A New Multi-Objective Hybrid Flow Shop Scheduling Method to Fully Utilize the Residual Forging Heat

QIANG CHENG,1,2, CHENFEI LIU,1,2, HONGYAN CHU,1,2, ZHIFENG LIU,1,2, WEI ZHANG3, and JUNJIE PAN3

1 Institute of Advanced Manufacturing and Intelligent Technology, Beijing University of Technology, Beijing 100124, China
2 Beijing Key Laboratory of Advanced Manufacturing Technology, Beijing University of Technology, Beijing 100124, China
3 Guizhou Anda Aviation Forging Company Ltd., Anshun 561005, China

Corresponding author: Hongyan Chu (chuhy2019@126.com)

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ABSTRACT This paper aims to solve the problem of high energy consumption in forging production through energy-saving scheduling. By analyzing the flow shop characteristics of a forging workshop, an energy-efficient hybrid flow shop scheduling problem with forging tempering (EEHFSP-FT) is proposed. An energy-efficient scheduling model is established to simultaneously minimize both the completion time and energy consumption. In the scheduling model, constraints such as heating furnace capacity, required forging temperature, and required quenching temperature are taken into consideration. An energy-saving strategy of heat treatment with residual forging heat is adopted to address the problem of energy underutilization after forging. In order to use multi-objective optimization algorithms to solve the scheduling problems of charging and machine selection in forging production, encoding and decoding rules and evolutionary search strategies are designed. Finally, a case study on the flow shop of an automated forging center is analyzed. The validity of the proposed model is demonstrated by testing cases of different scales in conjunction with three different evolutionary algorithms. By analyzing the performance of the three algorithms, the algorithm suitable for solving the proposed model is determined.

INDEX TERMS Energy saving scheduling, forging planning, multi-objective optimization, temperature constraint.

I. INTRODUCTION

During the past decades, the global energy consumption has been growing significantly, especially in the manufacturing industry, which accounts for almost half of the total consumption. Non-OECD (Non-organization for economic co-operation and development) economies consume 62% of the energy delivered in the industrial sector globally, and this percentage is expected to reach an average annual growth rate of 2.7% by 2030 [1]. In the manufacturing sector, hot-processing production consumes more energy than cold-processing production. Forging production is a typical type of thermal-processing production, which continuously consumes huge amounts of energy in stages such as heating and heat treatment. Therefore, in the context of sustainable development, research on energy-saving methods for forging production is necessary.

Currently, the research on energy-saving methods in forging production focuses mainly on process optimization [2], [3], innovative recycling technologies [4], forging machine improvement [5], [6], heating furnace reforming [7], and development of new materials [8], [9]. In recent years, with the continuous advances in the field of production scheduling, some researchers have achieved the goal of energy saving by solving the scheduling problems in forging production. However, most of these studies are aimed at single-machine scheduling of the furnace charging sequence. The research on energy-efficient scheduling (EES) of the forging production line that optimizes both makespan and energy consumption is still limited.
The forging production line is a typical flow shop due to its continuous process. One of the characteristics of forging production is that each heating furnace can accommodate several forgings. In the scheduling problem, it is understood that each machine can process multiple jobs at the same time. In order to match the job output quantity of the heating furnace, multiple parallel forging machines are generally equipped. Thus, the forging production line can be classified as a hybrid flow shop scheduling problem (HFSP). Some complex constraints need to be considered in forging production due to its special characteristics of processing, for example, in order to avoid energy loss, after the forging billet is heated, it must be kept in the heating furnace until the next stage of the machine is idle. There is no need to wait after the non-heating process is over. This processing mode is defined in the literature [16] as the mixed production mode considering continuous and intermittent processing. In addition, the temperature of the forging billet is an important indicator of forging production. Insufficient temperature of the forging billet may result in the forging billet not being processed at one time during the forging process. The forging billet needs to be tempered before it is returned to the forging press to continue forging. However, few scholars have integrated these factors into forging scheduling. It can be seen that the scheduling of the forging production line is complex and multi-constrained.

To the best of our knowledge, this study is the first attempt to propose and solve the energy-efficient hybrid flow shop scheduling problem with forging tempering (EEHFSP-FT). Based on the characteristics of forging production and a series of complex constraints, a new mathematical model for EEHFSP-FT was constructed. The energy-saving strategy of heat treatment with residual forging heat was used to address residual energy issues. Furthermore, encoding rules, decoding rules, and search strategies suitable for EEHFSP-FT were developed and applied to multi-objective evolutionary algorithms. The aim of this study is to solve the proposed scheduling problem and implement energy-efficient production in a real forging factory.

The rest of this paper is organized as follows: Section 2 describes the EEHFSP-FT and the scheduling model; in Section 3, the encoding and decoding rules and the search strategies are described in detail; in Section 4, a real flow shop case is presented and numerical results are provided; and Section 5 presents the conclusions.

II. LITERATURE REVIEW
A. SCHEDULING METHOD OF FORGING PRODUCTION
Existing studies related to energy-saving scheduling in forging production have mainly focused on furnace heating scheduling and charging scheduling. Regarding furnace heating scheduling, Jiang et al. [10] proposed a model aimed at minimizing the furnace capacity difference. The optimization considered the effect of forging size and shape on heating time. Based on Jiang’s study, He et al. [11] established a multi-objective furnace charging model with minimum capacity difference and minimum waiting time to tackle the continuous heating furnace charging problem. For charging scheduling, some scholars have regarded it as an infinite capacity charging model [12], while others considered it as a simple combinatorial optimization problem [13], [14] without taking into account the size of the forging billets or the capacity of the heating furnace. Tong et al. [15] optimized the charging plan by distinguishing between cold-work and hot-work scheduling. Liu et al. [16] optimized the charging plan based on a mixed production mode of intermittent machining and continuous processing with variable parameters. In summary, it can be seen that most studies have focused on single-machine scheduling of heating furnaces. However, there are only a few studies on scheduling problem of multi-machine collaboration in the forging production, and the steps of forging tempering have not been studied in depth in scheduling problems.

B. MODEL OF MULTI-MACHINE COLLABORATIVE
With respect to the studies on scheduling models related to multi-machine collaboration in the flow shop, Yu et al. [17] considered unrelated machines and machine eligibility constraint in a hybrid flow shop. Meng et al. [18] studied the energy-saving scheduling problem of hybrid flow shop with unrelated parallel machines. Ying et al. [19] evaluated various Mixed Integer Programming (MIP) formulations for solving single-machine and parallel-machine scheduling problems. In addition, some scholars have proposed irregular machine arrangements [20], branched-line operations involving reworking [21], and no-idle flow shop configuration [22]. However, such scheduling models are not suitable for forging production. In forging production, in order to prevent losing the heat energy of forging billets, forging pressing needs to be performed immediately after heating has been completed. However, in the subsequent stage, it should be decided whether the processing needs to be suspended according to the forging billet temperature. If the temperature of the forging billet does not meet the requirements of the process, the forging billet will be tempered. Consequently, few studies focus on the process of forging tempering in the construction of mathematical models, and researches on temperature constraints are still limited.

C. MODEL OF ENERGY-EFFICIENT SCHEDULING
Since the energy-efficient scheduling (EES) framework was first proposed, it has become a hot spot in the field of scheduling [23]. Many scholars have studied the EES flow shop model. In particular, Zhou et al. [24] proposed an energy-efficient model of hybrid flow shop to solve interval scheduling problem. Liu et al. [25] proposed a new mixed integer programming model for energy-efficient integration of process planning and scheduling. Wang et al. [26] studied an energy efficient model of two-stage hybrid flow shop and applied it to the scheduling method of glass production. Other scholars have considered multiprocessor collaboration [27], composite recycling [28], and energy saving...
strategies [29]–[31] in energy-consumption scheduling models. In most studies, the processing mode of flow shop scheduling problems is cold-work processing, and the effect of workpiece temperature is not taken into consideration in the scheduling model. However, in forging production, the forging billet temperature is an important factor in the production process, and the tempering process triggered by temperature constraints may cause extra energy consumption for transportation equipment and heating furnaces. Therefore, it is necessary to investigate an EEHFSP-FT model considering forging billet temperature. At the same time, energy-saving strategies related to temperature factors in forging production should also be further studied.

D. MOTIVATIONS

According to the above review, there is limited research on energy-efficient scheduling of multi-machine collaboration in forging production. The existing scheduling models cannot fully meet the needs of forging production in a hybrid flow shop. While the temperature factor is indispensable in forging production, current studies on flow shop related to the forging process do not consider the effect of forging billet temperature change on scheduling. With regard to energy-saving strategies, the application of heat treatment with residual forging heat in flow shop scheduling has not been investigated yet. In view of these inadequacies, this paper addresses the energy-efficient hybrid flow shop scheduling problem with forging tempering (EEHFSP-FT), and studies the model and energy-saving strategies of EEHFSP-FT.

III. PROBLEM DESCRIPTION

A. NOMENCLATURE AND ASSUMPTIONS

The detailed nomenclature used in this paper is given in the Appendix at the end of the manuscript. The assumptions of EEHFSP-FT are as follows:

1. Multiple jobs can be loaded into the furnace at the same time.
2. Jobs with the same material have the same cooling rate.
3. All machines and transport robots cannot fail.
4. When the forging billet cools, the ambient temperature is 15°C, and the temperature calculation does not take forging losses into consideration.

B. DESCRIPTION OF EEHFSP-FT

EEHFSP-FT can be described as follows: According to Fig. 1, there are \( N \) jobs processed in \( S \) serial stages. Each stage contains \( Ms \) parallel machines. Jobs can be allocated to any parallel machine. During the forging process, if the forging billet temperature does not suit the forging requirements, the tempering operation is performed. The heating furnace can heat more than one job at a time, with the total weight of the jobs not exceeding the capacity of the heating furnace. Machines other than heating furnaces can process only one job at a time. The optimization target of this problem is to minimize both the makespan and the total energy consumption. In this paper, the way to minimize makespan and energy consumption is to find the optimal processing sequence for all jobs and select the most suitable machine for each job.

The requirements related to the forging process are as follows:

1. The heating of forgings must meet the heating and heat preservation time of their materials, and not exceed the maximum allowable holding time of forgings at the initial forging temperature.
2. Forgings cannot be discharged from the heating furnace before forging, and can only be kept in the furnace for heat preservation.
3. The total mass of the forgings cannot exceed the maximum storage capacity of the furnace.
4. In the forging process, the forging temperature must be maintained above the malleable temperature. If the forging temperature is insufficient, the job will be transported to the tempering furnace to be tempered.

C. SCHEDULING PROCESS DETERMINATION

This study focuses on energy-efficient scheduling in forging production. Based on EEHFPP-FT, the whole forging process is categorized into three parts: heating, forging, and heat...
The key issues of this study will be explained based on Fig. 2.

In the heating stage, before the forging billet is charged into the heating furnace, it is necessary to determine whether the capacity of the selected heating furnace is sufficient. If the capacity is sufficient, the forging billet can be charged into the heating furnace; otherwise, it is necessary to wait until the heating furnace can accommodate the forging billet. After the forging billet has been heated to the initial forging temperature, the maximum heat preservation time cannot exceed the allowable heat preservation time of the forging billet. To ensure the holding time limit, the forging billet is transferred out of the furnace when the forging press selected for this forging billet is idle.

In the forging stage, if the forging billet temperature during the forging process is lower than the final forging temperature from the beginning to the end of forging, it needs to be transported to the tempering furnace to be tempered, as shown in Job 2 in Fig. 3 (a). If the forging billet temperature is always higher than the final forging temperature during forging, there is no need to perform tempering, as shown in Job 1 in Fig. 3 (a).

In the heat treatment stage, a traditional heat treatment method is employed: tempering after cooling and quenching after tempering. In order to reduce the energy loss of the forging during cooling, the energy-saving strategy of heat treatment with residual forging heat is adopted. First, the forging billet temperature after forging is compared with the temperature required for quenching. If the forging billet temperature is higher than the required quenching temperature, the forging billet is cooled, as shown in Job 3 in Fig. 3 (b). If the forging billet temperature is lower than the required quenching temperature, the forging billet is tempered, as shown in Job 4 in Fig. 3 (b).

**IV. MATHEMATICAL MODEL FOR EEHFSP-FT**

**A. MAKESPAN MODEL**

The total completion time of a batch of production tasks is measured by the makespan, which represents the time spent in the production tasks from the start of the first job until the end of the last job. Since the processing sequence of the job is uncertain, the makespan can be calculated as the maximum value in the array with the completion times of each job:

\[ T_{\text{max}} = \max \left\{ T_{\text{end} S, m, 1}, T_{\text{end} S, m, 2}, \ldots, T_{\text{end} S, m, N} \right\}, \quad m \in M^S \]  

1) **START PROCESSING TIME MODEL**

Before each processing task begins, the completion time of a task on the machine, the end time of a process in a job, and the transport time between processes need to be taken into consideration. The start processing time of the production

![FIGURE 2. The scheduling process of EEHFP-FT.](image)

![FIGURE 3. Relationship between forging temperature and tempering operation: (a) Forging stage; (b) Quenching stage.](image)
task can be calculated as follows:

\[
\sum_{n=1}^{N} x_{n,1} \cdot T_{1,m,n}^{\text{start}} = 0, \\
m = 1, 2, \ldots, m^1
\]

(2)

\[
\sum_{n=1}^{N} x_{n,1} \cdot T_{s,m,n}^{\text{start}} = \sum_{n=1}^{N} x_{n,1} \cdot (T_{s-1,m',n}^{\text{start}} + T_{s}^{\text{tr}}), \\
s = 2, 3, \ldots, S, m = 1, 2, \ldots, m^s, m' = 1, 2, \ldots, m^{s-1}
\]

(3)

Equation (2) gives the start processing time of the first stage of the job sequenced in the first position. Equation (3) gives the start processing time of stage \(s\) (\(s \geq 2\)) of the job sequenced in the first position. Equation (4) gives the start processing time of stage \(s\) (\(s \geq 2\)) of the job sequenced in the position \(l\) (\(l \geq 2\)).

The relevant constraints in the above calculation process can be expressed as follows:

\[
\sum_{s=1}^{S} \sum_{m=1}^{M^s} \sum_{n=1}^{N} z_{n,s,m} = 1, \quad \forall n
\]

(5)

\[
\sum_{n=1}^{N} x_{n,l} = 1, \quad \forall l
\]

(6)

\[
\sum_{l=1}^{N} x_{n,l} = 1, \quad \forall n
\]

(7)

\[
\sum_{n=1}^{N} x_{n,l+1} \cdot T_{s,m,n}^{\text{start}} \geq \sum_{n'=1}^{N} x_{n',l} \cdot T_{s,m,n'}^{\text{start}}, \quad \forall s, m, l
\]

(8)

\[
\sum_{l=1}^{N} x_{n,l} \cdot T_{s,m,n}^{\text{start}} \geq \sum_{n=1}^{N} x_{n,l} \cdot T_{s-1,m',n}^{\text{start}} + T_{s}^{\text{do}} + T_{s}^{\text{tr}}
\]

(9)

\(\forall s, m, m', l\),

\(x_{n,l} \in \{0, 1\}, \quad \forall n, l\)

(10)

\(z_{n,s,m} \in \{0, 1\}, \quad \forall l, s, m\)

(11)

Constraint (5) ensures that each job is assigned to only one machine in each stage. Constraint (6) ensures that each position of the job sequence can be occupied by one job at most. Constraint (7) ensures that each job can be assigned to only one position. Constraint (8) implies that the previous job is processed prior to the post-order jobs on the same machine. Constraint (9) implies that a job can be processed in the next stage only after it has been completed on the current machine. Constraints (10) and (11) define the value range of \(x_{n,l}\) and \(z_{n,s,m}\), respectively.

2) HEATING STAGE MODEL

In EEHFSP-FT, each heating furnace can accommodate several forging billets simultaneously, but it is ensured that its maximum capacity will not be exceeded. Therefore, the start processing time of each job is not only limited by the job sequence, but also by the capacity of the heating furnace. The model of the heating stage is as follows:

\[
\sum_{n=1}^{N} x_{n,l} \cdot T_{1,m,n}^{\text{start}} = \sum_{n=1}^{N} x_{n,l} \cdot T_{1,m,n}^{G}, \\
l = 1, 2, \ldots, N, \quad \tau = T_{1,m,n}
\]

(12)

\[
\sum_{n=1}^{N} x_{n,l} \cdot T_{s,m,n}^{\text{start}} = \sum_{n=1}^{N} x_{n,l} \cdot T_{s,m,n}^{G}, \\
l = 1, 2, \ldots, N, \quad \tau = T_{s,m,n}
\]

(13)

\[
y_{n}^{\tau} = \begin{cases} 1, & G_{m}^{-1} + G_{n} \geq G_{m}^{\text{max}}, \forall m, \tau, m \geq 2 \\ \infty, & G_{m}^{-1} + G_{n} \leq G_{m}^{\text{max}}, \forall m, \tau, m \geq 2 
\end{cases}
\]

(14)

\[
G_{m}^{\tau} = \sum_{n=1}^{N} x_{n,m,n} \cdot G_{n}, \forall \tau, m, G_{m}^{\tau} \leq G_{m}^{\text{max}}, \forall \tau, m
\]

(15)

(16)

Equation (12) gives the start processing time of the heating stage of the job sequenced in position \(l\) (\(l \geq 2\)). If the heating furnace capacity allows this job to be charged in the furnace at the same time as the previous job, the start processing time is the same as that of the previous job; otherwise, the start processing time is the time when the furnace capacity allows the job to be charged. Equation (13) determines the time when the furnace capacity allows the job to be charged. Equation (14) defines the value of \(y_{n}^{\tau}\), which determines whether the job can be charged to the machine at time \(\tau\). Equation (15) gives the total weight of the jobs in a machine at time \(\tau\). Constraint (16) ensures that the total weight of the jobs in a machine at time \(\tau\) does not exceed its maximum allowable capacity.

3) FORGING STAGE MODEL

In EEHFSP-FT, the forging billet temperature is an important parameter in the forging stage. When a job comes out of the furnace, its temperature begins to decrease. If the job has not been forged and pressed when its temperature drops below the final forging temperature, then it needs to be transported to the tempering furnace for tempering. After the job has been tempered, it is transported back to the forging press to continue processing. If the job is processed before its temperature drops below the final forging temperature, it can be directly transported to the next process without tempering.
The model of the forging stage is as follows:

\[ t_{n,\text{back}}^{F} =\sum_{n=1}^{N} x_{n,\text{back}}^{F} \] \( \forall n \) \hspace{1cm} (17)

\[ T_{s,\text{wait}}^{n} = \sum_{n=1}^{N} x_{n,\text{back}}^{F} \cdot T_{s,\text{start}}^{n,m,n} - \sum_{n=1}^{N} x_{n,\text{back}}^{F} \cdot T_{s,\text{end}}^{n,m',n} \] \( \forall s \geq 2, l \) \hspace{1cm} (18)

Equation (17) determines the time required for the job temperature to drop from the initial temperature to the final forging temperature after it has been released from the furnace. Equation (18) determines the time from after the job has been released from the furnace to the beginning of forging. Equation (19) gives the value of \( x_{n,\text{back}}^{F} \), which determines whether the job needs to be tempered. Equation (20) determines the tempering time of the job in the forging stage.

4) HEAT TREATMENT STAGE MODEL

In EEHFSP-FT, due to the adopted energy-saving strategy of heat treatment with residual forging heat, it is necessary to decide whether to perform a tempering or cooling operation based on the forging billet temperature after forging. After forging, if the job temperature is higher than the required quenching temperature, then the job needs to be cooled. On the other hand, if the job temperature after forging is lower than the required quenching temperature, the job needs to be transported to the tempering furnace for tempering. The model of the heat treatment stage is as follows:

\[ T_{n}^{\text{expo}} = \begin{cases} T_{n,\text{start}}^{\text{3,m,n}} - T_{n,\text{end}}^{\text{1,m',n}}, & T_{n,\text{back}}^{F} = 0 \\ T_{n,\text{start}}^{\text{3,m,n}} - T_{n,\text{back}}^{F} + T_{n}^{\text{F.back}}, & T_{n,\text{back}}^{F} = 1 \end{cases} \] \( \forall n \) \hspace{1cm} (21)

\[ H_{n}^{\text{before},Q} = \sum_{n=1}^{N} x_{n}^{Q} \cdot \left( T_{n}^{\text{expo}} \right) \] \( \forall n \) \hspace{1cm} (22)

\[ x_{n}^{Q,\text{back}} = \begin{cases} 1, & H_{n}^{\text{req}} > H_{n}^{\text{before},Q} \\ 0, & H_{n}^{\text{req}} \leq H_{n}^{\text{before},Q} \end{cases} \] \( \forall n \) \hspace{1cm} (23)

\[ T_{n}^{Q,\text{back}} = x_{n}^{Q,\text{back}} \cdot \left( H_{n}^{\text{req}} - H_{n}^{\text{before},Q} \right) + t_{\text{b.tr}} \] \( \forall n \) \hspace{1cm} (24)

\[ x_{n}^{Q,\text{cool}} = \begin{cases} 1, & H_{n}^{\text{req}} < H_{n}^{\text{before},Q} \\ 0, & H_{n}^{\text{req}} \geq H_{n}^{\text{before},Q} \end{cases} \] \( \forall n \) \hspace{1cm} (25)

\[ T_{n}^{Q,\text{cool}} = x_{n}^{Q,\text{cool}} \cdot \left( Z_{n}^{\text{line}} \left( H_{n}^{\text{req}} \right) + t_{\text{c.tr}} \right) \] \( \forall n \) \hspace{1cm} (26)

Equation (21) determines the duration of a job from the furnace to the completion of forging. If the job is tempered in the forging stage, the duration is from the end of tempering to the completion of forging. Equation (22) provides the temperature of the job after forging. Equation (23) gives the value of \( x_{n}^{Q,\text{back}} \), which determines whether the job needs to be tempered before quenching. Equation (24) determines the tempering time of the job after the forging stage. Equation (25) gives the value of \( x_{n}^{Q,\text{cool}} \), which determines whether the job needs to be cooled before quenching. Equation (26) provides the cooling time of the job after the forging stage.

5) COMPLETION TIME MODEL

Based on the above models for each stage, the completion time model can be defined as follows:

\[ T_{s,m,n}^{\text{end}} = T_{s,m,n}^{\text{start}} + T_{s,m,n}^{\text{do}} + \gamma \cdot T_{n}^{Q,\text{back}} + \delta \cdot \left( T_{n}^{Q,\text{back}} + T_{n}^{Q,\text{cool}} \right) \] \( \forall s, m, n \) \hspace{1cm} (27)

\[ \gamma = \begin{cases} 1, & s = 2 \\ 0, & s \neq 2 \end{cases} \] \hfill (28)

\[ \delta = \begin{cases} 1, & s = 3 \\ 0, & s \neq 3 \end{cases} \] \hfill (29)

\[ T_{1,m,n}^{\text{end}} = T_{n}^{\text{heat}} + T_{n}^{\text{preservation}} \] \( \forall m, n \) \hspace{1cm} (30)

Equation (27) gives the completion time of job \( n \) on machine \( m \) in stage \( s \). Equation (28) gives the value of \( \gamma \), which determines whether the current machining task is in the forging stage. Equation (29) gives the value of \( \delta \), which determines whether the current machining task is in the heat treatment stage. Equation (30) gives the processing time of job \( n \) on machine \( m \) in heating stage, \( T_{n}^{\text{heat}} \), \( T_{n}^{\text{preservation}} \) and \( T_{s,m,n}^{\text{do}} (s \geq 2) \) are known information.

In the heating stage, jobs cannot be transported out of the furnace until the selected forging machine is idle, so \( T_{1,m,n}^{\text{end}} \) needs to be corrected as follows:

\[ T_{1,m,n}^{\text{end}} = T_{1,m,n}^{\text{start}} + T_{n}^{\text{tr}} \] \( \forall n \) \hspace{1cm} (31)

Equation (31) gives the corrected completion time of job \( n \) on machine \( m \) in heating stage. Constraint (32) ensures that the heat preservation time of job \( n \) does not exceed its maximum heat preservation time. Otherwise, this solution is marked as invalid.

B. ENERGY CONSUMPTION MODEL

To minimize the total energy consumption, which is an important scheduling goal in EEHFSP-FT, the operating and idle energy consumption of the machine and the transportation energy consumption need to be calculated. As the heating furnace is in a continuous working state, the running time is calculated as the time from the first job that is charged to the last job that is taken out. The energy consumed by the tempering furnace is calculated as an extra consumption. The energy consumption model is as follows:

\[ T_{s,m,n}^{\text{m,ide}} = \sum_{n=1}^{N} x_{n}^{\text{m,ide}} \cdot T_{s,m,n} + \sum_{n=1}^{N} x_{n}^{\text{m,cool}} \cdot T_{s,m,n} \] \( s = 2, 3, \ldots, S, l = 1, 2, \ldots, l - 1 \) \hspace{1cm} (33)
E_{\text{total}} = \sum_{n=1}^{N} \sum_{s=2}^{S} \sum_{m=1}^{m^1} (E_{m}^{\text{run}} \cdot T_{s,m,n}^{d,o} + E_{m}^{\text{idle}} \cdot T_{s,m,n}^{d,i}) \quad (34)
\nonumber
\nonumber
\nonumber
E_{\text{total}}^{fu} = \sum_{m=1}^{m^1} \left( \sum_{n=1}^{N} x_{n,N} \cdot T_{m,n}^{d,e} - \sum_{n=1}^{N} x_{n,1} \cdot T_{m,n}^{d,st} \right) \cdot E_{m}^{f} \quad (35)
\nonumber
\nonumber
\nonumber
E_{\text{total}}^{extra} = \sum_{n=1}^{N} \left[ T_{n}^{f,b} + T_{n}^{Q,back} - (x_{n}^{F,back} + x_{n}^{Q,back}) \cdot (b, tr) \right] \cdot E_{bf} \quad (36)
\nonumber
\nonumber
\nonumber
E_{\text{total}}^{tr} = N \cdot \sum_{s=2}^{S} T_{s}^{tr} \cdot E_{tr}^{f} + \frac{N}{\sum_{n=1}^{N}} \left[ (x_{n}^{F,back} + x_{n}^{Q,back}) \cdot (b, tr) + x_{n}^{Q,cool} \cdot (c, tr) \right] \cdot E_{tr} \quad (37)
\nonumber
\nonumber
\nonumber
E_{\text{total}} = E_{\text{total}}^{machine} + E_{\text{total}}^{fu} + E_{\text{total}}^{extra} + E_{\text{total}}^{tr} \quad (38)
\nonumber
\nonumber
\nonumber
Equation (33) determines the idle time of the machine. Equations (34-38) determine the total energy consumption of the machine, furnace, tempering furnace, transportation, and schedule, respectively.

C. FORGING COOLING CURVE

In EEHFSP-FT, the job temperature is a key factor that determines whether the job needs to be tempered at the forging stage, or whether the job needs to be tempered or cooled before the heat treatment stage. The moment a job comes out of the furnace, it begins to cool. In actual production, a temperature measuring instrument can be used to measure the job temperature. However, in scheduling, the cooling curve of a job needs to be predicted in advance. Based on the material and size of a job, its heat dissipation rate can be calculated. The ambient temperature and heat dissipation rate were set in the De-form software to perform a cooling simulation in order to obtain the cooling curve of the operation.

V. MULTI-OBJECTIVE EVOLUTION ALGORITHM FOR EEHFSP-FT

A. MULTI-OBJECTIVE OPTIMIZATION PROBLEM

In EEHFSP-FT, due to the different production capacities of parallel machines and uncertain tempering factors, the two optimization goals conflict with each other. Multi-objective optimization methods aim to provide solutions that are as close as possible to the Pareto optimal front and uniformly distributed. Such methods have been shown to have good convergence and diversity [32].

Among multi-objective optimization algorithms, evolutionary-based algorithms (EA) are widely used. EAs are random search algorithms that simulate the survival and adaptation process of natural ecosystems. Currently, some excellent evolutionary algorithms have been proposed, such as PESA-II (Pareto Envelope-Based Selection) [33], SPEA2 (Strength Pareto Evolutionary) [34] and NSGA-II (Non-dominated Sorting Genetic) [35]. The main advantages of these algorithms are powerful adaptability and self-organization.

Considering that EAs are widely used to solve various practical problems, this paper uses the three classical multi-objective evolutionary algorithms mentioned above to solve the scheduling problem, and compares their performance.

All EAs include population iteration and evolution. In order to better solve the scheduling problem in this study, encoding and decoding rules suitable for EEHFSP-FT were designed, as well as evolutionary search strategies based on encoding rules. These rules and strategies were used to replace the corresponding parts of the original algorithm.

The optimization process can be described as follows. Firstly, generate a set of solutions according to the coding rules, and use the model to calculate the function values of the two objectives. Secondly, perform iterative optimization through a multi-objective optimization algorithm to find a set of optimal pareto solutions. Finally, decode the optimal solution to obtain the optimal processing sequence for all jobs and select the most suitable machine for each job.

B. ENCODING RULES

In order to better adjust the EA to solve the scheduling problem, encoding rules for EEHFSP-FT are designed. Each code is designed as a matrix with 3 rows and n columns, based on the problems of furnace charging and machine selection in EEHFSP-FT. The first row of the matrix represents the charging sequence, and each element represents the job serial number. The second row represents the machine selected for the job in forging stage, and the third row represents the machine selected for the job in heat treatment stage.

For example, if there are four jobs to be processed in the flow shop, the code shown in Fig. 4 means that the charging sequence is 1→2→3→4. Job 1 is assigned to machine 2 in the forging stage and to machine 5 in the heat treatment stage; job 2 is assigned to machine 3 in the forging stage and to machine 6 in the heat treatment stage; job 3 is assigned to machine 3 in the forging stage and to machine 5 in the heat treatment stage; and job 4 is assigned to machine 4 in the forging stage and to machine 7 in the heat treatment stage.

![FIGURE 4. Encoding rules.](image-url)
C. DECODING RULES

In EEHFSP-FT, there are some jobs that need to be tempered in the forging stage and, after tempering, return to the original forging machine to continue processing. As can be seen in the Gantt chart in Fig. 5(a), this Gantt chart reflects the scheduling of different jobs processed by specific machines. In the Gantt chart, abscissa represents time, ordinate represents equipment, and job \((n,s)\) represents the \(s\)-th processing stage of job \(n\). Since Job 1 needs to be tempered once during the forging process, the selected forging machine will be idle while the job 1 is tempered. To avoid this phenomenon, greedy rules are adopted. These rules can determine whether the subsequent job can be charged first, making effective use of this idle time, which can be seen in the Gantt chart in Fig. 5(b). If the forging processing time of Job 3 is less than the idle time of the forging machine, then the forging stage of Job 3 can be completed within this period of time. Pseudo code of the greedy rules is shown in Procedure 1.

Procedure 1 Greedy Rules

\[
\text{If } l \geq 2 \%
\]

- %l is the position of the job
- Find the idle time period of the machine selected before the start processing time of the job
- Generate the machine idle time matrix \(EI_m\)
- %A \(\in EI_m\), B is the processing time of the job
- If \(B < A\), then
  - Adjust the start processing time of the job and calculate the completion time
- End if

End if

In the heating stage, jobs cannot be transported out of the furnace until the selected forging machine is idle, which induces a large amount of additional holding time. This can be seen in the Gantt chart in Fig. 6(a), Job 2, Job 3 and Job 4 cannot be transported out of the furnace until the selected forging machine is idle. In order to effectively reduce the additional heat preservation time and the occupation time of furnace capacity, a delayed-charging rule is adopted, which is shown in Procedure 2. Fig. 6(b) shows the Gantt chart after the delayed-charging furnace rules and the greedy rules have been applied. Job 2, Job 3, and Job 4 are processed through delayed-charging furnace rules. After that, Job 4 is added to the processing before Job 2 through greedy rules.

Procedure 2 Delayed Charging Rules

\[
\text{If } s \geq 2\%
\]

- %s is the current processing stage of the job
- Find the start processing time \(T_{\text{start forging}}\) of the job in the forging stage
- Modify the furnace charging time to \(T_{\text{start forging}} - T_{\text{tr forging}} - T_{\text{do}}\)

End if

D. ITERATIVE SEARCH STRATEGY

Due to the special coding rules in EEHFSP-FT, the search process of EAs needs to be adjusted according to the encoding rules.

Based on the two key issues of furnace charging sequence and machine selection in scheduling, two crossover strategies were designed separately. Fig. 7(a) is a crossover strategy based on a furnace charging sequence, which randomly selects \(n\) jobs, keeps the positions of the selected jobs in the two parents, and crosses the remaining positions in order to generate two new children. Fig. 7(b) is a crossover strategy based on machine selection, which randomly selects \(n\) jobs, and swaps the selected machine for the selected jobs in the two parents to produce two children.

Similarly, two mutation strategies were designed for iterative search. Fig. 8(a) is a mutation strategy based on a furnace charging sequence, which randomly selects \(n\) (\(n \geq 2\)) jobs from a parent, randomly exchanges the sequence of the selected jobs, and ensures that the originally selected machine remains unchanged to generate one new child. Fig. 8(b) is a mutation strategy based on machine selection, which randomly selects \(n\) jobs and their selected machines, and replaces the selected machines with another optional machine to generate a new child.

In the evolution of each generation of the population, we will combine the four crossover and mutation strategies...
proposed above to generate new offspring, calculate the fitness value of each individual through the model, eliminate individuals with low fitness values, and select excellent individuals to join the population.

VI. CASE STUDY
A. CASE DESCRIPTION
In order to prove the effectiveness of the model proposed in this study, a case of a flow shop of a Chinese intelligent forging center was used. The layout of the flow shop is illustrated in Fig. 9.

As can be seen, the processes in the flow shop can be divided into the following sub-stages, where the machines used in each stage and the corresponding functions are as follows:

- Heating stage: a capacity ring furnace for heating forging billets
- Forging stage: two 1600 t screw presses used for forging processes
- Tempering stage: two box type heating furnaces for tempering forging billets
- Heat treatment stage: Three quenching machines

The working process is as follows: Robot R1 takes the forgings out of the feeding machine and puts them in the circular heating furnace, which can hold multiple forgings for continuous heating at the same time in turn. When the heating has been completed, robot R1 takes out the forgings and places them into the screw press for forging processes.

If the forging temperature during forging is insufficient, forgings will be taken out and sent back to the tempering furnace to be reheated, and then will be transported into the forging...
press by robots R2 and R3. After forging, the forgings are placed on the logistics roller table, which is charged into the heat treatment furnace for quenching, and then tempering heat preservation by robot R4. Although the performance of the two presses is different, the forging processes of all jobs can be completed.

The parameters used in the flow shop during processing, such as equipment energy consumption and transfer time, are given in Tables 1 and 2. In Table 2, S1 is the transfer time from the heating furnace to the forging press, S2 is the transfer time from the forging press to the tempering furnace, S3 is the transfer time from the forging press to quenching furnace, S4 is the transfer time for tempering before quenching, and S5 is the transfer time for cooling before quenching.

**B. EXPERIMENT SETTINGS**

In order to better solve EEHFSP-FT, this paper uses three evolutionary algorithms to solve the problem. In order to better analyze the adaptability of these algorithms in solving EEHFSP-FT, the quality of the algorithms’ direct solutions was compared through cases of different scales.

The EEHFSP-FT proposed in this paper is a novel scheduling approach in the field of forging scheduling. Its model is based on complex working conditions and multi-resource constraints. At present, there are no universal instances for experimentation. Therefore, the above case will be used as the background and some production tasks in several actual orders will be selected as instances. There are 8 different types of forging in one order. The related processing information and parameters are provided in Tables 3 and 4, respectively, and the temperature cooling curve of the forging billet is shown in Fig. 10 which represents the temperature change for all jobs. In order to simplify the calculation and make the scheduling results clear, the time of the heating stage was appropriately reduced.
In order to verify the performance of the proposed model combined with the EAs used to solve the problem, four orders from case factory were used as instances as follows: 8 jobs of 4 types; 8 jobs of 8 types; 16 jobs of 4 types; and 16 jobs of 8 types. For order 8 jobs of 4 types means 8 jobs are composed of 4 types of forgings. The specific parameters of these instances can be found in the Supplemental Material.

In order to compare the performance of the three algorithms, the following indicators obtained from [36] were adopted:

\( NS \): The number of non-dominated solutions in the Pareto archive, labelled \(|P|\).

\( CMetric \): Coverage metric; its calculation method is shown in (39), where \( C(P_1, P_2) \) represents the probability that the solution in \( P_2 \) is dominated by that in \( P_1 \). The larger the value of \( C(P_1, P_2) \), the more solutions are dominated in \( P_2 \).

\[
C(P_1, P_2) = \left| \left\{ b \in P_2 \mid \exists a \in P_1, a \succ b \right\} \right| \frac{1}{|P_2|} 
\] (39)

The three classical EAs were employed in the proposed model to solve the scheduling problem, and determine the most suitable algorithm. In PESA-II, SPEA2, and NSGA-II, each instance was run 10 times and the optimal Pareto solution set was recorded. Then, the above-described method was used to obtain the relationship between the three algorithms through the calculation of a coverage metric. The nonparametric Mann-Whitney test at 95% confidence level was used for numerical analysis.

### C. PARAMETER SETTINGS

The main parameters of the three EAs selected in this study were population size, number of iterations, crossover rate, and mutation rate. It is well known that the population size and number of iterations are positively correlated with the solution result, and negatively correlated with the algorithm solution time. Considering the solution quality and the solution time, the population was set to 100 and the number of iterations to 100. The combination of crossover rate and mutation rate resulted in four groups per algorithm (Table 5).

In order to investigate the effect of different parameters on algorithm performance. All 12 groups on 4 instances were run 10 times and the optimal Pareto solution set \( P_{i,j,k} \) was recorded separately. All combinations of \( P_{i,j,k} \) were examined to find the optimal Pareto solution set \( P_{best} \). Then, the coverage \( C(P_{i,j,k}, P_{best}) \) of each combination was calculated as the response value \( R_{value} \), according to

\[
R_{value} = \frac{\sum_{i=1}^{4} \sum_{j=1}^{10} C(P_{i,j,k}, P_{best})}{60}. 
\]

Fig. 11 shows the impact of the four parameter groups on each algorithm. Based on the test results, the combination chosen in PESA-II was \( \{C^r: 0.7, M^r: 0.3\} \), in SPEA2 was \( \{C^r: 0.7, M^r: 0.3\} \), and in NSGA-II was \( \{C^r: 0.7, M^r: 0.4\} \)

### TABLE 3. Processing times for the 8 different types of forgings in each stage.

| Type | Heating time (s) | Heat preservation (s) | Max heat preservation (s) | Forging time (s) | Heat treatment time (s) |
|------|-----------------|----------------------|---------------------------|-----------------|------------------------|
| 1    | 1800            | 1500                 | 2400                      | 1500            | 2340                   |
| 2    | 1520            | 1120                 | 1600                      | 1080            | 1680                   |
| 3    | 2600            | 2200                 | 2600                      | 2280            | 3420                   |
| 4    | 2440            | 2000                 | 2400                      | 2160            | 2880                   |
| 5    | 1600            | 1040                 | 1600                      | 1200            | 1800                   |
| 6    | 1400            | 1000                 | 1400                      | 1140            | 1740                   |
| 7    | 1650            | 1050                 | 1500                      | 1260            | 3000                   |
| 8    | 2650            | 2270                 | 2800                      | 2400            | 3600                   |

### TABLE 4. Parameters for 8 different types of forging.

| Type | Initial forging temperature (°C) | Final forging temperature (°C) | Diameter (m) | Length (m) | Weight (kg) |
|------|---------------------------------|--------------------------------|--------------|------------|-------------|
| 1    | 1200                            | 900                            | 0.22         | 0.25       | 43          |
| 2    | 1200                            | 900                            | 0.18         | 0.35       | 41          |
| 3    | 1200                            | 900                            | 0.18         | 0.48       | 55          |
| 4    | 1200                            | 900                            | 0.20         | 0.40       | 58          |
| 5    | 1200                            | 900                            | 0.18         | 0.33       | 38          |
| 6    | 1200                            | 900                            | 0.20         | 0.28       | 40          |
| 7    | 1200                            | 900                            | 0.20         | 0.30       | 42          |
| 8    | 1200                            | 900                            | 0.20         | 0.40       | 58          |

### TABLE 5. Combinations of crossover rate and mutation rate for each algorithm.

| Group | PESA-II | SPEA2 | NSGA-II |
|-------|---------|-------|---------|
|       | \( C^r \) | \( M^r \) | \( C^r \) | \( M^r \) | \( C^r \) | \( M^r \) |
| 1     | 0.8     | 0.3   | 0.8     | 0.3   | 0.8     | 0.3   |
| 2     | 0.8     | 0.4   | 0.8     | 0.4   | 0.8     | 0.4   |
| 3     | 0.7     | 0.3   | 0.7     | 0.3   | 0.7     | 0.3   |
| 4     | 0.7     | 0.4   | 0.7     | 0.4   | 0.7     | 0.4   |
TABLE 6. Comparison of coverage between the three algorithms.

| Case   | C (NSGA-II, SPEA2) | C (SPEA2, NSGA-II) | p-value | C (NSGA-II, SPEA2) | C (NSGA-II, SPEA2) | p-value | C (SPEA2, NSGA-II) | C (SPEA2, NSGA-II) | p-value |
|--------|-------------------|-------------------|---------|-------------------|-------------------|---------|-------------------|-------------------|---------|
| J8_T4  | 0.4295            | 0.4013            | 0.144   | 0.5075            | 0.2261            | 0.001   | 0.6191            | 0.2827            | 0.001   |
| J8_T8  | 0.4371            | 0.3900            | 0.182   | 0.3943            | 0.3855            | 0.162   | 0.5362            | 0.2748            | 0.005   |
| J15_T4 | 0.4960            | 0.2738            | 0.014   | 0.5761            | 0.1635            | 0.000   | 0.6561            | 0.2741            | 0.001   |
| J15_T8 | 0.5280            | 0.1704            | 0.003   | 0.4432            | 0.2069            | 0.005   | 0.5678            | 0.3293            | 0.005   |

**FIGURE 12.** Optimal solution set running 10 times for different scale cases on each algorithm.

**D. RESULT ANALYSIS**

A comparison of coverage between the three algorithms is given in Table 6. In the case of 8 jobs of 4 types and the case 8 jobs of 8 types, the performances of NSGA-II and SPEA2 were similar, the single solution quality of NSGA-II was slightly better than that of SPEA2, and both were better than PSEA-II. In the case of 16 jobs of 4 types and the case of 16 jobs of 8 types, NSGA-II was superior to the other two algorithms, and the p-value was less than 0.05. Fig. 12 compares the optimal solution set running 10 times for different scale cases on each algorithm. It was found that when the solved case had 8 jobs, the quality of the optimal solution set of the three algorithms was similar, with SPEA2 being slightly better than the other two algorithms. When the solved case had 16 jobs, the quality of the NSGA-II optimal solution set was superior to that of the other two algorithms. Under the condition of all algorithm parameters unchanged, as the number of processing jobs increased or the scale increased, it was found that the optimization results of NSGA-II were better than the other two algorithms. In practical applications, the appropriate algorithm can be selected based on the scale of the problem. In summary, when solving realistic problems, multiple methods can be adopted to seek the optimal solution.

Taking the case of 8 jobs of 8 types as an example, the scheduling result shown in Fig. 13 was selected from the solution set and expressed in the form of a Gantt chart. Each processing stage of each job and the machine selected in this stage were associated with the two numbers in brackets (n1, n2), where n1 is the processing stage of the job and n2 is the machine number in the processing stage. The completion time of this scheduling result was 293 min, and the energy consumption was 646 kW·h. Compared to the result without scheduling (completion time of 351 min and energy consumption of 741 kW·h), it can be seen that 16.52% of completion time and 12.82% of energy were saved. In the cases of other scales, the completion time and energy consumption were also effectively reduced.
The explanations of nomenclature for the proposed model was determined. Numerical calculations and analysis, the algorithm suitable time and energy consumption. Eventually, through adequate paper was able to effectively reduce both the completion original production scheme, the approach proposed in this paper was able to effectively reduce both the completion and energy consumption. Eventually, through adequate numerical calculations and analysis, the algorithm suitable for the proposed model was determined.

**APPENDIX**

The explanations of nomenclature

\[ N \] A set of \( N \) jobs, \( n = \{1, 2, \ldots, N\} \)

\[ S \] A set of \( S \) stages, \( s = \{1, 2, \ldots, S\} \)

\[ M^s \] A set of \( M^s \) machines in \( s\)-th stage, \( m = \{1, 2, \ldots, M^s\} \)

\[ l \] Processing sequence, \( l = \{1, 2, \ldots, N\} \)

\[ T_{\text{max}}, T_{\text{start}} \] Schedule makespan

\[ T_{\text{start}}, T_{\text{end}}, T_{\text{before}}, T_{\text{after}} \] Processing start time of job \( n \) at machine \( m \) in stage \( s \)

\[ T_{\text{before}}, T_{\text{after}} \] Completion time of job \( n \) at machine \( m \) in stage \( s \)

\[ T_{\text{before}} \] Transportation time before the \( s\)-th stage

\[ T_{\text{after}} \] Processing time of job \( n \) at machine \( m \) in stage \( s \)

\[ x_{n,l} \] Binary variable for job assignment. If job \( n \) assigned to position \( l \), then \( x_{n,l} = 1 \); otherwise, \( x_{n,l} = 0 \).

\[ z_{n,s,m} \] Binary variable for job assignment. If job \( n \) assigned to machine \( m \) at stage \( s \), then \( z_{n,s,m} = 1 \); otherwise, \( z_{n,s,m} = 0 \).

\[ y^r_n \] Binary variable for job loading. If job \( n \) cannot be loaded into machine \( m \) at time \( r \), then \( y^r_n = 1 \); otherwise, \( y^r_n = \infty \).

\[ T^G_{m,n} \] Time point when the furnace capacity allows the job \( n \) to be charged

\[ T^H_{n} \] Heating time of job \( n \)

\[ T^T_{n} \] Heat preservation time of job \( n \)

\[ T^P_{n} \] Maximum heat preservation time of job \( n \)

\[ G^x_{m} \] Total weight of the jobs in machine \( m \) at time \( r \)

\[ x^r_{n,m} \] Binary variable for weight calculation. If job \( n \) is assigned to machine \( m \) at time \( r \), then \( x^r_{n,m} = 1 \); otherwise, \( x^r_{n,m} = 0 \).

\[ G^\text{max}_{m} \] Maximum allowable weight of machine \( m \)

\[ t^\text{back}_n \] Time from the initial to the final forging temperature of job \( n \)

\[ Z^\text{line}_n \] Cooling curve function of job \( n \). It indicates the relationship between time and temperature after job \( n \) leaves the furnace

\[ H_{\text{init}} \] Initial forging temperature of job \( n \)

\[ H_{\text{final}} \] Final forging temperature of job \( n \)

\[ t^\text{wait}_n \] Wait time of job \( n \)

\[ x^F_{n,\text{back}} \] Binary variable for job tempering. If job \( n \) needs to be tempered during the forging process, then \( x^F_{n,\text{back}} = 1 \); otherwise, \( x^F_{n,\text{back}} = 0 \).

\[ T^F_{n,\text{back}} \] Tempering time of job \( n \) during the forging process

**FIGURE 13.** Gantt chart of the "8 jobs of 8 types" case.
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QIANG CHENG received the Ph.D. degree from the Industrial Engineering Department, Huazhong University of Science and Technology, Wuhan, China, in 2007.

He is currently the Dean of the Department of Mechanical Manufacturing and Automation, College of Mechanical Engineering and Applied Electronics Technology, Beijing University of Technology. His research interests include accuracy design of machine tool, reliability design, production line planning, intelligent scheduling, and so on.

CHENFEI LIU received the B.S. degree in mechanical engineering from the Beijing University of Technology, in 2018, where he is currently pursuing the master’s degree in mechanical engineering.

His research interests include intelligent scheduling, optimization algorithm, and production line planning.

ZHIFENG LIU received the Ph.D. degree in mechanical design from the College of Mechanical Engineering and Automation, Northeastern University, in 2001.

He is currently an Associate Dean of the College of Mechanical Engineering and Applied Electronics Technology, Beijing University of Technology, and the Director of the Beijing Key Laboratory of Advanced Manufacturing Technology. His research interests include digital design and manufacturing, manufacturing services, and manufacturing system information.

WEI ZHANG was born in Jingmen, Hubei, in December 1987. He received the bachelor’s degree, in 2011. Since 2016, he has been working with the Technical Room of Equipment Engineering Center, Guizhou Anda Aviation Forging Company Ltd. He has two life mottos. The first one is to do the truth. The second one is that luck always favors those who are prepared.

JUNJIE PAN is currently with the Technical Room of Equipment Engineering Center, Guizhou Anda Aviation Forging Company Ltd. He is mainly responsible for the research of forging technology.

HONGYAN CHU received the bachelor’s and master’s degrees in engineering from Jilin University, in 1994 and 1997, respectively, and the Ph.D. degree in mechanical design and theory from the Beijing University of Technology, in 2013.

She is currently the Dean of the Department of Mechanical Manufacturing and Automation, College of Mechanical Engineering and Applied Electronics Technology, Beijing University of Technology. Her research interests include intelligent manufacturing research, process simulation, and optimization research of hot machining process and digital design.

JUNJIE PAN is currently with the Technical Room of Equipment Engineering Center, Guizhou Anda Aviation Forging Company Ltd. He is mainly responsible for the research of forging technology.

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