Power quality comprehensive evaluation method based on fuzzy mathematics and cloud theory

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Abstract. In the comprehensive evaluation of power quality, there exist problems such as the boundary of evaluation index classification and weight distribution which is too subjective or objective. Based on the theory of fuzzy mathematics and cloud, the power quality comprehensive evaluation method was proposed to solve the two problems. This method improves the traditional fuzzy comprehensive evaluation method through variable weight theory and cloud theory, which are fixed weight calculation and membership function construction. Firstly, the superposition variable weight construction method of excitation type in variable weight theory is selected to calculate the weight vector. Secondly, the normal cloud model in cloud theory is used to construct the membership function to comprehensively evaluate power quality. Compared with the traditional fuzzy comprehensive evaluation method, the improved fuzzy comprehensive evaluation method solves the influence of the actual situation of evaluation factors on the index weight and avoids the influence of subjective factors in the membership function construction process of the traditional fuzzy comprehensive evaluation method, making the evaluation results more reasonable and credible. Finally, a practical example is given to prove that the model is more accurate and reasonable.

1. Introduction
In today's social and economic development, more and more new electrical equipment is put into use, which makes the load structure of the power system become more complex and diverse. Not only for the normal operation of electric power enterprises, safety and economic aspect, also is related to all the normal operation of electrical equipment. Therefore, reasonable and objective evaluation of power quality has become the trend of constructing power quality market today, and it is also a favorable guarantee for the cost reduction and benefit improvement of individual users and power enterprises. How to achieve a more scientific and objective comprehensive evaluation of power quality is of great significance [1].

At present, the commonly used assessment methods are roughly divided into four categories [2]. Methods based on intelligent optimization algorithm [3-5], probabilistic statistics principle [6-8], comprehensive mathematical theory [9-12], matter-element analysis [13], etc. Method based on intelligent optimization algorithm for to review object huge amount of data demand, the basis of the model constructed by the lack of physical significance. Method based on probability and statistics analysis, depended on the selection of basic value, poor results comprehensive and comprehensive. Based on the matter-element analysis method to deal with problems and things as a certain mathematical model, easy to ignore the handle and solve the randomness and fuzziness in the process of things. Literature [10] application of the theory of fuzzy mathematics comprehensive evaluation of power quality, using the
method of performance comprehensive evaluation of power quality, satisfactory results have been achieved.

Considering the influence of the actual situation of evaluation factors on the index weight, and the influence of subjective factors in the membership function construction of fuzzy comprehensive evaluation method. Therefore, a comprehensive power quality assessment method based on fuzzy mathematics and cloud theory is proposed. Firstly, according to the actual situation of the substation, the appropriate index value is selected and the variable weight vector is calculated by the superposition variable weight method. Then the normal cloud model in cloud theory is adopted to improve the membership function structure, determine the fuzzy relationship matrix; Comprehensive evaluation of power quality.

2. Evaluation index system
Determine the power quality evaluation index set \( X = (X_1, X_2, \cdots, X_{n-1}, X_n) \), rating \( Q = (Q_1, Q_2, \cdots, Q_{m-1}, Q_m) \) is based on the power quality standard formulated by the National Technical Committee of Standardization Administration of China (TC1) and the National Technical Committee on Electromagnetic Compatibility of Standardization Administration of China (TC246). In this paper, six measures of power quality in the standard research and analysis, frequency deviation \( X_1 \), voltage deviation \( X_2 \), voltage fluctuation \( X_3 \), voltage flicker \( X_4 \), harmonic voltage \( X_5 \), three-phase imbalance degree \( X_6 \), evaluation results are divided into I ~ V level.

In order to eliminate the influence of different dimensions or orders of magnitude among different indexes, it is necessary to standardize the range of evaluation indexes. Namely:

\[
    r_{ij} = \frac{\max(X_{ij}) - X_{ij}}{\max(X_{ij}) - \min(X_{ij})} \tag{1}
\]

In the formula, \( r_{ij} \) represents the standardized value of the j index in the data of group i, and \( X_{ij} \) represents the value of the j index in the data of group i.

3. Improve the comprehensive evaluation method of fuzzy mathematics
Fuzzy comprehensive evaluation method is mainly used in the study of fuzzy mathematics, which can make an overall evaluation of things under the condition that mathematical theory deals with complicated situations or under the influence of various factors. When using the fuzzy comprehensive evaluation method, the weight calculation mostly adopts the fixed weight method, and it is easy to ignore the influence of the actual situation on the index weight. Because the form of membership function in the traditional fuzzy comprehensive evaluation method has no clear definition and standard, it is mostly constructed subjectively, which makes the evaluation result more subjective.

In view of the shortcomings of the traditional comprehensive assessment method, the superposition variable weight \(^{[12]}\) construction model with consideration of incentives and the membership function construction based on the normal cloud model are introduced. First to determine the weight of each evaluation index vector, and then construct membership functions to determine the fuzzy relationship matrix and fuzzy comprehensive evaluation vector synthesis, through the evaluation vector calculation for power quality assessment.

The flow chart of improved fuzzy comprehensive evaluation method is shown in Figure 1.
3.1. Determine the weight

Step S021: Factor set \( F = \{ f_1, f_2, \ldots, f_n \} \) is considered to evaluate the power quality of the monitoring point in the evaluation object set \( U = \{ u_1, u_2, \ldots, u_m \} \). The evaluation matrix \( R = (r_{ij})_{m \times n} \) after normalized treatment of range is, where \( r_{ij} = f_j(u_i) \) represents the evaluation state value of \( u_i \) under factor \( f_j \).

Step S022: Observe the distribution characteristics of the normalized value of the range of the object to be evaluated, select the appropriate \( \rho_j \) and calculate \( \rho_j(r_{ij}), i=1,2,\ldots,m; j=1,2,\ldots,n \); Among them: \( \rho_j \) is expressed as the single factor we operator, \( \rho_j(r_{ij}) \) is expressed as the corresponding single factor \( r_{ij} \).

Step S023: mark the average goodness \( \bar{\rho}_i = \sum_{j=1}^{n} w_j \rho_j(r_{ij}) \) of \( u_i \), and determine the excitation weight adjustment operator \( \Delta^i_w(t) \) accordingly; Considering the relative importance factor of the object to be evaluated, the thought of deviation maximization is studied and analyzed to calculate the optimal weight \( w = (w_1, w_2, \ldots, w_n) \).

Step S024: Select the appropriate combined parameter \( \propto \) according to the actual situation, and calculate the superposition variable weight \( w_{ij}(X) = w_j + \propto w_j \Delta^i_w(\rho_j(r_{ij})) \), i=1,2,\ldots,m; j=1,2,\ldots,n \).

3.2. Determine the fuzzy relation matrix

Cloud theory \(^{[15]}\) can effectively solve the uncertainty of things as well as some fuzzy problems and concepts. The normal cloud model in the cloud theory has the characteristics of mathematical expression and widely used, so this paper is based on this model.

Definition 1 If any random number \( x(x \in U) \) satisfies: \( x \sim N(Ex, En^2) \), where \( En' \sim N(E_n, He^2) \), the certainty of qualitative concept \( C \) satisfies:

\[
\mu_c(x) = \exp \left[ \frac{-(x - Ex)^2}{2(En')^2} \right] \quad (2)
\]
Then the distribution of $x$ in the quantitative domain $U$ is called a normal cloud. The expected value $E_x$, entropy $E_n$ and super entropy $He$ are used to form the cloud membership function, and $E_x$ is expressed as the distribution center of normal cloud. $E_n$ the expression of the meaning uncertainty of the characteristics of the evaluation objects reflects the uncertainty of the selection of the evaluation objects and the description of the scope of the evaluation objects. $He$ is a measure of randomness and fuzziness in $E_n$, which reflects the correlation of uncertainty among the objects to be evaluated in the evaluation process.

According to the power quality standards formulated by The National Technical Committee of Voltage and Current Grade and Frequency of Standardization Administration of China (TC1) and The National Technical Committee on Electromagnetic Compatibility of Standardization Administration of China (TC246), the grade boundaries of the evaluation indexes are shown in Table 1.

| Table 1. Grade limit of evaluation indices. |          |
|-------------------------------------------|----------|
| Evaluation index | Class normal cloud model parameters $(E_x, E_n, He)$ |
| $X_1$ | $\leq 0.05$ | $\leq 0.10$ | $\leq 0.15$ | $\leq 0.20$ | $\leq 0.20$ |
| $X_2$ | $\leq 1.20$ | $\leq 3.00$ | $\leq 4.50$ | $\leq 7.00$ | $\leq 7.00$ |
| $X_3$ | $\leq 0.50$ | $\leq 1.00$ | $\leq 1.50$ | $\leq 2.00$ | $\leq 2.00$ |
| $X_4$ | $\leq 0.20$ | $\leq 0.50$ | $\leq 0.80$ | $\leq 1.00$ | $\leq 1.00$ |
| $X_5$ | $\leq 1.00$ | $\leq 2.00$ | $\leq 3.00$ | $\leq 5.00$ | $\leq 5.00$ |
| $X_6$ | $\leq 0.50$ | $\leq 1.00$ | $\leq 1.50$ | $\leq 2.00$ | $\leq 2.00$ |

Take the boundary of the numerical classification level of the index to be evaluated as the boundary $[a, b]$ $(a < b)$, and calculate the cloud parameters through $a$ and $b$. The calculation formula is as follows:

$$E_x = \frac{a + b}{2}$$  \hspace{1cm} (3)

$$E_n = \frac{b - a}{6}$$  \hspace{1cm} (4)

$$He = s$$  \hspace{1cm} (5)

Type: $s$ for constant, can be based on the characteristics and the evaluation index data adjusted by the actual situation. (He=0.0001 or 0.0002)

The hierarchical boundary cloud model is obtained through calculation and analysis of formulas (3), (4) and (5), as shown in Table 2.

| Table 2. Grade limit cloud model of evaluation indices. |          |
|-------------------------------------------|----------|
| Index | Class normal cloud model parameters $(E_x, E_n, He)$ |
| $X_1$ | (0.0255, 0.0795, 0.1795, 0.1795, 0.3000, 0.0001) |
| $X_2$ | (0.6050, 2.1450, 3.7050, 5.7950, 8.5000, 0.0001) |
| $X_3$ | (0.0850, 0.1017, 0.1017, 0.1017, 0.3667, 0.0001) |
Aiming at the problem that the membership function construction is relatively subjective in the traditional fuzzy comprehensive evaluation method, this paper combines the normal cloud model in cloud theory with the fuzzy mathematics theory to construct the membership function. The six power quality evaluation indexes stipulated by the state are regarded as cloud droplets, and a random number $E_n'$ that $E_n$ and $H_e$ obey the distribution law of normal cloud model can be obtained through analysis. Finally, the determination degree $k$ between the value $x$ and the normal cloud can be obtained. The calculation formula is as follows:

$$k = \exp\left[\frac{-(x - Ex)^2}{2(En')^2}\right]$$

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Thus, the fuzzy relation matrix $K$ can be obtained as follows:

$$K = \begin{bmatrix}
k_{11} & \cdots & k_{1m} \\
\vdots & \ddots & \vdots \\
k_{n1} & \cdots & k_{nm}
\end{bmatrix}$$

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Among them: $k_{ij}(1 \leq i \leq n, 1 \leq j \leq m)$ is the number of column $i$ of row $j$, and represents the membership degree of the corresponding grade of this index.

3.3. Determine the comprehensive evaluation result of power quality

The weight $W(X)$ and the fuzzy relation matrix $K$ are combined to obtain the fuzzy evaluation vector:

$$B = W(X) \ast K = [b_1, b_2, \ldots, b_m]$$

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According to the fuzzy evaluation vector $B$, the weighted average method is used to obtain the comprehensive evaluation value $r$ as follows:

$$r = \frac{\sum_{i=1}^{m} b_i f_i}{\sum_{i=1}^{n} b_i}$$

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Where, $f_i$ represents the score value of evaluation grade $i$, for $i=1,2,3,4,5$, respectively.

According to formula (7), there are random factors in the calculation of fuzzy relation matrix. Due to the influence of such random factors, the evaluation results need to be checked for confidence. After multiple operations, the expected $E_{rx}$ and standard deviation $E_{rn}$ of the comprehensive evaluation score can be obtained:

$$E_{rx} = \frac{r_1(x) + r_2(x) + \cdots + r_h(x)}{h}$$

10

$$E_{rn} = \sqrt{\frac{1}{h} \sum_{i=1}^{n} [r_i(x) - E_{rx}]^2}$$

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| $X_4$  | 0.1050, 0.3500, 0.6500, 0.9500, 1.5000, 0.0350, 0.0353, 0.0517, 0.2000, 0.0002 |
|-------|--------------------------------------------------|
|       | 0.001, 0.0002, 0.0002, 0.0002, 0.0002, 0.5500, 0.0533, 0.0002, 0.0002, 0.0002 |
|       | 0.2550, 0.7500, 1.2050, 1.7950, 3.0000, 0.2000, 0.1017, 0.1017, 0.1017, 0.3667, 0.0500, 0.1833, 0.0001, 0.0001, 0.0002, 0.0002 |
|       | 0.0850, 0.0850, 0.0850, 0.0850, 0.0850, 0.0850, 0.0850, 0.0850, 0.0850, 0.0850 |

5
Where, the value of $h$ is evaluated according to different situations, representing the number of operations needed (100 in this paper). $r_i(x)$ is the comprehensive evaluation value corresponding to the $i$th operation.

The $E_{rx}$ obtained by multiple operations is the evaluation value that can best represent the power quality level. $E_{rn}$ is a measure of the dispersion of the evaluation results. The larger $E_{rn}$ is, the more dispersed the results are. The confidence coefficient $\varepsilon$ is defined as:

$$\varepsilon = \frac{E_{rn}}{E_{rx}}$$  \hspace{1cm} (12)

The credibility of the evaluation result is reflected by the $\varepsilon$ value. The smaller the $\varepsilon$ value is, the greater the credibility will be. The reverse is true.

4. Case Study

Through the analysis of actual case to prove the effectiveness of the model and authenticity, the actual case is selected for analysis, and the evaluation results obtained by the model in this paper are compared with those obtained by other literature methods. The specific steps are as follows:

Step S01: Select the measured data verified in literature \cite{12} for research and analysis. The voltage level of the five monitoring points is 10kV. The data are shown in Table 3.

| Monitoring point | Frequency deviation/Hz | Voltage deviation/% | Voltage fluctuation/% | Voltage flicker/% | Voltage harmonic/% | Three-phase unbalance/% |
|------------------|------------------------|---------------------|----------------------|------------------|-------------------|----------------------|
| 1                | 0.09                   | 2.53                | 0.96                 | 0.22             | 1.12              | 0.88                 |
| 2                | 0.04                   | 1.66                | 1.05                 | 0.34             | 1.26              | 1.07                 |
| 3                | 0.19                   | 3.85                | 1.41                 | 0.47             | 1.18              | 0.83                 |
| 4                | 0.11                   | 2.01                | 0.85                 | 0.38             | 0.82              | 0.58                 |
| 5                | 0.07                   | 3.18                | 1.27                 | 0.53             | 1.35              | 1.23                 |

Step S02: Weight calculation.

Step S021: Evaluation matrix is obtained after range normalization treatment.

$$P = \begin{bmatrix} 0.552 & 0.481 & 0.791 & 1 & 0.316 & 0.415 \\ 1 & 1 & 0.525 & 0.492 & 0.108 & 0.162 \\ 0 & 0 & 0 & 0.124 & 0.220 & 0.495 \\ 0.410 & 0.766 & 1 & 0.362 & 1 & 1 \\ 0.713 & 0.208 & 0.165 & 0 & 0 & 0 \end{bmatrix}$$ \hspace{1cm} (13)

Step S023: Considering the relative importance of the object to be evaluated, study and analyze the idea of deviation maximization, solve the constant weight of the index to be evaluated, and establish the following model considering the known constraints:

$$\text{max } D = \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{k=1}^{m} w_j |r_{ij} - r_{kj}|$$ \hspace{1cm} (14)

$$\begin{cases} \sum_{j=1}^{n} w_j^2 = 1 \\ w_2 \geq w_1 \\ w_3 \geq w_4 \\ w_j \geq 0 \end{cases}, w \in \mathbb{C}, j = 1, \ldots, n$$ \hspace{1cm} (15)

The above model is solved by MATLAB software to obtain the constant weight of factors:

$$W = (0.1574, 0.1759, 0.1773, 0.1679, 0.1571, 0.1644)$$ \hspace{1cm} (16)
Step S024: Take the excitation weight operator $\Delta w_i(t) = t - \bar{\rho}_i$, $i=1,2,\ldots,m$, and calculate the excitation weight matrix:

$$\Delta W = \begin{bmatrix}
-0.033 & -0.104 & 0.134 & 0.415 & -0.269 & -0.170 \\
0.447 & 0.447 & -0.028 & -0.061 & -0.445 & -0.391 \\
-0.137 & -0.137 & -0.137 & -0.013 & 0.083 & 0.358 \\
-0.349 & 0.007 & 0.241 & -0.397 & 0.241 & 0.241 \\
0.535 & 0.030 & -0.013 & -0.178 & -0.178 & -0.178
\end{bmatrix}$$

(17)

Here, because the constant weight of the factor has high credibility, the parameter $\propto=0.1$, and the superposition variable weight can be obtained through calculation:

$$W(x) = \begin{bmatrix}
0.156 & 0.174 & 0.180 & 0.175 & 0.153 & 0.162 \\
0.164 & 0.184 & 0.177 & 0.167 & 0.150 & 0.158 \\
0.155 & 0.173 & 0.175 & 0.168 & 0.158 & 0.170 \\
0.152 & 0.176 & 0.181 & 0.161 & 0.161 & 0.168 \\
0.166 & 0.177 & 0.177 & 0.165 & 0.154 & 0.161
\end{bmatrix}$$

(18)

Step S03~S05: The membership degree is calculated by formula (7) to form a fuzzy relation matrix $K$, and then the fuzzy evaluation vector $B$ can be obtained by calculating the superposition weight $W(x)$, and then the comprehensive evaluation score and confidence coefficient of each observation point can be calculated by formula (10) (11).

In the calculation of fuzzy relation matrix, there are random factors between the evaluation index value and the normal cloud model, so the fuzzy relation matrix $K$ is not determined during the calculation, and the evaluation result needs to be evaluated with confidence calculation. In this paper, $h=100$ operations were carried out. This part of the operation process was calculated by MATLAB software. The main procedures are shown in Figure 2.
The final simulation results are shown in Figure 3:

![Figure 3. Comprehensive evaluation results of power quality.](image)

Table 4. Comparison of evaluation results

| Monitoring point | Comprehensive evaluation score | Confidence coefficient | Article rating | Literature [9] Rating | Literature [12] Rating | Literature [14] Rating |
|------------------|-------------------------------|------------------------|----------------|------------------------|------------------------|------------------------|
| 1                | 2.0226                        | 0.00015                | II             | II                     | II                     | II                     |
| 2                | 2.1795                        | 0.00149                | II             | II                     | II                     | II                     |
| 3                | 2.7946                        | 0.00155                | III            | III                    | III                    | II                     |
| 4                | 1.8997                        | 0.00007                | II             | II                     | II                     | II                     |
| 5                | 2.605                         | 0.00022                | III            | III                    | III                    | III                    |

As shown in Table 4, the power quality comprehensive assessment sequence of these 5 monitoring points is: 4>1>2>5>3. The results evaluated in this paper are consistent with the results of mutation decision analysis in literature [9] and variable weight comprehensive analysis in literature [12]. And compared with the literature [14] power quality comprehensive analysis and monitoring of 3 and 5 different sort, other observation points, the same variable weight synthesis in cloud theory in this paper on the basis of considering the monitoring of significant difference index, analysis of the monitoring points 3, the indicators on III level, closer to the power quality indicators qualified restrictions, judged III more reasonable level, so think 5 higher quality monitoring. In addition, after the evaluation results are obtained by the evaluation method in this paper, the credibility information is also given through the confidence coefficient. The confidence coefficient in this paper is small, which indicates that the smaller the degree of dispersion, the higher the credibility.

5. Conclusion

In this paper, the variable weight theory of superposition of the motivational variable weight vector is introduced into the fuzzy comprehensive evaluation method, solves the actual situation of evaluation factors on the influence of the index weight.

The evaluation model presented in this paper can not only obtain satisfactory comprehensive evaluation results but also provide confidence information of the evaluation results. Practical examples show that the evaluation results of this model are reasonable and reliable, and it can effectively evaluate the evaluation objects. Moreover, it is adaptable and easy to be programmed, which provides a new method for power quality assessment.
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