We analyze the recent result of the CHOOZ collaboration in the context of mixing and oscillations between all the three neutrino flavors. If one assumes the hierarchy among the vacuum mass eigenvalues $\delta_{21} \ll \delta_{31}$ where $\delta_{21} = \mu_2^2 - \mu_1^2$ and $\delta_{31} = \mu_3^2 - \mu_1^2$, then the CHOOZ result puts a strong constraint on the allowed values of the $(13)$ mixing angle $\phi$. It is also shown that in light of the CHOOZ result, the maximum contribution of the $\nu_\mu \leftrightarrow \nu_e$ oscillation channel to the atmospheric neutrino anomaly is less than 9 percent, thus demonstrating that the atmospheric neutrino anomaly is mainly due to $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. Most importantly the CHOOZ result now excludes a large part of the three flavor parameter space which was previously allowed as solutions to the solar and atmospheric neutrino problems.

PACS numbers: 14.60.Gh, 96.60.Kx, 95.30.Cq, 96.40.Tv
The CHOOZ collaboration, which searches for signals of $\bar{\nu}_e \rightarrow \bar{\nu}_x$ oscillations, where $x$ can be any other flavor, in the disappearance mode of the original flavor has recently reported the results of its first run [1]. They see no evidence of oscillations of the original flavor. They have analyzed their results assuming two flavor oscillations between $\nu_e$ and another flavor and gave an exclusion plot in the parameter space spanned by the mass squared difference $\Delta m^2$ and the mixing angle $\theta$. Their main result is that for $\Delta m^2 > 3 \times 10^{-3} eV^2$, $\sin^2(2\theta)$ must be less than 0.18. While this is a strong constraint, we remark that it has to be confirmed by an independent experiment. Nevertheless we may ask what are the consequences if we accept the CHOOZ result.

We reinterpret the CHOOZ result in terms of oscillations between the three active neutrino flavors. This is a more realistic framework because it is established that there are three light neutrino flavors whose interactions are prescribed the Standard Model. It is more natural to assume that all three of the light neutrinos mix with one another.

The flavor eigenstates are related to the mass eigenstates by

$$
\begin{bmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{bmatrix} = U
\begin{bmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{bmatrix},
$$

(1)

Here we can take, without loss of generality, that $m_3 > m_2 > m_1$. The unitary matrix $U$ can be parametrized as

$$
U = U^{23}(\psi) \times U^{\text{phase}} \times U^{13}(\phi) \times U^{12}(\omega),
$$

(2)

where $U^{ij}(\theta_{ij})$ is the two flavor mixing matrix between the $i$th and $j$th mass eigenstates with the mixing angle $\theta_{ij}$. For simplicity, we neglect the CP violation and set $U^{\text{phase}} = I$. The vacuum oscillation probability for a neutrino of flavor $\alpha$ to oscillate into a neutrino of flavor $\beta$ is given by

$$
P_{\alpha\beta}^0 = (U_{\alpha1}U_{\beta1})^2 + (U_{\alpha2}U_{\beta2})^2 + (U_{\alpha3}U_{\beta3})^2 + 2 U_{\alpha1}U_{\alpha2}U_{\beta1}U_{\beta2} \cos \left(2 \frac{\delta_{21}}{E}\right) +
2 U_{\alpha1}U_{\alpha3}U_{\beta1}U_{\beta3} \cos \left(2 \frac{\delta_{31}}{E}\right) + 2 U_{\alpha2}U_{\alpha3}U_{\beta2}U_{\beta3} \cos \left(2 \frac{\delta_{32}}{E}\right),
$$

(3)
where $d$ is the distance travelled in meters, $E$ is in MeV, and mass squared differences are in eV$^2$. We may also note the vacuum oscillation probabilities are same as in eq. (3) for the case of antineutrinos because CP violation is neglected. If we assume the hierarchy among the neutrino mass eigenstates $\delta_{31} \gg \delta_{21}$, and that $\delta_{21}$ is about $10^{-5}$eV$^2$, which is required to fit solar neutrino data [4], then the oscillatory term involving $\delta_{21}$ can be set to one. The oscillation probability relevant for the CHOOZ experiment is the electron neutrino survival probability $P_{ee}$ which is easily computed from eq. (3) to be

$$P_{ee} = 1 - \sin^2 2\phi \sin^2 \left(1.27 \frac{d \delta_{31}}{E}\right).$$

(4)

Notice the interesting point that this involves only the (13) mixing angle $\phi$, and because of the heirarchy the (12) mixing angle $\omega$ disappears from the probability. So we reinterpret the CHOOZ result [1], to be that for $\delta_{31} > 3 \times 10^{-3}$, $\sin^2(2\phi)$ must be less than 0.18, i.e $\phi < 12.5^\circ$.

We now estimate the maximum contribution of the $e - \mu$ channel to the atmospheric neutrino anomaly. Since the relevent $\delta_{31}$ is about $10^{-2}$eV$^2$, matter effects are negligible for the problem [3]. Hence the relevant probability is the vacuum $\nu_e \leftrightarrow \nu_\mu$ oscillation probability,

$$P_{\mu\bar{e}} = P_{\mu e} = \sin^2 2\phi \sin^2 \psi \sin^2 \left(1.27 \frac{d \delta_{31}}{E}\right).$$

(5)

Note that both $\phi$ and $\psi$ have to be non-zero for $P_{\mu e}$ to be non-zero, and also the oscillation length corresponding to $\delta_{21}$ does not contribute to the atmospheric neutrino problem [3]. Now solutions to Kamiokande atmospheric neutrino data [3][4] require a value of $\psi \geq 45^\circ$. The average contribution of the oscillatory term is 0.5. Therefore using the CHOOZ result that the maximum value of $\sin^2(2\phi)$ allowed is 0.18 we get

$$P_{\mu e}^{\max} \leq 1.0 \times 0.18 \times 0.5 = 0.09$$

(6)

which is less than 9 percent. Hence the atmospheric neutrino anomaly is driven almost completely by $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. The $\nu_e \leftrightarrow \nu_\tau$ conversion probability is given by

$$P_{e\bar{\tau}} = P_{e\tau} = \sin^2 2\phi \cos^2 \psi \sin^2 \left(1.27 \frac{d \delta_{31}}{E}\right).$$

(7)
Since $\psi > 45^\circ$, we find that the $e - \tau$ conversion probability is less than 5 percent, i.e. the electron neutrino flux is hardly converted to other flavors, which is what is experimentally observed.

Lastly we incorporate the CHOOZ constraints on our previous fits to solar and atmospheric neutrino data, and so we reproduce the plots from our earlier works, with the constraints coming from the CHOOZ results shown on them. In Fig.1, the light contours enclose the parameter region in $\phi - \psi$ plane allowed by the binned multi-GeV data of Kamiokande with $1.6 \sigma$ error bars. The present CHOOZ constraint has been shown as a thick vertical line, with the region to the right of it being excluded. Fig.2 shows the allowed region in the $\phi - \delta_{31}$ plane from the same analysis, with the CHOOZ constraint again being shown as a thick vertical line. Fig.3 and Fig.4 show the previously allowed regions by the solar neutrino data in $\phi - \omega$ and $\phi - \delta_{21}$ planes respectively along with the new constraint.

Note the fact that $\phi$ being the angle which connects the solar neutrino parameter space spanned by $\omega, \phi,$ and $\delta_{21}$ with the atmospheric neutrino space spanned by $\phi, \psi,$ and $\delta_{31}$, the constraint on $\phi$ also translates into a strong constraint in the solar neutrino parameter space. Now observe what is probably the most important consequence of the CHOOZ result. The fact that $\phi$, the link between the solar and the atmospheric neutrino problems is constrained to be small implies that the solar neutrino problem can be essentially viewed as a two flavor $\nu_e \leftrightarrow \nu_\mu$ oscillation phenomena, and the atmospheric neutrino problem essentially as a two flavor $\nu_\mu \leftrightarrow \nu_\tau$ oscillation phenomena even in a three flavor framework.

In conclusion the recent CHOOZ result limits the $\nu_\mu \leftrightarrow \nu_e$ contribution to the atmospheric neutrino anomaly as a function of the (13) mixing angle $\phi$, establishes the fact that the atmospheric neutrino anomaly is mainly $\nu_\mu \leftrightarrow \nu_\tau$ i.e vacuum oscillations, and excludes large parts of the parameter space previously allowed as solutions to solar and atmospheric neutrino data.

We thank M.V.N. Murthy and Rahul Sinha for discussions.
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FIGURES

FIG. 1. Allowed parameter region in $\phi - \psi$ plane by Kamiokande binned multi-GeV data with 1.6 $\sigma$ error bars (light lines) and the new constraint by CHOOZ (thick line).

FIG. 2. Allowed parameter region in $\phi - \delta_{31}$ plane by Kamiokande binned multi-GeV data with 1.6 $\sigma$ error bars (light lines) and the new constraint by CHOOZ (thick line).

FIG. 3. Allowed parameter region in $\phi - \omega$ plane by solar neutrino data with 1.6 $\sigma$ error bars (crosses) and the new constraint by CHOOZ (thick line).

FIG. 4. Allowed parameter region in $\phi - \delta_{21}$ plane by solar neutrino data with 1.6 $\sigma$ error bars (crosses) and the new constraint by CHOOZ (thick line).
EXCLUDED BY CHOOZ

Fig(2)
EXCLUDED BY CHOOZ

Fig(3)
EXCLUDED BY CHOOZ

Fig(4)