Dust-acoustic dispersion relation in three-dimensional complex plasmas under microgravity

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Abstract. A series of dedicated experiments with the Plasma Kristal Experiment (PKE)-Nefedov (Nefedov et al 2003 New J. Phys. 5 33) set-up were performed on board the International Space Station to measure the dispersion relation (DR) for the longitudinal dust-acoustic (DA) waves in quasi-isotropic three-dimensional (3D) complex plasmas. The waves were excited by applying ac electric modulation of variable frequency to the radio frequency (rf) electrodes. The amplitude of excitation was varied with frequency to ensure a ‘sufficiently linear’ regime of the dust density perturbations. The DR was obtained by measuring the induced density perturbations, revealing fairly good agreement with a simple multispecies theory of DA waves.

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1. Introduction

Complex (dusty) plasmas are low-pressure discharge plasmas with added microparticles (dust grains) that become ‘instantaneously’ charged once introduced into the discharge. It is a well recognized fact that, in addition to the usual plasma modes due to the electron/ion motion (Langmuir, ion acoustic, etc), charged microparticles in complex plasmas give rise to new low-frequency wave modes associated with the microparticle motion [1, 2]. Since the particles can be observed and tracked individually, this gives us an opportunity to investigate wave phenomena occurring in different phase states at the kinetic level.

Dust waves in three-dimensional (3D) complex plasmas have been observed in numerous experiments [3]–[10]. In most cases the waves were self-excited due to various instabilities operating in a plasma, and there were relatively few attempts to employ an external excitation. In all these experiments, however, the amplitude of the excited waves was too high for the linear response to be valid; moreover, when the waves were self-excited the actual dispersion relation (DR) could substantially—and sometimes unpredictably—deviate from the ‘equilibrium’ one. (Note that in 2D crystalline lattices the so-called dust-lattice waves were accurately investigated by measuring equilibrium spectra of thermally excited particle oscillations [11, 12].)

In this paper, we report on a recent series of dedicated ‘dust DR’ experiments performed with the Plasma Kristal Experiment (PKE)-Nefedov set-up [13] on board the International Space Station (ISS). Longitudinal dust-acoustic (DA) waves in quasi-isotropic 3D complex plasmas were externally driven by applying an ac electric signal of variable frequency to the rf electrodes. We made an attempt to minimize the amplitude of the dust waves by reaching a reasonable trade-off between wave nonlinearity and detectability. By measuring the induced dust density perturbations we deduced the DR in a relatively broad frequency range, compared it with existing theories, and showed that the simple multispecies theory [1] provides the best fit to the obtained data.

This paper is organized as follows. In section 2, the theoretical DR for DA waves is given; in section 3, the experimental set-up is described; the experimental data are analysed in section 4, and a comparison with the theoretical DR is given in section 5. In section 6, we draw our conclusions.
2. Theoretical DR

Complex plasmas can be in three basic phase states: ideal (or gaseous-like), strongly coupled (or liquid-like), and crystalline. The discriminating parameter between these states is the electrostatic coupling parameter

\[ \Gamma_s = \Gamma \exp (-\Delta / \lambda), \]

where \( \lambda \) is the wavelength and

\[ \Gamma = \frac{Z^2 e^2}{T_d \Delta} \]

is the Coulomb coupling scale. Here, \( Z \) is the dust particle charge number, \( e \) is the electron charge and \( \Delta = n_d^{-1/3} \) characterizes the interparticle spacing, \( n_d \) is the dust density and \( T_d \) is the dust temperature. The interactions between different species in complex plasmas are quite diverse and depend on the relations between the plasma characteristic parameters (see [2] and references therein). For \( \Gamma \ll 1 \), the plasma can be considered as ideal, whereas for \( \Gamma \gg 1 \), it is strongly coupled.

Both in ground-based and microgravity experiments, complex plasmas are usually in a strongly coupled regime. An uncorrelated gaseous-like phase can be seen when there is a strong energy influx into the sub-system of grains due to instabilities, which causes a substantial increase of \( T_d \) and, hence, decrease of \( \Gamma \). The instabilities can be induced by ion streaming [4, 7] or the spatial and/or temporal variations of grain charges [8]. In many cases the relatively simple hydrodynamic approach based on the analysis of the fluid equations allows us to catch the essential physics of these processes and, hence, to understand the major dynamical properties of complex plasmas.

The simplest approach to describe dust waves is as follows: considering the dust species as an ideal gas, it is possible to use the fluid equations for electrons, ions and dust, and write perturbations of all quantities as \( \propto \exp[i(\omega t + k \cdot \mathbf{r})] \), where \( \omega \) is the frequency, \( t \) is the time, and \( k \) the wavevector. By neglecting the variations of the grain charges and the possible anisotropy of a discharge (e.g. ambipolar electric fields), we essentially get two types of acoustic modes: the dust ion acoustic mode (DIA), i.e. the usual ion acoustic mode modified by the presence of dust and the DA wave [1], associated with the motion of the charged grains, whereas both electrons and ions provide the equilibrium neutralizing background. Assuming \( k v_T \), \( k v_T \gg \omega \) and \( k v_T \gg k v_T \), where \( v_{T_a} = \sqrt{T_a / m_a} \) is the thermal velocity and \( m_a \) is the mass (\( \alpha = d \) is for dust, \( \alpha = i \) for ions and \( \alpha = e \) for electrons), the DA DR is:

\[ \frac{\omega^2}{k^2} = \gamma_d v_{T_a}^2 + \frac{\omega_{pd}^2 \lambda_D^2}{1 + \lambda_D^2}, \]

where \( \gamma_d \) is the effective polytropic index, \( \omega_{pd} = \sqrt{4\pi Z^2 e^2 n_d / m_d} \) is the dust plasma frequency, and \( \lambda_D \) is the linearized Debye length, \( \lambda_D = \lambda_{De} = \frac{1}{m_a}, \) with \( \lambda_{De} = \sqrt{T_a / 4\pi e^2 n_a} \). If \( k v_T \ll \omega \), the phase velocity of the DA mode does not depend on the dust temperature and in the long wavelength limit \( k \lambda_D \ll 1 \) can be written as:

\[ C_{DA} = \omega_{pd} \lambda_D \equiv \sqrt{\frac{P \tau}{1 + (1 + P) \tau}} \frac{Z T_i}{m_d}, \]
where \( \tau = T_e/T_i \) and

\[
P = \frac{Z n_d}{n_e} \equiv \frac{n_i}{n_e} - 1,
\]

is the ‘Havnes parameter’ \([14]\) and characterizes the ion-to-electron density ratio. When collisions of microparticles with neutrals are taken into account—the so-called ‘Epstein drag’—the squared frequency in equation \((2)\) is replaced with \(\omega (\omega + i \gamma)\), where \(\gamma\) is the corresponding damping rate.

The dispersion properties of liquid plasmas significantly deviate from those of ideal gaseous plasmas discussed above, and there are several different theoretical approaches \([2]\). However, for typical experimental conditions, the neutral gas friction prevails over the coupling correction term and when the momentum exchange rate becomes comparable to \(\omega_{pd}\) the difference between the DR of ideal and strongly coupled plasmas can be washed away completely, especially when \(\omega \lesssim 0.6 \omega_{pd}\) \([15, 16]\).

Equation \((2)\) represents the simplest (multispecies) form of DA waves, but several other theories exist: the DR is modified in the presence of ion streaming and charge variations \([4, 7, 8]\), collisions can couple DA and DIA branches \([17]\), etc. Below we discuss the role of these factors in our experiment.

3. Experimental set-up

The plasma crystal experiment PKE-Nefedov \([13]\), on the ISS, allowed us to study basic phenomena in complex plasmas under microgravity conditions \([18]\). Dust particles of diameter \(3.4 \mu m\) and mass \(3 \times 10^{-11} \text{g}\) were injected by a dispenser into argon plasma produced in a symmetrically driven parallel plate capacitively coupled rf discharge. They were illuminated by a thin \(\simeq 150 \mu m\) laser sheet and recorded by a video camera, see figure \(1\), with frame rate 25 Hz and \(768 \times 576\) pixels spatial resolution.

\(\text{Figure 1.}\) Snapshot of the field seen by the overview camera, for \(f = 23\) Hz. The selected stripe used for the data analysis is given by equation \((5)\) with \(a = 0.6\) cm, \(b = 2.06\) cm and \(\epsilon = 0.04\) cm.

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For the wave excitation a low-frequency function generator (FG) voltage was applied to
the rf electrodes, in addition to the rf. The excitation frequency was increased between intervals
of 6 s: from \( f = 6 \) to 10 Hz the increase is \( \Delta f = 1 \) Hz, from \( f = 10 \) to 20 Hz, \( \Delta f = 2 \) Hz and
from \( f = 20 \) to 35 Hz, \( \Delta f = 2.5 \) Hz. Four different neutral pressures were explored: \( p = 12.2, 11, 9.5 \) and 8 Pa, for each pressure the FG amplitude was ramped up linearly with frequency,

\[
A(f) = A_{\text{max}} \frac{f - f_1}{f_2},
\]

with (a) \( f_1 = 3.2 \) Hz and \( f_2 = 18.8 \) Hz or (b) \( f_1 = 5.6 \) Hz and \( f_2 = 16.4 \) Hz, in both cases the
amplitude saturated at \( A_{\text{max}} = 26 \) V (due to technical limitations). All those studies were carried
out for two different densities of dust. (First, there was a sequence of three 2 s injections, the
eight frequency ramps described above are performed, then, after cleaning the chamber, there
was a sequence of four 2 s injections to reach higher density, and the eight ramps were repeated;
unfortunately, it was not possible to control the exact quantity of dust injected.)

As usual, a void [13] was formed in the centre of the dust cloud. Upon excitation, a
channel with excited DA waves was created—a ‘waveguide’. The formation of the waveguide
can be attributed to the fact that only the outer ring of the lower electrode was connected to the
FG [19]. Hence, waves in the cloud could only be driven in a region above the outer ring, which
is clearly seen in figure 1. Without excitation, no waves were visible, but a very slow vortex
(with angular velocity \( \sim 10^{-2} \) Hz) was situated in the position of the ‘waveguide’. The related
dynamics is not the object of the present work. An estimate of the value of \( n_d \) in the ‘waveguide’
is found by evaluating the number of dust particles in a rectangle defined by \( 1.8 < x < 2.3 \) cm
and \( 1.1 < y < 1.3 \) cm, i.e. inside the wave channel. Taking into account the thickness of the
laser sheet (\( \sim 150 \mu\text{m} \)), we have \( n_d \simeq 2 \times 10^5 \text{ cm}^{-3} \) (this value is taken when no excitation is
present, but similar values are found during the excitation).

4. Experimental DR

The data are analysed in the following way. Inside the wave channel, we select a ‘stripe’ of
thickness \( \epsilon \) that is approximately perpendicular to the wave front and whose lower line can be
written as:

\[
s(y) = \frac{b - y}{a},
\]

as plotted in figure 1. Next, we sum up the intensity \( I \) of each pixel located, within a given
approximation, along a segment perpendicular to \( s \) (that is parallel to the wave front) and inside
the stripe. Assuming that the intensity is proportional to the dust number density, in this way,
we evaluate the amplitude of the wave as a function of \( y \), or, equivalently, \( s \), as illustrated in
figure 2(a).

The choice of the datasets (frequency ramps) and the parameters \( a, b, \) and \( \epsilon \) (i.e. the region
were we study the waves) is determined by the requirements of being in a (quasi) linear regime
with reasonably resolved oscillations, namely the following.

(1) The range of frequencies where the waves are present and resolvable is about 18–33 Hz.
Only one or two crests are visible near the void border for \( f < 18 \) Hz and for \( f > 33 \) Hz
the resolution reduces significantly. This frequency range is strongly reduced for some of
the ramps, especially for those at lower dust density. Hence, we discard them and proceed
by analysing only the ramps at higher dust density.
Figure 2. (a) Profile of the wave along $y$ (or $s$), at frequency $f = 23$ Hz, as evaluated in the stripe shown in figure 1, at the same instant. (b) Intensity (dust number density) profiles in the stripe shown in figure 1 averaged over 3 s for $f = 0$ Hz and over 5 s for the others frequencies (in Hz). The void location is indicated. Note the different ranges in $y$.

(2) The assumption of constant background density $n_0$, or at least a length scale of the density variation much longer than the wavelength, should be satisfied. We evaluated the density along several stripes for various ramps, before the ramp starts and during the ramp, by averaging the resulting intensity (as shown in figure 2(a)), over a few seconds. In figure 2(b), the profile of the average intensity for three values of $f$ is presented for the same ramp as the previous figures, but the trend observed is typical. Before the excitation starts, a strong peak appears at the void boundary, this peak is then suppressed by the oscillations, leaving an approximately flat area. After the peak is suppressed, the intensity (density) changes slightly along $s$, with the characteristic inhomogeneity length being at least $\approx 20$ longer than the observed wavelength.

(3) The linear wave approximation is applicable provided that the amplitude of the waves is small enough, so that the observed intensity (density) variations are small, $\delta I/I \lesssim 1$. Most of the datasets exhibit $\delta I/I \simeq 0.2$–$0.4$ and the ratio decreases with increasing $f$, so the higher the frequency, the more linear the system (however, above $f \sim 32$ Hz the resolution becomes too low). The reason for this is that the amplitude of the particle oscillations and, hence, the density perturbations decrease with the frequency (for individual particles the velocity amplitude scales as $\propto f^{-1}$, provided $f \gg \gamma$). All the experiments performed before the one reported here had $\delta I/I > 1$ [2], this is the first attempt to have ‘purely linear’ DA waves. It can be shown that the corrections to the frequency due to nonlinear terms scale as $(\delta \omega/\omega) \sim (\delta I/I)$, so we can reasonably consider our values of $\delta I/I$ to be small enough.

(4) The last requirement is that in the range $\gamma \lesssim 2\pi f \lesssim \omega_{pd}$, the plot of the frequency versus the wavenumber is approximately linear and it passes through the origin. If this requirement is satisfied, we can construct an acoustic (linear) branch of the DR that has to have the form
\( \omega / k = \text{const.} \) We have plotted \( f \) versus the inverse wavelength \( \lambda^{-1} \) (equivalent to \( \omega \) versus \( k \)), for all the datasets that were well resolved. The wavelength \( \lambda \) was calculated in each frame, as shown in figure 2(a), and then averaged over 5 s for each frequency (we left out 1 s at the beginning of each step to exclude transient phenomena). The errors were estimated as the standard deviation over the time series.

All the ramps where analysed, the eight ones at lower density were discarded, as stated in point 1 above. Of the remaining eight, some where too noisy and some did not meet points 2 and/or 4 above. We eventually found that the best data were obtained in two ramps at \( p = 12.2 \text{ Pa} \). The first ramp is the one already treated in figures 1 and 2 with the stripe shown in figure 1. The FG amplitude is given by equation (4) with \( f_1 = 3.2 \) and \( f_2 = 18.8 \text{ Hz} \) (i.e. case (a)). Within that stripe, we took the interval \( 1.01 \leq y \leq 1.19 \text{ cm} \), where the intensity is approximately constant at the beginning and when \( f \) increases the inhomogeneity remains very small. We have also tried to take the region where the density peak was present and that becomes flat with the oscillations, however, the plot of \( f \) versus \( \lambda^{-1} \) does not pass through the origin in this case, probably due to strong nonlinearities caused by the neighbourhood of the void. The second ramp has the same experimental conditions as the first one, only \( f_1 = 5.6 \text{ Hz} \) and \( f_2 = 16.4 \text{ Hz} \) (case (b)). For this ramp, we found that the best stripe has \( a = 0.6 \text{ cm} \) (as before) and \( b = 2.19 \text{ cm} \), and we took the interval \( 1.22 \leq y \leq 1.30 \text{ cm} \); this interval is quite narrow, but outside it the background density cannot be considered flat.

The resulting DRs are plotted in figure 3. The slope of the line joining the experimental points is the experimental phase velocity \( C_{DA}^{\text{exp}} \). The two dashed lines delimit the angular sector including all the possible slopes. For figure 3(a), we found \( C_{DA}^{\text{exp}} \) between 2.0 and 2.5 cm s\(^{-1}\), whereas, for figure 3(b), \( C_{DA}^{\text{exp}} \) is between 1.7 and 2.2 cm s\(^{-1}\), but this second ramp apparently has a bigger influence of nonlinearities, as can be seen by the big error bars for some of the frequencies.

5. Comparison with the theory

For the present experimental situation, one can expect \( \tau \gg 1 \) and \( P \gtrsim 1 \), so that equation (3) reduces to

\[
C_{DA} \simeq \sqrt{Z T_i m_d}.
\]

(6)

With the phase velocity estimated above and room temperature ions, \( T_i \approx 0.025 \text{ eV} \), we deduce that the dust charge is \( Z \approx 3000 \). This appears to be a very reasonable value as the orbit motion model [2, 20] yields \( Z \) in the range 3000–9000 for electron temperature \( T_e \) between 1 and 3 eV, for the single particle, and those values reduce to \( Z \sim 1000 \) for \( P \approx 1 \). The effect of ion–neutral collisions [21] can give up to 50% reduction for the plasma conditions studied. With such estimates for the charge, assuming reasonable ion density \( n_i \approx 3 \times 10^8 \text{ cm}^{-3} \) and taking into account that \( n_e < n_i \), we find \( P = Z n_d / n_e \geq 2 \), which, considering all the approximations made, is in acceptable agreement with the theoretical predictions.

Now let us discuss the role of various factors that may cause deviation of the DR from the simplest acoustic regime, equation (3). First, the evaluation of the dust plasma frequency yields \( \omega_{pd} \approx 450 \text{ s}^{-1} \), so that \( \omega / \omega_{pd} \lesssim 0.5 \) and, therefore, the theory predicts no difference between the ideal and coupled regimes. Thus, we can safely apply equation (6) even if \( \Gamma \) is rather large.
Figure 3. Frequency $f$ plotted against $1/\lambda$ for the data for $p = 12.2$ Pa. The two dashed lines represent the extreme fit to the experimental data with the requirement of passing through the origin. (a) Ramp with the amplitude by equation (4), case (a), with the stripe by equation (5), $a = 0.6$ and $b = 2.06$ cm; (b) Ramp for case (b) with $a = 0.6$ and $b = 2.19$ cm.

(which is quite likely in our case). Also, the Epstein damping rate is $\gamma/2\pi \simeq 7$ Hz, which is 2–3 times smaller than the (smallest) excitation frequency used for the analysis. Furthermore, theories [8, 10] that take into account the influence of the charge variations and anisotropy on the DA waves show that the deviation from equation (3) occurs in a presence of substantial dc electric fields. The magnitude of the dc field should be $\simeq 5–10$ V cm$^{-1}$ (for our case), i.e. comparable to that necessary for the levitation of particles under gravity [10]. In our experiment, however, the particle cloud occupies a ‘quasi-isotropic’ region in the vicinity of bulk discharge, where the magnitude of weak ambipolar field is $\simeq T_e/eL \simeq 0.3–1$ V cm$^{-1}$ (here, $L = 3$ cm is the distance between the rf electrodes). An additional argument that the dc field in our experiment is indeed too small to affect the DR follows from the fact that particles noticeably reacted to the FG voltage of a few volts: even neglecting the, undoubtedly significant, screening in the plasma, this gives us a field of less than $\simeq 1$ V cm$^{-1}$.

Yet another argument that the magnitude of the dc field in our experiments was small derives from the fact that no self-excited waves—typical for ground-based experiments under similar conditions—were observed. This suggests that the field is substantially smaller than a few volts per cm. This fact makes a difference to many previous experiments (e.g. [6, 9]) where external dc fields caused substantial ion drift and presumably modified the resulting DR (as compared to the quasi-isotropic case investigated here).

6. Conclusions

We performed a series of experiments under microgravity conditions where longitudinal DA waves were excited in a quasi-isotropic 3D complex plasma cloud, by applying ac electric modulation of variable frequency. The amplitude of excitation was varied with frequency to
ensure sufficiently weak (‘linear’) perturbations of the dust density. The DR was obtained by measuring the induced density perturbations, revealing fairly good agreement with the simple multispecies theory [1].

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References

[1] Rao N N et al 1990 Planet. Space Sci. 38 543
[2] Fortov V E et al 2005 Phys. Rep. 421 1
[3] Chu J H et al 1994 J. Phys. D: Appl. Phys. 27 296
[4] Barkan A et al 1995 Phys. Plasmas 2 3563
[5] Thompson C et al 1997 Phys. Plasmas 4 2331
[6] Merlino R L et al 1998 Phys. Plasmas 5 1607
[7] Molotkov V I et al 1999 JETP 89 477
[8] Fortov V E et al 2000 Phys. Plasmas 7 1374
[9] Zobnin A V et al 2002 JETP 95 429
[10] Yaroshenko V V et al 2004 Phys. Rev. E 69 066401
[11] Nunomura S et al 2002 Phys. Rev. Lett. 89 035001
[12] Zhdanov S et al 2003 Phys. Rev. E 68 035401
[13] Nefedov A P et al 2003 New J. Phys. 5 33
[14] Havnes O et al 1987 J. Geophys. Res. A 92 2281
[15] Rosenberg M and Kalman G 1997 Phys. Rev. E 56 7166
[16] Kaw P K and Sen A 1998 Phys. Plasmas 5 3552
[17] Ivlev A V et al 1999 Phys. Plasmas 6 741
[18] Morfill G E et al 1999 Phys. Rev. Lett. 83 1598
[19] Ivlev A V et al 2003 Phys. Rev. Lett. 90 055003
[20] Allen J E 1992 Phys. Scr. 45 497
[21] Khrapak S et al 2005 Phys. Rev. E 72 016406