Optimal dynamic promotion strategies in the multiple competing supply chains

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ABSTRACT

In this paper, the optimal decision problem is investigated for a class of dynamic promotions in the multiple competing supply chains. Here, the dynamic advertising model is allowed to be influenced by horizontal joint promotions (HJP) and vertical joint promotions (VJP). We extend the classical Nerlove–Arrow model by considering the effect of competitive manufacturer’s VJP effort on brand goodwill. Also, the continuous-time demand is constructed with the effect of competitive retailer’s VJP and HJP efforts. Based on the Hamilton–Jacobi–Bellman equation, the optimal HJP and VJP strategies of supply chain members and the optimal promotion cost sharing rate of each manufacturer are obtained under the Decentralized–Decentralized and Decentralized–Centralized structures, respectively. Subsequently, the comparison between the two structures shows that the optimal VJP and HJP efforts of supply chain members increase with their marginal profits and each manufacturer is willing to pay for the retailer’s VJP effort level when the marginal profit of manufacturer exceeds the half of retailer’s marginal profit. Numerical examples are provided to show the change of profit difference with parameters $\alpha$ and $\beta$ under the different structures.

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1. Introduction

The recent financial crisis, as a common and inevitable phenomenon in the process of economic globalization, has gradually inflated to a world-wide economic crisis. To drive the incremental product sale and strengthen the brand goodwill, manufacturers and retailers may seek opportunities to cooperate with each other through joint promotion programs. To be specific, the joint promotion in a typical supply chain is divided into vertical joint promotions (VJP) and horizontal joint promotions (HJP). In VJP programs, a manufacturer is willing to pay a fraction of promotional expenditures for his downstream retailer (Zhang, Gou, Liang, & Huang, 2013). In addition, some members also offer a monetary support to their rivals in HJP programs recently (Karray, 2011). A great deal of work has been shown that the joint promotion programs bring more available choices to the consumers and higher profits to joint companies (Karray, 2015).

The VJP, also called cooperative advertising programs, are commonly promotional mechanisms provided by the manufacturer to influence the advertising investment of retailer. To be specific, cooperative advertising consists of the national advertising and the local advertising (Javid & Hoseinpour, 2011; Lamberti, 2014; Szmerekovsky & Zhang, 2009), where the manufacturer controls the national advertising to build up the brand goodwill of product and the retailer controls the local advertising to push up the product demand. Meanwhile, the manufacturer pays a portion of the retailer’s local advertising expenditures, which the fraction shared by the manufacturer is commonly defined as the participation rate. Due to the exclusive subsidies from manufacturer, the retailer may invest more local advertising expenditures and then supply chain would get more benefits from the VJP programs. As a result, the VJP programs have been extensively and successfully applied in many industry business, for example, it is estimated in Advertising Age (1999) that the cooperative advertising budget has accounted for approximated 50% of the sum of advertising budget for department store, approximated 75% of the grocer’s advertising. Moreover, it is estimated that the total expenditures on VJP in 2000 are believed to account for approximately 15 billion in the USA which are nearly quadrupled over 1970 in real term. Later, a recent report shows that the value of 2010s VJP expenditures exceeds 50 billion (Yan, 2010).

It is well known that HJP are partnerships in which several competitors share the expenditures of promotional activity. On the one hand, the HJP of manufacturers are called the generic advertising whose effect is to enhance demand for the category, mainly by attracting potential consumers to purchase product. Also, the generic...
advertising has widely used for many industries, for example, in the early stages of the life cycle of new agricultural product, the genetic advertising has been applied to create awareness or promote the new uses. On the other hand, the main aim of retailers’ HJP is to drive the incremental traffic in stores and increase the retailers’ visibility. As discussed in Pauwels (2007), a typical example, which the leading sporting goods retailers Bass Pro Shops and Marine Max implement joint promotional activity, shows that the HJP programs bring higher profits to the joint companies.

Motivated by the above discussions, taking a consecutive relationship between promotion decisions in each period into account, our paper uses differential game models to consider the time variation and addresses the above-mentioned issues by merging three research streams-VJP programs, HJP programs and competing supply chains with ‘multiple-manufacturer multiple-retailer’. Although several models have been developed in these disciplines, none explored the comprehensive effect of the whole three factors in one dynamic framework. Hence, the main contribution of this paper lies in that the optimal VJP and HJP strategies are given for the multiple competing supply chains by a dynamic way. Here, the game theoretic model is solved for Decentralized–Decentralized (DD) and Decentralized–Centralized (DC) structures. The focus is on how to answer the following three key questions: (1) Under what conditions do the manufacturers pay a portion of funds to support the local advertising? (2) How do the joint promotion programs influence players’ promotion strategies? (3) Under what conditions are the total profit of supply chain better off under the DC structure over DD structure? The reminder of this paper is organized as follows: In Section 2, the related previous literatures are briefly introduced. The problem description and the basic model are thoroughly described in Section 3. Subsequently, the optimal promotion strategies under two structures are presented in Section 4 to ensure the profit maximization. Some numerical examples are given to show the change of profit difference with coefficients $\alpha$ and $\beta$ under the different supply chain structures in Section 5. Finally, the conclusions are summarized.

2. Literature review

We review previous work in the following two streams that are relevant to this study: VJP and HJP programs. The issue of VJP programs will be firstly discussed in the supply chain and then, we pay attention to the discussions of HJP programs. We will briefly review the relevant results and also differentiate our study from previous literature.

Up to now, a great deal of efforts have been delivered to the cooperative advertising decision issues of supply chain members. These results, however, have been concerned with static model and dynamic model of cooperative advertising. To be specific, the cooperative advertising decision problem has been firstly investigated in Berger (1972) for a class of static model in the supply chain, which the manufacturer has paid a portion of advertising expenditures for retailer. Later, a great number of literature has been extended the work of Berger. Among them, Laskey and Nicholls have put particular emphasis on the impact of local advertising effort on demand (Roslow, Laskey, & Nicholls, 1993). In Huang and Li (2001), the optimal advertising strategies have been obtained under the following two scenarios: (i) the manufacturer and the retailer are leader and follower, respectively (ii) the players make decisions in a co-op partnership. Based on the work of literature (Huang & Li, 2001), the cooperative advertising problem of two-level supply chain has been studied by considering a price discount in demand elasticity market circumstance (Yue, Austin, Wang, & Huang, 2006). Moreover, the other extensions of static model have been mainly concerned with the cooperative advertising and price decision problem (SeyedEsfahani, Biazaran, & Gharakhani, 2011; Xie & Neyret, 2009), the competitive advertising decision problem (Wang, Zhou, Min, & Zhong, 2011) and the coordination problem of cooperative advertising (Cheng, Li, & Huang, 2006; Ghadimi, Szidarovszky, Farahani, & Yousefzadeh Khiabani, 2013; Wang et al., 2011).

Dynamic advertising models have gained more research attention in the context of supply chain. Note that many variables can be modelled by the differential equation and some characteristics of advertising effect can be closely described by dynamic models rather than by static models. One of the earliest models in this aspect is that of Nerlove and Arrow, where the time-dependent demand function has been related to some factors, such as brand goodwill, price and advertising investment (Nerlove & Arrow, 1962). Later, a rich body of literature has extended the Nerlove–Arrow (N-A) advertising model to the dynamic cooperative advertising problem (Jørgensen, Sigue, & Zaccour, 2000; Jørgensen, Taboubi, & Zaccour, 2003; Karray & Zaccour, 2005; Zhang & Zhang, 2006). Based on a common assumption that the local advertising has a positive effect on brand goodwill, the optimal advertising strategy problems have been discussed in Zhang and Zhang (2006) by utilizing the differential game theory. A special phenomenon has been discussed in Jørgensen et al. (2003), where the retailer’s promotion might damage the brand goodwill and Jørgensen has applied differential game theory to obtain the optimal advertising strategy. Additionally, Jørgensen has
considered the case where both players make long-term and short-term advertising efforts to increase demands and brand goodwill (Jørgensen et al., 2000). Extended the work of Jørgensen, the case, that a retailer simultaneously sells national brand products and their own products with private brand, has been considered in Karray and Zacour (2005). Very recently, the dynamic advertising models have been discussed in the ‘multiple-manufacturer single-retailer’ supply chain and ‘single-manufacturer multiple-retailer’ supply chain (Adida & DeMiguel, 2011; Cachon & Kòk, 2010; He, Gou, Wu, & Yue, 2013; Lu, Tsao, & Charoensiriwath, 2011). However, few results have been addressed a framework with multiple competing supply chains.

Moreover, a large number of results have been reported on the generic advertising (HJP of manufacturer) strategy problems. To mention a few, optimal generic advertising strategy problem has been proposed in Piga (1998), nor mentioned the brand advertising strategy. Later, many scholars have focused on the effect of generic and brand advertising on product demand by utilizing the differential game theory (Bass, Krishnamoorthy, Prasad, & Sethi, 2005; Qi, Ding, & Chen, 2008). Similar to the approach in Bass et al. (2005), three types of advertising, including defensive advertising, offensive advertising and generic advertising, have been taken into the duopolistic market and the optimal advertising strategy has been presented for each type advertising (Jørgensen & Sigu, 2015). However, the published results about retailers’ HJP are nearly insistent except the references (Karray, 2011, 2015) and the existent literature is primarily accounted for the static model of promotion strategy. To the best of author’s knowledge, there has been little work undertaken on the dynamic promotion strategies in the multiple competing supply chains.

To explore these issues, the problem is investigated for a class of dynamic promotions in two competing supply chains. Here, the brand goodwill is allowed to be influenced by the own and competitor’s national advertising efforts. By applying Hamilton–Jacobi–Bellman (HJB) equation, the optimal HJP and VJP strategies, the brand goodwill of product, the total supply chain profits and the optimal participation rate are obtained under the two structures: two competing decentralized systems (DD) and a decentralized system competing with a centralized system (DC). Numerical examples are provided to illustrate the change of profit difference with parameters \( \alpha \) and \( \beta \) under the different supply chain structures.

### 3. Notation and problem formulation

This section defines the notations and problem statement used throughout this paper.

#### 3.1. Notations

- \( G_i(t) \): Brand goodwill of the product \( i \) at time \( t \), \( i \in \{1, 2\} \)
- \( A_i(t) \): Manufacturer \( i \)'s national advertising effort at time \( t \)
- \( U_i(t) \): Retailer \( i \)'s local advertising effort at time \( t \)
- \( B_i(t) \): Retailer \( i \)'s HJP effort at time \( t \)
- \( S_i(t) \): Demand of product \( i \)
- \( \phi_i \in [0, 1] \): The participation rate of cooperative advertising
- \( \theta \): The HJP’s impact factor
- \( \lambda \): Discount rate of the manufacturers and the retailers
- \( \Pi_{Mi}, \Pi_{Ri}, \Pi_{ch} \): Profit functions for \( Mi, Ri \) and the total system, respectively
- \( J_{Mi}, J_{Ri} \): Current value of profit functions for \( Mi \) and \( Ri \), respectively

#### 3.2. Problem formulation

We consider two competing supply chains, where each supply chain is made of one manufacturer and one retailer. The manufactures produce the substitutable products with different brand described as \( i, j \in \{1, 2\} \) and sell the products to the end consumers through the retailers. To strength the brand goodwill and push up product demand, manufacturers and retailers would implement the VJP and HJP programs. We denote the manufacturer \( i \)'s national advertising effort over time as \( A_i(t) \) and the retailer \( i \)'s local advertising effort as \( U_i(t) \). Additionally, each retailer is also willing to cooperate with the competing retailer for HJP activity, in which the HJP effort is described as \( B_i(t) \) for retailer \( i \). In general, an assumption of similar economic strength is to ensure that its own advertising investment can still promote the accumulation of brand goodwill under the competitive circumstance. Each manufacturer controls his national advertising effort \( A_i(t) \) designed to build up the reputation or the stock of brand goodwill \( G_i(t) \). The evolution of brand goodwill is governed by the following differential equation from an extension of N–A model.

\[
\dot{G}_i(t) = A_i(t) - \alpha A_{3-i}(t) - \delta G_i(t), \quad G_i(0) = G_{i0},
\]

where \( G_{i0} > 0 \) is the initial value of brand goodwill for product \( i \). This specification deserves some comments. First, \( 0 < \alpha < 1 \) represents the negative effect of competitor’s national advertising on brand goodwill and we adopt the common assumption that the effect of competitor’s national advertising is lower than that of his own effect (He et al., 2013). Next, \( \delta > 0 \) is the decay rate or forgetting effect of brand goodwill which captures the idea
that consumers may forget the product to some extent. According to the literature (Jørgensen, Sigue, & Zac- 
cour, 2001), national advertising mainly focuses on the 
long-term growth of brand goodwill and local advertis-
ing on the immediate effect of product demand. Then, we 
extend the demand function in Jørgensen et al. (2001)t o 
the competitive circumstance and the product demand 
brought from VJP programs $S_{i,v}(t)$ is 

$$S_{i,v}(t) = U_i(t) \sqrt{G_i(t)} - \beta U_{3-i}(t) \sqrt{G_{3-i}(t)}. \quad (2)$$ 

In (2), $U_i(t) \sqrt{G_i(t)}$ and $\beta U_{3-i}(t) \sqrt{G_{3-i}(t)}$ measure the impact of own and competitor’s VJP programs on product demand, respectively. Although the two competing centralized 

$$S_{i,v}(t) = \theta[B_i(t) + B_{3-i}(t)]. \quad (3)$$ 

The increase in the demand as a result of HJP programs is assumed to be shared equally. Let $\theta$ denote the pro-
portion of demand increase that is transferred to both products. Hence, the total demand of product $i$ is $S_{i}(t) = S_{i,v}(t) + S_{i,h}(t)$. Adding Equations (2) and (3), the total effect of HJP and VJP programs on product demand is 

$$S_{i}(t) = U_i(t) \sqrt{G_i(t)} - \beta U_{3-i}(t) \sqrt{G_{3-i}(t)} \quad + \theta[B_i(t) + B_{3-i}(t)]. \quad (4)$$ 

Here, Equation (4) is intuitive in that the total demand is the sum of demand gain due to his VJP effort $U_i(t) \sqrt{G_i(t)}$, minus the demand loss due to the competitor’s VJP effort $\beta U_{3-i}(t) \sqrt{G_{3-i}(t)}$, plus the demand gain due to market expansion $\theta[B_i(t) + B_{3-i}(t)]$. The promotional expenditure is quadratic with respect to the corresponding promotional effort, that is 

$$C(A_i(t)) = \frac{1}{2} A_i^2(t), \quad C(U_i(t)) = \frac{1}{2} U_i^2(t), \quad C(B_i(t)) = \frac{1}{2} B_i^2(t). \quad (5)$$ 

As in most of the literature (Adida & DeMiguel, 2011; De Giovanni, 2011; Zhang et al., 2013), the promotional expenditures may be assumed by the means of convex and increasing function. Alternatively, one can use a linear promotional expenditure function and have appear as a square root in the state equations.

Without taking product cost into account, $\rho_{M}$ and $\rho_{R}$ are the margin profits of manufacturer and retailer, respectively. Although the two competing central-
ized systems (namely CC structure) have been considered

| Table 1. The profit functions under the DD and DC structures. |
|---------------------------------------------------------------|
| Chain | Profit | DD channel structure | DC channel structure |
|-------|--------|-----------------------|---------------------|
| 1     | $\pi_1$ | $\rho_M S_1(t) - \frac{1}{2} (1 - \phi_1) U_1^2(t) - \frac{1}{2} B_1^2(t)$ | $\rho_M S_1(t) - \frac{1}{2} (1 - \phi_1) U_1^2(t) - \frac{1}{2} A_1^2(t)$ |
| 2     | $\pi_2$ | $\rho_M S_2(t) - \frac{1}{2} (1 - \phi_2) U_2^2(t) - \frac{1}{2} B_2^2(t)$ | $\rho_M S_2(t) - \frac{1}{2} (1 - \phi_2) U_2^2(t) - \frac{1}{2} A_2^2(t)$ |
| 3     | $\pi_3$ | $\rho_M S_3(t) - \frac{1}{2} (1 - \phi_3) U_3^2(t) - \frac{1}{2} B_3^2(t)$ | $\rho_M S_3(t) - \frac{1}{2} (1 - \phi_3) U_3^2(t) - \frac{1}{2} A_3^2(t)$ |

In Karray (2011, 2015), we still investigate the optimal promotion strategies under the DD and DC structures. That is due partly to the fact that there exists no perfect cooperation among the practical supply chain systems (Harrison & New, 2002). Under the DD structure, the system consists of two competitive decentralized supply chain. The independent manufacturers sell similar products through independent retailers. Each supply chain member strives to maximize the present value of their own profits. In addition, under the DC structure, the system is formed of one decentralized supply chain and one centralized supply chain. The decentralized supply chain consists of manufacturer 1 and retailer 1, which both members independently control VJP and HJP strategies to maximize their own profits. In the centralized supply chain, the manufacturer 2 and retailer 2 jointly carry out VJP and HJP efforts to maximize the total profit of supply chain. Therefore, at time $t$, the profit functions of each retailer ($\pi_{R}$), manufacturer ($\pi_{M}$), and the total system ($\pi_{ch}$) are described in Table 1 under the DD and DC structures.

In Table 1, $\phi_i$ measures the amount that manufacturer is willing to provide for the VJP expenditures of the retailer. In the following sections, we will calculate the equilibrium promotion strategies under each structure.

### 4. Main results

Considering the different statuses of manufacturer and retailer in the supply chain, we use a Stackelberg game (the manufacturer as the leader and the retailer as the follower) to analyse the optimal promotion decisions under the different structures. The game sequence is given: the manufacturers separately decide their own national advertising efforts and the participate rates. This information is considered by the retailers who announce the local advertising efforts and HJP efforts. We use the superscript DD or DC to represent the optimal decisions obtained for each structure.
4.1. The optimal promotion strategies under the DD structure

Under the DD structure, each member strives to maximize the present value of its own profit. With a common discount rate \( \lambda > 0 \), the objective functions of manufacturer and retailer are expressed:

\[
J_{Mi} = \int_{0}^{\infty} e^{-\lambda t} \Pi_{Mi} \, dt, \\
J_{Ri} = \int_{0}^{\infty} e^{-\lambda t} \Pi_{Ri} \, dt, \quad i = 1, 2.
\]

To obtain the optimal promotion efforts and the participate rates under the DD structure, we solve the optimization problems for the objective functions \( J_{Mi} \) and \( J_{Ri} \) subject to the dynamic equation (1), respectively. We get the following propositions characterizing the equilibrium decisions for the DD structure.

**Proposition 4.1:** Under the DD structure, the optimal HJP and VJP efforts of supply chain members are given by:

\[
A_{i}^{DD} = \frac{\rho_{Mi} \rho_{Ri}}{(1 - \phi_{i}^{DD})(\lambda + \delta)} - \frac{\phi_{i}^{DD} \rho_{Ri}^2}{2(1 - \phi_{i}^{DD})^2(\lambda + \delta)} + \alpha \beta \rho_{Mi} \rho_{3-i} \left( \frac{1}{1 - \phi_{2-i}^{DD}} \right),
\]

\[
U_{i}^{DD} = \frac{\rho_{Mi} \sqrt{G_{i}(t)}}{1 - \phi_{i}^{DD}}, \quad \phi_{i}^{DD} = \theta \rho_{Ri},
\]

the participate rate of manufacturer \( i \) is:

\[
\phi_{i}^{DD} = \begin{cases} 
\frac{2 \rho_{Mi} - \rho_{Ri}}{2 \rho_{Mi} + \rho_{Ri}} \cdot \frac{\rho_{Mi}}{\rho_{Ri}}, & \frac{1}{2} \leq \frac{\rho_{Mi}}{\rho_{Ri}} < \frac{1}{2}, \\
0, & 0 < \frac{\rho_{Mi}}{\rho_{Ri}} \leq \frac{1}{2}.
\end{cases}
\]

**Proof:** Since the problem plays Stackelberg game, we first derive the retailers’ decision problems by applying backward induction. The Hamilton functions of retailers are then given by:

\[
H_{Ri}^{DD} = e^{-\lambda t} \left\{ \rho_{Ri} \left[ U_{i}(t) \sqrt{G_{i}(t)} - \beta U_{3-i}(t) \sqrt{G_{3-i}(t)} \right] + \theta (B_{i}(t) + B_{3-i}(t)) \right\} - \frac{1}{2} (1 - \phi_{i}^{DD}) U_{i}^{2}(t)
\]

\[
- \frac{1}{2} B_{i}^{2}(t) + p_{i} \left[ A_{i}(t) - \alpha A_{3-i}(t) - \delta G_{i}(t) \right] + p_{i,3-i} \left[ A_{3-i}(t) - \alpha A_{i}(t) - \delta G_{3-i}(t) \right].
\]

Performing the maximization of the right-hand side of Equation (9), one has:

\[
\frac{\rho_{Ri} \sqrt{G_{i}(t)} (1 - \phi_{i}) U_{i}(t)}{1 - \phi_{i}} = 0,
\]

\[
\frac{\theta \rho_{Ri} - B_{i}(t)}{1 - \phi_{i}} = 0.
\]

The optimal local advertising and HJP efforts are given by:

\[
U_{i}^{DD} = \frac{\rho_{Ri} \sqrt{G_{i}(t)}}{1 - \phi_{i}},
\]

\[
B_{i}^{DD} = \theta \rho_{Ri}.
\]

In addition, we will derive the manufacturers’ decision problems, the Hamilton functions of the manufacturers are:

\[
H_{Mi}^{DD} = e^{-\lambda t} \left\{ \rho_{Mi} \left[ U_{i}(t) \sqrt{G_{i}(t)} - \beta U_{3-i}(t) \sqrt{G_{3-i}(t)} \right] + \theta (B_{i}(t) + B_{3-i}(t)) \right\} - \frac{1}{2} \phi_{i} U_{i}^{2}(t)
\]

\[
- \frac{1}{2} A_{i}^{2}(t) + q_{i} \left[ A_{i}(t) - \alpha A_{3-i}(t) - \delta G_{i}(t) \right] + q_{i,3-i} \left[ A_{3-i}(t) - \alpha A_{i}(t) - \delta G_{3-i}(t) \right].
\]

Substituting Equations (12) and (13) into (14) and utilizing the necessary conditions of the Maximum principle, we have:

\[
\frac{\partial H_{Mi}}{\partial A_{i}} = 0,
\]

\[
\frac{\partial H_{Mi}}{\partial G_{i}} = 0,
\]

and the corresponding adjoint equations are:

\[
\frac{\partial H_{Mi}}{\partial G_{3-i}} = 0,
\]

\[
\frac{\partial H_{Mi}}{\partial \phi_{3-i}} = 0.
\]

From this, the first-order conditions for \( A_{i} \) and \( \phi_{i} \), yield

\[
A_{i} = e^{\lambda t} \left( q_{i} - \alpha q_{i,3-i} \right),
\]

\[
\phi_{i} = \frac{2 \rho_{Mi} - \rho_{Ri}}{2 \rho_{Mi} + \rho_{Ri}}.
\]

Due to the non-negative property of participation rate, we obtain the optimal participation rate:

\[
\phi_{i}^{DD} = \begin{cases} 
\frac{2 \rho_{Mi} - \rho_{Ri}}{2 \rho_{Mi} + \rho_{Ri}}, & \frac{1}{2} \leq \frac{\rho_{Mi}}{\rho_{Ri}} < \frac{1}{2}, \\
0, & 0 < \frac{\rho_{Mi}}{\rho_{Ri}} \leq \frac{1}{2}.
\end{cases}
\]

According to Equations (17)–(18), we get:

\[
\dot{q}_{ii} = \delta q_{ii} - \left[ \rho_{Mi} \rho_{Ri} \rho_{3-i} \frac{\rho_{Ri}^2}{2(1 - \phi_{3-i})^2} \right] e^{-\lambda t}
\]

\[
\dot{q}_{i,3-i} = \delta q_{i,3-i} + \frac{\beta \rho_{Mi} \rho_{3-i} \rho_{Ri}}{1 - \phi_{3-i}} e^{-\lambda t}.
\]
Solving the above differential equations, one has:

\[ q_{ii}(t) = c_{ii}e^{\delta t} + \left[ \frac{\rho_{Mi}\rho_{Ri}}{(1 - \phi_i)(\lambda + \delta)} - \frac{\phi_i\rho_{Ri}^2}{2(1 - \phi_i)^2(\lambda + \delta)} \right] e^{-\lambda t} \tag{22} \]

\[ q_{i3-i}(t) = c_{i3-i}e^{\delta t} - \frac{\beta\rho_{Mi}\rho_{R3-i}}{(1 - \phi_{3-i})(\lambda + \delta)} e^{-\lambda t} \tag{23} \]

then

\[ A_{i}^{DD} = (c_{ii} - \alpha c_{i3-i})e^{(\lambda + \delta)t} + \frac{\rho_{Mi}\rho_{Ri}}{(1 - \phi_i^{DD})(\lambda + \delta)} - \frac{\phi_i^{DD}\rho_{Ri}^2}{2(1 - \phi_i^{DD})^2(\lambda + \delta)} + \frac{\alpha\beta\rho_{Mi}\rho_{R3-i}}{(1 - \phi_{3-i}^{DD})(\lambda + \delta)}. \tag{24} \]

Once \( c_{ii} - \alpha c_{i3-i} \neq 0 \), the value of manufacturer advertising effort will tend to be infinite when \( t \rightarrow \infty \), which does not fit the current situation. Therefore, we obtain that \( c_{ii} - \alpha c_{i3-i} = 0 \), and combining the result in (24), the optimal advertising effort is written as:

\[ A_{i}^{DD} = \frac{\rho_{Mi}\rho_{Ri}}{(1 - \phi_i^{DD})(\lambda + \delta)} - \frac{\phi_i^{DD}\rho_{Ri}^2}{2(1 - \phi_i^{DD})^2(\lambda + \delta)} + \frac{\alpha\beta\rho_{Mi}\rho_{R3-i}}{(1 - \phi_{3-i}^{DD})(\lambda + \delta)}. \tag{25} \]

When there exists no VJP program between manufacturer and retailer (\( \rho_{Mi}/\rho_{Ri} < 0.5 \)), one has \( A_{i}^{DD} = \rho_{Mi}\rho_{Ri}(\lambda + \delta) + (\alpha\beta\rho_{Mi}\rho_{R3-i})(\lambda + \delta) \) and \( U_{i}^{DD} = \rho_{Ri}\sqrt{G_{i}(t)} \). That is meaning that, if the marginal profit of manufacturer is relatively lower or the marginal profit of retailer is relatively higher, the manufacturer has no enough ability to provide a monetary support for the retailer. As a result, the retailer would not invest more local advertising investment. Else, when there exists a VJP program between manufacturer and retailer (\( \rho_{Mi}/\rho_{Ri} > 0.5 \)), one has \( A_{i}^{DD} = ((2\rho_{Mi} + \rho_{Ri})^2 + 4\alpha\beta(2\rho_{Mi}\rho_{R3-i} + \rho_{R3-i}^2)\sqrt{G_{i}})/(8(\lambda + \delta)) \) and \( U_{i}^{DD} = (2\rho_{Mi} + \rho_{Ri})\sqrt{G_{i}}/2 \). The situation might lead to so more gain for each member who would invest more in publicizing product. These results are similar to the one described in He et al. (2013). In this paper, we mainly concern with the effect of HJP and VJP on supply chain strategy decision. Hence, we do not discuss the first situation here.

When each manufacturer provides a monetary support for the retailer, the following Proposition 4.2 characterizes the results under the DD structure.

**Proposition 4.2:** Under the DD structure, the following results are obtained at equilibrium for supply chain members:

(a) \( \frac{\partial A_{i}^{DD}}{\partial \rho_{Mi}} > 0, \frac{\partial A_{i}^{DD}}{\partial \rho_{Ri}} > 0, \frac{\partial A_{i}^{DD}}{\partial \rho_{Mi3-i}} > 0, \frac{\partial A_{i}^{DD}}{\partial \rho_{R3-i}} > 0, \frac{\partial U_{i}^{DD}}{\partial \rho_{Mi}} > 0, \frac{\partial U_{i}^{DD}}{\partial \rho_{Ri}} > 0, \frac{\partial U_{i}^{DD}}{\partial \rho_{R3-i}} > 0, \frac{\partial U_{i}^{DD}}{\partial \rho_{Mi3-i}} > 0 \)

(b) \( \frac{\partial U_{i}^{DD}}{\partial \rho_{Mi}} > 0, \frac{\partial U_{i}^{DD}}{\partial \rho_{Ri}} < 0 \).

Looking at the sensitivity of equilibrium strategies to changes in VJP and HJP efforts, Proposition 4.2 (a) shows that the VJP efforts increase with the marginal profits of supply chain members. This implies that the marginal profit is a major motivating factor for supply chain members to participate in the VJP programs. Further, to increase the margin profit, the manufacturer should reduce the production cost under the premise of ensuring product quality, while the retailer should increase the retail price or reduce the operation cost under the premise of ensuring product service. Moreover, the higher marginal profit will induce retailer to spend more funds in the HJP programs. However, the HJP effort of retailer does not change with the marginal profit of the competitive retailer.

**Remark 4.1:** One of the difficulties of this research is, in the multiple competing supply chain, under what conditions the manufacturer is willing to offer a monetary support to the downstream retailer. In summary, if the ratio between manufacturer’s marginal profit to retailer’s is more than 0.5, he is willing to pay for a portion of local advertising expenditures. Especially, without taking the promotional cost into account, if the manufacturer has the higher marginal profit, then he may invest more national advertising efforts and offers a higher participate rate to simulate advertising investments of the retailer. Else, when the retailer has a lower marginal profit, although the manufacturer is willing to undertake higher advertising expenditures of the retailer, she has no enough strength to spend more in local advertising. Hence, the manufacturer should consider the comprehensive power of downstream cooperator in order to gain more benefits from VJP programs. On the other hand, if the ratio between manufacturer’s marginal profit to retailer’s is less than 0.5, the manufacturer is not willing to pay for a portion of local advertising expenditures.

**Proposition 4.3:** At the equilibrium solutions, for the DD structure, the optimal brand goodwill for product \( i \) is given as follows:

\[ G_{i}^{DD}(t) = \frac{1}{\delta} \Omega_{i}^{DD} + (G_{i0} - \frac{1}{\delta} \Omega_{i}^{DD}) e^{-\delta t}. \tag{26} \]
The steady state of brand goodwill is

$$\lim_{t \to \infty} G_i^{DD}(t) = \frac{1}{\delta} \Omega_i^{DD},$$

(27)

where $\Omega_i^{DD} = A_i^{DD} - \alpha A_{3-i}^{DD}$.

It is worth noting that, in Proposition 4.3, the steady state of brand goodwill increases with the manufacturer’s VJP effort and decreases with the competitor’s VJP effort due to the competitive effect. In addition, when $\delta > \Omega_i^{DD}/G_{i0}$, the brand goodwill of product decreases over time, but the value of brand goodwill is always higher than $\Omega_i^{DD}/\delta$. However, when $\delta < \Omega_i^{DD}/G_{i0}$, the brand goodwill of product increases over time, but the brand goodwill value is always lower than $\Omega_i^{DD}/\delta$. Specifically, when the forgotten degree of product is relatively higher (lower), although the firms hold a high (low) initial goodwill, the brand goodwill of product would decreases (increases) without advertising behaviour.

**Proposition 4.4:** Under the DD structure, the optimal profit functions of manufacturers and retailers are then given by:

$$J_{Mi}^{DD} = \frac{\rho_{Mi}[2(1 - \phi_i^{DD})\rho_{Mi} - \phi_i^{DD}\rho_{RI}]}{2(1 - \phi_i^{DD})^2(\lambda + \delta)} G_{i0} - \frac{\beta \rho_{MI}\rho_{RI}}{(1 - \phi_3^{DD})(\lambda + \delta)} G_{3-i0} + \frac{\rho_{RI}[2(1 - \phi_i^{DD})\rho_{MI} - \phi_i^{DD}\rho_{RI}]}{2(1 - \phi_i^{DD})^2\lambda(\lambda + \delta)} \Omega_i^{DD} - \frac{\beta \rho_{MI}\rho_{RI}}{(1 - \phi_3^{DD})(\lambda + \delta)} \Omega_3^{DD-i} + \frac{\theta^2 \rho_{MI}(\rho_{RI} + \rho_{RI-3})}{2\lambda} - \frac{(A_i^{DD})^2}{\lambda}$$

$$J_{RI}^{DD} = \frac{\rho_{RI}^2}{2(1 - \phi_i^{DD})(\lambda + \delta)} G_{i0} - \frac{\beta \rho_{RI}\rho_{RI-3}}{(1 - \phi_3^{DD})(\lambda + \delta)} G_{3-i0} + \frac{\rho_{RI}^2}{2(1 - \phi_i^{DD})(\lambda + \delta)} \Omega_i^{DD} - \frac{\beta \rho_{RI}\rho_{RI-3}}{(1 - \phi_3^{DD})(\lambda + \delta)} \Omega_3^{DD-i} + \frac{\theta^2 \rho_{RI}(\rho_{RI} + 2\rho_{RI-3})}{2\lambda}.$$

### 4.2. The optimal promotion strategies under the DC structure

Under the DC structure, the decentralized system consists of manufacturer 1 and retailer 1 to maximize the present values of their own profits. Also, the manufacturer 2 integrates with the retailer 2 to maximize the total profits of centralized system. With a common discount rate $\lambda > 0$, the objective functions of supply chain members are expressed:

$$J_{M1} = \int_0^\infty e^{-\lambda t} \Pi_{M1} dt, \quad J_{R1} = \int_0^\infty e^{-\lambda t} \Pi_{R1} dt,$$

$$J_2 = \int_0^\infty e^{-\lambda t} \Pi_2 dt.$$

(28)

To obtain the optimal VJP and HJP efforts and the participate rate $\phi_1$ under the DC structure, we solve the optimization problems for the objective functions $J_{M1}, J_{R1}$ and $J_{R2}$ subject to the dynamic equation (1), respectively. Similar to the proof of Proposition 4.1, we get the following propositions to characterize the equilibrium decisions under the DC structure.

**Proposition 4.5:** Under the DC channel structure, the optimal HJP and VJP efforts for supply chain members are given by:

$$A_1^{DC} = \frac{\rho_{M1}\rho_{R1}}{1 - \phi_1^{DC}} + \frac{\phi_1^{DC} \rho_{R1}^2}{2(1 - \phi_1^{DC})^2(\lambda + \delta)} + \frac{\alpha \beta \rho_{M1}(\rho_{M2} + \rho_{R2})}{\lambda + \delta},$$

$$U_1^{DC} = \frac{\rho_{R1}}{1 - \phi_1^{DC}} G_1^{DC} = \theta \rho_{R1},$$

$$A_2^{DC} = \frac{(\rho_{M2} + \rho_{R2})^2}{2(\lambda + \delta)} + \frac{\alpha \beta \rho_{R1}(\rho_{M2} + \rho_{R2})}{1 - \phi_1^{DC}},$$

$$U_2^{DC} = (\rho_{R2} + \rho_{M2}) G_2^{DC} = \theta (\rho_{M2} + \rho_{R2})$$

the participate rate of manufacturer 1 is:

$$\phi_1^{DC} = \begin{cases} \frac{2\rho_{M1} - \rho_{R1}}{2\rho_{M2} + \rho_{R1}} \quad \frac{1}{2} \leq \frac{\rho_{M1}}{\rho_{R1}} \\ 0, \quad 0 < \frac{\rho_{M1}}{\rho_{R1}} < \frac{1}{2}. \end{cases}$$

(30)

**Proposition 4.6:** At the equilibrium solutions, for the DC structure, the optimal brand goodwill for product $i$ is satisfied as follows:

$$G_i^{DC}(t) = \frac{1}{\delta} \Omega_i^{DC} + (G_{i0} - \frac{1}{\delta} \Omega_i^{DC})e^{-\lambda t}.$$  

(31)

The steady state of brand goodwill is

$$\lim_{t \to \infty} G_i^{DC}(t) = \frac{1}{\delta} \Omega_i^{DC}.$$

(32)

where $\Omega_i^{DC} = A_i^{DC} - \alpha A_{3-i}^{DC}$. 
**Proposition 4.7:** Under the DC structure, the optimal profit functions of manufacturers and retailers are given by:

\[
J_{M1}^{DC} = \frac{\rho_{R1}[2(1 - \phi_1^{DC})\rho_{M1} - \phi_1^{DC} \rho_{R1}]}{2(1 - \phi_1^{DC})(\lambda + \delta)} G_{10} - \frac{\beta \rho_{M1} (\rho_{M2} + \rho_{R2})}{\lambda + \delta} G_{20} + \rho_{R1} (2(1 - \phi_1^{DC}) \rho_{M1} - \phi_1^{DC} \rho_{R1}) \Omega_1^{DC} - \frac{\beta \rho_{M1} (\rho_{M2} + \rho_{R2})}{\lambda + \delta} \Omega_2^{DC} + \frac{\theta^2 (\rho_{M2} + \rho_{R2}) (\rho_{M2} + \rho_{R2} + 2 \rho_{R1})}{2\lambda} - \frac{(A_1^{DC})^2}{2\lambda}.
\]

**Remark 4.2:** It is worth mentioning that, since there is a period of time between advertising investment and the advertising effect, the lagged effect are commonly occurred in the process of advertising (Aravindakshan & Naik, 2015). Accordingly, the phenomena of lagged effect, if not fully considered, might inevitably influence the performance of supply chain and will damage the benefits of firm. Then, when studying the dynamic promotion strategy problems, the lagged effect is necessary to be taken into the model. Moreover, as the other common phenomenon, the stochastic noise is often occurred in the process of promotion programs. The stochastic noise has two ways, that is, internal and external noise. In general, the internal random fluctuation originates in the industrial background or the humanitarian factor and the external noise originates in one or more uncontrollable variables of enterprises. Therefore, it is necessary to handle the stochastic noise to perfect the performance of firms. Motivated by the results in Prasad and Sethi (2004), Marinelli (2007), and Raman (2006), we are now researching into the stochastic optimal control problems for dynamic joint promotion programs in the multiple competing supply chain. The corresponding results will appear in the near future.

**4.3. Comparison between DD and DC structures**

Analysing the results of two structures, we utilize the conservations demonstrated in Proposition 4.8.

**Proposition 4.8:** Comparison of equilibrium decisions obtained under the DD and DC structures, we give:

(i) Each manufacturer’s national advertising effort is higher under the DC structure. Namely, \(A_1^{DC} > A_1^{DD}\).

(ii) Under the DC structure, if the brand goodwill is higher, then the retailer’s local advertising effort is higher. Namely, if \(G_1^{DC} > G_1^{DD}\) holds, then \(U_1^{DC} > U_1^{DD}\).

(iii) Each retailer’s HJP effort under the DC structure is no less than it under the DD structure. Namely, \(B_1^{DC} = B_1^{DD}\) and \(B_2^{DC} > B_2^{DD}\).

**Proof:** The extra national advertising effort of each manufacturer accrued from the DD and DC structures can be expressed by \(\Delta A_i = A_i^{DC} - A_i^{DD}\). Here, the national advertising effort \(A_i^{DD}\) and \(A_i^{DC}\) have been shown in Equations (25) and (29), respectively, and then we get the extra national advertising efforts.

\[
\Delta A_1 = \frac{\alpha \beta \rho_{M1} \rho_{R2}}{2(\lambda + \delta)} > 0, \quad \Delta A_2 = \frac{\alpha \beta \rho_{M1} \rho_{R2}}{2(1 - \phi_1^{DD})(\lambda + \delta)} + \frac{(1 - \phi_2^{DD}) (\rho_{M2} - \phi_2^{DD} \rho_{R2})^2 + (1 - \phi_2^{DD}) \rho_{R2}^2}{2(1 - \phi_1^{DD}) (\lambda + \delta)} > 0.
\]

Similarly, the extra local advertising efforts are given

\[
U_1^{DC} - U_1^{DD} = \frac{\rho_{R1}}{1 - \phi_1} \frac{G_1^{DC} - G_1^{DD}}{\sqrt{G_1^{DC}} + \sqrt{G_1^{DD}}}, \quad U_2^{DC} - U_2^{DD} = \frac{(\rho_{M2} + \rho_{R2})}{2} \frac{G_2^{DC} - G_2^{DD}}{\sqrt{G_2^{DC}} + \sqrt{G_2^{DD}}} + \frac{\rho_{R2}}{2} \sqrt{G_2^{DC}}.
\]

It is clearly shown that, for \(G_1^{DC} > G_1^{DD}\), we have \(U_1^{DC} > U_1^{DD}\). Otherwise, \(U_1^{DC} < U_1^{DD}\) holds. However, only the condition \(G_2^{DC} > G_2^{DD}\) is satisfied, then \(U_2^{DC} > U_2^{DD}\) holds.

Finally, the extra HJP efforts are obtained:

\[
B_1^{DC} = B_1^{DD}, \quad B_2^{DC} - B_2^{DD} = \theta \rho_{M2} > 0.
\]
Proposition 4.8 shows that the manufacturer and the retailer would invest non-decreasing expenditures in national advertising and HJP efforts under the DC structure. That is due to the fact that, taking the whole supply chain as the decision object can increase the marginal revenue of supply chain member and then the optimal national advertising effort and HJP efforts are higher under the DC structure. Moreover, the local advertising investment depends on not only the margin profits of manufacturer and retailer, but also the brand goodwill of product. To be specific, if the brand goodwill of product is higher under the DC structure, then the retailers will invest more expenditures in local advertising activity. And if the brand goodwill of product 1 is lower under the DC structure, then local advertising of retailer 1 is expected to decline but the retailer 2 still might invest more advertising expenditures.

We only analyse the extra profit of manufacturer 1, retailer 1 and supply chain 2 here in a special scenario, where the each manufacturer is willing to pay for the VJP expenditures of the downstream retailer. The following proposition characterizes the comparison result.

**Proposition 4.9:**
1. The profit of retailer 1 is higher under the DC structure if \( X > 0 \), with \( Y = -((βρ_{P1} ρ_{P2}/(2(λ + δ)))G_{20} + (((ρ_{P1} ρ_{P2}/(2(λ + δ)))\Delta Ω_1 - ((β ρ_{P1} ρ_{P2}/(2(λ + δ)))\Delta Ω_2 - (β ρ_{P1} ρ_{P2}/(2(λ + δ)))Ω_2^2 + (θ^2 ρ_{M1} ρ_{M2}/λ) - (1/2λ)Δ_1^DC(A_{11}^DC + A_{12}^DD)\).

2. The profit of manufacturer 1 is higher under the DC structure if \( Y < 0 \), with \( Z = -((β ρ_{P1} ρ_{P2}/(2(λ + δ)))G_{20} + (((2ρ_{P1} + ρ_{P1}))/(8λ(λ + δ)))\Delta Ω_1 - ((β ρ_{P1} ρ_{P2}/(2(λ + δ)))\Delta Ω_2 - (β ρ_{P1} ρ_{P2}/(2(λ + δ)))Ω_2^2 + (θ^2 ρ_{M1} ρ_{M2}/λ) - (1/2λ)Δ_1^DC(A_{11}^DC + A_{12}^DD)\).

3. The profit of supply chain 2 is higher under the DC structure if \( Z > 0 \), with \( Z = ρ_{P2}^2/(8(λ + δ))G_{20} + β(ρ_{P2} + ρ_{P2})(2ρ_{P1} + ρ_{P1})/(2(λ + δ))\ Δ_1^Ω_1 + ((θ^2 ρ_{M2} ρ_{M2} + ρ_{M2})/(2(λ + δ))\ Δ_2^Ω_2 + (θ^2 ρ_{M1} ρ_{M2}/λ) - (1/2λ)Δ_2^DC(A_{22}^DC + A_{22}^DD)\).

**Remark 4.3:** The other difficulties of this research are, in the multiple competing supply chain, which structure is benefit for each member. Proposition 9 shows that the choice of structure is closed related to the several parameters \( β, ρ_{P1}, ρ_{P1}, λ, δ \) and \( θ \). When \( X > 0, Y > 0, Z > 0 \) hold, each member is willing to accept the DC structure. Otherwise, we cannot get more intuitive results, then numerical analysis is given to show the choice of structure.

### 4.4. Numerical analysis

**Example 4.1:** Numerical analysis of \( ΔΠ_{R1}, ΔΠ_{M1} \) and \( ΔΠ_2 \).

Comparing the DD and DC structures, the effect of joint promotion on the extra benefit of retailer 1 (manufacturer 1 and supply chain 2) depends on the sign of the expression \( X (Y \) and \( Z) \). This expression is complex and it is related to the initial brand goodwill of product 2, parameters \( α \) and \( β \). To derive further insights, we resort to numerical analyses by varying the values of the parameters \( α, β \in [0, 1] \). The marginal profits are supposed as \( ρ_{P1} = 6, ρ_{P2} = 7, ρ_{P1} = 4 \) and \( ρ_{P2} = 3 \). The initial values of brand goodwill are \( G_{10} = G_{20} = 1.5 \) and the HJP parameter \( θ = 1.75 \). This shows that the expression \( X (Y \) and \( Z) \) can take positive and negative values depending on the parameters \( α, β \). Figures 1–3 show the extra profits for the retailer 1, manufacturer 1 and supply chain 2. We can see that \( X, Y \) and \( Z \) take negative values in most cases, except for very low effectiveness of the competitor’s VJP effort on demand (\( β \)).

For the higher HJP effect (\( θ \)), when the effect of national advertising is fixed, the advertising activities of
the retailer has a sensitive effect on the product demand, and then the DD structure will be more profitable for the supply chain members.

Example 4.2: The following parameter values are supposed in this example: $\lambda = 0.1$, $\delta = 0.1$, $\alpha = 0.1$, $\beta = 0.1$, $\theta = 1.2$, $G_1(0) = 50$, $G_2(0) = 40$. The marginal profit values are assumed as $\rho_{M1} = 7$, $\rho_{M2} = 6$, $\rho_{R1} = 3$ and $\rho_{R2} = 4$. Figures 4 and 5 shows the effect of $A_1$ on the profit in the different structures.

Figure 4 shows that (i) whether supply chain members adopt DD or DC structures, a change in the national advertising effort of manufacturer would cause a logarithmic linear tendency in the profit of the competitor; (ii) when the national advertising effort is increasing, the profit of manufacturer increases firstly and then decreases under two structures. If we change the advertising effort of manufacturer 2, similar results are also achieved and we ignore them in this discussion.

5. Conclusions

In this paper, the strategy problems have been investigated for a class of dynamic joint promotions in the multiple supply chains. Here, the dynamic advertising model is allowed to be influenced by HJP and VJP programs. By applying HJB equation, the optimal HJP and VJP strategies, the goodwill of product, the total supply chain profit as well as the optimal promotion cost sharing rate have been obtained under the DD and DC structures. And we provide a managerial vision about structure choice which helps improve the system performance. Looking at two competing supply chains with known marginal profit, when the effect of national advertising is fixed, the advertising investment of the retailer has a sensitive effect on the product demand, and then the DD structure will be more profitable for the supply chain members.

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