Beyond second-order convergence in simulations of magnetised binary neutron stars with realistic microphysics

Elias R. Most\textsuperscript{1,2}*, L. Jens Papenfort\textsuperscript{1}, L. Rezzolla\textsuperscript{1,3}

\textsuperscript{1} Institut für Theoretische Physik, Goethe Universität Frankfurt am Main, Germany
\textsuperscript{2} Center for Computational Astrophysics, Flatiron Institute, 162 Fifth Avenue, New York, NY 10010, USA
\textsuperscript{3} School of Mathematics, Trinity College, Dublin 2, Ireland

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

We investigate the impact of using high-order numerical methods to study the merger of magnetised neutron stars with finite-temperature microphysics and neutrino cooling in full general relativity. By implementing a fourth-order accurate conservative finite-difference scheme we model the inspiral together with the early post-merger and highlight the differences to traditional second-order approaches at the various stages of the simulation. We find that even for finite-temperature equations of state, convergence orders higher than second order can be achieved in the inspiral and post-merger for the gravitational-wave phase. We further demonstrate that the second-order scheme overestimates the amount of proton-rich shock-heated ejecta, which can have an impact on the modelling of the dynamical part of the kilonova emission. Finally, we show that already at low resolution the growth rate of the magnetic energy is consistently resolved by using a fourth-order scheme.

Key words: methods:numerical – MHD – gravitational waves – stars:neutron

1 INTRODUCTION

In the era of gravitational wave and multi-messenger astronomy of binary neutron stars accurate numerical modelling of neutron-star mergers and their remnants on long timescales $\approx 1$ s has never been more important. The coincident detection of a gravitational-wave signal from a neutron-star merger (The LIGO Scientific Collaboration & The Virgo Collaboration 2017) and an accompanying electromagnetic counterpart in form of a short gamma ray burst (LIGO Scientific Collaboration et al. 2017) and kilonova afterglow (Kasen et al. 2017; Drout et al. 2017) has established a firm connection with electromagnetic counterparts and highlights the need for multiphysics modelling of neutron-star mergers. A neutron-star merger consists of several stages starting from the late inspiral, where accurate numerical waveforms are needed to calibrate analytical models for waveforms, through merger (Kawaguchi et al. 2018; Nagar et al. 2018; Dietrich et al. 2019), which requires sophisticated microphysics in terms of finite temperature equations of state (EOS) satisfying recent observational constraints (Annala et al. 2018; Most et al. 2018; Tews et al. 2018; Burgio et al. 2018; Raithel et al. 2018) until the post-merger phase where neutrino and magnetic viscosity can drive large amounts of mass ejection (Just et al. 2015; Siegel & Metzger 2017, 2018; Fernández et al. 2018; Fujibayashi et al. 2018), that are needed to make a connection to the kilonova afterglow produced by the decay of heavy elements in the matter outflow. The modelling of this complicated multi-physics system requires both the use of numerical relativity (Baiotti & Rezzolla 2017; Duez & Zlochower 2019) and an accurate modelling of the fluid, the electromagnetic fields as well as the microphysics. Considerable effort has been placed on improved and highly accurate methods to solve Einstein field equations numerically (Baumgarte & Shapiro 2010; Shibata 2016) and to couple them with high-order methods for relativistic hydrodynamics (Radice et al. 2014a;Bernuzzi & Dietrich 2016). At the same time, mainly driven by an effort to model core-collapse supernovae, very sophisticated numerical schemes for neutrino transport have been developed (Ruffert et al. 1996; Buras et al. 2006; Shibata et al. 2011; Suniyoshi & Yamada 2012; Foucart et al. 2015; Just et al. 2015). When considering the late stages of the evolution of the system not only is it important to account for the various relevant physics contributions, such as neutrino interactions, but it is also crucial to understand how numerical errors at finite numerical resolution accumulate over time. This is even highlighted by the fact that current simulations of neutron-star mergers sometimes show non-convergent behaviour in the magnetic field evolution (Endrizzi et al. 2016; Ciolfi et al. 2017) even when small resolution changes are used. Notwithstanding, that with current computational ef-
ficiencies and available resources not even all physical scales involving magnetic turbulence can be resolved (Kiuchi et al. 2015b, 2018), studying the late time evolution of the remnants accretion disc is not only feasible but has been the subject of recent investigations (Siegel & Metzger 2017, 2018; Fernández et al. 2018). While all such simulations so far have used traditional second-order accurate finite volume schemes to model the evolution of the general-relativistic magnetohydrodynamics system (GRMHD), earlier works have already indicated the benefit of using more accurate high-order methods in this context (Del Zanna et al. 2007; Tchekhovskoy et al. 2007; Radice et al. 2014a), while even more recent studies have already started to consider advanced finite-element approaches (Kiddler et al. 2017; Fambri et al. 2018). Taking an intermediate approach similar to (Del Zanna et al. 2007; McCorquodale & Colella 2011; Chen et al. 2016; Felker & Stone 2018) we will consider the impact of using a fourth-order accurate numerical scheme to model the merger of magnetised binary neutron stars and show the advantages gained when additionally finite-temperature effects and neutrino cooling are included.

2 FORMULATION AND METHODS

In this study we solve the GRMHD equations in dynamical spacetimes (see Duez et al. 2005; Giacomazzo & Rezzolla 2007 for an overview), together with realistic microphysics. We will start by giving a brief overview of the system of equations and will then discuss details of the implementation.

The space-time is described by the metric $g_{\mu\nu}dx^\mu dx^\nu$ decomposed using the 3+1 split of space-time (Alcubierre 2000; Rezzolla & Zanotti 2013), that can be written as

$$ds^2 = \gamma_{\mu\nu}dx^\mu dx^\nu.$$  

The fluid is described by the baryonic rest-mass density $\rho$, the electron fraction $Y_e$ and the temperature $T$. Using these quantities the pressure $p(\rho, T, Y_e)$ and the specific internal energy $\epsilon(\rho, T, Y_e)$ can be computed from the equation of state (EOS), which closes the set of equations. Although this study highlights the benefit of high-order schemes for neutron-star simulations with realistic finite-temperature EOS, the latter equally applies also to approaches using simpler EOS.

The dynamics of the fluid is governed by the fluid four velocity $u^\mu = (u^0, \mathbf{v})$, $\mathbf{v}$ is the three velocity of the fluid and $W = \sqrt{1-v^2}$ the Lorentz factor.

The evolution of the electromagnetic field is described by the Faraday tensor $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$, where $A_\mu = (\Phi, A_i)$ is the vector potential, and using the Lorentz gauge (Etienne et al. 2012a) $\nabla_\mu F^{\mu\nu} = 0$ the Maxwell equations in the ideal MHD limit can be written as (Baumgarte & Shapiro 2003; Etienne et al. 2012a)

$$\partial_t A_i - \epsilon_{ijk} \mathbf{v}^k A^j = 0, \quad \partial_t (\sqrt{\gamma} \Phi) + \partial_j (\sqrt{\gamma} A^j - \beta^j \sqrt{\gamma} \Phi) = -\xi \beta \sqrt{\gamma} \Phi,$$

where $\sqrt{\gamma} \mathbf{B} = \epsilon^{ijk} \partial_j A_k$ is the magnetic field as seen by the normal observer, $\epsilon^{ijk}$ is the totally antisymmetric Levi-Civita tensor, $\gamma^i = \alpha^i - \beta^i$ is the transport velocity and $\xi^{-1}$ is the damping time scale of the gauge variable. It is also useful to define the magnetic field in the fluid rest-frame, i.e., $4\pi b_\mu = (g_{\mu\nu} + \mu_\mu u_\mu) B^\nu$.

The dynamics of the fluid is governed by the energy momentum tensor $T_{\mu\nu}$ (Baumgarte & Shapiro 2003; Shibata & Sekiguchi 2005)

$$T_{\mu\nu} = \left( \rho (1 + \epsilon) + b^2 + p \right) u_\mu u_\nu + \left( p + \frac{b_\mu b_\nu}{2} \right) g_{\mu\nu} - b_\mu b_\nu.$$  

which gives rise to the equations of hydrodynamics via $\nabla_\mu T^{\mu\nu} = 0$. Using the 3+1 split these equations can be recast as follows (Baumgarte & Shapiro 2003; Duez et al. 2005; Shibata & Sekiguchi 2005; Giacomazzo & Rezzolla 2007)

$$\partial_t \rho + \partial_j (\rho \mathbf{v}^j) = 0,$$

$$\partial_t \epsilon + \partial_j (\epsilon \mathbf{v}^j) = \frac{1}{\gamma} \partial_k \left( \nabla_\mu T^{\mu\nu} \partial_\nu \right),$$

$$\partial_t \tau + \partial_j (\tau \mathbf{v}^j) = \frac{1}{\gamma} \partial_k \left( \nabla_\mu T^{\mu\nu} \partial_\nu \right) + 2T^{0j} \partial_\nu \mathbf{B}^j + T^{ij} \partial_\nu K_{ij},$$

$$\partial_t \mathbf{v} + (\alpha \mathbf{v}^i - \beta^i \mathbf{v}) = \mathbf{0},$$

which are already written in conservative form using $\rho_s = \sqrt{\gamma} \rho W$, $S_i = \left( \rho_s h W + \sqrt{\gamma} B^2 \right) - \left( B^i \mathbf{v}^i \right)$, $\sqrt{\gamma} B_i$, $\tau = \rho_s (h W - 1) - \sqrt{\gamma} \rho + \sqrt{\gamma} B^2 - \frac{\gamma}{2} \left[ \mathbf{B}^2 \left( 1 - \gamma \right) + \left( B^0 \mathbf{v}^i \right)^2 \right]$, given as function of the primitive quantities $\rho, \mathbf{v}, \epsilon$. Here $h = 1 + \epsilon + p/\rho$ is the specific enthalpy.

2.1 Numerical methods

In the following we will briefly present the numerical methods used in this work. We will start by reviewing how to obtain high-order schemes for the GRMHD equation, before discussing the various parts necessary for numerically stable high-order GRMHD simulation involving realistic EOS and neutrino cooling.

2.1.1 High-order HRSC methods

The GRMHD equations need to be solved using high resolution shock capturing (HRSC) methods in order to address discontinuities that may appear both in the magnetic field as well as in the hydrodynamic variables, see Toro (2009) for an overview. Such equations are solved in discretized forms, i.e., an equation of the form

$$\partial_t U + \partial_i F = 0,$$

with state $U$ and flux $F$ is recast into

$$\partial_t \tilde{U} + \frac{1}{Ax} \left[ \tilde{F}_{i+1/2} - \tilde{F}_{i-1/2} \right] = 0,$$

where $\tilde{U}$ and $\tilde{F}$ are suitable discretisations on a computational grid $x^i$. Typically such equations are solved in a finite
are evaluated over a cell centered on $x^i$ and its adjacent faces, respectively. At second order the integral can be approximated by the value at the midpoints, i.e., $\hat{U}^i = U^i$ is evaluated at the centre $x^i$ and $F^{i+1/2}$ is computed at the interface between $x^i$ and $x^{i+1}$. The values of $U$ necessary to compute $F$ are then computed by simply reconstructing $U$ to the cell interface using a limited interpolation $\mathcal{R}$ that is shock-aware, and then applying a suitable Riemann solver to provide suitable upwinding in the computation of $\hat{F} = F(\mathcal{R}[U])$. This is the most common approach and is implemented in most simulation codes for compact systems (Etienne et al. 2015; Duez et al. 2005; Giacomazzo & Rezzolla 2007; Foucart et al. 2013; Liebling et al. 2010; Mősta et al. 2014). A different approach consists in interpreting (12) in a finite-difference sense, i.e.,

$$\hat{U} = U, \quad \hat{F} = \mathcal{R}[F],$$

where $\mathcal{R}$ is a limited interpolation, similar to the reconstruction of $\hat{U}$ in the finite volume approach, to which an additional upwinding formula similar to a Riemann solver in the computational volume is applied, such that the order of the method is solely set by the order of the interpolation routine $\mathcal{R}$, whereas in the finite volume sense the order is additionally set by the numerical approach used to evaluate the volume and surface integrals in (13) (McCorquodale & Colella 2011; Folker & Stone 2018).

Typically, such a finite difference approach requires a characteristic decomposition of $U$ at every step in order to reduce spurious oscillations (Shu 2003). We have in fact found that when not performing this step while using a realistic EOS, it can lead to large oscillations and unphysical states in low density regions. Instead, we opt for a different approach, the so-called ECHO scheme (Del Zanna et al. 2007, 2003; Del Zanna & Bucciantini 2002), in which (14) is split in a reconstruction and (high-order) derivative step $\mathcal{D}$, such that

$$\hat{F} = \mathcal{D}[F(\mathcal{R}[U])].$$

Here, first the primitive variables are reconstructed to the cell interfaces, then a suitable Riemann solver is used to compute the flux $F$ and an additional correction step $\mathcal{D}$ is applied, such that (12) is a high-order approximation to (11).

Most importantly, at second order this step can be omitted and the above finite volume scheme is recovered. At higher orders this is not the case and at fourth order one finds (Del Zanna et al. 2007)

$$\hat{F} = F(\mathcal{R}[U]) - \frac{\Delta x^2}{24} F''(\mathcal{R}[U]),$$

where the second derivative can be approximated using the standard finite difference expression

$$F''_{i+1/2} = \frac{F_{i+3/2} + F_{i-3/2} - 2F_{i+1/2}}{\Delta x^2}. \quad (17)$$

It should be remarked that this fixed stencil choice can in principle induce oscillations near shocks, but in practice is extremely robust as has been shown in (Del Zanna et al. 2007; Chen et al. 2016). On the other hand such a choice is necessary for constraint transport schemes as the flux gradient and the curl operator used to compute the magnetic field from the vector potential have to always cancel out to machine precision. Although it is possible to also use the fourth derivative $F^{(4)}$ to increase the formal convergence order to 6, we will restrict ourselves in this work to fourth-order as it has been shown in previous (formally) high-order purely hydrodynamical studies that e.g. convergence of the phase shift in gravitational waves converges at most at third order (Radice et al. 2014b, c; Bernuzzi & Dietrich 2016).

### 2.1.2 Implementation

We solve the coupled Einstein-GRMHD system using the Frankfurt/IllinoisGRMHD code (FIL), which is a high order extension of the publicly available IllinoisGRMHD code (Etienne et al. 2015) that is part of the Einstein Toolkit (Löffler et al. 2012). In the following we will give an overview of the numerical and implementation details.

To solve the Einstein equations FIL provides its own spacetime evolution module, which implements the Z4c (Hilditch et al. 2013; Bernuzzi & Hilditch 2010) and CCZ4 (Alic et al. 2012, 2013) formulations using forth order accurate finite differencing (Zochower et al. 2005) with different choices for the conformal factor. In this work we choose $\psi^{-2}$ and adopt the Z4c formulation with a damping coefficient $\kappa = 0.02$ (Weyhausen et al. 2012; Hilditch et al. 2013). The space-time gauges are evolved using the standard $1+\log$ slicing and shifting-shift Gamma driver conditions (Alcubierre 2008; Baumgarte & Shapiro 2010; Rezzolla & Zanotti 2013), where a uniform damping parameter of $\eta = 2/M$ is adopted. The implementation makes use of modern compile-time evaluated C++14 templates provided by the TensorTemplates library and can be vectorised using parallel datatypes (Kretz & Lindenstruth 2012).

The GRMHD equations (5) - (7) are solved using the ECHO scheme (Del Zanna et al. 2007) as given in (16), making our code overall formally fourth-order accurate. The fluxes are computed from the reconstructed primitive variables ($\rho, T, \chi, W, \nabla \times B$) using a HLLE Riemann solver (Harten et al. 1983). The reconstruction step $\mathcal{R}$ is performed using the WENO-Z method (Borges et al. 2008), with the optimal weights and stencils for a conservative finite difference scheme taken from (Del Zanna et al. 2007). We point out that these are different from those for finite volume or traditional finite difference approaches (Borges et al. 2008). The magnetic fields are evolved via a vector potential $A_\perp$ and the gauge field $\Phi$. The use of a vector potential trivially allows us to generalise the ECHO scheme to a multi-grid context, since the magnetic field is recomputed at every iteration from the vector potential $A_\perp$, thus preserving the divergence constraint. In order to maintain high-order convergence in this step the derivative correction $\mathcal{D}$ also needs...
to apply in this context, specifically we compute
\[
\sqrt{\mathcal{B}_{i+1, j, k}^x} = -\frac{\mathcal{D}[A_{i+1, j, k+\frac{1}{2}}] - \mathcal{D}[A_{i+1, j, k-\frac{1}{2}}]}{\Delta z} + \frac{\mathcal{D}[A_{i+\frac{1}{2}, j, k+\frac{1}{2}}] - \mathcal{D}[A_{i+\frac{1}{2}, j, k-\frac{1}{2}}]}{\Delta y}.
\]
(18)

Different from (Etienne et al. 2015) we implement the upwind constraint transport scheme as in (Del Zanna et al. 2007) in which the staggered magnetic fields \(\sqrt{\mathcal{B}}\) are reconstructed from two distinct directions to the cell edges, which we found greatly minimises diffusion related to a dimensional bias compared to the original implementation (Del Zanna et al. 2003) when used with a high-order scheme. The cell-centered magnetic fields are always interpolated from the staggered ones using fourth-order unlimited interpolation in the \(i\)-th direction for \(B^i\), in which this magnetic field component is continuous (Londrillo & Del Zanna 2004). A potential downside of high order schemes is that they can cause oscillations at sharp discontinuities, e.g., at the surface of the neutron star. In order to remedy such behaviours we follow the approach of (Radice et al. 2014a) and hybridise the high-order flux with a first order local Lax-Friedrichs flux based on a positivity preserving criterion for the conserved density \(\rho\) (Hu et al. 2013). It should be remarked that this step has to be done before the derivative correction (16) is applied, in order to be compatible with discrete \(\nabla\cdot B\)-constraint (Del Zanna et al. 2007). While this no longer exactly guarantees positivity of the conserved density, since the hybridisation coefficient is computed without accounting for the \(D\) correction, we find that stability is, nonetheless, improved and failures at the primitive inversion stage are greatly reduced allowing for a stable evolution of the neutron-star surface during the inspiral.

2.2 Microphysics

In light of the kilonova detection AT2017gfo (Cowperthwaite et al. 2017) modelling the mass ejection and its microphysical composition has become crucial for any study of neutron-star mergers including at least a basic treatment of weak interactions and finite-temperature EOS. Consequently, the FIL code includes a framework to handle tabulated finite-temperature EOS either in the StellarCollapse1 or in the CompOSE format. Any quantity \(Q(\rho, T, Y_e)\) in the EOS is then treated by means of a three-dimensional linear interpolation. Equilibrium weak interactions are computed following the approach laid out in (Ruffert et al. 1996; Rosswog & Liebendörfer 2003) and implemented in (O’Connor & Ott 2010; Galeazzi et al. 2013; Neilsen et al. 2014), including

- (inverse) \(\beta\)-decay
- electron-positron pair annihilation
- plasmon decay
- neutrino scattering on heavy nuclei and free nucleons
- electron-flavor neutrino absorption on free nucleons

Presently, neutrino interactions are included via a leakage approach (Rosswog & Liebendörfer 2003; Sekiguchi 2010a; Deaton et al. 2013; Galeazzi et al. 2013), which accounts for the effects of neutrino cooling by means of a neutrino emissivity \(Q_\nu\) and neutrino emission rates per baryon \(\nu_\nu\). Since these depend sensitively on the optical depth \(\tau_\nu\) we distinguish between optically thin (free-streaming) and optically thick (diffusive) regimes by computing a harmonic average of the rates following (Rosswog & Liebendörfer 2003). The optical depth \(\tau_\nu = \int \kappa_\nu \, dx\) is computed from the opacities \(\kappa_\nu\) by locally applying Fermat’s principle to the neighbouring cells on the computational grid (Neilsen et al. 2014; Poucext et al. 2013). The energy- and momentum-loss due to neutrino emission is then included by modifying the evolution equations accordingly (Sekiguchi 2010b; Deaton et al. 2013; Galeazzi et al. 2013).

2.3 Primitive inversion

A particularly delicate part is the non-analytical computation of the primitive variables from the evolved conserved variables (8) - (10). In the past a great effort has been devoted to designing stable and accurate inversion algorithms (Noble et al. 2006; Mignone & McKinney 2007; Del Zanna et al. 2007; Etienne et al. 2012b; Muhlberger et al. 2014; Newman & Hamlin 2014; Palenzuela et al. 2015; Nouri et al. 2018), similar to a recent review (Siegel et al. 2018) of these inversion schemes we perform a multi-stage inversion that consists of the following steps

(i) First check \(\varepsilon > B^2/\rho\), where \(\varepsilon\) is a very small number so that the contribution of \(B^2\) to \(\tau\) and \(S^2\) can be neglected to around machine precision if the above condition is fulfilled. In this case we are certainly in a purely hydrodynamical regime and can apply the algorithm of (Galeazzi et al. 2013) for general relativistic hydrodynamics for which an exact bracketing for the 1D-root finding problem can be provided and solved efficiently using Brent’s method (Brent 2002). If we have not found a root within the given accuracy we proceed to the last step.

(ii) If magnetic fields cannot be neglected we proceed with the single variable algorithm of (Palenzuela et al. 2015). Before performing the root-finding we apply the generalised solvability constraints (Palenzuela et al. 2015; Etienne et al. 2012b). If the root-finding has converged we assess the quality of the solution by computing (A4) of (Mignone & McKinney 2007) and check if the relative error on \(\tau\) is at least \(\sim 10^{-12}\). If the inversion was not successful or too inaccurate we proceed with the next step.

(iii) If we were not successful in the previous step we perform the fixed-point inversion of (Newman & Hamlin 2014). We have found that when disabling the Aitken acceleration step proposed in (Newman & Hamlin 2014) the robustness of the scheme increased in demanding situations. If a fixed-point has been found, perform the same consistency check on the solution as in the previous step.

(iv) Finally, if all previous attempts were unsuccessful we replace the energy variable \(\tau\) by the specific entropy \(s\) per

---

1. https://stellarcollapse.org
2. https://compose.obspm.fr
baryon that was independently evolved at every step as in (Nouri et al. 2018)
\[ \dot{\rho} (\rho, s) + \dot{\rho} \left( \rho, s, \beta^2 \right) = \frac{m_{\text{ne}} \sqrt{\gamma}}{k_B T} \left[ Q_v - R_e \rho \mu_e \right]. \] (23)
where \( m_{\text{ne}} \) is the neutron mass and \( \mu_e \) the electron neutrino chemical potential. Following (Nouri et al. 2018) we solve (A24) of (Mühlberger et al. 2014) using Brent’s method. Note that for this step the pressure \( P = P(\rho) \) is effectively a function of the density alone since for given \( \rho \) and \( Y_e \) in the iteration the temperature \( T = T(\rho, s, Y_e) \) can be trivially recovered and from it the pressure \( P = P(\rho, T, Y_e) \).

(v) If also the entropy cannot be recovered a point, we reset it to atmosphere values. This happens only on very rare occasions close to atmosphere values.

While the above procedure might seem in total rather involved and potentially expensive we find that the use of a fully high-order flux update greatly reduces the need for applying the entropy fix. This can be understood by considering the core part of every inversion algorithm, the computation of \( \epsilon \) at every step, see e.g., (Palenzuela et al. 2015) for an expression. While for example the ideal fluid \( p = \rho e (T - 1) \) every positive value of \( \epsilon \) corresponds to a physical pressure, this is no longer the case when using a finite range tabulated EOS, see Fig. 1 in (Galeazzi et al. 2013). Since at every intermediate step the recovered value of \( (\rho, T, Y_e) \) has to be inverted for the temperature \( T \), using a wrong and in most cases too low value for \( \rho \) results in the recovery of a wrong temperature \( T \) and hence of a wrong enthalpy \( h_{\text{recovered}} < h \), which is smaller than the actual enthalpy \( h \). We note that the same is also true when using the evolved entropy \( \tilde{s} \) as this always corresponds to a lower temperature \( T(\tilde{s}, \rho, Y_e) < T \), as its evolution equation (23) is only correct in the absence of shocks, and will hence always lead to smaller entropy values.

3 RESULTS
In the main part of this work we focus on the dynamics of an irrotational equal mass neutron-star binary with a total mass of \( 2.7 M_\odot \) constructed using the CCDQG code (Tsokaros et al. 2015; Tsokaros et al. 2018). The two stars are initially placed at a distance of 45 km and are equipped with a poloidal magnetic field with a maximum field strength \( B_r = 5 \times 10^{14} \text{G} \) confined to the interior of the two stars. The simulation domain is modelled by a series of seven nested boxes extending up to \( \sim 1500 \text{km} \). We consider a total of three resolutions \( 194 \text{m}, 262 \text{(2nd)}/295 \text{(4th) m} \) and \( 370 \text{m} \), in the following referred to as high (HR), medium (MR) and low (LR) resolutions. The composition of the stars is described by the soft TNYSTY EOS (Togashi et al. 2017), which is consistent with recent constraints on the EOS from GW170817 (Most et al. 2018).

In the following we will describe the inspiral and post-merger dynamics of the systems and highlight the differences between formally second and fourth-order convergent numerical schemes. In particular, we will compare two sets of simulations using exactly the same methods as described in Sec. 2.1.2, but where for one set the high order correction (16) is switched off. This approach using high order reconstruction combined with an HLL Riemann solver at overall second-order convergence is commonly used when modelling of neutron-star mergers and their remnants (Reisswig et al. 2013; Muhlberger et al. 2014; Foucart et al. 2016; Bovard et al. 2017; Nouri et al. 2018; Radice et al. 2018; Papenfort et al. 2018) and hence allows us to draw conclusion directly relevant for existing and future studies of neutron-star mergers.

The merger of a neutron-star binary can be separated into several stages, including the inspiral, early post-merger and long-term post-merger phase. Each stage gives rise to different observables and sets the initial conditions for the next stage of the evolution. Being able to accurately model each of these is a key task for numerical-relativity codes and in the following we will look at the first to stages in light of the impact of fully high-order numerical schemes.

3.1 Accurate modelling of neutron-star inspirals
An important aspect of numerical-relativity codes is their ability to compute gravitational waveforms for compact binaries. Since analytic waveform models rely on numerical-relativity input to account for tidal deformations in the last orbits, being able to accurately extract numerically convergent waveforms is crucial for any numerical-relativity code. In the following we will show how a fully high-order scheme improves the convergence of the inspiral and post-merger signal even when finite-temperature EOS are used. From our simulations we extract the \( \ell = m = 2 \) mode of the gravitational-wave strain at a radius of \( \sim 880 \text{km} \) from the merger site for all three resolutions considered in this work, for both the formally fourth- and second-order convergent schemes.

In the top panel of Fig. 2 we show the gravitational waveforms at different resolutions (upper panel) and the absolute error in the phase of the gravitational-wave signal, \( \Delta \phi_2 \) (bottom panel); the latter is either measured with respect to the highest resolution (top part) or against two neighbouring resolutions (bottom part). In essence, we find that when using the high-order scheme (solid lines) consistent phase evolutions are achieved even after merger. When instead using the second-order scheme (semi-transparent lines), we find that although convergence is obtained in the inspiral, the phase evolution shows non-convergent behaviour after merger. On the contrary, for the high-order scheme, consistent and convergent behaviour is also found for the lowest resolution.

To quantify the accuracy of the waveforms the right panel of Fig. 2 shows the point-wise self-convergence order for both schemes and the lower panel shows relative phase differences \( \Delta \phi_2^{\text{relative}} \) rescaled to a given convergence order consistent with the pointwise self convergence order in the right panel. In the case of the fourth-order scheme (solid line)

---

3 Ideally, since gravitational radiation is well defined only at future null infinity, the Cauchy evolution carried out here ought to be matched to a characteristic extraction evolving the radiation from a timelike boundary at finite radius over to future null infinity (see ? for a review). In practice, however, extracting at a finite radius and extrapolating to spatial infinity represents a very good approximation of the gravitational waveforms and serves the scopes of this paper.

4 The gravitational wave strain data can be downloaded from the publisher’s website.
we find that while in the inspiral a convergence order of only 2.5 is achieved, shortly after the merger transient, constant third-order convergence is established. We point out that this matches the convergence order of the employed Runge-Kutta time integration scheme, while our spatial scheme is formally fourth-order convergent. We attribute the reduced convergence order during the inspiral phase to a loss of accuracy at the sharp surface of the star, which we will discuss in the following section. Not being able to reach high-order convergence in the inspiral gravitational-wave signal is not unprecedented and has been studied in detail (Bernuzzi & Dietrich 2016).

In order to provide a better comparison to previous results and also to further assess our ability to compute highly accurate results, in App. A we show the results obtained for one model of (Bernuzzi & Dietrich 2016). Considering the second-order scheme, we indeed find second-order convergence in the inspiral, while the post-merger is no longer convergent as discussed before.

Previous studies of post-merger simulations have established that the frequencies of the post-merger signal obey universal relations (Bauswein & Janka 2012; Takami et al. 2014, 2015; Rezzolla & Takami 2016) and hence can be used to identify the EOS (Rose et al. 2018) once it will be detectable with future detectors. In the bottom panel of Fig. 2 we report the frequency spectrum for the gravitational-wave signals of our simulations, distinguishing again between results for the high-order scheme (top panel) and second-order scheme (bottom panel). The cyan solid line represents the sensitivity curve of advanced LIGO, while different symbols mark the frequency at merger and the frequency of the $f_2$ peak (Takami et al. 2014). Although all spectra agree well across all resolutions, small differences are visible and highlight the accuracy of the high-order numerical scheme. In particular when considering the minimum around 3 kHz and the second frequency peak around 4.5 kHz, one can see that the lowest resolution is not converged, whereas for the high-order scheme all resolutions yield consistent spectra across the relevant frequency range and with the main spectral features (i.e., the $f_1$, $f_2$ and $f_3$ peaks) clearly visible.

Apart from being able to extract convergent gravitational waveforms it is also important to set accurate initial conditions for the post-merger evolution and to clarify the effect of tidal interactions on the neutron stars, e.g., in the context of proposed pre-merger amplifications of the magnetic field (Ciolfi et al. 2017). One aspect of the inspiral different from the late-time evolution of the system is the presence of sharp gradients at the surfaces of the neutron stars. This region is prone to cause spurious mass outflows, as the hydrodynamical fluxes and the gravitational sources do not exactly balance, resulting in unphysical conserved states and inversion failures. As an example we show in Fig. 3 the rest-mass density distribution shortly before merger. We find that when using a second-order accurate scheme (right panel) that large amounts of spurious matter contaminate the domain close to the merger and numerical inversion failures at the surface break the initial symmetry of the equal mass system. We find that this seems to worsen when strong magnetic fields are present in the neutron stars and also when finite-temperature EOS are used. In contrast, when using a fully fourth-order convergent scheme, i.e., (16), we find that this spurious mass outflow is entirely absent. We also find that the surface of the two stars show no signs of inversion failures and the symmetry of the initial conditions is well preserved.

### 3.2 Merger dynamics: Mass ejection

With the detection of the neutron-star merger event GW170817 (Abbott et al. 2017) and the subsequent kilonova AT2017gfo (Cowperthwaite et al. 2017; Drout et al. 2017) modelling the mass ejection from merging binary neutron stars has become a very important part of numerical studies. The composition of the ejected mass depends sensitively on the precise inclusion of weak interactions in merger simulations and considerable effort has been aimed at studying the impact of different approaches to neutrino-transport.

---

**Figure 1.** (Top) Phase difference $\Delta \phi_{22}$ of the $t = m = 2$ mode to the high resolution model $\phi_{22}^{\text{HR}}$ and relative differences $\Delta \phi_{22}^{\text{rel}}$. The semi-transparent line denotes the second-order model. (Right) Self convergence order of the phase $\phi_{22}$ using the three available resolutions.
Gravitational-wave strain \( (f = m = 2) \) mode for a source at 100 Mpc extrapolated to spatial infinity for different resolutions ranging from 0.1328 – 0.25 \( M_\odot \). Bottom: Gravitational wave spectrum showing the effective strain \( h_{\text{eff}}(f) \) computed using the high-order scheme (top) and the second-order scheme (bottom). The cyan solid line represents the sensitivity curve of aLIGO.

In line with other works taking a more simplified approach (Palenzuela et al. 2015; Lehner et al. 2016; Bovard et al. 2017; Radice et al. 2018) we here explore the impact of having accurate fluid dynamics when using finite temperature EOS in the presence of magnetic fields. In particular, we will focus on the dynamical part of the mass ejection stemming from the first 20–30 ms after the merger. Although the dominant part of the mass ejection is produced on secular time scales > 200 ms, studying the impact of high-order methods on long-term disk evolutions is much more involved and requires a very careful analysis on its own (Porth et al. 2019), so that we reserve it for a future study.

During and shortly after the merger large amounts of matter are ejected either by tidal interactions of the two stars or by shock heating at and after the merger. The former will lead to mass outflows mainly in the equatorial plane, while the latter mass ejection is mainly isotropic. Given the small sizes of the two stars and the strong shocks involved in the process, the amount of ejected mass can quite sensitively depend on the accuracy of the numerical scheme, the resolution and the criterion to determine unbound material (Sekiguchi et al. 2016; Bovard & Rezzolla 2017; Bovard et al. 2017; Radice et al. 2018). In the following, we will compare the effects of resolution and of using the fourth-order scheme on the mass ejection of the equal mass system.

We start by looking at the shock front of dynamical mass ejection shortly after merger, shown with its entropy values in Fig. 4 in the meridional plane. It is easy to distinguish the tidally driven part of the ejection at low entropy, from the shock heated polar driven part at high entropy. We can also see that the ejection is preceded by a fast, low density shock front with very high entropy values. Comparing the top panel computed with the fourth-order scheme with the bottom panel using the second-order scheme, we find several important differences. Firstly, the outer shock front is Rayleigh-Taylor unstable in both cases, but the fourth-order scheme manages to resolve it very sharply even as the matter crosses refinement levels of decreasing numerical resolution. In the simulation using the second-order scheme, instead we find large scale instabilities in the fluid flow and some artificial low density outflow preceding the shock front, which is entirely absent in the high-order case. Secondly, starting from an equal mass binary introduces a symmetry in the system, and apart from symmetry breaking effects like an \( m = 0 \) instability (Paschalidis et al. 2015; East et al. 2016), the numerical scheme should be able to preserve it over time until large scale turbulence appears. Comparing the plumes of ejected matter in the polar direction we find that numerical error in the second-order scheme triggers a kink-like instability and produces asymmetries in the plume already < 10 ms after merger. On the contrary, the high order scheme produces a perfectly symmetric outflow and plume, although the symmetry is never manually imposed in the simulation.

In order to quantify the effects of the scheme on the properties of the mass ejection we consider the time integrated mass ejection on a sphere placed at \( \pm 738 \text{ km} \) from the origin. We find that the spatial distribution of the ejected matter is qualitatively the same for both schemes, although it appears to be slightly more smeared out in the case of the second-order scheme. Considering the composition of the ejected matter we find that the time and mass averaged electron fraction \( Y_e \) is significantly higher in polar regions for the second-order scheme, reflecting its drawbacks in the modelling of polar ejecta as anticipated from Fig. 4. To better understand and quantify this difference we consider the distributions of the ejected mass in terms of electron fraction \( Y_e \), entropy per baryon \( s \) and velocity \( v \). This is shown in Fig. 5, which reports from left to right: the electron fraction \( Y_e \), the entropy \( s \) per baryon and the velocity \( v \) as computed from the Lorentz factor of the outflow; the solid lines refer to results computed using the fourth-order scheme, dashed lines to results from the second-order
Figure 3. Rest-mass density distribution in the inspiral of an equal-mass neutron-star binary modelled using the finite temperature TNTYST EOS shortly before merger. The left panel shows the result using a fourth-order scheme, while the right panel shows only second-order accurate results.

Figure 4. Entropy per baryon s in the meridional plane < 10 ms after merger. The top panel shows the evolution of the system for the fourth-order accurate scheme, the bottom panel for the second-order scheme. The second-order scheme introduces asymmetries and fluid instabilities in the flow that are entirely absent when using the fourth-order scheme.

scheme. All simulations peak around $Y_e \approx 0.1$ corresponding to the tidally driven ejecta (see also green regions in Fig. 6). Additionally, we find that the distribution of the electron fraction for the second-order scheme has a second peak around $Y_e \approx 0.25$, which is entirely absent in the case of the high-order scheme. When comparing this feature at the two highest resolutions we find that not only is the overall distribution of the high-order scheme unchanged, but also that the second-order scheme even at high resolutions overestimates the amount of proton rich material. Further we find that the ejecta velocity are only correctly reproduced at the highest resolution for the second-order scheme, whereas the
Figure 5. Distributions of the composition features of the ejected material. From left to right: the electron fraction $Y_e$, the entropy $s$ per baryon and the velocity $v$ as computed from the Lorentz factor of the outflow. Solid lines refer to results computed using the fourth-order scheme, dashed lines to results from the second-order scheme.

Figure 6. Time integrated ejected mass $M_{ej}$ and mass weighted electron fraction $Y_e$ computed using the fourth-order (top) and second-order schemes (bottom).

The fourth-order scheme yields consistent distribution already at medium resolution. In line with the discussion of Fig. 4 we find that the second-order scheme produces a large tail of high entropy ejecta even at high resolution, which again is not present in the high-order simulations that feature a sharp cut-off around 60 k$_B$/baryon.

3.3 Magnetic field amplification in the merger remnant

After the merger a differentially rotating hypermassive neutron star (HMNS) is formed, which has a core that spins slower than its outer layers (Kastaun & Galeazzi 2015; Kastaun et al. 2016; Hanasuske et al. 2017). At the same time also a disk is formed around the HMNS and various magnetic field effects will cause an amplification of the magnetic field in different parts of the merger remnant. Examples include magnetic braking (Shapiro 2000) which removes differential rotation and generates a toroidal field inside the remnant, whereas the magneto-rotational instability (MRI) (Balbus & Hawley 1991, 1998) quickly leads to a magnetic field amplification and angular momentum transport in the disk. These effects in the early post-merger have been studied in great detail (Kuehne et al. 2013; Endrizzi et al. 2016; Ciolfi et al. 2017) and we here focus only on the benefits of using high-order methods to study the evolution of the remnant.

In Fig. 7 we report the evolution of the poloidal and toroidal components of the electromagnetic energy $E_{\text{EM}}$ and of the density weighted magnetic field

$$B_{\text{mean}} = \frac{\int dV \rho_e |B|}{\int dV \rho_e},$$

(24)

describing the evolution of the magnetic field inside the HMNS. Since we start with a poloidal field confined to the interior of the two stars, the field will simply be advected during the inspial and we do not find any sign of a proposed pre-merger amplification (Ciolfi et al. 2017). At the time of merger a magnetic field amplifying Kelvin-Helmholtz instability is expected to set in (Price & Rosswog 2006; Kiuchi et al. 2015a), which is, however, not present in any of our simulations, due to a lack of resolution. Shortly after the merger when the HMNS is formed magnetic braking sets in and reduces the amount of poloidal magnetic field, as can be seen from the mean magnetic field strength in the lower left panel. At the same time a toroidal magnetic field is generated and leads to an amplification of the overall magnetic field strength in the remnant. Comparing this behaviour with the exponential growth of the electromagnetic energy reveals that at the same time an amplification, presumably by the MRI, is active in the disk. For the different resolutions for the fourth-order scheme we find that for each resolution (even for the lowest one) consistent growth rates of the magnetic field strength and energy are observed in all cases. Taking a closer look at the poloidal energy, we can observe that an exponential growth is present that is only correctly reproduced at the highest resolution for the second-order scheme (red dash-dotted curve), whereas for the high-order scheme all resolutions yield the same behaviour. This could be the result of a minimum resolution requirement...
to resolve the MRI (Siegel et al. 2013), which is expected to be much weaker when more accurate high-order schemes are used. A comparison with the mean poloidal field inside the remnant reveals that this growth mainly originates from the disk, where lower resolutions are employed and hence high-order schemes are crucial in still capturing the essential features of the evolution of the system.

Regarding the evolution of the toroidal component we find that inside the remnant consistent growth rates and field strength are produced for both fourth- and second-order schemes, but the overall growth rates of the toroidal energy appear different for the second-and fourth order case. For the two highest resolutions the magnetic field strength amplification inside the remnant is actually almost the same as can be seen from the overlap of the red and blue curve in the bottom middle panel of Fig. 7, different from what has been found in similar studies using polytropic EOS and only second order accurate schemes (Endrizzi et al. 2016; Ciolfi et al. 2017). In order to better clarify the spatial differences in the magnetic field evolution Fig. 8 shows the magnetic field strength in the comoving frame, $b^2/\rho$, (left panel) and the magnetisation $b^2/\rho$ (right panel) for both the fourth-order (top) and the second-order (bottom) accurate schemes. As we have already seen from Fig. 7 the magnetisation and field strength inside the remnant is similar but there are large differences in the disk. As can be clearly seen on the right panel the high-order simulation produces a consistent magnetisation across the domain and specifically inside the disk on scales $> 100\, \text{km}$. In contrast, in the second-order scheme the disk close to the equatorial plane is several orders less magnetised indicating a strong lack of resolution even in the case of our highest resolution simulation.

4 CONCLUSIONS

We have investigated the impact of using a forth-order accurate numerical scheme in the simulation of merging magnetised neutron stars including finite-temperature EOS and neutrino cooling. We have presented the Frankfurt/IllinoisGRMHD (FIL) code, which extents the original IllinoisGRMHD code (Etienne et al. 2015) with a fully fourth-order numerical scheme based on the ECHO approach (Del Zanna et al. 2007) and implements a neutrino leakage scheme with finite temperature EOS support combined with improved primitive inversion methods. Since the fourth-order scheme requires the use of one additional ghost zone per direction and the computational of the associated flux, the fourth-order scheme is accompanied by an increased computational cost of $\sim 30\%$, as measured in our current implementation. However, these additional costs are easily compensated by the gains in accurateness and consistency of the solution.
Using FIL we have demonstrated first that we can capture the inspiral dynamics better by significantly reducing the amount of inversion failures at the surface of the neutron stars and second our ability to compute accurate gravitational waveforms even when employing realistic microphysics. More specifically, we have shown that using FIL can obtain a convergence order of $\approx 2.5$ for the phase of the gravitational-wave signal during the inspiral and reach third order convergence in the post-merger phase, which is most likely only limited by the strongly stability preserving third order time integration.

With the advent of multi-messenger astronomy of neutron-star mergers and more detections being expected soon to follow, accurate modelling of mass ejection has become a central part in the study of neutron-star mergers. We have investigated the impact the fourth-order accurate numerical scheme on the dynamical ejection of mass following the merger. We were able to show that the outgoing shock front is much more accurately captured, while the second-order simulations suffers from large scale Rayleigh-Taylor instabilities and do not well preserve the initial symmetry of the equal mass system. When considering the properties and composition of the ejected mass, the second-order scheme produces spurious high entropy ejecta and overestimates the amount of proton rich material.

Finally, when considering the magnetic field evolution we found that the fully fourth-order accurate approach allowed us to resolve poloidal and toroidal field amplification in the merger remnant showing consistent growth rates even at low resolution. The second-order simulation, on the other hand, showed no poloidal field amplification and saturated early in the toroidal field. While further investigations on longer timescales and at higher resolutions for the second-order scheme will be necessary, we believe that our results already indicate the importance of considering high-order schemes for GRMHD simulations of neutron stars, especially when considering long-term post-merger simulations.

One of the points not addressed in this study is the long-term evolution of the system, specifically of the disk. Such systems (Siegel & Metzger 2017, 2018; Fernández et al. 2018) are crucial for explaining the observed kilonova AT2017gfo as they provide large amounts of neutron rich ejecta in the equatorial plane. Since the timescale of this ejection is $\sim 1$ s, errors of the numerical scheme will accumulate over time. Hence, it will be important to assess the reliability of the current evolution scheme also for these systems, which we reserve for a future study.

ACKNOWLEDGEMENTS

ERM and LJP acknowledge support from HGS-HIRe. We thank Fabio Bacchini, Oliver Porth, Hector Olivares, and Bart Ripperda for useful discussions. Support comes in part from HGS-HIRe for FAIR; the LOEWE-Program in HIC for FAIR; “PHAROS”, COST Action CA16214 European Union’s Horizon 2020 Research and Innovation Programme (Grant 671698) (call FETHP-1-2014, project ExaHyPE); the ERC Synergy Grant “BlackHoleCam: Imaging the Event Horizon of Black Holes” (Grant No. 610058); The simulations were performed on SuperMUC at LRZ in Garching, on the GOETHE-HLR cluster at CSC in Frankfurt, and on the HazelHen cluster at HLRS in Stuttgart.

REFERENCES

Abbott B. P., et al., 2017, Phys. Rev. Lett., 119, 161101
Alcubierre M., 2008, Introduction to 3 + 1 Numerical Relativity. Oxford University Press, Oxford, UK, doi:10.1093/acprof:oso/9780199205677.001.0001
Alic D., Bona-Casas C., Bona C., Rezzolla L., Palenzuela C., 2012, Phys. Rev. D, 85, 064040
Alic D., Kastaun W., Rezzolla L., 2013, Phys. Rev. D, 88, 064049
Annala E., Görda T., Kurkela A., Vuorinen A., 2018, Phys. Rev.
Lett., 120, 172703
Balotti L., Rezzolla L., 2017, Rept. Prog. Phys., 80, 096901
Balbus S. A., Hawley J. F., 1991, Astrophys. J., 376, 214
Balbus S. A., Hawley J. F., 1998, Rev. Mod. Phys., 70, 1
Baumgarte T. W., Shapiro S. L., 2003, Astrophys. Journal, 585, 921
Baumgarte T. W., Shapiro S. L., 2010, Numerical Relativ-
ty: Solving Einstein’s Equations on the Com-
puter. Cambridge University Press, Cambridge, UK, doi:10.1017/coib781139195344
Bauswein A., Janka H.-T., 2012, Phys. Rev. Lett., 108, 011101
Bennuzi S., Dietrich T., 2016, Phys. Rev. D, 94, 064062
Bennuzi S., Hilditch D., 2010, Phys. Rev. D, 81, 084005
Borges R., Carmona M., Costa B., Don W., 2008, Journal of Computational Physics, 227, 3191
Bose S., Chakravarti K., Rezzolla L., Sathyaprakash B. S., Takami K., 2018, Phys. Rev. Lett., 120, 031102
Bovard L., Martin D., Guercilena F., Arcones A., Rezzolla L., 2018, Astrophys. J., 860, 139
Chen Y., Tóth G., Gombosi T. I., 2006, Astron. Astrophys., 378, 1177
Deaton M. B., et al., 2013, Astrophys. J., 776, 47
Douchin F., Haensel P., 2001, Astron. Astrophys., 380, 151
Del Zanna L., Zanotti O., Bucciantini N., Londrillo P., 2007, Astron. Astrophys., 475, 281
Duez M. D., Zlochower Y., 2019, Reports on Progress in Physics, 82, 016902
Duez M. D., Liu Y. T., Shapiro S. L., Stephens B. C., 2005, Phys. Rev. D, 72, 024028
East W. E., Paschalidis V., Pretorious F., Shapiro S. L., 2016, Phys. Rev. D, 93, 024041
Endrizzi A., Ciolfi R., Giacomazzo B., Kastaun W., Kawamura East W. E., Paschalidis V., Pretorius F., Shapiro S. L., 2016,
Fernández R., Tkachevskay A., Quataert E., Foucart F., Kasen D., 2018, Mon. Not. R. Astron. Soc.,
Foucart F., et al., 2013, Phys. Rev. D, 87, 084006
Foucart F., et al., 2015, Phys. Rev. D, 91, 124021
Foucart F., Chandra M., Gammie C. F., Quataert E., 2016, Mon. Not. R. Astron. Soc., 456, 1332
Fujibayashi S., Kiuchi K., Nishimura N., Sekiguchi Y., Shibata M., 2018, Astrophys. J., 860, 64
Galeazzi F., Kastaun W., Rezzolla L., Font J. A., 2013, Phys.
Rev. D, 88, 064009
Giacomazzo B., Rezzolla L., 2007, Class. Quantum Grav., 24, 235
Hanauske M., Takami K., Bovard L., Rezzolla L., Font J. A., Galeazzi F., Stocker H., 2017, Phys. Rev. D, 96, 043004
Harten A., Lax P. D., van Leer B., 1983, SIAM Rev., 25, 35
Hilditch D., Bernuzzi S., Thieferfeld M., Cao Z., Tichy W., Brüg-
mann B., 2013, Phys. Rev. D, 88, 084057
Hu X. Y., Adams N. A., Shu C.-W., 2013, Journal of Computational Physics, 242, 169
Just O., Bauswein A., Pulipollo R. A., Gorily S., Janka H.-T., 2015, Mon. Not. R. Astron. Soc., 448, 541
Kasen D., Metzger B., Barnes J., Quataert E., Ramirez-Ruiz E., 2017, Nature, 551, 80
Kastaun W., Galeazzi F., 2015, Phys. Rev. D, 91, 064027
Kastaun W., Ciolfi R., Giacomazzo B., 2016, Phys. Rev. D, 94, 044060
Kawaguchi K., Kiuchi K., Kyutoku K., Sekiguchi Y., Shibata M., Taniguchi K., 2018, preprint, (arXiv:1802.06518)
Kidder L., et al., 2017, Journal of Computational Physics, 335, 84
Kiuchi K., Kyutoku K., Hotokezaka K., Sekiguchi Y., Shibata M., 2013, in Pogorelov N. V., Audit E., Zank G. P., eds, Astro-
omical Society of the Pacific Conference Series Vol. 474, Nu-
erical Modeling of Space Plasma Flows (ASTRONUM2012).
Astronomical Society of the Pacific, San Francisco, CA, p. 84
Kiuchi K., Sekiguchi Y., Kyutoku K., Shibata M., Taniguchi K., Wada T., 2015a, Phys. Rev. D, 92, 064034
Kiuchi K., Cerda-Durán P., Kyutoku K., Sekiguchi Y., Shibata M., 2015b, Phys. Rev. D, 92, 124034
Kiuchi K., Kyutoku K., Sekiguchi Y., Shibata M., 2018, Phys. Rev. D, 97, 124039
Kretz M., Lindenstruth V., 2012, Software: Practice and Experience, 42, 1409
LIGO Scientific Collaboration Virgo Collaboration Gamma-Ray Burst Monitor F., INTEGRAL Journal. J. Lett., 848, L13
Lehner L., Liebling S. L., Palenzuela C., Caballero O. L., O’Connor E., Anderson M., Neilsen D., 2016, Classical and Quantum Gravity, 33, 184002
Liebling S. L., Lehner L., Neilsen D., Palenzuela C., 2010, Phys.
Rev. D, 81, 124023
Löffler F., et al., 2012, Class. Quantum Grav., 29, 115001
Londrillo P., Del Zanna L., 2004, Journal of Computational Physics, 195, 17
McQuigg D., Colella P., 2011, Communications in Applied
Mathematics and Computational Science, 6, 1
Mignone A., McKinney J. C., 2007, Mon. Not. R. Astron. Soc., 378, 1118
Mignone A., Tzeferacos P., Bodo G., 2010, Journal of Computational Physics, 229, 5896
Most E. R., Rezzolla L., Rezzolla L., Schaffer-Bielich J., 2018, Phys. Rev. Lett., 120, 261103
Möst P., et al., 2014, Classical and Quantum Gravity, 31, 015005
Mühlberger C. D., et al., 2014, Phys. Rev. D, 90, 104014
Nagar A., et al., 2018, Phys. Rev. D, 98, 104052
Neilsen D. W., Liebling S. L., Anderson M., Lehner L., O’Connor E., Palenzuela C., 2014, Phys. Rev. D, 89, 104029
Newman W., Hamlin N., 2014, SIAM Journal on Scientific Computing, 36, B661
Noble S. C., Gammie C. F., McKinney J. C., Del Zanna L., 2006, Astrophys. J., 641, 626
Nouri F. H., et al., 2018, Phys. Rev. D, 97, 083014
O’Connor E., Ott C. D., 2010, Class. Quantum Grav., 27, 114013
Palenzuela C., Liebling S. L., Neilsen D., Lehner L., Caballero O. L., O’Connor E., Anderson M., 2015, Phys. Rev. D, 92,
APPENDIX A: GRAVITATIONAL-WAVE CONVERGENCE: LONG INSPIRALS

In order to check the accuracy of the FIL code in extracting gravitational-wave signals, we study the inspiral of an equal mass binary with a total mass of 2.7$M_\odot$ using the SLy EOS (Douchin & Haensel 2001) that is initially placed in a quasi-circular orbit at a separation of 60 km (Bernuzzi & Dietrich 2016). The simulations were performed on a series of nested equally spaced grids extending up to $\approx 1500$ km, with four resolutions of $(0.1328, 0.16, 0.2, 0.25) M_\odot$ on the finest grid. In Fig. A1 we show the extracted gravitational-wave strain and the relative convergence order computed for the gravitational-wave phase at every time for two subsets of the resolution. We can see that the gravitational waveform is nicely convergent in the inspiral and difference only appear in the post-merger phase. Nonetheless, at all resolutions a BH is formed shortly after merger. We find that convergence in the inspiral is obtained even for the lowest resolution and that the same convergence order is obtained for the two subsets. Although our scheme is formally fourth-order accurate, we find that the convergence order as computed for two subsets of the resolution is only $\approx 2.5$, similar to what has been observed for the same configuration in a formally fifth-order convergent code (Bernuzzi & Dietrich 2016).

This paper has been typeset from a TeX/LaTeX file prepared by the author.
Figure A1. (Left) Gravitational-wave strain ($l = m = 2$ mode) extracted at $\approx 880$ km for different resolutions ranging from $0.1328 - 0.25 M_\odot$. (Right) Relative convergence order computed of the gravitational-wave phase using the $(0.16, 0.2, 0.25) M_\odot$ and $(0.1328, 0.16, 0.2) M_\odot$ (HR case).