Off-forward Matrix Elements in Light-Front Hamiltonian QCD

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(June 18, 2002)

Abstract

We investigate the off-forward matrix element of the light cone vector operator for a dressed quark state in light-front Hamiltonian perturbation theory. We obtain the corresponding splitting functions in a straightforward way. We show that the end point singularity is canceled by the contribution from the normalization of state. Considering mixing with the gluon operator, we verify the helicity sum rule in perturbation theory. We show that the quark mass effects are suppressed in the plus component of the matrix element but in the transverse component, they are not suppressed. We emphasize that this is a particularity of the off-forward matrix element and is absent in the forward case.

Keywords: Generalized parton distributions, Light-front Hamiltonian, Perturbation theory

1. Introduction

The hadronic matrix elements of quark and gluon operators appear in the description of all scattering processes. They are in general of two types: in the inclusive processes, one encounters diagonal matrix elements of bilocal operators. These matrix elements are related to parton distributions. On the other hand, in the elastic exclusive processes, one encounters form factors, which are off-diagonal matrix elements of local operators. The generalized parton distributions interpolate between these two types of matrix elements [1]. These are off-diagonal matrix elements of light-front bilocal operators. They play an important role in the deeply virtual Compton scattering amplitude [3] and electroproduction of mesons [4,5] (for reviews of hard exclusive reactions, see [6]). The off-forward matrix elements are the generalizations of the above two types of matrix elements; parton distributions are the forward limits of generalized parton distributions (GPD) and form factors are moments of them.

Recently, the generalized parton distributions have been investigated in the light-front formalism by several authors and an overlap representation for the plus component in terms

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of light-front wave functions has been given \cite{7,8}. Also GPD’s have been constructed using light cone model wave functions \cite{9}. The perpendicular and the minus components are somewhat more complicated because the operators in these cases involve interactions. They are the higher twist components. In this work, we calculate the the off-forward matrix elements of the plus component and the mass dependent helicity-flip part of the perpendicular component of the bilocal vector operator for a dressed quark target in light-front Hamiltonian QCD. Recently, we have made considerable progress in understanding polarized and unpolarized deep inelastic scattering (DIS) structure functions in this approach \cite{10} and we have shown that it gives an intuitive picture of DIS. It is suitable to calculate the forward matrix elements of transverse and minus components of the bilocal current operator. Also, the presence of quark mass does not cause any problem. Interference effects are straightforward to handle. The splitting functions are obtained easily and they agree with the well known expressions \cite{10}. It is possible to derive new sum rules which connect DIS structure functions to light-front QCD Poincare generators. In this work, we extend our approach to off-forward matrix elements.

2. Plus Component

We work in the so called symmetric frame \cite{7,8}. The momentum of the initial state is $P_{\mu}$ and that of the final state is $P'^{\mu}$. The average momentum between initial and final state is then $\bar{P}^\mu = \frac{P^\mu + P'^\mu}{2}$.

The momentum transfer is given by $\Delta^\mu = P'^\mu - P^\mu$, $P_{\perp} = -P_\perp = \frac{\Delta^\perp}{2}$, skewedness $\xi = -\frac{\Delta^+}{2P_{\perp}}$. Without any loss of generality, we take $\xi > 0$. We also get $\Delta^- = \xi P_{\perp}^2$.

The off-forward matrix element is given by,

$$F^{\mu}_{\lambda} = \int \frac{dz^-}{2\pi} e^{ip \cdot z^-} \langle P' \lambda' | \bar{\psi}(-\frac{z^-}{2}) \gamma_{\mu} \psi(\frac{z^-}{2}) | P \lambda \rangle.$$  \hspace{1cm} (1)

Fig. 1: Light-front time ordered diagrams considered in the calculations of Eqs. (1) and (19); only one time ordering is important in the kinematical region considered.
The + component of the matrix element is parametrized in terms of the off-forward distributions \( H(\vec{x}, \xi, t) \) and \( E(\vec{x}, \xi, t) \). However, the matrix element can be expressed directly in terms of overlaps of light-front wave functions. We calculate the plus component of this matrix element taking the target to be a dressed quark to order \( g^2 \) (see Fig. 1 (a)). We work in the light-front gauge, \( A^+ = 0 \), where the path-ordered exponential between the fermion fields in the bilocal operator is unity. For simplicity we suppress the flavor indices.

The Fock space expansion of the operator is given by,

\[
O^+ = 4 \sum_s \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3\sqrt{k^+}} \int \frac{dk'^+ d^2k'^\perp}{2(2\pi)^3\sqrt{k'^+}} \left[ \delta(2\vec{x} \hat{P}^+ - k'^+ - k^+)b^\dagger(k, s)b(k', s) + \delta(2\vec{x} \hat{P}^+ + k'^+ + k^+)d(k, -s)d(\bar{k}', -s) + \delta(2\vec{x} \hat{P}^+ + k^+ - k'^+)d(k, -s)b(\bar{k}', s) + \delta(2\vec{x} \hat{P}^+ + k^+ - k'^+)b^\dagger(k, s)d(\bar{k}', -s) \right].
\]

We have, \( k^+ > 0, k'^+ > 0, k^+ - k'^+ = p^+ - p'^+ = 2\xi \hat{p}^+ \). In the kinematical region, \( \xi < \bar{x} < 1 \), only the first term in Eq. (2) contributes \[8\]. We restrict ourselves to this kinematical region.

We take the state \(| P, \sigma \rangle\) to be a dressed quark consisting of bare states of a quark and a quark plus a gluon:

\[
| P, \sigma \rangle = \phi_1 b^\dagger(P, \sigma) | 0 \rangle + \sum_{\sigma_1, \lambda_2} \frac{dk_1^+ d^2k_1^\perp}{\sqrt{2(2\pi)^3k_1^+}} \int \frac{dk_2^+ d^2k_2^\perp}{\sqrt{2(2\pi)^3k_2^+}} \sqrt{2(2\pi)^3P^+} \delta^3(P - k_1 - k_2) \phi_2(P, \sigma | k_1, \sigma_1; k_2, \lambda_2) b^\dagger(k_1, \sigma_1) a^\dagger(k_2, \lambda_2) | 0 \rangle.
\]

Here \( a^\dagger \) and \( b^\dagger \) are bare gluon and quark creation operators respectively and \( \phi_1 \) and \( \phi_2 \) are the multiparton wave functions. They are the probability amplitudes to find one bare quark and one quark plus gluon inside the dressed quark state respectively. Up to one loop, if one considers all kinematical regions, there will be non-vanishing contributions from the overlap of 3-particle and one particle sectors of the state, this situation is similar to QED \[7\]. In the kinematical region we are considering, such kind of overlaps are absent and it is sufficient to consider dressing only by a single gluon. The state is normalized to one. \( \phi_1 \) actually gives the normalization constant of the state \[10\]:

\[
| \phi_1 |^2 = 1 - \frac{\alpha_s}{2\pi} C_f \int _\epsilon ^{1-\epsilon} dx \frac{1 + x^2}{1 - x} \log \frac{Q^2}{\mu^2},
\]

within order \( \alpha_s \). Here \( \epsilon \) is a small cutoff on \( x \).

The matrix element becomes,

\[
F^+ = 4\sqrt{P^+} \sqrt{\hat{P}^+} \left[ \phi_1^*(P') \phi_1(P) \delta(2\vec{x} \hat{P}^+ - 2\hat{P}^+) \right. \\
+ \left. \sum \int dp_1^+ dp_1^{\perp} \phi_2^*(P', \Delta + p_1, P - p_1) \phi_2(P, p_1, P - p_1) \delta(2\vec{x} \hat{P}^+ - \Delta^+ - 2p_1^+) \right]
\]

\( \Sigma \) denotes summation over helicities of the quark and gluon. The first term comes from one particle sector and it contributes only when \( \bar{x} = 1 \). The first term receives contribution upto order \( \alpha_s \) from the normalization condition of the state.
The other nontrivial contribution comes from the two particle sector given by the second term. We introduce Jacobi momenta \( x_i, q_i^\perp \) such that \( \sum_i x_i = 1 \) and \( \sum_i q_i^\perp = 0 \). Also, we introduce boost invariant wave functions,

\[
\psi_1 = \phi_1, \quad \psi_2(x_i, q_i^\perp) = \sqrt{\mathcal{F}^+} \phi(k_i^+, k_i^\perp). \tag{6}
\]

The contribution from the two-particle sector then becomes,

\[
F^+ = 2 \sum \int d^2 q^\perp \psi^*_2(\frac{x - \xi}{1 - \xi}, q^\perp + \frac{1 - x}{1 - \xi^2} \Delta^\perp) \psi_2(\frac{x + \xi}{1 + \xi}, q^\perp). \tag{7}
\]

The two particle wave function depends on the helicities of the quark and gluon and is given in terms of \( \psi_1 \) as,

\[
\psi_{2\sigma_1, \lambda}^\sigma(x, q^\perp) = -\frac{x(1 - x)}{(q^\perp)^2} T^a \frac{g}{\sqrt{(1 - x)}} \frac{\chi_{\sigma_1}^\dagger [2q^\perp - \bar{\sigma}^\dagger \cdot q^\perp]}{1 - x} + \frac{\bar{\sigma}^\dagger \cdot q^\perp}{x} \sigma^\dagger\lambda \chi_{\sigma}^\dagger \psi_1. \tag{8}
\]

Here \( \bar{\sigma}^\dagger = \sigma^2 \) and \( \bar{\sigma}^2 = -\sigma^1 \). Using Eq. (7) and Eq. (8), we see that the mass terms give suppressed contributions. Since mass terms in the vertex cause helicity flip, helicity flip parts of the matrix element are suppressed. We calculate the helicity non-flip part.

Using Eq. (8) we get, from Eq. (7),

\[
F^+ = \int d^2 q^\perp \frac{g^2}{(2\pi)^3} C_f \frac{(1 - \xi^2)^{\frac{1}{2}}}{(1 - \bar{x})} q^\perp \cdot (q^\perp + \frac{(1 - \bar{x})}{(1 - \xi^2)} \Delta^\perp) (1 - 2\xi^2 + \bar{x}^2), \tag{9}
\]

where \( C_f = \frac{(N^2 - 1)}{2N} \) for \( SU(N) \). The \( q^\perp \) integral is nontrivial and it is divergent for large \( q^\perp \).

Integration over the polar angle gives,

\[
\int d^2 q^\perp (q^\perp + \frac{(1 - \bar{x})}{(1 - \xi^2)} \Delta^\perp) = 2\pi \int_\mu^\Lambda dq \frac{1}{\sqrt{(q^2 + a^2 \Delta^2)^2 - 4a^2 \Delta^2 q^2}}
\]

\[
= \frac{\pi}{\mu} \int_{\mu}^{\Lambda} dq \left[ \frac{(q^2 + a^2 \Delta^2)}{\sqrt{(q^2 + a^2 \Delta^2)^2 - 4a^2 \Delta^2 q^2}} - 1 \right]
\]

\[= I_1 + I_2, \tag{10}\]

where \( a = \frac{(1 - \bar{x})}{(1 - \xi^2)} \). \( \Lambda \) is the large transverse momentum cutoff and \( \mu \) is the factorization scale separating hard and soft dynamics [11]. The divergence structure of the above integral can be seen by expanding the denominator of the integrand in the limit of small \( \Delta = |\Delta^\perp| \). We find that \( I_1 \) is logarithmically divergent in the limit \( \Lambda \to \infty \) but \( I_2 \) has no divergent part. The full divergent part is given by,

\[
F^+ = 2 \frac{g^2}{(2\pi)^3} C_f \frac{\sqrt{1 - \xi^2}}{(1 - \bar{x})} (1 - 2\xi^2 + \bar{x}^2) \pi \log \frac{Q^2}{\mu^2}, \tag{11}\]
where we have cut the transverse momentum integral at some scale $Q^2$. The splitting function can be easily extracted from the above expression:

$$P_{q q}^{}(\bar{x}, \xi) = C_f \frac{(1 + \bar{x}^2 - 2\xi^2)}{(1 - \bar{x})(1 - \xi^2)}. \quad (12)$$

This agrees with [2] (with $\xi$ replaced by $\xi^2$). The above expression contains endpoint singularity at $\bar{x} = 1$. However, this is canceled by the contribution from the normalization of the state to the single particle matrix element, so that the final result becomes

$$F^+ = 2\sqrt{1 - \xi^2}\left[\delta(1 - \bar{x}) + \frac{\alpha_s}{2\pi} C_f \log \frac{Q^2}{\mu^2} \frac{3}{2} \delta(1 - \bar{x}) + \frac{(1 + \bar{x}^2 - 2\xi^2)}{(1 - \bar{x})(1 - \xi^2)}\right]. \quad (13)$$

This result shows the importance of the normalization contribution to the single particle matrix element. In other words, it includes contributions from virtual gluon emission. In this approach, one uses probability amplitudes rather than probability densities in Altarelli-Parisi method and the effects due to both real and virtual gluon emissions are taken into account to the same order in $\alpha_s$ without any difficulty. In order to get the full scale evolution, one has to consider all the kinematical sectors which is beyond the scope of the present work. Also, one can see that the final result has no singularity at $\bar{x} = \xi$ which is as expected [7].

Next, we calculate the helicity flip part of the matrix element. The helicity flip contributions come from the mass term in the expression of the two-particle wave function. The form of the wave function shows that this contribution is suppressed.

Next, we parametrize the off-forward matrix element in terms of the generalized quark distributions,

$$F_{\lambda \lambda'}^\mu = \frac{1}{\bar{P}^+} \bar{U}_{\lambda'}(P') \left[H_q^{\mu}(\bar{x}, \xi, t) \gamma^\mu + E_q^{\mu}(\bar{x}, \xi, t) \frac{i}{2M} \sigma^{\mu\sigma} \Delta^\sigma\right] U_\lambda(P) + .... \quad (14)$$

where the ellipses indicate higher twist terms. $U_\lambda(P)$ is the quark spinor in our case. Using the explicit form of the light front spinors, we get for the plus component,

$$F^+ \delta_{\lambda \lambda} = 2\sqrt{1 - \xi^2} H_q^{\mu}(\bar{x}, \xi, t) - \frac{2\xi^2}{\sqrt{1 - \xi^2}} E_q^{\mu}(\bar{x}, \xi, t). \quad (15)$$

Here we have calculated the helicity non-flip part.

The helicity flip part becomes,

$$F^+ \delta_{\lambda' - \lambda} = \frac{-\Delta^1 + i\Delta^2}{m\sqrt{1 - \xi^2}} E_q^{\mu}(\bar{x}, \xi, t). \quad (16)$$

Since the helicity flip part of the matrix element is suppressed, we naturally find that $E$ is suppressed in perturbation theory. Here we have taken both $m$ and $|\Delta^\perp|$ to be small. In the limit of small $\xi$, we expand $H_q^{\mu}(\bar{x}, \xi, t) \approx H_q^{\mu}(\bar{x}, 0, 0) + \xi^2 H'_q^{\mu}(\bar{x}, 0, 0)$ in the kinematical

\[1\text{Here } \frac{1}{(1-\bar{x})_+} \text{ is the usual (principal value) plus prescription.} \]
region $\xi < \bar{x} < 1$. Using Eq. (13) and Eq. (15) and equating the coefficients of equal powers of $\xi$ on both sides, we get,

$$H_q(\bar{x}, 0, 0) = \delta(1 - \bar{x}) + \frac{\alpha_s}{2\pi} C_f \log \frac{Q^2}{\mu^2} \left( \frac{3}{2} \delta(1 - \bar{x}) + \frac{(1 + \bar{x}^2)}{(1 - \bar{x})_+} \right)$$  

$$H'_q(\bar{x}, 0, 0) = -\frac{\alpha_s}{2\pi} C_f \log \frac{Q^2}{\mu^2} (1 + \bar{x}).$$  

Eq. (17) gives the forward limit of the generalized quark distribution $H_q$ and it can be easily identified with the unpolarized quark distribution for a dressed quark state in perturbation theory [10].

Subsequently, we calculate the gluon distribution,

$$F^+_{g\lambda} = -\frac{1}{\bar{x} P^+} \int \frac{dz^-}{2\pi} e^{i z^- P^+} \langle P' | F^+(-\frac{z^-}{2}) F^+_{\alpha}(\frac{z^-}{2}) | P \lambda \rangle.$$  

The Fock space expansion of the relevant part of the operator is given by,

$$O_g = \frac{2}{\bar{x} P^+} \frac{1}{(2\pi)^3} \sum_{\lambda} \int dk_1^+ d^2 k_1^\perp \int dk_2^+ d^2 k_2^\perp a^\dagger(k_1, \lambda) a(k_2, \lambda) \delta(2\bar{x} P^+ - k_1^+ - k_2^+)$$  

We calculate the matrix element for a quark state dressed with a gluon (see Fig. 1(b)). The Fock space expansion of the state is given by Eq. (3).

The matrix element is given by,

$$F^+ = \frac{2}{\bar{x}} \sum \int d^2 q^\perp \psi_2^* \left( \frac{1 - \bar{x}}{1 - \xi}, q^\perp \right) \psi_2 \left( \frac{1 - \bar{x}}{1 + \xi}, q^\perp + \frac{1 - \bar{x}}{(1 - \xi^2)} \Delta \right) \sqrt{\bar{x}^2 - \xi^2}.$$  

Using the full form of the two particle wave function, we find that the helicity flip terms proportional to the quark mass give suppressed contribution and the helicity non-flip part is given by,

$$F^+ = 2 \sqrt{1 - \xi^2} \frac{\alpha_s}{2\pi} C_f \log \frac{Q^2}{\mu^2} \frac{(1 + (1 - \bar{x})^2 - \xi^2)}{\bar{x}(1 - \xi^2)}.$$  

The splitting function can be easily extracted and agrees with [4] with $\xi$ replaced by $\bar{x}$.

The gluon matrix element is parametrized in terms of the twist two distributions, $H_g$ and $E_g$,

$$F^\mu_{g\lambda \alpha} = \frac{1}{P^+} U_\lambda(P) \left[ H_g(\bar{x}, \xi, t) \gamma^\mu + E_g(\bar{x}, \xi, t) \frac{i}{2M} \sigma^{\mu \alpha} \Delta_{\alpha} \right] U_\lambda(P) + .......$$  

The fact that the helicity flip part of the matrix element is suppressed means that $E_g$ is also suppressed. In the limit of small $\xi$, we again expand $H_g(\bar{x}, \xi, t) \approx H_g(\bar{x}, 0, 0) + \xi^2 H'_g(\bar{x}, 0, 0)$ in the kinematical region $\xi < \bar{x} < 1$ and we obtain,

$$H_g(\bar{x}, 0, 0) = \frac{\alpha_s}{2\pi} C_f \log \frac{Q^2}{\mu^2} \frac{(1 + (1 - \bar{x})^2)}{\bar{x}},$$  

$$H'_g(\bar{x}, 0, 0) = -\frac{\alpha_s}{2\pi} C_f \log \frac{Q^2}{\mu^2} (1 + \bar{x}).$$
One can identify the above expression with the unpolarized gluon distribution for a dressed quark target \( [10] \).

\[
H'_g(\bar{x}, 0, 0) = \frac{\alpha_s}{2\pi} C_f \log \frac{Q^2 (1 - \bar{x})^2}{\mu^2}.
\] (25)

We next verify the helicity sum rule in perturbation theory. It has been shown \([11]\) that the light front helicity operator when expressed entirely in terms of the dynamical fields in the light front gauge, has the same form as in the free theory, provided we restrict ourselves to the topologically trivial sector (i.e., we take the dynamical fields to vanish at the boundary). This eliminates the residual gauge degrees of freedom and removes the surface terms. The helicity operator is given by,

\[
J^3 = J^3_{fi} + J^3_{fo} + J^3_{gi} + J^3_{go},
\] (26)

where, \( J^3_{fi} \) is the intrinsic quark helicity, \( J^3_{fo} \) is the orbital quark helicity, \( J^3_{gi} \) is the intrinsic gluon helicity and \( J^3_{go} \) is the orbital gluon helicity. The operators are given by,

\[
J^3_{fo} = \frac{1}{2} \int dx^- d^2 \bar{x} x^+ \left( x^+ \partial^2 - x^2 \partial^1 + x^2 \Sigma^3 \psi^+ \right),
\] (27)

\[
J^3_{fi} = \frac{1}{2} \int dx^- d^2 \bar{x} x^+ \left( x^+ \partial^2 + x^2 \partial^1 - x^2 \Sigma^3 \psi^+ \right),
\] (28)

\[
J^3_{gi} = \frac{1}{2} \int dx^- d^2 \bar{x} x^+ \left( A^1 \partial^+ A^2 - A^2 \partial^+ A^1 \right),
\] (29)

\[
J^3_{go} = \frac{1}{2} \int dx^- d^2 \bar{x} x^+ \left( A^1 \partial^+ A^2 + A^2 \partial^+ A^1 \right).
\] (30)

The color indices are implicit in the above expressions. We can check explicitly using the above expressions that the helicity sum rule for a dressed quark in perturbation theory is given entirely in terms of \( H(\bar{x}, 0, 0) \),

\[
\int_0^1 d\bar{x} \bar{x} H_q(\bar{x}, 0, 0) = 1 - \frac{\alpha_s}{2\pi} C_f \log \frac{Q^2}{\mu^2} \frac{4}{3} = \frac{2}{N} \langle P, \uparrow | J^3_{fi} | P, \uparrow \rangle + \frac{2}{N} \langle P, \uparrow | J^3_{fo} | P, \uparrow \rangle,
\] (31)

\[
\int_0^1 d\bar{x} \bar{x} H_g(\bar{x}, 0, 0) = \frac{\alpha_s}{2\pi} C_f \log \frac{Q^2}{\mu^2} \frac{4}{3} = \frac{2}{N} \langle P, \uparrow | J^3_{gi} | P, \uparrow \rangle + \frac{2}{N} \langle P, \uparrow | J^3_{go} | P, \uparrow \rangle,
\] (32)

where \( N \) is a normalization constant. This gives, for a dressed quark state in perturbation theory,

\[
\int_0^1 d\bar{x} \bar{x} (H_q(\bar{x}, 0, 0) + H_g(\bar{x}, 0, 0)) = \frac{1}{N} \langle P, \uparrow | 2J^3 | P, \uparrow \rangle = 1.
\] (33)

By comparing the \( lhs \) and \( rhs \) of Eqs. (31) and (32), one also verifies that in perturbation theory the total quark (gluon) momentum contribution is identical to the total quark (gluon)
helicity contribution. This result is expected from the fixed point solutions of the leading log evolution equations of $J_q$ and $J_g$ calculated in [12], which shows that the partition of the nucleon spin between quarks and gluons follows the partition of nucleon momentum.

3. Helicity Flip Part of the Transverse Component

Having studied the plus component of the matrix element of the bilocal operator, we now show the importance of quark mass in the matrix element of the transverse component of the bilocal vector operator. The matrix element of the transverse component is given by:

$$F_{\lambda \lambda}^\perp = \int \frac{dz^-}{2\pi} \bar{\psi}^{\perp}(\frac{z^-}{2}) \gamma^\perp \psi(\frac{z^-}{2}) \langle P' \lambda' | \bar{\psi}(\frac{z^-}{2}) | P \lambda \rangle. \tag{34}$$

The bilocal operator in this case can be written as,

$$O = \bar{\psi}^{\perp}(-\frac{z^-}{2}) \gamma^\perp \psi^{\perp}(\frac{z^-}{2}) = \psi^{\dagger/-}(\frac{z^-}{2}) \alpha^\perp \psi^{\perp}(\frac{z^-}{2}) + \psi^{-\dagger/(z^-)} \alpha^\perp \psi^{\perp}(\frac{z^-}{2}). \tag{35}$$

The operator involves the constrained field $\psi^-(-\frac{z^-}{2})$ and therefore it is the so-called bad component. $\psi^-$ can be eliminated using the constraint equation in light-front gauge,

$$i \partial^\perp \psi^- = [\alpha^\perp \cdot (i \partial^\perp + gA^\perp) + \gamma^0 m] \psi^+. \tag{36}$$

The terms linear in mass produce helicity-flip contributions in the matrix element. Quadratic mass terms generate helicity non-flip terms but it can be shown that they are suppressed.

It is known that in the forward case, for non-zero $P^\perp$, the above matrix element is related to the unpolarized DIS structure function $F_2$, as calculated through the plus component [13]. The contribution of the mass dependent helicity-flip part of the operator in this case cancels between the two terms of Eq. (35), as a result, quark mass effects are still suppressed here in the forward limit.

We calculate the matrix element of the helicity flip part of the operator in the off-forward case.

The operator is given by,

$$O = O_m + O_{k^\perp} + O_g, \tag{37}$$

where $O_m$ is the explicit mass dependent part of the operator, $O_{k^\perp}$ is the explicit $k^\perp$ dependent part and $O_g$ gives the interaction dependent part. The Fock space expansion of $O_m$ is given by,

$$O_m = -\psi^{\dagger/-}(\frac{z^-}{2}) \gamma^\perp m \frac{i}{\partial^\perp} \psi^{\perp}(\frac{z^-}{2}) + \frac{m}{i \partial^\perp} \psi^{-\dagger/-}(\frac{z^-}{2}) \gamma^\perp \psi^{\perp}(\frac{z^-}{2}) = 2 \sum \int \frac{dk^+ d^2 k^\perp}{2(2\pi)^3 \sqrt{k^+}} \int \frac{dk'^+ d^2 k'^\perp}{2(2\pi)^3 \sqrt{k'^+}} (im) \delta(2 \bar{x} P^+ - k^+ - k'^+) b^\dagger(k, s) b(k', s') \chi^\dagger \sigma^2 \chi s(\frac{1}{k^+} - \frac{1}{k'^+}). \tag{38}$$

Here $\chi$ is the two component spinor. This is the part of the operator that is relevant in the kinematical region $\xi < \bar{x} < 1$. We take the state to be a quark dressed with a gluon as before. The matrix element is given by,
\[ F_m^\perp = -\frac{2\xi}{P^+}(im)\chi^\dagger_\sigma^2\chi_\sigma' \frac{1}{\sqrt{1-\xi^2}} \left[ \delta(1 - \bar{x})\psi^\dagger_1 \psi_1 \right] + \int d^2 q^\perp \frac{1}{\bar{x}^2 - \xi^2} \psi^\dagger_2 q^\perp + \delta(1 - \bar{x})\psi^\dagger_2 \psi_2 \frac{2\bar{x} - 2\xi^2}{(1 - \bar{x})^2(\bar{x}^2 - \xi^2)}. \]  

(39)

Using the explicit form of the two particle wave function, and also using the normalization condition of the state, we write this as,

\[ F_m^\perp = -\frac{2\xi}{P^+}(im)\chi^\dagger_\sigma^2\chi_\sigma' \frac{1}{\sqrt{1-\xi^2}} \left[ \delta(1 - \bar{x}) \right] + \frac{\alpha_s}{2\pi} C_f \log \frac{Q^2}{\mu^2} \left( \frac{3}{2} \delta(1 - \bar{x}) + \frac{2\bar{x} - 2\xi^2}{(1 - \bar{x})^2(\bar{x}^2 - \xi^2)} \right). \]  

(40)

The other contribution to the helicity flip matrix element comes from,

\[ O_{k^\perp} = \xi^\dagger(-\frac{z^-}{2})(\partial^1 + i\sigma_3\partial^2)\frac{1}{\partial^+} \xi(-\frac{z^-}{2}) + (\partial^1 - i\sigma_3\partial^2)\frac{1}{\partial^+} \xi^\dagger(-\frac{z^-}{2}) \xi(-\frac{z^-}{2}). \]  

(41)

Here \( \xi \) is the two component fermion field. The Fock space expansion is given by,

\[ F_{k^\perp}^1 = \sum \int \frac{dk^+d^2k^\perp}{2(2\pi)^3\sqrt{k^+}} \int \frac{dk'^+d^2k'^\perp}{2(2\pi)^3\sqrt{k'^+}} \left[ \delta(2\bar{x}P^+ - k'^+ - k^+)b^\dagger(k, s)b(k', s) \frac{k^j}{k^+}(\delta_{ij} - i\sigma_3\epsilon_{ij}) + \frac{k'^j}{k'^+}(\delta_{ij} + i\sigma_3\epsilon_{ij}) \right]. \]  

(42)

The two particle contribution to the matrix element is of the form,

\[ F_{k^\perp}^1 = \frac{1}{P^+} \sum \int d^2 q^\perp \psi^\dagger_2 (\frac{\bar{x} - \xi}{1 - \xi}, q^\perp + \frac{1 - \bar{x}}{1 - \xi^2}\Delta^\perp) \psi_2 (\frac{\bar{x} + \xi}{1 + \xi}, q^\perp + \frac{2\bar{x}}{\bar{x}^2 - \xi^2}) + \frac{1}{P^+} \sum \int d^2 q^\perp \psi^\dagger_2 (\bar{x} - \xi, q^\perp + \frac{1 - \bar{x}}{1 - \xi^2}\Delta^\perp) \psi_2 (\frac{\bar{x} + \xi}{1 + \xi}, q^\perp + \frac{2\bar{x}}{\bar{x}^2 - \xi^2}). \]  

(43)

We have taken \(|\Delta^\perp|\) to be small. The terms linear in mass in \( \psi_2 \) give the helicity flip contribution given by,

\[ F_{k^\perp}^1 = \frac{2\xi}{P^+}(im)\chi^\dagger_\sigma^2\chi_\sigma' \frac{1}{\sqrt{1-\xi^2}} C_f \log \frac{Q^2}{\mu^2} \frac{2\alpha_s}{2\pi} \frac{2(1 - \bar{x})(\bar{x}^2 + \xi^2 + 2\bar{x})}{(\bar{x}^2 - \xi^2)(1 - \xi^2)}. \]  

(44)

The interaction part of the operator is given by,

\[ F_g^\perp = g\xi^\dagger(-\frac{z^-}{2})\frac{1}{i\partial^+} \left( A^1 + i\sigma_3A^2 \right) \xi(-\frac{z^-}{2}) + g\left[ \frac{1}{-i\partial^+} \xi^\dagger(-\frac{z^-}{2}) \left( A^1 - i\sigma_3A^2 \right) \right] \xi(-\frac{z^-}{2}). \]  

(45)

The Fock space expansion of the operator:

\[ F_{g}^1 = \sum_{s_1, s_2, \Lambda} \int \frac{dk^+_1 d^2k^\perp_1}{2(2\pi)^3\sqrt{k^+_1}} \int \frac{dk^+_2 d^2k^\perp_2}{2(2\pi)^3\sqrt{k^+_2}} \int \frac{dk^+_3 d^2k^\perp_3}{2(2\pi)^3\sqrt{k^+_3}} \left( b^\dagger(k_1, s_1)b(k_2, s_2)a(k_3, \lambda) \right) \left[ 4\delta(2\bar{x}P^+ - k^+_1 - k^+_2 + k^+_3) \chi^\dagger_{s_1} (\epsilon^1 - i\sigma_3\epsilon^2) \chi_{s_2} \right]. \]  

(43)
The terms containing $d$ and $d^\dagger$ will contribute in higher order. Out of the four terms, only the first and the fourth terms can contribute to the matrix element. These contributions are given in terms of the overlap of two-particle and one-particle wave functions. However, these contributions are zero since
\[ \sum_{\lambda} \chi^\dagger_{\lambda 1} (\epsilon^1 - i\sigma_3 \epsilon^2) \chi_{\lambda 2} = 0. \]

So we get, from Eq. (40) and (44), the helicity flip part of the matrix element:
\[
F^1 = -\frac{2\xi}{P^+(im)} \chi^\dagger_{\lambda 1} \sigma^2 \chi_{\lambda 2} \frac{1}{\sqrt{1 - \xi^2}} \left[ \frac{\delta(1 - \bar{x})}{\bar{x}^2 - \xi^2} \right. \\
+ \frac{\alpha_s}{2\pi} C_f \log \frac{Q^2}{\mu^2} \frac{3}{2} \delta(1 - \bar{x}) + \frac{2\bar{x} - 2\xi^2}{(1 - \bar{x})(\bar{x}^2 - \xi^2)} \\
- \frac{2\alpha_s}{2\pi} C_f \log \frac{Q^2}{\mu^2} \frac{\bar{x}^2 + \xi^2 + 2\bar{x}}{(1 - \xi^2)\bar{x}^2 - \xi^2} \right].
\]

So we see that in contrast to the forward case, the effect of quark mass is not suppressed in the matrix element of the transverse component of the bilocal current. It is clear that such a helicity flip contribution is zero in the forward limit because the above expression is proportional to $\xi$. So, this effect is a particularity of the off-forward matrix element only.

An interesting study using this approach will be to investigate the Wandzura-Wilczek relation [14] for the off-forward matrix elements of the transverse vector and axial vector operators in perturbation theory, for dressed quark states. We plan to undertake such studies in the near future.

To summarize, in this work, we have calculated the off-forward matrix element of the light-cone bilocal vector operator for a dressed quark state in perturbation theory. We have restricted ourselves in the kinematical region $\xi < \bar{x} < 1$. The contribution from the overlap of three and one particle wave functions is absent in this case. We have obtained the corresponding splitting functions directly. The end point singularity is canceled by the contribution from the normalization condition of the state. We have shown that the generalized parton distributions $E_q$ and $E_g$ are suppressed in perturbation theory. Furthermore, we have verified the helicity sum rule in perturbation theory for a dressed quark state. The terms linear in quark mass cause helicity flip. However, such terms are suppressed in the matrix element of the plus component. We have calculated the helicity flip part of the matrix element of the transverse component of the same operator and explicitly shown that quark mass effects are not suppressed. We point out that it is a feature of the off-forward case only and this term is absent in the forward limit.

We would like to thank A. Harindranath and M. V. Polyakov for helpful discussions.
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