Damped rotational vibrations of a long cylinder with a disk in the air flow

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Abstract. We consider the drag and the damped rotational oscillations of a cylinder. The cylinder length-to-diameter ratio is nine. At the upstream end of the cylinder, a disk is mounted coaxially on the leg. The disk reduces the drag of the cylinder. The effect of the disk on damping of oscillations is described by aerodynamic rotational derivatives. Experiment in the wind tunnel is carried out with the flow past a cylinder fixed on an elastic spring suspension. In experiment, the diameter of the coaxial disk, position of axis of rotation and the gap between the disk and the cylinder vary. Deflected from the equilibrium position, the cylinder performs damped oscillations. The dependence of the amplitude of angular oscillations on time is determined by the tensometric method. Aerodynamic coefficients of rotational derivatives are found.

1. Introduction
It is known that the disk mounted coaxially at the front end of the cylinder significantly reduces the drag [1]. However, the oscillations of cylinders with disks in the air flow are little studied. An exception is the study of rotational self-oscillations and damping oscillations of elastically fixed cylinders of small elongation with disks [2, 3]. These studies [2, 3] refer to cylinders whose aspect ratio (the ratio of length $L$ to diameter $D$) is equal to two. Elastically mounted cylinders of such aspect ratio without disks perform the steady-state angular oscillations in the air flow. The same cylinder with the disk can produce damped oscillations. In this paper, we consider a cylinder with aspect ratio 9. The effect of flow on damped oscillations is usually characterized by aerodynamic derivatives. To determine the aerodynamic derivatives, the methods of free or forced oscillations are used [4, 5, 6, 7]. We use a simple method of free oscillations.

2. Experimental method
The experiments were carried out at the wind tunnel AT-12 of Saint Petersburg University. Wind tunnel has an open test section. The diameter of the outlet part of the nozzle is 1.5 m. The wind tunnel is equipped with three-component balances with wire suspension. The same suspension was used in the experiment with cylinder oscillations. The suspension allows the cylinder to turn only around an axis perpendicular to the axis of the cylinder and the vector of the average velocity of the stream. The axis of rotation is located at a distance $x_a$ from the center of the cylinder. If the axis is shifted from the center upstream then $x_a > 0$. The diameter of the cylinder $D = 0.1$ m. Downstream, at a distance of 0.3 m from the axis of cylinder rotation, wires are attached to the top and bottom of the cylinder. The wires are connected to springs
located above and below the air jet. In the equilibrium position, the wires and the springs hold the cylinder horizontally (angle of attack $\alpha = 0$). One can see the scheme of the experiment in [8].

The tension of the lower spring is measured with semiconductor strain gauge. The output voltage from strain gauge is applied to a Velleman-PCS500A PC oscilloscope, which is connected to a computer. Digital signal is recorded in a file. During experiment the cylinder was deflected from the equilibrium position by an angle $\alpha_0$, then released. Oscillations are damped in all experiments. The time of single record is equal to 17 seconds. The spring tension values are read at a frequency of 100 Hz.

Calibration of the device is performed by suspending a known load to the cylinder at the point where the wires are attached. Then the angle of attack $\alpha$ and the change in the strain gauge readouts are determined. The reduced spring constant is also determined.

In our experiments, we vary the distance $x_a$, the diameter of the disk $d$ and the gap $g$ between the cylinder and the disk.

3. Mathematical model

The equation of motion of the cylinder is:

$$I_z \frac{d^2 \alpha}{dt^2} + r \frac{d \alpha}{dt} + k \alpha = (m_z^\omega + m_z^\alpha) \frac{q s L^2}{v} \frac{d \alpha}{dt} + m_z^\alpha q s L \alpha, \quad (1)$$

where $I_z$ is the moment of inertia, $\alpha$ is the angle of attack, $r$ is the friction coefficient, $q$ is the velocity pressure, $\rho$ is the air density, $k$ is the spring constant, $s$ is the characteristic area, $v$ is the air velocity, $m_z^\omega$, $m_z^\alpha$ and $m_z^\delta$ are the aerodynamic derivatives. In the right part of equation (1) there are terms that correspond to aerodynamic moment. Equation (1) can be simplified:

$$\frac{d^2 \alpha}{dt^2} + (\omega^2 - \Omega) \alpha = \frac{d \alpha}{dt}, \quad (2)$$

where

$$\omega^2 = k/I_z, \quad \Omega = m_z^\alpha q s L/I_z, \quad \mu = (m_z^\omega + m_z^\alpha) \frac{q s L^2}{v I_z} - r/I_z.$$ 

Let $\alpha = \alpha_0 \exp(\lambda t)$ be the solution of equation (2), where $\lambda = n + ip$ is complex value. Substituting the expression for $\alpha$ in equation (1) we get:

$$n = \mu/2, \quad p^2 = \omega^2 - \Omega - \mu^2/4. \quad (3)$$

Substituting the expression for $\mu$ in the first equation of system (3), we get that $n$ depends linearly on the flow velocity: $n = a + bv$. The coefficients $a$ and $b$ are calculated using the least squares method. Sum of rotational derivatives $m_z^\omega + m_z^\delta$ can be written as:

$$m_z^\omega + m_z^\delta = 4b I_z \frac{q s L^2}{\rho s \omega^2 L^2}. \quad (4)$$

One can write the expression for the coefficient of friction $r$:

$$r = 2 I_z a = \frac{2k a}{\omega^2}.$$ 

In our experiment, angular oscillations were studied. The angular velocity $\omega$ is equal to the derivative of the angle of attack. In our experiment, only the sum of the rotational derivatives $m_z^\omega + m_z^\delta$ can be determined. To find the values of each of the terms, you need to perform
another experiment. For example, it may be an experiment with translational oscillations in which the angular velocity is zero, but only the angle of attack of the body changes [9].

In the expression (4), the frequency of oscillations \( \omega \) is determined in experiments with damped vibrations when the wind tunnel fan is turned out. We assume that the amplitude \( A \) of the oscillations changes so slowly that during one period of oscillation, this change can be ignored in comparison with the amplitude. The result of measuring of the angle of attack \( \alpha_i \) at time \( t_i \) is expressed by the formula:

\[
\alpha_i = B \cos(pt_i) + C \sin(pt_i) + E + \xi_i,
\]

where \( \xi_i \) is a random variable; \( B, C \) and \( E \) are constants. Variable \( \xi_i \) corresponds to the measurement error. Let we have \( i = 1, 2, \ldots, N \). The parameters \( B, C \) and \( E \) can be determined using the least squares method. The minimum residual sum of squares is \( \sum_{i=1}^{N} \xi_i^2 \). Parameters \( B, C \) and \( E \) can be obtained by solving a system of linear equations:

\[
B \sum_{i=1}^{N} \cos^2 pt_i + C \sum_{i=1}^{N} \cos pt_i \sin pt_i + E \sum_{i=1}^{N} \cos pt_i = \sum_{i=1}^{N} \alpha_i \cos pt_i,
\]

\[
B \sum_{i=1}^{N} \cos pt_i \sin pt_i + C \sum_{i=1}^{N} \sin^2 pt_i + E \sum_{i=1}^{N} \sin pt_i t_i = \sum_{i=1}^{N} \alpha_i \sin pt_i,
\]

\[
B \sum_{i=1}^{N} \cos pt_i + C \sum_{i=1}^{N} \sin pt_i + EN = \sum_{i=1}^{N} \alpha_i.
\]

We selected a time interval which is equal to the oscillation period. If the number \( N \) is large, then the system of equations is simplified because several sums are close to zero. The oscillation amplitude \( A \) is determined by the formula \( A = (B^2 + C^2)^{1/2} \).

4. Results

Table 1 shows the difference between the drag coefficient of a cylinder with a disk and a cylinder without a disk \( C_D - C_{D0} \).

**Table 1.** Difference between drag coefficients of the cylinder with the disk and the drag of the cylinder without disk.

| \( d/D \) | \( g/D = 0.5 \) | \( g/D = 0.6 \) | \( g/D = 0.7 \) | \( g/D = 0.8 \) |
|---------|-------------|-------------|-------------|-------------|
| 1.25    | 0.63 ± 0.04 | 0.61 ± 0.04 | 0.62 ± 0.04 | 0.62 ± 0.04 |
| 1.0     | 0.03 ± 0.04 | 0.0 ± 0.04  | 0.03 ± 0.04 | 0.03 ± 0.04 |
| 0.875   | -0.31 ± 0.04| -0.32 ± 0.04| -0.34 ± 0.04| -0.31 ± 0.04|
| 0.75    | -0.50 ± 0.04| -0.48 ± 0.04| -0.48 ± 0.04| -0.44 ± 0.04|
| 0.625   | -0.53 ± 0.04| -0.51 ± 0.04| -0.49 ± 0.04| -0.40 ± 0.04|

If \( d/D = 1.25 \) then the drag of cylinder \( C_D > C_{D0} \). One can see that presence of the disk with \( d/D < 1 \) reduces the drag coefficient. The drag of a cylinder with a disk diameter \( d = 0.875D \) is less than that of a cylinder without a disk. Reducing the disk diameter down to \( d = 0.75D \) leads to a further reduction in drag.

Table 2 shows the values of aerodynamic derivatives \( m_x^\omega + m_z^\alpha \) calculated using the formula (4).
dependence the oscillation frequency on the square of the flow rate is close to linear function. The dependence of \( q \) does not exceed the thousandth part of \( m \).

Thus, the coefficient \( m \) of a cylinder without a disk at \( z = 0 \) is equal to 0.50 ± 0.06 for \( x_a = 0 \). Disks whose diameter are greater than or equal to the diameter of the cylinder do not have big effect on the damping of the cylinder oscillations. For a fixed gap between the disk and the cylinder, the dependencies of \( m \) on the disk diameter \( d \) have a maximum. Coefficient \( m \) of a cylinder without disk is equal to −0.88 ± 0.25 for \( x_a = 3 \).

From the dependence of the oscillation frequency on the air velocity we can determine another coefficient \( m \). We use the second equation of the system (3). The last term in the right part does not exceed the thousandth part of \( p^2 \). Therefore, this term can be ignored. The second term of the right part is proportional to the velocity pressure \( q \) which is proportional to the square of the velocity of the incoming flow. Experimental data confirm this. The dependence of the oscillation frequency on the square of the flow rate is close to linear function.

Thus, the coefficient \( m \) can be determined by the slope of the linear approximating the dependence \( p^2(v^2) \). The coefficient values are shown in Table 3. Approximation is carried out by the method of least squares.

### Table 2. Rotational derivatives \( m_{x}^{\alpha} + m_{z}^{\alpha} \) of the cylinder with the disk.

| \( x_a/D \) | \( d/D \) | \( g/D = 0.5 \) | \( g/D = 0.6 \) | \( g/D = 0.7 \) | \( g/D = 0.8 \) |
|----------------|--------|----------------|----------------|----------------|----------------|
| 0              | 1.25   | −0.48 ± 0.02   | −0.47 ± 0.02   | −0.46 ± 0.02   | −0.47 ± 0.02   |
| 0              | 1.0    | −0.42 ± 0.02   | −0.46 ± 0.02   | −0.46 ± 0.02   | −0.50 ± 0.02   |
| 0              | 0.875  | −0.56 ± 0.03   | −0.67 ± 0.03   | −0.74 ± 0.03   | −0.87 ± 0.03   |
| 0              | 0.75   | −0.67 ± 0.03   | −0.74 ± 0.03   | −0.77 ± 0.03   | −0.84 ± 0.03   |
| 0              | 0.625  | −0.85 ± 0.04   | −0.85 ± 0.04   | −0.75 ± 0.04   | −0.71 ± 0.04   |
| 3              | 1.25   | −0.92 ± 0.11   | −0.82 ± 0.11   | −0.83 ± 0.11   | −0.76 ± 0.11   |
| 3              | 1.0    | −0.78 ± 0.11   | −0.77 ± 0.11   | −0.83 ± 0.11   | −0.78 ± 0.11   |
| 3              | 0.875  | −0.85 ± 0.11   | −0.90 ± 0.11   | −1.01 ± 0.11   | −1.20 ± 0.11   |
| 3              | 0.75   | −0.88 ± 0.11   | −0.83 ± 0.11   | −0.80 ± 0.11   | −0.83 ± 0.11   |
| 3              | 0.625  | −0.91 ± 0.11   | −0.83 ± 0.11   | −0.84 ± 0.11   | −0.88 ± 0.11   |

The coefficient \( m_{x}^{\alpha} \) of a cylinder without a disk at \( x_a = 0 \) is 0.51 ± 0.09. Positive values of \( m_{x}^{\alpha} \)
indicate that a free-flying cylinder with the center of mass on the axis of rotation does not have static stability. To give it static stability, it is necessary to install stabilizers in the tail section. This also applies to the bodies carried by the helicopter on an external suspension.

The presence of a coaxial disk whose diameter exceeds the diameter of the cylinder reduces the coefficient \( m_z^\alpha \). Disks of smaller diameter increase this coefficient. For a fixed gap between the disk and the cylinder, the coefficient’s dependence on the disk diameter has a maximum at disk diameter \( d = 0.75D \). Increasing of the gap between the disk and the cylinder reduces the disk’s influence on \( m_z^\alpha \). The coefficient \( m_z^\alpha \) of a cylinder without a disk at \( x_a = 3 \) is \(-0.76 \pm 0.25\).

All the coefficients are given with 95% confidence intervals. The number of measurements for \( x_a = 3 \) is less than the number of measurements for \( x_a = 0 \). So confidence intervals differ.

There is another way to determine the coefficient \( m_z^\alpha \), based on direct measurement of the moment of pitch of the cylinder using aerodynamic balances at different angles of attack. An evaluation of the capabilities of the aerodynamic balances that our wind tunnel is equipped gives that this other method will be less accurate.

Coefficients \( m_z^\alpha < 0 \) in all cases. It means that cylinder has static stability. It is possible to calculate distances \( x_{an} \) that correspond to the neutral static stability.

It was supposed that moment of force at \( x_a = 3 \) is the sum of moment of force at \( x_a = 0 \) and a product of lift force and distance \( x_a = 3 \):

\[
m_z^\alpha |_{x_a=3} = m_z^\alpha |_{x_a=0} - (x_a/L)c_y^\alpha \alpha. \tag{5}\]

Aerodynamic derivative \( c_y^\alpha \) is calculated from equation (5). Table 4 shows derivative \( c_y^\alpha \).

**Table 4.** Derivatives \( c_y^\alpha \) of the cylinder with the disk.

| \( d/D \) | \( g/D = 0.5 \) | \( g/D = 0.6 \) | \( g/D = 0.7 \) | \( g/D = 0.8 \) |
|---------|---------|---------|---------|---------|
| 1.250   | 2.88    | 2.88    | 2.67    | 3.78    |
| 1.000   | 3.15    | 3.39    | 2.94    | 3.03    |
| 0.875   | 3.78    | 3.09    | 2.88    | 2.91    |
| 0.750   | 3.81    | 4.44    | 4.17    | 4.02    |
| 0.625   | 4.23    | 4.59    | 4.35    | 3.66    |

Coefficient \( c_y^\alpha \) of the cylinder without disk is equal to 3.82.

At position of neutral static stability \( m_z^\alpha |_{x_a=x_{an}} = 0 \). Hence \( x_{an} = Lm_z^\alpha |_{x_a=0}/c_y^\alpha \).

These distances are presented in Table 5. Maximum \( x_a/D \) corresponds to disk with diameter \( d = 0.875D \).

The aerodynamic moment of force is described by the right-hand side of equation (1). The angle of attack included in the second term is the sum of the angle of inclination and the angle of attack caused by the transverse movement of the cylinder center:

\[
\alpha + \frac{d\alpha}{dt} x_a v. \tag{6}\]

Thus, the equation of motion (1) for the case \( x_a = 3 \) has the form:

\[
I_z \frac{d^2\alpha}{dt^2} + r \frac{d\alpha}{dt} + k\alpha = \left( m_z^\omega + m_z^\alpha |_{x_a=0} + m_z^\alpha \frac{x_a}{L} \right) \frac{qsL^2}{v} \frac{d\alpha}{dt} + m_z^\alpha qsL\alpha.
\]
Table 5. Calculated $x_{an}/D$ for neutral static stability of the cylinder with the disk.

| $d/D$ | $g/D = 0.5$ | $g/D = 0.6$ | $g/D = 0.7$ | $g/D = 0.8$ |
|-------|-------------|-------------|-------------|-------------|
| 1.25  | 1.22        | 1.22        | 1.52        | 1.52        |
| 1.0   | 1.80        | 1.67        | 1.93        | 1.93        |
| 0.875 | 2.26        | 2.16        | 2.06        | 2.06        |
| 0.75  | 2.24        | 1.93        | 1.86        | 1.86        |
| 0.625 | 1.89        | 1.71        | 1.49        | 1.49        |

Hence it follows:

$$(m^w_z + m^a_z)|_{x_a=3} = (m^w_z + m^a_z)|_{x_a=0} + m^a_z x_a / L. \quad (7)$$

Difference of calculated derivative by equation (7) and measured derivative for $x_a = 3$ is presented in Table 6.

Table 6. Difference of calculated and measured rotational derivatives $m^w_z + m^a_z$.

| $d/D$ | $g/D = 0.5$ | $g/D = 0.6$ | $g/D = 0.7$ | $g/D = 0.8$ |
|-------|-------------|-------------|-------------|-------------|
| 1.25  | 0.31        | 0.22        | 0.22        | 0.22        |
| 1.00  | 0.15        | 0.10        | 0.16        | 0.16        |
| 0.875 | -0.03       | -0.02       | 0.05        | 0.05        |
| 0.750 | -0.11       | -0.23       | -0.26       | -0.26       |
| 0.625 | -0.24       | -0.31       | -0.15       | -0.15       |

One can see that assumption (6) is valid only for $d = 0.875D$.

5. Conclusions

A coaxially mounted disk, whose diameter is less than the diameter of the cylinder, significantly reduces the drag of the cylinder. The aerodynamic derivatives of the pitch coefficient $m^w_z + m^a_z$ and $m^w_z$ of a cylinder with a head part in the form of a coaxial disk affixed on a leg at the upstream end of the cylinder are determined. Disks whose diameter exceeds the diameter of the cylinder do not have a strong effect on the damping of oscillations.

References

[1] Koenig K and Roshko A 1985 J. Fluid Mech. 156 167–204
[2] Kiselev N and Ryabinin A 2018 AIP Conference Proceedings 1959 050016
[3] Ryabinin A N and Kiselev N 2017 ARPN J. Engin. Applied Science 12(23) 6803-08
[4] Adamov N P, Puzyrev L N, Kharitonov A M, Chasovnikov E A, Dyadkin A A and Krylov A N 2013 Thermophysics and Aeromechanics 20 729–38
[5] Adamov N P, Gurin A M and Chasovnikov E A 2018 AIP Conference Proceedings 2027 030151
[6] Liu J, Song Y and Hu J 2019 Investigation on dynamic derivative test technique in hypersonic wind tunnel Proc. 2019 Asia-Pacific Int. Symp. on Aerospace Technology, Lecture Notes in Electrical Engin. 459 883–91
[7] Zimper D and Huber K C 2020 CEAS Aeronautical J. 11 475-85
[8] Kaufman D V and Ryabinin A N 2020 IOP Conf. Series: Materials Science and Engineering 927 012002
[9] Belotserkovsky S M, Skripach B K and Tabachnikov V G 1971 A wing in an unsteady gas stream (Moscow: Nauka Publ.)