Partial Relay Selection for Secure Cooperative NOMA Networks

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Abstract In this paper, we investigate physical layer security of multi-relay non-orthogonal multiple access (NOMA) networks with partial relay selection considering decode-and-forward (DF) and amplify-and-forward (AF) protocols. We propose a partial relay scheme aiming to select the best relay based on the highest signal-to-noise-ratio (SNR) of the first link. We derive new exact and asymptotic expressions for strictly positive secrecy capacity (SPSC) and secrecy outage probability (SOP) considering Rayleigh fading channels. Numerical results demonstrate that AF and DF provide almost a similar secrecy performance. Moreover, they prove that partial relay selection improves SPSC and reduces SOP when the relay-cluster is closer to the legitimate receiver.

Keywords non-orthogonal multiple access (NOMA) · amplify-and-forward · decode-and-forward · strictly positive secrecy capacity · secrecy outage probability

1 Introduction

As a promising solution to enhance the spectral efficiency, the non-orthogonal multiple access (NOMA) has received a great interest in the wireless communication networks [1]. In NOMA transmission network, the superposition encoding is employed at the transmitter and successive interference cancellation (SIC) is implemented at the receivers [2]. Compared to conventional orthogonal multiple access (OMA) schemes, NOMA can share the same time and frequency resources among multiple users [3]. On the other hand, cooperative communication is an effective way to deal with the multipath propagation.
as well as ensure higher reliability for communication networks. The core concept is to integrate intermediate cooperating nodes between the source side and users side. Recently, cooperative communication is widely adopted in NOMA networks [4–10].

1.1 Related Work

In [4], the authors introduced a basic cooperative NOMA network in which the user with strong channel condition acts as an AF relay to transmit the signal to the user with poor channels condition. In [11, 12], the authors investigated a cooperative NOMA system in which the nearby users with better channel conditions are considered as a DF relay to improve the system reliability. In [5, 13, 14], cooperative NOMA networks with a single AF relay node were considered. Some performance metrics were determined such as outage probability and ergodic sum rate. The authors of [14] proposed a cooperative NOMA transmission system with one AF relay considering Nakagami-m fading channels. The authors of [15] addressed a NOMA relaying system with one DF relay node. They derived closed-form expressions for ergodic sum capacity and outage probability.

All these studies only considered one relay node. To obtain higher diversity gain, multiple relays were employed in cooperative NOMA networks [16–20]. In [16], cooperative NOMA networks with multiple DF relays were investigated considering Rayleigh fading channels. The authors of [17] proposed a relay selection scheme for cooperative NOMA systems with multiple AF relays. The authors of [19] and [21] considered a two-stage relay selection scenario with dynamic power allocation and fixed power allocation, respectively.

In the above mentioned work, the security issues of wireless networks are not considered. Because of the open nature of wireless environments, confidential message is vulnerable to be eavesdropped [22].

To improve the security of cooperative NOMA networks, physical layer security technology was proposed [23]. In this context, the authors of [24] investigated the physical layer security for cooperative NOMA networks with a single relay using both AF and DF protocols. Analytical expressions for SOP and SPSC were provided considering Rayleigh fading channels. The authors of [25] studied the secrecy performance of cooperative NOMA scheme with one AF relay under Rician fading channels. Asymptotic and exact expressions for SOP and SPSC were derived. In [26], a secure cooperative NOMA system with one relay was evaluated considering both AF and DF relaying protocols via Nakagami-m fading channels. The authors determined closed-form expressions for SPSC.

All the above mentioned contributions considered physical layer security for a cooperative NOMA system with a single relay. However, it is worth mentioning that to the best of the authors’ knowledge there is no published work concerning the secrecy performance of cooperative NOMA systems with multiple relays considering partial relay selection scheme and using both AF and DF relaying protocols.
1.2 Contributions

In this paper, we extend the study given in [24] to a secure cooperative NOMA system with multiple relays considering partial relay selection scenario. The main contributions of this paper are detailed as follows.

- We consider secure cooperative NOMA networks including one base station, two legitimate users, one eavesdropper and multiple relays. A partial relay selection method is proposed to select the best relay. In particular, the relay which has the highest received SNR will be selected to forward the information signal to the receiver nodes. The partial relay selection scheme can significantly increase the secrecy performance.

- We derive exact and asymptotic expressions for secrecy outage probability and strictly positive secrecy capacity considering both AF and DF relaying protocols.

- Numerical results show that the increase of number of relays considerably enhances the secrecy performance of both NOMA-DF and NOMA-AF relaying networks with partial relay selection. However, using more relays does not have a great impact on the secrecy performance for high average SNR of illegal link.

- Results also reveal that NOMA-AF and NOMA-DF networks with partial relay selection present a comparable secrecy performance at high SNR regime. Furthermore, the asymptotic results show that the secrecy performance does not change when employing several relays.

The rest of the paper will be structured as follows. The system model will be introduced in section 2. Exact and asymptotic expressions for strictly positive secrecy capacity of NOMA-AF and NOMA-DF systems with partial relay selection will be presented in section 3 and section 4, respectively. Exact and asymptotic expressions for secrecy outage probability of NOMA-AF and NOMA-DF networks with partial relay selection will be presented in section 5 and section 6, respectively. Section 7 will present various numerical results. Finally, section 8 will present the conclusion the paper.

2 System Model

As shown in Fig. 1, we consider a secure cooperative NOMA network including one base station (BS), multiple untrusted AF/DF relays \( \{R_k\}_{k=1}^M \), and two legitimate users, i.e., strong user (\( U_1 \)) and poor user (\( U_2 \)), and an eavesdropper (\( E \)). Moreover, the DF and AF protocols are employed at each relay and only one relay is chosen to transmit the messages to the users based on the maximum SNR of the first hop. As such, the eavesdropper \( E \) wants to intercept the information via decoding the received signal from the selected relay. In this system, we assume that the BS is placed at the origin of cell, and the users/eavesdropper are close to the cell edge. Hence, the transmission from BS to users is performed via a selected relay. We further assume that there are
no direct paths between $BS$ and both users/eavesdropper due to deep fading. Furthermore, all nodes in the system have single antennas and all the channels in the system are considered undergo quasi-static independent and identically Rayleigh fading.

During the first time slot, the BS forwards a superposed signal $\lambda_1 x_1 + \lambda_2 x_2$ to all untrusted relays where $x_i (i = 1, 2)$ stands for the unit power signal received by user $i$ and $\lambda_i (i = 1, 2)$ denotes the power allocation coefficient. We consider that $\lambda_1 \geq \lambda_2$ and $\lambda_1^2 + \lambda_2^2 = 1$ in order to provide better fairness and QoS requirements between the users [27]. Therefore, the received signal at the relay $R_k$ can be expressed as

$$y_{r_k} = \lambda_1 x_1 h_{r_k} \sqrt{P_s} + \lambda_2 x_2 h_{r_k} \sqrt{P_s} + n_{r_k}$$  \hspace{1cm} (1)$$

where $h_{r_k}$ denotes the channel gain between the base station and the $k^{th}$ relay and $n_{r_k}$ stands for the additive Gaussian noise (AWGN) with zero mean and variance $N_0$. We employ a partial relay selection based on the maximum SNR of the first link. Indeed, the relay with the highest received SNR is selected to transmit the information in the second hop. Hence, a partial selection criterion is adopted according to the following rule:

$$\gamma_j = \max_{k=1,...,M} (\gamma_{r_k})$$  \hspace{1cm} (2)$$
In order to determine the effects of the relaying protocols on the performance of secure NOMA networks with partial relay selection, the transmission schemes for both AF and DF are presented as the following subsections.

2.1 Amplify-and-Forward

During the second time slot, the selected relay transmits the amplified signal to both users. The received signal at the user \( i \) can be written as

\[
y_{r_j,i}^{AF} = G_j r_{r_j,i} + n_{r_j,i} \tag{3}
\]

where \( G_j \) denotes the amplifying gain defined as

\[
G_j = \sqrt{\frac{P_r}{\|h_{r_j}\|^2 + N_0}} \tag{28},
\]

\( h_{r_j,i} \) is the channel gain between the selected relay and user \( U_i \), \( n_{r_j,i} \) stands for the AWGN with zero mean and variance \( N_0 \) and \( P_r \) represents the transmit relay power. Assuming that \( P_r = P_s \), \( G_j = \frac{P_r}{\|h_{r_j}\|^2 + \frac{1}{\epsilon}} \) where \( \epsilon = \frac{P_s}{N_0} \) denotes the average SNR of legal links. \( h_{R_{j,i}} \) stands for the small-scale fading coefficient of the channel between the selected relay and user \( i \). The eavesdropper can overhear the relay signal due to the broadcast nature of wireless communications. Hence, the signal received by \( E \) can be expressed as

\[
y_{r_j,e}^{AF} = G_j r_{r_j,e} + n_{r_j,e} \tag{4}
\]

Where \( h_{r_j,e} \) is the channel gain between the selected relay and eavesdropper and \( n_{r_j,e} \) represents an AWGN with variance \( N_e \). The user \( i \) performs a successive interference cancellation (SIC) to detect its own signal \( x_i \) \([29, 30]\).

Therefore, the channel capacity from the chosen AF relay to user \( i \) can be written as

\[
C_{r_j,i}^{AF} = \frac{1}{2} \log_2 (1 + \Gamma_{r_j,i}^{AF}) \tag{5}
\]

where \( \Gamma_{r_j,i}^{AF} = \frac{|h_{r_j,i}|^2|\hat{h}_{r_j,i}|^2 \lambda_1^2}{\|h_{r_j,i}\|^2 + \|\hat{h}_{r_j,i}\|^2 + \frac{\lambda_1^2}{\epsilon}} \) represents the instantaneous signal-to-interference-pulse-noise ratio (SINR) at \( U_1 \) and \( \Gamma_{r_j,i}^{AF} = \frac{\epsilon |h_{r_j,e}|^2 |\hat{h}_{r_j,e}|^2 \lambda_2^2}{\|h_{r_j,e}\|^2 + \|\hat{h}_{r_j,e}\|^2 + \frac{\lambda_2^2}{\epsilon}} \) is the received SNR at user \( U_2 \).

We consider that the eavesdropper \( E \) has the multiuser detection ability \([30, 31]\). Specifically, a parallel interference cancellation (PIC) technology is employed by the eavesdropper to decode the signal of each legal user. Thus, the channel capacity from the selected relay to eavesdropper can be given as

\[
C_{e,i}^{AF} = \frac{1}{2} \log_2 (1 + \Gamma_{e,i}^{AF}) \tag{6}
\]

where \( \Gamma_{e,i}^{AF} = \frac{\alpha_e |h_{r_j,e}|^2 |\hat{h}_{r_j,e}|^2 \lambda_e^2}{\|h_{r_j,e}\|^2 + \|\hat{h}_{r_j,e}\|^2 + \frac{\lambda_e^2}{\epsilon_e}} \) is the received SNR by \( E \) and \( \epsilon_e = \frac{P_{r_e}}{N_e} \) stands for the average SNR of the illegal link. The secrecy rate of the NOMA-AF networks for user \( i \) can be defined as the difference between the secrecy rate
of the main channel and that of the eavesdropper channel. It can be written using (5) and (6) as

\[ C_{AF}^i = [C_{r_j,i}^{AF} - C_{e,i}^{AF}]^+ \]  

(7)

where \([x]^+ = \max(x, 0)\).

2.2 Decode-and-Forward

The selected relay decodes the received signal and then sends the re-encoded signal to users and eavesdroppers. The signals received by \(U_i\) and \(E\) can be given as

\[ y_{DF}^{r_j,\alpha} = h_{r_j,\alpha}(\lambda_1 x_1 + \lambda_2 x_2)\sqrt{P_s} + n_{r_j,\alpha} \]  

(8)

where \(\alpha \in \{i, e\}\).

The channel capacity of secure NOMA-DF relaying networks with partial relay selection is \(\min\{C_{bs,r_j}, C_{r_j,ui}\}\), where \(C_{bs,r_j}\) and \(C_{r_j,ui}\) denote the transmission capacity from base station to selected-relay and selected-relay to user \(U_i\), respectively. Hence, the capacity of the main channels can be written as

\[ C_{bs,ui}^{DF,i} = \frac{1}{2} \log_2(1 + \min\{\epsilon|h_{r_j}|^2 \lambda_1^2, \epsilon|h_{r_j,1}|^2 \lambda_2^2 \}) \]  

(9)

and

\[ C_{bs,ui}^{DF,j} = \frac{1}{2} \log_2(1 + \min\{\epsilon|h_{r_j}|^2 \lambda_2^2, \epsilon|h_{r_j,2}|^2 \lambda_2^2 \}) \]  

(10)

On the other hand, the capacity of the eavesdropper link can be written as

\[ C_{r_j,e}^{DF,i} = \frac{1}{2} \log_2(1 + \epsilon|h_{r_j,e}|^2 \lambda_2^2) \]  

(11)

Thus, the secrecy capacity of NOMA-DF relaying networks with partial relay selection considering Rayleigh fading channels can be given as

\[ C_i^{DF} = [C_{bs,ui}^{DF,i} - C_{r_j,e}^{DF,i}]^+ \]  

(12)

3 Strictly Positive Secrecy Capacity

In this section, we evaluate the secrecy performance of NOMA-AF and NOMA-DF networks with partial relay selection in terms of exact and closed-form expressions of SPSC.
3.1 Amplify-and-Forward

The strictly positive secrecy capacity for NOMA-AF relaying network with partial relay selection over Rayleigh fading channels can be written as

\[ \text{SPSC}^{AF} = \mathcal{P}(C_1^{AF} > 0, C_2^{AF} > 0) \]  

(13)

\( C_1^{AF} \) and \( C_2^{AF} \) are respectively expressed as

\[ C_1^{AF} = \frac{1 + \Gamma_{r,j}^{AF}}{\Gamma_{e}^{AF}} \]  

(14)

and

\[ C_2^{AF} = \frac{1 + \Gamma_{r,j}^{AF}}{\Gamma_{e}^{AF}} \]  

(15)

According to (14) and (15), \( \text{SPSC}^{AF} \) can be re-written as

\[ \text{SPSC}^{AF} = \mathcal{P}(\Gamma_{r,j}^{AF} > \Gamma_{e,1}^{AF}, \Gamma_{r,j}^{AF} > \Gamma_{e,2}^{AF}) \]  

(16)

The \( \Gamma_{r,j}^{AF} \), \( \Gamma_{e,1}^{AF} \) and \( \Gamma_{e,i}^{AF} (i = 1, 2) \) can be approximated for a high SNR regime as

\[ \Gamma_{r,j}^{AF} < \frac{\lambda_2^2}{\lambda_2^2} \]  

(17)

,\n
\[ \Gamma_{r,j}^{AF} < \frac{\epsilon |h_{r_j}|^2 |h_{r_j,2}|^2 \lambda_2^2}{\epsilon |h_{r_j}|^2 + \epsilon |h_{r_j,2}|^2} \]  

(18)

and

\[ \Gamma_{e,i}^{AF} < \frac{\epsilon \epsilon_e |h_{r_j}|^2 |h_{r_j,e}|^2 \lambda_2^2}{\epsilon |h_{r_j}|^2 + \epsilon_e |h_{r_j,e}|^2} \]  

(19)

respectively. The equations (18) and (19) can be written using the inequality \( xy/(x + y) \leq \min\{x, y\} \) as

\[ \Gamma_{r,j}^{AF} < \frac{\lambda_2^2 \min\{\epsilon |h_{r_j}|^2, \epsilon |h_{r_j,2}|^2\}}{\epsilon |h_{r_j}|^2 + \epsilon |h_{r_j,2}|^2} \]  

(20)

and

\[ \Gamma_{e,i}^{AF} < \frac{\lambda_2^2 \min\{\epsilon |h_{r_j}|^2, \epsilon_e |h_{r_j,e}|^2\}}{\epsilon |h_{r_j}|^2 + \epsilon_e |h_{r_j,e}|^2} \]  

(21)

respectively.

Therefore, \( \text{SPSC}^{AF} \) can be expressed as

\[ \text{SPSC}^{AF} = \mathcal{P}\left(\frac{1}{\epsilon_2} > \min(\epsilon |h_{r_j}|^2, \epsilon_e |h_{r_j,e}|^2), \min\{\epsilon |h_{r_j}|^2, \epsilon |h_{r_j,2}|^2\}\right) \]  

(22)

In order to simplify the mathematical analysis, we employ the equality \( \mathcal{P}({\tau_1}, {\tau_2}) = \mathcal{P}({\tau_1}) - \mathcal{P}({\tau_1}, {\tau_2}) \), such that \( \tau_2 \) stands for the complementary
event of $\tau_2$, $\tau_1 = \min\{\epsilon|h_{r_j}|^2, \epsilon|h_{r_j,\varepsilon}|^2\} > \min\{\epsilon|h_{r_j}|^2, \epsilon|h_{r_j,\varepsilon}|^2\}$ and $\tau_2 = \min\{\epsilon|h_{r_j}|^2, \epsilon|h_{r_j,\varepsilon}|^2\} < \frac{1}{\lambda_2^2}$.

Hence, $\mathcal{SPSC}^{AF}$ can be given as

$$\mathcal{SPSC}^{AF} = \mathcal{P}(\min\{\epsilon|h_{r_j}|^2, \epsilon|h_{r_j,\varepsilon}|^2\} > \min\{\epsilon|h_{r_j}|^2, \epsilon|h_{r_j,\varepsilon}|^2\})$$

- $\mathcal{P}(\min\{\epsilon|h_{r_j}|^2, \epsilon|h_{r_j,\varepsilon}|^2\} > \min\{\epsilon|h_{r_j}|^2, \epsilon|h_{r_j,\varepsilon}|^2\}, \min\{\epsilon|h_{r_j}|^2, \epsilon|h_{r_j,\varepsilon}|^2\} > \frac{1}{\lambda_2^2})$

We define $W_1 = \mathcal{P}(\min\{\epsilon|h_{r_j}|^2, \epsilon|h_{r_j,\varepsilon}|^2\} > \min\{\epsilon|h_{r_j}|^2, \epsilon|h_{r_j,\varepsilon}|^2\})$ and $W_2 = \mathcal{P}(\min\{\epsilon|h_{r_j}|^2, \epsilon|h_{r_j,\varepsilon}|^2\} > \min\{\epsilon|h_{r_j}|^2, \epsilon|h_{r_j,\varepsilon}|^2\}, \min\{\epsilon|h_{r_j}|^2, \epsilon|h_{r_j,\varepsilon}|^2\} > \frac{1}{\lambda_2^2})$.

$W_1$ can be re-written as

$$W_1 = \mathcal{P}(\min\{\epsilon|h_{r_j}|^2, \epsilon|h_{r_j,\varepsilon}|^2\} > \epsilon|h_{r_j,\varepsilon}|^2)$$

$$= \mathcal{P}(|h_{r_j}|^2 > \frac{\epsilon}{\epsilon}|h_{r_j,\varepsilon}|^2, |h_{r_j,\varepsilon}|^2 > \frac{\epsilon}{\epsilon}|h_{r_j,\varepsilon}|^2)$$

$$= \int_0^\infty \sum_{k=1}^M \left(\frac{M}{k}\right) (-1)^{k-1} \exp \left(-x\epsilon e \left(\frac{k\beta_0 + \beta_2}{\epsilon}\right)\right) \beta_c \exp \left(-x\beta_c\right) dx$$

$$= \sum_{k=1}^M \left(\frac{M}{k}\right) (-1)^{k-1} \beta_c \int_0^\infty \exp \left(-x\epsilon e \left(\frac{k\beta_0 + \beta_2}{\epsilon}\right)\right) \exp \left(-x\beta_c\right) dx$$

(24)

where $\beta_0$, $\beta_2$ and $\beta_c$ are respectively the Rayleigh channel parameters corresponding to $|h_{r_j}|^2$, $|h_{r_j,\varepsilon}|^2$ and $|h_{r_j,\varepsilon}|^2$. $W_1$ can be rewritten after calculating the above integral as

$$W_1 = \sum_{k=1}^M \left(\frac{M}{k}\right) (-1)^{k-1} \frac{\beta_c}{\frac{\epsilon\beta_0 k}{\epsilon} + \frac{\epsilon\beta_2}{\epsilon} + \beta_c}$$

(25)

$W_2$ can be given as

$$W_2 = \mathcal{P}(\min\{\epsilon|h_{r_j}|^2, \epsilon|h_{r_j,\varepsilon}|^2\} > \epsilon|h_{r_j,\varepsilon}|^2, \epsilon|h_{r_j,\varepsilon}|^2 > \frac{1}{\lambda_2^2})$$

$$= \mathcal{P}(|h_{r_j}|^2 > \frac{\epsilon}{\epsilon}|h_{r_j,\varepsilon}|^2, |h_{r_j,\varepsilon}|^2 > \frac{\epsilon}{\epsilon}|h_{r_j,\varepsilon}|^2, |h_{r_j,\varepsilon}|^2 > \frac{1}{\epsilon\lambda_2^2})$$

(26)

$W_2$ can be written considering Rayleigh fading channels as

$$W_2 = \int_0^\infty \frac{1}{\epsilon\lambda_2^2} \sum_{k=1}^M \left(\frac{M}{k}\right) (-1)^{k-1} \beta_c \exp \left(-x\epsilon e \left(\frac{k\beta_0 + \beta_2}{\epsilon}\right)\right) \exp \left(-x\beta_c\right) dx$$

(27)
After calculating the above integral, $W_2$ can be given as

$$W_2 = \sum_{k=1}^{M} \binom{M}{k} (-1)^{k-1} \frac{\beta_x}{\epsilon k \beta_0 + \epsilon \beta_2} \exp \left( -\frac{k \beta_0}{\epsilon \lambda_2^2} - \frac{\beta_2}{\epsilon \lambda_2^2} - \frac{\beta_x}{\epsilon \lambda_2^2} \right)$$

(28)

The strictly positive secrecy capacity for NOMA-AF relaying network with partial relay selection over Rayleigh fading channels can be expressed using (25) and (28) as

$$SPSC^{AF} = W_1 - W_2$$

$$= \sum_{k=1}^{M} \binom{M}{k} (-1)^{k-1} \frac{\beta_x}{\epsilon k \beta_0 + \epsilon \beta_2} \left[ 1 - \exp \left( -\frac{k \beta_0}{\epsilon \lambda_2^2} - \frac{\beta_2}{\epsilon \lambda_2^2} - \frac{\beta_x}{\epsilon \lambda_2^2} \right) \right]$$

(29)

3.2 Decode-and-Forward

The strictly positive secrecy capacity for NOMA-DF relaying network with partial relay selection considering Rayleigh fading channels can be written as

$$SPSC^{DF} = P(\epsilon |h_{r,j}|^2 \lambda_2^2 < 1, \min\{|h_{r,j}|^2, |h_{r,j,2}|^2\} > \frac{\epsilon}{\epsilon} |h_{r,j,2}|^2)$$

$$= P(|h_{r,j,2}|^2 < \frac{1}{\epsilon \lambda_2^2}, |h_{r,j}|^2 > \frac{\epsilon}{\epsilon} |h_{r,j,2}|^2, |h_{r,j,2}|^2 > \frac{\epsilon}{\epsilon} |h_{r,j,2}|^2)$$

(30)

$SPSC^{DF}$ can be given as

$$SPSC^{DF} = \sum_{k=1}^{M} \binom{M}{k} (-1)^{k-1} \int_{0}^{1} \frac{1}{\epsilon \lambda_2^2} \beta_x \exp \left( -\epsilon \beta_x \left( \frac{k \beta_0}{\epsilon} + \frac{\beta_2}{\epsilon} \right) \right) \exp (-\beta_x x) dx$$

(31)

The strictly positive secrecy capacity for NOMA-DF relaying system with partial relay selection via Rayleigh fading channels can be written after resolving the above integral as

$$SPSC^{DF} = \sum_{k=1}^{M} \binom{M}{k} (-1)^{k-1} \frac{\beta_x}{\epsilon k \beta_0 + \epsilon \beta_2} \left[ 1 - \exp \left( -\frac{k \beta_0}{\epsilon \lambda_2^2} - \frac{\beta_2}{\epsilon \lambda_2^2} - \frac{\beta_x}{\epsilon \lambda_2^2} \right) \right]$$

(32)
4 Asymptotic Strictly Positive Secrecy Capacity

4.1 Amplify-and-Forward

$\text{SPSC}^{AF}$ given in (29) can be written for high values of SNR ($\epsilon \to \infty$) as

\[
\text{SPSC}^{AF} = 1 - \exp\left(-\frac{\beta_\epsilon}{\epsilon_\epsilon_2}\right)
\]

(33)

4.2 Decode-and-Forward

For high SNR regime ($\epsilon \to \infty$), the strictly positive secrecy capacity can be approximated according to (32) as

\[
\text{SPSC}^{DF} = 1 - \exp\left(-\frac{\beta_\epsilon}{\epsilon_\epsilon_2}\right)
\]

(34)

5 Secrecy Outage Probability

In this section, we determine new closed-form expressions for secrecy outage probability of both NOMA-AF and NOMA-DF relaying systems with partial relay selection under Rayleigh fading channels.

5.1 Amplify-and-Forward

The secrecy outage probability for NOMA-AF network with partial relay selection can be written as

\[
\text{SOP}^{AF} = \mathcal{P}(C_1^{AF} < R_1 \text{ or } C_2^{AF} < R_2)
= 1 - \mathcal{P}\left(\frac{1 + \Gamma_1^{AF}}{1 + \Gamma_2^{AF}} > C_{th}^1, \frac{1 + \Gamma_2^{AF}}{1 + \Gamma_2^{AF}} > C_{th}^2\right)
= 1 - P_1
\]

(35)

where $R_i$ stands for the target data rate for user $i$ and $C_{th}^i = 2^{2^{R_i}}$. For high SNR regime, $P_1$ can be given as

\[
P_1 < \mathcal{P}\left(\frac{\epsilon e_\epsilon_1 |h_{r_1,j}|^2 |h_{r_j,e}|^2}{\epsilon |h_{r_j,e}|^2 + \epsilon_\epsilon |h_{r_j,e}|^2} < \zeta_1, \frac{\epsilon e_\epsilon_2 |h_{r_2,j}|^2 |h_{r_j,e}|^2}{\epsilon |h_{r_j,e}|^2 + \epsilon_\epsilon |h_{r_j,e}|^2} > \frac{C_{th}^2 e_\epsilon_1 |h_{r_2,j}|^2 |h_{r_j,e}|^2}{\epsilon |h_{r_j,e}|^2 + \epsilon_\epsilon |h_{r_j,e}|^2} + C_{th}^2 - 1\right)
\]

(36)
where \( \zeta_1 = \frac{1-\lambda_2^2C_{th}^2}{1+\lambda_2^2C_{th}^2} \) and \( \lambda_2^2 > \lambda_1^2 (C_{th}^2 - 1) \). This is due to the fact that \( \zeta_1 = \frac{\lambda_2^2-\lambda_1^2(C_{th}^2-1)}{\lambda_2^2C_{th}^2-1} \). Otherwise, \( SOP^{AF} = 1 \). \( P_1 \) can be approximated using the inequality \( xy/ (x+y) \leq \min \{ x, y \} \) as

\[
P_1 < \mathcal{P} (\min \{ |h_{r_1}|^2, |h_{r_2}|^2 \} < \zeta_1, \min \{ |h_{r_1}|^2, |h_{r_2}|^2 \})
\]

\[
\text{min}\{C_{th}^2|h_{r_1}|^2, \Delta|h_{r_2}|^2 \} + \Psi
\]

(37)

where \( \Delta = \frac{C_{th}^2 \epsilon_2}{\epsilon_1} \) and \( \Psi = \frac{C_{th}^2}{\epsilon_1} \). Obviously, it is difficult to resolve the above equation. Therefore, we use the equality presented after the equation (22), where \( \tau_1 = \min \{ |h_{r_1}|^2, |h_{r_2}|^2 \} > \min \{ C_{th}^2 |h_{r_1}|^2, \Delta|h_{r_2}|^2 \} + \Psi \) and \( \tau_2 = \min \{ |h_{r_1}|^2, |h_{r_2}|^2 \} < \zeta_1 \).

The equation (37) can be rewritten as

\[
P_1 < \mathcal{P} (\min \{ |h_{r_1}|^2, |h_{r_2}|^2 \} > \min \{ C_{th}^2 |h_{r_1}|^2, \Delta|h_{r_2}|^2 \} + \Psi)
\]

\[
- \mathcal{P} (\min \{ |h_{r_1}|^2, |h_{r_2}|^2 \} > \zeta_1, \min \{ |h_{r_1}|^2, |h_{r_2}|^2 \})
\]

\[
\text{min}\{C_{th}^2|h_{r_1}|^2, \Delta|h_{r_2}|^2 \} + \Psi) = P_{r1} - P_{r2}
\]

We consider that \( |h_{r_1}|^2 \) is smaller than \( C_{th}^2 |h_{r_1}|^2 + \Psi \). Therefore, the inequality \( \min \{ |h_{r_1}|^2, |h_{r_2}|^2 \} > \min \{ C_{th}^2 |h_{r_1}|^2, \Delta|h_{r_2}|^2 \} + \Psi \) can be expressed as

\[
\min \{ |h_{r_1}|^2, |h_{r_2}|^2 \} > \Delta|h_{r_2}|^2 + \Psi.
\]

In this case, \( P_{r1} \) can be given as

\[
P_{r1} = \mathcal{P} (|h_{r_1}|^2 > \Delta|h_{r_2}|^2 + \Psi, |h_{r_2}|^2 > \Delta|h_{r_2}|^2 + \Psi)
\]

\[
= \int_0^\infty \mathcal{P} (|h_{r_1}|^2 > \Delta x + \Psi, |h_{r_2}|^2 > \Delta x + \Psi) f_{|h_{r_2}|^2} (x) \ dx
\]

\[
= \sum_{k=1}^{M} \binom{M}{k} (-1)^{k-1} \int_0^\infty \exp \left( - (k \beta_0 + \beta_2) (\Delta x + \Psi) \right) \beta_e \exp \left( - \beta_e x \right) \ dx
\]

\[
= \sum_{k=1}^{M} \binom{M}{k} (-1)^{k-1} \frac{\beta_e \exp \left( - (k \beta_0 + \beta_2) \Psi \right)}{k \beta_0 \Delta + \beta_2 \Delta + \beta_e}
\]

(39)
$P_{r2}$ can be written as

$$P_{r2} = \mathcal{P} \left( \min \{ \epsilon |h_{rj}|^2, \epsilon_e |h_{r_j,e}|^2 \} > \zeta_1, \right. \\
\left. \min \{ |h_{rj}|^2, |h_{r_j,2}|^2 \} > \min \{ C_{h_0}^2 |h_{rj}|^2, \Delta |h_{r_j,e}|^2 \} + \Psi \right) \\
= \mathcal{P} \left( |h_{rj}|^2 > \frac{\zeta_1}{\epsilon}, |h_{r_j,e}|^2 > \frac{\zeta_1}{\epsilon_e}, |h_{rj}|^2 > \Delta |h_{r_j,e}|^2 \right. \\
+ \Psi, |h_{r_j,2}|^2 > \Delta |h_{r_j,e}|^2 + \Psi \right) \\
= \sum_{k=1}^{M} \binom{M}{k} (-1)^{k-1} \int_{\frac{\zeta_1}{\epsilon}}^{\infty} \exp \left( -(k \beta_0 + \beta_2)(\Delta x + \Psi) \right) \beta_e \exp \left( -\beta_e x \right) dx \\
= \sum_{k=1}^{M} \binom{M}{k} (-1)^{k-1} \frac{\beta_e \exp \left( -(k \beta_0 + \beta_2)\Psi \right)}{k \beta_0 \Delta + \beta_2 \Delta + \beta_e} \\
\exp \left[ -(k \beta_0 \Delta + \beta_2 \Delta + \beta_e) \frac{\zeta_1}{\epsilon_e} \right]$$

(40)

Thus, with (35), (38), (39) and (40), the secrecy outage probability for NOMA-AF relaying systems with partial relay selection considering Rayleigh fading channels can be given as

$$SOP^{AF} = 1 - P_{r1} + P_{r2} \\
= 1 + \sum_{k=1}^{M} \binom{M}{k} (-1)^{k-1} \frac{\beta_e \exp \left( -(k \beta_0 + \beta_2)\Psi \right)}{k \beta_0 \Delta + \beta_2 \Delta + \beta_e} \\
+ \frac{\beta_e \exp \left( -(k \beta_0 + \beta_2)\Psi \right)}{k \beta_0 \Delta + \beta_2 \Delta + \beta_e} \exp \left[ -(k \beta_0 \Delta + \beta_2 \Delta + \beta_e) \frac{\zeta_1}{\epsilon_e} \right]$$

(41)

5.2 Decode-and-Forward

The secrecy outage probability for NOMA-DF relaying systems with partial relay selection over Rayleigh fading channels can be expressed as

$$SOP^{DF} = \mathcal{P}(C_1^{DF} < R_1 \text{ or } C_2^{DF} < R_2) \\
= 1 - \mathcal{P}(C_1^{DF} > R_1, C_2^{DF} > R_2) \\
= 1 - P_2$$

(42)
Using a similar analysis presented in subsection 5.1, $P_2$ can be written as

$$P_2 < \mathcal{P} \left( |h_{r_j,e}|^2 < \frac{\zeta_1}{\epsilon_e}, \min\{|h_{r_j}|^2, |h_{r_j,e}|^2\} > \Delta |h_{r_j,e}|^2 + \Psi \right)$$

$$= \mathcal{P} \left( |h_{r_j,e}|^2 < \frac{\zeta_1}{\epsilon_e}, |h_{r_j}|^2 > \Delta |h_{r_j,e}|^2 + \Psi, |h_{r_j,e}|^2 > \Delta |h_{r_j,e}|^2 + \Psi \right)$$

$$= \sum_{k=1}^{M} \binom{M}{k} (-1)^{k-1} \int_{\frac{\Delta}{\beta_e}}^{\infty} \exp (-k\beta_0 (\Delta x + \Psi)) \exp (-\beta_2 (\Delta x + \Psi)) \beta_e \exp (-\beta_e x) \, dx$$

$$= \sum_{k=1}^{M} \binom{M}{k} (-1)^{k-1} \beta_e \exp (- (k\beta_0 + \beta_2) \Psi) \frac{\beta_e \exp (- (k\beta_0 + \beta_2) \Psi)}{k\beta_0 \Delta + \beta_2 \Delta + \beta_e}$$

$$\left( 1 - \exp \left[ - (k\beta_0 \Delta + \beta_2 \Delta + \beta_e) \frac{\zeta_1}{\epsilon_e} \right] \right)$$

By using (42) and (43), we obtain a closed-form expression for secrecy outage probability of NOMA-DF relaying system with partial relay selection under Rayleigh fading channels. The $SOP^{DF}$ can be expressed as

$$SOP^{DF} = 1 + \sum_{k=1}^{M} \binom{M}{k} (-1)^{k-1} \frac{-\beta_e \exp (- (k\beta_0 + \beta_2) \Psi)}{k\beta_0 \Delta + \beta_2 \Delta + \beta_e}$$

$$+ \frac{\beta_e \exp (- (k\beta_0 + \beta_2) \Psi)}{k\beta_0 \Delta + \beta_2 \Delta + \beta_e} \exp \left[ - (k\beta_0 \Delta + \beta_2 \Delta + \beta_e) \frac{\zeta_1}{\epsilon_e} \right]$$

$$= \exp \left[ - \frac{\beta e \zeta_1}{\epsilon_e} \right] \tag{44}$$

### 6 Asymptotic Secrecy Outage Probability

#### 6.1 Amplify-and-Forward

By using (41), the asymptotic secrecy outage probability for NOMA-AF relaying system can be expressed when $\Delta \to 0$ and $\Psi \to 0$ as

$$SOP^{AF} = \exp \left[ - \frac{\beta e \zeta_1}{\epsilon_e} \right] \tag{45}$$

#### 6.2 Decode-and-Forward

Similarly, $SOP^{DF}$ presented in (44) can be approximated as

$$SOP^{DF} = \exp \left[ - \frac{\beta e \zeta_1}{\epsilon_e} \right] \tag{46}$$
7 Numerical Results

In this section, numerical results are presented to evaluate the strictly positive secrecy capacity and secrecy outage probability of our proposed scheme in a Rayleigh fading environment. We set the power allocation coefficient to $\lambda_1 = 0.86$ and the Rayleigh channel parameters to $\beta_0 = \beta_2 = \beta_e = 1$.

Fig. 2 shows the strictly positive secrecy capacity of both NOMA-AF and NOMA-DF schemes versus SNR in dB for different number of relays $(M = 1, 2, 3, 5)$. We assume that $\epsilon_e = 7\, dB$. It is clear that for both scenarios

![Graph showing strictly positive secrecy capacity for NOMA-AF and NOMA-DF relaying systems](image)

Fig. 2: Strictly Positive Secrecy Capacity for NOMA-AF and NOMA-DF relaying systems when varying the number of relays $(M = 1, 2, 3, 5)$ and fixing $\lambda_1 = 0.86$, $\epsilon_e = 7\, dB$ and $\beta_0 = \beta_2 = \beta_e = 1$. 
NOMA-AF and NOMA-DF, the strictly positive secrecy capacity increases as the number of relays $M$ increases. It can be noted that the rise of number of relays has a remarkable effect on secrecy performance. From this figure, we can observe that multi-relay NOMA-AF and multi-relay NOMA-DF systems have the same secrecy performance for high values of SNR. Moreover, the asymptotic results show that the strictly positive secrecy capacity tends to become constant for high values of SNR. These results are well demonstrated in section 4. Fig.3 depicts the secrecy outage probability as a function of SNR in dB considering NOMA-AF and NOMA-DF protocols when varying the number of relays ($M = 1, 2, 3, 5$) and fixing the target data rates $R_1 = 0.1$ bit per channel use (BPCU), $R_2 = 0.5$ BPCU and the average SNR of the relay-eavesdropper link $\epsilon_e = 0dB$. It is clear that the increase in the number of relays worsens the secrecy outage probability and enhances the system performance.

We can also notice that the asymptotic outage probability does not vary when raising the number of relays and it tends to a fixed value. These observations are proved in section 6.

Fig.4 illustrates the strictly positive secrecy capacity versus SNR in dB when varying the value of the average SNR of the eavesdropper channel ($\epsilon_e = 0 dB, 10 dB$) and the number of relays ($M = 1, 2, 3, 5$) considering a partial relay selection scheme. We can see that the better strictly positive secrecy capacity is achieved for a high number of relays and a low value of $\epsilon_e$. It is also shown in Fig.4 that the secrecy performance loss is more severe when the average SNR of the non-legitimate link $\epsilon_e$ increases. Then, the improvement in the number of relays does not have a significant effect on strictly positive secrecy capacity for a high value of $\epsilon_e$. Fig.5 reveals the impact of the target data rates $R_1$ and $R_2$, and the average SNR of the eavesdropper channel $\epsilon_e$ on the secrecy outage probability for different number of relays ($M = 1, 2, 3, 5$).

In particular, we can observe that the lower the values of $R_1$, $R_2$ and $\epsilon_e$ are, the larger the secrecy improvement by using different number of relays will be. Therefore, we can deduce that the increase in the number of relays considerably improves the secrecy performance. We can further note that the decrease in the target data rates $R_1$ and $R_2$, the decrease in the average SNR of illegal link $\epsilon_e$, and the increase in the number of relays lead to a good secrecy performance.

### 8 Conclusion

In this paper, we analyzed NOMA-AF and NOMA-DF relaying networks in the presence of multiple relays under Rayleigh fading channels. A partial relay selection method was adopted to choose the best relay based on the highest SNR of the first link. The secrecy performance was evaluated in terms of secrecy outage probability and strictly positive secrecy capacity. We showed that the increase in the number of relays significantly raises the system performance. However, as the average SNR of the illegal link $\epsilon_e$ and the number of relays $M$ increase, the secrecy performance worsens. In fact, the increase
Fig. 3: Secrecy outage probability for NOMA-AF and NOMA-DF relaying systems for different number of relays \((M = 1, 2, 3, 5)\) assuming that \(R_1 = 0.1BPCU, R_2 = 0.5BPCU, \epsilon_e = 0 dB\) and \(\beta_0 = \beta_2 = \beta_e = 1\).

The increase in the number of relays does not have an important impact on the system performance especially for high values of \(\epsilon_e\). Furthermore, the increase in the number of relays has no effect on the asymptotic results.

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Fig. 4: Strictly Positive Secrecy Capacity for NOMA-AF and NOMA-DF relaying systems when varying $(\epsilon_e = 0 \, dB, 10 \, dB)$ for different number of relays $(M = 1, 2, 3, 5)$ when $\lambda_1 = 0.86$ and $\beta_0 = \beta_2 = \beta_e = 1$. 

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