Bell’s Theorem and Spacetime-Based Reformulations of Quantum Mechanics

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In this critical review of Bell’s Theorem, its implications for reformulations of quantum theory are considered. The assumptions of the theorem are set out explicitly, within a framework of mathematical models with well-defined inputs and outputs. Attention is drawn to the assumption that the mathematical quantities associated with a certain time and place can depend on past model inputs (such as preparation settings) but not on future inputs (such as measurement settings at later times). Keeping this time-asymmetric assumption leads to a substantial tension between quantum mechanics and relativity. Relaxing it, as should be considered for such no-go theorems, opens a category of Future-Input Dependent (FID) models, for which this tension need not occur. This option (often called “retrocausal”) has been repeatedly pointed out in the literature, but the exploration of explicit FID models capable of describing specific entanglement phenomena has begun only in the past decade. A brief survey of such models is included here. Unlike conventional quantum models, the FID model parameters needed to specify the state of a system do not grow exponentially with the number of entangled particles. The promise of generalizing FID models into a Lorentz-covariant account of all quantum phenomena is identified as a grand challenge.

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I. INTRODUCTION

In his Messenger Lectures, Feynman (1965) claimed that “nobody understands quantum mechanics”, and advocated for the development of alternative formulations to improve understanding. As an example, he noted that the astronomers of the Maya culture possessed a well-defined mathematical procedure for accurately predicting eclipses of the moon, without describing its path in space. Due to this success, he speculated that these experts would have discouraged any preliminary model of spatial orbits, despite the eventual advantages of a more physical reformulation.

In general, when we design or reformulate mathematical models of physical phenomena, the structure of those models is influenced by our physical intuitions and philosophical inclinations, which are in turn influenced by our successful models. One of the remarkable achievements of Bell’s Theorem (Bell, 1964) is that it strictly constrains these influences via a mathematical proof. Thanks to Bell, when we attempt to model correlations evident from quantum entanglement experiments, we know that there is a certain package of intuitive features that our models cannot contain. And to the extent that our successful mathematical models inform us about the physical universe, Bell’s Theorem is rightly thought to be telling us something profound.

Even so, Bell’s Theorem merely tells us what sorts of models will not be successful, leaving us with a range of possible models, with a corresponding range of physical implications. The purpose of this Colloquium is to survey the space of mathematical models consistent with both Quantum Mechanics (QM) and Bell’s Theorem, and to categorize those models based on features of the model’s spacetime-based parameters. (Such parameters are mathematical variables which are clearly associated with a specific time and place, such as the local values of classical fields, $Q(x, t)$. These localized parameters are what Bell’s Theorem was originally meant to address, and where the assumptions behind the theorem are most evidently applicable. The assumptions behind Bell’s Theorem have been greatly discussed in the literature [e.g., Goldstein et al. (2011) and Shimony (2017), see Mermin (1986) for an elementary exposition], and yet there remains significant confusion on the topic. Some physicists continue to be under the impression that the implications of Bell’s Theorem can be trivially dismissed by jettisoning some purported separate assumption (realism, or hidden variables, or determinism), a general misunderstanding that has required repeated refutations (Bell, 1981; Maudlin, 2014; Norsen, 2007, 2011, 2017). Another background assumption – the time-asymmetric assumption that model parameters must be independent of future inputs – is commonly made without a proper mention. Models which violate this assumption have seen a number of implementations over the past decade, but are generally unrepresented in discussions of Bell’s Theorem. This Colloquium will attempt to rectify the situation by emphasizing such Future-Input Dependent (FID) models.

We will take several steps to reduce the confusion on this topic. First, our framework will consider functional models, with well-defined inputs and outputs, not merely collections of probabilities and correlations. This is how QM is applied to systems in practice: certain parameters (say, preparation settings) are clearly external inputs, and other parameters are clearly outputs. Second, we require the same inputs and outputs as QM, setting aside approaches that do not recover QM’s predictive successes and general applicability. Third, we will stress the importance of the spacetime-based parameters, and exclusively refer to these parameters as we define a number of possible “locality” conditions. Finally, and most importantly, we will mathematically define each assumption, including the often unremarked distinction between future inputs and past inputs.

Despite conventional wisdom, one consequence of our analysis is that viable models can be in agreement with QM even if all parameters of the model are associated with locations in spacetime. Einstein once championed such models as the most obvious way to remove the apparent “action-at-a-distance” from conventional quantum theory. Following one of his arguments (Norsen 2007),

\[1\text{ For the importance of this viewpoint, see, e.g., Pearl (2009).}\]
2005), consider a single particle in a superposition of being located in one of two boxes:
\[
\psi = \frac{1}{\sqrt{2}} (\psi_1 + \psi_2),
\]
where \(\psi_1\) is fully localized in the first box, and \(\psi_2\) in the second. If one of the boxes is now opened, and the presence or absence of the particle is observed, the quantum wavefunction in both boxes immediately changes, and is now given by either \(\psi_1\) or \(\psi_2\). To Einstein, who rejected unmediated action-at-a-distance, this was already evidence of an underlying hidden-variable account. Indeed, a classical particle in a state of conventional uncertainty, known to be in one of the two boxes, would have that knowledge “collapse” upon observation in much the same way. The apparent nonlocality of the statistical description might then be understood through averaging over the different possibilities of a more detailed description, which could include the “hidden” parameter of the particle’s actual location.

But Bell’s Theorem shows that it is difficult to make a similar move for all of quantum theory, i.e., to find a more detailed, spacetime-based description that would be compatible with quantum predictions. We will reprise Bell’s analysis, discussing in detail the conditions that such models must violate if they hope to remain in agreement with QM. It is generally known that one such violation would be to introduce the very action-at-a-distance that Einstein was attempting to remove; we will indicate this option in our analysis. We will also demonstrate another option: FID models which allow future inputs to constrain unknown parameters in their past lightcone. This Lorentz-covariant approach would be consistent with the “block universe” view of spacetime from relativity, and may well allow a model with local interactions to succeed as a reformulation of QM.

Discovering reformulations of existing theories has historically been very useful, as demonstrated by the development of Lagrangian and Hamiltonian classical mechanics. Even in quantum theory, reformulations such as the path integral have already led to new insights and advances. And there is no indication that this strategy of searching for new reformulations has run its course. In particular, if it were possible to develop an alternative quantum model with parameters restored to functions on spacetime, instead of a multi-dimensional configuration space (or Hilbert space), the description of a particular realization of a system would grow linearly (rather than exponentially) with the extent of that system. There would also be much less of a disconnect between quantum theory and our best theories of relativistic spacetime. Any such reformulation could therefore have dramatic implications for topics ranging from quantum computation to black holes. The conventional alternative, giving up on ordinary spacetime as the setting for our model parameters, forces us to treat spacetime as somehow emergent, with the great task before us to quantize gravity. This may well be the correct path forward, despite the difficulties that this program has encountered. But logically, at least, it is not the only path forward. If quantum theory could be reformulated in terms of spacetime-based parameters, then spacetime would not necessarily have to be “quantized” at all.

The most common reason given for not pursuing such a research project is Bell’s Theorem itself, making it all the more important to carefully examine exactly what types of reformulations are compatible with Bell’s Theorem – the ultimate motivation for this Colloquium. Note that a successful reformulation of QM in terms of spacetime-based parameters would certainly not imply that quantum theory was incorrect. Quantum states could still represent our best possible knowledge about measurable aspects of those parameters, given accessible information. In this case, quantum states would be viewed as states of knowledge, a popular perspective in the field of quantum information (Caves et al., 2002; Leifer and Spekkens, 2013; Spekkens, 2007).

The next section will specify the framework of mathematical models we are considering in this review, and define the assumptions that lead to Bell’s Theorem. In Section III, reprising Bell’s Theorem, we will see that adopting all these assumptions leads to a family of models incapable of describing quantum phenomena. Sec-

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2 While partial-information Liouville descriptions of classical systems do grow exponentially with particle number, the underlying (complete) localized description does not.
tion IV frames the later discussion of “Bell-compatible” models that relax at least one of the relevant assumptions, dividing them into “Types” that are discussed in Sections V and VI. (A particularly enlightening example of a recently-developed FID model is reviewed in Section VI.A.2.) Section VII assesses the advantages of the various Types of models, and after setting aside some alternatives and misconceptions in Section VIII, Section IX provides the conclusion, encouraging future development of spacetime-based models compatible with Bell’s Theorem.

II. FRAMEWORK AND ASSUMPTIONS

Much has been written about what is ruled out by Bell’s Theorem; our concern here is with categorizing the models that are still ruled in. With this goal in mind, Section II.A lays out a broad definition of mathematical models that can be compared to physical observations in spacetime. Section II.B then lists reasonable but optional assumptions that could characterize such models.

A. Spacetime-Based Models

1. General Mathematical Models

In all of physics, one uses mathematical models to generate falsifiable predictions which can be compared with empirical observations. The sort of models that accomplish this are essentially functions that take some parameters as inputs and generate other parameters as outputs. We are therefore interested in models which come with well-defined input parameters (“inputs” for short), which will be denoted by the set $I$, and also well-defined output parameters (“outputs”), denoted by $O$. Models can have other parameters in addition to the inputs and outputs, and the set of these will be denoted by $U$. We will often discuss the set of all non-input parameters $Q$ (the union of $O$ and $U$). Parameters here are not limited to simple scalars – vectors, or more complicated mathematical constructs such as functions may be utilized.

Deterministic models are those for which specification of all inputs $I$ always exactly determines the non-input parameters $Q$. Stochastic models do not predict unique values for $Q$, but for any full set of inputs, the model assigns a probability for every possible combination of non-input parameters. Thus, a fully-specified mathematical model can always be written as $P_I(Q)$, a unique joint probability distribution function for the set of non-input parameters, given specific values for the inputs. For deterministic models, these distributions are $\delta$-functions, but the analysis is not limited to such cases.

This definition, which suffices for the present purposes, is minimal in the sense that it does not include the mathematical details of how one parameter is deduced from another within the model, nor details having to do with the physical interpretation of a model. By this definition, mathematical models are like mathematical functions, in that they have specified domains and ranges, and in that existence and uniqueness of $P_I(Q)$ is guaranteed. Cases for which this is not rigorously known to hold, such as the Navier-Stokes equation for turbulent flow, can not be considered as well-defined mathematical models. Such problematic cases do not arise in the context of our present discussion.

According to the standard rules for probabilities, the full joint probability distribution $P_I(Q)$ of all the non-input parameters of a model can be used to generate marginal distributions, $P_I(Q_1)$, for any subset $Q_1 \subset Q$. It also generates conditional probabilities, $P_I(Q_1|Q_2)$, where $q_2$ are specific values of parameters in another subset, $Q_2$. In some cases, a model may predict that $Q_1$ and $Q_2$ are statistically independent, meaning that $P_I(Q_1, Q_2) = P_I(Q_1)P_I(Q_2)$. When statistical independence holds, knowledge of the values of parameters in $Q_2$...
does not inform the marginal \( P_I(Q_1) \), as represented by the condition \( P_I(Q_1|Q_2) = P_I(Q_1) \).

Two models that use identical input and output sets \( I \) and \( O \) and also have the same marginal output probabilities \( P_I(O) \) are said to be in agreement: they always yield the same joint probability of the output parameters for a given set of inputs, even if they disagree at the level of all non-input parameters, \( P_I(Q) \). Models can be in agreement even if they utilize different parameters \( Q \); for example, in Classical Electromagnetism (CEM), one can add the electromagnetic potentials as additional parameters of the model, along with the electromagnetic fields. The discussion below treats any such parameter-changing reformulations as different models, because they might generally have different properties at the level of non-output parameters. For example, a version of CEM utilizing Coulomb-gauge potentials will generally not respect the Local Causality condition defined below, because the potentials can change instantaneously over all space.

In the following we will be interested primarily in models which are in QM-agreement (QMa), meaning that they are in agreement with standard QM, at least for a specific setup under consideration. Such models are guaranteed to share the empirical success of QM.

2. Spacetime-Based Parameters

As discussed in the Introduction, we are interested in models of spacetime-based parameters, each associated with a particular location in ordinary spacetime. Examples of such parameters include the values of physical fields, such as \( E(x, t) \) in classical electromagnetism and \( g^{\mu\nu}(x^\gamma) \) in general relativity. Other examples include instrument settings and measurement results, which are associated with definite regions rather than points in spacetime. These parameters correspond to what \( \text{Bell} \) (1976) called “local beables” (pronounced be-ables).

We restrict our discussion to models where all input parameters \( I \) are spacetime-based in this sense. We further restrict the non-input parameters \( Q \) to also be spacetime-based. (These restrictions enable the upcoming definitions to meaningfully capture notions of locality or action-at-a-distance.) It is standard for many of the inputs \( I \) and outputs \( O \) useful in physics to be spacetime-based parameters, corresponding to events in spacetime, but some models employ additional mathematical entities which are not spacetime-based. For example, the \( N \)-particle de Broglie-Bohm Pilot-Wave theory utilizes a configuration-space wavefunction, comprised of values that do not correspond to particular locations in spacetime if \( N > 1 \) (Bohm 1952). For the purposes of Bell’s analysis and the below discussion, non-localized or non-spacetime-based parameters such as configuration-space wavefunctions are simply omitted from \( Q \), even if they are mathematically utilized in a given model.

Of course, it is always possible to construct non-localized parameters out of spacetime-localized ones, such as the total energy of an extended system. Such non-localized parameters are not to be included as elements of \( I \) or \( Q \). One can also make the opposite move, taking mathematical entities that are not localized in spacetime, and mapping them onto different spacetime-based parameters. Any such move would introduce new localized parameters \( Q \), which should therefore be treated as a different model, possibly with different properties than the original.

Because of the specified inputs and outputs, there is more detail in a given mathematical model than can be discerned from the corresponding physical system. A system of interacting classical particles, for example, could be described by several distinct models corresponding to the same events. Model (A) might treat the parameters on the past and spatial boundaries as inputs, and apply differential equations for output generation, while Model (B) might use input parameters on the future and spatial boundaries, with the same equations (used in the opposite time direction) for output generation. Some Model (C) might even use input particle positions (but not velocities) on all boundaries (past, future, and spatial), and use action-extremization as the output-generation procedure. Because of the differences in the input parameters, these should be considered three different models, and
could not be directly compared at the level of $P_I(O)$.

B. Physical Assumptions

Next, we define a number of properties which may or may not hold for any specific mathematical model. This allows for a categorization of models into classes and subclasses, and sets the stage for the derivation of Bell’s Theorem. In order to maintain an appropriate scope, we will limit ourselves to defining only a few key properties which play significant roles in the discussion to follow. For example, the need to formally define the relativistic covariance of models does not arise here, although Minkowski spacetime and the associated lightcones will be used to motivate some of the definitions.

1. Continuity of Action (CA)

Instead of beginning with Bell’s approach to defining locality, we shall first define a weaker condition, Continuity of Action (CA), that encodes the spirit of no-action-at-a-distance without requiring any lightcone structure from relativity, or even a distinction between past and future.

As shown in Figure 1(a), consider spacetime regions 1 and 2, with 1 completely surrounded by a screening region S. Note that this is not merely a spatial region; S spans the past and future of 1 as well as its spatial extent. We will denote the set of all inputs in regions 1 and 2 by $I_1$ and $I_2$, respectively. If there are any additional inputs, besides $I_1$ and $I_2$, their values are assumed to be fixed in the definitions below. The non-input parameters in each region are denoted by the corresponding $Q_1$, $Q_2$, $Q_S$.

Loosely speaking, a mathematical model violates CA if it has unmediated “action-at-a-distance”, i.e., if changes in 2 can be associated with changes in 1 without being also associated with changes within S. For example, a CA-respecting model of a light switch in 2 correlated with a lamp in 1 must include a description of the correlating mechanism which passes through the intermediate screening region S. In such a model, knowledge of the values of all the parameters in S makes additional information regarding the state of the switch in 2 redundant, for the purpose of predicting what happens to the lamp in region 1. Note that the parameters in the screening region do not describe all that exists there – just what is included in the model describing the correlations, such as the currents in the wires connecting the switch to the lamp. As is standard in scientific modeling, this excludes most of the details of the laboratory equipment, as well as the human experimenters.

Mathematically, Continuity of Action (CA) corre-
sponds to the condition
\[ P_{t_1,t_2}(Q_1|Q_2, Q_S) = P_{t_1}(Q_1|Q_S), \tag{2} \]
for all combinations of the parameters in the regions depicted in Figure 1(a). This equation says that parameters in region 1 cannot be correlated with parameters in region 2 except via intermediate correlations with parameters in the screening region S. Specifically, Eqn. 2 indicates that \( P_{t_1}(Q_1|Q_S) \) is both statistically independent of \( Q_2 \), and functionally independent of \( I_2 \), in the sense that \( P_{t_1,t_2}(Q_1|Q_S) \) remains constant as \( I_2 \) is varied. When this occurs, we say that S “screens” 1 from 2. For CA models this equality is required to hold for all simply connected, non-overlapping regions 1, 2, and S, for which S completely separates 1 from 2 and is nowhere vanishingly thin. Because there is no essential difference between region 1 and region 2, a model with CA also must have S screen 2 from 1.

As an example of a model with CA, consider the stationary action principle for a particle trajectory with a given potential and time interval. One would generally have the initial and final positions as inputs and the initial and final velocities as outputs, with the full trajectory represented by additional parameters \( Q \). Such a model respects CA: any spacetime sub-region 1 bounded by S is determined by the requirements of the stationary action principle, so that a specification of the particle positions in S will determine \( Q_1 \) completely. No changes to parameters outside the screening region S could lead to a different \( P(Q_1) \), without also changing \( Q_S \).

2. No Future-Input Dependence (NFID)

There is a well-known tension between the time-symmetric equations characteristic of fundamental physical theories and the time-asymmetric manner in which those theories are utilized. For example, if one takes wavefunction “collapse” to be physically meaningful, this process defines a preferred direction of time, breaking the time symmetry evident in unitary evolution. More generally, a preferred direction of time is commonly chosen by limiting attention to models in which all events up to a time \( t' \) can be evaluated without regards to events in the future of \( t' \). Models of this sort obey a condition that we call No Future-Input Dependence (NFID):

A mathematical model \( P_I(Q) \) obeys NFID if, for any time \( t' \) included in the relevant spacetime region, there exists a restricted model \( P_I'(Q') \) where \( I' \) is the set of all inputs belonging to times up to \( t' \) and \( Q' \) is the set of all non-input parameters up to \( t' \), such that
\[ P_I(Q') = P_I'(Q'), \tag{3} \]
for all possible values of the parameters in \( I \) and \( Q' \).

Here the requirement is that the marginal \( P_I(Q') \) is functionally independent of future inputs.

When combined with CA, the assumption of NFID implies that there is no need to consider any parts of the screening region S that lie in the future of both regions 1 and 2. As shown in Figure 1(b), if CA holds for \( P \) and \( P' \) of an NFID-respecting model, then the smaller region \( S' \) also screens 1 from 2, \( P_{t_1,t_2}(Q_1|Q_2, Q_{S'}) = P_{t_1}(Q_1|Q_{S'}). \)

3. Bell’s Screening Assumption (BSA)

If one accepts both CA and NFID, and is furthermore interested in modeling only screening regions that remain applicable in all reference frames, it becomes appropriate to ignore any portion of S that is spacelike separated from both\[ \footnote{The word “shields” is often used in the literature, including \cite{bell1990b}, instead of “screens.”} \]regions 1 and 2. This leads to the smaller region \( S' \) shown in Figure 1(c). \cite{bell1990b} proposed that this smaller region, \( S' \), should screen region 1 from region 2:
\[ P_{t_1,t_2}(Q_1|Q_2, Q_{S''}) = P_{t_1}(Q_1|Q_{S''}). \tag{4} \]

It is important to note that this screening condition does not imply that parameters in 1 are independent of parameter values in 2 – merely that the latter values are
redundant, given the specification of all model parameters in $S''$. We will call Eqn. (4) Bell’s Screening Assumption, or BSA.

4. Local Causality (LC)

Models which conform to both BSA and NFID are unable to describe certain quantum phenomena, as will be shown in Section III. We will define this important combination of assumptions as Local Causality, or LC. This definition of LC may cause some initial confusion, because this term is often associated with Eqn. (4) in the literature, which is formally just BSA. However, in essentially all cases in which this is done, the authors are presupposing NFID, either explicitly or implicitly, and the addition of this assumption turns BSA into LC. [Bell (1990b)] himself introduced BSA after clearly assuming the past-to-future causal structure associated with NFID [see Figure 6.3 there], and used the term LC to convey essentially this combination, often using the shorter “locality” as a synonym.

The reader should be cautioned about interpreting the phrase “Local Causality” as being the simple conjunction of “locality” and “causality”. There are many different meanings that could be ascribed to both of these words, as will be discussed below (in fact, we have already seen three different notions of locality in Fig. 1 above). All that is needed in the present analysis is that LC implies both NFID and BSA, both being well-defined assumptions.

An important condition which follows from NFID (or from LC), but not BSA alone, can be derived by applying it to the $S''$ region from Figure 1(c). Requiring the probabilities of parameters to be independent of future inputs, and choosing $S''$ to lie entirely in the past of all of regions 1 and 2 (in some reference frame where NFID holds), one obtains the functional independence relation

$$P_{11,22}(Q_{S''}) = P(Q_{S''}).$$  

A variant of this condition will play an important role in the proof of Bell’s Theorem below.

C. Historical Interlude

At this point it is appropriate to emphasize how natural it is to assume that all of the above conditions, summarized by LC, should hold in any detailed model describing real physics. It is convenient to do so by referencing Einstein and Bohr.

If one views a single-particle wavefunction $\psi(x, t)$ as a set of spacetime-based parameters $Q(x, t)$, and does not permit any additional (hidden) parameters, the Einstein-boxes example from Section I already indicates a violation of BSA (and thus LC). In that example, the wavefunction changes over all space as a result of a local measurement. Given convincing evidence that the predictions of QM were empirically correct, Einstein argued that there must be an alternative formulation, which would conform to LC on one hand, and reproduce the results of QM on the other. For example, if the wavefunction in Eqn. (4) was not a combination of parameters in $Q$ but merely a state of knowledge, then correlated hidden parameters $Q_1$ and $Q_2$ (localized inside the boxes) could indicate the particle’s actual location.

In 1935, Einstein, Podolsky and Rosen (EPR) essentially applied the same LC assumptions to a two-particle system (of a type to be analyzed below), and reached the logical conclusion that LC-violations could only be avoided by adding new hidden parameters. If known, these new parameters would allow one to determine the outcomes in more detail than is possible within QM. EPR concluded that QM gave an incomplete description. (The reader is warned that this chain of reasoning cannot be trusted – Bell’s Theorem, to be proved explicitly below, shows that LC is inconsistent with the predictions of QM.)

EPR did not use the formal mathematical language of Bell’s analysis. Instead, they implied the existence of
spacetime-based parameters $Q$ that encoded “an element of physical reality” (italics in original, here and below), and deduced that hidden $Q$’s must be present in a complete theory, because in some cases it was possible to “predict with certainty ... the value of a physical quantity,” such as position or momentum, “without in any way disturbing a system.”

Bohr (1935) responded quickly to EPR, defending the completeness of QM on the basis of the notion of complementarity he had developed earlier in connection with the quantum uncertainty principle. He advocated “a radical revision of our attitude towards the problem of physical reality,” and argued that the phrase “without in any way disturbing a system” used by EPR “contains an ambiguity.”

Bohr considered in detail a situation in which the properties of a particle can be discerned by first allowing it to pass through a slit in a diaphragm, and later making a “free choice” of measuring either the momentum or the position of the diaphragm. (It is remarkable, especially in the context of the present work, that the guarantee for “without in any way disturbing” was spatial separation for EPR, but temporal order for Bohr.) “Of course there is ... no question of a mechanical disturbance ... during the last critical stage of the measuring procedure,” he wrote. But as one can only measure either the position or the momentum of the diaphragm, “even at this stage” there still might be “an influence on ... the possible types of predictions regarding the future behavior of the system.” Bohr thus advocated accepting some violations of LC which are present in the formalism of QM, while at the same time excluding other violations – those corresponding to a “mechanical disturbance”. (Similarly, his notion of “completeness” clearly differs from that of EPR.)

Most physicists simply adopted Bohr’s complementarity, either in its original form or a variant (Bell 1992), and continued to develop and apply QM to a variety of physical systems (Mann and Crease 1988, Mermin 1986). But Einstein was not convinced. Summarizing the situation in 1948, he wrote (Born 1971):

... those physicists who regard the descriptive methods of quantum mechanics as definitive in principle would ... drop the requirement ... for the independent existence of the physical reality present in different parts of space. ... when I consider the physical phenomena known to me, and especially those which are being so successfully encompassed by quantum mechanics, I still cannot find any fact anywhere which would make it appear likely that [that] requirement will have to be abandoned. I am therefore inclined to believe that the description of quantum mechanics ... has to be regarded as an incomplete and indirect description of reality, to be replaced at some later date by a more complete and direct one.

Here Einstein is essentially advocating for models to be built from spacetime-based parameters $Q$, while offering the opinion that other physicists had prematurely abandoned this possibility. But there was indeed a “fact” that he was not aware of, a theorem that would be proved by Bell in 1964 (sadly, after both Einstein and Bohr had passed away). We now turn to Bell’s Theorem, and the fact that all QMa models must violate the package of assumptions that is Local Causality. Subsequently, we will address the question of whether the localized-parameter models advocated by Einstein should still be pursued, even given LC-violation.

III. BELL’S THEOREM

This section provides a brief derivation of Bell’s Theorem, which in the present terminology reads:

No model conforming with Local Causality can be in agreement with QM.

(LC models cannot be QMa.) In other words, no model which meets Einstein’s minimal “reasonable” conditions can reproduce the success of QM, in the sense of making the same predictions, given the same inputs.

11 We believe it is appropriate to interpret Bohr in this manner, but acknowledge that it is probably impossible to uncontroversially translate his writing into the formal language introduced later.
It is to be emphasized that the disagreement is not only with the predictions of QM, but also with the results of empirical observations – experiments which have been performed. The proof below is based on the Clauser-Horne-Shimony-Holt (CHSH) inequality (Clauser et al., 1969), which concerns a particular application of QM to the experimental scenario shown in Figure 2. Specifically, a source emits a pair of particles, and these are later analyzed and detected in spacelike-separated regions 1 and 2.

Mathematical models describing such situations will have an input parameter $c$ specifying the particular settings/arrangement of the common source of the two particles. Additional input parameters $a$ and $b$ specify the settings/arrangement of the detectors in region 1 and region 2 respectively. The results of the detections are the output parameters $A$ in region 1 and $B$ in region 2. The set of all the model’s spacetime-based parameters in region $\Lambda$ is denoted by $\lambda$. The parameters $a$, $b$ and $c$ are inputs, and $A$ and $B$ are outputs, just as in QM. The set $\lambda$ can be quite general, with possibilities ranging from complex combinations of functions and operators to the simplest possibility: the empty set, for the case with no local parameters residing in the region $\Lambda$.

The proof of Bell’s Theorem, following Bell (1976, 1990b, 1981), proceeds in the next two subsections by only assuming LC, deriving the CHSH inequality without making any reference to QM. The third subsection then compares the inequality with quantum theory and experiments. (Difficulties with Bell’s original 1964 proof are discussed in Section VIII.B below.)

### A. Bell’s Separability Condition

Any mathematical model capable of producing predictions for the setup of Figure 2 will provide a joint probability distribution $P_{a,b,c}(A, B, \lambda)$. The marginal distribution $P_{a,b,c}(A, B)$ can be compared with experiment and with QM. Models will generically also have other parameters, located between the designated regions (and some models may also have parameters not localized in spacetime), but these are not necessary for the main argument.

From the assumption of Local Causality, specifically from BSA of Eqn. (4), it follows that

$$P_{a,b,c}(A | \lambda, B) = P_{a,c}(A | \lambda),$$

because $\Lambda$ screens 1 from 2, in the sense that the necessary $S''$ region can be chosen to be fully contained in $\Lambda$. Similarly, $P_{a,b,c}(B | \lambda, A) = P_{b,c}(B | \lambda)$. It also follows from NFID that any model-generated probabilities of $\lambda$ must be independent of the settings $a$ and $b$, because those settings lie in the future of $\lambda$. In equation form, following (5), this reads

$$P_{a,b,c}(\lambda) = P_c(\lambda).$$

Eqn. (7) is often known as “measurement independence,” a term that unfortunately obscures the input nature of the measurement settings in any QMa model. It is clearer to call this condition $\lambda$-independence, as the

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**FIG. 2** The essential geometry of a Bell-type experiment. The parameters $a, b, c$ are inputs; the arrows indicate that their values come from outside the model. The parameters $A$ and $B$ are observable outputs. $\lambda$ is the set of all localized model parameters in the region $\Lambda$, which screens regions 1 and 2 from the overlap of their backward lightcones.
equation specifies that $\lambda$ is independent of the inputs $a, b$, via a direct application of NFID.\footnote{Another perspective results if one departs from the QMa-model framework and denies free-input-parameter status to the measurement settings, treating $a$ and $b$ as stochastic variables instead. This is the “supersdeterministic” scenario, to be discussed in Section VIII.A which allows a version of Eqn. (7) to be considered as a “no conspiracy”, “freedom of choice”, or even a “free will” condition, while still assuming NFID.}

Basic probability theory provides a product expression for the joint conditional probability: $P_{a,b,c}(A,B|\lambda) = P_{a,b,c}(A|B, \lambda) P_{a,b,c}(B|\lambda)$. With the above applications of BSA, Eqn. (6), this can be written

$$P_{a,b,c}(A,B|\lambda) = P_{a,c}(A|\lambda) P_{b,c}(B|\lambda).$$ \hspace{1cm} (8)

Since $\lambda$ is hidden, the observable joint probability is found by summing Eqn. (8) over all possible values of $\lambda$:

$$P_{a,b,c}(A,B) = \int d\lambda P_{a,b,c}(\lambda) P_{a,c}(A|\lambda) P_{b,c}(B|\lambda),$$ \hspace{1cm} (9)

where the integral is understood as a sum if $\lambda$ is a discrete variable, or a functional integral if $\lambda$ is a function (and suitably generalized if $\lambda$ is more involved). Finally, using NFID by substituting (7) yields Bell’s “Separability Condition”:

$$P_{a,b,c}(A,B) = \int d\lambda P_{c}(\lambda) P_{a,c}(A|\lambda) P_{b,c}(B|\lambda).$$ \hspace{1cm} (10)

This must hold for every applicable model which obeys Local Causality.

B. The CHSH Inequality

This inequality, from Clauser et al.\footnote{See, e.g., \cite{Bell1971} for details.} (1969), is a generalized version\footnote{The proof generalizes immediately to measurements with continuous results, provided their ranges are restricted, $|A|, |B| \leq 1.$} of Bell’s original inequality \cite{Bell1964}, and follows from Bell’s Separability Condition, Eqn. (10).

It applies to models for which the output parameters in regions 1 and 2, \textit{i.e.}, the outcomes $A$ and $B$, have two possible values, assigned to $\pm 1$ on each side. The product $AB$ must then also be $\pm 1$, and its expectation value for given inputs, \textit{i.e.}, the correlator of the outcomes, is denoted by:

$$E_{a,b,c} = \sum_{A,B} AB P_{a,b,c}(A,B).$$ \hspace{1cm} (11)

Calculating this probability from Eqn. (10) involves averaging separately over the conditional probabilities:

$$\tilde{A}_{a,c}(\lambda) \equiv \sum_{A} A P_{a,c}(A|\lambda),$$ \hspace{1cm} (12)

$$\tilde{B}_{b,c}(\lambda) \equiv \sum_{B} B P_{b,c}(B|\lambda),$$ \hspace{1cm} (13)

where the overbars denote conditional averages.

The CHSH inequality restricts the values of a combination of correlators, which involves two of the possible settings of the input parameter $a$ in region 1, labelled $a$ and $a'$, and two possibilities for the input setting in region 2, labelled $b$ and $b'$. The source input setting $c$ is held constant while the four possible combinations of inputs are manipulated, and will be suppressed from here on (we will later consider only particular Bell states, for which only one value of $c$ is relevant). With this notation, the CHSH inequality can be obtained directly from Bell separability, through the following steps.

First, substituting (10) in (11) and using (12) and (13) gives

$$E_{a,b} = \int_{\lambda} P(\lambda) \tilde{A}_{a}(\lambda) \tilde{B}_{b}(\lambda),$$ \hspace{1cm} (14)

implying integration over $\lambda$. Next, consider the combinations

$$E_{a,b} + E_{a,b'} = \int_{\lambda} P(\lambda) \tilde{A}_{a}(\lambda) [\tilde{B}_{b}(\lambda) + \tilde{B}_{b'}(\lambda)],$$ \hspace{1cm} (15)

$$E_{a',b} - E_{a',b'} = \int_{\lambda} P(\lambda) \tilde{A}_{a'}(\lambda) [\tilde{B}_{b}(\lambda) - \tilde{B}_{b'}(\lambda)].$$ \hspace{1cm} (16)

Taking the absolute value of each side, and noting that taking the absolute value of the integrand rather than the integral, and setting $|\tilde{A}| \to 1$ can only increase the right-hand side, yields the inequalities

$$|E_{a,b} + E_{a,b'}| \leq \int_{\lambda} P(\lambda) |\tilde{B}_{b}(\lambda) + \tilde{B}_{b'}(\lambda)|,$$ \hspace{1cm} (17)

$$|E_{a',b} - E_{a',b'}| \leq \int_{\lambda} P(\lambda) |\tilde{B}_{b}(\lambda) - \tilde{B}_{b'}(\lambda)|.$$ \hspace{1cm} (18)

Adding these two equations, two of the $\tilde{B}$’s will cancel for each specific value of $\lambda$, and the other two will double. A similar substitution $2|\tilde{B}| \to 2$ can again only increase the right-hand side. Removing the absolute-value operations on the left-hand side can only decrease it. Then, given that $\int_{\lambda} P(\lambda) = 1$, one finds

$$E_{a,b} + E_{a,b'} + E_{a',b} - E_{a',b'} \leq 2.$$ \hspace{1cm} (19)

This is the CHSH inequality.
C. Contradiction with QM and Experiment

When the Bell inequalities were first derived, they were shown to be in conflict with the predictions of QM. Now, they can be shown to be in direct conflict with actual experiments (Giustina et al., 2015; Hensen et al., 2015; Rosenfeld et al., 2017; Shalm et al., 2015), independent of the formalism of QM, demonstrating the failure of Local Causality. Clearly, this incompatibility of LC with the observed results would follow even if QM had never been developed.

It is simple to demonstrate that at least some QM predictions violate the CHSH inequality, Eqn. (19). Consider two photons entangled in a spin-zero Bell state, as in several of the early experiments (Aspect et al., 1981; Clauser and Shimony, 1978) (equivalently, two spin-1/2 particles can be analyzed). Suppose each photon encounters a polarizing beamsplitter, with outputs directed onto two single-photon detectors. The two beamsplitters are aligned at angles $a$ and $b$ in regions 1 and 2 respectively (these are the measurement settings, defined modulo $\pi$). For the outcome parameters $A$ and $B$, assign a value of $+1$ when the detectors imply a measured polarization aligned with the setting, and $-1$ for a measurement of the perpendicular polarization. The predictions of QM are then given by the probabilities

$$p_{a,b}(A, B) = \frac{1}{4}[1 + AB \cos(2a - 2b)].$$

The expectation value of the product $AB$ is therefore $E_{a,b} = \cos(2a - 2b)$.

For certain combinations of settings, this violates the CHSH inequality by a wide margin. The largest violation obtains for $a = 0, a' = \frac{\pi}{4}, b = -b' = \frac{\pi}{8}$, for which the left hand side of (19) is $2\sqrt{2}$ (each of the four terms contributes $+1/\sqrt{2}$). These non-classical correlations between the two photons served historically as an early and striking example of the much wider family of phenomena associated with quantum entanglement [see, e.g., Brunner et al. (2014) and Streltsov et al. (2017)].

The observed violations of the inequalities are by impressive margins, greatly exceeding the experimental accuracy. Indeed, as an empirical test of a mathematical model or a class of models, the confidence with which the CHSH inequality is rejected approaches the certainty of a mathematical proof. For example, the experimental results of Giustina et al. (2015) boast a value less than $3.7 \cdot 10^{-31}$ for the probability that the results could be obtained under the assumption of LC, according to the standard statistical analysis. Furthermore, this result belongs to the recent generation of “loophole-free” experiments (those cited above), which are free from all of the simplifying assumptions which were necessary for Bell tests with earlier technology. The observations not only violate the CHSH inequality – the quantitative results follow the predictions of QM in fine detail. We now turn to models that can be consistent with these observations, Bell’s Theorem notwithstanding.

IV. DISCUSSION OVERVIEW

The upshot of Bell’s Theorem is that there is no longer any hope of finding a reformulation of QM which respects LC (Local Causality). But the use of spacetime-based parameters has not been ruled out altogether, and the motivations for using such parameters in a reformulation remain intact. Given the empirical successes of QM, the rest of this Colloquium is dedicated to an analysis of the possibilities for developing LC-violating models that are QMa (in agreement with QM).

Additional preparatory material for this analysis is arranged in the following three subsections. The first proposes a categorization scheme for QMa models which violate LC. This serves to outline the structure of the remaining sections, which are dedicated to an in-depth discussion of the different categories. The second examines some relevant concepts of “locality” and “causality”. The third subsection provides guidance for the discussion to follow by suggesting a list of desirable features for future reformulations of QM, features which could quite possibly be achieved if spacetime-based parameters play a dominant role.

15 This program is distinct from further experimental testing of QM, which often involves analysis and empirical refutation of non-QMa theories or families of models [e.g., Gröblacher et al. (2007); Leggett and Garg (1985); and Salart et al. (2008)].
A. Categories of Bell-Compatible Models

Bell’s Theorem dictates that no QMa model can respect LC, which is the conjunction of two assumptions: **NFID** (No Future-Input Dependence) and **BSA** (Bell’s Screening Assumption). Bell-compatible models must violate at least one of these in some non-trivial manner, such that the CHSH inequality can also be violated. Of these two assumptions, we argue that the primary one for categorization purposes should be **NFID**, because the motivation for **BSA** in Section II built off of a **NFID**-based assumption. Specifically, the argumentation from the time-neutral screening condition (**CA**) to **BSA**, as depicted in Figure 1, passed through an intermediate step where **NFID** was assumed. If **NFID** is relaxed, it follows that the appropriate screening condition should be **CA**, not **BSA**.

We therefore propose the following categorization of QMa models:

- **Type I**: Respect **NFID**, violate **BSA**
- **Type II**: Violate **NFID**, respect **CA**
- **Type III**: Violate both **NFID** and **CA**

From the definition of **NFID**, Type I models allow for the calculation of all spacetime-based parameters in temporal order, using inputs that enter into the calculation in that same order. But because of the necessary **BSA** violation, such models cannot adhere to the traditional light-cone-constrained Cauchy problem found in classical physics. Conventional QM in the Schroedinger picture falls in the Type I category. We will discuss further examples and recent developments in this category in Section VI.

Type II and Type III models violate **NFID**, so they are not temporally-sequential calculations. As indicated in the Introduction, we will call such models Future-Input-Dependent, or **FID** models. For example, any model used to calculate a closed-timelike-curve in the framework of general relativity has to solve for the full history “all at once”. Other examples of **FID** models include action principles, where the inputs include final constraints that are utilized to calculate the full action-extremized solution. With such well-known examples, it is clear that **FID** models cannot be trivially dismissed. Further discussion, including explicit examples of Type II models which reproduce the Bell state correlations of QM, can be found in Section VI.

B. Causality and Local Causality

The failure of LC leads naturally to the question: In what sense, if at all, does **Local Causality** correspond to assumptions of causality or locality? We will now discuss these issues, while indicating relevant criteria that any QMa model must meet.

1. Cause and Effect

The definition of **NFID** in Section II.B uses the distinction between input- and non-input-parameters, rather than the words “cause” and “effect.” Nevertheless, the **NFID** condition is closely related to a definition of causality which arises naturally within the modern account of “interventionist” causation, where causes are identified as interventions [Pearl 2009, Woodward 2005]. If the input parameters in question are deemed to be controllable parameters, then it is appropriate to identify them as causes, according to this interventionist account. QM itself clearly adopts this connection between inputs and controllable parameters: the mathematical formalism of QM is a procedure for making operational predictions for observations, given the values of the controllable inputs. As our goal is to discuss QMa models, it is natural for us to adopt this approach. Such models limit the inputs $I$ to the parameters that QM tells us can be externally controlled. Given this connection between “controllable inputs” and “causes”, one can identify different possible causal structures. In models that respect **NFID**, non-input parameters are typically functionally dependent on past inputs, but are always functionally independent of future inputs. This “forward-causal” structure is clearly what Bell had in mind when he used the terms “causality” and “causal structure,” with the controllable inputs called “free variables” or “free elements”
FID models, on the other hand, do not have a forward-causal structure. In other words, they cannot generally compute a given parameter \( q(t') \) (or its probability distribution) without specifying certain inputs in the future of \( t' \). In the framework of interventionist causality, if those future inputs are controllable, the FID models are “retrocausal”\(^\text{16}\)

Some FID models, such as the classical action principles already mentioned, are not retrocausal. In those cases, the final boundary constraints are required mathematical inputs, but not controllable inputs, and so are not considered causes. Analysis of the causal structure of such a theory involves inverting the functional relation between some of the inputs and some of the outputs, so that a different model is obtained – a model in which all inputs are controllable. Such complications do not arise in QMa models, because we are limiting the inputs to QM’s controllable parameters. Note that the identification of inputs as controllable is not part of the mathematics of a model, but the requirement of agreement with QM leaves no ambiguity on this point.

At the time of Bell’s work, the interventionist approach to causation had not yet been well-developed. An older approach was taken for granted, dictating that if two parameters exhibit cause-effect correlations, it is appropriate to refer to the one earlier in time as a cause, and the later one as an effect, regardless of which one can be externally controlled.\(^\text{17}\) This is one topic where one’s definition of causation directly impacts the types of mathematical models that one views as acceptable. Applied to the \( \lambda \)-independence condition, any violation of Eqn. (7) would be viewed as retrocausal in the framework of interventionist causation, an instance of FID. But if one instead assumed that \( \lambda \) was the cause of the settings \( a, b \), because \( \lambda \) occurs before \( a, b \) were chosen, one would have to conclude that the settings were effects, and could not be treated as free inputs. The model would then not be QMa. This point will be expanded upon in Section \( \text{VIII.A} \)

2. Signals

Just as QM restricts the inputs \( I \) to be controllable, it also specifies that the outputs \( O \) are observable (we are here restricting \( O \) to only spacetime-based observables). If \( I \) is controllable and \( O \) is observable, \( P_I(O) \) summarizes all possible signals. And as QM does not allow signals to be sent back in time, it follows that for QMa models the outputs \( O \) cannot depend on future inputs. We shall call this requirement signal causality, or explicitly,

\[
P_I(O') = P'_I(O'),
\]

where the primed sets of parameters are all those associated with times up to \( t' \), as in the similar Eqn. (3).

Comparison with Eqn. (3) indicates that any violation of NFID in a QMa model must be at the level of unobservable (hidden) parameters in \( Q \). In other words, if some input \( I_2 \) is correlated with some earlier parameter \( Q_1 \) via some non-trivial relationship \( P_{I_2}(Q_1) \) (with the other inputs fixed), it must be that \( Q_1 \subset U = Q \setminus O \). Such an FID model would be retrocausal (at a hidden level), but would not violate signal causality.\(^\text{18}\)

Motivated by special relativity, it is natural to formulate a stronger restriction on signaling. This condition, called signal locality, limits signals to traveling no faster than light, so that signals associated with a particular controlled input are limited to outputs in its future lightcone, and signals associated with a particular observable are limited to inputs in its past lightcone. For outputs \( O_1 \) localized in region \( 1 \), the relevant inputs \( I'' \) should thus lie in the past lightcone of \( 1 \), and the signal

\(^{16}\) There is an early exception, in Bell (1964), where “causality” was used to imply “complete causality,” i.e., determinism.

\(^{17}\) The word “retrocausal” conventionally implies there are some future causes of some past parameters, not a purely-reverse-causal structure.

\(^{18}\) For example, Bell (1976, 1990b) gave inputs and statistical variables an equal status in his notation, \( \{A(a, \lambda) \} \) for the distribution of \( A \) given the values of \( a \) and \( \lambda \). In contrast, we use \( P_a(A|\lambda) \), taking advantage of the modern approach to causation.

\(^{19}\) If one demanded not only that “causes” are identified with controllable inputs but also that “effects” are identified with observable outputs, one would be led to take Eqn. (2) as representing the causal arrow of time. However, the term retrocausal in the literature does not signify violations of signal causality. We use the more technical term NFID, which explicitly focuses on inputs, in order to minimize confusion.
**locality** requirement corresponds to the existence of a restricted model \( P'' \) such that
\[
P_I(O_1) = P''_I(O_1).
\]
(22)

This condition also holds in QM, and must thus be maintained for any QMa model.

These signal-based definitions of "locality" and "causality" are operational, in the sense of involving only controllable inputs and observable outputs. But Bell’s Theorem indicates that QMa models must violate either a distinct notion of "locality" (BSA), or a distinct notion of "causality" (NFID). These are not defined operationally, as they refer to hidden model parameters, not signals. Because of these different definitions, models can be local in one sense, but not in another.  

**C. Reformulation Goals**

Before describing and discussing Bell-compatible models in the next sections, we consider the potential features that one could hope to achieve in a full reformulation of QM. There is no consensus as to which of the following are the most important, or even if all of them should be considered positive features. Nevertheless, displaying them at this stage will help to better frame the later discussion.

The motivation for these goals generally stems from known concerns with conventional QM. Additional motivation comes from our model framework (Section II), where a conventional probability \( P_I(Q) \) is associated with each complete configuration of spacetime-based parameters \( Q \). This particularly informs the first two goals in the list below. Further goals are motivated by the successes of special and general relativity.

**Physical Interpretation**: The parameters of the model should correspond to physical events in some evident manner. Furthermore, the model should not crucially depend on parameters that do not correspond to physical events. This would be an improvement over the framework of conventional QM, which is notoriously difficult to interpret in such terms.

**Linear Scaling**: In classical field theory the number of parameters involved in specifying the actual configuration of a system is linear with the size of the spacetime region, and classical particle theory scales linearly with particle number. This stands in contrast with the traditional specification of a "quantum state", which grows exponentially. A reformulation might restore linear scaling to the most fundamental mathematical description, even if the probability space of possible configurations was still exponential, as in classical statistical mechanics.

**Local Interactions**: The connections between model parameters should respect the local metric structure of conventional spacetime. This would imply a "screening" condition for distant correlations (see Figure 1 of Section II), but would not necessarily lead to BSA.

**Lorentz Covariance**: The model (or an appropriate variant of it) should utilize parameters \( Q \) that are Lorentz covariant. The rules of the model should also be covariant, applicable in all valid reference frames.

**Time Symmetry**: The model should be clearly time-symmetric, as appropriate for descriptions of microscopic degrees of freedom. This important symmetry is evident in classical physics, and also to some extent in QM. However, in order to respect signal causality, the model must incorporate some asymmetry, possibly imposed only by an asymmetric specification of input and output variables. This asymmetry need not necessarily lead to NFID.

**Resolved Measurement Problem**: There should be no special mathematical rules pertaining exclusively to "measurements". Measurements should be treated as interactions, with the measurement apparatuses treated as large systems, and should lead to consistent results if applied at different scales.

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20 The literature on Bell’s Theorem involves quite a few additional "locality" conditions [see, e.g., Wiseman (2014)], but these are not needed for the present discussion.

21 Despite describing time-symmetric phenomena, the mathematical formalism of QM is not as time-symmetric as one would or should expect (Aharonov et al. 1964; Leifer and Pusey 2017; Price 2012), indicating room for improvement. We will return to this point in Section VII below.
Even though the above italicized concepts form an incomplete list and do not fully define an ideal reformulation, they will be useful in assessing various approaches for modeling CHSH-inequality violations, both for reformulations which have been developed in the past and for approaches which may be proposed in the future. We now turn to a discussion of the different types of models that can in principle explain observed entanglement correlations, Bell’s Theorem notwithstanding.

V. NFID MODELS (TYPE I)

Any model which can reproduce the CHSH correlations without violating NFID falls under Type I. These models must violate BSA in some essential manner, either using faster-than-light mediating parameters or literal action-at-a-distance. It is generally considered useful to distinguish between these two options, and we formalize this by considering whether the most basic screening condition, CA, is respected by the model. This leads to the following sub-categorization:

- Type Ia: Respect NFID, violate BSA, respect CA
- Type Ib: Respect NFID, violate both BSA and CA

Either of these Type I approaches would allow the settings on one arm of the entanglement experiment to influence future parameters on the other side, a process which could easily explain any observable quantum correlation. In principle, this style of influence could even be used to model stronger-than-quantum correlations or faster-than-light signals, sharpening the question of why these are generically forbidden by QM (Wood and Spekkens, 2015).

In order to preserve CA, Type Ia models require a mathematical description of faster-than-light mediators between the two measurement regions in an entanglement experiment, bypassing the screening region $S''$ from Figure 1(c) but passing through the larger screening region $S'$ from Figure 1(b). This means that the step in the above derivation, $P_{a,b,c}(A|B,\lambda) = P_{a,c}(A|\lambda)$, Eqn. (6) would be incorrect; the outcome $A$ could depend on the setting $b$ and the outcome $B$. In order for this not to also violate NFID in a different reference frame, one might propose a special reference frame in which the model uniquely applies.

In practice, Type Ib models generally utilize mathematical intermediaries $R$ to connect distant spacetime-based parameters. Recall that we have defined the model parameters $(I, Q)$ to be spacetime-based parameters, associated with particular places and times. But a mathematical model could introduce non-spacetime-based parameters $R$, that would not show up in these sets. If distant $I$’s and $O$’s were both coupled to the same $R$ via the rules of the model, then this would provide a mathematical mechanism for “action-at-a-distance” between $I$ and $O$. (Since $R$ would not be localized in spacetime, it could not be considered as residing in any intermediate screening region.)

A. Examples

The most prominent example of a Type Ib model is conventional Schroedinger-picture QM itself. The many-body wavefunction from an entanglement experiment plays the role of $R$ in this case; it is defined on configuration space, and is not comprised of spacetime-based parameters. The conventional QM account therefore involves no parameters localized in the region $\Lambda$. Since $\lambda$ is then an empty set, BSA as implemented in Eqn. (6) would imply that the outcomes $A$ and $B$ should be completely uncorrelated. In fact, the outcomes can be significantly correlated, via the configuration-space wavefunction, directly violating both BSA and CA.

Another type Ib model, which explicitly motivated Bell’s studies (Bell, 1982), is the Pilot-Wave theory (Bohm, 1952). In this reformulation, particle positions are added to the wavefunction of QM; these positions are initially distributed according to a stochastic rule, but later evolve according to a deterministic dynamics guided by the wavefunction, and the outcomes are also determined without additional stochastic elements. Unlike conventional QM, there are now unknown localized parameters $\lambda$ – the particle positions – but the Bell correlations indicate that BSA must be violated. The mech-
anism of this violation still involves the configuration-space wavefunction, but now the consequences are even more explicit, as the choice of measurement setting a in region 1 directly affects the acceleration of the particle in region 2 [see, e.g., Dewdney et al. (1987)]. On the positive side, the fact that measurement devices are also comprised of particles means that this approach has a Resolved Measurement Problem.

Other NFID models for which the correlations are directly enforced via a non-spacetime-based wavefunction also fall into Type Ib, almost regardless of what the model uses as localized parameters. Spontaneous-collapse models (Ghirardi et al., 1986), first introduced in order to resolve the measurement problem, also enforce distant correlations via the configuration-space wavefunction. So-called “flash” models have parameters in spacetime (the flashes) generated from the wavefunction, but there are no intermediate screening parameters (Tu- multka, 2006). And while a purely empirical approach (one that takes only the measurement outcomes to be parameters) can claim to be agnostic about what lies in the screening region, such models formally violate all the screening conditions from Section II.

An alternate parameterization that can essentially convert conventional QM into a Type Ia model is to introduce new spacetime-based parameters corresponding to the entire configuration space wavefunction. First, identify “local copies” of the full wavefunction \(|\psi(t)\rangle\) as spacetime-based parameters \(Q\) that exist at every point in space with time coordinate \(t\) (Spekkens, 2015). Then, implement a literal instantaneous collapse of this \(Q\) upon measurement, such that \(|\psi(t)\rangle\) updates everywhere in space. This updated wavefunction could be thought of as transferring information from one region to another at an infinite speed, bypassing the \(S''\) region of Figure 1(c), while passing through the upper boundary of the region \(S'\) in Figure 1(b). This conversion could presumably be performed for any Type Ib model that uses a wavefunction, but the Linear Scaling goal would not be achieved, due to the use of parameters which are exponentially complicated.

Unfortunately, besides this generic conversion, there are very few Type Ia models in the literature. Perhaps the only attempt to develop an explicit example of such a model is by Norsen (2010), representing the many-body wavefunction of a Pilot-Wave-like approach through an infinite number of spacetime-based fields in a preferred reference frame. Although this achieves the goal of employing exclusively spacetime-based parameters, it is not clear that intermediate screening parameters are always available to conform with CA, and the dynamical rules of the model have the local field evolution explicitly influenced by the locations of other, distant particles. So despite being fully spacetime-based, the model does not have Local Interactions.

B. Discussion

Despite the lack of explicit Type Ia models, the general concept has been promoted by various authors, including Bell (1981) himself. The possibility of a hidden faster-than-light mediator with finite propagation speed in some special reference frame (ether) has not been ruled out by existing experiments (Salart et al., 2008), but it would require an instantaneous mediation in the frame of the ether to be fully QMa in all cases. Either way, Lorentz Covariance would seem to be impossible. We will note that such a mediator would not only be unmeasureable, but also unattenuated by increasing distance or obstacles, and incapable of carrying a signal. Furthermore, this would imply an asymmetry between the two entangled particles, in that one of them would be sending the hidden mediator to the other – and this asymmetry would have to switch directions as the timing of the two measurements was reversed in the frame of the ether. This problem afflicts Type Ib models as well.

One apparent advantage of Type Ia models over Type Ib models is that they restore the mediators to spacetime-based parameters, and might better allow a Physical Interpretation. But if such models require the reintroduction of a special reference frame, any success in restoring the role of spacetime might be seen as beside the point. At the very least, the spacetime that would be restored would not be conventional spacetime, but rather something else, with all the inherent problems of the 19th century ether. Still, Bell at one point imagined
that adding an ether might be the “cheapest resolution” to BSA-violation [Bell 1981].

Type Ib models, on the other hand, violate CA and therefore have no intermediate screening region. There may still be a mathematical continuity in some higher-dimensional configuration space, but this should not be treated as a stand-in for a physical continuity in ordinary spacetime. Such models explicitly give up on Local Interactions, instead somehow enforcing correlations across spacelike separations. This would seem to still require a preferred reference frame, retaining some of the threat to Lorentz Covariance, but arguably less severe than Type Ia (Gisin 2010). Also, by retreating to non-spacetime-based parameters, Type Ib models would provide an explanation of why obstacles in the intermediate spacetime region cannot diminish the correlations.

VI. FID MODELS (TYPES II AND III)

Bell-compatible FID (Future-Input Dependent) models are underrepresented in the literature on Bell’s Theorem [22]. Therefore, this section will provide a rather thorough discussion of such models.

The essential strategy behind Type II models of entanglement is to allow a violation of $\lambda$-independence, Eqn. (7), such that $P(a,b)(\lambda)$ is not independent of the input settings $a, b$. The relevant $\lambda$ lies in the past light-cones of $a, b$, so that such models are technically “retrocausal” as defined in Section IV.B.1. But as noted there, any correlations with future settings must be sequestered in hidden variables, not observable outputs, if agreement with QM is to be maintained. Therefore there is no question of signals being sent back in time in QMa models, and there is no concern about paradoxes (other concerns with such models will be addressed below).

The promise of Type II models is that, in any given case, there exist parameters $\lambda$ that can act as local mediators of the actual correlations. It is always simple to find a distribution of shared parameters $\lambda$ that will produce a given correlation for a particular measurement setting; Bell showed that the problem was getting the same $P(\lambda)$ distribution to consistently work for all measurement settings. But for models $P(a,b)(\lambda)$ where the distributions can be different for different settings, Bell’s consistency problem disappears. This means that it is possible to retain BSA in some FID models, or at least the weaker locality condition CA.

Unlike Type II models, Type III models do not respect any of the locality conditions from Section II, as in general they do not have the necessary spacetime-based parameters $\lambda$. For this reason Type III models are assumed to be of less interest, but will be addressed when relevant. (Similarly, models with parameters which are directly dependent on distant events, and thus violate Local Interactions, should be of less interest, even if they manage to fulfill the formal locality conditions involved in Bell’s Theorem.)

After giving several explicit examples of Type II models below (and one which is arguably of Type III), we will respond to some arguments against them in the literature. Then we will outline some promising future directions for this general research program.

A. Examples

At the current stage of development of Type II models, there are none which are applicable to a wide range of quantum phenomena. Accordingly, we will discuss some of the existing example models, which aim at reproducing merely the known correlations for a Bell state, Eqn. (20), or are similarly limited in scope.

1. Proof-of-Principle Models

Although the idea of using Future-Input Dependence to explain entanglement had been around for a long time (Costa de Beauregard 1953, 1977, 1979; Cramer 1980; Price 1984, 1997; Sutherland 1983), it was explicitly cast in the mathematical terms of a Type II QMa model only in the last decade (Argaman 2010). Using the terminology of Section III.C, where $a$ and $b$ represent the an...
gle settings of polarizers, the Bell state correlations were achieved via the following toy-model.

First, constrain the two photons to both be initially polarized at an unknown angle \(\lambda\), and distribute \(\lambda\) according to

\[
P_{a,b}(\lambda) = \frac{1}{4} \left\{ \delta(a - \lambda) + \delta \left( a + \frac{\pi}{2} - \lambda \right) + \delta(b - \lambda) + \delta \left( b + \frac{\pi}{2} - \lambda \right) \right\}.
\]

(Here \(\lambda \in [0, \pi]\) and the \(\delta\)-functions are modulo \(\pi\).) Thus \(\lambda\) is somehow constrained by the future settings to be either \(a\), \(a + \pi/2\), \(b\) or \(b + \pi/2\) with equal probabilities, and identified as the polarization of the photons as they leave the source. Then, an application of Malus’ law for the results of the single-photon measurements, \(A\) and \(B\) [e.g., \(P_A(A = 1|\lambda) = \cos^2(a - \lambda)]\), using Eqn. (9), reproduces the QM probabilities for the spin-zero Bell state, Eqn. (20). While this model is clearly very schematic, it demonstrates that only mediation along the spacetime paths of the particles is required. Unlike a faster-than-light connection between two regions, such mediations can be viewed in a Lorentz Covariant manner.

Notice that such models may be formulated with the model parameter \(\lambda\) associated with different spacetime regions. For example, a model identical with the above except for having \(\lambda\) not associated with times earlier than \(a\) and \(b\), does not represent an FID model. In fact, this is precisely the model suggested by Di Lorenzo (2012).

A number of additional Type II schematic models follow a similar strategy. They consist of two components: (i) a specification of the sample space of the hidden variables and their distributions \(P_{a,b}(\lambda)\), and (ii) models for the measurement outcomes, \(p_{a,[\lambda]}(A)\) and \(p_{b,[\lambda]}(B)\), such that the combination of (i) and (ii) per Eqn. (9) gives a QMa model for a specific setup of interest. (The notation \([\lambda]\) emphasizes that although \(\lambda\) is an input for the second component, it is not an external input.)

For want of space, we will provide the details of just one additional example, that of Hall (2010). The version adapted to photon polarizations Argaman (2018) has:

\[
P_{a,b}^{\text{Hall}}(\lambda) = \frac{1}{\pi} \frac{1 + \hat{A}\hat{B}\cos(2a - 2b)}{1 + \hat{A}\hat{B}(1 - z)}.
\]

where \(\hat{A} = \text{sign}[\cos(2a - 2\lambda)]\), \(\hat{B} = \text{sign}[\cos(2b - 2\lambda)]\) and \(z = \frac{1}{2}|a - b|\) are abbreviations. This model is deterministic in the sense that \(A\) is fully determined by \(a\) and \(\lambda\) through \(p_{a,[\lambda]}(A) = \delta_{\lambda,\lambda}\), with the same expression relating \(B\) to \(b\) and \(\lambda\). It reproduces the results of QM for the Bell state, Eqn. (20). Here, knowledge of \(\lambda\) provides only a very rough idea of what \(a\) and \(b\) are. When properly quantified, the information about \(a\) and \(b\) which can be gleaned from the past parameter \(\lambda\) amounts to less than 0.07 bits per entangled pair Hall (2016). In this sense, it is appropriate to view the toy-model of Eqn. (24) as a dramatic improvement over that of Eqn. (23).

Still, such proof-of-principle schematic models raise immediate concerns. The connection between the settings and \(\lambda\) is simply asserted, without a proposed mechanism or explanation. One natural justification for such a connection is Time Symmetry: one could argue that the symmetry exhibited by micro-scale phenomena implies an equal role for both past and future parameters. This would make the future settings \((a, b)\) just as important as the initial state preparation \(c\) when modeling \(\lambda\). But this justification seems inapplicable to these schematic models, because they do not possess Time Symmetry in any sense.

Conventional QM has a similar conflict with Time Symmetry. For example, consider the polarization of a photon which is known to have passed through two consecutive polarizers set at angles \(\theta_1\) and \(\theta_2\). The conventional description associates the angle \(\theta_1\) with the polarization of the photon between the two polarizers, but time-reversal symmetry implies that \(\theta_2\) should be just as relevant to the intermediate description. Any time-symmetric account of the intermediate photon should therefore be a FID model. We now turn to an example that builds off this very strategy of restoring microscopic Time Symmetry, leading to a Bell-compatible FID model that proposes a mechanism for an appropriate \(P_{a,b}(\lambda)\) distribution.

2. The Schulman Model

A more time-symmetric approach to the above two-polarizer problem has been developed by Schulman (1997 2012), using a time-varying polarization angle
Schulman considered the possibility that \( q(t) \) could be perturbed by microscopic rotations \( dq \) (“kicks”) so that \( q(t) \) evolves from \( \theta_1 \) to \( \theta_2 \) (or \( \theta_2 + n\pi \)) between the polarizers, without requiring a collapse at the last instant. If the magnitude of each microscopic kick is normally distributed (or has a finite second moment) one would obtain diffusive behavior, which is inappropriate. However, if \( q(t) \) describes a Lévy flight, e.g., if the magnitudes of the kicks are distributed according to the Cauchy (Lorentzian) distribution, \( \propto d\gamma/[(dq)^2 + (d\gamma)^2] \) with a small width \( d\gamma \), the net rotation \( \Delta q \) has a similar probability distribution:

\[
P(\Delta q) = \frac{1}{\pi} \frac{\gamma}{(\Delta q)^2 + \gamma^2},
\]

(25)

where \( \gamma \) is the sum of all the \( d\gamma \) widths of all the kicks along the path.

With \( q(t) \) constrained to \( \theta_1 \) at the time of the initial polarizer and to \( \theta_2 \) at the final time, \( q(t) \) provides an appealing time-symmetric description of the dynamics (constrained by initial and final boundaries). Moreover, and this is the main point of Schulman’s derivation, the model correctly predicts the outcome probabilities for a single photon in the limit \( \gamma \to 0 \), if the measurement acts as a boundary constraint corresponding to discrete possibilities, requiring the photon polarization to either be aligned or perpendicular to the polarizer angle (either \( \theta_2 \) or \( \theta_2 + \pi/2 \)). Adding all the equivalent contributions corresponding to \( \theta_2 + n\pi \) per Eqn. (25) gives a result \( \propto 1/\sin^2(\theta_1 - \theta_2) \). Comparing this to the sum over \( \theta_2 + (n+\frac{1}{2})\pi \) reproduces, upon normalization, Malus’ law: the probability for a photon of initial polarization \( \theta_1 \) to align with a polarizer oriented at \( \theta_2 \) is \( \cos^2(\theta_1 - \theta_2) \). Details of the proof can be found in the Appendix.

Note that for small \( \gamma \) the path \( q(t) \) is very close to being a constant, but the initial and final requirements enforce at least one significant “kick”, with a distribution \( \propto d\gamma/(dq)^2 \). In the \( \gamma \to 0 \) limit the paths with a single kick dominate. This means that there is an event which corresponds to “collapse” in this description (unless \( \theta_1 = \theta_2 \) or \( \theta_1 = \theta_2 + \pi/2 \)), but it happens at an arbitrary time between preparation and measurement, rather than at the time of the measurement, and thus respects Time Symmetry.

This model can be trivially extended to the case of two maximally entangled photons, by combining two copies of the single-particle model, \( q_1(t) \) and \( q_2(t) \), and constraining their unknown initial polarization angles to be identical [Almada et al. 2016 Wharton 2014]. Identifying this initial polarization as the hidden parameter \( \lambda \) reproduces precisely the probability distribution of the first toy-model above, Eqn. (23). This follows because the most probable scenario is to have only one kick in the combination of the two paths, but this in turn requires \( \lambda \) to match one of the two future settings. While the schematic model omitted the details of why \( \lambda \) might be correlated with the future settings, the Schulman model provides a “master rule” which naturally generates the appropriate connection.

In this model, the screening region \( S'' \) from Figure 1(c) contains the parameters \( q_1(t) \). Nothing on the other arm of the experiment can affect the probability of the measured outcome \( q_1(t) \) without also affecting the earlier values \( q_1(t) \), thus respecting BSA. The earlier schematic models also respect BSA for similar reasons. The mechanism by which the correlations are enforced is NFID-violating: the future settings \( (a, b) \) constrain the full histories \( q_1(t) \) and \( q_2(t) \), including the possible value of the initial hidden parameter \( q_1(t_1) = q_2(t_1) \). This explicitly violates NFID and Eqn. (7), violating LC. In other words, this model shows explicitly how CA and BSA can be preserved, by allowing violations of NFID.

This model resolves the concerns raised by the proof-of-principle models above. Indeed, this two-particle toy-model is currently the most sophisticated example of how a model can yield the correct Bell state correlations while retaining the BSA form of locality. It demonstrates an available spacetime-based mechanism for the correlations involved in entanglement, utilizing the entire history rather than instantaneous “states”. By assigning probabilities to histories rather than states, this type of mechanism avoids the tension between entanglement effects and relativistic covariance. In general, NFID violations need not conflict with relativity, because all of

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\(^{23}\) Schulman’s discussion of spin-1/2 particles is here adapted to photons.
the mediation is by parameters that reside on timelike or lightlike worldlines. If the relevant parameters \( \lambda \) reside on the classical worldlines of the entangled particles, this would look essentially similar in every reference frame, no matter which particle was measured “first.” Furthermore, the mediators in these accounts would no longer be “unblockable” as in Type I models – blocking the particle worldlines would clearly destroy any expected correlations.

In addition to the above, the same probability rule explains not only the proper two-photon correlations but also provides the appropriate Malus-Law probabilities for a single photon. Furthermore, assuming that the boundary constraint at measurement would also be imposed for additional intermediate measurements, trying to devise ways to measure the hidden parameters \( \lambda \) would necessarily fail to reveal the mechanism behind the correlations. Indeed, adding a new intermediate measurement would change the entire history of the experiment, requiring two ‘kicks’ instead of one, reproducing again the often-perplexing results of standard QM.

The two-particle Schulman model can also be trivially generalized from the spin-zero state to any maximally entangled two-qubit state [Almada et al., 2016]. Further generalizations to scenarios with several particles [Bennett et al., 1993; Pan et al., 1998] might no longer respect BSA if some of the correlating parameters are localized on connected zigzagging worldlines, but this would continue to respect CA and would still be Type II. The challenge of extending this type of model to partially-entangled states remains an open problem.

3. The-Two-State Vector Formalism

A well-known FID model is the two-state-vector formalism introduced by Aharonov and Vaidman (1991). For single-particle cases, it proposes that the ordinary wavefunction \( \psi(x,t) \) shares spacetime with another wavefunction \( \phi(x,t) \). This second wavefunction is determined by the setting and the outcome of the next strong measurement on the particle (in the future of \( t \)). Specifically, one can find a solution \( \phi \) to the Schrödinger equation consistent with the next strongly measured eigenfunction (essentially a future boundary constraint), and treat \( \phi \) as a spacetime-based field that will unitarily evolve into that eigenfunction. Together, \( \phi \) and \( \psi \) can be used to generate weak values that can be tested experimentally (by post-selecting and averaging many weak measurements), but these two fields do not formally interact.

It is conventional within this approach to take \( \phi \) to be determined by the future outcome, rather than the future setting, but for the model to be QMa the outcome must be an output parameter, included in \( Q \) [Aharonov and Vaidman, 2008]. With this adjustment, the resulting model is FID because the choice of future measurement setting restricts the space of possible eigenfunctions, encoded at earlier times in the possible values of \( \phi(x,t) \). Vaidman (2013) argues that it gives a reasonable account of single-particle interference scenarios.

Unfortunately, for entanglement scenarios with more than one particle, the relevant state vectors are conventional configuration-space wavefunctions, \( \psi(x_1,x_2,t) \) and \( \phi(x_1,x_2,t) \), and these entangled two-particle wavefunctions cannot easily be mapped onto spacetime-based fields. While these wavefunctions are not spacetime-based, they are at least \( \text{time-based} \) parameters, and so they still exhibit an essential violation of the ideas behind NFID. But having departed from spacetime, they no longer have any localized screening parameters, and so violate CA. It is therefore fair to categorize this model as Type III, while the earlier examples were all Type II.

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24 The general strategy of reducing a two-particle entanglement problem to two single-particle problems can be extended to all maximally-entangled bipartite states [Wharton et al., 2011].

25 The present discussion also applies to the well-known Transactional Interpretation [Cramer, 1980, 2016], where the individual “confirmation” waves correspond to \( \phi \).

26 He also considers treating the weak values of local operators as parameters (in \( Q \)), but demonstrates that such a description has clear deficiencies when applied to a certain nested interferometer.

27 Sutherland (2017) has made progress in restoring spacetime-based fields to a two-state-style model, but this promising effort is as yet unable to generate probabilities without utilization of the configuration space wavefunction.
B. Objections to FID Models

Despite the existence of the FID models presented above, much of the contemporary discussion of Bell’s Theorem fails to recognize such models as a possibility. For example, in the recent round of loophole-free experiments (Giustina et al., 2015; Hensen et al., 2015; Rosenfeld et al., 2017; Shalm et al., 2015), not one mentioned the possibility of FID models. In the rare case where experimental papers mention a retrocausal option, it is typically relegated to a mere footnote (Handsteiner et al., 2017; Rauch et al., 2018).

With this lack of attention, there are few published concerns about FID models in the recent literature, although a number of “intuitive” objections are likely to occur to most physicists upon first encountering these models. The most common such concerns will be addressed first, immediately below. Going beyond intuition, a few researchers have developed specific formal reasons why NFID-violating models might not be promising. These arguments in the literature will also be briefly summarized and discussed.

1. Intuitive Objections

One common objection to FID models is that they violate some principle of “causality”, in that they would lead to logical difficulties such as time-travel paradoxes. But time-travel paradoxes require communication with the past, with at least some level of observable signal, and this is forbidden in QMa models which conform to signal causality, Eqn. (21) above. For any FID model in agreement with QM, the future-input dependence is always at the level of the hidden parameters, $\lambda$, and therefore would not allow retro-signalling.

Another common concern is that FID models imply future inputs must “exist” to constrain hidden parameters in the past, and some find this block-universe view problematic (Kastner, 2017; Sorkin, 2007). But it appears ill-advised to avoid developing a theory for such reasons – it would have been a pity, for example, if Newton were to avoid developing the Law of Universal Gravitation because he perceived its nonlocality to be unacceptable. Furthermore, treating future events as valid model parameters and analyzing entire spacetime regions “all at once” is common in physics; consider the examples of general relativity, action principles, and Wick rotations. And in any case, one can always wait until the whole relevant spacetime region is in the past, and perform the model analysis retrospectively. We set aside this objection as an essentially anthropocentric restriction on mathematical models (Wharton, 2015).

As a related objection, some might take the view that because QM conforms to signal causality there should never be any reason to consider FID reformulations of QM in particular. However, as we have seen above, Bell’s Theorem (perhaps surprisingly) provides just such a reason. And again, such a restriction is routinely ignored by physicists in practice. Histories approaches such as Griffiths (2001), and path integrals in general, insist on considering the past and future together as one unit, violating the spirit of NFID. In some applications of Heisenberg’s Matrix Mechanics, measurement operators are effectively evolved back in time to the previous measurement. And some analyses of “delayed-choice” experiments, such as that of Bohr (1935) briefly described in Section II.C, allow one to make incompatible inferences about past events for different future measurements. If those past events are parameterized, this also violates NFID.

2. Formal Objections

An early technical argument against FID models is due to Maudlin (1994), adapted here to apply to measurements on two particles in a Bell state. Consider the case with one of the measurements (yielding outcome $A$) performed early enough so that the result $A$ can be sent ahead of the other particle (say, via a laser signal) to the other measurement device. The measurement outcome $A$ could then be used to determine the other setting $b$, via some algorithm $b = f(A)$. The challenge is one of self-consistency: if one uses a model that requires $b$ as an input to generate $\lambda$, and then uses $\lambda$ to generate the outcomes $A, B$, the function $f(A)$ might be found to disagree with the value of $b$ utilized in the calculation. This
is of a particular concern for “schematic” models (such as those in Section VI.A.1) that have been designed with one experiment in mind, because this set-up is essentially a different experimental configuration.

But it is unreasonable to expect precisely the same model, with the same inputs and outputs, to apply to this new configuration. In our version of Maudlin’s challenge, we have eliminated the setting parameter \( b \) from being an input to the model (it cannot be freely set, given this configuration), so an analysis of this new experiment would require an FID model of the form \( P_{\alpha}(Q) \), rather than the original \( P_{\alpha,b}(Q) \). The Schulman model of Section VI.A.2 is general enough to handle this new experimental configuration, because the boundary constraints imposed by the future measurements are still enforced in the global solution, no matter whether the settings are free inputs or calculated parameters. So long as the solution is calculated “all at once” – assigning probabilities to entire histories rather than states – every intermediate solution is self-consistent, by definition (Berkovitz 2008; Lewis 2013; Wharton 2014).28

A more recent objection, that applies even to all-at-once accounts, has appeared in Wood and Spekkens (2015) – although, notably, this stands as an objection to all causal accounts of entanglement phenomena, not specifically FID models. The essential point is that causal channels are typically accompanied by signal channels, absent some special “fine tuning” of the underlying model. Such fine-tuning would require additional explanation. In any causal account of entanglement, such as the faster-than-light options discussed in Section V, signal causality (the inability to send a spacelike signal) must be the result of some perfect cancellation in the marginal probabilities. This is said to be “fine-tuned” because even a slight deviation would lead to spacelike signaling. The situation might appear to lead to an additional challenge to FID models, where causal channels exist directly into the past, because another fine-tuning argument can be applied to signal causality (the inability to send signals into the past).

But a more careful analysis reveals that the fine-tuning objection is not significantly worse for FID models than it is for NFID models, because spacelike signaling violates signal causality in some reference frame. Further analysis of the Schulman model has revealed that the appearance of signal locality follows from a basic symmetry (Almada et al. 2016), providing just the sort of explanation (from symmetry) that is most often used to explain fine-tunings in particle physics. A more comprehensive explanation of both signal locality and signal causality has also recently been proposed by Adlam (2018).

There is also a flip-side to the Wood-Spekkens fine-tuning argument. If an underlying physics model indeed breaks time symmetry according to the NFID condition, it would take a very finely balanced restriction to make microscopic physics look as time-symmetric as it does. Leifer and Pusey (2017) weigh this argument against the Wood-Spekkens fine-tuning argument, and propose that the time symmetry argument is stronger.

We briefly mention another recent analysis of FID models which has indicated that they must be “process contextual” in a particularly-defined sense (Shrapnel and Costa 2018). This would imply that a Type II model could require distinct hidden variable descriptions for two different situations that could not be distinguished by ordinary quantum theory. But these processes concern mixed states, and therefore imply that knowable information is discarded. The necessity of including such information in FID models deserves more study, but this is clearly not a no-go result for Type II models in general.

C. Discussion of Type II Models

The examples described above demonstrate that a number of Type II FID models can successfully account for the Bell state correlations.29 Thus, Bell’s Theorem

28 The QMa status of the original \( P_{\alpha,b}(Q) \) guarantees through signal causality that its operational version, \( P_{\alpha,b}(O) \), can be restricted to times up to the first measurement, yielding \( P_{\alpha}(A) \); subsequently, the full applicable model can be reconstructed: \( P_{\alpha}(Q) = \sum_{A} P_{\alpha}(A) P_{\alpha,f(A)}(Q|A) \).

29 Other recent examples in this category include retrocausal models generated by machine learning (Weinstein 2017, 2018), and a Pilot-Wave-style account that is explicitly FID (Sen 2019).
cannot be said to stand in the way of a spacetime-based reformulation of QM. In particular, the Schulman model admirably achieves the goals of *Physical Interpretation*, *Local Interactions* and *Time Symmetry* prescribed in Section IV.C.

This last point deserves further attention, for the way that the Schulman model reconciles time-symmetric rules with the time-asymmetry of *signal causality* is instructive. The only asymmetry in the model enters via the distinction between inputs and outputs (preparations and measurements). For preparations, it is assumed that one can select the initial polarization (or the initial correlation between two polarizations, for the case of entanglement). But the measurement settings do not allow this level of control; one can set the polarizer angle, but not the output polarization. This empirically-based distinction between input control and output non-control provides the symmetry-breaking mechanism which leads to the appearance of *signal causality* in the model. Everything else about the model respects *Time Symmetry* – most notably, the intermediate account between preparation and measurement.

Generalizing this compatibility of *Time Symmetry* and *signal causality* to more complicated systems has been considered by [Price (1997)](#), emphasizing the role of “agents,” who select which of the available parameters to use as inputs to a model and which to use as outputs. As these agents are themselves subject to an arrow of time, they are unable to influence a particular setup if they learn of outcomes at its final time (a typical measurement), whereas they can influence it, e.g., by blocking a certain particle path, if they learn of certain outcomes at its initial time (a typical preparation). Other than the role of agents, Price takes physical phenomena to be strictly *Time Symmetric*.

As an alternative, consider the following. It is well-known that fixing initial conditions in a model with time-symmetric *deterministic* dynamics, such as Maxwell’s equations for the electromagnetic field, leads to *NFID* – an external perturbation, such as a dipole oscillating for a brief interval of time beginning at $t$, produces electromagnetic waves only at later times. The time-symmetry-breaking effect of fixing the initial conditions (treating them as early-time inputs, in addition to the inputs associated with the external perturbation at the later time $t$) suffices to produce these *NFID* results.

The results are different if the degrees of freedom of the model are subject to stochastic time-symmetric rules, rather than classical deterministic differential equations. Fixing the initial conditions would still break time symmetry, but would no longer necessarily lead to *NFID*. Such stochastic models are necessarily subject to an uncertainty principle [known as the “No-Cloning Theorem” ([Daffertshofer et al. 2002]; see also [Spekkens 2007]), because the precise microscopic value of a degree of freedom is perturbed by any interaction with a measurement device, and the stochasticity of the rules generally prevents a precise reconstruction of its value or its distribution.

But this does not preclude the existence of more macroscopic, “coarse-grained” variables, for which the values can reliably be copied or measured (for example, the macroscopic variables may represent the basin of attraction in which the microscopic variables propagate). An intriguing possibility, called “lenient causality” in [Argaman (2018)], is that in some models of this type the symmetry-breaking could impose *signal causality* for the macroscopic parameters, without leading to *NFID* for the microscopic ones. In fact, one may speculate that within such a theory, it would transpire that the appropriate microscopic degrees of freedom are fields – subject to quantum fluctuations and uncertainty – while the particle aspects represent the macroscopically available information.

Concerning the other reformulation goals discussed in Section IV.C it appears likely that a suitably generalized Type II approach could achieve *Lorentz Covariance*, as it avoids the need for special reference frames or connections across spacelike distances. And by using only spacetime-based parameters $Q$, the model $P_t(Q)$ has an evident *Physical Interpretation*; the model specifies the probability of each possible set of events in spacetime $Q$, but only one particular configuration actually occurs.

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30 Better still, it could lead to a condition known as Information Causality ([Pawlowski et al. 2009]), which would then automatically guarantee observance of the Tsirelson bound ([Cirelson 1980]).
Similarly, *Linear Scaling* would be a plausible feature of any model built from entirely spacetime-based parameters; doubling the size of the modeled spacetime region would double the relevant model parameters. The Schulman model provides a clear example, in that the parameters required for a two-particle experiment are merely two copies of the single particle case.

The exponential growth of the conventional wavefunction $\psi(t)$ with particle number might lead one to think *Linear Scaling* was an impossible goal, especially if one viewed the information contained in $\psi(t)$ as some physical entity which had to be translated into parameters $Q(t)$. But note that $\psi(t)$ contains information about all possible measurement outcomes which might occur, for all possible future measurement settings. In an FID model, $Q(t)$ can be a function of those future settings, and therefore need only inform the outcomes for the actual future measurement, vastly reducing the required number of parameters [for further analysis, see Wharton (2014)].

At the present stage of development of Type II models, the Resolved Measurement Problem goal is not yet clearly in sight. Still, the Type II Schulman model does help resolve a portion of this issue in two ways. First, measurements do not correspond to any sudden collapse, so they look more like an ordinary interaction (the collapse-like event occurs somewhere between preparation and measurement). Second, there is no confusion about whether the size of the relevant configuration space should expand (as in a QM interaction) or be reduced (as in a QM measurement), because nothing lives in configuration space; all parameters exist in spacetime.

A future Type II theory should provide an explanation for why the interaction between some large systems (measurement devices) and some smaller systems (such as the measured particles) can be described effectively by imposing boundary constraints on the smaller systems. It is worth noting that in general, such behavior is evident near large conductors in electromagnetism and thermal reservoirs in classical thermodynamics.

The connection to the lenient-causality ideas mentioned earlier is also of interest. According to this proposal, the measureable parameters $Q$ of a model would not be partitioned merely according to which serve as outputs, but according to their microscopic or macroscopic behavior, i.e., whether or not they are inevitably disturbed when they are subject to an interaction – their ability to be “cloned.” For a specific model, outputs can then be selected at will out of this set of parameters, as all of them are subject to the signal causality and signal locality conditions, whether or not there are agents which use them for signaling. As in the relation between statistical mechanics and thermodynamics, mesoscopic systems would exhibit an intermediate behavior, only very rarely violating these conditions.

Returning to shorter-term goals, beyond Bell state correlations there are plenty of other quantum phenomena that must be addressed to approach a full reformulation of QM. Other recent FID models have tackled some of these issues, including single-particle interference (Wharton, 2018), position measurements of entangled particles (Sen, 2019), and formal relativistic covariance (Heaney, 2013; Sutherland, 2017; Wharton, 2010). Presumably more models will be developed in the coming years, addressing additional issues such as 3-particle and partial entanglement phenomena.

**VII. COMPARISON OF TYPE I AND TYPE II APPROACHES**

In this section, we take stock of the results obtained within our model framework, Section II.A, comparing and contrasting the various Bell-compatible approaches. Other approaches outside our framework will be briefly discussed in the next section.

The central distinction between the approaches covered in Section III and those of Section VI is the NFID assumption (No Future-Input Dependence) of Eqn. 3. Specifically, for every time $t'$, NFID requires that a compliant model $P_t(Q)$ must be compatible with a restricted probability distribution, $P_{t'}(Q')$, which does not involve any parameters to the future of $t'$. In other words it

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31 It is of interest to compare these speculative expectations with the careful analysis of Zurek (2003), within standard QM, of the relation between measurements and dissipative environments.
requires a full description of the past which is independent of future inputs. This is a very natural assumption, following from the intuitive notion that “the past is fixed”, but Bell’s Theorem proves that there is a steep price for enforcing it. Specifically, any NFID model in agreement with QM must violate BSA (Bell’s Screening Assumption). We are forced to give up at least one of these assumptions, but Bell’s Theorem on its own does not indicate which path to pursue.

Instead, we propose assessing the various approaches with respect to the goals in Section IV.C for guidance. Concerning an evident Physical Interpretation, we note that every Type I model utilized some non-spacetime-based parameters (e.g., in Hilbert space), while no type II model required such parameters. As described in the previous section, any model \( P_I(Q) \) in the latter category has a trivial interpretation; if this is judged to be an important model property, Type II models would be generally superior. This same advantage carries over to Linear Scaling, where an entirely spacetime-based parameter set naturally avoids QM’s exponential parameter growth with particle number. Type I models do not accomplish this, since they cannot restrict their required parameters to the case of the actual future measurement setting. Instead, the same wavefunction is used to calculate for all possible future measurement settings, which increase exponentially with particle number.

Most of the above Type I models did not respect any type of screening condition, violating Local Interactions. The reason for this was again the non-spacetime-based parameters, which effectively acted “at-a-distance” between spacelike-separated regions in most Type I approaches. Models without Local Interactions have had a long history in physics, including, e.g., Newton’s original proposal of universal gravitation. Newton recognized this aspect as problematic from the outset, and the removal of action-at-a-distance by general relativity is universally regarded as a highly productive step forward. This would indicate another advantage for Type II models, where there is no indication that any violation of Local Interactions would be required.

Another goal for which Type II models have a clear advantage is that of Time Symmetry, as has been discussed extensively. Leifer and Pusey (2017) have recently examined the non-signaling sector of quantum theory, where Time Symmetry should be evident in its strongest form, as signal causality is not applicable. They were able to prove that retaining Time Symmetry in the quantum description is impossible in Type I models [as opposed to the situation in classical physics (Price 2012), again indicating the need to develop FID approaches. But physicists who are inclined to demand the time-asymmetric premise of NFID presumably are not persuaded that Time Symmetry is a particularly important goal in the first place. On the other hand, physicists who highly weight the importance of microscopic Time Symmetry are probably more inclined to consider FID models.

Of all the goals introduced in Section IV.C Lorentz Covariance is arguably the most important for making further progress in fundamental physics. As Bell himself noted, “we have an apparent incompatibility, at the deepest level, between the two fundamental pillars [QM and relativity] of contemporary theory” (Bell 1986a), a difficulty which continues to be discussed to the present [see, e.g., Albert and Galchen (2009)]. Resolving this incompatibility – at the model level of \( P_I(Q) \), not merely the outcomes \( P_I(O) \) – would therefore be quite significant. Type I models – the models which Bell was considering – have yet to achieve this goal. While not formally Lorentz Covariant, the above analysis of the recent Type II models indicates all the essential elements for success: the use of exclusively spacetime-based parameters, mediators confined to timelike (or light-like) worldlines, and an insensitivity as to which distant measurement is made first.

The final goal from Section IV.C – the need for a Resolved Measurement Problem – currently favors a subset of Type I models. While standard QM suffers from serious difficulties with these issues (Bell 1990a; Zurek 2003), there exists two other Type I theories – the Pilot-Wave approach and that of “spontaneous” collapse (Ghirardi et al. 1986) – which suggest different resolutions. Indeed, when talking about directions for future research on the foundations of QM, Bell (1990a) wondered whether these two could be generalized to Lorentz Covariant versions. Developments of Type II models
have yet to tackle this issue – all of the above examples require imposing special rules or boundary constraints at the spacetime locations corresponding to “measurements”. Still, for reasons discussed in Section VI.C, this issue appears to be somewhat less problematic for the Schulman model as compared to standard QM.

VIII. ALTERNATIVES AND MISCONCEPTIONS

Over the years, many possible descriptions of quantum phenomena which deviate from our model framework have been put forward. For example, negative probabilities, as in the Wigner distribution function (Wigner, 1932), have been considered. If conventional probability theory is invalid in this manner, the derivation of Eqns. (17) and (18) would certainly be suspect. However, probability theory is based on sound logic, and there is no loophole here concerning Bell’s Theorem. Such approaches suggest some potentially interesting and novel techniques – in this case, quasi-probability distributions – but can in no way produce QMa models with LC.

Another potential alternative is the Many Worlds Interpretation (Everett, 1957), sometimes claimed as a way to avoid Bell’s framework because all possible measurement outcomes are represented in a never-collapsed wavefunction. In this approach, the measurement problem is avoided by removing the Born rule from the fundamental description, but then the empirical success of QM, the \( P_I(O) \), is apparently removed as well. Maudlin (2010) notes that “Bell discussed only theories that make The Predictions ... of standard quantum theory. Because the many worlds interpretation fails to make The Predictions, Bell’s Theorem has nothing to say about it.” This may be going too far, as there are certainly proponents of Many Worlds who would argue that at an effective level, a version of the Born rule is still applicable. However, at that level, the theory becomes a Type I approach, in essentially the same category as conventional QM.

There are other well-established methods that utilize a more spacetime-based viewpoint, but not in a manner which falls into our model framework or clearly achieve the goals of Section IV.C. For example, path-integral accounts of QM utilize spacetime-localized paths. It might be tempting to think that each path might be represented by a set of parameters \( Q \), but the path integral cannot be parsed into probabilities \( P_I(Q) \) where only one path \( Q \) exists.\(^{32}\) Similarly, Quantum Field Theory can be viewed as assigning a complex amplitude to all possible field configurations in spacetime, but each of these configurations cannot be assigned a probability. Even approaches such as Stochastic Mechanics (Nelson, 1966, 2012) rely on an ensemble of exponentially many paths to make calculations, harming the goals of Physical Interpretation and Linear Scaling. Similar concerns apply to Stochastic Quantization (Damgaard and H"uffel, 1987).

That said, these approaches might be excellent starting points for building new FID models, as they incorporate Time Symmetry to a large extent. Each one has a symmetry-breaking assumption\(^{33}\) – presumably introduced with the goal of recovering NFID – which could perhaps be replaced by an alternative.

The most prominent alternative model framework in the Bell’s Theorem literature, commonly known as superdeterminism, will be addressed next. Following that discussion, we move to general misconceptions about Bell’s Theorem.

A. Superdeterminism

The deviation from the framework which appears most frequently in the recent literature [perhaps because it was discussed repeatedly by Bell (1977, 1990b, 1981)] is “superdeterminism”, which retains the implicit NFID assumption while considering violations of the “\( \lambda \)-independence” assumption, Eqn. (7). This cannot be done within our model framework (Section II.A), which treats the measurement settings \((a, b)\) as input parameters, corresponding to the mathematical concept of free variables. But if the settings are changed to statistical non-input parameters, \( \lambda \)-independence is transformed\(^{32}\) Introducing an FID viewpoint, along with a different parsing of \( Q \), might potentially resolve this problem (Wharton, 2016).

\(^{33}\) For example, the use of retarded (rather than time-ordered) Green’s functions explicitly breaks the symmetry, as does dismissing “negative frequency” solutions to second order equations.
into the related condition:

\[ P_r(\lambda | a, b) = P_r(\lambda), \]  

(26)

where \( c \) encodes the free preparation setting, still treated as an input. This is a statistical-independence relation, and permits a Bayesian inversion to an equation sometimes known as the “no conspiracies” assumption:

\[ P_c(a, b | \lambda) = P_c(a, b). \]  

(27)

Violations of this condition can then be pursued, by expanding \( \lambda \) (or using additional variables) to include the systems that choose the measurement settings. Notice that this approach is only coherent to the extent that it makes sense to talk about the probabilities of the settings, \( a \) and \( b \).

The ideas behind superdeterminism originally arose in a discussion between Bell and Shimony et al. (1976), who raised the possibility that the results of QM could be produced in a conspiratorial setup which does not violate LC. They suggested that a list of measurement settings and outcomes could be “concocted” in advance, and the experimenters cajoled into using the settings on the list (the conspiracy would necessarily include the manufacturer of the measurement devices, which would be designed to produce the results on the list regardless of the properties of any particles produced by the source). The aim of Shimony et al. was to emphasize the importance of the free-variable assumption, not to suggest that it could sensibly be violated. Indeed, they argued that the “enterprise of discovering the laws of nature” by “scientific experimentation” necessarily involves the assumption that “hidden conspiracies of this sort do not occur.”

In his reply, Bell (1977) considered the possibility of a future theory which would have no input parameters other than initial conditions. Such a theory would have to include a model of whatever it is in a particular setup which selects the measurement settings. Rather than considering the inner workings of a human being, Bell considered the possibility that the settings would be chosen by a mechanical pseudorandom generator, and concluded that its output would be “effectively free for the purpose at hand.” After all, the mere failure of Eqn. (27) is necessary but not sufficient to explain the CHSH-inequality violations observed in experiment. Given the required amount of \( (a, b, \lambda) \) correlations, even poor random number generators for \( a \) and \( b \) could easily make repeatable CHSH-violations essentially impossible without remarkable “conspiracies” in the seeds of the random numbers.

Furthermore, if there existed some superdeterministic model which predicted a correlated probability output distribution \( P(a, b, \lambda) \) from some given past input, then this predicted output could be marginalized to produce a prediction for the settings alone, \( P(a, b) \). But this means that two different experimenters, who chose two different distributions of setting probabilities, \( P_1(a, b) \) and \( P_2(a, b) \), cannot be modeled with the same superdeterministic model; one would need an essentially different model for each distribution. The only practical way around this dilemma is to treat the settings \( (a, b) \) as free input parameters, and use them to infer the correlations with some model \( P_{a,b}(\lambda) \). But this is precisely the class of FID models already discussed in Section VI.

Despite these strong arguments, violations of the input status of the settings \( a \) and \( b \) have been seriously considered in the literature [e.g., 't Hooft (2016)]. However, in making suggestions for concrete models, modeling of the random number generators was not included, and so there are no true superdeterministic models available. In practice, the proposed hidden variables \( \lambda_0 \) in Brans (1988), and \( \mu \) in Hall (2016) simply contain copies of the measurement setting parameters. As a result, the other elements of these models [which are essentially a copy of standard QM for Brans (1988), and Eqn. (24) for Hall (2016)] actually constitute FID models with a prescribed \( P_{a,b}(\lambda) \). In other words, regardless of the discussion indicating the contrary, \( a \) and \( b \) are still treated as inputs in these models [Argaman (2018)].

For these reasons, we have included in our definition of agreement with QM not only the predictions for the outputs, but also the requirement that they use the same input parameters as QM. This was the point of Shimony et al. (1976), and Bell himself continued to emphasize in his later writing that for the Theorem to hold, the measurement settings must be treated as free inputs [Bell 1990b, 1981]. Even if one day a full superdeterministic model is developed (modelling the settings as outputs,
not inputs), it would still not be QMa.

B. Misconceptions

It has been claimed, and is still often claimed, that Bell’s Theorem is based on additional assumptions which have not been identified in the present derivation, including determinism and realism. These erroneous claims have already been well-addressed in the literature (Maudlin 2010, 2014; Norsen 2011, 2017), but the issues bear repetition.

Bell did not originally present his proof as we have outlined in Section III; this unified approach only came later. The original paper (Bell 1964), to which many physicists have turned to understand Bell’s Theorem, instead built upon the EPR argument (as explicitly noted in its title). The EPR paper (see Section II.C) demonstrated that certain perfect correlations between distant measurements, which are predicted by QM, clearly violate LC (Local Causality), unless one adds deterministic hidden parameters. Bell built upon this result, and demonstrated that even with deterministic hidden parameters LC could not be saved, in the sense that other predictions of QM could not be obtained.

Norsen (2017) has recently expanded this point to include other misconceptions:

At the end of the day, Bell’s Theorem tells us absolutely nothing about “hidden variables” or determinism or counter-factual definiteness or “realism” or whether the moon is there or not when you aren’t looking or any of these sorts of things that people say it is fundamentally about. Everybody is simply wrong here, because they have ... forgotten the EPR argument – which, remember, is supposed to be a proof that deterministic hidden variables are required, in the first place, precisely in order to avoid non-locality.

Unfortunately, the argumentation of EPR contained several additional elements which made it appear paradoxical even before Bell’s work, and the notion that Bohr (1935) had refuted it was widespread among physicists. Furthermore, Bell did not make the two-step nature of his argument as clear as he could have – in 1964, he was interested in proving the new result, and did not go through the EPR part of the argument in detail. Therefore many physicists have looked at Bell (1964) and concluded that all implications of Bell’s Theorem could be avoided by simply not postulating hidden parameters in the first place, or by not requiring them to be “deterministic” (or “realistic”, or “counterfactual definite”, or some other stipulation). But such moves do not save LC, for reasons described in the EPR paper itself. Without shared localized parameters to deterministically enforce distant correlations, the existence of QM’s perfect correlations between distant objects would immediately violate LC directly. Bell himself later wrote: “It is remarkably difficult to get this point across, that determinism is not a presupposition of the analysis” (Bell 1981).

Adding to the confusion, the first derivation of the CHSH inequality (Clauser et al. 1969) did not use the EPR argument as part of the derivation, and began with deterministic hidden parameters as a bare assumption. (That same paper, the very first to take a significant step beyond Bell’s seminal work, presented the EPR argument as a paradox to which Bohr had suggested a resolution.) Bell realized immediately that the same CHSH inequality could be derived for indeterministic local hidden-variable models as well [see footnote 10 of Bell (1971), and a detailed presentation was given by Clauser and Horne (1974). However, the idea that one can resolve the problem of Bell’s Theorem by considering non-deterministic models is still a widespread misunderstanding, even though the later proofs of the CHSH inequality [as in Section III above, or in Clauser and Horne (1974)] do not require any assumption of determinism.

Another source of misunderstanding is due to the common use of the phrase “local realism” to refer to LC. One might misinterpret this phrase as the conjunction of two assumptions, locality and “realism”, and conclude that it was possible to save LC by giving up “realism”. But whatever one might mean by this term [see Norsen (2007) for the various possibilities], it is impossible to give it up without rendering the concepts behind LC meaningless.
(both BSA and NFID). Therefore this is not a coherent strategy for saving LC – at the mathematical level, local realism involves no assumptions beyond LC. Maudlin (2010) has made the point that other purported alternatives to LC-violation encounter a similar problem.

Our above identification of NFID as an originally-unremarked assumption inside of LC is not subject to the same criticisms. For one, there is no doubt that it is a common assumption that future input settings should not be correlated with past model parameters. For another, all mathematical proofs of Bell’s Theorem indicate the formal need for the NFID-related Eqn. (7), which cannot be derived from BSA alone (even with the absence of conspiracies implicitly assumed). And finally, the examples in Section VI.A make it clear that it is possible to violate NFID without violating BSA in at least some cases, demonstrating that they are distinct assumptions.

IX. CONCLUSIONS

We have discussed Bell’s Theorem, which demonstrates that mathematical models obeying Local Causality are incompatible with QM and with observed quantum-entanglement phenomena. To the extent we demand that the parameters in our mathematical models should correspond to physical events, this result is quite significant. Einstein described this physical view of Local Causality in a 1948 letter (Born, 1971):

If one asks what, irrespective of quantum mechanics, is characteristic of the world of ideas of physics, one is first of all struck by the following: the concepts of physics relate to a real outside world ... It is further characteristic of these physical objects that they are thought of as arranged in a space-time continuum. An essential aspect of this arrangement of things in physics is that they lay claim, at a certain time, to an existence independent of one another, provided these objects ‘are situated in different parts of space’.

The following idea characterizes the relative independence of objects far apart in space (A and B): external influence on A has no direct influence on B ...

Bell showed that this line of thinking leads to limitations on distant correlations which are in direct conflict with QM. The outcomes of spatially-separated experiments are correlated in a manner which cannot be explained only in terms of common past causes. Therefore Local Causality is not tenable. Still, it does not follow that our only option is to throw out the entirety of Einstein’s analysis, giving up on “physical objects... arranged in a space-time continuum”. At least one of the assumptions that make up Local Causality needs to go, but spacetime-based parameters might still be retained.

The present Colloquium clarified these general statements by defining a model framework for such parameters, emphasizing that agreement with the formalism of QM requires not only reproducing the same predictions for the outputs, but also sharing the same well-defined inputs. (The conspiratorial scenario known as “superdeterminism” is thus set aside.) It was also demonstrated that the screening assumption called BSA (Bell’s Screening Assumption; see Figure 1 above) is not equivalent to Local Causality, as the additional time-symmetry-breaking assumption of NFID (No Future-Input Dependence) is necessary for a proof of Bell’s Theorem. The manner in which the latter breaks time-symmetry represents a strong arrow-of-time assumption, which is nevertheless often taken for granted rather than explicitly recognized. As a result of this analysis, it becomes natural to categorize all potential reformulations of QM by whether they only violate a screening assumption (Type I), only violate the strong arrow-of-time assumption (Type II), or violate both (Type III).

In standard Schroedinger-picture QM (and other Type I models) effective connections between space-like distant events are mathematically mediated via many-particle wavefunctions $\psi(x_1, x_2, ..., x_N, t)$, generalized variables

---

34 Recall that Bell identified LC with BSA alone, and many other authors have done the same, using implicit causal reasoning rather than a mathematically formalized version of NFID or some similar arrow-of-time assumption.
external to spacetime. Despite not being localized in space, these wavefunctions are still functions of time. If they interact with localized parameters that are space-like separated, this requires a notion of simultaneity, and can lead to severe difficulties with Lorentz Covariance. A standard resolution to this, often used in the context of Quantum Field Theory, is to calculate transition amplitudes “all-at-once”, which requires the settings of all measurements as input parameters. Such approaches might violate NFID, amounting to the use of a Type III model.

However, the possibility of Type II models is also available. Such models have all the effective connections associated with the particle world-lines, either within the lightcones or on the lightcones for photons (i.e., there are no direct space-like connections). Dropping the NFID arrow-of-time assumption implicit in the above Einstein quote allows these to be two-way connections. Using this strategy, the “independence of objects far apart in space” can be softened without requiring connections which violate the spirit of relativity. In particular, this view accommodates entanglement scenarios by allowing an external influence on A to have an indirect influence on B, via events in the intersection of their past lightcones, without raising any difficulties with Lorentz Covariance. (As discussed in Section VI.B this need not lead to inconsistencies.)

Physics models explicitly violating the NFID assumption have been developed already in the context of Classical Electrodynamics [Wheeler and Feynman 1945 1949], and their relevance to Bell-like scenarios was pointed out even before Bell’s Theorem emerged [Costa de Beaufondregard 1953], and then repeatedly since [e.g., Price 1997]. Despite this, the development of explicit Type II models of entanglement phenomena has only recently begun, and is currently limited to only a few particular applications, most notably the Bell state correlations which typically serve to demonstrate the issue of Bell’s Theorem. The detailed discussion of the proof-of-principle examples of such models in Section VI is hoped to introduce these possibilities to a wider audience, and includes indications of several possible avenues for future developments. This would include describing more complicated entanglement scenarios and achieving a fully Resolved Measurement Problem.

It is emphasized that while FID (Future-Input Dependent, or retrocausal) models of QM can have an underlying structure that is as Time Symmetric as classical physics, all such models must have a mechanism to recover the time-asymmetric condition known as signal causality. Two possibilities for such a mechanism have been suggested above. The first [Price 1997] emphasizes the role of time-asymmetric “agents” employing the theory: they select which parameters of a theory to use as inputs of a specific model and which as outputs, and this asymmetric choice suffices to impose signal causality. The second considers the possibility that a physical principle, possibly only a specification of some initial conditions, would break the symmetry [in the cosmological context, this could well be the Weyl-Curvature hypothesis (Penrose 2009)]. As a result of this mild symmetry breaking, irreversibility could appear in the thermodynamic limit, and with it, signal causality.

A successful Type II reformulation of QM would employ only spacetime-based parameters and would associate conventional probabilities with each fully-specified configuration. An appropriate interpretation would take only one of these possibilities to actually occur in Nature. In other words, the number of parameters describing a system would grow only linearly with its extent. This stands as a dramatic advantage over existing approaches, where the number of necessary parameters scales exponentially with the number of particles in the system. Combined with Lorentz Covariance, this could greatly alleviate the disconnect between quantum theory and general relativity.

Such a reformulation would also shed light on an unresolved issue in quantum foundations – how to interpret the conventional wavefunction \( \psi \) and the collapse postulate. Although \( \psi \) is not included in the underlying model, it could still represent available knowledge about the actual parameters – a viewpoint that has been

35 It is interesting to note that Bell (1990b) already asked: “Could it be that causal structure emerges only in something like a ‘thermodynamic’ approximation?” But his tentative answer was negative, possibly due to his taking NFID for granted.
come known as "ψ-epistemic" (Spekkens 2007). Such states of incomplete knowledge naturally reside in configuration space (as in classical statistical mechanics), as they have to represent a large number of possible correlations. Unitary evolution of these states would then correspond to time-evolving the available information, in analogy to Liouville dynamics. Learning additional information about future settings and future outcomes would then lead to a Bayesian updating of ψ, corresponding to a (non-physical) collapse. This is essentially the style of model advocated by Einstein, where the actual state of the system was not ψ, but rather something more fundamental (Harrigan and Spekkens 2010).

While the present work is focused on Bell’s Theorem, additional lines of research are also converging on the promise of FID models. As discussed above, Leifer and Pusey (2017) motivate such models via Time Symmetry. Another argument is motivated by the much-discussed Pusey-Barrett-Rudolph (PBR) Theorem (Pusey et al. 2012), recently reviewed by Leifer (2014). One of Leifer’s conclusions exactly matches ours, promoting the development of "retrocausal … models that posit a deeper reality underlying quantum theory that does not include the quantum state." Our localized parameters Q would mathematically represent this “deeper reality”. Fully realizing this goal remains an open challenge.

Acknowledgements: The authors warmly thank J. Finkelstein, S. Friederich and R. Sutherland for useful comments on a draft of the manuscript. This work is supported in part by the Fetzer Franklin Fund of the John E. Fetzer Memorial Trust.

APPENDIX: DERIVATION OF THE SCHULMAN MODEL

Schulman’s original single-particle model applies to a single spin-1/2 particle; here we convert it to a photon polarization problem. The photon’s classical trajectory is known, and it has a real (hidden) polarization direction \( q(t) \) everywhere on its trajectory. The photon is prepared and measured by passing through two polarization cubes, with the first set at an angle \( \theta_1 \) and the second set at \( \theta_2 \). The initial polarization is constrained, \( q(t_1) = \theta_1 \), as is usual for initial boundary conditions. Schulman enforced a similar final boundary condition at measurement, where the final polarization was constrained to be either \( q(t_2) = \theta_2 \) or \( q(t_2) = \theta_2 + \pi/2 \).

This final constraint is controllable (modulo \( \pi/2 \)) and the model is FID. The time-asymmetry (modulo \( \pi/2 \) at the output, but modulo \( \pi \) at the input) is external: an experimenter can choose to block a photon with an unwanted input polarization, but does not know the output polarization until it is too late to interfere. Otherwise, everything in this model is fully time-symmetric.

Such two-time-boundary problems can only be solved “all-at-once,” with probabilities assigned to entire histories, \( q(t) \), not instantaneous states. (One can extract the latter probabilities from the former.) Defining a net rotation

\[
\Delta q \equiv \int_{t_1}^{t_2} dq(t) = \int_{t_1}^{t_2} dt \int dq(t) dt
\]

(which is permitted to be larger than \( 2\pi \) for multiple rotations), the convolution of Schulman’s proposed Cauchy kicks imply the probability assignment of Eqn. (28):

\[
P(\Delta q) \propto \frac{1}{(\Delta q)^2 + \gamma^2}.
\]

Remarkably, this distribution recovers Malus’s Law as \( \gamma \to 0 \). Seeing this requires adding the probabilities for all the rotations which end at the same polarization angle (modulo \( \pi \), and normalization.

The evaluation requires summing over all the possibilities of getting from \( \theta_1 \) to \( \theta_2 \mod \pi \), allowing for rotations through angles larger than \( \pi \) in both directions. The sum,

\[
\sum_{n=-\infty}^{\infty} \frac{1}{\pi(\Delta \theta + n\pi)}
\]

with \( \Delta \theta = \theta_1 - \theta_2 \), can be calculated, similarly to Euler’s solution of the Basel problem (\( \sum_{n=1}^{\infty} \frac{1}{n^2} \)), by equating two different families of polynomial approximations to the same function, in this case, \( f(x) = \sin(\Delta \theta + x) \sin(\Delta \theta - x) \). One family is the Taylor expansion, and as \( f(x) = \frac{1}{2} (\cos(2x) - \cos(2\Delta \theta)) \), the coefficient of \( x^2 \) is \( -1 \), yielding \( f(x) = \sin^2(\Delta \theta) - x^2 + O(x^4) \). The other polynomial approximation scheme is obtained by multiplying the value of the function at \( x = 0 \) by a factor of \( (1 - x/z_k) \) for each of the zeros \( z_k \) of the original function (a specific approximation is obtained by including only roots up to a certain absolute magnitude).
Treating the roots in pairs, \(z_n = -z_n' = \Delta \theta + n\pi\), gives

\[
f(x) = \sin^2(\Delta \theta) \prod_{n=-\infty}^{\infty} \left(1 - \frac{x^2}{(\Delta \theta + n\pi)^2}\right),
\]

and expanding only up to terms quadratic in \(x\) gives the necessary sum:

\[
\sum_{n=-\infty}^{\infty} \frac{1}{(\Delta \theta + n\pi)^2} = \frac{1}{\sin^2(\Delta \theta)}.
\]

(30)

Normalizing the probabilities for either \(q(t_2) = \theta_2\) or \(q(t_2) = \theta_2 + \pi/2\), is achieved by simply multiplying by the product of the corresponding denominators on the right-hand side of Eqn. (30), yielding Malus’ Law:

\[
p = \cos^2(\Delta \theta),\text{ as required for Section VI.A.2.}
\]

Remarkably, Schulman used this idea to prove the Born rule, in the sense of showing that probabilities \(\propto |\psi|^2\) are compatible with this idea of multiple kicks only for \(x = 2\) (whether or not the Cauchy-Lorentz distribution is used for each kick).

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