Survey on the kinematics of a 6RSS parallel robot based on manipulability indices

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Abstract. The study of the kinematic performances of a parallel mechanisms an important aspect in terms of analysis and design. This paper presents a study on the kinematic manipulability and kinematic efficiency of a 6RSS parallel mechanism. The parallel mechanism consists of a fixed plate and a mobile platform connected by six independent kinematic chains. Each of the six kinematic chains has an actuated rotational joint and two spherical joints. The motion equations for the six kinematic chains of the parallel mechanism are defined when the characteristic point $P$ moves on the spatial curve with constant speed. Based on the global Jacobian matrix determinant and the angular speeds of actuated arms was graphically represented the variation of the two kinematic indices. Kinematic indices allow for assessing the quality of the mechanism motion; they also help us avoid those critical configurations encountered in the case of parallel mechanisms.

1. Introduction

Parallel mechanisms provide a number of advantages over similar series mechanisms. These advantages usually include higher speed and acceleration, improved positional accuracy and increased rigidity \cite{1-3}. With the closed-loop structure, parallel mechanisms gradually gain wide application in different industrial branch \cite{4-6}. Thus, in the past few years, parallel mechanisms such as flight simulators, telescopes, radar antennas, pointing devices have been introduced in several technical applications. Following the analysis of the results obtained in the application of parallel structures in various fields of industry, parallel kinematic machines have been found to fully satisfy the new modern requirements, adding a significant increase in speed, machining accuracy and complex surface generation possibilities. Due to the need for better control of the kinematics of the parallel mechanisms, manipulability indices have been introduced. Various researchers have rated manipulability as a measure of the ability to move the actuating arms that determine the positioning and orientation of the end effector. Thus, the final motion imposed on it, as a sum of the movements of the robot actuators, is controlled based on the kinematic indices throughout the motion. Based on these kinematics indices, we can therefore speak of the "quality" of the movement.

Also, based on the kinematic indices, a number of researches have designed avoiding singular configurations \cite{7}. Thus manipulability is used as a real-time optimization function during the operation of the mechanism. The function can detect those critical positions and can command the robot to modify a particular trajectory before reaching a limit position.

According to some researchers, manipulability quantifies the capacities of transmitting the speed of manipulation or, in other words, the robot's dexterity \cite{8}. In order to better determine the
manipulability of a mechanism, it was proposed to separate its movements into: translational movements and rotation movements [9].

2. Manipulability concept for 6RSS parallel mechanisms

The concept of manipulability associated with mechanisms has been introduced by Yoshikawa since 1985. He considers that, when the manipulability index \( \mu \) reaches the maximum value, the mechanism is at the furthest position from a singular configuration. Manipulability for redundant mechanisms is defined by the expression [10]:

\[
\mu = \sqrt{\det(J\cdot[J]^T)}
\]

in which \([J]\) is the global Jacobian matrix of the mechanism, and \([J]^T\) is transposed. The manipulability denoted by \( \mu \) measures how much the mobile platform is moving when is giving an infinitesimal motion to the actuating joints. It is the absolute value of the global Jacobian matrix determinant of the parallel mechanism in a current position. For non-redundant mechanisms, the manipulability index \( \mu \) is given by the relation [11]:

\[
\mu = \left| \det(J) \right|
\]

For the kinematic point of view of quantitative evaluation of the parallel mechanisms, the concept of manipulability ellipsoid was introduced. This ellipsoid is determined by the field of speeds \( \mathbf{v} \) of the mobile platform, which satisfies the condition:

\[
\mathbf{q} \leq 1
\]

This ellipsoid belongs to the euclidean space of the mechanism with \( m \) dimension. Thus, in the direction of the major axis of the ellipsoid, the end effector will move at high speed, and in the direction of the ellipsoid's minimum axis the end effector will move at low speed. In the particular case where this ellipsoid is a sphere, the mobile platform will move uniformly in all directions.

The parallel mechanism with structural formula 6RSS (figure 1) consists of a fixed plate and a mobile platform connected to each other by six rods \( AA_k \) that transmit the motion from the actuated arms \( OA_k \) to the mobile platform.

![Figure 1](image.png)

**Figure 1.** Parallel mechanism 6RSS with highlighting kinematics joints.

The link between the fixed plate and branches is done by six rotational joints \( C_3 \) while the connection between branches \( r_k \) and rods \( l_k \) is achieved by means of spherical joints \( C_3 \). The rods \( l_k \), cause the mobile platform to move through other six spherical joints. Also in the figure above it is
schematically represented the general case of the kinematic chains $k$ of the parallel mechanism, with 
kinematic rotational joints at point $O_k$ and spherical kinematic joints at points $A_k$ and $B_k$ respectively. 
The actuating arms $r_k$ are rotated by an angle $\theta^k_1$ around the axis by unit vector $u_k$, passing through 
the point $O_k$. The matrix expression of the kinematic model of the mechanism is given by the relation:

$$
\begin{bmatrix}
  w_1 & 0 & 0 & 0 & 0 \\
  0 & w_2 & 0 & 0 & 0 \\
  0 & 0 & w_3 & 0 & 0 \\
  0 & 0 & 0 & w_4 & 0 \\
  0 & 0 & 0 & 0 & w_6 \\
\end{bmatrix}
\begin{bmatrix}
  \omega_1 \\
  \omega_2 \\
  \omega_3 \\
  \omega_4 \\
  \omega_5 \\
\end{bmatrix}
=
\begin{bmatrix}
  a_1 & b_1 & c_1 & s_1 & r_1 & t_1 \\
  a_2 & b_2 & c_2 & s_2 & r_2 & t_2 \\
  a_3 & b_3 & c_3 & s_3 & r_3 & t_3 \\
  a_4 & b_4 & c_4 & s_4 & r_4 & t_4 \\
  a_5 & b_5 & c_5 & s_5 & r_5 & t_5 \\
\end{bmatrix}
\begin{bmatrix}
  v_{r_x} \\
  v_{r_y} \\
  v_{r_z} \\
  v_{\omega_x} \\
  v_{\omega_y} \\
  v_{\omega_z} \\
\end{bmatrix}
\quad (4)
$$

From the relationship (4) we have the expressions for the direct kinematic Jacobian $[B]$ and the 
inverse kinematic Jacobian $[A]$ of the mechanism:

$$
[B] = \begin{bmatrix}
  w_1 & 0 & L & 0 \\
  0 & O & 0 & M \\
  M & 0 & 0 & 0 \\
  0 & L & 0 & w_{\beta_k}^{(6,6)} \\
\end{bmatrix}
\quad \text{and} \quad
[A] = \begin{bmatrix}
  a_1 & b_1 & L & t_1 \\
  a_2 & b_2 & L & t_2 \\
  M & M & M & M \\
  a_6 & b_6 & L & t_{\alpha_k}^{(6,6)} \\
\end{bmatrix}
$$
in which:

$$
\begin{align*}
(x_{B_k} - x_{A_k}) &= a_1; \\
(y_{B_k} - y_{O_k}) &= b_1; \\
(z_{B_k} - z_{A_k}) &= c_1; \\
\left[(y_{B_k} - y_{O_k}) \left(z_{B_k} - z_{A_k}\right) - \left(z_{B_k} - z_{O_k}\right) \left(y_{B_k} - y_{O_k}\right)\right] &= s_k; \\
\left[(x_{B_k} - x_{O_k}) \left(z_{B_k} - z_{A_k}\right) + \left(z_{B_k} - z_{O_k}\right) \left(x_{B_k} - x_{A_k}\right)\right] &= r_k; \\
\left[(y_{B_k} - y_{O_k}) \left(x_{B_k} - x_{A_k}\right) - \left(x_{B_k} - x_{O_k}\right) \left(y_{B_k} - y_{O_k}\right)\right] &= t_k; \\
\left[\left[z_{B_k} - z_{O_k}\right] - u_k \left(y_{B_k} - y_{O_k}\right)\right] \left(x_{B_k} - x_{A_k}\right) + \\
\left[\left[z_{B_k} - z_{O_k}\right] \left(x_{B_k} - x_{O_k}\right) - u_k \left(z_{B_k} - z_{O_k}\right)\right] \left(y_{B_k} - y_{O_k}\right) + \\
\left[\left[y_{B_k} - y_{O_k}\right] - u_k \left(z_{B_k} - z_{O_k}\right)\right] \left(z_{B_k} - z_{A_k}\right) &= w_k;
\end{align*}
$$

The relationship (4) written in compact form becomes:

$$
[B] \cdot \dot{\mathbf{q}} = [A] \cdot \mathbf{\tau}
$$

in which:

$$
\dot{\mathbf{q}} = \begin{bmatrix}
  \omega_1 \\
  \omega_2 \\
  M \\
  \omega_6^{(6,1)} \\
\end{bmatrix}
\quad \text{and} \quad
\mathbf{\tau} = \begin{bmatrix}
  v_{r_x} \\
  v_{r_y} \\
  v_{r_z} \\
  v_{\omega_x} \\
\end{bmatrix}
$$
In the case of the 6RSS mechanism by imposing for \( \dot{q} \) a uniform vector norm in the generalized velocity space we obtain the relation:

\[
|\dot{q}| = \dot{q} \cdot \dot{q} = g_1^2 + g_2^2 + g_3^2 + g_4^2 + g_5^2 + g_6^2 = 1 \tag{6}
\]

Based on the direct kinematic model given by the relationship:

\[
[J][q] = \tau \tag{7}
\]

relative to the condition (6), it results:

\[
\dot{q}^T = \tau^T \cdot ([J]^{-1})^T \tag{8}
\]

and:

\[
\dot{q} = [J]^{-1} \cdot \tau \tag{9}
\]

From relations (8) and (9) based on relation (6) we have the expression:

\[
[\tau]^T \cdot ([J]^{-1})^T \cdot [J]^{-1} \cdot [\tau] = 1 \tag{10}
\]

If we denote:

\[
[K] = ([J]^{-1})^T \cdot [J]^{-1} \tag{11}
\]

Thereby equation (10) can be written as a matrix:

\[
\begin{bmatrix}
\dot{v}_x & \dot{v}_y & L & \omega_\gamma
\end{bmatrix}_{(6 \times 6)} \cdot \begin{bmatrix}
k_{11} & k_{12} & L & k_{16} \\
k_{21} & k_{22} & L & k_{26} \\
M & M & M & M \\
k_{61} & k_{62} & L & k_{66}
\end{bmatrix}_{(6 \times 6)} \cdot \begin{bmatrix}
v_{p_x} \\
v_{p_y} \\
M \\
\omega_\gamma
\end{bmatrix}_{(6 \times 1)} = 1 \tag{12}
\]

in which \( k_{ij} \) are the elements of the matrix \([K]\).

We denote by \( p_{ij} \) the elements of the matrix product \([\tau]^T\) and \([K]\) which are given by the expressions:

\[
\begin{align*}
&v_{p_x} k_{11} + v_{p_y} k_{21} + v_{p_z} k_{31} + \omega_\alpha k_{41} + \omega_\beta k_{51} + \omega_\gamma k_{61} = p_{11} \\
v_{p_x} k_{12} + v_{p_y} k_{22} + v_{p_z} k_{32} + \omega_\alpha k_{42} + \omega_\beta k_{52} + \omega_\gamma k_{62} = p_{12} \\
&v_{p_x} k_{16} + v_{p_y} k_{26} + v_{p_z} k_{36} + \omega_\alpha k_{46} + \omega_\beta k_{56} + \omega_\gamma k_{66} = p_{16}
\end{align*} \tag{13}
\]

From (12) and (13) result an expression of the form:

\[
\sum_{j=1}^{15} e_j \cdot \dot{u}_j^2 + \sum_{j=1}^{15} (\omega_n \cdot \omega_n) f_j \cdot \dot{u}_n \cdot \dot{u}_n + \sum_{j=1}^{15} g_j \cdot \dot{u}_j + d = 0 \tag{14}
\]

in which \( v_{p_x}, v_{p_y}, \ldots, \omega_\gamma \), \( e_j, f_j, g_j \) represent the coefficients that belong to the matrices \([K]\) of the geometric parameters of the parallel mechanism and \( d \) is the free term. Expression (14) represents the second degree algebraic equation of an ellipsoid in the six-dimensional space of the mechanism. For our mechanism it is useful to split the global Jacobian \([J]\), (belongs to the six-

dimensional Euclidean space $\mathbb{R}^6$), in two Jacobian matrices $[J_\omega]$ and $[J_v]$ belonging to the tri-dimensional Euclidean space $\mathbb{R}^3$.

$$[J] = \begin{bmatrix} [J_\omega] \\ [J_v] \end{bmatrix} \in \mathbb{R}^6$$  \hspace{1cm} (15)

For any configuration of the mechanism, $[J_\omega]$ can be used to create angular velocity manipulability ellipsoids and $[J_v]$ can be used to create linear velocity ellipsoids. The main axis of these ellipsoids indicates the direction along which the mechanism has greater freedom of motion, while the minor axis indicates the direction along which the motion of the mechanism is restricted.

To characterize the motion generation capability for non-redundancy robots, the "kinematic efficiency" parameter $e_k$ is defined by the relationship:

$$e_k = \frac{v_P}{\sqrt{\omega_1^2 + \omega_2^2 + \ldots + \omega_6^2}}$$  \hspace{1cm} (16)

where $\omega_j$ are the angular speeds of the actuators and $v_P$ is the speed of the characteristic point $P$.

The kinematic efficiency parameter $e_k$, express the structural quality of the parallel mechanism to generate motion, independent of the magnitude of the characteristic point speed for a given configuration. This definition gives this parameter a general character independent of the kinematic regime.

3. Defining the concept of kinematic efficiency for a given application

To simulating the motion of the mechanism we have used an independent CAD product dedicated (DMU Kinematics Simulator) from the CATIA program. We have defined the motion equations for the six kinematic chains of the parallel mechanism when the characteristic point $P$ is moving on the spatial curve on which 72 points are equidistantly distributed. As a particularity of this application we have the Euler angles $\beta \neq \gamma \neq 0 = ct(constant)$ and variable value for $\alpha$. In this application, the central axis of the mobile platform is throughout the motion normal to the trajectory and the point $P$ is in constant contact with it. Another particularity of the application is that the point $P$ is moving at constant speed along the spatial curve. Based on the values $\omega_{j,P}$ for the $P_j (j=1...72)$ positions of the characteristic point, their variation graphs were plotted (figure 2).

![Figure 2. Variation of angular speeds $\omega_{1..6}$.](image)

Also based on the manipulability index $\mu$ and the kinematic efficiency index $e_k$, their variation diagrams were plotted. An important aspect is that with the manipulability index can be follow the final position of the characteristic point, i. e. the variation of the manipulability measure (figure 3). In
other words, when the characteristic point is far from zero value, the mechanism is far from a critical position. It can be observed (figure 4) that the value of the kinematic efficiency index \( e_k \) is inversely proportional to the amplitude of the actuating arms motion.

4. Conclusions
The main contribution in this paper is determination of the kinematic indices and motion ellipsoid in the six-dimensional space of the mechanism. Through the concept of manipulability and kinematic efficiency the final user can better understand both the kinematic capabilities and the limits of the mechanism. Based on the kinematic manipulability analysis, it is possible to determine in which areas of the robot workspace it is more convenient to perform a certain motion and how closer is the characteristic point to a critical position. Also, the motion ellipsoid providing a better understanding the kinematics of the mechanism, allowing to avoid singularity positions of its.

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