Interplay between superconductivity and pseudogap state in bilayer cuprate superconductors

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The interplay between the superconducting gap and normal-state pseudogap in the bilayer cuprate superconductors is studied based on the kinetic energy driven superconducting mechanism. It is shown that the charge carrier interaction directly from the interlayer coherent hopping in the kinetic energy by exchanging spin excitations does not provide the contribution to the normal-state pseudogap in the particle-hole channel and superconducting gap in the particle-particle channel, while only the charge carrier interaction directly from the intralayer hopping in the kinetic energy by exchanging spin excitations induces the normal-state pseudogap in the particle-hole channel and superconducting gap in the particle-particle channel, and then the two-gap behavior is a universal feature for the single layer and bilayer cuprate superconductors.

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The conventional superconductors are characterized by the energy gap, which exists in the excitation spectrum below the superconducting (SC) transition temperature $T_c$, and therefore is corresponding to the energy for breaking a Cooper pair of the charge carriers and creating two quasiparticles. However, in the cuprate superconductors, an energy gap called the normal-state pseudogap exists above $T_c$ but below the pseudogap crossover temperature $T^*$, which is associated with some anomalous properties. Although the charge carrier pair gap in the cuprate superconductors has a domelike shape of the doping dependence, the magnitude of the normal-state pseudogap is much larger than that of the charge carrier pair gap in the underdoped regime, then it smoothly decreases upon increasing doping, and seems to merge with the charge carrier pair gap in the overdoped regime, eventually disappearing together with superconductivity at the end of the SC dome. In this case, the charge carrier pair gap and normal-state pseudogap are thus two fundamental parameters of the cuprate superconductors whose variation as a function of doping and temperature provides important information crucial to understanding the details of superconductivity.

Experimentally, a large body of experimental data obtained by using different measurement techniques have provided rather detailed information on the low-energy excitations of the single layer and bilayer cuprate superconductors, where the Bogoliubov-quasiparticle nature of the low-energy excitations is unambiguously established. However, there are numerous anomalies for the bilayer cuprate superconductors, which complicate the physical properties of the low-energy excitations in the bilayer cuprate superconductors. This follows a fact that the bilayer splitting (BS) has been observed in the bilayer cuprate superconductors in a wide doping range, which derives the low-energy excitation spectrum into the bonding and antibonding components due to the presence of the bilayer blocks in the unit cell. In particular, it has been argued that this BS may play an important role in the form of the well pronounced peak-dip-hump structure in the low-energy excitation spectrum of the bilayer cuprate superconductors. In this case, an important issue is whether the behavior of the normal-state pseudogap observed in the low-energy excitation spectrum as a suppression of the spectral weight is universal or not. Within the framework of the kinetic energy driven SC mechanism, the interplay between the SC gap and normal-state pseudogap in the single layer cuprate superconductors has been studied recently, where the interaction between charge carriers and spins directly from the kinetic energy by exchanging spin excitations induces the normal-state pseudogap state in the particle-hole channel and SC-state in the particle-particle channel, then there is a coexistence of the SC gap and normal-state pseudogap in the whole SC dome. In particular, this normal-state pseudogap is closely related to the quasiparticle coherent weight, and both the normal-state pseudogap and SC gap are dominated by one energy scale. In this paper, we study the interplay between the SC gap and normal-state pseudogap in the bilayer cuprate superconductors along with this line. We show explicitly that the weak charge carrier interaction directly from the interlayer coherent hopping in the kinetic energy by exchanging spin excitations does not provide the contribution to the normal-state pseudogap in the particle-hole channel and SC gap in the particle-particle channel, while only the strong charge carrier interaction directly from the intralayer hopping in the kinetic energy by exchanging spin excitations induces the normal-state pseudogap in the particle-hole channel and SC gap in the particle-particle channel, and then the two-gap behavior is a universal feature for the single layer and bilayer cuprate superconductors.

The single common feature in the layered crystal structure of the cuprate superconductors is the presence of the two-dimensional CuO$_2$ plane, and then it is believed that the unconventional physics properties of the cuprate superconductors is closely related to the doped...
CuO$_2$ planes. In this case, it is commonly accepted that the essential physics of the doped CuO$_2$ planes is captured by the $t$-$J$ model on a square lattice. However, for discussions of the interplay between the SC gap and normal-state pseudogap in the bilayer cuprate superconductors, the $t$-$J$ model can be extended by including the bilayer interaction as follows:

$$H = \sum_{ij\sigma} C_{ia}^\dagger C_{ib} + t \sum_{i\sigma} C_{i\sigma}^\dagger C_{i+\sigma},$$

where the spin configuration rearrangements due to the pseudogap degree of freedom together with some effects of the electron single occupancy local constraint is satisfied by the full charge carrier normal and anomalous Green’s functions are obtained as,

where $J_{\text{eff}} = J(1 + \delta)^2$, $J_{\text{eff}} \perp = J_L (1 + \delta)^2$, and $\delta = \langle h_{ia}^\dagger h_{ia} \rangle / \langle h_{ia}^\dagger h_{ia} \rangle$ is the doping concentration.

For the bilayer cuprate superconductors, there are two coupled CuO$_2$ planes in one unit cell. In this case, the SC order parameter for the electron Cooper pair is a matrix $\Delta = \Delta_L + \sigma_z \Delta_T$, with $\Delta_L$ and $\Delta_T$ are the corresponding longitudinal and transverse parts, respectively.

In the doped regime without an antiferromagnetic long-range order (AFLRO), the charge carriers move in the background of the disordered spin liquid state, and then the longitudinal and transverse SC order parameters can be expressed in the CSS fermion-spin representation as,

where $\Delta_{hL} = -\chi_L \Delta_{hL}$ and $\Delta_T = -\chi_L \Delta_{hT}$, with $\chi_L$ and $\chi_T$ are the corresponding longitudinal and transverse parts of the charge carrier pair gap parameter, respectively, and the spin correlation functions $\langle S^{\dagger}_{ia} S^{\dagger}_{i+\sigma} \rangle = \langle S^{\dagger}_{ia} S^{\dagger}_{i+\sigma \sigma} \rangle = \chi_L$, and $\langle S^{\dagger}_{ia} S^{\dagger}_{i+\sigma \sigma} \rangle = \chi_T$. The result in Eq. (4) shows that as in the single layer case, the SC gap parameter in the bilayer cuprate superconductors is also closely related to the corresponding charge carrier pair gap parameter, and therefore the essential physics in the SC-state is dominated by the corresponding one in the charge carrier pairing state.

Within the framework of the kinetic energy driven SC mechanism, the electronic structure of the bilayer cuprate superconductors has been discussed, and the result shows that the low-energy excitation spectrum is split into the bonding and antibonding components due to the presence of BS, then the observed peak-dip-hump structure is mainly caused by BS, with the peak being related to the antibonding component, and the hump being formed by the bonding component. Following our previous discussions, the self-consistent equations that satisfy the full charge carrier normal and anomalous Green’s functions are obtained as,

respectively, where the full charge carrier normal Green’s function $g(k, \omega) = \Sigma^{(b)}(k, \omega)$, and the full charge carrier anomalous Green’s function $\Sigma^{(b)}(k, \omega) = \Sigma^{(b)}(k, \omega)$, the charge carrier self-energies $\Sigma^{(b)}(k, \omega) = \Sigma^{(b)}(k, \omega) + \sigma_\perp \Sigma^{(b)}(k, \omega)$ and $\Sigma^{(b)}(k, \omega) = \chi_T^{(b)}(k, \omega)$ have been obtained as.
\[ g_L^{(0)}(k, \omega) = \frac{1}{2} \sum_{\alpha=1,2} \frac{1}{\omega - \xi_{\alpha k}}, \quad (6a) \]
\[ g_T^{(0)}(k, \omega) = \frac{1}{2} \sum_{\alpha=1,2} \frac{(-1)^{\alpha+1}}{\omega - \xi_{\alpha k}}, \quad (6b) \]

respectively, where \( \alpha = 1, 2 \), the MF charge carrier spectrum \( \xi_{\alpha k} = Zt \chi_1^{(k)} - Zt \chi_2^{(k)} - \mu + (-1)^{\alpha+1} \chi_{L}(k) \), the spin correlation function \( \chi_2 = \langle S^z_{\nu k} S^z_{\nu + \alpha k} \rangle, \quad \forall k \in (1/Z) \sum_{\eta} s_k^{\eta} s_{k+\eta} \rangle, \gamma_{k}^{(k)} = (1/Z) \sum_{\xi} e_k^{\xi - \eta} \), and \( Z \) is the number of the nearest neighbor or next nearest neighbor sites. However, in the bilayer coupling case, the more appropriate classification is in terms of the normal and anomalous Green's functions within the basis of the bonding and antibonding components, i.e., the full charge carrier normal and anomalous Green's functions can be rewritten in the bonding-antibonding representation as,

\[ g_{\nu}(k, \omega) = g_{L}(k, \omega) + (-1)^{\nu+1} g_{T}(k, \omega), \quad (7a) \]
\[ \Sigma_{\nu}^{L}(k, \omega) = \Sigma_{\nu}^{L}(k, \omega) + (-1)^{\nu+1} \Sigma_{\nu}^{T}(k, \omega), \quad (7b) \]

respectively, where \( \nu = 1, 2 \), with \( \nu = 1 (\nu = 2) \) represents the corresponding bonding (antibonding) component, then the bonding and antibonding components of the self-energies \( \Sigma_{\nu}^{L}(k, \omega) \) and \( \Sigma_{\nu}^{T}(k, \omega) \) can be obtained from the spin bubble as,

\[
\Sigma_{\nu}^{L}(k, i\omega_n) = \Sigma_{\nu}^{L}(k, i\omega_n) + (-1)^{\nu+1} \Sigma_{\nu}^{T}(k, i\omega_n) = \frac{1}{8N^2} \sum_{pq} \sum_{\nu' \nu_2 \nu_3} \Lambda_{\nu' \nu_2 \nu_3}^{\nu \nu_1} \sum_{ipm} g_{\nu_2}(p + k, ip_m + i\omega_n) \Pi_{\nu_1 \nu_3}(p, q, ip_m), \quad (8a)
\]
\[
\Sigma_{\nu}^{T}(k, i\omega_n) = \Sigma_{\nu}^{T}(k, i\omega_n) + (-1)^{\nu+1} \Sigma_{\nu}^{L}(k, i\omega_n) = \frac{1}{8N^2} \sum_{pq} \sum_{\nu' \nu_2 \nu_3} \Lambda_{\nu' \nu_2 \nu_3}^{\nu \nu_1} \sum_{ipm} \times \Sigma_{\nu}^{L}(p - k, -ip_m - i\omega_n) \Pi_{\nu_2 \nu_3}(p, q, ip_m), \quad (8b)
\]

respectively, with \( \Lambda_{\nu' \nu_2 \nu_3}^{\nu \nu_1} = \frac{1}{2} \sum_{iqm} \sum_{p} D_{\nu_2}(q, ip_m) \times D_{\nu_3}(p + q, ip_m + q), \quad (9) \)

where the MF spin Green's functions \( D_{\nu_2}(q, ip_m) = B_{\nu_2 p}/(i\omega_n)^2 - \omega_n \), with the MF spin excitation spectrum \( \omega_n \) and function \( B_{\nu_2 p} \) have been given in Ref.\(^{8,16}\).

As in the single layer case, the pairing force and charge carrier pair gap are incorporated into the self-energy \( \Sigma_{\nu}^{L}(k, \omega) \), then it is called as the effective charge carrier pair gap \( \Delta^{(e)}(k, \omega) = \Sigma_{\nu}^{L}(k, \omega) \). On the other hand, the self-energy \( \Sigma_{\nu}^{T}(k, \omega) \) renormalizes the MF charge carrier spectrum\(^{8,16}\). Moreover, \( \Sigma_{\nu}^{T}(k, \omega) \) is an even function of \( \omega \), while \( \Sigma_{\nu}^{L}(k, \omega) \) is not. For a convenience, \( \Sigma_{\nu}^{L}(k, \omega) \) can be broken up into its symmetric and antisymmetric parts as, \( \Sigma_{\nu}^{L}(k, \omega) = \Sigma_{\nu}^{L}(k, \omega) + \Sigma_{\nu}^{L}(k, \omega) \), then both \( \Sigma_{\nu}^{L}(k, \omega) \) and \( \Sigma_{\nu}^{L}(k, \omega) \) are an even function of \( \omega \). As in the conventional superconductors\(^{12}\), the retarded function \( \text{Re} \Sigma^{L}(k, \omega) \) may be a constant, independent of \( \omega \). It just renormalizes the chemical potential, and therefore can be neglected. Now we define the charge carrier coherent weight as \( Z_{\nu}^{(i)}(k, \omega) = 1 - \text{Re} \Sigma_{\nu}^{L}(k, \omega) \), and then in the static limit approximation, i.e., \( Z_{\nu}^{(i)}(k, \omega) = 1 - \text{Re} \Sigma_{\nu}^{L}(k, \omega) = 0 \), the, and \( \Delta^{(e)}(k) = \text{Re} \Sigma^{L}(k, \omega) = \text{Re} \Sigma^{L}(k, \omega) = 0 \), then \( \Delta_{\nu}^{(e)}(k) = (\Delta^{(e)}(k) + (-1)^{\nu+1} \Delta_{\nu}^{(e)}(k)) + (-1)^{\nu+1} \Delta_{\nu}^{(e)}(k) + (-1)^{\nu+1} \Delta_{\nu}^{(e)}(k) \), with \( \Delta_{\nu}^{(e)}(k) = \Delta_{\nu}^{(e)}(k) = \Delta_{\nu}^{(e)}(k) = \Delta_{\nu}^{(e)}(k) \), then \( \gamma_{k}^{(e)} = (\cos k_x - \cos k_y)/2 \), and \( \Delta_{\nu}^{(e)}(k) \) can obtain the full charge carrier normal and anomalous Green's functions of the bilayer cuprate superconductors. In this case, with the help of these full charge carrier normal and anomalous Green's functions, the self-energy \( \Sigma^{(e)}(k, \omega) \) and effective charge carrier pair gap \( \Delta^{(e)}(k) \) in Eq.\(^{8}\) can be evaluated explicitly as,

\[
\Sigma_{\nu}^{L}(k, \omega) = \frac{1}{N^2} \sum_{pq} \sum_{\nu' \nu_2 \nu_3} \sum_{\sigma_1 \sigma_2 \sigma_3} \Lambda_{\nu' \nu_2 \nu_3}^{\nu \nu_1} \frac{B_{\nu_2 p} B_{\nu_2 q} + \nu_2 p + \nu_2 q}{6 \omega_n \nu_2 q \nu_2 p}, \quad (10a)
\]

\[
\Delta^{(e)}(k) = \frac{1}{N^2} \sum_{pq} \sum_{\nu' \nu_2 \nu_3} \sum_{\sigma_1 \sigma_2 \sigma_3} \Lambda_{\nu' \nu_2 \nu_3}^{\nu \nu_1} \frac{B_{\nu_2 p} B_{\nu_2 q} + \nu_2 p + \nu_2 q}{6 \omega_n \nu_2 q \nu_2 p}, \quad (10b)
\]
interlayer result shows that although there is a single electron charge carrier pair gap in the particle-particle channel, the charge carrier quasiparticle spectrum $\xi_{\nu,k} = Z_{\nu,k}^{(\nu)}$, the renormalized charge carrier pair gap $\Delta_{h}(k) = \frac{E_{\nu,k}^{(\nu)} - E_{\nu,k}^{(\nu)}}{E_{\nu,k}^{(\nu)}}$, which reflects that within the framework of the kinetic energy by exchanging spin excitations, induces superconductivity in the particle-particle channel, and then the charge carrier pair gap is dominated by the corresponding longitudinal part, i.e., $\Delta^{(1)}_h \approx \Delta^{(2)}_h \approx \Delta_{hl}$. This result is also consistent with the experimental results of the bilayer cuprate superconductor Bi$_2$(Pb)$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$, where the SC gap separately for the bonding and antibonding components has been measured, and it is found that both the antibonding and bonding components are identical within the experimental uncertainties.

Now we discuss the interplay between the SC-gap and normal-state pseudogap in the bilayer cuprate superconductors. As in the single layer case, the self-energy $\Sigma^{(b)}_1(k,\omega)$ in Eq. (10a) in the particle-hole channel can also be rewritten approximately as,

$$\Sigma^{(b)}_1(k,\omega) \approx \frac{[2\Delta_{pg}(k)]^2}{\omega + M_k},$$

where $M_k = M_{lk} + \sigma_x M_{Tk}$ is the energy spectrum of $\Sigma^{(b)}_1(k,\omega)$. As in the case of the effective charge carrier pair gap, the interaction force and normal-state pseudogap have been incorporated into $\Delta_{pg}(k) = \Delta_{pgL}(k) + \sigma_x \Delta_{pgT}(k)$, and therefore it is called as the effective normal-state pseudogap. In the bonding-antibonding representation, the self-energy in Eq. (11) can be expressed as,

$$\Sigma^{(b)}_1(k,\omega) \approx \frac{[2\Delta_{pg}(k)]^2}{\omega + M_k},$$

with $M_{k} = M_{lk} + (-1)^{\nu+1} M_{Tk}$, and $\Delta_{pg}(k) = \Delta_{pgL}(k) + (-1)^{\nu+1} \Delta_{pgT}(k)$. Substituting $\Sigma^{(b)}_1(k,\omega)$ in Eq. (12) into Eq. (13), the full charge carrier normal and anomalous Green’s functions can be obtained straightforwardly as,

$$g_{\nu}(k,\omega) = \frac{1}{\omega - \xi_{\nu,k} - \Sigma^{(b)}_1(k,\omega) - [\Delta^{(b)}_{h}^{(\nu)}(k)]^2/\omega + \xi_{\nu,k} + \Sigma^{(b)}_1(-k,\omega)} = \frac{[U_{1hk}^{(\nu)}]^2}{\omega - E_{1hk}^{(\nu)}} + \frac{[V_{1hk}^{(\nu)}]^2}{\omega + E_{1hk}^{(\nu)}},$$

$$\Sigma^{(b)}_1(k,\omega) = \frac{\Delta_{h}^{(b)}(k)}{2E_{1hk}^{(\nu)}} (k) \left( \frac{1}{\omega - E_{1hk}^{(\nu)}} - \frac{1}{\omega + E_{1hk}^{(\nu)}} \right) + \frac{\alpha_{2k}^{(b)} \Delta_{h}^{(b)}(k)}{2E_{2hk}^{(\nu)}} (k) \left( \frac{1}{\omega - E_{2hk}^{(\nu)}} - \frac{1}{\omega + E_{2hk}^{(\nu)}} \right),$$

respectively, where $\alpha_{1k}^{(\nu)} = \{[E_{1hk}^{(\nu)}]^2 - M_{1k}^2\}/\{[E_{1hk}^{(\nu)}]^2 - [E_{1hk}^{(\nu)}]^2\}$, $\alpha_{2k}^{(\nu)} = \{[E_{2hk}^{(\nu)}]^2 - M_{2k}^2\}/\{[E_{2hk}^{(\nu)}]^2 - [E_{2hk}^{(\nu)}]^2\}$.
where $\beta_{1k} = \xi_{k}^{2} - M_{k}^{2} + 2|\Delta_{h}(k)|^{2}$, $\beta_{2k} = (\xi_{k} - M_{k})^{2} + |\Delta_{h}(k)|^{2}$, while the coherence factors,

$$(U_{1h})^{2}_{1k} = \frac{1}{2} \left\{ \alpha_{1k}^{(v)} \left[ 1 + \frac{\xi_{k}}{E_{1h}} \right] - \alpha_{3k}^{(v)} \left[ 1 + \frac{M_{k}}{E_{1h}} \right] \right\},$$

$$(V_{1h})^{2}_{1k} = \frac{1}{2} \left\{ \alpha_{1k}^{(v)} \left[ 1 - \frac{\xi_{k}}{E_{1h}} \right] - \alpha_{3k}^{(v)} \left[ 1 - \frac{M_{k}}{E_{1h}} \right] \right\},$$

$$(U_{2h})^{2}_{2k} = \frac{1}{2} \left\{ \alpha_{2k}^{(v)} \left[ 1 + \frac{\xi_{k}}{E_{2h}} \right] - \alpha_{3k}^{(v)} \left[ 1 + \frac{M_{k}}{E_{2h}} \right] \right\},$$

$$(V_{2h})^{2}_{2k} = \frac{1}{2} \left\{ \alpha_{2k}^{(v)} \left[ 1 - \frac{\xi_{k}}{E_{2h}} \right] - \alpha_{3k}^{(v)} \left[ 1 - \frac{M_{k}}{E_{2h}} \right] \right\},$$

satisfy the sum rule: $|U_{1h}^{(v)}|^{2} + |U_{2h}^{(v)}|^{2} + |V_{1h}^{(v)}|^{2} + |V_{2h}^{(v)}|^{2} = 1$, with $\alpha_{3k}^{(v)} = \left[ 2|\Delta_{pg}^{(v)}(k)|^{2} \right] / \left[ \left\{ E_{1h}^{(v)} - E_{2h}^{(v)} \right\}^{2} \right]$, and then the corresponding effective normal-state pseudogap $\Delta_{pg}^{(v)}(k)$ and energy spectra $M_{k}$ can be obtained explicitly in terms of the self-energies $\Sigma_{1h}^{(h)}(k, \omega)$ in Eq. (10a) as,

$$\Delta_{pg}^{(v)}(k) = \frac{L_{2}^{(v)}(k)}{2 \sqrt{L_{1}^{(v)}(k)}},$$

$$M_{k} = \frac{L_{2}^{(v)}(k)}{L_{1}^{(v)}(k)}.$$
sults of the bilayer cuprate superconductors, since only one normal-state pseudogap is observed in the bilayer cuprate superconductors by using different measurement techniques. In combination with the previous result of the single layer case, our present study suggests that the single-layer model is sufficient for capturing the two-gap feature in cuprate superconductors.

It is well known that the many-body correlation and the related quasiparticle coherence in solids are closely related to the electron self-energy. In particular, the positions of the low-energy quasiparticle peaks in the low-energy excitation spectrum are determined by the electron self-energy. However, in the previous discussions of the electronic structure based on the kinetic energy driven SC mechanism for both the single layer and bilayer cuprate superconductors, the treatment of the charge carrier self-energy in the particle-hole channel is oversimplified, i.e., in the static limit approximation, the charge carrier self-energy in the particle-hole channel is replaced by the charge carrier coherent weight, then some subtle many-body effects from the normal-state pseudogap is abandoned, which leads to that the peak-dip-hump structure in the low-energy excitation spectrum is absent from the single layer cuprate superconductors, while the peak-dip-hump structure in the bilayer case is mainly induced by BS. Recently, the electronic structure of the single layer cuprate superconductors has been reexamined based on the kinetic energy driven SC mechanism by considering the normal-state pseudogap effect (then the many-body correlation) beyond the previous static limit approximation for the charge carrier self-energy in the particle-hole channel, and the result shows that even in the single layer cuprate superconductors, there is an obvious peak-dip-hump structure due to the presence of the normal-state pseudogap, in qualitative agreement with the numerical result based on the dynamical MF theory. In combination this result for the single layer cuprate superconductors and the previous result for the bilayer case, it suggests that both the normal-state pseudogap and BS induce the peak-dip-hump structure in the bilayer cuprate superconductors, however, the notable peak-dip-hump structure in the bilayer cuprate superconductors may be mainly dominated by BS.

The essential physics of the two-gap feature in the bilayer cuprate superconductors is the same as in the single layer case, and can be attributed to the doping and temperature dependence of the charge carrier interactions in the particle-hole and particle-particle channels directly from the kinetic energy by exchanging spin excitations. Our present results also indicate that although BS due to the presence of the interlayer coherent hopping can play an important role in the form of the peak-dip-hump structure around the antinodal point, it may have not an impact on the overall global feature for the SC gap and normal-state pseudogap parameters. This follows a fact that BS is maximum around the antinodal point, and it vanishes along the nodal direction. As an result, this momentum dependence of BS has an impact on the momentum dependence of the peak-dip-hump structure, while it does no has an effect on the momentum independence of the SC gap and normal-state pseudogap parameters. Furthermore, in the present bilayer case, we have also calculated the doping dependence of the coupling strength $V_{\text{eff}}$, and the result shows that as in the single layer case, the coupling strength $V_{\text{eff}}$ smoothly decreases upon increasing the doping concentration from a strong-coupling case in the underdoped regime to a weak-coupling side in the overdoped regime. Since the charge carrier interactions in both the particle-hole and particle-particle channels are mediated by the same spin excitations as shown in Eq. (8), therefore all these charge carrier interactions are controlled by the same magnetic interaction $J$. In this sense, both the normal-state pseudogap and SC gap in the phase diagram of the bilayer cuprate superconductors are dominated by one energy scale. This is why both $\bar{\Delta}_{\text{pgT}} \approx 0$ (then $\bar{\Delta}_{\text{pg}}^{(1)} \approx \bar{\Delta}_{\text{pg}}^{(2)}$, $\bar{\Delta}_{\text{pgL}}$) and $\bar{\Delta}_{\text{hT}} \approx 0$ (then $\bar{\Delta}_{\text{h}}^{(1)} \approx \bar{\Delta}_{\text{h}}^{(2)}$, $\bar{\Delta}_{\text{hL}}$) simultaneously in the bilayer cuprate superconductors, and then the two-gap behavior is a universal feature for the single layer and bilayer cuprate superconductors.

In conclusion, we have discussed the interplay between the SC gap and normal-state pseudogap in the bilayer cuprate superconductors based on the framework of the kinetic energy driven SC mechanism. Our results show that the single-layer model is sufficient for capturing the two-gap feature in cuprate superconductors. The weak charge carrier interaction directly from the interlayer coherent hopping in the kinetic energy by exchanging spin excitations does not provide the contribution to the normal-state pseudogap in the particle-hole channel and charge carrier pair gap in the particle-particle channel, which leads to that the transverse parts of the effective normal-state pseudogap parameter $\bar{\Delta}_{\text{pgT}} \approx 0$ and effective charge carrier pair gap parameter $\bar{\Delta}_{\text{hT}} \approx 0$ simultaneously, while only the strong charge carrier interaction directly from the intralayer hopping in the kinetic energy by exchanging spin excitations therefore induces the normal-state pseudogap in the particle-hole channel and charge carrier pair gap in the particle-particle channel, and then the normal-state pseudogap and charge carrier pair gap are dominated by the corresponding longitudinal parts, i.e., $\bar{\Delta}_{\text{pg}}^{(1)} \approx \bar{\Delta}_{\text{pg}}^{(2)}$, $\bar{\Delta}_{\text{pgL}}$, and $\bar{\Delta}_{\text{h}}^{(1)} \approx \bar{\Delta}_{\text{h}}^{(2)}$, $\bar{\Delta}_{\text{hL}}$.

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