Anderson transition: a novel route to high thermoelectric performance

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Anderson transition: a novel route to high thermoelectric performance

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Discovered exactly 200 years ago in 1821, thermoelectricity is nowadays of global interest as it allows to directly interconvert thermal and electrical energy via the Seebeck/Peltier effect, which could be exploited to enhance energy efficiency. In their seminal work, Mahan and Sofo mathematically derived the conditions for 'the best thermoelectric' - a delta-distribution-shaped electronic transport function, where charge carriers contribute to transport only in an infinitely narrow energy interval. So far, however, only approximations to this concept were expected to really exist in nature. Here, we propose as a physical realisation of this scenario the Anderson transition in an impurity band, i.e. the transition from Anderson-localised to extended quantum states. We obtained a significant enhancement and dramatic change of the thermoelectric properties from $p$-type to $n$-type in the stoichiometric Heusler compound Fe$_2$VAl, which we assign to a narrow region of delocalised electrons in the energy spectrum near the Fermi energy. We achieved this through an innovative approach of driving the Anderson transition via continuous disorder tuning: variable amounts of atomic defects are induced in a controlled fashion by thermal quenching from high temperatures (950 – 1380 °C). Based on our experimental electronic transport and magnetisation results, supported by Monte-Carlo and density functional theory calculations, we demonstrate a universal enhancement strategy towards colossal thermoelectric performance that is applicable to diverse material classes.

1 Thermoelectric (TE) devices are capable of converting wasted heat into useful electrical energy or act as Peltier coolers. Facing an increasing worldwide demand for efficient energy utilisation, the immense diversity of potential technologocial applications has sparked great interest. Still, TE devices are currently restrained in their applicability due to their limited efficiency. The dimensionless figure of merit $ZT = S^2σT/(κ_e + κ_ph)$, which is closely related to the conversion efficiency, comprises three material-dependent parameters. These are the thermopower $S$, the electrical conductivity $σ$ and the thermal conductivity $κ$, consisting of a contribution from electrons $κ_e$ and phonons $κ_{ph}$, which are the quanta of lattice vibrations. While considerable progress towards achieving high $ZT$ has been achieved so far by reducing $κ_{ph}$, increasing the electronic part of $ZT$ is a much more formidable, yet necessary task and new exotic concepts for enhancement are required.

2 In 1996, Mahan and Sofo mathematically identified 'the best thermoelectric' as an ideal system, characterised by an infinitely narrow delta-distribution-shaped transport function $Σ(E)$.

3 Here, we propose that this seemingly unrealisable mathematical concept becomes actually realised in real materials at the Anderson transition in an impurity band, as predicted theoretically. As sketched in Fig.1, such a transition occurs when the number of randomly distributed impurities increases above a critical value $x_c$, known as quantum percolation. Below $x_c$, all impurity states are Anderson-localised due to disorder. A singularity of the transport function occurs at $x_c$ when an infinitesimally small region of states in the density of states (DOS) becomes delocalised. This was explained by Mott in 1967 through the concept of 'mobility edges', which are two critical energies $E_{c1,2}$ that appear at the centre of an impurity band, separating localised states in the band tails from delocalised, extended states in the centre. Far above $x_c$, $E_{c1}$ and $E_{c2}$ shift towards the band edges, eventually delocalising all impurity states.

4 Our study is focused on the TE transport properties across the Anderson transition in an impurity band with

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Fig. 1: Schematic of the Anderson transition in an impurity band with increasing impurity concentration. When the number of randomly distributed impurities in a periodically ordered crystal increases, the impurity electrons remain localised below a threshold value due to Anderson localisation. At the Anderson transition, the critical density of impurities allows for delocalisation of an infinitely narrow energy region of extended states inside the localised impurity states. The delocalised impurity band is marked by two mobility edges $E_{c1}, E_{c2}$, which are critical energies that separate the localised from delocalised states.

two mobility edges. We experimentally realised such a scenario in undoped, stoichiometric bulk Fe$_2$VAl by controlling the degree of lattice disorder via thermal quenching. This Heusler compound recently became an excellent candidate for studying new TE optimisation strategies [12–14]. Our measurements of the electronic transport and magnetisation in this work, supported by Monte-Carlo and density functional theory (DFT) simulations, show clear evidence for a significant enhancement of the TE performance, which we attribute to the Anderson transition. In the following, we describe the structural, electronic and magnetic properties of disorder-tuned Fe$_2$VAl as obtained by our experiments and simulations; finally, we show the TE properties of the material at the Anderson transition.

Structural and electronic properties

Ternary Fe$_2$VAl forms a fully ordered L2$_1$ structure at low temperatures that undergoes two second-order structural phase transitions (see Fig. 2a) into the partly disordered B2 structure at $T_{B2} \approx 1100^\circ$C and fully disordered A2 structure at $T_{A2} \approx 1250^\circ$C [15]. Our Monte-Carlo simulations based on effective cluster interactions show how the degree of atomic disorder in bulk Fe$_2$VAl can be controlled by temperature, finding a remarkable agreement with the experimental L2$_1$–B2 transition temperature [16] (the B2–A2 transition temperature was overestimated as discussed in Methods). This allows us to semiquantitatively assess the concentration of Fe, V and Al atoms on the respective sublattices as depicted in Fig. 2b. Note that while the V/Al sublattice is fully disordered in the B2 phase, there is already a significant site exchange on the Fe sublattice (5–25% Fe antisites). The large amount of antisite defects obtained at high temperatures as a result of the thermal excitations can be partly frozen by ultrafast quenching our samples.

To illustrate the nature of localised Fe/V antisite electronic states near the Fermi energy $E_F$, we calculated the spin-polarised DOS by making use of the exact muffin-tin orbital coherent potential approximation method (EMT-CPA). This method allows to calculate the DOS of a single-impurity embedded in an infinitely large and ordered effective medium, mimicking the electronic and structural properties of an alloy in the dilute limit of antisite concentration $x_{AS} \rightarrow 0$. Figs 2c,d show the occurrence of sharp, hydrogen-like impurity states near $E_F$ for both FeV and V$_{Fe}$ defects, as compared to the fully ordered compound. Similar results are obtained for Fe$_{Al}$ impurity states (see Extended Data Fig. 2). Furthermore, the spin degeneracy...
is removed due to the strong correlation of Fe–d electrons, which leads to isolated magnetic impurities in the non-magnetic, ordered host matrix. With increasing quenching temperature and thus increasing antisite concentration, the randomly distributed isolated defects form a continuum of clusters with different sizes leading to a broadening of the localised electronic states (see Extended Data Fig.3). Eventually, a delocalised impurity band forms, i.e. the Anderson transition, as sketched in Fig.1. However, neither the CPA nor the supercell approach can determine the critical concentration for the Anderson delocalisation transition. To overcome this difficulty, more effortful methods like the typical local density of states calculation could be used as a means of possibly identifying such transitions in future works.

Magnetic properties

The formation of magnetic clusters predicted by our simulations shown in the previous section (for details see Methods) can be confirmed by our magnetisation measurements shown in Fig.3. Fig.3a exhibits the field-dependent magnetisation $M$ at $T = 4$ K for stoichiometric Fe$_2$VAI, heat-treated at different conditions. Measurements of the magnetisation have previously shown to be an effective way of probing Anderson-localised states in other semiconductors such as Si. The immediate saturation of $M$ at small fields observed in Fig.3, the absence of hysteresis as well as the strong curvature of isothermal Arrot plots (see Extended Data Fig.7) are strong indications that the magnetic properties are dominated by the localised antisite electrons, in line with our ab initio calculations. In Fig.3b, we compare the saturation magnetisation $M_{\text{sat}}$ of our samples with their quenching temperature $T_{\text{quench}}$. It can be clearly seen that $M_{\text{sat}}$ consistently increases for higher $T_{\text{quench}}$, corroborating the picture drawn by our ab initio Monte-Carlo calculations. Moreover, both $M_{\text{sat}}$ and the calculated concentration of Fe antisite defects, when rescaled to the experimental transition temperatures, increase in a similar fashion showing an abrupt increase near $T_{\text{A2}}$. This demonstrates that the rapid quenching method could successfully induce the magnetic antisites in these samples, which is in excellent qualitative agreement with our ab initio calculations.

Thermoelectric properties

Fig.3 shows the temperature-dependent electrical resistivity $\rho(T)$ and thermopower $S(T)$ of Fe$_2$VAI, measured in a wide temperature range from 4 to 800 K for samples heat-treated and quenched at various temperatures. Above 400–500 K, a semiconductor-like behaviour of the resistivity, $d\rho/dT < 0$, is found for all samples (see Fig.1a). This can be attributed to the intrinsic pseudogap of the compound. At lower temperatures, the behaviour modifies from semiconductor-like to metallic, $d\rho/dT > 0$, demonstrating the Anderson-Mott-type insulator-metal transition due to the delocalisation of antisite electrons with increasing $T_{\text{quench}}$. Indeed, the residual resistivity $\rho_0$ decreases by an order of magnitude with increasing $T_{\text{quench}}$, which also manifests itself by a substantial increase of the Hall carrier concentration (see Extended Data Fig.8a), over-compensating the increased number of scattering centres due to disorder. Furthermore, the appearance of metallic transport goes hand in hand with the development of a local maximum in $\rho(T)$ at a temperature $T_{\rho,\text{max}}$, which shifts to higher temperatures as $T_{\text{quench}}$ increases. In Fig.3b, $S(T)$ is shown from 4 to 800 K. As-cast and furnace-cooled samples display positive values of $S(T)$ and a pronounced maximum at $\approx 200$ K, consistent with the
narrow pseudogap band structure, where $E_F$ is situated near the valence band edge. As $T_{\text{quench}}$ and the antisite concentration increase, $S(T)$ becomes consistently smaller at all measured temperatures and even exhibits a sign reversal for $T_{\text{quench}} > 1000^\circ\text{C}$ over the whole temperature range. This implies a substantial negative contribution of the antisite electrons to $S(T)$ in order to account for the dramatic change of the thermopower, from large $p$-type to large $n$-type values.

Further evidence for the presence of Anderson-localised states near $E_F$ is given by a more detailed analysis of the temperature-dependent transport properties. As-cast and furnace-cooled samples, according to the respective magnetic measurements, represent the dilute limit of antisites, where $E_F$ is expected to be situated within the localised states. Indeed, the low-temperature resistivity of these samples (see upper panel Fig.4c) can be well described by phonon-assisted variable-range hopping (VRH) conduction:\(^{22}\)

$$\rho(T) \propto \exp \left( \left( \frac{T_0}{T} \right)^{1/(d+1)} \right),$$  \hspace{1cm} (1)

which specifies in 3D to $\rho(T) \propto \exp \left( \left( \frac{T_0}{T} \right)^{1/4} \right)$, where $T_0$ is the characteristic Mott temperature. $T_0$ inversely depends on the localisation length $\xi_L$, which diverges at the insulator-metal transition. The fitted values of $T_0$ are about 2 – 4 mK, which are at least five orders of magnitude lower than for VRH between localised donor and acceptor states in marginally doped semiconductors.\(^{23,24}\) Again, this low $T_0$ corroborates the picture of Anderson-localised states near $E_F$.\(^{25,26}\) The temperature-dependent behaviour of the thermopower $S(T) \propto T^{1/2}$ at low temperatures (see Fig.4c centre panel) is also consistent with VRH in 3D, corroborating the resistivity data. Finally, even the low-temperature Hall mobility $\mu_H(T)$ shows an unusual, almost constant, slightly increasing temperature dependence (see Fig.4c lower panel), consistent with localisation of charge carriers near $E_F$.\(^{28}\) This picture of Anderson-localised states close to the Fermi level also reconciles many other peculiar properties of this compound, e.g. metallic thermodynamic and photoemission data in spite of semiconductor-like transport properties\(^{20}\), negative magnetoresistance\(^{29}\), anomalous Hall effect\(^{30}\) etc., which have been an ongoing discussion over the past three decades.\(^{14,20,31}\)

**Discussion**

Fig.5 shows in detail the evolution of the electronic transport across the Anderson transition as well as the enormous TE performance that can be attributed to the contribution of the delocalised impurity band. In Fig.5, we plotted the
residual resistivity $\rho_0$ at 4 K and $T_{\rho,\text{max}}$ as a function of $T_{\text{quench}}$. With the spontaneous appearance of $T_{\rho,\text{max}} > 0$ around $T_{\text{quench}}^* = 1000 - 1050^\circ\text{C}$, $\rho_0$ simultaneously shows a pronounced kink. In Fig.5a, we show the peak values of the thermopower $S_{\text{max}}$ as well as the residual conductivity $\sigma_0$. Again around $T_{\text{quench}}^*$, $S(T)$ displays a sign reversal and $\sigma_0$ deviates from a linear scaling behaviour. These anomalies, together with the monotonous increase of $T_{\rho,\text{max}}$ and decrease of $\rho_0$, clearly indicate the continuous delocalisation of the impurity band (IB), as sketched in the insets of Fig.5b. 

Fig.5c shows the power factor (PF) of as-cast and 1380 $^\circ\text{C}$-quenched Fe$_2$VAl. Usually, the optimisation of thermoelectric materials involves changing the position of the Fermi level, i.e. the total number of electrons, while leaving the electronic structure unchanged, which is called rigid-band doping. Despite being undoped and not yet optimised, the maximum power factor of 1380 $^\circ\text{C}$-quenched Fe$_2$VAl is already 7.6 mW/mK$^2$, which is an enhancement by an order of magnitude compared to the as-cast sample and 40% higher than the best PF for optimised rigid-band doping in this system. Considering that the total thermopower $S_{\text{tot}}$ and total conductivity $\sigma_{\text{tot}}$ in a material with multiple electronic bands can be written as 

$$S_{\text{tot}} = \sum_i S_i \sigma_i,$$

$$\sigma_{\text{tot}} = \sum_i \sigma_i,$$

with $S_i, \sigma_i$ being the respective single-band contributions, we can estimate the contribution of the impurity band which led to the dramatic change in TE transport. Here, the index ($i = \{\text{pristine, impurity}\}$) refers to contributions from the pristine band structure and the delocalised impurity band. Bearing in mind that the transport properties of the as-cast sample with purely localised impurity states is mainly dominated by the pristine band structure, we can calculate the additional delocalised impurity contribution $S_{\text{imp}}, \sigma_{\text{imp}}$ from our measured data by solving the system of Eqs.2,3. The contribution of the impurity band to the total measured power factor of the 1380 $^\circ\text{C}$-quenched sample is plotted by red squares in Fig.5c. An extremely large PF of more than 18 mW/mK$^2$ at 400 K is found for a stoichiometric sample not yet optimally doped, exceeding that of the pristine compound by a factor of 30. It is noteworthy to mention that $\kappa_{\text{ph}}$ was also reduced by a factor of $2^{-3}$ due to the disorder introduced by quenching (see Extended Data Fig.8b). Consequently, this means that the disorder induced by thermal quenching could be a strategy that can enhance all thermoelectric properties at the same time, which is not achievable by conventional doping strategies. We expect that $ZT$ should be further greatly enhanced by optimising the position of $E_F$ and by reducing the background DOS of Fe$_2$VAl, which can be achieved by appropriate
co-substitutions with e.g. Si and Ta \cite{33}.

To make a prediction about potential ZT values in such systems near the Anderson transition, we developed a charge transport model for two mobility edges in an impurity band (see Extended Data Fig.9a). By least-squares-fitting $S(T)$ and $\rho(T)$ of 1380 °C-quenched Fe$_2$VAI, we found that a narrow impurity band, with a bandwidth of 0.03 eV, accounts best for the measured temperature dependencies (see Extended Data Fig.9b). The predictions of the model are shown in Fig.9. They reveal that if the chemical potential is placed optimally near the mobility edge, a colossal ZT > 9 could be achieved if $\kappa_{ph} \approx 2.3 \text{ Wm}^{-1}\text{K}^{-1}$, currently achievable in Fe$_2$VAI by thin film deposition \cite{33}.

Considering that this strategy might be well applicable to other thermoelectric materials, which generally have even smaller values $\kappa_{ph} \approx 1 \text{ Wm}^{-1}\text{K}^{-1}$ or lower, huge values of the figure of merit ZT $> 20$ are within reach. Such performances in terms of ZT excel those of current state-of-the-art thermoelectrics by an order of magnitude, paving the way for a paradigm shift in energy and cooling applications.

In summary, we theoretically and experimentally demonstrated how the interplay of electronic correlation, Anderson localisation and defect thermodynamics can significantly boost the TE performance in undoped bulk Fe$_2$VAI by passing across the Anderson transition, where electrons are delocalised but occupy only a narrow interval in energy space. Indeed, we showed that Mahan and Sŏfo’s best thermoelectric is not just a mathematical construct, but can be implemented in real materials by exploiting charge carriers at the Anderson transition in an impurity band. Moreover, controlling the level of disorder allows us to tune the optimal width of the energy-dependent transport function directly, which is not achievable by other band engineering strategies employed so far. Although disorder and charge localisation have been mostly considered as detrimental for thermoelectricity, our work discloses a novel paradigm to improve thermoelectric materials and devices via impurity conduction, employing temperature-induced disorder as a new tuning and control parameter.

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