Scale dependences of local form non-Gaussianity parameters from a DBI isocurvature field

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Abstract: We derive the spectral indices and their runnings of local form $f_{NL}$ and $g_{NL}$ from a DBI isocurvature field and we find that the indices are suppressed by the sound speed $c_s$. This effect can be interpreted by the Lorentz boost from the viewpoint in the frame where brane is moving.

Keywords: inflation, non-Gaussianity.

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1. Introduction

Non-Gaussianity [1] has become a very important probe to the physics in the early universe. It is helpful to figure out the mechanism for generating primordial curvature perturbation. A well-defined non-Gaussianity is the so-called local form non-Gaussianity which says that the curvature perturbation can be expanded to the non-linear orders at the same spatial point

$$\zeta(x) = \zeta_g(x) + \frac{3}{5} f_{NL} \zeta_g^2(x) + \frac{9}{25} g_{NL} \zeta_g^3(x) + \ldots,$$

(1.1)

where $\zeta_g$ is the Gaussian part of curvature perturbation, $f_{NL}$ and $g_{NL}$ are the non-Gaussianity parameters which characterize the sizes of local form bispectrum and trispectrum respectively. Single-field inflation model predicts $f_{NL} \sim O(n_s - 1) \sim O(10^{-2})$ [2]. A convincing detection of a large local form non-Gaussianity is the smoking gun for the multi-field inflation in which the quantum fluctuations of the isocurvature field contribute to the final curvature perturbation on the super-horizon scales. See, for example, [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20] etc.

In the literatures the non-Gaussianity parameters are taken as a constant. However, recently there are some hints at a possible scale dependence of $f_{NL}$ come from the numerous observations of massive high-redshift clusters [21, 22, 23, 24] which seems in greater abundances than expected from a Gaussian statistics [25, 26]. In fact, $g_{NL}$ may also be scale dependent. The scale dependences of $f_{NL}$ and $g_{NL}$ are measured by their spectral indices $n_{fNL}$ and $n_{gNL}$ and their runnings $\alpha_{fNL}$ and $\alpha_{gNL}$ which are respectively defined by

$$f_{NL}(k) = f_{NL}(k_p) \left( \frac{k}{k_p} \right)^{n_{fNL} + \frac{1}{2} \alpha_{fNL} \ln \frac{k}{k_p}},$$

(1.2)

$$g_{NL}(k) = g_{NL}(k_p) \left( \frac{k}{k_p} \right)^{n_{gNL} + \frac{1}{2} \alpha_{gNL} \ln \frac{k}{k_p}},$$

(1.3)

where $k_p$ is a pivot scale. The authors in [27] showed that Planck [28] and CMBPol [29] are able to provide a 1-σ uncertainty on the spectral index of $f_{NL}$ as follows

$$\Delta n_{fNL} \simeq 0.1 \frac{50}{f_{NL}} \frac{1}{f_{sky}}$$

for Planck,
\[ \Delta n_{f_{NL}} \simeq 0.05 \frac{50}{f_{NL}} \frac{1}{\sqrt{f_{sky}}} \text{ for CMBPol,} \quad (1.5) \]

where \( f_{sky} \) is the sky fraction. The effects on the halo bias from the scale dependent \( f_{NL} \) are discussed in \[30, 31\]. The studies on fingerprints of the scale-dependent \( g_{NL} \) in CMB and large-scale structure are called for in the near future.

Actually the scale-independent non-Gaussianity parameters are not generic predictions of inflation model. The large scale dependences of non-Gaussianity parameters can be obtained in the axion Nflation with different decay constants for different axion fields \[32\] and in the model with self-interacting canonical isocurvature field \[33, 34, 35, 36, 37, 38, 39\]. It is interesting for us to ask how the results should be modified for the non-canonical isocurvature field. In particular, we are curious that the scale dependence is enhanced or suppressed by the sound speed. For simplicity, we focus on the case with a DBI isocurvature field.

Our paper is organized as follows. In Sec. 2 we will discuss the dynamics and quantum fluctuation of a DBI isocurvature field. In Sec. 3 the spectral indices of \( f_{NL} \) and \( g_{NL} \) from a DBI isocurvature field will be calculated. Some discussions are contained in Sec. 4.

### 2. Dynamics and quantum fluctuation of a DBI isocurvature field

Brane inflation \[40\] is considered to be a popular inflation model embedded into string theory. A more realistic setup is that a D3-brane moves in the internal six-dimensional Calabi-Yau manifold which contains one or more throats in Type IIB string theory \[41\]. For simplicity, we consider a D3-brane which is mobile in a throat whose metric is given by

\[ ds^2 = h^{-1/2} g_{\mu\nu} dx^\mu dx^\nu + h^{1/2} \left[ dr^2 + b^2(r) d\theta^2 + \cdots \right], \quad (2.1) \]

Here \( \theta \) is an angular coordinate which is transverse to the radius direction \( r \) and \( b(r) \) is the radius of throat at \( r \). In this paper, we choose signature \((-,+,+)+)\). The action for the mobile D3-brane is given by

\[ S = -T_3 \int d^4x \left[ \sqrt{-\det \left[ h^{-1/2} (g_{\mu\nu} + h \partial_\mu r \partial_\nu r + h b^2 \partial_\mu \theta \partial_\nu \theta) \right]} - h^{-1} \int d^4x \sqrt{-g} V(r, \theta) \right]. \quad (2.2) \]

It is convenient to define two canonical fields in the slow-rolling limit as follows

\[ \phi = \sqrt{T_3 r}, \quad (2.3) \]
\[ \sigma = \sqrt{T_3 b \theta}. \quad (2.4) \]

Here we consider \( \dot{\sigma} \ll \dot{\phi} \), and then \( \phi \) and \( \sigma \) are taken to be the adiabatic and entropic direction during inflation respectively.

In the limit of \( f \dot{\phi}^2 \ll 1 \) and \( f \dot{\sigma}^2 \ll 1 \), these two-field inflation is reduced to the canonical case, where \( f = h/T_3 \). In this paper we focus on another limit in which

\[ c_s \simeq \sqrt{1 - f \dot{\phi}^2} \ll 1, \quad (2.5) \]

and

\[ \dot{\sigma}^2/\dot{\phi}^2 \ll c_s^2. \quad (2.6) \]
From the above action the equations of motion for $\phi$ and $\sigma$ are roughly given by

$$\ddot{\phi} + 3H(1 - \kappa/3)\dot{\phi} \simeq \frac{1}{2f^2}\frac{\partial f}{\partial \phi} - c_s\frac{\partial V(\phi, \sigma)}{\partial \phi},$$  

(2.7)

$$3H\dot{\sigma} \simeq c_s\left(3\frac{\eta_b}{c_s}H^2\sigma - \frac{\partial V(\phi, \sigma)}{\partial \sigma}\right),$$  

(2.8)

where

$$\kappa \equiv \frac{\dot{c}_s}{H c_s},$$  

(2.9)

$$\eta_b \equiv \frac{\dot{b}}{H b},$$  

(2.10)

and both $\kappa$ and $\eta_b$ are assumed to be much smaller than unity. For simplicity, the cross-coupling between $\phi$ and $\sigma$ is assumed to be negligibly small, and Eq. (2.8) is valid when

$$\left|\frac{V''(\sigma)}{3H^2}\right| \ll c_s^{-1}.$$  

(2.11)

This is the slow-roll condition for the isocurvature field $\sigma$. Now the dynamics of $\sigma$ becomes

$$3H\dot{\sigma} \simeq -c_s\left[\tilde{m}^2\sigma + V'(\sigma)\right],$$  

(2.12)

where

$$\tilde{m}^2 \equiv -3\eta_bH^2/c_s.$$  

(2.13)

The slow variation of the radius of throat induces an effective mass for the isocurvature field along the angular direction.

The quantum fluctuations of $\phi$ and $\sigma$ have been well studied in [12]. See also [18, 43, 44]. Here we don’t want to repeat the computations. We will only briefly recall the results in [12]. The canonically normalized quantum fluctuations of $\phi$ and $\sigma$ are respectively given by

$$v_\phi = \frac{a}{c_s}\frac{\delta \phi}{\sqrt{2}}, \quad v_\sigma = \frac{a}{\sqrt{c_s}}\frac{\delta \sigma}{\sqrt{2}},$$  

(2.14)

whose Fourier modes corresponding to the Minkowski-like vacuum on very small scales are given by

$$v_{\phi,k} \simeq v_{\sigma,k} \simeq \frac{1}{\sqrt{2k c_s}}e^{-ikc_s\chi}\left(1 - \frac{i}{kc_s\chi}\right),$$  

(2.15)

where

$$\chi = \int \frac{dt}{a(t)}$$  

(2.16)

is the conformal time. The perturbation mode of $k$ exits horizon at the time of $c_s k = aH$. Therefore the power spectra for $\delta \phi$ and $\delta \sigma$ are

$$P_{\delta \phi} \simeq \left(\frac{H}{2\pi}\right)^2,$$  

(2.17)

$$P_{\delta \sigma} \simeq \left(\frac{H}{2\pi c_s}\right)^2.$$  

(2.18)
The amplitude of $\delta \sigma$ is amplified by a factor of $1/c_s$ compared to the canonical one.

Before closing this section, we want to estimate the typical value of $\sigma$ during inflation. The quantity of $\langle \sigma^2 \rangle$ coming from its quantum fluctuations is

$$\langle \sigma^2 \rangle = \frac{1}{(2\pi)^3} \int \left| \delta \sigma_k \right|^2 d^3 k = \frac{1}{(2\pi)^3} \int \left( \frac{1}{2a^2 k} + \frac{H^2}{2k^3 c_s^2} \right) d^3 k. \quad (2.19)$$

Considering that the physical momentum $p$ is related to $k$ by $p = k/a = e^{-Ht} k$, we have

$$\langle \sigma^2 \rangle = \frac{1}{(2\pi)^3} \int \frac{d^3 p}{p} \left( \frac{1}{2} + \frac{H^2}{2p^2 c_s^2} \right). \quad (2.20)$$

The first term is contributed from vacuum fluctuations in Minkowski space and it can be eliminated by renormalization. In addition, the physical momenta for the modes of quantum fluctuations we concern in the time interval between 0 and $t$ are those from $c_s^{-1} H$ to $c_s^{-1} He^{-Ht}$. Therefore

$$\langle \sigma^2 \rangle = \frac{H^2}{4\pi^2 c_s^2} \int_{c_s^{-1} H e^{-Ht}}^{c_s^{-1} H} \frac{dp}{p} = \frac{H^3}{4\pi^2 c_s^2} t. \quad (2.21)$$

It indicates that the quantum fluctuation of the field $\sigma$ in the inflation epoch can be modeled by a random walk with step size $H/2\pi c_s$ per Hubble time. However the vacuum expectation value of $\sigma$ cannot go like $t$ for $t \to \infty$ if $\sigma$ has a potential. For example, let’s consider the potential of $\sigma$ is

$$V(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \lambda \sigma^n. \quad (2.22)$$

From Eq. (2.12), the dynamics of $\sigma$ is described by

$$3H \dot{\sigma} = -c_s m^2 \sigma(1 + n\lambda \sigma^{n-2}/m^2), \quad (2.23)$$

where

$$m^2 = m_\sigma^2 + \tilde{m}^2. \quad (2.24)$$

Similar to [45], combining the contributions from quantum fluctuation and classical equation of motion, we have

$$\frac{d\langle \sigma^2 \rangle}{dt} = \frac{H^3}{4\pi^2 c_s^2} - c_s \frac{2m^2}{3H} \langle \sigma^2 \rangle \left[ 1 + \frac{n\lambda}{m^2} \langle \sigma^2 \rangle^{n-1} \right]. \quad (2.25)$$

The above differential equation approaches a constant equilibrium value, namely

$$\langle \sigma^2 \rangle = \frac{3H^4}{8\pi^2 m^2 c_s^3} \frac{1}{1 + \frac{2}{s} s}, \quad (2.26)$$

where

$$s \equiv 2\lambda \langle \sigma^2 \rangle^{n-1}/m^2. \quad (2.27)$$

The typical value of $\sigma$ during inflation is $\sigma_* = \sqrt{\langle \sigma^2 \rangle} \sim c_s^{-3/2}$ which is enhanced for $c_s \ll 1$. 

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3. Scale dependence of local form non-Gaussianity parameters

In this paper we consider that the curvature perturbation is generated by the isocurvature field \( \sigma \) at the end of inflation or deep in the radiation dominated era, and the curvature perturbation can be expanded to the non-linear orders by using the \( \delta N \) formalism [46]:

\[
\zeta(t_f, x) = N_\sigma(t_f, t_i) \delta \sigma(t_i, x) + \frac{1}{2} N_{\sigma\sigma} \delta \sigma^2(t_i, x) + \frac{1}{6} N_{\sigma\sigma\sigma} \delta \sigma^3(t_i, x) + \ldots, \tag{3.1}
\]

where \( N_\sigma, N_{\sigma\sigma} \) and \( N_{\sigma\sigma\sigma} \) are the first, second and third order derivatives of the number of e-folds with respect to \( \sigma \) respectively. Here \( t_f \) denotes a final uniform energy density hypersurface and \( t_i \) labels any spatially flat hypersurface after the horizon exit of a given mode. Similar to [36, 38], \( t_i \) is set to be \( t_*(k) \) which is determined by \( k = a(t_*) H_* \) for a given mode with comoving wavenumber \( k \). \( H_* \) denotes the Hubble parameter during inflation from now on.

From Eq. (3.1), the amplitude of curvature perturbation is given by

\[
\Delta_R^2 = N_\sigma^2(t_*) \left( \frac{H_*}{2\pi c_s} \right)^2. \tag{3.2}
\]

The amplitude of Gravitational waves perturbation only depends on the energy scale of inflation as follows

\[
\Delta_T^2 = \frac{H_*^2}{\pi^2/2}. \tag{3.3}
\]

Here we work on the unit of \( M_p = 1 \). The scale dependence of gravitational wave perturbation is measured by \( n_T \) which is defined by

\[
n_T \equiv \frac{d\Delta_T^2}{d\ln k} = -2\epsilon_H, \tag{3.4}
\]

where

\[
\epsilon_H \equiv -\frac{\dot{H}_*}{H_*^2}. \tag{3.5}
\]

For convenience, the tensor-scalar ratio \( r_T \) is introduced to measure the amplitude of gravitational waves:

\[
r_T \equiv \frac{\Delta_T^2}{\Delta_R^2} = \frac{8\epsilon_s^2}{N_\sigma^2(t_*)}. \tag{3.6}
\]

Comparing Eq. (3.1) to (1.1), the non-Gaussianity parameters are given by

\[
f_{NL} = \frac{5}{6} \frac{N_{\sigma\sigma}(t_*)}{N_\sigma^2(t_*)}, \tag{3.7}
\]

and

\[
g_{NL} = \frac{25}{54} \frac{N_{\sigma\sigma\sigma}(t_*)}{N_\sigma^3(t_*)}. \tag{3.8}
\]

Following the method in [36, 38], we introduce a new time \( t_r(t > t_*) \) which is chosen as a time soon after all the modes of interest exit the horizon during inflation and keep it fixed. The value of \( \sigma \) at \( t_r \) is related to that at time \( t_* \) and the time \( t_* \) by

\[
\int_{\sigma_*}^{\sigma_r} \frac{d\sigma}{V_{\text{eff}}(\sigma)} = - \int_{t_*}^{t_r} \frac{c_s(t)}{3H(t)} dt, \tag{3.9}
\]
where
\[ V_{\text{eff}} = \frac{1}{2} \dot{\sigma}^2 + V(\sigma). \] (3.10)

The equation of motion in Eq. (2.12) can be rewritten by
\[ 3H\dot{\sigma} = -c_s V'_{\text{eff}}. \] (3.11)

Therefore we have
\[ \frac{\partial \sigma_r}{\partial \sigma_s}(t_*) = \frac{V'_{\text{eff}}(\sigma_r)}{V'_{\text{eff}}(\sigma_s)}, \] (3.12)
\[ \frac{\partial \sigma_r}{\partial t_*}(\sigma_r) = \frac{c_s(t_*) V'_{\text{eff}}(\sigma_r)}{3H(t_*)}. \] (3.13)

Considering
\[ \frac{d}{dt_*} F(\sigma_r) = \frac{\partial F(\sigma_r)}{\partial \sigma_r}(\dot{\sigma} \frac{\partial \sigma_r}{\partial \sigma_s} + \frac{\partial \sigma_r}{\partial t_*}) \] (3.14)
and Eq. (3.11), one finds
\[ \frac{d}{d \ln k} F(\sigma_r) = \frac{d}{H_* dt_*} F(\sigma_r) = 0, \] (3.15)
which implies that $F(\sigma_r)$ is scale independent. \(^1\) Taking into account that $\sigma_r$ is a function of $\sigma_s$, we have

\[ N_{\sigma}(t_*) = \frac{\partial \sigma_r}{\partial \sigma_s} \frac{\partial N(\sigma_r)}{\partial \sigma_r}, \] (3.16)
\[ N_{\sigma\sigma}(t_*) = \frac{\partial^2 \sigma_r}{\partial \sigma_s^2} \frac{\partial N(\sigma_r)}{\partial \sigma_r} + \left( \frac{\partial \sigma_r}{\partial \sigma_s} \right)^2 \frac{\partial^2 N(\sigma_r)}{\partial \sigma_r^2}, \] (3.17)
\[ N_{\sigma\sigma\sigma}(t_*) = \frac{\partial^3 \sigma_r}{\partial \sigma_s^3} \frac{\partial N(\sigma_r)}{\partial \sigma_r} + 3 \frac{\partial \sigma_r}{\partial \sigma_s} \frac{\partial^2 \sigma_r}{\partial \sigma_s^2} \frac{\partial^2 N(\sigma_r)}{\partial \sigma_r^2} + \left( \frac{\partial \sigma_r}{\partial \sigma_s} \right)^3 \frac{\partial^3 N(\sigma_r)}{\partial \sigma_r^3}. \] (3.18)

Since $\frac{\partial N(\sigma_r)}{\partial \sigma_s}$, $\frac{\partial^2 N(\sigma_r)}{\partial \sigma_s^2}$ and $\frac{\partial^3 N(\sigma_r)}{\partial \sigma_s^3}$ are scale independent, one obtains
\[ \frac{d \ln N_{\sigma}(t_*)}{d \ln k} = c_s \eta_{\sigma}, \] (3.19)
\[ \frac{d \ln N_{\sigma\sigma}(t_*)}{d \ln k} = 2 c_s \eta_{\sigma\sigma} + c_s \beta_3 N_{\sigma\sigma}(t_*), \] (3.20)
\[ \frac{d \ln N_{\sigma\sigma\sigma}(t_*)}{d \ln k} = 3 c_s \eta_{\sigma\sigma\sigma} + 3 c_s \beta_3 N_{\sigma\sigma}(t_*) + c_s \xi_4 N_{\sigma\sigma\sigma}(t_*). \] (3.21)

where the slow-roll equation of motion for $\sigma$ is considered and
\[ \eta_{\sigma\sigma} = \frac{V''_{\text{eff}}(\sigma_s)}{3H_*^2}, \quad \beta_3 = \frac{V''''(\sigma_s)}{3H_*^4}, \quad \xi_4 = \frac{V''''(\sigma_s)}{3H_*^6}. \] (3.22)

From the above results, the spectral indices of $\Delta_R^2$, $f_{\text{NL}}$ and $g_{\text{NL}}$ are respectively given by
\[ n_s \equiv 1 + \frac{d \ln \Delta_R^2}{d \ln k} = 1 - 2\epsilon_H - 2\kappa + 2c_s \eta_{\sigma\sigma}, \] (3.23)

\(^1\) More precisely, the perturbation mode with $k$ exits horizon when $c_s k = a_* H_*$, and then $d \ln k = (1 - \epsilon_H - \kappa) H_* dt_* \simeq H_* dt_*$.
\begin{equation}
  n_{fNL} \equiv \frac{d \ln |f_{NL}|}{d \ln k} = c_{s} \eta_{3} N_{\sigma}(t_{s})/N_{\sigma\sigma}(t_{s}),
\end{equation}

and
\begin{equation}
  n_{gNL} \equiv \frac{d \ln |g_{NL}|}{d \ln k} = 3c_{s} \eta_{3} N_{\sigma\sigma}(t_{s})/N_{\sigma\sigma\sigma}(t_{s}) + c_{s} \xi_{4} N_{\sigma}(t_{s})/N_{\sigma\sigma\sigma}(t_{s}).
\end{equation}

We see that the spectral indices of both $f_{NL}$ and $g_{NL}$ are suppressed by a factor of $c_{s}$ for $c_{s} \ll 1$.

Similar to [36, 38], the running of spectral indices of $f_{NL}$ and $g_{NL}$ are defined by
\begin{equation}
  \alpha_{fNL} \equiv \frac{dn_{fNL}}{d \ln k} = (\kappa + 2 \epsilon_{H} - c_{s}\eta_{\sigma\sigma} - c_{s}\eta_{4})n_{fNL} - n_{fNL}^2,
\end{equation}
\begin{equation}
  \alpha_{gNL} \equiv \frac{dn_{gNL}}{d \ln k} = (\kappa + 2 \epsilon_{H} - 2c_{s}\eta_{\sigma\sigma})n_{gNL} - n_{gNL}^2
  + \frac{1}{g_{NL}} \left[ 2f_{NL}^2(c_{s}\eta_{\sigma\sigma} - c_{s}\eta_{4} + n_{fNL})n_{fNL} - \frac{25}{432} \xi_{5} \eta_{T} \right],
\end{equation}
where
\begin{equation}
  \eta_{4} = \frac{V_{eff}^{(4)}}{3H^2 V^{(4)}}, \quad \xi_{5} = \frac{V_{eff}^{(5)}}{3H^2 V^{(4)}}.
\end{equation}

The spectral indices of $f_{NL}$ and $g_{NL}$ are nice parameters to characterize the scale dependences of these two non-Gaussianity parameters only when $n_{fNL}$ and $n_{gNL}$ are much less than unity.

3.1 DBI isocurvature field with polynomial potential

In this subsection we will consider a simple example in which the DBI isocurvature field has an effective polynomial potential
\begin{equation}
  V_{eff}(\sigma) = \frac{1}{2} m^2 \sigma^2 + \lambda \sigma^n,
\end{equation}
and then
\begin{equation}
  \eta_{\sigma\sigma} = \frac{m^2}{3H_*^2} \left[ 1 + n(n-1)s/2 \right],
\end{equation}
\begin{equation}
  \eta_{3} = \frac{\eta_{\sigma\sigma} n(n-1)(n-2)s/2}{\sigma^* \left( 1 + n(n-1)s/2 \right)}.
\end{equation}

Combing the normalization of curvature perturbation, we find
\begin{equation}
  n_{fNL} f_{NL} = \text{sign}(N_{\sigma}) \frac{5}{6\Delta_{R}} \left( c_{s} \eta_{\sigma\sigma} \right) \frac{H_{s}/2\pi c_{s} n(n-1)(n-2)s/2}{\sigma^* \left( 1 + n(n-1)s/2 \right)},
\end{equation}
Here $\Delta_{R}$ is normalized to be $4.96 \times 10^{-5}$ by WMAP in [47]. The slow-roll condition for $\sigma$ is $c_{s}\eta_{\sigma\sigma} \ll 1$. $H_{s}/2\pi c_{s}$ is the amplitude of quantum fluctuation of $\sigma$ and it should be less than $\sigma_{s}$. For $c_{s}\eta_{\sigma\sigma} \sim 10^{-2}$, $H_{s}/2\pi c_{s} \sim 10^{-1}$ and $s \gtrsim 10^{-1}$, we have $|n_{fNL} f_{NL}| \sim O(10)$ which is detectable by PLANCK. In this case, if $c_{s} \lesssim 10^{-2}$, $\eta_{\sigma\sigma} \gtrsim 1$ which implies that the effective mass of $\sigma$ is not less than the Hubble parameter during inflation.

Taking into account the typical value of $\sigma$ in Sec. 2, the above equation becomes
\begin{equation}
  n_{fNL} f_{NL} = \text{sign}(N_{\sigma}) (c_{s} \eta_{\sigma\sigma})^{3/2} d(s),
\end{equation}
where
\begin{equation}
  d(s) = \frac{5\sqrt{2}}{6\Delta_{R}} \sqrt{\frac{1 + ns/2}{1 + n(n-1)s/2}} \frac{n(n-1)(n-2)s/2}{1 + n(n-1)s/2}
\end{equation}
which is illustrated in Fig. II. For $n = 4$ and $s \gg 1$, $d(s) \simeq 2.7 \times 10^{4}$, the scale dependence of $f_{NL}$ can be detected by PLANCK if $c_{s}\eta_{\sigma\sigma} \gtrsim 0.003$. 

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4. Discussions

In this paper we calculate the spectral indices of $f_{NL}$ and $g_{NL}$ and their runnings generated by a DBI isocurvature field on the super-horizon scales. We find that the indices are suppressed by the speed of sound $c_s$. This suppression effect is not surprising and one can understand it from the effect of Lorentz boost. Here $1/c_s$ is nothing but the Lorentz boost factor. The time interval in the frame where brane is moving is dilated by a factor $1/c_s$ compared to that in the frame where brane is at rest. Because the coordinate $\sigma$ is transverse to the motion direction of brane, the value of $\sigma$ does not change. Therefore the velocity of $\sigma$ is suppressed by a factor $c_s$. That is why there is a factor $c_s$ on the right hand side of Eq. (2.13). On the other hand, the scale dependences of the non-Gaussianity parameters come from the small variation of isocurvature field $\sigma$ from $t_k$ to $t_{k+dk}$, where $t_k$ and $t_{k+dk}$ correspond to the time when perturbation modes $k$ and $k+dk$ exit horizon during inflation respectively. Therefore the variation of isocurvature field $\sigma$ is suppressed by $c_s$, and hence the spectral indices of $f_{NL}$ and $g_{NL}$ are suppressed by $c_s$ as well.

Even though the scale dependences of non-Gaussianity parameters generated by the DBI isocurvature field are probably detectable, its effective of mass is required to be comparable or larger than the Hubble parameter during inflation if $c_s \ll 1$.

There are so many models which can produce a large local form non-Gaussianity and an accurate measurement of $f_{NL}$ cannot help us to distinguish them. But once the scale dependences of the non-Gaussianity parameters are detected, we can get more information about the isocurvature field, such as how it interacts with itself.

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