Probing Low-x QCD With Very High Energy Prompt Muons

Namit Mahajan$^{a}$ and Sukanta Panda$^{b}$

$^{a}$Department of Physics, National Taiwan University, Taipei 10617, Taiwan (ROC).
$^{b}$Departamento de Fisica Teorica C-XI and Instituto de Fisica Teorica C-XVI, Universidad Autonoma de Madrid, Cantoblanco, E-28049 Madrid, Spain.

E-mail: *nmahajan@phys.ntu.edu.tw, †sukanta@delta.ft.uam.es

We explore the possibility of utilizing the prompt muon fluxes at very high energies in order to discriminate various models/parametrizations of low-x QCD behaviour of hadronic cross-sections relevant at such energies. We find that the pair meter technique for measuring high energy prompt muons can be very efficient in such an endeavor. As a by product, it allows to cleanly probe the change in composition of the primary cosmic rays expected at high energies.

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Introduction

The cosmic ray (CR) spectrum is characterized by a sharply falling power law behaviour, $\frac{dN}{dx} \sim E^{-(\gamma+1)}$ [1]. The spectrum gets more steeper around $10^6$ GeV with the spectral index $\gamma$ changing from 1.7 to 2.1 - this region is called the knee. Around $E \sim 5 \times 10^9$ GeV, one observes a flattening of the spectrum, with the spectral index $\gamma$ falling between 1.4 and 1.7. This is the so called ankle. The change in the slope of the spectrum at these two places is a puzzling issue. The region beyond the ankle is the regime of ultra high energy cosmic rays. There is not much data available in that region and no clear consensus exists on the composition or the particle content in this region [2]. It is generally believed that the change in the slope around the knee is astrophysical in nature rather than any specific change in hadronic properties and/or interactions. Though not conclusive, there appears to be some evidence [3] that the composition is heavier above the knee region. If true, then a significant suppression of the very high energy ($\geq 10^5$ GeV) lepton fluxes is expected due to CR interactions during the journey downwards [4].

There is a sharp reduction of the lepton flux from pions and kaons (this component is called the conventional flux) above a few TeV [4]. This is due to the increasing competition between their interaction and decay lengths. Therefore, at very high energies, almost all the lepton flux is expected to arise from the semi-leptonic decays of heavy hadrons (this component is called the prompt flux), most noticeably the charmed hadrons. It must be mentioned that the B-hadrons can also contribute significantly to the lepton fluxes in this energy range. This perhaps becomes an important issue for the $\nu_{\tau}$ because the only source of $\nu_{\tau}$ in the charm sector is $D_s$ while there can be many $B$-hadron decay channels giving rise to $\tau$ and $\nu_{\tau}$. Another point of importance is the fact that the neutrinos propagating at such high energies have almost vanishing probabilities to oscillate to another flavour. Therefore, measurement of neutrino flux in this regime requires no corrections due to the oscillation phenomenon. Further, it is known that the prompt muon flux is only about 10% smaller than the prompt $\nu_{\mu}$ flux at the surface of the earth. Therefore, measurement of the prompt muon flux at high energies will act as a normalization for the prompt neutrino flux and a direct comparison of the two is both desirable and also necessary. The prompt muon flux, thus, is an object of great importance, not just from the above mentioned points but also from the fact that such muons are a major background to various neutrino experiments.

To this end, we require precise theoretical predictions for the prompt lepton fluxes. However, in reality the situation is drastically the opposite. Various phenomenological predictions for the prompt lepton fluxes span over two decades of interval [6, 7, 8, 9, 10, 11, 12]. The reason is mainly due to the vastly different choices for the charm production cross-section - perturbative QCD (pQCD) with a $K$ factor [6], next-to-leading order (NLO) pQCD [8, 11], quark-gluon string models and recombination quark-parton models [10]. Within the QCD based models, the problem can be traced back to the fact that the available data from accelerator/collider experiments is used as an input for the charm production (and also for other hadrons) at those experiments. This is then extrapolated to high energies and low-x values. Typical values sampled in high energy cosmic ray events, relevant for prompt muon flux via the charm production, correspond to $x \sim 10^{-5}$ and maybe smaller. There is no data available in this region and one is forced to extrapolate without clear guidelines. The gluon distribution function $g(x)$ extracted from the available data, and naively extrapolated to higher energies and lower $x$ values, is known to grow indefinitely, thereby causing concern regarding unitarity. Physically, it is expected that some mechanism should tame this sharply rising behaviour, though none is conclusively known at present. Often, the theoretical models assume a power law behaviour for the gluon distribution function, $xg(x) \sim x^{-\lambda}$, with $\lambda$ varying in the range $0 - 0.5$. The fluxes therefore depend strongly on the chosen value of $\lambda$. The data from $ep$ scattering at HERA, albeit at a very different energy and low-x regime, show a rise in the gluon distribution function. On the other hand, RHIC data at higher energies and for different compositions of the colliding particles (also later HERA data) gives some indication of partial taming of the rising distribution function. This is the so called saturation of gluon density at very low-x. Roughly speaking, at very low-x values, one can expect that the non-linear effects, recombination effects to become significant, thereby leading to the saturation or taming of the gluon density. For an early discussion of RHIC results and theoretical expectations, see [13]. The saturation models have been shown to fit the HERA data as well. An important lesson is to be very cautious in extrapolating the existing accelerator
data to cosmic ray energies. The saturation picture can be expected to be applicable at cosmic ray energies \[14\]. For a general overview of various theoretical issues in low-x and saturation physics see \[13\]. An overview of hadronic interactions and cosmic rays can be found in \[10]\.

The experimental situation is not very precise either at this stage. Various experiments \[17\] provide upper limits on the muon and neutrino fluxes in the energy range of interest. At present, these limits allow a large variation in the prompt fluxes. One can therefore expect that better measurements of muon fluxes can play a definitive role in selecting the charm production models, and thereby, also providing invaluable information about parton densities at such low-x and high energy values. Another related source of large theoretical uncertainties is strong dependence of the hadronic cross sections on the renormalization and factorization scales. This is partly related to the naive extrapolation of parton distribution functions to very different energy and x-values. For the case of conventional fluxes originating from the pions and kaons, these issues are in much better control and therefore the predictions stand on a sound footing.

In the present letter, we explore the possibility of utilizing the high energy prompt muon flux(es) in order to investigate whether the general expectations expressed above can in practice help in selecting the charm production model/parametrization. The range of models or parameterizations employed in the literature is too vast to be covered in this work. We choose some of the models often used and compare the predictions, incorporating the saturation model of Golec-Biernat and Wüthoff \[18\]. Also, it should be mentioned that for the case of muons and $\nu_\mu$, the flux from the beauty hadrons is not more than a few percent of that from the charm hadrons. Therefore, we ignore the beauty contributions in this work. However, for $\nu_\tau$ flux, there is almost 40% enhancement \[12\] and should be considered.

**Prompt muons and their detection** At higher and higher energies, the (prompt) muon flux is expected to reflect the onset of contributions from the production of heavy hadrons - the charmed and the beauty. The energy dependence of prompt muons (the flux is isotropic) follows the parent cosmic ray spectrum and is therefore harder. In contrast, the conventional flux energy spectrum is more steeply falling (almost by one extra power), caused due to the tension between the decay and interaction of the parent mesons while passing through the atmosphere. A schematic picture of the entire process, very similar to the conventional lepton flux, is as follows:

- **CR Flux** ($X = 0$) → **Flux** ($X$) → **Decay** → **Prompt Flux** → **To detector**

Of all the ingredients entering the calculation, the model for charm production has the biggest uncertainties as discussed above. Before proceeding further, let us have a look at the expression for the lepton flux. Assuming infinite isothermal atmospheric depth, the final expression for the flux, for lepton energies $E_l < \epsilon^{crit}_{charm} \approx 10^7$ GeV is (for zenith angle $< 60^\circ$, $\epsilon^{crit}_{i} = \frac{m_i h_0}{\epsilon_{i}^{crit}}$, $h_0 = 6.4$ km where $m_i, \tau_i$ are the rest mass and life time of the ith particle)

$$\phi_i(E_l) = Z_{Y_{i,l}}(E) \frac{\Lambda_Y(E)}{\Lambda_{N_{i,l}}(E)} N_{i,l} \frac{\phi_{N_{i,l}}(E, 0)}{\text{depletion}}$$

where $\Lambda_Y(E) = \frac{\lambda_{\nu_Y}(E)}{\lambda_{\nu_{N_{i,l}}}(E)}$ and $\lambda_{\nu_Y}(E) = \frac{\sigma_{\nu_Y}(X=0)}{\sigma_{\nu_Y}(X=H)}$ are the attenuation and interaction length of the nucleon in the atmosphere. $\phi_{N_{i,l}}(E, 0)$ is flux of the nucleon at depth $X=0$. $Y_{i,l}$ denotes various charm hadrons in the sequence - $D^0(\pm)$, $D_s$, $\Lambda_c$. For the total Nucleon-Air cross section, we adopt the parametrization \[12\]

$$\sigma_{NA}(E) = \left[ 280 - 8.7 \ln \frac{E}{\text{GeV}} \right] \text{mb}$$

(2)

Assuming scaling, spectrum weighted moments for a power law initial flux, $\phi_N(E, X = 0) = \phi_0 N^{- (\gamma + 1)}$, are defined as

$$Z_{N,j}(E) = \int_0^1 dxe^{(\gamma - 1)} \frac{1}{\sigma_{NA}(E)} \frac{d\sigma_{NA \rightarrow j}(E, x)}{dx}$$

(3)

where $x \approx \frac{E}{E_j}$ and $j$ denotes any of the final species. The charm hadron Z moments are related to the partonic level charm Z moments through their relative yields \[12, 20\], $Z_{N,j}(E) = f_j Z_{Nc}(E)$. The lepton production Z moments can be defined in an analogous fashion by replacing the production cross-section by the (differential) decay rates and plugging in the corresponding branching fractions \[21\].

It is therefore quite clear from the above expressions that the lepton fluxes at the end are quite sensitive to the charm production cross section. Till the knee, the cosmic ray flux and composition is rather established. The direct measurement of such high energy muons will require impractically large detectors with, presumably, very strong magnetic fields. Also the error in magnetic deflection of the muons at such large energies become comparable to the total angular deflection itself. However, there exists a clean method capable of capturing the energy information of the incoming high energy muons. The method, \textit{pair meter technique} \[22\], is based on measuring the individual muon energy via the energy deposition by fast muons in the dense matter. It is quasi well known that above a few TeVs, the dominant energy loss mechanism for muons while passing through the atmosphere is pair production, valid for the whole range of momentum transfer values. Further, at TeV energy or above, the differential energy loss, $dE_\mu/dx$ is linear in $E_\mu$, providing a clean estimation of muon energy. This technique has been successfully employed for a limited cosmic ray data \[23\] and the study confirms that the same method is capable of being used for even high energy muons. The requirement for such a measurement is to have the muons pass through a dense material like iron (or some other heavier material). In this
regard, it is worthwhile to emphasize that the typical expressions employed in most cosmic ray studies are Born cross sections. However, for heavier materials, there is a need to consider the Coulomb corrections arising due to all order resummation effects coming from $Z_{\text{atom}}$, where $Z$ is the atomic number of the material [21]. One can easily verify that for the case of iron, even at such large muon energies and for small momentum transfer, these corrections are at most a couple of percent. For lead, on the contrary, the Coulomb corrections are at most a couple of percent. For lead, on the contrary, the Coulomb corrections are at most a couple of percent. For lead, on the contrary, the Coulomb corrections are at most a couple of percent.

As our last representative model, we consider flux calculation within the saturation model proposed by Golec-Biernat and Wusthoff [18]. We follow [12] in parameterizing the charm production within this model. Within 10% accuracy, we can adopt the following parametrization

$$ x \frac{d\sigma}{dx}(p + \text{air} \to c + ..) = A x^\beta (1 - x^{1.2})^n $$

where $\beta = 0.05 - \ln(E/10 \text{ TeV})$ and

$$ n = \begin{cases} 7.6 + 0.025 \ln(E/10 \text{ TeV}) & 10^4 < E < 10^8 \text{ GeV} \\ 7.6 + 0.012 \ln(E/10 \text{ TeV}) & 10^8 < E < 10^{11} \text{ GeV} \end{cases} \quad (7) $$

$$ A = \begin{cases} 140 + (11 \ln(E/0.1 \text{ TeV}))^{1.65} & 10^4 < E < 10^8 \text{ GeV} \\ 4100 + 254 \ln(E/10^5 \text{ TeV}) & 10^8 < E < 10^{11} \text{ GeV} \end{cases} \quad (8) $$

A is expressed in nano-barns. For this model, we consider two cases: GBW1 - where the protons are taken to be the primary and GBW2 - where we include the effect of heavy elements also. On the basis of arguments presented earlier, these two cases should have a different nature, with the expectation that GBW2 should have a decreased number of muons.

Figure 1 shows the results for muon fluxes in various models. In Figure 2, we present the number of muons entering a typical detector per solid angle in a span of five years. To have a quick comparison, we present the same results in Table 2.

![FIG. 1: $E_\mu^3 \times$ Flux vs. muon energy $E_\mu$ in different flux models entering the underground detector after passing through the rock depth of $3.5 \times 10^3 \text{gm/cm}^2$.](image-url)
are quoted as number per steradian. However, a better quantity would be the total number of muons. Very naively, for the down-going muons that we are interested in, this would require multiplying these numbers by a factor $2\pi$. However, in reality this would perhaps be overestimating the number of muons. Therefore, a conservative multiplicative factor can be $\#\pi$ where $1 < \# < 2$. From the figures and the table, it is clear that direct measurement of muon spectrum using the pair meter technique is capable of distinguishing between various models of charm production. In particular, as expected, making the composition heavier reduces the number of prompt muons (the curve GBW2 in Fig 2).

The conventional contribution is dominant in this region. However, due to the fact that the conventional flux is much better known and the theoretical uncertainties are in better control there, this should not be a serious trouble. As far as the prompt muons are concerned, the conventional one should be viewed as a calculable background. Further, from Table 2 we notice that $E_\mu \sim 200$ TeV seems as a practical cut-off within the set up assumed here. However, let us remind ourselves that $E_{\mu\text{surface}} \sim 5E_\mu$ and $E_{CR} \sim 20E_{\mu\text{surface}}$ and the fact that very high energy muons do not lose significant amount of energy while traveling downwards. Therefore, a muon of given energy entering the detector is actually probing the parent cosmic ray of energy almost two orders of magnitude higher. We can thus expect to probe the hadron dynamics in the region around the knee, and also the possible change in composition, very easily with this technique. Before concluding we would like to mention that there is an indication of strangeness enhancement in high energy heavy ion collisions from RHIC and CERN-SPS (for a survey of results, see [27]). Taking clue from this, we expect the contribution from the strange hadrons to increase. In particular, for the heavy primary composition, the kaon component of the conventional flux and prompt component from $D_s$ is expected to change, though a detailed calculation is needed to quantify it.

In conclusion, we have explored the possibility of using pair meter technique to measure the prompt muon spectrum. This can be used to gain information about the charm production mechanism at these energies and therefore yield invaluable information about low-$x$ behaviour of gluon distributions. The range probed here is very different from what is expected to be probed at the LHC and therefore this will provide an important set of data points in the gluon distribution evolution plot. The results obtained are quite encouraging and indicate that prompt muons (leptons in general) have the capability of probing the charm (heavy hadron) production models and also the hypothesis of composition change around the knee region.

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[1] T. K. Gaisser, *Cosmic Rays and Particle Physics*, (Cambridge University Press, Cambridge) 1990; M. S. Longair, *High Energy Astrophysics*, (Cambridge University Press, Cambridge) 1992.
[2] M. Nagano and A. A. Watson, Rev. Mod. Phys. 72, 689 (2000); P. Bhattacharjee and G. Sigl, Phys. Reports 327, 109 (2000); Astropart. Phys. 16, 373, (2000).
[3] K. H. Kampert et al (KASCADE Collaboration) *Proceedings of the ICRC26*, OG.1.2.11; Acta Phys.Polon. B 35 1799, (2004) [astro-ph/0405608]; M. Aglietta et al (EAS-TOP Collaboration and MACRO Collaboration), Astropart. Phys. 20, 641, (2004).
slavskaya, Nuovo Cim. C12, 41, (1989).

[11] G. Gelmini, P. Gondolo and G. Varieschi Phys. Rev. D61, 036005, (2000); Phys. Rev. D61, 056011, (2000); Phys. Rev. D 67, 017301, (2003).

[12] A. Martin, M. Ryskin and A. Stasto, Acta Phys.Polon. B34, 3273, (2003) [hep-ph/0302140]

[13] J. Adams et. al., STAR Collaboration, Nucl. Phys. A757, 102 (2005).

[14] H. J. Drescher, A. Dumitru, M. Strikman, Phys.Rev.Lett. 94, 231801 (2005); hep-ph/0408073

[15] L. Frankfurt, M. Strikman and C. Weiss, Ann. Rev. Nucl. Part. Sci 55, 403 (2005); J. Jalilian-Marian, Y. V. Kovchegov, Prog. Part. Nucl. Phys. 56, 104 (2006).

[16] S. Ostapchenko, [hep-ph/0612068] R. Engel, Nucl. Phys. Proc. Suppl. 151, 437 (2006).

[17] M. Aglietta et. al., LVD Collaboration, Phys. Rev. D60 112001 (1999); M. Nagano et. al., Journ. of Physics G: Nucl. Phys. 12 69, (1986). [http://amanda.berkeley.edu/]

[18] K. Golec-Biernat, M. Wusthoff, Phys.Rev. D 59, 014017 (1999); K. Golec-Biernat, M. Wusthoff, Phys.Rev. D60, 114023 (1999).

[19] H. H. Mielke et. al, J. Phys. G 20, 637 (1994).

[20] S. Frixione et. al, Nucl. Phys. B 431, 453 (1994).

[21] E. V. Bugaev et al Phys. Rev. D 58, 054001, (1998).

[22] I. S. Alekseev and G. T. Zatsepin Proceedings of the ICRC1, 326, (1960); R. P. Kokoulin and A. A. Petrukhin, Nucl. Instrum. Methods Phys. Res. A 263, 468 (1988); R. P. Kokoulin and A. A. Petrukhin, Sov. J. Part. Nucl 21(3), 332 (1990).

[23] A. P. Chikkatur et al, NuTeV/CCFR Collaboration, Z. Phys. C74, 279, 1997.

[24] D. Ivanov, E. A. Kuraev, A. Schiller, V. G. Serbo, Phys.Lett. B 442, 453, 1998.

[25] See [http://www.imsc.res.in/~ino]

[26] [http://baikalweb.jinr.ru/]

[27] J. Rafelski, J. Letessier, [hep-ph/0610106]