Reversible thermal diode and energy harvester with a superconducting quantum interference single-electron transistor

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The density of states of proximitized normal nanowires interrupting superconducting rings can be tuned by the magnetic flux piercing the loop. Using these as the contacts of a single-electron transistor allows to control the energetic mirror asymmetry of the conductor, this way introducing rectification properties. In particular, we show that the system works as a diode that rectifies both charge and heat currents and whose polarity can be reversed by the magnetic field and a gate voltage. We emphasize the role of dissipation at the island. The coupling to substrate phonons enhances the effect and furthermore introduces a channel for phase tunable conversion of heat exchanged with the environment into electrical current.

Nanometric electronic devices demand on-chip components that are driven by temperature gradients or that manage thermal flows. Indeed, nanoscale conductors are interesting in this sense because of their intrinsic nonlinearities, strong spectral features and tunability$^5$. However, progress in this direction has been slow due to difficulties in the precise experimental detection of heat currents. Only very recently, great advances have been achieved in various mesoscopic configurations$^2$–$^7$. They open the way to realize theoretical proposals of heat current converters$^8$–$^{16}$, refrigerators$^{17}$–$^{25}$, thermal transistors$^{26,27}$ and diodes$^{28}$–$^{31}$ and valves$^{32}$ in the lab.

A thermal rectifier is a system which responds to reversed temperature gradients with currents of different magnitude$^{33}$. For it to work as a thermal diode, forward and backward flows must be of different orders of magnitude. In electronic systems, this is the case for two terminal mesoscopic junctions with strong non-linearities due to Coulomb interactions$^{28,34}$–$^{37}$ or coupled to an additional thermal bath with which it exchanges energy but no charge$^{38}$–$^{42}$. The performance of the device is then controlled by an external parameter, typically a gate voltage. A requirement is the absence of mirror symmetry, which can also be introduced in the spectral properties of the contacts$^{29}$–$^{31,43,44}$.

At low temperatures, hybrid metallic-superconducting junctions are interesting candidates as one can make use of the properties of single-electron tunneling in strongly interacting islands$^{15}$ and of the energy filtering introduced by the gap of the superconducting contacts. Here we investigate a superconducting quantum interference single-electron transistor (SQUISET), sketched in Fig. 1(a). Similar setups have recently been proposed$^{46}$ and implemented$^{47}$ as single-particle sources and heat valves$^{48}$. It consists on a normal metal island in the strong Coulomb blockade regime such that its occupation fluctuates between $n=0,1$ extra electrons. It can be controlled by a voltage $V_g$ of a plunger gate coupled to it via a capacitance $C_g$. The island is tunnel-coupled to two short wires that close two respective superconducting rings serving as contacts. Due to the proximity effect, the wires acquire a minigap$^{49,50}$ that is controlled by the magnetic flux, $\Phi_1$, piercing the corresponding ring$^{46}$:

$$\Delta_l = \Delta_0 \left| \cos \frac{\pi \Phi_l}{\Phi_0} \right|, \quad (1)$$

with $l=1,2$ and where $\Delta_0$ is the gap of the superconducting rings and $\Phi_0$ is the flux quantum. This way, the island is effectively coupled to contacts whose spectral properties can be tuned with an external magnetic field. Importantly for our purposes, their asymmetry can also be controlled if the size of the rings is different, as shown in Fig. 1(b). In particular, one can tune it quite flexibly from configurations with $\Delta_1 \gg \Delta_2$ to the opposite. This introduces the possibility to find a thermal diode whose polarity can be furthermore reverted on chip. The interplay between extrinsic magnetic and intrinsic elec-

![Fig. 1. SQUISET rectifier. (a) A metallic island (controlled by a gate voltage $V_g$) is tunnel coupled to the weak links of two superconducting loops held at different temperatures, $T$ and $T+\Delta T$, but the same chemical potential $\mu$. (b) The magnetic flux piercing each loop controls the gap of the proximitized contacts, $\Delta_l$, yielding asymmetric spectra when their area is different, such that $\Phi_2 \neq \Phi_1$ (here, $\Phi_1=4\Phi$). Heat exchanged with a phonon bath at temperature $T_p$ defines the island temperature, $T_i$. Energy filtering due to the superconducting gaps affect differently the (c) forward and (d) backward currents if $\Delta_1 \neq \Delta_2$.]

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trostatic control (via $\Phi$ and $V_g$, respectively) allows for an enhanced tunability of the device both for heat and thermoelectric currents responding to longitudinal temperature gradients $\Delta T$ between terminals 1 and 2. We assume they are at the same chemical potential, $\mu=0$.

Thermalization in the island plays a central role as, on one hand, it establishes a well defined electronic distribution and, on the other hand, it dissipates part of the heat injected from the leads. The forward and backward currents hence are given by the response of the barrier separating the island, at temperature $T_1$, and the corresponding cold reservoir, at temperature $T$. If $\Delta_1=\Delta_2$ they will be different, i.e. transport is rectified. We emphasize that $T_1$ depends on the currents injected from the two terminals. For instance, electrons with an energy lying in one of the windows $\pm [\Delta_1-\Delta_2]$ can tunnel only through one barrier, see Fig. 1(d). As they thermalize, they inject heat in the island, leading to a different $T_1$ for the forward and backward configurations and thus affecting the rectification effect, see Figs. 1(c) and (d).

As heat dissipated at the island is important, the coupling to substrate phonons, at a temperature $T_p$, cannot be disregarded\cite{51}. It in turn helps to increase the rectification. Interestingly, it also makes the system work as an energy harvester that converts heat exchanged with the phonon bath into an electric current\cite{52}.

The metallic island is described by the simple model\cite{45} $H = E_1(n-n_g)^2$, with a charging energy $E_1 = e^2/(2C)$ given by its capacitance, $C$, and the elementary charge $e$. The charge of the island can be externally tuned via $n_g = C_g V_g/e$. Sequential tunneling events through terminal $l$ into the island are described by the rates:

$$\Gamma_l^+ = \frac{1}{e^2 R_l} \int dE N_l(E) f_l(E) [1 - f_l(E-U)]$$

with the associated resistances $R_l$, and the Fermi functions $f_l(E) = (1+e^{E/k_BT_l})^{-1}$. The difference of chemical potentials for this transition is $U = E_1 (1 - 2 n_g)$. This defines the charge degeneracy point at $n_g = 1/2$. Tunneling out rates, $\Gamma_l^-$, are obtained by replacing $f_l(E) \rightarrow 1 - f_l(E)$ in Eq. (2). At finite temperatures, subgap states appear in the superconducting leads, which are taken into account by the Dynes density of states:\cite{53}

$$N_l(E) = \text{Re} \left( \frac{E + i\gamma}{\sqrt{(E+i\gamma)^2 - \Delta_l^2}} \right),$$

where $\gamma/h$ is the inverse quasiparticle lifetime.

With the particle tunneling rates $\Gamma_l^\pm$, one obtains the steady state occupation of the island $p_n$ via the stationary solution of the rate equations:

$$\dot{p}_0 = -\dot{p}_1 = \sum_l \left( \Gamma_l^- p_1 - \Gamma_l^+ p_0 \right),$$

with $p_0 + p_1 = 1$. The charge and heat currents into $l$ are hence respectively given by:

$$I_l = -e \left[ \Gamma_l^- p_1 - \Gamma_l^+ p_0 \right],$$

$$\dot{Q}_l = \Gamma_l^- p_1 - \Gamma_l^+ p_0, \quad (5)$$

where $\Gamma_l^\pm$ are obtained by inserting $E$ in the integral of the corresponding $\Gamma_l^\pm$. As no Joule heating is generated in the system, energy conservation is maintained by the heat absorbed at the island: $\dot{Q}_1 = -\dot{Q}_1 - \dot{Q}_2$. The temperature of the island is then given by the balance of the heat injected from the leads and the heat exchange with the phononic environment via the equation:

$$\dot{Q}_1(T_l) = \lambda (T_p^5 - T_l^5), \quad (7)$$

where $\lambda$ depends on the dimensions of the island and its material. At the superconducting contacts, this exchange is suppressed by the gap.

The diode performance is characterized by a difference of the currents under opposite thermal bias. We thus write the forward, $I_f = I_f(T_1=T+\Delta T, T_2=T)$, and backward, $I_b = I_f(T_1=T, T_2=T+\Delta T)$, charge currents, and

FIG. 2. Forward (a,b) and backward (c,d) charge and heat currents as functions of the gating, $n_g$, and the magnetic field, $\Phi=\Phi_0=\Phi_1/4$. The typical sign change of thermoelectric currents and double peak structure of heat currents around $n_g=1/2$ of single-electron transistors is modulated by the magnetic field. (e,f) Rectification coefficients for the corresponding currents. They change sign when $\Delta_1=\Delta_2$. (g) Cuts of $R_h$ in (f) for different $n_g$ show its change of polarity with gating and flux. Parameters: $E_1=0.25$ meV, $R_1=R_2=R_0=100\,k\Omega$, $T=116\,mK$, $\Delta T=2T$, $T_p=58\,mK$, $\lambda=\lambda_0=0.45\,\text{meV}^2\text{K}^{-5}$, and $\gamma=10^{-3}\,\text{meV}$.


accordingly for the heat currents $\dot{Q}_f$ and $\dot{Q}_b$. With these, we define the rectification coefficients:

$$R_c = \frac{|I_f| - |I_b|}{\max(|I_f|, |I_b|)} \quad \text{and} \quad R_h = \frac{|\dot{Q}_f| - |\dot{Q}_b|}{\max(|\dot{Q}_f|, |\dot{Q}_b|)}.$$  \hspace{1cm} (8)

Note that the sign of $R_c$ informs of the polarity of the diode, but this is not necessarily related to the sign of the currents. In particular, for the charge currents, both forward and backward flows change sign around the electron-hole symmetric configurations with $n_g = 1/2$, as expected for thermoelectric currents in Coulomb blockade systems\textsuperscript{34–57}, see Fig. 2. However their amplitude depends on the gap of the contacts, and hence $R_c$ oscillates and changes sign with the magnetic flux, vanishing when $\Delta_1 = \Delta_2$, see Figs. 2(a), (c) and (e). On the other hand, if the phonons do not introduce an additional thermal gradient, $\dot{Q}_f$ and $\dot{Q}_b$ are always positive, see Figs. 2(b) and (d). Note that due to heat flowing into the island, $R_h$ also changes its polarity with $n_g$, as shown in Figs. 2(f) and (g). This is not the case for $R_c$ due to charge conservation in the leads ($I_1 + I_2 = 0$).

The maximal rectification coefficients are found close to the electron-hole symmetric configurations. However, additional maxima can be found by tuning $n_g$ as an effect of the border of the superconducting gap. Note that $R_c$ is not well defined at $n_g = 1/2$, as no thermoelectric currents can be generated in electron-hole symmetric configurations.

We now investigate the crucial effect of the phonon bath. Let us consider first the case with $T_p = T$, see Figs 3(a) and (b). If the coupling to the phonons is strong, the island dissipates the incoming heat from the leads into the environment, so its temperature $T_1$ is not affected (as if it were connected to a thermostat). The rectification effects are then maximal and high coefficients $R_c$ and $R_h$ are achieved, see Figs. 3(a) and (b). With less effective couplings the island increases its temperature, which is detrimental for the diode performance. Note also that a strong coupling to the phonons can also revert the sign of the charge rectification coefficient by keeping the island close to the base temperature, $T$. Even in that case, there is a temperature gradient $\Delta T$ at which the forward and backward charge currents cross and $R_c$ changes sign, see Fig. 3(a).

In real configurations, electrons and phonons are not necessarily at the same temperature. The presence of an additional temperature gradient $T_p - T$ introduces unexpected effects in the rectification properties of the system, due to the local breaking of detailed balance. This introduces crossed thermoelectric effects: the phonon bath behaves as a third terminal with which the electronic system exchanges heat (and of course, no charge). This heat exchange can be converted into a contribution of the charge current whose sign is defined by both mirror and electron-hole asymmetries\textsuperscript{58,59}, even if $\Delta T = 0$. This additional current can flow in the opposite direction as the backward current and eventually cancel it. At that point, only the forward current is finite and we get $R_c = 1$, as shown in Fig. 3(c) for two different configurations of the magnetic flux and the gate voltage.

The behaviour of the heat currents is different, see Fig. 3(d). At low temperature gradients $\Delta T$, heat injected from the phonon bath dominates and reverts the forward current, while the backward one is strongly suppressed (due to the gap $\Delta_1$). This leads to $R_h \approx -1$. As $\Delta T$ increases, the longitudinal heat current dominates, so the forward flow changes sign. At that point, as only the forward current flows, $R_h \approx -1$. The position of this sharp feature changes with the temperature of the electrons.

Let us finally further explore the charge current generated by the temperature difference with the phonon bath. For this, we consider the case $T_1 = T_2 = T$ such that there is no longitudinal thermoelectric effect. The conductor is in equilibrium, except for the coupling to phonons at the island. The electrons in the island thermalize at a temperature given by Eq. (7). Thus a local temperature difference appears in the electronic system: In the likely case that $T_p < T$, the island hence becomes a cold spot, with $T_1 < T$. The effect of having broken electron-hole symmetry (for $n_g \neq 1/2$) and broken mirror symmetry (due to the superconducting gaps) breaks local detailed balance at both barriers, i.e. $\Gamma^1/\Gamma_0 \neq \Gamma^1/\Gamma_0^1$. According to Eq. (5), a net charge current is generated, as sketched in Fig. 4(a) and shown if Fig. 4(b). In this sense, it is a single-electron analogue of a thermocouple in which the electron-hole asymmetry is externally
tunable. Contrary to usual thermoelectric heat engines, where heat from a hot bath is transformed into power, the generated charge current is due to heat emitted into a colder environment. Furthermore, the sign of the current can in this case be controlled either by tuning the gate voltage or the magnetic flux, see Fig. 4(c). As a function of $n_g$, the current has a maximum when the chemical potential of the island is close to the gap, such that the asymmetry is larger. For higher chemical potentials the asymmetry is larger. For higher chemical potentials the asymmetry is larger.

We emphasize that this way, the system has a dual performance as a three-terminal converter of environmental heat. The phonon bath plays the role of the third terminal with which the electronic conductor exchanges heat but no charge. Differently from all-normal devices, the presence of the superconductors, apart from introducing the required asymmetric energy filtering, ensures that heat is exchanged with the phonon bath only at the mesoscopic region (the island, in our case): in the leads, it is strongly suppressed by the gap. This is expected to increase the efficiency. As the temperature difference with the phonons is in principle small, and so are the tunneling couplings, we do not expect the system to be a powerful heat converter. However, this effect can be used as a probe of the environmental temperature.

To conclude, we have investigated the response of a superconducting quantum interference single-electron transistor to temperature gradients. The interplay of Coulomb interactions in the island and phase coherence in the superconducting rings leads to the rectification of both charge and heat currents. The system then works as a phase and gate tunable thermal diode whose polarity can furthermore be reversed in various configurations. We find that electron-phonon coupling at the metallic island plays an essential role, with heat exchanged with the phonon bath enhancing the diode performance. Transport is sensitive to temperature differences with the phonons. This leads to salient features such as the vanishing of forward or backward flows, resulting in optimal diode behaviours with $R_c/h = ±1$, or in the generation of charge currents in a conductor which is otherwise in equilibrium (with $T_1 = T_2$). The device is then proposed as a versatile and efficient element for heat management in the nanoscale.

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1G. Benenti, G. Casati, K. Saito, and R. S. Whitney, Phys. Rep. 694, 1 (2017).
2F. Giazotto and M. J. Martínez-Pérez, Nature 492, 401 (2012).
3M. J. Martínez-Pérez, P. Solinas, and F. Giazotto, J. Low Temp. Phys. 175, 813 (2014).
4S. Jezouin, F. D. Parmentier, A. Anthore, U. Gennser, A. Cavanna, Y. Jin, and F. Pierre, Science 342, 601 (2013).
5C. Riha, P. Miechowski, S. S. Buchholz, O. Chiatti, A. D. Wieck, D. Reuter, and S. F. Fischer, Appl. Phys. Lett. 106, 083102 (2015).
6L. Cui, W. Jeong, S. Hur, M. Matt, J. C. Klöckner, F. Pauly, P. Nielaba, J. C. Cuevas, E. Meyhofer, and P. Reddy, Science 355, 1192 (2017).
7B. Dutta, J. T. Peltonen, D. S. Antonenko, M. Meschke, M. A. Skvortsov, B. Kubala, J. König, C. B. Winkelmann, H. Courtois, and J. P. Pekola, Phys. Rev. Lett. 119, 077701 (2017).
8R. Sánchez and M. Büttiker, Phys. Rev. B 83, 085428 (2011).
9H. Thierschmann, R. Sánchez, B. Sothmann, F. Arnold, C. Heyn, W. Hansen, H. Buhmann, and L. W. Molenkamp, Nature Nanotechnology 10, 854 (2015).
10B. Sothmann, R. Sánchez, A. N. Jordan, and M. Büttiker, Phys. Rev. B 85, 205301 (2012).
11F. Hartmann, P. Pfeffer, S. Höfling, M. Kamp, and L. Worschech, Phys. Rev. Lett. 114, 146805 (2015).
12B. Roche, P. Roulleau, T. Jullien, Y. Jompol, I. Farrer, D. A. Ritchie, and D. C. Glattli, Nature Communications 6, 6738 (2015).
13T. E. Humphrey, R. Newbury, R. P. Taylor, and H. Linke, Phys. Rev. Lett. 89, 116801 (2002).
14M. Josefsson, A. Sivilans, A. M. Burke, E. A. Hoffmann, S. Fahvik, C. Thelander, M. Leijnse, and H. Linke, Nat. Nanotechnol. 13, 920 (2018).
15A. N. Jordan, B. Sothmann, R. Sánchez, and M. Büttiker, Phys. Rev. B 87, 075312 (2013).
16G. Jalil, R. K. Puddy, R. Sánchez, A. N. Jordan, B. Sothmann, I. Farrer, J. P. Griffiths, D. A. Ritchie, and C. G. Smith, arXiv (2019), 1901.10561.
