Theory and Phenomenology of Heavy Flavor at RHIC

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Abstract. We review the problem of heavy-quark diffusion in the Quark-Gluon Plasma and its ramifications for heavy-quark spectra in heavy-ion collisions at RHIC. In particular, we attempt to reconcile underlying mechanisms of several seemingly different approaches that have been put forward to explain the large suppression and elliptic flow of non-photonic electron spectra. We also emphasize the importance of a quantitative description of the bulk medium evolution to extract reliable values for the heavy-quark diffusion coefficient.

1. Introduction

The heavy-quark (HQ) mass, $m_Q \gg T_c$, brings a large scale into the problem of probing strongly interacting matter in the vicinity of its (pseudo-) critical temperature, $T_c$. This implies that in high-energy heavy-ion collisions charm and bottom quarks ($Q = c, b$) are mostly produced upon initial impact of the incoming nucleons, on a timescale $\tau_{\text{prod}} \approx 1/m_Q \leq 0.1 \text{ fm}/c$, while their abundance is expected to be frozen thereafter (which is supported by data [1]). Therefore, changes in HQ momentum spectra due to reinteractions in the produced medium become an excellent (since rather direct) probe of the latter. The large HQ mass also implies that (the approach to) thermalization is delayed thus providing a more sensitive means to study the in-medium interactions responsible for equilibration. The (approximate) “factorization” of $Q\bar{Q}$ production and rescattering renders HQ observables a well-defined probe over the entire range of transverse momentum ($p_T$), enabling a comprehensive study of (a) thermalization at low $p_T$, (b) a kinetic regime at intermediate $p_T$ and (c) “jet-quenching” at high $p_T$. The conservation of individual charm and bottom quantum numbers allows to further test quark coalescence mechanisms in the hadronization transition. Theoretically, $m_Q \gg T$ opens the possibility to describe (low-$p_T$) HQ motion in the QGP within a diffusion equation [2] which facilitates the extraction of pertinent transport coefficients from heavy-ion data. The transition from the elastic to the radiative scattering regime is, of course, a key issue which is closely related to items (a)-(c) above. Finally, the availability of rather accurate lattice QCD (lQCD) computations of HQ free energies at

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finite temperature has spurred the hope of being able to define and extract interaction potentials which may not only govern the properties of heavy-quark ($Q \bar{Q}$) bound states but also heavy-quark diffusion. The nonperturbative nature of the latter has been exhibited in Refs. [3, 4] (see Ref. [5] for a recent review).

Our article is organized into a discussion of HQ diffusion coefficients (Sec. 2), their applications to HQ spectra and observables at RHIC (Sec. 2) and conclusions (Sec. 4).

2. Heavy Quark Diffusion in the Quark-Gluon Plasma

Brownian motion of a heavy quark in a fluid of light partons is described by a Fokker-Planck (FP) equation, schematically written as

$$\frac{\partial f_Q}{\partial t} = \gamma \frac{\partial p f_Q}{\partial p} + D \frac{\partial^2 f_Q}{\partial p^2},$$

(1)

which follows from the Boltzmann equation to second order in the momentum transfer, $k$, to the heavy quark. The friction ($\gamma$) and momentum diffusion ($D$) coefficients,

$$\gamma p = \int d^3k \ w(k, p) \ k, \quad D = \frac{1}{2} \int d^3k \ w(k, p) \ k^2,$$

(2)

are given in terms of transition rates, $w$, which are typically computed from the elastic scattering amplitude of the heavy quark with the (light) medium constituents. Both terms in Eq. (1) are essential for the HQ distribution function, $f_Q$, to approach thermal equilibrium. This feature is highlighted by the Einstein relation, $T = D/(\gamma m_Q)$, which also shows that the FP equation involves only one independent transport coefficient. The latter is often quoted in terms of the spatial diffusion constant, $D_s = T/(\gamma m_Q)$.

Early estimates of HQ diffusion in the QGP have utilized “leading-order” (LO) perturbative QCD (pQCD) which is dominated by $t$-channel gluon exchange with thermal partons (quarks and especially gluons) [2, 3, 5]. The infrared divergence for forward scattering is regulated by a thermal Debye mass, $\mu_D \sim gT$, resulting in a total cross section $\sigma_{Qp} \sim \alpha_s/\mu_D^2$. However, due to the forward-angle dominated scattering, the transport cross section is much smaller; at temperatures around $T=300$ MeV and for a strong coupling constant, $\alpha_s=0.4$, the thermal relaxation time for $c$-quarks amounts to $\tau = \gamma^{-1} \simeq 15-20$ fm/c (larger by a factor of $\sim m_b/m_c$ for $b$ quarks). This is much longer than the expected QGP lifetime at RHIC and would therefore lead to small modifications of HQ spectra.

In Ref. [6] it was suggested that mesonic resonances in the QGP could be operative in thermalizing heavy quarks. Pertinent effective Lagrangians where constructed utilizing HQ effective theory with a chirally symmetric set of $D$- and $B$-like resonances for $c$-$\bar{q}$ and $b$-$\bar{q}$ scattering. Within an estimated range of the underlying 2 parameters (coupling constant and resonance mass), the HQ relaxation times where found to be reduced by a factor $\sim 3$ compared to the LO-pQCD results. Pertinent Langevin simulations within an expanding QGP at RHIC, augmented with quark coalescence at $T_c$, resulted in a reasonable description [9] of non-photonic electron ($e^\pm$) observables at RHIC [11, 10] (cf. left panel of Fig. 5 below). The assumption of resonances calls for a
microscopic treatment to eliminate the free parameters of the effective Lagrangian and to assess their temperature dependence. In Ref. [4], a $T$-matrix equation for heavy-light quark scattering was set up, which, after partial wave expansion ($L=0,1,...$), takes the form

$$T^a_L(E_{cm};q,q') = V^a_L(q,q') + \int dk k^2 V^a_L(q,k) G_{QQ}(E_{cm},k) T^a_L(E_{cm};k,q') ;$$ (3)

$G_{QQ}$ denotes the intermediate 2-particle propagator which in principle contains in-medium selfenergies (mass corrections and width) of the individual heavy and light quark. The main idea is to implement the interaction kernel, $V^a_L$, in a parameter-free way by utilizing first-principle information from lattice QCD computations in terms of the HQ free energy at finite temperature,

$$F_{QQ}(T, r) = U_{QQ}(T, r) - TS_{QQ}(T, r) .$$ (4)

The same approach has been applied in recent years to compute heavy quarkonium spectral functions in the QGP, with fair success in reproducing independent lQCD results for Euclidean-time correlation functions [11] [12] [13] [14]. In the vacuum, $F_{QQ}(r) = U_{QQ}(r)$ closely resembles the phenomenological Cornell potential (Coulomb+confinement) which has been very successful in quarkonium spectroscopy and is now understood as the low-momentum limit of QCD with heavy quarks (so-called “potential QCD”) [15]. The appropriate potential definition at finite temperature is currently an open question. An upper limit may be obtained by using the (subtracted) internal energy, $V_{QQ}(r;T)=U_{QQ}(r;T) - U_{QQ}(r=\infty;T)$, which in the quarkonium sector provides the largest binding [14]. The subtraction is required to ensure convergence of the Fourier transform of the potential in momentum space in Eq. (3). Two examples of pertinent potential extractions are shown in Fig. 1. Their application in the $T$-matrix equation (3) additionally includes a relativistic correction to simulate color-magnetic interactions [17]. The Born approximation to the $T$-matrix, $T^a_L = V^a_L$, recovers the results from LO-pQCD within $\sim 10\%$ above $E_{cm}=4$ GeV (for the same value of $\alpha_s$). The different color channels, $a$, are accounted for by Casimir scaling of the potentials. It

Figure 1. Heavy-quark internal energies as extracted from fits [16] [17] to quenched (left) and $N_f=2$ (right) free energies computed in thermal lattice QCD [18].
turns out that the resulting in-medium $T$-matrices are dominated by $S$-wave scattering in the attractive color-singlet (meson) and -antitriplet (diquark) channels, supporting resonance-like structures close the $qQ$ threshold up to $T \approx 1.7 T_c$ and $1.4 T_c$, respectively. Close to $T_c$, the corresponding friction coefficients are a factor $\sim 3$ larger than LO-pQCD (see Fig. 2), but they decrease as temperature increases due to the weakening (color-screening) of the IQCD-based potential in the left panel of Fig. 1 (for the potential in the right panel of Fig. 1 a slight increase of $\gamma(T)$ is found). It is interesting to note that the collisional dissociation of $D$ and $B$ mesons in the QGP evaluated in Ref. [20] is based on a similar Cornell-type potential as the $T$-matrix [4], but without the inclusion of medium effects in the potential. The effect of a reduced formation time of $B$ relative to $D$ mesons [20] is encoded in the $T$-matrix calculations as a mass effect leading to a stronger binding (analogous to the heavy quarkonium sector [14]).

Perturbative evaluations of HQ transport have recently been revisited in Ref. [19], where it is argued that the infrared regulator in the $t$-channel gluon propagator becomes operative at a significantly softer scale than the Debye mass. In addition, a running of the strong coupling constant to small scales is implemented. The combined effect on the HQ friction coefficient is an approximately ten-fold increase over the Born result with Debye-mass regulator, cf. Fig. 2. Under these circumstances the perturbative treatment should be revisited and presumably augmented by resummonations.

HQ diffusion has also been evaluated in conformal gauge theories where nonperturbative results can be inferred using a correspondence to string theory (AdS/CFT) [21]. Defining the friction coefficient as the (inverse) timescale of momentum degradation, $dp/dt = -\gamma p$, cf. Eq. (1), one finds for a $\mathcal{N}=4$ Super-Yang-Mills plasma:

$$\gamma_{\text{AdS/CFT}} = \frac{\pi \sqrt{\lambda T_{\text{SYM}}^2}}{2m_Q} \quad \leftrightarrow \quad \gamma_{\text{QCD}} = C \left(\frac{T^2}{m_Q}\right).$$

(5)
Figure 3. Spatial diffusion coefficient, \( D_s = T / (\gamma m_Q) \), for c (left) and b quarks (right) in a QGP for: LO-pQCD with fixed \( \alpha_s = 0.4 \) (dashed lines), effective resonance model + LO-pQCD (bands for \( \Gamma_{D,B} = 0.4-0.75 \text{GeV} \)) [6], T-matrix approach + LO-pQCD (gluons only) [4] and pQCD with running \( \alpha_s \) (dash-dotted line) [19]. The AdS/CFT result, Eq. (5), corresponds to \( 2\pi T D_s = 2\pi / C \approx 1.5-4 \) (not shown in the plots).

The relation to a QCD plasma (second expression in Eq. (5)) requires a careful translation of the temperature scale and coupling constant; matching the energy densities of the SYM plasma and QGP to identify \( T \) and using the static HQ force from lQCD to identify the coupling strength, \( \lambda \), one finds \( C \approx 1.5-4 \). Note that this identification makes direct contact with the lQCD HQ free energy, as in the \( T \)-matrix approach.

The right panel of Fig. 2 compiles the temperature dependence of the friction coefficient, \( \gamma (p=0) \), in the approaches discussed above. The pQCD+running-\( \alpha_s \) and AdS/CFT results are quite comparable but significantly (much) larger than the ones from the \( T \)-matrix (LO-pQCD), except close to \( T_c \). However, it might be in the vicinity of \( T_c \) where the QGP is most strongly coupled providing favorable conditions for the validity of the strong-coupling limit. The \( T \)-matrix approach exhibits the weakest temperature dependence of the displayed curves, with a moderate uncertainty induced by different version of the internal energy (however, using the free energy as potential would give results closer to LO-pQCD).

Finally, we compare in Fig. 3 the spatial diffusion coefficients (in units of the thermal Compton wavelength, \( 1/(2\pi T) \)) at zero 3-momentum (which may be thought of as being proportional to the ratio of shear viscosity to entropy density) for charm and bottom quarks. All approaches give results fairly independent of temperature and HQ mass, except for the lQCD-based \( T \)-matrix calculation where the increase with temperature indicates a significant loss of coupling strength in the QGP.

3. Heavy-Flavor Spectra at RHIC

Several groups have applied the Brownian motion framework to simulate HQ diffusion in Au-Au collisions at RHIC, using hydrodynamic [7, 10, 23] or expanding fireball [9] models. Obviously, a realistic description of the bulk medium (temperature and flow evolution) is mandatory for a quantitative extraction of diffusion coefficients from data.
Figure 4. Time evolution of nuclear modification factor (central collisions, left panel) and elliptic flow (semicentral collisions, right panel) in a QGP fireball at RHIC. [5].

Let us first address the time evolution of HQ $p_T$-spectra, characterized by the nuclear modification factor $R_{AA}(p_T) = (dN_{AA}/dp_T)/(N_{coll}dN_{pp}/dp_T)$, and elliptic flow coefficient, $v_2(p_T)$. The results in Fig. 4, computed in a thermal fireball evolution with HQ transport based on resonance+LO-pQCD interactions [9], suggest that the high-$p_T$ suppression is built up significantly earlier than the elliptic flow. The former feature is quite reminiscent to a recent analysis [24] of the empirical system-size dependence of high-$p_T$ hadron suppression, arguing that parton energy-loss is predominantly operative in the first 2-3 fm/c. On the contrary, the bulk $v_2$ in hydrodynamic models (used to construct the fireball evolution) requires a duration of at least $\Delta \tau \approx 2-3$ fm/c to build up most of its strength, thus “delaying” the transfer to heavy quarks.

Next we compare Langevin simulations for charm quarks in semicentral Au-Au collisions at RHIC. Representative values obtained for $v_2$ and $R_{AA}$ in 3 different evolution codes (2 hydro, 1 fireball), implementing 3 of the approaches discussed above for HQ transport coefficients, are summarized in Tab. 1. Given the complete independence of

| Model [Ref.]   | $D_s(2\pi T)$ | $b$ [fm] | $v_2^{\text{max}}$ | $R_{AA}(p_T=5 \text{ GeV})$ |
|----------------|---------------|----------|---------------------|-----------------------------|
| hydro + LO-pQCD [7] | 24            | 6.5      | 1.5 %               | 0.7                         |
| hydro + LO-pQCD [7] | 6             | 6.5      | 5 %                 | 0.25                        |
| fireball + LO-pQCD [9] | ~30           | 7        | 2 %                 | 0.65                        |
| fireball + reso+LO-pQCD [2] | ~6            | 7        | 6 %                 | 0.3                         |
| hydro + Eq. (5) [23] | 21            | 7.1      | 1.5-2 %             | ~0.7                        |
| hydro + Eq. (5) [23] | $2\pi$        | 7.1      | 4 %                 | ~0.3                        |

Table 1. Overview of model approaches (1. column) and input parameters (2. column: spatial charm-quark diffusion coefficient, 3. column: nuclear impact parameter) for Langevin simulations of charm-quark spectra in Au-Au collisions at RHIC; selected values for the resulting elliptic flow ($v_2^{\text{max}} \approx v_2(p_T=5 \text{ GeV})$) and nuclear modification factor are quoted in columns 4 and 5.
the calculations, the agreement on the \( \sim 30\% \) level is encouraging (it might improve when accounting for finer details, e.g., the results of Ref. [23] correspond to terminating the evolution in the middle of the mixed phase while in Refs. [7, 9] it is run to the end of the mixed phase, cf. right panel of Fig. 4).

Finally, theoretical predictions for non-photonic electron spectra are compared to RHIC data in Fig. 5. This requires the conversion of the modified \( c \) - and \( b \)-quark spectra into meson spectra followed by their semileptonic decay. For the radiative energy-loss calculations of Ref. [26] and the Langevin simulations of Ref. [7] hadronization is treated via independent fragmentation, while in Refs. [9, 4] coalescence processes with light quarks from the medium are accounted for. The latter enhance the hadron (and electron) \( v_2 \) while reducing the suppression (see dashed lines in the right panel of Fig. 5). While the radiative energy-loss calculations (with upscaled pQCD transport coefficient) roughly account for the observed suppression, the corresponding elliptic flow is underpredicted. This underlines the importance of accounting for the collectivity of the expanding QGP medium, transferred to heavy quarks via the diffusion term in Eq. (1). The current data-theory comparison points at HQ diffusion coefficients in the range of \( D_s(2\pi T) \approx 4-6 \).
4. Conclusions

Several calculations of HQ diffusion in a QGP at moderate temperatures have been conducted recently but do not (yet?) show satisfactory agreement, neither in magnitude nor in their temperature dependence. We have argued, however, that there is significant overlap in the underlying physics, which in most cases is based on a (in-medium) color-Coulomb type interaction as implicit in both one-gluon exchange and potential models (or in matching to conformal gauge theories). The challenge is thus to reconcile the different approaches (perturbative, $T$-matrix, AdS/CFT, etc.) by revisiting their regimes of applicability. The inclusion of radiative processes in a diffusion framework remains another challenge. On the empirical front, electron spectra at RHIC indicate a small HQ diffusion constant, but also here further scrutiny in the implementation of different medium evolution models (including the hadronic phase) needs to be exerted.

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