Abstract—Parallel least mean square-partial parallel interference cancelation (PLMS-PPIC) is a partial interference cancelation which employs adaptive multistage structure [1]. In this algorithm the channel phases for all users are assumed to be known. Having only their quarters in \((0, 2\pi)\), a modified version of PLMS-PPIC is proposed in this paper to simultaneously estimate the channel phases and the cancelation weights. Simulation examples are given in the cases of balanced, unbalanced and time varying channels to show the performance of the modified PLMS-PPIC method.

I. INTRODUCTION

The multiple access interferences (MAI) is the root of user limitation in CDMA systems [2], [5]. The parallel least mean square partial parallel interference cancelation (PLMS-PPIC) method is a multuser detector for code division multiple access (CDMA) receivers which reduces the effect of MAI in bit detection. In this method and similar to its former versions like LMS-PPIC [6] (see also [7]), a weighted value of the MAI of other users is subtracted before making the decision for a specific user in different stages [1]. In both of these methods, the normalized least mean square (NLMS) algorithm is engaged [8]. The \(m\)th element of the weight vector in each stage is the true transmitted binary value of the \(m\)th user divided by its hard estimate value from the previous stage. The magnitude of all weight elements in all stages are equal to unity. Unlike the LMS-PPIC, the PLMS-PPIC method tries to keep this property in each iteration by using a set of NLMS algorithms with different step-sizes instead of one NLMS algorithm used in LMS-PPIC. In each iteration, the parameter estimate of the NLMS algorithm is chosen whose element magnitudes of cancelation weight estimate have the best match with unity. In PLMS-PPIC implementation it is assumed that the receiver knows the phases of all user channels. However in practice, these phases are not known and should be estimated. In this paper we improve the PLMS-PPIC procedure [1] in such a way that when there is only a partial information of the channel phases, this modified version simultaneously estimates the phases and the cancelation weights. The partial information is the quarter of each channel phase in \((0, 2\pi)\).

The rest of the paper is organized as follows: In section \[II\] the modified version of PLMS-PPIC with capability of channel phase estimation is introduced. In section \[III\] some simulation examples illustrate the results of the proposed method. Finally the paper is concluded in section \[IV\].

II. MULTISTAGE PARALLEL INTERFERENCE CANCELATION: MODIFIED PLMS-PPIC METHOD

We assume \(M\) users synchronously send their symbols \(\alpha_1, \alpha_2, \cdots, \alpha_M\) via a base-band CDMA transmission system where \(\alpha_m \in \{-1, 1\}\). The \(m\)th user has its own code \(p_m(.)\) of length \(N\), where \(p_m(n) \in \{-1, 1\}\), for all \(n\). It means that for each symbol \(N\) bits are transmitted by each user and the processing gain is equal to \(N\). At the receiver we assume that perfect power control scheme is applied. Without loss of generality, we also assume that the power gains of all channels are equal to unity and users’ channels do not change during each symbol transmission (it can change from one symbol transmission to the next one) and the channel phase \(\phi_m\) of \(m\)th user is unknown for all \(m = 1, 2, \cdots, M\) (see [1] for coherent transmission). According to the above assumptions the received signal is

\[
r(n) = \sum_{m=1}^{M} \alpha_m e^{j\phi_m} p_m(n) + v(n), \quad n = 1, 2, \cdots, N, \quad (1)
\]

where \(v(n)\) is the additive white Gaussian noise with zero mean and variance \(\sigma^2\). Multistage parallel interference cancelation method uses \(\alpha_1^{s-1}, \alpha_2^{s-1}, \cdots, \alpha_M^{s-1}\), the bit estimates outputs of the previous stage, \(s-1\), to estimate the related MAI of each user. It then subtracts it from the received signal \(r(n)\) and makes a new decision on each user variable individually to make a new variable set \(\alpha_1^s, \alpha_2^s, \cdots, \alpha_M^s\) for the current stage \(s\). Usually the variable set of the first stage (stage 0) is the output of a conventional detector. The output of the last stage is considered as the final estimate of transmitted bits. In the following we explain the structure of a modified version of the PLMS-PPIC method [1] with simultaneous capability of estimating the cancelation weights and the channel phases.

Assume \(\alpha_m^{(s-1)} \in \{-1, 1\}\) is a given estimate of \(\alpha_m\) from stage \(s-1\). Define

\[
w_m^s = \frac{\alpha_m}{\alpha_m^{(s-1)}} e^{j\phi_m}. \quad (2)
\]
From (1) and (2) we have
\[
\hat{r}(n) = \sum_{m=1}^{M} w_m^s \alpha_m^{(s-1)} p_m(n) + v(n). \tag{3}
\]

Define
\[
W_s^x = [w_1^s, w_2^s, \cdots, w_M^s]^T, \tag{4a}
\]
\[
X_s(n) = [\alpha_1^{(s-1)} p_1(n), \alpha_2^{(s-1)} p_2(n), \cdots, \alpha_M^{(s-1)} p_M(n)], \tag{4b}
\]
where \( T \) stands for transposition. From equations (3), (4a) and (4b), we have
\[
r(n) = W_s^x X_s(n) + v(n). \tag{5}
\]

Given the observations \( \{r(n), X_s(n)\}_{n=1}^{N} \), in modified PLMS-PPIC, like the PLMS-PPIC \([1]\), a set of NLMS adaptive algorithms are used to compute
\[
W_s^x(N) = [w_1^s(N), w_2^s(N), \cdots, w_M^s(N)]^T, \tag{6}
\]
which is an estimate of \( W_s^x \) after iteration \( N \). To do so, from (3), we have
\[
|w_m^s| = 1 \quad m = 1, 2, \cdots, M, \tag{7}
\]
which is equivalent to
\[
\sum_{m=1}^{M} |w_m^s| - 1 = 0. \tag{8}
\]

We divide \( \Psi = \left( 0, 1 - \sqrt{\frac{M-1}{M}} \right) \), a sharp range for \( \mu \) (the step-size of the NLMS algorithm) given in [9], into \( L \) subintervals and consider \( L \) individual step-sizes \( \Theta = \{ \mu_1, \mu_2, \cdots, \mu_L \} \), where \( \mu_1 = 1 - \sqrt{\frac{M-1}{M}} \), \( \mu_2 = 2\mu_1 \), \cdots, and \( \mu_L = L\mu_1 \). In each stage, \( L \) individual NLMS algorithms are executed (\( \mu \) is the step-size of the \( l \)th algorithm). In stage \( s \) and at iteration \( n \), if \( W_{k_s}^x(n) = [w_{1,k_s}^s, \cdots, w_{M,k_s}^s]^T \), the parameter estimate of the \( k \)th algorithm, minimizes our criteria, then it is considered as the parameter estimate at time iteration \( n \). In other words if the next equation holds
\[
W_{k_s}^x(n) = \arg \min_{W_s^x(n) \in W_s^x} \left\{ \sum_{m=1}^{M} |w_m^s(n)| - 1 \right\}, \tag{9}
\]
where \( W_s^x(n) = W_s^x(n-1) + \mu \frac{X_s(n)}{\|X_s(n)\|^2} e(n), \quad l = 1, 2, \cdots, k, \cdots, L-1, L \) and \( I_{W_{s}^x} = \{W_1^x(n), \cdots, W_L^x(n)\} \), then we have \( W_s^x(n) = W_{k_s}^x(n) \), and therefore all other algorithms replace their weight estimate by \( W_{k_s}^x(n) \). At time instant \( n = N \), this procedure gives \( W_s^x(N) \), the final estimate of \( W_s^x \), as the true parameter of stage \( s \).

Now consider \( R = (0, 2\pi) \) and divide it into four equal parts \( R_1 = (0, \frac{\pi}{2}), R_2 = (\frac{\pi}{2}, \pi), R_3 = (\pi, \frac{3\pi}{2}) \) and \( R_4 = (\frac{3\pi}{2}, 2\pi) \). The partial information of channel phases (given by the receiver) is in a way that it shows each \( \phi_m \) \((m = 1, 2, \cdots, M)\) belongs to which one of the four quarters \( R_i \), \( i = 1, 2, 3, 4 \).

Assume \( W_s^x(N) = [w_1^s(N), w_2^s(N), \cdots, w_M^s(N)]^T \) is the weight estimate of the modified algorithm PLMS-PPIC at time instant \( N \) of the stage \( s \). From equation (2) we have
\[
\phi_m = \angle(\frac{w_m^s}{w_m^s} N_m). \tag{10}
\]
We estimate \( \phi_m \) by \( \hat{\phi}_m \), where
\[
\hat{\phi}_m^s = \angle(\frac{w_m^s}{w_m^s} N_m). \tag{11}
\]
Because \( \angle(\frac{w_m^s}{w_m^s} N_m) = 1 \) or \(-1\), we have
\[
\hat{\phi}_m^s = \begin{cases}
\angle w_m^s(N) & \text{if } \frac{\angle w_m^s(N)}{\angle w_m^s(N)} = 1 \\
\pm \pi + \angle w_m^s(N) & \text{if } \frac{\angle w_m^s(N)}{\angle w_m^s(N)} = -1 
\end{cases} \tag{12}
\]
Hence \( \hat{\phi}_m^s \in P^s = \{\angle w_m^s(N), \angle w_m^s(N) + \pi, \angle w_m^s(N) - \pi\} \).

If \( w_m^s(N) \) sufficiently converges to its true value \( w_m^* \), the same region for \( \hat{\phi}_m^s \) and \( \phi_m \) is expected. In this case only one of the three members of \( P^s \) has the same region as \( \phi_m \).

For example if \( \phi_m \in (0, \frac{\pi}{2}) \), then \( \hat{\phi}_m^s \in (0, \frac{\pi}{2}) \) and therefore only \( \angle w_m^s(N) \) or \( \angle w_m^s(N) + \pi \) or \( \angle w_m^s(N) - \pi \) belongs to \((0, \frac{\pi}{2})\). If, for example, \( \angle w_m^s(N) + \pi \) is such a member between all three members of \( P^s \), it is the best candidate for phase estimation. In other words,
\[
\phi_m \approx \hat{\phi}_m^s = \angle w_m^s(N) + \pi.
\]
We admit that when there is a member of \( P^s \) in the quarter of \( \phi_m \), then \( w_m^s(N) \) converges. What would happen when none of the members of \( P^s \) has the same quarter as \( \phi_m \)? This situation will happen when the absolute difference between \( \angle w_m^s(N) \) and \( \phi_m \) is greater than \( \pi \). It means that \( w_m^s(N) \) has not converged yet. In this case where we can not count on \( w_m^s(N) \), the expected value is the optimum choice for the channel phase estimation, e.g. if \( \phi_m \in (0, \frac{\pi}{2}) \) then \( \hat{\phi}_m^s \) is the estimation of the channel phase \( \phi_m \), or if \( \hat{\phi}_m^s \in (\frac{\pi}{2}, \pi) \) then \( \hat{\phi}_m^s \) is the estimation of the channel phase \( \phi_m \). The results of the above discussion are summarized in the next equation
\[
\hat{\phi}_m^s = \begin{cases}
\angle w_m^s(N) & \text{if } \angle w_m^s(N), \phi_m \in R_1, \quad i = 1, 2, 3, 4 \\
\angle w_m^s(N) + \pi & \text{if } \angle w_m^s(N) + \pi, \phi_m \in R_1, \quad i = 1, 2, 3, 4 \\
\angle w_m^s(N) - \pi & \text{if } \angle w_m^s(N) - \pi, \phi_m \in R_1, \quad i = 1, 2, 3, 4 \\
\angle w_m^s(N) \pm \pi & \text{if } \phi_m \in R_i, \quad \angle w_m^s(N) \pm \pi \notin R_i,
\end{cases}
\]
Having an estimation of the channel phases, the rest of the proposed method is given by estimating \( a_m^s \) as follows:
\[
\alpha_m^s = \text{sign} \left\{ \text{real} \left( \sum_{n=1}^{N} q_m^s(n)e^{j\hat{\phi}_m^s}p_m(n) \right) \right\}, \tag{13}
\]
where
\[
q_m^s(n) = r(n) - \sum_{m'=1, m' \neq m}^{M} w_m^{s'}(N)\alpha_m^{(s-1)} p_{m'}(n). \tag{14}
\]
The inputs of the first stage \( \{\alpha_0^0 m_{m=1}^{M}\} \) (needed for computing \( X^1(n) \)) are given by
\[
\alpha_0^0 = \text{sign} \left\{ \text{real} \left( \sum_{n=1}^{N} r(n)e^{-j\hat{\phi}_m^0}p_m(n) \right) \right\}. \tag{15}
\]
Assuming $\phi_m \in \mathbb{R}$, then
\[ \hat{\phi}_m^0 = \frac{(i - 1)\pi + i\pi}{4}. \]

Table I shows the structure of the modified PLMS-PPIC method. It is to be notified that
- Equation (15) shows the conventional bit detection method when the receiver only knows the quarter of channel phase in $(0, 2\pi)$.
- With $L = 1$ (i.e., only one NLMS algorithm), the modified PLMS-PPIC can be thought as a modified version of the LMS-PPIC method.

In the following section some examples are given to illustrate the effectiveness of the proposed method.

### III. Simulations

In this section we have considered some simulation examples. Examples [1][3] compare the conventional, the modified LMS-PPIC and the modified PLMS-PPIC methods in three cases: balanced channels, unbalanced channels and time varying channels. In all examples, the receivers have only the quarter of each channel phase. Example 1 is given to compare the modified LMS-PPIC and the PLMS-PPIC in the case of balanced channels.

**Example 1: Balanced channels:** Consider the system model in which $M$ users synchronously send their bits to the receiver through their channels. It is assumed that each user’s information consists of codes of length $N$. It is also assumed that the signal to noise ratio (SNR) is 0dB. In this example there is no power-unbalanced or channel loss is assumed. The step-size of the NLMS algorithm in modified LMS-PPIC method is $\mu = 0.1(1 - \sqrt{\frac{M-1}{M}})$ and the set of step-sizes of the parallel NLMS algorithms in modified PLMS-PPIC method are $\Theta = \{0.01, 0.05, 0.1, 0.2, \cdots, 1\}(1 - \sqrt{\frac{M-1}{M}})$, i.e. $\mu_1 = 0.01(1 - \sqrt{\frac{M-1}{M}}), \cdots, \mu_4 = 0.2(1 - \sqrt{\frac{M-1}{M}}), \cdots, \mu_M = (1 - \sqrt{\frac{M-1}{M}})$. Figure 1 illustrates the bit error rate (BER) for the case of two stages and for $N = 64$ and $N = 256$. Simulations also show that there is no remarkable difference between results in two stage and three stage scenarios. Table I compares the average channel phase estimate of the first user in each stage and over 10 runs of modified LMS-PPIC and PLMS-PPIC, when the the number of users is $M = 15$.

### IV. Conclusion

In this paper, parallel interference cancelation using adaptive multistage structure and employing a set of NLMS algorithms with different step-sizes is proposed, when just the quarter of the channel phase of each user is known. In fact, the algorithm has been proposed for coherent transmission with full information on channel phases in [1]. This paper is a modification on the previously proposed algorithm. Simulation results show that the new method has a remarkable performance for different scenarios including Rayleigh fading channels even if the channel is unbalanced.

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Fig. 1. The BER of the conventional, the modified LMS-PPIC and the modified PLMS-PPIC methods versus the system load in balanced channel, using two stages for $N = 64$ and $N = 256$.

Fig. 2. The BER of the conventional, the modified LMS-PPIC and the modified PLMS-PPIC methods versus the system load in unbalanced channel, using two stages for $N = 64$ and $N = 256$.

Fig. 3. The BER of the conventional, the modified LMS-PPIC and the modified PLMS-PPIC methods versus the system load in time varying Rayleigh fading channel, using two stages for $N = 64$ and $N = 256$.

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TABLE III
THE PROCEDURE OF THE MODIFIED PLMS-PPIC METHOD

| Initial Values | for $m = 1, 2, \cdots, M$ | $\phi_m \in R_i, \quad i = 1, 2, 3, 4$ $\implies$ |
|----------------|-------------------------|------------------|
|                |                         | $\hat{\phi}_m^0 = \frac{(i-1)\pi + i\pi}{4}$ |
|                |                         | $\alpha_{m}^0 = \text{sign} \left\{ \text{real} \left\{ \sum_{n=1}^{N} r(n)e^{-j\hat{\phi}_m^0 p_m(n)} \right\} \right\}$ |
| for $s = 1, 2, \cdots, S$ | $W^s(0) = [w_1^s(0), \cdots, w_M^s(0)]^T$ = (0, 0, \cdots) |
| PNLMS algorithm | for $n = 1, 2, \cdots, N$ | $X^s(n) = [\alpha_1^{(s-1)c_1(n)}, \alpha_2^{(s-1)c_2(n)}, \cdots, \alpha_M^{(s-1)c_M(n)}]^T$ |
|                |                         | $e(n) = r(n) - W^s(n-1)X^s(n)$ |
|                |                         | $Z(n) = \frac{X^s(n)}{\|X^s(n)\|}e(n)$ |
|                |                         | $\min = \infty, l = 1$ |
| for $k = 1, 2, \cdots, L$ | $W_k^s(n) = W^s(n-1) + \mu_k Z(n)$ |
|                |                         | if $\sum_{m=1}^{M} ||w_{m,k}^s(n)|| - 1 < \min :$ |
|                |                         | $\min = \sum_{m=1}^{M} ||w_{m,k}^s(n)|| - 1$ |
|                |                         | $l = k$ |
|                | $W^s(n) = W_k^s(n)$ |
| Phase Estimation | for $m = 1, 2, \cdots, M$ | $\hat{\phi}_m^s = \angle w_m^s(N)$ $\quad$ if $\angle w_m^s(N), \phi_m \in R_i$ |
|                |                         | $\hat{\phi}_m^s = \angle w_m^s(N) + \pi$ $\quad$ if $\angle w_m^s(N) + \pi, \phi_m \in R_i$ |
|                |                         | $\hat{\phi}_m^s = \angle w_m^s(N) - \pi$ $\quad$ if $\angle w_m^s(N) - \pi, \phi_m \in R_i$ |
|                |                         | $\hat{\phi}_m^s = \frac{(i-1)\pi + i\pi}{4}$ $\quad$ if $\phi_m \in R_i, \angle w_m^s(N), \angle w_m^s(N) \pm \pi \notin R_i$ |
| for $m = 1, 2, \cdots, M$ | $q_m^s(n) = r(n) - \sum_{m'=1, m' \neq m}^{M} w_{m'}^s(N)\alpha_{m'}^{(s-1)} p_{m'}(n)$ |
|                |                         | $\alpha_{m}^s = \text{sign} \left\{ \text{real} \left\{ \sum_{n=1}^{N} q_{m}^s(n)e^{-j\hat{\phi}_m^s p_m(n)} \right\} \right\}$ |