Class field theory for strictly quasilocal fields
with Henselian discrete valuations

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1. Introduction

The purpose of this paper is to describe the norm groups of the fields pointed out in
the title. Our starting point is the fact that a field $K$ is strictly quasilocal, i.e. its finite
extensions are strictly primarily quasilocal (abbreviated, strictly PQL) fields if and
only if these extensions admit one-dimensional local class field theory (cf. [5, Sect. 3]).
Assuming that $K$ is strictly quasilocal and has a Henselian discrete valuation $v$, we
first show that the norm group $N(R/K)$ of each finite separable extension $R$ of $K$ is
of index $i(R/K)$ (in the multiplicative group $K^*$ of $K$) dividing the degree $[R:K]$.
We say that $R$ is a class field of $N(R/K)$, if $i(R/K) = [R:K]$. The present paper
shows that $N(R/K)$ possesses a class field $\text{cl}(N(R/K))$ which is uniquely determined
by $N(R/K)$, up-to a $K$-isomorphism. It proves that $\text{cl}(N(R/K))$ includes as a
subfield the maximal abelian extension $R_{ab}$ of $K$ in $R$. Also, we show that
$\text{cl}(N(R/K))$ embeds in $R$ as a $K$-subalgebra and is presentable as a compositum of
extensions of $K$ of primary degrees. This gives rise to a canonical bijection $\omega$ of the
set of isomorphism classes of class fields of $K$ upon the set $Nr(K)$ of norm groups
of finite separable extensions of $K$. Our main results describe the basic properties of
$\omega$ and eventually enable one to obtain a complete characterization of the elements
of $Nr(K)$ in the set of subgroups of $K^*$. They indicate that $K^*$ can be endowed
with a structure of a topological group with respect to which $Nr(K)$ is a system of
neighbourhoods of unity. This topology on $K^*$ turns out to be coarser than the one
induced by $v$ unless the residue field $\overline{K}$ of $(K,v)$ is finite or of zero characteristic

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(when they are equivalent). The present research plays a role in clarifying some general aspects of one-dimensional abstract local class field theory, such as the scope of validity of the classical norm limitation theorem (cf. [11, Ch. 6, Theorem 8]) for strictly PQL ground fields, and the possibility of reducing the study of norm groups of quasilocal fields to the special case of finite abelian extensions (see Remark 3.4, [3] and [6]).

The main field-theoretic notions needed for describing the main results of this paper are the same as those in [6]. Our basic terminology and notation concerning valuation theory, simple algebras and Brauer groups, profinite groups, field extensions and Galois theory are standard (and can be found, for example, in [9; 15; 17] and [21]). As usual, Galois groups are regarded as profinite with respect to the Krull topology. For convenience of the reader, we define the notion of a field with (one-dimensional) local class field theory in Section 2. For each field $E$, $E_{sep}$ denotes a separable closure of $E$, $G_E := G(E_{sep}/E)$ is the absolute Galois group of $E$ and $P(E)$ is the set of those prime numbers $p$ for which $E$ is properly included in its maximal $p$-extension $E(p)$ in $E_{sep}$. Recall also that $E$ is said to be PQL, if every cyclic extension $F$ of $E$ is embeddable as an $E$-subalgebra in each central division $E$-algebra $D$ of Schur index $\text{ind}(D)$ divisible by the degree $[F:E]$. When this occurs, we say that $E$ is a strictly PQL-field, if the $p$-component $\text{Br}(E)_p$ of the Brauer group $\text{Br}(E)$ is nontrivial, for every $p \in P(E)$. Let us note that PQL-fields and quasilocal fields are naturally singled out by the study of some of the basic types of stable fields with Henselian valuations (see [1, 5] and the references there). It is also worth mentioning that they admit satisfactory inner characterizations which are fairly complete when the considered ground fields belong to some actively studied and frequently used special classes (see Section 2 and the observations at the end of [5, Sect. 4], for more details). The research in this area, however, is primarily motivated by the fact that strictly PQL-fields admit local class field theory, and by the validity of the converse in all presently known cases (cf. [5, Theorem 1 and Sect. 3]). This applies particularly to the noted place of strictly quasilocal fields in this theory. As to the choice of our main topic, it is determined by the fact that the structure of strictly quasilocal fields with Henselian discrete valuations is known (cf. [4, Sect. 3]) and sheds light on essential general properties of arbitrary quasilocal fields. The main results of this paper aim at extending the traditional basis of one-dimensional abstract local class field theory (cf. [25] and [2]). They can be stated as follows:

**Theorem 1.1.** Let $(K,v)$ be a Henselian discrete valued strictly quasilocal field with a residue field $\hat{K}$. Then class fields and norm groups of $K$ are related as follows:

(i) For each $U \in \text{Nr}(K)$, there exists a class field $\text{cl}(U)$ which is uniquely determined,
up-to a $K$-isomorphism; the extension $\text{cl}(U)/K$ is abelian if and only if $P(\hat{K})$ contains the prime divisors of the index of $U$ in $K^*$.

(ii) A class field $\text{cl}(U)$ of a group $U \in \text{Nr}(K)$ embeds as a $K$-subalgebra in a finite extension $R$ of $K$ in $K_{\text{sep}}$ if and only if $N(R/K)$ is included in $U$; furthermore, if $N(R/K) = U$, then the $K$-isomorphic copy of $\text{cl}(U)$ in $R$ is unique and includes $R_{\text{ab}}$.

(iii) There exists a set $\{\Phi_U: U \in \text{Nr}(K)\}$ of extensions of $K$ in $K_{\text{sep}}$, such that $\Phi_U$ is a class field of $U$, $U \in \text{Nr}(K)$, and for each $U_1, U_2 \in \text{Nr}(K)$, $\Phi(U_1 \cap U_2)$ equals the compositum $\Phi(U_1) \Phi(U_2)$ and $\Phi(U_1 U_2) = \Phi(U_1) \cap \Phi(U_2)$.

Theorem 1.2. Assume that $K$, $v$ and $\hat{K}$ satisfy the conditions of Theorem 1.1, $\text{Op}(K)$ is the set of open subgroups of $K^*$ of finite indices, and $\Sigma(K)$ is the set of subgroups of $K^*$ of finite indices not divisible by $\text{char}(\hat{K})$. Then $\text{Nr}(K)$, $\text{Op}(K)$ and $\Sigma(K)$ have the following properties:

(i) The intersection of finitely many groups from $\text{Nr}(K)$ lies in $\text{Nr}(K)$; also, if $V$ is a subgroup of $K^*$ including a group $U \in \text{Nr}(K)$, then $V \in \text{Nr}(K)$;

(ii) $K^n \in \Sigma(K)$, for each positive integer $n$ not divisible by $\text{char}(\hat{K})$;

(iii) $\text{Nr}(K)$ is a subset of $\text{Op}(K)$ including $\Sigma(K)$; in order that $\text{Nr}(K) = \text{Op}(K)$ it is necessary and sufficient that $\text{char}(\hat{K}) = 0$ or $\hat{K}$ is a finite field.

The paper is organized as follows: Section 2 includes preliminaries used in the sequel. Section 3 contains the proof of Theorem 1.1 (i) and a characterization of class fields among finite separable extensions of a given Henselian discrete valued strictly quasilocal field. Theorems 1.1 (ii), (iii) and 1.2 are proved in Section 4.

2. Preliminaries

Let $E$ be a field, $\text{Nr}(E)$ the set of norm groups of finite extensions of $E$ in $E_{\text{sep}}$, and $\Omega(E)$ the set of finite abelian extensions of $E$ in $E_{\text{sep}}$. We say that $E$ admits (one-dimensional) local class field theory, if the mapping $\pi$ of $\Omega(E)$ into $\text{Nr}(E)$ defined by the rule $\pi(F) = N(F/E): F \in \Omega(E)$, is injective and satisfies the following two conditions, for each pair $(M_1, M_2) \in \Omega(E) \times \Omega(E)$:

The norm group of the compositum $M_1 M_2$ is equal to the intersection $N(M_1/E) \cap N(M_2/E)$ and $N((M_1 \cap M_2)/E)$ equals the inner group product $N(M_1/E)N(M_2/E)$.

Our approach to the study of fields with such a theory is based on the following lemma (proved e.g. in [6]).

Lemma 2.1. Let $E$ be a field and $L/E$ a finite extension, such that $L = L_1 L_2$, for
some extensions $L_1$ and $L_2$ of $E$ of relatively prime degrees. Then $N(L/E) = N(L_1/E) \cap N(L_2/E)$, $N(L_1/E) = E^* \cap N(L/L_2)$ and there is a group isomorphism $E^*/N(L/E) \cong (E^*/N(L_1/E)) \times (E^*/N(L_2/E))$.

The following generalization of the norm limitation theorem for local fields (found in [3]) is used in Section 3 for proving the existence in the former part of Theorem 1.1 (i).

**Proposition 2.2.** Assume that $E$ is a quasilocal field and $p$ is a prime number, for which the natural Brauer group homomorphism $Br(E) \to Br(L)$ maps the $p$-component $Br(E)_p$ surjectively on $Br(L)_p$, for every finite extension $L$ of $E$. Also, let $R/E$ be a finite separable extension, $R_{ab,p} = R_{ab} \cap E(p)$ and $N_p(R/E)$ the set of those $\alpha \in E^*$ for which the co-set $\alpha N(R/E)$ is a $p$-element in $E^*/N(R/E)$. Then $N(R/E) = N_p(R/E) \cap N(R_{ab,p}/E)$.

In the rest of this paper, $\overline{P}$ denotes the set of prime numbers, and for each field $E$, $P_0(E)$ is the subset of those $p \in \overline{P}$ for which the polynomial $\sum_{u=0}^{p-1} X^u$ has a zero in $E$ (i.e. $p = \text{char}(E)$ or $E$ contains a primitive $p$-th root of unity). Also, we assume that $P_1(E) = \{p' \in (\overline{P} \setminus P_0(E)) : E^* \neq E^{*p'}\}$ and $P_2(E) = \overline{P} \setminus (P_0(E) \cup P_1(E))$. Every finite extension $L$ of a field $K$ with a Henselian discrete valuation $v$ is considered with its valuation extending $v$, this prolongation is also denoted by $v$. We write $U(L), \hat{L}$ and $e(L/K)$ for the multiplicative group of the valuation ring of $(L, v)$, the residue field of $(L, v)$ and the ramification index of $L/K$, respectively. As usual, $U(L)^\nu := \{\lambda^\nu : \lambda \in U(L)\}$, for each $\nu \in \mathbb{N}$. The study of the basic types of PQL-fields with Henselian discrete valuations relies on the following results (see [2, Sect. 2] and [4, Sects. 2 and 3]):

(2.1) (i) $K$ is PQL if and only if $\hat{K}$ is a perfect PQL-field, and for each $p \in P(\hat{K})$, $\hat{K}(p)/\hat{K}$ is a $\mathbb{Z}_p$-extension;
(ii) $K$ is strictly PQL if and only if it is PQL and $P(\hat{K}) = P(K)$;
(iii) $K$ is quasi-local if and only if $\hat{K}$ is perfect and $G_{\hat{K}}$ is metabelian of cohomological dimension $\text{cd}(G_{\hat{K}}) \leq 1$; conversely, every profinite metabelian group $G$ satisfying the inequality $\text{cd}(G) \leq 1$ is continuously isomorphic to the absolute Galois group of the residue field of some Henselian discrete valued quasi-local field $K(G)$;
(iv) $K$ is strictly quasi-local if and only if the following two conditions hold: (α) $\hat{K}$ is perfect with $G_{\hat{K}}$ metabelian of cohomological $p$-dimension $\text{cd}_p(G_{\hat{K}}) = 1$, for each $p \in \overline{P}$; (β) $P_0(\hat{L}) \subseteq P(\hat{L})$, for every finite extension $\hat{L}$ of $\hat{K}$. Conversely, if $G$ is a metabelian profinite group, such that $\text{cd}_p(G) = 1$, for all $p \in \overline{P}$, then $G$ is realizable as an absolute Galois group of the residue field of a strictly quasi-local field $K(G)$ with a Henselian discrete valuation.
The fulfillment of (2.1) (i) ensures that K is nonreal (cf. [14, Theorem 3.16]) and
the assumption that K is strictly quasilocal implies the following:

(2.2) (i) \( P_0(K) \setminus \{ \text{char}(\hat{K}) \} = P_0(\hat{K}) \setminus \{ \text{char}(\hat{K}) \} \); 
(ii) The group \( G_K \) is prosolvable (see [2, Proposition 3.1]); 
(iii) If \( \tilde{L}/\hat{K} \) is a finite extension, then the quotient group \( \tilde{L}^*/\tilde{L}^{*p^\nu} \) is cyclic 
of order \( p^\nu \), for every \( \nu \in \mathbb{N} \) and each \( p \in P_1(\hat{K}) \); this is also true in case 
\( p \in (P_0(\hat{K}) \setminus \{ \text{char}(\hat{K}) \}) \) and \( \nu \) is a positive integer for which \( \hat{K} \) contains a primitive 
\( p^\nu \)-th root of unity; 
(iv) \( Br(\tilde{L}) = \{ 0 \} \) and \( Br(L)_p \) is isomorphic to the quasicyclic \( p \)-group \( \mathbb{Z}(p^\infty) \), for 
every finite extension \( L \) of \( K \) and each \( p \in P(\hat{K}) \) (apply [21, Ch. II, Proposition 6 (b)] and Scharlau’s generalization of Witt’s theorem [19]).

It is well-known (cf. [15, Ch. VIII, Sect. 3]) that if \( \varepsilon_p \) is a primitive \( p \)-th root of unity 
in \( \hat{K}_{\text{sep}} \), for a given number \( p \in (P \setminus \{ \text{char}(\hat{K}) \}) \), then the degree \( [\hat{K}(\varepsilon_p):\hat{K}] \) divides 
\( p - 1 \). Our next result (proved in [3]) shows that this is the only restriction on the possible values of the sequence \( [\hat{K}(\varepsilon_p):\hat{K}] : p \in P \). Combined with the former part of 
(2.1) (iv), it also describes the behaviour of the sets \( P(\hat{K}) \) and \( P_j(\hat{K}) : j = 0, 1, 2 \), when \( (K, v) \) runs across the class of Henselian discrete valued strictly quasilocal fields with \( \text{char}(\hat{K}) = 0 \).

**Proposition 2.3.** Let \( P_0, P_1, P_2 \) and \( P \) be subsets of \( P \), such that \( P_0 \cup P_1 \cup P_2 = P \), \( 2 \in P_0 \), \( P_i \cap P_j = \phi : 0 \leq i < j \leq 2 \), and \( P_0 \subset P \subset (P_0 \cup P_2) \). For each 
\( p \in (P_1 \cup P_2) \), let \( \gamma_p \) be an integer \( \geq 2 \) dividing \( p - 1 \) and not divisible by any 
element of \( P \). Assume also that \( \gamma_p \geq 3 \) in case \( p \in (P_2 \setminus P) \). Then there exists a Henselian discrete valued strictly quasilocal field \( (K, v) \) with the property that 
\( P_j(\hat{K}) = P_j : j = 0, 1, 2 \), \( P(\hat{K}) = P \), and for each \( p \in (P_1 \cup P_2) \), \( \gamma_p \) equals the degree 
\( [K(\varepsilon_p):K] \), where \( \varepsilon_p \) is a primitive \( p \)-th root of unity in \( K_{\text{sep}} \).

**Remark 2.4.** It should be pointed out that an abelian torsion group \( T \) is realizable 
as a Brauer group of a strictly PQL-field with a Henselian discrete valuation if and 
only if \( T \) is divisible with a 2-component \( T_2 \cong \mathbb{Z}(2^\infty) \) (cf. [4, Sect. 4]). This, 
compared with (2.2) (iv), shows that strictly PQL-fields form a substantial extension 
of the class of strictly quasilocal fields. Such a conclusion can also be drawn from 
the study in [7] of the basic types of the considered fields in the class \( \text{Alg}(E_0) \) of 
algebraic extensions of a global field \( E_0 \). Besides characterizations and a general 
existence theorem for such fields, the preprint [7] contains a description of \( N(R/E) \), 
for a given finite extension \( R \) of a strictly PQL-field \( E \in \text{Alg}(E_0) \). Similarly to (2.1), 
Proposition 2.3 and the results of the present paper, this enables one not only to find 
various nontrivial examples of PQL-fields but also to combine the formal approach
to one-dimensional class field theory (followed in [5], [6, Sect. 3] and [3, Sects. 3-4]) with efficient constructive methods (see [3, Sect. 5] and [6, Sect. 4]).

3. Existence and uniqueness of class fields

The purpose of this Section is to prove Theorem 1.1 (i). Its main result shows that the validity of the former part of (2.1) (iv) guarantees the existence of class fields presentable as compositums of extensions of primary degrees over the ground fields. It also sheds light on the role of Proposition 2.3 in the study of norm groups of quasilocal fields.

**Theorem 3.1.** Assume that \((K, v)\) is a Henselian discrete valued strictly quasilocal field and \(R\) is a finite extension of \(K\) in \(K_{\text{sep}}\). Then \(R/K\) possesses an intermediate field \(R_1\) satisfying the following conditions:

(i) The sets of prime divisors of \(e(R_1/K), [\hat{R}_1: \hat{K}]\) and \([\hat{R}: \hat{R}_1]\) are included in \(P_1(\hat{K}), \overline{P} \setminus P(\hat{K}) \) and \(P(\hat{K})\), respectively;

(ii) \(N(R/K) = N((R_{ab}R_1)/K)\) and \(K^*/N(R/K)\) is isomorphic to the direct sum \(G(R_{ab}/K) \times (K^*/N(R_1/K))\);

(iii) \(R_{ab}R_1\) is a class field of \(N(R/K)\) and \([R_{ab}R_1]: K] = [R_{ab}: K] \times [R_1: K]\).

**Proof.** Let \(R'\) be the maximal inertial extension of \(K\) in \(R\), i.e. the inertial lift of \(\hat{R}\) in \(R\) over \(K\) (cf. [12, Theorems 2.8 and 2.9]). Note first that \(R'\) contains as a subfield an extension of \(K\) of degree \(n_0\), for each \(n_0 \in \mathbb{N}\) dividing \([R': K]\). Indeed, by (2.1) (iv), \(G_{\hat{R}}\) is metabelian with \(\text{cd}_p(G_{\hat{R}}) = 1: p \in \overline{P}\), and by [2, Lemma 1.2], this means that the Sylow pro-\(p\)-subgroups of \(G_{\hat{R}}\) are continuously isomorphic to \(\mathbb{Z}_p\), for each \(p \in \overline{P}\). Therefore, the Sylow subgroups of the Galois groups of finite Galois extensions of \(\hat{K}\) are cyclic. Observing also that if \(M\) is the normal closure of \(R'\) in \(K_{\text{sep}}\) over \(K\), then \(M/K\) is inertial with \(G(M/K) \cong G(\hat{M}/\hat{K})\) (cf. [12, page 135]), one deduces our assertion from Galois theory and the following lemma.

**Lemma 3.2.** Assume that \(G\) is a nontrivial finite group whose Sylow subgroups are cyclic, \(H\) is a subgroup of \(G\) of order \(n\), and \(n_1\) is a positive integer divisible by \(n\) and dividing the order \(o(G)\) of \(G\). Then \(G\) possesses a subgroup \(H_1\) of order \(n_1\), such that \(H \subseteq H_1\).

**Proof.** Denote by \(p\) the greatest prime divisor of \(o(G)\). Our assumptions show that \(G\) is a supersolvable group, whence it has a normal Sylow \(p\)-subgroup \(G_p\) as well as a subgroup \(A_p\) isomorphic to \(G/G_p\) (cf. [13, Ch. 7, Sects. 1 and 2]). In view of the supersolvability of the subgroups of \(G\), this allows one to prove by induction
on \( o(G) \) that \( G \) has a subgroup \( \hat{H}_1 \) of order \( n_1 \). Taking finally into account that \( H \) is conjugate in \( G \) to a subgroup of \( \hat{H}_1 \) [18] (see also [22, Theorem 18.7]), one completes the proof of the lemma.

Suppose now that \( R'_1 \) is the maximal tamely ramified extension of \( K \) in \( R \), \([R'_1:R] = n\) and \( P \) is the set of prime numbers dividing \([R:K]\). For each \( p \in P \), let \( f(p) \) and \( g(p) \) be the greatest integers for which \( p^{f(p)}[R:K] \) and \( p^{g(p)}[R':K] \).

As noted above, Lemma 3.2 indicates that there is an extension \( R_p \) of \( K \) in \( R' \) of degree \( p^{f(p)} \), for each \( p \in P \). Observing that \( \alpha \in U(R') \), provided that \( \alpha \in R' \) and \( v(\alpha - 1) > 0 \), one obtains from [16, Ch. II, Proposition 12] that \( R'_1 = R'(\theta) \), where \( \theta \) is an \( n \)-th root of \( \pi \rho \), for some \( \rho \in U(R') \). Suppose now that \( p \in (P_0(\hat{K}) \cup P_1(\hat{K})) \) and \( p \neq \text{char} (\hat{K}) \). Since \( p \) does not divide \([R':R_p]\), statement (2.2) (iii) implies the existence of an element \( \rho_p \in U(R_p) \), such that \( \rho_p \rho^{-1} \) is a \( p^{(f(p) - g(p))} \)-th power in \( U(R') \). Therefore, the binomial \( X^{p^{(f(p) - g(p))}} - \pi \rho_p \) has a zero \( \theta_p \in R'_1 \). Summing up these results, one proves the following:

(3.1) For each \( p \in P \cap (P_0(\hat{K}) \cup P_1(\hat{K})) \), \( p \neq \text{char} (\hat{K}) \), there exists an extension \( T_p \) of \( K \) in \( R'_1 \) of degree \( p^{f(p)} \); moreover, if \( p \in P_0(\hat{K}) \), then the normal closure of \( T_p \) in \( K_{\text{sep}} \) over \( K \) is a \( p \)-extension.

Let \( R_1 \) be the compositum of the fields \( R_p \): \( p \in (P_2(\hat{K}) \setminus P(\hat{K})) \), and \( T_p \): \( p \in P_1(\hat{K}) \). Clearly, then \( R_1 \) satisfies the conditions of Theorem 3.1 (i) and embeds as a \( K \)-subalgebra in the field \( R'(\theta^\mu) := L' \), where \( \mu \) is the greatest integer dividing \([R'_1:R']\) and not divisible by any element of \( \mathcal{P} \setminus \mathcal{P}(\hat{K}) \). Assuming further that \( R_1 \subseteq L' \), put \( L = R'R_1 \), \( T = R_{ab}R_1 \) and \( T' = T(\theta^\mu) \). It is easily verified that \( R' \subseteq T \) and \( T' = R_{ab}L' \). As g.c.d. \( ([R_{ab}:K],[R_1:K]) = 1 \), and by (2.2) (iv), \( \text{Br}(\hat{K})_p \cong \mathbb{Z}(p^\infty) \), for all \( p \in P(\hat{K}) \), Lemma 2.1 and [5, Theorem 3.1] indicate that \( K^*/N(T/K) \cong G(R_{ab}/K) \times (K^*/N(R_1/K)) \). One also sees that \( e(T/K) = e(R_{ab}/K)e(R_1/K) \), \( T = R_{ab}L \) and the extensions \( L'/L \) and \( T'/T \) are tamely totally ramified of degree \( \prod \left[ p^{f(p) - g(p)} \right] \), where \( p \) runs through the set \( P \cap (P_2(\hat{K}) \setminus P(\hat{K})) \). In order to complete the proof of Theorem 3.1, it remains to be shown that \( N(T/K) = N(R/K) \) and \( K^*/N(R_1/K) \) is a group of order \([R_1:K]\). Our argument is based on the following two statements:

(3.2) (i) The natural homomorphism of \( \text{Br}(K)_p \) into \( \text{Br}(Y)_p \) is surjective, for each \( p \in P(\hat{K}) \) and every finite extension \( Y \) of \( K \) in \( K_{\text{sep}} \);
(ii) \( N(T'/K) = N(T/K) \) and \( r\pi^{[R'/K]} \in N(R/K) \), for some \( r \in U(K) \).

Statement (3.2) (i) is implied by the final assertion of (2.2) (iv) and the well-known result (cf. [17, Sects. 13.4 and 14.4]) that the relative Brauer group \( \text{Br}(Y/E) \)
is of exponent dividing \([Y:E]\). The rest of the proof of (3.2) relies on the assumption that \(R'\) is the maximal inertial extension of \(K\) in \(R\). In particular, \(R\) is totally ramified over \(R'\), which means that \(U(R')\) contains an element \(\rho\), such that \(\rho\pi \in N(R/R')\). Therefore, the latter part of (3.2) (ii) applies to the element \(r = N_{K/R'}(\rho)\). In view of (2.1) (iii) and Galois cohomology (cf. [21, Ch. II, Proposition 6 (b)]), we have \(N(\hat{R}/\hat{K}) = \hat{K}^*\), so it follows from the Henselian property of \(v\) that \(N(R'/K) = U(K)\langle r\pi^{[R':K]} \rangle = U(K)\langle \pi^{[R':K]} \rangle\). This implies that \(N(R'/K)\) is a subgroup of \(K^*\) of index \([R':K]\). These observations, combined with the fact that \(R' \subseteq T \subseteq T' \subseteq R_1\) and the fields \(T, T'\) are tamely and totally ramified over \(R'\), show that \(N(T/K) = U(K)e^{(T/K)}\langle r\pi^{[R':K]} \rangle\) and \(N(T'/K) = U(K)e^{(T'/K)}\langle r\pi^{[R':K]} \rangle\). As proved above, \([T':T]\) is not divisible by any \(p \in (P(\hat{K}) \cup P_1(\hat{K}))\), whereas \(e(T/K) = e(R_{ab}/K)e(R_1/K)\), so it turns out that \(g.c.d. ([T':T], e(T/K)) = 1\), \(U(K)e^{(T/K)} = U(K)e^{(T'/K)}\) and \(N(T/K) = N(T'/K)\). Arguing in a similar manner, one obtains that \(N(R_1/K) = U(K)e^{(R_1/K)}\langle r_{0}\pi^{[R_0:K]} \rangle\), where \(R_0 = R' \cap R_1\) and \(r_0\) is the norm over \(K\) of a suitably chosen element of \(U(R_0)\). Since prime divisors of \(e(R_1/K)\) lie in \(P_1(\hat{K})\), the former part of (2.2) (iii) and the Henselian property of \((K, v)\) imply that \(U(K)/U(K)e^{(R_1/K)}\) is a cyclic group of order \(e(R_1/K)\). The obtained results enable one to establish the required properties of \(K^*/N(R_1/K)\) as a consequence of the well-known equality \([R_1:K] = [R_1:\hat{K}]e(R_1/K)\) (following from Ostrowski’s theorem and from [23, Propositions 2.2 and 3.1]).

It remains to be seen that \(N(R/K) = N(T/K)\). The inclusion \(N(R/K) \subseteq N(T/K)\) is evident, so we prove the converse one. Consider an arbitrary element \(\beta\) of \(U(K) \cap N(T/K)\), put \([R:T'] = m\), and for each \(p \in (P \cap P(\hat{K}))\), let \(R_{ab,p} = R_{ab} \cap K(p)\) and \(\rho_p\) be the greatest integer dividing \([R:p]\) and not divisible by \(p\). It follows from the inclusion \(N(T/K) \subseteq N(R_{ab,p}/K)\), statement (3.2) (i) and Proposition 2.2 that \(\beta^{\rho_p} \in N(R/K)\), for each \(p \in (P \cap P(\hat{K}))\). At the same time, the equality \(N(T/K) = N(T'/K)\) implies that \(\beta^{m} \in N(R/K)\). Observing now that the prime divisors of \(m\) lie in \(P(\hat{K})\), one obtains that \(g.c.d. \{m, \rho_p; p \in (P \cap P(\hat{K}))\} = 1\), and therefore, \(\beta \in N(R/K)\). Since \(N(T/K) = U(K)e^{(T/K)}\langle r\pi^{[R':K]} \rangle\) and \(r\pi^{[R':K]} \in N(R/K)\), this means that \(N(T/K) \subseteq N(R/K)\), so Theorem 3.1 is proved.

The former part of Theorem 1.1 (i) is contained in Theorem 3.1 and the following lemma.

**Lemma 3.3.** Let \((K, v)\) be a Henselian discrete valued strictly quasilocal field, and let \(L_1\) and \(L_2\) be finite extensions of \(K\) in \(K_{sep}\). Assume also that \(L_1\) and \(L_2\) are class fields of one and the same group \(N \in Nr(K)\). Then \(L_1\) and \(L_2\) are isomorphic over \(K\).
Proof. It follows from Theorem 3.1, statement (3.1) and the observations preceding Lemma 3.2 that one may consider only the special case in which \([L_i : K]\) is a \(p\)-primary number, for some \(p \in \left(\mathcal{P} \setminus \mathcal{P}(\hat{K})\right)\). Suppose first that \(p \in \left(\mathcal{P}_2(\hat{K}) \setminus \mathcal{P}(\hat{K})\right)\) and denote by \(M\) the minimal normal extension of \(K\) in \(K_{\text{sep}}\) including \(L_1\) and \(L_2\). Then \(L_1\) and \(L_2\) are inertial over \(K\), whence \(M\) has the same property. In view of (2.1) (iii) and [2, Lemma 1.2], this implies that the Sylow subgroups of \(G(M/K)\) are cyclic, so it follows from [22, Theorem 18.7] that \(G(M/L_1)\) and \(G(M/L_2)\) are conjugate in \(G(M/K)\). Hence, by Galois theory, there exists a \(K\)-isomorphism \(L_1 \cong L_2\). Assume now that \(p \in \mathcal{P}_1(\hat{K})\), fix a primitive \(p^{\text{th}}\) root of unity \(\varepsilon \in K_{\text{sep}}\), and put \(L'_i = L(\varepsilon)\), for each finite extension \(L\) of \(K\) in \(K_{\text{sep}}\). It is clear from (2.1) (iii), the condition on \(p\) and the general properties of cyclotomic extensions (cf. [24, Lemma 1]) that \(K'\) contains a primitive \(p^m\)-th root of unity for every \(m \in \mathbb{N}\). Since \([K' : K]\) divides \(p - 1\) (cf. [15, Ch. VIII, Sect. 3]), our argument also shows that \(K'^* = K^*K^*p^m\) and \([L'_i : K'] = [L_i : K]\). These observations, combined with Lemma 2.1, imply that the natural embedding of \(K^*\) into \(K'^*/N\) induces an isomorphism of \(K^*/N\) on \(K'^*/N(L'_i/K')\). As \(L_1\) and \(L_2\) are class fields of \(N\), the obtained result leads to the conclusion that \(N(L'_i/K') = N(L'_2/K')\). At the same time, (3.1) indicates that the normal closure of \(L'_i\) in \(K_{\text{sep}}\) over \(K'\) is a \(p\)-extension, so it follows from [6, Theorem 1.1] and the established properties of \(N(L'_i/K')\) that \(L'_i/K'\) is abelian and \(L'_i/K\) is normal. As \(K'\) admits local class field theory, one also sees that \(L'_1 = L'_2\). Note finally that \(G(L'_i/K)\) is solvable and \(G(L'_i/L_1)\) and \(G(L'_i/L_2)\) are subgroups of \(G(L'_i/K)\) of order \([K' : K]\) and index \([L_1 : K]\). Hence, by P. Hall’s theorem (cf. [13, Ch. 7]), they are conjugate in \(G(L'_i/K)\), i.e. \(L_1\) and \(L_2\) are isomorphic over \(K\), so Lemma 3.3 is proved.

Proof of the latter part of Theorem 1.1 (i). It is clear from (2.2) (i) and Galois theory that if \(A/K\) is a finite abelian extension, then \([A : K]\) is not divisible by any \(p \in \left(\mathcal{P} \setminus \mathcal{P}(\hat{K})\right)\). Conversely, let \(A\) be a finite extension of \(K\) in \(K_{\text{sep}}\), such that the prime divisors of \([A : K]\) are contained in \(\mathcal{P}(\hat{K})\). Then it follows from (3.2) and Proposition 2.2 that \(N(A/E) = N(A_{\text{ab}}/E)\). Thus the latter part of Theorem 1.1 (i) reduces to a consequence of the former one.

Remark 3.4. Theorem 3.1 indicates that if \((K, v)\) is a Henselian discrete valued strictly quasilocal field, \(R/K\) is a finite separable extension and \(\text{cl}(R/K)\) is the class field of \(N(R/K)\) in \(R\), then \(R\) is totally ramified over \(\text{cl}(R/K)\) and \([R : \text{cl}(R/K)]\) is not divisible by any \(p \in \mathcal{P}_1(\hat{K})\). In addition, it is easily obtained from (2.1) (iii), [2, Lemma 1.2] and Galois theory that \(\hat{R}\) is abelian over \(\hat{K}\) if and only if \(P(\hat{K})\) contains all prime divisors of \([\hat{R} : \hat{K}]\). These results enable one to come to the following conclusions:
(i) $R = \text{cl}(R/K)$ if and only if the sets of prime divisors of $[R: R_{ab}]$ and $e(R/K)$ are included in $\mathcal{P} \setminus \mathcal{P}(\widehat{K})$ and $P_1(\widehat{K}) \cup P(\widehat{K})$, respectively;

(ii) $N(R/K) = N(R_{ab}/K)$ if and only if $\widehat{R}/\widehat{K}$ is abelian and $[R: K]$ is not divisible by any $p \in P_1(\widehat{K})$.

4. Proofs of Theorems 1.1 (ii), (iii) and 1.2

Our objective in this Section is to complete the proof of Theorems 1.1 and 1.2. In what follows, we assume that $K$ is a strictly quasilocal field with a Henselian discrete valuation $v$. Note first that the former part of Theorem 1.2 (i) is implied by Theorem 1.1 (iii). The presentation of the rest of our argument is divided into three main parts.

Proof of Theorem 1.1 (ii) and the latter part of Theorem 1.2 (i). Let $U$ be a group from $N_{r}(K)$, $V$ a subgroup of $K^*$ including $U$, $j(u)$ the index of $U$ in $K^*$, and $L_1$ a class field of $U$ in $K_{sep}$. Clearly, it is sufficient to prove that $V \in N_{r}(K)$ in the special case where $j(u)$ is a $p$-primary number. Firstly, if $p \in P(\widehat{K})$, this is implied by Theorem 3.1, Galois theory and the availability of a local class field theory on $K$. Secondly, if $p \in (P_2(\widehat{K}) \setminus P(\widehat{K}))$, then $L_1$ is inertial over $K$ (see Remark 3.4 (i)). As shown at the beginning of the proof of Theorem 3.1, this indicates that $K$ has an extension $\Psi_\tau$ in $L_1$ of degree $\tau$, for each positive integer $\tau$ dividing $[L_1: K]$. Now the assertion that $V \in N_{r}(K)$ follows directly from the computation of $N(L'/K)$ carried out in the process of proving (3.2) (ii). Suppose finally that $p \in P_1(\widehat{K})$, fix a primitive $p$-th root of unity $\varepsilon_p$ in $K_{sep}$, and put $K' = K(\varepsilon_p)$ and $L'_1 = L'_1(\varepsilon_p)$. Analyzing the proof of Lemma 3.3, one obtains that there is a bijection of the set of intermediate fields of $L'_1/K'$ on the set of subgroups of $K'^*$ including $N(L'_1/K')$, and also, that these intermediate fields are normal over $K$. Thus it turns out that if $L'$ is the extension of $K'$ in $L'_1$ corresponding to $V.N(L'/K')$, then $V = N(L/K)$, where $L = L' \cap L_1$. In particular, $V \in N_{r}(K)$, as claimed by the latter part of Theorem 1.2 (i). In view of Lemma 3.3, this also proves the former part of Theorem 1.1 (ii).

Assume now that $R$ is a finite extension of $K$ in $K_{sep}$ with $N(R/K) = U$, $P$ is the set of prime numbers dividing $[R: K]$, $R_1$ is a subfield of $R$ determined as required by Theorem 3.1, and $\Phi$ is a class field of $U$ in $R$. By Lemma 3.3, there is a $K$-isomorphism $\Phi \cong R_{ab}R_1$, so $R_{ab}$ is a subfield of $\Phi$. Furthermore, it follows from Galois theory and the definition of $R_1$ that $\Phi$ is presentable as a compositum of extensions $\Phi_p$ of $K$ of $p$-primary degrees, with $p$ running across $P$. We show that $\Phi = R_{ab}R_1$. It is clearly sufficient to consider only the special case in which $R \neq R_{ab}$ and to establish the equality $R_{ab}\Phi_p = R_{ab}\Theta_p$, for an arbitrary $p \in (P \setminus P(\widehat{K}))$ and
a given $K$-isomorphic copy $\Theta_p$ of $\Phi_p$ in $R_{ab}R_1$. Suppose first that $p \in P_1(\hat{K})$. It follows from the proof of (3.1) that if $R_{ab}\Phi_p \neq R_{ab}\Theta_p$, then the set $R \setminus R_{ab}$ must contain a primitive $p$-th root of unity $\varepsilon_p$. However, since the extension $K(\varepsilon_p)/K$ is abelian (cf. [15, Ch. VIII, Sect. 3]), this is impossible, so we have $R_{ab}\Phi_p = R_{ab}\Theta_p$.

Let now $p \in (P_2(\hat{K}) \setminus P(\hat{K}))$ and $(R_{ab}\Theta_p) \cap (R_{ab}\Phi_p) = V_p$. In this case, $\Phi_p$ and $\Theta_p$ are inertial over $K$, whence $R_{ab}\Phi_p$ and $R_{ab}\Theta_p$ are inertial over $V_p$. In view of (2.1) (iv) and Lemma 3.2, this means that if $R_{ab}\Phi_p \neq R_{ab}\Theta_p$, then $R$ possesses distinct subfields $W_1$ and $W_2$, which are $V_p$-isomorphic extensions of $V_p$ of degree $p$. Denote by $W_3$ the minimal normal extension of $V_p$ in $K_{sep}$ including $W_1$ and $W_2$. Clearly, $W_3/V_p$ is a nonabelian Galois extension, and because of the prosolvability of $G_K$ (and the inclusion $(W_1 \cup W_2) \subseteq R$), $W_3$ is a subfield of $R$ of degree $pm$ over $V_p$, for some integer $m$ dividing $p - 1$. In addition, it is easily seen that $W_3$ contains as a subfield a cyclic extension of $V_p$ of degree $m$. Observing also that $W_1$, $W_2$ and $W_3$ are inertial over $V_p$ (see [12, page 135]), one deduces from (2.1) (iii) and Galois theory that $m$ is divisible by at least one number $\mu \in P(\hat{K})$. Thus the hypothesis that $R_{ab}\Phi_p \neq R_{ab}\Theta_p$ leads to the conclusion that $V_p$ admits an inertial cyclic extension $Y_p$ in $R$ of degree $\mu$. Since $\mu \in P(\hat{K})$, statement (2.1) (i) (applied to $K$ and $V_p$) implies the existence of an inertial cyclic extension $Y_p'$ of $K$ with the property that $Y_p'V_p = Y_p$. The obtained result, however, contradicts the inclusions $Y_p' \subseteq R_{ab} \subseteq V_p$, so the equality $R_{ab}\Phi_p = R_{ab}\Theta_p$ and the latter part of Theorem 1.1 (ii) are proved.

**Proof of Theorem 1.1 (iii).** By (2.2) (ii), $G_K$ is prosolvable, whence it possesses a closed Hall pro-$\Pi$-subgroup $H_\Pi$ (uniquely determined, up-to conjugacy in $G_K$), for each subset $\Pi$ of $\mathcal{P}$. Denote by $K_{ur}$ the compositum of inertial finite extensions of $K$ in $K_{sep}$, and for each $p \in \mathcal{P}$, assume that $\Lambda_p$ is the extension of $K$ in $K_{sep}$ corresponding by Galois theory to a given Hall pro-$\{p\}$-subgroup of $G_K$, $\Omega_p = K_{ur} \cap \Lambda_p$, $K_{ab}(p)$ is the maximal abelian extension of $K$ in $K(p)$, and $N_p$ is the set of all groups $X_p \in N_R(K)$ of $p$-primary indices in $K^*$. Let $I_p$ be the set of finite extensions of $K$ in $\Lambda_p$, $\Omega_p$ or $K_{ab}(p)$, depending on whether $p \in P_1(\hat{K})$, $P_2(\hat{K}) \setminus P(\hat{K})$ or $P(\hat{K})$, respectively. Returning to the proof of Lemma 3.3 and taking into account that $K$ admits local class field theory (as well as the conjugacy of the Hall pro-$\{p\}$-subgroups of $G_K$), one obtains the following result:

(4.1) The mapping of $I_p$ into $N_p$, defined by the rule $\Delta_p \to N(\Delta_p/K)$:

$\Delta_p \in I_p$, is bijective, for each $p \in \mathcal{P}$. It transforms field compositums into group intersections and field intersections into inner group products. Also, every finite extension of $K$ in $K_{sep}$ of $p$-primary degree is $K$-isomorphic to a field from $I_p$.

Suppose now that $U \in N_R(K)$, $U \neq K^*$, and $P_U$ is the set of prime divisors of the
index of $U$ in $K^*$. Then there exists a unique set $\{U_p : p \in P_U\}$ of subgroups of $K^*$, such that $\cap_{p \in P_U} U_p = U$ and each $U_p$ is of $p$-primary index in $K^*$. Hence, by the latter part of Theorem 1.2 (i), $U_p \in N_p$, and by (4.1), there is a unique field $\Phi_p(U) \in I_p$ with $N(\Phi_p(U)/K) = U_p$, for each $p \in P_U$. Denote by $\Phi_U$ the compositums of the fields $\Phi_p(U) : p \in P_U$. Applying (4.1) and Lemma 2.1, one obtains that the set $\{\Phi_U : U \in \text{Nr}(K)\}$, where $\Phi_K^* = K^*$, has the properties required by Theorem 1.1 (iii).

Proof of Theorem 1.2 (ii) and (iii). Let $n$ be a positive integer not divisible by char ($\hat{K}$), $n_0$ the greatest divisor of $n$ for which $\hat{K}$ contains a primitive $n_0$-th root of unity, $n_1$ the greatest divisor of $n$ not divisible by any $p \in (P \setminus P_1(\hat{K}))$, and $C_t$ a cyclic group of order $t$, for any $t \in \mathbb{N}$. It is easily deduced from (2.1) (iv) and (2.2) (iii) that $\hat{K}^{*n} = \hat{K}^{*n'}$ and $K^*/K^{*n}$ is isomorphic to the direct product $C_n \times C_{n'}$, where $n' = n_0n_1$. In particular, it becomes clear that $K^{*n} \in \Sigma(K)$, as claimed by Theorem 1.2 (ii). One also sees that $K^{*n} = N((R_nT_{n'})/K)$ and $R_nT_{n'}$ is a class field of $K^{*n}$, provided that $R_n$ and $T_{n'}$ are extensions of $K$ in $K_{\text{sep}}$, such that $R_n$ is inertial, $T_{n'}$ is totally ramified, $[R_n : K] = n$ and $[T_{n'} : K] = n'$. This, combined with Theorem 1.2 (i), proves that $\Sigma(K) \subseteq \text{Nr}(K)$. As to the inclusion $\text{Nr}(K) \subseteq \text{Op}(K)$, it can be viewed as a consequence of Theorem 3.1, since it is well-known that the groups from $\text{Nr}(K)$ are open in $K^*$. Note also that the classical existence theorem yields $\text{Nr}(K) = \text{Op}(K)$ in case $\hat{K}$ is a finite field (cf. [11, Ch. 6] or [9, Ch. IV]), so it remains to be seen that $\text{Nr}(K) \neq \text{Op}(K)$ whenever $\hat{K}$ is infinite of characteristic $q > 0$ and cardinality $\kappa$. Arguing as in the proof of Propositions 3 and 4 of [25, Part IV] (or of [9, Ch. V, (3.6)]), one obtains this result by showing that $\text{Nr}(K)$ and the set of finite abelian extensions of $K$ in $K(q)$ are of cardinality $\kappa$ whereas $\text{Op}(K)$ is of cardinality $2^\kappa$. Thus Theorems 1.1 and 1.2 are proved.

Remark 4.1. It is easily obtained from (2.2) (iii) that if $U$ is a subgroup of $K^*$ of finite index $n$ not divisible by char ($\hat{K}$), then $K^*/U$ is isomorphic to $C_e \times C_{n/e}$ and $n$ divides $e^2$, where $e$ is the exponent of $K^*/U$.

Note finally that Theorem 1.2 fully characterizes the elements of $\text{Nr}(K)$ in the set of subgroups of $K^*$, provided that char ($\hat{K}$) = 0. If char ($\hat{K}$) = $q > 0$ and $L$ is a finite extension of $K$ in $K_{\text{sep}}$, then Theorem 3.1 and Lemma 2.1 imply that $N(L/K) = N(L_{\text{ab},q}/K) \cap N(L_0/K)$, for some extension $L_0$ of $K$ in $L$ of degree not divisible by $q$. This, combined with Hazewinkel’s existence theorem [10] concerning totally ramified abelian $q$-extensions of $K$ (see also [8, 3.5 and 3.7] and [20]), allows one to obtain a satisfactory inner characterization of the groups from $\text{Nr}(K)$.

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