Coordinated control of active and reactive power of distribution network with distributed PV cluster via model predictive control

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Abstract. A coordinated optimal control method of active and reactive power of distribution network with distributed PV cluster based on model predictive control is proposed in this paper. The method divides the control process into long-time scale optimal control and short-time scale optimal control with multi-step optimization. The models are transformed into a second-order cone programming problem due to the non-convex and nonlinear of the optimal models which are hard to be solved. An improved IEEE 33-bus distribution network system is used to analyse the feasibility and the effectiveness of the proposed control method.

1. Introduction

With the development of distributed power generation technology, the permeability of distributed photovoltaic in the distribution network is increasing, and this brings a new challenge to the safe and stable operation of the distribution network \cite{1-4}.

Due to the randomness and fluctuation of the PV output and load demand, the prediction error increases with the advance of the forecast time, the difficulty of the optimal control of the distribution system is increasing. Model Predictive Control (MPC) can overcome the uncertainty of the system, which is an effective way to solve the above problems. It is widely used in the optimization of power system \cite{5-9}. In \cite{6-7}, voltage control strategy of power system with distributed generation based on MPC is proposed. In \cite{8}, the model predictive control is used in the active dispatch control of large-scale wind power integration, and the wind turbine and the traditional unit are optimized to improve the economy of the system.

The R/X ratio of the distribution network is large, and the coupling of active and reactive power is strong. Both of them have great influence on the line loss and voltage quality. For the distribution network with high PV permeability, active and reactive decoupling optimization control method is no longer applicable, an active and reactive power coordinated optimal control method need to be considered. In \cite{9}, an active and reactive multi-time optimal model of distribution network is established, which is optimized by adjusting the distributed power supply, energy storage device and capacitor bank.

To simplify the control of the PV units with the high penetration, there is an effective method to divide the PV units connected into different PCCs into several clusters, accordingly the number of controlled units can be reduced significantly. Based on PV cluster, an active-reactive power coordination control method of distribution network with distributed PV cluster based on MPC is presented.
2. Optimal control strategy based on MPC

2.1. Theory of MPC

MPC is a finite time domain closed-loop optimal control algorithm based on model. It has the advantages of easy modelling, good control effect and strong robustness, and can effectively deal with the nonlinearity, time-varying and uncertainty of the system. It has been used in the process industries in chemical plants, oil refineries and power systems. MPC is composed of three parts: forecasting model, rolling optimization and feedback correction. The mechanism of MPC is as follows: at the time of each sampling, based on the current system state and the forecast results of future state according to the forecasting model, on-line solving a finite time-long optimal control problem, get the control behaviour of current time and future time, only the first step of the control strategy is implemented. At the next sampling time, based on new system state and new measurement information, the calculations are repeated starting from the new current state.

2.2. Optimal control strategy based on MPC

Based on MPC, the active power regulation is completed by adjusting the energy storage system (ESS), the reactive power regulation is completed by adjusting the on-load tap changer (OLTC), fixed capacitor (FC), static VAR compensation (SVC) and distributed PV cluster. And the control process is divided into long-time scale optimal control and short-time scale optimal control. The long-time scale optimal control ensures the economical operation of the system, and the short-time scale optimal control ensures the safety operation of the system. The two perform separate model predictive control based on their respective optimal targets. The long-time scale optimal control takes the minimum of system network loss as the optimal target, based on the forecasting data of PV output and load demand, solving the active and reactive power of the controllable devices in the future \( M \Delta T \) time by \( \Delta T \).

According to the current system operating state and the forecasting data of PV output and load demand in smaller time scale, the short-time scale optimal control solves the increment of active and reactive power of the controllable devices in the future \( N \Delta t \) time by \( \Delta t \) (\( \Delta t < \Delta T \)). The structure of control strategy based on MPC is shown in figure 1.

![Figure 1. Structure of control strategy based on MPC.](image)

3. Optimal control model based on MPC

3.1. Long-time scale optimal control model

In long-time scale optimal control model, the tap of OLTC and FC, reactive power output of SVC and distributed PV cluster and charge and discharge power of ESS are chosen as the control variables, current operation state of system is taken as the initial value, the forecasting data of PV output and load demand is taken as the input variable.

3.1.1. Optimization target

In order to ensure the economical operation of the system and reduce the loss of system, the optimization target of long-time scale optimal control is minimal the loss of system.
\[
\begin{align*}
\min F &= \min_{t_0 \in M, \Delta T} \sum_{i=1}^{n} \sum_{j \in c(i)} r_{ij}^2 |I_{ij}|^2 \\
I_{ij,t} &= f(K_{ij,t}, H_{ij,t}, P_{ch,j,t}, P_{dis,j,t}, Q_{DG,j,t}, Q_{SVC,j,t})
\end{align*}
\]

(1)

Where \( t_0 \) is the start time of long-time scale optimal control, \( \Delta T \) is the time interval of control, \( M \) is the step-size of control, \( n \) is the count of node, \( c(i) \) is the collection of the frontier points of the line with \( i \) as the head node of the line, \( r_{ij} \) is the resistance of line \( ij \), \( I_{ij,t} \) is the square of the current of line \( ij \) at time \( t \) and expressed as a function of controllable variables, \( K_{ij,t} \) is the tap of OLTC, \( H_{ij,t} \) is the tap of FC, \( P_{ch,j,t} \) and \( P_{dis,j,t} \) is the charge power and discharge power of ESS, \( Q_{DG,j,t} \) is the reactive power output of distributed PV cluster, \( Q_{SVC,j,t} \) is the reactive power output of SVC.

3.1.2. Constraint

(1) Constraint of power flow

In this paper, the power flow equation of the radiation network is expressed by Distflow method, the single line topology of radiation network is shown in Figure 2.

![Figure 2. Branch flow model.](image)

(2) Constraint of voltage

\[
U_{i,t}^{\min} \leq U_{i,t} \leq U_{i,t}^{\max}
\]

(3) Constraint of current

\[
I_{ij,t} \leq I_{ij,t}^{\max}
\]

Where \( I_{ij,t}^{\max} \) is the upper bound of the current of line \( ij \).

(4) Constraint of distributed PV cluster

Where \( r_{ij} + jx_{ij} \) is the total impedance of line \( ij \) and OLTC on the line \( ij \). \( I_{ij,t} \) is the current amplitude of line \( ij \) at time \( t \). \( U_{i,t} \) is the ratio of OLTC at time \( t \).
\[ P_{DG,i,t} = P_{DG,i,t}^{pre} \] (8)

\[ Q_{DG,i,t}^{min} \leq Q_{DG,i,t} \leq Q_{DG,i,t}^{max} \] (9)

\[ Q_{DG,i,t}^{max} = -Q_{DG,i,t}^{min} = \sqrt{(S_{DG,i})^2 - (P_{DG,i,t})^2} \] (10)

Where \( P_{DG,i,t}^{pre} \) is the forecasting value of the active power output of distributed PV cluster at time \( t \), \( Q_{DG,i,t}^{max} \) and \( Q_{DG,i,t}^{min} \) are the upper and lower bounds of the reactive power output of distributed PV cluster at time \( t \), \( S_{DG,i} \) is the capacity of distributed PV cluster.

(5) Constraint of OLTC

\[ k_{i,t} = k_0 + K_{i,t} \Delta k_{i} \] (11)

\[ K_{i}^{min} \leq K_{i,t} \leq K_{i}^{max} \] (12)

Where \( k_0 \) is the standard ratio of OLTC, \( \Delta k_{i} \) is the adjustment step, \( K_{i,t} \) is the tap of OLTC at time \( t \), \( K_{i}^{min} \) and \( K_{i}^{max} \) are the upper and lower bounds of the tap of OLTC.

(6) Constraint of SVC

\[ Q_{SVC,i,t}^{min} \leq Q_{SVC,i,t} \leq Q_{SVC,i,t}^{max} \] (13)

Where \( Q_{SVC,i,t}^{min} \) and \( Q_{SVC,i,t}^{max} \) are the upper and lower bounds of the reactive power output of SVC.

(7) Constraint of FC

\[ \begin{align*}
H_{i,t} \Delta Q_{c,i,t} & = Q_{c,i,t} \\
0 \leq H_{i,t} & \leq H_{max} \quad (H_{i,t} \in Z)
\end{align*} \] (14)

Where \( H_{i,t} \) is the tap of FC, \( \Delta Q_{c,i,t} \) is the adjustment step of FC, \( H_{max} \) is the maximum tap of FC.

(8) Constraint of ESS

\[ E_{SOC,i,t} + P_{ch,i,t} \eta_{ch} \Delta T - P_{dis,i,t} \eta_{dis} \Delta T = E_{SOC,i,t+\Delta T} \] (15)

\[ 20\% \times E_{SOC,i,t}^{max} \leq E_{SOC,i,t} \leq 80\% \times E_{SOC,i}^{max} \] (16)

\[ \begin{align*}
0 \leq P_{ch,i,t} & \leq P_{ch,i,t}^{max} D_{ch,i,t} \\
0 \leq P_{dis,i,t} & \leq P_{dis,i,t}^{max} D_{dis,i,t} \\
D_{ch,i,t} + D_{dis,i,t} & \leq 1
\end{align*} \] (17)

Where \( E_{i,t} \) is the power of ESS, \( P_{ch,i,t} \) and \( P_{dis,i,t} \) are the charge and discharge power of ESS, \( \eta_{ch} \) and \( \eta_{dis} \) are the charge and discharge efficiency of ESS, \( P_{ch,i,t}^{max} \) and \( P_{dis,i,t}^{max} \) are the maximum charge and discharge power of ESS, \( D_{ch,i,t} \) and \( D_{dis,i,t} \) are 0-1 variables, equation(18) ensures the charging and discharging will not occur simultaneously, \( E_{SOC,i}^{max} \) is the maximum capacity of ESS, and the state of charge (SOC) of ESS is limited between 20\% to 80\%.

3.2. Short-time scale optimal control model

Due to the fluctuation and randomness of distributed PV cluster output and load demand, the forecasting data of long-time scale is poor of precision. It is necessary to carry out the short-time scale optimal control to adjust the control results of long-time scale based on the current operating state of the system and the forecasting data of smaller time scale. In short-time scale optimal control, the increment of active and reactive power of the controllable devices will be solved in the future \( N \Delta T \) time by \( \Delta T \) ( \( \Delta T < N \Delta T \) ). The response speed of OLTC and FC is relatively slow, they are not appropriate to be adjusted frequently. Therefore, the control variables of short-time scale optimal control are the increments of the charge and discharge power of ESS, the reactive power of SVC and PV cluster.
3.2.1. Optimization target In order to cope with the fluctuation and randomness of the PV out and the loads, ensure the safety of the system operation, and the consistency with the optimal direction of the long-time scale, the optimal target of the short-time scale optimal control is the minimum power loss.

$$\begin{align*}
\min F &= \min_{t=0}^{i=N/N_{\text{soc}}} \sum_{i=1}^{N} \sum_{j=1}^{P_{\text{soc}}} c_{ij}(I_{ij})^2 \\
(I_{ij})^2 &= f(P_{\text{soc}} + \Delta P_{\text{soc}} + P_{\text{soc}} + \Delta P_{\text{soc}} + P_{\text{soc}} + \Delta P_{\text{soc}})
\end{align*}$$

Where $\Delta t$ is the time interval of short-time scale optimal control, $N$ is the step-size of control, $P_{\text{soc}}$, $P_{\text{soc}}$, $Q_{\text{soc}}$, $Q_{\text{soc}}$ are the charge and discharge power of ESS, the reactive power output of distributed PV cluster and the reactive power output of SVC which are calculated in long-time scale optimization control, $\Delta P_{\text{soc}}$, $\Delta P_{\text{soc}}$, $\Delta Q_{\text{soc}}$, $\Delta Q_{\text{soc}}$ indicate increments of the charge and discharge power of ESS, the increments of the reactive power of distributed PV cluster and SVC.

3.2.2. Constraint

1. Equality constraints of power flow:

$$\begin{align*}
P_{ij} &= P_{\text{DG},ij} = P_{\text{load},ij} + (P_{\text{soc},ij} + \Delta P_{\text{soc},ij}) \\
Q_{ij} &= (Q_{\text{DG},ij} + \Delta Q_{\text{DG},ij}) + (Q_{\text{soc},ij} + \Delta Q_{\text{soc},ij}) - Q_{\text{load},ij}
\end{align*}$$

2. Inequality constraints:

$$\begin{align*}
U_{ij}^{\text{min}} &\leq U_{ij} \leq U_{ij}^{\text{max}} \\
I_{ij} &\leq I_{ij}^{\text{max}} \\
Q_{\text{DG},ij}^{\text{min}} &\leq (Q_{\text{DG},ij} + \Delta Q_{\text{DG},ij}) \leq Q_{\text{DG},ij}^{\text{max}} \\
Q_{\text{soc},ij}^{\text{min}} &\leq (Q_{\text{soc},ij} + \Delta Q_{\text{soc},ij}) \leq Q_{\text{soc},ij}^{\text{max}} \\
20\% \times E_{\text{SOC},ij}^{\text{max}} - E_{\text{SOC},ij} &\leq 80\% \times E_{\text{SOC},ij}^{\text{max}} \\
0 &\leq P_{\text{soc},ij} + \Delta P_{\text{soc},ij} \leq P_{\text{soc},ij}^{\text{max}} \\
0 &\leq P_{\text{soc},ij} + \Delta P_{\text{soc},ij} \leq P_{\text{soc},ij}^{\text{max}} \\
D_{\text{soc},ij} &\leq 1
\end{align*}$$

3.3. SOCP description of optimal model

In this paper, the optimization problem proposed contains continuous variables and integer variables. Its mathematical essence is a mixed integer non-convex and nonlinear optimization problem. It is difficult to find the optimal solution. In this paper, the optimization problem is transformed into the second-order cone programming (SOCP) which can be solved efficiently.

3.3.1. Standard SOCP The standard form of SOCP is as follows:

$$\min_{x} \{ c^{T} x | A x = b, x_{i} \in K, i = 1, 2, \ldots N \}$$

Variable $x \in \mathbb{R}^{N}$; constant $b \in \mathbb{R}_{M}$, $c \in \mathbb{R}^{N}$, $A \in \mathbb{R}^{M \times N}$; $K$ is a second-order cone or a rotated second-order cone as shown in equation(22) and (23).

$$K = \{ x_{i} \in \mathbb{R}^{N} | x_{i}^{2} \geq \sum_{i=2}^{N} x_{i}^{2}, x_{i} \geq 0 \}$$

$$K = \{ x_{i} \in \mathbb{R}^{N} | 2x_{i}x_{j} \geq \sum_{i=3}^{N} x_{i}^{2}, x_{i}, x_{j} \geq 0 \}$$
3.3.2. SOCP description of optimization model  Taking the long-time scale optimal control model as an example, equation (3) and (4) are non-convex and nonlinear, the optimization target function and other constraints are linear equations.

Equation (3) and (12) are the constraints of OLTC. In this paper, an exact linearization modeling method based on piecewise linearization is used to model the OLTC. (3) and equation (12) are transformed into linear constraints.

Equation (4) is relaxed by using second-order cone method, and it can be rewritten as:

\[ U_{ij} I_{ij} \geq \sqrt{P_{ij}^2 + Q_{ij}^2} \]  

Rewritten as a standard form of second-order cone:

\[ \begin{bmatrix} 2P_{ij} & 2Q_{ij} & (I_{ij})^2 - (U_{ij})^2 \end{bmatrix} \begin{bmatrix} (I_{ij})^2 + (U_{ij})^2 \end{bmatrix} \leq \begin{bmatrix} (I_{ij})^2 + (U_{ij})^2 \end{bmatrix} \]

The long-time scale optimal control model contains discrete variables and continuous variables, it was converted to a mixed integer second-order cone programming model. The short-time scale optimal control model contains continuous variables, it was converted to second-order cone programming model.

4. Simulation analysis

4.1. Simulation system

In this paper, an improved IEEE 33-bus distribution network system is used for simulation analysis, the structure of the system is shown in figure 3. The voltage adjustment range of OLTC is [0.95, 1.05] p.u., and its step-size of control is 0.0125pu, 8 taps adjustable. The reactive power adjustment range of FC is 0–300kvar, its adjustment step-size is 50kvar. The reactive power adjustment range of SVC is 0–500kvar. The capacity of ESS is 1000kW•h, maximum charge and discharge power is 200kW, and its initial capacity is 500kW•h. There are three 1000kW PV clusters set in the example.

![Figure 3. Modified IEEE 33-bus system.](image)

In this example, the active and reactive power output of the controllable device in the next 4 h will be calculated as the time interval is 1h in long-time scale optimal control. In short-time scale optimal control, 15min is taken as the time interval, the active and reactive power output increment controllable device in next 1h will be calculated.

4.2. Simulation analysis

The forecasting curve of PV output and load demand is shown in Figure 4. In order to verify the effectiveness of the proposed method in response to the fluctuation of PV output and load demand, a set of (−5%~5%) random sequences were added to each long-time scale optimal and short-time scale optimal control process, simulating the disturbance of the PV output and load demand as the input of model predictive control.
Figure 4. PV output and load demand curve.

Figure 5(a) shows the taps adjustment of OLTC and FC. Limited by the device performance and the response time, OLTC and FC can only be adjusted in the long-time scale optimal control. Figure 5(b) shows the reactive power output of SVC. Figure 5(c) shows the charge and discharge power and the state of charge of ESS. Figure 5(d) shows the reactive power output of distributed PV cluster.

Figure 5. Control results: (a) Adjustment of the tap of OLTC and FC, (b) Reactive power output of SVC, (c) Charge state of ESS, (d) Reactive power output of distributed PV cluster.

Figure 6(a) shows the power curve of root node before and after optimization. Figure 6(b) shows that the proposed method can effectively reduce the reactive power obtained from the main network, thus reducing the long-distance transmission of reactive power.

Figure 6. Power curve of root node before and after optimal control: (a) Active power of root node, (b) Reactive power of root node.

Figure 7 shows the comparison of the node voltage of the system before and after optimal control in two typical scenes. In this paper, the node voltage is limited to [0.95,1.05] pu. Scenario 1 shows the PV output is high and the load demand is relatively low at 12:00, the power loss of the system is obviously reduced after optimization with the safe node voltage. Scenario 2 shows the PV output is low.
and the load demand is high at 20:00, the node voltage is over the lower limit, the node voltage is within the safe range after optimization.

Figure 7. Node voltage of the system before and after optimal control: (a) Scenario 1, (b) Scenario 2.

Figure 8 is the comparison of loss of the system before and after optimal control. It can be seen that after 18:00, due to the decrease of PV output and the increase of load demand, the loss of system is higher than other time. By adjusting the controllable devices, the optimization results are better than other time. It can be found that the loss of the system reduced significantly after optimization.

Figure 8. Loss of the system before and after optimal control.

5. Conclusions
The key work of the paper about the active and reactive power control of distribution network with distributed PV cluster contains:
(1) Propose a different time scale optimal control method which performs separate model predictive control for each control target and control variable. The multi-step rolling optimization is used to solve the active and reactive power output of FC, SVC, ESS and distributed PV cluster.
(2) Apply the second-order cone relaxation method to solve the nonlinear optimization problem, and ensure the convergence of the algorithm.

The next work is aimed at the transient control of PV cluster in the emergency case, which is most important for the safe operation of distributed network with the high penetration of PV.

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