Nucleon Momentum Decomposition in QCD

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Abstract: Based on the gauge invariant quark canonical momentum we construct two theoretically possible decompositions of nucleon momentum to those of quarks and gluons. We predict that either 6% or 21% of nucleon momentum is carried by gluons, depending on what type of gluons are in nucleons. We clarify the existing confusions on this problem and discuss the physical implications of our result on the proton spin crisis problem.

PACS numbers: 11.15.-q, 14.20.Dh, 12.38.-t, 12.20.-m

Keywords: canonical and kinematic quark momentum, gauge invariant canonical quark momentum, gauge invariant decomposition of nucleon momentum, gluon momentum in nucleon

An important problem in nuclear physics is to find out how much fraction of nucleon momentum is carried by the gluons. It has generally been believed that gluons carry about a half of nucleon momentum [1]. But recently there has been a new assertion that only about one-fifth of the nucleon momentum should be carried by the gluons [2]. This has created considerable controversy and confusion in the literature [3, 4].

To resolve this problem one has to know how to decompose the momentum of nucleon to those of its constituents. At the first glance this problem seems to be simple enough. But in gauge theories it is very difficult to obtain a gauge invariant decomposition of momentum or spin to those of the constituents. In fact it has long been suggested that this is impossible in gauge theories. The reason is that the gauge interaction makes a gauge invariant decomposition of the total momentum (and spin) to those of the constituents very difficult [5, 6]. The purpose of this Letter is to clarify the confusion on this problem and provide new nucleon momentum decompositions to predict the fraction of gluon momentum in nucleons.

To understand the problem, consider the canonical decomposition of the momentum of positronium in QED

\[ P^{(\text{can})}_{\mu} = P^e_{\mu} + P^\gamma_{\mu} = i \int \bar{\psi} \gamma^\mu D^\alpha \psi d^3x \]

This does provide a decomposition of momentum to those of the constituents, but is not gauge invariant. We can change it to the popular gauge invariant decomposition adding a surface term [2, 4]

\[ P^{(\text{gau})}_{\mu} = \bar{P}^e_{\mu} + \bar{P}^\gamma_{\mu} = i \int \bar{\psi} \gamma^\mu D^\alpha \psi d^3x + \int (F_{\mu}^a F^a_{\alpha\beta} + \frac{1}{4} \delta^\mu_\alpha F^2_{\alpha\beta}) d^3x. \] (2)

But this also may not be the desired decomposition because the first term involves both electron and photon.

The problem stems from the fact that charged particles have two momentums, the “canonical” one given by \(-i\partial_\mu\) and the “kinematic” one given by \(-iD_\mu\), but neither is suitable for the momentum decomposition of composite particles [3, 4]. This is because the canonical momentum is not gauge invariant, and the kinematic momentum contains the gauge field. Moreover, there are actually two different issues in this problem. The first is theoretical: How to make a gauge invariant decomposition of the total momentum. The second is experimental: How to make a measurable (and gauge invariant) decomposition of the total momentum. This is more subtle because here we must figure out what are the measurable momentums of the constituents.

To obtain a gauge invariant decomposition of the positronium momentum, we first decompose the photon field to the vacuum part \(\Omega_\mu\) and the physical part \(X_\mu\) [7],

\[ A_\mu = \Omega_\mu + X_\mu, \quad \Omega_\mu = \partial_\mu \theta, \quad \partial_\mu X_\mu = 0. \] (3)

Notice that this decomposition is gauge independent. Moreover, the gauge transformation affects only the pure gauge part, so that the physical part remains gauge invariant. In particular, the physical part here becomes

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a Lorentz covariant four-vector, so that we can make $X_0 = 0$ choosing a proper Lorentz frame (as far as $X_\mu$ is space-like).

Now, adding a surface term to (2), we can easily change it to

$$P_\mu = i \int \bar{\psi} \gamma^0 \hat{D}_\mu \psi d^3 x + \int [(\partial_\mu X_\alpha) F^{\alpha 0}$$
$$+ \frac{1}{4} \delta_\mu F^{2}_{\alpha \beta}] d^3 x, \quad \hat{D}_\mu = \partial_\mu - i e \Omega_\mu. \quad (4)$$

Unlike (1) or (2), each term now is gauge invariant and at the same time involves only one constituent. So, theoretically it does describe a gauge invariant decomposition of the total momentum.

It has generally been believed that the kinematic momentum is what experiments measure, because this is gauge invariant. This has made (2) a popular decomposition. But here we have shown that the canonical momentum can also be expressed by a covariant derivative. So there are actually two gauge invariant momentums that we can construct and thus can possibly measure. If so, which momentum is measurable and why is that so?

Classically it appears that the conserved momentum of a charged particle moving in an electromagnetic field is the sum of the kinematic momentum of the particle and the electromagnetic momentum (the Poynting vector) \( \mathbf{F} \). This favors the kinematic momentum. But quantum mechanically the kinematic momentum operators do not satisfy the canonical momentum commutation relation, since they do not commute. Moreover, the canonical momentum $\hat{D}_\mu$ defined by the vacuum potential is gauge invariant and at the same time satisfies the canonical commutation relation. This strongly implies that (4) is the correct momentum decomposition.

In QCD the conserved momentum obtained by Noether’s theorem is given by

$$P_\mu^{(\text{QCD})} = i \int \bar{\psi} \gamma^0 \partial_\mu \psi d^3 x$$
$$+ \int [(\partial_\mu \tilde{A}_\alpha) \cdot \tilde{F}^{\alpha 0} + \frac{1}{4} \delta_\mu F^{2}_{\alpha \beta}] d^3 x. \quad (5)$$

Adding a surface term we can change it to the popular gauge invariant decomposition

$$P_\mu^{(\text{QCD})} = i \int \bar{\psi} \gamma^0 D_\mu \psi d^3 x$$
$$+ \int [F_{\mu \alpha} \cdot \tilde{F}^{\alpha 0} + \frac{1}{4} \delta_\mu F^{2}_{\alpha \beta}] d^3 x. \quad (6)$$

But again the first term contains quarks and gluons. To cure this defect we first have to find out the gauge covariant canonical momentum operator which does not include gluons.

To construct such momentum operator we must have a gauge independent decomposition of the non-Abelian gauge potential to the vacuum part $\hat{\Omega}_\mu$ and the physical part $\tilde{Z}_\mu$ similar to (3). Consider SU(2) QCD for simplicity, and let $\hat{n}_i (i = 1, 2, 3)$ be a gauge covariant right-handed orthonormal basis in SU(2) space. Then imposing the vacuum condition to the potential

$$\forall_i \quad D_\mu \hat{n}_i = (\partial_\mu + g \hat{A}_\mu \times) \hat{n}_i = 0. \quad (\hat{n}_i^2 = 1) \quad (7)$$

we obtain the most general vacuum $\hat{\Omega}_\mu$,

$$\hat{A}_\mu \rightarrow \hat{\Omega}_\mu = \frac{1}{2g} \epsilon_{ijk} (\hat{n}_i \cdot \partial_\mu \hat{n}_j) \hat{n}_k. \quad (8)$$

Next, we make the decomposition

$$\tilde{A}_\mu = \hat{\Omega}_\mu + \tilde{Z}_\mu, \quad (9)$$

and find that under the gauge transformation we have

$$\delta \hat{\Omega}_\mu = \frac{1}{g} \hat{D}_\mu \tilde{a}, \quad \delta \tilde{Z}_\mu = - \tilde{a} \times \tilde{Z}_\mu;$$

$$\hat{D}_\mu = \partial_\mu + g \hat{\Omega}_\mu \times. \quad (10)$$

where $\tilde{a}$ is the (infinitesimal) gauge parameter. Notice that $\tilde{Z}_\mu$ becomes a Lorentz covariant (as well as gauge covariant) four-vector. Finally, we impose the transversality condition to $\tilde{Z}_\mu$ to make it physical

$$\hat{\tilde{D}}_\mu \tilde{Z}_\mu = 0. \quad (11)$$

Notice that this is not a gauge condition, because it applies to any gauge. Obviously this is the generalization of (3) to QCD which provides the desired decomposition.

Now, we can modify (5) to

$$P_\mu^{(\text{QCD})} = i \int \bar{\psi} \gamma^0 \tilde{D}_\mu \psi d^3 x$$
$$+ \int [(\tilde{D}_\mu \tilde{Z}_\alpha) \cdot \tilde{F}^{\alpha 0} + \frac{1}{4} \delta_\mu F^{2}_{\alpha \beta}] d^3 x, \quad (12)$$

adding a surface term

$$- \int (\hat{\tilde{D}}_\mu \tilde{Z}_\mu \cdot \tilde{F}^{\alpha 0}) d^3 x. \quad (13)$$

Clearly this provides a gauge invariant decomposition of total momentum to those of the quarks and gluons. But this may not be the desired decomposition that we are looking for. The reason is that QCD has two types of gluons, so that we have to figure out which become the constituents of nucleons (10, 12).

To see this one has to understand that QCD allows the Abelian decomposition which separates the gluons to the colorless binding gluons and the colored valence gluons gauge independently. Because of this we have two types of QCD, the restricted QCD (RCD) made of the binding gluons and the standard QCD made of all gluons. Moreover, QCD can be viewed as RCD which has the valence...
gluons as the colored source \[8, 9\]. So the valence gluons (just like the quarks) become another colored source which has to be confined. This means that RCD plays the crucial role in confinement, which is known as the Abelian dominance in QCD \[11, 12\]. This interpretation has been confirmed numerically in a series of lattice QCD calculations \[13, 14\].

Most importantly, the quark model of hadrons tells that nucleons (in particular the low-lying nucleons) are made of three quarks which are colored, not quarks and colored gluons \[15\]. The colored gluons make up glueballs. This implies that valence gluons have no place in those of the constituents, one in QED and two in QCD. The reason why we have two competing decompositions in QCD is because we have two types of gluons. If nucleons contain only the binding gluons, \[12\] or \[21\] must be the correct one. But if they contain all gluons, we must have \[6\] or \[19\].

To find which decomposition is correct, suppose only the kinematic momentum is measurable. In this case we have in the asymptotic limit \[1\] the kinematic momentum is measurable. In this case we have two gauge invariant electron momentums, the canonical \(-i\tilde{D}_\mu\) and the kinematic \(-iD_\mu\). But in QCD we have three. To see this notice that we can express \[12\] by

\[
P^{(rcd)}_\mu = i \int \bar{\psi}\gamma^\mu \psi d^3x + \int \left[ (\tilde{D}_\mu \tilde{B}_\alpha) \cdot \tilde{F}^{\alpha 0} + \frac{1}{4} \delta^0_{\alpha \beta} \tilde{F}^{2}_{\alpha \beta} \right] d^3x.
\]

From this we can obtain \[12\] adding a surface term.

Now, we come back to the difficult question: What are the quark and gluon momentums in nucleon? In QED we have two gauge invariant electron momentums, the canonical \(-i\tilde{D}_\mu\) and the kinematic \(-iD_\mu\). But in QCD we have three. To see this notice that we can express \[12\] by

\[
P^{(rcd)}_\mu = i \int \bar{\psi}\gamma^\mu \psi d^3x + \int \left[ \tilde{F}_{\mu \alpha} \cdot \tilde{F}^{\alpha \alpha} + \frac{1}{4} \delta^0_{\alpha \beta} \tilde{F}^{2}_{\alpha \beta} \right] d^3x.
\]

adding a surface term. Notice that the first term represents the quark kinematic momentum, but this contains only the binding gluons. This tells that there are two gauge invariant quark kinematic momentums, \(-i\tilde{D}_\mu\) and \(-iD_\mu\), on top of the gauge invariant canonical momentum \(-iD_\mu\). So here we can not just say that it is the kinematic momentum that experiments measure.

The above analysis tells us the followings. First, in gauge theories there is a gauge invariant decomposition of total momentum (and spin) of a composite particle to those of the constituents, one in QED and two in QCD. But these decompositions involve the canonical momentum which may or may not be measurable by experiment. Second, if the canonical momentum is not measurable, there is no gauge invariant decomposition of total momentum (and spin) to those of constituents in the strict sense. But we still have “partial” decompositions which involve the kinematic momentum, again one in QED and two in QCD.
Now, the difference between (21) and the popular (6) is that (21) includes only the binding gluons \((n_g = 2)\) but (6) includes all gluons \((n_g = 8)\). So (6) gives the well-known prediction (with \(n_f = 5\) as usual) that about 51\% of nucleon momentum is carried by gluons \([1]\). In contrast (21) tells that only about 21\% of nucleon momentum must be carried by gluons.

Now, suppose only the canonical momentum is measurable. In this case (22) must change, and it has been proposed that (22) be replaced by

\[
P_{\mu}^{g} = \frac{n_g}{n_g + 6n_f} p_{\mu}^{tot}.
\]

This should be confirmed by an independent calculation, but suppose this is true. Then (12) which assumes that nucleons contain all gluons predicts that gluons carry about 21\% of nucleon momentum \([2]\). Notice the strange coincidence between this prediction and that of (21) based on (22). They have the same prediction, but totally different physics. If nucleons have only binding gluons, however, (19) tells that only about 6\% of momentum must be carried by gluons. Notice that the fraction of gluon momentum becomes less if nucleons has only binding gluons, for obvious reason.

Exactly the same argument applies to the nucleon spin crisis problem \([3, 16, 17]\). Here again there are three (one canonical and two kinematic) quark orbital angular momentums. Moreover, assuming that only the canonical angular momentum is measurable, we have two nucleon spin decompositions depending on which gluons are in nucleons \([7]\). And only one of them can describe the correct nucleon spin decomposition.

Independent of the details the essence of our analysis can be summarized as follows. \textit{First, there exist more than one logically acceptable gauge invariant quark and gluon momentums in QCD.} Indeed quarks have three and gluons have four such momentums, as we have shown in (6), (12), (19), and (21). This is because QCD potential always been believed that all gluons are in nucleons \([2–4, 16, 17]\). But the Abelian decomposition tells that QCD has two types of gluons, and the quark model implies that only the binding gluons are in nucleons \([8, 9, 15]\). Certainly this is against the common wisdom \([2–4, 16, 17]\). Second, we must know which gluons are in nucleons to have a correct momentum (and spin) decomposition. So far this point has completely been ignored, because it has always been believed that all gluons are in nucleons \([2–4, 16, 17]\). But the Abelian decomposition tells that QCD has two types of gluons, and the quark model implies that only the binding gluons are in nucleons \([8, 9, 15]\). Clearly this is against the common wisdom \([2–4, 16, 17]\).

We hope that our analysis will help to settle the current controversies on nucleon momentum and spin decomposition \([2–4, 16, 17]\). A detailed discussion on this and related issues will be presented elsewhere \([2, 18]\).

\section*{Acknowledgement}

The work is supported in part by National Research Foundation (Grant 2010-002-1564) of Korea and by Natural Science Foundation (Grants 10604024, 11035006, and 11075077) of China.

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