Nature of Supersonic Turbulence and Density Distribution Function in the Multiphase Interstellar Medium

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Abstract

Supersonic flows in the interstellar medium (ISM) are believed to be a key driver of the molecular cloud formation and evolution. Among molecular clouds’ properties, the ratio between the solenoidal and compressive modes of turbulence plays important roles in determining the star formation efficiency. We use numerical simulations of supersonic converging flows of the warm neutral medium (WNM) resolving the thermal instability to calculate the early phase of molecular cloud formation, and we investigate the turbulence structure and the density probability distribution function (density PDF) of the multiphase ISM. We find that both the solenoidal and compressive modes have their power spectrum similar to the Kolmogorov spectrum. The solenoidal (compressive) modes account for $\gtrsim 80\%$ ($\lesssim 20\%$) of the total turbulence power. When we consider both the cold neutral medium (CNM) and the thermally unstable neutral medium (UNM) up to $T \lesssim 400$ K, the density PDF follows the lognormal distribution, whose width $\sigma_\text{b}$ is well explained by the known relation from the isothermal turbulence as $\sigma_\text{b} = \ln(1 + b^2 M^2)$ (where $b$ is the parameter representing the turbulence mode ratio and $M$ is the turbulent Mach number). The density PDF of the CNM component alone ($T \lesssim 50$ K), however, exhibits a narrower $\sigma_\text{b}$ by a factor of $\sim 2$. These results suggest that observational estimations of $b$ based on the CNM density PDF requires the internal turbulence within each CNM clump but not the interclump relative velocity, the latter of which is instead powered by the WNM/UNM turbulence.

Unified Astronomy Thesaurus concepts: Interstellar medium (847); Warm neutral medium (1789); Cold neutral medium (266); Interstellar dynamics (839)

1. Introduction

Molecular clouds are progenitors of stars (Kennicutt & Evans 2012), and their physical states set the initial condition to determine the star formation rate (SFR)/star formation efficiency (SFE). In particular, the density structure is important where the densest volume of molecular clouds eventually collapses gravitationally to form stars (Krumholz & McKee 2005).

One of the key statistics is the (column) density probability distribution function (PDF). Based on analytic models and one-dimensional simulations, Passot & Vázquez-Semadeni (1998) show that isothermal turbulence produces a lognormal density PDF, which is a compilation of density enhancement by multiple shocks. Other analytic and numerical studies (e.g., Padoan et al. 1997; Nordlund & Padoan 1999) also find such a lognormal density PDF and discuss its relation to the functional form of the stellar initial mass function (IMF). Two- and three-dimensional simulations later suggest that the lognormal density PDF ubiquitously exists under the driven turbulence (e.g., Federrath et al. 2008), where the tails can deviate from the lognormal PDF owing to nonisothermality (Passot & Vázquez-Semadeni 1998; Scalo et al. 1998), self-gravity (Klessen 2000), and magnetization effects (Li et al. 2008; see also the review by Elmegreen & Scalo 2004). Molecular clouds in the Milky Way indeed exhibit lognormal PDF’s of (column) densities at the low-$A_v$ regime, followed by a power-law tail at the high-density end (Lombardi et al. 2011; Schneider et al. 2013; Alves de Oliveira et al. 2014; Schneider et al. 2016; but see also Lombardi et al. 2015).

There have been theoretical studies to estimate how the SFR varies based on the formation of the densest parts of such density PDF’s within molecular clouds, by assuming some SFE per freefall time in individual molecular cloud cores (e.g., Krumholz & McKee 2005; Federrath et al. 2008; Padoan & Nordlund 2011). In particular, the suite of isothermal simulations with driven turbulence shows that the turbulence mode ratio between solenoidal and compressive modes and the turbulent Mach number are important parameters that control the width of the density PDF (e.g., Federrath et al. 2008, 2010). For example, the SFR varies from by a factor of one to by one order of magnitude depending on the turbulence mode ratio (Federrath & Klessen 2012). This suggests that different density PDF’s and the turbulence mode ratios result in various SFRs of star-forming clouds even under a fixed turbulence power spectrum. Meanwhile, detailed simulations of radiative feedback from massive stars suggest that the SFE depends strongly on the initial column density of the cloud (e.g., Kim et al. 2018; Fukushima et al. 2020; Fukushima & Yajima 2021), and the turbulent structure likely impacts the stellar initial mass function as well (Padoan & Nordlund 2002). Therefore, it is important to reveal the turbulence structure and resultant density structures achieved in molecular clouds under realistic conditions of molecular cloud formation and evolution.
Supersonic flows in the interstellar medium (ISM) are one of the key driving sources of the turbulence. In galactic disk regions of star-forming galaxies (like the Milky Way), the typical interval between the passages of successive supersonic flows at any volume of the ISM is about 1 Myr (McKee & Ostriker 1977), where those flows originate in supernovae, expanding H II regions, galactic spirals, etc. The typical molecular cloud lifetime is also believed to be about a few × 10 Myr (Kawamura et al. 2009; see also Meidt et al. 2015); where molecular clouds finally evaporate within 10 Myr owing to the stellar feedback once massive stars form (Hosokawa & Inutsuka 2006; Kruisjes et al. 2019; Fukushima et al. 2020; Kim et al. 2021). The interaction between molecular clouds and supersonic flows is naturally expected to occur multiple times during the cloud lifetime. Matsumoto et al. (2015) perform simulations of an isothermal ISM to investigate such interactions and show that the turbulent velocity within molecular clouds does not significantly differ between with and without the colliding flows if averaged over 1–3 freefall times of the cloud (see also Seifried et al. 2018), but the turbulent mode ratio and the density PDF do differ, resulting in a rapid star formation due to the colliding flows. Padoan et al. (2016) perform simulations of multiple supernovae on a 100 pc scale and find that the expansion of multiple supernova remnants leads to driving the solenoidal modes more strongly than the compressive modes by a factor of about 5.

Supersonic flows drive not only the turbulence but also the evolution of the thermal sates by triggering the phase transition from the warm neutral medium (WNM) to the cold neutral medium (CNM) through the thermal instability (e.g., Field 1965; Balbus 1986). This transition from the WNM to the CNM increases the density and is believed to be an important first step of molecular cloud formation (Koyama & Inutsuka 2002; Audit & Hennebelle 2005). Therefore, simultaneous investigations of both the turbulent structure and thermal states provide the comprehensive understanding of the evolution of the density structure within molecular clouds and the resultant SFR. However, such studies are still limited and focus on the later stages of molecular cloud evolution, where molecular clouds are massive enough to be self-gravitating (e.g., Körtgen et al. 2017).

As a complementary study to understand the molecular cloud evolution comprehensively from its early phase along with the thermal evolution, we perform hydrodynamics simulations of supersonic converging WNM flows, by resolving the thermal instability and the formation of the multiphase ISM as a precursor of molecular clouds. We investigate the density PDF and turbulence structure in this multiphase ISM. We measure the turbulence power spectrum, velocity structure functions, CNM clump size and mass spectra, the density PDF as a function of temperature, and the turbulence mode ratio on 0.01–10 pc scales, based on which we show the range in which the isothermal turbulence framework explains the density PDF in the multiphase ISM.

The rest of this article is organized as follows. In Section 2, we explain the method of our simulations and analyses. In Section 3, we explain the basic equations that describe the density PDF and the known relation in the isothermal turbulence theory framework, as well as the definition of the solenoidal/compressive modes. We present the main results in Section 4 by using the data from the entire simulation domain, whereas we focus on the results from nine subvolumes in Section 5. In Section 6, we discuss the implications from our results and future prospects. We summarize our results in Section 7, followed by appendices explaining the convergence with respect to the spatial resolution and additional information of the technical details in our analyses.

2. Method

2.1. Basic Equations and Setups

We utilize the simulation code and setups from our previous study (Kobayashi et al. 2020). Our code is based on the hydrodynamics part from the magnetohydrodynamics code originally developed in Inoue & Inutsuka (2008). This code employs the second-order Godunov scheme (van Leer 1979) and explicitly time-integrates the heating and cooling with the second-order accuracy. We solve the following basic equations:

$$\frac{\partial \rho}{\partial t} + \nabla_i (\rho v_i) = 0,$$

$$\frac{\partial \rho v_i}{\partial t} + \nabla_j (\rho v_j v_i) = 0,$$

$$\frac{\partial \rho e}{\partial t} + \nabla_i (\rho e v_i) = \nabla_i (\kappa(T) \nabla_i T) - \rho \mathcal{L}(T).$$

Here $\rho$ is the mass density, $v$ represents the velocity, $P$ represents the thermal pressure, $T$ is the temperature, and $\nabla_i = \partial / \partial x_i$, where $x_i$ spans $x, y, \text{ and } z$. $\delta_i$ is the identity matrix. We calculate the total energy density, $\rho e$, as $\rho e = P / (\gamma - 1) + \rho v^2 / 2$, where $\gamma = 5/3$ is the ratio of the specific heat. We implement the thermal conductivity, $\kappa$, as $\kappa(T) = 2.5 \times 10^5 T^{0.5} \text{erg cm}^{-1} \text{s}^{-1} \text{K}^{-1}$, representing collisions between hydrogen atoms (Parker 1953).

The thermal state of the gas is controlled by the balance between the cooling and heating, which is implemented in the net cooling rate per mass $\mathcal{L}(T)$. We use the functional form of $\mathcal{L}(T)$ that we develop in our previous study (Kobayashi et al. 2020). This is the combination of the results from Koyama & Inutsuka (2002) in $T \leq 14,577$ K and Cox & Tucker (1969) and Dalgarno & McCrory (1972) in $T > 14,577$ K by considering the cooling rate due to Ly$\alpha$, CII, He, C, O, N, Ne, Si, Fe, and Mg lines with the photoelectric heating, which corresponds to the typical ISM condition in the Milky Way. In general, the shock-heated WNM experiences the thermally unstable phase ($\partial (\mathcal{L}(T) / \partial T)_p < 0$; see, e.g., Balbus 1986, 1995), leading to the formation of the CNM clumps. The multiple phases that appear in our simulations roughly correspond to the WNM as $T \geq 5000$ K, the thermally unstable neutral medium (UNM) as $100$ K $\leq T < 5000$ K, and the CNM as $T < 100$ K based on $\partial (\mathcal{L}(T) / \partial T)_p$. We refer the readers to Kobayashi et al. (2020) for the exact form of this $\mathcal{L}(T)$.

We prepare a three-dimensional simulation domain with its size of $L_{x,y,z} = 20, 10, 10$ pc and calculate the head-on collision of supersonic WNM converging flows continuously injected through the two $x$ boundaries over $3$ Myr. We employ the periodic boundary condition on both $y$ and $z$ boundaries. This flow creates a shock-compressed layer sandwiched by two shock fronts at the box center $x \approx L_x/2$, where the two flows collide. The flow velocity is fixed as $20$ km s$^{-1}$, representing common situations in galactic disks (e.g., the late phase of supernova remnant expansion, H II region expansion, and normal shocks due to galactic spirals). The injected WNM flow is thermally stable with the mean number density $n_0 = 0.57$ cm$^{-3}$ and
pressure $P_0/k_B = 3500 \, \text{K} \, \text{cm}^{-3}$. This corresponds to the mean temperature of 6141 K, the sound speed of $C_s = 8.16 \, \text{km} \, \text{s}^{-1}$, and the ram pressure of the converging flow as $P/k_B = 2.6 \times 10^4 \, \text{K} \, \text{cm}^{-3}$.

The density of the injected WNM flow has a fluctuation with the Kolmogorov power spectrum $P_{fl}(k) \propto k^{-11/3}$ (Kolmogorov 1941; Armstrong et al. 1995). The highest frequency is $k/2\pi = 12.5 \, \text{pc}^{-1}$ (i.e., the wavelength of 0.08 pc). The amplitude is set such that the mean dispersion is $\sqrt{\langle \delta n^2 \rangle}/n_0 = 1$ as the most realistic parameter in our calculation to describe the density structure in star-forming regions (Kobayashi et al. 2020). The interaction between this density inhomogeneity and shock fronts induces turbulence in the shock-compressed layer (e.g., Inoue & Inutsuka 2012; Carroll-Nellenback et al. 2014).

We employ uniform spatial resolution of 0.01 pc in fiducial analyses, with additional resolutions of 0.02, 0.04, and 0.08 pc for comparison (Appendix A). This 0.01 pc is motivated by our previous systematic study over a wide parameter range of $\langle \delta n^2 \rangle/\bar{n}_0$ by Kobayashi et al. (2020). In Kobayashi et al. (2020), we show that, with the thermal instability, the spatial resolution with $\lesssim 0.02$ pc is required to fully resolve the cooling length of the UNM and have the convergence in the mean density of the shock-compressed layer. This spatial resolution resolves the highest frequency of the injected density fluctuation with at least eight cells. In Appendix A, we also show that CNM clump properties have their convergence with the 0.01 pc resolution compared with 0.02 and 0.04 pc (such as mean density, size, and mass spectra).

The calculation results are sampled every 0.1 Myr, and we identify the shock front position by $P > 1.3P_0$ to define the shock-compressed layer in each sampled time. Figure 1 shows an example of the two-dimensional slices of the density distribution (panel (a)) and the temperature distribution (panel (b)) at 2.5 Myr. The central region sandwiched by the two shock fronts is the progenitor of molecular clouds, within which the multiphase ISM forms through the thermal instability. The effective typical cooling length peaks at $\sim 1$ pc for the UNM and at 0.02–0.2 pc for the CNM (see panel (a) of Figure 10). Therefore, through the growth of the thermal instability, the CNM structures develop on subparsec scales as seen as clumpy structures on $\sim 0.1$ pc in Figure 1.

2.2. The Decomposition into the Solenoidal and Compressive Modes and Our Choice of Analyzed Volume

We apply the Fourier transformation to the shock-compressed layer to obtain the Fourier component of the velocity field by using the Fast Fourier Transform in the West (FFTW 3.3; Frigo & Johnson 2005), and we decompose this into the solenoidal modes and compressive modes (see Section 3). We perform this decomposition both on the entire volume of the shock-compressed layer and on nine subvolumes of $(2.56 \, \text{pc})^3$ size within the shock-compressed layer. We summarize the technical details involved in this decomposition in Appendix B. We employ five temperature thresholds to extract volumes with various thermal states and investigate the difference in the turbulent structure and density PDF between the WNM+UNM and CNM: $T \lesssim 50 \, \text{K}$, $T \lesssim 200 \, \text{K}$, $T \lesssim 400 \, \text{K}$, $T \lesssim 600$, and $T \lesssim 1000 \, \text{K}$.

3. Density PDF in Isothermal Turbulence

The lognormal density PDF is defined as

$$p_s(s) = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left(-\frac{(s - s_0)^2}{2\sigma_s^2}\right),$$

where $s$ is the natural logarithm of density. In the framework of the isothermal turbulence, essentially three parameters determine the SFR by controlling the fraction of dense volume that gravitationally collapses to form stars. One is the mean density, another is the mean Mach number of turbulence, and the other is the mode ratio between the solenoidal/compressive modes. In the case of such isothermal turbulence, it is known that the width of this lognormal density PDF $\sigma_s$ is well described by the combination of the forcing parameter $b$ and the mean Mach number $M$ as

$$\sigma_s^2 = \ln(1 + b^2M^2)$$

(see, e.g., Federrath et al. 2010). Here $b$ represents the compressibility defined by the compressive ratio $\chi$ (Pan et al. 2016) as

$$b = \frac{\chi}{\sqrt{1 + \chi}}.$$
Figure 2. (a) The one-dimensional averaged velocity power spectrum at 3 Myr over the entire volume of the shock-compressed layer. The black line corresponds to the total power, whereas the colored lines show the breakdown as the solenoidal modes (blue) and the compressive modes (red). The solenoidal mode amounts to ≥80% of the total power on all scales (shown in panel (b)), so that the “total” (black) is almost overlapped with “solenoidal” (blue) in this figure. The shade shows the Poisson noise based on the sample counts in each k-vector bin. The vertical gray lines show k/2π = 0.1 pc−1 (left solid; the proxy of the lowest frequency of the shock-compressed layer; see Appendix B), k/2π = 5.12 pc−1 (middle thin dotted; the numerical diffusion likely impacts the power on k/2π > 5.12 pc−1), and the Nyquist frequency k/2π = 51.2 pc−1 (right solid). We use the powers on 0.1 pc−1 ≤ k/2π ≤ 5.12 pc−1 in the analyses hereafter. We overplot two reference lines representing power laws of ∝k−5/3 and ∝k−6/3. The k-space is linearly binned where the highest frequency corresponds to the Nyquist frequency. (b) Same as panel (a), but the solenoidal/compressive mode fraction of the total turbulent power.

\[ \chi = \frac{\langle \tilde{v}_{\text{comp}}^2 \rangle}{\langle \tilde{v}_{\text{sol}}^2 \rangle} \]  

Here \( \tilde{v}_{\text{comp}} \) and \( \tilde{v}_{\text{sol}} \) mean the compressive and solenoidal modes of the turbulent velocity field, respectively.

Any velocity field is in principle able to be decomposed into the solenoidal (transverse) and compressive (longitudinal) modes through the Helmholtz decomposition as

\[ \tilde{v}_{\text{sol}}(\hat{k}) = (\hat{k} \times \tilde{v}) \times \hat{k} \]  

\[ \tilde{v}_{\text{comp}}(\hat{k}) = (\hat{k} \cdot \tilde{v})\hat{k}. \]  

Here \( \hat{k} \) is a unit wavevector and \( \tilde{v} \) is the Fourier component of the velocity field. In the following analyses, we will measure \( b, M_t \), and \( \sigma_s \) from our simulations and evaluate \( \sigma_{s,\text{theory}} \) based on Equation (5) to investigate the difference between \( \sigma_s \) and \( \sigma_{s,\text{theory}} \) in various thermal states.

4. Results I: The Entire Volume of the Shock-compressed Layer

Hereafter the spatial frequency \( k/2\pi \) denotes the inverse of the wavelength of a corresponding wave, such that \( k/2\pi = 0.1 \) pc−1 corresponds to 10 pc.

4.1. Solenoidal and Compressive Mode Fraction

Panel (a) of Figure 2 shows the one-dimensional averaged velocity power spectrum. The overall spectrum follows the Kolmogorov spectrum \( \propto k^{-5/3} \), where the solenoidal modes dominate over the compressive mode on all scales. The spectrum of the solenoidal modes exhibits a break at \( k/2\pi \sim 5.0 \) pc−1, and so does the total spectrum. This is attributed to the numerical diffusion, which we evaluate further in Appendix A. Given that the typical cooling length of the UNM peaks at ~1 pc (panel (a) of Figure 10 in Appendix A) and that the break of the compressive mode power is limited compared to that of the solenoidal-mode power, our spatial resolution is high enough to resolve the cooling length of the UNM to follow the dynamical condensation through the thermal instability (but also see Section 6.2).

Panel (b) of Figure 2 shows the solenoidal/compressive mode fraction of the total turbulent power. This shows that the solenoidal modes account for 80%–90% on most scales.

Figure 3 shows the time evolution of the total solenoidal/compressive mode fraction integrated from \( k/2\pi = 0.1 \) to 5.12 pc−1 over the entire volume of the shock-compressed layer. A converging flow configuration itself is a compressive motion, and thus the compressive mode initially accounts for 40% of the turbulent power in the shock-compressed layer. Along with the turbulent generation, the solenoidal modes increase to reach 80%–90%. After 0.5 Myr, the fractions into the solenoidal and compressive modes are quasi-steady with 80%–90% and 10%–20%, respectively. In this quasi-steady state, the mass fraction between WNM, UNM, and CNM is also steady, and the expansion speed of the shock-compressed layer is almost constant (Kobayashi et al. 2020). At the final time step, 3 Myr, the fractions of the compressive and solenoidal modes are \( f_{\text{comp}} = 0.10 \) and \( f_{\text{sol}} = 0.90 \), respectively. The corresponding mode ratio \( \chi \) becomes 0.11, so that the forcing parameter is \( b = 0.32 \).

4.2. Density PDF and Its Width with Various Thermal States

Panel (a) of Figure 4 shows the thermal state within the shock-compressed layer at 3 Myr. This shows that the shock-heated WNM at \( n = 1–10 \) cm−3 and \( T \sim 6000 \) K cools through the thermally unstable regime, and their densest parts reach the CNM state at 100–1000 cm−3 and \( T \lesssim 50 \) K. The thermal states in the shock-compressed layer widely distribute on this
show that the fraction of the compressive (solenoidal) mode powers is 23% (77%). This is still solenoidal mode dominated and is similar to the averaged $b = 0.32$ that we obtain here on the entire shock-compressed layer. In this section, therefore, we simply employ $b = 0.32$ for all the temperature ranges when evaluating $\sigma_{s,\text{theory}}$.

Table 1 shows that (1) the measured width of the density PDF $\sigma_{s,\text{sim}}$ and the expected width $\sigma_{s,\text{theory}}$ are close to each other when we combine both the UNM and CNM components, especially at $T < 400$ K, (2) but the difference becomes larger toward the higher temperature range ($T < 1000$ K) and the lower temperature range ($T < 50$ K) up to a factor of $\sim 2$. The difference is attributed to the evaluation of $b$ and $M$ in $\sigma_{s,\text{theory}}$ (see Equation (5)). In the following subsections, we will show that $\sigma_{s,\text{sim}}$ and $\sigma_{s,\text{theory}}$ are close to each other when we take into account the thermal states properly to select the consistent $\langle C_s(T_f) \rangle$, $\sqrt{\langle \delta v^2(T_f) \rangle}$ and $\sigma_{s,\text{sim}}$ responsible for each other; thus, the difference between $\sigma_{s,\text{sim}}$ and $\sigma_{s,\text{theory}}$ is a natural consequence of the multiphase nature of this system.

4.2.1. Warmer Region with $T \sim 1000$ K

In the warmer regions, $\sigma_{s,\text{sim}}$ is wider than $\sigma_{s,\text{theory}}$-$\sqrt{\langle \delta v^2(T_f) \rangle}$ and $\langle C_s(T_f) \rangle$ are volume-weighted quantities, so that they tend to depend more on diffuse (i.e., warmer) components when we expand the temperature threshold to the warmer regime. For example, $\sigma_{s,\text{theory}}$ at $T < 1000$ K depends more on WNM and UNM at $T \sim 1000$ K that just starts to cool. On the other hand, the measured $\sigma_{s,\text{sim}}$ tries to take into account the cooler UNM and CNM components as well ($T < 400$ K), so that $\sigma_{s,\text{sim}}$ at $T < 1000$ K is wider than $\sigma_{s,\text{theory}}$.

In panel (b) of Figure 4, we additionally show the breakdown of density PDF, such as $600 < T[K] < 1000$ (labeled as (B)) and $200 < T[K] < 400$ (labeled as (D)). If we restrict ourselves to the distribution (B) alone, we find that $\sigma_{s,\text{sim}}(600 < T[K] < 1000) = 0.629$. Meanwhile, $\langle C_s(600 < T[K] < 1000) \rangle = 2.96$ km s$^{-1}$ and $\sqrt{\langle \delta v^2(600 < T[K] < 1000) \rangle} = 4.10$ km s$^{-1}$, so that $M_{\text{sim}}(600 < T[K] < 1000) = 1.39$ and $\sigma_{s,\text{theory}}(600 < T[K] < 1000) = 0.51$. This shows that $\sigma_{s,\text{sim}}$ and $\sigma_{s,\text{theory}}$ are closer to each other at $600 < T[K] < 1000$ rather than at the entire $T[K] < 1000$, suggesting that the original values of $\sqrt{\langle \delta v^2 \rangle}$, $\langle C_s \rangle$, $M_{\text{sim}}$, and corresponding $\sigma_{s,\text{theory}}$ at $T < 1000$ K are weighted to warmer components.

4.2.2. Cold Region with $T < 50$ K: CNM Clamp Statistics

Contrarily to the warmer regimes, $\sigma_{s,\text{theory}}$ is wider than $\sigma_{s,\text{sim}}$ at colder regimes. In $T < 50$ K, for example, $b \approx 0.1$ is required to have $\sigma_{s,\text{sim}} = \sigma_{s,\text{theory}}$ with the given $M = 5.99$. Such a small $b$ less than 0.3 is difficult to achieve in reality, even with a purely solenoidal turbulence (Federrath et al. 2008). Figure 2 already indicated that the Kolmogorov spectrum extends to smaller scales with the almost constant $b \sim 0.3$, even when we perform simulations with a higher resolution to remove the numerical diffusion effect. The generation of the compressive mode due to the thermal instability also takes place on small scales. Therefore, our results suggest that the evaluation of $M$ plays a more significant role to explain the difference between

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6 Note that, although the histograms are well fitted by lognormal functions, the heating/cooling processes always induce deviation from the lognormal functions because the temperature and density in the thermally stable state have correlations. For example, the density PDF of $T < 50$ K has a strong transition at 50–100 cm$^{-3}$, where the thermally stable state has $T = 50$ K so that most of the gas with $T < 50$ K is restricted to $n \gtrsim 100$ cm$^{-3}$ (see panel (a) of Figure 4).

7 The total density PDF of the whole temperature range is bimodal between WNM and CNM, which can be seen in the thick black histogram in panel (b) of Figure 4 labeled ‘(A)’. The lognormal fitting is close to the peak of warmer components that have more volume than the cold components.
\[ \sigma_{s,\text{sim}} \text{ and } \sigma_{s,\text{theory}}. \] 
As seen in Section 2.1, the subparsec-scale CNM clumps form through the thermal instability, and they are spatially disconnected to each other. Our straightforward evaluation of \( \sqrt{\langle \delta n^2(T) \rangle} \) (Equation (11)) therefore reflects the relative velocity within CNM clumps more than the internal velocity dispersion within individual CNM clumps.

To further confirm the above discussions, we identify CNM clumps and measure their physical properties. We define a CNM clump as a connected volume with the temperature \( T < 50 \text{ K} \), which typically has its density \( n \geq 50 \text{ cm}^{-3} \) (see panel (a) of Figure 4). In the following analyses, we restrict ourselves to the clumps with \( \geq 512 \) cells to reduce the effect of numerical noises, and we analyze 2403 such CNM clumps.

Panels (a) and (b) of Figure 5 show the size and mass histograms at 3 Myr. We use \( \sqrt{I_{\text{max}}/M} \) as the size of individual clumps, where each clump has \( I_{\text{max}} \) as the maximum eigenvalue of its inertia matrix and \( M \) as its mass. The size and mass peak at \( \sim 0.08 \text{ pc} \) and \( 6 \times 10^{-3} \text{M}_\odot \), respectively. The volume-weighted mean density of a clump is \( \langle n_{\text{clump}} \rangle \simeq 189 \text{ cm}^{-3} \). As the mass spectrum of the CNM clumps is \( dn/dm \propto m^{-1.8} \), more massive clumps dominate the mass budget of CNM components. This spectrum is often reported by other authors investigating converging flows (e.g., Inoue & Inutsuka 2012) and consistent with a theoretically expected spectrum of \( dn/dm \propto m^{-1.78} \) under the thermal instability growth (see, e.g., Hennebelle & Audit 2007).

Panel (c) of Figure 5 shows the relation between the size and internal velocity dispersion within each CNM clump at 3 Myr. The mean internal velocity dispersion and the mean sound speed are \( \sqrt{\langle \delta v^2_{\text{clump}} \rangle/\nu} = 1.05 \text{ km s}^{-1} \) and \( \langle C_{s, \text{ clump}} \rangle/\nu = 0.43 \text{ km s}^{-1} \), so that the mean Mach number within CNM clumps is \( M_{\text{clump}} \sim 2.44 \). This internal velocity dispersion is consistent with \( \lesssim 1 \text{ km s}^{-1} \) suggested from the second-order velocity structure function on the 0.1 pc scale (shown later in Section 5.2 as Figure 8).

In order to evaluate the mode parameter within CNM clumps, \( b_{\text{clump}} \), we measure the powers of the grid-scale turbulent modes as \( \langle (\nabla_g \times \nu)^2_{\text{clump}} \rangle /\nu \) for the compressive mode and \( \langle (\nabla_g \times \nu)^2_{\text{clump}} \rangle /\nu \) for the solenoidal mode (Kida & Orszag 1990a, 1990b). Here \( \nu \) is the velocity field in the real space, and \( \nabla_g \) is the spatial differential operator on each numerical cell scale. The fraction of the compressive (solenoidal) mode powers is 23\% (77\%), and the corresponding mode parameter on the grid scale within CNM clumps is \( b_{\text{clump}} \sim 0.48 \). Therefore, \( b_{\text{clump}} \) is indeed larger than 0.1 as we speculated in the first paragraph of this subsection. Here \( b_{\text{clump}} \sim 0.48 \) also suggests that the mode fraction at the Nyquist frequency alone in the Fourier analysis (the gray vertical line in panel (a) of Figure 2) already indicated a fraction close to \( b_{\text{clump}} \) even though such a Fourier power is dominated rather by the WNM and UNM than by the CNM.

Equation (5) with \( M_{\text{clump}} \sim 2.44 \) and \( b_{\text{clump}} \sim 0.48 \) suggests that the density PDF width of the CNM component alone is \( \sigma_{s,\text{theory}} \sim 0.61 \), which is consistent with \( \sigma_{s,\text{sim}} \) = 0.61.

In conclusion of Sections 4.2.1 and 4.2.2, our results suggest that (1) when we focus on the volume with \( T < 1000 \text{ K} \), turbulence is more powered by diffuse warm components, such as the WNM and UNM that just start to cool, and thus the isothermal theory with the volume-weighted turbulence explains the width of the density PDF of such warmer components without the cold tails; (2) the coldest part of the density PDF (\( T < 50 \text{ K} \)) is governed by the turbulence within current clumps.
individual CNM clumps rather than by the strong turbulence on the entire cloud scale, which determines the relative velocity between CNM clumps, and the turbulent \( M \) within individual CNM clumps is more important to evaluate the density PDF of the CNM component alone; and (3) Equation (5) from the isothermal theory is directly applicable when we compile all the volumes with \( T < 400 \) K.

5. Results II: The Subvolumes of the Shock-compressed Layer

In this section, we analyze the nine subvolumes to investigate the local variations. The nine subvolumes are selected so that their volume-centered position is \((L_x/2, L_y/2, L_z/2)\) for the most-centered one and \((L_x/2 \pm 2.56 \text{ pc}, L_y/2 \pm 2.56 \text{ pc}, L_z/2 \pm 2.56 \text{ pc})\) for the other eight. We assume the periodicity in the Fourier analysis and also expect that the boundary condition does not significantly affect the turbulent powers on the scales of \( k/2\pi \lesssim 5.12 \text{ pc}^{-1} \) (see Section B.2).

5.1. Turbulence Modes and Density PDF

Panel (a) of Figure 6 shows the averaged velocity power spectrum from the nine subvolumes. Although the wavelength ranges affected by the numerical diffusion have a significant fraction in these subvolumes, the overall power spectrum shows that the Kolmogorov spectrum and the solenoidal modes always dominate the turbulence power, similar to the result of the entire volume of the shock-compressed layer (Section 4.1). Panel (b) of Figure 6 shows the time evolution of the solenoidal and compressive modes, which shows that, at later stages of \( \gtrsim 1.7 \) Myr, the variation among the nine regions is \( \lesssim 10\% \) and the solenoidal modes amount to \( \sim 80\% \) of the total turbulence power.\(^\text{10}\) At the final time step, 3 Myr, the fractions of the compressive and solenoidal modes are \( f_{\text{comp}} = 0.15 \) and \( f_{\text{sol}} = 0.85 \), respectively. Parameter \( \chi \) becomes 0.18, so that the forcing parameter is \( b = 0.39 \).

Figure 7 shows the sum of the density PDF from the nine subvolumes, overplotted with the corresponding lognormal fittings. We summarize the measured properties in Table 2. Similar to Table 1, \( \sigma_{s,\text{sim}} \) and \( \sigma_{s,\text{theory}} \) are comparable when we combine both the UNM and CNM components, especially at \( T \lesssim 400 \) K.

In conclusion, the power spectrum and the density PDF suggest that the local variation of the mode fraction exists but is limited at about a 10\% level, so that the solenoidal modes are always dominant.

5.2. Second-order Velocity Structure Function

To investigate the connection between the various thermal states in our simulations and the observations of the ISM turbulence, we measure the second-order velocity structure function within individual temperature thresholds:

\[
S(r) = \langle |v(x + r) - v(x)|^2 \rangle, \tag{12}
\]

where \( r = |r| \). This analysis does not require any stringent boundary condition compared with the power spectrum analysis and is capable of directly analyzing individual thermal states even when the volumes of the focused thermal states are clumpy and spatially disconnected from each other. For example, the structure function of the CNM component alone can cover from the turbulence within CNM clumps to the relative motion between CNM clumps on 10 pc scales. However, the internal velocity dispersions are affected by the numerical diffusion as we have discussed so far. Therefore, let us focus on only large scales \( \gtrsim 0.2 \) pc when we have quantitative discussions in the following.

Figure 8 shows the measured structure function. As a representative of different thermal states, we employ \( T < 50 \) K, \( 200 \) K \( < T < 400 \) K, and \( 600 \) K \( < T < 1000 \) K, similar to the density PDF analyses. The analyzed volume in each panel of Figure 8 is limited to the one within the corresponding temperature range, so that panel (a) represents the CNM and panels (b) and (c) mostly represent the UNM. We overplot the power laws of Kolmogorov \( S(r) \propto r^{2/3} \) and Larson \( S(r) \propto r^4 \). The figure indicates that the UNM components exhibit the spectrum close to the Kolmogorov one (e.g., \( 600 < T |K| < 1000 \)), while the CNM components exhibit the spectrum close to the Larson one (e.g., \( T < 50 \) K). Therefore, the early phase of molecular cloud formation through converging flows exhibits Larson’s law in the CNM components, which is consistent with observed supersonic turbulence (e.g., Larson 1981; Heyer & Brunt 2004), while the overall

\(^\text{10}\) Note that the significant variation at early phases is likely affected by the choice of analyzed volume, so that focusing on only after 1 Myr is fair when we discuss the variation between the subvolumes; see Section B.2.
CNM clump statistics

Figure 5. The statistics of 2403 CNM clumps: panel (a) shows the size histogram, panel (b) shows the mass histogram, and panel (c) shows the size—internal velocity dispersion relation at 3 Myr. In panel (c), each small circle represents individual CNM clumps, and the black solid line is the least-squares fitting result, which is $b_{\text{clump}} \propto l_{\text{size}}^{0.52}$.

6. Discussions

6.1. Comparison with Previous Studies and the Importance of Self-gravity

We calculate up to 3 Myr, and the accumulated mass is limited to $2.6 \times 10^8 M_\odot$. We ignore the self-gravity, which is subdominant compared with the ram pressure of the converging flow and the thermal/turbulent energy until $\sim 40$ Myr (see Kobayashi et al. 2020). Our calculation therefore corresponds to early phases of the cloud formation and evolution.

Compared with our simulations, other previous studies focus on later stages of the cloud evolution and show the impact of the self-gravity. For example, Matsumoto et al. (2015) perform hydrodynamics simulations to calculate 3 Myr evolution of an isothermal molecular cloud, whose initial mean density (mass) is already $10^2$ to $10^3$ cm$^{-3}$ ($10^3$ to $10^4 M_\odot$) and whose initial turbulence is with the solenoidal modes alone. They report that the compressive mode fraction remains $\lesssim 20\%$ without the self-gravity, which is close to our value of 10%, while the compressive mode generation due to the self-gravity is able to enhance the fraction more than 20%. Körtgen et al. (2017) also perform cloud formation simulations with heating and cooling similar to our studies, but they focus more on massive clouds ($\sim 10^2 M_\odot$) on a larger scale (256 pc). They investigate a long-term evolution (until 30 Myr) by employing a refinement technique with the spatial resolution from 62 to 0.03 pc at highest, and they measure the turbulent driving parameter $b$. Their results show that the parameter $b$ evolves from 0.3 to 0.8 owing to the self-gravitational collapse of the cloud.

Our current results of $b \sim 0.3$ is consistent with the early stages of these previous studies. It is, however, still important to investigate the evolution of self-gravitating clouds by resolving the thermal instability coherently from the beginning to later stages $\sim 40$ Myr. This is left for future studies.

6.2. Limitations: Internal Velocity Dispersion of CNM Clumps

In Section 4.2.2, we show that, to determine the density PDF of the CNM component, the internal velocity dispersion in individual CNM clumps is more responsible than the relative velocity between the CNM clumps. Higher resolution is still ideal to perform a more precise quantitative comparison between $\sigma_{v,\text{sim}}$ and $\sigma_{v,\text{theory}}$ because the $\sigma_v$ dependence on the Mach number is $M^2$ as shown in Equation (4), and also because the spatially smallest growing mode of the thermal instability is limited by the size of numerical cells and our current calculations are affected by the numerical diffusion on the scales below CNM clumps $\lesssim 0.1$ pc. Such calculations are left for future studies, but we expect that this will be important when compared with (future) observations (see Section 6.4).

Nevertheless, our current simulations already obtained the mean power of the internal turbulence well enough to discuss its importance for the density PDF. In Appendix A, we will investigate the resolution dependence to show that our current simulation with 0.1 pc resolution has the convergence. For example, the internal velocity dispersion of CNM clumps is 1.05, 1.02, and 1.44 km s$^{-1}$ when the spatial resolution is 0.01, 0.02, and 0.04 pc, respectively.

This convergence originates in the fact that the power of the internal turbulence is stronger on larger scales (as suggested in panel (a) of Figure 8). This means that, in the evaluation of the velocity dispersion responsible for the density PDF, it is important to resolve the eddies whose sizes are close to CNM clumps’ sizes. The CNM clumps’ sizes ($\sim 0.1$ pc) are typically comparable to the spatial scale at which the numerical diffusion starts to play in our current simulation ($\sim 0.2$ pc). The structure function at 0.1 pc in panel (a) is still close to the continuation of the $r^2$ scale dependence from the larger scales, which is free from the numerical diffusion ($\gtrsim 0.2$ pc). This suggests that the impact of the numerical diffusion is still limited for the turbulent eddy whose size is $\sim 0.1$ pc.

Therefore, although it is still ideal to perform simulations with a higher spatial resolution to investigate more precise
scale dependence of the velocity dispersion within CNM clumps, we expect that such simulations do not significantly change our current conclusion that the internal velocity dispersion is important to control the density PDF rather than the interclump relative velocity. Our convergence study in Appendix A suggests that resolving the CNM clump size with \( \geq 5 \) cells is the typical criterion to have the convergence in CNM clump mean properties.

### 6.3. Caveats: Other Conditions and Physical Processes

The flow velocity that we employ in the current simulations is \( \approx 20 \text{ km s}^{-1} \), a typical velocity of the shock propagation at late stages of supernova remnant expansion. However, a faster velocity \( \approx 100 \text{ km s}^{-1} \) also exists in galaxy–galaxy tidal interactions or mergers (e.g., Fukui et al. 2017; Maeda et al. 2021). Such fast flows generate stronger shear motion in the shock-compressed layer and can change the turbulent mode ratio and/or CNM mass fraction by preventing the thermal instability. This is beyond our scope in this article, but the dependence on flow velocity has to be investigated to understand more intense star-forming regions (see Kim & Ryu 2005; Federrath & Klessen 2012, for the Mach number dependence of the velocity/density power spectrum and the density PDF).

Magnetic fields also play important roles in cloud formation and evolution through the magnetic pressure (e.g., Hennebelle & Pérault 2000; Inoue & Inutsuka 2008; Heitsch et al. 2009; Vázquez-Semadeni et al. 2011; Inoue & Inutsuka 2012; Iwasaki et al. 2019) and the formation of filamentary structures that host a protostar (André et al. 2010; Inoue & Fukui 2013). The thermal instability is able to grow along the magnetic field lines to form multiphase structure. For examples, Inoue & Inutsuka (2012) and Iwasaki et al. (2019) show that, even in magnetized cases, the converging flows are able to create highly turbulent two-phase molecular clouds like the results in this article, especially when the angle between the converging flow direction and magnetic fields is close to parallel. We therefore should investigate whether our understanding is valid in the magnetohydrodynamics regime by comparing the known relation of the density PDF width in magnetized isothermal gas (e.g., Federrath & Klessen 2012). This is beyond the scope at this moment, but we are planning to extend our simulations toward this direction.

It is also important to calculate chemical networks (either on the fly or post-process) followed by a synthetic observation to directly compare with molecular line observations. The WNM and CNM volumes are, in principle, accessible by H1 and CO line observations, whereas we expect that the intermediate temperature range of a few hundred kelvin is an interesting frontier traceable by the combination of the OH lines (e.g.,
Ebisawa et al. (2020) and by the H I line, with ALMA, SKA, ngVLA, and so on. Such calculations are left for future studies at this moment.

6.4. Implications to Observational Estimations in Interstellar Solenoidal/Compressive Mode Ratio

Direct observations of the solenoidal/compressive mode ratio parameter \( b \) in actual molecular clouds are difficult because observations are limited to the position–velocity space. Nevertheless, there have been attempts to estimate the solenoidal/compressive mode ratio by using Equation (5) (or its magnetized version) based on the measured Mach number and the width of logarithmic density PDF (e.g., Brunt 2010; Federrath et al. 2016; Sharda et al. 2022). For example, Federrath et al. (2016) find that typical molecular clouds have their \( b \) greater than 0.4, with a significant compressive mode contribution exceeding the natural mixing, whereas the G0.253+0.016 cloud at the Central Molecular Zone exhibits \( b = 0.22 \), dominated by the solenoidal modes. The authors suspect that the origin of this variation is in the difference of the driving mechanism; the strong shear motion is invoked by its recent galactic-pericenter passage by Sgr A* (Longmore et al. 2013b), which explains the low SFE at the Galactic center (Longmore et al. 2013a).

The exact value of \( b \), however, needs careful analyses. For example, Brunt (2010) discusses how the subresolution structures below 0.1 pc impact the \( b \) estimation by assuming that Larson’s law continues to the infinitely small spatial scale (Heyer et al. 2001) and concludes that such structures induce the underestimation in \( b \) by a factor of \( \sqrt{2} \) in the case of the Taurus molecular cloud, and this has to be tested by observations with a higher angular resolution. Such a subresolution effect is indeed important because the thermal instability grows on wide spatial ranges even below 0.1 pc (Koyama & Inutsuka 2000). Furthermore, even with a high angular resolution, observational estimations of the Mach number using molecular lines (e.g., CO, HNCO) have a chance to probe the relative velocity between CNM clumps along the line of sight. This relative velocity is powered more by the strong turbulence of the WNM and UNM and does not determine the density PDF at the CNM temperature/density. Therefore, to address the variation of the mode ratio parameter \( b \) across various star-forming environments, we have to take the multiphase nature of the ISM into account, by selecting the turbulent motion within the dense structures. Higher spectral resolution and higher spatial resolution are important (see also Federrath 2013b for the importance of the Mach number estimation as an origin of the variation in SFR at a given gas column density).

7. Summary

Toward the understanding of the turbulence properties realized in the multiphase ISM, we perform a series of hydrodynamics simulations of converging WNM flows on a 10 pc scale with heating and cooling by which the thermally UNM and the clumpy CNM form. We list our main findings as follows.

1. The overall velocity power spectrum follows the Kolmogorov law, where the solenoidal (compressive) modes account for >80% (<20%) of the turbulence power on all scales.
2. When we consider both the UNM and CNM components, the density PDF has its width, \( \sigma_x \), close to the one expected from the framework of the isothermal turbulence theory as \( \sigma_{x,\text{theory}} = \ln(1 + b^2 M^2) \) (where \( M \) and \( b \) are the turbulent Mach number and the turbulence parameter representing the turbulence mode ratio, respectively). Meanwhile, the width of the CNM component alone is narrower than the width that the isothermal theory predicts, if the isothermal theory employs the interclump relative velocity between CNM clumps as the main driver of turbulence responsible to control the CNM density PDF when we consider only the CNM component.
3. These results suggest that the coldest part of the density PDF (i.e., clumpy CNM structures) is governed by the thermal instability and the resultant weak turbulence of \( \lesssim 1 \) km s\(^{-1} \) within individual CNM clumps rather than by the strong turbulence on the entire cloud scale with 2–5 km s\(^{-1} \), which determines the relative velocity between CNM clumps. Observational interpretations of the CNM density PDF require careful analyses to evaluate the turbulence within each CNM clump but not the relative velocity between CNM clumps; otherwise, the Mach number of the turbulence can be overestimated and the corresponding forcing parameter \( b \) is underestimated accordingly. \( \sigma_{x,\text{theory}} \) is indeed close to \( \sigma_x \) when the isothermal theory uses the mean Mach number of the internal turbulence within CNM clumps to derive \( \sigma_{x,\text{theory}} \).
4. The second-order velocity structure function suggests that the overall volume follows the Kolmogorov spectrum, as already shown by Figure 2, and that the limited volume of the CNM component alone follows more closely Larson’s spectrum. If we observationally measure the density PDF and the second-order velocity structure function in multiple thermal states, they provide estimations of the turbulent forcing parameter $b$ more consistent with the thermal evolution of molecular clouds.

5. Our resolution study suggests that resolving the CNM clump size with $\geq 5$ cells is the typical criterion to have the convergence in the mean properties of CNM clumps.

These results have to be further investigated by including magnetic fields. We also hope that upcoming observations (e.g., ALMA, SKA, ngVLA) will measure the UNM/CNM (column) density PDF and the velocity structure function with multiple lines (HI, OH, CO) to investigate our results.

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**Appendix A**

**Numerical Diffusion and Ideal Spatial Resolution**

As shown in Section 4.1, the turbulence power spectrum exhibits a break at an intermediate spatial scale of $k/2\pi \sim 5.12$ pc$^{-1}$. Such a break widely appears in various previous simulations as well, which is likely due to numerical diffusion (e.g., Federrath 2013a; Padoan et al. 2016; see also Hennebelle & Audit 2007; high-resolution albeit two-dimensional). To investigate the possible responsibility of the numerical diffusion, we perform additional simulations with multiple resolutions and compare the velocity power spectrum, which is shown in Figure 9. Panel (a) shows the spectrum from fiducial runs with multiple spatial resolutions, and panel (b) shows the spectrum from reference runs where we calculate the adiabatic converging flows with the specific heat ratio of 1.05. The choice of 1.05 here aims to investigate the evolution of the almost isothermal ISM, free from the heating and cooling effect. These panels show that the break scale shifts at about 1/10 of the Nyquist frequency as we change the spatial resolution, suggesting that this decrease of the turbulence power at high frequencies is due to the numerical diffusion. The relative increase of the compressive modes below this break scale is due to the numerical diffusion accordingly. We therefore employ powers above this break scale to evaluate the turbulence mode ratio $\chi$ and the forcing parameter $b$. For example, when we analyze the entire volume of the shock-compressed layer in Section 4, we integrate the turbulence power from $k/2\pi = 0.1$ to 5.12 to 25.6 pc$^{-1}$. Note that, in general, resolving vortex motions likely requires more numerical cells than resolving contracting/expanding motions, so that the solenoidal mode tends to lose its power more than the compressive mode.

In our previous study (Kobayashi et al. 2020), we show that our current spatial resolution of 0.01 pc is already high enough compared with the typical cooling length of the UNM ~ 1 pc and it is capable of following the dynamical condensation motion from the UNM to the CNM due to the thermal instability. Indeed, Kobayashi et al. (2020) show that the CNM mass fraction is converged with the spatial resolution of 0.02 pc or higher. Panel (a) of Figure 10 shows the histogram of the effective cooling length $\lambda_{\text{cool,eff}}$ in our current simulation, where we calculate $\lambda_{\text{cool,eff}} = eC_p/[(\rho\mathcal{L} + P\nabla\nu)]$ in each numerical cell. The transition from the WNM to CNM appears at ~0.1 pc, which is consistent with the peak of CNM clumps’ sizes in panel (a) of Figure 5. Panel (a) of Figure 10 suggests that the spatial resolution of 0.01 pc is a scale where the CNM dominates the cooling length. This panel also suggests that the largest scale of the numerical diffusion (~0.2 pc) resolves the typical cooling length of the UNM (~1 pc) and is close to the transition from the WNM to CNM. Therefore, our current high-resolution simulations are adequate to follow the dynamical condensation motion from the UNM to the CNM.
simulations indeed follow the dynamical condensation motion from the WNM/UNM to the CNM. Our current 0.01 pc resolution, however, limits the highest frequency mode of the thermal instability growth. CNM clumps are typically resolved with 10 cells per dimension (i.e., 0.1 pc/0.01 pc); thus, it is difficult to follow the detailed evolution of the turbulent structure within individual clumps. In a self-gravitating system where the gravitational collapse enhances the turbulence, resolving the Jeans length by more than 32 cells is suggested as the criterion to have the convergence in the turbulent power of the solenoidal modes (e.g., Federrath et al. 2011; Higashi et al. 2021). Our simulations presented in this article do not include self-gravity but include heating/cooling processes, which invoke the dynamical condensation motion by the thermal instability and resultant turbulence on smallest scales. The gas temperature decreases during this condensation motion, so that the 32-cell Jeans criterion of isothermal collapsing gas is not directly applicable, but this criterion suggests that we ideally should have \( \sim 3 \times 10^{-3} \) pc or higher spatial resolution to fully resolve the turbulent motions within individual CNM clumps of \( \sim 0.1 \) pc sizes. In our current simulations, the fastest-growing mode of the thermal instability should exist at the scale \( \sim 7 \times 10^{-3} \) pc, based on the typical sound speed (0.65 km s\(^{-1}\)) and the cooling time (0.011 Myr) of CNM clumps with 40 K and 189 cm\(^{-3}\). The spatial resolution \( \gtrsim 3 \times 10^{-3} \) pc is, therefore, desired to fully follow the formation of tiny CNM clumps or tiny structure within CNM clumps on the scale of \( \sim 7 \times 10^{-3} \) pc. Such a high-resolution simulation is difficult to perform in three dimensions unless we ignore the cloud-scale evolution \( \sim 10 \) pc (i.e., WNM and UNM components) and focus on local structures within molecular clouds, which is left for future studies.

Nevertheless, the mass spectrum in panel (b) of Figure 5 shows that the CNM mass budget is dominated by massive/large clumps, as is the density PDF. This is consistent with the

\[ \text{Figure 9.} \text{ The velocity power spectrum for our fiducial calculation with heating and cooling (panel (a)), and that for our reference calculation with adiabatic converging flow of } \gamma = 1.05 \text{ (panel (b)). The black lines represent the total power, and the red lines represent the power of the compressive mode. The four lines correspond to the spatial resolution of 0.08, 0.04, 0.02, and 0.01 pc. For visualization purposes, we do not plot the solenoidal modes here, whose power is almost the same as the total power. Panel (a) is at 3 Myr, whereas panel (b) is at 1.5 Myr; the shock-compressed layer hits the x boundaries at } \approx 2 \text{ Myr in the adiabatic case, so that for panel (b) we choose the snapshot early enough to avoid the effect of the boundary.} \]

\[ \text{Figure 10.} \text{ (a) The histogram of the effective cooling length } \lambda_{\text{cool,eff}} = c_s^2 / (\rho L + P \nabla T) \text{ at 3 Myr. The black line shows the total histogram, whereas the red/green/blue lines represent the histogram of the WNM/UNM/CNM. (b, c) The CNM clump size histogram with 0.02 pc spatial resolution and with 0.04 pc spatial resolution, respectively.} \]
### Table 3

| Resolution (pc) | Number of CNM clumps |
|-----------------|-----------------------|
|                 | 0.01                  |
|                 | 0.02                  |
|                 | 0.04                  |
| (n_{\text{damp}})_V (cm^{-3}) | 189                  |
| (C_{\text{damp}})_V (km s^{-1}) | 0.65                |
| \sqrt{(\langle V_x^2 \rangle_{\text{damp}})}_V (km s^{-1}) | 1.02                |
| M_{\text{damp}} = \sqrt{(\langle V_x^2 \rangle_{\text{clump}})}_V / (C_{\text{clump}})_V | 1.61                |
| \chi_{V_x} (%) | 22.5                |
| \chi_{V_x} (%) | 77.5                |

Note. Top row: the spatial resolution and number of identified CNM clumps. Middle row: the mean number density of a CNM clump. The second line, (C_{\text{clump}})_V, corresponds to the mean sound speed of a CNM clump. The third line, \sqrt{(\langle V_x^2 \rangle_{\text{clump}})}_V, corresponds to the mean internal velocity dispersion of a CNM clump. The fourth line, M_{\text{damp}}, is the mean effective Mach number within CNM clumps. The fifth and sixth lines, \chi_{V_x} (%) and \chi_{V_x} (%), show the grid-scale fraction of the compressive (solenoidal) mode out of the total turbulence power, based on the measurements of (\langle V_x^2 \rangle_{\text{damp}})_V and (\langle V_x^2 \rangle_{\text{clump}})_V.

### Appendix B

#### Assumptions in the Fourier Transformation and Technical Procedures to Derive the Power Spectrum

In the evaluation of the mode ratio \chi and the forcing parameter \beta, we choose 1/10 of the Nyquist frequency as the highest frequency (i.e., k/2\pi = 5.12 pc^{-1}), below which is the turbulent inertial range. Meanwhile, the choice of the lowest frequency and the estimation of corresponding power on that scale are not trivial. In this appendix, we summarize the assumptions and the technical procedures involved in these analyses.

##### B.1. The Frequency Range: Entire Shock-compressed Layer

As already shown in Figure 1, the shock-compressed layer is significantly deformed, which evolves in time, and thus performing the Fourier transformation on such an arbitrary volume is not trivial. Ideally we should apply some methods to remove the effect of such geometry. There are already several methods proposed in the astrophysical context, such as the cosmic microwave background analysis, whose mask region onto the galactic foreground has complicated shapes (e.g., Planck Collaboration et al. 2014), or large galactic surveys, where the observed area does not have any periodicity (e.g., Pseudo-spectrum method; Brown et al. 2005; Hikage et al. 2011, 2019). However, this high accuracy is beyond our scope at this moment.

We instead opt to fill the velocity fields outside the shock-compressed layer uniformly by the mean bulk velocity of the shock-compressed layer, so that the powers of low frequencies are reduced, which are otherwise dominated by the converging flow itself. The converging flow configuration naturally results in a mean bulk velocity of the shock-compressed layer of 0 (in other words, the shock-compressed layer is by definition in the rest frame of the contact discontinuity). We therefore use \nu = 0 as this temporally filling velocity outside the shock-compressed layer and perform the Fourier transform over the entire simulation domain. Before the actual applications, we perform simple demonstrations. We first generate turbulence of solenoidal mode only (compressive mode only) in the 10 pc \times 10 pc \times 10 pc domain by superposing 32 sinusoidal modes with the Kolmogorov power spectrum. We locate this velocity field at the center of the 20 pc \times 10 pc \times 10 pc domain, with the velocity field outside the central (10 pc)^3 being 0. We perform the Fourier transform over the entire volume of the 20 pc \times 10 pc \times 10 pc domain and find that the power on the k/2\pi = 1/20 pc^{-1} scale is successfully suppressed more than one order of magnitude compared to the power on the k/2\pi = 1/10 pc^{-1} scale, and we obtain the Kolmogorov power spectrum, where the artificial mode conversion due to the \nu = 0 procedure is limited to \leq 5%.

In actual calculations, the shock-compressed layer is significantly deformed and is not as simple as our test calculations. Nevertheless, the physical width of the shock-compressed layer is 10–12 pc after 1 Myr, and the mean bulk velocity of the shock-compressed layer is limited to <1 km s^{-1}, at least one order of magnitude smaller than the turbulent velocity. We therefore use this method in our analysis and employ k/2\pi = 0.1 pc^{-1} as the proxy of the lowest frequency representing the shock-compressed layer throughout this article (e.g., when we measure \chi and \beta).

##### B.2. The Frequency Range: Subvolumes

In the case of the Fourier analysis of the subvolumes, we always choose the lowest frequency as the frequency corresponds to 2.56 pc because we analyze the cubic of (2.56 pc)^3. We here did not apply any special care of \nu = 0 as we did above in Section B.1. This fixed volume and fixed frequency range affect the large scatter in the mode ratio among these subvolumes at early stages (panel (b) of Figure 6). Each subvolume has a timing at which they become completely embedded within the shock-compressed layer. The layer initially expands almost adiabatically with its width to 2 pc at 0.5 Myr, and then the expansion gradually slows down owing to the cooling with the width to 8 pc at 1 Myr, which corresponds to the separation between the most separated subvolumes. Therefore, it is fair to focus on the evolution after 1 Myr when we discuss the variation among the subvolumes, and we report most of our results from 3 Myr in Section 5.
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