The reduced $\ell^p$-cohomology in degree 1 and the Poisson boundary

by Antoine Gournay

Abstract: Reduced $\ell^p$-cohomology in degree 1 (for short "LpR1") is a useful quasi-isometry invariant of graphs [of bounded valency] whose definition is relatively simple. On a graph, there is a natural gradient operator from functions to vertices to functions on edges defined by looking at the difference of the value on the extremities of the edge. Simply put, this cohomology is the quotient of functions with gradient in $\ell^p$ (of the edges) by functions who are themselves in $\ell^p$ (of the vertices).

In this talk, I will explain how, under some assumptions on the isoperimetric profile one can identify "LpR1" with a subspace of the space of harmonic functions and yields some interesting corollaries on the space of harmonic functions (for example, on lamplighter graphs). In fact, the "interesting part" of "LpR1" can be identified with a subspace of the Poisson boundary. The transport cost comes up naturally as a key ingredient in the proof.

Cutoff for non-backtracking random walks on sparse random graphs

by Justin Salez

Abstract: An ergodic Markov chain is said to exhibit cutoff if its distance to stationarity drops abruptly from near 1 to near 0 over a negligible time period. Discovered in the context of card shuffling (Aldous-Diaconis, 1986), this phenomenon is now believed to be rather typical among fast mixing Markov chains. Here we consider
non-backtracking random walks on large random graphs. Under a sparsity condition, we establish the cutoff phenomenon and determine its precise window in terms of the degree distribution. In addition, we prove that the cutoff shape converges to a remarkably simple universal limit. This is joint work with Anna Ben-Hamou.

**A new Proof of Friedman’s second eigenvalue Theorem**

by Charles Bordenave

Abstract: It was conjectured by Alon and proved by Friedman that a random d-regular graph has nearly the largest possible spectral gap, more precisely, the largest absolute value of the non-trivial eigenvalues of its adjacency matrix is at most $2\sqrt{d-1} + o(1)$ with probability tending to one as the size of the graph tends to infinity. We will discuss a new method to prove this important statement.

**On the number of zeros of linear combinations of independent characteristic polynomials of random unitary matrices**

by Joseph Najnudel

Abstract: We consider a given linear combination of characteristic polynomials of independent matrices, uniform on the unitary group in dimension $N$. In a paper with Barhoumi, Hughes and Nikeghbali, we show that the average proportion of zeros of the polynomial which is obtained tends to 1 when $N$ goes to infinity. This result is the random matrix analogue of an earlier result by Bombieri and Hejhal on the distribution of zeros of linear combinations of $L$-functions, thus being consistent with the conjectured links between the value distribution of the characteristic polynomial of random unitary matrices and the value distribution of $L$-functions on the critical line.

**Graph convergence and entropy**

by Miklós Abért

Abstract: I will talk about connections between Benjamini-Schramm convergence of finite graphs and entropy. In particular, I will outline a new approach with Benjy Weiss to sofic entropy theory of probability measure preserving group actions and a conjecture with Balazs Szegedy on the entropy of cellular automata that would imply that Luck approximation holds mod $p$. 
Cumulants and triangles in Erdos-Rényi random graphs

by Valentin Féray

Abstract: In this talk, we are interested in the number of triangles (or more generally copies of a given subgraph) in a random graph distributed with Erdos-Rényi model $G(n, p)$. It is known (Rucinski, 1988) that, whenever this number is nonzero with probability 1, then it satisfies a central limit theorem. A powerful tool to establish this result is the theory of joint (or mixed) cumulants. In particular, the key fact is that joint cumulants of independent random variables vanish. In this talk, we will explain these ideas and present two extensions. The first one consists in proving sharper bounds on cumulants, leading to an extension of the domain of validity of the central limit theorem (joint work with Pierre-Loïc Méliot and Ashkan Nikeghbali). In the second one, we weaken the independence assumption. As a consequence, we can prove a similar central limit theorem in Erdos-Rényi model $G(n, M)$ (in which presence of edges are no longer independent events). This result had been obtained with a different method by Janson (1995). These two extensions rely on the combinatorial structure of joint cumulants, in terms of set-partitions.

Existence of Ramanujan families of graphs of arbitrary degree (after Marcus-Spielman-Srivastava)

by Alain Valette

Abstract: Ramanujan graphs are $d$-regular graphs which are optimal expanders from the spectral point of view. Explicit families of Ramanujan graphs where constructed in 1986 by Lubotzky-Phillips-Sarnak and Margulis, using deep number theory. A drawback of the number-theoretic nature of the graphs was restriction on the degree: $d = 1 + (\text{primepower})$. Until may 2013, the existence of Ramanujan families for other values of $d$ was an open question. Marcus-Spielman-Srivastava then gave a probabilistic, hence purely existential proof of the existence of families of $d$-regular bipartite Ramanujan graphs, for every $d \geq 3$.

Anisotropic random matrices

by Antti Knowles

Abstract: Most of the literature on random matrices focuses on matrix models that are isotropic in the sense that their Green functions are with high probability close
to a multiple of the identity matrix. Such models include Wigner matrices, Erdos-Renyi graphs, and random band matrices. I will talk about matrix models that are not isotropic. An important family of examples is provided by sample covariance matrices whose underlying population has nontrivial correlations. I present a new method that yields local laws for such anisotropic models. Applications include the proof of the Tracy-Widom-Airy statistics near the soft edges, as well as the distribution of the eigenvectors. These results apply in the single-cut and the multi-cut cases.