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Maximum Likelihood Localization Method With MIMO-OFDM Transmission

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ABSTRACT In this research, we propose to estimate the location of mobile users by using the maximum likelihood (ML) method with statistical properties of the transmission signal angle of departure (AOD) and received signal strength (RSS) from access points (APs) to user equipment (UE). Location estimation (LE) is performed at each UE by using a signal from a multiple-input, multiple-output (MIMO) antenna system at the AP, which transmits specially designed, MIMO-orthogonal frequency division multiplexing (MIMO-OFDM), beamforming signals. The ML localization method is derived from statistical models of AOD and RSS of the OFDM signal. We also derive the theoretical root mean square error (RMSE) given the statistical models. Based on the results, the ML with the AOD and RSS methods has a lower RMSE than the other methods and can achieve close to the theoretical RMSE. The RMSE can also be significantly reduced by using a higher number of APs along with proper AP placement. In addition, the LE performance increases as the number of antennas and the number of subcarriers increases but with diminishing effectiveness. The developed RMSE calculation tool in this paper can be an important instrument to investigate and plan the deployment of APs for localization and can be further extended into larger-scale studies.

INDEX TERMS Localization, wireless, MIMO, OFDM, maximum likelihood.

I. INTRODUCTION

Localization or positioning systems are well-known technologies to detect the locations of mobile devices such as smartphones. Localization also plays a crucial part in the rise of technologies such as unmanned aerial vehicles, automated driving, automated factories, and the Internet of Things. It is expected that low-latency, centimeter-level accuracy, positioning systems will be an important key for further advancement of these technologies [1]. Even though the importance of localization is indisputable, its overdependence on global positioning systems (GPSs) [2] can limit its capability to function effectively in certain conditions. For example, satellite-based GPS signals suffer from significant attenuation when they arrive at a location with obstructions such as tall buildings in a dense city area or blocked by walls in an indoor location [3], [4].

Various solutions have been investigated to improve the performance of localization technology [4]. Wireless communication systems such as cellular networks have been considered to be potential candidates to enhance localization [5]. Using this approach, access points (APs) or base stations (BSs) are proposed to be used as both communication and positioning systems. Due to the ubiquitous nature of cellular networks, this method enables transmitters to be physically closer to receivers, which increases the likelihood of achieving line of sight (LOS) transmission along with stronger received signal strength (RSS) than satellite GPS signals in certain settings. These advantages are further amplified by the advancement of communication technology, which is moving toward high-density AP systems, such as small cells [6], cell-free systems [7] and heterogeneous networks [8].

Despite these advantages, it may not be practical to use APs or BSs from cellular networks to achieve localization using a similar strategy to that of the satellite-based GPS method. GPS uses sophisticated equipment to synchronize the signal transmission from satellites, which enables the user equipment UE to calculate its position by using the difference between the time of arrival of multiple GPS signals [2].
Since radio waves travel at the speed of light, the signal transmission needs to be synchronized with very stable atomic clocks; otherwise, there will be significant error in the positioning calculation. Clearly, it is not economically viable to apply similar satellite-based GPS technology to the AP-based localization method. One way to solve this issue is to use additional parameters to assist the location estimation (LE), namely, by using the signal power parameters [9]–[11] and transmission angle parameters [12], [13]. For the signal power parameters, LE is performed by using a received signal strength (RSS) acquired at the UE from the APs. From the RSS, the distance between the transmitter and receiver can be estimated with a suitable power-distance, path-loss model [9]. The RSS method can be improved by using a massive MIMO system or a very large number of antenna systems [15]. This is because a massive MIMO system can use a beamforming strategy to focus the transmission power at a certain target while reducing interference in nontarget directions, which in effect increases the signal-to-noise ratio and interference ratio (SNIR). In addition, a massive MIMO system can also improve angle-based LE. The basic idea of this method is to use the signal angle of departure (AOD) or angle of arrival (AOA) between the APs and the UE to triangulate the position of the UE. Since a massive MIMO system is known to have higher beamforming accuracy than a conventional MIMO system [12], this indicates that it can further improve the accuracy of the estimated angle parameters. Various LE approaches using massive MIMO systems have been investigated [12]–[18]. In [12], [13], the location and orientation of UE could be estimated by using one BS. The disadvantage of this method is that it must have a massive MIMO system at both the transmitter and receiver to estimate the transmission angle accurately. Due to space limitations, the mobile device or the UE must have a small number of antennas, while massive arrays can be placed at the BS or AP. In [14], massive MIMO BSs could estimate the UE location by processing the signal AOA at the BSs. Due to its cooperative nature, this indicates that it requires information exchange between multiple BSs to perform LE for each UE. This can be a problem if there are many UEs present in the area, which results in high calculation complexity. In [15]–[18], a massive MIMO system was used for LE with fingerprinting methods. The main concern of the fingerprinting or machine learning approach is that it requires data training and complex pattern recognition algorithms such as deep learning [16]. Hence, the calculation complexities can be a problem with such an approach. In [19], massive MIMO with an OFDM signal was designed to perform LE at the UE using joint RSS and AOD parameters. This method does not use the statistical properties of the RSS and AOD of the signal to improve the LE.

The main contribution of this paper is to investigate maximum likelihood LE using MIMO-OFDM signal transmission from APs to UEs. We derive the maximum-likelihood (ML) LE formula based on statistical models of RSS and AOD of the MIMO-OFDM signals with unique beamforming angles allocated to the subcarriers. Using the statistical models, we also derive the theoretical root mean square error (RMSE) of the LE and compare the results with the proposed maximum likelihood LE. Even though [19] uses RSS and AOD approach, it does not derive maximum likelihood LE to minimize RMSE given the statistical models. In addition, [19] does not derive the theoretical RMSE that enables performance calculation without using the more computationally heavy Monte Carlo simulation. To the best of our knowledge, no previous research has provided a similar calculation tool for this specific design of MIMO-OFDM signals. An additional contribution of this paper is to investigate the impact of various parameters on the performance of the proposed LE, namely, the number of APs, the position of APs, the number of antennas at each AP and the number of OFDM signal subcarriers.

Notation: $|\cdot|$, $\text{Im}(\cdot)$, $\text{Re}(\cdot)$ and $\angle(\cdot)$ are elementwise absolute value, imaginary value, real value, and angle of complex number, respectively. Boldface variables represent vectors, $\mathcal{N}(a, b)$ and $\mathcal{CN}(a, b)$ are Gaussian and complex Gaussian distributions, respectively, with mean $a$ and variance $b$, $\{\cdot\}_x$ is the $x$-th element of the vector or the set, $(\cdot)^H$ is a Hermitian transpose, $\circ$ is a Hadamard product, $j$ is an imaginary number and $\exp(\cdot)$ is an exponential value.
where $P_B$ is the average transmit power from the AP normalized to the average noise power. $g_v$ is a $1 \times M$ complex vector that represents the channel response between the AP and UE at delay $\tau$, $s_{n-\tau}$ is a $M \times 1$ OFDM signal transmitted at time $n-\tau \geq 0$, $w$ is a complex random Gaussian noise signal with a $CN(0,1)$ distribution, and $T$ is a set of delay values.

We use a block fading channel model where the channel response for each delay can be approximated as constant within a single coherence time. The channel response in (1) can be described as

$$g_v = \beta_v h_v$$

(2)

where $\beta_v$ is the large-scale fading component and $h_v$ is the $1 \times M$ small-scale fading vector. We separate the channel response (2) into two categories: LOS and NLOS. Each of these categories has different properties. We set the LOS signal as the reference delay time, which has a $\tau = 0$ delay, which indicates that any signal with $\tau > 0$ is considered NLOS. This makes sense because NLOS must take a longer propagation path than LOS to arrive at the receiver. The large-scale fading $\beta_v$ depends on the path loss model. The models for $\beta_{v=0}$ (LOS) and $\beta_{v>0}$ (NLOS) are discussed in the results section. The small-scale fading vector $h_v$ is affected by the time delay, antenna arrangement and transmission angle [19]. For the LOS signal, there is only one direct signal path between the transmitter and receiver and no attenuation from scattering effects. In the case of NLOS, there are multiple signal paths that depart from the AP antennas from multiple angles, as illustrated in Fig. 1, along with attenuation from scattering effects. In the multiple antenna system, different antennas have different signal phases. The phase differences between the antennas depend on the relative positions between them. The AOD of the signal and the delay of signal arrival can also affect the signal phase. As a result, the small-scale fading component in (2) can be written as steering vectors of multiple antenna systems as follows:

$$h(\tau) = \exp(j\phi(a,\theta,\tau)) \quad \text{for} \; \tau = 0,$$

(3)

$$h(\tau) = \frac{1}{\sqrt{L}} \sum_{l=1}^{L} \alpha_{v,l} \exp(j\phi(a,\theta,\tau)) \quad \text{for} \; \tau > 0.$$

(4)

where $\phi(a,\theta,\tau)$ is a $1 \times M$ phase function that depends on $1 \times M$ antenna positions of $a$, AOD of $\theta$ and signal arrival delay of $\tau$. $\alpha_{v,l}$ is a complex valued channel attenuation based on an exponential decay model, and values of $\tau$ for each path are generated based on a random Poisson distribution [21]. To simplify the notation in this paper, we use a complex number to represent the 2-dimensional (2D) coordinate system. The location of an antenna with index number $m$ is defined as

$$a_m = a_{m,x} + j a_{m,y},$$

(5)

where $a_{m,x}$ and $a_{m,y}$ are the x and y coordinates of antenna $m$, respectively, with the center of mass of the antenna system as the origin. The coordinates of all antennas for a certain AP are grouped into a vector form as

$$a = [a_1, a_2, \ldots, a_M].$$

(6)

Assuming that the same signal propagation path has the same AOD for all antennas of an AP, then the phase function, $\phi$, in (3) and (4) can be written as

$$\phi(a,\theta,\tau) = 2\pi f_C \left[ -|a| \cos(\angle(a) - \theta) \frac{1}{c} \tau + \tau \right].$$

(7)

where $f_C$ is the carrier frequency and $c$ is the speed of light. See [19] for the derivation. In this paper, we use a circular antenna arrangement with half wavelength spacing, $\lambda/2$. Therefore, using basic trigonometry, the vector $a$ can be expressed as

$$a = \left( \sin \left( \frac{\pi}{M} \right) \right)^{-1} \frac{\lambda}{4} \exp \left( \frac{2\pi j}{M} [1, 2, \ldots, M] \right).$$

(8)

### B. SIGNAL BEAMFORMING DESIGN

In this section, we describe the MIMO-OFDM signal beamforming design at the transmitter (AP), which enables LE at the receiver (UE). As discussed previously, in addition to using the RSS, the receiver must also be able to estimate the AOD of the signal from the transmitter. To achieve this, the OFDM signals are simultaneously beamformed toward isotropic horizontal directions with equal angle spacing, with each beamforming angle allocated to a unique subcarrier, as follows:

$$\theta = [\theta_1, \theta_2, \ldots, \theta_k, \ldots, \theta_N].$$

(9)

where $\theta_k = \frac{2\pi k}{N}$ is the beamforming angle for subcarrier index number $k$, and $N$ is the total number of beamforming angles, which is also equal to the total number of subcarriers that are used for beamforming. Note that the allocation of the beamforming angle, $\theta_k$, to the subcarrier index, $k$, does not necessarily need to be in linear order, as in (9). The arrangement can be at any random order if appropriate reordering is performed at the receiver. Once we allocate each beamforming angle to the specific subcarrier, then the beamforming vector at angle $\theta_k$ can be generated using

$$b_k = \exp(j\phi(a,\theta_k,0))^H.$$

(10)

where $\phi$ is defined in (7). $b_k$ is a $M \times 1$ vector to represent $M$ antennas in the beamforming transmission from the AP. This indicates that the $m$-th element of the vector, $[b_k]_m$, is an OFDM symbol that is allocated to the subcarrier $k$ at antenna number $m$.

The next step of OFDM transmission is to convert the signal from the subcarrier domain to the time domain. Using the inverse discrete Fourier transform (IDFT), the signal transmission vector from the AP with $M$ antennas at discrete time $n$ can be written as [22]

$$s_n = \frac{1}{N} \sum_{k=0}^{N-1} b_k \exp(j2\pi kn/N),$$

(11)

where $s_n$ is a $M \times 1$ subcarrier-domain signal vector. Since we aim to beamform the signal at angles defined in (9), this indicates that the value of $x_k$ is set to $x_k = b_k$ from (10). Note that (11) is the OFDM signal at the transmitter AP.
The OFDM signal that arrives at the UE can be acquired by substituting $s_n$ from (11) into (1). We assume that there is no intersymbol interference or inter-AP interference.

III. LOCALIZATION USING AOD AND RSS

In this section, we discuss the background concept of LE [19] without using the maximum likelihood method. The LE in this research is achieved using two parameters: the distance and the angle between the transmitter and receiver. The distance is estimated from the RSS using the path loss model, while the angle or AOD is estimated using the beamforming property of the MIMO-OFDM signal.

A. LOCALIZATION USING AOD

After the OFDM signal from the AP (11) is transmitted through the propagation channel Model (1), the signal is then processed at the UE from the time domain to the subcarrier domain by applying DFT. The processed signal at subcarrier $f$ can be written as [22]

$$X_f = \sum_{n=0}^{N-1} y_n \exp(-j2\pi fn/N).$$

where $y_n$ is the received signal at time $n$ (1). All symbols $X_f$ for different subcarrier values $f$ in (12) can also be compiled in vector form as $X = [X_1, X_2, \ldots, X_f, \ldots X_F]$.

This research depends on a LOS signal for LE. We expect that the beamformed signal angle that has the highest amplitude is the LOS signal angle because unlike the NLOS signal, the LOS signal energy is not absorbed by any surrounding material excluding the air through which it propagates. Since each subcarrier is allocated to a certain beamforming angle, we can find the AOD of the LOS by finding the subcarrier that has the highest amplitude, as follows:

$$k' = \arg\max_{k'} \{ |X|_k \}. \tag{13}$$

Using the subcarrier number $k'$ (15), the AOD can be estimated as

$$\hat{\theta} = \{ \theta \}_{k'}, \tag{14}$$

where $\theta$ is defined at (9).

To estimate the location of UE using (14), we need at least two AODs from two LOS signals that create intersection points at the UE, as shown in Fig. 2.

From Fig. 2, the estimated location of UE, $\hat{p}$, in terms of the complex number coordinate system, can be solved by using trigonometry as follows:

$$\begin{bmatrix} \text{Im} (\hat{p}) \\ \text{Re} (\hat{p}) \end{bmatrix} = \begin{bmatrix} 1, & -\tan (\hat{\theta}_1) \\ 1, & -\tan (\hat{\theta}_2) \end{bmatrix}^{-1} \times \left[ \begin{bmatrix} \text{Im} (q_1) - \tan (\hat{\theta}_1) \text{Re} (q_1) \\ \text{Im} (q_2) - \tan (\hat{\theta}_2) \text{Re} (q_2) \end{bmatrix} \right], \tag{15}$$

where $\hat{p} = \text{Re} (\hat{p}) + j\text{Im} (\hat{p})$, $\hat{\theta}_1$ and $\hat{\theta}_2$ are the estimated AODs from AP 1 and 2, respectively, and $q_1$ and $q_2$ are the coordinates for AP 1 and AP 2 using complex numbers, respectively.

B. LOCALIZATION USING RSS

The localization of UE using RSS can be achieved by using the OFDM signal received from at least one AP. Unlike the RSS-based localization methods that use large-scale path loss values [9]–[11], this research uses the RSS of OFDM subcarriers with LOS beamforming. Since each OFDM subcarrier is set to have a different beamforming direction, this indicates that the subcarrier with LOS propagation must have a higher amplitude than that of subcarriers with NLOS propagation. Therefore, using the processed signal at the receiver (12), the RSS of the LOS subcarrier signal can be estimated as

$$r = \max (X)/(MP_B). \tag{16}$$

The numerators in (16), $MP_B$, are parameters to normalize the RSS with transmit power and multiple antenna effects.

Next, the distance can be estimated using the path-loss model

$$d = wr^v, \tag{17}$$

where $w$ and $v$ are path loss parameters that depend on statistical observation. Then, we use $\hat{d}$ along with $\hat{\theta}$ from (14) to solve the UE location using AP location $q_1$ using simple trigonometry as follows:

$$\begin{bmatrix} \text{Im} (\hat{\hat{p}}) \\ \text{Re} (\hat{\hat{p}}) \end{bmatrix} = \begin{bmatrix} \text{Im} (q_1) + \hat{d} \sin (\hat{\theta}) \\ \text{Re} (q_1) + \hat{d} \cos (\hat{\theta}) \end{bmatrix}. \tag{18}$$

Technically, the calculation in (18) requires both AOD and RSS to be used to estimate the location. However, for simplicity, we termed this method the RSS method. The main difference between the AOD method in (15) and the method in (18) is that the former method does not require the RSS value.

IV. LOCALIZATION WITH MAXIMUM LIKELIHOOD

In section III, LE is achieved without considering statistical information of AOD and RSS, such as the standard deviation of measurement error. In this section, we extend the LE concept in section III with the maximum likelihood approach, which requires us to develop certain statistical models.
A. Statistical Models

In theory, if the relationship between distance, angle, RSS and AOD can be known and measured without error and interference, then location estimation can also be achieved without error. However, it is not possible to practically estimate these parameters without error. The error can arise due to various factors, such as interference from the multipath NLOS signal and imprecision of the MIMO-OFDM beamforming signal. By obtaining the probability distribution using the statistical analysis of the measurement error, we can use the maximum likelihood function to estimate the position of the device. Hence, in this section, we derive the statistical relationships of the measured RSS and estimated AOD with the actual distance and the actual AOD between the transmitter and receiver. We assume that the RSS and AOD between all APs and UEs have the same statistical properties.

In the case of RSS, \( r \), versus distance, \( d \), the relationship can be expressed as follows:

\[
\log r = a_1 \log d + a_2 + \epsilon,
\]

where \( r \) is the RSS from AP to UE measured in (16), \( d \) is the actual distance between AP and UE, and \( a_1 \) and \( a_2 \) are constants. \( \epsilon \) is the random variable error with mean 0 and standard deviation \( \sigma \). We calculate the values of \( a_1, a_2 \) and the statistical properties of \( \epsilon \) using numerical analysis. Specifically, we simulate the channel and signal model from section II across many channel realizations and use curve fitting to calculate \( a_1, a_2 \) and \( \sigma \).

Based on our simulation of the channel model, the standard deviation of \( \epsilon \), \( \sigma \), varies linearly with the log-distance, \( \sigma = b_1 \log d + b_2 \). In addition, \( a_1, a_2 \) and \( \sigma \) are affected by the number of antennas, \( M \), and the number of subcarriers of the signal, \( N \). This is expected because \( M \) and \( N \) can affect the beamforming precision, which also affects the distance measurement accuracy. For notation simplicity, we exclude the dependence on these parameters from the equation. However, we will discuss their effect in the results section. If we approximate \( \epsilon \) to have a normal distribution, then we can re-write (19) as a normal distribution function as follows:

\[
\log r = N(\mu_d, \sigma_d),
\]

where

\[
a_1 = \mu_d = a_1 \log d + a_2,
\]

\[
a_2 = b_1 \log d + b_2.
\]

In the case of the estimated AOD, \( \hat{\theta} \), versus distance, \( d \), the relationship can be written as

\[
\hat{\theta} = \theta + \epsilon,
\]

where \( \epsilon \) is the random variable error with 0 mean and \( \varphi \) standard deviation. Based on the simulation of the channel and signal model from section II and the estimated AOD (14), we observe that log \( \varphi \) varies linearly with log distance, log \( \varphi_d = c_1 \log d + c_2 \). Similar to \( r \) versus \( d \), \( \hat{\theta} \) versus \( \theta \) is also affected by parameters \( M \) and \( N \). For notation simplicity, we also exclude the dependence on these parameters from the equation, but we will include their effect in the results section. If we approximate \( \epsilon \) to have a normal distribution, then we can re-write (24) as a normal distribution function as follows:

\[
\hat{\theta} = N(\theta, \varphi_d),
\]

where

\[
\log \varphi_d = c_1 \log d + c_2.
\]

c_1 and \( c_2 \) are constants acquired from curve fitting of log \( \varphi_d \) versus log \( d \) using the simulated channel and signal model in section II and AOD estimation from section III. From (25), the PDF of the estimated AOD \( \hat{\theta} \) given actual AOD \( \theta \) at distance \( d \) can be expressed as

\[
\text{pdf}(\hat{\theta}|\theta, d) = \frac{1}{\varphi_d \sqrt{2\pi}} \exp\left(-\frac{(\hat{\theta} - \theta)^2}{2\varphi_d^2}\right).
\]

B. Maximum Likelihood Estimation

Theorem 1: Based on the statistical models in (23) and (27), the maximum likelihood of UE location given \( r_{i \in Q}, \hat{\theta}_{i \in Q}, q_{i \in Q} \) can be expressed as

\[
\hat{p} = \arg \max_p \left\{ L\left(p|r_{i \in Q}, \hat{\theta}_{i \in Q}, q_{i \in Q}\right) \right\},
\]

where

\[
L(p|r_{i \in Q}, \hat{\theta}_{i \in Q}, q_{i \in Q}) = \prod_{i \in Q} \frac{1}{2\pi \sigma_{p,q_i} \varphi_{p,q_i}} \times \exp\left(-\frac{(r_i - \mu_{p,q_i})^2}{2\sigma_{p,q_i}^2} - \frac{(\hat{\theta} - \theta_{p,q_i})^2}{2\varphi_{p,q_i}^2}\right).
\]

Proof: See Appendix A. Note that we have not found the closed-form solution for (28). However, the value of \( p \) that maximizes function (29) can still be acquired by using a maximum search algorithm such as gradient ascent.
In case the LE is performed by using RSS information only, then from (29), the likelihood $p$ is

$$L(p|\hat{r}_i, q_i) = \prod_{i \in Q} \frac{1}{2\pi \sigma_{r,q_i}} \exp\left(-\frac{(\log r_i - \mu_{r,p,q_i})^2}{2\sigma_{r,q_i}^2}\right).$$

(30)

In case the LE is performed by using AOD information only, then from (29), the likelihood of $p$ is

$$L(p|\hat{\theta}_i, q_i) = \prod_{i \in Q} \frac{1}{2\pi \psi_{r,q_i}} \exp\left(-\frac{(\hat{\theta} - \theta_{p,q_i})^2}{2\psi_{r,q_i}^2}\right).$$

(31)

Since the LE calculation is performed at each UE, preliminary statistical information must be known by the UEs. The UEs can acquire this information from the APs. Since there are a small number of parameters and the statistical properties of wireless channels do not change rapidly [23], the amount of preliminary data that need to be acquired by the UE to perform LE should be low. The calculation complexity of the proposed LE (28) is $O(\mathbf{AK}/\alpha)$, where $A$ is the search area to find the maximum value of (29), $K$ is the number of APs used in the LE and $\alpha$ is the accuracy level of the maximum search. Since [19] does not depend on the maximum search method, it has the calculation complexity of $O(K)$, which is less complex than the proposed method. However, generally mobile users do move very far from previous location within a single time step calculation. Therefore, the complexity of (28) can be reduced by limiting the search range $A$ within the vicinity of the previously estimated location.

C. THEORETICAL RMSE

In this section, we derive the theoretical RMSE based on probability error distributions derived from the statistical models of RSS and AOD. This will enable us to acquire the RMSE results without having to perform time-consuming Monte Carlo simulations. This derivation is considered an approximation since we assume that the PDF distributions of the relevant statistical models follow a normal distribution.

**Theorem 2:** The theoretical RMSE given the UE position of $p$ with statistical models (23) and (27) can be expressed as

$$\text{RMSE}_p = \sqrt{\int_{-\infty}^{\infty} (\hat{p} - p)^2 \text{pdf}(\hat{p}|p) \, d\hat{p}}.$$  

(32)

where $\text{pdf}(\hat{p}|p) = \prod_{i \in Q} \text{pdf}(\theta_i|p,q_i)$ and $\text{pdf}(\theta_i|p,q_i)$ is defined in (47). Proof: See Appendix B.

Due to the complexity of the statistical models in section III-A, we have not found an analytical solution for integral operations in (32). However, the integrals can be calculated using numerical method approximation. This is performed by approximating the integrals in (32) with summation terms, as follows:

$$\text{RMSE}_p = \sqrt{\frac{\sum_{\hat{p} \in P} (\hat{p} - p)^2 \text{pdf}(\hat{p}|p) \Delta\hat{p}}{\sum_{\hat{p} \in P} \text{pdf}(\hat{p}|p) \Delta\hat{p}]].}$$  

(33)

where $P$ is a set of all possible UE coordinates within a finite 2D range and $\Delta\hat{p}$ is the small constant area of intervals between adjacent coordinates in $P$. To obtain high accuracy RMSE simulation results, $P$ must have a large distance range, whereas $\Delta\hat{p}$ must have a small value. For this research, we choose a range of $p \pm 50m$ in both the $x$- and $y$-axis directions for $P$, and we use $10^{-4}m^2$ for $\Delta\hat{p}$.

V. RESULTS

For the simulation results, we set the antennas of each AP to have a circular arrangement with a half wavelength separation between adjacent antennas (9). The path loss exponent calculation between AP and UE is based on the ITU model [24], which is defined as

$$\text{Loss}(d) = 20 \log_{10} fc + N \log_{10} (d) + P_f - 28 \text{ dB},$$  

(34)

where $f_c$ is the carrier frequency in megahertz, $N$ is the distance power loss coefficient, $d$ is the distance between BS and UE and $P_f$ is the floor loss penetration factor ($P_f = 0$ for the same floor). For $N$, we set it to 20 for LOS (free space path loss) and 22 for NLOS (indoor commercial area) [24]. The carrier frequency is $f_c = 2$ Ghz. From the path loss exponent (34), we can calculate the large-scale fading between transmitters $\beta = 10^{-\text{Loss}(d)}$. The noise power at the receiver is standardized based on the Boltzmann formula [25]. The small-scale fading channel realization for NLOS signals is based on multipath channels that are generated according to the statistical indoor channel model from [21]. We set the multipath channels to contain 10 clusters of paths with an arrival rate of 1/17 ns and decay rate of 34 ns [14]. In addition, each cluster has 10 subpaths with an arrival rate of 1/5 ns and decay rate of 29 ns. Each NLOS signal cluster has random AODs. The number of useful subcarriers equals the number of discrete beamforming angles (9), which is $N = 2048$.

In the first test, the locations of APs and UEs are shown in Fig. 3.
In Fig. 3, the UEs are arranged between $-20 \text{ m}$ and $20 \text{ m}$ across the x-axis, and the spacing between the APs is set to $80 \text{ m}$ [14]. To test the performance of the proposed method, we measure the LE accuracy performance by using RMSE, which is based on the error between the estimated location and true location. The lower the RMSE is, the better the accuracy of LE. We use Monte Carlo simulation to acquire the RMSE for ML-AOD-RSS (29), ML-AOD (30), and ML-RSS (31) across many realizations of the channel Model (1) for each UE position. Additional comparisons are also performed with the AOD (15) and RSS (18) methods [19]. The theoretical RMSE can be calculated directly using statistical parameters without using Monte Carlo simulation (33). The results are given in Fig. 4.

![FIGURE 4. RMSE of various LE methods with certain UE positions and 2 APs located based on Fig. 3.](image)

Fig. 4 shows the performance measured in terms of RMSE for various LE methods using 2 APs. In general, the ML-RSS method performs the worst except at the center of the x-axis (approximately $0 \text{ m}$ at the x-axis). At the center, the AOD method performs the worst, followed by ML-AOD, which is the second worst. This is because in this region, the AOD and ML-AOD methods suffer from the parallel LOS signal effect, in which a small AOD estimation error in this region creates a large RMSE, as shown in Fig. 5.

![FIGURE 5. Parallel LOS effect that results in high estimation error.](image)

Fig. 5 shows that even though the angle deviation between the estimated and actual position 1 is higher than that of position 2, position 2 still has a higher LE error because the LOSs for position 2 are close to being parallel. In this position, even a small angle variation can create a large LE error. The ML-AOD also has less error than the AOD because the ML-AOD uses statistical information in addition to the estimated AOD to calculate the maximum likelihood of the UE position. The RSS method has a lower RMSE than the ML-RSS method. This is because the RSS method from [19] uses both RSS and AOD to find the estimated position, as discussed in section III-B, while the ML-RSS method only uses RSS. In general, the ML-AOD-RSS outperforms all other methods. The RMSE of this method is also very close to that of the theoretical RMSE. This indicates that the best performance can be achieved by combining AOD and RSS with statistical information using maximum likelihood calculations.

In the next test, we repeat the simulation from Fig. 4 by replacing 2 APs with 4 APs, as shown in Fig. 6.

![FIGURE 6. The positions of UEs and 4 APs.](image)

Note that using more than 2 APs to perform LE indicates that the AOD method may have more than one line of intersections to estimate the UE position. Since this will result in multiple estimated positions, we choose to use the median of the estimated positions. The results are given in Fig. 7.

![FIGURE 7. RMSE of various LE methods for various UE positions with 4 APs located based on Fig. 6.](image)

Fig. 7 shows that in general, using 4 APs has a lower RMSE than using 2 APs (Fig. 4) for all tested methods.
This is as expected since using 4 APs indicates that there is a more diverse source of signals to estimate the position, which results in less LE error. Significant improvement can be observed in the AOD and ML-AOD methods using 4 APs. This is because the parallel LOS effect that occurs in the AP arrangement in Fig. 3 can be avoided by using the AP arrangement in Fig. 6. The reason for this is that in the latter arrangement, there must be at least two combinations of APs that do not experience the parallel LOS effect for all UE locations with a reasonable distance from the APs. Hence, the high RMSE at the centre of the x-axis can be avoided using an appropriate number of APs along with the placement of the APs. Fig. 7 also shows that the performance of ML-AOD is very close to that of ML-AOD-RSS. This is very different compared to the Fig. 4 where adding RSS parameter can significantly improve the performance. This is because the angle-based LE method suffers substantial loss of performance due to parallel LOS effect which we have discussed in Fig. 5. Since RSS method is not affected by parallel LOS effect, the RSS in this case can improve the ML-AOD method. However, if there is a low parallel LOS effect such as in Fig. 6 arrangement, the ML-AOD method can perform well without the RSS, as shown in Fig. 7.

Thus far, we have seen LE performance by varying the UE position across 1-dimensional space, which is the x-axis. Next, to obtain clearer performance results within the area, we simulate the RMSE results by varying the UE position across 2D space, namely, the x- and y-axes. Since the results in Figs. 4 and 7 show that the theoretical RMSE is achievable by using the ML-AOD-RSS method, we will focus on the performance analysis using the theoretical RMSE in the next simulation. The results for the 2 AP and 4 AP arrangements are given in Figs. 8 and 9, respectively. The results for 4 APs with alternative arrangements are also provided in Fig. 10.

Figs. 8 and 9 show that using 4 APs has a more consistent RMSE and lower RMSE than using 2 APs across various x- and y-axis positions. Fig. 8 also confirms that the issue of the parallel LOS effect when using two AP arrangements persists at various y-axis positions. Fig. 10 has higher RMSE peaks between the APs than Fig. 9 despite using the same number of APs. This is due to the parallel LOS effect occurring between the APs in the arrangement in Fig. 10. To summarize, the 2D RMSE analysis enables us to plan for AP deployment and to optimize the positioning of the APs based on the simulation method.

To provide a more comprehensive analysis, we next simulate the theoretical RMSE for various numbers of antennas and subcarriers. Note that changing these parameters can affect the statistical parameter values for RSS and AOD, as mentioned in section III. The results using the 2 AP arrangements are given in Figs. 11 and 12.

Fig. 11 shows that a higher number of antennas results in a lower RMSE. This is expected since as the number of antennas increases, the beamforming signal is more focused (less dispersion) toward each beamforming angle. This indicates that there is less AOD estimation error, which results in higher
and the estimated location of the method. The theoretical observed based on RMSE measured between the true location can perform LE using the estimated AOD and RSS along
MIMO-OFDM signals with unique beamforming angles allo-
derived from statistical models of AOD and RSS of
In this paper, we investigate the ML localization method
these variables become very large.
results also show that the RMSE reduction using more anten-
as and more subcarriers can become less effective when
accuracy of LE is also improved by using more APs with
In addition, a higher number of antennas and a higher number of subcarriers lead to better LE but with diminishing efficacy. The RMSE calculation method of the proposed method can be a convenient tool to gain important insights into the performance of the proposed LE, and future studies can be extended to develop AP placement and parameter optimization strategies on a larger scale.

**APPENDIX**

**A. PROOF OF MAXIMUM LIKELIHOOD ESTIMATION**

The distance between AP and UE can also be written in terms of AP and UE coordinates. For example, if the coordinate of a certain AP $i$ is given as $q_i$ (since there can be multiple APs, we use $i$ as the index number to differentiate multiple APs) and the UE coordinate is given as $p$, then the distance is $d_i = |p - q_i|$. The angle of AP with coordinates $q_i$ from the UE with coordinates $p$ can be written as $\theta_{p,q_i} = \angle (p - q_i)$. This indicates that the likelihood functions can be derived from PDFs (23) and (27) in terms of variables $p$ and $q_i$ as

\[
L(p|r_i,q_i) = \frac{1}{\sigma_{p,q_i}\sqrt{2\pi}} \exp\left(-\frac{(\log r_i - \mu_{p,q_i})^2}{2\sigma_{p,q_i}^2}\right), \quad (35)
\]

\[
L(p|\hat{\theta}_i,q_i) = \frac{1}{\varphi_{p,q_i}\sqrt{2\pi}} \exp\left(-\frac{(\hat{\theta}_i - \theta_{p,q_i})^2}{2\varphi_{p,q_i}^2}\right), \quad (36)
\]

where

\[
\mu_{p,q_i} = a_1 \log |p - q_i| + a_2, \quad (37)
\]

\[
\sigma_{p,q_i} = b_1 \log |p - q_i| + b_2, \quad (38)
\]

\[
\theta_{p,q_i} = \angle (p - q_i), \quad (39)
\]

\[
\log \varphi_{p,q_i} = c_1 \log |p - q_i| + c_2. \quad (40)
\]

(37), (38) and (40) are derived from (21), (22) and (26), respectively, by changing their dependency from variable $d$ to variables $p$ and $q_i$. Using the joint likelihood and assuming that the error distribution of the RSS is independent from that of the AOD distribution, the joint likelihood of location estimation at $p$, given measured parameters $r_i$, $\hat{\theta}_i$ and AP $i$ location $q_i$, can be expressed as

\[
L(p|r_i,\hat{\theta}_i,q_i) = L(p|r_i,q_i)L(p|\hat{\theta}_i,q_i). \quad (41)
\]

Note that (41) is the likelihood derived for a single AP $i$. If the UE estimates its location using multiple APs ($i \in Q$), then (41) can be extended as follows:

\[
L(p|r_{i\epsilon Q},\hat{\theta}_{i\epsilon Q},q_{i\epsilon Q}) = \prod_{i\epsilon Q}L(p|r_i,\hat{\theta}_i,q_i). \quad (42)
\]

Substituting (35) and (36) into (41) and (41) into (42), the likelihood function is (29). To obtain ML estimation, we need
B. PROOF OF THE THEORETICAL RMSE

To obtain the RMSE, we first need to find the PDF of the estimated distance, \( \hat{d} \), and estimated AOD, \( \hat{\theta} \), given certain values of true distance, \( d \), and true AOD, \( \theta \). Since \( \hat{d} \) is estimated from \( r \) (17), this means that log \( r = a_1 \log \hat{d} + a_2 \).

Using (23), the PDF of \( \hat{d} \) given \( d \) is

\[
\text{pdf} \left( \hat{d} | d \right) = \frac{1}{\sigma_d \sqrt{2\pi}} \exp \left( -\frac{(a_1 \log \hat{d} + a_2 - \mu_d)^2}{2\sigma_d^2} \right).
\]

From (27) and (43), the joint PDF distribution of \( \hat{d} \) and \( \hat{\theta} \), given \( d \) and \( \theta \), can be expressed as

\[
\text{pdf} \left( \hat{d}, \hat{\theta} | d, \theta \right) = \frac{1}{\sigma_d \sigma_{d|\theta} \sqrt{2\pi}} \exp \left( -\frac{(a_1 \log \hat{d} + a_2 - \mu_d)^2}{2\sigma_d^2} - \frac{(\hat{\theta} - \theta)^2}{2\sigma_{d|\theta}^2} \right).
\]

Since \( \hat{d} \) and \( \hat{\theta} \) are the estimated distance and AOD from a certain AP, these variables can also be represented in terms of the estimated location of the device,

\[
\hat{d}_i = |\hat{p} - q_i|, \quad \hat{\theta}_i = \angle (\hat{p} - q_i),
\]

where \( \hat{p} \) is the estimated location of the device and \( q_i \) is the location of AP \( i \). Using (44), (45) and (46), the PDF of estimated UE location \( \hat{p} \), given the true UE location of \( p \) and AP \( i \) location \( q_i \), can be expressed as

\[
\text{pdf} \left( \hat{p} | p, q_i \right) = \frac{1}{2\pi \sigma_{p,q_i} \sqrt{\sigma_{p,q_i}}} \exp \left( -\frac{(a_1 \log |\hat{p} - q_i| + a_2 - \mu_{p,q_i})^2}{2\sigma_{p,q_i}^2} - \frac{\angle (\hat{p} - q_i) - \theta_{p,q_i})^2}{2\sigma_{p,q_i}^2} \right).
\]

where \( \mu_{p,q_i}, \sigma_{p,q_i}, \theta_{p,q_i} \), and \( \sigma_{p,q_i} \) are defined in (37), (38), (39) and (40), respectively. If multiple APs are used to estimate the location, (47) can be extended to

\[
\text{pdf} \left( \hat{p} | p, q_{i\in Q} \right) = \prod_{i\in Q} \text{pdf} \left( \hat{p} | p, q_i \right).
\]

The square error of the estimation is defined as \( |\hat{p} - p|^2 \).

Using the expected value calculation of the continuous probability function [26], the RMSE given location \( p \) can be written as

\[
\text{RMSE}|p = \sqrt{\int_{-\infty}^{\infty} \left| \hat{p} - p \right|^2 \text{pdf} \left( \hat{p} | p \right) d\hat{p}}.
\]

Substituting (48) into (49), Theorem 2 is proven.

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