Heavy-quark expansion for $D$ and $B$ mesons in nuclear matter

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Abstract. The planned experiments at FAIR enable the study of medium modifications of $D$ and $B$ mesons in (dense) nuclear matter. Evaluating QCD sum rules as a theoretical prerequisite for such investigations encounters heavy-light four-quark condensates. We utilize an extended heavy-quark expansion to cope with the condensation of heavy quarks.

1 Introduction

The forthcoming experimental perspectives for in-medium heavy-light quark (i.e. $D$ and $B$) meson spectroscopy, in particular at FAIR, are accompanied by the need for sophisticated theoretical analyses, e.g. [1–6]. When utilizing QCD sum rules [7–9], this requires a thorough discussion of heavy-quark condensates in general, and, in particular, in the nuclear medium [10]. Therefore, the heavy-quark expansion (HQE), originally developed for the heavy two-quark condensate $\langle \bar{Q}Q \rangle$ in vacuum, is extended here to four-quark condensates and to the in-medium case, thus going beyond previous approaches, e.g. [11]. Specific formulas are derived and presented which provide important pieces for the complete QCD sum rule analysis of $D$ and $B$ mesons in nuclear matter.

2 Recollection: HQE in vacuum

In [12], a general method is introduced for vacuum condensates involving heavy quarks $Q$ with mass $m_Q$. The heavy-quark condensate is considered as the one-point function

$$
\langle 0 | \bar{Q}Q | 0 \rangle = -i \int \frac{d^4 p}{(2\pi)^4} \langle 0 | Tr_{c,D} S_Q(p) | 0 \rangle
$$

expressed by the heavy-quark propagator $S_Q$ in a weak classical gluonic background field in Fock-Schwinger gauge, $S_Q(p) = \sum_{k=0}^{\infty} S_Q^{(k)}(p)$ with $S_Q^{(k)}(p) = (-1)^k S_{Q(0)}^{(k)}(p) \gamma^\mu \tilde{A}_\mu S_{Q(0)}^{(k)}(p) \ldots \gamma^\mu \tilde{A}_\mu S_{Q(0)}^{(k)}(p)$, incorporating the free heavy-quark propagator $S_{Q(0)}(p) = (\gamma^\mu p_\mu + m_Q)/(p^2 - m_Q^2)$ and the derivative operator $\tilde{A}$ emerging from a Fourier transform defined as $\tilde{A}_\mu = \sum_{m=0}^{\infty} \tilde{A}^{(m)}_\mu$ with $\tilde{A}^{(m)}_\mu = \frac{i}{m^2 + 2} \left( D_{\mu_1} \ldots D_{\mu_m} G_{\mu_1}(x) \right)_{x=0} \partial^\nu \partial^{\mu_1} \ldots \partial^{\mu_m}$ [13, 14]. In this way, the heavy-quark propagator interacts with the complex QCD ground state via soft gluons generating a series expansion in the inverse
heavy-quark mass. The compact notation (1) differs from [12], but provides a comprehensive scheme easily extendable to in-medium condensates. The first HQE terms of the heavy two-quark condensate (1) reproduce [12]:

$$
\langle 0|\bar{Q}Q|0\rangle = -\frac{g^2}{48\pi^2 m_Q} \langle G^2 \rangle - \frac{g^3}{1440\pi^2 m_Q^3} \langle G^3 \rangle - \frac{g^4}{120\pi^2 m_Q^3} \langle (DG)^2 \rangle + \ldots
$$

(2)

with the notation

$$
\langle G^2 \rangle = \langle 0| G^A_{\mu \nu} G^A_{\mu \nu} |0\rangle,
$$

(3)

$$
\langle G^3 \rangle = \langle 0| f^{ABC} G^A_{\mu \nu} G^B_{\nu \lambda} G^C_{\lambda \mu} |0\rangle,
$$

(4)

$$
\langle (DG)^2 \rangle = \langle 0| \left( \sum_f \bar{q}_f \gamma_{\mu} t^A q_f \right)^2 |0\rangle.
$$

(5)

The graphic interpretation of the terms in (2) is depicted too: the solid lines denote the free heavy-quark propagators and the curly lines are for soft gluons whose condensation is symbolized by the crosses, whereas the heavy quark-condensate is symbolized by the crossed circles [15]. An analogous expression for the mixed heavy-quark gluon condensate can be obtained along those lines which contains, however, a term proportional to $m_Q$. The leading-order term in (2) was employed already in [7] in evaluating the sum rule for charmonia.

The vacuum HQE method was rendered free of UV divergent results for higher mass-dimension heavy-quark condensates by requiring at least one condensing gluon per condensed heavy-quark [15, 16], which prevents unphysical results, where the condensation probability of heavy-quark condensates rises for an increasing heavy-quark mass.

3 Application of HQE to in-medium heavy-light four-quark condensates

The above method can be extended to in-medium situations. Our approach contains two new aspects: (i) formulas analogous to equation (1) are to be derived for heavy-quark condensates, e.g. $\langle \bar{Q}fQ \rangle$, $\langle \bar{Q}f G Q \rangle$, $\langle \bar{q} t^A q \bar{Q} \rangle$, which additionally contribute to the in-medium operator product expansion (OPE) and (ii) medium-specific gluonic condensates, e.g. $\langle G^2/4 - (vG)^2/v^2 \rangle$, $\langle G^3/4 - f^{ABC} G^A_{\mu \nu} G^B_{\nu \lambda} G^C_{\lambda \mu} v^2 \rangle$, enter the HQE of heavy-quark condensates for both, vacuum and additional medium condensates, where $\langle \ldots \rangle$ denotes Gibbs averaging.

We are especially interested in heavy-light four-quark condensates entering the OPE of $D$ and $B$ mesons, inter alia, in terms corresponding to the next-to-leading-order perturbative diagrams with one light-quark ($q$) and one heavy-quark ($\bar{Q}$) line cut. There are 24 two-flavour four-quark condensates in the nuclear medium [17] represented here in a compact notation by $\langle q \Gamma T^A q \bar{Q} \Gamma' T'^A Q \rangle$, where $\Gamma$ and $\Gamma'$ denote Dirac structures and $T^A$ with $A = 0, \ldots, 8$ are the generators of $SU(3)$ supplemented by the unit element ($A = 0$). We obtain the analogous formula to (1) for heavy-light four-quark condensates:

$$
\langle \bar{q} \Gamma T^A q \bar{Q} \Gamma' T'^A Q \rangle = -i \int \frac{d^4 p}{(2\pi)^4} \langle \bar{q} \Gamma T^A q \text{Tr}_{c,D} \left[ \Gamma' T'^A S_Q(p) \right] \rangle.
$$

(6)
The leading-order terms of this HQE are obtained for the heavy-quark propagators $S_Q^{(1)}$ containing $\tilde{A}^{(1)}_\mu$ and $S_Q^{(2)}$ with leading-order background fields $\tilde{A}^{(0)}_\mu$:

$$
\langle \bar{q} \Gamma^T A_q \bar{Q} \Gamma^T A Q \rangle = -i \int \frac{d^4 p}{(2\pi)^4} \langle \bar{q} \Gamma^T A_q \text{Tr}_{c,D} \left[ \Gamma^T A_q \left( S_Q^{(1)}(p) + S_Q^{(2)}(p) + \ldots \right) \right] \rangle \quad (7)
$$

$$
= \langle \bar{q} \Gamma^T A_q \bar{Q} \Gamma^T A Q \rangle^{(0)} + \langle \bar{q} \Gamma^T A_q \bar{Q} \Gamma^T A Q \rangle^{(1)} + \ldots \quad (8)
$$

Evaluation of the first term of the expansion (8) for the complete list of two-flavour four-quark condensates in [17] gives three non-zero results:

$$
\langle \bar{q} \gamma^\nu r^A_q \bar{Q} \gamma_\nu r^A Q \rangle^{(0)} = -\frac{2}{3} \frac{g^2}{(4\pi)^2} \left( \log \frac{\mu^2}{m_Q^2} + \frac{1}{2} \right) \langle \bar{q} \gamma^\nu q \sum_f \bar{q}_f \gamma_\nu q_f \rangle, \quad (9)
$$

$$
\langle \bar{q} \gamma^\nu r^A_q \bar{Q} \gamma_\nu r^A Q \rangle^{(0)} = -\frac{2}{3} \frac{g^2}{(4\pi)^2} \left( \log \frac{\mu^2}{m_Q^2} + \frac{2}{3} \right) \langle \bar{q} \gamma^\nu q \sum_f \bar{q}_f \gamma_\nu q_f \rangle, \quad (10)
$$

$$
\langle \bar{q} r^A_q \bar{Q} r^A Q \rangle^{(0)} = -4 \frac{g^2}{3(4\pi)^2} \left( \log \frac{\mu^2}{m_Q^2} - \frac{1}{8} \right) \langle \bar{q} r^A q \sum_f \bar{q}_f \gamma_\nu q_f \rangle, \quad (11)
$$

where logarithmic singularities are calculated in the $\overline{\text{MS}}$ scheme, $\mu$ is the renormalization scale, and $r^A = T^A$ for $A = 1, \ldots, 8$. The non-zero contributions for the second term of (8) read

$$
\langle \bar{q} q \bar{Q} \rangle^{(1)} = -\frac{1}{3(4\pi)^2} \frac{g^2}{m_Q} \langle \bar{q} q G^{A_{\mu\nu}} G_{\mu\nu} \rangle, \quad (12)
$$

$$
\langle \bar{q} r^A_q \bar{Q} r^A Q \rangle^{(1)} = -\frac{1}{6(4\pi)^2} \frac{g^2}{m_Q} \langle d^{ABC} \bar{q} r^A_q G^{B_{\mu\nu}} G^{C_{\mu\nu}} \rangle, \quad (13)
$$

$$
\langle \bar{q} \gamma^\nu s \bar{Q} \gamma^\nu s Q \rangle^{(1)} = -\frac{1}{4(4\pi)^2} \frac{g^2}{m_Q} \langle i \bar{q} \gamma^\nu s q G^{A_{\mu\nu}} G_{\mu\nu} \rangle, \quad (14)
$$

$$
\langle \bar{q} \gamma^\nu r^A_q \bar{Q} \gamma^\nu r^A Q \rangle^{(1)} = -\frac{1}{8(4\pi)^2} \frac{g^2}{m_Q} \langle i d^{ABC} \bar{q} \gamma^\nu r^A_q G^{B_{\mu\nu}} G^{C_{\mu\nu}} \rangle, \quad (15)
$$

$$
\langle \bar{q} \gamma^\nu s \bar{Q} \gamma^\nu s Q \rangle^{(1)} = -\frac{1}{3(4\pi)^2} \frac{g^2}{m_Q} \langle \bar{q} \gamma^\nu q G^{A_{\mu\nu}} G_{\mu\nu} \rangle, \quad (16)
$$

$$
\langle \bar{q} \gamma^\nu r^A_q \bar{Q} \gamma^\nu r^A Q \rangle^{(1)} = -\frac{1}{3(4\pi)^2} \frac{g^2}{m_Q} \langle d^{ABC} \bar{q} \gamma^\nu r^A_q G^{B_{\mu\nu}} G^{C_{\mu\nu}} \rangle, \quad (17)
$$

$$
\langle \bar{q} \sigma_{\mu\nu} r^A_q \bar{Q} \sigma^{\mu\nu} r^A Q \rangle^{(1)} = -\frac{5}{6(4\pi)^2} \frac{g^2}{m_Q} \langle f^{ABC} \bar{q} \sigma_{\mu\nu} r^A_q G^{B_{\lambda\mu}} G^{C_{\lambda\nu}} \rangle, \quad (18)
$$

$$
\langle \bar{q} \sigma_{\mu\nu} r^A_q \bar{Q} \sigma^{\mu\nu} r^A Q \rangle^{(1)} = -\frac{5}{6(4\pi)^2} \frac{g^2}{m_Q} \langle f^{ABC} \bar{q} \sigma_{\mu\nu} r^A_q G^{B_{\lambda\mu}} G^{C_{\lambda\nu}} \rangle, \quad (19)
$$

$$
\langle \bar{q} \gamma^\nu s \bar{Q} \gamma^\nu s Q \rangle^{(1)} = \frac{5}{6(4\pi)^2} \frac{g^2}{m_Q} \langle f^{ABC} \bar{q} \gamma^\nu s q G^{B_{\mu\nu}} G^{C_{\mu\nu}} \rangle, \quad (20)
$$

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where \( f^{ABC} \) is the anti-symmetric structure constant of the color group and the corresponding symmetric object \( d^{ABC} \) is defined by the anti-commutator \( \{ t^A, t^B \} = \delta^{AB}/4 + d^{ABC} t^C \).

4 Summary and conclusions

The extension of the OPE for QCD sum rules of \( \bar{q}Q \) and \( \bar{Q}q \) mesons by four-quark condensates to mass dimension 6 yields heavy-light condensate contributions requiring HQE in a nuclear medium. The necessary steps to generalize the vacuum HQE [12] to cover in-medium situations are described and a general formula for the HQE of in-medium heavy-light four-quark condensates is presented. The two leading-order terms of this expansion for the complete list of two-flavour four-quark condensates [17] have been evaluated. In leading-order the results contain known condensate structures, thus, reducing the number of condensates entering the sum rule evaluation of mesons composed of a heavy and a light quark. It can be seen that the series does not exhibit a simple expansion in \( 1/m_Q \), not even in vacuum. Therefore, the lowest order terms are not suppressed by inverse powers of \( m_Q \) as for \( \langle \bar{q}Q \rangle \), challenging the omission of heavy-light four-quark condensates, as often done in previous sum rule analyses.

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