Mathematical modelling and simulation machining of CNC milling of double-arc spiral internal bevel gear with nutation drive

Daizhi Xie, Ligang Yao*, Zhenya Wang and Gaosong Li
School of Mechanical Engineering and Automation, Fuzhou University, Fuzhou, Fujian, 350108, PR China
*Corresponding author’s e-mail: ylgyao@fzu.edu.cn

Abstract. The machining method of double-arc spiral internal bevel gear with nutation drive-by milling cutter was proposed in this paper. The mathematical modelling of milling cutter trajectory and axial cut-off of forming milling cutter was established, the special machining tool model of bevel gear was further developed by Solidcam, the simulation machining of bevel gear was carried out. Then, the gear model obtained after simulation was compared and analyzed with the theoretical gear model, and the ideal experimental results, confirmed the CNC modelling milling method processing technology could be used to process bevel gears, the research is very important to improve the development of nutation dynamic transmission.

1. Introduction
Nutation drive is a new form of mechanical transmission based on the principle of celestial planetary motion, nutation gear drive has the characteristics of high transmission efficiency, large transmission ratio, and smooth transmission [1], and is expected to be widely used in various types of transmission devices such as machine tools, instruments, petrochemicals, and aviation [2]. High efficiency, high precision, and high flexibility manufacturing technology is the goal pursued by the manufacturing industry [3]. Much of the current paper focuses on the structural design, gear mesh analysis, strength analysis, and transmission dynamics of chapter drive [4], while face milling is an important method for machining spiral bevel gears due to its high productivity and critical importance for producing high quality gears and minimizing manufacturing costs, but the high complexity of process kinematics has led to little research in this area [5]. As shown in Figure 1, due to the bending and twisting of the bevel gear tooth surface geometry, which may lead to the tool cutting surface embedded in the theoretical tooth surface, the material within the theoretical tooth surface will be removed, that is, the tool cutting surface on the theoretical tooth surface produced overcutting phenomenon. Although it has the commonality of tooth milling, but has its unique structure of individuality [6], the actual process of machining the parts is easy to cut the completed surface again, resulting in secondary cutting, which affects the smooth processing of tooth cutting and reduces the working performance of the gear.

2. Principle of CNC modelling and milling method processing technology
In this paper, the tooth grooves of the gear being machined are milled one by one using the CNC shaping method machining technique. The CNC shaping method machining technique [7-8] is to preset the machining feed rate, milling speed, etc., so that the geometric centre of the tool moves in a defined attitude along the already determined tool path to machine the tooth profile of the gear.
Figure 1. Internal bevel gear 3D model structure.

Figure 2. Basic tooth profile with double arc normal surface.

Figure 3. Double arc spiral bevel gear tooth surface diagram.

$O_0$ is the geometric centre point of the gear pitch plane, circular curves $bb'$ and $cc'$ are the plane gear tooth lines of the internal bevel gear, and curves $22'$ and $44'$ are the tool paths of the tool geometric centre reference point $nO$. As shown in Figure 4, the tool reference point $O_n$ moves along the curve $22'$ to the machine tooth line $bb'$. Then, the tool reference point $O_n$ moves along the curve $44'$ to the machine tooth line $cc'$ on the other side of the tooth surface. In Solidcam simulation programming, only one part of the program needs to be written, and the other part is obtained directly using the mirroring function. The expressions of tooth lines $bb'$ and $cc'$ can be calculated by the gear 3D model, and then combined with the structural parameters of the forming tool, to obtain the curves $22'$ and $44'$ equidistant from the tooth lines, and through Matlab numerical calculation, the coordinate points of the tool walking trajectory are obtained, combined with the coordinate changes, and finally the curves $22'$ and $44'$ are converted to the knuckle cone of the internal bevel gear that meshes with the circular plane [9], as shown in Figure 5, to form the actual simulated machining tool path.

3. Equation of tool trajectory

The tooth direction line is transformed by coordinates to the internal bevel gear to form the tool path line.

As shown in Figure 6, on the coronation gear pitch taper, the tooth orientation line is:

$$
\begin{align*}
    x_z &= e^{\rho \cos \beta} \cos(\theta - \theta_1 + \frac{\pi}{2}) \\
    y_z &= e^{\rho \cos \beta} \sin(\theta - \theta_1 + \frac{\pi}{2})
\end{align*}
$$
As shown in Figure 7, a fixed reference coordinate system \( S_0(i_0, j_0, k_0) \) and a dynamic coordinate system \( S_c(i_c, j_c, k_c) \) are established on the production wheel; a fixed reference coordinate system \( S_m(i_m, j_m, k_m) \) and a dynamic coordinate system \( S_2(i_2, j_2, k_2) \) are established on the inner bevel gear. The fixed reference coordinate system is used as the initial reference position of the dynamic coordinate system, the rotation speed of the production wheel is \( \omega_c \) and the rotation angle is \( \varphi_c \); the rotation speed of the internal bevel gear is \( \omega_2 \) and the rotation angle is \( \varphi_2 \). The direction of the axis \( i_m \) is opposite to that of the axis \( i_0 \) and the angle between the axis \( k_c \) and \( j_2 \) is \( \delta_2 \).

The transformation matrix of the dynamic coordinate system \( S_c(i_c, j_c, k_c) \) on the production wheel to the dynamic coordinate system \( S_2(i_2, j_2, k_2) \) on the internal bevel gear is:

\[
R_c^2 = \begin{bmatrix}
    b_{11} & b_{12} & b_{13} & 0 \\
    b_{21} & b_{22} & b_{23} & 0 \\
    b_{31} & b_{32} & b_{33} & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix}
\]

Among them, \( b_1 = -\cos \varphi_c \cos \varphi_c - \sin \varphi_2 \sin \delta_2 \sin \varphi_c \); \( b_2 = -\cos \varphi_2 \sin \varphi_c + \sin \varphi_2 \sin \delta_2 \cos \varphi_c \); \( b_{13} = -\sin \varphi_2 \cos \delta_2 \); \( b_{21} = \sin \varphi_2 \cos \varphi_c - \cos \varphi_2 \sin \delta_2 \sin \varphi_c \); \( b_{22} = \sin \varphi_2 \sin \varphi_c + \cos \varphi_2 \sin \delta_2 \cos \varphi_c \); \( b_{23} = -\cos \varphi_2 \cos \delta_2 \); \( b_{31} = \cos \delta_2 \sin \varphi_c \); \( b_{32} = -\cos \delta_2 \cos \varphi_c \); \( b_{33} = -\sin \delta_2 \).

This gives the tool path equation:

\[
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} = R_c^2 \begin{bmatrix}
x_2 \\
y_2 \\
0 \\
1
\end{bmatrix}
\]
4. Solution of cut-off shape of forming milling cutter

In the process of machining bevel gears, the milling face of the tool and the tooth face are spatially meshed. The forming milling cutter rotates around its axis and its reference point $O_n$ moves along the bevel bus, while the bevel gear rotates around its axis, and the relative velocity of the tool and tooth face at the contact point is perpendicular to the common normal of both contact points. The contact condition equation of forming milling is obtained, and then the contact condition equation is associated with the equation of tooth surface in the tool coordinate system, and the contact line expression of forming milling cutter and bevel gear is obtained by substituting the tooth surface parameters, and finally, the three-dimensional model of the milling cutter is obtained by rotating around the axis of the milling cutter.

The mathematical model for milling bevel gears with finger milling cutters is shown in Figure 8:

- Spatial fixed coordinate system $(o-i,j,k)$,
- Gear coordinate system $(o_i-i,j,k)$,
- Normal plane coordinate system $(o_n-i,j,k)$,
- Auxiliary coordinate system $(o_f-i,j,k)$,
- Tool coordinate system $(o_g-i,j,k)$,
- Auxiliary coordinate system $(o_r-i,j,k)$,
- Tool coordinate system $(o_m-i,j,k)$.

The tool reference point $o_n$ moves at equal speed from the large end of the gear to the small end along the knuckle cone bus with speed $c_v$. The distance between $o_n$ and $O$ is denoted by $c_R$. The angular velocities of the bevel gear and the tool rotating around their own axes are $i$ and $g$, respectively, and the angles of $f_k$ and $g_k$ of the tool are denoted by $\gamma$. The contact point $M$ between the bevel gear tooth surface and the cutting surface of the tool has a vector diameter $(g_r)$ in the tool coordinate system $(o_g-i,j,k)$ and a vector diameter $(i_R)$ in the gear coordinate system $(o_i-i,j,k)$.

For ease of calculation, let the third-order matrices of the transformation matrix $M_{on}, M_{wo}, M_{fo}, M_{go}, M_{fg}, M_{gf}$ be respectively $T_{on}, T_{wo}, T_{fo}, T_{go}, T_{fg}, T_{gf}$.

### Speed of tool movement:

$$v_c^{(e)} = \frac{dR_c}{dt} \vec{k}_c$$  \hspace{2cm} (3)

### Tool rotation speed:

$$v_{mg}^{(g)} = \alpha_g \vec{k}_g \times r_g$$  \hspace{2cm} (4)

### The speed at the point $M$ on the milling face of the tool:

$$v_m^{(g)} = v_c^{(g)} + v_{mg}^{(g)}$$  \hspace{2cm} (5)

### Velocity at the point $M$ on the tooth surface of the bevel gear:

$$v_i^{(g)} = T_{fg}^{-1}T_{gf}^{-1}(\alpha_f \times M_{mg}R_f^{(i)})$$  \hspace{2cm} (6)

### The relative velocity of tool and tooth surface at contact point $M$:

$$v_{mi}^{(g)} = v_m^{(g)} - v_i^{(g)}$$  \hspace{2cm} (7)

### The normal vector of point $M$ of the tooth surface in the tool coordinate system:

$$n^{(e)} = T_{fg}^{-1}T_{gf}^{-1}T_{fo}^{-1} \begin{pmatrix} n_{xi} \\ n_{yi} \\ n_{zi} \end{pmatrix}$$  \hspace{2cm} (8)

### Contact condition equation for form milling:

$$n^{(g)}_m \cdot v_{mi}^{(g)} = 0$$  \hspace{2cm} (9)
The contact condition equation and the tooth equation in the tool coordinate system are combined, and then the contact line expression between the milling cutter and the bevel gear can be obtained by substituting the tooth parameters, and the axial truncation can be obtained after numerical calculation as shown in Figure 9.

The mathematical equation for the axial truncation of the forming milling cutter:

\[
\begin{align*}
R &= \sqrt{x_g^2 + y_g^2} \\
z_g &= z_g
\end{align*}
\]

(10)

5. Gear 3D modelling
The main design parameters were obtained by searching national standards and performing theoretical calculations, as shown in Table I. Using MATLAB software, the bevel gear tooth equations, large-end bevel equations, and small-end bevel equations [10] are numerically calculated to obtain a series of coordinate points, and then imported into Solidworks, and through tooth trimming and post-processing, the accurate tooth profile of the double-arc spiral bevel gear is obtained, and combined with the main design parameters of the bevel gear, thus completing the accurate establishment of the three-dimensional model of the internal bevel gear. As shown in Figure 1.

6. Tools and their machining area
Figure 10(a) shows the flat-bottom cutter for initial tooth profile machining, leaving machining allowance for milling machining, and the milling machining parameters are set as shown in Table 2. Figure 10(b) shows the left and right convex tooth surface of forming cutter No. 1, Figure 10(c) shows the left and right transition arc of forming cutter No. 2, Figure 10(d) shows the left and right concave tooth surface of forming cutter No. 3, and Figure 10(e) shows the left and right tooth root arc of forming cutter No. 4. In the simulation process, different milling tools move along different tool trajectory lines, thus milling different tooth surfaces and finally completing the milling of the whole tooth profile.

| Table 1. Main design parameters of double arc spiral bevel gear. |
|---------------------------------------------------------------|
| **Parameter Name**               | **Numerical value**  |
| Nutation dynamic angle         | 5°                   |
| Spiral angle                   | 25°                  |
| Internal bevel gear knuckle taper angle | 127.81°            |
| Spiral cone tooth taper        | 50 mm                |
| Number of internal bevel gear teeth | 28                  |
| Spiral bevel gear small end normal face modulus | 2 mm                |
Table 2. Milling parameters setting.

| Projects                                                   | Parameters    |
|------------------------------------------------------------|---------------|
| Spindle speed                                              | 3500 r/min    |
| Retracting tool feed                                       | 2000 mm/min   |
| Retracting distance                                        | 20 mm         |
| Safe distance                                              | 2 mm          |
| Retreating knife safety distance                           | 2 mm          |

(a) Preliminary tooth profile machining for flat-bottom cutters.
(b) No.1 shaper for right convex tooth surface.
(c) No.2 forming tool for right transition circle surface.
(d) No.3 shaper for right concave tooth surface.
(e) No.4 shaping tool for the right tooth root rounding surface.

Figure 10. Different tools for different areas.
7. Preparation and implementation of simulation programs
Firstly, the machine tool model is built, the tool library model is established, then the theoretical 3D model of the bevel gear and the 3D model of the tooth embryo are imported, the geometric center of the tool is set, and finally, the appropriate machining origin is selected, the machining process is programmed, and the tooth embryo is milled. The errors between the simulation results and the theoretical model are analyzed to verify that the CNC profile milling method can be used for the machining of internal bevel gear tooth profiles.

Figure 11 shows the general structure of the simulation system. The system is composed of modules for gear model input, tooth embryo model input (as shown in Figure 12), machining coordinate system setting (as shown in Figure 13), machine tool building (as shown in Figure 14), tool library generation, machining parameter input, geometry pre-setting, operation procedure writing, machining process simulation, and simulation result processing.

8. Analysis of simulation results
During the milling simulation process, there was no interference, collision and secondary cutting, etc. When all the tooth profiles were milled and machined, the ideal internal bevel gear was obtained, as shown in Figure 15. Through the comparison operation between the blank and the workpiece, we can see that the error between the two is 0.01mm, the orange part is produced overcut and the green part is produced undercut. The method meets the requirements of national machining standards, from which
it can be concluded that the CNC forming and milling method machining technology can be used to machine bevel gears. During the machining process, the trajectory of the tool, the profile accuracy of the forming milling cutter, and the machine tool error [11] all affect the machining accuracy of the gears.

Figure 15. The results of the analysis of the blanks compared with the workpiece. Methods to reduce the error: use a reasonable calculation method to reduce the error of numerical calculation, thus reducing the motion trajectory, forming the error of the milling cutter profile; the motion trajectory of the milling cutter is achieved by the CNC machine tool of each CNC axis linkage, take more tool position points to ensure that the actual machining motion trajectory is close to the theoretical motion trajectory.

9. Conclusions
(1) It is known from the previous introduction: internal bevel gears are a very complex class of curved parts, which are difficult to design and process, and are processed by CNC molding and milling method to reverse the accuracy of 3D modeling. This provides accurate three-dimensional geometric models for tooth contact analysis, finite element stress analysis, and CNC machining of gears.
(2) Milling simulation processing through CNC molding milling method, instead of actual trial cutting, can carry out reasonable setting of processing parameters and verify the reasonableness of machine tool parameter debugging, and play the effect of optimizing the processing process, thus making it possible to save processing time, reduce production costs and improve processing efficiency in actual processing.
(3) In this paper, a unique shaping milling cutter is designed, and the ideal tooth surface is successfully machined, which proves the feasibility of the tool design. Compared with the method of using spherical milling cutter to machine complex surfaces, the machining error is reduced from the principle and the machining efficiency is improved at the same time.

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