Gravitational Waves Constraint over Asymptotic Inflation in a Nonminimal Derivative Coupling Cosmology

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Abstract

In this paper, we review the nonminimal derivative coupling (NDC) between gravity and a scalar field, from the point of view of the constraint imposed by the detection of gravitational waves. We study in detail the relation between the asymptotic inflationary solution obtained from NDC and its relation with the constraint, by means of a dynamical system analysis, with two different sets of variables: the original ones and new variables that allow us to study asymptotic limits as points in phase space. In this sense, we thus show that the asymptotic inflationary solutions from NDC are incompatible with the gravitational waves speed constraint.

Keywords: Modified Gravity, Inflation, Gravitational Waves

1. Introduction

Scalar-tensor theories became part of the main components of early universe cosmology, since they are present, for instance, in both inflationary and quantum cosmology approaches. Even though there are many covariant theories with a scalar field, the most common one used for this purpose is the so-called minimal coupling, for several reasons\textsuperscript{1}. This theory consists of a canonical scalar field coupled to gravity, which can be represented by the Lagrangian density below:

\[
L[\phi, g_{\mu\nu}] = \sqrt{-g} \left[ -\frac{1}{16\pi G} R - \frac{1}{2} g_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi - V(\phi) \right], \tag{1}
\]

where \( R \) is Ricci scalar, \( G \) is Newton’s constant, \( g_{\mu\nu} \) is a metric, \( \phi \) is the scalar field and \( V(\phi) \) is a scalar potential.

An important fact about (1) is that yet a large class of gravitational theories seem quite different from it at first, they are in fact equivalent to it up to a conformal transformation\textsuperscript{1}. Some interesting examples are \( \dot{R}^2 \) inflation, Brans-Dicke theory, and Higgs inflation\textsuperscript{2}.

Besides the well known advandtages of canonical scalar field inflation, some of its aspects still need a special attention, such as the initial singularity and the very specific character of the potential \( V \), which is considered by some authors as a fine-tuning problem. The singularity problem can be handled with quantum corrections, for instance. For the potential issue, we can aim to avoid it by replacing scalar potential \( V(\phi) \) by some other contribution, for example. Thus, if such an alternative theory is not conformally equivalent to (1), we can then investigate if it is able to describe an inflationary scenario. If such a structure exists, then it would be an alternative to minimal coupling inflation, without the fine-tuning, for it has no potential. This is one of the main motivations to investigate a scalar-tensor theory substantially different from (1).

An important example of such an alternative is the Nonminimal Derivative Coupling, as defined by the covariant Lagrangian below:

\[
L[\phi, g_{\mu\nu}] = \sqrt{-g} \left[ -\frac{R}{8\pi} - g_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi - \kappa G_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi \right], \tag{2}
\]

where \( \kappa > 0 \) is the non-minimal coupling constant, with dimension of time squared, and \( G_{\mu\nu} \) is the Einstein tensor. The main term in (2) is \( \kappa G_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi \), the coupling between Einstein tensor and the covariant derivatives of the scalar field. This coupling is called nonminimal because it is not conformally equivalent to (1), which is a result valid for a broader class of theories, as shown in\textsuperscript{3}. This strong distinction between (1) and (2) is what makes one expect that some new dynamics would follow from (2).

Theory (2) and other similar derivative couplings where investigated in various papers, with several different approaches and applications. Most of them (including (2)) can also be seen as particular cases of the bigger Horndeski theory\textsuperscript{4,5}. The term \( G_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi \) is also present in the so-called Fab Four theory\textsuperscript{6}. Just to mention some of the studies about that kind of couplings, see\textsuperscript{7,11}, for instance.

The particular form (2) above was studied in\textsuperscript{15}, where it was shown that it predicts an asymptotic accelerated expansion. This is the main result from\textsuperscript{15}, since it is
an indication that an actual inflationary theory could be build up based on that. In this sense, it was shown in [16] that such a solution matches the duration of inflation if the coupling constant is set to be $\kappa = 10^{-74}$ s$^2$. Because of that, we will focus on this paper in studying the consequences of that specific asymptotic solution, as we will describe below.

In this scenario, the revolutionary first detection of gravitational waves has imposed a very strong constraint over their speed $c_{GW}$, which was shown to be really close to that of light [17] [19]. This means that now all of our theories must deal with this observational data. Thus, this detection was immediately followed by a massive revision of gravitational theories. In particular, it was soon realized that NDC and similar nonminimal couplings seem to be incompatible with that constraint [5] [20]. But this does not seem to be an established fact, since some authors, such as [21], disagree.

In this letter, we aim to show that there is a precise criterion to decide that for the specific theory [2], in the way it was presented in [15]. Such criterion consists in studying the dependence of the theoretical value of $c_{GW}$ in terms of the scalar field and comparing this with the predicted solutions and the constraints. We show here the exact allowed range for $\dot{\phi}$ and its speed $c$, which means that there is no inflationary solution of [2] in accordance with gravitational waves speed constraint.

To apply this idea, we first analyze [2] in detail, in two different points of view. We first review that theory, showing four different asymptotic solutions, with a special attention to the inflationary solution proposed in [13]. Then we introduce a new set of variables that allow us to see the most important asymptotic solutions as points in phase space, from which we can see which ones are sources and attractors. We present that in Section 2.

In Section 3, we show the precise behavior of $c_{GW}$, which can be seen as a function of the product $\sqrt{\kappa \dot{\phi}}$ only, in order to establish the actual constraint over $\dot{\phi}$, which is Eq. (27). Now, once we compare this with the results from Section 2, we show how this leads to the fact that no asymptotic inflationary solution from [2] can agree with the $c_{GW}$ constraint.

Finally, in Section 4 we make some conclusions about what can be done to have a deeper understanding of this result in future works.

2. Nonminimal Derivative Coupling Cosmology

Even though there are many theories called derivative couplings, we will refer only to [2] as the Nonminimal Derivative Coupling theory (hereafter called just NDC) through this letter. We now briefly review its basic properties, in the way they were presented in [15], but we also present some new asymptotic analysis.

Let us just set some conventions first. We are considering the usual 4-dimensional spacetime, with greek indices running from 0 to 3 and latin indices running from 1 to 3. The background geometry is given by the flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$ds^2 = -N^2dt^2 + a^2\delta_{ij}dx^idx^j,$$

where $a(t)$ is the scale factor and $N(t)$ is the lapse function [22]. We temporarily keep $N$ because it may be useful for future works that involve any quantization process. We are using units such that the speed of light is $c = 1$, yet we can write $c$ explicitly sometimes just to emphasize.

From [3], we find the usual components of the Ricci tensor:

$$R_{00} = 3\frac{\dot{a}}{a}N - 3\frac{\ddot{a}}{a},$$
$$R_{0i} = 0,$$
$$R_{ij} = \delta_{ij}\frac{a^2}{N^2}\left(2\dot{\phi}^2 - \frac{\ddot{a}}{a} - \frac{\dot{a}N}{aN}\right),$$

from which we find the expression for $\frac{L}{N} = e^{3\alpha} \left( - \frac{3\dot{\alpha}^2}{4\pi N} + \frac{\dot{\phi}^2}{N} - 3\frac{\kappa a^2 \dot{\phi}^2}{N^3} \right)$.

Now we can study the equations of motion which describe the cosmological evolution generated by NDC, on the minisuperspace form [5]. The variation with respect to $N$ gives the constraint below, setting $N = 1$ after deriving the equation:

$$\frac{3}{4\pi} \dot{\alpha}^2 - \dot{\phi}^2 + 9\kappa a^2 \dot{\phi}^2 = 0.$$  

The variation with respect to $\alpha$ and $\dot{\phi}$, respectively, gives the equations of motion below, for $N = 1$:

$$2\ddot{\alpha} + 3\dot{\alpha}^2 + 4\dot{\phi}^2[1 + \kappa(2\ddot{\alpha} + 3\dot{\alpha}^2 + 4\dot{\phi}^2\dot{\phi}^{-1})] = 0,$$
$$3\ddot{\phi} + 3\dot{\alpha}\dot{\phi} - 3\kappa(\dot{\alpha}^2 \ddot{\phi} + 2\dot{\alpha}\dot{\phi} + 3\dot{\phi}^3) = 0.$$  

Equations (9), (10a), and (10b) were derived in [15]. Solving for $\dot{\alpha}$ and $\dot{\phi}$, the second-order system (10) can be rewritten as

$$\ddot{\alpha} = \frac{3\dot{\alpha}^2 - 9\kappa \dot{\alpha}^4 + 4\kappa \dot{\phi}^2 - 48\pi \kappa \dot{\alpha}^2 \dot{\phi}^2 + 108\pi \kappa^2 \dot{\alpha}^4 \dot{\phi}^2}{2(1 - 3\kappa a^2 + 4\pi \kappa \dot{\phi}^2 + 36\pi \kappa^2 \dot{\alpha}^2 \dot{\phi}^2)},$$
$$\ddot{\phi} = -\frac{3\dot{\alpha}\dot{\phi}(1 + 8\pi \kappa \dot{\phi}^2)}{1 - 3\kappa a^2 + 4\pi \kappa \dot{\phi}^2 + 36\pi \kappa^2 \dot{\alpha}^2 \dot{\phi}^2}. $$
Figure 1: The phase portrait (a) represents the dynamical system \((11)\) for \(\sqrt{\bar{\kappa}} \phi \times \sqrt{\bar{\kappa}} \dot{\alpha} \). The direction of the arrows indicates time evolution; the three black dots represent the critical points \((12)\), all of which are unstable; the thick curves in blue represent the constraint \((9)\); the dashed blue curve is just an example of solution, for which the initial conditions are \(\dot{\alpha}(0) = 0.1/\sqrt{\bar{\kappa}}, \phi(0) = -1/\sqrt{2\bar{\pi}\bar{\kappa}}\). The shaded region represents the allowed range of values \(|\dot{\alpha}| < 1/\sqrt{\bar{\kappa}}\), a limitation imposed by \((9)\); the dash-dotted lines represent the singularities, for which the denominator of \((11)\) vanishes. As for phase portrait (b), it represents the dynamical system \((16)\), where the variables are now \(x\) and \(y\), defined in \((11)\). The shaded region is the range of values \(|y| < 1\), which is equivalent to \(|\dot{\alpha}| < 1/\sqrt{\bar{\kappa}}\). The black dots represent the critical points relevant for the present discussion, which are \((0, 0)\), \((\pm 1, 0)\), and \((0, \pm 1)\) (the other critical points lie outside the constraint circle); the blue thick line represents the constraint \((15)\): the dashed line is an example of solution, with initial conditions \(x(0) = -0.3, y(0) = 0.3\), which is the same example shown in (a).

Note that this system is not the same originally presented in \([15]\), but they are equivalent. Now, we start analysing NDC theory based on \((11)\). The structure of system \((11)\) shows that it can be seen as an autonomous dynamical system for which \(\dot{\alpha}\) and \(\phi\) are the independent variables. The phase portrait of that system is shown in Fig. 1. Let us first discuss critical points and asymptotic solutions, because this leads to the inflationary mechanism claimed in \([15]\).

There are three relevant critical points \((\dot{\phi}_C, \dot{\alpha}_C)\) of \((11)\):

\[
(0, 0), \quad (0, 1/\sqrt{3\bar{\kappa}}), \quad \text{and} \quad (0, -1/\sqrt{3\bar{\kappa}}). \quad (12)
\]

The point \((0, 0)\) is a trivial solution. The point \((0, 1/\sqrt{3\bar{\kappa}})\) represents an exponential acceleration, because \(\dot{\alpha}\) is the same as the Hubble factor. And \((0, -1/\sqrt{3\bar{\kappa}})\) represents an exponential deceleration.

After some algebra, we can identify four asymptotic solutions for the scale factor:

- (S1) for \(|\dot{\phi}| \rightarrow \infty\) and \(\dot{\alpha} > 0\), \(a \sim e^{t/\sqrt{9\bar{\kappa}}};\)
- (S2) for \(|\dot{\phi}| \rightarrow \infty\) and \(\dot{\alpha} < 0\), \(a \sim e^{-t/\sqrt{9\bar{\kappa}}};\)
- (S3) for \(\dot{\phi} \rightarrow 0\), \(a \sim t^{2/3};\)
- (S4) for \(\kappa \rightarrow 0\), \(a \sim t^{1/3};\)

Let us now show how those solutions can be obtained by taking some limits on \((11)\). Solutions (S1) and (S2) are obtained by rewriting \((9)\) as

\[
\frac{3}{4\pi} \dot{\alpha}^2 - 1 + 9\bar{\kappa}\dot{\alpha}^2 = 0 \quad (13)
\]

and then taking \(|\dot{\phi}| \rightarrow \infty\); (S3) comes from the simplification of \((11a)\) when \(\dot{\phi} \rightarrow 0\); finally, (S4) is just the canonical scalar field solution when there is no potential. The phase portrait on the left handed side of Fig. 1 helps us interpreting those solutions.

The solution (S1) describes the asymptotic behavior in the early times for a solution like the one represented by the blue dashed line, and also both sides of the upper half of the constraint line in Figure 1 (a). This is the inflationary solution presented in \([15]\), where the coupling constant is set to be \(\kappa = 10^{-74} s^2\), so that it fits in the duration of inflation, as explained in \([16]\). Note that, in any case, all solutions like (S1) lie inside the allowed \(\dot{\alpha}\) range, which is \(-1/\sqrt{9\bar{\kappa}} < \dot{\alpha} < 1/\sqrt{9\bar{\kappa}}\), according to \((11)\). Hence, if we follow the path of any solution like (S1), it comes from \(t \rightarrow -\infty\) by an asymptotic de Sitter solution, without breaking the condition \(-1/\sqrt{9\bar{\kappa}} < \dot{\alpha} < 1/\sqrt{9\bar{\kappa}}\). Thus, (S1) represents the inflationary solution constructed...
in [15].

As for the other solutions: from (S2), a contracting universe is possible according to NDC; from (S3), there is a matter-dominated era when \( \dot{\phi} \approx 0 \); finally, we see from (S4) that the universe is dominated by stiff matter when the nonminimal coupling term is negligible.

Some of those asymptotic solutions become points if we define convenient variables. Let us take

\[
    x \equiv \sqrt{\frac{3}{4\pi}} \frac{\dot{\phi}}{\phi}, \quad y \equiv \sqrt{9\kappa \dot{\alpha}}.
\]  

(14)

Now, as we shall see in more detail below, the asymptotic solutions in the phase space.

Thus, we gain new information, since now we can easily see which asymptotic solution is an attractor and which one is a source.

For \( x \) and \( y \), constraint (10) becomes the unit circle

\[
    x^2 + y^2 = 1
\]

(15)

and the equations of motion, after some algebra, become the following dynamical system:

\[
\begin{align*}
    \dot{x} &= -\frac{x y (y^4 - x^2 y^2 - 8 y^2 - 3 x^2 + 3)}{2\sqrt{\kappa} (y^4 - x^2 y^2 + y^2 + 3 x^2)} , \\
    \dot{y} &= -\frac{y^6 - x^2 y^4 - 4 y^4 + 3 x^2 y^2 + 3 y^2}{2\sqrt{\kappa} (y^2 - x^2 y^2 + y^2 + 3 x^2)} .
\end{align*}
\]

(16a, 16b)

The phase portrait of (16) is shown in Fig. 1 (b). Let us now study its main critical points and solutions.

The critical points (\( x_C, y_C \)) of (16) are

\[
    (0, 0) , \quad (\pm 1, 0) , \quad (0, \pm 1) , \quad \text{and} \quad (0, \pm \sqrt{3}) .
\]

(17)

From Figure 1 (b), (0, 1) is an unstable node and (0, -1) is an attractor. Thus, from (13), we can see the actual meaning of those points: (0, 1) corresponds to (S1), the de Sitter solution \( a \sim e^{t/\sqrt{\kappa}} \); hence it is a source of dynamical solutions in the phase space.

We find solution (S2), which is \( a \sim e^{-t/\sqrt{\kappa}} \), at the point (0, -1) in the \( xy \) plane. As we can see in Figure 1(b), this is the final attractor. Note that both the asymptotic limits \( \phi \to \pm \infty \) are condensate in \( x = 0 \) (for any \( \dot{\phi} \neq 0 \)).

In particular, then, we can take the dashed curve in Figure 1(b), which corresponds to a dynamics that comes from \( |\phi| \to +\infty \), passes through \( \dot{\phi} = 0 \) and then evolves back to \( |\phi| \to +\infty \), and this last behavior is actually an attractor, meaning that all relevant solutions necessarily evolve to \( |\phi| \to +\infty \). In the next Section, we will discuss the precise relation between this particular behavior and gravitational waves.

As a final comment in this Section, we should mention that, since the origin in the \( xy \) phase space is a critical point, in principle a solution that goes to (0, 0) never actually comes from this state. But this is not true. Take, for instance, the blue dashed curve in Figure 1(a). For the original variables \( \sqrt{\kappa} \dot{\phi} \) and \( \sqrt{\kappa} \dot{\alpha} \), we virtually see all its behavior: it asymptotically comes from (S1) and then, after it approaches \( \dot{\alpha} = 0 \), it starts to evolve to the asymptotic solution (S2). And there is no singularity in between. Thus, when we write the same solution, the blue dashed curve, in terms of \( x \) and \( y \), the point (0, 0) in \( xy \) plane is not an actual physical critical point, it is just a consequence of the definition of the variables \( x \) and \( y \) (see (14)).

In other words, it does make sense to say that the solutions inside the unitary disk in Fig. 1 (b) come from the source point (0, 1), which represents (S1), and then they evolve to the attractor (0, -1), which represents (S2), as we claimed above.

3. Gravitational Waves Constraint over NDC

As we know, the recent detection of gravitational waves has imposed a very tight constraint over gravitational theories [17][19]: if \( c \) is the speed of light and \( c_{GW} \) is the speed of gravitational waves, then:

\[
    -3 \times 10^{-15} < c_{GW}/c - 1 \leq 7 \times 10^{-16} .
\]

(18)

One way to describe how this constrains NDC is to notice that NDC itself is just a particular case of Horndeski theory [4], which is represented by the action

\[
    S_H[\phi, g_{\mu\nu}] = \int d^4 x \sqrt{-g} (L_2 + L_3 + L_4 + L_5) ,
\]

(19)

where

\[
    L_2 = K(\phi, X) ,
\]

(20)

\[
    L_3 = -G_3(\phi, X)\Box \phi ,
\]

(21)

\[
    L_4 = G_4(\phi, X) R + G_{4, X}(\phi, X)(\Box \phi)^2
    - \nabla^\mu \nabla^\nu \phi \nabla_\mu \nabla_\nu \phi ,
\]

(22)

\[
    L_5 = G_5(\phi, X) G^\mu\nu \nabla_\mu \nabla_\nu \phi - \frac{1}{4} G_{5, X}(\phi, X)(\Box \phi)^3
    - 3 \Box \phi \nabla^\mu \nabla^\nu \phi \nabla_\mu \nabla_\nu \phi
    + 2 \Box \nabla_\nu \phi \nabla_\lambda \nabla^\mu \phi \nabla^\nu \nabla^\lambda \phi .
\]

(23)

The functions \( K(\phi, X) \) and \( G_i(\phi, X) \) are generic; \( X \equiv -\frac{1}{2} \nabla^\mu \phi \nabla_\mu \phi \) is the kinetic term; the notation \( G_{i, X} \) represents the derivative of \( G_i \) with respect to \( X \). Thus, we see that NDC is the particular case of Horndeski for which \( K(\phi, X) = 2X \), \( G_3 = 0 \), \( G_4 = 1/8\pi \), and \( G_5 = G_5(\phi) \) is such that \( dG_5/d\phi = \kappa \). This last equality is obtained by integrating by parts the \( G_5 \) term of Horndeski when \( G_5 = G_5(\phi) \) and comparing it with [2], discarding surfaces terms.

For recent reviews of Horndeski theory, see [5] [20]. For the present purposes, we must remember that Horndeski theory is the most general covariant scalar-tensor gravitational theory in four dimensions with second-order equations of motion. This means that in four dimensions, including only one extra degree of freedom, which is the scalar field \( \phi \), under very reasonable assumptions we are lead unavoidably to Horndeski theory. The second-order
equations of motion guarantee that Ostrogradsky’s instability is avoided. Hence, in particular, those are features of NDC as well.

\[ c^2_{GW} = \frac{1 - 4\pi\kappa\dot{\phi}^2}{1 + 4\pi\kappa\dot{\phi}^2}. \]  

In Figure 2(a), we can see \( c^2_{GW} \) as a function of \( \sqrt{4\pi\kappa} \dot{\phi} \). Since \( \kappa > 0 \), the last expression immediately implies that

(i) if \( \sqrt{4\pi\kappa} |\dot{\phi}| < 1 \), then \( c_{GW} > 0 \) is well defined;
(ii) if \( \sqrt{4\pi\kappa} |\dot{\phi}| = 1 \), then \( c_{GW} = 0 \);
(iii) if \( \sqrt{4\pi\kappa} |\dot{\phi}| > 1 \), then \( c_{GW} \) is imaginary.

Therefore, even before we take into account the observational data, we can see that NDC theory [2] is not in full agreement with the description of gravitational waves, at least classically. It follows from (i) that NDC can describe the propagation of tensor perturbations with an actual real and positive speed \( c_{GW} \) if, and only if, \( |\dot{\phi}| < (4\pi\kappa)^{-1/2} \). But that is not sufficient for a consistent gravitational wave, in view of [18], which imposes the tighter constraint below:

\[ 1 - 3 \times 10^{-15} < \left( \frac{1 - 4\pi\kappa\dot{\phi}^2}{1 + 4\pi\kappa\dot{\phi}^2} \right)^{1/2} \leq 1 + 7 \times 10^{-16}. \]  

The inequality on the right-hand side is automatically satisfied since the maximum value of (25) is 1, for any \( \dot{\phi} \) and any \( \kappa > 0 \). Solving the inequality on the left-hand side for \( \dot{\phi} \), we find:

\[ \sqrt{4\pi\kappa} |\dot{\phi}| \lesssim 3 \times 10^{-8}. \]  

In Figure 2(b), we can see this range (green region) and the correspondent values of the deviation \( c_{GW} - 1 \) (green region). The intersection of the two shaded regions represents the actual allowed range, given by both [18] and [27].

Now, it follows from the previous Section that any solution of the system [11] goes to or comes from \( |\dot{\phi}| \to \infty \). Therefore, they all break the gravitational waves constraint [27], at some point. In other words, there cannot be an asymptotic inflationary solution of NDC in accordance with [27].

In particular, the inflationary solution for which \( \kappa = 10^{-74}s^2 \), doesn’t satisfy [27]. It is true that when \( \kappa \) gets smaller, the constrained range for \( |\dot{\phi}| \) becomes wider, in view of [27]. However, no matter how big the range of \( |\dot{\phi}| \) is, there is an unavoidable incompatibility between the limitation imposed by this range and the fact that any inflationary solution is taken asymptotically when \( |\dot{\phi}| \) goes to infinity.

4. Conclusion

In summary, we can say that there really exists a range of values for the scalar field allowed by gravitational waves constraint, but it is not wide enough to be compatible with the asymptotic inflationary solution from [12]. Hence, those solutions are actually incompatible with gravitational waves constraint.
A possible way to avoid that issue may be done with quantum methods, such as Bohmian quantum cosmology, since they can be seen as small corrections of classical theories. We aim to investigate such possibility in a future work.

Acknowledgments

We thank to Felipe Tovar Falciano, Ingrid Ferreira da Costa, Júlio César Fabris, and Nelson Pinto-Neto for very important discussions about this paper. This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) - Finance Code 001, and also by CNPq and FAPES, from Brazil.

References

[1] V. Faraoni, Cosmology in scalar tensor gravity, Vol. 139, 2004. doi:10.1007/978-1-4020-1989-0
[2] P. Zyla, et al., Review of Particle Physics, PTEP 2020 (8) (2020) 083C01. doi:10.1093/ptep/ptaa104
[3] L. Amendola, Cosmology with nonminimal derivative couplings, Physics Letters B 301 (2) (1993) 175–182. doi:https://doi.org/10.1016/0370-2693(93)90685-B
URL https://www.sciencedirect.com/science/article/pii/037026939390685B
[4] G.W. Horndeski, Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space, Int. J. Theor. Phys. 10 (1974) 363–384. doi:10.1007/BF01807638
[5] T. Kobayashi, Horndeski theory and beyond: a review, Reports on Progress in Physics 82 (8) (2019) 086901. arXiv:1901.07183 doi:10.1088/1361-6636/ab2429
[6] C. Charmousis, E.J. Copeland, A. Padilla, P.M. Saffin, General Second-Order Scalar-Tensor Theory and Self-tuning, Phys. Rev. Lett. 108 (2012) 051101. arXiv:1106.2090 doi:10.1103/PhysRevLett.108.051101
[7] E. N. Saridakis, S. V. Sushkov, Quintessence and phantom cosmology with nonminimal derivative coupling, Phys. Rev. D 81 (2010) 083510. doi:10.1103/PhysRevD.81.083510
URL https://link.aps.org/doi/10.1103/PhysRevD.81.083510
[8] J. B. Dent, S. Dutta, E. N. Saridakis, J.-Q. Xia, Cosmology with non-minimal derivative couplings: perturbation analysis and observational constraints, Journal of Cosmology and Astroparticle Physics 2013 (11) (2013) 058–058. doi:10.1088/1475-7516/2013/11/058.
URL https://doi.org/10.1088/1475-7516/2013/11/058
[9] E. Balbi, C. Charmousis, Dressing a black hole with a time-dependent Galileon, JHEP 08 (2014) 106. arXiv:1312.3204 doi:10.1007/JHEP08(2014)106
[10] B. Gumjudpai, P. Rangdee, Non-minimal derivative coupling gravity in cosmology, Gen. Rel. Grav. 47 (11) (2015) 140. arXiv:1511.00491 doi:10.1007/s10714-015-1985-2
[11] N. Kawskhao, B. Gumjudpai, Cosmology of non-minimal derivative coupling to gravity in palatini formalism and its chaotic inflation of the Dark Universe 20 (2018) 20–27. doi:https://doi.org/10.1016/j.dark.2018.02.004
URL https://www.sciencedirect.com/science/article/pii/S0370269318300074
[12] S. Shahidi, Cosmology of a higher derivative scalar theory with non-minimal Maxwell coupling, Eur. Phys. J. C 79 (6) (2019) 448. arXiv:1811.10170 doi:10.1140/epjc/s10052-019-6960-8
[13] I. Torres, J.C. Fabris, O.F. Piattella, Classical and quantum cosmology of Fab Four John theories, Physics Letters B 798 (2020) 135063. arXiv:1811.08852 doi:https://doi.org/10.1016/j.physletb.2019.135003
[14] I. Torres, J.C. Fabris, O.F. Piattella, A.B. Batista, Quantum Cosmology of Fab Four John Theory with Conformable Fractional Derivative, Universe 6 (4) (2020) 50. arXiv:2001.07680 doi:10.3390/universe6040050
[15] S.V. Sushkov, Exact Cosmological Solutions with Nonminimal Derivative Coupling, Phys. Rev. D 80 (2009) 103505. arXiv:0910.0980 doi:10.1103/PhysRevD.80.103505
[16] S.V. Sushkov, Realistic Cosmological Scenario with Nonminimal Kinetic Coupling, Phys. Rev. D 85 (2012) 123520. arXiv:1204.6372 doi:10.1103/PhysRevD.85.123520
[17] LIGO Scientific Collaboration and Virgo Collaboration. (B.P. Abbott et al.), GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral, Phys. Rev. Lett. 119 (2017) 161101. arXiv:1710.05832 doi:10.1103/PhysRevLett.119.161101
[18] A. Goldstein et al., An Ordinary Short Gamma-Ray Burst with Extraordinary Implications: Fermi-GBM Detection of GRB 170817A, The Astrophysical Journal 848 (2) (2017) L14. arXiv:1710.05446 doi:10.3847/2041-8213/aa8f41
[19] B. P. Abbott et al., Gravitational Waves and Gamma-Rays from a Binary Neutron Star Merger: GW170817 and GRB 170817A, The Astrophysical Journal 848 (2) (2017) L13. arXiv:1710.05834 doi:10.3847/2041-8213/aa920c
[20] L. Heisenberg, A systematic approach to generalisations of general relativity and their cosmological implications, Physics Reports 796 (2019) 1–113, a systematic approach to generalisations of General Relativity and their cosmological implications. doi:https://doi.org/10.1016/j.physrep.2018.11.006
URL http://www.sciencedirect.com/science/article/pii/S0370157319307257
[21] Y. Gong, E. Papantonopoulos, Z. Yi, Constraints on scalar-tensor theory of gravity by the recent observational results on gravitational waves, The European Physical Journal C. 78 (9) (2018) 738. doi:10.1140/epjc/s10052-018-6227-9
[22] G. Calcagni, Classical and Quantum Cosmology, Graduate Texts in Physics, Springer, 2017. doi:10.1007/978-3-319-41127-9
[23] M. Bojowald, Quantum Cosmology: A Fundamental Description of the Universe, Lecture Notes in Physics, Springer New York, 2011.
[24] A. D. Felice, S. Tsujikawa, Conditions for the cosmological viability of the most general scalar-tensor theories and their applications to extended galileon dark energy models, Journal of Cosmology and Astroparticle Physics 2012 (02) (2012) 607–607. doi:10.3847/2041-8213/aa8f36
[25] E. Bellini, I. Sawicki, Maximal freedom at minimum cost: linear large-scale structure in general modifications of gravity, Journal of Cosmology and Astroparticle Physics 2014 (07) (2014) 050–050. arXiv:1404.3713 doi:10.1088/1475-7516/2014/07/050