Additional studies of the probability that the events with a superjet observed by CDF are consistent with the SM prediction

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In the $W+2,3$ jet data collected by CDF during the 1992-1995 Fermilab collider run, 13 events were observed to contain a superjet when $4.4 \pm 0.6$ events are expected. A previous article detailed the selection and the kinematical properties of these events. The present paper provides estimates of the probability that the kinematics of these 13 events is statistically consistent with the standard model prediction.

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I. INTRODUCTION

The CDF experiment has reported an excess of events in the $W+2$ and $W+3$ jet topologies in which the presumed heavy-flavor jet contains a soft lepton (SLT tag) in
addition to a secondary vertex (SECVTX tag). The rate of these events (13 observed) is larger than what is predicted by a simulation of known standard model (SM) processes (4.4 ± 0.6 events expected, including single and pair production of top quarks). Various kinematical distributions of these events are compared in Ref. [1] to what is expected if the excess were simply due to a statistical fluctuation of the SM contributions. The simulation is cross-checked by comparing to a complementary sample of 42 $W + 2$ and $W + 3$ jet events with SECVTX tags but no supertags. According to the simulation [1], events with a superjet and the complementary data set have quite similar heavy flavor composition. A set of 18 kinematical variables was chosen a priori to look for differences between data and simulation. Each data distribution is compared to the SM expectation using a Kolmogorov-Smirnov (K-S) test [2,3]. The probability $P$ that each distribution is consistent with the SM simulation is derived with Monte Carlo pseudo-experiments which include Poisson fluctuations and Gaussian uncertainties in the prediction of each standard model contribution.

In Ref. [1], a subset of 9 kinematical variables is selected a posteriori to illustrate the main differences between the data and the simulation: $E_T^l$ and $\eta^l$, the transverse energy and pseudo-rapidity of the primary lepton (l); $E_T^{suj}$ and $\eta^{suj}$, the transverse energy and pseudo-rapidity of the superjet (suj); $E_T^b$ and $\eta^b$, the transverse energy and pseudo-rapidity of the additional jets (b) in the event; $E_T^{l+b+suj}$ and $y^{l+b+suj}$, the transverse energy and rapidity of the system $l + b + suj$; and $\delta \phi^{l,b+suj}$, the azimuthal angle between between the primary lepton and the system $b + suj$ composed by the superjet and the other jets in the events. The first 8 variables test if the production cross sections $\frac{d^2\sigma}{dp_Td\eta}$ of each object in the final state is consistent with the SM simulation and the ninth variable tests if the data are consistent with the production and decay of $W$ bosons from known sources. Table I summarizes the probabilities of these comparisons. The SM simulation models correctly the complementary sample of data, but has a systematically low probability of being consistent.

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1Such a double tag is called supertag in Ref. [1]; jets with a supertag are referred to as superjets.
with the kinematic distributions of the events with a superjet. The use of this subset of variables is well motivated by the fact that it provides a simple way to describe in full the kinematics of the final state with relatively modest correlations. However, it is not the only possible choice.

Table II lists the result of the K-S test of the other 9 kinematical distributions inspected: 
- \(E_{T}\), the corrected transverse missing energy;
- \(M_W\), the \(W\) transverse mass calculated using the primary lepton and \(E_{T}\);
- \(M_{b+suj}\), \(y_{b+suj}\), and \(E_{T}^{b+suj}\), the invariant mass, rapidity, and transverse energy of the system \(b + suj\) respectively;
- \(M_{l+b+suj}\), the invariant mass of the system \(l + b + suj\); \(\delta \theta^{b,suj}\) and \(\delta \phi^{b,suj}\), the angle and the azimuthal angle between the superjet and the \(b\)-jets, respectively; and \(\delta \theta^{l,b+suj}\), the angle between the primary lepton and the system \(b + suj\). The simulation models correctly these distributions for the complementary sample. The probabilities for the events with a superjet are systematically lower, but the disagreement between data and simulation is much reduced for this second set of variables. This second set of 9 distributions would have been better suited to find differences if, for example, events with a superjet were produced by the two-body decay of a massive object produced in association with a \(W\) boson or by the three-body decay of a massive object produced in association with large \(E_{T}\).

In Sect. II, we first evaluate the combined probability that the data are statistically consistent with the simulation using different methods in order to estimate the effect of possible correlations between kinematic variables. We then study the effect of the bias introduced by the choice of particular sets of kinematical variables which were not motivated by a specific model or by the analysis of an independent data sample. Section III summarizes our conclusions.
TABLE I. Results of the K-S comparison between data and simulation for the first set of 9 kinematical variables. $P$ is the probability of making an observation with a K-S distance no smaller than that of the data.

| Variable     | Events with a superjet | Complementary sample |
|--------------|-------------------------|-----------------------|
| $E_T^l$      | 2.6                     | 70.9                  |
| $\eta^l$    | 0.10                    | 72.7                  |
| $E_T^{suj}$  | 11.1                    | 43.0                  |
| $\eta^{suj}$| 15.2                    | 73.4                  |
| $E_T^b$      | 6.7                     | 8.6                   |
| $\eta^b$    | 6.8                     | 80.0                  |
| $E_T^{l+b+suj}$ | 2.5                   | 18.8                  |
| $y^{l+b+suj}$| 13.8                    | 7.8                   |
| $\delta\phi^{l,b+suj}$ | 1.0                   | 77.9                  |

TABLE II. Results of the K-S comparison between data and simulation for the second set of 9 kinematical variables.

| Variable     | Events with a superjet | Complementary sample |
|--------------|-------------------------|-----------------------|
| $E_T$        | 27.1                    | 57.1                  |
| $M_W$        | 13.1                    | 38.2                  |
| $M^{b+suj}$  | 4.0                     | 58.9                  |
| $y^{b+suj}$  | 7.1                     | 34.9                  |
| $E_T^{b+suj}$| 24.0                    | 60.1                  |
| $M^{l+b+suj}$| 21.0                    | 33.6                  |
| $\delta\theta^{b,suj}$ | 30.1                   | 41.1                  |
| $\delta\phi^{b,suj}$ | 15.3                   | 83.8                  |
| $\delta\theta^{l,b+suj}$ | 37.3                   | 35.7                  |
II. EVALUATION OF THE COMBINED PROBABILITY

Using the results of the previous section, we first evaluate the combined probability that the data are statistically consistent with the simulation using the set of 9 kinematical variables listed in Table II. The combined probability is evaluated with three different approaches in order to test the sensitivity of the result to the correlations between kinematical variables.

In the simplest method, we evaluate the probability of observing a value of \( \Pi = \prod_i P_i \), where \( n \) is the number of kinematic variables, no larger than that of the data (\( \Pi^0 \)). If the kinematical variables are uncorrelated, this probability is \( \Pi_T = \Pi^0 \sum_{k=0}^{n-1} \frac{(-\ln \Pi^0)^k}{k!} \). This method yields \( \Pi_T = 0.46 \) for the complementary sample and \( \Pi_T = 1.6 \times 10^{-6} \) for events with a superjet.

In the second method, which accounts for the effect of correlations between variables, we perform a large number of Monte Carlo pseudo-experiments. In each experiment, we form a set of 8 \( W + 2 \) jet and 5 \( W + 3 \) jet different events randomly extracted from the simulations of the 12 processes listed in Tables V and VI of Ref. [I]. In each experiment, we first randomly determine \( N_i \), the number of events contributed by each process \( i \), separately for the 2 and 3 jet bin. This is done using as probabilities the ratios \( \sigma_i/\sigma \), where the contribution \( \sigma_i \) of each process \( i \) (as listed in Tables V and VI of Ref. [I]) is smeared, in each experiment, by its error using a Gaussian distribution and \( \sigma = \sum_{i=1}^{12} \sigma_i \). We then randomly extract \( N_i \) events from the simulation of each process \( i \) to form a sample of 13 events (8 with 2 jets and 5 with 3 jets). We compare the distribution of the nine kinematical variables to the SM templates by using the same K-S test of Ref. [I] and derive the product \( \Pi \) of the probabilities \( P_i \) for each experiment. The combined probability that the data are consistent with the SM simulation is given by \( \Pi_C \), the fraction of pseudo-experiments which have a probability \( \Pi \) no larger than \( \Pi^0 \). The distribution of the probability product \( \Pi \) resulting from \( 10^7 \) pseudo-experiments which use simulated events is shown in Figure [I]. We find 16 pseudo-experiments with a product of probabilities no larger than that observed for the superjet data. This corresponds to a combined probability \( \Pi_C = (1.6 \pm 0.4) \times 10^{-6} \) (4.8 \( \sigma \))
effect).

We have also performed pseudo-experiments in which we compare the SM simulation to 13 different events extracted randomly from the complementary sample of data consisting of 42 events. For each experiment, we compare the kinematical distributions of each sample to the SM templates and derive the product of probabilities $\Pi$. Figure 2 shows the $\Pi$ distribution of the $10^7$ pseudo-experiments. The probability that 13 events randomly extracted from the control sample have a product $\Pi$ no larger than the data is $(1.4 \pm 0.4) \times 10^{-6}$. In other words, it is very hard to find, among these particular 42 events, a subsample of 13 events that disagrees with the SM simulation as much as the superjet sample.

We have studied a few effects which might influence the low value of the combined probability.

As observed in Section VD of Ref. [1], the rapidity distributions of the objects in the final state are quite asymmetric. Since we know of no physics process that would produce such asymmetries, it is possible that they are due to an obscure detector problem, not seen in other data samples, or to a low probability statistical fluctuation. Therefore, it is of interest to understand the effect of these asymmetries on the low value of the combined probability. We have done this by comparing the 9 observed and simulated distributions using the pseudo-rapidity absolute values. This test also yields a small value of the combined probability ($\Pi_T = 4.5 \times 10^{-6}$).

The combined probability value depends on the estimate of the contribution of each SM process and its uncertainty. We have studied the effect of varying the fraction of $t\bar{t}$ events. If we make the hypothesis that the data are contributed only by $t\bar{t}$ events, $\Pi_T$ grows to $1.2 \times 10^{-5}$ for the events with a superjet and decreases to $0.8 \times 10^{-2}$ for the complementary sample.

We next study the bias due to the use of a particular set of kinematical variables which, while quite reasonable and well motivated, was not chosen \textit{a priori}. For example, we could have evaluated the combined probability using a slightly different set of 8 kinematic variables: $E_T', \eta', E_T^{suj}, \eta^{suj}, E_T^b, \eta^b, E_T$, and $M_W^T$. This set does not describe the kinematics of all
objects in the final state as completely as the previous one, but it contains some variables
which are more intuitive. In this case, we derive the following combined probabilities:
\( \Pi_T = 7.4 \times 10^{-5} \) and \( \Pi_C = (2.5 \pm 0.5) \times 10^{-5} \) (4.2 \( \sigma \) effect).

Events with a superjet are not very anomalous when using the set of 9 kinematical
variables listed in Table II. Using this set of variables, the combined probabilities for events
with a superjet are \( \Pi_T = 1.9 \times 10^{-2} \) and \( \Pi_C = 2.3 \times 10^{-2} \), respectively.

The effect of the bias due to the \textit{a posteriori} choice of a particular set of kinematical vari-
ables is removed by evaluating the combined probability for all the 18 kinematical variables
inspected. In this case, the probability that the data are consistent with the simulation is
\( \Pi_T = 0.67 \) for the complementary sample and \( \Pi_T = 6.0 \times 10^{-7} \) for events with a superjet.

This estimate of the combined probability does not account for the effect of large correlations
between a few of the 18 kinematical variables. With \( 10^6 \) pseudo-experiments which use simu-
lated events we evaluate that in this case the combined probability for events with a superjet
is \( \Pi_C = (3.4 \pm 0.6) \times 10^{-5} \). The \( \Pi \) distribution resulting from these pseudo-experiments is
shown in Figure B.

![Figure 1](image-url)

**FIG. 1.** Distribution of the product \( \Pi \) of 9 probabilities obtained with \( 10^7 \) pseudo-experiments
which use 13 events randomly extracted from the SM simulation (see text). The arrow indicates
the \( \Pi \) value of the data. The inset shows the \( \Pi \) distribution in full.
FIG. 2. Distribution of the product $\Pi$ of 9 probabilities of 13 events extracted randomly from the complementary sample of 42 events. The arrow indicates the $\Pi$ value of the data.

FIG. 3. Distribution of the product $\Pi$ of 18 probabilities obtained with $10^6$ pseudo-experiments which use 13 events randomly extracted from the SM simulation (see text). The arrow indicates the $\Pi$ value of the data.
III. CONCLUSIONS

Having taken into account the correlations between kinematical variables, we estimate that the combined probability that the events with a superjet, reported by the CDF collaboration in Ref. [1], are statistically consistent with the SM simulation is $(1.6 \pm 0.4) \times 10^{-6}$ (4.8 $\sigma$ effect). This probability is derived using a particular set of 9 kinematical variables, selected a posteriori from a larger set of 18, which was chosen a priori in order to search for differences between data and simulation. The effect of the bias due to the a posteriori selection of particular sets of variables cannot be univocally assessed. We have therefore evaluated the combined probability that these events are consistent with the simulation using all kinematical variables which have been inspected. We find that the combined probability remains low $[(3.4 \pm 0.6) \times 10^{-5}$ (4.1 $\sigma$ effect)].

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