Angular Fulde-Ferrell-Larkin-Ovchinnikov superfluid due to self-one-dimensionalization in imbalanced cold fermion gases

Youichi Yanase

Department of Physics, University of Tokyo, Tokyo 113-0033, Japan
and Department of Physics, Niigata University, Niigata 950-2181, Japan

Abstract

We study the angular Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state, in which the rotation symmetry is spontaneously broken, in population imbalanced fermion gases. The superfluid gases at near \( T = 0 \) are investigated on the basis of the Bogoliubov-de Gennes (BdG) equation. We find that the angular FFLO state is stabilized in the gases confined in the toroidal trap, but not in the harmonic trap. We discuss the mechanism of the angular FFLO state based on the self-one-dimensionalization of the superfluid gas.

1. Introduction

Cold fermion gases provide vast opportunities to study novel quantum condensed states [1]. One of the goals of current studies is the realization of the FFLO state [2, 3] in population imbalanced superfluid gases [4, 5]. However, no experimental evidence has been obtained for the FFLO state in cold fermion gases. This is mainly because the cold atom gases lack the translation symmetry owing to the trap potential, and therefore the spontaneous breaking of the space symmetry has never been observed. Then, it is difficult to differentiate the FFLO state from the phase separation [6].

Although the translation symmetry is absent in cold fermion gases, the rotation symmetry can be present. Therefore, it is highly desired to produce and study the FFLO state with broken rotation symmetry. For this purpose, we have studied the angular FFLO (A-FFLO) state in which the rotation symmetry is spontaneously broken [7, 8]. We proposed that the A-FFLO state is realized in the toroidal trap, although the harmonic trap cannot produce the A-FFLO state. In other words, the FFLO state has not been observed in cold fermion gases, because the experiments were carried out by using the harmonic trap. Therefore, it is highly desired to investigate the superfluidity in the toroidal trap. In this paper, we clarify the mechanism of the A-FFLO state in the toroidal trap, and discuss why the A-FFLO state is not realized in the harmonic trap.

2. Model and Results

To discuss the imbalanced fermion gases in the trap, we adopt the two-dimensional lattice Hamiltonian given as

\[
H = -t \sum_{\langle r, r' \rangle, \sigma} \hat{c}^\dagger_{r, \sigma} \hat{c}_{r', \sigma} + \sum_{\sigma} (V(j' - \tilde{r}_0) - \mu_{\sigma}) n_{\sigma, \sigma}
\]

where \( \sigma = 1, 2 \) denote two hyperfine states, \( \tilde{r}_0 \) is the center of the trap, and \( n_{\sigma, \sigma} = \hat{c}^\dagger_{\sigma, \sigma} \hat{c}_{\sigma, \sigma} \) is the number operator of \( \sigma \) particles. The two-dimensional gas is produced by the one-dimensional optical lattice along the axial direction as well as by the pancake potential \( \omega_2 \gg \omega_1 \) with \( \omega_2 \) and \( \omega_1 \) being the harmonic trap frequency along the axial and radial direction, respectively. We take the unit \( \hbar = c = 1 \). The symbol \( \langle \vec{r}, \vec{r}' \rangle \) denotes the summation over nearest neighbour sites. The chemical potential \( \mu_{\sigma} \) for \( \sigma \) particles is determined so that the particle number of each state is \( N_{\sigma} \). The particle number and the imbalance are expressed as \( N = N_1 + N_2 \) and \( P = (N_1 - N_2)(N_1 + N_2) \), respectively. The lattice model is adopted here for simplicity, but we have confirmed that the discreteness of the lattice is negligible by assuming the small particle density \( N/N_{\text{latt}} \leq 0.1 \), where \( N_{\text{latt}} = L \times L \) is the number of lattice sites. Therefore, the following results are valid for continuous systems without lattices in the two-dimensional space. We take the unit of length \( d \) so that \( 1/2m d^2 = 1 \), where \( m \) is the mass of atoms. The last term of eq. (1) describes the s-wave attractive interaction. We assume \( U/t = -5 \) throughout this paper.

The trap potential is assumed to be \( V(r) = \frac{1}{2} \omega_{\text{ho}} (r/r_{\text{ho}})^2 + \omega_\perp \exp(-r/\xi) \) with \( \xi = 5 \). This potential describes the harmonic trap for \( \omega_\perp = 0 \) and the toroidal trap for \( \omega_\perp \neq 0 \). We found that the A-FFLO state is stabilized for any \( \omega_\perp/\omega_{\text{ho}} > 0 \), whose reason will be discussed later. We here show the results for the toroidally trapped system with \( \omega_\perp/\omega_{\text{ho}} = \frac{1}{4} \) and \( \omega_\perp/\omega_{\text{ho}} = \frac{1}{2} \) and for the harmonically trapped system with \( \omega_\perp/\omega_{\text{ho}} = 0 \).

We analyze the model on the basis of the mean field BdG equation. Details of the calculation have been shown in Ref. 7. It has been shown that the effects of the fluctuation beyond the BdG equation do not alter the qualitative results [7].

Figure 1 shows the spatial dependence of the superfluid order...
The rotation symmetry is spontaneously broken in the A-FFLO state and therefore the A-FFLO state is stabilized in the highly imbalanced gases. Since this mechanism is relevant even for a significantly weak toroidal trap \( \omega_{tr}/\omega_{ho} > 0 \), the presence of the A-FFLO state is generally expected in the toroidal trap.

Because the self-one-dimensionalization discussed above does not occur in the harmonic trap with \( \omega_{tr}/\omega_{ho} = 0 \), the A-FFLO state is not stabilized in the harmonic trap. We stress again that this is the reason why the FFLO state has never been observed in the cold fermion gases. Figures. 1(g-i) show that the rotation symmetry is not broken in the harmonic trap. We understand this result by considering the distribution of fermion atoms. The particle density is the largest at the center of the harmonic trap. Therefore, the superfluid phase is stable at the trap core, while the superfluidity is suppressed around the trap edge by increasing the population imbalance. Since the order parameter at the trap core vanishes in the A-FFLO state, the A-FFLO state is highly unstable in the harmonic trap. Thus, the harmonic trap does not produce the broken spatial symmetry in the imbalanced superfluid gases.

3. Summary and Discussion

We have shown that the A-FFLO state is stabilized in the population imbalanced fermion gases confined in the toroidal trap, but not in the harmonic trap. The formation of the R-FFLO state leads to the self-one-dimensionalization of the superfluid gas and stabilizes the A-FFLO state in the highly imbalanced gases. The search of the FFLO state in cold fermion gases has been fruitless probably because the experiments were carried out for the harmonically trapped gases. It is difficult to detect the FFLO state in the harmonic trap since no space symmetry breaking occurs. On the other hand, the rotation symmetry is spontaneously broken in the A-FFLO state. We suggest that the experiment in the toroidal trap will realize the FFLO state with a broken rotation symmetry and will obtain the unambiguous evidence for the FFLO state which has been searched for more than 40 years after the theoretical predictions [2, 3].

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