Conformal Cosmological Model Test with Distant SNIa Data

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Abstract

Assuming that supernovae type Ia (SNe Ia) are standard candles one could use them to test cosmological theories. The Hubble Space Telescope team analyzed 186 SNe Ia [1] to test the standard cosmological model (SC) and evaluate its parameters. We use the same sample to determine parameters of Conformal Cosmological models (CC). We concluded, that really the test is extremely useful and allows to evaluate parameters of the model. From a formal statistical point of view the best fit of the CC model is almost the same quality approximation as the best fit of SC model with $\Omega_\Lambda = 0.72, \Omega_m = 0.28$. As it was noted earlier, for CC models, a rigid matter component could substitute the $\Lambda$-term (or quintessence) existing in the SC model.

Key words: General Relativity and Gravitation; Cosmology; Observational Cosmology

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1. Introduction

Now there is enormous progress in observational and theoretical cosmology and even it is typically accepted that cosmology enters into an era of precise science (it means that a typical accuracy of standard parameter determination is about few percents), despite, there are different approaches including alternative theories of gravity to fit observational data (see recent reviews [2] for references). Some classes of theories could be constrained by Solar system
data even if they passed cosmological tests. Thus, all the theories should pass all possible tests including cosmological ones.

Since the end of the last century distant supernovae data is a widespread test for all theoretical cosmological models in spite of the fact the correctness of the hypothesis about SNe Ia as the perfect standard candles is still not proven. However, the first observational conclusion about accelerating Universe and existence of non-vanishing Λ-term was done with the cosmological SNe Ia data.

Therefore, typically standard (and alternative) cosmological approaches are checked with the test.

Conformal cosmological models (CC) are also discussed among other possibilities. First attempts to analyze SN data to evaluate parameters of CC models were done, so it was used only 42 high redshift type Ia SNe, but after that it was analyzed a slightly extended sample. In spite of a small size of the samples used in previous attempts to fit CC model parameters, it was concluded that if Ω_{m0} is significant in respect to the critical density, CC models could fit SN Ia observational data with a reasonable accuracy. An aim of the paper is to check and clarify previous conclusions about possible bands for CC parameters with a more extended (and more accurate) sample used commonly to check standard and alternative cosmological models. The HST cosmological SNe Ia team have corrected data of previous smaller samples as well and also considered possible non-cosmological but astronomical ways to fit observational ways and concluded that some of them such a replenishing dust (with Ω_m = 1, Ω_Λ = 0.) could fit observational data pretty well even in respect to the best fit cosmological model.

The content of the paper is the following. In Section 2, the basic CC relations are reminded. In Section 3, a magnitude-redshift relation for distant SNe is discussed. In Section 4, results of fitting procedure for CC models with the "gold" and "silver" sample and the "gold" subsample are given. Conclusions are presented in Section 5.

2. Conformal Cosmology Relations

We will remind basic relations between observational data and CC model parameters. The correspondence between the SC and the CC is determined by the evolution of the dilaton

\[(\varphi')^2 = \rho(\varphi),\]  

(1)
where the prime denotes the derivative with respect to the conformal time \( \eta \) and the time, the density \( \rho(\varphi) \), and the Hubble parameter \( H_0 \) are treated as measurable quantities. The standard cosmological definitions of the redshift and the density parameter are the following

\[
1 + z \equiv \frac{1}{a(\eta)} = \frac{\varphi_0}{\varphi(\eta)}, \quad \Omega(z) = \frac{\rho(\varphi)}{\rho(\varphi_0)},
\]

where \( \Omega(0) = 1 \) is assumed. The density parameter \( \Omega(z) \) is determined

\[
\Omega(z) = \Omega_{\text{rig}}(1 + z)^2 + \Omega_{\text{rad}} + \frac{\Omega_m}{(1 + z)} + \frac{\Omega_{\Lambda}}{(1 + z)^4}.
\]

We note here that all the equations of state that are known in the standard cosmology: the rigid state \( (p_{\text{rig}} = \rho_{\text{rig}}(\varphi) = \text{const}/\varphi^2) \), the radiation state \( (p_{\text{rad}} = \rho_{\text{rad}}/3 = \text{const}) \), and the matter state \( (p_m = 0, \ \rho_m = \text{const} \cdot \varphi) \). Then the equation (1) takes the form

\[
H_0 \frac{d\eta}{dz} = \frac{1}{(1 + z)^2} \frac{1}{\sqrt{\Omega(z)}},
\]

and determines the dependence of the conformal time on the redshift factor. This equation is valid also for the conformal time - redshift relation in the SC where this conformal time is used for description of a light ray.

A light ray traces a null geodesic, i.e. a path for which the conformal interval \((ds^L)^2 = 0\) thus satisfying the equation \(dr/d\eta = 1\). As a result we obtain for the coordinate distance as a function of the redshift

\[
H_0 r(z) = \int_0^z \frac{dz'}{(1 + z')^2} \frac{1}{\sqrt{\Omega(z')}}.
\]

The equation (5) coincides with the similar relation between coordinate distance and redshift in SC.

In the comparison with the stationary space in SC and stationary masses in CC, a part of photons is lost. To restore the full luminosity in both SC and CC we should multiply the coordinate distance by the factor \((1 + z)^2\). This factor comes from the evolution of the angular size of the light cone of emitted photons in SC, and from the increase of the angular size of the light cone of absorbed photons in CC.

However, in SC we have an additional factor \((1 + z)^{-1}\) due to the expansion of the universe, as measurable distances in SC are related to measurable distances
in CC (that coincide with the coordinate ones) by the relation

\[ \ell = a \int \frac{dt}{a} = \frac{r}{1 + z}. \]  

(6)

Thus, we obtain the relations

\[ \ell_{\text{SC}}(z) = (1 + z)^2 \ell = (1 + z)r(z), \]  

(7)

\[ \ell_{\text{CC}}(z) = (1 + z)^2 r(z). \]  

(8)

This means that the observational data are described by different regimes in SC and CC.

3. Magnitude-Redshift Relation

Typically to test cosmological theories one should check a relation between an apparent magnitude and a redshift. In both SC and CC models it should be valid the effective magnitude-redshift relation:

\[ \mu(z) \equiv m(z) - M = 5 \log [H_0 \ell(z)] + \mathcal{M}, \]  

(9)

where \( m(z) \) is an observed magnitude, \( M \) is the absolute magnitude, \( \mathcal{M} \) is a constant with recent experimental data for distant SNe. Values of \( \mu_i, z_i \) and \( \sigma_i \) could be taken from observations of a detected supernova with index \( i \) (\( \sigma_i^2 \) is a dispersion for the \( \mu_i \) evaluation). Since we deal with observational data we should choose model parameters to satisfy an array of relations (9) by the best way because usually, a number of relations is much more than a number of model parameters and there are errors in both theory and observations (as usual we introduce indices for the relations corresponding to all objects). Typically, \( \chi^2 \)-criterium is used to solve the problem, namely, we calculate

\[ \chi^2 = \sum_i \frac{(\mu_i^{\text{theor}} - \mu_i)^2}{\sigma_i^2}, \]  

(10)

where \( \mu_i^{\text{theor}} \) are calculated for given \( z_i \) with the assumed theoretical model and after that we can evaluate the best fit model parameters minimizing \( \chi^2 \)-function.
Fig. 1. $\mu(z)$-dependence for cosmological models in SC and CC. The data points include 186 SN Ia (the "gold" and "silver" sample) used by the cosmological supernova HST team [1]. For a reference we use the best fit for the flat standard cosmology model with $\Omega_m = 0.27, \Omega_\Lambda = 0.73$ (the thick dashed line), the best fit for CC is shown with the thick solid line. For this CC model we do not put any constraints on $\Omega_m$.

4. Model Fits

4.1. Total sample analysis

For the standard cosmological model for the 186 SNe (the "gold" and "silver" sample)\(^1\) a minimum of the $\chi^2$-function gives us $\Omega_m = 0.28$ ($\chi^2_{SC\ flat} = 232.4$) and $\Omega_m = 0.31, \Omega_\Lambda = 0.80$ assuming $|\Omega_k| \leq 0.11$ ($\chi^2_{SC\ flat} = 231.0$). Since other cosmological tests dictate that the Universe should be almost flat and $\Omega_m = 0.28$ is an acceptable value [2], we choose the flat SC model for a reference.

In Fig. 1 we compare the SC and CC fits for the effective magnitude-redshift relation if we will not put any constraint on $\Omega_m$ (in this case we assume that SNe Ia data is the only cosmological test for CC models we obtain the best fit expressed in the first row in Table 1). Analyzing the curves corresponding to the best fits for SC and CC models one can say that they almost non-distinguishable, moreover the best fit CC provide even better the $\chi^2$ value (see first row in Table 1). We would not claim that we discovered a cosmological

\(^1\) To express differences in quality of spectroscopic and photometric data the supernovae were separated into "high-confidence" ("gold") and "likely but not certain" ("silver") subsets [1].
model with negative $\Omega_m$, but we would like to note that the best CC and SC fits are almost non-distinguishable from a formal statistical point of view (the thick solid and long dashed lines, respectively in Fig. 1). Sometimes new physical phenomena are introduced qualitatively with the same statistical arguments (such as an introduction of the phantom energy, for instance), but if we should follow a more conservative approach, we could conclude that in this case we should simply put extra constraints on $\Omega_m$ to have no contradictions to other cosmological (and astronomical) tests. So, if we put "natural" constraints on $\Omega_m \geq 0$, the best fit parameters for CC model are presented in second row in Table 1. In this case the $\chi^2$ difference between two CC models ($\Delta \chi^2 \approx 16$) is not very high and a difference between this fit and the SC best fit for a flat model is about $\Delta \chi^2 \approx 10$ (or less than 5%), it means the CC fit is at an acceptable level. For references, we plotted also pure flat CC models, so that rigid, matter, lambda and radiation models are shown with thin dotted, short dashed, dot dash, dash dot dot dot lines, respectively. Corresponding $\chi^2$ values are given in Table 2. One can see that only pure flat rigid CC model has relatively low $\chi^2$ values (and it could be accepted as a rough and relatively good fit for cosmological SNe Ia data), but other models should be definitely ruled out by the observational data.

So, if we put further constraints on $0.2 \leq \Omega_m \leq 0.3$ based on measurements of clusters of galaxies and other cosmological arguments [11], the best fit parameters for CC model are presented in third row in Table 1. In this case the $\chi^2$ difference between two CC models ($\Delta \chi^2 \approx 18$) is not very high also and a difference between $\chi^2$ for the CC and SC models is about $\Delta \chi^2 \approx 12$ (or about 5%), it means the CC fit is at an acceptable level. Dependence of $\chi^2$ on $\Omega_m$ is

| Constraints on $\Omega_m$ | $\Omega_m$ | $\Omega_\Lambda$ | $\Omega_{\text{rad}}$ | $\Omega_{\text{rig}}$ | $\chi^2$ |
|--------------------------|-----------|------------------|----------------------|----------------------|---------|
| No constraints           | -4.13     | 3.05             | 0.05                 | 2.085                | 226.64  |
| $\Omega_m \geq 0.$       | 0.        | 0.18             | 0.                   | 0.80                 | 242.76  |
| $0.2 \leq \Omega_m \leq 0.3$ | 0.2      | 0.013            | 0.                   | 0.75                 | 244.67  |
| $0.2 \leq \Omega_m \leq 0.3$ | 0.29     | 0.               | 0.                   | 0.7                  | 246.58  |
| $0.2 \leq \Omega_m \leq 0.3$ | 0.27     | 0.               | 0.                   | 0.72                 | 245.66  |

Table 1
The fits for CC models for the total sample with different constraints on $\Omega_m$ (the best fits are shown in first, second and third rows, two almost best fits are presented in fourth and fifth rows).

| Model types | $\Omega_m = 1$ | $\Omega_\Lambda = 1$ | $\Omega_{\text{rad}} = 1$ | $\Omega_{\text{rig}} = 1$ |
|-------------|----------------|-----------------------|---------------------------|---------------------------|
| $\chi^2$    | 924.27         | 4087.93               | 478.42                    | 276.71                    |

Table 2
The $\chi^2$ values for pure flat CC models for the total sample. The models are shown in Figs. 1,2 as references.
4.2. Analysis of the "Gold" Subset

We also did the same calculations for "gold" subset of SNe Ia data (157 objects). The best fit for SC flat model corresponds to $\Omega_m = 0.285$ and $\chi^2 = 177.10$ (if we try to find the best fit for SC model with the constraint $|\Omega_k| \leq 0.11$ we have $\Omega_m = 0.32$, $\Omega_\Lambda = 0.79$ and $\chi^2 = 176.07$), however as in the previous case for the total sample, we have selected generally accepted the SC flat model for a reference with $\Omega_m = 0.285$.

The parameters for the best fits for CC models are given in first row in Table 3 (the corresponding plot is shown in Fig. 3). One can note here that in spite of the previous case for the "gold" and "silver" sample the best fit for CC model does not need so exotic negative $\Omega_m$. Analyzing $\chi^2$ presented in Table 3 (last column), one can say that like in the previous case, dependence of $\chi^2$ function on $\Omega_m$ is very weak and SNe Ia data is not very good way to evaluate precisely
Fig. 3. $\mu(z)$-dependence for cosmological models in SC and CC. The data points include 157 SN Ia (the "gold" subsample) used by the cosmological supernova HST team [1]. For a reference we use the best fit for the flat standard cosmology model with $\Omega_m = 0.285, \Omega_\Lambda = 0.715$ (the thick dashed line), the best fit for CC is shown with the thick solid line. For this CC model we do not put any constraints on $\Omega_m$.

| Constraints on $\Omega_m$ | $\Omega_m$ | $\Omega_\Lambda$ | $\Omega_{\text{rad}}$ | $\Omega_{\text{rig}}$ | $\chi^2$ |
|--------------------------|------------|-------------------|------------------------|------------------------|----------|
| No constraints           | 0.16       | 0.0               | 0.0                    | 0.76                   | 187.20   |
| $0.2 \leq \Omega_m \leq 0.3$ | 0.2       | 0.0               | 0.0                    | 0.74                   | 187.50   |
| $0.2 \leq \Omega_m \leq 0.3$ | 0.264     | 0.0               | 0.0                    | 0.73                   | 188.94   |

Table 3
The fits for CC models for the "gold" subsample with different constraints on $\Omega_m$ (the best fits are shown in first, second rows, a good fit is presented in third row).

| Model types | $\Omega_m = 1$ | $\Omega_\Lambda = 1$ | $\Omega_{\text{rad}} = 1$ | $\Omega_{\text{rig}} = 1$ | $\chi^2$ |
|-------------|----------------|----------------------|---------------------------|---------------------------|----------|
| $\chi^2$    | 809.79         | 3631.55              | 407.02                    | 210.0                     |          |

Table 4
The $\chi^2$ values for pure flat CC models for the the "gold" subsample. The models are shown in Figs. 3, 4 as references.

As for the total sample, we obtain that for the "gold" subset a $\chi^2$ difference between $\chi^2$ for the CC and SC models is about $\Delta \chi^2 \approx 12$ (or about 5%), it means the CC fit is at an acceptable level, however, the best SC flat model is still a little bit more preferable.
The data points include 157 SN Ia (the "gold" subsample). For a reference we use the best fit for the flat standard cosmology model with $\Omega_m = 0.285, \Omega_\Lambda = 0.715$ (the thick dashed line), the best fit for CC is shown with the thick solid line. For this CC model we assume $\Omega_m \in [0.2, 0.3]$.

5. Conclusions

Using "gold" and "silver" 186 SNe Ia [1] we confirm in general and clarify previous conclusions about CC model parameters, done earlier with analysis of smaller sample of SNe Ia data [6, 7] that the pure flat rigid CC model could fit the data relatively well since $\Delta \chi^2 \approx 44.3$ (or less than 20 %) in respect of the standard cosmology flat model with $\Omega_m = 0.28$. Other pure flat CC models should be ruled out since their $\chi^2$ values are too high.

For the total sample, if we consider CC models with a "realistic" constraint $0.2 \leq \Omega_m \leq 0.3$ based on other astronomical or cosmological arguments except SNe Ia data, we conclude that the standard cosmology flat model with $\Omega_m = 0.28$ is still preferable in respect to the fits for the CC models (with $\Omega_m = 0.2$ and $\Omega_{\text{rig}} = 0.75$ or $\Omega_m = 0.27$ and $\Omega_{\text{rig}} = 0.72$, for instance, see third and fifth rows in Table 1), but the preference is not very high (about 5 % in relative units of $\chi^2$ value), so the CC models could be adopted as acceptable ones taking into account possible sources of errors in the sample and systematics.

With the presented analysis for the "gold" subsample we re-confirm results given with the total sample except the absence of the best fit for CC models with the negative $\Omega_m$ as it was obtained with the total sample. The best fits for CC models without constraints and with "natural" constraints on $\Omega_m$ such as $\Omega_m \in [0.2, 0.3]$ give almost the same curves for magnitude-redshift
dependences. For the "gold" subsample, a $\chi^2$ difference is about then 5 \% for the best fit CC and SC models, it means that the conclusion obtained earlier for the total ("gold" and "silver") sample is also correct for the "gold" subsample.

Thus, for CC model fits calculated with SNe Ia data, in some sense, a rigid equation of state could substitute the $\Lambda$-term (or quintessence) in the Universe content.

The best CC models provide almost the same quality fits of SNe Ia data as the best fit for the SC flat model, however the last (generally accepted) model is more preferable.

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References

[1] A.D. Riess, L.-G. Strolger, J. Tonry et al., Astrophys. J., 607 (2004) 665.
[2] C. Will, Living Rev. Relativity, 9 (2006) 3;
S. Bludman, astro-ph/0605198;
E.J. Copeland, M. Sami, S. Tsujikawa, astro-ph/0603057;
N. Straumann, Mod. Phys. Lett. A, 21, (2006) 1083.
[3] A.F. Zakharov, A.A. Nucita, F. De Paolis, G. Ingrosso, Phys. Rev. D 74, (2006) 107101;
X. Jin, D. Liu, X. Li, astro-ph/0610854.
[4] N. Panagia, astro-ph/0502247;
[5] D.B. Blaschke, S.I. Vinitsky, A.A. Gusev, V.N. Pervushin, and D.V. Proskurin, Phys. Atom. Nucl. 67, (2004) 1050;
B.M. Barbashov, V.N. Pervushin, A.F. Zakharov, V.A. Zinchuk, Int. J. Mod. Phys. A (accepted), astro-ph/0511824;
B.M. Barbashov, V.N. Pervushin, A.F. Zakharov, V.A. Zinchuk, Int. J. Geom. Meth. Mod. Phys. (accepted); hep-th/0606054;
B.M. Barbashov, V.N. Pervushin, A.F. Zakharov, V.A. Zinchuk, Phys. Atom. Nucl. (accepted); astro-ph/0507368;
A.F. Zakharov, V.N. Pervushin, V.A. Zinchuk, Phys. Part. and Nucl. 37 (2006) 104;
B.M. Barbashov, V.N. Pervushin, A.F. Zakharov, V.A. Zinchuk, Physics
Letters B 633, 438 (2006); V.N. Pervushin, A.F. Zakharov, V.A. Zinchuk, in Proceeding of the ITAS Summer School and International Conference "New Trends in High-Energy Physics (experiment, phenomenology, theory)”, Yalta, Crimea, Ukraine, 2005, Bogoliubov Institute for Theoretical Physics, National Academy of Sciences of Ukraine, Joint Institute for Nuclear Research (Dubna), p. 271; B.M. Barbashov, V.N. Pervushin, A.F. Zakharov, V.A. Zinchuk, in Proc. "Nuclear Science and Safety in Europe”, T. Cechak el al. (eds.), Springer, 2006, p. 125; [astro-ph/0511824] V.N. Pervushin, A.F. Zakharov, V.A. Zinchuk, in Proceedings "Nuclear Science and Safety in Europe”, T. Cechak el al. (eds.), Springer, 2006, p. 201; B.M. Barbashov, V.N. Pervushin, A.F. Zakharov, V.A. Zinchuk, in Proc. of the 8th International Workshop Relativistic Nuclear Physics: from Hundreds MeV to TeV, JINR, Dubna, Russia, p. 11, 2006; B.M. Barbashov, V.N. Pervushin, A.F. Zakharov, V.A. Zinchuk, in Proc. of the XXVIII Spanish Relativity Meeting E.R.E. "A Century of Relativity Physics” Oviedo (Asturias) Spain, American Institute of Physics, v. 841, p. 362 (2006).

[6] D. Behnke, D.B. Blaschke, V.N. Pervushin, D.V. Proskurin, Phys. Lett. B 530, (2002).
[7] D. Behnke, Conformal Cosmology Approach to the Problem of Dark Matter, PhD Thesis, Rostock Report MPG-VT-UR 248/04 (2004).
[8] A.G. Riess et al., Astron. J. 116, (1998) 1009; S. Perlmutter et al., Astrophys. J. 517, (1999) 565 (1999).
[9] V.N. Pervushin, V.I. Smirichinski, J. Phys. A: Math. Gen. 32 (1999) 6191.
[10] M. Pawlowski, V.N. Pervushin, Int. J. Mod. Phys. 16 (2001) 1715.
[11] N.A. Bahcall, X. Fan, Publ. Nat. Academy of Science, 95, (1998) 5956; N.A. Bahcall, X. Fan, Astrophys. J. 504, (1998) 1; D.N. Spergel, R. Bean, O. Dore’ et al. [astro-ph/0603449]; U. Seljak, A. Slosar, P. McDonald, J. Cosm. Astroparticle Phys. 10 (2006) 14.