Effects of Jamming Attacks on Wireless Networked Control Systems Under Disturbance

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Abstract—Jamming attacks on wireless networked control systems are investigated for the scenarios where the system dynamics face exogenous disturbance. In particular, the control input packets are assumed to be transmitted from a controller to a remotely located linear plant over an insecure wireless communication channel that is subject to jamming attacks. The time-varying likelihood of transmission failures on this channel depends on the power of the jamming interference signal emitted by an attacker. We show that jamming attacks can prevent stability when the system faces disturbance, even if the attacked system without disturbance is stable. We also show that stability under jamming and disturbance can be achieved if the average jamming interference power is restricted in a certain way that we characterize in the paper. We illustrate our results on an example networked control system with a fading wireless channel, where the outage probability is affected by jamming attacks.

Index Terms—Networked control, cyber-security, wireless networks, jamming interference, disturbance

I. INTRODUCTION

As the Internet of Things is gaining popularity, the use of wireless communication channels and the Internet is increasing in remote control applications. These communication technologies are easy to set up and they provide efficiency in the transmission of measurement and control data, but they can create major cyber-security issues. In the framework of cyber-physical systems, researchers have identified a range of potential cyber attacks with different properties [2]–[4]. For instance, an attacker who is knowledgeable about the system dynamics can disrupt control operation by injecting false data into the system or altering measurement and control data [5], [6]. Attackers with limited information can also cause cyber-security issues by means of denial-of-service (DoS) attacks to prevent communication over networks. For example, a jamming attacker can effectively prevent transmission of packets over wireless channels by emitting sufficiently strong interference signals, [7]. Jamming attacks may result in performance issues and instability in wireless networked control systems.

The effects of jamming and other DoS attacks in control systems have recently been investigated (see [8] for an overview). In those works, various attack models have been considered. For instance, [9] considered a model where the attacker conducts cycles of sleeping and jamming in a repetitive fashion. Moreover, the works [10]–[13] considered models that allow the timing of attack strategies to be arbitrary as long as the average attack duration and the average frequency of attacks satisfy certain bounds. It was first observed in [10] that when a control system is subject to disturbance, duration and frequency conditions for attacks need to be stronger to guarantee stability in comparison to the case without disturbance.

In this paper, our goal is to investigate the effects of jamming attacks specifically for wireless networked control problems that are subject to disturbance. In particular, we consider the control problem over a wireless channel, where the transmission failure model can be characterized through the time-dependent Signal-to-Interference-plus-Noise-Ratio (SINR), which is the ratio of the transmission power of the signal to the jamming attacker’s interference power summed with the channel noise power. We consider channel models explored in the wireless communications literature [14], [15]. In those models, the effect of SINR on transmission failures is described through probabilistic relations. A jamming signal with a strong interference power results in a smaller SINR, which ends up increasing the likelihood of a transmission failure. For instance, in wireless channels with fading, small SINR increases the so-called outage probability, as explored in [16]–[18]. Jamming can affect practical wireless communication networks. For instance, effects of jamming attacks on SINR, packet decoding errors, and failures in IEEE 802.11 communication networks are investigated through experiments in [19].

Previously, SINR-based channel models were used by [20]–[22] for game-theoretic analysis of remote state estimation problems under jamming attacks. Moreover, in [23], a probabilistic channel model was considered in a networked control problem setting and optimal attack policies were explored for the case where the total number of attacks in a fixed interval is bounded. In [24], we used an SINR-based probabilistic model to investigate a discrete-time networked stabilization problem for scenarios where there is no disturbance, but a jamming attacker can jam the wireless channel at each time instant with a different interference power level that is unknown a priori. Our results in [24] indicate that stabilization can be achieved if the average interference power is bounded in the long run even if the power can be very large at certain times.

In this paper, we consider scenarios where the jamming attacker can potentially change the interference power levels at each time, as in [24]. However, differently from [24], we now consider disturbance, and through stochastic analysis, we show that when the dynamics is subject to disturbance, jamming attacks can potentially become more dangerous. Our results indicate that a strategic attacker may take advantage of the disturbance to cause instability even if the attacked system without disturbance is stable. Specifically, the attacker can cause the state norm to grow to arbitrarily large values with arbitrarily high probabilities, while keeping the average jamming interference power below a threshold in the long run. Thus, as in the deterministic case discussed in [10], a restriction is also needed in this paper. We consider a probabilistic model and the attacker can only partially affect the occurrence probability of a transmission failure. We show that when jamming attacks are restricted so that the wireless channel is not subject to long consecutive emissions of high powered interference signals, then the first moment of the state stays bounded. Interestingly, even under such restrictions, the wireless channel may be attacked at all time instants with small interference powers and thus for any finite interval, there is always a positive probability that all transmission attempts may fail. In this aspect, our setting differs from the deterministic case, where the maximum possible length of
is successful, then the transmitted control command is applied at the plant side. If, on the other hand, there is a transmission failure, then the control input at the plant side is set to 0. In this setting, the dynamics of the plant is given by

\[ x(t + 1) = Ax(t) + (1 - l(t))Bu_C(t) + w_P(t), \]

(1)

where \( x(t) \in \mathbb{R}^n \) is the state, \( u_C(t) \in \mathbb{R}^m \) is the control command that is attempted to be transmitted by the controller to the plant at time \( t \), \( w_P(t) \in \mathbb{R}^n \) is the disturbance, and \( l(t) \in [0, 1] \) represents the transmission status (with \( l(t) = 1 \) indicating failure and \( l(t) = 0 \) indicating success). Moreover, \( A \in \mathbb{R}^{n \times n} \) is the unstable system matrix and \( B^{n \times m} \) is the input matrix.

In this paper, we investigate the networked stabilization of the plant (1) through a state-feedback controller, where the control command transmitted by the controller is given by

\[ u_C(t) = Kx(t) + w_C(t), \quad t \in \mathbb{N}_0, \]

(2)

where \( K \in \mathbb{R}^{n \times n} \) denotes the feedback gain, and \( w_C(t) \in \mathbb{R}^m \) is used for describing disturbances on the control command.

A. Closed-Loop System Dynamics

With \( w(t) \triangleq w_P(t) + (1 - l(t))Bu_C(t) \), the closed-loop networked control system (1), (2) becomes

\[ x(t + 1) = Ax(t) + (1 - l(t))BKx(t) + w(t), \quad t \in \mathbb{N}_0. \]

(3)

The vector \( w(t) \) in (3) represents the overall disturbance in the control system dynamics and it is not related to the jamming signal emitted by the attacker. In our problem setting, the jamming action affects the probability of successful/failed delivery of control commands (as we will explain below more precisely). In this sense, our problem setting is similar to those in [10], [20], [22], which involve DoS attacks causing packet losses. We note that there are other problem settings in the literature, where the notion of “jamming” is used for describing the noise on the transmitted data (see, e.g., [29]–[31]); there, the received data is the sum of the original data and the jamming noise. In contrast, in our work, when there is a successful delivery, the control command \( u_C(t) \), which is transmitted from the controller, is assumed to be received by the plant (and applied as an input) without any change.

Remark 2.1: In this paper, we present our results in terms of the overall disturbance \( w(t) \) in (3), which includes exogenous disturbances on the plant modeled with \( w_P(t) \), as well as potentially network-related disturbances on the controller modeled with \( w_C(t) \). For the scenarios where the state measurement is noisy, the effects of noise can also be represented through the process \( w_C(t) \). In such cases, the control command is given by \( K \tilde{x}(t) \), where \( \tilde{x}(t) = x(t) + \eta(t) \) is the measured state and \( \eta(t) \in \mathbb{R}^n \) represents the measurement noise. This situation is represented through (3) by setting \( w_C(t) \triangleq K\eta(t) \). We also note that in our analysis, \( w(t) \) is considered as a stochastic process. However, we do not assume to know its distribution.

Remark 2.2: While we derived the closed-loop system (3) using a static state-feedback controller, the form of the dynamics in (3) also allows representing closed-loop systems under other control architectures. For instance, one can consider a dynamic controller

\[ x_C(t + 1) = A_Cx_C(t) + B_Cx(t), \]

\[ u_C(t) = C_Cx_C(t) + D_Cx(t) + w_C(t), \]

where \( x(t) \) is the state of the plant (1), \( x_C(t) \in \mathbb{R}^{n_C} \) is the internal state of the controller, \( u_C(t) \in \mathbb{R}^m \) is the control command transmitted from the controller, and \( A_C, B_C, C_C, D_C \) are matrices that characterize the controller’s dynamics. The closed-loop system
under this dynamic controller can be described by an equation similar to (3). Specifically, by setting $\tau(t) = [x^T(t), w^T_C(t)]^T$, we have
\[
\tau(t + 1) = \tau(t) + (1 - l(t))B \kappa \tau(t) + \kappa(t),
\]
where $\tau(t) = [w^T_C(t) + (1 - l(t))(Bw_C(t))^T, 0_{1 \times n_C}]^T$ and
\[
A = \begin{bmatrix} 0 & 1 \\
B & 0
\end{bmatrix}, \quad B = \begin{bmatrix} B \\
0_{n_C \times m}
\end{bmatrix}, \quad K = \begin{bmatrix} D_C & C_C
\end{bmatrix}.
\]
Output-feedback controllers can also be described similarly. We note that dynamic controllers are shown to be advantageous in anytime-control frameworks and soft real-time control systems.

Equation (1) represents the setting where the input of the plant is set to 0 whenever there is a transmission failure. Similarly, we can consider the setting where the plant uses the previous input value if there is a failure. In that case, the plant dynamics is given by
\[
x(t + 1) = Ax(t) + Bv(t)l(t) + (1 - l(t))Bw_C(t) + w_I(t),
\]
where $v(t) \in \mathbb{R}^m$ represents the last control command that was successfully transmitted from the controller. In the case of the state-feedback controller (2), we can let $\tau(t) = [x^T(t), w^T_C(t)]^T$ and describe the dynamics of the closed-loop system using $\hat{A}$, where
\[
\hat{A} = \begin{bmatrix} A & B \\
0_{m \times n_c} & I_m
\end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B \\
0_{n_c \times m}
\end{bmatrix}, \quad \hat{K} = \begin{bmatrix} K & -I_m
\end{bmatrix},
\]
with $I_m$ denoting the identity matrix in $\mathbb{R}^{m \times m}$ and $\tau(t) = [w^T_C(t) + (1 - l(t))(Bw_C(t))^T, (1 - l(t))w^T_I(t)]^T$.

### B. Transmission Failure Model

In our problem setting, the likelihood of a transmission failure depends on the power of the jamming interference. If the interference power is large, then a transmission failure may likely occur. In particular, with $\phi(t) \in [0, \infty)$ denoting the interference power at time $t$, the transmission failure indicator $l(t)$ in (1) is given by
\[
l(t) = I[r(t) \leq p(\phi(t))], \quad t \in \mathbb{N}_0,
\]
where $p : [0, \infty) \to [0, 1]$ is a Borel-measurable, nondecreasing function, and $r(0), r(1), \ldots$ are independent random variables that are distributed uniformly in $[0, 1]$. Furthermore, $\{r(t) \in [0, 1] : t \in \mathbb{N}_0$ and $\{\phi(t) \in [0, \infty) : t \in \mathbb{N}_0$ are assumed to be mutually independent processes. Notice that for a fixed scalar $\phi$, we represent by $p(\phi)$ the conditional probability of a transmission failure given that the jamming interference power is set to $\phi$. In particular, (5) implies
\[
\mathbb{P}[l(t) = 1|\phi(t) = \phi] = \mathbb{P}[r(t) \leq p(\phi)|\phi(t) = \phi] = \int_0^p \mathbb{P}[r(t) \leq \phi|\phi(t) = \phi] \, d\phi.
\]
Observe that, if $\phi(t)$ is large so that $p(\phi(t))$ is close to 1, then it becomes more likely that $r(t) \leq p(\phi(t))$, and hence by (5), a transmission failure is likely to occur. We note that the attacker controls the power level $\phi(t)$ of jamming signals, but not the jamming signals themselves.

Note also that transmission failures at different times are conditionally independent given the interference powers at those times. Namely, for every $t_1 < t_2 < \cdots < t_k, k \in \mathbb{N}$,
\[
\mathbb{P}[l(t) = 1, \ldots, l(t_k) = 1|\phi(t_1) = \phi_1, \ldots, \phi(t_k) = \phi_k] = \prod_{i=1}^k \mathbb{P}[l(t_i) = 1|\phi(t_i) = \phi_i] = \prod_{i=1}^k p(\phi_i).
\]

The characterization in (5) enables us to describe security properties of different wireless channel models, as illustrated below.

**Example 2.1 (Outage probability):** The function $p$ can be used for describing the outage probability in wireless channels with fading. Outage occurs when the SINR at the receiver side (the plant in this paper) goes below a threshold due to fading (see Section 14.2 in [14] and Section 12.2.3 in [15]). Outage probability has been used in different problem settings that involve jamming attacks [16, 17]. Here we present two examples. First, in the case of a Rayleigh-fading channel considered in [17], the outage probability is given by
\[
p(\phi) = 1 - e^{-\gamma(1/2b_1^2)/(2b_2^2)},
\]
where $\gamma \in (0, \infty)$ and $\sigma \in (0, \infty)$ are constants associated respectively with the transmission power and the power of the channel noise. The scalars $b_1, b_2 \in (0, \infty)$ depend on the distances of the jamming attacker and the controller from the plant. They are constant in our setup, since the geographical locations of the jamming attacker, the controller, and the plant are fixed. The scalar $\gamma$ represents the SINR-threshold. As the second example, we can also investigate the approximate outage probability considered in [16]–[18] by setting
\[
p(\phi) = 1 - e^{-2\gamma/\phi},
\]
where $\gamma = \frac{b_2}{b_1^2} \phi$ is the SINR and $\gamma$ is its threshold for outage. The scenarios in [16], [18] involve moving transmitters and interference sources. Our setup is closer to [17] in that the jamming attacker is not mobile, but capable of changing the power of emitted interference.

**Example 2.2:** Additive white Gaussian noise channel models considered in [21, 22] can be represented by appropriately choosing $p$. For instance, a special case of the model in [21] with fixed channel gains can be represented with
\[
p(\phi) = 1 - (1 - Q(\sqrt{2s}(\phi + s)))^L,
\]
where $Q(y) \triangleq \frac{1}{\sqrt{2\pi}} \int_y^\infty e^{-x^2/2} \, dx$, $L \in \mathbb{N}$ denotes the length of packet being transmitted, and the positive constants $\xi$ and $\sigma$ respectively denote the transmission and the channel noise powers.

The transmission failure probability function $p$ in (5) plays an important role in the analysis presented in the next section. In particular, if $p$ is a concave function and the average power of jamming interference is upper-bounded by a scalar $\bar{c}$ (as we explain later), then $p(\bar{c})$ can be used in the stability analysis as an upper bound on the long-run average number of transmission failures (i.e., $\lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{n-1} l(i) \leq p(\bar{c})$, almost surely). If $p$ is not concave, then a concave function that upper-bounds $p$ can be used for the same purpose. To this end, in this paper we use a continuous, nondecreasing, and concave function $\hat{p} : [0, \infty) \to [0, 1]$ such that
\[
\hat{p}(\phi) \geq p(\phi), \quad \phi \in [0, \infty).
\]
Notice that such a function $\hat{p}$ always exists. The work [24] discusses methods of finding tight concave upper-bounding functions $\hat{p}$. Furthermore, in the case of the transmission failure probability functions in (6) and (7) from Example 2.1, it suffices to choose $\hat{p}$ same as $p$, since in both cases $p$ is continuous, nondecreasing, and concave (by having a nonpositive second derivative).

### III. Analysis of Networked Stabilization

In this section, we first provide a quick look at the stability of networked control system (3) in the disturbance-free case. Then we discuss how a strategic jamming attacker can take advantage of the presence of disturbance to prevent stabilization. Finally, we obtain conditions of stability under disturbance.
A. Stabilization in the Disturbance-Free Case

A networked control system under jamming attacks but without disturbance \((w(t) = 0, t \in \mathbb{N}_0)\) was studied in [24]. There, it was noted that emitting jamming interference signals is a costly action due to its large energy requirements [7]. The following assumption on the attacker’s interference power was considered in that work as a natural way to describe the energy constraints of an attacker.

**Assumption 3.1:** There exist scalars \(\overline{\pi} \geq 0\), \(\overline{\phi}_j \geq 0\) such that

\[
P\left(\sum_{i=0}^{t-1} \phi_j(i) \leq \overline{\pi} + \overline{\phi}_j t\right) = 1, \quad t \in \mathbb{N}. \tag{10}\]

Here, the scalar \(\overline{\pi}\) models the attacker’s initial capabilities. Large \(\overline{\pi}\) values describe attackers with large initial energy resources capable of setting \(\phi_j(t)\) to large values for a few initial time instants. On the other hand, \(\overline{\phi}_j\) is an upper bound on the long-run average interference power (i.e., \(\limsup_{k \to \infty} \frac{1}{k} \sum_{i=0}^{k-1} \phi_j(i) \leq \overline{\phi}_j\)) describing the overall attack strength. The scalar \(\overline{\phi}_j\) is typically strictly smaller than the maximum possible power of the interference that can be physically emitted from the attacker. However, by waiting sufficiently long without attacking, the attacker can preserve energy and emit strong interference signals with power levels larger than \(\overline{\phi}_j\) for certain durations while still satisfying (10).

The analysis in [24] indicates that if the long-run average power bound \(\overline{\phi}_j\) is sufficiently small, then the closed-loop system (3) is asymptotically stable almost surely, implying \(P[\lim_{t \to \infty} \|x(t)\|_2 = 0] = 1\). A similar conclusion is drawn in Proposition A.1 in the Appendix for the first-moment asymptotic stability, implying \(\lim_{t \to \infty} \mathbb{E}[\|x(t)\|_2] = 0\).

B. Effects of Jamming Attacks on Systems Under Disturbance

For certain systems that face disturbance, jamming attacks can become more dangerous. Even if the disturbance is very small and the attacker has very limited resources, there still exist attack strategies that can destabilize the system while satisfying Assumption [3.1] with very small \(\overline{\phi}_j\). We illustrate this idea in the following example.

**Example 3.1:** Consider a scalar networked control system (9), (10) with \(x_0 > 0\), \(A + BK \in [0, 1]\), \(\lambda > 1\), and a constant disturbance \(w(t) = w^* > 0\), \(t \in \mathbb{N}_0\), as a dynamic effect that is unrelated to jamming. Suppose that the conditional probability \(p\) of transmission failures is a strictly increasing function (e.g., \(p\) given by the outage probability (6)). For this setup, an attacker can wait sufficiently long and then attack for a duration with a sufficiently large interference power level so that the state norm grows to large values but the average interference power does not go above \(\overline{\phi}_j\). In particular, for any \(\overline{\phi}_j > 0\), \(x_0 > 0\), \(z > 0\), and \(\rho \in (0, 1)\), the attack strategy

\[
\phi_j(t) = \begin{cases} \phi_j^*, & t \in \{t_1, \ldots, t_1 + t_2 - 1\}, \\ 0, & \text{otherwise}, \end{cases} \quad \tag{11}
\]

with \(\phi_j^* = p^{-1}(\rho^{\frac{1}{2}}) + 1\), \(t_1 \geq \left[\max\{\phi_j^*, \overline{\phi}_j, 0, 0\}\right] + 1\), \(t_2 \geq \left[\log_2 \left(\frac{\rho^2 - \overline{\phi}_j}{\phi_j^* - \overline{\phi}_j}\right)\right] + 1\) guarantees that Assumption 3.1 is satisfied and the state exceeds the value \(\rho\) at time \(t_1 = t_1 + t_2\), i.e., \(P[x(t) > \rho] > \rho\).

To show this, consider

\[
E(\tau_1, \tau_2) \triangleq \{\omega \in \Omega : (t = 1, t \in \{\tau_1, \ldots, \tau_1 + \tau_2 - 1\}) \in \mathcal{F}, \tag{12}
\]

which represents the case where all packet transmissions during \(t \in \{\tau_1, \ldots, \tau_1 + \tau_2 - 1\}\) fail. By (11), we have \(P[E(\tau_1, \tau_2)] = p^{\tau_2}(\phi_j^*)\).

Now, since \(x_0 > 0\), \(A > 1\), and \(w^* > 0\), we obtain \(x(t) \geq w^*\), \(t \in \mathbb{N}\). Therefore,

\[
P[x(\tau > z)] \geq P[x(\tau) > z | E(\tau_1, \tau_2)]P[E(\tau_1, \tau_2)]
\]

\[
\geq P[A^\tau_2 x(\tau_1) + \sum_{i=0}^{\tau_2-1} A^i w^* > z | E(\tau_1, \tau_2)]p^{\tau_2}(\phi_j^*)
\]

\[
\geq P[A^\tau_2 w^* > z | E(\tau_1, \tau_2)]p^{\tau_2}(\phi_j^*) > 1 - \rho^{\frac{\rho^2 - \overline{\phi}_j}{\phi_j^* - \overline{\phi}_j}} = \rho.
\]

Furthermore, the attack strategy (11) satisfies Assumption [3.1] for \(\overline{\pi} = 0\), because \(\tau_1 \geq \max\{\phi_j^*, \overline{\phi}_j\}\), \(\phi_j^* > 0\), and thus, \(\sum_{i=0}^{\tau_2-1} \phi_j(i) = \phi_j^* t_2 \leq \phi_j^* (\tau_1 + t_2) = \phi_j^* \tau_2\).

The attack strategy (11) can make the state grow arbitrarily large even if the interference power bound \(\overline{\phi}_j\) is small. This attack strategy is effective, because even if the attacker initially waits for a long duration without attacking, the state never reaches a small neighborhood of zero due to the disturbance. Hence, after waiting for a while, the attacker can consecutively attack with high interference powers to cause many transmission failures and make the state norm grow to large values. This is further illustrated in Section IV.

**Remark 3.1:** Attack strategies similar to the one discussed in Example 3.1 may not always be able to leverage the existence of disturbance to cause instability, if the disturbance only affects a stable mode of the system that does not get influenced by jamming-related transmission failures. For instance, consider (9), (10) where

\[
A = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad K = [0 \ -1.5]. \tag{12}
\]

In this case, consider a disturbance process \(w(t) = [w_1(t), 0]^T\). Here, the disturbance only affects the first state and the jamming attacks only affect the second state. The first state remains bounded under bounded disturbances regardless of jamming. On the other hand, sufficiently frequent transmission failures due to jamming attacks can cause the second state to diverge. However, differently from Example 3.1, the attacker needs to spend considerably more resources to cause instability, because disturbance and jamming affect different parts of the dynamics. In the following sections, we provide conditions of stabilization by considering the worst-case scenarios as in Example 3.1. Direct application of such conditions can be conservative for systems similar to (12). However, in some cases, conservativeness can be reduced. For instance, if a system has components that are influenced by both the jamming and the disturbance, partitioning the system to apply the results to only those components can help reduce conservativeness.

C. Jamming Interference and Bounded Disturbance

To ensure stability under both disturbance and jamming, the attacks need to be restricted in a way that high jamming interference powers at consecutive times are not allowed. To this end, we consider the following assumption.

**Assumption 3.2:** There exist scalars \(\hat{\kappa} \geq 0\) and \(\hat{\phi}_j \geq 0\) such that

\[
P\left[\sum_{i=t_1}^{t_2-1} \phi_j(i) \leq \hat{\kappa} + \phi_j^*(t_2 - t_1)\right] = 1, \quad \tag{13}
\]

for all \(t_1, t_2 \in \mathbb{N}_0\) with \(t_1 < t_2\).

Notice that (13) implies (10) (with \(\overline{\pi} = \hat{\kappa}\) and \(\overline{\phi}_j = \hat{\phi}_j\)), but the converse is not true. Assumption 3.2 is thus more restrictive than Assumption 3.1. In particular, under Assumption 3.2 the attacker can attack with a jamming interference power \(\phi_j^* > \hat{\phi}_j\) consecutively for at most \(\left[\hat{\kappa}/(\phi_j^* - \hat{\phi}_j)\right]\) time steps; hence, the destabilizing attacks discussed in Example 5.1 are avoided. Furthermore, setting \(\hat{\kappa}/(\phi_j^* - \hat{\phi}_j)\) as 1 and solving for \(\phi_j^*\) provide us a hard constraint \(\phi_j^*(t) \leq \phi_j^*\) for every \(t \in \mathbb{N}_0\) on the interference.
power level. The value $\phi_{j}^{\max}$ is related to the physical limits of interference generation in wireless jamming units.

Assumption 3.2 is related to other characterizations of attacks. In particular, in the continuous-time deterministic DoS attack characterization of [10], the number of attacks in a given time frame as well as the total duration of those attacks are bounded by certain ratios of the length of that time frame. Under that characterization, the maximum possible length of a continuous attack duration is bounded, which enables analysis of input-to-state stability under disturbance. The restriction on jamming through Assumption 3.2 is similar, since long consecutive emissions of high-powered interference signals are not allowed. We note, however, that Assumption 3.2 does not allow the case where the channel is attacked at all times if the attacker’s interference power for certain times is small.

In this section, we investigate the networked control system (3) under bounded disturbances. The analysis is then extended in Section 3.3 to the case where the disturbance has finite second moments but its norm may not be bounded by a fixed scalar.

In this paper, we consider scenarios where the norm of the disturbance does not approach zero, and hence the state or its moments may not converge to the origin. Instead of exploring asymptotic stability, our goal is to obtain conditions for the first moment of the state to stay bounded. To this end, let $A(t) \triangleq \Lambda(t)A + (1 - \Lambda(t))(A + BK)$, $t \in \mathbb{N}_0$, and moreover, for every $t_1, t_2 \in \mathbb{N}_0$ with $t_1 \leq t_2$, let

$$F(t_2, t_1) = \begin{cases} A(t_2), & t_1 = t_2, \\ A(t_2) - A(t_1), & t_1 < t_2. \end{cases}$$

For the closed-loop system (3), we have $x(t) = F(t - 1, 0)x_0 + \sum_{j=0}^{t-2} F(t - 1, j + 1)w(j) + w(t - 1)$, for $t \in \mathbb{N}$. Therefore, for any induced norm $\|\cdot\|$, it follows from the triangle inequality and the submultiplicativity property that $\|x(t)\| \leq \left( \prod_{i=0}^{t-1} \|A(i)\| \right)\|x_0\| + \sum_{j=0}^{t-2} \left( \prod_{i=j+1}^{t-1} \|A(i)\|\right)\|w(j)\| + \|w(t-1)\|$. Here, we have $\|A(i)\| = \Lambda(i)\|A\| + (1 - \Lambda(i))\|A + BK\|$, $i \in \mathbb{N}_0$. Hence, by setting

$$\zeta_1 \triangleq \|A\| - \|A + BK\|, \quad \zeta_0 \triangleq \|A + BK\|,$$  

we obtain for $t \in \mathbb{N}$, $\|x(t)\| \leq \left( \prod_{i=0}^{t-1} (\zeta_1\|A\| + \zeta_0) \right)\|x_0\| + \sum_{j=0}^{t-2} \left( \prod_{i=j+1}^{t-1} (\zeta_1\|A\| + \zeta_0) \right)\|w(j)\| + \|w(t-1)\|$. By using this inequality, we can obtain an upper bound of the Euclidean norm of the state. Specifically, by Corollary 5.4.5 of [28], there exist $c_1 > 0$ and $c_2 > c_1$ such that

$$c_1\|y\| \leq \|y\|_2 \leq c_2\|y\|, \quad y \in \mathbb{R}^n.$$  

Therefore, we have

$$\|x(t)\|_2 \leq \frac{c_2}{c_1} \left( \left( \prod_{i=0}^{t-1} (\zeta_1\|A\| + \zeta_0) \right)\|x_0\|_2 + \sum_{j=0}^{t-2} \left( \prod_{i=j+1}^{t-1} (\zeta_1\|A\| + \zeta_0) \right)\|w(j)\|_2 + \|w(t-1)\|_2 \right).$$  

Notice here that the particular values of $c_1$ and $c_2$ depend on the choice of the vector norm that induces the matrix norm $\|\cdot\|$. We use (13) to provide bounds on the first moment $\mathbb{E}[\|x(t)\|_2]$. First, in the following result, we consider the case where the disturbance is bounded and the jamming attacks satisfy Assumption 3.2.

Theorem 3.2: Consider the closed-loop networked control system (3). Suppose that the attacker’s interference power process $\{\phi_j(t) \in [0, \infty)\}_{t \in \mathbb{N}_0}$ satisfies Assumption 3.2. Furthermore, suppose that there exists $\varpi \geq 0$ such that

$$\mathbb{P}[\|w(t)\|_2 \leq \varpi] = 1, \quad t \in \mathbb{N}. \quad (17)$$

If

$$(1 - \bar{p}(\hat{\phi}_1))\|A + BK\| + \bar{p}(\hat{\phi}_2)\|A\| < 1,$$  

then there exist $\hat{\mu} \geq 0, \hat{\theta} \in (0, 1)$, and $\hat{\epsilon} \geq 0$ such that

$$\mathbb{E}[\|x(t)\|_2] \leq \hat{\mu}^d \|x_0\|_2 + \hat{\epsilon} \varpi, \quad t \in \mathbb{N}. \quad (19)$$

The proof of Theorem 3.2 is given later in the paper. Theorem 3.2 shows that if jamming attacks satisfy Assumption 3.2 with a sufficiently small $\phi_j$ such that (15) holds, then the first moment of the state stays bounded. Furthermore, the upper bound given in (19) is geometrically decreasing towards the constant $\varpi$, where $\varpi$ is an upper bound on the Euclidean norm of disturbance $w(t)$.

Notice that the condition (13) of Theorem 3.2 and the condition (5) in the disturbance-free case in Proposition A.1 are in the same form, but use different scalars $\phi_1$ and $\phi_2$ due to the difference of the jamming interference characterizations in Assumptions 3.1 and 3.2.

We remark that for attacks that satisfy both assumptions, we have $\bar{p}_j \leq \hat{\phi}_j$.

As we establish later in the proof of Theorem 3.2, the first moment upper bound in (19) depends on parameters $\bar{\epsilon}$ and $\hat{\phi}_j$. In particular, the values of $\bar{\epsilon}$ and $\hat{\phi}_j$ are large when $\bar{\epsilon}$ and $\hat{\phi}_j$ take large values. Moreover, the scalar $\bar{\theta}$ is directly related to the term $(1 - \bar{p}(\hat{\phi}_1))\|A + BK\| + \bar{p}(\hat{\phi}_2)\|A\|$ on the left-hand side of (15). If this term is close to zero, then $\bar{\theta}$ is close to zero, which indicates faster convergence of the bound in (19) towards the constant $\varpi$. We note that $(1 - \bar{p}(\hat{\phi}_1))\|A + BK\| + \bar{p}(\hat{\phi}_2)\|A\|$ represents the behavior of the overall networked control system and it is composed of the convex combination of the terms $\|A + BK\|$ and $\|A\|$ weighted respectively with the lower bound $(1 - \bar{p}(\hat{\phi}_1))$ of the long-term ratio of successful transmissions and the upper bound $\bar{p}(\hat{\phi}_2)$ of the long-term ratio of failed transmissions.

Our analytical approach differs from the more classical approaches used when the transmission failure indicator process $\{l(t)\}_{t \in \mathbb{N}_0}$ is a Bernoulli process or a Markov chain. In those cases, stability analysis can rely on the possibility of failures $\mathbb{P}[l(t) = 1]$ and conditional failure probabilities $\mathbb{P}[l(t) = q|l(t - 1) = r], q, r \in \{0, 1\}$, (see [33], [34]). In our case, precise information of such probability terms is not available due to the uncertainty in the generation of attacks. Specifically, the interference power $\phi_j(t)$ at a given time $t$ is part of attacker’s strategy and cannot be known with certainty. As a result, the transmission failure probability at that time is also uncertain and cannot be used in the analysis. Note, however, that Bernoulli-type packet losses are a special case where $\phi_j$ is a constant function. In this paper, we are interested in the cases where the interference power level $\phi_j$ is time-varying and the attacker designs its progression so as to leverage the disturbance to cause instability as in Example 3.1.

A crucial role in our analysis is played by the following lemma, where we investigate the products of affine functions that involve the transmission failure indicator $l(t)$ and obtain some upper bounds for their expected values. As shown later in the proof of Theorem 3.2 such upper bounds allow us to conduct stability analysis without relying on transmission failure probabilities for each time step. In the derivation of these bounds, an essential step is to exploit the concavity of the upper-bounding function $\bar{p}$ given in (9).

Lemma 3.3: Suppose that the attacker’s interference power process $\{\phi_j(t) \in [0, \infty)\}_{t \in \mathbb{N}_0}$ satisfies Assumption 3.2. Then for every $\alpha_1 \geq 0, \alpha_0 \geq 0$ that satisfy

$$\alpha_1 \bar{p}(\hat{\phi}_1) + \alpha_0 < 1,$$  

the product $\alpha_1 \bar{p}(\hat{\phi}_1)+\alpha_0$ is related to the physical limits of interference generation in wireless jamming units.
there exist scalars \( \mu \geq 0 \) and \( \theta \in (0, 1) \) such that
\[
E\left[ \prod_{i=1}^{t_2-1} (\alpha_1 l(i) + \alpha_0) \right] \leq \mu \theta^{t_2-t_1},
\]
(21)
for \( t_1, t_2 \in \mathbb{N}_0 \) with \( t_1 < t_2 \).

Proof: For the case where \( \alpha_1 + \alpha_0 = 0 \), (21) holds for any \( \mu \geq 0 \) and \( \theta \in (0, 1) \). In the following, we consider the case where \( \alpha_1 + \alpha_0 > 0 \). First, by Lemma 2.1 of [24],
\[
E\left[ \prod_{i=1}^{t_2-1} (\alpha_1 l(i) + \alpha_0) \right] = E\left[ \prod_{i=1}^{t_2-1} (\alpha_1 p(l(i)) + \alpha_0) \right],
\]
(22)
Next, by (22), \( \alpha_1 \geq 0 \), and \( p(l) \leq \bar{p}(l), \phi \in [0, \infty) \), we get
\[
E\left[ \prod_{i=1}^{t_2-1} (\alpha_1 l(i) + \alpha_0) \right] \leq E\left[ \prod_{i=1}^{t_2-1} h(\phi(i)) \right],
\]
(23)
where \( h(\phi) \triangleq \alpha_1 \bar{p}(l) + \alpha_0 \). We note that \( h(\cdot) \) is nondecreasing, concave, and continuous, as \( \bar{p}(\cdot) \) also has such properties and \( \alpha_1 \geq 0 \). To obtain a bound for \( E\left[ \prod_{i=1}^{t_2-1} h(\phi(i)) \right] \) in (23), we first show
\[
\prod_{i=1}^{t_2-1} h(\phi(i)) \leq h^{t_2-t_1}(\frac{1}{t_2-t_1} \sum_{i=t_1}^{t_2-1} \ln h(\phi(i))).
\]
(24)
We note that (24) holds if \( h(\phi(i)) = 0 \) for some \( i \in \{t_1, \ldots, t_2-1\} \). Now, consider the case where \( h(\phi(i)) > 0 \) for all \( i \in \{t_1, \ldots, t_2-1\} \). For this case, we have
\[
\prod_{i=1}^{t_2-1} h(\phi(i)) = (t_2-t_1)(\frac{1}{t_2-t_1} \sum_{i=t_1}^{t_2-1} \ln h(\phi(i))).
\]
(25)
Here, \( h(\cdot) \) is concave, since it is the composition of a nondecreasing concave function \( \ln(\cdot) \) and a concave function \( h(\cdot) \) (see Proposition 2.16 in [35] and Section 3.2.4 in [36]). Thus, by (25),
\[
\prod_{i=1}^{t_2-1} h(\phi(i)) \leq (t_2-t_1) \ln h(\frac{1}{t_2-t_1} \sum_{i=t_1}^{t_2-1} \phi(i)),
\]
(26)
which implies (24). The interference power process \( \phi(l) \) satisfies (22) in Assumption 3.2 and hence,
\[
\prod_{i=1}^{t_2-1} h(\phi(i)) \leq h^{t_2-t_1}(\frac{1}{t_2-t_1} \sum_{i=t_1}^{t_2-1} \phi(i)),
\]
(27)
Now, by (20), we get \( \langle \phi(l) \rangle < 1 \). Therefore, by the continuity of \( h(\cdot) \), there exists \( \delta > 0 \) such that \( h(\delta + \phi(l)) < 1 \). As a result, for sufficiently large values of \( t_2-t_1 \), we have \( h(\frac{1}{t_2-t_1} + \phi(l)) < 1 \).

Let \( T^* \) be a positive integer such that \( h(\frac{1}{T^*} + \phi(l)) < 1 \) and let
\[
\theta \triangleq h(\frac{1}{T^*} + \phi(l)).
\]
(28)
It follows from (24) that
\[
E\left[ \prod_{i=1}^{t_2-1} h(\phi(i)) \right] \leq \theta^{t_2-t_1},
\]
(29)
for all \( t_1, t_2 \in \mathbb{N}_0 \) such that \( t_2-t_1 \geq T^* \). If \( T^* = 1 \), then (21) holds, by (29). If, on the other hand, \( T^* > 1 \), then by using \( h(\phi(l)) \leq \alpha_1 + \alpha_0, \forall l, t \in \mathbb{N}_0 \), we obtain
\[
E\left[ \prod_{i=1}^{t_2-1} h(\phi(i)) \right] \leq (\alpha_1 + \alpha_0)^{t_2-t_1} \leq (\alpha_1 + \alpha_0)^{T^*-1},
\]
(30)
for all \( t_1, t_2 \in \mathbb{N}_0 \) such that \( 0 < t_2 - t_1 < T^* \). Letting
\[
\mu \triangleq (\alpha_1 + \alpha_0)^{T^*-1} \theta^{-(T^*-1)},
\]
(31)
we obtain (21), by (29) and (30).

Lemma 3.3 shows that under Assumption 3.2, the expectation term
\[
E\left[ \prod_{i=1}^{t_2-1} (\alpha_1 l(i) + \alpha_0) \right]
\]
converges to zero at a geometric rate. By using this lemma, we obtain the following result.

Lemma 3.4: Suppose that the attacker’s interference power process \( \{\phi(l) \in [0, \infty)\}_{l \in \mathbb{N}_0} \) satisfies Assumption 3.2. Then for every \( \alpha_1 \geq 0, \alpha_0 \geq 0 \) satisfying (20), there exists a scalar \( d \geq 0 \) such that
\[
\sum_{j=0}^{t-2} E\left[ \prod_{i=j+1}^{t_1} (\alpha_1 l(i) + \alpha_0) \right] \leq d, \quad t \in \{2, 3, \ldots\}.
\]
(32)
Proof: Since (20) holds, it follows from Lemma 3.3 that
\[
E\left[ \prod_{i=j+1}^{t_1} (\alpha_1 l(i) + \alpha_0) \right] \leq \mu \theta^j, \quad \mu \geq 0, \quad \theta \in (0, 1),
\]
are scalars that depend on \( \alpha_1 \) and \( \alpha_0 \). Letting
\[
d \leq \mu/(1-\theta),
\]
(33)
we obtain\( \sum_{j=0}^{t-2} E\left[ \prod_{i=j+1}^{t_1} (\alpha_1 l(i) + \alpha_0) \right] \leq \sum_{j=0}^{t-2} \mu \theta^j = d \), which completes the proof.

In Lemmas 3.3 and 3.4, we obtained the upper-bounding inequalities (21) and (32) concerning the transmission failure indicator process \( \{1 \in \mathbb{N}_0 \} \). In our proof of Theorem 3.2 given below, we utilize these inequalities.

Proof of Theorem 3.2: By (19) and (21), \( \|x(t)\|_2 \leq \frac{c_1}{1} \left( \prod_{i=0}^{t-1} (\zeta_1 l(i) + \zeta_0) \right) \|x_0\|_2 + \frac{c_1}{1} \left( \sum_{j=0}^{t-2} \prod_{i=j+1}^{t_1} (\zeta_1 l(i) + \zeta_0) + 1 \right) \|x_0\|_2 \)
(34)
must always exist. An attacker with large resources can cause the state norm to grow large. This is also indicated in the upper bound for the first moment in (19).
If $\hat{\eta}$ in Assumption 3.2 is large, then $T^*$ is large, which makes $\hat{\mu}$ large, as $\hat{\mu}$ is an increasing function of $T^*$. Further, since
\[
\frac{c_2}{\mu} \left( 1 - \frac{\hat{\mu} (\hat{\phi}_1)}{A + BK} \right) + 1 \leq \hat{d},
\]
we observe that $\hat{d}$ is large for large values of $\hat{\mu}$ and $\hat{\phi}_1$. On the other hand, for large values of $T^*$, $\theta$ is close to $(1 - \hat{\mu} (\hat{\phi}_1)) |A + BK| + \hat{\mu} (\hat{\phi}_1) |A|$. If the upper bound $\hat{d} \phi_1$ of average interference powers is large, then $\hat{\mu} \theta^t$ in (19) converges slowly, since $\theta$ is close to 1.

**D. Jamming and Disturbance with Finite Second Moment**

In Theorem 3.2, we explored the case where the disturbance norm is bounded. Next, we investigate scenarios where the disturbance may not be bounded. We obtain a relation between the state and the disturbance similar to those used for noise-to-state stability analysis of stochastic systems (e.g., [25, 26]). Specifically, in the next result, we provide an upper bound for the first moment of the state by using the second moment of the disturbance.

**Theorem 3.6:** Consider the closed-loop networked control system (3). Suppose that the attacker’s interference power process $\{\phi_3(t) \in [0, \infty)\}_{t \in \mathbb{N}}$ satisfies Assumption 3.2. Furthermore, suppose $\mathbb{E} [||w(t)||^2] < \infty$, $t \in \mathbb{N}$. If
\[
(1 - \hat{\mu} (\hat{\phi}_1)) |A + BK| + \hat{\mu} (\hat{\phi}_1) |A|^2 < 1,
\]
then there exist $\hat{\mu}, \hat{f} > 0$, and $\hat{\theta} \in (0, 1)$ such that for $t \in \mathbb{N}$,
\[
\mathbb{E} [||x(t)||^2] \leq \hat{\mu} \theta^t ||x_0||^2 + \hat{f} \max_{\hat{\phi} \in (0, \ldots, t-1)} \mathbb{E} [||w(i)||^2]^\frac{1}{2}.
\]

**Proof:** The proof of this result relies on the following lemma.

**Lemma 3.7:** Suppose that the attacker’s interference power process $\{\phi_3(t) \in [0, \infty)\}_{t \in \mathbb{N}}$ satisfies Assumption 3.2. Then for every $\gamma_i \geq 0$, $\gamma_0 \geq 0$ that satisfy
\[
(\gamma_i^2 + 2\gamma_i \gamma_0) \hat{\mu} \hat{\phi}_1 + \gamma_0^2 < 1,
\]
there exists a scalar $\hat{f} \geq 0$ such that
\[
\sum_{j=0}^{t-2} \mathbb{E} \left[ \left( \prod_{i=j+1}^{t-1} \gamma_i \right)^2 \right] \frac{1}{2} \leq \hat{f}, \quad t \in \{2, 3, \ldots\}.
\]

**Proof:** For every $j \in \{0, 1, \ldots, t-2\}$, $t \in \{2, 3, \ldots\}$, we have $\left( \prod_{i=j+1}^{t-1} \gamma_i \right)^2 = \prod_{i=j+1}^{t-1} \left( \frac{\gamma_i^2 (\gamma_i)}{2} + 2\gamma_i \gamma_0 (\gamma_i) + \gamma_0^2 \right)$. Let $\alpha_1 \triangleq \gamma_i^2 + 2\gamma_1 \gamma_0$ and $\alpha_0 \triangleq \gamma_0^2$. Since $t^2(i) = l(i)$, it follows that
\[
\mathbb{E} \left[ \left( \prod_{i=j+1}^{t-1} \gamma_i \right)^2 \right] = \mathbb{E} \left[ \prod_{i=j+1}^{t-1} \left( \alpha_1 \right) + \alpha_0 \right].
\]

By (41), (40) holds. Therefore, it follows from Lemma 3.3 that
\[
\mathbb{E} \left[ \prod_{i=j+1}^{t-1} \left( \alpha_1 \right) + \alpha_0 \right] \leq \hat{\mu} \theta^t (1 - \frac{1}{2}),
\]
where $\mu \geq 0$ and $\theta \in (0, 1)$ depend on $\gamma_1$ and $\gamma_0$. Next, we apply Lemma 3.7 to find an upper bound of the summation term $\sum_{j=0}^{t-2} \mathbb{E} \left[ \left( \prod_{i=j+1}^{t-1} \gamma_i \right)^2 \right]^{\frac{1}{2}}$. Specifically, let $\gamma_1 \triangleq \gamma_1$ and $\gamma_0 \triangleq \gamma_0$. By (40), we have (41). Noting that $\gamma_1 > 0$ and $\gamma_0 \in (0, 1)$, we obtain by Lemma 3.7 that
\[
\sum_{j=0}^{t-2} \mathbb{E} \left[ \left( \prod_{i=j+1}^{t-1} \gamma_i \right)^2 \right]^{\frac{1}{2}} \leq f,
\]
where $f \geq 0$ depends on $\gamma_1$ and $\gamma_0$. Now, by letting
\[
\hat{\theta} \triangleq \theta, \quad \hat{\mu} \triangleq \mu \theta / c_1, \quad \hat{f} \triangleq (f + 1) c_2 / c_1,
\]
we obtain (40) from (49)–(51).

Theorem 3.6 shows that if the jamming attacks satisfy Assumption 3.2 with a sufficiently small $\hat{\phi}_1$ such that (49) holds, then the first moment of the state satisfies the bound in (40).
Remark 3.8: The constants $\hat{\theta}$ and $\hat{\mu}$ of the first-moment inequality (40) are the same as those provided in Remark 5.5 for the bounded-disturbance case. Specifically, $\hat{\theta}$ and $\hat{\mu}$ are given respectively by (40) and (52). Furthermore, $\hat{f} \geq 0$ in (40) can be obtained from (43) and (52) as

$$
\hat{f} = \frac{\phi}{c_1} \left( \frac{\|A\|^2(\hat{T}^2-1)}{1 - \hat{\theta}^2/2} + 1 \right),
$$

where $\hat{\theta} \triangleq (1 - \hat{\mu}(\hat{\phi_1} + \hat{\phi}))\|A + BK\|^2 + \hat{\mu}(\hat{\phi_1} + \hat{\phi})\|A\|^2$, and $\hat{T}$ is a positive integer that satisfies $(1 - \hat{\mu}(\hat{\phi_1} + \hat{\phi}))\|A + BK\|^2 + \hat{\mu}(\hat{\phi_1} + \hat{\phi})\|A\|^2 < 1$.

Theorem 3.6 is applicable to scenarios where the condition (17) of Theorem 3.5 may fail to hold. In particular, if disturbance distributions have infinite support, then (12) does not hold (e.g., Gaussian distribution with $w(t) \sim \mathcal{N}(m, \Sigma)$ where $m \in \mathbb{R}^n$ and $\Sigma \in \mathbb{R}^{n \times n}$ is a positive-definite matrix). In such cases, Theorem 3.6 can be utilized. If $\mathbb{E}[\|w(t)\|_2^2] \leq \tilde{w}$ holds for all $t \in \mathbb{N}_0$ with a scalar $\tilde{w} \geq 0$, then it follows from (40) that $\limsup_{t \to \infty} \mathbb{E}[\|x(t)\|_2^2] \leq \tilde{f} \tilde{w}^2$, denoting the long-run boundedness of expected state norm.

Although Theorem 3.6 is applicable to a wider range of scenarios in terms of the disturbance, the condition (39) is more restrictive than the condition (18) of Theorem 3.2. In particular, we have $\{(1 - \hat{\mu}(\hat{\phi_1}))(A + BK) + \hat{\mu}(\hat{\phi_1})\|A\|^2 < (1 - \hat{\mu}(\hat{\phi_1}))(A + BK) + \hat{\mu}(\hat{\phi_1})\|A\|^2 \}$ for $\hat{\phi_1} \in (0, 1)$ indicating that (39) implies (18), but not vice versa. We also note that the finite second-moment condition in Theorem 3.6 holds in many control engineering scenarios. A particular example is the Gaussian measurement noise setting.

It is interesting that both Theorems 3.2 and 3.6 can be used for assessing stability in the scenarios where the transmission failure indicator process $\{\hat{\tau}(t)\}_{t \in \mathbb{N}_0}$ and the disturbance process $\{w(t)\}_{t \in \mathbb{N}_0}$ are not independent of each other. This is the case, e.g., when the state measurements received by the controller are subject to noise. Note also that the conditions in both theorems can be checked using different induced matrix norms $\|\cdot\|$. Certain norms can provide less conservative results, as illustrated in Section IV.

IV. NUMERICAL EXAMPLE

Consider the networked control system (3) with

$$
A = \begin{bmatrix} 0.1 & -1 \\ 1.1 & 1.8 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad K = [-0.9277 -1.2615],
$$

and the channel model from Example 2.1 with outage probability $p$ given by (6), where $\xi = 2, \sigma = 0.05, \gamma = 2$, and $b_1 = b_2 = 1$.

We first investigate the disturbance-free case ($w(t) \equiv 0$). Noting that $p$ is a concave, continuous, and nondecreasing function, we set $\hat{p}(\phi) \triangleq p(\phi)$, which satisfies (9). By Theorem 3.5 of [24], the system is almost surely asymptotically stable under any attacks that satisfy Assumption 3.1 with $\phi_1 \leq 0.62$. The analysis in [24] is Lyapunov-based, and for the case with $\phi_1 = 0.62$, it uses the Lyapunov-like function $V(x) \equiv x^T P \|x\|^2$ with the positive-definite matrix

$$
P = \begin{bmatrix} 0.7728 & 0.8554 \\ 0.8554 & 3.2649 \end{bmatrix}.
$$

The matrix $P$ is also useful for the first-moment stability analysis. In particular, we can use the matrix norm $\|\cdot\|_P$ induced by the vector norm $\|x\|_P \equiv \sqrt{x^T P x}$. By using this matrix norm, the stability condition in Proposition A.1 is satisfied for $\phi_1 \leq 0.27$. This indicates that the networked control system (3) without disturbance is first-moment geometrically stable under jamming attacks that satisfy Assumption 3.2 with $\phi_1 \leq 0.27$. Hence, in the disturbance-free case $\mathbb{E}[\|x(t)\|_2]$ converges to zero with a geometric rate. The choice of the matrix norm is important for stability analysis. For instance, in this example, the stability condition in Proposition A.1 does not hold with matrix norms induced by 1-norm, Euclidean-norm, or infinity norm, because for those norms, $\|A + BK\| > 1$.

1) Disturbance-free scenario: As discussed in Section III-B, Assumption 3.1 allows the attacker to jam the channel with very large interference powers after waiting without attacking for sufficiently long durations. For instance, for the attack strategy considered in (13) with $\tau_1 = 960, \tau_2 = 40, \phi_1 = 6.75$, Assumption 3.1 is satisfied with $\pi = 0$ and $\sigma_1 = 0.27$. In the disturbance-free case, this attack strategy does not create a problem for stability since $\sigma_1$ is sufficiently small. In particular, after the long duration $\tau_1$ without attacks, the state norm gets very close to zero, and as a result, the state norm after the attack period of $\tau_2$ time steps is also small.

2) Scenarios with disturbance: By contrast, in the case with disturbance, the attack strategy (13) makes the state norm grow at time $\tau_1 + \tau_2$. This is because, even after the long attack-free duration, the state norm cannot get close to zero due to the disturbance. This is shown in the top part of Fig. 2 with the disturbance given by

$$
w(t) = \begin{bmatrix} \cos(\theta) \tilde{w}(t) \\ \sin(\theta) \tilde{w}(t) \end{bmatrix}^T, \quad t \in \mathbb{N}_0,
$$

where $\theta = \pi/2$ and $\tilde{w}(t) \in \mathbb{R}$ at each time $t$ is uniformly distributed in $[-0.5, 0.5]$. Under disturbance, the length $\tau_2$ of the attack period directly affects the growth of the state norm. The attacker can increase the waiting time $\tau_1$ to attack with a longer duration $\tau_2$ with the same high interference power $\phi_1$ to make the state norm grow, while still satisfying Assumption 3.1.

In the bottom part of Fig. 2 we see that for the same disturbance but with $\tau_2 = 60$, the state is driven to larger values. Notice that with $\tau_1 = 1440, \tau_2 = 60, \phi_1 = 6.75$, Assumption 3.1 is also satisfied with $\pi = 0$ and $\sigma_1 = 0.27$. Although after the time $\tau_1 + \tau_2$, the effect of the attack diminishes, the attacker can repeat cycles of sleeping and jamming, and the state norm may grow if the attacker uses higher interference powers for longer durations. To guarantee a predetermined bound on the expected state norm, interference power levels need to be restricted. This is achieved by Assumption 3.2. Under Assumption 3.2 the attacker can attack with a jamming interference power $\phi_1 > \phi_1$ consecutively for at most $\lfloor \kappa/\phi_1 \rfloor$ time steps. For instance, with $\kappa = 250$ and $\phi_1 = 0.27$, the jamming attacks in the top part of Fig. 2 satisfy Assumption 3.2. However, the jamming attacks in the bottom part do not satisfy Assumption 3.2 with the same $\kappa$ and $\phi_1$ due to the longer attack duration. For a duration of $60$ time steps, the maximum allowed...
interference power is $\phi_1 = 4.59$. We remark that the parameters $\hat{\kappa}$ and $\hat{\phi}_J$ can be selected to reflect the capabilities of the attacker.

If the jamming strategy satisfies Assumption 3.2 with $\hat{\phi}_J \leq 0.27$, then by Theorem 3.2 the first moment of the state satisfies the bound in (19) for any bounded disturbance. If the disturbance is not bounded, then Theorem 3.6 can be applied; by Theorem 3.6 the bound in (40) holds if the attacker is less powerful with $\hat{\phi}_J \leq 0.1$.

The first moment bound provided in (19) can be evaluated using the values of $\theta$, $J$, and $d$ given in Remark 3.5. We note that the bound is not tight for this example. This is partly because the inequalities (16) and (19), which relate packet transmission failure indicators to the norm of the state and its first-moment, are not tight for multi-dimensional systems. Another factor is that the disturbance in this example is a stochastic process and does not necessarily increase the state norm at each time.

In Figs. 3 and 4 we show plots for the first moment bound in (19) with the values of $\theta$, $J$, and $d$ provided in Remark 3.5. These plots show that the bound becomes larger for more powerful attacks. Notice that in Fig. 3 the bounds are not visibly decreasing. This is because $\theta \in (0,1)$ is close to 1 and the term $\dot{\mu}^T [\ddot{x}_0]_2$ is smaller than $\ddot{\sigma}_T$.

Next, we explore the effects of different disturbance realizations. In particular, we run simulations for different values of $\vartheta \in [0, \pi]$ in (35) and evaluate $\mathbb{E}[\|x(\tau_1 + \tau_2)\|_2]$ as a performance index. The evaluations are done under the attack strategy (11). This strategy was also used for obtaining Fig. 2 for the particular value $\vartheta = \pi/2$ corresponding to the situation where the disturbance only affects the second state. Different values of $\vartheta$ result in different levels of disturbance on the first and the second states. Fig. 5 shows that $\mathbb{E}[\|x(\tau_1 + \tau_2)\|_2]$ can vary largely depending on $\vartheta$ even though the disturbance magnitude $\|u(t)\| = |\tilde{w}(t)|$ does not depend on how $\vartheta$ is chosen. Here the value of $\mathbb{E}[\|x(\tau_1 + \tau_2)\|_2]$ is $\pi$-periodic, because the distribution of disturbance as a function of $\vartheta$ is $\pi$-periodic. Variations in $\mathbb{E}[\|x(\tau_1 + \tau_2)\|_2]$ indicate that jamming attacks can be more/less effective depending on how the disturbance enters in the dynamics.

3) Countermeasures against jamming: The damaging effects of the jamming attacks can be reduced by adjusting the transmission power ($\xi$ in (6)). Consider the setup where successful communications are replied with acknowledgement messages. In the case where communication fails, the controller would receive no acknowledgement, which indicates the failure. In this setup, the controller can improve the overall performance by increasing the transmission power when there are many consecutive failures. This countermeasure against jamming can be described as follows. If $l(t-\tau) = 1$ for each $i \in \{1, \ldots, N_C\}$ (representing $N_C$ total consecutive failures), then at time $t$, the transmission power $\xi(t)$ is set to a value $\xi_C$ (larger than the nominal value $\xi$ used above) for a duration of $T_C$ time steps. Thus, at those time steps, failures become less likely. After $T_C$ time steps, the transmission power is set back to its nominal (lower) value and the countermeasure system restarts counting consecutive failures. We explore the effectiveness of this countermeasure against the attacks that are illustrated in the bottom part of Fig. 4. Specifically, Fig. 6 shows the expected total state norm $\sum_{i=0}^{\tau} \mathbb{E}[\|x(i)\|_2]$ and expected total transmission power $\sum_{i=0}^{\tau} \mathbb{E}[\xi(i)]$ approximated through 500 simulations for different parameter values $\xi_C \in \{4, 8\}$, $N_C \in \{4, 8\}$, $T_C \in \{4, 8\}$. The results indicate that the effects of jamming can be mitigated by temporarily increasing transmission powers, and the performance gets better with larger total transmission power use.

V. CONCLUSION

We explored the networked control problem under jamming attacks with time-varying interference power. Specifically, we investigated the effects of jamming attacks on systems that are subject to disturbance, and obtained conditions under which the first moment of the state stays bounded. Our results indicate that if the disturbance is known to be bounded, stability of a system can be guaranteed under larger average jamming interference powers.

Our results can be extended for the case where multiple wireless channels are used for the transmission of state and control data.
such cases, increasing the number of channels through which the plant and the controller communicate can increase the level of tolerance against certain jamming attack scenarios.

One of our future research directions is to increase robustness properties of the overall system by utilizing predictive control approaches proposed previously in [13]. Another future work is to provide an analysis of the networked control system under time-varying transmission powers. In this line of research, for a wireless networked control problem without attacks, [38] recently explored stability and energy-efficiency under time-varying transmission powers.

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APPENDIX

Here we provide an analysis of moment stability of the networked control system under Assumption 3.1 In particular, the following result provides a condition under which the first-moment of the state \( \mathbb{E}[\|x(t)\|_2^2] \) of system 3 converges to zero at a geometric rate.

**Proposition A.1:** Consider the closed-loop networked control system 3 for the case where \( w(t) = 0, t \in \mathbb{N}_0 \). Suppose that the attacker’s interference power process \( \{\phi(t) \in [0, \infty)\}_{t \in \mathbb{N}_0} \) satisfies Assumption 3.1 Moreover, assume

\[
(1 - \hat{p}(\mathbf{J}_1))|A + B\mathbf{K}| + \hat{p}(\mathbf{J}_2)|A| < 1. \tag{56}
\]

Then the closed-loop system 3 is first-moment geometrically stable, that is, there exist \( \gamma \geq 0 \) and \( 
T \in (0, 1) \) such that

\[
\mathbb{E}[\|x(t)\|_2^2] \leq \gamma^t \|x_0\|_2^2, \quad t \in \mathbb{N}. \tag{57}
\]

The proof of Proposition A.1 is based on the following result.

**Lemma A.1:** Suppose that the attacker’s interference power process \( \{\phi(t) \in [0, \infty)\}_{t \in \mathbb{N}_0} \) satisfies Assumption 3.1 Then for every \( \alpha_0 \geq 0, \alpha_0 \geq 0 \) that satisfy

\[
\alpha_1 \hat{p}(\mathbf{J}_1) + \alpha_0 < 1, \tag{58}
\]

Note: The page number and volume information are not relevant to the content of the document and have been removed. The page number 10 at the beginning of the document indicates that the reference to the journal title is a placeholder and not relevant to the actual content.
there exist scalars \( \mu \geq 0 \) and \( \theta \in (0, 1) \) such that

\[
\mathbb{E} \prod_{i=0}^{t-1} (\alpha_1 l(i) + \alpha_0) \leq \mu \theta^t, \quad t \in \mathbb{N}.
\]  

(59)

**Proof:** The proof is similar to that of Lemma 3.3. In particular, we have (23) and (24) with \( t_1 = 0, t_2 = t, h(v) \triangleq \alpha_1 \hat{p}(v) + \alpha_0, \) and hence, \( \mathbb{E} \prod_{i=0}^{t-1} (\alpha_1 l(i) + \alpha_0) \leq \mathbb{E} [h^t(\frac{1}{t} \sum_{i=0}^{t-1} \phi_j(i))] \). By Assumption 3.1, we then obtain \( \mathbb{E} [h^t(\frac{1}{t} \sum_{i=0}^{t-1} \phi_j(i))] \leq h^t(\frac{1}{t} \sum_{i=0}^{t-1} \phi_j) \).

Therefore, by (58), after letting \( T^* \) be a positive integer such that \( h(\frac{1}{T^*} + \overline{\phi}_j) < 1 \) and defining

\[
\theta \triangleq h(\frac{1}{T^*} + \overline{\phi}_j), \quad \mu \triangleq (\alpha_1 + \alpha_0)^{T^* - 1} \theta^{-(T^* - 1)},
\]  

(60)

we obtain (59).

**Proof of Proposition A.1** By (16) with \( w(t) = 0, t \in \mathbb{N}_0, \) we have

\[
\mathbb{E} \|x(t)\|_2 \leq \frac{c_2}{c_1} \mathbb{E} \left[ \prod_{i=0}^{t-1} (\zeta_1 l(i) + \zeta_0) \right] \|x_0\|_2.
\]  

(61)

Next, we apply Lemma A.1. First, since \( \|A\| > 1 \), (56) implies \( \|A + BK\| \leq 0, 1 \), and thus, \( \zeta_1 > 0, \zeta_0 \in [0, 1] \). With \( \alpha_1 = \zeta_1 \) and \( \alpha_0 = \zeta_0 \), (56) implies (58). Therefore, by Lemma A.1, we have \( \mathbb{E} \prod_{i=0}^{t-1} (\zeta_1 l(i) + \zeta_0) \leq \mu \theta^t \), where \( \mu \geq 0 \) and \( \theta \in (0, 1) \). Hence, by (61), the inequality (57) holds with \( \overline{\mu} \triangleq \frac{c_2}{c_1} \mu \) and \( \overline{\theta} = \theta \). ■

Proposition A.1 provides a method to check the first-moment geometric stability of the system (1), (2) under jamming attacks that satisfy Assumption 3.1. The scalars \( \|A + BK\| \) and \( \|A\| \) in condition (56) respectively represent the behavior of the closed-loop dynamics under successful transmissions and the open-loop dynamics under failed transmissions. Here, we select the matrix norm \( \| \cdot \| \) to ensure \( \|A + BK\| < 1 \). This is possible since the feedback gain \( K \) is designed to make \( A + BK \) a Schur matrix, for which such a matrix norm can be constructed (see Corollary 9.3.4 of [28]). On the other hand, for unstable open-loop dynamics, we have \( \|A\| > 1 \). Notice that the inequality in (56) holds if the upper bound \( \overline{\phi}_j \) of the average jamming interference power is sufficiently small so that \( \hat{p}(\overline{\phi}_j) \) is sufficiently close to zero. In such cases, transmission failures happen sufficiently rarely in average, and thus the overall networked control system frequently follows the stable behavior of the closed-loop dynamics and the geometric convergence of the first-moment of the state as in (57) can be guaranteed. As we establish in the proof, the scalar \( \overline{\theta} \) in (57) represents the rate of convergence, and it depends on \( \hat{p}(\overline{\phi}_j) \) as well as the scalars \( \|A + BK\| \) and \( \|A\| \). In particular, if the bound \( \overline{\phi}_j \) on the long run average jamming interference power is small, then \( \overline{\theta} \) is also small, indicating faster convergence of the first-moment.

**First-moment geometric stability** discussed in Proposition A.1 is a stronger notion of stochastic stability in comparison to almost-sure asymptotic stability explored in [24]. As expected, first-moment geometric stability condition (57) is more restrictive with respect to the attack parameter \( \overline{\phi}_j \). Specifically, the almost-sure asymptotic stability condition presented in Theorem 3.5 of [24] reduces to

\[
(1 - \hat{p}(\overline{\phi}_j)) \ln \|A + BK\| + \hat{p}(\overline{\phi}_j) \ln \|A\| < 0,
\]  

(62)

with \( \| \cdot \| \) denoting the matrix norm induced by the vector norm \( \|x\| = \sqrt{x^T P x} \) where \( P \in \mathbb{R}^{n \times n} \) is a positive-definite matrix. For this matrix norm, (56) implies (62).