Abstract

We consider lepton-antilepton annihilation into a fermion-antifermion pair at variable c.m. energy. We propose for this process a simple parametrization of the virtual effects of the most general model of new physics of universal type. This parametrization is based on a recent approach, that uses the experimental results of LEP1, SLC as theoretical input. It introduces three functions whose energy dependence is argued to be smooth and, in first approximation, negligible. A couple of representative models of new physics are considered, as a support of the previous claim. Explicit bounds are then derived for this type of new physics from the available LEP2 data, and a discussion is given of the relevance in this respect of the different experimental measurements. The method is then extended to treat the case of two particularly simple models of non universal type, for which it is possible to draw analogous conclusions.
1 Introduction.

One of the most interesting consequences of the high precision results obtained at LEP1, SLC \[1\] has been the determination of bounds on the virtual effects of models of new physics. This had led in some cases to drastic conclusions from which very useful indications about the possible "surviving" models have been drawn.

One fundamental step in this process of derivation of bounds has been the introduction of “simple” parametrizations of the new physics effects. In this respect, we feel, both the original idea of Peskin and Takeuchi of introducing two \((S, T)\) parameters \[2\], and the later more rigorous proposal of Altarelli and Barbieri, leading to the definition of \(\epsilon_1, \epsilon_3\) \[3\], have played a fundamental role for any meaningful investigation of new physics effects at the \(Z\) resonance \[4\].

Strictly speaking, the previous two-parameter (e.g. \(\epsilon_1, \epsilon_3\)) description of new physics is only possible if the latter is of universal type, i.e. independent of the flavour of the final fermion-antifermion pair. At the \(Z\) peak, the remarkable simplification occurs that box diagrams can be ignored, being kinematically suppressed. Therefore, the only possible non universal effects may come from the final vertices (the most remarkable example being that of the \(b\bar{b}\) vertex). When these are carefully taken into account, the theoretical description of new physics becomes less simple, but still “relatively” straightforward. This is also due to the fact that, at the \(Z\) peak, the s-channel photon exchange, with all its new physics content, can be safely neglected.

An important question that arises in a natural way is that of whether the previous, reasonably simple, picture will continue to be valid at the higher energy electron-positron accelerators, in particular at LEP2, but also, looking at a more distant future, at a linear electron-positron collider (LC)\[5\] or at a muon-muon collider\[6\]. Here all the previous simplifications occurring on the \(Z\) resonance are no longer valid. In particular, both the photon exchange with its universal and non universal components and the non universal box diagrams at one loop cannot be now neglected \[7\]. Also, another technical advantage of the experiments at the \(Z\) peak is lost: present and future measurements in the two fermion production process will be spread out over a range of different center of mass energies, leading in general to a diluted experimental precision in the investigation of small, \(q^2\) dependent, deviations from the Standard Model predictions. This certainly gives the impression that the theoretical description of new physics effects might be, in general, much more cumbersome than at the \(Z\) peak.

The aim of this short paper is that of showing that, for what concerns the process of final fermion-antifermion production, the situation is still promising if one restricts the investigation to effects of new physics of universal type. Here, under "reasonable" assumptions, a situation that appears as the immediate generalization of that met at the \(Z\) peak, with only three parameters rather than two (the extra one being an obvious relic of the photon exchange contribution), will be proposed and justified with some examples.

The starting point of our analysis will be the use of a theoretical description of the process of electron-positron annihilation into a charged fermion-antifermion pair (with the exclusion for the moment of Bhabha scattering that will be treated in a separate forthcoming paper) that was called the "\(Z\)-peak subtracted" approach. This has been exhaustively illustrated and discussed in previous references \[8\] and here we shall neither
repeat its motivations nor recall the relevant features. The only technical detail that we shall use is the expression of the differential unpolarized cross section at variable squared c.m. energy $q^2$ and scattering angle $\theta$; for our purposes, it will be convenient to write it in the following form ($f$ denotes the final fermion of the considered process, that will be either a lepton ($\mu, \tau$) or a quark, more precisely a "light" ($u, d, s, c, b$) one; $\ell$ indicates the initial lepton, which is for the purposes of this paper an electron):

\[
\frac{d\sigma_{if}}{d\cos \theta} = \frac{4\pi}{3} N_f q^2 \left( \frac{3}{8} (1 + \cos^2 \theta) U_{11} + \frac{3}{4} \cos \theta U_{12} \right)
\]

where

\[
U_{11} = \frac{\alpha^2(0) Q_f^2}{q^4} [1 + 2 \tilde{\Delta}_{\alpha,f}(q^2, \theta)] + 2[\alpha(0)|Q_f|] q^2 - M_Z^2 \left( [3\Gamma_{l1}\left[\frac{3\Gamma_f}{N_f M_Z}\right]}^{1/2} \frac{\tilde{\nu}_f}{(1 + \tilde{\nu}_f^2)^{1/2}} + \frac{\tilde{\nu}_l}{(1 + \tilde{\nu}_l^2)^{1/2}} \right)
\]

\[
\times [1 + \tilde{\Delta}_{\alpha,f}(q^2, \theta) - R_{lf}(q^2, \theta) - 4\tilde{\gamma}_l \tilde{\gamma}_f \{ \frac{1}{\tilde{\nu}_l} V_{lf}^{\gamma Z}(q^2, \theta) + \frac{|Q_f|}{\tilde{\nu}_f} V_{lf}^{\gamma f}(q^2, \theta) \}]
\]

\[
U_{12} = 2[\alpha(0)|Q_f|] q^2 - M_Z^2 \left( [3\Gamma_{l1}\left[\frac{3\Gamma_f}{N_f M_Z}\right]}^{1/2} \frac{1}{(1 + \tilde{\nu}_l^2)^{1/2}} \frac{1}{(1 + \tilde{\nu}_f^2)^{1/2}} \right)
\]

\[
\times [1 + \tilde{\Delta}_{\alpha,f}(q^2, \theta) - R_{lf}(q^2, \theta)]
\]

\[
+ \frac{[3\Gamma_{lf}\left[\frac{3\Gamma_f}{N_f M_Z}\right]}^{1/2} \frac{4\tilde{\nu}_f}{(1 + \tilde{\nu}_f^2)^{1/2}}} q^2 - M_Z^2 \left( [3\Gamma_{l1}\left[\frac{3\Gamma_f}{N_f M_Z}\right]}^{1/2} \frac{1}{(1 + \tilde{\nu}_l^2)^{1/2}} \frac{1}{(1 + \tilde{\nu}_f^2)^{1/2}} \right)
\]

\[
\times [1 - 2 R_{lf}(q^2, \theta) - 4\tilde{\gamma}_l \tilde{\gamma}_f \{ \frac{1}{\tilde{\nu}_l} V_{lf}^{\gamma Z}(q^2, \theta) + \frac{|Q_f|}{\tilde{\nu}_f} V_{lf}^{\gamma f}(q^2, \theta) \}]
\]

In the previous equations, $\Gamma_{l,f}$ are the partial $Z$ widths into a final lepton ($l\bar{l}$) or into a general fermion ($f\bar{f}$) pair; $\tilde{s}_l^2 = 1 - \tilde{c}_l^2$ is the effective weak angle $\sin^2 \theta_W$, (for simplicity we neglect here very small differences between parameters entering at the one loop level); $\tilde{\nu}_l = 1 - 4\tilde{s}_l^2$; $\tilde{\nu}_f = 1 - 4|Q_f|\tilde{s}_l^2$; $N_f$ is the colour factor including QCD corrections.

In the SM, the four functions $\tilde{\Delta}_{\alpha,f}$, $R_{lf}$, $V_{lf}^{\gamma Z}$, $V_{lf}^{\gamma f}$ that appear in the various expressions are gauge-invariant combinations of one-loop self-energies, vertices and boxes. They determine the $\theta$ integrated expression of the various unpolarized cross sections and asymmetries in a way that it is immediate to derive for self-energies and vertices, and requires a more complicated numerical calculation in the case of boxes, that are generally $\theta$ dependent.
When one considers models of new physics (NP) of electroweak type, whose virtual effects on the considered fermion-antifermion production process do not add extra Lorentz structures to the scattering amplitude at one loop, the previous description of Eqs. (1)-(3) still remains valid, with the formal replacement

$$\tilde{\Delta}_{\alpha,lf}(q^2, \theta) \rightarrow \tilde{\Delta}_{\alpha,lf}(q^2, \theta) + \tilde{\Delta}_{\alpha,lf, NP}(q^2, \theta)$$ (4)

(similarly for $R_{lf}$, $V_{lf}^{\gamma Z}$ and $V_{lf}^{Z\gamma}$). This leads to a straightforward modification of the expressions of the various observables where the effect of the extra model can be estimated, at each $q^2$, once the expressions of the various functions $(\tilde{\Delta}_{\alpha,lf}^{NP}, R_{lf}^{NP}, V_{lf}^{\gamma Z, NP}, V_{lf}^{Z\gamma, NP})$ have been given. One might then proceed to the determination of bounds for the new physics effects, at each $q^2$ value that was considered, after a procedure that implies the angular integration. The latter will leave, in general, several different functions of $q^2$ generated by the different powers of $\cos \theta$ that appear in the integrand. A typical example of such a situation would be the determination of the effects due to SUSY boxes, that were already discussed in a special case, at the LEP2 energies, in a previous paper [9]. For these cases, the determination of the bounds would require a dedicated computational program, in agreement with the expectation that was anticipated in this paper.

A first simplification occurs for models of new physics that do not have any $\theta$-dependence. In such a case, the extra contribution to the observables can be very easily expressed in terms of four different functions of $q^2$ only, to be indicated in this paper as $\tilde{\Delta}_{\alpha,lf}^{NP}(q^2)$, $R_{lf}^{NP}(q^2)$, $V_{lf}^{\gamma Z, NP}(q^2)$, $V_{lf}^{Z\gamma, NP}(q^2)$. For what concerns these functions, it would become possible to derive bounds for their values at each different $q^2$ that is experimentally relevant. This would require a separate analysis for each different model of new physics type, that would also depend on the details of the final state on which the four functions might depend.

A second remarkable simplification occurs for models of new physics whose effects are of universal type (UNP). In such cases, only three functions of $q^2$ would remain in the theoretical expression, since $V_{lf}^{\gamma Z, NP} = V_{lf}^{Z\gamma, NP}$ in this case [8]. These functions will be called from now on $\tilde{\Delta}_{\alpha,lf}^{UNP}(q^2)$, $R_{lf}^{UNP}(q^2)$, $V_{lf}^{UNP}(q^2)$.

The previous scenario is particularly appealing, not only because of the gain in conceptual simplicity achieved through the reduction of the overall number of unknown functions, but also because the measurements of fermion pair production for different final flavours can be combined to determine the best fit values for the NP contributions, leading to bounds that will still be $q^2$ dependent.

A final and drastic simplification occurs if we introduce the physical assumption that the models of new physics that are considered contain an intrinsic scale that is essentially larger than the $q^2$ values at which the measurements are performed. In the $Z$-peak subtracted approach, this assumption can be exploited in a very useful way. In fact, one of the characteristic features of this approach is that the three functions $\tilde{\Delta}_{\alpha,lf}^{UNP}(q^2)$, $R_{lf}^{UNP}(q^2)$, $V_{lf}^{UNP}(q^2)$ must always vanish, respectively, at the $q^2$ values that correspond to the photon or to the $Z$ mass. In other words, one has by construction:

$$\tilde{\Delta}_{\alpha,lf}^{UNP}(q^2 = 0) = R_{lf}^{UNP}(q^2 = M_Z^2) = V_{lf}^{UNP}(q^2 = M_Z^2) = 0$$ (5)
As an immediate consequence of Eq.(5), one is entitled to write:

\[ R_{\text{UNP}}(q^2) = \frac{(q^2 - M_Z^2)}{M_Z^2} \delta_z \]  

(6)

\[ V_{\text{UNP}}(q^2) = \frac{(q^2 - M_Z^2)}{M_Z^2} \delta_s \]  

(7)

\[ \Delta_{\alpha}^{\text{UNP}}(q^2) = \frac{q^2}{M_Z^2} \delta_{\gamma} \]  

(8)

where \( \delta_{z,s,\gamma} \) are dimensionless functions of \( q^2 \). As indicated by the indices, \( \delta_z \) refers to the modification of the \( Z \) coupling, \( \delta_{\gamma} \) to the modification of the photon coupling and \( \delta_s \) to that of the effective weak angle \[ \delta \].

For values of the intrinsic new physics scale sufficiently larger than the c.m. energies at which the experimental searches are performed, we can reasonably expect that the \( q^2 \) dependence of \( \delta_{z,s,\gamma} \) is smooth. In this case, it will be reasonable to approximate the \( \delta_i \) by the lowest order terms in their \( q^2 \) expansion. This procedure will reduce the functions \( \delta_{s,\gamma,z} \) to three constants, i.e. the coefficients of the lowest \( q^2 \) power. This will correspond to writing \( \delta_i(q^2) \approx \delta_i(0) \equiv \delta_i \) in all cases where \( \delta_i(0) \neq 0 \).

For the class of new physics models that meet all the previous requirements (that, as we shall explicitly show with some examples, is not empty) the effect on the various observables is rather simple. We have only considered in this paper the case of unpolarized initial beams and concentrated our attention on five different observables, that are measured at LEP2. These are: the cross section for muon (or tau) production \( \sigma_\mu \); the related forward-backward asymmetry \( A_{FB,\mu} \); the cross section for five "light" \( (u, d, s, c, b) \) quark production \( \sigma_5 \); the cross section for \( (b\bar{b}) \) production \( \sigma_b \) and the related forward-backward asymmetry \( A_{FB,b} \). In terms of the three parameters \( \delta_z, \delta_s, \delta_{\gamma} \), the effects of the most general model of new physics of universal type can be written as follows (we use systematically, for a given observable \( O_i \), the notation \( O_i = O_i^{\text{SM}}[1 + dO_i^{\text{UNP}}/O_i^{\text{SM}}] \) while the experimental value and errors is denoted by \( O_i^{\text{meas}} \pm \sigma_i \):

\[
\frac{d\sigma_{\mu}^{\text{UNP}}}{\sigma_\mu} = \frac{2}{M_Z^2[7.99(q^2 - M_Z^2)^2 + q^4]} \left\{ \delta_\gamma \left[ 7.99q^2(q^2 - M_Z^2)^2 \right] - \delta_z \left( q^2 - M_Z^2 \right)q^4 - \delta_s \left( q^2 - M_Z^2 \right) \left[ 0.70q^2(q^2 - M_Z^2) + 0.25q^4 \right] \right\} \]  

(9)

\[
\frac{dA_{FB,\mu}^{\text{UNP}}}{A_{FB,\mu}} = \frac{1}{M_Z^2[7.99(q^2 - M_Z^2)^2 + q^4]} \left\{ (\delta_\gamma q^2 + \delta_z \left( q^2 - M_Z^2 \right)) \left[ q^4 - 7.99(q^2 - M_Z^2)^2 \right] + \delta_s \left( q^2 - M_Z^2 \right) \left[ 0.49q^4 - 0.18 \frac{q^6}{q^2 - M_Z^2} \right] \right\} \]  

(10)

\[
\frac{d\sigma_5^{\text{UNP}}}{\sigma_5} = \frac{1}{M_Z^2[0.81q^4 + 0.06q^2(q^2 - M_Z^2) + (q^2 - M_Z^2)^2]} \times \]
\[ \left\{ \delta_s q^2 \left[ 2(q^2 - M_Z^2)^2 + 0.06q^2(q^2 - M_Z^2) \right] - \delta_z (q^2 - M_Z^2) \left[ 0.06q^2(q^2 - M_Z^2) + 1.62q^4 \right] \\
- \delta_s (q^2 - M_Z^2) \left[ 1.46q^2(q^2 - M_Z^2) + 0.89q^4 \right] \right\} \]

\[
\frac{d\sigma_{b}^{UNP}}{\sigma_b} = \frac{1}{M_Z^2[5.53(q^2 - M_Z^2)^2 + 0.59q^2(q^2 - M_Z^2) + 9q^4]} \times \left\{ \delta_s q^2 \left[ 5.53(q^2 - M_Z^2)^2 + 0.30q^2(q^2 - M_Z^2) \right] - \delta_z (q^2 - M_Z^2) \left[ 0.30q^2(q^2 - M_Z^2) + 9q^4 \right] \\
- \delta_s (q^2 - M_Z^2) \left[ 6.99q^2(q^2 - M_Z^2) + 3.48q^4 \right] \right\} \]

\[
\frac{dA_{FB,b}^{UNP}}{A_{FB,b}} = \frac{d\sigma_{FB,b}^{UNP}}{\sigma_{FB,b}} - \frac{d\sigma_b^{UNP}}{\sigma_b} \quad \text{(13)}
\]

where, in the various kinematical coefficients, the values of the various \(Z\) peak inputs that enter Eqs. (13-14) have been taken by the most recent experimental review [1].

In order to demonstrate the predictivity of the proposed parametrization of virtual NP effects, we used a set of preliminary results [11] from the LEP collaborations obtained with the largest available sample of data collected at a single center of mass energy, i.e. the data of the 1998 LEP run at \(\sqrt{s} = 189\) GeV. The most probable values of the \(\delta\) parameters are obtained from the minimization of a \(\chi^2\) defined in terms of the deviations of the present measurements with respect to the SM predictions and of the residual NP contributions to the theoretical expressions of the observables:

\[ \chi^2 = \sum_{i=1}^{N} \left( \frac{O_{i}^{\text{meas}} - O_{i}^{\text{SM}} - dO_{i}^{UNP}}{\sigma_i} \right)^2 \]

where the index \(i\) corresponds to a specific observable measured in the two fermion process and the theoretical deviations with respect to the SM are expressed as a function of the \(\delta\) parameters according to Eqs. (13-14). Since the experimental precision of the measurements determines the ultimate constraints on the UNP parameters, the present accuracy on \(R_b\) and \(A_{FB,b}\) makes their contribution to the overall \(\chi^2\) negligible with respect to the leptonic cross sections and asymmetries and to the \(q\bar{q}\) production. Therefore, although they will carry useful information in a final analysis, where the whole LEP2 data set can be exploited, we will not use them in the following discussions.

Fig.1 shows that the measurements of \(\sigma_\mu\), \(\sigma_\tau\), \(\sigma_5\), \(A_{FB,\mu}\) and \(A_{FB,\tau}\) from the four LEP experiments [11] lead to simultaneous determinations of the \(\delta\) parameters which appear statistically consistent among them and in agreement, within the experimental uncertainty, with the SM expectations. There is no need to stress here the relevance of a
combination of the results of the different experiments, which, accounting for the possible correlations of some sources of systematic experimental uncertainties, allows to improve the sensitivity of this kind of analysis to potential hints of NP. The results of a first combination \cite{12}, which is far from being trivial due to the different signal definitions adopted by each experiment, have been used in the following in order to obtain a determination of $\delta_z$, $\delta_s$, and $\delta_\gamma$ from the whole data sample collected at $\sqrt{s} = 189$ GeV. Taking in mind the premature nature of this combination and the preliminary results used as inputs, the resulting determination, shown in Fig.1 is mainly intended to evidentiate the size of the ultimate constraints on our general parameterization of UNP effects allowed by the use of the entire set of experimental results obtained at $\sqrt{s} = 189$ GeV, rather than to give the final bounds on the $\delta$ parameters.

The hypothesis of smooth $q^2$ dependence of the NP effects can be exploited to further improve these constraints with the measurements of the two fermions observables at other center of mass energies. Table 1 shows the size of the 95\% C.L. exclusion regions obtained by a simultaneous minimization of the $\chi^2$ with respect to the three parameters when, for each observable, the experimental precision of the combination \cite{12} of the data from the four LEP2 experiments at $\sqrt{s} = 189$ GeV and $\sqrt{s} = 183$ GeV is assumed. Since the sensitivity of the two fermion observables to $\delta_i$ increases in our approach with the $q^2$ of the process, further significant improvement of these constraints may be envisaged in the near future from the data collected at higher center of mass energies in the last two years of LEP2 operation.

Before attempting an estimate of the final combined LEP2 sensitivity to $\delta_i$, we illustrate a couple of examples of New Physics of typical scales $\Lambda$ larger than the electroweak one, whose residual virtual effects at LEP2 energies can be parameterized according to the scheme proposed in this paper. In such examples the specificity of the models will also entail a further reduction of the number of free parameters, thus allowing for more stringent constraints to be derived from the experimental data.

The first case that we considered was that of Anomalous Gauge Couplings (AGC). We used the framework of Ref. \cite{13} in which the effective Lagrangian is constructed with dimension six operators respecting $SU(2) \times U(1)$ and $CP$ invariance. As shown in Ref. \cite{14}, only two parameters ($f_{DW}$ and $f_{DB}$) survive in the $Z$-peak subtracted approach. The explicit expression of the UNP contribution to $\delta_z$, $\delta_s$ and $\delta_\gamma$ are

$$
\delta_z = 8\pi\alpha M_Z^2 \frac{f_{DW}}{\Lambda^2} \left( \frac{c_l^2}{s_l} f_{DW} + \frac{s_l^2}{c_l} f_{DB} \right),
$$

$$
\delta_s = 8\pi\alpha M_Z^2 \frac{c_l}{s_l} f_{DW} - \frac{s_l}{c_l} f_{DB},
$$

$$
\delta_\gamma = -8\pi\alpha M_Z^2 \left( f_{DW} + f_{DB} \right),
$$

They satisfy the linear constraint:

$$
\delta_z - \frac{1 - 2s_l^2}{s_l c_l} \delta_s + \delta_\gamma = 0.
$$
The second considered model was one of Technicolour type with two families of strongly coupled resonances \((V \text{ and } A)\) \[15\]. The typical UNP parameters are the two ratios \(F_A/M_A\) and \(F_V/M_V\) where \(F_{AV}\) and \(M_{AV}\) are the couplings and the masses (that in this case play the role of the new physics scale \(\Lambda_{TC} >> q^2\)) of the lightest axial and vector resonances. The contribution to \(\delta_z, \delta_s\) and \(\delta_\gamma\) are

\[
\delta_z = M^2_Z \frac{\pi \alpha}{\tilde{s}_l \tilde{c}_l} \left(1 - 2\tilde{s}_l^2\right)^2 \frac{F^2_V}{M^4_V} + \frac{F^2_A}{M^4_A},
\]

\[
\delta_s = M^2_Z \frac{2\pi \alpha}{\tilde{s}_l \tilde{c}_l} \left(1 - 2\tilde{s}_l^2\right) \frac{F^2_V}{M^4_V},
\]

\[
\delta_\gamma = -4\pi \alpha M^2_Z \frac{F^2_V}{M^4_V}.
\]

Again, we have a linear constraint in the \((\delta_z, \delta_s, \delta_\gamma)\) space:

\[
\delta_s = -\left(\frac{1 - 2\tilde{s}_l^2}{2\tilde{s}_l \tilde{c}_l}\right) \delta_\gamma,
\]

and the constraints

\[
\delta_{z,s} > 0 \quad \delta_\gamma < 0
\]

Fig. 2 shows, together with the projections on the three coordinate planes of the allowed three-dimensional region for the \(\delta\) parameters in the general case, the improved determination of \(\delta_z, \delta_s\) and \(\delta_\gamma\) achieved when the NP underlying the \(\delta\) parameters is assumed to be of AGC type. Technically, this hypothesis cuts the three-dimensional region, in which the \(\delta\) are likely to lie, with a plane corresponding to the linear constraint in eq. \[18\], thus effectively reducing the number of free parameters in the model to two. The data used here are \(\sigma_\mu, \sigma_\tau, \sigma_5, A_{FB,\mu}, A_{FB,\tau}\) measured by the four LEP experiments at \(\sqrt{s} = 189\) GeV. The different contributions of the leptonic and hadronic cross sections to the combined constraints in the case of the two free parameter fit (AGC model) is shown in the plot by the enlarged contours which correspond to the result of the \(\chi^2\) minimization when one of the two experimental contraints is released. Since the measurements of \(\sigma_5\) and \(\sigma_{\mu,\tau}\) provide in such model constraints almost mutually orthogonal, and since the relative experimental uncertainty affecting the lepton forward-backward asymmetries is larger than that in the cross section measurements, \(A_{FB,\mu,\tau}\) do not contribute significatively to the overall contraint. The situation is different in the case on NP effects originated by the TC model discussed above. The improved 1\(\sigma\) contours in the \(\delta_z - \delta_s\) and \(\delta_z - \delta_\gamma\) planes obtained when the TC model is considered are shown in Fig. 3 (the same experimental inputs used for the limits in Fig. 2 are used here). Although the statistical sensitivity of the test is not yet very stringent\[1\], one might incidentally notice that, since this specific model of NP requires \(\delta_z\) to be positive and \(\delta_\gamma\) negative, the determination of the \(\delta\) parameters from the data does not seem to favour such a model.

\[1\] The contour displayed in Fig. 3 for the TC model corresponds to a C.L. of 34\% for the values of two free parameters to fall simultaneously in the ellipse.
At this preliminary stage, we should note that, while no significative deviation from the SM prediction is observed, a systematic shift of the experimental results seems to appear in the three plans of pair of parameters $\delta_{z,s,\gamma}$, with definite correlation signs. Although this is not statistically significative, it shows how our representation may reveal some features which are not immediately visible on each observable separately. This is an illustration of how it may be possible to get some hints on NP properties.

The very successful operation of LEP in the recent years allows to foresee that the final expected precision [16] on the observables measured in two-fermion production processes will be reached and eventually overcome. Therefore, as a final work, we made the exercise of simulating the final precision on the $\delta$ parameters that might be conceivably achieved at the end of the LEP operation. With this aim, we assumed that the measurements of $\sigma_\mu$, $\sigma_\tau$, $\sigma_5$, $A_{FB_\mu}$ and $A_{FB_\tau}$ at $\sqrt{s} = 183$ GeV and $\sqrt{s} = 189$ GeV, with the experimental precision arising from the combination of the data from the four LEP experiments, can be complemented by four independent measurements of each variable with statistical precision corresponding to an integrated luminosity of 400 pb$^{-1}$ collected at an average center of mass energy of 200 GeV. No attempt of estimating the effects of systematic uncertainties, which might also be correlated between different measurements, experiments and center of mass energies, was made. Furthermore, since we just aim to estimate the final contribution of LEP2 to the constraints on the parameterization of general NP model, we assumed that the final combined results will not show deviations with respect to the SM.

The width of the 95% C.L. intervals on each parameter (as obtained in a simultaneous fit) are listed in Table 1 and the projections of the $\chi^2 < \chi^2_{\text{min}} + 1$ three-dimensional region are shown in Fig.4 and compared with the corresponding sensitivity derived from the data sets collected in the past and already analysed. The restricted domains allowed in the parameter space, when the specific models of NP corresponding to AGC and TC are assumed, are also displayed for comparison.

One notes from inspection of Table 1 that the available bound for the parameter $\delta_\gamma$ is more stringent than those for $\delta_z, \delta_s$. This is due to the fact that $\delta_\gamma$ contains the new physics effects on the photon exchange, that is largely dominant in the muon cross section. The remaining parameters $\delta_z, \delta_s$ contain the effect of the $Z$ exchange and of the $\gamma Z$ interference, and there is no unpolarized observable that privileges one of them. This would be the case of the longitudinal polarization asymmetry $A_{LR}$, that is drastically affected by the $\gamma Z$ interference and would therefore represent a special test for $\delta_s$.

A final remark that can be added is that Table 2, which summarizes the correlations between the $\delta$ parameters in the general three parameter fit, suggests that other observables, affected by different combinations of the NP relic $\delta_i$, might be useful to disentangle to contributions of the different effects in a final analysis. A promising candidate at LEP2 is the Bhabha scattering, to be considered in a forthcoming paper.

Our analysis of possible effects of new physics is, at this point, concluded. As explained in the Introduction, we have only considered models that are both $\theta$ independent and of universal "smooth" type, thus achieving remarkable simplifications, particularly in our $Z$ peak subtracted approach where the existence of privileged $q^2$ values ($q^2 = 0$ and $q^2 = M_Z^2$) can be usefully exploited (see Eqs. (6)-(8)) to reduce the number of parameters.
to a triplet of constants. There exist, though, interesting models of new physics that do not meet all the previous requests but nevertheless involve a very small number of parameters, so that the analysis of their effects could proceed in a simple way even in a ”standard” treatment. Using our approach, though, automatically takes into account one loop contributions (like e.g. $\Delta \rho$) already included in the used $Z$ peak inputs. This is why, in this final part, we have retained our approach and extended our analysis to the study of two simple models, that we list here following the order in which they violate our simplicity conditions.

a) Contact interactions.

The following interaction
\[ \mathcal{L} = \frac{G}{\Lambda^2} \bar{\Psi} \gamma^\mu (a_e - b_e \gamma^5) \Psi \bar{\Psi} \gamma^\mu (a_f - b_f \gamma^5) \Psi \]  
was first introduced with the idea of compositeness [17], but it applies to any virtual NP effect (for example higher vector boson exchanges) satisfying chirality conservation (Vector and Axial Lorentz structures) and whose effective scale $\Lambda$ is high enough so that one can restrict oneself to $dim = 6$ operators.

The parameters $a, b, a_f, b_f$ can be adjusted in order to describe all kinds of chiral couplings. For each choice of pair of chiralities among $L(a = b = 1/2)$, $R(a = -b = 1/2)$, $V(a = 1, b = 0)$, $A(a = 0, b = 1)$, there is only one free parameter.

These models are not of universal type, but retain the property of being $\theta$ independent. Their contribution to all observables can be computed in our approach, by straightforward ”projection” on the four $q^2$ dependent functions $\tilde{\Delta}_a$, $R$, $V^{Z\gamma}$, $V^{Z\gamma}$. By construction the model satisfies automatically the parametrization of eqs.(6)-(8), with three constants $\delta_z, \delta_{\gamma Z}$, $\delta_{\gamma}$ and one new constant $\delta_{Z\gamma}$ that takes into account its non universality, and have the expressions:

\[
\begin{align*}
\delta_{z,ef} &= -\left(\frac{GM^2_Z}{\Lambda^2}\right)\frac{4 \tilde{s}_e^2 \tilde{c}_f^2 b_e b_f}{e^2 I_{3e} I_{3f}} \\
\delta_{\gamma Z} &= -\left(\frac{GM^2_Z}{\Lambda^2}\right)\frac{4 \tilde{s}_e \tilde{c}_f (a_e - b_e \tilde{v}_f) b_f}{e^2 Q_e I_{3f}} \\
\delta_{Z\gamma} &= -\left(\frac{GM^2_Z}{\Lambda^2}\right)\frac{4 \tilde{s}_e \tilde{c}_f (a_f - b_f \tilde{v}_f) b_e}{e^2 Q_f I_{3e}} \\
\delta_{\gamma,ef} &= \left(\frac{GM^2_Z}{\Lambda^2}\right)\frac{(a_e - b_e \tilde{v}_f)(a_f - b_f \tilde{v}_f)}{e^2 Q_e Q_f}
\end{align*}
\]  

Note that the appearance of the non universal extra quantity $\delta_{Z\gamma}$ requires the use of the more complete Eqs. [1,2,3] with the four functions being expressed in terms of the $\delta_i$ through Eqs. [4]-[8].

b) Manifestations of extra dimensions.

Recently, an intense activity has been developed on possible low energy effects of graviton
exchange. The following matrix element for the 4-fermion process $e^+e^- \rightarrow \bar{f}f$ is predicted:

$$\frac{\lambda}{\Lambda^4}[\bar{e}\gamma^\mu e \bar{f}\gamma^\nu f(p_2 - p_1)(p_4 - p_3) - \bar{e}\gamma^\mu e \bar{f}\gamma^\nu f(p_2 - p_1)\nu(p_4 - p_3)\mu]$$  \hspace{1cm} (26)

This model can be formally treated in our formalism in a way that is analogous to that used in the previous case. One easily arrives, after a few steps, to the formal definitions:

$$\delta_{z,ef} = \left(\frac{\lambda M_Z^2 q^2}{\Lambda^4}\right)\frac{4\bar{s}_l\bar{c}_f}{e^2 I_{3e} I_{3f}}$$

$$\delta_{\gamma Z,ef} = \left(\frac{\lambda M_Z^2 q^2}{\Lambda^4}\right)\frac{2\bar{s}_l\bar{c}_f}{e^2 Q_e I_{3f}}$$

$$\delta_{Z\gamma,ef} = \left(\frac{\lambda M_Z^2 q^2}{\Lambda^4}\right)\frac{2\bar{s}_l\bar{c}_f}{e^2 Q_f I_{3e}}$$

$$\delta_{\gamma,ef} = \left(\frac{\lambda M_Z^2 q^2}{\Lambda^4}\right)(\bar{v}_l\bar{v}_f - 2\cos^2\theta)$$  \hspace{1cm} (27)

Comparing the above expressions with Eq. (25), one sees that a first difference is that now the four quantities $\delta_z, \delta_{Z\gamma,ef}, \delta_{\gamma Z,ef}, \delta_{\gamma,ef}$ are all proportional to $q^2$. This is simply a kinematical manifestation of the higher dimension of the Lagrangian, that shifts the lowest order coefficients of the $q^2$ expansion to the first order in $q^2$ without changing the philosophy of our approach.

The second difference is the appearance of a $\theta$ dependence in the modification of the $\gamma$-coupling, as shown by Eq. (27). This cannot be reabsorbed or eliminated, and forces one to restart from the more general Eqs. (12,13) performing the full $\cos \vartheta$ integration.

In the final Table 3 we have shown the present and (optimistic) future LEP2 bounds on the two considered (a, b) models. We have verified that a substantial agreement exists with the available “standard” calculations of the present bounds recently performed by LEP collaborations [19]. This is particularly encouraging since our approach is of a totally independent nature.

A final comment is related to the weight of the different measurements in the two considered cases (a,b). In the first model, the situation that appears is essentially the same as in the universal examples that we have considered, in the sense that the bulk of the information is provided by the two cross sections, with $\sigma_\mu$ still playing a major role. A totally different picture appears in the case of extra dimensions. Here, all the information is provided by the muon asymmetry. The reason is not difficult to understand: the relevant parameter $\delta_z$ is now proportional (neglecting the small $\bar{v}_l^2$ term ) to $\cos \theta$. As a consequence of this fact, it only contributes $A_{FB,\mu}$ and not $\sigma_{\mu,5}$. This shows that, at least in the specific case of one interesting model, the role of the forward-backward muon asymmetry can become essential.

In conclusion, we have proposed a simple parametrization of the virtual effects of a class of models of new physics in the present and future four-fermion processes. This is
particularly suited to treat the case of effects of $\theta$-independent, smooth, universal type, but can be easily extended to some cases of models that do not meet all previous requests, by means of simple numerical programs that exist and are available.


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Figure Captions

**Fig.1**: Projections of the region in the $\delta_z, \delta_s, \delta_{\gamma}$ parameter space defined by the condition $\chi^2 < \chi^2_{\text{min}} + 1$. The experimental data used in the fit are the cross sections for quark, muon and tau pair production and the forward-backward asymmetries of muon and tau pairs. The preliminary measurements, presented by the LEP collaborations at the EPS-HEP’99, obtained at $\sqrt{s} = 189$ GeV have been used. The combination of the results of the four experiments (shaded area) is shown to sensibly improve the precision of the determination of the $\delta$ parameters.

**Fig.2**: Projections of the $\chi^2 < \chi^2_{\text{min}} + 1$ region in the $\delta_z, \delta_s, \delta_{\gamma}$ parameter space obtained from the combined LEP results for $\sigma_{\mu}, \sigma_{\tau}, \sigma_5, A_{\text{FB}\mu}$ and $A_{\text{FB}\tau}$ at $\sqrt{s} = 189$ GeV. The small ellipses represent the projections on each plane of the intersection between the three dimensional “allowed region for the $\delta$ parameters” and the plain corresponding to the linear constrain between them, arising from the AGC model discussed in the text.

**Fig.3**: The reduction of the allowed parameter space for the $\delta$ parameters in the hypothesis of virtual effects driven by Technicolor type of New Physics is shown here. The definition of the projected ellipses and the experimental constraints applied in the $\chi^2$ are as in Fig. (2).

**Fig.4**: Envisaged final constraints on the $\delta$ parameters from the whole LEP data set. The contours, corresponding to $\chi^2 = \chi^2_{\text{min}} + 1$, are derived by assuming that the present available measurements of $\sigma_{\mu}, \sigma_{\tau}, \sigma_5, A_{\text{FB}\mu}$ and $A_{\text{FB}\tau}$ at 183 GeV and 189 GeV might be complemented by analogous measurements from a total integrated luminosity of 400 pb$^{-1}$ collected at an average center of mass energy of 200 GeV. Statistical uncertainties only have been assessed on the future measurements and agreement of the final combined results with respect to the Standard Model predictions has been assumed. For comparison the sensitivity achieved with the currently available data is shown. The light and dark shaded areas represent the final constraints in the AGC and TC models.
Table 1: 95% C.L. bounds on $\delta_z$, $\delta_s$ and $\delta_\gamma$ from the combined three free parameter fit of the general theoretical expressions for $\sigma_\mu$, $\sigma_\tau$, $\sigma_5$, $A_{FB\mu}$ and $A_{FB\tau}$ to the present and future measurements of fermion pair production at LEP2. The 95% C.L. bounds obtained from a two free parameters fit to to the specific models of AGC and TC, are also reported.

| Model | DATA | $\delta_z$ | $\delta_s$ | $\delta_\gamma$ |
|-------|------|------------|-------------|-----------------|
| 3 free par.s | $\sqrt{s} = 189$ GeV | $-0.0027^{+0.0036}_{-0.0030}$ | $-0.0020^{+0.0031}_{-0.0031}$ | $-0.0026^{+0.0094}_{-0.0094}$ |
|          | $\sqrt{s} = 183 - 189$ GeV | $-0.0011^{+0.0034}_{-0.0031}$ | $-0.0033^{+0.0037}_{-0.0027}$ | $-0.0022^{+0.0081}_{-0.0081}$ |
|          | Final LEP2 | $\pm 0.016$ | $\pm 0.014$ | $\pm 0.0043$ |
| AGC | $\sqrt{s} = 189$ GeV | $-0.0014^{+0.0037}_{-0.0030}$ | $-0.0031^{+0.0074}_{-0.0074}$ | $-0.0026^{+0.0082}_{-0.0082}$ |
|          | $\sqrt{s} = 183 - 189$ GeV | $-0.0015^{+0.0032}_{-0.0032}$ | $-0.0029^{+0.0064}_{-0.0064}$ | $-0.0022^{+0.0071}_{-0.0071}$ |
|          | Final LEP2 | $\pm 0.0016$ | $\pm 0.0033$ | $\pm 0.0037$ |
| TC | $\sqrt{s} = 189$ GeV | $-0.0061^{+0.0015}_{-0.0015}$ | $0.0014^{+0.0047}_{-0.0047}$ | $-0.0021^{+0.0075}_{-0.0075}$ |
|          | $\sqrt{s} = 183 - 189$ GeV | $-0.0055^{+0.0013}_{-0.0013}$ | $0.0010^{+0.0044}_{-0.0044}$ | $-0.0016^{+0.0064}_{-0.0064}$ |
|          | Final LEP2 | $\pm 0.0066$ | $\pm 0.0021$ | $\pm 0.0034$ |

Table 2: Correlations between the best fit values of the $\delta$ parameters obtained using the experimental constraints from $\sigma_\mu$, $\sigma_\tau$, $\sigma_5$, $A_{FB\mu}$ and $A_{FB\tau}$.

|          | $\delta_z$ | $\delta_s$ | $\delta_\gamma$ |
|----------|------------|-------------|-----------------|
| 3 free par.s | $\delta_z$ | 1.00 | -0.94 | -0.01 |
|          | $\delta_s$ | -0.94 | 1.00 | 0.28 |
|          | $\delta_\gamma$ | -0.01 | 0.28 | 1.00 |
| AGC | $\delta_z$ | 1.00 | 0.47 | |
|          | $\delta_s$ | 0.47 | 1.00 | |
| AGC | $\delta_z$ | 1.00 | - | 0.12 |
|          | $\delta_\gamma$ | 0.12 | - | 1.00 |
| AGC | $\delta_s$ | - | 1.00 | 0.93 |
|          | $\delta_\gamma$ | - | 0.93 | 1.00 |
| TC | $\delta_z$ | 1.00 | -0.89 | |
|          | $\delta_s$ | -0.89 | 1.00 | |
| TC | $\delta_z$ | 1.00 | - | 0.89 |
|          | $\delta_\gamma$ | 0.89 | - | 1.00 |
Table 3: The following two tables show respectively the present and (optimistic) future LEP2 95\% C.L. bounds on the models \((a,b)\) as provided by using all measurements \((\sigma_l, \sigma_5\) and \(A_{FB,l})\) and by releasing one of the measurements.

| \(\Lambda_{CT} \) (TeV) | All   | no \(\sigma_l\) | no \(\sigma_5\) | no \(A_{FB,l}\) |
|-------------------------|-------|-----------------|-----------------|-----------------|
| LL                      | 2.9   | 1.8             | 2.8             | 2.9             |
| RR                      | 2.7   | 1.6             | 2.7             | 2.7             |
| VV                      | 4.7   | 2.7             | 4.6             | 4.6             |
| AA                      | 4.1   | 3.8             | 4.0             | 3.3             |

| \(\Lambda_{ED} \) (TeV) | All   | no \(\sigma_l\) | no \(\sigma_5\) | no \(A_{FB,l}\) |
|-------------------------|-------|-----------------|-----------------|-----------------|
| LL                      | 4.0   | 2.5             | 3.9             | 3.9             |
| RR                      | 3.7   | 2.1             | 3.7             | 3.7             |
| VV                      | 6.4   | 3.6             | 6.3             | 6.3             |
| AA                      | 5.5   | 5.0             | 5.2             | 4.7             |

| \(\Lambda_{CT} \) (TeV) | All   | no \(\sigma_l\) | no \(\sigma_5\) | no \(A_{FB,l}\) |
|-------------------------|-------|-----------------|-----------------|-----------------|
| LL                      | 0.78  | 0.78            | 0.78            | 0.25            |
| RR                      | 0.78  | 0.78            | 0.78            | 0.25            |
| VV                      | 0.78  | 0.78            | 0.78            | 0.25            |
| AA                      | 0.78  | 0.78            | 0.78            | 0.25            |
Figure 1:
Figure 2:
Figure 3:
Figure 4: