Prototyping a Six-Axis Positioning Device

M V Smirnov¹, A V Gritsenko¹², V D Shepelev¹

¹South Ural State University (national research university), Department of Automobile Transport, 76 Lenin prospekt, Chelyabinsk 454080, Russia
²South Ural State Agrarian University, Department of Engineering and Technology, 75 Lenin prospekt, Chelyabinsk 454080, Russia

E-mail: alexgrits13@mail.ru

Abstract. In this paper, we describe prototyping a six-axis positioning device (SPD) with parallel kinematics in a new structural design. Each new design requires the development of a mathematical model. This is necessary to implement the algorithm for controlling this device, as well as analyzing its workspace. Each design variant can be used for various tasks and different areas, as it will have its own unique shape and size of the workspace. The paper scientific relevance deals with developing a new structural design of SPD, as well as obtaining its mathematical model. Based on the obtained mathematical model, it is possible in the future to model the workspace in order to obtain an idea of its shape and size.

1. Introduction
Mechanisms with parallel kinematics are promising for widespread use. They are used on CNC machines, manipulators, robots, dynamic platforms, etc. The objective of these mechanisms is to move the working bodies such as tools in machines, tools or objects in robots and manipulators, or the center of mass of the structure in dynamic platforms [1, 2]. Mechanisms with parallel kinematics in various design versions appear increasingly [3, 4].

Each version has its own advantages and disadvantages compared to existing versions. The workspace is interesting not only as a volume, but also as a form. It is the shape of the workspace that determines the scope of equipment application for solving various process tasks. When calculating the workspace, it is necessary to take into account the limitations and generalized coordinates, joint-hinge angle values, possible collisions, and the presence of special positions [5, 6].

The purpose of this study is to develop a prototype of a six-axis device (SD) in a new design. To achieve this goal, it was necessary to solve a set of problems: to develop a kinematic scheme of the SD new structural design and its mathematical model; to calculate the deflection limit in the joints; to assemble the SD prototype; to analyze the feedback sensors.

2. Methods and results

2.1. Structural features
SD is designed to change the center-of-gravity position of the aircraft to maintain the stability of the device during its flight and landing. SPD belongs to the class of devices with parallel kinematics, due to its advantages [7, 8].
The device has a mobile element that can move along 6 independent coordinates. A change in the position of the mobile element results in shifting the center of gravity of the device.

The SPD consists of fixed platforms and a moving body. Fixed platforms are connected to the body using 6 sectionalized bars. Each bar consists of a constant-length bar and a variable-length bar of a kinematic screw-nut pair. The bar has a triple hinge joint on both sides.

The hinges of the lower fixed platform are displaced relative to each other by 120 °. The hinges of the upper platform are displaced relative to the lower platform by 60 ° [3].

2.2. Development of the kinematic model
Building the kinematic model of the device is necessary to ensure its correct positioning and proper operation. The kinematic model describes the relationship between input (controlled) and output parameters [9, 10].

The design of the device should provide the six-axis movement of the moving element: three coordinates (x, y, z) are linear and three (ψ, θ, γ) are angular. The linear coordinates are the coordinates of the pole (we call it O') of the moving element in the fixed coordinate system (CS) O'X'Y'Z'.

The angular coordinates determine the rotation of the moving element around the pole; the Krylov angles are used as such coordinates. These angles describe the rotation of the moving coordinate system attached to the moving element relative to the fixed coordinate system OXYZ. The six coordinates described above are the output coordinates for the kinematic model of the device.

Figure 1 shows the kinematic diagram of the positioning device with parallel kinematics.

![Figure 1. Kinematic diagram of the device. A1-A6 is the ball hinge, B1-B6 is the ball hinge, C1-C6 is the screw-nut pair, ME is the moving element, UFP is the upper fixed platform, LFP is the lower fixed platform, L is the constant-length bar, d is the variable-length bar, R is radius of the moving element end plate, O' is the pole, O'X'Y'Z' is the moving coordinate system, OXYZ is the fixed coordinate system).](image)

Building a kinematic model is reduced to writing equations that relate the lengths of variable-length bars (input coordinates) and the coordinates of the pole O'. The obtained equations allow us to determine the spatial position of the moving element relative to fixed platforms in the fixed OXYZ CS. The relationship between the fixed platforms and the moving element is carried out through ball hinges A and B, as well as constant-length bars. The equation for the relationship between elements is as follows:

\[ L_i^2 = (x_{A_i} - x_{B_i})^2 + (y_{A_i} - y_{B_i})^2 + (z_{A_i} - z_{B_i})^2, \quad L_i = L = \text{const}, \quad i = 1..6 \] (1)
where \( x_{A_i}, y_{A_i}, z_{A_i} \) are the coordinates of the \( i \)-th ball hinge \( A; x_{B_i}, y_{B_i}, z_{B_i} \) are the coordinates of the \( i \)-th ball hinge \( B; L \) is the constant-length bar.

The coordinates of the \( B \) ball hinges are much easier to determine in the moving coordinate system since in this coordinate system, the \( B \) hinges change only \( z \)-coordinate. The coordinates of the \( B \) hinge in the moving \( O'X'Y'Z' \) CS are:

\[
\begin{align*}
x'_{B_i} &= R \cdot \cos \varphi, \\
y'_{B_i} &= R \cdot \sin \varphi, \\
z'_{B_i} &= \left( \frac{h}{2} \right) + R^2 - d_i
\end{align*}
\]

(2)

The coordinates of spherical hinges \( B_i \) related to the moving CS are recalculated to the fixed \( OXYZ \) CS by multiplying the coordinates by the directional cosines:

\[
\begin{align*}
x_{B_i} &= l_1 \cdot x'_{B_i} + l_2 \cdot y'_{B_i} + l_3 \cdot z'_{B_i} + x_{O'}, \\
y_{B_i} &= m_1 \cdot x'_{B_i} + m_2 \cdot y'_{B_i} + m_3 \cdot z'_{B_i} + y_{O'}, \\
z_{B_i} &= n_1 \cdot x'_{B_i} + n_2 \cdot y'_{B_i} + n_3 \cdot z'_{B_i} + z_{O'}
\end{align*}
\]

(3)

where \( x_{O'}, y_{O'}, z_{O'} \) are the coordinates of the pole \( O' \).

The directional cosines \( l_j, m_j, n_j \) \( (j=1...3) \) are determined through the output angular coordinates (the Krylov angles):

\[
\begin{align*}
l_1 &= \cos \psi \cdot \cos \gamma + \sin \psi \cdot \sin \gamma \cdot \sin \vartheta, \\
m_1 &= -\sin \psi \cdot \cos \gamma + \sin \vartheta \cdot \cos \psi \cdot \sin \gamma, \\
n_1 &= \cos \vartheta \cdot \sin \gamma, \\
l_2 &= \cos \vartheta \cdot \sin \psi, \\
m_2 &= \cos \vartheta \cdot \cos \psi, \\
n_2 &= -\sin \vartheta, \\
l_3 &= -\cos \psi \cdot \sin \gamma + \sin \psi \cdot \sin \vartheta \cdot \cos \gamma, \\
m_3 &= \sin \psi \cdot \sin \gamma + \cos \psi \cdot \sin \vartheta \cdot \cos \gamma, \\
n_3 &= \cos \vartheta \cdot \cos \gamma.
\end{align*}
\]

Substituting the Krylov angles instead of the output angular coordinates, we obtain the following expression:

\[
\begin{align*}
L_i^2 &= x_{A_i}^2 + y_{A_i}^2 + z_{A_i}^2 + \left( x_{O'} - x_{A_i} \right)^2 + \left( y_{O'} - y_{A_i} \right)^2 + \left( z_{O'} - z_{A_i} \right)^2 + \\
&+ 2\left[ x_{B_i} (C\psi \cdot C\gamma + S\psi \cdot S\vartheta \cdot S\gamma) + y_{B_i} \cdot S\psi \cdot C\vartheta + z_{B_i} \left( S\psi \cdot S\vartheta \cdot C\gamma - C\psi \cdot S\gamma \right) \right] \times \left( x_{O'} - x_{A_i} \right) + \\
&+ 2\left[ x_{B_i} (C\psi \cdot S\vartheta \cdot S\gamma - S\psi \cdot C\gamma) + y_{B_i} \cdot C\psi \cdot C\vartheta + z_{B_i} \left( S\psi \cdot S\gamma - C\psi \cdot S\vartheta \cdot C\gamma \right) \right] \times \left( y_{O'} - y_{A_i} \right) + \\
&+ 2\left[ x_{B_i} \cdot C\vartheta \cdot S\gamma - y_{B_i} \cdot S\vartheta + z_{B_i} \cdot C\vartheta \cdot C\gamma \right] \times \left( z_{O'} - z_{A_i} \right), \quad i=1...6
\end{align*}
\]

(5)

where \( S\psi = \sin \vartheta, C\psi = \cos \vartheta \), etc.

Five equations (1-5) link all six input and six output coordinates of the device in question. The system of these equations is a kinematic model of the device.

The positioning of the moving element is limited by design features of the six-axis device. One of these limitations is the bar deflection limit in ball joints [11, 12, 13].

3
2.3. Positioning the moving element

The SPD is designed for positioning (i.e. giving a certain position) the moving element relative to the stationary body. The “Housing” body consists of a frame and two fixed platforms, the upper and lower.

The “Moving element” body is a cylinder. Round metal plates are attached to the ends of the cylinder, on which six measuring units and six electric motors are installed (three units and three electric motors on each plate). The measuring unit is designed to determine the current length of the bar. In this device, the measuring unit operates as a feedback sensor (FB). The electric motors in this device are designed to change the length of the bars.

Figures 2 and 3 show the position of the measuring units and electric motors on the plates.

![Figure 2. Measuring units and electric motors on the upper plate: 1 are ball joints; 2 is the constant-length bar; 3 is the variable-length bar; 4 is the measuring unit; 5 is electric motor.](image)

![Figure 3. Measuring units and electric motors on the lower plate.](image)

The connection of the movable element with the housing is carried out by six constant-length bars of and six variable-length bars.

The measuring unit is the MDC-25MX Mitutoyo digital micrometer [2]. 6 LS-16GA030-50 low-power gear motors are used as electric motors [3].

3. Experimental study

The process of positioning a moving element can be considered by the example of varying the length of one of the bars. When the motor shaft rotates, the torque is transmitted to the rod of the measuring unit by means of a cylindrical gear transmission. Further, the torque is transmitted from the rod of the measuring unit to the variable-length bar. When rotating, the bar changes its length, and, therefore, changes the position of the moving element.

The feedback sensor has an output interface. Waveforms of the output interface signals are presented in Figure 5.
Due to the obtained dependence of the length on the display and on the sensor, we can easily get the current length value by reading the information from the FB sensor.

4. Results and conclusion
Based on the resulting mathematical model, we can further simulate the workspace to imagine its shape and size. In addition, in the future, we will develop a control system for SPD based on the mathematical model and the dependence of the length obtained from the FB sensor.

5. References
[1] Artobolevsky I I 1988 Theory of Mechanisms and Machines (Moscow: Nauka Publ)
[2] Korendyasev A I 2006 Theoretical Foundations of Robotics (Moscow: Nauka Publ)
[3] Kolovsky M Z, Evgrafov A N, Semenov Yu A and Slouchch A V 2000 Advanced Theory of Mechanisms and Machines (New York: Springer - Verlag Berlin Heidelberg)
[4] Gritsenko A, Shepelev V, Zadorozhnaya E and Shubenkova K 2020 FME Transactions 48 1 46-52
[5] Semenov Yu A and Semenova N S 2004 Theory of mechanisms and machines 1 3 26-41
[6] Semenov Yu A and Semenova N S 2003 Theory of mechanisms and machines 2 3-14
[7] Salmanzadeh G Y, Karpovich S E, Rymko V M and Drogun E A 2019 Actual issues of
machinery 8 71-75

[8] Karpovich S E, Kuznetsov V V, Polyakovskiy V V and Fortan M M 2016 Information systems and technologies: management and security 4 74-80

[9] Sulatskaya E Yu and Petrova L N 2009 Bulletin of the South Ural State University. Series: Mechanical Engineering Industry 11 144 42-45

[10] Li L, Fang Y and Wang L 2020 Mech Mach Theory 148

[11] Ye W, Chai X and Zhang K 2020 Mech Mach Theory 149

[12] Chen M, Zhang Q, Qin X and Sun Y 2019 International Conference on Advances in Construction Machinery and Vehicle Engineering (Changsha) (IEEE)

[13] Jin X, Fang Y and Zhang D 2019 Mech Mach Theory 134 117-134

Acknowledgments
The work was supported by Act 211 Government of the Russian Federation, contract № 02.A03.21.