Unification of Cosmology and Second Law of Thermodynamics: Solving Cosmological Constant Problem, and Inflation

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We seek here to unify the second law of thermodynamics with the other laws, or at least to put up a law behind the second law of thermodynamics. Assuming no fine tuning, concretely by a random Hamiltonian, we argue just from equations of motion – but without second law – that entropy cannot go first up and then down again except with the rather strict restriction $S_{\text{large}} \leq S_{\text{small1}} + S_{\text{small2}}$. Here $S_{\text{large}}$ is the “large” entropy in the middle era while $S_{\text{small1}}$ and $S_{\text{small2}}$ are the entropies at certain times before and after the $S_{\text{large}}$ era respectively. From this theorem of “no strong maximum for the entropy” a cyclic time $S^1$ model world could have entropy at the most varying by a factor two and would not be phenomenologically realistic. With an open ended time axis $(-\infty, \infty) = \mathbb{R}$ some law behind the second law of thermodynamics is needed if we do not obtain as the most likely happening that the entropy is maximal (i.e. the heat death having already occurred from the start). We express such a law behind the second law – or unification of second law with the other ones – by assigning a probability weight $P$ for finding the world/the system in various places in phase space. In such a model $P$ is almost unified with the rest as $P = \exp(-2S_{\text{tiny}})$ with $S_{\text{tiny}}$ going in as the imaginary part of the action. We derive quite naturally the second law for practical purposes, a Big Bang with two sided time directions and a need for a bottom in the Hamiltonian density. Assuming the cosmological constant is a dynamical variable in the sense that it is counted as "initial condition" we even solve in our model the cosmological constant problem without any allusion to anthropic principle.

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I. INTRODUCTION

It is the purpose of the present article to make a sort of unification of the second law of thermodynamics with the other laws in physics and cosmology, or rather we want to discuss how one may attempt to set up a law behind this second law. That is to say we want to make a formulation of a law at the macro physics level that can lead to and explain the second law of thermodynamics. Really we shall discuss such a potential law behind the second law of thermodynamics being of the form of there existing a by fundamental law given probability density $P(\text{"path") assigning a probability density – in phase space – to all possible solutions to the equations of motion, called here abstractly “path”. By assuming only mild generic properties of this probability density $P$ we then hope to get out not only approximately the second law of thermodynamics, but also a bit of cosmological and physical results. Most optimistically we believe our approach will play a role in solving the cosmological constant problem.

Some of the promising attempts to solve the cosmological constant problem – including the baby universe theory attempt – as to why the cosmological constant $\Lambda$ phenomenologically is so exceedingly small compared to an a priori fundamental scale for energy densities, using the Planck units as the fundamental units, make use of getting this cosmological constant become rather a dynamical variable than a constant of nature only. For instance is the effective cosmological constant $\Lambda_{\text{eff}}$ corrected by influence coming in via the baby universes and thus has become dependent on ordinary dynamical variables. In Guendelmann’s solution to the cosmological constant problem the effective cosmological constant is also dynamical in the sense that it is some field expressions shown to be constant in both space and time from what is essentially equations of motion or constraints. Even in order to obtain the vanishing cosmological constant by means of anthropic principle it is also needed since the cosmological constant $\Lambda$ at least has the possibility of taking other values than just a single one. Even in the derivation of the effective cosmological constant going to zero as shown in the work(s) by R. Woodard and N. C. Tsamis there are such gravitational fields around in what may look like the vacuum in practice that we in some sense even in this model can claim that the effective cosmological constant came to depend on fields.
and thus could be declared “dynamical”.

If indeed one has a model in which the status of the cosmological constant \( \Lambda \) has become on an equal footing with dynamical variables such as fields or positions of particles, then the argumentation for its value becomes in principle of the same nature as to argue about what the initial conditions for fields and particle positions are. Now it is the case since the separation between initial conditions and laws of nature by Newton, that the usual mechanical laws and Maxwell equations etc. do not tell about the initial conditions. It is rather so that we define the “initial conditions” to be the information which these laws for time development do not deliver. It is only the second law of thermodynamics that says something about the initial state conditions, which otherwise are left over for purely observational determination.

From this logic it looks, that if we use as a solution of the cosmological constant problem one of the type in which \( \Lambda \) is “dynamical”, we need to invoke some knowledge about initial state conditions. Potentially we need some extension or model behind the second law of thermodynamics.

Now such a model behind the second law is at least a priori upset a bit by the famous problem of the arrow of time \[12\]. Really the point is that mysteriously enough the second law violates drastically the time reversal symmetry! which seems to be an at least approximate symmetry, and indeed it holds to a very good approximation among the other laws in the Standard Model we could say.

This mysterious fact means that either we must live with one strange law not obeying time reversal invariance, and really not even CPT-symmetry, or we should have some sort of spontaneous breaking of time reversal invariance. Our cosmological models would indeed be strangely disturbed if we attempted to make second law be CPT-invariant i.e. for example, letting antiparticles have decreasing entropy. Here we used the words “some sort of spontaneous breakdown” to just mean that the fundamental laws have the symmetry in question – time reversal or CPT – but what goes on in Nature, the initial conditions, are not invariant under the symmetry. Since usually initial conditions do not share the symmetries with the laws of nature there is no real reason that is against spontaneous breaking in the abstract sense mentioned above. Also indeed it is one of the purposes of this article to present such a spontaneous-breakdown-like scenario for obtaining the second law.

At first glance it may seem that the Hartle-Hawking \[14\] no boundary way of letting the universe development start looks like the model, we describe into our formalism from the fifth paragraph in section 2 below. This means essentially Boltzmann’s way \[15\] which has a very special start. However due to the imaginary time really used by Hartle-Hawking and the quantum mechanics inducing probability the Hartle-Hawking model for the actual development of the universe may in fact come much closer \[16\], to picture of ours in the later part of the present article. Our main unification analogy of letting the actual path of the development be given by an imaginary part of the action may indeed be considered a generalization to a more general form of imaginary part of the action than the one obtained in the special way of using complex or imaginary time in the Hartle-Hawking scheme. \[16\]

In the following section 2 we shall discuss unification of the second law with the rest, the time development laws and introduce a probability density \( P \) over phase space as a replacement – or behind-law – for the second law. In section 3 we put forward our attitude of avoiding fine tuning by thinking more concretely on a “random Hamiltonian model.” Immediately we use this avoidance to produce a theorem restricting how much entropy can grow and next diminish again. – Of course such a discussion with diminishing entropy only makes sense when second law is not assumed; but remember we seek to derive if rather –. In section 4 we take the philosophy of a rather “random” probability density \( P \) – as introduced in section 2 – and show that even without too much details we easily get suggestive arguments for the hope for “spontaneous-breakdown like effect.” A bit more detail is developed in section 5, where we stress the assumption of translational invariance. Finally we argue for an infinitely long time axis. Then in section 6 we study the asymptotic behavior in time. Then in section 7 we discuss the benefit of having a short era with lower entropy and in section 8 and we really get the second law out.

In section 9 we return to the asymptotics from section 7 and argue about the Hubble expansion having to go to zero for \( t \to \pm \infty \) and thus solve the cosmological constant problem, provided the cosmological constant is assumed “dynamical”.

In section 10 we present some further results from our model: existence of approximate big bang and a bottom in the Hamiltonian density. In section 11 we conclude and come with some further outlooks.

II. A LAW BEHIND THE SECOND LAW OF THERMODYNAMICS

Since as mentioned the second law seems somewhat outside the company of the other laws a unification of second law with the other laws seems a priori a special challenge.

It is clear that since the second law tells about initial conditions or simply something about which solutions to the equations of motion are realized – at least in some statistical sense – a law behind the second law must be some law that assigns a probability to each of all the possible equation of motion solutions. Since in a phase space description – we here ignore quantum mechanics and work in the classical approximation in order not to invoke lots of further complications and philosophical troubles – allows us to describe the solutions by their phase space point at say time \( t = 0 \), a weight of probability for solutions could be described as a probability...
distribution over phase space destined to be used for the state at time $t = 0$, $(q(0), p(0))$. Putting the existence of such a probability distribution over phase space at a specific moment would be empty until we specify something about which weight to use. We can namely by such a weight describe any probability assignment to different solutions that would at all be possible. Unless the second law were in some way logically contradictory it should therefore also be possible to describe the knowledge somehow along such lines.

It is the purpose of the present article to seek to unify the second law with the other laws by imposing the symmetries of the other laws – especially time translation invariance and time reversal invariance for example – on the weight $P$ of probability function over the space of solutions to the equations of motion. Such symmetry requirements are not an empty information input. Rather it seems to severely endanger the chances of getting the second law out.

Even just imposing on $P$ that the probability of the different eq. of motion solutions should depend on these solutions in a time translational invariant way seems a dangerous assumption. It would then seem that if we can conclude from a little bit of honey picked out of a bottle that there is high probability for being a lot more honey in the bottle, then we might also think that honey tends to keep together in big bottles in the future and that by throwing some honey into a bottle we can make it likely that it will contain a lot of honey.

A model that seems to roughly work in formulating the second law – but not caring so much for e.g. time translational invariance – would be to take a model in which the phase space point in the “first moment” $t = t_{\text{first}}$, $(q(t_{\text{first}}), p(t_{\text{first}}))$ is used for the description.

Now we assume that we can in practice divide the phase space for the whole world described by its micro degrees of freedom which are fields into various subsets called “macro states”. Each macro state considered a subset should then contain those micro states ( = phase space points) that by some macroscopic observation could be distinguished as that macro state. The macro state should then have to consider one state (from the macroscopic point of view.)

Now our expectations are in practice close to the following model: In the early past i.e. $t = t_{\text{first}}$ there is no huge probability difference between different macro states – because they are all in practice “possible” – thus if the volume $V(B)$ in phase space of a macro state $B$ is, as is well-known, given as

$$V(B) = u_s^{2N} \exp(S(B)), \quad (1)$$

we just need to, very crudely, have

$$P(q(t_{\text{first}}), p(t_{\text{first}})) \sim e^{-S(B)}, \quad (2)$$

for $(q(t_{\text{first}}), p(t_{\text{first}})) \in B$.

Here $u_s$ is a kind of discretization cutoff in phase space that could represent Heisenberg’s uncertainty relation, say $u_s = \sqrt{\hbar}$. The number of degrees of freedom of the system is called $N$, and $S(B)$ denotes entropy of the macro state $B$.

It is crucial for a good second law to come out that we do not assume the same weight dependence at a later time, but only for $t = t_{\text{first}}$. This means that the proposed $P$-ansatz (2) is very non-time reversal invariant. At first it may seem hopeless to invent a time reversal invariant $P$ to reproduce the violently non-time reversal invariant second law $\dot{S} \geq 0$ which under time reversal transformation changes totally sign and in the nontrivial case $\dot{S} > 0$ becomes directly contradictory with itself. Nevertheless we want in the present article to even argue that we can obtain the second law for practical purposes from a $P$ that obeys a series of usual symmetries obeyed by the usual laws. Among these symmetries we even have time reversal invariance. The escape from the obvious contradiction is that we postulate below that in practice we only notice half the time axis so that under a strict time reversal inversion our world and the era would go into an era “before Big Bang” which we do not in practice take seriously. In other words we and our time would under time reversal go into a negative time $t < 0 (= t_o)$ era which is not supposed to exist in usual thinking, but exists in our model; in this era only, existing in our model one has the opposite second law, i.e. $\dot{S} \leq 0$. Even the time translation symmetry would be a threat of course if it were realized even as a symmetry of the state of the universe so that the state were constant since then $\dot{S} = 0$. But here one just has to have that the solution realized does not have to share all the symmetries of the fundamental law, in this case e.g. $P$. That is a phenomenon much like spontaneous breakdown meaning that even vacuum does not share the symmetries of the laws of nature, so let alone the states of the universe.

To make the probability density $P$ as a function of the solution have a form compatible with the symmetry and locality properties of the other laws it is good to think of the analogy with the action $S(\text{path})$ which is also – like $P$ a function of the path, the solution – when it is extremized –. The locality in time of the action means that it is additive in contributions from different regions in time. Really it is of the integral form

$$S(\text{path}) = \int dl. \quad (3)$$

This means in the Feynmann path integral $\int D\phi e^{iS(\text{path})}$ a factorization, so that we have for the integrand $e^{iS}$ factors coming from the different time regions. In order that effects of separate times should mean at first that one time could be treated independently of the other ones, the probability density described by the path-dependent function $P(\text{path})$ should be factored into factors depending only on what happens for the path in question in the different time intervals. In this sense $P(\text{path})$ is to be thought of as analogous to $e^{iS(\text{path})}$. Since we in quantum mechanics have the rule of squaring numerically the amplitude which in first approximation is $e^{iS(\text{path})}$ hav-
ing numerical square $|e^{iS(path)}|^2$, it is actually seen that $P(path)$ could be interpreted as an exponential of minus twice an imaginary part of the action expressed by $S_{lm}$

$$P(path) = e^{-2S_{lm}(path)}.$$  \hspace{1cm} (4)

In general the locality in time meaning the just mentioned factorization of $P(path)$ should mean that $P(path)$ should take the form

$$P(path) = \exp(-\int \mathcal{P}(q(t), p(t))dt) \hspace{1cm} (5)$$

$$= \exp(-\int \mathcal{P}(q(t), \dot{q}(t))dt)$$

and we would even see the formal identification with imaginary part of the Lagrangian $L_{lm}$

$$\mathcal{P} = 2L_{lm}$$  \hspace{1cm} (6)

where $L$ stands for Lagrangian.

It must be immediately admitted that introducing such a probability weight $P(path)$ which depends on the behavior of the paths of all times is in immediate danger of giving effects that make the model phenomenologically non-viable. We postpone till section 12 the discussions giving effects that make the model phenomenologically non-viable. We can with the interpretation of Lorentz invariance and mass protection analogous to the Standard Model. We can thus accept generic solutions as truly existing.

Under the regime of this generic philosophy we shall now show the following theorem (or lemma):

**Theorem:** Under the assumed random Hamiltonian setting: If a system passes successively in time and during a reasonable time through a series of macro states among which three macro states have the entropies $S_1, S_2, S_3$ respectively and are reached in this order $S_1, S_2, S_3$ in time, we have the restriction

$$S_2 \leq S_1 + S_3 \hspace{1cm} (8)$$

alone from the equations of motion.

Here we introduced the somewhat cryptical notation of “during a reasonable time”. By this extra requirement is meant that the time intervals are supposed to be extremely small compared to numbers as $e^{S_1}, e^{S_2}$ and $e^{S_3}$ which describe the volumes of the phase spaces of the macro states in question.

Now the proof: Let us denote the biggest acceptable “reasonable time” denoted by $T_r$, to be allowed for passage $T_r$. Then we have assumed

$$T_r \ll e^{S_1}, e^{S_2}, e^{S_3}. \hspace{1cm} (9)$$

Really we even assume $\log T_r \ll S_1, S_2, S_3$.

We can now ask for the volume of the region inside the macro state with entropy $S_2$ which has any chance of being reached from any micro state (meaning phase space point) in the first of the three macro states, namely the one with entropy $S_1$. If we denote by $v$ a typical “velocity” of a point in phase space (It may be easiest to just take $v$ as unit and ignore if $v = 1$, then the volume we asked for reachable from the $S_1$-macro state is estimated to

$"Reachable volume" = T_r \cdot v \cdot e^{S_1}u_2^{2N}. \hspace{1cm} (10)$

The $u_2^{2N}$ means a size for a Heisenberg uncertainty allowed phase space volume $u_2^{2N} = \hbar^N$, where $\hbar = 2\pi\hbar$ is the Planck constant and $N$ is the number of degrees of freedom of the world-system considered. Really this $u_2^{2N} = \sqrt{\hbar^{2N}}$ factor is only introduced to make dimensions be correct without giving the entropies very strange dimensions. For simplicity of our arguments it is really best to put $u_2 = 1$.

Analogously we calculate the volume in phase space – and it is also placed inside the macro state with entropy through the various “macro states” just by using equations of motion and some avoidance of fine tuning. Really we use a philosophy of the Hamiltonian being under some restrictions to be considered random as our way of avoiding fine tuning. We shall indeed take it that the Hamiltonian or at least a part of it is a random function on phase space with the restriction, however, that it (= the random part) does not cause the system to go from one macro state to another. That is to say the random part has zero Poisson-bracket with the macro variables, the variables characterizing a macro state. Really our point of view is that we only accept generic solutions as truly existing.

III. RESTRICTIONS ON ENTROPY GOING UP AND THEN DOWN

It is the purpose of the present section to show without at all assuming to the second law some restriction on how the entropy can vary when the system passes...
$S_2$ – of all those micro states that can reach possibly the final macro state of the three ones, namely the one with entropy $S_3$.

“reachable part of macro state with $S_3$”

$$= T_r \cdot v \cdot u_s^{2N} \cdot e^{S_3}$$

(11)

Now we shall remember that the volume of the part of the phase space corresponding to the macro state with $S_2$ as its entropy, the time-wise middle one, is

“Vol. of macrostate with $S_2$” = $u_s^{2N} \cdot e^{-S_2}$.  

(12)

Then we use the philosophy of random Hamiltonian to argue that this means that the fraction of $u_s^{2N}$-volumes – taken as a sort of cutoff in phase space – that can be reached from the $S_1$-macro state is

$$\frac{T_r \cdot v \cdot e^{S_3}}{e^{S_2}} = T_r \cdot v \cdot e^{S_1 - S_2}.$$  

(13)

Statistically we expect therefore that there out of the

$$T_r \cdot v \cdot e^{S_3}$$  

(14)

cells of size $u_s^{2N}$ that possibly can reach the $S_3$-macro state are

$$(T_r \cdot v \cdot e^{S_3}) \cdot T_r \cdot ve^{S_1 - S_2} = (T_r \cdot v)^2 \cdot e^{S_3 + S_1 - S_2}$$

(15)

that come from the $S_1$-macro state. Since even the log$(T_r \cdot v)$ is negligible compared to the entropies – of course measure in Boltzmann constant as unit – the condition for this number to not to be much smaller than unity is that

$$S_3 + S_1 - S_2 \geq 0$$  

(16)

or

$$S_2 \leq S_1 + S_3$$  

(17)

which were the condition we should prove. It is of course needed that at least one cell of the cutoff volume $u_s^{2N}$ should be able to come through. End of the proof.

Formally this theorem seems to exclude the possibility of having both a big bang and a big crunch with very low entropy and special states. Such a scenario could at first seem quite attractive in the light of Hawking’s connection of the Hubble expansion time arrow and the second law of thermodynamics arrow. S. E. Rugh and one of us and later Hartle and Gellmann proposed a two-entropy picture for such a scenario with both big crunch and big bang, but from the present theorem this kind of scenarios seem to have trouble.

IV. TIME-AXIS DISCUSSION

If we make a very strong assumption of time translation invariance we are forced to consider the set of all moments of time – the time axis so to speak – to be either the set of all real numbers $\mathbf{R}$ or a compactified circular “time axis” isomorphic to the $S^1$ circle. If we have the perhaps phenomenologically more realistic half interval with a certain starting time (creation), Big Bang, we do not strictly speaking have time translational invariance. With this excuse we shall only treat the mentioned two possibilities, $\mathbf{R}$ and $S^1$. We shall quickly exclude $S^1$ and the model put forward in the present article as our viable model is one with time axis $\mathbf{R}$. We thus strictly speaking have the problem of needing a bouncing back Big Bang instead of having the singularities needed by the Hawking-Penrose singularity theorem. In the light of the fact that we do not really know quantum gravity this possible trouble by needing a bounce should not be considered a so extremely severe problem, especially not in the very abstract formulation used in the present article. Let us first get over the possibility of the compactified time forming an $S^1$-circle, i.e. a world with an intrinsic period $T$ say imposed. In such a world the periodicity requirements are in number equal to the number of “initial state” describing variables, namely twice the number of degrees of freedom, the dimension of phase space. This means that in the generic philosophy – random Hamiltonian – there will only be a discrete set of solutions to the equations of motion. This means that there is, apart from the discrete possibilities, a unique solution, so that any extra assumptions about “initial conditions” become in first approximation superfluous. So long as we consider the solution essentially uniquely determined there would in the $S^1$-circle case be no place for the in section 2 introduced probability weight. In the $S^1$-circle case we could therefore at best hope for the second law of thermodynamics simple to come out by itself in this approximation, without any behind-law being put in. It also looks in the first go as if the compactified time axis is indeed helpful to get in the direction of the second law, in as far as we indeed obtained that the entropy in compact time world model is about equally likely to different entropy values. That is to say that the periodicity requirement is very likely to enforce the likely solution to have less entropy than maximal entropy. This is already to be considered a step in the right direction in as far as a random point with the natural phase space measure as probability distribution (i.e. $P = 1$ in our notation of section 2) would with very high probability be in the one of the macro state(s) with the maximal entropy – the heat death macro state. – Therefore already the fact that one does not get the heat death as prediction means that the compact world model even without any P-probability density imposed provides the answer to what is already a mystery: Why is universe not, now say, in the heat death macro state? However we found that the $S^1$-compact time world also leads to the entropy being totally constant as a function of time all the circle around. That there is at least strong limits to how much entropy could possibly vary in compact time or intrinsic periodicity model can be seen by applying our theorem in section 3. Since the time is
on the circle we can in fact always choose to consider the highest entropy $S_h$ reached in time in between the lowest $S_l$ on both sides. Thus our restriction from the theorem will tell us
\[ S_h \leq 2S_l \] (18)
so that alone from this theorem the highest entropy $S_h$ reached along the time-circle could not possibly be more than at most twice the lowest one $S_l$ reached.

However, if we reconsider the proof of our theorem above with the compactified time $S^1$ in mind we easily find that the chance for finding at least one cell of $u_2^{2N}$ size in the macro state with the highest entropy $S_h$ that comes from and goes to the same micro state cell in the low entropy $S_l$ macro state is crudely by ignoring factors $T^o$ etc.

\[ e^{S_l - S_h}. \] (19)
The equation of motion development backward and forward from the $S_l$-entropy macro state have to “accidentally” meet inside the $S_h$-entropy macro state phase space volume $u_2^{2N} e^{S_h}$. Thus the generalization of our theorem to the compact time case really leads to
\[ S_h \leq S_l \] (20)
which of course implies that entropy must be totally constant $\dot{S} = 0$ as we already claimed in \[20\].

Phenomenologically we have often $S > 0$ strictly and so that $S^1$-time system does not fit to this non-trivial realization of the second law of thermodynamics.

Keeping to the assumption of so strong time translational invariance that we even shall be able to find a moment on the time axis time translated by an arbitrary time-distance relative to any time-moment we are therefore driven to the time axis being the set of all real numbers.

In such a world – with the time axis $(-\infty, +\infty) = \mathbb{R}$ – we shall certainly not get anything like second law without imposing a drastically varying phase space probability density $P$. Indeed if we look $P \sim 1$ we would get the heat death prediction which is a priori the first expectation. With the two infinite time direction states left free we do not as in the $S^1$-time axis case have any restrictions that can prevent the maximal entropy $S_m$ or heat death macro state.

In the $\mathbb{R}$-time axis model – which we shall use in the bulk of this article – we thus need the probability density $P$ law as law behind the second law of thermodynamics.

\section{The Behavior for Time $t \to \pm \infty$}

Let us now consider the infinite time axis model with some probability density $P$. Even if some entropy variation occurs one will expect asymptotic behaviors for $t \to \pm \infty$, which we should study first, i.e. in this section.

We have already put forward a concept of macro states to describe subregions of the phase space in which at least to some degree the system can remain for some time. However some of these macro states may be much more stable than others. Potentially we might find classes of macro states which could be counted together and function together as a more stable macro state in which the systems stays longer. In the asymptotic times we would expect to find the system/universe in a class of macro states which is very stable. We must with a probability weight really expect that in the $t \to \pm \infty$ time regions the stable class of macro states chosen by Nature must be the one with on the average the highest value of $< P \cdot e^{S} >$. By this $e^S$ in our a bit formal expression $< P e^S >$ we mean the phase space volume, really it is $u_2^{2N} e^S$, for the set of states counted to the macro states together in the class of macro states that together were stable. The average sign $< \ldots >$ means that we take the average over associated micro states in the macro state(s).

We expect that the macro variables in such a most stable class of macro states are either moving periodically or have basically stopped moving.

As one of the major macro variables we can phenomenologically think of the size $a(t)$ of the universe as it occurs in usual Robertson-Walker cosmological model. If as we have just suggested the $< P e^S >$ being maximized determines the asymptotic values or oscillation intervals for the macro variables, we might expect also that the universe size variable $a(t)$ would in the asymptotic stop. That should mean that Hubble expansion should essentially stop asymptotically.

Phenomenologically it is at least true that compared to the suspected fundamental scale for time, the Plank time $10^{-43}$s(second), the Hubble constant $H$ is already exceedingly small compared to the inverse fundamental time $10^{43} s^{-1}$. In this – a bit thin – way we can claim that the universe already develops very slowly.

But why should the universe not continue to inflate strongly (say on Planck scale), since that would just make the density even lower and the stability even better in later times? This could be explained if there – with a 50% accident – were a term making log $P$ fall as the universe radius grows. Then namely it would not “pay” in terms of $< P > e^S$ to let the universe grow faster than needed to satisfy enough stability.

\section{Deriving Second Law of Thermodynamics}

As we shall argue for in section\[\text{[VII]}\] below, it is very likely that there shall be a short era with very small entropy. Then there be transition times between this era and the asymptotic eras described in section\[\text{[VI]}\].

Our major point is that in these transition times we will – almost certainly – have variation of the entropy. Indeed to find the for the short time interval to be used state(s) will most likely mean finding and realizing a very
small phase space region meaning a low entropy macro state. One can easily imagine that by some accident certain low entropy state is a very high $P$ state. We will therefore very likely find that in the time direction away from the time, $t_0$ say, when the huge $P$ but “unstable” macro state(s) is/are realized the entropy will increase. That is to say:

\[ \dot{S} > 0, \quad \text{for} \quad t > t_0 \]
\[ \dot{S} < 0, \quad \text{for} \quad t < t_0 \] (21)

If indeed – as is very likely – the entropy around the time $t_0$ is very small compared to entropies achieved at other times, our theorem in section 3 will essentially come to mean that on each side of $t_0$ the time axis we must have that entropy becomes a monotonous function of time.

Formally this is very easily seen by using the very likely happening that $S(t_0)$ very small. Then two times later than $t_0$, say

\[ t_0 < t_1 < t_2 \] (22)

we use our theorem to derive

\[ S(t_1) \leq S(t_2) + S(t_0) \] (23)

which for $S(t_0) \ll S(t_2), S(t_1)$ means

\[ S(t_1) \approx S(t_2). \] (24)

That is to say that in the region $t > t_0$ we have increasing entropy

\[ \dot{S}(t) \geq 0, \quad \text{for} \quad t \geq t_0. \] (25)

Similarly we see again by using our theorem and the approximation that $S(t_0)$ is also very small compared to the entropies in the earlier than $t_0$ times $t < t_0$, i.e. that

\[ S(t_0) \ll S(t) \] (26)

also for $t < t_0$, that if

\[ t_2 < t_1 < t_0 \] (27)

then

\[ S(t_1) \leq S(t_2) + S(t_0) \approx S(t_2). \] (28)

Thus in the $t < t_0$ times we have

\[ \dot{S}(t) \leq 0, \quad \text{for} \quad t < (t_0) \] (29)

i.e. falling entropy. Of course we would just invert the time axis and claim as usual that we have increasing entropy and thus that usual second law holds also if we happened to live in the era $t < t_0$.

In this sense we have derived as almost unavoidable that in a model with a strong probability density variation over phase space the most likely, in the $P$-sense, solution(s) to equations of motion will effectively look as having the second law, provided though the following small caveats of interpretation:

1) We shall choose the time axis so that positive $t$-direction is counted away from the $t_0$-time. We redefine for this purpose a new time $t_{new}$ as

\[ t_{new} = t - t_0 \quad \text{for} \quad t > t_0 \]
\[ t_{new} = t_0 - t \quad \text{for} \quad t < t_0. \] (30)

2) We only derived the $\dot{S} = \frac{dS}{dt_{new}} \geq 0$ for one half of the time axis. To have it – the second law – we must ignore what goes on on the other side of the time $t_0$.

3) Our derivation only worked under the though suggestive assumption that we could argue for – or generally expect – that the entropy enforced at the time $t_0$ was indeed so small that it were negligible compared to the other entropies. Really though this seems to be almost unavoidable a correct assumption, but it might deserve further theoretical study.

Phenomenologically of course the time $t_0$ is being identified with the Big Bang time. But notice then that our model suggests that there be a bounce from a contracting universe, though one with opposite second law.

VII. DO WE OBTAIN BIG BANG?

In the light of that it is unavoidable with essentially, whatever the probability density function $P$ might be, that we should have the universe behaving corresponding to highest $< Pe^S >$ stable class of macro states, we might now – somewhat worried from phenomenological point of view – ask whether the totally most favoured, most probable with $P$, development would not be to have the approximately static universe all through from $-\infty$ to $+\infty$ in time.

The answer, we shall show is that such an all the time like the asymptotic region behaving universe is not likely to be the most likely outcome. The point is that we can very easily risk that it would pay probabilitywise to get even for limited time interval the system in a state with a huge $P$ compared to the asymptotic average. Since we in seeking the macro states for the asymptotic only could look for the subclass of “stable” classes of macro states, it is indeed very likely – almost surely – so that there exist “unstable” macro states outside the usable classes of stable collections of macro states for which the $< Pe^S >$ is much higher than for any “stable” competitor. If this is so a more likely universe development than the one being in the “Stable class” all along the time axis would be one in which the more high $Pe^S$ macro state is reached during some finite time while we still have the already described asymptotic behaviors. Then of course there is needed on both sides on the time axis some development between the “unstable” utterly special $P$-favoured state and the asymptotic behaviors.
VIII. THE COSMOLOGICAL CONSTANT PROBLEM

As mentioned in the introduction a good start on solving the cosmological constant problem is to make the cosmological constant become a dynamical variable on an equal footing with the fields or positions of particles. However, logically this completely alone cannot be the whole story for even if the cosmological constant is in some model part of the dynamical variable of the system, why should it be extremely small, almost zero, for that reason? It is at this stage that our present work can have a role to play. We shall thus simply assume that by some mechanism on the market we have got to the effective cosmological constant being a function of variables in the phase space of states of the universe. Or we could let it, the cosmological constant, simply be one of the variables, one of the generalized coordinates. We must of course then in some way or the other assume or obtain that it be a constant in time. That is to say we must somehow achieve \( \Lambda = 0 \) where \( \Lambda \) is the cosmological constant. However, it may well be allowed that our probability density function \( P \) can depend somewhat on \( \Lambda \), and most importantly the value of \( \Lambda \) has great significance for the development of the other dynamical variables in the model.

We already argued in section 6 that the Hubble expansion should end up being small so as to lead to an asymptotic state or class of macro states. Since ignoring matter one has in gravitationally determined units

\[
H^2 \approx \Lambda \tag{31}
\]

a small \( H \) already suggests a small \( \Lambda \). This was, however, only true ignoring matter. There are however reasons to believe that requiring the “stable” macro states or classes of them called for in section 6 the universe does the best job of stability by having an exceedingly low mass density. In the very long run the formation of black holes could be an important mechanism for producing entropy since by means of them you can liberate even the free energy contained in the baryon masses. But if it happens that the large black hole states are not so favoured with respect to \( <Pe^S > \) they should be avoided in the asymptotic most favoured stable states. That might be achieved by having low matter density, because it then simply becomes impossible for enough matter to collect at one spot so as to form black holes. In this way we may understand or derive that stability against falling into a possibly less high \( <Pe^S > \) state with black hole(s) will require a low matter density. Thus we may see that the asymptotic state which should be the most favoured \( <Pe^S > \) among the stable states must be a low matter density one since otherwise it would not be “stable”.

But then we see that both matter density \( \rho_m \) and Hubble expansion rate must be small in the asymptotic state. Thus we see that indeed with a “dynamical” cosmological constant it must be small.

This we can consider a solution to the cosmological constant problem, but we must admit of course that we only succeeded under the assumption that the cosmological constant were influencable at all.

On the other hand we must also remark that it looks extremely hard to give an explanation to the smallness – of what we really have to – of the effective, or dressed, cosmological constant without any chance that it is influencable. Precisely because it is the dressed or effective cosmological constant rather than the bare one that should be zero or small, it depends on all the vacuum energy density corrections. It is very hard to see how all these – huge– corrections should be just cancelled by terms in the theory for the bare cosmological constant unless it can become influenced somehow from the vacuum having all the contributions of different forces. Thus it is on general grounds extremely hard to see, how one could get the cosmological constant problem solved if the cosmological can not be influenced some way or an other, unless somehow the corrections to the vacuum energy density is zero or some very simple value. This latter is the possibility of global supersymmetry in which the corrections are zero and also the cosmological constant. But as is well known the pure global SUSY explanation for the vacuum energy density cannot solve the cosmological constant problem with sufficient accuracy because the SUSY-breaking needed for phenomenology is much bigger than the actual value for this constant.

IX. INFLATION AND BOUNCING SCENARIO

It is actually very natural and easy to obtain in our model an inflation scenario \[21\] – as is well known to be strongly called for phenomenologically – because the function \( P \) in terms of which a suggestive form for the probability measure density \( P \) is written can easily be a function of some scalar field \( \phi(x) \), the inflaton field. It would then be extremely likely because \( P \) would be big to have a period era in which the various scalar fields would take values just favouring \( P \) to be as small as possible and the \( P \) large.

Such an inflation with \( \phi \) near the low \( P \) value would normally not be stable but the inflation field would roll down and “re” heating \[22\] would occur causing a creation of a lot of entropy. With the bit of adjustment of cosmological constant and the expansion rate resulting we could get approximately the asymptotic conditions called for to have a “stable” class of macro states.

With the \( P(\phi(x),...) \) being taken the same all over space time would make the same all over space time would make the same inflation starting value \( \phi(x) = \phi_{\text{start}} \). So the inflation scenario is rather a simple inflation picture homogeneous from the start than a chaotic inflation picture.

An interesting point that turns out also to concern inflation is the prediction above that we get two half time axis worlds only separately having a second law. This picture calls for a rebounce scenario rather than a big
bang singularity.

On the other hand if we let the $Pe^S$ favoured state that so to speak has to be realized – around the moment we call $t_0$ – be the inflating state, then there would be no call from $Pe^S$ for a pre-inflation state of the universe. Just inflation would be enough.

Luckily enough for the working of our model: In spite of that it is hard to get rebouncing – Penrose-Hawking theorem – a model dominated by an inflation state just with an essentially constant scalar field – or several scalar fields – would simply be a de Sitter model. A de Sitter universe actually does precisely rebounce provided the universe has a positive spatial curvature so that $K = +1$.

We have namely the Einstein equation in the Friedmann model:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left(\frac{\rho}{\rho_{\text{crit}}} - \frac{k_0}{a^2} + \lambda_0\right)$$

(32)

showing that as $a$ approaches $\sqrt{\frac{k_0}{a^2}}$ the Hubble expansion $H = \frac{\dot{a}}{a}$ goes to zero.

Here $\lambda_0$ is given by

$$\lambda_0 = \frac{\Lambda c^3}{3H_0^2}$$

(33)

$$H_0 = \left(\frac{\dot{a}}{a}\right)_0$$

where the suffix 0 means the value at present time.

$$\rho_{\text{crit}}^0 = \frac{3c^2 H_0^2}{8\pi G}$$

(34)

is the critical energy density. The curvature parameter $k_0$ is given by $k_0 = \frac{\Lambda c^2}{H_0^2}$.

When both $k$ and the cosmological constant, which in gravity specified units is $\lambda_0$, are positive, the bounce, $H = 0$, occurs for a finite real value of the radius $a$ of the universe. So if we take it that infinite universes with $k = -1$ or $k = 0$ are “unpleasant” philosophically we will get very nicely a rebounce in the inflational time itself. This liberates our model from a potentially great problem: rebouncing could be very difficult to achieve.

**X. FURTHER RESULTS**

We should here stress that our model in addition to the major results already mentioned: the cosmological constant being small and the effective second law, for the half time axis in which we live, further leads to:

A) The existence of an approximate big bang in the sense that the universe once were very much smaller than today. We find, however, not a true starting singularity but rather a rebouncing in the inflational period. There is no singularity around the time $t_0$ (of course in usual cosmologies taken as $t_0 = 0$), but rather that an $S^3$-spherical universe with at that moment an effective positive cosmological constant due to some inflation field(s) rebounce smoothly. The Hubble constant smoothly goes from negative to positive.

B) In order that there can be the stability of the asymptotic state needed there must effectively be a lower bound for the Hamiltonian density, a bottom in the Hamiltonian density.

**XI. WOULD OUR MODEL GIVE UNACCEPTABLE EFFECTS ?: CAVEATS**

A. General risk of influence from future

At first it seems extremely dangerous for getting very strange miraculous effects to introduce a probability function $P(path)$ which is sensitive to what the path does at all times and not only in the far past. Such a probability density would namely select the path(s) to be realized not only with special properties (typically low entropy) in the far past (say around the effective Big Bang time called $t_0$ above) but also in say the near future. If really the path were selected – as in our model – also with respect to what happens in the (near) future, such arrangement would be recognized as a foresight or the hand of God or a miracle. Nevertheless such signs of foresight are so seldom empirically that many people do not believe in their existence at all. For our model to be phenomenologically viable it is therefore needed that such effects of foresight adjustments of the path realized are extremely weak and can in practice be ignored.

Now there is first to be taken into account that we imagine the $P$ factors coming from the inflation period, i.e. $\int P\sqrt{g} dx$ from this period, to be very strong so that the realized track is very dominantly determined to make the factor from that period large and from the infinitely long time periods $t \rightarrow \pm \infty$. The influence from the relatively short time span over which humans have the capacity for observing the foresight or miraculous effects is relatively less significant. Alone for that reason the effect is significantly suppressed. We should remember that the special form of inflation – the special inflation field and the value of it during inflation – were first of all selected to contribute highest possible to log $P(path)$, so if it comes to dominate it is not so surprising again. But at least we are able only to collect information about miracles, foresight etc. over say of the order of thousands of years while the universe up to now has at least existed for milliards of year, the observable effects are in any case down by a factor a million.

Now there is, however, also an a bit more sophisticated reason for the suppression of such effects coming from the nature of the Standard Model. This effect is described here below:
B. Suppression of influence from future:

First let us remark that a part in $\hat{P}$ having the same form as the kinetic term in the Lagrangian will vanish if the free equation of motion for the field is obeyed. If we talk about a massless particle due to some mass protection conservation law there will be no mass parameter, and all the non-interaction term is already fixed. For such a mass-protected theory a field-bilinear-expression can only have one form and thus must be the same – in reality only an $i = \sqrt{-1}$ is the difference – form for the Lagrangian density $L_{bil}$ and for the bilinear part of the $\hat{P}$, called $\hat{P}_{bil}$, so that at least to this bilinear approximation – no interaction approximation – the $\hat{P}_{bil}$ vanishes on shell. According to this argument you can for mass protected particles in the free approximation, see no effect of $P(path)$. This result may also be seen – may be more convincingly – by simply using Lorents invariance to tell that since there exists no way of having a Lorents invariant length of the time track of a massless particle, we cannot possibly obtain a sensible form of the contribution to $\log P$ from a passing around massless particle in the approximation of the free path dominating. For massive particles on the other hand there can be defined a time track length, since it is then possible to use the eigentime.

We may estimate the contribution coming from a Higgs particle time track going along. For dimensional reasons we need a mass factor in front of the eigentime to get a contribution to the dimensionless $\log P$. Even if we take the mass to be the compared to fundamental scales – say Planck scale – a very small mass $m_H$ of the Higgs itself say, the typical life time $\tau_H \approx \frac{1}{m_H}$ for a Higgs would give a contribution to $\log P$ of the order of magnitude

$$\log P \Big|_{Higgs \; flight} \approx \tau_H m_H \approx \frac{m_H}{\Gamma_H} \approx 100 \text{GeV} \Gamma_H \approx 100 \text{MeV} \approx 10^3$$.

(35)

If the equations of motion are not so strongly guided by what happens at inflation time or at far future, that a factor $e^{-10^3}$ in $P$ is not felt, this would e.g. mean that the production of a Higgs would somehow miraculously be prevented to the level of only happening once out of a priori $e^{-10^3}$ expected cases. How it would happen is not so clear, but terrorists on LHC might be very effective.

These problems deserve further investigation, but until now nobody saw safely the Higgs. So a total prevention of Higgs production by mysterious foresight effects is not out of question experimentally.

XII. CONCLUSION AND OUTLOOK

We have sought to build up crudely a model behind the second law of thermodynamics by speculating about the properties of a “probability weight function” $P(path)$ which assigns to all the paths through phase space of the universe (everything in classical thinking) a probability density relative to the phase space density. We actually suggested that we could obtain a reasonable scenario without making exceedingly specific assumptions about the form of the function of “probability density function” $P(path)$. Basically we actually made use of the following:

A) We used a time factorized form (Locality in time),

B) We used that one could distinguish some macro states or classes of them which were stable.

C) We used that the variation in the size of $P(path)$ were indeed huge.

D) Especially we assumed that a small or fundamental scale universe in inflating state would have highest $P e^S$. Very likely an inflational universe would be favoured because the scalar fields could easily dominate $\hat{P}$ giving $P \sim \exp(- \int \hat{P} \sqrt{g} d^4x)$.

The most concrete picture of a model unifying the second law with the other laws would be given by a probability density with the logarithm of the form

$$\log P(path) = \int \hat{P}(\varphi, \partial_\mu \varphi, g_{\mu\nu}, \partial_\rho g_{\mu\nu}, \cdots) \sqrt{g} d^4x \tag{36}$$

so that it is very much like an imaginary part of the complex action:

$$\hat{P} = 2 \text{Im} \mathcal{L}_{compl.} \tag{37}$$

However, most of the structure of this expression $P(path)$ were not really used since the details were not so important.

Our main results were:

1. That in an infinite time axis model $t \in (-\infty, \infty)$ we could indeed obtain two “half time axes” each of which had for itself a second law of thermodynamics (provided that we choose the appropriate sign on the time axis separately for the two half axes.)

2. We obtain a small cosmological constant under the assumption that first the cosmological constant could be considered part of the “initial conditions” and secondly that the universe develops towards a stationary asymptotic macro state. This means that we need a universe developing as little as possible.

3. In an intermediate step of the argument it also meant that the density of matter should be very low compared say to the “fundamental density”.

4. Further we got for keeping the stability that a bottom in the Hamiltonian density should be called for and adjusted to exist (approximately at least).
5. We can even claim that our model had it very naturally that there should be an effective Big Bang; but here it is an important point that a true Big Bang singularity at which space-time starts is not suggested by our model. Rather the suggested scenario has the “two sides in time” which means that we seen from our branch have a half infinite pre-big-bang era with falling entropy. Rather than any singularity we then argued for the inflationary era from the pre-big-bang times \((t < t_0)\) – in which with our time arrow convention the universe contracted “inflationally” – going smoothly over into the growing universe inflation for \(t > t_0\). Really we have in the time around \(t_0\) – the simulated two sided Big Bang – just a positive curvature world developing as a de Sitter universe with the quite smooth and natural rebounce in this case. There is at least approximately a time reversal invariance mirror symmetry around \(t = t_0\).

This rebouncing in the inflationary stage is quite sufficient for Big Bang model phenomenology because all the details on which we have phenomenological check only goes back to the last 68 or so e-foldings in the inflational period.

In conclusion our picture has thus several good results: second law of thermodynamics, small cosmological constant, bottom in Hamiltonian density, practical Big Bang, but in reality there is no true Big Bang. There is rather a rebounce from a pre-big-bang time taking place in the inflation era.

With all these good features phenomenologically we have however to admit there are also strange effects being predicted: With the philosophy that there is a more fundamental path dependent probability density \(P(path)\) – even having special dependence on what happen today say – there is a danger of \(P\) functioning as a source of effects which obvious to the observer, would be governed from the future (may be even the near future). That would look like there being a foresight. A crude discussion leads to that a suggestive place to look for such foresight effects would be to investigate if the foresight would seek to prevent Higgs particles being produced (or perhaps instead favour Higgs production, depending on the sign). Would the foresight make an accident attacking the start of \(LHC\)?

We think that these foresight effects that could severely threaten the viability of our model behind the second law deserves further calculations as to whether the effects will be strong enough to be really seen.

One could also imagine that because the true path realized would be guided towards a sort of maximum in the probability density the derivatives of \(\log P(path)\) near that path will be small and that such an effect of derivatives at maximum being zero could lead to a reduction to zero of the coefficients determining say how unlikely a path becomes by having a Higgs particle produced.

Since we got the second law without much details assumed about \(P\) we can consider the present work a “random dynamics” derivation of the second law of thermodynamics. That is to say we would say that second law came almost surely, almost unavoidably.

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[1] S. Carnot, “Réflexions sur la Puissance Motrice du Feu et sur les Machines Propres a Developper cette Puissance”, Bechelier, Paris 1824; see “The Second Law of Thermodynamics”, translated and edited by W. F. Magie, Harper and Brothers, New York 1899, pp.3-61.
[2] R. Clausius, Ann. Phys. Chem., 79(1850)368-397, 500-524; ibid., 93(1854)418-506; ibid., 125(1865)353-400, see “The Mechanical Theory of Heat”, translated by W. R. Browne, MacMillan and co., London, 1879; see also W. F. Magie [1] pp.65-108
[3] W. Thomson, Mathematical and Physical Papers, University Press, Cambridge, 1882, Vol.1, (1848) pp.100-106; (1849)113-155; (1851-1854, 1878)174-332; with J. P. Joule), (1852-1862)333-455.
[4] I. Prigogine: “Thermodynamics of irreversible processes”, Interscience, A division of Wiley and Sons, NY London 1961; see also J.Kirkwood, J. Chem. Phys. 14, p.180(1946)
M. Born and H. S. Green, Proc. Roy. Soc. A 190, p.455(1947) G. Klein and I. Prigogine, Physica 13, p.74, 89(1957)
[5] S. Coleman, Nucl. Phys. B 307, 867(1988)
[6] S. Coleman, Nucl. Phys. B 310, 643 (1988).
[7] T. Banks, Nucl. Phys. B 309, 493 (1988).
[8] S. W. Hawking, Phy. Lett. 134B, 403 (1984).
[9] E. I. Guendelman and A. B. Kaganovich, “Dynamical measure and field theory models free of the cosmological constant problem”, gr-qc/9905029 (1999), Phys. Rev. D55, 7205 (1996); see also in: Proceedings of the third Alexander Friedmann International Seminar on Gravity and Cosmology, edited by Yu. N. Gnedin, A. A. Grib, V. N. Nostepanenko (Friedmann Laboratory Publishing, St. Petersburg, 1995); Phys. Rev. D55, 5971 (1997); Mod. Phys. Lett. A12, 2421 (1997); Phys. Rev. D56, 3548 (1997); Hadronic Journal 21, 19 (1998); Mod. Phys. Lett. A13, 1583 (1998).
[10] S. Weinberg, “The Cosmological Constant Problem”, Rev. Mod. Phys. 61, 1-123 (1989).
[11] N. C. Tsamis and R. Woodard, Phys. Lett. B301, 351-357 (1993).
[12] S. W. Hawking, Phys. Rev. D47, 5342-5356 (1993), gr-qc/9301017.
[13] S. W. Hawking, Phys. Rev. D32, 2489 (1985).
[14] J. B. Hartle and S. W. Hawking, Phys. Rev. D28, 2960-2975 (1983).
[15] L. Boltzmann, Ann. Physik 60, 392 (1897), translated in S. G. Brush, Kinetic Theory (Pergamon Press, New York, 1965) Quoted in: James B. Hartle, Quantum Theory of Gravity, World Scientific, Singapore, 1983.
[16] S. W. Hawking and Thomas Hertog, Why Does Inflation Start at the Top of the Hill?, hep-th/0204212.
[17] H. B. Nielsen and S. E. Rugh, “Arrows of time and Hawking’s no-boundary proposal”, Neils Bohr Institute Activity Report 1995, Murray Gell-Mann and James B. Hartle, Time Symmetry and Asymmetry in Quantum Mechanics and Quantum Cosmology, arXiv: gr-qc/9905029.
[18] P. J. Steinhardt, Neil Turok, hep-th/0110098.
[19] H. B. Nielsen and M. Ninomiya, Compactified Time and Likely Entropy – World Inside a Time Machine: Closed time-like Curve. – To appear in Proceedings for “What comes beyond the Standard Model?” held in Bled, Slovenia, 18th-29th of July 2005. YITP-05-38 and OIQP-05-06.
[20] A. H. Guth, The Inflationary Universe: A possible Solution to the Horizon and Flatness Problems, Phys. Rev. D23, 347 (1981). A. D. Linde, A New Inflationary Universe Scenario: A possible Solution of the Horizon, Flatness, Homogeneity, Isotropy, and Primordial Monopole Problems, Phys. Lett. B108, 389 (1982); A. Albrecht and P. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982); A. A. Starobinski, Phys. Lett. B91, 99 (1980); A. D. Linde, Phys. Lett. B129, 177 (1983); D. La and D. J. Steinhardt, Phys. Rev. Lett. 62, 376 (1989).
[21] Since it is a peculiarity of our model that from $t_0$ on it was cold all the time during inflation “reheating” as one usually calls the heating after inflation is strictly speaking a wrong terminology.