A new lattice measurement for potentials between static $SU(3)$ sources

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Abstract. In this article, a new calculation of static potentials between sources of different representations in $SU(3)$ gauge group is presented. The results of author’s previous study $^1$ at the smallest lattice spacing $a_s \approx 0.11$ fm are shown to have been affected by finite volume effects. Within statistical errors, the new results obtained here are still in agreement with both, Casimir scaling and flux tube counting. There is also no contradiction to the results obtained in Ref. $^2$ which however exclude flux counting.

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1 Introduction

Studying the potential between static sources in QCD is still an interesting subject. At large distances, linearity of the potential leads to the confinement which is one of the most challenging issues in QCD. There are various confinement hypotheses. The Casimir scaling, sine scaling, and flux tube counting are among those hypotheses which try to describe the potential between static sources in different representations. The question of the Casimir scaling/sine scaling/flux tube counting is essential for our understanding of the confinement mechanism and to clarify the correspondence between quantum field theories at a large number of colors and string theories on certain compactified manifolds.

At short distances, potentials between two quarks can be calculated using perturbation theory. For this regime, called asymptotic freedom, the coupling constant is small enough to use perturbation theory. As the distance between quarks increases, a tube of chromoelectric flux is formed between them and a potential proportional to the quark separation is expected. Perturbation theory does not work for this region. Lattice gauge theory is one of the most successful methods which works well for this low energy regime where the potential may be defined as:

$$V(r) \simeq A/r + K r + C$$  \hspace{1cm} (1)

where $r$ and $K$ indicate the quark separation and the string tension, respectively. The first term shows the Coulombic potential which is the result of one gluon exchange at short distances. The confinement is understood from the second term which shows that the potential between the pair of quark-antiquark increases by distance. Quite a few lattice studies have devoted to the calculations of string tensions in the closed string sector (torelons) $^3$, yet only two authors, S. Deldar $^1$ and G. Bali $^2$, have recently systematically studied the situation in the open string sector and have measured the potentials between static sources for a variety of representations. Both calculations have shown good agreement with the Casimir scaling, especially based on Bali’s paper, within an accuracy of 5 percent, no violation to the Casimir scaling is detected. Here, the Casimir scaling means that the string tension for each representation is to be roughly proportional to the eigenvalue of the quadratic Casimir operator in that representation. The Casimir scaling regime is expected to exist for intermediate distances, perhaps extending from the onset of confinement to the onset of screening $^4$. There is another argument about the linear part of the potential at intermediate distances which claims that the string tension at this region is proportional to the number of fundamental flux tubes embedded into the representation $^5$. The fundamental flux or string is the one that connects a fundamental heavy quark with an anti-quark. This idea is called “flux tube counting”. In general, at very large $N$ strings do not interact, therefore, one would expect that the tension of meta-stable strings is proportional to the number of flux tubes. Of course, there are $\frac{1}{N}$ corrections. And then, if one waits for long enough, these meta-stable strings will decay into the stable strings with the given $N$-ality, whose tension is likely to be described by the sine formula or the Casimir scaling in some approximation which is not yet known so far.

The confining string with $N$-ality $k$, is usually called k-string and $\sigma_k$ is the corresponding string tension. For $SU(N)$ gauge sources at large distances, where the strings are stable and do not decay, there exist some different theories about the ratio of $\left( \frac{\sigma_k}{\sigma_f} \right)$ where $\sigma_f$ and $\sigma_k$ are the...
string tensions of the fundamental quarks and k-sources, respectively. The most trivial idea is, to assume that the total flux is carried by k independent fundamental tubes:

\[ \sigma_k = k \sigma_f \]  
\[ \text{(2)} \]

Because of charge conjugation, \( \sigma_k = \sigma_{N-k} \). Thus, for \( SU(3) \) gauge group, \( \sigma_2 = \sigma_1 \) and one universal string tension is obtained. In this case, string tensions of non-zero triality representations will be equal to the string tension of fundamental quarks at large distances. Asymptotic Casimir scaling is another theory about k-string ratios [4]:

\[ \frac{\sigma_k}{\sigma_f} = \frac{k(N-k)}{N-1} \]  
\[ \text{(3)} \]

Another hypothesis is based on calculations in brane M-theory [7] or Sine-law scaling:

\[ \frac{\sigma_k}{\sigma_f} = \frac{\sin \frac{N\pi}{k}}{\sin \frac{\pi}{k}} \]  
\[ \text{(4)} \]

In the large \( N \) limit, the Casimir and sine-law scaling will be the same and equal to \( k \).

Back to \( SU(3) \) gauge group, at large distances, larger than 1.2 fm, where the potential between two sources is large enough, a pair of adjoint sources releases from the vacuum and string breaking may happen. Then, based on the triality of the representation, we expect to see screening or change of the slope of the potential to the slope of the potential of the fundamental representation. Since there is only one independent string tension for \( SU(3) \) gauge field which is the string tension of the fundamental representation, potentials of zero triality representations like the adjoint representation are expected to be screened at large enough distances which means that the string tensions will be equal to zero and for non-zero triality representations, one expects to see the string tension of the fundamental representation.

Although in this study, potentials for distances larger than 1.2 fm are calculated, no string breaking and therefore no sign of the screening or change of the slope of the potentials to that of the fundamental representation is observed. As mentioned in the previous paper [1], it is probably because of the fact that the Wilson loops do not couple well to the screened representations. As will be explained in the next section, potentials are calculated by measuring the Wilson loops. Therefore, through this article, the intermediate string tensions and two of the theories which may be applied to this region are discussed. These theories are flux tube counting and the Casimir scaling for intermediate distances. Equations [2] [8] and [4] for stable strings which may be applied to large distances are not investigated. I should mention that no comparison with MQCD or sine scaling could be done, since as far as the author knows, no calculations in that framework are done for the meta-stable strings [5]. There are comparison with both the Casimir scaling and sine-scaling in the studies which measure the string tensions in the closed string sector (torelons) [8].

In spite of the good agreement between author’s previous calculations and the Casimir scaling, the results of one of the lattices have not been scaled well with the others. The simulations were done on a couple of anisotropic lattices with spatial lattice spacings of 0.43, 0.25 and 0.11 fm. The results for lattice spacing 0.43 fm and 0.25 fm have been in good agreement but the data for the finest lattice have not been scaled well with others, especially for higher representations. A new lattice spacing has been examined to probe the disagreement. This measurement is really important since in the author’s previous calculations, the non-scaled potentials were the ones belonged to the finer lattice. In fact, in order to get the continuum, one has to use finer lattices; therefore, the results of finer lattice should be more reliable. This is in contrary to the author’s previous calculations where the finer lattice has not behaved properly. In this paper, it will be shown that the problem with the finer lattice has been the significant smaller volume compared with others, which leads to the finite volume effect error that is one of the most important errors of lattice gauge theory calculations. On the other hand, with the new lattice spacing, \( a_s = 0.19 \) fm, the lattice is still considered fine and the volume is large enough to not encounter the finite volume error.

The hypotheses of the Casimir scaling and the flux tube counting for the meta-stable strings are also investigated and it will be shown that string tensions are roughly in agreement with both theories.

The paper is organized as the following: in section two, the Wilson loops and the potentials and in section three, the action and the lattice are discussed. Results, scaling behavior, the string tensions and their features, and the conclusion are discussed in the next sections, respectively.

## 2 Wilson loops and the static potentials

The potential between two static quarks is found by measuring the Wilson loop and looking for the area law fall-off for large \( t \):

\[ W(r, t) \simeq \exp[-V(r)t] \]  
\[ \text{(5)} \]

\( W(r, t) \) is the Wilson loop as a function of \( r \), the distance between two sources, and \( t \), the propagation time. The potential at distance \( r \) is determined from the asymptotic behavior of the Wilson loop, \( W(r, t) \):

\[ V(r) \simeq \lim_{t \to +\infty} \ln \left( \frac{W(r, t)}{W(r, t + 1)} \right) \]  
\[ \text{(6)} \]

The string tension and the coefficient of the Coulombic term may be obtained by fitting \( V(r) \) to equation [4] \( V(r) \)'s of different \( r \) are obtained from equation [6] Wilson loops of higher representations (\( R_s \)), \( W_{R_s} \), are calculated from the fundamental Wilson loop, \( U \), by the tensor product method. Trace of \( W_{R_s} \) for representations 6, 8, 10, 15 symmetric, 15 antisymmetric and 27 which are the representations studied in this paper are as the following:

\[ \text{tr}(W_6) = 1/2 \left[ (trU)^2 + trU^2 \right], \]  
\[ \text{(7)} \]
by since its value in perturbation theory differs from unity. A 16 \times 24 Action and the lattice

\[ tr(W_8) = tr^* tr - 1, \]

\[ tr(W_{10}) = 1/6 \left[ (trU)^3 + 2(trU^3) + 3trU trU^2 \right], \]

\[ tr(W_{15s}) = 1/24 \left[ (trU)^4 + 6(trU)^2 trU^2 \\
+ 8trU(trU^3) + 3(trU^2)^2 + 6trU^4 \right], \]

\[ tr(W_{15a}) = 1/2tr^* \left[ (trU)^2 + trU^2 \right] - trU, \]

\[ tr(W_{27}) = 1/4[trU^2 + (trU)^3] \left[ (tr(U^*)^2 + (trU^*)^2 \right] \\
- trU trU^* \] 

\[ \text{3 Action and the lattice} \]

A 16^3 \times 24 lattice has been used for this new measurement. The coupling constant is 2.7 and the ratio of the spatial lattice spacing to the temporal one, \( \xi = a_s/a_t \), is equal to 2.

The improved action used for this lattice is [9]:

\[ S = \beta \left\{ \frac{5}{3} \Omega_{sp} + \frac{4}{3} \xi \Omega_{tp} - \frac{1}{12} \Omega_{sr} + \frac{1}{12} \xi \Omega_{str} \right\}, \]

where \( \beta = g^2/6 \), \( g \) is the QCD coupling, and \( \xi \) is the aspect ratio (\( \xi = a_s/a_t \) at tree level in perturbation theory). \( \Omega_{sp} \) and \( \Omega_{tp} \) include the sum over spatial and temporal plaquettes; \( \Omega_{sr} \) and \( \Omega_{str} \) include the sum over 2 \times 1 spatial rectangular and short temporal rectangular (one temporal and two spatial links), respectively. For \( a_t \ll a_s \) the discretization error of this action is \( O(a_t^4) \). The coefficients are determined at tadpole-improved tree level [10]. The spatial mean link, \( u_s \), is given by:

\[ \frac{1}{3} ReTr P_{ss'} \]

where \( P_{ss'} \) denotes the spatial plaquette. In general the temporal link \( u_t \), can be determined from:

\[ u_t = \sqrt{\frac{4(ReTr P_{st})}{u_s}}, \]

where \( P_{st} \) is the spatial-temporal plaquette. When \( a_t \ll a_s, u_t \), the temporal mean link can be fixed to \( u_t = 1 \), since its value in perturbation theory differs from unity by \( O(a_t^4) \). To minimize the excited state contamination in correlation functions, spatial links are smeared [11], (APE smearing).

\[ \text{4 Results} \]

Using equation [8] and measuring the Wilson loops for a variety of \( t's \) for fixed \( r's \), the potential \( V(r) \) can be found. This process is repeated for several \( r's \) and the optimum \( V(r) \) for each \( r \) is extracted from plots like figure [1]. More details may be found in [12]. Figure [2] shows \( V(r) \) versus \( r \) for all representations using the new coupling constant. The potentials have been fitted to equation [1] which has a linear plus a Coulombic term. The error bars on the points are the sum in quadratic of statistical and systematic errors. The systematic errors are due to the change of the fit range of \( V \) versus \( r \). From the plot, one can see that the potentials are linear at intermediate distances and thus quarks are confined at this regime. At short distances, potentials are proportional to \( \frac{1}{r} \). The slopes of the potentials of higher representations are expected to decrease at large distances so that zero triality representations are screened and the slope of the potentials of quarks with non-zero triality representations changes to the slope of the potential between quarks in the fundamental representation. Change of the slope for the potential between gluons (quarks in the adjoint representation) is expected to happen at \( r \approx 1.2 \) fm, where the potential is equal to the potential energy of glueballs. The gluon anti-gluon released from the vacuum, couple to the initial sources and screening would happen. For the coupling constant used in this calculation which is \( \beta = 2.7 \), the lattice spatial spacing is about 0.19 fm and the maximum lattice distance is \( r = 8 \). Thus the maximum physical distance between static sources is about 1.5 fm which is not really large.
Sedigheh Deldar: A new lattice measurement for potentials between static $SU(3)$ sources

Fig. 2. Potentials for the fundamental, 6, 8, 10, 15a, 15s and 27 representations. The fits are based on 12800 measurements. Rough agreement with the Casimir scaling is observed at the intermediate distances. $K_{r_0}$, the dimensionless string tension for each representation is indicated in the plot. $r_0$ is the hadronic scale which is defined in terms of the force between static quarks at intermediate distance.

enough to observe significant bending of screened potentials. In fact, even in the previous measurements where the maximum lattice distance was about 2.2 fm, this change of the slope of the potential or string breaking has not been observed. As discussed in reference [1], it is probably because of the well known fact that the Wilson loops do not couple well to screened states. It should be mentioned that the errors of the large distances potentials are large enough to not have a significant role in the fitting.

5 Scaling behavior

The actual measurement by lattice simulations is represented in lattice units. To convert the units into the physical units, one has to look for a physical quantity with a known value. Then this reference quantity is measured on the lattice and by comparing the two values, the lattice spacing is extracted in physical units. The hadronic scale $r_0$, defined in terms of the force between static quarks at intermediate distance, [12] can be used as the reference quantity:

$$[r^2dV/dr]_{r=r_0} = 1.65$$  \hspace{1cm} (16)

where $V(r)$ is the static quark potential in the fundamental representation. The definition of Eq. (16) gives $r_0 \approx 0.5$ fm in a phenomenological potential model. $r_0^{-1} = 410 \pm 20$ MeV determined by C. Morningstar et al. [13] is used in this study. To set the scale and find $a_s$, I use equation (1) and the hadronic scale equation, equation (16):

$$\frac{r_0}{a_s} = \sqrt{\frac{1.65 - A}{Ka_s^2}}.$$  \hspace{1cm} (17)

For the anisotropic lattice, $Ka_s a_t$ may be found from the fits. The input aspect ratio $\xi = \frac{a_s}{a_t} = 2$ has been chosen since the difference between the input value and lattice simulations is expected to vanish in the continuum limit. As shown by previous calculations, the good scaling behavior of the fundamental string tension is a good evidence that this assumption is correct. It is now possible to show the results of different lattice simulations, using the scaled potentials and lattice distances in terms of the hadronic scale $r_0$. Figure 3 shows the static quark potential $V(r)$ in terms of hadronic scale $r_0$ for the fundamental representation. Potentials of all four measurements are in agreement. $Q$ represents the confidence level of the fit.

Fig. 3. The static quark potential $V(r)$ in terms of hadronic scale $r_0$ for the fundamental representation. Potentials of all four measurements are in agreement. $Q$ represents the confidence level of the fit.

As shown by previous calculations, the good scaling behavior of the fundamental string tension is a good evidence that this assumption is correct. It is now possible to show the results of different lattice simulations, using the scaled potentials and lattice distances in terms of the hadronic scale $r_0$. Figure 3 shows the static quark potential versus hadronic scale $r_0$ for the following four lattices: $10^3 \times 24$, $18^3 \times 24$, $16^3 \times 24$, and $16^3 \times 24$ with spatial lattice spacings 0.45 fm, 0.25 fm, 0.11 fm and 0.19 fm, respectively. The new lattice is the one with lattice spacing equal to 0.19 fm. Proper scaling for all couplings for the fundamental representation is observed. In contrast to the fundamental representation where a good scaling is obtained for all four lattices, as the dimension of representation increases, the finest lattice, $a_s = 0.11$ fm, violates the scaling. Figure 4 shows the potential for representation 6. The finest lattice
does not scale well but the data for other three measurements are in good agreement. Figure 8 which shows the potentials for representations 15a, is another confirmation of this fact. The potentials of the finest lattice do not show good scaling behavior. Probably this happens because of the smaller lattice volume of this coupling. Table 4 shows lattice parameters including lattice volumes and lattice spacings for the four lattices of this study. The volume of the finest lattice, \( a_s = 0.11 \) fm, is significantly smaller than others. Therefore if one does not want to encounter the error of finite volume effect should use larger lattice volume. This may happen by increasing the number of lattice points in each dimension or increasing the lattice spacing. The former increases the running time and is not economic. Thus the second option has been chosen and \( \beta = 2.7 \) is selected which gives the larger lattice spacing, 0.19 fm with compared to the previous finer lattice where the lattice spacing was equal to 0.11 fm. It is still finer than the two others with lattice spacings 0.45 fm and 0.25 fm. I recall that these are spatial lattice spacings and temporal lattice spacing has been kept the same by choosing appropriate aspect ratios. Potentials from this new measurement scales with the two previous measurements.

Using this new coupling constant and the two previous ones: \( \beta = 1.7 \) and \( \beta = 2.4 \), potentials between static sources are studied and parameters of potentials are obtained. Table 2 shows string tensions from the three scaled measurements. \( K_{\beta}^{2} \), the dimensionless string tension of each lattice measurement and the best estimate for each representation are indicated. The best estimate is found by the weighted average of the three lattice measurements. The first error in the best string tension is the statistical error (from the weighted average) and the second one is the systematic error of discretization determined by the standard deviation. The ratio of the string tension is proportional to the ratio of the Casimir scaling of the 7th column. Table 4 shows the coefficient of the Coulombic term obtained from each lattice measurement. Again the ratio of the coefficient of each representation to that of the fundamental representation is brought for comparison with the ratio of the Casimir scaling.

6 Discussion on string tensions

Lattice calculations 11 and 12 show that the potentials between static \( SU(3) \) sources are linear and proportional to the Casimir operator of each representation, means proportional to the eigenvalue of the quadratic Casimir operator of that representation. The proportionality of the potentials with the Casimir operator is expected at short distances where the force between heavy quarks may be described by one gluon exchange. But this behavior has not been understood for intermediate distances even though it is observed not only for \( SU(3) \) but also for all \( SU(N) \) gauge groups (examples are references 3 and 14 and references in them). Another scenario which tries to explain the linear potentials at intermediate distances, is flux tube counting. The idea is that the string tension of higher representation sources can be obtained by multiplying the number of fundamental strings times the string tension of the quarks in the fundamental representation 15. A fundamental string is a string which connects a fundamental heavy quark to an antiquark. The last column of table 4 shows the number of the fundamental fluxes of each representation. Figure 7 shows the data of lattice calculation and the thick center vortices model 15. Thick center vortices model is one of the phenomenological models which tries to describe the behavior of the linear part of the static sources potentials. Cross signs show the string ratios obtained from lattice calculation of this paper. The Casimir ratios and the number of fundamental flux tubes are indicated by circles and diamonds, respectively. Square signs represent the string tensions ratios obtained from thick center vortices model. As it is observed, lattice calculations and thick center vortices results agree qualitatively with both the Casimir scaling and flux tube counting. The possible reasoning that the string tension is larger than the number of fluxes is discussed in the second reference of 15 and also in 15. This might happen because at intermediate distances, the fundamental fluxes overlap and a positive energy is added to the binding en-
Table 1. Lattice parameters for the four lattice measurements. $\beta$ is the coupling constant; $a_s$ indicates the spatial lattice spacing and $\xi$ shows the ratio of spatial spacing to the temporal one. Number of configurations is brought in the last column. The spatial lattice volume for the finer lattice is significantly smaller than others. Since possibly, the finite volume effect error destroys the measurement, it has been excluded from further calculations.

| Lattice | $\beta$ | $\xi = \frac{a_s}{a_t}$ | $a_s(fm)$ | Spatial Volume ($fm^3$) | No configurations |
|---------|---------|----------------|----------|------------------------|-------------------|
| $10^3 \times 24$ | 1.7    | 5.0           | .43      | 79.5                   | 22400             |
| $18^3 \times 24$ | 2.4    | 3.0           | .25      | 91.1                   | 21620             |
| $16^3 \times 24$ | 2.7    | 3.0           | .19      | 28.1                   | 12800             |
| $16^3 \times 24$ | 3.1    | 1.5           | .11      | 5.5                    | 18200             |

Table 2. String tensions in terms of $r_0$ for different coupling constants, lattice sizes, and the best estimate. Ratios of string tension of each representation to that of the fundamental are roughly qualitatively in agreement with the ratios of the corresponding the Casimir numbers as well as the number of fluxes.

| Rep. | $K_{r0}^2(\beta = 1.7)$ | $K_{r0}^2(\beta = 2.4)$ | $K_{r0}^2(\beta = 2.7)$ | best estimate | $\frac{r_{C}}{t}$ | $\frac{r_{C}}{t}$ | flux No. |
|------|-------------------------|-------------------------|-------------------------|---------------|-------------------|-------------------|----------|
| 3    | 1.25(8)                 | 1.32(1)                 | 1.294(2)                | 1.295(2)(36)  | 1                 | 1                 | 1        |
| 8    | 2.60(1)                 | 2.60(3)                 | 2.88(2)                 | 2.65(1)(17)   | 2.05(1)(14)      | 2.25              | 2        |
| 6    | 2.9(2)                  | 3.00(3)                 | 3.102(5)                | 3.10(1)(16)   | 2.39(1)(14)      | 2.5               | 2        |
| 15a  | 4.4(2)                  | 4.6(1)                  | 4.68(5)                 | 4.65(4)(18)   | 3.59(3)(17)      | 4.0               | 3        |
| 10   | 4.9(3)                  | 5.4(2)                  | 5.35(2)                 | 5.35(2)(32)   | 4.13(2)(27)      | 4.5               | 3        |
| 27   | 5.9(5)                  | 6.62(6)                 | 7.48(3)                 | 7.3(1)(10)    | 5.64(2)(79)      | 6                 | 4        |
| 15s  | 7.1(5)                  | 7.6(2)                  | 8.1(1)                  | 7.97(9)(67)   | 6.15(7)(54)      | 7                 | 4        |

Table 3. Coulombic coefficients found by different lattice calculations and the best estimate. Rough agreement with the Casimir ratios are observed.

| Rep. | $A(\beta = 1.7)$ | $A(\beta = 2.4)$ | $A(\beta = 2.7)$ | best estimate | $\frac{r_{C}}{t}$ | $\frac{r_{C}}{t}$ |
|------|-----------------|-----------------|-----------------|---------------|-------------------|-------------------|
| $10^3 \times 24$ |                 |                 |                 |               |                   |                   |
| $18^3 \times 24$ |                 |                 |                 |               |                   |                   |
| $16^3 \times 24$ |                 |                 |                 |               |                   |                   |
| 3    | -.40(8)         | -.330(9)        | -.356(4)        | -.352(4)(37)  | 1                 | 1                 |
| 8    | -.60(5)         | -.93(3)         | -.69(1)         | -.71(1)(18)   | 2.02(3)(54)      | 2.25              |
| 6    | -.54(9)         | -.69(6)         | -.798(2)        | -.80(1)(20)   | 2.27(3)(61)      | 2.5               |
| 15a  | -.84(1)         | -.12(2)         | -.118(4)        | -.86(2)(33)   | 2.44(6)(97)      | 4.0               |
| 10   | -.50(2)         | -.5(2)          | -.43(1)         | -.124(1)(76)  | 3.5(1)(22)       | 4.5               |
| 27   | -.19(5)         | -.17(6)         | -.182(1)        | -.182(1)(96)  | 5.1(1)(28)       | 6                 |
| 15s  | -.16(4)         | -.21(2)         | -.228(4)        | -.227(4)(49)  | 6.5(1)(16)       | 7                 |

energy of fluxes and makes the string tension larger than the number of fluxes times the fundamental string tension.

Since the error due to the hadronic scale uncertainty is not considered in our lattice data, the lattice errors are larger than what is reported in the figure. In addition, the potential is supposed to be measured from the area fall-off at large $t$ from equation $W(r,t) \simeq \exp^{-V(r)t}$. Since for large $r$ values — especially for higher representations — the Wilson loops get too small for large $t$, and the error due to statistical fluctuations makes the measurements meaningless; therefore, calculation of the potentials using smaller $t$’s is essential. Even though a systematic error by changing the fit range or by comparing with $V$ of smaller $r$’s is obtained for potential of each representation, it seems that the string tension is overestimated expecting a larger error especially for higher representations. Figure 4 shows potential between fundamental quarks versus $t$ for $r = 3$, where $r$ indicates the lattice distance. Fitting range is shown by the solid line. As mentioned before, for higher representations, especially for large $r$, large $t$’s could not be used. Looking at this plot, it seems that one overestimates the potentials if one measures the potentials using small $t$’s.

7 Conclusion

Using a new coupling constant, SU(3) potentials between static sources for a variety of representations are obtained. String tensions are found using this new measurement and the author’s previous calculations which have shown good scaling behavior. The data for the finest lattice have been excluded since by comparison, it is observed that finite volume effect destroys the measurement.

String tensions still remain qualitatively in agreement with both the Casimir scaling and flux tube counting. In fact, the errors of the author’s lattice data of this study are still too large to discriminate between the two hypotheses.
The results of this study is in agreement with the data of [2] which have found that the potentials are proportional to the Casimir scaling but, however, clearly exclude flux tube counting. Furthermore, since the Wilson loops do not couple well with screened representation, investigation about k-string picture for large distances or stable strings could not be done.

The author would like to emphasize that even though there are some evidences of proportionality of the string tensions with the Casimir scaling, at intermediate distances, for SU(N) gauge groups, the Casimir scaling is still a puzzle. Understanding the physics of string tensions and confinement is still an open and interesting subject.

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