Gauge Fluctuations in Superconducting Films

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In this paper we consider a superconducting film modeled by the Ginzburg-Landau model, confined between two parallel planes a distance $L$ apart from one another. Our approach is based on the Gaussian effective potential in the transverse unitarity gauge, which allows to treat gauge contributions in a compact form. Using techniques from dimensional and zeta-function regularizations, modified by the confinement conditions, we investigate the critical temperature as a function of the film thickness $L$. The contributions from the scalar self-interaction and from the gauge fluctuations are clearly identified. The model suggests the existence of a minimal critical thickness below which superconductivity is suppressed.

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I. INTRODUCTION

It is currently assumed to be a good approximation to neglect magnetic thermal fluctuations in the Ginzburg-Landau (GL) model, to explore general features of superconducting transitions. However, this approximation excludes the study of the so called charged phase transitions. They only can be investigated when fluctuations of the gauge field are taken into account, which make explicitly appear in the thermodynamic quantities the coupling constant for the interaction between the scalar field and the gauge field (the charge of the boson). This is a hard problem to be directly faced and many attempts have been done to go beyond the pioneering work of ref. [1]. In what concerns physical situations already present in the literature, a large amount of work has been done on the Ginzburg-Landau model applied to the study of superconductors, both in the single component and in the $N$-component versions of the model, using the renormalization group approach. The interested reader can find an account on the state of the subject for both type-I and type-II superconductors and related topics in Refs. [2–8].

In this paper, we intend to take into account gauge fluctuations in a study of superconducting films. We consider the Ginzburg-Landau model, the system being submitted to the constraint of confinement between two parallel planes a distance $L$ apart from one another. From a physical point of view, for dimension $d = 3$ and introducing temperature by means of the mass term in the Hamiltonian, this corresponds to a film-like material. We investigate the behaviour of the system taking into account gauge fluctuations, which means that charged transitions are included in our work. We are particularly interested in the problem of how the critical behaviour depends on the film thickness $L$. This study is done by means of the Gaussian Effective Potential (GEP) as developed in Refs. [9–12], together with a spatial compactification mechanism introduced in recent publications [13,14].

Effects of spatial boundaries on the behaviour of physical systems appears in several forms in the literature. For instance, there are systems that present defects, as domain walls created for instance in the process of crystal growth by some prepared circumstances. At the level of effective field theories, in many cases this can be modeled by considering a Dirac fermionic field whose mass changes sign as it crosses the defect, what means that the domain wall can be interpreted as a kind of a critical boundary [15,16]. Questions concerning stability and the existence of phase transitions may also be raised if we enquire about the behaviour of field theories as function of spatial boundaries. The existence of phase transitions would be in this case also associated to some spatial parameters describing the breaking of translational invariance, in our case the distance $L$ between planes confining the system (a superconducting film of thickness $L$). In particular the question of how the superconducting critical temperature could depend on the thickness of the film can be raised.

In the next Section, we apply the functional approach of the Gaussian effective potential formalism to the Ginzburg-Landau model, obtaining the mass, which obeys a generalized ”Gaussian” Dyson-Schwinger equation. In Section III, we extend the formalism of Section II to the GL model, confined between two parallel planes and a study of the critical behaviour of the system is performed. In particular, we obtain an expression for the critical temperature as a function of of the spacing between the planes (the film thickness). Finally, we summarize our results in Section IV and a qualitative comparison with some experimental observations is done.

II. THE GAUSSIAN EFFECTIVE POTENTIAL FOR THE GINZBURG-LANDAU MODEL

We begin by briefly presenting the study of the U(1) Scalar Electrodynamics in the transverse unitarity gauge, along the lines developed in ref. [17]. We start from the Hamiltonian density of the GL model in Euclidean $d$-dimensional space written in the form [18],

$$\mathcal{H}' = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (\partial_{\mu} - i e A_{\mu}) \Psi |^2$$
$$+ \frac{1}{2} m_0^2 |\Psi|^2 + \lambda (|\Psi|^2)^2, \quad (2.1)$$

where $\Psi$ is a complex field, and $m_0$ is the bare mass. The components of the transverse magnetic field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ (\mu, \nu = 1, ..., d)$ are related to the $d$-dimensional potential vector by the well known equation,

$$\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = |\nabla \times A|^2. \quad (2.2)$$

In order to obtain only physical degrees of freedom, we can introduce two real fields instead of the complex field $\Psi$, assuming a transverse unitarity gauge. We can define the field in terms of two real fields, as $\Psi = \phi e^{i\gamma}$, together with the gauge transformation $A \to A - 1/e \nabla \gamma$. The unitarity gauge makes the original transverse field acquire a longitudinal component $A_L$ proportional to $\nabla \gamma$. Then the original functional integration over $\Psi$ and $\Psi^*$ in the generating functional of correlation functions, becomes an integration over $\phi, A_T$ and $A_L$. The longitudinal component of the vector potential can be integrated out, leading to the generating functional (up to constant terms),

$$Z[j] = \int D\phi \ D A_T \exp \left[ - \int d^d x \mathcal{H} + \int d^d x \ j \phi \right], \quad (2.3)$$

where the Hamiltonian is

$$\mathcal{H} = \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m_0^2 \phi^2 + \lambda \phi^4$$

$$+ \frac{1}{2} e^2 \phi^2 A^2 + \frac{1}{2} (\nabla \times A)^2 + \frac{1}{2e} (\nabla \cdot A)^2. \quad (2.4)$$

We have introduced above a gauge fixing term, the limit $\epsilon \to 0$ taken later on after the calculations have been done. In Eq.(2.4) and in what follows, unless explicitly stated, $A$ stands for the transverse gauge field.

The Gaussian effective potential can be obtained from Eq.(2.4), performing a shift in the scalar field in the form $\phi = \phi + \varphi$, which allows to write the Hamiltonian in the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{int}}, \quad (2.5)$$

with $\mathcal{H}_0$ being the free part and $\mathcal{H}_{\text{int}}$ the interaction part, given respectively by

$$\mathcal{H}_0 = \left[ \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m_0^2 \phi^2 \right]$$

$$+ \frac{1}{2} (\nabla \times A)^2 + \frac{1}{2} \Delta^2 A_\mu A^\mu + \frac{1}{2e} (\nabla \cdot A)^2, \quad (2.6)$$

and

$$\mathcal{H}_{\text{int}} = \sum_{n=0}^4 v_n \phi^n + \frac{1}{2} (e^2 \varphi^2 - \Delta^2) A_\mu A^\mu$$

$$+ \frac{1}{2} e^2 \phi A_\mu A^\mu \varphi + \frac{1}{2} e^2 A_\mu A^\mu \varphi^2, \quad (2.7)$$

where

$$v_0 = \frac{1}{2} m_0^2 \varphi^2 + \lambda \varphi^4, \quad (2.8)$$

$$v_1 = m_0^2 \varphi + 4 \lambda \varphi^3, \quad (2.9)$$

$$v_2 = \frac{1}{2} m_0^2 \varphi^2 + 6 \lambda \varphi^2 - \frac{1}{2} \Omega^2, \quad (2.10)$$

$$v_3 = 4 \lambda \varphi, \quad (2.11)$$

$$v_4 = \lambda. \quad (2.12)$$

It is clear from Eqs. (2.5, 2.6) and (2.7), that $\mathcal{H}$ describes two interacting fields, a real scalar field $\phi$ of mass $\Omega$ and a real vector gauge field $A$ of mass $\Delta$.

The effective potential, which is defined by

$$V_{\text{eff}}[\varphi] = \frac{1}{V} \left[ -\ln Z + \int d^d x j \varphi \right], \quad (2.13)$$

where $V$ is the total volume, can be obtained at first order from standard methods from perturbation theory. One can find, from Eqs. (2.3),(2.6) and (2.7),

$$V_{\text{eff}}[\varphi] = I^d_0(\Omega) + 2I^d_1(\Delta) + \frac{1}{2} m_0^2 \varphi^2 + \lambda \varphi^4$$

$$+ \frac{1}{2} \left[ m_0^2 - \Omega^2 + 12 \lambda \varphi^2 + 6 \lambda I^d_0(\Omega) \right] I^d_0(\Omega)$$

$$+ \left[ e^2 (\varphi^2 + I^d_0(\Omega)) - \Delta^2 \right] I^d_0(\Delta), \quad (2.14)$$

where,

$$I^d_0(M) = \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 + M^2}, \quad (2.15)$$

$$I^d_1(M) = \frac{1}{2} \int \frac{d^d k}{(2\pi)^d} \ln(k^2 + M^2), \quad (2.16)$$

with $k = (k_1, ..., k_d)$ being the $d$-dimensional momentum.

The Gaussian effective potential is derived by the requirement that $V_{\text{eff}}[\varphi]$ must be stationary under variations of the masses $\Delta$ and $\Omega$. This means that values of $\overline{\Omega}$ and $\overline{\Delta}$ for the masses $\Omega$ and $\Delta$ should be found such that the conditions,

$$\frac{\partial V_{\text{eff}}}{\partial \Omega} \bigg|_{\Omega^2 = \Omega^2} = 0, \quad (2.17)$$

$$\frac{\partial V_{\text{eff}}}{\partial \Delta} \bigg|_{\Delta^2 = \Delta^2} = 0, \quad (2.18)$$

be simultaneously satisfied. These conditions generate the gap equations,

$$\overline{\Omega} = m_0^2 + 12 \lambda \varphi^2 + 12 \lambda I^d_0(\overline{\Omega}) + 2e^2 I^d_0(\overline{\Delta}), \quad (2.19)$$

$$\overline{\Delta} = e^2 \varphi^2 + e^2 I^d_0(\overline{\Omega}). \quad (2.20)$$

Replacing $\Omega$ and $\Delta$ in Eq.(2.14) by the solutions $\overline{\Omega}$ and $\overline{\Delta}$, of Eqs.(2.19) and (2.20) we obtain for the GEP the formal expression,
\[
\n\nabla_{\text{eff}}[\varphi] = I_{1}^{d}(\Omega) + 2I_{1}^{d}(\Xi) + \frac{1}{2}m_{0}^{2}\varphi^{2} + \lambda\varphi^{4} - 3\lambda[I_{0}^{d}(\Omega)]^{2} - e^{2}I_{0}^{d}(\Omega)I_{1}^{d}(\Xi).
\]

Notice that Eqs.(2.19) and (2.20) are a pair of very involved coupled equations, and no analytical solution for them has been found, they can be solved only by numerical methods. Later on we will see that this difficulty, in the limit of criticality can be surmounted.

Next we intend to write an expression for the Gaussian mass, \( m_{\varphi} \), obtained in our case from the standard prescription, as the second derivative of \( \nabla_{\text{eff}} \) with respect to \( \varphi \), we remark from Eqs.(2.19) and (2.20) that \( \Omega^{2} \) and \( \Xi^{2} \) also depend on \( \varphi \) according to the relations

\[
\frac{d\Omega^{2}}{d\varphi} = \frac{24\lambda\varphi - 6\varphi I_{1}^{d}(\Xi)}{1 + 6\lambda I_{1}^{d}(\Omega)}, \quad \frac{d\Xi^{2}}{d\varphi} = 2e\varphi - \frac{1}{2}e^{2}I_{1}^{d}(\Xi),
\]

where

\[
I_{1}^{d}(M) = 2\int \frac{d^{d}k}{(2\pi)^{d}} \frac{1}{(k^{2} + M^{2})^{2}}.
\]

Replacing Eq.(2.23) in (2.22) we get,

\[
\frac{d\Omega^{2}}{d\varphi} = \frac{24\lambda - 6\varphi^{2} I_{1}^{d}(\Xi)}{1 + 6\lambda I_{1}^{d}(\Omega)} I_{1}^{d}(\Omega),
\]

and the second derivative of the GEP with respect to \( \varphi \) is given by,

\[
\frac{d^{2}\nabla_{\text{eff}}}{d\varphi^{2}} = m_{0}^{2} + 12\lambda\varphi^{2}
+ 12\lambda I_{0}^{d}(\Omega) + 2e^{2}I_{0}^{d}(\Xi) + 2e^{4}\varphi^{2}I_{1}^{d}(\Xi)
- \frac{[6\lambda + \frac{1}{2}e^{4}I_{1}^{d}(\Xi)][24\lambda - 6\varphi I_{1}^{d}(\Xi)]}{1 + [6\lambda - \frac{1}{2}e^{4}I_{1}^{d}(\Omega)] I_{1}^{d}(\Omega)} \varphi^{2}.
\]

Thus we have the formula for the Gaussian mass,

\[
\bar{m}^{2} = \left. \frac{d^{2}V_{\text{eff}}}{d\varphi^{2}} \right|_{\varphi = 0} = m_{0}^{2} + 12\lambda I_{0}^{d}(\Omega) + 2e^{2}I_{0}^{d}(\Xi),
\]

where \( \Omega_{0} \) and \( \Xi_{0} \) are respectively solutions for \( \Omega \) and \( \Xi \) at \( \varphi = 0 \), explicitly,

\[
\Omega_{0}^{2} = m_{0}^{2} + 12\lambda I_{0}^{d}(\Omega), \quad \Xi_{0}^{2} = e^{2}I_{0}^{d}(\Xi).
\]

Therefore, from Eqs.(2.27) and (2.28) we get simply,

\[
\bar{m}^{2} = \Omega_{0}^{2}.
\]

Hence, we see from the gap equation (2.27) that the Gaussian mass obeys a generalized "Gaussian" Dyson-Schwinger equation,

\[
\bar{m}^{2} = m_{0}^{2} + 12\lambda I_{0}^{d}(\Omega) + 2e^{2}I_{0}^{d}(\Xi) \left( \sqrt{e^{2}I_{0}^{d}(\Xi)} \right).
\]

This expression will be used later to describe the system in the neighbourhood of criticality.

III. CRITICAL BEHAVIOUR OF THE CONFINED GINZBURG-LANDAU MODEL

A. The effect of confinement

Let us now consider the system confined between two parallel planes, normal to the \( x_{d} \)-axis, a distance \( L \) apart from one another and use Cartesian coordinates \( r = (x_{d}, z) \), where \( z \) is a \( (d-1) \)-dimensional vector, with corresponding momenta \( k = (k_{d}, \mathbf{q}) \), \( \mathbf{q} \) being a \( (d-1) \)-dimensional vector in momenta space. In this case, the model is supposed to describe a superconducting material in the form of a film. Under these conditions the field \( \phi(x_{d}, z) \) satisfies the condition of confinement along the \( x_{d} \)-axis, \( \phi(x_{d} = 0, z) = \phi(x_{d} = L, z) = 0 \), and should have a mixed series-integral Fourier expansion of the form,

\[
\phi(x_{d}, z) = \sum_{n = -\infty}^{\infty} c_{n} \int d^{d-1}q \ b(q) e^{-i\omega_{n}x_{d} - i\mathbf{q} \cdot \mathbf{z}} \varphi(\omega_{n}, \mathbf{q}),
\]

(3.1)

where \( \omega_{n} = 2\pi n/L \) and the coefficients \( c_{n} \) and \( b(q) \) correspond respectively to the Fourier series representation over \( x_{d} \) and to the Fourier integral representation over the \( (d-1) \)-dimensional \( z \)-space. The above conditions of confinement of the \( x_{d} \)-dependence of the field to a segment of length \( L \), allow us to proceed with respect to the \( x_{d} \)-coordinate, in a manner analogous as it is done in the imaginary-time Matsubara formalism in field theory. The Feynman rules should be modified following the prescription,

\[
\int \frac{dk_{d}}{2\pi} \rightarrow \frac{1}{L} \sum_{n = -\infty}^{+\infty}, \quad k_{d} \rightarrow \frac{2n\pi}{L} \equiv \omega_{n}.
\]

(3.2)

We emphasize however, that here we are considering an Euclidean field theory in \( d \) purely spatial dimensions, we are not working in the framework of finite temperature field theory. Temperature is introduced in the mass term of the Hamiltonian by means of the usual Ginzburg-Landau prescription.

For our purposes we only need the calculation of the integral given in Eq.(2.15) in the situation of confinement of the present section. With the prescription (3.2),
the equation corresponding to Eq.(2.15) for the confined system can be written in the form,

\[ I_0^d(M) = \frac{1}{4\pi^2 L} \sum_{n=-\infty}^{+\infty} \int \frac{d^{d-1}q}{q^2 + a n^2 + c^2}, \tag{3.3} \]

where \( q_i = k_i/2\pi, a = 1/L^2 \) and \( c^2 = M^2/4\pi^2 \).

Eq.(3.3) can be treated within the framework of the formalism developed in Refs. [13], [14]. Using a well known regularization formula [19], we can write Eq.(3.3) in the form

\[ I_0^d(M) = \frac{\sqrt{a}}{4\pi^2 d/2} \Gamma \left( 2 - \frac{d}{2} \right) A_1^2 \left( \frac{3 - d}{2}, a \right), \tag{3.4} \]

where \( A_1^2 \left( \frac{3-d}{2}, a \right) \) is one of the Epstein-Hurwitz zeta-functions, defined by [20]

\[ A_K^\nu(\nu; a_i) = \sum_{n_1, \ldots, n_K = -\infty}^{+\infty} (a_1 n_1^2 + \ldots + a_K n_K^2 + c^2)^{-\nu}, \tag{3.5} \]

with \( \text{Re}(\nu) > K/2 \) (in our case \( \text{Re}(d) < 2 \)).

The Epstein-Hurwitz zeta-function can be extended as a meromorphic function to the whole complex \( \nu \)-plane (for us, to all values of the dimension \( d \)), and we obtain after some rather long but straightforward manipulations described in detail in [13], the expression,

\[ I_0^d(M) = 2^{-\frac{d}{2}} \pi^{1-\frac{d}{2}} \left[ 2^{1-\frac{d}{2}} \Gamma \left( 1 - \frac{d}{2} \right) M^{-2+d} \right] + 2 \sum_{n=1}^{\infty} \left( \frac{M}{n L} \right)^{-1+\frac{d}{2}} K_{-1+\frac{d}{2}}(M Ln), \tag{3.6} \]

where \( K_{\nu} \) are the Bessel functions of third kind.

### B. Critical behaviour

Let us take \( M = \overline{m} \) in Eq.(3.6) and let us restrict ourselves to the neighbourhood of criticality, that is, to the region defined by \( \overline{m} \approx 0 \). Then the asymptotic formula,

\[ K_{\nu}(z) \approx \frac{1}{2} \Gamma(\nu) \left( \frac{z}{2} \right)^{-\nu}, \quad (z \sim 0) \tag{3.7} \]

allows to write Eq.(3.6) in the form

\[ I_0^d(\overline{m} \approx 0) \approx \frac{\pi^{1-\frac{d}{2}}}{2} \Gamma \left( 1 - \frac{d}{2} \right) \frac{1}{L^{d-2}} \zeta(d-2), \tag{3.8} \]

where \( \zeta(d-2) \) is the Riemann zeta-function, \( \zeta(d-2) = \sum_{n=1}^{\infty} (1/n^{d-2}) \), defined for \( d > 3 \). For \( d \geq 3 \), in the sense of the analytic continuation in dimension of the Epstein-Hurwitz zeta-functions, we obtain the expression,

\[ I_0^d(\overline{m} \approx 0) \approx \frac{1}{2\sqrt{\pi}} \frac{1}{L} \zeta(d-2). \tag{3.9} \]

The integral \( I_0^d(\overline{m} = \sqrt{c^2 I_0^d(\overline{m})}) \), which enters Eq.(2.31), must be considered carefully. For a dimension \( d \geq 3 \), we get,

\[ I_0^d(\overline{m}) = 2^{-\frac{d}{2}} \pi^{1-\frac{d}{2}} \left[ 2^{1-\frac{d}{2}} \Gamma \left( 1 - \frac{d}{2} \right) \sum_{n=1}^{\infty} \left( \frac{\overline{m}}{n L} \right)^{\frac{d}{2}} K_{\frac{d}{2}}(\overline{m} Ln) \right], \tag{3.10} \]

or, using the exact expression for the summation in the above equation,

\[ \sum_{n=1}^{\infty} \left( \frac{\overline{m}}{n L} \right)^{\frac{d}{2}} K_{\frac{d}{2}}(\overline{m} Ln) = -\sqrt{\frac{\pi}{2}} \frac{1}{L} \ln \left( 1 - e^{-\overline{m} L} \right), \tag{3.11} \]

Eq.(3.10) becomes (with \( \overline{m} = \sqrt{c^2 I_0^d(\overline{m})} \))

\[ I_0^d \left( \sqrt{c^2 I_0^d(\overline{m})} \right) \approx \frac{1}{2\sqrt{\pi}} \left[ \frac{1}{2} \Gamma \left( 1 - \frac{d}{2} \right) \sqrt{c^2 I_0^d(\overline{m})} \right] - \sqrt{2\pi} \frac{1}{L} \ln \left( 1 - e^{-\sqrt{c^2 I_0^d(\overline{m}) L}} \right). \tag{3.12} \]

However, notice that if we are in the limit \( \overline{m} \approx 0 \), we see replacing \( I_0^d(\overline{m}) \) from Eq.(3.9) in the exponential, that the logarithm in the last term of Eq.(3.12) will disappear at \( d = 3 \), due to the divergence of \( \zeta(d-2) \) as \( d \to 3 \). Hence, Eq.(3.12) becomes simply, for a dimension \( d \geq 3 \),

\[ I_0^d \left( \sqrt{c^2 I_0^d(\overline{m} \approx 0)} \right) \approx \frac{e}{2\pi^{1/2} \sqrt{2}} \frac{1}{L^{d/2}} \zeta(d-2). \tag{3.13} \]

Thus we can write the Gaussian gap equation (2.31) in the neighbourhood of criticality in the form,

\[ \overline{m}^2 \approx m_0^2 + \frac{24}{\sqrt{\pi}} \lambda L \zeta(d-2) - \frac{1}{\pi^{1/2} \sqrt{2} \frac{1}{L^{d/2}}} \zeta(d-2). \tag{3.14} \]

For \( \overline{m} = 0 \), Eq.(3.14) defines a critical equation for \( d \geq 3 \). But it is well known that the only singularity of the zeta-function \( \zeta(z) \) is a pole at \( z = 1 \), which would make Eq.(3.14) meaningless as it stands for \( d = 3 \), just the physically interesting situation.

However, we can give a physical sense to Eq.(3.14) for \( d = 3 \), by means of a regularization procedure. This can be done using the formula,

\[ \lim_{z \to 1} \left[ \zeta(z) - \frac{1}{1 - z} \right] = \gamma, \tag{3.15} \]
where $\gamma$ is the Euler constant, to define for $d \geq 3$ a new, dressed mass $m$, related to the former bare mass by,

$$m^2 = m_0^2 - \frac{24\lambda}{\sqrt{\pi}L(d-3)} + \frac{e^3}{2\pi^{1/4}\sqrt{2L}} \sum_{p=1}^{\infty} C_p^{d/2} \gamma^{d/2-p} \frac{(-1)^p}{(d-3)^p}. \quad (3.16)$$

where $C_p^{d/2}$ are appropriate generalizations of the coefficients of the binomial expansion for a fractional power.

Then replacing the above equation in Eq.(3.14) and using the binomial formula to expand $\alpha L^{\gamma}$ we have

$$T_c = T_0 - \frac{24\beta \gamma}{\alpha \sqrt{\pi}L} + \frac{e^3 \sqrt{\gamma}}{\alpha^{1/4} \sqrt{2L}} \approx \frac{24\beta \gamma}{\alpha \sqrt{\pi}L} + \frac{e^3 \sqrt{\gamma}}{\alpha^{1/4} \sqrt{2L}}. \quad (3.18)$$

where, in agreement with the standard notation, we have introduced $\beta = \lambda$, the Ginzburg-Landau parameter. This is an equation that describes the behaviour of the critical temperature of a superconducting film of thickness $L$, taking into account the gauge fluctuations. Of course when $L \to \infty$, we recover the case of the material in bulk form.

We see clearly two separated contributions in the expression to the critical temperature given in Eq.(3.18). The first one due to the self-interaction of the scalar field, and the other coming from the interaction between the scalar and gauge fields. This last one would characterize a charged phase transition. It should be noticed that the self interaction contribution to the critical temperature depends on the inverse of the film thickness, while the charged contribution goes with the inverse of the square root of $L$. Taking $T_c = 0$ in Eq.(3.18), we obtain a positive solution for $L$,

$$L(0) = \left( \frac{48\beta \gamma}{\alpha} \right) \left[ \frac{\sqrt{\gamma}}{2\pi^{1/4}} e^3 + \left( \frac{\gamma}{\alpha(2)\sqrt{\pi} e^6} \right)^{-2} \right]. \quad (3.19)$$

For $L < L(0)$, the critical temperature (in absolute units) becomes negative, meaning that $L(0)$ is the minimal physically allowed film thickness, below which the superconducting transition is suppressed.

IV. CONCLUSIONS

In this paper we have considered the Ginzburg-Landau model, confined between two parallel planes, and in the transverse unitarity gauge, as a model to describe a superconducting film. To generate the contributions from gauge fluctuations, we have used the Gaussian effective potential, which allows to obtain a gap equation that can be treated with the method of recent developments [13,14]. We have derived a critical equation that describes the changes in the critical temperature due to confinement. Independent contributions from the self interaction of the scalar field and from the gauge field fluctuations are found. Our approach suggests a minimal film thickness for the existence of both charged and non-charged superconducting transitions. The behaviour described in Eqs.(3.18) and (3.19) may be contrasted with the linear decreasing of $T_c$ with the inverse of the film thickness, that has been found experimentally in materials containing transition metals, for example, in Nb [21], in W-Re alloys [22] and in epitaxial MgB$_2$ films [23]; for some of these cases, this behaviour has been explained in terms of proximity, localization and Coulomb-interaction effects. With our formalism such a linear decreasing of $T_c$ with the inverse film thickness, can be directly obtained from our Eq.(3.18), simply taking $e = 0$ (this could mean that the transitions observed in the above quoted references are non-charged). Also a comparison may be done with recent theoretical results for type II superconductors [24], where a similar behaviour of the critical temperature with the film thickness has been found for non-charged transitions. Moreover we would like to emphasize, that our results do not depend on microscopic details of the material involved nor account for the influence of manufacturing aspects, like the kind of substrate on which the film is deposited. In other words, our results emerge solely as a property that appears in the context of the Gaussian effective potential formalism and as a topological effect of the compactification of the Ginzburg-Landau model in one direction. Finally we would like to remark that our approach includes both charged and non-charged transitions. Our results are in qualitative agreement with the experimentally observed behaviours mentioned above, under the assumption that they correspond to non-charged phase transitions.

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[1] L. Halperin, T. C. Lubensky, S-K. Ma, Phys. Rev. Lett. 32 (1974) 292.
[2] I.D. Lawrie, Phys. Rev. B 50 (1994) 9456.
[3] I.D. Lawrie, Phys. Rev. Lett. 79 (1997) 131.
[4] I. Afleck, E. Brézin, Nucl. Phys. 257 (1985) 451.
[5] E. Brézin, D.R. Nelson, A. Thiaville, Phys. Rev. B 31 (1985) 7124.
[6] L. Radzihovsky, Phys. Rev. Lett. 74 (1995) 4722.
[7] M.A. Moore, T.J. Newman, A.J. Bray, S.-K. Chin, Phys. Rev. B 58 (1998) 936.
[8] C. de Calan, A.P.C. Malbouisson, F.S. Nogueira, Phys. Rev. B 64 (2001) 212502.
[9] P. M. Stevenson, Phys. Rev. D 30 (1984) 1712; P. M. Stevenson, Phys. Rev. D 32 (1985) 1389.
[10] P. M. Stevenson, I. Roditi, Phys. Rev. D 33 (1986) 2305.
[11] I. Roditi, Phys. Lett. 169B (1986) 264; I. Roditi, Phys. Lett. 177B (1986) 85;
[12] A. Okopińska, Phys. Rev. D 35 (1987) 1835; I. Stancu, P. M. Stevenson, Phys. Rev. D 42 (1990) 2710. R. Ibañez-Meier, I. Stancu, P. M. Stevenson, Z. Phys. C 70 (1996) 307.
[13] A. P. C. Malbouisson, J. M. C. Malbouisson, A. E. Santana, Nucl. Phys. B 631, 83 (2002).
[14] A. P. C. Malbouisson, J. M. C. Malbouisson, J. Phys. A: Math. Gen. 35, 2263 (2002).
[15] L. Da Rold, C.D. Fosco, A.P.C. Malbouisson, Nucl. Phys. B 624 (2002) 485.
[16] C.D. Fosco, A. Lopez, Nucl. Phys. B 538 (1999) 685.
[17] M. Camarda, G. G. N. Angilella, R. Pucci and F. Siringo, cond-mat/0211523.
[18] H. Kleinert, Gauge Fields in Condensed Matter, Vol. 1: Superflow and Vortex Lines, World Scientific, Singapore, 1989.
[19] Pierre Ramond, Field Theory - a modern primer", Benjamin Cummings (1981), Appendix B
[20] E. Elizalde, E. Romeo, J. Math. Phys. 30, 1133, (1989); E. Elizalde, "Ten physical applications of spectral zeta functions", Lecture Notes in Physics, Springer-Verlag, Berlin, (1995)
[21] M. S. M. Minhaj, S. Meepagala, J. T. Chen, and L. E. Wenger, Phys. Rev. B 49, 15235 (1994).
[22] H. Raffy, R. B. Laibowitz, P. Chaudhari, and S. Maekawa, Phys. Rev. B 28, R6007 (1983).
[23] A. V. Pogrebnjakov, J. M. Redwing, J. E. Jones, X. X. Xi, S. Y. Xu, Q. Li, V. Vaithyanathan, D. G. Schlom, cond-mat/0304164.
[24] L.M. Abreu, A.P.C. Malbouisson, J.M.C. Malbouisson, A.E. Santana, cond-mat/0304664, to appear in Phys. Rev. B (2003)