Stabilizing Dilaton and Moduli Vacua in String and M–Theory

Cosmology

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Abstract

We show how non-trivial form fields can induce an effective potential for the dilaton and metric moduli in compactifications of type II string theory and M–theory. For particular configurations, the potential can have a stable minimum. In cosmological compactifications of type II theories, we demonstrate that, if the metric moduli become fixed, this mechanism can then lead to the stabilization of the dilaton vacuum. Furthermore, we show that for certain cosmological M–theory solutions, non-trivial forms lead to the stabilization of moduli. We present a number of examples, including cosmological solutions with two solitonic forms and examples corresponding to the infinite throat of certain p–branes.

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1 Introduction

Recent advances in understanding the structure underlying string theory have renewed interest in type II and eleven-dimensional supergravities and the role played by the form-field degrees of freedom in these theories. It is now understood that there are strong-weak coupling dualities relating each of the known supersymmetric string theories together with a putative theory in eleven-dimensions, M–theory. This suggests that the corresponding low-energy effective theories, which are supergravities, may be directly relevant to particle physics and cosmology. A second development has been the discovery of D–brane states in open string theory as sources of Ramond-Ramond (RR) form field charge in type II supergravities. These states have proved central to the recent understanding of some of the statistical dynamics of black holes.

Given this new perspective, it becomes interesting to ask what role these form fields might play in compactifications of string theory and M-theory. Phenomenologically, probably the most relevant case in M–theory is the compactification on a Calabi-Yau three-fold cross the orbifold \( S_1/Z_2 \), considered by Witten, which describes the strong coupling limit of heterotic string theory. Some of the particle physics and cosmological implications of this limit have been discussed by Banks and Dine and Horava. In each of these compactifications, the three-form potential in eleven-dimensional supergravity is excited. In fact, it can be shown that it cannot be set to zero. Compactifications of type II theory with non-trivial form fields have been considered in \( \mathbb{R}^4 \). In two recent papers, we studied cosmological solutions with compact, but dynamic, internal spaces and non-trivial form fields. Assuming that the internal space was a product of maximally symmetric subspaces, the general solutions are found to be closely related to non-extremal black \( p \)-brane solutions, though with the role of the radial and time coordinates exchanged. A class of solutions, with spherical subspaces, correspond directly to the interior of black \( p \)-brane solutions. Solutions with a non-trivial Neveu-Schwarz (NS) two-form field had been considered previously by various authors. Other authors have also subsequently considered solutions with non-trivial RR forms. One of our initial examples was presented at almost the same time in a paper by Kaloper. A later paper by Lü et al. gave a further, broad class of solutions. The singularity-free cosmologies arising inside black holes were first discussed by Behrndt and Förste and then with RR fields by Poppe and Schwager. That these solutions really corresponded to the interiors of black \( p \)-brane solutions was stressed recently by Larsen and Wilczek, who also gave some further examples and commented on the relation to D–branes.

The purpose of the present paper is to show that the presence of non-trivial form fields can actually stabilize the vacua of the dilaton and the moduli that arise in the compactification of type II string theory or M–theory. We will work within the context of the cosmological solutions that we recently introduced in \( \mathbb{R}^4 \), since understanding the stabilization of the dilaton and moduli is particularly physically relevant in cosmology. However the mechanism is more widely applicable.
We give a simple argument as to how an effective potential for both the dilaton and the moduli arises, and show that this potential can have stable minima. The value of the fields at the minimum is controlled by the form field charges. We then discuss, within the context of type II and M-theory, the mechanism by which our cosmological solutions are attracted to, and at late time stabilized at, a finite minimum of the potential.

In the case of type II string theory, we find that the effect of exciting non-trivial form fields is to introduce an effective potential for both the dilaton and the geometrical moduli. Generically, the potential has flat directions or, even if there are no flat directions, it may not have a minimum. Thus, typically, not all the moduli are stabilized. However, in general, there will be other contribution to the effective potential. In particular, Tseytlin and Vafa [20] have argued in the cosmological context that including a gas of string matter in toroidal compactifications leads to an effective non-perturbative potential for the geometric moduli, which tends to stabilize them near the string scale. This non-perturbative potential is independent, however, of the dilaton. Using this mechanism, we show in detail how including non-trivial NS and RR form fields can then lead to a complete stabilization of the dilaton.

In the case of eleven-dimensional supergravity coming from M–theory, the dilaton appears geometrically. Compactifying one direction on a circle (or \( S_1/Z_2 \) orbifold), connects M–theory to type IIA string theory (or heterotic string theory), where the dilaton is related to the radius of the compact direction. We show that, in general, it is possible to stabilize the moduli of several eleven-dimensional compactifications simply by using form-fields. We also briefly investigate the possibility of stabilizing those configurations where the theory contains a dilaton.

The layout of the paper is as follows. In section two, we explain the mechanism by which non-trivial form fields can produce an effective potential, by considering a simple toy example. In order to place this idea in a cosmological context, in section three we summarize the structure of the cosmological solutions found in [8, 9]. Using spacetimes of this form, where the internal space is a product of spheres and tori, we show explicitly how an effective potential for the dilaton and geometrical moduli describing the radii of the internal spaces can arise. In section four, we show that, using a non-perturbative mechanism to stabilize the geometrical moduli, the dilaton vacuum can be fixed by exciting a pair of form fields. The fifth section is a discussion of how, in the context of compactified supergravity, we are able to circumvent the usual scaling argument that the dilaton cannot be stabilized. In the sixth section, we discuss examples where the vacua of the geometrical moduli arising from compactifying \( M \)-theory are fixed by this mechanism. We briefly present our conclusions in section seven.
2 A simple example

To understand how a non-trivial form field can lead to an effective moduli potential, consider a simple model of a scalar field coupled to an electromagnetic field strength in four dimensions, which mimics the dilaton coupling in ten-dimensional supergravity,

\[ S = \int d^4x \left[ -\frac{1}{4} e^{2\phi} F^2 - \frac{1}{2} \left( \partial \phi \right)^2 \right]. \quad (1) \]

As in the ten-dimensional theory the action has a symmetry under \( \phi \rightarrow \phi + \ln \lambda \) together with a rescaling of the gauge potential \( A_\mu \rightarrow \lambda^{-1} A_\mu \). Classically, the scalar field is the massless Goldstone boson of this symmetry and so it would appear that it has a flat effective potential. On the other hand the \( \phi \) equation of motion reads

\[ \partial^2 \phi = \frac{1}{2} e^{2\phi} F^2, \quad (2) \]

so that \( F \) could clearly supply an effective potential for the scalar field, though naively it would appear not to have a minimum. Suppose that the \( x \)- and \( y \)-directions are compactified so that the spacetime is a two dimensional Minkowski space cross an “internal space” torus \( T_2 \). The gauge field equation of motion and Bianchi identity can be written as

\[ d \left( *e^{2\phi} F \right) = 0, \quad dF = 0, \quad (3) \]

respectively. Since there are no electric or magnetic charges present, the flux of electric and magnetic field across the \( x-y \) plane must be conserved, so we have the conserved charges

\[ e = \int_{T_2} *e^{2\phi} F, \quad g = \int_{T_2} F. \quad (4) \]

The simplest configuration with these charges is a uniform electric and magnetic field pointing along the \( z \)-direction, with the magnitudes

\[ E_z = e^{-2\phi} \frac{e}{A}, \quad B_z = \frac{g}{A}, \quad (5) \]

where \( A \) is the area of the torus. Substituting into eq. (2), we find

\[ \partial^2 \phi = 2e^{2\phi} F^2 = e^{2\phi} \left( B_z^2 - E_z^2 \right) = g^2 \frac{e^{2\phi}}{A^2} - e^2 \frac{e^{-2\phi}}{A^2} = \frac{dV_{\text{eff}}}{d\phi}. \quad (6) \]

where we define an effective potential for \( \phi \) of the form

\[ V_{\text{eff}} = \frac{g^2}{2A^2} e^{2\phi} + \frac{e^2}{2A^2} e^{-2\phi}. \quad (7) \]

Note that, because of the factor of \( \exp(2\phi) \) which enters the expression for the conserved electric charge (4), the contribution to the effective potential from the electric field has the opposite dependence on \( \phi \) from what might be expected from the form of the action (1). Clearly, the potential has
a stable minimum at $<\phi> = \frac{1}{4} \ln (e^2/g^2)$. Remarkably, exciting an electric and a magnetic field has not only provided an effective potential, but, when both are present, has apparently stabilized the scalar. The potential is, of course, a little naive. One might wonder whether the electric and magnetic fields might not vary with time in a way which removes this apparent stability. However, the charges (4) must always be conserved, so, if the fields do not depend on the compact directions, the only solution for $E_z$ and $B_z$ is that given by eqs. (5). From a compactification point of view, allowing dependence on the $(x, y)$-directions corresponds to exciting massive Kaluza-Klein modes, which would generally be expected to raise rather than lower the vacuum energy. In fact, for a fixed charge, the minimum contribution to the effective potential comes from the case where the electric and magnetic fields are uniform. It is natural to ask how we have avoided the flat direction implied by the scaling symmetry of the original action (1). The point is that the scaling symmetry does not preserve the conserved charges (4). Rather, we find $e \to \lambda e$ and $g \to \lambda^{-1}g$. Thus, although making a scaling does provide a way of generating new solutions, dynamically the new solution can never be reached from the old one, since this would imply a violation of charge conservation. We note that there is still remnant of the scaling symmetry in the arbitrariness of the amount of electric and magnetic charge in the solution. However, once these charges are chosen, the value of the scalar field is stable and fixed.

In summary, there are two parts to the stabilization. Without any electric or magnetic field the theory has a scaling symmetry which implies that the scalar field is always massless. Turning on one field, with its corresponding conserved charge, breaks the symmetry and introduces an effective potential for the scalar. However the potential has no stable minimum. Only if the second field is excited do we get a potential which stabilizes the scalar. Here the minimum arose from balancing the magnetic and electric contributions of a single gauge field. However, it is equally possible to produce the stabilization using two different gauge fields. For instance, if the Lagrangian has two terms $e^{2\phi}F^2 + e^{-2\phi}F'^2$, two magnetic fields can produce a minimum because of the different coupling each term has to the scalar field. Such a theory would be analogous to stabilizing between the NS and RR terms in type II theories.

With this simple example in mind, we can now see how this stabilization would work in a compactification of ten- or eleven-dimensional supergravity. As usual, we take spacetime to be a product of a four-dimensional space $M_4$ with some compact internal space $K$. (In our examples $M_4$ will be a Robertson-Walker cosmology, while $K$ is a product of spheres and tori.) Considered as a theory in four dimensions, in addition to the dilaton there are moduli $\alpha_i$ describing the internal space (for instance, the radii of the internal spheres and tori in our examples). These degrees of freedom appear in the metric for the internal space and so will also couple to the form-field $F^2$ terms in the supergravity action. In our simple examples they appear as exponential coefficients, exactly analogous to the dilaton coupling. This coupling implies that, as we will see, exciting non-trivial form fields can provide an effective potential for the metric moduli of the internal space as well as
for the dilaton.

What form field orientations should we take? We start by noting that, for all the examples we consider, the contribution from possible Chern-Simons in the supergravity theory are zero. Thus we may effectively drop these terms from the action. Suppose there is a $\delta$-form potential which appears in the action as $\exp(-a\phi) F_\delta^2$. We can again form conserved electric and magnetic charges

$$e_\delta = \int_{\Sigma_{\tilde{\delta}+1}} *e^{-a\phi} F_\delta, \quad g_\delta = \int_{\Sigma_{\tilde{\delta}+1}} F_\delta,$$

where $D$ is the total spacetime dimension and $\tilde{\delta} = D - \delta - 2$. The integrals are over compact subspaces $\Sigma_{\tilde{\delta}+1}$ and $\Sigma_{\delta+1}$ which, by analogy to the four-dimensional case, lie only in the internal space $K$. Thus, we are interested in the cases where either $F_\delta$ or $*F_\delta$ lies only in $K$. It is immediately clear that we can only have an electric charge if $\delta > 2$, since otherwise $*F_\delta$ must lie partly in $M_4$. This implies, for instance, that we cannot excite an electric charge in heterotic string theory, since there are no massless form fields with $\delta > 2$. Likewise, though less relevantly, for a magnetic charge $\delta < D - 4$. In the effective four-dimensional theory, a given ten-dimensional form field appears as of four-dimensional form of varying degree, depending on how many components of the form span the internal space. Our condition on $F_\delta$ implies that in four-dimensions we have either a 0-form (magnetic case) or a four-form (electric case). Such dimensional reductions, though with only magnetic charges, appear in the derivation of “massive” supergravity theories given in [14]. In either case, the field has no four-dimensional dynamics and simply provides an effective potential for the moduli and dilaton. In fact, in both cases, since we assume that the dilaton depends only on the external coordinates, the equations of motion for the form field reduce to

$$d * G = dG = 0,$$

where the forms, exterior derivative and Hodge star are now restricted to the internal subspace $K$. Here $G$ is the projection of $F_\delta$ onto $K$ in the magnetic case, while in the electric case it is the projection of $*F_\delta$. The conditions imply that $G$ is harmonic in $K$. For each independent solution, we can fix either the electric or magnetic charge. In our examples, we will often refer to these two cases as “fundamental” (for an electric charge) and “solitonic” (for a magnetic charge) by analogy with the fields surrounding fundamental and solitonic $p$-branes. With this formalism in mind, we now give a detailed analysis of dilaton and moduli vacuum stabilization within the context of cosmological type II and M–theory solutions.
3 Cosmological framework

We are interested in cosmological solutions of a low energy action, which can describe the bosonic modes of type II or eleven-dimensional supergravity, written in the Einstein frame,

\[ S = \int d^Dx \sqrt{-g} \left[ R - \frac{4}{D-2} (\partial \phi)^2 - \sum_r \frac{1}{2(\delta_r + 1)!} e^{-a(\delta_r)\phi} F_r^2 - \Lambda_8 e^{-a\Lambda_8 \phi} \right] \]  

(10)

Here \( g_{MN} \) is the \( D \)-dimensional metric, \( \phi \) is the dilaton and \( F_r = dA_r \) are \( \delta_r \)-forms. The forms encompass the NS two-form as well as the RR forms present in type II theories. We have also included a cosmological constant \( \Lambda_8 \) which appears in the massive extension of type IIA supergravity [18] and can be interpreted as a RR 9-form coupling to 8 branes [19]. Here we adopt the Ansatz that none of our solutions include contributions from Chern-Simons terms, and so, such terms can therefore be dropped from the action. The various types of forms are distinguished from each other by the dilaton couplings \( a(\delta_r) \) which are given by

\[
a(\delta_r) = \begin{cases} 
\frac{8}{D-2} & \text{NS 2 - form} \\
\frac{4\delta_r - 2(D-2)}{D-2} & \text{RR } \delta_r \text{- form}
\end{cases}
\]

(11)

and

\[ a_{\Lambda_8} = -\frac{2D}{D-2}. \]

(12)

Note that the couplings for the NS form and the RR forms have opposite signs. The above action can account for a wide range of cosmological solutions in type II theories and M-theory, and a large class of such solutions has been constructed in [9]. Since it is this class which we will use to illustrate our main idea, let us briefly review some of its properties.

Our Ansatz for the metric is characterized by a split of the total \( D \)-dimensional space into \( n \) maximally symmetric \( d_i \) dimensional spatial subspaces with scale factors \( \bar{\alpha}_i, i = 0, ..., n-1 \). The corresponding metric reads

\[ ds^2 = -N^2(\tau) d\tau^2 + \sum_{i=0}^{n-1} e^{2\bar{\alpha}_i(\tau)} d\Omega_{K_i}^2, \]

(13)

where \( d\Omega_{K_i}^2 \) is the metric of a \( d_i \) dimensional space with constant curvature \( K_i = -1, 0 \) or \(+1\). The dilaton should also depend only on time, \( \phi = \phi(\tau) \). We have in mind that three of the spatial directions should be identified with the spatial part of the observed universe. Typically, these three directions correspond to one subspace to ensure homogeneity as well as isotropy. One might, however, also allow for a further split up of this three dimensional part, if the resulting anisotropy disappears asymptotically in time. The other directions should be viewed as a compact internal space and the corresponding scale factors are interpreted as moduli.

The structure of eq. (13) allows two different types of Ansätze for the forms, which we call “elementary” and “solitonic”. This terminology is motivated by a close analogy to p-brane solutions
which has been explained in ref. [8, 9]. The nonvanishing components of their field strengths are given by

- elementary: if \( \sum_i d_i = \delta_r \) for some of the spatial subspaces \( i \) we may set

\[
(F_r)_{\mu_1...\mu_\delta_r} = A_r(\bar{\alpha}) f'_r(\tau) \epsilon_{\mu_1...\mu_\delta_r}, \quad A_r(\bar{\alpha}) = e^{-2 \sum_i d_i \bar{\alpha}_i} \tag{14}
\]

where \( \mu_1...\mu_\delta_r \) refer to the coordinates of these subspaces, while \( \epsilon_{\mu_1...\mu_\delta_r} \) is a totally antisymmetric tensor density spanning these subspaces, \( f_r(\tau) \) is an arbitrary function to be fixed by the form field equation of motion, and the prime denotes the derivative with respect to \( \tau \).

Note that the sum over \( i \) in the exponent runs only over those subspaces which are spanned by the form. The “electric” configurations discussed in the last section correspond to an elementary form which spans all the external subspaces.

- solitonic: if \( \sum_i d_i = \delta_r + 1 \) for some of the spatial subspaces \( i \) we may allow for

\[
(F_r)_{\mu_1...\mu_{\delta_r+1}} = B_r(\bar{\alpha}) w_r \epsilon_{\mu_1...\mu_{\delta_r+1}}, \quad B_r(\bar{\alpha}) = e^{-2 \sum_i d_i \bar{\alpha}_i} \tag{15}
\]

where \( \mu_1...\mu_{\delta_r+1} \) refer to the coordinates of these subspaces and \( w_r \) is an arbitrary constant.

As for the elementary Ansatz, the sum over \( i \) in the exponent runs over the subspaces spanned by the form. It is easy to check that this Ansatz already solves the form equation of motion. The “magnetic” configurations discussed in the last section correspond to a solitonic form which spans no part of the external subspaces.

From the form of the action (10), it is clear that the two Ansätze for the forms (14) and (15) generate an effective potential for the scale factors as well as for the dilaton. Also, terms resulting from curved subspaces can be incorporated into this potential. In ref. [8] this has been made precise by deriving an effective action for the vector \( \bar{\alpha} = (\bar{\alpha}_I) = (\bar{\alpha}_i, \phi) \)

\[
\mathcal{L} = \frac{1}{2} E \bar{\alpha}^T \bar{G} \bar{\alpha}' - E^{-1} U . \tag{16}
\]

where the metric \( \bar{G}_{IJ} \) on the \( \bar{\alpha} \) space is defined by

\[
\bar{G}_{ij} = 2(d_i \delta_{ij} - d_i d_j) \quad \bar{G}_{in} = \bar{G}_{ni} = 0 \quad \bar{G}_{nn} = \frac{8}{D-2} . \tag{17}
\]

and

\[
E = \frac{1}{N} \exp(\bar{d} \cdot \bar{\alpha}) \tag{18}
\]

with the dimension vector \( \bar{d} = (d_i, 0) \). The effective potential \( U \) can be written as a sum

\[
U = \frac{1}{2} \sum_{r=1}^m u_r^2 \exp(\bar{q}_r \cdot \bar{\alpha}) \tag{19}
\]
over all elementary and solitonic form configurations as well as all curvature terms. The type of a certain term \( r \) is specified by the vector \( \vec{q}_r \). For an elementary \( \delta \)-form it is given by

\[
\vec{q}^{(\text{el})} = (2\epsilon_i d_i, a(\delta)) , \quad \epsilon_i = 0, 1 , \quad \delta = \sum_{i=0}^{n-1} \epsilon_i d_i
\]  

(20)

with \( \epsilon_i = 1 \) if the form is nonvanishing in the subspace \( i \) and \( \epsilon_i = 0 \) otherwise. The dilaton couplings \( a(\delta) \) have been defined in eq. (11). For a solitonic \( \delta \)-form it reads

\[
\vec{q}^{(\text{sol})} = (2\tilde{\epsilon}_i d_i, -a(\delta)) , \quad \tilde{\epsilon}_i \equiv 1 - \epsilon_i = 0, 1 , \quad \tilde{\delta} \equiv D - 2 - \delta = \sum_{i=0}^{n-1} \tilde{\epsilon}_i d_i
\]  

(21)

with \( \tilde{\epsilon}_i = 1 \) if the form vanishes in the subspace \( i \) and \( \tilde{\epsilon}_i = 0 \) otherwise. In both cases, the constant \( u_r^2 \) in potential (19) is a positive integration constant, proportional to the square of the conserved electric of magnetic form-field charge. Finally, curvature in the \( k \)th subspace leads to a potential term characterized by

\[
\vec{q}^{(\text{curv})}_k = (2(d_i - \delta_{ik}), 0) .
\]  

(22)

In this case the constant \( u_r^2 \) is determined by the curvature, \( u_r^2 = -2K_k \), and can be of either sign.

The dynamical properties of models specified by Lagrangian (16) and potential (19) have been studied at length in refs. [9]. In particular, the general solution for models with only one term in the potential and the solution for those models related to Toda theory have been found. Here, though, we concentrate on the question of dilaton and moduli vacuum stabilization.

So far, we have treated all scale factors on the same footing. Physically, however, it is useful to distinguish scale factors of the observable universe from internal moduli arising from compactification, and to transform to a new basis in which these two types of fields decouple.

To do this, let us split up the \( \vec{\alpha} \) space into an external observable universe part, an internal moduli part and the dilaton as \( \vec{\alpha} = (\vec{\alpha}_\beta^{(e)}, \vec{\alpha}_b^{(i)}, \phi) \). Note that we use indices \( \beta, \gamma, \ldots \) to specify the scale factor(s) of the universe and indices \( b, c, \ldots \) to specify the moduli. Of course, we have a split into \( 3+1 \) external dimensions and either 6 (string theory) or 7 (M–theory) internal dimensions in mind; that is \( D(e) \equiv \sum_{\beta} d_{\beta} = 3 \) and \( D(i) \equiv \sum_b d_b = 6 \) or 7. All other vectors will be split correspondingly, for example \( \vec{q} = (\vec{q}^{(e)}, \vec{q}^{(i)}, \vec{q}) \). As it stands, Lagrangian (16) mixes the external and the internal spaces since the metric \( \bar{G}_{ij} \), eq. (17), is completely off–diagonal. To decouple these spaces we should, instead of action (10), consider its dimensional reduction \( \sqrt{-g}_{10} R_{10} \rightarrow \sqrt{-g}_4 R_4 + \text{moduli} \). In order to do this reduction one has to perform a Weyl rotation on the external metric. Within our framework this Weyl rotation can be simply described by a linear transformation \( \vec{\alpha} = P^{-1}\vec{\alpha} \) to a new basis \( \vec{\alpha} \) and a corresponding transformation of the gauge parameter \( N \). In the basis \( \vec{\alpha} \) the new metric \( G = P^T \bar{G} P \) is block diagonal in the internal and external parts. It turns out that the transformation is given by

\[
\vec{\alpha}_\beta = \alpha_\beta - \frac{1}{D(e) - 1} \sum_b d_b \alpha_b
\]
\[\tilde{\alpha}_b = \alpha_b \] (23)
\[\tilde{\phi} = \phi .\]
\[\tilde{N} = e^{-\sum_e d_e\alpha_e/(D^{(e)}-1)}N\]

Comoving time \(t\) in the new frame is defined by setting \(N = 1\). The new metric \(G\) is explicitly given by
\[G = \begin{pmatrix} G^{(e)} & 0 & 0 \\ 0 & G^{(i)} & 0 \\ 0 & 0 & \frac{8}{D-2} \end{pmatrix}\] (24)
with
\[G^{(e)}_{\beta\gamma} = 2(d_\beta \delta_{\beta\gamma} - d_\beta d_\gamma)\] (25)
\[G^{(i)}_{bc} = 2\left(d_b \delta_{bc} + \frac{1}{D^{(e)}-1}d_b d_c\right)\] (26)

Note that the external part of the metric \(G^{(e)}\) is unchanged, as it should be. Correspondingly, one should compute the vectors \(q = P^T \bar{q}\) in the new basis. This can be worked out in a straightforward way, and eq. (23) shows that only the internal part of these vectors changes. We find that elementary and solitonic forms are now characterized by
\[q^{(el)} = \begin{pmatrix} 2\epsilon_{\beta} d_{\beta} \\ 2\left(\epsilon_b - \frac{\delta^{(e)}}{D^{(e)}-1}\right) d_b, a(\delta) \end{pmatrix}\] (27)
\[q^{(sol)} = \begin{pmatrix} 2\tilde{\epsilon}_{\beta} d_{\beta} \\ 2\left(\tilde{\epsilon}_b - \frac{\tilde{\delta}^{(e)}}{D^{(e)}-1}\right) d_b, -a(\delta) \end{pmatrix}\] (28)
where \(\delta^{(e)} = \sum_\beta \epsilon_\beta d_\beta\) is the “overlap” of an elementary form with the external space and \(\tilde{\delta}^{(e)} = \sum_\beta \tilde{\epsilon}_\beta d_\beta\) is the complementary quantity for a solitonic form. The vectors specifying curvature in the external and internal space are given by
\[q^{\text{(curv)}}_{\beta} = \begin{pmatrix} 2(d_\gamma - \delta_{\beta\gamma}) d_\beta \\ 2d_\gamma, 2\left(\frac{1}{D^{(e)}-1}d_c - \delta_{cb}\right) \end{pmatrix} .\] (29)
\[q^{\text{(curv)}}_b = \begin{pmatrix} 2d_\gamma, 2\left(-\frac{1}{D^{(e)}-1}d_c - \delta_{cb}\right) \\ 0 \end{pmatrix}.\] (30)

The quantity \(E\) defined in eq. (15) should be rewritten in terms of the transformed quantities as
\[E = \frac{1}{N} \exp(d \cdot \alpha) .\] (31)

From transformation (23) we read off the new dimension vector
\[d = (d_\beta, 0, 0) .\] (32)

Note that the internal components of this vector vanish. This could have been anticipated from the fact that we are actually performing a dimensional reduction, so \(E\) should depend on the reduced metric only.
We can now rewrite Lagrangian (16) in terms of the new, unbarred quantities as

\[ L = \frac{1}{2}E^α T^'G α' - E^{-1}U \]  

(33)

where, as before, \( U = \frac{1}{2} \sum_{r=1}^{m} u_r^2 \exp(q_r \cdot α) \). Since the new metric \( G \) is block diagonal, we can separate the equations of motion into an external, an internal and a dilaton part as

\[
\frac{d}{dτ} \left( E G^{(e)} α^{(e)} \right) + E^{-1} \frac{∂U}{∂α^{(e)}} = 0 \quad (34)
\]

\[
\frac{d}{dτ} \left( E G^{(i)} α^{(i)} \right) + E^{-1} \frac{∂U}{∂α^{(i)}} = 0 \quad (35)
\]

\[
\frac{d}{dτ} \left( E - 8 \frac{D}{D - 2} φ' \right) + E^{-1} \frac{∂U}{∂φ} = 0 \quad (36)
\]

\[
\frac{1}{2} E α^{(e)} T^G α^{(e)} + \frac{1}{2} E α^{(i)} T^G α^{(i)} + E^{-1}U = 0 \ . \quad (37)
\]

The last equation is a constraint which arises as the equation of motion for the gauge parameter \( N \). It is useful to rewrite the equations of motion for the moduli and the dilaton in terms of the comoving time \( t \) defined by \( N = 1 \). Defining a modified potential \( V \) by

\[ V = \exp(-2d^{(e)} \cdot α^{(e)})U \]  

(38)

we get from eq. (33), (34), (31) and (32) that

\[ G^{(i)} \dot{α}^{(i)} + (d^{(e)} \cdot \dot{α}^{(e)}) G^{(i)} \dot{α}^{(i)} + \frac{∂V}{∂α^{(i)}} = 0 \]  

(39)

\[ \ddot{φ} + (d^{(e)} \cdot \dot{α}^{(e)}) \dot{φ} + \frac{∂V}{∂φ} = 0 \]  

(40)

where the dot denotes the derivative with respect to \( t \). The potential \( V \) is explicitly given by

\[ V = \frac{1}{2} \sum_{r=1}^{m} u_r^2 \exp((q_r^{(e)} - 2d^{(e)} \cdot α^{(e)}) \exp(q_r^{(i)} \cdot α^{(i)}) \exp(q_r φ) \ . \]  

(41)

The potential \( V \) can be interpreted as the effective moduli and dilaton potential in the dimensionally reduced 4–dimensional external space action. Correspondingly, the above equations are exactly those of scalar fields with a potential \( V \) in an expanding universe. The potential is provided by the forms and the curvature terms. There is, however, one difference from the ordinary case. Unlike a usual scalar field potential, \( V \) can also depend on the external scale factors \( α^{(e)} \) so that, in general, its shape changes due to the evolution of the universe. Let us analyze this in detail. The terms in potential (41) without \( q_r^{(e)} = 2d^{(e)} \) have no dependence on external scale factors and can be viewed as the “true” potential. A comparison of the \( q \) vectors in eq. (27), (28), (29), (30) with \( d \) in eq. (32) shows that the entries of \( q_r^{(e)} \) are always smaller or equal to \( 2d^{(e)} \). Therefore, all other terms with \( q_r^{(e)} \neq 2d^{(e)} \) are suppressed at late time if the universe expands. In terms of the reduced four-dimensional theory, these suppressed terms correspond to exciting four-dimensional matter in
the form of 0-, 1- or 2-form potentials. The suppression implies that, at late time, we have

$$V \simeq \frac{1}{2} \sum_{q(e) = 2d(e)} u^2 \exp(q(e) \cdot \alpha^{(i)}) \exp(q_r \phi) ,$$

(42)

where, as indicated, the sum now runs over all terms with $q(e) = 2d(e)$. Which forms and curvature terms can actually contribute to this late time potential? Eq. (27) shows that an elementary form should occupy the whole external space to meet this requirement. As such, it corresponds exactly to the “electric” configuration discussed in the previous section. If the external space is 3-dimensional, this can be done with the 3-form of type IIA or M-theory. On the other hand, from eq. (28), a solitonic form should have nonvanishing components in the internal space only, and corresponds to the “magnetic” configuration discussed in the previous section. Finally, a curvature term contributes to the late time potential if it describes a curved internal space, as can be seen from eqs. (29) and (30).

We see that there are a number of possible sources for the asymptotic potential at late time within our framework. It is conceivable that this can be used to stabilize the dilaton and/or the moduli at a finite minimum of $V$. We will now address this question in detail, distinguishing two cases. As the first case, we assume that the dilaton is not the modulus of any compactification but acts as the string coupling constant only. This is the pure type II (string) theory point of view. As the second case, we assume that the dilaton is on the same footing as the moduli; that is, it is a modulus itself (related to the compactification from $D = 11$ to $D = 10$). This is the M-theory point of view.

4 Stabilizing the dilaton in type II

Let us consider the first case, when $\phi$ is not a geometrical modulus. In general, the potential provided by the forms is not sufficient to fix all the moduli and the dilaton. However, we would like to show that it can fix the dilaton vacuum once the moduli vacua have been fixed. To fix the moduli, we assume the existence of a nonperturbative potential $V_{np}(\alpha^{(i)})$, which depends on the moduli only. This is added to $V$,

$$V_T = V + V_{np}(\alpha^{(i)}) ,$$

(43)

and should have a minimum to which a sufficiently large set of trajectories is attracted at late time. A concrete realization is, for example, provided by the mechanism discussed by Tseytlin and Vafa [20]. They have shown that the inclusion of string matter both in the form of momentum and winding modes around a compact direction can stabilize a modulus if both types of matter fail to annihilate. The momentum modes prevent the compact direction from collapsing and the winding modes around that direction prevent expansion. Clearly, since we assume that the dilaton is not a
geometrical modulus and therefore does not correspond to a compact direction, such a mechanism cannot be invoked to provide dilaton stabilization.

We now analyze under what conditions the dilaton can be stabilized. At early times, the moduli, as well as the dilaton, will be displaced from their minima and, finally, oscillate around them. Since we are mainly interested in the vacuum of the dilaton in the present epoch, we will not address this early period, but rather attempt to find a late time asymptotic solution.

First, write out the total late time potential $V_T$ as

$$V_T \simeq \frac{1}{2} \sum_{q_r^{(\epsilon)}=2d^{(\epsilon)}} u_r^2 \exp(q_r^{(i)} \cdot \alpha^{(i)}) \exp(q_r \phi) + V_{np}(\alpha^{(i)}) . \tag{44}$$

The nonperturbative potential $V_{np}$ has been included to stabilize the moduli and we have assumed that it is of the appropriate form to do so. We have, however, to guarantee that a constant moduli solution survives if the whole potential $V_T$ is taken into account. Let us make the consistency assumption that the dilaton is fixed (to be verified later). The total potential $V_T$ should then still have a minimum with a sufficiently large basin of attraction. This is, for example, true if $V_{np} \to \infty$ for $|\alpha^{(i)}| \to \infty$ (a requirement which is fulfilled by the mechanism of Tseytlin and Vafa) and the form and curvature potential is bounded from below (This is true for all possible sources except for positive curvature subspaces. These diverge for small scale factors and have to be balanced by a positive form contribution to fulfill the requirement). These conditions allow us to assume the existence of a well defined moduli minimum $< \alpha^{(i)} >$ for $V_T$. Then $\alpha^{(i)} = < \alpha^{(i)} >$ fulfills the moduli equation of motion (39) and potential (44) turns into

$$V_T \simeq \frac{1}{2} \sum_{q_r^{(\epsilon)}=2d^{(\epsilon)}} \tilde{u}_r^2 \exp(q_r \phi) + \Lambda_{np} , \tag{45}$$

where

$$\tilde{u}_r^2 = u_r^2 \exp(q_r^{(i)} \cdot < \alpha^{(i)} >) \tag{46}$$

$$\Lambda_{np} = V_{np}(< \alpha^{(i)} >). \tag{47}$$

Note that $\Lambda_{np}$ is the contribution to the cosmological constant which results from the nonperturbative moduli potential. Its actual value depends on the specific mechanism which has been invoked to create $V_{np}$.

In order for the dilaton to have a minimum, the sum in (45) should contain at least two terms with opposite sign of the dilaton coupling $q_r$. The analog of the simple toy example given in section two, would be to excite a solitonic orientation and a fundamental orientation of the same form, since, from eqs. (27), (28), we see the two orientations do have different signs in the dilaton coupling. Note, however, that the elementary part has to cover the full external space in order to get a “real” potential term which is not suppressed for a large observable universe. Though the
NS 2–form could provide both a solitonic and an elementary Ansatz, the elementary part does not fully cover a 3 + 1–dimensional external space. Therefore the corresponding potential term drops as the universe expands and the dilaton cannot be stabilized. However, an elementary RR 3–form fits into a 3 + 1 dimensional external space. With an additional solitonic 3–form entirely in the internal space the potential indeed has a stable minimum. The problem is that as a result the Chern-Simons contribution to the IIA equations of motion does not vanish. This takes us outside our Ansatz, and, for this reason, while such a configuration may provide a way of stabilizing the dilaton, we will ignore this possibility from here on.

The remaining possibility with opposite sign dilaton couplings is to turn on a solitonic NS 2–form and a solitonic RR form in the internal space, as can be seen from the dilaton couplings (11). Note that, in order to have the opposite sign of their dilaton couplings, it is crucial to have a RR form turned on in addition to the NS form. Under this condition we indeed have a solution \( \phi = \langle \phi \rangle \) for the dilaton equation of motion (10), where \( \langle \phi \rangle \) is the minimum of (15). The value of \( \langle \phi \rangle \) will consequently be controlled by the strengths of the form fields given by the appropriate \( u_r \) parameters and the vacuum values of the moduli. To conclude, under very mild restrictions on the forms and the structure of the nonperturbative potential, we have found that the dilaton approaches a constant value \( \langle \phi \rangle \) at late time. With the fixed dilaton, the late time potential (45) turns into a pure cosmological constant

\[
\Lambda = \Lambda_f + \Lambda_{\text{np}},
\]

where

\[
\Lambda_f = \frac{1}{2} \sum_{q_r^{(e)} \neq 2d^{(e)}} \tilde{u}_r^2 \exp(q_r \cdot \langle \phi \rangle)
\]

is the contribution to the cosmological constant arising from the forms and curvature terms. The only negative contribution to \( \Lambda_f \) arises from internal subspaces with positive curvature. If they are absent, \( \Lambda_f \) is positive, otherwise it can be of either sign or it can vanish.

As the last step in constructing a consistent late time solution, we should analyze the behaviour of the external scale factors \( \alpha^{(e)} \). To do so, we need the effective potential \( U_T = \exp(2d^{(e)} \cdot \alpha^{(e)})V_T \) which arises after fixing the moduli and the dilaton. Inserting this into eq. (37) then determines the evolution of the external scale factors. From eq. (41) we have

\[
U_T = \Lambda \exp(2d^{(e)} \cdot \alpha^{(e)}) + \frac{1}{2} \sum_{q^{(e)} \neq 2d^{(e)}} \tilde{u}_r^2 \exp(q_r \cdot \alpha^{(e)})
\]

with \( \tilde{u}_r = \tilde{u}_r^2 \exp(q_r \cdot \langle \phi \rangle) \). Note that the first term represents the cosmological constant which, as discussed above, arises from the late time potential. In addition, we have all those terms from eq. (41) with \( q^{(e)} \neq 2d^{(e)} \) that decay at late time if the universe expands. Clearly, these terms can
be neglected at late time if the cosmological constant is positive, $\Lambda > 0$. Let us consider this case first.

For simplicity, assume that the external space is spatially isotropic; that is $\alpha^{(e)} = (\alpha_0)$ and $d^{(e)} = (3)$. Then, from eq. (25), we have $G^{(e)} = (-12)$ and from eq. (31) it follows that $E = \exp(3\alpha_0)$ in the comoving gauge $N = 1$. Inserting these quantities into eq. (37), we find $\dot{\alpha}_0 = \sqrt{\Lambda/6}$. This corresponds to a de Sitter spacetime with inflationary expansion.

It might be possible to tune the two contributions to $\Lambda$ such that $\Lambda = 0$. In that case, if there is no other source of energy density, the universe is static, $\dot{\alpha}^{(e)} = 0$. Energy density for an expansion could be provided by the other terms in eq. (50) related to forms or curvature terms which are “nontrivial” in the external space. In terms of the reduced four-dimensional action, such terms correspond to exciting form-field matter, or curving the spatial part of the four-dimensional metric. It results in a radiation–like expansion. Let us again consider the case of an isotropic external space. Then the only possibility to have $q^{(e)} \neq 2d^{(e)}$ is $q^{(e)} = (0)$. In the reduced theory this corresponds to exciting the kinetic terms of 0-form, that is scalar, matter. From eq. (50) we find $U = \text{const}$ and eq. (37) can be readily solved to give $\alpha_0 = \ln t/3 + c$, where $c$ is a constant related to $U$. As expected, this subluminal expansion with a power $1/3$ is characteristic for an expansion driven by scalar field kinetic energy. If the external space is non-isotropic, one may have nonzero vectors $q^{(e)}$ with $q^{(e)} \neq 2d^{(e)}$ and the solution is more complicated. Its general form, if only one of those terms appears in the potential (50), has been given in ref. [9]. In this case, the expansion is subluminal, radiation–like but the expansion powers, though always smaller than one, may depart from the value $1/3$. Alternatively, one could add radiation to the model which one expects to arise from the decay of the coherent moduli and dilaton oscillations at early times. This would yield a true radiation dominated phase with an expansion power $1/2$.

To summarize, we have shown that, within our Ansatz, in order to stabilize the dilaton vacuum it is essential to have a solitonic NS 2–form and a solitonic RR form both turned on in the internal space. The value of the dilaton vacuum is controlled by the charges of the form fields and the vacuum values of the moduli. Having additional forms does not change this result. We emphasize, that the existence of RR forms is crucial for this mechanism to work because of their different couplings to the dilaton in action (10).

Let us illustrate the above mechanism with a concrete $D = 10$, type IIA example. We choose an external $1 + 3$–dimensional space ($D^{(e)} = 3, d^{(e)} = (6)$) with scale factor $\alpha^{(e)} = (\alpha_0)$. The internal 6–dimensional space ($D^{(i)} = 6$) is split up as $3 + 2 + 1$ so that we are dealing with three moduli $\alpha^{(i)} = (\alpha_1, \alpha_2, \alpha_3)$. For simplicity, we take all spatial subspaces to be flat.

The internal space has been split in this particular way so as to place a solitonic NS 2–form in the 3–dimensional subspace and a solitonic RR 1–form in the 2–dimensional subspace.
eq. (28) we find that these two forms are described by the vectors \( q^{NS} = (6, -9, -2, -1, -1) \) and \( q^{RR} = (6, -3, -6, -1, 3/2) \). Note that for both vectors \( q^{NS(e)} = 2d^{(e)} = 6 \), so that they are maximal on the external space and contribute to the late time potential. Furthermore, we have the internal vectors \( q^{(i)NS} = (-9, -2), q^{(i)RR} = (-3, -6, -1) \) and the dilaton couplings \( q^{NS} = -1, q^{RR} = 3/2 \). Then, from eq. (41) we find the potential

\[
V = \frac{1}{2} \left( u^{2}_{NS} e^{-2\alpha_1 - 2\alpha_2 - 3\alpha_3} + u^{2}_{RR} e^{-3\alpha_1 - 6\alpha_2 - 3\alpha_3} \right). 
\]

(51)

Clearly, this potential has a minimum in the dilaton direction. As it stands, however, it drives the moduli to infinity and their variation in time then also renders the dilaton minimum time dependent. Therefore, we assume that the moduli \( \alpha_1, \alpha_2, \alpha_3 \) (but not the dilaton!) are stabilized by some nonperturbative potential \( V_{np}(\alpha^{(i)}) \). (The possibility of stabilizing all fields, the dilaton and the moduli, without invoking any nonperturbative effects will be analyzed below). After a sufficiently long time, oscillations are damped out and the moduli have settled down to their minimum \( \alpha^{(i)} = \langle \alpha^{(i)} \rangle \). Then potential (51) turns into

\[
V = \frac{1}{2} \left( \tilde{u}^{2}_{NS} e^{-\phi} + \tilde{u}^{2}_{RR} e^{3\phi/2} \right). 
\]

(52)

The constants \( \tilde{u}^{2}_{NS} \) and \( \tilde{u}^{2}_{RR} \) are defined as in eq. (46). This potential has a dilaton minimum at

\[
\langle \phi \rangle = \frac{2}{5} \ln \left( \frac{2\tilde{u}^{2}_{NS}}{3\tilde{u}^{2}_{RR}} \right) 
\]

(53)

with positive cosmological constant

\[
\Lambda_f = \frac{5}{6} \left( \frac{2}{3} \right)^{-2/5} \left( \tilde{u}^{2}_{NS} \right)^{3/5} \left( \tilde{u}^{2}_{RR} \right)^{2/5}. 
\]

(54)

If the total cosmological constant \( \Lambda = \Lambda_f + \Lambda_{np} \) is positive, the external space expands in a de Sitter phase. If \( \Lambda = 0 \) the external space is static. By adding radiation (for example from the decay of the early time oscillations) to our model we can also get a radiation dominated phase.

5 A scaling argument

It is useful, at this point, to present a more physical explanation of why the dilaton vacuum can be determined, along the lines of the discussion for the simple model given in section two. For clarity, we focus on the specific example just discussed which will graphically illustrate our main point. Any other solution can be analyzed in a similar manner. Consider the action (10) restricted to this specific example. The relevant fields are, in addition to the metric and dilaton, a NS 2–form \( B_{\mu\nu} \) and a RR \( \delta–form \ A_{\mu_1...\mu_3} \) each living in the internal space only. We will restrict the form fields to be solitonic, in line with the example, though we will set \( \delta = 1 \) only later. We make the physical
assumption that the observable space continuously expands but that the compactified space, after a period of contraction, becomes fixed. Let us scale the fields according to

\[ \phi \rightarrow \phi + s \ln \lambda \]
\[ B_{\mu\nu} \rightarrow \lambda \tilde{B}_{\mu\nu} \]
\[ A_{\mu_1 \cdots \mu_5} \rightarrow \lambda^{\tilde{r}} \left( \frac{5-d}{2} \right) \tilde{A}_{\mu_1 \cdots \mu_5} . \]

Furthermore, we scale the 3 + 1–dimensional part of the metric as

\[ g_{\mu\nu} \rightarrow \lambda^{-r} g_{\mu\nu} \]

but hold the 6–dimensional internal space metric fixed. The action is, up to an overall factor, invariant under these transformations for arbitrary values of \( r, s \) and, hence, so are the equations of motion.

It follows from the invariance of the action under the Abelian gauge transformation \( B \rightarrow B + d\Lambda_2 \) that there exists a conserved gauge current and, hence, a conserved electric charge, associated with fundamental string sources, given by

\[ e_2^B = \int_{\Sigma_7} * e^{-\phi} H , \]

where \( H = dB \) and \( \Sigma_7 \) is a compact 7-dimensional space. In our example, this charge vanishes. However, there also exists a magnetic charge, associated with solitonic 5–brane sources,

\[ g_6^B = \int_{\Sigma_3} H . \]

In our example there is a non-zero magnetic charge when the integral is taken over the internal 3-dimensional subspace. Under the non–compact scaling transformations in (55), (56) these conserved charges transform as

\[ e_2^B \rightarrow \lambda^{-\tilde{r}} \tilde{e}_2^B \]
\[ g_6^B \rightarrow \lambda^{\tilde{r}} \tilde{g}_6^B , \]

respectively. Similarly, the invariance of the action under the Abelian gauge transformation \( A_\delta \rightarrow A_\delta + d\Lambda_\delta \) leads to two conserved charges

\[ e_\delta^A = \int_{\Sigma_{\delta+1}} * e^{-\left( \frac{d-4}{2} \right) \phi} F_\delta \]
\[ g_\delta^A = \int_{\Sigma_{\delta+1}} F_\delta \]

associated with elementary \( \delta - 1 \) brane and solitonic \( \tilde{\delta} - 1 \) brane sources, respectively, where \( F_\delta = dA_\delta \) and \( \tilde{\delta} = 8 - \delta \). Again, in our example the electric charge is zero, but the magnetic charge
is non-zero, when the integral is taken over the internal 2-dimensional subspace. We find that

\[ e_\delta^A \to \lambda^{\frac{-\delta - 4}{2}} \left( \frac{\delta + 4}{2} \right) \frac{e_\delta^A}{g_\delta^A} \]  
(63)

\[ g_\delta^A \to \lambda^{\frac{-\delta + 4}{2}} \left( \frac{\delta - 4}{2} \right) \frac{g_\delta^A}{e_\delta^A} \]  
(64)

under the scaling transformation \( \delta \leadsto \lambda \).  

Let us first consider solutions for which all electric and magnetic charges vanish. In this case, the effective four-dimensional theory governing these solutions must exhibit the full scaling symmetry specified by \( r \) and \( s \). This symmetry tells us that the dilaton potential must be flat, with any value of the dilaton being an allowed vacuum. As we will see shortly, this is indeed the case. Since the dilaton can take any value in this flat potential, we conclude that this theory does not fix the dilaton vacuum. Now consider solutions for which all electric and magnetic charges vanish except for a single NS charge. To be specific, assume, as in the example, that \( g_B^6 \neq 0 \). All solutions of the associated effective theory must preserve this charge. Note from expression (60) that \( g_B^6 \) is, in general, not preserved under scaling transformations. However, \( g_B^6 \) will be preserved under the one–parameter subset of scaling transformations specified by

\[ r = -s. \]  
(65)

It follows that the effective theory still must exhibit a scaling symmetry, now specified by \( s \) only. However, this reduced symmetry implies that either the potential is flat, or it is non–flat with no stable finite vacuum of the dilaton. In this case, as we will see below, the vacuum degeneracy is lifted, but the dilaton runs off to infinity. We conclude that this theory still cannot fix the dilaton vacuum.

Now, however, consider the case that, in addition to the nonvanishing NS solitonic charge \( g_B^6 \), there is also a non–vanishing RR solitonic charge \( g_\delta^A \). We see from eq. (64) that this charge will be scale invariant only if

\[ r = -\frac{\delta - 4}{2} s. \]  
(66)

Since in a type IIA theory \( \delta \leq 3 \), this expression is never compatible with (65). Therefore, scale invariance is completely broken. It follows that the dilaton vacuum degeneracy must be lifted. This is a first, and necessary, step toward stabilizing the dilaton vacuum. Again, it is not in itself sufficient because the vacuum may still run off to infinity and never stabilize at a finite value. In this case, however, it is possible that the dilaton potential has a finite, non–degenerate minimum. We now show that the existence of two non-vanishing solitonic forms, one NS and one RR, actually stabilizes the dilaton vacuum. Note that the coefficients \( a(\delta_r) \) in action (10), which control the coupling of the dilaton to \( \delta_r \)–forms, are given by \( a = 1 \) for the NS 2–form and \( a(\delta) = (\delta - 4)/2 \) for a RR \( \delta \)–form in type IIA. Since \( \delta \leq 3 \) for the RR form, it follows that \( a(\delta) < 0 \), opposite in sign from the NS coefficient. Inserting the two solitons into action (10) leads, in the reduced four-dimensional
effective theory, to a potential energy for the dilaton of the generic form

$$V_{\text{eff}} = \frac{1}{2} \left( A^2 e^{-\phi} + B^2 e^{\left( \frac{1}{2} - \delta \right) \phi} \right),$$

(67)

where $A^2$ and $B^2$ are positive real numbers, related to the NS and RR form magnetic charges. For theories with no charges, $A = B = 0$ and the dilaton potential is flat, as we argued from scaling invariance. Theories with all charges zero except $g_6^B$ have $A \neq 0$, $B = 0$, thus admitting the first term in eq. (67) only. The potential is no longer flat, but the dilaton runs off to infinity. However, when $g_6^B$ and $d_6^A$ are non-vanishing, both $A$ and $B$ are non-zero, and the potential has a stable vacuum at

$$\langle \phi \rangle = \frac{2}{6 - \delta} \ln \left( \frac{A^2}{B^2} \frac{2}{4 - \delta} \right).$$

(68)

Setting $\delta = 1$ yields the potential and the dilaton vacuum of the above example, given in (52) and (53), respectively. We conclude, that theories of this type exhibit a stable dilaton vacuum. The reason for this stability is first, the complete breaking of scale invariance by the topological charges, which must be conserved, and second, the fixing of the dilaton vacuum at a finite value due to the different sign of the dilaton coupling to the NS and RR forms.

### 6 Stabilizing moduli in M–theory

Next, we would like to discuss the case where the dilaton is viewed as a modulus. This is the appropriate point of view if one considers M–theory where the dilaton arises as the compactification radius of the eleventh dimension. Since the dilaton does not play a special role from that perspective, there is no reason why one should invoke a nonperturbative mechanism to stabilize the moduli but not the dilaton. We should therefore ask the more ambitious question whether the potential provided by the forms and curvature terms allows for a stabilization of all moduli $\alpha_i$. Of course, there are other moduli, corresponding to further deformations of the spherical or toroidal subspaces and zero modes of the form fields, which we have not included. Thus, in the examples that follow, we will strictly be searching for solutions which stabilize a subset of the moduli, including, hopefully, the dilaton.

The low-energy limit of M–theory is 11-dimensional supergravity, which contains a single three-form potential. The action is of our general form (10), if we take $D = 11$ and set the dilaton and cosmological constant to zero. The full supergravity action also includes a Chern-Simons term, describing the self-coupling of the three-form field. However, as stated above, for all the configurations we will consider the contribution from this term is zero, and so we can drop it from the action. That there might be solutions with all moduli stabilized, is suggested by the original seven-sphere compactifications of eleven-dimensional supergravity discussed by Duff [21]. In these solutions spacetime is a product of a four-dimensional anti-deSitter space and a seven-sphere of
fixed radius. The three-form potential is excited, so that the corresponding four-form field strength spans the four-dimensional space. The radius of the seven-sphere is directly related to the charge of the form field. Considered as a compactification to four-dimensions, the radius is a modulus field which appears to have been stabilized by the presence of the three-form charge.

We can see this stabilization directly by rewriting the solution in the framework given at the beginning of this paper. By doing so we will also show that it can sensibly be interpreted as the asymptotic limit of a dynamical cosmological solution. That is, if the radius of the seven sphere is a little away from its stabilized value, there is a smooth solution where the external four-space continues to evolve while the radius settles down into the minimum. First we note that the space has been split into an external three-space and an internal seven-space, so that $\tilde{d} = (3, 7)$ and we have a metric of the form

$$ds^2 = -N^2(\tau)d\tau^2 + e^{2\tilde{\alpha}_0}d\Omega^2_{K_0} + e^{2\tilde{\alpha}_1}d\Omega^2_{K_1},$$

(69)

where $\tilde{\alpha}_0$ describes the curvature of the external space while $\tilde{\alpha}_1$ is the modulus describing the radius of the internal space. Since the internal space is a seven-sphere, we must have $K_1 = 1$, while the external space is anti-de Sitter, so must have a negatively curved spatial subspace implying $K_0 = -1$. These curvatures contribute to the effective potential given in eq. (13), with $\tilde{q}_r$ vectors given by $\tilde{q}_{K_0} = (6, 12)$ and $\tilde{q}_{K_1} = (4, 14)$ respectively. The different signs of the curvatures imply that the terms in the effective potential also differ by a sign. For the seven sphere the coefficient of the exponential is $u^2 = -2$, while for the external three space $u^2 = 2$. The form field spans the external space and has a time-like component. Thus it corresponds to an elementary Ansatz, and gives the vector $\tilde{q} = (6, 0)$. Collecting all this together we find that the effective potential (13) is given by

$$U = \frac{1}{2}u^2 e^{6\tilde{\alpha}_0} - 2e^{6\tilde{\alpha}_0 + 12\tilde{\alpha}_1} + 2e^{4\tilde{\alpha}_0 + 12\tilde{\alpha}_1},$$

(70)

the three terms corresponding to the form field, the seven-sphere and the curvature of the external space respectively. As discussed above, we must make a Weyl rescaling in order to put the Einstein-Hilbert action for the external, four-dimensional part of the metric in canonical form, and so diagonalize the metric $G$ in the $\tilde{\alpha}$ space. The general transformation is given in eq. (23) and here simply corresponds to introducing $\alpha_0 = \tilde{\alpha}_0 + \frac{7}{2}\tilde{\alpha}_1$ and $\alpha_1 = \tilde{\alpha}_1$. The potential can then be written as

$$U = e^{6\tilde{\alpha}_0} \left( \frac{1}{2}u^2 e^{-21\alpha_1} - 2e^{-9\alpha_1} \right) + 2e^{4\tilde{\alpha}_0}.$$

(71)

We note first that the last term, which comes from the curvature of the external space, now no longer depends on the internal modulus $\alpha_1$. This is as is expected since it is a property of the external space alone. The other two terms provide a potential for $\alpha_1$. The point here is that this potential has a minimum, which fixes the radius of the internal sphere at $\alpha_1 = <\alpha_1> = \frac{1}{12}\ln(7u^2/12)$. There is a balance between the contribution to the potential from the curvature energy of the internal
seven-sphere, which increases with radius, and the field energy due to the form field, which decreases with radius. This balance is the origin of the stabilization of the radius modulus. It is important that, in calculating the dependence of the form-field energy on radius, we recall that the charge of the solution cannot change dynamically. This translates into the condition that the flux of the form field across the seven-sphere, that is $\int_{S^7} *F$, which is proportional to $u$, must remain fixed.

To really show stability we must be a little more careful because there is a dynamical prefactor $\exp(6\alpha_0)$ in the relevant terms in the potential (71). Following our previous discussion, we must write out the equation of motion for $\alpha_1$ in comoving time, defined by $N = \exp(7\alpha_1/2) \bar{N} = 1$. From eqs. (38) and (39) we find that the relevant potential, which is just the effective potential in the reduced four-dimensional theory, is then

$$V = \exp(-6\alpha_0)U = \left(\frac{1}{2}u^2 e^{-21\alpha_1} - 2e^{-9\alpha_1}\right) + 2e^{-2\alpha_0}, \quad (72)$$

and the prefactor disappears. Thus we can conclude that minimizing the term in parentheses truly represents a stabilization of the radius of the internal space. To complete the description of the solution, we note that, at the minimum, the value of this term is negative and so provides a negative cosmological constant. This is the reason why the solution for $\alpha_0$ then gives a four-dimensional anti-de Sitter space.

Two further comments are worth making about this solution. First, it is completely supersymmetric, preserving the full $N = 8$ supersymmetry in four dimensions. Secondly, it also represents the infinite throat inside the membrane solution of eleven-dimensional supergravity, as first discussed by Gibbons and Townsend [22]. In fact, as we will discuss below, there are a number of other $p$-brane solutions with an infinite throat which lead to cosmological solutions with stable moduli.

While compactifying on a seven sphere provides an interesting example of a pure supergravity solution with stable moduli it is not very physical. It is more natural to look for solutions which have one internal direction compactified on a circle. This radius can then be related to the dilaton of string theory.

Let us assume that the spacetime is split into an external $(3 + 1)$–space with a scale factor $\alpha_0$ and an internal seven-space which is further split into maximally-symmetric subspaces, one of which is a circle. Any stabilization of the moduli will be controlled by the potential $U$. Referring to eqs. (38) and (39), we recall that the potential that actually enters the canonical moduli equations of motion is $V = \exp(-6\alpha_0)U$, the effective potential in the reduced four-dimensional theory. To be sure of a stable solution, the part of $V$ which has a minimum for the moduli must be independent of the external scale factor $\alpha_0$. This is equivalent to the statement that, in the reduced effective four-dimensional theory, the excited form field strengths appear either as 0-forms or as 4-forms, and so are not dynamical, but contribute only to the effective potential. They correspond to the “electric”
and “magnetic” configurations discussed in section two. One also notes that the components of the vectors $q$ which control the exponentials in the sum of terms which enter $U$, are always negative or zero. This implies that it is impossible to stabilize all the moduli with a such a potential unless at least one of the coefficients $u^2$ is negative. The only way this is possible is to have some positive curvature in the internal space.

Having made these general observations let us consider a simple case where we split the internal space into two three-spheres (providing the necessary curvature) and a circle. In addition, we include two solitonic orientations of the form field. One spans one three-sphere and the circle, the other spans the other three sphere and the circle. We also include an external space curvature, $K = 0, \pm 1$, in case the stable solution leads to a non-zero four-dimensional cosmological constant. We will write $\alpha_1$, $\alpha_2$ and $\alpha_3$ for the moduli of the two three-spheres and the circle respectively, and keep $\alpha_0$ for the external space. Using the expressions for the relevant $q$ vectors (27), (28), (29) and (30), we find that the potential $V$ is given by

\[ V = \frac{1}{2} u_1^2 e^{-9\alpha_1 - 3\alpha_2 - 3\alpha_3} + \frac{1}{2} u_2^2 e^{-9\alpha_1 - 3\alpha_2 - 3\alpha_3} - 2e^{-5\alpha_1 - 3\alpha_2 - \alpha_3} - 2e^{-3\alpha_1 - 5\alpha_2 - \alpha_3} - 2Ke^{-2\alpha_0}, \quad (73) \]

where $u_1$ and $u_2$ are the charges of the two solitonic orientations of the form field. The last term represents the external space curvature and does not effect the stabilization. To see if there is a minimum of $V$, it is convenient to introduce new variables

\[ x = 2\alpha_1 + 2\alpha_2 + \alpha_3, \quad y = \alpha_1 - \alpha_2, \quad z = \alpha_3. \quad (74) \]

The potential then reads

\[ V = \frac{1}{2} e^{-3x} \left( u_1^2 e^{-3y} + u_2^2 e^{3y} \right) - 2e^{z-2x} (e^y + e^{-y}) - 2Ke^{-2\alpha_0}. \quad (75) \]

It is then clear the potential is not stabilized in the $z$ direction but rather goes to negative infinity as $z$ increase. For fixed $z$ there is however a minimum in $x$ and $y$. In this sense the potential “stabilizes” two of the moduli.

This is, in fact, a generic result. Using our simple Ansatz with maximally symmetric subspaces, it is not possible to find a solution with one modulus describing a circle and all the moduli stabilized. However, as in the case of the string theories discussed previously, if one of the moduli gets stabilized by some other mechanism, the presence of non-trivial form fields can then lead to stabilization of all the other moduli. Two further points are worth making. First, we only chose configurations which did not excite the Chern-Simons term in the supergravity action. It is possible that relaxing this condition provides the freedom necessary to stabilize all the moduli. Secondly, in the most physical scenario, corresponding to strong coupling limit of the heterotic string, one dimension is compactified on an orbifold rather than a circle, and, further, the presence of gauge fields living on the ten-dimensional fixed points of the orbifold leads to sources for the form field $B$. Including either these effects would take us outside the types of field configurations considered here.
Even within our Ansatz of maximally-symmetric subspaces, many other solutions with stable moduli exist, especially if one relaxes the condition that the external space is four-dimensional. Some of these solutions preserve some fraction of the supersymmetry. As an example, consider a spacetime split into an external three-dimensional space and an internal space which is the product of a three-sphere with a four-torus and a circle. We excite a solitonic form across the sphere and the circle and a fundamental form across the external space and the circle. Let us assume that the external space is negatively curved. If we write $\alpha_0$ for the external scale factor and $\alpha_1$, $\alpha_2$ and $\alpha_3$ for the scale factors of the internal sphere, torus and circle respectively, then, using eqns. (27), (28), (29) and (30), we find the effective three-dimensional potential is given by

$$V = \frac{1}{2} u_1^2 e^{-12\alpha_1 - 8\alpha_2 - 4\alpha_3} + \frac{1}{2} u_2^2 e^{-12\alpha_1 - 16\alpha_2 - 2\alpha_3} - 2 e^{-8\alpha_1 - 8\alpha_2 - 2\alpha_3} + 2 e^{-2\alpha_0}.$$  

(76)

Here $u_1$ is proportional to the charge of the solitonic form, while $u_2$ is proportional to the charge of the fundamental form. It is convenient to introduce two new variables

$$x = 4\alpha_2 - \alpha_3, \quad z = 4\alpha_1 + 4\alpha_2 + \alpha_3,$$

(77)

so that the effective potential can be rewritten as

$$V = \frac{1}{2} e^{-3z} \left( u_1^2 e^x + u_2^2 e^{-x} \right) - 2 e^{-2z} + 2 e^{-2\alpha_0}.$$  

(78)

Thus we find that the potential depends only on two of the three moduli; that is to say, there is a flat direction. Moreover there is a stable minimum at

$$x = \ln |u_2/u_1|, \quad z = \ln |3u_1u_2/4|.$$  

(79)

Thus two of the three moduli are stabilized while the third corresponds to a flat direction and so can take on any constant value, implying that we have a consistent cosmological solution with fixed moduli. The value of $V$ at this minimum is negative so that the external space is in a de Sitter phase.

Furthermore, this solution corresponds to the infinite throat of the intersecting membrane-fivebrane solution of M–theory, in the degenerate limit where the brane charges are equal (so we take $u_1 = u_2$). As such it preserves one-quarter of the supersymmetry. Other solutions can similarly be identified with, for instance, the infinite throats of the single fivebrane solution and the degenerate triple fivebrane solution, preserving all and one-eighth of the supersymmetry respectively. Likewise, there are cosmological solutions corresponding to the throats of $p$-brane solutions in type II and heterotic solutions. Again, they describe the stabilization of several moduli, though, in general the dilaton is not fixed in these solutions. Similar behavior, of an effective potential with enhanced supersymmetry at points where the moduli become fixed, has been observed in work on black holes.

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7 Conclusion

In this paper, we have shown that non-trivial form fields of type II and M–theory provide an effective potential for the dilaton and moduli, which can fix these fields to a finite, stable minimum during their cosmological evolution. The value of the fields at the minimum is controlled by the strength of the form field charges. The structure of this potential is such that after a short period of oscillations around the minimum, which are damped by the expansion of the universe and a possible decay of the coherent modes, the moduli and dilaton settle down to what should be interpreted as the vacuum of low-energy particle physics. Furthermore, this process is consistent with an ongoing expansion of the observable universe.

More specifically, we have addressed cosmological dynamics in type II theories. The dilaton can be fixed by turning on solitonic NS and RR forms in the internal space, once the geometrical moduli are stabilized by an additional nonperturbative potential. A physical understanding of this can be obtained by analyzing the scaling symmetries of the theory. We argued that these symmetries, which normally prevent a dilaton stabilization, are broken by the conserved form field charges.

In M–theory, we asked the more ambitious question of whether the moduli can be consistently fixed without invoking additional nonperturbative effects. It turned out that this is indeed possible in simple examples by turning on solitonic forms in the internal space (or an elementary form which covers the full external space) and by using positively curved internal spaces. Moreover, these examples show that part of the supersymmetry, which we generically expect to be completely broken during the early period of the moduli evolution, can be restored once the moduli have settled down to their vacuum. Therefore our mechanism can be consistent with the idea of low energy supersymmetry.

Some properties of this cosmological scenario are reminiscent of phenomena observed in the context of string black holes [24, 25, 26, 27]. There it has been noted that certain scalar fields are attracted to fixed points once the radial coordinate approaches the black hole horizon. Moreover, at these fixed points supersymmetry is restored. These analogies between the time evolution of cosmological models and the radial dependence of black holes are not surprising given the fact that a subclass of the types of cosmological solutions we consider corresponds to the interior solutions of black p–branes where the radius coordinate becomes timelike [8, 12, 13, 14]. However, it should be stressed that the mechanism discussed in this paper is not restricted to cosmological models related to black holes, but applies to a much wider class.

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