DETERMINING OPTIMAL MARKETING AND PRICING POLICIES BY CONSIDERING CUSTOMER LIFETIME NETWORK VALUE IN OLIGOPOLY MARKETS

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Abstract. Nowadays, the customer network effects are becoming a central issue for the companies, while the previous studies have been only limited to customer lifetime value and its related models. Therefore, this paper aims to presents a model for calculating customer lifetime value, and simultaneously the network effects are considered. For this purpose, an oligopoly market is considered in which companies compete with each other. The companies individually have a number of buyers and sellers. Interestingly, their policy is based on offering services to their buyers free, and receiving the membership fees from the sellers instead. Customers influence each other, and their word-of-mouth marketing leads to a change in the number of companies’ customers. This interaction is also observed among the sellers. In the absence of buyers, the presence of sellers is meaningless. In other words, there is a remarkable proportion between the number of buyers and sellers, which directly affects the companies’ profitability. Each company seeks to determine the optimal marketing and pricing policies, considering the effects of the network. By applying differential game theory, the companies are able to receive the market share, advertising, and pricing strategy. The Genetic Algorithm is employed to solve the model. Finally, a numerical example and model validation are provided to demonstrate the proposed model capabilities.

1. Introduction. Over the past decade, or more, Customer Relationship Management (CRM) has been the strategic approach that most companies had taken in trying to figure out how to supervise their customers’ behavior [24, 30, 18, 4]. Customer lifetime value (CLV) is one of customer relationship management’s fundamental issues that aims to evaluate customer profitability. Moreover, in recent years there has been growing interest in CLV [14, 23, 31, 5-7]. In 1974, Kotler was a pioneer in the field of customer lifetime value and introduced this issue as the present value of the future profit stream expected over a given time horizon of transacting with the customer [21]. Overall, since the companies prosper considerably by having high CLV, they seek to find ways to raise their customer life cycle value. There are various models for calculating CLV, including RFM [28, 29], probabilistic [32, 17], econometric, persistence, diffusion/Growth, etc [8]. A key problem with much of the literature in relation to CLV is that most of the authors generally ignored the...
customer network effects, and the invisible correlation between the customers has been overlooked. As a matter of fact, one customer's value is assumed to be independent of the other ones in the previous studies. In accordance with most of the models presented for CLV, the companies typically pay much attention to the customers' benefits rather than the number of customers being absorbed by the existing ones. Therefore, in the light of recent events in CLV, there is now considerable concern about the communication networks[3]. Some companies, such as online stores with the Marketplace business model (example, e-Bay), offer their services to the buyers free and receive membership fees from companies and vendor brands in return. According to traditional CLV models, the buyers' CLV in these stores is negative (due to the costs incurred by these buyers to the company, like advertising and the lack of direct income from them). However, having the sellers without the buyers is pointless. Actually, the rise in the number of buyers leads to increasing the number of sellers. Since The companies intend to increase the number of their buyers by conducting marketing activities, ignoring network communications turns out to be even more problematic. Therefore, traditional models cannot be used for these companies, and there is a need to provide a model to consider network communications. Moreover, a company’s decisions affect its competitive decisions in oligopoly markets.

Regardless of the vast range of studies conducted on CLV models and the methods to calculate them, the present paper aims to calculate CLV in a space such as a Marketplace (in the presence of buyers, sellers, and competing companies) concerning the network relations and the communication occurring between them. The primary objectives of this research are summarized as follows:

- Providing a model for calculating customer lifetime value by considering the relationships between buyers and companies in oligopoly markets.
- Determining optimal marketing (for buyers) and pricing strategies (for sellers) by considering the relationships between buyers and companies in oligopoly markets.
- Analyzing the invisible relationship between the buyers and sellers according to customer lifetime network value.

The required variables and input parameters are presented in the beginning, then the proposed model and its analysis are demonstrated. Optimal pricing and advertising are calculated using the optimal control model and based on the buyers' and sellers' growth constraints. Besides, we develop the monopoly model given by Gupta et al. (2006) into the oligopoly model providing explicit closed-loop solutions [8]. An infinite horizon differential game is solved to derive the feedback Nash equilibrium and consider the network’s impact on the cost of advertising, market share, and profitability. Besides, this study applies differential game theory to capture the carryover dynamics of advertising, pricing, and competitive interactions. This theory is also useful to understand the best course of action for each firm while considering its competitors’ responses. Since the problem of this study is NP-hard, the Genetic Algorithm is used to solve the model. With respect to the recent studies, the Genetic Algorithm outperforms the other ones among meta-heuristic algorithms, and it is very efficient in solving optimal control problems [12, 22]. The proposed model is simulated by considering three companies, and the purpose of this simulation is to maximize the customers’ obtained profit for Company 1.

A numerical example is presented to clarify the proposed model's application. Moreover, for the sake of validation, we exploited the data of a particular company.
Some of these data are employed to solve the model; the validation is conducted based on the rest.

The rest of this study is organized as follows: Section 2 gives the literature review concerning the contributions that have been conducted recently about CLV. In the third section, the model and assumptions are presented. A numerical example and model validation are outlined in the fourth section. Moreover, some conclusions are drawn in the final section.

2. Literature review. In this section, we summarized the primary researches conducted regarding CLV and its fundamental models. According to Table 1, the significant similarities and differences of this paper with the previous ones are considered. The differences are related to the parameters that are not mentioned in this study. The similarities are about the analyses and investigations that are common between our study and the previous ones.

As mentioned in the Introduction, in 1974, Kotler introduced CLV as the present value of the future profit stream expected over a given time horizon of transacting with the customer [21]. In fact, one of the methods of customer profitability evaluation entails calculating the customer lifetime value (CLV). Regarding the CLV calculation, there is a considerable body of literature on the equations to calculate it [19, 5]. For instance, Berger and Nasr proposed a valuable equation to calculate CLV [2]. Besides, some equations are used to calculate CLV in the previous studies that bear a close resemblance to the equation Berger and Nasr proposed. According to the previous studies, one of the main common issues between them and our work is considering CLV and paying attention to the companies’ profitability.

Moreover, the invisible interaction between the buyers and sellers is considered, but the direct effects between them are considered, which is among the most commonly discussed topics. This effect can include the interaction between the buyers or sellers. For instance, the buyers affect each other, and their behavior is changed.

The model employed in this paper is inspired by the model that Gupta et al. proposed for the monopoly market. Gupta et al. have considered a company and attempted to maximize its profits by taking into account the network effects between customers and sellers. However, the main drawback of their analysis is that their model is only limited to monopoly markets. With respect to oligopoly markets, a company’s decisions affect its competitive decisions, and its pricing strategies cannot be considered independent of its rivals. Hence, our proposed model is the developed form of their model and considers an oligopoly market [10].

According to the research conducted by Prasad et al. in 2012, in addition, to market churn, each company’s market share is dependent upon its own and competitors’ advertising decisions in oligopoly markets [25]. This idea is also taken into consideration in this study. As mentioned earlier, our model considers the oligopoly markets, i.e., a company’s decision depends on the other companies’ decision. This is the main feature in this study that makes our work distinct from others. A more detailed look at Table 1 reveals the fact that few researchers have considered network effects into general CLV computational models. Previous research can only be considered the first step towards a more profound understanding of customer lifetime network value in oligopoly markets. As a matter of fact, we aim to determine optimal marketing and pricing policies with respect to this value. The significant differences of our study with the previous ones can be summarized as follows:
Table 1. Literature review

| Authors | Considered elements / defined model | The main similarities | The main differences |
|---------|-------------------------------------|-----------------------|----------------------|
| Berger and Nasr (1998) [2] | • Two steps should be taken to calculate CLV: 1. Projecting the net cash flows that the company is likely to take from customers. 2. The present value computation of that stream of cash flows. | CLV is investigated. | 1. Network effects are not considered. 2. The net cash flows are examined. 3. Mathematical models are considered. 4. Differences between monopoly market and oligopoly market regarding CLV and network effects are not clarified. |
| Rust et al. (2000) [27] | • A method to determine CLV that incorporates customer-specific brand switching metrics. • The Markov model is employed in this study to model the customer’s probability of switching from one brand to another by a transition matrix. | CLV calculation is considered. | 1. Network effects are not considered. 2. Differences between the monopoly market and oligopoly market regarding CLV and network effects are not clarified. |
| Blattberg et al. (2001) [1] | • Customer lifetime value is the sum of three components, which are: return on the acquisition, return on retention, and return on cross-selling | 1. CLV is investigated. 2. A method is proposed to raise the companies’ profitability. | 1. Network effects are not considered. 2. Differences between the monopoly market and oligopoly market regarding CLV and network effects are not clarified. |
| Authors                  | Considered elements / defined model                                                                 | The main similarities                                                                 | The main differences                                                                 |
|-------------------------|------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|
| Hogan et al. (2003)[13] | • They showed that after losing a customer, a company will not only lose the cash flow it can earn from that customer in the future but will also lose the cash flow that can be gained from other customers due to less customer attraction as a result of reduced social impact. | 1. Network effects are considered. 2. The target is increasing the companies’ profitability. | 1. The Genetic Algorithm is not used to solve the model. 2. Differences between the monopoly market and oligopoly market regarding CLV and network effects are not clarified. |
| Hwang et al. (2004) [16] | • Three factors are considered. According to the studies conducted regarding CLV, these factors have been neglected. The factors are past profit contribution, Potential benefit, and defection probabilities of the customer.  
• Besides, a framework is presented to analyze customer values and segment them according to their values. | 1. CLV calculation is considered. 2. The target is increasing the companies’ profitability. | 1. Network effects are not considered. 2. This paper considers customer retention as the most critical issue. 3. Differences between the monopoly market and oligopoly market regarding CLV and network effects are not clarified. |
| Fader et al. (2005) [6] | • Customers are grouped based on three factors of RFM.                                                | CLV calculation is considered.                                                        | 1. Network effects are not considered. 2. Differences between the monopoly market and oligopoly market regarding CLV and network effects are not clarified. |
| Authors | Considered elements / defined model | The main similarities | The main differences |
|---------|--------------------------------------|-----------------------|---------------------|
| Gupta et al. (2006) [10] | • A joint model of buyer and seller growth is developed to calculate the customers' value.  
• Three sources, including marketing actions (price and advertising), direct network effects (such as buyer to buyer effects), and indirect network effects (such as buyer to seller effects), constitute this growth.  
• The company's concern is related to optimal pricing determination and advertising according to the customer growth limitations. The growth model is used to solve this problem simultaneously. | 1. Network effects are Considered.  
2. CLV calculation is considered.  
3. The target is to increase the companies' profitability. | 1. The Genetic Algorithm is not used to solve the model.  
2. Differences between the monopoly market and oligopoly market regarding CLV and network effects are not clarified.  
3. Oligopoly market is not considered. |
| Kim et al. (2006) [20] | • A framework is provided to analyze the customer's value and segment them according to their value.  
• After the segmentation, strategies related to each segment are proposed.  
• Customer defection and cross-selling opportunities are considered in this article. | The customer value is analyzed. | 1. Network effects are not Considered.  
2. The customers are segmented according to their value.  
3. Differences between the monopoly market and the oligopoly market regarding CLV and network effects are not clarified. |
| Authors | Considered elements / defined model | The main similarities | The main differences |
|---------|-------------------------------------|-----------------------|----------------------|
| Haenlein et al. (2007) [11] | • A model is proposed for customer lifetime value calculation.  
• This calculation is conducted according to the Markov chain model and classification, and regression tree.  
CLV calculation is considered. | | 1. Network effects are not considered.  
2. Differences between the monopoly market and the oligopoly market regarding CLV and network effects are not clarified. |
| Yeh et al. (2009) [34] | • Considering the time parameter based on the first purchase and churn probability and developing the RFM model.  
They expanded a model to consider their desired parameters. | | 1. Network effects are not considered.  
2. Differences between the monopoly market and the oligopoly market regarding CLV and network effects are not clarified. |
| Prasad et al. (2012) [25] | • The churn effects are considered.  
• The companies’ market share is related to market churn, the advertising decisions they made, and their competitors’ advertising decisions.  
• Differential game theory is applied to extract a feedback Nash equilibrium according to the symmetric and asymmetric competition.  
1. The presented model regarding advertising in an oligopoly is considered dynamically.  
2. The target is increasing the companies’ profitability.  
3. Counteracting the churn effect is considered. | | 1. Churn effect in dynamic oligopoly markets is considered.  
2. Differences between the monopoly market and oligopoly market regarding CLV and network effects are not clarified.  
3. Counteracting the churn effect is considered. |
| Authors                        | Considered elements / defined model                                                                 | The main similarities                                                                 | The main differences                                                                                   |
|-------------------------------|-----------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------|
| Klier et al. (2014) [35]      | • A model for customer valuation is developed.                                                      | 1. Network effects are Considered.                                                      | The difference between the monopoly market and the oligopoly market regarding CLV and network effects is not clarified. |
|                               | • This model is based on customer lifetime network value.                                           | 2. A real-world dataset of a company is employed to show the application customer lifetime network value. |                                                                                        |
| Das et al. (2017) [3]         | • Customer lifetime network value is divided into two parts:                                         | 1. Network effects are Considered.                                                      |                                                                                        |
|                               | 1. The present value of individual cash flows and                                                  | 2. A real-world dataset of a company is employed to show the application customer lifetime network value. |                                                                                        |
|                               | 2. The present value of network contribution                                                          |                                                                                        |                                                                                        |
| Grossmann et al. (2019) [7]   | • This paper seeks to show how much the word-of-mouth effects are significant and play a prominent role in estimating CLV in start-up businesses. | 1. Network effects are Considered.                                                      | Differences between the monopoly market and the oligopoly market regarding CLV and network effects are not clarified. |
|                               |                                                                                        | 2. CLV calculation is considered.                                                        |                                                                                        |

- The previous studies have been generally limited to the customer’s profit investigation. The word-of-mouth marketing and interaction between the customers that lead to increasing them and consequently increasing the companies’ profitability has been neglected. In fact, the researchers only attempted to calculate CLV and examined its models regardless of the customer network effects.
- In particular, no study, to our knowledge, has considered customer lifetime network value in oligopoly markets.
- No study to date has exploited the Genetic Algorithm used in this research to solve the presented model.
- The model is dynamically solved in this paper. In other words, the parameters vary based on the time.
3. **Notation and problem formulation.** This section introduces all states and control variables, input parameters, assumptions, and conditions considered in the proposed model.

3.1. **Indices.** i (i = 1, 2, ..., n) Set of Companies

3.2. **States variables.** The states variables considered in this research are as follows:

- $N_{i}^{b}$: The number of buyers of the company i at time t
- $N_{i}^{s}$: The number of sellers of the company i at time t

3.3. **Control variables.** The control variables used here are as follows:

- $A_{i}^{t}$: Company i’s marketing cost at time t
- $P_{i}^{t}$: Company i’s price at time t

3.4. **Input parameters.** The input parameters considered in the proposed model are as follows:

- $p_{t}$: The price is given by a consumer at time t
- $c_{t}$: The direct cost of servicing the customer at time t
- $I$: Discount rate or cost of capital for the firm
- $r_{t}$: Probability of customer repeat buying or being “alive” at time t
- $AC$: Acquisition cost
- $T$: The time horizon for estimating CLV
- $C_{i}^{c}$: Company i’s service cost at time t
- $M_{i}^{B}$: The potential market size of buyers
- $M_{i}^{S}$: The potential market size of Sellers
- $b_{1}$: The strength of direct network effects between Sellers
- $b_{2}$: The strength of direct network effects between Buyers
- $c_{1}$: The strength of indirect network effects (sellers on buyers)
- $c_{2}$: The strength of indirect network effects (buyers on sellers)
- $r_{i}$: Company i’s discount rate

3.5. **Assumptions.** The proposed model is based on the following assumptions:

1. The planning horizon is infinite.
2. Parameters are deterministic.
3. The term $a(A_{t})$ realizes that through advertising, a company can speed up its buyers’ growth. This term can be formulated in the form of $α(A_{t}) = x + hl_nA_{t}$ [15] (Gupta et al. 2006) [10]. Here, diminishing marginal return from advertising is considered, and according to the equation above, h specifies buyers’ responsiveness to the company’s advertising. In fact, advertising depends on the time and is known as a decision variable for the company.
4. The term $(P_{t})$ represents a striking correlation between the sellers’ growth and the amount of money the companies ask them. Thus, their growth is not independent of this amount of money. This term is assumed to be in the form of $α(P_{t}) = \theta − w lnP_{t}$ (Gupta et al. 2006) [10]. According to this equation, w denotes the sellers’ sensitivity to the company’s pricing.
5. Total sellers are relatively stable. A seller leaves one company to join another [25].
6. The sales of the market are relatively stable. A buyer leaves one company to join another [25].
With respect to Assumption 5 and 6, the industry sales are considered stable, i.e., the customer churn from the whole industry is unlikely to occur. Since a buyer definitely buys from one of the industry’s companies, he cannot switch from all of this industry. This assumption is also considered for the sellers.

3.6. Model formulation. As mentioned in the literature review, Berger and Nasr proposed a practical equation to calculate CLV, which is demonstrated below [2]:

$$CLV = \sum_{i=1}^{n} \frac{(R_i - C_i)}{(1 + d)^i}$$  \hspace{1cm} (1)

According to Equation (1), \(i\) corresponds to the time horizon, \(R_i\) denotes the acquired income from the customer over a period \(i\), \(C_i\) represents the total customer costs over a period \(i\), and \(n\) is the number of periods. Reinartz and Kumar (2003) have defined CLV as following [26]:

$$CLV = \sum_{t=0}^{r} \frac{(p_t - C_t)r_t}{(1 + i)^t} - AC$$  \hspace{1cm} (2)

As outlined in the literature review, in an oligopoly market, a company’s decisions have an undeniable impact on its rivals’ decisions. Thus, their pricing strategies are not independent of the other companies’ ones. When a company decides or determines an advertising strategy or pricing strategy, the impact of its decision or strategies on its rivals is remarkably important since they are competing with each other in an oligopoly market. As a result, the decisions and strategies are competitively considered.

According to Figure 1, this model aims to evaluate customer value when interacting with two parallel populations (for instance, buyers and sellers). In addition, the direct and indirect effects on the network are considered. In this condition, a set of
customers (e.g., sellers) typically offer a direct financial return to the company. For example, sellers offer commissions to real estate agencies. However, the companies must acquire and retain another set of customers (e.g., buyers). These customers are “free” since they bring no direct revenue to the companies. The main target here is to develop a model to evaluate the value of both types of customers. Moreover, determining these companies’ market strategies is a dynamic decision and depends on the customers’ CLV. As demonstrated in Figure 1, n companies competing with each other within an oligopoly market are considered in the beginning. Each company has various buyers and various sellers and offers its services to its buyers free and consequently receives a membership fee from the sellers. These n companies compete with each other in order to attract more sellers and buyers. The total number of buyers in the industry is $M_B$, which is divided between n companies. The total number of sellers in the industry is equal to $M_S$, and this parameter is also divided between n companies.

The model proposed in this paper is an optimal control model, and its complexity has made us unable to use classical and accurate methods to solve it. Therefore, metaheuristic algorithms are employed to tackle this problem and obtain the desired results.

An optimal control model is presented based on the followings:

1. Communication between:
   - Buyers and buyers (each company)
   - Buyers and buyers (competitors of each company)
   - Buyers and sellers (each company)
   - Buyers and sellers (competitors of each company)
   - Sellers and sellers (each company)
   - Sellers and sellers (competitors of each company)

2. pricing strategy (each company)

3. marketing strategy (each company)

4. pricing strategy (competitors of each company)

5. marketing strategy (competitors of each company)

Company $i$’s objective is to maximize its long-run discounted profit (Equation 3)

$$\max v_i = \max CLV = \int_{t=0}^{\infty} ((P_i^t - C_i^t)N_i^{s_i} - A_i^t) e^{-r_j^t} dt$$

Subject to market share dynamics given by:

$$\frac{dN_i^{s_i}}{dt} = \alpha (P_i^t) - \sum_{j \neq i} \alpha (p_j^t) + b_1 \left( \frac{N_i^{s_i}}{M^s} \right) + c_1 \left( \frac{N_i^{b_i}}{M^b} \right) \left( M^s - N_t^{s_i} \right)$$

$$\frac{dN_i^{s_i}}{dt} = \alpha (A_i^t) - \sum_{j \neq i} \alpha (A_j^t) + b_2 \left( \frac{N_i^{b_i}}{M^b} \right) + c_2 \left( \frac{N_i^{s_i}}{M^s} \right) \left( M^B - N_t^{B_i} \right)$$

- Equation 3 is a definition of CLV derived from the Gupta model (Gupta et al. 2006) [10].
The term \((P_i^t)\) in Equation 4 indicates that the amount of money a company asks the sellers has a considerable impact on the seller increase (Gupta et al. 2006) [10].

Similarly, the term \((A_i^t)\) in Equation 5 shows that a company is able to speed up the growth of its buyers by advertising (Gupta et al. 2006).

The second term in Equation 4 represents the market share loss according to competitive pricing.

Similarly, the second term in Equation 5 represents the market share loss due to competitive advertising.

The direct network or word-of-mouth effect between sellers is denoted by the third term in Equation 4. (Gupta et al. 2006) [10].

Similarly, the direct network or word-of-mouth effect between buyers is represented by the third term in Equation 5 (Gupta et al. 2006) [10]

The more company obtains buyers, the more the sellers tend to join this company. On the other hand, when a company has more sellers, the buyers become more interested in this company. This network effect is known as the indirect network effect, and it is represented by the fourth term in Equations 4 and 5 [9].

Direct and indirect effects between competitors’ buyers and sellers are captured by fifth and sixth terms in Equation 4 and Equation 5.

4. Analysis and results. In this section, a numerical example and model validation are provided to demonstrate the proposed model capabilities.

4.1. Numerical example. To illustrate the application of the model, a triopoly is examined. Input parameters are highlighted in Table 2. A set of hypothetical data is employed in this numerical example and. In order to examine the correctness of our model, the data used in this section are extracted from three online stores. The Genetic Algorithm is considered to solve the problem. The Genetic Algorithm is a potential search process for optimization developed based on a natural selection mechanism. In this optimization, each member of the population represents an answer for a problem under examination, and each member is called a chromosome. A chromosome grows in a repeated sequential process called generation. Based on each new generation, the chromosome is transformed by the crossover and mutation operators and evaluated by the fitness function. During this process, the generations converge towards the best chromosome due to the chromosomes’ fitness function. Finally, this best chromosome is accepted as the desired answer to the problem. In fact, quickness in decision-making has led us to use the metaheuristic GA method.
to solve the model instead of conventional methods and deterministic optimization methods. The procedure of GA is illustrated in Figure 2.

**Chromosome:** In this problem, each chromosome means an answer to the problem, consisting of the sum of decision variables. The eight model variables are considered in this study as total.

**Fitness function:** In optimization problems, the objective function is a fitness function that is considered to maximize profits in this study.

**Generation:** The repetition of an algorithm that leads to creating a new population is called a generation.

**Population:** A set of chromosomes is called a population. A variety of populations with 100, 200, 500, 1000, and 10000 chromosomes are employed to find the best possible answer.

**Genetic operators:** A vital part of the genetic algorithm is creating new chromosomes called children through some old chromosomes called parents. This critical process is carried out by genetic operators, which are as follows:

- **Intersection operator:** It is considered the primary operator that leads to creating new chromosomes in GA. The intersection operators used in this study consist of Scattered, Two-point, and Heuristics.
- **Mutation operator:** The mutation operators employed in this study consist of Constraint dependent, Gaussian, and Adaptive feasible.
Table 2. The input parameters value

| Parameter | Value |
|-----------|-------|
| $M_A$ | 1,000,000 |
| $M_S$ | 300,000 |
| $B_1$ | 0.5 |
| $b_2$ | 0.5 |
| $c_1$ | 0.5 |
| $c_2$ | 0.5 |

$N_{B_1}^0 = N_{B_2}^0 = N_{B_3}^0 = 20$

$N_{S_1}^0 = N_{S_2}^0 = N_{S_3}^0 = 20$

$r_1 = r_2 = r_3 = 0.6$

$\alpha_P = 0.1$

$\alpha_A = 0.1$

$\alpha_{P_i} = 1$

$\alpha_{A_i} = 1$

Figure 3. Number of buyers and the number of sellers of company 1 in 30 months

With the aim of maximizing the profit of Company 1, the simulation is conducted by using the Genetic Algorithm. Figures 3, 4, 5, 6 demonstrate the results of this simulation.

The results demonstrate that in a period of 30 months, the cost of advertising and pricing is not stable, and it is also completely variable over time. As it can be seen from the results, this cost depends on the number of sellers, buyers of the company, competitors, and the pricing and marketing strategies of competitors. This is an essential issue that the companies’ managers must pay significant attention to.

4.2. Model validation. To illustrate the model validation, three online stores’ data have been used.

• First, Based on the change in marketing and pricing values, the values of $b_1, b_2, c_1, c_2$, etc., have been estimated.
Second, estimated parameters are used as model input parameters (See Figure 7)

As shown in Figure 7, a good agreement between the actual data and simulation results is observed. Thus, the practicality of the proposed model and its feasibility has been proved. This model can also be used for many applications.

5. Conclusion. In summary, this paper argued that the customers’ network effects are not less important than the customer lifetime value. However, customer retention is playing a prominent role in the company’s profitability; the number of customers attracted by the existing customers is worth taking into account. Thus, a model is presented to calculate customer lifetime value by considering network effects. For this purpose, an oligopoly market is considered in which the companies compete with each other. Each company has some buyers and sellers, and offers the services to the buyers free, and receives membership fees from the companies and vendor brands. An optimal control model is presented based on the communication between Buyers and Buyers, Buyers and Sellers, Sellers and Sellers, Pricing, and Marketing Strategy for each company and Its Competitors. Through the use of differential game theory and Genetic Algorithm, the marketing and pricing policies of each company are determined. To illustrate the application of the model, a triopoly market is examined. Besides, a numerical example is considered, and some hypothetical data are employed to prove the proposed model’s correctness. The main results regarding this paper are as follows:
• The cost of advertising and pricing depends on the time, and it is not a stable parameter (See Figures 3,4,5,6).
• The number of sellers, buyers of a company and competitors, pricing, and marketing strategies of competitors, are considered the main parameters that affect the cost of advertising and pricing (See Figures 3,4,5,6).
• Since a good agreement has been observed between the actual data and simulation results, the proposed model is reliable in terms of correctness, accuracy, and practicality.
• Since the importance of customer effects is highlighted in this paper, in order to raise a company’s profitability, the manager can also consider this effect.
• Since a company’s managerial decisions in an oligopoly market influence the other companies’ decisions and the proposed model correctness has been proved, this model can be of interest to the managers aiming to be successful in this competing market.

Since, in this study, the total number of buyers and sellers in a fixed industry is considered. Future research can consider the potential effects of buyers’ churn and sellers’ churn from the industry. Also, in this research, companies and buyers are considered quite similar. It is suggested that the next researches focus on different sellers and buyers. By relaxing constraints on growth by one customer, his/ her lifetime value can impute to the firm. In addition, this model can also be extended to multi-sided markets. For example, YouTube is a three-sided market with viewers, content providers, and advertisers.
Figure 6. Number of buyers and the number of sellers of company 3 in 30 months
Figure 7. Number of buyers and the number of sellers of company 3 in 30 months
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