Enhancing the hydraulic bulge-test using full-field DIC data

M Rossi, A Lattanzi and D Amodio
Faculty of Engineering, Università Politecnica delle Marche, via breccie bianche, 60131, Ancona, Italy
E-mail: m.rossi@univpm.it

Abstract. The bulge test is an important experimental technique to identify the properties of materials and, in particular, metals. One of the main advantages of such technique, in testing metals, is the possibility of reaching high levels of deformation before fracture. For instance, a tensile test allows to evaluate the hardening curve only up the occurrence of necking, i.e. at the maximum force in the load vs. displacement curve. A bulge test can be performed using mechanical punches or the hydraulic pressure of a fluid. When hydraulic pressure is used, the test is not affected by friction. The hydraulic bulge test (HBT) is therefore a very efficient method to evaluate the properties of metals at large strains. Usually, the outcome of the HBT is the hardening curve in the equi-biaxial stress state that occurs at the top of the dome, during the expansion. In order to obtain the stress value is important to measure the curvature of the dome, this involve a second derivative of the vertical displacement field that can lead to measurement errors, especially if an optical technique is used to obtain the shape of the bulge. If DIC is employed as full-field measurement technique to evaluate the displacement and the strain field of the sheet metal during the HBT, only few measurement points in the center of the dome are actually used in the evaluation of the stress-strain curve. Accordingly, there is a large part of information that is not used in the identification procedure In this paper, the whole full-field measurement is exploited to derive the hardening behavior using an adaptation of the Virtual Fields Method (VFM). Suitable virtual fields are used to write the equilibrium equation in a large zone of the sheet metal during the test. All the strain data measured with DIC in such zone are then used in the identification procedure. The method is illustrated through simulated experiments on stainless steel sheet metals.

1. Introduction

The hydraulic bulge test (HBT) represents an effective experimental protocol to establish the equi-biaxial stress state, that is often required to calibrate non-quadratic anisotropic yield functions [1, 2]. In fact, when the bulge is formed through the application of a pressure on the blank sheet, a membrane stress state of a thin-walled spherical vessel can be assumed in proximity of the dome apex, permitting the calculation of stress-strain curve in the condition of equi-biaxial stress state [3, 4].

In particular, the achievement of biaxial stress-strain curve requires the measurement of three quantities: the forming pressure of fluid, the bulge curvature, and the through-thickness strain. Full-field measurement with stereo-DIC allows to obtain the latter two inputs, since it returns both shape and displacement fields; the in-plane strains can be derived from displacements fields, and, then, the through-thickness strain component can be obtained according to the hypothesis
of volume conservation, valid during finite plastic deformation [5]. However, only one point is used in the identification while the full-field information coming from the DIC is not exploited, moreover the curvature computation is affected by noise since a double derivative is involved. In this paper, the whole full-field information is included in the identification procedure through the virtual fields method (VFM). Furthermore, the curvature information is not anymore required.

2. Method description

Under the assumption of plane stress and neglecting the bending stress, the HBT can be viewed as a 2D problem, where the membrane stress/strain is distributed over a curved surface. The non-linear VFM [6] can be applied to this surface to evaluate the constitutive parameters. The method consists in minimizing a cost function $\Psi (\xi)$ where $\xi$ is the set of constitutive parameters that have to be identified. The cost function assumes this form:

$$\Psi (\xi) = \frac{1}{N_v N_t} \sum_{i=1}^{N_v} \sum_{j=1}^{N_t} \psi (\xi, \delta v_i, t_j)$$

(1)

where $\delta v_i$ are $N_v$ kinematically admissible virtual fields and $t_j$ are $N_t$ load steps of the test. The function $\psi$, evaluated for each virtual fields and load step, is a balance between the internal and the external virtual work, according to the principle of virtual work (PVW). In large deformation, the PVW has different formulations, in particular, here, it is convenient writing the virtual fields in terms of the reference configuration using the following form:

$$\psi (\xi, \delta v, t) = \left| \int_{\partial B_0} T^{1PK} \cdot \delta F \cdot dv_0 - \int_{\partial B_0} (T^{1PK} n_0) \cdot \delta v \cdot da_0 \right|$$

(2)

where $\delta F \cdot$ is the gradient of $\delta v$ in the Lagrangian description:

$$\delta F \cdot = \text{Grad} \delta v (x_0, t)$$

(3)

and $T^{1PK}$ is the first Piola-Kirchhoff stress tensor, defined as:

$$T^{1PK} = \det (F) \cdot T F^{-T}$$

(4)

where $F$ is the deformation gradient. In this case, since the problem is reported to a 2D situation, the curved surface that the sheet metal assumes during deformation has to be remapped over an equivalent planar surface. To this purpose different methods can be used [7], which are commonly employed, for instance, in computer vision to evaluate the strain field in sheet metal forming applications.

The deformation gradient can be evaluated as:

$$F = F_1 F_0^{-1}$$

(5)

where $F_0$ and $F_1$ are $3 \times 2$ matrices that maps a point from a 2D reference plane to a 3D surface, in the undeformed and deformed configuration, respectively. If we consider a point $p$ with coordinates $(X, Y)$ in the reference plane and the points $p_0$ and $p_1$ in the undeformed and deformed 3D configuration, there will be two functions $f_0$ and $f_1$ so that:

$$p_0 = f_0(p) \quad \text{and} \quad p_1 = f_1(p)$$

(6)

it follows:

$$F_{0ij} = \frac{f_{0i}}{\partial X_j} \quad \text{and} \quad F_{1ij} = \frac{f_{1i}}{\partial X_j}$$

(7)
with $i \in [x, y, z]$ and $j \in [X, Y]$. If the initial configuration corresponds to the reference plane, as occurs in the HBT, where the process starts from a planar blank sheet, the planar deformation can be expressed in terms of a reduced Cauchy-Green right tensor $C_1$:

$$C_1 = F_1^T F_1$$

and the stretch tensor can be obtained as:

$$U_1 = \sqrt{F_1}$$

If we assume the volume constancy during deformation, then $\det(F) = 1$. Furthermore, in the reference 2D plane under the plane stress hypothesis, the first Piola-Kirchhoff tensor of Eq. 4 can be rewritten as:

$$T^{1PK} = T U_1^{-T}$$

where $T$ is now a $2 \times 2$ matrix with the planar component of the stress (all other components are zero). The corresponding strain tensor can be computed in terms of logarithmic strain as:

$$E = \log(U_1)$$

The tensors $F_1$, $C_1$ and $U_1$ are computed from the measured surface displacement field, obtained for instance with stereo-DIC, and the logarithmic strain can be used to evaluate the stress tensor as a function of the constitutive parameters $\xi$ [8, 9]. The computed stress is then used in Eq. 2 for non-linear VFM.

![Figure 1. Schematic of the initial and deformed sheet, the used coordinate system and the pressure distribution are illustrated.](image)

In Figure 1, a schematic of the bulge test in the initial and deformed configuration is illustrated. Let us apply the VFM using as initial placement $B_0$ (see Eq. 2) a circle with diameter $R_0$ and the following virtual field:

$$\delta \mathbf{v} = \begin{cases} 
\delta v_x = x \\
\delta v_y = y
\end{cases} \quad \text{with} \quad \delta \mathbf{F}^{(1)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

If we call the first term within the absolute value of Eq. 2 internal virtual work $W_i$ and the second term external virtual work $W_e$, according to the used VF, it follows:
\[ W_i = \int_{\mathcal{B}_0} (T_{xx}^{1PK} + T_{yy}^{1PK}) \, dv_0 \quad \text{and} \quad W_e = R_0 \int_{\partial \mathcal{B}_0} (T_{\mathbf{n}0}^{1PK}) \, da_0 = R_0 \int_{\partial \mathcal{B}} \mathbf{t} \, da \] (13)

where, according to the properties of the stress tensors, \( W_e \) can be rewritten in terms of the traction vector \( \mathbf{t} \) applied in the deformed configuration \( \mathcal{B} \). This property is useful, because, using the global equilibrium equation of the body, we know that:

\[ \int_{\partial \mathcal{B}} \mathbf{t} \sin \theta \, da = P \pi R^2 \] (14)

where \( P \) is the internal pressure and \( \theta \) is the normal angle respect to \( z \), as illustrated in Figure 1.

If the angle \( \theta \) remains almost constant along the circumference of \( R \), it can be put out of the integral and Eq. 14 used to compute \( W_e \). After some manipulation, Eq. 2 becomes:

\[ \psi(\xi, t) = \left| \int_{\mathcal{B}_0} (T_{xx}^{1PK} + T_{yy}^{1PK}) \, dv_0 - P \frac{\pi R^2 R_0}{\sin \theta} \right| \] (15)

Such equation can be used to identify for instance the flow curve using the minimization of a cost function or a linear method as in [10]. With the same principle, other virtual fields could be generated keeping fixed the external boundary condition.

3. Results and discussion

In order to validate the method, a numerical model of a HBT was built using ABAQUS/Standard. The 3D displacement field obtained from the FE model was used as input and used to identify the hardening law of the material.

3.1. Numerical model

The adopted HBT numerical model is composed by two main parts: the circular die, modeled as a 3D analytic rigid shell, and the blank sheet, which is represented by a deformable shell. Fixed boundary condition is imposed at the external circumference of both parts, while a maximum forming pressure of 80 bar is applied to the blank sheet internal surface. Then, a frictional contact is imposed between the blank sheet and the die, assuming a frictional coefficient of \( \mu_s = 0.16 \), typical of steel-steel lubricated surfaces.

**Figure 2.** FE model of the HBT, vertical displacement.
Table 1. Characteristics of the FE model

| Geometry of HBT       | FEM characteristics                  |
|-----------------------|---------------------------------------|
| Blank size 300 mm     | Type of element S4R (4-nodes, reduced int.) |
| Die diameter 200 mm   | Number of elements 9716               |
| Thickness 1.5 mm      | Hardening law                         |
|                       | $\sigma = K (\bar{\varepsilon} + \varepsilon_0)^N$ |
|                       | $K = 1000$ MPa                        |
|                       | $\varepsilon_0 = 0.02$               |
|                       | $N = 0.5$                            |

Concerning the material model, isotropic von Mises plasticity is adopted, and the isotropic hardening regulated by the Swift law. All information about the geometry of the HBT and the characteristic of the used FEM model are listed in Table 1. The displacement field obtained from the FE model was remapped on a regular grid of $65 \times 65$ points with a uniform initial distance of 2.5 mm. This reproduce a typical output from a DIC measurement.

3.2. VFM implementation

The VFM was applied using a radius $R_0 = 70$ mm in Eq. 15. Figure 3 illustrates the distribution of angle $\theta$ in the last step of the simulated test. The angle $\theta$ can be considered constant along the circumference with radius equal to $R_0$. For each test step, the angle $\theta$ and the deformed dimension of the radius $R$ can be retrieved from the full-field measurement, as illustrated in Figure 4. It is worth noting that the angle $\theta$ can be obtained as derivation of the vertical displacement, and is less affected by noise if compared with the curvature.

The VFM was used to identify two parameters of the Swift law, i.e. $K$ and $N$, see Table 1. The parameters $\varepsilon_0$ has a minor impact in the hardening law and was set equal to 0.02. Figure 5 shows results of the minimization process using two sets of initial values: $(K_1 = 1500, N_1 = 0.1)$
and \((K_1 = 500, N_1 = 0.8)\). At the end of the minimization process a perfect balance is obtained between the internal and the external virtual works. Figure 6 illustrates the identified parameters and the resulting stress-strain curves, a good agreement is found with the reference one.

Figure 5. Balance of the internal and external virtual works with two initial guesses.

Figure 6. Comparison between the reference stress strain curve and the identified ones.

4. Conclusion
The non-linear VFM at large strain was used to identify the hardening behaviour of a HBT. With respect to the conventional method, this approach has the advantage of using the whole full-field measurement information and avoid the computation of the curvature, which requires a double derivative of experimental data. The method was applied on a simulated experiment of HBT. Further studies and the application to real experiments are necessary to validate the proposed method.

References
[1] Barlat F, Aretz H, Yoon J-W, Karabin M E, Brem J C, and Dick R E 2005 Linear transformation-based anisotropic yield functions Int. J. Plasticity 21 1009
[2] Banabic D, Bunge H-J, Pöhlandt, and Tekkaya A E 2000 Formability of metallic materials. (Springer Berlin)
[3] Gutsher G, Wu H-C, Ngaile G, and Altan T 2004 Determination of flow stress for sheet metal forming using the viscous pressure bulge (vpb) test J. Mater. Process. Technol. 146 1
[4] Lee J Y, Barlat F, Wagoner R H, and Lee M G 2013 Balanced biaxial testing of advanced high strength steels in warm conditions Exp. Mech. 53 1681
[5] ISO 16808:2014 Determination of biaxial stress-strain curve by means of bulge test with optical measuring systems
[6] Pierron F and Grédiac M 2012 The Virtual Fields Method. (Springer New York)
[7] Chung T J 1988 Continuum mechanics. (Prentice-Hall Englewood Cliffs)
[8] Rossi M and Pierron F 2012 Identification of plastic constitutive parameters at large deformations from three dimensional displacement fields Comput. Mech., 49 53
[9] Rossi M, Pierron F, and Štamborská S 2016 Application of the virtual fields method to large strain anisotropic plasticity Int. J. Solids Struct., 97 322
[10] Rossi M, Lattanzi A, and Barlat F 2018 A general linear method to evaluate the hardening behaviour of metals at large strain with full-field measurements Strain In press