Neutrinoless Double Beta Decay and $H^{\pm\pm} \rightarrow l'^{\pm}l'^{\pm}$ Decays in the Higgs Triplet Model

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Abstract

The connection between the neutrinoless double beta ($\beta\beta_{0\nu}$) decay effective Majorana mass, $|M_{ee}|$, and the branching ratios of the decays $H^{\pm\pm} \rightarrow l^{\pm}l'^{\pm}$, $l, l' = e, \mu$, of the doubly charged Higgs boson $H^{\pm\pm}$ is analysed within the Higgs Triplet Model of neutrino mass generation. We work in the version of the model with explicit breaking of the total lepton charge conservation, in which $H^{\pm\pm} \rightarrow l^{\pm}l'^{\pm}$, $l, l' = e, \mu, \tau$, are the dominant decay modes of $H^{\pm\pm}$. It is assumed also that $H^{\pm\pm}$ are relatively light so that they can be produced at LHC and the branching ratios of interest measured. Taking into account the current and prospective uncertainties in the values of the neutrino mixing parameters most relevant for the problem studied - the atmospheric neutrino mixing angle $\theta_{23}$ and the CHOOZ angle $\theta_{13}$, and allowing the lightest neutrino mass and the CP violating Dirac and Majorana phases to vary in the intervals $[0, 0.3 \text{ eV}]$ and $[0, 2\pi]$, respectively, we derive the regions of values of $\text{BR}(H^{\pm\pm} \rightarrow e'^{\pm}e'^{\pm})$ and $\text{BR}(H^{\pm\pm} \rightarrow e'^{\pm}\mu'^{\pm})$ for which $|M_{ee}| \geq 0.05 \text{ eV}$, or $|M_{ee}| < 0.05 \text{ eV}$. This is done for neutrino mass spectrum with normal ordering, inverted ordering and in the case when the type of the spectrum is not known, and i) without using the possible additional data on $\text{BR}(H^{\pm\pm} \rightarrow \mu'^{\pm}\mu'^{\pm})$, ii) using prospective data on $\text{BR}(H^{\pm\pm} \rightarrow \mu'^{\pm}\mu'^{\pm})$. In the latter case results for several values of $\text{BR}(H^{\pm\pm} \rightarrow \mu'^{\pm}\mu'^{\pm})$ are presented.

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1 Introduction

Determining the origin of neutrino masses and mixing is one of the major challenges of future research in neutrino physics. It is well known that the existence of nonzero neutrino masses can be related to the presence of more complicated Higgs sector in the Standard Theory, involving additional Higgs fields beyond the single doublet field. Actually, it was realised a long time ago \[1, 2, 3\] that a Majorana mass term for the left-handed (LH) flavour neutrino fields can be generated by $SU(2)_L \times U(1)_Y$ invariant Yukawa couplings of two lepton doublet fields to a Higgs triplet field, carrying two units of the weak hyperchange, $|Y| = 2$. Such a Higgs field has an electrically neutral, singly charged and doubly charged components. The Majorana mass term for the active flavour neutrinos arises when the neutral component of the Higgs triplet field acquires a nonzero vacuum expectation value (vev), breaking the $SU(2)_L \times U(1)_Y$ symmetry. There are several possible realisations of this scenario. The realisation in which the global $U(1)_L$ symmetry associated with the conservation of the total lepton charge $L$ is broken only spontaneously by the Higgs triplet vev \[4\], was ruled out by the LEP data on the invisible decay width of the $Z^0$-boson. If, however, the $U(1)_L$ symmetry is broken explicitly in a manner that leads to a nonzero vacuum expectation value of the neutral component of the Higgs triplet field (see, e.g. \[5, 6\]) one obtains a viable model of neutrino mass generation. This model has been investigated in detail recently \[7, 8, 9\] and was shown to have a rich and physically interesting phenomenology owing to the fact that i) the couplings of the doubly and singly charged Higgs fields to the flavour neutrinos and charged leptons are proportional to the elements of the Majorana mass matrix of the (flavour) neutrinos, $M_{
u L}$, and can be relatively large, and that ii) the physical doubly charged and singly charged Higgs fields, $H^{\pm \pm}$ and $H^{\pm}$, can have masses in the range from $\sim 100$ GeV to $\sim 1$ TeV and thus can, in principle, be produced and observed at LHC. Point i) implies that the indicated couplings are determined essentially by the elements of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix \[10\] and by the neutrino masses. In \[7, 8, 9\] it was shown that by studying the decays $H^{\pm \pm} \rightarrow l^\pm l'^\pm$, $l, l' = e, \mu, \tau$, it might be possible to obtain information on the absolute neutrino mass scale, on the type of neutrino mass spectrum (which can be, e.g. normal hierarchical (NH), inverted hierarchical (IH) and quasi-degenerate (QD)), and on the Majorana CP violating phases \[11\] present in the neutrino mixing matrix.

In the present article we investigate the possibility to use the information on the neutrino mass spectrum and the Majorana CP violating phases from the measurements of the $H^{\pm \pm} \rightarrow l^\pm l'^\pm$ decay branching ratios, $\text{BR}(H^{\pm \pm} \rightarrow l^\pm l'^\pm)$, $l, l' = e, \mu$, in order to obtain predictions for the effective Majorana mass in neutrinoless double beta ($\beta\beta_{0\nu}$) decay, $|M_{ee}|$ (see, e.g. \[12\]). Our study is motivated by the fact that most probably the searches for the doubly charged scalars $H^{\pm \pm}$ and the decays $H^{\pm \pm} \rightarrow l^\pm l'^\pm$ will be carried out at LHC before the next generation of $(\beta\beta_{0\nu})$-decay experiments will be operative. Among the different decay channels $H^{\pm \pm} \rightarrow l^\pm l'^\pm$, $l, l' = e, \mu, \tau$, the easier to observe are those with two electrons (positrons), two muons (antimuons), or an electron (positron) and a muon (antimuon), $e^\pm e^\pm$, $\mu^\pm \mu^\pm$ and $e^\pm \mu^\pm$, in the final state. If the mass of $H^{\pm \pm}$ does not exceed approximately 400 GeV, the branching ratios of the $H^{\pm \pm}$ decays into
$e^\pm e^\pm$, $e^\pm \mu^\pm$ and $\mu^\pm \mu^\pm$ can be measured at LHC with a few percent error [13]. We will show that if the doubly charged Higgs bosons $H^{\pm\pm}$ will be discovered at LHC and at least the three branching ratios $\text{BR}(H^{\pm\pm} \to e^\pm e^\pm)$, $\text{BR}(H^{\pm\pm} \to e^\pm \mu^\pm)$ and $\text{BR}(H^{\pm\pm} \to \mu^\pm \mu^\pm)$ will be measured with a sufficient accuracy, one can obtain unique information on the $(\beta\beta)_{0\nu}$-decay effective Majorana mass $|M_{ee}|$. This information will be extremely important, in particular, for the upcoming next generation of $(\beta\beta)_{0\nu}$-decay experiments.

## 2 The Higgs Triplet Model

In the Higgs Triplet Model (HTM) [1 2 3] a $I = 1, Y = 2$ complex $SU(2)_L$ triplet of Higgs scalar fields is added to the Standard Model (SM) Lagrangian. In the $2 \times 2$ representation, the Higgs triplet field has the form:

$$
\Delta = \begin{pmatrix}
\Delta^+/\sqrt{2} & \Delta^{++} \\
\Delta^0 & -\Delta^+/\sqrt{2}
\end{pmatrix}
$$

(1)

where $\Delta^0$, $\Delta^+$ and $\Delta^{++}$ are neutral, singly charged and doubly charged scalar fields. In the flavour basis in which the charged lepton mass matrix is diagonal we are going to use throughout this article, a Majorana mass term for the LH flavour neutrino fields can be generated (without the introduction of a right-handed neutrino fields) by the $SU(2)_L \times U(1)_Y$ gauge invariant Yukawa interaction:

$$
\mathcal{L} = h_{\nu l} \psi_{\nu L}^T C (i\tau_2) \Delta \psi_{\nu L} + h.c.
$$

(2)

Here $h_{\nu l} = h_{\nu l}'$, $l', l = e, \mu, \tau$, are complex Yukawa couplings forming a symmetric matrix $h$, $C$ is the charge conjugation matrix, $\tau_2$ is a Pauli matrix for $SU(2)_L$ indices, and $\psi_{\nu L}^T = (\nu_L \ l_L)^T$, $l = e, \mu, \tau$, is the LH lepton doublet field. A non-zero triplet vacuum expectation value, $\langle \Delta^0 \rangle \equiv v_\Delta \neq 0$, gives rise to a Majorana mass matrix $M$ for the LH flavour neutrino fields $\nu_{\nu L}$:

$$
M_{\nu l} = 2 h_{\nu l} \langle \Delta^0 \rangle = \sqrt{2} h_{\nu l} v_\Delta, \quad l', l = e, \mu, \tau.
$$

(3)

The requisite $v_\Delta \neq 0$ arises from the minimisation of the most general $SU(2) \times U(1)_Y$ invariant Higgs potential [5 6]:

$$
V = m^2 (\Phi^\dagger \Phi) + \lambda_1 (\Phi^\dagger \Phi)^2 + M_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) + \lambda_2 \left[\text{Tr}(\Delta^\dagger \Delta)^2 + \lambda_3 \text{Det}(\Delta^\dagger \Delta)\right]
+
\lambda_4 (\Phi^\dagger \Phi) \text{Tr}(\Delta^\dagger \Delta) + \lambda_5 (\Phi^\dagger \tau_1 \Phi) \text{Tr}(\Delta^\dagger \tau_1 \Delta) + \left(\frac{1}{\sqrt{2}} \mu (\Phi^T i\tau_2 \Delta^\dagger \Phi) + h.c\right),
$$

(4)

$\Phi^T = (\phi^+ \ phi^0)^T$ being the SM Higgs doublet field. In eq. (4), $M_\Delta^2 > 0$ is the common mass of the triplet scalars. The choice $m^2 < 0$ ensures that $\langle \phi^0 \rangle = v/\sqrt{2} \neq 0$, which breaks spontaneously $SU(2) \times U(1)_Y$ to $U(1)_Q$. In the model of Gelmini-Roncadelli [4] the term $\mu (\Phi^T i\tau_2 \Delta^\dagger \Phi)$ is absent, which leads for $M_\Delta^2 < 0$ to a spontaneous breaking of the global $U(1)_L$ symmetry associated with the conservation of the total lepton number. The resulting Higgs spectrum contains a massless Goldstone boson - the triplet scalar Majoron, $J$, and another light scalar, $H^0$. The decay $Z^0 \to H^0 J$ would give too large a contribution to the invisible decay width of the $Z^0$-boson.
and this model was excluded by the LEP data. The inclusion of the term $\mu (\Phi^T i\tau_2 \Delta^\dagger \Phi)$ explicitly breaks the lepton number conservation when $\Delta$ is assigned two units of the total lepton charge $L$, and therefore avoids the presence of a Goldstone boson - the Majoron, in the model [2, 3]. Thus, the scalar potential in eq. (1) together with the triplet Yukawa interaction of eq. (2) lead to a phenomenologically viable model of neutrino mass generation.

The expression for $v_\Delta$ resulting from the minimisation of the potential $V$, eq. (1), reads:

$$v_\Delta \simeq \frac{\mu v^2}{2M_\Delta^2 + (\lambda_4 + \lambda_5)v^2}, \quad \text{for } v_\Delta \ll v. \quad (5)$$

In the scenario of relatively light triplet scalars within the discovery reach of the LHC we will be interested in, one has $M_\Delta \approx v$ and eq. (5) leads to $v_\Delta \approx \mu$. In extensions of the HTM, the term $\mu (\Phi^T i\tau_2 \Delta^\dagger \Phi)$ can arise in various ways: i) through the vev of a Higgs singlet field [14, 15]; ii) can be generated at higher orders in perturbation theory [6]; or iii) can appear in the context of theories with extra dimensions [5, 16].

An upper limit on $v_\Delta$ can be obtained from considering its effect on the parameter $\rho = M_H^2/M_Z^2 \cos^2 \theta_W$. In the SM, $\rho = 1$ at tree-level, while in the HTM one has

$$\rho \equiv 1 + \delta \rho = 1 + \frac{2x^2}{1 + 4x^2}, \quad x \equiv v_\Delta/v. \quad (6)$$

The measurement $\rho \approx 1$ leads to the bound $v_\Delta/v \lesssim 0.03$, or $v_\Delta < 8$ GeV. At the 1-loop level $v_\Delta$ must be renormalised and explicit analyses lead to bounds on its magnitude similar to those derived from the tree-level analysis [17].

The HTM has seven physical Higgs scalar particles ($H^{++}, H^{--}, H^+, H^-, H^0, A^0, h^0$). The doubly charged Higgs field $H^{++}$ coincides with the triplet scalar field $\Delta^{++}$. The remaining Higgs mass-eigenstate fields are in general mixtures of the doublet and triplet fields. The corresponding mixing parameter is proportional to the ratio of triplet and doublet vevs, $v_\Delta/v$, and hence is small even if $v_\Delta$ assumes its largest value of a few GeV. Therefore $H^+, H^0, A^0$ are predominantly composed of the triplet fields, while $h^0$ is predominantly composed of the doublet field and plays the role of the SM Higgs boson. The masses of $H^{\pm\pm}, H^{\pm}, H^0, A^0$ are of order $M_\Delta$ with splittings of order $\lambda_5 v$. For $M_\Delta < 1$ TeV of interest for direct searches for the Higgs bosons at the LHC, the couplings $h_{\ell\ell}$ are constrained to be $O(0.1)$ or less by a variety of processes such as $\mu \rightarrow eee, \tau \rightarrow lll$ etc. These constraints are reviewed in [18, 19].

In this article we will be interested, in particular, in the decays of $H^{\pm\pm}$ into a pair of same-sign charged leptons, $H^{\pm\pm} \rightarrow l^\pm l'^\pm$, $l, l' = e, \mu, \tau$, which give clear signals even in hadron colliders like the LHC. These decays are important not only for the searches of $H^{\pm\pm}$, but also because of the very interesting possibility that their lepton flavour dependence can be directly related to the Majorana mass matrix of the LH flavour neutrinos.

In our analysis we will assume that $M_{H^{\pm\pm}} \leq M_{H^{\pm}}$ and $v_\Delta \lesssim 1$ MeV. Under these conditions the decay $H^{\pm\pm} \rightarrow H^{\pm} W^{\pm}$ is forbidden, while the decay $H^{\pm\pm} \rightarrow W^\pm W^\pm$ is sufficiently strongly
suppressed. In this case the branching ratios of the decays $H^{±±} \rightarrow l^±l^±$ are given by the following simple expressions (see, e.g. [7, 8, 9]):

$$BR_{l^±l^±} \equiv BR(H^{±±} \rightarrow l^±l^±) = \frac{2}{1 + \delta_{ll}} \frac{|h_{l^±}|^2}{\sum_{l^±} |h_{l^±}|^2}$$

(7)

$$= \frac{2}{1 + \delta_{ll}} \frac{|M_{l^±l^±}|^2}{\sum_i m_i^2},$$

(8)

where $\delta_{ll}$ is the Kronecker delta. Note that the branching ratios depend only on the parameters of neutrino mass matrix. The measurement of $BR_{l^±l^±}$ can give significant information on the elements of the neutrino mass matrix $|M_{l^±l^±}|$, and therefore, e.g. on the absolute neutrino mass scale (i.e. lightest neutrino mass), type of neutrino mass spectrum, Majorana CP violating phases in the neutrino mixing matrix, etc.

3 The Neutrino Masses, Mixing and the $(\beta\beta)_{0\nu}$-Decay

We work in the flavour basis in which the mass matrix of the charged leptons is diagonal. As we have shown, in the Higgs triplet model of interest, the LH flavour neutrino fields $\nu_{ll}$ acquire a Majorana mass term. The corresponding Majorana mass matrix $M_{ll}$ is diagonalised with the help of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix [10]:

$$M_{ll} = [U_{PMNS} \text{diag}(m_1, m_2, m_3) U_{PMNS}^T]_{ll},$$

(9)

where $m_j, j = 1, 2, 3$, are the real positive eigenvalues of $M_{ll}$ - the masses of the Majorana neutrinos $\chi_j$ with definite mass. In what follows we will use the standard parametrisation of the PMNS matrix (see, e.g. [20, 21]):

$$U_{PMNS} \equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \text{diag}(1, e^{i\alpha_{21}}, e^{i\alpha_{31}}),$$

(10)

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$, the angles $\theta_{ij} = [0, \pi/2]$ ($i < j = 1, 2, 3$), $\delta = [0, 2\pi]$ is the Dirac CP-violating phase, and $\alpha_{21}$ and $\alpha_{31}$ are two Majorana CP-violation phases [11, 22]. The phases $\alpha_{21}$ and $\alpha_{31}$ can vary in the interval $[0, 2\pi]$. It proves useful for our further discussion to define also the difference of the two Majorana phases: $\alpha_{32} \equiv \alpha_{31} - \alpha_{21}$. Let us add that at present we do not have experimental information on $\delta$, $\alpha_{21}$ and $\alpha_{31}$.

The existing neutrino oscillation data [23, 24, 25, 26, 27] allow to determine with a rather good precision the mixing angles and neutrino mass squared differences which drive the solar neutrino and the dominant atmospheric neutrino oscillations, $\sin^2 2\theta_{12}, \Delta m^2_{21}$ and $\sin^2 2\theta_{23}, |\Delta m^2_{31}|(\simeq |\Delta m^2_{32}|)$, and to obtain a rather stringent limit on the CHOOZ angle $\theta_{13}$. In our analysis we will
use the following best fit values of $\sin^2 \theta_{12}$, $\Delta m^2_{21}$, $\sin^2 \theta_{23}$ and $|\Delta m^2_{31}|$ \cite{28,29,30}: 

\begin{align}
\Delta m^2_{21} &= 7.6 \times 10^{-5} \text{ eV}^2, \quad \sin^2 2\theta_{12} = 0.87, \\
|\Delta m^2_{31}| &= 2.4 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{25} = 1. 
\end{align}

(11)  

(12) 

The upper limit on $\sin^2 2\theta_{13}$ obtained in CHOOZ reactor anti-neutrino experiment \cite{26} reads:

$$\sin^2 2\theta_{13} < 0.14.$$  

(13)  

From the global analyses of the neutrino oscillation data one finds (see, e.g. \cite{30}):

$$\sin^2 \theta_{13} < 0.056, \quad 99.73\% \text{ C.L.}$$  

(14)  

The next generation of experiments with reactor $\bar{\nu}_e$, which are under preparation, Double CHOOZ \cite{31}, Daya Bay \cite{32}, RENO \cite{33}, can improve the currently reached sensitivity to the value of $\sin^2 2\theta_{13}$ by a factor of (5-10) (see, e.g. \cite{33}), while future long baseline experiments aim at measuring values of $\sin^2 2\theta_{13}$ as small as $10^{-4}-10^{-3}$ (see, e.g. \cite{35}).

Let us note that the uncertainty in the experimental determination of $\sin^2 2\theta_{23}$ corresponds to a rather large interval of allowed values of $s^2_{23}$ \cite{23}: 0.38 $\leq s^2_{23}$ $\leq$ 0.62. We will take into account this uncertainty in our numerical analysis. It should be added that the accuracy on $\sin^2 2\theta_{23}$ is planned to be improved considerably in future long baseline experiments. The uncertainty in $\sin^2 2\theta_{23}$ is foreseen to be reduced in the T2K experiment \cite{36}, for instance, to $\sin^2 2\theta_{23} > 0.99$ (0.45 $< s^2_{23} <$ 0.55), if the true value of $\sin^2 2\theta_{23} = 1$. As we will see, the correlations between the branching ratios of the decays $H^{\pm \pm}$ $\rightarrow$ $l^{\pm}l'^{\pm}$, BR$_{ll'}$, $l, l'$ = e, $\mu, \tau$, and the effective Majorana mass in neutrinoless double beta (($\beta\beta$)$_{0\nu}$) decay, $|\langle m \rangle|$ $\equiv$ $M_{ee}$, which is the main subject of our study, depend not only on the elements of the neutrino mixing matrix, but also on the type of neutrino mass spectrum and on the absolute scale of neutrino masses.

As is well known, owing to the fact that the sign of $\Delta m^2_{31}$, cannot be determined from the existing data, there are two possible types of neutrino mass spectrum compatible with the data - with normal ordering and with inverted ordering. In the standardly used convention we are also going to employ, the two spectra correspond to:

- $m_1 < m_2 < m_3$, $\Delta m^2_{31} > 0$, normal ordering (NO),
- $m_3 < m_1 < m_2$, $\Delta m^2_{31} < 0$, inverted ordering (IO).

The $\nu$-mass spectrum can be: i) normal hierarchical (NH), $m_1 \ll m_2 < m_3$, with $m_2 \cong \sqrt{\Delta m^2_{21}} \cong 8.8 \times 10^{-3}$ eV, $m_3 \cong \sqrt{\Delta m^2_{31}} \cong 4.9 \times 10^{-2}$ eV; ii) inverted hierarchical (IH), $m_3 \ll m_1 < m_2$, with $m_2 \cong \sqrt{\Delta m^2_{23}} \cong 4.9 \times 10^{-2}$ eV; $m_1 \cong \sqrt{\Delta m^2_{31} - \Delta m^2_{21}} \cong 4.8 \times 10^{-2}$ eV, and iii) quasi-degenerate (QD), $m_1 \cong m_2 \cong m_3$, $m^2_{1,2,3} \gg |\Delta m^2_{31}|$. In the latter case one has: $m_j \gtrsim 0.10$ eV.

\footnote{Varying $\sin^2 \theta_{12}$ in the $3\sigma$ interval of allowed values of $\sin^2 \theta_{12}$ \cite{28,30}, 0.25 $\lesssim \sin^2 \theta_{12} \lesssim$ 0.37, has essentially negligible effect on the results of our analysis.}
The type of neutrino mass spectrum, i.e. \( \text{sgn}(\Delta m^2_{31}) \), can be determined by studying oscillations of neutrinos and antineutrinos, say, \( \nu_\mu \leftrightarrow \nu_e \) and \( \bar{\nu}_\mu \leftrightarrow \bar{\nu}_e \), in which matter effects are sufficiently large. This can be done in long base-line \( \nu \)-oscillation experiments (see, e.g. [35]). If \( \sin^2 2\theta_{13} \gtrsim 0.05 \) and \( \sin^2 2\theta_{23} \gtrsim 0.50 \), information on \( \text{sgn}(\Delta m^2_{31}) \) might be obtained in atmospheric neutrino experiments by investigating the effects of the subdominant transitions \( \nu_\mu(e) \rightarrow \nu_e(\mu) \) and \( \bar{\nu}_\mu(e) \rightarrow \bar{\nu}_e(\mu) \) of atmospheric neutrinos which traverse the Earth [37]. For \( \nu_\mu(e) \) (or \( \bar{\nu}_\mu(e) \)) crossing the Earth core, new type of resonance-like enhancement of the indicated transitions takes place due to the (Earth) mantle-core constructive interference effect (neutrino oscillation length resonance (NOLR)) [38]. For \( \Delta m^2_{31} > 0 \), the neutrino transitions \( \nu_\mu(e) \rightarrow \nu_e(\mu) \) are enhanced, while for \( \Delta m^2_{31} < 0 \) the enhancement of antineutrino transitions \( \bar{\nu}_\mu(e) \rightarrow \bar{\nu}_e(\mu) \) takes place, which might allow to determine \( \text{sgn}(\Delta m^2_{31}) \). If \( \sin^2 2\theta_{13} \) is sufficiently large, the sign of \( \Delta m^2_{31} \) can also be determined by studying the oscillations of reactor \( \bar{\nu}_e \) on distances of \( \sim (20 - 40) \) km [42]. An experiment with reactor \( \bar{\nu}_e \), which, in particular, might have the capabilities to measure \( \text{sgn}(\Delta m^2_{31}) \), was proposed recently in [43] (see also [44]). Information on the type of neutrino mass spectrum can also be obtained in \( \beta \)-decay experiments having a sensitivity to neutrino masses \( \sim \sqrt{|\Delta m^2_{31}|} \approx 5 \times 10^{-2} \) eV [45] (i.e. by a factor of \( \sim 4 \) better than the planned sensitivity of the KATRIN experiment [46], see below).

Direct information on the absolute neutrino mass scale can be derived in \( ^3 \)He \( \beta \)-decay experiments [47, 48, 46]. The most stringent upper bounds on the \( \bar{\nu}_e \) mass were obtained in the Troitzk [48] and Mainz [46] experiments:

\[
m_{\bar{\nu}_e} < 2.3 \text{ eV}, \quad 95\% \text{ C.L.}
\]

We have \( m_{\bar{\nu}_e} \approx m_{1,2,3} \) in the case of the QD \( \nu \)-mass spectrum. The KATRIN experiment [46], which is under preparation, is planned to reach a sensitivity of \( m_{\bar{\nu}_e} \sim 0.20 \) eV, i.e. it will probe the region of the QD spectrum.

The CMB data of the WMAP experiment [49], combined with data from large scale structure surveys (2dFGRS, SDSS), lead to the following upper limit on the sum of neutrino masses (see, e.g. [50]):

\[
\sum_j m_j \equiv \Sigma < (0.4 - 1.7) \text{ eV}, \quad 95\% \text{ C.L.}
\]

Data on weak lensing of galaxies, combined with data from the WMAP and PLANCK experiments, may allow \( \Sigma \) to be determined with an uncertainty of \( \sim 0.04 \) eV [50, 51].

In our analysis we will consider both types of neutrino mass spectrum - with normal and with inverted ordering, as well as the specific cases of normal hierarchical (NH), inverted hierarchical (IH) and quasi-degenerate (QD) spectra. Correspondingly, the lightest neutrino mass \( \min(m_j) \equiv m_1 \) as a consequence of this effect, the corresponding \( \nu_{\mu(e)} \) (or \( \bar{\nu}_{\mu(e)} \)) transition probabilities can be maximal [39] (for the precise conditions of the mantle-core (NOLR) enhancement see [38, 39]). Let us note that the Earth mantle-core (NOLR) enhancement of neutrino transitions differs [38] from the MSW one. The conditions of the Earth mantle-core enhancement [38, 39] also differ [40] from the conditions of the parametric resonance enhancement of the neutrino transitions discussed in the articles [41].
Figure 1: Diagrams for the neutrinoless double beta decay. The diagram (a) is the standard and dominant one, and (b)-(f) are possible but negligible in the HTM.

\( m_0 \), which determines the absolute neutrino mass scale, will be varied in the interval:

\[
0 \leq m_0 \leq 0.3 \text{ eV}, \quad m_0 \equiv \min(m_j), \quad j = 1, 2, 3.
\]  

(17)

As we will show, the results we obtain essentially do not depend on the maximal value \( m_0 \) as long as the latter is not smaller than 0.3 eV. The reason is that for \( m_0 \gtrsim 0.3 \text{ eV} \) (i.e. in the QD region), the branching ratios \( \text{BR}_{\nu l} \) we are interested in practically do not depend on the neutrino masses:

\[
\text{BR}_{\nu l} \approx \frac{2}{3(1 + \delta_{\nu l})} \left| \sum_j U_{\nu j}^* U_{l j} \right|^2, \quad m_0 \gtrsim 0.3 \text{ eV}.
\]  

(18)

Neutrinoless Double Beta decay

In the Higgs triplet model the massive neutrinos are Majorana particles. Determining the nature of massive neutrinos is of fundamental importance for understanding the origin of neutrino masses and, more generally, for understanding the symmetries governing the particle interactions. The existence of massive Majorana neutrinos is associated with non-conservation of the total lepton charge. In this case the neutrinoless double beta decay \((A, Z) \rightarrow (A, Z + 2) + 2e^-\) is allowed (see, e.g. [12, 52, 53]). Assuming that the dominant mechanism for the decay is the exchange of light Majorana neutrinos (Fig. 1(a)), the half-life \( T_{1/2}^{0\nu} \) for the decay is given by

\[
T_{1/2}^{0\nu} = \left( G^{0\nu} |M^{0\nu}|^2 |M_{ee}|^2 \right)^{-1},
\]  

(19)

where \( G^{0\nu} \) is a phase space factor and \( M^{0\nu} \) is the nuclear matrix element of the process. All the dependence of \( T_{1/2}^{0\nu} \) on the neutrino masses and mixing parameters factorises into the \((\beta\beta)_{0\nu}\)-decay
effective Majorana mass $M_{ee}$:

$$|M_{ee}| = |c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13} m_2 e^{i\alpha_{21}} + s_{13}^2 m_3 e^{i(\alpha_{31} - \delta)}|.$$  \hspace{1cm} (20)

The most stringent upper bound on $|M_{ee}|$, $|M_{ee}| < (0.35 - 1.05) \text{ eV}$ was obtained by using the lower limit $T_{1/2}^{0\nu} > 1.9 \times 10^{25}$ yr (90% C.L.) found in the Heidelberg-Moscow $^{76}$Ge experiment [55]. The IGEX collaboration has obtained the result $T_{1/2}^{0\nu} > 1.57 \times 10^{25}$ yr (90% C.L.), from which the limit $|M_{ee}| < (0.33 - 1.35) \text{ eV}$ was derived [56]. A positive $(\beta\beta)_{0\nu}$-decay signal at $> 3\sigma$, corresponding to $T_{1/2}^{0\nu} = (0.69 - 4.18) \times 10^{25}$ yr (99.73% C.L.) and implying $|M_{ee}| = (0.1 - 0.9) \text{ eV}$, is claimed to be observed in [57], while a recent analysis reports evidence at $6\sigma$ of $(\beta\beta)_{0\nu}$-decay with $|M_{ee}| = 0.32 \pm 0.03 \text{ eV}$ [58]. Two experiments, NEMO3 (with $^{100}$Mo, $^{82}$Se, etc.) [59] and CUORICINO (with $^{130}$Te) [60], designed to reach a sensitivity to $|M_{ee}| \sim (0.2 - 0.3) \text{ eV}$, set the limits: $|M_{ee}| < (0.61 - 1.26) \text{ eV}$ [59] and $|M_{ee}| < (0.19 - 0.68) \text{ eV}$ [60] (90% C.L.), where estimated uncertainties in the NME are accounted for. The two upper limits were derived from the experimental lower limits on the half-lives of $^{100}$Mo and $^{130}$Te, $T_{1/2}^{0\nu} > 5.8 \times 10^{23}$ yr (90% C.L.) [59] and $T_{1/2}^{0\nu} > 3.0 \times 10^{24}$ yr (90% C.L.) [60]. Most importantly, a large number of projects aim at a sensitivity to $|M_{ee}| \sim (0.01 - 0.05) \text{ eV}$ [61]: CUORE ($^{130}$Te), GERDA ($^{76}$Ge), SuperNEMO, EXO ($^{130}$Xe), MAJORANA ($^{76}$Ge), MOON ($^{100}$Mo), COBRA ($^{116}$Cd), XMASS ($^{136}$Xe), CANDLES ($^{48}$Ca), etc. These experiments, in particular, will test the positive result claimed in [57].

The predicted value of $|M_{ee}|$ depends strongly on the type of $\nu$–mass spectrum [62, 20], more precisely, on the type of hierarchy neutrino masses obey. The existence of significant and robust lower bounds on $|M_{ee}|$ in the cases of IH and QD spectra [62] (see also [63]), given respectively by $|M_{ee}| \gtrsim 0.01 \text{ eV}$ and $|M_{ee}| \gtrsim 0.03 \text{ eV}$, which lie either partially (IH spectrum) or completely (QD spectrum) within the range of sensitivity of the next generation of $(\beta\beta)_{0\nu}$-decay experiments, is one of the most important features of the predictions of $|M_{ee}|$. At the same time we have $|M_{ee}| \lesssim 5 \times 10^{-3} \text{ eV}$ in the case of NH spectrum [64]. The fact that max$(|M_{ee}|)$ in the case of NH spectrum is considerably smaller than min$(|M_{ee}|)$ for the IH and QD spectrum opens the possibility of obtaining information about the type of $\nu$-mass spectrum from a measurement of $|M_{ee}| \neq 0$ [62]. More specifically, a positive result in the future generation of $(\beta\beta)_{0\nu}$-decay experiments with $|M_{ee}| > 0.01 \text{ eV}$ would imply that the NH spectrum is strongly disfavoured (if not excluded). For $\Delta m_{31}^2 > 0$, such a result would mean that the neutrino mass spectrum is with normal ordering, but is not hierarchical. If $\Delta m_{31}^2 < 0$, the neutrino mass spectrum should be either IH or QD.
4 Prediction for $|M_{ee}|$ from Measurements of $\text{BR}(H^{\pm} \to l'^{\pm}l^\pm)$

In this Section we investigate within the HTM the predictions one can obtain for the $(\beta\beta)_{0\nu}$-decay effective Majorana mass $|M_{ee}|$ by using data on BR$_{\ell\ell}$. The dominant mechanism of $(\beta\beta)_{0\nu}$-decay - the light Majorana neutrino exchange, corresponds to the diagram in Fig. 1(a), the contributions from the diagrams in Figs. 1(b)-(f) being negligible [63, 64].

We use three branching ratios, BR$_{ee}$, BR$_{e\mu}$, and BR$_{\mu\mu}$, in our analysis. The expressions for these branching ratios in terms of neutrino masses, neutrino mixing angles and CP violating phases read (see [7, 8, 9]):

\[
(\sum_i m_i^2) \text{BR}_{ee} = |M_{ee}|^2 = \left| c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13} e^{i\alpha_{21}} + s_{13}^2 m_3 e^{i(\alpha_{31}-2\delta)} \right|^2, \tag{21}
\]

\[
(\sum_i m_i^2) \text{BR}_{e\mu} = 2 \left| c_{12} c_{13} (-s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta}) m_1 
+ s_{12} c_{13} (c_{12} c_{23} - s_{23} s_{13} e^{i\delta}) m_2 e^{i\alpha_{21}} + s_{23} c_{13} s_{13} m_3 e^{i(\alpha_{31}-\delta)} \right|^2, \tag{22}
\]

\[
(\sum_i m_i^2) \text{BR}_{\mu\mu} = \left| (-s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta})^2 m_1 
+ (c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta})^2 m_2 e^{i\alpha_{21}} + s_{23}^2 c_{13}^2 m_3 e^{i\alpha_{31}} \right|^2. \tag{23}
\]

Given the solar and atmospheric neutrino oscillation parameters and the CHOOZ angle, $\sin^2 \theta_{12}$, $\Delta m_{21}^2$, $\sin^2 \theta_{23}$, $|\Delta m_{31}^2|$ and $\theta_{13}$, $|M_{ee}|$ depends on $m_0 \equiv m(m_j)$, $\alpha_{21}$, $\alpha_{31}$, $\delta$ and on the type of neutrino mass spectrum (NO or IO). In the case of spectrum with IO or of QD type, the dependence of $|M_{ee}|$ on $\theta_{13}$ is relatively weak and can be neglected, as long as $\cos 2\theta_{12} \gg \sin^2 \theta_{13}$. In this case $|M_{ee}|$ does not depend on the Majorana phase $\alpha_{31}$ and on the Dirac phase $\delta$. From the measurement of the three observables, BR$_{ee}$, BR$_{e\mu}$, and BR$_{\mu\mu}$, three parameters, say, $m_0$, $\alpha_{21}$ and $\alpha_{31}$, can, in principle, be determined and information on the type of neutrino mass spectrum - with NH, IH or QD can be obtained [7, 8, 9]. This would allow to tightly constrain $|M_{ee}|$. Let us review briefly the predictions for BR$_{ee}$, BR$_{e\mu}$, and BR$_{\mu\mu}$ in the cases of NH, IH and QD spectrum (see also [7,8,9]).

a) NH spectrum, $m_1 \ll m_2 < m_3$.

Using the best fit values of the neutrino oscillation parameters one finds \cite{21} that in this case BR$_{ee}$, BR$_{e\mu}$, and BR$_{\mu\mu}$ can take values in the following intervals: $0 \lesssim \text{BR}_{ee}^{\text{NH}} \lesssim 10^{-2}$, $0 \lesssim \text{BR}_{e\mu}^{\text{NH}} \lesssim 0.08$, $0.16 \lesssim \text{BR}_{\mu\mu}^{\text{NH}} \lesssim 0.31$. We get BR$_{ee}^{\text{NH}} = 0$ for $(\alpha_{32} - 2\delta) = \pi$ and $s_{13}^2 = s_{12}^2 \sqrt{\Delta m_{21}^2/\Delta m_{31}^2} \approx 0.05$, while BR$_{e\mu}^{\text{NH}} = 0$ for $(\alpha_{32} - \delta) = \pi$ and $s_{13}^2 = s_{12}^2 c_{12}^2 \cot^2 \theta_{23} (\Delta m_{21}^2/\Delta m_{31}^2) \approx 6.9 \times 10^{-3}$. The

\footnote{The inequality $\cos 2\theta_{12} \gg \sin^2 \theta_{13}$ is fulfilled for the 2$\sigma$ experimentally allowed ranges of values of $\cos 2\theta_{12}$ and $\sin^2 \theta_{13}$, see, e.g. [23, 30]. If one uses the 3$\sigma$ ranges, one obtains $\sin^2 \theta_{13}/\cos 2\theta_{12} \lesssim 0.22$.}

\footnote{The limiting values quoted in this paragraph are obtained for the best fit values of the neutrinos oscillation parameters and for $\sin^2 2\theta_{13} \lesssim 0.14$.}
minimal and maximal values of $\text{BR}^{\text{NH}}_{\mu\mu}$ depend weakly on $s_{13}^2$. Neglecting this dependence, one obtains a simple expression for the Majorana phase (difference) $\alpha_{32}$ in terms of $\text{BR}^{\text{NH}}_{\mu\mu}$:

$$\cos \alpha_{32} \approx \frac{1}{2} \left( \frac{\Delta m_{31}^2}{\Delta m_{21}^2} \right) \frac{1}{2} \frac{\text{BR}^{\text{NH}}_{\mu\mu} - s_{23}^4}{c_{13}^2 c_{23}^2 s_{23}^2}$$  \hspace{1cm} (24)

b) IH spectrum, $m_3 \ll m_1 < m_2$.

We find very different results in this case: $0.5 c_{13}^4 \cos^2 2\theta_{12} \lesssim \text{BR}^{\text{IH}}_{ee} \lesssim 0.5 c_{13}^4$ or $0.06 \lesssim \text{BR}^{\text{IH}}_{ee} \lesssim 0.5$, $0 \lesssim \text{BR}^{\text{IH}}_{e\mu} \lesssim 0.48$, $0 \lesssim \text{BR}^{\text{IH}}_{\mu\mu} \lesssim 0.14$. One has $\text{BR}^{\text{IH}}_{e\mu} = 0$ for $\delta = 0$, $\alpha_{21} = \pi$ and $s_{13}^2 \approx 0.036$. Now $\text{BR}^{\text{IH}}_{ee}$ exhibits a very weak dependence on $s_{13}^2$. For the Majorana phase $\alpha_{21}$ we obtain in terms of $\text{BR}^{\text{IH}}_{ee}$:

$$\cos \alpha_{21} \approx 1 - \frac{c_{13}^4 - 2\text{BR}^{\text{IH}}_{ee}}{2c_{13}^4 c_{23}^2 s_{21}^2}.$$  \hspace{1cm} (25)

c) QD spectrum, $m_{1,2,3} \gtrsim 0.1$ eV.

It is not difficult to convince oneself that the branching ratios of interest for the QD spectrum to a good approximation can take values in the following intervals: $\cos^2 2\theta_{12}/3 \lesssim \text{BR}^{\text{QD}}_{ee} \lesssim 1/3$ or $0.03 \lesssim \text{BR}^{\text{QD}}_{ee} \lesssim 0.33$, $0 \lesssim \text{BR}^{\text{QD}}_{e\mu} \lesssim 0.46$, $\cos^2 2\theta_{23}/3 \lesssim \text{BR}^{\text{QD}}_{\mu\mu} \lesssim 0.33$. Actually, we have up to small corrections $\text{BR}^{\text{QD}}_{ee} \approx (2/3)\text{BR}^{\text{IH}}_{ee}$. For the Majorana phase $\alpha_{21}$ in this case we get:

$$\cos \alpha_{21} \approx 1 - \frac{c_{13}^4 - 3\text{BR}^{\text{QD}}_{ee}}{2c_{13}^4 c_{23}^2 s_{21}^2}.$$  \hspace{1cm} (26)

Given $\alpha_{21}$ and a sufficiently large $s_{13}$, information about the Dirac phase $\delta$ and the Majorana phase $\alpha_{31}$ can be obtained from the knowledge of $\text{BR}^{\text{QD}}_{e\mu}$ and $\text{BR}^{\text{QD}}_{\mu\mu}$. If, however, a stringent limit on $s_{13}$ will be obtained, $\alpha_{31}$ can be determined using $\text{BR}^{\text{QD}}_{\mu\mu}$ and the knowledge of $\alpha_{21}$.

It is clear from the above simple analysis that the measurement of the three branching ratios $\text{BR}_{ee}$, $\text{BR}_{e\mu}$, and $\text{BR}_{\mu\mu}$ would provide information about the type of neutrino mass spectrum and the Majorana phases. If, for instance, it is experimentally established that $\text{BR}_{ee} > 0.01$, the neutrino mass spectrum of NH type will be excluded. The spectrum can either be of IH or QD type. If in addition $\text{BR}_{\mu\mu}$ is determined to be $\text{BR}_{\mu\mu} > 0.14$, the IH spectrum will be ruled out. If, however, the neutrino mass spectrum will turn out to be QD, it will be very difficult (if not practically impossible) to distinguish between the spectrum with NO and that with IO, i.e. to get information about the sign of $\Delta m_{31}^2$.

Consider next the more general case of $m_0$ having an arbitrary value. First, let us set $\theta_{13} = 0$ for simplicity. We will consider the case of $\theta_{13} \neq 0$ later. For $\theta_{13} = 0$, the main uncertainty in the prediction of $|M_{ee}|$ comes from the lack of knowledge of $m_0$ and $\alpha_{21}$. Note that in this case $\text{BR}_{ee}$ and $\text{BR}_{e\mu}$ are independent of $\alpha_{31}$, similarly to $M_{ee}$\footnote{BR$_{ee}$ is independent of $\alpha_{31}$ as well, but this mode is more difficult to measure than the two modes we are discussing.}. Knowing these two branching ratios allows
to determine \( m_0 \) and \( \cos \alpha_{21} \). In the case of spectrum with NO we have for the lightest neutrino mass:

\[
m_1^2 = \frac{(\Delta m_{21}^2 + \Delta m_{31}^2)(2c_{23}^2 BR_{ee} + BR_{e\mu}) - 2c_{12}^2 c_{23}^2 \Delta m_{21}^2}{2c_{23}^2 - 6c_{23}^2 BR_{ee} - 3BR_{e\mu}}, \tag{27}
\]

\[
\cos \alpha_{21} = \frac{m_1}{\sqrt{m_1^2 + \Delta m_{21}^2}} \pm \frac{\Delta m_{21}^2 (2c_{12}^2 c_{23}^2 BR_{ee} - s_{12}^2 BR_{e\mu}) - m_1^2 BR_{e\mu}}{2c_{12}^2 s_{12}^2 m_1 \sqrt{m_1^2 + \Delta m_{21}^2} (BR_{e\mu} + 2BR_{ee} c_{23}^2)}. \tag{28}
\]

Equation (27) is valid also for the second to lightest neutrino mass \( m_1 \) in the case of spectrum with IO. The expression for \( \cos \alpha_{21} \) obviously cannot be used to determine \( \cos \alpha_{21} \) for \( m_1 = 0 \): for \( \theta_{13} = 0 \) and \( m_1 = 0 \), neither \( BR_{ee} \) nor \( BR_{e\mu} \) depend on \( \alpha_{21} \). In the case of spectrum with IO (inverted ordering) we obtain:

\[
m_3^2 = \frac{(2\Delta m_{23}^2 - \Delta m_{31}^2)(2c_{23}^2 BR_{ee} + BR_{e\mu}) - 2c_{23}^2 (\Delta m_{23}^2 - c_{12}^2 \Delta m_{21}^2)}{2c_{23}^2 - 6c_{23}^2 BR_{ee} - 3BR_{e\mu}}, \tag{29}
\]

\[
\cos \alpha_{21} = 1 - \frac{BR_{e\mu}}{2c_{12}^2 s_{12}^2 (BR_{e\mu} + 2BR_{ee} c_{23}^2)} + O \left( \frac{\Delta m_{21}^2}{\Delta m_{23}^2} \right). \tag{30}
\]

As can be expected, for \( m_1^2 \gg \Delta m_{21}^2 \), the expression for \( \cos \alpha_{21} \) in the case of spectrum with NO coincides with that for spectrum with IO. Using eq. (27), we get a universal expression for \(|M_{ee}|\) valid for both types of spectrum - with NO and IO and any hierarchy between neutrino masses:

\[
|M_{ee}|^2 = \left( \sum_i m_i^2 \right) BR_{ee} = \frac{2c_{23} \Delta m_{21}^2 - 2c_{23}^2 (3s_{12}^2 - 1) \Delta m_{21}^2}{2c_{23}^2 - 6c_{23}^2 BR_{ee} - 3BR_{e\mu}} BR_{ee} \tag{31}
\]

\[
\simeq \frac{\text{sgn}(\Delta m_{21}^2) \times BR_{ee}}{1 - 3BR_{ee} - 3BR_{e\mu}} \times 2.4 \times 10^{-3} \text{eV}^2, \tag{32}
\]

where in the last equation we have used the best fit value of \( \theta_{23} \) and have neglected the term \( \sim (3s_{12}^2 - 1)\Delta m_{21}^2/\Delta m_{23}^2 \). Note that the denominator in the expression for \(|M_{ee}|\), eq. (31), does not go through zero since we have:

\[
2c_{23}^2 BR_{ee} + BR_{e\mu} = \frac{2c_{23}^2 (m_1^2 + s_{12}^2 \Delta m_{21}^2)}{3m_1^2 + \Delta m_{21}^2 + \Delta m_{23}^2}, \quad \text{NO spectrum}, \tag{33}
\]

\[
2c_{23}^2 BR_{ee} + BR_{e\mu} = \frac{2c_{23}^2 (m_2^2 + \Delta m_{23}^2)}{3m_3^2 + 2\Delta m_{23}^2}, \quad \text{IO spectrum}, \tag{34}
\]

where we have neglected terms \( \sim (\Delta m_{21}^2/\Delta m_{23}^2) \) in the second equation. We see that in the QD region, where \(|M_{ee}|\) has a relatively large value, one has \( (2c_{23}^2 BR_{ee} + BR_{e\mu}) \cong (2/3)c_{23}^2 (1 + \Delta m_{23}^2/(3m_0^2)) \).

It follows from eq. (32) that \(|M_{ee}| > 0.05 \text{eV} \simeq \sqrt{\Delta m_{21}^2} |\) can be predicted without the knowledge of \( \text{sgn}(\Delta m_{31}^2) \), if the collider experiments show that the branching ratios \( BR_{ee} \) and \( BR_{e\mu} \) satisfy

\[
- \frac{4c_{23}^2}{3} BR_{ee} + \frac{2c_{23}^2}{3} \gtrsim BR_{e\mu} \gtrsim - \frac{8c_{23}^2}{3} BR_{ee} + \frac{2c_{23}^2}{3} \tag{35}
\]
If indeed $\sin^2 \theta_{13}$ is negligibly small and these conditions are satisfied by the measured $\text{BR}_{ee}$ and $\text{BR}_{e\mu}$, a positive result can be expected in the next generation of $\beta\beta_{0\nu}$-decay experiments having a sensitivity to $|M_{ee}| \geq 0.05$ eV. Note that the magnitude of the left and right sides of the inequality is very sensitive to the value of $c_{23}^2$. Note also that these conditions do not depend explicitly on $\text{BR}_{\mu\mu}$. For this reason we will first obtain constraints on $|M_{ee}|$ using only the branching ratios $\text{BR}_{ee}$ and $\text{BR}_{e\mu}$.

Next, we analyse the case of $\theta_{13} \neq 0$ numerically. We calculated $\text{BR}_{ee}$, $\text{BR}_{e\mu}$, and $M_{ee}$ by using $|\Delta m_{31}^2|$, $\Delta m_{21}^2$, and $\sin^2 2\theta_{12}$ given in eqs. (11) and (12). We allow $m_0$ to vary in the interval in eq. (17), while the other parameters are varied in the following ranges reflecting the uncertainties in their knowledge or lack of any constraints:

$$\sin^2 2\theta_{23} > 0.94, \quad \sin^2 2\theta_{13} < 0.14, \quad \delta, \alpha_{21}, \alpha_{31} = 0 - 2\pi. \quad (36)$$

Later we will present results for the prospective smaller uncertainties in $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{13}$, corresponding to $\sin^2 2\theta_{23} > 0.99$ and $\sin^2 2\theta_{13} < 0.04$.

In Fig. 2 we show the regions in the $\text{BR}_{ee}-\text{BR}_{e\mu}$ plane where we definitely have $|M_{ee}| \geq 0.05$ eV or $|M_{ee}| < 0.05$ eV. More specifically, the solid (red) line determines the complete allowed region in the HTM, corresponding to $\Delta m_{21}^2$, $|\Delta m_{31}^2|$ and $\sin^2 2\theta_{12}$ given in eqs. (11) and (12), and values of $m_0$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, $\delta$, $\alpha_{21}$ and $\alpha_{31}$, which were allowed to vary in the ranges specified in eqs. (17) and (36). The dashed blue (dash-dotted green) lines determine the black (grey) regions where $|M_{ee}|$ is definitely larger (smaller) than 0.05 eV in the HTM when $m_0$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, $\delta$, $\alpha_{21}$ and $\alpha_{31}$, are varied within the indicated intervals (i.e. eqs. (17) and (36)). For values of $\text{BR}_{ee}$ and $\text{BR}_{e\mu}$ from the region depicted in white (and located between those shown in black and in grey), the determination of $|M_{ee}|$ is not unambiguous: both values of $|M_{ee}| \geq 0.05$ eV and $|M_{ee}| < 0.05$ eV are possible. This degeneracy can be lifted to certain extent, but not completely, by using additional information on $\text{BR}_{\mu\mu}$ (see further). The dotted black line in Fig. 2 corresponds to $\text{BR}_{ee} + \text{BR}_{e\mu} = 1$. We show it only to indicate the boundary of the region of possible values of $\text{BR}_{ee}$ and $\text{BR}_{e\mu}$: the area above this line is unphysical. The results in Fig. 2 are obtained without using the possible additional data on $\text{BR}_{e\mu}$. The left and middle panels correspond to NO and IO spectrum, respectively, while the results shown in the right panel were obtained assuming that the sign of $\sin(\Delta m_{31}^2)$ (i.e. the type of the neutrino mass spectrum) is unknown. The black area where $|M_{ee}|$ is, e.g. definitely larger than 0.05 eV in the right panel corresponds to the intersection of the black areas in left and middle panels. Note that we can have $|M_{ee}| \gtrsim 0.05$ eV also in the region shown in white and located between the grey areas in the right panel of Fig. 2. This cannot be unambiguously predicted, however, knowing only the values of $\text{BR}_{ee}$ and $\text{BR}_{e\mu}$ which lie in the white area.

Next we show how the results discussed above change when we add information on $\text{BR}_{\mu\mu}$. The $\mu^{\pm}\mu^{\pm}$ decay mode of $H^{\pm\pm}$ is relatively easy to measure at LHC by virtue of the two same sign muons in the final state. We present results for $\text{BR}_{\mu\mu} = 0; 0.1; 0.2; 0.3$ in Figs. 3, 4, 5 and 6 respectively, where $\text{BR}_{\mu\mu} = 0$ in practice corresponds to $\text{BR}_{\mu\mu} < 0.01$. When we quote a

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*We recall that in this analysis we do not use possible data on $\text{BR}_{\tau\tau}$, $\text{BR}_{e\tau}$ and $\text{BR}_{e\tau}$.
Figure 2: Values of $\text{BR}_{ee}$ and $\text{BR}_{e\mu}$, for which $|M_{ee}| > 0.05 \text{ eV}$ (black areas limited by the dashed blue lines) or $|M_{ee}| < 0.05 \text{ eV}$ (grey areas limited by the dash-dotted green lines) in the HTM. The solid (red) line shows the entire region of allowed values of $\text{BR}_{ee}$ and $\text{BR}_{e\mu}$ in the HTM (black and grey areas and the white area between the coloured one). The results shown are obtained by varying $m_0$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, $\delta$, $\alpha_{21}$ and $\alpha_{31}$, in the ranges given in eqs. (17) and (36). The left and middle panels correspond to NO and IO spectrum, respectively, while the right panel was obtained assuming that $\text{sgn}(\Delta m^2_{31})$ is unknown. The dotted line corresponds to $\text{BR}_{ee} + \text{BR}_{e\mu} = 1$. The region above this line is unphysical. See text for further details.

Figure 3: The same as in Fig. 2 but assuming that the experimentally determined $\text{BR}_{\mu\mu} = 0$. The dotted line corresponds to $\text{BR}_{ee} + \text{BR}_{e\mu} + \text{BR}_{\mu\mu} = 1$; the region above the line is unphysical. See text for further details.
Figure 4: The same as in Fig. 3 but for BR_{\mu\mu} = 0.1.

Figure 5: The same as in Fig. 3 but for BR_{\mu\mu} = 0.2.

Figure 6: The same as in Fig. 3 but for BR_{\mu\mu} = 0.3.
Figure 7: The same as in Fig. 2 but allowing $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{13}$ to vary in the following more narrow intervals: $\sin^2 2\theta_{23} > 0.99$ and $\sin^2 2\theta_{13} < 0.04$. No information on $\text{BR}_{\mu\mu}$ was used in deriving the results shown in the figure. See text for further details.

Figure 8: The same as in Fig. 7 but assuming that the experimentally determined $\text{BR}_{\mu\mu} = 0$. The dotted line corresponds to $\text{BR}_{ee} + \text{BR}_{e\mu} + \text{BR}_{\mu\mu} = 1$; the region above the line is unphysical.

Figure 9: The same as in Fig. 8 but for $\text{BR}_{\mu\mu} = 0.2$. 

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specific value of $BR_{\mu\mu} = x$, we include an uncertainty of ±0.01 in $x$, i.e. we use $BR_{\mu\mu} = x \pm 0.01$ in the numerical calculations. The dotted lines in Figs. 3-6 correspond to $BR_{ee} + BR_{e\mu} + BR_{\mu\mu} = 1$. We do not use this constraint: the line represents the boundary of the physical region of values of $BR_{ee}$, $BR_{e\mu}$ and $BR_{\mu\mu}$.

It is clear from Fig. 3 that the measurement of $BR_{\mu\mu}$ can improve the predictability of $|M_{ee}|$: the relative magnitude of the white “degeneracy” region, in general, is smaller than in the case when no information on $BR_{e\mu}$ is available. This is not the case, however, for IO spectrum and values of $BR_{\mu\mu} = 0.1; 0.2$ (Figs. 4 and 5, middle panels).

Figures 2-6 show also the regions (dash-dotted line) where one definitely has $|M_{ee}| < 0.05$ eV. If the measured values of $BR_{ee}$, $BR_{e\mu}$ and $BR_{\mu\mu}$ lie in one of these regions, the observation of $(\beta\beta)_{0v}$-decay in the next generation of experiments can be extremely challenging. Even in such a case, however, searches for the $(\beta\beta)_{0v}$-decay are important and necessary also as a test of the HTM itself. If the $(\beta\beta)_{0v}$-decay is observed while the measured values of the $H^{\pm\pm}$ lepton decaying branching ratios imply, e.g. a negative result of the searches for $(\beta\beta)_{0v}$-decay, we will be led to conclude that $M_{\nu l}$ and $h_{\nu l}$ are not directly related: $M_{\nu l} \neq \sqrt{2}h_{\nu l}v_\Delta$. Such a situation can arise, for instance, if $v_\Delta = 0$, or in models with $H^{++}$ which is not an $SU(2)_L$ triplet, but, e.g. is a $Y = 4$ singlet [67] with couplings to the charged leptons given by $h_{\nu l} (l_R^c)^c l_R H^{++}$.

We have performed the same analysis, but with reduced uncertainties in $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{13}$: $\sin^2 2\theta_{23} > 0.99$ and $\sin^2 2\theta_{13} < 0.04$. The indicated precisions (or better ones) in the determination of $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{13}$ are expected to be achieved in the upcoming T2K [36] and reactor antineutrino experiments Double CHOOZ [31], Daya Bay [32] and RENO [33], respectively. The results of the analysis are shown graphically in Figs. 7-10. The notations in Figs. 7-10 are the same as in Figs. 2-6. We see from Figs. 7-10 that improving the precision on $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{13}$ leads to a noticeable reduction of the regions of values of $BR_{ee}$ and $BR_{e\mu}$, for which it is impossible to determine whether $|M_{ee}| \geq 0.05$ eV or $|M_{ee}| < 0.05$ eV. As Fig. 8 demonstrates, the reduction will be particularly significant if the measured $BR_{\mu\mu} < 10^{-2}$ (which we remind the reader corresponds to the case denoted by us as $BR_{\mu\mu} = 0$).
5 Conclusions

We have investigated the connection between the \((\beta\beta)_{0\nu}\)-decay effective Majorana mass \(|M_{ee}|\), and the branching ratios of the decays \(H^{\pm\pm} \to l^\pm l'^\pm\), \(l, l' = e,\mu\), of the doubly charged Higgs boson \(H^{\pm\pm}\) within the Higgs Triplet Model (HTM) of neutrino mass generation. Our analysis was performed within the version of the model with explicit breaking of the total lepton charge conservation, in which \(H^{\pm\pm} \to l^\pm l'^\pm\), \(l, l' = e,\mu,\tau\), are the dominant decay modes of \(H^{\pm\pm}\). In this model the couplings of the doubly charged Higgs field \(H^{++}\) to the flavour neutrinos and charged leptons are proportional to the elements of the Majorana mass matrix of the (flavour) neutrinos, \(M_{\nu}\), and the branching ratios \(\text{BR}(H^{\pm\pm} \to l^\pm l'^\pm)\) are entirely determined by the elements of the PMNS matrix and neutrino masses. The latter possibility is realised if the mass of the doubly charged Higgs scalar does not exceed the mass of the singly charged one, \(M_{H^{\pm\pm}} \leq M_{H^\pm}\), and if the vacuum expectation value of the neutral component of the Higgs triplet field satisfies \(v_\Delta \lesssim 1\) MeV. The model under discussion was shown [7] [8] [9] to have a rich and physically interesting phenomenology owing to the fact that the physical doubly charged and singly charged Higgs fields, \(H^{\pm\pm}\) and \(H^\pm\), can have masses in the range from \(\sim 100\) GeV to \(\sim 1\) TeV and thus, can, in principle, be produced and observed at LHC. More importantly, by studying the decays \(H^{\pm\pm} \to l^\pm l'^\pm\), \(l, l' = e,\mu,\tau\), it might be possible to obtain information on the absolute neutrino mass scale, on the type of neutrino mass spectrum and on the Majorana CP violating phases present in the neutrino mixing matrix [7] [8] [9].

In the present article we have investigated the possibility to use the information on the neutrino mass spectrum and the Majorana CP violating phases from the measurements of the \(H^{\pm\pm} \to l^\pm l'^\pm\) decay branching ratios, \(\text{BR}(H^{\pm\pm} \to l^\pm l'^\pm), l, l' = e,\mu\), in order to obtain predictions for the effective Majorana mass in neutrinoless double beta \((\beta\beta)_{0\nu}\) decay, \(|M_{ee}|\). Among the different decay channels \(H^{\pm\pm} \to l^\pm l'^\pm\), \(l, l' = e,\mu,\tau\), the easier to observe and measure the corresponding branching ratios with high precision are those with two electrons (positrons), two muons (antimuons), or an electron (positron) and a muon (antimuon), \(e^\pm e^\pm, e^\pm\mu^\pm\) and \(\mu^\pm\mu^\pm\), in the final state. If the mass of \(H^{\pm\pm}\) does not exceed approximately 400 GeV, the branching ratios of the \(H^{\pm\pm}\) decays into \(e^\pm e^\pm, e^\pm\mu^\pm\) and \(\mu^\pm\mu^\pm\) can be measured at LHC with a few percent error [13].

Taking into account the current and prospective uncertainties in the values of the neutrino mixing parameters most relevant for the problem studied - the atmospheric neutrino mixing angle \(\theta_{23}\) and the CHOOZ angle \(\theta_{13}\), and allowing the lightest neutrino mass and the CP violating Dirac and Majorana phases to vary in the intervals \([0, 0.3\) eV\)] and \([0, 2\pi]\), respectively, we have derived the regions of values of \(\text{BR}(H^{\pm\pm} \to e^\pm e^\pm)\) and \(\text{BR}(H^{\pm\pm} \to e^\pm\mu^\pm)\) for which we definitely have \(|M_{ee}| \geq 0.05\) eV, or \(|M_{ee}| < 0.05\) eV. This is done for neutrino mass spectrum with normal ordering (NO), inverted ordering (IO) and in the case when the type of the spectrum is not known. In what concerns the branching ratio \(\text{BR}(H^{\pm\pm} \to \mu^\pm\mu^\pm)\), we have considered two cases: i) the possible data on \(\text{BR}(H^{\pm\pm} \to \mu^\pm\mu^\pm)\) is not used as an additional constraint in the analysis, ii) the possible data on \(\text{BR}(H^{\pm\pm} \to \mu^\pm\mu^\pm)\) is included in the analysis. In the latter case, results
for several values of $\text{BR}(H^{\pm\pm} \rightarrow \mu^{\pm}\mu^{\pm})$ have been obtained.

Our results are presented graphically in Figs. 2-10. They show that if the doubly charged Higgs bosons $H^{\pm\pm}$ will be discovered at LHC and at least the two branching ratios $\text{BR}(H^{\pm\pm} \rightarrow e^{\pm}e^{\pm})$ and $\text{BR}(H^{\pm\pm} \rightarrow e^{\pm}\mu^{\pm})$ will be measured with a sufficient accuracy, one can obtain important information on the $(\beta\beta)_{0\nu}$-decay effective Majorana mass $|M_{ee}|$. In the various cases considered, we have identified the regions values of $\text{BR}(H^{\pm\pm} \rightarrow e^{\pm}e^{\pm})$ and $\text{BR}(H^{\pm\pm} \rightarrow e^{\pm}\mu^{\pm})$, for which $|M_{ee}|$ is definitely bigger or smaller than 0.05 eV (Fig. 2). We have shown also that due to i) the uncertainties in the determination of $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{13}$, ii) the absence of data on the CP violating phases in the neutrino mixing matrix, and iii) the existing rather loose upper bound on the absolute neutrino mass scale, there exist also noticeable regions of values of $\text{BR}(H^{\pm\pm} \rightarrow e^{\pm}e^{\pm})$ and $\text{BR}(H^{\pm\pm} \rightarrow e^{\pm}\mu^{\pm})$ for which it is impossible to determine unambiguously whether $|M_{ee}| \geq 0.05$ eV or $|M_{ee}| < 0.05$ eV (Fig. 2). This “degeneracy” can be partially lifted by using the additional information from a measurement of $\text{BR}_{\mu\mu}$ (Figs. 3-6).

The same analysis was performed with reduced uncertainties in $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{13}$ corresponding to $\sin^2 2\theta_{23} > 0.99$ and $\sin^2 2\theta_{13} < 0.04$. The results are presented graphically in Figs. 7-10. They show that improving the precision on $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{13}$ leads to a noticeable reduction of the regions of values of $\text{BR}_{ee}$ and $\text{BR}_{e\mu}$ for which it is impossible to determine whether $|M_{ee}| \geq 0.05$ eV or $|M_{ee}| < 0.05$ eV. The reduction will be particularly significant if the measured $\text{BR}_{\mu\mu} < 10^{-2}$.

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References

[1] W. Konetschny and W. Kummer, Phy. Lett. B 70 (1977) 433.

[2] T. P. Cheng and L. F. Li, Phys. Rev. D 22, 2860 (1980).

[3] J. Schechter and J.W.F. Valle, Phys. Rev. D 22 (1980) 2227.

[4] G. B. Gelmini and M. Roncadelli, Phys. Lett. B 99, 411 (1981).

[5] E. Ma, M. Raidal and U. Sarkar, Phys. Rev. Lett. 85, 3769 (2000); E. Ma, M. Raidal and U. Sarkar, Nucl. Phys. B 615, 313 (2001).

[6] E. J. Chun, K. Y. Lee and S. C. Park, Phys. Lett. B 566, 142 (2003).

[7] A. G. Akeroyd, M. Aoki and H. Sugiyama, Phys. Rev. D 77, 075010 (2008).

[8] J. Garayoa and T. Schwetz, JHEP 0803, 009 (2008).

[9] M. Kadastik, M. Raidal and L. Rebane, Phys. Rev. D 77, 115023 (2008).

[10] B. Pontecorvo, Zh. Eksp. Teor. Fiz. (JETP) 33 (1957) 549; *ibid.* 34 (1958) 247; *ibid.* 53 (1967) 1717; Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28 (1962) 870.

[11] S.M. Bilenky, J. Hosek and S.T. Petcov, Phys. Lett. B 94 (1980) 495.

[12] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. 59 (1987) 67.

[13] P. Fileviez Perez, T. Han, G. y. Huang, T. Li and K. Wang, Phys. Rev. D 78, 015018 (2008).

[14] J. Schechter and J. W. F. Valle, Phys. Rev. D 25, 774 (1982).

[15] M. A. Diaz, M. A. Garcia-Jareno, D. A. Restrepo and J. W. F. Valle, Nucl. Phys. B 527, 44 (1998).

[16] M. C. Chen, Phys. Rev. D 71, 113010 (2005).

[17] T. Blank and W. Hollik, Nucl. Phys. B 514, 113 (1998); M. Czakon, M. Zralek and J. Gluza, Nucl. Phys. B 573, 57 (2000); M. Czakon, J. Gluza, F. Jegerlehner and M. Zralek, Eur. Phys. J. C 13, 275 (2000); J. R. Forshaw, D. A. Ross and B. E. White, JHEP 0110, 007 (2001). M. C. Chen and S. Dawson, Phys. Rev. D 70, 015003 (2004); M. C. Chen, S. Dawson and T. Krupovnickas, Int. J. Mod. Phys. A 21, 4045 (2006); M. C. Chen, S. Dawson and T. Krupovnickas, Phys. Rev. D 74, 035001 (2006).

[18] J. F. Gunion, J. Grifols, A. Mendez, B. Kayser and F. I. Olness, Phys. Rev. D 40, 1546 (1989).

[19] F. Cuypers and S. Davidson, Eur. Phys. J. C 2, 503 (1998).

[20] S.M. Bilenky, S. Pascoli and S.T. Petcov, Phys. Rev. D64 (2001) 053010 and 113003.

[21] S.T. Petcov, Nucl. Phys. B (Proc. Suppl.) 143 (2005) 159 (hep-ph/0412410).

[22] J. Schechter and J.W.F. Valle, Phys. Rev. D 22 (1980) 2227; M. Doi et al., Phys. Lett. B 102 (1981) 323.
[23] B. T. Cleveland et al., Astrophys. J. 496, 505 (1998); W. Hampel et al. [GALLEX Collaboration], Phys. Lett. B 447, 127 (1999); J. N. Abdurashitov et al. [SAGE Collaboration], J. Exp. Theor. Phys. 95, 181 (2002) [Zh. Eksp. Teor. Fiz. 122, 211 (2002)] [arXiv:astro-ph/0204245].
B. Aharim et al. [SNO Collaboration], Phys. Rev. D 73, 112001 (2006); J. Hosaka et al. [Super-Kamiokande Collaboration], Phys. Rev. D 78, 033010.

[24] Y. Ashie et al. [Super-Kamiokande Collaboration], Phys. Rev. D 71, 112005 (2005) [arXiv:hep-ex/0501064]; J. Raaf, a talk presented in the XXIII International Conference on Neutrino Physics and Astrophysics (Neutrino 2008), May 25-31, 2008, Christchurch, New Zealand.

[25] M. H. Ahn et al. [K2K Collaboration], Phys. Rev. D 74, 072003 (2006); P. Adamson et al. [MINOS Collaboration], Phys. Rev. Lett. 101, 131802 (2008).

[26] M. Apollonio et al. [CHOOZ Collaboration], Eur. Phys. J. C 27, 331 (2003).

[27] S. Abe et al. [KAMLAND Collaboration], Phys. Rev. Lett. 100, 221803 (2008).

[28] A. Bandyopadhyay et al., Phys. Lett. B 608 (2005) 115, and [arXiv:0804.4857]

[29] G.L. Fogli et al., Phys. Rev. D 78 (2008) 033010.

[30] T. Schwetz, M. Törnala and J. W. F. Valle, New J. Phys. 10, 113011 (2008).

[31] F. Ardellier et al. [Double Chooz Collaboration], hep-ex/0606025.

[32] See, e.g., K. M. Heeger, talk given at Neutrino’06 International Conference, June 13 - 19, 2006, Sant Fe, U.S.A., and the Daya Bay homepage [http://dayawane.ihep.ac.cn/]

[33] S.-B. Kim et al. [RENO Collaboration], Journal of Physics: Conference Series 120 (2008) 052025.

[34] K. Anderson et al., hep-ex/0402041 and the references quoted therein.

[35] A. Bandyopadhyay et al. [ISS Physics Working Group], arXiv:0710.4947 [hep-ph] and the references quoted therein.

[36] Y. Itow et al. [The T2K Collaboration], arXiv:hep-ex/0106019 For an updated version, see: [http://neutrino.kek.jp/jhfnu/loi/loi.v2.030528.pdf]

[37] M.V. Chizhov, M. Maris and S.T. Petcov, hep-ph/9810501; J. Bernabéu, S. Palomares-Ruiz and S.T. Petcov, Nucl. Phys. B 669 (2003) 255; S. Palomares-Ruiz and S.T. Petcov, Nucl. Phys. B 712 (2005) 392.

[38] S. T. Petcov, Phys. Lett. B 434, 321 (1998), (E) ibid. B 444, 584 (1998).

[39] M. V. Chizhov and S. T. Petcov, Phys. Rev. Lett. 83 (1999) 1096, and Phys. Rev. D 63 (2001) 073003.

[40] M. V. Chizhov and S. T. Petcov, Phys. Rev. Lett. 85, 3979 (2000).

[41] V. K. Ermiilova et al., Short Notices of the Lebedev Institute 5, 26 (1986); E. Kh. Akhmedov, Yad. Fiz. 47, 475 (1988); P. I. Krastev and A. Yu. Smirnov, Phys. Lett. B226, 341 (1989).

[42] S. T. Petcov and M. Piai, Phys. Lett. B 533, 94 (2002); S. Choubey, S. T. Petcov and M. Piai, Phys. Rev. D 68, 113006 (2003).

[43] J. Learned et al., Phys. Rev. D 78, 071302 (2008) and arXiv:0810.4975. M. Batygov et al., arXiv:0810.2580
[44] L. Zhan et al., Phys. Rev. D 78 (2008) 111103; arXiv:0901.2976.
[45] S.M. Bilenky, M.D. Mateev and S.T. Petcov, Phys. Lett. B 639 (2006) 312.
[46] K. Eitel et al., Nucl. Phys. Proc. Suppl. 143 (2005) 197.
[47] F. Perrin, Comptes Rendus 197 (1933) 868; E. Fermi, Nuovo Cim. 11 (1934) 1.
[48] V. Lobashev et al., Nucl. Phys. A 719 (2003) 153c.
[49] D.N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 148 (2003) 175.
[50] M. Tegmark, Phys. Scripta T121 (2005) 153; S. Hannestad, H. Tu and Y. Y. Y. Wong, JCAP 0606 (2006) 025.
[51] W. Hu and M. Tegmark, Astrophys. J. Lett. 514 (1999) 65.
[52] C. Aalseth et al., hep-ph/0412300.
[53] S.T. Petcov, New J. Phys. 6 (2004) 109 (http://stacks.iop.org/1367-2630/6/109); Physica Scripta T121 (2005) 94 (hep-ph/0504110); S. Pascoli and S.T. Petcov, hep-ph/0308034.
[54] V.A. Rodin et al., Nucl. Phys. A 766 (2006) 107, [Erratum-ibid. A 793, 213 (2007)]; Poves, talk given at the NDM06 International Symposium, September 3-8, 2006, Paris; E. Caurier et al., Phys. Rev. Lett. 100, 052503 (2008) and Eur. Phys. J. A 36, 195 (2008)
[55] H.V. Klapdor-Kleingrothaus et al., Nucl. Phys. Proc. Suppl. 100 (2001) 309.
[56] C.E. Aalseth et al., Phys. Atomic Nuclei 63 (2000) 1225.
[57] H. V. Klapdor-Kleingrothaus et al., Phys. Lett. B 586 (2004) 198.
[58] H. V. Klapdor-Kleingrothaus et al., Mod. Phys. Lett. A 16 (2001) 2409.
[59] A. S. Barabash [NEMO Collaboration], arXiv:0807.2336 [nucl-ex].
[60] C. Arnaboldi et al. [CUORICINO Collaboration], Phys. Rev. C 78, 035502 (2008).
[61] F. Avignone, Nucl. Phys. Proc. Suppl. 143 (2005) 233.
[62] S. Pascoli and S.T. Petcov, Phys. Lett. B 544 (2002) 239; ibid. B 580 (2004) 280.
[63] S. Pascoli, S. T. Petcov and L. Wolfenstein, Phys. Lett. B 524, 319 (2002); S. Pascoli and S.T. Petcov, hep-ph/0111203.
[64] S. Pascoli, S. T. Petcov and T. Schwetz, Nucl. Phys. B 734, 24 (2006); S. Pascoli and S. T. Petcov, Phys. Rev. D 77 (2008) 113003.
[65] J. Schechter and J. W. F. Valle, Phys. Rev. D 25, 2951 (1982).
[66] R. N. Mohapatra and J. D. Vergados, Phys. Rev. Lett. 47, 1713 (1981); W. C. Haxton, S. P. Rosen and G. J. Stephenson, Phys. Rev. D 26, 1805 (1982); L. Wolfenstein, Phys. Rev. D 26, 2507 (1982).
[67] A. Zee, Nucl. Phys. B 264, 99 (1986); K. S. Babu, Phys. Lett. B 203, 132 (1988).