Confinement & Chiral Symmetry Breaking:
The fundamental problems of hadron physics

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Some of the difficulties arising when one tries to understand confinement as well as dynamical and anomalous chiral symmetry breaking are briefly reviewed. Criteria to be fulfilled by a successful and complete picture of these phenomena are presented, and a few of the suggested explanations are listed.

1. Phenomenology versus Theory

The most intriguing phenomena of hadron physics are confinement as well as dynamical and anomalous chiral symmetry breaking. Despite the fact that the theory of the Strong Interactions, Quantum Chromodynamics (QCD), is known since decades we still lack a fundamental understanding of the corresponding physics.

As a phenomenon confinement is easily described. On one hand, representing the Strong Interaction by a local Quantum Field Theory (i.e. by QCD) necessitates to introduce fundamental fields with a new quantum number, namely quarks and gluons with some “colour”. The advantage of this approach is twofold: It provides a mathematical framework, and it orders the plethora of hadrons into a clearly arranged pattern. On the other hand, quarks and gluons have never been detected as particles, i.e. nobody has ever seen quarks and gluons making a track in a detector. The confinement hypothesis can therefore be formulated as: the colour-neutral hadrons, being a kind of bound states of coloured quarks and gluons, are the only strongly interacting particles, no “coloured” particles exist. This hypothesis has been extremely successful. The colour-charge version of ionization does plainly not occur. Even more, the concept of mutual forces by mutual polarization, the van-der-Waals forces, also does not have a colour-charge analogue. Thus as a phenomenon confinement seems to be plain and simple.

As a theoretical concept confinement is astonishingly hard to put into precise terms. Even the question how to obtain a concise definition of ‘charge’ did undergo some severe discussions when trying to find a theoretically unequivocal definition of confinement. E.g. the Wilson loop provides an order parameter only in the absence of fundamental charges, i.e. quarks. Despite all efforts such an order parameter has not been found in the real world with light quarks, a satisfactory and detailed description of the underlying mechanisms of confinement stays elusive. The fact that for charges in higher representations there are common aspects with the Higgs mechanism complicates the issue even further.

The situation is not drastically different when addressing dynamical Chiral Symmetry Breaking (\chiSB) and the \textit{U}_A(1) anomaly. As phenomena they are clearly identifiable, the first because of the relatively small pion mass and several patterns in the interaction of pions with themselves and other hadrons. The latter because of the large \eta' mass.

When it comes to theoretically understanding \chiSB we also lack a lot of basic knowledge. We know that dynamical \chiSB comes along with the dynamical generation of “constituent”quark masses (which, however, depend on the momentum of the quarks). One may explain dynamical \chiSB and the \textit{U}_A(1) anomaly with two seemingly different approaches. One approach starts by considering quark zero modes in topologically non-trivial field configurations. A non-vanishing
density of such zero modes in the limit of infinite volume signals dynamical $\chi_{SB}$ [1]. The non-vanishing topological susceptibility provides the explicit $U_A(1)$ symmetry breaking, see e.g. [2] and references therein.

The other approach rests on a supercritical effective interaction between quarks [3,4], usually described in a covariant Green’s function approach see e.g. [5,6,7,8,9,10] and references therein. The mass generating mechanism becomes then similar to the generation of a gap in superconductors. Especially, if this interaction is infrared divergent the effective coupling always exceeds the critical one and therefore dynamical $\chi_{SB}$ occurs. What is more astonishing is the fact that a confining-type infrared divergence in the effective quark-quark interaction results in a non-vanishing $\eta'$ mass [11,12]. Therefore it may well be possible that these two so differently appearing approaches are merely two distinct but correct ways of describing the related physics and aspects thereof.

As we have no commonly accepted complete picture of the strongly interacting domain of QCD the relation between confinement on the one hand and dynamical, resp., anomalous, $\chi_{SB}$ on the other hand is not firmly established. However, there are important hints that quark confinement and $\chi_{SB}$ are closely related. Even beyond the debated question whether the corresponding phase transition(s) occur(s) at the same temperature (see e.g. [13] and references therein) an analysis of the so-called dual quark condensate and dressed Polyakov loops points to such a close relation [14,15,16] via linking confinement to spectral properties of the Dirac operator [17]. Again such a close relation can be found in the approaches mentioned above: Either when investigating topologically non-trivial, confining field configurations [18,19,20] or when studying the infrared behaviour of QCD Green functions, and hereby especially the quark-gluon vertex in Landau gauge [21,22]. But despite all evidence for a deep connection between confinement and $\chi_{SB}$ the situation is not conclusive yet.

2. Remarks on Quantum Field Theory

According to my understanding QCD is a local Quantum Field Theory as expressed in the quote from Haag’s book [23] in a clear way as follows:

“The role of fields is to implement the principle of locality. The number and the nature of different basic fields needed in the theory is related to the charge structure, not to the empirical spectrum of particles.”

To put this understanding in a more precise setting: I assume validity of the Osterwalder-Schrader axioms [24] except reflection positivity. This provides a well-defined mathematical framework as described in refs. [23,24] and a number of other monographs. It is important to note that all methods in Quantum Field Theory, including perturbation theory, lattice field theory, and functional approaches, rely on this framework. If it were true that QCD is not a local theory more or less all attempts to understand hadron physics from QCD are questionable. Fortunately, the results obtained from QCD provide evidence for the validity of locality.

Gaining an understanding of physics is quite often related to develop intuitive pictures. In the case of confinement such a picture will be preferentially formulated with the help of the fundamental fields, the gluons and quarks. But these are only valid elements of the theory after gauge-fixing. Of course, confinement as an observable phenomenon exists without reference to any gauge, and in different gauges picturing confinement might result in quite different scenarios. However, this is exactly the point. Everybody will agree that the hydrogen atom can be described by quantum mechanics independent of the gauge chosen for electromagnetism. For gaining an understanding of the laws of Quantum Mechanics, however, it was of utmost importance that the spectrum of the hydrogen atom can be easiest understood when choosing Coulomb gauge. To gain knowledge in which gauge confinement will be explained easiest would be a tremendous step forward. Consequently, fixing the gauge is likely to be helpful for an understanding of confinement.

As already mentioned the Wilson loop gives
only a clear criterion in the absence of quarks. So, what are the possibilities for a theoretically sound definition of confinement? A potential procedure may look like:

- Construct a colour charge operator, *e.g.* as described in [25],
- demonstrate it to be well-defined ("unbroken charge"), and
- check for a mass gap in the physical state space.

In case one obtains a well-defined charge with unbroken global symmetry and a mass gap in the physical state space one has confinement [26]. As pictorially presented\(^\text{1}\) in fig. 1 the unbroken global symmetry without a mass gap provides the Coulomb-type phase whereas broken global symmetry with mass gap gives the Higgs phase.

![Diagram](image)

Figure 1. A pictorial presentation how the field around a test charge and the (non-)existence of a mass gap allows to distinguish between the Coulomb, confinement and Higgs phase [24].

To conclude this section let me emphasize the rôle of the Becchi-Rouet-Stora–Tyutin (BRST) symmetry in gauge-fixed quantum gauge field theories. The existence of BRST quartets and the construction of a BRST cohomology does not only allow the generalization of the Gupta-Bleuler mechanism of QED to QCD but also very likely is substantial in constructing the physical state space. The distinction between the complete and the positive-definite state space is hereby absolutely crucial in understanding the mathematical framework of quantum gauge field theories. An introduction to the subject can be found in ref. [25], a short summary on how this may relate to the confinement problem in ref. [27].

3. Requirements for an investigation of Confinement

First, confinement in four-dimensional field theories requires the dynamical generation of a physical mass scale. In presence of such a mass scale, however, the renormalisation group (RG) equations imply the existence of essential singularities in physical quantities (such as the \(S\)-matrix) as functions of the coupling at \(g = 0\). This is due to the dependence of the RG invariant confinement scale on the coupling and the renormalisation scale \(\mu\) near the ultraviolet fixed point as given by

\[
\Lambda = \mu \exp \left( - \int \frac{dg'}{\beta(g')} \right),
\]

\[
g \to 0 \quad \mu \exp \left( - \frac{1}{2\beta_0 g'} \right),
\]

\(\beta_0 > 0\). Therefore a truly non-perturbative method is needed for the study of confinement.

Second, in some scenarios confinement is related to severe infrared divergences, *i.e.*, divergences which cannot be removed from physical cross sections by a suitable summation over degenerate states as in QED\(^\text{2}\). In any finite volume these infrared divergences could be detected only by a careful extrapolation to infinite volume. Therefore either such an analysis of lattice results and/or an ab initio continuum approach is needed for an understanding of such confinement scenarios.

\(\text{\textsuperscript{2}}\)See, however, ref. [28] which shows that confinement criteria can be fulfilled without infrared divergences.
Third, confinement implies the suppression of long-wavelength propagation. Phrased otherwise, confinement is a true quantum phenomenon. Therefore a purely (semi-)classical description is necessarily incomplete, a quantum theoretical picture is needed for an investigation of confinement.

4. Criteria for a Confinement picture

A successful confinement scenario should explain many properties either deduced from hadron physics or lattice calculations. One of them is

- **string formation.**

There are two distinct sorts of representation dependence of the static quark potential, depending on the static source separation:

- **Casimir Scaling.** Initially the slope of the linear potential – the string tension – is proportional to the quadratic Casimir of the group representation.
- **N-ality Dependence.** Asymptotically, the force between charged fields in an SU(N) gauge theory depends only on the so-called “N-ality” of the group representation, given by the number of boxes mod N in the Young tableau of the representation.

Another such property is the

- **absence of van-der-Waals forces**

as discussed in the introductory section.

Related to the issue of the mathematical framework of the theory is the property of

- **positivity violation**

and hereby

- **the BRST quartet mechanism** for tree-level positive fields.
- **antiscreening beyond perturbation theory** as expressed in the Oehme–Zimmermann superconvergence relations.\(^3\)

And, last but not least, as a successful theory of confinement is a theory of Infrared QCD it should include a description of

- **dynamical χSB**

and the

- **U\(_A\)(1) anomaly.**

5. Candidates for a Confinement picture

There are many proposals for the confinement mechanism. It is impossible to provide an exhaustive list in a short article, so I will cite only those proposals which according to my opinion seem best supported by existing numerical studies or other arguments.\(^4\)

A line of thought is that the QCD functional integral is dominated by some special class of field configurations which cause the expectation value of a large Wilson loop to fall off exponentially with the minimal area of the loop. The leading candidates for these special configurations are magnetic monopoles [18], dyons /calorons [20,31] or center vortices [19], although other objects have been suggested.

A different approach is based on the special properties of quantum fields in Coulomb gauge, and thereby the existence of the gauge-fixing ambiguity, the Gribov problem, and the existence of a Gribov horizon plays a special role [32,33].

Another idea is, preferentially in Landau gauge, to solve non-perturbatively for quark and gluon propagators and vertex functions, analytically by an infrared expansion of the complete set of Schwinger-Dyson and Exact Renormalization Group equations, and numerically by solving a truncated set of these equations, see e.g. [34,35], [36,37] and references therein.

Finally, there is a fascinating relationship between gauge theory in \(D = 4\) dimensions and string theory quantized in a special ten-dimensional background geometry known as anti-DeSitter space. This is the AdS-CFT correspondence, see refs. [38,39] and many others.

It has turned out that a number of these suggestions are related in interesting ways: monopole

\(^3\)See e.g. refs. [29,30].

\(^4\)Some of these arguments are briefly reviewed in ref. [30].
wordlines are found to lie on center vortex world-sheets, and center vortex worldsheets appear to be crucial in some ways to the confinement scenario in Coulomb gauge. Both Coulomb and Landau gauge investigations emphasize the importance of the Faddeev-Popov operator, and the infrared properties of the ghost propagator.

6. Outlook

In this contribution to a lively on-going discussion I tried to describe what are the difficulties encountered in the endeavour of studying infrared QCD. It is striking that after decades of effort we do not understand how the Strong Interaction really works at long distances. Nevertheless, there has been appreciable progress in this subject. Step by step we uncover surprising details about confinement and dynamical, resp., anomalous, chiral symmetry breaking.

Between the existing approaches there are not yet understood relations. Although many details are still missing these relations make plain that the different confinement pictures are definitely not mutually exclusive. Maybe we will learn that a non-trivial merger of all these scenarios of Infrared QCD will eventually fulfill all the criteria required for a consistent and convincing description.

Even if this will constitute the major breakthrough for theory one should keep in mind that even then there is still a tough challenge left: Find an experimentally accessible hadron observable to verify/falsify the presented picture of confinement and chiral symmetry breaking.

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