Abstract

Agreement plays a central role in distributed systems working on a common task. The increasing size of modern distributed systems makes them more susceptible to single component failures. Fault-tolerant distributed agreement protocols rely for the most part on leader-based atomic broadcast algorithms, such as Paxos. Such protocols are mostly used for data replication, which requires only a small number of servers to reach agreement. Yet, their centralized nature makes them ill-suited for distributed agreement at large scales. The recently introduced atomic broadcast algorithm ALLCUR enables high throughput for distributed agreement while being completely decentralized. In this paper, we extend the work on ALLCUR in two ways. First, we provide a formal specification of ALLCUR that enables a better understanding of the algorithm. Second, we formally prove ALLCUR’s safety property on the basis of this specification. Therefore, our work not only ensures operators safe usage of ALLCUR, but also facilitates the further improvement of distributed agreement protocols based on ALLCUR.

Keywords
Distributed Agreement; Atomic Broadcast; Design Specification; Safety Proof; TLA+

I. INTRODUCTION
Distributed systems working on a common task often have a shared state. Typically, the ordering of updates to this shared state is relevant, i.e. only identical ordering guarantees identical resulting states. Thus, to ensure identical distributed states, distributed agreement is required. Both for systems handling critical data and very large systems, the ability to sustain failures is crucial. For the former, inconsistencies are unacceptable, for the latter, failures become so common, that the lack of fault tolerance becomes a performance issue. Atomic broadcast algorithms enable fault-tolerant distributed agreement.

In this paper we formally specify ALLCUR [25], a protocol that provides distributed agreement through a leaderless concurrent atomic broadcast algorithm under the assumption of partial synchrony (II-B). ALLCUR enables decentralized distributed agreement among a group of servers that communicate over an overlay network described by a sparse digraph. Moreover, it requires subquadratic work per server for every agreement instance and it significantly reduces the expected agreement time by employing an early termination mechanism. ALLCUR’s specification is based on the original description of the protocol (III). In addition, we provide a mechanically verifiable proof of ALLCUR’s safety (IV); the proof follows the steps of the informal proof described in the original paper [25].

A. Related work and motivation
Atomic broadcast plays a central role in fault-tolerant distributed systems; for instance, it enables the implementation of both state machine replication [27], [13] and distributed agreement [25], [29]. As a result, the atomic broadcast problem sparked numerous proposals for algorithms [8]. Many of the proposed algorithms rely on a distinguished server (i.e., a leader) to provide total order; yet, the leader may become a bottleneck, especially at large scale. As an alternative, total order can be achieved by destinations agreement [8], [5], [25]. On the one hand, destinations agreement enables decentralized distributed agreement; on the other hand, it entails agreement on the set of delivered messages and, thus, it requires consensus.

Most consensus algorithms and implementations are leader-based [14], [15], [17], [22], [3], [24], [4], [6]; thus, one server is on the critical path for all communication. Several attempts were made to increase performance by adopting a multi-leader approach [21], [18], [19]. Still, such approaches assume the overlay network is described by a complete digraph, i.e., each server can send messages to any other server. In general, leader-based consensus algorithms are intended for data replication, where the number of replicas (i.e., servers) are bounded by the required level of data reliability [24]. Distributed agreement has no such bound—the number of agreeing servers is an input parameter and it can be in the range of thousands or more. Thus, solutions that rely (for communication) on a complete digraph are not suitable for distributed agreement.

ALLCUR provides leaderless consensus and, thus, leaderless atomic broadcast, by using (as overlay network) any digraph with a vertex-connectivity exceeding the maximum number of tolerated failures. The original paper provides both a detailed

1We use ALLCUR to refer to both the distributed agreement protocol and the atomic broadcast algorithm.
A. The atomic broadcast problem

To formally specify and prove that \( A \) is a distributed agreement protocol that relies on a leaderless concurrent atomic broadcast algorithm (III); a formal proof of \( A \)'s safety property (IV).

II. OVERVIEW

This section describes the system model we consider for solving the atomic broadcast problem (II-A) and it provides an overview of \( A \) and its early termination mechanism (II-B).

A. The atomic broadcast problem

We consider \( n \) servers that are subject to a maximum of \( f \) fail-stop failures. The servers communicate via messages according to an overlay network, described by a digraph \( G \)—server \( p \) can send a message to another server \( q \) if \( G \) has an edge \((p, q)\). The communication is reliable, i.e., messages cannot be lost, only delayed; also, message order is preserved by both edges and nodes. We use the terms node and server interchangeably.

To formally define atomic broadcast, we use the notations from Chandra and Toueg [5]: \( m \) is a message (that is uniquely identified); \( A\text{-broadcast}(m) \) and \( A\text{-deliver}(m) \) are communication primitives for broadcasting and delivering messages atomically; and \( \text{sender}(m) \) denotes the server that \( A \)-broadcasts \( m \). Then, any (non-uniform) atomic broadcast algorithm must satisfy four properties [5], [11]:

- (Validity) If a non-faulty server \( A \)-broadcasts \( m \), then it eventually \( A \)-delivers \( m \).
- (Agreement) If a non-faulty server \( A \)-delivers \( m \), then all non-faulty servers eventually \( A \)-deliver \( m \).
- (Integrity) For any message \( m \), every non-faulty server \( A \)-delivers \( m \) at most once, and only if \( m \) was previously \( A \)-broadcast by \( \text{sender}(m) \).
- (Total order) If two non-faulty servers \( p \) and \( q \) \( A \)-deliver messages \( m_1 \) and \( m_2 \), then \( p \) \( A \)-delivers \( m_1 \) before \( m_2 \), if and only if \( q \) \( A \)-delivers \( m_1 \) before \( m_2 \).

Integrity and total order are safety properties; validity and agreement are liveness property (III-D). If validity, agreement and integrity hold, the broadcast is reliable [11], [5]. We consider reliable broadcast algorithms that use (as overlay networks) digraphs with the vertex-connectivity exceeding \( f \); thus, despite any \( f \) failures, the servers remain connected. Moreover, we consider atomic broadcast algorithms that provide total order through destinations agreement [8]—all non-faulty servers reach consensus on the set of messages to be \( A \)-delivered in a deterministic order.

In a synchronous round-based model [2, Chapter 2], consensus requires (in the worst case) at least \( f + 1 \) rounds [1]. Clearly, if \( G \) is used as overlay network, consensus requires (in the worst case) \( f + D_f(G, f) \) rounds, where \( D_f(G, f) \) denotes \( G \)'s fault diameter—\( G \)'s diameter after removing any \( f \) nodes [12]. Yet, always assuming the worst case is inefficient: It is very unlikely for the number of rounds to exceed \( D_f(G, f) \), if the mean time between failures is long compared to the length of rounds [25].

B. ALLCONCUR

We consider ALLCONCUR [25]—a leaderless concurrent atomic broadcast algorithm that adopts a novel early termination mechanism to avoid assuming always the worst case. ALLCONCUR is round-based: In every round, every server \( A \)-broadcasts a (possibly empty) message and then \( A \)-delivers all known messages in a deterministic order. Since a synchronous model is impractical, while an asynchronous model makes solving consensus impossible [9], ALLCONCUR assumes the following model of partial synchrony—message delays can be approximated by a known distribution [25]. Under this assumption, a heartbeat-based failure detector (FD) [5]—every server sends heartbeats to its successors and, if it fails, the successors detects the lack of heartbeats—can be treated (with a certain probability) as a perfect FD. Specifically, completeness (i.e., all failures are eventually detected) is deterministically guaranteed, while accuracy (i.e., no server is falsely suspected to have failed) is probabilistically guaranteed [25].

ALLCONCUR's early termination mechanism uses failure notifications to track \( A \)-broadcast messages. Every server maintains a tracking digraph for every \( A \)-broadcast message. The nodes of a tracking digraph are the servers suspected to have the \( A \)-broadcast message, while the edges indicate the suspicion of how the message was transmitted. To stop tracking an \( A \)-broadcast message, a server must either receive the message or suspect only failed servers to have the message. A server can safely \( A \)-deliver its known messages once it stops tracking all \( A \)-broadcast messages (IV).

The early termination mechanism relies on the following proposition:
Fig. 1: (a) Overlay network connecting nine servers. (b) Server $p_0$ tracking $p_0$’s message $m_0$ within a $G_S(9, 3)$ digraph. Dotted red nodes indicate failed servers. Dashed gray nodes indicate servers from which $p_0$ received failures notifications (i.e., dashed red edges). Solid green nodes indicate servers suspected to have $m_0$.

Fig. 2: ALLCONCUR system design from the perspective of a server $p$. Boxes depict the state (i.e., the variables), while arrows indicate the flow of information.

**Proposition 1.** If a non-faulty server receives an A-broadcast message and, subsequently, reliably broadcasts a failure notification, any other non-faulty server receives the A-broadcast message prior to the failure notification.

The reason Proposition 1 holds is twofold: (1) a non-faulty server sends further (to its successors) any message it receives for the first time; and (2) message order is preserved by both edges and nodes.

To illustrate the early termination mechanism we consider an example similar to the one in the ALLCONCUR paper [25]: $n = 9$ servers ($p_0, \ldots, p_8$) connected through a $G_S(n, d)$ digraph [28] with $d = 3$ the digraph’s degree (see Figure 1a). The digraph is regular and optimally connected [20], i.e., the vertex-connectivity equals the degree; hence, $f = 2$. Figure 1b shows the changes to the tracking digraph used by $p_0$ to track $p_0$’s message $m_0$ under the following failure scenario: $p_0$ fails after sending $m_0$ only to $p_5$, which receives $m_0$, but fails without sending it further. When $p_0$ receives the first notification of $p_0$’s failure, for example originating from $p_4$, it adds to the digraph all of $p_0$’s successors except for $p_4$, the sender of the notification—$p_6$ suspects that, before $p_0$ failed, it sent $m_0$ to all its successors, except $p_4$, which could not have received $m_0$ from $p_0$. Had $p_4$ received $m_0$ from $p_0$, then $m_0$ would have arrived to $p_0$ before the failure notification (cf. Proposition 1). Also, $p_0$ tracks $m_0$ until it only suspects failed servers to have it; only then is $p_0$ sure no non-faulty server has $m_0$ and stops tracking it.

III. ALLCONCUR: DESIGN SPECIFICATION

We use TLA+ [16] to provide a formal design specification of ALLCONCUR [23]. In addition to the number of servers $n$ and the fault tolerance $f$ (II-A), we define $S$ to be the set of servers and $E$ the set of directed edges describing the overlay network; clearly, $E \subseteq S \times S$. To define the digraph $G$ (i.e., the overlay network), we use the Graphs module [16]: $G$ is defined as a record whose node field is $S$ and edge field is $E$. We assume that $G$’s vertex-connectivity is larger than $f$.

We split the design of ALLCONCUR into three modules: (1) an atomic broadcast (AB) module; (2) a networking (NET) module; and (3) a failure detector (FD) module. Figure 2 illustrates the three modules together with the variables describing ALLCONCUR’s state; moreover, the arrows indicate the flow of information, e.g., receiving a failure notification updates the set $F[p]$, which leads to the update of the tracking digraphs in $g[p]$ (III-A). The AB module is the core of ALLCONCUR’s design (III-A). It exposes two interfaces at every server $p \in S$: the input interface $\text{Abcast}(p)$, to A-broadcast a message; and the output interface $\text{Adeliver}(p)$, to A-deliver all known A-broadcast messages. The AB module relies on the other two modules for interactions between servers: the NET module (III-B) provides an interface for asynchronous message-based communication; and the FD module (III-C) provides information about faulty servers [5].
TABLE I: The effect of the next-state relations on ALLCONCUR’s state. The brackets [] indicate what elements of the variables are modified; [*] indicates that the entire variable is modified. Also, \( p^+ (G) \) denotes the set of successors of \( p \) in \( G \); m.o denotes the server that first sent message \( m \); and \( m.t \) denotes the server targeted by a failure notification \( m \), i.e., the failed server.

| \( \text{Abcast}(p) \) | \( M \) | \( g \) | \( F \) | \( nf \) | \( ab \) | \( done \) | \( recvBuf \) | \( recvMsg \) | \( FD \) |
|--------------------------|-------|--------|------|--------|--------|--------|------------|------------|--------|
| \( \text{Adeliver}(p) \) | \( p \) | \( p[p] \) | - | - | - | - | - | - | - |
| \( \text{RecvBCAST}(p,m) \) | \( p \) | \( p[m.o] \) | - | - | - | - | - | - | - |
| \( \text{RecvFAIL}(p,m) \) | \( - \) | \( p \) | \( [p,m.t] \) | - | - | - | - | - | - |
| \( \text{Fail}(p) \) | \( - \) | \( p \) | - | - | - | - | - | - | - |
| \( \text{SendMsg}(p,...) \) | - | - | - | - | \( p \) | - | - | - | - |
| \( \text{TXMsg}(p) \) | - | - | - | - | - | \( p \) | \( q \in p^+ (G) \) | - | - |
| \( \text{DeliverMsg}(p) \) | - | - | - | - | - | - | \( p \) | - | - |
| \( \text{DetectFail}(p,q) \) | - | - | - | - | - | - | \( p \) | \( p[q] \) | - |

A. The atomic broadcast module

Let \( p \in S \) be any server. Then, the state of the AB module is described by the values of four variables (see Figure 2): (1) \( M[p] \) is the set of A-broadcast messages known by \( p \); actually, \( M[p] \subseteq S \). (2) \( g[p] \) is an array of \( n \) tracking digraphs, one per server; the digraph \( g[p][q] \) is used by \( p \) to track the message A-broadcast by \( q \). (3) \( F[p] \) is an array of \( n \) sets, one per server; the set \( F[p][q] \) contains all servers from which \( p \) received notifications of \( q \)’s failure. (4) \( \text{flag}(p) \) is a record with three binary fields—\( nf \) indicates whether \( p \) is non-faulty, \( ab \) indicates whether \( p \) A-broadcast its message, and \( done \) indicates whether \( p \) terminated. For ease of notation, we omit the \( flag \) prefix and refer to the three fields as \( nf[p] \), \( ab[p] \) and \( done[p] \), respectively.

**Initial state.** Initially, all sets in both \( M[p] \) and \( F[p] \) are empty—\( p \) neither knows of any A-broadcast message nor has received any failure notifications. For any \( q \in S \), \( g[p][q] \) contains only \( q \) as node—\( p \) suspects that each message is only known by their owner. Also, \( p \) is initially non-faulty and it has neither A-broadcast its message nor terminated.

**Next-state relations.** The AB module defines six operators that specify all the possible state transitions (see Table I). In addition to the two exposed interfaces, i.e., \( \text{Abcast}(p) \) and \( \text{Adeliver}(p) \), \( p \) can perform the following four actions: (1) receive a message, i.e., \( \text{ReceiveMessage}(p) \); (2) invoke the NET module for transmitting a message, i.e., \( \text{TXMsg}(p) \) (III-B); (3) fail, i.e., \( \text{Fail}(p) \); and (4) invoke the FD module for detecting the failure of a predecessor \( q \in S \), i.e., \( \text{DetectFail}(p,q) \) (III-C).

The \( \text{Abcast}(p) \) operator updates \( M[p] \), \( g[p] \) and \( ab[p] \)—it adds \( p \) to \( M[p] \); it removes all servers from \( g[p][p].node \) and it sets the \( ab[p] \) flag. Also, it sends \( p \)’s message further by invoking the \( \text{SendMsg} \) operator of the NET module (III-B). The main precondition of the operator is that \( p \) has not A-broadcast its message already; hence, a message can be A-broadcast at most once.

The \( \text{Adeliver}(p) \) operator sets the \( done[p] \) flag; as a result, \( p \) can A-deliver the messages in \( M[p] \) in a deterministic order. The main precondition of the operator is that all \( p \)’s tracking digraphs are empty, i.e., \( g[p][q].node = \emptyset \) \( \forall q \in S \). In Section IV, we show that this precondition is sufficient for safety.

The \( \text{ReceiveMessage}(p) \) operator invokes the \( \text{ReceiveMsg} \) operator of the NET module; as a result, the least recent message from the \( recvBuf \) is stored into \( recvMsg \) (III-B). ALLCONCUR distinguishes between A-broadcast messages and failure notifications. For both, the \( o \) field indicates the server that first sent the message, i.e., the owner; also, the \( t \) field of a failure notification indicates the server suspected to have failed, i.e., the target. When \( p \) receives a message \( m \), it invokes one of the following operators—\( \text{RecvBCAST}(p,m) \) or \( \text{RecvFAIL}(p,m) \). To avoid resends, both operators are enabled only if \( p \) has not already received \( m \).

The \( \text{RecvBCAST}(p,m) \) operator updates both \( M[p] \) and \( g[p] \)—it adds \( m.o \) to \( M[p] \); and it removes all servers from \( g[p][m.o].node \). Also, it sends \( m \) further by invoking the \( \text{SendMsg} \) operator of the NET module (III-B). In addition, if \( p \) has not A-broadcast its message, receiving \( m \) triggers the \( \text{Abcast}(p) \) operator. One of the precondition of the operator is that \( m \) was A-broadcast by its owner; although this condition is ensured by design, it facilitates the proof of the integrity property (IV).

The \( \text{RecvFAIL}(p,m) \) operator updates both \( F[p] \) and \( g[p] \)—it adds \( m.o \) to \( F[p][m.t] \) and updates every tracking digraph, in \( g[p] \), that contains \( m.t \). Updating the tracking digraphs is the core of ALLCONCUR’s early termination mechanism and we describe it in details in Section III-A1.

The \( \text{Fail}(p) \) operator clears the \( nf[p] \) flag. As a result, all of \( p \)’s operators are disabled—ALLCONCUR assumes a fail-stop model. The main precondition of the operator is that less than \( f \) servers have already failed.

\( \text{TXMsg}(p) \) and \( \text{DetectFail}(p,q) \) are discussed in Section III-B and Section III-C, respectively.

1) **Updating the tracking digraphs:** Tracking digraphs are trivially updated when \( p \) adds an A-broadcast message to the set \( M[p] \) and, as a result, removes all the servers from the digraph used (by \( p \)) to track this message. Updating the tracking digraphs is the core of ALLCONCUR’s early termination mechanism and we describe it in details in Section III-A1.
digraphs becomes more involved when \( p \) receives a failure notification. Let \( p_t \) be the target and \( p_o \) the owner of a received failure notification, i.e., \( p_o \) detected \( p_t \)'s failure. Then, \( p \) updates every tracking digraph, in \( g[p] \), that contains \( p_t \). Let \( g[p][p_s] \) be such a digraph: if, after the update, \( g[p][p_s] \) contains only servers known by \( p \) to have failed, then it is completely pruned—\( p \) is certain no non-faulty server can have \( m_s \). We specify two approaches to update \( g[p][p_s] \). The first approach follows the algorithm described in the ALLCONCUR paper [25]; yet, due to its recursive specification, it is not suitable for the TLA+ Proof System (TLAPS) [7]. The second approach constructs the tracking digraph from scratch, using the failure notifications from \( F[p] \); yet, it requires the TLA+ Model Checker [16] to enumerate all paths of a digraph.

**First approach—Recursive specification.** Let \( p^+_i (g[p][p_s]) \) be the set of \( p_i \)'s successors in \( g[p][p_s] \). Then, if \( p^+_i (g[p][p_s]) \neq \emptyset \), then \( p \) already received another notification of \( p_i \)'s failure, which resulted in \( p \) suspecting \( p_i \) to have sent (before failing) \( m_s \) to its other successors, including \( p_o \); thus, after receiving the failure notification, \( p \) removes the edge \((p_o, p_i)\) from \( g[p][p_s] \). Removing an edge, may disconnect some servers from the root \( p_s \); intuitively, these servers cannot have received \( m_s \) from any of the other suspected servers and thus, are removed from \( g[p][p_s] \).

Yet, if \( p^+_i (g[p][p_s]) = \emptyset \) (i.e., this is the first notification of \( p_i \)'s failure \( p \) receives), then, we update \( g[p][p_s] \) using a recursive function that takes two arguments—a FIFO queue \( Q \) and a digraph \( td \). Initially, \( Q \) contains all the edges in \( G \) connecting \( p_t \) to its successors (except \( p_o \)) and \( td = g[p][p_s] \). Let \((x, y)\) be an edge extracted from \( Q \). Then, if \( y \in g[p][p_s] \) (i.e., \( y \) is suspected by \( p \) to have \( m_s \)), the edge \((x, y)\) is added to \( td \) (i.e., \( p \) suspects \( y \) got \( m_s \) from \( x \)). If \( y \notin g[p][p_s] \), \( y \) is also added to \( td \) (i.e., \( p \) suspects \( y \) has \( m_s \)). In addition, if \( F[p][y] \neq \emptyset \) (i.e., \( y \) is known by \( p \) to have failed), the edges connecting \( y \) to its successors (except those known to have failed) are added to \( Q \). Finally, the function is recalled with the updated \( Q \) and \( td \). The recursion ends when \( Q \) is empty.

**Properties of tracking digraphs.** From the recursive specification, we deduce the following four invariants that uniquely define a non-empty tracking digraph, denoted by \( TD(p, p_s) \):

1. \( TD(p, p_s).node \) contains its root, i.e., \( p_s \in TD(p, p_s).node; \)
2. \( TD(p, p_s).node \) contains all the successors of every server (in the digraph) known to have failed, except those successors from which failure notifications were received, i.e.,
   \[ \forall TD(p, p_s).node : F[p][q] \neq \emptyset \Rightarrow \forall q_s \in q^+(G) \setminus F[p][q] : q_s \in TD(p, p_s).node; \]
3. \( TD(p, p_s).edge \) contains only edges that connect a server (in the digraph) known to have failed to all its successors, except those successors from which failure notifications were received, i.e.,
   \[ \forall (e_1, e_2) \in TD(p, p_s).edge : (F[p][e_1] \neq \emptyset \land e_2 \in e_1^+(G) \setminus F[p][e_1]); \]
4. \( TD(p, p_s).node \) contains only servers that are either the root or the successor of another server (in the digraph) known to have failed, except those successors from which failure notifications were received, i.e.,
   \[ \forall q \in TD(p, p_s).node : (q = p_s \lor (\exists q_p \in TD(p, p_s).node : \land F[p][q_p] \neq \emptyset \land q \in q_p^+(G) \setminus F[p][q_p])). \]

The intuition behind invariant \( I_1 \) is straightforward—while tracking \( p_s \)'s message, \( p \) always suspects \( p_u \) to have it. Invariants \( I_2 \) and \( I_3 \) describe how a tracking digraph expands. The successors of any server \( q \), which is both suspected to have \( m_s \) and known to have failed, are suspected (by \( p \)) to have received \( m_s \) directly from \( q \) before it failed. Yet, there is one exception—successors from which \( p \) already received notifications of \( q \)'s failure. Receiving a notification of \( q \)'s failure before receiving \( m_s \) entails the sender of the notification could not have received \( m_s \) directly from \( q \) (cf. Proposition 1).

\( I_1, I_2 \) and \( I_3 \) are necessary but not sufficient for a non-empty digraph to be a tracking digraph. As an example, we consider nine servers connected through a \( G_S(9, 3) \) digraph [28] (see Figure 3a). While \( p_0 \) is tracking \( m_0 \), it receives two notification, one from \( p_4 \) indicating \( p_0 \)'s failure and another from \( p_T \) indicating \( p_2 \)'s failure; i.e., \( F[p_0][p_4] = \{p_4\} \) and \( F[p_0][p_T] = \{p_T\} \). Clearly, the digraph \( KD(p_0) \) illustrated in Figure 3b satisfies the first three invariants. Yet, there is no reason for \( p_0 \) to suspect that \( p_2 \) has \( m_0 \).

For sufficiency, invariant \( I_4 \) is needed. Together with \( I_1, I_4 \) requires that \( p \) suspects only those servers that are connected through failures to the root \( p_s \). In other words, for \( p \) to suspect a server \( q \), there must be a sequence of servers starting with \( p_s \) and ending with \( q \) such that every server preceding \( q \) is both known to have failed and suspected to have sent \( m_s \) to the subsequent server. Note that a server is always connected through failures to itself. Figure 3c shows the actual digraph used by \( p_0 \) to track \( m_0 \) in the above example.

**Second approach—TLAPS specification.** We use the above invariants to provide a non-recursive specification of a non-empty tracking digraph. Let \( K_n \) be a complete digraph with \( n \) nodes; clearly, \( K_n \) satisfies \( I_1 \) and \( I_2 \). Let \( K_n(p) \) be a digraph obtained from \( K_n \) by removing all edges not satisfying \( I_3 \) (see Figure 3b for nine servers connected through a \( G_S(9, 3) \) digraph). Then, the set \( TD(p, p_s).node \) contains any node in \( K_n(p) \) that is either \( p_s \) or (according to \( p \)) is connected to \( p_s \).
Fig. 3: (a) An overlay network connecting nine servers; (b) a digraph that satisfies $I_1$, $I_2$ and $I_3$; and (c) the digraph used by $p_6$ to track $m_6$. Both digraphs consider nine servers connected through a $G_S(9, 3)$ digraph [28] and are based on two failure notifications received by $p_6$ (indicated by dashed edges), one from $p_4$ indicating $p_0$’s failure and another from $p_7$ indicating $p_2$’s failure. Note that $p_4 \notin g[p_0][p_0]$.

through failures, i.e.,

\[
\{ q \in \bar{K}_n(p).node : \forall q = p_\ast \\
\exists \pi_{p_\ast, q}(\bar{K}_n(p)) : F[p][x] \neq 0, \forall x \in \pi_{p_\ast, q}(\bar{K}_n(p)) \setminus \{ q \} \},
\]

where $\pi_{p_\ast, q}(\bar{K}_n(p))$ is a path in $\bar{K}_n(p)$ from $p_\ast$ to $q$. Note that when removing a node from $\bar{K}_n(p)$ we also remove all the edges incident on that node. Using TLAPS, we prove that this specification satisfies all four invariants [23].

B. The networking module

The NET module specifies an interface for asynchronous message-based communication; the module assumes that servers communicate through an overlay network. The interface considers three constants: (1) $S$, the set of servers; (2) $G$, the digraph that describes the overlay network; and (3) $Message$, the set of existing messages. We assume that every message has a field $\sigma$ indicating the server that first sent it.

Let $p \in S$ be any server. Then, the state of the NET module is described by the values of three variables (see Figure 2): (1) $sendBuf[p]$, is $p$’s sending buffer; (2) $recvBuf[p]$, is $p$’s receiving buffer; and (3) $recvMsg$, is the latest received message. Note that while $recvBuf[p]$ is a sequence of received messages, $sendBuf[p]$ is a sequence of tuples, with each tuple consisting of a message and a sequence of destination servers. Also, both buffers act as FIFO queues. In the initial state, the buffers are empty sequences; the initial value of $recvMsg$ is irrelevant.

Next-state relations. The NET module defines three operators that specify all the possible state transitions (see Table I). The operators consists of the three main actions performed in message-based communication—sending, transmitting and delivering a message. To describe the three operators, let $msgs$ be a sequence of messages and $nf$ a mapping $S \rightarrow \{0, 1\}$ indicating the non-faulty servers.

The $SendMsg(p, msgs, nf)$ operator, updates $sendBuf[p]$ by appending tuples consisting of messages from $msgs$ with their destinations; for every message $m$, the set of destinations consist of $p$’s non-faulty successors, except for $m.o$. Note that the $SendMsg$ operator has no precondition.

The $TXMsg(p)$ operator sends $m$, the next message from $sendBuf[p]$, to $q$, one of $m$’s destinations; $m$’s sequence of destinations is updated by removing $q$; when there are no more destinations, $m$ is removed from $sendBuf[p]$. Also, $m$ is appended to $recvBuf[q]$. As a precondition, the send buffer of $p$ must not be empty.

The $DeliverMsg(p)$ operator updates $recvBuf[p]$ by removing a message (i.e., the least-recent received) and storing it in $recvMsg$. Note that $recvMsg$ is only a temporary variable used by the AB module to access the delivered message (i.e., the ReceiveMessage operator). As a precondition, the receive buffer of $p$ must not be empty.

C. The failure detector module

The FD module provides information about faulty servers. It specifies a FD that guarantees both completeness and accuracy, i.e., a perfect FD [5]. The specification assumes a heartbeat-based FD with local detection: Every server sends heartbeats to its successors; once it fails, its successors detect the lack of heartbeats. The module considers two constants: (1) $S$, the set of servers; and (2) $G$, the digraph that describes the overlay network.

Let $p \in S$ be any server. Then, the state of the FD module is described by the values of one variable (see Figure 2)—$FD[p]$, is an array of $n$ flags, one per server; $FD[p][q]$ indicates whether $p$ has detected $q$’s failure. Clearly, $FD[p][q] = 1 \Rightarrow q \in p^+(G)$. Initially, all flags in $FD[p]$ are cleared.
Next-state relation. The FD module defines only one operator, DetectFail\((p, q)\), that specifies the state transition when \(p\) detects \(q\)’s failure; i.e., the \(FD[p][q]\) flag is set (see Table I). The operator has a set of preconditions. First, \(p\) must be both non-faulty and a successor of \(q\). Second, \(q\) must be faulty, i.e., \(nf[q] = 0\); this condition guarantees the accuracy property required by a perfect FD [5].

Once a failure is detected, the AB module must be informed. The FD module invokes the NET module to append a notification of \(q\)’s failure to recvBuf\([p]\) (see Figure 2). This ensures that any A-broadcast messages sent by \(q\) to \(p\) that were already transmitted (i.e., added to recvBuf\([p]\)) are delivered by \(p\) before its own notification of \(q\)’s failure. Thus, Proposition 1 holds: If \(p\) receives from \(q_s \in q^+(G)\) a notification of \(q\)’s failure, then \(q_s\) has not received from \(q\) any message that \(p\) did not already receive. In the above scenario, \(q_s = p\).

D. Safety and liveness properties

Using the above specification, we define both safety and liveness properties. First, ALLCONCUR relies on a perfect FD for detecting faulty servers; hence, it guarantees both accuracy and completeness [5]. Accuracy is a safety property: It requires that no server is suspected to have failed before actually failing, i.e.,

\(\forall q \in S : nf[q] = 1 \Rightarrow \forall p \in S : FD[p][q] = 0\).

Completeness is a liveness property: It requires that all failures are eventually detected, i.e.,

\(\forall q \in S : nf[q] = 0 \Rightarrow (\forall p \in q^+(G) : nf[p] = 1 \Rightarrow FD[p][q] = 1)\),

where \(X \sim Y\) asserts that whenever \(X\) is true, \(Y\) is eventually true [16].

Second, any atomic broadcast algorithm must satisfy four properties—validity, agreement, integrity, and total order [11], [5]. Integrity and total order are safety properties. Integrity requires for any message \(m\), every non-faulty server to A-deliver \(m\) at most once, and only if \(m\) was previously A-broadcast by its owner \(q\), i.e.,

\(\forall p \in S : nf[p] = 1 \Rightarrow \forall q \in M[p] : ab[q] = 1\).

Note that the requirement that a server A-delivers \(m\) at most once is ensured by construction, i.e., \(M[p]\) is a set.

Total order asserts that if two non-faulty servers \(p\) and \(q\) A-deliver messages \(m_1\) and \(m_2\), then \(p\) A-delivers \(m_1\) before \(m_2\), if and only if \(q\) A-delivers \(m_1\) before \(m_2\). Since \(p\) A-delivered messages in \(M[p]\) in a deterministic order, we replace total order with set agreement: Let \(p\) and \(q\) be any two non-faulty servers, then, after termination, \(M[p] = M[q]\), i.e.,

\(\forall p, q \in S : (nf[p] = 1 \land done[p] = 1 \land nf[q] = 1 \land done[q] = 1) \Rightarrow M[p] = M[q]\).

Validity and agreement are liveness properties. Validity asserts that if a non-faulty server A-broadcasts a message, then it eventually A-delivers it, i.e.,

\(\forall p \in S : (\Box(nf[p] = 1) \land ab[p] = 1) \Rightarrow a\text{-deliver}(p, p),\)

where \(\Box X\) asserts that \(X\) is always true [16]; also, \(a\text{-deliver}(p, q) = q \in M[p] \land done[p] = 1\) asserts the conditions necessary for \(p\) to A-deliver the message A-broadcast by \(q\).

Agreement asserts that if a non-faulty server A-delivers a message A-broadcast by any server, then all non-faulty servers eventually also A-deliver the message.

\(\forall p, q, s \in S : (\Box(nf[q] = 1) \land a\text{-deliver}(p, s)) \Rightarrow (nf[q] = 1 \Rightarrow a\text{-deliver}(q, s)).\)

To verify that all the above properties hold, we use the TLA+ Model Checker [16], hence, the need for a tracking digraph specification that does not enumerate all paths of a digraph (III-A1). For a small number of servers, e.g., \(n = 3\), the model checker can do an exhaustive search of all reachable states. Yet, for larger values of \(n\) the exhaustive search becomes intractable. As an alternative, we use the model checker to randomly generate state sequences that satisfy both the initial state and the next-state relations. In the model, we consider the overlay network is described by a \(G_S(n, d)\) digraph [28]. When choosing \(G_S(n, d)\)’s degree, i.e., its fault tolerance (II-B), we require a reliability target of 6-nines; the reliability is estimated over a period of 24 hours according to the data from the TSUBAME2.5 system failure history [26], [10], i.e., server MTTF \(\approx 2\) years.

In addition, we use TLAPS to formally prove the safety properties—the FD’s accuracy and the atomic broadcast’s integrity and set agreement (IV). TLAPS does not allow for liveness proofs. However, validity and agreement require ALLCONCUR to terminate; termination is informally proven in the ALLCONCUR paper [25].
IV. ALLCONCUR: FORMAL PROOF OF SAFETY

Atomic broadcast has two safety properties—integrity and total order. In ALLCONCUR, total order can be replaced by set agreement (III-D). In addition, ALLCONCUR relies on a perfect FD for information about faulty servers; thus, for safety, accuracy must also hold. For all three safety properties, we use TLAPS [16] to provide mechanically verifiable proofs [23]. All three proofs follow the same pattern: We consider each property to be an invariant that holds for the initial state and is preserved by the next-state relations.

Accuracy. The FD's accuracy is straightforward to prove. Initially, all flags in FD are cleared; hence, the property holds. Then, according to the specification of the DetectFail operator (III-C), setting the flag FD[p][q] for \( \forall p, q \in S \) is preconditioned by \( q \) previously failing, i.e., \( n_f[q] = 0 \). Moreover, due to the fail-stop assumption, a faulty server cannot become subsequently non-faulty. As a result, accuracy is preserved by the next-state relations.

Integrity. Integrity is also straightforward to prove. Initially, all the sets in M are empty; hence, the property holds. Then, the only two operators that update M are Abcast and RecvBCAST (see Table I). The Abcast(p) operator adds p to M[p] and also sets the ab[p] flag; hence, integrity is preserved by Abcast. The RecvBCAST(p, m) operator adds m.o to M[p]. Clearly, receiving a message m entails the existence of a path from m.o to p such that each server on the path has m in its M set (see Lemma IV.1); this also includes m.o. When m.o adds its message m to M[m.o], it also sets the ab[m.o] flag (according to the Abcast operator). Thus, integrity is preserved also by RecvBCAST. In practice, to simplify the TLAPS proof, we introduce an additional precondition for the RecvBCAST(p, m) operator—ab[m.o] = 1 (III-A).

A. Set agreement

The set agreement property is the essence of ALLCONCUR—it guarantees the total order of broadcast messages. Clearly, set agreement holds in the initial state, since all done flags are cleared. Moreover, the done flags are set only by Adeliver operator (see Table I); thus, we only need to prove set agreement is preserved by Adeliver. We follow the informal proof provided in the ALLCONCUR paper [25]. We introduce the following lemmas:

**Lemma IV.1**. Let p be a server that receives \( p_\star \)'s message \( m_\star \); then, there is a path (in G) from \( p_\star \) to p such that every server on the path has received \( m_\star \) from its predecessor on the path, i.e.,

\[
\forall p, p_\star \in S : \exists \pi_{p, \star}(G) = (a_1, \ldots, a_\lambda) : \\
\land \forall k \in \{1, \ldots, \lambda\} : p_k \in M[a_k] \land \forall q \in S : (\exists k \in \{1, \ldots, (\lambda - 1)\} : a_{k+1} \in F\{q|a_k\}) \Rightarrow p_\star \in M[q].
\]

**Proof**: Equation (2a) is straightforward. Equation (2b) ensures that every server on the path (except \( p_\star \)) received \( m_\star \) from its predecessor. Any server \( q \) that received a failure notification from a server on the path (except \( p_\star \)) targeting its predecessor on the path also received \( m_\star \) (cf. Proposition 1).

**Lemma IV.2**. Let p be a non-faulty server that does not receive \( p_\star \)'s message \( m_\star \); let \( \pi_{p_\star, a_i}(G) = (a_1, \ldots, a_\lambda) \) be a path (in G) on which \( a_i \) receives \( m_\star \); let \( (a_1, \ldots, a_i) \) also be a path in g[p][p_\star]. Then, done[p] = 1 \( \Rightarrow (\forall q \in g[p][p_\star], \text{node} : F\{q|g[p][p_\star]\} \neq \emptyset) \).

**Proof**: A necessary condition for p to terminate is to remove every server \( a_j, \forall 1 \leq j \leq i \) from \( g[p][p_\star] \). According to ALLCONCUR's specification (III-A1), a server \( a_j \) can be removed from \( g[p][p_\star] \) in one of the following scenarios: (1) \( p_\star \in M[p]; \) (2) \( \not\exists \pi_{p_\star, a_k}(g[p][p_\star]) \); and (3) \( \forall q \in g[p][p_\star], \text{node} : F\{q|g[p][p_\star]\} \neq \emptyset \). Clearly, \( p_\star \notin M[p] \). Also, \( \not\exists \pi_{p_\star, a_k}(g[p][p_\star]) \) entails at least the removal of an edge from the \( (a_1, \ldots, a_i) \) path. Let \( (a_i, a_{i+1}), 1 \leq i < j \) be one of the removed edges. Then, \( a_{i+1} \in F\{p|a_i\} \), which entails \( p_\star \in M[p] \) (cf. Lemma IV.1). Thus, p terminates only if \( \forall q \in g[p][p_\star], \text{node} : F\{q|g[p][p_\star]\} \neq \emptyset \).

**Lemma IV.3**. Let p, q be two non-faulty servers such that \( p_\star \in M[p] \), but \( p_\star \notin M[q] \); let \( \pi_{p_\star, a_i}(G) = (a_1, \ldots, a_\lambda) \) be the path on which \( p \) receives \( m_\star \); let \( a_k \) be a server on \( \pi_{p_\star, a_i}(G) \) such that \( n_f[a_k] = 1 \) and \( n_f[a_i] = 0, \forall 1 \leq i < k \). Then, \( (a_1, \ldots, a_i), \forall 1 \leq i \leq k \) is a path in \( g[q][p]\), i.e., \( (a_1, \ldots, a_i) \in g[q][p]\).

**Proof**: We use mathematical induction: The basic case is given by invariant I_1 (III-A1), \( a_1 = p_\star \in g[q][p\star], \text{node} \). For the inductive step, we assume \( (a_1, \ldots, a_i) \in g[q][p\star] \). Due to the FD's completeness property (III-D), the failure of \( a_j, \forall 1 \leq j \leq i \) is eventually detected; due to the vertex-connectivity of G, q eventually receives the failure notification of every \( a_j \). Moreover, q cannot remove \( a_j, \forall 1 \leq j \leq i \) from \( g[q][p\star] \) before it receive failure notifications of every \( a_j \) (cf. Lemma IV.2). Thus, eventually \( F\{q|a_i\} \neq \emptyset \) and, since \( a_{i+1} \notin F\{q|a_i\} \) (cf. Lemma IV.1), \( a_{i+1} \in g[q][p\star], \text{node} \) and \( (a_i, a_{i+1}) \in g[q][p\star], \text{edge} \) (according to invariants I_2 and I_3).

**Theorem IV.4**. ALLCONCUR's specification guarantees set agreement.

**Proof**: Let \( p_\star \in M[p] \), but \( p_\star \notin M[q] \). Then, \( \exists \pi_{p_\star, a_i}(G) = (a_1, \ldots, a_\lambda) \) on which \( m_\star \) first arrives at p (cf. Lemma IV.1). Let \( a_k \) be a server on \( \pi_{p_\star, a_i}(G) \) such that \( n_f[a_k] = 1 \) and \( n_f[a_i] = 0, \forall 1 \leq i < k \); the existence of \( a_\lambda \) is given by the existence of \( p_\star \), a server that is both non-faulty and on \( \pi_{p_\star, a_i}(G \). The path \( \pi_{p_\star, a_i}(G \) is illustrated in Figure 4; the faulty servers are indicated by...
dotted red boxes. Before \( q \) terminates, \( a_k \in g[q][p_*] \).node (cf. Lemma IV.3). However, due to the precondition of Adeliver\((q)\), i.e., done\((q) = 1 \Rightarrow g[q][s].node = \emptyset, \forall s \in S \) ( III-A), \( a_k \) is subsequently removed from \( g[q][p_*] \). Since \( m_* \not\in M[q] \) and \( n_f[a_k] = 1 \), \( a_k \) must have been disconnected from \( p_* \) ( III-A1). Thus, \( q \) removed at least one edge from \( (a_1, \ldots, a_k) \); let \( (a_i, a_{i+1}), 1 \leq i < k \) be one of the removed edges. Then, \( a_{i+1} \in F[q][a_i] \) (cf. I.3), which entails \( p_* \in M[q] \) (cf. Lemma IV.1).

1) Reconstructed tracking digraphs: The set agreement proof relies on the following property of tracking digraphs ( III-A1): If a server \( q \) was removed from \( g[p][p_*] \), then one of the following is true—(1) \( p_* \in M[p] \); (2) \( F[p][q] \neq \emptyset \); and (3) \( \exists p_{s}, p_{r}(g[p][p_*]) \). Yet, when the Adeliver\((p)\) operator is enabled, \( g[p][p_*].node = \emptyset \); checking the existence of a path \( \pi_{p_{s}, p_{r}}(g[p][p_*]) \) is not possible. Thus, we reconstruct \( g[p][p_*] \) from the failure notifications received by \( p \); we denote the result digraph by \( \text{RTD}(p, p_*) \).

RTD nodes. Constructing the set \( \text{RTD}(p, p_*) \).node is similar to the TLAPS specification of updating tracking digraphs ( III-A1). The difference is twofold. First, if \( p \) receives from \( p_0 \) a notification of \( p_0 \)'s failure, with \( p_0 \not\in \text{RTD}(p, p_*) \), then \( p \) adds all \( p_0 \)'s successors to \( \text{RTD}(p, p_*) \)—including \( p_0 \). Second, servers are never removed from \( \text{RTD}(p, p_*) \). Clearly, every server added at any point to \( g[p][p_*] \) is also in \( \text{RTD}(p, p_*) \).

RTD edges. To construct the set \( \text{RTD}(p, p_*) \).edge, we first connect (in \( \text{RTD}(p, p_*) \)) each server known to have failed to its successors. Then, we remove all the edges on which we are certain \( p_* \)'s message \( m_* \) was not transmitted. To identify these edges we use Proposition 1: If \( p \) received from \( e_2 \) a notification of \( e_1 \) failure without previously receiving \( m_* \), then the edge \( (e_1, e_2) \) is not part of \( \text{RTD}(p, p_*) \), i.e.,

\[
\forall e_1, e_2 \in \text{RTD}(p, p_*) \).node : e_2 \in F[p][e_1] \land p_* \not\in M[p] \Rightarrow (e_1, e_2) \not\in \text{RTD}(p, p_*) \).edge.
\]

Some of the remaining edges in \( \text{RTD}(p, p_*) \) were still not used for transmitting \( m_* \), i.e., the edges for which \( e_2 \in F[p][e_1] \) occurred before \( p_* \in M[p] \). Since the Adeliver\((p)\) operator has no access to the history of state updates modifying either \( F[p][e_1] \) or \( M[p] \), we cannot identify these edges. Yet, since \( p_* \in M[p] \), these edges play no role in proving set agreement. Clearly, every edge added at any point to \( g[p][p_*] \) is also in \( \text{RTD}(p, p_*) \).

RTD invariant. Using reconstructed tracking digraphs, we redefine the above property of tracking digraphs as an invariant, referred to as the RTD invariant:

\[
\forall p, q, p_* \in S : (q \in \text{RTD}(p, p_*) \).node \land q \not\in g[p][p_*] \).node \Rightarrow p_* \in M[p] \lor F[p][q] \neq \emptyset \lor \exists p_{s}, p_{r}(\text{RTD}(p, p_*)) \).
\]

The RTD invariant enables us to formulate the prove set agreement using TLAPS [23]. Clearly, when Adeliver\((q)\) is enabled, \( a_k \) (from the proof of Theorem IV.4) is in \( \text{RTD}(q, p_*), \) but not in \( g[q][p_*] \). According to the initial assumptions, \( p_* \not\in M[q] \) and \( F[q][a_k] = \emptyset \). Moreover, \( \pi_{p_{s}, a_*} = (a_1, \ldots, a_k) \) was at some point a path in \( g[q][p_*] \) (cf. Lemma IV.3); hence, \( \pi_{p_{s}, a_*} \) is also a path in \( \text{RTD}(p, p_*) \) (follows from RTD’s construction), which contradicts the RTD invariant.

2) Proving the RTD invariant: To prove the RTD invariant, we follow the same pattern as before: We prove that the invariant holds for the initial state (since \( \text{RTD}(p, p_*) = g[p][p_*], \forall p, p_* \)) and is preserved by the only three operators that update the \( M, G \) or \( F \) variables—Abcast; RecvBCAST; RecvFAIL (see Table I). In the following proofs, \( X' \) denotes the updated value of a variable \( X \) after applying an operator [16].

Theorem IV.5. Both Abcast and RecvBCAST operators preserve the RTD invariant.

**Proof:** According to the specifications of both Abcast and RecvBCAST, \( g[p][p_*] \not\in g'[p][p_*] \Rightarrow p_* \in M'[p] \) ( III-A); thus, we assume \( g[p][p_*] = g'[p][p_*] \). In addition, since \( F[p][p_*] = F'[p][p_*] \), \( \text{RTD}(p, p_*) \).node = \( \text{RTD}(p, p_*) \).node. Thus, since \( M[p] \subseteq M'[p] \), the only possibility for the RTD invariant to not be preserved by either operators is \( \exists \pi_{p_{s}, p_{r}}(\text{RTD}(p, p_*)) \). Let \( (e_1, e_2) \) be one of the edges that enables such a path, i.e., \( (e_1, e_2) \not\in \text{RTD}(p, p_*) \).edge \land (e_1, e_2) \in \text{RTD}(p, p_*) \).edge. As a result, \( e_2 \in F[p][e_1] \land p_* \not\in M[p] \lor (e_2 \not\in F'[p][e_1] \lor p_* \in M'[p]) \) (cf. Equation (3)), which is equivalent to \( p_* \in M'[p] \).

Theorem IV.6. The RecvFAIL operator preserves the RTD invariant.

\[2F[p][q] \neq \emptyset \) is necessary, but not sufficient to remove \( q \) from \( g[p][p_*] \).\]
Proof: We consider $\text{RecvFAIL}(s, m)$, with $s \in S$; clearly, $M = M'$ (see Table I). Also, if $s \neq p$, the RTD invariant is preserved, since also $F[p] = F'[p]$ and $g[p] = g'[p]$. Thus, the proof is concerned only with $\text{RecvFAIL}(p, m)$. Aside from adding $m \otimes F'[p][m.t]$, $F = F'$. Clearly, $q = m.t \Rightarrow F'[p][q] \neq \emptyset$. Thus, we make the following assumptions, i.e., the only non-trivial case (cf. RTD invariant):

\begin{align*}
(A_1) & \quad q \neq m.t; \\
(A_2) & \quad q \in \text{RTD}(p, p_*, \text{node}); \\
(A_3) & \quad F'[p][q] = \emptyset; \\
(A_4) & \quad \exists p' \in M'[p]; \\
(A_5) & \quad \exists p_{\pi, q}(\text{RTD}(p, p_*, \text{node})).
\end{align*}

We split the remainder of the proof in three steps.

Step 1. We show that $q$ is in $\text{TD}(p, p_*, \text{node})$. Let $\pi_{p, q}(\text{RTD}(p, p_*, \text{node})) = (a_1, \ldots, a_\gamma)$ (cf. (A6)); then $(a_0, a_{k+1}) \in \text{TD}(p, p_*, \text{node}). \forall k < \gamma$, since $\pi \notin M'[p]$ (cf. (A4)) and no edges are removed from $\text{RTD}(p, p_*, \text{node})$ (cf. Equation (3)). Thus, $\forall k \in \text{TD}(p, p_*, \text{node}). \forall k < \gamma$ (cf. Equation (1)), which entails the following assumption:

$$(A_\gamma) \quad q \in \text{TD}(p, p_*, \text{node}).$$

Step 2. We show that $g[p][p_*]$ is not updated. If $g'[p][p_*] \neq g[p][p_*]$, then $g'[p][p_*]$ is either $\text{TD}(p, p_*, \text{node})$ or an empty digraph obtained after completely pruning $\text{TD}(p, p_*, \text{node})$. Since the latter option requires $F'[\pi][s] \neq \emptyset, \forall \pi \in \text{TD}(p, p_*, \text{node})$ (III-A1), which contradicts (A3) and (A7), we assume $g'[p][p_*] = \text{TD}(p, p_*, \text{node})$. Yet, this entails $q \in g[p][p_*], \text{node} (\text{cf. A7})$, which contradicts (A3). Thus, we make the following two equivalent assumptions (III-A1):

$$(A_3) \quad g[p][p_*] = g'[p][p_*]; \quad (A_9) \quad m.t \notin g[p][p_*], \text{node}.$$  

Step 3. We distinguish between three scenarios—according to the specification (III-A1), $g[p][p_*]$ can be in one of the following three states: (1) an initial state, i.e., $g[p][p_*], \text{node} = \{p_*\}$, with $F[p][p_*] = \emptyset$; (2) a final state, i.e., $g[p][p_*], \text{node} = \emptyset$; and (3) an intermediary state, i.e., $g[p][p_*] = \text{TD}(p, p_*)$.

The initial state entails $g'[p][p_*], \text{node} = \{p_*\}$ (cf. (A3)); thus, $p_*$ is neither $m.t$ (cf. (A9)) nor $q$ (cf. (A3)). Clearly, $p_* \neq m.t \Rightarrow F[p][p_*] = F'[p][p_*]$. Moreover, according to (A9) and the construction of $\text{RTD}(p, p_*, \text{node})$, $p_* \neq q$ entails $F[p][p_*] = \emptyset$. Yet, this contradicts the condition of the initial state, i.e., $F[p][p_*] = \emptyset$.

The final state, i.e., $g[p][p_*], \text{node} = \emptyset$, can be reached only by completely pruning the tracking digraph (cf. (A4) and $M'[p] \subseteq M'[p]$); i.e., $F[p][s] = \emptyset, \forall s \in \text{TD}(p, p_*, \text{node})$. According to invariant I2, no server can be added to $\text{TD}(p, p_*, \text{node})$, regardless of the received failure notification. As a result, $q \in \text{TD}(p, p_*, \text{node})$ (cf. (A7)) and thus, $F[p][q] = \emptyset$. Yet, since $F[p][q] = F'[p][q]$ (cf. (A1)), this contradicts (A9).

The intermediary state entails $g'[p][p_*] = \text{TD}(p, p_*)$ (cf. (A3)); thus, $\exists p_{\pi, q}(\text{TD}(p, p_*)$ (cf. (A3)). Nonetheless, $\exists p_{\pi, q}(\text{TD}(p, p_*) = (b_1, \ldots, b_k), \forall i < k < \gamma$ (cf. (A7) and Equation (1)). Moreover, $\exists i < k < \gamma : (b_k, b_{k+1}) \notin \text{TD}(p, p_*), \text{edge}$. Let $b_k$ be the smallest index satisfying this property. Then, either $F[p][b_k] = \emptyset$ (cf. invariant I1). If $F[p][b_k] = \emptyset$, then $b_k = m.t$ (since $F'[p][b_k] = \emptyset$). Also, $(b_1, \ldots, b_k)$ is a path in $\text{TD}(p, p_*)$ (cf. invariants I1, I2 and I3) and, thus, $b_k \in \text{TD}(p, p_*, \text{node})$, which entails $m.t \notin g'[p][p_*]$. Yet, this contradicts (A3) and (A9). If $b_{k+1} \notin F[p][b_k]$, then $b_{k+1} \in F'[p][b_k]$ (i.e., $F[p][b_k] \subseteq F'[p][b_k]$), which contradicts $(b_k, b_{k+1}) \notin \text{TD}(p, p_*, \text{node})$.

V. CONCLUSION

We have provided both a formal design specification and a formal proof of safety of ALLCONCUR, a leaderless concurrent atomic broadcast algorithm. Previous work shows the advantage of using ALLCONCUR over classic leader-based approaches, such as Paxos—it enables higher throughput for distributed agreement while being completely decentralized [25]. This work builds on ALLCONCUR by both improving the understanding of the algorithm, through a TLA+ design specification, and formally proving the algorithm's safety property, using the TLA+ Proof System.

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