Hamiltonian structure and noncommutativity in $p$-brane models with exotic supersymmetry

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Abstract

The Hamiltonian of the simplest super $p$-brane model preserving $3/4$ of the $D = 4$ $N = 1$ supersymmetry in the centrally extended symplectic superspace is derived and its symmetries are described. The constraints of the model are covariantly separated into the first- and the second-class sets and the Dirac brackets (D.B.) are constructed. We show the D.B. noncommutativity of the super $p$-brane coordinates and find the D.B. realization of the $OSp(1\mid8)$ superalgebra. Established is the coincidence of the D.B. and Poisson bracket realizations of the $OSp(1\mid8)$ superalgebra on the constraint surface and the absence there of anomaly terms in the commutation relations for the quantized generators of the superalgebra.

1 Introduction

As shown in [1] exotic BPS states preserving $\frac{M-1}{M}$ fraction of $N = 1$ supersymmetry can be realized by static configurations of free tensionless super $p$-branes ($p = 1, 2, \ldots$) with the action linear in derivatives$^1$. These static configurations were described by general solutions of the equations of motion of super $p$-branes evolving in superspace extended by tensor central charge (TCC) coordinates. Because of the $OSp(1\mid2M)$ global symmetry of the model, its static $p$-brane solution was formulated in terms of symplectic supertwistors previously used while studying superparticle models [3], [4], [5] and forming a subspace of the $Sp(2M)$ invariant symplectic space [6], [7]. As a result, the static form of the discussed supertwistor representation of the BPS brane solution is not static in terms of the original superspace-time and TCC coordinates. It is static only modulo transformations of enhanced $\kappa$-symmetry and its accompanying local symmetries, since the supertwistor components are invariant under

$^1$New Wess-Zumino like super $p$-brane models nonlinear in derivatives and preserving $\frac{M-1}{M}$ fraction of supersymmetry were recently proposed in [2].
these gauge symmetries, as shown in [8]. The unphysical \( p\)-brane motions related to the gauge symmetries were geometrically realized as the abelian shifts \([8]\) of the space-time and TCC coordinates by the Lorentz bivectors (generally multivectors) generalizing vector light-like Penrose shifts of the standard space-time coordinates [9]. Being inessential on the classical level of consideration, these shifts may turn out to be essential in the quantum dynamics of strings and branes. This necessitates quantum treatment of the model \([1]\) in the original variables that belong to the superspace extended by TCC coordinates and auxiliary spinor fields. An interest to this problem is stimulated by a conjectured relation of the tensionless strings with higher spin theories and free conformal SYM theories [10], [11], [7], as well as by the presence of higher spins in the quantized \( OSp(1|2M)\) invariant model of superparticle [12].

The Dirac analysis of the Hamiltonian structure of tensionless extended objects permits to outline some peculiarities of their quantum dynamics [13], [14], [15]. For the case of dynamical systems including the second-class constraints, such information is accumulated in the Dirac brackets used during the quantization procedure. The brane model \([1]\) contains the first- and the second-class constraints in view of the linear character of the Lagrangian in the world-volume time derivatives and the presence of the auxiliary spinor field parametrizing the string/brane momenta. Construction of the Dirac brackets for the discussed super \( p\)-brane model implies its Hamiltonian analysis with a covariant division of the first- and the second-class constraints.

As an example we solve this problem here for the \( D = 4\) \( N = 1\) super \( p\)-brane model and construct its Hamiltonian and Lorentz covariant Dirac brackets. We find that the Hamiltonian symplectic structure of the brane model encoded in the Dirac brackets is parametrized by only one dynamical variable \( \rho^\tau \) describing the proper time component of the vector density \( \rho^\mu \). A covariant reduction of the phase space excluding \( \rho^\tau \) leads to the appearance of a nonlocal factor in the Dirac brackets depending on the light-like projection of the super \( p\)-brane momentum. It exposes an important distinction of the brane dynamics from the superparticle dynamics which may turn out to be essential in the quantum picture. We start the investigation of this problem and find the Dirac bracket (D.B.) noncommutativity between the space-time, TCC and auxiliary brane coordinates. To study effects of the noncommutativity on the algebraic level we construct the D.B. realization of the \( OSp(1|8)\) superalgebra of the global symmetry of the model. Established is the coincidence between the D.B. and P.B. realizations of the superalgebra, but only on the primary constraint surface. Applying the \( \hat{q}\hat{p}\) ordering prescription, previously studied in [13], we consider a quantum realization of the \( OSp(1|8)\) superalgebra and establish that its commutation relations are anomaly free on the constraint surface.

## 2 Lagrangians for strings and branes with enhanced supersymmetry and symplectic twistor

A new simple model \([1]\) describes tensionless strings and \( p\)-branes spreading in the symplectic superspace \( \mathcal{M}^{susy}_M \). For \( M = 2^{[D]}_M \) \( (D = 2,3,4 mod 8)\) this superspace naturally associates with \( D\)-dimensional Minkowski space-time extended by the Majorana spinor \( \theta_a \) \( (a = 1,2,...,2^{[D]}_M)\) and the tensor central charge coordinates \( z_{ab} \) additively unified with the standard \( x_{ab} = x^m (\gamma_m C^{-1})_{ab} \) space-time coordinates in the symmetric spin-tensor \( Y_{ab} \). The supersymmetric and reparametrization invariant action of the model \([1]\)

\[
S_p = \frac{1}{2} \int d\tau d^p \sigma \rho^a U^a W_{\mu ab} U^b
\]  

(1)
includes the world-volume pullback
\[ W_{\mu ab} = \partial_\mu Y_{ab} - 2i(\partial_\mu \theta_a \theta_b + \partial_\mu \theta_b \theta_a) \]  
(2)
of the supersymmetric Cartan differential one-form \( W_{ab} = W_{\mu ab} d\xi^\mu \), where \( \partial_\mu \equiv \frac{\partial}{\partial \xi^\mu} \) and \( \xi^\mu = (\tau, \sigma^M) \), \( M = 1, 2, \ldots, p \) are world-volume coordinates. The local auxiliary Majorana spinor \( U^a(\tau, \sigma^M) \) parametrizes the generalized momentum \( P^{ab} = \rho^\mu U^a U^b \) of the tensionless \( p \)-brane and \( \rho^\mu(\tau, \sigma^M) \) is the world-volume vector density providing the reparametrization invariance of \( S_p \). This action has \((M-1)\) \( \kappa \)--symmetries and consequently preserves \( \frac{M-1}{M} \) fraction of the original global supersymmetry.

By the generalized Penrose transformation of variables
\[ Y_{ab} U^b = i\tilde{Y}_a + \tilde{\eta} \theta_a, \quad \tilde{\eta} = -2i(U^a \theta_a), \]  
(3)
where \( \tilde{\eta} \) is real Goldstone fermion associated with the spontaneous breakdown of \( \frac{1}{M} \) supersymmetry, the differential one-form \( U^a W_{ab} U^b \) is presented as
\[ U^a W_{ab} U^b = i\{U^a d\tilde{Y}_a - dU^a \tilde{Y}_a + d\tilde{\eta} \tilde{\eta}\} \equiv dY^\Lambda G_{\Lambda \Sigma} Y^\Sigma. \]  
(4)
The new object \( Y^\Lambda = (iU^a, \tilde{Y}_a, \tilde{\eta}) \) in (3), (4) is \( OSp(1|2M) \) supertwistor and \( G_{\Lambda \Sigma} = (-)^{\Lambda \Sigma + 1} G_{\Sigma \Lambda} \) is \( OSp(1|2M) \) invariant supersymplectic metric
\[ G_{\Lambda \Sigma} = \begin{pmatrix} \omega^{(2M)} & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\delta_a^b \\ \delta^a_b & 0 \end{pmatrix} \]  
(5)
which is the supersymmetric generalization of \( Sp(2M) \) symplectic metric \( \omega^{(2M)} \). In view of (3) and (4), the action \( S_p \) is presented in the supertwistor form
\[ S_p = \frac{1}{2} \int d\tau d\sigma \rho^\mu \partial_\mu Y^\Lambda G_{\Lambda \Sigma} Y^\Sigma \]  
(6)
that is apparently invariant under the global generalized superconformal \( OSp(1|2M) \) symmetry. For the particular case of \( D = 11 \) the action (6) invariant under the \( OSp(1|64) \) symmetry was considered in \( \text{[10]} \).

The original action (11) is invariant under \((M-1)\) \( \kappa \)--symmetries since the transformation parameter \( \kappa_a \) is restricted by only one real condition
\[ U^a \kappa_a = 0, \]  
(7)
as it follows from the transformation rules of the primary variables
\[ \delta_\kappa \theta_a = \kappa_a, \quad \delta_\kappa Y_{ab} = -2i(\theta_a \kappa_b + \theta_b \kappa_a), \quad \delta_\kappa U^a = 0. \]  
(8)
It is easy to show that all components of the supertwistor \( Y^\Lambda = (iU^a, \tilde{Y}_a, \tilde{\eta}) \) are invariant under \( \kappa \)--symmetry transformations (7), (8)
\[ \delta_\kappa \tilde{Y}_a = 0, \quad \delta_\kappa \tilde{\eta} = 0, \quad \delta_\kappa U^a = 0, \]  
(9)
so that the new representation of \( S_p \) includes only \( \kappa \)--invariant variables. Note that in 4--dimensional space-time \( Y^\Lambda \) contains only 9 real variables that is twice less than the number of the original variables \( Y_{ab}, \theta_a, U^a \).
3 Example of the $OSp(1|8)$ invariant string/brane model. Primary constraints

$OSp(1|8)$ is the global supersymmetry of the massless fields of all spins in $D = 4$ space-time extended by TCC coordinates \[3\]. Therefore, we study $D = 4$ example of the string/brane model \[4\] formulated in generalized $(4 + 6)$-dimensional space $M_{4+6}$ extended by the Grassmannian Majorana bispinor $\theta_a$. In this case the $D = 4$ $N = 1$ superalgebra

$$\{Q_a, Q_b\} = (\gamma^m C^{-1})_{ab} P_m + i(\gamma^m C^{-1})_{ab} Z_{mn}$$  \[10\]

includes the TCC two-form $Z_{mn}$, and the matrix coordinates $Y_{ab}$ are

$$Y_{ab} = x_{ab} + z_{ab},$$  \[11\]

where

$$x_{ab} = x_m (\gamma^m C^{-1})_{ab}, \quad z_{ab} = z_{mn} (\gamma^m C^{-1})_{ab}$$  \[12\]

with the charge conjugation matrix $C$ chosen to be imaginary in the Majorana representation. Here we use the same agreements about the spinor algebra as in \[5\].

In the Weyl basis real symmetric $4 \times 4$ matrix $Y_{ab}$ is presented as

$$Y^b_a = Y_{ad} C^{db} = \begin{pmatrix} z^\alpha_\beta & x^\alpha_\beta \\ \bar{x}^{\dot{\alpha}_\beta} & \bar{z}^{\dot{\alpha}_\beta} \end{pmatrix},$$  \[13\]

$$C^{ab} = \begin{pmatrix} \epsilon^{\alpha\beta} & 0 \\ 0 & \epsilon_{\dot{\alpha}\dot{\beta}} \end{pmatrix}.$$  \[14\]

In the $D = 4$ case the auxiliary Majorana spinor $U^a(\tau, \sigma^M)$ together with other auxiliary Majorana spinors $V^a(\tau, \sigma^M)$, $(\gamma_5 U)_a$ and $(\gamma_5 V)_a$ form a local spinor basis

$$U_a = \begin{pmatrix} u_\alpha \\ \overline{\theta}_\alpha \end{pmatrix}, \quad V_a = \begin{pmatrix} v_\alpha \\ \overline{\theta}_\alpha \end{pmatrix}, \quad (U \gamma_5 V) = -2i, \quad (UV) = 0$$  \[15\]

attached to string/brane world volume and the $\gamma_5$-matrix is

$$(\gamma_5)_a^b = \begin{pmatrix} -i\delta^\beta_\alpha & 0 \\ 0 & i\delta^\beta_\alpha \end{pmatrix}.$$  \[16\]

Respectively the linear independent Weyl spinors $u^n$ and $v^n$ may be identified with the local Neuman-Penrose dyad \[6\]

$$u^n u^\alpha \equiv u^n \varepsilon_{\alpha\beta} u^\beta = 1, \quad u^n u_\alpha = v^n v_\alpha = 0.$$  \[17\]

In the Weyl basis the action \[7\] acquires the form

$$S_p = \frac{1}{2} \int d\tau d\rho \rho^\mu \left( 2u^n \omega_{\mu\alpha\dot{\alpha}} u^\dot{\alpha} + u^n \omega_{\mu\alpha\beta} u^\beta + \bar{u}^\dot{\alpha} \bar{\omega}_{\mu\dot{\alpha}\dot{\beta}} \bar{u}^\dot{\beta} \right),$$  \[18\]

where the supersymmetric one-forms $\omega_{\mu\alpha\dot{\alpha}}$ and $\omega_{\mu\alpha\beta}$ are

$$\omega_{\mu\alpha\dot{\alpha}} = \partial_\mu x_{\alpha\dot{\alpha}} + 2i(\partial_\mu \theta_\alpha \bar{\theta}_{\dot{\alpha}} + \partial_\mu \bar{\theta}_\dot{\alpha} \theta_\alpha),$$

$$\omega_{\mu\alpha\beta} = -\partial_\mu z_{\alpha\beta} - 2i(\partial_\mu \theta_\alpha \theta_\beta + \partial_\mu \bar{\theta}_\dot{\beta} \theta_\alpha),$$

$$\bar{\omega}_{\mu\dot{\alpha}\dot{\beta}} = -\partial_\mu \bar{z}_{\dot{\alpha}\dot{\beta}} - 2i(\partial_\mu \bar{\theta}_{\dot{\alpha}} \bar{\theta}_{\dot{\beta}} + \partial_\mu \theta_{\alpha} \bar{\theta}_{\dot{\beta}}).$$  \[19\]
The momenta densities $\mathcal{P}^{\text{gr}}(\tau, \sigma^M)$

$$\mathcal{P}^{\text{gr}} = \frac{\partial L}{\partial \dot{Q}^{\text{gr}}} = (P^{\alpha \beta}, \pi^\alpha, \bar{\pi}^\beta, \bar{\pi}^{\dot{\alpha}}, \bar{\pi}^{\dot{\beta}}, P^\alpha, \bar{P}_u^\bar{\alpha}, \bar{P}_v^{\dot{\beta}}, P^{(\rho)})$$

are canonically conjugate to the coordinates

$$Q^{\text{gr}} = (x_{\alpha \dot{\alpha}}, z_{\alpha \dot{\beta}}, \bar{z}_{\dot{\alpha}}, u_\alpha, \bar{u}_{\dot{\alpha}}, v_\alpha, \bar{v}_{\dot{\beta}}, \rho^\mu)$$

with respect to the Poisson brackets

$$\{ \mathcal{P}^{\text{gr}}(\bar{\sigma}), Q^{\text{gr}}(\bar{\sigma}') \}_{P.B.} = \delta_3^{\text{gr}} \delta^p(\bar{\sigma} - \bar{\sigma}')$$

with the periodic $\delta-$function $\delta^p(\bar{\sigma} - \bar{\sigma}')$, where $\bar{\sigma} = (\sigma^1, ..., \sigma^p)$, for the case of closed string/brane studied here.

As far as $S^p$ (13) is linear in the proper time derivatives, it is characterized by the presence of the primary constraints. These constraints may be divided into four sectors.

The constraints from the Grassmannian $\Psi-$sector, where $\Psi = (\Psi^\alpha, \bar{\Psi}^{\dot{\alpha}})$, are given by

$$\Psi^\alpha = \pi^\alpha - 2i \theta_\dot{\alpha} P^{\dot{\alpha} \alpha} - 4i \pi^{\dot{\alpha} \dot{\beta}} \theta_\beta \approx 0,$$

$$\bar{\Psi}^{\dot{\alpha}} = - (\Psi^\alpha)^* = \bar{\pi}^{\dot{\alpha}} - 2i \bar{P}^{\dot{\alpha} \alpha} \bar{\theta}_\alpha - 4i \bar{\pi}^{\dot{\alpha} \dot{\beta}} \bar{\theta}_\beta \approx 0.$$  

The dyad or $(u, v)-$sector is formed by the constraints

$$P_u^\alpha \approx 0, \quad \bar{P}_u^{\dot{\alpha}} \approx 0, \quad P_v^\alpha \approx 0, \quad \bar{P}_v^{\dot{\alpha}} \approx 0,$$

$$\Xi \equiv u^\alpha u_\alpha - 1 \approx 0, \quad \bar{\Xi} \equiv \bar{u}^{\dot{\alpha}} \bar{u}_{\dot{\alpha}} - 1 \approx 0.$$  

Finally, the $\rho-$sector includes the constraints

$$P^{(\rho)}_{\mu} \approx 0, \quad \mu = (\tau, M), \quad M = (1, ..., p).$$

The constraints forming $\Phi-$sector have zero Poisson brackets (P.B.) among themselves and with the $\Psi-$sector constraints

$$\{ \Phi(\bar{\sigma}), \Phi(\bar{\sigma}') \}_{P.B.} = 0, \quad \{ \Phi(\bar{\sigma}), \Psi(\bar{\sigma}') \}_{P.B.} = 0.$$  

They also P.B. commute with $P_v^\alpha, \bar{P}_v^{\dot{\alpha}}, \Xi, \bar{\Xi}$ and $P^{(\rho)}_{M}$

$$\{ \Phi(\bar{\sigma}), P_v^\alpha(\bar{\sigma}') \}_{P.B.} = 0, \quad \{ \Phi(\bar{\sigma}), \Xi(\bar{\sigma}') \}_{P.B.} = 0, \quad \{ \Phi(\bar{\sigma}), P^{(\rho)}_{M}(\bar{\sigma}') \}_{P.B.} = 0.$$  

The $\Psi-$constraints have zero P.B. with the constraints from all other sectors, but have nonzero P.B. among themselves. Note that the constraints (23)- (26) do not contain the space-time $x_{\alpha \dot{\alpha}}$ and TCC $z_{\alpha \dot{\beta}}, \bar{z}_{\dot{\alpha}}^{\dot{\beta}}$ coordinates as well as the $\rho^M$ components.

To find all local symmetries of the brane action, it is necessary to split the constraints (23)- (26) into the first- and second-class constraints. Then the P.B. of the first-class constraints with the brane coordinates will generate the local symmetries in accordance with the Dirac prescription.
4 \( \Psi \)–sector: \((3 \oplus 1)\)–splitting and first-class constraints generating enhanced \( \kappa \)–symmetry

Here we find the Hamiltonian realization of the enhanced \( \kappa \)–symmetry generators. The generators of \( \kappa \)–symmetry are included into the \( \Psi \)–sector \[24\] of the primary constraints due to their Grassmannian graduation. To derive these constraints it is enough to study only Poisson brackets among the \( \Psi \)–constraints

\[
\{ \Psi^\alpha(\bar{\sigma}), \Psi^\beta(\bar{\sigma}') \}_\text{P.B.} = -8i\pi^{\alpha\beta}\delta^p(\bar{\sigma} - \bar{\sigma}'),
\]

\[
\{ \Psi^\alpha(\bar{\sigma}), \bar{\Psi}^\beta(\bar{\sigma}') \}_\text{P.B.} = -4i\Phi^{\beta\alpha}\delta^p(\bar{\sigma} - \bar{\sigma}'),
\]

because of the commutativity of the \( \Psi \)–sector with the others. Then upon the multiplication of \[24\], \[30\] by \( u_\beta(\tau, \bar{\sigma}') \) and \( \bar{u}_\beta(\tau, \bar{\sigma}') \), respectively, and using \[17\] we find

\[
\{ \Psi^\alpha(\bar{\sigma}), \Psi^{(u)}(\bar{\sigma}') \}_\text{P.B.} = -8i\Phi^{\alpha\beta}u_\beta\delta^p(\bar{\sigma} - \bar{\sigma}'),
\]

\[
\{ \Psi^\alpha(\bar{\sigma}), \bar{\Psi}^{(u)}(\bar{\sigma}') \}_\text{P.B.} = -4i\bar{u}_\beta\Phi^{\beta\alpha}\delta^p(\bar{\sigma} - \bar{\sigma}').
\]

This means that \( \Psi^{(u)} \) and \( \bar{\Psi}^{(u)} \)

\[
\Psi^{(u)} \equiv \Psi^\alpha u_\alpha \approx 0,
\]

\[
\bar{\Psi}^{(u)} \equiv \bar{\Psi}^{\alpha} \bar{u}_\alpha \approx 0
\]

are the first-class constraints. The Poisson brackets between the \( \Psi^{(u)} \), \( \bar{\Psi}^{(u)} \) are the following

\[
\{ \Psi^{(u)}(\bar{\sigma}), \bar{\Psi}^{(u)}(\bar{\sigma}') \}_\text{P.B.} = -4i\Phi^{(u)}(\bar{\sigma} - \bar{\sigma}') \approx 0,
\]

\[
\{ \Psi^{(u)}(\bar{\sigma}), \Psi^{(u)}(\bar{\sigma}') \}_\text{P.B.} = -8iT^{(u)}(\bar{\sigma} - \bar{\sigma}') \approx 0,
\]

\[
\{ \bar{\Psi}^{(u)}(\bar{\sigma}), \bar{\Psi}^{(u)}(\bar{\sigma}') \}_\text{P.B.} = -8i\bar{T}^{(u)}(\bar{\sigma} - \bar{\sigma}') \approx 0,
\]

where \( T^{(u)} \) and \( \Phi^{(u)} \) are the constraints

\[
T^{(u)} \equiv u_\alpha \Phi^{\alpha\beta}u_\beta \approx 0,
\]

\[
\bar{T}^{(u)} = (T^{(u)})^*,
\]

\[
\Phi^{(u)} \equiv \bar{u}_\beta \Phi^{\beta\alpha}u_\alpha.
\]

The remaining two constraints from the \( \Psi \)–sector are presented by the projections

\[
\Psi^{(v)} \equiv \Psi^\alpha v_\alpha,
\]

\[
\bar{\Psi}^{(v)} \equiv \bar{\Psi}^{\alpha} \bar{v}_\alpha.
\]

Projecting the Poisson brackets \[24\], \[30\] on the spinors \( v_\beta(\tau, \bar{\sigma}') \) and \( \bar{v}_\beta(\tau, \bar{\sigma}') \) and summing up the resulting expressions we find

\[
\{ \Psi^\alpha(\bar{\sigma}), \Psi_R^{(v)}(\bar{\sigma}') \}_\text{P.B.} = -4i(\bar{v}_\beta \Phi^{\beta\alpha}v_\beta + 2\Psi^{\alpha\beta}v_\beta)\delta^p(\bar{\sigma} - \bar{\sigma}') \approx 0,
\]

where the real constraint \( \Psi_R^{(v)} \) is defined by the sum

\[
\Psi_R^{(v)} \equiv \Psi^{(v)} + \bar{\Psi}^{(v)} \approx 0.
\]

The Poisson brackets \[36\] show that \( \Psi_R^{(v)} \) is the real first-class constraint.

Multiplying \[36\] by \( u_\alpha(\tau, \bar{\sigma}) \) we find

\[
\{ \Psi^{(u)}(\bar{\sigma}), \Psi_R^{(v)}(\bar{\sigma}') \}_\text{P.B.} = -4i(\bar{v}_\beta \Phi^{\beta\alpha}u_\alpha + 2u_\alpha \Phi^{\alpha\beta}v_\beta)\delta^p(\bar{\sigma} - \bar{\sigma}') \approx 0,
\]

\[
\{ \bar{\Psi}^{(u)}(\bar{\sigma}), \Psi_R^{(v)}(\bar{\sigma}') \}_\text{P.B.} = -4i(\bar{u}_\beta \Phi^{\beta\alpha}v_\alpha + 2\bar{u}_\alpha \Phi^{\alpha\beta}\bar{v}_\beta)\delta^p(\bar{\sigma} - \bar{\sigma}') \approx 0.
\]
The Poisson brackets for $Ψ_R^{(v)}$ with itself are
\[ \{Ψ_R^{(v)}(\bar{σ}), Ψ_R^{(v)}(\bar{σ}')\}_{P.B.} = -8iT_R^{(v)}δ^p(\bar{σ} - \bar{σ}') ≈ 0, \]
where the real constraint $T_R^{(v)}$ from the $Φ$-sector is defined as
\[ T_R^{(v)} ≡ v_αΦ^αβv_β, \]
\[ T^{(v)} ≡ v_αΦ^αβv_β, \quad Ψ^{(v)} = \tilde{Ψ}^v_αv_α. \]
(40)

Thus, the $Ψ$-sector constraints are split into the three real constraints $Ψ^{(u)}$, $\tilde{Ψ}^{(u)}$ and $Ψ_R^{(v)}$ belonging to the first-class and one real constraint $Ψ_I^{(v)}$ given by the imaginary part of $Ψ^{(v)}$
\[ Ψ_I^{(v)} = i(Ψ^αv_α - \tilde{Ψ}^α\tilde{v}_α) ≈ 0. \]
(41)

The calculation of the Poisson brackets for $Ψ_I^{(v)}$ with itself yields
\[ \{Ψ_I^{(v)}(\bar{σ}), Ψ_I^{(v)}(\bar{σ}')\}_{P.B.} = 8i(T_R^{(v)} - Ψ^{(v)} - 2ρ^τ)δ^p(\bar{σ} - \bar{σ}'), \]
resulting to the weak equality
\[ \{Ψ_I^{(v)}(\bar{σ}), Ψ_I^{(v)}(\bar{σ}')\}_{P.B.} ≈ -16iρ^τδ^p(\bar{σ} - \bar{σ}'), \]
(42)
proving that $Ψ_I^{(v)}$ is the second-class constraint. Therefore, value of the nonzero world-volume field $ρ^τ$ measures breaking of the fourth $κ$-symmetry of the brane model.

Using the definition of canonical Poisson brackets
\[ \{π^α(\bar{σ}), θ_β(\bar{σ}')\}_{P.B.} = δ^α_βδ^p(\bar{σ} - \bar{σ}'), \quad \{\tilde{π}^α(\bar{σ}), \tilde{θ}_β(\bar{σ}')\}_{P.B.} = δ^α_βδ^p(\bar{σ} - \bar{σ}'), \]
(43)
we find the transformations of the $θ$-coordinates under the charges corresponding to the first-class constraints $Ψ^{(u)}$, $\tilde{Ψ}^{(u)}$ and $Ψ_R^{(v)}$
\[ δ_κθ_α = \{∫ d^pσ'κΨ^{(u)}(\bar{σ}'), θ_α(\bar{σ})\}_{P.B.} = κu_α, \]
\[ δ_κ\tilde{θ}_α = \{∫ d^pσ'κΨ^{(u)}(\bar{σ}'), \tilde{θ}_α(\bar{σ})\}_{P.B.} = \tilde{κ}u_α, \]
\[ δ_κx_{α\bar{α}} = -2i(κu_α\tilde{θ}_\bar{α} + \tilde{κ}u_\bar{α}θ_α), \quad δ_κz_{αβ} = -2i(u_αθ_β + u_βθ_α); \]
\[ δ_κθ_α = \{∫ d^pσ'κΨ_R^{(v)}(\bar{σ}'), θ_α(\bar{σ})\}_{P.B.} = κ_Rv_α, \]
\[ δ_κ\tilde{θ}_α = \{∫ d^pσ'κΨ_R^{(v)}(\bar{σ}'), \tilde{θ}_α(\bar{σ})\}_{P.B.} = \tilde{κ}_R\tilde{v}_α, \]
\[ δ_κx_{α\bar{α}} = -2iκ_R(v_α\tilde{θ}_\bar{α} + \tilde{v}_\bar{α}θ_α), \quad δ_κz_{αβ} = -2iκ_R(v_αθ_β + v_βθ_α). \]
(45)

To connect the transformations (45) with the original $κ$-symmetry transformations, let us expand $κ$ in the dyad basis
\[ κ_α = -(κ_βv^β)u_α + (κ_βu^β)v_α = -(κ_βv^β)u_α + [Re(κ_βu^β) + iIm(κ_βu^β)]v_α. \]
(46)

Then the transformations (45) are presented in the form
\[ δ_κθ_α = -(κ_βv^β)u_α + Re(κ_βu^β)v_α, \]
(47)
in view of the condition $Im(κ_βu^β) = 0$ equivalent to (7). Comparison of (45) and (47) shows that the transformations (45) are the original $κ$-symmetry transformations with the complex parameter $κ$ and the real parameter $κ_R$ connected with the original parameters $κ_α$ by the relations
\[ κ = -κ_αv^α, \quad κ_R = Re(κ_αu^α). \]
(48)

Therefore, we proved that the first-class constraints $Ψ^{(u)}$, $Ψ^{(u)}$ and $Ψ_R^{(v)}$ are the generators of three $κ$-symmetries since their Poisson brackets are closed by the constraints from the $Φ$-sector. We shall comment these Poisson brackets in the next section, where the division of the $Φ$-sector into the first- and the second-class constraints will be considered.
5  $\Phi$–sector: $(6 \oplus 4)$—splitting and first-class constraints generating new local symmetries

The Poisson brackets \([33]\) of the $\kappa$–symmetry constraints $\Psi^{(u)}$ and $\bar{\Psi}^{(u)}$ are closed by the constraints $\Phi^{(u)}$ and $T^{(u)}$ \([34]\) from the $\Phi$–sector. Let us show that the latter constraints are also the first-class ones. It is easy to see that the constraint $\Phi^{(u)}$ transforms the $x$–coordinates but leaves all other variables in $S_p$ \([18]\) intact. The transformation of the $x$–coordinates is

$$
\delta_{\Phi^{(u)}} x_{\alpha\dot{\alpha}} = \left\{ \int d^p \sigma' \epsilon_{\Phi^{(u)}}(\bar{\sigma}'), x_{\alpha\dot{\alpha}}(\bar{\sigma}) \right\}_{P.B.} = \epsilon_{\Phi^{(u)}} u_\alpha \bar{u}_{\dot{\alpha}}
$$

(49)

and is the local symmetry of the action $S_p$ \([18]\)

$$
\delta_{\Phi^{(u)}} S_p = \int d\tau d^p \sigma \rho^\mu \partial_\mu (\epsilon_{\Phi^{(u)}} u_\alpha \bar{u}_{\dot{\alpha}}) \bar{u}^\alpha = 0
$$

(50)

due to the relation \([17]\) $u^\alpha u_\alpha = 0$. Consequently, the constraint $\Phi^{(u)}$ is the first-class one. The transformation \([19]\) is the local shift of $x_m$ along the light-like 4–vector $(u\sigma_m \bar{u})$

$$
\delta_{\Phi^{(u)}} x_m = -\frac{1}{2} \epsilon_{\Phi^{(u)}}(u\sigma_m \bar{u})
$$

(51)

and can be rewritten as a weak equality

$$
\delta_{\Phi^{(u)}} x_m \approx \tilde{\epsilon}_{\Phi^{(u)}} P_m,
$$

(52)

where $P_m = -2 \tilde{P}^{\alpha\dot{\alpha}} \sigma_{m\alpha\dot{\alpha}}$ and $\tilde{\epsilon}_{\Phi^{(u)}} = \frac{\epsilon_{\Phi^{(u)}}}{\rho^2}$. On the contrary, the constraints $T^{(u)}$ and $\tilde{T}^{(u)}$ change only the TCC coordinates $z_{\alpha\beta}$

$$
\delta_{T^{(u)}} z_{\alpha\beta} = \epsilon_{T^{(u)}} u_\alpha u_\beta, \quad \delta_{\tilde{T}^{(u)}} z_{\alpha\beta} = \tilde{\epsilon}_{\tilde{T}^{(u)}} \bar{u}_{\dot{\alpha}} \bar{u}_{\dot{\beta}},
$$

(53)

as it follows after utilization of the canonical Poisson brackets

$$
\{\pi^{\alpha\beta}(\bar{\sigma}), z_{\gamma\delta}(\bar{\sigma}')\}_{P.B.} = \frac{1}{2}(\delta^{\alpha}_{\gamma} \delta^{\beta}_{\delta} + \delta^{\beta}_{\gamma} \delta^{\alpha}_{\delta}) \delta^\mu_{\bar{\sigma} - \bar{\sigma}'}.
$$

(54)

The transformations \([33]\) do not change the action \([18]\)

$$
\delta_{T^{(u)}} S_p = \frac{1}{2} \int d\tau d^p \sigma \rho^\mu [u^\alpha \partial_\mu (\epsilon_{T^{(u)}} u_\alpha u_\beta)] u^\beta + c.c.] = 0
$$

(55)

and, consequently, are the first-class constraints too.

To establish the geometric sense of the transformations \([53]\) let us multiply them by $(\sigma_{mn} \varepsilon)^{\alpha\beta}$ and $(\bar{\sigma}_{mn} \varepsilon)^{\dot{\alpha}\dot{\beta}}$, respectively, and sum up the results. Then we find

$$
\delta_{T^{(u)}} z_{mn} = -\frac{i}{4} [\epsilon_{T^{(R)}}(u\sigma_{mn} u + \bar{u}\bar{\sigma}_{mn} \bar{u}) + \epsilon_{T^{(I)}}(u\sigma_{mn} u - \bar{u}\bar{\sigma}_{mn} \bar{u})]
$$

(56)

using the relation \([17]\)

$$
z_{mn} = -\frac{i}{4} [z_{\alpha\beta} \sigma_{mn} \alpha^{\beta} + z_{\dot{\alpha}\dot{\beta}} \bar{\sigma}_{mn} \dot{\alpha}^{\dot{\beta}}]
$$

(57)

connecting the spinor representation $z_{\alpha\beta}$ of the TCC coordinates with the tensor representation) $z_{mn} = -z_{nm}$. In terms of the Majorana spinor $U_\alpha$ \([15]\) the transformation \([56]\) is presented as

$$
\delta_{T^{(u)}} z_{mn} = \frac{i}{8} [\epsilon_{T^{(R)}}(U \gamma_{mn} U) + \epsilon_{T^{(I)}}(U \gamma_{mn} \gamma_5 U)],
$$

(58)
with the real parameters \( \epsilon^{(R)}_{T(u)} \) and \( \epsilon^{(I)}_{T(u)} \)
\[
\epsilon^{(R)}_{T(u)} = \frac{1}{2}(\epsilon_{T(u)} + \bar{\epsilon}_{T(u)}), \quad \epsilon^{(I)}_{T(u)} = \frac{1}{2i}(\epsilon_{T(u)} - \bar{\epsilon}_{T(u)}).
\]

Now let us take into account the observation \(^{18}\) that the bivectors \((\bar{U} \gamma_{mn}U)\) and \((\bar{U} \gamma_{mn} \gamma_5 U)\) are null tensors, i.e.
\[
(\bar{U} \gamma_{mn}U)^2 = 0, \quad (\bar{U} \gamma_{mn} \gamma_5 U)^2 = 0.
\]

This means that \( T^{(u)} \) and \( \bar{T}^{(u)} \) generate the local shifts of \( z_{mn} \) along the isotropic bivectors \(^{60}\) and these shifts are a natural generalization of the vector light-like shift \(^{51}\). On the other hand, these shifts may be presented as the local shifts along the TCC momentum
\[
\delta_T^{(u)} z_{\alpha\beta} \approx -2\bar{\epsilon}_{T^{(u)}} \pi_{\alpha\beta}, \quad \delta_{\bar{T}}^{(u)} \bar{z}_{\dot{\alpha}\dot{\beta}} \approx -2\bar{\epsilon}_{\bar{T}^{(u)}} \bar{\pi}_{\dot{\alpha}\dot{\beta}}
\]
if the primary constraints \(^{23}\) are taken into account. Thus, the Poisson brackets \(^{33}\) can be presented in the form including only the first-class constraints
\[
\{\Psi^{(u)}(\vec{\sigma}), \Psi^{(u)}(\vec{\sigma}')\}_{P.B.} = -8i(u \pi u) \delta^p(\vec{\sigma} - \vec{\sigma}'), \\
\{\tilde{\Psi}^{(u)}(\vec{\sigma}), \Psi^{(u)}(\vec{\sigma}')\}_{P.B.} = -4i(u \pi u) \delta^p(\vec{\sigma} - \vec{\sigma}'), \\
\{\tilde{\Psi}^{(u)}(\vec{\sigma}), \tilde{\Psi}^{(u)}(\vec{\sigma}')\}_{P.B.} = -4i(u \pi u) \delta^p(\vec{\sigma} - \vec{\sigma}'),
\]
where the r.h.s. of \(^{62}\) are the vector \( P^{\dot{\alpha}\alpha} \) and the tensor \( \pi^{\alpha\beta} \) momenta projections on the isotropic (bi)vectors.

The next first-class constraint from the \( \Phi \)-sector is the constraint \(^{10}\) \( \bar{T}_R^{(v)} \), which closes the Poisson brackets \(^{39}\) for the extra \( \kappa \)-symmetry generator \( \Psi_R^{(v)} \). To prove this observation let us note that \( \bar{T}_R^{(v)} \) transforms only the variables from the \( \Phi \)-sector \( x_{\alpha\dot{\alpha}}, z_{\alpha\beta}, \bar{z}_{\dot{\alpha}\dot{\beta}} \)
\[
\delta_{\bar{T}_R^{(v)}} x_{\alpha\dot{\alpha}} = \epsilon_{\bar{T}_R^{(v)}} v_{\alpha\dot{\alpha}}, \quad \delta_{\bar{T}_R^{(v)}} z_{\alpha\beta} = \epsilon_{\bar{T}_R^{(v)}} v_{\alpha\beta}, \quad \delta_{\bar{T}_R^{(v)}} \bar{z}_{\dot{\alpha}\dot{\beta}} = \epsilon_{\bar{T}_R^{(v)}} \bar{v}_{\dot{\alpha}\dot{\beta}}.
\]
Using the transformations \(^{63}\) we find
\[
\delta_{\bar{T}_R^{(v)}} S_\rho = \int d\tau d^p \partial' \partial_{\rho} \epsilon_{\bar{T}_R^{(v)}} [u^\alpha v_\alpha]^2 = 0
\]
and conclude that this transformation is a local symmetry of the brane action \(^{18}\). The symmetry transformation \(^{63}\) describes local shifts of \( x_m \) and \( z_{mn} \) coordinates along the second light-like direction \( (v \sigma_m \bar{v}) \) formed by the dyad \( v_{\alpha} \). But, unlike the light-like shifts \(^{51}\), \(^{33}\), the shifts \(^{63}\) are admissible only due to the mutual cancellation between the \( x \) and \( z \) contributions into the action variation \(^{63}\).

This result hints that shifts in the directions transversal to the light-like ones \( u_{\alpha} \bar{u}_{\dot{\alpha}}, v_{\alpha} \bar{v}_{\dot{\alpha}} \) and \( u_{\alpha} u_{\beta}, v_{\alpha} v_{\beta} \) may be the symmetries of the action \(^{18}\) too. To answer this question let us remind that the real basic orts \( m^{(\pm)}_n \) of the local tetrad which are orthogonal to the real light-like orts \( n^{(\pm)}_n \) are given by \(^{17},^{18}\)
\[
m^{(+)\alpha\dot{\alpha}} = u_{\alpha} \bar{v}_{\dot{\alpha}} + v_{\alpha} \bar{u}_{\dot{\alpha}}, \quad m^{(-)\alpha\dot{\alpha}} = i(u_{\alpha} \bar{v}_{\dot{\alpha}} - v_{\alpha} \bar{u}_{\dot{\alpha}}).
\]

The local shifts of the \( x \)-coordinates in the transverse directions \( m^{(\pm)\alpha\dot{\alpha}} \)
\[
\delta_{\Phi^{(+)\alpha\dot{\alpha}}} x^{(+)\alpha\dot{\alpha}} = \epsilon_{\Phi^{(+)\alpha\dot{\alpha}}} m^{(+)\alpha\dot{\alpha}}, \quad \delta_{\Phi^{(-)\alpha\dot{\alpha}}} x^{(-)\alpha\dot{\alpha}} = \epsilon_{\Phi^{(-)\alpha\dot{\alpha}}} m^{(-)\alpha\dot{\alpha}}
\]
generated by the constraints $\Phi^{(\pm)}$

$$\Phi^{(+)} \equiv \Phi^{a} m_{\alpha a}^{(+)} \approx 0, \quad \Phi^{(-)} \equiv \Phi^{a} m_{\alpha a}^{(-)} \approx 0 \quad (67)$$

change the action $[18]$ 

$$\delta_{\Phi^{(+)}} S_p = \int d\tau d^p \sigma \rho^k \epsilon_{\Phi^{(+)}} (u^a \partial_\mu u_\alpha + \bar{u}^\alpha \partial_\mu \bar{u}_\alpha),$$

$$\delta_{\Phi^{(-)}} S_p = i \int d\tau d^p \sigma \rho^k \epsilon_{\Phi^{(-)}} (u^a \partial_\mu u_\alpha - \bar{u}^\alpha \partial_\mu \bar{u}_\alpha). \quad (68)$$

However, these variations may be compensated by the corresponding transformations of the TCC coordinates $z_{\alpha \beta}$

$$\delta_{T^{(+)}} z_{\alpha \beta} = \epsilon_{\Phi^{(+)}} u_{\alpha} \beta, \quad \delta_{T^{(+)}} \bar{z}_{\alpha \beta} = \epsilon_{\Phi^{(+)}} \bar{u}_{\alpha} \beta;$$

$$\delta_{T^{(-)}} z_{\alpha \beta} = i \epsilon_{\Phi^{(-)}} u_{\alpha} \beta, \quad \delta_{T^{(-)}} \bar{z}_{\alpha \beta} = -i \epsilon_{\Phi^{(-)}} \bar{u}_{\alpha} \beta \quad (69)$$

generated by the constraints $T^{(+)}$ and $T^{(-)}$

$$T^{(+)} \equiv \Phi^{a} \beta u_{\alpha} \beta + \Phi^{a} \beta \bar{u}_{\alpha} \beta = \Phi^{a} \beta u_{\alpha} \beta + \Phi^{a} \beta \bar{u}_{\alpha} \beta \approx 0,$$

$$T^{(-)} \equiv i[\Phi^{a} \beta u_{\alpha} \beta] - \Phi^{a} \beta \bar{u}_{\alpha} \beta = i[\Phi^{a} \beta u_{\alpha} \beta] - \Phi^{a} \beta \bar{u}_{\alpha} \beta \approx 0, \quad (70)$$

where $u_{\alpha} \beta = \frac{1}{2} (u_{\alpha} \beta + \beta u_{\alpha} \beta)$. In fact, the variations of the action $[18]$ generated by the doubled constraints $2T^{(+)}$ and $2T^{(-)}$

$$\delta_{2T^{(+)}} S = - \int d\tau d^p \sigma \rho^k \epsilon_{\Phi^{(+)}} (u^a \partial_\mu u_\alpha + \bar{u}^\alpha \partial_\mu \bar{u}_\alpha),$$

$$\delta_{2T^{(-)}} S = -i \int d\tau d^p \sigma \rho^k \epsilon_{\Phi^{(-)}} (u^a \partial_\mu u_\alpha - \bar{u}^\alpha \partial_\mu \bar{u}_\alpha) \quad (71)$$

exactly compensate the variations $[68]$. It proves that the two real constraints $\bar{T}^{(\pm)}$ belong to the first class

$$\bar{T}^{(+)} \equiv \Phi^{(+)} + 2T^{(+)} = \Phi^{a} \alpha (u_{\alpha} \bar{v}_\alpha + v_{\alpha} \bar{u}_\alpha) + 2 \left( \Phi^{a} \beta u_{\alpha} \beta + \Phi^{a} \beta \bar{u}_{\alpha} \beta \right) \approx 0,$$

$$\bar{T}^{(-)} \equiv \Phi^{(-)} + 2T^{(-)} = i \left[ \Phi^{a} \alpha (u_{\alpha} \bar{v}_\alpha - v_{\alpha} \bar{u}_\alpha) + 2 \left( \Phi^{a} \beta u_{\alpha} \beta - \Phi^{a} \beta \bar{u}_{\alpha} \beta \right) \right] \approx 0. \quad (72)$$

Thus, we constructed six real bosonic constraints $\Phi^{(u)}$, $T^{(u)}$, $\bar{T}^{(u)}$, $\tilde{T}^{(u)}$, $\tilde{T}^{(R)}$ and $\tilde{T}^{(+)}$ belonging to the first class out of the ten primary constraints $[23]$ of the $\Phi-$sector. Further we find additional first-class constraints that are certain linear combinations of the constraints from the $\Phi-$, $(u, v)$- and $\rho$-sectors.

### 6 Dyad sector: $2 \oplus 8-$splitting and first-class constraints

By analogy with the $\Psi$- and $\Phi$-sectors one can assume that the first-class constraints from the $(u, v)$-sector describe the local symmetries related to dyad shifts along themselves. The shifts of $v_{\alpha}$ along $u_{\alpha}$

$$\delta_{\epsilon} v_{\alpha} = \epsilon u_{\alpha}, \quad \delta_{\epsilon} \bar{v}_{\alpha} = \bar{\epsilon} \bar{u}_{\alpha} \quad (73)$$

are generated by the constraints $P^{(u)}_v$ and $\bar{P}^{(u)}_v$

$$P^{(u)}_v \equiv P^{a} u_{\alpha} \approx 0, \quad \bar{P}^{(u)}_v \equiv \bar{P}^{a} \bar{u}_{\alpha} \approx 0. \quad (74)$$
These shifts supply obvious local symmetry of the action \( (18) \) since \( S_p \) does not depend on \( v_\alpha \). The same is true for the primary constraints from \( \Phi, \Psi \) and \( \rho \)-sectors

\[
\{ P^{(u)}_v (\bar{\sigma}), \Phi (\bar{\sigma}') \}_{P.B.} = 0, \quad \{ P^{(u)}_v (\bar{\sigma}), \Psi (\bar{\sigma}') \}_{P.B.} = 0, \quad \{ P^{(u)}_v (\bar{\sigma}), P^{(\omega)}_\mu (\bar{\sigma}') \}_{P.B.} = 0. \tag{75}
\]

Moreover, these shifts do not change the \( \Xi \) and \( \bar{\Xi} \) constraints that depend on \( v_\alpha \) and \( \bar{v}_\alpha \)

\[
\{ P^{(u)}_v (\bar{\sigma}), \Xi (\bar{\sigma}') \}_{P.B.} = 0, \quad \{ \bar{P}^{(u)}_v (\bar{\sigma}), \bar{\Xi} (\bar{\sigma}') \}_{P.B.} = 0 \tag{76}
\]

as it follows from the canonical relations

\[
\{ P^{(u)}_v (\bar{\sigma}), v_\beta (\bar{\sigma}') \}_{P.B.} = \delta_\beta^\alpha \delta^p (\bar{\sigma} - \bar{\sigma}'), \quad \{ \bar{P}^{(u)}_v (\bar{\sigma}), \bar{v}_\beta (\bar{\sigma}') \}_{P.B.} = \delta_\beta^\alpha \delta^p (\bar{\sigma} - \bar{\sigma}') \tag{77}
\]

and their complex conjugate. Therefore, the two real constraints \( (74) \) are the first-class constraints, and they have zero Poisson brackets between themselves

\[
\{ P^{(u)}_v (\bar{\sigma}), P^{(u)}_v (\bar{\sigma}') \}_{P.B.} = 0, \quad \{ P^{(u)}_v (\bar{\sigma}), \bar{P}^{(u)}_v (\bar{\sigma}') \}_{P.B.} = 0. \tag{78}
\]

However, the \( v \)-shifts along themselves generated by the constraints \( P^{(v)}_v \) and \( \bar{P}^{(v)}_v \)

\[
P^{(v)}_v \equiv P^{(v)}_v v_\alpha \approx 0, \quad \bar{P}^{(v)}_v \equiv \bar{P}^{(v)}_v \bar{v}_\alpha \approx 0 \tag{79}
\]

do not preserve the constraints \( \Xi \) and \( \bar{\Xi} \)

\[
\{ P^{(v)}_v (\bar{\sigma}), \Xi (\bar{\sigma}') \}_{P.B.} = (1 + \Xi) \delta^p (\bar{\sigma} - \bar{\sigma}'), \quad \{ \bar{P}^{(v)}_v (\bar{\sigma}), \bar{\Xi} (\bar{\sigma}') \}_{P.B.} = (1 + \bar{\Xi}) \delta^p (\bar{\sigma} - \bar{\sigma}') \tag{80}
\]

As a result, the constraints \( (79) \), as well as the constraints \( P^{(u)}_u \) and \( \bar{P}^{(u)}_u \)

\[
P^{(u)}_u \equiv P^{(u)}_u u_\alpha \approx 0, \quad \bar{P}^{(u)}_u \equiv \bar{P}^{(u)}_u \bar{u}_\alpha \approx 0 \tag{81}
\]

are not the first-class ones.

Moreover, the \( u_\alpha \)-shifts along \( v_\alpha \), generated by the constraints \( P^{(v)}_u \) and \( \bar{P}^{(v)}_u \)

\[
P^{(v)}_u \equiv P^{(v)}_u v_\alpha \approx 0, \quad \bar{P}^{(v)}_u \equiv \bar{P}^{(v)}_u \bar{v}_\alpha \approx 0 \tag{82}
\]

which also have zero Poisson brackets with \( \Xi \) and \( \bar{\Xi} \)

\[
\{ P^{(v)}_u (\bar{\sigma}), \Xi (\bar{\sigma}') \}_{P.B.} = 0, \quad \{ \bar{P}^{(v)}_u (\bar{\sigma}), \bar{\Xi} (\bar{\sigma}') \}_{P.B.} = 0, \tag{83}
\]

are not the symmetries of \( S_p \). These constraints have also nonzero Poisson brackets with the \( \Phi \)-sector constraints

\[
\{ \Phi^{\alpha\beta} (\bar{\sigma}), P^{\beta} (\bar{\sigma}') \}_{P.B.} = \epsilon^{\alpha\beta \rho} u^\rho \bar{u}_\alpha \delta^p (\bar{\sigma} - \bar{\sigma}'), \\
\{ \Phi^{\alpha\beta} (\bar{\sigma}), P^{\alpha} (\bar{\sigma}') \}_{P.B.} = -\frac{1}{2} \rho^\alpha \epsilon^{\alpha\beta \gamma} v^\gamma (u^\beta + u^\beta u^\gamma) \delta^p (\bar{\sigma} - \bar{\sigma}'). \tag{84}
\]

where the canonical P.B. relations

\[
\{ u^\alpha (\bar{\sigma}), P^{\alpha} (\bar{\sigma}') \}_{P.B.} = -\epsilon^{\alpha\beta \rho} \delta^p (\bar{\sigma} - \bar{\sigma}') \tag{85}
\]

were used. After multiplication of \( (84) \) by \( v_\beta (\tau, \bar{\sigma}') \) and \( v_\gamma (\tau, \bar{\sigma}') \), respectively, we find

\[
\{ \Phi^{\alpha\beta} (\bar{\sigma}), P^{(v)}_u (\bar{\sigma}') \}_{P.B.} = \rho^\tau v^\alpha u^\beta \bar{u}_\alpha \delta^p (\bar{\sigma} - \bar{\sigma}'), \\
\{ \Phi^{\alpha\beta} (\bar{\sigma}), P^{(v)}_u (\bar{\sigma}') \}_{P.B.} = -\frac{1}{2} \rho^\tau (v^\alpha u^\beta + v^\beta u^\gamma) \delta^p (\bar{\sigma} - \bar{\sigma}'). \tag{86}
\]

where the r.h.s. of \( (86) \) do not belong to the primary constraints. So, we resume that the \((u, v)\)-sector itself includes only two real first-class constraints, but, as we shall see below, some of the considered shifts may be compensated by the transformations from the \( \rho \)-sector.
7 \( \rho \)-sector: \( p \oplus 1 \)-splitting and first-class constraints

The \( \rho \)-sector of the \( p \)-brane constraints \(^{(26)}\) includes \( p \) constraints of the first-class

\[ P_M^{(\rho)} \approx 0, \quad M = (1, \ldots, p). \] (87)

It follows from the observation that the corresponding canonically conjugate variables \( \rho^M \)

\[ \{ P_M^{(\rho)}(\bar{\sigma}), \rho^N(\bar{\sigma}') \}_{\text{P.B.}} = \delta_M^N \delta^p(\bar{\sigma} - \bar{\sigma}') \] (88)
do not enter the primary constraints \(^{(23)-(26)}\) and, consequently, \( P_M^{(\rho)} \) have zero Poisson brackets with all these constraints. \( P_M^{(\rho)} \) constraints correspond to redefinition of \( p \) space components \( \rho^M \) of the \((p + 1)\)-dimensional world-volume vector density \( \rho^\mu(\tau, \bar{\sigma}) \)

\[ \delta_\epsilon \rho^M = \epsilon^M(\tau, \bar{\sigma}). \] (89)

The transformations \(^{(89)}\) are new local symmetries of the action \( S_p \) due to arbitrariness in the definition of \( \rho^M \). On the other hand, the world-volume time component \( \rho^\tau \) enters in the \( \Phi \)-sector constraints and the remaining component \( P_\tau^{(\rho)} \) from the \( \rho \)-sector has nonzero Poisson brackets with this sector constraints

\[ \{ P_\tau^{(\rho)}(\bar{\sigma}), \Phi^\alpha(\bar{\sigma}') \}_{\text{P.B.}} = -u^\alpha \bar{u}^\dot{\alpha} \delta^p(\bar{\sigma} - \bar{\sigma}'), \]
\[ \{ P_\tau^{(\rho)}(\bar{\sigma}), \Phi^\alpha(\bar{\sigma}') \}_{\text{P.B.}} = \frac{1}{2} u^\alpha \bar{u}^\dot{\alpha} \delta^p(\bar{\sigma} - \bar{\sigma}'), \]
\[ \{ P_\tau^{(\rho)}(\bar{\sigma}), \Phi^\alpha(\bar{\sigma}') \}_{\text{P.B.}} = \frac{1}{2} \bar{u}^\dot{\alpha} \bar{u}^\dot{\beta} \delta^p(\bar{\sigma} - \bar{\sigma}'). \] (90)

However, the transformation of \( \rho^\tau \)

\[ \delta_\epsilon \rho^\tau = \epsilon^{\tau}(\tau, \bar{\sigma}) \] (91)
generated by the constraint \( P_\tau^{(\rho)} \), which is not the first-class one, may be compensated by the corresponding transformation of dyads as we shall show below.

8 The Weyl symmetry constraint

Here we find the first-class constraint showing the presence of the local Weyl symmetry of the brane action. Because the \( \Phi \)-sector constraints contain the dyad \( u_\alpha \), the transformation \(^{(91)}\) can be compensated by the following transformation of \( u_\alpha \)

\[ \delta_\epsilon u_\alpha = -\frac{\epsilon^\tau}{2 \rho^\tau} u_\alpha, \quad \delta_\epsilon \bar{u}^{\dot{\alpha}} = -\frac{\epsilon^\tau}{2 \rho^\tau} \bar{u}^{\dot{\alpha}}, \] (92)

generated by the constraint \( \Delta^{(u)} \)

\[ \Delta^{(u)} \equiv P_\alpha^{(u)} u_\alpha + \bar{P}_\dot{\alpha}^{(u)} \bar{u}^{\dot{\alpha}} - 2 \rho^\tau P_\tau^{(\rho)} \approx 0. \] (93)

However, the new constraint \(^{(93)}\) has nonzero Poisson brackets with \( \Xi \) and \( \bar{\Xi} \)

\[ \{ \Delta^{(u)}(\bar{\sigma}), \Xi(\bar{\sigma}') \}_{\text{P.B.}} = (1 + \Xi) \delta^p(\bar{\sigma} - \bar{\sigma}'), \quad \{ \Delta^{(u)}(\bar{\sigma}), \bar{\Xi}(\bar{\sigma}') \}_{\text{P.B.}} = (1 + \bar{\Xi}) \delta^p(\bar{\sigma} - \bar{\sigma}'). \] (94)

This noncommutativity can be easily corrected by the compensating transformation of \( v_\alpha \)

\[ \delta_\epsilon v_\alpha = \frac{\epsilon^\tau}{2 \rho^\tau} v_\alpha, \quad \delta_\epsilon \bar{v}^{\dot{\alpha}} = \frac{\epsilon^\tau}{2 \rho^\tau} \bar{v}^{\dot{\alpha}}. \] (95)
It implies the generalization of $\Delta^{(u)}$ into the new constraint $\Delta_W'$

$$\Delta_W' \equiv (P^\alpha u_\alpha + \bar{P}_u \bar{u}_\alpha) - (P^\alpha v_\alpha + \bar{P}_v \bar{v}_\alpha) - 2\rho^\tau P^{(\rho)}_\tau \approx 0$$

which has zero Poisson brackets with the $\Phi-$sector. Moreover, the Poisson brackets of $\Delta_W'$ with the primary constraints are equal to zero on the constraint surface. These properties of the constraint $\Delta_W'$ will be preserved after addition of the first-class constraints which transform $\Delta_W'$ to

$$\Delta_W \equiv (P^\alpha u_\alpha + \bar{P}_u \bar{u}_\alpha) - (P^\alpha v_\alpha + \bar{P}_v \bar{v}_\alpha) - 2\rho^\mu P^{(\rho)}_\mu \approx 0.$$ (97)

The local transformation generated by $\Delta_W$ is the dilation affecting only the auxiliary fields $u_\alpha, v_\alpha$ and $\rho^\mu$

$$\rho^{\mu} = e^{-2\Lambda} \rho^\mu, \quad u'_\alpha = e^{\Lambda} u_\alpha, \quad v'_\alpha = e^{-\Lambda} v_\alpha, \quad x'_{\alpha\dot{\alpha}} = x_{\alpha\dot{\alpha}}, \quad z'_{\alpha\beta} = z_{\alpha\beta}, \quad \theta'_{\alpha} = \theta_{\alpha}.$$ (98)

The transformation is identified with the Weyl symmetry of the $p-$brane action. From the string point of view, the Weyl invariants $\rho^\mu u_\alpha \bar{u}_{\dot{\alpha}}$ and $\rho^\mu u_\alpha u_\beta$ constructed from auxiliary world-volume fields are similar to the conventional Weyl invariant of tensile string

$$\sqrt{-g}g^{\mu\nu} \Leftrightarrow \rho^\mu u_\alpha \bar{u}_{\dot{\alpha}},$$ (99)

but here is the symmetry the tensionless super $p-$brane action.

The transformations can be used for the gauge fixing

$$\rho^M = 0.$$ (100)

Then the Weyl symmetry may be used to fix $\rho^\tau$ by the gauge condition

$$\rho^\tau (\tau, \bar{\sigma}) = 0,$$ (101)

or $\rho^\tau = \rho^0 = \text{const}.$

Below we shall show that the $\rho^M$ translations are supplemented by reparametrization transformation of the world-volume coordinates $\sigma_M$ generated by $p$ first-class constraints formed by intermixing of the primary constraints.

9 **Secondary constraints: $p$ space-like reparametrizations**

To find the first-class constraints associated with the reparametrizations of $p$ world-volume coordinates $\sigma_M$, let us consider the canonical Hamiltonian for $p-$brane action.

Using the standard definition of the canonical Hamiltonian density

$$H_0 = \dot{Q}_{29D^p - L} = \dot{x}_{\alpha\dot{\alpha}} P^{\alpha\dot{\alpha}} + \dot{\bar{z}}_{\alpha\dot{\alpha}} \bar{\pi}^{\alpha\dot{\alpha}} + \dot{z}_{\alpha\beta} \bar{\pi}^{\alpha\beta} + \dot{\bar{\pi}}^{\alpha\dot{\alpha}} + \dot{\bar{\pi}}^{\alpha\beta} + \bar{\pi}^{\alpha\dot{\alpha}} + (\dot{u}_\alpha P^\alpha + \dot{v}_\alpha P^\alpha + c.c.) + \rho^\mu P^{(\rho)}_\mu - L$$ (102)

and the $p-$brane Lagrangian we find

$$H_0 \approx -\rho^M [u^\alpha \omega_{M\alpha\dot{\alpha}} \bar{u}^{\dot{\alpha}} + \frac{1}{2} u^\alpha \omega_{M\alpha\beta} u^\beta + \frac{1}{2} \bar{u}^{\dot{\alpha}} \bar{\omega}_{M\dot{\alpha}\dot{\beta}} \bar{u}^{\dot{\beta}}],$$ (103)
where we omitted the momenta from the \((u, v)-\) and \(\rho\)-sectors equal to zero. Taking into account the definitions (23) one obtains

\[
H_0 \approx -\frac{\rho^M}{\rho^r} \left[ -(\Phi^{\alpha} - P^{\hat{\alpha}}) \omega_{Ma\hat{\alpha}} + (\Phi^{\alpha} - \pi^{\alpha\beta}) \omega_{Ma\beta} + (\Phi^{\hat{\alpha}} - \pi^{\hat{\alpha}\hat{\beta}}) \omega_{Ma\hat{\beta}} \right]
\]

\[
\approx -\frac{\rho^M}{\rho^r} \left[ P^{\hat{\alpha}} \omega_{Ma\hat{\alpha}} - \pi^{\alpha\beta} \omega_{Ma\beta} - \pi^{\hat{\alpha}\hat{\beta}} \omega_{Ma\hat{\beta}} \right].
\]

Due to the \(\Psi-\)constraints (24) there is the equality

\[
l^{(\Phi, \Psi)}_M \equiv P^{\hat{\alpha}} \partial_M x_{a\hat{\alpha}} + \pi^{\alpha\beta} \partial_M z_{a\beta} + \pi^{\hat{\alpha}\hat{\beta}} \partial_M \bar{z}_{\hat{\alpha}\hat{\beta}} + \partial_M \theta_a \pi^a + \partial_M \bar{\theta}_a \bar{\pi}^a,
\]

where \(l^{(\Phi, \Psi)}_M\) is the reparametrization generator of the \(\Phi-\) and \(\Psi-\)sector coordinates. The total reparametrization generator \(L_M\) is connected with \(l^{(\Phi, \Psi)}_M\) by the equality

\[
L_M = l^{(\Phi, \Psi)}_M + l^{(u,v,\rho)}_M,
\]

where \(l^{(u,v,\rho)}_M\) is the reparametrization generator for the coordinates \(u, v, \rho\)

\[
l^{(u,v,\rho)}_M \equiv \left( P^a \partial_M u_a + P^v \partial_M v_a \right) + \left( \tilde{P}^{\hat{a}} \partial_M \bar{u}_{\hat{a}} + \tilde{P}^{\hat{\alpha}} \partial_M \bar{v}_{\hat{\alpha}} \right) - \partial_M P^{(\rho)}_\rho \rho' \approx 0.
\]

It follows from Eqs. (105)-(107) that \(H_0\) (103) may be presented in the form

\[
H_0 \approx -\frac{\rho^M}{\rho^r} L_M, \quad (M = 1, ..., p).
\]

As a result of the Dirac selfconsistency condition for the constraints (87)

\[
\hat{p}^{(\rho)}_M = \int d^p \sigma' \{ H_T(\bar{\sigma}'), P^{(\rho)}_M(\bar{\sigma}') \} P.B. = \int d^p \sigma' \{ H_0(\bar{\sigma}'), P^{(\rho)}_M(\bar{\sigma}') \} P.B. = \frac{1}{\rho^r} L_M \approx 0
\]

we find \(p\) new constraints \(L_M\)

\[
L_M = P^{\hat{\alpha}} \omega_{Ma\hat{\alpha}} - \pi^{\alpha\beta} \omega_{Ma\beta} - \pi^{\hat{\alpha}\hat{\beta}} \omega_{Ma\hat{\beta}} + \partial_M \theta_a \pi^a + \partial_M \bar{\theta}_a \bar{\pi}^a + \left( P^a \partial_M u_a + P^v \partial_M v_a \right) + \left( \tilde{P}^{\hat{a}} \partial_M \bar{u}_{\hat{a}} + \tilde{P}^{\hat{\alpha}} \partial_M \bar{v}_{\hat{\alpha}} \right) - \partial_M P^{(\rho)}_\rho \rho' \approx 0.
\]

One can check that \(L_M\) have Poisson brackets with the primary constraints from all sectors weakly equal to zero. Thus, these constraints realize the \(\sigma_M\) transformations of the reparametrization invariance of the brane action. These secondary constraints complete realization of the Dirac procedure of the first-class constraints construction. The remaining time-like \(\tau\)-reparametrization constraint is not independent and is constructed from the constraints \(\Phi^{(u)}, T^{(u)}\) (33) for \(x_{a\hat{a}}, z_{a\beta}\) and some algebraic combinations of other first-class constraints for the remaining generalized coordinates.

## 10 \(Y^\Lambda\) supertwistor as an invariant of local symmetries

Here we shall show that \((8_B + 1_F)\) real components of the supertwistor \(Y^\Lambda = (iU^a, \bar{Y}_a, \bar{\eta})\) in 4-dimensional Minkowski space extended by 6 TCC coordinates are the invariants of \((8_B + 3_F)\) local symmetries generated by the first-class constraints from the \(\Phi-, \Psi-\) and dyad sectors. We prove that the transition from the original representation of the action (11) (or
in terms of \((16_B + 4_F)\) variables \((x, z, u, v; \theta)\) to the \(Y^A\) supertwistor representation \((8_B + 1_F)\), including \((8_B + 1_F)\) components, preserves all the local symmetries of \(S_p\) \((1)\). It will follow from the observation that \(Y^A\) forms a representation of the Weyl symmetry \((158)\), the space-like reparametrizations \((100)\) and its invariance under the \(\rho^M\)-shifts \((89)\). All these \((2p + 1)\) symmetries remain local symmetries of the supertwistor representation \((10)\).

Starting the proof we observe that the supertwistor component \(U_\alpha = (\bar{\Psi}_\alpha)\) is a trivial invariant of all the 11 symmetries generated by the \(\Psi_\alpha\), \(\Phi_\alpha\) and \((u, v)\)-sectors. The fermionic component \(\tilde{\eta} = -2i(U^\alpha \theta_\alpha) = -2i(u^\alpha \theta_\alpha + u_\alpha \bar{\theta}^\alpha)\) is not transformed under symmetry transformations from the \(\Phi_\alpha\) and \((u, v)\)-sectors. Moreover, its invariance under the \(\kappa\)-symmetry generated by \(\bar{\Psi}_\alpha\), \(\overline{\Psi}_\alpha\) and \(\Psi_{\alpha R}^{(v)}\) \((45)\) follows from the relations

\[
\delta_{\kappa} \bar{\eta} = \frac{2}{\imath} \kappa u^\alpha u_\alpha = 0, \quad \delta_{\kappa} \eta = -\frac{2}{\imath} \bar{\kappa} \bar{u}^\alpha \bar{u}_\alpha = 0, \quad \delta_{\kappa_R} \eta = \frac{2}{\imath} \bar{\kappa}_R [u^\alpha v_\alpha + \bar{u}_\alpha \bar{v}^\alpha] \approx 0. \tag{111}
\]

Thus, it remains to prove the invariance of the \(\bar{Y}_\alpha\) components of the supertwistor

\[
i \bar{Y}_\alpha = Y_{ab} U^b - \bar{\eta} \theta_\alpha = \left( -z_{\alpha \beta} u^\beta + x_{\alpha \bar{\alpha}} \bar{u}^\bar{\alpha} - \bar{\eta} \theta_\alpha \right). \tag{112}
\]

Due to the invariance of \(U_\alpha\) and \(\tilde{\eta}\) variables the variation of \(\bar{Y}_\alpha\) is given by

\[
i \delta \bar{Y}_\alpha = \delta Y_{ab} U^b - \tilde{\eta} \delta \theta_\alpha = \left( -\delta z_{\alpha \beta} u^\beta + \delta x_{\alpha \bar{\alpha}} \bar{u}^\bar{\alpha} - \tilde{\eta} \delta \theta_\alpha \right) \quad \tag{113}
\]

and it is enough to verify the invariance of the upper Weyl component \(i \bar{y}_\alpha\). Using the relations \((45)\) we find that

\[
i \delta_{\kappa} y_\alpha = -2i \kappa u_\alpha [u^\beta \theta_\beta - \bar{u}^\beta \bar{\theta}_\beta - \frac{\imath}{12} \bar{\eta}] = 0, \quad i \delta_{\kappa_R} y_\alpha = -2i \bar{\kappa}_R v_\alpha [u^\beta \theta_\beta - \bar{u}^\beta \bar{\theta}_\beta - \frac{\imath}{12} \bar{\eta}] = 0. \tag{114}
\]

So, we confirmed the invariant character of \(\bar{Y}_\alpha\) under the \(\kappa\)-symmetry transformations.

The invariance of \(\bar{Y}_\alpha\) under the symmetry transformations \((73)\) from the \((u, v)\)-sector is evident, because \(\bar{Y}_\alpha\) does not include \(v_\alpha\).

The transformations of \(\bar{Y}_\alpha\) under the symmetries of the \(\Phi\)-sector are defined by

\[
i \delta \bar{Y}_\alpha = \left( -\delta z_{\alpha \beta} u^\beta + \delta x_{\alpha \bar{\alpha}} \bar{u}^\bar{\alpha} \right) \tag{115}
\]

because \(\theta_\alpha\) and \(\tilde{\eta}\) are invariants of these symmetries. The invariance of \(\bar{Y}_\alpha\) under the symmetries \((49)\) and \((53)\) is evident because of the relations

\[
\delta_{\phi(u)} x_{\alpha \bar{\alpha}} \bar{u}^\bar{\alpha} = \epsilon_{\phi(u)} u_\alpha (u_\alpha \bar{u}^\bar{\alpha}) = 0, \quad \delta_{T(u)} z_{\alpha \beta} u^\beta = \epsilon_{T(u)} u_\alpha (u_\beta u^\beta) = 0 \tag{116}
\]

and their complex conjugate.

The invariance under the symmetry \((63)\) follows from the cancellation between the \(x\) and \(z\) contributions

\[
\delta_{\bar{T}(v)} x_{\alpha \bar{\alpha}} \bar{u}^\bar{\alpha} - \delta_{\bar{T}(v)} z_{\alpha \beta} u^\beta \approx \epsilon_{\bar{T}(v)} u_\alpha - \epsilon_{\bar{T}(v)} v_\alpha = 0. \tag{117}
\]

Analogous cancellations also take place with respect to the transformations \((66), (69)\)

\[
\begin{align*}
\delta_{\bar{T}(v)} x_{\alpha \bar{\alpha}} \bar{u}^\bar{\alpha} - \delta_{\bar{T}(v)} z_{\alpha \beta} u^\beta &= \epsilon_{\bar{T}(v)} [m_{\alpha \bar{\alpha}}^{(+)} \bar{u}^\bar{\alpha} - u_{\alpha v_\beta} u^\beta] \approx 0, \\
\delta_{\bar{T}(v)} x_{\alpha \bar{\alpha}} \bar{u}^\bar{\alpha} - \delta_{\bar{T}(v)} z_{\alpha \beta} u^\beta &= \epsilon_{\bar{T}(v)} [m_{\alpha \bar{\alpha}}^{(-)} \bar{u}^\bar{\alpha} - i u_{\alpha v_\beta} u^\beta] \approx 0 \tag{118}
\end{align*}
\]
and their complex conjugate. It completes the proof of invariant character of $Y^\Lambda$ under $(8_B + 3_F)$ local symmetries generated by the $\Psi$-, $\Phi$- and $(u,v)$-sectors. These symmetries show the pure gauge character of $(8_B + 3_F)$ variables among $(16_B + 4_F)$ primary variables $(x, z, u, v; \theta)$. The transition to the supertwistor representation including $(8_B + 1_F)$ invariant variables $Y_\Lambda$ just encodes these $(8_B + 3_F)$ degrees of freedom

$$(16_B + 4_F) - (8_B + 3_F) = (8_B + 1_F),$$

so we get the next scheme of the reduction of the original brane variables:

$$\kappa - \text{symmetry:} \quad \theta_\alpha, \bar{\theta}_{\dot{\alpha}} \Rightarrow \tilde{\eta},$$

$$\text{shifts:} \quad x_{a\dot{a}}, z_{a\dot{a}}, \bar{z}_{\dot{\alpha}j}; v_\alpha, \bar{v}_{\dot{\alpha}} \Rightarrow (Y_\alpha, \bar{Y}_{\dot{\alpha}}).$$

Taking into account the Weyl symmetry transformations (98) one can find the Weyl transformation of the supertwistor

$$\dot{Y}^\Sigma = e^{A(\tau, \bar{\tau})} Y^\Sigma.$$  \hspace{1cm} (121)

Using this result one verifies that the supertwistor representation (6) of $S_p$ is invariant under the Weyl symmetry. Indeed, the transformed action (6) is

$$S_p' = \frac{1}{2} \int d\tau d^p \sigma \rho^\mu \left[ \partial_\mu Y_\Sigma G_{\Sigma \Xi} Y^{\Xi} - \partial_\mu \Lambda(Y_\Sigma G_{\Sigma \Xi} Y^{\Xi}) \right]$$

$$= S_p - \frac{1}{2} \int d\tau d^p \sigma \rho^\mu \partial_\mu \Lambda(Y_\Sigma G_{\Sigma \Xi} Y^{\Xi}),$$

but the last term equals zero, because $Y_\Sigma G_{\Sigma \Xi} Y^{\Xi} = 0$ and $S_p' = S_p$. Also, the supertwistor action is invariant under $p$ remaining local symmetries (89) from $\rho$–sector and $p$ world-volume reparametrizations generated by the secondary first-class constraints (110).

Thus, we conclude that the strong reduction of the number of variables during the transition to the symplectic supertwistor representation (6) is a consequence of the described local symmetries. The pure gauge degrees of freedom are eliminated by the change of variables without any gauge fixing.

11 The Hamiltonian

Having completed the Dirac prescription of the first-class constraints construction we present the total Hamiltonian density of the super $p$-brane in the form of their linear combination

$$H_T = \kappa_u \Psi^{(u)} + \bar{\kappa}_u \bar{\Psi}^{(u)} + \kappa_R \Psi^{(v)}_R$$

$$+ a_u \Phi^{(u)} + b_u T^{(u)} + \bar{b}_u \bar{T}^{(u)}$$

$$+ c^{(\pm)} \bar{T}^{(\pm)} + c^{(-)} \bar{T}^{(-)} + c^{(v)} \bar{T}^{(v)}$$

$$+ e P^{(u)} + \bar{e} \bar{P}^{(u)} + \omega \Delta_W$$

$$+ f^M P_M^{(\rho)} + \bar{\rho}^M L_M \approx 0,$$  \hspace{1cm} (123)

where the functions $\kappa, a, b, c, e, f, \omega$ and $\bar{\rho}$ form the set of $(9 + 2p)_B + 3_F$ real Lagrange multipliers.

In the Hamiltonian formalism the second-class constraints are taken into account by the transition to the Dirac brackets (D.B.) which have to be changed by the (anti)commutators in the quantum dynamics. To construct the D.B. we need to find the second-class constraints remaining after the first-class constraint separation. This problem will be solved below.
12 Second-class constraints and the Dirac brackets

To find the second-class constraints we shall follow to the applied above projection method \[13, 17, 18\]. In the $\Psi$-sector only one constraint $\Psi^{(v)}_I$ \[14\] has remained which is the second-class constraint, because its P.B.

$$\{\Psi^{(v)}_I(\vec{\sigma}), \Psi^{(v)}_I(\vec{\sigma}')\}_{P.B.} = 8i\{v_\alpha \Phi^{\alpha\beta} v_\beta + \bar{v}_\alpha \tilde{\Phi}^{\alpha\dot{\beta}} \bar{v}_{\dot{\beta}} - \bar{v}_\beta \tilde{\Phi}^{\dot{\beta}\alpha} v_\alpha\} - 2\rho^\tau \delta^\rho(\vec{\sigma} - \vec{\sigma}')$$  \[124\]

are nonzero in the weak sense

$$\{\Psi^{(v)}_I(\vec{\sigma}), \Psi^{(v)}_I(\vec{\sigma}')\}_{P.B.} \approx -16i\rho^\tau \delta^\rho(\vec{\sigma} - \vec{\sigma}').$$  \[125\]

In the $\Phi$-sector there are four second-class constraints that may be constructed either from the $\Phi^{\alpha\beta}$-subsector alone or from the $\Phi^{\alpha\dot{\beta}}$- and $\Phi^{\dot{\alpha}\alpha}$-subsectors. We choose the projections of the three remaining constraints from the $\Phi^{\alpha\alpha}$-subsector

$$\Phi \equiv \Phi^{\dot{\alpha}\alpha}(m^+) - im^(-) = 2\bar{v}_{\dot{\alpha}} \Phi^{\dot{\alpha}\alpha} v_\alpha \approx 0$$

$$\Phi^\star \equiv (\Phi)^* \equiv \Phi^{\dot{\alpha}\alpha}(m^+) + im^(-) = 2\bar{u}_{\alpha} \Phi^{\alpha\dot{\alpha}} v_\alpha \approx 0,$$

$$\Phi^{(v)} \equiv \bar{v}_\alpha \Phi^{\alpha\dot{\alpha}} v_\alpha \approx 0$$  \[126\]

to be the second-class constraints. Then the additional fourth second-class constraint $T^{(v)}_I$ is supplied by the $\Phi^{\alpha\beta}$-subsector

$$T^{(v)}_I \equiv i(T^{(v)} - \bar{T}^{(v)}) = i(v_\alpha \Phi^{\alpha\beta} v_\beta - \bar{v}_\alpha \tilde{\Phi}^{\alpha\dot{\beta}} \bar{v}_{\dot{\beta}}) \approx 0.$$  \[127\]

The second-class constraints \[126, 127\] have non-zero P.B. with the constraints belonging to the dyad and $\rho$-sectors. Therefore, we shall seek for such linear combinations from these sectors that form canonically conjugate pairs with the constraints \[126, 127\].

Then we find the following four second-class constraints

$$P^{(v)}_{\tau}(\vec{\sigma}) \approx 0, \quad \{P^{(v)}_{\tau}(\vec{\sigma}), \Phi^{(v)}(\vec{\sigma}')\}_{P.B.} \approx \delta^\rho(\vec{\sigma} - \vec{\sigma}'),$$  \[128\]

$$P^{(v)}_{\alpha}(\vec{\sigma}) \equiv v_\alpha \Phi^{\alpha\beta} v_\beta \approx 0, \quad \{P^{(v)}_{\alpha}(\vec{\sigma}), \Phi^{(v)}(\vec{\sigma}')\}_{P.B.} \approx 2\rho^\tau \delta^\rho(\vec{\sigma} - \vec{\sigma}'),$$  \[129\]

$$P^{(v)}_{\dot{\alpha}}(\vec{\sigma}) \equiv \bar{v}_{\dot{\alpha}} \tilde{\Phi}^{\dot{\alpha}\alpha} v_\alpha \approx 0, \quad \{P^{(v)}_{\dot{\alpha}}(\vec{\sigma}), \Phi^{(v)}(\vec{\sigma}')\}_{P.B.} \approx 2\rho^\tau \delta^\rho(\vec{\sigma} - \vec{\sigma}').$$  \[129\]

and

$$\Delta_I \equiv \Delta^{(v)}_I - \Delta^{(u)}_I \equiv i(P^{(v)}_v - P^{(v)}_{\bar{v}}) - i(P^{(u)}_u - P^{(u)}_{\bar{u}})$$

$$\equiv i(P^{\alpha\beta} v_\alpha - \bar{P}^{\dot{\alpha}\dot{\beta}} \bar{v}_{\dot{\beta}}) - i(P^{\alpha\beta} u_\alpha - \bar{P}^{\dot{\alpha}\dot{\beta}} \bar{u}_{\dot{\beta}}) \approx 0,$$

$$\{\Delta_I(\vec{\sigma}), T^{(v)}_I(\vec{\sigma}')\}_{P.B.} \approx 2\rho^\tau \delta^\rho(\vec{\sigma} - \vec{\sigma}').$$  \[130\]

The remaining four second-class constraints forming canonically conjugate pairs are supplied by the constraints belonging to the dyad and $\rho$-sectors

$$\Delta \equiv P^{(v)}_u + P^{(v)}_{\bar{v}} - \rho^\tau P^{(\rho)}_\tau - \frac{i}{2} \Delta_I \approx 0, \quad \Xi \approx 0,$$

$$\{\Delta(\vec{\sigma}), \Xi(\vec{\sigma}')\}_{P.B.} = 2\delta^\rho(\vec{\sigma} - \vec{\sigma}').$$  \[131\]
and their complex conjugate. As a result, the P.B.'s. of 12 bosonic constraints (120) - (131) and one fermionic constraint (11) form the Dirac matrix $\hat{C}$ having the symplectic form

$$\hat{C} = \begin{pmatrix}
P_\tau^{(\rho)} & P_\tau^{(v)} & \Phi & \Phi & \Delta & T_\tau^v & \Delta & \Xi & \bar{\Delta} & \bar{\Xi} & \Psi_I^{(v)} \\
P_\tau^{(v)} & 0 & -1 & 1 & 0 \\
P_u^{(v)} & 0 & 2\rho^\tau & -2\rho^\tau & 0 & 0 \\
\bar{P}_u^{(v)} & -2\rho^\tau & 0 & 0 & 2\rho^\tau & -2\rho^\tau & 0 \\
\Delta & 0 & 0 & 0 & 2 & -2 \\
\Xi & 0 & -2 & 0 \\
\bar{\Xi} & 0 & 0 & 0 & 0 & 0 \\
\bar{\Delta} & 0 & -2 & 0 \\
\Psi_I^v & -16i\rho^\tau \\
\end{pmatrix}$$

(132)

multiplied by $\delta^p(\bar{\sigma} - \sigma')$. Then we find the determinant of the matrix $\hat{C}$

$$\text{det}\hat{C} = i(4\rho^\tau)^7.$$

(133)

The inverse matrix $\hat{C}^{-1}$ is used to construct the Dirac brackets

$$\{f(\sigma), \ g(\sigma')\}_{D,B.} = \{f(\sigma), \ g(\sigma')\}_{P.B.} - \sum \int dp\sigma'' \{f(\sigma), \ F(\sigma'')\}_{P.B.}(\hat{C}^{-1}) F G(\sigma'') \{G(\sigma''), \ g(\sigma')\}_{P.B.},$$

(134)

where $F$ and $G$ contain the second-class constraint set forming $\hat{C}$.

The inverse Dirac matrix $\hat{C}^{-1}$ is given by

$$\hat{C}^{-1} = \begin{pmatrix}
P_\tau^{(\rho)} & P_\tau^{(v)} & \Phi & \Phi & \bar{\Phi} & \Delta & T_\tau^v & \Delta & \Xi & \bar{\Delta} & \bar{\Xi} & \Psi_I^v \\
P_\tau^{(v)} & 0 & 1 & -1 & 0 \\
P_u^{(v)} & 0 & -\frac{1}{2\rho^\tau} & \frac{1}{2\rho^\tau} & 0 \\
\bar{P}_u^{(v)} & -\frac{1}{2\rho^\tau} & 0 & 0 & -\frac{1}{2\rho^\tau} \\
\Phi & \frac{1}{2\rho^\tau} & 0 & 0 & 0 \\
\bar{\Phi} & 0 & -\frac{1}{2\rho^\tau} & \frac{1}{2\rho^\tau} & 0 \\
\Delta & 0 & 0 & 0 & \frac{1}{2} \\
\Xi & -\frac{1}{2} & 0 & 0 \\
\bar{\Xi} & 0 & \frac{1}{2} & 0 \\
\bar{\Delta} & 0 & 0 & 0 \\
\Psi_I^v & \frac{i}{16\rho^\tau} \\
\end{pmatrix}$$

(135)

multiplied by $\delta^p(\bar{\sigma} - \sigma')$.

The matrices $\hat{C}$ (132) and $\hat{C}^{-1}$ (135) define the Hamiltonian symplectic structure in the total phase space of the original variables (20) and (21). It is easy to see that this phase
space can be covariantly reduced by solving the constraint \( \Phi^{(v)} \) which gives the following representation for the coordinate \( \rho^\tau \)

\[
\rho^\tau = (vP\bar{v}).
\]  

(136)

As a result, the canonical pair \((\rho^\tau, P^\tau_\rho)\) may be excluded from the original phase space, because \( P^\tau_\rho \approx 0 \). This reduction does not change the matrix \( \hat{C} \), but only strikes out the upper \( 2 \times 2 \) submatrix and substitutes \((vP\bar{v})\) for \( \rho^\tau \) in the remaining diagonal blocks. Then the partially reduced matrix \( \hat{C}_{\text{red}}^{-1} \) is substituted for \( \hat{C}^{-1} \). As a result of the reduction the matrix \( \hat{C}_{\text{red}}^{-1} \) contains the nonlocal factor \( \frac{1}{P^0} \) absent in the D.B.'s. of the superparticle dynamics. This peculiarity of the brane’s D.B.'s. may lead to principal distinctions between the string/brane and particle quantum descriptions.

13 Noncommutativity of coordinates under the Dirac brackets

The D.B. \([133]\) defines the new commutation relations between the canonical variables encoding the second-class constraint presence.

Analysis of the matrix \( \hat{C}^{-1} \) \([135]\) structure shows that the dyad-sector coordinates \( u_\alpha, v_\beta \) commute among themselves

\[
\{u_\alpha(\vec{\sigma}), u_\beta(\vec{\sigma}')\}_{D.B.} = \{v_\alpha(\vec{\sigma}), v_\beta(\vec{\sigma}')\}_{D.B.} = 0
\]

and with \( \rho^\tau \) and \( \theta_\alpha \)

\[
\{u_\alpha(\vec{\sigma}), \rho^\tau(\vec{\sigma}')\}_{D.B.} = 0, \quad \{v_\alpha(\vec{\sigma}), \rho^\tau(\vec{\sigma}')\}_{D.B.} = 0,
\]

\[
\{u_\alpha(\vec{\sigma}), \theta_\beta(\vec{\sigma}')\}_{D.B.} = 0, \quad \{v_\alpha(\vec{\sigma}), \theta_\beta(\vec{\sigma}')\}_{D.B.} = 0.
\]

(138)

However, they do not commute with \( x_{\alpha\bar{\alpha}} \) and \( z_{\alpha\beta} \) coordinates

\[
\{x_{\alpha\bar{\alpha}}(\vec{\sigma}), u_\beta(\vec{\sigma}')\}_{D.B.} = \frac{1}{\rho^\tau} u_\alpha \bar{\nu}_\alpha v_\beta \delta^p(\vec{\sigma} - \vec{\sigma}'),
\]

\[
\{x_{\alpha\bar{\alpha}}(\vec{\sigma}), v_\beta(\vec{\sigma}')\}_{D.B.} = 0,
\]

\[
\{z_{\alpha\beta}(\vec{\sigma}), u_\gamma(\vec{\sigma}')\}_{D.B.} = \frac{1}{2\rho^\tau} v_\alpha v_\beta u_\gamma \delta^p(\vec{\sigma} - \vec{\sigma}'),
\]

\[
\{z_{\alpha\beta}(\vec{\sigma}), v_\gamma(\vec{\sigma}')\}_{D.B.} = -\frac{1}{2\rho^\tau} v_\alpha v_\beta v_\gamma \delta^p(\vec{\sigma} - \vec{\sigma}').
\]

(139)

The coordinates \( \theta_\alpha \) have non-zero D.B.'s among themselves

\[
\{\theta_\alpha(\vec{\sigma}), \theta_\beta(\vec{\sigma}')\}_{D.B.} = \frac{i}{16\rho^\tau} v_\alpha v_\beta \delta^\rho(\vec{\sigma} - \vec{\sigma}'),
\]

\[
\{\theta_\alpha(\vec{\sigma}), \bar{\theta}_\beta(\vec{\sigma}')\}_{D.B.} = -\frac{i}{16\rho^\tau} v_\alpha \bar{\nu}_\beta \delta^\rho(\vec{\sigma} - \vec{\sigma}'),
\]

\[
\{\bar{\theta}_\alpha(\vec{\sigma}), \bar{\theta}_\beta(\vec{\sigma}')\}_{D.B.} = \frac{i}{16\rho^\tau} \bar{\nu}_\alpha \bar{\nu}_\beta \delta^\rho(\vec{\sigma} - \vec{\sigma}').
\]

(140)

and this results in the noncommutativity of the Goldstone fermion \( \tilde{\eta} \) \([14]\)

\[
\{\theta_\alpha(\vec{\sigma}), \tilde{\eta}(\vec{\sigma}')\}_{D.B.} = \frac{1}{4\rho^\tau} v_\alpha \delta^\rho(\vec{\sigma} - \vec{\sigma}'),
\]

\[
\{\tilde{\eta}(\vec{\sigma}), \bar{\eta}(\vec{\sigma}')\}_{D.B.} = -\frac{i}{\rho^\tau} \delta^\rho(\vec{\sigma} - \vec{\sigma}').
\]

(141)

But, the projection \((v^\alpha \theta_\alpha)\) associated with the unbroken directions commutes with \( \theta_\beta \) and \( \tilde{\eta} \)

\[
\{v^\alpha \theta_\alpha(\vec{\sigma}), \theta_\beta(\vec{\sigma}')\}_{D.B.} = 0, \quad \{v^\alpha \theta_\alpha(\vec{\sigma}), \tilde{\eta}(\vec{\sigma}')\}_{D.B.} = 0
\]

(142)
The coordinates $\theta_\alpha$ have nonzero D.B.'s. with $x_{\alpha\bar{\alpha}}$ and $z_{\alpha\beta}$ either:

\[ \{\theta_\alpha(\bar{\sigma}), x_{\beta\bar{\beta}}(\bar{\sigma}')\}_\text{D.B.} = \frac{1}{8p^{\alpha}} v_\alpha (v_\beta \bar{\theta}_\beta - \theta_\beta \bar{v}_\beta) \bar{\delta}^p(\bar{\sigma} - \bar{\sigma}'), \]
\[ \{\theta_\alpha(\bar{\sigma}), z_{\beta\bar{\delta}}(\bar{\sigma}')\}_\text{D.B.} = \frac{1}{8p^{\alpha}} v_\alpha (\theta_\beta v_\delta + v_\beta \theta_\delta) \bar{\delta}^p(\bar{\sigma} - \bar{\sigma}'), \]
\[ \{\theta_\alpha(\bar{\sigma}), \bar{z}_{\bar{\beta}\bar{\delta}}(\bar{\sigma}')\}_\text{D.B.} = -\frac{1}{8p^{\alpha}} v_\alpha (\bar{\theta}_\beta \bar{v}_\delta + \bar{v}_\beta \bar{\theta}_\delta) \bar{\delta}^p(\bar{\sigma} - \bar{\sigma}'). \]

However, the projection \( (v^\alpha \theta_\alpha) \) has zero D.B.'s. with the $x_{\alpha\bar{\alpha}}$ coordinates

\[ \{v^\alpha \theta_\alpha(\bar{\sigma}), x_{\beta\bar{\beta}}(\bar{\sigma}')\}_\text{D.B.} = 0, \]

but does not commute with the TCC coordinates $z_{\beta\bar{\delta}}$

\[ \{v^\alpha \theta_\alpha(\bar{\sigma}), z_{\beta\bar{\delta}}(\bar{\sigma}')\}_\text{D.B.} = \frac{1}{2p^{\alpha}} (v^\alpha \theta_\alpha) v_\beta v_\delta \bar{\delta}^p(\bar{\sigma} - \bar{\sigma}'). \]

The space-time $x_{\alpha\bar{\alpha}}$ and TCC coordinates $z_{\alpha\beta}$ have nonzero D.B.'s among themselves

\[ \{x_{\alpha\bar{\alpha}}(\bar{\sigma}), x_{\beta\bar{\beta}}(\bar{\sigma}')\}_\text{D.B.} = \frac{1}{4p^{\alpha}} (\theta_\beta \bar{v}_\alpha - v_\alpha \bar{\theta}_\beta)(\theta_\beta \bar{v}_\beta - v_\beta \bar{\theta}_\beta) \bar{\delta}^p(\bar{\sigma} - \bar{\sigma}'), \]
\[ \{z_{\alpha\beta}(\bar{\sigma}), z_{\gamma\delta}(\bar{\sigma}')\}_\text{D.B.} = \frac{1}{4p^{\alpha}} (\theta_\alpha v_\beta + \theta_\beta v_\alpha)(\theta_\gamma v_\delta + \theta_\delta v_\gamma) \bar{\delta}^p(\bar{\sigma} - \bar{\sigma}'), \]
\[ \{x_{\alpha\bar{\alpha}}(\bar{\sigma}), z_{\beta\gamma}(\bar{\sigma}')\}_\text{D.B.} = -\frac{1}{4p^{\alpha}} (\theta_\alpha \bar{v}_\beta - v_\alpha \bar{\theta}_\beta)(\theta_\beta v_\gamma + \theta_\gamma v_\beta) \bar{\delta}^p(\bar{\sigma} - \bar{\sigma}'). \]

The D.B. noncommutativity (146) has gauge-dependent character and one can show that the r.h.s. of (146) vanishes in the partially fixed $\kappa$-symmetry gauge

\[ \theta^{(v)} \equiv \theta^v v_\alpha = 0. \]

To prove this we use the decomposition of $\theta_\alpha$ in the dyad basis

\[ \theta_\alpha = \theta^{(v)} u_\alpha - \theta^{(u)} v_\alpha = \theta^{(v)} u_\alpha - (\text{Re}\theta^{(u)} - i\bar{\eta}) v_\alpha \]

and present the multipliers entering the r.h.s. of (146) as

\[ (\theta_\beta \bar{v}_\alpha - v_\alpha \bar{\theta}_\beta) = \frac{i}{2} \bar{\eta} \bar{v}_\alpha v_\alpha + i(I\text{m}\theta^{(v)} m^{(v)}_{\alpha\bar{\alpha}} - \text{Re}\theta^{(v)} m^{(v)}_{\alpha\bar{\alpha}}), \]
\[ (\theta_\beta v_\gamma + \theta_\gamma v_\beta) = \frac{i}{2} \overline{\eta} v_\beta v_\gamma = 2\text{Re}\theta^{(v)} v_\beta v_\gamma + \theta^{(v)} (u_\beta v_\gamma + u_\gamma v_\beta). \]

In the gauge (147) the representations (149) reduce to

\[ (\theta_\beta \bar{v}_\alpha - v_\alpha \bar{\theta}_\beta)|_{\text{gauge (147)}} = \frac{i}{2} \overline{\eta} \bar{v}_\alpha v_\alpha, \]
\[ (\theta_\beta v_\gamma + \theta_\gamma v_\beta)|_{\text{gauge (147)}} = \frac{i}{2} \overline{\eta} v_\beta v_\gamma \]

and the substitution of (150) into (146) yields

\[ \{x_{\alpha\bar{\alpha}}(\bar{\sigma}), x_{\beta\bar{\beta}}(\bar{\sigma}')\}_\text{D.B.}|_{\text{gauge (147)}} = \{z_{\alpha\beta}(\bar{\sigma}), z_{\gamma\delta}(\bar{\sigma}')\}_\text{D.B.}|_{\text{gauge (147)}} = 0, \]
\[ \{x_{\alpha\bar{\alpha}}(\bar{\sigma}), z_{\beta\gamma}(\bar{\sigma}')\}_\text{D.B.}|_{\text{gauge (147)}} = 0, \]

in view of the relation $\overline{\eta}^2 = 0$. This proves the gauge-dependent character of the D.B. noncommutativity of the coordinates $x$ and $z$ between themselves.

In contrast to this picture, the D.B. noncommutativity of $\theta_\alpha$ with $x_{\alpha\bar{\alpha}}$ and $z_{\alpha\beta}$ coordinates cannot be removed by gauge fixing since

\[ \{\theta_\alpha(\bar{\sigma}), x_{\beta\bar{\beta}}(\bar{\sigma}')\}_\text{D.B.}|_{\text{gauge (147)}} = \frac{i\overline{\eta}}{16p^{\alpha}} v_\alpha v_\beta \bar{v}_\beta \delta^p(\bar{\sigma} - \bar{\sigma}'), \]
\[ \{\theta_\alpha(\bar{\sigma}), z_{\beta\gamma}(\bar{\sigma}')\}_\text{D.B.}|_{\text{gauge (147)}} = \frac{i\overline{\eta}}{16p^{\alpha}} v_\alpha v_\beta v_\gamma \delta^p(\bar{\sigma} - \bar{\sigma}'). \]
The same conclusion takes place for the D.B.'s of the light-like projection \((ux\bar{u})\) with the transverse coordinates \(x^{(+)}\) and \(x^{(-)}\)
\[
\{(ux\bar{u})(\bar{\sigma}), \{x^{(+))(\bar{\sigma})}\}_{D.B.} = \left(\frac{1}{\rho'}\bar{\sigma}^{(+)}) + \frac{\tilde{a}}{4} Re\theta^{(\nu)}\delta^{\rho}(\bar{\sigma} - \bar{\sigma}')\right),
\]
\[
\{(ux\bar{u})(\bar{\sigma}), \{x^{(-)}(\bar{\sigma}')\}_{D.B.} = \left(\frac{1}{\rho'}\bar{\sigma}^{(-)} - \frac{\tilde{a}}{4} Im\theta^{(\nu)}\delta^{\rho}(\bar{\sigma} - \bar{\sigma}'),
\]
whereas
\[
\{x^{(+))(\bar{\sigma}), \{x^{(-)}(\bar{\sigma}')\}_{D.B.} = \frac{1}{\rho'} Re\theta^{(\nu)} Im\theta^{(\nu)} \delta^{\rho}(\bar{\sigma} - \bar{\sigma}'),
\]
which vanishes in the gauge \((1\text{4})\). The D.B.'s \((1\text{53})\), \((1\text{54})\) are accompanied with zero D.B.
\[
\{\{v\bar{x}\bar{v}\}(\bar{\sigma}), x^{(\pm)}(\bar{\sigma}')\}_{D.B.} = 0,
\]
\[
\{x^{(+)}(\bar{\sigma}), x^{(+)}(\bar{\sigma}')\}_{D.B.} = \{x^{(-)}(\bar{\sigma}), x^{(-)}(\bar{\sigma}')\}_{D.B.} = 0,
\]
\[
\{\{ux\bar{u}\}(\bar{\sigma}), (ux\bar{u})(\bar{\sigma}')\}_{D.B.} = \{\{v\bar{x}v\}(\bar{\sigma}), (v\bar{x}v)(\bar{\sigma}')\}_{D.B.} = 0,
\]
\[
\{(ux\bar{u})(\bar{\sigma}), (ux\bar{u})(\bar{\sigma}')\}_{D.B.} = \{0\}.
\]
Note also that the \(x_{\alpha\bar{\alpha}}\) components have nonzero D.B.'s. with \(\rho^{(\tau, \bar{\sigma})}\)
\[
\{x_{\alpha\bar{\alpha}}(\bar{\sigma}), \rho^{(\tau)}(\bar{\sigma}')\}_{D.B.} = v_{\alpha\bar{\alpha}}\delta^{\rho}(\bar{\sigma} - \bar{\sigma}'),
\]
in contrast to the TCC coordinates
\[
\{z_{\alpha\bar{\beta}}(\bar{\sigma}), \rho^{(\tau)}(\bar{\sigma}')\}_{D.B.} = 0.
\]
It shows that the classical variable \(\rho^{\tau}\) has to be changed by the operator projection \((v\bar{P}\bar{v})\) after quantization in the reduced phase space, as it follows from the constraint \((1\text{3}6)\). As a result, in the quantum brane dynamics some commutation relations between brane coordinates will be proportional to the non-local factor \(\frac{1}{(v\bar{P}\bar{v})}\) due to the substitution of (anti)commutators for the D.B.'s. The vector \(v_{\alpha}\bar{v}_{\bar{\alpha}}\) fixing the projection direction is a light-like vector, so the projection \((v\bar{P}\bar{v})\) has physical sense analogous to \(p_-\) (or \(p_+\)) for the Green-Schwarz superstring \((1\text{1}\text{9})\). The non-local factor proportional to \(\frac{1}{p}\) appears in the two-point function of the energy-momentum tensor for the Green-Schwarz superstring, where it is associated with the Weyl anomaly presence. Note also that the presence of \(\frac{1}{(v\bar{P}\bar{v})}\) in the brane model correlates with the anomalous factor appearing in the quantum algebra \((1\text{3}4)\) of the conformal symmetry of tensionless string quantized in the light-cone gauge. This correlation is not so evident, because our consideration is free of any gauge fixing.

In view of the above mentioned observations and \(OSp(1,8)\) invariance of our brane model, the D.B. and commutator realizations of the \(OSp(1,8)\) superalgebra deserve to be carefully studied. The next section will be devoted to this goal.

### 14 Dirac bracket realization of the \(OSp(1|8)\) superalgebra

Having proved the \(OSp(1|8)\) symmetry of the super \(p\)-brane action \((1\text{8})\) one can construct it P.B. and D.B. realizations and to observe the position of the non-local factor \(\frac{1}{(v\bar{P}\bar{v})}\) in its structural functions.

The generator densities \(Q^{\alpha}\) and \(\bar{Q}^{\dot{\alpha}}\) of the \(N = 1\) global supersymmetry are given by
\[
Q^{\alpha}(\tau, \bar{\sigma}) = \pi^{\alpha} + 2i\theta^{\alpha}\bar{P}^{\dot{\alpha}} + 4i\pi^{\alpha\dot{\beta}}\theta^{\dot{\beta}},
\]
\[
\bar{Q}^{\dot{\alpha}}(\tau, \bar{\sigma}) = \bar{\pi}^{\dot{\alpha}} + 2i\bar{P}^{\dot{\alpha}}\theta_{\alpha} + 4i\bar{\pi}^{\dot{\alpha}\dot{\beta}}\bar{\theta}^{\dot{\beta}}.
\]
and their P.B’s. have the standard form
\[
\{Q^\alpha(\bar{\sigma}), Q^\beta(\bar{\sigma}')\}_{P.B.} = 4iP^\alpha\delta^\beta(\bar{\sigma} - \bar{\sigma}'),
\{Q^\alpha(\bar{\sigma}), Q^\beta(\bar{\sigma}')\}_{P.B.} = 8i\eta^{\alpha\beta}\delta^\gamma(\bar{\sigma} - \bar{\sigma}).
\]
\tag{159}

The “square roots” \(S_\gamma\) and \(\bar{S}_\gamma\) of the generalized conformal boost densities \(K_{\gamma\dot{\gamma}}\), \(K_{\gamma\lambda\dot{\gamma}}\)
are given by
\[
\{S_\gamma(\bar{\sigma}), \bar{S}_\rho(\bar{\sigma}')\}_{P.B.} = 4iK_{\gamma\dot{\rho}}\delta^\rho(\bar{\sigma} - \bar{\sigma}'),
\{S_\gamma(\bar{\sigma}), S_\lambda(\bar{\sigma}')\}_{P.B.} = 4iK_{\gamma\lambda}\delta^\rho(\bar{\sigma} - \bar{\sigma}').
\tag{160}
\]

The generalized conformal boost densities \(K_{\gamma\lambda}, K_{\gamma\dot{\gamma}}\) are respectively presented as
\[
\begin{align*}
K_{\gamma\lambda}(\tau, \bar{\sigma}) &= 2z_{\gamma\beta}z_{\lambda\delta}\pi^\beta\delta + 2x_{\gamma\beta}x_{\lambda\delta}\pi^\beta\delta + z_{\gamma\beta}x_{\lambda\delta}\pi^\beta\delta + x_{\gamma\beta}z_{\lambda\delta}\pi^\beta\delta \\
&+ \theta_{\lambda}(z_{\gamma\delta}\pi^\delta + x_{\gamma\delta}\pi^\delta) + \theta_{\gamma}(z_{\lambda\delta}\pi^\delta + x_{\lambda\delta}\pi^\delta) \\
&+ (u^\delta z_{\delta\lambda} - \bar{u}^\delta x_{\delta\lambda})P_{\gamma\lambda} + (u^\delta z_{\delta\gamma} - \bar{u}^\delta x_{\delta\gamma})P_{\lambda\gamma} \\
& - 2i(u^\delta \theta_{\delta} - \bar{u}^\delta \theta_{\delta})(\theta_{\gamma}\gamma + \theta_{\lambda}\lambda).
\end{align*}
\tag{162}
\]

Using the matrix multiplication agreement one can present \eqref{162} and \eqref{163} in more compact form
\[
\begin{align*}
K_{\gamma\lambda}(\tau, \bar{\sigma}) &= 2(z\pi z)_{\gamma\lambda} + 2(x\pi x)_{\gamma\lambda} + (xPz)_{\gamma\lambda} + (xPz)_{\lambda\gamma} \\
&- [(z\pi)_{\gamma}\theta_{\lambda} + (z\pi)_{\lambda}\theta_{\gamma}] - [(x\pi)_{\gamma}\theta_{\lambda} + (x\pi)_{\lambda}\theta_{\gamma}] \\
&+ [P_{\gamma\lambda}(zu) + P_{u\lambda}(zu)] - [P_{u\gamma}(zu) + P_{\gamma\lambda}(zu)] \\
&- 2i(u\theta - (u\bar{\theta}))(\theta_{\gamma}\gamma + \theta_{\lambda}\lambda).
\end{align*}
\tag{164}
\]

and respectively
\[
\begin{align*}
K_{\gamma\dot{\gamma}}(\tau, \bar{\sigma}) &= (\bar{z}Pz)_{\gamma\dot{\gamma}} + (xPx)_{\gamma\dot{\gamma}} + 2(z\pi x + x\pi \bar{z})_{\gamma\dot{\gamma}} \\
&+ \theta_{\gamma}(\pi x + \bar{z}\pi)_{\dot{\gamma}} + \theta_{\dot{\gamma}}(z\pi + x\pi)_{\gamma} \\
&+ P_{u\gamma}(ux - \bar{u}\bar{z})_{\dot{\gamma}} + (xu - zu)_{\gamma}P_{u\dot{\gamma}} \\
&- 2i((u\theta - (u\bar{\theta}))(\theta_{\gamma}\gamma + \theta_{\dot{\gamma}}\dot{\gamma}).
\end{align*}
\tag{165}
\]

The remaining 16 generator densities of the \(Sp(8)\) algebra \(L_{\alpha\beta}, L_{\gamma\dot{\rho}}\) (together with their complex conjugate) are defined by the P.B’s.
\[
\{Q^\gamma(\bar{\sigma}), P_{\rho}(\bar{\sigma}')\}_{P.B.} = 4iL^\gamma\rho\delta^\rho(\bar{\sigma} - \bar{\sigma}'),
\{Q^\gamma(\bar{\sigma}), \bar{S}_\rho(\bar{\sigma}')\}_{P.B.} = 4iL^\gamma\rho\delta^\rho(\bar{\sigma} - \bar{\sigma}').
\tag{166}
\]
and have the form

\[
L_\alpha^\beta (\tau, \tilde{\sigma}) = P^{\beta\alpha} x_{\beta\gamma} + 2\pi^{\alpha\gamma} z_{\gamma\beta} - \pi^{\alpha} \theta_{\beta} + u^\alpha P_{u\beta}, \\
L_\alpha^\beta (\tau, \tilde{\sigma}) = 2\pi^{\alpha\gamma} x_{\gamma\beta} + P^{\beta\alpha} \bar{z}_{\gamma\beta} - \pi^{\alpha} \bar{\theta}_{\beta} - u^\alpha \bar{P}_{u\beta}
\]

(167)

completed by their complex conjugate P.B’s. The derived P.B realization of the \( OSp(1|8) \) superalgebra together with the definition of the Dirac brackets (134) are enough for construction of the D.B. realization of the \( OSp(1|8) \) superalgebra.

Keeping in mind the results [14] one can wait the anomaly presence in the commutator of the operator densities \( K_{\gamma\tilde{\gamma}} \) and \( L_\alpha^\beta \). To this end let us in the first place calculate the Dirac bracket of the generalized superconformal boost density \( K_{\gamma\tilde{\gamma}} \) with \( L_\alpha^\beta \). Then we find

\[
\{ K_{\gamma\tilde{\gamma}} (\tilde{\sigma}), L_\alpha^\beta (\tilde{\sigma}') \}_{D.B.} = -\delta^\alpha_\beta K_{\beta\gamma} \delta^\rho (\tilde{\sigma} - \tilde{\sigma}') \\
\quad - \int d^p\sigma'' \{ K_{\gamma\tilde{\gamma}} (\tilde{\sigma}), \Phi (\tilde{\sigma}'') \}_{P.B.} \left( \frac{1}{\rho^p (\tilde{\sigma}'')} \right) \{ P_u^{(v)} (\tilde{\sigma}''), L_\alpha^\beta (\tilde{\sigma}') \}_{P.B.} \\
\quad + \int d^p\sigma'' \{ K_{\gamma\tilde{\gamma}} (\tilde{\sigma}), P_u^{(v)} (\tilde{\sigma}'') \}_{P.B.} \left( \frac{1}{\rho^p (\tilde{\sigma}'')} \right) \{ \Phi (\tilde{\sigma}''), L_\alpha^\beta (\tilde{\sigma}') \}_{P.B.} \\
\quad + \int d^p\sigma'' \{ K_{\gamma\tilde{\gamma}} (\tilde{\sigma}), \Delta I (\tilde{\sigma}'') \}_{P.B.} \left( \frac{1}{\rho^p (\tilde{\sigma}'')} \right) \{ T_I^{(v)} (\tilde{\sigma}''), L_\alpha^\beta (\tilde{\sigma}') \}_{P.B.} \\
\quad - \int d^p\sigma'' \{ K_{\gamma\tilde{\gamma}} (\tilde{\sigma}), \Psi_I^{(v)} (\tilde{\sigma}'') \}_{P.B.} \left( \frac{i}{16\rho^p (\tilde{\sigma}'')} \right) \{ \Psi_I^{(v)} (\tilde{\sigma}''), L_\alpha^\beta (\tilde{\sigma}') \}_{P.B.}
\]

(168)

with the omitted terms vanishing in the strong sense. The terms in the r.h.s. of the D.B. (168) containing the non-local factor \( \frac{1}{\rho^p} \) have a special structure in view of which each of them is proportional to some of the constraints of the model. As a result, we find that the contribution of the factor \( \frac{1}{\rho^p} \) will be vanishing on the constraint surface.

To prove this fact we note that the second term in (168) includes the two multipliers

\[
\{ K_{\gamma\tilde{\gamma}} (\tilde{\sigma}), \Phi (\tilde{\sigma}'') \}_{P.B.} = 2[\rho^p u_\gamma u^\delta (x_{\beta\gamma} - 2i\theta^{\beta\tilde{\gamma}}_{\gamma}) - \rho^p u_\gamma \bar{u}^\delta (\bar{z}_{\beta\bar{\gamma}} - 2i\bar{\theta}^{\beta\bar{\gamma}}_{\bar{\gamma})} \\
= u_\gamma \bar{v}^\delta \bar{L}_{\gamma}^\delta - \bar{v}_\gamma u_\delta L^\delta_{\gamma} \delta^p (\tilde{\sigma} - \tilde{\sigma}''),
\]

(169)

where \( \bar{L}_{\gamma}^\delta \) is the Lorentz generator complex conjugate to \( L^{\delta}_{\gamma} \). One can see that the third multiplier is vanished, because of the primary constraint \( P_{u\beta} \approx 0 \) presence.

The third term in (168) also includes the multipliers equal

\[
\{ K_{\gamma\tilde{\gamma}} (\tilde{\sigma}), P_u^{(v)} (\tilde{\sigma}'') \}_{P.B.} = [\bar{v}^\delta (\bar{z}_{\delta\gamma} - 2i\bar{\theta}^{\delta}_{\gamma}) P_{u\gamma} - \bar{v}^\delta (x_{\gamma\delta} - 2i\theta^{\delta}_{\gamma}) \bar{P}_{u\gamma}] \delta^p (\tilde{\sigma} - \tilde{\sigma}''), \\
\{ \Phi (\tilde{\sigma}''), L_\alpha^\beta (\tilde{\sigma}') \}_{P.B.} = 2v_\beta \bar{u}^\lambda \bar{P}_{u\gamma} \delta^p (\tilde{\sigma} - \tilde{\sigma}')
\]

(170)

and the former is proportional to the primary constraint \( P_{u\gamma} \approx 0 \) and it complex conjugate. The same story is repeated for the fourth term in (168) which multipliers are equal to

\[
\{ K_{\gamma\tilde{\gamma}} (\tilde{\sigma}), \Delta I (\tilde{\sigma}'') \}_{P.B.} = 2i[\bar{u}^\delta (\bar{z}_{\delta\gamma} - 2i\bar{\theta}^{\delta}_{\gamma}) P_{u\gamma} - u^\delta (z_{\gamma\delta} - 2i\theta^{\delta}_{\gamma}) \bar{P}_{u\gamma}] \delta^p (\tilde{\sigma} - \tilde{\sigma}''), \\
\{ T_I^{(v)} (\tilde{\sigma}''), L_\alpha^\beta (\tilde{\sigma}') \}_{P.B.} = 2i v_\beta v_\lambda \pi^{\lambda\alpha} \delta^p (\tilde{\sigma} - \tilde{\sigma}');
\]

(171)

and the first of them is vanished, because of the constraints \( P_{u\gamma} \approx 0 \) and \( P_{u\gamma} \approx 0 \) presence.
Finally, the latter term in \((168)\) including the multipliers
\[
\{K_{\gamma\gamma}(\bar{\sigma}), \Psi_{I}^{(\alpha)}(\bar{\sigma}^{\prime}\prime)\}_{P.B.} = \left[ 8(\theta_{\gamma} \bar{P}_{w\gamma} - \bar{\theta}_{\gamma} P_{w\gamma}) - iv_{\gamma} \left( (z_{\gamma\delta} + 2i\bar{\theta}_{\gamma}\theta_{\delta})\bar{\Psi}^{\delta} + (x_{\gamma\delta} + 2i\theta_{\gamma}\bar{\theta}_{\delta})\Psi^{\delta} \right) \right] \delta^{\gamma}(\bar{\sigma} - \bar{\sigma}^{\prime}),
\]
\[
\{\Psi_{I}^{(\alpha)}(\bar{\sigma}^{\prime}), L_{\alpha\beta}(\bar{\sigma}^{\prime})\}_{P.B.} = iv_{\alpha}\Psi^{\beta}(\bar{\sigma}^{\prime} - \bar{\sigma}^{\prime})
\]
(172)
is also vanished, because of the constraints \(\Psi^{\alpha} \approx 0\) and \(P_{w\gamma} \approx 0\) presence there.

Conclusion is that the D.B. of \(K_{\gamma\gamma}\) and \(L_{\alpha\beta}\) on the surface of the primary constraints coincides with the Poisson bracket and is equal to
\[
\{K_{\gamma\gamma}(\bar{\sigma}), L_{\alpha\beta}(\bar{\sigma}^{\prime})\}_{D.B.|\text{constraint surface}} = -\delta^{\alpha}_{\gamma} K_{\beta\gamma}\delta^{\eta}(\bar{\sigma} - \bar{\sigma}^{\prime}).
\]
(173)

Similar analysis can be done for the D.B.’s. of the remaining generators of the \(OSp(1|8)\) superalgebra and, as a result, one can find that the D.B and P.B realizations of the superalgebra coincide on the primary constraint surface.

In the quantum dynamics the commutator \([\hat{K}_{\gamma\gamma}, \hat{L}_{\alpha\beta}]\) has to be substituted for the D.B. \((173)\) and all the coordinates and momenta should be presented by the correspondent operators. Moreover, the generators of the \(OSp(1|8)\) quantum superalgebra have to include sums of the ordered products of the coordinates and momenta. One can choose, e.g. \(\hat{Q}\hat{P}\)-ordering, where \(\hat{Q}\) is a condensed notation for a chain of coordinate operators \(\hat{q}\) and, respectively, \(\hat{P}\) for a chain of momentum operators \(\hat{p}\). Then, taking into account that any, but fixed \(\hat{p}\)-operator is not contained two or more times into the \(\hat{P}\)-chains, forming the operators \(\hat{K}_{\gamma\gamma}\) and \(\hat{L}_{\alpha\beta}\), we find
\[
[\hat{K}_{\gamma\gamma}(\bar{\sigma}), \hat{L}_{\alpha\beta}(\bar{\sigma}^{\prime})]|_{\text{constraint surface}} = -\delta^{\alpha}_{\gamma} \hat{K}_{\beta\gamma}\delta^{\eta}(\bar{\sigma} - \bar{\sigma}^{\prime}),
\]
(174)
because the operator ordering was not broken during the calculation of the commutator \((174)\). Thus, this commutation relation is anomalous free on the primary constraint surface.

The same \(\hat{Q}\hat{P}\)-ordering procedure was applied to construct the remaining quantum generators of the \(OSp(1|8)\) superalgebra and we observed that all (anti)commutators of the superalgebra generators are anomalous free on the surface of some of the primary constraints.

However, this observation is not yet enough to exclude a possibility of quantum anomaly because, the above discussed subset of the primary constraints contains not only the second-class constraints but, also the first-class constraints which equal zero only in the weak sense. So, more careful investigation of this problem needs to be done and it will be presented in another place.

15 Conclusion

The Hamiltonian structure of the simplest \(D = 4\ N = 1\) super \(p\)-brane model of which general solution describes the BPS state with exotic fraction of supersymmetry equal to \(3/4\) was studied here. The covariant division of the brane constraints into the first and second classes was found. As a result, the generators of the local symmetries and the covariant realization of the Dirac matrix \(\hat{C}\) were constructed. The matrix \(\hat{C}\) was diagonalized and presented in the symplectic form parametrized by the component \(\rho^{\mu}(\tau, \bar{\sigma})\) of the brane world-volume vector density \(\rho^{\mu}(\tau, \bar{\sigma})\). The corresponding D.B. encoding the Hamiltonian symplectic structure of the constrained super \(p\)-brane dynamics were constructed. The D.B. commutation relations between the original \(p\)-brane coordinates in the centrally extended superspace were calculated and their D.B. noncommutativity was established. The D.B.
noncommutativity in the subspace of the space-time and TCC coordinates was shown to have a gauge dependent character and can be removed by complete gauge fixing of the exotic $\kappa$-symmetry. On the contrary, the D.B. noncommutativity of the Grassmannian $\theta$ coordinates among them and with the space-time and TCC coordinates was shown to be gauge independent. The constructed Dirac brackets revealed a deep dynamical role of the original auxiliary spinor variables which manifests itself in their noncommutativity with the space-time and TCC coordinates. The same effect was established for the D.B.’s. of the space-time coordinates with $\rho^\tau$. Exclusion of the canonical pair $(\rho^\tau, P_\tau^{(\rho)})$ from the total phase space was shown to result in the appearance of the non-local factor $\frac{1}{(vP_\tau-v)}$ in the Dirac matrix $\hat{C}_{\text{red}}$ and, consequently, in the Dirac brackets. This peculiarity of the Dirac brackets changed by (anti)commutators may turn out to be principal in the quantum picture of the string/brane dynamics and we started the investigation of the nonlocality problem and calculated the Dirac bracket realization of the global $OSp(1|8)$ superalgebra. Moreover, the $\hat{Q}\hat{P}$-ordering procedure for the superalgebra generators was applied and shown was that the potentially dangerous terms in the superalgebra commutators vanish on the primary constraint surface. Due to the first-class constraints presence among these primary constraints a room for the anomaly presence is not yet excluded and a little bit more further investigation has to be done. This is an objective of our paper under preparation [20], where the BRST procedure for the quantization of the considered string/brane model is studied. Note also, that our analysis is naturally applied for the super $p$-brane models preserving $\frac{M-1}{M}$ fraction of $N = 1$ supersymmetry in higher dimensions. In particular, this concerns the centrally extended $D = 11$ superspace, where $31/32$ fraction of $N = 1$ supersymmetry for the studied here model of tensionless super $p$-brane is preserved [16]. Some additional details arising at the transition from $D = 4$ to $D = 11$ can be found in the recent paper [21], where superstring model preserving less number fractions of $D = 11$ $N = 1$ supersymmetry was considered.

16 Acknowledgements

A.Z. thanks Fysikum at the Stockholm University and Mittag-Leffler Institute for kind hospitality and I. Bengtsson, O. Laudal, U. Lindstrom and B. Sundborg for useful discussions. The work was supported in part by the Royal Swedish Academy of Sciences and SFFR of Ukraine under project 02.07/276.

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