ANISOTROPIC INTERMITTENCY OF MAGNETOHYDRODYNAMIC TURBULENCE

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ABSTRACT

A higher-order multiscale analysis of spatial anisotropy in inertial range magnetohydrodynamic turbulence is presented using measurements from the STEREO spacecraft in fast ambient solar wind. We show for the first time that, when measuring parallel to the local magnetic field direction, the full statistical signature of the magnetic and Elsässer field fluctuations is that of a non-Gaussian globally scale-invariant process. This is distinct from the classic multiexponent statistics observed when the local magnetic field is perpendicular to the flow direction. These observations are interpreted as evidence for the weakness, or absence, of a parallel magnetofluid turbulence energy cascade. As such, these results present strong observational constraints on the statistical nature of intermittency in turbulent plasmas.

Key words: magnetohydrodynamics (MHD) – plasmas – solar wind – turbulence

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1. INTRODUCTION

Turbulence is a universal fluid phenomenon that generates intermittent fluctuations (Bruno & Carbone 2013). The solar wind provides an ideal laboratory for the in situ study of plasma turbulence, wherein intermittent fluctuations have been analyzed in considerable detail. These have been linked to non-uniform plasma heating (Osman et al. 2011a, 2012b; Wu et al. 2013) and enhanced turbulent dissipation (Wan et al. 2012; Karimabadi et al. 2013; TenBarge & Howes 2013). In addition, there is evidence to suggest that increased alpha particle (Perrone et al. 2013), proton (Servidio et al. 2012), and electron (Haynes et al. 2014) temperature anisotropies are associated with intermittent structures. The same structures can cause particle velocity distribution functions to deviate from local thermal equilibrium (Greco et al. 2012) and have been preferentially found in plasma unstable to microinstabilities (Osman et al. 2012a). A subset of non-Gaussian intermittent structures correspond to active magnetic reconnection sites (Servidio et al. 2011) which can in turn generate fluctuations that exhibit the hallmarks of intermittency (Leonards et al. 2013). These spatial structures may also be related to trapping boundaries that delineate dropouts of energetic particle flux as seen in solar energetic particle data (Ruffolo et al. 2003). Indeed, recent work suggests that these structures contribute to the acceleration and transport of interplanetary superthermal particles (Tessein et al. 2013). Intermittent fluctuations are the reason why turbulence can enhance the transport of particles, heat, momentum, and current in laboratory plasmas. The intermittent structures in plasmas share striking similarities with fluctuations found in turbulent neutral fluids (Frisch 1995; Sreenivasan & Antonia 1997). Therefore, quantifying intermittency is central to understanding and interpreting a large body of observations in turbulent systems.

Intermittency lies at the heart of turbulence theory. The classical signatures of intermittency in both neutral fluid and magnetohydrodynamic (MHD) turbulence are a non-Gaussian probability distribution function (PDF) of fluctuations and multifractal scaling in the higher-order statistics (Bruno & Carbone 2013). However, non-fluid phenomenology such as the kinetic range of plasma turbulence can have a monofractal scaling (Kiyani et al. 2009). These different scaling types imply different underlying physics. We will quantify both the non-Gaussian behavior of fluctuations and their statistical scaling in the turbulent solar wind. Intermittency is related to the emergence of small-scale coherent structures that are responsible for enhanced dissipation. Hence, the most fundamental approach to the study of intermittency is to examine the dissipation rate PDF. However, the Kolmogorov refined similarity hypothesis (hereafter KRSH; Kolmogorov 1962; Obukhov 1962) allows local averages of the dissipation rate to be related to increments of the velocity field calculated on different spatial scales, $r$. The PDF of velocity increments is then linked to intermittency (Anselmet et al. 1984), where departures from a normal distribution occur on small spatial scales while large-scale features are uncorrelated and converge toward a Gaussian distribution. This non-Gaussian behavior is also observed in the turbulent solar wind magnetic field (Sorriso-Valvo et al. 1999). A method to quantify intermittency is based on computing a sequence of $m$th order moments of the magnetic or velocity field increments. For an increment scale $r$, the moments have power-law scalings, $\propto r^{\xi}$, where the exponents $\xi$ depend on the moment order $m$. Here the physical meaning lies in the sensitivity of higher-order moments to concentrations of dissipation and, from KRSH, to large increments. The behavior of these exponents is also connected to known fractal and multifractal models (Frisch et al. 1978; She & Leveque 1994; Politano & Pouquet 1995). This Letter presents novel observational results from a higher-order analysis that examines the statistical properties of MHD turbulence in the spatially anisotropic solar wind. We find for the first time that these statistical properties depend on the angle of the local magnetic field direction to the (radial) solar wind flow. This provides strong constraints on the physics and phenomenology of inertial range turbulence in collisionless plasmas.

The presence of a magnetic field in plasma turbulence breaks the isotropy found in hydrodynamics and orders the fluctuations (Horbury et al. 2005). In the solar wind, fluctuation components parallel and transverse to the background magnetic field display differences in dynamics and statistics (Chapman...
& Hnat 2007). However, this “variance” anisotropy is not reflected in the higher-order moments and both components display a multifractal intermittent scaling (Kiyani et al. 2013). The distribution of energy over the full three-dimensional (3D) space of wavevectors is also anisotropic (Oughton et al. 1994). This spatial anisotropy has been observed in second-order statistics such as the power spectral density (PSD; Osman & Horbury 2009b) and correlation function (Matthaeus et al. 1990; Osman & Horbury 2007, 2009a). It has also been found in third-order statistics (Osman et al. 2011b). A higher-order analysis of wavevector anisotropy would provide a direct test of theoretical predictions regarding the statistical nature of intermittency and, more broadly, the phenomenology of the turbulent cascade. However, an investigation into the wavevector anisotropy of intermittent fluctuations has not, to the best of our knowledge, been conducted.

2. ANALYSIS

We use 8 Hz magnetic field measurements from the IMPACT instrument (Acuña et al. 2008; Luhmann et al. 2008) and 1 minute resolution proton plasma data from the PLASTIC instrument (Galvin et al. 2008) on board the two STEREO spacecraft in the ecliptic. The solar wind intervals used here are all in high-speed streams and contain no sector crossings. These are listed in Table 1 and are identical to those used by Podesta (2009). It has been suggested (e.g., Chapman & Hnat 2007; Horbury et al. 2008) that a local scale-dependent mean magnetic field and associated scale-dependent fluctuations, rather than a large-scale global field (Matthaeus et al. 2012), should be used in anisotropy studies of plasma turbulence. Hence, we use the undecimated discrete wavelet transform method described in Kiyani et al. (2013) to decompose the magnetic field into a local scale-dependent background and fluctuations, \( B_t(t, f) \) and \( \delta B(t, f) \), where \( f \) explicitly shows the frequency or scale dependence. These fluctuations are binned according to the angle of the local magnetic field to the (radial) flow, \( \theta_{\text{VR}} \). Here we focus on fluctuations in the \( \theta_{\text{VR}} = 0^\circ - 10^\circ \) and \( 80^\circ - 90^\circ \) bins, which correspond to wavenumbers using Taylor’s hypothesis (Taylor 1938) that are, respectively, nearly field-parallel \( \delta B(k||) \) and nearly field-perpendicular \( \delta B(k\perp) \).

3. RESULTS

We present a detailed analysis of interval eight listed in Table 1, which is typical of the stationary fast solar wind intervals used in this study. The PSD is independent of the azimuthal angle about the local magnetic field for all relevant spacecraft frame frequencies (Podesta 2009). Hence, the spacecraft frame wavelet PSD depends only on the angle of the magnetic field to the flow direction:

\[
\text{PSD}(f, \theta_{\text{VR}}) = \frac{2A}{N} \sum_{j=1}^{N} \delta B^2(t_j, f, \theta_{\text{VR}}).
\]

where \( \delta B(t_j, f, \theta_{\text{VR}}) \) is the magnitude of the trace fluctuations at time \( t_j \) and frequency \( f \). \( A \) is the sampling time between consecutive measurements, and \( N \) is the sample size at each frequency. Figure 1 shows the PSD for two angular bins, \( \theta_{\text{VR}} = 0^\circ - 10^\circ \) and \( 80^\circ - 90^\circ \), which correspond to wavevectors roughly parallel and perpendicular to the local field, respectively. These are both well described by power laws. The power levels are lower and the spectral slope is steeper for \( \delta B(k||) \) compared to \( \delta B(k\perp) \), which is consistent with previous studies (e.g., Horbury et al. 2008; Podesta 2009). Hence, the statistical behavior of fluctuations in wavevectors at large angles to the magnetic field would have dominated all previous estimates of the inertial range intermittency, since these contain the most power. The dashed vertical lines define the typical range of timescales used in the higher-order analysis. However, this range of timescales is varied slightly for each higher-order analysis in order to minimize the errors associated with the computed scaling exponents.

In order to determine the higher-order scaling of fluctuations for different \( \theta_{\text{VR}} \), we compute the absolute moments of the magnetic field increments, \( \delta B(t, \tau) = B(t + \tau) - B(t) \). The \( m \)th order wavelet structure function is given by

\[
S^m(\tau, \theta_{\text{VR}}) = \frac{1}{N} \sum_{j=1}^{N} \left| \frac{\delta B(t_j, \tau, \theta_{\text{VR}})}{\sqrt{\tau}} \right|^m,
\]

where \( \tau = 2^i \Delta : i \in \{0, 1, 2, 3, \ldots\} \) is the dyadic timescale parameter related to the central frequency \( f \). Note that wavelets change the regular expressions for structure functions (Kiyani et al. 2013). The higher-order structure functions increasingly capture the more intermittent fluctuations. In hydrodynamics, these large fluctuations represent the spatial gradients responsible for dissipating the turbulent cascade energy. However, there is growing evidence to suggest these intermittent structures are also associated with non-uniform heating (Osman et al. 2011a, ...
order scaling of the magnetic field-parallel and -perpendicular turbulence (Frisch 1995) and solar wind turbulence at MHD multiexponent scaling, which is characteristic of hydrodynamic \( \theta \) (Kiyani et al. 2006). The \( \theta \) fluctuations in plasma turbulence.

Here the focus will be on the scaling behavior of structure functions, where scale invariance is indicated by

\[
S^m(\tau) \propto \tau^{\zeta(m)}
\]

and \( \zeta(m) \) are the scaling exponents. Figure 2 shows the scaling exponents for both the \( \theta_{VB} = 0°-10° \) and \( 80°-90° \) angular bins. When Equation (3) is satisfied, \( \zeta(2) \) is directly related to the spectral index \( \alpha \) by \( \zeta(2) = \alpha - 1 \) (Monin & Yaglom 1975). Here \( \alpha(\theta_{VB} = 0°-10°) = 2.05 \pm 0.14 \) and \( \alpha(\theta_{VB} = 80°-90°) = 1.58 \pm 0.04 \), which are consistent with the spectral indices obtained in Figure 1 from the slope of the PSD. The higher-order scaling of the magnetic field-parallel and -perpendicular fluctuations are distinct; this is a novel result. The \( \theta_{VB} = 80°-90° \) fluctuations have a nonlinear \( \zeta(m) \) that indicates a multifractal scaling, which is characteristic of hydrodynamic turbulence (Frisch 1995) and solar wind turbulence at MHD scales (Bruno & Carbone 2013). In contrast, the \( \theta_{VB} = 0°-10° \) fluctuations are characterized by a linear \( \zeta(m) = Hm \) with a single exponent \( H \), which indicates monoscaling. The errors on \( \zeta(m) \) shown in Figure 2 were obtained from the sum of the regression error when using Equation (3), and from variations in \( \zeta(m) \) that resulted from repeating the same regression over a subinterval of the original scaling range (Kiyani et al. 2006). The scaling range for the \( \theta_{VB} = 0°-10° \) and \( 80°-90° \) fluctuations are nearly identical, but were selected independently to minimize the errors on \( \zeta(m) \). In order to confirm the robustness of this result, the analysis was repeated for all nine intervals listed in Table 1 and the same \( \theta_{VB} \) dependent intermittency was obtained.

The statistical analysis is completed by examining scale-by-scale the PDF for the \( \theta_{VB} = 0°-10° \) fluctuations. A component of the trace magnetic field fluctuations is selected in order to show any symmetric or asymmetric behavior in the fluctuations. Here we use one of the components transverse to the local field, although the behavior is identical for all three vector components of the fluctuations. Since global scale invariance of the structure functions implies that the PDF of the increments at a scale \( \tau \) should collapse onto a unique scaling function \( P_\tau \), we use the self-affine scaling operation \( P_\tau(\delta B\tau^{-1}) = \sigma P(\delta B, \tau) \) to rescale the fluctuations by their standard deviation. Figure 3 shows PDFs corresponding to \( \tau \in \{2, 4, 8, 16, 32\} \) s that are rescaled and overlaid, where the central \( \tau \) is plotted in red and shows the associated errors on the PDFs. There is an excellent collapse onto a single curve, although the largest events in the tails of the distribution are not statistically well-sampled as indicated by the large errors, which is an unavoidable consequence of heavy-tailed distributions. In addition, a fitted Gaussian distribution illustrates the highly non-Gaussian nature of the PDF tails and reflects the presence of rare large amplitude fluctuations.

While the higher-order analysis has focused exclusively on magnetic field fluctuations, it is the total energy (magnetic and velocity) that is cascaded from large to small scales by plasma turbulence. Therefore, it is instructive to examine Elsässer fluctuations, \( \delta z^+ = \delta V \pm \delta B \), since dynamic couplings produce structure in both magnetic and velocity fields. Here the magnetic field has been normalized to Alfvén velocity units, \( \delta B / \sqrt{\mu_0 \mu_m \rho V_F} \), and the fluctuations have been sector rectified such that \( \delta z^+ \) is Sunward and \( \delta z^- \) is anti-Sunward. Figure 4 shows the scaling exponents for the anti-Sunward Elsässer variable in both the \( \theta_{VB} = 0°-10° \) and \( 80°-90° \) angular bins. The higher-order scaling of the fluctuations in both these bins is...
similar to those in Figure 2 for the magnetic field fluctuations. The $\theta_{m} = 80^\circ $–$90^\circ$ Elsässer fluctuations have a nonlinear $\zeta (m)$, which is typical of MHD scale solar wind turbulence. This behavior is also associated with the energy dissipation intensity being distributed on a fractal (Bruno & Carbone 2013). The $\theta_{m} = 0^\circ $–$10^\circ$ fluctuations have a linear $\zeta (m)$, which is characteristic of global scale invariance. In theories of turbulence, this scaling behavior is associated with the energy dissipation intensity being distributed on a fractal. This analysis was repeated for the Sunward and anti-Sunward Elsässer fluctuations in all nine intervals listed in Table 1 and the same $\theta_{m}$ dependent intermittent scaling was obtained. However, the Sunward fluctuations have greater associated errors since these are a minority and contain the least power (Gogoberidze et al. 2012).

4. DISCUSSION

We have presented the first direct observation that higher-order scaling of the magnetic and Elsässer field fluctuations depends on the angle of the local magnetic field direction to the (radial) flow. In fluctuations with wavevectors parallel to the local magnetic field direction, global scale invariance is a robust feature of inertial range collisionless plasma turbulence in the fast ambient solar wind. This is distinct from the multifractal scaling that is characteristic of neutral fluid turbulence and MHD fluctuations with wavevectors perpendicular to the local field. These properties must be included in any successful theory that attempts to explain inertial range intermittency.

A process that has multifractal properties generates fluctuations through a multiplicative sequence such as an energy cascade of eddies in turbulent flows, while monofractal processes generate fluctuations through additive sequences. Therefore, the solar wind MHD turbulence cascade proceeds from smaller to larger wavenumbers that are mainly perpendicular to the local magnetic field direction. The presence of monoscaling parallel to the local field indicates that the cascade in this direction does not proceed in the well-understood classic fluid turbulence manner. It could be that kinetic physics is important in the parallel cascade even on what is typically considered MHD scales, and thus behavior associated with kinetic range turbulence such as monoscaling (Kiyani et al. 2009) is observed. Alternatively, the monofractal scaling may be evidence for a weak turbulent cascade. This is consistent with incompressible 3D MHD simulations that found weak (rapid) spectral transfer into wavevectors parallel (perpendicular) to the mean magnetic field (Oughton et al. 1994). This can be understood in terms of resonant three-wave interactions (Shebalin et al. 1983). A weak parallel cascade interpretation would also be consistent with several theories and models of collisionless plasma turbulence (Oughton & Matthaeus 2005) such as reduced MHD (e.g., Montgomery 1982), “critical balance” (Goldreich & Sridhar 1995), and gyrokinetics (Schekochihin et al. 2009). However, while our results imply that the parallel and perpendicular wavevector cascades proceed with different physics, further work is required to determine the exact nature of these differences.

The present analysis applies to intermittent turbulence exclusively in fast ambient solar wind, and further investigation is required to determine whether the phenomenon of spatial anisotropic intermittency is universal. Hence, similar studies will be conducted in different solar wind streams and plasma environments, such as planetary shocks and magnetospheres, with the aim of reproducing the current results. In addition, work has already begun on investigating the presence of similar spatial anisotropy in the higher-order multiscale analysis of dissipation range turbulence, where a dominant monoscaling has already been observed (Kiyani et al. 2009).

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