A Secure User Authentication Protocol for Heterogeneous Mobile Environments

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ABSTRACT Mobile devices have become very important for our daily needs. The user authentication protocols with the key agreement are required to deal with the security issues that arise from the use of mobile devices through Internet applications. However, existing user authentication protocols are only suitable if the client and the server use a similar cryptographic approach. Therefore, it is important to develop an authentication protocol for mobile environments with heterogeneous cryptographic approaches. In this paper, an efficient user authentication and key agreement protocol is proposed for a heterogeneous client-server mobile environment. The security of the proposed scheme is formally proved under the \( q \)-strong Diffie-Hellman problem (\( q \)-SDH), the \( q \)-bilinear Diffie-Hellman inversion problem (\( q \)-BDHI), and the modified bilinear Diffie-Hellman inversion problem (mBDHI), respectively. Our scheme has reasonable processing costs and communication costs on the client and server sides. Moreover, our scheme is suitable for applications that use different cryptographic approaches. In particular, the proposed protocol can work when the client applies the identity-based cryptosystem and the server applies the certificateless cryptosystem.

INDEX TERMS User authentication, key agreement, random oracle model, heterogeneous environment.

I. INTRODUCTION

Previously, larger devices such as laptops and PCs were preferred over smaller devices such as cell phones and tablets. Today, however, people prefer mobile devices because they can be used for a variety of applications such as e-commerce, e-banking, e-healthcare, and e-government [1]–[3]. Mobile applications can work in different architectures, for example, in a client-server environment and in a multi-server environment [4], [5]. In a client-server architecture, the mobile device represents the client for accessing the services offered by the server. Separately, mobile device applications operate in an open network by using the Internet, which raises security issues such as authentication, key agreement, and confidentiality [6]–[8]. The client and server must authorize each other. In addition, the two parties must agree on a session key for subsequent connections [9].

To overcome the drawbacks of authentication, key agreement, and confidentiality, various public-key cryptosystems have been proposed, such as the public-key infrastructure cryptosystem (PKI) [10], the identity-based cryptosystem (IBC) [11], and the certificateless cryptosystem (CLC) [12]. However, PKI is very computationally intensive due to certificate management and is therefore not suitable for mobile...
environments. IBC appears to reduce the computational overhead of PKI and is more suitable for mobile environments, but lacks efficiency in key escrow. The CLC, on the other hand, eliminates both the need for the certificate in PKI and the key escrow problem in IBC [13].

Certain user authentication schemes with key agreement have been proposed to solve the user authentication and key agreement issues [14]–[16]. However, all protocols are homogeneous, i.e., the client and the server all belong to a similar cryptographic approach. For example, the clients get their keys from the server, which mainly uses IBC or CLC. With the rapid growth of mobile applications and the daily needs of users, it is important to have heterogeneous architectures that can work with different cryptographic approaches. For example, clients get their keys from the server using IBC and servers get their keys from the server using CLC. In this context, mobile applications using the Internet with different cryptographic environments may need to communicate with each other to access services.

Recently, Li et al. [17] proposed a heterogeneous user authentication scheme that takes into account when the client and the server use different cryptosystems. In their protocol, the client uses the IBC environment and the server is from PKI. However, their scheme has the disadvantage of causing certificate management overhead, which is unsuitable for mobile devices with limited memory and power. Therefore, existing systems need to be vastly improved or new ones created. For example, some applications require the client to request an IBC while the server requests a CLC. In this case, it is highly desirable to use a user authentication protocol that meets the requirements of this environment. In this paper, we propose a secure user authentication protocol for heterogeneous mobile environments.

II. RELATED WORKS

To tackle the authentication issues in mobile environments, several identity-based user authentication schemes have been suggested without mutual authentication and key agreement [18], [19].

To address authentication issues in mobile environments, several identity-based user authentication schemes without mutual authentication and key agreement have been proposed [18], [19]. Similarly, many identity-based user authentication protocols with the key agreement have been introduced to overcome the weaknesses of the previous schemes in terms of computational cost, communication cost, and security vulnerabilities. Wu and Tseng [14] have presented a new user authentication protocol based on IBC and provided a formal proof of their protocol. In addition, their protocol provides key agreement and mutual authentication. In terms of computational cost improvement, He [20] proposed another user authentication protocol with formal proof that has the same security properties as Wu and Tseng’s scheme [14], resulting in a computationally effective and efficient scheme compared to that of Wu and Tseng’s [14]. Chou et al. [21] proposed a two-party scheme AKA for the mobile application environment. This scheme is based on identity with ECC.

In addition, they extended their work to design another three-party scheme AKA for initiating the session key between trusted service providers and users. They claimed that their proposed scheme resists common attacks on user authentication schemes, such as perfect forward secrecy, known key security, and key identity compromise impersonation [22]. Later, Farash and Attari [23] found that Chou’s schemes et al. [21] are not as resistant to the above attacks as they claimed. They introduced an improved identity-based AKA scheme to overcome the drawbacks of Chou et al.’s scheme [21].

Tsai and Lo [24] considered the anonymous property of the client identity which led them to design anonymous user authentication protocol for mobile device applications. Furthermore, their scheme has less processing cost on the client side than He’s scheme. Tseng et al. [25] also considered ephemeral secret leakage (ESL) attacks which resulted in designing a user authentication protocol which resists these attacks. In all the schemes above, the client and the server belong to the IBC. Because the private keys in IBC are generated by a trusted third party called the private key generator (PKG), these schemes suffer from the key escrow problem. Consequently, if the PKG is compromised by an adversary, then all clients’ private keys are jeopardized.

Since the certificateless cryptosystem (CLC) [26] is proposed to overcome the key escrow problem in IBC, Hou et al. [27] suggested the scheme of user authentication protocol without achieving mutual authentication which used CLC. The proposed protocols should be secure against the adversary type I and adversary type II as discussed in [26]. Aftermath, Hassan et al. [16] presented a new protocol based on CLC that offers key agreement and mutual authentication. They claimed that their scheme is secured from adversary types I and II respectively. Nevertheless, Hassan et al. [28] found that the protocol in [16] does not resist adversary type II. Then, they proposed another user authenticated key agreement protocol with mutual authentication which is indeed secure against adversary types I and II. In all the schemes above, the client and the server remains on the CLC.

After looking at all the previous user authentication protocols, we found that the client and the server belong to the same cryptosystem environment. For example, the client and the server belong to the IBC or the CLC. However, if an application needs to work in different cryptosystem environments (e.g., the client belongs to the PKI and the server belongs to CLC), it doesn’t require such user authentication protocol. Recently, Li et al. [17] proposed a heterogeneous user authentication protocol of mobile client-server. In their scheme, the client belongs to the IBC and the server belongs to the PKI. In this paper, user authentication with the key agreement protocol is presented for the heterogeneous mobile client-server environment. In our proposed scheme, the client applies to the IBC, while the server applies to the CLC.
A. ORGANIZATION

Our paper is presented as follows: The preliminaries are introduced in Section III while our protocol is proposed in Section IV. The security of the presented protocol is displayed in Section V and the analysis of the protocol’s performance is demonstrated in Section VI with the conclusions in Section VII.

III. PRELIMINARIES

A. BILINEAR PAIRINGS

Let \( G_1 \) be a cyclic additive group and \( G_2 \) be a cyclic multiplicative group with a large prime order \( q \). Let \( P \) be a generator of \( G_1 \). Therefore, a bilinear map \( e : G_1 \times G_1 \rightarrow G_2 \) realizes the following properties [29], [30]:

1) Bilinearity : \( \forall X, Y \in \mathbb{Z}_q^* \) and \( \forall Q, R \in G_1 \), \( e(xQ, yR) = e(Q, R)^{xy} \).
2) Non-degeneracy: Let \( P \) be a generator of \( G_1 \), \( e(P, P) \neq 1_{G_2} \). Where \( 1_{G_2} \) is identity element of \( G_2 \).
3) Computability: \( \forall Q, R \in G_1 \), the \( e(Q, R) \) is processed efficiently.

1) \( q \)-STRONG DIFFIE-HELLMAN (\( q \)-SDH) PROBLEM

Given \( (P, \alpha P, \alpha^2 P, ..., \alpha^q P) \), the \( q \)-strong Diffie-Hellman (\( q \)-SHD) problem in \( (G_1, G_2) \) is to produce a pair \( (w, \frac{1}{\alpha + w}) \) where \( w, \alpha \in \mathbb{Z}_q^* \) and \( P \in G_1 \).

2) \( q \)-BILINEAR DIFFIE-HELLMAN INVERSION PROBLEM

Given \( (P, \alpha P, \alpha^2 P, ..., \alpha^q P) \), the \( q \)-bilinear Diffie-Hellman inversion problem (\( q \)-BDHIP) in \( (G_1, G_2) \) is to produce \( e(P, P)^{\frac{x}{y}} \) as hard assumption.

3) MODIFIED BILINEAR DIFFIE-HELLMAN INVERSION PROBLEM

Given \( (P, \alpha P, \gamma P) \), the modified bilinear Diffie-Hellman inversion problem (mBDHIP) in \( (G_1, G_2) \) is to produce \( e(P, P)^{\frac{x}{y}} \) as hard assumption.

B. NETWORK ARCHITECTURE

Figure 1 depicts our scheme’s network architecture. The architecture comprises the client, a service provider (SP), and an Internet server. The client employs the IBC to communicate with the server, while the server employs the CLC. In IBC, the private key generator (PKG) generates the private keys of the clients, while the clients’ partial private keys are generated by the key generator center (KGC). Here, the SP plays the role of PKG for the clients and the role of the KGC for the server. Our protocol is capable of working in this environment. It is assumed that the SP is trusted and cannot be attacked [17].

C. SECURITY MODEL

In this subsection, the security authentication and key agreement as well as the capabilities of the adversary \( A \) are presented. The symbol \( \Pi^A_u \) denotes as an instance \( \lambda \) of a member \( u \). In the following, we show how the challenger \( C \) reacts with the adversary \( A \):

1) Setup(\( 1^\lambda \)): Taking as input a security parameter \( \lambda \), \( C \) runs the Setup algorithm to create the master secret key \( x \in \mathbb{Z}_q^* \), the master public key \( P_{pub} \) as well as the system parameters \( params \). Afterwards, \( C \) sends \( params \) to \( A \) and remains \( x \) secret.

2) Probing: At any time \( A \) can request the following queries in an adaptive manner:

   a) Extract partial private key queries: For any identity \( ID \), \( A \) can request the partial private key. The corresponding partial private key \( D_{ID} \) is calculated by \( C \) and is reverted to \( A \) consequently.

   b) Extract private key queries: \( A \) allows to request the private key for any \( ID \). The corresponding private key is computed by \( C \) and is returned to \( A \) consequently.

   c) Request public key queries: Upon \( A \) requests the public key for any \( ID \), \( C \) calculates public key \( PK_{ID} \) associated with \( ID \) and returns \( PK_{ID} \) to \( A \).

   d) Replace public key queries: a new secret value \( \nu' \) for any \( ID \), can be chosen by \( A \). Then, using \( \nu' \) a new public key is computed by \( A \) consequently. Thus, the \( PK_{ID} \) can be replaced by \( A \) with \( PK'_{ID} \).

   e) Send(\( \Pi^A_u, m \)) queries: while a plaintext \( m \) is submitted depending on the introduced protocol from \( A \) to \( C \), the estimation and the response to \( A \) are made by \( C \).

   f) Reveal(\( \Pi^A_u \)) queries: \( A \) may access the session key \( sk \) from \( C \), when \( C \) accepts. If \( C \) does not accept \( A \)’s session key, \( C \) returns a null.

   g) Corrupt(u) queries: When a member’s \( u \) private key is remitting, \( A \) makes a Corrupt query to \( C \).

   h) Test(\( \Pi^A_u \)) queries: After \( A \) sends one Test query, \( C \) throws a coin \( b \). If \( b = 1 \), \( A \) obtains the session key. Otherwise, an arbitrary string is returned. This query gives the semantic security of the session key.
Afterwards, $A$ generates $b'$ as estimation for $b$. Mathematically, the advantage of $A$ is represented by $Adv(A)$. Therefore, $Adv(A) = |Pr[b'= b] - 1/2|$, where $Pr[b'= b]$ represents the probability of $b'= b$.

In our security proof, $Extract$ private key and $Send$ queries are used to show that our scheme has client-to-server authentication. For the key agreement, we show that our scheme is secure against adversary type I and adversary type II due to the server belongs to the CLC. $Extract$ partial key, $Extract$ private key, $Request$ public key, $Replace$ public key, $Send$, $Reveal$, $Corrupt$, and $Test$ queries are used for adversary type I. $Extract$ private key, $Request$ public key, $Send$, $Reveal$, $Corrupt$, and $Test$ queries are used for adversary type II.

### IV. PROPOSED PROTOCOL

The symbols used in our paper are displayed in Table 1. Our scheme is illustrated by the phases: Setup, Key Extraction, as well as User Authentication with Key agreement. The proposed scheme lets the client and the server register by their identity in the SP. Note that the client applies the IBC and the server applies the CLC. To be sure that our scheme provides the user authentication and key agreement requirement, the proposed scheme follows the following steps. First, in the Setup phase, the SP prepares the public parameters that will be used in the communication between the client and the server. Second, the SP runs the extract phase to generate the corresponding keys for the client and the server respectively. Third, the client computes a value and sends this value with his identity to the server. Then, the server checks this value which is correct or not. Fourth, after verify the value sent by the client, the server computes other values and sends it to the client. By this step, the server can know whether is communicating with the right server or not. Fifth, the client receives the server’s values and uses them to compute the session key and generate a signature and send it to the server. Finally, the server receives the client’s signature and checks if it’s correct will accept the client otherwise reject the client.

### A. SETUP PHASE

The service provider (SP) executes this phase by the following steps:

1) Take $\lambda$ as input and calculate the $\text{params}$.
2) Pick two cyclic groups $G_1$ and $G_2$, where $G_1$ is additive group and $G_2$ is multiplicative group with the same prime order $q$ and the bilinear pairing $e : G_1 \times G_1 \rightarrow G_2$. Let $P$ be a generator of $G_1$.
3) Pick the master secret key $x \in Z_q^*$ randomly and let $P_{pub} = xP$, and pick secure hash functions $H_1 : [0, 1]^* \rightarrow G_1$, $H_2 : G_1 \rightarrow Z_q^*$ as well as $H_3 : [0, 1]^* \times [0, 1]^* \times G_1 \times G_2 \times Z_q^* \rightarrow Z_q^*$.
4) Issue the $\text{params} = \{G_1, G_2, q, e, P, P_{pub}, g, H_1$, $H_2, H_3\}$, where $g = e(P, P)$, as the public parameters and remain $x$ secret.

### B. KEY EXTRACTION

This algorithm is carried out by the SP as follows:

- **IBC-KG**: A client sends his identity $ID_c$ to the SP. Then, the SP responds with $SK_{ID_c} = (1/H_1(ID_c) + x)P$ as a client’s private key. The client uses his identity $ID_c$ as public key.
- **CLC-KG**: A server sends his identity $ID_s$ to the SP. Then, the SP responds with $DK_{ID_s} = (1/H_1(ID_s) + x)P$ as a server’s partial private key. Then, the server with identity $ID_s$ chooses randomly $v \in Z_q^*$ as secret value to compute his public key as follows $PK_v = v(H_1(ID_s)P + P_{pub})$. When the $DK_{ID_s}$ and the $v$ are given, the server computes his full private key as follows $SK_{ID_s} = \frac{1}{v}DK_{ID_s} + H_2(PK_v)$.

### C. USER AUTHENTICATION WITH KEY AGREEMENT

Figure 2 depicts the collaboration among the client and the server to do the verification. Here, this phase gives the ability to the client who applies the IBC to authenticate from himself/herself in the server, while the server applies the CLC. Indeed, this step illustrates the authentication in heterogeneous cryptographic approaches. This phase can be completed by the following steps:

- While a client private key $SK_c$, a client public key $ID_c$, and the server public key $PK_v$ are given, the client responds as follows:
  1. Select $\varphi$ randomly from $Z_q^*$.
  2. Compute $r = g^{\varphi}$.
  3. Compute $T = \varphi(PK_v + H_2(PK_v)(H_1(ID_c)P + P_{pub}))$ and return $ID_c$ and $T$ to the server.

Note that the values of $r$ and $T$ are precomputed off-line by the client.

- After $ID_c$ and $T$ are received, the server responds by the following:
  1. Calculate $r = e(T, SK_c)$.
  2. Choose $\beta \in Z_q^*$.
  3. Compute $h = H_3(ID_c, ID_s, PK_v, r, \beta)$.
  4. Return $\beta$ and $h$ to the client.
To insure that the proposed scheme is correct, first we have
\[ T = \varphi(PK_s + H_2(PK_s)(H_1(ID_s)P + P_{pub})) \]

Since \( \beta \) and \( h \) are received, the client responds as follows:
1) Verify if \( h \) is similar to the computed \( H_3(ID_c, ID_s, PK_s, \beta, r) \) by the client.
2) Compute \( sk = H_3(ID_c, ID_s, PK_s, r, \beta) \) as the session key that will be used between the client and the server for the future communication.
3) Compute \( S = (\varphi + h)SK_c \) and return it to the server.

Since \( S \) is received, the server verifies from \( S \) using the following equation \( r = e(S, H_1(ID_c)P + P_{pub})g^{-h} \). Then, the session key \( sk = H_3(ID_c, ID_s, PK_s, r, \beta) \) is computed.

To insure that the proposed scheme is correct, first we have
\[ r = e(T, SK_c) = e(\varphi PK_s, \frac{1}{v + H_2(PK_s)H_1(ID_s)P + P_{pub}}), \]
\[ r = e(\varphi(PK_s + H_2(PK_s)(H_1(ID_s)P + P_{pub})), \frac{1}{v + H_2(PK_s)H_1(ID_s) + x}P) \]
\[ r = e((\varphi + H_2(PK_s))(H_1(ID_s)P + P_{pub})), \frac{1}{v + H_2(PK_s)H_1(ID_s) + x}P ) \]
\[ r = e(\varphi, P, P)\]
\[ r = g^\varphi \]
\[ r = r \]

Then, we verify the correctness of the following equation
\[ r = e(S, H_1(ID_c)P + P_{pub})g^{-h} \]
\[ r = e((\varphi + h)SK_c, H_1(ID_c)P + P_{pub})g^{-h} \]

\[ r = e((\varphi + h) \frac{1}{H_1(ID_c) + x}P, H_1(ID_c)P + P_{pub})g^{-h} \]
\[ r = e((\varphi + h) \frac{1}{H_1(ID_c) + x}P, (H_1(ID_c) + x)P)g^{-h} \]
\[ r = e(P, P)g^{-h} \]
\[ r = g^\varphi \]
\[ r = r \]

V. SECURITY ANALYSIS

This section depicts that our protocol can achieve the client-to-server authentication (CSA), key agreement, and server-to-client authentication (SCA) by using the random oracle model [31]. We employed the logic in [14] to do the security analysis.

A. ANALYSIS OF CLIENT-TO-SERVER AUTHENTICATION

In theorem 1, \( A \) unable to use the client to complete the communication with the server. The \( A \) can not solve the \( q \)-SDH problem which is known as hard.

Theorem 1: Supposing that \( A \) takes a non-negligible advantage \( \varepsilon \) for breaking the CSA security. Also, at most \( q_5 \) queries to the server's oracle \( \Pi_5^j \), \( q_c \) queries to the client's oracle \( \Pi_c^i \). Extract private key queries and \( q_{H_i} \) queries on \( H_i \) oracle \( \forall i \in \{1, 2, 3\} \) are made by a challenger \( C \). Therefore \( C \) solves the \( q \)-SDH problem by having a non-negligible probability.

Proof: Figure 3 depicts the proof structure of this theorem. As the proof in [30], the challenger \( C \) takes as input \((P, \alpha P, \alpha^2 P, \ldots \alpha^p P)\) and attempts to extract \((w_1, \frac{1}{w_1 + \alpha^p} P)\) from its communication with \( A \), where \( w_i, \alpha \in \mathbb{Z}_q^* \).

Initialization: In IBC-KG preparation phase, \( C \) chooses randomly \( w_1, w_2, \ldots, w_{p-1} \in \mathbb{Z}_q^* \). As in the proof technique of [30], \( C \) sets up a generator \( Q \in \mathbb{G}_1 \) and the public key \( Q_{pub} = \alpha P \in \mathbb{G}_1 \), such that it knows \( p - 1 \) pairs \((w_i, k_i = \frac{1}{w_i + \alpha^p} Q)\) for \( i \in \{1, 2, 3, \ldots, p - 1\} \). To do so,
the polynomial $f(z) = \prod_{i=1}^{p-1}(z + w_i)$ is expanded by $C$ to acquire the coefficients of $f(z)$, then $f(z) = \sum_{i=0}^{p-1} l_i z^i$ where $\{l_0, l_1, l_3, \ldots, l_{p-1}\} \in \mathbb{Z}_q^*$. After that, $Q$ and $Q_{pub}$ are computed by $C$ using the following equations:

$$Q = \sum_{i=0}^{p-1} l_i (\alpha^i P) = f(\alpha)P \in \mathbb{G}_1$$

and

$$Q_{pub} = \sum_{i=1}^{p} l_{i-1}(\alpha^{i-1} P) = \alpha f(\alpha)P = \alpha Q$$

The pair $(w_i, k_i)$ can be acquired similar to the proof in [30]. The following equations $f_i(z)$ are expanded by $C$ to get

$$f_i(z) = \sum_{i=0}^{p-2} d_i z^i$$

and $k_i$ is taken as

$$k_i = \sum_{i=0}^{p-2} d_i(\alpha^i P) = f_i(\alpha)P = \frac{f(\alpha)}{\alpha + w_i}P = \frac{1}{\alpha + w_i}Q$$

2) $H_2$ queries: When $A$ submits this query on $PK_s$, First of all $C$ verifies if the value of $H_2$ was predefined for $PK_s$. If it was defined, $C$ returns the value that was defined previously. Otherwise, $C$ randomly chooses $h_{2,i} \in \mathbb{Z}_q^*$. Then, $C$ returns $h_{2,i}$ to $A$ and updates $L_{H_2}$ with ($PK_s$, $h_{2,i}$).

3) $H_3$ queries: When $A$ sends this query on ($ID_c$, $ID_s$, $PK_i$, $r_i$, $\beta_i$), $C$ firstly looks at $L_{H_3}$ to verify that the value of $H_3$ was predefined assigned to the ($ID_c$, $ID_s$, $PK_i$, $r_i$, $\beta_i$). If yes, the precedent value is returned by $C$ to $A$. Otherwise, $C$ randomly chooses $h_{3,i} \in \mathbb{Z}_q^*$. Then, $C$ returns $h_{3,i}$ to $A$ and updates $L_{H_3}$ with ($ID_c$, $ID_s$, $PK_i$, $r_i$, $\beta_i$).

4) Extract private key queries: When $A$ forwards this query to get the private key of $ID_c$, $C$ does as follows:
   a) If $ID = ID_c$, $C$ fails and stops the simulation.
   b) If $ID \neq ID_c$, the private key of the client is known and is computed by the $C$ with $k_i = 1/(\alpha + w_i)Q$.

5) Send queries:
   a) When $A$ submits $Send$ ($\Pi_{i=1}^n \mu_i$ “start”) query with the client identity $ID_c$. If $ID \neq ID_c$, $\mu_i \in \mathbb{Z}_q^*$ is chosen randomly by $C$. Then, $r = g^\mu$ and $T = \mu (w_iQ + Q_{pub}) - h(PK_q + h_{2,i}(w_iQ + Q_{pub}))$ are computed. Finally, $C$ returns $ID_c$ and $T$ to $A$. If $ID = ID_c$, $C$ fails and stops the simulation.
   b) Whenever $A$ forwards $Send$ ($\Pi_{i=1}^n \beta_i$, ($ID_c$, $T$)) query with ($ID_c$, $T$) to the server, if $ID \neq ID_c$, $C$ computes $r = e(T, SK_e)$. Then, $\beta_i \in \mathbb{Z}_q^*$ is chosen randomly and $h = H_3(ID_c, ID_s, PK_r, r, \beta_i)$ is computed by $C$. Finally, $\beta$ and $h$ are returned to $A$. Otherwise, $C$ fails and stops the simulation.
   c) When $A$ submits $Send$ ($\Pi_{i=1}^n \beta_i$, ($h$, $h'))$ query to the client, If $ID \neq ID_c$, $H_2(ID_c, ID_s, PK_s, \beta, r, \beta')$ is computed again by $C$ to check whether it is equal to the $h$ that is received from the server or not. Then, $S = \mu SK_e$ and $sk = H_3(ID_c, ID_s, PK_s, r, \beta')$ are computed by $C$. Finally, $S$ is returned to $A$. If $ID = ID_c$, $C$ fails due to the $A$ can not get the correct value of $h'$.
   d) Whenever $A$ forwards $Send$ ($\Pi_{i=1}^n \beta_i$, $S$) query to the server, if $ID \neq ID_c$, $e(S, w_i(Q + Q_{pub}))g^{-h}$ is computed by $C$ to check whether it is equal to $r$. If the condition is satisfied, $C$ accepts and computes $sk = H_3(ID_c, ID_s, PK_s, r, \beta')$. Otherwise, the simulation is stopped by $C$.

Analysis: Turing machine $A'$ can be constructed if $A$ is a forger in the above simulation, according to the forking lemma [32]. Then, $A'$ is able to sign messages ($ID^*, h, S$) and ($ID^*, h^*, S^*$) with $h \neq h^*$ and send them to the challenger. A machine $C$ is constructed to solve the $q$-SDHP by using $A'$ as follows:

- $C$ finds two various signatures ($ID^*, h, S$) and ($ID^*, h^*, S^*$) by running $A'$.
- $k^* = (S - S^*)(h - h^*)^{-1} = 1/(\alpha + w*)Q = f(\alpha)/\alpha + w*P$ is computed by $C$. 

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The long division is used and the polynomial \( f(z) = \Psi(z) = (z + w^*) + \Psi - 1 \) by \( C \) for several \( \Psi(z) = \sum_{i=0}^{p-2} \Psi_i z^i \) and some \( \Psi - 1 \in \mathbb{Z}_q^* \). Then \( f(z) \) can be computed as

\[
\frac{f(z)}{z + w^*} = \Psi(z) + \frac{\Psi - 1}{z + w^*} = \sum_{i=0}^{p-2} \Psi_i z^i + \frac{\Psi - 1}{z + w^*}
\]

Then, \( \frac{1}{z + w^*} P \) is computed by \( C \) as follows

\[
\frac{1}{z + w^*} P = \frac{1}{\Psi(z)} (\frac{\Psi - 1}{z + w^*} - \sum_{i=0}^{p-2} \Psi_i (x^i P))
\]

The pair \((w^*, \frac{1}{z + w^*} P)\) is outputted by \( C \) as answer of \( q\)-SDHP.

By the forking lemma [32], if \( A \) is succeeded in a time \( t \) with probability \( \epsilon \geq \frac{16q_3 + 1}{q_3} \); then, the \( q\)-SDHP is solved by \( C \) with expected time

\[
t' \leq 120686 q_H q_{H_2} \frac{t + O(q_x t_p)}{(1 - 1/2^k)(1 - q/2^k)} + O(q_z^2 t_m).
\]

Indeed, our scheme provides a client-to-server authentication under the \( q\)-SDHP.

### B. ANALYSIS OF KEY AGREEMENT

**Theorem 2:** Suppose that the Test query coin can be predicted by \( A_{\ell} \) and \( A_{\ell+1} \) with a non-negligible advantage \( \epsilon \) and at most \( q_{SD} \) queries to the server’s oracle \( \Pi'_C, q_C \) queries to the client’s oracle \( \Pi'_C \), Extract private key queries, Extract private key queries, Request public key queries, Replace public key queries, Send queries, Corrupt queries, Reveal queries, Test queries and \( q_H \) queries to \( H \). Then, \( C \) solves the \( q\)-BDHI Problem and \( \text{mBDHI} \) problem with a non-negligible probability.

**Proof:**

**Lemma 1:** Our protocol resists to the adversary type I \( A_{\ell} \) under \( q\)-BDHI problem in the random oracle model.

**Proof:** Figure 4 depicts the proof structure of this theorem. When a \( \text{Test} \) query is submitted, adversary can guess the value of coin \( b \) correctly with probability less than 1/2. Let \( b \) be the adversary’s guess for the coin. As a result, adversary can request the Test query coin \( \frac{\Psi - 1}{z + w^*} \) to acquire the correct session key. Here, some symbols are employed in our proof. We use the symbol \( E \) to show the event of acquiring the correct session key by the adversary. In this proof, the symbols \( \text{Test}(C') \) and \( \text{Test}(S') \) are used to explain the correct test queries that are made to the client and to the server separately. The symbol \( E^{C2S} \) represents the event of breaking the CSA security. By supposing that the Test query can be submitted to the client as well as to the server by adversary, subsequently this probability, i.e., some \( i \) and \( j \), is:

\[
\text{Pr}[\text{Test}(\Pi'_C) \land E^{C2S}] + \text{Pr}[\text{Test}(\Pi'_C) \land \neg E^{C2S}]
\]

Assuming the possibility of breaking the CSA security is denoted by \( \text{Pr}_{C2S} \). Then, the following probability is computed into some \( i \) and \( j \):

\[
\text{Pr}[\text{Test}(\Pi'_C)] \geq \frac{\epsilon}{2}.
\]

**Initialization:** In preparation phase, \( C \) chooses a random element \( e \in \mathbb{Z}_q^* \) and \( w_1, w_2, \ldots, w_{\gamma - 1}, w_{\gamma - 2}, w_p \in \mathbb{Z}_q^* \). Then, \( \gamma \) is computed by \( C \) for \( i \in \{1, 2, 3, \ldots, \gamma - 1, \gamma - 2, \ldots, p\} \). As in the proof technique of [30], \( C \) sets up a generator \( Q \in G_1 \) and \( Y = \alpha Q \in G_1, \) such that it knows \( p - 1 \) pairs \((w_i, k_i) = \frac{1}{w_{i+\gamma}} Q\) for \( i \in \{1, 2, 3, \ldots, \gamma - 1, \gamma - 2, \ldots, p\} \). Then \( C \) can be computed by \( C \) by the following equations:

\[
Q = \sum_{i=0}^{p-1} \alpha^{i} P \in G_1
\]

and

\[
Y = \sum_{i=1}^{p} \alpha^{i-1} \alpha^{i} P = \alpha f(\alpha) P = \alpha Q
\]

The pairs \((w_i, k_i)\) can be acquired similar to the proof in [30]. The following equations \( f(z) \) are expanded by \( C \) to get

\[
f(z) = \frac{f(z)}{z + w_i} = \sum_{i=0}^{p-2} d_i z^i
\]

and

\[
k_i = \sum_{i=0}^{p-2} d_i \alpha^{i} P = f(\alpha) P = \frac{f(\alpha)}{\alpha + w_j} P = \frac{1}{\alpha + w_j} Q
\]

\(Q_{\text{pub}}\) and \( x = (\alpha - e)\gamma \) are chosen as the master public key and master secret key of the SP respectively.
$Q = (-\alpha - e_y)$, $Q_{pub} = (-Y - e_yQ)$ and $g = e(Q, Q)$ are sent to the $A_j$ by the $C$. $\forall i \in \{1, 2, 3, \ldots, p\}$ except $\gamma$, then we have $(e_i, - \kappa_i) = \left(\frac{1}{e_{i+\gamma}}Q\right)$. The public key and private key of the server $PK_s, SK_s$ are computed by the $C$ using the CLC-KG algorithm and are sent to $A_j$.

The algorithm $C$ generates the system parameters $\{\mathcal{G}_1, \mathcal{G}_2, q, e, P, g, P_{pub}, H_1, H_2, H_3\}$ and sends them with the public key of the server $PK_s = \nu(H_1(ID_s)P + P_{pub})$ to $A_j$. $C$ selects $ID_t$ randomly. $C$ simulates the random oracles of $H_1, H_2$ and $H_3$ with lists regarding to avoid collision and consistency. For oracles queries and responses, $C$ prepares four lists $L_{H_1}, L_{H_2}, L_{H_3}$ and $L_k$ to keep the public keys. We assume $H_1(ID)$ query is completed first then the other queries are created.

1) $H_1$ queries: When $A_j$ sends this query on $ID$, $C$ chooses randomly $e_j \in \mathbb{Z}_q^+$ as well as sends it to $A_j$. However, if $ID = ID_1, C$ returns $e_j$ to $A_j$. Then, $C$ updates $L_{H_1}$ with $(ID, h_{1i})$.

2) $H_2$ queries: When $A_j$ submits this query on $PK_s$. First of all $C$ verifies if the value of $H_2$ was previously defined for $PK_s$. If it was defined, $C$ returns the value that was defined previously. Otherwise, $C$ randomly chooses $h_{2i} \in \mathbb{Z}_q^+$. Then, $C$ returns $h_{2i}$ to $A_j$ and updates $L_{H_2}$ with $(PK_s, h_{2i})$.

3) $H_3$ queries: When $A_j$ submits this query on $(ID, ID_3, PK_i, r_i, h_i)$. $C$ firstly looks at $L_{H_3}$ to verify that the value of $H_3$ was previously assigned to the $(ID, ID_3, PK_i, r_i, h_i)$. If yes, the precedent value is returned by $C$ to $A_j$. Otherwise, $C$ chooses randomly $h_{3j} \in \mathbb{Z}_q^+, \xi^* = r_i.e(Q, Q)^{h_{3j}}$ and returns it to $A_j$. $C$ updates $L_{H_3}$ with $(ID, ID_3, PK_i, r_i, h_{3j}, \xi^*)$.

4) Extract partial private key queries: This query can be asked by $A_j$ on identity ID. If $ID = ID_1, C$ fails and stops the simulation. Else, the value of $H_1(ID) = e_i$ is known by $C$ that can be used to compute the partial private key $\kappa_i = \frac{1}{e_{i+\gamma}}P$ and is returned to $A_j$.

5) Extract private key queries: $A_j$ can submit this query on $ID_3$ requesting for the private key. If $ID = ID_3, C$ fails and stops the simulation. If $ID \neq ID_3$, the partial private key is known by $C$. Then, $C$ looks up at the $L_k$ for the inputs $(ID_3, PK_i, v_i)$ (if these inputs do not already exist in $L_k$, a new user key information will be generated by $C$) and $SK_i = -(1/v_i)\kappa_i$ is returned by $C$ to $A_j$.

6) Request public key queries: An identity $ID_1$ can be selected by $A_j$ and is sent to $C$. Then, $C$ checks whether $L_k$ has these tuples $(ID_3, PK_i, v_i)$ if yes, the $PK_i$ is returned to $A_j$. If it has not, a random $v_i \in \mathbb{Z}_q^+$ is selected and $PK_i = v_i(e_iQ + Q_{pub})$ is set. Finally, $C$ updates $L_k$ with $(ID_3, PK_i, v_i)$ and returns $PK_i$ to $A_j$.

7) Replace public key queries: The public key $PK_i$ may be replaced with chosen value by $A_j$. For this query on $(ID_3, PK_i)$ the $L_k$ is updated by $C$ with $(ID_3, PK_i, \perp)$ where $\perp$ means that the value is unknown.

8) Send queries:

   a) When $A_j$ submits $Send(\Pi_1^L, \text{“start”})$ query with the client identity $ID_s$. If $ID_s \neq ID_1, \mu \in \mathbb{Z}_q$ is chosen randomly by $C$. Then, $r = g^\mu$ and $T = \mu(v(Q + Q_{pub}) - h(PK_s + h_2(Q + Q_{pub}))$ are computed. Finally, $C$ returns $ID_t$ and $T$ to $A_j$.

   b) When $A_j$ submits $Send((\Pi_2^L, (ID_c, T)))$ query to the server with $(ID_c, T)$, if $ID \neq ID_3, C$ computes $r = e(T, SK_s)$. Then, $\beta \in \mathbb{Z}_q^+$ is chosen randomly and $h = H_3(ID_c, ID_3, PK_s, r, \beta)$ is computed by $C$. Finally, $\beta$ and $h$ are returned to $A_j$. Otherwise, $C$ fails and stops the simulation.

   c) When $A_j$ submits $Send((\Pi_3^L, (\beta, h))$ query to the client, If $ID \neq ID_s, H_2(ID_c, ID_3, PK_s, r, \beta)$ is computed again by $C$ to check whether it is equal to the $h$ that is received from the server or not. Then, $S = \mu SK_s$ and $sk = H_3(ID_c, ID_3, PK_s, r, \beta)$ are computed by $C$. Finally, $S$ is returned to $A$. If $ID = ID_s, C$ fails due to the $A_j$ can not get the correct value of $h'$.

   d) When $A_j$ submits $Send((\Pi_4^L, S))$ query to the server, If $ID \neq ID_1, e(S, e(Q + Q_{pub})g^{-h})$ is computed by $C$ to check whether is equal to $r$. If the condition is satisfied, $C$ accepts and computes $sk = H_3(ID_c, ID_3, PK_s, r, \beta)$. Otherwise, the simulation is stopped by $C$.

9) Corrupt queries: Whenever this query on $ID_c$ is sent by $A_j$, $-k_i$ is returned by $C$.

10) Reveal queries: Whenever this query on $ID_c$ is submitted by $A_j$, If $ID_s \neq ID_1$, $C$ fails and stops the simulation. Otherwise, $\lambda \in \mathbb{Z}_q^+$ is chosen and $T^* = -\lambda Q$ is computed by $C$. $T^*$ is returned to $A_j$ to get the session key. Assume that $\Gamma = \frac{1}{a}$ and we know that $x = (-\alpha - e_i)$, so then we can get

$$T^* = -\lambda v_iQ - \lambda h(2, s)Q$$
$$= -\Gamma v_i Q - \Gamma ah(2, s)Q$$
$$= (e_i + x)\Gamma v_i Q + (e_i + x)\Gamma h(2, s)Q$$
$$= \Gamma v_i e_i Q + \Gamma v_i Q_{pub} + (e_i + x)\Gamma h(2, s)Q$$
$$= \Gamma v_i e_i Q + Q_{pub} + e_i\Gamma h(2, s)Q + \Gamma h_2 Q_{pub}$$
$$= \Gamma v_i (e_iQ + Q_{pub}) + \Gamma h_2 (e_iQ + Q_{pub})$$
$$= \Gamma (PK_s + h_2 (e_iQ + Q_{pub}))$$

Then, $A_j$ is unable to get the correct session key unless it makes $H_3$ query on $e(P, P)^\Gamma$.

11) Test queries: Whenever this query is sent by $A_j$, a fair coin $b$ is generated by $C$. A random input $(ID_c, ID_3, PK_i, r_i, h_i, j_i)$ is selected from $L_{H_3}$ by $C$ that contains no more than $q_{H_3}$ inputs. If $b = 1$ the session key is returned to $A_j$ by $C$. If $b = 0, C$ sends $\perp$ to $A_j$ where $\perp$ denotes a random string.

On the other hand, it is observed by $C$ that these two events $\exists i, Esk \wedge Test(\Pi_C^L)$ and $\exists j, Esk \wedge Test(\Pi_C^L) \wedge \neg E_C^{2S}$ are equivalent. As result we know this probability $Pr[Esk \wedge Test(\Pi_C^L)] \geq \frac{\xi}{2} - Pr_C^{2S}$. In addition, by simulating the queries that are submitted to the client, we can get this
probability
\[
Pr \left[ sk = H_3(ID_c, ID_s, PK_s, r, \beta) \mid r \in \mathbb{G}_2^2 \right] \geq \frac{\epsilon}{2} - Pr_{C^{25}}
\]
It is known that Pr_{C^{25}} is negligible from the proof in Theorem 1. Furthermore, whether \( \epsilon \) is non-negligible, then we know that \( \frac{\epsilon}{2} - Pr_{C^{25}} \) is a non-negligible. Hence, to obtain the session key, the adversary \( A_1 \) needs to select inputs containing the correct element of \( r_i = e(P, P)^i \) with probability \( \frac{1}{q_{h_i}} \). \( A_1 \) allows to make the previous queries with the following limitations:

1) \( A_1 \) cannot submit Extract partial private key query on ID_1 when the public key of the ID_1 has been replaced.
2) \( A_1 \) cannot submit Extract private key query on ID_1.

Indeed, as the proof in [33] the value of \( (e(Q, Q)^{i-1}) \) is computed by \( C \) as a solution for q-BDHI problem when \( \epsilon \) is \( e(P, P)^{i-1} \) with the following equation:
\[
e(Q, Q)^{1/\alpha} = \sum_{j=0}^{p-2} e(Q, eP_{j+1}(\alpha^j P))
\]

Hence, our scheme achieves a key agreement protocol under q-BDHI problem against adversary type I.

Lemma 2: Our scheme is resists to \( A_{II} \) under mBDHI problem in the random oracle model.

Proof: When a Test query is submitted, the adversary can guess the value of coin \( c \) correctly with probability not less than \( 1/2 \). Let suppose adversary is able to guess the coin with \( e \). As result, the adversary with advantage \( Pr[Esk] \geq e/2 \) enables to acquire the correct session key. Here, some symbols are employed in our proof. We use the symbol Esk to show the event of acquiring the correct session key by adversary. In addition, these symbols Test\((C')\) and Test\((S')\) are used to explain the correct test queries that are made to the client and to the server separately. The symbol \( E_{C^{25}} \) represents the event of breaking the CSA security. By supposing that the Test query can be submitted to the client as well as to the server by adversary, then this probability, into some \( i \) and \( j \), is:
\[
Pr[Esk \land Test(P_i) \land E_{C^{25}}] + Pr[Esk \land Test(P_i) \land \neg E_{C^{25}}] + Pr[Esk \land Test(P_i)] \geq \frac{\epsilon}{2}
\]
Assuming the possibility of breaking the CSA security is denoted by \( Pr_{C^{25}} \). Then, the following probability is computed into some \( i \) and \( j \):
\[
Pr[Esk \land Test(P_i) \land \neg E_{C^{25}}] + Pr[Esk \land Test(P_i)] \geq \frac{\epsilon}{2} - Pr_{C^{25}}
\]
As the proof in [30], the challenger \( C \) takes as input \( (P, aP, a^2P, \ldots, a^\rho P) \) and attempts to extract \( (w_1, \frac{1}{w_1 + \alpha} P) \) from its communication with \( A_{II} \), where \( w_1, \alpha \in \mathbb{Z}_q^* \) and \( P_{pub} = \alpha x \) is computed. The algorithm \( C \) generates the system parameters \( \{G_1, G_2, q, e, P, g, P_{pub}, H_1, H_2, H_3\} \) and forwards to \( A_{II} \). \( C \) selects \( ID_1 \) randomly. \( C \) simulates the random oracles of \( H_1, H_2 \), and \( H_3 \) with lists regarding to avoid collision and consistency. For oracles queries and responses, \( C \) prepares four lists \( L_{i1}, L_{i2}, L_{i3}, L_{iK} \) to keep the public keys. We assume \( H(T, ID_1) \) is computed is first query then the other queries are created.

1) \( H_1 \) queries: When \( A_{II} \) submits this query on ID, \( C \) checks whether \( L_1 \) has these inputs \( (ID_1, h_{11}) \). If it has them, \( e_i \) is returned to \( A_{II} \) and \( L_{i1} \) is updated with \( (ID, e_i) \). However, if \( ID = ID_1 \), \( C \) returns \( H_1(ID) = e_i \) to \( A_{II} \) when \( A_{II} \) submits this query on \( ID_1 \).

2) \( H_2 \) queries: When \( A_{II} \) submits this query on \( PK_s \), first of all \( C \) verifies if the value of \( H_2 \) was previously defined for \( PK_2 \). If it was defined, \( C \) returns the value that was defined previously. Otherwise, \( C \) verifies if \( PK_i = e_iP + xH_2 \). If it is satisfied, \( C \) returns \( h_{2i} = \gamma \) and updates \( L_{i1} \) with \( (PK_S, \gamma) \). If it is not satisfied, \( C \) randomly chooses \( h_{2j}, e \in \mathbb{Z}_q^* \). Then, \( C \) returns \( h_{2i}, h_{2j} \) to \( A_{II} \) and updates \( L_{i1} \) with \( (PK_s, h_{2i}, h_{2j}) \).

3) \( H_3 \) queries: When \( A_{II} \) submits this query on \( ID_1, ID_3, PK_i, r_i, \beta_i \), \( C \) firstly looks at \( L_{i1} \) to verify that the value of \( H_3 \) was previously assigned to the \( (ID_1, ID_3, PK_i, r_i, \beta_i) \). If yes, the precedent value is returned by \( C \) to \( A_{II} \). Otherwise, \( C \) randomly chooses \( h_{3i}, \beta \in \mathbb{Z}_q^* \). Then, \( C \) returns \( h_{2i}, h_{2j}, \beta \) to \( A_{II} \) and \( h_{2i}, h_{2j}, \beta \) are computed and is returned to \( A_{II} \). \( C \) updates \( L_{i1} \) with \( (ID_1, ID_3, PK_i, r_i, \beta_i, h_{3i}, \beta) \).

4) Extract private key queries: \( A_{II} \) can submit this query on \( ID_1 \) requesting for the private key. \( C \) checks the value of \( ID_1 \). If \( ID \neq ID_1 \), \( C \) fails and stops the simulation. If \( ID = ID_1 \), \( C \) looks up at the \( L_k \) for the inputs \( (ID_1, PK_i, v_1) \) (if these inputs do not already exist in \( L_k \) a new user information will be generated by \( C \) and \( SK_i = \frac{1}{v_3} \frac{1}{v_3} + \frac{1}{v_3} + \frac{1}{v_3} \) is computed and is returned to \( A_{II} \) by \( C \) to \( A_{II} \). 5) Request public key queries: An identity ID can be selected by \( A_{II} \) and is sent to \( C \). If \( ID \neq ID_1 \), \( v \in \mathbb{Z}_q^* \) is selected randomly by \( C \) and \( PK_k = v(e_iP + P_{pub}) \) is set as public key. Then, \( C \) updates \( L_k \) with \( (ID_1, PK_i, v_1) \) and \( PK_k \) is returned to \( A_{II} \). If \( ID = ID_1 \), the public key \( PK_i = e_iP + xP_{pub} \) is set and is returned to \( A_{II} \). Finally, \( C \) updates \( L_k \) with \( (ID_1, PK_s, \bot) \).

6) Send queries:

a) When \( A_{II} \) submits Send\((P_i, (ID_c, T)) \) query to the server with \( (ID_c, T) \). If \( ID_1 \) is not \( ID_1 \), \( C \) computes \( r = e(T, SK_i) \). Then, \( \beta \in \mathbb{Z}_q^* \) is chosen randomly and \( h = H_3(ID_c, ID_3, PK_s, r, \beta) \) is computed by
Finally, $\beta$ and $h$ are returned to $A_H$. Otherwise, $C$ fails and stops the simulation.

c) When $A_H$ submits $\text{Send}$ ($\Pi'_1$, $(\beta, h)$) query to the client, if $ID \neq ID_t$, $H_3(ID_c, ID_s, PK_s, r, \beta)$ is computed again by $C$ to check whether it is equal to the $h$ that is received from the server or not. Then, $S = \mu SK_s$ and $sk = H_3(ID_c, ID_s, PK_s, r, \beta)$ are computed by $C$. Finally, $S$ is returned to $A_H$. If $ID = ID_t$, $C$ fails due to the $A_H$ cannot get the correct value of $h'$.

d) When $A_H$ submits $\text{Send}$ ($\Pi'_2$, $S$) query to the server, if $ID \neq ID_t$, $e(S, eP + P_{\text{pub}})g^{-h}$ is computed by $C$ to check whether it is equal to $r$. If the condition is satisfied, $C$ accepts and computes $sk = H_3(ID_c, ID_s, PK_s, r, \beta)$. Otherwise, the simulation is stopped by $C$.

7) $\text{Corrupt}$ queries: Whenever $\text{Corrupt}$ query on $ID_c$ is sent by $A_H$, $D_{ID_c}$ is returned by $C$.

8) $\text{Reveal}$ queries: Whenever $\text{Reveal}$ query on $ID_c$ is sent by $A_H$, if $ID \neq ID_t$, $C$ fails and stops the simulation. Otherwise, $\alpha \in \mathbb{Z}_p^*$ is chosen and $T^* = aP$ is processed by $C$. The $T^*$ is returned to $A_H$ to get the session key. Then, $A_H$ is unable to get the correct $r$ unless it makes $H_3$ query on $e(T^*, SK_s)$ to get the session key.

9) $\text{Test}$ queries: Whenever this query is sent by $A_H$, a fair coin $b$ is generated by $C$. A random input ($ID_c, ID_s, PK_s, r_i, \beta_i, h_{12}, \varsigma_i$) is selected from $L_{H_3}$ by $C$ that contains no more than $q_{H_3}$ inputs. If $b = 1$ the session key is returned to $A_H$ by $C$. If $b = 0$, $C$ sends $\bot$ to $A_H$ where $\bot$ denotes a random string.

On the other hand, it is observed by $C$ that these two events $\exists i, E_{sk} \land \text{Test}((\Pi'_1))$ and $\exists j, E_{sk} \land \text{Test}((\Pi'_2)) \land \neg E_{C25}$ are equivalent. As a result, we know this probability $\Pr[E_{sk} \land \text{Test}((\Pi'_1)) \geq \frac{\epsilon}{2} - Pr_{C25}$. In addition, by simulating the queries that are submitted to the client we can get this probability

$$\Pr \left[ sk = H_3(ID_c, ID_s, PK_s, r_i, \beta_i)e^{2\mathbb{Z}_p} \right] \geq \frac{\epsilon}{2} - Pr_{C25}$$

It is known that $Pr_{C25}$ is negligible from the proof in Theorem 1. Furthermore, whether $\epsilon$ is non-negligible, then we know that $\frac{\epsilon}{2} - Pr_{C25}$ is a non-negligible. Hence, to have the key agreement the adversary should select inputs containing the correct element of $r_i = e(T^*, SK_s)$ with probability $\frac{1}{q_{H_3}}$. $A_H$ allows to make the previous queries, which can not submit $\text{Extract private key}$ query on $ID_t$. Indeed, the mBDHI problem can be delivered by the following equation

$$r_i = e(T^*, SK_s) = e(\lambda P, \frac{1}{\alpha + \gamma} e_i + x)$$

then we have

$$e(P, e^{\frac{1}{\alpha + \gamma}}) = r_i^{e_i + x}$$

Indeed, our scheme achieves a key agreement protocol under mBDHI problem against adversary type II.

C. ANALYSIS OF SERVER-TO-CLIENT AUTHENTICATION

The server can not be compromised by the adversary to interact with the client by Theorem 3 under $q$-BDHI problem.

Theorem 3: Suppose that $A$ takes a non-negligible advantage $\epsilon$ for breaking the SCA security. Also, at most $q_{S}$ queries to the server’s oracle $\Omega_S$, $q_{C}$ queries to the client’s oracle $\Omega_C$, and $q_{H_3}$ queries on $H_3$ oracle $\forall i \in \{1, 2, 3\}$ are made by $A$. Therefore a challenger $C$ solves the $q$-BDHI problem by having a non-negligible probability.

Proof: Figure 5 depicts the proof structure of this theorem. As it is mentioned in Theorem 2, the simulation is working correctly since the $E_{C25}$ happens. We denote the event of breaking the server-to-client authentication with this symbol $E_{C25}$. The event $E_{C25}$ happens during the simulation when $(ID_c, T)$ are sent by the client to the server. In addition, $(\beta, h)$ are received by the client while are not issued from the correct server. The previous situation occurs in one of the following conditions:

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Client time.}
\end{figure}
The value of $h$ is guessed by adversary $A$ with probability less than $q_C/2^k$.

2) The value $T$ is happened in a further session with a probability $q_C/q \times (q_C - 1)$ less than $q_C^2/q$.

3) $H_1(Id_t)$ is asked by adversary $A$ with a probability $Pr[\{ ID_c, ID_s, PK_s, r, \beta \} | \beta \in R Z_q^*, r = e(T, SK_s)]$.

Then, we have

$$Pr[E^{\mathbb{G}_2} \rightarrow E^{\mathbb{G}_1}] \leq Pr[\{ ID_c, ID_s, PK_s, r, \beta \} | \beta \in R Z_q^*, r = e(T, SK_s)] + \frac{q_C}{2^k} + \frac{q_C^2}{q}$$

The algorithm $C$ generates the system parameters $\{ G_1, G_2, q, e, P, g, P_{pub}, H_1, H_2, H_3 \}$ and sends them with the public key of the server $PK_s = v(H_1(ID_s)P + P_{pub})$ to $A$. $C$ selects $ID_t$ randomly. $C$ simulates the random oracles of $H_1$, $H_2$ and $H_3$ with lists regarding to avoid collision and consistency. For oracles queries and responses, $C$ prepares four lists $L_{H_1}$, $L_{H_2}$, $L_{H_3}$ and $L_K$ to keep the public keys.

Hence, to break our server-to-client authentication, the adversary needs to select inputs containing the correct element of $r_i = e(T^*, SK_i)$ with probability $1/q_C$. Indeed, the $q$-BDHI problem can be delivered by the following equation

$$r_i = e(T^*, SK_i) = e(\lambda P, \frac{1}{\alpha + h_{2,i}} e_i + x)$$

subsequently we get

$$e(P, P)^{\frac{-1}{q_C^2}} = r_i^{q_C^2}$$

Indeed, our scheme achieves mutual authentication under $q$-BDHI problem.

VI. PERFORMANCE

This section illustrates the advantages of the proposed protocol compared to existing protocols. To this regard, the evaluation with consideration to security and the performance is conducted. The performance evaluation shows the computational cost and the communication overhead in the client and server for the compared protocols. This comparison is conduct with the following protocols He (HE) [20], Tseng et al. (TW) [25], and Hassan et al. (HA) [28]. For the performance evaluation, we use these notations $T_p$, $T_m$, $T_{ad}$, and $T_{inv}$ to explain the bilinear pairing operation time, multiplication time in $G_1$, inversion operation time, and addition in $G_1$ time respectively.

The theoretical analysis is given regarding to the computational cost in Table 3. From Table 2, We use $Y$ in Table 3 to indicate that the protocol satisfies specific security requirement. We find that the proposed scheme can be employed when the client belong to IBC and the server belong to CLC. Hence, the proposed scheme owns the advantage of working with applications using various cryptosystem environments compared with the existing protocols.

To evaluate the real computational costs, four schemes [20], [25] [28] including our proposed scheme are implemented using Java pairing-based cryptography (JPBC).
Library [34]. Our experimental is done on a machine with CPU Intel Core i 7-3537U dual core (2.00 and 2.50) GHz and RAM 4 GB for the server, while Huawei Mate 8 with CPU Hisilicon Kirin 950 and RAM 4.0 GB is used for the client.

The curve $y^2 = x^3 + x$ over the field $F_p$ is used to construct the Type A pairings for $p = 3$ mod 4. The experimental, entailed 80-bit AES key size security level while the size of $p = 1024$ bits and size of $q = 160$ bits [35]. The processing costs of the mobile side and the server side are shown in Figure 6 and Figure 7 respectively. Our scheme has better processing cost in the client side while it is better than HE [20] and HA [28] schemes in the server side. Indeed, the proposed scheme is suitable for the mobile devices that have limited storage and power.

The communication costs of the schemes are displayed in Figure 8. We use the following notations $|id| = 20$ bytes, the elliptic carve with $q = 160$ bytes according to the work introduced in [36], to calculate the exact costs for each scheme.

Here, we give the communication cost for the compared schemes HE [20], TW [25], HA [28], and the proposed scheme using the following calculations $|id| + 2|Z_q^*| + 2|G_1| = 10 + 2 \times 20 + 3 \times 65 = 245$ bytes, $|id| + 2|Z_q^*| + 4|G_1| = 10 + 2 \times 20 + 4 \times 65 = 310$ bytes, $|id| + 2|Z_q^*| + 2|G_1| = 10 + 2 \times 20 + 2 \times 65 = 180$ bytes, $|id| + 2|Z_q^*| + 2|G_1| = 10 + 2 \times 20 + 2 \times 65 = 180$ bytes respectively.

Figure 8 exhibits that our scheme has better communication overhead than HE [20] and TW [25], while it has the same communication cost with HA [28].

VII. CONCLUSION

In this paper, we present a scheme for user authentication with a key agreement in heterogeneous client-server systems. The security of our scheme is proved in the random oracle model. We also use the $q$-strong Diffie-Hellman problem ($q$-SDH), the $q$-bilinear Diffie-Hellman inversion problem ($q$-BDHI), and the modified bilinear Diffie-Hellman inversion problem (mBDDH) as hard assumptions in our security proof. Our protocol can be used in heterogeneous environments when the client relies on IBC and the server relies on CLC. However, exciting protocols cannot be used in this environment, which emphasizes the importance of our scheme. Moreover, our protocol can be used in both client-server architectures and multi-server architectures. In the future, we plan to improve the efficiency of our protocol by using post-quantum lattice-based cryptography. We will also address privacy preservation in the development of user authentication protocols for mobile requirements.

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