On the Triviality of Textbook Quantum Electrodynamics

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Abstract

By adding a small, irrelevant four fermi interaction to the action of lattice Quantum Electrodynamics (QED), the theory can be simulated with massless quarks in a vacuum free of lattice monopoles. This allows an ab initio high precision, controlled study of the existence of ”textbook” Quantum Electrodynamics with several species of fermions. The lattice theory possesses a second order chiral phase transition which we show is logarithmically trivial. The logarithms of triviality, which modify mean field scaling laws, are pinpointed in several observables. The result supports Landau’s contention that perturbative QED suffers from complete screening and would have a vanishing fine structure constant in the absence of a cutoff.
Attempts to solve conventional field theory problems by modern lattice gauge theory techniques run up against myriad problems. Two of those are: 1. lattice field configurations may possess unphysical artifacts of the finite short distance cutoff; and, 2. simulations with realistic physical properties, such as (almost) vanishing fermion masses, are almost impossible [1], [2].

These problems can be solved in large part by adding a small, irrelevant four fermi interaction to standard lattice actions with staggered fermions. The resulting lattice action can be simulated directly in the chiral limit (massless fermions) because an auxiliary scalar field $\sigma$ (essentially the chiral condensate $\langle \bar{\psi} \psi \rangle$) acts as a dynamical mass term for the quarks insuring that the inversion of the Dirac operator will be successful and very fast. In addition, in the case of lattice QED supplemented with a four fermi interaction, there is a second order chiral transition where a continuum field theory may exist and the gauge field configurations for couplings near the transition are free of lattice artifacts, such as monopoles and Dirac strings, etc.

Consider the $U(1)$–gauged Nambu Jona Lasinio (GNJL) model with four species of fermions. Because of the irrelevance of the pure four fermi interaction, this model will make "textbook" QED accessible and this paper will address the classic problem of whether QED suffers from complete charge screening. Our measurements will show that the theory is logarithmically trivial.

The Lagrangian for the continuum GNJL model is,

$$L = \bar{\psi}(i\gamma \partial - e\gamma A - m)\psi - \frac{1}{2}G^2(\bar{\psi}\psi)^2 - \frac{1}{4}F^2$$

The Lagrangian has an electromagnetic interaction with continuous chiral invariance ($\psi \to e^{i\alpha \gamma_5} \psi$, where $\tau$ is the appropriate flavor matrix) and a four fermi interaction with discrete ($Z_2$) chiral invariance ($\psi \to \gamma_5 \psi$). The mass term $m\bar{\psi}\psi$ breaks the chiral symmetries and will be set to zero. The pure NJL model has been solved at large $N$ by gap equation methods [3], and an accurate simulation study of it has been presented [4].

To begin, we introduce an auxiliary random field $\sigma$ by adding $-\frac{G^2}{2}(\bar{\psi}\psi - \frac{\sigma}{G^2})^2$ to the
Lagrangian. This makes the Lagrangian a quadratic form in the fermion field so it can be analyzed and simulated by conventional methods. The model is then discretized by using staggered fermions. The lattice Action reads:

$$S = \sum_{x,y} \bar{\psi}(x)(M_{xy} + D_{xy})\psi(y) + \frac{1}{2G^2} \sum_{\tilde{x}} \sigma^2(\tilde{x}) + \frac{1}{2e^2} \sum_{x,\mu,\nu} F_{\mu\nu}^2(x)$$

where

$$F_{\mu\nu}(x) = \theta_\mu(x) + \theta_\nu(x + \hat{\mu}) + \theta_{-\mu}(x + \hat{\mu} + \hat{\nu}) + \theta_{-\nu}(x + \hat{\nu})$$

$$M_{xy} = (m + \frac{1}{16} \sum_{<x,\tilde{x}>} \sigma(\tilde{x}))\delta_{xy}$$

$$D_{xy} = \frac{1}{2} \sum_\mu \eta_\mu(x)(e^{i\theta_\mu(x)}\delta_{x+\hat{\mu},y} - e^{-i\theta_\mu(y)}\delta_{x-\hat{\mu},y})$$

In this formulation $\sigma$ is defined on the sites of the dual lattice $\tilde{x}$, and the symbol $<x,\tilde{x}>$ denotes the set of the 16 lattice sites surrounding the direct site $x$. The factors $e^{\pm i\theta_\mu}$ are the gauge connections and $\eta_\mu(x)$ are the staggered phases, the lattice analogs of the Dirac matrices. $\psi$ is a staggered fermion field and $m$ is the bare fermion mass, which will be set to 0. Note that the lattice expression for $F_{\mu\nu}$ is non-compact in the lattice field $\theta_\mu$, while the gauge field couples to the fermion field through compact phase factors which guarantee local gauge invariance.

The global discrete symmetry of the Action (2) reads:

$$\psi(x) \rightarrow (-1)^{x_1+x_2+x_3+x_4}\psi(x)$$

$$\bar{\psi}(x) \rightarrow -\bar{\psi}(x)(-1)^{x_1+x_2+x_3+x_4}$$

$$\sigma \rightarrow -\sigma.$$

where $(-1)^{x_1+x_2+x_3+x_4}$ is the lattice representation of $\gamma_5$.

A rough estimate of the improved performance of the algorithm can be made. To invert the lattice Dirac operator having a minimum eigenvalue $\lambda_{\text{min}}$ to an accuracy $R$ (residual), takes a number of sweeps $N$ of a conjugate gradient algorithm which scales as $R \propto \exp(-a\lambda_{\text{min}}N)$. But $\lambda_{\text{min}}$ is typically proportional to $\sigma + m$, where $\sigma$ is the average
of the $\sigma(x)$ field over the configuration. If in a simulation $m$ is 0.01 while $\sigma$ is 0.10, then the new algorithm is a full order of magnitude faster than the old. Our computer experiments of QCD with four fermi interactions as well as GNJL have proven to be at least as good as this $[3]$.

We studied this model using the Hybrid Molecular Dynamics algorithm because this method can treat the number of fermion flavors as a continuous variable. The standard staggered fermion algorithm produces eight flavors in the continuum limit. The Hybrid Molecular Dynamics algorithm can take the square root of the fermion determinant and produce a theory with four species of fermions in the continuum limit. This is the theory we chose to study. The trick used in simulations of QCD where one reduces the number of fermion species by a factor of two by placing lattice pseudo-fermion fields only on even sites does not apply to the NJL term in this action. Since the Hybrid Molecular Dynamics algorithm is not exact, we carefully monitored our simulations for systematic errors $[4]$.

Interesting limiting cases of the above Action are the pure $Z_2$ NJL model ($e = 0$), which has a phase transition at $G^2 \simeq 2$ $[3]$ and the pure lattice QED ($G = 0$) limit, whose chiral phase transition is near $\beta_e \equiv 1/e^2 = .204$ for four flavors $[8], [9]$. The pure QED ($G = 0$) model also has a monopole percolation transition which is probably coincident with its chiral transition at $\beta_e = .204$ $[11]$. Past simulations of this lattice model have led to contradictory results $[9], [11], [12]$. Since the GNJL model can be simulated at $m = 0$ for all couplings, the results reported here will be much more precise and decisive than those of the pure lattice QED ($G = 0$) limit.

We scanned the 2 dimensional parameter space ($\beta_e, G^2$) using the Hybrid Molecular Dynamics algorithm and measured the chiral condensate and monopole susceptibility as a function of $\beta_e$ and $G^2$. Recall that non-compact lattice QED possesses monopole excitations and Dirac strings $[13]$. We are particularly interested, however, in simulating the model in regions of its parameter space where these topological excitations are non-critical so they do not contribute to the model’s continuum limit. We found that as we increased $G^2$ and moved off the $G = 0$ axis, the peak of the monopole susceptibility shifted from $\beta_e = .204$ at
$G = 0$ to $\beta_e = .244$ at large $G$. By contrast the chiral transition point shifted to a larger $\beta_e$ than the monopole percolation transition for a given value of $G$ and became distinct from the monopole percolation point as soon as $G$ became nonzero. The movement of the monopole percolation peak in the $(\beta_e, G^2)$ plane can be understood by noticing that $\sigma$ in the Action plays the role of a site dependent mass term (Eq. 4). When the fluctuations of the $\sigma$ field are not important, as is the case at large $G$, the gauge field dynamics becomes equivalent to QED with a bare mass given by the constant $\sigma$ value. So, as $G^2$ increases and $\sigma$ grows, the theory approaches the large $m$ limit of QED, i.e. quenched QED, which has a monopole percolation transition at $\beta_e = .244$ \[13\]. This result was confirmed quantitatively in the simulation. In conclusion, the chiral transition line extends from $(\beta_e, G^2) = (.204, 0)$ to $(\infty, \simeq 2)$, while the monopole percolation line extends from (.204, 0) to (.244, $\infty$). The two transitions only coincide at the ”pure” QED point, $G = 0$. Thus, the gauged NJL model makes it possible to study the triviality of conventional $U(1)$ gauge interactions without topological excitations, an important physics problem which has bedeviled field theory for decades. Landau originally concluded in the context of perturbation theory that QED would be a free field theory because fermion vacuum polarization would screen the electric charge completely. Alternatively, if one renormalized the theory holding the renormalized charge fixed, then the effective charge measured on a particular smaller length scale would diverge (Landau’s ghost). Landau’s argument was originally made in the context of pure QED, but its conclusion is not effected by adding irrelevant operators into the Lagrangian since irrelevant operators do not effect the long distance, physical content of the theory. In fact, in the context of lattice QED and the GNJL model, the $G = 0$ lattice model is not ”pure QED”. Since the discrete differences of the lattice formulation differ from continuum derivatives, irrelevant operators distinguish the two field theories. In the formulation studied here where the weak four fermi term is explicitly introduced for technical reasons, we are always addressing the same physical issues of the more familiar case of ”pure QED”.

We have made accurate measurements on the chiral critical line for many choices of couplings $(\beta_e, G^2)$ and lattice sizes ranging from $8^4$ to $20^4$. Here we shall discuss highlights
of our data collected varying $\beta_e = 0.15 - 0.30$ at fixed $G^2 = 1/4$ on a $16^4$ lattice and leave a more thorough presentation to another, lengthier presentation. Finite size effects, efficiency and errors in the algorithm, measurement statistics and error analysis, as well as other scaling laws and critical indices will be dealt with elsewhere [7].

In Fig.1 we show the data for the chiral condensate $<\bar{\psi}\psi>$, at fixed $G^2 = 1/4$ and variable $\beta_e$. The superb accuracy of this data, as compared to typical lattice QCD data, will allow us to see, with a minimum of analysis, that the chiral critical point is logarithmically trivial. Consider the most conventional fitting forms for this data. In mean field theory one predicts the equation of state $\beta_c - \beta_e = a\sigma^{1/\beta_{mag}}$, with the critical index $\beta_{mag} = 1/2$. This scaling law is modified by logarithms in trivial four dimensional models: in the two component $\phi^4$ model, $\beta_c - \beta_e = a\sigma^2/\ln(b/\sigma)$ [14], and in the $Z_2$ NJL model, $\beta_c - \beta_e = a\sigma^2\ln(b/\sigma)$ [4]. In both of these simple models the interaction strength falls to zero logarithmically as the cutoff is taken to infinity and this slow vanishing of the interactions causes the logarithmic effects in the equation of state for each model. It is interesting that the logarithms enter differently in both equations of state, so we will be able to distinguish between $\phi^4$ triviality and NJL triviality. In fact $\phi^4$ triviality is almost always assumed for ”textbook” QED, but we shall find that NJL triviality is the actual answer.

The data shown in Fig.1 has been fit to a form which can accomodate either $\phi^4$ or NJL triviality: $\beta_c - \beta_e = a\sigma^2(\ln(b/\sigma))^p$, where the parameter $p$, the critical point $\beta_c$, the amplitude $a$ and the scale $b$ are determined by the fitting routine. For the scaling window of gauge couplings $\beta_e$ between .18 and .225, we found the parameters $\beta_c = .2350(1)$, $a = 34.3(3.9)$, $\ln b = 1.55(10)$ and $p = 1.00(8)$ with a confidence level of 34 percent. This is the fit shown in the figure. Since the uncertainty in the power of the logarithm $p = 1.00(8)$ is so small, we have superb evidence for the triviality of ”textbook” QED. In fact, these simulations also measured topological observables for the system’s vacuum and we confirmed that monopoles and related objects were not critical near the chiral transition $\beta_c = .2350(1)$, $G^2 = 1/4$. ( We measured that the monopole percolation transition is very narrow and occurs at $\beta_e = .2175(25)$ for $G^2 = 1/4$ [7]. )
The importance of the logarithm in the equation of state can be seen explicitly if we plot the data and the fit as shown in Fig. 2., $|\beta_c - \beta_e|/\sigma^2$ vs. $\ln(1/\sigma)$. If mean field theory were true, this plot would be flat; if $\phi^4$ triviality applied, it would fall; and, if NJL triviality applied, it would rise linearly. Fig. 2 shows that the third possibility is chosen decisively. The dashed line is the previous fit redrawn in this format.
Finally, in Fig.3 we show the inverse of the longitudinal susceptibility of the auxiliary field $\sigma$ at fixed $G^2 = 1/4$ and variable $\beta_e$. We plot the inverse of the susceptibility rather than the susceptibility itself, because in mean field theory, the singular piece of the longitudinal susceptibility $\chi$ diverges at the critical point $\beta_c$ as $\chi_+ = c_+|t|^{-\gamma}$, $t \equiv (\beta_c - \beta_e)/\beta_c$, as $t$ approaches zero from above in the broken phase, and as $\chi_- = c_-|t|^{-\gamma}$ in the symmetric phase [14]. The critical index $\gamma$ is exactly unity in mean field theory. As we see in the figure, the linear scaling law works well in the scaling windows on both sides of the critical point where the inverse susceptibility vanishes.

![FIG. 3. Inverse Susceptibility vs. Coupling $\beta_e$](image)

The plot picks out a critical point $\beta_c = .2358(5)$ and is consistent with the mean field value of the critical index $\gamma = 1.0$.

Another prediction of mean field theory is that the universal amplitude ratio $c_-/c_+$ is exactly 2.0. However, in logarithmically trivial models $\gamma$ remains unity, but the amplitudes $c_+$ and $c_-$ develop weak logarithmic dependences [14]. In the two component $\phi^4$ model, $c_-/c_+ = 2 + \frac{2}{3}/\ln(b \sigma)$, while in the $Z_2$ NJL model, $c_-/c_+ = 2 - 1/\ln(b \sigma)$ [4], where the scale $b$ was determined in the order parameter fit. So, if $\phi^4$ triviality applied here we should find the amplitude ratio slightly larger than 2, and if NJL triviality applied we should find
the amplitude ratio slightly smaller than 2. In fact, the constrained linear fits to the data shown in the figure produced the amplitude ratio \( c_-/c_+ = 1.74(10) \). Since \( \sigma \) varies from .0953(1) to .0367(2) over the \( \beta_e \) range .18 - .225 of the scaling window in the broken phase, the theoretical prediction of the NJL model states that \( c_-/c_+ \) should range from 1.75 to 1.79. Again, the agreement between the simulation data and theory is very good.

We have checked [7] that the four fermi term is irrelevant as we get log-improved mean field theory in each of our runs at various \( G^2 \neq 0 \), ranging from \( G^2 = 1/8 \) to \( G^2 = 1 \).

Taken together Figs. 1-3 give a nicely consistent picture of the triviality of the four species \( U(1) \) gauged Nambu Jona Lasinio model with a \( Z_2 \) chiral group. We find no support for the approximate analytic schemes discussed in [15].

It would be worthwhile to continue this work in several directions. One could calculate the theory’s renormalized couplings and their RG trajectories in the chiral limit, extending the work of [12]. One could also simulate the model with the \( Z_2 \) chiral group replaced by a continuous group so the model would have Goldstone bosons even on a coarse lattice. Finally, it would be interesting to simulate compact QED with a small four fermi term and study the interplay of monopoles, charges and chiral symmetry breaking. Since the \( G = 0 \) limit of the compact model is known to have a first order transition [14], generalizations of the action will be needed to find a continuous transition where a continuum limit of the lattice theory might exist.

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