Optimization of Laminate Topology for Multilayered Composite Structures Subjected To Buckling and Failure Constraints

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Abstract. In the present paper the combinatorial optimization problems dealing with laminate stacking sequence optimization are formulated and studied. Two types of design variables are introduced: 1) classical, discrete and 2) new, continuous ones. In order to obtain the optimal solution a classical genetic algorithm is applied. We are looking for the maximal buckling loads for 2-D multilayered structures subjected to bimodality constraints and FPF constraints. Two special buckling problems are analyzed in details, i.e. buckling of compressed rectangular plates and of cylindrical shells under an external pressure. The results demonstrate the effectiveness and advantages of the use of continuous design variables.

1. Introduction

Material properties of composite materials strongly depend on fibre orientations in each layer, so that the optimization of laminate topology becomes one of the most important problems. On the other hand, for isotropic structures a large number of failure modes exist comparing with those for isotropic constructions. For instance, one can distinguish the global failure mode i.e. buckling and the local modes in the form of first-ply-failure, delamination with local buckling and matrix or fibre breakage. Nowadays, it is possible to find the optimal topology of the laminate independently for each failure form mentioned above. The problem of searching for the optimal laminate topology, when several failure modes are taken into account is still unsolved in a general manner. The current work is devoted to this question. We will present with details different mathematical formulation of the optimization problem. This problem will be stated as single-modal and multi-modal with various constraints. The optimization will be formulated for buckling of plates and composite shells with different failure forms (FPF and delamination), that occurred at the same time. The influence of modal and multi-modal buckling forms on optimal laminate configuration will be also presented.

The aim of the present work is two-fold:

• To discuss various formulations of different optimization problems with the aid of new, continuous type of design variables introduced instead of classical fibre orientations and stacking sequences describing mechanical properties of multilayered composite laminates,

• To obtain the optimal solution using Borland BILDER C++ and genetic algorithms provided by the commercial Evolver package in order to demonstrate the possibility of solving completely different problems in the identical way.

We shall not dwell herein on the repetition of the existing literature in this area. In our opinion it is worth to mention only Refs. [1, 2] showing the optimization methods used in the problems of composite laminates as well as the general ideas of genetic algorithms.
For isotropic and multilayered composite structures different problems dealing with stacking sequence optimization of 2D (beams, plates, shells) problems are formulated, discussed in details and solved in Refs [3-7]. A special attention is focused on the reduction of total number of design variables characterizing optimization objectives.

2. Definition of design variables
In 2-D approach a laminate configuration is expressed uniquely by the appropriate terms of the A, B and D matrices. Topological variables defining the connectivity of particular structural elements in the structure; in the paper it denotes the stacking sequence of the individual layers in the laminate, are understood in the sense of the sequence of layers having prescribed discrete fibre orientations \( \theta_i \) in each individual ply; commonly, it is assumed that the thickness of individual plies are identical, i.e. \( t_i = t/N \), where \( t \) denotes the total thickness of the laminate, whereas \( N \) means the total number of plies, and \( i = 1,2, \ldots, N \). In order to assure a great flexibility and generality in formulation of various optimization problems different types of the above-mentioned discrete design variables must be represented in a similar unified manner, i.e. each design variable must be coded as a finite string of digits in the classical GA. Let us note that the angle-ply anti-symmetric laminates are considered only, however, it can be easily extended for arbitrary laminate configuration. Using the classical method of coding 1 represents \( 0^\circ \), 2 – \( \pm 45^\circ \), 3 – \( 90^\circ \). Each design variable \( s \) representing fibre orientation (i.e. 1, 2, and 3) is coded as binary number and called as a gene. The sequence \{1,2,3\} is called as a chromosome. Such a representation is not very convenient for optimization problems since there are a lot of design variables (increasing with the total number of plies \( N \)) and in addition various stacking sequences are described by the identical values of the A, B, D matrices. Therefore, we propose to adopt herein a special type of continuous variables \( x_r^{[A,D]} \) introduced by Muc [3, 4] that are completely different than those used by Miki [8]. The new design variables represent triangles in the design space, however there is no unique mapping between spaces \( x_r^A \) and \( x_r^D \). For the assumed laminate configuration the B matrix is identically equal to zero whereas the terms of the A and D matrices can be written in the following way:

\[
A_1 = t(U_1 - U_3) + \frac{4t}{N}U_2(x_1^A - x_3^A) + \frac{8t}{N}U_3(x_1^A + x_3^A),
\]

\[
D_1 = \frac{t^3}{12}U_1 - U_3 + \frac{4t}{N^3}U_2(x_1^A - x_3^A) + \frac{1}{60} \frac{4t}{N^3}U_3(x_1^A + x_3^A),
\]

\[
x_r^{[A,D]} = \sum_{k=1}^{N/4} \left\{ \frac{3(k-1)+1}{3} \right\} \cos \theta \Xi(\alpha_r), \quad \Xi(\alpha_r) = \begin{cases} 1 & \alpha_r = \theta \\ 0 & \alpha_r \neq \theta \end{cases}, \quad \alpha_r = 90^\circ \frac{r-1}{3}, \quad r = 1, 2, 3
\]

3. Formulation of the optimization problem
The optimization problem is formulated as follows:

\[
\max_{S} \left\{ \min_{m,n} \lambda_b \right\},
\]

where \( \lambda_b \) is a critical multiplier of loading (understood in the sense of a global buckling), \( S \) denotes the vector of design variables representing fibre orientations in the laminate in a discrete or continuous manner given by relations (1), and \( m, n \) are numbers of half-waves in two perpendicular directions corresponding to a plate/shell co-ordinate system. The analysed problem may be subjected to various subsidiary constraints written in the following form:

Bimodal constraints: \( \lambda_b[m,n] \leq \lambda_b[m+1(m),n(n+1)] \),

\[
\text{(3)}
\]
Failure constraints: 
\[ \lambda_{\text{failure}} \leq \lambda_b \] (4)

The constraint (3) presents two conditions for each wave number in buckling independently, whereas the relations (4) demonstrates the possibility of introduction of various failure modes in the form of the matrix cracking, fibre breakage, delaminations etc. The definition of the \( \lambda_{\text{failure}} \) multiplier is directly connected with the analyses failure mode.

To illustrate the effectiveness of the proposed method of solution two buckling problems will be solved in details. The first one deals with the buckling problem of bi-axially compressed, rectangular plates, where (Ref. [9]):

\[
\lambda_b = \frac{(m\pi / a)^2}{P_y(1 + k\beta_m^2)} \left[ D_{11} + 2(D_{12} + 2D_{66})\beta_m^2 + D_{22}\beta_m^4 \right], \quad \beta_m = (na)/(mb), \quad k = P_y / P_x,
\] (5)

and in the second buckling of cylindrical shells under external pressure is considered (Ref. [10]):

\[
\lambda_{b,1} \frac{n^2}{R} = K_{11} + \frac{2K_{11}K_{22} - K_{11}^2K_{22} - K_{22}^2K_{11}}{K_{11}K_{22} - K_{12}^2},
\]

\[
K_{11} = A_{11}\left( \frac{m\pi}{L} \right)^2 + A_{46}\left( \frac{n}{R} \right)^2, \quad K_{12} = (A_{11} + A_{46})\left( \frac{m\pi}{L} \right)\left( \frac{n}{R} \right), \quad K_{22} = A_{22}\left( \frac{n}{R} \right)^2 + A_{46}\left( \frac{m\pi}{L} \right)^2,
\]

\[
K_{33} = D_{11}\left( \frac{m\pi}{L} \right)^4 + 2(D_{12} + 2D_{66})\left( \frac{m\pi}{L} \right)^2\left( \frac{n}{R} \right)^2 + D_{22}\left( \frac{n}{R} \right)^4 + A_{22}^2\left( \frac{m\pi}{L} \right)^2
\]

where \( p \) denotes an external, uniform pressure, and \( R \) and \( L \) are the shell radius and length, respectively.

4. Numerical solutions

To find the optimal solutions of the problem given by equations (2)-(4) in the most general manner the package Evolver have been employed. It is conjugated with the C++ Builder to evaluate different forms of the buckling load parameter as well as constraint conditions. The analysis has been started from the discrete definition of the design variables that are assumed to take the following form: \( 0^\circ \) or \( \pm 45^\circ \) or \( 90^\circ \). The laminate of the thickness equal to 6.096 mm is made of 48 individual plies (but according to the assumed laminate symmetry 12 of them only constitute the vector of design variables) having the following material properties: \( E_1=127.6 \) GPa, \( E_2=13.03 \) GPa, \( G_{12}=6.41 \) GPa, \( v_{12}=0.3 \). In all cases considered the population have been identical and possessed 70 individuals; the one-point crossover probability is equal to 0.5, whereas the mutation parameter is equal to 0.01. To compare the optimal solutions the initial population has been identical and made of genes having orientations \( 0^\circ \) only. The results of 100 independent searching for the optimal stacking sequence of rectangular plates are presented in Figure 1. As it may be seen a scatter of optimal values of buckling load factor is observed and in addition the maximal value obtained with the use of continuous variables (5) does not correspond exactly to those evaluated in the numerical way (the classical GA).
Figure 1. The bar chart plot of optimal results for rectangular plates ($a/b=3$, $n=1$, $m=3$) subjected to bimodal and first-ply-failure constraints.

Figure 2. The contour plot of the membrane part of the buckling load factor – a cylindrical shell ($L/D=5/3$, $n=6$, $m=1$) under external pressure.

This is a typical result which can be noticed using classical GA for multilayered structures. The terms of the D matrix are responsible for buckling results, and the A matrix for the plate first-ply-failure. Therefore it is impossible to obtain always identical optimal stacking sequences, mainly due to the lack of the unique mapping between the A and D matrices terms.

It is better demonstrated in the contour plots of the buckling load factor for cylindrical shells under external uniform pressure – see Figure 2 and 3. Figure 2 presents the distributions of the part of the buckling load factor corresponding to the membrane terms in the relation (6), whereas Figure 3 is a plot of the latter bending part. Even looking for the optimum given by equation (2) with no constraints it is impossible to find directly the optimal laminate configuration.

Figure 3. The contour plot of the bending part of the buckling load factor – a cylindrical shell ($L/D=5/3$, $n=6$, $m=1$) under external pressure.

Figure 4. The bar chart plot of optimal results for cylindrical shells.
However, the optimum occurs at the boundary of the design variable domain. Similarly as previously for multilayered plates a scatter of the maximal buckling load factor for cylinders is also observed – see Figure 4 but the location of the optimum (at the boundary) results in much better correlation with the case as the continuous variables (5) are used in the analysis. The plot has been obtained after 100 runs of the Evolver package using the classical GA as the optimization algorithm. The number of optimum results corresponding to the exact one is much higher than in the previous case of stacking sequence optimization for multilayered plates.

5. Discussion of the results
The possibility of a general approach to various types of discrete stacking sequence optimization problems with the use of discrete and continuous design variables have been formulated and studied. The presented results demonstrate the effectiveness of the proposed method of the solution with the use of the classical discrete design variables combined with the Evolver package. The accuracy of the results has been compared with those obtained by Haftka [10]. However, the above analysis shows evidently that the use of the classical GA is not always very convenient for our optimization problem (2)-(4) since it is impossible to answer whether the global optimum is reached or not. As it may be observed as the number of constraints increases the effectiveness of the method decreases – compare Figure 1 and Figure 4. Therefore, one can conclude that it is better to change the optimization algorithm but in our opinion it is much better to replace the ordinary set of discrete design variables by a continuous one introduced by equation (1). Briefly speaking it results in the replacement of the wheel roulette method by the elite one. In fact such a change of design variables, according to the nomenclature used in the optimization problems, is equivalent to the introduction of a new algorithm called as evolutionary algorithms.

6. Conclusion
The proposed method of the solution with the use of the classical discrete design variables combined with the Evolver package is an effective and it results in the replacement of the wheel roulette method by the elite one. A change of design variables, according to the nomenclature used in the optimization problems, is equivalent to the introduction of a new algorithm called as evolutionary algorithms.

7. References
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