MREIT conductivity imaging based on the local harmonic $B_z$ algorithm: animal experiments

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Abstract. From numerous numerical and phantom experiments, MREIT conductivity imaging based on harmonic $B_z$ algorithm shows that it could be yet another useful medical imaging modality. However, in animal experiments, the conventional harmonic $B_z$ algorithm gives poor results near boundaries of problematic regions such as bones, lungs, and gas-filled stomach, and the subject boundary where electrodes are not attached. Since the amount of injected current is low enough for the safety for in vivo animal, the measured $B_z$ data is defected by severe noise. In order to handle such problems, we use the recently developed local harmonic $B_z$ algorithm to obtain conductivity images in our ROI(region of interest) without concerning the defected regions. Furthermore we adopt a denoising algorithm that preserves the ramp structure of $B_z$ data, which informs of the location and size of anomaly. Incorporating these efficient techniques, we provide the conductivity imaging of post-mortem and in vivo animal experiments with high spatial resolution.

1. Introduction
Magnetic resonance electrical impedance tomography (MREIT) attempts to provide static conductivity images of an electrically conducting object with high spatial resolution [1–7]. We sequentially inject multiple currents through chosen pairs of surface electrodes to produce internal current density $\mathbf{J} = (J_x, J_y, J_z)$ and also magnetic flux density $\mathbf{B} = (B_x, B_y, B_z)$ distributions inside the object. In order to reconstruct conductivity images, MREIT uses an MRI scanner to measure a set of $B_z$ data where $z$ is the direction of the main magnetic field of the scanner [8]. The conductivity image reconstruction in MREIT is based on the relationship among the conductivity distribution $\sigma$, the applied current $I$, and the measured $B_z$ data:

$$B_z(r) = \frac{\mu_0}{4\pi} \int_{\Omega} \frac{\sigma(r') \left[(x-x') \frac{\partial}{\partial r}(r')-(y-y') \frac{\partial}{\partial r'}(r')\right]}{|r-r'|^3} \, dr' + \text{lead wire effects}$$

where $u$ is the electrical potential generated by the injection current $I$ that satisfies the following mixed boundary value problem:

$$\begin{cases}
\nabla \cdot (\sigma \nabla u) = 0 & \text{in } \Omega \\
I = \int_{\mathcal{E}^+} \sigma \frac{\partial u}{\partial n} \, ds = - \int_{\mathcal{E}^-} \sigma \frac{\partial u}{\partial n} \, ds \\
\nabla u \times \mathbf{n} = 0 & \text{on } \mathcal{E}^+ \cup \mathcal{E}^- \\
\sigma \frac{\partial u}{\partial n} = 0 & \text{on } \partial\Omega \setminus (\mathcal{E}^+ \cup \mathcal{E}^-)
\end{cases}$$

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Here, \( \mathbf{n} \) is the unit outward normal vector to \( \partial \Omega \), \( \mathbf{r} = (x, y, z) \), \( \frac{\partial u}{\partial \mathbf{n}} = \nabla u \cdot \mathbf{n} \), and \( \mu_0 \) the magnetic permeability. The first \( B_2 \)-based MREIT reconstruction algorithm called the harmonic \( B_2 \) algorithm was proposed in [9]. Numerous numerical and experimental studies have shown that it is quite successful in producing high resolution conductivity images. However, in animal experiments, the conventional harmonic \( B_2 \)-algorithm shows poor performance in providing accurate conductivity reconstruction near boundaries of problematic regions including bones, lungs, air-filled stomach, and the subject boundary. The recent MREIT algorithm called local harmonic \( B_2 \)-algorithm [10] was designed to deal with image defects caused by the presence of those problematic regions. In the local harmonic \( B_2 \)-algorithm, the quality of the conductivity image is mainly dependent upon the quality of the \( B_2 \) data. Recently a new denoising method [11] was proposed to preserve ramp structures in the \( B_2 \) data, that shows the change of conductivity.

In this paper, we examine the local harmonic \( B_2 \)-algorithm with the ramp preserving denoising method for the \( B_2 \)-data from \textit{in vivo} and post-mortem animal experiments.

2. Methods

2.1. The local harmonic \( B_2 \)-algorithm: sub-regional reconstruction

Let the subject to be imaged occupy a three-dimensional bounded domain \( \Omega \subset \mathbb{R}^3 \) with a smooth connected boundary \( \partial \Omega \). We assume that the subject \( \Omega \) contains a region with low conductivity. In practice, the region may include bones, lungs, and an air-filled stomach. We denote by \( \Omega_{z_0} := \Omega \cap \{z = z_0\} \subset \mathbb{R}^2 \), the slice of \( \Omega \) cut by the plane \( \{z = z_0\} \). We assume that the conductivity \( \sigma \) is isotropic. We inject two independent electrical currents \( I_1 \) and \( I_2 \) through two pairs of surface electrodes \( E_1^\pm \) and \( E_2^\pm \), respectively. For \( j = 1 \) and \( 2 \), let \( u_j[\sigma] \) be a solution of (2) corresponding to \( I_j \). The \( z \)-component of the curl of the Ampere law \( \nabla \times \mathbf{J} = \frac{1}{\mu_0} \nabla \times \nabla \times \mathbf{B} \) is

\[
\langle \nabla \sigma \ , \ \mathbf{L} \nabla u_j[\sigma] \rangle = \frac{1}{\mu_0} \nabla^2 B_{z,j}
\]

where \( \mathbf{L} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \). The above identity means that the data \( \frac{1}{\mu_0} \nabla^2 B_{z,j} \) can convey any local change of \( \sigma \) along the direction \( \mathbf{L} \nabla u_j[\sigma] \). We know that the vector field \( \sigma \nabla u_j[\sigma] \) is dictated mainly by injection current \( I_j \) and the global geometry of the domain \( \partial \Omega \) instead of the local distribution of \( \sigma \) provided the magnitude of \( \nabla \sigma \) is small. Indeed, we may approximate \( \sigma \nabla u_j[\sigma] \approx \nabla u_j[\sigma_0] \) where \( \sigma_0 = 1 \). Taking account of this approximation of the vector field \( \sigma \nabla u_j[\sigma] \) in our mind, it is desirable that (3) is rewritten in the following from:

\[
\langle \nabla \ln \sigma \ , \ \mathbf{L} \sigma \nabla u_j[\sigma] \rangle = \frac{1}{\mu_0} \nabla^2 B_{z,j} .
\]

Combining the identity (4) for \( j = 1, 2 \), we have

\[
\begin{bmatrix}
\frac{\partial \ln \sigma}{\partial x}
\frac{\partial \ln \sigma}{\partial y}
\frac{\partial \ln \sigma}{\partial z}
\end{bmatrix} = \frac{1}{\mu_0} \left( \mathcal{A}[\sigma](\mathbf{r}) \right)^{-1} \begin{bmatrix}
\nabla^2 B_{z,1}(\mathbf{r}) \\
\nabla^2 B_{z,2}(\mathbf{r})
\end{bmatrix}, \quad \mathbf{r} \in \Omega
\]

where \( \mathcal{A}[\sigma](\mathbf{r}) = \begin{bmatrix}
\sigma(\mathbf{r}) \frac{\partial u_1[\sigma]}{\partial x}(\mathbf{r}) & -\sigma(\mathbf{r}) \frac{\partial u_2[\sigma]}{\partial x}(\mathbf{r}) \\
\sigma(\mathbf{r}) \frac{\partial u_1[\sigma]}{\partial y}(\mathbf{r}) & -\sigma(\mathbf{r}) \frac{\partial u_2[\sigma]}{\partial y}(\mathbf{r}) \\
\sigma(\mathbf{r}) \frac{\partial u_1[\sigma]}{\partial z}(\mathbf{r}) & -\sigma(\mathbf{r}) \frac{\partial u_2[\sigma]}{\partial z}(\mathbf{r})
\end{bmatrix} \). Unfortunately, in \textit{in vivo} animal experiments, \( B_2 \) data is very noisy due to the requirement of low amplitude of injection currents. Hence, before solving the equation (5), we need pre-processing such as harmonic fill-in of defected regions, denoising of \( B_2 \), and so on.

2.2. Preprocessing

2.2.1. Harmonic fill-in of defected regions. We first deal with locating defected region \( R \) where MR signal is very low. Numerous animal experiments show that the conventional inpainting
method [12] of solving Dirichlet problem in the segmented domain $R$ is not appropriate since it causes some artifacts near $\partial R$ and requires additional segmentation process which is highly dependent on user’s manual work. One way is to take isotropic diffusion into $R$ from the reliable data in the neighborhood of $R$. This method does not require to get the boundary $\partial R$ via segmentation for solving the harmonic equation.

2.2.2. Ramp preserving denoising. In [11], authors proposed a new ramp preserving denoising method which utilizes the structure tensor of the image gradient to identify ramp structure of the data image. This method takes advantage of eigenvalue analysis of the structure tensor to eliminate salt-pepper type noise while preserving ramp structure of the noisy data.

3. Results
Based on the reconstruction method mentioned in the previous section, we performed animal experiments with in vivo canine head and post-mortem canine pelvis using a 3.0 Tesla MRI scanner. Figure 1 shows the conventional MR image, segmentation with region $R$, $B_z$ data, and its denoised $B_z$ data of the in vivo canine head. Figure 2-(a) shows an autoscale image of the reconstructed conductivity distribution of the in vivo canine head by the conventional harmonic $B_z$-algorithm. Because of serious reconstruction errors near the boundary and in the defected region, the autoscale image in Figure 2-(a) cannot provide any useful information of the conductivity distribution inside the domain. Figure 2-(b) is its scaled image adjusted for the best view. Figure 2-(c) is the result of the local harmonic $B_z$-algorithm with denoising and the harmonic fill-in of $R$. In Figure 2-(c), we see the brain of in vivo canine through a local window. We see that the local harmonic $B_z$ algorithm combined with denoising and harmonic fill-in provides better image of canine brain than the conventional harmonic $B_z$ algorithm.
Figure 3. (a) Conventional MR image, (b) segmentation with the region $R$, (c) $B_z$ data of which injection current direction is horizontal, (d) its denoised $B_z$ data.

Figure 4. (a) Reconstructed image using the harmonic $B_z$-algorithm, (b) Its scaled version for the best view, (c) Reconstructed image using the local harmonic $B_z$-algorithm.

Figure 3 shows the conventional MR image, segmentation with region $R$, $B_z$ data, and its denoised $B_z$ data of the post-mortem canine pelvis. Figure 4-(a) shows a reconstructed image by the harmonic $B_z$-algorithm. Because of the boundary artifact, we cannot see the scaled conductivity distribution information in the domain. Figure 4-(b) is its scaled image adjusted for the best view. Figure 4-(c) is the result of the local harmonic $B_z$-algorithm with denoising and the harmonic fill-in of $R$.

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References
[1] Woo E J, Lee S Y and Mun C W 1994 SPIE 2299 377–385
[2] Kwon O, Woo E J, Yoon J R and Seo J K 2002 IEEE Trans. Biomed. Eng. 49 160–167
[3] Oh S H, Lee B I, Woo E J, Lee S Y, Cho M H, Kwon O and Seo J K 2003 Phys. Med. Biol. 48 3101–3116
[4] Ider Y Z, Onart S and Lionheart W R B 2003 Physiol. Meas. 24 591–604
[5] Park C, Kwon O, Woo E J and Seo J K 2004 IEEE Trans. Med. Imag. 23 388–394
[6] Park C, Park E J, Woo E J, Kwon O and Seo J K 2004 Physiol. Meas. 25 257–269
[7] Liu J J, Seo J K, Sini M and Woo E J 2007 SIAM J. Appl. Maths 67 1259–1282
[8] Scott G, R Armstrong M J and Henkelman R 1992 J. Mag. Res. 97 235–254
[9] Seo J K, Yoon J R, Woo E J and Kwon O 2003 IEEE Trans. Biomed. Eng. 50 1121–1124
[10] Seo J K, Kim S W, Kim S, Liu J, Woo E J, Jeon K and Lee C-O 2008 IEEE Trans. Med. Imag. 27 1754–1761
[11] Lee C-O, Ahn S and Jeon K 2009 Denoising of $B_z$ data for conductivity reconstruction in magnetic resonance electrical impedance tomography (MREIT) (Preprint KAIST DMS BK21 Research Report Series/09-16)
[12] Lee S, Seo J K, Park C, Lee B I, Woo E J, Lee S Y, Kwon O and Hahn J 2006 IEEE Trans. Med. Imag. 25 168–176