Manipulation of optomechanically induced transparency and absorption by indirectly coupling to an auxiliary cavity mode

Guo-qing Qin,1 Hong Yang,1 Xuan Mao,1 Jing-wei Wen,1 Min Wang,1 Dong Ruan,†,‡ and Gui-lu Long1,2,3,

1State Key Laboratory of Low-Dimensional Quantum Physics and Department of Physics, Tsinghua University, Beijing 100084, P.R.China
2Beijing Information Science and Technology National Research Center, Beijing 100084, China
3Beijing Academy of Quantum Information Sciences, Beijing 100193, China
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We theoretically study the optomechanically induced transparency (OMIT) and absorption (OMIA) phenomena in a single microcavity optomechanical system, assisted by an indirectly-coupled auxiliary cavity mode. We show that the interference effect between the two optical modes plays an important role and can be used to control the multiple-pathway induced destructive or constructive interference effect. The three-pathway interference could induce an absorption dip within the transparent window in the red sideband driving regime, while we can switch back and forth between OMIT and OMIA with the four-pathway interference. The conversion between the transparency peak and absorption dip can be achieved by tuning the relative amplitude and phase of the multiple light paths interference. Our system proposes a new platform to realize multiple pathways induced transparency and absorption in a single microcavity and a feasible way for realizing all-optical information processing.

I. INTRODUCTION

In the past decades, electromagnetically induced transparency (EIT) has been studied both theoretically and experimentally Ref. [1, 2]. And the potential applications range from ultraslow light propagation, quantum information storage, to the enhancement of nonlinear processes. Normally, EIT is a coherent phenomenon due to the destructive quantum interference of two excitation pathways in a three-level system. On the other hand, the electromagnetically induced absorption (EIA) is the result of constructive interference between different pathways. Recently, the EIT effects are widely studied in the optical microresonators, i.e. the so-called EIT-like effect. Usually, this effect can be generated through a high quality factor (Q) cavity mode directly or indirectly coupling to a low Q one. In this case, a sharp transparent window at their original resonant frequency region appears when the two modes are frequency overlapped.

Based on the strong optical and mechanical interaction, optomechanical systems have potential applications in the fundamental research and provide a promising platform for exploring quantum nonlinear phenomena, such as ground state cooling, entanglement in cavity optomechanical system. Meanwhile, the cavity optomechanics could enable the exploration of a variety of optical processes. The system has been studied and applied in OMIT Ref. [50], OMIA Ref. [51, 52], quantum information processing Ref. [62] and amplification Ref. [63].

Analog to the EIT (EIA) in the atomic system, the OMIT (OMIA) effect is the result of destructive (constructive) interference between different pathways in optomechanical system. The OMIT effect has been investigated both theoretically and experimentally Ref. [50, 51, 57, 69, 70]. Compared with OMIT, the OMIA system generally consists of more degrees of freedom or more than two transition pathways for the probe Ref. [13, 61, 71–75]. The OMIT could be switched to OMIA through multiple-pathway interference effects, accomplished by coupling to an additional microcavity or adding more mechanical resonators. However, these approaches make the system more complicated and fragile to control.

In this paper, we study the optomechanical system and multiple-pathway interference in a single microcavity, in which one cavity optical mode indirectly couples to an auxiliary cavity mode. As shown schematically in Fig.1, the optomechanical system contains two optical modes and one mechanical mode. The auxiliary cavity mode $a_2$ is adjacent to the dominant optical mode $a_1$ in frequency domain and can be tuned to overlap with $a_1$. Differ from the directly coupled cavity case Ref. [71, 72], the two optical modes here have no direct couple. The interference effect between the two optical modes are mediated by the tapered fiber and could induce the EIT-like phenomenon without the optomechanical interaction. Combining the EIT-like and OMIT effect, we can easily realize multiple-pathway interference. When the auxiliary cavity mode decouples to the phonons, the three-pathway interference will induce constructive interference effect. We find in the red sideband driving regime, the transmission rate of the system is sensitive to the auxiliary cavity mode. While the auxiliary cavity mode couples to the mechanical mode, the four-pathway interference could induce destructive or constructive interference effect. And the bi-directional transformations between OMIT and OMIA can be achieved. We also study the transmission of the three-pathway interference case in the blue side-
The auxiliary cavity mode $a_2$ can be tuned to overlap with the dominant optical mode $a_1$ in frequency domain. The optical mode $a_2$ can be excited by the waveguide simultaneously with two optical modes and a mechanical resonator. The coupling between the auxiliary cavity mode and the mechanical mode has the frequency $\omega_{20}$ can couple to the mechanical mode. And we discuss the optical response of probe field both in red and blue sideband regime. The optical amplification mainly depends on the optomechanical gain and could enhanced by the auxiliary cavity mode. Therefore, the output spectrum can be precisely controlled by tuning the auxiliary cavity mode and the optomechanical coupling strength. The paper is organized as follows. In Sec II, we introduce the theoretical model of the optomechanical system shown in Fig.1. In Sec III, we study the physical origins of the three light propagation paths when the auxiliary cavity mode decouples to the mechanical mode. And we discuss the optical response of probe field both in red and blue sideband driving regime. In Sec IV, the bi-directional conversion between OMIT and OMIA can be achieved through the four-pathway interference effect. In Sec V, the conclusion is given.

II. MODEL

As schematically shown in Fig.1, the system we considered here is a whispering-gallery-mode optomechanical microresonator, where two optical modes and one mechanical mode have been excited. The dominant optical mode $a_1$ has the resonant frequency $\omega_1$ and the internal loss $\kappa_{10}$. The auxiliary cavity mode $a_2$ has the resonant frequency $\omega_2$ and the internal loss $\kappa_{20}$. The mechanical resonator has the frequency $\omega_m$ with the effective mass $m$. The optical modes $a_1$ and $a_2$ are two independent WGMs. However, the cavity mode $a_1$ can couple to the mode $a_2$ indirectly through the waveguide [17]. The interference between the two optical pathways results in EIT-like line shape. Both optical modes could couple to the mechanical resonator with different optomechanical coupling rate $g_1$ and $g_2$. As shown in Fig.1 (c) and (d), the optomechanical coupling strength between the auxiliary cavity mode and the mechanical mode is weak. When the optomechanical coupling $g_2$ can be ignored, the system shows three pathways interference. Without loss of generality, the system shows four pathways interference when both optical modes couple to the mechanical mode. The Hamiltonian can be described as

$$H = \hbar \omega_1 a_1^\dagger a_1 + \hbar \omega_2 a_2^\dagger a_2 + \frac{1}{2} m \omega_m^2 x^2 + \frac{p^2}{2m} + \int_{-\infty}^{+\infty} \hbar \omega c^\dagger (\omega) c (\omega) d\omega + \hbar g_1 x a_1^\dagger a_1 + \hbar g_2 x a_2^\dagger a_2$$

$$+ \hbar \sum_{j=1,2} \kappa_{ex,j} (\omega) [c^\dagger (\omega) a_j - a_j^\dagger c (\omega)] d\omega$$

(1)

where the operators $x$ and $P$ represent the position and momentum of the mechanical mode, respectively. The annihilation operator $c(\omega)$ indicates the waveguide mode, which satisfies $[c(\omega), c^\dagger (\omega')] = \delta (\omega - \omega')$. $\kappa_{ex,j} (\omega) (j=1,2)$ describes the coupling constant between the cavity mode $a_j$ and waveguide modes. The first four terms represent the free Hamiltonians of the optical and mechanical modes. The fifth term is to describe the Hamiltonian of the waveguide modes. And the last term indicates the coupling between the optical modes and the waveguide modes. In our scheme, the system is driven by a strong control laser field with the amplitudes $\varepsilon_s$ and frequency $\omega_s$, is applied on the system. The dynamics of the optomechanical system can thus be described in the rotating frame at the pump frequency $\omega_p$

$$\frac{da_1}{dt} = i \Delta_1 a_1 - \frac{\kappa_1}{2} a_1 - ig_1 x a_1 - \sqrt{\kappa_{ex1} \kappa_{ex2}} \frac{a_2}{2}$$

$$+ \sqrt{\kappa_{ex1} \varepsilon_p} + \sqrt{\kappa_{ex1} \varepsilon_s} e^{-i \delta t} + \sqrt{\kappa_{ex1} \xi_1}$$

(2)

$$\frac{da_2}{dt} = i \Delta_2 a_2 - \frac{\kappa_2}{2} a_2 - ig_2 x a_2 - \sqrt{\kappa_{ex2} \kappa_{ex2}} \frac{a_1}{2}$$

$$+ \sqrt{\kappa_{ex2} \varepsilon_p} + \sqrt{\kappa_{ex2} \varepsilon_s} e^{-i \delta t} + \sqrt{\kappa_{ex2} \xi_2}$$

(3)

$$\frac{dx}{dt} = \frac{P}{m}$$

(4)

$$\frac{dP}{dt} = -m \omega_m^2 x - \hbar (g_1 a_1^\dagger a_1 + g_2 a_2^\dagger a_2) - \frac{\Gamma_m}{2} P + \delta F$$

(5)

where $\Delta_1 = \omega_p - \omega_1$ and $\Delta_2 = \omega_p - \omega_2$ represent the detunings of optical modes with respect to the driving field. $\kappa_1 = \kappa_{10} + \kappa_{ex1}$ indicates the total decay rate of optical mode $a_1 (j=1,2)$. $\delta = \omega_p - \omega_g$ is the frequency detuning between the probe field and the control field. And $\xi_1$ ($\xi_2$) is the external noise of the optical mode $a_1$ ($a_2$) introduced by the waveguide. $\delta F$ is the thermal noise of the mechanical oscillator and $\Gamma_m$ is the decay rate of
the mechanical mode. In our case, the pump laser field is much stronger than the probe field. By using the linearization approach, thus the Heisenberg operators can be divided into the steady parts and the fluctuation ones, i.e., \( a_j = \tilde{a}_j + \delta a_j \) (\( j = 1, 2 \)) and \( x = \tilde{x} + \delta x \). Substituting the division forms into Eq. (2), the steady solutions of the above dynamical equations can be obtained as

\[
\begin{align*}
\tilde{a}_1 &= \frac{-\sqrt{\kappa_{ex1}}}{i\Delta_1 - ig_1\tilde{x} - \kappa_1/2 - \sqrt{\kappa_{ex1}\kappa_{ex2}}/2} \delta a_2 \\
\tilde{a}_2 &= \frac{-\sqrt{\kappa_{ex2}}}{i\Delta_2 - ig_2\tilde{x} - \kappa_2/2} \delta a_1 \\
\tilde{x} &= -\hbar g_1|\tilde{a}_1|^2 + g_2|\tilde{a}_2|^2/m\omega_m
\end{align*}
\]

Then, we only keep the first-order terms in the small fluctuation ones \( \delta a_1, \delta a_1^\dagger, \delta a_2, \delta a_2^\dagger \) and \( \delta x \). Under these conditions, we can obtain the linearized Langevin equations as follows:

\[
\frac{d\delta a_1}{dt} = (i\Delta_1 - \frac{\kappa_1}{2})\delta a_1 - ig_1\tilde{a}_1\delta x - \frac{\sqrt{\kappa_{ex1}\kappa_{ex2}}}{2} \delta a_2 + \sqrt{\kappa_{ex1}\kappa_{ex2}} e^{-i\delta t} + \sqrt{\kappa_{ex1}}\xi_1
\]

\[
\frac{d\delta a_2}{dt} = (i\Delta_2 - \frac{\kappa_2}{2})\delta a_2 - ig_2\tilde{a}_2\delta x - \frac{\sqrt{\kappa_{ex1}\kappa_{ex2}}}{2} \delta a_1 + \sqrt{\kappa_{ex1}\kappa_{ex2}} e^{-i\delta t} + \sqrt{\kappa_{ex2}}\xi_2
\]

\[
\frac{d\delta a_1^\dagger}{dt} = (i\Delta_2 - \frac{\kappa_2}{2})\delta a_1^\dagger - ig_2\tilde{a}_1^\dagger\delta x - \frac{\sqrt{\kappa_{ex1}\kappa_{ex2}}}{2} \delta a_2^\dagger + \sqrt{\kappa_{ex2}\kappa_{ex1}} e^{i\delta t} + \sqrt{\kappa_{ex2}}\xi_2^\dagger
\]

\[
m\frac{d^2\delta x}{dt^2} = -\frac{m\Gamma_m}{2} \delta x - m\omega_0^2\delta x - \hbar g_1\tilde{a}_1(\delta a_1 + \delta a_1^\dagger) - \hbar g_2\tilde{a}_2(\delta a_2 + \delta a_2^\dagger) + \delta F
\]

where \( \Delta_1 = \Delta_1 - g_1\tilde{x} \) and \( \Delta_2 = \Delta_2 - g_2\tilde{x} \) denote the effective detuning between the cavity modes and the control laser beam, including the frequency shift caused by the mechanical motion.

Combining the above analysis and Fig. 1 (c) and (d), we can better understand the multiple light paths for interference. Fig. 1 displays a schematic of quantum interference between different light paths. The photons at the output port could come from four different path. The different paths are: (1) the probe photons excite the cavity mode \( a_1 \) and pass to the output port; (2) the photons generated by the sideband transition through the optomechanical interaction in mode \( a_1 \) are coupled out the cavity through the waveguide; (3) the probe field passes through the cavity mode \( a_2 \) directly. In our case, the system only shows three different pathways when the optomechanical coupling rate \( g_2 \) can be neglected. Otherwise, the fourth pathway is: (4) the photons generated by anti-Stokes process in the optical mode \( a_2 \) are coupled out the cavity through the waveguide. Thus the photons in the output beam is the sum of four different paths. For convenience, we introduce \( x = x_{ZPF}(b + b^\dagger) \) and \( P = -i\hbar\omega_{m}\gamma_{ZPF}(b - b^\dagger) \) to get the transmission of the system, where \( x_{ZPF} \) is the zero point fluctuation. We also neglect the quantum noise \( \xi_1, \xi_2 \) and \( \delta F \). Without loss of generality, in the following discussions the frequency of the auxiliary cavity mode is tuned to satisfy \( \Delta_1 = \Delta_2 = \Delta \). According to the input-output theory[79], we can get the transmission of field \( a_{out} = a_0 - \sqrt{\kappa_{ex1}}\tilde{a}_1 - \sqrt{\kappa_{ex2}}\tilde{a}_2 \). With neglecting the high order sideband effect, the normalized transmission coefficient in the red sideband driving regime can be simplified to

\[
l_r = \frac{(L_1 - \kappa_{ex1})(L_2 - \kappa_{ex2}) - (A - \sqrt{\kappa_{ex1}\kappa_{ex2}})^2}{L_2(L_1 - \frac{A^2}{L_2})}
\]

where \( G_1 \) and \( G_2 \) are the effective optomechanical coupling strength, \( L_1 = i(\Delta - \delta) + \frac{\kappa_1}{2} + \sqrt{\kappa_{ex1}} \), \( L_2 = i(\Delta - \delta) + \frac{\kappa_2}{2} + \sqrt{\kappa_{ex2}} \), and \( A = \frac{G_1G_2}{i(\Delta - \delta) - \frac{g_1^2}{2}} + \frac{\sqrt{\kappa_{ex1}\kappa_{ex2}}}{2} \). When the system is driven by the blue-detuned pump field, the normalized transmission coefficient is

\[
l_p = \frac{(R_1 + \kappa_{ex1})(R_2 + \kappa_{ex2}) - (B - \sqrt{\kappa_{ex1}\kappa_{ex2}})^2}{R_2(R_1 - \frac{B^2}{R_2})}
\]

where \( R_1 = i(\Delta + \delta) - \frac{\kappa_1}{2} - \frac{G_1^2}{i(\Delta + \delta) - \frac{g_1^2}{2}} \), \( R_2 = i(\Delta + \delta) - \frac{\kappa_2}{2} - \frac{G_2^2}{i(\Delta + \delta) - \frac{g_2^2}{2}} \), and \( B = \frac{G_1G_2}{i(\Delta + \delta) - \frac{g_1^2}{2}} + \frac{\sqrt{\kappa_{ex1}\kappa_{ex2}}}{2} \). And the corresponding power transmission coefficient is given by \( T = |r|^2 \).

### III. TRANSMISSION RATE WITH THREE PATHWAYS INTERFERENCE

Before we discuss the three-pathway interference effect, the two-pathway induced EIT-like effect is considered. The all-optical analogues of the EIT effect are widely studied in optical resonators systems. And this EIT-like effect contains two coupling mechanism: directly coupled-resonator-induced transparency(DCRIT) [13, 15] and indirectly coupled-resonator-induced transparency(ICRIT) [17, 19]. By combining the DCRIT and OMIT effect, the three-pathway induced EIT and EIA effects has been studied in different system [13, 71, 75–77]. For the coupled cavity system, the two cavities should be tuned precisely to couple, requiring the complicated control and fabrication for experimentns. Thus here we study the three-pathway interference effect by combining the ICRIT effect and optomechanical interaction in a single cavity. In this section, we consider the situation that the auxiliary cavity mode \( a_0 \) decouples to the mechanical mode, i.e., \( g_2 = 0 \). We study the power transmission coefficient with the effect of the auxiliary cavity mode both
in the red and blue sideband driving regime. We find that the three mode interaction could be tuned by mode $a_2$, which can be used to control the transmission rate.

![Figure 2](image1.png)

**FIG. 2.** The transmission rate are plotted with the different external coupling strength $\kappa_{ex2}$ of the auxiliary cavity mode $a_2$ in the red sideband regime. The parameters used here are: $\kappa_1 = 5$ MHz, $\kappa_{ex1} = 15$ MHz, $\kappa_{20} = 40$ MHz, $\Gamma_m = 50$ KHz, $\omega_m = 100$ MHz and $\Delta = -\omega_m$.

![Figure 3](image2.png)

**FIG. 3.** The transmission rate are plotted with the different optomechanical coupling rate between the optical modes and the phonons. The parameters used here are: $\kappa_{ex1} = 15$ MHz, $\kappa_{20} = 40$ MHz, $\kappa_{ex2} = 60$ MHz, $\Gamma_m = 50$ KHz, $\omega_m = 100$ MHz and $\Delta = -\omega_m$.

**A. Three-pathway interference with red-detuned driving**

We have discussed the physical origins of the three pathway interference phenomena in Fig[1](c). The photons at the output port with frequency equaling to the probe signal come from the three pathways. When one light path is changed, the relative amplitude and phase of the multiple light paths interference is adjusted. This influence of the three-pathway interference effect will change the output spectrum. To investigate the influence of path 3 on the three-pathway interference effect, we plot the output spectrum in Fig[2](a) with different external coupling rate $\kappa_{ex2}$. In Fig[2](a), the transmission shows the OMIT window without the auxiliary mode. This is the result of the two-pathway destructive interference effect. When the optical mode $a_2$ couples with tapered fiber, the transparent peak maintains but becomes really minute plotted by blue line (@ $G_1/\kappa_{10} = 0.7$) in (b). While the absorption dip starts appearing represent by the red solid line with the weaker optomechanical coupling rate $G_1 = 1.5$ MHz. Further increasing the external coupling rate $\kappa_{ex2}$ of the auxiliary mode to 40 MHz in

![Figure 4](image3.png)

**FIG. 4.** The transmission is plotted as the function of the optomechanical coupling rate $G_1$ and the external coupling strength $\kappa_{ex2}$ of the auxiliary cavity mode. The parameters used here are: $\kappa_1 = 5$ MHz, $\kappa_{ex1} = 15$ MHz, $\kappa_{20} = 40$ MHz, $\Gamma_m = 50$ KHz, $\omega_m = 100$ MHz and $\Delta = -\omega_m$.

**FIG[2](c),** the output spectra show the evident absorption dips within a EIT-like window for both lines. The power transmission drops to zero due to the constructive interference. By tuning the external coupling rate $\kappa_{ex2}/\kappa_{10}$ to 75, the tapered fiber mediated interference effect is strong compared with the optomechanical coupling strength. Both the transmission spectrum and the absorption window become wider. While the transmis-
sion \( \delta = \omega_m \) increases to 0.33 even with the absorption dip.

Form Fig.2 we can obtain that the interference between the two optical modes could lead the transmission rate form OMIT peak to OMIA dip. The reason for the absorption windows is the constructive interference effect induced by the auxiliary cavity mode. Without the auxiliary cavity mode \( a_2 \), the output field only connects with mode \( a_1 \) and shows a OMIT peak in Fig.2 (a). When the auxiliary cavity mode is introduced, the photons with frequency equaling to the probe signal come from the optical modes \( a_1 \) and \( a_2 \) both. While the anti-stokes process leads to destructive interference for the intracavity field \( a_1 \), suppressing the photons population of \( a_1 \). The constructive interference leads an absorption dip and the transmission rate of central dip drops to zero.

Then we study the influence of the optomechanical interaction on the three-pathway interference. The transmission rate is plotted in Fig.3 with the different optomechanical coupling strength \( G_1 \). In Fig.3(a), (b) and (c), the external coupling strength \( \kappa_{ex2} \) is fixed to 60 MHz and \( G_2 = 0 \). The transmission displays the EIT-like window with \( G_1 = 0 \) plotted by the blue dashed lines. When the optomechanical coupling strength \( G_1 \) is increased to 0.4 MHz , there is a small absorption window within the EIT window in Fig.3 (a). In Fig.3 (c), the transmission of the central dip drops to 0.1 with stronger optomechanical coupling rate \( G_1 \). It is obvious that with increasing the optomechanical coupling rate, the transmission shows more prominent absorption effect.

From the Fig.2 and Fig.3, we can obtain that the three-pathway interference effect could vary from the transparent window to an absorption dip in our system. The output spectrum is sensitive to the auxiliary cavity mode in Fig.2 which can be used to control the three mode interaction. While in Fig.3 the depth of the absorption dip can be manipulated by the path (2). This system offers two effective methods to tune the optical response. The transmission is plotted in Fig.4 as the function of the optomechanical coupling rate \( G_1 \) and the external coupling strength \( \kappa_{ex2} \) of the auxiliary cavity mode. It can be obtained that with the weak optomechanical coupling rate, the effect of two optical modes interference becomes more evident and leads to transparency-like window. While the transmission shows OMIT window when \( \kappa_{ex2} \) is weak. Thus the transmission can be manipulated through tuning the relative amplitudes and phases of the three-pathway interference.

B. Three-pathway induced amplification with blue-detuned driving

In our system, the photons generated form the optomechanical interaction will directly affect the intracavity probe photons population, which could result in OMIA dip in the red sideband pumping regime. Now we consider the situation that the system is driven by a blue-detuned driving field and the auxiliary optical mode decouples to the mechanical mode. In this scheme we limit our parameters to the stable driving regime. And the stability condition can be obtained by analyzing the Lyapunov exponents [30] of the Jacobian matrix. We have plotted the border between the stable and unstable condition in the inset of Fig.5(c). In Fig.5 we have shown the the normalized output spectra of optomechanical sys-
tem with a blue-detuned pump field. In Fig.5 (a), we plot the transmission rate as a function of $\delta$ for different values of external coupling rate $\kappa_{ex2}$. The red solid line indicates the transmission with optomechanical coupling rate $G_1 = 0.3$ MHz while $\kappa_{ex2} = 0$. Without the auxiliary optical mode, it is clear that the transmission line shows an amplified peak. With the enhancement of the two optical mode interference through increasing $\kappa_{ex2}$, the transmission becomes broader and shows the EIT-like window. As the inset of Fig.5 (a) shows, the transmission at the central peak could enhanced with stronger external coupling strength of the auxiliary optical mode.

We also plot the transmission amplification effect for different values of the coupling strength $G_1$ in Fig.5 (b). Here $\kappa_{ex2}$ is fixed to 60 MHz, and $G_1$ is increased to 0.3 MHz. The red solid line shows a EIT-like window without optomechanical interaction. It is clear that the transmission rate of central peak increases to 1.75 with $G_1 = 0.3$ MHz compared with red line. In Fig.5 (b), the transmission amplification effect of the probe field becomes stronger with increasing the optomechanical coupling rate $G_1$. Fig.5 (c) plots the transmission rate of the central peak as a function of the coupling rate $G_1$ when the probe frequency detuning $\delta = \omega_m$. The blue (red) solid line plots the amplification effect when $\kappa_{ex2}$ equals to 10 (60) MHz. When $G_1$ is small, the fiber mediated two optical interference is strong, the transmission rate depends on $\kappa_{ex2}$. For strong optomechanical coupling rate, the amplification depends on $G_1$, while the transmission can be affected by the auxiliary cavity mode.

By increasing the optomechanical coupling rate $G_1$, the stokes process is enhanced and emits photons and phonons. The photons generated by the optomechanical interaction are degenerate with the probe field, which accelerates the circulating power of $a_1$. The two optical interference is weak compared with the optomechanical coupling. Thus the optomechanical amplification plays a dominant role in the three pathways. The closer to the stability border, the more prominent the optomechanical gain effect is. However, even the central peak is not sensitive to $\kappa_{ex2}$, the amplification effect could be enhanced by the two optical interference.

IV. TRANSMISSION RATE WITH FOUR-PATHWAY INTERFERENCE EFFECT

In this section, we will consider the situation that the auxiliary cavity mode couples to the mechanical mode. Here we assume that the effective optomechanical coupling strength $G_1$ and $G_2$ can be tuned separately by using the extra tapered fiber to introduce the extra pump field.

To investigate the effect of optomechanical coupling rate $G_1$, $G_2$ on the transmission spectrum, we plot the transmission with the different optomechanical coupling rates $G_1$ and $G_2$ in Fig.6 with the red-detuned driving. Under the three pathways situation in red sideband regime, the transmission rates show an absorption window, which have been presented in Fig.5. Through increasing the optomechanical coupling rate $G_1$, the absorption dip becomes deeper and wider. Different with three pathways induced absorption, four pathways interference effect can switch the transmission spectrum back
and forth between OMIT and OMIA depending on the coupling strength $G_1$ and $G_2$. In Fig. 6(a), when $G_2 = 0$, the transmission of the central dip is 0.13 which has been presented by the blue dashed line. While the absorption dip becomes more evident in the middle of transmission window when $G_1 = 0.6$ MHz and $G_2 = 0.2$ MHz. The transmission drops to 0 due to constructive interference effect. By keeping $G_1 = 0.6$ MHz unchanged, the transmission rate is plotted in Fig. 6(b) when the coupling rate $G_2$ is enlarged to 1 MHz. In contrast to the absorption dip for the coupling rate $G_2 = 0.2$ MHz, here the OMIT peak appears for the resonant case, making the transmission go up to 0.75. By increasing the coupling rate $G_2$ to 1.8 MHz, the constructive interference between different pathways results in the OMIA again. To show the manipulation between OMIA and OMIT through tuning the ratio between $G_1$ and $G_2$, the height of the central peak is plotted in Fig. 6(c). It is clear that when the optomechanical coupling strength $G_2$ is weak, the four-pathway interference will enhance the constructive interference and lead to the OMIA effect. Further increasing $G_2$, the four pathways will result in the destructive interference and OMIT window. While $G_2$ is larger than 1.5 MHz, the constructive interference occurs, resulting in the absorption window again. In Fig. 6(d), we plot the transmission rate as a function of different coupling rate $G_1$ and $G_2$. We can obtain that the conversion between the constructive and destructive interference can be achieved by tuning the coupling strength of $G_1$ and $G_2$. Compared with $T = 0.51$ when the coupling rate $G_1 = G_2 = 0$, transmission can be switched to 0 or 1 due to the OMIA or OMIT effect.

V. CONCLUSION

In summary, we have explored an optomechanical system with multiple light paths interference effect in a single optical resonator. We give the explicit physical explanations and detailed calculations in this paper. By combining the EIT-like and OMIT effect, the system shows destructive or constructive interference effect under different conditions. The auxiliary cavity mode offers the additional light pathway to control the output spectrum. Through enhancing the optomechanical coupling strength or the external coupling rate of the auxiliary cavity mode, we can tune the optical response of the probe field effectively. Experimentally, the system has no limitation on the quality factor of the auxiliary cavity mode, making it easy to implement. Moreover, our model paves an easy way for realization of multiple interference and manipulation of OMIT and OMIA.

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