An Analytical Approach for the Design of Innovative Hairpin Winding Layouts
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Abstract — This work deals with an analytical approach aimed at accurately predicting Joule losses in innovative hairpin winding layouts. While hairpin windings are seeing an ever-increasing use in automotive and aerospace applications due to their inherently high slot fill factor, they also present drawbacks such as the non-uniform current distribution potentially occurring across their cross section. This phenomenon is emphasized at high frequencies, leading to a significant increase of the effective conductor resistance and, consequently, of copper losses. Hence, particular attention has to be given to the design of electrical machines employing hairpin conductors, aiming to reduce the high-frequency losses as much as possible.

In this paper, an analytical model based on previous investigations is updated and modified in order to increase the degrees of freedom in the design and analysis of hairpin windings. With the developed analytical model, the copper losses associated to innovative hairpin configurations can be accurately predicted. The findings also confirm that such alternative layouts can effectively reduce the Joule losses when compared to traditional hairpin technologies.

Index Terms— hairpin, analytical model, AC losses, asymmetric windings, segmented windings, high-speed motors

I. INTRODUCTION

In recent years, the trend in electric motors (EMs) production is to design smaller and more powerful, i.e., high power density, devices to meet stringent requirements demanded nowadays especially by the traction-drive world.

There are several ways to increase power density in an EM [1]. A very effective way consists in maximizing the amount of copper within the EM’s slot or, in other words, the slot fill factor. This is defined as the ratio between the amount of electric conductor material and the available winding space. Increasing this ratio means that, for a given slot area, more copper material can be inserted within the slot, thus resulting in a number of possible improvement implications (e.g., higher number of conductors, reduced machine size, reduced resistance, etc.) which finally would lead to more power dense devices.

Therefore, several works have focused on increasing the fill factor in EMs equipping random wound, round windings. Orthocyclic and layer windings can achieve fill factors in the range of 65-70% [1], but they need specific and expensive machineries. Other techniques have been proposed, such as pressing tooth-wound coils teeth [3], obtaining fill factors up to 75%, but they are limited to concentrated windings.

On the other hand, using conductors with rectangular cross sections inherently represents a means to achieve high fill factors in EMs. In fact, when the slot features parallel sides, the rectangular shape of the conductors perfectly matches that of the slot. This technology, already in use in automotive applications [4], besides ensuring high fill factors, provides a good industrial automation for mass production [5].

The need for large production scale is forcing the automotive industry to push towards fully automated processes for the manufacturing of EMs. In this context, hairpin windings match this high automation requirement and can be a valid solution for mass production replacing random windings [6], [7]. Although hairpin windings enable the aforementioned opportunities, they present several limitations from both an electromagnetic [8] and technological point of view [9], [10].

In fact, due to their large cross section, hairpins suffer at high frequencies of high (AC) losses due to skin [11] and proximity effects [12], [13]. These high-frequency, parasitic effects, combined together, make the current flow through the area farthest from the center of the conductor, thus reducing the effective conductor’s cross section and, consequently, increasing the winding resistance and losses.

As an example, Fig. 1 shows a finite-element (FE) visualization of the coexistence of these two parasitic phenomena in wires featuring a round cross section, and how frequency can affect the non-uniform current distribution.

![Fig. 1 Non uniform current distribution due to the combined effects of skin and proximity phenomena: a) 500Hz b) 1000Hz.](image)

However, while round conductors can be subdivided in several sub-conductors (e.g., litz wires) to reduce such effects, the technological restrictions of hairpin conductors pose a
constraint on the physical size, i.e. their height cannot be smaller than ≈1mm. Such technological limitations are accentuated by the high number of bending and welding points needed to ensure the electric continuity of the winding [14].

Aiming to address the above challenges, a lot of research effort has been dedicated to the characterization of these windings and to the estimation of the associated AC copper losses [15], [16].

However, to date, there is no configuration able to drastically improve hairpin winding performance at high frequencies, and the available tools are only able to predict losses within conventional hairpin structures. In this work, a more flexible analytical tool is proposed, allowing to accurately evaluate losses within non-conventional hairpin structures, which eventually permit overcoming the current challenges associated to such type of winding.

II. CLASSICAL ANALYTICAL MODEL

As mentioned in Section I, analytical models aimed at predicting losses in hairpin conductors are already available in literature [15]-[17].

The classical model considers the basic scenario reported in Fig. 2, consisting of a rectangular solid conductor (outlined by the orange line) carrying sinusoidal current, surrounded on three sides by ferromagnetic material featuring infinite permeability and representing the slot (outlined by the black lines). The current creates a magnetic field strength $H$ and leakage flux, which crosses the slot and the conductor rectilinearly (i.e. along the x-axis) as shown in the figure, i.e. the magnetic field strength $H$ and the flux density $B$ have only $x$-axis components within the conductor, whereas the current density $J$ and the electric field $E$ have only $z$-axis components.

Generally speaking, AC losses in a rectangular conductor are defined as in (1), where $b_c$ and $h_c$ are respectively width and height of the conductor, $\sigma_c$ is its conductivity and $I$ is the active conductor (and core) length. Equation (1) shows that AC losses are strictly connected to the current density $J$. This means that reducing $J$ is equivalent to reducing motor losses.

$$P_{AC} = \int_0^{h_c} (jb_c \, dy)^2 \frac{l}{\sigma_c b_c \, dy} = \frac{b_c l}{\sigma_c} \int_0^{h_c} J^2 \, dy \quad (1)$$

When $k$ conductors fill the slot drawn in Fig. 1, the classical model returns a current density distribution along the y-axis (i.e. along the conductor height) defined as in (2), with $a$ and $\delta$ expressed as in (3) and (4). This model assumes that the current $I$ flowing inside the $k$ conductors is always the same, and this is a significant limitation when parallel paths are used.

$$J_{zk}(y) = a \frac{l}{b_c} \left[ k \cosh(ay) - (k - 1) \cosh(a(y - h_c)) \right] \quad (2)$$

$$a = \frac{(1 + i)}{\delta} \quad (3)$$

$$\delta = \frac{1}{\sqrt{\pi f \sigma \mu}} \quad (4)$$

To evaluate AC losses, usually a coefficient is used to correlate them to the DC losses, i.e. those associated to the “real” (i.e. at low frequencies) value of the conductor’s resistance. This coefficient, namely $K_r$, is defined in (5) as the ratio of AC to DC resistances.

$$K_{rk} = \frac{R_{AC}}{R_{DC}} = \frac{P_{AC}}{P_{DC}} = \frac{b_c^2 h_c}{I^2} \int_0^{h_c} J^2 \, dy \quad (5)$$

To simplify the study, in [17], the equivalent expression (6) for $K_r$ is provided, considering series connected conductors. In (6), the two terms $\varphi$ and $\psi$ are introduced and their expressions are given in (7) and (8). These two quantities are expressed as a function of the variable $\zeta$, which is defined in (9), where $\omega$ is angular speed and $\mu_0$ is the permeability of vacuum.

$$K_{rk} = k(k - 1) \psi(\zeta \varphi(\zeta)) + \psi(\zeta) \varphi(\zeta) \quad (6)$$

$$\varphi(\zeta) = \frac{\sinh(2\zeta) + \sin(2\zeta)}{\cosh(2\zeta) - \cos(2\zeta)} \quad (7)$$

$$\psi(\zeta) = \frac{\sinh(\zeta) - \sin(\zeta)}{\cosh(\zeta) + \cos(\zeta)} \quad (8)$$

$$\zeta = \frac{1}{2\omega \mu_0 \sigma_c} \frac{b_c}{b} \cdot h_c = \alpha \cdot h_c \quad (9)$$

The model presented above is normally used for predicting current densities and losses in solid conductors, thus including conventional hairpin windings. However, it is based on the assumption of having all the slot conductors connected in series, i.e. they carry the same current. This offers a low flexibility in terms of hairpin design, which indeed can be imagined differently from their conventional layout.

In the next section, two innovative hairpin configurations are first presented as possible means to reduce the AC losses. Then, an update of the classical analytical model is proposed taking into account such non-standard structures.

III. UPDATED ANALYTICAL MODEL

To reduce losses in hairpin windings several methods have been already developed, such as decreasing the height of the conductors without adding extra conductors or decreasing their size while increasing the number of conductors within a slot. The first solution (Fig. 3a) is not ideal for meeting the fill...
factor maximization target, since part of the slot is left empty [15]. The second option (Fig. 4) increases the manufacturing complexity, i.e. the number of conductors to bend and weld in the motor is higher [20].

Transposition of conductors is also often used to reduce losses, as studied in [20]. In Fig. 4, a comparison between non-transposed (in blue) and transposed (in red) windings is carried out in terms of current density, using the classical analytical model recalled in Section II. Fig. 4 demonstrates that when the conductors are not transposed (or transposed incorrectly), they behave as one bigger conductor with ensuing higher current densities. This effect obviously must be avoided since introduces unacceptable current densities which lead to additional losses and heat generation.

Additional techniques and winding layouts are currently under investigation at the University of Modena and Reggio Emilia, aiming to mitigate the negative aspects related to hairpin technologies. These include:

1. Hairpins with variable cross sections, i.e. variable thicknesses along the slot height in order to guarantee a constant value of $K_e$ inside the slot;
2. Hairpins split in parallel paths in proximity of the slot openings in order to reduce the current density in the most critical conductors.

To analyze such configurations, the classical analytical model needs to be slightly modified. More specifically, in (1) $h_i$ should not be considered a constant value anymore when referring to option 1, while (2) should account for different current values according to number of parallel paths selected, when referring to option 2. This means that the updated analytical model is based on the same foundation as the classical one, but the boundary conditions should vary for each conductor. This also implies that the model presented in (6) is no longer valid due to the changed boundary conditions. In light of this, it is much simpler to start from the general solution of the current density reported in (10), where $a$ is the quantity introduced in (9). To calculate the two constants $C_1$ and $C_2$, firstly (10) is differentiated thus leading to (11). Then (12), which derives from elaborating Maxwell’s equations, is used and evaluated using suitable boundary conditions.

$$J = C_1 e^{(1+j)ay} + C_2 e^{-(1+j)ay}$$  \hspace{1cm} (10)

$$\frac{\partial J}{\partial y} = (1+j)ae^{(1+j)ay}C_1 - (1+j)ae^{-(1+j)ay}C_2$$  \hspace{1cm} (11)

$$\frac{\partial J}{\partial y} = -j\omega \mu_0 \sigma c H$$  \hspace{1cm} (12)

In particular, the boundary conditions are at $y=0$, where $H=0$, and at $y=h_{c1}$, with $h_{c1}$ being the incremental height corresponding to the $i^\text{th}$ hairpin of the slot. At $y=h_{c1}$, $H$ must be evaluated taking into account that, of the $k$ hairpins inside the slot, $c$ of them are split in $d$ parallel paths, thus the current flowing inside $n=k-c$ hairpins is $I$, whereas the current $I/d$ flows in the remaining $c$ hairpins. Finally, equating (11) and (12) at $y=0$ and at $y=h_{c1}$, a system of 2 equations in the 2 unknowns (i.e. $C_1$ and $C_2$) is obtained and then solved.

$$A = \begin{pmatrix} (1+j)a & -(1+j)a \\ (1+j)ae^{(1+j)ah_{c1}} & -(1+j)ae^{-(1+j)ah_{c1}} \end{pmatrix}$$  \hspace{1cm} (13)

$$g = \begin{pmatrix} -j\omega \mu_0 \sigma c H |_{y=0} \\ -j\omega \mu_0 \sigma c H |_{y=h_{c1}} \end{pmatrix} = \begin{pmatrix} 0 \\ -j\omega \mu_0 \sigma c H_{c1} \end{pmatrix}$$  \hspace{1cm} (14)

In this way every conductor has its own calculation for the current density and this allows to envision and analyse more flexible winding layouts. A flowchart illustrating the developed methodology is shown in Fig. 5 for the sake of completeness.
IV. VALIDATION VIA FE ANALYSIS

To validate the updated analytical model, an FE analysis is carried out using MagNet® software. In particular, the aim of this section is firstly that of proving the limits of the classical approach when unconventional structures are employed. Then, the effectiveness of the modifications applied to such classical model is proven by analysing the alternative winding layouts mentioned in Section III.

A. Brief description of the model

To verify the model accuracy many geometries and winding layouts are tested. Two-slot models are built to simulate all the case-studies. Particular attention is devoted to the elementary hairpins: each hairpin is modelled as a solid coil to take into account the skin and proximity effects possibly taking place at high frequencies; also, the mesh used is particularly fine inside the conductors. A 2D representation of the built FE model is shown in Fig. 6. During the realization of the windings, the basic connection rules [20] of hairpin windings have been used with particular attention to their transposition. The FE simulations are carried out using the time-harmonic solver. These evaluations, performed at a fixed frequency, return the current density along the conductors.

The FE solutions have been compared with those obtained with both the classical and the updated approaches.

Fig. 6 Two-slot FE model: a) geometry and 2) corresponding mesh

B. Results

As mentioned above, the aims of this section are to prove that the classical analytical model cannot correctly predict losses in certain cases and that the upgraded model is able to return losses in good agreement with FE, even when unconventional hairpin arrangements are considered.

All the simulation results presented hereafter are performed using the same slot geometry, the same supply frequency and current and the same hairpin width. These parameters are listed in Table I.

| TABLE I SIMULATION PARAMETERS |
| Parameter | Value |
| Frequency [Hz] | 1000 |
| Slot width [mm] | 3 |
| Slot height [mm] | 10 |
| Current [A] | 100 |
| Conductor width [mm] | 2.75 |

1) Classical model: analytical vs. FE results

When the slot conductors are all connected in series and feature identical cross sections, the classical hairpin winding is achieved, and the classical analytical approach described in Section II can be utilized for the loss evaluation. In addition, with such model, it is possible to predict losses in hairpins with variable cross section along the slot height, but only when the conductors are series-connected. Fig. 7 shows the case of 4 series-connected conductors with identical height equal to 2.25mm, i.e. the conventional hairpin winding. As expected, the classical analytical model is able to predict losses in a very accurate way when a conventional hairpin winding is analysed. In fact, the analytical results are in excellent agreement with the FE ones.

Fig. 7 Conventional hairpins: classical analytical model vs. FE results

By varying \( h_c \) along the slot height, i.e. hairpins with variable cross sections, and using the opportune \( h_c \) values in (2), the classical model, slightly manipulated, can be also used. Fig. 8 shows the comparison in terms of current density between analytical and FE results, when 4 slot conductors featuring variable cross section are investigated. Here, the two conductors in proximity of the slot bottom have a bigger cross section, i.e. 2.75x3mm, whereas the second group of two hairpins are thinner, with an area of 1.5x2.75mm. Also in this case, analytical and FE results are in excellent agreement.

Fig. 8 Hairpins with variable cross section: classical analytical model (slightly updated) vs. FE results

Unfortunately, when parallel paths for some of the elementary hairpins are considered, the classical model needs a further modification to allow for an accurate estimation of the current density, as explained in Section III. In particular, always considering the case with 4 series-connected conductors within the slot and splitting 2 of them (i.e. those in proximity of the slot opening) in 2 parallel paths, the classical model is still accurate for the non-split conductors, while it significantly overestimates the current density (and consequently losses) in the “segmented” hairpins. This concept is proven in Fig. 9, while a detail of the connection concept is reported in Fig. 10 for the sake of clarity. Besides
noticing that the classical model is inaccurate when considering such unconventional layout, it is also worth highlighting that this kind of winding is very promising in terms of loss reduction (see the FE curve in Fig. 9). Therefore, an upgrade of the classical analytical model is needed.

Fig. 9 Hairpins with parallel paths: classical analytical model vs. FE

Fig. 10 Winding connections for 4 series-connected equivalent hairpins, where the last two are split in 2 parallel paths

2) Updated analytical model: analytical vs. FE results
First the updated model described in Section III has been evaluated on the conventional hairpin configuration studied in Fig. 7. A very good match between FE and analytical results can be observed in Fig. 11.

Fig. 11 Conventional hairpins: updated analytical model vs. FE results

Fig. 12 Hairpins with parallel paths: updated analytical model vs. FE results

Moving to the analysis of the segmented hairpin layout, Fig. 12 proves that the new model can very accurately predict the current density within each conductor, including those split and connected in parallel. In Fig. 12, where the same case study as in Fig. 9 has been used, an almost perfect match between analytical and FE results is achieved, thus finally demonstrating the effectiveness of the updates applied to the classical approach.

V. CONCLUSIONS
In this study, updates and variations to the classical analytical model used for estimating current densities and AC losses within rectangular conductors were proposed.

The proposed model is still based on the classical approach but allows to flexibly vary the boundary conditions when unconventional winding layouts are investigated. With these modifications, the analytic model can accurately predict losses even in such innovative winding configurations. To prove its validity, several comparisons against FE results were shown.

The importance of having a fast, flexible and accurate model is further strengthened by the innovative winding arrangements presented in this paper as a means to overcome the challenges related to conventional hairpin windings, i.e. high AC losses. In particular, hairpins with variable cross sections or with parallel paths were investigated, showing very promising results in this perspective. Hairpins with parallel paths, when also transposed, are characterized by a smaller cross section and by a smaller current than the others. As a result, the segmented conductors near the slot opening feature a lower current density and, as a consequence, lower AC losses than the conventional hairpin windings.

Future researches will focus on the optimization and the practical feasibility of these unconventional winding layouts, with the final goal of reducing the AC losses and push the frequency limit at which hairpin windings can be employed.

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