Superfluidity vs. prethermalisation in a nonlinear Floquet system

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Abstract – We show that superfluidity can be used to prevent thermalisation in a nonlinear Floquet system. Generically, periodic driving boils an interacting system to a featureless infinite temperature state. Fast driving is a known strategy to postpone Floquet heating with a large but always finite boiling time. In contrast, using a nonlinear periodically driven system on a lattice, we show the existence of a continuous class of initial states which do not thermalise at all. This absence of thermalisation is associated to the existence and persistence of a stable superflow motion.

Introduction. – Thermalisation of isolated quantum systems has been the subject of intensive research during the past decade. In this context, it is important to distinguish ergodic systems, which dynamically explore all possible energetically accessible states, from nonergodic ones which can only explore a limited fraction of such states. Beside fine-tuned integrable systems whose dynamics is constrained by constants of motion, a seminal example of a nontrivial nonergodic behavior is many-body localisation happening in strongly disordered quantum interacting systems [1,2]. Other notable ergodicity breaking phenomena include quantum many-body scars and Hilbert space fragmentation [3,4].

In conservative systems, the dynamics can only take place within the energy shell since energy is conserved. In time-dependent systems, this is no longer true and time-periodic driving has been used to realize new non-equilibrium phases of matter, e.g., Floquet topological insulator and discrete time crystal [5,6]. Generically, periodically driven systems subject to interaction tend to evolve toward a featureless state akin to infinite temperature at long times [7–9]. Different strategies have been devised to escape or hamper this heating mechanism due to energy exchanges. A first example is achieving many-body localisation through strong disorder in quantum systems [10–13]. Another example is to bring the system into a long-lived prethermalization state where Floquet energy transfers are largely suppressed and heating is postponed to exponentially large times. This prethermal state can be achieved by driving the system sufficiently fast. This mechanism is interesting because it is universal as it applies to both quantum [14–20] and classical systems [21–24] and does not rely on strong disorder. Nevertheless fast driving does not prevent ultimately the system from boiling to an infinite temperature state.

In this letter, we investigate the robustness of superfluidity in interacting Floquet systems as it could offer an escape route against prethermalisation. Indeed, in conservative systems, a superfluid at zero temperature [25] is immune to small enough perturbations. As long as its velocity remains smaller than a well-defined critical velocity [26], scattering by impurities is suppressed and internal excitations cannot be activated, preventing energy redistribution and thus thermalisation. It is only above this threshold that the system may enter a route toward wave thermalisation [27] and eventually reach a statistical equilibrium with an effective temperature. First discovered in liquid helium [28,29], superfluidity was later shown to be more universal and was observed in various quantum fluids [30–33].

Recent studies show that quenched superfluids generally evolve to a thermal equilibrium state at large times, see, e.g., [34,35]. In this letter, we show that, under suitable conditions, superfluidity can be maintained at all times in

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Floquet systems and escape prethermalisation and heating. Using a lattice model with periodically kicked nonlinear interaction and onsite potentials, we build a driving protocol allowing the system to remain superfluid despite energy injection from the kicks, and show that the time \( t_{\text{boil}} \) needed to boil the system diverges at the prethermal to superfluid transition.

Our paper is organized as follows. We first introduce our periodically driven system and the new driving protocol allowing for superfluidity to emerge dynamically. We discuss its mapping to, and difference with, conservative superfluids. Superfluidity and heating features are then characterized by gradually increasing the complexity of the model. In the clean case, we perform a Bogoliubov analysis to unveil two important characteristic scales, the sound velocity and the healing length. We also describe the Floquet heating instability due to the presence of a nonlinearity. We then study the effect of a single impurity. Two distinct phases are found, a superfluid and a prethermal phase, exhibiting dramatically different dynamical behaviours. A superfluid flow is observed up to a critical velocity given by a Landau criterion [26,36,37]. Above that, the system reaches a prethermal phase with a large but finite boiling time. Strikingly, we find that the boiling time diverges at the transition between the superfluid and prethermal regions (up to the longest times accessible numerically). Thus, thermalisation is absent in the superfluid phase. Finally, we show that this phenomenology remains valid in the presence of disorder. We describe the statistical properties of the critical velocity using extreme value statistics and we find a very good agreement with numerical results. Technical details are given in the Supplementary Material (SM).

Model. – Driven systems with periodically kicked onsite potentials have attracted extensive attention to study the dynamical localization transition and engineer exotic topological phases of matter [38–41]. There has also been a rising interest with temporally modulated interactions to design synthetic gauge fields or modify the transport properties [42–46]. In the same spirit, we consider here bosonic particles hopping in a one-dimensional lattice (with unit lattice constant \( a = 1 \)) comprising \( N \) sites (with periodic boundary conditions) where both the onsite potential and the mean-field (repulsive) interaction terms are periodically kicked with the same time sequence (time period \( T = 1 \) set to unity, driving frequency \( \Omega = 2\pi \)). Setting \( \hbar = 1 \), the dynamics of our system is then governed by the following time-dependent Gross-Pitaevskii (GP) lattice equation:

\[
\imath \partial_t \psi_x = -\frac{J}{2} (\psi_{x+1} + \psi_{x-1}) + \text{III}(t)(V_x + \tilde{g} N_x |\psi_x|^2) \psi_x, \tag{1}
\]

where \( \text{III}(t) = \sum_{n \in \mathbb{Z}} \delta(t-n) \) is the Dirac comb, \( x \) labels the lattice sites, \( J > 0 \) is the nearest-neighbour hopping amplitude, \( V_x \) the onsite potential, \( N_x \) the number of particles and \( \tilde{g} > 0 \) the two-body interaction strength. The wave function is normalized to \( \sum_{x=1}^{N} |\psi_x(t)|^2 = 1 \). In the following, we will consider \( V_x = 0 \) (clean case), \( V_x = W \delta_{x,x_0} \) (single-site impurity with strength \( W \) located at some given site \( x_0 \)), and \( V_x \in [-W/2,W/2] \) (site-uncorrelated uniformly distributed random potential). For future purposes, we define the renormalised nonlinear interaction strength \( g = \tilde{g} N_a / N \). This model system could possibly be realized with cold atoms hopping in an optical lattice in the tight-binding regime. An additional optical potential (disordered or not) would be periodically flashed on the atoms. The interaction could also possibly be repeatedly switched on and off using a Feshbach resonance [47].

Noteworthy, the system described by eq. (1) is formally equivalent to a nonlinear quantum kicked rotor (QKR) [48]. The QKR is a driven system exhibiting a rich phenomenology, from quantum chaos [49,50] to topological effects [51,52]. In particular, the QKR displays dynamical localisation [53], a phenomenon analogous to Anderson localisation [54] but in momentum space [55]. This system has been successfully implemented with cold atoms, see, e.g., [56–58]. A variant of the QKR includes mean field GP interactions [48,59,60] and has been used to study the breakdown of dynamical localisation [48,61–63] and more recently prethermalization and wave condensation [64].

Importantly, we consider here a driving protocol where the potential \( V_x \) is turned on adiabatically instead of abruptly. Sudden quench effects have been recently studied with the nonlinear QKR model in the regime of strong nonlinearities. It has been found that the system shows interesting prethermal properties before reaching an infinite-temperature thermal behavior at large times [65]. By contrast, we focus on an adiabatic driving protocol where an additional controlling parameter \( a_t \) is coupled to \( V_x \), whose strength is slowly ramped up as \( a_t = \tanh(t/\tau) \) with \( t \in \mathbb{Z} \) and \( \tau = 10^3 \) unless specified otherwise. We will consider the subsequent dynamics at times \( t \gg \tau \) where the onsite potential has reached its stationary value. Defining \( \psi^n_x = \psi_x(t = n + 0^+) \) and \( \psi^n_k = \psi_k(t = n + 0^+) \) with \( \psi^n_k = \sum_x \psi^n_x e^{i k x} \), where \( k \in [-\pi,\pi] \) is the quasi-momentum, the dynamics of our system is obtained by iterating the following nonlinear map:

\[
\begin{align*}
\psi^{n+1+\text{c}}_k &= e^{i J \cos k} \psi^{n+\text{c}}_k, \\
\psi^{n+1-}_x &= e^{-i a_n V_x} e^{-i g |\psi^{n+\text{c}}_x|^2} \psi^{n+1-}_x. \tag{2}
\end{align*}
\]

At this stage, it is interesting to comment on the connection of our model with the well-known conservative case. Our model, eq. (2), is explicitly time dependent, and we study its dynamics at stroboscopic times. This gives rise to the existence of Floquet quasi-energy bands. Like in [64], we will consider a situation where these quasi-energy bands are well separated from each other so that their coupling, due to interactions, is weak. This implies constraints on the parameters of the model which we will discuss later. In this regime, it is also possible to work in the low quasi-energy sector, i.e., at the edge of a quasi-energy band. This is achieved by considering initial
Superfluid properties in the clean case. – We will now show that our model shares some important formal features with the usual conservative superfluids, in particular the concepts of sound velocity and healing length. Adapting Bogoliubov theory to our periodically driven system, we derive a low-energy excitation spectrum with a linear dispersion relation at low momenta that supports sound waves. Along this analysis, a threshold on the hopping amplitude $J$ is obtained to avoid fast Floquet heating [64,66].

Superfluidity can be characterized by a stability analysis of the initial plane-wave mode $\psi(t) = 0 = \sqrt{N} e^{-i k_0 x}$. In the clean situation, the onsite potential is zero and the plane wave $\psi(x,t) = \sqrt{\frac{1}{N}} e^{-i k_0 x}$ is an exact solution when $\phi(t) = g \Pi(t) - J \cos k_0$. In order to obtain the low-energy excitation spectrum, we perform a linear stability analysis with the ansatz $\psi(x,t) = \psi^0(x,t)(1 + \delta \psi(x,t))$. We decompose the perturbation $\delta \psi(x,t) = \sum_q u(q,t) e^{-iqx} + v^*(q,t)e^{iqx}$ into different plane-wave modes labeled by $q \in [-\pi, \pi)$. Linearizing eq. (1), we obtain the time-dependent Bogoliubov-de Gennes equation for the perturbation,

$$i \partial_t \begin{pmatrix} u \\ v \end{pmatrix} = \mathcal{M}(q,t) \begin{pmatrix} u \\ v \end{pmatrix}.$$  

(3)

Introducing $\lambda = 2 J \sin(q) \sin(q + k_0)$ and the Pauli matrices, we have $\mathcal{M}(q,t) = \lambda \sigma_z + g \Pi(t)(\sigma_x + i \sigma_y)$. Since the operator $\mathcal{M}(q,t)$ is periodic in time, we consider the one-period evolution operator associated with it, also known as the Floquet operator, given by the time-ordered integration $U(q) = T e^{-i \int_{0}^{T} dt \mathcal{M}(q,t)}$, that is $U(q) = e^{-i \lambda \sigma_z} e^{-i g(q,\sigma_x + i \sigma_y)}$, 

(4)

We next employ the Baker-Campbell-Hausdorff formula [67] to approximate the effective Floquet Hamiltonian for $U(q)$ order by order in both $g$ and $\lambda$, i.e., $H_F(q) = -\ln U(q) \approx H_F^{(1)}(q) + H_F^{(2)}(q) + \cdots$. At first order in $g$ and $\lambda$, we find

$$H_F^{(1)}(q) = (\lambda + g) \sigma_z + ig \sigma_y,$$

(5)

with eigenvalues $\omega_{\pm}^{(1)}(q) = \pm \omega(q)$ where $\omega(q) = \sqrt{(\lambda + g)^2 - g^2}$. At $k_0 = 0$, we get

$$\omega(q) = 2 \sqrt{gJ} \left| \sin \frac{q}{2} \right| \sqrt{\frac{J}{g} \sin^2 \frac{q}{2} + 1}.$$  

(6)

At small momenta, $\sin(q/2) \sim q/2$ and this excitation spectrum adopts a form similar to the Bogoliubov spectrum for time-independent systems with a quadratic kinetic energy term with particle mass $m \sim 1/J$. In the limit $q \to 0$, we recover a linear dispersion relation $\omega(q) \sim cq$ where $c = \sqrt{gJ}$ plays the role of a sound velocity. We also see from the square-root term in eq. (6) that $\xi = \sqrt{J/g}$ is a length scale playing the same role as the healing length in usual superfluid systems.

At this level of approximation, the excitation dynamics of our periodically driven system is identical to that for a time-independent system described by $H_F^{(1)}(q)$. In particular, Floquet heating is totally absent. It will arise when taking into account higher order terms in the expansion of $H_F(q)$. As shown in the SM, it is possible to exactly diagonalise $U(q)$ in eq. (4) and look for dynamical instabilities (eigenvalues of the Floquet operator outside the unit disk in the complex plane). At $k_0 = 0$, the first unstable mode is $q = \pi$ and appears when $J + g > \pi/2$ which defines a threshold for Floquet heating. Actually, this criterion has a simple interpretation when considering the relation between the driving frequency $\Omega = 2\pi$ and the effective bandwidth $E_0$ of our model [15,64,68]. The single-particle bandwidth is given by $E_0 = 2 J$, and the correction from nonlinearity results in $E_0 = 2(J + g)$ for small $g$. An estimate for suppressing the direct inter Floquet band transition is achieved by setting $E_0 = 2(J + g) < \pi$ to avoid fast Floquet heating. In the following, we will set the hopping amplitude $J < (\pi/2 - g)$ and $g = 0.1$.

Superfluid flow across a single impurity. – Here, we investigate further the analogy between our periodically driven system and a conservative superfluid by studying the fate of an initial plane wave in the presence of a single impurity of strength $W$ located at some given site $x_0$, $V_x = W \delta_{x,x_0}$, that is ramped up adiabatically to avoid sudden quench effects. In the following, since we work with periodic boundary conditions, we set $x_0 = N/2$ without any loss of generality. We show that, below a certain critical velocity threshold, the system maintains its superfluid properties, i.e., can flow through the impurity without any dissipation or back scattering.

In traditional superfluids, the nature of the flow interacting with a single localized impurity is governed by two independent dimensionless parameters (see [36,37] and SM). The first one is the ratio between the velocity of the fluid and the sound velocity, also known as the Mach number $M$. The second parameter is the dimensionless impurity strength $\Lambda$ (see SM). In this $(M, \Lambda)$ parameter space, there is a critical line that separates the superfluid phase existing at low Mach numbers from a non superfluid phase existing at higher Mach numbers. In the superfluid regime, the impurity cannot induce any excitation in the fluid and the flow is stationary. It therefore lasts forever. Above the critical line, the flow is slowed down by energy transfers from the coherent motion to internal excitations and enters a weak turbulent regime characterized by erratic and complex dynamics.

Such a critical line also exists in our periodically driven system. In our case, since $J \sim 1/m$ and $h = 1$, the velocity corresponds to $Jk_0$ and the Mach number simply reads

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Fig. 1: Stroboscopic temporal dynamics of the population $ρ(k_0,t)$ of the initial mode for different initial momenta $k_0$ (Mach number $M = k_0ξ$, where $ξ$ is the healing length). System parameters values are $J = 1.3, g = 0.1(ξ ≈ 3.6, c ≈ 0.36), Λ = W/c = 0.41$ and $N = 1024$. This population remains almost unchanged when $k_0$ is below a certain critical value while a rapid drop of the population takes place when $k_0$ is above that critical value.

$M = Jk_0/c = k_0ξ$ whereas the effective impurity strength is given by $Λ = Wξ/J = W/c$, see table 1 in the SM. Concretely, we initialize the system in the single plane-wave mode $ψ_x(t = 0) = 1/√N e^{-ik_0x}$ with quasi-momentum $k_0$. We then iterate the quantum map eq. (2) with $V_x = Wδ_{x,x_0}$ for a given $W$. The nature of the flow is then established by looking at the evolution of the population of the initial mode $ρ(k_0,t) = |ψ_{k_0}(t)|^2$ over a very long time. Figure 1 illustrates our numerical experiment for a given set of parameters. Remarkably, $ρ(k_0,t ≥ τ)$ shows two different behaviors. For initial momenta $k_0$ below a certain critical threshold $k_c$, the population of the initial plane-wave remains very close to unity, meaning that a superflow is preserved. Above $k_c$, $ρ(k_0,t)$ drops to zero after a certain time, meaning that the initial state is only marginally populated compared to other modes and the superflow is lost.

Next, we repeat the numerical simulation for different impurity strength $W$ to qualitatively extract the critical line in the $(M,Λ)$-plane, see fig. 2. The agreement between our data, obtained for a Floquet system, and the analytical prediction for this critical line in a conservative quantum fluid [36,37], namely

$$\Lambda_c(M) = \frac{\sqrt{2}}{4M} \sqrt{-8M^4 - 20M^2 + 1 + (1 + 8M^2)^{3/2}},$$

is truly remarkable.

**Superfluid to prethermal transition.** – Above the critical velocity, the initial state is unstable and the flow enters a regime analogous to the weak turbulent regime in time-independent systems [36,37]. Following [64], we expect our system to enter a long-lived prethermal state before Floquet heating drives the system into an infinite temperature state after some boiling time $τ_{boil}$. A natural question that then arises is whether, below the critical line, $τ_{boil}$ is finite (albeit larger than the observation time considered in fig. 1), in which case we would have a metastable superfluid state, or infinite, in which case we would have a true superfluid state. In this section, we show that superfluidity prevents thermalisation at all times in our periodically driven nonlinear system.

The boiling process induced by Floquet heating can be conveniently characterised by the temporal evolution of the variance $σ^2_k(t) = \langle k^2 \rangle - \langle k \rangle^2$ of the quasi-momentum distribution [64]. In the prethermal regime, $σ^2_k(t)$ shows a plateau for $t ∼ τ_{boil}$ while at $t ≫ τ_{boil}$ it saturates to the value $π^2/3$ obtained for a uniform momentum distribution, a signature of an infinite temperature state. The transition from these two behaviors is sharp and the time at which this happens defines the boiling time. Figures 3(b) and (c) present two examples of simulations in, respectively, the superfluid regime and the prethermal regime. In the first case, the variance remains small and no boiling time could be defined up to the maximal integration time, i.e., $10^8$ periods. On the contrary, inset (c) demonstrates the existence of a finite boiling time although it is exponentially large. Note that, though the variance remains small before $τ_{boil}$ in both insets (b) and (c), the underlying dynamical behaviors are different, see SM. This distinction is hardly visible on inset (c), where the prethermal plateau is reached for $log t > 4$ only, because the effective temperature is very small for this set of parameters.

We now present one of our most important results. In fig. 3(a), we plot the evolution of $τ_{boil}$ as a function of the
Mach number $M = k_0 \xi$ at fixed dimensionless impurity strength $\Lambda = W/c$. Strikingly, we find that $\tau_{\text{boil}}$ jumps discontinuously when $M$ crosses the critical value $M_c$. In the prethermal phase, $\tau_{\text{boil}}$ is always finite, while, in the superfluid phase, thermalisation of the system does not take place up to the longest times considered ($t = 10^5$).

The only difference is that the critical impurity remains entirely correct in the presence of a disordered potential. This is reflected in fig. 2, where the color map in the phase diagram represents the critical velocity as it is explained in the SM. This approach gives a rather accurate description of the statistical properties of $M_c$, as demonstrated below. Our central result is the cumulative distribution of the critical $M_c$ which is given by the following formula:

$$\Phi_N(M_c) = 1 - \left[ 1 + \text{erf} \left( \frac{\Lambda_0(M_c) - \Lambda_0}{\alpha \xi} \right) \right] \frac{N}{2} \tau_c,$$  

with $M_c = k_0 \xi$, $\Lambda_0 = \sqrt{\frac{2}{\alpha}} \Lambda = \sqrt{\frac{2}{\alpha} W}$, where $\alpha = 2.78$ (see SM). Then the probability distribution function (PDF) $P_N(M_c)$ of the critical $M_c$ is simply the derivative of $\Phi_N$, $P_N(M_c) = \Phi'_N(M_c)$. Note that this simple expression contains nontrivial scaling properties of $M_c$ with respect to the system size $N$, the disorder strength $W$, the hopping amplitude $J$ and the nonlinearity strength $g$ via $c$ and $\xi$.

Figure 4 shows the comparison between numerically extracted histograms of the critical $M_c$ and our analytical predictions.

**Disordered case.** We now address the dynamical behavior of our system in the presence of a site-uncorrelated disordered potential with uniform distribution $V_c \in [-W/2,W/2]$. As expected from [69], our numerical investigations (data not shown) indicate that the phenomenology observed in figs. 2 and 3 with a single impurity remains entirely correct in the presence of a disordered potential. The only difference is that the critical line obtained in fig. 2 now depends on the configuration of disorder considered and $M_c$ becomes a random variable.
predictions for different values of the disorder strength $W$. Our statistical model remarkably reproduces not only the typical critical $M_c$ and its fluctuations but also the full probability distribution. In order to quantify the typical value of the critical $M_c$, we have computed its median $M_c$ defined as $\Phi_N(M_c) = 1/2$, as a function of the disorder strength $W$ and the results are shown in fig. 5. One can see that it is substantially below the critical Mach number predicted for a single impurity but very well captured by the effective disorder computed from extreme value arguments. In addition, we have also checked the scaling with the system size $N$ (see SM) and interaction strength $g$ (data not shown) which give the same level of agreement. Altogether, these results demonstrate that the superfluid to prethermal transition in the disordered case can be understood and accurately described by a coarse-graining procedure. In other words, the single-site impurity case can be seen as an effective description of the graining procedure. In other words, the single-site impurity case can be understood and accurately described by a coarse-graining procedure. In other words, the single-site impurity case can be understood and accurately described by a coarse-graining procedure. In other words, the single-site impurity case can be understood and accurately described by a coarse-graining procedure. In other words, the single-site impurity case can be understood and accurately described by a coarse-graining procedure.

Last, we have also addressed the Floquet heating instability with disorder. We find that the distribution of the critical $M_c$ determined above predicts very well the onset of a diverging $\tau_{\text{boil}}$ (see SM). For a superfluid moving through disorder with initial $M$ below its renormalised critical $M_c$, thermalisation is not seen to occur under periodic driving.

**Conclusion.**—In this letter, we have shown that superfluidity can be used to prevent thermalisation in a nonlinear Floquet system. While fast driving is a well-known mechanism to postpone Floquet heating to exponentially long times, we have demonstrated a model where the boiling time diverges at the prethermal to superfluid transition. This promotes superfluidity as a new mechanism for ergodicity breaking in Floquet systems.

This opens interesting avenues to study Floquet topological superfluids [70] and dynamical phase transitions [71] such as the recently proposed dynamical Berezinskii-Kosterlitz-Thouless phase transition [72].

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*Data availability statement:* The data that support the findings of this study are available upon reasonable request from the authors.

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