CP Violation from the Neutrino Sector: A Case for the Superweak Model

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Abstract

We discuss how CP violation originating in the right-handed neutrino sector can feed into the quark sector, in an otherwise CP invariant theory. The dominant effects are superweak, and we suggest that this may yield a natural resolution of the strong CP problem. This work builds on and extends a previously proposed model of quark and lepton masses, based on a new strong flavor interaction above the weak scale.

In this paper we will propose that CP violation arises dynamically in association with the breakdown of lepton-number, as manifested in right-handed neutrino condensates. We will discuss how the “leakage” of CP violation into the quark sector can then be small, and by showing up in 4-quark operators, result in the classic superweak model of CP violation [1]. The deviations from purely real quark mass matrices may also be small enough to naturally resolve the strong CP problem. Of most immediate interest for this picture is the prediction of the near absence of CP violation in the $b$ system.

Our discussion takes place in the context of dynamical symmetry breaking, but the picture is somewhat different from a standard extended-technicolor picture. There is a fourth family of fermions (not technifermions) whose dynamical masses are related to electroweak symmetry breaking. There is also a new strong flavor gauge interaction which acts on the four families and which first breaks at a scale $\Lambda$ in the 100 to 1000 TeV range.

When we consider the operators in the effective theory below the scale $\Lambda$, we find that those which can feed CP violation into the quark sector are lepton-number

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\footnote{There have been other proposed resolutions of the strong CP problem in the context of the superweak model [4].}
violating, 6-fermion operators. For example, a CP-violating $\Delta S = 2$ operator could be of the form

$$\bar{d}_R s_L d_R s_L \nu_{\tau'} \nu_{\tau'}.$$  

This is a piece of an $SU(2)_L \times U(1)_Y$ invariant operator and $\nu_{\tau'}$ is the heavy fourth-family left-handed neutrino. The presence of both quarks and leptons in this operator reflects the fact that both quarks and leptons couple to the flavor gauge interaction. If the coefficient of this operator is of order $1/\Lambda^5$ and $\langle \nu_{\tau'} \nu_{\tau'} \rangle \approx \Lambda^3_{EW}$ then the coefficient of the resulting $\bar{d}_R s_L d_R s_L$ operator is of order $\Lambda^3_{EW}/\Lambda^5$. As we will see, this can be the appropriate size.

A theory of CP violation should also be a theory of mass, and so a substantial fraction of this paper must be devoted to that subject. In next section we describe the new flavor interactions and how they can give rise to a class of operators required to generate quark and lepton masses. In section 2 we consider the CP violation in the right-handed neutrino sector and show how it feeds into the quark sector via this same class of operators. Finally in section 3 we describe in detail how the quark and lepton mass spectrum can arise.

1 Preliminaries

A minimal flavor-gauge symmetry, $U(2)_V \equiv SU(2)_V \times U(1)_V$, has been described in [3]. This leads to a four family model where pairs of same-charge fermions from two of the families transform as a 2 under $U(2)_V$ and pairs from the other two families transform as a $\bar{2}$. We label the quarks and leptons in these four families as $[Q_1, L_1]$, $[Q_2, L_2]$, $[\bar{Q}_1, \bar{L}_1]$, $[\bar{Q}_2, \bar{L}_2]$, respectively. The $V$ will remind us that $U(2)_V$ is a vector symmetry with respect to these fields, which are not necessarily the mass eigenstates.

All right-handed neutrinos are assumed to have a dynamical Majorana mass of order the flavor-physics scale $\Lambda$. They are the only fermions to receive mass at the flavor scale, and their condensates will serve as the order parameters for the breakdown of $U(2)_V$ to $U(1)_X$. If $SU(2)_R \times U(1)_{B-L}$ is part of the weak gauge symmetry at the flavor scale, it will also be broken in the appropriate way to $U(1)_Y$ by these neutrino condensates. $U(1)_X$ is defined such that the $[Q_2, L_2]$ and $[\bar{Q}_2, \bar{L}_2]$ families are $U(1)_X$ neutral (the light two families) while $[Q_1, L_1]$ and $[\bar{Q}_1, \bar{L}_1]$ have equal and opposite $U(1)_X$ charges (the heavy two families). $U(1)_X$ breaks close to the weak scale as we describe below, and it should play a role in the generation of the fourth-family masses which in turn break the electroweak symmetry.

[The fermion content of the theory and the flavor symmetry could be larger, for
example the two sets of families could transform as $n_f$ and $\pi_f$ under $SU(n_f)$ for $n_f \geq 4$. In place of the breakdown $U(2)_V \rightarrow U(1)_X \rightarrow$ nothing, we would have $SU(n_f) \rightarrow SU(n_f - 1) \rightarrow SU(n_f - 2)$. Since we are not concerned here with trying to understand the dynamical implications of these different choices, we will consider $U(2)_V$ as the complete flavor symmetry for simplicity.]

As described in [3], the dynamical fourth family masses are as follows; the $t'$ and $b'$ quarks correspond to the mass term $\overline{Q}_{L1}Q_{R1}$ (the hermitian conjugate term will always be implicit), the $\tau'$ corresponds to $\overline{E}_{L1}E_{R1}$, and the left-handed $\nu_{\tau'}$ corresponds to $\overline{N}_{L1}$. The $(t', b')$ masses could be close to a TeV, while $\nu_{\tau'}$ may have a mass in the few hundred GeV range, with the $\tau'$ mass roughly twice as large. The leptons with such masses make only small contributions (perhaps negative) to $S[4]$ and $T[5]$. The gauge dynamics generating the $(t', b')$ masses is isospin symmetric, and the small amount of $t' - b'$ mass splitting implied by the $t - b$ mass splitting gives only a small contribution to $T$, since it is suppressed by $(m_t/m_{t'})^4$ [3]. And finally there is the $(t', b')$ contribution to $S$. But since we are suggesting that the gauge dynamics generating the $(t', b')$ masses is itself breaking down, the theory is quite unlike QCD (i.e. there is no $\rho$-like resonance), and the contribution to $S$ is uncertain. Given all this it seems that a fourth family with dynamical mass can still be consistent with precision electroweak measurements.

We note that there is one additional symmetry of the flavor physics, as we have described it. That symmetry is a $U(1)_A$ under which the $[Q, L]$ and $[Q, L]$ fields have equal and opposite axial charge. Either this is a gauged symmetry which is broken at the flavor scale, or the symmetry is already broken by 4-fermion interactions originating at a higher scale. In either case these additional interactions can serve to make the flavor interactions chiral, and thus resistant to the formation of mass.

The aspect of strong flavor dynamics crucial to our picture of quark and lepton masses is the generation of nonperturbative multi-fermion condensates. Given the presence of strong interactions, it is not unnatural to expect that condensates allowed by the unbroken symmetries will form. Their presence is especially significant when most fermions are not receiving dynamical masses (as long as $SU(2)_L \times U(1)_Y$ is an unbroken symmetry), since in that case the condensates will imply the existence of multi-fermion operators in the effective theory below the flavor scale.

In the presence of the fourth family masses, these operators make contributions to the lighter quark and charged-lepton masses. When we consider these contributions we find that the dominant contributions should come from a particular subset of possible 4-fermion operators. Besides being singlets under $SU(3)_C \times SU(2)_L \times U(1)_Y$,
the interesting operators have the following properties.

- They have the chiral structure $\overline{\psi}_L \psi_R \overline{\psi}_L \psi_R$, where each $\psi$ denotes any quark or lepton.

- They preserve $SU(2)_V$ and CP.

- At least some or perhaps all display maximal $SU(2)_R$ breaking.

The fact that these condensates are singlets under $SU(2)_V$ makes dynamical sense, since it implies that they are in an attractive channel with respect to these strong interactions. By maximal $SU(2)_R$ breaking we mean for example that $\overline{Q}_L D_R \overline{Q}_L U_R$ is dynamically generated but not $\overline{Q}_L U_R \overline{Q}_L D_R$. The latter can be induced from the former, though, via an $SU(2)_R$ gauge boson exchange; this will be our mechanism for producing the $t$–$b$ mass ratio.

Why should operators of the LRLR form dominate? One might speculate that instanton dynamics will play a role in the generation of condensates of the LRLR form, as opposed for example to condensates of the $\overline{\psi}_L \psi_R \overline{\psi}_R \psi_L$ form. Nevertheless some operators of the alternative LRRL form will be induced by tying together a LRLR operator with the conjugate of another LRLR operator in a loop. But even these effects may be suppressed due to factors of $4\pi$. The coefficients of LRLR operators are expected to be of order $1/f^2 \equiv g^2/M^2 \approx 4\pi/M^2$ where $M$ is the mass of a gauge boson and the strong coupling is $g^2/4\pi \approx 1$. Loop effects are then suppressed if we take $M$ as the ultraviolet cutoff on loop integrations and use a factor of $1/(4\pi)^2$ for each loop.

Four-fermion operators may be composed of $SU(2)_V$-invariant scalars like $Q_L Q_R$ which preserve $U(1)_V$ and scalars like $Q_L Q_R \varepsilon_{ij}$ which do not. Four-fermion condensates which break $U(1)_V$ will also break $U(1)_X$, and we assume that the resulting $m_X/g_X$ is in the TeV range. The hierarchy between the $U(2)_V/U(1)_X$ gauge boson masses and the $X$ mass corresponds to our expectation that contributions to gauge boson masses are larger when coming from the 2-fermion (Majorana neutrino) condensates than when coming from the 4-fermion condensates. The contribution from a 4-fermion condensate involves tying the condensate together with its conjugate (three loops), and the same loop analysis as before indicates that this is suppressed.

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2One can of course construct potentials for scalar fields where the analog of this breaking pattern would occur for a range of parameters (see the appendix in [6]). In a similar way we could illustrate the naturalness of various other dynamical assumptions made in this paper.

3With respect to the larger possible flavor symmetry $SU(n_f)$ mentioned above, $Q_L Q_R \varepsilon_{ij}$ with $i, j = 1, 2$ is again in an attractive channel, and condensates containing it would break $SU(n_f - 1)$ to $SU(n_f - 2)$. 
2 CP Violation

Above the flavor scale we assume that we have a CP invariant gauge theory of massless fermions. We then assume that the flavor dynamics is such that CP violation, lepton-number violation, and $SU(2)_V$ breaking all originate in the right-handed neutrino condensates (both bilinear and multilinear). CP violation for example would be reflected in the phases of the Majorana mass condensates $\langle N_{R2}^2 \rangle$, $\langle N_{R2}^2 \rangle$, $\langle N_{R1} N_{R1} - N_{R2} N_{R2} \rangle$ and $\langle N_{R1} N_{R1} + N_{R2} N_{R2} \rangle$, which are the most general allowed by the breaking $U(2)_V \rightarrow U(1)_X$. Note that only the first three break $SU(2)_V$, and by combining a neutrino mass and a conjugate neutrino mass there are also amplitudes which break $SU(2)_V$ and not lepton-number. We will argue that when the right-handed neutrinos are integrated out, the only CP-violating operators in the effective theory must violate lepton-number or $SU(2)_V$ or both.

Let us write $\langle N_{R2}^2 \rangle = a e^{i\alpha}$, $\langle N_{R2}^2 \rangle = b e^{i\beta}$, $\langle N_{R2} N_{R2} \rangle = c e^{i\chi}$ where the various constants appearing here are real. Let us consider combinations of these condensates which could appear internally in diagrams after the right-handed neutrinos have been integrated out. Let us first consider combinations which preserve $N_R$ number and $\overline{N}_R$ number. Some of these combinations would be intrinsically real, such as when a condensate and its complex conjugate appear in a loop. When there are four condensates in a loop the phases need not cancel, and for example one combination would be proportional to $abc^2 e^{i(\alpha + \beta - 2\chi)}$. But there is another diagram in which all condensates are replaced by their complex conjugates, and so the sum is proportional to $\cos(\alpha + \beta - 2\chi)$. The sum is thus CP conserving, i.e. invariant under reversing the signs of all phases simultaneously.

A similar argument applies to any combination of bilinear and multilinear neutrino condensates. To preserve $N_R$ number and $\overline{N}_R$ number, every neutrino line from a condensate must be paired with an antineutrino line of the same flavor from another condensate. Each such combination of condensates is either intrinsically real, or when it is not there is another combination in which all condensates are replaced by their complex conjugates so that the sum is real. Thus to find CP violation we must consider combinations of condensates which do not preserve $N_R$ number and/or $\overline{N}_R$ number. These combinations produce amplitudes which break lepton-number but not $SU(2)_V$ (such as $N_{R1} \overline{N}_{R1} + N_{R2} \overline{N}_{R2}$), or break $SU(2)_V$ completely but not lepton number (such as $\overline{N}_{R2} \overline{N}_{R2}$), or break both lepton-number and $SU(2)_V$ (such

\footnote{The dynamical breakdown of CP naively leads to a domain wall problem, but various resolutions of this problem have now been proposed \cite{7, 8, 9}. We learn from these references that the issue is more complex and probably less serious than once thought.}
as $N^2_{R2}$). Since nothing else at the flavor scale breaks lepton number or $SU(2)_V$, the implication is that in the effective theory below the flavor breaking scale the only CP-violating operators are one of these three types.

We digress briefly to comment on the origin of the dynamical breakdown of CP. We can expect a term proportional to $\cos(\alpha + \beta - 2\chi)$ (phases defined above) in some effective potential constructed to describe the neutrino condensation. If this term has the appropriate sign, then minimization of this one term implies that $\alpha + \beta - 2\chi = \pi$. The only CP-conserving solution has the condensates real with $\langle N^2_{R2} \rangle$ and $\langle N^2_{L1} \rangle$ opposite in sign. The CP-violating solutions allow $\langle N^2_{R2} \rangle$ and $\langle N^2_{L1} \rangle$ to be equal but complex. Other terms in the effective action, such as those involving multi-neutrino condensates, can potentially pick out the latter solution.

We now consider the lepton-number and $SU(2)_V$ violating operators in the effective theory after the right-handed neutrinos have been integrated out. The lowest dimension $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant operators are of dimension 9, and interesting examples are the following.

\[
\begin{align*}
\overline{D}_{L2}D_{R2}\overline{D}_{L2}D_{R2}N_{L1}N_{L1} & \quad (2) \\
\overline{D}_{L2}D_{R2}\overline{D}_{L2}D_{R2}N_{L1}N_{L1} & \quad (3) \\
\overline{D}_{L2}D_{R1}\overline{D}_{L2}D_{R1}N_{L1}N_{L1} & \quad (4) \\
\overline{D}_{L1}D_{R2}\overline{D}_{L1}D_{R2}N_{L1}N_{L1} & \quad (5)
\end{align*}
\]

They can be seen to arise from the $SU(2)_V$-preserving operators, $\overline{D}_{Li}D_{Ri}N_{Lk}N_{Rl}\varepsilon_{kl}$, $\overline{D}_{Li}D_{Ri}N_{Lk}N_{Rl}\varepsilon_{kl}$, and $\overline{D}_{Li}D_{Rj}\varepsilon_{ij}N_{Lk}N_{Rl}\varepsilon_{kl}$, along with an insertion of the $N_{R2}$ mass. $N_{L1}$ is the fourth left-handed neutrino $\nu_{\tau}'$ which has a Majorana mass close to the weak scale. The result below the weak scale is an effective CP-violating 4-quark operator. In particular the operators in (4) and (5) turn out to be essentially the $\Delta S = 2$ operators $(\bar{d}_{LsR})^2$ and $(\bar{s}_{LdR})^2$ respectively. This will become clear from the quark mass matrices given in the next section.

Assuming a CP-violating phase of order unity, the coefficients of these $\Delta S = 2$ operators (which contain pseudoscalar pseudoscalar pieces) should be order $10^{-10}$ TeV$^{-2}$ to recover the known value of $\varepsilon$ in the neutral kaon system \cite{1}. If the coefficients of the $SU(2)_V$-preserving 4-fermion operators are $\approx \Lambda^{-2}$ and the $N_{R2}$ mass is $m_N$, then the coefficients of the operators in (2) and (3) are $\approx 1/(\Lambda^4 m_N)$. It is reasonable that this is of order $1/(100 \text{ TeV})^5$ and $\langle N^2_{L1} \rangle \approx (1 \text{ TeV})^3$, in which case the coefficient of the $\Delta S = 2$ operator is of the desired size.

We have recovered the classic superweak model \cite{1} which accounts for CP violation in $K-\bar{K}$ mixing. CP-violating $\Delta S = 1$ operators can be induced from those in (2).
and (3) by mass mixing between the $d$ and the $s$, but this produces a negligible contribution to $\varepsilon'$. $\Delta b = 2$ CP violation arises from the operators in (4) and (5) which generate $(\bar{d}_L b_R)^2$ and $(\bar{b}_L d_R)^2$ operators respectively. These latter effects would have to be $10^3$ to $10^4$ times larger than the $\Delta s = 2$ CP violation in order to match the standard model prediction. This is very unlikely, thus making the nonobservation of standard model CP-violating effects in the $b$ system a signature of our picture.

Another possible signal of CP violation in the quark sector is in the QCD vacuum angle $\theta$. In the underlying CP-invariant theory of massless fermions the QCD vacuum angle $\theta$ vanishes, but a nonzero $\theta$ can be generated if CP violation in the neutrino sector feeds into the quark mass matrix. In fact a possibly dangerous contribution arises if the operator $\mathcal{E}_{L1} \mathcal{E}_{R1} \Sigma_{L1} N_{R2}$ is dynamically generated. Along with the operators considered above it would generate the following 6-fermion operators.

$$
\mathcal{D}_{L2} D_{R2} \mathcal{E}_{L1} \mathcal{E}_{R1} \Sigma_{L1} \Sigma_{L1} \tag{6}
$$
$$
\mathcal{D}_{L2} D_{R2} \mathcal{E}_{L1} \mathcal{E}_{R1} \Sigma_{L1} \Sigma_{L1} \tag{7}
$$
$$
\mathcal{D}_{L1} D_{Rj} \varepsilon_{ij} \mathcal{E}_{L1} \mathcal{E}_{R1} \Sigma_{L1} \Sigma_{L1} \tag{8}
$$

$\mathcal{E}_{L1} \mathcal{E}_{R1}$ corresponds to the $\tau'$ mass, and so along with the $\nu_{\tau'}$ mass these operators could make CP-violating contributions to the $\bar{d}_{LS_R}$, $\bar{s}_{L} d_R$, $\bar{d}_L b_R$ or $\bar{b}_L d_R$ off-diagonal mass terms. If the coefficients of these operators are of order $1/(100 \text{ TeV})^5$ and $\langle \mathcal{E}_{L1} \mathcal{E}_{R1} \rangle \approx (1 \text{ TeV})^3$ then the contribution to the imaginary parts of these mass terms could be as large as roughly 100 eV. By comparing to light quark masses we see that the suppression arising from the small size of generic 6-fermion operators may not sufficiently suppress $\theta$.

The largest contribution to $\theta$ will likely come from the $d-s$ mass elements. Given that the diagonal elements of the down quark mass matrix dominate the determinant (see next section), we have

$$
\bar{\theta} \approx -\frac{\text{Re}(m_{ds}) \text{Im}(m_{sd}) + \text{Re}(m_{sd}) \text{Im}(m_{ds})}{m_dm_s}. \tag{9}
$$

We can now identify additional possible sources of suppression which make an acceptable value for $\bar{\theta}$ fairly plausible.

- In the next section we will see that $\text{Re}(m_{sd})$ and $\text{Re}(m_{ds})$ are suppressed because they can only be generated by 4-fermion operators of the suppressed LRRL form. (In the up-sector on the other hand, LRLR operators contribute to the off-diagonal terms, which could then be the origin of most of the Cabbibo mixing.)

\footnote{The current experimental upper bound on the neutron EDM is satisfied for $\bar{\theta} \approx 10^{-9}$ \cite{11}.}
• The offending $E_{L1} E_{R1} N_{L1} N_{R2}$ operator may be one of the operators disfavored due to the maximal breakdown of $SU(2)_R$, in which case it may only arise as a radiative correction to the operator $E_{L1} N_{R1} N_{L1} E_{R2}$. These two operators would then be analogous to the $\tilde{D}$ and $D$ 4-quark operators appearing in the next section.

• As for CP violation leaking into the up-sector masses, in addition to $E_{L1} E_{R1} N_{L1} N_{R2}$ we would need operators like $U_{R2} U_{L2} N_{L1} N_{R2}$ which are of the suppressed LRRL form. The generation of such operators may be further suppressed due to the maximal breakdown of $SU(2)_R$.

• Due to the absence of color interactions it is conceivable that purely leptonic operators (or at least those which break $U(1)_V$ and $U(1)_X$) are generated only through loops (at least two) involving other LRLR operators. In fact we will see that purely leptonic operators of dynamical origin are not required for the generation of quark or charged-lepton masses.

3 Quark and Lepton Masses

We first describe the quark masses in a manner similar to, but not identical to, a previous description [3]. We then turn to a description of lepton masses which is essentially new. We will see that quarks and charged-lepton masses may be completely described in terms of operators of the LRLR form. We will also highlight the interplay between the quark and lepton sectors.

We first consider 4-quark operators. In the following list we have labeled those pieces of $SU(2)_V$-invariant operators which make important contributions to the quark masses. Only the $B$ and $\tilde{B}$ operators preserve both $U(1)_V$ and $U(1)_A$.

$$\begin{align*}
&\bar{U}_{L1} D_{R1} \bar{D}_{L1} U_{R1} \quad B \\
&\bar{D}_{L1} U_{R1} \bar{U}_{L1} D_{R1} \quad \tilde{B} \\
&\bar{U}_{L1} D_{R1} \bar{U}_{L1} U_{R2} \quad C \\
&\bar{D}_{L1} U_{R1} \bar{U}_{L1} D_{R2} \quad \tilde{C} \\
&\bar{U}_{L2} D_{R1} \bar{D}_{L1} U_{R1} \quad D \\
&\bar{D}_{L2} U_{R1} \bar{U}_{L1} D_{R1} \quad \tilde{D} \\
&\bar{Q}_{L1} U_{Rj} \varepsilon_{ij} \bar{Q}_{Lk} D_{Ri} \varepsilon_{kl} \quad E \\
&\bar{Q}_{L1} U_{Rj} \varepsilon_{ij} \bar{Q}_{Lk} D_{Ri} \varepsilon_{kl} \quad F
\end{align*}$$

These operators feed mass down to the known three families of quarks from the $t'$ and $b'$ masses ($\bar{U}_{L1} U_{R1}$ and $\bar{D}_{L1} D_{R1}$) except for the $F$ operator, which feeds mass down from the $t$ mass ($\bar{U}_{L1} U_{R1}$). The $t'$ and $b'$ masses have to be close to degenerate
and so the $t$–$b$ mass ratio must be due to $SU(2)_R$ breaking in these operators. If there is a dynamical breakdown of $SU(2)_R$ then we could suppose that the $B$, $C$ and $D$ operators are generated but not the $\tilde{B}$, $\tilde{C}$ and $\tilde{D}$ operators. If $SU(2)_R$ is a weak gauge symmetry at the flavor scale then the latter operators will be induced from the former operators by an $SU(2)_R$ gauge boson exchange. In this way the $b$ mass arises as a radiative correction to the $t$ mass.

Important contributions to the quark masses will also feed in from the lepton sector. The following mixed quark-lepton operators feed mass down from the $\tau'$ mass $(\tilde{E}_{L1}\tilde{E}_{R1})$. Only the $G$ operators preserve both $U(1)_V$ and $U(1)_A$.

\[
\begin{bmatrix}
E_{L1} & E_{R1} & U_{L1} & U_{R1} & G_1 \\
E_{L1} & E_{R1} & U_{L2} & U_{R2} & G_2 \\
E_{L1} & E_{R1} & U_{L1} & U_{R1} & H_1 \\
E_{L1} & E_{R1} & U_{L2} & U_{R2} & H_2 \\
E_{L1} & E_{R1} & U_{L1} & U_{Rj} & I \\
E_{L1} & E_{R1} & U_{L1} & U_{Rj} & J
\end{bmatrix}
\]

(11)

We write the quark mass matrices in terms of the original fields as follows, where the $t'$ and $b'$ masses correspond to the bottom right corner.

\[
\begin{bmatrix}
\overline{Q}_{L2} & \overline{Q}_{R2} & \overline{Q}_{L2} & \overline{Q}_{R1} & \overline{Q}_{L2} & \overline{Q}_{R1} \\
\overline{Q}_{L2} & \overline{Q}_{R2} & \overline{Q}_{L2} & \overline{Q}_{R1} & \overline{Q}_{L2} & \overline{Q}_{R1} \\
\overline{Q}_{L1} & \overline{Q}_{R2} & \overline{Q}_{L1} & \overline{Q}_{R1} & \overline{Q}_{L1} & \overline{Q}_{R1} \\
\overline{Q}_{L1} & \overline{Q}_{R2} & \overline{Q}_{L1} & \overline{Q}_{R1} & \overline{Q}_{L1} & \overline{Q}_{R1}
\end{bmatrix}
\]

(12)

Here then are the contributions from the various operators.

\[
M_u = \begin{bmatrix}
0 & G_2 & \mathcal{I} & 0 \\
\mathcal{H}_2 & \mathcal{E} & \mathcal{D} & J \\
\mathcal{I} & C & B & G_1 \\
0 & J & \mathcal{H}_1 & A
\end{bmatrix}
\]

(13)

\[
M_d = \begin{bmatrix}
\mathcal{F} & 0 & 0 & 0 \\
0 & \mathcal{E} & \tilde{D} & 0 \\
0 & \tilde{C} & \tilde{B} & 0 \\
0 & 0 & 0 & \mathcal{A}
\end{bmatrix}
\]

(14)

None of the zero entries are exactly zero; in $M_u$ these entries are too small to have any significance while in $M_d$ some could be significant, but they must be generated by operators of the suppressed LRRL form.

The following points are relevant to understanding the various hierarchies.

- The operators have different transformation properties under the strong $U(1)_X$, and this will cause different anomalous power-law scaling enhancements as the
operators are run down from the flavor scale to a TeV.

\[ \mathcal{B} > \mathcal{C}, \mathcal{D} > \mathcal{E} \]  
\[ \mathcal{G}_1, \mathcal{H}_1 > \mathcal{I}, \mathcal{J} > \mathcal{G}_2, \mathcal{H}_2 \]  

- There are different heavy masses, \( m_{\nu, \nu'} > m_{\tau'} > m_t \), being fed down.

\[ \mathcal{E} > \mathcal{F} \]  
\[ \mathcal{B} > \mathcal{G}_1, \mathcal{H}_1 \]  
\[ \mathcal{C}, \mathcal{D} > \mathcal{I}, \mathcal{J} \]  

- \( \tilde{\mathcal{B}}, \tilde{\mathcal{C}} \) and \( \tilde{\mathcal{D}} \) arise from weak radiative corrections.

\[ \mathcal{B}, \mathcal{C}, \mathcal{D} > \tilde{\mathcal{B}}, \tilde{\mathcal{C}}, \tilde{\mathcal{D}} \]  

- Some operators break \( U(1)_A \) while others do not. Thus, for example,

\[ \mathcal{G} > \mathcal{H}. \]  

We note that the \( \mathcal{E} \) entry is the same in the two matrices, since that operator is intrinsically \( SU(2)_R \) conserving. If this entry determines the \( s \) mass then the \( \mathcal{C} \) and \( \mathcal{D} \) entries must be responsible for the \( c \) mass, by causing mixing with the \( t \). Similar in size to the \( \mathcal{E} \) operator is the \( \mathcal{F} \) operator, which feeds mass from the \( t \) to the \( d \). We thus expect that

\[ \frac{m_d}{m_s} \approx \frac{m_t}{m_{\nu'}}. \]  

Examples of matrices which give realistic masses\(^6\) and mixings are the following.

\[
M_u = \begin{bmatrix}
  0 & .1 & 1 & 0 \\
  -.025 & .1 & 10 & 1 \\
  -1 & -10 & 160 & 10 \\
  0 & -1 & -2.5 & 1000
\end{bmatrix}
\]

\[
M_d = \begin{bmatrix}
  .005 & 0 & 0 & 0 \\
  0 & .1 & .07 & 0 \\
  0 & -.07 & 3 & 0 \\
  0 & 0 & 0 & 1000
\end{bmatrix}
\]

\(^6\)The up-type masses are (.002, 74, 160, 1000) GeV and the down-type masses are basically the diagonal entries; these values are appropriate for masses renormalized at a TeV.
We now turn to the charged-lepton masses, where the mixed quark-lepton operators again play an essential role. The following operators will feed mass down from the $t'$,

\[
\begin{bmatrix}
E_{L1}U_{R1}\overline{U}_{L1}E_{R1} & B_{\ell} \\
E_{L1}U_{R1}\overline{U}_{L1}E_{R2} & C_{\ell} \\
E_{L2}U_{R1}\overline{U}_{L1}E_{R1} & D_{\ell} \\
E_{L2}U_{R1}\overline{U}_{L1}E_{R2} & E_{\ell}
\end{bmatrix}
\] (25)

while the following operators will feed mass down from the $t$.

\[
\begin{bmatrix}
E_{L1}U_{R1}\overline{U}_{L1}E_{R1} & F_{\ell} \\
E_{L2}U_{R1}\overline{U}_{L1}E_{R1} & G_{\ell} \\
E_{L1}U_{R1}\overline{U}_{L1}E_{R2} & H_{\ell} \\
E_{L2}U_{R1}\overline{U}_{L1}E_{R2} & I_{\ell}
\end{bmatrix}
\] (26)

Only the $B_{\ell}$ and $F_{\ell}$ operators preserve both $U(1)_V$ and $U(1)_A$. There are also purely leptonic operators of interest which, unlike all the other operators we have considered in this paper, are generated by the exchange of massive $SU(2)_V$ gauge bosons. We will label two operators of this type, $E_{L1}E_{R1}E_{R2}E_{L2}$ and $E_{L1}E_{R1}E_{R2}E_{L2}$, by $J_{\ell}$ and $K_{\ell}$.

We write the charged-lepton mass matrix as follows, where the large $\tau'$ mass is in the bottom right corner.

\[
\begin{bmatrix}
E_{L2}E_{R2} & E_{L2}E_{R2} & E_{L2}E_{R1} & E_{L3}E_{R1} \\
E_{L2}E_{R2} & E_{L2}E_{R2} & E_{L2}E_{R1} & E_{L3}E_{R1} \\
E_{L1}E_{R2} & E_{L1}E_{R2} & E_{L1}E_{R1} & E_{L1}E_{R1} \\
E_{L1}E_{R2} & E_{L1}E_{R2} & E_{L1}E_{R1} & E_{L1}E_{R1}
\end{bmatrix}
\] (27)

The various operators contribute as follows.

\[
M_{\ell} = 
\begin{bmatrix}
\mathcal{K}_{\ell} & \mathcal{I}_{\ell} & \mathcal{G}_{\ell} & 0 \\
\mathcal{E}_{\ell} & \mathcal{J}_{\ell} & 0 & \mathcal{D}_{\ell} \\
\mathcal{C}_{\ell} & 0 & 0 & \mathcal{B}_{\ell} \\
0 & \mathcal{H}_{\ell} & \mathcal{F}_{\ell} & \mathcal{A}_{\ell}
\end{bmatrix}
\] (28)

We see that the $B_{\ell}$ and $F_{\ell}$ operators are essential for the generation of the $\tau$ mass, and we note that these operators are the analog of the dominant $B$ operator in the quark sector which generated the $t$ mass. The $\mathcal{K}_{\ell}$ operator then feeds the resulting $\tau$ mass down to the electron mass. It seems reasonable for the $\mathcal{J}_{\ell}$ operator to give the $\mu$ mass, since its coefficient would have to be $\approx 1/(100 \text{ TeV})^2$ assuming that $\langle E_{L1}E_{R1} \rangle \approx (1 \text{ TeV})^3$. If in fact the $\mathcal{J}_{\ell}$ and $\mathcal{K}_{\ell}$ are the dominant contributions to the $\mu$ and $e$ masses then we expect that

\[
\frac{m_e}{m_\mu} \approx \frac{m_\tau}{m_{\tau'}}.
\] (29)
The remaining zeros in the charged-lepton mass matrix would be filled in by operators of the suppressed LRRL form.

Remaining to be discussed are the three light left-handed neutrinos, $\nu_e$, $\nu_\mu$, $\nu_\tau$. Their Majorana masses are generated from 6-fermion operators, which leads to a natural suppression of these masses compared to all other masses. Such operators are generated from purely leptonic $SU(2)_V$-invariant 4-fermion operators; for example two $\overline{E}_{L1}E_{R1}N_{L2}N_{R2}$ operators along with the large $N_{R2}$ mass can produce the operator

$$\overline{E}_{L1}E_{R1}\overline{E}_{L1}E_{R1}N_{L2}N_{L2}. \quad (30)$$

This, in the presence of the $\tau'$ mass, produces a small $N_{L2}$ (i.e. $\nu_e$) mass. This neutrino mass is naively of the same order ($\approx 100$ eV) as the CP-violating contributions to the quark masses, although some of the additional sources of suppression mentioned there can also apply here.

We will summarize the possible combinations of 4-fermion operators and right-handed neutrino masses which produce left-handed neutrino masses. We write the left-handed neutrino mass matrix as follows.

$$
\begin{pmatrix}
N_{L2}^2 & N_{L2}N_{L2} & N_{L2}N_{L1} & N_{L2}N_{L1} \\
N_{L2}N_{L2} & N_{L2}^2 & N_{L2}N_{L1} & N_{L2}N_{L1} \\
N_{L2}N_{L1} & N_{L2}N_{L1} & N_{L1}^2 & N_{L1}N_{L1} \\
N_{L2}N_{L1} & N_{L2}N_{L1} & N_{L1}N_{L1} & N_{L1}^2 \\
\end{pmatrix} \quad (31)
$$

The large $\nu_{\tau'}$ mass in the bottom right corner essentially decouples from the rest, and so we will just consider the operators relevant to the remaining $3 \times 3$ matrix.

$$
\begin{pmatrix}
\overline{E}_{L1}E_{R1}N_{L2}N_{R2} & B_\nu \\
\overline{E}_{L1}E_{R1}N_{L2}N_{R2} & C_\nu \\
\overline{E}_{L1}E_{R1}N_{L1}N_{R2} & D_\nu \\
\overline{E}_{L1}E_{R1}N_{L1}N_{R1} & E_\nu \\
\overline{E}_{L1}E_{R1}N_{L2}N_{R1} & F_\nu \\
\overline{E}_{L1}E_{R1}N_{L2}N_{R1} & G_\nu \\
\end{pmatrix} \quad (32)
$$

We label the right-handed neutrino masses as follows.

$$
\begin{pmatrix}
N_{R2}^2 & m_1 \\
N_{R2}^2 & m_2 \\
N_{R2}N_{R2} & m_3 \\
N_{R2}N_{R1} & m_4 \\
\end{pmatrix} \quad (33)
$$

The left-handed masses then arise from the following combinations of operators and
right-handed neutrino masses.

\[
\begin{pmatrix}
\frac{B_\nu^2}{m_1} & B_\nu C_\nu & F_\nu G_\nu & B_\nu D_\nu & \frac{E_\nu G_\nu}{m_4} \\
\frac{B_\nu C_\nu}{m_3} + \frac{F_\nu G_\nu}{m_4} & \frac{C_\nu^2}{m_2} & \frac{C_\nu D_\nu}{m_2} & \frac{D_\nu^2}{m_2} \\
\frac{B_\nu D_\nu}{m_3} + \frac{E_\nu G_\nu}{m_4} & \frac{C_\nu D_\nu}{m_2} & \frac{D_\nu^2}{m_2} & \frac{D_\nu^2}{m_2}
\end{pmatrix}
\]

(34)

This matrix can take a very different form from the quark and charged-lepton mass matrices. For example it would not be unnatural to assume that the masses \(m_1, m_2, m_3, m_4\) are similar, and that \(B_\nu\) and \(C_\nu\) are similar, in which case \(\nu_e\) and \(\nu_\mu\) could have similar mass and enjoy large mixing. \(D_\nu\) could be smaller than \(B_\nu\) and \(C_\nu\), in which case \(\nu_\tau\) could be lighter than \(\nu_e\) and \(\nu_\mu\). We also note that the \(E_\nu\) operator, which contributes to \(\nu_e-\nu_\tau\) mixing, enjoys the most enhancement from \(U(1)_X\) scaling. And finally we note that since the right-handed neutrino masses are involved in generating this matrix, large CP-violating phases can be present.

We now briefly discuss other flavor-changing effects, all of which appear to be at suitably suppressed levels.

- We have mentioned above that CP violation could also show up in lepton-number conserving, \(SU(2)_V\)-violating operators. For example a \(N_R\) to \(\bar{N}_R\) transition inside a loop involving a \(W_R\) could induce a \(\mu-e-\gamma\) coupling, which along with \(\mu-e\) mass mixing could generate electron and muon electric dipole moments. Even ignoring \(\mu-e\) mass mixing suppression, the moments are sufficiently suppressed by the large masses of right-handed neutrinos and \(W_R\). The decay \(\mu \rightarrow e\gamma\), as well as \(\mu \rightarrow 3e\) and \(\mu-e\) conversion from the \(\mu-e-Z\) coupling, are well below current bounds for the same reason.

- \(K-\bar{K}\) mixing could arise from the operator \(\overline{D}_{L2} D_{R2} D_{R2} E_{L2}\) which corresponds to \(\overline{d}_L s_R \bar{d}_R s_L\). \(K \rightarrow e^-\mu^+\) could arise from the operators \(\overline{E}_{L2} D_{R2} D_{R2} E_{L2}\) and \(\overline{E}_{R2} D_{L2} D_{L2} E_{R2}\), which correspond to \(\overline{e}_L s_R \bar{d}_R \mu_L\) and \(\overline{e}_R d_L \bar{s}_L \mu_R\). All these operators break \(U(1)_A\) and are of the suppressed LRRL form. They also receive no enhancement from \(U(1)_X\) scaling.

- The exchange of an \(U(2)_V\) gauge boson produces the \(\overline{s}_L s_R \bar{s}_R s_L\) and \(\overline{\mu}_L s_R \bar{s}_R \mu_L\) operators for example, which can give rise to \(K-\bar{K}\) mixing and \(K \rightarrow e^-\mu^+\) in the presence of appropriate mass mixing in the down-quark and charged-lepton sectors. But we have seen how mass mixings in these sectors are suppressed.
Since there is more mass mixing in the up-quark sector the corresponding effects for $D-\bar{D}$ mixing should be somewhat larger.

- $B_d-\bar{B}_d$ mixing could arise from the operator $\overline{D}_{L1} D_{R2} \overline{D}_{R1} D_{L2}$ which corresponds to $\overline{b}_L d_R \overline{b}_R d_L$. This is again of the suppressed LRRL form, although it would be $U(1)_X$ enhanced. Lastly, $X$ gauge boson exchange can give rise to $B_d-\overline{B}_d$, $B_s-\overline{B}_s$ and $D-\overline{D}$ mixing given the appropriate mass mixings (which are suppressed for the $b$).

In summary we have explored some implications of new flavor interactions at a scale a few orders of magnitude larger than the weak scale. When the broken flavor gauge interaction is strong it can be expected to generate a diverse set of multi-fermion operators in the low energy theory. We have highlighted the role of mixed quark-lepton operators in the generation of quark and lepton masses. A superweak theory of CP violation emerges very naturally, in a manner of some relevance to the strong CP problem. In this picture the smallness of CP violation in the quark sector and the smallness of neutrino masses are related, since they both arise from effective 6-fermion operators.

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