Dynamic response analysis of submarine tunnel under different wave loads

Zhiyong Ouyang, Peijie Li*, Ruofan Luo and Zhuojun Feng

College of Civil Engineering, Jiaying University, Meizhou, 510405, China

*Corresponding author’s e-mail: 52424848@qq.com

Abstract. Based on theory of Biot's dynamic consolidation and the elastic dynamics, this paper considers seabed soil and the contact effect of the tunnel, the seabed soil along the depth direction variability and viscoelastic boundary. The two-dimensional finite element model is established to compare the wave force acting on the seabed surface, which causes the internal stress of the tunnel and its dynamic response of the seabed in the first-order elliptic linear wave, cosine wave and the second order stokes wave. Results show that when the relative water depth is larger, the first-order elliptical cosine wave theory used to calculate the dynamic response of the seabed-tunnel will underestimate the seabed soil liquefaction resistance and tunnel resistance to deformation ability. And when the relative water depth is smaller, the linear wave theory to solve the dynamic response of the seabed-tunnel will overestimate the seabed soil liquefaction resistance and the ability to resist deformation of the tunnel. With the increase of L/d and T wave parameter, the first-order elliptical cosine wave and the degree of nonlinear second order stokes wave are in increase. Compared with the linear wave, the seabed soil around the tunnel of pore water pressure, displacement and the internal stress amplitude of the tunnel are obviously increasing.

1. Introduction

With the rapid development of economic construction and the acceleration of urbanization in China's coastal areas, various coastal cities are forced to get close to form urban agglomerations. Therefore, the construction of cross-river and cross-sea tunnels connecting major coastal economic areas will play a decisive role in the national or regional economic and social development. The first cross-harbour immersed tube tunnel was built in Hong Kong in 1972; the Kaohsiung subsea tunnel was built in Taiwan in 1984; the Pearl River immersed tube tunnel was built in Guangzhou in 1993, and the Yongjiang immersed tube tunnel was built in Ningbo in 1996. The completion of three immersed tube tunnels were respectively located at Changhong, Hangzhou Bay and Shanghai Outer Ring road in 21st century, which indicates that the cross-river immersed tube tunnel will become an important part of China's major infrastructure construction, and its stability will have an important impact on China's future economic development. Apart from the submarine earthquake, wave load is one of the most important factors that affect the stability of cross-river and cross-sea tunnels. At present, the calculation of seabed wave force is mostly based on the elliptical cosine theory, linear wave theory and Stokes wave theory. Linear wave theory is widely used because of its simple expression and easy solution. LAINW[1] uses finite difference method and finite element method to analyse the pressure distribution around the oil pipeline under the action of linear waves; LIUPLF[2] uses the boundary element method to analyse the wave forces on pipelines in seabed under linear wave loads. It is assumed in the document[3-5] that pipelines are elastic deformable bodies, and pore water and soil are compressible. At the same time, considering the contact
effect between soil and pipeline, the internal stress of submarine pipeline and the dynamic response of surrounding seabed under linear wave load are studied. For the offshore area, the wave propagates from the deep water area to the shallow water area. Influenced by submarine topography, depth and other factors, the wave gradually changes from symmetry to asymmetry, and becomes a wave between the oscillatory waves and the push wave. The shallower of the water depth, the more significant the wave surface deformation becomes (for waves very close to the static level, the peak value is much higher than the static level and the wave trough is very flat), which means that the wave nonlinear characteristics are stronger. Therefore, using linear wave theory to solve the interaction between seabed and structure in offshore area (shallow water area) may bring great errors to the calculation results. At present, the nonlinear wave theories used in offshore areas mainly include solitary wave theory, hyperbolic wave theory and elliptical cosine theory, and the first two are actually approximations of the latter[6]. In the study of the interaction between seabed and seabed structure, the seabed is mostly considered to be homogeneous at present. Most of the force solutions are based on linear wave theory and do not consider the influence of boundary conditions.

This paper is based on Boit's dynamic consolidation theory and elastic dynamics theory. Besides, the inertial effect of seabed soil and tunnel, the contact effect of seabed soil and tunnel, and the variability of seabed soil along depth direction and viscoelastic boundary are also considered. Thus, a two-dimensional finite element model is established to compare the internal stress of the tunnel and dynamic response of the surrounding seabed which are respectively affected by three different wave forces acting on the seabed surface. The forces are respectively calculated by first-order elliptic cosine wave theory, linear wave theory and second-order Stokes wave theory. It’s systematically discussing the influence of internal stress in tunnel and dynamic response of surrounding seabed, which cause by the ratio of wavelength to depth (L/d) and the change of T. It’s systematically discussing the influence of internal stress in tunnel and dynamic response of surrounding seabed, which cause by the ratio of wave-length to depth (L/d) and the change of T.

2. Boundary value

Figure. 1 is the schematic diagram of wave-tunnel-seabed interaction. As shown in Figure. 1, it is assumed that the thickness of the earth covering on the impermeable rigid basement is s, and the water depth is d. The pipeline whose radius is r is buried in the saturated pore seabed, and the buried depth is b. Due to space limitations, the setting of artificial boundaries in the model, the calculation of the example and the setting of the pore pressure can be referred to the literature 7.

![Figure.1 Seabed-tunnel interaction](image)

2.1 Governing equations

Due to the sedimentation and consolidation of seabed soil, the actual seabed is mostly anisotropic to a certain extent. Anisotropy includes hydraulic anisotropy and mechanical anisotropy. Hydraulic anisotropy is mainly manifested by different permeability coefficients in horizontal and vertical directions. Anisotropy of mechanical mostly shows transverse anisotropy, which means that mechanical parameters (such as elastic modulus and Poisson's ratio) are different in horizontal and vertical directions.
The anisotropy of mechanical properties considered here is limited to the relatively simple case of transverse anisotropy.

\[
C_{hh}E_{h}\frac{\partial^{2}z}{\partial x^{2}} + G_{v}\frac{\partial^{2}z}{\partial z^{2}} + (C_{vh}E_{h} + G_{v})\frac{\partial^{2}z}{\partial x\partial z} + \frac{\partial G_{v}}{\partial x}\left(\frac{\partial z}{\partial x}\right)^{2} + \frac{\partial G_{v}}{\partial z}\left(\frac{\partial z}{\partial z}\right)^{2} = \frac{\partial p}{\partial x} + \rho \frac{\partial^{2}z}{\partial t^{2}}
\]

\[
G_{v}\frac{\partial^{2}z}{\partial x^{2}} + C_{vv}\frac{\partial^{2}z}{\partial z^{2}} + (C_{hv}E_{h} + G_{v})\frac{\partial^{2}z}{\partial x\partial z} + \frac{\partial C_{vv}}{\partial x}\left(\frac{\partial z}{\partial x}\right)^{2} + \frac{\partial C_{vv}}{\partial z}\left(\frac{\partial z}{\partial z}\right)^{2} = \frac{\partial p}{\partial z} + \rho \frac{\partial^{2}z}{\partial t^{2}}
\]

\[
k_{h} \frac{\partial^{2}p}{\gamma_{w}\partial x^{2}} + \frac{k_{v} \partial^{2}p}{\gamma_{w}\partial z^{2}} - \frac{1}{g} \frac{\partial^{2}}{\partial t^{2}} \left(\frac{K_{h}}{\gamma_{w}} \frac{\partial z}{\partial x} + \frac{K_{v}}{\gamma_{w}} \frac{\partial z}{\partial z}\right) + \frac{1}{\gamma_{w}} \frac{\partial k_{v}}{\partial z} \frac{\partial z}{\partial z} - \frac{1}{\gamma_{w}} k_{v} \frac{\partial^{2}z}{\partial t^{2}} = n \beta \frac{\partial p}{\partial t} + \frac{\partial z}{\partial t}
\]

In terms of, \(\frac{\partial k_{v}}{\partial z}\) and \(\frac{\partial G_{v}}{\partial z}\) in equations (1) and (2) are due to non-uniformity of shear modulus (elastic modulus), and two of the equations (3) containing \(\frac{\partial k_{v}}{\partial z}\) are due to the non-uniformity of the permeability coefficient. If the influence of two-phase acceleration of soil skeleton and pore fluid are ignored, \(\frac{\partial^{2}z}{\partial t^{2}}\) of the formula (1) and \(\frac{\partial^{2}z}{\partial t^{2}}\) of the formula (2) and in equation (3), \(\frac{1}{g} \frac{\partial^{2}}{\partial t^{2}} \left(\frac{K_{h}}{\gamma_{w}} \frac{\partial z}{\partial x} + K_{v} \frac{\partial z}{\partial z}\right)\) and \(\frac{1}{\gamma_{w}} \frac{\partial k_{v}}{\partial z} \frac{\partial z}{\partial z}\) and other items will not exist. \(E_{h}\), \(E_{v}\) are the elastic modulus in the horizontal and vertical directions respectively; \(G_{v}\) is the shear modulus in the vertical direction; \(K_{h}\) and \(K_{v}\) are the elastic modulus in the horizontal direction and the vertical direction respectively; \(C_{hh}\), \(C_{vh}\), \(C_{hv}\) and \(C_{vv}\) represent the soil anisotropic parameters of mechanical properties.

2.2 Wave Pressure on the Surface of the Seabed

When solving the wave pressure on the surface of the seabed, it is generally assumed that water is the ideal fluid; the wave height does not decay during the propagation of the wave; the seepage of the water in the seabed satisfies Darcy’s law, and does not consider the friction between the seabed and the waves, and is regarded as a smooth boundary. Assuming that \(d\) represents water depth; \(h\) is the thickness of the seabed; \(H\) is the height of the wave; \(L\) is the wavelength; \(\omega\) is the circular frequency of the wave; and \(k\) is the wave number.

2.2.1 Linear Regular Wave Theory

When the friction between seawater and seabed soil is ignored, the vertical effective stress and shear stress on the surface of the seabed can be ignored, either. Therefore, the pore water pressure exerted by the linear wave on the surface of the bed is equal to the wave pressure caused by the wave. The formula is:

\[
p(x, t) = \frac{r_{w}H}{2 \cos(kd)} \cos(kx - \omega t)
\]

2.2.2 The Stokes Second-order Nonlinear Wave Theory

According to the Stokes second-order nonlinear wave theory, it can be concluded that the wave pressure of the finite-amplitude propulsion wave acting on the surface of the seabed is:

\[
p(x, t) = \frac{n_{w}H^{2}}{4L \sin(2kd)} + \frac{r_{w}H}{2 \cos(kd)} \cos(kx - \omega t) + \frac{3n_{w}H^{2}}{4L \sin(2kd)} \cos(2(kx - \omega t))
\]

2.2.3 First-order Approximate Elliptical Cosine Nonlinear Wave Theory

The wave front of the first-order elliptical cosine wave can be expressed by the following equation:

\[
\eta = H \left(\frac{1}{m^{2}} - 1 - \frac{E}{m^{2}K}\right) + Hcn^{2} \left[2K \left(\frac{x}{T} \right) \right]
\]

In the formula: \(cn(\cdot)\) is the Jacobian elliptical cosine function; \(x\) is the coordinate of the wave propagation direction; \(T\) is the period; \(m\) is the elliptical cosine die; \(K\) and \(E\) are the first and second types of complete elliptic integrals respectively, and the formulas are as follows:

\[
K = \int_{0}^{\pi/2} \frac{1}{\sqrt{1 - m^{2} \sin^{2} \varphi}} \varphi \, d\varphi
\]

Therefore, the wave front generated by the wave front of the first-order elliptical cosine wave on the seabed surface is:
\[ p(x, t) = H \left( \frac{1}{m^2} - 1 - \frac{e(m)}{m^2 K(m)} \right) + Hcn^2 \left[ 2K(m) \left( \frac{\xi}{L} - \frac{\tau}{T} \right) \right] \]  

\[ L \sqrt{\frac{3H}{16d^3}} = mK(m) \]  

2.3 Example Verification

Example 1: Since the first-order elliptical cosine wave theory contains the first-class and second-class complete elliptic integrals and the complex Jacobian elliptic function, there are two main methods for solving the elliptical cosine wave: the chart analysis method \cite{11} and numerical analysis methods \cite{12}, in which the chart analysis method mainly uses the calculated calculation curve to obtain the calculation parameters of the elliptical cosine wave corresponding to different periods, water depths and wave heights. The advantage is that the calculation is simple, but the table lookup time is long and the calculation accuracy is not high. When using the numerical analysis method, it can be known that the accuracy of solving the elliptic cosine wave theory depends on the value of the elliptic integral norm \cite{13}. In this paper, the direct iterative method and the arithmetic-geometric average method are used to solve the elliptical cosine wave theory. The values of the elliptic integral mode are given by graph analysis and numerical analysis in the case of known water depth, wave height and wavelength. Check the accuracy of the integral norm of Table 1. The water depth, wave height, wavelength and integral norm in the table can be replaced by formula (10) to check whether the equation is true or within the error range, the calculation shows that the accuracy of the numerical analysis method is better than that of the chart analysis method.

| Calculation parameter | Chart analysis (Integral Module (m)) | Numerical analysis (Integral Module (m)) |
|-----------------------|-------------------------------------|----------------------------------------|
| (Wave Depth, Wave Height, Wavelength (m)) | | |
| 5.7, 1,  153.045 | 0.992 | 0.9995359 |
| 5.7, 1,  129.96 | 0.989 | 0.9979738 |
| 5.7, 1,  107.445 | 0.987 | 0.9916954 |
| 5.7, 1,  82.764 | 0.958 | 0.9633329 |
| 5.7, 1,  57.285 | 0.851 | 0.8565085 |

3. Numerical calculation and analysis

Figure 2 The encryption of mesh grid and the distribution of stratified soil around tunnel

In the numerical model of this paper, the length of the seabed is 140m and the thickness is 50m. Due to the large radius of the tunnel (R=6), viscoelastic boundary conditions are set on both sides of the seabed to avoid boundary effects. In order to accurately analyze the interaction between the seabed and the
tunnel, a contact unit (μ=0.6) is set at the contact between the tunnel and the seabed, and the mesh is locally encrypted. Besides, for calculating the circumferential normal stress and shear stress of the tunnel after numerical analysis, local polar coordinates are established with the center of the tunnel as the origin, as shown in Figure. 2. Point A is $0^\circ$ and point B is $90^\circ$. Due to the sedimentation and consolidation of seabed soil, most of the actual seabed presents a certain degree of non-uniform anisotropy. It is assumed that the shear modulus and permeability coefficient of the seabed soil vary linearly with depth, and the formula is $G_3(z)=G_0\left[1-9\left(\frac{z}{h}\right)^2\right]$. The shear modulus at the bottom of the seabed is ten times that of the surface of the seabed, and the permeability coefficient is $k_3(z)=k_0\left[1+0.9\left(\frac{z}{h}\right)^2\right]$. The permeability coefficient at the bottom of the seabed is only one tenth of the surface of the seabed, as shown in Figure.2. The parameters of sandy seabed soil are as follows: the density $\rho=1700$ Kg*m$^{-3}$, reference shear modulus $G_0=11.5$MPa, the Poisson's ratio $\nu=0.35$, porosity $n=0.25$, the horizontal permeability and the vertical permeability are $k_0=0.001$ m/s. The buried depth of the tunnel is 2m, the Poisson's ratio is $\nu=0.2$, the density $\rho=2400$ Kg*m$^{-3}$, the Young's modulus is $E_0=3000$ MPa.

![Figure 3 The Pore water pressures of seabed](image)

![Figure 4 The hoop normal stress of tunnel](image)

![Figure 5 The hoop shear stress of tunnel](image)

![Figure 6 The displacement of seabed](image)

**Calculation Condition 1**: The wave parameters are as follows: $L=57.285m$, $L/d=10$, $T=8.16s$, $H=1m$, and the calculation results are shown in Figure.3 to Figure.6. It can be seen from document 9 that $L/d=10$ belongs to the case of relatively large water depth. As shown in Figure.3 to Figure.6, the linear wave and the stokes second-order wave cause the pore water pressure, displacement and internal stress of the seabed soil around the tunnel to be similar when the relative water depth is large, thus it can be seen that the nonlinear effect of stokes second-order wave is not significant at this time. However, the amplitude of pore water pressure, displacement and tunnel internal stress of seabed soil caused by the first-order elliptic cosine wave theory is larger than that of linear wave and second-order stokes wave, especially the differential value between pore water pressure and displacement of seabed soil is about 1.5 times. If the first-order elliptic cosine wave theory is used to solve the pore water pressure, displacement and internal stress of seabed soil around the tunnel in the case of relatively large water depth, it will
underestimate the anti-liquefaction capability of seabed soil and the anti-deformation capability of the tunnel, and bringing about relatively errors.

**Figure 7 The Pore water pressures of seabed**

**Figure 8 The hoop normal stress of tunnel**

**Figure 9 The hoop shear stress of tunnel**

**Figure 10 The displacement of seabed**

**Calculation Condition 2:** Under the condition that other parameters are the same as those in working condition 1, and the nonlinearity of elliptic cosine wave and the stokes second-order wave is more significant by changing wavelength and period. The parameters are as follows: L=166.44m, L/d=29.2, T=21.5s, and the numerical results are shown in Figure 7 to Figure 10. When L/d=29.2, which means that when the relative water depth is small, the results calculated by the three different wave theories are quite different. As can be seen from the following figure, the pore water pressure, displacement and internal stress amplitude of seabed soil around the tunnel calculated by the stokes second-order wave theory are the largest, while the amplitude calculated by the linear wave theory is the smallest. From Figure 7 and Figure 8, it can be seen that the pore water pressure of seabed soil around the tunnel and the amplitude of the tunnel circumferential normal stress calculated by the stokes second-order wave theory are 2.5-3.5 times that of the linear wave theory, while the calculation results working out by the first-order elliptic cosine wave theory are between the result working out by the stokes second-order wave theory and the linear wave theory. Therefore, in the case of relatively small water depth (offshore area), the pore water pressure, displacement and internal stress of seabed soil around the tunnel are solved by linear wave theory, which will overestimate the anti-liquefaction ability of the seabed soil and the deformation resistance of the tunnel. However, when the relative water depth is small, using the stokes second-order wave theory will lead to the opposite conclusion.
This paper is based on Boit's dynamic consolidation theory and elastic dynamics theory. It also considers the inertia effect of seabed soil and tunnel, the contact effect of seabed soil and tunnel, the variability of seabed soil along the depth direction and the viscoelastic boundary. Based on this, a two-dimensional finite element model is established to compare the internal stress of the tunnel and dynamic response of (2019) 022044


doi:10.1088/1755-1315/371/2/022044

4. Conclusions

This paper is based on Boit's dynamic consolidation theory and elastic dynamics theory. It also considers the inertia effect of seabed soil and tunnel, the contact effect of seabed soil and tunnel, the variability of seabed soil along the depth direction and the viscoelastic boundary. Based on this, a two-dimensional finite element model is established to compare the internal stress of the tunnel and dynamic response of
the surrounding seabed which are affected by three different wave forces acting on the seabed surface. By comparing the ratio of the wavelength of the variable parameter to the water depth (L/d) and T, the following results can be obtained:

1) In the case of relatively large water depth, using the first-order elliptical cosine wave theory to calculate the pore water pressure, displacement and internal stress of the seabed soil around the tunnel will underestimate the anti-liquefaction ability of the seabed soil and the ability of the tunnel to resist deformation, and it will bring large errors. However, calculation result of the linear wave is close to the calculation result of the stokes second-order wave.

2) In the case of relatively small water depth, using the linear wave theory to solve the pore water pressure, displacement and internal stress of seabed soil around the tunnel will overestimate the liquefaction resistance of seabed soil and the deformation resistance of the tunnel, while using the stokes second-order wave theory is the opposite.

3) When the wave parameters L/d and T increase, which means that the wave propagates from the deep water region to the offshore region, the nonlinear degree of the first-order elliptical cosine wave and the stokes second-order wave increase. Compared with the linear wave, the amplitude of pore water pressure, displacement and internal stress of the seabed soil around the tunnel increase obviously. Therefore, the wave theory should be reasonably used and the nonlinear characteristics of the wave should be considered for the tunnel engineering along the coast.

4) The calculation of the variable wave parameters L/d and T shows that the relationship between the three types of wave theory to calculate the dynamic response of pore seabed-tunnel is not unique, but depends on the combination of wave parameters.

Acknowledgements
This work was supported by the the National Natural Science Foundation of China (Grant No. 51808259); Jiaying College Science and Technology Project (2019KJY08).

References
[1] LAINW, DOMINGUEZRF, DUNLAPWA. (1974)Numerical solution for determining wave-induced pressure distribution around buried pipelines. Texas: Texas A & M University.
[2] LIU P L F, O’DONNELL T P. (1979) Wave-induced forces on buried pipelines in permeable seabed. Proceedings of the Civil Engineering in the Oceans IV Conference:111-121.
[3] Maotian Luan, Peng Qu, Ying Guo etc. (2007) Numerical analysis of dynamic response of saturated porous seabed-pipeline interaction under wave loading. The Ocean Engineering, 25(2) : 43-51.
[4] Haifeng Deng, Zhongtao Wang, Peng LIU. (2014)Study on seabed-pipeline interaction under random wave loading. Journal of Water Resources and Architectural Engineering,12 (4) : 43-49.
[5] Liu Jing, Minggao Li.( 2009) Study on Interaction between Submarine Buried Pipeline and Porous Elastic Seabed. Water Resources and Power, 27(5) :159-162.
[6] Rujian Ma, Zhonggong Jia, Huili Xing. (1999)Shallow water wave forces acting on the structure calculation and experimental study. China Offshore Oil and Gas(Engineering), 11 (6) :23-27.
[7] Zhiyong Ouyang, Jie Cui, Yadong Li, Dandan Yuan. (2016) Interaction Between non-homogeneous Anisotropic Porous Seabed and Tunnel Under the Action of Wave[J]. Electronic Journal of Geotechnical Engineering,Vol.21, Bund. 16, 5371-5388
[8] Leina Hua. (2009)Dynamic Interaction among Wave, Structure and Seabed. Beijing: Tsinghua University.
[9] Daihong Qiu. (1986)Wave theory and its application in Engineering .BeiJing: Higher Education
[10] ZHANG Liaojun, ZHANG Huixing, WANG Dasheng etc. (2008) The application of artificial viscous-spring boundary in ADINA. World Earthquake Engineering, 24(1): 50-60.

[11] Wiegel R L. (1960) A presentation of cnoidal wave theory for practical application. Fluid Mech, 7: 273-286.

[12] Jialong Gu, Xianrong Shen. (1989) The elliptical cosine wave numerical calculation and its features. Ocean Engineering, 1: 30-40.

[13] Taojun Hu, Yincan Ye. (2007) Solution of the wave forces acting on the submarine slope stability analysis[J]. Journal of Ocean, 2(6): 120-125.