Neutron Stars in a Chiral Model with Finite Temperature

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Abstract. Neutron star matter is investigated in a hadronic chiral model approach using the lowest flavor-SU(3) multiplets for baryons and mesons. The parameters are determined to yield consistent results for saturated nuclear matter as well as for finite nuclei. The influence of baryonic resonances is discussed. The global properties of a neutron star such as its mass and radius are determined. Proto-neutron star properties are studied by taking into account trapped neutrinos, temperature and entropy effects.
1. Introduction

Although the early stages of the life of a neutron star, called proto-neutron star, last for just a few seconds, their properties have consequences that dictate the properties of the neutron star many years after its formation. For example, the baryon number of a proto-neutron star is extremely important because it is a limit for the baryon mass of the neutron star it will form, since it is known that when the maximum mass predicted by the Einstein’s equations of General Relativity is exceeded the star will collapse into a black hole [1]. During the life of the proto-neutron star, all the electron neutrinos trapped during the supernova explosion escape carrying thermal energy out of the stellar system so that the temperature of the star decreases very quickly during this process. Luckily the time the changes in the composition of the star takes to occur is more than three orders of magnitude bigger than the dynamical timescale of the readjustment of pressure and gravity forces, so that the system can still be treated in a quasistatic approximation.

An efficient way to describe infinite nuclear matter with hadronic degrees of freedom is through the use of effective theories because they allow us to work in a specific energy scale, practically ignoring other degrees of freedom of the system. Since not much is known about matter at very high densities, the results are first compared to nuclear matter properties in order to constrain the values of the coupling constants of the model and, as a second step, the macroscopic properties of the stars (such as mass and radius) are used to select the most acceptable models.

The matter inside neutron stars has densities up to several times the nuclear saturation density; at this point the scalar condensates responsible for chiral symmetry breaking of vacuum are strongly reduced and chirality is largely restored therefore. Based on this feature, we conclude that it is important to have a chiral invariant lagrangian density [2]. It is also important to analyze in which way chiral symmetry restoration occurs.

2. The Chiral Model

Because of the high densities nuclear matter inside neutron stars can reach it is possible to find not only nucleons, but also hyperons ($\Lambda$, $\Sigma$, $\Xi$) and resonances ($\Delta$, $\Sigma^*$, $\Xi^*$, $\Omega$) in the core of the star. This system can be described by a nonlinear chiral model in the mean-field approximation. Besides the kinetic energy part, the Lagrangian density contains terms that describe the interaction between baryons and scalar mesons and between baryon and vector mesons, self interactions of scalar and vector mesons and a term that breaks chiral symmetry explicitly, which is responsible for the pseudo-scalar mesons masses. Besides the usual mesons ($\sigma$, $\delta$, $\zeta$, $\omega$, $\phi$, $\rho$), the dilaton field ($\chi$), acting as the effective gluon condensate, is also included. Electrons and muons are considered for charge neutrality. Therefore, the Lagrangian density of our model is:

$$L_{MFT} = L_{Kin} + L_{Bscal} + L_{Bvec} + L_{scal} + L_{vec} + L_{SB},$$

(1)
\[ L_{Bscal} + L_{Bvec} = - \sum_i \bar{\psi}_i [g_i \omega \gamma_0 \omega + g_i \phi \gamma_0 \phi + g_i \rho \gamma_0 \tau_3 \rho + m_i^*] \psi_i, \]  
\text{(2)}

\[ L_{vec} = - \frac{1}{2} (m_\omega^2 \omega^2 + m_\rho^2 \rho^2) \frac{\chi^2}{\chi_0^2} - \frac{1}{2} m_\phi^2 \phi^2 \frac{\chi^2}{\chi_0^2} - g_4 \left[ \omega^4 + 6 \rho^2 \omega^2 + \rho^4 + 2 \phi^4 \right], \]  
\text{(3)}

\[ L_{scal} = \frac{1}{2} k_0 \chi^2 (\sigma^2 + \zeta^2 + \delta^2) - k_1 (\sigma^2 + \zeta^2 + \delta^2)^2 \]  
\[ - k_2 \left( \frac{\sigma^4}{2} + \frac{\delta^4}{2} + 3 \sigma^2 \delta^2 + \zeta^4 \right) - k_3 \chi (\sigma^2 - \delta^2) \zeta \]  
\[ + k_4 \chi^4 + \frac{1}{4} \chi^4 \ln \frac{\chi^4}{\chi_0^4} - \epsilon \chi^4 \ln \frac{(\sigma^2 - \delta^2) \zeta}{\sigma_0^2 \delta_0}. \]  
\text{(4)}

\[ L_{SB} = \left( \frac{\chi}{\chi_0} \right)^2 \left[ m_{\pi}^2 f_{\pi} \sigma + \left( \sqrt{2} m_{\pi}^2 f_{\pi} - \frac{1}{2} m_{\pi}^2 f_{\pi} \right) \right]. \]  
\text{(5)}

The baryon masses are entirely generated by the scalar fields except for a small explicit mass term \( \delta m \). The effective masses decrease at high densities, since the scalar fields tend to zero in this limit; so, the effective masses reproduce the measured baryon masses at low densities:

\[ m^* = g_{iso} \sigma + g_{iso} \delta + g_{iso} \zeta + g_{iso} \chi + \delta m. \]  
\text{(6)}

### 3. Results

At small densities, the star contains only neutrons. When density increases towards the center of the star, other particles appear. First, protons and electrons are populated at the same rate in order to keep charge neutrality. At higher densities, the charge neutrality must be achieved taking into account the presence of hyperons (\( \Lambda, \Sigma \) and \( \Xi \)) in the system (figure [1]). The appearance of these particles can also be seen in the compressibility function plot (figure [2]), first calculated in [3]. The compressibility function curve exhibits a bump at each density when a new hyperon species is populated; that is because compressibility, being a second order derivative, is a measure of the rate of increase of energy as a function of density:

\[ K = 9 \frac{dP}{d\rho} = 9 \rho \frac{d^2 \epsilon}{d\rho^2}. \]  
\text{(7)}

The negative values the compressibility function assumes for very small densities are a consequence of the liquid-gas phase transition. It can also be seen that the mass that the star can hold against gravitational collapse is larger for systems composed by higher compressibility nuclear matter, as shown in figure [3] (here, besides the model applied for the interior of the star, an inner crust, an outer crust and an atmosphere are also considered). In comparison with a system composed of nucleons and electrons solely, the maximum star mass predicted when muons and the whole baryon octet are included is smaller because of the new degrees of freedom available. The effect of the
resonances is the same, but in this case the star mass predicted is so small that it can not describe the very massive stars that have been observed lately.

For proto-neutron stars, the maximum mass increases with temperature up to a certain limit (40 MeV). After that the layer formed by electron-positron pairs that surround the star becomes too big, which requires a more careful treatment.

Up to this point of our analysis, the whole star is considered to be at the same constant temperature, what is unrealistic since it is known that the stars are hotter in the center. A more natural approach [4] would be to consider a star with constant entropy (figure 4). In this case, in an early stage (S/baryon=2) the temperature can reach 60 MeV and in a second stage (S/baryon=1) the temperature can reach 30 MeV in the center, but at all stages the star has got a low temperature on the border. During the evolution of the proto-neutron star, both temperature and entropy decrease as the neutrinos leave the star, so the lepton number, defined as

\[ Y_l = \frac{\rho_e + \rho_{\nu_e}}{\rho_B}, \]  

(8)
can be expected to decrease as well. This feature is shown in figure 3, where it can be seen that the maximum mass of the star decreases with time. There are two reasons for that: the first is that higher entropy implies higher temperature at any density and the second is that higher lepton number implies more negative charge, and in this case there is no need of many hyperons in order to achieve charge neutrality (and the presence of hyperons tend to reduce the maximum mass of the star). Even for the SU(2) case, which contains no hyperons, the effect is the same. This fact demonstrates the relevance of thermal effects in this model.

4. Conclusion

The chosen set of parameters allows us to model a massive neutron star which contains the complete octet of baryons. The maximum mass predicted by this model is $M = 1.86M_\odot$ while the heaviest star observed has a mass of $M = 2.1^{+0.4}_{-0.5}M_\odot$ [5].

The description of proto-neutron stars is done by fixing entropy and lepton number,
and allowing higher temperatures in the center of the star. At a first moment, right after the supernova explosion, approximately described by $S = 2$ and $Y_l = 0.4$ the star contains many neutrinos. After several seconds a considerable fraction of these neutrinos escapes carrying thermal energy out of the star; as a consequence, the maximum mass of the star decreases about 4%. In a final stage the cold star without neutrinos has a maximum mass about 6% smaller than in the initial case. This shows the influence both temperature and the amount of leptons have in the binding effects that determine the mass of a proto-neutron star.

5. References

[1] T. Takatsuka, Nucl. Phys. A 588, 365 (1995).
[2] S. Schramm, Phys. Lett. B 560, 164 (2003) [arXiv:nucl-th/0210053].
[3] V. A. Dexheimer, C. A. Z. Vasconcellos and B. E. J. Bodmann, Int. J. Mod. Phys. D 16, 269 (2007).
[4] D. Gondek, P. Haensel and J. L. Zdunik, Astron. Astrophys. 325, 217 (1997) [arXiv:astro-ph/9705157].
[5] D. J. Nice, E. M. Splaver, I. H. Stairs, O. Loehmer, A. Jessner, M. Kramer and J. M. Cordes, Astrophys. J. 634, 1242 (2005) [arXiv:astro-ph/0508050].