The Ground State of the “Frozen” Electron Phase in Two-Dimensional Narrow-Band Conductors with a Long-Range Interelectron Repulsion. Stripe Formation and Effective Lowering of Dimension.

A. A. Slutskin, V. V. Slavin, and H. A. Kovtun.

Institute for Low Temperature Physics and Engineering, 47 Lenin Ave., Kharkov, Ukraine.

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In narrow-band conductors a weakly screened Coulomb interelectron repulsion can suppress narrow-band electrons’ hopping, resulting in formation of a “frozen” electron phase which differs principally from any known macroscopic self-localized electron state including the Wigner crystal. In a zero-bandwidth limit the “frozen” electron phase is a classical lattice system with a long-range interparticle repulsion. The ground state of such systems has been considered in the case of two dimensions for an isotropic pair potential of the mutual particle repulsion. It has been shown that particle ordering into stripes and effective lowering of dimension universally resides in the ground state for any physically reasonable pair potential and for any geometry of the conductor lattice. On the basis of this fact a rigorous general procedure to fully describe the ground state has been formulated. Arguments have been adduced that charge ordering in High-\(T_c\) superconductors \cite{11,12} testifies to presence of a “frozen” electron phase in these systems.

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1. Introduction.

High-\(T_c\) superconductors studies have caused a surge of interest in properties of narrow-band layered and two-dimensional (2D) conductors. An important consequence of the layeriness is substantial weakening of the screening of a Coulomb interaction between the charge carriers. (The screening radius cannot, under any circumstances, be less than the interlayer distance). Besides, in layered conductors it is possible to well separate the charge carriers (for the sake of definiteness, we consider them electrons) from the donors, so that the mean energy, \(u_{ee}\), of the long-ranged interelectron repulsion prevails over the energy of an electron attraction to the donors. Under these conditions it is the mutual repulsion of narrow-band electrons that can suppress their tunneling between the equivalent orbits of the conductor lattice, resulting in formation of a “frozen” electron phase (FEP) which differs principally from any known macroscopic self-localized electron state including the Wigner crystal \cite{1}. The FEP occurs when the electron bandwidth, \(t\), is less than \(\delta u = (a/r_{ee})u_{ee}\), where \(\delta u\) is the typical change in the narrow-band electron Coulomb energy in electron hopping, \(a\) is the range of hopping, \(r_{ee}\) is the mean electron separation. The high-\(T_c\) cuprates, grain boundaries of polycrystal electroceramic materials \cite{3}, as well as some art 2D conductors \cite{13,14,15} appear to be most favorable for 2D FEP coming to existence.

Macroscopic behavior of the 2D FEP is rather unconventional. Its distinctive features are rooted in properties of its ground state (GS) at \(t \ll \delta u\). In the limit \(t/\delta u \rightarrow 0\) the GS of the 2D FEP is much the same as that of other 2D lattice systems with a long-ranged interparticle repulsion. (An example is an ensemble of adsorbed atoms strongly interacting with their substrate and mutually repelling each other \cite{2}). As far as we know, neither the thermodynamics nor the GS of such systems have been studied adequately. Here we offer a unified approach to the description of the GS of the 2D zero-bandwidth FEP (and similar lattice systems) with an isotropic pair potential of the interelectron repulsion, \(v(r)\) (\(r\) is the distance between interacting electrons). The key point of our consideration is a new phenomenon — a zero-temperature effective lowering of dimension (LOD) — which we have revealed to underlay (despite the pair potential isotropy) the main GS properties of the 2D FEP for: i/ arbitrary arrangement of the sites which can be occupied by electrons provided the sites constitute a primitive lattice (it is called host lattice below); ii/ any filling factor, \(\rho = N/N\) (\(N \rightarrow \infty\) and \(N \rightarrow \infty\) are the total numbers of the electrons and the host-lattice sites respectively); iii/ any physically reasonable \(v(r) > 0\). We take the term LOD to mean that the GS of the 2D FEP is a set of different effective 1D FEP whose “particles” are periodic stripes on the lattice of the 2D conductor. For each 1D system of the set there is its own \(\rho\) interval where this 1D FEP represents the 2D one, the whole range, \(0 \leq \rho \leq 1\), comprising all the intervals. The LOD enables us to offer a rigorous analytical procedure for the 2D FEP GS description, using the exact results of the general theory of the 1D lattice systems with a long-ranged interparticle repulsion \cite{7,8,9}.

2. Hamiltonian. S-crystals.

The Hamiltonian, \(\mathcal{H}\), of the system under consideration has the form
\[ \mathcal{H}\{n(\vec{r})\} = \frac{1}{2} \sum_{\vec{r} \neq \vec{r}'} v \left( |\vec{r} - \vec{r}'| \right) n(\vec{r}) n(\vec{r}') , \]

where \( \vec{r} = m_1 \vec{a}_1 + m_2 \vec{a}_2 \) are radius vectors of the host-lattice sites, \( m_{1,2} \) are integers, \( \vec{a}_{1,2} \) are host-lattice primitive translation vectors (PTVs); the occupation numbers of the host-lattice sites, \( n(\vec{r}) = 0 \) or 1, are microscopic variables \( \mathbb{I} \); the sum is taken over the whole host lattice. The pair potential is assumed to be an everywhere-convex function with the form \( v(r) = \tilde{v}(r)/r \), where function \( \tilde{v}(r) \) depends on the character of the screening medium and its position with respect to the 2D FEP. In any case \( \tilde{v}(r) \) tends to zero as \( r^{-2} \) or faster when \( r \to \infty \); \( \tilde{v}(0) = e^2/\kappa \) (\( e \) is the electron charge, \( \kappa \) is the dielectric permittivity). Otherwise \( \tilde{v}(r) \) can be reckoned as arbitrary: as will be shown below, its specific form is immaterial to our approach.

Among the GS configurations \( \{n(\vec{r})\} \) with different \( \rho \) the simplest ones are 2D crystals with one electron per cell (“S-crystals”). Their inverse \( \rho \) values make up an infinite set of integers \( Q_j = |\det(m_{\kappa \lambda}(\vec{a}_1, \vec{a}_2))| \), where \( j \) indexes S-crystals, integers \( m_{\kappa \lambda} \) \((\kappa, \lambda = 1, 2)\) are components of S-crystal PTVs in the \( \vec{a}_k \) basis; \( Q_j \) is the \( j \)-th S-crystal elementary-cell area measured in units of that of the host lattice, \( \sigma_0 = |\vec{a}_1 \times \vec{a}_2| \).

Our strategy is to derive the full description of the GS for any \( \tilde{a}_{1,2} \), starting with consideration of small vicinities of \( \rho = 1/Q_j \). Since specific \( m_{\kappa \lambda} \) values are irrelevant to this reasoning, we drop index \( j \) at \( Q \) and at other characteristics of the S-crystals for a while.

Due to discreteness of the system with the Hamiltonian \( \mathbb{I} \) a macroscopically small change, \( \delta \rho \), in \( \rho \) \((\delta \rho \to 0, N \to \infty \) when \( N \to \infty \)) produces only isolated defects in an S-crystal, the space structure of the defects essentially depending on whether they result from an increase or a decrease in \( \rho \). This fact is expressed by the identity

\[ E_g (N \pm |\delta N|, \mathcal{N} \pm |\delta \mathcal{N}|) - E_g (N, \mathcal{N}) = \pm \mu_{\pm} |\delta N| \mp P_{\pm} |\delta \mathcal{N}| , \]

where \( E_g \) is the GS energy, \( \delta N \) and \( \delta \mathcal{N} \) are changes in \( N \) and \( \mathcal{N} \) producing \( \delta \rho \). The proportionality coefficients, \( \mu_- < \mu_+ \), \( P_- < P_+ \), are the values of the chemical potential, \( \mu \), and the pressure, \( P \), which are the endpoints of the \( \mu \) and \( P \) intervals of S-crystal existence. They are determined by the energies of formation of corresponding defects. Thus, in some vicinity of \( \rho = 1/Q \) the GS is bound to be a superstructure of the defects. Our next step is to find them.

3. Zero-dimensional defects and their coalescence.

Adding to or removing from an S-crystal one electron results in formation of a zero-dimensional defect, “\(+\text{-defecton}” or “\(-\text{-defecton}” respectively. One can be inclined to think that \( \pm \delta N \) should be identified exactly with the total number of \( \pm \text{defectons} \) spatially separated, \( \pm \mu_{\pm} \) being simply the energy of \( \pm \text{defecton} \) formation, \( \epsilon_{\pm} \). However, this seemingly evident statement is actually incorrect due to a coalescence of defectons of the same “sign”. In other words, if the number, \( |\nu| \), of S-crystal electrons removed \((\nu < 0)\) or added \((\nu > 0)\) is more than one, a bound state of \( |\nu| \) defectons arises whose energy is less than \( |\nu| \epsilon_{\pm} \). We have revealed the coalescence by computation, using a “dipole” description of the GS with \( \nu = \pm 1, \pm 2, \ldots \), which we have specially worked out for this purpose. The dipole approach offers a clear view of how the defectons’ bound state arises despite the fact that the defectons of the same sign repel each other, being widely spaced.

At \( \nu \neq 0 \) a perturbed S-crystal is formed where beside electrons placed at host-lattice sites in the interstices of the S-crystal \((\nu > 0)\) or empty S-crystal sites, “holes”, \((\nu < 0)\) there are generally a certain number of S-crystal electrons shifted from their native S-crystal sites. The latter can be considered as “antiparticles” whose charge is equal to the electron one in magnitude but is opposite in sign, a pair “an electron shifted by a vector \( \vec{\xi} + \) its antiparticle located at an S-crystal site \( \vec{r}’ \) being the \( \vec{r}, \vec{\xi}\)-dipole”. Thus, the perturbation of the S-crystal can be envisioned as an ensemble consisting of several dipoles and \( |\nu| \) interstitial particles/holes (IP/Hs). The dipoles interact with the IP/Hs and with each other. The energy of interaction between the IP/H \((\vec{r} = 0)\) and \( \vec{r'} \)-dipole is \( u_{\vec{\xi}}(\vec{r} = 0) = \nu \vec{v}(\vec{r} = 0) \vec{\xi} \equiv \nu \vec{\xi} \vec{v}(\vec{r} = 0) \). The energy of interaction between \( \vec{r}, \vec{\xi} \)- and \( \vec{r'}, \vec{\xi}'\)-dipole is \( u_{\vec{\xi}}(\vec{r} = \vec{r'}) = \Delta_{\vec{\xi}} \Delta_{\vec{\xi}'} \vec{v}(\vec{r} = \vec{r'}) \). The IP/Hs, in turn, undergo a mutual repulsion and are exposed to an “external” field, \( u(\vec{r}) \), which is equal to \(-2u_0\) for holes (here and further on \( u_{\nu} \) of the S-crystal is denoted by \( u_0 \)), and for IPs it is the field produced at a point \( \vec{r} \) by all electrons of the S-crystal. In these terms the change in the GS energy at a given \( \nu \), \( U_{\text{GS}}(\nu) \), takes the form

\[ U_{\text{GS}}(\nu) = \min \left( V_{\text{rep}} + U_d + U_{\text{exc}} + U \right) \]

Here \( V_{\text{rep}} = \sum_{\alpha < \beta} v_0(\vec{r}_{\alpha \beta}) \) is the energy of the mutual repulsion of the IP/Hs; \( U_d = \sum_{\alpha \neq \beta} u_{\vec{\xi}}(\vec{r}_{\alpha \beta}) \) is the energy of their interaction with the dipoles; \( U_{\text{exc}} = \sum_{\alpha \neq \beta} \delta u_{\vec{\xi}} + \sum_{\alpha \neq \beta} u_{\vec{\xi}}(\vec{r}_{\alpha \beta}) > 0 \) is the excitation energy of S-crystal with \( n_d \) dipoles at \( \nu = 0 \); \( \delta u_{\vec{\xi}} \sim \nu_0 \xi^2/\epsilon_{\xi} > 0 \) is the energy of formation of one dipole; \( U = \sum_{\alpha \neq \beta} u(\vec{r}_{\alpha \beta}) \) is the energy of the IP/Hs in the external field mentioned; indexes \( \alpha = 1, \ldots, |\nu| \) and \( i = 1, \ldots, n_d \) enumerate the IP/H radius-vectors and dipoles respectively, \( n_d \) is the total number of the dipoles; \( \vec{r}_{\alpha \beta} \equiv \vec{r}_\alpha - \vec{r}_\beta \). The minimum is taken in respect to \( n_d \), the dipole variables, \( \vec{r}_\alpha \), \( \vec{\xi}_\alpha \), and \( \vec{r}_{\alpha \beta} \). Therefore, the dipole approach allows to work with only a few discrete variables. This facilitates considerably
the Monte-Carlo computer simulation of the ± defectons \((U_{GS}(±1) = ± e^±)\) and their coalescence at \(|\nu| > 1\).

The mechanism of the coalescence can be elucidated by the following heuristic arguments. The GS total dipole energy, \(E_d(\nu) = U_{exc}(\nu) + U_d(\nu)\), is negative, so that for any \(|\nu|\) the GS space structure is determined by an interplay between negative \(U_d\) and positive \(U_{exc}, V_{rep}\). The IP/H – dipole interaction gives the maximal gain in energy when each IP/H is embedded in a “shell” of four dipoles which are attracted to it, the dipoles’ antiparticles forming a parallelogram of a size \(\sim r_{cc} \sim Q^{1/2}\) (Fig.1). The shells of neighboring IP/Hs are bound to share some of their dipoles for \(U_{exc}\) (and hence \(n_d\)) to be as small as possible. This requirement can be fulfilled only when all IP/Hs are aligned in a row, the near-neighbor IP/Hs being shifted relative to one another by the same S-crystal PTV with the modulus \(|\nu|\) (section 2) such that \(\sim (0)\). This \(\gamma\) is the criterion of the coalescence for any \(|\nu|\) (and any \(d_1,2\)).

Criterion 3 is for the most part fulfilled. It holds for any \(\tilde{\nu}(r)\) (section 3) such that \(\tilde{\nu}(0) = \tilde{\nu}(r_{cc}) = \tilde{\nu}(0)\). This case will be the focus of our attention from here on. Parameter \(\gamma\) becomes \(\gg 1\) if \(\tilde{\nu}(r)\) decreases substantially only for \(r\) which are exponentially large in \(\gamma\). In this limit the mutual repulsion of the IP/Hs disrupts their row, and there is no coalescence, at least for sufficiently large \(|\nu|\). However, in section 3 it is outlined that the LOD governs the GS in this rather special case, too.

4. The lowering of dimension.

4.1. The elementary stripes in the 2D FEP. As follows from the aforesaid, the bound state of \(|\nu|\) defectons is transformed into a periodic stripe-like structure with an infinite increase in \(|\nu|\). (Fig. 1). It consists of elementary 1D defects which, as will be shown below, repel each other. Therefore, it is the simplest 1D defects that are expected to form the GS superstructure. An arbitrary 1D defect of such a type is a stripe of rarefaction or compression which arises when an S-crystal part adjacent to a line of electrons with some PTV, \(d\), is shifted as a whole relative to the other one by a host-lattice translation vector, \(\xi\). Formation of one stripe of length \(L_n\) changes \(N\) by \(\delta N = ± \sigma L_n\) (\(\sigma = |d|\xi|\)). \(L_n\) is measured in units of \(|d|\). The corresponding change in energy, \(\delta E\), is proportional to \(\delta N\):

\[
\delta E/|\delta N| = \varepsilon(d, \xi) = \sigma^{-1} \sum_{n=1}^{\infty} \sum_{r} u_{\xi}(r - n\bar{f}). \tag{5}
\]

Here \(\sum_r^\prime\) means summation over the S-crystal semiplane \(r = k\bar{d} + l\bar{f} (\sim -k < k < \infty, \sim -\infty < l < 0)\); \(\bar{f}\) is any S-crystal PTV other than \(\bar{d}\). The GS is realized by the stripes with \(\bar{d} = d_±\) and \(\xi = \xi_±\) which minimize \(\varepsilon(d, \xi)\) at a given sign of \(-\delta N\) (– or + symbolizes rarefaction or compression respectively). We will call these stripes “–stripes” or “+stripes”.

The energies \(\varepsilon_± = |\varepsilon(d±, \xi±)|\) are the quantities \(P_±\) (see (3)) associated with ±stripes formation. The corresponding \(\mu_±\), as follows from general thermodynamic considerations, are

\[
\varepsilon_± = \mu_0 + Q\varepsilon_±. \tag{6}
\]

Lest there be no contradiction with the fact of the coalescence, energies \(\varepsilon_±\) and \(\varepsilon_±\) are bound to satisfy inequalities

\[
\varepsilon_+ < \varepsilon_- < \varepsilon_+ + \varepsilon_-< \varepsilon_. \tag{7}
\]

When \(Q \gg 1\) and \(v(r)\) goes to zero over distances \(R \ll r_{cc} \sim Q^{1/2}\), they follow from simple estimates. Taking into account that \(|\xi_±| \sim a_0\), and, correspondingly, \(|d±_± × \xi±_±| \sim Q^{1/2}a_0\), from Eq. (3) we obtain: \(\varepsilon_± \sim (a_0Q^{1/2}/R)u_0\). On the other hand, \(|\varepsilon_+| \sim u_0 \sim v(r_{cc})\), and hence, \(\varepsilon_+ \gg |\varepsilon_-|\). In the case under consideration the GS is realized by the IP and the S-crystal sites. The energy is much more than \(\varepsilon_+\) as \(R \ll r_{cc}\).

To make sure that the inequalities (6) hold for other \(v(r)\) and \(R/r_{cc}\) we have computed \(\varepsilon_±\) (basing on Eq. (3) and Eq. (5)) in parallel to the Monte-Carlo computer studies of the coalescence. They have confirmed that the inequalities are really the case for all \(v(r)\) under consideration and for all \(Q\), maybe except \(Q = 2\).

Together with the mutual repulsion of ±stripes of the same sign the inequalities (5) lead to the conclusion that +stripes or –stripes do constitute the GS superstructure in the vicinity of 1/Q. The position of each ± stripe – a constituent of the superstructure – is determined by the stripe “coordinate”, \(l\), which is the total number of particle lines (with the PTV \(d±\)) between this stripe and some fixed one (\(l = 0\)). A set of these coordinates determines uniquely the 2D FEP space structure. Therein lies the LOD.

4.2. The GS superstructure of stripes. The GS arrangement of the ±stripes is governed by the pair potential of the stripe-stripe interaction,
\[ V_{\infty}^\pm(l) = \sum_{n=+1}^{\infty} \sum_{r} u_{\xi_\pm, r} (r - n f_\pm - \xi_\pm) \] (8)

where inter-stripe “distance” \( l = 1, 2, \ldots \): \( f_\pm \) is an S-crystal PTV other than \( d_\pm \), \( \Sigma' \) means the same as in Eq. (\ref{eq:superscript}) \( (d, f = d_\pm, f_\pm) \). For all \( v(r) \) under consideration \( \Sigma'(n) > 0 \), and \( \Sigma'(n) = \Sigma'(n+1) \). Hence, \( V_{\infty}(l) > 0 \) is a convex function of \( l \). This enables us to describe the \( \pm \)stripes superstructure at \( \vartheta = Q \neq 0 \) \( (\vartheta = 1/\rho) \) on the basis of the universal 1D algorithm \( \ref{eq:universal} \) \ref{eq:universal} \ref{eq:universal}, considering the stripes as the “particles” of an effective 1D FEP:

\[ l_m = \lfloor m/c_\pm \rfloor; \quad c_\pm = |\vartheta - Q|/\sigma_\pm, \quad \sigma_\pm = |d_\pm \times \xi_\pm| \] (9)

where \([ \cdot ]\) is the integral part of a number, \( m \) enumerates the \( \pm \)stripes; integer \( l_m \) is the coordinate of \( m \)-th stripe, which is a pair of neighboring lines of electrons \( r_{m,1}(k) = k d_\pm + l_m f_\pm + m \xi_\pm \) and \( r_{m,2}(k) = r_{m,1}(k) + f_\pm + \xi_\pm \) \( (k = 0, \pm 1, \ldots) \). The superstructure described by Eq. (\ref{eq:universal}) is thus a mixture of \( \pm \)stripes \( (\vartheta = Q > 0) \) and unperturbed stripes of the S-crystal which are parallel to \( d_\pm \), so that \( c_\pm = N_s/N_c \) is the concentration of the \( \pm \)stripes; \( N_s \) is their number; \( N_c \) is the total number of the \( \pm \)stripes and the S-crystal ones. The number of unperturbed stripes between \( m \)-th and \( m+1 \)-th \( \pm \)stripes equals \( l_{m+1} - l_m - 1 \).

4.3. An algorithm for the determination of \( \pm \)stripes. S-crystals with \( f_\pm = q_\pm \xi_\pm \), where \( q_\pm = Q/\sigma_\pm \) is an integer are of frequent occurrence. Particularly, this occurs for a triangular host lattice (section \ref{sec:triangular}), and also for \( \sigma_\pm = \sigma_0 \), as is typical of S-crystals on a host lattice of a lower symmetry. In such a case the above-mentioned electron lines of both types, \( r_{m,1}(k) \) and \( r_{m,2}(k) \), fall into the class of electron lines \( k d_\pm + l \xi_\pm \) \( (k = 0, \pm 1, \ldots; l \) is an integer), which can be considered as 1D “particles” with “coordinates” \( l \). Their arrangement, as follows from Eq. (\ref{eq:universal}), obeys the algorithm:

\[ l_m = \lfloor s m \rfloor; \quad s = q_\pm \mp c_\pm, \]

where \( l_m \) is the “coordinate” of the \( m \)-th line, \( s \) is the mean line separation measured in units of \( |\xi_\pm| \).

5. Devil staircase.

The dependence of \( c_\pm \) (or \( \rho \)) on \( \mu \), much the same to the 1D FEP \ref{eq:universal} \ref{eq:universal} \ref{eq:universal}, is a well-developed fractal structure, a devil staircase whose steps occur at all \( \text{rational} \) \( c_\pm = M/L \leq 1 \) \( (M, L \) are coprime integers). At given \( M, L \) the GS configuration of the 2D FEP is thus a “FEP crystal” with \( L \) electrons per cell and with PTVs \( d_\pm, f_\pm, \xi_\pm \).

In the commonly occurring case that \( f_\pm \) is a multiple of \( \xi_\pm \) (section \ref{sec:multiple}) the steps’ widths, \( \Delta \mu = \Delta \mu(M/L) \), can be found by direct application of the 1D theory \ref{eq:universal} \ref{eq:universal} \ref{eq:universal}, considering the energy of the line-line repulsion,

\[ \mathcal{V}(l) = \sum_{k=-\infty}^{\infty} v(|k d_\pm \mp l \xi_\pm|) \]

\( (l \) is the distance between interacting lines, as the 1D pair potential. This produces

\[ \Delta \mu = \mathcal{L} \sum_{m=1}^{\infty} \sigma \left( |L m - 1| - 2 \sigma(L m) + \sigma(L m + 1) \right), \]

where \( \mathcal{L} = q_\pm L \mp M \) is the period of the lines’ pattern. The expression in the brackets is positive since in the case under consideration \( \mathcal{V}(l) \) is a convex function. Generally, \( \Delta \mu(M/L) \) are expressed in terms of \( V_{\infty}(l) \) by a slight modification of the 1D theory.

6. \( j \) branches and first-order transitions in the ground state of the 2D FEP.

The algorithm \ref{eq:universal} can be extended over the whole \( 0 < c_\pm < 1 \), provided the crystal with one particle per cell (“S’-crystal” with PTVs \( d_\pm, f_\pm + \xi_\pm \) which arises at \( c_\pm = 1 \) \( (\vartheta = Q \mp \sigma_\pm \) is stable (i.e. it is another S-crystal) or metastable. This follows from the fact that \( \vartheta \) owing to the coalescence of defects macroscopically small variations in \( \vartheta \) generate, at any \( c_\pm \), 1D defects only; ii/ these 1D defects, according to our computer calculations, have the same PTV, \( d_\pm \), for all \( c_\pm \).

Moreover, due to (meta)stability of the S’-crystal the algorithm \ref{eq:universal} holds over a \( \vartheta \) range adjacent to the interval \( [Q - \sigma_\pm, Q + \sigma_\pm] \). In such a case Eq. (\ref{eq:universal}) determines a mixture of stripes of new geometry which are characterized by a new triple of vectors, \( d_\pm, f_\pm, \xi_\pm \), the analogues of \( d_\pm, f_\pm, \xi_\pm \), and the \( \pm \)stripes concentration \( c_\pm = |\vartheta - Q \pm \sigma_\pm|/|d_\pm \times \xi_\pm| \). Transition from one geometry to another is continuous in \( \vartheta \) since \( c_\pm \) goes to zero when \( \vartheta \to Q \pm \sigma_\pm \).

Continuously extending the algorithm \ref{eq:universal} in the manner shown above, we obtain the “\( j \)-branch” (we introduce the index \( j \) again) which comprises all (meta)stable structures Eq. (\ref{eq:universal}) connected in continuity with the starting S-crystal. The corresponding energy, \( E_j(\vartheta) \), can be easily found in terms of \( V_{\infty}(l) \), using Eq. (\ref{eq:universal}). As a rule, there exist different S-crystals belonging to the same \( j \)-branch. On the other hand, as we have computed, intersections of different \( E_j(\vartheta) \), and hence, zero-temperature first-order transitions in variables \( \mu \) or \( P \) (a type of polymorphism), are universally present in the 2D FEP. (See example in section \ref{sec:example}.) The dependence of \( E_j \) on \( \vartheta \) is the function which comprises all stable portions of all \( E_j(\vartheta) \).

Thus, owing to the LOD described above the GS of the 2D FEP is fully determined by the S-crystals PTVs, \( m_\nu l \),
the "directors", $\vec{d}_{ij}^{kl}$, and the displacement vectors, $\xi_{\pm}$, together with the set of $E_j(\bar{\theta})$ intersection points which are the only GS characteristics changing on small variations in $v(r)$. All these quantities can be computed on the basis of Eq. (8) and Eq. (9) by a self-consistent procedure, finding the S-crystals together with the $j$-branches. We have found the GS for triangular and square host lattices as well as for a number of those with central symmetry only. The computation has not revealed principal differences between GS properties of 2D FEP with different geometry of host lattices, at least for those which are not significantly anisotropic.

7. Example.

Here we illustrate the above general results with a triangular host lattice (THL). All triangular lattices on the THL are necessarily S-crystals. This follows from the fact that it is the triangular lattice that realizes the absolute energy minimum of the system whose electrons are free to move. Such S-crystals are "$p,q$-crystals" with PTVs $p\bar{a}_1 + q\bar{a}_2$, $p\bar{a}_2 + q\bar{a}_3$ and $\bar{\theta} = p^2 + q^2 - pq$ ($p,q$ are arbitrary integers, $\bar{a}_{1,2,3}$ is a triple of THL PTVs which are equal in the modulus and form an angle of $120^\circ$ with each other). Using the procedure discussed in section 6, we have found that all $0, q$-crystals belong to the same $j$-branch (the main branch), which covers the range $4 \leq \bar{\theta} < \infty$. The $S'$-crystals of the $0, q$-ones are S-crystals too. They occur at $\bar{\theta} = q(q + 1)$ ($2 \leq q < \infty$) and have PTVs $q\bar{a}_k, (q + 1)\bar{a}_\lambda$ ($\kappa, \lambda = 1, 2, 3; \kappa \neq \lambda$). The stripe structures (9) have the same PTV, $q\bar{a}_k$, for all $\bar{\theta}$ of the interval $[q(q - 1), q(q + 1)]$, their $\xi_{\pm}$ being $\pm \bar{a}_\lambda$ ($\kappa \neq \lambda$).

When $p, q \neq 0$, $j$-branches of different $p, q$-crystals are distinct. They do not have mutual intersections, but all intersect the main branch, the intersections occurring at rather small concentrations of the $p, q$-crystals' ±stripes. In other words, the intervals of $p, q$-crystals stability ($p, q \neq 0$), and correspondingly main-branch metastability, turn out to be narrow.

8. The limit of $\gamma \gg 1$.

So far the case of $\gamma \lesssim 1$ (section 3) has been discussed. Here we outline the limiting case $\gamma \gg 1$. It is realized when the Coulomb interelectron forces are screened by conductors which are at distances $\gg r_{ee}$ from the 2D FEP. Modelling such a situation by the potential $v(r) \propto r^{-1} \exp(-r/R)$ with $R \gg r_{ee}$, we have computed that the energies $\varepsilon_{\pm}$, $\varepsilon_{\pm}$ satisfy the inequalities $\varepsilon_- < \varepsilon_+ < \varepsilon_{\pm}$, which are opposite to those of Eq. (8). Due to this fact it is separated zero-dimensional defects of the S-crystal that form the GS superstructure for $\rho$ which are sufficiently close to $1/Q$. We have revealed that these zero-dimensional defects are "bidefectons", which are complexes consisting of two bound defectons. Well-separated bidefectons can be considered as new particles on the S-crystal as the host lattice, the mean bidefecton separation, $r_d$, being equal to $\sim |2(\rho - 1/Q)|^{-1/2}$.

The effective pair potential of a mutual bidefecton repulsion is characterized by the same space parameter, $R$, as $v(r)$. If $r_d \gg R$, the bidefectons, according to the general results of sections 3, 4 are bound to be ordered into stripes arranged by the algorithm (1). Extention of this reasoning to the case of $R \gg r_d$ leads to new stripe-like superstructures consisting of zero-dimensional defects of "new" S-crystals, and so on. Eventually a well-developed fractal arises. Though details of its structure are still to be determined, it is safe to say now that the LOD does take place for $\gamma \gg 1$, too.

9. Summary.

The above consideration shows that the electron ordering into stripes and the effective lowering of dimension reside in the 2D FEP universally. In essence, a combination of discreteness of electrons’ positions with a long-ranged interelectron repulsion is the only factor which gives rise to this phenomenon. For this reason it is also bound to arise with an external disorder present, the stripes being fractured and pinned by the disorder. Thus, stripe formation in 2D and layered narrow-band conductors can be considered to be the principle signature of a 2D FEP.

9.1. The charge ordering in cuprates as a manifestation of a 2D FEP. From the above standpoint the charge ordering in CuO$_2$ planes of high-temperature superconductors (cuprates) (neutron scattering), (channeling) is of especial interest. The fact that it takes place even with very low doping suggests that a 2D FEP might be present in these systems primordially. One can envision that formation of ionized oxygen molecules, $O_2^-$, in oxygen planes gives a certain energy gain even in cuprates of the stoichiometric composition. In consequence, a part of electrons leaves the oxygen planes for $s$-orbits of $Cu^{++}$ ions in CuO$_2$ planes, resulting in formation of a number of ions $Cu^+$. Since the amplitude of electron hopping $Cu^+ \leftrightarrow Cu^{++}$ is relatively small, the $Cu^+$ ensemble should be expected to be a 2D FEP, the concentration of the $Cu^+$ and, correspondingly, of the $O_2^-$ being determined by thermodynamic equilibrium between the 2D FEP and the ensemble of the $O_2^-$. It is evident that stripe formation in the 2D FEP of $Cu^+$ ions inevitably brings to existence $O^-$ superstructures in CuO$_2$ planes. Their PTVs are likely to be the same as that of the $Cu^+$ FEP.

In the connection with the aforesaid it should be noted that a simple explanation of the high-temperature superconductivity can be offered in terms of the 2D FEP taking into account the finiteness of the bandwidth. It lies in...
the fact that a virtual exchange of 2D FEP elementary excitations between oxygen holes (which are known to be free charge carriers in the doped cuprates) leads inevitably to a mutual effective attraction of the holes and thereby to superconductivity (of purely Coulomb origin) with high $T_c$. Our preliminary studies have shown that the lowest-energy elementary excitations in the cuprate 2D FEP are kinks on the disorder-fractured stripes.

9.2. Some expectable features of the 2D FEP thermodynamics and conductivity as a consequence of the stripe formation. Our preliminary studies have shown that the effective lowering of dimension in the ground state of the 2D FEP accounts for a fairly interesting and unusual low-temperature thermodynamics. It is characterized by first-order transitions in $T$, $\mu$-plane ($T$ is the temperature) from the FEP crystals (section 3) slightly perturbed by an ideal gas of separate defectons (they arise due to thermal activation) to a strongly correlated liquid of thermally fractured stripes (“FEP liquid”) where there is no long-ranged order. The melting temperature as the function of $\mu$ turns out to be reduced to zero at the endpoints of the intervals of the devil staircase. Therefore, at any $T \neq 0$ there is a set of alternating $\mu$ intervals which correspond to the FEP crystals or the FEP liquid.

Conduction in the 2D FEP liquid is expected to be by movement of kinks of the fractured stripes, each kink carrying a fractal charge (measured in units of $e$). That in the FEP crystals is of the common Drude type, the charge carriers being $\pm$ defectons with the charge $\pm e$. With a change in $\mu$ (at a fixed $T$) these conduction mechanisms alternate, resulting in pronounced 2D FEP resistivity oscillations which reflect the ground-state devil-staircase dependence of $\rho$ on $\mu$: the oscillations’ peaks are bound to occur close to the rational filling factors of the FEP crystals which survive at a given $T$. This phenomenon is yet another distinctive mark of the 2D FEP. We have found it to be very similar to the resistivity oscillations of a conductive sheet in a system metal – n-type semiconductor – p-type semiconductor, which still remain to be explained. We are going to publish the results concerning this issue in the near future.

It is remarkable that an artificially created external perturbation localized within a small region can block up conduction over all FEP liquid, pinning only one stripe. The most appropriate systems to test this experimentally are perhaps granular thin films like those described in [5]. A similar phenomenon was reported in [4]. Yet granular films used in these experimental studies were highly disordered, and it is unclear now whether the above theory works in such a situation.

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