Abstract

It is pointed out that heavy top quark mass can be attributed to a singular normalization of its kinetic term, in which rescaling into canonical one yields large top Yukawa coupling. We pursue this novel possibility in a democratic mass matrix model where only the normalization of the third generation can be different from that of the other two generations. With diagonal breaking of democratic $S_3$ symmetry, we show that the singular normalization for the top quark is essential to reproduce observed quark masses and mixing angles. We also briefly argue other applications of this mechanism.

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I. INTRODUCTION

The origin of the flavor structure is a big puzzle in particle physics. In the standard model, the masses and mixing of quarks and leptons are just parameters given by hand. Obviously this is unsatisfactory, and there have been many attempts to explain the fermion mass matrices in the literature [1–8].

There are some characteristic features in the quark and lepton mass matrices. As for the mixing among different generations, the quark sector has all small mixing angles whereas the neutrino sector has two large mixings and one small one. Another important property concerns the mass hierarchy of the quarks and leptons. The third generation of the quarks and charged leptons are heavier than the other generations. In particular, the top quark is much heavier than the others.

Here we shall point out that the large top quark mass can be accounted for by a singular normalization of its kinetic term. After rescaling the fields to get the canonical normalization, one can obtain a large Yukawa coupling and thus a large mass of the top quark. We shall demonstrate this interesting possibility in the framework of a democratic fermion mass matrix model [9–14]. In the democratic $S_3$ symmetry, the fermions belong to 3 dimensional reducible representations, which are decomposed into 1 and 2 dimensional representations. The trivial representation is identified with the field in the third generation. This group theoretical structure allows the third generation to have different normalization from the other two generations [15]. We will show that, with diagonal breaking of the $S_3$ symmetry, the singular normalization of the top quark is essential to reproduce observed quark masses and mixing angles. We will also discuss other examples where this mechanism may be important.

II. THE MODEL

The model we are considering was described in Refs. [9–14]. Non-canonical Kähler potential was introduced in Ref. [15]. We would like to briefly review essential points of our model. Here we focus on the quark sector. See Refs. [11–14] for the successful extensions to the lepton sector, including neutrino masses and mixing.

In the quark sector, there are a product of permutation symmetry groups $S_3(Q_L) \times S_3(U_R) \times S_3(D_R)$. Under $S_3(Q_L)$, the doublet quarks $Q_{Li} (i = 1, 2, 3)$ transform as 3 dimensional representation, $\mathbf{3}$, which is decomposed into two irreducible representations: $\mathbf{3} = \mathbf{1} + \mathbf{2}$. Here $\mathbf{1}$ is a trivial representation and $\mathbf{2}$ is two dimensional one. In fact, $(Q_{L1} + Q_{L2} + Q_{L3})/\sqrt{3}$ does not change under the permutation. The other two combinations will constitute a basis of $\mathbf{2}$. Similar arguments can apply for $SU(2)_L$ singlet quarks $U_{Ri}$ and $D_{Ri}$.

In the following we consider the minimal supersymmetric standard model (MSSM) to illustrate our points, and use the terminology of supersymmetry, such as Kähler potential and superpotential. Supersymmetry is, however, not essential for our subsequent argument.

To begin with we will develop some formalism. Let us introduce a $\mathbf{3}$ representation, $X_i$, which will be identified with three families of the MSSM matter fields later. The $S_3$ invariant Kähler potential is written
\[ K = [Z^X_i I + Z^X_j J]_{ij} X_i^\dagger X_j, \]  

(1)

where

\[ I \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad J \equiv \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \]  

(2)

Reflecting the fact that the above Kähler potential is a bilinear function of \( X_i \), there arise two invariants: the universal matrix \( I \) and the democratic matrix \( J \). Here \( Z^X_i \) and \( Z^X_j \) are functions of fields in general and we omit terms which have no relation to mass matrices. We assume that some dynamics fixes vacuum expectation values (vevs),

\[ \langle Z^X_{i,j} \rangle = z^X_{i,j}. \]  

(3)

Existence of the democratic part \( z^X_j \) plays an essential role in our arguments. In fact, the non-universal kinetic terms stem from \( z^X_j \),

\[ K_{ij}^{\text{dem}} = [z^X_i I + z^X_j J]_{ij}. \]  

(4)

Using the following matrix:

\[ A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix}, \]  

(5)

\( K^{\text{dem}} \) is diagonalized as

\[ K^{\text{diag}} = A^T K^{\text{dem}} A = z^X_i I + z^X_j T, \]  

(6)

where

\[ T \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \]  

(7)

Hereafter we will call the field basis obtained this way the diagonal basis. The kinetic terms are written explicitly

\[ K_{\text{kin}} = \begin{pmatrix} z^X_i & 0 & 0 \\ 0 & z^X_j & 0 \\ 0 & 0 & z^X_j + z^X_j \end{pmatrix} X_i^{\text{diag}} X_j^{\text{diag}} + z^X_j T, \quad X^{\text{diag}} = A^T X. \]  

(8)

In this basis, it becomes clear that the reducible 3 representation consists of the trivial 1 representation and the irreducible 2 representation. After rescaling the fields by using the diagonal matrix

\[ C_X = \text{diag} \left( \frac{1}{\sqrt{z^X_i}}, \frac{1}{\sqrt{z^X_j}}, \frac{1}{\sqrt{z^X_j (1 + r_X)}} \right), \quad r_X \equiv z^X_j / z^X_i, \]  

(9)
we obtain canonically normalized kinetic terms:

\[ K_{\text{kin}} = X_i^{\text{can}\dagger} X_i^{\text{can}}, \quad X^{\text{can}} = C_X^{-1} X^{\text{diag}}. \] (10)

We will call this field basis the canonical basis.

Let us consider Yukawa interaction which consists of two matter fields \( X_L, X_R^c \) and a Higgs field \( H \),

\[ W = Y^\text{dem}_{ij} X_L^i X_R^{c_j} H, \] (11)

where \( Y^\text{dem} \) represent a Yukawa coupling matrix. The \( S_3(X_L) \times S_3(X_R^c) \) symmetry allows \( Y^\text{dem} \) to have only the democratic matrix:

\[ Y^\text{dem} = y_0 J, \] (12)

where \( y_0 \) is a constant. It is obvious that

\[ Y^{\text{diag}} = y_0 T \] (13)

in the diagonal basis and that

\[ Y^{\text{can}} = C_L Y^{\text{diag}} C_R = \text{diag} \left( 0, 0, \frac{y_0}{\sqrt{z_L z_R^c (1 + r_L)(1 + r_R)}}, 0 \right) \] (14)

in the canonical basis, and thus no mixing angle arises. Notice that only the \((3,3)\) element in \( Y^{\text{diag}} \) and \( Y^{\text{can}} \) survives since both of the 1 representation of \( X_L \) and the one of \( X_R^c \) must be involved in order for couplings to be invariant. Therefore, there appear two massless fields and one massive field after the Higgs develops a vev, which, roughly speaking, simulates our world \(^1\).

\( S_3 \) breaking parameters must be involved in order to make our model realistic. Let us assume that the small \( S_3 \) breaking matrix possesses a diagonal form:

\[ Y^\text{dem} = y_0 \begin{bmatrix} J + \begin{pmatrix} -\epsilon & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \delta \end{pmatrix} \end{bmatrix}. \] (15)

Diagonalization of the kinetic terms using \( A \) followed by rescaling the fields using \( C_X \) drives the Yukawa matrix to the following form:

\[ Y^{\text{can}} = y_0 C_L \begin{bmatrix} T + A^T \begin{pmatrix} -\epsilon & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \delta \end{pmatrix} A \end{bmatrix} C_R \] (16)

in the canonical basis. One is convinced that \( Y^{\text{can}} \) is almost diagonal and that resulting mixing angles will be small. After diagonalization of \( Y^{\text{can}} \),

\(^1\)Democratic structure of Yukawa interactions can be also realized in brane-world models [16].
we obtain

\[ y_1 = Y_0 \left( \frac{\Delta}{3} - \frac{\Xi}{6} \right), \quad y_2 = Y_0 \left( \frac{\Delta}{3} + \frac{\Xi}{6} \right), \quad y_3 = Y_0 \left( 1 + \frac{\delta}{3} \right) \]

where

\[ Y_0 = \frac{y_0}{\sqrt{z_L^L z_R^R (1 + r_L)(1 + r_R)}}, \quad \Xi = 2(\Delta^2 + 3E^2)^{1/2}, \]

\[ \Delta = \sqrt{(1 + r_L)(1 + r_R)\delta}, \quad E = \sqrt{(1 + r_L)(1 + r_R)\epsilon}. \]

Here we choose \( \delta \) and \( \epsilon \) to be real and ignore higher orders of \( \delta \) and \( \epsilon \). The unitary matrices \( U_L \) and \( U_R \) are given by

\[
U_{L,R} = \begin{pmatrix}
\cos \theta & -\sin \theta & -\Lambda_{L,R} \sin 2\theta \\
\sin \theta & \cos \theta & -\Lambda_{L,R} \cos 2\theta \\
\Lambda_{L,R} \sin 3\theta & \Lambda_{L,R} \cos 3\theta & 1
\end{pmatrix},
\]

where

\[ \tan 2\theta = \frac{\sqrt{3}\epsilon}{\delta}, \quad \Lambda_{L,R} = \sqrt{1 + r_{L,R}} \frac{\xi}{3\sqrt{2}}, \quad \xi = 2\delta \left( 1 + \frac{3\epsilon^2}{\delta^2} \right)^{1/2}. \]

Notice that in the case where \( E \ll \Delta \ll O(1) \), a hierarchical spectrum is derived:

\[ y_1 \sim -\frac{E^2}{2\Delta} Y_0, \quad y_2 \sim \frac{2\Delta}{3} Y_0, \quad y_3 \sim Y_0. \]

Hereafter we assume this type of hierarchy since it matches with the observed quark mass hierarchy as we will see shortly.

Let us apply the above discussion to the quark sector in the MSSM, in which the left-handed quark doublets \( Q_i \), the charge-conjugated right-handed up-type quarks \( U^{c}_{i} \) and the charge-conjugated right-handed down-type quarks \( D^{c}_{i} \) exist. The up-type Yukawa coupling matrix and the down-type one are diagonalized as

\[
U_{L}^{uT} Y_{u}^{\text{can}} U_{L} = \text{diag}(y_{1}, y_{2}, y_{3}), \quad U_{Q}^{dT} Y_{d}^{\text{can}} U_{D} = \text{diag}(y_{d}, y_{s}, y_{b})
\]

respectively. The Cabibbo-Kobayashi-Maskawa (CKM) matrix is given by

\[
V_{\text{KM}} = U_{Q}^{uT} U_{Q}^{dT}
\]

where

\[ \theta_c = \theta_u - \theta_d, \quad \theta_t = 2\theta_u + \theta_d, \quad \theta_b = \theta_u + 2\theta_d. \]
One finds that the small $S_3$ breaking parameters induce small mixing angles. Notice that our model naturally includes the model discussed in Ref. [10], where non-universal kinetic terms are absent.

We can express the CKM matrix elements in terms of the mass eigenvalues. The Cabibbo angle is determined solely by the observed values as

$$|V_{us}| = |\sin \theta_c| \sim \sqrt{\frac{m_d}{m_s} \pm \sqrt{\frac{m_u}{m_c}}}.$$  \hspace{1cm} (26)

This Fritzsch relation have been known to be successful [17]. $V_{cb}$ and $V_{ub}$ are given by

$$|V_{cb}|^2 + |V_{ub}|^2 = \frac{\bar{\Lambda}_u^2}{1 + r_U} \left( 1 + \frac{1}{x^2} - \frac{2 \cos 2\theta_c}{x} \right),$$

$$\frac{V_{ub}}{V_{cb}} = \frac{\sin(3\theta_u - 2\theta_c) - x \sin 3\theta_u}{\cos(3\theta_u - 2\theta_c) - x \cos 3\theta_u}.$$  \hspace{1cm} (27)

where

$$x = \frac{\sqrt{1 + r_D} \bar{\Lambda}_u}{\sqrt{1 + r_U} \bar{\Lambda}_d},$$

$$\bar{\Lambda}_u = \sqrt{(1 + r_Q)(1 + r_U)} \frac{\xi_u}{\sqrt{3}} = \sqrt{(1 + r_U)} \Lambda_u^Q,$$

$$\bar{\Lambda}_d = \sqrt{(1 + r_Q)(1 + r_D)} \frac{\xi_d}{\sqrt{3}} = \sqrt{(1 + r_D)} \Lambda_d^Q.$$  \hspace{1cm} (28)

Notice that in eq. (27) values of $\theta_u$ and $\bar{\Lambda}_u$ are almost fixed by the empirical masses as

$$|\theta_u| \sim \sqrt{\frac{m_u}{m_c}}, \quad \bar{\Lambda}_u^2 \sim \frac{m_u^2}{2m_t^2}.$$  \hspace{1cm} (29)

(See eqs. (18) and (20)). Thus, parameters determined from $V_{cb}$ and $V_{us}$ are $x$ and $1 + r_U$. $1 + r_Q$ does not play an essential role.

**III. ANALYSIS**

Let us perform crude estimation before making numerical analysis, elucidating the importance of unusual structure of kinetic terms. For the present, we adopt the following representative values:

$$|\sin \theta_c| = 0.22, \quad |\theta_u| = 5 \times 10^{-2}.$$  \hspace{1cm} (30)

Taking into account that

$$|x| \sim O(0.1) \frac{\sqrt{1 + r_D}}{\sqrt{1 + r_U}},$$  \hspace{1cm} (31)

the area where $|x|$ itself is close to $O(0.1)$ is preferable. From the constraint $0.057 \leq |V_{ub}/V_{cb}| \leq 0.126$ one finds that the most favorable range is $\theta_u \theta_c > 0$ and
\[-1.14 \leq x \leq -0.61. \quad (32)\]

We discard the case where \(\theta_u\theta_c < 0\) since rather large \(|x|\) is demanded. Combining this inequality (eq. (32)) with the constraint \(1.44 \times 10^{-3} \leq |V_{ub}|^2 + |V_{cb}|^2 \leq 1.94 \times 10^{-3}\), we obtain

\[0.013 \leq 1 + r_U \leq 0.034. \quad (33)\]

for \(m_c = 677\) MeV and \(m_t = 175\) GeV. Thus, we conclude that considerably suppressed \(1 + r_U\) is necessary in order that our \(S_3\) model with the diagonal breaking matrices explains the empirical masses and mixings. Recalling that

\[y_t \sim \frac{y_{0u}}{\sqrt{z_I^Q z_I^U (1 + r_Q)(1 + r_U)}}, \quad (34)\]

this conclusion highlights a possibility that extremely large top mass is attributed to singular structure of normalization of the fields in the case of small \(y_{0u}\).

Put another way, we draw a plot of the allowed value of the top quark mass \(m_t\) as a function of \(1 + r_U\) when the other quantities are fit with their experimental values as above. The allowed region is depicted in Fig. 1. One finds that the allowed top mass is rather sensitive to the choice of \(1 + r_U\). It is interesting to observe that the case of universal kinetic terms examined in Ref. [10] is completely ruled out and thus the inclusion of the singular normalization with extremely small \(1 + r_U\) is essential to make the model realistic.

We now exhibit numerical analysis based on the diagonal breaking matrices. For simplicity we set \(1 + r_Q = 1 + r_D = 1\), which does not alter our conclusion. The following parameter set,

\[1 + r_U = 2.53 \times 10^{-2},\]
\[y_{0u}(H_u)/\sqrt{z_I^Q z_I^U} = 27.5\ \text{GeV}, \quad \delta_u = 3.76 \times 10^{-2}, \quad \epsilon_u = 2.36 \times 10^{-3},\]
\[y_{0d}(H_d)/\sqrt{z_I^Q z_I^D} = 3.09\ \text{GeV}, \quad \delta_d = -4.37 \times 10^{-2}, \quad \epsilon_d = 9.10 \times 10^{-3} \quad (35)\]

reproduces mass eigenvalues

\[m_u = 2.26\ \text{MeV}, \quad m_c = 683\ \text{MeV}, \quad m_t = 175\ \text{GeV},\]
\[m_d = 2.47\ \text{MeV}, \quad m_s = 94.0\ \text{MeV}, \quad m_b = 3.05\ \text{GeV} \quad (36)\]

and magnitudes of the CKM matrix elements

\[|V_{us}| = 0.22, \quad |V_{cb}| = 0.038, \quad |V_{ub}| = 0.0036, \quad (37)\]

which should be compared with the experimental data evaluated at the \(Z\)-boson mass scale [18].

we have made the comparison at the \(Z\)-boson mass scale. The same conclusion that extremely small \(1 + r_U\) is required holds even if we made this comparison at the GUT scale in the MSSM, though the value \(1 + r_U\) itself is different. This is because the effect of running comes from the top Yukawa coupling, which appears in the wave-function renormalization of the top quark in supersymmetry.

Although we have so far supposed that the parameters which are responsible for quark masses and mixing angles are real, the observed \(CP\) asymmetry can be also explained in this framework by taking complex \(\epsilon\) and \(\delta\).
IV. SUMMARY AND DISCUSSION

In this paper, we have pointed out the possibility that the wave function normalization of the top quark can be singular, which makes it very heavy. We have illustrated this in the framework of the democratic fermion ansatz. There the fermion mass structure is controlled by the $S_3$ permutation symmetries, with diagonal breaking introduced. In this case, we can make even a stronger statement: the singular normalization is essential to reproduce the top quark mass and the other quark masses as well as their mixing.

This investigation presented here is indeed based on a particular form of the $S_3$ breaking. However, we emphasize that the large top quark mass can be realized by invoking singular structure of the corresponding kinetic terms. This argument is irrespective of the form of the $S_3$ breaking matrices. This new idea of highly different normalization of fields can save theories in which Yukawa couplings would otherwise be suppressed and thus would fail to reproduce the large top mass. Examples include

- Brane-world scenarios in which quark and/or Higgs fields propagate in the bulk. In this case Yukawa coupling constants in four dimensions are volume-suppressed by some powers of $M_c/M_*$, where $M_c$ denotes the compactification scale and $M_*$ the fundamental one;

- $SU(6)$ GUTs. An attractive feature of the $SU(6)$ GUTs is that the MSSM Higgs doublets can arise as pseudo-Goldstone multiplets [19]. A natural extension to $SU(6)$ from $SU(5)$ suggests to introduce $15$, $6^*$ and $6'^*$ as quarks and leptons, and $6$ and $6^*$ as Higgs multiplets. However with these matter contents one cannot write down $SU(6)$ invariant Yukawa couplings for up-quark masses at the renormalizable level. Up-type Yukawa couplings arise from non-renormalizable operators and are suppressed by powers of $M_{GUT}/M_*$, where $M_{GUT}$ represents the GUT scale. Thus one conventionally introduce $20$ to obtain the large top Yukawa coupling. Our mechanism of the Yukawa enhancement due to the singular wave-function normalization, however, may allow us to construct a simpler model without introducing the $20$ multiplet.

Throughout this paper, we have assumed that the singular normalization for the top quark is realized by some means, but we have not specified a possible mechanism. A particularly interesting possibility is that the singular normalization is realized dynamically associated with, for example, the electroweak symmetry breaking. Although a naive inspection suggests that one needs a very flat potential for a field responsible for the wave-function normalization, which is thus unlikely, further study along this line is interesting and should be encouraged.

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FIG. 1. The top quark mass $m_t$ evaluated at the $Z$-boson mass scale as a function of $1 + r_U$. The hatched region is allowed by the constraints from $|V_{cb}|$ and $|V_{ub}|$. The 1σ region of the experimentally measured top mass ($170 \text{ GeV} \leq m_t \leq 180 \text{ GeV}$) is also shown by the two horizontal lines. Here we use $-1.14 \leq x \leq -0.61$ and $m_c = 677 \text{ MeV}$. 