ON THE BROKEN GAUGE, CONFORMAL AND DISCRETE
SYMMETRIES IN PARTICLE PHYSICS

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Summary.— Relationships between gauge, conformal and discrete symmetries in particle physics are analysed. We study also the effect of the electroweak mixing on the cancellation of SU(2) anomalous actions. It is shown that the relation $\theta_W = 2(\theta_{12} + \theta_{23} + \theta_{13})$ between the Weinberg angle and the Cabibbo-Kobayashi-Maskawa angles should be satisfied and this effect is completely defined by the mixing of Dirac fermions. We compare two mechanisms of the spontaneous breaking of gauge symmetry, discuss the renormalizability of theories, and argue for the existence of the Majorana fermions necessary to remove the SU(2) anomalous action. The fate of the majoron and the spontaneously broken lepton number is discussed. We also show the compatibility of the boson and fermion mixings with Dyson-Schwinger equations.

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"393. ... Nec Geometrica argumenta quidquam probant in mea Theoria pro divisibilitate ultra eum limitem; posteaquam enim deventum fuerit ad intervalla minora, quam sit distantia duorum punctorum, sectiones ulteriores secabunt intervalla ipsa vacua, non materiam."

P. Rogerio Josepho Boscovich Societatis Jesu, Theoria Philosophiae Naturalis, Pars Tertia, Venetiis MDCCLXIII.
I. INTRODUCTION

In this paper we want to reanalyse gauge theories in such a way as to try to understand relations between gauge, conformal and discrete symmetry breaking observed in particle physics. Two questions arise very naturally: (1) Is it a coincidence that discrete symmetry breaking happens together with the SU(2) gauge symmetry breaking? (2) Can we explain the breaking of essentially spacetime symmetries, such as conformal and discrete symmetries in particle physics, by some characteristic of the physical spacetime and relate it to the gauge symmetry breaking? We try to answer to these questions, studying nonperturbative anomalies, conformal symmetry and bootstrap equations.

In the analysis of gauge quantum field theories (QFT) anomalies play a crucial role setting the consistency requirements on the theory. Calculating quantum fermion loops with pseudoscalar or axial-vector couplings, one can account for all perturbative anomalies. The axial anomaly (Adler-Bell-Jackiw anomaly)\[1\] was first discovered within the effective chiral theories and it could account for the nonvanishing $\pi_0 \rightarrow 2\gamma$ amplitude. On the contrary, the Adler-Bardeen anomaly \[2\], found as an anomalous part of the Ward-Takahashi identities, should vanish in order to preserve the renormalizability of the theory. In the standard electroweak model\[3\], the anomalous part comes from the axial-vector coupling and it was shown that higher loops neither affected nor altered the form of the anomaly. The anomalous terms could be removed by satisfying the constraint on the sum of hypercharge assignments of weak doublets in the model.

In this paper, however, we want to analyze essentially nonperturbative anomalies arising from quantum fluctuations in the generating functional. In Sec. 2 we try to understand the source of this type of anomalies and how to remove the unwanted terms. In Sec. 3 we discuss the role of zero-mode fermion eigenstates in the cancellation of anomalous actions, resulting in new unexpected results peculiar to the SU(2) symmetry. Section 4 is devoted to the study of the quantization and renormalizability of the Higgs- and non-Higgs-type spontaneously broken gauge theories. In Section 5 we study trace anomaly, whereas in Section 6 we resolve the problem of the spontaneously broken lepton number accompanied with the existence or nonexistence of the majoron. Finally, the electroweak mixing is introduced nonperturbatively in the generating functional for the electroweak theory (Sec. 7) and the compatibility of the angle relation with Dyson-Schwinger equations is proved. The concluding section
presents a comparison of the phenomenology with theoretical constraints and predictions, and some general remarks.

II. SU(2) GLOBAL ANOMALY

Perturbative anomalies of the Green functions test the local gauge symmetry of the QFT, whereas nonperturbative anomalies test the global gauge symmetry through actions and generating functionals. Since the central point in the gauge QFT is always the problem of symmetry breaking, the possible impact of nonperturbative anomalies on the model building and symmetry-breaking mechanisms might be very important.

The first nonperturbative study of anomalies was performed by Fujikawa [4]. He also realized the necessity of referring to the index theorems [5] of the Weyl and Dirac operators. Thus one is then immediately faced with the Wess-Zumino actions introduced earlier [6] into the study of anomalies, and with the role of homotopy pointed out by Witten [7].

We now repeat some relevant results. To apply mathematical theorems, considerations are carried out in the Euclidean space with possible compactification to a sphere or a disc. One of the central achievements in the physical cognition is the following statement: the index of the Dirac operator in D+2 dimensions is equal to the index of the Weyl operator in D dimensions [8, 9]. The gauge-orbit parameters perfectly fit into the scheme as two additional dimensions. If the difference between the number of left- and right-handed zero modes does not vanish, it will cause the nonvanishing of the Wess-Zumino term. However, the meaning of this anomalous term is physically clear: if the global gauge symmetry is broken, this term cannot vanish. We can now raise a few questions: (a) What is the relationship between the anomalous terms of the fermion and boson operators? (b) Is there any problem with unitarity when the anomalous actions appear in the theory? (c) What is the role of homotopy in the study of the nonperturbative anomaly?

Similarly to the construction of the fermion Lagrangian density in the six-dimensional space, one can build the gauge boson density which is cubic in the field strength [8, 10]. The calculus of quantum fluctuations in the fermion and boson determinants then reduces to completely geometrical [8] calculations using Cartan’s forms, cohomology, and Stokes’ theorem. However, we should be very cautious about the relative sign between two anomalous actions because the integration over the Grassmannian (fermionic, anticommutative)
variables yields an additional negative sign.

Gauge non-invariant Wess-Zumino actions are nonhermitian structures that should disappear because they spoil the unitarity at the quantum-loop level \([9, 10]\). The most natural way of solving this problem is to choose the same gauge fields in the boson and fermion part of the action. Then the cancellation of the Wess-Zumino actions will be automatic owing to the additional sign obtained after the Grassmannian integration:

\[
\text{ind}(F^3) = -\text{ind}(D_6) = -\text{ind}(\partial_4 + A_1 \frac{1}{2}(1 - \gamma_5)),
\]

\[
\text{ind}(D_6) = \frac{i^3}{(2\pi)^3} \int_{S^5} [\omega_5(A^g, F^g) - \omega_5(A, F)],
\]

\[
\omega_5(A^g, F^g) = \omega_5(A, F) + d\alpha_4 + \frac{1}{10} \text{Tr}(g^{-1} dg)^5,
\]

where \(F = dA + A^2\), \(g \in SU(2)\) on the boundary of \(S^5\).

The most detailed information about the topology comes from the homotopy groups. For considerations in the gauge QFT, we need to know the homotopy groups and their sequences of unitary symmetries in the four- and higher-dimensional spaces. Let us recall, for example, the following homotopy sequence\([11]\):

\[
\pi_5(SU(3)) \xrightarrow{j} \pi_5(SU(3)/SU(2)) \xrightarrow{\partial} \pi_4(SU(2)) \xrightarrow{i} \pi_4(SU(3)),
\]

\(j = \text{inclusion map}, \ i = \text{group homomorphism}, \ \partial = \text{boundary homomorphism}.

This sequence shows that the SU(2) subsymmetry of the SU(3) symmetry will be broken, because the topological defects in the two-dimensional gauge orbit space (zero modes) affect the SU(2) subsymmetry only. We have chosen this example because in the standard model we are faced with the broken SU(2) symmetry\([3]\). If we start with the SU(3) family\([12]\) in the six-dimensional space\([8, 9]\), where gauge fields have six components, we have sufficient gauge freedom to develop the structure of the whole standard model. Evidently, we can obtain:\{\(A_a^c; c=0,1,2,3,5,6; a=1,...,8; \text{number of the gauge degrees of freedom}=6 \times 8 = 48\}\} and in the standard model in the Minkowski space\{\(U(1) : 4 \times 1; SU(2) : 4 \times 3; SU(3) : 4 \times 8\);\(SU(3) \times SU(2) \times U(1): 4 + 12 + 32 = 48\}\}. There is presently at our disposal\([13]\) a mechanism that can generate the local gauge theory of subsymmetry, called the mechanism of hidden local symmetries. Whereas a detailed analysis of this issue can be found elsewhere\([13]\), here
we point out the necessary ingredients: (a) the local symmetry and subsymmetries of the basic group $G$ are generated as hidden local symmetries, but at the same time preserving the global $G$ symmetry, (b) the interaction of fermions with gauge bosons proceeds in the standard way, (c) the appearance of fermion and gauge boson masses just breaks the global symmetry drawn from the homotopy sequence. By choosing $G=SU(3)$ as the basic group, one is able to recover the standard model at the tree level, because the embedding of the $SU(2)\times U(1)$ in $SU(3)$ with the fermion representations (color singlet and triplet) exists and it is unique; the $SU(3)$ global symmetry is preserved as it should be in QCD; if the $SU(2)$ symmetry is broken, fermions are coupled chirally asymmetrically to $SU(2)$ bosons:

$$SU(3) : H_1, H_2 \text{ generators commuting with all others},$$

$$E(\vec{\mu}) : \vec{\mu} = \vec{\alpha}, -\vec{\alpha}, \vec{\beta}, -\vec{\beta}, \vec{\gamma}, -\vec{\gamma} \text{ (} \vec{\mu} = \text{ root vectors)},$$

$$\vec{\alpha} = \frac{1}{\sqrt{3}} \vec{e}_1, \quad \vec{\beta} = \frac{1}{2\sqrt{3}} \vec{e}_1 + \frac{1}{2} \vec{e}_2, \quad \vec{\gamma} = -\frac{1}{2\sqrt{3}} \vec{e}_1 + \frac{1}{2} \vec{e}_2, \quad (3)$$

$SU(2)\times U(1)$ embedding:

$$SU(2) : \sqrt{3}H_1, \sqrt{6}E(\pm \vec{\alpha}); \quad U(1) : H_2. \quad (4)$$

Thus we have a perfect unification scheme that has no problem with unitarity, and it is not a surprise that it is realized in nature. It is worthwhile to notice that the six-dimensional conformal space fits perfectly as Minkowski space plus gauge-orbit two-dimensional space.

### III. THE SU(2) ZERO MODES

In this section we show that for the $SU(2)$ symmetry, the Dirac zero mode fermion eigenstates cannot contribute to the index of the Weyl operator. To prove this, we closely follow the discussion in Ref. where a relation is established between the index of $iD_{2n+2}$ and the winding number of the phase of the 2n-dimensional Weyl operator $i \mathcal{D}^{\ell, \theta}_{\pm} = i \mathcal{D}^{\ell, \theta}_{2n+2}(1 - \gamma_5)$. Instead of the operator $iD_{2n+2}$, the authors of Ref. studied the following deformed operator with the same zero mode space:
\[ i \mathcal{D}_{2n+2} = \frac{1}{\epsilon^2} \sum_{\mu=1}^{2n} D_\mu \Gamma^\mu + i \mathcal{D}_2, \]

as a matter of fact, the square of this operator:

\[
H_\epsilon \equiv (i \mathcal{D}_{2n+2})^2 = \frac{1}{\epsilon^2} (iD_\mu \Gamma^\mu)^2 + (iD_i \Gamma^i)^2 + \frac{i^2}{\epsilon} \Gamma^\mu (D_i D_\mu - D_\mu D_i).
\]

The zero modes with a definite chirality (\(\chi\)) take the form

\[
\chi = +1 : \begin{pmatrix} \psi \\ 0 \end{pmatrix}; \quad \chi = -1 : \begin{pmatrix} 0 \\ \psi \end{pmatrix}.
\]

The locally Euclidean coordinates \(\phi_1, \phi_2\) are introduced in the vicinity of each zero eigenvalue of the operator \(i \mathcal{D}_{2n} \theta\) and the space of zero modes is spanned by two states:

\[
\psi_\pm (x) \equiv \frac{1}{2}(1 \mp \gamma_5)\psi_{\phi_i=0}(x).
\]

We need to investigate an operator in the vicinity of the zero mode, so its perturbation near \(\phi_i = 0\) is of the form

\[
\delta(i \mathcal{D}_{2n}^{\phi_1, \phi_2}) = \sum_j (i \partial_j A) \phi_j,
\]

where \(\partial_j A \equiv (\partial/\partial \phi_j) A^{\phi_1, \phi_2} |_{\phi_i=0},\)

and its matrix elements in the \((\psi_+, \psi_-)\) basis are

\[
\begin{pmatrix}
0 & \sum_i z_i^* \phi_i \\
\sum_i z_i \phi_i & 0
\end{pmatrix}, \quad \text{where } z_i \equiv (\psi_+, i \partial_i A \psi_+), \text{ are complex constants.} \quad (5)
\]

Then in the vicinity of \(\phi_i = 0\) two zero modes have eigenvalues:

\[
i \mathcal{D}_{2n}^\phi \Psi_\lambda^\phi (x) = \lambda(\phi_i) \Psi_\lambda^\phi (x),
\]

\[
\lambda(\phi_i) = \pm |\phi_1 + \phi_2|.
\]
After further considerations the authors of Ref. [9] obtained an expression for the chirality of the zero mode:

\[ \chi = \frac{|z_1| |z_2|}{\text{Im}(z_1^* z_2)}. \]  

(6)

Now let us repeat how Dirac and Majorana states are formed from two-component Weyl spinors. For a \(2 \times 2\) unimodular matrix \(L\) belonging to \(\text{SL}(2,\mathbb{C})\), Weyl spinors transform under Lorentz transformation as

\[ \eta_a \rightarrow \eta'_a = L^b_a \eta_b, \]
\[ \dot{\eta}_{\dot{a}} \rightarrow \dot{\eta}'_{\dot{a}} = L^b_{\dot{a}} \dot{\eta}_b. \]

It is important to stress that \(L\) and \(L^*\) are not equivalent representations of \(\text{SL}(2,\mathbb{C})\) [17].

A four-component Dirac spinor can now be formed from two Weyl spinors:

\[ \Psi_D = \begin{pmatrix} \eta_a \\ \dot{\eta}_{\dot{a}} \end{pmatrix}, \]

and its charge-conjugate spinor is

\[ \Psi_D^c = C \bar{\Psi}_D^T = \begin{pmatrix} \xi_a \\ \dot{\xi}_{\dot{a}} \end{pmatrix}. \]

Majorana states are defined by the Majorana condition:

\[ \Psi_M = \begin{pmatrix} \eta_a \\ \dot{\eta}_{\dot{a}} \end{pmatrix}, \quad (\Psi_M)^c = C (\bar{\Psi}_M)^T = \Psi_M, \]
\[ C = \begin{bmatrix} \epsilon_{ab} & 0 \\ 0 & \epsilon^a_{\dot{b}} \end{bmatrix}, \quad \eta^a = \epsilon^{ab} \eta_b, \quad \dot{\eta}^{\dot{a}} = \epsilon^a_{\dot{b}} \dot{\eta}_{\dot{b}}, \]
\[ \epsilon^{12} = 1, \quad \epsilon_{12} = -1. \]

Let us now see chiralities of the \(\text{SU}(2)\) zero modes. The calculation of the index takes into account the summation over the chiralities of all independent zero modes. If a Dirac state is a zero mode of some gauge symmetry, then its charge-conjugate state is not necessarily a zero
mode. However, the charge-conjugate SU(2) Weyl states are equivalent. This is peculiar to the SU(2) group [17], and this equivalence could be easily understood because the generators $\tau^i$ and its complex-conjugates $\tau^{i*}$ are connected through the similarity transformation. As a consequence, a charge-conjugate SU(2) Dirac spinor is also a zero mode if its appropriate Dirac spinor is a zero mode:

$$\Psi_D \equiv \begin{pmatrix} \eta_a \\ \xi^a \end{pmatrix}, \quad \Psi^c_D = \begin{pmatrix} \xi_a \\ \eta^a \end{pmatrix} \overset{\text{equiv.}}{\simeq} \begin{pmatrix} \xi^a \\ \eta_a \end{pmatrix}. $$

In the $(\psi_+, \psi_-)$ basis, the perturbation of the $i D_{2n}^{\phi_1, \phi_2}$ operator for the charge-conjugate Dirac spinor is of the form

$$\begin{pmatrix} 0 & \sum_i z_i \phi_i \\ \sum_i z_i^* \phi_i & 0 \end{pmatrix}. $$

(7)

From the formula for the chirality of the zero mode it follows:

$$\chi(\Psi_D) = \frac{|z_1| |z_2|}{Im(z_1^* z_2)} = - \frac{|z_1| |z_2|}{Im(z_1 z_2^*)} = -\chi(\Psi^c_D).$$

(8)

Thus the total contribution of the SU(2) Dirac zero modes always vanishes. On the contrary, a Majorana state by definition does not have an independent charge-conjugate spinor, but it has a definite chirality. The index theorem and the necessity of the exact cancellation of SU(2) anomalous actions (Sec. 2) forces the existence of the surplus of the left-handed (positive chirality) zero modes.

**IV. QUANTIZATION OF SPONTANEOUSLY BROKEN GAUGE THEORIES**

The path integral or the canonical quantization of spontaneously broken non-Abelian gauge theories have been consistently carried out in the past quarter of the century. However, in the light of the results obtained in the preceding section, we also want to include into consideration Majorana particles. The second reason for reconsideration lies in our general unsatisfaction with the Higgs mechanism, as an example of the spontaneously broken gauge theory realized in the electroweak sector of the Standard Model(SM).
Let us write the most general form of the electroweak Lagrangian with leptons, together with the gauge-fixing and the Faddeev-Popov (FP) terms, which should be included for the correct quantization of the spontaneously broken non-Abelian gauge theory \[18, 19\]:

\[
\mathcal{L} = \mathcal{L}_{\text{lep}} + \mathcal{L}_{\text{g.bos}} + \mathcal{L}_{\text{scal}} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\text{M.Yuk}} + \mathcal{L}_{\text{g.fix}} + \mathcal{L}_{\text{FP}},
\]

\[
\mathcal{L}_{\text{lep}} = \bar{R}i\gamma^\mu (\partial_\mu + ig'B_\mu) R + \bar{L}i\gamma^\mu (\partial_\mu + ig\gamma^5 A_\mu^i) L + \bar{\psi}_R i\gamma^\mu \partial_\mu \psi_R,
\]

\[
\mathcal{L}_{\text{g.bos}} = -\frac{1}{4} F_{\mu\nu}^i F^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu},
\]

\[
\mathcal{L}_{\text{scal}} = (\partial_\mu \Phi^\dagger + i\gamma^5 B_\mu \Phi^\dagger + i\gamma^5 \tau^i A_\mu^i \Phi^\dagger)(\partial^\mu \Phi - i\gamma^5 B^\mu \Phi - i\gamma^5 \tau^i A^{i\mu} \Phi) - V(\Phi),
\]

\[
\mathcal{L}_{\text{Yuk}} = -Y e \bar{D} \Phi R - Y \psi \bar{D} \tilde{\Phi} \psi_R + \text{h.c.},
\]

\[
\mathcal{L}_{\text{M.Yuk}} = -Y \psi M \bar{L} \tilde{\Phi} \psi_R + \text{h.c.},
\]

\[
\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \\ \phi^- \end{pmatrix}, \quad \Phi = i\tau^2 \Phi^* = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix},
\]

\[
\phi^0(x) = (v + H(x) + i\chi^0(x))/\sqrt{2}, \quad v = \text{symmetry breaking parameter},
\]

\[
H = \text{physical Higgs scalar}, \quad \phi^+, \chi^0 = \text{Nambu–Goldstone scalars}.
\]

The Majorana mass terms for charged fermions are forbidden by charge conservation. If the scalar doublets carry no lepton number, then the Majorana mass term is excluded, provided that only the spontaneously broken lepton number is allowed. This is the situation in the SM where only Dirac fermions receive masses through the Higgs mechanism. However, in this case it is not possible to have a Majorana zero mode to break the \(SU(2)_L\) gauge symmetry at the tree level by the mass terms. As a consequence, the SM seems to be anomalous with respect to the nonperturbative \(SU(2)\) anomaly. Of course, it does not affect the perturbative unitarity and renormalizability of the model.

On the other hand, if the scalar doublets carry the lepton number \[20\]

\[
\tilde{\Phi} \to e^{2i\alpha} \tilde{\Phi}, \quad L \to e^{i\alpha} L, \quad R \to e^{i\alpha} R, \quad \psi \to e^{i\alpha} \psi,
\]

10
then its interactions with Dirac fermions are forbidden and no tree-level Dirac masses can be generated. On the contrary, the Majorana masses are generated, but the lepton number as the global $U(1)$ symmetry of the Lagrangian is also spontaneously broken in addition to the local $SU(2)_L$ gauge symmetry.

The fate of the Nambu-Goldstone particle (majoron) will be discussed in Section 6. The theory is now unitary with respect to the nonperturbative $SU(2)$ anomaly. Even the Majorana masses could be generated through the Higgs mechanism, but in this paper we want to refer also to the mechanism proposed in Ref. [12]. Namely, it is possible to generate masses to the $SU(2)$ bosons and the tree-level mass to the fermion by analyzing the trace anomaly under the hypothesis of noncontractible space (see Sec. 5)[12]:

\[ M_W = M_Z = \sqrt{6} \frac{g}{\pi} \frac{\hbar}{2cd}, \]
\[ \hbar = \frac{\hbar}{2\pi}, \quad d \approx 6 \times 10^{-17} \text{ cm}, \]
\[ m_{\text{fermion (tree level)}} = \frac{3}{2} \frac{\hbar}{2cd} = \frac{3}{2} \Lambda = m_0. \]

Now, the question is whether this mechanism represents a mechanism of spontaneous gauge symmetry breaking. Very general results of Kugo and Ojima [18] claim that massive gauge bosons always require spontaneously broken global charges in a one-to-one correspondence. Moreover, it was explicitly proved that not only the Higgs mechanism, but also the symmetry breaking in the Schwinger model [21] and the Nambu-Jona-Lasinio model [22] occurred spontaneously. It could be easily seen that if we put the physical Higgs scalar field and the Higgs potential equal to zero, quantize the theory in the $R_\xi$-gauge [23] with inclusion of the symmetry-breaking parameter $v$, and if we proceed smoothly to the U-gauge, then we obtain the Lagrangian with degenerate massive bosons and Majorana fermions of the UV-finite theory for $v = \sqrt{6}\Lambda/\pi$. The presence of the Nambu-Goldstone scalars corresponds to spontaneous symmetry breaking, thus a noncontractible spacetime as a symmetry breaking mechanism satisfies the Kugo-Ojima theorem ([18], theorem 6.6).

Let us now discuss the issue of the renormalizability of the theories with and without the Higgs scalar. The necessary condition for the spontaneously broken gauge theory to be renormalizable is to have dimensionless gauge coupling constants, linear Yukawa coupling of scalars to fermions and up to quartic self-coupling of scalars [24]. However, it is well known
that this is not sufficient to preserve the renormalizability, namely the sufficient condition requires various identities between Green functions. If the quantized theory possesses BRST symmetry, then, as a consequence, the generalized Ward-Takahashi and Slavnov-Taylor identities are valid. The analysis of Kugo and Ojima of the Higgs-Kibble SU(2) model shows that even in the model with the physical Higgs scalar, the BRST transformations for the gauge vector, Faddeev-Popov ghosts, Nakanishi-Laudrup and Nambu-Goldstone asymptotic fields (formulae (4.34) of Ref. [18]) form a closed set of field transformations without the inclusion of the physical Higgs scalar. Thus the massive non-Abelian gauge theory is renormalizable even without the inclusion of the Higgs scalar because the quantized theory preserves the BRST symmetry. The inclusion of fermions or the electroweak mixing does not alter the conclusion. In the case of the Abelian theory, FP ghosts are decoupled from the gauge vector and NG scalar fields and then we need the Higgs scalar to preserve the renormalizability (see Ref. [26]).

Thus in Eq.(9) we have displayed the Lagrangian for the four renormalizable electroweak theories and let us denote them by (AX), (AY), (BX), and (BY) with the following notations: (A)=scalar doublet does not carry lepton number, (B)=scalar doublet carries lepton number=−2, (X)=Higgs scalar and its quartic self-coupling are present, (Y)=absence of the Higgs scalar and the presence of the hypothesis of noncontractible spacetime. In the next sections we study the (BY) theory.

V. TRACE ANOMALY AND NONCONTRACTIBLE SPACETIME

The hypothesis of the fundamental length attracted much attention in the past, but with poor success. Yukawa and Heisenberg touched the essential problems with the ultraviolet singularity and bootstrap equations in QFT, but it would have been premature to expect resolutions at that time when gauge QFT had been confirmed and established neither experimentally nor theoretically.

The fundamental (elementary) length appears as a universal parameter in nonlocal QFT to make interactions nonlocal through some nonlocal functionals, as in the work of Efimov. This theory does not improve QFT but introduces new arbitrariness owing to the free choice of elementary-length-dependent functionals.

Insisting on local gauge theory, Kadyshevsky et al. developed a theory positioned in
the five-dimensional de Sitter space where the fundamental length appeared as the upper bound to the masses, and the fifth coordinate was fixed by the "maximon" mass. However, it seems very dubious to set any upper bound to all the masses (timelike region) by the fundamental length (cutoff in spacelike region).

We should mention lattice gauge theories\[29\] with discretized spacetime. In these theories the size of hypercubes is a technical rather than a physical parameter that theorists want to remove from their nonperturbative calculations. The doubling of the fermion species on the lattice represents an insurmountable task for the treatment of lattice gauge theories.

Contrary to the hypotheses mentioned above, we propose the following\[12\]: (1) The physical spacetime is a continuum in the sense that spacetime variables can be continuously changed, (2) the physical spacetime is noncontractible in the sense that spacelike intervals have a lower bound, the universal nonvanishing minimal distance ($d \neq 0$).

The question arising here concerns the correspondence between these postulates and local relativistic quantum field theory. First, it must be clear that the concept of field discards the concept of point particle. We describe the quantum field as a wave-packet system, i.e. a very nonlocal structure. However, the interaction between the fields could be local in the sense that only coordinates of the centre of motion of the fields (particles) coincide. The nonlocal structure of the fields and the fields in interaction should be defined by the complete structure of dressed Green functions (propagators and vertices).

Contrary to the perturbation theory, the calculation of the trace anomaly is essentially a nonperturbative field identity, valid to arbitrary quantum loop order. We shall study the matrix elements of the trace of the energy-momentum tensor. Let us consider massless spin-$\frac{1}{2}$ fermions coupled to gauge bosons. A part with the spinor of the trace operator is of the form\[12\]:

$$\theta^{\mu} = -3i\bar{\psi}\gamma^\mu \partial_\mu \psi + \frac{3}{2}i\partial^\mu [\bar{\psi}\gamma_\mu \psi], \tag{12}$$

where we have discarded the terms linear in vector gauge fields because these cannot contribute to the spinor matrix element of the trace at the tree level. We search for the nonvanishing contribution to the mass term and only terms with the canonical momenta could contribute. Owing to translational invariance we can choose the reference point $x_\mu = 0$ of the trace operator (in the QFT only canonical variables are correlated):
\[ \theta_\mu^\mu(0) = -\frac{3}{2} t (\psi^\dagger(0) \dot{\psi}(0) - \dot{\psi}^\dagger(0) \psi(0)), \]  

(13)

where \( \dot{\psi}(0) = \min \Delta \psi(0)/\min \Delta t, \ \Delta \psi(t) = t (\partial/\partial t) \psi(t) = -i t \vec{p}^\mu \psi(t). \) Taking into account Lorentz invariance and the noncontractibility of the physical space, it follows that \( \Delta t \geq d/c \quad (c = \text{velocity of light}) \)

\[ \theta_\mu^\mu(0) = -\frac{3}{2} \frac{1}{cd} \min [\psi^\dagger(t \vec{p}^\mu + \vec{p}^0 t) \psi](0) \]

\[ = -\frac{3}{2} \frac{1}{cd} \min [\psi^\dagger(t \vec{p}^\mu - \vec{p}^0 t) \psi](0). \]  

(14)

Planck principle of quantum action (or Heisenberg uncertainty principle) leads us further to

\[ < 1 \text{fermion} \mid \theta_\mu^\mu(0) \mid 1 \text{fermion} > = \frac{3}{2} \frac{\hbar}{cd}. \]  

(15)

Otherwise, in local gauge QFT the following relation is valid for massive fermions coupled to arbitrary gauge fields

\[ < 1 \text{fermion} \mid \theta_\mu^\mu(0) \mid 1 \text{fermion} > = m(1 + \gamma), \]  

(16)

where \( m \) is the physical mass of the fermion and \( \gamma \) is the sum of anomalous dimensions of the fermion bilinear operator with respect to the coupled vector gauge fields.

Obviously, from (15) and (16) we may conclude that \( m_0 = \frac{3}{2} \frac{\hbar}{cd}, \) because at the tree level \( \gamma = 0 \) by definition. The finiteness of distances, velocities and energies is essential for the derivation of the mass. The role and the meaning of this tree-level mass we discussed in Sec. 4.

On the mass-shell matrix elements of \( \theta_\mu^\mu \) (or equivalently \( \partial_\mu D^\mu \)) of spin-$\frac{1}{2}$ fermions simultaneously measure nonvanishing energy densities and the noncontractibility of space. Generalizing this consideration to all elementary particles and taking into account physical dimensions, one would expect
\begin{align}
<1 \text{ fermion} | \theta^\mu_\nu(0) | 1 \text{ fermion} > & \propto \frac{\hbar}{cd}, \\
<1 \text{ vector} | \theta^\mu_\nu(0) | 1 \text{ vector} > & \propto \left(\frac{\hbar}{cd}\right)^2. 
\end{align}

Massless gluons can satisfy (18) mainly because of the non-Abelian character of gauge symmetry, Wick’s theorem applied to the terms quartic in $G^a_\mu$ and the running coupling. Actually, the following nonperturbative relation for the trace is valid \cite{30}:

\begin{align}
\theta^\mu_\mu &= \frac{\beta(g)}{2g} G^a_\mu G^{a\mu}, \\
where \beta(g) &= \lambda \frac{\partial g(\lambda)}{\partial \lambda}, \quad G^a_\mu G^{a\mu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu.
\end{align}

In the preceding chapters on anomalies we have emphasized the necessity of having massive weak bosons to remove the SU(2) chiral anomaly by the SU(2) boson anomaly. Proceeding as for gluons, the trace anomaly for SU(2) bosons is of the form \cite{12,18}

\begin{align}
\theta^\mu_\mu &= 2 F^i_\mu F^{i\mu} , \quad i = 1, 2, 3, \\
F^i_\mu &= \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g \epsilon^{ikl} A^k_\mu A^l_\nu.
\end{align}

Only the quartic self-coupling terms in (20) can generate the mass term. Therefore, applying Wick’s theorem to the time-ordered product, one obtains \cite{12}

\begin{align}
\theta^\mu_\mu(x) &= 2g^2 \epsilon^{ijk} \epsilon^{ilm} A^i_\nu(x) A^j_\lambda(x) A^{kl}(x) + ... \\
&= 2g^2 \epsilon^{ijk} \epsilon^{ilm} \sum_{P.E.R.M.} : A^i_\nu(x) A^j_\lambda(x) : \overline{A^{il}(x) A^{jm}(x)} + ... , \\
A^{il}(x) A^{jm}(x) &= < T(A^{il}(x) A^{jm}(x)) > = i g^{\nu\lambda} D^\nu_0(0) \delta^{lm}, \\
D^\nu_0(0) &= -(2\pi)^{-4} \int k^{-2} d^4 k, \quad \text{(Wick’s contraction)}. \tag{21}
\end{align}

From (21) it follows that

\begin{align}
\theta^\mu_\mu(x) &= 24g^2 i D^\nu_0(0) : A^i_\nu(x) A^{i\mu}(x) : + ... . \tag{22}
\end{align}

After performing Wick’s rotation and integration in $D^\nu_0(0)$ in Euclidean space, one obtains \cite{12}.
\[ \theta_\mu^\nu(x) = \frac{3g^2}{2\pi^2}\Lambda^2 : A^i_\mu(x)A^{i\mu}(x) : +... \] (23)

where \( \Lambda = \hbar/cd \) is the ultraviolet (UV) cut-off in UV finite theory.

Comparison with the trace anomaly of massive bosons:

\[ \theta_\mu^\nu(x) = M^2_W : A^i_\mu(x)A^{i\mu}(x) : +... \] (24)

immediately gives \[12]\]

\[ M_W = \frac{\sqrt{6}g}{\pi} \frac{\hbar}{2cd}, \]

\[ \hbar = \frac{\hbar}{2\pi}, \quad d \simeq 6 \times 10^{-17} \text{cm}. \] (25)

We have successfully generated masses at the tree level, but masses of weak bosons are degenerate, which is in disagreement with reality: \( M_W \neq M_Z \).

Owing to the absence of self-coupling in Abelian gauge QFT it is apparently impossible to satisfy (18). However, there is a unique resolution of this problem: SU(2) × U(1) is a subsymmetry of SU(3) and one can propose mixing in the neutral sector \[3\]:

\[ A_\mu = \cos\theta_W B_\mu + \sin\theta_W A^3_\mu, \]

\[ Z_\mu = \sin\theta_W B_\mu - \cos\theta_W A^3_\mu, \quad \theta_W \neq 0. \] (26)

Redefinition of the neutral SU(2) boson and the U(1) boson in (26) makes a drastic change in the spectrum and the couplings. Substitution of (26) into the SU(2) × U(1) lagrangian with weak \( g \) and electromagnetic \( g' \) couplings recovers the standard Glashow-Salam-Weinberg model and as a consequence \[3\]

\[ \cos \theta_W = M_W/M_Z, \quad e = g \sin\theta_W = g' \cos\theta_W. \] (27)

Owing to (24) the physical photon field \( A_\mu \) in (24) fulfills the condition (18)

\[ < 1 ~ \text{photon} \mid \theta_\mu^\nu(0) \mid 1 ~ \text{photon} > = \sin^2\theta_W M^2_W \neq 0. \] (28)

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VI. SPONTANEOUSLY BROKEN LEPTON NUMBER AND THE MAJORON

In this Section we want to show that the fate of the majoron is in complete analogy with the fate of the axion in QCD[18]. We have formed the electroweak Lagrangian invariant under the global $U(1)$ transformation of the lepton number. The Adler-Bell-Jackiw anomaly of leptons coupled chirally-asymmetrically to the external SU(2) fields in the triangular graph breaks the lepton number-conservation at the quantum-loop level[31]:

$$\partial_\mu j^{LN}_\mu = -\frac{g^2 N_{wd}}{32\pi^2} \epsilon_{\mu\rho\sigma\tau} F^\mu_{\rho i} F^\rho_{\sigma i}, \quad i = 1, 2, 3,$$

$$j^{LN}_\mu = \bar{L} \gamma_\mu L + \bar{R} \gamma_\mu R + \ldots, \quad N_{wd} = \text{number of lepton weak doublets.} \quad (29)$$

However, the situation can be improved by redefinition of the lepton-number vector current to restore the conservation of the current, but spoiling the gauge invariance of the current[18]:

$$j^{LN}_\mu = j^{LN}_\mu + X^\mu,$$

$$X^\mu = \frac{g^2 N_{wd}}{16\pi^2} \epsilon^{\mu\rho\sigma\tau} \left[ A_i^\rho F^i_{\sigma} - \frac{g}{3} \epsilon_{ijkl} A^i_\rho A^j_\sigma A^k_\mu \right]. \quad (30)$$

Let us calculate the following vacuum expectation value:

$$\partial_\mu \langle 0 \mid T(J^{LN}_\mu (x) \bar{\psi}_L (\psi^c)_R (\bar{\psi}^c)_R \psi_L (0) \mid 0 \rangle = 2 \delta^4 (x) \langle 0 \mid (\bar{\psi}_L (\psi^c)_R (\bar{\psi}^c)_R \psi_L (0) \mid 0 \rangle \neq 0.$$

This matrix element does not vanish because of the spontaneously broken $SU(2)_L$ gauge symmetry through the Majorana mass. Now we apply the Goldstone theorem which asserts that the spontaneously broken global $U(1)$ symmetry leads inevitably to the existence of the Goldstone boson:

$$J^{LN}_\mu (x) \xrightarrow{x_0 \to +\infty} \partial_\mu \chi^{\text{out}} (x) + \ldots; \quad \chi = \text{Goldstone boson} = \text{majoron.} \quad (31)$$

The central question whether this Nambu-Goldstone particle is physical or not, is answered within the Kugo-Ojima quartet mechanism[18], and a straightforward construction of all members of the quartet gives:
\( [Q_B, \chi(x)] = -i\gamma(x); \ Q_B = BRST \ charge \ operator, \)

\[ C^\mu(x) \equiv [iQ_B, J_{LN}^\mu(x)] = \frac{g^2 N_w}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} \partial_\nu \bar{c}^i \partial_\rho A_i^\sigma(x), \]

\[ C^\mu(x) \xrightarrow{x_0 \to \infty} \partial^\mu \gamma(x) + \ldots, \]

similar construction gives: \( \tilde{C}^\mu(x) = \frac{g^2 N_w}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} \partial_\nu \bar{c}^i \partial_\rho A_i^\sigma(x); \)

\[ B^\mu(x) = \{Q_B, \tilde{C}^\mu(x)\}, \ c^i, \bar{c}^i = \text{Faddeev – Popov ghosts}, \]

\[ \tilde{C}^\mu(x) \xrightarrow{x_0 \to \infty} \partial^\mu \gamma(x) + \ldots, \ B^\mu(x) \xrightarrow{x_0 \to \infty} \partial^\mu \beta(x) + \ldots, \]

\[ \{Q_B, \gamma(x)\} = \beta(x), \ (\chi, \beta, \gamma, \bar{\gamma}) = \text{Goldstone quartet}. \]

Since any member of the quartet appears only in the zero-norm combination, the majoron is an unphysical particle until the weak interaction is described by a non-Abelian gauge symmetry.

**VII. COMPATIBILITY OF THE ELECTROWEAK MIXING WITH BOOTSTRAP EQUATIONS**

Absence of fermion and weak boson mixing (mass degeneracy) should be the only condition for exact cancellation of the anomalous actions. The electroweak mixing affects the \( SU(2) \) global charges and the only appropriate object for nonperturbative study is the generating functional of the electroweak theory:

\[ Z = \int [dA_\mu][d\psi][d\bar{\psi}] e^{i \int L_{EW}(A_\mu, \psi, \bar{\psi})} d^4 x. \]  

(33)

One can only ad hoc introduce the mixing into the functional measure and the Lagrangian density. However, this cannot be done completely arbitrarily. In fact, the total mixing in the boson sector should exactly cancel the total mixing in the Dirac fermion sector of the functional measure:

\[ [dA_\mu][d\psi][d\bar{\psi}] = [dA_\mu]_{W^\pm}[d\psi]_{u,c,t}[d\bar{\psi}]_{\pi,\tau,\eta} \]

\[ \times V(\theta_W) \begin{pmatrix} [dA_\mu]_\gamma \\ [dA_\mu]_Z \end{pmatrix} V_{CKM}^{-1} \begin{pmatrix} [d\psi]_d \\ [d\psi]_s \\ [d\psi]_b \end{pmatrix} V_{CKM}^{-1} \begin{pmatrix} [d\bar{\psi}]_\tau \\ [d\bar{\psi}]_\pi \\ [d\bar{\psi}]_\eta \end{pmatrix}; \]  

(34)

\[ V(\theta_W)(TV_{CKM}^{-2}S)_{2 \times 2} = 1, \quad T, S = \text{unitary matrices}; \]

(35)
\[ TV_{CKM}^{-2} S = \begin{pmatrix} V(-2(\theta_c + \theta_{23} + \theta_{13})) & 0 \\ 0 & 1 \end{pmatrix}, \]

\[ V(\theta_W) = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix}; \]

⇒ \[ \theta_W = 2(\theta_c + \theta_{23} + \theta_{13}). \] (37)

We propose the existence of three fermion families. The relation between boson and fermion mixing angles should be valid for both quark and lepton Dirac species. One cannot distinguish particle from antiparticle fermions in the measure, and what figures is only the total rotation of the Dirac-fermion variables deduced by a biunitary transformation and reduction to a 2×2 mixing matrix.

The introduction of rotated fields into the Lagrangian density restores the structure of the electroweak sector of the SM: (1) It does not affect neutral currents \[3\] and brings the CKM matrix into the charged currents \[32\], but now with a constrained sum on the Euler angles (see Eq.(37)) for quarks and leptons. (2) Rotated electroweak neutral gauge bosons account for masses as in the SM \[3\].

If we accept the Higgs mechanism for Dirac particles as in the SM, the theory contains no Majorana zero modes and so it is anomalous with respect to the nonperturbative SU(2) anomaly. If we accept the Higgs mechanism for Majorana particles, it is not anomalous, but it remains to be UV-singular in a nonperturbative sense. Thus one can only calculate perturbative (radiative) corrections to mass terms. It is most favorable to choose a noncontractible spacetime as a symmetry breaking mechanism \[12\] when the theory is not only renormalizable, but also nonanomalous and nonsingular (BY theory).

The study of UV-finite theory allows nonperturbative calculations. In fact, we have a well-defined set of equations for elementary particles: Dyson-Schwinger (DS) equations \[33\] and, for bound states, Bethe-Salpeter (BS) equations \[34\]. Any of two-, three-, and four-point Green’s functions satisfies some bootstrap equations that are coupled and connected with Ward-Takahashi and Slavnov-Taylor \[35\] identities. Using the UV cut-off, \(M_W, M_Z\), the fine-structure constant \(\alpha_e\), the tree-level fermion mass, the angle constraint (37), and the QCD vertices, we should be able to find lepton and quark masses (see Figs. 1 and 2).

The most important features of bootstrap systems can be summarized as follows: (1) The structure and mass singularities of fermion propagators in the timelike region are defined by
the evolution from the spacelike region \[36\]. (2) It is essential for fermions to couple to the massive gauge boson in order to develop a fermion mass singularity of the propagator \[36, 37\]. (3) The tree-level fermion mass appears only in the equations for Majorana particles. (4) The appearance of three fermion families could only be interpreted as zero-, one-, and two-node solutions of the DS equations \[36, 38\]. (5) Mass gaps between fermion families are in accordance with the behavior of solutions with different number of nodes, especially with the slopes at \(p^2 = 0\) of the mass functions \[38\]. (6) The complete mass matrix for neutral leptons for the three families is a 6×6 matrix of the form \[39, 40\]

\[
(f, F) \begin{bmatrix}
M_L & M_D \\
M_D & M_R \\
\end{bmatrix}
\begin{bmatrix}
f \\
F \\
\end{bmatrix},
\]

\[f = \frac{1}{\sqrt{2}}(\psi_L + (\psi_L)^c), \quad F = \frac{1}{\sqrt{2}}(\psi_R + (\psi_R)^c). \tag{38}\]

\(M_D = \) Dirac mass matrix, \(M_{L,R} = \) Majorana mass matrices.

(7) Because of the absence of the right-handed currents, the Majorana mass matrix \(M_R\) vanishes. Only the Dirac sector is responsible for flavor mixing and the Majorana sector is flavor diagonal. Thus, after diagonalizing the off-diagonal Dirac mass matrix, we obtain six Majorana physical states for which the seesaw relation \[40\] is valid for each family separately:

\[
m_D \ll m_N, \quad m_\nu \simeq \frac{m_D^2}{m_N},
\]

\[
\psi_N \simeq f - \frac{m_D}{m_N} F, \quad \psi_\nu \simeq \gamma_5 \left(F + \frac{m_D}{m_N} f\right),
\]

\(\psi_N, \psi_\nu = \) Majorana particles. \(\tag{39}\)

(8) The light Majorana particles, namely, neutrinos, can interact with the left-handed currents because \((\psi^c)_L = (\psi_R)^c, \quad \psi^c = C \bar{\psi}^T\) (one can also envisage this in terms of Weyl’s spinors). (9) The Dirac masses are much smaller than the Majorana masses \(m_N\) because of no tree-level mass and smaller effective couplings \((\theta_W = 0 \text{ implies } m_D = \text{diag}(m_1, m_2, m_3))\). (10) The Dirac masses of the first family are \(\mathcal{O}(0.1\text{MeV})\). However, because of the strong interaction we cannot measure quark flavour masses and there are no quark-mass singularities \[41\] (massless gluons prevent their appearance), which is interpreted as confinement. In QCD, one uses values of quark current mass functions which

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are much higher than flavour-mass eigenstates (see Fig. 2). (11) An essential contribution to the heavy Majorana fermion masses of the second and third family comes from the longitudinal strongly interacting part in quantum loops (Nambu-Goldstone mode), with \( g_{eff}^2 = 1/16 \sin^2 \theta_W \times \alpha_e / \pi (m_N/M_W)^2 \), \( m_N \gg M_W \), but the heavy Majorana lepton of the first family acquires the mass \( m_{N_e} \approx m_0 = 485 \text{ GeV} \). (12) The electron neutrino has the mass \( m_{\nu_e} = \mathcal{O}(0.1 \text{ eV}) \). (13) Considerable progress has recently been made in solving the BS equations for the quarkonium, based on modeling propagators [42]. (14) In QED, perturbation theory can still be successfully applied to the BS system for the positronium [43]. (15) The theory is free of the Adler-Bardeen anomaly [2] and it contains 60 elementary particles, 48 fermions, and 12 vectors, so in fact there is an equal number of spin-\( \frac{1}{2} \) and spin-0 species (12 vectors are equivalent to \( 12 \times 4 = 48 \) scalars) [12]

\[
\begin{pmatrix}
A_{\mu} \\
W^-_\mu \\
W^+_\mu \\
Z_\mu
\end{pmatrix} \otimes 
\begin{pmatrix}
\nu_l \\
l^- \\
l^+
\end{pmatrix}, \, l = e, \mu, \tau \, ; \,
\begin{pmatrix}
\bar{\nu}_f \\
\bar{d}_g \\
\bar{u}_f
\end{pmatrix}, \, \left( \begin{array}{c}
f \\
g
\end{array} \right) = \left( \begin{array}{c}
u_l \\
l^-
\end{array} \right), \, k = 1, 2, 3; \, \left[ G^a_{\mu} \right], \, a = 1, ..., 8.
\]

VIII. CONCLUSIONS

In this paper we have made an attempt to understand and resolve the problem of the \( SU(2) \) nonperturbative anomaly and its possible connection with the electroweak mixing. The resolution of the problem does not favour the SM, but the theory with Majorana leptons. A qualitative study of the spectrum of the UV-finite theory using the gauge symmetry-breaking mechanism based on the characteristic of the physical spacetime, namely, its noncontractibility, leads to surprisingly successful results. It could be plausible that the breaking of gauge, discrete [44], and conformal symmetries has a common source, i.e. noncontractible spacetime. Whereas the weak-interaction scale defines the measure of noncontractibility, the conformal space with the \( SU(3) \) symmetry describes the complete vector gauge structure of the SM. A study of UV-finite bootstrap equations can explain the appearance of fermion families, flavor mixing, small neutrino masses, mass gaps between fermion families, and mass gaps between Majorana and Dirac fermions.
In the quark sector it is possible to check an angle constraint

\[
\begin{align*}
\sin \theta_c &= 0.221 \pm 0.003 \\
\sin \theta_{23} &= 0.040 \pm 0.008 \\
\sin \theta_{13} &= 0.0035 \pm 0.0015
\end{align*}
\]

at 90% CL

\[
\sin^2 \theta_W = 0.2319 \pm 0.0007
\]

define \( \Delta \theta_{BF} = \theta_W - 2(\theta_c + \theta_{23} + \theta_{13}) \),

then at 99% CL \( \Delta \theta_{BF} = -0.0405 \pm 0.0402 \).

The large error comes from the poor knowledge of the Bethe-Salpeter wave functions of mesons in the calculation of processes with large momentum transfer necessary to evaluate the \( \theta_{23} \) angle. Recent calculations lower this angle significantly: heavy quark effective theory gives \( \theta_{23} \approx 0.0267 \) (Ref. [47]) and Salpeter solutions \( \theta_{23} \approx 0.032 \) (Ref. [48]).

Measurements of the solar neutrino flux and its observed deficiency could be explained by the Mikheyev-Smirnov-Wolfenstein mechanism [49], which requires massive neutrinos with flavor mixing. The structure-formation analysis with cold and hot dark matter referring to the COBE data always contains massive neutrinos. Rotational curves and gravitational lensing measurements give strong evidence for dark matter, and cosmologically stable light and heavy neutrinos are perfect particle candidates for dark matter [50]. Presently, only astronomical and astrophysical measurements provide some evidence for massive neutrinos, whereas terrestrial experiments only set bounds on the masses and mixing angles. However, interesting results may be expected in the near future from new experiments on the neutrino mixing at CERN (CHORUS, NOMAD), LANL (LSND) [51], neutrinoless double beta decay, etc.

LEP 1 data could be fairly well fitted by the perturbation theory of the SM [52], but perturbative calculations could be easily provided in the UV-finite theory without Higgs scalars except that instead of regularization one has to integrate up to the UV-cutoff. The sensitivity of the observables to the Higgs mass or to the UV-cutoff is only logarithmic, so one has to wait for more and better data from LEP 2. A recent SM model fit of LEP data cannot explain \( b \) and \( c \) quark channel decay widths of the Z-boson [53]. One can also notice a problem to fit Tevatron data for the cross section for jets at the transverse energies greater than 200 GeV by perturbative QCD [54]. A definite answer about the source of
symmetry breaking will be given by LHC, which is also capable to detect heavy Majorana fermions up to 700 GeV\[55\], or by some future linear $e^+e^-$ collider.

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Fig. 1. - (a)-(c) Dyson-Schwinger equations for Majorana, neutral and charged Dirac leptons, respectively. Dashed bubbles denote proper self-energy and vertex parts. Electroweak loops with physical gauge vector fields or unphysical Goldstone scalars are marked by W, Z and γ.
Fig. 2. – Dyson-Schwinger equations for quarks.