Precision allocation method of large-scale CNC hobbing machine based on precision-cost comprehensive optimization

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Abstract
In modern machine tool design, precision is an important index to characterize machine tool performance and precision allocation has become a key task. Since the middle of the twentieth century, precision allocation methods using optimization technology to balance manufacturing cost and precision level have gradually developed, but most methods mainly take the cost minimization as the goal to optimize the precision allocation. As the precision and manufacturing costs are a pair of factors to be comprehensively considered, balance between them is needed to meet different design requirements. This paper proposes a comprehensive optimization method to trade-off between precision and cost. A multi-object precision allocation optimization model aiming at minimizing fuzzy manufacturing cost and comprehensive precision of machine tool is constructed. A multi-object optimization algorithm to solve the model is designed, combining the multi-objective gray wolf optimization algorithm with multi-objective decision analysis method Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS). A case study based on a large-scale hobbing machine shows that the comprehensive optimization of manufacturing cost and machining precision is realized by using the proposed multi-object precision allocation optimization method.

Keywords Gear hobbing machine · Comprehensive error model · Precision allocation · Precision-cost model · Multi-object optimization

List of Symbols

\( S_x, S_y, S_z \)

Displacements of linear motion axes \( X, Y, \) and \( Z \)

\( \theta_A, \theta_C, \theta_M \)
Rotation angles of rotating motion axes \( A, C, \) and \( M \)

\( x \delta_1(S_x), x \delta_2(S_x), x \delta_3(S_x) \)
Position errors of axis \( X \) at \( S_x \)

\( y \delta_1(S_y), y \delta_2(S_y), y \delta_3(S_y) \)
Position errors of axis \( Y \) at \( S_y \)

\( z \delta_1(S_z), z \delta_2(S_z), z \delta_3(S_z) \)
Position errors of axis \( Z \) at \( S_z \)

\( x \epsilon_1(A_x), x \epsilon_2(A_x), x \epsilon_3(A_x) \)
Position errors of axis \( A \) at \( A \)

\( y \epsilon_1(A_y), y \epsilon_2(A_y), y \epsilon_3(A_y) \)
Position errors of axis \( A \) at \( A \)

\( z \epsilon_1(A_z), z \epsilon_2(A_z), z \epsilon_3(A_z) \)
Position errors of axis \( A \) at \( A \)

\( c \delta_1(\theta_C), c \delta_2(\theta_C), c \delta_3(\theta_C) \)
Position errors of axis \( C \) at \( \theta_C \)

\( c \epsilon_1(\theta_M), c \epsilon_2(\theta_M), c \epsilon_3(\theta_M) \)
Position errors of axis \( M \) at \( \theta_M \)

\( x \delta_1(A_x), x \delta_2(A_x), x \delta_3(A_x) \)
Angle errors of axis \( X \) at \( A_x \)

\( y \delta_1(A_y), y \delta_2(A_y), y \delta_3(A_y) \)
Angle errors of axis \( Y \) at \( A_y \)

\( z \delta_1(A_z), z \delta_2(A_z), z \delta_3(A_z) \)
Angle errors of axis \( Z \) at \( A_z \)

\( x \epsilon_1(A_x), x \epsilon_2(A_x), x \epsilon_3(A_x) \)
Angle errors of axis \( A \) at \( A_x \)

\( y \epsilon_1(A_y), y \epsilon_2(A_y), y \epsilon_3(A_y) \)
Angle errors of axis \( A \) at \( A_y \)

\( z \epsilon_1(A_z), z \epsilon_2(A_z), z \epsilon_3(A_z) \)
Angle errors of axis \( A \) at \( A_z \)

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1 Introduction

Precision is an important index to characterize machine tool performance. The overall precision of machine tool is determined by the precision of moving parts. Different parts have different effects on the overall precision. Precision allocation is to optimize the precision design of main parts on the basis of meeting overall precision design requirements. Precision allocation has become a key task in modern machine tool design.

Early designers generally use trial-and-error method to allocate precision according to experimental data, professional knowledge, and expert experience, or use some simple principles such as principle of similarity, equal tolerance, equal influence, and equal precision [1]. These methods mainly aim to meet the design precision of machine tools. Manufacturing cost impacts are rarely considered when allocating precision. However, the manufacturing cost is closely related to machine tool precision. Improving precision design requirements may increase the manufacturing cost. Excessive improving precision pursuit will lead to unnecessary cost increase. A good machine tool design needs to find the best balance between them.

Since the middle of the twentieth century, the precision allocation method using optimization technology to balance manufacturing cost and precision has gradually developed [2]. At present, the main method is to transform the precision allocation into an optimization problem, with the goal of minimizing the manufacturing cost and the constraint of meeting precision design, and establish a precision-cost model to get the optimal precision allocation. In the production process and actual design, the manufacturing cost is related to many factors. It is difficult to accurately estimate the manufacturing cost with a certain precision, and it is impossible to establish a real cost function. Therefore, a relative relationship model between precision and manufacturing cost is generally established to solve the optimal precision allocation problem. At present, the commonly used models include model of exponential, power exponential, negative square, cubic or quartic polynomial, exponential power exponential composite, and linear exponential composite.

Sheng Hongliang et al. put forward a value analysis method of mechanism precision allocation considering three factors including function, cost, and value [3]. A cost-error model with the empirical data of typical production process was established by Dong Z et al. [4]. Feng C X et al. presented a precision allocation design method with the goal of the lowest cost by using the random integer programming method [5].

Diplaris S C et al. presented an analytical cost-tolerance model by considering the size of tolerance, tolerance dimension, initial tolerance, and workpiece surface, to produce results closer to industrial practice [6]. Rao S et al. proposed a precision allocation optimization method, which can minimize the given objective function on the premise of meeting required function and constraints [7]. Krishna A G and Rao established an optimization model with the goal of minimizing total manufacturing cost for simultaneously manufacturing tolerances and allocating design [8]. Huang X et al. proposed a global precision allocation optimization method of machine tool component precision by combining BP neural network and genetic algorithm [9]. Kang Fang et al. optimized the error parameters using genetic algorithm and established the precision allocation model of machine tool with the goal of minimum manufacturing cost [10]. Muthu et al. considered the quality loss and manufacturing cost of each component, and established a nonlinear integer model aiming at minimizing manufacturing cost [11]. Sanz-Lobera A et al. established a cost-tolerance model to establish individual relation for each tolerance by considering manufacturing resource existing variabilities of each moment [12].

\[
Y \delta_x, Y \delta_y, Y \delta_z \quad \text{Position errors between } M \text{ and } Y \text{ axes}
\]

\[
Y \epsilon_x, Y \epsilon_y, Y \epsilon_z \quad \text{Angle errors between } M \text{ and } Y \text{ axes}
\]

\[
M_{1,2}, M_{2,3}, M_{3,4}, M_{4,5}, M_{5,6}, M_{6,7} \quad \text{Motion transformation matrices}
\]

\[
E_{1,2,3,4,5,6,7} \quad \text{Motion error transformation matrices}
\]

\[
E_{XZ, YZ, ZA, Y, Y, Z} \quad \text{Inter axis error transformation matrices}
\]

\[
M_{1,7} \quad \text{Ideal motion transformation matrix of hobbing}
\]

\[
M_{1,7}' \quad \text{Motion transformation matrix of hobbing considering errors}
\]

\[
E \quad \text{Total error matrix of hobbing machine}
\]

\[
\delta x, \delta y, \delta z, \epsilon x, \epsilon y, \epsilon z \quad \text{Total errors of hobbing machine}
\]

\[
\sigma_{\delta x}, \sigma_{\delta y}, \sigma_{\delta z}, \sigma_{\epsilon x}, \sigma_{\epsilon y}, \sigma_{\epsilon z} \quad \text{Error distribution variances}
\]

\[
I_{\delta x}, I_{\delta y}, I_{\delta z}, I_{\epsilon x}, I_{\epsilon y}, I_{\epsilon z} \quad \text{Precisions at } 3\sigma
\]

\[
F_X, F_Y, F_Z, F_A, F_C, F_M, FA_y \quad \text{Fuzzy manufacturing costs of moving axes}
\]

\[
a, b, c, d, m_{ij} \quad \text{Precision-cost function coefficients}
\]

\[
F(Error) \quad \text{Fuzzy cost optimization objective}
\]

\[
Ip(Error), Ia(Error) \quad \text{Precision optimization objectives}
\]

\[
W_{Ip}, W_{Ia}, W_F \quad \text{Optimize decision weights}
\]
With the goal of minimizing manufacturing cost and motion error, Sarina adopted multi-objective nonlinear optimization method to realize precision design optimization [13]. According to reliability theory, Yu Zhimin et al. established a reliability limit state function for machining precision to meet design requirements, and proposed a precision allocation method for large-scale NC machine tools [14]. According to multi-body system (MBS) theory, Xing Yuan et al. established a machining quality approximate model under comprehensive actions of machine tool geometric errors, and proposed a precision reverse design method of NC machine tools [15]. Cai L et al. established both reliability and sensitivity model for machining precision under multiple failure modes, and proposed a precision allocation method to improve machine tool machining precision reliability under multiple failure modes [16]. Cheng Q et al. developed a precise allocation method to optimize the allocation of manufacturing and assembly tolerances and to minimize the cost of controlling errors and nonconformities [17]. Guo J et al. established a state space model considering error transfers and geometric errors of each part in the assembly process, and realized the optimization of precision allocation [18]. According to the cubic transformation function of fault mode and impact analysis, Yang Z et al. proposed a comprehensive reliability allocation method by considering severity and incidence of faults [19].

By taking total cost of service quality loss and assembly as objective function, Y. M. Zhao et al. established a product tolerance optimization model with constraint condition of the tolerance superposition and economic machining tolerance [20], Zhang Y et al. proposed a manufacturing easiness index that can not only evaluate manufacturing difficulty, but also indirectly reflect manufacturing cost. On the premise of meeting the quality objectives and manufacturing constraints, an optimization problem aiming at maximizing the manufacturability index was proposed [21]. Zhang Z et al. established a reliability prediction model of machine tools and an error parameter identification and optimization model that have a great impact on reliability, which can determine the allowable level of each geometric error parameter and optimize the processing cost [22]. Liu Peng et al. established a machine tool assembly precision allocation model based on state space model with total machining cost minimization as the objective function [23]. Cai L et al. took machine reliability as constraint and the minimum failure possibility and cost as criterion, and put forward an optimization method for machining precision retention based on robust design [24]. Taking the minimal cost as optimization object and machining precision reliability as constraint, Zhang Z et al. proposed a geometric error budget method [25]. Balamurugan C et al. took the time cost of product degradation and quality loss into consideration, and established a tolerance allocation optimization model to minimize total loss and cost of production. Results show that the longer planning cycle will lead to the increase of tolerance cost and quality loss [26].

Cheng Bin Bin et al. established an optimization model of assembly error distribution under actual working conditions based on the modified Jacobian spinor model [27]. In order to optimize the total cost and reliability, Zhang Z proposed a precision allocation method under the geometric and operational constraints of machine tools [28]. Aiming at maximizing the interval width of geometric error sources, Tlija M et al. presented an economic tolerance allocation method considering difficulty coefficient evaluation and Lagrange multiplier [29]. Wang et al. introduced interval theory into the kinematic modeling of static volume error of heavy machine tools, and proposed a tolerance analysis method of heavy machine tools based on interval uncertainty [30]. He C et al. presented a statistical tolerance allocation method for mechanical products considering shape error. By using deep Q-learning with reward function considering target function requirements, consistency, cost of precision maintenance, and processing, an optimal solution is obtained [30]. Based on finite element analysis, Fan J et al. established a tolerance allocation optimal method which can reduce total manufacturing cost about 11.5% considering small deformation of five axis machine tools as constraints [32].

These researches provide effective methods for machine tool precision allocation, but these methods mainly take the comprehensive error less than or equal to the precision design requirements as the constraint condition, and take the cost minimization as the goal to optimize the precision allocation. However, in machine tool design, precision and manufacturing costs are a pair of factors to be comprehensively considered, such as the necessary precision improvement at a small cost, or the valuable manufacturing cost reduction at a tolerable precision loss. This balance between precision and cost can meet different design requirements.

Gear hobbing machine has high machining efficiency. It can not only cut straight and helical cylindrical gears, but also process worm gears and sprockets. It is the most widely used gear processing machine tool. Large gear hobbing machine is an indispensable and important equipment for processing large and high-precision gears used in wind power, ships, construction machinery, and heavy-duty vehicles. Compared with small and medium-sized machine tools, the dimensions of the body, column, and worktable of large-scale gear hobbing machine are larger. In order to meet the requirements of high speed and large tool walking amount on the high rigidity of the machine tools, the bed and column adopt large plane rectangular steel inlaid guide rails, and the worktable adopts static pressure guide rails with multi-cavity synchronous control. On the other hand, because the
workpiece of gear blank has large mass, large inertia and low speed, the worktable adopts worm and worm gear pair transmission, and the tool spindle adopts gear pair transmission, that makes the transmission chain longer. These characteristics of guide rail structure and transmission mode make the geometric errors have a greater impact on the gear tooth surface errors. The precision allocation is more important for the design of large-scale gear hobbing machine.

In this paper, taking a computer numerical control large-scale gear hobbing machine (CNC-LGHM) as the research object, a multi-objective precision allocation optimization model with the goal of minimizing fuzzy manufacturing cost and overall precision of machine tools is constructed, based on the idea of comprehensive optimization of precision and cost. The precision of each component of the hobbing machine is optimized, and the comprehensive optimization of manufacturing cost and machining precision is realized.

In the following sections, the paper structure is arranged as: First, a geometric error model of a typical CNC-LGHM is presented based on the MBS theory in Section 2. Then, a multi-objective precision allocation optimization model is established in Section 3. Based on MOGWO (multi-objective gray wolf optimizer) and TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution), a solution and decision-making method of precision allocation optimization model is proposed in Section 4. A case study and discussion are made on a CNC-LGHM using the proposed method in Section 5. Finally, the full text is summarized.

2 Geometric error model of CNC-LGHM

Key parts geometric error modeling is the precision allocation basis. Many methods can be used to establish comprehensive error models, such as method of matrix transformation [33], error matrix [34], rigid body kinematics [35], D-H method [36], screw theory [37], differential transform [38], and modeling based on MBS theory [39]. Among these methods, the method which takes homogeneous coordinate transformation and MBS theory as its foundation has advantages of less modeling assumptions, standardized process, strong formalization, good generality, and easy computer automatic modeling. In recent years, it has become the preferred method of machine tool error modeling [40–42].

In this paper, a CNC-LGHM is taken as the object of study, and its geometric error model is established based on MBS.

2.1 Typical structure and multi-body definition of CNC-LGHM

As shown in Fig. 1, a typical CNC-LGHM is composed of worktable, bed, column, longitudinal pallet, rotary pallet, tangential pallet, and hob. It has 6 moving axes, including the hob radial feeding axis (X), the hob tangential feeding axis (Y), the hob axial feeding axis (Z), the hob yaw axis (A), the hob spindle (M), and the workpiece table rotary axis (C). In the process of gear hobbing, the column moves in a straight line along the X-axis on the bed, the longitudinal pallet moves in a straight line along the Z-axis on the column, the rotary pallet rotates around the A-axis on the longitudinal pallet, and the tangential pallet moves in a straight line along the Y-axis on the rotary pallet. The hob is installed on the hob frame and rotates at high speed around the M-axis. The workpiece is placed on the worktable, which drives it to rotate around the C-axis.

Multi-body system (MBS) is an abstract representation of a system composed of multiple physical entities in engineering applications. It is a common method to study and analyze the kinematic and dynamic characteristics of complex mechanical systems. Body is the basic unit of MBS for the abstract description of a physical entity. The topological structure formed by all bodies in MBS is an abstract description of the adjacency relationship of physical entities.
The definition of MBS of the CNC-LGHM is illustrated in Fig. 2. As the workpiece is fixed on the worktable and rotates with it during machining, the worktable and workpiece are defined as \( B_1 \) as a whole. We establish the global coordinate system \( O_0X_0Y_0Z_0 \) of the MBS and the local coordinate system \( O_iX_iY_iZ_i \) of each body to describe the relative pose relationship of the moving parts of the gear hobbing machine. The coordinate axis direction is the same as the \( X-Y-Z \) direction shown in Fig. 1. Without losing generality, the initial position of the origin of all coordinate systems is the same.

The relative pose relationship of two adjacent bodies \( B_i \) and \( B_j \) in MBS can be described by the transformation relationship of their local coordinate systems. Any complex transformation relationship can be decomposed into six basic transformations, namely three translation transformations and three rotation transformations around the \( X, Y, \) and \( Z \) coordinate axes. As shown in Fig. 3, the local coordinate system \( B_i \) would change to \( B_j \) by translation transformations \( \delta_x, \delta_y, \) and \( \delta_z \) and rotation transformations \( \varepsilon_x, \varepsilon_y, \) and \( \varepsilon_z. \)

Each basic transformation can be described by a homogeneous transformation matrix, as shown in Table 1. Then, the transformation from \( O_iX_iY_iZ_i \) to \( O_jX_jY_jZ_j \) in Fig. 3 can be represented by Eq. (1).

\[
M_{ij} = R_x(\varepsilon_z)R_y(\varepsilon_y)R_z(\varepsilon_x)T_x(\delta_x)T_y(\delta_y)T_z(\delta_z) \tag{1}
\]

### 2.2 Geometric error analysis of moving axis

When the moving parts of the gear hobbing machine move along the linear axis or around the rotating axis, the deviation between the actual positional posture and the ideal positional posture is the geometric error of the axis.

The geometric errors of linear axes are related to the current spatial position of moving parts. Taking the \( X \)-axis as an example, the column moves along the \( X \)-axis on the guide rail of the bed. The error of \( X \)-axis is caused by the straightness error of the guide rail and the positioning error of column movement. As shown in Fig. 4, in the initial position, the local coordinate system of the column coincides with the global coordinate system \( O_0X_0Y_0Z_0 \). In the ideal situation, when the column translates \( S_x \) along the \( X \)-axis, the local coordinate system \( O_3X_3Y_3Z_3 \) of the column translates \( S_x \) in

![Fig. 3 The reference coordinate system of \( B_i \) and \( B_j \)](image)

![Fig. 4 Moving displacement and errors of \( X \)-axis](image)

| Table 1 Basic transformation matrix of coordinate system |
|-------------------------------------------------------|
| **Coordinate axis** | **Translation transformation** | **Rotation transformation** |
| \( X \) | \( T_x(\delta_x) = \begin{bmatrix} 1 & 0 & 0 & \delta_x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) | \( R_x(\varepsilon_x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\varepsilon_x) & -\sin(\varepsilon_x) & 0 \\ 0 & \sin(\varepsilon_x) & \cos(\varepsilon_x) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) |
| \( Y \) | \( T_y(\delta_y) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \delta_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) | \( R_y(\varepsilon_y) = \begin{bmatrix} \cos(\varepsilon_y) & 0 & \sin(\varepsilon_y) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\varepsilon_y) & 0 & \cos(\varepsilon_y) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) |
| \( Z \) | \( T_z(\delta_z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \) | \( R_z(\varepsilon_z) = \begin{bmatrix} \cos(\varepsilon_z) & -\sin(\varepsilon_z) & 0 & 0 \\ \sin(\varepsilon_z) & \cos(\varepsilon_z) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) |
the $X_0$ direction of $O_0X_0Y_0Z_0$. Due to the error of $X$-axis, the actual local coordinate system $O^e_1X^e_1Y^e_1Z^e_1$ of the column deviates from the ideal local coordinate system $O_0X_0Y_0Z_0$. The relative positional posture error between them is the geometric error of $X$-axis at $S_e$. It can be divided into 3 basic position errors $x_1\delta(S_1)$, $x_2\delta(S_2)$, and $x_3\delta(S_3)$ and 3 basic angle errors $x_1\varepsilon_x(S_1)$, $x_2\varepsilon_x(S_2)$, and $x_3\varepsilon_x(S_3)$.

The geometric errors of the rotation axes are related to the current rotation angle of the moving parts. Taking the $C$-axis as an example, the worktable rotates around the $C$-axis, and the errors of the $C$-axis are generated by the errors of the worktable guide rail and the rotation positioning error of the worktable. As shown in Fig. 5, in the initial position, the local coordinate system of the worktable coincides with the global coordinate system $O_0X_0Y_0Z_0$. In the ideal situation, when the worktable rotates $\theta_C$ around the $C$-axis, the local coordinate system $O_1X_1Y_1Z_1$ of the worktable rotates accordingly. Due to the error of $C$-axis, the actual local coordinate system $O^e_1X^e_1Y^e_1Z^e_1$ of the worktable deviates from the ideal local coordinate system $O_1X_1Y_1Z_1$. The relative positional posture error between them is the geometric error of $C$-axis at $\theta_C$. It can be decomposed into 3 position errors $c_2\delta_2(\theta_C)$, $c_3\delta_3(\theta_C)$, and $c\delta_3(\theta_C)$ and 3 angle errors $c\varepsilon_x(\theta_C)$, $c\varepsilon_y(\theta_C)$, and $c\varepsilon_z(\theta_C)$.

Geometric errors of 6 axes of a gear hobbing machine are listed in Table 2, where $S_x$, $S_y$, and $S_z$ are the current traveling positions of the $X$-, $Y$-, and $Z$-axis, and $\theta_A$, $\theta_C$, and $\theta_M$ are the current rotation angles of the $A$-, $C$-, and $M$-axis, respectively.

The assembly errors between two adjacent axes are mainly caused by the verticality error, parallelism error, and intersecting error of each axis in the process of assembling. The assembly errors between axes reflect the relative static positional posture deviation between moving axes, which are independent of the spatial position of moving parts. Taking the $X$- and $C$-axes as an example, after the assembly is completed, the local coordinate system $O_1X_1Y_1Z_1$ of the worktable coincides with the local coordinate system $O_2X_2Y_2Z_2$ of the bed in ideal situation. In case of assembly errors, the actual coordinate system $O^e_2X^e_2Y^e_2Z^e_2$ of the bed deviates from the ideal coordinate system $O_2X_2Y_2Z_2$. The relative positional posture error between them is the assembly error between $C$- and $X$-axis. It can be present by three position errors $x\delta_x$, $X\delta_x$, and 3 angular errors $x\varepsilon_x$, $x\varepsilon_y$, and $x\varepsilon_z$, as shown in Fig. 6.

Assembly errors between 6 moving axes of a large gear hobbing machine are listed in Table 3.

---

**Table 2: Geometric errors of each axis of a CNC-LGHM**

| Motion axis | Position errors (mm) | Angular errors (rad) |
|-------------|----------------------|----------------------|
| X           | $x\delta_x(S_x), x\delta_y(S_y), x\delta_z(S_z)$ | $x\varepsilon_x(S_x), x\varepsilon_y(S_y), x\varepsilon_z(S_z)$ |
| Y           | $y\delta_x(S_x), y\delta_y(S_y), y\delta_z(S_z)$ | $y\varepsilon_x(S_x), y\varepsilon_y(S_y), y\varepsilon_z(S_z)$ |
| Z           | $z\delta_x(S_x), z\delta_y(S_y), z\delta_z(S_z)$ | $z\varepsilon_x(S_x), z\varepsilon_y(S_y), z\varepsilon_z(S_z)$ |
| A           | $A_x(\theta_A), A_y(\theta_A), A_z(\theta_A)$ | $A_x(\theta_A), A_y(\theta_A), A_z(\theta_A)$ |
| C           | $C_x(\theta_C), C_y(\theta_C), C_z(\theta_C)$ | $C_x(\theta_C), C_y(\theta_C), C_z(\theta_C)$ |
| M           | $M_x(\theta_M), M_y(\theta_M), M_z(\theta_M)$ | $M_x(\theta_M), M_y(\theta_M), M_z(\theta_M)$ |

---

![Fig. 5 Rotation angle and error of C-axis](image1)

![Fig. 6 Assembly errors between X- and C-axis](image2)
2.3 Geometric error modeling of gear hobbing machine

According to the analysis in Section 2.2, the ideal motion transformation matrix of each motion axis can be obtained by using the homogeneous transformation matrix, as shown in Eqs. (2)–(7).

\[
M_{1,2} = R_z(\theta_C) = \begin{bmatrix}
\cos(\theta_C) & -\sin(\theta_C) & 0 & 0 \\
\sin(\theta_C) & \cos(\theta_C) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(2)

\[
M_{2,3} = T_z(S_z) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(3)

\[
M_{3,4} = T_z(S_z) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(4)

\[
M_{4,5} = R_A(\theta_A) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(\theta_A) & -\sin(\theta_A) & 0 \\
0 & \sin(\theta_A) & \cos(\theta_A) & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(5)

\[
M_{5,6} = T_y(S_y) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(6)

\[
M_{6,7} = R_M(\theta_M) = \begin{bmatrix}
\cos(\theta_M) & 0 & \sin(\theta_M) & 0 \\
0 & 1 & 0 & 0 \\
-\sin(\theta_M) & 0 & \cos(\theta_M) & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(7)

Considering the error of a moving axis as a small motion of the moving part, the error transformation matrix of the moving axis can be obtained. As the error values are very small, items above the second order are ignored in matrix operation. The motion axis error transformation matrices are shown in Eqs. (8)–(13). For simplicity, the motion displacement is omitted here, such as \(x_\delta_\alpha(S_\alpha)\) is simplified to \(x_\delta_\alpha\).

\[
E_{m,1,2} = \begin{bmatrix}
1 & -c_\varepsilon_\alpha & c_\varepsilon_\alpha & 0 \\
c_\varepsilon_\alpha & 1 & -c_\varepsilon_\alpha & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(8)

\[
E_{m,2,3} = \begin{bmatrix}
1 & -c_\varepsilon_y & c_\varepsilon_y & 0 \\
c_\varepsilon_y & 1 & -c_\varepsilon_y & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(9)

\[
E_{m,3,4} = \begin{bmatrix}
1 & -z_\varepsilon_y & z_\varepsilon_y & z_\delta_\alpha \\
z_\varepsilon_y & 1 & -z_\varepsilon_y & z_\delta_\alpha \\
0 & 0 & 1 & z_\delta_\alpha \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(10)

\[
E_{m,4,5} = \begin{bmatrix}
1 & -z_\varepsilon_y & z_\varepsilon_y & z_\delta_\alpha \\
z_\varepsilon_y & 1 & -z_\varepsilon_y & z_\delta_\alpha \\
0 & 0 & 1 & z_\delta_\alpha \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(11)

\[
E_{m,5,6} = \begin{bmatrix}
1 & -y_\varepsilon_x & y_\varepsilon_x & y_\delta_\alpha \\
y_\varepsilon_x & 1 & -y_\varepsilon_x & y_\delta_\alpha \\
0 & 0 & 1 & y_\delta_\alpha \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(12)

\[
E_{m,6,7} = \begin{bmatrix}
1 & -M_\varepsilon_y & M_\varepsilon_y & M_\delta_\alpha \\
M_\varepsilon_y & 1 & -M_\varepsilon_y & M_\delta_\alpha \\
0 & 0 & 1 & M_\delta_\alpha \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(13)
And the assembly error matrices can be expressed as (14)–(18).

\[
E_{C_X} = R_x(C_ε_x)R_y(C_ε_y)T_x(C_δ_x)T_y(C_δ_y)T_z(C_δ_z)
\]
\[
= \begin{bmatrix}
-\delta_x & 1 & 0 & 0 \\
-\delta_y & 0 & 1 & 0 \\
-\delta_z & 0 & 0 & 1 \\
\end{bmatrix}
\]

(14)

\[
E_{X_Z} = R_z(x_ε_z)R_y(z_ε_y)T_x(z_δ_x)T_y(z_δ_y)T_z(z_δ_z)
\]
\[
= \begin{bmatrix}
1 & -z_δ_y & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
\]

(15)

\[
E_{Z_A} = R_z(A_ε_z)R_y(A_ε_y)R_z(A_δ_z)T_x(A_δ_x)T_y(A_δ_y)T_z(A_δ_z)
\]
\[
= \begin{bmatrix}
1 & -A_δ_y & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
\]

(16)

\[
E_{A_Y} = R_y(A_ε_y)R_x(A_ε_x)T_y(A_δ_y)T_x(A_δ_x)
\]
\[
= \begin{bmatrix}
1 & -A_δ_x & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
\]

(17)

\[
E_{M_7} = R_x(M_7)R_y(M_7)T_x(M_7)T_y(M_7)T_z(M_7)
\]
\[
= \begin{bmatrix}
1 & -M_7 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
\]

(18)

The comprehensive positional posture error \( E \) can also be decomposed into 3 basic position errors \( \delta_x, \delta_y, \) and \( \delta_z, \) and 3 angular errors \( \epsilon_x, \epsilon_y, \) and \( \epsilon_z \).

\[
E = \begin{bmatrix}
1 & -\epsilon_x & \epsilon_y & \delta_x \\
\epsilon_z & 1 & -\epsilon_x & \delta_y \\
-\epsilon_y & \epsilon_x & 1 & \delta_z \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(23)

3 Multi-object precision allocation optimization

Precision allocation is an important work in machine tool design. Its main purpose is to reasonably distribute the design and manufacturing precision of each part of the transmission chain, so that the overall precision and manufacturing cost of the machine tool can meet the design requirements. Obviously, high precision and low cost are a pair of contradictory design objectives. Improving the precision would result in the manufacturing cost rise. On the contrary, reducing the manufacturing cost would decrease the precision. In machine tool design, precision and manufacturing costs are a pair of factors that need to be comprehensively considered. It is necessary for designers to know how to improve the necessary precision at a small cost or reduce the valuable manufacturing cost with tolerable precision loss. Therefore, this paper takes the overall precision improvement and manufacturing cost decrease of the machine tool as the optimization goal to optimize the precision of each part of the transmission chain, so as to obtain the optimal scheme by reasonably determining the weight of the precision and cost in the machine tool design according to the design requirements.

3.1 Design variables and precision model

The first step in establishing the optimization model is to design the optimization variables. The purpose of machine tool precision allocation is to determine the precision of each component of the machine tool after determining the overall design precision constraints.

The comprehensive precision of machine tools is affected by many factors, such as component manufacturing errors, assembly errors, thermal deformation errors, and force deformation errors. The design of machine tools is carried out in stages, such as structural design, precision design, and thermal design. The component manufacturing errors and assembly errors are determined by the machine tool design and manufacturing process, and can be regarded as static errors, which are the main factors considered in precision allocation. The thermal deformation errors and force
deformation errors are related to the material characteristics, machining process, and machining environment of the machined gear. They are dynamic errors in the machining state of the machine tool. These influencing factors are carried out in other design stages, such as that thermal deformation errors are mainly considered in the thermal design stage of machine tools.

Therefore, the precision here mainly includes two kinds: one is the manufacturing precision of the parts themselves, and the other is the installation precision of machine tool parts. The precision of each component is its maximum allowable error, and the component installation precision is its maximum allowable installation error. Therefore, the precision distribution optimization model of CNC-LGHM takes 36 motion axis errors and 30 inter-axis assembly errors of its 6 axes as design variables as shown in Tables 2 and 3.

Many researches and experiments show that the machine tool geometric errors are approximately normal distribution [43–45]. Relative pose errors between workpiece and tool are geometric errors. Here, it is assumed that they are normal distribution variables, and their mean value is 0.

\[
\begin{align*}
\delta x &\sim \left(0, \sigma_{\delta x}^2\right) \\
\delta y &\sim \left(0, \sigma_{\delta y}^2\right) \\
\delta z &\sim \left(0, \sigma_{\delta z}^2\right) \\
e_x &\sim \left(0, \left(\delta x / 3\right)^2\right) \\
e_y &\sim \left(0, \left(\delta y / 3\right)^2\right) \\
e_z &\sim \left(0, \left(\delta z / 3\right)^2\right)
\end{align*}
\] (24)

According to the normal distribution 3σ law, the probability of a normally distributed data set distributed in the interval (μ-3σ, μ+3σ) is 0.9973. In practical engineering, 3σ precision generally means that the standard deviation of the data set is less than or equal to 1/3 of the precision, that is:

\[e \sim \left(0, \left(\frac{1}{3}\right)^2\right)\] (25)

Then, the precision model of the hobbing machine can be obtained:

\[
\begin{align*}
I_{\delta x} &= 3\sigma_{\delta x} \\
I_{\delta y} &= 3\sigma_{\delta y} \\
I_{\delta z} &= 3\sigma_{\delta z} \\
e_x &= 3\delta x \\
e_y &= 3\delta y \\
e_z &= 3\delta z
\end{align*}
\] (26)

Motion axes errors and assembly errors are also independent normal distribution variables. According to the hobbing machine comprehensive error models (22), \(\delta x, \delta y, \delta z, e_x, e_y, e_z\) and assembly errors. According to the nature of normal distribution, their standard deviations \(\sigma_{\delta x}, \sigma_{\delta y}, \sigma_{\delta z}, \sigma_{e_x}, \sigma_{e_y}, \sigma_{e_z}\), and \(\sigma_{\delta x}, \sigma_{\delta y}, \sigma_{\delta z}\), can be obtained from the variance of motion axes errors and assembly errors.

### 3.2 Precision-cost modeling

This paper establishes the precision-cost model based on power exponential model. Considering that constant terms in cost function are not affected by the precision and structure of machine tool, this paper ignores the constant term in calculating the cost.

The cost of machine tools discussed here includes manufacturing cost and assembly cost. Manufacturing cost is divided into linear axis cost and rotating axis cost. These are discussed separately below.

#### 3.2.1 Precision-cost function of linear axis

The geometric errors of a linear axis are mainly caused by moving part manufacturing precision, and the position error in its moving direction is also affected by the positioning precision of control system. Therefore, the two kinds of error-related costs are considered separately. The fuzzy cost-coefficient related to the position error in the moving direction of a linear axis is set as \(a\), and the fuzzy cost-coefficient related to other errors is set as \(b\). The precision-cost functions of linear axes are shown in formula (27).

\[
\begin{align*}
F_X &= \frac{a}{x^3}\delta x + b\left(\frac{1}{x^3}\delta x^3 + \frac{1}{x^2}\delta x^2 + \frac{1}{x}\delta x + \frac{1}{1}\right) \\
F_Y &= \frac{a}{y^3}\delta y + b\left(\frac{1}{y^3}\delta y^3 + \frac{1}{y^2}\delta y^2 + \frac{1}{y}\delta y + \frac{1}{1}\right) \\
F_Z &= \frac{a}{z^3}\delta z + b\left(\frac{1}{z^3}\delta z^3 + \frac{1}{z^2}\delta z^2 + \frac{1}{z}\delta z + \frac{1}{1}\right)
\end{align*}
\] (27)

#### 3.2.2 Precision-cost function of rotating axis

The geometric errors of a rotating axis are mainly caused by moving part manufacturing precision, and the angular error in its rotation direction is also affected by positioning precision of the machine tool control system. Therefore, the two kinds of error-related costs are considered separately. The fuzzy cost-coefficient related to the angular error in the rotating axis rotation direction is defined as \(c\), and the fuzzy cost-coefficient related to other errors is defined as \(d\). The precision-cost functions of rotating axes are shown in formula (28).
The assembly error in installing and debugging mainly comes from the manufacturing precision and assembly precision of the mating surface. The assembly precision-cost function is shown in formula (29).

\[ FA_g = m_i \left( \frac{1}{j \delta_x^2} + \frac{1}{j \delta_y^2} + \frac{1}{j \delta_z^2} + \frac{1}{j \kappa_x^2} + \frac{1}{j \kappa_y^2} + \frac{1}{j \kappa_z^2} \right) \]  

where \( i \) and \( j \) are the symbols of two adjacent axes, and \( m_i \) is the fuzzy cost-coefficient related to the assembly precision of two adjacent axes.

The above three cost functions are integrated to establish a comprehensive cost function, as shown in formula (30).

\[ F(\text{Error}) = (F_X + F_Y + F_Z) + (F_C + F_A + F_M) + \sum FA_{ij} \]  

where Error represents the 66 kinematic and assembly errors.

### 3.2.4 Determination of model coefficient

There are several undetermined fuzzy cost-coefficients in the above precision-cost model, which reflect the influence of relevant error terms on cost. An effective method is needed to determine these coefficients reasonably.

In 1965, the American scientist L.A. Zadeh put forward the fuzzy set theory. Based on this theory, the fuzzy comprehensive evaluation method (FCEM) is derived. Nowadays, it has been widely used in many fields. The FCEM is a reasonable evaluation method with the comparison of the advantages and disadvantages of the research object under the present conditions.

To determine the values of coefficients in the cost functions, the FCEM is adopted. The following are the main steps:

1) Set the evaluation object set \( X \)

The evaluation object in this paper is the coefficients in the precision-cost functions.

### 3.2.3 Precision-cost function of axes assembly

The assembly error in installing and debugging mainly comes from the manufacturing precision and assembly precision of the mating surface. The assembly precision-cost function is shown in formula (28).

\[
\begin{align*}
F_C &= \frac{c}{c \delta_x^2} + d \left( \frac{1}{c \delta_y^2} + \frac{1}{c \delta_z^2} + \frac{1}{c \kappa_x^2} + \frac{1}{c \kappa_y^2} + \frac{1}{c \kappa_z^2} \right) \\
F_A &= \frac{c}{c \delta_x^2} + d \left( \frac{1}{c \delta_y^2} + \frac{1}{c \delta_z^2} + \frac{1}{c \kappa_x^2} + \frac{1}{c \kappa_y^2} + \frac{1}{c \kappa_z^2} \right) \\
F_M &= \frac{c}{c \delta_x^2} + d \left( \frac{1}{c \delta_y^2} + \frac{1}{c \delta_z^2} + \frac{1}{c \kappa_x^2} + \frac{1}{c \kappa_y^2} + \frac{1}{c \kappa_z^2} \right)
\end{align*}
\]

2) Set the evaluation factor set \( U \)

\( U = \{u_i\} \) is composed of various factors that affect the evaluation object. The element \( u_i \) represents the \( i \)-th factor which affecting the evaluation object and usually has different degrees of fuzziness.

In this paper, \( u_1 \) is the impact of improving the precision of parts or the control system positioning precision on the cost, \( u_2 \) is the impact of improving the assembly precision of parts on the cost, \( u_3 \) is the precision failure probability in the use of machine tools, and \( u_4 \) is the difficulty of solving the failure problem. Coefficients \( b \) and \( d \) are related to the component size, and an evaluation factor \( u_5 \) is added for the influence of the component size on the relationship between precision and cost.

3) Set the evaluation grade set \( V \) and the score vector \( S \)

\( V = \{v_i\} \) is composed of all kinds of results that may be made to the evaluation object, and the element \( v_i \) represents the \( i \)-th evaluation grade. Each evaluation grade in \( V \) is given a score to form a score vector \( S \) with \( s_i \) as the weight of evaluation grade \( v_i \) in \( V \). The evaluation grade and score of each factor are shown in Table 4. Here, we have:

| Factor | Evaluation grade       | Score |
|--------|------------------------|-------|
| \( u_1 \) | Most cost increase     | 5     |
| \( u_2 \) | More cost increase     | 4     |
| \( u_3 \) | Moderate cost increase | 3     |
| \( u_4 \) | Little cost increase   | 2     |
| \( u_5 \) | Very high failure probability | 5     |
| \( u_6 \) | High failure probability | 4     |
| \( u_7 \) | General failure probability | 3     |
| \( u_8 \) | Low failure probability | 2     |
| \( u_9 \) | Very low failure probability | 1     |
| \( u_{10} \) | Very difficult         | 5     |
| \( u_{11} \) | Difficult              | 4     |
| \( u_{12} \) | Moderate               | 3     |
| \( u_{13} \) | Easy                   | 2     |
| \( u_{14} \) | Very easy              | 1     |
| \( u_{15} \) | Very great impact      | 5     |
| \( u_{16} \) | Great impact           | 4     |
| \( u_{17} \) | General impact         | 3     |
| \( u_{18} \) | Small impact           | 2     |
| \( u_{19} \) | Very small impact      | 1     |
The precision allocation optimization in this paper aims at improving the precision, so the optimization goal is the overall precision of the machine tool. As described in Section 3.1, there are 6 parameters describing the machining precision of a large NC hobbing machine tool, i.e., $I_{\delta x}$, $I_{\delta y}$, $I_{\delta z}$, $I_{e x}$, $I_{e y}$, and $I_{e z}$. As the unit and value of $I_{\delta x}$, $I_{\delta y}$, and $I_{\delta z}$ are quite different from that of $I_{e x}$, $I_{e y}$, and $I_{e z}$, the two kinds of precision parameters could not be directly and simply superimposed when constructing the objective function. Therefore, two objective functions are constructed.

$$I_{p}(\text{Error}) = \sqrt{(I_{\delta x})^2 + (I_{\delta y})^2 + (I_{\delta z})^2}$$  \hspace{1cm} (37)$$

$$I_{a}(\text{Error}) = \sqrt{(I_{e x})^2 + (I_{e y})^2 + (I_{e z})^2}$$  \hspace{1cm} (38)

3.3.2 Optimal cost object function

The manufacturing cost is an inevitable problem in the production of machine tool. It is unrealistic to increase the manufacturing cost indefinitely for the purpose of improving precision, so the manufacturing cost of machine tool also needs to be controlled and optimized. The cost optimization object function is $F(\text{Error})$ shown in formula (30).

Then, the precision allocation optimization model of transmission chain of CNC-LGHM based on precision-cost comprehensive optimization can be established as:

$$V - \min f(\text{Error}) = [I_{p}(\text{Error}), I_{a}(\text{Error}), F(\text{Error})]^T$$

Subject to:

$$S_x \in (0, S_X)$$
$$S_y \in \left[-\frac{S_Y}{2}, \frac{S_Y}{2}\right]$$
$$S_z \in (0, S_Z)$$
$$\theta_A \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$$

where $S_X$, $S_Y$, and $S_Z$ represent the maximum stroke of $X$, $Y$, and $Z$ axes, respectively; $S_x$, $S_y$, $S_z$, and $\theta_A$ are the moving position of machine tool moving axes. Since the machine tool hob spindle and the $C$-axis of the worktable rotate 360 degrees in the machining process, there is no need to restrict these two items.

3.3 Multi-objective optimization model of precision allocation

So far, the comprehensive error model and fuzzy precision-cost model of hobbing machine have been established. The machine tools precision allocation is realized with precision-cost objective comprehensive optimization. Therefore, two kinds of objective function of optimal precision and optimal cost should be established respectively to construct a multi-objective optimization model.

3.3.1 Optimal precision object function

The precision allocation optimization in this paper aims at improving the precision, so the optimization goal is the
This paper adopts the multi-objective gray wolf optimization (MOGWO) algorithm to solve the precision allocation optimization model, and the TOPSIS multi-objective decision-making method for selecting optimal solutions to meet the actual needs from the obtained Pareto solution set.

### 4.1 Optimization model solution

Mirjalili proposed GWO which is an intelligent optimization algorithm in 2014 [46]. There is a cooperative mechanism in the predation of gray wolves in nature. GWO simulates this behavior for optimization purposes. GWO algorithm is simple in structure and easy to implement, with less parameter adjustment. In order to achieve the balance between global search and local optimization, GWO adopts information feedback mechanism and sets adaptive convergence factor, which makes it have good accuracy and convergence speed in solving problems.

Wolves in GWO include four kinds of wolves called $\alpha$, $\beta$, $\delta$, and $\omega$ wolfs. The $\alpha$, $\beta$, and $\delta$ wolf are the top three solutions, and $\omega$ wolves are the candidate solutions in the search space. The process of solving the optimal solution, all $\omega$ wolves tracking the location of $\alpha$, $\beta$, and $\delta$ wolf to gradually approach the optimal solution, as shown in Fig. 7.

In 2016, the MOGWO algorithm was presented by Mirjalili et al. [47]. The Pareto optimal solution is saved and retrieved by a fixed size external archive on the basis of GWO. In the multi-objective search space, the gray wolf hunting behavior is simulated with the archive to define the social level to resolve multi-objective optimization problem.

In this paper, MOGWO is adopted as a solving method for the precision-cost multi-objective optimization model established in Section 3. Each gray wolf in the gray wolf group represents a precision allocation. $Ip$, $Ia$, and $F$ are calculated by using the comprehensive error model and fuzzy cost model to evaluate the fitness of each gray wolf, select the current non-dominated solution, and finally obtain the Pareto solution set of precision allocation after continuous updating and iteration.

In particular, according to the comprehensive error model, $Ip$ and $Ia$ are related not only to the precision, but also to the position of moving axes. For a precision allocation, i.e., a gray wolf, the worst-case machine tool comprehensive errors caused by them must be considered. Therefore, when solving $Ip$ and $Ia$, it is necessary to search the space position that makes $Ip$ and $Ia$ reach the maximum value in the effective workspace of the machine tool moving axes, and take the maximum $Ip$ and $Ia$ as the fitness of the gray wolf. In this work, GWO algorithm is used to solve $Ip$ and $Ia$ with the axes position $(S_X, S_Y, S_Z, \theta_A, \theta_C, \theta_M)$ as variables.

### 4.2 Decision-making of optimization

Because the solution of the multi-objective model is a solution set, and the results in the solution set have their own advantages and disadvantages, a systematic method is needed to select the solution. For multi-objective decision analysis, TOPSIS which first presented by C.L. Hwang et al. in 1981 is an effective and commonly used method. It is a ranking method based on the proximity between idealized objectives and evaluation objects to evaluate the existing objects with relative advantages or disadvantages. This method only needs to guarantee the monotonicity of each utility function to approach the ideal solution.

This paper selects the non-dominated solutions based on TOPSIS method. According to the multi-objective model, the decision scheme set $D = \{d_1, d_2, \ldots, d_m\}$ is defined by the Pareto solution set. The variables to measure the scheme attributes include fuzzy cost $F$, precision index $Ip$, and $Ia$. For each scheme $d_i$ in set $D$, three attribute values composed a vector $[a_{i1}, a_{i2}, a_{i3}]$, which uniquely represents a scheme.

The main decision-making steps are as follows:

#### 4.2.1 Setting up the decision matrix $A$

$$A = [a_{ij}]_{m \times 3} \quad (40)$$

Decision-making results and evaluation results will be affected by the differences of decision-making attribute types, attribute value sizes, and attribute dimensions, so it is necessary to standardize the attribute values first. This paper adopts the linear normalization method. The normalized decision matrix is set as:

$$B = [b_{ij}]_{m \times 3} \quad (41)$$

**Fig. 7** Wolf $\omega$ location update

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where:

\[ b_{ij} = 1 - \frac{a_{ij}}{\max_i a_{ij}} \]  \hspace{2cm} (42)

4.2.2 Construction of the weighted canonical matrix C

\[ C = [c_{ij}]_{n \times 3} \]  \hspace{2cm} (43)

\[ c_{ij} = w_j \cdot b_{ij} \quad (i = 1, 2, ..., m; j = 1, 2, 3) \]  \hspace{2cm} (44)

where \( W = [w_1, w_2, w_3]^T \) is the weight vector given by decision-makers.

4.2.3 Calculation of ideal solutions of positive and negative

\[ c_j^* = \min_i c_{ij} \]  \hspace{2cm} (45)

\[ c_j^0 = \max_i c_{ij} \]  \hspace{2cm} (46)

4.2.4 Calculation of the distance between schemes and ideal solutions

\[ s_j^* = \sqrt{\sum_{i=1}^{n} (c_{ij} - c_j^*)^2} \quad (i = 1, 2, ..., m) \]  \hspace{2cm} (47)

\[ s_j^0 = \sqrt{\sum_{i=1}^{n} (c_{ij} - c_j^0)^2} \quad (i = 1, 2, ..., m) \]  \hspace{2cm} (48)

4.2.5 Comprehensive evaluation value calculation

\[ f_i^* = \frac{s_j^0}{s_j^0 + s_i^*} \quad (i = 1, 2, ..., m) \]  \hspace{2cm} (49)

4.2.6 Make the decision

Sort all the schemes according to the comprehensive evaluation value from large to small. Generally, select the scheme with the largest comprehensive evaluation value as the optimal precision allocation scheme.

4.3 Solution and decision algorithm

Based on the multi-object optimization model of precision allocation established in Section 3, the precision-cost comprehensive optimization solution and decision algorithm for precision allocation can be constructed with MOGWO and TOPSIS. First, the multi-object optimization model was solved with MOGWO to obtain a set of Pareto optimal solution. Each solution is a precision allocation scheme. Then, determine the cost and precision weights according to the design requirement and calculate the evaluation value of each scheme to select an optimal precision allocation scheme.

The flowchart of the solution and decision algorithm of precision-cost multi-objective optimization model is shown in Fig. 8.

5 Case study and discussion

5.1 Case description

In the previous sections, the precision-cost comprehensive optimization model of hobbing machine is established, and a multi-objective optimization and decision-making algorithm for precision allocation of hobbing machine is proposed. This section presents an example of a large gear hobbing machine with model 31120 to demonstrate the feasibility and effectiveness of the proposed precision-cost comprehensive optimization allocation method, as shown in Fig. 9. The gear hobbing machine is applicable to the processing of spur gear, helical gear, and worm gear. The maximum diameter of the gear can be 1600 mm and the module can be 24 mm. The main technical parameters of the machine tool are shown in Table 5.

The design nominal precision of motion axes of the gear hobbing machine is shown in Table 6.

Now, the manufacturer wants to improve the precision of the hobbing machine to get higher cost performance. Improving the machining precision of the hobbing machine will result in higher manufacturing cost. Therefore, designers need to know the impact of the improvement of precision on the cost, and obtain the maximum precision improvement at the lowest cost. In order to achieve this goal, this case study takes the minimum machining error of hobbing machine and the minimum fuzzy manufacturing cost as the optimization goal, and uses the multi-objective precision-cost comprehensive optimization modeling and solution method proposed in this paper to obtain the candidate optimization solutions of precision allocation of moving axes of the hobbing machine. Then, setting a group of decision weights of machine precision and manufacturing cost, the evaluation value of each candidate solution is calculated to select the optimal solution. Finally, the precision allocation value of each moving axis is obtained.

5.2 Modeling and solution

5.2.1 Precision-cost comprehensive optimization model

The first step is to establish a precision-cost comprehensive optimization model. Combined Eqs. (23), (26), (30), (37), and (38), the position precision \( l_a(\text{Error}) \),
angle precision $I_p(\text{Error})$, and fuzzy manufacturing cost $F(\text{Error})$ of the large-scale CNC hobbing machine can be obtained. It can be seen from Table 5 that SX = 900 mm, SY = 300 mm, and SZ = 1300 mm in this case. Then, the precision-cost comprehensive optimization model can be obtained from Eq. (39).

$$V - \min f(\text{Error}) = [I_p(\text{Error}), I_a(\text{Error}), F(\text{Error})]^T$$

Subject to:

- $S_x \in (0, 900)$
- $S_y \in (-150, 150)$
- $S_z \in (0, 1300)$
- $\theta_A \in \left(\frac{\pi}{4}, \frac{\pi}{4}\right)$

$$V - \min f(\text{Error}) = [I_p(\text{Error}), I_a(\text{Error}), F(\text{Error})]^T$$

Subject to:

- $S_x \in (0, 900)$
- $S_y \in (-150, 150)$
- $S_z \in (0, 1300)$
- $\theta_A \in \left(\frac{\pi}{4}, \frac{\pi}{4}\right)$

(50)
In model (50), the cost function $F(\text{Error})$ has several undetermined coefficients. According to the coefficient determination method proposed in Section 3.2, the weight vector $A$ and comprehensive evaluation matrix $R$ of each evaluation factor are set according to the relevant field knowledge and expert experience of machine tool design and manufacturing, and the model coefficients are calculated by fuzzy comprehensive evaluation method (FCEM).

Taking the coefficient $a$ as an example, according to the structural characteristics of CNC-LGHM and the opinions of relevant experts in design and maintenance, the evaluation weight vector $A$ for coefficient $a$ is set as:

$$A = [0.4, 0.2, 0.1, 0.3]$$

and the comprehensive evaluation matrix $R$ is set as:

$$R = \begin{bmatrix}
0 & 0 & 0.2 & 0.2 & 0.6 \\
0 & 0 & 0.2 & 0.4 & 0.4 \\
0 & 0 & 0.6 & 0.4 & 0 \\
0 & 0.2 & 0.2 & 0.4 & 0.2
\end{bmatrix}$$

Then, it can be obtained:

$$a = F = B \cdot S^T = 1.98$$

In the same way, other coefficients in the precision-cost model can be obtained. All the results are shown in Table 7.

### 5.2.2 Model solving

The second step is to use MOGWO to solve the precision-cost comprehensive optimization model (50). The basic algorithm parameters are as follows: the iteration number is 500, the size of gray wolf population is 100, and the number of non-dominated solutions stored by external files is 20. The results are shown in Fig. 10 and Table 8. Because the orders of magnitude of $I_p$ and $I_a$ are very different, the relationship between $I_p$, $I_a$, and fuzzy cost $F$ is drawn in Fig. 10 to better show the relationship between them. Thus, we obtain 20 optimal candidate solutions for precision-cost comprehensive optimization, and each solution represents a precision allocation scheme.
Fig. 10 MOGWO operation results

(a) Fuzzy cost vs Ip
(b) Fuzzy cost vs Ia

Table 8 Pareto solution set of fuzzy manufacturing cost and precisions

| Solution | Ip        | Ia        | Fuzzy cost | Solution | Ip        | Ia        | Fuzzy cost |
|----------|-----------|-----------|------------|----------|-----------|-----------|------------|
| 1        | 0.12808   | 1.5623E-04| 8.6103E+10 | 11       | 0.12893   | 1.5708E-04| 8.5118E+10 |
| 2        | 0.12847   | 1.5656E-04| 8.5683E+10 | 12       | 0.12911   | 1.5733E-04| 8.4890E+10 |
| 3        | 0.12847   | 1.5663E-04| 8.5668E+10 | 13       | 0.12920   | 1.5754E-04| 8.4712E+10 |
| 4        | 0.12854   | 1.5662E-04| 8.5623E+10 | 14       | 0.12935   | 1.5782E-04| 8.4845E+10 |
| 5        | 0.12856   | 1.5670E-04| 8.5569E+10 | 15       | 0.12940   | 1.5775E-04| 8.4470E+10 |
| 6        | 0.12864   | 1.5686E-04| 8.5435E+10 | 16       | 0.12940   | 1.5775E-04| 8.4470E+10 |
| 7        | 0.12879   | 1.5704E-04| 8.5283E+10 | 17       | 0.12943   | 1.5768E-04| 8.4487E+10 |
| 8        | 0.12889   | 1.5706E-04| 8.5180E+10 | 18       | 0.12944   | 1.5788E-04| 8.4397E+10 |
| 9        | 0.12891   | 1.5713E-04| 8.5171E+10 | 19       | 0.12957   | 1.5804E-04| 8.4227E+10 |
| 10       | 0.12892   | 1.5705E-04| 8.5143E+10 | 20       | 0.12961   | 1.5811E-04| 8.4166E+10 |

Table 9 Evaluation value of 20 candidate solutions

| Candidate solution | Evaluation value | Candidate solution | Evaluation value | Candidate solution | Evaluation value | Candidate solution | Evaluation value |
|--------------------|------------------|--------------------|------------------|--------------------|------------------|--------------------|------------------|
| 1                  | 0.464933         | 6                  | 0.477962         | 11                 | 0.503346         | 16                 | 0.535143         |
| 2                  | 0.470693         | 7                  | 0.479325         | 12                 | 0.515458         | 17                 | 0.534841         |
| 3                  | 0.467165         | 8                  | 0.493045         | 13                 | 0.527095         | 18                 | 0.530305         |
| 4                  | 0.469770         | 9                  | 0.500684         | 14                 | 0.531554         | 19                 | 0.535054         |
| 5                  | 0.471649         | 10                 | 0.500856         | 15                 | 0.535143         | 20                 | 0.535067         |

5.2.3 Optimal solution selection

The third step is to use TOPSIS to select the optimal solution that meets the requirements according to the weight of precision and cost in the optimal design of machine tool. Here, we assume that the cost weight $W_F$ is 0.3, and the position precision weight $W_{Ip}$ and angle precision weight $W_{Ia}$ are 0.35. According to the method proposed in Section 4.2, the evaluation values of 20 optimal candidate solutions can be obtained from Eqs. (40)–(49), as shown in Table 9. Obviously, solution 15 and 16 have the largest evaluation value. Here, we select solution 15 as the final optimal solution under the above weight setting.

5.2.4 Optimal allocation results

For optimal solution 15, the corresponding precision optimization allocation results are shown in Table 10. Compared with the initial precision allocation shown in Table 6, it can be seen that after multi-objective optimization design, 35 precision items are relaxed and 31 ones tightened. The relaxed items include 18 moving axes geometric precision items $(x\delta_x, x\delta_y, x\delta_z, y\delta_x, y\delta_y, y\delta_z, z\delta_x, z\delta_y, z\delta_z, A\delta_x, A\delta_y, A\delta_z, C\delta_x, C\delta_y, C\delta_z, X\varepsilon_x, Y\varepsilon_y, Y\varepsilon_z, Z\varepsilon_x, Z\varepsilon_y, Z\varepsilon_z, A\varepsilon_x, A\varepsilon_y, A\varepsilon_z, X^t\varepsilon_x, Y^t\varepsilon_y, Z^t\varepsilon_z)$ and 17 assembly precision items $(x, x^t\delta_x, x^t\delta_y, x^t\delta_z, y, y^t\delta_x, y^t\delta_y, z, z^t\delta_x, z^t\delta_y, z^t\delta_z, M, M^t\delta_x, M^t\delta_y, M^t\delta_z, A^t\delta_x, A^t\delta_y, A^t\delta_z, X^t\varepsilon_x, X^t\varepsilon_y, X^t\varepsilon_z, Y^t\varepsilon_x, Y^t\varepsilon_y, Y^t\varepsilon_z, Z^t\varepsilon_x, Z^t\varepsilon_y, Z^t\varepsilon_z)$. The others are tightened.
The comprehensive precision of the hobbing machine is \( \text{Ip} = 0.12940 \) and \( \text{Ia} = 1.5775E - 4 \); the fuzzy manufacturing cost is \( F = 8.4470E + 10 \).

### 5.3 Discussion

The optimization results obtained in Section 5.2 are discussed in this section. Firstly, the results of precision optimization allocation are analyzed and its influence on the precision and manufacturing cost of gear hobbing machine is discussed. Then, comparison with other methods is made to verify the effectiveness and advantages of the proposed method. Thirdly, the influence of precision and cost weight on optimization decision-making is deeply discussed. Finally, the universality of the method is discussed.

#### 5.3.1 Analysis of optimization results

According to Table 8, the relationship between fuzzy cost and precision can be drawn, as shown in Fig. 11. In the figure, the abscissa is precision \( \text{Ip} \) or \( \text{Ia} \). It can be seen that with the increase of the \( \text{Ip} \) or \( \text{Ia} \), which means the machining precisions get worse, the fuzzy cost shows a downward trend.

![Fuzzy Cost vs Ip](image1)

![Fuzzy Cost vs Ia](image2)

**Fig. 11** Relationship between fuzzy cost and precision
The non-optimization method is obviously high. The results are shown in Table 11.

|                | Non-optimal method | Proposed method |
|----------------|-------------------|-----------------|
| Ip             | 0.12976           | 0.1294          |
| Ia             | 1.5510E-4         | 1.5775E-04      |
| Fuzzy cost     | 1.1590E+11        | 8.4470E+10      |

This is repeated until the expected precision requirements are met and the precision allocation is completed. Then, calculate the overall precision of the machine again. If it does not meet the expected design requirements, the sensitivity analysis or analytic hierarchy process method is used to find out the precision items that have a great impact on the overall precision, and improve its precision setting value. Then, calculate the overall precision of the machine again. This is repeated until the expected precision requirements are met and the precision allocation is completed.

According to the weight setting \( (W_g, W_{ae}, W_p) = (0.35, 0.35, 0.3) \), the optimal solution obtained at this time is \( (Ip, Ia, F) = (0.12940, 1.5775E - 4, 4.84470E + 10) \). The precision allocation value of each moving axis is shown in Table 10. Firstly, we use the similar precision requirements, i.e., \( (Ip, Ia) = (0.13, 1.58E - 4) \), and carry out the precision allocation using the non-optimization method shown. The results are shown in Table 11.

It can be seen that the manufacturing cost obtained by the non-optimization method is obviously high. The results show that the manufacturing cost obtained by the proposed method is 27.12% lower than that of the non-optimization method, which means that the manufacturing cost is effectively reduced. This verifies the effectiveness of the method proposed in this paper.

### 5.3.3 Comparison with common single objective optimization methods

The common single objective optimization precision allocation method in the literature is generally used for the cost optimization after the expected design value of precision is determined. Here, the precision allocation is also carried out with the same precision requirements, i.e., \( (Ip, Ia) = (0.13, 1.58E - 4) \). The single objective optimization model is shown in Eq. (55).

\[
V = min F(\text{Error})
\]

Subject to:

\[
\begin{align*}
S_x & \in (0, 900) \\
S_y & \in (150, 150) \\
S_z & \in (0, 1300) \\
\theta_a & \in \left( -\frac{\pi}{4}, \frac{\pi}{4} \right) \\
Ip & \leq 0.13 \\
Ia & \leq 1.58E - 04
\end{align*}
\] (55)

NSGA-II optimization method commonly used in single objective optimization precision allocation is used, and the results are shown in Table 12.

It can be seen that under the predetermined precision requirements, the single objective optimization method and the proposed multi-objective optimization method can obtain basically the same optimization results, which verifies the effectiveness of the method proposed in this paper from another aspect. However, compared with the single objective optimization method, the multi-objective optimization proposed in this paper has more advantages. The single objective optimization precision allocation method is generally used for the cost optimization after the precision is determined. In the case described in Section 5.1, the designer wants to seek the most economical and reasonable scheme for improving the precision of the hobbing machine. Using the single objective method, it is necessary to set an initial value of precision, change it by a certain step, and iteratively get an optimization solution, which is time-consuming and laborious. Moreover, affected by the initial value of the assumed precision value and the iteration step, the optimal solution may not be found. The proposed method can give the Pareto optimal solution set at one time, and then make optimization decisions according to the precision-cost weights, which can effectively help designers reasonably determine the optimal scheme.
To sum up, the comparison shows that the precision-cost comprehensive optimization method presented in this paper well balances two machine tool factors, precision and manufacturing cost, and obtains a more economical and reasonable precision allocation result.

5.3.4 Influence of decision weight on optimal allocation selection

In the third step of precision-cost comprehensive optimization, the decision weight of precision and cost should be set according to the design requirements, and the TOPSIS decision method should be used to select the optimal scheme meeting the requirements from the Pareto solution set. The weights to be set include the position precision weight $W_{Ip}$, the angle precision weight $W_{Ia}$, and the fuzzy manufacturing cost weight $W_f$. Here, we analyze the influence of weights on decision-making results, so that designers can use the proposed method for precision cost optimization more conveniently.

Since $I_p$ and $I_a$ are precision and $F$ is fuzzy cost, they belong to two different categories. When setting the weights, first set the cost weight $W_f$ as 0, 0.25, 0.5, 0.75, and 1, respectively, and then set different $W_{Ip}$ and $W_{Ia}$ combinations for each $W_f$. A total of 13 groups of weight combinations ($W_{Ip}$, $W_{Ia}$, $W_f$) are set, as shown in Table 13. This setting is mainly to examine the influence of different cost weights on determining the optimal solution, and the influence of different position and angle precision weights on the decision results under the same cost weight.

The evaluation values of 20 candidate optimization solutions are calculated for each group of weight combination, and the results are shown in Fig. 12 in which the ordinate is evaluation value and the abscissa is the weight combination. There are 13 groups of histograms. Each group is the evaluation value of 20 candidate solutions calculated according to the weight combination. The maximum evaluation value and optimal candidate solution in each group are shown in Table 13. This result is analyzed below.

First look at the histograms of group 1–3 in Fig. 12, where $W_f = 0$, $W_{Ip}$, and $W_{Ia}$ take different values, respectively. It can be seen that the evaluation values of 20 candidate solutions have little difference. Similarly, histograms of groups 4–5, 7–9, and 10–12 showed similar characteristics. This shows that after the cost weight is determined, the different

![Fig. 12 Evaluation value for different weight combination](image)

| $W_{Ip}$ | $W_{Ia}$ | $W_f$ | Max evaluation value | Optimal allocation |
|--------|--------|------|----------------------|-------------------|
| 1    | 0.25   | 0.75 | 0  | 1  | 1 |
| 2    | 0.5    | 0.5  | 0  | 1  | 1 |
| 3    | 0.75   | 0.25 | 0  | 1  | 1 |
| 4    | 0.1875 | 0.5625 | 0.25 | 0.561579 | 2 |
| 5    | 0.375  | 0.375 | 0.25 | 0.527675 | 1 |
| 6    | 0.5625 | 0.1875 | 0.25 | 0.554651 | 1 |
| 7    | 0.125  | 0.375 | 0.5 | 0.705829 | 19 |
| 8    | 0.25   | 0.25  | 0.5 | 0.728653 | 20 |
| 9    | 0.375  | 0.125 | 0.5 | 0.706642 | 20 |
| 10   | 0.0625 | 0.1875 | 0.75 | 0.87782 | 20 |
| 11   | 0.125  | 0.125 | 0.75 | 0.889575 | 20 |
| 12   | 0.1875 | 0.0625 | 0.75 | 0.87844 | 20 |
| 13   | 0      | 0     | 1  | 20 | 20 |
values of $W_{Ip}$ and $W_{Ia}$ have little impact on the evaluation results. The reason is that the position precision $Ip$ and angle precision $Ia$ reflect two different aspects of machine tool precision. They are essentially one. When conducting multi-objective optimization, the optimization direction of their increase or decrease is also the same. Therefore, it will not have a significant impact on the evaluation results due to different weight setting. This also tells us that $W_{Ip}$ and $W_{Ia}$ can be set to the same value when making optimization decisions.

Then, analyze the impact of cost weight on the optimization decision-making results. When the cost weight is small (group 1–6), the evaluation value decreases with the decrease of cost. The smaller the cost weight, the faster the evaluation value decreases. The candidate solution 1 or 2 with the largest cost is selected as the optimal solution. When the cost weight is large (group 7–13), the evaluation value increases with the decrease of cost. The larger the cost weight, the faster the evaluation value increases. The candidate solution 19 or 20 with the lowest cost is selected as the optimal solution. This means that the optimal candidate solution will gradually transfer from high-cost solution to low-cost solution with the gradual increase of cost weight. Therefore, when making decisions, the main need to determine is the proportion of cost weight in the whole decision-making process.

The above analysis shows that TOPSIS method can effectively select the appropriate optimization scheme in Pareto solution set for multi-objective optimization according to different requirements of precision and cost in machine tool design.

5.3.5 The generality of proposed method

As its high machining efficiency and ability to process various gears, large gear hobbing machine is widely used in processing large and high-precision gears for wind power, ships, construction machinery, and heavy-duty vehicles. This paper takes CNC-LGHM as the object of study and proposes a precision-cost comprehensive optimization method. Despite the differences between large-scale gear hobbing machines and small and medium-sized machine tools in specific guide rail structure and transmission mode, the basic structure and transmission principle of their main moving axes are basically the same as that of multi-axis machine tools. There is no essential difference in error generation and transmission principle and the influence mechanism of error on gear machining precision between them. Therefore, the method proposed in this paper has certain generality.

To optimize the precision and cost of a specific machine tool, the same method can be used to establish the precision-cost comprehensive optimization model. The difference is that the model coefficient should be determined according to the characteristics of the machine tool itself. After the model is established, it can be solved by the model solution and decision algorithm proposed in this paper, but the precision and cost decision weight should be determined according to the characteristics and design requirements of the machine tool.

For CNC-LGHM or other large-medium size machine tools, the geometric errors caused by structure, size, and other characteristics have a relatively large impact on the comprehensive machining errors of machine tools. The proposed method is of great significance and applicable to this kind of machine tools. However, for small size machine tools whose geometric errors have a relatively small impact on the comprehensive machining errors, the generality and practicability of the proposed method need to be studied and discussed further.

6 Conclusion

Precision allocation is an important task in modern machine tool design. At present, most precision allocation methods mainly consider how to reduce the manufacturing cost or failure cost through the precision optimal allocation of machine tool parts under the premise of given machine tool precision. In these methods, the precision design of machine tool and cost optimization are considered separately. The machine tool design precision is determined first, and then the cost is optimized through precision allocation. The optimization effect is limited. Considering the reasonable balance between precision and cost, this paper presents a comprehensive optimization method for machine tool precision and manufacturing cost, which can better meet different design requirements.

This paper mainly does the following work. First, according to MBS theory, a geometric error model of CNC-LGHM is established. This model describes the relationship between gear hobbing machine comprehensive errors and geometric errors of motion axes and assembly errors between axes. Then, a multi-objective optimization model aiming at minimizing manufacturing cost and machining precision is established. In the precision-cost model, the influence of component size on the relationship between precision and manufacturing cost is considered. The model parameters are determined by the fuzzy comprehensive evaluation method, and a new precision-cost model is obtained which describe the relationship between precision and cost more accurately. Thirdly, a precision-cost multi-objective optimization and decision algorithm is proposed. MOGWO is adopted to solve multi-objective optimization model. Pareto solution set of the optimal precision allocation scheme is obtained. Then, the multi-objective decision analysis method TOPSIS is used for auxiliary decision-making. The decision weights
of precision and cost are set according to the design requirements to evaluate the optimal non-dominated solutions, and the optimal allocation scheme meeting the requirements is selected according to the evaluation results. At last, a case study is carried out based on a large-scale hobbing machine to verify the effectiveness of the proposed precision-cost comprehensive optimization modeling and solving method.

By analyzing the results of the case study, the following conclusions can be drawn:

(1) The proposed multi-objective optimization solution and decision-making method can give a reasonable precision optimization allocation according to the design requirements and expert knowledge, realize the comprehensive optimization of machine tool manufacturing cost and precision, and provide an effective precision allocation method for machine tool designers. Compared with the original scheme, the precision is improved by 20% at the cost of 11% increase in cost. Compared with the conventional non-optimization method, under the same expected design precision of the machine tool, the fuzzy manufacturing cost can be reduced by 27.12% by using the proposed precision allocation optimization method.

(2) Compared with the common single objective optimization method in the literature, under the same expected design precision of machine tool, the proposed optimization method can obtain similar precision allocation optimization results. In the case study, the fuzzy manufacturing cost of the single objective optimal allocation is 8.5031E+10, and that of the optimal allocation of the proposed method is 8.4470E+10. However, the multi-objective optimization method has more advantages than the single objective optimization method. Especially in the early stage of machine tool design, it is more convenient to obtain a variety of optimization schemes, and obtain the optimal scheme that meets the requirements according to the weight setting of precision and cost in the design, which can provide more decision support for machine tool designers.

(3) Due to the different units and large order of magnitude difference between position precision and angle precision, they are divided into two optimization objectives $Ip$ and $Ia$ in modeling, but they are essentially two different aspects of machine tool precision and are integrated in essence. The experimental results also show that the same weight value can be set for them.

(4) The proposed method is of great significance and applicable to CNC-LGHM and other large-medium size machine tools, whose geometric errors have a relatively large impact on the comprehensive machining errors of machine tools. Further investigation needs to be made for the generality and practicability of the proposed method to small size machine tools whose geometric errors have a relatively small impact on the comprehensive machining errors.

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Declarations

Conflict of interest The authors declare no competing interests.

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