Three-body Coulomb breakup of $^{11}\text{Li}$ in the complex scaling method

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Coulomb breakup strengths of $^{11}\text{Li}$ into a three-body $^{9}\text{Li}+n+n$ system are studied in the complex scaling method. We decompose the transition strengths into the contributions from three-body resonances, two-body $^{9}\text{Li+n+n}$ and three-body $^{9}\text{Li+n+n+n}$ continuum states. In the calculated results, we cannot find the dipole resonances with a sharp decay width in $^{11}\text{Li}$. There is a low energy enhancement in the breakup strength, which is produced by both the two- and three-body continuum states. The enhancement given by the three-body continuum states is found to have a strong connection to the halo structure of $^{11}\text{Li}$. The calculated breakup strength distribution is compared with the experimental data from MSU, RIKEN and GSI.

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Studies of unstable nuclei have obtained much attention with the development of radioactive beams. The $^{11}\text{Li}$ nucleus is known as a typical Borromean system, in which the $^{9}\text{Li}+n$ and $n+n$ subsystems do not have any bound states, but the total $^{9}\text{Li}+n+n$ system has a bound state. This Borromean mechanism is considered to play an important role in the formation of a halo, but it is not yet fully understood. The observed large matter radius of $^{11}\text{Li}$, which is an evidence of the halo, suggests a large mixing of the $\left(1s_{1/2}\right)^{2}$ neutron component in addition to the $\left(0p_{1/2}\right)^{2}$ one. Another interesting problem related to the halo structure of $^{11}\text{Li}$ is a characteristic property of the excitation mode. For the excited states of $^{11}\text{Li}$, the so-called soft dipole resonance is expected in the low-energy region. In the shell model picture, the major component of the soft dipole resonance is described as $\left(1s_{1/2}\right)\left(0p_{1/2}\right)$. Thus, the behavior of the $1s$- and $0p$-orbit of valence neutrons is very crucial to understand the halo structure and the excited states in $^{11}\text{Li}$.

Experimentally, measurements of the Coulomb breakup strength distributions of $^{11}\text{Li}$ have been carried out by three groups at MSU, RIKEN and GSI. The low energy enhancement of the strength seems to indicate the existence of the soft dipole resonance, although the shapes of distributions obtained by the three experiments at different incident energies do not coincide with each other. In addition to these measurements, observations of the two-body correlation provide us with a key to discuss the mechanism of the breakup reaction. The measured invariant mass spectrum of $^{9}\text{Li}+n$ shows the low energy enhancement. This result implies the existence of the low-lying $1s$-orbit near the $^{9}\text{Li}+n$ threshold energy in $^{10}\text{Li}$. Such a $1s$-orbit in the $^{9}\text{Li}+n$ system is expected to provide “a continuum structure” in the three-body breakup reaction of $^{11}\text{Li}$. On the other hand, in the case of the two-body breakup reaction of $^{11}\text{Be}$, the low energy enhancement observed in the strength is understood as “a continuum response” of a large low-momentum component in the halo structure of the ground state. In order to understand the observed enhancement of the $^{11}\text{Li}$ breakup strength, therefore, we must consider the continuum structure of the $^{10}\text{Li}+n$ binary component and the continuum response of $^{9}\text{Li}+n+n$ in addition to the three-body resonance.

Theoretically, many methods such as the Faddeev method, the hyperspherical harmonics approach and sophisticated variational methods have been developed to solve the Borromean systems. However, there is a discrepancy between some theoretical results for the soft dipole resonances in $^{11}\text{Li}$. Garrido et al. calculated the dipole strength distribution, and predicted at least three dipole resonances, but we did not obtain any resonant solution with a sharp width ($\Gamma/2 < E_r$) in the variational method. This discrepancy is considered to come from a difference on the $^{9}\text{Li}-n$ potential. The dipole strength distribution calculated by Garrido et al. does not agree with the observed strengths within the experimental ambiguity. The peak energy is slightly lower and the width is much sharper than the experimental one. Thus, it is strongly desirable to obtain detailed information about the $^{9}\text{Li}-n$ potential, the excited resonant states and the continuum responses through analyses of the Coulomb breakup reaction of $^{11}\text{Li}$.

For this purpose, we have been developing the applicability of the complex scaling method (CSM) which has recently received much attention for finding three-body resonances. It is a big advantage of CSM that for an unbound system conduces to the separation not only between resonances and continuum states but

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also between different kinds of continuum states starting from different thresholds\(^{14}\). This advantage of CSM is exploited in the calculation of transition strengths of unbound states beyond the two-body systems. We showed a successful result for the three-body Coulomb breakup reaction of a simpler two-neutron halo nucleus \(^{6}\text{He}\)\(^{14}\). The results for \(^{6}\text{He}\) are summarized as (i) no resonance peak corresponding to the soft dipole mode is obtained, and (ii) the \(^{6}\text{He}(3/2^-) + n\) binary component dominates the \(E1\) strength distribution and the responses of the other components, such as three-body continuum states of \(^{4}\text{He}+n+n\), are very small.

In this letter, we extend this method to \(^{11}\text{Li}\) and investigate the continuum structures and responses through the Coulomb breakup reaction. We briefly explain an extended \(^{9}\text{Li}+n+n\) model for \(^{11}\text{Li}\), and report the results of the Coulomb breakup strength distributions comparing with experimental data. From the obtained results, we discuss the mechanism of the breakup processes.

We describe \(^{11}\text{Li}\) with an extended \(^{9}\text{Li}+n+n\) three-body model\(^{15}\). The Hamiltonian of this model is given in the orthogonality condition model\(^{16}\) as follows:

\[
H^{(11}\text{Li}) = H^{(9}\text{Li}) + \sum_{i=1}^{3} t_i - T_G + \sum_{i=1}^{2} V_{c,n,i} + V_{nn} + \lambda_{PF} |\phi_{PF}\rangle \langle \phi_{PF}|,
\]

where \(H^{(9}\text{Li}\rangle\), \(t_i\) and \(T_G\) are the internal Hamiltonian of \(^{9}\text{Li}\), the kinetic energy of each cluster and the center-of-mass of the three-body system, respectively. The \(^{9}\text{Li}-n\) potential, \(V_{c,n}\), is given by a folding-type one with MHN interaction\(^{15}\). For the potential \(V_{nn}\) for two valence neutrons, the Minnesota potential\(^{18}\) is used with parameter \(u=0.95\). The last term in Eq. (1) is a projection operator to remove the Pauli forbidden (PF) states from the \(^{9}\text{Li}-n\) relative motion\(^{15}\). The definition of the PF states is also given in Ref. \(^{15}\), where the value of \(\lambda_{PF}\) is taken as 10 MeV in the present calculation.

The folding-type \(^{9}\text{Li}-n\) potential was originally constructed so as to produce energy splittings in the \(^{10}\text{Li}\) spectra, such as \(1^- - 2^+\) (for the \(p_{1/2}\)-neutron) and \(1^- - 2^-\) (for the \(s_{1/2}\)-neutron) due to the coupling between spins of the valence neutron and \(^{9}\text{Li}(3/2)\)\(^{14}\). However, in the study of \(^{11}\text{Li}\)\(^{15}\), we discussed to add a phenomenological tail potential to the original folding-type potential to improve the behaviour of the tail part of the \(^{9}\text{Li}-n\) potential. The behavior of the \(s\)-wave state near the threshold is very sensitive to the tail part of the potential due to the spatial extension of the wave function. The tail potential also plays an important role in lowering the energy of the \((1s_{1/2})^2\) component with respect to that of the \((0p_{1/2})^2\)-component in \(^{11}\text{Li}(9\text{Li}+n+n)\). Then, we have two parameters in the \(^{9}\text{Li}-n\) potential; one is the \(\delta\) to change the strength of the second range of the folding part, and another the strength of phenomenological tail potential given by a Yukawa-form\(^{15}\). They are determined so as to reproduce the \(1^+\) resonance of \(^{10}\text{Li}\) at 0.42 MeV\(^{20}\) and the \(s\)-wave property which is a virtual state showing a large negative scattering length.

The wave function of \(^{11}\text{Li}\) is given as

\[
\Psi^{J}(^{11}\text{Li}) = \sum_{i} \mathcal{A} \left\{ \phi_{J/2}^{+}\left(\mathcal{C}_{i}\right), \chi_{(nn)}^{j}\right\},
\]

where \(\mathcal{A}\) and \(\mathcal{C}_{i}\) are V-type and T-type coordinate sets, respectively. The radial part of the relative wave function is expanded with a finite number of Gaussian basis functions centered at the origin.

By employing \(C_1 = (0p_{3/2})_v^2\) and \(C_2 = (0p_{3/2})_c^2(0p_{1/2})_v^2\) for \(p\)-shell neutrons in \(^{9}\text{Li}\) (\(N_c\) is 2), we solve a coupled-channel \(^{9}\text{Li}+n+n\) three-body problem. We use the MHN interaction\(^{17}\) to calculate the neutron pairing correlation in \(^{9}\text{Li}\), which leads to the \(|a_2|^2 = 15\%\) of the pairing excitation\(^{15}\). When the valence neutrons approach to \(^{9}\text{Li}\), the coupling between the motion of the valence neutrons and the pairing correlation in \(^{9}\text{Li}\) becomes stronger. It was discussed that this dynamical coupling in \(^{10}\text{Li}(9\text{Li}+n)\) provides the so-called pairing-blocking effect\(^{22}\) which explains the lowering of \(1s\)-orbits in \(^{10}\text{Li}\). For \(^{11}\text{Li}\), we adjust the \((0p)^2-(1s)^2\) pairing coupling between valence neutrons to reproduce the observed binding energy of \(^{11}\text{Li}\) (0.31 MeV)\(^{22}\).

In the analysis of excited states in \(^{11}\text{Li}\), we prepare the three types of the \(^{11}\text{Li}\) wave function; P-1, P-2 and P-3, which are characterized by the \((1s_{1/2})^2\) probability in the ground state. In Table \(I\) we list the properties of the prepared \(^{10}\text{Li}\) and \(^{11}\text{Li}\) wave functions. The \(s\)-orbit properties are different among them, which affect on the scattering length of the \(^{10}\text{Li}\) \(s\)-state and the size of the halo structure of the \(^{11}\text{Li}\) ground state. We also pay attention to the effects of the halo structure on the breakup strength. They can be seen from the responses of resonances and continuum states in \(^{11}\text{Li}\).

We calculate the unbound states of \(^{11}\text{Li}\) applying CSM to the extended \(^{9}\text{Li}+n+n\) system, where the relative co-
TABLE I: Results for the three types of wave functions of the present model; (upper) Scattering lengths $a_s$ and energies $E$ of the virtual state $(2^-,1^+)$ of $^{10}\text{Li}$, (lower) $(1s_{1/2})^2$ probability $P$ and matter radius $R_m$ of the $^{11}\text{Li}$ ground state.

| $^{10}\text{Li}$ | P-1 | P-2 | P-3 |
|-----------------|-----|-----|-----|
| $a_s(2^-)$ [fm] | -12.7 | -17.0 | -21.7 |
| $E(2^-)$ [MeV]  | -0.05 | -0.03 | -0.02 |
| $a_s(1^-)$ [fm] | -6.5  | -8.6  | -10.7 |
| $E(1^-)$ [MeV]  | -0.08 | -0.06 | -0.05 |

| $^{11}\text{Li}$ | P-1 | P-2 | P-3 | exp. |
|-----------------|-----|-----|-----|-----|
| $P[(1s_{1/2})^2]$ [%] | 21.0 | 29.4 | 38.8 | 3.12±0.16$^a$ |
| $R_m$ [fm] | 3.33 | 3.58 | 3.85 | 3.53±0.06$^b$ |

*Reference$^{[22]}$, $^a$Reference$^{[23]}$.

The ordinates between $^9\text{Li}$ and the two neutrons are transformed with a scaling angle $\theta$ as

$$\xi_{V,T} \rightarrow \xi_{V,T} e^{i\theta}. \quad (4)$$

The momenta corresponding to the asymptotic channel $\alpha$ of $^{9}\text{Li}+n+n$ and $^{10}\text{Li}(\nu)+n$ are also transformed as

$$k_{\alpha} \rightarrow k_{\alpha} e^{-i\theta}. \quad (5)$$

Here, we notice that $2\theta$ corresponds to a rotation angle of the cuts in the Riemann sheets of complex energies, and the angular part of the wave function does not change in CSM. In Fig. 1 we show a schematic energy eigenvalue distribution of the complex-scaled $^{9}\text{Li}+n+n$ system governed by the ABC-theorem$^{[13]}$. When $\theta = 0$, (unbound) scattering states are obtained on the real energy axis, which includes all components of resonances and continuum states. For a finite value of $\theta$, the continuum states are obtained on the Riemann cuts rotated down by $2\theta$, and hereafter we call these rotated continuum states as the continuum ones. When we take a large $\theta$, as shown in Fig. 1 in addition to the three-body bound states (3BB), we obtain (i) discrete three-body resonances (3BR), (ii) two-body continuum states (2BC) of $^{10}\text{Li}(1^+, 2^+)+n$, and (iii) three-body continuum states (3BC) of $^{9}\text{Li}+n+n$, which are decomposed from the three-body scattering states. Two-body continuum states of $^{10}\text{Li}+n+n$ are expressed by the two straight lines whose origins are resonance positions of $^{10}\text{Li}(1^+, 2^+)$. Since the virtual states of $^{10}\text{Li}(1^-, 2^-)$ and of $2n$ cannot be located in CSM due to the limitation of the scaling angle $\theta$, the channels of $^{10}\text{Li}(1^-, 2^-)+n$ and $^{9}\text{Li}+2n$ components are included in 3BC of $^{9}\text{Li}+n+n$.

Using the Green’s function, the strength function for the operator $O_\lambda$ with rank $\lambda$ is expressed as

$$S_\lambda(E) = -\frac{1}{\pi} \text{Im} \left[ \int d\xi d\xi' \bar{\Psi}_\lambda^\dagger(\xi) \hat{O}_\lambda \mathcal{G}(E, \xi, \xi') \right]. \quad (6)$$

where $\Psi_i(\xi)$ is the initial wave function of $^{11}\text{Li}$. We apply the complex scaling to the right hand side of Eq. (6):

$$S_\lambda(E) = -\frac{1}{\pi} \text{Im} \left[ \int d\xi d\xi' \left[ \bar{\Psi}_\lambda^\dagger(\xi) \hat{O}_\lambda \mathcal{G}(E, \xi, \xi') \right] \times \hat{O}_\lambda^\dagger \Psi_i(\xi') \right]. \quad (7)$$

In this expansion, $E^\nu_\theta$ and $\Psi^\nu_\theta(\xi)$ ($\bar{\Psi}^\nu_\theta(\xi)$) are the energy eigenvalues and eigenfunctions (bi-orthogonal eigenfunctions$^{[14, 22, 27]}$) of the complex-scaled Hamiltonian $H(\theta)$, respectively. Therefore, summation and/or integration are taken over $\nu$ of the solutions of $H(\theta)$ including 3BR, 2BC of $^{10}\text{Li}(1^+, 2^+)+n$ and 3BC of $^{9}\text{Li}+n+n$ (There is no bound state except for the ground state).

Inserting the complex-scaled Green’s function in Eq. (7), we obtain the strength function decomposed into each component $S_{\lambda,\nu}(E)$ for the final state $\nu$ as

$$S_\lambda(E) = \sum_{\nu} S_{\lambda,\nu}(E). \quad (10)$$

$$S_{\lambda,\nu}(E) = -\frac{1}{\pi} \text{Im} \left[ \frac{\langle \Psi_i | (\hat{O}_\lambda)^\dagger \Psi^\nu_\theta \rangle \langle \bar{\Psi}^\nu_\theta | \hat{O}_\lambda \Psi_i \rangle}{E - E^\nu_\theta} \right]. \quad (11)$$

It is noted that the total strength $S_{\lambda}(E)$ is an observable and positive definite for any energy and independent of $\theta$. On the other hand, the partial strengths $S_{\lambda,\nu}(E)$ of resonance and continuum components are not necessarily positive definite, and in fact show sometimes
negative values. A detailed explanation is discussed in Refs. [14, 27]. Since the number of resonances obtained in the calculation depends on θ and jumps at a certain θ, $S_{1,ν}(E)$ has a discontinuity at some θ values [27].

In the present calculation, we solve the eigenvalue problem of the complex-scaled Hamiltonian with the hybrid-TV model, and the discretized approximation is adopted for continuum states. We use 25 Gaussian basis functions for one relative motion and set their maximum range about 40 fm. We numerically checked the reliability of the discretized representation of continuum states and the stability of the calculated strengths by changing the parameters of the basis functions.

In Fig. 2, we show the eigenvalue distributions of $1/2^+$, $3/2^+$ and $5/2^+$ states in a complex energy plane at $θ = 28$ degrees. These are dipole excited states ($j = 1$) of $^{11}$Li from the ground state ($J^* = 3/2^−$). We obtain all eigenvalues along three lines of rotated Riemann cuts corresponding to two 2BC of $^{10}$Li$(1^+,2^+)−n$ and one 3BC of $^{9}$Li$+n−n$ as discussed in Fig. 1. There is no dipole resonance which is located between the real energy axis and the rotated continuum lines of the excited states. In other words, we cannot find any resonances with a sharp width at least $(Γ/2E_r < tan−1 2θ; θ = 28°)$. This result means that the dipole strengths are exhausted by continuum states of $^{11}$Li. Since the wave function of s-wave valence neutrons is neutrally extended, dipole resonances including s-wave components in $^{11}$Li may tend to decay easily, and so resonances have large decay widths, whose poles are located below the rotated continuum states. The effect of such resonances is included in the continuum spectra in this calculation.

Using the solutions of the continuum spectra of the dipole excited states, we calculate the dipole transition from the ground state. In Fig. 3(a), we show the results of the dipole strength functions summing up the contributions from the $1/2^+$, $3/2^+$ and $5/2^+$ states for the three types of the ground state wave function. The energy is measured from the $^{9}$Li$+n−n$ threshold. It is found that the strengths show the low energy enhancement whose height is sensitive to the $(1s_{1/2})^2$ probability of the $^{11}$Li ground state. This enhancement is interpreted as a threshold effect coming from the continuum states and reflects the halo structure of $^{11}$Li. We compare our results to the experimental data of MSU [14]. The position of the enhancement almost agrees with the data, but a disagreement of the shape is seen in the strength above the energy 1 MeV. We also compare our results with the calculation (denoted as DR in Fig. 3(a)) by Garrido et al. [10].

In Figs. 3(b) and 3(c), we derive the cross sections of the $^{11}$Li breakup by multiplying the transition strength and the virtual photon number in the equivalent photon method [28], where the target is Pb in both cases. We can see a good agreement with the data of RIKEN [2] for the P-2 wave function with the $(1s_{1/2})^2$ probability being around 30%. However, the magnitude of the cross section is not observed. It would be desirable to determine its magnitude. For the data of GSI [3], the P-1 wave function with the $(1s_{1/2})^2$ probability being around 20% gives a good result. However, the experimental error bars are still large. Further experimental data with high resolu-

FIG. 2: Energy eigenvalues of three dipole excited states ($1/2^+$, $3/2^+$, $5/2^+$) of $^{11}$Li with $θ$=28 degree in CSM. Squares and triangles indicate 2BC of $^{10}$Li$(1^+,2^+)−n$ and $^{10}$Li$(2^+)−n$, respectively. Circles indicate 3BC of $^{9}$Li$+n−n$.

FIG. 3: Calculated dipole strengths and cross sections of $^{11}$Li in comparison to the theory (denoted as DR, Fig. 5 (a) of Ref. [10]) in (a) and the experimental data (a) [3], (b) [2], and (c) [4], where we take into account the convolution.
tion and statistics would be required.

In Fig. 4 we show the result of the separation of the $E1$ transition strength of $^{11}$Li into two-body and three-body continuum components for the three types of the $(1s_{1/2})^2$ probability in the ground state. It is found that the two-body continuum component of $^{10}$Li$(1^+)+n$ shows a low energy enhancement in each panel, whose peak position is just above the two-body threshold (0.42 MeV) of $^{10}$Li$(1^+)+n$. Another two-body continuum component of $^{10}$Li$(2^+)+n$ shows a broader structure because a larger decay width of the $2^+$ state than that of the $1^+$ state in $^{10}$Li broadens the strength. The transition strengths into the $^{10}$Li$(1^+,2^+)+n$ 2BC correspond to the following two kinds of physical situations: First one is that one of the valence neutrons of $(0p_{1/2})^2$ in the $^{11}$Li ground state is excited to a continuum state of $s$- or $d$-waves by the $E1$ external field, and a remaining $p$-orbital valence neutron forms resonances of $^{10}$Li with $^9$Li. This situation is similar to the case of $^6$He breakup reactions because the $^6$He ground state is almost dominated by $(0p_{1/2})^2$ of two valence neutrons. Second one is that one of the valence neutrons of $(1s_{1/2})^2$ in the $^{11}$Li ground state is excited to a $p$-orbit and forms resonances of $^{10}$Li$(1^+,2^+)$ with $^9$Li, and a remaining $s$-orbital valence neutron becomes a continuum state. This is the characteristic situation of $^{11}$Li. The contribution of this process can be seen as the differences between the strengths of 2BC of P-1, P-2, P-3 in Fig. 4. The low energy enhancement in two-body continuum components is interpreted as a threshold effect of 2BC of the $^{10}$Li$(1^+,2^+)+(s$-wave neutron) channels.

From Fig. 4, it is also found that the contribution from 3BC of $^9$Li+n+n increases as the $(1s_{1/2})^2$ probability increases. The strength distribution of 3BC has a peak at the low energy region ($\sim$ 0.5 MeV), and slowly decreases with energies. At energies higher than about 2 MeV, the strength of 3BC becomes dominant. The difference between the 3BC and 2BC strength distributions may be understood from the level density of the continuum states. The level density of 3BC is widely distributed, because 3BC have two degrees of freedom of the relative motions, but 2BC has one degree of freedom.

With the increasing of the $(1s_{1/2})^2$ probability in the $^{11}$Li ground state, the magnitude of the strengths of 2BC increases, but the shapes do not change much. On the other hand, the strength of 3BC shows a sharper peak at a low energy, and its magnitude increases markedly with the $(1s_{1/2})^2$ probability in comparison to the case of 2BC. This result indicates that the structure of 3BC depends strongly on the halo structure of the ground state. To see it more clearly, we evaluate the integrated strengths up to 5 MeV for the three types of $(1s_{1/2})^2$ probabilities, which exhaust about 65% of the non-energy weighted sum rule value (NEW-SRV) as shown in Table II. We also list the ratios of the each component of 2BC and 3BC to the total integrated strength. It is found that the ratios of the contribution of 3BC increase proportionally with the $(1s_{1/2})^2$ probability.

These results mean that the components of 2BC and 3BC, contribute comparably in the Coulomb breakup reaction of $^{11}$Li, and that, in particular, 3BC depends strongly on the halo structure of the ground state. In fact, a large contribution of 3BC cannot be seen in the case of $^6$He breakup reactions. In the three-body model of $^6$He, the $(1s_{1/2})^2$ probability of two valence neutrons is 2.4% which is much smaller than that of $^{11}$Li. These results indicate that the mechanisms of the breakup reactions of $^6$He and $^{11}$Li are different, and this fact is caused by the different $(1s_{1/2})^2$ probabilities in $^6$He and $^{11}$Li.

![FIG. 4: Decomposition of the dipole transition strength of $^{11}$Li with the three types of the $(1s_{1/2})^2$ probability in the ground state, as described in the text.](image-url)

| Integral $[e^2 fm^2]$ | P-1 | P-2 | P-3 |
|------------------------|-----|-----|-----|
| NEW-SRV $[e^2 fm^2]$   | 2.215 | 2.856 | 3.593 |
| $^9$Li+n+n             | 0.392 | 0.522 | 0.658 |
| $^{10}$Li$(1^+)+n$     | 0.306 | 0.262 | 0.216 |
| $^{10}$Li$(2^+)+n$     | 0.302 | 0.216 | 0.126 |
there their ground states.

There may be broad resonances including virtual states which are located under the Riemann cuts of 2BC and 3BC calculated here. However, their effects are considered to produce no remarkable structure, although they are seen to give a considerable strength. In the present calculation, we cannot conclude how much they could contribute to the strength.

In summary, we investigate the three-body Coulomb breakup reaction of $^{11}\text{Li}$ employing the complex scaling method (CSM) to describe the three-body unbound states and the $^{3}\text{Li}+n+n$ three-body continuum states. Here, the strength of three-body continuum states include a contribution from the virtual states in $^{19}\text{Li}$, which we do not consider in this study. It is interesting to investigate the effect of virtual states on the breakup reaction in the future works.

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