Nonfactorizable Effects in Exclusive Charmless $B$ Decays

Hai-Yang Cheng
Institute of Physics, Academia Sinica
Taipei, Taiwan 115, Republic of China

B. Tseng
Department of Physics, National Taiwan University
Taipei, Taiwan 106, Republic of China

Abstract

Nonfactorizable effects in charmless $B \to PP, VP$ decays can be lumped into the effective parameters $a_i$ that are linear combinations of Wilson coefficients, or equivalently absorbed into the effective number of colors $N_c^{\text{eff}}$. Naive factorization with $N_c^{\text{eff}} = 3$ fails to explain the CLEO data of $B^\pm \to \omega K^\pm$, indicating the first evidence for the importance of nonfactorizable contributions to the penguin amplitude. The decays $B^\pm \to \omega \pi^\pm$ dominated by tree amplitudes are sensitive to the interference between external and internal $W$-emission diagrams. Destructive interference implied by $N_c^{\text{eff}} = \infty$ leads to a prediction of $B(B^\pm \to \omega \pi^\pm)$ which is about $2\sigma$ too small compared to experiment. Therefore, the CLEO data of $B^\pm \to \omega K^\pm, \omega \pi^\pm$ suffice to rule out $N_c^{\text{eff}} = 3$ and strongly disfavor $N_c^{\text{eff}} = \infty$ for rare charmless $B$ decays. Factorization based on $N_c^{\text{eff}} \approx 2$ can accommodate the data of $B^\pm \to \omega K^\pm$, but it predicts a slightly smaller branching ratio of $B^\pm \to \omega \pi^\pm$ with $B(B^\pm \to \omega \pi^\pm)/B(B^\pm \to \omega K^\pm) = 0.6$. We briefly explain why the $1/N_c$ expansion is still applicable to the $B$ meson decay once the correct large-$N_c$ counting rule for the Wilson coefficient $c_2(m_b)$ is applied.
an effective contributions, especially for class-II decays, to render the color suppression of internal difficulty indicates that it is inevitable and mandatory to take into account nonfactorizable evidence of Wilson coefficients does not get compensation from the matrix elements. The first
emission ineffective. In principle, the second difficulty also should not occur since the matrix elements of four-quark operators ought to be evaluated in the same renormalization scheme as that for Wilson coefficients and renormalized at the same scale \( \mu \).

Because there is only one single form factor (or Lorentz scalar) involved in the class-I or class II decay amplitude of \( B (D) \rightarrow PP, PV \) decays (\( P: \) pseudoscalar meson, \( V: \) vector meson), the effects of nonfactorization can be lumped into the effective parameters \( a_1 \) and \( a_2 \) \[2\]:

\[
a_{1}^{\text{eff}} = c_1(\mu) + c_2(\mu) \left( \frac{1}{N_c} + \chi_1(\mu) \right), \quad a_{2}^{\text{eff}} = c_2(\mu) + c_1(\mu) \left( \frac{1}{N_c} + \chi_2(\mu) \right),
\]

where nonfactorizable contributions are characterized by the parameters \( \chi_1 \) and \( \chi_2 \). Taking the decay \( B^- \rightarrow D^0\pi^- \) as an example, we have \[3\] \[4\] \[5\]

\[
\chi_1(\mu) = \varepsilon_8^{(BD,\pi)}(\mu) + \frac{a_1}{c_2} \varepsilon_1^{(BD,\pi)}(\mu), \quad \chi_2(\mu) = \varepsilon_8^{(B\pi,D)}(\mu) + \frac{a_2}{c_1} \varepsilon_1^{(B\pi,D)}(\mu),
\]

where

\[
\varepsilon_1^{(BD,\pi)}(\mu) = \frac{\langle D^0\pi^-|\langle \bar{d}u \rangle_{V-A}\langle \bar{c}b \rangle_{V-A}|B^- \rangle_{nf}}{\langle D^0\pi^-|\langle \bar{d}u \rangle_{V-A}\langle \bar{c}b \rangle_{V-A}|B^- \rangle_f} = \frac{\langle D^0\pi^-|\langle \bar{d}u \rangle_{V-A}\langle \bar{c}b \rangle_{V-A}|B^- \rangle}{\langle \pi^-|\langle \bar{d}u \rangle_{V-A}|0 \rangle \langle D^0|\langle \bar{c}b \rangle_{V-A}|B^- \rangle} - 1,
\]

\[
\varepsilon_8^{(BD,\pi)}(\mu) = \frac{1}{2} \frac{\langle D^0\pi^-|\langle \bar{d}\lambda^a u \rangle_{V-A}\langle \bar{c}\lambda^aq b \rangle_{V-A}|B^- \rangle}{\langle \pi^-|\langle \bar{d}u \rangle_{V-A}|0 \rangle \langle D^0|\langle \bar{c}b \rangle_{V-A}|B^- \rangle},
\]

are nonfactorizable terms originated from color-singlet and color-octet currents, respectively, \( (\bar{q}_1q_2)_{V-A} \equiv \bar{q}_1\gamma_\mu(1 - \gamma_5)q_2 \), and \( (\bar{q}_1\lambda^a q_2)_{V-A} \equiv \bar{q}_1\lambda^a\gamma_\mu(1 - \gamma_5)q_2 \). The subscript ‘f’ and

\[\footnotemark[1]: \] As pointed out in \[3\], the general amplitude of \( B(D) \rightarrow VV \) decay consists of three independent Lorentz scalars, corresponding to \( S-, P- \) and \( D- \) wave amplitudes. Consequently, it is in general not possible to define an effective \( a_1 \) or \( a_2 \) unless nonfactorizable terms contribute in equal weight to all partial wave amplitudes.
‘nf’ in Eq. (3) stand for factorizable and nonfactorizable contributions, respectively, and the superscript \((BD, \pi)\) in Eq. (2) means that the pion is factored out in the factorizable amplitude of \(B \to D\pi\) and likewise for the superscript \((B\pi, D)\). In the large-\(N_c\) limit, \(\varepsilon_1 = \mathcal{O}(1/N_c^2)\) and \(\varepsilon_8 = \mathcal{O}(1/N_c)\) [3]. Therefore, the nonfactorizable term \(\chi\) in the \(N_c \to \infty\) limit is dominated by color-octet current operators. Since \(|c_1/c_2| \gg 1\), it is evident from Eq. (1) that even a small amount of nonfactorizable contributions will have a significant effect on the color-suppressed class-II amplitude. Note that the effective parameters \(a_{\text{eff}}^i\) include all the contributions to the matrix elements and hence are \(\mu\) independent [6]. If \(\chi_{1,2}\) are universal (i.e. channel independent) in charm or bottom decays, then we still have a new factorization scheme in which the decay amplitude is expressed in terms of factorizable contributions multiplied by the universal effective parameters \(a_{\text{eff}}^{1,2}\). The first systematical study of nonleptonic weak decays of heavy mesons within the framework of the improved factorization was carried out by Bauer, Stech, and Wirbel [7]. Phenomenological analyses of two-body decay data of \(D\) and \(B\) mesons indicate that while the generalized factorization hypothesis in general works reasonably well, the effective parameters \(a_{\text{eff}}^{1,2}\) do show some variation from channel to channel, especially for the weak decays of charmed mesons [2, 5, 8]. An eminent feature emerged from the data analysis is that \(a_{\text{eff}}^2\) is negative in charm decay, whereas it becomes positive in bottom decay [2, 9, 6]:

\[
a_{\text{eff}}^2(D \to \bar{K}\pi) \sim -0.50, \quad a_{\text{eff}}^2(B \to D\pi) \sim 0.26. \tag{4}
\]

It should be stressed that since the magnitude of \(a_{1,2}\) depends on the model results for form factors, the above values of \(a_2\) should be considered as representative ones. The sign of \(a_{\text{eff}}^2\) is fixed by the observed destructive interference in \(D^+ \to \bar{K}^0\pi^+\) and constructive interference in \(B^- \to D^0\pi^-\). Eq. (4) then leads to

\[
\chi_2(\mu \sim m_c; D \to \bar{K}\pi) \sim -0.36, \quad \chi_2(\mu \sim m_b; B \to D\pi) \sim 0.11. \tag{5}
\]

In general the determination of \(\chi_2\) is easier and more reliable than \(\chi_1\). The observation \(|\chi_2(B)| \ll |\chi_2(D)|\) is consistent with the intuitive picture that soft gluon effects become stronger when the final-state particles move slower, allowing more time for significant final-state interactions after hadronization [4].

Phenomenologically, it is often to treat the number of colors \(N_c\) as a free parameter and fit it to the data. Theoretically, this amounts to defining an effective number of colors by

\[
1/N_c^{\text{eff}} \equiv (1/N_c) + \chi. \tag{6}
\]

It is clear from Eq. (5) that

\[
N_c^{\text{eff}}(D \to \bar{K}\pi) \gg 3, \quad N_c^{\text{eff}}(B \to D\pi) \sim 2. \tag{7}
\]

Consequently, the empirical rule of discarding subleading \(1/N_c\) terms formulated in the large-\(N_c\) approach [10] is justified for exclusive charm decay; the dynamical origin of the \(1/N_c\) expansion comes from the fact that the Fierz \(1/N_c\) terms are largely compensated by
nonfactorizable effects in charm decay. Since the large-$N_c$ approach implies $a_2^{\text{eff}} \sim c_2$ and since $a_2^{\text{eff}}$ is observed to be positive in $B^- \to D^{(*)}\pi^-(\rho^-)$ decays, one may wonder why is the $1/N_c$ expansion no longer applicable to the $B$ meson? Contrary to the common belief, a careful study shows this is not the case. As pointed out in [3], the large-$N_c$ color counting rule for the Wilson coefficient $c_2(\mu)$ is different at $\mu \sim m_b$ and $\mu \sim m_c$ due to the presence of the large logarithm at $\mu \sim m_c$. More specifically, $c_2(m_b) = O(1/N_c)$ and $c_2(m_c) = O(1)$. Recalling that $c_1 = O(1)$, it follows that in the large-$N_c$ limit [3]:

$$a_2^{\text{eff}} = \begin{cases} c_2(m_c) + O(1/N_c) & \text{for the } D \text{ meson,} \\ c_2(m_b) + c_1(m_b) \left( \frac{1}{N_c} + \varepsilon_8(m_b) \right) + O(1/N_c^3) & \text{for the } B \text{ meson.} \end{cases}$$

Therefore, a priori the $1/N_c$ expansion does not demand a negative $a_2^{\text{eff}}$ for bottom decay! and $N_c^{\text{eff}}(B \to D\pi) \sim 2$ is not in conflict with the large-$N_c$ approach! It should be remarked that although $\chi_2$ is positive in two-body decays of the $B$ meson, some theoretical argument suggests that it may become negative for high multiplicity decay modes [3].

Thus far the nonfactorization effect is discussed at the purely phenomenological level. It is thus important to have a theoretical estimate of $\chi_i$ even approximately. Unfortunately, all existing theoretical calculations based on the QCD sum rule [11], though confirm the cancellation between the $1/N_c$ Fierz terms and nonfactorizable soft gluon effects [12], tend to predict a negative $\chi$ in $B^0 \to D^+\pi^-$, $D^0\pi^0$ and $B \to J/\psi K(K^*)$ decays. This tantalizing issue should be clarified and resolved in the near future. It is interesting to remark that, relying on a different approach, namely, the three-scale PQCD factorization theorem, to tackle the nonfactorization effect, one of us and Li [13] are able to explain the sign change of $\chi_2$ from bottom to charm decays.

For $B$ meson decay, the effective parameters $a_{1,2}^{\text{eff}}$ have been determined so far only for $B \to D^{(*)}\pi(\rho)$ and $B \to J/\psi K^{(*)}$ where nonfactorizable effects amount to having $N_c^{\text{eff}} \sim 2$. Recently, several exclusive charmless rare $B$ decay modes have been reported for the first time by CLEO [14] and many of them are dominated by the penguin mechanism. It is thus important to know (i) does the constructive interference of tree amplitudes persist in class-III charmless $B$ decay? (class-III transitions receive contributions from both external and internal $W$ emissions), and (ii) is $N_c^{\text{eff}} \sim 2$ still applicable to the penguin amplitude? Whether $N_c^{\text{eff}} \sim 2$ or $N_c^{\text{eff}} \sim \infty$ for $B$ decay to two light mesons is still under debate. For example, predictions for exclusive charmless $B$ decay are presented in [13] for $N_c^{\text{eff}} = \infty$. Recently, it was argued in [10] that the CLEO data of two-body charmless $B$ decays can be accommodated by $0 \leq 1/N_c^{\text{eff}} \leq 0.5$ with $N_c^{\text{eff}} = \infty$ being more preferred. In this Letter we shall demonstrate that naive factorization (i.e. $N_c^{\text{eff}} = 3$) is ruled out by the CLEO data of $B^{\pm} \to \omega K^{\pm}$ and $N_c^{\text{eff}} = \infty$ is strongly disfavored by the data of $B^{\pm} \to \omega\pi^{\pm}$. This implies the applicability of $N_c^{\text{eff}} \sim 2$ to the rare charmless $B$ decays.

2. In this section we will consider the decay modes dominated by penguin diagrams in order to study their $N_c^{\text{eff}}$ dependence. It was pointed out in [17] that the parameters $a_2$, $a_3$ and $a_5$ are strongly dependent on $N_c^{\text{eff}}$ and the rates dominated by these coefficients can have large variation. For example, the decay widths of $B^- \to \omega K^{(*)-}$, $B^0 \to \omega K^0$, $\rho K^{*-}$, $\omega K^{*-}$, $\omega\pi^-$ and $\omega\pi^+$ are strongly dependent on $N_c^{\text{eff}}$. In this Letter we shall demonstrate that naive factorization (i.e. $N_c^{\text{eff}} = 3$) is ruled out by the CLEO data of $B^{\pm} \to \omega K^{\pm}$ and $N_c^{\text{eff}} = \infty$ is strongly disfavored by the data of $B^{\pm} \to \omega\pi^{\pm}$. This implies the applicability of $N_c^{\text{eff}} \sim 2$ to the rare charmless $B$ decays.
$B_s \rightarrow \eta \omega, \eta \phi, \omega \phi, \cdots$, etc. have strong $N_c$ dependence \[7\]. We shall see that the branching ratio of $B^- \rightarrow \omega K^-$ has its lowest value near $N_c^\text{eff} \sim 3 - 4$ and hence the naive factorization with $N_c^\text{eff} = 3$ is ruled out by experiment.

Before proceeding we briefly sketch the calculational framework. The relevant effective $\Delta B = 1$ weak Hamiltonian is

\[ \mathcal{H}^\text{eff}(\Delta B = 1) = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{cb}^* (c_1 O_1^u + c_2 O_2^u) + V_{ub} V_{cb}^* (c_1 O_1^c + c_2 O_2^c) - V_{tb} V_{tb}^* \sum_{i=3}^{10} c_i O_i \right] + \text{h.c.}, \]

where $q = u, d, s$, and

\[ O_1^u = \langle \bar{u} b \rangle_{V-A} \langle \bar{q} u \rangle_{V-A}, \quad O_2^u = \langle \bar{q} b \rangle_{V-A} \langle \bar{u} u \rangle_{V-A}, \]

\[ O_3(5) = \langle \bar{q} b \rangle_{V-A} \sum_{q'} (\bar{q}' q')_{V-A(V+A)}, \quad O_4(6) = \langle \bar{q} a b \rangle_{V-A} \sum_q (\bar{q}' g a)_{V-A(V+A)}, \]

\[ O_7(9) = \frac{3}{2} \langle \bar{q} b \rangle_{V-A} \sum_{q'} e_{q'} (\bar{q}' q')_{V-A(V-A)}, \quad O_8(10) = \frac{3}{2} \langle \bar{q} a b \rangle_{V-A} \sum_{q'} e_{q'} (\bar{q}' g a)_{V-A(V-A)}, \]

where $O_3$-$O_6$ are QCD penguin operators and $O_7$-$O_{10}$ originate from electroweak penguin diagrams. As noted in passing, in order to ensure the renormalization-scale and -scheme independence for the physical amplitude, the matrix elements of 4-quark operators have to be evaluated in the same renormalization scheme as that for Wilson coefficients and renormalized at the same scale $\mu$. Before utilizing factorization, it is necessary to take into account QCD and electroweak corrections to matrix elements:

\[ \langle O_i(\mu) \rangle = \left[ 1 + \frac{\alpha_s(\mu)}{4\pi} \hat{m}_s(\mu) + \frac{\alpha}{4\pi} \hat{m}_c(\mu) \right] \langle O_i^\text{tree} \rangle, \]

so that $c_i(\mu) \langle O_i(\mu) \rangle = \tilde{c}_i \langle O_i^\text{tree} \rangle$, where

\[ \tilde{c}_i = \left[ 1 + \frac{\alpha_s(\mu)}{4\pi} \hat{m}_s(\mu) + \frac{\alpha}{4\pi} \hat{m}_c(\mu) \right] c_j(\mu). \]

Then the factorization approximation is applied to the hadronic matrix elements of the tree operator $O^\text{tree}$. Perturbative QCD and electroweak corrections to the matrices $\hat{m}_s$ and $\hat{m}_c$ have been calculated in \[13\], \[14\], \[15\], \[16\]. Using the next-to-leading order $\Delta B = 1$ Wilson coefficients obtained in the ’t Hooft-Veltman scheme and the naive dimension regularization scheme at $\mu = 4.4$ GeV, $\Lambda^{(5)}_{\overline{MS}} = 225$ MeV and $m_t = 170$ GeV in Table 22 of \[20\], we obtain the renormalization-scheme and -scale independent Wilson coefficients $\tilde{c}_i$ at $k^2 = m_b^2/2$: \[2\]

\[ \tilde{c}_1 = 1.187, \quad \tilde{c}_2 = -0.312, \]

\[ \tilde{c}_3 = 0.0236 + i0.0048, \quad \tilde{c}_4 = -0.0547 - i0.0143, \]

\[ \tilde{c}_5 = 0.0164 + i0.0048, \quad \tilde{c}_6 = -0.0640 - i0.0143, \]

\[ \tilde{c}_7 = -(0.0757 + i0.0558)\alpha, \quad \tilde{c}_8 = 0.057 \alpha, \]

\[ \tilde{c}_9 = -(1.3648 + i0.0558)\alpha, \quad \tilde{c}_{10} = 0.264 \alpha. \]
We now apply the Hamiltonian (9) and factorization to the decay $B^- \rightarrow \omega K^-$ and obtain

\[
A(B^- \rightarrow \omega K^-) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{us}^* \left( a_1 X_1 + a_2 X_{2u} + a_3 X_3 \right) - V_{ub} V_{ts}^* \left( a_4 + a_{10} - 2(a_6 + a_8) \frac{m_K^2}{(m_s + m_u)(m_b + m_u)} X_1 \right) + \frac{1}{2} (4a_3 + 4a_5 + a_7 + a_9) X_{2u} + \left( a_4 + a_{10} - 2(a_6 + a_8) \frac{m_B^2}{(m_s + m_u)(m_b + m_u)} X_3 \right) \right\},
\]

(14)

where $a_{2i} = \tilde{c}_{2i} + \frac{1}{N_c} \tilde{c}_{2i-1}$, $a_{2i-1} = \tilde{c}_{2i-1} + \frac{1}{N_c} \tilde{c}_{2i}$, and $X_i$ are factorizable terms:

\[
\begin{align*}
X_1 &= \langle K^-|(\bar{s}u)_{\nu-A}|0\rangle \langle \omega|(\bar{u}b)_{\nu-A}|B^-\rangle = -i \sqrt{2} f_K m_\omega A_0^{B\omega}(m_K^2) (\varepsilon \cdot p_B), \\
X_{2q} &= \langle \omega|((\bar{q}q)_{\nu-A}|0\rangle \langle K^-|(\bar{s}b)_{\nu-A}|B^-\rangle = -i \sqrt{2} f_\omega m_{\omega} F_1^{BK}(m_\omega^2) (\varepsilon \cdot p_B), \\
X_3 &= \langle \omega K^-|(\bar{s}u)_{\nu-A}|0\rangle \langle 0|(\bar{u}b)_{\nu-A}|B^-\rangle,
\end{align*}
\]

(15)

with $\varepsilon$ the polarization vector of the $\omega$ meson, and $A_1$, $F_i$ the form factors defined in [22]. Just as in the case of tree amplitudes, one can show that nonfactorizable effects in the penguin amplitudes of $B \rightarrow PP$, $VP$ decays can be absorbed into the effective penguin coefficients. This amounts to replacing $N_c$ in the penguin coefficients $a_i$ ($i = 3, \ldots, 10$) by $(N_c^{\text{eff}})_i$. (It must be emphasized that the factor of $N_c$ appearing in any place other than $a_i$ should not be replaced by $N_c^{\text{eff}}$.) For simplicity, we will assume $(N_c^{\text{eff}})_1 \approx (N_c^{\text{eff}})_2 \cdots \approx (N_c^{\text{eff}})_{10}$ so that the subscript $i$ can be dropped. For $N_c^{\text{eff}} = 3$, the QCD-penguin Wilson coefficients are numerically given by $\text{Re} a_3 = 0.0054, \text{Re} a_4 = -0.0468, \text{Re} a_5 = -0.0049$, and $\text{Re} a_6 = -0.0585$. From Eq. (14) we see that a large cancellation occurs in the QCD penguin amplitude due to the large compensation between $a_3$ and $a_5, a_4$ and $a_6$. Since $|V_{ub} V_{ts}^*| \gg |V_{ub} V_{us}^*|$, the decay rate of $B^\pm \rightarrow \omega K^\pm$ has its minimum around $N_c^{\text{eff}} \sim 3 - 4$ (see Fig. 1), as noticed in [17, 13] and analyzed in detail in [16].

Neglecting the $W$-annihilation contribution denoted by $X_3$, and using $f_K = 160$ MeV, $f_\omega = 195$ MeV for decay constants, $A_0^{B\omega}(0) = 0.28, F_1^{BK}(0) = 0.34$ [22], and dipole $q^2$ dependence for form factors $A_0$ and $F_1$ [22], $m_u = 5$ MeV, $m_d = 10$ MeV, $m_s = 175$ MeV, $m_b = 5$ GeV for quark masses, $\tau(B^\pm) = (1.66 \pm 0.04)$ ps [24] for the charged $B$ lifetime, and $A = 0.804, \lambda = 0.22, \eta = 0.30, \rho = 0.30$ [21] for Wolfenstein parameters [25], we obtain the averaged branching ratios of $B^\pm \rightarrow \omega K^\pm$ defined by

\[
B(B^\pm \rightarrow \omega K^\pm) \equiv \frac{1}{2} \left[ B(B^+ \rightarrow \omega K^+) + B(B^- \rightarrow \omega K^-) \right],
\]

(16)

in Table I and Fig. 1. We see that the prediction $B(B^\pm \rightarrow \omega K^\pm) = 1.44 \times 10^{-6}$ at $N_c^{\text{eff}} = 3$ and $\rho = -0.30$ is off by $2\sigma$ from the experimental result [14]

\[
B(B^\pm \rightarrow \omega K^\pm) \approx \left( 1.2_{-0.5}^{+0.7} \pm 0.2 \right) \times 10^{-5}.
\]

(17)

This shows that naive factorization with $N_c^{\text{eff}} = 3$ (or $\chi = 0$) fails to explain the decay rate of $B^\pm \rightarrow \omega K^\pm$. It is clear from Table I or Fig. 1 that, for $\rho < 0$, $N_c^{\text{eff}} = 2$ is slightly better than
Table I. Averaged branching ratios for charmless $B$ decays, where “Tree” refers to branching ratios from tree diagrams only, “Tree+QCD” from tree and QCD penguin diagrams, and “Tree+QCD+QED” from tree, QCD and electroweak (EW) penguin diagrams. Predictions are made for $k^2 = m_K^2/2$, $\eta = 0.30$, $\rho = 0.30$ (the first number in parentheses) and $\rho = -0.30$ (the second number in parentheses).

| Decay                  | $N_c^{\text{eff}}$ | Tree       | Tree+QCD  | Tree+QCD+EW | Exp. [14]            |
|------------------------|---------------------|------------|-----------|-------------|----------------------|
| $B^\pm \to \omega K^\pm$ | 2                   | $6.53 \times 10^{-7}$ | $(2.43, 6.82) \times 10^{-6}$ | $(3.28, 8.31) \times 10^{-6}$ | $(1.2^{+0.7}_{-0.5} \pm 0.2) \times 10^{-5}$ |
|                        | 3                   | $4.49 \times 10^{-7}$ | $(2.58, 9.63) \times 10^{-7}$ | $(0.27, 1.44) \times 10^{-6}$ |                      |
|                        | $\infty$            | $1.56 \times 10^{-7}$ | $(9.49, 6.44) \times 10^{-6}$ | $(8.46, 5.61) \times 10^{-6}$ |                      |
| $B^0 \to \omega K^0$   | 2                   | $5.25 \times 10^{-8}$ | $(3.24, 4.46) \times 10^{-6}$ | $(3.78, 5.09) \times 10^{-6}$ |                      |
|                        | 3                   | $4.64 \times 10^{-9}$ | $(1.37, 2.10) \times 10^{-7}$ | $(2.89, 3.96) \times 10^{-7}$ |                      |
|                        | $\infty$            | $6.45 \times 10^{-8}$ | $(6.28, 8.16) \times 10^{-6}$ | $(5.18, 6.89) \times 10^{-6}$ |                      |
| $B^\pm \to \omega \pi^\pm$ | 2                   | $9.71 \times 10^{-6}$ | $(1.15, 0.53) \times 10^{-5}$ | $(1.16, 0.52) \times 10^{-5}$ | $(1.2^{+0.7}_{-0.5} \pm 0.2) \times 10^{-5}$ |
|                        | 3                   | $6.23 \times 10^{-6}$ | $(7.07, 3.93) \times 10^{-6}$ | $(7.16, 3.76) \times 10^{-6}$ |                      |
|                        | $\infty$            | $1.57 \times 10^{-6}$ | $(1.49, 1.93) \times 10^{-6}$ | $(1.52, 1.78) \times 10^{-6}$ |                      |

$N_c^{\text{eff}} = \infty$ and yields a branching ratio of order $8.3 \times 10^{-6}$, in agreement with experiment. \footnote{Our conclusions for $N_c^{\text{eff}} = 2$ and $N_c^{\text{eff}} = \infty$ are different from that in [16] which claimed that a value of $1/N_c^{\text{eff}}$ in the range $0.15 \leq 1/N_c^{\text{eff}} \leq 0.55$ is disfavored by the data of $B^\pm \to \omega K^\pm$ and that $0 \leq 1/N_c^{\text{eff}} \leq 0.15$ is preferred.}

Note that the electroweak penguin contribution is constructive at $\rho > 0$, destructive at $\rho < 0$, and negligible in the region $1 < 1/N_c^{\text{eff}} \leq 2$. However, we have shown in [21] that a positive $\rho$ is quite disfavored by data as it predicts $B(B^\pm \to \eta \pi^\pm) \sim 1 \times 10^{-5}$, which is marginally larger than the CLEO measured upper limit: $0.8 \times 10^{-5}$ [14].

A similar strong $N_c^{\text{eff}}$ dependence also can be observed in $B_d^0 \to \omega K^0$ decay. \footnote{As noted before, nonfactorizable effects in $B \to VV$ generally cannot be absorbed into the effective parameters $a_i$. Hence, we will not discuss $B \to \omega K^*$, $\rho K^*$ decays.}

The expression of its factorizable amplitude is simpler than the charged $B$ meson:

$$A(B_d^0 \to \omega K^0) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{us}^* a_2 X_{2u} - V_{tb} V_{ts}^* \left[ \frac{1}{2} (4a_3 + 4a_5 + a_7 + a_9) X_{2u} + \left( a_4 - \frac{1}{2} a_{10} - (2a_6 - a_8) \frac{m_K^2}{(m_s + m_d)(m_s + m_d)} \right) X_1 \right] \right\}.$$

The averaged decay rate $\Gamma(B^0 \to \omega K^0) \equiv \frac{1}{2} [\Gamma(B^0 \to \omega K^0) + \Gamma(\bar{B}^0 \to \omega \bar{K}^0)]$ is minimal near $N_c^{\text{eff}} \sim 4$ (see Fig. 2). The branching ratio predicted by $N_c^{\text{eff}} = \infty$ is slightly larger than that by $N_c^{\text{eff}} = 2$ (see also Table I).
between $N_{c}^{\text{eff}} = \infty$ and $N_{c}^{\text{eff}} = 2$ from $B \to \omega K$ decays, though the latter can explain the data of $B^\pm \to \omega K^\pm$ and is more preferred. It is thus important to have a more decisive test on $N_{c}^{\text{eff}}$. For this purpose, we shall focus in this section the decay modes dominated by the tree diagrams and sensitive to the interference between external and internal $W$-emission amplitudes. The fact that $N_{c}^{\text{eff}} = 2$ ($N_{c}^{\text{eff}} = \infty$) implies constructive (destructive) interference will enable us to differentiate between them. Good examples are the class-III emission amplitudes. The fact that $N_{c}^{\text{eff}}$ has been measured by CLEO, we will first investigate this mode.

Under factorization, the decay amplitude of $B^- \to \omega \pi^-$ is given by

$$A(B^- \to \omega \pi^-) = \frac{G_F}{\sqrt{2}} \left[ V_{ub}V_{ud}^* (a_1X_1' + a_2X_2' + a_3X_3') - V_{tb}V_{td}^* \left( (a_4 + a_10 - 2(a_6 + a_8)\frac{m^2_{\pi}}{(m_b + m_u)(m_u + m_d)})X_1' + \frac{1}{2}(4a_3 + 2a_4 + 4a_5 + a_7 + a_9 - a_{10})X_2' + \left( a_4 + a_{10} - 2(a_6 + a_8)\frac{m^2_{B}}{(m_b + m_u)(m_u + m_d)})X_3' \right) \right],$$

(19)

with the expressions of $X_i'$ similar to (15). Since

$$V_{ub}V_{ud}^* = A\lambda^3(\rho - i\eta), \quad V_{cb}V_{cd}^* = -A\lambda^3, \quad V_{tb}V_{td}^* = A\lambda^3(1 - \rho + i\eta),$$

(20)
in terms of the Wolfenstein parametrization [25], are of the same order of magnitude, it is clear that $B^- \to \omega \pi^-$ is dominated by external and internal $W$ emissions and that penguin contributions are suppressed by the smallness of penguin coefficients. In the limit of $N_{c}^{\text{eff}} \to \infty$, we have $a_1 = c_1$ and $a_2 = c_2$, which in turn imply a destructive interference of tree amplitudes in $B^\pm \to \omega \pi^\pm$. It is easily seen from Eq. (13) that the interference becomes constructive when $N_{c}^{\text{eff}} < 3.8$. From Fig. 3 or Table I we see that the averaged branching ratio of $B^\pm \to \omega \pi^\pm$ has its lowest value of order $2 \times 10^{-6}$ at $N_{c}^{\text{eff}} = \infty$ and then increases with $1/N_{c}^{\text{eff}}$. Since experimentally [14]

$$B(B^\pm \to \omega \pi^\pm) = \left( 1.2^{+0.7}_{-0.5} \pm 0.2 \right) \times 10^{-5},$$

(21)
it is evident that $N_{c}^{\text{eff}} = \infty$ is strongly disfavored by the data. Note that though the predicted branching ratio $B(B^\pm \to \omega \pi^\pm) = 1.16 \times 10^{-5}$ for $N_{c}^{\text{eff}} = 2$ and $\rho = 0.30$ (see Table I) is in good agreement with experiment, we have discussed in passing that a positive $\rho$ seems to be ruled out [21]. For $\rho < 0$, our prediction $B(B^\pm \to \omega \pi^\pm) = 0.52 \times 10^{-5}$ is on the verge of the lower side of the CLEO data. Of course, the CLEO measurement can be more satisfactorily explained by having a much smaller $N_{c}^{\text{eff}}$, but this possibility is very unlikely as it implies a large nonfactorization effect in $B^\pm \to \omega \pi^\pm$. Recalling that the magnitude of nonfactorizable term is $\chi \sim 0.1$ in $B \to D\pi$ decay and that the energy release in the process $B \to \omega \pi$ is larger than that in $B \to D\pi$, it is thus expected physically that $\chi \lesssim 0.1$ for the former.
Using $N_e^{\text{eff}} = 2$, we find a slightly smaller branching ratio for $B^\pm \to \omega \pi^\pm$ with
\[ \frac{\mathcal{B}(B^\pm \to \omega \pi^\pm)}{\mathcal{B}(B^\pm \to \omega K^\pm)} = 0.61. \] (22)

Since theoretically it is difficult to see how the branching ratio of $B^\pm \to \omega \pi^\pm$ can be enhanced from $0.5 \times 10^{-5}$ to $1.2 \times 10^{-5}$, it is thus important to have a refined and improved measurement of this decay mode.

In analogue to the decays $B \to D^{(*)} \pi(\rho)$, the interference effect of tree amplitudes in class-III charmless $B$ decay can be tested by measuring the ratios:
\[ R_1 \equiv 2 \frac{\mathcal{B}(B^- \to \pi^- \pi^0)}{\mathcal{B}(B^0 \to \pi^- \pi^+)} , \quad R_2 \equiv 2 \frac{\mathcal{B}(B^- \to \rho^- \pi^0)}{\mathcal{B}(B^0 \to \rho^- \pi^+)} , \quad R_3 \equiv 2 \frac{\mathcal{B}(B^- \to \pi^- \rho^0)}{\mathcal{B}(B^0 \to \pi^- \rho^+)} . \] (23)

Since penguin contributions are very small, to a good approximation we have
\[ R_1 = \frac{\tau(B^-)}{\tau(B^0)} \left( 1 + \frac{a_2}{a_1} \right)^2 , \]
\[ R_2 = \frac{\tau(B^-)}{\tau(B^0)} \left( 1 + \frac{f_\pi A_0^{B\rho} (m_{\pi^0}^2) a_2}{f_\rho F_{1B\pi} (m_{\rho}^2) a_1} \right)^2 , \]
\[ R_3 = \frac{\tau(B^-)}{\tau(B^0)} \left( 1 + \frac{f_\rho F_{1B\pi} (m_{\rho}^2) a_2}{f_\pi A_0^{B\rho} (m_{\pi^0}^2) a_1} \right)^2 . \] (24)

Evidently, the ratios $R_i$ are greater (less) than unity when the interference is constructive (destructive). Numerically we find
\[ R_1 = \begin{cases} 1.74, & \text{for } N_e^{\text{eff}} = 2, \\ 0.58, & \text{for } N_e^{\text{eff}} = \infty, \end{cases} \]
\[ R_2 = \begin{cases} 1.40, & \text{for } N_e^{\text{eff}} = 2, \\ 0.80, & \text{for } N_e^{\text{eff}} = \infty, \end{cases} \]
\[ R_3 = \begin{cases} 2.50, & \text{for } N_e^{\text{eff}} = 2, \\ 0.26, & \text{for } N_e^{\text{eff}} = \infty, \end{cases} \] (25)

where use of $\tau(B^0) = (1.55 \pm 0.04) \text{ ps}$ [24], $f_\rho = 216 \text{ MeV}$, $A_0^{B\rho}(0) = 0.28$ [23] has been made. Hence, a measurement of $R_i$ (in particular $R_3$), which has the advantage of being independent of the parameters $\rho$ and $\eta$, will constitute a very useful test on the effective number of colors $N_e^{\text{eff}}$.

We would like to stress once again that the observation that $N_e^{\text{eff}} = \infty$ is very likely to be ruled out in charmless $B$ decay does not imply the inapplicability of the large-$N_c$ approach to the $B$ meson case. As explained before, the correct large-$N_c$ counting rule for the Wilson coefficient $c_2(m_b)$ is proportional to $1/N_c$. Consequently, a nontrivial $a_2^{\text{eff}}$, given by $c_2(m_b) + \frac{1}{N_e^{\text{eff}}} c_1(m_b)$, starts at the order of $1/N_c$ and hence $N_e^{\text{eff}}$ cannot go to infinity.

4. To conclude, by absorbing the nonfactorizable effects into the effective number of colors $N_e^{\text{eff}}$, we have shown that $N_e^{\text{eff}} = 3$ is ruled out by the CLEO data of $B^\pm \to \omega K^\pm$, implying the inapplicability of naive factorization to charmless $B$ decays, and that $N_e^{\text{eff}} = \infty$ is strongly disfavored by the experimental measurement of $B^\pm \to \omega \pi^\pm$, indicating a constructive interference in class-III charmless $B$ decays.
Since the energy release in charmless two-body decays of the $B$ meson is generally slightly larger than that in $B \to D^{(*)}\pi$, $D^{(*)}\rho$, it is natural to expect that $N_{\text{c}}^{\text{eff}}$ for the $B$ decay into two light mesons is close to $N_{\text{c}}^{\text{eff}}(B \to D\pi) \approx 2$. We have shown that $N_{\text{c}}^{\text{eff}} \approx 2$ can accommodate the data of $B^{\pm} \to \omega K^{\pm}$, but it predicts a slightly smaller branching ratio of $B^{\pm} \to \omega \pi^{\pm}$ with $B(B^{\pm} \to \omega \pi^{\pm})/B(B^{\pm} \to \omega K^{\pm}) = 0.6$. Thus far, the discussion of nonfactorization in rare $B$ decay is at the purely phenomenological level. It remains to be a challenge, especially in the framework of the QCD sum rule, to compute it theoretically.

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Figure 1: The branching ratio of $B^\pm \to \omega K^\pm$ vs $1/N_c^{\text{eff}}$. The solid and dashed curves are for $\rho = -0.30$ and $\rho = 0.30$ respectively.

Figure 2: Same as Fig. 1 except for $B^0 \to \omega K^0$. 


Figure 3: Same as Fig. 1 except for $B^\pm \rightarrow \omega \pi^\pm$. 