Topological edge modes in non-Hermitian plasmonic waveguide arrays

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Abstract: We investigate the topological edge modes of surface plasmon polaritons (SPPs) in a non-Hermitian system composed of graphene pair arrays with alternating gain and loss. The topological edge modes emerge when two topologically distinct graphene arrays are connected. The gain and loss present in the system provide additional ways to control the propagation loss and field distributions of the topological edge modes. Moreover, the existence of the topological edge modes is related to the broken parity-time (PT) symmetry. We show the beam diffraction can be steered by tuning the chemical potential of graphene. Thanks to the strong confinement of SPPs, the topological edge modes can be squeezed into a lateral width of ~λ/70. We also show such modes can be realized in lossy graphene waveguides without gain. The study provides a promising approach to realizing robust light transport and optical switches on a deep-subwavelength scale.

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OCIS codes: (240.6680) Surface plasmons; (230.7370) Waveguides.

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1. Introduction

Metallic waveguides can support surface plasmon polaritons (SPPs) and have sparked enormous interest in manipulating light to circumvent the diffraction limit and may find
application in compact devices and nanoscale resolution imaging [1–5]. By tuning the local SPP dispersion relation in metallic waveguide arrays through changing the refractive index and the waveguide spacing, the diffraction of SPPs can be controlled and the diffraction-free beam propagation and negative refraction can be realized [6].

On the other hand, topological photonics characterize the collective behavior of the wave functions on the band, providing new and robust way to control the light flows [7]. A topological edge mode emerges at the interface between topological trivial and non-trivial structures characterized by an integer-valued quantity [8]. For example, the winding number is used to characterize the topology of the one-dimensional dimer chains, the so-called Su-Schrieffer-Heeger (SSH) model [9]. According to the ratio between intra- and inter-layer couplings, the winding number is either zero or unity separated by the Dirac point, the spectral degeneracies in Hermitian systems [10]. Such modes remain stable against disorders as the structure topology is hard to destroy [7]. The SSH model has been studied in dielectric [9], metallic [10, 11], and graphene waveguide arrays [12].

As the optical loss of SPPs is remarkable and thus hampers the practical application, one has used gain to compensate or even reverse the propagation loss. The gain can be realized by extra optical pumping or depositing quantum dots to the waveguides [13–16]. The system associated with gain or loss is non-Hermitian [17]. When the gain-loss distribution is an odd function of position, the system may possess all real or complex eigenvalues, corresponding to unbroken and broken parity-time (PT) symmetries [18]. The two phases should be divided by non-Hermitian degeneracies, known as exceptional points (EPs) [19, 20]. The topological edge states can exist in non-Hermitian systems [21, 22]. It is revealed that they are associated with the EPs [23]. The topological edge modes with non-Hermitivity are theoretically and experimentally investigated in dielectric waveguides [24].

In this work, we investigate the topological edge plasmons in the PT-symmetric graphene pair arrays. Graphene has recently attracted much attention as it can support SPPs in a wide spectrum ranging from terahertz to infrared frequencies [25–27]. The SPPs in graphene are confined more than two orders smaller than that of the free-space wavelength, providing new possibility to steer light propagation on deep-subwavelength scale [28–30]. Thanks to its unique electronic band structures, the surface current of graphene can be flexible tuned by electrostatic and chemical doping [31–35]. The high carrier mobility in graphene makes it suitable for ultrafast switching [36–39]. One has shown that periodically patterned graphene can have large topological bandgaps and thus sustains remarkably stable topological edge modes [40]. These properties enable graphene a promising platform to investigate the topological properties of SPPs.

The remainder of this work is organized as follows. Section 2 presents the configuration and the dispersion relation of the proposed periodic graphene pair array. The unbroken and broken PT symmetry according to the coupled mode equation are specified in Sec. 3. The topological invariants are also discussed. Section 4 discusses the topological edge modes in finite graphene multilayers and the influence of different gain and loss distributions, which are followed in Sec. 5 by a discussion about the existence of the topological edge modes. Section 6 gives insights on beam dynamics by accessing topological edge modes. In addition, the topological edge modes in passive PT-symmetric graphene waveguides are studied in Sec. 7. Section 8 provides final conclusions of the whole work.

2. Bloch modes in graphene pair arrays

We start from investigating the dispersion relation of graphene pair arrays. Figure 1 shows the geometry of graphene pair arrays under consideration. The structure is periodic along \( x \) direction. There are two graphene sheets in each unit cell with alternating gain and loss. The arrays are embedded in the dielectrics with permittivity denoted as \( \varepsilon_d \). Here they are initially assumed to be freestanding in air with \( \varepsilon_d = 1 \). The surface conductivities are denoted as \( \sigma_1 \) and \( \sigma_2 \), which can be determined by the Kubo formula [30]. They are related to temperature \( T \),
chemical potential $\mu_c$, momentum relaxation time $\tau$, and photon frequency $\omega$. The intra- and interlayer spacing is denoted as $d_1$ and $d_2$, respectively.

Fig. 1. Schematic of graphene pair arrays. The structure is periodic along $x$ direction. In each unit cell, there are two graphene sheets with alternating gain (red) and loss (blue).

We only consider the TM polarized SPPs that propagate along the $z$ direction and thus the field has only three components $H_y$, $E_x$, and $E_z$. The field should decay as increasing distance from the graphene sheets. According to Maxwell equations and Bloch theorem, the dispersion relation is given by

$$\cos(\phi) = \frac{\xi_1 \xi_2 \kappa^2}{2} \sin(\kappa d_1) \sin(\kappa d_2) - \frac{\xi_1 + \xi_2}{2} \sin(\kappa d) + \cos(\kappa d). \quad (1)$$

where $\kappa = (\beta^2 - \varepsilon_0 k^2) 0^{1/2}$ with $\beta$ the wavevector of SPPs in the $z$ direction and $k_0$ being the free space wavevector, $\phi = k_x d$ represents the Bloch momentum with $k_x$ the lateral Bloch wave vector along the $x$ direction and $d = d_1 + d_2$ denoting the period, and $\xi_i = \eta \sigma_i/(i \varepsilon_0 k_0)$ with $\eta_0$ representing the free space impedance. In this work, the incident wavelength of $\lambda = 10 \mu m$ is considered. The surface conductivities are $\sigma_{2,1} = (1.20 \times 10^{-6} + 7.68 \times 10^{-4}i)$ S with the lossy sheet corresponding to $\mu_c = 0.15$ eV and $\tau = 0.5$ ps at room temperature [1, 30]. The real parts of surface conductivities represent the loss and gain. The gain in graphene can be obtained through the optical pumping [13, 15, 16, 41–43] or carrier injection [44]. Taking optical pumping as an example, graphene can absorb photons and results in population inversion [42]. This is because the electron in graphene has the linear dispersion around the Dirac point and thus the intraband relaxation is faster than the recombination of electron-hole pairs. The interband stimulated emissions then take place under a strong optical excitation, leading to the negative conductivity of graphene [15, 43].

Figures 2(a) and 2(b) shows the dispersion relations in the first Brillouin zone for different interlayer spacing $d_2$. The intralayer spacing is fixed at $d_1 = 80$ nm. There are two bands in the diagram, corresponding to two kinds of Bloch modes in the array. As shown in Fig. 2(a), the system is PT symmetric while the propagation constants are real across the Brillouin zone in spite of the loss and gain. However, when $d_2 = d_1$, as displayed in Fig. 2(b), an EP emerges at the Brillouin boundary as the propagation constants turn to be complex. Since the spectra possess mirror symmetry, that is, $\beta(\phi) = \beta(-\phi)$, the EPs always appear in pairs. The $\beta(\phi)$ curves as a whole indicate broken PT symmetry. Figures 2(c)-(f) show the normalized magnetic field distributions corresponding to Fig. 2(a) for $\phi = 0$ and $\phi = \pi$. The magnetic fields have real and imaginary parts due to the loss and gain in graphene. The real parts are symmetric for upper band in each unit cell as illustrated in Figs. 2(c) and 2(e), while they are anti-symmetric for lower band as shown in Figs. 2(d) and 2(f). The fields possess a phase difference of $\phi$ between adjacent cells according to Bloch theorem. Therefore, the mode profiles between adjacent periods are in phase as $\phi = 0$ and out of phase as $\phi = \pi$. In contrast to Hermitian cases, the real parts of fields at two graphene sheets in each unit cell possess slightly different amplitudes. It is because that the two graphene sheets have different real parts of surface conductivity. The loss and gain can be further used to tailor the energy distributions. The amplitude at one of graphene sheets can be completely suppressed [34].
Fig. 2. The dispersion relation and mode profile of SPPs in the PT-symmetric graphene pair arrays. (a) and (b) Real (blue line) and imaginary (red line) parts of the propagation constants for (a) \( d_2 = 120 \text{ nm} \) and (b) \( d_2 = 80 \text{ nm} \). In all cases, \( d_1 = 80 \text{ nm} \). (c)-(f) The field distributions \((H_y)\) at \( \varphi = 0 \) and \( \varphi = \pi \) correspond to (a). The real part of the modes is illustrated in blue and the imaginary part in red.

3. PT symmetry and topological invariants

The Bloch modes of graphene pair arrays can be further understood by the coupled mode equation. For the parameters used in the study, the SPP mode width of a single graphene sheet is \( \text{Re}(\zeta) = 46 \text{ nm} \), which is smaller than the waveguide spacing. Therefore, the graphene sheets are weakly coupled and the system Hamiltonian is given by

\[
H = \begin{pmatrix}
\beta_0 - i\gamma/2 & c_1 + c_2 e^{-i\varphi} \\
\c_1 + c_2 e^{i\varphi} & \beta_0 + i\gamma/2
\end{pmatrix},
\]

where \( \beta_0 \) denotes the real part of propagation constants in single waveguide, \( \pm \gamma/2 \) is the modal loss or gain, \( c_1 \) and \( c_2 \) represent the intra- and interlayer coupling coefficients. The SPP propagation constant in single graphene sheet is given by \( \beta_{\text{SPP}} = (\epsilon k^2 \theta + 4/\zeta^2)^{1/2} \) and we can get \( \beta_0 = 43.43 \text{ \mu m}^{-1} \) and \( \gamma = 1.36 \text{ \mu m}^{-1} \). The eigenvalues of pair arrays read

\[
\beta_\pm = \beta_0 \pm \sqrt{c_1^2 + c_2^2 + 2c_1c_2 \cos \varphi - \gamma^2/4}.
\]

where the signs indicate the upper and lower bands. According to Eq. (3), the system possesses real eigenvalue spectrum in spite of the gain and loss as \( |c_1 - c_2| \geq \gamma/2 \). However, as \( |c_1 - c_2| < \gamma/2 \), the spectrum turns to be complex at the edge of Brillouin zone. The EP arises as the radicand equals to zero.
The solution of the coupled mode equation agrees well with the data calculated by Eq. (1). The coupling coefficients should have the form \( c_i = c_0 \exp(-\kappa d_i) \) with \( c_0 \) being a constant [12]. When inter- and intralayer couplings are equal, that is, \( c_1 = c_2 \), the constant reads as

\[
\begin{aligned}
    c_0 &= \frac{\exp(\kappa d_1) \sqrt{\beta_0^2(0) - \beta_0^2(\pi)} / 2}{c_1 - \gamma / 2}
    \end{aligned}
\]

where \( \beta_0(0) \) and \( \beta_0(\pi) \) are the propagation constants for \( \phi = 0 \) and \( \phi = \pi \), respectively. As the propagation constants can be numerically calculated from Eq. (1), we get \( c_0 = 36 \, \mu m^{-1} \) and \( c_1 = 1.1 \, \mu m^{-1} \) at \( d_1 = d_2 = 80 \, nm \). The range of broken PT symmetry is for interlayer spacing between

\[
\log\left(\frac{c_0}{c_1 + \gamma / 2}\right) / \kappa < d_2 < \log\left(\frac{c_0}{c_1 - \gamma / 2}\right) / \kappa,
\]

that is, \( 69 \, nm < d_2 < 101 \, nm \). Figures 3(a) and 3(b) show the dispersion relation with respect to different \( d_2 \) values. The spectra are complex within the range of \( 70 \, nm < d_2 < 96 \, nm \), which is coincident with the value predicted above. When \( d_2 = d_1 \), the system has most complex spectrum and the EPs locate at \( \phi_{EP} = \pm |\pi - \gamma/(2c_1)| = \pm 0.81\pi \), which approach the numerical values \( \phi_{EP} = \pm 0.83\pi \) as well.

As shown in Fig. 3(a), the real parts of dispersion relation are open in two PT symmetric regions, which are separated by the closed band with broken PT symmetry. According to general topological band theory in Hermitian system, the topological phase changes when a bandgap closes at a Dirac point and then reopens when the system is further modified [7, 8]. One may expect the topological phase transition takes place when the PT symmetry is broken. The topology of one-dimensional waveguide arrays can be characterized by the winding number given by [8, 24]

\[
W_h = \frac{1}{\pi} \int_{-\pi/d}^{\pi/d} dk_x \langle \Psi_m(k_x) | \frac{\partial}{\partial k_x} | \Psi_m(k_x) \rangle,
\]

where \( \Psi_m \) is the eigenfunction (mode profile) with \( m \) indicating the index of the band \( (m = 1, 2) \). The numerical results are shown in Figs. 3(c) and 3(d), where intralayer spacing is fixed at \( d_1 = 80 \, nm \). In the PT symmetric regime, the winding number is the same with that in the lossless SSH structure. It is found \( W_h = 1 \) for \( d_2 < 70 \, nm \) and \( W_h = 0 \) for \( d_2 > 96 \, nm \). However, when the PT symmetry is broken, the winding number is complex and varies as \( d_2 \) changes,
which is different from the lossless case as the phase transition point is right at $d_2 = d_1$. The complex winding number originates from the non-orthogonal eigenstates in non-Hermitian systems. On the other hand, when the interlayer spacing is fixed at $d_2 = 80$ nm, the broken PT symmetry is also within 70 nm $< d_1 < 96$ nm. However, the winding number now is $W_h = 0$ for $d_1 < 70$ nm and $W_h = 1$ for $d_1 > 96$ nm. The topological edge states will take place at the interface between two structures with different topological invariants.

4. Topological edge plasmon modes

Figures 4(a) and 4(b) show the structures of finite layers of graphene waveguides. The gain and loss present in the system can be settled in different ways to control the modal propagation loss. We first consider the graphene sheet at the center with gain. For the periodic waveguide arrays, the array sequence is “long-short-long-short-long-short-long”. For the aperiodic structure shown in Fig. 4(a), we insert a double-short defect in the center and the sequence now is “long-short-long-short-long-short-long-short-long”. The structure also can be regarded as two different arrays connecting. The intra- and interlayer spacing of left (right) array are denoted by $d_{L1}$ and $d_{L2}$ ($d_{R1}$ and $d_{R2}$), respectively. They are $d_{L1} = d_{R2} = 60$ nm and $d_{L2} = d_{R1} = 80$ nm, and thus the two structures should have different topologies. The left plus center graphene sheet is trivial with $W_h = 0$, while the right side has non-trivial topology with $W_h = 1$. Therefore, it must possess a topological edge state. The total number of graphene multilayers is set to be $N = 41$. The supermodes can be calculated using transfer matrix method [1].

![Fig. 4. Two arrays connect through (a) a short-short and (b) a long-long defects. The center graphene sheet is with gain. The green arrows denote the unit cell. (c) and (d) are the eigenvalues of supermode corresponding to (a) and (b), respectively. (e) and (f) show the mode profiles ($|H|$) of the topological edge modes for (a) and (b), respectively. Two trivial defect modes (purple and green) are also shown in Fig. 4(e).](image)

As shown in Fig. 4(c), there is evidently a SPP mode in the band gap with propagation constant $\beta = 43.7 + 0.66i$ $\mu$m$^{-1}$, almost equaling to that in single lossy waveguide. The SPP wavelength $\text{Re}(2\pi/\beta)$ is about 70 times smaller than the incident wavelength. The topological edge mode locates at the center defect, plotted with black curve in Fig. 4(e). Most of the energy is confined in the lossy graphene sheet. There are also two other topological edge modes with $\beta = 43.51 - 0.67i$ $\mu$m$^{-1}$, locating at the left and right edges of the structure. In
addition, the system also allows for two trivial defect states, represented by the purple and green dots in Fig. 4(c). They are formed as the three graphene sheet around the defect constitutes a region of higher refractive index than the rest of the structure. The two modes are also located around the center as shown in Fig. 4(e), which will influence the beam dynamics shown later.

When the spacing becomes \( dL_1 = dR_2 = 120 \text{ nm} \) and \( dL_2 = dR_1 = 80 \text{ nm} \), a long-long defect is formed and the sequence now is “short-long-short-long-short-long-short”. The defect also can be seen as an interface between two structures that have different topological invariants, that is, the left half-space possesses \( W_h = 1 \) and the right \( W_h = 0 \). As displayed in Fig. 4(d), there is a topological edge mode within the band gap with the propagation constant \( \beta = 43.43 - 0.68i \mu \text{m}^{-1} \). Figure 4(f) shows the mode profile of the topological edge mode. The field is well confined at the interface. While the other modes in the system are lossless, the topological mode with gain can be selectively amplified. Importantly, if the distributions of gain and loss are reversed, the imaginary parts of propagation constants of edge modes are also changed.

![Fig. 5. The band diagram and the mode profile for a different gain and loss distribution. The center graphene sheet now is lossless. (a) and (b) show the structures with short-short and long-long defects, respectively. (c) and (d) are the eigenvalues of all supermodes. (e) and (f) show the mode profiles (|\( \mathbf{H} \)|) of the topological edge modes and two trivial defect modes (purple and green) at the interface.](image)

The lossless topological edge mode can also be realized by another loss and gain distribution. A lossless graphene sheet is placed at the center (black line), as shown in Figs. 5(a) and 5(b). The surface current distributions satisfy a PT-invariant fashion [17] as the gain-loss distribution has to be an odd function of position around the given origin \( x = 0 \). The structure parameters are the same with that used in Fig. 4. For the short-short defect, as illustrated in Fig. 5(c), the spectrum possesses a lossless topological edge mode with a propagation constant \( \beta = 43.7 \mu \text{m}^{-1} \). The mode locates at the center defect, displayed as the black line in Fig. 5(e). Similarly, there are also two other topological edge modes with complex propagation constants locating at the edge of the graphene multilayers. In addition, the system also allows for two trivial defect states, represented by the purple and green dots in Fig. 5(c). The two modes are also located around the interface as shown in Fig. 5(e). For the
long-long defect, as displayed in Fig. 5(d), there is only a topological edge mode in the band gap. Moreover, the spectrum preserves entirely real eigenvalues. Figure 3(f) shows the mode profile of the topological edge mode. The field is well confined at the interface. Therefore, we have shown topological edge modes can be amplified, dissipated, or lossless by introducing the non-Hermitivity in the system.

5. Emergence condition of topological edge modes

As the winding number of periodic waveguide arrays is only stable in the PT-symmetric regime, one may expect that the topological modes only exist when PT symmetry holds. Figure 6 shows the numerically calculated eigenvalues for different spacing $dL$ $1(dR = dL 1)$, while the spacing $dL 2$ and $dR 1$ are fixed at 80 nm. The results in Figs. 6(a) and 6(b) correspond to the structures shown Figs. 4 and 5, respectively. The eigenvalues for the short-short and long-long defects are plotted using different vertical ordinates to clearly illustrate the band gaps. The result shows that the band gap narrows when approaching the regions of broken PT symmetry. At the same time, the topological edge states will experience a weaker confinement. The band gap finally closes at $dL 1 = dR 2 = 70$ nm for the short-short defect and $dL 1 = dR 2 = 96$ nm for the long-long defect. Therefore, there are no topological edge modes when the PT symmetry is broken. This distinguishes from the lossless cases where the band gap disappears exactly at $dL 1 = dR 2 = 80$ nm. On the other hand, as the waveguide spacing changes, the eigenvalues of the topological modes remain stable while the eigenvalues of trivial defect modes change.

![Fig. 6. The real part of eigenvalues for different waveguide spacing $d_2$. Here $d_2$ refers to $dL 1 = dR 2$. The spacing $dL 2$ and $dR 1$ are fixed at 80 nm. (a) is for the structure shown in Fig. 4 and (b) for Fig. 5. The blue (red) dots represent the eigenvalues for the short-short (long-long) defects. When $70$ nm $< d_2 < 96$ nm, the PT symmetry is broken (green regions).](image)

The bound states, including trivial and non-trivial edge modes locating at the center defects, are governed by the coupled mode equation. The filed amplitudes for the five sheets at the center of the structure abide by the relation

$$
\begin{align*}
\beta A_{-1} &= (\beta_0 - i \frac{\gamma}{2}) A_{-1} + c_1 B_{-1} + c_2 B_{-2}, \\
\beta B_{-1} &= (\beta_0 + i \frac{\gamma}{2}) B_{-1} + c_2 A_{-1} + c_1 A_{-1}, \\
\beta A_0 &= (\beta_0 - i \frac{\gamma}{2}) A_0 + c_2 B_{-1} + c_2 B_0, \\
\beta B_0 &= (\beta_0 + i \frac{\gamma}{2}) B_0 + c_2 A_0 + c_1 A_0, \\
\beta A_1 &= (\beta_0 - i \frac{\gamma}{2}) A_1 + c_1 B_0 + c_2 B_2,
\end{align*}
$$

where $A_n$ and $B_n$ denote the amplitudes in the respect graphene sheets and $c_{1,2}$ is the coupling coefficients. If there were bound states, the amplitudes should attenuate from the center and the mode amplitudes can be written as
with $0 < |\sigma| < 1$. We first discuss the bound mode for the structure shown in Fig. 4(a), which possesses an amplifying graphene sheet at the center and a short-short defect ($|c_2| > |c_1|$). Recalling the eigenvalue of the topological edge mode is $\beta = \beta_0 + i\gamma/2$, one can find $a = 0$, $b = -c$ and $\sigma = -c_1/c_2$. The relations imply the bound states has vanishing amplitude on every A site. For the long-long defect ($|c_2| < |c_1|$), the eigenvalue of the topological edge mode is $\beta = \beta_0 - i\gamma/2$. Then one can find the relations $b = c = 0$ and $\sigma = -c_2/c_1$, indicating the bound mode has vanishing amplitude on every B site. The field amplitudes agree well with the numerical results shown in Figs. 4(e) and 4(f). The results can be applied in both PT symmetric and broken regions. However, in broken PT symmetry phase, the bound state is trivial as there is no band gap.

For the structures shown in Fig. 5, which possess a lossless graphene sheet at the center, the propagation constants of the topological edge mode confined at the short-short and long-long defects are both $\beta = \beta_0$. Using the similar method, one can find the relations $b = -c = i\gamma a/(2c_1 + 2c_2 \sigma)$ and $\sigma^2 + 2w\sigma + 1 = 0$ with $w = (c_1^2 + c_2^2 - \gamma^2/4)/(2c_1c_2)$. Considering $0 < |\sigma| < 1$, the condition for a bound state is $|c_1 - c_2| > \gamma/2$ or $|c_1 + c_2| < \gamma/2$. As the coupling $c_1 = 1.1 \mu$m$^{-1}$ is always larger than $\gamma/2 = 0.68 \mu$m$^{-1}$ for the parameters used in this study, the latter condition never happens. The condition $|c_1 - c_2| > \gamma/2$ is exactly the same as the regions such that the periodic graphene arrays are PT-symmetric. Therefore, there are no bound states when the PT symmetry is broken. The bound states possess amplitudes on both A and B sites, which distinguish from the results shown in Fig. 4. Therefore, the field distributions of topological plasmons can be controlled by applying different gain and loss distributions.

6. Beam dynamics

The topological and trivial defect modes greatly influence the beam propagations. The wave evolutions are simulated using the finite element method (FEM) performed by Comsol Multiphysics. Graphene is modeled by employing the surface current boundary condition [27, 35]. All domains have been discretized less than 1/12 of SPP wavelength. The waveguides are excited from the single graphene sheet at the center.

![Fig. 7. The light evolutions for a single-waveguide excitation corresponding to the structures shown in Fig. 5. (a) Two graphene arrays connected by a short-short defect. (b) The arrays with equidistance. (c) The arrays connected by a long-long defect.](image-url)
beating pattern as the trivial and non-trivial defect modes are excited simultaneously. When the waveguides are of equidistant spacing $d_L 1 = d_L 2 = d_R 1 = d_R 2 = 80$ nm, the PT symmetry is broken and there is no bound mode at the interface. Therefore, the beam spreads during the propagation as displayed in Fig. 7(b). Figure 7(c) is for the waveguide that has a long-long defect. The field localizes around the defect because only the topological mode is excited.

![Fig. 8. The influences of chemical potential on the beam propagations. (a) The regions of broken PT symmetry as a function of chemical potential for the infinite graphene arrays. The intra-layer spacing is fixed at $d_1 = 80$ nm. (b) and (c) are the light evolutions for $\mu_c = 0.16$ eV and $\mu_c = 0.137$ eV, respectively. The structures are the same as that shown in Fig. 7(c).](image)

Importantly, the topological edge states can be controlled as the regions of broken PT symmetry relate to the chemical potential $\mu_c$. Figure 8(a) shows the broken region for the infinite periodic graphene arrays as $\mu_c$ is varying. The broken region enlarges as chemical potential decreases. For example, as $\mu_c = 0.16$ eV, the region is within $72.1$ nm $< d_2 < 91.2$ nm. The topological edge mode can still exist in the structures. However, as chemical potential decreases to $0.137$ eV, the broken region now is $67$ nm $< d_2 < 132$ nm. The topological edge mode will disappear when PT symmetry is broken. The beam propagations for two different $\mu_c$ are illustrated in Figs. 8(b) and 8(c). Other parameters are the same as that used in Fig. 7(c). As $\mu_c = 0.16$ eV, the beam can be confined well at the center defect during the propagation as shown in Fig. 8(b). However, as displayed in Fig. 8(c), the beam spreads when $\mu_c = 0.137$ eV. Therefore, the light transports in the non-Hermitian SSH lattices are controllable by actively tuning the chemical potential.

The similar phenomenon takes place by altering the incident wavelength. The region of broken PT symmetry also enlarges as the wavelength $\lambda$ decreases for fixed chemical potential. When chemical potential is $\mu_c = 0.15$ eV, the broken region is $72$ nm $< d_2 < 91$ nm for $\lambda = 10.5$ $\mu$m and $67.9$ nm $< d_2 < 113.9$ nm for $\lambda = 9.5$ $\mu$m. The topological edge modes can still exist when the incident wavelength is slightly altered. However, as operating wavelength decreases to $9.4$ $\mu$m, the broken region now is $67.2$ nm $< d_2 < 123.1$ nm. For the structures used in Fig. 7(c), there are no topological edge modes.
7. Passive PT symmetric structures

As the optical gain is not easy to realize, the PT symmetry is usually realized in passive devices. We further show topological edge modes discussed above can exists in lossy graphene arrays without gain. The intrinsic loss of graphene mainly relates to the momentum relaxation time $\tau$, which can extend into the picosecond in high-quality graphene [39]. Here we choose $\tau = 1$ ps, 0.67 ps, and 0.5 ps for three different kinds of graphene. Hence, the SPP propagation constants in single graphene are $\beta_0 = 43.43 + 0.34\mu m^{-1}$, $43.43 + 0.51i\mu m^{-1}$, and $43.43 + 0.68i\mu m^{-1}$, which are of equidifferent losses. They refer to the “gain”, “lossless”, and “loss” waveguide as the “lossless” waveguide now sustains a loss of $0.51i$, which corresponds to the passive PT symmetry [18, 19].

![Figure 9](image_url)

Fig. 9. The topological edge modes in passive PT-symmetric graphene multilayers. (a) and (b) are the spectra of the supermodes for the structure shown in Figs. 4(b) and 5(b), respectively. The “lossy” waveguide now is with $\tau = 0.5$ ps, the “gain” with $\tau = 1$ ps, and the “lossless” waveguide with $\tau = 0.67$ ps.

Figure 9(a) shows the eigenvalues of supermode in the lossy waveguides. The structure is the same as that used in Fig. 4(b). The results shows that there is indeed a mode inside the band gap with $\beta = 43.44 + 0.34i\mu m^{-1}$. The whole spectrum is similar to that shown in Fig. 4(d) as the loss of the topological edge mode is smaller than other modes present in the system. Figure 9(b) shows the eigenvalues of supermode in the lossy waveguides corresponding to Fig. 5(b). The topological edge mode now has a propagation constants $\beta = 43.43 + 0.51i\mu m^{-1}$, while the propagation losses of other modes also approach to $\text{Im}(\beta) = 0.51i\mu m^{-1}$. The results demonstrate the PT-symmetric topological edge mode can be observed in lossy graphene waveguides.

8. Conclusions

In conclusion, we have studied the infrared topological edge modes in PT-symmetric graphene pair waveguide arrays. The periodic graphene pair arrays show distinct winding numbers in two PT-symmetric regions separated by the EPs. Subsequently, we construct a certain structure by connecting two structures with different topologies, resulting in topological edge modes confined at the interface. The gain and loss present in the system greatly influence the propagation losses, the field distributions, and emergence condition of the topological edge modes. There are no topological edge modes in the regions as the PT symmetry is broken. As the broken ranges can be tuned via the chemical potential of graphene, the topological edge mode and the beam diffraction in the waveguide can be controlled, which cannot be realized in lossless cases. The results are still valid for longer wavelengths at THz frequencies. We also show such topological edge mode can be observed in lossy graphene waveguides that maintain passive PT symmetry. The results provide a robust way to control light flow that immunes to environmental alterations and may find application in optical switches on a deep-subwavelength scale.

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