A Study of Hopf and Amplitude-Controlled Phase Oscillators for Snake Robot Locomotion

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Abstract: In this paper, a snake-like robot locomotion control is proposed with Center Pattern Generators (CPGs). The snake robot is designed in SolidWorks environment and composed of a head, seven-link propulsion mechanism and a tail connected to the last link. The designed model is also adapted to Matlab/SimMechanics environment in order to observe two-dimensional motion and analyze the constructed CPG networks. The CPG networks are carried out with Amplitude-Controlled Phase (ACP) and Hopf oscillators to create rhythmic, stable, oscillatory and robust locomotion patterns. Both CPG structures are constructed with bidirectional chain models. In order to analyze the locomotion of the snake-like robot, S-type forward and C-type turning motions are performed. The simulation results show that ACP oscillator with bidirectional chain network structure gives effective, smooth and robust control performance for both forward and turning motions of the snake robot. Thus, a base of more intelligent locomotion control approach with ACP oscillator is provided and some critical comparisons are performed.

Keywords: Amplitude-Controlled Phase Oscillator, Center Pattern Generator, Hopf Oscillator, Locomotion Control, Snake Robot

1. Introduction

Nowadays, robots and robotics technologies have begun to be used in extreme environments which are too dangerous to access directly by human such as observations and operations at the base of oceans, space, or nuclear plants. One of the important issue is the adaptability of robot to such environments and many developed robotic systems have begun to be inspired from the nature for the solution. Especially, novel robotic studies inspired by animals increase due to the fact that animals are adapted to environmental changes over both short and long periods of time. For example, snake-like creeps can move on flat and inclined terrains, under water, water surface and on trees. Thus, snake-like robots have become very popular designs because of their high locomotion capabilities. Due to their very large number of Degrees of Freedom (DoF), the locomotion control for snake robots are difficult. Recently, a solution which uses neural oscillators takes attention to manage the interaction between environments and robot. It has been widely shown that Central Pattern Generator (CPG) plays an important role in the creation and coordination of animal locomotion. CPG is a neural network that exists in spinal cords of living forms which is utilized as activation signal for muscle contractions. By applying CPG concept as a control mechanism of biomimetic robots, rhythmic locomotion patterns such as walking, crawling and swimming can be generated [1-6].

Some CPG models are used for motion control of robots in the literature. Hu et al. examined an adaptive CPG network capable of different locomotor patterns for a multi-joint robotic fish. In order to generate swimming patterns, CPG network was modeled as a chain of coupled Hopf oscillators and a teaching signal used in the designed model [7]. Hu et al. presented a biomimetic thunniform robotic fish and the robot was controlled with two unidirectional coupled Hopf oscillators [8]. Also, Hu et al. observed a numerical method for parameter synthesis of a CPG network to acquire desired locomotor patterns. The CPG network was modeled as a chain of unidirectional and bidirectional coupled Hopf oscillators with a novel coupling scheme that eliminates the influence of afferent signals on amplitude of the oscillators [9]. In another study, Kassim et al. controlled ten-joint bio-inspired robot with Matsuoka oscillators [10]. Wu and Ma developed a sensory feedback CPG model to control collision avoidance in a snake robot [11]. Matsuo and Ishii also developed an adaptive control system based on Matsuoka oscillator and applied to motion control of a snake-like robot [12]. Wu and Ma proposed a bio-inspired control system imitating a neural nervous network to control a snake-like robot. They analyzed the CPG-based locomotion control with experiments and simulations [13]. Matsuo and Ishii presented a design method for a phase lag adjustment system using neural oscillators for a snake-like robot and developed a sensory feedback mechanism [14]. Nor and Ma examined the locomotion control of a snake-like robot with unidirectional coupling oscillators and a single parameter was controlled for forward and backward motions [15]. Yang used a unified configuration description method based on graph theory and Amplitude-Controlled Phase (ACP) oscillator-based CPG model was proposed [16]. Nor and Ma also turning motions with sensory feedback CPG model [17]. Sergienko and Chen presented an adaptive head stabilization system for a snake robot with a chain of coupled Hopf oscillators [18]. Wang et al. also proposed a double chain CPG network of coupled Hopf oscillators as an artificial generator for a snake robot with 2-DOF joints. In this paper, bidirectional chain CPG network of coupled ACP oscillators is presented for snake robots.
oscillators is presented as an artificial spinal cord model for a snake robot and ACP oscillator network is compared with the Hopf phase oscillators. The snake-like robot is composed of a head, a seven-link propulsion mechanism and a tip pointed tail fixed to the last link. The designed robot is modelled in MATLAB/SimMechanics environment with two-dimensional motion abilities. An important point addressed in the paper is that the designed robot has a very large number of DoF and the locomotion control is difficult and challenging for multi-DoF jointed robots. The proposed CPG model has the ability to produce stable, robust and rhythmic patterns applied in the sidewinding locomotion of snake robots. Also, CPG model has explicit control parameters including not only frequency and amplitude of oscillation but also phase differences between the neighbor oscillators. Thus, CPG gives an easy and high applicable solutions for locomotion control of the snake robot. Moreover, the proposed CPG model is also suitable for locomotion control in other types of biomimetic robots such as robotic fish, salamander robots, hexapods or quadruped robots of which various locomotion patterns can be obtained with determining the phase differences of multiple links or legs.

This article is organized as follows: The network structures and mathematical model of the CPGs are presented in Section 2. In Section 3, the effects of each motion parameter on the output are analyzed and simulation results showing the effectiveness of the CPG controlled snake-like robot are given. Finally, conclusions are given in Section 4.

2. The Constructed CPG Networks

The basic motions of living forms such as flying, swimming, walking, running, crawling and creep are provided with CPGs in the spinal cord. In these basic patterns, CPGs can produce significant responses to disturbances coming from external environment and they can continue the rhythmic behaviors. Thus, an artificial CPG controller plays an important role to generate biomimetic snake-like robot locomotion [19]. The designed CPGs developing for the multiple links based robots can be classified into three basic levels as biophysical, connectionist and abstract models. Biophysical models generally generate rhythmic outputs that carry the ion charge properties and they can mimic cell membrane potentials. Connectionist models generate the rhythmic outputs with inhibitory/excitatory synaptic interactions. Abstract models differ from other structures. These models are defined by mathematical expressions of the basic oscillators and they are constructed by coupling these oscillators [20-22].

In this paper, bidirectional chain network model of coupled ACP oscillator is designed and constructed. In order to prove the locomotion patterns of ACP, the basic patterns and CPG outputs are compared with the coupled Hopf oscillators because of their limit cycle characteristics. It is noted that all CPG networks are constructed with open loop structure and eight bidirectional coupled oscillators.

2.1. Mathematical Model of ACP Oscillator

Inspired by the Kuramoto oscillators proposed by Yoshiki Kuramoto in 1975, ACP oscillators are developed as given in Fig. 1 [20,21]. The mathematical equations of the ACP can be expressed as follows:

\[ \dot{\phi}_i = \omega_i + \sum_{j}^{N} (\alpha_{ij} r_j \sin(\phi_j - \phi_i - \varphi_{ij})) \]  

\[ \dot{r}_i = a_r (\frac{\omega_i}{r_i} (R_i - r_i) - \dot{r}_i ) \]  

\[ \dot{\theta}_i = x_i + r_i \cos(\theta_i) \]  

Where, \( \phi_i \) is \( i \)th oscillation phase, \( \omega_i \) is desired frequency, \( N \) is number of oscillators, \( \alpha_{ij} \) is coupling weight between \( i \)th and \( j \)th oscillators, and \( \varphi \) is the phase difference. The \( R_i \) and \( X_i \) are the desired amplitude and bias. The \( r_i \) and \( x_i \) are also state variables that determine the oscillation amplitude and bias, respectively. \( a_r \) and \( a_x \) are positive constants which adjust the convergence speed of the output. \( \theta_i \) indicates the oscillator output.

With this CPG model, a coupled oscillator outputs exhibit limit cycle behaviors and the outputs does not effect from external perturbations in steady-state. According to control parameters as \( \omega_i, R_i \) and \( X_i \), the oscillator output can be smoothly changed and adapted to the desired behavior. Feedback terms can also be applied to the designed CPG model in order to respond to environmental influences.

2.2. Mathematical Model of Hopf Oscillator

Hopf oscillators based on abstract level features can produce oscillatory rhythmic outputs against the external perturbations. In Fig. 2, the Hopf oscillator structure is given by two half-centered neurons in the Cartesian Coordinate Plane.

The dynamics of a Hopf Oscillator is expressed as follows:

\[ \dot{x} = y \sqrt{\mu - r^2} (x - \mu_1) - \omega (y - \mu_2) - a_{ij} x_j \]  

\[ \dot{y} = x \sqrt{\mu - r^2} (y - \mu_2) + \omega (x - \mu_1) - a_{ij} y_j \]  

\[ r = \sqrt{(x - \mu_1)^2 + (y - \mu_2)^2} \]  

Here, \( x \) and \( y \) are oscillator outputs and \( y \) is the convergence rate. \( \sqrt{\mu} \) and \( \omega \) indicate the amplitude and frequency, respectively. \( \mu_1 \) and \( \mu_2 \) are feedback parameters. \( a_{ij} \) also represent the coupling weight between \( i \)th and \( j \)th oscillators. The steady-state respond of the oscillator can be also represented as:

\[ x_{\infty}(t) = \sqrt{\mu} \cos (wt + \phi_{ij}) \]
\[ y_{oc}(t) = \sqrt{2} \sin (wt + \varphi_{oc}) \]  
(9)

Where, \( \varphi_{oc} \) and \( \varphi_{oc} \) are initial phases in the transient-state time. It can be known that the excellent feature of the Hopf is the response of against external perturbations [20,21].

2.3. The Designed Snake Robot with CPG Network

The detailed configuration of the snake robot model with modular parts is illustrated in Fig. 3. It can be seen that there is an anterior rigid head, a seven-link propulsion mechanism, which is actuated with eight RC servo motors, and a tip pointed tail fixed to the last link. The motion direction occurs in two-dimensional x-y plane. Each link is connected to previous joint and this structure creates a serial chain link mechanism. Center of Gravity (CoG) point represents the motion according to the Earth-fixed Frame.

![Fig. 3. The designed snake robot model](image)

Fig. 3. The designed snake robot model

Fig. 4 also shows the MATLAB/SimMechanics model of a link of the robot. In each link, -90° is added to servo motor for initial position.

![Fig. 4. MATLAB/SimMechanics model of a link of the snake robot](image)

The designed CPG network with the bidirectional coupled structure is given in Fig. 5. In this network structure, both CPG models are constructed in the same configuration. In order to analyze the motion effects, ACP oscillators are performed and compared with the same structure of coupled Hopf oscillators.

\[ \{\sqrt{2}(\mu_1, \mu_2)\} = \{R, X\} \]  

As shown in Fig. 5, the input frequencies (\( \omega \)) are the same for Hopf and ACP but the amplitude and bias parameters are not the same. The Hopf oscillator amplitude and bias are \((\sqrt{2}, (\mu_1, \mu_2))\) while the ACP oscillator amplitude and bias are \((R_1), (X_1)\), respectively. In this way, the CPG networks determine the joint angles of the snake robot. Therefore, phase differences and control of joint angles have very significant role on snake locomotion. It is noted that the snake robot consists of eight joints. The locomotion is also realized by the interaction of the robot with the ground.

3. Analysis and Simulation Results

According to snake robot locomotion, two main gaits as S-type forward and C-type turning is provided in this paper. Bidirectional chain type coupled ACP oscillators are realized and it is proved with Hopf oscillators. The comparison of oscillator results is also given in this section.

3.1. Forward Motion Analysis

CPG parameters for forward motion of the snake robot are given in Table 1. Parameters given in Table 1 have the same values for two oscillators. The phase difference value is chosen as 45° and the bias is chosen as 0, which is the key parameter of the turning motion with the cause of spin of the snake robot. In Fig. 6, joint angles of the snake robot are given, which indicate the amplitude, phase differences and biases for the forward motion.

![Fig. 5. CPG network structure with bidirectional coupled scheme](image)

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reduced, the oscillation waves can increase from head to tail. While the frequency is increasing, the robot moves faster and while the amplitude is increasing or decreasing, the S-type motion of the robot becomes smaller or larger, respectively.

### Table 1. Selected parameters of CPG network for forward motion

| CPG Parameters | Values |
|----------------|--------|
| Amplitude (\(\mu, Ri\)) | 18° |
| Flapping Frequency (\(f\)) | 0.4 Hz |
| Phase difference (\(\Phi\)) | 45° |
| Biases (\(\mu_x, \mu_2, X_i\)) | 0 |

The SimMechanics snapshots of the forward motion for ACP oscillator are illustrated in Fig. 7 during one flapping period.

![Fig. 7](image_url)

**Fig. 7.** Snapshots of SimMechanics analysis for forward motion of the snake robot

It is seen from Fig. 7 that the forward motion performance of the robot is also satisfactory with smooth oscillation patterns.

### 3.2. Turning Motion Analysis

CPG parameters for turning motion of the snake robot are given in Table 2. The phase difference value is chosen as 45°. According to the turning motion, key parameter is bias and it is not 0 for turning motion of the snake robot. Joint angles are also given in Fig. 8, which indicate the amplitude, phase differences and biases.

### Table 2. Selected parameters of CPG network for turning motion

| CPG Parameters | Values |
|----------------|--------|
| Amplitude (\(\tilde{\mu}, Ri\)) | 18 |
| Flapping Frequency (\(f\)) | 0.4 |
| Phase difference (\(\Phi\)) | 45 |
| Hopf Bias (\(\mu_x, \mu_2\)) | 10.5 |
| ACP Bias (\(X_i\)) | 1.25 |

While the amplitude value is increasing or decreasing, the body of robot is taken to C-type and this shape can be larger or narrower. The turning motion is generated with different biases of the CPGs and the bias values are chosen as 1.25 and 10.5 for ACP and Hopf oscillators, respectively. As shown in Fig. 8, the transient-state time of Hopf and ACP oscillators are nearly 22.6 s and 0.15 s, respectively.

![Fig. 8](image_url)

**Fig. 8.** Joint angles of the turning motion: (a) Hopf, (b) ACP

As shown in Fig. 9, snapshots of the ACP oscillator with better turning performance are illustrated during one flapping period.

![Fig. 9](image_url)

**Fig. 9.** Snapshots of SimMechanics analysis for turning motion of the snake robot

It is clear that ACP oscillator has significant performance for turning as well as the forward motion, while Hopf oscillator does not provide satisfactory performance with its transient-state time although smaller bias values than ACP oscillator. ACP oscillator provides appropriate solution for turning motions even if high bias values. Thus, the ACP oscillator is reached the steady-state more quickly.

### 3.3. Comparison of the ACP and Hopf Oscillators

According to two-dimensional motion, ACP oscillator results are performed and proved with Hopf oscillators. In addition, three critical factors are also determined for the performance analysis of the CPGs as Computational Complexity (CC), Determination of Parameters (DP) and Sensory Feedback (SF). These factors are examined with both oscillators. In order to determine the differences, the CC levels are selected as low, medium, and high; DP levels are selected as easy, medium and difficult; SF levels are...
selected as weak, moderate and good. Also, unidirectional and bidirectional chain coupled network topologies are compared according to the transient-state times. This detailed comparison is illustrated in Table 3.

| CPG Model | Level | CC  | DP | SF   |
|-----------|-------|-----|----|------|
| ACP       | Abstract | Low | Easy | Weak |
| Hopf      | Abstract | Medium | Medium | Weak |

As shown in Table 3, SF levels are the same in both oscillators. CC is low and DP is easy for ACP oscillators which have better performance than Hopf. In the terms of network topologies, ACP oscillators have the best results for all coupled types and motions. In addition, forward and turning motions are also performed together in SimMechanics environment during 12 s with ACP oscillators. Amplitude is chosen as 18 and bias is chosen as 0. After 10 s, bias is set to 1.25 and snapshots are given in Fig. 10.

![Fig. 10. Snapshots of SimMechanics for forward and turning motions of the snake robot. It is noticed that After 10 s, bias is set to 1.25](image)

From all simulation analysis for forward and turning motions, ACP oscillators with bidirectional chain network structure give effective, smooth and robust control performance to solve multi-link locomotion problems of the snake robot. Thus, a base of more intelligent locomotion control approach with ACP network is provided and some critical comparisons are performed.

### 4. Conclusions

In this paper, the designed snake robot with a multi-link tail mechanism is analyzed with CPG based control architecture. CPG models are realized with ACP and and Hopf oscillators to develop rhythmic and robust locomotion patterns. The designed snake robot is modelled in SolidWorks and it is also converted to SimMechanics model to analyze the S-type forward and C-type turning motions. The network models are developed as bidirectional chain types and each oscillator outputs are applied to joints, which are actuated with RC servo motors. The optimum results are obtained by determining the appropriate CPG parameter values. Simulation results show that ACP oscillator with chain type bidirectional network exhibits smooth, effective and robust control performance for forward and turning motions. Thus, an effective locomotion CPG controller is proposed with its simple applicable structure.

In future works, a multi-link snake-like robot will be designed and implemented with three-dimensional motion patterns. Also, different types of neural CPGs will be applied to increase the locomotion abilities.

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