Laser-assisted doubly charged Higgs pair production in Higgs triplet model (HTM)

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Abstract In the framework of Higgs triplet model (HTM), we study the pair production process of doubly charged Higgs bosons via e+e− annihilation in the presence of a laser field with circular polarization. We begin our work by presenting the theoretical calculation of the differential cross section in the centre of mass frame including both Z and γ diagrams. Then, from the numerical analysis of the production cross section’s dependence on the laser field parameters, we have shown that the laser-assisted total cross section decreases as far as the electromagnetic field intensity enhances or by decreasing its frequency. Finally, we analyse the variation of the total cross section versus the mass of the doubly charged Higgs boson by fixing the laser field parameters and the centre of mass energy, and we have found that the order of magnitude of the cross section decreases with the increment of \(M_{H^{\pm \pm}}\). 

1 Introduction

After the discovery of the laser technology in 1960 [1], the study of laser-assisted processes has been an active area of both theoretical [2–6] and experimental [7–10] research. This is due not only to its fundamental importance in collision physics but also to its role in assisting our understanding and interpreting a wide range of scientific phenomena and technological applications [11–13]. The study of laser-assisted processes provides important information about behaviour of particles and their properties. In this respect, a significant deal of theoretical works has been consecrated to the study of weak decay and scattering processes [14–16], and it is found that the circularly polarized laser field prolongs the particle’s lifetime and enhances its modes decay. In the past few years, there has been particular interest in the study of the electron–positron interactions in the presence of an external field even though there are no experimental evidences. Moreover, the effect of the electromagnetic field with circular polarization is that it decreases the total cross section. Detailed reports on laser-assisted elementary particles’ production via electron–positron annihilation can be found in our previous papers [17,18] and in some papers of other groups that are interested in this field of research [19–23]. In addition, the laser-assisted electron–positron annihilation allows the observation of a variety of new phenomena related to high-energy physics such as the standard model of particle physics and beyond. In ref [24], we have found that the electromagnetic field reduces the total cross section of the production of a charged Higgs boson in association with a charged weak W-boson (\(H^{\pm\pm} W^{\mp}\)).

The LHC milestone discovery [25,26] of the Higgs boson in July 2012 with a mass 125 GeV has been a great mutation in modern particle physics because it provides us with a deep understanding of the electroweak symmetry breaking mechanism (EWSB) which is outlined in the Standard Model of particle physics by Brout–Englert–Higgs mechanism [27–29]. After some years of accumulated data, the properties of this new particle are in excellent agreement with the prediction of the standard model [30]. However, this current experimental result raises the question of whether there are other fundamental scalars that may solve some of the open questions in particle physics such as the origin of neutrino mass and the existence of dark matter. An alternative philosophy is based on adding the minimal required number of new fields to the Standard Model in order to address these problems. One example is the SM extended by two real scalar fields which can provide a viable dark matter candidate [31,32], and it is also very rich in terms of its collider phenomenology [33].

The Higgs triplet model (HTM) [34,35] is one of the simplest models beyond the SM which postulates that the SM Higgs sector should be extended by a triplet field \(\Delta\) with \(Y = 1\). The spectrum of the HTM contains two doubly charged \(H^{\pm\pm}\), two singly charged \(H^{\pm}\), one CP-odd \(A^{0}\) and two CP-even \(h^{0}\) and \(H^{0}\). The characteristic feature of such model with Higgs triplet is that it proposes the most direct way to explain neutrino masses [36,37]. In addition, it is well known that the doubly charged Higgs bosons \(H^{\pm\pm}\) can be seen as the typical particles in this model. In the HTM, there are two main decay modes for \(H^{\pm\pm}\): the same-sign diboson decay (\(H^{\pm\pm} \to W^{\pm}W^{\mp}\)) and the same-sign dilepton decay (\(H^{\pm\pm} \to l^{\pm}l^{\pm}\)). The collider phenomenology of a doubly charged scalar has been discussed in [38]. If a doubly charged Higgs boson is discovered at LHC, it will be critical to determine its couplings at the future high-energy linear colliders such as the compact linear collider (CLIC) [39] and the international linear collider (ILC).

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In the presence of a laser field. The Feynman diagrams of this process are shown in Fig. 1.

The couplings of the doubly charged scalars, which are related to our calculation, can be written as \[48,49\]:

\[
G = \begin{pmatrix}
\cos \beta & \sin \beta \\
\sin \beta & -\cos \beta 
\end{pmatrix}
\]

using the following two unitary rotations, \[R_1, R_2\]. The short notations \(G\) of doubly charged Higgs bosons \(\Phi^\pm\) after an orthogonal rotation by using the mixing matrix defined by \(R_\Delta\), and they are given by:

\[\Phi^\pm = \begin{pmatrix}
\Phi^+ \\
\Phi^0 \\
\Phi^- 
\end{pmatrix}\]

In the above potential, \(\mu_H\) and \(\mu_\Delta\) stand for mass squared parameters. \(\lambda, \lambda_{i=1,4}\) are dimensionless couplings, while \(\mu\) denotes dimensionless coupling that mixes two Higgs fields. After the electroweak symmetry breaking, the neutral components, \(\Phi^0\) and \(\delta^0\), can acquire vacuum expectation values such that:

\[H = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 \\
v_\Phi \\
0
\end{pmatrix}, \quad \Delta = \frac{1}{\sqrt{2}} \begin{pmatrix}
\delta^+ \\
0 \\
\delta^-
\end{pmatrix}\]

The most general renormalizable and gauge invariant potential is given by [47]:

\[V(H, \Delta) = -\mu_H^2 H^\dagger H + \frac{\lambda}{4} (H^\dagger H)^2 + \mu_\Delta^2 Tr(\Delta^\dagger \Delta) + \lambda_1 (H^\dagger H) Tr(\Delta^\dagger \Delta) + \lambda_2 (Tr \Delta^\dagger \Delta)^2 + \lambda_3 Tr(\Delta^\dagger \Delta)^2 + \lambda_4 H^\dagger \Delta^\dagger \Delta + [\mu (H^\dagger i_\tau_2 \Delta^\dagger H) + h.c.]\]

(2)

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0 \\
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0
\end{pmatrix}, \quad \Delta = \frac{1}{\sqrt{2}} \begin{pmatrix}
\delta^+ \\
0 \\
\delta^-
\end{pmatrix}\]

(3)

with \(v^2 = v_\Phi^2 + 2v_\Delta^2 = (246 GeV)^2\). The spectrum of the scalar potential will have seven scalar particles. Indeed, in addition to the pair of doubly charged Higgs bosons \(H^{\pm\pm}\), the HTM provides a pair of charged Higgs bosons \(H^{\pm}\) that appear together with the charged Goldstone \(G^\pm\) after an orthogonal rotation by using the mixing matrix defined by \(R_\beta\), where \(\beta\) denotes the angle between the non-physical fields \(\Phi^\pm\) and \(\delta^\pm\) such that \(\tan \beta_\pm = \sqrt{2} v_\Delta/v_\Phi\). Analagously, the two CP-even neutral scalars \(h^0, H^0\) and the two CP-odd neutral pseudo-scalars \(G^0, A^0\) are obtained by an orthogonal transformation using the following two unitary rotations, \(R_e\) and \(R_\beta\), respectively (see [46] for more details). Moreover, in the HTM, the gauge couplings of the doubly charged scalars, which are related to our calculation, can be written as [48,49]:

\[A_\nu H^{++} H^{--} = -2i e (p_3 - p_4) \nu, \quad Z_\nu H^{++} H^{--} = -i e (1 - 2 S^2_W) (p_3 - p_4) \nu\]

(4)

The short notations \(C_\nu\) and \(S_\nu\) represent successively \(\cos(\theta_W)\) and \(\sin(\theta_W)\), where \(\theta_W\) is the Weinberg angle. \(p_3\) and \(p_4\) denote the four-momentum of \(H^{++}\) and \(H^{--}\) outside the laser field, respectively.

2.2 Laser-assisted cross section

In this section, we perform a theoretical calculation of the differential cross section of the process \(e^+ e^- \rightarrow H^{++} H^{--}\) at tree-level in the presence of a laser field. The Feynman diagrams of this process are shown in Fig. 1.

In this study, we have considered both the produced Higgs bosons \(H^{\pm\pm}\) and the incident \(e^+ e^-\) beam inside a laser field which is considered as a plane, monochromatic and circularly polarized electromagnetic wave, and it is given by the following classical four-potential:

\[A^\mu = \eta_1^\mu \cos \phi + \eta_2^\mu \sin \phi\]

(5)

Here, \(\phi = k \cdot x\) is the phase of the laser field. \(k = (\omega, 0, 0, \omega)\) is the wave four vector, with \(\omega\) is the laser frequency. \(\eta_1 = (0, \eta_1, 0, 0)\) and \(\eta_2 = (0, 0, \eta_2, 0)\) are the polarization four vectors, and they satisfy the following equations: \(\eta_1^2 = \eta_2^2 = \eta_3^2 = \eta_4^2 = 0\).

\(\eta_1 = \eta_2 = \eta_3 = \eta_4 = 0\).
\[-|\eta|^2 = -(\varepsilon_0/\omega)^2\] where \(\varepsilon_0\) is the electric field strength. The application of Lorentz condition, \(\partial_{\mu} A^\mu = 0\), implies that \(k.\eta_1 = 0\) and \(k.\eta_2 = 0\). In the presence of an electromagnetic field, the incident particles are described by Dirac–Volkov functions \[50\]. The latter are written as follows:
\[
\begin{align*}
\Psi_{p_1,s_1}(x) &= \left[ 1 + \frac{e^2 k}{2(k.p_1)} \right] \frac{u(p_1,s_1)}{\sqrt{2Q_{1}V}} \exp [ S(q_1,s_1) ] \\
\Psi_{p_2,s_2}(x) &= \left[ 1 + \frac{e^2 k}{2(k.p_2)} \right] \frac{v(p_2,s_2)}{\sqrt{2Q_{2}V}} \exp [ S(q_2,s_2) ],
\end{align*}
\]
where the first term stands for the electron’s state, and the second one represents the positron’s state. \(x\) indicates the space time coordinate of both the electron and positron. \(u(p_1,s_1)\) and \(v(p_2,s_2)\) are their Dirac bispinors where \(s_i (i = 1, 2)\) denote their spins which satisfy the following sum rules \(\sum_s u(p_1,s) \bar{u}(p_1,s) = (p_1 - m_e)\) and \(\sum_s v(p_2,s) \bar{v}(p_2,s) = (p_2 + m_e)\). \(p_1 = (E_1, |p_1|, 0, 0)\) and \(p_2 = (E_2, -|p_1|, 0, 0)\) are referring to their corresponding free four-momentum in the centre of mass frame. In the presence of the electromagnetic field, the electron and positron get non-vanishing effective energy given by \(Q_i (i = 1, 2)\). We define the arguments of the exponential terms in equation (6) such as:
\[
\begin{align*}
S(q_1,s_1) &= -q_1 x + \frac{e(p_1.p_1)}{k.p_1} \sin \phi - \frac{e(p_2.p_1)}{k.p_1} \cos \phi \\
S(q_2,s_2) &= +q_2 x + \frac{e(p_1.p_2)}{k.p_2} \sin \phi - \frac{e(p_2.p_2)}{k.p_2} \cos \phi,
\end{align*}
\]
with \(q_i (i = 1, 2) = p_1 + e^2 \eta^2 / 2(k.p_i)k\) represents the effective momentum of the electron and positron inside the laser field such that:
\[
q_i^2 = m_e^2 = m_e^2 + e^2 \eta^2.
\]
Here, \(m_e^*\) is the effective mass of the incident particles. \(\eta\) is the charge of the electron, and \(m_e\) denotes its mass. \(Q_i = E_i + e^2 \eta^2 / 2k.p_i \omega\) is the time component of \(q_i\) where \(E_i\) is the particle’s energy outside the laser field. The produced Higgs pair \(H^{++}\) and \(H^{--}\) are charged boson particles, and inside a laser field \(A(\phi)\), the Klein–Gordon Hamiltonian for a boson particle with a mass \(m\) and electric charge \(e\) is expressed as follows:
\[
H(y) = -\Pi_{\mu} \Pi^{\mu} + m^2, \quad \Pi_{\mu} = i \partial_{\mu} - e A_{\mu}(\phi),
\]
where the Volkov solutions of the following equation:
\[
H(y) \varphi(y) = 0,
\]
are expressed as Klein–Gordon–Volkov states \[51\] for \(H^{++}\) and \(H^{--}\) as follows:
\[
\varphi_{p_3}(y) = \frac{1}{\sqrt{2Q_{H^{--}}V}} \exp [ S(q_3,s_3) ] \varphi_{p_4}(y) = \frac{1}{\sqrt{2Q_{H^{++}}V}} \exp [ S(q_4,s_4) ] .
\]
Here, \(y\) is the space time coordinate of outgoing particles. \(q_i (i = 3, 4) = p_1 + \beta e^2 \eta^2 / 2(k.p_i)k\) is the effective momentum of the doubly charged Higgs boson where \(p_3 = (E_{H^{--}}, |p_3| \cos \theta, |p_3| \sin \theta, 0)\) and \(p_4 = (E_{H^{++}}, -|p_4| \cos \theta, -|p_4| \sin \theta, 0)\) are the free four momentums of the doubly charged Higgs bosons, with \(E_{H^{--}}\) and \(E_{H^{++}}\) are their corresponding energies outside the laser field. Inside the laser field, the doubly charged Higgs bosons \(H^{--}\) and \(H^{++}\) acquire the energies \(Q_{H^{--}}\) and \(Q_{H^{++}}\), respectively. In equation (11), the arguments of the exponential term is given by:
\[
\begin{align*}
S(q_3,s_3) &= -q_3 x + \frac{\beta e(p_1.p_3)}{k.p_3} \sin \phi - \frac{\beta e(p_2.p_3)}{k.p_3} \cos \phi \\
S(q_4,s_4) &= +q_4 x + \frac{\beta e(p_1.p_4)}{k.p_4} \sin \phi - \frac{\beta e(p_2.p_4)}{k.p_4} \cos \phi,
\end{align*}
\]
where \(\beta = 2\) denotes the number of charges of the doubly charged Higgs boson. Inside the electromagnetic field, the pair of charged Higgs bosons couplings to the electroweak gauge bosons is given by:
\[
A_{\mu} H^{++} H^{--} = -2ie \left[ (q_3 - q_4)^\nu - \beta e k^\nu \right].
\]
and
\[ Z_v H^{++} H^{-+} = -i \frac{e(1 - 2S_W^2)}{2C_W S_W} \left[ (q_3 - q_4)^\nu - \beta e \Delta k^\nu \right]. \] (14)

The quantity \( \Delta \) is expressed as follows:
\[ \Delta = \left[ \frac{p_3 \cdot \eta_1}{k \cdot p_3} + \frac{p_4 \cdot \eta_1}{k \cdot p_4} \right] \cos(\phi) + \left[ \frac{p_3 \cdot \eta_2}{k \cdot p_3} + \frac{p_4 \cdot \eta_2}{k \cdot p_4} \right] \sin(\phi). \] (15)

By using the gauge couplings given by equations (13, 14), the scattering matrix element \[ 52\] of the laser-assisted doubly charged Higgs boson pair production in HTM can be written as:
\[ S_{fi}(e^+ e^- \rightarrow H^{++} H^{-+}) = \int d^4x \int d^4y \left\{ \psi_{p_{2z}, 2s}(x) \left( \frac{-i e}{2C_W S_W} \gamma^\mu (g_\mu - g_a \gamma^5) \right) \psi_{p_{1z}, 1s}(x) D_{\mu \nu}(x - y) \times \phi_p^a(y)(-i e(1 - 2S_W^2)) \left( (q_3 - q_4)^\nu - \beta e \Delta k^\nu \phi_p(y) + \bar{\psi}_{p_{2z}, 2s}(x)(-i e \gamma^\mu) \times \phi_{p_{1z}, 1s}(x) G_{\mu \nu}(x - y) \phi_p(y) \right), \] (16)

where \( g_\mu^e = -1 + 4 \sin^2(\theta_W) \) and \( g_a^e = 1 \) stand, respectively, for the vector and axial vector coupling constants. The factors \( D_{\mu \nu}(x - y) \) and \( G_{\mu \nu}(x - y) \) are the Feynman propagators for Z-boson and \( \gamma \)-boson, respectively. Their expressions are given by:
\[ D_{\mu \nu}(x - y) = \int \frac{d^4q}{(2\pi)^4} \left( \frac{e^{-iq(x-y)}}{q^2 - M_Z^2} - ig_{\mu \nu} q_i q_i M_Z^2 \right), \] (17)
\[ G_{\mu \nu}(x - y) = \int \frac{d^4q}{(2\pi)^4} \left( \frac{e^{-iq(x-y)}}{q^2} - ig_{\mu \nu} q_i q_i \right), \] (18)

with \( \xi = 1 \) for the Feynman gauge or \( \xi = 0 \) for the Lorentz gauge. \( q \) indicates the four-momentum of the off-shell \( V \) \((V = Z \text{ or } \gamma)\). After introducing a short description of the procedure used to analyse the effects of laser field on the process 1, we substitute the expressions of wave functions (equations (6) and (11)) and Feynman propagators (equations (17) and (18)) into the equation (16). Thus, we obtain:
\[ S_{fi}^{n,n'}(e^+ e^- \rightarrow H^{++} H^{-+}) = \frac{(2\pi)^4 \delta^4(q_3 + q_4 - q_1 - q_2 - (n + n')k)}{4V^2 \sqrt{Q_1 Q_2 Q_{n} Q_{n'}}} \left( M_{Z}^{n,n'} + M_{\gamma}^{n,n'} \right). \] (19)

In the above scattering matrix element expression (equation 19), \( M_{Z}^{n,n'} \) denotes the total scattering amplitude that is coming from the contribution of the Z-boson, while \( M_{\gamma}^{n,n'} \) is coming from the \( \gamma \)-exchange inside the electromagnetic field, and they are given by:
\[ M_{Z}^{n,n'} = \frac{e^2}{2C_W S_W} \left( \frac{1 - 2S_W^2}{S_W C_W} \right) \left( q_1 + q_2 + (n + n')k \right)^2 \left[ \Lambda_0 \phi_n(z) e^{-in' \phi_0} \right. \]
\[ + \frac{1}{2} \Lambda_1 \phi_{n+1}(z) e^{-i(n+1) \phi_0} + J_{n-1}(z) e^{-i(n-1) \phi_0} \]
\[ + \frac{1}{2} \Lambda_2 \phi_{n+1}(z) e^{-i(n+1) \phi_0} - J_{n-1}(z) e^{-i(n-1) \phi_0} \]
\[ \times \left[ \kappa^0 \phi_n(z) e^{-in \phi_0} + \frac{1}{2} \kappa^1 \phi_{n+1}(z) e^{-i(n+1) \phi_0} + J_{n-1}(z) e^{-i(n-1) \phi_0} \right. \]
\[ + \frac{1}{2} \kappa^2 \phi_{n+1}(z) e^{-i(n+1) \phi_0} - J_{n-1}(z) e^{-i(n-1) \phi_0} \left. \right] \bar{u}(p_1, s_1). \] (20)
\[ M_{\gamma}^{n,n'} = \frac{2e^2}{(q_1 + q_2 + (n + n')k)^2} \left[ \Lambda_0 \phi_n(z) e^{-in' \phi_0} + \frac{1}{2} \Lambda_1 \phi_{n+1}(z) e^{-i(n+1) \phi_0} \right. \]
\[ + J_{n-1}(z) e^{-i(n-1) \phi_0} \]
\[ + \frac{1}{2} \Lambda_2 \phi_{n+1}(z) e^{-i(n+1) \phi_0} - J_{n-1}(z) e^{-i(n-1) \phi_0} \]
\[ \times \left[ \lambda_0 \phi_n(z) e^{-in \phi_0} + \frac{1}{2} \lambda_1 \phi_{n+1}(z) e^{-i(n+1) \phi_0} + J_{n-1}(z) e^{-i(n-1) \phi_0} \right. \]
\[ + \frac{1}{2} \lambda_2 \phi_{n+1}(z) e^{-i(n+1) \phi_0} - J_{n-1}(z) e^{-i(n-1) \phi_0} \left. \right] \bar{u}(p_1, s_1). \] (21)
$n$ is interpreted as the number of exchanged photons between the $e^+e^-$ beam and the laser field, while $n'$ is the transferred photons number with the produced $H^{++}H^{--}$. The eight quantities that appear in Eqs. 20 and 21 are expressed as follows:

$$
\begin{align*}
    \lambda_0 & = \gamma + 2b_{p_1} b_{p_2} q \kappa \frac{1}{\sqrt{2}} (g' - g''), \\
    \lambda_1 & = b_{p_1} \gamma (g' - g''), \\
    \lambda_2 & = b_{p_1} \gamma (g' - g''), \\
    \lambda_0' & = \gamma + 2b_{p_1} b_{p_2} q \kappa \frac{1}{\sqrt{2}} (g' - g''), \\
    \lambda_1' & = b_{p_1} \gamma (g' - g''), \\
    \lambda_2' & = b_{p_1} \gamma (g' - g''), \\
    \Lambda_0 & = (q_3 - q_4), \\
    \Lambda_1 & = -\beta e \left( \frac{p_3 \cdot n_1}{k \cdot p_3} + \frac{p_4 \cdot n_1}{k \cdot p_4} \right) k_\mu, \\
    \Lambda_2 & = -\beta e \left( \frac{p_3 \cdot n_2}{k \cdot p_3} + \frac{p_4 \cdot n_2}{k \cdot p_4} \right) k_\mu.
\end{align*}
$$

(22)

where $b_{p_{1,2}} = e/(2(k \cdot p_i))$. We have used the generating function of Bessel functions defined by:

$$
\begin{align*}
    \alpha_1 & = e \left( \frac{(n_1 \cdot p_1)}{(k \cdot p_1)} - \frac{(n_1 \cdot p_2)}{(k \cdot p_2)} \right), \\
    \alpha_2 & = e \left( \frac{(n_2 \cdot p_1)}{(k \cdot p_1)} - \frac{(n_2 \cdot p_2)}{(k \cdot p_2)} \right), \\
    \alpha_3 & = \beta e \left( \frac{(n_1 \cdot p_1)}{(k \cdot p_1)} - \frac{(n_1 \cdot p_4)}{(k \cdot p_4)} \right), \\
    \alpha_4 & = \beta e \left( \frac{(n_2 \cdot p_3)}{(k \cdot p_3)} - \frac{(n_2 \cdot p_4)}{(k \cdot p_4)} \right).
\end{align*}
$$

(26)

(27)

As it is known, to determine the differential cross section in the centre of mass frame, we divide the squared matrix element by $VT$ and by the density of particles $\rho = V^{-1}$, then by the current of the incoming particles given by $|J_{inc}| = (\sqrt{(q_1 q_2)^2 - m^2_e} / Q_1 Q_2 V)$. We obtain the expression of the partial differential cross section as:

$$
\frac{dx}{y} \rho \int d^3 q_3 (2\pi)^3 V \int d^3 q_4 (2\pi)^3 \rho V F. \quad (28)
$$

By averaging over the initial spins and summing over the final ones, and after some algebraic calculations, the differential cross section will be as follows:

$$
\int d^4 x f(x) \delta(g(x)) = \frac{f(x)}{g(x) |g(x)|_{g(x)=0}}.
$$

(30)
Finally, the differential cross section corresponding to the doubly charged Higgs bosons pair production via $e^+e^-$ annihilation is given by:

$$
\frac{d\sigma_{n,n'}}{d\Omega}(e^+e^- \rightarrow H^+H^-) = \frac{1}{16\sqrt{(q-q_+)^2-m^2_e}} \left| M_Y^{n,n'} + M_Z^{n,n'} \right|^2 \frac{2|q_3|^2}{(2\pi)^2 Q_H} \frac{1}{g'(|q_3|)|g'(q_3)|=0},
$$

where

$$
|g'(|q_3|)| = -2 \left[ (\sqrt{s} + (n+n')\omega + \frac{e^2\eta^2}{2\sqrt{s}}) \right] \frac{|q_3|}{\sqrt{|q_3|^2 + M_{H^\pm}^2}},
$$

with $M_{H^\pm}^2 = M_{H^\pm}^2 + \beta e^2\eta^2$ is the mass of the doubly charged Higgs $H^\pm$ inside the laser field. To find the expression of the quantity $|M_Y^{n,n'} + M_Z^{n,n'}|^2$ given in equations 29 and 31, we have used the FeynCalc program [53,54], and its expression is given in Appendix. We have checked the gauge invariance of the total cross section, and for simplicity reasons, we have considered the Feynman gauge, $\xi = 1$, in our calculations. It is known that the effect of the laser field on charged particle is caused by its effective mass acquired inside the laser field. In addition, this acquired mass is important as much as the particle mass is small and by increasing the laser field strength. We illustrate in Table 1 the effect of the laser field intensity on both the electron’s mass and the doubly charged Higgs mass.

| Frequency | $\epsilon_0 [V \cdot cm^{-1}]$ | $m_e^2 [GeV]$ | $M_{H^\pm}^2 [GeV]$ |
|-----------|-----------------|---------------|-----------------|
| $\omega = 0.117 eV$ | 10 | $0.511 \times 10^{-3}$ | 300 |
| $\omega = 1.17 eV$ | 10 | $0.511 \times 10^{-3}$ | 300 |

**Table 1** Effective masses of incident and scattered particles inside a laser field for different laser field strengths and frequencies

It is obvious from this table that, for the intensities considered in this paper, the electromagnetic field has no effect on the doubly charged Higgs mass. For this reason, we will only consider the $e^+e^-$ beam inside the laser field. As a consequence, for the intensities and Higgs masses considered in this study, there will be a linear relation between the leading-order total cross section of the considered process with that obtained for laser-assisted charged Higgs pair production in IHDM [55]. In this case, if we choose the same laser parameters as well as the same Higgs mass and centre of mass energies as in [55], we will get a total cross section which is four times higher than the one obtained in [55], and this is due to the difference between charges of the produced Higgs bosons.

### 3 Results and discussion

In this section, we present and analyse our finding results for the doubly charged Higgs boson production process through $e^+e^-$ annihilation in the centre of mass frame. Our calculations focus on the case in which the wave 4-vector of the laser field is taken to be parallel to the z-axis. Moreover, we can obtain the total cross section by performing a numerical integration of the differential cross section, given by equation (31), over the solid angle $d\Omega$. For the numerical analysis, we take the SM parameters from PDG [56] such...
that \( m_e = 0.511 \text{MeV}, M_Z = 91.1875 \text{GeV}, \) and the mixing angle \( \sin^2 \theta_W = 0.23126. \) As we have shown in equation 31, besides the mass of the doubly charged Higgs boson \( M_{H^{++}} \) and \( e^+ e^- \) colliding energy, the cross section depends on the electromagnetic field parameters such as the number of exchanged photons \( n \), the laser field strength \( \varepsilon_0 \) and its frequency \( \omega \). Before presenting the results of our scan, we want to test our numerical calculations for the obtained laser-assisted total cross section. Therefore, we have focused our analysis on the effect of a circularly polarized electromagnetic field on the scattering process. We start our discussion with a very important point which concerns the behaviour of the partial total cross section that corresponds to each four-momentum conservation \( \delta(p_3 + p_4 = q_1 + q_2 - n k) = 1 \), versus the number of exchanged photons.

In Fig. 2, we compare the laser-assisted total cross section of the process \( e^+ e^- \rightarrow H^{++}H^{--} \) in the presence of a circularly polarized laser field with its corresponding laser-free total cross section [48]. The presence of a laser field implies a long and complicated calculation. This makes it difficult for us to verify our calculations analytically, and this comparison technique provides the possibility of testing the validity of our results. Figure 2 clearly shows that, if we consider the laser field parameters as zero, the two total cross sections are very similar, and this result is in excellent agreement with our theoretical calculation. Now, we will focus our analysis on the effect of a circularly polarized electromagnetic field on the scattering process. We start our discussion with a very important point which concerns the behaviour of the partial total cross section that corresponds to each four-momentum conservation \( \delta(p_3 + p_4 = q_1 + q_2 - n k) = 1 \), versus the number of exchanged photons.

![Fig. 2](https://example.com/figure2.png)

**Table 2** Laser-assisted total cross section as a function of the number of exchanged photons for different laser field strengths and frequencies. The centre of mass energy and the doubly charged Higgs mass are chosen as \( \sqrt{s} = 1000 \text{GeV} \) and \( M_{H^{++}} = 300 \text{GeV} \), respectively.

| \( \varepsilon_0 (V \cdot \text{cm}^{-1}) \) | \( n \) | \( \sigma \) [fb] | \( \sigma \) [fb] | \( \sigma \) [fb] |
|---|---|---|---|---|
| CO\(_2\) Laser \( \omega = 0.117 \text{eV} \) | | | | |
| \( 10^5 \) | \( \pm 1300 \) | \( 0 \) | \( \pm 18 \) | \( 0 \) | \( \pm 8 \) | \( 0 \) |
| \( \pm 1050 \) | \( 0 \) | \( \pm 15 \) | \( 0 \) | \( \pm 6 \) | \( 0 \) |
| \( \pm 900 \) | \( 0.00043251 \) | \( \pm 12 \) | \( 0.319588 \) | \( \pm 4 \) | \( 2.41077 \) |
| \( \pm 600 \) | \( 0.0426101 \) | \( \pm 8 \) | \( 5.45433 \) | \( \pm 3 \) | \( 8.64928 \) |
| \( \pm 300 \) | \( 0.0268137 \) | \( \pm 4 \) | \( 1.95167 \) | \( \pm 2 \) | \( 12.1979 \) |
| \( 0 \) | \( 0.0347302 \) | \( 0 \) | \( 3.60327 \) | \( 0 \) | \( 8.35336 \) |
| \( 10^6 \) | \( \pm 6000 \) | \( 0 \) | \( \pm 150 \) | \( 0 \) | \( \pm 50 \) | \( 0 \) |
| \( \pm 5100 \) | \( 0 \) | \( \pm 120 \) | \( 0 \) | \( \pm 40 \) | \( 0 \) |
| \( \pm 4000 \) | \( 0.000585962 \) | \( \pm 90 \) | \( 0.756126 \) | \( \pm 30 \) | \( 0.660953 \) |
| \( \pm 2000 \) | \( 0.000651189 \) | \( \pm 60 \) | \( 0.425914 \) | \( \pm 20 \) | \( 0.666273 \) |
| \( \pm 1000 \) | \( 0.000741295 \) | \( \pm 30 \) | \( 0.122119 \) | \( \pm 10 \) | \( 0.255696 \) |
| \( 0 \) | \( 0.0030297 \) | \( 0 \) | \( 0.145686 \) | \( 0 \) | \( 0.915362 \) |
| \( 10^7 \) | \( \pm 12000 \) | \( 0 \) | \( \pm 1300 \) | \( 0 \) | \( \pm 500 \) | \( 0 \) |
| \( \pm 9150 \) | \( 0 \) | \( \pm 1100 \) | \( 0 \) | \( \pm 400 \) | \( 0 \) |
| \( \pm 6000 \) | \( 0.000360217 \) | \( \pm 900 \) | \( 0.000432513 \) | \( \pm 300 \) | \( 0.0122968 \) |
| \( \pm 3000 \) | \( 0.000294922 \) | \( \pm 600 \) | \( 0.0426101 \) | \( \pm 200 \) | \( 0.0996274 \) |
| \( \pm 1000 \) | \( 0.000268565 \) | \( \pm 300 \) | \( 0.0268137 \) | \( \pm 100 \) | \( 0.0261197 \) |
| \( 0 \) | \( 0.000347827 \) | \( 0 \) | \( 0.0347302 \) | \( 0 \) | \( 0.0989345 \) |
Fig. 3 Laser-assisted total cross section as a function of the number of exchanged photons by taking the centre of mass energy and the charged Higgs mass as $\sqrt{s} = 1000GeV$ and $M_{H^\pm\pm} = 300GeV$, respectively. The He:Ne Laser ($\omega = 2eV$) is used in both figures with $\epsilon_0 = 10^7 V.cm^{-1}$ (left panel) and $\epsilon_0 = 10^6 V.cm^{-1}$ (right panel).

(a) (b) (c)

Fig. 4 Laser-assisted total cross section of $e^+e^- \rightarrow H^{++}H^{--}$ as a function of the centre of mass energy for different exchanged photons number and by taking $M_{H^\pm\pm} = 300GeV$. The laser field strength and its frequency are chosen as: $\epsilon_0 = 10^5 V.cm^{-1}$ and ($\omega = 0.117eV$) in (a); $\epsilon_0 = 10^6 V.cm^{-1}$ and ($\omega = 1.17eV$) in (b); $\epsilon_0 = 10^7 V.cm^{-1}$ and ($\omega = 2eV$) in (c).

We illustrate, in Table 2, the variation of the partial total cross section of the process $e^+e^- \rightarrow H^{++}H^{--}$ versus the number of exchanged photons $n$. This variation is presented for different laser field strengths and frequencies. The transfer of photons between the laser field and the scattering process indicates that the incoming particles are interacting with the laser field. In addition, we see that for the laser field strength, $\epsilon_0 = 10^7 V.cm^{-1}$, and for a specific laser frequency $\omega = 0.117eV$, a significant number of photons can be exchanged ($\pm 1050$ photons) between the laser field and the colliding physical system. Moreover, this number of exchanged photons is enhanced with the increase of the laser field strength. For instance, for $\epsilon_0 = 10^6 V.cm^{-1}$, the greatest value of photons number that can be exchanged is equal to $\pm 5100$, while the cutoff number is $\pm 9150$ for the case where $\epsilon_0 = 10^7 V.cm^{-1}$. This result indicates that the incident particles interact strongly with high laser field strengths. Thus, the effect of electromagnetic field on the electron–positron scattering process becomes prominent and important, and as a consequence, the partial total cross section will be affected and changed. As an example, Fig. 3 shows the partial cross section as a function of the number of laser photons absorbed ($n > 0$) or emitted ($n < 0$) for $\omega = 2eV$ and for two different known laser strengths. One can see in Fig. 3 (left panel) that the maximum number of photons that can be transferred and for which the partial total cross section vanishes for $\epsilon_0 = 10^7 V.cm^{-1}$ is greater than that corresponds to $\epsilon_0 = 10^6 V.cm^{-1}$ (right panel). Therefore, these results are in good agreement with those given in Table 2.
Fig. 5 Laser-assisted total cross section of the process $e^+e^- \rightarrow H^+H^-$ as a function of the doubly charged Higgs mass and $e^+e^-$ colliding energy by summing over $n$ from $-20$ to $20$ and by taking the laser field strength and its frequency as $\varepsilon_0 = 10^6\text{V.cm}^{-1}$ and $\omega = 1.17\text{eV}$, respectively.

Table 3 Laser-assisted total cross section of $H^+H^-$ production at $\sqrt{s}=1000\text{GeV}$ for some typical values of $M_{H^{\pm\pm}}$. The laser’s parameters and the number of exchanged photons are chosen as: $\varepsilon_0 = 10^6\text{V.cm}^{-1}$, $\omega = 1.17\text{eV}$ and $n = \pm 20$

| $M_{H^{\pm\pm}}$ [GeV] | $\sigma$ [fb] |
|-------------------------|--------------|
| 320                     | 7.021        |
| 340                     | 6.399        |
| 360                     | 5.754        |
| 380                     | 5.089        |
| 400                     | 4.407        |
| 420                     | 3.709        |
| 440                     | 2.992        |
| 460                     | 2.249        |
| 480                     | 1.443        |
| 500                     | 1.132        |

In Fig. 4, we plot the dependence of the laser-assisted total cross section $\sigma$ on the centre of mass energy for different laser parameters. We use different colours to clearly show the influence of the laser field on the order of magnitude of the total cross section. As it can be seen, due to the phase space suppression, the total cross section $\sigma$ declines by increasing $\sqrt{s}$. Moreover, for all cases, the exchange of a large number of photons will always give a quite large values of cross section until $n$ reaches ± cutoff. For instance, in Fig. 4b, in which $\varepsilon_0 = 10^6\text{V.cm}^{-1}$ and $\omega = 1.17\text{eV}$, the total cross section remains under $24\text{ fb}$ for $n = \pm 60$, while it reaches up to $40\text{ fb}$ for $n = \pm 90$. The summation over ± cutoff number is called sum-rule [57, 58], and it leads to a cross section which is equal to its corresponding laser-free cross section in all centre of mass energies. Another important point which should be discussed, here, is that the laser parameters have a great effect on the order of magnitude of the cross section. By comparing Figs. 4a and c, for the same number of exchanged photons such as $n = \pm 300$, we observe that the maximum value of the cross section mostly remains under $12\text{ fb}$ for $\varepsilon_0 = 10^5\text{V.cm}^{-1}$ and $\omega = 0.117\text{eV}$, and it increases to about $38\text{ fb}$ for $\varepsilon_0 = 10^7\text{V.cm}^{-1}$ and $\omega = 2\text{eV}$.

In order to study the dependence of the total cross section $\sigma$ on the centre of mass energy and the mass of doubly charged Higgs, we present in Fig. 5 the total laser-assisted cross section in the $(M_{H^{\pm\pm}}, \sqrt{s})$ plane. The numerical results for some typical values of $M_{H^{\pm\pm}}$ are also given in Table 3 where $\sqrt{s} = 1000\text{GeV}$. From Fig. 5, we can see that for light doubly charged Higgs mass, the total cross section is rather important. This imposes a sever constraint on $\sqrt{s}$. Namely, for $M_{H^{\pm\pm}} < 310\text{GeV}$, $\sqrt{s}$ is obliged to be in the range of $1000 \sim 1200\text{GeV}$. The range of $\sqrt{s}$ becomes more large as long as the doubly charged Higgs mass increases. Consequently, the total cross section declines. It is remarkable that there exists a small region of light doubly charged Higgs boson, i.e. the doubly charged Higgs boson mass can be around $300\text{GeV}$, and with a large $\sqrt{s}$ ($\sqrt{s} > 2900\text{GeV}$) in which the total cross section is less than $1\text{ fb}$. Table 3 shows that the total laser-assisted cross section decreases from about $7.02\text{ fb}$ to around $1.13\text{ fb}$ as $M_{H^{\pm\pm}}$ increases from $320$ to $500\text{GeV}$. We mention that the chosen doubly charged Higgs mass in Table 3 is consistent with the HTM constraints [48]. Before ending this section, we present in Fig. 6 the influence of the laser field amplitude on the cross section for different known frequencies and different number of exchanged photons which are indicated in each plot by the same type of lines. From this plot, it is obvious that, for small laser intensities $\varepsilon_0 < 10^5\text{V.cm}^{-1}$, all curves are very similar. This means that the impact of laser field on the incoming particles is strongly suppressed. Consequently, no photon will be transferred between the laser field and the colliding physical system. From Fig. 6a, in which $\omega = 0.117\text{eV}$, both curves correspond to $n = \pm 50$ and $n = \pm 100$ begin to deviate from $\varepsilon_0 \geq 10^3\text{V.cm}^{-1}$, while the other curves begin to deviate from each other when $\varepsilon_0 \geq 10^4\text{V.cm}^{-1}$. Therefore, the laser-assisted total cross section decreases until it becomes zero. Moreover, by comparing Fig. 6a and b, we remark that, for the same number of exchanged photons such as $n = \pm 900$, the order of magnitude of the total cross section vanishes as much as the laser field strength is very close to $10^7\text{V.cm}^{-1}$ and $10^9\text{V.cm}^{-1}$ for $\omega = 0.117\text{eV}$ and $\omega = 1.17\text{eV}$, respectively.
4 Conclusion

Searching for new Higgs boson state may reveal first signs of new physics beyond the standard model (BSM). In this work, we study in detail the doubly charged Higgs pair production at future electron–positron colliders in the presence of a circularly polarized electromagnetic field within the framework of HTM. We calculate the total cross section of the process \( e^+e^- \rightarrow H^{++}H^{--} \) inside the laser field. Then, we study the dependence of the production cross section on \( M_{H^{+\pm}} \) and \( e^+e^- \) colliding energy as well as on the laser field parameters such as the number of exchanged photons, the laser field strength and its frequency. The numerical results show that there is a correlation between the number of exchanged photons and the laser field strength. More accurately, this number enhances as long as the laser field strength increases. In addition, as far as the number \( n \) takes high values, the total laser-assisted cross section increases until it becomes equal to its corresponding laser-free cross section. We have provided some numerical results for \( \sigma \) as a function of \( M_{H^{+\pm}} \) for different known frequencies at \( \sqrt{s} = 1000GeV \). We have found that the total cross section varies in the range of \( 7 \sim 1.13 \text{ fb} \) with the increment of \( M_{H^{+\pm}} \) from 300 to 500GeV. Finally, we indicate that the behaviour of the total cross section does not change for small laser intensities while it is affected and changed for high laser field strengths.

5 Appendix

In this appendix, we give the expression of the quantity \( M_f^2 = \left| M_V^{n,n'} + M_Z^{n,n'} \right|^2 \) that appears in equation (31).

\[
M_f^2 = \frac{1}{4} \sum_{n=-\infty}^{\infty} \sum_{n'=\infty}^{\infty} \left| M_V^{n,n'} + M_Z^{n,n'} \right|^2 = \frac{1}{4} \sum_{n=-\infty}^{\infty} \sum_{n'=\infty}^{\infty} \left[ \frac{4e^4}{(q_1 + q_2 + (n + n')k)} \right] \text{Tr} \left[ \Lambda_0 \mu_0 \left( J_{n+1}(z') e^{-i(n+1)0^+} + J_{n-1}(z') e^{-(n-1)0^+} \right) + \Lambda_2 \mu_2 \left( J_{n+1}(z') e^{-i(n+1)0^+} - J_{n-1}(z') e^{-(n-1)0^+} \right) \right] \nonumber \\
\times \left( \left( J_{n+1}(z) e^{-i(n+1)0^+} + J_{n-1}(z) e^{-(n-1)0^+} \right) + \Lambda_4 \mu_4 \left( J_{n+1}(z) e^{-i(n+1)0^+} - J_{n-1}(z) e^{-(n-1)0^+} \right) \right) 
\]
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