The Gauged Unparticle Action

A. Lewis Licht
Dept. of Physics
U. of Illinois at Chicago
Chicago, Illinois 60607

We show that the unparticle action that is made gauge invariant by the inclusion of an open Wilson line factor can be transformed into the integral-differential operator action that avoids the use of the Wilson line factor. The two forms of the action should therefore give the same Feynman diagrams. We also show that it is relatively easy to construct Feynman diagrams using the operator action.

1. INTRODUCTION

One approach to making Georgi’s [1] [2] unparticle action gauge invariant has been to introduce an open Wilson line factor that obeys the Mandelstam condition [3] on its derivative [4] [5]. Recent work [6] [7] [8] has shown that the Mandelstam condition in this context is mathematically inconsistent. Another approach to gauging the unparticle action has been found [9] that uses an integral-differential operator method and avoids the Mandelstam derivative. In the following we show that these two methods are equivalent in that the action using the Wilson line can be transformed into the operator action. They should therefore give the same unparticle-gauge field Feynman diagrams.

In Section 2 we review briefly the Wilson line unparticle action. In Section 3 we review the operator action. In section 4 we show their equivalence. The use of the operator action to find gaugeon-unparticle vertexes is discussed in section 5 and the single gaugeon vertex is derived. The double gaugeon vertexes are derived in section 6 and a brief discussion of general Gaugeon-Unparticle diagrams is given in section 7.

2. THE WILSON LINE UNPARTICLE ACTION

The unparticle action introduced by Georgi [1] [2] has been extended by Terning et al [4] [5] to

\[ I = \int d^4x d^4y \Phi^\dagger_u(x) K(x,y) W_\Lambda(x,y) \Phi_u(y) \]  

(1)

where

\[ K(x,y) \equiv \left[ - (\partial_\mu \partial^\mu) x + i \varepsilon \right]^{2-d_u} \delta^4(x-y) \]

(2)

and

\[ A_{d_u} = \frac{16 \pi^{5/2}}{(2\pi)^{2d_u} \Gamma(d_u - 1) \Gamma(2d_u)} \]

(3)

Here \( W_\Lambda \) is a path ordered Wilson line, involving an integral of the gauge field over the path \( \Lambda \), introduced to make the action gauge invariant:

\[ W_\Lambda(x,y) = P \exp \left[ +ig \int_y^x A_\alpha(\zeta) d\zeta^\alpha \right] \]

(4)

* licht@uic.edu
The path ordering is from $x$ on the left to $y$ on the right. $W_{\lambda}$ is assumed to satisfy the Mandelstam condition on its derivative,

$$\frac{\partial}{\partial x^\nu} W_{\lambda}(x, y) = +igA_\nu(x) W_{\lambda}(x, y)$$

(5)

In Ref. [8] it was shown that for an actual Wilson line there should be an extra term on the right hand side of Eq. (5). In the following however it is shown that if one assumes there does exist some sort of factor that obeys this Mandelstam condition, then the Unparticle action can be transformed into a form that does not require the $W_{\lambda}$.

### 3. THE OPERATOR UNPARTICLE ACTION

We review here the differential-integral operator notation for action integrals introduced in Ref. [9]. We introduce kets $|x, i\rangle$ that are the eigenkets of an abstract position operator $X^\mu$, and also carry the gauge representation index $i$, satisfying

$$X^\mu |x, i\rangle = x^\mu |x, i\rangle$$

$$\langle x, i| x', j\rangle = \delta^4 (x - x') \delta_{ij}$$

(6)

and kets $|p, i\rangle$, eigenkets of the conjugate operator $P^\mu$,

$$P^\nu |p, i\rangle = p^\nu |p, i\rangle$$

$$\langle p, i| p', j\rangle = \delta^4 (p - p') \delta_{ij}$$

$$\langle x, i| p, j\rangle = \frac{e^{-ipx}}{(2\pi)^2} \delta_{ij}$$

(7)

with

$$[X^\mu, P^\nu] = -i\delta^\mu_\nu$$

(8)

We identify the unparticle fields $\Phi_u$ with Hilbert space vectors

$$|\Phi_u\rangle = \int d^4x |x, i\rangle \Phi^i_u(x)$$

(9)

Note that it follows from the above that

$$P_\mu |x, i\rangle = -i\partial_\mu |x, i\rangle$$

$$P_\mu |\Phi_u\rangle = \int d^4x |x, i\rangle i\partial_\mu \Phi^i_u(x)$$

$$\langle \Phi_u| P_\mu = \int d^4x \Phi^i_{\mu\dagger}(x) \left(-i\partial_\mu\right) \langle x, i|$$

(10)

Using the branch integral formula

$$z^n = -\frac{e^{i\pi n}}{\pi} \sin (\pi n) \int_0^\infty dx \frac{x^n}{x - z}$$

(11)

we can write the ungauged unparticle action as
\[ I = \langle \Phi_u | K | \Phi_u \rangle \]  

(12)

where

\[ K = K_0 \int_0^\infty dM^2 \frac{(M^2)^{2-d_u}}{M^2 - P^2 - i\varepsilon} \]  

(13)

with

\[ K_0 = \frac{2 \sin^2 (\pi d_u)}{\pi A_{du}} \]  

(14)

The gauge field \( A_\mu \) can be considered as an operator on this Hilbert space, with

\[ A_\mu = \int d^4x |x, i\rangle A^a_\mu (x) T^a_{ij} \langle x, j| \]  

(15)

Under the gauge transform

\[ |\Phi_u\rangle \rightarrow |\Phi'_u\rangle = e^{+ig\Lambda} |\Phi_u\rangle \]  

(16)

\[ A_\mu \rightarrow A'_\mu = e^{+ig\Lambda} A_\mu e^{-ig\Lambda} + \frac{1}{g} [e^{+ig\Lambda}, P_\mu] e^{-ig\Lambda} \]  

(17)

The combination

\[ D_\mu = P_\mu + gA_\mu \]  

(18)

transforms as

\[ D_\mu \rightarrow D'_\mu = e^{+ig\Lambda} D_\mu e^{-ig\Lambda} \]  

(19)

so that replacing \( P_\mu \) by \( D_\mu \) gives us the gauge invariant action,

\[
I = K_0 \int_0^\infty dM^2 (M^2)^{2-d_u} \langle \Phi_u | \frac{1}{M^2 - D^2 - i\varepsilon} | \Phi_u \rangle \\
= K_0 \int_0^\infty dM^2 (M^2)^{2-d_u} \langle \Phi_u | \frac{1}{M^2 - P^2 - g \{ P_\mu, A_\mu \} - g^2 A_\mu A^\mu - i\varepsilon} | \Phi_u \rangle
\]  

(20)

4. THE EQUIVALENCE BETWEEN THE ACTIONS.

We will assume here that there actually is a factor \( W(x,y) \) such that

\[ \frac{\partial}{\partial x^\mu} W(x,y) = + igA_\mu (x) W(x,y) \]  

(21)

and that

\[ W(x,x) = 1 \]  

(22)
Using the branch cut integral of Eq. (11), the kernel of Eq. (1) can be written as

$$K(x, y) = K_0 \int_0^\infty dM^2 \left( \frac{M^2}{M^2 + \partial_x^2 - i\varepsilon} \right)^{2-d_u} e^{-ip(x-y)}$$

$$= K_0 \int_0^\infty dM^2 \left( \frac{M^2}{M^2 + \partial_x^2 - i\varepsilon} \right)^{2-d_u} \delta^4(x-y)$$

$$= \sum_{n=0}^\infty (-1)^n E_n \left( \partial_x \partial_\mu - i\varepsilon \right)^n \delta^4(x-y)$$

with the parameters $E_n$ given by

$$E_n = \lim_{m \to 0} K_0 \int_0^\infty dM^2 \left( \frac{M^2}{M^2 + \partial_x^2 - i\varepsilon} \right)^{1-d_u-n}$$

where the limit is to be taken after the summation.

Now, the integral

$$I_n = (-1)^n E_n \int d^4 x d^4 y \Phi^\dagger_u(x) \left( \partial_x \partial_\mu - i\varepsilon \right)^n \delta^4(x-y) W(x,y) \Phi_u(y)$$

$$= (-1)^n E_n \int d^4 x d^4 y \Phi^\dagger_u(x) \left[ \left( -\partial_x \partial_\mu - igA_\mu \right) \left( -\partial_x \partial_\mu - igA^\mu \right) - i\varepsilon \right]^n \Phi_u(x)$$

after integrating by parts and using Eq. (21). Integrating out $y$ and using Eq. (22) makes this

$$I_n = (-1)^n E_n \int d^4 x \Phi^\dagger_u(x) \left[ \left( -\partial_x \partial_\mu + gA_\mu \right) \left( -\partial_x \partial_\mu + gA^\mu \right) + i\varepsilon \right]^n \Phi_u(x)$$

Using Eq. (10) this can be written in the operator notation of Section (3) as

$$I_n = E_n \langle \Phi_u | (D^2 + i\varepsilon)^n | \Phi_u \rangle$$

Summing, we get the action of Eq. (20).

5. GAUGEON-UNPARTICLE VERTEXES

The operator method gives a relatively simple way of deriving unparticle-gauge field vertexes. Let

$$G_M(P) = \frac{1}{M^2 - P^2 - i\varepsilon}$$

Then Eq. (20) can be written as

$$I = \sum_{n=0}^\infty K_0 \int_0^\infty dM^2 \left( M^2 \right)^{2-d_u} \langle \Phi_u | G_M(P) [ZG_M(P)]^n | \Phi_u \rangle$$

where
\begin{equation}
Z = g \{ P^\mu, A_\mu \} + g^2 A_\mu A^\mu \tag{30}
\end{equation}

Recall that the propagator for the gauge field in Feynman gauge is

\begin{align}
\langle \Omega | T A^a_\alpha (x) A^b_\beta (y) | \Omega \rangle_0 &= iD_A (x - y) \eta_{\alpha\beta}\delta^{ab} \\
&= -\eta_{\alpha\beta}\delta^{ab} \int d^4p e^{-ip \cdot (x - y)} \frac{1}{(2\pi)^4 - p^2 - i\varepsilon} \\
&= i\eta_{\alpha\beta}\delta^{ab} \int d^4p e^{-ip \cdot (x - y)} \frac{1}{(2\pi)^4} \tilde{D}_A (p) \tag{31}
\end{align}

Where we denote the particle vacuum state by \( \Omega \) and the zeroth order propagator by a zero subscript.

The propagator for the unparticle is

\begin{align}
\langle \Omega | T \Phi^i_u (x) \Phi^{ij}_u (y) | \Omega \rangle_0 &= i\delta^{ij} S (x - y) \\
&= i\delta^{ij} \int d^4p e^{-ip \cdot (x - y)} \frac{1}{(2\pi)^4} \tilde{S}_i (p) \\
&= i\delta^{ij} \int d^4p e^{-ip \cdot (x - y)} \frac{1}{(2\pi)^4} \int_0^\infty dM^2 (M^2)^{d_u - 2} - iA_{du} G_M (p) \tag{32}
\end{align}

We will use here the notation

\begin{align}
\Phi^i_u (x) &= \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot x} \tilde{\Phi}^i_u (p) \\
A^a_\alpha (x) &= \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot x} \tilde{A}^a_\alpha (p) \tag{33}
\end{align}

Then

\begin{align}
| \Phi_u \rangle &= \int d^4x | x, i \rangle \Phi^i_u (x) \\
&= \int \frac{d^4p}{(2\pi)^4} | p, i \rangle \tilde{\Phi}^i_u (p) \tag{34}
\end{align}

and

\begin{align}
A_\alpha &= \int d^4x | x, i \rangle A^a_\alpha (x) T^a_{ij} \langle x, i | \\
&= \int \frac{d^4qd^4k}{(2\pi)^8} | q + k, i \rangle A^a_\alpha (q) T^a_{ij} \langle k, i | \tag{35}
\end{align}

We consider first the single gaugeon vertex. The single gauge field interaction can be seen from Eqs. (29) and (30) to be

\begin{equation}
I_1 = gK_0 \int_0^\infty dM^2 (M^2)^{2 - d_u} \langle \Phi_u | G_M (P) \{ P^\mu, A_\mu \} G_M (P) | \Phi_u \rangle \tag{36}
\end{equation}

Which can be written now as
\[ I_1 = gK_0 \int_0^\infty dM^2 (M^2)^{2-d_u} \int \frac{d^4p'd^4q'd^4\ell d^4p}{(2\pi)^8} \tilde{\Phi}_u(p') (p'|G_M(p')|q + \ell)(q + 2\ell) \]

\[ \hat{A}_\mu(q) (\ell|G_M(p)|p) \tilde{\Phi}_u(p) \]

\[ = gK_0 \int_0^\infty dM^2 (M^2)^{2-d_u} \int \frac{d^4p'd^4q'd^4p}{(2\pi)^8} \delta (p' - q - p) \tilde{\Phi}_u(p') G_M(p') (p' + p) \]

\[ \hat{A}_\mu(q) G_M(p) \tilde{\Phi}_u(p) \]

Where we have suppressed the representation indices.

We consider now the vacuum expectation value of the Heisenberg fields:

\[ V^{ij\alpha}(u, v, w) = \langle \Omega | T (\Phi^i_u(u) A^{\alpha}_a(v) \Phi^j_u(w)) | \Omega \rangle \tag{38} \]

To lowest order in \( g \), this would be given by

\[ V^{ij\alpha}(u, v, w) = i^3 \int d^4u'd^4v'd^4w' S(u - u') D_A(v' - v) S(w' - w) \frac{\delta}{\delta \Phi^i_u(u')} \frac{\delta}{\delta A^{\alpha}_a(v')} \frac{\delta}{\delta \Phi^j_u(w')} iI_1 \tag{39} \]

Now inverting Eq. (33), we have

\[ \tilde{\Phi}_u(p) = \int d^4x e^{ip \cdot x} \Phi_u(x) \]

\[ \tilde{A}^\alpha_a(p) = \int d^4x e^{ip \cdot x} A^\alpha_a(x) \tag{40} \]

From which we get,

\[ \frac{\delta}{\delta \Phi^i_u(x)} = \int d^4p \frac{\delta \tilde{\Phi}^i_u(p)}{\delta \Phi^i_u(x)} \frac{\delta}{\delta \tilde{\Phi}^i_u(p)} \]

\[ = \int d^4pe^{ip \cdot x} \frac{\delta}{\delta \tilde{\Phi}^i_u(p)} \tag{41} \]

and similarly,

\[ \frac{\delta}{\delta A^\alpha_a(x)} = \int d^4pe^{ip \cdot x} \frac{\delta}{\delta A^\alpha_a(p)} \tag{42} \]

Equivalently,

\[ \frac{\delta}{\delta \Phi^i_u(p)} = \int d^4x e^{-ip \cdot x} \frac{\delta}{\delta \tilde{\Phi}^i_u(x)} \frac{\delta}{\delta \Phi^i_u(x)} \]

\[ = \int \frac{d^4x}{(2\pi)^4} e^{-ip \cdot x} \frac{\delta}{\delta \tilde{\Phi}^i_u(x)} \frac{\delta}{\delta \Phi^i_u(x)} \tag{43} \]

\[ \frac{\delta}{\delta A^\alpha_a(p)} = \int \frac{d^4x}{(2\pi)^4} e^{-ip \cdot x} \frac{\delta}{\delta A^\alpha_a(x)} \]

The vacuum expectation value in momentum space is

\[ \tilde{V}_{\alpha}^{ij\alpha}(p', q, p) = \int d^4u d^4v d^4w e^{i(p' \cdot u - q \cdot v - p \cdot w)} V^{ij\alpha}(u, v, w) \tag{44} \]
Which becomes

$$\tilde{V}_{ija}^{\alpha} (p', q, p) = i^3 \int d^4u' d^4v' d^4w' \tilde{S} (p') \tilde{D}_A (q) \tilde{S} (p) e^{i(p' - w' - q - v')} \frac{\delta}{\delta \Phi^i_u (u')} \frac{\delta}{\delta \tilde{A}^{\alpha a} (v')} \frac{\delta}{\delta \Phi^i_u (w')} i I_1$$

(45)

Eq. (45) makes this into

$$\tilde{V}_{ija}^{\alpha} (p', q, p) = i^3 \tilde{S} (p') \tilde{D}_A (q) \tilde{S} (p) \left( 2\pi \right)^4 \left( \begin{array}{cc} \delta & \delta \tilde{S} (p') \delta \tilde{D}_A (q) \delta \tilde{S} (p) \end{array} \right) i I_1$$

(46)

The vertex function is defined as

$$\tilde{V}_{ija}^{\alpha} (p', q, p) = i^3 \tilde{S} (p') \tilde{D}_A (q) \tilde{S} (p) (2\pi)^4 \delta (p' - q - p) \Gamma_{ija}^{\alpha} (p', q, p)$$

(47)

The one gaugeon vertex is thus

$$i g (2\pi)^4 \delta (p' - q - p) \Gamma_{ija}^{\alpha} (p', q, p) = (2\pi)^4 \delta (p' - q - p) \Gamma_{ija}^{\alpha} (p', q, p)$$

(48)

Which becomes, using Eq. (37),

$$\Gamma_{ija}^{\alpha} (p', q, p) = K_0 (p' + p)^{\alpha} T^a_{ij} \int_0^\infty dM^2 (M^2)^{2-d_a} G_M (p') G_M (p)$$

(49)

It can easily be shown that

$$G_M (p') G_M (p) = \frac{1}{p'^2 - p^2} [G_M (p') - G_M (p)]$$

(50)

which makes the vertex into

$$\Gamma_{ija}^{\alpha} (p', q, p) = \frac{(p' + p)^{\alpha}}{p'^2 - p^2} T^a_{ij} \left[ \tilde{S}^{-1} (p') - \tilde{S}^{-1} (p) \right]$$

(51)

Exactly the form found in Ref. [4].

6. THE TWO GAUGEON VERTEXES.

We consider here the general case of an n gaugeon vertex, and give specific results for the n= 2 case. Eq. (50) is a special case of a general result that is useful for the multiple gaugeon vertexes.

$$\prod_{k=1}^{n} \frac{1}{M^2 - p_k^2 - i \epsilon} = \sum_{k=1}^{n} \frac{1}{M^2 - p_k^2 - i \epsilon} \prod_{j \neq k} \frac{1}{p_k^2 - p_j^2}$$

(52)

The left hand side of this equation is an analytic function $F_L (M^2)$ with poles at the points $M^2 = p_k^2$ and the right hand side is another analytic function $F_R (M^2)$ with poles at the same points, and with the same residues. The difference, $F (M^2) = F_L (M^2) - F_R (M^2)$ is an analytic function with no poles, and goes to zero as $M^2$ goes to infinity. It is therefore zero by Liouville’s theorem [10], which proves Eq. (52).

The n-gaugeon vertex can be expressed as
\[ \frac{i g^n (2\pi)^d}{2} \delta \left( p' - \sum q_k - p \right) \Gamma^{ij\alpha_1 \cdots \alpha_n}_{\alpha_1 \cdots \alpha_n} (p', q_1 \cdots q_n, p) = \]
\[ \frac{(2\pi)^{4(n+2)}}{\delta \Phi_1^\dagger (p')} \left( \prod_{k=1}^n \frac{\delta}{\delta A^{\alpha_k a_k} (q_k)} \right) \frac{\delta}{\delta \Phi_a^i (p)} I_n \]

which yields an expression of the form

\[ \Gamma^{ij\alpha_1 \cdots \alpha_n}_{\alpha_1 \cdots \alpha_n} (p', q_1 \cdots q_n, p) = K_0 \int_0^\infty dM^2 \left( M^2 \right)^{2-d_a} \left\{ \sum G_M (p') \prod (ZG_M (k)) \right\}_n \]

where the sum is over the various permutations of the external gaugeon lines and the term with n total gaugeon lines is to be taken.

For example, we give the explicit expansion for \( n = 2 \).

\[ \Gamma^{\alpha \beta \gamma \delta}_{\gamma \delta \alpha \beta} (p', q', q, p) = K_0 \int_0^\infty dM^2 \left( M^2 \right)^{2-d_a} G_M (p') \left\{ \sum G_M (p') (p' + p + q)^{\alpha} (2p + q)^{\beta} T^a T^b \right. \]
\[ + G_M (p + q') (p' + p + q')^{\beta} (2p + q')^{\alpha} T^b T^a + \left\{ T^a, T^b \right\} \eta^{\alpha \beta} \left\} \right. G_M (p) \]

Using Eq. (52) this can be expressed as

\[ \Gamma^{\alpha \beta \gamma \delta}_{\gamma \delta \alpha \beta} (p', q', q, p) = (p' + p + q)^{\alpha} (2p + q)^{\beta} T^a T^b \]
\[ \left[ \frac{S^{-1} (p')}{(p'^2 - (p + q)^2) (p'^2 - p^2)} \right] \]
\[ + \frac{S^{-1} (p)}{(p^2 - (p + q)^2) (p^2 - p'^2)} + \frac{S^{-1} (p + q)}{(p^2 - (p + q)^2) (p'^2 - (p + q)^2)} \]
\[ + (p' + p + q')^{\beta} (2p + q')^{\alpha} T^b T^a \]
\[ \left[ \frac{S^{-1} (p')}{(p'^2 - (p + q')^2) (p'^2 - p^2)} \right] \]
\[ + \frac{S^{-1} (p)}{(p^2 - (p + q')^2) (p^2 - p'^2)} + \frac{S^{-1} (p + q')}{(p^2 - (p + q'^2) (p'^2 - (p + q')^2)} \]
\[ + \left\{ T^a, T^b \right\} \eta^{\alpha \beta} \frac{S^{-1} (p') - S^{-1} (p)}{p'^2 - p^2} \]

These vertexes have also been derived using the Terning method by Liao [11] [12].

### 7. Diagrams.

We will use diagrams to express these gaugeon-unparticle vertexes in which the integral over the unparticle mass is indicated by a heavy line extending between two small circles. The gaugeons will be indicated by dashed lines. The segments of the unparticle line between intersections with gaugeon lines correspond to \( G_M \) factors. The single gaugeon vertex is then shown in Figure 1. The two main contributions to the double gaugeon vertexes are shown in Figures 2 and 3. A third contribution to the unparticle-double gaugeon vertex, \( \Gamma_2 \), that comes from two single gaugeon vertexes connected by an unparticle propagator is shown in Figure 4. Reading it off from the figure and symmetrising with respect to the gaugeons gives
\[
\Gamma_{2a\alpha\beta}^{a\alpha\beta} (p', q', q, p) = -\frac{(p' + p + q)^\alpha}{p'^2 - (p + q)^2} T^a \left[ S^{-1} (p') - S^{-1} (p + q) \right] S (p + q) \\
\times \frac{(2p + q)^\beta}{(p + q)^2 - p^2} T^b \left[ S^{-1} (p + q) - S^{-1} (p) \right] \\
- \frac{(p' + p + q')^\beta}{p'^2 - (p + q')^2} T^b \left[ S^{-1} (p') - S^{-1} (p + q') \right] S (p + q') \\
\times \frac{(2p + q')^\alpha}{(p + q')^2 - p^2} T^a \left[ S^{-1} (p + q') - S^{-1} (p) \right]
\] (57)

FIG. 1: The one gaugeon vertex

FIG. 2: The \( A^2 \) contribution to the double gaugeon-unparticle vertex.

FIG. 3: The \( \{P, A\} G_M \{P, A\} \) contribution to the double gaugeon vertex.
8. CONCLUSIONS

We have shown that the bosonic unparticle action made gauge invariant by an open Wilson line factor can be transformed into the differential-integral operator unparticle action. The two actions should therefore give the same vertexes. This is done by ignoring, as is usually done, the extra term that should be present on the right hand side of Eq. (5). It is conceivable that the vertexes would be different if this extra term was actually taken into account.

We have also shown that there is a diagramatic technique that allows one to read off fairly easily the unparticle-gaugeon vertexes from the differential-integral operator unparticle action.

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