Description of the weak interactions within the framework of electrodynamics

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Abstract

The weak interactions are described within the framework of electrodynamics. The massive vector field is considered as photon with the effective mass defined in the second order of the perturbation theory. Muon and tau-lepton are considered to be composite particles having the structure electron-positron-electron. The mass of muon is defined by the section of two-photon annihilation. The mass of tau-lepton is defined by the section of three-photon annihilation.

1 Introduction

As known [1], the weak interactions can be described by the Lagrangian

$$L \rightarrow -\frac{e^2}{m^2} J_\mu(x) J_\nu(x) \rightarrow -G_F J_\mu(x) J_\nu(x).$$ (1)

From this the weak interactions can be considered within the framework of the theory with the massive vector field where the mass of the vector field is defined by the Fermi constant

$$m \sim \left( \frac{e^2}{G_F} \right)^{1/2}. \quad (2)$$

Since the theory with the massive vector fields is nonrenormalizable, this must follow from some renormalizable theory.

In the $SU(2) \times U(1)$ electroweak theory [2], original bosonic and fermionic fields are massless. These acquire masses due to the Higgs mechanism. The scalar field $\varphi$ is introduced, with its potential is given by

$$V = -\lambda^2 (|\varphi|^2 - \eta^2)^2. \quad (3)$$

Interaction of the vector fields with the scalar field gives the masses to the vector fields $Z, W$

$$m_{Z,W} \sim e\eta. \quad (4)$$

Also the massive scalar Higgs field $H$ arises, with its mass is given by

$$m_H = 2\lambda \eta. \quad (5)$$

Masses of the fermions, leptons and quarks, arise due to the Yukawa interaction

$$f(\bar{\psi}_L \psi_R \phi + \bar{\psi}_R \psi_L \phi) \quad (6)$$
where the couplings $f$ are different for each fermion.

In the $SU(2) \times U(1)$ theory, the value of $\eta$ is defined by the Fermi constant

$$\eta \sim (G_F)^{-1/2}. \tag{7}$$

The value of $\lambda$ is not defined. Yukawa couplings are defined by the masses of the fermions

$$f \sim m_{\text{fer}}. \tag{8}$$

Thus the $SU(2) \times U(1)$ theory do not define the mass of the Higgs field and do not explain the hierarchy of the leptons and quarks masses.

## 2 Massive vector field

In the electrodynamics, in the second order of the perturbation theory, the transition of photon into itself via the birth and annihilation of the electron-positron pair occurs [4]. At the energy of photon equal to the mass of electron when the formation of the real electron-positron pair is possible, the matrix element of transition is given by

$$M \sim \frac{\alpha^2}{m_e}. \tag{9}$$

This can be interpreted as that photon acquires an effective mass

$$m \sim \alpha^{-2}m_e. \tag{10}$$

In this way we can consider interactions with the massive vector field where the mass is given by eq. [11]

$$L \rightarrow -\frac{e^2}{m_e} \alpha^4 (x) J_{\mu}(x) J_{\nu}(x). \tag{11}$$

Let us assume that the weak interactions can be described within the framework of electrodynamics. $Z$-, $W$-bosons can be considered as photons with effective masses defined in the second order of the perturbation theory. Then the weak interactions can be described by the Lagrangian [11]. The theory with the Lagrangian [11] is renormalizable, since this emerges within the framework of electrodynamics.

Consider the process of scattering of photon by photon proceeding in the second order of the perturbation theory

$$\gamma\gamma \rightarrow e^- e^+ e^- e^+ \rightarrow \gamma\gamma. \tag{12}$$

Determine the effective mass of photon from the section of the reaction [12]. At $h\omega < m_e c^2$, the section of scattering for non-polarized photons grows as [2]

$$\sigma = 0.031 \alpha^2 \frac{r^2}{e^4} \left( \frac{h\omega}{m_e c^2} \right)^6. \tag{13}$$
At $\hbar \omega >> m_e c^2$, the section of scattering for non-polarized photons diminishes as 

$$\sigma = 4.7 \alpha^4 \left( \frac{\epsilon}{\omega} \right)^2. \quad (14)$$

In the limiting case $\hbar \omega = m_e c^2$ when the formation of the real electron-positron pair is possible, the section of scattering is

$$\sigma = 0.031 \alpha^2 r_e^2. \quad (15)$$

This gives the radius of interaction

$$r = \left( \frac{0.031}{\pi} \right)^{1/2} \alpha r_e \quad (16)$$

and the effective mass of photon

$$m = \left( \frac{\pi}{0.031} \right)^{1/2} \frac{\hbar}{\alpha r_e c} = \left( \frac{\pi}{0.031} \right)^{1/2} \frac{m_e}{\alpha^2}. \quad (17)$$

According to eqs. (16), (17), the radius of interaction is $r \sim 2 \cdot 10^{-16}$ cm, and the effective mass of photon is $m = 96.5 \cdot 10^2$ GeV. This value is close to the mass of Z-boson equal to $m_Z = 93 \cdot 10^2$ GeV \[3\].

### 3 Mass hierarchy between lepton generations

Let us consider the problem of mass hierarchy between lepton generations within the framework of electrodynamics. The mass of electron is a parameter defined from the experimental data. The theory have to define the masses of muon and tau-lepton.

Let us assume that muon and tau-lepton are composite particles. Namely muon and tau-lepton have the following structure

$$\mu^- \equiv e^- e^+ e^- \quad (18)$$

$$\tau^- \equiv e^- e^+ e^- \quad (19)$$

Muon arises due to the reaction

$$e^- + 2\gamma \rightarrow e^- e^+ e^- \quad (20)$$

and tau-lepton arises due to the reaction

$$e^- + 3\gamma \rightarrow e^- e^+ e^- \quad (21)$$

The mass of muon is defined by the energy required to bear electron-positron pair in the process of two-photon annihilation. Since two-photon annihilation is characterized by the classical radius of electron $r_e$, the mass of muon is of order

$$m_\mu \sim \frac{\hbar}{\alpha r_e} \quad (22)$$
According to eq. (22), the mass of muon is $m_\mu \sim 70$ MeV that agrees with the experimental value $m_\mu = 106$ MeV [3].

The mass of tau-lepton is defined by the energy required to bear electron-positron pair in the process of three-photon annihilation. Let us determine the radius which characterizes three-photon annihilation. The section of two-photon annihilation in the non-relativistic limit is given by [2]

$$\bar{\sigma}_{2\gamma} = \pi \left( \frac{e^2}{m_e c^2} \right)^2 \frac{c}{v}, \quad (23)$$

The section of three-photon annihilation in the non-relativistic limit is given by [2]

$$\bar{\sigma}_{3\gamma} = \frac{4(\pi^2 - 9)}{3} \alpha \left( \frac{e^2}{m_e c^2} \right)^2 \frac{c}{v}, \quad (24)$$

From this the radius which characterizes three-photon annihilation is

$$r_{3\gamma} = r_e \left( \frac{4(\pi^2 - 9)}{3\pi \alpha} \right)^{1/2}. \quad (25)$$

Then the mass of tau-lepton is of order

$$m_\tau \sim \frac{\hbar}{c r_{3\gamma}}. \quad (26)$$

According to eq. (26), the mass of tau-lepton is $m_\tau \sim 2200$ MeV that agrees with the experimental value $m_\tau = 1784$ MeV [3].

In order to describe the decays of muon and tau-lepton

$$\mu^- \rightarrow e^- + \bar{\nu}_e \nu_\mu \quad (27)$$

$$\tau^- \rightarrow e^- + \bar{\nu}_e \nu_\tau \quad (28)$$

within the framework of electrodynamics let us assume that the following reactions occur

$$\gamma \rightarrow \bar{\nu}_e \nu_e \quad (29)$$

$$\bar{\nu}_e + \gamma \rightarrow \nu_\mu \quad (30)$$

$$\bar{\nu}_e + 2\gamma \rightarrow \nu_\tau. \quad (31)$$

Then combining eqs. (18), (20), (29), (30), (31) we obtain the reaction for the decay of muon

$$\mu^- \equiv e^- e^+ e^- \rightarrow e^- + 2\gamma \rightarrow e^- + \gamma \bar{\nu}_e \nu_e \rightarrow e^- + \bar{\nu}_e \nu_\mu. \quad (32)$$

Combining eqs. (19), (21), (29), (31) we obtain the reaction for the decay of tau-lepton

$$\tau^- \equiv e^- e^+ e^- \rightarrow e^- + 3\gamma \rightarrow e^- + 2\gamma \bar{\nu}_e \nu_e \rightarrow e^- + \bar{\nu}_e \nu_\tau. \quad (33)$$
References

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[3] M. Aguilar-Benitez et al. [Particle Data Group], Review of particle properties, Phys. Lett., 170B, 1986.