Bitwise Retransmission Schemes for Resources Constrained Uplink Sensor Networks

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Abstract—Novel bitwise retransmission schemes are devised which retransmit only the bits received with small reliability. The retransmissions are used to accumulate the reliabilities of individual bits. Unlike the conventional automatic repeat request (ARQ) schemes, the proposed scheme does not require a checksum for the error detection. The bits to be retransmitted are reported as a combination number, or two synchronized random number generators (RNGs) at the transmitter and receiver are used to greatly compress the feedback message. The bitwise retransmission decisions and/or combining can be performed after the demodulation or after the channel decoding at the receiver. The bit-error rate (BER) expressions are derived for the case of one and two retransmissions, and verified by computer simulations. Assuming three specific retransmission strategies, the scheme parameters are optimized to minimize the overall BER. For the same number of retransmissions and packet length, the proposed schemes always outperform the frequently used stop-and-wait ARQ. The impact of feedback errors is also considered. Finally, practical designs of the bitwise retransmissions for data fusion from sensor nodes in Zigbee, Wifi and Bluetooth networks are presented.

Index Terms—Automatic repeat request, data fusion protocol, feedback signaling, performance analysis, retransmissions.

I. INTRODUCTION

It is well-known that feedback cannot improve the information theoretic capacity of memoryless channels [1]. However, the availability of feedback can greatly simplify the encoding and decoding complexity [2]. A good example of such schemes with the reduced implementation complexity due to feedback are the ARQ retransmission schemes [3]. [4]. The retransmission schemes are optimized to trade-off the reliability (e.g., the BER), the throughput (or equivalently, delay) and the implementation complexity [5] or the energy consumption [6]. For instance, the retransmissions can comprise a smaller number of bits than in the original packet. The incremental redundancy hybrid ARQ (IR-HARQ) or type-II HARQ schemes progressively reduce the coding rate of the forward error correction (FEC) code with each additional retransmission at the expense of increasing the decoding delay and reducing the throughput [7]. The retransmission decision delays in HARQ schemes were reduced in [8] by exploiting the structure of tail-biting convolutional codes. The permutations of bits in the retransmitted packets are used in [9] and [10] to improve the reliability of ARQ schemes. A holistic design of the complexity-constrained type-II hybrid ARQ schemes with turbo codes is considered in [11]. The joint design of FEC coding for forward data delivery and reverse feedback signaling is studied in [12] and [13]. The IR diversity and the time-repetition (TR) diversity are compared in [14]. The time and superposition-coding packet sharing between two independent information flows is optimized in [15], the latter is shown to have a slightly better performance at the expense of larger design and implementation complexity. Such transmission sharing strategies can significantly outperform the conventional HARQ schemes when signal-to-noise ratio (SNR) is sufficiently large [15]. Assuming Gaussian transmission codebooks, the achievable throughput of the HARQ schemes are compared to the ergodic channel capacity in [15], and to the delay-limited channel capacity in [14]. The transmission powers of ARQ schemes are optimized in [14].

Since the received packets typically contain only a few transmission errors, the retransmission efficiency can be improved by the partial ARQ schemes [16]. A truncated type II hybrid ARQ over block fading channels is considered in [17]. In addition to ARQ schemes exploiting a variable number of retransmissions, the variable rate ARQ schemes optimize the number of bits in each retransmission [14], [18], [19]. The number of retransmission bits required for a successful decoding is estimated from the mutual information in [19] and [15]. The pre-defined retransmission patterns are assumed in [16]. The reactive rate-adaptive ARQ strategies [2] usually outperform the proactive strategies [18]. The FEC coding to recover from the transmission errors may be preferred to the ARQ retransmissions in case of multimedia [20]. Except the stop-and-wait ARQ schemes, the go-back-N ARQ retransmissions are optimized, for example, in [21]. A multi-bit feedback signaling to improve the ARQ performance is considered in [22] and [23]. Other papers account for more realistic design constraints such as a noisy feedback [14], [17], and the channel estimation in [16]. It is shown in [14] that the errors of 1 and 2-bit feedback messages can be neglected if their bit error probability is less than $10^{-3}$, or if these errors are compensated by a non-uniform allocation of the transmission powers. Furthermore, the repetition diversity appears to be more robust to feedback errors than the IR-HARQ [14].

In our conference paper [22], we assumed a multi-bit error-free feedback to evaluate the performance trade-offs between the throughput and the reliability of the segmentation-based and the bitwise ARQ retransmission schemes. Both schemes were found to outperform the stop-and-wait ARQ with the bitwise ARQ providing larger reliability gains than...
the segmentation-based ARQ, albeit at the expense of a greater complexity of the feedback signaling. In this paper, we revisit the selective bitwise retransmission scheme proposed in [22] to carry out more rigorous performance analysis and to optimize its design. Recall that the proposed scheme aims to accumulate the reliabilities of the least reliable individual bits in the received packet, so it does not require the cyclic redundancy check (CRC) bits to make the retransmission decisions. It can be combined with other FEC coding schemes where the bit reliability is evaluated either before or after the FEC decoding. From the implementation point of view, it is useful to keep the number of retransmitted bits as well as the number of retransmissions constant, for example, to maintain a constant transmission delay and throughput for each data packet. However, a variable rate scheme that retransmits all bits having their reliability below a given threshold is also investigated. The bits with small reliabilities are reported back to the sender using either a binomial combination number, or using a deterministic sequence of bit-permutations. Our analysis assumes multi-bit error free feedback signaling, however, we also evaluate the conditions when such assumption is justifiable.

Finally, we consider a more specific system-level design of the proposed bitwise retransmission scheme to be employed in an uplink data collection scenario from the resources-constrained sensors to a data fusion access point assuming time-sharing of the transmission channel among the network nodes. The resulting transmission protocol creates fully occupied packets with the retransmitted bits having a higher priority than the newly arrived information bits.

The rest of this paper is organized as follows. System model is introduced in Section II. The mathematical theoretic analysis of the proposed bitwise retransmission scheme is carried out in Section III. The retransmission protocols are presented and optimized in Section IV. The uplink data fusion bitwise ARQ scheme is designed in Section V. Conclusions are given in Section VI.

II. SYSTEM MODEL

The design and analysis of the proposed bitwise retransmission scheme assumes a point-to-point duplex communication link between a source node and a destination node. As shown in [14], the feedback errors in ARQ schemes can be neglected provided that their probability of error is sufficiently small. Hence, in our analysis, we assume the error-free feedback, and in Section IV, we estimate the acceptable probability of feedback bit errors when such assumption is justified in practice.

Assuming that \( M \)-ary modulation is used for transmission from the source to the destination node, prior to the FEC decoding, the reliability of the received bit \( b_i \) can be calculated as

\[
\Lambda(b_i|y) = \log \frac{\Pr(b_i = 0|y)}{\Pr(b_i = 1|y)}, \quad i = 1,2,\ldots,M
\]

where \( y \) is the received \( M \)-ary modulation symbol. For equally probable modulation symbols, assuming coherent detection in a channel with additive white Gaussian noise (AWGN), the reception of modulation symbols can be mathematically modeled as,

\[
y = h \cdot s + w
\]

where \( h \) is a residual attenuation, possibly after the (multipath) equalization, and \( w \) is a zero-mean sample of AWGN. The log-likelihood ratio (LLR) (1) can be rewritten as,

\[
\Lambda(b_i|y) = \log \frac{\sum_{s \in S(b_i = 0)} \exp \left(-\frac{|y - hs|^2}{N_0} \right)}{\sum_{s \in S(b_i = 1)} \exp \left(-\frac{|y - hs|^2}{N_0} \right)} \tag{2}
\]

where the \( M \)-ary modulation constellation is partitioned as, \( S = S(b_i = 0) \cup S(b_i = 1) \), depending on the value of the \( i \)-th bit \( b_i \) in modulation symbols, \( N_0 \) is the power spectral density of the AWGN, and \(|·|\) denotes the absolute value.

The LLR (1) can be also used as the soft-input decisions to the FEC decoder. On the other hand, we can also use the soft-output decisions produced by the decoder to make the retransmission decisions. In this case, the sums in (2) are done over the corresponding codewords mapped to sequences of \( M \)-ary modulation symbols, and the sums in (2) are often approximated using the formula, \( \log \sum \exp(-a) \approx \min(a) \).

In order to simplify the analysis and illustrate the main concepts, in the sequel, we consider a binary antipodal modulation. Generalization to higher order modulations is straightforward by scaling the demodulated binary symbols by appropriate constants computed for a given \( M \)-ary modulation scheme. Hence, assume the transmitted binary symbols \( S_1 = \sqrt{E_b} \) and \( S_2 = -\sqrt{E_b} \), so the received \( N \)-bit packet can be written as,

\[
r_i = S_{ji} + w_i, \quad i = 1,2,\ldots,N
\]

where \( S_{ji} \in \{S_1,S_2\} \), and the zero-mean AWGN samples \( w_i \) have the constant variance, \( \text{E}[|w_i|^2] = \sigma_w^2 = N_0/(2|h|^2) \).

Then, the reliabilities of the received bits are

\[
\Lambda(b_i|r_i) \propto |r_i/\sigma_w| \equiv |\bar{r_i}|.
\]

Finally, the SNR per binary modulation symbol is defined as \( \gamma_b = E_b/N_0 \). If \( R_f \) denotes a fraction of information bits in the sequence of all bits transmitted from the source to the destination, for a fair comparison of different schemes with various rates \( R_f \), we assume the SNR per bit, \( \gamma_b = E_b R_f/N_0 \).

A. Bitwise retransmissions

After the initial transmission of \( N \)-bit data packet, the destination uses the received bit reliabilities to decide which \( W_d \) binary symbols among \( N \) received, \( 0 \leq W_d \leq N \), should be retransmitted, where \( d = 1,2,\ldots,D \) denotes the retransmission index. The retransmission request is a feedback message of \( C_d \geq 1 \) bits sent to the source node from the destination node via a reverse (feedback) link. Consequently, after \( D \) retransmissions, the total number of bits sent over the forward link to the destination is,

\[
N_f = N + D \sum_{d=1}^{D} W_d
\]
whereas the total number of bits sent from the destination to the source over the reverse link is,

$$N_i = \sum_{d=1}^{D} C_d.$$  

For instance, the conventional stop-and-wait ARQ scheme has the parameters $W_d = N$ and $C_d = 1$ (ACK or NACK). The retransmitted bits are combined using a maximum ratio (MRC) or other combining method to improve their reliability. The timings of the bitwise retransmission protocol are shown in Fig. 1 where $T_d$ represents the delay until the start of transmission of the window of $W_d$ retransmitted bits. The forward transmission rate is then defined as,

$$R_f(D) = \frac{N}{N + \sum_{d=1}^{D} W_d} = \frac{N}{N_i} \quad (3)$$

and the reverse transmission rate is defined as,

$$R_r(D) = \frac{\sum_{d=1}^{D} C_d}{N + \sum_{d=1}^{D} C_d} = \frac{N_i}{N + N_i}.$$  

Note that the feedback delays $T_d$ are not included in the definition of these rates, since during these time intervals, both forward and reverse links are possibly available for other transmissions.

In order to simplify the notation, let $W_1 = W_2 = \ldots = W_D = W$, and $C_1 = C_2 = \ldots = C_D = C$, and we drop the bit index within the packet unless stated otherwise. The $W$ least reliable bits requested for the next retransmission can be identified by sorting the received bits by their reliabilities. A typical sorting algorithm has the complexity $O(N \log N)$ \cite{22}. The locations of these $W$ bits within the packet of $N$ bits can be reported back to the sender by the corresponding binomial number. The number of bits representing such feedback message is,

$$C = \left[ \log_2 \binom{N}{W} \right]$$

where $\binom{N}{W}$ is the binomial number, and $[.]$ is the ceiling function. More importantly, even though typically $C > W$, we always have that, $R_f \gg R_r$, as in many other ARQ protocols. Thus, the multi-bit feedback can be very beneficial to improve the performance of ARQ protocols \cite{22}.

III. PERFORMANCE ANALYSIS

Given the values of $N, W, C, D$ and $\gamma_b$, we now derive the average BER of the proposed bitwise retransmission scheme. We first assume the case of a single retransmission, i.e., $D = 1$. We optimize the number of feedback message bits $C$ to maximize the BER, and compare our scheme to the conventional stop-and-wait ARQ. We then perform the similar analysis for $D = 2$ retransmissions before generalizing the obtained BER expressions to the case of $D > 2$ retransmissions.

Without any retransmissions (i.e., immediately after the new data packet of $N$ bits was received), the conditional probability density function (PDF) of the received bit reliability $\bar{r}$ can be written as,

$$f_0(\bar{r}|S_1, D = 0) = \frac{1}{2} \sqrt{\frac{N_0}{\pi}} e^{-\frac{(N_0 - 2\bar{r}n_0)^2}{4N_0}}$$

and

$$f_0(\bar{r}|S_2, D = 0) = \frac{1}{2} \sqrt{\frac{N_0}{\pi}} e^{-\frac{(N_0 - 2\bar{r}n_0)^2}{4N_0}}.$$  

Since the bits are selected for retransmission based on their reliability, we first obtain the BER conditioned on the reliability interval, $L_D \leq |\bar{r}| \leq U_D$, for some positive constants $U_D \geq L_D \geq 0$. This BER is equal to the probability of the error event ‘e’ that $S_1$ was transmitted, however, $S_2$ is decided at the receiver when $-U_D \leq \bar{r} \leq -L_D$. With no retransmissions, the probability of such error event is equal to,

$$\Pr(e|S_1, D = 0) = \Pr(-U_D \leq \bar{r} \leq -L_D|S_1)$$

$$= \int_{-U_D \sqrt{\frac{N_0}{2\gamma_b}}}^{-L_D \sqrt{\frac{N_0}{2\gamma_b}}} f_0(\bar{r}|S_1, D = 0) \, d\bar{r}$$

$$= Q\left(\sqrt{2\gamma_b} \left(\frac{L_D}{2\gamma_b} + 1\right)\right) - Q\left(\sqrt{2\gamma_b} \left(\frac{U_D}{2\gamma_b} + 1\right)\right)$$

where $Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \, dt$ is the Q-function, and due to symmetry, we have,$\Pr(e|S_1, D = 0) = \Pr(e|S_2, D = 0)$. Assuming the a priori transmission probabilities, $\Pr(S_1) = \Pr(S_2) = 1/2$, the overall average BER for $L_D = 0$ becomes,

$$\text{BER}_0 = Q\left(\sqrt{2\gamma_b}\right) - Q\left(\sqrt{2\gamma_b} \left(\frac{U_0}{2\gamma_b} + 1\right)\right). \quad (4)$$

Note that, for $U_0 \to \infty$, the BER \cite{3} corresponds to the BER of unencoded binary antipodal signaling, i.e., $\text{BER} = Q\left(\sqrt{2\gamma_b}\right) \cite{3}$. Subsequently, the probability that the reliability of the received bit is in the interval $L_D \leq |\bar{r}| \leq U_D$ without any retransmissions is computed as,

$$P_{0|S_1} = \Pr(L_D \leq |\bar{r}| \leq U_D|S_1)$$

$$= \int_{-L_D \sqrt{\frac{N_0}{2\gamma_b}}}^{U_D \sqrt{\frac{N_0}{2\gamma_b}}} f_0(\bar{r}|S_1, D = 0) \, d\bar{r}$$

$$= Q\left(\sqrt{2\gamma_b} \left(\frac{L_D}{2\gamma_b} + 1\right)\right) - Q\left(\sqrt{2\gamma_b} \left(\frac{U_D}{2\gamma_b} + 1\right)\right) +$$

$$Q\left(\sqrt{2\gamma_b} \left(\frac{L_D}{2\gamma_b} - 1\right)\right) - Q\left(\sqrt{2\gamma_b} \left(\frac{U_D}{2\gamma_b} - 1\right)\right).$$

For $L_D = 0$, this probability is equal to,

$$P_{0|S_1} = 1 - Q\left(\sqrt{2\gamma_b} \left(\frac{U_0}{2\gamma_b} + 1\right)\right) - Q\left(\sqrt{2\gamma_b} \left(\frac{U_0}{2\gamma_b} - 1\right)\right)$$

and $P_{0|S_1} \to 1$ for $U_0 \to \infty$. Due to symmetry, and for equally probable symbols $S_i$, we get, $P_0 = P_{0|S_1} = P_{0|S_2}$. Furthermore, if $Z$ is the (random) number of bits having the reliabilities in the interval, $0 \leq |\bar{r}| \leq U_0$, the mean value of $Z$, $E[Z] = N P_0$, and $\gamma_b$, can be used to optimize the retransmission window size. For example, by letting $W = E[Z]$, we can control the target BER after the first retransmission.
In general, it is convenient to re-scale the received bits after every retransmission and subsequent MRC combining to maintain the constant average energy of the received symbols. In particular, the received symbol after combining the initially received sample $\bar{r}_0$ with the retransmitted and scaled samples $\bar{r}_d$, $d = 1, 2, \ldots, D$, can be written as,

$$\bar{r}_{MRC} = \frac{1}{D+1} \left( \bar{r}_0 + \sum_{d=1}^{D} \bar{r}_d \right).$$

In the sequel, let $L_d = 0$ for $\forall d \geq 0$, and, without loss of generality, we assume that the packet of $N$ symbols $S_1$ was transmitted.

A. BER with one retransmission

After the first transmission of the packet of $N$ bits, only those bits that have the reliabilities in the interval $0 \leq |\bar{r}| \leq U_0$ are requested to be retransmitted. After the first retransmission, the bit samples are combined as,

$$\bar{r}_{MRC} = \left\{ \begin{array}{ll}
\frac{1}{2}(\bar{r}_0 + \bar{r}_1) & \text{for } |\bar{r}_0| \leq U_0 \\
\bar{r}_0 & \text{for } |\bar{r}_0| > U_0,
\end{array} \right.$$  

The random variable $\bar{r}_0$ has the PDF $f_0(\bar{r}|S_1, D = 0)$, whereas the PDF of the random variable $\bar{r}_1$ can be obtained by conditioning on $|\bar{r}_0| \leq U_0$. Since the random received samples $\bar{r}_0$ and $\bar{r}_1$ are statistically independent, the PDF of $\bar{r}_{MRC}$ is given by the convolution (denoted as $*$), i.e.,

$$f_{MRC}(\bar{r}|S_1, D = 1) = \frac{f_0(\bar{r}|S_1, d = 0) \phi_1(\bar{r}, U_0)}{\Pr(|\bar{r}_0| \leq U_0 | S_1, D = 0)} * f_1(\bar{r}|S_1, d = 0)$$

where

$$\phi_1(\bar{r}, U_0) = \eta(\bar{r} + U_0)(1 - \eta(\bar{r} - U_0))$$

and $\eta(\bar{r})$ is the unit-step function, i.e., $\eta(\bar{r}) = 1$ if $\bar{r} \geq 0$, and 0 otherwise. After some manipulations, the PDF of $\bar{r}_{MRC}$ can be written as,

$$f_{MRC}(\bar{r}|S_1, D = 1) = \frac{\chi_1(\bar{r}, U_0)}{\Pr(|\bar{r}_0| \leq U_0 | S_1, D = 0)}$$

where we defined the function,

$$\chi_d(\bar{r}, U_0) = \frac{1}{4\sqrt{(d+1)N_0}} e^{-\frac{(\bar{r}^2 - 4\frac{N_0}{d+1})}{4(d+1)N_0}} \times \left\{ \begin{array}{ll}
\text{erf}\left(\frac{\sqrt{(d+1)N_0}}{4d}(U_0 - \bar{r})\right) & \text{for } U_0 < \bar{r} \\
\text{erf}\left(\frac{\sqrt{(d+1)N_0}}{4d}(U_0 + \bar{r})\right) & \text{for } \bar{r} < U_0
\end{array} \right.$$

and erf$(\cdot)$ denotes the error function $[3]$.

Assuming that all bits with the reliabilities $|\bar{r}_0| \leq U_0$ are retransmitted, and then combined using the MRC, the overall BER after the first retransmission is computed as $[3]$.

$$\text{BER}_1 = \Pr(e|d = 0, S_1) \Pr(d = 0|S_1) + \Pr(e|d = 1, S_1) \Pr(d = 1|S_1) = \int_{-\infty}^{-U_0} f_1(\bar{r}|S_1, d = 0) \, d\bar{r} + \int_{-U_0}^{0} \chi_1(\bar{r}, U_0) \, d\bar{r}. $$

In the appendix, we show that BER$_1$ can be accurately approximated as,

$$\text{BER}_1 \approx Q\left(\sqrt{2\gamma_b} (U_0 + 1)\right) + Q\left(\sqrt{4\gamma_b}\right) - \sum_{k=1}^{2} \frac{A_k}{\sqrt{1 + 2B_k}} \left\{ e^{-\alpha_k^{(1)}(U_0) \gamma_b} Q\left(\beta_k^{+}(U_0) \sqrt{\gamma_b}\right) + e^{-\alpha_k^{(1)}(U_0) \gamma_b} Q\left(\beta_k^{-}(U_0) \sqrt{\gamma_b}\right) \right\}$$

where the coefficients $A_k$ and $B_k$, are given in the appendix, and the auxiliary functions,

$$\alpha_{k,d}(U) = \frac{B_k(d+1)(2 - U/\gamma_b)^2}{2d(1 + \frac{2U}{\gamma_b})},$$

$$\beta_{k,d}(U) = \frac{1 - \frac{B_kU}{\gamma_b}}{\sqrt{\frac{1}{2(d+1)}}},$$

$$\beta_k^{+}(U) = \frac{1 + \frac{B_kU}{\gamma_b}}{\sqrt{\frac{1}{2(d+1)}}}.$$ 

B. BER with two retransmissions

Consider now the case with two retransmissions. After the second retransmission, the received bits having the reliabilities $|\bar{r}| \leq U_1$ are combined with the retransmitted samples $\bar{r}_2$. The threshold value $U_1$ is chosen to be greater than $U_0$ due to improvement of the bit reliabilities after the first retransmission. More generally, we assume that,

$$U_{D-1} \geq U_{D-2} \ldots \geq U_1 \geq U_0.$$ 

Hence, the bit-samples after two retransmissions can be written as,

$$\bar{r}_{MRC} = \left\{ \begin{array}{ll}
\frac{1}{2}(\bar{r}_0 + \bar{r}_1 + \bar{r}_2) & \text{for } |\bar{r}_0| \leq U_0, \frac{1}{2}(\bar{r}_0 + \bar{r}_1) \leq U_1 \\
\frac{1}{2}(\bar{r}_0 + \bar{r}_1) & \text{for } |\bar{r}_0| \leq U_0, \frac{1}{2}(\bar{r}_0 + \bar{r}_1) > U_1 \\
\frac{1}{2}(\bar{r}_0 + \bar{r}_2) & \text{for } U_0 < |\bar{r}_0| \leq U_1 \\
\bar{r}_0 & \text{for } |\bar{r}_0| > U_1.
\end{array} \right.$$  

(5)

However, in order to make the analysis mathematically tractable, we merge the first two conditions in (5), and consider instead the retransmission scheme,

$$\bar{r}_{MRC} = \left\{ \begin{array}{ll}
\frac{1}{2}(\bar{r}_0 + \bar{r}_1 + \bar{r}_2) & \text{for } |\bar{r}_0| \leq U_0 \\
\frac{1}{2}(\bar{r}_0 + \bar{r}_2) & \text{for } U_0 < |\bar{r}_0| \leq U_1 \\
\bar{r}_0 & \text{for } |\bar{r}_0| > U_1.
\end{array} \right.$$  

(6)

having the upper-bound performance than the scheme (5). The scheme (6) can be interpreted as making the decision about the number of retransmissions for each bit already after receiving the initial data packet of $N$ bits. Even though the scheme (6) may unnecessarily retransmit some bits even if their reliability have already reached the desired threshold, our numerical results indicate that, for $D = 2$, the performance difference is not significant (less than 1 dB). Consequently, the number of retransmissions for each bit can be determined by quantization of the initial reliabilities $|\bar{r}_0|$ using the thresholds $U_0$ and $U_1$. The bits having the reliabilities $|\bar{r}_0| \leq U_0$ are always combined with two other retransmitted bits, whereas the received bits
with the reliabilities \( U_0 < |\bar{r}_0| \leq U_1 \) are combined with exactly one retransmission.

The PDF \( f_{\text{MRC}} (\bar{r}|S_1, d = 2) \) of \( \bar{r}_{\text{MRC}} \), for \( |\bar{r}| \leq U_0 \), is given as,

\[
f_{\text{MRC}} (\bar{r}|S_1, d = 2) = \frac{\chi_1 (\bar{r}, U_0)}{\Pr (|\bar{r}_0| \leq U_0 | S_1, d = 0)}
\]

and for \( U_0 < |\bar{r}_0| \leq U_1 \), it is given as,

\[
f_{\text{MRC}} (\bar{r}|S_1, d = 2) = \frac{\lambda_1 (\bar{r}, U_1, U_0)}{\Pr (U_0 < |\bar{r}_0| \leq U_1 | S_1, d = 0)}
\]

where

\[
\lambda_d (\bar{r}, U_d, U_{d-1}) = \frac{1}{4} \sqrt{(d+1)N_o \pi} \exp \left( -\left( \frac{\bar{r} - 2 \sqrt{2\gamma}}{\sqrt{(d+1)N_o}} \right)^2 \right)
\]

\[
\left\{ \begin{array}{l}
\text{erf} \left( \frac{(d+1)N_o}{4 \bar{r}} (U_d + \bar{r}) \right) + \text{erf} \left( \frac{(d+1)N_o}{4 \bar{r}} (U_d - \bar{r}) \right) \\
\text{erf} \left( \frac{(d+1)N_o}{4 \bar{r}} (U_{d-1} + \bar{r}) \right) - \text{erf} \left( \frac{(d+1)N_o}{4 \bar{r}} (U_{d-1} - \bar{r}) \right)
\end{array} \right.
\]

For \( |\bar{r}_0| > U_1 \), the PDF \( f_{\text{MRC}} (\bar{r}|S_1, d = 2) \) is given by the PDF \( f_1 (\bar{r}|S_1, d = 0) \). The overall BER is obtained using the law of the total probability, i.e.,

\[
\text{BER}_2 = \sum_{d=0}^{2} \Pr (d|S_1) \Pr (d|S_1) = \int_{-U_0}^{U_1} f_1 (\bar{r}|S_1, D = 0) \, d\bar{r}
\]

where the probability of the second retransmission of the same bit is, \( \Pr (d = 2 | S_1) = \Pr (f \leq -U_1 | S_1, d = 1) \). As for one retransmission, BER_2 can be approximated (see the appendix),

\[
\text{BER}_2 \approx Q \left( \sqrt{2\gamma}b_+ \right) + Q \left( \sqrt{2\gamma}b_- \right) - \sum_{k=1}^{2} \frac{A_k}{\sqrt{1+2\beta}} \left[ e^{-\alpha \gamma_k (U_0)} Q \left( b_{+k} (U_0) \sqrt{2\gamma} \right) + e^{-\alpha \gamma_k (U_0)} Q \left( b_{-k} (U_0) \sqrt{2\gamma} \right) \right]
\]

\[
+ \frac{A_k}{\sqrt{1+2\beta}} \left[ e^{-\alpha \gamma_k (U_0)} Q \left( b_{+k} (U_0) \sqrt{2\gamma} \right) + e^{-\alpha \gamma_k (U_0)} Q \left( b_{-k} (U_0) \sqrt{2\gamma} \right) \right]
\]

The probability that the received reliability is in the interval \( U_0 \leq |\bar{r}| \leq U_1 \) is evaluated as,

\[
P_1 = \Pr (|\bar{r}| \leq U_1 | S_1, d = 1) \Pr (d = 0 | S_1) + \Pr (|\bar{r}| \leq U_1 | S_1, d = 1) \Pr (d = 1 | S_1)
\]

This probability can be efficiently approximated as,

\[
P_1 \approx Q \left( \sqrt{2\gamma}b_+ + 1 \right) + Q \left( \sqrt{2\gamma}b_- + 1 \right) - Q \left( \sqrt{2\gamma}b_+ - 1 \right) - Q \left( \sqrt{2\gamma}b_- - 1 \right)
\]

where the auxiliary functions,

\[
\Theta_{k,d}^{+} (U) = \frac{U^2}{\sqrt{2\gamma}b_+ + 1} + \frac{U^2}{\sqrt{2\gamma}b_- + 1} + \frac{b_{+k} (U_d - U)}{2 \sqrt{2\gamma}b_+}
\]

\[
\Theta_{k,d}^{-} (U) = \frac{U^2}{\sqrt{2\gamma}b_+ - 1} + \frac{U^2}{\sqrt{2\gamma}b_- - 1} + \frac{b_{-k} (U_d - U)}{2 \sqrt{2\gamma}b_-}
\]

\[
\Theta_{k,d}^{+} (U) = \frac{U^2}{\sqrt{2\gamma}b_+ - 1} + \frac{U^2}{\sqrt{2\gamma}b_- - 1} + \frac{b_{+k} (U_d + U)}{2 \sqrt{2\gamma}b_+}
\]

\[
\Theta_{k,d}^{-} (U) = \frac{U^2}{\sqrt{2\gamma}b_+ + 1} + \frac{U^2}{\sqrt{2\gamma}b_- + 1} + \frac{b_{-k} (U_d + U)}{2 \sqrt{2\gamma}b_-}
\]

\[
\mu(U) = \frac{U_D}{\sqrt{2\gamma}b_+} - \frac{2b_U}{\sqrt{2\gamma}b_-} \left( 1 + \frac{2B_k}{d} \right)
\]

and \( \text{sign} (\cdot) \) is the sign function.

C. BER with multiple retransmissions

For \( D \geq 1 \) retransmissions, the overall BER is calculated as,

\[
\text{BER}_D = \sum_{d=0}^{D} \Pr (d|S_1) \Pr (d|S_1)
\]

Recall that, for the sake of mathematical tractability, we assume that the received bits with their initial reliability within the interval \( |\bar{r}| \leq U_0 \) will be retransmitted \( D \) times, the bits with the reliability in the interval \( U_0 < |\bar{r}| \leq U_1 \) will be retransmitted \( D - 1 \) times and so on. The number of the received copies for each reliability range after \( D \) retransmissions is shown in Fig. 2. Consequently, the overall BER can be expressed using the functions \( \chi_D (\bar{r}, U_{D-1}) \) and \( \lambda_D (\bar{r}, U_0, U_{D-1}) \) as,

\[
\text{BER}_D = \int_{-\infty}^{U_0} \chi_D (\bar{r}, U_{D-1}) \, d\bar{r} + \sum_{D-1}^{D} \int_{-\infty}^{U_0} \chi_D (\bar{r}, U_{D-1}) \, d\bar{r} + \int_{U_0}^{\infty} \chi_D (\bar{r}, U_0) \, d\bar{r}
\]

The probability that the received reliability is in the interval \( U_{D-1} \leq |\bar{r}| \leq U_D \) is evaluated as,

\[
P_D = \int_{U_{D-1}}^{U_D} \chi_D (\bar{r}, U_{D-1}) \, d\bar{r} - \int_{U_0}^{U_{D-1}} \chi_D (\bar{r}, U_{D-1}) \, d\bar{r}
\]

This probability can be efficiently approximated as,
with the third term being zero for \( D = 1 \) retransmission. It is again possible to obtain the approximation of the probability \( P_D \) for \( D > 2 \). However, the resulting expression is much more involved than in the case of \( P_1 \) and \( P_2 \), so it is not presented here.

### D. BER for slowly varying fading channels

Since the wireless links are typically subject to time-varying fading, we can average the BER expressions obtained in the previous subsection over the time-varying received power, for example, following the chi-square distribution. In this case, the received SNR has the PDF, \( f_{\gamma_b}(\gamma_b) = \frac{1}{\gamma_b} \exp \left( -\frac{\theta}{\gamma_b} \right) \), where \( \gamma_b = E[\gamma_b] \) is the average SNR. Moreover, we assume that the SNR variations are sufficiently slow (e.g., assuming that the nodes are nomadic or even stationary), so the SNR can be considered to be constant during the first packet transmission as well as during the subsequent \( D \) retransmissions. Then, the long-term average BER is evaluated using the expectation \( H \),

\[
\text{BER}_D = \int_0^\infty \text{BER}(\gamma_b) f_{\gamma_b}(\gamma_b) d\gamma_b.
\]

The computationally efficient formula for calculating \( \text{BER}_D \) is displayed below Fig. 2 on the next page. Similarly, the average BER is evaluated as,

\[
P_D = \int_0^\infty P_D(\gamma_b) f_{\gamma_b}(\gamma_b) d\gamma_b.
\]

### IV. BITWISE RETRANSMISSION PROTOCOLS

We consider three specific bitwise retransmission strategies and optimize their parameters to maximize the transmission reliability rather than to maximize their throughput. We also verify mathematical formulas derived in the previous section by computer simulations assuming error-free feedback. This assumption is revisited at the end of this section.

#### A. Fixed rate technique

In this design, we assume a constant retransmission window size \( W \) determined as,

\[
W = \left\lfloor \frac{N}{D} \left( \frac{1}{R_f} - 1 \right) \right\rfloor
\]

for a priori given parameters \( N \), \( D \) and the forward rate \( R_f \). Since the window size \( W \) is constrained as \( 1 \leq W \leq N \), the possible forward rates are restricted to the interval, \( \frac{1}{1+D} < R_f \leq \frac{N}{D} \). Thus, for \( W = N \), the rate \( R_f = \frac{1}{1+D} \), and the proposed retransmission scheme corresponds to a block repetition code (BRC) with \( D \) repetitions of the original packet. We have the following proposition.

**Proposition 1:** For a given SNR, the fixed rate retransmission technique achieves the minimum BER for some specific value of the rate \( R_f \). The optimum value of \( R_f \) minimizing the BER increases with the SNR.

Prop. 1 can be proven by letting the derivative \( \left( \frac{d}{dR_f} \right) \text{BER}_D \) to be equal to zero. For large values of SNR and \( N \), the forward rate \( R_f \) approaches its maximum value of 1, so in such case, the overhead due to retransmissions can be neglected. Moreover, when \( N \) is large or \( W = N \), the BER of the fixed rate retransmission scheme approaches the BER of the BRC.

Fig. 3 and Fig. 4 show the BER of the forward link versus the forward rate \( R_f \) for the different number of retransmissions \( D \) at two SNR values \( \gamma_b = 0 \) dB and \( \gamma_b = 5 \) dB, respectively. We observe that the minimum BER value is more pronounced (i.e., the optimization is more important) for larger values of SNR. The BER curves in Fig. 3 and Fig. 4 also compare the simulations with the approximate expressions given in the previous section; for \( D > 1 \), the difference between the approximate expressions and the simulations is negligible.

The forward rates \( R_f \) yielding the minimum BER are shown in Fig. 5 for the different number of retransmissions \( D \). Finally, Fig. 6 shows the BER versus the SNR \( \gamma_b \) for the different number of retransmissions \( D \) assuming that the forward rate \( R_f \) is optimized for each SNR value \( \gamma_b \) to minimize the achieved BER. Note again that such optimization becomes more effective if the SNR is increased.

#### B. Fixed window technique

As for the fixed rate technique, the retransmission window size is fixed, and it is determined as,

\[
W = \left\lfloor \frac{N}{D} \right\rfloor
\]

where the retransmission decision thresholds \( U_d \) are set, so that the probabilities \( P = P_0 = P_1 = \ldots = P_{D-1} \), which have been obtained in the previous section, have the same value. Then, given \( N \), \( D \), and \( W \), the forward rate is calculated as,

\[
R_f = \frac{1}{1 + DW}.
\]

We have the following proposition.

**Proposition 2:** For a given SNR, the fixed window technique achieves the minimum BER for some value of the retransmission window size. The optimum window size value decreases with the SNR.

Prop. 2 can be again proved by letting the derivative, \( \left( \frac{d}{dW} \right) \text{BER}_D \) to be equal to zero. When the ratio \( W/N \) approaches unity, the BER of the fixed window technique approaches the BER of the BRC. On the other hand, when the ratio \( W/N \) approaches zero, the retransmission overhead can be neglected.

Fig. 7 and Fig. 8 show the BER versus the normalized window size \( W/N \) for the different number of retransmissions \( D \) and the SNR values \( \gamma_b = 0 \) dB and \( \gamma_b = 5 \) dB, respectively. We again observe a negligible difference between the approximate and the simulated BER curves, especially for larger values of \( D \). In addition, the minimum BER values are more pronounced when the SNR is increased.

Fig. 9 shows the normalized window size \( W/N \) corresponding to the minimum BER for the different number of retransmissions \( D \). Fig. 10 shows the BER versus the SNR \( \gamma_b \) for the different number of retransmissions \( D \) assuming the optimum value of \( W/N \) for each \( \gamma_b \), minimizing the BER. Note that such minimization of the BER is more effective when the SNR is large. Finally, for the different number of retransmissions \( D \), Fig. 11 shows the BER for a slowly varying chi-square distributed, unit-mean received power, provided that the parameters are optimized to minimize the BER.
C. Fixed threshold technique

Unlike the previous two techniques, the fixed threshold technique allows for different retransmission window sizes while assuming the single constant reliability threshold, $U_0 = U_1 = \ldots = U_{D-1} = U$, during each retransmission. Given $U$, we obtain the probabilities $P_{d}$, $d = 1, 2, \ldots, D$, defined previously, and calculate the retransmission window sizes as,

$$W_d = \lfloor NP_d \rfloor.$$

The corresponding forward rate is given as,

$$R_f = \frac{1}{1 + \sum_{d=1}^{D} W_d / N}.$$

We have the following proposition.

**Proposition 3:** For a given SNR, the fixed threshold technique achieves the minimum BER value for some specific threshold $U$. This optimum threshold value is increasing with the SNR.

Prop. 3 can be again proved by letting the derivative, $(d/dU) \text{BER}_D$, to be equal to zero.

Fig. [12] and Fig. [13] show the BER versus the normalized threshold $U/\sqrt{E_b}$ for the different number of retransmissions $D$ and the SNR values $\gamma_0 = 0 \text{ dB}$ and $\gamma_0 = 5 \text{ dB}$, respectively. Since the approximate BER expressions were already verified for the other two techniques considered, the BER curves in Fig. [12] and Fig. [13] only show these derived expressions. We again observe that the minimum BER values are more apparent when the SNR is increased. Fig. [14] shows the normalized thresholds $U/\sqrt{E_b}$ having the minimum BER for the different number of retransmissions $D$. Finally, Fig. [15] shows the BER versus the SNR $\gamma_b$ for different $D$ assuming the optimum thresholds $U/\sqrt{E_b}$ for each SNR $\gamma_b$ that achieves the minimum BER. We again find that minimization of the BER by optimizing the threshold $U/\sqrt{E_b}$ is more effective when the SNR is increased.

D. Feedback signaling

There are two main issues when designing practical feedback signaling schemes. The first issue is how to constrain the
number of feedback bits. The second issue is the transmission errors of feedback bits.

Sending a small number of feedback bits in a dedicated packet is very inefficient due to the associated protocol overheads. In practice, it is common to reserve a few bits within the packet payload for the feedback signaling, so the packet overhead is shared by the feedback as well as data. In such case, it is beneficial to minimize the number of feedback bits in order to increase the data payload. Here, we reduce the number of feedback bits sent from the destination to the source over the reverse link by considering a deterministic sequence of bit permutations synchronously generated at the transmitter and at the receiver to encode the feedback message. Such sequence is conveniently generated as pseudo-random permutations of $N$-tuples, $(1, 2, \ldots, N)$, using the two synchronized RNGs. In addition, the transmitter and the receiver RNGs synchronously advance by exactly $2^C_1$ permutations every symbol period. For every generated permutation, the receiver checks whether a sufficient number of bits with small reliabilities fall into a predefined window of $W$ bits; for instance, we can assume that the retransmission window is represented by the first $W$ bit positions. When such permutation is found, the feedback message to notify the source is a binary representation of the permutation order modulo $2^C_1$. The source then selects the corresponding $W$ bits in the packet, and retransmits them to the destination. In particular, for the window size $W$, the number of permutations $K$ searched until the desired one has been found can be expressed as,

$$K = 2^C_1 I + (K \mod 2^C_1)$$
where $I$ is an integer number of the idle symbol periods. The key property is that,

$$(K \mod 2^C_1) \ll \left(\frac{N}{W}\right)$$

whereas, on average, $E[K] = \left(\frac{N}{W}\right)$. Hence, the feedback message is only represented by $C_1$ bits. The value of $C_1$ is a design parameter, and it trade-offs the feedback message size, and the required delay until the start of the next retransmission [22].

More importantly, given some target BER, the window size $W$ and the average number of searched permutations $E[K]$ are decreasing with the SNR. As shown in Fig. 16, $E[K]$ is only about 30 for all $N \leq 64$ when the SNR is at least 10 dB, and thus, the feedback of $C_1 = 5$ bits can be used to find the desired permutation during one symbol period.

The average forward throughput with one retransmission can be defined as,

$$\zeta_1 = \frac{N}{E[I+1]}$$

By simulations, we found that there exists an optimum value $C_1^*$ which minimizes the expected delay $E[I+1]$, i.e., which maximizes the throughput $\zeta_1$. This optimum value is given as,

$$C_1^* = \left\lceil -0.5 + \log_2 E[K] \right\rceil$$

and it is plotted in Fig. 17 for different values of the packet length $N$. The corresponding maximum throughput $\zeta_1^*$ is shown in Fig. 18 For comparison, the throughput $1/2$ of the BRC corresponding to the conventional stop-and-wait ARQ with one retransmission is also shown in Fig. 18. More importantly, we observe that the proposed scheme can achieve better...
throughput than the rate $1/D$ repetition code for the same number of retransmissions $D$, especially at the medium to large values of SNR. Furthermore, we can show that the throughput of the fixed window technique in the limit of very large SNR converges to, $\lim_{\gamma_b \to \infty} \xi_1 \approx \frac{N}{1 + \frac{W}{N}}$, since $\lim_{\gamma_b \to \infty} \frac{W}{N} = 0$.

Finally, we reconsider the assumption of the error-free feedback which is often adopted in many papers concerning the ARQ retransmission schemes. Recall that it was shown in [14] that errors of 1 and 2-bit feedback messages can be neglected if their bit error probability is less than $10^{-3}$. This result can be readily modified for the case of multi-bit feedback messages which are utilized in our bitwise retransmission schemes. In particular, assuming the feedback errors are independent, and they are occurring with the probability $p_r$, we have the following proposition.

\textit{Proposition 4:} The errors in the feedback messages of $C$ bits can be tolerated, provided that their bit-error probability is bounded as,

$$p_r \leq 1 - P_{\min}^{1/C}$$

where the required minimum probability of the error-free feedback messages was established in [14] to be, $P_{\min} = 1 - 10^{-3} = 99.9\%$.

The proof of Prop. 4 follows from the binomial distribution of independent and equally probable errors. Hence, the longer the feedback message, the smaller the feedback bit error probability $p_r$ is required.
We now illustrate the use of the proposed bitwise retransmission schemes in a practical scenario of data fusion from a group of \( L \) sensor nodes into a single central access point (AP). A time-division multiple access (TDMA) protocol with \( (L+1) \) time slots is used to share the communication channel. The time-division duplex (TDD) protocol further divides the available time slots into \( L \) uplink time slots to transmit data from the \( L \) sensor nodes, and one time slot is allocated for the feedback signaling from the AP. Each time slot can carry at most \( N \) bits of information including the protocol overhead. The AP is assumed not to be battery powered, so the transmit power in the downlink can be much larger than in the battery constrained uplink.

More specifically, we consider the following three sensor node technologies: Zigbee 802.15.4, Wifi 802.11b and Bluetooth v. 4.2 802.15.1. The parameters of these three technologies are summarized in Table I. Their BER expressions have been obtained by fitting the sum of exponentials (the Prony method, \[25\]) to the performance curves reported in \[26\]. It is obvious that Zigbee is the most energy efficient technology for the sensor nodes, and it can operate at small SNR values.

The specific design examples of the bitwise retransmission schemes with a constant retransmission window size employing the binomial number feedback and the packet segmentation are listed in Table II. Therein, \( p_f \) is the BER of the forward link, \( p_r \) is the BER of the reverse link, \( N_{\text{seg}} \) is the number of segments of the disjoint partitioning of the \( N \) bit packet, \( W_{\text{seg}} \) is the retransmission window size per segment, and \( C_{\text{tot}} \) is the total length of the feedback message (in bits) required.
for the whole packet. Hence, $N/N_{\text{seg}}$ is the segment length, $W_{\text{seg}}N_{\text{seg}} = W$ is the total number of retransmitted bits, and $C_{\text{tot}}/N_{\text{seg}}$ is the length of the feedback message per segment. Furthermore, $P_f$ is the probability that there are at most $W_{\text{seg}}$ errors in any segment (i.e., the closer the value of $P_f$ to 1.0, the better), and $P_r$ is the probability that there is at least one error among the $C_{\text{tot}}$ feedback bits received at the source (i.e., the probability that the feedback message is received incorrectly at the source). As shown in the previous section, it is required that $P_f < P_{\text{min}} = 10^{-3}$ in order to neglect the effect of feedback errors on the performance. Other system parameters are given in Table I including the packet size $N$ and the required SNR for given values of $p_f$.

We found that, to obtain efficient designs of the proposed bitwise retransmission schemes, the BER $p_f$ of the forward link should be at most $10^{-3}$, and the BER $p_r$ of the reverse link should be at most $10^{-5}$. In terms of the minimum required SNR for bidirectional connections, the Zigbee requires $-1.16$ dB and 0.96 dB, the Wifi requires 5.43 dB and 7.59 dB, and the Bluetooth requires 10.93 dB and 13.34 dB, respectively. Such SNR levels can be satisfied for all three wireless technologies considered, provided that the reverse link has 3 dB larger transmit power than the forward link. Such transmission power unbalances can be readily obtained in the sensor networks with the centralized mains-powered AP.

Next, we consider scheduling of the packet contents for our single-cell TDMA/TDD multiple access protocol. We assume that the parameters $W$, $D$, $N$, $L$ and $C_{\text{tot}}$ are constant, even though they can be optimized for the uplink and downlink BERs $p_f$ and $p_r$. Recall that all packets have the maximum length of $N$ bits. In the uplink, the nodes send their data as well as schedule the retransmitted bits for the previously transmitted data packets. In the downlink, the AP broadcasts the retransmission requests to all sensor nodes at once. The contents in the uplink packets are scheduled following the following two rules.

1) Include the retransmitted sequences in the order corre-

| Table I |
| --- |
| TRANSMISSION PARAMETERS OF THE THREE SENSOR NODE TECHNOLOGIES |
| | Zigbee | Wifi | Bluetooth |
| BER | $P_b(N_h) = 1.5203e^{-0.5617h}$ | $P_b(N_h) = 10.0e^{-3.4535h}$ | $P_b(N_h) = 0.2436e^{-0.4997h}$ |
| $10^{-2}$ | 2.79 | 3.88 | 8.91 |
| $10^{-3}$ | 1.16 | 5.43 | 10.93 |
| $10^{-4}$ | 0.03 | 6.63 | 12.30 |
| $10^{-5}$ | 0.96 | 7.59 | 13.34 |
| $10^{-6}$ | 1.73 | 8.37 | 14.18 |
| Packet [bytes] | header | preamble+header | header+CRC |
| | 15-24 | 2+2 | 2+2 |
| | 127 | 1500 | 252 |
| $N$ [bits] | (6+127)×8 = 1064 | (24+1500)×8 = 12192 | (4+252)×8 = 2048 |
| Segments | 14×76, 19×56, 28×38 | 16×762, 32×381, 48×254 | 16×128, 32×64, 64×32 |
| | 38×28, 56×19, 76×14 | 96×127, 127×96, 254×48 | 128×126, 256×8, 512×4 |
| | 133×8, 152×7, 266×4 | 381×32, 508×24, 762×16 | 1024×2, 2048×1 |
| | 532×2, 1064×1 | 1016×12, 1524×8, 2032×6 | 3048×4, 4064×3, 6096×2 |

| Table II |
| --- |
| THE BITWISE RETRANSMISSION DESIGNS WITH THE CONSTANT WINDOW SIZE AND THE PACKET SEGMENTATION |
| | $p_f$ | $p_r$ | $N_{\text{seg}}$ | $W_{\text{seg}}$ | $C_{\text{tot}}$ | $P_f$ | $P_r$ |
| Zigbee | $10^{-3}$ | $10^{-5}$ | 2 | 3 | 50 | 0.9978 | 5.0×10^{-4} |
| | $10^{-3}$ | $10^{-5}$ | 1 | 4 | 36 | 0.9953 | 3.6×10^{-4} |
| | $10^{-3}$ | $10^{-5}$ | 1 | 5 | 44 | 0.9992 | 4.4×10^{-4} |
| | $10^{-4}$ | $10^{-5}$ | 4 | 1 | 36 | 0.9997 | 3.6×10^{-4} |
| | $10^{-4}$ | $10^{-5}$ | 2 | 1 | 20 | 0.9986 | 2.0×10^{-4} |
| | $10^{-4}$ | $10^{-5}$ | 2 | 2 | 36 | 1.0000 | 3.6×10^{-4} |
| | $10^{-4}$ | $10^{-5}$ | 2 | 3 | 50 | 1.0000 | 5.0×10^{-4} |
| | $10^{-4}$ | $10^{-5}$ | 1 | 1 | 11 | 0.9947 | 1.1×10^{-4} |
| | $10^{-4}$ | $10^{-5}$ | 1 | 2 | 20 | 0.9998 | 2.0×10^{-4} |
| | $10^{-4}$ | $10^{-5}$ | 1 | 3 | 28 | 1.0000 | 2.8×10^{-4} |
| | $10^{-4}$ | $10^{-5}$ | 1 | 4 | 36 | 1.0000 | 3.6×10^{-4} |
| Wifi | $10^{-3}$ | $10^{-6}$ | 1 | 21 | 220 | 0.9928 | 2.2×10^{-4} |
| | $10^{-3}$ | $10^{-6}$ | 4 | 2 | 92 | 0.9962 | 9.2×10^{-5} |
| | $10^{-4}$ | $10^{-6}$ | 3 | 2 | 69 | 0.9917 | 6.9×10^{-5} |
| | $10^{-4}$ | $10^{-6}$ | 2 | 3 | 72 | 0.9965 | 7.2×10^{-5} |
| | $10^{-4}$ | $10^{-6}$ | 2 | 4 | 92 | 0.9996 | 9.2×10^{-5} |
| | $10^{-4}$ | $10^{-6}$ | 1 | 4 | 50 | 0.9917 | 5.0×10^{-5} |
| | $10^{-4}$ | $10^{-6}$ | 1 | 5 | 61 | 0.9984 | 6.1×10^{-5} |
| | $10^{-4}$ | $10^{-6}$ | 1 | 6 | 72 | 0.9997 | 7.2×10^{-5} |
| | $10^{-4}$ | $10^{-6}$ | 1 | 7 | 83 | 1.0000 | 8.3×10^{-5} |
| | $10^{-4}$ | $10^{-6}$ | 1 | 8 | 94 | 1.0000 | 9.4×10^{-5} |
| Bluetooth | $10^{-2}$ | $10^{-6}$ | 2 | 18 | 256 | 0.9913 | 2.6×10^{-4} |
| | $10^{-3}$ | $10^{-6}$ | 2 | 4 | 72 | 0.9960 | 7.2×10^{-5} |
| | $10^{-3}$ | $10^{-6}$ | 1 | 6 | 57 | 0.9949 | 5.7×10^{-5} |
| | $10^{-3}$ | $10^{-6}$ | 1 | 7 | 65 | 0.9987 | 6.5×10^{-5} |
| | $10^{-4}$ | $10^{-6}$ | 1 | 8 | 73 | 0.9997 | 7.3×10^{-5} |
| | $10^{-4}$ | $10^{-6}$ | 4 | 1 | 36 | 0.9987 | 3.6×10^{-4} |
| | $10^{-4}$ | $10^{-6}$ | 2 | 1 | 20 | 0.9951 | 2.0×10^{-4} |
| | $10^{-4}$ | $10^{-6}$ | 2 | 2 | 38 | 0.9998 | 3.8×10^{-4} |
| | $10^{-4}$ | $10^{-6}$ | 1 | 2 | 21 | 0.9988 | 2.1×10^{-4} |
| | $10^{-4}$ | $10^{-6}$ | 1 | 3 | 31 | 0.9999 | 3.1×10^{-4} |
| | $10^{-4}$ | $10^{-6}$ | 1 | 4 | 40 | 1.0000 | 4.0×10^{-4} |
fill in the whole packet of $N$ bits.

Hence, the transmitted packet can contain retransmitted bits for multiple previously transmitted packets, and also data bits from multiple information blocks. As an example, assuming first-in first-out (FIFO) buffering of the information blocks of $N_{buf} = N = 1064$ bits (Zigbee protocol) with $N_{seg} = 2$ segments, $W_{seg} = 2$ bits, i.e., the retransmission window of $W = 2 \times 2 = 4$ bits, $D = 3$ retransmissions, and in total $L_{pac} = 10$ packets to be transmitted, the packets content schedule is shown in Table III. Therein, we use the notation $D_l(n)$ to denote a sequence of $n$ bits belonging to the $l$-th information block, and $R_{l,d}(m)$ is the sequence of $m$ retransmitted bits in the $d$-th retransmission for the $l$-th information block where the sequence indexes, $1 \leq l \leq L_{pac}$, $1 \leq n \leq N$, $1 \leq d \leq D$ and $1 \leq m \leq W$. We have the following proposition.

**Proposition 5:** For $N \gg DW$, $N_{buf} = N$ and $L_{pac} \geq 1$, only the very first information block is completely transmitted in the first packet whereas all other information blocks are split into exactly 2 subsequent packets. The $D$ retransmissions are scheduled into $D$ subsequent packets immediately after the transmission of the corresponding information block was completed. Moreover, only the first $L_{pac}$ transmitted packets are fully occupied with $N$ bits. The available transport capacity in the last $(D+1)$ transmitted packets can be used to transmit additional information blocks with the progressively shorter block lengths and/or smaller number of retransmissions.

The main assumption required in Prop. 5 is that the total number of retransmitted bits $D \cdot W$ is much smaller than the block length $N$. If this condition is not satisfied, the packet structure of the retransmission protocol is less predictable.

We conclude our discussion about the bitwise retransmission scheme for the uplink data fusion from $L$ sensor nodes by considering the packets structure in the downlink. According to Prop. 5, the AP (the data fusion center) sends the retransmission requests for the $l$-th information block during the time slots \{1, 2, $\ldots$, $D_l$\}, if $l = 1$, and \{$l+1$, $l+2$, $\ldots$, $(l+D_l)$\}, if $l \geq 2$. However, since the information blocks with the index $l \geq 2$ are transmitted exactly in 2 subsequent packets with the indexes $l$ and $l+1$, the number of retransmission requests contained in the downlink packet first raises to the maximum value of $D$ requests per sensor node. The number of the requests then remain constant until it is gradually decremented to 1 request in the last $(D - 1)$ transmissions. Consequently, we have the last proposition.

**Proposition 6:** The maximum number of sensor nodes $L_{max}$ which can be supported by the proposed retransmission scheme is bounded as,

$$L_{max} \leq \frac{N - N_{ovh}}{D \cdot C_{tot}}$$

where $N_{ovh}$ is the protocol overhead, and $C_{tot}$ is the total number of feedback bits per retransmission and sensor node. For the example presented in Table III assuming $N_{ovh} = 106$ bits (10% of $N$), we have $C_{tot} = 36$ bits, so $L_{max} \leq 8$. A larger number of sensor nodes can be supported by trading off the number of feedback bits $C_{tot}$ with the number of retransmissions. In addition, it is possible that different sensor nodes set their retransmission parameters differently, for example, to match the SNR they experience in the uplink and downlink. In this case, the value of $L_{max}$ in Prop. 6 can be calculated by assuming the maximum total number of feedback bits, $\max(D \cdot C_{tot})$, allowed per any sensor node.

**VI. CONCLUSIONS**

A novel bitwise retransmission scheme was presented to selectively retransmit only the bits which were received with a small reliability. The bitwise retransmission decisions as well as combining can be done either immediately after demodulating the received symbols, or after the channel decoding. In case of the turbo decoding, the bit reliabilities are available as the soft-input (i.e., channel output) or the soft-output values. The locations of the bits to be retransmitted are reported to the source as a binomial combination number. Since the proposed scheme does not involve any complex operations, it does not limit the processing throughput at the receiver nor at the transmitter.

The analysis presented in the paper assumes uncoded binary modulation for mathematical tractability and clarity of the presentation. However, the proposed scheme can be readily generalized to non-binary modulations by appropriately scaling the demodulated bits. We derived the overall BER conditioned on the error-free feedback link. The accurate closed-form BER expressions as well as their computationally efficient approximations were presented assuming one and two retransmissions. The approximations were verified by computer simulations. We showed that the bitwise retransmissions can be optimized for the given SNR, especially in the forward link, since the BER curves are convex in the transmission rates as well as in the reliability threshold. In addition to minimizing the BER as investigated in this paper, it is possible to maximize the throughput instead.

We next compared the BER and throughput performances of the three specific retransmission strategies which are referred to as the fixed rate technique, the fixed window technique, and the fixed threshold technique. It was shown that, for the same number of retransmissions, and the same packet length, the proposed schemes always outperform the repetition diversity, and, in some cases including the transmissions over time-varying channels, the performance improvement can be
significant. Furthermore, in order to reduce the number of feedback bits, we proposed to use two synchronized RNGs at the transmitter and at the receiver which can greatly compress the transmitted binary feedback sequences at the expense of larger delays. We also derived a condition when the impact of feedback errors to the overall performance can be neglected.

We then considered practical design issues of multi-user bitwise retransmission schemes for data fusion applications where the sensor nodes forward data packets in the uplink into a centralized AP. The retransmission requests are broadcasted to the sensor nodes from the AP in the downlink. Assuming TDMA/TDD and the Zigbee, WiFi and Bluetooth protocols, we presented the design examples of retransmission parameters. These examples suggest that the efficient designs of the bitwise retransmission schemes can be obtained provided that the BER of the forward link is below $10^{-3}$ and the BER of the reverse link is below $10^{-5}$. If the bidirectional link has the same or similar BERs in both directions, the smaller BER in the reverse link can be readily achieved by increasing the transmit power in the downlink (i.e., at the AP). We also devised scheduling based on the Prony approximation of the $H(t)$ function [25],

$$Q(x) = \sum_{k=1}^{2} A_k e^{-B_k x^2}$$

where $A_1 = 0.208$, $A_2 = 0.147$, $B_1 = 0.971$, and $B_2 = 0.525$.

Since $\text{erf}(x) = 1 - 2Q\left(\frac{x}{\sqrt{2}}\right)$, assuming positive constants $H > 0$, and $h_i > 0$, $i = 1, 2, 3, 4, 5$, the expressions for BER$_D$ and $P_D$ can be approximated using the following expressions. These approximations are used and verified numerically in the figures presented in Section IV.

$$f_{H \sim h_1 e^{-\frac{(h-b)^2}{2}}} Q(h_4 (h_5 - \bar{r})) \, d\bar{r}$$

$$\approx \sum_{k=1}^{4} f_{H \sim h_1 e^{-\frac{(h-b)^2}{2}}} A_k e^{-B_k (h_4 (h_5 - \bar{r}))^2} \, d\bar{r}$$

$$\approx \frac{h_1 A_k}{2 \sqrt{h_3 + h_k^2 B_k}} e^{-\frac{h_1^2 h_4^2 (h_5 - \bar{r})^2}{h_3 (1+h_k^2 B_k)}} \sqrt{\pi} \text{erfc}\left(\frac{h_3 h_4^2 h_k^2 B_k}{\sqrt{h_3 (1+h_k^2 B_k)}}\right)$$

$$f_{H \sim h_1 e^{-\frac{(h-b)^2}{2}}} Q(h_4 (h_5 + \bar{r})) \, d\bar{r}$$

$$\approx \sum_{k=1}^{4} f_{H \sim h_1 e^{-\frac{(h-b)^2}{2}}} A_k e^{-B_k (h_4 (h_5 + \bar{r}))^2} \, d\bar{r}$$

$$\approx \frac{h_1 A_k}{2 \sqrt{h_3 + h_k^2 B_k}} e^{-\frac{h_1^2 h_4^2 (h_5 + \bar{r})^2}{h_3 (1+h_k^2 B_k)}} \sqrt{\pi} \text{erfc}\left(\frac{h_3 h_4^2 h_k^2 B_k}{\sqrt{h_3 (1+h_k^2 B_k)}}\right)$$

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