Dynamic phase transitions of a driven Ising chain in a dissipative cavity

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We study the nonequilibrium quantum phase transition of an Ising chain in a dissipative cavity driven by an external transverse light field. When driving and dissipation are in balance, the system can reach a nonequilibrium steady state which undergoes a super-radiant phase transition as the driving strength increases. Interestingly, the super-radiant field changes the effective bias of the Ising chain in return and drives its own transition between the ferromagnetic and paramagnetic phase. We study the rich physics in this system with sophisticated behavior, and investigate important issues in its dynamics such as the stability of the system and criticality of the phase transition.

PACS numbers:

A collection of two-level atoms in a cavity is a classical system for studying atom-field interactions known as the Dicke model [1]. It hosts many interesting physical effects including the unusual super-radiant phase transition [2] when the strength of the atom-field interaction becomes so strong that it is comparable to the atomic level splitting. Once the coupling strength exceeds a critical value, the ground state of the system changes from vacuum to a state with nonzero macroscopic photon occupation. There has been much interest in studying the super-radiant phenomena in various contemporary context such as critical entanglement [3], finite-size scaling [4], and quantum chaos [5, 6]. Experimentally, it has been observed in a system of laser-driven BEC coupled to an optical cavity [7]. Though it has an origin in atomic and optical physics, with the development of new technologies the Dicke model has found its place in many other physical systems such as iron traps [9] and solid-state qubits [10, 12], though, it is possible to induce appreciable interactions between the two-level entities [21] that play the role of atoms in the original Dicke model. From a theoretical point of view, introduction of strong interactions between the two-level entities is an interesting addition to the system that can give rise to nontrivial new physics [27–30]. Not only can it have an impact on the interaction between the two-level entities and the cavity field, but it greatly enriches the physics of the system of the two-level entities itself. Under such a consideration, in this work we study the dynamical nonequilibrium quantum phase transition in a generalized Dicke model with cavity leakage and Ising interactions between the two-level entities. Specifically, as shown in Fig. 1 (a), our system consists of N identical atoms located inside a cavity and also driven by a transverse external light field. Though we have used atoms to describe our system, in reality they can be other entities such as ions [10] and solid-state qubits [12] depending on the specific physical system [9]. It is assumed that the atoms have two hyperfine ground states with an energy splitting of δ that are coupled by the cavity mode and external driving field via two Raman processes [31] depicted in Fig. 1 (b). Further, atoms are arranged in a 1D chain structure with nearest-neighbor Ising interactions, which can be accomplished using a quasi-1D optical lattice potential for atoms [32] or simply by controlled ion-trap potential for ion qubits [10] and controlled fabrication for solid-state qubits [12]. Such a system is described by the following Hamiltonian of a coupled spin-field system with Ising interactions,

\[
H(t) = \left(\omega_a - \frac{g^2}{\Delta_c}\right) a^\dagger a + \frac{g\beta(t)}{\Delta_c} a^\dagger a + \frac{g\beta^*(t)}{\Delta_c} a^\dagger a + \sum_i \sigma_i^z - \delta \sum_i \sigma_i^z - J \sum_i \sigma_i^y \sigma_{i+1}^y, \tag{1}
\]

where \(a^\dagger\) (\(a\)) is the annihilation (creation) operator for...
the cavity field with frequency $\omega_a$, and $\sigma_x^i$, $\sigma_y^i$, and $\sigma_z^i$ are Pauli matrices for an effective spin whose up and down states are the two ground states in the atomic energy levels in Fig. 1 (b). $\beta(t) = \beta_0 \exp(i \omega_b t + i \varphi)$ describes the effect of the driving field with frequency $\omega_b$ and phase $\varphi$, and $\Delta_c$ is the detuning of the Raman processes in Fig. 1 (b). $g$ characterizes the effective spin-field coupling strength, and $J$ is the strength of Ising interactions between neighboring spins. In these parameters, both the spin’s energy splitting $\delta$ and the spin-field coupling strength $g$ are much smaller than the frequency of the driving field and cavity mode, $\omega_a, \omega_b \gg \delta, g \beta_0 / \Delta_c$. $J$. However, the detuning $\omega_a - \omega_b$ can be comparable to the spin’s energy splitting $\delta$ and interaction strength $J$.

To study the nonequilibrium phase transition in our system, we first shift to the rotating frame defined by the external driving field by applying the operator $U(t) = \exp(-i \omega_b a^\dagger a t)$. After the Rotating Wave Approximation (RWA), we obtain the following effective many-body time-independent Hamiltonian,

$$H' = (\Delta - i \frac{\kappa}{2}) a^\dagger a + \frac{g_0}{\sqrt{N}}(a^\dagger e^{i \varphi} + ae^{-i \varphi}) \sum_i \sigma_z^i - \delta \sum_i \sigma_x^i - J \sum_i \sigma_x^i \sigma_x^{i+1},$$

where $\Delta = \omega_a - \frac{g^2}{\Delta_c^2} - \omega_b$ is the mismatch between the cavity and external driving fields, $\kappa$ is the photon loss rate, and $g_0 = \sqrt{N} g \beta_0 / \Delta_c$. The Hamiltonian in Eq. (2), which contains both Ising interactions between the spins and photon leakage out of the cavity, is an interesting and useful model for studying dynamic phase transitions that has not been studied so far.

In Eq. (2), the second term dictates the interaction between the spin chain and the cavity field assisted by the driving of the external field. Because of the leakage of photons out of the cavity, the system evolves irreversibly into a steady state until the effect of driving and dissipation reaches a dynamical equilibrium. Obviously, this steady state has a decisive impact on the physics of both the cavity field and Ising chain spin. To determine the steady state, we expand the field operator $a$ and the spin-chain operator $\sum_i \sigma_z^i$ around their mean field values $\phi_s$ and $S_x$ in the thermodynamic limit. This allows us to approximate the spin-field interaction term as

$$\frac{(a^\dagger e^{i \varphi} + ae^{-i \varphi})}{\sqrt{N}} \sum_i \sigma_z^i \approx \sqrt{N} (a^\dagger e^{i \varphi} + ae^{-i \varphi}) S_x + 2 \phi_s \sum_i \sigma_z^i - 2 \phi_s S_x N,$$

where $S_x = \frac{1}{N} \langle \sum_i \sigma_x^i \rangle_{ss}$ and $\phi_s = \frac{1}{2 \sqrt{N}} (a^\dagger e^{i \varphi} + ae^{-i \varphi})_{ss}$ are average values evaluated under the steady state. The effective Hamiltonian can then be written as

$$H_{\text{eff}} = (\Delta - i \frac{\kappa}{2}) a^\dagger a + g_0 \sqrt{N} (a^\dagger e^{i \varphi} + ae^{-i \varphi}) S_x - 2 g_0 \phi_s N S_x - \sum_i \left( \delta \sigma_z^i - 2 g_0 \phi_s \sigma_x^i + J \sigma_y^i \sigma_y^{i+1} \right).$$

We can now derive the equations of motion for the field operators from $H_{\text{eff}}$:

$$\dot{a} = -i \Delta a - i g_0 \sqrt{N} S_x e^{-i \varphi} - \frac{\kappa}{2} a,$$

$$\dot{a}^\dagger = i \Delta a^\dagger + i g_0 \sqrt{N} S_x e^{i \varphi} - \frac{\kappa}{2} a^\dagger.$$

In the steady state ($\frac{d}{dt} a = 0$), the mean photon field in cavity is

$$\phi_s = -\frac{\Delta g_0 S_x}{\Delta^2 + \kappa^2 / 4},$$

which is related to the ground state average $S_x$ of the spin part of the Hamiltonian [29]. Eq. (6) reveals a critical relation between the field and spin-chain behavior resulting...
from the dynamic equilibrium. Specifically, macroscopic occupation of cavity photons and macroscopic polarization of the spins in the $x$ direction occur at the same time. Thus, a super-radiant phase transition, if it occurs, may be accompanied by a phase transition in the spin chain between the ferromagnetic and paramagnetic phase. To see how it dictates the system characteristics, we notice that the mean field $\phi_s$ in turn has a direct impact on the spin part of the Hamiltonian

$$H_{\text{Ising}} = -\sum_i \delta \sigma_i^z - 2g_0 \phi_s \sigma_i^x + J \sigma_i^y \sigma_{i+1}^y \tag{7}$$

by modulating the transverse field of the Ising model. The transverse field Ising Hamiltonian in Eq. (7), which has a ferromagnetic phase for a strong Ising interaction $J > B_0 = \sqrt{\delta^2 + 4g_0^2 \phi_s^2}$, and a paramagnetic phase for a strong transverse field $B_0 > J$, can be solved exactly by the Jordan-Wigner transformation [36]. Its many-body ground state wave function, $|\Psi_g(\phi_s)\rangle$, is dependent on the transverse field $B_0$ of the Ising model and hence on $\phi_s$. From $|\Psi_g(\phi_s)\rangle$, we can calculate the macroscopic polarization in the $x$ direction,

$$S_x(\phi_s) = \frac{1}{N} \langle \Psi_g(\phi_s) \sum_i \sigma_i^x | \Psi_g(\phi_s) \rangle, \tag{8}$$

which is also dependent on $\phi_s$. By solving Eqs. (6) and (8), we can then self-consistently determine the values of $\phi_s$ and $S_x$ in the dynamical equilibrium.

In Figs. 2 (a) and (b), we plot the solved values for $\phi_s$ and $S_x$ against the driving strength $g_0$ when the spin energy splitting $\delta$ is smaller than the Ising interaction strength $J$. It is seen that, when the coupling is weak, $\phi_s = 0$, there is no macroscopic photon occupation in the cavity. By Eq. (6), $S_x = 0$, the spin-chain does not exhibit a macroscopic polarization in the $x$ direction either. When the coupling strength $g_0$ increases, $\phi_s$ becomes nonzero, indicating that a super-radiant phase transition occurs. However, the solutions of $\phi_s$ and $S_x$ are not single-valued in a range of driving strength $g_0$. The question which solutions are physical can be resolved with a stability analysis.

To study the stability of the solutions for $\phi_s$ and $S_x$, we consider the time evolution of deviations from their mean field value. We split the field and spin operators into their steady state mean values and small fluctuations

$$a(t) = a_s + \delta a(t) \tag{9a}$$

$$\frac{1}{N} \sum_t \sigma_i^x(t) = S_x + \delta S_x(t), \tag{9b}$$

with $a_s = \langle a \rangle_{ss}$. Assuming a small fluctuation term for the field value, we have $\phi = \phi_s + \delta \phi$ with $\delta \phi = \frac{1}{2\sqrt{N}} (\delta e^{i\phi} + \delta e^{-i\phi})$, and $\delta S_x(t) = \frac{\partial S_x}{\partial \phi} \delta \phi \frac{\partial \phi}{\partial t} \tag{35}$. Using the equation of motion for the field operator in Eq. 6, we have

$$\dot{\delta \phi} = -\frac{\kappa}{2} C_s \delta \phi, \tag{10}$$

where

$$C_s = 1 + \frac{g_0}{\Delta^2 + \kappa^2/4} \frac{\partial S_x}{\partial \phi} \tag{11}$$

is the stability coefficient. A solution for $\phi_s$ is stable if and only if $C_s > 0$. Therefore, from the plot of $C_s$ in Fig. 2(c), we conclude that the branches in solid lines for the solutions of $\phi_s$ and $S_x$ are stable, whereas the branches in dashed lines are unstable.

From this stability analysis, we can infer that there is a hysteresis in the solutions for $\phi_s$ and $S_x$ with bi-stability as indicated in Figs. 2 (a) and (b). When the spin-field driving strength $g$ is increased above a critical value $g_2$, the value of $\phi_s$ jumps from 0 to a nonzero value, and the system makes a discontinuous transition to the super-radiant phase. Likewise, the Ising spin chain experiences a discontinuous phase transition at $g_2$ from the ferromagnetic phase to paramagnetic phase, as shown in Fig. 2.
the vacuum state at a coupling strength driving strength is decreased, it makes a transition to
When the system is in the super-radiant phase and the system are first-order in nature. This is notably differ-
tent than the conventional transverse field Ising model, in
in Fig. 2. Other parameters are the same as in Fig. 2 and
δ/J greater than 1, indicating the disappearance of the hysteresis in Fig. 2. Other parameters are the same as in Fig. 2 and J is the energy unit.

(b). Therefore, the phase transitions in our dissipative system are first-order in nature. This is notably differ-
ent than the conventional transverse field Ising model, in
which the quantum phase transition is continuous. When the system is in the super-radiant phase and the driving strength is decreased, it makes a transition to the vacuum state at a coupling strength g1 lower than g2, giving rise to the hysteresis in Fig. 2.

Studying the impact of the system’s key parameters on the nature and characteristics of phase transitions can help gain deep insight into our dissipative system. First, we calculate how the critical coupling strengths g1 and g2 for the phase transitions are dependent on ∆, the effective photon frequency in the effective Hamiltonian in Eq. (2). For a non-dissipative Dicke system, the coupling strength required for the super-radiant phase transition decreases monotonically with the photon frequency.

As shown in Fig. 3 (a), the behavior of our dissipative system is quite different. Not only do g1 and g2 reach their minima at a finite value of ∆, but they diverge when ∆ → 0. This can be understood from Eq. (3), which suggests that the super-radiant phase transition cannot occur for ∆ = 0 since φs = 0. As for the minimum values of g1 and g2, we can prove that they occur at ∆ = κ/2. According to Eq. (6), the values of g1(∆ = κ/2) and g2(∆ = κ/2) are determined by solving

\[ \phi_s = \frac{g_0 S_x}{\kappa/2}, \]

with Sx determined by the ground state of

\[ H_{\text{Ising}} = -\sum_i \delta \sigma_i^z - 2g_0 \phi_s \sigma_i^x + J \sigma_i^y \sigma_{i+1}^y. \]

For a value of ∆ slightly different, ∆ = κ/2 ± ε (ε ≪ κ), g1(∆ = κ/2 ± ε) and g2(∆ = κ/2 ± ε) are determined by solving

\[ \phi_{s}' = \frac{g_0 S_x}{\kappa/2}, \]

with Sx determined by the ground state of

\[ H_{\text{Ising}} = -\sum_i \delta \sigma_i^z - 2g_0 \phi_{s}' \sigma_i^x + J \sigma_i^y \sigma_{i+1}^y, \]

where g_0 and φ_s' are obtained by expansion of Eq. (6) to second order of ε with the results

\[ g_0 = g_0 \sqrt{1 - 2\epsilon^2/\kappa^2} \]

and

\[ \phi_{s}' = \frac{\phi_s}{\sqrt{1 - 2\epsilon^2/\kappa^2}}. \]

Notice that Eqs. (14) (15) are in the same form with Eqs. (12) (13), indicating that their solutions are related by a proper scaling. Specifically,

\[ g_1'(\Delta = \kappa/2 \pm \epsilon) = g_1(\Delta = \kappa/2) \]

with

\[ g_1'(\Delta = \kappa/2 \pm \epsilon) \equiv g_1(\Delta = \kappa/2 \pm \epsilon) \sqrt{1 - 2\epsilon^2/\kappa^2}. \]

Thus we have

\[ g_1(\Delta = \kappa/2 \pm \epsilon) = \frac{g_1(\Delta = \kappa/2)}{\sqrt{1 - 2\epsilon^2/\kappa^2}} > g_1(\Delta = \kappa/2), \]

and we can conclude that the values of g1 and g2 are the minimal at ∆ = κ/2.

Then, we investigate the effect of dissipation rate κ on the critical coupling strengths g1 and g2 for the phase transitions. As shown in Fig. 3 (b), g1 and g2 increase monotonically with the dissipation rate κ, suggesting that the phase transitions occur at stronger couplings.
The number of fluctuating photon number \( \langle a^\dagger a \rangle \) in the steady state becomes significant, which signals that the photon field enters a normal phase from a super-radiant phase. Likewise, when the driving strength increases and approaches the critical point \( g_1 \) of the system, the field fluctuations grow quickly. In Figs. 4(e) and (f), we plot the photon fluctuations on a log scale as a function of deviation of the driving strength from the critical point \( g_1 \) and \( g_2 \). The critical exponent when the external driving and cavity loss is fluctuations because of the crucial role they play in phase transitions \[37, 38\]. For simplicity, we focus on the fluctuation of the cavity field, and assume that the Ising chain always stays in the ground state corresponding to the cavity field. Under such an assumption, the fluctuation in the spin chain’s polarization in the \( x \) direction, \( S_x \), is \( \delta S_x(t) = \frac{\partial S_x}{\partial \phi} \delta \phi \) to first order in the cavity field fluctuation \( \delta \phi \) \[35\]. Denoting the field fluctuations with \( V = [\delta a, \delta a^\dagger]^T \), and using the equations of motion for the field operators, we have

\[
\frac{\partial}{\partial t} V = M V + \hat{\xi},
\]

where \( \hat{\xi} = (\xi, \xi^\dagger)^T \) is the vector of operators for the quantum noise which has zero mean values and whose only non-vanishing correlation is \( \langle \xi(t)\xi^\dagger(t') \rangle = \kappa \delta(t - t') \). \( M \) is the linear stability matrix of the mean field solution

\[
M = \begin{bmatrix}
-i\Delta - \kappa & -\frac{\kappa}{2} \\
\kappa & i\Delta - \kappa
\end{bmatrix}
\]

The matrix \( M \) is non-normal, therefore it has different left and right eigenvectors \( R, L \) \[20\], which form a biorthogonal system \( LR = I \). \( M \) has two eigenvalues,

\[
\omega_{m1} = -\kappa - i\sqrt{4\Delta^2 + 4\Delta g^0 \partial S_x / \partial \phi} / 2
\]

\[
\omega_{m2} = -\kappa + i\sqrt{4\Delta^2 + 4\Delta g^0 \partial S_x / \partial \phi} / 2
\]

which are plotted as a function of driving strength in Figs. 4(a) and (b). When the real part of the eigenvalues is negative, the system is stable \[20\]. Following the method discussed in Ref. \[20\], we solve for the number of fluctuating photon number which is

\[
\langle \delta a^\dagger \delta a \rangle = -\sum_{i,j} \frac{\kappa}{\omega_{m1} + \omega_{m2}} \left[ L_{i,1} L_{j,2} R_{2,j} R_{1,i} \right]
\]

where \( \omega_{m1}, \omega_{m2} \) are the eigenvalues of matrix \( M \). As shown in Fig. 4(d), when the driving strength decreases and approaches the critical point \( g_1 \) of the system, the photon number fluctuation in the steady state becomes significant, which signals that the photon field enters a normal phase from a super-radiant phase. Likewise, when the driving strength increases and approaches the critical value \( g_2 \), the fluctuation grows quickly. In Figs. 4(e) and (f), we plot the photon fluctuations on a log scale as a function of deviation of the driving strength from the critical point \( g_1 \) and \( g_2 \). The critical exponent can be read from these plots. Near \( g_2 \), it is \(-1.0\), consistent with the super-radiant phase transition in a conventional Dicke model without Ising interaction. Near \( g_1 \), the exponent is about \(-0.75\).

In summary, we have studied dynamic phase transitions in an open system consisting of an Ising chain in a lossy cavity. When the external driving and cavity dissipation is more severe, this is understandable, since a higher rate of dissipation must be balanced by stronger driving to support the super-radiant phase. Finally, in Fig. 3(c), we show how \( g_1 \) and \( g_2 \) change with the relative magnitude of the spin energy splitting \( \delta \) and Ising interaction strength \( J \). It is seen that, when \( \delta \) is smaller than \( J \), \( g_1 \) and \( g_2 \) have different values, which suggests discontinuous first-order phase transitions. However, when \( \delta \) is greater than \( J \), \( g_1 \) and \( g_2 \) are equal. This suggests that the hysteresis in Fig. 2 disappears. Closer examination of the values of \( \varphi_s \) and \( S_z \) reveals that the phase transitions have become continuous. This behavior is similar to that in closed Dicke systems without dissipation \[27, 28\], except that \( \delta/J \) needs to be slightly larger than 1 for the phase transitions to become continuous due to the influence of the dissipation.
leakage reach a dynamic equilibrium, the cavity can undergo a super-radiant phase transition when the driving strength for the Ising chain is strong enough. Interestingly, because of the mutual impact between the Ising chain and cavity field, it is accompanied by a transition between ferromagnetic and paramagnetic phase in the Ising chain. Under certain conditions, the phase transitions are hysteretic and discontinuous, and exhibit different characteristics than those in conventional closed systems. By studying important issues of the system such as stability and fluctuations, we gained valuable insights in the unique properties of quantum phase transitions in driven systems in dynamic equilibrium.

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