General theory of electromagnetic fluctuations near a homogeneous surface, in terms of its reflection amplitudes

Giuseppe Bimonte and Enrico Santamato

Dipartimento di Scienze Fisiche Università di Napoli Federico II
Complesso Universitario MSA Via Cintia I-80126 Napoli Italy;
INFN, Sezione di Napoli, Napoli, ITALY

(Dated: February 1, 2008)

We derive new general expressions for the fluctuating electromagnetic field outside a homogeneous material surface. The analysis is based on general results from the thermodynamics of irreversible processes, and requires no consideration of the material interior, as it only uses knowledge of the reflection amplitudes for its surface. Therefore, our results are valid for all homogeneous surfaces, including layered systems and metamaterials, at all temperatures. In particular, we obtain new formulae for the near-field region, which are important for interpreting the numerous current experiments probing proximity effects for macroscopic and/or microscopic bodies separated by small empty gaps. By use of Onsager’s reciprocity relations, we obtain also the general symmetry properties that must be satisfied by the reflection matrix of any material.

PACS numbers: 42.50.Lc, 78.20.Ci, 05.70.Ln, 74.45+c
Keywords: fluctuations, irreversible, reflection, Onsager, Casimir, heat transfer.

I. INTRODUCTION

The study of electromagnetic (e.m.) fluctuations has received much attention over the years, because of its fundamental importance in diverse areas of physics. While the classical problem of determining the thermal spectrum of e.m. fluctuations in a black body is at the origin of quantum theory, the concept of zero-point e.m. quantum fluctuations leads to new quantum phenomena, like the Lamb shift, and the existence of van der Waals forces between atoms and/or macroscopic bodies. Electromagnetic fluctuation-driven forces become appreciable in the submicron range and rapidly increase in the nanometer scale.

It should be observed that while general thermodynamical theorems (Kirchhoff’s law) give much information on the radiation field far from the body surface, no analogously general results are available so far, on the problem of characterizing the e.m. field very close to a surface. A detailed study of the near-field region is of great importance in numerous physical phenomena that depend on proximity effects between macroscopic and/or microscopic bodies separated by small empty gaps. Important examples of such phenomena are the Casimir effect [1], radiative heat transfer between closely spaced dielectric bodies. While appealing for its conceptual simplicity, a drawback of Rytov’s approach is that it requires detailed knowledge of the e.m. fields in the body’s interior. In particular, the theory requires as inputs the dielectric tensor $\varepsilon_\parallel (\omega, \mathbf{x}, \mathbf{x}')$ and/or the magnetic permittivity $\mu_\parallel (\omega, \mathbf{x}, \mathbf{x}')$ of the material, together with the boundary conditions satisfied by the electric and magnetic fields at the interface. Therefore, with this approach, the problem arises of understanding to what extent the final expressions for the fluctuating e.m. field outside the body depend on the assumptions made for the permittivities and the chosen boundary conditions, which is a very delicate issue in the case, say, of spatially non-local materials.

The spirit of our approach is radically different, as we never consider the e.m. field or the microscopic fluctuating currents in the interior of the material body. Therefore, we do not make any use of either the constitutive equations for the material, or the boundary conditions for the e.m. field across the interface. We regard the surface as fully characterized in terms of a set of reflection amplitudes $r_{\lambda\mu} = r_{\lambda\mu}(\omega, \mathbf{k}_\perp)$, depending on the frequency $\omega$ and the projection $\mathbf{k}_\perp$ of the wave-vector onto the surface. These amplitudes are considered by us as data, obtained either by experiments or by an independent computation, which is of no concern to us. We shall
see that knowledge of the reflection amplitudes is sufficient to determine, in a completely general way, the correlators for the e.m. field outside the body. The possibility of expressing the correlators of the e.m. field outside a material surface in terms of its reflection amplitudes is not entirely new. It is clearly suggested by Lifshitz formula [9], as in its final form it expresses the interaction between two nearby parallel dielectric slabs in terms of their respective reflection amplitudes. The general validity of Lifshitz formula, once expressed in terms of reflection amplitudes, has been recognized by several authors, and it has been recently demonstrated using the scattering approach [10, 11]. The idea that field correlators can be given in terms of reflection amplitudes was also utilized in the study of radiative heat transfer, including evanescent contributions, in Ref. [4]. The distinctive feature of our approach is that it relies on general results from the thermodynamics of irreversible processes, and requires no consideration about the field in the interior of the body. The process of reflection is considered as a macroscopic irreversible process, entirely described in terms of the reflection amplitudes at the surface. Therefore, our results are of a very general nature, and they apply to arbitrary homogeneous surfaces at all temperatures, thus providing a generalization of previously available results, that includes the case of non-diagonal reflection matrices, i.e., mixing TE and TM modes. While for PW we recover the well-known Kirchhoff’s formula, Eq. (11) below, we obtain a new general and simple formula which constitutes the main result of this paper. By exploiting Onsager’s reciprocity thermodynamical relations, we obtain also new general symmetry properties that must be satisfied by the reflection matrix of any material.

II. THE E.M. FIELD OUTSIDE A HOMOGENEOUS SURFACE

In this Section we derive the expression for the correlators of the e.m. field outside a homogeneous surface. The procedure we follow is a generalization of the method used in Ref. [12] to obtain the Planck radiation law. In that paper the authors considered an electric dipole in thermal equilibrium with the radiation field of a black body. Apart from the intra-dipole binding force, the dipole is subjected to two forces of e.m. origin: one of them is a systematic damping force, representing the reaction to the radiation emitted by the dipole itself, and the other is a random force due to the fluctuating electric field of the black body (to linear order, magnetic interactions can be neglected). The damping force is computed using Maxwell’s Equations, and then one can use the general fluctuation-dissipation theorem to obtain the power spectrum of the fluctuating e.m. field, which gives the Planck’s radiation law. In our case, we are interested in determining how the presence of a surface influences the radiation field in the vacuum region bounded by the surface (for simplicity we limit our analysis to flat homogeneous surfaces), and we shall see that the problem can be solved in full generality by means of a similar procedure, but considering, instead of one, two electric dipoles placed on a plane parallel to surface.

To be definite, let us suppose that the body occupies the $z > 0$ half-space, such that its surface is at $z = 0$. By invariance under translations in the $x, y$ plane, the fluctuating electric field in the vacuum region outside the body (i.e. in the $z < 0$ half-space) can be written in general as:

$$E^{(\text{free})}(x_{\perp}, z) = \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d\mathbf{k}_\perp}{(2\pi)^2} \left( E^{(\infty)}(z; \omega, \mathbf{k}_\perp) + E^{(S)}(z; \omega, \mathbf{k}_\perp) \right) e^{i(k_{\perp} \cdot x_{\perp} - \omega t)} + \text{c.c.}, \quad (1)$$

where

$$E^{(\infty)}(z; \omega, \mathbf{k}_\perp) = \left[ a_s^{(\infty)}(\omega, \mathbf{k}_\perp) \mathbf{e}_\perp + a_p^{(\infty)}(\omega, \mathbf{k}_\perp) \frac{c}{\omega}(\mathbf{k} \times \mathbf{e}_\perp) \right] e^{ik_z z} +$$
$$+ \left[ a_s^{(\infty)}(\omega, \mathbf{k}_\perp) \left[ r_s r_s \mathbf{e}_\perp + r_p \frac{c}{\omega}(\mathbf{k}^{(r)} \times \mathbf{e}_\perp) \right] + a_p^{(\infty)}(\omega, \mathbf{k}_\perp) \left[ r_p r_p \frac{c}{\omega}(\mathbf{k}^{(r)} \times \mathbf{e}_\perp) + r_s r_p \mathbf{e}_\perp \right] \right] e^{-ik_z z}, \quad (2)$$

$$E^{(S)}(z; \omega, \mathbf{k}_\perp) = \left[ a_s^{(S)}(\omega, \mathbf{k}_\perp) \mathbf{e}_\perp + a_p^{(S)}(\omega, \mathbf{k}_\perp) \frac{c}{\omega}(\mathbf{k}^{(r)} \times \mathbf{e}_\perp) \right] e^{-ik_z z}. \quad (3)$$

In the above Equations, $\mathbf{k} = \mathbf{k}_\perp + k_z \mathbf{\hat{z}}$ and $\mathbf{k}^{(r)} = \mathbf{k}_\perp - k_z \mathbf{\hat{z}}$ denote, respectively, the wave-vectors for fields propagating towards and away from the surface, $k_z = \sqrt{\omega^2/c^2 - k_\perp^2}$, and we take the square root such that...
$k'_z \geq 0$, $k''_z \geq 0$ (with prime and double prime denoting real and imaginary parts, respectively). We recall that real $k_z$ correspond to PW waves, while imaginary $k'_z$ describe EW. Moreover, $x_\perp = x \hat{x} + y \hat{y}$, $e_\perp = \hat{z} \times k_\perp$, the subscripts $s$ and $p$ denote $TE$ and $TM$ polarizations, respectively, and $r_{\lambda\mu} = r_{\lambda\mu}(\omega, k_\perp)$ are the reflection amplitudes for an incident field with polarization $\mu$ to be reflected as a field with polarization $\lambda$. The field $E(\infty)$ represents an incoming field from $z = -\infty$, that gets reflected by the surface, while we can think of $E(S)$ as the field radiated by the surface. Since the expansion for the magnetic field is obtained from Eqs. (1-3) by integrating Maxwell's equations ($\mathbf{B} = -c \int dt \nabla \times \mathbf{E}$), the fluctuating e.m. field is fully described by the correlators among the amplitudes $a^{(\infty/S)}(\omega, k_\perp)$. We let $a^{(\alpha)}$, $\alpha = \infty, S$ a column vector with elements $(a^{(\alpha)}_s, a^{(\alpha)}_p)$, and $a^{(\alpha)\dagger}$ a row vector with elements $(a^{(\alpha)s\dagger}, a^{(\alpha)p\dagger})$. From invariance under time translations, and by homogeneity in the $x,y$ plane, the correlators must be of the form:

$$
\langle a^{(\alpha)}(\omega, k_\perp), a^{(\beta)\dagger}(\omega', k'_\perp) \rangle = (2\pi)^3 C^{(\alpha\beta)}(\omega, k_\perp) \delta(\omega - \omega') \delta^{(2)}(k_\perp - k'_\perp)
$$

(4)

with all other correlators vanishing. Therefore, the problem reduces to computing the four matrices $C^{(\alpha\beta)}(\omega, k_\perp)$. To do this, we imagine placing in the radiation field two electric dipoles. We let $q^{(A)}$ and $\xi^{(A)}$, $A = 1,2$ their respective charges and displacements, and we assume that they are placed at arbitrary points $x_\perp^{(1)}$ and $x_\perp^{(2)}$ on a plane $z = w < 0$ in the empty region below the surface.

$$
m^{(A)} \ddot{\xi}^{(A)}(t) + k^{(A)} \dot{\xi}^{(A)}(t) - q^{(A)} \sum_{B=1,2} \int_{-\infty}^{t} dt' E^{\text{(ret)}}_{\perp}(t - t', x_\perp^{(A)} - x_\perp^{(B)}, w) \dot{\xi}^{(B)}(t') = q^{(A)} E^{(\text{flu})}_{\perp}(t, x_\perp^{(A)}, w)
$$

(5)

In this Equation, $k^{(A)}$ are the elastic constants for the restoring intra-dipolar forces, while the non-local tensor kernel $E^{\text{(ret)}}_{\perp}(t - t', x_\perp^{(A)} - x_\perp^{(B)}, w)$ represents projection onto the $x,y$ plane of the retarded electric field produced by dipole $B$ at the position of dipole $A$, which produces a systematic frictional force. Finally, the force on the r.h.s. is a random term arising from the fluctuating electric field. The expression of the time Fourier transform of the kernel $E^{\text{(ret)}}_{\perp}(t - t', x_\perp^{(A)} - x_\perp^{(B)}, w)$ is found by resolving Maxwell Equations in the empty region outside the body, a task that is easily accomplished by exploiting translational invariance along the surface. Upon performing a space-Fourier transform of the fields in the $z$-plane, we obtain:

$$
E^{\text{(ret)}}_{\perp}(\omega, x_\perp^{(A)} - x_\perp^{(B)}, w) = \frac{2\pi}{e} \int \frac{d k_\perp}{(2\pi)^2} \left\{ \frac{\omega}{c k_z} (1 + r_{ss} e^{-2ik_z w}) \mathbf{e}_\perp - r_{pp} e^{-2ik_z w} \hat{k}_\perp \right\} \otimes \mathbf{e}_\perp + \left[ \frac{c k_z}{\omega} (-1 + r_{pp} e^{-2ik_z w}) \hat{k}_\perp + r_{pp} e^{-2ik_z w} \mathbf{e}_\perp \right] e^{ik_z(x_\perp^{(A)} - x_\perp^{(B)})}
$$

(6)

Now, Eq. (6) is a generalized Langevin Equation, of the type commonly used in the theory of irreversible processes [13]. At thermal equilibrium, it implies a set of general relations, collectively known as fluctuation-dissipation theorems [13], involving the frictional forces on one side, and correlators of random forces or of dynamical variables of the system, on the other. Of particular interest to us is the so-called second fluctuation-dissipation theorem [13], which in our case reduces to the following relation between the correlator of the ran-
dom force and the time Fourier transform of the damping

\[ \int_0^\infty dt e^{i\omega t} \left( E^{(\text{fluc})}_r(t_0 + t, x_\perp^{(A)}), E^{(\text{fluc})}_r(t_0 + t, x_\perp^{(B)}) \right) = -F(\omega, T) \begin{pmatrix} e^{(\text{ret})}_{ij}(\omega; \mathbf{x}_\perp^{(A)} - \mathbf{x}_\perp^{(B)}, w) \end{pmatrix} \]

where \( F(\omega, T) \) is the quantity

\[ F(\omega, T) = \frac{\hbar \omega}{2} \coth \left( \frac{\hbar \omega}{2k_B T} \right) , \]  

with \( T \) the temperature, and \( k_B \) Boltzmann’s constant.

We note that Eq. (7) takes account of quantum effects, and includes the contribution from zero-point fluctuations. The expressions for the quantities \( C^{(a/b)}(\omega, k_\perp) \) can be now determined by inserting Eqs. (1-3) into the l.h.s. of Eq. (7), and requiring that the two members are equal for arbitrary values of \( \mathbf{x}^{(A)}, \mathbf{x}^{(B)} \) and \( w \). After some tedious but straightforward computations one obtains for \( C^{(a/b)}(\omega, k_\perp) \) the following expressions:

\[ C^{(\infty\infty)} = F(\omega, T) \frac{2\pi \omega}{c^2} \operatorname{Re} \left( \frac{1}{k_z} \right) , \]  

\[ C^{(\infty S)} = C^{(S\infty)} = 0 \ . \]  

Eq. (9) coincides with the emission spectrum of a black surface, and Eqs. (10) show that this radiation is uncorrelated with that emitted by the surface. As for \( C^{(SS)} \), when \( k_z \) is real, i.e. for PW, we find:

\[ C^{(SS)} = \frac{2\pi \omega}{c^2 k_z} F(\omega, T) (1 - R R^\dagger) \ , \]  

while for \( k_z \) imaginary, i.e. for EW, we obtain:

\[ C^{(SS)} = -i \frac{2\pi \omega}{c^2 |k_z|} F(\omega, T) (R - R^\dagger) \ , \]  

where \( R \) is the reflection matrix \( R_{\lambda\mu} = r_{\lambda\mu} \). Eqs. (11) and (12) describe the emission from the surface. It should be observed that the contribution from PW, Eq. (11) is equivalent to Kirchhoff’s law, and it is obviously possible to derive it by simply imposing detailed energy balance between the radiation flux from the black surface at infinity and the flux radiated by the surface. The same type of argument, however, is useless to determine the EW contribution, Eq. (12), because the associated average flux of energy is zero. Therefore, Eq. (12) should be regarded as a non-trivial extension of Kirchhoff’s formula to the near-field region, and Eqs. (11) and (12) together provide the complete description of the e.m. field outside the body. It should be noted that EW fluctuations are zero when the reflection matrix \( R \) is hermitean.

We remark an interesting consequence of the symmetry properties of the correlation functions, in Eq. (7), in the absence of applied external magnetic fields, the correlators on the l.h.s. of Eq. (7) are invariant under time reversal, and therefore the quantities \( E_{ij} \) on the r.h.s. satisfy the following symmetry property:

\[ E_{ij}^{(\text{ret})}(\omega; \mathbf{x}_\perp^{(A)} - \mathbf{x}_\perp^{(B)}, w) = E_{ji}^{(\text{ret})}(\omega; \mathbf{x}_\perp^{(B)} - \mathbf{x}_\perp^{(A)}, w) \ . \]  

These relations constitute an example of Onsager’s reciprocity relations. Upon using Eq. (6), and recalling that Eq. (13) must be satisfied for all \( \mathbf{x}^{(A)}, \mathbf{x}^{(B)} \) and \( w \), one finds that Eq. (13) are equivalent to the following symmetry conditions for the reflection amplitudes:

\[ r_{ss}(\omega, \tilde{k}_\perp) = r_{ss}(\omega, -\tilde{k}_\perp) , \]  

\[ r_{pp}(\omega, \tilde{k}_\perp) = r_{pp}(\omega, -\tilde{k}_\perp) , \]  

\[ r_{sp}(\omega, \tilde{k}_\perp) = -r_{ps}(\omega, -\tilde{k}_\perp) . \]  

The above relations can be used a criterion to reject specific models for the reflection amplitudes. An interesting example of this sort is provided by the Drude-Born model for chiral materials, which is based on the following constitutive Equations (in the frequency domain):

\[ D = \varepsilon \mathbf{E} - \mathbf{f} \nabla \times \mathbf{E} , \quad \mathbf{B} = \mathbf{H} . \]  

It can be shown that the reflection amplitudes implied by this model do not satisfy Eqs. (16), and therefore this model must be rejected. However, the model for chiral materials proposed by Fedorov [14]

\[ D = \varepsilon (\mathbf{E} + \beta \nabla \times \mathbf{E}) , \quad \mathbf{B} = \mu (\mathbf{H} + \beta \nabla \times \mathbf{H}) \]  

satisfies the above conditions, and is therefore viable from the point of view of thermodynamics. Moreover, it is worth noting that chiral Fedorov’s materials, even if transparent, have a non hermitean reflection matrix \( R \). Proximity effects related to EW near the surface of chiral media are therefore expected.

**III. CONCLUSIONS AND DISCUSSION**

In conclusion, using general arguments from the thermodynamics of irreversible processes, we have obtained general formulae for the fluctuating e.m. fields outside a homogeneous surface. Our results are valid for all materials, at all temperatures, and provide a generalization of Kirchhoff’s law to the near field region. In particular,
they apply to non-diagonal reflection matrices, that mix TE and TM modes. Moreover, by use of Onsager’s reciprocity relations, we have also obtained the general symmetry properties that reflection amplitudes must satisfy, in order to be consistent with the requirements of thermodynamics. The same methods can be also applied to cavities bounded by homogeneous surfaces, and to systems out of thermal equilibrium, but we postpone to a separate paper the discussion of these problems.

[1] H.B.G. Casimir, Proc. K. Ned. Akad. Wet. Rev. 51, 793 (1948); S.K. Lamoreaux, Phys. Rev. Lett. 78, 5 (1997); U. Mohideen and A. Roy, ibid. 81, 4549 (1998); G. Bressi, G. Carugno, R. Onofrio and G. Ruoso, ibid. 88 041804 (2002); R.S. Decca, D. López, E. Fischbach, and D.E. Krause, ibid. 91, 050402 (2003); G.Bimonte, E. Calloni, G. Esposito, L. Milano and L. Rosa, ibid. 94, 180402 (2005).

[2] S.M. Rytov, *Theory of Electrical Fluctuations and Thermal Radiation*, Publishing House, Academy os Sciences, USSR (1953).

[3] D. Polder and M. Van Hove, Phys. Rev. B 4, 3303 (1971).

[4] A. I. Volokitin and B. N. J. Persson, Phys. Rev. B 63, 205404 (2001); G. Domingues, S. Volz, K. Joulain and J.J. Greffet, Phys. Rev. Lett. 94, 085901 (2005); G. Bimonte, Phys. Rev. Lett. 96, 160401 (2006).

[5] J. B. Pendry, J. Phys.: Condens. Matter 9, 10301 (1997); B.C. Stipe, H.J. Mamin, T.D. Stowe, T.W. Kenny and D. Rugar, Phys. Rev. Lett. 87, 096801 (2001); A. I. Volokitin and B. N. J. Persson, ibid. 91, 106101 (2003); J.R. Zurita-Sanchez, J.-J. Greffet and L. Novotny, Phys. Rev. A 69, 022902 (2004); A.I. Volokitin and B.N.J. Persson, Phys. Rev. Lett. 94, 086104 (2005).

[6] H.B. Chan, V.A. Aksyuk, R.N. Kleinman, D.J. Bishop and F. Capasso, Science 291, 1941 (2001); Phys. Rev. Lett. 87, 211801 (2001).

[7] Y.J. Lin, I. Teper, C. Chin and V. Vuletic, Phys. Rev. Lett. 92, 050404 (2004); J. M. McGuirk, D.M. Harber, J.M. Obrecht and E.A. Cornell, Phys. Rev. A 69, 062905 (2004); J.F. Babb, G.L. Klimchitskaya and V.M. Mostepanenko, ibid. A 70, 042901 (2004); M. Antezza, L.P. Pitaevskii and S. Stringari, ibid., 053619 (2004).

[8] I. Carusotto, L. Pitaevskii, S. Stringari, G. Modugno and M. Inguscio, Phys. Rev. Lett. 95, 093202 (2005).

[9] E.M. Lifshitz, Sov. Phys. JETP 2, 73 (1956); E.M. Lifshitz and L.P. Pitaevskii, *Landau and Lifshitz Course of Theoretical Physics: Statistical Physics Part II* (Butterworth-Heinemann, 1980.)

[10] M. T. Jaekel and S. Reynaud, J. Phys. 1, 1395 (1991); C. Genet, A. Lambrecht and S. Reynaud, Phys. Rev. A 67, 043811 (2003).

[11] R. Esquivel, C. Villareal and W. L. Mochan, Phys. Rev. A 68, 052103 (2003); ibid. 71, 029904 (2005); A. M. Contreras-Reyes and W. L. Mochan, ibid. 72, 034102 (2005).

[12] H. B. Callen and T. A. Welton, Phys. Rev. 83, 34 (1951).

[13] R. Kubo, Rep. Prog. Phys. 29, 255 (1966).

[14] A. Lakhtakia, *Beltrami Fields in Chiral Media* (World Scientific, 1994.)