Constraints on Kinematic Model from Recent Cosmic Observations: SN Ia, BAO and Observational Hubble Data

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In this paper, linear first order expansion of deceleration parameter \( q(z) = q_0 + q_1(1 - a) \) (\( M_1 \)), constant jerk \( j = j_0 \) (\( M_2 \)) and third order expansion of luminosity distance (\( M_3 \)) are confronted with cosmic observations: SCP 307 SN Ia, BAO and observational Hubble data (OHD). Likelihood is implemented to find the best fit model parameters. All these models give the same prediction of the evolution of the universe which is undergoing accelerated expansion currently and experiences a transition from decelerated expansion to accelerated expansion. But, the transition redshift depends on the concrete parameterized form of the model assumed. \( M_1 \) and \( M_2 \) give value of transition redshift about \( z_t \sim 0.6 \). \( M_3 \) gives a larger one, say \( z_t \sim 1 \). The \( \chi^2/dof \) implies almost the same goodness of the models. But, for its badness of evolution of deceleration parameter at high redshift \( z > 1 \), \( M_1 \) can not be reliable. \( M_1 \) and \( M_2 \) are compatible with \( \Lambda \)CDM model at the 2\( \sigma \) and 1\( \sigma \) confidence levels respectively. \( M_3 \) is not compatible with \( \Lambda \)CDM model at 2\( \sigma \) confidence level. From \( M_1 \) and \( M_2 \) models, one can conclude that the cosmic data favor a cosmological model having \( j_0 < -1 \).

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I. INTRODUCTION

The expansion of the universe or kinematics of the universe is described by the expansion rate \( H = \dot{a}/a \), a dimensionless parameter \( q = -a\ddot{a}/\dot{a}^2 \), named deceleration parameter and jerk parameter \( j = -a\dddot{a}/(\dot{a}^2) \), where \( a \) is the scale factor in Friedmann-Roberson-Walker (FRW) metric. The front negative sign of \( q \) is added to obtain a positive parameter when one considers a decelerated expansion universe dominated by matter fields with attractive force, for example dark matter dominated universe with \( q = 1/2 \). However, the observations of Type Ia Supernovae (SN Ia) from two teams \([1, 2]\) imply that our universe is undergoing an accelerated expansion at present. Whereafter, this result is confirmed by the observations from WMAP \([3, 4]\) and Large Scale Structure survey \([5, 6]\). Then at present, the deceleration parameter would be a negative number. In fact, to explain the accelerated expansion of the universe, a flood of various cosmological models have been explored, please see \([7]\) for recent reviews.

In the literatures, two approaches have been taken into accounts. One is that an extra energy component, dubbed dark energy, is introduced, e.g. cosmological constant, quintessence \([8, 9, 10, 11]\), phantom \([12]\) and quintom \([13]\), etc. The other is that the accelerated expansion of the universe is due to the modification of the gravity theory at large scale, e.g. modified gravity theory, Brans-Dicke theory and higher dimensional theory, etc. These correspond to the dynamics of the universe. In general, different dark energy models would predict different expansion histories of the universe, for example quintessence dark energy model with various scaler potentials \([14]\) will have different values of the transition redshift \( z_T \) from decelerated expansion to accelerated expansion. These different models would have different dynamics. However, a kinematic approach will be held regardless of the underlying cosmic dynamics \([15, 16]\).

In particular, the jerk parameter can provide us the simplest approach to search for departures from the cosmic concordance model, for its constant value \( j = -1 \) for cosmological constant. This approach is called cosmography \([17, 18]\), cosmokinetics \([19]\), or Friedmannless cosmology \([20, 21]\).

Some explore the accelerated expansion of the universe by using different parameterized forms of deceleration parameters \( q(z) \) \([22, 23, 24]\) in the so-called model independent way, for examples constant model \( q = constant \), linear model with variable \( z \) (\( q(z) = q_0 + q_1 z \)) and linear model with variable \( a \) (\( q(a) = q_0 + q_1(1 - a) \)), etc. Recently, the authors of \([16, 21]\) investigated constraints on some kinematic models by employing a Bayesian marginal likelihood.

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II. KINEMATIC MODELS

The metric of a flat FRW cosmological model containing dark matter and dark energy is written as

\[ ds^2 = -dt^2 + a^2(t)dx^2. \]  

(1)

where \( a(t) \) is the scale factor, which describes the whole history of the universe evolution, and has the relations with redshift \( z \) in terms of \( a = (1 + z)^{-1} \) (\( a_0 = 1 \) is normalized). The Hubble parameter

\[ H \equiv \frac{\dot{a}}{a}, \]  

(2)

and deceleration parameter

\[ q \equiv -\frac{1}{H^2} \frac{\ddot{a}}{a} = \frac{1}{2}(1 + z)\frac{[H(z)^2]''}{H(z)^2} - 1. \]  

(3)

are defined as the rate of expansion and accelerated expansion. By using the relation \( a_0/a = 1 + z \) and the relations between \( H \) and \( q \), i.e., one can rewrite Eq. (3) in its integration form

\[ H(z) = H_0 \exp \left[ \int_0^z [1 + q(u)] d\ln(1 + u) \right]. \]  

(4)

Similarly, the jerk parameter is defined as

\[ j \equiv -\frac{1}{H^3} \frac{\dot{a}}{a} = \left[ \frac{1}{2} (1 + z)^2 \frac{[H(z)^2]'''}{H(z)^2} - (1 + z) \frac{[H(z)^2]''}{H(z)^2} \right] + 1. \]  

(5)

Easily, one can find that the deceleration parameter and jerk parameter have the relations

\[ j = - \left[ q + 2q^2 + (1 + z) \frac{dq}{dz} \right], \]  

(6)

which will be used when the parameterized forms of \( q(z) \) are going to be tested. By using these definitions Eq. (2), Eq. (3) and Eq. (5), one can describe the recent cosmic expansion with their current values

\[ a(t) = 1 + H_0(t - t_0) - \frac{1}{2}q_0H_0^2(t - t_0)^2 - \frac{1}{3!}j_0H_0^3(t - t_0)^3 + O[(t - t_0)^4], \]  

(7)

from which the luminosity distance can be expanded as

\[ d_L(z) = \frac{c}{H_0} \left[ z + \frac{1}{2}(1 - q_0)z^2 - \frac{1}{6}(1 - q_0 - 3q_0^2 - 5j_0)z^3 \right] + O(z^4). \]  

(8)

Also, one can find the relations of Hubble parameter \( H(z) \) and luminosity distance \( d_L(z) \)

\[ H^{-1}(z) = -(1 + z) \frac{dt}{dz} = \frac{d}{dz} [(1 + z)^{-1}d_L(z)], \]  

(9)

which will be useful when the observational Hubble data are used as cosmic observation constraint.

The basic aim in this paper is to examine some simple kinematic models for the cosmic expansion based on specific parameterizations for \( q(z) \) in Eq. (3) (variable jerk parameter), a constant jerk parameter and their comparison with the expansion (7).
The first model, \( M_1 \), is given by linear expansion of the scale factor \( a, \) \( q(a) = q_0 + q_1(1-a) \) in terms of \( a \), which can be rewritten in the terms redshift \( z, q(z) = q_0 + q_1z/(1+z) \). The second model, \( M_2 \), is a constant jerk parametrization model, \( j(z) = j_0 \), for detecting the departure from the flat \( \Lambda \)CDM scenario, for which \( j(z) = j_0 = -1 \). Model \( M_3 \) is the expansion \( \Box \), which has as free parameters \( q_0 \) and \( j_0 \). In the Appendix A see also [16], one can find the basic analytical expressions between the Hubble parameter \( H(z) \), deceleration parameter \( q(z) \) and jerk parameter \( j(z) \). One can take this paper as a generalization and complement to [16] where only the 307 SN Ia data points are used. Here, the BAO and OHD datastes are also included as useful cosmic constraints. One would notice that all of them do not include \( \Omega_m \) explicitly.

III. COSMIC OBSERVATION DATA SETS AND STATISTICAL RESULTS

A. SN Ia

We constrain the parameters with the Supernovae Cosmology Project (SCP) Union sample including 307 SN Ia [26], which distributed over the redshift interval \( 0.015 \leq z \leq 1.551 \). Constraints from SN Ia can be obtained by fitting the distance modulus \( \mu(z) \)

\[
\mu_{\text{th}}(z) = 5 \log_{10}(D_L(z)) + \mu_0,
\]

where, \( D_L(z) \) is the Hubble free luminosity distance \( H_0 d_L(z)/c \)

\[
d_L(z) = c(1 + z) \int_0^z \frac{dz'}{H(z')},
\]

\[
\mu_0 = 42.38 - 5 \log_{10} h,
\]

where \( H_0 \) is the Hubble constant which is denoted in a re-normalized quantity \( h \) defined as \( H_0 = 100h \) km s\(^{-1}\)Mpc\(^{-1}\). The observed distance moduli \( \mu_{\text{obs}}(z_i) \) of SN Ia at \( z_i \) is

\[
\mu_{\text{obs}}(z_i) = m_{\text{obs}}(z_i) - M,
\]

where \( M \) is their absolute magnitudes.

For SN Ia dataset, the best fit values of parameters in a model can be determined by the likelihood analysis is based on the calculation of

\[
\chi^2(p_s, m_0) = \sum_{SN Ia} \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(p_s, z_i)]^2}{\sigma_i^2}
\]

\[
= \sum_{SN Ia} \frac{[5 \log_{10}(D_L(p_s, z_i)) - m_{\text{obs}}(z_i) + m_0]^2}{\sigma_i^2},
\]

(14)

where \( m_0 = \mu_0 + M \) is a nuisance parameter (containing the absolute magnitude and \( H_0 \)) that we analytically marginalize over [27],

\[
\tilde{\chi}^2(p_s) = -2 \ln \int_{-\infty}^{+\infty} \exp \left[ -\frac{1}{2} \chi^2(p_s, m_0) \right] dm_0,
\]

(15)

to obtain

\[
\tilde{\chi}^2 = A - \frac{B^2}{C} + \ln \left( \frac{C}{2\pi} \right),
\]

(16)

where

\[
A = \sum_{SN Ia} \frac{[5 \log_{10}(D_L(p_s, z_i)) - m_{\text{obs}}(z_i)]^2}{\sigma_i^2},
\]

(17)

\[
B = \sum_{SN Ia} \frac{5 \log_{10}(D_L(p_s, z_i)) - m_{\text{obs}}(z_i)}{\sigma_i^2},
\]

(18)
\[ C = \sum_{SN1a} \frac{1}{\sigma_i}, \] (19)

The Eq. (14) has a minimum at the nuisance parameter value \( m_0 = B/C \). Sometimes, the expression
\[ \chi^2_{SN1a}(p_s, B/C) = A - (B^2/C) \] (20)
is used instead of Eq. (16) to perform the likelihood analysis. They are equivalent, when the prior for \( m_0 \) is flat, as is implied in (15), and the errors \( \sigma_i \) are model independent, what also is the case here. Obviously, from the value \( m_0 = B/C \), one can obtain the best-fit value of \( h \) when \( M \) is known.

To determine the best fit parameters for each model, we minimize \( \chi^2(p_s, B/C) \) which is equivalent to maximizing the likelihood
\[ \mathcal{L}(p_s) \propto e^{-\chi^2(p_s, B/C)/2}. \] (21)

**B. BAO**

The BAO are detected in the clustering of the combined 2dFGRS and SDSS main galaxy samples, and measure the distance-redshift relation at \( z = 0.2 \). BAO in the clustering of the SDSS luminous red galaxies measure the distance-redshift relation at \( z = 0.35 \). The observed scale of the BAO calculated from these samples and from the combined sample are jointly analyzed using estimates of the correlated errors, to constrain the form of the distance measure \( D_V(z) \) \[28, 29, 30\].

\[ D_V(z) = \left[ (1 + z)^2 D_A^2(z) \frac{cz}{H(z)} \right]^{1/3}, \] (22)

where \( D_A(z) \) is the proper (not comoving) angular diameter distance which has the following relation with \( d_L(z) \)
\[ D_A(z) = \frac{d_L(z)}{(1 + z)^2}. \] (23)

Matching the BAO to have the same measured scale at all redshifts then gives \[30\]
\[ D_V(0.35)/D_V(0.2) = 1.812 \pm 0.060. \] (24)

Then, the \( \chi^2_{BAO}(p_s) \) is given as
\[ \chi^2_{BAO}(p_s) = \frac{\left[ D_V(0.35)/D_V(0.2) - 1.812 \right]^2}{0.060^2}. \] (25)

**C. OHD**

The observational Hubble data are based on differential ages of the galaxies \[31\]. In \[32\], Jimenez et al. obtained an independent estimate for the Hubble parameter using the method developed in \[31\], and used it to constrain the EOS of dark energy. The Hubble parameter depending on the differential ages as a function of redshift \( z \) can be written in the form of
\[ H(z) = -\frac{1}{1 + z} \frac{dz}{dt}. \] (26)

So, once \( dz/dt \) is known, \( H(z) \) is obtained directly \[33\]. By using the differential ages of passively-evolving galaxies from the Gemini Deep Deep Survey (GDDS) \[34\] and archival data \[35, 36, 37, 38, 39, 40\], Simon et al. obtained \( H(z) \) in the range of \( 0 \lesssim z \lesssim 1.8 \) \[33\]. The observational Hubble data from \[33\] are list in Table 1.

The best fit values of the model parameters from observational Hubble data \[33\] are determined by minimizing
\[ \chi^2_{Hub}(p_s) = \sum_{i=1}^{9} \frac{[H_{th}(p_s; z_i) - H_{obs}(z_i)]^2}{\sigma^2(z_i)}, \] (27)

where \( p_s \) denotes the parameters contained in the model, \( H_{th} \) is the predicted value for the Hubble parameter, \( H_{obs} \) is the observed value, \( \sigma(z_i) \) is the standard deviation measurement uncertainty, and the summation is over the 9 observational Hubble data points at redshifts \( z_i \).
Observational Hubble data (OHD). Here, we give three simple examples: Taylor expansion of value. It can be seen from Fig. 2, the best fit ΛCDM model is out of the range of 1.2, the ΛCDM model result is included as a short line segment denoting 1 parameterized form one assumed. Here for its bad behavior at relative high redshift, say hM confidence levels with expansion of luminosity distance. The results imply a universe with explicit. Then, the results may not depend on the dynamic variables, say ΩM of h only the kinematic variables, say Ωm, and gravitation theory. Here more datasets are included to constrain the model parameters than that in Ref. [16] where the SCP 307 is used alone. After the calculation as described above, the result is listed in Table II.

\[
\chi^2 = \chi^2_{SN Ia} + \chi^2_{BAO} + \chi^2_{Hub},
\]

where \(\chi^2_{SN Ia}, \chi^2_{BAO}\) and \(\chi^2_{Hub}\) are the ones described in Eq. (20), Eq. (25) and Eq. (27) respectively. It is clear that only the kinematic variables, say \(h_0, q_0, j_0\), are contained in all \(\chi^2\)s equations where \(\Omega_m\) does not appear explicitly. Then, the results may not depend on the dynamic variables, say \(\Omega_m\), and gravitation theory. Here more datasets are included to constrain the model parameters than that in Ref. [16] where the SCP 307 is used alone. After the calculation as described above, the result is listed in Table II.

| Models | \(\chi^2_{min}\) | \(q_0(1\sigma)\) | \(j_0(1\sigma)\) | \(h_0(1\sigma)\) | \(z_t(1\sigma)\) | \(\chi^2/dof\) |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|
| \(M_1\) | 326.599 | \(-0.715^{+0.045}_{-0.045}\) | \(-2.196^{+0.254}_{-0.244}\) | \(0.716^{+0.053}_{-0.053}\) | \(0.609^{+0.110}_{-0.107}\) | 1.043 |
| \(M_2\) | 326.442 | \(-0.658^{+0.061}_{-0.057}\) | \(-1.382^{+0.219}_{-0.225}\) | \(0.709^{+0.053}_{-0.053}\) | \(0.592^{+0.099}_{-0.096}\) | 1.040 |
| \(M_3\) | 330.733 | \(-0.461^{+0.031}_{-0.033}\) | \(-0.147^{+0.110}_{-0.1064}\) | \(0.716^{+0.053}_{-0.053}\) | \(0.946^{+0.110}_{-0.081}\) | 1.053 |

The results are compatible with that of Ref. [16] except the model of \(M_3\) which is the worst one among these three models in the viewpoint of \(\chi^2/dof\). This is very different from the results of \(M_3\) obtained in Ref. [16] where SCP 307 is used alone. For ΛCDM model, using the datasets of SN Ia, BAO and OHD, we found the best fit values: \(\Omega_m = 0.283^{+0.033}_{-0.031}\) and \(h_0 = 0.732^{+0.044}_{-0.031}\). And the corresponding minimum value of \(\chi^2\) is \(\chi^2_{min} = 327.246\) \((\chi^2/dof = 1.039)\). But, it is clear all the models give the same prediction that the universe is undergoing accelerated expansion and experience an transition from decelerated expansion to accelerated expansion. The transition redshift \(z_t\) depends on the concrete parameterized form one assumed. Here \(M_1\) and \(M_2\) give the value about 0.6. However, the luminosity expansion model \(M_3\) give large transition redshift. From the right panel of Fig. II one can not take the expansion model seriously for its bad behavior at relative high redshift, say \(z > 1\).

The 1σ and 2σ contour plots in \(q_0 - j_0\) and \(q_0 - h_0\) planes are plotted in Fig. 2 and Fig. 3. In \(q_0 - j_0\) contour plots the ΛCDM model result is included as a short line segment denoting 1σ interval where the dot denotes the best fit value. It can be seen from Fig. 2 the best fit ΛCDM model is out of the range of 1σ region in \(M_1\) and 2σ region in \(M_3\), but it is in the range of 1σ region in \(M_2\). It means that the best fit ΛCDM model is compatible at 1σ and 2σ confidence levels with \(M_2\) and \(M_1\) respectively. The results are compatible with that of Ref. [16] except the model of expansion of luminosity distance. The results imply a universe with \(j_0 < -1\) from \(M_1\) and \(M_2\).

### IV. Conclusions

In this paper, kinematic models are constrained by recent cosmic observations which include SN Ia, BAO and observational Hubble data (OHD). Here, we give three simple examples: Taylor expansion of \(q(a)\) at present \(a_0 = 1\) \((M_1)\), constant jerk parameter \(j = j_0\) \((M_2)\) and expansion of luminosity distance \((M_3)\). All of the models give the
FIG. 1: The evolutions of deceleration parameters $q(z)$ with respect to redshift $z$ in $1\sigma$ error regions. The left, center and right panels correspond to $M_1$ ($q(z) = q_0 + q_1 z/(1 + z)$), $M_2$ ($j = j_0$) and $M_3$ (expansion of luminosity distance) respectively.

FIG. 2: The contour plots of $q_0 - j_0$ with $1\sigma$ and $2\sigma$ regions. The left, center and right panels correspond to $M_1$ ($q(z) = q_0 + q_1 z/(1 + z)$), $M_2$ ($j = j_0$) and $M_3$ (expansion of luminosity distance) respectively, where the dots denote the best fit values of the parameters. The $\Lambda$CDM model result is included as a short line segment denoting $1\sigma$ interval.

The same prediction of the evolution of the universe which is undergoing accelerated expansion at current and experiences a transition from decelerated expansion to accelerated expansion. But the transition redshift depends on the concrete parameterized forms of the models. The best fit values of the parameter of $M_1$ and $M_2$ predict the transition redshift is about $z_t \sim 0.6$. However, $M_3$ predict a large transition redshift $z_t \sim 1$. The $\chi^2/dof$ imply the same goodness of the models. But, from evolution curves of deceleration parameter as plotted in Fig. 1 and the knowledges of the history

FIG. 3: The contour plots of $q_0 - h_0$ with $1\sigma$ and $2\sigma$ regions. The left, center and right panels correspond to $M_1$ ($q(z) = q_0 + q_1 z/(1 + z)$), $M_2$ ($j = j_0$) and $M_3$ (expansion of luminosity distance) respectively, where the dots denote the best fit values of the parameters.
of the universe, one found that $M_1$ and $M_2$ are better than $M_3$ for the badness of behavior at high redshift of $M_3$, say $z > 1$.

The $1\sigma$ and $2\sigma$ confidence contours in $q_0 - j_0$ and $q_0 - h_0$ planes are plotted in Fig. 2 and Fig. 3 where the best fit value of $\Lambda$CDM model is denoted by a line segment, $1\sigma$ interval. From the Fig. 2 one can find that $M_1$ and $M_2$ are compatible with $\Lambda$CDM models at $2\sigma$ and $1\sigma$ confidence levels respectively. $M_3$ is not compatible with $\Lambda$CDM model at $2\sigma$ confidence level. For its badness of the evolution history of model $M_3$ at high redshift. One can not take it seriously and treat it as a unreliable model. Then, one can conclude that the cosmic data favors the model having the value $j_0 < -1$ for the large parts of the confidence regions is under the line $j_0 = -1$. It is compatible with the conclusion obtained in Ref. [16].

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**APPENDIX A: MODELS AND USEFUL RELATIONS**

In this appendix, one can find the analytical expressions between the Hubble parameter $H(z)$, deceleration parameter $q(z)$ and jerk parameter $j(z)$.

**M$_1$**

\[
q(z) = q_0 + q_1 \frac{z}{1+z} \tag{A1}
\]

\[
H(z) = H_0 (1+z)^{-q_0+q_1} \exp \left( -\frac{q_1 z}{1+z} \right) \tag{A2}
\]

\[
j(z) = -q_0 - \frac{q_1 z}{1+z} - (1+z) \left( -\frac{q_1 z}{(1+z)^2} + \frac{q_1}{1+z} \right) - 2 \left( q_0 + \frac{q_1 z}{1+z} \right)^2 \tag{A3}
\]

**M$_2$**

\[
H(z) = H_0 [c_1 (1 + z)^{\alpha_1} + c_2 (1 + z)^{\alpha_2}]^{\frac{1}{\alpha_1 - \alpha_2}} \tag{A4}
\]

\[
q(z) = \frac{c_1 (1 + z)^{\alpha_1} (\frac{\alpha_1}{\alpha_2} - 1) + c_2 (1 + z)^{\alpha_2} (\frac{\alpha_2}{\alpha_1} - 1)}{c_1 (1 + z)^{\alpha_1} + c_2 (1 + z)^{\alpha_2}} \tag{A5}
\]

\[
j(z) = j_0 \tag{A6}
\]

\[
z_t = \left[ \frac{c_2 \alpha_2 - 2}{c_1 \alpha_1 - 2} \right]^{\frac{1}{\alpha_1 - \alpha_2}} - 1 \tag{A7}
\]

where

\[
\alpha_{1,2} = \frac{3}{2} \pm \sqrt{\frac{9}{4} - 2(1+j_0)} \tag{A8}
\]

\[
c_1 = \frac{2(1+q_0) - \alpha_2}{\alpha_1 - \alpha_2} \quad \text{and} \quad c_2 = 1 - c_1 \tag{A9}
\]

From (A8) we see that $j_0 < \frac{1}{8}$.

**M$_3$** - defined by the expanded luminosity distance Eq. (8),

\[
d_L(z) = \frac{H_0}{H_0} (z + Az^2 + Bz^3), \text{ where } A = (1-q_0)/2 \text{ and } B = -(1 - q_0 - 3q_0^2 - j_0)/6.
\]

\[
H(z) = H_0 \left[ \frac{(1+z)^2}{1 + 2Az + (A + 3B)z^2 + 2Bz^3} \right] \tag{A10}
\]
Note that the expressions for ΛCDM can be easily obtained from (A4-A7), putting $j = -1$ into (A8).
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