A NEW PROBE OF THE DISTRIBUTION OF DARK MATTER IN GALAXIES

SUKANYA CHAKRABARTI
School of Physics and Astronomy, Rochester Institute of Technology, 84 Lomb Memorial Drive, Rochester, NY 14623, USA; chakrabarti@astro.rit.edu

Received 2011 December 6; accepted 2013 March 25; published 2013 June 21

ABSTRACT
The scale radius of dark matter halos is a critical parameter for specifying the density distribution of dark matter, and is therefore a fundamental parameter for modeling galaxies. We develop here a novel, observationally motivated probe to quantitatively infer its value within the context of the Navarro, Frenk, & White profile. We demonstrate that disturbances in the extended atomic hydrogen gas disks of galaxies can be analyzed to infer the scale radius of the dark matter halo. Our primary metric is the phase (i.e., the shape of the spiral) of the $m = 1$ mode of the disturbance in the outskirts of the gas disk, which we take to be produced by a tidal interaction. We apply the method to the Whirlpool galaxy, which has an optically visible satellite. We infer a scale radius of $\sim 17$ kpc for M51, which is consistent with expectations from dissipationless cosmological simulations. We explore potential degeneracies due to orbital inclination, initial conditions, satellite mass, and pericenter approach distance, and find our results to be relatively insensitive to these considerations. Our method of tracing the dark potential well through observed disturbances in outer gas disks is complementary to gravitational lensing.

Key words: dark matter – galaxies: dwarf – galaxies: evolution – galaxies: kinematics and dynamics – galaxies: spiral – methods: numerical

Online-only material: color figures

1. INTRODUCTION

The cold dark matter (CDM) paradigm of structure formation is successful at recovering the basic skeletal structure of the universe—the large-scale distribution of galaxies (Springel et al. 2006). However, the agreement between theory and observation is less secure when this model is applied to galactic (and sub-galactic) scales. While the existence of dark matter halos in galaxies was observationally inferred in the 1970s (Rubin et al. 1977; Bosma 1978), there are few observational probes that can be used to infer the details of the distribution of dark matter. Simulations have not yet been able to resolve the mitigating effects of gas cooling (Gnedin et al. 2004) versus outflows (Governato et al. 2010), which can act to decrease the distribution of dark matter in galaxies. In the coming decades, the CDM paradigm will be increasingly vetted against detailed observations of the innards of galaxies that will delve even more deeply into galaxy evolution.

Navarro et al. (1996, 1997; NFW) found that dark matter halos formed through dissipationless hierarchical clustering have a universal density profile. Due to the absence of scale in this hierarchical structure formation scenario, dark matter haloes are scaled versions of each other, with the total mass being the scaling parameter. The NFW density profile is given by

$$\rho(r) = \frac{\delta_c \rho_c}{(r/R_s)(1+r/R_s)^2},$$

where $\rho_c$ is the critical density for closure of the universe, and $\delta_c$ is a characteristic overdensity for the halo. The scale radius, $R_s$, is a characteristic radius where $d \log(\rho)/d \log(r) = -2$. The density switches its behavior at the scale radius, from $r^{-1}$ for $r < R_s$ to $r^{-3}$ for $r \gg R_s$, passing through the transitional $r^{-2}$ region for $r \sim R_s$. Within the NFW model, the scale radius is given by $R_s = R_{200}/c$, where $c$ is the concentration parameter that relates this characteristic radius and $R_{200}$, which is the radius at which the mean enclosed dark matter density is 200 times the critical density. The characteristic overdensity of the halo, $\delta_c$, is simply a function of the concentration parameter $c$ (NFW). The NFW profile differs from the Hernquist (1990) profile in its asymptotic behavior, scaling as $r^{-3}$ rather than $r^{-4}$ for $r/R_s \gg 1$. The scale radius of the dark matter halo along with the concentration parameter specify the density distribution for a given halo mass in the NFW formalism. Currently, these parameters are prescribed from large cosmological N-body simulations (Bullock et al. 2001; Maccio et al. 2008). It is not currently clear how we can determine these values for individual galaxies. Our aim in this paper is to develop an independent, observationally motivated probe of the scale radius of dark matter halos. We begin here by considering the NFW model, and apply it to the well-known Whirlpool galaxy (M51) to infer its scale radius.

The extended atomic hydrogen (H I) disks of galaxies provide a unique probe of galaxy evolution. They are ideal tracers of tidal interactions with satellites and the galactic gravitational potential well. We recently developed a novel method whereby one can infer the mass and relative position (in radius and azimuth) of satellites from analysis of observed disturbances in outer gas disks, without requiring knowledge of their optical light (Chakrabarti & Blitz 2009, henceforth CB09; Chakrabarti & Blitz 2011, henceforth CB11; Chakrabarti et al. 2011, henceforth CBCB; Chang & Chakrabarti 2011, henceforth CC11). We applied this method to M51 and inferred that its satellite has a mass one-third that of the primary galaxy, with a pericentric approach distance of 15 kpc. We found these estimates to be corroborated by observations (Smith et al. 1990) and recent simulation studies (Salo & Laurikainen 2000; Dobbs et al. 2010). Moreover, at the time when our simulations achieve the best fit to the H I data, the azimuth of the satellite in the simulations agrees closely with its observed location. CBCB note that the derivation of these numbers is uncertain at the factor of two level due to variations in the initial conditions of
the simulated M51 galaxy, orbital inclination and orbital velocity of the satellite. We call this method “Tidal Analysis.”

We frame the problem here in the following way—for galaxies like M51, for which we have demonstrated that our Tidal Analysis method allows us to calculate the satellite mass and pericentric approach distance, can we determine the scale radius of the dark matter halo? It is worth noting that such a procedure has viable and immediate applicability. The THINGS survey produced H I maps of local spiral galaxies (Walter et al. 2008), many of which display disturbances in the outskirts (Bigiel et al. 2010). Attempting to constrain the dark matter halo in this way is similar in spirit to earlier work on analysis of stellar tidal tails by Mihos et al. (1998). If the satellite mass and pericentric distance can be characterized, as for M51 (CBCB), then we find that the phase of the \( m = 1 \) mode directly yields the scale radius of the dark matter halo.

Our method of probing the dark matter mass distribution is independent of the stellar light and is complementary to strong gravitational lensing, which primarily probes regions interior to the Einstein radius, \( \sim 10 \) kpc in spirals (Wright & Brainerd 2000; Treu & Koopmans 2002). Our method yields constraints on the dark matter distribution in the outskirts (which are dark-matter dominated). The weak lensing signal has also been exploited by stacking the surface density contrast over large numbers of galaxies to produce mean density profiles for clusters (Sheldon et al. 2009) and total galaxy masses (Mandelbaum et al. 2006).

This paper is organized as follows: in Section 2, we review the simulation setup and the Tidal Analysis method. In Section 3, we demonstrate that the phase of the \( m = 1 \) mode can be used to quantitatively infer the scale radius of dark matter halos and apply the method to M51. We also assess the possible dependence of this metric on a range of parameters (including orbital inclination, gas fraction, bulge fraction, \( R_{200} \), equation of state (EQS), satellite mass, and pericenter approach distance). We discuss caveats and future work (including generalizations beyond the NFW model) in Section 4, and conclude in Section 5.

2. SIMULATION SETUP

The simulations we discuss here have the same setup as in CBCB, and are part of an ongoing effort to characterize dark matter and dark-matter sub-structure from analysis of disturbances in outer gas disks. We carry out smoothed particle hydrodynamic simulations using the GADGET-2 code (Springel 2005) of M51 interacting with its companion. These simulations have gravitational softening lengths of 100 pc for the gas and stars, and 200 pc for the halo. The number of gas, stellar, and halo particles in the primary galaxy is \( 4 \times 10^5, 4 \times 10^5 \), and \( 1.2 \times 10^6 \) respectively for our fiducial case. The initial structure of the dark matter halo is taken to follow a Hernquist profile that is associated with an equivalent NFW profile with the same dark matter mass within \( R_{200} \) (Springel et al. 2005). The Hernquist (1990) profile is given by \( \rho_{\text{Hernquist}}(r) = \left(M_{\text{Hernquist}}/2\pi r^2 \right) \left[ a/(r + a)^2 \right] \). Since we also require the inner density profile to be equal \( \rho_{\text{Hernquist}} = \rho_{\text{NFW}} \) for \( r < R_{200} \), this implies a relation between \( a \) and the scale length of the NFW profile. This relation is \( a = R_{\text{in}} (2 \ln(1 + c) - c/(1 + c))^1/2 \), where \( c \) is the concentration parameter. Springel et al. (2005) show that with this setup, the Hernquist and NFW density profiles are nearly identical out to \( r \sim R_{200} \).

Our models include an exponential disk of stars and gas, with a flat extended H I disk, as found in surveys of spirals (Bigiel et al. 2010). A specified fraction \( f_{\text{gas}} \) of the mass of the disk is in a gaseous component, where \( f_{\text{gas}} = 0.142 \) for the fiducial model. The mass fraction of the extended H I disk relative to the total gas mass is equal to 0.52, and its scale length is 20 times that of the exponential disk of gas and stars. These simulations employ an effective EQS of the gas following the model of Springel & Hernquist (2003; SH03). In this model, the effective pressure is related to the adiabatic index of the gas and the density and specific thermal energy for a multiphase interstellar medium (ISM) (composed of hot and cold clouds). The medium can be pictured as a fluid composed of condensed clouds in pressure equilibrium with an ambient hot gas. SH03 describe the mass exchange between the phases, which can depend on supernova feedback, cooling, and star formation. Springel et al. (2005) describe the implementation of this effective EQS in detail, and show how the pressure varies as a function of density as the effective EQS is varied. This is done by interpolating between the isothermal EQS and the pressure–density relation for the multiphase ISM; EQS = 0 corresponds to the “soft” isothermal equation of state, and EQS = 1 corresponds to a “stiff” pressure–density relation (depicted in Figure 4 of Springel et al. 2005). An intermediate value of EQS (such as EQS = 0.25) linearly interpolates between the isothermal EQS and the multiphase ISM.

In this and earlier papers on the Tidal Analysis method (CB09; CB11; CBCB; CC11), we have focused our analysis on the outskirts of galaxies \( (r > r_{\text{life}}, \text{i.e., beyond the outer Lindblad resonance} \sim 15 \) kpc for M51), where the effects of external perturbers are predominant over the intrinsic response of the gas, due to stellar spiral arms churning the gas. In the outskirts, most of the gas is in the form of H I (Wong & Blitz 2002; Bigiel et al. 2010) and as such there is little fuel available for star formation. Therefore, the incidence of supernova feedback (the rate of supernovae is taken to be proportional to the star formation rate in the SH03 model) and its effect on the multiphase ISM are correspondingly much lower in the outskirts of galaxies. As such, while processes like star formation and feedback can in general affect the temperature structure of the gas (and they do in the inner regions), in the outskirts these processes have considerably less effect. The temperature for the ambient gas in the outskirts is on average \( \sim 10^{4} \) K, which is the minimum temperature the gas can reach as a result of radiative cooling processes in these simulations (we do not include molecular cooling or metals).

The Appendix reviews how changes in the EQS (varying the EQS from isothermal to a multiphase ISM that includes heating from supernovae) may affect the phase (we find there is little change in the outskirts) and compares runs of low and high resolution that show that our metric of choice yields robust results on \( \sim 1 \) kpc scales. We cannot reliably follow the evolution of structures in these simulations on scales less than \( \sim 100 \) pc, but our resolution is sufficient for us to analyze the observed H I map of M51.

Table 1 presents the simulations discussed in this paper. Specifically, for a given concentration-scale radius value, we consider (1) variations in the gas fraction of the disk (from \( f_{\text{gas}} = 0.142 \) for the fiducial model to \( f_{\text{gas}} = 0.25 \)); (2) variations in the bulge mass fraction (we consider a fiducial model that does not include a bulge as well as a model with \( M_{\text{bulge}} = 0.012 \)); (3) variations in the orbital inclination of the satellite (from a coplanar inclination to an inclined orbit; the “e” inclination refers to an inclined orbit, with the same nomenclature as in Cox et al. 2006); (4) variations in the EQS, from an isothermal equation of state (EQS = 0) to a
multiphase ISM that includes energy injection from supernovae (EQS = 0.25); and (5) variations in the satellite mass and pericenter approach distance to within the range determined by CBCB that would allow us to fit the Fourier amplitudes. To check that our results are reasonably robust to varying $R_{200}$, we also vary the concentration parameter and $R_{200}$, holding $R_1$ constant, for the $R_1 = 17$ h$^{-1}$ kpc case. We set $h = 0.7$, which is the value of the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$. While this is not an exhaustive list of simulations (which would be computationally prohibitive), we can analyze these simulations to understand the effects of changing the scale radius on the disturbances in the outer H I disk, as well as the effects of initial conditions on this problem.

### 2.1. Review of Tidal Analysis

CBCB carried out a simulation parameter survey and compared the resultant Fourier amplitudes of the low-order modes of the projected gas surface density with the observed H I data of M51 to derive the satellite mass and pericentric approach distance. They computed the Fourier amplitudes, $a_m(r, t)$:

$$a_m(r, t) = \frac{1}{2\pi} \int_0^{2\pi} \Sigma(r, \phi, t) e^{-im\phi} d\phi$$

of the data and of the simulations as a function of time, where $\Sigma(r, \phi, t)$ is the projected gas surface density at time $t$. They calculated the residuals of the $m = 0$--4 modes for a given simulation relative to the data, for $r \gtrsim 15$ kpc, as follows:

$$S_{1-4} = \sum_m [a_{1,D}(r) - a_{1,S}(r, t)]^2 + [a_{2,D}(r) - a_{2,S}(r, t)]^2 + [a_{3,D}(r) - a_{3,S}(r, t)]^2 + [a_{4,D}(r) - a_{4,S}(r, t)]^2.$$  

Here, $a_{m,D}$ and $a_{m,S}$ denote the Fourier amplitudes for the data ($D$) and the simulation ($S$) respectively, normalized to the axisymmetric mode. The quantity $S_1 = \sum_m [a_{1,D}(r) - a_{1,S}(r, t)]^2$ is the residual of the $m = 1$ mode only. The best-fit time snapshot is that which minimizes $S_1$ and $S_{1-4}$ for a given simulation. The entire simulation set was searched accordingly. CBCB found that placing simulations on a variance versus variance plot, i.e., on an $S_1$ versus $S_{1-4}$ plot, made the best-fit simulations visually apparent. They found that the best fit to the Fourier amplitudes of the H I data of M51 occurred for a 1:3 mass ratio satellite with a pericentric distance of 15 kpc, parameters that are corroborated observationally and are in agreement with other simulation studies (Smith et al. 1990; Dobbs et al. 2010; Salo & Laurikainen 2000). Moreover, they showed that initial conditions (such as bulge fraction, gas fraction, EQS of the gas), or orbital inclination and orbital velocity of the satellite, did not significantly affect the global metric they adopted (i.e., $S_1$ and $S_{1-4}$), which allowed them to characterize the satellite mass and pericentric approach distance to within a factor of two.

CBCB found that the azimuthal location of the companion of the best-fit simulation agrees very closely with the observed location of M51’s companion, at the time when the Fourier amplitudes most closely match the data. They calculated the azimuthal location from the relative offset of the phase of the $m = 1$ mode in the simulation relative to the data. For projected gas surface density denoted $\Sigma(r, \phi)$, we calculate the phase of individual modes “$m$” by taking the fast Fourier transform (FFT), where below we have set $m = 1$:

$$\phi(r, m = 1) = \arctan \left[ \frac{\text{Imag FFT} \Sigma(r, \phi)}{\text{Re FFT} \Sigma(r, \phi)} \right].$$

The angle of the perturber in the observational frame is determined by the relative offset in the phase as follows:
\[ \Phi_{\text{perturber}} = \Phi_{\text{sat}}^{\text{sim}} - \Phi_{\text{sat}}^{\text{sim}} + \Phi_{\text{data}}^{\text{sim}}, \]
where \( \Phi_{\text{sat}}^{\text{sim}} \) and \( \Phi_{\text{data}}^{\text{sim}} \) are the phase of the \( m = 1 \) mode of the simulation and data respectively, and \( \Phi_{\text{sat}}^{\text{sim}} \) is the azimuthal location of the satellite in the simulation. This calculation is necessary as we cannot assume that the simulations and observations are exactly aligned. The quantity \( [\Phi_{\text{sim}}^{\text{sat}} + \Phi_{\text{data}}^{\text{sat}}] \) tells us how we have to rotate the coordinate system (modulo \( 2\pi \) or \( 2\pi \) for \( m = 1 \)). This procedure is similar to visual matching of (dominant) features between the simulations and the data.

In principle, the Fourier amplitudes and the phase are independent quantities. For our best-fit tidally interacting models, we find that they both independently indicate the same time of encounter (i.e., either through a minimum in \( S_1 - S_{1-4} \), or agreement in the shape of the phase). We found that this is the case for M51 (CBCB). Earlier work in this series of papers presented the basic reasoning as to why the mass-pericentric approach degeneracy in the tidal force can be broken when the time integrated response of the primary galaxy is considered (CB09), and described the method to find the azimuth of galactic satellites from the phase of the modes (CB11).

3. RESULTS

The main goal of this paper is to determine whether the scale radius of the dark matter halo in the primary galaxy can be inferred from analysis of observed disturbances in the extended H I disk of M51. Since we earlier characterized the mass and pericentric approach distance of M51’s companion, we begin by taking these quantities as inputs in our study here. (Section 3.1 presents our results on the variation of the phase of the \( m = 1 \) mode when the allowed range in satellite mass and pericenter approach, as determined by CBCB, is considered.) We begin by varying the density profile (and hence the potential depth) of the dark matter halo of M51 to see if varying the scale radius will be reflected cleanly enough in the disturbances of the extended H I disk to allow us to infer its value. We study the gas density response as M51 interacts with a 1:3 mass ratio satellite with a pericentric distance of 15 kpc (parameters we derived in CBCB), while we vary the scale radius of the dark matter halo from low to high values (\( R_s = 11-26 \ h^{-1} \) kpc).

In the NFW model, the scale radius is related to the concentration parameter and the outer radius of the dark matter halo, i.e., \( R_s = R_{200}/c \). Therefore, we can either hold \( R_{200} \) constant and vary the concentration parameter, or hold the concentration parameter constant and vary \( R_{200} \). The mass of the dark matter halo scales as \( M_3 = 200 \rho S R_{200}^3 \), while the concentration parameter is related to the angular momentum of the halo as motivated by the Mo et al. (1998) formalism (Springel et al. 2005), and affects the size of the baryonic disk.

Our earlier models (CBCB) were based on the \( R_{200} = 160 \ h^{-1} \) kpc case, which gives a galaxy mass consistent with observational estimates (Leroy et al. 2008). We first set \( R_{200} = 160 \ h^{-1} \) kpc and vary the concentration parameter, which is equivalent to varying the scale radius of the dark matter halo. Below we show that varying the concentration parameter and \( R_{200} \) such that the scale radius is constant gives nearly identical results for the phase of the \( m = 1 \) mode in the outskirts, which demonstrates that the critical parameter that governs the formation of these disturbances (once we have an observational handle on the mass of the galaxy from the rotation curve) is the scale radius. Thus, the two cases we focus on here are: (1) holding \( R_{200} \) constant and varying \( c \), which corresponds to varying \( R_s \), which means we hold the mass of the dark halo constant while we vary the scale radius; and (2) holding \( R_s \) constant and varying \( R_{200} \) (which will then also vary \( c \)) and therefore the mass of the dark halo.

Figure 1 shows the resultant gas density response of M51 when we vary the scale radius (concentration) of the dark matter halo of M51 from a small (large) value (\( R_s = 11 \ h^{-1} \) kpc, \( c = 14 \)), to the fiducial value used earlier by CBCB (\( R_s = 17 \ h^{-1} \) kpc, \( c = 9.4 \)), to a large (low) value (\( R_s = 26 \ h^{-1} \) kpc, \( c = 6 \)). Varying the scale radius varies the potential depth of M51 and will therefore dramatically impact the formation of tidal features, as we see clearly from Figure 1.
Figure 2. Phase of the $m = 1$ mode as the scale radius of the dark matter halo is varied, shown at the same times as the gas density images in Figure 1. The three cases correspond to probing regions inside the scale radius (the large scale radius case), close to the scale radius (the $c = 9.4$, $R_s = 17$ $h^{-1}$ kpc case), and outside the scale radius (the low scale radius case). $R_{200}$ is held fixed here as the scale radius (concentration) is varied.

(A color version of this figure is available in the online journal.)

Steep potential wells, which are produced by steeper density profiles, are more effective at holding onto the gas (Mihos et al. 1998). Therefore, the $R_s = 11$ $h^{-1}$ kpc, $c = 14$ case yields more tightly wound structures relative to the fiducial value of the scale radius ($R_s = 17$ $h^{-1}$ kpc, $c = 9.4$), as well as of course the largest scale radius we consider here ($R_s = 26$ $h^{-1}$ kpc, $c = 6$), where the density profile follows the shallow slope of $r^{-1}$ over much of the extended H I disk of M51. The location of the satellite was shown in CBCB (Figure 2 of that paper) for the fiducial $R_s = 17$ $h^{-1}$ kpc, $c = 9.4$ case.

Figure 2 shows how the phase of the $m = 1$ mode varies as we vary the scale radius of the dark matter halo for the three cases shown in Figure 1. The phase of the modes contains information on the shape of the spiral planform, i.e., tightly wrapped spirals will have a sharp gradient in the phase, while open spirals will have a flatter profile (Shu 1984; Chakrabarti & Blitz 2011). For the fiducial value of $R_s = 17$ $h^{-1}$ kpc (c = 9.4) used by CBCB (shown in the solid green line), the phase of the $m = 1$ mode in the simulation is fairly flat, as is that of the H I data of M51, shown in the dashed green line. However, for the other cases, i.e., $R_s = 11$ $h^{-1}$ kpc and $R_s = 26$ $h^{-1}$ kpc, the phase is either steeply falling or rising. For $R_s = 26$ $h^{-1}$ kpc, the density profile follows a shallow $r^{-1}$ slope over much of the H I disk of M51, and as such this is the loosest spiral (nearly a straight line) produced. The other extreme is the $R_s = 11$ $h^{-1}$ kpc case where the density follows a $r^{-3}$ profile in the radial range shown in Figure 2. Since the density profile is much steeper for the latter case, it is harder to “unwrap” the spiral, leading to a tighter spiral planform and positive gradient in the phase of the $m = 1$ mode. The radial variation of the phase, therefore, gives us a handle on the scale radius. We have examined smaller and larger scale radii than presented here, and the trends are the same.

There are three radial regions of interest in the behavior of the phase. We mark them explicitly in Figure 3, where we plot the phase as a function of the dimensionless variable $r/R_s$ for the $R_s = 17$ $h^{-1}$ kpc case. We see that $d/dr(\phi_1) < 0$ when $r/R_s < 1$; there is a transition region for $r/R_s \sim 1$, and $d/dr(\phi_1) > 0$ when $r/R_s > 1$, until the edge of the gas disk is reached. In Section 3.1, we show that the uncertainty in the phase that arises due to variations in initial conditions (the detailed structure of the simulated galaxy and range of values of the satellite mass and pericentric approach distance) and orbital inclination does not preclude us from differentiating between the cases shown in Figure 2. If one can observe the transitional region, where the gradient in the phase switches from declining to increasing, one can constrain the value of the scale radius. The shape of the phase of the $R_s = 17$ $h^{-1}$ kpc case is quite flat, as is that of the H I data for M51, indicating that the scale radius of the dark matter halo in the Whirlpool galaxy is $\sim 17$ kpc.

Our inference of a scale radius of 17 kpc for M51 is comparable to values expected from dissipationless cosmological simulations (Bullock et al. 2001). These simulations predict a 68% spread in concentration values from 9.2 to 21.3 for virial mass halos of $\sim 10^{12} M_{\odot}$ having virial radii $R_{200} \sim 200$ $h^{-1}$ kpc, which would yield $R_s \sim 13$–31 kpc. More recent work (Maccio et al. 2008) finds similar concentration–mass relations. The NFW profile emerges in simulations that do not include baryonic physics. One may well expect that baryonic processes, such as gas cooling or supernova feedback (Gnedin et al. 2004; Governato et al. 2010), would alter the density profile of the dark matter halo. It is worth noting that derivation of the scale radius for a local spiral galaxy with this new method (that relies on an analysis of disturbances in the extended gas disk) yields a value that is consistent with the expected range from dissipationless, cosmological simulations.

It is important to emphasize that for $r/R_s > 1$, our results depend essentially on the scale radius of the dark matter halo, and not on the concentration parameter. We demonstrate this in Figure 4, where we plot the phase of the $m = 1$ mode for three cases where we vary the concentration parameter and $R_{200}$, but hold the scale radius fixed. As is clear, for $R_s > 1$, the phase depends primarily on the scale radius (until we reach the edge of the gas disk; the size of the gas disk does depend on the concentration parameter, which is why there are small differences at the very edge). For $R_s < 1$, the concentration parameter affects the size of the disk, and hence phase in the inner regions as well.

Ideally, one would observe the transitional region where the phase of the $m = 1$ mode switches from $d/dr(\phi_1) < 0$ for $R_s > 1$ to $d/dr(\phi_1) > 0$ for $R_s > 1$ to infer the value of the scale radius for a specific tidally interacting galaxy. If $R_s$ is so large that it is comparable to the extent of the H I
Figures 5(a)–(c) depict the variation (the spread is shaded in yellow) of the phase of the $m = 1$ mode as the concentration is varied (by varying $R_{200}$) while holding the scale radius of the dark matter halo constant. The solid black line marks the $c = 9.4$, $R_s = 17 \ h^{-1} \ kpc$ case, the dotted line marks the $c = 10.5$, $R_s = 17 \ h^{-1} \ kpc$, and the dashed line the $c = 11.7$, $R_s = 17 \ h^{-1} \ kpc$. As is clear, the behavior of all three is very similar for $r/R_s > 1$, i.e., in the outskirts. For $r/R_s < 1$, the different concentration ($R_{200}$) parameters yield different disk sizes, which is why the phase is different in the inner regions for these three cases.

Figure 4. The variation of the phase of the $m = 1$ mode as the concentration is varied (by varying $R_{200}$) while holding the scale radius of the dark matter halo constant. The solid black line marks the $c = 9.4$, $R_s = 17 \ h^{-1} \ kpc$ case, the dotted line marks the $c = 10.5$, $R_s = 17 \ h^{-1} \ kpc$, and the dashed line the $c = 11.7$, $R_s = 17 \ h^{-1} \ kpc$. As is clear, the behavior of all three is very similar for $r/R_s > 1$, i.e., in the outskirts. For $r/R_s < 1$, the different concentration ($R_{200}$) parameters yield different disk sizes, which is why the phase is different in the inner regions for these three cases.

Figure 5. Phase of the $m = 1$ mode as the scale radius of the dark matter halo is varied from (a) top: $R_s = 26 \ h^{-1} \ kpc$, $c = 6$; (b) middle: $R_s = 17 \ h^{-1} \ kpc$, $c = 9.4$, the fiducial case; and (c) bottom: $R_s = 11 \ h^{-1} \ kpc$, $c = 14$, shown at the same times as the gas density images. Each figure displays the range in the phase as initial conditions (specifically, the gas and bulge fraction, along with the gas equation of state of the simulated M51 galaxy, and orbital inclination of the satellite) are varied for a given concentration-scale radius value. The phase of the data of M51 is over-plotted in each figure as a dot-dashed green line. (A color version of this figure is available in the online journal.)

Figure 6. Phase of the $m = 1$ mode as the satellite mass and pericenter approach distances are varied (within the range allowed by the Fourier amplitude constraint) for (a) $R_s = 26 \ h^{-1} \ kpc$, $c = 6$; (b) $R_s = 17 \ h^{-1} \ kpc$, $c = 9.4$, the fiducial case; and (c) $R_s = 11 \ h^{-1} \ kpc$, $c = 14$. The phase of the data of M51 is overplotted in each figure as a dot-dashed green line. The $R_s = 17 \ h^{-1} \ kpc$, $c = 9.4$ model provides a better fit to M51’s phase than the other simulation models, even when the satellite mass and pericenter approach distance variation is considered. (A color version of this figure is available in the online journal.)

disk ($R_{H1}$), then one can at best put a lower limit on the scale radius, i.e., if we observe that the phase of the $m = 1$ mode continues to have a negative gradient out to $R_{H1}$, then $R_e$ is greater than $R_{H1}$. H I disks are generally more extended than the optical radius (Wong & Blitz 2002), so the simulation modeling of the baryonic regions can in general be informed by optical or near-infrared photometry to yield stellar masses and disk sizes.

3.1. Dependence on Initial Conditions

We have investigated the dependence of the phase of the $m = 1$ mode at the best-fit time on the initial conditions. Figures 5(a)–(c) depict the variation (the spread is shaded in yellow) of the phase of the $m = 1$ mode for a given concentration-scale radius value as the EQS for the gas and bulge and gas fraction of the simulated M51 galaxy are varied, along with the orbital inclination of the satellite, for the three scale radius-concentration cases we have considered, namely, (a) $R_s = 26 \ h^{-1} \ kpc$, $c = 6$, to (b) $R_s = 17 \ h^{-1} \ kpc$, $c = 9.4$, to (c) $R_s = 11 \ h^{-1} \ kpc$, $c = 14$. Specifically, we consider here the fiducial model from CBCB that does not include a bulge, as well as a case with a bulge mass fraction $m_b = M_{bulge}/M_{total} = 0.012$. We consider gas fractions $f_g = m_{gas}/m_{disk}$ (where $m_{disk}$ is the disk mass) of $f_g = 0.142$ (the fiducial case) and $f_g = 0.25$. We include the fiducial case from CBCB of a coplanar orbit for M51’s satellite, as well as an inclined orbit (the “e” orbit as denoted in CBCB). We also vary the equation of state of the gas from isothermal (EQS = 0) to a multiphase ISM with energy injection from supernovae (EQS = 0.25). In each panel, the phase of the $m = 1$ mode of the H I data of M51 is overplotted as a dot-dashed green line for comparison. As is clear, the dependence on these other parameters does have some effect on the phase. However, the different scale radii cases are still distinct from each other, and the $R_s = 17 \ h^{-1} \ kpc$, $c = 9.4$ provides the best fit to the phase of M51, even when the dependence on initial conditions and orbital inclination is considered. Thus, these other parameters do not significantly affect this metric.

Figure 6 explores the question whether one could obtain a better fit to the phase of M51 by varying the satellite mass and pericenter approach distance along with the scale radius. Since we must also satisfy the Fourier amplitude constraint, we consider satellite masses and pericenter distances that lie close to the minimum in the variance versus variance plot produced by CBCB. Specifically, we choose three cases to illustrate our results: the standard 3R15 case (a 1:3 mass ratio satellite with a pericenter distance of 15 kpc), the 2R15 case (a 1:2 mass ratio satellite with a pericenter distance of 15 kpc), and the 4R7 case (a 1:4 mass ratio satellite with a pericenter distance of 7 kpc), for each of the scale radii cases we have focused on ($R_s = 11, 17, 26 \ h^{-1} \ kpc$). Varying the satellite mass and pericenter approach distance does affect the resultant
phase of the disturbances quantitatively. However, we see that despite this variation, the best fit is still produced by the fiducial $R_s = 17 \, h^{-1} \, \text{kpc}$, $c = 9.4$ case, which in fact yields the best fit to the phase of the H I data for all three satellite mass and pericenter approach distances considered here.

As is clear, while varying initial conditions (satellite mass, pericenter approach distance, orbital inclination, gas fraction, EQS, etc.) has some effect on the phase, it does not change the essential character of the phase to the point that it would preclude us from discriminating from the different scale radius cases (at a factor of two level). Moreover, it does not lead to the smaller or large scale radii cases yielding a better fit when the variation from these other parameters is considered. We found earlier in CBCB that modestly varying the orbital velocity of the satellite does not significantly affect our results, and we do not vary this parameter here. In the future, we will carry out a larger parameter survey and study the detailed effects of these and other parameters on this metric. Our initial study here does show that this metric is reasonably robust to the variation of other parameters, i.e., the gradient of the phase is sufficiently distinct to allow us to characterize the scale radius of the dark matter halo.

We have shown the phase behavior here at the time when a given simulation best fits the Fourier amplitudes of the H I data. CB09 and CBCB have outlined how the best-fit simulation from a simulation parameter survey can be displayed on a variance versus variance plot. Disturbances in the gas disk dissipate on the order of a dynamical time. As such, even the best-fit simulation traces a trajectory on the variance versus variance plot, with the minimum yielding the best-fit time. We have examined the time variation of these simulations as well. We do not find that the time variation of the phase would allow distinct scale radius cases to mimic each other. We conclude therefore that the phase of the $m = 1$ mode allows us to discriminate between different scale radii within a factor of $\sim 1.5$ (i.e., we can discriminate the $R_s = 17 \, h^{-1} \, \text{kpc}$ case from the $R_s = 11 \, h^{-1} \, \text{kpc}$ case and the $R_s = 17 \, h^{-1} \, \text{kpc}$ case from the $R_s = 26 \, h^{-1} \, \text{kpc}$ case).

4. CAVEATS AND FUTURE WORK

There are some caveats in our work here that are worth noting for the application of this method in generality. We have not investigated in this paper the effect of non-spherical halos on either the Fourier amplitudes or the phase. Recent work has studied the effects of prolate halos (Banerjee & Jog 2011) on the vertical structure of the Milky Way’s H I disk. We defer a detailed investigation of these effects to a future paper. Preliminary work (S. Chakrabarti et al., in preparation) finds that the evolution of the halo shape (and thereby the strength of the Fourier amplitudes in the outskirts of the gas disk as determined by intrinsic processes) is significantly affected by gas cooling and angular momentum transport from the gas to the halo. Debattista et al. (2008) found earlier that the presence of a baryonic (stellar) disk affected the shape of the dark matter halo, leading to a rounder halo as the disk is grown.

We have recently examined similar simulations which include a gaseous component (S. Chakrabarti et al., in preparation) and find that the presence of the gaseous component renders halo shapes more spherical over time after the onset of gas cooling. The shapes of halos are considerably rounder close to present day, when the stellar to dark matter masses are comparable to local spirals ($M_*/M_{DM} \sim 0.03$; Leroy et al. 2008), relative to $z \sim 2$. Examination of the Fourier amplitudes of these simulations close to present day indicates that the Fourier amplitudes in the outskirts are $\lesssim 10\%$, which suggests that non-spherical halos would not be the primary contributor to the observed strength of the Fourier amplitudes in local spirals. This is certainly valid for M51, which is known to be a tidally interacting system (CBCB; Salo & Laurikainen 2000). Local spirals that display small ($\lesssim 10\%$ in the Fourier amplitudes, as defined in CB09, relative to the axisymmetric mode) perturbations in the outskirts and have no visible tidally interacting companion may require a more careful treatment of the effect of halo shapes. Nonetheless, the presence of a cold dissipative gaseous component needs to be modeled to more comprehensively understand the evolution of halo shapes. While cosmological simulations (Maccio et al. 2008) find halo shapes rendered non-spherical (in many cases triaxial) by repeated mergers, this effect may be mitigated in reality by a cold gaseous component transferring its abundance of angular momentum to the dark halo, thereby rendering it more spherical.

Our observationally motivated inference of the scale radius of the dark matter halo may be useful for a number of reasons. First, the NFW density profile has some dependence on the spectral index of primordial density fluctuations (NFW96; NFW97). Second, this profile emerges within the context of dissipationless, cosmological simulations, and baryonic effects may well serve to alter this profile (Governato et al. 2010; Gnedin et al. 2004). Therefore, an independent observationally motivated probe of the density profile for specific spiral galaxies can serve to test the theoretical bases of this so-called universal density profile, wherein galaxies have grown hierarchically from primordial density fluctuations.

We should also note that while recent high-resolution simulations find qualitatively similar results to the NFW profile, there are important quantitative differences, particularly in the inner regions. In the highest resolution simulations to date (Springel et al. 2008; Stadel et al. 2009), the profile reaches a logarithmic slope of $\sim -0.8$ in the innermost numerically converged radius, close to 0.05% of the virial radius. Merritt et al. (2006) find that the Einasto profile provides a better fit to dissipationless cosmological simulations (see also Navarro et al. 2010), and we plan to consider this profile in a future paper. Part of the debate on the inner density profile of dark matter halos stems from the apparent inconsistency of the NFW cusp with the core of low surface brightness galaxies (de Blok et al. 2001). Gravitational lensing studies (Treu & Koopmans 2002) find the real structure of dark matter halos may not be well described by the NFW profile, which may be due to baryonic effects in the inner regions. However, there are other observational studies that do find the density profile of dark matter halos to be consistent with the NFW model (Rines et al. 2000; Rines et al. 2004).

We adopt the NFW profile here largely as a toy model, as done in lensing studies (Mandelbaum et al. 2006). It is beyond the scope of this paper to determine what the true density profile of dark matter in real galaxies is. Indeed, the real variation in the scale radius is certainly expected to be larger than what we find here assuming an NFW profile. Our assumption of an NFW profile is a limitation of this work. Nonetheless, our work does provide a new and useful complement to lensing studies that adopt the NFW profile in a similar vein (e.g., Mandelbaum et al. 2006).

In the future, we plan to apply this method to a large sample of spirals from the THINGS survey to arrive at a statistical determination of the scale radii of spiral galaxies in the Local Volume. Such a study can be performed to zeroth-order using the
scaling relations in Chang & Chakrabarti (2011) that allow one to infer the satellite mass from the observed H\textsc{i} map, and refined with numerical calculation. We also plan to generalize the linear perturbation analysis of Chang & Chakrabarti (2011) to determine scaling relations between the underlying dark matter density profile and the shape of the spiral planform. Another important tracer of the mass distribution is the observed rotation curve (Bosma 1978; Rubin et al. 1982; Blitz 1979). The rotation curve for many spirals cannot be determined beyond $\sim$10 disk scale lengths. We leave a comparison of this method to the rotation curve to a future paper. It would also be worthwhile to compare our approach to gravitational lensing (Treu & Koopmans 2002; Mandelbaum et al. 2006), which also serves to probe mass distributions independent of the stellar light. Our method is currently ideally suited for analysis of disturbances in the outskirts of spiral galaxies in the Local Volume, and can in the future be applied to a higher redshift population with observations from instruments such as the Square Kilometer Array.

5. CONCLUSION

1. We find that the shape of disturbances in the fragile, extended H\textsc{i} disks of galaxies reflects the underlying density profile of the dark matter halo in spiral galaxies. We employ the phase of the $m = 1$ mode as our primary metric of comparison to observed H\textsc{i} data. Building on prior results from CBCB where the satellite mass and pericentric approach distance were determined for M51, we find that the scale radius of the dark matter halo can be determined from the phase of the $m = 1$ mode.

2. The three regimes of interest for the phase of the $m = 1$ mode that delineate the variation of the scale radius and how it is reflected in the phase are: $d/dr(\phi_1) < 0$ for $r/R_1 < 1$, a transitional region at $r/R_1 \sim 1$, and $d/dr(\phi_1) > 0$ for $r/R_1 > 1$ till the edge of the gas disk. Long-baseline H\textsc{i} observations that can reveal this transitional region between the $r^{-1}$ shallow profile (that would produce more loosely wrapped spirals) and the steeper $r^{-3}$ profile (that would produce more tightly wrapped spirals) will allow us to quantitatively infer the scale radius of dark matter halos of specific spiral galaxies. The uncertainty in the phase that arises due to variations in initial conditions and orbits does not preclude us from inferring the value of the scale radius to within a factor of $\sim 1.5$. We have discussed preliminary work that suggests that other effects (such as intrinsic processes arising from non-spherical halos) will not significantly affect our results here.

3. Application of this method to the Whirlpool galaxy yields a scale radius of $\sim$17 kpc, which is consistent with the expected range of scale radii of $\sim10^{12} M_\odot$ halos in dissipationless cosmological simulations. It should be noted however that we derive this result assuming an NFW profile, and as such the real uncertainty in the scale radius may be larger than the factor of $\sim 1.5$ that we find here.

4. We have also demonstrated that for $r/R_1 > 1$, our results depend essentially on the scale radius and not on the concentration parameter. These results suggest that we can utilize observed disturbances in the extended H\textsc{i} disks of galaxies to characterize the density profile of the dark matter halo in spiral galaxies. In the future, we will apply this method to a large sample of spirals to obtain a statistical determination of the scale radii of spiral galaxies in the Local Volume.

Figure 7. Phase of the $m = 1$ mode as (a) the equation of state is varied from isothermal, EQS = 0 (green line) to a multiphase ISM with energy injection from supernovae, EQS = 0.25 (black line), the fiducial case (b) as resolution is varied: for the low resolution (red line), standard resolution (green line), high resolution (black line).

We thank Lars Hernquist, Chris McKee, Phil Chang, Pedro Marronetti, Leo Blitz, Jay Gallagher, Risa Wechsler, and Daniel Holz for helpful discussions. We also thank the anonymous referee for helpful comments that have improved the presentation of this paper.

APPENDIX

It is important to note that our analysis focuses on the outskirts of the gas disk ($r \gtrsim r_\text{OLR}$), where the effects of external perturbers are predominant over the intrinsic response. The outskirts are deficient in molecular gas (Wong & Blitz 2002; Bigiel et al. 2010), and one may therefore expect that the outskirts are less subject to the effects of supernova feedback and star formation than the inner regions. These processes can affect the temperature of the gas as modeled in simulations, so we examine here the effects of varying the EQS, which Springel & Hernquist (2003) employ to characterize these effects in a spatially averaged way. Springel & Hernquist (2003) describe the temperature structure of the multiphase model as a function of the baryonic overdensity. Below a density $\delta \sim 10^6$, star formation does not occur, and the gas is essentially a single temperature fluid with a temperature of $\sim 10^4$ K. When star formation sets in, a hot ambient medium develops, and cold clouds are present at a temperature of $10^7$ K. The mass-weighted effective temperature can be interpreted in terms of
an effective pressure. Mass exchange between the hot and cold clouds is driven by star formation, cloud evaporation arising from supernovae, and cloud growth caused by cooling. The effective EQS parameterizes these effects, interpolating between an isothermal equation of state (EQS = 0) and a multiphase ISM that can have energy injection from supernovae (EQS = 0.25). Springel & Hernquist (2003) take the heating rate from supernovae to be proportional to the star formation rate. Since the latter is low in the outskirts, we see little change in the phase of the $m = 1$ mode (Figure 7(a)) between an isothermal EQS and the multiphase ISM.

Finally, to show our adopted numerical resolution is sufficient for us to analyze the observed H\text{I} map, we consider a low- and high-resolution case, along with our standard resolution. The low resolution has a factor of two fewer particles (with the softening length adjusted as $N^{1/3}$) relative to the standard resolution; the high resolution has a factor of two more particles relative to the standard. Thus, these three cases span a factor of eight in particle number. As Figure 7(b) shows, there is no difference in the resultant phase on kpc scales between these cases. It should be noted that with our adopted resolution, we cannot reliably follow structures on scales $\lesssim 100$ pc. In their studies of galaxy mergers, Cox et al. (2006) adopt a particle number that is a factor of several lower than what we use and find converged results on scales of several kpc. As previously emphasized in the text, while it may be desirable to go to higher particle number if one desires to investigate structures on scales of $\lesssim 100$ pc, this is not our goal here. The resolution of the Very Large Array map of M51 is about a kiloparsec, and therefore our adopted resolution is sufficient to analyze the observed structure of M51’s gas disk.

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