The study of hadron spectroscopy is a useful approach to deepen our understanding of the nonperturbative behavior of strong interaction. In the past decade, big progress has been made on the observation of bottom baryons, which is a crucial step to construct a complete $b$-hadron spectroscopy. Among these observations, the $\Lambda_b(5912), \Lambda_b(5920), \Lambda_b(6072), \Lambda_b(6146), \Lambda_b(6152), \Xi_b(6100), \Xi_b(6327),$ and $\Xi_b(6333)$ which were reported in Refs. [1–6] have a close relation to the low-lying $\Lambda_b$ and $\Xi_b$ baryons. Meanwhile, these $P$-wave $\Sigma_b, \Sigma'_b,$ and $\Omega_b$ states were also established one by one which are mainly due to the effort from LHCb [7–10]. In contrast to the situation of single bottom baryons, none of the double bottom baryons has been discovered, which is becoming not only a challenge but also an opportunity for the future experiments. If further checking the status of bottom and bottom-strange mesons, we can find that the families of bottom and bottom-strange mesons are far from being established since there are only seven states collected in each of the bottom and bottom-strange meson families, which can be referred to the Particle Data Group (PDG) [14]. Thus, constructing the spectroscopy of bottom and bottom-strange mesons is still on the way.

Before introducing the motivation of the present work, we should mention the lesson from the investigation of single heavy baryons. Lanzhou group has systematically studied the single heavy baryons by the nonrelativistic constituent quark models [15–20], where the diquark picture was employed. These studies not only depicted the newly observed single heavy baryons, but also have stood the test of later experimental observations. For examples, the predicted properties of $P$-wave $\Omega_b$ [19] and $D$-wave $\Xi_b$ [20] states were confirmed by the LHCb experiments [6, 10]. To some extent, our works could be regarded as a typical example of applying the diquark picture to hadron spectroscopy.

For the double bottom baryon, we may also treat this interesting system as a quasi-two-body problem if putting two bottom quarks together which is also a typical diquark. Indeed, the heavy-diquark-light-quark picture has been taken into account when calculating the mass spectrum of double bottom baryons [21–23]. In the following, we should mention the similarity between the double bottom baryon and the bottom meson if the diquark picture is considered. As illustrated in Fig. 1, the double bottom baryon system could be simplified as a quasi-two-body system in the diquark picture, where the heavy diquark has the same color structure as a heavy antiquark in the bottom meson. Thus, it is naturally expect that the $\lambda$-mode excited $bbq$ baryons have the similar dynamics to the bottom mesons. So it provides a possibility to carry a combined study of the mass spectrum of these two kinds of $b$-hadrons.
Besides giving their mass spectrum, in this work, we also focus on the strong decay properties of the bottom meson and double bottom baryon together. As shown later, most of the low-excited \( \lambda \)-mode \( \bar{b}q \) states are expected to be below the \( \Lambda_bB \) and \( \Xi_bB \) thresholds. Thus, for the strong decays of these double bottom baryon excitations, two bottom quarks which transit into a final \( \bar{b}q \) state could be treated as a whole (see Fig. 2). So the heavy diquark is just a spectator in the decay process, which is similar to the strong decay of an excited \( \bar{b}q \) meson. Therefore, the similarity should exist not only in the spectroscopy of low-lying \( \bar{b}q \) and \( \bar{b}bq \) states, but also in their strong decays. It means that these two kinds of \( \bar{b} \)-hadrons can be investigated in the same theoretical scheme for their masses and strong decays. In this way, the model parameters fitted by the known \( \bar{b}q \) mesons can provide a valuable reference to the \( \bar{b}bq \) system since none of \( \bar{b}bq \) states has been discovered. The proposed approach is different from most of studies in the literature, where the \( \bar{b}q \) and \( \bar{b}bq \) states were studied individually.

The paper is organized as follows. After the introduction, the theoretical scheme is introduced in details in Sec. II. The mass spectra and strong decay behaviors of these low-lying \( \bar{b}q \) and \( \bar{b}bq \) states are investigated in Sec. III and IV, respectively. The similarities of the \( \bar{b}q \) and \( \bar{b}bq \) systems which are implied in the predicted masses and strong decays are further discussed in Sec. V. Finally, the paper ends with the conclusion and outlook in Sec. VI.

II. THEORETICAL SCHEME

It has been a long time since Savage and Wise proposed the superflavor symmetry which can be related to the discussion of the properties of the \( \bar{Q}q \) mesons and \( \bar{Q}Qq \) baryons [24]. The emergence of superflavor symmetry is a consequence of the heavy quark limit. In the limit of \( m_Q \to \infty \), two heavy quarks in the \( \bar{Q}Qq \) baryonic system may form a small weakly bound color triplet subsystem. In literature [25], the \( \{\bar{Q}Q\} \) subsystem is also referred to as the heavy diquark. As an approximation, this heavy diquark could be regarded as a static source of color, which plays essentially the same role as the heavy antiquark in a heavy-light meson.

The superflavor symmetry has been developed and applied by Cohen [26], Roberts [27, 28], Ma [29] and their collaborators. An important application of the superflavor symmetry is to obtain the properties of these unknown double heavy baryons from the well-measured heavy-light mesons. The superflavor symmetry was used in Ref. [26] to predict the mass difference of two ground double heavy baryons with \( J^P = 1/2^+ \) and \( J^P = 3/2^+ \), respectively, where the masses of two ground heavy-light mesons (the pseudoscalar and vector mesons) were taken as the input. Based on the superflavor symmetry, the selection rule and the spin-counting relation for the strong decays of the double heavy baryon were studied in Ref. [28]. The superflavor symmetry was incorporated into an effective Lagrangian approach in Ref. [29]. Then the masses and decays of ground and \( 1P \) (with \( J^P = 1/2^- \) and \( J^P = 3/2^- \)) doubly heavy baryons were predicted. Recently, the superflavor symmetry was also extended to investigate the tetraquark system \( Q_iQ_i'\bar{q}_i\bar{q}_i' \) [30].

In this work, we are also dedicated to the application of superflavor symmetry. Differently, we shall take the concrete dynamical model to systematically investigate the mass spectrum and decay behavior of these low-lying \( \bar{Q}q \) mesons and \( \bar{Q}Qq \) baryons together, where the superflavor symmetry will be considered. Concretely, a nonrelativistic quark potential model\(^1\) is employed to calculate the mass spectra of the \( \bar{b}q \) mesonic and \( \bar{b}bq \) baryonic states. Here, we take the Cornell potential [33] to phenomenologically depict the confining interaction between a \( 3 \) color component\(^2\) and the light quark. Based on this consideration, we may construct the Schrödinger equation for the discussed system, i.e.,

\[
\left(-\frac{\nabla^2}{2m_p} + \frac{4\alpha}{3r} + br - C + \frac{32\alpha\sigma^3}{9\sqrt{\pi}m_b m_q} \frac{s_{\bar{b}} \cdot s_q}{r^3}\right)\psi_{nl} = E\psi_{nl}.
\]

Here, \( m_b \) and \( s_q \) denote the mass and the spin of the heavy component \( 1 \) in the \( \bar{b}q/\bar{b}bq \) hadrons. Equation (1) has included the spin-spin contact hyperfine interaction between the heavy component \( 1 \) and the light quark. For the \( \bar{E}_{gg} \) and \( \bar{Q}_{gg} \) baryons \( s_b = 1 \) is determined, while for the \( B \) and \( B^* \) mesons, we take \( s_b = 1/2 \). The parameters \( \alpha, b, \) and \( C \) stand for the strength of the color Coulomb potential, the strength of linear confinement, and a mass-renormalization constant, respectively. By solving the Schrödinger equation, the average masses of the \( \bar{b}q \) and \( \bar{b}bq \) baryons are obtained. When the following spin-dependent interactions are further incorporated, we can further obtain the mass of these discussed \( \bar{b}q/\bar{b}bq \) states.

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\(^1\) In principle, the dynamics of light quark \( q \) in the \( \bar{Q}q \) and \( \bar{Q}Qq \) systems should be depicted by the relativistic models. When one tries to calculate the masses of low-lying hadron states, however, it is believed that a nonrelativistic model could include many relativistic corrections by a redefinition of its parameters [31], such as the constituent quark masses, the effective parameters in the static potential [see Eq. (1)]. So it can explain why the mass spectra obtained by the relativized quark model are qualitatively similar to those of the usual nonrelativistic model [32]. For the doubly heavy baryons, the excited energies given by the relativistic quark model [33] and the nonrelativistic quark model [21, 22] are also nearly equal.

\(^2\) It corresponds to the \( (bb) \) diquark in a \( \bar{b}bq \) baryon state or the antiquark \( \bar{b} \) in a \( \bar{b}q \) meson state, which is denoted by the symbol \( 1 \) in the following discussion.
first spin-dependent interaction
\[ H_T = \frac{4\alpha_s}{3m_b m_q} \frac{1}{r^3} \left( \frac{3s_{bq} \cdot r s_{aq} \cdot r}{r^2} - s_b \cdot s_q \right), \]  
(2)
is a tensor term for depicting the magnetic-dipole-magnetic-dipole color hyperfine interaction. The second spin-dependent interaction is the spin-orbit interaction
\[ H_{SO} = \left[ \frac{2\alpha_s}{3r^3} - \frac{b}{2r} \right] \frac{1}{m_b^2} s_b \cdot L + \left[ \frac{2\alpha_s}{3r^3} - \frac{b}{2r} \right] \frac{1}{m_q^2} s_q \cdot L + \frac{4\alpha_s}{3r^3} m_b m_q \left( S \cdot L + \delta_T \right), \]
(3)
which arises from both the short-range one-gluon exchange contribution and the long-range Thomas-Precession term.

As shown in Eqs. (2)-(3), the “s_b, L” coupling and the tensor interaction will vanish in the limit of \( m_b \to \infty \). Thus, the spin-dependent interactions which are denoted in Eqs. (2)-(3) could be further expressed as
\[ H_{SO} = \frac{1}{m_b^2} \left( \frac{2\alpha_s}{3r^3} - \frac{b}{2r} \right) s_b \cdot L + \frac{4\alpha_s}{3r^3} m_b m_q \left( S \cdot L + \delta_T \right), \]
(4)
where \( \delta_T \) and \( S \) are defined as \( \delta_T = (3s_{bq} \cdot r s_{aq} \cdot r)/r^2 - s_b \cdot s_q \) and \( S = s_q + s_b \), respectively. For the \( bq \) and \( bbq \) hadrons, the \( m_b \) is much larger than the \( m_q \) (\( m_b \gg m_q \)). Then the first term in the right-hand side of Eq. (4) is predominant for the \( bq/bbbq \) systems.

Accordingly, the practical calculation of matrix mass is often performed in the \( jj \) coupling scheme, where the basis is defined as \( |s_j, L, j_q, s_b, J \rangle \) with \( j_q = s_q + L \) and \( J = j_q + s_b \). For the sake of convenience, the basis \( |s_j, L, j_q, s_b, J \rangle \) could be abbreviated as \( |j_q, J^P \rangle \) without any confusions. The particles of the \( bq \) meson and \( \lambda \)-mode excited \( bbq \) baryon are fixed as \( P = (-1)^{j_q+1} \) and \( P = (-1)^J \), respectively. As an example, the basis of \( D \)-wave \( bbq \) mesons could be defined as the \( |3/2, 1/2^+, 3/2, 3/2^+ \), \( |3/2, 5/2^+ \), \( |5/2, 3/2^+ \), \( |5/2, 5/2^+ \), and \( |5/2, 7/2^+ \rangle \), which could be grouped into the \( j_q = 3/2 \) and \( j_q = 5/2 \) triplets. The multiplets of all low-lying \( bq/bbbq \) states are presented in Fig. 3.

In reality, the masses of the antiquark \( \bar{b} \) and the diquark \( \{bb\} \) are finite. Then, the degeneracy of the states in a multiplet is broken, and there exist mixing of the states with same \( J^P \) in the different multiplets. Due to \( m_b \gg m_q \), however, the mixing effect of the \( bq/bbbq \) states with the same \( J^P \) is not obvious. It means that the basis \( |j_q, J^P \rangle \) defined in the heavy quark limit could represent the physical state approximately.

The superflavor symmetry is reflected not only in the mass spectrum of the \( bq \) and \( bbq \) systems, but also in their strong decay behavior. Since the dynamics of light quark \( q \) in the \( bq \) and \( bbq \) systems is independent of the spin and mass of the heavy component \( \bar{b} \) and only the light quark \( q \) takes an active part in the strong decay process, the states of the corresponding multiplets (see Fig. 3) with the same \( j_q \) but different \( J^P \) should have the similar decay behavior. We may take the \( P \)-wave \( bq \) and \( bbq \) states as an example. These four \( P \)-wave \( bq \) mesons can be arranged into two doublets, i.e., the \( (0^+, 1^+)_{j_q=1/2} \) and \( (1^+, 2^+)_{j_q=3/2} \). While these five \( P \)-wave \( bbq \) baryons could be grouped in a doublet and a triplet, i.e., the \( (1/2^-, 3/2^-)_{j_q=1/2} \) doublet and the \( (1/2^-, 3/2^-, 5/2^-)_{j_q=3/2} \) triplet. Since the \( \bar{b}q \) mesons in the \( (1^+, 2^+)_{j_q=3/2} \) doublet have been measured to be the narrow states [14], the superflavor symmetry indicates that the \( P \)-wave \( \Xi_{bbq} \) and \( \Xi_{bb} \) baryons in the \( (1/2^-, 3/2^-, 5/2^-)_{j_q=3/2} \) triplet should also be the narrow states. This phenomenon can be reflected by the practical calculations which will be presented in Sec. IV.

In this work, it is reasonable to extend the decay formula which was proposed by Eichten, Hill, and Quigg in Ref. [34] to study the decay of the double bottom baryons. This strong decay formula (or named as the EHQ formula) incorporates the heavy quark symmetry for depicting a decay process of an excited heavy-light meson [35]. The EHQ formula, which has been used to explain the decays of the charm mesons and charm baryons in our previous works [16, 36], could be improved as
\[ \Gamma^{A \rightarrow BC}_{j_q, j_f} = \xi \left| C_{j_q, j_f, j_a}^{s_b, s_a} M^{s_b, s_a}_{j_q, j_a} (p/\beta) \right|^2 p e^{-p^2/m^2}. \]

In this way, the EHQ formula can be used to study the decay properties of both the bottom mesons and the \( \lambda \)-mode excited double bottom baryons. The only difference of the \( bbq \) and \( bq \) states is that the spin of the diquark \( \{bb\} \) in a \( bbq \) baryon is 1 (\( s_{bb} = 1 \)), while the spin of the antiquark \( b \) in a \( bq \) meson is 1/2 (\( s_b = 1/2 \)). Then, the spin of the heavy quark \( s_q \) in the original EHQ formula [34] has been replaced by \( s_b \) in Eq. (5).

And \( A \) and \( B \) in Eq. (5) represent the initial and final heavy-light hadrons, respectively, and \( C \) denotes the light flavor meson. The magnitude of three-momentum for a final state is denoted as \( p \) in the rest frame of the initial state. The flavor factor \( \xi \) in Eq. (5) has been given in Ref. [37]. The symbols \( s_c \) and \( j_f \) represent the spin of the light hadron \( C \) and the orbital angular momentum relative to \( B \), respectively. The normalized
coefficient $C_{j_c, j_A, J_A}^{s_B, j_B, J_B}$ is rewritten as

$$C_{j_c, j_A, J_A}^{s_B, j_B, J_B} = (-1)^{j_A + j_B + j_c} \sqrt{(2J_A + 1)(2J_B + 1)}$$

$$\times \left\{ \begin{array}{c} s_B \quad j_B \quad J_B \\ j_c \quad J_A \quad J_A \end{array} \right\},$$

(6)

where $j_c \equiv s_c + \bar{\ell}$. The $C_{j_c, j_A, J_A}^{s_B, j_B, J_B}$ denoted by Eq. (6) reflects the requirement of the heavy quark symmetry. The transition factors $N_{j_c, j_A, J_A}^{s_B, j_B, J_B}(p/\beta)$, which is relevant to the nonperturbative dynamics, could be obtained by the various phenomenological models. In our previous work [16, 36], the transition factors were extracted by the $3P_0$ model. Here, we directly borrow the transition factors from Ref. [36] for the calculation. The parameter $\beta$ denotes the scale of harmonic oscillator wave function for the hadrons involved in the discussed transitions. Since the $bqq$ baryon has been simplified as a quasitwo-body system in the diquark picture, the strong decays of the low-excited $bqq$ and $\bar{b}q$ states have the same $N_{j_c, j_A, J_A}^{s_B, j_B, J_B}(p/\beta)$. This is nothing but the consequence of superflavor symmetry which has been stressed above.

The parameters involved in the adopted potential model are collected in Table II. The dimensionless parameter $\gamma$ in the $3P_0$ model was fixed as 0.125 by the decay width of the $P$-wave $D_s(2640)^0$ state [36], where the value of $\beta$ is taken as 0.38 GeV. One may notice that the mass of $bqq$ diquark in Table I is more than twice of the antiquark $\bar{b}$. According to the results in Ref. [41], the effective mass of $b$ quark which was fixed by the discovered bottom baryons is larger than the mass of antiquark $\bar{b}$ in the bottom mesons. So it implies that the constituent quark mass of $b$ quark in a bottom baryon could be different from the mass of $\bar{b}$ in a bottom meson. In the calculation, the mass of antiquark $\bar{b}$ is fixed as 4.64 GeV by the measured bottom mesons. On the other hand, the mass of $bqq$ diquark is taken as 9.55 GeV, which is larger than the mass of $\bar{b}q$ state ($m_{\bar{b}q} = 9.46$ GeV). This is consistent with the expectation of quark potential models. In fact, the masses of lowest $bqq$ diquark (the $1S_J$ state) in Refs. [21–23] were also larger than the mass of $\bar{b}q$ state.

With these parameters as the input, the mass spectrum, the decay width, and the corresponding branching ratio of the discussed $\bar{b}q$ and $bqq$ hadrons are obtained in the following sections.

### III. BOTTOM AND BOTTOM-STRANGE MESONS

#### A. The $B$ meson

| $B$ | $B_s$ |
| --- | --- |
| $B(5280)/B^*(5325)$ | $B(5367)/B_s^*(5416)$ |
| $B_s^*(5732)/B_s(5721)/B_s^*(5747)$ | $B_s^*(5850)/B_s(5830)/B_s^*(5840)$ |
| $B(5840)/B_s(5970)$ | $B_{sJ}(6064)/B_{sJ}(6114)$ |

As shown in Table II, the low-lying $B$ mesons are far from being well established. At present, only the $B(5280)$, $B^*(5325)$, $B_s(5721)$, $B_s^*(5747)$, $B_s^*(5840)$, and $B_s(5970)$ are collected by the PDG [14]. Among them, the $B(5280)$, $B^*(5325)$, $B_s(5721)$, and $B_s^*(5747)$ were established in experiment without controversy. However, two broad $1P$ bottom mesons are not established. As a disputed candidate of the $1P$ $B$ meson, the $B_s^*(5732)$ was reported in Refs. [42–45], where the measured resonant parameters from different experiments are listed in Table III.

Exploring higher excited $B$ mesons was continuing. The L3 Collaboration reported a bottom meson which could be a $2S$ or $1D$ candidate in the hadronic decay process of the $Z$ boson [45]. Its mass and decay width were measured to be $5937 \pm 21 \pm 4$ MeV and $50 \pm 22 \pm 5$ MeV, respectively. However, this $B$ state has never been confirmed by other experiments and none of the other the higher excited $B$ state was found in the next many years. In 2013, the CDF Collaboration found a state $B(5970)$ in the $B\pi$ final states [11]. Two years later, the LHCb Collaboration reported two $B$ resonances, the $B_{sJ}(5840)$ and $B_{sJ}(5960)$ [12]. So far, the spin-parity quantum numbers of these reported $B$ mesons are still underdetermined (see the discussions in Ref. [46]). In this work, we try to identify their properties by combing our theoretical result with the experimental data.

| Parameters | $m_q$ (GeV) | $m_h$ (GeV) | $\alpha$ | $b$ (GeV$^2$) | $C$ (GeV) |
| --- | --- | --- | --- | --- | --- |
| $B$ | 0.45 | 4.64 | 0.50 | 0.138 | 0.135 |
| $B_s$ | 0.54 | 4.64 | 0.50 | 0.138 | 0.077 |
| $\Xi_{bb}$ | 0.45 | 9.55 | 0.42 | 0.130 | 0.190 |
| $\Omega_{bb}$ | 0.54 | 9.55 | 0.42 | 0.130 | 0.130 |

#### 1. The $1P$ states

The $1P$ bottom mesons $B_s(5721)$ and $B_s^*(5747)$ have been established by experiments [14]. As shown in Table IV, the
TABLE IV: A comparison of our predicted masses of the $B$ and $B_s$ mesons with other results Refs. [47–50] and the experimental data [13, 14] (in MeV).

| State | Expt. | Our | Ref. [47] | Ref. [48] | Ref. [49] | Ref. [50] | Expt. | Our | Ref. [47] | Ref. [48] | Ref. [49] | Ref. [50] |
|-------|-------|-----|-----------|-----------|-----------|-----------|-------|-----|-----------|-----------|-----------|-----------|
| $1^3S_0$ | 5280 | 5279 | 5279 | 5280 | 5312 | 5268 | 5367 | 5368 | 5373 | 5372 | 5394 | 5377 |
| $1^3S_1$ | 5325 | 5325 | 5324 | 5326 | 5371 | 5329 | 5416 | 5414 | 5421 | 5414 | 5440 | 5422 |
| $2^3S_0$ | 5889 | 5903 | 5886 | 5890 | 5904 | 5877 | 5977 | 5985 | 5978 | 5976 | 5984 | 5929 |
| $2^3S_1$ | 5929 | 5920 | 5906 | 5933 | 5905 | 6001 | 6019 | 5992 | 6012 | 5949 | 6012 | 5949 |
| $3^3S_0$ | 6357 | 6357 | 6320 | 6379 | 6335 | 6288 | 6413 | 6421 | 6467 | 6467 | 6410 | 6305 |
| $3^3S_1$ | 6378 | 6347 | 6387 | 6355 | 6308 | 6431 | 6449 | 6475 | 6429 | 6319 | 6429 | 6319 |
| $1^1P_0$ | 5724 | 5706 | 5749 | 5756 | 5704 | 5795 | 5804 | 5833 | 5831 | 5770 | 5831 | 5770 |
| $1P_1$ | 5759 | 5742 | 5774 | 5784 | 5739 | 5834 | 5842 | 5865 | 5861 | 5801 | 5861 | 5801 |
| $1P_2$ | 5726 | 5726 | 5700 | 5723 | 5777 | 5755 | 5829 | 5819 | 5805 | 5831 | 5857 | 5803 |
| $2^1P_2$ | 5740 | 5740 | 5714 | 5741 | 5797 | 5769 | 5840 | 5833 | 5820 | 5842 | 5876 | 5822 |
| $2^1P_0$ | 6181 | 6163 | 6221 | 6213 | 6129 | 6236 | 6264 | 6318 | 6279 | 6160 | 6279 | 6160 |
| $2^3P_1$ | 6218 | 6194 | 6281 | 6228 | 6161 | 6268 | 6296 | 6345 | 6296 | 6166 | 6296 | 6166 |
| $2^3P_2$ | 6202 | 6175 | 6209 | 6197 | 6175 | 6278 | 6278 | 6321 | 6279 | 6196 | 6279 | 6196 |
| $1^1D_1$ | 6099 | 6052 | 6119 | 6110 | 6022 | 6114 | 6153 | 6127 | 6209 | 6182 | 6057 | 6182 | 6057 |
| $1D_2$ | 6110 | 6037 | 6121 | 6124 | 6026 | 6164 | 6140 | 6218 | 6196 | 6059 | 6218 | 6196 |
| $1D_3$ | 5993 | 5985 | 6103 | 6095 | 6031 | 6080 | 6095 | 6120 | 6169 | 6064 | 6169 | 6064 |
| $1^3F_2$ | 6376 | 6264 | 6412 | 6387 | 6259 | 6415 | 6369 | 6501 | 6454 | 6273 | 6454 | 6273 |
| $1F_2$ | 6382 | 6271 | 6420 | 6396 | 6249 | 6421 | 6376 | 6515 | 6462 | 6277 | 6462 | 6277 |
| $1F_3$ | 6223 | 6220 | 6391 | 6358 | 6264 | 6307 | 6332 | 6468 | 6425 | 6265 | 6425 | 6265 |
| $1F_4$ | 6226 | 6226 | 6380 | 6364 | 6252 | 6311 | 6337 | 6475 | 6432 | 6267 | 6432 | 6267 |

Thus, the study of the decay behavior of the $B_s'(5747)$ enforces the conclusion of the $B_s'(5747)$ as a $B(1^3P_2)$ meson. If assigning the $B_1(5726)$ to be a pure $1P_1$ meson with $j_q = 3/2$, its total decay width is predicted to be 17.8 MeV in the heavy quark limit, which indicates the $B_1(5726)$ to be a narrow state. In reality, the mass of the $b$ quark is not infinite, which makes that the heavy quark symmetry is broken slightly. Thus, the $B_1(5726)$ state may contain a small $1P(1/2, 1^+)$ component, i.e., two $1P_1$ and $1P_2$ states of the B meson listed in Table IV should be as the mixtures between the $1P(1/2, 1^+)$ and $1P(3/2, 1^+)$ states

$$
\left( \begin{array}{c} B_1'(5759) \\ B_1(5726) \end{array} \right) = \left( \begin{array}{cc} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{array} \right) \left( \begin{array}{c} |1^+ , 1/2 \rangle \\ |1^+ , 3/2 \rangle \end{array} \right).
$$

The mixing angle $\theta$ is fixed to be $-175.3^\circ$ by the nonrelativistic quark potential model. Thus, the physical state $B_1(5726)$ has the dominant component of the $1P(3/2, 1^+)$ state, while the undiscovered $B_1'(5759)$ is predominantly the $1P(1/2, 1^+)$ state. When considering this mixing effect, the decay width of the $B_1(5726)$ increases to 30.9 MeV as shown in Table V, which is consistent with the LHCb result [12]. In a word, it is reasonable to categorize the $B_1(5721)$ and $B_2'(5747)$ as the $P$-wave bottom mesons.

We also investigated the other two $1P$ states of the $B$ meson. Here, the masses of the $1^3P_0$ and $1^3P_1$ states of the $B$ meson are predicted to be 5724 MeV and 5759 MeV, respec-
tively, which are comparable with the former theoretical results reported in Refs. [47–51]. As shown in Table V, the total decay widths of the $1^3P_0$ and $1^P_1$ states of the $B$ meson are given as 255.6 MeV and 245.1 MeV, respectively, in line with expectations of the $B(1^3P_0)$ and $B(1^P_1)$ states having the broad widths as suggested by other theoretical approaches like the $^3P_0$ model [52], the chiral quark model [53], and the QCD sum rule [54]. Since the $B(1^3P_0)$ and $B(1^P_1)$ states have the broad widths around 200 MeV, experimentally identifying these two $P$-wave states is not an easy task, which may naturally explain why two members in the $1P(0^+, 1^+_{1/2})$ doublet were not yet established in experiment. At present, it is too early to assign the $B_J(5732)$ state, which was observed by the OPAL [42], DELPHI [43], ALEPH [44], and L3 [45] collaborations, to a member of the $1P(0^+, 1^+_{1/2})$ doublet before achieving more precise data.

### 2. The $2S$ states

Precisely measuring the resonance parameters of $2S$ heavy-light mesons is full of challenges. We may take the $2S$ charmed mesons as an example to illustrate this point. As the $2S$ states of $D$ meson, the $D_0(2550)^0$ [55–57] and $D_1^*(2600)^0$ [55–58] have been reported for many years. However, the measurements based on the different production processes gave the different results for the resonance parameters (see the review paper [59] for more details). The main reason is that distinguishing the broad resonances $D_0(2550)^0$ and $D_1^*(2600)^0$ in the $D^\pi$ invariant mass spectrum is difficult. A similar situation can happen for the $2S$ states of the bottom meson (see the predicted widths listed in Table VI). And, a low-energy photon from the $B^* \to BY$ decay was not reconstructed in the realistic analysis of the CDF [11] and LHCb [12] experiments, which makes the results of the $B_J(5840)$ and $B_J(5970)$ to be more uncertain. Thus, the precise measurements are desired for the $2S$ $B$ mesons in the future. In this work, the following ratio of the partial widths of the $B\pi$ and $B^*\pi$ channels

$$R[\B^*(2S^1_1)] = \frac{\mathcal{B}(\B^*(2S^1_1) \to B\pi)}{\mathcal{B}(\B^*(2S^1_1) \to B\pi)} = 1.94,$$

is predicted for the $B(2S^1_1)$ meson, which can be examined in the future experiment.

If comparing our result with the measurement of the $B_J(5840)$ and $B_J(5970)$, we find that the $B_J(5840)$ could be interpreted as a good candidate of the $2S$ state, while the possibility of the $B_J(5970)$ as a $2S$ state could be preliminarily excluded due to its relatively narrow width. In fact, the mass difference between the $B_J(5840)$ and the $B_J(5970)$ also disfavors the assignment of the $B_J(5970)$ as a $2S$ candidate when the $B_J(5840)$ has been assigned as a $2S$ candidate (see the obtained mass splitting of two $2S$ $B$ mesons in Table IV).

### 3. The $1D$ states

As shown in Fig. 3, four $1D$ states of the $B$ meson could be grouped into two doublets $1D(1^-, 2^-_{3/2})$ and $1D(2^-, 3^-_{3/2})$. The average mass of two members in the $1D(1^-, 2^-_{3/2})$ doublet is around 6100 MeV, which is about 100 MeV larger than the states in the $1D(2^-, 3^-_{3/2})$ doublet (see Table IV). As shown in Table VII, the decay properties of these states in different doublets are quite different, while the states in the same doublet have similar decay behavior. The $1D_1$ and $1D_2$ states are expected to be broad, while the $1D_3$ and $1D_4$ states are much narrower. Here, the $1D_1$, $1D_3$, and $1D_4$ states decay into the $B^*\pi$ channel through $p$-wave, while the $B^*\pi$ decays of the $1D_2$ states occur via $s$-wave. In addition, the processes $B(1D_1) \to B(5726)\pi$ and $B(1D_2) \to B(5740)\pi$ proceed via $s$-wave, while the corresponding strong decays of the $1D_3$ and $1D_4$ states occur via $d$-wave. Obviously, the phase space of decay processes for the $1D_1$ and $1D_2$ $B$ states are much larger than that of the $1D_3$ and $1D_4$ states.

By comparing the measured resonance parameters with the calculated results in Table VII, we find that the $B_J(5970)$ state could be explained as a member of $1D(2^-, 3^-_{3/2})$ doublet since the measured mass of the $B_J(5970)$ is comparable with the predicted value. Since the photon emitted from the state $B^*(5325)$ in the decay process $B_J(5970) \to B^*(5325) + \pi \to B\gamma + \pi$ cannot be reconstructed in experiment [11, 12], the enhancement structure of the $B_J(5970)$ state may contain the signals of the $1D_3$, and $1D_4$ states. Our result naturally explains why the measured width of $B_J(5970)$ is about twice times larger than the theoretical result in Table VII. In this work, we further give the following ratios of the partial widths of the $B^*\pi$ and $B\pi$ decay modes

$$R[\B (1^D_1)] = \frac{\mathcal{B}(\B (1^D_1) \to B^*\pi)}{\mathcal{B}(\B (1^D_1) \to B\pi)} = 0.56,$$

and

$$R[\B (1^D_3)] = \frac{\mathcal{B}(\B (1^D_3) \to B^*\pi)}{\mathcal{B}(\B (1^D_3) \to B\pi)} = 0.90.$$
for the $1^3D_1$ and $1^3D_3$ $B$ mesons, respectively.

### B. The $B_s$ mesons

The low-lying $B_s$ mesons are also far from being established. Up to now, only the $B_s(5367)$, $B_s'(5415)$, $B_{s1}(5830)$, $B_{s2}^0(5840)$, and $B_{s2}^+(5850)$ are collected by the PDG [14]. Among them, the $B_s(5367)$, $B_s'(5415)$ [two 1S states], $B_{s1}(5830)$, and $B_{s2}^0(5840)$ [two narrow 1P states in the 1P($1^+, 2^+\)_{3/2}$ doublet) have been well established in experiment. However, more efforts should be paid for establishing two $P$-wave $B_s$ states in the 1P($0^+$, $1^+\)_{1/2}$ doublet, where assigning the $B_{s2}^+(5850)$ reported by OPAL [42] as a 1P state is still open to dispute. Recently, two higher $B_s$ resonance structures, the $B_{s2}(6044)$ and $B_{s2}(6114)$, were found by the LHCb Collaboration [13]. Decoding the properties of these observed $B_{s2}^+(5850)$, $B_{s3}(6064)$, and $B_{s3}(6114)$ is a task of this work.

#### 1. The 1P states

As shown in Tables IV and VIII, the resonance parameters of $B_{s1}(5830)$ and $B_{s2}^0(5840)$ can be reproduced in our theoretical scheme if they are treated as the members of the 1P($1^+, 2^+\)_{3/2}$ doublet. The partial width ratios of the $B_s^+K^-$ and $B_s^+K^-$ decay channels

$$R[B_s^+(1^P_2)] = \frac{\mathcal{B}(B_s^0(1^P_2) \to B_s^+K^-)}{\mathcal{B}(B_s^0(1^P_2) \to B_s^+K^-)} = 8.6\%,$$

is obtained by the EHQ formula, which is comparable with the experimental value $\Gamma(B_s^0K^-)/\Gamma(B_s^+K^-) = (9.3 \pm 1.8)\%$ [14]. As a counterpart of the $B_s(5721)$ in the $B_s$ sector, the $B_{sJ}(5830)$ also contains a small 1P($1^+, 1^+\) component due to the slight breaking of heavy quark symmetry. Thus, the mixing effect should be considered. If taking the mixing angle of $B_s(5721)$ meson as an input, the total decay width of the $B_{sJ}(5830)$ is evaluated to be 1.33 MeV, which is comparable with the experimental data [11, 60].

#### 2. The 2S states

The average mass of the $2S$ $B_s$ mesons are calculated to be around 6.0 GeV as given in Table IV. Their total decay widths are evaluated to be more than 100 MeV, which are listed in Table IX. These results indicate that the $2S$ bottom-strange mesons are two broad resonances. If roughly comparing the measured mass of the newly observed $B_{sJ}(6064)$ with the predictions of the $2S$ $B_s$ mesons, it seems possible to explain the $B_{sJ}(6064)$ as a $2S$ candidate. However, there still exists a difficulty for this assignment. The width of the $B_{sJ}(6064)$ was measured to be $13$ [13]

$$\Gamma(B_{sJ}(6064)) = 26 \pm 4(\text{stat}) \pm 4(\text{syst}) \text{ MeV},$$

is obtained by the EHQ formula, which is comparable with the experimental value $\Gamma(B_s^0K^-)/\Gamma(B_s^+K^-) = (9.3 \pm 1.8)\%$ [14]. As a counterpart of the $B_s(5721)$ in the $B_s$ sector, the $B_{sJ}(5830)$ also contains a small 1P($1^+, 1^+\) component due to the slight breaking of heavy quark symmetry. Thus, the mixing effect should be considered. If taking the mixing angle of $B_s(5721)$ meson as an input, the total decay width of the $B_{sJ}(5830)$ is evaluated to be 1.33 MeV, which is comparable with the experimental data [11, 60].

### TABLE VII: The calculated decay properties of the $1D$ states of the $B$ meson and the measured width of the $B_s(5970)$ [12] (in MeV).

| Decay modes | 1$^3D_1$ | 1$^3D_2$ | 1$^3D_3$ |
|-------------|----------|----------|----------|
| $B\pi$      | 39.2     | $\times$ | 14.3     |
| $B\pi'$     | 22.1     | 64.7     | 12.9     |
| $B\eta$     | 9.7      | $\times$ | 0.2      |
| $B\eta'$    | 4.4      | 13.7     | 0.1      |
| $B_sK$      | 8.0      | $\times$ | 0.1      |
| $B_s^*K$    | 3.3      | 10.5     | 0.0      |
| $B_p\rho$   | 4.9      | 4.1      | $-$      |
| $B_p\omega$ | 1.3      | 0.7      | $-$      |

The thresholds of the $B_s^0$ meson are expected to be broad. The similar results were also obtained by other quenched potential models [46, 52, 53].

### TABLE VIII: The strong decay behavior of the 1P states of the $B_s$ meson (in MeV).

| Decay modes | 1$^3P_0$ | 1$^3P_1$ | 1$^3P_2$ |
|-------------|----------|----------|----------|
| $B_s(5795)$ | $B_s(5835)$ | $B_s(5819)$ | $B_s(5833)$ |
| $B\pi$      | 226.8    | $\times$ | 1.52     |
| $B\pi'$     | $\times$ | 189.7    | 0.11     |
| Total       | 226.8    | 189.7    | 1.63     |

The properties of the 1$^3P_0$ and 1$^3P_1$ states of the $B_s$ meson are still in debate since no clear signal of these states has been observed in experiment. As shown in Table VIII, the 1$^3P_0$ and 1$^3P_1$ states of the $B_s$ meson are expected to be broad. The similar results were also obtained by other quenched potential models [46, 52, 53].

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4 The thresholds of the $s$-wave channels $BK$ and $B_s^0K^-$ are about 10–20 MeV below the predicted bare masses of the 1$^3P_0$ and 1$^3P_1$ $B_s$ states. So the effect of nearby closed channels should be important for the 1$^3P_0$ and 1$^3P_1$ $B_s$ states. In fact, the 1$^3P_0$ and 1$^3P_1$ $B_s$ states were found to be the bound states below the corresponding thresholds when the nontrivial coupled channel effect was considered [61, 62]. So the experiment may find the $B_s$ states in the 1P($0^+$, $1^+\)_{1/2}$ doublet through the $B_s^0\rho^0$ and $B_s^0\pi^0$ decay channels, respectively, if they are below the $B^0K$ thresholds. Obviously, it is premature to regard the $B_{s2}^+(5850)$ as a candidate of the 1P($0^+$, $1^+\)_{1/2}$ doublet.
which is much smaller than the theoretical expectation (see Table IX). The possibility of $B_{sJ}(6114)$ as a $B(2S)$ state can also be excluded since its mass is too heavy to be regarded as the $2S$ candidate. In the future, our experimental colleague should pay more effort to perform the search for the $B_s(2S)$ mesons.

TABLE IX: The decay widths of the $B_s(2S)$ mesons (in MeV).

| Decay modes | $BK$ | $B^*K$ | $B_{sJ}$ | $B_{sJ}^*$ | Total |
|-------------|------|--------|---------|----------|-------|
| $B(5977)$  | ×    | 118.5  | ×       | −        | 118.5 |
| $B^*(6001)$| 62.8 | 96.4   | 3.0     | 0.3      | 162.5 |

More valuable information of the $B_s(2^3S_1)$ meson can be provided to the further experimental exploration. Based on the partial widths of $BK$ and $B^*K$ channels in Table IX, the following partial width ratios

$$R[B_s^*(2^3S_1)] = \frac{\Gamma(B_s^*(1^3D_1) \rightarrow B^{*+}K^-)}{\Gamma(B_s^*(1^3D_1) \rightarrow B^{*+}K^-)} = 1.54,$$  \hspace{1cm} (13)

is obtained.

3. The $1D$ states

The masses of the $1^3D_1$ and $1^3D_2$ $B_s$ mesons are predicted to be $6153$ MeV and $6164$ MeV, respectively, while the masses of the $1^2D_2$ and $1^2D_3$ $B_s$ mesons are expected to be $6080$ MeV and $6086$ MeV. Mass spectrum analysis supports the $B_{sJ}(6064)$ state as a member in the $1D(2^{-}, 3^{-})_{5/2}$ doublet and the $B_{sJ}(6114)$ state as a member in the $1D(1^{-}, 2^{-})_{3/2}$ doublet. Besides, we also study the strong decays of the $D$-wave $B_s$ mesons, as listed in Table X. The predicted decay behaviors may enforce to the assignments of $B_{sJ}(6064)$ and $B_{sJ}(6114)$ above. Concretely, the narrow $B_{sJ}(6064)$ could be regarded a candidate of a $1D_2$ or $1^3D_1$ $B_s$ state, while the $B_{sJ}(6114)$ state could be a $1^3D_1$ or $1^3D_2$ $B_s$ meson.

TABLE X: The partial and total decay widths of the $1D$ states of the $B_s$ meson (in MeV).

| Decay modes | $1^3D_1$ | $1^3D_2$ | $1^2D_2$ | $1^2D_3$ |
|-------------|---------|---------|---------|---------|
| $B(6153)$  | $B_{sJ}$ | $B(6164)$ | $B_{sJ}$ | $B(6080)$ | $B_{sJ}$ | $B(6086)$ | $B_{sJ}$ |
| $BK$       | 64.1    | 33.1    | ×       | 12.2    | 5.2     | 114.6   | 26.8    |
| $B^*K$     | ×       | 99.1    | 33.1    | 11.8    | 0.2     | 115.4   | 25.8    |
| $B_{sJ}$   | ×       | ×       | 99.1    | 11.8    | 0.2     | 115.4   | 25.8    |
| $B_{sJ}^*$ | ×       | ×       | ×       | 99.1    | 0.2     | 115.4   | 25.8    |
| $B_{sJ}^*$ | ×       | ×       | ×       | ×       | 99.1    | 0.2     | 115.4   | 25.8    |
| Total      | 114.6   | 115.4   | 12.0    | 17.6    |         |         |         |
| Expt. [13] | 72±43   | 64±8    |         |         |         |         |         |

In the future, the experiment may measure the following partial width ratios

$$R[B_s^*(1^3D_1)] = \frac{\Gamma(B_s^*(1^3D_1) \rightarrow B^{*+}K^-)}{\Gamma(B_s^*(1^3D_1) \rightarrow B^{*+}K^-)} = 0.52,$$  \hspace{1cm} (14)

and

$$R[B_s^*(1^3D_3)] = \frac{\Gamma(B_s^*(1^3D_3) \rightarrow B^{*+}K^-)}{\Gamma(B_s^*(1^3D_3) \rightarrow B^{*+}K^-)} = 0.76,$$  \hspace{1cm} (15)

to examine the properties of the $1^3D_1$ and $1^3D_3$ $B_s$ states.

IV. DOUBLE BOTTOM BARYONS

A. The $\Xi_{bb}$ baryons

1. The $nS$ states of the $\Xi_{bb}(nS)$ baryon

The masses of the ground $\Xi_{bb}$ baryons, including the $\Xi_{bb}^*$ $(J^P = 1/2^+)$ and $\Xi_{bb}^{*+}$ $(J^P = 3/2^+)$ states, have been investigated by the different methods or models (see Refs. [63, 64] and references therein).

With the quark potential model introduced in Sec. II, the mass of the $\Xi_{bb}^*$ state is predicted to be $10171$ MeV, which is comparable with the results from Refs. [23, 41, 65–67]. The complete mass spectra of these low-lying $\Xi_{bb}$ states are listed in Table XI. The mass difference of the $\Xi_{bb}^*(1S)$ and $\Xi_{bb}^{*+}(1S)$ states is predicted to be $24$ MeV, which is comparable with these results from Refs. [23, 41, 74, 75]. There only exists the weak decays for the $\Xi_{bb}^*$ state, while the $\Xi_{bb}^{*+}$ state can transit into the $\Xi_{bb}^{*+}$ state by emitting a photon.

The masses of the $2S$ states of the $\Xi_{bb}$ baryon are predicted to be about $550$ MeV above the ground $\Xi_{bb}$ states, which allows the $2S$ $\Xi_{bb}$ states to decay into a ground or $1P$ $\Xi_{bb}$ state plus a light meson. The strong decay properties of these two $2S$ states of the $\Xi_{bb}$ baryon are given in Table XII. Our result indicates that the $2S$ states of the $\Xi_{bb}$ baryon are two broad resonances, and the $\Xi_{bb}^*\pi$ and $\Xi_{bb}^{*+}\pi$ are their main decay modes. Finally, we predict the partial width ratios of $\Xi_{bb}^*\pi$ and $\Xi_{bb}^{*+}\pi$

$$R[\Xi_{bb}^*(10738)] = \frac{\Gamma(\Xi_{bb}^*(10738) \rightarrow \Xi_{bb}^*\pi)}{\Gamma(\Xi_{bb}^*(10738) \rightarrow \Xi_{bb}^{*+}\pi)} = 7.64,$$  \hspace{1cm} (16)

and

$$R[\Xi_{bb}^{*+}(10738)] = \frac{\Gamma(\Xi_{bb}^{*+}(10738) \rightarrow \Xi_{bb}^{*+}\pi)}{\Gamma(\Xi_{bb}^{*+}(10738) \rightarrow \Xi_{bb}^*\pi)} = 1.20,$$  \hspace{1cm} (17)

for the $2S$ states of the $\Xi_{bb}$ baryon, which could be tested in the future experiment.

\footnote{For the $QQb$ baryon, there only exists the experimental observation of the double charm baryon $\Xi_c^*(3621)$ from the LHCb Collaboration [68–70]. Before the discovery of $\Xi_c^*(3621)$, some theoretical groups successfully predicted its mass [23, 41, 65–67]. These groups also predicted the $\Xi_{bb}$ state in the mass range of $10.14–10.20$ GeV. The nearly equal predictions were also achieved for the mass of $\Xi_{bb}$ state by other approaches including the QCD sum rule [71, 72], the Salpeter model with AdS/QCD inspired potential [73], and the extended chromomagnetic model [74].}
Fig. 3). The predicted average mass of 1
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TABLE XII: The decay widths of the 2S states of the Ξbb baryon (in MeV).

| Decay modes | 2S (1/2 + 3/2) | 2S (1/2 + 3/2) | (Continued) | Ξbb baryon | Ξbb baryon |
|-------------|----------------|----------------|--------------|-------------|-------------|
|             | Ξbb(10738)    | Ξbb(10753)    |              |             |             |
| Ξbbπ       | 19.4          | 79.3          | Ξbb(10593)π | 1.2         | ×           |
| Ξbbη       | 148.3         | 95.5          | Ξbb(10606)π | × 1.3       |             |
| Ξbbη       | 0.2           | 1.6           | Ξbb(10547)π | × 0.1       |             |
| Ξbbη       | –             | 0.3           | Ξbb(10561)π | 0.0 0.1     |             |
| Ξbbη       | –             | 0.3           | Ξbb(10560)π | 0.1 0.1     |             |
| Total       | 169.2         | 178.3         |              |             |             |

2. The 1P state of the Ξbb baryon

Different from the case of the \(\bar{b}q\) meson, there are five excited states for the \(P\)-wave Ξbb baryons. According to their light degrees of freedom \(j_q\), one can categorize these five 1P Ξbb states into one doublet and one triplet, which are denoted as 1P(1/2\(^+\), 3/2\(^−\))\(_{1/2}\) and 1P(1/2\(^+\), 3/2\(^−\), 3/2\(^−\))\(_{3/2}\) (see Fig. 3). The predicted average mass of 1P Ξbb baryons is about 380 MeV higher than that of the 1S Ξbb states, which agrees with the expectations in Refs. [75–77]. However, the predictions of “\(\frac{1}{2}\) (1P) – \(\frac{1}{2}\) (1S)” for the Ξbb baryons, which were predicted in Refs. [21, 23], are about 50–70 MeV larger than our result. Then these different expectations should be tested in a future experiment.

The Ξbbπ and Ξbbπ are the main decay channels of the 1P Ξbb baryons. According to our results in Table XIII, two states in the 1P(1/2\(^−\), 3/2\(^−\))\(_{1/2}\) doublet have broad widths, while the states in the 1P(1/2\(^−\), 3/2\(^−\), 3/2\(^−\))\(_{3/2}\) triplet are much narrow. Thus, finding the Ξbb(10547), Ξbb(10561), and Ξbb(10560) in the decay channels Ξbbπ and Ξbbπ is suggested. Our conclusion for the decay behaviors of the 1P Ξbb baryons is consistent to the former work [79]. We further give the following partial width ratios of the Ξbbπ and Ξbbπ channels

\[ R[\Xi_{bb}(10561)] = \frac{\Gamma(\Xi_{bb}(10561) \rightarrow \Xi_{bb}^0 \pi)}{\Gamma(\Xi_{bb}(10561) \rightarrow \Xi_{bb}^0 \pi')} = 2.85, \quad (18) \]

and

\[ R[\Xi_{bb}(10560)] = \frac{\Gamma(\Xi_{bb}(10560) \rightarrow \Xi_{bb}^0 \pi)}{\Gamma(\Xi_{bb}(10560) \rightarrow \Xi_{bb}^0 \pi')} = 0.62, \quad (19) \]

for the 1P(3/2, 3/2\(^−\)) and 1P(3/2, 5/2\(^−\)) states of Ξbb baryon.

TABLE XIII: The decay widths of the 1P states of the Ξbb baryon (in MeV).

| Ξbb(10593) | Ξbb(10606) | Ξbb(10547) | Ξbb(10561) | Ξbb(10560) |
|------------|------------|------------|------------|------------|
| Ξbbπ       | 262.5      | ×          | 3.3        | 8.7        |
| Ξbbπ       | ×          | 266.6      | 9.5        | 9.4        |
| Total      | 262.5      | 266.6      | 9.5        | 12.7       | 14.1      |

3. The 1D states of the Ξbb baryon

As shown in Fig. 3, six D-wave Ξbb states can be grouped into two triplets, which are distinguished by their light degrees of freedom \(j_q\). Interestingly, the average mass of these Ξbb baryons in the 1D(1/2\(^−\), 3/2\(^−\), 3/2\(^−\))\(_{3/2}\) triplet is about 100 MeV higher than the members in the 1D(3/2\(^−\), 5/2\(^−\), 7/2\(^−\))\(_{3/2}\) triplet. Furthermore, the mass differences of these states in the same triplet are only several MeV. Accordingly, the mixing effect of the 1D Ξbb baryons which have the same \(J^P\) is not obvious in the \(jj\) coupling scheme. Here, the mixing angle of two \(J^P = 3/2^−\) states fixed by the quark potential model is no more than one degree. There exist the similar result for the case of two \(J^P = 5/2^−\) Ξbb states.
TABLE XIV: The decay widths of the 1D states of the Ξ_{bb} baryon (in MeV). The P-wave Ξ_{bb} baryon, as a daughter state in the decay process, is denoted as Ξ_{bb}^{(i)}(J^P)\). The mass of P-wave Ξ_{bb} baryon has been given in Table XI.

| State             | Ξ_{bb}(10913) | Ξ_{bb}(10918) | Ξ_{bb}(10921) | Ξ_{bb}(10798) | Ξ_{bb}(10803) | Ξ_{bb}(10805) | Ξ_{bb}(2S) | Ξ_{bb}^* K | Ξ_{bb} K | Ξ_{bb}^* / Ξ_{bb} K |
|-------------------|--------------|--------------|--------------|--------------|--------------|--------------|-----------|-----------|---------|----------------|
| Ξ_{bb}(10913)     | 60.3         | 7.9          | 11.0         | 1.2          | 7.5          | 0.8          | ×         | 0.2       | 2.2     | 0.6           | 0.0           | 0.0       | 277.6     |
| Ξ_{bb}(10918)     | 37.3         | 31.2         | 7.0          | 5.1          | 4.8          | 3.2          | 0.1       | 0.2       | 1.5     | 1.7           | 0.0           | 0.0       | 279.7     |
| Ξ_{bb}(10921)     | ×            | 69.8         | ×            | 11.6         | ×            | 7.4          | 0.2       | 0.1       | 0.3     | 1.2           | ×            | 0.0       | 279.1     |
| Ξ_{bb}(10798)     | ×            | 13.5         | ×            | 0.0          | ×            | 0.0          | 0.0       | 0.2       | 0.2     | 0.0           | ×            | 13.9      |
| Ξ_{bb}(10803)     | 4.3          | 10.9         | 0.0          | 0.0          | 0.0          | 0.0          | 0.2       | 0.1       | 0.2     | 0.2           | –            | –         | 15.7      |
| Ξ_{bb}(10805)     | 10.6         | 6.2          | 0.0          | 0.0          | 0.0          | 0.0          | ×         | 0.0       | 0.1     | 0.3           | –            | –         | 17.2      |

The small mixing angle of the 1D states of the Ξ_{bb} baryon with the same J^P also results in that three members in one triplet have the similar decay widths. As the members of the 1D(1/2+), 3/2+, 5/2+)\rangle_{1/2} triplet, the Ξ_{bb}(10913), Ξ_{bb}(10918), and Ξ_{bb}(10921) states are predicted to be very broad.\(^6\) The s-wave decay channels, including the Ξ_{bb}^{(1/2)}(1/2−)π, Ξ_{bb}^{(3/2)}(3−)π, and Ξ_{bb}^{(5/2)}(5−)π, mainly contribute to the total widths of the Ξ_{bb}(10913), Ξ_{bb}(10918), and Ξ_{bb}(10921) (see Table XIII). The Ξ_{bb}(10798), Ξ_{bb}(10803), and Ξ_{bb}(10805) in the 1D(3/2+, 5/2+, 7/2+)\rangle_{1/2} triplet are much narrower since their total decay widths are around 10–20 MeV.

Our study shows that the Ξ_{bb}(10913) could be found by analyzing the Ξ_{bb}^{*}π and Ξ_{bb}^{(10547)}[J_q = 3/2, J_P = 1/2 + ] + π channels. The Ξ_{bb}(10918) could be searched in the Ξ_{bb}^{*}π, Ξ_{bb}π, and Ξ_{bb}(10561)[J_q = 3/2, J_P = 3/2 + ] + π channels. The Ξ_{bb}(10921) could be observed in its Ξ_{bb}^{*}π and Ξ_{bb}(10560)[J_q = 3/2, J_P = 5/2 − ] + π channels. The Ξ_{bb}^{*}π and Ξ_{bb}π are the main decay modes of these narrow states in the 1D(3/2+, 5/2+, 7/2+)\rangle_{1/2} triplet. Specifically, the Ξ_{bb}(10798) could be detected in the Ξ_{bb}^{*}π channel, while the Ξ_{bb}(10803) and Ξ_{bb}(10805) could be found in their Ξ_{bb}^{*}π and Ξ_{bb}π decay channels. We also obtain the following ratios

\[
R[Ξ_{bb}(10913)] = \frac{Γ(Ξ_{bb}(10913) → Ξ_{bb}^{*}π)}{Γ(Ξ_{bb}(10913) → Ξ_{bb}π)} = 0.13,
\]

\[
R[Ξ_{bb}(10918)] = \frac{Γ(Ξ_{bb}(10918) → Ξ_{bb}^{*}π)}{Γ(Ξ_{bb}(10918) → Ξ_{bb}π)} = 0.84,
\]

\[
R[Ξ_{bb}(10803)] = \frac{Γ(Ξ_{bb}(10803) → Ξ_{bb}^{*}π)}{Γ(Ξ_{bb}(10803) → Ξ_{bb}π)} = 2.53,
\]

\[
R[Ξ_{bb}(10805)] = \frac{Γ(Ξ_{bb}(10805) → Ξ_{bb}^{*}π)}{Γ(Ξ_{bb}(10805) → Ξ_{bb}π)} = 0.58,
\]

for the 1D Ξ_{bb} states.

\(^6\) According to our result, the Ξ_{bb}(10913) and Ξ_{bb}(10918) states can also decay into the B_{bb} channel via the p-wave. Since these two Ξ_{bb} states are just above the threshold of the B_{bb} channel, the phase spaces of these decay processes are tiny. Thus, the partial decay widths of the B_{bb} decay channel are not large for the broad Ξ_{bb}(10913) and Ξ_{bb}(10918) states.

B. The Ω_{bb} baryons

1. The nS states of the Ω_{bb} baryon (n = 1, 2)

TABLE XV: The decay widths of the 2S and 1D states of the Ω_{bb} baryon (in MeV).

| State             | Ω_{bb}K | Ω_{bb}^* K | Ω_{bb}πη | Ω_{bb}^* πη | Total |
|-------------------|---------|------------|----------|-------------|-------|
| Ω_{bb}(10816)     | 16.8    | 109.9      | 0.0      | –           | 126.7 |
| Ω_{bb}(10830)     | 73.7    | 77.6       | 0.7      | –           | 152.0 |
| Ω_{bb}(10971)     | 88.3    | 10.9       | 12.2     | 1.3         | 112.7 |
| Ω_{bb}(10975)     | 55.2    | 43.7       | 7.8      | 5.3         | 112.0 |
| Ω_{bb}(10979)     | ×       | 98.6       | ×        | 12.2        | 110.8 |
| Ω_{bb}(10891)     | ×       | 5.4        | ×        | 0.0         | 5.4   |
| Ω_{bb}(10896)     | 2.1     | 4.5        | 0.0      | 0.0         | 6.6   |
| Ω_{bb}(10898)     | 5.3     | 2.6        | 0.0      | 0.0         | 7.9   |

The experimental measurements\(^{[14]}\) indicate that the B_s (or D_s) state is about 100 MeV heavier than the corresponding B (or D) state. As shown in Table XI, the masses of ground Ω_{bb} states are expected to be about 100 MeV larger than the ground Ξ_{bb} states, which is similar to the case of the heavy-light mesons. Searching for the Ω_{bb} state via its weak decay processes is suggested, while the Ω_{bb}^* can be found in the decay channel Ω_{bb}^* → Ω_{bb}^* + υ_s.

The predicted masses of the 2S Ω_{bb} states are given in Table XI, which are about 540 MeV higher than the average mass of the ground states of the Ω_{bb} baryon. Thus, the decay modes Ω_{bb}^* K, Ω_{bb}^{*} K, and Ω_{bb}^{*} πη are allowed for the 2S states of the Ω_{bb} baryon. Here, the total decay widths of two 2S states of the Ω_{bb} baryon are predicted to be larger than 100 MeV (see Table XV). It is obvious that the discussed Ω_{bb}(10816) and Ω_{bb}^*(10830) are two broad resonances, which can mainly decay into the Ξ_{bb}^* K and Ξ_{bb}^* K channels. Furthermore, the following partial width ratios

\[
R[Ω_{bb}(10816)] = \frac{B(Ω_{bb}(10816) → Ξ_{bb}^* K)}{B(Ω_{bb}(10816) → Ξ_{bb}^* K)} = 6.54,
\]

\[
R[Ω_{bb}(10830)] = \frac{B(Ω_{bb}(10830) → Ξ_{bb}^* K)}{B(Ω_{bb}(10830) → Ξ_{bb}^* K)} = 1.05,
\]

are calculated for the 2S states of the Ω_{bb} baryon.
2. The 1P states of the Ωbb baryon

Table XVI: The decay widths of the 1P states of the Ωbb baryon (in MeV).

| State          | Width (MeV) |
|----------------|-------------|
| Ωbb(10669)     | 96.2        |
| Ωbb(10681)     | ×           |
| Ωbb(10641)     | ×           |
| Ωbb(10656)     | ×           |
| Ωbb(10655)     | ×           |

The obtained masses of these five 1P Ωbb states are listed in Table XI, which are about 470 MeV above the ground Ξbb states. Then, the OZI-allowed strong decays are forbidden for most of 1P states of the Ωbb baryon since they are located below the thresholds of the Ξbb(1S)K channel. As indicated by our results in Table XVI, only the 1P(1/2−, 1/2+) state (the Ωbb(10669) in Table XI) can proceed via its OZI-allowed decay process.7

3. The 1D states of the Ωbb baryon

As shown in Table XI, the mass gap between the 1D(1/2+, 3/2+, 5/2+)S/2 and 1D(3/2+, 5/2+, 7/2+)(5/2) triplets of the 1D Ωbb baryons is predicted to be about 80 MeV. Similar to the case for the Ξbb baryon, the mixing angles of the D-wave Ωbb states with the same J^P are very small due to the large mass of heavy diquark (bb). In Table XV, we collect the predicted decay behaviors of these 1D states of the Ωbb baryon which are also similar to that of the corresponding Ξbb counterparts. The Ωbb(10971), Ωbb(10975), and Ωbb(10979) in the 1D(1/2+, 3/2+, 5/2+)(5/2) triplet are expected to be the broad resonances, while the Ωbb(10891), Ωbb(10896), and Ωbb(10898) states in another triplet have the narrow total decay widths. The ΞbbK and ΞbbK are their main decay channels, which could be as the ideal channels to search for these discussed D-wave excited Ωbb states. The following

7 It should be careful to the conclusion obtained by the quenched quark model, since the bare masses of the Ωbb(10669) and Ωbb(10681) states are near to the thresholds of their s-quark channel ΞbbK and ΞbbK, respectively. So the nontrivial coupled channel effect should be important for the 1P(1/2, 1/2−) and 1P(1/2, 3/2−) Ωbb states. Further theoretical study on this issue should be paid. As a matter of fact, the masses of the Ωbb(10669) and Ωbb(10681) could be shifted down tens MeV due to the coupled channel effect. Thus, the physical 1P(1/2, 1/2−) and 1P(1/2, 3/2−) Ωbb states would be below the thresholds of their OZI-allowed decay channels, which implies that both 1P(1/2, 1/2−) and 1P(1/2, 3/2−) Ωbb baryons are the narrow states. In a word, the five 1P Ωbb baryons are probably the narrow states. This interesting scenario could be tested by the future experiment.

R[Ωbb(10971)] = \frac{\Gamma(Ωbb(10971) \rightarrow Ξbb K)}{\Gamma(Ωbb(10971) \rightarrow Ξbb K)} = 0.12,

R[Ωbb(10975)] = \frac{\Gamma(Ωbb(10975) \rightarrow Ξbb K)}{\Gamma(Ωbb(10975) \rightarrow Ξbb K)} = 0.79,

R[Ωbb(10896)] = \frac{\Gamma(Ωbb(10896) \rightarrow Ξbb K)}{\Gamma(Ωbb(10896) \rightarrow Ξbb K)} = 2.14,

R[Ωbb(10898)] = \frac{\Gamma(Ωbb(10898) \rightarrow Ξbb K)}{\Gamma(Ωbb(10898) \rightarrow Ξbb K)} = 0.49,

are predicted, which could be applied to further distinguish the different Ωbb states in the same triplet. The partial width ratios of the Ωbb(10971), Ωbb(10975), Ωbb(10896), and Ωbb(10898) are comparable with those of D-wave Ξbb states in Eq. (20), which may provide a direct evidence to reflect the dynamical similarity of the Ξbb and Ωbb systems.

V. FURTHER DISCUSSIONS OF THE DYNAMICAL SIMILARITIES BETWEEN THE bq AND bbq SYSTEMS

For a clarity, the main results for the mass spectrum and strong decay behavior of these discussed low-lying bq and bbq states are presented in Fig. 4. In the following, we discuss the dynamical similarities between the bq and bbq systems behind our results. For the mass spectra, we first define the following ratios

\[ R_1 = \frac{M_{2S} - M_{1S}}{M_{1P} - M_{1S}}, \quad R_2 = \frac{M_{1D} - M_{1S}}{M_{1P} - M_{1S}}. \]

Here, the \( M_{1S}, M_{1P}, M_{2S}, \) and \( M_{1D} \) represent the average masses of the 1S, 1P, 2S, and 1D states in the B, B∗, Ξbb, and Ωbb sectors. As shown in Table XVII, we find that \( R_1 \) or \( R_2 \) for the B, B∗, Ξbb, and Ωbb systems almost have the same values. Since the 1P, 2S, and 1D states of the B/B∗ mesons are not well established and none of the bbq baryons have been found in experiment, we should take the predicted \( R_1 \) and \( R_2 \) of the bq/bbq systems to make a comparison with these Λ′ baryons which could also be regarded as typical heavy-light hadrons in the diquark picture. With the measured masses of Λbb(5620), Λbb(5912), Λbb(5920), Λbb(6070), Λbb(6146), Λbb(6152) states, the \( R_1 \) and \( R_2 \) values [80] are also listed in Table XVII. Obviously, the predicted \( R_1 \) or \( R_2 \) of the bq/bbq states are comparable with the measurements of Λbb baryons. The nearly equal values of \( R_1 \) or \( R_2 \) indicate that the bottom mesons, the single bottom baryons, and the double bottom baryons have the similar dynamics. This novel phenomenon may reflect that the superflavor symmetry is an effective symmetry for these different kinds of bottom hadron systems.

In the following, we should mention the spin-orbit inversion phenomenon of the highly orbital excitations of the bq and bbq systems, which may also reflect the dynamical similarity between the bq and bbq hadrons. The spin-orbit inversion of these P-wave heavy-light mesons has been discussed in Refs. [81–83] for many years. Since the P-wave bottom
mesons are not established, the spin-orbit inversion of the P-wave heavy-light mesons remains inconclusive. According to our predictions in Table IV and XI, the spin-orbit inversion should appear in the D-wave $bq$ and $bbq$ states since the members in the $j_q = 5/2$ multiplet are about 70–100 MeV lower than the $j_q = 3/2$ multiplet (see Fig. 4). We try to give a qualitative explanation to this phenomenon. For the D-wave excited $bq/bbq$ states, the contribution from the spin-dependent interactions arising from the short-range one-gluon exchange contribution becomes smaller, while the contribution from the Thomas-precession term from the long-range confining potential becomes dominant. Thus, the first term in Eq. (4), as the mainly spin-dependent interaction for the orbitally excited potential becomes dominant. Thus, the first term in Eq. (4), as

\[ \text{Thomas-precession term} \]

contribution becomes smaller, while the contribution from the $l$ multiplet. This phenomenon can be reflected from our results in Tables V, VII, X, XIII, XIV, and XV, where the predicted decay widths of these states in the $j_q = L + \frac{1}{2}$ multiplet are at least one order smaller than those of the states in the $j_q = L - \frac{1}{2}$ multiplet. In fact, this phenomenon has been confirmed by the measured decay widths of the 1P states of the D meson [14]. Here, the decay widths of the $D_1(2420)$ and $D_1^*(2460)$, which belong to the $J^P = \frac{3}{2}^+$ doublet, were measured to be 25–50 MeV. In contrast, the decay widths of the $D_0^*(2300)$ and $D_0(2430)$, the states in the $J^P = \frac{1}{2}^+$ doublet, are larger than 200 MeV. For enforcing the conclusion for the decay widths of the $j_q = L \pm \frac{1}{2}$ multiplets, we expect more data of these discussed low-lying $bq$ and $bbq$ states to be accumulated in the future experiment.

VI. CONCLUSION AND OUTLOOK

The dynamical similarities between the $bq$ and $bbq$ systems provide us a possibility to carry a combined study of their properties in the same theoretical framework. In this work, we systematically investigate the mass spectra and strong decays of these low-lying $B$, $B_s$, $\Xi_{bb}$, and $\Omega_{bb}$ states. Our result not only decodes these newly observed states $B_s(5840)$, $B_s(5970)$, $B_{sJ}(6064)$, and $B_{sJ}(6114)$, but also reveals the similarities of the $bq$ and $bbq$ systems existing in their mass spectra and strong decays.

According to our results, the $B_s(5840)$ could be a $2S$ state with $J^P = 0^-$ or $J^P = 1^-$, while the $B_s(5970)$ could be a candidate of the $1D(2^-, 3^-)_{J_q=5/2}$ doublet. Since a low-energy photon from the $B^+ \to B^0$ decay was not reconstructed in the realistic measurements of the CDF [11] and LHCb [12] collaborations, the $B_s(5840)$ signal can be resulted by two $2S$ states of the $B$ meson. Similar phenomenon may also happen for the $B_s(5970)$ signal which could contain two states of the $1D(2^-, 3^-)_{J_q=5/2}$ doublet. Thus, we suggest the experiment to further identify the spin-parity quantum numbers for these states, or measure the ratio $\Gamma(B^*\pi)/\Gamma(B\pi)$, which is helpful to clarify the nature of $B_s(5840)$ and $B_s(5970)$. By compar-

TABLE XVII: The predicted ratios of $\mathcal{R}_1$ and $\mathcal{R}_2$ defined in Eq. (23) for the $bq$ and $bbq$ states and a comparison with the measured $\mathcal{R}_1$ and $\mathcal{R}_2$ of the $\Lambda_b$ baryons.

| Ratios | $\Lambda_b$ | $B$ | $B_s$ | $\Xi_{bb}$ | $\Omega_{bb}$ |
|--------|-------------|-----|-------|-------------|-------------|
| $\mathcal{R}_1$ | 1.520 | 1.428 | 1.399 | 1.453 | 1.430 |
| $\mathcal{R}_2$ | 1.780 | 1.699 | 1.680 | 1.715 | 1.702 |

FIG. 4: The predicted masses and decay properties of these discussed low-lying $B$, $B_s$, $\Xi_{bb}$, and $\Omega_{bb}$ states. The superscripts “$n$” and “$b$” in the brackets represent the predicted narrow and broad states. The important thresholds of the $B^*\Lambda, B_\pi, \Xi_{bb},$ and $\Omega_{bb}$ channels are presented for the $B$, $\Xi_{bb}$, and $\Omega_{bb}$ states. The newly observed $B_s(5840), B_s(5970), B_{sJ}(6064)$, and $B_{sJ}(6114)$ are also listed for a comparison.
ing to our predicted masses and decays of the $B_1$ mesons, the $B_{sJ}(6064)$ and $B_{sJ}(6114)$ states which were newly observed by the LHCb Collaboration [13] could be grouped into the $1D(1^{-}, 2^{+})_{J=3/2}$ and $1D(2^{-}, 3^{+})_{J=5/2}$ doublets, respectively. Experimental measurement of the ratio $Γ(B^∗K)/Γ(BK)$ for the $B_{sJ}(6064)$ and $B_{sJ}(6114)$ states can further provide the valuable information. For other unknown $1P$, $2S$, and $1D B/B_1$ mesons, we present a complete prediction for their masses and decay properties, which is useful for the further exploration in the experiment.

The undiscovered $bbq$ baryons are also investigated in this work, where the mass spectra and strong decays of these low-lying $Ξ_{bb}$ and $Ω_{bb}$ baryons, including the $1S$, $2S$, $1P$, and $1D$ states, are presented. These predictions may provide some clues for the experiment to search for these double bottom baryons in future.

We further point out the similarities between the $bq$ and $bbq$ systems which are implied in their mass spectrum and strong decay. As the first similarity of the $bq$ and $bbq$ systems, we define two ratios in Eq. (23) and show that the predicted $R_1$ and $R_2$ values are nearly equal for the $B$, $B_s$, $Ξ_{bb}$, and $Ξ_{bb}$ states (see Table XVII). We further point out that the predicted $R_1$ and $R_2$ of the $bq$ and $bbq$ systems are also comparable with the measured result of the $D$ meson. As the second similarity, we find a spin-orbit inversion may occur in the $D$-wave $bq$ and $bbq$ states. Finally, we point out that the $bq$ and $bbq$ states in the $j_q = L - \frac{1}{2}$ multiplet are much broader than those in the $j_q = L + \frac{1}{2}$ multiplet, which could be regarded as the third similarity of $bq$ and $bbq$ systems.

With the accumulation of experimental data, more and more new hadrons have been discovered in the past decades [84, 85]. Especially, with the running of LHCb, the studies of heavy flavor hadron enter a new era [86]. When facing on this issue, we have reasons to believe that more progresses on the heavy flavor hadron will be made in the near future. Surely, the present study may provide some valuable information for the experiments in the next stage.

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