Flavor and electroweak symmetry breaking at the TeV scale

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We present a unified picture of flavor and electroweak symmetry breaking at the TeV scale. Flavor and Higgs bosons arise as pseudo-Goldstone modes in a nonlinear sigma model. Explicit collective symmetry breaking yields stable vacuum expectation values and masses protected at one loop by the little-Higgs mechanism. The coupling to the fermions through a Yukawa lagrangian with a $U(1)$ global flavor symmetry generates well-defined mass textures that correctly reproduce the mass hierarchies and mixings of quarks and leptons. The model is more constrained than usual little-Higgs models because of bounds on weak and flavor physics. The main experimental signatures testable at the LHC are a rather large mass $m_{h^0} = 317 \pm 80$ GeV for the (lightest) Higgs boson and a characteristic spectrum of new bosons and fermions with masses around the TeV scale.

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I. MOTIVATIONS AND BACKGROUND

The requirement of naturalness for the standard model with a light Higgs boson seems to demand new physics at or around the 1 TeV scale. On the other hand, precision measurements do not show any departure from standard physics up to roughly 10 TeV. This little hierarchy problem can be solved in little-Higgs models [1–3] by introducing new particles near 1 TeV, the effect of which is however sufficiently hidden not to show in the precision tests.

The Higgs sector of the standard model also contains the physics of flavor, that is, the hierarchy in the fermion masses and the mixing among different generations. What happens if we try to include flavor symmetry and its breaking in the little-Higgs models? At first sight, this seems impossible because of the much higher scale of the order of $10^4$ TeV at which flavor symmetry breaking should take place if we want it to agree with the known bounds on flavor changing neutral currents. However, these bounds depend on the specific realization of the symmetry breaking and they are not necessarily so strong if the flavor symmetry is only global and there are no flavor charged gauge bosons [4].

As we shall show, it is possible to take closely related breaking scales for both the electroweak and flavor symmetries and thus unify the two into a single little-Higgs model. Thanks to this unification we are able to both solve the little hierarchy problem and provide stable textures in the mass matrices of a well-defined type that correctly reproduce the mass hierarchies and mixings of quarks and leptons. The model has a characteristic spectrum testable at the LHC of new particles, in addition to those of the standard model, and a lightest Higgs boson mass more constrained than in the usual, electroweak only, little-Higgs models that turns out to be heavy, in the sense of preferring values larger than 200 GeV.

Some of the work in this paper has been already presented in Letter form [10]. This paper includes additional results and a more detailed discussion of the model and its consequences.

A. A stable standard model with a natural cut off around 10 TeV

For the standard model to be valid up to a scale around 10 TeV (as indicated by precision measurements), the amount of fine-tuning of the value of the bare mass required in order to keep the mass of the Higgs boson to its value of about a hundred GeV is of the order of 1% for 1-loop (quadratically divergent) radiative corrections coming from top-quark loops:

$$\frac{\lambda t \Lambda^2}{(4\pi)^2} \simeq (1 \text{ TeV})^2,$$

where $\Lambda = 4\pi f^2$ is the effective cut off and $f \simeq 1$ TeV the scale of the electroweak symmetry breaking, and 10% for similar loop corrections coming from the gauge bosons and the Higgs itself:

$$\frac{g^2 A^2}{(4\pi)^2} \simeq \frac{\lambda ^2 A^2}{(4\pi)^2} \simeq (700 \text{ GeV})^2.$$


This fine-tuning (the little hierarchy problem) can be considered a hint to new physics taking place at or around the 1 TeV scale. For instance, the problem would be greatly ameliorated if additional degrees of freedom were to cancel the 1-loop divergence and the first contribution were to arise only at the 2-loop order, thus providing an additional factor of roughly $(4\pi)^2$ in the suppression of these corrections and therefore bringing the amount of fine-tuning to an acceptable level.

This idea has provided the motivation for recent work on the so-called little Higgs models [1–3]. In these models the Higgs boson is first thought as a Goldstone boson and therefore exactly massless and with no potential. Its effective potential, and therefore mass and vacuum expectation value, is generated by an explicit symmetry breaking that however can only proceed by simultaneously breaking more than one symmetry. Because of this collective symmetry breaking, the effective potential does not contain quadratically divergent mass terms—in particular from the gauge and heavy top quark loops—and therefore the value of the Higgs mass is made stable against 1-loop radiative corrections with very little tuning of the value of the bare mass.

Many examples of little Higgs models have been introduced and discussed in the literature [1–3] and some of their experimental signatures (and constraints) have been already extensively discussed [5]. Even though some of these constraints are rather strong and tend to reintroduce some amount of fine tuning, we shall not be overly concerned with their impact since we are looking more for a consistent framework than with its detailed realization (see, however, ref. [6] for a simple way to lessen the constraints). In what follows, we are particularly interested in the so-called littlest Higgs model [1], in which the Higgs boson is a pseudo-Goldstone boson of the spontaneous breaking of a $SU(5)$ symmetry down to $SO(5)$; at low energies, the model contains only one Higgs doublet. The littlest Higgs model is of relevance in what follows because—as we shall argue in section II G—it is embedded in the flavorless limit of the model presented here.

B. Flavons and textures

One of the most tantalizing clue for physics beyond the standard model that we know of comes from the Higgs sector and the closely related flavor structure. Data on particle masses and mixing angles present us with a wealth of information not too dissimilar to that once offered by Mendeleev’s period table and seem to beg for a dynamical explanation of their regularities. These data are encoded in the standard model into the Yukawa lagrangian which gives mass to the fermions and shapes their mixing and mass hierarchies. This lagrangian is thus controlled by a (large) number of parameters that appear to be arbitrary insofar as their values are chosen by hand to match the experimental data; moreover, their values must be chosen in a precise manner and many of them vary across several orders of magnitude. The stability against radiative corrections of these patterns and hierarchies seems to require some amount of fine tuning. While any fine tuning of the parameters can always be seen as a mere coincidence (or, perhaps more speculatively, as anthropic selection at work), we take here the point of view that its explanation—like that for the little hierarchy—calls for new physics.

A first step in the direction of improving our understanding of the flavor structure of the standard model can be taken by re-organizing the parameters and considering the mass matrices of quarks and leptons not as $3 \times 3$ arbitrary matrices but as matrices having well-defined textures controlled by one or at most few parameters. In this picture, the mass matrices have entries that are powers of these few parameters modulated by dimensionless coefficients of order one—and thus requiring no further explanation. While this is not yet a dynamical model—it is really just “kinematics”—it helps in providing a framework in which to bring the dynamics eventually.

At least part of this dynamics comes from identifying the small parameters, the powers of which give rise to the textures, with the vacuum expectation values of some scalar fields with quantum numbers running across the horizontal family structure of fermions. In this picture—usually referred to as the Froggart-Nielsen mechanism [7]—the emerging textures are due to different charge assignments for the fermions, and therefore from the different powers of the small parameters, associated to the vacua of the scalar fields, making up the mass matrices arising from the Yukawa lagrangian.

The next, and crucial step consists in providing stability against radiative corrections for the patterns thus generated and therefore explaining away the apparent fine tuning of parameters encountered. This problem can be rephrased in terms of the naturalness of the dimensionful parameters of the model that must be protected against corrections that tends to bring all of them to the highest mass scale in the problem, usually the cut off of the effective field theory (all dimensionless parameters are then assumed of order 1 and therefore natural). Such a naturalness—in the ’t Hooft’s sense—is achieved by identifying the one or more symmetries that would be recovered in the limit of vanishing interactions and vacuum expectation values.

This problem of fine tuning and overall stability is present in all models trying to describe the mass matrices of the fermions, and the textures by which they are characterized, in terms of the vacuum expectation value $v$ of one, or more, scalar fields, the flavons [7, 8]. In these models, the texture is written in terms of the ratio $\varepsilon = v/f$, where
$f$ is now the flavor symmetry breaking scale and $\varepsilon$ is the small parameter of the texture typically of the order of the Cabibbo angle. These patterns tend however to be washed out by the quadratically divergent radiative corrections to the mass term $\mu^2$ that make the vacuum expectation value, for a generic quartic potential proportional to a parameter $\lambda \simeq 1$,

$$v^2 \simeq \mu^2 \simeq f^2$$

and therefore $\varepsilon = 1$.

A small $\varepsilon$ comes in a natural manner if the mass term is protected at the one loop level and only logarithmically divergent so that

$$v^2 = \mu^2 \simeq \frac{\log(A^2/f^2)}{(4\pi)^2} f^2$$

and we have $\varepsilon^2 \ll 1$ independently of the scale $f$. For this reason, the same collective symmetry breaking mechanism as in the little Higgs mechanism and the identification of the flavons as pseudo-Goldstone bosons of some horizontal symmetry were successfully applied in a recent series of papers [4, 9] to the problem of flavor physics.

In this approach—referred to, for obvious reasons, as the little flavon model—an SU(6) symmetry is spontaneously broken to Sp(6) thus giving rise to 14 (real) Goldstone bosons (a similar model has been discussed before in the context of weak physics in ref. [2]). Four subgroups [SU(2) × U(1)]^2 are gauged and their gauge couplings explicitly break the global SU(6) symmetry thus giving rise to a potential for the, at this point, two pseudo-Goldstone bosons that are doublets under the flavor symmetry. The quadratically divergent contribution to the 1-loop potential are however forbidden by the little-Higgs mechanism of collective breaking for which only logarithmic divergencies are allowed.

The surviving flavor symmetry is SU(2) × U(1), the spontaneous symmetry breaking of which comes from the Yukawa coupling to the right-handed neutrinos. The Yukawa lagrangian couples the little flavons to the fermions, accordingly with the charge assignment chosen, and, after spontaneous symmetry breaking, gives raise to the mass matrices with the desired textures in terms of the vacuum expectation values of the two flavor doublets. It is shown in [9] that such a model gives rise to characteristic textures that correctly reproduces all fermion masses and mixing angles.

### C. Little Higgs meets little flavon

As viable as the little flavon model is, it leaves open the question of how the flavons and the weak Higgs field can be accomodated within an unified picture. Since the mass textures are but a modulation of the vacuum expectation value of the Higgs boson, the interplay between electroweak and flavor symmetries must lead us toward an unified picture of the two symmetries and their spontaneous breaking at closely related, if not the same, scales.

The energy scale of any horizontal flavor symmetry breaking is usually thought as well separated from that of the electroweak symmetry breaking mainly because of the constraints on neutral flavor changing currents. The experimental bounds on flavor changing processes set rather stringent constraints on the value of the scale $f$ at which flavor symmetry must be broken. In the case of the little flavon model, this scale turns out [4] to be between $10^3$ and $10^4$ TeV. This bound comes from processes—like $K^0\bar{K}^0$ mixing—mediated by the flavor gauge bosons. It puts the flavor scale several order of magnitude higher than that of electroweak physics and makes it difficult to think of them in a unified manner. Moreover, radiative corrections from the flavor to the electroweak sector become dangerously large and bring back into the picture some unwelcome fine tuning. However, this result is heavily based on assuming the horizontal symmetry to be local and therefore having to include the effect of the corresponding gauge bosons. In the absence of these, the constraint can be relaxed and, depending on the specific model, the energy scale made closer and even the same as that of electroweak physics.

Interesting enough, contrary to those for the flavor gauge bosons, direct bounds on the effect of the scalar flavons are not very restrictive, giving, at least for some specific model, a scale $f$ of the order of the TeV. This observation suggests to make the flavor symmetry into a global (rather than local) symmetry and thus avoid the more stringent bounds on the gauge flavor bosons (that do not exist any longer) and bring the flavor symmetry breaking scale closer

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1. This global symmetry, as well as those of the little Higgs model, must be thought as arising at same intermediate scale, well below that of string theory where all symmetries are necessarily local.
to that of electroweak physics. The unification of flavor and electroweak symmetry is thus made possible and an explicit example of it is the main result of this work.

Needless to say, the above scenario—that is going to be realized in the model that follows—is still far from being a complete theory of flavor. In particular, it leaves open the question of the absolute value of the fermion masses, most notably the large difference between those of neutrinos and heavy quarks; this problem, and the much larger hierarchy implied, clearly requires a much deeper understanding of the dynamics in the ultraviolet and beyond the cut off of the model. Nevertheless, the model we discuss does set the scene for a more profound dynamical understanding of the physics of flavor by creating a framework (with a special, and rather restrictive, choice of textures for the mass matrices of the fermions) and identifying the relevant symmetries and degrees of freedom at or around the TeV scale that make an unification between flavor and electroweak physics possible within a natural model. In doing so, it says something specific about physics in the range to be explored by LHC, giving a (lightest) Higgs boson mass in a well defined range and particles in addition to those of the standard model to be discovered.

II. THE FLHIGGS MODEL [10]

In order to have a single, unified model à la little Higgs describing the entire flavor structure as well as the electroweak symmetry breaking, the Higgs boson and the flavons must be the pseudo-Goldstone bosons of the same spontaneously broken global symmetry. These pseudo-Goldstone bosons—we shall call them flhiggs—should transform under both flavor and electroweak symmetries. The symmetry breaking should leave the electroweak (or an extended symmetry, a subgroup of which is the electroweak) and the flavor symmetry unbroken. In a further step, the flavor and the electroweak symmetries break leaving, as in the standard model, the electric charge $U(1)_{Q}$ as the only unbroken symmetry.

To construct such a model, it is necessary first to identify the flavor and electroweak symmetry subgroups at the scale $f$ of the spontaneous symmetry breaking of the global symmetry. The simplest choice would seem to be a product of the flavor symmetry $G_{f}$ and the electroweak symmetry $[SU(2) \times U(1)]_{W}$, where for the flavor group $G_{f}$ we can take, without loss of generality, $U(N)_{F}$. In this case the flhiggs bosons should transform, for example if we take $U(N)_{F}$ to be $U(2) \simeq SU(2) \times U(1)$, as doublets in the fundamental representations of the two $SU(2)$ groups, the electroweak and the $SU(2)$ in $U(2)_{F}$. However we have to reject this choice since with the scalar fields as doublets in both groups the scale of flavor breaking will necessarily coincide with that of the electroweak breaking with undesirable consequences for the phenomenology of the model.

This holds true for any choice of $G_{f}$. We are therefore necessarily lead to extend the electroweak symmetry and the minimal extention gives us $[U(N)]_{F} \times [SU(3) \times U(1)]_{W}$. In this case if the flhiggs bosons transform in the fundamental representations of both groups, the breaking of the flavor symmetry can happen at a scale different from the electroweak, that is, there is a limit in which the flavor symmetry is broken and its breaking induces the breaking of the $[SU(3) \times U(1)]_{W}$ electroweak symmetry to the standard $[SU(2) \times U(1)]_{W}$. This extension brings into the model an extra neutral gauge boson and exotic fermion states necessary to complete the weak doublets. These additional states give rise to new physics with crucial phenomenological consequences for the model. The mass and mixing of the extra gauge boson affect the neutral currents and impose rather severe bounds on the parameters of the model. Moreover, the masses and mixing of the exotic with standard fermions must be controlled by some additional symmetry that we take for simplicity to be an ableian $U(1)_{f}$. The exotic fermions, being charged under this abelian symmetry, only weakly couple to the standard fermions and acquire heavier masses.

A. What is the horizontal flavor symmetry?

The flavor symmetry could, in principle be abelian or nonabelian, that is a $U(2)$ since, for three generation at least, a $U(3)$ would introduce no differentiation. Let then consider the nonabelian $SU(2)$ case. ² The flhiggs bosons arising as pseudo-Goldstone bosons in a model of this kind are in the fundamental representations of both the flavor and the weak $SU(2)$ groups. Therefore, they transform as $(2,3)$ and $(2,\bar{3})$ under the flavor-electroweak symmetry.

In order to construct flavor-electroweak invariant Yukawa term we have to choose the representations for the standard fermions. The left-handed fermions have to transform as a 3 of $[SU(3) \times U(1)]_{W}$ since we want a doublet when $[SU(3) \times U(1)]_{W}$ is broken to $[SU(2) \times U(1)]_{W}$. As already noticed, we are obliged to introduce at least one exotic left-handed fields for each quark and lepton family. On the contrary, the right-handed ones could be singlets of

² Which is the symmetry discussed in the little flavon model of ref. [9].
SU(3) or the third component of a triplet (anti-triplet) of weak SU(3). Notice that in the latter case we would have to introduce other two exotic right-handed fermions for each quark and lepton family.

We still have to assign the representations with respect to the flavor group. We could have singlets, doublets or triplets. We reject the last case since it is impossible to reproduce the right hierarchies by this choice for either left-handed or right-handed fermions or both. If we use singlets and doublets we have, for instance, that two left-handed fermion families form a flavor doublet and the third is a singlet. In this case the choice for the left-handed fermion representations severely restricts that of the right-handed while there is no mixing between the doublet and the singlet.

Consider for example the Yukawa term for the charged leptons and suppose that the second and the third family are in a flavor doublet, while the first family is a singlet with respect to the flavor symmetry. This assignment is motivated by the results obtained in [9]. Each left-handed lepton family forms a triplet of SU(3) , so we have

\[ L_L = \begin{pmatrix} v^e_L \\ e_L \end{pmatrix} = (1,3)_L \quad E_L = \begin{pmatrix} \nu^e_L \\ \tilde{e}_L \end{pmatrix} = (2,3)_L. \tag{5} \]

with \( i = 1,2 \), \( e^1 = \mu \) and \( e^2 = \tau \) and an exotic lepton for each family. In eq. (6) we have indicated in the brackets the fields representations with respect to flavor SU(2) and weak SU(3), respectively. There are only two possible choices for the representations of the right-handed charged leptons in order to have a Yukawa term involving \( L_L^T \) that give mass to the electron and these are

\[ \tau_R = (1,3) \rightarrow (1,\bar{3})_R (2,3)_\phi (2,\bar{3})_\phi (1,3)_L \]
\[ \bar{E}_R = (2,1) \rightarrow (2,1)_R (2,\bar{3})_\phi (1,3)_L, \tag{6} \]

and analogously for \( E_L \) with other two possibilities

\[ \tau_R = (1,1) \rightarrow (1,1)_R (2,\bar{3})_\phi (2,3)_L \]
\[ \bar{E}_R = (2,\bar{3}) \rightarrow (2,\bar{3})_R (2,3)_\phi (2,3)_L. \tag{7} \]

In eq. (7) we have indicated the fields by the indices: \( \phi \) stands for the pseudo-Goldstone bosons, \( R \) and \( L \) for right-handed and left-handed leptons respectively. By comparing eq. (6) with eq. (7) we see that there is no choice for the right-handed fermions representations that permits mixing between the first family charged lepton and the other two. As it happens, we have the same problem in the neutrino sector, and this means that it is impossible to reproduce the experimental lepton mixing matrix since we cannot have an angle different from zero between the first and the second family. In conclusion, we are forced to use only flavor singlets. Notice that we would arrive at the same conclusion if we had started with a doublet composed by the first and the second family or if we had considered the quark sector.

The previous analysis shows that the introduction of a nonabelian flavor symmetry is not helpful since we are forced to use only flavor singlets as representations of the standard fermions if we want to reproduce the correct textures in the mixing matrices. Such a symmetry could in principle be useful if the pseudo-Goldstone bosons content would be enlarged by making the fHiggs belong to different representations of SU(3) and the flavor group, for instance, by having weak singlets in addition to doublets. Our aim is to build a model as simple as possible and therefore we try avoiding such an enlargement. This leads us to taking the abelian group \( U(1) \) as our flavor symmetry.

### B. Spontaneous symmetry breaking

Our discussion so far has lead us to identify the low-energy symmetry we expect to see realized in the model as \( \text{SU}(3) \times U(1)_{W} \times U(1)_F \) plus the additional symmetry, which we take to be \( U(1)_X \), that controls the exotic fermions.

Once chosen the symmetry at the lower scale, we have to identify the minimal global symmetry, the spontaneous breaking of which gives rise to the pseudo-Goldstone bosons to be identified with flavons and Higgs boson. Since we need at least two copies of \( SU(3) \times U(1) \), plus two copies of an extra \( U(1) \) to control the masses of the exotic fermions, we end up with a group of rank 9, that we take to be \( SU(10) \).

The \( SU(10) \) global symmetry is spontaneously broken to \( SO(10) \) at the scale \( f \). This provides us with an effective theory with a cut off at the scale \( \Lambda = 4\pi f \). Fifty-four generators of \( SU(10) \) are broken giving 54 real Goldstone bosons we parametrize in a non-linear sigma model fashion as

\[ \Sigma(x) = \exp \left[ i \Pi(x)/f \right] \Sigma_0, \tag{8} \]
with \( \Pi(x) = t^a \pi^a(x) \), where \( t^a \) are the broken generators of \( SU(10) \), \( \pi^a(x) \) the fluctuations around the vacuum \( \Sigma_0 \) given by

\[
\Sigma_0 \equiv \langle \Sigma \rangle = \begin{pmatrix}
0 & I_{4 \times 4} & 0 & 0 \\
I_{4 \times 4} & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{pmatrix}.
\]  

(9)

The vacuum state (9) can be rotated into its canonical form \( I_{10 \times 10} \) by a change of basis. In this basis the breaking pattern is more evident but the sigma model dynamics more involved.

Within \( SU(10) \) we identify seven subgroups

\[
SU(10) \supset U(1)_F \times [SU(3) \times U(1)]_W^2 \times [U(1)_X]^2,
\]

(10)

where the \( U(1)_F \) is the global flavor symmetry while the \( [SU(3) \times U(1)]_W^2 \) are two copies of an extended electroweak gauge symmetry, the need of which we discussed in the previous section. The groups \( [U(1)_X]^2 \) are two copies of an extra gauge symmetry we need in order to separate standard fermions from the exotic fermions the model requires because of the enlarged \( SU(3) \) symmetry that turns the weak doublets into triplets.

As discussed in sect. I, we want the flavor symmetry proper to be global so as not to have in the theory flavor charged gauge bosons that would make impossible for flavor and weak symmetry breaking to be of the same order. On the other hand, all the other symmetries in addition to those of the standard model are local so as to reduce the number of Goldstone bosons in the physical spectrum.

The generators of the five \( U(1) \) are taken to be

\[
Y_{F_1} = \text{diag}(0, 0, 0, 1, 0, 0, 0, -1, 0, 0)/2 \\
Y_{W_1} = \text{diag}(0, 0, 0, 0, 1, 1, 1, 0, 0, 0) / \sqrt{6} \\
Y_{W_2} = \text{diag}(1, 1, 1, 0, 0, 0, 0, 0, 0, 0) / \sqrt{6}, \\
Y_{X_1} = \text{diag}(0, 0, 0, 0, 0, 0, 0, 0, 1, 0) / \sqrt{2}, \\
Y_{X_2} = \text{diag}(0, 0, 0, 0, 0, 0, 0, 0, 0, 1) / \sqrt{2},
\]

(11)

while the generators of the two copies of \( SU(3)_W \) can be identified with the corresponding generators \( Q^a_1 \) and \( Q^a_2 \) with \( a = 1, \ldots, 8 \) within \( SU(10) \). Note that \( Y_{F_1} \) and \( Y_{W_{1,2}} \) are generators of \( SU(10) \), while \( Y_{X_{1,2}} \) are not and their normalization is chosen for simple convenience.

The breaking of \( SU(10) \) into \( SO(10) \) also breaks the subgroups \( [SU(3) \times U(1)]_W^2 \times [U(1)_X]^2 \) and only a diagonal combination survives. On the contrary, the flavor symmetry \( U(1)_F \) survives the breaking and we eventually have that

\[
U(1)_F \times [SU(3) \times U(1)]_W^2 \times [U(1)_X]^2 \rightarrow U(1)_F \times [SU(3) \times U(1)]_W \times U(1)_X.
\]

(12)

The breaking in the gauge sector \( [SU(3) \times U(1)]_W^2 \times [U(1)_X]^2 \rightarrow [SU(3) \times U(1)]_W \times U(1)_X \) leaves 10 gauge bosons massive after eating 10 of the 54 real Goldstone bosons. The remaining 44 Goldstone bosons can be labeled according to representations of \( U(1)_F \times [SU(3) \times U(1)]_W \times U(1)_X \) symmetry:

- 2 complex fields \( \Phi_1 [3_{[1,1/2,0]}] \) and \( \Phi_2 [3_{[1,-1/2,0]}] \), accounting for 12 degrees of freedom. They transform as triplets of \( [SU(3)]_W \) and have the same \( U(1)_W \) and opposite \( U(1)_F \) charges. They are not charged under the exotic gauge symmetry \( U(1)_X \).
• 2 complex fields $\Phi_3 [3_{1,0,1/2}]$ and $\Phi_4 [3_{1,0,-1/2}]$, accounting for other 12 degrees of freedom. They transform as triplets of $[SU(3)]_W$ and have the same $U(1)_W$ and opposite $U(1)_X$ charge. They are not charged under the flavor symmetry $U(1)_F$.

• a sextet of complex fields $z_{ij} [6_{2,0,0}]$, for 12 degrees of freedom,

• 4 complex fields $s [1_{0,-1,0}]$, $s_1 [1_{0,-1/2,1/2}]$, $s_2 [1_{0,1/2,1/2}]$, $s_3 [1_{0,0,-1}]$, for the remaining 8 degrees of freedom.

In the above notation, the representations with respect to the $SU(3)_W$ are indicated between square brackets and the indexes are the $U(1)$ charges: the first refers to the weak group, the second to the flavor group and the third to the exotic.

In terms of these representations, the field $\Pi(x)$ can then be written as

$$
\Pi(x) = \begin{pmatrix}
0 & 0 & 0 & \Phi_1/\sqrt{2} & z_{ij} & \Phi_2/\sqrt{2} & s & s_1/2 & s_2/2 \\
0 & 0 & 0 & \Phi_1/\sqrt{2} & 0 & \Phi_2/\sqrt{2} & s & s_1/2 & s_2/2 \\
0 & 0 & 0 & \Phi_1/\sqrt{2} & 0 & 0 & \Phi_1/\sqrt{2} & \Phi_3/\sqrt{2} & \Phi_4/\sqrt{2} \\
z_{ij}^* & \Phi_2^*/\sqrt{2} & 0 & 0 & 0 & \Phi_1^*/\sqrt{2} & \Phi_3^*/\sqrt{2} & \Phi_4^*/\sqrt{2} & \Phi_5^*/\sqrt{2} \\
\Phi_1^*/\sqrt{2} & s^* & \Phi_1/\sqrt{2} & 0 & s_1^*/2 & s_2^*/2 \\
\Phi_2^*/\sqrt{2} & s_1^*/2 & \Phi_4^*/\sqrt{2} & s_2/2 & 0 & s_3 \\
\Phi_3^*/\sqrt{2} & s_2^*/2 & \Phi_3^*/\sqrt{2} & s_1/2 & s_3^*/2 & 0
\end{pmatrix},
$$

(13)

where we have put zeros for the components that are going to be eaten by the gauge fields becoming massive. All these fields are still Goldstone bosons with no potential; their potential arises after the explicit breaking of the symmetry to which we now turn.

C. Explicit collective symmetry breaking

The effective lagrangian of the pseudo-Goldstone bosons must contain terms that explicitly break the $SU(10)$ global symmetry. These terms provide masses of the order of the scale $f$ for the $s$, $s_i$ and $z_{ij}$ fields. However, each term separately preserve enough symmetry to keep the higgs fields $\Phi_i$ exact Goldstone bosons. Only the simultaneous action of two or more of the terms (collective breaking) turns them into pseudo-Goldstone with a potential, even though there is still no mass term. Quadratic terms for the higgs will come from the coupling to right-handed neutrino, as we shall discuss presently.

The effective lagrangian is given by the kinetic term

$$
\mathcal{L}_0 = \frac{f^2}{2} \text{Tr} (D^\mu \Sigma)(D_\mu \Sigma)^* ,
$$

(14)

the covariant derivative of which couples the pseudo-Goldstone bosons to the gauge fields:

$$
D_\mu \Sigma = \partial_\mu + ig_i W^a_i (Q^a_i \Sigma + \Sigma Q^a_i T) + ig_i y_i B_{i\mu} (Y_{W_i} \Sigma + \Sigma Y_{W_i}^T) + ik_i X_{i\mu} (Y_X \Sigma + \Sigma Y_X^T) \quad i = 1, 2,
$$

(15)

where $W^a_i, B_{i\mu}$ and $X_{i\mu}$ are the gauge bosons of the $SU(3)_{W_i}$, $U(1)_{W_i}$ and $U(1)_{X_i}$ respectively, $Q^a_i$, $Y_{W_i}$ and $Y_X$, their generators and $y_i$, the $U(1)_{W_i}$ charges.

The lagrangian in eq. (15) gives mass to the $z_{ij}$ and $s_3$ fields. On the other hand, each term of index $i$ preserves a $SU(3)$ symmetry so that only when taken together they can give a contribution to the potential of the higgs fields.

At this point the fields $s$, $s_1$ and $s_2$ are still massless. They play no important role in the model but cannot remain massless. To give them a mass, we introduce plaquette terms—terms made out of components of the $\Sigma$ field that preserve enough symmetry not to induce masses for the higgs fields.

As an example, one of these plaquette term can be written by looking at the Goldstone fields in the matrix eq. (13) after having rotate it by the vacuum $\Sigma_0$. We select the field $s^*$ to which we want to give mass in the components $(8, 8)$ and $(4, 4)$ of the matrix $\Sigma_0 \Pi(x)$. Both these choices leave a different $SU(9)$ symmetry acting on the remaining columns and rows that then prevents further terms to the potential of the fields that transform in the coset of $SU(10)/SU(9)$.
The other possible plaquette terms are given by choosing by the same token the two couples (4, 10) and (8, 9) and (4, 9) and (8, 10) components to give masses to the $s_1$ and $s_2$ respectively. Together they induce (harmless) terms and corrections into the coefficients of the $\text{flhiggs}$ potential.

After adding the plaquette terms, we therefore have the effective lagrangian:

$$
\mathcal{L} = \mathcal{L}_0 + a_i^2 f^2 \Sigma_{4,4} \Sigma_{4,4}^* + a_i^2 f^2 \Sigma_{8,8} \Sigma_{8,8}^* + a_i^2 f^2 \Sigma_{4,9} \Sigma_{4,9}^* \\
+ a_i^2 f^2 \Sigma_{8,10} \Sigma_{8,10}^* + a_i^2 f^2 \Sigma_{4,10} \Sigma_{4,10}^* + a_i^2 f^2 \Sigma_{8,9} \Sigma_{8,9}^*,
$$

where $a_i$ are coefficients of $O(1)$. The relative signs of the plaquette terms are in principle arbitrary and presumably fixed by the ulterior completion of the theory. At this level we simply require $m_{s_a}^2 > 0$.

In sec. (II B) we said that in the breaking of $[SU(3) \times U(1)]_V \rightarrow [SU(3) \times U(1)]_W$ nine gauge bosons become massive. We now see that their masses are given by

$$
M_{W^*_a}^2 = \frac{(g_1^2 + g_2^2)}{2} f^2 \quad M_{B^*_a}^2 = \frac{(g_1^2 + g_2^2)}{2} f^2 \quad M_{\chi}^2 = \frac{(k_1^2 + k_2^2)}{2} f^2,
$$

where $a = 1, \ldots, 8$.

These heavy gauge bosons—because of their mixing with those with lighter masses to be identified with the standard model gauge bosons—induce corrections on many observables that we know to be constrained by high-precision measurements, mainly coming from low-energy physics (like atomic parity violation and neutrino-hadron scattering). Their presence is the major constrain on the scale $f$ and, accordingly, the naturalness of the model, as discussed for the littlest-Higgs model in [5]. We shall come back to them when we discuss these constrains in the flhiggs model in section II.E.

The effective potential for the flhiggs fields is given by the tree-level contribution coming from the plaquettes and the one-loop Coleman-Weinberg effective potential arising from the gauge interactions:

$$
\frac{\Lambda^2}{16\pi^2} \text{Tr} \left[ M^2(\Sigma) \right] + \frac{3}{64\pi^2} \text{Tr} \left[ M^4(\Sigma) \left( \log \frac{M^2(\Sigma)}{\Lambda^2} + \text{const.} \right) \right],
$$

where the second, logarithmic term is very much suppressed and is not included in what follows.

The effective potential $O(f^{-2})$ is obtained by expanding the sigma-model field $\Sigma$ and is given by

$$
\mathcal{V}_0[\Phi_i, z_{ij}, s, s_1] = \frac{2}{3} y_i^2 f^2 |z_{ij}|^2 - \frac{i}{3} \frac{2}{\sqrt{2} f} (\Phi_1, \Phi_2, + \Phi_3, \Phi_4)^2 + \frac{2}{3} y_i^2 f^2 |z_{ij}|^2 + \frac{i}{3} \frac{2}{\sqrt{2} f} (\Phi_1, \Phi_2, + \Phi_3, \Phi_4)^2 \\
+ \frac{1}{3} y_i^2 f^2 |z_{ij}|^2 - \frac{i}{3} \frac{2}{\sqrt{2} f} (\Phi_1, \Phi_2, + \Phi_3, \Phi_4)^2 + \frac{1}{3} y_i^2 f^2 |z_{ij}|^2 + \frac{i}{3} \frac{2}{\sqrt{2} f} (\Phi_1, \Phi_2, + \Phi_3, \Phi_4)^2 \\
+ \frac{1}{4} y_i^3 f^2 |s_3 - \frac{1}{2} \frac{s_1 s_2}{2} + \Phi_1 \Phi_2 |^2 + a_2^2 f^2 |s - \frac{1}{2} \frac{s_1 s_2}{2} + \Phi_1 \Phi_2 |^2 \\
+ \frac{1}{4} y_i^3 f^2 |s_1 - \frac{1}{2} \frac{s_1 s_2}{2} + \Phi_1 \Phi_2 |^2 + \frac{1}{4} y_i^3 f^2 |s_2 - \frac{1}{2} \frac{s_1 s_2}{2} + \Phi_1 \Phi_2 |^2 \frac{1}{4} a_3^2 f^2 |s_3 - \frac{1}{2} \frac{s_1 s_2}{2} + \Phi_1 \Phi_2 |^2.
$$

From eq. (19) we see by inspection that the effective potential gives mass to the scalar fields $s$, $s_1$, $s_2$, $s_3$ and $z$, their masses given by

$$
m_s^2 = \frac{(2g_1^2 + 2g_2^2 + g_3^2 + g_4^2)}{6} f^2 \quad m_{s_1}^2 = \frac{(k_1^2 + k_2^2)}{2} f^2 \\
m_{s_2}^2 = \frac{(a_1^2 + a_2^2)}{2} f^2 \quad m_{s_3}^2 = \frac{(a_1^2 + a_2^2)}{2} f^2,
$$

respectively. The effect of these states must be included in the study of the low-energy observables together with that of the heavy gauge bosons.

After integrating out the massive states by means of their equations of motion, the potential, the four pseudo-Goldstone bosons $\Phi_i$, the flhiggs, is made of only quartic terms

$$
\mathcal{V}_1[\Phi_i] = \lambda_1 (\Phi_1^4 \Phi_1 + \Phi_2^4 \Phi_2) + \lambda_2 (\Phi_1^4 \Phi_3 + \Phi_4^4 \Phi_4) + \lambda_3 (\Phi_1^4 \Phi_3 + \Phi_4^4 \Phi_4) \\
+ \lambda_4 (\Phi_2^4 \Phi_2 + \Phi_4^4 \Phi_4) + \lambda_5 (\Phi_2^4 \Phi_2 + \Phi_4^4 \Phi_4) \\
+ \lambda_6 (\Phi_2^4 \Phi_2 + \Phi_4^4 \Phi_4) + \lambda_7 (\Phi_2^4 \Phi_2 + \Phi_4^4 \Phi_4).
$$

(21)
where the coefficients are given by

\[ \lambda_1 = \lambda_2 = \frac{(2g_1^2 + g_2^4)(2g_1^2 + g_2^4)}{2g_1^2 + g_2^2 + g_3^2} \]
\[ \lambda_3 = \lambda_4 = \frac{a_1^2a_2^2}{a_3^2 + a_4^2} \]
\[ \lambda_5 = \lambda_6 = \frac{a_5^2a_6^2}{a_5^2 + a_6^2} \]  

(22)

and

\[ \xi_1 = \frac{(2g_1^2 + g_2^2)(2g_1^2 + g_2^2)}{2g_1^2 + g_2^2 + g_3^2} + \frac{a_2a_3}{a_4^2 + a_3^2} \]
\[ \xi_2 = \frac{k_1^2k_2^2}{k_1^2 + k_2^2} + \frac{a_5^2a_6^2}{a_5^2 + a_6^2} \]
\[ \xi_3 = \frac{(2g_1^2 + g_2^2)(2g_1^2 + g_2^2)}{2g_1^2 + g_2^2 + g_3^2} + \frac{a_2a_3^2}{a_4^2 + a_3^2} \]
\[ \xi_4 = \frac{(2g_1^2 + g_2^2)(2g_1^2 + g_2^2)}{2g_1^2 + g_2^2 + g_3^2} + \frac{a_5^2a_6^2}{a_5^2 + a_6^2}. \]  

(23)

The coefficients \( \xi_1, \xi_3 \) and \( \xi_4 \) differ only by the plaquette contributions. Notice that we can take them equal if we assume the plaquette coefficients to be equal as well.

Quadratic terms

\[ \mathcal{V}_2[\Phi_1] = \mu_1^2(\Phi_1^4\Phi_1) + \mu_2^2(\Phi_2^4\Phi_2) + \mu_3^2(\Phi_3^4\Phi_3) + \mu_4^2(\Phi_4^4\Phi_4) \]  

(25)

that are necessary to induce vacuum expectation values for the fHiggs fields, and quartic terms of the type

\[ \mathcal{V}_4[\Phi_1] = \chi_1(\Phi_1^4\Phi_1)^2 + \chi_2(\Phi_2^4\Phi_2)^2 + \chi_3(\Phi_3^4\Phi_3)^2 + \chi_4(\Phi_4^4\Phi_4)^2 \]  

(26)

are not generated at one–loop in the bosonic sector discussed so far. In order to introduce them we couple the pseudo-Goldstone bosons to right-handed neutrinos with masses at the scale \( f \). This means that the flavor and electroweak symmetry breaking of the model is triggered by the right-handed neutrinos. This is done again along the lines of the little-Higgs collective symmetry breaking: to prevent quadratically divergent mass term for \( \Phi_1 \)—and thus render useless what done up to this point—the Yukawa Lagrangian of the right-handed neutrinos sector is constructed by terms that taken separately leave invariant some subgroups of the approximate global symmetry \( SU(10) \). In this way the fHiggs bosons receive a mass term only from diagrams in which all the approximate global symmetries of the Yukawa Lagrangian are broken. Because of this collective breaking, the one-loop contributions to the fHiggs masses are only logarithmic divergent.

The right handed neutrino sector is given by sixteen 10-components multiplets

\[ N^1_R = \begin{pmatrix} 0 & \nu^1_R \\ \nu^1_R & 0 \end{pmatrix} \]
\[ N^2_R = \begin{pmatrix} 0 & 0 \\ 0 & \nu^2_R \end{pmatrix} \]
\[ N^3_R = \begin{pmatrix} 0 & 0 \\ \nu^3_R & 0 \end{pmatrix} \]
\[ N^4_R = \begin{pmatrix} 0 & 0 \\ \nu^4_R & 0 \end{pmatrix} \]
\[ N^5_R = \begin{pmatrix} \nu^5_R & 0 \\ \nu^5_R & \nu^5_R \end{pmatrix} \]
\[ N^6_R = \begin{pmatrix} \nu^6_R & 0 \\ \nu^6_R & \nu^6_R \end{pmatrix} \]
\[ N^7_R = \begin{pmatrix} \nu^7_R & 0 \\ \nu^7_R & \nu^7_R \end{pmatrix} \]
\[ N^8_R = \begin{pmatrix} \nu^8_R & 0 \\ \nu^8_R & \nu^8_R \end{pmatrix} \]
\[ N^9_R = \begin{pmatrix} \nu^9_R & 0 \\ \nu^9_R & \nu^9_R \end{pmatrix} \]
\[ N^{10}_R = \begin{pmatrix} \nu^{10}_R & 0 \\ \nu^{10}_R & \nu^{10}_R \end{pmatrix} \]
\[ N^{11}_R = \begin{pmatrix} \nu^{11}_R & 0 \\ \nu^{11}_R & \nu^{11}_R \end{pmatrix} \]
\[ N^{12}_R = \begin{pmatrix} \nu^{12}_R & 0 \\ \nu^{12}_R & \nu^{12}_R \end{pmatrix} \]
\[ N_{13}^R = \begin{pmatrix} \nu_{13}^{R,\alpha} \\ \nu_{13}^{R,\beta} \end{pmatrix}, \quad N_{14}^R = \begin{pmatrix} 0 \\ \nu_{14}^{R,\alpha} \end{pmatrix}, \quad N_{15}^R = \begin{pmatrix} 0 \nu_{15}^{R,\beta} \end{pmatrix}, \quad N_{16}^R = \begin{pmatrix} 0 \\ \nu_{16}^{R,\alpha} \end{pmatrix}, \]

where \( \alpha = 1, 2, 3 \) and \( \eta = (0, 0, 0)^T \).

Only the \( N_{1i}^R \) with \( i = 1, \ldots, 8 \) couple directly to the fermions. The reason why we introduce so many fields is that we eventually want different, and independent, mass terms \( \mu_i \) to be induced in the effective potential by the right-handed neutrino sector and also we do not want right-handed neutrinos massless.

The Yukawa lagrangian for the right-handed neutrinos can be written in a \( SU(10) \)-invariant manner as

\[
\mathcal{L}_{Y}^{\nu_R} = \eta_1 f \left( \frac{\Phi^1_{R} \Phi_1}{2 f^2} - \frac{\Phi^1_{R} \Phi_2}{2 f^2} \right) \left( \nu_{R}^1 \Phi^5_{R} + \nu_{R}^1 \nu^5_{R} + \nu_{R}^1 \nu^5_{R} + \nu_{R}^1 \nu^5_{R} \right) + \eta_2 f \left[ \frac{\Phi^2_{R} \Phi_1}{2 f^2} - \frac{\Phi^2_{R} \Phi_2}{2 f^2} \right] \left( -2 s^2 s + s^2 s + s^2 s \right) + \eta_3 f \left[ \frac{\Phi^3_{R} \Phi_1}{2 f^2} - \frac{\Phi^3_{R} \Phi_2}{2 f^2} \right] \left( -2 s^2 s + s^2 s + s^2 s \right) + \eta_4 f \left[ \Phi^3_{R} \nu^7_{R} + \Phi^3_{R} \nu^7_{R} + \nu^7_{R} \nu^7_{R} \right] + \eta_5 f \left[ \nu^7_{R} \nu^7_{R} + \nu^7_{R} \nu^7_{R} + \nu^7_{R} \nu^7_{R} \right] + \eta_6 f \left[ \nu^7_{R} \nu^7_{R} + \nu^7_{R} \nu^7_{R} + \nu^7_{R} \nu^7_{R} \right] + \eta_7 f \left[ \nu^7_{R} \nu^7_{R} + \nu^7_{R} \nu^7_{R} + \nu^7_{R} \nu^7_{R} \right] + \eta_8 f \left[ \nu^7_{R} \nu^7_{R} + \nu^7_{R} \nu^7_{R} + \nu^7_{R} \nu^7_{R} \right].
\]

From eq. (29) we see that, after integrating out the neutrinos, the divergent one-loop contributions to the pseudo-Goldstone bosons masses in the effective potential \( \mathcal{V}_{3} \) of eq. (25) are given by

\[
\begin{align*}
\mu_1^2 & \simeq \left( \frac{\eta_1^2 \eta_5^2 + \eta_1^2 \eta_7^2}{4 \pi^2} \right) f^2 \log \frac{\Lambda^2}{M_\eta} \\
\mu_2^2 & \simeq \left( \frac{\eta_1^2 \eta_6^2 + \eta_2^2 \eta_7^2}{4 \pi^2} \right) f^2 \log \frac{\Lambda^2}{M_\eta} \\
\mu_3^2 & \simeq \left( \frac{\eta_3^2 \eta_6^2 + \eta_3^2 \eta_1^2}{4 \pi^2} \right) f^2 \log \frac{\Lambda^2}{M_\eta} \\
\mu_4^2 & \simeq \left( \frac{\eta_3^2 \eta_9^2 + \eta_3^2 \eta_2^2}{4 \pi^2} \right) f^2 \log \frac{\Lambda^2}{M_\eta},
\end{align*}
\]

where in the logarithm of eq. (30) we have generically indicated the mass of right handed neutrinos with \( M_\eta \simeq \eta f \).

From eq. (29), we can also estimate the one-loop divergent contributions to the quartic terms in the effective potential \( \mathcal{V}_{3} \) of eq. (26) coming from the right-handed neutrino sector. The coefficients \( \chi_i \) turn out to be logarithmically
divergent and proportional to four, not necessarily different, powers of $\eta$:
\[
\chi_i \simeq \eta_i^4 \frac{f^2}{(4\pi)^2} \log \frac{\Lambda^2}{M^2}.
\]  
(31)

They only play a minor role in what follows.

D. Vacuum expectation value

The effective potential for the pseudo Goldstone bosons is therefore made of the sum of eqs. (22), (25) and (26)
\[
V[\Phi] = V_1 + V_2 + V_3.
\]  
(32)

We want to find vacuum expectation values for the Higgs fields $\Phi$. We take the solution in which $\xi_1$ and $\xi_2$ are two vacuum values and simpler expressions for them in terms of the parameters. On the other hand, we do want to impose for simplicity that $v_1 = v_2 = v_3$. Under these assumptions, the field configuration of eq. (33) becomes
\[
\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_W/2 \\ v_F/2 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_W/2 \\ -v_F/2 \end{pmatrix}, \quad \langle \Phi_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ v_{X_1}/2 \end{pmatrix}, \quad \langle \Phi_4 \rangle = \begin{pmatrix} 0 \\ 0 \\ v_{X_2}/2 \end{pmatrix}.
\]  
(33)

The conditions to be satisfied, in order for eq. (33) to be a minimum, are the vanishing of the 24 first derivatives:
\[
\frac{\partial V[\Phi]}{\partial \Phi_i} |_{\Phi_i = \langle \Phi_i \rangle} = 0.
\]  
(34)

Substituting the field configuration of eq. (33) in eq. (34), we have 16 equations satisfied and eight conditions that reduce to the electric charge group $U(1)_Q$. Such a vacuum is, in general, given by the field configurations
\[
\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_W/2 \\ v_F/2 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_W/2 \\ -v_F/2 \end{pmatrix}, \quad \langle \Phi_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ v_{X_1}/2 \end{pmatrix}, \quad \langle \Phi_4 \rangle = \begin{pmatrix} 0 \\ 0 \\ v_{X_2}/2 \end{pmatrix}.
\]  
(36)

We take the solution in which $\xi_4 = \xi_3$ and $v_{F_1} = -v_{F_2}$. This solution is quite natural if, as pointed out in sec. II C, we consider all plaquette terms to come with equal strengths. We also impose for simplicity that $v_{X_1} = v_{X_2} = v_X$ and $v_{F_1} = v_{F_2} = v_F$. The values of $v_X$ and $v_F$ need not be equal but we shall identify them to obtain a model with only two vacuum values and simpler expressions for them in terms of the parameters. On the other hand, we do want to keep $v_F$ distinct from $v_W$ because otherwise the increase in symmetry would lead to the presence of extra Goldstone bosons and other undesirable phenomenological consequences for the model.

Under these assumptions, the field configuration of eq. (33) becomes
\[
\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_W/2 \\ v_F/2 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_W/2 \\ -v_F/2 \end{pmatrix}, \quad \langle \Phi_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ v_{X_1}/2 \end{pmatrix}, \quad \langle \Phi_4 \rangle = \begin{pmatrix} 0 \\ 0 \\ v_{X_2}/2 \end{pmatrix},
\]  
(36)

that is the vacuum expectation value we are going to use in what follows.

At this point we are left with six independent conditions that reduce to four if
\[
\xi_3 = \left(1 - \frac{v_W^2}{v_F^2}\right) \xi_1,
\]  
(37)

The four remaining equations yield the following expressions for the vacua as function of the coefficients of the effective potential:
\[
v_W^2 = \frac{(\lambda_3 + \lambda_6 - \lambda_4 - \lambda_5)(\mu_1^2 - \mu_2^2 + \mu_3^2 - \mu_4^2) + 2(\chi_2 - \chi_1)(\mu_1^2 - \mu_3^2) + 2(\chi_4 - \chi_3)(\mu_1^2 - \mu_2^2)}{4(\chi_1 - \chi_2)(\chi_3 - \chi_4) - (\lambda_4 - \lambda_3)^2 - (\lambda_5 - \lambda_6)^2},
\]  
(38)
and also yield the following conditions on the $\mu_i^2$ since we have reduced the number of degrees of freedom by imposing the previous equalities.

\[
\begin{align*}
-\mu_1^2(2\xi_1 - \xi_2 - 2\chi_3 - \lambda_2 - \lambda_6 - \lambda_3) + \mu_2^2(\xi_1 - 2\chi_2 - \lambda_1 - \lambda_4 - \lambda_6) \\
(\lambda_1 + \xi_1 + 2\chi_2)(2\xi_1 - \chi_2 - 2\chi_3 - \lambda_2 - \lambda_6 - \lambda_3) = (\lambda_6 + \lambda_3)(\xi_1 - 2\chi_2 - \lambda_1 - \lambda_4 - \lambda_6) \\
+ (\mu_1^2 + \mu_2^2)(-2\chi_3 + 2\chi_4 + \lambda_1 + \lambda_4 - \lambda_6 - \lambda_3) + (\mu_2^2 - \mu_3^2)(-2\chi_4 + 2\chi_1 + \lambda_4 - \lambda_5 + \lambda_6 - \lambda_3)
\end{align*}
\]

Also notice that all the relationships discussed can only be approximate since the coupling of the scalar fields to the fermions introduces small corrections.

The vacuum in eq. (36) breaks the global symmetry $U(1)_F$ and there seem to be a Goldstone boson in the spectrum. It can be removed by a mass term introduced by hand at an intermediate scale between $v_F$ and $f$. However, as it is possible to see after fermion will be introduced in the model, this global symmetry is actually anomalous. This means that the would-be Goldstone is not part of the physical spectrum. 3 Also notice that similar anomalies in the gauge groups are automatically compensated by the Goldstone bosons, as it always happens in spontaneously broken gauge theories [12]; they however reappear above the scale $f$ and may help in the determination of the UV completion of the theory.

In order to give a back-of-the-envelope estimate of this solution—and to see that it satisfies the requirements outlined in the introduction—it is useful to make a few approximations. Let us for instance take

\[
\xi_i \simeq \chi_i \simeq \lambda_1 = \lambda_2 = \lambda_3 = \chi \quad \lambda_4 = \lambda_5 = \lambda_6 = \lambda
\]

(40)

to reduce the number of coefficients. These approximations are rather natural and do not introduce any fine-tuning. Accordingly, the vacuum of the potential in eq. (32) can be given as

\[
v_F^2 = \frac{\mu_1^2 - \mu_2^2}{\lambda - \chi} \quad \text{and} \quad v_W^2 = \frac{\mu_2^2 - \mu_3^2 + \mu_4^2 - \mu_5^2}{\lambda - \chi}.
\]

(41)

In this simplified case, and by further taking $\lambda - \chi \simeq 1$, $\mu_1^2 - \mu_2^2 \simeq -\mu^2/2$ and $\mu_2^2 - \mu_3^2 \simeq 3\mu^2/8$ (and $\chi \simeq 1/2, \mu_3^2 \simeq 2\mu^2$ to satisfy eq. (38)) we obtain that

\[
v_W^2 = -\mu^2/4 \quad \text{and} \quad v_F^2 = -\mu^2
\]

(42)

so that for the electroweak vacuum given by its experimental value $v_W = -\mu/2 = 246$ GeV, we find $v_F \simeq 500$ GeV. For $f \simeq 1$ TeV, the parameter $k = v_W^2/f^2$ in the mass textures turns out to be small and of the order of the Cabibbo angle.

In section IV we will come back to the vacuum solution in eq. (33) and study it for arbitrary parameters to show the range of masses allowed for the scalar particles as well as for the other states of the model. Before that, we must study the gauge boson sector. As we are about to see, this sector is severely constrained and its consistency with precision electroweak data constrains the possible values of $v_F$ and $g'$ and therefore of $f$ if we want to keep the texture parameter small enough.

E. Gauge bosons and currents

After symmetry breaking, the model is described at low-energy by a set of gauge and scalar bosons. We discuss first the gauge boson sector. Its structure is complicated by the mixing of the standard model gauge bosons to the

---

3 Alternatively, one can think of the anomaly as an effective mass for the would-be Goldstone boson that, like the $y'$ of the $U(1)_A$ symmetry of chiral perturbation theory, becomes massive with a mass of the order of the symmetry breaking. In our case, this process would make the mass of the would-be Goldstone boson heavier than those of the other Higgs.
new states we have introduced. Our general strategy is to impose that the charged currents of the model coincide with those of the standard model. This done, we are essentially left with the theory of the standard model with the addition of a massive neutral gauge boson $Z'$ and we must check that its presence affects the $\rho$ parameter, the Weinberg angle $\theta_W$, the tree level coefficients of the neutral current and that the value of the mass of the $Z'$ are all within the experimental bounds. In this way two of the free parameters of the model, namely $v_F$ and $g'$ are fixed.

At the scale $f$, the symmetry surviving the spontaneous breaking of $SU(10)$ into $SO(10)$ is $SU(3)_V \times U(1)_W \times U(1)_X \times U(1)_F$ and we can write the effective kinetic lagrangian for the four scalar triplets $\Phi_i$ as

$$L^K_P = (D_\mu \Phi_1)^\dagger (D_\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D_\mu \Phi_2) + (D_\mu \Phi_3)^\dagger (D_\mu \Phi_3) + (D_\mu \Phi_4)^\dagger (D_\mu \Phi_4)$$

where the covariant derivatives are given by

$$D_\mu = \partial_\mu + i g W_\mu t^a + ig' x_\Phi B_\mu$$
$$D'_\mu = \partial_\mu + i g W_\mu t^a + ig' x_\Phi B_\mu \pm \frac{i}{2} X_\mu ,$$

with, as before in eq. (15), $W_\mu^a$ the gauge bosons of the $SU(3)$ electroweak group, $t^a$ its generators, $B_\mu$ the gauge boson of the $U(1)$ electroweak symmetry, $X_\mu$ that of the exotic $U(1)$ gauge symmetry and $g$, $g'$ and $k$ their coupling respectively, while $x_\Phi$ is the $U(1)$ extended electroweak charge of the triplets $\Phi_i$. Since the $SU(3) \times U(1) \times U(1)$ gauge symmetry at the low scale is the diagonal combination surviving the spontaneous breaking of $SU(10)$ into $SO(10)$ their couplings are given, respectively, by

$$g^2 = \frac{g^2 v^2}{g^2 + g'^2}, \quad g'^2 = \frac{g^2 g'^2}{g^2 + g'^2} \quad \text{and} \quad k^2 = \frac{k_1^2 k_2^2}{k_1^2 + k_2^2} .$$

When the triplets acquire the vacuum expectation values given by eq. (33), we are left with nine massive and one massless gauge boson; this latter being the photon.

The eight massive gauge bosons can be written as 3 complex and 3 real gauge bosons. The lightest complex fields and the lightest real can be identified with the standard model weak gauge bosons $W$ and $Z$. The remaining complex bosons are new massive charged gauge particles $\tilde{W}_{1,2}$. The masses of these complex gauge bosons are given by

$$m_W^2 = \frac{1}{4} g^2 v_W^2 , \quad m_{\tilde{W}_1}^2 = \frac{1}{2} g^2 v_F^2 \quad \text{and} \quad m_{\tilde{W}_2}^2 = \frac{1}{2} g^2 (v_F^2 + \frac{v_W^2}{2}) ,$$

respectively. The charged $W$ gauge bosons behave exactly like those of the standard model and can be directly identified with them. Contrary to the heavy gauge bosons in eq. (17), the gauge bosons $\tilde{W}_{1,2}$ do not mix with $W$ and therefore do not induce additional effective operators in the low-energy theory. Similarly, the gauge boson of the exotic $U(1)$ gauge symmetry does not mix and acquires a mass given by

$$m_X^2 = \frac{1}{4} k^2 v_F^2 .$$

The other three real gauge bosons, those associated to the diagonal generators of the $SU(3)$, $W_3^8$ and $W_8^3$, and the gauge boson of $U(1)_W$, $B$, do mix, and their mass matrix is given by

$$M_{WB}^2 = \left( \begin{array}{ccc} \frac{g^2 v_W^2}{4} & -g^2 v_W^2/4 \sqrt{3} & -g g' v_W^2/2 \\ -g^2 v_W^2/4 \sqrt{3} & g^2 (v_W^2/12 + 2 v_F^2/3) & g g' (v_W^2 - 4 v_F^2)/2 \sqrt{3} \\ -g g' v_W^2/2 & g g' (v_W^2 - 4 v_F^2)/2 \sqrt{3} & g'^2 (v_W^2 + 2 v_F^2) \end{array} \right) ,$$

where in eq. (48) $g' = g' x_\Phi$. The $3 \times 3$ mixing arises because of the $SU(3)$ weak group we started with and leads to the most characteristic (and constrained) new physics in the model.

One eigenvalue of the matrix $M_{WB}^2$ in eq. (48) is zero and corresponds to the photon, the other two depend on the values $v_F$ and $g'$, the lightest mass to be identified with that of the standard model $Z$, the heaviest with an extra gauge boson $Z'$.

The mixing between $W_3$, $W_8$ and $B$ is delicate since it gives rise to electric and neutral currents for the standard fermions. We fix the value of $v_F$ and $g'$ by imposing that the electric and neutral currents in our model coincide with those of the standard model. In order to analyze the neutral currents, consider the orthogonal matrix $U_W$ that diagonalize $M_{WB}^2$ according to

$$\text{diag}(0, M_Z^2, M_Z'^2) = U_W^T M_{WB}^2 U_W .$$
Once $g$ and $v_W$ are fixed by their standard model values, the entries of the matrix $U_W$—three of which are independent variables—depend on the parameters $\hat{g}'$ and $v_T$ that we are going to determine by requiring consistency with the experimental data.

Consider now the interactions between a fermion triplet (antitriplet) of $SU(3)_C$, $Q_L$ ($Q^*_L$), of $U(1)_W$ charge $x_L$, and two fermion singlets of $SU(3)_C$, $\psi^{1,2}_R$, of $U(1)_W$ charge $y_R^{1,2}$ respectively and a fermion singlet of $SU(3)_C$, $\psi_R$, of $U(1)_W$ charge $y_R$ and of $U(1)_X$ charge $-1/2$, with the electroweak gauge bosons, that is we neglect the exotic $X$-current. The first two components of the left-handed triplet (antitriplet) $Q_L (Q^*_L), \psi_R$ and $\psi^2_L$ form a $SU(2)_W$ Standard Model doublet (antidoublet), and when $SU(3)_C \times U(1)_W \times U(1)_X$ is broken into $U(1)_Q$, $\psi^j = \psi^j_L + \psi^j_R$ has electric charge $Q_f$, with $j = 1, 2$. At the same time, the third component of the triplet (antitriplet) $Q_L (Q^*_L), \psi_L$ and the exotic $SU(3)_C$ singlet $\psi_R$ give rise to an electric charged fermion $\tilde{\psi} = \tilde{\psi}_L + \tilde{\psi}_R$, with charged $Q_{f_1}$, where the index 2 refers to the second component of the triplet (antitriplet) $Q_L (Q^*_L)$. Dividing the Standard model doublet (antidoublet) component,

The kinetic lagrangian is given by

$$L_K^f = \overline{Q_L} \gamma \cdot D Q_L + \overline{\psi_R} \gamma \cdot D \psi_R + \overline{\psi^2_L} \gamma \cdot D \tilde{\psi}_R,$$

for a triplet and

$$L_K^{f*} = \overline{Q^*_L} \gamma \cdot D^* Q^*_L + \overline{\psi_R} \gamma \cdot D \psi_R + \overline{\psi^2_L} \gamma \cdot D \tilde{\psi}_R,$$

for an antitriplet, with

$$D_\mu = \partial_\mu + igW^a_\mu t^a + ig' x^j_{L,R} B_\mu,$$

where in eq. (52) have been used the same notations as in eq. (44). Consider only the terms in eqs. (50)–(51) that give rise to the electromagnetic and the neutral current for all the fermions, that is

$$L_K = \overline{\psi^j_L} \gamma^\mu \left( \partial_\mu + ig T_3 f_J W^{3\mu} + ig' x^j_{L,R} B_\mu \right) \psi^j_L$$  

$$+ \overline{\psi_R} \gamma^\mu \left( \partial_\mu + ig' x^j_{L,R} B_\mu \right) \psi^j_R$$  

$$+ \overline{\psi^2_L} \gamma^\mu \left( \partial_\mu + ig W^8_\mu + ig' x^j_{L,R} B_\mu \right) \tilde{\psi}_L + \overline{\psi_R} \gamma^\mu \left( \partial_\mu + ig' \tilde{x}_R B_\mu \right) \tilde{\psi}_R,$$

where we have explicit the standard model doublet (antidoublet) components $\psi^{1,2}_L$ and the exotic fermion $\tilde{\psi}_L$ and where $p$ is equal to 1 or $-1$ for the left handed fermion coming from a triplet or an antitriplet respectively.

The gauge bosons $W^3_\mu, W^8_\mu$ and $B_\mu$ mix through the $U_W$ of eq. (49) giving the photon, $A_\mu$ the gauge boson $Z_\mu$ and an heavy $Z'$-type gauge boson, $Z'_\mu$, in particular we have

$$\begin{pmatrix} W_3 \\ W_8 \\ B \end{pmatrix} = U_W \begin{pmatrix} A \\ Z \\ Z' \end{pmatrix}.$$  

Substituting the expressions coming from eq. (54) in eq. (53) we can write the electric, and the neutral and the extra neutral currents using the parametrization given in [15]

$$L' = -e Q_f \overline{\psi^j} \gamma^\mu \psi^j A_\mu - \frac{e}{2swcw} \left(1 + \frac{\alpha T}{2}\right) \overline{\psi \gamma^\mu} (g\gamma^3 - g' A \gamma^5) \psi^j Z_\mu$$  

$$- \frac{e}{2swcw} \overline{\psi^j} \gamma^\mu (\tilde{h}^j - h^j A \gamma^5) \psi^j Z'_\mu$$  

$$- eQ_2 \overline{\psi \gamma^\mu} \tilde{A}_\mu - \frac{e}{2swcw} \overline{\psi \gamma^\mu} (g\gamma^3 - g' A \gamma^5) \overline{\psi} Z_\mu$$  

$$- \frac{e}{2swcw} \overline{\psi \gamma^\mu} (\tilde{h}^3 - h^3 A \gamma^5) \overline{\psi} Z'_\mu,$$

where $sw$ and $cw$ are sine and cosine of the Weinberg angle $\theta_W$, $T$ is one of the oblique parameters,

$$g_{V,A} = g_{V,A}^{SM} + \tilde{g}_{V,A},$$
with

$$g^{i\, SM}_V = T_{3j} - 2Q_js^2 \quad \text{and} \quad g^{i\, SM}_A = T_{3j},$$

(57)

and $g^{i\, V,A}_V$ and $h^{3\, V,A}_V$ are related to the neutral currents of the exotic fermions $\tilde{\psi}$, as we shall show in section III below are only weakly coupled to the standard model states. The coefficients $\tilde{g}_{V,A}$ contain the deviation from the standard model, $\tilde{h}_{V,A}$ the strength of the coupling of $Z'$ to the standard model fermions.

First of all, the entries of the orthogonal matrix $U_W$ have to satisfy the following conditions in order to have in eq. (55) the correct electric charge current for the standard and exotic fermions:

$$\begin{align*}
\frac{U_{W_{11}}}{U_{W_{11}}} &= s_W \\
\frac{U_{W_{21}}}{U_{W_{11}}} &= \frac{1}{\sqrt{3}} \\
\frac{1}{U_{W_{11}}}(U_{W_{21}}, \frac{p}{2\sqrt{3}} + x_L \frac{g'}{g} U_{W_{11}}) &= x_L^{SM} \\
g' \frac{x^l_R}{g} U_{W_{11}} &= Q_3 \\
g' \frac{x^r_R}{g} U_{W_{11}} &= Q_2,
\end{align*}$$

(58)

where $x_L$ and $x_L^r$ are the extra $U(1)_W$ fermion charges, while $x_L^{SM}$ is the $U(1)_W$ standard model charge of the electroweak doublet $\psi_L$. The conditions of eq. (58) together with

$$\frac{1}{U_{W_{11}}}(U_{W_{21}}, \frac{1}{\sqrt{12}} + \frac{g'}{g} U_{W_{11}}) = 0,$$

(59)

that follows by inserting eq. (58) in eq. (43) and imposing zero electric charge for the second and the third components of the triplets $\Phi_i$, completely determines all the independent parameters of the matrix $U_W$.

Since the mass matrix of eq. (48) depends on $g' = g'x_\Phi$ the other three conditions give us the values of $x_L/x_\Phi$, $x_L^r/x_\Phi$ and $x_R/x_\Phi$, that is the fermion charges in units of the triplet $\Phi_i$ charges, with the further constrain on $U_{W_{31}}$ of giving rational numbers for the charges.

By equating now the neutral currents, we obtain that

$$\begin{align*}
(1 + \frac{\alpha T}{2}) &= c_W U_{W_{12}}(1 - \frac{U_{W_{32}}}{U_{W_{12}}}) \\
(1 + \frac{\alpha T}{2}) s^2 &= -c_W \frac{U_{W_{32}}}{U_{W_{31}}} U_{W_{12}} \\
(1 + \frac{\alpha T}{2}) \tilde{g}_V &= (1 + \frac{\alpha T}{2}) \tilde{g}_A = p c_W \frac{U_{W_{32}}}{2\sqrt{3}} (1 - \frac{U_{W_{32}}}{U_{W_{12}}}) \\
\tilde{h}_V &= T_{3j} U_{W_{13}} \left(1 - \frac{U_{W_{33}}}{U_{W_{13}}}ight) + 2 \frac{U_{W_{33}}}{U_{W_{13}}} U_{W_{12}} Q_j + p \frac{U_{W_{32}}}{\sqrt{12}} \left(1 - \frac{U_{W_{33}}}{U_{W_{13}}}ight) \\
\tilde{h}_A &= T_{3j} U_{W_{13}} \left(1 - \frac{U_{W_{33}}}{U_{W_{13}}}ight) + \frac{U_{W_{33}}}{2\sqrt{3}} \left(1 - \frac{U_{W_{33}}}{U_{W_{13}}}ight),
\end{align*}$$

(60)

A more complete analysis would require that also the corrections arising from the effective operators induced by the heavy gauge bosons in eq. (17) be included. They are important because they violate the $SU(2)$ custodial symmetry of the standard model. They affect the relationships in eq. (60) to $O(v_W^2/f^2)$ and, in the littlest Higgs model of ref. [1], force the scale $f$ to be above 4 TeV [5]. As already mentioned, these constrains can be lessen by introducing an additional discrete symmetry [6]. Notice that the overall fit of these corrections against the experimental electroweak data can in principle be improved in the presence of the fhiggs model of the additional parameters in eq. (60) thus lowering the scale $f$ with respect to that found in the case of the littlest Higgs model.

As we said in section I, we are more interested in the consistency of the framework than in its detailed realization and therefore we neglect, in this work, these $O(v_W^2/f^2)$ corrections and consider eq. (60) as it stands.

The parameters and coefficients in eq. (60) are constrained by precision measurements of neutral currents in low-energy observables like atomic parity violation in atoms and neutrino-hadron scattering. The mass of the $Z'$ gauge
boson is bounded by data on Drell-Yan production (with subsequent decay into charged leptons) in $p\bar{p}$ scattering to be larger than 690 GeV \cite{16} and this constrain must be included as well. We require that deviation in the $\rho$-parameter

$$\rho = 1 + \alpha T$$

and the Weinberg angle be within $10^{-3}$ while the $\tilde{g}$ coefficients in eq. \eqref{60} be less than $10^{-2}$. This choice put these deviations in the tree-level parameters in the ball park of standard-model radiative corrections.

The importance of these constraints resides in their fixing the values of the free parameters $v_F$ and $\tilde{g}' = g' x_\Phi$. The bound on the $\rho$ parameter essentially fixes the effective gauge coupling

$$\tilde{g}' \simeq 0.13.$$  

For simplicity we take $x_\Phi = 1$ so we have $\tilde{g}' = g'$.

Once $\tilde{g}'$ has been fixed to this value, the bound on the mass of $Z'$ requires

$$v_F \gtrsim 1260 \text{ GeV}.$$  

We would like to have $v_F$ as close to $v_W$ as possible but the phenomenological constraints force it to a higher scale.

The rather large value we must take for $v_F$ does imply unfortunately that some amount of fine tuning in the parameters of the potential in eq. \eqref{19} is present. If we go back to our back-of-the-envelope estimate in section II D, we see that while there we had $v_f \approx 2v_W$ with no fine tuning (that is, the coefficient were chosen with a tuning of one out of four or 25\%) on the values of the $\mu_i$ coefficients, we now must have $v_f \approx 4v_W$ that can be obtained by taking, for instance, $\mu_1^2 - \mu_2^2 \approx -\mu^2/2$ and $\mu_3^2 - \mu_4^2 \approx 24\mu^2/50$ that means 1 out of 25, that is a fine tuning of 4\%. The actual fine-tuning in the model is however less than this because of the larger number of parameters involved and roughly of 10\% or less.

**F. The scalar sector and the lightest flhiggs boson**

We now turn to the scalar sector of the model. The number of scalar bosons can readily be computed: the number of degrees of freedom of 4 complex triplets is 24. Of these 9 are eaten by the gauge fields, while 1—the would-be Goldstone boson of the spontaneous breaking of the $U(1)_F$ global symmetry—is eliminated, after introducing the fermions in the model, by the anomaly. Therefore, the scalar sector contains $24 - 10 = 14$ massive fields. To describe them, we parametrize the $\Phi_i$ triplets with respect to these fifteen fields as follows

\[
\Phi_i = \begin{pmatrix}
\frac{u_{11}^\rho \rho_1}{2} \\
\frac{(v_W + u_{1i}^\delta \delta_i + u_{1j}^\varphi \varphi_j)/2}{2} \\
\frac{(v_F + u_{2i}^\delta \delta_i + u_{2j}^\varphi \varphi_j)/2}{2}
\end{pmatrix}
\]

\[
\Phi_2 = \begin{pmatrix}
\frac{u_{12}^\rho \rho_2}{2} \\
\frac{(v_W + u_{3i}^\delta \delta_i + u_{3j}^\varphi \varphi_j)/2}{2} \\
\frac{(v_F + u_{4i}^\delta \delta_i + u_{4j}^\varphi \varphi_j)/2}{2}
\end{pmatrix}
\]

\[
\Phi_3 = \begin{pmatrix}
\frac{u_{22}^\rho \rho_2}{2} \\
\frac{(v_W + u_{5i}^\delta \delta_i + u_{5j}^\varphi \varphi_j)/2}{2} \\
\frac{(v_F + u_{6i}^\delta \delta_i + u_{6j}^\varphi \varphi_j)/2}{2}
\end{pmatrix}
\]

\[
\Phi_4 = \begin{pmatrix}
\frac{u_{32}^\rho \rho_2}{2} \\
\frac{(v_W + u_{6i}^\delta \delta_i + u_{6j}^\varphi \varphi_j)/2}{2} \\
\frac{(v_F + u_{7i}^\delta \delta_i + u_{7j}^\varphi \varphi_j)/2}{2}
\end{pmatrix}
\]

with $i = 1, \ldots, 4$ and $j = 1, \ldots, 7$ and where $u_{ij}^{\rho,\delta,\varphi}$ are the entries of the unitary matrix which diagonalizes the mass matrix defined as

$$M_{\Phi}^{2} = \frac{\partial^2 V[\Phi]}{\partial \Phi_i \partial \Phi_j} \bigg|_{\Phi = \langle \Phi \rangle},$$

and can be written in terms of the coefficients of the effective potential.

The scalar fields $\rho_k$, $\delta_i$, and $\varphi_j$ are the Higgs-like components of the flhiggs fields and the most interesting experimental signature of the model.
The fields $\rho_{1,2}$ are electrically charged, their masses given respectively by

$$m_{\rho_1}^2 = 4 \xi_1 (v_F^2 - v_W^2) \quad \text{and} \quad m_{\rho_2}^2 = (\xi_1 - \xi_2) v_F^2 - \xi_1 v_W^2.$$ \hfill (66)

We shall call the lightest of the two $h^{\pm}$.

The masses of the neutral $\delta_i$ fields are given by

$$m_{\delta_1}^2 = 4 \xi_1 (v_F^2 - v_W^2)$$

$$m_{\delta_3}^2 = 2 \xi_1 v_F^2 - \xi_1 v_W^2 \left(1 - \frac{v_W^2}{v_F^2}\right)$$

$$m_{\delta_4}^2 = (\xi_1 - \xi_2) v_F^2 - \xi_1 v_W^2.$$ \hfill (67)

The missing $\delta_2$ field, that in the diagonalization appears as a massless state, is the would-be Goldstone boson eliminated by the anomaly.

The masses of the fields $\varphi_j$ are obtained by diagonalization of the remaining sub-matrix. This sub-matrix written in terms of the vacua and the coefficients of the potential has a cumbersome form that is not particularly inspiring and that we do not include. We do not have an exact diagonalization for it but it must contain the lightest neutral scalar boson that we call $h^0$. This can be understood by thinking at one single Higgs triplet for which the $\delta_i$ part would correspond to the imaginary component and the $\phi_j$ to the real part and therefore Higgs-like component.

We study the scalar sector spectrum numerically in section IV to obtain an estimate of the allowed values for $m_{h^0}$ and $m_{h^{\pm}}$ for arbitrary $O(1)$ coefficients in the potential and $v_F$ fixed to the values determined in the previous section.

G. The flavorless limit

In the limit in which we factorize out the flavor part by taking $v_F = f$, the model has the littlest Higgs model of ref. [1] embedded inside. We can identify within the global symmetry $SU(10)$ a reduced symmetry $SU(5)$. The two f-Higgs fields $\Phi_3, \Phi_4$ decouple from this subsector that feels no $U(1)_X$ symmetry. In the notation of [1], the f-Higgs fields $\Phi_{1,2}$ go into the Higgs boson $h$, while the fields $z_{ij}$ go into the weak triplet field $\phi$. Clearly, all the Yukawa coupling of the next section become trivial and the fermion masses degenerate if we take the Yukawa coefficients to be all of $O(1)$.

III. INTRODUCING THE FERMIONS

According to the rules of the little-Higgs mechanism, the coupling of the fermions to the f-Higgs must proceed by preserving enough symmetry not to give rise to 1-loop quadratic divergent contributions to their masses. This means that for every fermion with an Yukawa coupling of $O(1)$ we must introduce one (or even more) state to cancel the divergent diagram. This procedure brings in two more sets of fermions, one for the standard model fermions with large Yukawa couplings and one for the exotic states we introduced to complete the $SU(3)$ triplets.

Even though the introduction of new fermions seems to lead us to a structure of Baroque richness, notice that these states nicely fall into the fundamental representations of $SU(10)$ giving a natural structure to the Yukawa interactions in terms of the larger symmetry group that can be written in general, and by neglecting for the moment the flavor group, as

$$\mathcal{L}_Y \simeq \lambda_1 \chi X + \lambda_2 \bar{\chi} \Sigma \chi.$$ \hfill (68)

where $X$ is a decuplet of fermions in the fundamental representation of $SU(10)$.

The Yukawa lagrangians at the scale $f$ is obtained by writing the $SU(3)_W \times U(1)_X \times U(1)_F$ invariant terms involving the four triplets $\Phi_1, \Phi_2, \Phi_3$ and $\Phi_4$, and the fermions. Standard model left-handed doublets are members of an $SU(3)_W$ triplet, the third component being an exotic fermion.

To help the reader in keeping track of the various terms, Tables I–IV contain the representations and the charge assignments with respect to the exotic, the flavor and the electroweak groups of all fermions and f-Higgs bosons, the latter having been determined solving eq. (58) and eq. (59).
TABLE I: Representations and charges assignments for the higgs bosons.

|   | $U(1)x$ | $U(1)_F$ | $SU(3)_W$ | $U(1)_W$ |
|---|---------|----------|-----------|----------|
| $\Phi_1$ | 0       | 1/2      | 3         | 1        |
| $\Phi_2$ | 0       | $-1/2$   | 3         | 1        |
| $\Phi_3$ | 1/2     | 0        | 3         | 1        |
| $\Phi_4$ | $-1/2$  | 0        | 3         | 1        |

TABLE II: Representations and charges assignments for the quarks. Different families run over the index $i$; they differ only for the flavor charges that are written as $(q_1, q_2, q_3)$ for, respectively, the first, second and third family. $U(1)_W$ charges are determined by the data constrains (see text of main body).

|   | $U(1)x$ | $U(1)_F$ | $SU(3)_W$ | $U(1)_W$ |
|---|---------|----------|-----------|----------|
| $Q_L^i$ | $\left( \begin{array}{l} Q_L^i \\ 0 \\ U_L^i \\ 0 \end{array} \right)$ | $\begin{cases} d_L \\ u_L \\ \tilde{u}_L \\ 0 \end{cases}$ | 0 | $\left( \begin{array}{c} 9/2, 7/2, 2/3 \end{array} \right)$ | $\bar{3}$ | 1 |
| $\tilde{Q}_L^i$ | $\begin{cases} m_L^i \\ n_L^i \end{cases}$ | $\begin{cases} 0 \\ (9/2, 7/2, 2/3) \end{cases}$ | 3 | 3 |
| $U_L^i$ | $\begin{cases} 0 \\ (3, 1, 0) \end{cases}$ | $U_L^i$ | $1/2$ | $\left( \begin{array}{c} 9/2, 7/2, 2/3 \end{array} \right)$ | 1 | 2 |
| $U_L^i$ | $\begin{cases} 0 \\ (4, 3, 1) \end{cases}$ | 1 | 2 |
| $u_L^i$ | $\begin{cases} 0 \\ 0 \end{cases}$ | $u_L^i$ | $0$ | $(3, 1, 0)$ | 1 | $-2$ |
| $\tilde{u}_L^i$ | $\begin{cases} 0 \\ 0 \end{cases}$ | $\tilde{u}_L^i$ | $-1/2$ | $\left( \begin{array}{c} -9/2, -7/2, -3/2 \end{array} \right)$ | 1 | $-2$ |
| $\tilde{d}_L^i$ | $\begin{cases} 0 \\ 0 \end{cases}$ | $\tilde{d}_L^i$ | $0$ | $(7/2, 5/2, 5/2)$ | 1 | 1 |

A. Quarks

In order to avoid large quadratic corrections to the higgs masses induced by divergent one-loop contributions from the heaviest fermions present in the model, that is the top and the exotic quarks (and leptons) that complete the electroweak triplets, we introduce for each family a number of colored Weyl fermions, both triplets of the $SU(3)_W$ electroweak gauge group and singlets. Their charges are all summarized in tab. (II). The number of multiplets and singlets introduced is the smallest number that permit us to write a quark Yukawa lagangian composed by terms that singularly preserve enough symmetry in order to keep the four triplets $\Phi_1, \Phi_2, \Phi_3$ and $\Phi_4$ massless. In this way quadratic divergent contributions to the higgs masses arise only at two-loops.
The Yukawa lagrangian for the quarks is given by

\[
\mathcal{L}_q^q = \lambda^{u1}_{ab} f U_L^a \Sigma Q_L^b \left( \Sigma_{4,4} \right) |y_{Q1} \pm y_{Q2}| + \lambda^{u2}_{ab} f u^a \Sigma_{4,4} \left( \Sigma_{4,4} \right) |y_{Q1} \pm y_{Q2}|
\]

\[
+ \tilde{\lambda}^{u1}_{ab} f U_L^a \Sigma Q_L^b \left( \Sigma_{4,4} \right) |y_{Q1} \pm y_{Q2}| + \tilde{\lambda}^{u2}_{ab} f u^a \Sigma_{4,4} \left( \Sigma_{4,4} \right) |y_{Q1} \pm y_{Q2}|
\]

\[
+ \eta^{u}_{ab} f \tilde{Q}_L^a \tilde{Q}_L^b \left( \Sigma_{4,4} \right) |y_{Q1} \pm y_{Q2}| + \lambda^{d}_{ab} d^a \left( \epsilon_{ijk} Q_L^b \Sigma_{4,4} \Sigma_{8,4} \Sigma_{8,4} \right) \left( \Sigma_{4,4} \right) |y_{Q1} \pm y_{Q2}| \tag{69}
\]

with \(a = 1, 2, 3\) and \(d^{1,2,3}_{4,4} = d^i L, s^i L, b^i L\). \(y_{Q1}^q\) are the flavor charges of the \(Q^a\) and \(\tilde{Q}^a\) triplets, now members of the \(Q^a\) multiplets defined in tab. (II), \(y_{Q2}^q\) that of the \(U^a\) multiplets and \(y_{Q3}^q\) the flavor charges of the Weyl fermions \(d^a\). In eq. (69) the terms with the coefficients \(\lambda^{u1}_{ab}\) preserve an \(SU(8)\) subgroup of the approximate global symmetry \(SU(10)\) while the terms with the coefficients \(\tilde{\lambda}^{u1}_{ab}\) break it but preserve an \(SU(9)\) subgroup of \(SU(10)\). Analogously, the terms with coefficients \(\lambda^{u2}_{ab}\) and \(\tilde{\lambda}^{u2}_{ab}\) preserve different subgroups of \(SU(10)\) making possible the protection of the higgs masses through the collective symmetry breaking mechanism.

The \((4, 4)\) component of the sigma model field \(\Sigma\) that appears in eq. (69) is the only \(SU(8)\) and \(SU(9)\) singlet and the only possible field we can include to balance the flavor charges to make eq. (69) invariant.

Expanding the \(\Sigma\) and keeping only the terms involving the \(\Phi_i\) in eq. (69) yields

\[
\mathcal{L}_q^q = \lambda^{u1}_{ab} f \left[ u^a U_L^b \left( U_L^{a} \right)^T \left( \frac{1}{f} \Phi_1 + \frac{1}{f} \Phi_2 \right) \right] |y_{Q1} \pm y_{Q2}| + \lambda^{u2}_{ab} f u^a U_L^b \left( U_L^{a} \right)^T |y_{Q1} \pm y_{Q2}|
\]

\[
+ \tilde{\lambda}^{u1}_{ab} f \left[ \tilde{u}^a U_L^b \left( U_L^{a} \right)^T \left( \frac{1}{f} \Phi_1 + \frac{1}{f} \Phi_2 \right) \right] |y_{Q1} \pm y_{Q2}| + \tilde{\lambda}^{u2}_{ab} f \tilde{u}^a U_L^b \left( U_L^{a} \right)^T |y_{Q1} \pm y_{Q2}|
\]

\[
+ \eta^{u}_{ab} f \tilde{Q}_L^a \tilde{Q}_L^b \left( U_L^a \right)^T \left( \frac{1}{f} \Phi_1 + \frac{1}{f} \Phi_2 \right) |y_{Q1} \pm y_{Q2}|
\]

\[
+ \lambda^{d}_{ab} d^a \left( \epsilon_{ijk} Q_L^b \right) \left( U_L^a \right)^T \left( \frac{1}{f} \Phi_1 + \frac{1}{f} \Phi_2 \right) |y_{Q1} \pm y_{Q2}| \tag{70}
\]

After the symmetry breaking (36) that for convenience we rewrite here

\[
\langle \Phi_1 \rangle = \left( \begin{array}{c} 0 \\ v_W/2 \\ v_F/2 \end{array} \right) \quad \langle \Phi_2 \rangle = \left( \begin{array}{c} 0 \\ v_W/2 \\ -v_F/2 \end{array} \right) \quad \langle \Phi_3 \rangle = \left( \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right) \quad \langle \Phi_4 \rangle = \left( \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right) \tag{71}
\]

the Yukawa lagrangian eq. (70) at the leading order becomes

\[
\mathcal{L}_q^q = \lambda^{u1}_{ab} f \left[ u^a U_L^b + u^a \left( \frac{v_W}{f} u_L^b - \frac{v_F}{f} u_L^b + \frac{v_W}{f} n_L^b + \frac{v_F}{f} n_L^b \right) \right] |y_{Q1} \pm y_{Q2}| + \lambda^{u2}_{ab} f u^a U_L^b \left( k \right)^{|n_{Q1} - n_{Q2}|} + \lambda^{d}_{ab} d^a \left( \epsilon_{ijk} Q_L^b \right) \left( U_L^a \right)^T \left( \frac{1}{f} \Phi_1 + \frac{1}{f} \Phi_2 \right) |y_{Q1} \pm y_{Q2}| \tag{72}
\]

where \(k = v_F^2/f^2\) is the parameter in terms of which we write the mass textures.

From the lagrangian in eq. (72), we can read off the mass matrices for the quarks (see also eq. (81) below). These matrices and their textures are discussed in section III.D.

\[\text{B. Collective breaking in the up-quark sector and decoupling of the exotic fermions}\]

Let us now pause for a moment and show how the collective breaking mechanism works in preventing 1-loop quadratically divergent corrections to the higgs masses. Consider only the terms of the type “up” components of
the third family
\[
\mathcal{L}_{\text{top}}^Y = \lambda_{33}^1 f \left[ t'^c_L T_L + t'^{c'}_L \left( \frac{v_F}{f} t_L + \frac{v_W}{F} \tilde{t}_L + \frac{v_F}{f} \tilde{n}_L + \frac{v_F}{f} \tilde{n}_L^2 \right) \right] - k) + \lambda_{33}^2 f \left[ t''^c_L T_L \left( - k \right) \right. + \lambda_{33}^1 f \left[ \tilde{t}^{c}_L \tilde{T}_L + \tilde{t}^{c'}_L \left( \frac{v_F}{f} \tilde{T}_L + \frac{v_F}{f} \tilde{n}_L \right) \right] + \lambda_{33}^2 f \left[ \tilde{t}^{c'}_L \tilde{T}_L \right. + n^a_{\text{h}} f \left( m^a_{\text{h}} m^a_{\text{L}} + n^a_{\text{h}} n^a_{\text{L}} + n^a_{\text{h}}^2 n^a_{\text{L}}^2 \right). \tag{73}
\]
We see that \( t'^c_L \) and \( t''^c_L \) mix into a heavy and light combination, the latter being the standard top quark \( t^c_L \). Since the mixing is given by
\[
t^c_L = \frac{\lambda_{33}^1}{\sqrt{(\lambda_{33}^1)^2 + (\lambda_{33}^2)^2}} t'^c_L - \frac{\lambda_{33}^2}{\sqrt{(\lambda_{33}^1)^2 + (\lambda_{33}^2)^2}} t''^c_L,
\]
the top Yukawa coupling is
\[
\lambda_{33}^a = \frac{\lambda_{33}^1 \lambda_{33}^2}{\sqrt{(\lambda_{33}^1)^2 + (\lambda_{33}^2)^2}}. \tag{75}
\]
Similarly, the exotic \( \tilde{t}^{c}_L \) and \( \tilde{t}^{c'}_L \) mix into a heavy and a light combination, giving rise to the exotic quarks \( \tilde{t}^c_L \) and \( \tilde{t}^{c'}_L \):
\[
\tilde{t}^c_L = \frac{\tilde{\lambda}_{33}^1}{\sqrt{(\tilde{\lambda}_{33}^1)^2 + (\tilde{\lambda}_{33}^2)^2}} \tilde{t}'^c_L - \frac{\tilde{\lambda}_{33}^2}{\sqrt{(\tilde{\lambda}_{33}^1)^2 + (\tilde{\lambda}_{33}^2)^2}} \tilde{t}''^c_L,
\]
and the exotic top Yukawa coupling is given by
\[
\tilde{\lambda}_{33}^a = \frac{\tilde{\lambda}_{33}^1 \tilde{\lambda}_{33}^2}{\sqrt{(\tilde{\lambda}_{33}^1)^2 + (\tilde{\lambda}_{33}^2)^2}}. \tag{77}
\]
We can neglect the mixing between the top, the exotic quark and the components of the triplets \( \tilde{Q}^3_L \) since they are much heavier thanks to the explicit mass term in eq. \((74)\). Therefore, eq. \((74)\) becomes
\[
\mathcal{L}_{\text{top}}^Y = \lambda_{33}^0 v_W t^c_L t_L \left( - k \right) - \lambda_{33}^1 v_F t^c_L \tilde{t}_L \left( - k \right) - \lambda_{33}^2 v_F \tilde{t}^c_L \tilde{T}_L \left( - k \right) + n_{\text{h}} \tilde{t}^c_L \tilde{T}_L \left( - k \right), \tag{78}
\]
where we have neglected the terms involving the exotic triplets \( \tilde{Q}^i_L \). In eq. \((78)\) there is mixing between the standard top \( t_L \) and the exotic one \( \tilde{t}_L \), which is however very much suppressed, as we shall show shortly.

The same argument should in principle be applied to the first and second family. However in these cases we can neglect altogether the mixing between the \( u'^1, 2^c \) and \( u''^1, 2^c \) because it is strongly suppressed. Considering for example the second family, we have
\[
c^L = \frac{\lambda_{22}^{u1} (-e)^3}{\sqrt{\lambda_{22}^{u1} (-e)^3)^2 + (\lambda_{22}^{u2})^2}} c^{1c}_L - \frac{\lambda_{22}^{u2}}{\sqrt{\lambda_{22}^{u1} (-e)^3)^2 + (\lambda_{22}^{u2})^2}} c^{2c}_L, \tag{79}
\]
and from eq. \((79)\) follows that
\[
c^c_L \simeq c^{1c}_L, \quad c^c_L \simeq c^{2c}_L. \tag{80}
\]
In the exotic sector the situation follows what happens in the case of the top quark, and we define a light and a heavy exotic quark for both families, \( \tilde{c}^L_2 \) and \( \tilde{c}^L_1 \) for the second and \( \tilde{u}^L_2 \) and \( \tilde{u}^L_1 \) for the first one. At the end, the Yukawa lagrangian for the lightest quarks, both standard and exotic is given by

\[
\mathcal{L}^u_Y = \lambda^u_{ab} v_W u^a_L u^b_L ( - k ) |y_0^u + y_0^L| - \lambda^{u*}_{ab} v_F u^a_L \tilde{u}^b_L ( - k ) |y_0^u + y_0^L| + \lambda^d_{ab} v_F \tilde{u}^a_L \tilde{u}^b_L ( - k ) |y_0^d + y_0^L| + \lambda^d_{ab} d^a_L d^b_L + 2 v_W ( - k ) |y_0^d + y_0^L|. \tag{81}
\]

To see that the mixing between the standard and the exotic fermions is negligible, consider the mass matrix at the leading order, that is by taking all the parameter \( \lambda \) equal to 1. We have the following 6 \( \times \) 6 mass matrix

\[
M^{u}_{RL} = \begin{pmatrix}
v_v^k k^7 & v_v^k k^6 & v_f^k k^6 & v_f^k k^6 & v_f^k k^6 & v_f^k k^6 \\
v_v^k k^5 & v_v^k k^4 & v_f^k k^4 & v_f^k k^4 & v_f^k k^4 & v_f^k k^4 \\
v_v^k k^3 & v_v^k k^3 & v_f^k k^3 & v_f^k k^3 & v_f^k k^3 & v_f^k k^3 \\
0 & 0 & 0 & v_f & v_f & v_f \\
0 & 0 & 0 & v_f & v_f & v_f \\
0 & 0 & 0 & v_f & v_f & v_f
\end{pmatrix}. \tag{82}
\]

To give an estimate of the mixing between standard and exotic fermions we have to consider \( M^d \) and \( M^d \), that is

\[
M^{u*}_{RL} M^{u}_{RL} = \begin{pmatrix}
v_v^8 & v_v^7 & v_v^5 & v_v^5 & v_v^5 & v_v^5 \\
v_v^7 & v_v^6 & v_v^4 & v_v^4 & v_v^4 & v_v^4 \\
v_v^5 & v_v^4 & v_v^2 & v_v^2 & v_v^2 & v_v^2 \\
v_v^5 & v_v^4 & v_v^2 & v_v^2 & v_v^2 & v_v^2 \\
v_v^5 & v_v^4 & v_v^2 & v_v^2 & v_v^2 & v_v^2 \\
v_v^5 & v_v^4 & v_v^2 & v_v^2 & v_v^2 & v_v^2
\end{pmatrix}. \tag{83}
\]

Nine angles out of the 15 parametrizing the unitary matrix that diagonalize the mass matrix of eq. (83) contribute to the mixing between standard and exotic fermions. Let us call \( \theta_{ij} \) with \( i = 1, 2, 3 \) and \( j = 4, 5, 6 \) one of these nine angles, we see that

\[
\tan 2 \theta_{ij} \simeq -2 M_{ij}/M_{jj}, \tag{84}
\]

that is

\[
\theta_{ij} \simeq -k^{ij} v_W/v_F, \tag{85}
\]

so the largest mixing angle between the standard and exotic fermions is \( \theta_{y_6} \simeq -k^2 v_W/v_F \simeq 10^{-2} \), while that with the first two standard families, are completely negligible and well beyond any current bound [17].

C. Leptons

While standard model quark doublets are put in \( SU(3) \) electromawkew antitriples, standard model left-handed leptons are embedded in \( SU(3) \) triplets. Since leptons are lighter than quarks we should worry only about the divergent quadratic one-loop corrections to the higgs masses coming from the exotic leptons. The lepton content of each family is given in table III.

The right-handed neutrinos \( N_i \) with \( i = 1, \ldots, 8 \) couple to the left-handed triplets. In order to see their effect on the low-energy lagrangian, it is sufficient to consider a pair of them, for instance, \( \nu^L_1 \) and \( \nu^L_5 \) since the equal coupling of the remaining three pairs only renormalizes the overall Yukawa coupling.

The Yukawa lagrangian for the leptons is given at the leading order for each term by

\[
\mathcal{L}_Y = \frac{\eta L}{2} ( v_L^+ v_R^5 + v_L^+ v_R^6 ) + \frac{\eta L}{2} ( v_L^+ v_R^5 + v_L^+ v_R^6 ) + \lambda_{\nu^5} f \nu_R^L \left( \nu_{ij} L^i_L \Sigma_{i,\Sigma_{4,4}} \right) \left( \Sigma_{4,4} \right) |y_L^5|^2 + \lambda_{\nu^6} f \nu_R^L \left( \nu_{ij} L^i_L \Sigma_{i,\Sigma_{4,4}} \right) \left( \Sigma_{4,4} \right) |y_L^6|^2 + \lambda_{\nu^5} f \nu_R^L \left( \nu_{ij} L^i_L \Sigma_{i,\Sigma_{4,4}} \right) \left( \Sigma_{4,4} \right) |y_L^5|^2 + \lambda_{\nu^6} f \nu_R^L \left( \nu_{ij} L^i_L \Sigma_{i,\Sigma_{4,4}} \right) \left( \Sigma_{4,4} \right) |y_L^6|^2.
\]
TABLE III: Representations and charges assignments for the leptons. Different families run over the index $i$; they differ only for the flavor charges that are written as $(q_1, q_2, q_3)$ for, respectively, the first, second and third family. $U(1)_W$ charges are determined by the data constrains (see text of main body).

| | $U(1)_X$ | $U(1)_F$ | $SU(3)_W$ | $U(1)_W$ |
|---|---|---|---|---|
| $\mathcal{L}^i_L$ | $(\tilde{L}^i_L)$ | \[
\tilde{L}^i_L = \begin{pmatrix} z^i_L & w^i_L \\ w^i_L & \end{pmatrix} \]
| | $L^i_L$ | \[
L^i_L = \begin{pmatrix} \nu^i_L \\ \tilde{e}^i_L \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \]
| | $\tilde{E}^i_L$ | \[
\tilde{E}^i_L = \begin{pmatrix} \tilde{e}^i_L \\ 0 \\ 0 \\ 0 \end{pmatrix} \]
| $\mathcal{E}^e_L$ | \[
\mathcal{E}^e_L = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \]
| | $\tilde{\mathcal{E}}^e_L$ | \[
\tilde{\mathcal{E}}^e_L = \begin{pmatrix} \tilde{e}^e_L \\ 0 \\ 0 \end{pmatrix} \]

TABLE IV: Representations and charges assignments for the two right-handed neutrinos.

| | $U(1)_X$ | $U(1)_F$ | $SU(3)_W$ | $U(1)_W$ |
|---|---|---|---|---|
| $\nu^i_R$ | \[
\nu^i_R = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \]
| $\tilde{\nu}^i_R$ | \[
\tilde{\nu}^i_R = \begin{pmatrix} \tilde{\nu}^i_R \\ 0 \\ 0 \\ 0 \end{pmatrix} \]

\[
\begin{align*}
&+ \chi^a_{bc} f \mathcal{E}^a_L \sum \mathcal{L}^b_L \left( \Sigma_{4,4} \right) y^b_L - y^a_L \nonumber \\
&+ \tilde{\chi}^a_{bc} f \tilde{\mathcal{E}}^a_L \sum \mathcal{L}^b_L \left( \Sigma_{4,4} \right) y^b_L - y^a_L \nonumber \\
&\quad + \chi^a_{eab} f \tilde{\mathcal{E}}^a_L \tilde{E}^b_L \left( \Sigma_{4,4} \right) |y^b_L - y^a_L| \nonumber \\
&\quad + \tilde{\chi}^a_{eab} f \tilde{\mathcal{E}}^a_L \tilde{E}^b_L \left( \Sigma_{4,4} \right) |y^b_L - y^a_L|,
\end{align*}
\]  

(86)

with $a = 1, 2, 3$, $L^a_L$ defined in tab. (III), $\nu^{1,2,3}_R = \nu^1_R, \nu^2_R, \nu^3_R$, $\tilde{\nu}^{1,2,3}_R = \tilde{\nu}^1_R, \tilde{\nu}^2_R, \tilde{\nu}^3_R$, $\tau^1, \tau^2, \tau^3 = \overline{\tau^1}, \overline{\tau^2}, \overline{\tau^3}$. $y^a_L$ are the flavor charges of the $L^a_L$ and $\tilde{L}^a_L$ triplets members of the multiplets $\mathcal{L}^a_L$ defined in tabs. (III)–(IV). $y^a_\nu$ of the $\mathcal{E}^a_L$ multiplets, while the two right handed neutrinos, $\nu^1_R$ and $\tilde{\nu}^5_R$, flavor charges are $-1/2$ and $1/2$ respectively. In eq. (86) we have only two terms that preserve two different subgroups of the approximate global symmetry $SU(10)$, that is the terms with coefficients $\tilde{\chi}^{(1,2)}$. This permit us to protect the flavilogs $\Phi_{4,4}$ masses from the one-loop quadratic divergent contributions coming from the lepton triplets, since they couple to them with a large Yukawa coupling. As in eq. (69) for the quarks, the component $\Sigma_{4,4}$ is the only group singlet of the approximated global symmetries that can be introduced to make the lagrangian flavor invariant.
Like for the quarks, the exotic leptons $\tilde{e}^ν_L$ and $\tilde{\nu}^ν_L$ mix giving a light and a heavy exotic leptons, $\tilde{e}^ν_L$ and $\tilde{\nu}^ν_L$. In terms of the standard leptons, of the light exotic leptons and of the $Φ_i$, eq. (86) becomes

$$L^ν = \frac{ην}{f}(\nu^ν_L \nu^ν_R^c + \nu^ν_R^c \nu^ν_R) + \frac{ην}{f}(\nu^ν_c \nu^ν_R^c + \nu^ν_R^c \nu^ν_R^c) + \frac{λ^ν}{f^2} \tilde{ν}^ν_R \Phi_j \Phi_k \left( \frac{Φ_1^ν}{f^2} \right) |y^ν_L|^2$$

$$+ \frac{λ^ν}{f} \tilde{ν}^ν_R \left( Φ_1^ν \Phi_1^ν \right) \left( \frac{Φ_1^ν}{f^2} \right) |y^ν_L|^2 + \frac{λ^ν}{f} \tilde{ν}^ν_R \left( Φ_1^ν \Phi_1^ν \right) \left( \frac{Φ_1^ν}{f^2} \right) |y^ν_L|^2$$

$$+ \frac{λ^ν}{f^2} \tilde{ν}^ν_R \Phi_1^ν \left( \frac{Φ_1^ν}{f^2} \right) \left( \frac{Φ_1^ν}{f^2} \right) |y^ν_L|^2 + H.c.$$ (87)

The neutrino sector in eq. (91) is given by four Majorana right-handed neutrinos (two copies of them, actually) and three left-handed neutrinos, these latter being the standard neutrinos. Right-handed neutrinos are heavy, since their masses is of the same order of the scale $f$, we can integrate them out to obtain a Majorana mass matrix for the left-handed ones through the see-saw mechanism [13]. If we define the neutrino Dirac mass matrix, $M^ν_{RL,ab}$ through

$$\tilde{ν}^ν_R M^ν_{RL,ab} ν^ν_L ≈ \lambda^ν_{ab} \tilde{ν}^ν_R \left( Φ_1 \Phi_1 \right) \left( \frac{Φ_1^ν}{f^2} \right) |y^ν_L|^2$$

and the right-handed Majorana mass matrix, $M^ν_{RR,ij}$ by

$$\tilde{ν}^ν_R M^ν_{RR,ij} ν^ν_R = \frac{ην}{f}(\nu^ν_L \nu^ν_R^c + \nu^ν_R^c \nu^ν_R) + \frac{ην}{f}(\nu^ν_c \nu^ν_R^c + \nu^ν_R^c \nu^ν_R^c)$$

where we have defined $ν^ν_R = (ν^ν_R^1, ν^ν_R^2, ν^ν_R^3)$ and $y^ν_{Rw}$ the right-handed neutrinos flavor charges as reported in tab. (IV), we have

$$M^ν_{LL,ab} = M^µ_{RL,ab} M^ν_{RR,ij} M^ν_{RR,ij} M^ν_{RL,ab}.$$ (90)

After the symmetry breakings in eq. (71) and after having integrating out the right-handed neutrinos , eq. (87) becomes

$$L^ν = \tilde{ν}^ν_L \left( ν^ν_L^c ν^ν_R + ν^ν_R^c ν^ν_R \right) + \frac{λ^ν}{f^2} \tilde{ν}^ν_R \left( Φ_1^ν \Phi_1^ν \right) \left( \frac{Φ_1^ν}{f^2} \right) |y^ν_L|^2$$

$$+ \frac{λ^ν}{f} \tilde{ν}^ν_R \left( Φ_1^ν \Phi_1^ν \right) \left( \frac{Φ_1^ν}{f^2} \right) |y^ν_L|^2$$

$$+ \frac{λ^ν}{f^2} \tilde{ν}^ν_R \left( Φ_1^ν \Phi_1^ν \right) \left( \frac{Φ_1^ν}{f^2} \right) |y^ν_L|^2 + H.c.$$ (91)

where we can easily read the left-handed Majorana mass matrix of eq. (90) and where $O(λ^ν_{ab}) = O(λ^ν_{ab})^2$.

As for the quarks, the mixing between the standard charged leptons and the exotic one is negligible and in the discussion of the textures we will consider only the three standard lepton families.

The see-saw in eq. (90) is at low energy and therefore provides only a small part of the suppression of the neutrino Yukawa coefficient with respect to the others fermions. The problem of the absolute smallness of neutrino masses is left unsolved in the flihiggs model which only addresses the relative hierarchy in the fermion masses.

D. Fermion masses and mixing matrices

The fermion mass matrices are obtained from eq. (81) and eq. (91), respectively for quarks and leptons.

The quark mass matrices can be read off from eq. (81) by inserting the charges of all fermions according to Table II. They are given by

$$M^u_{RL} = λ^u ν wk^6 \left( \begin{array}{ccc}
λ^u_{11} & λ^u_{12} & λ^u_{13} \\
λ^u_{21} & λ^u_{22} & λ^u_{23} \\
λ^u_{31} & λ^u_{32} & λ^u_{33}
\end{array} \right)$$ (92)

and

$$M^d_{RL} = λ^d ν wk^3 \left( \begin{array}{ccc}
λ^d_{11} & λ^d_{12} & λ^d_{13} \\
λ^d_{21} & λ^d_{22} & λ^d_{23} \\
λ^d_{31} & λ^d_{32} & λ^d_{33}
\end{array} \right) ,$$ (93)
where, we recall, the texture parameter is given by $k = v_f^2/f^2$. We have written the mass matrices by extracting an overall coefficient for each matrix according and then treating the ratios of Yukawa couplings as a set of arbitrary parameters to be varied within a $O(1)$ range.

The essential feature of these mass matrices is that the fundamental textures are determined by the vacuum structure alone—that is that obtained by taking all Yukawa couplings $\lambda_{ij}^{u,d}$ of $O(1)$. In fact, by computing the corresponding CKM matrix one finds in first approximation

$$V_{\text{CKM}} = \begin{pmatrix} 1 & O(k) & O(k^3) \\ O(k) & 1 & O(k^2) \\ O(k^3) & O(k^2) & 1 \end{pmatrix},$$

(94)

that is roughly of the correct form and, moreover, suggests a value of $k \simeq \sin \theta_C \simeq 0.2$, as anticipated.

At the same time it is possible to extract from (92) and (93) approximated mass ratios:

$$\frac{m_u}{m_c} \simeq \frac{m_c}{m_t} \simeq O(k^3) \quad \text{and} \quad \frac{m_d}{m_s} \simeq \frac{m_s}{m_b} \simeq O(k^3),$$

(95)

which again roughly agree with the experimental values.

These results show that the quark masses and mixing angles can be reproduced by our textures. While a rough agreement is already obtained by taking all Yukawa couplings to be equal, the precise agreement with the experimental data depends on the actual choice of the Yukawa couplings $\lambda_{ij}^{u,d}$. Their values can be taken all of the same order, as we shall see in the appendix, and therefore the naturalness of the model is preserved.

Turning now to the leptons, eq. (91) yields the mass matrices

$$M_{\nu}^{LL} = (\lambda^\nu)^2 \frac{v^2_W}{m_f} \begin{pmatrix} \lambda_{11}^\nu k^2 & \lambda_{12}^\nu k & \lambda_{13}^\nu k \\ \lambda_{12}^\nu k & \lambda_{22}^\nu k^2 & \lambda_{23}^\nu k \\ \lambda_{13}^\nu k & \lambda_{23}^\nu k & \lambda_{33}^\nu k \end{pmatrix},$$

(96)

where $\lambda^\nu$ is an overall factor of the order of the yukawa coupling of the neutrino’s Dirac mass matrix, and

$$M_{\nu}^{RL} = \lambda^\nu v_W \begin{pmatrix} \lambda_{11}^e k^3 & \lambda_{12}^e k^5 & \lambda_{13}^e k^4 \\ \lambda_{21}^e k & \lambda_{22}^e k^2 & \lambda_{23}^e k \\ \lambda_{31}^e k & \lambda_{32}^e k & \lambda_{33}^e k \end{pmatrix},$$

(97)

where again we have extracted the overall factors and written the matrices in terms of the ratios of Yukawa couplings divided by the overall coefficients.

The matrices in eqs. (96)–(97) reduce—at the order $O(k)$, and up to overall factors—to

$$M^{(\nu)} = \begin{pmatrix} 0 & 1 & O(k) \\ 1 & 0 & O(k) \\ O(k) & O(k) & 1 \end{pmatrix} \quad \text{and} \quad M^{(l)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & O(k) & O(k) \\ O(k) & O(k) & 1 \end{pmatrix},$$

(98)

where, as before in the case of the quarks, the 1 stands for $O(1)$ coefficients.

The eigenvalues of $M^{(l)}$ can be computed by diagonalizing $M^{(l)\dagger} M^{(l)}$. This product is—again for each entry to leading order in $k$:

$$M^{(l)\dagger} M^{(l)} = \begin{pmatrix} 0 & 0 & O(k) \\ 0 & 1 & O(k) \\ O(k) & O(k) & 1 \end{pmatrix}.$$

(99)

By inspection of the $2 \times 2$ sub-blocks, the matrix eq. (99) is diagonalized by three rotations with angles, respectively, $\theta_{21}^\nu \simeq \pi/4$ and $\theta_{12}^\nu \simeq \theta_{13}^\nu \ll 1$, leading to one maximal mixing angle and two minimal. On the other hand, the neutrino mass matrix in eq. (98) is diagonalized by three rotations with angles, respectively, $\tan 2\theta_{12}^\nu \simeq 2/k^2$ and $\theta_{23}^\nu \simeq \theta_{13}^\nu \ll 1$ (the label 3 denotes the heaviest eigenstate). Therefore, the textures in the mass matrices in eqs. (96)–(97) give rise to a PMNS mixing matrix [14]—that is the combination of the the two rotations above—in which $\theta_{23}$ is maximal, $\theta_{12}$ is large (up to maximal), while $\theta_{13}$ remains small.
The natural prediction when taking all coefficients $O(1)$ is then: a large atmospheric mixing angle $\theta_{23}$, possibly maximal, another large solar mixing angle $\theta_{12}$, and a small $\theta_{13}$ mixing angle; at the same time, the mass spectrum includes one light ($O(k^2)$) and two heavy states ($O(1)$) in the charged lepton sector ($m_e$, $m_{\mu}$ and $m_\tau$ respectively), two light states ($O(k^2)$) and one heavy ($O(1)$) in the neutrino sector, thus predicting a neutrino spectrum with normal hierarchy.

While the quark textures are the same as those discussed in ref. [9], those for the leptons are slightly different because of the different flavor symmetry, an abelian $U(1)$ in the flhiggs model as opposed to the $SU(2)$ of [9].

We have included in the appendix a numerical analysis in which all the experimental data for both quarks and leptons are reproduced by a random choice of the rescaled Yukawa coefficients $\lambda_{ij}^{u,d,e,\nu}$ of order 1. This analysis shows that we need the texture parameter to be $k = 0.14$ and therefore $f \simeq 3.4$ TeV for $v_F \simeq 1.3$ TeV.

IV. EXPERIMENTAL SIGNATURES

The model contains many new particles. As explained, they are necessary in order to implement the collective symmetry breaking that solve the little hierarchy problem. Some live at the scale $f$, others at the lower scale $v_F$ and all the way to reach the weak scale $v_W$ below which the standard model particles live. In the low-energy range, these new states affect electroweak precision measurements and, as discussed in sec. II E, this essentially fixes the scale $v_F$ of flavor symmetry breaking which cannot be lowered more than about the TeV. They also affect the overall fit of these precision data and can be included together with standard model radiative corrections.

The range of energies from $v_F$ and $f$ is going to be explored in the next few years by LHC. Let us here summarize these new particles predicted by the model and briefly discuss their main experimental signatures.

| TABLE V: Particles and energy spectrum of the model |
| energy scale | states |
| --- | --- |
| $f \simeq 3$ TeV | $\tilde{z}_{1j}, s, s_{1,2,3}, W_{1}^{\pm}, Z^{0}, X^{i}, \nu_{R}^{1-16}, \hat{Q}_{f}, \hat{l}_{f}$ |
| $v_F \simeq 1$ TeV | $\tilde{W}_{1,2}^{\pm}, \tilde{Z}, \tilde{X}, \tilde{\tilde{q}}_{f}, \tilde{\tilde{l}}_{f}, \tilde{\tilde{l}}_{2, f}$ |
| between $v_W$ and $v_F$ | $\rho_{1}^{0}, \delta_{3,4}, \phi_{3,4,6}, \hat{h}_{0}(\phi_{7}), \tilde{h}^{\pm}(\rho_{2}^{0})$ |
| below $v_W = 246$ GeV | $\gamma, W^{\pm}, Z, q_{f}, l_{f}$ |

The most interesting experimental signature for LHC is in the scalar boson sector. The flhiggs model contains 12 scalar bosons, ten of which are neutral, two charged. For arbitrary coefficients of the potential we lack an analytic result for all their masses (see eqs. (66)–(67) for the analytically known part). Their values depend on $v_F$ and $f$ and, after having fixed them, they are a function of the parameters of the potential. These parameters $\xi_{i}$, $\chi_{i}$ and $\lambda_{i}$ can assume any value as long as they remain of order 1. To obtain an estimate of these masses, we vary the numerical value of the coefficients in the potential by a Gaussian distribution around the natural value 1 with a spread of 20% (that is $\sigma = 0.2$). This procedure gives us average values of these masses with a conservative error and we can consider the result the natural prediction of the model. The error is large enough to cover the uncertainty due to higher loop corrections.

For each solution we verify that all bounds on flavor changing neutral currents are satisfied. The most stringent of these is the potential contribution of the flhiggs fields to the $K^{0}-\bar{K}^{0}$ $\Delta S = 2$ amplitude. The presence of the flavor-charged flhiggs fields at such a low energy scale is possible because the relevant effective operators induced by their exchange are suppressed by powers of the fermion masses over $f$ [4, 9].

The lightest neutral scalar boson (what would be called the Higgs boson in the standard model) turns out to have a mass

$$m_{h^0} = 317 \pm 80 \text{ GeV}.$$  (100)

This is a rather heavy Higgs mass due to the value of $v_F \simeq 1$ TeV we were forced to take in order to satisfy the bounds on the $Z'$ mass. It is still inside the stability bound for a cut off of around a few TeVs. It is a value that only partially overlaps with the 95% CL of the overall fit of the electroweak precision data that gives $m_{h^0} < 237$ GeV [18] and gives the most characteristic prediction of the flhiggs model: a heavy Higgs boson (that is, with a mass larger than 200 GeV).

Notice that for a heavy Higgs boson like that we have found, and a cut off $f$ that we take around 3 TeV in order to generate the correct mass textures, we would have a little hierarchy problem with a fine tuning of 1% that justifies the little-Higgs mechanism we have implemented in order to be solved.
Above the lightest, the other scalar boson masses are spread, the heaviest of them reaching above $f$. The lightest charged Higgs bosons have a mass $m_{h^±} = 560 \pm 192$ GeV.

Like all little-Higgs models, the presence of the heavy gauge bosons and the additional top-like quarks can be used as signatures in the experimental searches. In addition, the flhiggs model has also a number of exotic fermionic states of known masses and coupling. They couple only weakly with standard fermions, as explained in section III. They can be used as further experimental signatures for the model.

Table V lists all the particles present in the flhiggs model ordered by the energy scale at which they live.

### A. Estimating the residual fine tuning

Even though the model was conceived to provide a framework for electroweak and flavor physics free of fine tuning of the parameters, the requirement of having $v_F \gtrsim 1$ TeV together with that of having the texture parameter $k$ of the order of the Cabibbo angle—and therefore $f \simeq 3$ TeV—reintroduce some amount of fine tuning.

The bound on $v_F$ implies relationships on the coefficients of the effective potential that, as already discussed, in turn give a fine tuning of about 10%. We find the same amount of fine tuning by considering the effect of having $f \simeq 3$ TeV and therefore of having the exotic quarks related to the top with masses of that order. They give a contribution to the (lightest) Higgs boson mass of the order of

$$-rac{3f^2\lambda}{16\pi^2}\log \frac{\Lambda^2}{f^2}$$

which, for $m_{h^0} \simeq 300$ GeV is a correction to be cancelled by the bare mass at the 10% level.

We conclude that while the flhiggs model has still a certain amount of fine tuning in its parameters, this is substantially less than in the standard model with a light Higgs boson.

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### APPENDIX A: NUMERICAL ANALYSIS OF TEXTURES

In order to show that the mass textures we found reproduce in a natural manner all the experimental data we retain the first non-vanishing contribution to each entry in all mass matrices and then—having extracted an overall coefficient for each matrix according to eqs. (92)–(93) and eqs. (96)–(97)—treat the ratios of Yukawa couplings as a set of arbitrary parameters to be varied within a $O(1)$ range. The absolute value of $f$ is immaterial to the textures that only depend on the ratio $k = v_F^2/f^2$. We keep the value of this texture parameter fixed and equal to $k = 0.14$. It corresponds in our fit to the values of $v_F = 1260$ GeV and $f = 3.4$ TeV.

In practice, we generated for the quark and lepton matrices many sets of Yukawa parameters whose moduli differ by at most a factor 10 and accepted those that reproduces the known masses and mixings.

For the leptonic sector, we generate random sets of 14 real parameters. Lacking experimental signature of CP violation in the leptonic sector, we have neglected, for the purpose of illustration, leptonic phases in the numerical exercise.

We obtain that for the representative choice

$$\begin{bmatrix}
\lambda_{11} & \lambda_{12} & \lambda_{13} \\
\lambda_{21} & \lambda_{22} & \lambda_{23} \\
\lambda_{31} & \lambda_{32} & \lambda_{33}
\end{bmatrix} = \begin{bmatrix}
1.6 & -2.9 & 1.0 \\
-2.9 & 0.55 & -0.40 \\
1.0 & -0.40 & 2.9
\end{bmatrix}$$

(A1)

with $\lambda_\nu = O(10^{-5})$ in eq. (96), and

$$\begin{bmatrix}
\lambda_{11} & \lambda_{12} & \lambda_{13} \\
\lambda_{21} & \lambda_{22} & \lambda_{23} \\
\lambda_{31} & \lambda_{32} & \lambda_{33}
\end{bmatrix} = \begin{bmatrix}
-0.26 & -0.83 & 1.0 \\
-0.48 & -1.7 & -0.13 \\
-1.2 & 2.6 & -1.1
\end{bmatrix}$$

(A2)
TABLE VI: Experimental data vs. the result of our numerical analysis based on a representative set of Yukawa couplings of order one (see text) and $k = 0.14$. Uncertainties in the experimental data are explained in ref. [9]

|                | exp  | numerical results |
|----------------|------|------------------|
| $|V_{us}|$       | 0.219 - 0.226 | 0.22            |
| $|V_{ub}|$       | 0.002 - 0.005 | 0.003           |
| $|V_{cb}|$       | 0.037 - 0.043 | 0.04            |
| $|V_{td}|$       | 0.004 - 0.014 | 0.007           |
| $|V_{ts}|$       | 0.035 - 0.043 | 0.04            |
| $\delta$       | 61.5$^\circ$ ± 7$^\circ$ | 53$^\circ$      |
| $\sin 2\beta$  | 0.705$^{+0.042}_{-0.032}$ | 0.71            |
| $m_t/m_c$       | 248 ± 70    | 222             |
| $m_c/m_u$       | 325 ± 200   | 369             |
| $m_b/m_s$       | 40 ± 10     | 40              |
| $m_s/m_d$       | 23 ± 10     | 17              |
| $\tan^2 \theta_\odot$ | 0.23 - 0.69 | 0.67            |
| $\sin^2 2\theta_\odot$ | 0.8 - 1.0  | 0.9             |
| $\sin^2 \theta_{13}$ | < 0.09   | 0.03            |
| $\Delta m^2_{31}/\Delta m^2_{21}$ | 0.014 - 0.12 | 0.06           |
| $m_{\tau}/m_\mu$ | 17         | 17              |
| $m_{\mu}/m_e$   | 207         | 190             |

with $\lambda^e = O(10^{-2})$ in eq. (97), the experimental values are well reproduced.

We can see by inspection that there is a certain amount of tension between the request of a maximal mixing angle in the (2,3) sector and the mass splitting between the $\mu$ and $\tau$ that forces an unnatural ratio of about 25 between the smallest and the largest of these ratios of Yukawa coefficients. This was already pointed out in [9] and is a necessary feature of most textures discussed in the literature.

We proceed in a similar manner in the quark sector by generating this time 18 random complex parameters.

We obtain that for the representative choice

$$
\begin{align*}
&\begin{pmatrix}
\lambda^u_{11} & \lambda^u_{12} & \lambda^u_{13} \\
\lambda^u_{21} & \lambda^u_{22} & \lambda^u_{23} \\
\lambda^u_{31} & \lambda^u_{32} & \lambda^u_{33}
\end{pmatrix} =
\begin{pmatrix}
-1.1 + 1.3i & 0.37 + 0.37i & 0.36 + 0.42i \\
-0.22 - 1.6i & -0.39 - 1.2i & 1.0 - 0.56i \\
-0.16 + 1.2i & 0.39 - 1.1i & -1.3 - 0.22i
\end{pmatrix}
\end{align*}
$$

(A3)

with $\lambda^u = O(k^{-1})$ in eq. (92), and

$$
\begin{align*}
&\begin{pmatrix}
\lambda^d_{11} & \lambda^d_{12} & \lambda^d_{13} \\
\lambda^d_{21} & \lambda^d_{22} & \lambda^d_{23} \\
\lambda^d_{31} & \lambda^d_{32} & \lambda^d_{33}
\end{pmatrix} =
\begin{pmatrix}
-0.54 + 1.4i & -0.38 + 0.98i & -0.85 - 0.09i \\
1.3 - 0.43i & -0.65 + 0.52i & 0.51 - 1.2i \\
-0.62 - 1.0i & 0.43 + 0.37i & -0.02 - 0.54i
\end{pmatrix}
\end{align*}
$$

(A4)

with $\lambda^d = O(k^{-1})$ in eq. (93), the experimental values are well reproduced.

Table A summarizes the experimental data and compares them to the result of the above procedure. The agreement is quite impressive. While the values of the overall constants (which are related to the scale of the heaviest state in the mass matrices) are not explained by the model, the hierarchy among the mass eigenvalues and the mixing angles are given in first approximation by the flavor symmetry and the flavor vacuum so that, within each mass matrix, the Yukawa couplings remain in a natural range.

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