The Natural Explanation of the Muon Anomalous Magnetic Moment via the Electroweak Supersymmetry from the GmSUGRA in the MSSM

Waqas Ahmed,1 Intiaz Khan,2,3 Jinmian Li,4 Tianjun Li,2,3 Shabbar Raza,5 and Wenxing Zhang6

1School of Physics, Nankai University, No.94 Weijin Road, Nankai District, Tianjin, China
2CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
3School of Physical Sciences, University of Chinese Academy of Sciences, No. 19A Yuquan Road, Beijing 100049, China
4College of Physics, Sichuan University, Chengdu 610065, China
5Department of Physics, Federal Urdu University of Arts, Science and Technology, Karachi 75300, Pakistan
6Tsung-Dao Lee Institute and School of Physics and Astronomy, Shanghai Jiao Tong University, 800 Dongchuan Rd., Minhang, Shanghai 200240, China

The Fermi-Lab Collaboration has announced the results for the measurement of muon anomalous magnetic moment. Combining with the previous results by the BNL experiment, we have 4.2σ deviation from the Standard Model (SM), which strongly implies the new physics around 1 TeV. To explain the muon anomalous magnetic moment naturally, we analyze the corresponding five Feynman diagrams in the Supersymmetric SMs (SSMs), and show that the Electroweak Supersymmetry (EWSUSY) is definitely needed. We realize the EWSUSY in the Minimal SSM (MSSM) with Generalized Minimal Supergravity (GmSUGRA). We find large viable parameter space, which is consistent with all the current experimental constraints. In particular, the Lightest Supersymmetric Particle (LSP) neutralino can be at least as heavy as 550 GeV. Most of the viable parameter space can be probed at the future HL-LHC, while we do need the future HE-LHC to probe some viable parameter space. However, it might still be challenge if R-parity is violated.

Introduction.— After Higgs particle was discovered at the LHC, the Standard Model (SM) has been confirmed. However, there exists some problems, and thus we need to study the new physics beyond the SM. It is well-known that Supersymmetry (SUSY) is the most natural solution to the gauge hierarchy problem. In the supersymmetric SMs (SSMs) with R-parity, we can achieve gauge coupling unification [1], have the Lightest Supersymmetric Particle (LSP) like the lightest neutralino as dark matter (DM) candidate [2], and break the electroweak (EW) gauge symmetry radiatively because of the large top quark Yukawa coupling, etc. Moreover, gauge coupling unification strongly implies the Grand Unified Theories (GUTs) [3–7], and the SSMs and SUSY GUTs can be constructed from superstring theory. Therefore, supersymmetry provides a bridge between the low energy phenomenology and high-energy fundamental physics, and thus is the most promising new physics beyond the SM.

However, after the second run at the Large Hadron Collider (LHC), we still did not have any SUSY signals, and then the LHC SUSY searches have already given strong constraints on the SSMs. For instance, the masses of the gluino, first-two generation squarks, stop, and sbottom must be larger than about 2.3 TeV, 1.9 TeV, 1.25 TeV, and 1.5 TeV, respectively [8–12]. Thus, at least the colored supersymmetric particles (sparticles) must be heavy around a few TeV.

Interestingly, a well-known long-standing deviation is a 3.7 σ discrepancy for the muon anomalous magnetic moment $a_\mu = (g_\mu - 2)/2$ between the experimental results $[13,14]$ and theoretical predictions $[15,18]$.

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (27.4 \pm 7.3) \times 10^{-10} .$$

(1)

Computing the hadronic light-by-light contribution with all errors under control by using lattice QCD, several groups have tried to improve the precision of the SM predictions $[19,22]$. And the $\Delta a_\mu$ discrepancy has been confirmed by the recent lattice calculation for the hadronic light-by-light scattering contribution $[23]$, and then a new physics explanation is needed. Also, the ongoing experiment at Fermilab $[24,25]$ and one planned at J-PARC $[26]$ will try to reduce the uncertainty.

To escape the LHC SUSY search constraints, explain the muon anomalous magnetic moment, and be consistent with various experimental results, some of us proposed the Electroweak Supersymmetry (EWSUSY) $[27,29]$, where the squarks and/or gluinos are around a few TeV while the sleptons, sneutrinos, Bino and Winos are within about 1 TeV. The Higgsinos (or say the Higgs bilinear $\mu$ term) can be either heavy or light. Especially, the EWSUSY can be realized in the Generalized Minimal Supergravity (GmSUGRA) $[30,31]$.

Recently, the Fermi-Lab Collaboration has announced the results for the measurement of the anomalous magnetic moment of the muon. Combining with the previous results by the Brookhaven National Lab (BNL) experiment, we have 4.2σ deviation from the SM $[32]$.

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (25.1 \pm 5.9) \times 10^{-10} .$$

(2)

Because it strongly suggests the new physics around 1 TeV, we shall explain the muon anomalous magnetic moment in the SSMs in this paper. Here, we will not consider the SSMs where the supersymmetry breaking soft terms are introduced at the low scale since such kind of scenarios has more freedoms and might not be consistent with the GUTs and string models. To explain
the muon anomalous magnetic moment naturally, we analyze five corresponding Feynman diagrams, and show that the EWSUSY is definitely needed. We realize the EWSUSY in the Minimal SSM (MSSM) with the GmSUGRA. We find large viable parameter space, which is consistent with all the current experimental constraints. In particular, the LSP neutralino can be at least as heavy as 550 GeV. Most of the viable parameter space can be probed by the future High Luminosity-LHC (HL-LHC), while we do need the future High Energy-LHC (HE-LHC) with center-of-mass energy 27 TeV to probe some viable parameter space. However, it may be a big challenge if R-parity is violated.

FIG. 1: Feynman diagrams that give the dominant SUSY contributions to the $a_\mu$.

**Muon Anomalous Magnetic Moment and EWSUSY.**— There are five Feynman diagrams in the SSMs which will contribute to $\Delta a_\mu$ [33], as given in Fig. 1. First, for $M_2 < M_1$ and $M_1 > 0$, diagrams (a), (b), and (c) give positive contributions, while diagrams (d) and (e) give negative contributions. Second, if Higgsino is very heavy, only the diagram (b) will give dominant contribution. Third, if the mass splitting between the muon neutrino and left-handed smuon are small as in our study, the sum of the diagrams (a) and (d) is positive, i.e., the contributions from diagram (a) is generically larger. Fourth, the contribution from diagram (c) is always relatively smaller compared to these from diagrams (a) and (b) in our study. Of course, the contribution from diagram (c) can be dominant if we choose light Bino, Higgsino, and left-handed smuon, as well as heavy $M_2$ and right-handed smuon by hand at low energy, and we have confirmed it numerically. However, this is not consistent with GUTs and string models since larger $M_2$ will increase the left-handed smuon mass due to the renormalization group equation (RGE) running. Fifth, we find that the contribution to $\Delta a_\mu$ from diagram (e) is smaller than $6 \times 10^{-10}$ in our study, and is generically smaller than $10 \times 10^{-10}$, i.e., out of $2\sigma$ region. In short, within $2\sigma$ region, we can only explain the muon anomalous magnetic moment via diagrams (a) and (b). And then we obtain that the sleptons, sneutrinos, Bino and Wino must be light and cannot be much heavier than 1 TeV. Also, if diagram (b) gives dominant contribution, the Higgsinos can be very heavy, while Wino cannot be very heavy since it will contribute to the left-handed smuon mass due to the RGE running. Therefore, we have shown that EWSUSY is definitely needed to explain the muon anomalous magnetic moment.

The EWSUSY from the GmSUGRA in the MSSM.— The EWSUSY can be realized in the GmSUGRA [30, 31], where the sleptons and electroweakinos (charginos, Bino, Wino, and/or Higgsinos) are within about 1 TeV while squarks and/or gluinos can be in several TeV mass ranges [27–29]. The supersymmetry breaking soft (SBS) terms in the GmSUGRA [30, 31] are

\[
M_3 = \frac{5}{2} M_1 - \frac{3}{2} M_2 ,
\]

\[
m_{\tilde{Q}_i}^2 = \frac{5}{6} \left( m_{\tilde{U}_i}^2 + \frac{1}{3} m_{\tilde{E}_i}^2 \right) ,
\]

\[
m_{\tilde{D}_i}^2 = \frac{5}{3} \left( m_{\tilde{U}_i}^2 - \frac{2}{3} m_{\tilde{E}_i}^2 \right) ,
\]

\[
m_{\tilde{L}_i}^2 = \frac{5}{3} \left( m_{\tilde{D}_i}^2 - \frac{2}{3} m_{\tilde{L}_i}^2 \right) ,
\]

where $M_1$, $M_2$, $M_3$ are the masses for Bino, Wino, and gluino at the GUT scale, respectively, as well as $m_{\tilde{Q}_i}$, $m_{\tilde{U}_i}$, $m_{\tilde{D}_i}$, and $m_{\tilde{L}_i}$ represent the scalar masses of the left-handed squark doublets, right-handed up-type squarks, right-handed down-type squarks, left-handed sleptons, and right-handed sleptons, respectively, while $m_{\tilde{L}_i}$ is the universal scalar mass, as in the mSUGRA. In the EWSUSY, $m_{\tilde{L}_i}$ and $m_{\tilde{E}_i}$ are both within about 1 TeV, resulting in light sleptons. Especially, in the limit $m_{\tilde{L}_i} \gg m_{\tilde{L}/\tilde{E}_i}$, we have the approximated relations for
Also, we consider the following parameter ranges (see [34]). We have performed the random scans for the parameter space given below. In this package, the weak-scale values of the gauge and third generation Yukawa couplings are evolved to \( M_{GUT} \) via the MSSM RGEs in the \( \overline{DR} \) regularization scheme. We do not strictly enforce the unification condition \( g_3 = g_1 = g_2 \) at the GUT scale \( M_{GUT} \), since a few percent deviation from unification can be assigned to the unknown GUT-scale threshold corrections [35]. With the boundary conditions given at \( M_{GUT} \), all the SBS parameters, along with the gauge and Yukawa couplings, are evolved back to the weak scale \( M_Z \) (for more detail see [34]). We have performed the random scans for the following parameter ranges

\[
100 \text{ GeV} \leq m_{U} \leq 1000 \text{ GeV} ,
100 \text{ GeV} \leq |M_1| \leq 1600 \text{ GeV} ,
100 \text{ GeV} \leq |M_2| \leq 10000 \text{ GeV} ,
100 \text{ GeV} \leq m_{L} \leq 10000 \text{ GeV} ,
100 \text{ GeV} \leq m_{E} \leq 10000 \text{ GeV} ,
-10000 \text{ GeV} \leq m_{\tilde{H}_{u,d}} \leq 10000 \text{ GeV} ,
-10000 \text{ GeV} \leq A_{U} = A_{D} \leq 10000 \text{ GeV} ,
-10000 \text{ GeV} \leq A_{E} \leq 10000 \text{ GeV} ,
2 \leq \tan \beta \leq 60 . \tag{7}
\]

Also, we consider \( \mu > 0 \), and use \( m_t = 173.3 \text{ GeV} \) and \( m_{\tilde{D}_R}(M_Z) = 2.83 \) GeV [36]. Note that our results are not too sensitive to one or two sigma variations in the value of \( m_t \) [37]. Also, we will use the notations \( A_c, A_b, A_t \) for \( A_{U}, A_{D} \) and \( A_{E} \), respectively. In scanning the parameter space, we employ the Metropolis-Hastings algorithm as described in [35]. In particular, we also perform some focus scans inspired from the diagrams (a) and (b), which do give us better viable parameter spaces to explain the \( \Delta a_{\mu} \). The collected data points all satisfy the requirement of the Radiative Electroweak Symmetry Breaking (REWSB), with the lightest neutralino being the LSP.

**Scanning Process.**—We employ the ISAJET 7.85 package [34] to perform random scans over the parameter space given below. In this package, the weak-scale values of the gauge and third generation Yukawa couplings are evolved to \( M_{GUT} \) via the MSSM RGEs in the \( \overline{DR} \) regularization scheme. We do not strictly enforce the unification condition \( g_3 = g_1 = g_2 \) at the GUT scale \( M_{GUT} \), since a few percent deviation from unification can be assigned to the unknown GUT-scale threshold corrections [35]. With the boundary conditions given at \( M_{GUT} \), all the SBS parameters, along with the gauge and Yukawa couplings, are evolved back to the weak scale \( M_Z \) (for more detail see [34]). We have performed the random scans for the following parameter ranges

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100 \text{ GeV} \leq |M_2| \leq 10000 \text{ GeV} ,
100 \text{ GeV} \leq m_{L} \leq 10000 \text{ GeV} ,
100 \text{ GeV} \leq m_{E} \leq 10000 \text{ GeV} ,
-10000 \text{ GeV} \leq m_{\tilde{H}_{u,d}} \leq 10000 \text{ GeV} ,
-10000 \text{ GeV} \leq A_{U} = A_{D} \leq 10000 \text{ GeV} ,
-10000 \text{ GeV} \leq A_{E} \leq 10000 \text{ GeV} ,
2 \leq \tan \beta \leq 60 . \tag{7}
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Also, we consider \( \mu > 0 \), and use \( m_t = 173.3 \text{ GeV} \) and \( m_{\tilde{D}_R}(M_Z) = 2.83 \) GeV [36]. Note that our results are not too sensitive to one or two sigma variations in the value of \( m_t \) [37]. Also, we will use the notations \( A_c, A_b, A_t \) for \( A_{U}, A_{D} \) and \( A_{E} \), respectively. In scanning the parameter space, we employ the Metropolis-Hastings algorithm as described in [35]. In particular, we also perform some focus scans inspired from the diagrams (a) and (b), which do give us better viable parameter spaces to explain the \( \Delta a_{\mu} \). The collected data points all satisfy the requirement of the Radiative Electroweak Symmetry Breaking (REWSB), with the lightest neutralino being the LSP.

**Constraints.**—After collecting the data, we impose the bounds that the LEP2 experiments set on charged sparticle masses \( (\gtrsim 100 \text{ GeV}) \) [39], for Higgs mass bounds [40] due to an estimated 2 GeV theoretical uncertainty in the calculation of \( m_h \) in the MSSM – see e.g. [41] – we apply the constraint from the Higgs boson mass to our results as \( m_h = [122, 128] \text{ GeV} \). In addition, based on [8-12], we consider the constraints on gluino \( m_{\tilde{g}} \gtrsim 2.2 \text{ TeV} \). The constraints from rare decay processes \( B_s \to \mu^+\mu^- \), \( b \to s \gamma \) [43], and \( B_u \to \tau\nu\tau \) [44]. We also require the relic abundance of the LSP neutralino to satisfy the Planck bound within \( 5\sigma \) [45]. More explicitly, we set

\[
m_h = 122 - 128 \text{ GeV} \tag{8}
\]

\[
m_{\tilde{g}} \geq 2.2 \text{ TeV} \tag{9}
\]

\[
0.8 \times 10^{-9} \leq \text{BR}(B_s \to \mu^+\mu^-) \leq 6.2 \times 10^{-9} \text{ (2}\sigma) \tag{10}
\]

\[
2.99 \times 10^{-4} \leq \text{BR}(b \to s\gamma) \leq 3.87 \times 10^{-4} \text{ (2}\sigma) \tag{11}
\]

\[
0.15 \leq \frac{\text{BR}(B_u \to \tau\nu\tau)_{\text{MSSM}}}{\text{BR}(B_u \to \tau\nu\tau)_{\text{SM}}} \leq 2.41 \text{ (3}\sigma) \tag{12}
\]

\[
0.114 \leq \Omega_{\text{CDM}} h^2 \text{(Planck)} \leq 0.126 \text{ (5}\sigma) \tag{13}
\]
Results.—We shall discuss results of the scans in the following. In Fig. 2 we display plot in $m_{\tilde{\chi}_0^\pm} - \Delta a_\mu$ plane. So in our model, we can explain $\Delta a_\mu$ very easily. In particular, some viable parameter spaces including the viable parameter spaces with the correct dark matter relic density have $\Delta a_\mu$ around the central value $25.1 \times 10^{-10}$. Note that the peak at $m_{\tilde{\chi}_0^\pm} \sim 300$ GeV is not the unique feature but the artifact of focused scans. Another important point to be noted is that the solutions with relatively heavy LSP neutralino $\sim 600$ GeV are still consistent within $2\sigma$ values of $\Delta a_\mu$. As far as we know, no one has found such kind of viable parameter space before. Thus, it is very interesting. However, we want to make a comment here about the solutions with $m_{\tilde{\chi}_0} > 600$ GeV: we have checked some points and found that some of them might not be numerically stable.

In Fig. 3 we plot in $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_0^\pm}$ and $m_{\tilde{\chi}_2^\pm} - m_{\tilde{\chi}_1^\pm}$. The solid black and red curves are the 95% CL exclusion limits on $\chi_1^\pm \chi_1^\mp$ and $\chi_2^\pm \chi_2^\mp$ from pair productions with $\ell$-mediated decays as a function of the $\tilde{\chi}_1^\pm$, $\tilde{\chi}_2^\pm$ and $\tilde{\chi}_0^\pm$ masses [46]. Purple and green patches as stated before are due to the focused scans. In the left panel, we see that we have two sets of solutions which satisfy $\Delta a_\mu$ within $1\sigma$ and $2\sigma$ that is the compressed region where $\tilde{\chi}_0^\pm$ and $\tilde{\chi}_1^\pm$ are almost degenerate in mass and region where $\tilde{\chi}_0^\pm$ and $\tilde{\chi}_2^\pm$ mass splitting is greater than 500 GeV. Green solutions in the compressed region mostly belongs to diagram (a) of Fig. 4 and the solutions with heavy charginos masses but light $m_{\tilde{\chi}_0}$ belongs to diagram (b) of Fig. 4. We show three benchmark points in Table I. Point 1 represents a set of solutions with relatively heavy neutralino $m_{\tilde{\chi}_0} > 400$ GeV. Because Wino and Higgsinos are heavy in Points 1 and 2, these points belong to the diagram (b) explanation. Point 3 is an example where Bino, Wino and sneutrino are light but Higgsinos are heavy, so diagrams (a) and (b) both contribute here.

Collider Searches: Bounds and Prospects.—The SUSY has been searched inclusively at the LHC. The light EW sector of the EWSUSY model that is used to address the $\Delta a_\mu$ will have been excluded by those searches, except that the spectrum is compressed or the mass scale is high. All benchmark points are featured by large mass splitting between the left-handed and right-handed sleptons. For the benchmark points 1 and 2, the left-handed sleptons are relatively heavy and the right-handed sleptons have masses close to the LSP (it is opposite for benchmark point 3), such that the direct slepton search at 13 TeV 139 fb$^{-1}$ [47, 48] cannot probe them. The most sensitive search for those two points is the trilepton search [49] for Wino production $\tilde{\chi}_2^0 \tilde{\chi}_1^\pm \rightarrow (\ell\ell)X$ and $\tilde{\chi}_1^0 \ell \rightarrow (\ell\ell)\ell X$. Those two benchmark points just fall beyond the current bound for this channel and may be probed/excluded in the near future, when the same analysis is performed on higher integrated luminosity dataset. For benchmark point 3, although the $\tilde{\chi}_2^0$ has relatively large mass splitting with the LSP and decays through $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \ell \ell$ producing leptons in the final state, it is Bino dominate and its production rate at the LHC is suppressed. To probe the light compressed wino for this benchmark point, the HE-LHC with collision energy 27 TeV is required [50].

Discussion and Conclusion.—Combining the BNL and Fermi-Lab experimental results for the measurements of muon anomalous magnetic moment, we have 4.2$\sigma$ deviation from the SM, which strongly suggests the new physics around 1 TeV. We analyzed the corresponding Feynman diagrams in the SSMs, and showed that the EWSUSY is definitely needed. We realized the EW-SUSY in the MSSM with the GmSUGRA. We found large parameter spaces including the viable parameter space. The benchmark points, which have been studied in this work, are based on the assumption of R-parity conservation. If the R-parity is broken, the corresponding collider bounds may become much weaker and then the wider classes of benchmark points become viable. As a result, testing the EWSUSY which can explain the $\Delta a_\mu$ naturally at the future HL-LHC and HE-LHC will become a big challenge.

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