Runaway Domain Wall and Space-time Varying $\alpha$

Takeshi Chiba
Department of Physics,
College of Humanities and Sciences,
Nihon University,
Tokyo 156-8550, Japan

Masahide Yamaguchi
Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan

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Abstract. Recently spatial as well as temporal variations of the fine structure constant $\alpha$ have been reported. We show that a "runaway domain wall", which arises for the scalar field potential without minima, can account for such variations simultaneously. The time variation is induced by a runaway potential and the spatial variation is induced by the formation of a domain wall. The model is consistent with the current cosmological data and can be tested by the future experiments to test the equivalence principle.
1. Introduction

String theory is the most promising approach to unify all the fundamental forces in nature. It is believed that in string theory all the coupling constants and parameters (except the string tension) in nature are derived quantities and are determined by the vacuum expectation values of the dilaton and moduli. However, only few mechanisms (see for example, [1]) are known how and when to fix the dilaton/moduli. On the other hand, we know that the Universe is expanding. Then it is no wonder to imagine the possibility of the variation of the constants of nature during the evolution of the Universe.

In fact, it is argued that the effective potentials of dilaton or moduli induced by nonperturbative effects may exhibit runaway structure; they asymptote zero for the weak coupling limit where dilaton becomes minus infinity or internal radius becomes infinity and symmetries are restored in the limit [2, 3]. Thus it is expected that as these fields vary, the natural “constants” may change in time and moreover the violation of the weak equivalence principle may be induced [3, 4, 5, 6, 7].

Hence, any detection or nondetection of such variations at various cosmological epochs could provide useful information about the physics beyond the standard model. In this respect, the recent claims of the detection of the time variation [8, 9] as well as the spatial variation [11] of the fine structure constant \( \alpha \) may hint towards new physics.

Narrow lines in quasar spectra are produced by absorption of radiation in intervening clouds of gas, many of which are enriched with heavy elements. Because quasar spectra contain doublet absorption lines at a number of redshifts, it is possible to check for time variation in \( \alpha \) simply by looking for changes in the doublet separation of alkaline-type ions with one outer electron as a function of redshift.

Webb et al. [8] introduced a new technique (called many-multiplet (MM) method) that compares the absorption wavelengths of magnesium and iron atoms in the same absorbing cloud, which is far more sensitive than the alkaline-doublet method. From the latest analysis of Keck/HIRES (High Resolution Echelle Spectrometer) 143 absorption systems for \( 0.2 < z < 3.7 \), they found that \( \alpha \) was smaller in the past [10]:

\[
\frac{\Delta \alpha}{\alpha} = (-0.543 \pm 0.116) \times 10^{-5}.
\]

Moreover, recently, Webb et al. analyzed a dataset from the ESO Very Large Telescope (VLT) and found the opposite trend: \( \alpha \) was larger in the past [11]. Combined with the Keck samples, they claimed the spatial variation of \( \alpha \) [11]:

\[
\frac{\Delta \alpha}{\alpha} = (1.10 \pm 0.25) \times 10^{-6} (r/\text{Glyr}) \cos \theta,
\]

where \( r \) is the look-back time \( r = ct(z) \) and \( \theta \) is the angle between the direction of the measurement and the axis of best-fit dipole.

However, concerning the claimed time variation of \( \alpha \), similar observations from VLT/UVES (Ultraviolet and Visual Echelle Spectrograph) have not been able to
duplicate these results \[12, 13, 14, 15, 16\]. It is to be noted, however, that the analysis by Srianand et al. \[12\] may suffer from several flaws. For example, the uncertainty in wavelength calibration in \[12\] may not be consistent with the error in \(\Delta \alpha/\alpha\) \[17\]. According to the analysis of the fundamental noise limitation \[17\], the systematic errors in \[12\] may be several times underestimated. Recent detailed re-analysis of Srianand et al. and Chand et al. confirms these concerns: flawed parameter estimation methods in a \(\chi^2\) minimization analysis \[18\] (see however, \[19\]) and systematic errors in the UVES wavelength calibration \[20\].

Overall, although the claims of the detection of the variations are not confirmed by independent methods (however see \[21\]), the claims are also not disputed seriously. We regard the claims of the spatial/temporal variations currently refuse to deny or confirm. In this paper, we take the claims of the spatial/temporal variations of \(\alpha\) seriously and attempt to explain the data qualitatively.

One major difficulty in explaining both temporal (Eq. (1)) and spatial (Eq. (2)) variations is that the spatial variation across the horizon scale (\(\Delta \alpha/\alpha \simeq 10^{-5}\) at \(r \sim 14\)Glyr) is as large as the time variation during the Hubble time. If the time variation is induced by a scalar field \[22, 23, 24, 25, 26\], such a cosmologically time evolving scalar field is very light with its mass being comparable to the Hubble parameter and hence its relative fluctuation is very small. So it is almost impossible to explain simultaneously both temporal and spatial variations by the light scalar field and its fluctuation. One possible solution to this problem is to consider nonlinear objects like topological defects \[27\] or giant voids \[28\]. In this paper, we consider the former possibility since the scalar field is dispersive and does not trace the matter density perturbation much. Ref. \[30\] considered a domain wall to explain the spatial variations only but did not consider the time variations. We point out a certain type of domain walls can be utilized to explain not only the time variations of \(\alpha\) but also the spatial variations of \(\alpha\) of the same order of magnitude as the time variations.

The paper is organized as follows: In Sec. 2, we detail our model and then in Sec. 3 we study several constraints on the model parameters. Sec. 4 is devoted to summary.

2. Runaway Domain Wall and Varying \(\alpha\)

For definiteness, we consider the theory described by the following action

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_{pl}^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - V(\phi) - \frac{1}{16\pi \alpha_0} B(\phi) F_{\mu\nu} F^{\mu\nu} \right] + S_m(3)
\]

Here \(M_{pl} = 1/\sqrt{8\pi G} = 2.4 \times 10^{18}\)GeV is the reduced Planck mass, \(\phi\) is the real symmetry breaking field, \(\alpha_0\) is the bare fine structure constant and \(S_m\) denotes the action of other matter (relativistic/non-relativistic particles and dark energy). We consider the

\[\dagger\] It is possible to induce the spatial variations by the environmental dependence \[24, 29\], but it is difficult to explain the variation of dipole type like Eq. \[2\].
following scalar field potential of runaway type:

\[ V(\phi) = \frac{M^{2p+4}}{(\phi^2 + \sigma^2)^p}. \tag{4} \]

Even if the potential has no minimum, the discrete symmetry \( \phi \leftrightarrow -\phi \) can be broken dynamically, which results in the formation of a domain wall. Such defects are dubbed "vacuumless" defects in [31], but we prefer to call them "runaway" defects since they arise from the potential of runaway type.

For \( \phi \gg \sigma \), from the balance between the kinetic energy and the potential energy, one finds \( \phi \simeq (M^{p+2} x)^{1/(p+1)} \) and the energy density is proportional to \( x^{-2p/(p+1)} \), where \( x \) is the distance from the wall [31]. We assume \( p > 1 \) so that the tension of the wall is finite. Then, the width of the wall is estimated as \( \delta \simeq \sigma^{p+1}/M^{p+2} \) and the tension of the wall is given by

\[ \mu \simeq \frac{M^{2p+4}}{\sigma^{2p-1}}, \delta \simeq \frac{M^{p+2}}{\sigma^{p+1}}, \tag{5} \]

The profile of such a runaway domain wall solution is shown in Fig. 1. In the cosmological situation, we replace \( x \) with the Hubble distance \( H^{-1} \) so that

\[ \phi \propto a^{3/2(p+1)} \tag{6} \]

during the matter dominated era in accord with the tracker solution for \( V \propto \phi^{-2p} \) [32]. This scaling solution is useful to account for the cosmological time variation of \( \alpha \); in the opposite case of \( \phi < \sigma \), the scalar field near the local maximum exhibits thawing behavior [33] and moves very slowly, and hence it is difficult to explain the cosmological time variation of \( \alpha \).

The coupling function \( B(\phi) \) in front of the electromagnetic kinetic term induces the spatio-temporal variations of \( \alpha \) since \( \alpha(\phi) = \alpha_0/B(\phi) \). We consider the following coupling function:

\[ B(\phi) = e^{-\xi \phi/M_{pl}}. \tag{7} \]

Since the effective \( \alpha \) for small \( \xi \) is given by

\[ \alpha(\phi) \simeq \alpha_0 \left( 1 + \frac{\xi \phi}{M_{pl}} \right), \tag{8} \]

the time variation in either side of the wall is given by

\[ \frac{\dot{\alpha}}{\alpha_0} = \xi \frac{\dot{\phi}}{M_{pl}} = \pm \frac{3}{2(p+1)} \xi \frac{|\phi|}{M_{pl}} H, \tag{9} \]

where we have used Eq. (6), and the spatial variation across the wall is given by

\[ \frac{\Delta \alpha}{\alpha_0} = \frac{\xi \Delta \phi}{M_{pl}} = 2\xi \frac{\phi}{M_{pl}}. \tag{10} \]

Thus, the opposite time variation of \( \alpha \) between the Keck (increasing \( \alpha \)) and the VLT (decreasing \( \alpha \)) as well as the spatial variation of the same order of magnitude as the time variation during the Hubble time \( (\Delta \alpha/\alpha_0 \sim |\dot{\alpha}|/\alpha_0 H^{-1}) \) are accommodated in this model.
Figure 1. $\phi$ (upper) and the energy density (lower) of a runaway domain wall for $p = 2$ in a flat space.

For definiteness, we shall choose $p = 2$ henceforth so that $\phi \propto a^{1/2}$. Then the measured $\alpha$ by the QSO absorption lines ($[8, 9, 10, 14, 15, 17, 18]$) are fitted as a function of $a^{1/2}$ as $(\alpha(a) - \alpha_0)/\alpha_0 \simeq -6.2 \times 10^{-6}(1 - a^{1/2})$ and hence

$$\frac{\dot{\alpha}}{\alpha_0} \simeq 3.1 \times 10^{-6}a^{1/2}H. \quad (11)$$

Using this fit in Eq. (9) at $z \simeq 2$, we find that

$$\xi \frac{\phi}{M_{pl}} \simeq 3 \times 10^{-6}. \quad (12)$$

Putting this into Eq. (10), it implies that

$$\frac{\Delta \alpha}{\alpha_0} \simeq 7 \times 10^{-6}, \quad (13)$$

which explains naturally the largeness of the spatial variation.

Before closing this section, we would like to comment on the formation and dynamics of domain walls. For $|\phi| \ll \sigma$, the (tree-level) potential can be expanded
as
\[ V(\phi) \simeq V_0 - \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{4} \lambda \phi^4 + \cdots, \] (14)
where \( V_0 = \frac{M^8}{\sigma^4}, \ m_\phi^2 = 4M^8/\sigma^6, \) and \( \lambda = 12M^8/\sigma^8. \) By taking into account the finite temperature effects, the effective potential around the origin reads
\[ V_T(\phi) = V(\phi) + \xi^2 \frac{T^4}{M_{\text{pl}}^2} \phi^2 + \frac{\lambda}{8} T^2 \phi^2 + \cdots, \] (15)
Thus, the phase transition occurs around the critical temperature \( T_c \sim \frac{2}{\sqrt{3}} \sigma \) so that domain walls are formed. After some relaxation period, domain walls evolve according to the linear scaling solution \[ T_1 \], in which typical scale of the domain wall is comparable to the Hubble scale and its energy density is roughly given by
\[ \rho_{\text{wall}} \sim \mu H \propto \frac{1}{t}. \] (16)

3. Experimental Constraints

Let us now discuss the observational and experimental constraints on the parameters.

\textit{Sachs-Wolfe Effect:} A domain wall induces the temperature anisotropy by the Sachs Wolfe effect \[ \text{[27].} \] The gravitational potential due to the wall at the horizon scale is \( 2\pi G\mu H_0^{-1} \simeq (1/4)M_{\text{pl}}^{-2}\mu H_0^{-1} \) which induces the temperature anisotropy via the Sachs-Wolfe effect. The requirement that this should be less than \( 10^{-5} \) gives
\[ M < 30\text{GeV} \left( \frac{\sigma}{10^{15}\text{GeV}} \right)^{1/4}. \] (17)
The present energy density of a runaway domain wall within the horizon scale is estimated as \( \mu H_0^{-2}/(4\pi H_0^{-3}/3) \), which should be much less than the critical density \( 3M_{\text{pl}}^2H_0^2 \). This also gives a similar bound as Eq. \[ 17. \]

\textit{The Violation of the Weak Equivalence Principle:} Since the effective mass is very light: \( \sqrt{V''} \simeq M^4/\phi^3 < 10^{-41}\text{GeV}(M/10\text{GeV})^4(\sigma/10^{15}\text{GeV})^{-3} \) for \( \phi > \sigma, \) the scalar \( \phi \) mediates a long-range force via the coupling to nucleons, leading to the violation of the weak equivalence principle \[ \text{[22, 23].} \] The modification of the nucleon mass follows from the electromagnetic corrections. To leading order in \( \alpha \) these corrections are given by \[ 35, \]
\[ \delta m_p = B_p \delta \alpha/\alpha_0 = 0.63\text{MeV}\delta \alpha/\alpha_0, \] (18)
\[ \delta m_n = B_n \delta \alpha/\alpha_0 = -0.13\text{MeV}\delta \alpha/\alpha_0, \] (19)
\( \text{§ The friction force due to the thermal plasma is estimated as } m_\phi^2T^2v (v : \text{wall velocity}) \) and is always subdominant in comparison to a force per unit area \( \sim \sigma/t \) coming from the curvature \( \sim t. \) Thus, the friction effects on the domain wall dynamics due to the thermal plasma are negligible. This is simply because the field \( \phi \) consisting domain walls only weakly interacts with the thermal plasma.
where \(m_p\) and \(m_n\) are the proton and the neutron masses, and \(B_p \simeq 0.63\) MeV and \(B_n \simeq -0.13\) GeV are the Born terms for the proton and the neutron, respectively. Hence, from Eq. (19), the exchange of \(\phi\) induces a composition dependent long-range force. A test body of mass \(m\) experiences the acceleration induced by the \(\phi\)-exchange force \[a_\phi = \frac{\xi^2}{4\pi M_p^2 m r^2} \left(\frac{N^E_p B_p}{N^E_p + N^E_n} + \frac{N^E_n B_n}{N^E_p + N^E_n}\right) \left(\frac{N^E_p B_p + N^E_n B_n}{N_{p,1} + N_{n,1}}\right),\] where \(N^E_{p,n}(N_{p,n})\) are numbers of protons and neutrons in the Earth (the test body), in addition to the usual Newtonian acceleration due to the Earth mass \(M_E\):

\[
a_g = \frac{M_E}{8\pi M_p^2 m r^2}.
\]

The difference in accelerations between the two test bodies in Eötvös-Dicke-Braginsky type experiments is parametrized by the Eötvös ratio:

\[
\eta = \frac{2|a_1 - a_2|}{|a_1 + a_2|},
\]

where \(a_1\) and \(a_2\) are the accelerations of two bodies. Here we assume that the test bodies have almost equal masses \(m_1 \simeq m_2\), which implies \(N_{p,1} + N_{n,1} \simeq N_{p,2} + N_{n,2}\). In the present case, \(\eta\) is estimated as

\[
\eta \simeq 3 \times 10^{-14} \left(\frac{\xi}{10^{-3}}\right)^2.
\]

This should be smaller than the current experimental bounds \(\eta < 2 \times 10^{-13}\) \[36\], which gives

\[
\xi < 2.6 \times 10^{-3}.
\]

**Allowed Parameter Region:** The present value of \(\phi\) is estimated as

\[
\phi_0 \simeq (M^4 H_0^{-1})^{1/3}.
\]

From Eq. (12) and Eq. (24), we obtain

\[
M \simeq 30\text{GeV} \left(\frac{\xi}{10^{-3}}\right)^{-3/4}.
\]

Moreover, the requirement of \(\phi > \sigma\) to account for the time variation of \(\alpha\), from Eq. (24), leads to

\[
M > 6\text{GeV} \left(\frac{\sigma}{10^{15}\text{GeV}}\right)^{3/4}.
\]

For example, for \(\sigma \simeq 10^{15}\text{GeV}, M \simeq 30\text{GeV}\) and \(\xi \simeq 10^{-3}\) satisfy Eqs. (17), (25), and (26). This gives \(\phi_0 \simeq 8 \times 10^{15}\text{GeV}\). In Fig. 2 we show the allowed range of the parameters \(M\) and \(\sigma\) together with the relation Eq. (25).
Figure 2. Allowed parameter space. Upper region is excluded due to the Sachs-Wolfe effect (or large density parameter) Eq. (17); lower region is excluded because of $\phi < \sigma$ and the absence of the scaling solution, Eq. (26). Dotted lines explain the QSO data (Eq. (25)) with $\xi = 2 \times 10^{-3}, 10^{-3}, 3 \times 10^{-4}, 10^{-4}$ from bottom to top.

Window for Future Experiments: From Eq. (17) and Eq. (26), upper bounds on $M$ and $\sigma$ are found:

$$M < 70 \text{GeV} \quad \text{and} \quad \sigma < 2 \times 10^{16} \text{GeV},$$

which imply a lower bound on $\xi$ from Eq. (25):

$$3 \times 10^{-4} < \xi < 2.6 \times 10^{-3}.$$  \hspace{1cm} (28)

This in turn provides a window for $\eta$ from Eq. (22):

$$3 \times 10^{-15} < \eta < 2 \times 10^{-13}.$$  \hspace{1cm} (29)

Therefore, orders of magnitude improvements of the experimental limits on the weak equivalence principle by proposed experiments (such as MICROSCOPE [37], SR-POEM [38], Galileo Galilei [39] and STEP [40]) could lead to the detection of the violation of the weak equivalence principle induced by the scalar exchange force or refute this.
model. These experiments (in particular, MICROSCOPE launched in 2012) can test the violation of the equivalence principle better than $\eta = 10^{-15}$. Therefore, the model can be tested within a few years by these gravitational experiments.

4. Summary

Motivated by possible detections of spatial and temporal variations of $\alpha$, we have proposed a model based on a domain wall of runaway type. We have found that it is possible to construct a model to explain both variations simultaneously. We have studied the cosmological constraints on the model and found that the model can be made consistent with the current cosmological data and can be falsified by the future experiments to test the equivalence principle. We note that a model is not limited to a runaway potential, but we can construct a model with local minima so that the vacuum expectation value is determined by a runaway dilaton [41].

We have focused on $\alpha$ in this paper since our prime purpose was to provide a existence proof of a model. However, unless forbidden by symmetry, the direct couplings of $\phi$ to fermionic matter should exist, which result in the violation of the equivalence principle and in spatio-temporal variations of the proton-to-electron mass ratio. There are some indications of a non-zero value of a spatial variation of it [42]. It would be interesting to study the consequences of such matter couplings further.

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