Role of Three-Body Forces in Proton and Heavy-Ion Scatterings

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Abstract. The three-body forces (TBF) effect is known to play an important role on the nuclear saturation property, which can be demonstrated typically in the Brueckner (G-matrix) theory. Recently, we have proposed new complex G-matrix interactions CEG07, from which nucleon-nucleus (NA) and nucleus-nucleus (AA) folding potentials are obtained. The CEG07 G-matrices are derived from the free-space nucleon-nucleon interaction, the Extended Soft Core (ESC) model, including the TBF contributions composed of the three-body repulsive (TBR) and attractive (TBA) components. Using CEG07, we have analyzed the elastic scattering of NA and AA systems. For NA systems, we have tested the optical potentials obtained by the single-folding procedure with CEG07 in the cases of the proton elastic scattering. We have further applied the CEG07 G-matrix to the AA systems in the framework of the double-folding model and analyzed the elastic scattering of complex nuclei systems at $E/A = 70 \sim 135$ MeV. The TBF (especially TBR) effect is clearly seen in all cases investigated.

1. Introduction
The role of nuclear three-body force (TBF) in complex nuclear systems is one of the key issue not only in nuclear physics but also in nuclear astrophysics relevant to high-density nuclear matter in neutron stars and supernova explosions. It is well known that the empirical saturation point of nuclear matter (the binding energy per nucleon $E/A \approx 16$ MeV at a saturation density $\rho_0 \approx 0.17$ fm$^{-3}$) cannot be reproduced by using only two-body nucleon-nucleon (NN) interactions [1]. In order to obtain the reasonable saturation curve, it is indispensable to take into account the additional contributions of the TBF which contains the two parts of the three-body attraction (TBA) and the three-body repulsion (TBR). It is important here that the saturation curve in high-density region is strongly pushed up by the TBR contribution and the nuclear-matter
incompressibility becomes large as a result [2, 3, 4, 5]. This effect is intimately related to our problem.

It is a longstanding and fundamental subject to understand nucleon-nucleus (NA) and nucleus-nucleus (AA) interactions microscopically starting from basic NN interaction. In order to solve a complicated many-body problem in nuclear reactions, one needs to rely upon some realistic approach based on reasonable approximations. One of the promising approaches would be to derive the NA and AA folding potentials on the basis of the lowest-order Brueckner theory. Here, the NN G-matrix interactions are obtained in infinite nuclear matter, and folded into NA and AA density distributions with the local-density approximation. The G-matrix equation is solved for a NN pair in medium, one of which corresponds to an inside nucleon, under the scattering boundary condition. The obtained G-matrix interaction is composed of real and imaginary parts, being dependent on the incident energy and the nuclear-matter density. As noted here, the G-matrix is considered to be an effective NN interaction in folding procedures, where short-range singularities in a free-space NN interaction are smoothed out.

Recently, the present authors have proposed a new G-matrix interaction CEG07 [6] derived from the extended soft-core (ESC) model [7, 8]. The ESC model is designed so as to give a consistent description of interactions not only for NN system but also for nucleon-hyperon and hyperon-hyperon systems, where the TBR effect is taken into account effectively by changing vector-meson masses in a density-dependent way. On the other hand, the TBA part is typically due to two-pion exchange with excitation of an intermediate Δ-resonance, that is the Fujita-Miyazawa diagram, which gives an important contribution at low densities. In Ref. [6], the TBA effect was included as an effective two-body interaction according to the formalism of Ref. [9] and added to the G-matrix obtained from the two-body interaction. Although the saturation curve of nuclear matter can be produced reasonably as combined contributions of TBA and TBR, it is decisively important in our results that the TBR contribution becomes more and more remarkable as the density increases. Following Ref. [6], we call the G-matrix interaction derived from the ESC part only as CEG07a and the version including further the effect of TBF (both the TBA and TBR components) as CEG07b. The CEG07 models were first applied to the analysis of proton-nucleus elastic scattering over the wide range of incident energy and target nucleus with a remarkable success [6]. Although the inclusion of TBF effect, in general, gave rise to only minor change of pA elastic-scattering cross sections as expected from the above discussion, it was demonstrated that the inclusion of TBR effect clearly improved the fit to forward-angle analyzing power data in some energy region. Second application is done in the systems of the nucleus-nucleus elastic scattering [10, 11]. The TBR effect is clearly seen in the cross section of several systems (\(^{16}\text{O} +^{16}\text{O}\), \(^{16}\text{O} +^{12}\text{C}\), \(^{16}\text{O} +^{28}\text{Si}\), \(^{16}\text{O} +^{40}\text{Ca}\) and \(^{12}\text{C} +^{12}\text{C}\)) in the energy region from 70 to 135 MeV/u.

In this paper, we make use of the CEG07 G-matrix interactions with and without the TBF effect and analyze the proton and heavy-ion elastic scatterings to clarify how the TBF effect plays an important role in the NA and AA scattering systems.

2. Formalism

Now, we apply the complex G-matrix interaction, CEG07, to investigate the effect of three-body force in the proton and heavy-ion scatterings through the folding procedure. The optical model potential (OMP) is composed of the real and imaginary parts as,

\[
U_{\text{OMP}} = V + iW. \tag{1}
\]

We then give the optical potential in the framework of the single and double folding models for the proton and heavy-ion scattering systems, respectively. Because we use the complex G-matrix interaction, the folding model potential also becomes complex. The central and spin-orbit potentials for the proton scattering are derived in the same manner as Ref. [6]. The central part
of the optical potential for the heavy-ion scattering is derived in the same manner as Ref. [11]. The optical model potential based on the folding model is written as,

\[ U_{\text{OMP}}^{(NA)} = V_{C}^{(SF)} + iW_{C}^{(SF)} + (V_{LS}^{(SF)} + iW_{LS}^{(SF)})\ell \cdot \sigma, \]

\[ U_{\text{OMP}}^{(AA)} = V_{C}^{(DF)} + iW_{C}^{(DF)}, \]

where \( U_{\text{OMP}}^{(NA)} \) and \( U_{\text{OMP}}^{(AA)} \) are the optical potential derived from the folding models of the proton and heavy-ion scattering systems, respectively. \( V_{C}^{(SF,DF)} \) and \( W_{C}^{(SF,DF)} \) denote the real and imaginary components of the central part of folding model potential, while \( V_{LS}^{(SF)} \) and \( W_{LS}^{(SF)} \) are the real and imaginary ones of the spin-orbit part of the single-folding model.

3. Results

Here, we introduce the renormalization factor, \( N_W \), for the imaginary part before the folding model potential is applied to the analysis of the proton and heavy-ion scatterings, as

\[ U_{\text{OMP}}^{(NA)} = V_{C}^{(SF)} + iN_WW_{C}^{(SF)} + (V_{LS}^{(SF)} + iN_WW_{LS}^{(SF)})\ell \cdot \sigma, \]

\[ U_{\text{OMP}}^{(AA)} = V_{C}^{(DF)} + iN_WW_{C}^{(DF)}. \]

We fix the factor \( N_W \) so that the renormalized folding potential reproduces the experimental data of proton-nucleus total reaction cross sections in the same as Ref. [6]. On the other hand, in the case of nucleus-nucleus systems, we adjust the renormalization factor so as to attain optimum fits to the experimental data for elastic-scattering cross sections because of lack of the experimental data.

![Figure 1](image_url)

**Figure 1.** Elastic cross section and analyzing power of incident proton by the \(^{12}\text{C}\) target nucleus at \( E_p = 122 \text{ MeV} \). The experimental data is taken from Ref. [19].
data. The results of the analyzing power with CEG07b which include TBF show apparently better agreement with the experimental data at forward angles than that with CEG07a. This result is a clear indication of the important role of TBF in the proton elastic scattering.

It should however be noted that the large effect of TBF on the analyzing power is not due to the change of the spin-orbit part of folding potential with the inclusion of TBF but mainly due to the change of the real-central component of the folding potential, the strength of which is most strongly affected by the inclusion of TBF, particularly the TBR contribution. In fact, a “modified CEG07b” potential in which only the real-central component is artificially replaced by that obtained by the CEG07a interaction produces the analyzing powers which are very close to the CEG07a results. One should note that the analyzing power is composed of the product of vector amplitude which is mainly governed by the spin-orbit potential and the scalar amplitude which is mainly governed by the central potential and, in the present case, the large change of the real-central component of folding potential with the inclusion of TBF is the main source of the drastic improvement of the analyzing power at forward angles.

![Figure 2. Folding model potential for the $^{16}$O + $^{16}$O system at $E/A = 70$ MeV.](image)

Figures 2 and 3 show the real and imaginary parts of the folding potential and the $^{16}$O + $^{16}$O elastic scattering cross sections calculated with CEG07a and CEG07b at $E/A = 70$ MeV. The solid and dotted curves are the results with the use of CEG07b and CEG07a interactions. Here, we take $N_W$ to be 0.8 so that the solid curve (with the TBF effect) gives an optimum fit to the data. The solid curve with the TBF effect well reproduce the experimental data up to backward angles, while the dotted curve with CEG07a (without the TBF) overshoots the experimental data at middle and backward angles reflecting the too deep strength of the real part of the folding model potential. It should be emphasized that no reasonable fit to the data is obtained by the folding model potential with CEG07a (without the TBF) no matter how the imaginary part of the FMP is modified, for example by changing the value of $N_W$ [11]. Thus, the large difference between the solid and dotted curves in Fig. 3 clearly shows an evidence of the decisive role of the TBF on elastic scattering of the $^{16}$O + $^{16}$O system.

Figures 4 and 5 show the elastic-scattering cross sections of incident $^{16}$O particle for the $^{12}$C, $^{28}$Si and $^{40}$Ca targets at $E/A = 93.9$ MeV and for the $^{12}$C + $^{12}$C system at $E/A = 135$ MeV, ...
Figure 4. Elastic cross section of incident $^{16}\text{O}$ nucleus by the several targets at $E/A = 93.9$ MeV. The experimental data is taken from Ref. [21]

respectively. The dotted and solid curves are the results with CEG07a and CEG07b, respectively. The CEG07b well reproduces the elastic cross sections up to the backward angles. The effect of the TBF is clearly seen in the cross sections as in the case of the $^{16}\text{O} + ^{16}\text{O}$ scattering shown in Figs. 3. For the $^{28}\text{Si}$ target, the fit to the experimental data is not necessarily perfect at large angles. It may be related to the fact that the $^{28}\text{Si}$ nucleus presents a slightly stronger absorption ($N_W = 0.9$) to the incident $^{16}\text{O}$ nucleus compared with other target nuclei ($N_W \approx 0.75$). Since the $^{28}\text{Si}$ nucleus is known to be a very deformed nucleus that shows a typical rotational band in the excitation spectrum, it may be reasonable to expect that an additional absorption should be induced dynamically by collective excitation of the $^{28}\text{Si}$ nucleus in the collision with $^{16}\text{O}$. This kind of dynamical effect may not be represented by the imaginary part of the $G$-matrix interaction evaluated in the nuclear matter. This would be one of the reason of a slightly larger value of the optimum $N_W$ for the $^{28}\text{Si}$ target ($N_W = 0.9$) compared with that for other targets ($N_W \approx 0.75$). In fact, we have confirmed that a better fit up to the backward angles is obtained by a coupled-channel (CC) calculation based on the present FMP potential in the case of the $^{28}\text{Si}$ target with a smaller value of $N_W$.

4. Summary and conclusion

In this paper, we have successfully applied the new complex $G$-matrix, CEG07, to both proton-nucleus and nucleus-nucleus elastic scattering at intermediate energies through the framework of the microscopic single- and double-folding models, respectively. We have revealed an essential importance of the repulsive component of the three-body force (TBF) in understanding the elastic scattering of finite nuclear systems. The repulsive TBF is believed to play an essential role to make the saturation curve of nuclear matter in high-density region realistic and consistent with the upper-limit of the mass of neutron stars.

For colliding finite nuclear systems, the repulsive TBF strongly reduces the strength of the
real part of calculated folding potentials at short distances due to the high local densities. Its effect is, however, not so crucial in the case of proton-nucleus scattering because the local density does not exceed the saturation density even at short distances and its effect is only visible in the calculated vector analyzing powers. On the other hand, in the case of nucleus-nucleus systems, the local density evaluated by the frozen-density prescription [11] exceeds the saturation value and reaches as large as twice its value when two nuclear systems fully overlap to each other. This leads to a drastic reduction of the real potential strength at short distances due to the repulsive TBF contribution at high-density region. In fact, we have shown that the calculated folding potentials with the TBF effects (i.e. with CEG07b) perfectly reproduce all the experimental data for nucleus-nucleus elastic scattering investigated here and that no reasonable fit to the data cannot be obtained without introducing the TBF contribution (particularly the TBR one).

These results imply that we now have a reliable tool to extract the realistic saturation curve of infinite nuclear matter such as those in neutron stars, particularly in high-density region, in terms of the elastic scattering of finite nuclear systems in laboratorys on the earth.

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