoff transformation [40], the operators $\Sigma_\pm = \Sigma_+ \pm i\Sigma_-$ and $\Sigma_3$ are expressed as $\Sigma_\pm = B^\dagger(N_\pm - N_\mp i)/2$, $\Sigma_3 = (N_+ - N_-)/2B$ and $\Sigma_3 = -N_3 + 2N_3M$, where $N_\pm$ is the total number of spins, and where $B^\dagger B = N_3$ is a number operator. If the operator $B$ satisfies the bosoncic relation $[B, B^\dagger] = 1$, then the following holds: $[\Sigma_2, \Sigma_3] = 2\Sigma_2$, $[\Sigma_2, \Sigma_\pm] = -2\Sigma_\pm$ and $[\Sigma_+, \Sigma_-] = \Sigma_2$. The approximation $(N_3M/N_\pm) \ll N^1/2_\pm$ leads to $\Sigma = \Sigma_2 = N^1/2_\pm(B^\dagger uA_\pm + B^\dagger uA_{-}) + 2N^2M$, where $u_{A\pm} = [(v_\Lambda \cdot \hat{\mathbf{x}} \pm i(v_\Lambda \cdot \hat{\mathbf{y}})]/2$, $u_{A_\pm} = \vec{u}_\Lambda \cdot \hat{\mathbf{z}}$, and the magnon number operator $N_3M$ is defined by $N_3M = N^2_\pm - N/2$. This approximation is valid near the bistability onset provided that $\gamma_c/(|K_3M|N_\pm) \ll 1$. For YIG, the spin density is $\rho_s = 4.2 \times 10^{21}$ cm$^{-3}$, thus for a sphere of radius $R_\rho = 125 \mu$m the number of spins is $N_\pm = V_\rho \rho_s = 3.4 \times 10^{16}$, hence for the current experiment $\gamma_c/(|K_3M|N_\pm) \approx 10^{-1}$. This estimate suggests that inaccuracy originating from this approximation may be significant for the current experiment near and above the bistability threshold.

Second-order anisotropy gives rise to a quartic nonlinear term in the Hamiltonian (3) with a coefficient $Q_3 \approx (K_{22}/K_{11})|\xi_2/\omega_\xi|K_{11}$ (the exact value depends on the angle $\phi$ between the magnetization vector and the easy axis). Near or below the bistability onset the quartic term can be safely disregarded provided that $(K_{22}/K_{11})|\xi_2/\omega_\xi|K_{11}) < 1$. When this condition is satisfied, the Hamiltonian (3) for the case where $\xi$ is parallel to $\hat{\mathbf{z}}$ (i.e., $u_{A_\pm} = 1$ and $u_{A_\pm} = 0$) approximates becomes

$$\hbar^2 H_M = \omega_c N_3 M + K_3 N^2_3 M.$$  

The term proportional to $K_3M$ represents the anisotropy-induced Kerr nonlinearity.

For YIG $M_s = 140$ kA/m, $K_{11} = -610$ J/m$^3$ at 297 K (room temperature), hence for a sphere of radius $R_\rho = 125 \mu$m the expected value of the Kerr coefficient is given by $K_3M/(2\pi) = -2 \times 10^{-9}$ Hz. This value well agrees with the above estimation of $K_3M/(2\pi)$ based on the measured input power at the bistability onset. For YIG $K_{22}/K_{11} = 4.8 \times 10^{-2}$ ($K_{22}/K_{11} = 4.3 \times 10^{-2}$) at a temperature of $T = 4.2$ K ($T = 294$ K) [6]. Based on these values one finds that for the sphere resonators used in the current experiment $(K_{22}/K_{11})|\xi_2/\omega_\xi|K_{11}) \approx 10^{-6}$, hence the second-order anisotropy term (proportional to $K_{22}$) in eq. (3) can be safely disregarded in the vicinity of the bistability onset.

**Stable spiral and stable node.** The technique of intermodulation is commonly employed for studying nonlinear systems. In this section we analyze the intermodulation conversion gain of a resonator having Kerr nonlinearity, and find that the gain can be obtained from the linearized equations of motion. The analysis is mainly focused on bifurcations between different types of fixed points, and the experimental detection of these bifurcations using intermodulation measurements.

To explore the regime of weak nonlinear response, consider a resonator being driven by a monochromatic pump tone having amplitude $b_0$ and angular frequency $\omega_p$. The time evolution in a frame rotating at the pump driving frequency is assumed to have the form

$$\frac{dC}{dt} + \Theta_C = F_C,$$

where the operator $C$ is related to the resonator’s anhilation operator $A_k$ by $C_k = A_k e^{i\omega_p t}$, the term $\Theta_C = \Theta_1(C_2, C_1^*)$, which is expressed as a function of both $C_2$ and $C_1^*$, is assumed to be time independent, and $F_C$ is a noise term having a vanishing expectation value. The complex number $B_k$ represents a fixed point, for which $\Theta_1(B_k, B_k^*) = 0$. By expressing the solution as $C_k = B_k + c_k$, and considering the operator $c_k$ as small, one obtains a linearized equation of motion from eq. (5) given by

$$\frac{dc}{dt} = W_1 c_1 + W_2 c_1^* = F_c,$$

where $W_1 = \partial \Theta_1/\partial C_1$ and $W_2 = \partial \Theta_1/\partial C_1^*$ (both derivatives are evaluated at the fixed point $C_k = B_k$).

The stability properties of the fixed point depend on the eigenvalues $\lambda_1$ and $\lambda_2$ of the $2 \times 2$ matrix $W$, whose elements are given by $W_{11} = W_{22} = W_1$ and $W_{12} = W_{21} = W_2$ (see eq. (6)). In terms of the trace $T_W = W_1 + W_2$ and the determinant $D_W = |W_1|^2 - |W_2|^2$ of the matrix $W$, the eigenvalues are given by $\lambda_1 = T_W/2 + i\nu_W$ and $\lambda_2 = T_W/2 - i\nu_W$, where the coefficient $\nu_W$ is given by $\nu_W = \sqrt{(T_W/2)^2 - D_W}$. Note that in the linear regime, i.e., when $\nu_W = 0$, the eigenvalues become $\lambda_1 = W_1$ and $\lambda_2 = W_1^*$. For the general case, when both $\lambda_1$ and $\lambda_2$ have a positive real part, the fixed point is locally stable. Two types of stable fixed points can be identified. For the so-called stable spiral, the coefficient $\nu_W$ is purely imaginary (i.e., $(T_W/2)^2 - D_W < 0$), and consequently $\lambda_2 = \lambda_1^*$, whereas both $\lambda_1$ and $\lambda_2$ are purely real for the so-called stable node, for which $\nu_W$ is pure real. A bifurcation between a stable spiral and a stable node occurs when $\nu_W$ vanishes.

Further insight can be gained by geometrically analyzing the dynamics near an attractor. To that end the operators $c_k$ and $F_C$ are treated as complex numbers. The equation of motion (6) for the complex variable $c_k$ can be rewritten as $d\xi/dt = W^\dagger \xi$, where $\xi = (\text{Real}(c_k e^{i\phi}), \text{Imag}(c_k e^{i\phi}))^T$ and $\vec{f} = (\text{Real}(F(e^{i\phi})), \text{Imag}(F(e^{i\phi})))^T$ are both two-dimensional real vectors, and where the rotation angle $\phi$ is real. Transformation into the so-called system of principle axes is obtained when the angle $\phi$ is taken to be given by $e^{2i\phi} = W_1 W_2^*/|W_1| W_2|$. For the case the $2 \times 2$ real matrix $W'$ becomes

$$W' = \begin{pmatrix} \cos \theta_1 & -\sin \theta_1 \\ \sin \theta_1 & \cos \theta_1 \end{pmatrix} \begin{pmatrix} W_1 & 0 \\ 0 & W_2 \end{pmatrix},$$

where $\theta_1 = \text{arg}(W_1)$ and where $W_\pm = |W_1| \pm |W_2|$. Thus, multiplication by the matrix $W'$ can be interpreted for
this case as a squeezing with coefficients \( W' \pm \) followed by a rotation by the angle \( \theta_1 \).

The flow near an attractor is governed by the eigenvectors of the \( 2 \times 2 \) real matrix \( W' \). For the case where \( \nu_W \) is purely real the angle \( \alpha_W \) between these eigenvectors is found to be given by \( \sin \alpha_W = \nu_W / |W'| \). Thus, at the bifurcation between a stable spiral and a stable node, i.e., when \( \nu_W = 0 \), the two eigenvectors of \( W' \) become parallel to one another. In the opposite limit, when \( \nu_W = |W'| \), i.e., when \( W_1 \) becomes real, and consequently the matrix \( W \) becomes Hermitian, the two eigenvectors become orthogonal to one another (i.e., \( \alpha_W = \pi/2 \)). Flow maps near different types of attractors, including a stable spiral and a stable node, are depicted by figs. 14–17 of ref. [41].

The bifurcation between a stable spiral and a stable node can be observed by measuring the intermodulation conversion gain \( G_1 \) of the resonator. This is done by injecting another input tone (in addition to the pump tone), which is commonly referred to as the signal, at angular frequency \( \omega_s + \omega_d \). The intermodulation gain is defined by

\[
G_1(\omega_{ds}) = |g_i(\omega_{ds})|^2,
\]

where \( g_i(\omega_{ds}) \) is the ratio between the output tone at angular frequency \( \omega_s - \omega_d \), which is commonly referred to as the idler, and the input signal at angular frequency \( \omega_p + \omega_d \). In terms of the eigenvalues \( \lambda_{c1} \) and \( \lambda_{c2} \) the gain \( G_1 \) is given by

\[
G_1 = \left| \frac{2\gamma_{c1}W_2}{(\lambda_{c1} - i\omega_{ds})(\lambda_{c2} - i\omega_{ds})} \right|^2, \tag{8}
\]

where \( \gamma_{c1} \) is the coupling coefficient (in units of rate) between the feed line that is used to deliver the input and output signals and the resonator. For the case of a stable spiral, i.e., when \( \lambda_{c2} = \lambda_{c1}^2 \), one has \(|(\lambda_{c1} - i\omega_{ds})(\lambda_{c2} - i\omega_{ds})|^2 = |\lambda' - (\lambda'' - i\omega_{ds})|^2|\lambda'' + (\lambda'' + i\omega_{ds})|^2\), where \( \lambda' = \text{Re}\lambda_{c1} \) and \( \lambda'' = \text{Im}\lambda_{c1} \) (i.e., \( \lambda = \lambda' + i\lambda'' \)), whereas for the case of a stable node, i.e., when both \( \lambda_{c1} \) and \( \lambda_{c2} \) are purely real, one has \(|(\lambda_{c1} - i\omega_{ds})(\lambda_{c2} - i\omega_{ds})|^2 = (\lambda_{c1}^2 + \omega_{ds}^2)(\lambda_{c2}^2 + \omega_{ds}^2)\).

For the case of a resonator having Kerr nonlinearity and cubic nonlinear damping \( \Theta_c \), is given by \( \Theta_c = |i\Delta_c + \gamma_c + (iK_c + \gamma_{c3})N_c|C_c + i\sqrt{2\gamma_{c1}}e^{i\phi_{c1}}b_c \), where \( \Delta_c = \omega_c - \omega_p \) is the driving detuning, the total rate of linear damping is \( \gamma_c = \gamma_{c1} + \gamma_{c2} \), the rate \( \gamma_{c1} \) characterizes the coupling coefficient between the feed line and the resonator, \( \gamma_{c2} \) is the rate of internal linear damping, \( \gamma_{c3} \) is the rate of internal cubic damping, \( K_c \) is the Kerr coefficient, \( N_c = A'_cA_c \) is the resonator number operator, and \( \phi_{c1} \) is a phase coefficient characterizing the coupling between the feed line and the resonator [38]. The rates \( W_1 \) and \( W_2 \) are given by

\[
W_1 = i\Delta_c + \gamma_c + 2(iK_c + \gamma_{c3})B_c^2, \quad W_2 = (iK_c + \gamma_{c3})B_c^2.
\]

The condition \( \Theta_c(B_c, B_c^*) = 0 \) can be expressed as a cubic polynomial equation for the number of magnons \( E_z = |B_c|^2 \), given by

\[
|E_z| + K_cE_z^2 + (\gamma_c + \gamma_{c3}E_z)^2|E_z| = 2\gamma_{c1}|b_c|^2.
\]

The eigenvalues can be expressed in terms of \( E_z \), as \( \lambda_{c1,2} = T_W/2 \pm i\nu_W \), where

\[
T_W/2 = \gamma_c + 2\gamma_{c3}E_z \quad \text{and} \quad \nu_W = \sqrt{(\Delta_c - \Delta_{c1})(\Delta_c + \Delta_{c1})},
\]

where \( \Delta_{c1} = (\sqrt{1 + (\gamma_{c3}/K_c)^2 \pm 2K_cE_z} \). The stability map of the system is shown in fig. 4. Both driving detuning \( \Delta_c \) and driving amplitude \( b_c \) are normalized with the corresponding values at the bistability onset point (BOP) \((\Delta_{c1,BOP}, b_{c,BOP})\) (see eqs. (46) and (47) of ref. [38]). Inside the regions “C” and “R” of mono-stability (“CC,” “CR” and “RR” of bistability) the resonator has one (two) locally stable attractors. A stable spiral (node), for which \( \lambda_{c2} = \lambda_{c1}^2 \) (both \( \lambda_{c1} \) and \( \lambda_{c2} \) are purely real), is labeled by “C” (“R”).

In the bistable region, the cubic polynomial equation has 3 real solutions for \( E_z \). The corresponding values of the complex amplitude \( B_c \) are labeled as \( C_1 \), \( C_2 \) and \( C_3 \). In the flow map shown in fig. 5, which is obtained by numerically integrating the equation of motion (5) for the noiseless case \( E_z = 0 \), the point \( C_1 \) is a stable node, the point \( C_2 \) is a saddle point and the point \( C_3 \) is a stable spiral. The red and blue lines represent flow toward the stable node attractor at \( C_1 \) and the stable spiral attractor at \( C_3 \), respectively. The green line is the separatrix, namely the boundary between the basins of attraction of the attractors at \( C_1 \) and \( C_3 \). A closer view of the region near \( C_1 \) and \( C_2 \) is shown in fig. 5(b).

The intermodulation conversion gain \( G_1 \) induced by the Kerr nonlinearity is measured with the ferrimagnetic resonator DUT (see fig. 3(a)), and the results are compared with the theoretical prediction given by eq. (8). In these measurements the pump frequency \( \omega_p \) is tuned close to the resonance frequency \( \omega_c \). The measured gain

Fig. 4: Stability map of a driven Duffing-Kerr resonator (the case \( K_c > 0 \) is assumed). The BOP is the point \((\Delta_{c1,BOP}, b_{c,BOP}) = (-1, 1) \). In both “C” (dark blue) and “R” (light blue) regions, there is a single locally stable attractor, whereas there are two in the regions “CC” (light green), “CR” (orange) and “RR” (red). The letter “C” is used to label a stable spiral, whereas the letter “R” labels a stable node.
$G_1$ is shown in the color-coded plots in fig. 6 (for three different values of the pump frequency $\omega_p$) as a function of the detuning between the signal and pump frequencies $\omega_{ds}/(2\pi)$ and the pump power $P_p$

The overlaid black dashed lines in fig. 6 indicate the calculated values of the imaginary part of the eigenvalues $\lambda'' = \text{Im}\lambda_{c1}$ and $-\lambda'' = \text{Im}\lambda_{c2}$. The calculation is based on the above-discussed Duffing-Kerr model. At the point where $\lambda''$ vanishes, a bifurcation from stable spiral to stable node occurs. As can be seen from comparing panels (a), (b) and (c) of fig. 6, the pump power $P_p$ at which this bifurcation occurs depends on the pump frequency $\omega_p$. This bifurcation represents the transition between the regions “CC” and “CR” in the stability map shown in fig. 4. A bifurcation from the bistable to the monostable regions occurs at a higher value of the pump power $P_p$. This bifurcation gives rise to the sudden change in the measured response shown in fig. 6. In the stability map shown in fig. 4, this bifurcation corresponds to the transition between the regions “CR” and “C”.

**Conclusion.** – We present two nonlinear effects that can be used for signal sensing and amplification. The first one is based on the so-called Landau-Zener-Stuckelberg process [30] of frequency mixing between transverse and longitudinal driving tones that are simultaneously applied to the magnon resonator. This process can be employed for frequency conversion between the RF and the MW bands. The second nonlinear effect, which originates from magnetization anisotropy, can be exploited for developing intermodulation receivers in the MW band. Measurements of the intermodulation response near the onset of the Duffing-Kerr bistability reveal a bifurcation between a stable spiral attractor and a stable node attractor. Above this bifurcation, i.e., where the attractor becomes a stable node, the technique of noise squeezing can be employed in order to enhance the signal-to-noise ratio [38].

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Fig. 5: Flow map of a Duffing oscillator in the region of bistability. The point $C_1$ is a stable node, the point $C_2$ is a saddle point, and the point $C_3$ is a stable spiral. A closer view of the region near $C_1$ and $C_2$ is shown in (b).

Fig. 6: Intermodulation gain $G_I$ as a function of detuning between the signal and pump frequencies $\omega_{ds}/(2\pi)$ and pump power $P_p$ (in dBm units). The plots in the top raw (a)–(c) exhibit the measured gain and the plots in the bottom raw (d)–(f) exhibit the calculated gain (according to eq. (8)). The pump frequency $\omega_p/(2\pi)$ is: (a), (d) 3.8674 GHz; (b), (e) 3.8704 GHz; and (c), (f) 3.8734 GHz. The signal power is $-15$ dBm. Note that $G_I$ is measured with $\omega_{ds} > 0$ only, and the plots are generated by mirror reflection of the data around the point $\omega_{ds} = 0$. Note also that for clarity the region near the pump frequency, i.e., close to $\omega_{ds} = 0$, has been removed from the plot. The width of this region, in which the intense pump peak is observed, depends on the resolution bandwidth setting of the spectrum analyzer. The black dashed lines indicate the calculated values of the imaginary part of the eigenvalues $\lambda'' = \text{Im}\lambda_{c1}$ and $-\lambda'' = \text{Im}\lambda_{c2}$. The pump amplitude $b_c$ and pump detuning $\Delta_c$ used for the calculation of the eigenvalues are determined from the measured value of $P_p = 0.4$ dBm for the pump power and the value of $\Delta_c/(2\pi) = 1.3$ MHz for the pump detuning at the BOP. The region where two symmetric branches of the black dashed line are shown corresponds to the region of stable spiral, for which $|\lambda''| > 0$. 

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