Calculable CP-violating Phases in the Minimal Seesaw Model of Leptogenesis and Neutrino Mixing

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Abstract

We show that all nontrivial CP-violating phases can be determined in terms of three lepton flavor mixing angles and the ratio of $\Delta m^2_{\text{sun}}$ to $\Delta m^2_{\text{atm}}$ in the minimal seesaw model in which the Frampton-Glashow-Yanagida (FGY) ansatz is incorporated. This important point allows us to make very specific predictions for the cosmological baryon number asymmetry and CP violation in neutrino oscillations. A measurement of the smallest neutrino mixing angle will sensitively test the FGY ansatz, in particular in the case that three light neutrinos have a normal mass hierarchy.

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I. INTRODUCTION

In the minimal standard model of electroweak interactions, the lepton number conservation is assumed and neutrinos are exactly massless Weyl particles. However, today’s Super-Kamiokande [1], SNO [2], KamLAND [3] and K2K [4] neutrino oscillation experiments have provided us with very strong evidence that neutrinos are actually massive and lepton flavor mixing does exist. The most economical modification of the minimal standard model, which can both accommodate neutrino masses and allow lepton number violation to explain the cosmological baryon asymmetry via leptogenesis [5], is to introduce two heavy right-handed neutrinos $N_1, N_2$ and keep the Lagrangian of electroweak interactions invariant under the $SU(2)_L \times U(1)_Y$ gauge transformation [6,7]. After the spontaneous electroweak symmetry breaking, this simple but interesting model yields the following neutrino mass term:

$$-\mathcal{L}_{\text{mass}} = (\nu_e, \nu_\mu, \nu_\tau) M_D \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} + \frac{1}{2} (N^c_1, N^c_2) M_R \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} + \text{h.c.},$$

(1)

where $N^c_i \equiv C \bar{N}^T_i$ with $C$ being the charge-conjugation operator; and $(\nu_e, \nu_\mu, \nu_\tau)$ denote the left-handed neutrinos. The Dirac neutrino mass matrix $M_D$ is a $3 \times 2$ rectangular matrix, and the Majorana neutrino mass matrix $M_R$ is a $2 \times 2$ symmetric matrix. The scale of $M_D$ is characterized by the electroweak scale $v = 174$ GeV. In contrast, the scale of $M_R$ can be much higher than $v$, because $N_1$ and $N_2$ are $SU(2)_L$ singlets and their corresponding mass term is not subject to the scale of gauge symmetry breaking. Then one may obtain the effective (light and left-handed) neutrino mass matrix $M_\nu$ via the well-known seesaw mechanism [8]:

$$M_\nu \approx M_D M_R^{-1} M_D^T.$$  

(2)

Without loss of generality, both $M_R$ and the charged lepton mass matrix $M_l$ can be taken to be diagonal, real and positive; i.e., $M_R = \text{Diag} \{M_1, M_2\}$ and $M_l = \text{Diag} \{m_e, m_\mu, m_\tau\}$, where $M_{1,2}$ denote the masses of two heavy Majorana neutrinos. In such a specific flavor basis, the low-energy neutrino phenomenology is governed by $M_\nu$, while the cosmological baryon number asymmetry is associated with $M_D$ via the leptogenesis mechanism.

Unfortunately, the model itself has no restriction on the structure of $M_D$. In Ref. [6], Frampton, Glashow and Yanagida (FGY) have conjectured that $M_D$ may take the form

$$M_D = \begin{pmatrix} a & 0 \\ a' & b \\ 0 & b' \end{pmatrix},$$

(3)

or

$$M_D = \begin{pmatrix} a & 0 \\ 0 & b \\ a' & b' \end{pmatrix}.$$  

(4)

The texture zeros in $M_D$ could stem from an underlying horizontal flavor symmetry. With the help of Eq. (2), one may straightforwardly arrive at
\[
M_\nu = \begin{pmatrix}
\frac{a^2}{M_1} & \frac{aa'}{M_1} & 0 \\
\frac{aa'}{M_1} & (a')^2 + \frac{b^2}{M_2} & \frac{bb'}{M_2} \\
0 & \frac{bb'}{M_2} & (b')^2 + \frac{(b')^2}{M_2}
\end{pmatrix}
\] (5)

from Eq. (3); or

\[
M_\nu = \begin{pmatrix}
\frac{a^2}{M_1} & 0 & \frac{aa'}{M_1} \\
0 & \frac{b^2}{M_2} & \frac{bb'}{M_2} \\
\frac{aa'}{M_1} & \frac{bb'}{M_2} & (a')^2 + \frac{(b')^2}{M_2}
\end{pmatrix}
\] (6)

from Eq. (4). Note that \(\text{Det}(M_\nu) = 0\) holds in either case \(^1\). Note also that \(|\text{Det}(M_\nu)| = m_1 m_2 m_3\) holds in the chosen flavor basis, where \(m_i\) (for \(i = 1, 2, 3\)) denote the masses of three light neutrinos. Thus one of three neutrino masses must vanish. As the solar neutrino oscillation data have set \(m_2 > m_1\) \(^2\), we are left with either \(m_1 = 0\) (normal hierarchy) or \(m_3 = 0\) (inverted hierarchy). In Ref. [9], the Majorana neutrino mass matrix with one texture zero and one vanishing eigenvalue has been classified and discussed in some detail.

The main purpose of this paper is to reveal a very striking feature of the minimal seesaw model in which the FGY ansatz is incorporated: all nontrivial CP-violating phases can be calculated in terms of the lepton flavor mixing angles \((\theta_x, \theta_y, \theta_z)\) and the ratio of \(\Delta m^2_{\text{sun}}\) to \(\Delta m^2_{\text{atm}}\), where \(\Delta m^2_{\text{sun}}\) and \(\Delta m^2_{\text{atm}}\) stand respectively for the typical mass-squared differences of solar and atmospheric neutrino oscillations. This important point, which was not observed in the previous analyses of the minimal seesaw model \([6,7,10]\), implies that a stringent test of the FGY ansatz can simply be realized once the smallest mixing angle \(\theta_z\) is measured or constrained to a better degree of accuracy. Considering both normal and inverted mass hierarchies of three light neutrinos, we obtain very specific predictions for the cosmological baryon number asymmetry, the effective mass of neutrinoless double beta decay and CP violation in neutrino oscillations.

II. DETERMINATION OF CP-VIOLATING PHASES

Without loss of generality, one may always redefine the phases of charged lepton fields to make \(a, b\) and \(b'\) of \(M_D\) real and positive \([6]\). In other words, only \(a'\) is complex and its phase \(\phi \equiv \text{arg}(a')\) is the sole source of CP violation in the model under discussion. Because

\(^1\)It has been shown in Ref. [9] that \(\text{Det}(M_\nu) = 0\) is independent of the specific texture zeros taken in Eq. (3) or (4). In other words, the minimal seesaw model itself guarantees that \(\text{Det}(M_\nu) = 0\) holds automatically.
both $M_l$ and $M_R$ have been taken to be diagonal, real and positive, $M_\nu$ can in general be parametrized as follows:

$$M_\nu = (PUQ) \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} (PUQ)^T,$$

where $P = \text{Diag}\{e^{i\alpha}, e^{i\beta}, e^{i\gamma}\}$ and $Q = \text{Diag}\{e^{i\rho}, e^{i\sigma}, e^{i\omega}\}$ are two phase matrices, and $U$ is given by

$$U = \begin{pmatrix} c_x c_z & s_x c_z & s_z \\ -c_x s_y s_z - s_x c_y e^{-i\delta} & -s_x s_y s_z + c_x c_y e^{-i\delta} & s_y c_z \\ -c_x c_y s_z + s_x s_y e^{-i\delta} & -s_x c_y s_z - c_x s_y e^{-i\delta} & c_y c_z \end{pmatrix}$$

with $s_x \equiv \sin \theta_x$, $c_x \equiv \cos \theta_x$ and so on. The phase parameter of $U$ (Dirac phase) governs the strength of CP violation in neutrino oscillations, while two independent phase parameters of $Q$ (Majorana phases) are relevant to the neutrinoless double beta decay [11]. The phases of $P$ cannot be neglected in the parametrization of $M_\nu$ — their essential role is to fulfil a complete match between the phases of $M_\nu$ in Eq. (5) or (6) and those defined in Eqs. (7) and (8). It is then obvious that the nontrivial phases of $P$, $U$ and $Q$ should have definite relations with $\phi$.

Note that three mixing angles of $U$ can directly be given in terms of the mixing angles of solar, atmospheric and reactor [12] neutrino oscillations. Namely, $\theta_x \approx \theta_{\text{sun}}$, $\theta_y \approx \theta_{\text{atm}}$ and $\theta_z \approx \theta_{\text{ch}}$ hold as a good approximation. In view of current experimental data, we have $\theta_x \approx 32^{\circ}$ and $\theta_y \approx 45^{\circ}$ (best-fit values [13]) as well as $\theta_z < 12^{\circ}$. The mass-squared differences of solar and atmospheric neutrino oscillations are defined respectively as $\Delta m^2_{\text{sun}} \equiv m^2_2 - m^2_1$ and $\Delta m^2_{\text{atm}} \equiv |m^2_3 - m^2_2|$. Their best-fit values read $\Delta m^2_{\text{sun}} \approx 7.13 \times 10^{-5}$ eV$^2$ and $\Delta m^2_{\text{atm}} \approx 2.6 \times 10^{-3}$ eV$^2$ [13]. These typical numbers will be used in our numerical calculations.

To be more specific, we shall concentrate on the FGY ansatz for $M_D$ in Eq. (3) or equivalently $M_\nu$ in Eq. (5). Some brief comments will be given on the consequences of $M_D$ in Eq. (4) or equivalently $M_\nu$ in Eq. (6). Indeed, both possibilities lead to very similar phenomenological results.

### A. Normal neutrino mass hierarchy ($m_1 = 0$)

If $m_1 = 0$ holds, we obtain $m_2 = \sqrt{\Delta m^2_{\text{sun}}} \approx 8.4 \times 10^{-3}$ eV and $m_3 = \sqrt{\Delta m^2_{\text{sun}} + \Delta m^2_{\text{atm}}} \approx 5.2 \times 10^{-2}$ eV. In this case, only a single Majorana phase of CP violation is physically nontrivial. Hence the phase matrix $Q$ can be simplified to $Q = \text{Diag}\{1, e^{i\sigma}, 1\}$ and six independent matrix elements of $M_\nu$ can be written as

$$(M_\nu)_{11} = e^{2i\alpha} \left[ m_2 s_x^2 c_z^2 e^{2i\sigma} + m_3 s_z^2 \right],$$

$$(M_\nu)_{22} = e^{2i\beta} \left[ m_2 \left( -s_x s_y s_z - c_x c_y e^{-i\delta} \right)^2 e^{2i\sigma} + m_3 s_y^2 c_z^2 \right],$$

$$(M_\nu)_{33} = e^{2i\gamma} \left[ m_2 \left( s_x c_y s_z + c_x s_y e^{-i\delta} \right)^2 e^{2i\sigma} + m_3 c_y^2 c_z^2 \right];$$

(9)
and

\[
(M_\nu)_{12} = e^{i(\alpha + \beta)} \left[ m_2 s_x c_z \left( -s_x s_y s_z + c_x c_y e^{-i\theta} \right) e^{2i\sigma} + m_3 s_y s_z c_z \right], \\
(M_\nu)_{13} = e^{i(\alpha + \gamma)} \left[ -m_2 s_x c_z \left( s_x c_y s_z + c_x s_y e^{-i\theta} \right) e^{2i\sigma} + m_3 c_y s_z c_z \right], \\
(M_\nu)_{23} = e^{i(\beta + \gamma)} \left[ -m_2 \left( s_x c_y s_z + c_x s_y e^{-i\theta} \right) \left( -s_x s_y s_z + c_x c_y e^{-i\theta} \right) e^{2i\sigma} + m_3 s_y c_y c_z^2 \right].
\]

(10)

Because of \((M_\nu)_{13} = 0\) as shown in Eq. (5), we straightforwardly obtain

\[
\delta = \pm \arccos \left[ \frac{c_x s_y s_z - \xi s_x^2 s_z^2}{2 \xi s_x^2 c_x s_y c_y s_z} \right], \\
\sigma = \frac{1}{2} \arctan \left[ \frac{c_x s_y s_z + c_x s_y \cos \delta}{s_x c_y s_z} \right],
\]

(11)

where \(\xi \equiv m_2/m_3 \approx 0.16\). This result implies that both \(\delta\) and \(\sigma\) can definitely be determined, if and only if the smallest mixing angle \(\theta_2\) is measured.

To establish the relationship between \(\phi\) and \(\delta\), we need to figure out \(\alpha, \beta\) and \(\gamma\). Because \(a, b\) and \(b'\) are real and positive, \((M_\nu)_{11}, (M_\nu)_{23}\) and \((M_\nu)_{33}\) must be real and positive. Then \(\alpha, \beta\) and \(\gamma\) can be derived from Eqs. (10) and (11):

\[
\alpha = -\frac{1}{2} \arctan \left[ \frac{\xi s_x^2 c_z^2 \sin 2\sigma}{s_x^2 + \xi s_x^2 c_z^2 \cos 2\sigma} \right], \\
\beta = -\gamma - \arctan \left[ \frac{c_x c_y s_z \sin \delta}{s_x s_y - c_x c_y s_z \cos \delta} \right], \\
\gamma = +\frac{1}{2} \arctan \left[ \frac{s_x^2 \sin 2\sigma}{\xi s_x^2 c_z^2 + s_x^2 \cos 2\sigma} \right].
\]

(12)

Then the overall phase of \((M_\nu)_{12}\), which is equal to the phase of \(a'\), is given by

\[
\phi = \alpha + \beta - \arctan \left[ \frac{s_x c_y s_z \sin \delta}{c_x s_y + s_x c_y s_z \cos \delta} \right].
\]

(13)

Now that all six phase parameters \((\delta, \sigma, \phi, \alpha, \beta, \gamma)\) have been determined in terms of \(\xi, \theta_x, \theta_y\) and \(\theta_z\), a measurement of the unknown angle \(\theta_z\) becomes crucial to test the model.

### B. Inverted neutrino mass hierarchy \((m_3 = 0)\)

If \(m_3 = 0\) holds, we arrive at \(m_1 = \sqrt{\Delta m^2_{\text{atm}} - \Delta m^2_{\text{sol}}} \approx 5.0 \times 10^{-2} \text{ eV}\) and \(m_2 = \sqrt{\Delta m^2_{\text{atm}}} \approx 5.1 \times 10^{-2} \text{ eV}\). Since only a single Majorana phase of CP violation is physically nontrivial, we can always simplify the phase matrix \(Q\) to \(Q = \text{Diag}\{1, e^{i\sigma}, 1\}\). In this case, six independent matrix elements of \(M_\nu\) turn out to be

\[
(M_\nu)_{11} = e^{2i\alpha} \left[ m_1 c_x^2 c_z^2 + m_2 s_x^2 c_z e^{2i\sigma} \right], \\
(M_\nu)_{22} = e^{2i\beta} \left[ m_1 \left( c_x s_y s_z + s_x c_y e^{-i\theta} \right)^2 + m_2 \left( -s_x s_y s_z + c_x c_y e^{-i\theta} \right)^2 e^{2i\sigma} \right], \\
(M_\nu)_{33} = e^{2i\gamma} \left[ m_1 \left( -c_x c_y s_z + s_x s_y e^{-i\theta} \right)^2 + m_2 \left( s_x c_y s_z + c_x s_y e^{-i\theta} \right)^2 e^{2i\sigma} \right].
\]

(14)
and

\[(M_\nu)_{12} = e^{i(\alpha + \beta)} \left[ -m_1 c_x c_z (c_x s_y s_z + s_x c_y e^{-i\delta}) + m_2 s_x c_z (s_x s_y s_z + c_x c_y e^{-i\delta}) e^{2i\sigma} \right], \]
\[(M_\nu)_{13} = e^{i(\alpha + \gamma)} \left[ m_1 c_x c_z (-c_x c_y s_z + s_x s_y e^{-i\delta}) - m_2 s_x c_z (s_x c_y s_z + c_x s_y e^{-i\delta}) e^{2i\sigma} \right], \]
\[(M_\nu)_{23} = e^{i(\beta + \gamma)} \left[ -m_1 (c_x s_y s_z + s_x c_y e^{-i\delta}) (-c_x c_y s_z + s_x s_y e^{-i\delta}) - m_2 (-s_x s_y s_z + c_x c_y e^{-i\delta}) (s_x c_y s_z + c_x s_y e^{-i\delta}) e^{2i\sigma} \right]. \] (15)

Because of $(M_\nu)_{13} = 0$ as shown in Eq. (5), we get

\[
\delta = \pm \arccos \left[ \frac{(\zeta^2 c_x^4 - s_x^4) c_y^2 s_z^2 + (\zeta^2 - 1) s_x^2 c_y^2 s_z^2}{2 s_x c_x (s_y^2 + \zeta^2 c_x^2) s_y c_y s_z} \right],
\]
\[
\sigma = -\frac{1}{2} \arctan \left[ \frac{s_y c_y s_z \sin \delta}{s_x c_x (s_y^2 - c_y^2 s_z^2) + (s_x^2 - c_x^2) s_y c_y s_z \cos \delta} \right], \] (16)

where $\zeta \equiv m_1/m_2 \approx 0.98$. Once the smallest mixing angle $\theta_z$ is observed, one may determine both $\delta$ and $\sigma$ with the help of Eq. (16).

The phase parameters $\alpha$, $\beta$ and $\gamma$ can be fixed by taking account of the positiveness of $(M_\nu)_{11}$, $(M_\nu)_{23}$ and $(M_\nu)_{33}$. With the help of Eqs. (14) and (15), we obtain

\[
\alpha = -\frac{1}{2} \arctan \left[ \frac{s_x^2 \sin 2\sigma}{\zeta c_x^2 + s_x^2 \cos 2\sigma} \right], \\
\beta = \gamma + \pi, \\
\gamma = \frac{\delta}{2} + \frac{1}{2} \arctan \left[ \frac{s_x s_y \sin \delta}{s_x s_y \cos \delta - c_x c_y s_z} \right] - \pi. \] (17)

Then the overall phase of $(M_\nu)_{12}$, which is equal to the phase of $\alpha'$, is given by

\[
\phi = \alpha + \beta - \arctan \left[ \frac{s_x c_y s_z \sin \delta}{c_x s_y + s_x c_y s_z \cos \delta} \right] - \pi. \] (18)

Again, a measurement of the unknown mixing angle $\theta_z$ will allow us to determine all six phase parameters ($\delta$, $\sigma$, $\phi$, $\alpha$, $\beta$ and $\gamma$).

**C. Numerical dependence of ($\delta, \sigma, \phi, \alpha, \beta, \gamma$) on $\theta_z$**

Using the best-fit values of $\Delta m^2_{\text{sun}}$, $\Delta m^2_{\text{atm}}$, $\theta_x$ and $\theta_y$, we illustrate the numerical dependence of six phase parameters ($\delta, \sigma, \phi, \alpha, \beta, \gamma$) on the smallest mixing angle $\theta_z$ in Fig. 1(a) and Fig. 1(b) for the $m_1 = 0$ case and in Fig. 2(a) and Fig. 2(b) for the $m_3 = 0$ case. Some discussions are in order.

(1) In the $m_1 = 0$ case, $\theta_z$ is restricted to a very narrow range $4.0^\circ \lesssim \theta_z \lesssim 4.4^\circ$ (namely, $0.070 \lesssim s_z \lesssim 0.077$). This result implies that the FGY ansatz with $m_1 = 0$ is highly sensitive to $\theta_z$ and can easily be ruled out if the experimental value of $\theta_z$ does not really lie in the predicted region. To a good degree of accuracy, we obtain $\delta \approx 2\sigma$, $\phi \approx \alpha \approx -\sigma$, $\beta \approx -\gamma$, $\gamma \approx 0$, $\alpha \approx -\pi$, $\beta \approx -\pi$, and $\phi \approx -\pi$. We note that the signs of $\alpha$ and $\beta$ are to be assigned according to the sign of $\theta_z$.
and $\gamma \approx 0$. These instructive relations can essentially be observed from Eqs. (11), (12) and (13), because of $s_z \ll 1$. Note that we have only shown the dependence of $\delta$ on $\theta_z$ in the range $0 < \delta < \pi$. The reason is simply that only this range can lead to $Y_B > 0$ (i.e., the positive cosmological baryon number asymmetry), as one can see later on.

(2) In the $m_3 = 0$ case, there is no strong constraint on $\theta_z$ except that $\theta_z > 0.36^\circ$ (or equivalently $s_z > 0.0063$) must hold. We see that $\delta \approx \beta \approx \phi + \pi$, $\phi \approx \gamma$, $\sigma \approx -\alpha$ and $\alpha \approx 0$ hold to a good degree of accuracy. These results can also be observed from Eqs. (16), (17) and (18) by taking account of $s_z \ll 1$. Again, we have used the positive sign of $Y_B$ to constrain the allowed range of $\delta$. The other phase parameters are required to take possible values between $-\pi$ and $+\pi$.

As useful by-products, the Jarlskog parameter of CP violation ($J_{CP}$ [14]) and the effective mass of neutrinoless double beta decay ($\langle m_{ee} \rangle$ [15]) can be calculated. We show the numerical results of $J_{CP}$ versus $\langle m_{ee} \rangle$ in Fig. 1(c) for $m_1 = 0$ and in Fig. 2(c) for $m_3 = 0$, respectively. It is clear that $0 < J_{CP} \lesssim 0.016$ and $2.1$ meV $\lesssim \langle m_{ee} \rangle \lesssim 2.7$ meV in the $m_1 = 0$ case, while $0 < J_{CP} \lesssim 0.035$ and $\langle m_{ee} \rangle \approx \sqrt{\Delta m^2_{3\text{ atm}}} \approx 0.05$ eV in the $m_3 = 0$ case. The present experimental upper bound of $\langle m_{ee} \rangle$ is $\langle m_{ee} \rangle < 0.35$ eV at the 90% confidence level [16].

### III. BARYON ASYMMETRY VIA LEPTOGENESIS

Because of lepton number violation, two heavy Majorana neutrinos $N_i$ (for $i = 1$ and 2) may decay into $lH$ and its CP-conjugate state, where $l$ denotes the left-handed lepton doublet and $H$ stands for the Higgs-bozon weak isodoublet. The decay occurs at both the tree level and the one-loop level (via self-energy and vertex corrections), and their interference leads to a CP-violating asymmetry $\varepsilon_i$ between the CP-conjugated $N_i \to l + H$ and $N_i \to \bar{l} + H^*$ processes [5]. If the masses of $N_1$ and $N_2$ are hierarchical (i.e., $M_1 \ll M_2$), the interactions of $N_1$ can be in thermal equilibrium when $N_2$ decays. The asymmetry $\varepsilon_2$ is therefore erased before $N_1$ decays, and only the asymmetry $\varepsilon_1$ produced by the out-of-equilibrium decay of $N_1$ survives. In the flavor basis chosen above, we have

\[
\varepsilon_1 \equiv \frac{\Gamma(N_1 \to l + H) - \Gamma(N_1 \to \bar{l} + H^*)}{\Gamma(N_1 \to l + H) + \Gamma(N_1 \to \bar{l} + H^*)} \\
\approx -\frac{3}{16\pi v^2} \cdot \frac{M_1}{M_2} \cdot \frac{\text{Im} \left[ (M_D^b M_D^D)_{12} \right]^2}{(M_D^D M_D^D)_{11}} \\
= \frac{3}{16\pi v^2} \cdot \frac{M_1 |(M_\nu)_{12}|^2 |(M_\nu)_{23}|^2 \sin 2\phi}{||(M_\nu)_{11}|^2 + |(M_\nu)_{12}|^2 |(M_\nu)_{33}|}.
\]

In deriving this formula, we have used $(M_D^b M_D^D)_{11} = a^2 + |a'|^2$ and $(M_D^b M_D^D)_{12} = (a')^* b$ as well as

\[
a^2 = M_1 |(M_\nu)_{11}|, \quad |a'|^2 = M_1 \frac{|(M_\nu)_{12}|^2}{|(M_\nu)_{11}|}, \\
b^2 = M_2 \frac{|(M_\nu)_{23}|^2}{|(M_\nu)_{33}|}, \quad (b')^2 = M_2 |(M_\nu)_{33}|.
\]

7
One can see that \( \varepsilon_1 \) is independent of \( M_2 \), as long as \( M_2 \gg M_1 \) is satisfied. A nonvanishing \( \varepsilon_1 \) may result in a net lepton number asymmetry \( Y_L \equiv n_L/s = d \varepsilon_1/g_s \), where \( g_s = 106.75 \) is an effective number characterizing the relativistic degrees of freedom which contribute to the lepton-number-violating wash-out processes [17]. If the effective neutrino mass parameter \( \tilde{m}_1 \equiv (M_D^\dagger M_D)_{11}/M_1 \) [18] lies in the range \( 10^{-2} \) eV \( \lesssim \tilde{m}_1 \lesssim 10^3 \) eV, one may estimate the value of \( d \) by use of the following approximate formula [19] 2:

\[
\begin{align*}
        d & \approx 0.3 \left( \frac{10^{-3} \text{ eV}}{\tilde{m}_1} \right) \left( \ln \left( \frac{\tilde{m}_1}{10^{-3} \text{ eV}} \right) \right)^{-0.6} .
\end{align*}
\]

The lepton number asymmetry \( Y_L \) is eventually converted into a net baryon number asymmetry \( Y_B \) via the nonperturbative sphaleron processes [21]: \( Y_B \equiv n_B/s \approx -0.55 Y_L \). A generous range \( 0.7 \times 10^{-10} \lesssim Y_B \lesssim 1.0 \times 10^{-10} \) has been drawn from the recent WMAP observational data [22].

It is clear that \( \varepsilon_1 \) and \( Y_B \) only involve two free parameters: \( M_1 \) and \( \phi \). Because \( \phi \) is associated with the unknown flavor mixing angle \( \theta_z \), one may analyze the dependence of \( Y_B \) on \( \theta_z \) for given values of \( M_1 \). For \( m_1 = 0 \) and \( m_3 = 0 \) cases, we plot the numerical results of \( Y_B \) in Fig. 1(d) and Fig. 2(d) respectively. Some comments are in order.

1. In the \( m_1 = 0 \) case, current observational data of \( Y_B \) require \( M_1 \geq 2.9 \times 10^{10} \) GeV for the allowed ranges of \( s_z \). Once \( s_z \) is precisely measured, it is possible to fix the value of \( M_1 \) in most cases (e.g., \( M_1 = 10^{11} \) GeV will be ruled out, if \( s_z \approx 0.074 \) holds).

2. In the \( m_3 = 0 \) case, \( M_1 \geq 2.7 \times 10^{13} \) GeV is required by current observational data of \( Y_B \). Although \( \theta_z \) is less restricted in this scenario, it remains possible to determine the value of \( M_1 \) once \( \theta_z \) is measured (e.g., \( M_1 \approx 5 \times 10^{13} \) GeV is expected, if \( s_z \approx 0.1 \) holds).

3. One can carry out a similar analysis of \( Y_B \) in the framework of supersymmetric seesaw and leptogenesis models. However, the FGY ansatz does not favor \( M_1 \lesssim 10^8 \) GeV, which crucially affects the maximum reheating temperature of the universe after inflation in the generic supergravity models [17].

We remark that the cosmological baryon number asymmetry is closely correlated with the Jarlskog parameter of CP violation in the FGY ansatz 3. For illustration, we plot the numerical correlation between \( Y_B \) and \( J_{CP} \) in Fig. 3, where \( M_1 = 5 \times 10^{10} \) GeV for the \( m_1 = 0 \) case and \( M_1 = 5 \times 10^{13} \) GeV for the \( m_3 = 0 \) case have typically been taken. One can see that the observationally-allowed range of \( Y_B \) corresponds to \( J_{CP} \sim 1\% \) in the \( m_1 = 0 \)

\[ \text{For } M_1 \ll 10^{14} \text{ GeV, Giudice et al have presented a different approximate formula for the dilution factor (denoted as } \eta [20]: \frac{1}{\eta} \approx 3.3 \times 10^{-3} \text{ eV} \left( \frac{\tilde{m}_1}{5.5 \times 10^{-4} \text{ eV}} \right)^{1.16} \right. \] 2. We find that the ratio \( \eta/d \) will vary in the range \( 0.9 \lesssim \eta/d \lesssim 2.0 \), if \( \tilde{m}_1 \) takes values in the region \( 10^{-2} \text{ eV} \lesssim \tilde{m}_1 \lesssim 10^3 \text{ eV} \). Thus there is no significant inconsistency between two empirical formulas.

3. A similar point has been discussed by Endoh et al [7] in a more general way for the minimal seesaw model with leptogenesis. See, also, Ref. [23] for a possible seesaw bridge between leptogenesis and CP violation at low energies beyond the minimal seesaw model.
case and $J_{CP} \sim 2\%$ in the $m_3 = 0$ case. The correlation between $Y_B$ and $J_{CP}$ is so strong that a measurement of the latter in the long-baseline neutrino oscillation experiments could shed some light on the ball-park magnitude of $M_1$.

It is worth mentioning that we have neglected possible renormalization-group running effects of neutrino masses and lepton flavor mixing parameters between the scales of $v$ and $M_1$ [24]. Such an approximation is expected to be safe in the $m_1 = 0$ case, in which three light neutrinos have a clear mass hierarchy. In the $m_3 = 0$ case, in which $m_1 \approx m_2$ holds, a careful analysis of possible running effects on the FGY ansatz is needed [25] but it is beyond the scope of this paper.

Finally let us comment on possible phenomenological consequences of the FGY ansatz for $M_D$ in Eq. (4) or equivalently $M_\nu$ in Eq. (6). In the $m_1 = 0$ case, one can make use of Eqs. (9) and (10) to calculate relevant phase parameters by setting $(M_\nu)_{12} = 0$. A similar analysis can be done for the $m_3 = 0$ case by using Eqs. (14) and (15) and taking $(M_\nu)_{12} = 0$. We find that the simple replacements $\delta \rightarrow \delta - \pi$ and $\theta_y \rightarrow \pi/2 - \theta_y$ may allow us to write out the expressions of $\sigma$, $\phi$, $\alpha$, $\beta$ and $\gamma$ in the $(M_\nu)_{12} = 0$ case directly from Eqs. (11)–(13) and (16)–(18). It turns out that the numerical results of $\sigma$, $\phi$ and $\alpha$ are essentially unchanged, but those of $\beta$, $\gamma$ and $J_{CP}$ require the replacements $\beta \leftrightarrow \gamma$ and $J_{CP} \rightarrow -J_{CP}$. The results for $Y_B$ are essentially identical in $(M_\nu)_{12} = 0$ and $(M_\nu)_{13} = 0$ cases.

IV. SUMMARY

We have analyzed the minimal seesaw model for leptogenesis and neutrino mixing, in which the FGY ansatz is incorporated. We point out a very striking feature of this model: all nontrivial CP-violating phases can be determined in terms of the lepton flavor mixing angles and the ratio of $\Delta m^2_{sun}$ to $\Delta m^2_{atm}$. This important observation allows us to make very specific and testable predictions for the cosmological baryon number asymmetry, the effective mass of neutrinoless double beta decay and CP violation in neutrino oscillations. A precise measurement of the smallest mixing angle in reactor- and accelerator-based neutrino oscillation experiments will be extremely helpful to examine the FGY ansatz and other presently viable ansätze of lepton mass matrices.

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FIG. 1. Numerical results for the $m_1 = 0$ case: (a) dependence of $\delta$, $\sigma$ and $\phi$ on $\sin \theta_z$; (b) dependence of $\alpha$, $\beta$ and $\gamma$ on $\sin \theta_z$; (c) allowed ranges of $J_{\text{CP}}$ and $\langle m \rangle_{ee}$; (d) dependence of $Y_B$ on $M_1$ and $\sin \theta_z$. The region between two dashed lines in (d) corresponds to the range of $Y_B$ allowed by current observational data.
FIG. 2. Numerical results for the \( m_3 = 0 \) case: (a) dependence of \( \delta, \sigma \) and \( \phi \) on \( \sin \theta_z \); (b) dependence of \( \alpha, \beta \) and \( \gamma \) on \( \sin \theta_z \); (c) allowed ranges of \( J_{\text{CP}} \) and \( \langle m \rangle_{ee} \); (d) dependence of \( Y_B \) on \( M_1 \) and \( \sin \theta_z \). The region between two dashed lines in (d) corresponds to the range of \( Y_B \) allowed by current observational data.
FIG. 3. Numerical illustration of the correlation between $Y_B$ and $J_{CP}$: (a) in the $m_1 = 0$ case with $M_1 = 5 \times 10^{10}$ GeV; and (b) in the $m_3 = 0$ case with $M_1 = 5 \times 10^{13}$ GeV. The region between two dashed lines in (a) or (b) corresponds to the range of $Y_B$ allowed by current observational data.