The MONET code for the evaluation of the dose in hadrontherapy

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Abstract. The MONET is a code for the computation of the 3D dose distribution for protons in water. For the lateral profile, MONET is based on the Molière theory of multiple Coulomb scattering. To take into account also the nuclear interactions, we add to this theory a Cauchy-Lorentz function, where the two parameters are obtained by a fit to a FLUKA simulation. We have implemented the Papoulis algorithm for the passage from the projected to a 2D lateral distribution. For the longitudinal profile, we have implemented a new calculation of the energy loss that is in good agreement with simulations. The inclusion of the straggling is based on the convolution of energy loss with a Gaussian function. In order to complete the longitudinal profile, also the nuclear contributions are included using a linear parametrization. The total dose profile is calculated in a 3D mesh by evaluating at each depth the 2D lateral distributions and by scaling them at the value of the energy deposition. We have compared MONET with FLUKA in two cases: a single Gaussian beam and a lateral scan. In both cases, we have obtained a good agreement for different energies of protons in water.

1. Introduction
Hadrontherapy is an oncological technique that uses heavy particles, in particular protons and Carbon ions, for the tumors treatment [1]. The advantage of using heavy particles instead of photons is due to the depth dose profile. In conventional radiotherapy, the photons lose most of their energy in the entrance region. On the contrary, the depth dose profile of heavy particles is relatively small at the beginning and reaches a maximum at the end of their range, followed by a decrease to almost zero values. This feature is known as Bragg Peak [2] and it allows, by setting an appropriate energy, to concentrate the dose to the tumor tissue. Therefore, the accurate evaluation of the dose distribution is important to delimit the prescribed dose to the cancer volume, saving as much as possible the surrounding healthy tissue. In this study, we have evaluated the energy deposited of proton beams in water with the new MONET code comparing the results with the Monte Carlo (MC) simulations.

2. The MONET code
MONET (Model of ion dosE for Therapy) is a fast and accurate code for the evaluation of the 3D dose distribution for protons in water introduced recently by our group [3]. The input variables are the energy of proton and the beam shape: for example in this study we use Gaussian beams of energy 100, 150 and 200 MeV. The first step of the MONET code is the evaluation of lateral distribution taking into account the Multiple Coulomb scattering and
the nuclear interaction. Afterwards, starting from the 1D lateral profile the model reconstructs the 2D radial distribution. The last step is the calculation of energy deposition and the scale of the 2D lateral distribution at this value. In this way, our code is able to evaluate a 3D dose distribution. The free parameters for the nuclear interaction are only four compared with other model [4]: two parameters for the lateral profile (Cauchy-Lorentz function) and two for the linear parametrization in the longitudinal profile.

All results are compared with the FLUKA code, which is currently used in many hadrontherapy centers [5] and has been recently validated with experimental data [6].

2.1. Lateral profile
The first step of the implementation of MONET code is the evaluation of lateral distribution. The shape of the lateral profile comes from the combination of two processes: the multiple Coulomb scattering and nuclear interactions. The MONET code is based on the Molière theory of multiple Coulomb scattering [6, 7]. To take into account also of the nuclear interaction, we have added to Molière distribution the Cauchy-Lorentz function, that describes the nuclear tail of distribution. The total lateral distribution is given by [6]:

\[ f_x(x) = W_p f_M(x) + (1 - W_p) \frac{t(x)}{\int t(u)du}. \]  

(1)

The Cauchy-Lorentz distribution \( t(x) \) has only two free parameters, that are obtained by a fit to FLUKA simulation. The Molière distribution is weighted for the primary particle weight \( W_p \), evaluated using the formula present in the Ulmer paper [8], while the Cauchy-Lorentz distribution is multiplied by \( 1 - W_p \), the nuclear contribution [3].

In Fig. 1 (left), the total lateral distribution is in agreement with FLUKA simulation.

Figure 1. Lateral distributions (left) and radial distributions (right) for 150 MeV protons in water at a depth \( z=11 \) cm.

Now, the next step is the passage from the projected lateral distribution to a 2D radial distribution.

The projected distribution \( f_x \) and \( f_y \) are uncorrelated but not independent [9]. Therefore, in this case is not possible to use the factorization theorem for independent probability distribution. A solution is given by the Papouli theorem [10] that allows, in case of cylindrical symmetry, to
rebuild the radial distribution starting from the projected one. We have implemented the Papoulis algorithm in the code and the comparison with simulation is good (Fig. 1 right).

2.2. Longitudinal profile
The second part of MONET code is the evaluation of energy loss process in the longitudinal way. The longitudinal profile is mainly determined by the stopping process of protons due to inelastic collisions with the atomic electrons. In our model, we use the following expression for the average energy loss:

\[ E_K(z) = -m + \frac{F(z)}{2} + \sqrt{m^2 + \frac{F^2(z)}{4}}, \quad F(z) = p\beta \left(1 - \frac{z}{R}\right)^{k/2}, \quad k = 1.07. \quad (2) \]

Our formula has a similar behaviour to the relation already present in the literature [11, 12]. We have analyzed different therapeutic energies and our formula is in good agreement with FLUKA simulations within 1%.

Due to the statistical nature of the energy loss process, the longitudinal profile is characterized by straggling, that produces a widening of the Bragg peak. A good approximation for the straggling is given by the convolution of the energy loss with a Gaussian function [11, 13]:

\[ \hat{E}_K(z)0 = \int_0^R E_K(\bar{z}) \frac{e^{-\left(z-\bar{z}\right)^2/2\sigma^2}}{\sqrt{2\pi}\sigma} d\bar{z}, \quad \sigma \approx 0.012R^{0.935}. \quad (3) \]

The inclusion of straggling effects in our model is still in agreement with simulations. Also nuclear interactions play an important role by reducing the fluence of the primary protons, in the order about 1% per centimeter in water. In order to complete the longitudinal dose profile, the nuclear contributions are included in the calculation using a linear parametrization where the coefficients are obtained by fitting FLUKA simulations.

\[ f(z) = W_p\hat{E}_K(z) + (1 - W_p)E_N(z) \quad (4) \]

where \( E_N(z) \) is a linear parametrization for the nuclear contribution \( E_N(z) = az + b \), with the coefficients \( a \) and \( b \) are obtained by fitting FLUKA simulations.

The average energy loss is weighted for the primary particle weight [8], while the nuclear part is multiplied for nuclear contribution, using the same approach of the lateral profile. In Fig. 2, the comparison between the evaluation of total energy deposition and the FLUKA simulation shows a good agreement (1%).

In conclusion, the MONET code is able to evaluate the total dose profile in a three dimensional mesh with only four free parameters, by calculating at each depth the longitudinal deposited energy and by distributing it laterally on the transverse plane. Therefore the dose is the product between the longitudinal energy deposition and the radial probability distribution in the transverse plane.

3. Results
To assess the accuracy of our model, we have evaluated the dose in two conditions: a single beam with Gaussian profile and a lateral scan. We have analysed 3 energies by evaluating the dose with the model in a 3D mesh with voxels of 1 mm dimensions. The results are obtained by selecting transverse plane at fixed depths [3].

In Fig. 3, the comparison between FLUKA and MONET profile for proton of 100 MeV at 4 cm water depth is shown. The relative error is evaluated in a central cross section and is about
Figure 2. Bragg curves including straggling effects and nuclear contributions for $E = 100, 150$ and $200$ MeV.

Figure 3. Energy 100 MeV at depth $z=4$ cm.

3%. We also present the quantile-quantile (QQ) plot to compare the quantiles of the model and MC distribution. The QQ plot is a 45° line in case of perfect match and in our case this is well verified.

In Fig.4, we have obtained a similar result for proton of 150 MeV: the agreement is good, the relative error is still 3% and also the QQ plot shows a correspondence between model and MC simulation.
Figure 4. Energy 150 MeV at depth z=11 cm.

We have analyzed all depths for any energy and the maximum relative error in the worst case is lower than 10% in a voxel of 1 mm dimensions. It is important to note that this discrepancy once considered on the absolute energy deposition, is in a range of negligible clinical relevance. In order to estimate the accuracy of the model focusing on the tails of the distribution that give rise to the low-dose envelope, we have reproduced a lateral scan as a sum of many pencil beams. In this case, the code calculates the energy deposition for a single beam. Afterwards the dose profile is translated in order to obtain a lateral scan and the total energy deposition will be given by a sum of all single beams.

We have considered the field size factor for our model and for MC simulations. The field size factor is the ratio between the total dose deposition inside of concentric square fields of increasing size and the same quantity in a square reference

$$D_{ref}, f = 2, 4, 6, 8, 10, 12.$$  (5)

In Fig.5, the FSF variations as a function of square size are reported. Note that the scale in y is much expanded and the agreement between FLUKA and the model is still good. The maximum difference is about 5%, of the same order of accuracy of the single beam tests. These results indicate a good description of the low dose contribution.

Another advantage of the MONET code is the fast calculation time. Indicatively, for each depth the calculation time is $\sim 2$ seconds for the single beam and $\sim 4$ seconds for the full lateral scan. These times are competitive compared to the simulation time ($\sim 1 – 10$ hours).
4. Conclusions
The MONET code accounts for all the physical interaction of protons with water and is based on well known and validated theories. In general, the pencil beam algorithm uses pre-calculated look-up tables for the depth dose evaluation, while the lateral profile is modelled with a Gaussian or double Gaussian distribution. On the contrary, MONET evaluates the lateral and longitudinal terms starting from first principles. The advantages of our model are the small number of parameters, the fast calculation time and the accuracy. A possible development of this study is the creation of a dose database of interest for the clinical use and an online/in-room fast dose evaluation tool.

Currently, we are extending the model to the case of Helium beam. Helium is a possible new particle for hadrontherapy: it is a good compromise between the proton and carbon ions, because it has a smaller lateral profile compared to protons and a smaller contribution of energy deposition behind the Bragg peak compared to Carbon ions.

For the lateral profile, the results of the model for Helium is in good agreement with FLUKA simulation and the accuracy is better than the parameterization just used for proton and carbon ions [14]. The last step for the implementation of MONET for $^4He$ beams, renamed MONETα will be the evaluation of energy deposition in the longitudinal profile.

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