Study of Accretion Processes on Black Holes: Fifty Years of Developments

Sandip K. Chakrabarti
S.N. Bose National Centre for Basic Sciences
JD-Block, Sector-III, Salt Lake, Kolkata 700098, INDIA

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Abstract

Fifty years ago, in 1952, the first significant paper on accretion flows was written by Bondi. The subject has grown exponentially since then. In fact, today many of the satellites engaged in space physics research look for signatures of accretion processes in whatever objects are studied. In this review, I will touch upon the significant developments in these years in this subject, emphasizing mainly on accretion onto black holes. Since winds and accretions are generally studied under similar framework, some references of the winds/outflows studies will also be made.

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1 Introduction

In astrophysical context, accretion is a process by which matter is collected around a central object. Normally, when one talks about accretion in binary systems, one star is tidally deformed and matter flows out from it to the compact companion. When one deals with an isolated object, it may accrete from the interstellar medium at a very low rate. In many of the galactic centers there are evidence of supermassive black holes. There are no companions, but matter is accreted from winds of surrounding stars. In these cases, stars may also be tidally disrupted if they come very close to the black hole and the matter would be accreted from the disrupted star to the central black hole.

In the present review, we give an overall development of the subject which took place over the last fifty years. It is clearly a Herculean task and it would be impossible to cover the whole subject in these few pages. We will first consider significant developments in last five decades and at the end discuss in detail about the very modern developments.

Though the subject began with the study of accretion onto ordinary stars, very quickly the similar method was found to be useful for the study of accretion onto compact objects, such as white dwarfs, neutron stars and black holes. We mainly review the accretion flows on black holes. Since winds are produced from compact stars and accretion disks around black holes, and interestingly, are studied from the same set of equations, the development of the subject of winds and outflows began almost at the same time. So it is inevitable that some aspects of the outflows would be discussed simultaneously.

*Also at Centre for Space Physics P-61 Southend Gardens, Kolkata, 700084
†e-mail: chakraba@boson.bose.res.in
2 Accretion and outflows problem over the decades

2.1 1950s: Toddler Days - era of curiosities

The simplest problem of axisymmetric particle accretion through a shock front was done by Hoyle and Lyttleton back in 1939 [1]. The problem was to study how much matter would accrete on a star moving through an interstellar medium. The work was not satisfactory as pressure effects were ignored. In 1952, the classic paper of Bondi [2] was published. There he computed the mass accretion rate on a star ‘in rest in an infinite cloud of gas’ by including the pressure effects. The solution showed clearly that matter, originating subsonically at a large distance can become supersonic on a star surface. Bondi [2] already anticipated some directions in which the subject should proceed: first, compressional heat may be lost and pressure at the inner edge may be diminished causing much larger inflow rate. Due to this non-adiabaticity, the polytropic index $\gamma$ should become close to unity near the star. Second, a correct understanding may require a time-dependent solution.

Almost simultaneously, Biermann [3] suggested that the behaviour of cometary tails could be explained by assuming outflows from the solar surface. This was later explained by Parker [4] who showed that one of the sub-sonic branches crosses to become supersonic and in fact, could explain the velocity of solar wind at the Earth’s orbit. This thermal pressure driven wind solution was thus found to be important in the subject of solar and stellar winds, and indeed for astrophysical jets, in general. Parker’s solution was thus similar to the Bondi-flow solution except that the flow originates at smaller radius and goes to infinity. Also, initial models were discussed for isothermal gas.

From the observational point of view, 1950s saw a large number of pioneering results, obviously with ground based instruments. Jennison and Das Gupta [5] first resolved two radio blobs of Cyg A and thus double radio sources were discovered. Baade and Minkowski [6] termed the train of optical knots seen in M87 as ‘jets’ and proclaimed that they must be ejected from the nucleus of the galaxy. These paved the way to a completely new subject of cosmic radio jets and the physics around black holes. These powerful radio emitting regions turned out to be indirect evidences of massive black holes at the centers of active galaxies. In the 90s, evidences for such activities around smaller mass black holes (so-called galactic microquasars) were also discovered.

2.2 1960s: Childhood - era of dare devils

Having powered by very satisfying accretion model of Bondi [2], workers became engaged in applying this model to explain various known observations of star-gas interactions. With improved instruments in the post-war period radio observations became very popular and several important discoveries were made. However, Solar wind observations were much easier to do and thus more advancements were made in improving wind solution by additional physics.

In 1963, first QSO 3C272 was discovered [7, 8]. Using radio occultation of 3C273 by the moon, Hazard, Mackay and Shimmins [7] using Parkes 210 feet radio telescope found two distinct radio sources and located the position. This was communicated to M. Schmidt at Caltech who obtained the redshift by optical measurements [8]. This showed that the need to exchange data was already appreciated at this early age. In the first Texas Symposium of 1963 [9] various explanations were proposed all converging to the idea that the enormous energy radiated by a QSO must be gravitational in nature. In the following year, Salpeter [10] used Schwarzschild solution of 1916 and interpreted that the luminosity could be due to
Bondi accretion on very massive compact objects ($> 10^7 M_\odot$). These objects were termed as ‘collapsed matter’ or collapsars at that time since the phrase ‘black hole’ was coined by J. Wheeler only in 1967 [11]. The computed luminosity was found to be on the order of $10^{47}$ ergs per sec for reasonable parameters. The problem of local vs. cosmological nature of QSOs persisted, however. Hoyle and Burbidge [12] debated over the possibility of cosmological distance since it was difficult to explain rapid variation (40 percent in two years) of intensity at 8GHz emission from 3C273.

Russian school primarily headed by Zeldovich and Sunyaev had contributed immensely in our understanding of the accretion processes. Indeed even today the so-called standard model is originated from this school. In 1966, Zeldovich and Guseynov [13] pointed out that there are several binary systems in which the companion is ‘unseen’ yet massive indicating that collapsed stars (later known as black holes) should exist in these systems. ‘An unambiguous proof of the existence of a collapsed star would naturally be of the great interest.’ write Zeldovich and Guseynov [13]. Today the situation has improved (see, [14]) and we have several very good galactic and extragalactic black hole candidates. Because Quasars have typical luminosities of $10^{44-47}$ ergs sec$^{-1}$, (which means destruction of $\sim 0.002 - 2M_\odot$ yr$^{-1}$ completely at each Quasar) energy must be coming from gravitational energy. Based on this, Lynden-Bell [15] made a strong argument that accretion close to Eddington rate on super-massive black holes must be responsible for this tremendous activity.

In 1963, a major discovery on the theoretical front was the vacuum solution of Einstein equation around a compact rotating collapsed matter by R.P. Kerr [16]. Immediately after that there was an explosion of activities in Cambridge, Princeton and other places to understand the implication of this new solution. A large number of papers were published to study behaviour of matter (mostly particle) trajectories (see, [17]). Today, these objects are called Kerr black holes and are almost universally thought to be playing central role in black hole astrophysics.

As observations were carried out, it became clear that QSOs are just special cases of Active Galactic Nuclei which come in a variety of forms: line-less BL Lacs, fainter Seyfert galaxies, Blazers, Quasars, Optically Violent Variables, Liners etc. In the 90s active efforts were made to unify these various classes.

Among other major discoveries in the 60s which are related to accretion/winds are the pulsars and X-ray sources. Antony Hewish discovered regular pulses of Radio waves towards the end of 1967 [18]. Among interesting models which were proposed immediately were emission from hot spots on rotating compact stars by Ostriker [19] and gravitational lensing model (binary neutron stars with one occulting and amplifying other’s radiation) of Barnothy and Barnothy [20].

Unlike radio observations which were carried out in the forties and in the fifties, X-rays from accretion onto compact objects could not be studied from ground. After the rocket technology was improved, experiments started with Aerobee rockets. In 1962, Giacconi and his team discovered X-Ray sources by rocket measurements. During 1962-1966, they concentrated on Cygnus region, Sco X-1 and Crab Nebula, and determined their positions. Cyg X-1, the first black hole candidate was discovered in these early attempts in 1962 [21]. From simple binary accretion model, soon it was realized that the companion should exist and optical search was also made in Palomar plates. These were the first of the experiments and the avenues opened by these pioneers are still followed though the technology has advanced radically to observe much fainter sources for much longer periods using balloon and satellite technology.

Apart from professional astrophysicists and astronomers who are interested in accretion
physics, many others got interested in the black hole accretion as well. For instance, Salpeter [10] acknowledges discussions with R. Feynmann, M. Schwarzschild and L. Spitzer. Even energy extraction by feeding rotating black holes with negative energy through accreting matter was evoked by Penrose [22] in this decade. The prospect of Quasars being cosmological became brighter each day leaving cosmologists such as George Gamow to wonder [9]: Twinkle, twinkle quasi-star/Biggest puzzle from afar/How unlike the other ones/Brighter than a billion suns/Twinkle, twinkle quasi-star/How I wonder what you are. Thus all the best brains of the day were working to sort out puzzle uncovered by the great discoveries of the 60s.

By early 1960s, the phrase ‘solar wind’ became common and works were carried out to study interaction between solar wind and the earth and comets. The proximity of the Sun drove many astrophysicists to understand wind mechanisms better. Parker [23] improved on the study of his earlier work [4] on solar and stellar winds. Important ingredients such as angular motion and magnetic field of the sun was added at this stage by Pneuman [24], Weber and Devis [25] etc. Weber and Devis [25] worked out the effects of angular momentum and radial-azimuthal field lines on the equatorial plane and computed the torque exerted on the sun and how the sun can be slowed down. Work advanced a great deal in this direction with heating, cooling, conduction etc. For instance, Eisler [26] includes conductive heat flux in spherical geometry. Weber [27] discussed solution with conducting heat flux in flows with angular momentum and magnetic field. Whang, Liu and Chang [28] included viscosity and thermal conduction. Interestingly, they also discuss the ratio of the viscous stress to thermal pressure, later known to be Shakura-Sunyaev viscosity parameter [29] in the context of accretion disks. In a classic paper by Axford and Newman [30] solved the Bondi accretion flow and winds simultaneously after including viscosity and thermal conduction. They studies weak shocks as well. More complex interaction which included bow-shock waves were also studied in this decade [31]. Works with this vigour were not carried out in accretion around black holes for another decade or so.

2.3 1970s: Adolescence - Era of Confusion and Exploration

In the 1960s, Pandora’s box was opened on all fronts: Radio and X-ray observations, Theory of black holes, even simple theoretical models of accretion flows. Models proposed were diverse: simplistic yet bold. The confusion persisted in the 1970s since the technology was mature to make satellite observations and competition to build realistic models was too much. Hectic search for a quick-fix was thus a necessity.

Models of accretion was going in several directions: the spherical accretion was simple and had a plenty of room for improvements. Detailed spectra of stars and QSOs demanded that angular momentum should be included and mechanism of radiation from the disks should be more efficient than that of a spherical accretion. Thus came the standard Keplerian disks of Shakura and Sunyaev [29]. Cosmic radio jets were observed extensively and along with it came the realization that there must be powerful engines sitting at the galactic centers to produce them. Since black holes have no hard surfaces, jets must be produced from the accretion itself. Disk models evolved specifically to produce jets efficiently and thick accretion disk models began which were perfected and studied in detail in next decade. Below we discuss briefly these models. From the work of Lynden-Bell [15] it was realized that the QSO emission must be related to accretion onto black holes. In 1970, from the study of absorption lines, Sturrock [32] suggested that the QSO themselves must have formed these jets and outflows. So, naturally there was an extra emphasis to understand the origin of large luminosity and the origin of jets simultaneously. This has only recently
been understood in full detail which will be discussed later.

It is to be noted that though black holes themselves were new at this time, numerous authors (e.g. [33]) already were engaged in looking for possibilities to detect them from Doppler shifts of line emissions. So, many of the ingredients of what to look for (such that double-horn pattern of line-emission found in black hole accretion flow) were already emerging. Bardeen [34] suggested that by the time the mass of the accreted matter is more than fifty percent of the initially non-rotating black hole mass, the black hole would likely to become extremal Kerr black hole ($a \sim 1$) though radiation effects could slow it down to more like $a = 0.998$ [35]. A treatise on general relativity and particle and light behaviour around black holes which was compiled by Misner, Thorne and Wheeler in 1970, published in 1973 [36], even tried to convince reader through a dialogue between Sagredus and Salvatius why the phrase ‘black hole’ is the most appropriate for the collapsed stars.

Pringle and Rees [38] first attempted quantitative analysis of the nature of the X-ray radiation which are emerged from an accretion disk around a black hole or a neutron star. Even the nature of accretion on a neutron star with magnetic field and origin of pulsed emissions were discussed. They conjectured that both for high and low accretion rates, the inner part of the disk should flare up because of high radiation pressure and because of low efficiency of radiation respectively. This work is clearly precursor of radiation pressure dominated thick disks and ion pressure dominated accretion tori respectively. In the same year, in 1972, Shakura and Sunyaev [29] wrote the classic paper on the standard disk model, in which they assumed that the angular momentum distribution is Keplerian throughout the disk. For stability reason, the inner edge of the disk was truncated at $6GM/c^2$, i.e., 3 Schwarzschild radii. Viscosity was assumed to be responsible to transport angular momentum outwards to give rise to such a distribution. The heat generated by viscosity was assumed to be radiated away instantaneously from the surface. The viscous stress was assumed to be dominated by the $w_{r\phi}$ component (which is a reasonable assumption for a geometrically thin disk) and whether it is turbulence viscosity or magnetic viscosity or both, the viscous stress is considered to be proportional to the local pressure by the relation $w_{r\phi} = -\alpha \rho v_s^2$, where $\alpha$ is a constant, $\rho$ is density and $v_s$ is the isothermal sound velocity. For causality, $\alpha < 1$ always. The disk was assumed to be in vertical equilibrium and the vertical thickness was computed accordingly. In presence of strong radial motion $v_r$, Chakrabarti and Molteni [38] suggested that the above prescription should be modified to $w_{r\phi} = -\alpha \rho (v_s^2 + v_r^2)$. This would create a smooth variation of the viscous stress across a shock transition and will not transport angular momentum unnecessarily by an axisymmetric shock.

Shakura and Sunyaev [29] presented analytical solutions of the disk variables such as temperature, density etc. as a function of the radial distance from the axis. They also computed radiation spectrum by adding black body contribution from successive annulus of the disk. This so-called multicolour black body spectrum became the hallmark of the standard disk model. The above work was carried out using Newtonian equations. Novikov and Thorne [39], subsequently extended this work to include general relativity. Along with the continuum emission, workers began to observe line emissions from a rotating accretion disk [40] around a black hole. The Doppler broadening, Doppler shift and gravitational broadening were also studied in this work. At roughly the same time another study envisaged outflow driven by X-ray radiation on the disk [41].

Meanwhile in December 1970, the UHURU satellite was launched. This was the first Astrophysics related satellite and first few things which were looked at included the black hole candidate Cyg X-1. First few papers [42] showed that X-rays of energy larger than
100 keV was emitted by this source. There were contradictory reports on whether this source showed periodic variability with 73 ms variability or not [43]. Only recently, confirmed very low-frequency (0.05-0.1 Hz) Quasi-Periodic oscillation has been reported from this candidate [44].

Soon after the standard disk model was published, Lightman and Eardley [45] pointed out that in a radiation pressure dominated region, viscosity prescription of Shakura and Sunyaev was inconsistent if $\alpha$ remains constant. In fact, they show that the disk would break into thin rings. Subsequently, Thorne and Price [46] argued that Shakura-Sunyaev 1973 disk model is unable to explain the high energy X-rays found in Cyg X-1. Eardley, Lightman and Shapiro [47] had a very successful model with a Keplerian disk which flares into a thick disk closer to the inner edge. They considered two temperature flow and it was a single component disk with a small accretion rate. The spectrum of Cyg X-1 was fitted reasonably accurately. Later, Pringle [48] showed that such a disk is thermally unstable. Observed spectra of the black hole candidates, such as Cyg X-1 [49] indicated that the spectrum consists of two distinct components. Observations similar to Cyg X-1 were made for active galaxies and Quasars [50]. The soft X-ray bump in these spectra could be explained by multicolour black body emission from a Keplerian disk. The power-law component of the spectra was explained by Comptonization of softer photons by hot electrons from a magnetic Corona on an accretion disk [51]. Not only does the spectrum consist of two distinct components, it was apparent from X-Ray observations [52] that Cyg X-1 actually has a ‘soft state’ (when X-ray power is emitted in soft X-rays) and a ‘hard state’ (when X-ray power is emitted in hard X-rays). Ichimaru [53] pointed out that the two states observed in Cyg X-1 could be due to two distinct solutions that the equations yield – one is an optically thin branch which emits hard X-rays and the other is an optically thick branch which emits soft X-rays. The work was done using Newtonian equations and thus the inner boundary conditions were not appropriately chosen. The optically thin solution was later termed as Advection Dominated Flow or ADAF [54]. Hoshi and Inoue [55] considered X-ray irradiated disk and also found similar transition from optically thin to optically thick solution. The bimodal behaviour was claimed to be responsible for the state transition. Today, it is understood that the hard and soft components of the spectrum are emitted at the same time and can vary independently and these problems are easily explained by two-component advective flow (TCAF) model discussed later [56] in this review.

Progress in general relativistic accretion disks occur greatly by the efforts of Bardeen and Patterson [57] who studied disk accretion in an inclined plane at a large distance and showed that the flow is dragged in the equatorial plane of the black hole in the region between $100GM/c^2$ and $10^4GM/c^2$. This kind of tilted disk was envisaged sometime back by Wilson [58] who suggested that during asymmetric explosion of a supernovae, resulting compact object may change its spin axis from that of the normal on the binary plane. The effect of Kerr geometry on the accretion disks was explored in great detail in works of Cunningham [59]. For instance, how the spectrum is modified due to gravitational red-shift was investigated. Another curious work is that of Luminet [60] who computed how an accretion disk around a black hole itself would look like if photographed from the vicinity. Due to the light bending effect, the disk would have an interesting shape — the region away from the observer would be bent and even lower side of the disk could be seen. Matter on one side of the disk moving away from the observers would emit the red-shifted light, and that from the other side would emit the blue-shifted light.

Along with the accretion disk model, efforts were being made to compute the properties of spherical symmetric Bondi accretion in presence of heating and cooling effects. Michel [61]
extended the original adiabatic work in general relativity. Shapiro [62] included electron-electron and electron-proton bremsstrahlung effects. Using the suggestion of Shvartsman [63] and Zeldovich and Novikov [64] Shapiro [65] included effects of tangled magnetic field whose energy density could be as high as gravitational energy density and found that the synchrotron radiation will be predominantly emitted in the infra-red region. An important addition to such study is the effect of pre-heating on the luminosity [66] of the spherical emission. They find that if the infalling gas intercepts X-rays emitted at the inner edge of the disk, then it will be heated up if the sound speed exceeds the escape velocity, the gas accretion is decreased resulting in decrease in the luminosity limit orders of magnitude lower compared to the Eddington limit. If the efficiency $\eta \ll 1.7 \times 10^{-4}$, the results agree with those of Shapiro [62], but as $\eta \to 1.7 \times 10^{-4}$ their limit is higher compared to Shapiro [62] who ignored the effects of pre-heating. Later, Bisnovatyi-Kogan and Blinnikov [67] repeated this calculation and found that solution exists for any efficiency of energy release, including those typical for accretion on neutron stars, for any luminosity below the Eddington limit. These conclusions [66, 67] were questioned by Stellingwerf and Buff [68] who found a much smaller forbidden region and showed that the flow is disrupted only if the unheated Bondi luminosity exceeds the Eddington Luminosity. Spherical Bondi flow was also studied on neutron stars as the procedure was the same, but the inner boundary condition varied. The boundary layer of a neutron star where the flow settles down to zero velocity was first studied by Shapiro and Salpeter [69]. Thus the importance of the inner boundary condition was recognized, and particularly it was realized that the boundary layer is an extension of the disk flow itself. Though for black holes there are no hard surfaces, this concept could be extended [70] for black holes also, especially if matter possesses angular momentum, albeit small. This will be discussed later.

Yet another type of study included giving a pre-assigned velocity and density profiles and obtaining the corresponding variation in temperature as a function of radial distance in presence of bremsstrahlung, synchrotron etc. [71]. This type of work was extended to two temperature flows later by Maraschi, Roasio and Treves [72]. In the absence of a fully self-consistent study, these solutions gave pretty good idea about how the electron temperature gradually deviated from the proton temperature due to faster cooling processes. Eardley and Lightman [73] subsequently solved two temperature Keplerian disk. Till today, there are debates on whether electrons and protons should have separate temperatures or not, particularly that there are processes (Ohmic heating) which may heat up the electrons otherwise cooled down by synchrotron emission [74].

With the understanding that general relativity should be essential to explain behaviour of matter and radiation around a black hole both the accretion and wind models were generalized, first by assuming predominantly rotating flow [39] and then by assuming predominantly radial flow [75]. The later work was essentially the generalization of the Bondi flow and Parker winds already described. Moreover, the shock transitions as described by Holzer and Axford [76] in the context of solar winds were also studied here in the context of accretion. A supersonic flow must have a shock transition if the fluid pressure in the upstream has a finite value. Thus accretion on a neutron star, or winds propagating supersonically to a medium of finite pressure must have a shock. In the case of black hole accretion, the pressure is essentially zero on the horizon, and hence a spherically accreting adiabatic flow endowed with a single sonic point does not produce a shock in accretion.

The decade of the 70s saw another set of studies which were essentially computer simulations. As new and faster computers came in laboratories, one started carrying out numerical experiments by computers. This is because it is understood that it is impossible to perform
real experiments in Astrophysics. Time dependent general relativistic flows were studied in this decade by pioneers such as Wilson [77]. Wilson used his skill of simulating collapse of stars and supernovae explosion and applied them to study accretion of rotating matter around a Kerr black hole. He found the existence of propagating shocks inside the disk when angular momentum is significant. He kept the angular momentum to be constant, contrary to standard works [29, 39] which immediately followed his work. Modification of the equations of Shakura and Sunyaev [29] clearly included addition of the pressure terms and advection terms and more realistic heating and cooling effects. Maraschi, Reina and Treves [78] included radiation pressure and drew a few important conclusions. They found that as accretion rate is increased even higher than the Eddington rate, the accretion solution still exists. Furthermore, with the radiation pressure, the flow is deviated from a Keplerian flow. This work was, however, carried out using Newtonian geometry. Lynden-Bell [79] worked out the steady state structure of the flow with constant angular momentum in general relativistic flows and predicted that the flow would develop giant vortices along the axis, simply because centrifugal force would keep matter away from the axis. Here pressure terms were included as well (which was unimportant in Keplerian disks).

While this decade was full of theoretical advancements, one should not forget that space based experiments were already in place and major results were arriving almost on a daily basis. Similarly Radio telescopes were in place since 1940s and 50s which were being pointed at the Quasars and active galaxies. Cosmic radio jets, which were found to be ejected from galactic centers were highly puzzling and it was already realized that they must be related to the central engines of the active galaxies and specifically to the accretion disks. First set of serious models of cosmic jet formation were given by Blandford and Rees [80] and Scheuer [81]. Blandford and Rees [80] envisaged that somehow matter would bore ‘de-Laval’ type nozzles through ambient medium and propagate supersonically on either side of the galactic plane. Observations of these jets put pressure on theorists to modify their models so that super-luminal and well collimated jets could be produced.

Meanwhile, it was realized by several workers [46, 47, 82] that radiation pressure may cause disks to thicken due to thermal and other instabilities and an era of thick accretion disks followed. Key issues which motivated to construct this new disk models were: (a) pressure, either due to radiation or due to hot ion, must be included which was ignored in the standard Keplerian disk models of Shakura and Sunyaev [29] (b) matter must accrete very rapidly towards a black hole and thus the infall velocity should be included and finally (c) formation and collimation of cosmic jets must be explained. Lynden-Bell [79] simply assumed constant angular momentum flow and found that disks thicken up closer to the black hole. This work was extended and thick accretion disk models were constructed by Paczyński and his collaborators [83-85] by choosing various combinations of angular momentum distribution. In these so-called radiation pressure supported ‘thick-disk’ models, radial velocity was ignored totally. These disks had ‘vortices’ or ‘funnels’ which were hoped to be suitable for launching jets and outflows. These disk models were not self-consistent, in that they had an element of ad-hoc angular momentum distribution, but their importance lie in that they were pre-cursors of more complete models which include both rotation and advection. These are discussed later.

### 2.4 1980s: Youthful days - era of enlightenment

In the 70s, a large number of theoretical fronts were opened. In the 80s most of these fronts were consolidated and bits and pieces were put together to have a more global view, which is, after all, the goal of all the model builders. Using simple polytropic equation of state,
the spherical accretion solution in Schwarzschild geometry was solved by Begelman [86] who showed that there should be one critical point. This was farther extended to relativistic equation of state by Brinkman [87]. Moncrief [88] showed that the transonic spherical flow is stable as well. Works proceeded on non-adiabatic flows also. Pre-heating was already discussed [66]. Effects of dissipation heating by magnetic field was studied by Scharlemann [89] who showed that sound speed can become so large that the Bondi flow would cease to be supersonic. Of course, this would lead to shock waves or non-steady flows. Yahel and Brinkman [90] studied the emergent spectrum from a spherical Bondi flow and also studied the effect of $e^- - e^+$ on the emergent spectrum and found a sharp cut-off at around 1MeV due to $\gamma - \gamma$ interaction. Thorne and his collaborators [91] presented detailed general relativistic photo-hydrodynamical spherical accretion in optically thick regime. Flammang [92] studied the nature of critical points in presence of radiative transfer. Ipser and Price [93] considered the effect of synchrotron cooling in Schwarzschild geometry and classified different regimes of accretion according to whether synchrotron self-absorption is important or not. Payne and Blandford [94] studied the nature of the emitted spectrum of the optically thick converging flow in Newtonian geometry and found that the energy spectral index $\alpha$ ($F \sim \nu^{-\alpha}$) would be more like 2. With general relativistic considerations this index becomes 1.5 and would turn out to be the most convincing signature of a black hole accretion [56]. A more general solution, though in Newtonian geometry, was put forward by Wandel, Yahil and Milgrom [95] and Colpi, Maraschi and Treves [96] who studied single temperature solution of non-adiabatic flow and showed that Quasar luminosity increases with the accretion rate and the luminosity is a few percentage of the Eddington luminosity. A two temperature spherical accretion solution with bremsstrahlung, synchrotron cooling, Coulomb coupling between protons and electrons and and Comptonization was constructed. It was seen that the protons could be hotter than the electrons by a factor of a thousand or so. However the spectrum of Seyfert galaxies and Quasars where clear excess of radiation in ultraviolet was seen [97] could not be explained by spherical or converging flows. Fortunately, Shakura-Sunyaev Keplerian disk [29] is known to emit the multi-colour blackbody radiation in this region and several cases were fitted and satisfactory results were found [98]. The mass of the black hole was found to be about $10^8 M_\odot$. Subsequently, Wandel and Petrosian [99] showed that basically two parameters, namely the mass of the black hole and the accretion rate can describe the spectrum very well and the masses of black holes of the Quasar and Seyferts fall in the range of $10^8 - 10^{9.5} M_\odot$ and $10^{7.5} - 10^{8.5} M_\odot$ respectively. Ross, Fabian and Mineshige [100] improved the disk model by properly treating the inner edge of the accretion disk and showed that even for disks around a massive black hole, a significant radiation could be in soft X-rays. Generally, active galactic nuclei also show line emissions along with continuum emissions. These lines are believed to be emitted from rapidly moving clouds on both sides of an accretion disk. Measurement of the motion of the cloud from the Doppler shift and the distances of the cloud from reverberation mapping [101] method can give an estimate of the mass of the central object. In this method, the time lag between certain variation in the continuum spectra and the line emission is used to measure the distance of the broad line emitters. Masses of a few active galaxies have been measured this way: NGC 5548 ($8.8 \times 10^7 M_\odot$), NGC 3227 ($3.8 \times 10^7 M_\odot$).

There was some degree of uneasiness from several fronts: Astrophysicists having expertise in stellar accretion and winds would have liked to understand the black hole accretion as well, but not everyone was an expert in general relativity. For instance, studies of oscillations of stars were extended to accretion disks around white dwarfs by Cox [102]. This was done to explain quasi-periodic oscillations (QPOs) of dwarf-novae seen before. The
extension to this work to relativistic disk was done much later by Wagoner and his collaborators [103] and topic disk-seismology was thus introduced. Other interested individuals in the oscillations and QPOs were Kato [104] and Livio and Shaviv [105]. While Kato, Honma and Matsumoto [106] later considered trapped oscillations as potential candidates for Quasi-Periodic oscillations, Langer, Chanmugam and Shaviv [107] considered shock oscillation in the accretion column to be the serious candidate. It will be shown later that QPOs in black hole oscillations are likely to be due to shock oscillations in accretion disks [108].

Paczyński and Wiita [85] while studying accurate description of thick accretion disks presented a potential $\phi = -GM/(r-2GM/c^2)$, where $G$ and $c$ are the gravitational constant and velocity of light respectively, $M$ is the mass of the central black hole. This potential has the feature that Keplerian distribution of angular momentum is exactly the same as that around a Schwarzschild black hole. Thus it was expected that work done using Newtonian formalism with this so-called pseudo-Newtonian potential might yield results similar to those obtained around a black hole. This has indeed been found to be the case since the error occurring is at the most a few percent [109] and perhaps within the observational error bars. A similar potential was constructed by Chakrabarti and Khanna [110] which was much improved by Chakrabarti and Lu [111] to work in Kerr geometry. A second problem was that workers already realized (e.g., [112]) that the disk must not remain Keplerian close to a black hole as the radial velocity becomes very large below the marginally stable orbit ($r = r_{ms}$). Both the approaches [85, 112] assumed that either the thick disk or the sub-Keplerian flow are generated from a Keplerian disk nearby. More work along this line, this time including radial velocity around the Paczyński-Wiita potential, were carried out by Paczyński and his collaborators [113] The global solutions were not found properly, but the indications were clear that the flow has to cross a sonic point close to a black hole. Similar study to find a consistent solution for $r < r_{ms}$ was also attempted by [114]. Abramowicz et al. [115] in the so-called ‘slim’ accretion disk tried to introduce transonicity in the global solution and was only partly successful and many un-physical solutions were also obtained. More complete and global solution of the slim disks were obtained by Chen and Taam [116] who showed that a significant heat generated by viscous friction is carried away inside the black hole.

At the same time the radiation pressure supported disk models were being explored, Rees and his collaborators [117] argued that for small accretion rates, the disk should be so inefficiently cooled that the gas temperature could rise to almost virial temperature and the ion pressure could puff up the disk to form a so-called ‘ion-pressure supported’ thick accretion disk. Since thick disks have a toroidal ‘center’ where the pressure is maximum, they behave like very hot stars, only toroidal in shape. Compared to stars where the central temperature is around $10^7$K, disks can have temperatures around $10^8$−$10^{11}$K, though densities are much lower in disks than in stars. However, nuclear reaction rates are very sensitive to temperatures and thus even if the infall time scale in disks are low (in the hot regions in stellar black holes, this time is around a few milliseconds) the nuclear reaction could be significant to modify the composition of the infalling matter. Rees [118] first suggested that hot ion tori can have some nuclear reactions and energy generation. Chakrabarti [119] generalized the thick accretion disk study in any axisymmetric spacetime and showed that the disks and the pre-jet matters could be studied using the same theoretical formalism. The angular momentum distribution was based on the properties of the von-Zeipel surfaces on which angular momentum and angular velocities remain constant. Using this model [120], Chakrabarti and collaborators [121] computed the nuclear reactions inside thick disks in
detail and showed that thick accretion disks with a very low viscosity could have significant
nucleosynthesis and a part of the newly formed elements could be ejected by jets and are
dispersed in the interstellar space creating some metalicities. These works were further
followed up by several workers and similar results were reported [121]. Recently, with the
perfection of the transonic flow solution (namely, with the emergence of the two component
advective flow [TCAF] paradigm [56]) this problem was taken up once more [122] and it
was observed that in presence of standing shocks the possibility of nucleosynthesis is much
more. It was also observed that the flow indeed remains stable in most of the parameter
space even when nuclear reaction is turned on.

Since the properties of thick accretion disks are closely linked to the deviation of the
angular momentum distribution from the Keplerian distribution, some curiosities arose as
to whether self-gravity of the thick disk itself would cause any further deviation. After all,
the disk has a center (of toroidal topology) and matter between the black hole and the
center could require lesser angular momentum to stay in stable orbits, while matter outside
that of the centre would require more angular momentum than Keplerian due to combined
gravity of the black hole and the disk. This was first analyzed in Newtonian geometry [123].
In general relativity exact solution of the space time metric is possible in presence of an
axisymmetric ring [124] and several works in this connection were carried out later [125].

Just as the progress in thick disk was being made, a parallel line of approach was adopted
which abandoned the concept of a Keplerian disk altogether and consider a self-consistent
solution with a simple angular momentum distribution. Liang and Thompson [126] using
the Paczyński-Wiita potential argued that unlike a Bondi flow, a thin, rotating, adiabatic
flow can have three sonic points, two are of ‘Saddle’ type and one is of ‘Center’ type. From
this, one notices that in order that such an adiabatic flow passes through a sonic point the
flow must have angular momentum less than a Keplerian flow [127]. However, it was wrongly
concluded that because there were two saddle points, the flow would not know which one
to use, and possibly exhibit bi-stability behaviour [127]. This confusion was removed later
by Chakrabarti [128] where it was shown that the entropy densities at these two saddle
points are completely different and such bi-stabilities should not occur. Possession of more
than one saddle type sonic point prompted the study of shock waves in the disks just as
there were studied in solar winds [76]. Already Chang and Ostriker [129] pointed out that
the presence of pre-heating in a spherical flow would change the speed of sound in the flow,
causing the flow to have more than one sonic point giving rise to standing shocks. These
shocks were very weak and located very far out. Nevertheless, they increased the efficiency
of emission in spherical flows. Fukue [130] pointed out that indeed shock solution in rotating
accretion flows is a possibility. Chakrabarti [128] studied in detail such flows and found that
both accretion and winds can have shocks in a large region of the parameter space spanned
by specific energy and angular momentum. He separated the parameter space according to
whether shocks could form or not. One of the most interesting aspects of these solutions is
that perfectly stable and global solutions seem to be existing even without viscosity. This
was because for a black hole accretion, the gravity eventually wins and allows matter to
sink in even with constant angular momentum. Another important point is that like a
Bondi-flow, the energy need not be dissipated at all along the way to a black hole. The
entire amount of energy which the flow possesses at infinity can enter inside a black hole
without radiating anything! This aspect is very important since the event horizon sucks
in matter and energy and a large accretion rate not necessarily translates into luminosity.
This is in contrast with solutions around neutron stars (or any other star for that matter)
where half of the kinetic energy is released on the boundary layer. Recent observations [131]
suggest that black hole candidates indeed show very low luminosity for a given accretion rate (particularly for a low accretion rate for which intrinsic cooling is low) and therefore testifying the correctness of the advective flow solutions of Chakrabarti [128] and its more generalized version developed by him in the 1990s. In any case, formation and study of shock waves in accretion disks turned out to be very important in black hole astrophysics and we plan to discuss about them later when we describe achievements in the 1990s.

Progress in the direction of numerical simulation of accretion disks were also very good in the 80s. Pringle [132] demonstrated how viscosity can transport angular momentum and allow matter to sink inside a Keplerian disk. Vitello [133] considered time dependent simulation of spherical flows and found the solution to quickly converge to the steady state flow. The net escaping luminosity was found to be very small compared to the Eddington luminosity. We already pointed out that as far back as 1972, Wilson considered the simulation of rotating adiabatic disk-like flows and found propagating shocks. With the improved numerical skills and computational power, Hawley, Smarr and Wilson [134] considered the same problem and found very strong accretion shocks forming near the centrifugal barrier. However, the codes were dissipative and parameters in which shocks could form were not known until the work of Chakrabarti [128]. As a result, in all the numerical works these shocks were found to propagate backward. Also, the theoretical was not known, and therefore the only available solutions to compare the numerical results were those of thick accretion disks which had no radial motion and no standing shocks. Thus, the comparisons were also poor. Another set of simulations were carried out with radiative flows by Eggum, Coroniti and Katz [135]. They carried out simulations for both the sub- and super-Eddington accretions. For sub-critically accreting flow, they find that initially radiation pressure supported disks quickly become gas pressure supported and collapsed on the equatorial plane, as expected from works of Lightman and Eardley [45]. A super-critically accreting flow, on the contrary, was found to form a thick disk with outflows along the vertical axis.

Although we focussed our discussion on numerical simulations on the disk-type accretion flows, a significant number of papers have been written on axisymmetric flows which form because of motion of a star through a medium. The original problem goes back to Hoyle and Lyttleton [1]. Numerical simulation study of such axisymmetric, but non-rotating, accretion was carried out by Matsuda et al. [136] and Taam and Fryxell [137]. The solution was seen to be unsteady and accretion seemed to be occurring with a ‘flip-flop’ instability. These simulations were carried out in two dimensions, but later work in the 90s in three dimensions and higher resolutions do not show these instabilities very much [138].

Theoretical results of the 70s were quantified and actual line emission profiles of a relativistically rotating moving accretion disk around black hole was calculated by Garbal and Pelat [139]. It was shown that each emission line should be split into two lines, one blue-shifted and the other red-shifted, the blue shifted one being more intense due to Doppler boosting. Similar calculations were carried out by Smak [140] and elaborated and extended later by Horne and his collaborators [141]. Doppler Tomography technique introduced by these authors is able to determine the velocity profile on the accretion disk very accurately, and is a good tool to verify if the motion is strictly Keplerian or not. In some of the broad line radio galaxies, such as 3C390.3 and ARP102B [142] the double peaks were found to vary with time in a manner that is perhaps inconsistent with a relativistic Keplerian disk and may require spiral shocks [143]. Similarly, in the ionized disk of M87, spiral patterns were observed [144] which could also be modeled with non-Keplerian disk [145].

As in the case of thin accretion disks in the 70s, thick accretion disks were also found
to be unstable under lower mode non-axisymmetric perturbation due to strong shear [146], especially when the angular momentum is constant or nearly constant. Numerical simulations devoid of radial motion did show formation of four planet-like structures due to balance of pressure gradient force and Coriolis force in such accretion disks [147]. However, a small radial motion was seen to damp out the perturbation [148]. More realistic numerical simulation of advective thick disks show that these disks are stable under axisymmetric and non-axisymmetric instabilities [149].

A frustrating experience of all the model builders had been to pinpoint the exact cause of viscosity in an accretion disk which is supposed to transport the angular momentum radially so that matter may fall in. Since radiative and molecular (and ionic) viscosities may be very small, especially for radiation pressure supported disks, one was forced to look for anomalous sources, such as magnetic turbulence [150], non-axisymmetric instabilities [146], spiral shock waves [151] and amplification of magnetic fields due to shear instabilities [152]. All these models essentially try to obtain a more realistic value of size of the interacting units (e.g., turbulent cells vis-a-vis ions) and mean velocity of these units (e.g., turbulent velocity vis-a-vis mean thermal velocity). For instance, in Coroniti’s model, magnetic flux tubes are sheared and reconnected repeatedly by the differentially rotating disks and each of the components separated by reconnection acts as a transporting agent. At spiral shocks, the normal component of the flow is squeezed while the tangential component remains unchanged thereby bending the flow direction. This process repeatedly removes angular momentum. In the case of the study of magnetic amplification made by Balbus and Hawley, an initially vertical field when perturbed radially would tend to bend rapidly due to differential motion and amplify due to shear. This process has been shown to grow in dynamical timescale. Recent simulations indicate that the resulting transport may not be in outward direction alone, and as a result the viscous mechanism may not be very efficient. All these processes indicate that $\alpha \lesssim 0.01$. In the advective disk paradigm (discussed later) solution topologies depend very strongly on the viscosity parameter, particularly for such low viscosity shocks may form if other conditions are satisfied.

There were tremendous progress in the study of jet formation in this decade. These studies generally concentrated on the wind type outflow from the funnel of thick accretion disks [153] or by magnetohydrodynamic process from all over the disk [154], or due to hydromagnetic energy extraction from black holes [155]. (For a contemporary review in jet formation from thick disks see, [156].) Numerical simulations of jets (not from disks) were also carried out with high speed super-computers to understand how the jets are collimated, how they interact with the surroundings, and under what condition cocoons, internal shocks, etc. are produced [157]. At some point it was believed that these internal shocks which manifest themselves as hot spots (in M87, for instance), could be due to periodic outbursts as in dwarf novae [158]. The explanations of the interesting behaviour at the interface of the cocoon and the jet usually lie in the combinations of the Kelvin-Helmholz and Rayleigh-Taylor instabilities [157-159]. Jet production from accretion disks which includes radiative transfer was also studied by Eggum, Coroniti and Katz [160]. Among many important results, they found that 80 percent of the energy is trapped and advected to the black hole and around 0.4 percent of the inflowing matter actually participates in jets. Shields, Mckee, Lin and Begelman [161] considered wind formation throughout the disk and as long as the the outflow rate is increased with the inflow rate and in fact larger compared to the inflow rate, they found that the X-ray luminosity heats up the corona and can cause instability in the incoming accretion rate. The source was assumed to be at the centre. Königl [162] extended self-similar models of Blandford and Payne to include self-consistently the finite
thickness of the disk itself and successfully produced self-collimated jets. Following this, Chakrabarti and Bhaskaran later showed that it is easier to produce magnetically collimated jets if the disk itself is sub-Keplerian [163]. Today, study of the properties of the jets in microquasars such as GRS1915+105 verify this that hard states which is dominated by sub-Keplerian flow close to a black hole also has dominating wind.

Though Lightman and Eardley considered Comptonization to be the mechanism of production of hard X-rays in the 70s, more detailed computation of Comptonization applicable to more general cases came later. Sunyaev and Titarchuk [164] argued that this must be emitted due to Comptonization from a hot electron cloud. Nature of emitted power-law distribution as a function of the electron temperature and optical depth of the electron cloud was derived. Its angular distribution and polarization properties were further extended for various emitting geometries [165]. Phillips and Mészáros [166] showed that disk polarization could be as high as 20 percent with harder radiation having larger degree of polarization. Success in explaining hard radiation by hot electron clouds is followed by efforts to identify the ‘Compton Cloud’, the source of the hot electrons. Burm [167] considered formation of magnetically active corona above a disk. Others considered continuous injection of hot electrons and and soft photons and showed that hard X-ray spectrum in AGNs could be explained by the Comptonization process. They also included effects of pair production [168]. Kazanas and Ellison [169] considered formation of accretion shock supported by pair plasma and the effect on the AGN spectrum was studied by Blondin and Königl [170]. If pair remains coupled to accreted matter (which is likely in presence of small magnetic field), then pair opacity would reduce the effective Eddington luminosity. As a result the emitted radiation could contribute strongly to the support the shock. Blondin and Königl [170] demonstrated this by using a simple minded model of a shock that is mediated by Fermi-accelerated relativistic protons and radiation and in which $e^+ - e^-$ pair are produced through photon-photonistic collisions. With the launching of EXOSAT in 1983 and GINGA in 1987, a large amount of data in X-Rays were available, especially from Low Mass X-ray binaries (LMXRB). A number of important discoveries were made, important among them are the Quasi-periodic Oscillations (QPOs) in in LMXRBs (in both the neutron star and black hole candidates) and X-ray pulsars. Frantic efforts followed to model these QPOs of neutron star candidates. Many galactic and extragalactic sources were also studied [171]. The observations clearly show that one requires a source of hot electrons for Compton scattering as indicated by Sunyaev and Titarchuk [164]. A number of sources such as Cyg X-1, LMC X-3, A0620-00, which were already identified to be black hole candidates by McClintock and Remillard [172] by dynamical considerations also showed similar variation of spectral properties where a blackbody plus a power-law components were needed to fit the data (c.f., [173] who consider only modified blackbody models to fit the spectra).

2.5 1990s till today: maturity - era of perfection

Accretion disk theory since 1990 till today saw perfecting a new paradigm called the Advective Disk Paradigm. The goal was to achieve a single paradigm so that observations could be explained within a single framework. There were too many ‘models’ each suitable for a specific purpose, or to explain a specific observation. We have not reached to that stage of the grand, grand unified, i.e., THE MOST GENERAL solution (necessarily time-dependent, because variabilities observed in all the time scales) which would include every possible physical processes, such as, heating, cooling, pair-production etc. and yet, carried out in totally general relativistic framework in three-dimensions. But there are every indications that ‘we are getting there’. The Advective Disk Paradigm is perhaps closest to
reality and today all the observations are explained by using some or the other predictions of this paradigm. Being proponents of this paradigm, we shall naturally spend some time on this. A number of branches in accretion physics which started in earlier decades but whose studies were extended to this decade have already been discussed earlier and will not be repeated here.

Major shift in accretion disk modeling was due to the perception that the radial motion must be included since the radial velocity becomes velocity of light on the horizon. Thus the inertial force is as important as the pressure gradient term and centrifugal pressure terms etc. Some work on spherical accretion model (which is advective any way, but not disk-like) still continued [174] partly because it is easier to handle spherical flows. Nobili, Turolla and Zampieri [175] presented very general relativistic spherical flow in which various heating/cooling and dissipation were included. Solutions of generalized rotating flow is clearly needed. In an earlier work [128] the complete classification of global solutions of an inviscid, polytropic transonic flow was presented which showed that in some region of the parameter space, the flow will have multiple sonic points [126]. Within this region, there is a sub-class of solutions where Rankine-Hugoniot shock conditions are satisfied and standing shock waves are formed due to the centrifugal barrier. Four locations, namely, $x_{si}$, ($i = 1..4$) were identified where these shocks could formally be possible, but it was pointed out that only $x_{s2}$ and $x_{s3}$ were important for accretion on black holes since the flow has to be supersonic on a black hole horizon and $x_{s1}$ could also be important for a neutron star accretion while $x_{s4}$ was purely a formal shock location. In Chakrabarti [176-177] viscosity was also added and the complete set of global solutions in isothermal VTFs with and without shocks, were presented. In the language of Shakura-Sunyaev [29] viscosity parameter $\alpha$, it was shown that if viscosity parameter is less than some critical value $\alpha_{cr}$, the incoming flow may either have a continuous solution passing through the outer sonic point, or, it can have standing shock waves at $x_{s3}$ if the flow allows such a solution in accretion. For $\alpha > \alpha_{cr}$, a standing shock wave at $x_{s2}$ persisted, but the flow now had two continuous solutions — one passed through the inner sonic point, and the other through the outer sonic point. The one passing through the inner sonic point is clearly slowly moving in most of the regions, and therefore could be optically thick when accretion rate is large enough. The one passing through the outer sonic point becomes optically thin at a large distance. A standing shock can connect these two pieces if conditions are favourable. Later analytical [178] and numerical works [179, 38, 180] showed that $x_{s3}$ is stable (indeed, at attempt of fitting AGN spectra by including $X_{s3}$ was made at this stage Chakrabarti and Wiita [181], Most importantly, these solutions show that they could join (though not quite smoothly, since viscosity parameter was chosen to be constant) with a Keplerian disk at some distances, depending on viscosity and angular momentum [176]. A two-component advective flow model (TCAF) was constructed with higher viscosity flows on the equatorial plane and lower viscosity flow away from the equatorial plane [182-184, 56] It is now widely believed that an inflow onto the black holes indeed have two components (e.g., [185]). Incidentally, numerical work of Chakrabarti and Molteni [179] was the first to establish that the non-linear solutions which included shock waves were really stable and were accurately described by Chakrabarti [128, 176] solutions. Molteni, Ryu and Chakrabarti [186] also tested these solutions with different codes. It is clear that one could use these non-linear solutions to benchmark one’s codes to check if the numerical code is dissipative or not.

In the mid 90s, some hope was raised by Narayan and his collaborators that a self-similar solution of the same equations used since 1980s may have a ‘new’ solution [54]. These were termed as Advection Dominated Accretion Flow (ADAF) which apparently advects away
most of the energies of the disk. In order to investigate if ADAF really represents any new solution branch, Chakrabarti [183] found that even when the isothermality condition is dropped, the flow topologies remained the same as in [176, 177]. The behaviour of the solution with viscosity was found to be intriguing. At very high viscosities the flow does not have shocks, and sub-Keplerian, optically thin solutions emerge out of the Keplerian disks, very similar to the Ichimaru [53] solution. However, when the viscosity is intermediate, the solution passes through a standing shock for some region of the parameter space when \( \gamma < 1.5 \). It became clear that ADAF does not represent any new solution, and is, in fact, only a special case of the transonic/advective disk solutions. ADAF’s criterion was that it should be radiatively inefficient. In that respect, by definition all the solutions of Chakrabarti [128] are faithfully one hundred percent ADAF since energy is conserved till the horizon and the entropy generated at the shock is also advected into the black hole. Naturally, some confusions followed. Chakrabarti [187-188] wrote more detail why ADAFs are not only not new, they are incorrect solutions (as obtained in [54]) as well. Actually those who could not find solutions with shocks, did not look for them. For instance, workers like Peitz and Appl [189] did not even look for shock solutions. On the contrary, those who looked for this uniquely stable and most relevant solution found them without any problem [e.g., 180, 190-191] On the one hand Narayan writes [192] “The Global Solutions described in \( \S 2 \). are free of shocks. However, Chakrabarti and his collaborators (cf. Chakrabarti and Titarchuk, 1995) have claimed, that shocks are generic to ADAFs. Initially Chakrabarti (1990) considered viscous flows under isothermal conditions, but more recently, with increased interest in ADAFs, he has switched his attention to adiabatic flows. The clearest accounts of his results are found in Chakrabarti and Titarchuk (1995) where the authors claim (i) low \( \alpha \) flows (\( \alpha \lesssim 0.01 \)) have shocks and high \( \alpha \gtrsim 0.01 \)) flows do not and (ii) low \( \alpha \) flows have sub-Keplerian rotation at large radii while high \( \alpha \) flows are Keplerian at all radii except very close to the black holes. Neither statement is confirmed in the work described in \( \S 2.4 \). In particular, no shocks are seen in any global solutions, which span a wide range of parameter values: \( \alpha \) ranging from \( 10^{-3} \) to 0.3, and \( \gamma \) ranging from \( 4/3 \) to \( 5/3 \). What is the source of the discrepancy? Narayan et al. (1997a) suggested that it may lie in different philosophies regarding angular momentum parameter \( j \) in equation (2.3). Chakrabarti simply assigns a value to \( j \) (in fact, different values in different papers) whereas this parameter ought to be treated as an eigenvalue and determined self-consistently through boundary conditions.” On the other hand, it was shown [193, 194] that indeed Chakrabarti’s solutions are not only correct, correct ADAF solution is recoverable from it with lesser difficulty if Chakrabarti’s method was followed. For instance [193] writes ‘We numerically solve the set of dynamical equations describing advection-dominated flows (ADAFs) around black holes, using a method to similar to Chakrabarti. ... . We recover the ADAF-thin disk solutions constructed by Narayan, Kato and Honma in a paper representative of previous works on global ADAF solutions, ... . Chakrabarti and his collaborators introduced a very clever procedure (e.g., Chakrabarti, 1996a). The difficulty of finding eigenvalues was simply avoided: \( R_s \) and \( j \) were to be free-parameters ... Very recently, adopting a procedure very similar to Chakrabarti’s, Igumenshchev et al. (1998) obtained global ADAF solutions .... By varying the values of two free parameters \( R_s \) and \( j \), we can find all the possible solutions: although some of the solutions constructed this way may not be physically acceptable, no physical solutions will be missed if such a ‘carpet bombing’ approach is used.” and [194] writes ‘To avoid this difficulty, Chakrabarti and his collaborators (see Chakrabarti, 1996 and references therein) introduced a very clever mathematical trick. They assumed a different set of boundary conditions, which are not only at the outer boundary, but also at the
sonic radius, and near to the black hole horizon. In this way, the most difficult part of the problem – finding the eigenvalue – is trivially solved. The lesson is therefore ADAF cannot be anything except a special case of advective/transonic solutions [176]. Paczyński, a veteran from the thick accretion disk era, also wonders in 1998, if ADAFs were new solutions [195]: “The purpose of this paper is to present a toy model of a disk accreting onto a black hole but not radiating, i.e., advection dominated, presented in the spirit of the early 1980s which seems to be simpler and more transparent than the spirit dominating the late 1990s.” Though non-advecting thick disk models were un-physical, the solutions with constant energy indeed existed before ADAF, after all. Recent efforts are on include various effects into ADAF (see, e.g., [196, 197]) to bring it back to known advective solutions. For instance, advective disks (not ADAFs) always had funnels, otherwise the flow would be unstable. This is pointed out recently [14, 196, 197].

So far, there is no observation which suggests that a new solution other what is in the advective disk paradigm, was required to explain it. The only observation [131] that the luminosity of black hole candidates should be lower compared to that of the neutron stars, which ADAFs claim to be in their support is by definition a triviality: there were black hole solutions with a 100 per cent advection of energy [128] and thus in principle, for a given accretion rate the ratio of these luminosities could be as close to zero as possible. There is no limit! Today this new terminology has created a major confusion in the subject, and any time a solution has a radial term (whatever be its accretion rate!) it is termed as an ADAF, even though it was proven that ADAF, with original sense of Narayan and Yi [54] is unstable unless accretion rate in excruciatingly low (basically no accretion). Today, workers lost track of whether a solution should be called a transonic/advective flow or simply ADAF (reference too many to cite) so that both are blissfully cited. Original proponents still think it is a new solution and stick to the definition of ADAF as the shock-free solutions of Chakrabarti [176]. Indeed readers will miss nothing in the subject if they ignore ADAF-ZDAF altogether.

Extensive numerical simulations of quasi-spherical, inviscid, adiabatic accretion flows [179, 186, 198-201] show that shocks form very close to the location where vertically averaged model of adiabatic flows predict them [128]. The flow energy is conserved and the entropy generated at the shock is totally advected into the black hole allowing the flow to pass through the inner sonic point. Flows with positive energy and higher entropy form supersonic winds. In presence of viscosity also, very little energy radiates away if the accretion rate is low. Having satisfied with the stability of these solutions [38, 179, 198-201] a unified scheme of advective accretion disks was proposed [38, 202] which combined the physics of formation of sub-Keplerian disks with and without shock waves depending on viscosity parameters and angular momentum at the inner edge. In this review on historical developments it is worth quoting from [202]: “This findings are very significant as they propose a unifying view of the accretion disks. This incorporates two extreme disk models into a single framework: for inviscid disks, strong shocks are produced, and for disks with high enough viscosity, the stable shock disappears altogether and angular momentum distribution can become Keplerian.” The solutions remained equally valid for both the black hole and neutron star accretions as long as appropriate inner boundary conditions are employed. A large number of numerical simulations with various codes (mostly in disguise of ADAF solution) have since then been done but there is no new result. However, very recently, a large number of numerical simulation results have been reported by Hawley and collaborators [203] where magnetic fields have been included. In an axisymmetric ‘cylindrical disk’ the viscous stress is even found to be close to 0.1 – 0.2.
Having advanced a great deal in the subject over last two decades with Newtonian or Pseudo-Newtonian models, it was time to pause and verify major results in Kerr geometry. Prescription to study Keplerian disks was presented by Rieffert and Harold [204] and detailed structures were computed using self-consistent vertical height by [205]. Study of transonic flows and standing shocks in Kerr geometry was made in Chakrabarti [206]. Two dimensional solutions of advective disks in Kerr geometry also showed the presence of funnels along the axis [207] very similar to the thick accretion disks. Some other advective solutions with relativistic equation of state in a different range in parameters were obtained by Gammie and Popham [208]. Such funnels were non-existent in ADAF solutions (see, Chakrabarti [14, 188] for comparison of solutions.). Similar conclusions, that the flow cannot maintain its structure (from the radiative transfer point of view) along the pole has been reached by others [195-197]. They showed that there is no way matter can be kept hot along the pole and matter must cool down. Indeed, in [208] it was shown that a single component so-called ADAF flow becomes cooled by the soft-photons from a Keplerian disk of the same accretion rate. That is why for most of the explanations of the observations of black hole candidates one requires two components [56].

Several other developments were taking place in the theory of advective flows particularly in the MHD limit and when the flow is non-axisymmetric. Chakrabarti [209] found all the global solutions of Weber-Devis type of equations. In this case, the nature of the magnetic field is predefined to be radial and azimuthal, but nevertheless the solutions indicated new types of sonic points in both accretion and winds. Instead of three sonic points (two X-types and one ‘O’ type), one obtains five sonic points (three X-type: slow magnetosonic, Alfvénic and fast magnetosonic and two ‘O’ type). Shocks were also found analytically for the first time. Since then similar solutions have been found for cold flows where pressure and gravity effects were ignored (see, [210] and subsequent works of this group). However, till today, no new solution has been found other than those in [209].

With the advent of detailed and accurate observations which demanded more accurate solutions for proper explanations, it became clear that the flow with standing [128] or oscillating shocks [108] are more relevant than ever before to describe spectral properties of accreting black holes. In fact, the property of the Centrifugal pressure supported Boundary Layer (or, CENBOL for short), be it emission of radiation or driving winds and outflows is closely linked to what is observed from the vicinity of a black hole candidate. We refer to the readers a few reviews written to summarize the properties of advective disk paradigm [70, 211-213] where ‘basic building blocks’ of all possible solutions (as well as non-solutions which are clearly marked) are presented and we do not go into details in them. In realistic cases, neither the viscosity parameter is constant, nor the accretion rate remains constant. More importantly, one needs to consider processes such as Coulomb coupling between ions and electrons, bremsstrahlung, Comptonization etc. Also important is the availability of the driving forces (such as thermal pressure, centrifugal, magnetic etc.) to form outflows. Depending on these factors, a realistic flow would be made up of combinations of these basic building blocks of the inflow and outflow.

One relevant matter in this regard is the ratio of the outflow rate to the inflow rate $R_{\dot{m}}$. Already in the 80s numerical simulations were carried out to show that very little matter comes out of the disk. Kusunose [214] considered two temperature Keplerian disks and found that in some cases more than 50% mass loss is possible. In more recent works, Chakrabarti [212, 215] gave analytical estimate of mass loss rate as a function of the shock compression ratio and Das and Chakrabarti [216] carried out this analysis using sonic point method. They showed that $R_{\dot{m}}$ depend on the strength of the shock, when it forms [212,
One could argue that in soft states, when the accretion rate is high, the inner sub-Keplerian region is cooled down [56] and the shock is as good as non-existent and there is no driving force to produce outflows [217]. In the hard states, on the contrary, when the accretion rate is low, the stronger shocks also cause a very low outflow rate. In the absence of shocks, driving force is weak, and the outflow may be negligible (in the absence of strong magnetic fields). For intermediate shock strength, the outflow rate is high and it could be cooled down by soft photons from the pre-shock Keplerian disk. Once the flow is cooled down, it would be super-sonic and fly away but the terminal velocity would be low. This process may take place repeatedly [217, 218] and cause low frequency variations of the light curve. In this so called ‘flare’ state, the outflow would be ‘blobby’. These works therefore directly indicated that there should be direct relation of the outflow rate with the spectral states. As far as the radiative acceleration of the outflows goes, the situation is far from clear. On the one hand radiation from the CENBOL is bound to deposit momentum on the outflow, but quick acceleration will also cause it to slow down due to radiation drag effect. Acceleration of particles are difficult from a Keplerian Disk [219] but could be a bit more efficient when the photon is focussed, for instance, from the CENBOL [220] of an advective disk.

Two major types of oscillations of the centrifugal pressure supported boundary layer is discussed in the literature, and it is possible that both are important. In one case the steady shock solution is absent even when the inner sonic point exists [200]. In this case, the flow searches for a steady solution by first generating entropy through turbulence and then trying to pass through the inner sonic point non-existent in a steady flow. In another type [108] the steady shock solution exists but the cooling time in the post-shock/corona region roughly agrees with the infall time in this region. An oscillation of similar kind is seen at the transition radius between Keplerian and sub-Keplerian flows as some matter is removed as winds from this region [187, 188, 221]. This is present even when no shocks are produced. A fourth type of oscillation may be set in due to the quasi-periodic cooling of the outflow (provided it is high as is the case for average shock strengths $R \sim 2.5 - 3$ see, [212, 215]) due to enhanced interception of the soft-photons emitted from a Keplerian disk. It can be assumed that the quasi-periodic oscillation (QPO) in X-rays observed in black hole candidates is the result of one or more of these different types of oscillations since other types of oscillations, such as those due to trapped oscillations [106] or disko-seismology [103] are incapable of modulating X-rays with large scale amplitude. Recently, Molteni et al. [222] have discovered, through yet another numerical simulation where matter is injected both from the upper and lower halves, that the inflow interacts with the outflow and bends like an ‘warped’ disk. This has some resemblance with the suggestion of Pringle [223] where warping at the outer edge could form due to the interaction with radiation emitted at the central region.

Up until 7-8 years ago, it was believed that the hard radiation is the result of Comptonization of soft photons by ‘Compton Clouds’ floating around the disk or by hot corona. Haardt and Maraschi [224] considered two phase accretion in AGNs, one with cold component along the equatorial plane, and the other with hot component above the cooler component. Chakrabarti and Titarchuk [56] for the first time pointed out that the so-called ‘Compton Cloud’ is the sub-Keplerian inner edge of the disk itself which is puffed up due to heat in CENBOL! Here the higher viscosity Keplerian disk flows on the equatorial plane inside a sub-Keplerian flow which may have standing or oscillating shock waves (This ‘two component advective flow’ [TCAF] model is to be contrasted with the earlier models [47, 53] which considered only the Keplerian disks.) Thus, a new paradigm of accretion
disks, based on actual solution of advective flows, started. Today, this picture is universally adopted in most of the models of accretion flows. In the hard state, Keplerian flow rate could be very small while the sub-Keplerian rate could be very high [56]. In the soft state, it is the opposite. The de-segregation of matter into these two types of rates are believed to be due to the fact that the flow closer to the equatorial plane is likely to be more viscous and with larger Shakura-Sunyaev parameter $\alpha$, and therefore is likely to be Keplerian. However, flows away from the equatorial plane may have smaller $\alpha$ and therefore they deviate from a Keplerian disk farther out [70, 183, 211]. Contribution to this sub-Keplerian flow may also have come from wind accretion. This sub-Keplerian flow may form shocks at around $10 - 20R_g$. In the soft state, shocks may be nominally present, but would be cooler due to Comptonization, and would be as good as non-existent. When the viscosity and flow accretion rate is large, the sub-Keplerian region shrinks as it is cooled down by thermal Comptonization. Only the bulk motion Comptonization will take place in flows at around $1 - 3R_g$ thereby producing hard tails in soft states [56, 225]. Outflow also softens the spectra mimicking those from bulk motion Comptonization [226], but as discussed above, it is unlikely that there would be strong outflows in the soft state due to the lack of driving forces. Now-a-days there are very strong evidences in black hole candidates such as GRS1915+105 that during outflows spectral softening would take place and when some of the failed outflow falls back, the spectrum is hardened [227].

Similar to Quasars, observers early in this decade discovered that some of the galactic black hole candidates such as GRS 1915+105 also have relativistically moving outflows [228] which were termed as micro-quasars. A shift in understanding that the outflow may actually be coming from a small boundary layer (CENBOL) of a black hole rather than from all over the disk got observational supports from variability data of this microquasar where it is seen that radio variabilities in the jet are strongly correlated with that of the Comptonising region. In active galaxies, even the direct observations [229] showed that the base of the jet in M87 is very narrow, only a few tens of the Schwarzschild radii. In certain epochs, when the anti-correlation was observed in the X-ray and radio intensities in the black hole candidate GRS 1915+105 such concept of the localization of the jet formation was first proposed [70, 182]. It was suggested that a strong field in a hot gas feels magnetic tension (‘rubber-band effect’) and is contracted catastrophically evacuating the post-shock flow, i.e., the inner part of the accretion disk. Quoting [70]: “Since the inner part of the accretion disk could literally disappear by this magnetic process, radio flares should accompany reduction of X-ray flux in this objects. Since the physical process is generic, such processes could also be responsible for the formation of jets in active galaxies and similar anti-correlation may be expected, though time delay effects are to be incorporated for a detailed modeling.” Recently this has been quantified [230] and it was shown that the magnetic flux tubes indeed collapse with Alfven speed inside the CENBOL region.

As discussed, effort is on to understand various phenomena using a single advective disk paradigm perhaps to ‘wrap up’. We present here some of the observations and show that these can be addressed by the advective disk paradigm quite satisfactorily.

*Sub-Keplerian motion on a large scale:* Since shocks are transitions from supersonic to subsonic motion, and since supersonic flows are sub-Keplerian [70, 183], any presence of shocks in a disk would indicate sub-Keplerian motions. Sub-Keplerian flows rotate slower, and velocity predicted from Doppler shifted disk emission lines would correspond to a higher central mass. Chakrabarti [145] pointed out that the disk around M87 contained spiral shocks and therefore the flow must be sub-Keplerian. As a result, the mass of the central object was found to be around $4 \times 10^9 M_\odot$ rather than $2 \times 10^9 M_\odot$ as predicted by Harms
et al. [144] (see, [231] for references) with calculations purely based on Keplerian motion. The fact that shock-ionization causes the emission processes on M87 disk has been stressed recently by several others [232].

**Sub-Keplerian motion on a small scale:** It is the usual practice to assume that the inner edge of a Keplerian disk extends till $3R_g$, the marginally stable orbit. However, the advective disk models [70, 183] show that the inner edge could extend to $\sim 10 - 20R_g$ where the CENBOL should form. There are overwhelming evidence today that this is indeed the case [233].

**Power-law hard radiation in very high states:** Chakrabarti and Titarchuk [56] pointed out that when the accretion rate is relatively high, the electrons in the sub-Keplerian region become cooler and this region becomes practically indistinguishable from that of Keplerian disk. However, very close to the black hole horizon, matter moves with almost velocity of light and deposits its bulk momentum onto the photons thereby energizing these photons to very high energy forming a power-law. This power-law is the hallmark of all the known black holes [234]. The success of this model crucially hangs on the transonic flow solution which utilizes the fact that the inner boundary condition is independent of the history of incoming matter.

**Quasi-Periodic Oscillations from black hole candidates:** X-rays from galactic black hole candidates often show persistent oscillations which are quasi-periodic in nature. Advective disk solutions do allow oscillations especially in the X-ray emitting regions. The observation of these oscillations during the transition of states is the triumph of the advective disk model (e.g., [235]). Chakrabarti and Manickam [218] proves that the oscillation occurs in hard X-rays only strongly pointing to the shock-oscillation origin, as in this model post-region acts as the Comptonising region [56].

**Outflows and their effects on the spectral properties:** Recent high resolution observations of jets in M87 strongly suggest that they are produced within a few tens of Schwarzschild radii of the horizon [229], strongly rejecting ADAF (and its variants such as ADIOS [236]) models for the outflows, which has no special length scale at these distances. Given an inflow rate, one is now capable of computing the outflow rate when the compression ratio at the shock surface is provided [212, 215]. This solution naturally predicts that the outflow must form at the CENBOL. The Globally complete Inflow-Outflow Solutions (GIOS) were also found [216, 236]. In presence of winds, the spectra is modified: hardening of the soft-state and softening the hard state is predicted and is observed in GRS1915+105 [227].

**Quiescence states of black holes:** Chakrabarti [188] and Das and Chakrabarti [216], pointed out that in some regions of the parameter space, the outflow could be so high that it evacuates the disk and forms what is known as the quiescence states of black holes. A well known example is the starving black hole at Sgr A* at our galactic centre whose mass is $\sim 2.6 \times 10^6 M_\odot$ and the accretion rate is supposed to be around $\sim 10^{-5} M_\odot$ yr$^{-1}$ which is much smaller compared to the Eddington rate. Quiescence states are also seen in stellar mass black hole candidates such as A0620-00 and V404 Cygni. Another way of producing these states is to use well known viscous instability in an accretion disk as used in models of dwarf-novae outbursts [56, 238].

**On and Off-states during QPOs in black holes:** The black hole candidates GRS1915+105 displays a variety of behaviour: usual high frequency QPOs are frequently interrupted by low frequency oscillations. While the QPO frequency can be explained by the shock oscillations, the switching of on and off states is explained by the duration in which extended corona becomes optically thick. It is possible that outflows are slowed down by this process and the matter falls back to CENBOL, extending the duration of the ‘on’ state (which,
if exists, is found to be comparable to the duration of the ‘off’ state [239]). The QPO frequency does evolve during this time scale and it is suggested that this is due to the steady movement of the inner edge of the Keplerian accretion disk [239] or the steady movement of the shock itself in viscous time scale [218]. The correlation between the duration of the off-state and the QPO frequency has been found to agree with the observations [218].

Quick state transition in GRS 1915+105: Very accurate observations of the galactic black hole candidate GRS 1915+105 brought a large number of challenges along with it. QPO frequencies changed everyday in unpredictable ways. Classification of light curves were carried out by Belloni et al. [241] and subsequently rearranged in the order by Nandi et al. [242]. In particular Belloni et al. [241] pointed out that GRS1915+105 likes to go from a harder state (C) to softer state (B) through another state (A). Using the advective disk paradigm, such behaviour could be fully understood [243, 227]. So far, no alternative explanations exist of this curious behaviour.

Relationship of outflows and the black hole states: It has been already pointed out that in advective disks hard states (shock strength is high) should have smaller outflow rates than the intermediate states (intermediate shock strength), but there should be virtually no outflow in the very soft states (shock non-existent). This picture has gained some support as well from observations [244].

Polarization of AGNs: Several studies of polarization suggests that in AGN, polarization matches with thick disk models more accurately than those obtained from the thin disk models [245]. Kartje and Königl [246] found that due to multiple reflection on the funnel wall polarization could be as high as 10% and suggested that polarization of X-ray selected BL Lac objects may be due to this disk geometry. A post-shock flow is almost as good as a thick disk, only better since the radial motion is included. Even without shocks, funnel walls are produced in advective disks of weaker viscosity. Thus perhaps signatures of such disks have been observed by this method.

The 90s also saw a few major modifications of the standard accretion disk model itself. Even though inner disk needed major modifications by introduction of the radial motion, the Keplerian disk is still a major source soft photons in the spectrum from AGNs and X-Ray binaries and such modifications were thus quite natural and welcome additions. For instance, Wandel and Liang [247] gave analytical solutions for two distinct states of a Keplerian disk, one being cool (≈10^6 K) and other hot (≈10^9 K). Both single temperature and two temperature solutions were considered. Another study was to compute spectral hardening factor f. If electron scattering is the dominating opacity source and if the X-Ray spectrum is affected by Comptonization the local flux in the X-ray range is expected to form a Wien peak whose intensity is approximated by a diluted black body \( F_\nu \sim \pi B_\nu(fT_{eff})/f^4 \) where \( \nu, T_{eff}, f, \) and \( B_\nu \) are the frequency, effective temperature, spectral hardening factor and Planck function respectively. This work was extended to include transonic disks as well [248].

Shimura and Takahara [249] showed that the emitted radiation spectrum from a Keplerian disk around a black hole could be described by a diluted black body spectrum with a hardening factor \( f \sim 1.8 - 2.0 \) for rates close to the Eddington rate and \( \sim 1.7 \) for rates close to 10 percent of the Eddington rate. These factors must be taken into account while estimating the mass of a black hole.

There was remarkable improvements in understanding the nature of our Galactic Centre in this decade. Kinematic considerations fitting [Ne II] line map showing continuous variation of velocity along a spiral pattern showed that the mass of the black hole at the Center could be around \( 2 \pm 0.5 \times 10^6 M_\odot \) [250]. Preliminary fits of its spectrum by dis-
sipating spherical accretion model also argued for its mass about this high [251]. More recent studies of motion of stars around the center yields the estimate of the central mass to be converging to $2.65 \times 10^6 M_\odot$ [252]. From polarization measurements any of the ADAF solutions has been completely ruled out [253]. It seems to be a clear case of an advective disk with a very small accretion rate.

It was pointed out [254, 255] that since advective disks close to a black hole need not be Keplerian, it would affect the gravitational wave properties of a coalescing binary. The angular momentum loss to the disk by the companion in comparison to the angular momentum loss to gravitation wave was found to be very significant [255]. The variation of the signal is shown in [212]. One of the exciting predictions of this scenario is that since the spectrum of an accretion disk contains a large number of informations (e.g. mass of the central black holes, distance of the black hole, accretion rate, and viscosity parameter) a simultaneous observation of the electromagnetic spectrum from the disk and the gravity wave spectrum (which also must depend on those parameters, except possibly the distance) should tighten the parameters very strongly. Recently, ADAF model of extremely low accretion rate was used to repeat these computations, and not-surprisingly, no significant change in gravitational signal was found [256]. If little matter is accreting, it is as good as having no accretion disk at all. ADAF model ideally valid for zero accretion rate systems and not found to be true in any of the realistic accreting systems. As discussed earlier, evidences of the oscillating shocks, bulk motions etc. (which are absent in ADAF) are abundant. Thus it is likely that the gravitational signal would be affected exactly in a way computed by [255].

In the 80s iron lines were observed in galactic and extra-galactic sources [171]. The observers also interpreted that the iron-line may be emitted from the inner part of the accretion disk, where the Shakura-Sunyaev disk may be cooler [257]. They consider shining cooler disk of high mass accretion rate with extended sources of X-rays which photo-ionize cooler disk in order to produce observed equivalent width of several hundreds of electron volts. Recently one of the wings has been seen to have been strongly red-shifted giving impression that the emission is extremely close to a black hole. Details of computation of the nature of the line profile is in [258]. While it is unclear if disks around a stellar mass black hole could be as cooler as is required for these lines, in AGNs the prospect is better, where variation of these lines can give informations about inhomogeneities of the disk. Placing the X-ray source to excite the lines is a major problem. Some suggestions as to how the correlation of short time-scale X-ray variabilities with variation of iron line emission could be used to infer the mass and spin of AGNs are given by Reynolds et al. [259]. The interpretation of disk origin of the iron-line is not unique however, since in the modern understanding of the accretion disk model, standard Keplerian disk very often does not extend to the marginally stable orbit, but is truncated farther out. It is believed that one could produce iron-line in outflows as well [56].

Resonance lines of iron have been seen in several black hole candidates [260, 261]. Usually one of the two observed wings is found to be stretched compared to the other and it is explained to be due to the combination of the Doppler shift and the gravitational red-shift. Generally, it is difficult to explain very large equivalent width of the lines in this models. This problem can be circumvented if the lines are assumed to be coming from outflowing winds. The stretched wing would then be due down-scattered emission lines [56]. The idea of line emissions from the winds is finding supports by other workers as well [261-263].
3 Concluding remarks

It is clear that since a ‘carpet bombing’ method was used, solutions of accretion disks presented in [56], [176] and [183] are the most general ones and the method of obtaining them are the least difficult of all. All the outstanding issues related to the black hole accretion, namely, the disk-jet connection, spectra of disks, relatinship between the spectral states and jet formation quasi-periodic oscillations, variations of the Comptonizing region, etc. are explained with such solutions or the time-dependent solutions (e.g. [108], [222] etc.). Concerted future efforts should be made to understand the observations on the basic of these general solutions rather than making new ‘models’ which are at best hand-waiving. To keep up with competition, observers very often tend to ‘listen and learn’ the basic results from theoretical solutions and make new models with these salient features without ever referring to the original theory! Particularly latter has slowed down the progress considerably in recent years since it is causing confusion especially for the new-comers.

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