Novel Analysis Results of Impulse Current Mutual Inductor Circuits

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Abstract. The impulse magnetic current $i_m(t)$ was generally neglected in conventional analysis of impulse current mutual inductor circuits. According to analytic results of the impulse magnetic current $i_m(t)$, there were actually several positive components included in magnetic current $i_m(t)$ and could not be ignored. When the impulse current mutual inductor circuits are operating stably, the impulse magnetic current $i_m(t)$ is in proportion to duty cycle $D$. Therefore, these relationships and curves could be used to instruct and verify the design and engineering application of the unidirectional and bi-directional impulse current mutual inductor circuits.

1. Introduction
The unidirectional impulse current mutual inductor circuit (UICMIC) and bi-directional impulse current mutual inductor circuit (BICMIC), which are shown in the figure 1 and figure 2 respectively, are always used in the Direct Current to Direct Current Convertor [1, 2] of the spacecraft. They can measure the values of high frequency current through transferring the high frequency current $i(t)$ into the high frequency voltage $V_o(t)$ [3].

In order to simplify the model, the impulse magnetic current $i_m(t)$ was usually all or partly neglected in the general analysis of impulse current mutual inductor circuits [4-7]. Actually, if the input winding turns are supposed as 1 and the input current $i(t)=I_0+\beta t$, where $I_0$ and $\beta$ are all positive constant, $t$ is time and output winding turns are $N$, it can be proved that there are also two positive components included in the impulse magnetic current $i_m(t)$ except the $\beta t$. Therefore, the $i_m(t)$ can’t be ignored.

The expressions of output voltage $V_o(t)$, circuit parameters (figure 1(b) and figure 2(b)) and impulse current parameters (figure 1(a) and figure 2(a)) are presented when circuits are operating stably during $0<t\leq DT$ and $0\leq t\leq DT/2$. Therefore, the impulse waveforms of output voltage can be achieved which have three states such as exponential rising state, parallel state and exponential declining state.

The UICMIC and BICMIC will be discussed respectively in the following passage.

2. Two kinds of impulse current mutual inductor circuits
Figure 1 is the impulse current waveform and UICMIC circuits. The figure 1 (a) is the impulse current waveform of UICMIC, the figure 1 (b) is the UICMIC circuit in period of $DT$ and the figure 1 (c) is the UICMIC circuit in period of $(1-D)T$. 
### Figure 1. Impulse current waveform of UICMIC.

(a) Impulse current waveform of UICMIC.

(b) UICMIC circuit in the period of $DT$.

(c) UICMIC circuit in the period of $(1-D)T$.

**Figure 2.** Impulse current waveform and BICMIC.

(a) Impulse current waveform of BICMIC.

(b) BICMIC circuit in the period of $DT/2$.

3. **UICMIC in period of $DT$**

   Due to $i_2(t) = i_1(t) + i_0(t)$ in the figure 1 (b), the following Equation can be achieved.

   $$\frac{I_0 + \beta t - i_m(t)}{N} = \frac{V_o(t) + 0.5}{\alpha R_o} + \frac{V_o(t)}{R_o}$$  \hspace{1cm} (1)

   where 0.5 is supposed of the forward voltage drop of diode in volts. According to Equation (1),

   $$V_o(t) = \frac{1}{1+\alpha} \left( \alpha \left( I_0 + \beta t - i_m(t) \right) \frac{R_o}{N} - 0.5 \right)$$  \hspace{1cm} (2)
In addition, according to the law of electromagnetic induction [8, 9], then

\[ N_l \frac{di_m}{dt} = V_o(t) + 0.5 \]  

(3)

combining the Equation (2) and Equation (3), the operation equation of \( i_m(t) \) in period of \( DT \) is shown as following after standardizing,

\[ \frac{di_m}{dt} = -\alpha_i i_m + \alpha_i \frac{N}{R_o} \left( 0.5 + \frac{R_o}{N} (I_o + \beta t) \right) \]

(4)

where \( \alpha_i = \frac{\alpha}{1 + \alpha} N l \).

Quoting the Equation of [10], then

\[ i_m(t) = e^{-\alpha t} \left( \int \frac{\alpha N}{R_o} \left( 0.5 + \frac{R_o}{N} (I_o + \beta t) \right) e^{\alpha t} dt + C \right) \]

(5)

where \( C \) is an undetermined constant.

According to the result of Equation (5), then

\[ i_m(t) = \beta t + I_1 + Ce^{-\alpha t} \]

(6)

where \( I_1 = I_o + 0.5 \frac{N}{R_o} - \frac{\beta}{\alpha} \).

Supposing \( t=0 \), Equation (6) will be,

\[ C = i_m(0) - I_1 \]

(7)

combining the Equation (6) and Equation (7), then

\[ i_m(t) = \beta t + I_1 \left( 1 - e^{-\alpha t} \right) + i_m(0) e^{-\alpha t} \]

(8)

Therefore, there are three components in the Equation (8),

- \( \beta t \) is the part of \( i(t) \) above the line of \( I_o \) in figure 1 (a) and is also one component of \( i_m(t) \).
- \( I_1 \left( 1 - e^{-\alpha t} \right) \) is an exponential rising variable from zero.
- \( i_m(0) e^{-\alpha t} \) is an exponential decline variable from \( i_m(0) \).

In other words, there are three components in the impulse magnetic current \( i_m(t) \), which means the \( i_m(t) \) can’t be ignored. The three components only exist in the input winding of impulse current mutual inductor and will not be present in the output winding.

When \( t=DT \), the Equation (8) can be transformed as following,

\[ i_m(DT) = \beta DT + I_1 \left( 1 - e^{-\alpha DT} \right) + i_m(0) e^{-\alpha DT} \]

(9)

4. UICMIC in Period of (1-D)T

Figure 1 (c) shows the current operation character of UICMIC in period of (1-D)T.

According to the law of electromagnetic induction, the operation equation of \( i_m(t) \) in period of (1-D)T is shown as following after standardizing,
\[
\frac{di_z}{dt} + \frac{\alpha R_n}{N^2 L_p} i_n = 0
\]  \hspace{1cm} (10)

Equation (9) is the initial conditions of Equation (10). Quoting the Equation of [2], then

\[
i_n(t) = C e^{-\alpha z}
\]  \hspace{1cm} (11)

where C is an undetermined constant, \( \alpha = \frac{\alpha R_n}{N^2 L_p} \) and \( DT \leq t \leq T \).

Supposing \( t=DT \) and combining the Equation (9) and Equation (11), the impulse magnetic current \( i_m(t) \) will be,

\[
i_m(t) = \frac{DT \beta + I_1 \left(1 - e^{-\alpha DT}\right)}{e^{-\alpha DT}} + i_m(0) e^{-\alpha DT}
\]  \hspace{1cm} (12)

Supposing \( t=T \) and Equation (12) can be transformed as following,

\[
i_m(T) = e^{\alpha(1-D)D} \left( DT \beta + I_1 \left(1 - e^{-\alpha DT}\right) + i_m(0) e^{-\alpha DT}\right)
\]  \hspace{1cm} (13)

5. Equation of \( i_m(0) \) as UICMIC Operating Stably

When UICMIC is operating stably, Equation (13) will be equal to \( i_m(0) \). Then \( i_m(0) \) is shown as following,

\[
i_m(0) = e^{\alpha(1-D)D} \left( DT \beta + I_1 \left(1 - e^{-\alpha DT}\right) + i_m(0) e^{-\alpha DT}\right)
\]  \hspace{1cm} (14)

from the Equation (14), then

\[
i_m(0) = \frac{e^{\alpha(1-D)D}}{1-e^{-\alpha \frac{D}{1+\alpha}}} \times \left( DT \beta + I_1 \times \left(1 - e^{-\alpha DT}\right)\right)
\]  \hspace{1cm} (15)

6. Calculating \( i_m(0) \) as UICMIC Operating Stably

Supposing \( N=100, R_0=100 \Omega, N^2 L_p=2.5 \times 10^{-3} \text{H}, T=5 \times 10^{-6} \text{s} \) and \( \alpha=20 \) in Equation (15), the relationship between \( i_m(0) \) and duty cycle \( D \) is shown as following table 1,

| \( D \) | \( i_m(0) \) |
|---|---|
| 0.00 | 0.00000(I_0+0.5)+0.0000x10^{-6}\beta |
| 0.10 | 0.0005(I_0+0.5)+0.0001x10^{-6}\beta |
| 0.20 | 0.0016(I_0+0.5)+0.0008x10^{-6}\beta |
| 0.30 | 0.0036(I_0+0.5)+0.0030x10^{-6}\beta |
| 0.40 | 0.0091(I_0+0.5)+0.0091x10^{-6}\beta |
| 0.50 | 0.0140(I_0+0.5)+0.0180x10^{-6}\beta |
| 0.60 | 0.0266(I_0+0.5)+0.0410x10^{-6}\beta |
| 0.70 | 0.0510(I_0+0.5)+0.0910x10^{-6}\beta |
| 0.80 | 0.1034(I_0+0.5)+0.2120x10^{-6}\beta |

Quoted symbols \( f_1(D) \times (I_0+0.5 \times (N/R_0)) + f_2(D)\beta \)

Due to \( f_1(D) > f_2(D) \), the relationship between \( i_m(0) \) and \( D \) is shown as figure 3.
Figure 3. The relationship between \( f_1(D) \) and \( D \)

7. Output Impulse Voltage and Waveforms as UICMIC Operating Stably

After the derivative of Equation (8) to \( t \), both sides of Equation (8) is multiplied by \( NL_p \), then

\[
\frac{di}{dt} = \beta - \frac{\left(NL_p \beta - \alpha_1 \frac{N}{R_0} \left(I_t + \frac{\beta}{\alpha_1} - i_n(0)\right)\right)e^{-\alpha_1 t}}{NL_p}
\] (16)

Combining the Equation (3) and Equation (16), then

\[
V_o(t) = NL_p \beta - 0.5 - \left(NL_p \beta - \alpha_1 \frac{N}{R_0} \left(I_t + \frac{\beta}{\alpha_1} - i_n(0)\right)\right)e^{-\alpha_1 t}
\] (17)

If \( f_2(D) \) is neglected, the value of \( V_d(0) \) is shown as following,

\[
V_o(0) = \frac{\alpha}{1+\alpha} \left(0.5 + R_0 \frac{I_n}{N}\right)(1 - f_1(D)) - 0.5
\] (18)

Equation (17) minuses Equation (18), then

\[
V_o(t) - V_o(0) = \left(NL_p \beta - \delta_i (1 - f_1(D))\right)(1 - e^{-\alpha_1 t})
\] (19)

where \( \delta_i = \frac{\alpha}{1+\alpha} \left(0.5 + R_0 \frac{I_n}{N}\right) \).

Therefore, the amplitude of \( V_d(t) \) can be achieved by Equation (18) in the coordinate of \( V_d(t) \sim t \). The waveform of \( (V_d(t) - V_d(0)) \) can be achieved by Equation (19).

When \( NL_p \beta - \delta_i (1 - f_1(D)) > 0 \), the waveform of \( (V_d(t) - V_d(0)) \) is an exponential rising state as shown in figure 4.

When \( NL_p \beta - \delta_i (1 - f_1(D)) = 0 \), the waveform of \( (V_d(t) - V_d(0)) \) is a parallel state as shown in figure 4.

When \( NL_p \beta - \delta_i (1 - f_1(D)) < 0 \), the waveform of \( (V_d(t) - V_d(0)) \) is an exponential declining state as shown in figure 4.
8. $i_{m}(t)$ Waveforms as BICMIC Operating Stably

Figure 5 shows the $i_{m}(t)$ waveforms as BICMIC operating stably according to the following principles,

- In period of $DT/2$, the direction of $i_{m}(t)$ can be changed because there is current exiting in the input winding.
- In period of $(1-D)T/2$, the direction of $i_{m}(t)$ can't be changed and the amplitude of $i_{m}(t)$ is declining in the same direction because there is no current exiting in the input winding.
- When the impulse current mutual inductor circuits are operating stably, the values of $i_{m}$ at the beginning and ending of each cycle $T$ will be same.

9. BICMIC in Period of $DT/2$

Due to $i_{2}(t)=i_{1}(t)+i_{d}(t)$ in the figure 2 (b), the following Equation can be achieved.

$$\frac{I_{o} + \beta t - i_{m}(t)}{N} = \frac{V_{o}(t) + 1}{\alpha R_{o}} + \frac{V_{o}(t)}{R_{o}}$$

(20)

where $I_{o}$ is supposed of the forward voltage drop of diode in volts. According to Equation (21),

$$V_{o}(t) = \frac{1}{1+\alpha} \left( \alpha (I_{o} + \beta t - i_{m}(t)) R_{o} - 1 \right)$$

(21)

In addition, according to the law of electromagnetic induction,
\[ N L \frac{di_m}{dt} = V_i(t) + 1 \]  \hspace{1cm} (22)

Combining the Equation (21) and Equation (22), the operation equation of \( i_m(t) \) in period of \( DT / 2 \) is shown as following after standardizing,

\[ \frac{di_m}{dt} = -\alpha_t i_m + \alpha_t \frac{N}{R_0} \left( 1 + \frac{R_u}{N} (I_0 + \beta t) \right) \]  \hspace{1cm} (23)

Quoting the Equation of [2], then

\[ i_m(t) = e^{\alpha t} \left( \int \frac{\alpha_t}{R_0} \left( 1 + \frac{R_u}{N} (I_0 + \beta t) \right) e^{\alpha t} dt + C \right) \]  \hspace{1cm} (24)

where \( C \) is an undetermined constant.

According to integration results of Equation (24), then

\[ i_m(t) = \beta t + I_z + Ce^{-\alpha t} \]  \hspace{1cm} (25)

where \( I_z = I_0 + \frac{N}{R_0} \frac{\beta}{\alpha_t} \).

According to the same reasoning in section 2, then

\[ i_m(t) = \beta t + I_z \left( 1 - e^{-\alpha t} \right) + i_m(0) e^{-\alpha t} \]  \hspace{1cm} (26)

When \( t=DT/2 \), then

\[ i_m \left( \frac{DT}{2} \right) = \frac{1}{2} DT \beta + I_z \left( 1 - e^{-\alpha \frac{DT}{2}} \right) + i_m(0) e^{-\alpha \frac{DT}{2}} \]  \hspace{1cm} (27)

10. BICMIC in Period of \((1 - D)T/2\)

According to the same reasoning in section 3, the operation equation of \( i_m(t) \) in period of \((1-D)T / 2\) is shown as following,

\[ i_m(t) = \frac{DT \beta}{2} + I_z \left( 1 - e^{-\alpha \frac{DT}{2}} \right) + i_m(0) e^{-\alpha \frac{DT}{2}} \]  \hspace{1cm} (28)

where \( \frac{DT}{2} \leq t \leq \frac{T}{2} \).

Supposing \( t=T/2 \), then

\[ i_m \left( \frac{T}{2} \right) = e^{-\alpha \frac{(1-D)T}{2}} \times \left( \frac{DT \beta}{2} + I_z \left( 1 - e^{-\alpha \frac{DT}{2}} \right) + i_m(0) e^{-\alpha \frac{DT}{2}} \right) \]  \hspace{1cm} (29)

11. Equation of \( i_m(0) \) As BICMIC Operating Stably

When BICMIC is operating stably, Equation (29) will be equal to \(-i_m(0)\) according to the figure 5. Then the equation of \( i_m(0) \) is shown as following,
\[ i_m(0) = \frac{-e^{-\alpha \frac{(1-D)^2}{2}}}{1 + e^{-\alpha \frac{(1-D)^2}{2}}} \left( \frac{1}{2} DT \beta + I_z \left( 1 - e^{-\alpha \frac{DT}{2}} \right) \right) \] 

(30)

12. Calculating \( i_m(0) \) as BICMIC Operating Stably

Supposing \( N=100 \), \( R_0=100\Omega \), \( N^2L_p=2.5\times10^{-3}\)H, \( T=5\times10^{-5} \)s and \( \alpha=20 \) in the Equation (30), then the relationship between \( i_m(0) \) and \( D \) is shown as following table 2,

| \( D \)  | \( i_m(0) \)                      |
|--------|----------------------------------|
| 0.00   | 0.00000(I_0+0.5)+0.0000\times10^6\beta |
| 0.10   | 0.0005(I_0+0.5)+0.0001\times10^6\beta |
| 0.20   | 0.0014(I_0+0.5)+0.0010\times10^6\beta |
| 0.30   | 0.0032(I_0+0.5)+0.0024\times10^6\beta |
| 0.40   | 0.0061(I_0+0.5)+0.0060\times10^6\beta |
| 0.50   | 0.0110(I_0+0.5)+0.0130\times10^6\beta |
| 0.60   | 0.0185(I_0+0.5)+0.0280\times10^6\beta |
| 0.70   | 0.0297(I_0+0.5)+0.0530\times10^6\beta |
| 0.80   | 0.0458(I_0+0.5)+0.0940\times10^6\beta |
| 0.90   | 0.0675(I_0+0.5)+0.1560\times10^6\beta |
| 1.00   | 0.0950(I_0+0.5)+0.2450\times10^6\beta |

Quoted symbols \( f_1(D) \times (I_0+1\times(N/R_0))+f_2(D)\beta \)

Due to \( f_1(D) >> f_2(D) \), the relationship between \( i_m(0) \) and \( D \) is shown as figure 6.

![Figure 6. The relationship between \( f_1(D) \) and \( D \)](image)

13. Output Impulse Voltage and Waveforms as UICMIC Operating Stably

After the derivative of Equation (26) to \( t \), both sides of Equation (26) is multiplied by \( NL_p \), then

\[ \frac{di_m}{dt} = \beta - \left( NL_p \beta - \frac{N}{R_0} \left( I_n + \frac{N}{R_0} - i_m(0) \right) \right) e^{-\alpha t} \] 

(31)

combining the Equation (22) and Equation (31), then
\[ V_0(t) = N_l \beta \ln \left( 1 + \frac{N_l \beta - \alpha}{R_0} \left( I_o + \frac{N}{R_0} - I_m(0) \right) \right) e^{-\alpha t} \]  \hspace{1cm} (32)\]

If \( f_2(D) \) is neglected, the value of \( V_0(0) \) is shown as following,

\[ V_0(0) = \frac{\alpha}{1+\alpha} \left( 1 + R_0 \frac{I_o}{N} \right) \left( 1 + f_1(D) \right) - 1 \]  \hspace{1cm} (33)\]

Equation (32) minus Equation (33), then

\[ V_0(t) - V_0(0) = \left( N_l \beta - \delta \left( 1 + f_1(D) \right) \right) \times \left( 1 - e^{-\alpha t} \right) \]  \hspace{1cm} (34)\]

where \( \delta = \frac{\alpha}{1+\alpha} \left( 1 + R_0 \frac{I_o}{N} \right) \).

Therefore, the amplitude of \( V_o(t) \) can be achieved by Equation (33) in the coordinate of \( V_o(t)-t \). The waveform of \( (V_o(t)-V_o(0)) \) can be achieved by Equation (34).

When \( N_l \beta - \delta \left( 1 + f_1(D) \right) > 0 \), the waveform of \( (V_o(t)-V_o(0)) \) is an exponential rising state as shown in figure 7.

When \( N_l \beta - \delta \left( 1 + f_1(D) \right) = 0 \), the waveform of \( (V_o(t)-V_o(0)) \) is a parallel state as shown in figure 7.

When \( N_l \beta - \delta \left( 1 + f_1(D) \right) < 0 \), the waveform of \( (V_o(t)-V_o(0)) \) is an exponential declining state as shown in figure 7.

![Figure 7](image_url)

**Figure 7.** The waveforms of output voltage \( V_o(t) \) as BICMIC operating stably

14. Conclusion

By means of theoretical analysis, there are several positive components in the impulse magnetic current \( i_m(t) \) which can’t be ignored. Based on these calculation results, the relationship between the impulse magnetic current \( i_m(t) \) and duty cycle \( D \) were achieved. Moreover, the waveforms of the output voltage \( V_o(t) \) of two impulse current mutual inductor circuits were also achieved.

These theoretical analysis results and waveforms can be used to instruct and verify the engineering design of unidirectional and bi-directional impulse current mutual inductor circuits.

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