Buckling load of an infinitely long cylindrical shell interacting with the soil environment

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Abstract. The article considers the problems of numerical computational stability analysis for an infinitely long cylindrical shell interacting with the soil environment. Several spatial computational models of "the shell – the soil" system have been compiled to determine the buckling load and the buckling mode. The interaction of the shell and the soil environment is provided by contact pairs located on the respective surfaces. Contact pairs with a rigid connection between bodies and contact pairs with the detachment possibility of interacting bodies are used. Contact pairs with and without the friction coefficient are also used. The problem is solved in linear and nonlinear statement. The materials of the shell and the soil environment are given by elastic-plastic models of Mohr-Coulomb and Drucker-Prager. The buckling loads are defined relative to the effective gravity load. The buckling loads and the buckling modes of the cylindrical shell are obtained as a result of numerical computational stability analysis. The comparative analysis of the received buckling loads is presented and the corresponding conclusions are given.

1. Introduction
Cylindrical shells are widely used in the engineering structures for various purposes and can be both an independent structure and part of a more complex structure. During the operation of these structures, the shell experiences a complex stress-strain state, the definition of which is necessary at the design stage of the object. This work considers circular cylindrical shells interacting with the soil environment and simulating an underground tunnel. Cylindrical shells, working in underground structures, are under load from their own weight and from the weight of the soil layers. However, the buckling load at which the cylindrical shell structure loses stability is of particular interest. The size of this load will allow to assess "stability margin" of the structure, which will avoid accidents during the operation of the structure.

The buckling loads and the buckling mode of a circular cylindrical shell are determined by numerical analysis of the spatial calculation model in linear and nonlinear statement. The buckling load is presented as a multiple of the natural acting load for the convenience of determining the coefficient of "stability margin". Based on obtained results, a comparative analysis is presented.
2. Numerical analysis of the spatial model for "the shell- the soil" system. The case of an infinitely long cylindrical shell

Numerical analysis of the spatial model for "the shell-the soil" system in the case of an infinitely long cylindrical shell is made in the ANSYS software package [1,2,3,4]. For the calculation of the infinitely long cylindrical shell and the soil environment selected spatial band width of 1 m. The calculation model is made using the two-dimensional flat four-node elements of the shell and three-dimensional tetrahedral ten-node elements of the soil environment. The interaction between the shell and the soil environment is provided by contact pairs located on the respective surfaces. The calculation is made for the spatial model in linear and nonlinear statements under different interaction conditions of the shell with the soil environment:

- Linear analysis with two-way connections between the shell and the soil environment. The model assumes full contact between the shell and the soil environment, not taking into account the possibility of contact disruption (detachment). The materials are given linearly elastic.
- Nonlinear analysis with one-way connections between the shell and the soil environment. The shell interacts with the soil using contact pairs without taking into account the friction coefficient. The materials are given linearly elastic.
- Nonlinear analysis with one-way connections between the shell and the soil environment. The shell interacts with the soil using contact pairs taking into account the friction coefficient. The materials are given linearly elastic.
- Nonlinear analysis with one-way connections between the shell and the soil environment. The shell interacts with the soil using contact pairs without taking into account the friction coefficient. The materials are given by elastic-plastic models.
- Nonlinear analysis with one-way connections between the shell and the soil environment. The shell interacts with the soil using contact pairs taking into account the friction coefficient. The materials are given by elastic-plastic models.
- Nonlinear analysis with one-way connections between the shell and the soil environment. The shell interacts with the soil using contact pairs taking into account the friction coefficient. The materials are given by elastic-plastic models. Computational model with horizontal exciting force applied to the top point of the shell.

2.1. Linear analysis of the spatial model for “the shell-the soil” system with two-way connections.

Linear elastic material

The shell diameter is \( D = 5 \text{ m} \), the cross section is rectangular \( 1 \text{ m} \times 0.25 \text{ m} \). Distance from the shell edge to the side model edges is \( L = 5D \), where \( D \) – the shell diameter. The distance from the shell edge to the upper model boundary is \( 3D \), to the lower model boundary – \( 1D \) (on the assumption that below is nondeformable rocky ground). The model is under load from the own weight of the soil and the shell \( g = 9.81 \text{ m}/\text{c}^2 \). Side and lower boundaries of the soil environment are fixed from normal displacements to surfaces. The shell at the edges has similar constraints. This ensures stability of geometrical shape of the computational model. The finite element mesh of the shell and the soil environment are connected "node to node". In figure 1 is shown the general view of the calculation model.
The linear elastic model is taken for the material. The physical and mechanical properties of the shell correspond to the concrete properties: elastic modulus $E = 3.0\cdot10^{10}$ Pa, Poisson’s ratio $\mu = 0.18$, density $\rho = 2300$ kg/m$^3$. Physical and mechanical properties of the soil environment: deformation modulus $E_{\text{def}} = 14.0\cdot10^{6}$ Pa, Poisson’s ratio $\mu = 0.30$, density $\rho = 1600$ kg/m$^3$.

In figure 2 is shown the buckling mode of the spatial model for "the shell-the soil" system in the case of an infinitely long cylindrical shell. The loss of stability occurs on the right side of the shell. The buckling load value is $36477.0\gamma z_f$.

The results showed that the linear analysis of the spatial model for "the shell-the soil" system with two-way connections leads to overestimate values of the buckling load relative to the natural load.

2.2. Nonlinear analysis of the spatial model for “the shell-the soil” system with one-way connections without taking into account the friction coefficient. Linear elastic material

As against the previous model, nonlinear analysis is applied here. Nonlinearity allows using an iterative process to find the contact zone of the elements (the detachment area of the shell from the soil) and determine the time-varying position of the shell. A linear elastic model of the material is used. The material characteristics are taken from the previous paragraph.

In figure 3 is shown the total displacements of the spatial model for “the shell-the soil” system in the case of an infinitely long cylindrical shell when the buckling load is achieved. The buckling load value is $4.3\gamma z_f$.

The results showed that if the friction coefficient is not taken into account, the shell loses of stability at a low buckling load.
2.3. Nonlinear analysis of the spatial model for “the shell-the soil” system with one-way connections taking into account the friction coefficient. Linear elastic material
This model takes into account the friction coefficient between the shell and the soil environment. The friction coefficient is $f = 0.6$, since the physical and mechanical properties of the soil are accepted as for dry soil, and the properties of the shell as for concrete. The calculation is made in a nonlinear statement. Nonlinearity allows using an iterative process to find the contact zone of the elements (the detachment area of the shell from the soil) and determine the time-varying position of the shell. A linear elastic model of the material is used. The material characteristics are taken from the previous paragraph.
In figure 4 is shown the total displacements of the spatial model for “the shell-the soil” system in the case of an infinitely long cylindrical shell when the buckling load is achieved. The nonlinear analysis stopped at the load increment step, at which solution convergence is not achieved. The buckling load value is $39.9\gamma_z$.
The results showed that if the friction coefficient is taken into account, the system does not lose stability in the early stages of load growth.

2.4. Nonlinear analysis of the spatial model for “the shell-the soil” system with one-way connections without taking into account the friction coefficient. Elastic-plastic material
It is known that the ultimate compressive stress of concrete is much higher than the ultimate tensile stress of concrete. In the following calculation case, an attempt is made to take into account the difference between the ultimate compression stress and the ultimate tensile stress in the shell material. To do this, the shell has an elastic-plastic material on the Drucker-Prager model. The soil environment has an elastoplastic material according to the Mohr-Coulomb model. The calculation is made in a nonlinear statement. Nonlinearity allows using an iterative process to find the contact zone of the
elements (the detachment area of the shell from the soil) and determine the time-varying position of the shell. Also nonlinearity allows to determine the plastic collapse in the body of the shell and the soil environment [5,6].

The physical and mechanical properties of the shell are given by the Drucker-Prager model with the following parameters: elastic modulus $E = 3.0 \times 10^{10}$ Pa, Poisson’s ratio $\mu = 0.18$, density $\rho = 2300$ kg/m$^3$, uniaxial tensile strength $R_t = 2.5$ MPa, uniaxial compressive strength $R_c = 42$ MPa, biaxial compressive strength $R_b = 50$ MPa. The physical and mechanical properties of the soil environment are given by the Mohr-Coulomb model with the following parameters: deformation modulus $E_{def} = 14 \times 10^6$ Pa, Poisson’s ratio $\mu = 0.30$, density $\rho = 1600$ kg/m$^3$, friction angle $\varphi = 20^\circ$, cohesion $C_u = 10$ kPa.

In figure 5 is shown the total displacements of the spatial model for "the shell-the soil" system in the case of an infinitely long cylindrical shell when the buckling load is achieved. The nonlinear analysis stopped at the load increment step, at which solution convergence is not achieved. The buckling load value is $2.2 \gamma z_1$.

The results showed that if the friction coefficient is not taken into account, the shell loses of stability at a low buckling load, as in the case of a linear elastic material.

![Figure 5](image_url)

**Figure 5.** Fields of total displacements of the spatial model for "the shell-the soil" system when the buckling load is achieved.

2.5. Nonlinear analysis of the spatial model for “the shell-the soil” system with one-way connections taking into account the friction coefficient. Elastic-plastic material

This model takes into account the friction coefficient between the shell and the soil environment. The friction coefficient is $f = 0.6$, since the physical and mechanical properties of the soil are accepted as for dry soil, and the properties of the shell as for concrete. The calculation is made in a nonlinear statement. Nonlinearity allows using an iterative process to find the contact zone of the elements (the detachment area of the shell from the soil) and determine the time-varying position of the shell. Also nonlinearity allows to determine the plastic collapse in the body of the shell and the soil environment.

The shell has an elastic-plastic material on the Drucker-Prager model. The soil environment has an elastoplastic material according to the Mohr-Coulomb model. The material characteristics are taken from the previous paragraph.

In figure 6 is shown the total displacements of the spatial model for "the shell-the soil" system in the case of an infinitely long cylindrical shell when the buckling load is achieved. The nonlinear analysis stopped at the load increment step, at which solution convergence is not achieved. In figure 7 is shown the plastic collapse in the body of the soil environment. In figure 8 is shown the plastic collapse in the body of the shell. The buckling load value is $5.3 \gamma z_1$.

The results showed that if the friction coefficient is taken into account, the system does not lose stability in the early stages of load growth. However, the use of elastic-plastic material model does not allow to achieve values of buckling loads obtained by linear elastic material model.
The maximum principal stress [7,8] in the shell body is 2.7 MPa in the lower arch of the cylindrical shell. The minimum principal stress in the shell body is 55.4 MPa in the lower arch of the cylindrical shell. In figure 9 is shown the distribution of the maximum principal stresses in the shell body. In figure 10 is shown the distribution of the minimum principal stresses in the shell body. And in figure 11 is shown the displacement curve of the top point of the shell depending on the loading step.

**Figure 6.** Fields of total displacements of the spatial model for "the shell-the soil" system when the buckling load is achieved

**Figure 7.** The plastic collapse in the body of the soil environment

**Figure 8.** The plastic collapse in the body of the shell
Figure 9. The distribution of the maximum principal stresses in the shell body

Figure 10. The distribution of the minimum principal stresses in the shell body

Figure 11. The displacement curve of the top point of the shell depending on the loading step
2.6. **Nonlinear analysis of the spatial model for “the shell-the soil” system with one-way connections taking into account the friction coefficient. Elastic-plastic material. Computational model with horizontal exciting force applied to the top point of the shell**

The horizontal exciting force is added to this calculation model. The exciting force is applied to the top point of the shell. This force coerces the system to react to the bifurcation point. The buckling load may decrease relative to the previous calculation case. The horizontal exciting force value is 500 N.

In figure 12 is shown the total displacements of the spatial model for "the shell-the soil" system in the case of an infinitely long cylindrical shell when the buckling load is achieved. The buckling load value is $5.3\gamma z_1$.

The results showed that the buckling load value did not decrease with the addition of the horizontal exciting force.

![Figure 12](image)

**Figure 12.** Fields of total displacements of the spatial model for "the shell-the soil" system when the buckling load is achieved

3. **Conclusion**

Table 1 shows the values of the buckling load from the five calculated cases of the spatial model for "the shell-the soil" system in the case of an infinitely long cylindrical shell.

| Type of analysis | The value of the buckling load |
|------------------|--------------------------------|
| Linear analysis with two-way connections. Linear elastic material | $36477.0\gamma z_1$ |
| Nonlinear analysis with one-way connections without taking into account the friction coefficient. Linear elastic material | $4.3\gamma z_1$ |
| Nonlinear analysis with one-way connections taking into account the friction coefficient. Linear elastic material | $39.9\gamma z_1$ |
| Nonlinear analysis with one-way connections without taking into account the friction coefficient. Elastic-plastic material | $2.2\gamma z_1$ |
| Nonlinear analysis with one-way connections taking into account the friction coefficient. Elastic-plastic material | $5.3\gamma z_1$ |
| Nonlinear analysis with one-way connections taking into account the friction coefficient. Elastic-plastic material. Computational model with horizontal exciting force | $5.3\gamma z_1$ |

The results of the first calculation case showed that the linear analysis of the spatial model for "the shell-the soil" system with two-way connections leads to overestimate values of the buckling load relative to the natural load. The results of the next calculation case with one-way connections showed
that if the friction coefficient is not taken into account, the shell loses of stability at a low buckling load. The results showed that if the friction coefficient is taken into account, the system does not lose stability in the early stages of load growth and the value of the buckling load is significantly increased. The transition to elastic-plastic models of the material led to significantly lower values of the buckling load. These values of the buckling load are closer to the values of the natural load. The calculation case with horizontal exciting force is also made. This force coerces the system to react to the bifurcation point. However, the buckling load value did not decrease with the addition of the horizontal exciting force.

It should also be noted that the coefficient of "stability margin" - 5.3, obtained in the last calculation case, in relation to the natural load is not fully sufficient for the structures in use.

References
[1] ANSYS Mechanical 17.0 Tutorials. Canonsburg, 2016
[2] CHigarev A V, Kravchuk A S and Smaltuk A F 2004 ANSYS for engineers: reference guide (Moscow: Mashinostroenie) p 512
[3] Fedorova N N, Valger S A, Danilov M N and Zakharova Iu V 2017 Basics of ANSYS 17 (Moscow: DMK Press) p 128
[4] Basov K A 2005 ANSYS: User reference (Moscow: DMK Press) p 640
[5] Timoshenko S P and Gere J 1963 Theory of Elastic Stability. 2nd Edition (New York: McGraw-Hill) p 278
[6] Kosytsyn S B and Akulich V Yu 2019 Stress-strain state of a cylindrical shell of a tunnel using construction stage analysis Communications – Scientific letters of the University of Zilina. Vol. 21. 3. p 72
[7] Zienkiewicz O C and Taylor R L 2000 The finite element method. Vol. 2: Solid mechanics fifth edition (Oxford: Butterworth-Heinemann) p 479
[8] Aleksandrov A V and Potapov V D 1990 Fundamental theory of elasticity and plasticity (Moscow: Higher school) p 400