Independent Vector Extraction for Joint Blind Source Separation and Dereverberation

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Abstract—We address a blind source separation (BSS) problem in a noisy reverberant environment in which the number of microphones $M$ is greater than the number of sources of interest, and the other noise components can be approximated as stationary and Gaussian distributed. Conventional BSS algorithms for the optimization of a multi-input multi-output convolutional beamformer have suffered from a huge computational cost when $M$ is large. We here propose a computationally efficient method that integrates a weighted prediction error (WPE) dereverberation method and a fast BSS method called independent vector extraction (IVE), which has been developed for less reverberant environments. We show that the optimization problem of the new method can be reduced to that of IVE by exploiting the stationary condition, which makes the optimization easy to handle and computationally efficient. An experiment of speech signal separation shows that, compared to a conventional method that integrates WPE and independent vector analysis, our proposed algorithm has significantly faster convergence speeds while maintaining its separation performance.

Index Terms—Blind source separation, dereverberation, independent vector analysis, block coordinate descent method

I. INTRODUCTION

When multiple speech signals are observed by distant microphones (e.g., in a conference room), they are contaminated with reverberation and background noise. The problem of extracting each speech signal and removing the reverberation and background noise from only the observed signal, which is a linear convolutive mixture, is called (convolutive) blind source separation (BSS) [1]–[3]. Here, we consider BSE in the short-term Fourier transform (STFT) domain under the following two conditions:

- The reverberation time ($RT_{60}$) is larger than the frame length of the STFT, and the mixture should be treated as a convolutive mixture in the STFT domain as well. In the experiment carried out in Section V, $RT_{60}$ in an office room is 780 ms, and the frame length is set to 128 ms.
- The number of microphones $M$ is greater than the number of speech signals $K$ and there can be background noise.

To cope with reverberation, one can apply a dereverberation method [4] such as weighted prediction error (WPE) [5]–[7] as preprocessing of BSE for instantaneous mixtures (called BSE-inst in this paper). We then apply some BSE-inst methods such as independent vector analysis (IVA) [8]–[10] and independent vector extraction (IVE) [11]–[17] developed for less reverberant environments, to extract $K$ speech signals. Such a cascade configuration of WPE and IVA/IVE has a low computational cost, but the WPE dereverberation filter is estimated without considering the separation attained by IVA/IVE following WPE.

To jointly optimize the WPE dereverberation and separation filters through a unified optimization, methods that integrate WPE and several BSE-inst methods have been proposed [6], [7], [18]–[20], and it has been reported that these methods can give higher separation performance than the cascade configuration of WPE and BSE-inst (see, e.g., [18]). However, the computational cost of optimizing both WPE and BSE-inst models becomes huge when $M$ is large.

To reduce the computational cost of the conventional joint optimization methods while maintaining their separation performance, we propose a new BSE method called IVE for convolutive mixtures (IVE-conv), which integrates WPE and IVE (Section III). We show that the IVE-conv optimization problem can be reduced to the IVE optimization problem by exploiting the stationary condition, and this reduction is not computationally intensive due to the stationary Gaussian approximation for the background noise (Section IV-A). The IVE optimization problem can be solved fast [13]–[17], and so can the IVE-conv optimization problem (Section IV-B). We also propose another new algorithm for IVE-conv that alternately optimizes WPE and IVE (Section IV-C). Similar algorithms have already been developed in [6]. [7], but our proposed one significantly reduces the computational time complexity of the conventional ones (see Table I). In a numerical experiment in which two speech signals are extracted from mixtures, we show the effectiveness of our new approach (Section V).

II. BLIND SOURCE EXTRACTION PROBLEM

Let $M$ be the number of microphones. Suppose that an observed mixture $x := \{x(f,t)\}_{f,t} \subset \mathbb{C}^M$ in the time-frequency domain is a convolutive mixture of $K$ nonstationary source signals $N_x := M - K$ background noise signals:

$$x(f,t) = \sum_{\tau=0}^{N_x} \sum_{i=1}^{K} a_i(f,\tau) s_i(f,t-\tau) + A_x(f,\tau) z(f,t-\tau)$$

$$a_i(f,\tau) \in \mathbb{C}^M, \quad s_i(f,t) \in \mathbb{C}, \quad i \in \{1, \ldots, K\},$$

$$A_x(f,\tau) \in \mathbb{C}^{M \times N_x}, \quad z(f,t) \in \mathbb{C}^{N_x}.$$  

(1) (2)

Here, $f = 1, \ldots, F$ and $t = 1, \ldots, T$ denote the frequency bin and time-frame indexes, respectively. Also, $s_i(f,t) \in \mathbb{C}$ and $z(f,t) \in \mathbb{C}^{N_x}$ are the signals of the target source $i = 1, \ldots, K$ and the background noises, respectively. $\{a_i(f,\tau)\}_{\tau=0}^{N_x}$ and

$1$ The assumption that the dimension of the noise signal is $M - K$ concerns the rigorous development of efficient algorithms and can be violated to some extent when applied in practice (see numerical experiments in Section V).
\{A_k(f, \tau)\}_{k=0}^{N_r-1} \) are the acoustic transfer functions (or FIRs) for the corresponding sources, where \( N_r + 1 \) is the length of the FIRs. The BSE problem addressed in this paper is defined as the problem of estimating the sources of interest, i.e., \( \{s_i(t)\}_{i,f,t} \). We assume that \( K \) is given and the background noises are more stationary than the sources of interest.

### III. Probabilistic model

We present a model of our proposed IVE-conv to solve BSE. Suppose for each frequency bin \( f = 1, \ldots, F \) that there exists a convolutional filter \( W(f) \in \mathbb{C}^{(M+L) \times M} \) with \( L := M|\Delta| \) and \( \Delta = \{\tau_1, \ldots, \tau_{|\Delta|}\} \subset \mathbb{N}_{>0} \) satisfying

\[
s_i(f, t) = \hat{w}_i(f, t) \hat{x}(f, t) \in \mathbb{C}, \quad i \in \{1, \ldots, K\},
\]

where \( \hat{x}(f, t) \) is the vector obtained by stacking \( x(f, t) \) and the past observed signals \( \{x(f, t - \delta)\}_{\delta \in \Delta} \). Note that \( \Delta \subset \mathbb{N}_{>0} \) determines the length of the filters, which is a hyperparameter.

We also assume that the original source signals are mutually independent and that the target source (resp. noise) signals obey time-dependent (resp. time-independent) complex Gaussian distributions in the same way as in IVE [11]–[17]:

\[
\begin{align*}
\hat{s}_i(t) &:= \left[ s_i(1,t), \ldots, s_i(F,t) \right]^T \in \mathbb{C}^F, \\
\hat{s}_i(t) &\sim \mathcal{CN}(0_F, \hat{v}_i(t) I_F), \quad \hat{v}_i(t) \in \mathbb{R}_{>0}, \\
\hat{z}(f, t) &\sim \mathcal{CN}(0_N, \Omega(f)), \quad \Omega(f) \in \mathbb{C}^{N_s \times N_s}, \\
\{\hat{s}_i(t), \hat{z}(f, t)\}_{i,f,t} &\text{ are mutually independent}.
\end{align*}
\]

Here, \( I_d \in \mathbb{C}^{d \times d} \) is the identity matrix, \( O_d \in \mathbb{C}^d \) is the zero vector, and \( S_k^{d \times d} \) denotes the set of all Hermitian positive definite matrices of size \( d \times d \). As we will see, assumption [9] that the noise signal is stationary Gaussian distributed is essential for developing computationally efficient algorithms.

The model of IVE-conv is defined by [3]–[10]. In the model, the variables \( \hat{W} := \{W(f)\}_f, \hat{v} := \{v_i(t)\}_{i,t}, \text{ and } \Omega := \{\Omega(f)\}_f \), which can be optimized by solving the following problem of minimizing the negative log-likelihood \( \hat{g}(\hat{W}, \hat{v}, \Omega) \) (see, e.g., [20] for the derivation):

\[
\begin{align*}
(\hat{W}, \hat{v}, \hat{\Omega}) = \arg \min \hat{g}(\hat{W}, \hat{v}, \hat{\Omega}), \\
\hat{g}(\hat{W}, \hat{v}, \hat{\Omega}) &= \frac{1}{T} \sum_{t=1}^{T} \left[ \sum_{i=1}^{K} \frac{|s_i(t)|^2}{v_i(t)} + F \log \hat{v}_i(t) \right] \\
+ \frac{1}{T} \sum_{f=1}^{F} \sum_{t=1}^{T} \left[ \frac{1}{\hat{\Omega}(f)} \hat{z}(f, t) \hat{z}(f, t)^H \right] + \log \det \hat{\Omega}(f) \\
- \frac{1}{T} \sum_{f=1}^{F} \log |\det W(f)|^2.
\end{align*}
\]

Here, \( W(f) \in \mathbb{C}^{M \times M} \) is defined as the upper \( M \times M \) submatrix of \( W(f) \) as shown in Eqs. (12) and (13) below. It is known that \( W(f) \) serves as the separation matrix of ICA (see Remark 1 for details).

**Remark 1.** As mentioned in [21], [22], the convolutional filter \( W(f) \) can be decomposed into the prediction matrix \( G(f) \in \mathbb{C}^{L \times M} \) of the dereverberation method WPE [6], [7] and the ICA separation matrix \( W(f) \in \mathbb{C}^{M \times M} \) in the following way:

\[
\hat{W}(f) = \begin{bmatrix} I_M & -G(f) \end{bmatrix} W(f) \in \mathbb{C}^{(M+L) \times M}.
\]

Therefore, the proposed IVE-conv can be understood as an integration of WPE [1], [7] and IVE [11]–[17]. If we replace IVE with ICA, IVA, or independent low-rank matrix analysis (ILRMA) [23], then the IVE-conv turns out to be the method that integrates WPE with ICA [6], [7], WPE with IVA (IVA-conv) [18], or WPE with ILRMA [19], [20], respectively. In this sense, the novelty of the IVE-conv model might seem limited. However, if \( M \) gets large, computationally efficient algorithms can be developed only for IVE-conv, which is our main contribution in this letter (see Table I for the computational time complexity of each method).

### IV. Optimization algorithm

To obtain a local optimal solution of (11), two new block coordinate descent (BCD) algorithms will be developed in Sections IV-B and IV-C. The proposed BCDs are summarized in Table II with a comparison to conventional methods.

All the algorithms shown in Table II update \( v \) and \( (\hat{W}, \hat{\Omega}) \) alternately. When \( (\hat{W}, \hat{\Omega}) \) is kept fixed, \( v \) can be globally optimized as \( v_i(t) = \frac{1}{T} \sum_{t=1}^{T} s_i(t) \).

In what follows, we will develop two BCD algorithms to optimize \( (\hat{W}, \hat{\Omega}) \) while keeping \( v \) fixed. Because this subproblem can be addressed independently for each frequency bin, we focus only on optimizing \( \hat{W}(f) \) and \( \Omega(f) \), and the frequency bin index \( f \) is dropped off to ease the notation. In this case, objective function \( \hat{g}(\hat{W}, \hat{v}, \Omega) \) can be expressed as

\[
\hat{g}(\hat{W}, \hat{v}, \Omega) = \frac{1}{T} \sum_{t=1}^{T} \left[ \frac{1}{\Omega(t)} \hat{v}(t)^2 + \log \det \Omega(t) \right] - \frac{1}{T} \sum_{t=1}^{T} \log |\det W(f)|^2,
\]

where we define \( v_i(t) = 1 \) for every \( t = 1, \ldots, T \).

Interestingly, if \( L = 0 \), then this objective function has the same form as the counterparts of ICA, IVA, and IVE for instantaneous mixtures, which has been discussed extensively in the literature [13]–[17], [20]–[24]. For \( L \geq 1, K = M, \text{ and } N_s = 0 \), the problem has been discussed explicitly in [18], [20] and implicitly in [6], [7], [19]. In what follows, we denote the submatrices of \( W \) and \( \hat{R}_i, i \in \{1, \ldots, K, z\} \) as

\[
\hat{W} = \begin{bmatrix} W_1 & \cdots & W_K \end{bmatrix}, \quad \hat{W}_z = \begin{bmatrix} W_1 & \cdots & w_1 \end{bmatrix}, \quad \hat{R}_i = \begin{bmatrix} \hat{W}_1 & \cdots & \hat{R}_i \end{bmatrix} \in \mathbb{C}^{M \times L}, \quad \hat{R}_i = \begin{bmatrix} \hat{W}_1 & \cdots & \hat{R}_i \end{bmatrix} \in \mathbb{C}^{M \times L}.
\]

This letter is based on our work [24] reported in a domestic workshop in which an algorithm similar to but less efficient than Algorithm 1 (proposed in Section IV-B) was first presented. Recently, as follow-up research of our previous work [24], a method has been developed [25] that replaces the IVE-conv spectrum model [1] with a model using nonnegative matrix factorization (NMF) [26]–[28]. In contrast, here we develop a more efficient Algorithm 1 in a rigorous way by providing new insight into the IVE-conv optimization problem in Section IV-A. In addition, Algorithm 2 proposed in Section IV-C is completely new.
A. Reduction from IVE-conv to IVE

Before developing the algorithms, we show that the problem of minimizing \( \hat{g} \) with respect to \( \bar{W} \) and \( \Omega \) (when \( v \) is remained fixed), i.e.,

\[
(W, \Omega) \in \text{argmin}_{W, \Omega} g(W, \Omega, v),
\]

(14)

can be reduced to problem (17) below that has been addressed in the study of IVE [13]–[17].

Every optimal \( \bar{W} \) (the lower part of \( W \)) in problem (14) satisfies the stationary condition [15], which is computed as

\[
\frac{\partial \hat{g}}{\partial w_i} = 0_L \quad \iff \quad P_i \bar{w}_i + \hat{R}_i \bar{w}_i = 0_L \in \mathbb{C}^L,
\]

\[
\iff \quad \bar{w}_i = -\hat{R}_i^{-1} P_i \bar{w}_i \in \mathbb{C}^L,
\]

(15)

\[
\frac{\partial \hat{g}}{\partial W_z} = 0 \quad \iff \quad P_z \bar{W}_z + \hat{R}_z \bar{W}_z = 0 \in \mathbb{C}^L \times N_z,
\]

\[
\iff \quad \bar{W}_z = -\hat{R}_z^{-1} P_z \bar{W}_z \in \mathbb{C}^L \times N_z,
\]

(16)

where * denotes the element-wise conjugate. Eqs. (15) and (16) imply that the upper part \( W \) of the variable \( \bar{W} \) is a function of \( W \) and that the variable \( \bar{W} \) can be removed from \( \hat{g} \) by substituting (15) and (16).

In other words, problem (14) is equivalent to the following problem through (15) and (16):

\[
(W, \Omega) \in \text{argmin}_{W, \Omega} g(W, \Omega, v),
\]

(17)

\[
g(W, \Omega) = \sum_{i=1}^{K} w_i^h V_i w_i^h + \text{tr}(\bar{W}_z^h V_z \bar{W}_z \Omega^{-1})
\]

\[
\quad + \log \det \Omega - \log |\det W|^2,
\]

(18)

\[
V_i := R_i - P_i^h \hat{R}_i^{-1} P_i \in S_{++}^M, \quad i \in \{1, \ldots, K, z\}.
\]

Since problem (17) is nothing but the problem addressed in the study of IVE, we can directly apply efficient algorithms that have already been developed for IVE [13]–[17]. Our new algorithm developed in Section IV-B is based on this observation.

B. Algorithm 1: Update each convolutional filter one by one

To solve problem (14), we propose a cyclic BCD algorithm that updates \( \bar{w}_1 \rightarrow (W_z, \Omega) \rightarrow \cdots \rightarrow \bar{w}_K \rightarrow (W_z, \Omega) \) one by one by solving the following subproblems:

\[
\bar{w}_i \in \text{argmin}_{\bar{w}_i} \hat{g}(\bar{w}_1, \ldots, \bar{w}_K, \bar{W}_z, \Omega, v),
\]

(20)

\[
\bar{W}_z, \Omega \in \text{argmin}_{\bar{W}_z, \Omega} \hat{g}(\bar{w}_1, \ldots, \bar{w}_K, \bar{W}_z, \Omega, v).
\]

(21)

From the observation given in Section IV-A, these subproblems can be equivalently transformed to

\[
\bar{w}_i \in \text{argmin}_{\bar{w}_i} g_i(\bar{w}_i) \equiv \bar{w}_i^h V_i \bar{w}_i - 2 \log |\det W_z|,
\]

(22)

\[
(W_z, \Omega) \in \text{argmin}_{W_z, \Omega} g_z(W_z, \Omega),
\]

(23)

\[
g_z(W_z, \Omega) = \text{tr}(W_z^h V_z W_z \Omega^{-1}) + \log \det \Omega - \log |\det W|^2
\]

through (15) and (16), respectively. Here, \( V_i \) and \( v_i \) are defined by (19). As shown in (34), problem (22) can be solved as

\[
u_i \leftarrow (W_i^h V_i)^{-1} e_i \in \mathbb{C}^M,
\]

(24)

\[
w_i \leftarrow u_i (u_i^h V_i u_i)^{-\frac{1}{2}} \in \mathbb{C}^M,
\]

(25)

where \( e_i \) is the i-th column of \( I_M \). On the other hand, as shown in [16] Proposition 4, problem (23) can be solved as

\[
W_z \leftarrow \left[ (W_z^h V_z E_z)^{-1} (W_z^h V_z E_z) \right] \in \mathbb{C}^{M \times N_z},
\]

\[
\Omega \leftarrow W_z^h V_z W_z \in S_{++}^{N_z},
\]

(26)

where \( \Omega := [w_1, \ldots, w_K] \in \mathbb{C}^{M \times K} \), \( E_z \in \mathbb{C}^{M \times K} \) is the first \( K \) columns of \( I_M \), and \( E_z \in \mathbb{C}^{M \times N_z} \) is the last \( N_z \) columns of \( I_M \), i.e., \([E_z]_z = I_M\).

Remark 2. Note that the update formula for \( w_i \), i.e., (15), (19), and (25), has already been developed in our previous paper [18], [24] in a different manner. In this subsection, we reveal that it can also be developed by exploiting the stationary condition of the problem. The update formula for \( W_z \), i.e., (16), (19), (26), and (27), is newly developed in this subsection.

C. Algorithm 2: Alternate update of WPE and ICA

In Remark 1, we recalled that convolutional filter \( \bar{W} \) can be decomposed into WPE prediction matrix \( G \) and ICA separation matrix \( W \) for instantaneous mixtures. Here, we develop a new cyclic BCD algorithm that updates \( G \rightarrow w_1 \rightarrow W_z \rightarrow \cdots \rightarrow w_K \rightarrow W_z \rightarrow \cdots \rightarrow w_K \rightarrow W_z \) one by one by solving the following subproblems:

\[
G \in \text{argmin}_{G} \hat{g}(G, w_1, \ldots, w_K, W_z, \Omega, v),
\]

(28)

\[
w_i \in \text{argmin}_{w_i} \hat{g}(G, w_1, \ldots, w_K, W_z, \Omega, v),
\]

(29)

\[
(W_z, \Omega) \in \text{argmin}_{W_z, \Omega} \hat{g}(G, w_1, \ldots, w_K, W_z, \Omega, v).
\]

(30)

When \( K = M \) and there are no noise components, problems (28) and (29) have already been discussed in [6], [7], [18], [19]. However, the conventional algorithms to solve (28), which were developed in [6], [7], [18], [19], suffer from huge computational cost as shown in Table IV. We thus propose a more computationally efficient algorithm.

1) Algorithm to solve problems (29) and (30): We first explain how to solve problems (29) and (30). By substituting Eqs. (3), (4), and (12) into objective function \( \hat{g} \), these problems can be simply expressed as problems (22) and (23), respectively, except that \( V_i \) is replaced by the following \( V'_i \):

\[
V'_i = \frac{1}{2} \sum_{t=1}^{T} u(t) y(t) \in S_{++}^M, \quad i \in \{1, \ldots, K, z\},
\]

\[
y(t) = \left[ I_M^h \bar{G} \right] x(t) \in \mathbb{C}^M.
\]

Here, \( y(t) \) is the signal dereverberated by WPE. Thus, in the same way as in the previous subsection, problem (29) can be solved as (24)–(25), where \( \bar{V}_i \) is replaced by \( V'_i \). Also, problem (30) can be solved as (26)–(27), where \( \bar{V}_i \) is replaced by \( V'_i \). 2) Algorithm to solve problem (28): We next propose an algorithm to solve (28) with less computational time complexity than conventional ones. Every optimal \( G \in \mathbb{C}^{L \times M} \) of problem (28) (when \( W, \Omega, \) and \( v \) are kept fixed) satisfies the stationary condition, which can be computed as

\[
O_{L,M} = \frac{\partial \hat{g}}{\partial G} = -\frac{\partial \hat{g}}{\partial W} \bigg|_{W = -\bar{G} W^h},
\]

(31)
The oracle spatial images were obtained by truncating the RIRs at 32 ms (i.e., the points after 32 ms were replaced by zeros). We used the notations $W = [w_{K+1}, \ldots, w_M] \in \mathbb{C}^{M \times (M-K)}$ and $W = [w_{K+1}, \ldots, w_M] \in \mathbb{C}^{M \times (M-K)}$.

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The generated mixtures consisted of $K = 2$ speech signals and six noise signals randomly chosen from the above dataset. The SNR of each mixture was adjusted to $\text{SNR} = 10 \log_{10} \frac{\lambda_2^{(s)} + \lambda_2^{(n)}}{\lambda_2^{(s)} + \lambda_2^{(n)}}/2 = 5$ [dB], where $\lambda_2^{(s)}$ and $\lambda_2^{(n)}$ denote the sample variances of the $i$-th speech signal ($i = 1, 2$) and the $j$-th noise signal ($j = 1, \ldots, 6$).

Criteria: Using museval [39], we measured the signal-to-distortion ratio (SDR) [40] between the separated and oracle spatial images of the speech signals at the first microphone. The oracle spatial images were obtained by truncating the RIRs at 32 ms (i.e., the points after 32 ms were replaced by zeros) and convolving them with the speech signals.

Conditions: For all methods, we initialized the convolutional filter as $W(f) = -I_M$ and $W(f) = G(f) = O$. The sampling rate was 16 kHz, the frame length was 2048 (128 ms), and the frame shift was 512 (32 ms).

V. EXPERIMENT

In this numerical experiment, we evaluated the signal extraction and runtime performance of the four methods described in Table I.

Dataset: We generated synthesized convolutive noisy mixtures of two speech signals. We obtained speech signals from the test set of the TIMIT corpus [36] and concatenated them so that the length of each signal exceeded 10 seconds. We used the test set of the TIMIT corpus [36] and concatenated them to reach a length exceeding 10 seconds.

The generated mixtures consisted of $K = 2$ speech signals and six noise signals randomly chosen from the above dataset. The SNR of each mixture was adjusted to $\text{SNR} = 10 \log_{10} \frac{\lambda_2^{(s)} + \lambda_2^{(n)}}{\lambda_2^{(s)} + \lambda_2^{(n)}}/2 = 5$ [dB], where $\lambda_2^{(s)}$ and $\lambda_2^{(n)}$ denote the sample variances of the $i$-th speech signal ($i = 1, 2$) and the $j$-th noise signal ($j = 1, \ldots, 6$).

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Conditions: For all methods, we initialized the convolutional filter as $W(f) = -I_M$ and $W(f) = G(f) = O$. The sampling rate was 16 kHz, the frame length was 2048 (128 ms), and the frame shift was 512 (32 ms).

VI. CONCLUSION

To achieve joint source separation and dereverberation with a small computational cost, we proposed IVE-conv, which is an integration of IVE and WPE. We also developed two efficient BCD algorithms for optimizing IVE-conv. The experimental results showed that IVE-conv yields significantly faster convergence speed than the integration of IVA and WPE while maintaining its separation performance.
