Stabilized NMSSM without Domain Walls

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Abstract
We reconsider the Next to Minimal Supersymmetric Standard Model (NMSSM) as a natural solution to the $\mu$-problem and show that both the stability and the cosmological domain wall problems are eliminated if we impose a $\mathbb{Z}_2$ $R$-symmetry on the non-renormalizable operators.

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The $N = 1$ supersymmetric extension of the Standard Model provides a well defined framework for the study of new physics beyond it [1]. The low energy data support the unification of gauge couplings in the supersymmetric case in contrast to the standard case. The Minimal Supersymmetric extension of the Standard Model (MSSM) is defined by promoting each standard field into a superfield, doubling the higgs fields and imposing $R$-parity conservation. The most viable scenario for the breaking of supersymmetry at some low scale $m_s$, no larger than $\sim 1 \text{ TeV}$, is the one based on spontaneously broken supergravity. Although this scenario does not employ purely gravitational forces but could require the appearance of gaugino condensates in some hidden sector, it is usually referred to as gravitationally induced supersymmetry breaking. The resulting broken theory, independently of the details of the underlying high energy theory, contains a number of soft supersymmetry (susy) breaking terms proportional to powers of the scale $m_s$. Probably the most attractive feature of the MSSM is that it realizes a version of “dimensional transmutation” where radiative corrections generate a new scale, namely the electroweak breaking scale $M_W$. This is a highly desirable, but also non-trivial, property that is equivalent to deriving $M_W$ from the supersymmetry breaking scale as opposed to putting it by hand as an extra arbitrary parameter. Unfortunately, a realistic utilization of radiative symmetry breaking [2] in MSSM requires the presence of the so called $\mu$-term coupling directly the higgs fields $H_1$ and $H_2$, namely $\mu H_1 H_2$, with values of the theoretically arbitrary parameter $\mu$ close to $m_s$ or $M_W$. This nullifies all merits of radiative symmetry breaking since it reintroduces an extra arbitrary scale from the back door. Of course, there exist explanations for the values of the $\mu$-term, alas, all in extended settings [3].

At first glance, the most natural solution to the $\mu$-problem would be to introduce a massless gauge singlet field $S$, coupled to the higgs fields as $\lambda S H_1 H_2$, whose vacuum expectation value (vev) would turn out to be of the order of the other scales floating around, namely $m_s$ and $M_W$. This leads to the simplest extension of the MSSM the so called “Next to Minimal” SSM or NMSSM [4] with a cubic (renormalizable) superpotential

$$W_{ren} = \lambda S H_1 H_2 + \frac{\kappa}{3} S^3 + Y^{(u)} Q U^c H_1 + Y^{(d)} Q D^c H_2 + Y^{(e)} L E^c H_2.$$  \hspace{1cm} (1)

Unfortunately, the above scenario runs into difficulties. As can be readily seen the NMSSM at the renormalizable level possesses a discrete non-anomalous $Z_3$ global symmetry under which all superfields are multiplied by $e^{2\pi i/3}$. The discrete symmetry is broken during the phase transition associated with the electroweak symmetry breaking in the early universe and cosmologically dangerous domain walls are produced. These walls would be harmless provided they disappear effectively before nucleosynthesis which, roughly, requires the presence in the effective potential of $Z_3$-breaking terms of magnitude

$$\delta V \sim O(1 \text{ MeV})^4 \sim 10^{-12} \text{ GeV}^4.$$  

Such an estimate is not very different from the more elaborate one [5]

$$\delta V \sim 10^{-7} v^3 M_W^2 / M_P,$$

where $v$ is the scale of spontaneous breaking of the discrete symmetry and $M_P \sim 1.2 \times 10^{19} \text{ GeV}$ is the Planck mass. The above magnitude of $Z_3$-breaking seems to correspond
to the presence in the superpotential or in the Kähler potential of $Z_3$-breaking operators suppressed by one inverse power of the Planck mass. However, these $Z_3$-breaking non-renormalizable terms involving the singlet $S$ were shown \[3\] to induce quadratically divergent corrections\[3\] which give rise to quadratically divergent tadpoles for the singlet \[3\]. Their generic form, cut-off at $M_P$, is

$$\xi m_s^2 M_P (S + S^*),$$

where $m_s$ is the scale of supersymmetry breaking in the visible sector. The value of $\xi$ depends on the loop order of the associated graph (two or three in this case) which, in turn, depends on the particular non-renormalizable term that gives rise to the tadpole. Such terms lead to a vev for the light singlet $S$ much larger than the electroweak scale. Thus, it seems that the non-renormalizable terms that are able to make the walls disappear before nucleosynthesis are the ones that destabilize the hierarchy.

The purpose of the present article is to address the two problems of domain walls and destabilization that arise in the NMSSM and show that, despite the impass that the previous arguments seem to indicate, there is a simple way out rendering the model a viable solution to the $\mu$-problem. The crucial observation is that due to the divergent tadpoles a $Z_3$-breaking operator could have a much larger effect on the vacuum than its dimension naively indicates. Thus, it is conceivable that non-renormalizable terms suppressed by more than one inverse powers of $M_P$ are able to generate linear terms in the effective potential which are strong enough to eliminate the domain wall problem although, at the same time, they are too weak to upset the gauge hierarchy. Clearly, it would be very helpful to obtain a better understanding of both the symmetries that could be imposed on the model and the magnitude of destabilization that the various non-renormalizable operators generate.

The renormalizable part of the NMSSM superpotential (1) possesses the following global symmetries:

$$U(1)_B : Q(\frac{1}{3}), U^c(-\frac{1}{3}), D^c(-\frac{1}{3}), L(0), E^c(0), H_1(0), H_2(0), S(0)$$

$$U(1)_L : Q(0), U^c(0), D^c(0), L(1), E^c(-1), H_1(0), H_2(0), S(0)$$

$$U(1)_R : Q(1), U^c(1), D^c(1), L(1), E^c(1), H_1(1), H_2(1), S(1)$$

(where in parenthesis is given the charge of the superfield under the corresponding symmetry). The last $U(1)$ is an anomalous $R$-symmetry under which the renormalizable superpotential $\mathcal{W}_{ren}$ has charge 3. The soft trilinear susy-breaking terms break the continuous $R$-symmetry $U(1)_R$ down to its $Z_3$ subgroup that we mentioned earlier which, however, is not an $R$-symmetry. We see that the renormalizable part of the model possesses a genuinely discrete symmetry whose spontaneous breakdown produces domain walls.

Of course, one does not have to impose all the above continuous symmetries in order to obtain the renormalizable superpotential $\mathcal{W}_{ren}$ of the NMSSM. The same $\mathcal{W}_{ren}$ can be

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1These non-renormalizable terms appear either as $D$-terms in the Kähler potential or as $F$-terms in the superpotential. The natural setting for these interactions is $N = 1$ Supergravity spontaneously broken by a set of hidden sector fields.
obtained if we impose a discrete symmetry. There are various choices among which it is useful to consider two interesting possibilities:

a) $\mathbb{Z}_2^{MP} \times \mathbb{Z}_3$. The matter parity $\mathbb{Z}_2^{MP}$ is generated by

$$
\mathbb{Z}_2^{MP} : (Q, U^c, D^c, L, E^c) \rightarrow -(Q, U^c, D^c, L, E^c), \quad (H_1, H_2, S) \rightarrow (H_1, H_2, S)
$$

and the $\mathbb{Z}_3$ symmetry by

$$
\mathbb{Z}_3 : (Q, U^c, D^c, L, E^c, H_1, H_2, S) \rightarrow e^{2\pi i/3}(Q, U^c, D^c, L, E^c, H_1, H_2, S).
$$

Note that $\mathbb{Z}_3 \subset U(1)_R$, as already mentioned. Both $\mathbb{Z}_2^{MP}$ and $\mathbb{Z}_3$ are not $R$-symmetries ($\cal W \rightarrow \cal W$).

b) $\mathbb{Z}_2^{MP} \times \mathbb{Z}_4^{(R)}$. The matter parity $\mathbb{Z}_2^{MP}$ generator is defined as in the previous case. The $\mathbb{Z}_4$ $R$-symmetry $\mathbb{Z}_4^{(R)} \subset U(1)_R$ generator is defined by

$$
\mathbb{Z}_4^{(R)} : (Q, U^c, D^c, L, E^c, H_1, H_2, S) \rightarrow i(Q, U^c, D^c, L, E^c, H_1, H_2, S), \quad \cal W \rightarrow -i\cal W.
$$

Although it makes no difference which of the above symmetries are imposed on the renormalizable superpotential, we should make sure that the $\mathbb{Z}_3$ symmetry, or any other symmetry containing it, is not a symmetry of the non-renormalizable operators. If $\mathbb{Z}_3$ invariance is imposed on the complete theory the domain walls will not disappear. In contrast, the $\mathbb{Z}_4^{(R)}$ symmetry can be imposed on the non-renormalizable operators and no domain walls associated with its breaking will form because the soft susy-breaking terms break $\mathbb{Z}_4^{(R)}$ completely.

Let us now move to the other important issue that has to be addressed in the presence of the gauge singlet superfield $S$, namely the destabilization of the electroweak scale due to quadratically divergent tadpole diagrams involving non-renormalizable operators which generate in the effective action linear terms of the type (2). As mentioned, such terms lead to a vev for the light singlet which, in general, is much larger than the electroweak scale. Abel [7] has shown that the potentially harmful non-renormalizable terms are either even superpotential terms or odd Kähler potential ones. Such terms are easily avoided if we impose on the non-renormalizable operators a $\mathbb{Z}_2$ $R$-symmetry $\mathbb{Z}_2^{(R)}$ under which the superpotential as well as all superfields flip sign. This symmetry is a subgroup of both $U(1)_R$ and $\mathbb{Z}_4^{(R)}$. Therefore, one has the option of imposing on all operators a symmetry $\mathbb{Z}_2^{MP} \times \mathbb{Z}_4^{(R)}$ or $\mathbb{Z}_2^{MP} \times \mathbb{Z}_2^{(R)}$ or just $\mathbb{Z}_2^{(R)}$ assuming in the last two cases that the renormalizable superpotential has accidentally a larger symmetry.

Notice that the non-renormalizable terms allowed by $\mathbb{Z}_2^{(R)}$ or $\mathbb{Z}_4^{(R)}$, although not harmful to the gauge hierarchy, are still able to solve the $\mathbb{Z}_3$-domain wall problem since they generate in the effective action through $n$-loop tadpole diagrams linear terms of the form

$$
\delta V \sim (16\pi^2)^{-n}m_3^3(S + S^*).
$$

These terms are small to upset the gauge hierarchy but large enough to break the $\mathbb{Z}_3$ symmetry and eliminate the domain wall problem. For example, the presence of the term $S^7/M_\chi^4$
in the superpotential, allowed by both symmetries $\mathbb{Z}_2^{(R)}$ and $\mathbb{Z}_4^{(R)}$, is able to generate at four loops such a harmless linear term, as shown by Abel [7].

Combining all the above we see that by adopting the renormalizable superpotential (1) of the NMSSM and imposing on the non-renormalizable operators just a $\mathbb{Z}_2^R$-symmetry $\mathbb{Z}_2^{(R)}$ we are able to solve both the cosmological and the stability problems of the model[^2]. Thus, NMSSM can be finally regarded as a solution to the $\mu$-problem of the MSSM without invoking non-minimal Kähler potentials coupling directly visible and hidden sector fields.

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References

[1] H.-P. Nilles, Phys. Rep. 110 (1984) 1; H. E. Haber and G. L. Kane, Phys. Rep. 117 (1985) 75; A. B. Lahanas and D. V. Nanopoulos, Phys. Rep. 145 (1987) 1.

[2] L. E. Ibañez and G. G. Ross, Phys. Lett. 110 (1982) 215; K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Progr. Theor. Phys. 68 (1982) 927, 71 (1984) 96; J. Ellis, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B121 (1983) 123; L. E. Ibañez, Nucl. Phys. B218 (1983) 514; L. Alvarez-Gaumé, J. Polchinski and M. Wise, Nucl. Phys. B221 (1983) 495; J. Ellis, J.S. Hagelin, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. B125 (1983) 275; L. Alvarez-Gaumé, M. Claudson and M. Wise, Nucl. Phys. B207 (1983) 96; C. Kounnas, A. B. Lahanas, D. V. Nanopoulos and M. Quiros, Phys. Lett. B132 (1983) 95; Nucl. Phys. B236 (1984) 438; L. E. Ibañez and C. E. Lopez, Phys. Lett. B126 (1983) 54, Nucl. Phys. B233 (1984) 511.

[3] L. Hall, J. Lykken and S. Weinberg, Phys. Rev. D27 (1983) 2359; J. E. Kim and H.-P. Nilles, Phys. Lett. B138 (1984) 150; G. F. Giudice and A. Masiero, Phys. Lett. B206 (1988) 480; E. J. Chun, J. E. Kim and H.-P. Nilles, Nucl. Phys. B370 (1992) 105; I. Antoniadis, E. Gava, K. S. Narain and T. R. Taylor, Nucl. Phys. B432 (1994) 187; C. Kolda, S. Pokorski and N. Polonsky, hep-ph/9803310.

[4] P. Fayet, Nucl. Phys. B90 (1975) 104; H.-P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B120 (1983) 346; J.-M. Frere, D. R. T. Jones and S. Raby, Nucl. Phys. B222 (1983) 11; J.-P. Derendinger and C. A. Savoy, Nucl. Phys. B237 (1984) 307; B. R. Greene and P. J. Miron, Phys. Lett. B168 (1986) 226; J. Ellis, K. Enqvist, D. V. Nanopoulos,

[^2]: Incidentally notice that $\mathbb{Z}_2^{(R)}$ (or $\mathbb{Z}_4^{(R)}$) eliminates all dimension five operators leading to fast proton decay.
K. A. Olive, M. Quiros and F. Zwirner, Phys. Lett. B176 (1986) 403; L. Durand and J. L. Lopez, Phys. Lett. B217 (1989) 463; M. Drees, Intern. J. Mod. Phys. A4 (1989) 3645; J. Ellis, J. Gunion, H. Haber, L. Roszkowski and F. Zwirner, Phys. Rev. D39 (1989) 844;

U. Ellwanger, M. Rausch de Traubenberg and C. A. Savoy, Phys. Lett. B315 (1993) 331, Z. Phys. C67 (1995) 665, Nucl. Phys. B492 (1997) 21; T. Elliott, S. F. King and P. L. White, Phys. Lett. B351 (1995) 213; S. F. King and P. L. White, Phys. Rev. D52 (1995) 4183.

[5] S. A. Abel, S. Sarkar and P. L. White, Nucl. Phys. B454 (1995) 663.

[6] H.-P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B124 (1983) 337; A. B. Lahanas, Phys. Lett. B124 (1983) 341; U. Ellwanger, Phys. Lett. B133 (1983) 187; J. Bagger and E. Poppitz, Phys. Rev. Lett. 71 (1993) 2380; J. Bagger, E. Poppitz and L. Randall, Nucl. Phys. B455 (1995) 59; V. Jain, Phys. Lett. B351 (1995) 481.

[7] S. A. Abel, Nucl. Phys. B480 (1996) 55.