TEST OF UNIVERSALITY HYPOTHESIS FOR SCALAR CONFINING POTENTIAL BETWEEN QUARKS

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1 Introduction

Abstract

The universality hypothesis for scalar confining potential between quark and antiquark with different flavours is confirmed within 0.05 level of relativity errors.

Characteristic feature of quarks interaction potential is its growth at high distances. This behaviour of potential requires the clarification at least the following questions: how long the increase of the potential takes place, what are its transformational properties according to Lorentz group transformations, how the form of the increasing potential depends on the quark
flavours? In the present work we shall consider in the main the last question, supposing, and it is sufficiently well founded, that one can believe the confinement potential in some domain of radius $\sim 1 \, \text{Fm}$ to be linearly growing and scalar according to Lorentz group transformations [1-3]. It is well known, that direct application of QCD for determination of spectra of bound states of quarks and/or antiquarks leads to a number of difficulties, therefore the compound quark models are frequently used, in which frameworks the sufficiently simple calculations, in particular, of masses and decay widths can be produced (for example,[1-3]). The most simple, from the models, used for description of spectroscopic characteristics of hadrons, are the potential ones and, especially, nonrelativistic potential models, which turned out to be exceptionally succesive for the description of heavy quarkonia characteristics. However, the presence of a number of nonperturbative effects, connected, for instance, with the complicated structure of QCD vacuum, and also, with the necessity to take into consideration the nonrelativistic corrections, led to the complication of the formulation of these models, and, as a rule, to the change to the worse of the precision of obtained results. Undoubtedly, those models present the greatest interest, which do not contradict the basic principles of QCD, have the minimal number of phenomenological parameters, promote the clearing up of the interaction form in nonperturbative domain and keep in calculation of both light and heavy hadrons characteristics the high level of precision, which have been achieved in nonrelativistic models of heavy quarkonia. In the present work the phenomenological model of hadrons, proposed in refs [4-5] is used, the basic statements of the model are given briefly in section 2. As has been shown in section 3, the results of calculations light mesons masses built of $u-$, $d-$ quarks and antiquarks in the framework of this model, in the approximation of zero current masses of quarks and absence of static quasi-Coulomb interaction, are in good agreement with experimental data. In the section 4 on the base of numerical solution of the basic equation of the model with the account for the quasi-Coulomb interaction the values of the strong interaction constants in the perturbative domain in the presence of the quasi-Coulomb interaction and in nonperturbative domain in the presence of scalar linearly growing potential are found. In the last section, taking into account the obtained results, the application of the universality hypothesis of the scalar confining potential in the framework of the considered model is discussed.
2 Phenomenological model of generalized quark fields for hadrons

In the framework of the model proposed in refs [4-5], hadron is the compound system which consists of the general self-consistent confining field, this field, likely, includes the production and annihilation effects of sea $(q\bar{q})$-pairs and some number of valent quarks and/or antiquarks. For the sake of simplicity, in further computations we shall suppose, that confining field is spherically symmetric and its space-time motion is given by the centre position with coordinates $q_0^\mu, \mu = 0, 1, 2, 3$ The coordinates of valent quarks and/or antiquarks we shall write as $q_i^\mu, \mu = 0, 1, 2, 3; i = 1, \ldots, n$, where $n$ is the total number of valent fermion constituents. We shall work in quasi-independent particles approximation, when the wave function of the whole system is represented as a product of its components wave functions, dependent in general on coordinates of two or more constituents.

$$\Psi_i(q_0, q_1, \ldots, q_n) = \Psi_0(q_0, q_1, \ldots, q_n)\Psi_1(q_0, q_1)\ldots\Psi_n(q_0, q_n)$$ (1)

where wave function $\Psi_0(q_0, q_1, \ldots, q_n)$ describes the general field state with the centre in point $q_0^\mu$, and $\Psi_i(q_0, q_i)$-is the state of the $i$-th valent fermion particle in this general field. It should be noticed, that we consider the $i$-th particle not to be free, but bound with the others by means of general field. Making the further simplifications, we shall suppose, that the origin of the general field is due only to the fermion components and combine the inertia centre of fermion components system $Q^\mu$, cannonically conjugate to their total momentum, with the point $q_0^\mu$. We shall restrict the consideration of ordinary hadrons to the case [4], when the behaviour of $\Psi_i(Q, q_i)$ in the confinement region is determined, in the main, by the Dirac equation with the scalar potential $U_i((q_i - Q)^2)$:

$$\left(i\gamma^\mu \frac{\partial}{\partial(q_i^\mu - Q^\mu)} - U_i((q_i - Q)^2) - \mu_i\right)\Psi_i(Q, q_i) = 0$$ (2)

The dependence of $\Psi_i(Q, q_i)$ on $q_i - Q$, is the only important one for masses spectra calculation. In the hadron rest-frame we shall use the well known in many-particles theory single-time approximation,when $Q^0 = q_0^0 = \ldots = q_1^0$ In that case the potentials $U_i((q_i - Q)^2)$ do not depend on time and it is possible to come to the standard Dirac equations solution for $i = 1, \ldots, n$ Thus, the hadron state in the given model is determined by the state of the each quark, situated in its orbital, and by the state of confining field,
which interaction with the $i$-th quark is given by the potential $U_i((\vec{q}_i - \vec{Q}_i)$. It should be noticed, that the general field is assumed to be "white", unlike the colour gluon field, the quarks interact with on small distances, and which contribution in general should be determined by perturbation theory. Setting the energy value of general gluon field as (this value is unknown and therefore is the phenomenological parameter), we shall obtain, that in the hadron rest-frame its mass equals:

$$M_0 = \epsilon_0 + \epsilon_1 + \ldots + \epsilon_n$$

where $\epsilon_i, i = 1, \ldots, n$ are defined as eigenvalues of Dirac stationary equations (2) with potentials $U_i((\vec{q}_i - \vec{Q}_i)\text{and quark current masses } \mu_i \text{ values. In the quasi-independent generalized quark fields } \Psi_i(Q, q_i) \text{ approximation we shall consider the simplest from the present hadron systems - two-particles mesons. All the more that the produced numerical calculations of mass spectra in the framework of the given model can be compared with a great number of very precise experimental data [6]. Let us make use of the supposed spherical symmetry of scalar potential and distinguish the stationary states of fermion components with the definite values of total momentum $j_i$. Thus, the $i$-th quark/antiquark state in the hadron will be determined by its spherical orbital $\Psi_{j_i}$. The main difficulty in the present model consists in the reduction of a number of a possible states with different values of $j_1$ and $j_2$. For two-particles mesons with different values of $J_{PC}$, built of quark and antiquark with masses $\mu_1$ and $\mu_2$ the following selection rules have been found [4] :

$$j_1 = j_2 = J + 1/2 \quad \text{if } J = L + S;$$
$$j_1 = j_2 + 1 = J + 3/2 \quad \text{if } J \neq L + S, \mu_1 \leq \mu_2$$

(4)

The eigenenergy of the gluon field is also found to be dependent on relative orbital momentum $L$ of quark and antiquark and their total spin $S$. Dividing it in two parts between quark and antiquark, we can define the mass spectral function (term) of quark and antiquark in the meson in the following way:

$$E_i(n_i, j_i) = c[1 + (-1)^{L+j_i-1/2}] + E_i'(n_i, j_i)$$

(5)

where $c = 0.07 \text{ GeV}$ and the value of $E_i'(n_i, j_i)$ is found from the approximate solution of squared Dirac equation (2) with the accont for the phenomenological constants $L' = L$ and $\delta(J) \sim J$. For two-particles mesons in the
basic equation (2) with the scalar interaction $S_i(r)$ one can include the vector interaction $V_i(R)$, where $r$ is the relative distance between quark and antiquark, that is, make the substitution in the equation (2):

$$U_i(r) \rightarrow \gamma_0 V_i(r) + S_i(r)$$

With the account for the selection rules (4), the mass of the $n^{2S+1}L_J$-meson state is calculated with the help of the formula:

$$M(n^{2S+1}L_J) = E_1(n, J_1) + E_2(n, j_2),$$

where $n = 1 + n_r$, $n_r$ is the radial quantum number, moreover, the quantum numbers of quark and antiquark coincide.

### 3 Light mesons mass spectra in the approximation of quarks zero current masses and absence of quasi-Coulomb interaction

In some cases one can solve the equation (6) exactly and find the analytical expression for quark and antiquark terms in mesons. So, for example, it turns out well in the approximation of zero values of quark current masses and absence of quasi-Coulomb interaction between them. Let us consider in this section the calculation of mass spectra of $n^{2S+1}L_J$-mesons, built of $u-, d-$ quarks and antiquarks. In this case one should expect, that a good approximation will be the following one: $\mu_1 \simeq \mu_2 \simeq 0$, where $\mu_1, \mu_2$ - current values of quark and antiquark masses. Let us also take into account, that one should keep the quasi-Coulomb perturbative term $-4\alpha_s/3r$ only at small distances, therefore we can reject it for comparatively large dimensions of bound states. In this case the equality $L' \simeq L$ takes place. Then it is not difficult to obtain from equation (6) for quark and antiquark spectral function the following formula:

$$E(n, j) = c[1 + (-1)^{L+j-1/2}] + \sqrt{\sigma(2n + L + j - 1/2)}$$

As far as we know, the formula (8) is the best one according to its precision and fitting parameters number for the light mesons and reproduces, together with the selection rules (4), the whole spectrum of states built of $u-, d-$ quarks and antiquarks [5]. The relative error of calculations has a range of $\sim 1\%$ practically for all mesons, and only in some cases it reaches 3-4%, that can be recognized as a good agreement, taking into account the
approximate character of formula (8). For the radial excitations of mesons
this formula in some approximation corresponds to the Veneziano-Namby
formula [6-8]. As can be seen from formula (8) the simple linear dependence
of Chew-Frautschi [9] for Regge trajectories is valid only for some meson
states, for example for vector $1^{--}$-mesons and their orbital excitations. In
this case $\alpha'_R = 1/8\sigma$. The same connection between $\alpha'_R$ and $\sigma$ from other
considerations was obtained in the work [10]. Let us remind, that in the
model of Nambu-Goto strings for the hadrons $\alpha'_R$ and $\sigma$ are connected by
the relation: $\alpha'_R = 1/2\pi\sigma$

4 Calculation of the parameters for the interaction potential between quark and antiquark

In the general case it is necessary to make use of the numerical meth-
ods for the solution if the basic equation of the model. The main aim in
the present work is the determination of the parameters of the interaction
potential between quark and antiquark for all the known flavours, taking
into account quasi-Coulomb interaction. Without taking into consideration
quasi-Coulomb interaction, as it was noted in ref.[4], there had been ob-
served a systematic decrease of the slope parameter of the linearly rising
potential, when one proceeds from light to heavy quarks. This could be
connected both with the neglect of the contribution of the quasi-Coulomb
interaction, and with an explicit dependence of the slope parameter of the
linearly rising potential on quark and antiquark flavour. With the help of
the numerical method of the solution of the problem on the eigenvalues of
the basic equation and following calculation of the masses of both light and
heavy vector mesons, we have found, that in the range of 1-2% of the rela-
tive error, which in most cases does not exceeds the experimental one, with
which mesons masses spectra are determined, the hypothesis of the univer-

torsality for the slope parameter $\sigma$ of the scalar linearly rising parameter takes
place. The value of $\sigma$, have been obtained, is found to be 0.21 GeV$^2$, that
corresponds to 1.06 GeV/Fm. The $\alpha_s$ value, which enters as a multiplier
in the quasi-Coulomb term was $\sim$0.35 and weekly decreases when one pro-
cceeds from light to heavy quarks. This value is consistent with the the
generally accepted $\alpha_s$ values in sufficiently broad domain of the distances
between interacting quark and antiquark, however the further calculations
are necessary for the confirmation of the logarithmic dependence of $\alpha_s$ on
the distance. The quarks current masses thus also correspond to the values,
found earlier in the majority of works: $m_{u,d} = 8 \text{ MeV}$, $m_s = 1380 \text{ MeV}$, $m_b = 4730 \text{ MeV}$.

5 Conclusion

In the framework of the model with the phenomenological selection rules, proposed in work [4] the parameters of the interaction potential between quark and antiquark for all known up to now quark flavours are found. It is found, that in the range of experimental errors, with which the mesons masses are obtained, the universality hypothesis for the slope parameter of the linearly rising potential, proposed earlier in works [11-13], takes place. The $\sigma$ value is found to be 0.21 GeV$^2$. The increase of the interaction constant on small distances $\alpha_s$ when one proceeds from heavy to light quarks have been observed. The obtained results confirm the approximate analytic formulae (8) for the mesons masses spectra, found in the works [4] and at the same time it show, that the obtained on the base of these formulae the parameter $\sigma$ values are have been diminished ones and effectively take into account the contribution of the quasi-Coulomb interaction. The authors are grateful to V.I. Savrin and A.M. Snigirev for useful discussions.

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