Improved method of analysis for Recoil Distance Measurements of nuclear lifetimes

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Abstract. The Recoil Distance Method is known as a very powerful tool for measuring the lifetimes of excited nuclear levels in the picosecond region. Usually, for each level, coincidences with the shifted part of a feeding transition are used to avoid the bias from the accumulated delay in the higher lying cascade, by means of the Differential Decay Curve Method. Here, an integral version of the Decay Curve Method is proposed and discussed. This technique has several noticeable advantages, particularly when the overall statistics of the results is limited owing to the weak beam current, as expected with radioactive ion beams.

1. Introduction
Present and future facilities for radioactive ion beams have been reported in a number of talks in this conference. They will provide excellent opportunities for nuclear spectroscopy, but to exploit them at best we must be ready to refine our current methods of measurement and analysis. Here, I will consider the measurements of nuclear lifetimes in the picosecond region, with the Recoil Distance Method (RDM).

In this case, the excited nuclei, usually produced by fusion-evaporation, have enough momentum to exit from the target and fly in the vacuum, until they reach a stopper at a variable distance $x$ from the target. Decays in flight can be distinguished from those at rest in the stopper, due to their Doppler shift $\Delta E = E_0\beta \cos \theta$, where $\beta = v/c$ and $\theta$ is the angle between the direction of the emitted $\gamma$ and the velocity of the decaying nucleus.

If there was only one significant lifetime, the fraction of decays at rest would show the typical exponential decay with respect to the distance $x$. This is not the case when the accumulated delay in the feeding cascade is comparable with the lifetime to be measured, as the unshifted intensity is given by the convolution of the exponential decay with the feeding-time distribution $f(t)$. In this case, the shifted and unshifted components of the relevant $\gamma$ ray are measured in coincidence with the Doppler shifted part of a feeding transition, and the results are analyzed with the Differential Decay Curve Method (DDCM) [1, 2].

If all nuclei move with the same velocity in the direction of the beam, a simple relation holds between the unshifted intensity, the time derivative of the shifted intensity and the lifetime.
In fact,

\[ I_u^{(s)}(t) = \int_0^t f(t') \exp \left( \frac{t - t'}{\tau} \right) dt' \]  

\[ I_s^{(s)}(t) = \int_0^t f(t') \left[ 1 - \exp \left( \frac{t - t'}{\tau} \right) \right] dt' \]

By deriving Eq. 2 with respect to \( t \) one obtains

\[ I_u^{(s)}(t) = \tau \frac{dI_s^{(s)}(t)}{dt} = v \tau \frac{dI_s^{(s)}(x)}{dx} \]

where \( x = vt \) and the velocity \( v \) is assumed to be constant. A typical example of data analysis with DDCM is shown in Fig. 1. The curve of shifted intensity \( I_s(x) \) is fitted with a quadratic spline, whose derivative is used to deduce the lifetime from the unshifted intensity \( I_u \) at each measurement point. As we can see, in this case the result is remarkably stable in a rather wide region of confidence. However, when the errors on the individual points increase (as would be the case with lower statistical accuracy) or the slope of the shifted curve has a less regular trend, the error in the evaluation of the derivative can become rather large, especially at the two borders of the explored region. Moreover, working with very weak currents, one will be obliged to use a rather thick target. As a consequence, nuclei produced at different depths in the target will experience different energy loss before exiting in the vacuum, and a rather wide distribution of velocities of nuclei flying in the vacuum implies that their average velocity \( \bar{v} \) will be different for the different values of the target-to-stopper distance \( x \): faster nuclei, in fact, arrive to the stopper earlier, and contribute less to the average velocity. In this situation (that could be the rule for measurements with the weak beams of radioactive ions) the simple strategy of the DDCM (which assumes a constant value of \( \bar{v} \)) needs to be revised.

**Figure 1.** Example of analysis of RDM results with the DDCM. (\( J^\pi = 9^- \) level of \(^{146}\)Sm). The trend of the Doppler shifted intensities \( I_s \) (part a) is fitted with a quadratic spline (whose nodes are indicated by vertical lines). The unshifted intensities \( I_u \) at the different distances (part b) are divided by the corresponding derivatives of the spline and by the average velocity \( v \), to obtain the values of the mean life depicted in the part c. The resulting value of the mean life is \( \tau = 0.97(5) \) ns. From [3], Il Nuovo Cimento A, 111, 797 (1998), with kind permission of Società Italiana di Fisica.
2. The integral form of the Decay Curve Method

It is preferable, in such a situation, to use in the place of the DDCM Eq. 3, the one obtained by integration of it with respect to the time:

\[ \int_{t_1}^{t_2} I_u(t) \, dt = \tau [I_s(t_2) - I_s(t_1)] \]  

(4)

or its more general extension, which takes into account the possible dependence on \( x \) of the average velocity \( \bar{v}_x \) (along the beam direction) of nuclei decaying in flight:

\[ \int_{x_1}^{x_2} I_u(x) \, dx = \tau [\bar{v}_x(x_2) I_s(x_2) - \bar{v}_x(x_1) I_s(x_1)] \]  

(5)

It is important to note that Eq. 5 remains valid independently of the velocity distribution of nuclei moving in the vacuum, provided that \( \bar{v}(x) \) be deduced from the observed Doppler Shifts at the distances \( x_1 \) and \( x_2 \). In fact, let us assume that \( g(x, v_x)dv_x \, dx \) gives the number of nuclei with velocity component (along the beam direction) in the interval \( v_x, v_x + dv_x \), whose decay feeds the relevant level while they lie in the space interval \( x, x + dx \). The Doppler shifted intensity \( I_s \) and the unshifted intensity \( I_u \) are given by

\[ I_u(x) = \int_0^x dx' \int_0^\infty dv_x \, g(x', v_x) \exp \left( -\frac{x - x'}{v_x \tau} \right) \]  

(6)

\[ I_s(x) = \int_0^x dx' \int_0^\infty dv_x \, g(x', v_x) \left[ 1 - \exp \left( -\frac{x - x'}{v_x \tau} \right) \right] \]  

(7)

We integrate \( I_u \) from 0 to a generic distance \( X_0 \):

\[ \int_0^{X_0} I_u(x) \, dx = \int_0^{X_0} dx' \int_0^\infty dv_x g(x', v_x) \int_0^{X_0} \, dx \exp \left( -\frac{x - x'}{v_x \tau} \right) \]  

\[ = \int_0^{X_0} dx' \int_0^\infty dv_x \, g(x', v_x) \, v_x \tau \left[ 1 - \exp \left( -\frac{X_0 - x'}{v_x \tau} \right) \right] \]  

(8)

where

\[ \bar{v}_x(X_0) = \frac{\int_0^{X_0} dx' \int_0^\infty dv_x \, v_x \, g(x', v_x) \left[ 1 - \exp \left( -\frac{x - x'}{v_x \tau} \right) \right] \int_0^{X_0} \, dx \exp \left( -\frac{x - x'}{v_x \tau} \right)}{\int_0^{X_0} dx' \int_0^\infty dv_x \, g(x', v_x) \left[ 1 - \exp \left( -\frac{x - x'}{v_x \tau} \right) \right]} \]  

(9)

is exactly the one which can be deduced from the measured Doppler shift for a \( \gamma \) transition in coincidence with the same coincidence gate used to determine \( I_s \) and \( I_u \). To obtain Eq. 5, it is sufficient to take the difference of the results of Eq. 8 for \( X_0 = x_2 \) and \( X_0 = x_1 \).

In principle, one could use also the differential form

\[ I_u(x) = \tau \frac{d[\bar{v}_x(x) I_s(x)]}{dx} \]  

(10)

which can be obtained from Eq. 8 by derivation of its second line with respect to \( X_0 \). Also in this form, the result is independent of the particular shape of the velocity distribution. The differential form can be less useful than the integral one in many practical cases, particularly when the overall statistics is limited, but it can be interesting to compare the results one would obtain from it to those obtained from the DDCM formula 3, assuming (erroneously) a constant value of \( \bar{v}_x \). A simple example is discussed in Section 3.
As a consequence of the finite target thickness, the velocities $v_x$ of nuclei emerging from the target are statistically distributed in a finite interval around their average value $v_0$. The consequences for the DDCM analysis have been evaluated with a simple numerical simulation, whose results are shown in Fig. 2. The delayed feeding of the relevant level, assumed to have mean life $\tau$, is attributed to two cascading transitions having mean lives $\tau_1 = \frac{2}{3}\tau$ and $\tau_2 = \frac{5}{6}\tau$. The assumed statistical distribution of the velocity $v_x$ is rather broad, extending from $\frac{1}{2}v_0$ to $\frac{3}{2}v_0$. The average value $\bar{v}$ of velocities $v_x$ for nuclei decaying in flight (dashed curve in Fig. 2a) approaches $v_0$ at large values of the distance $x$ and decreases appreciably when this distance decreases. As we can see, the difference between the two curves of part a, showing the values of $\bar{v}I_s$ with the calculated values of $\bar{v}(x)$ or assuming a constant $\bar{v} = v_0$, is rather small, but it is significantly enhanced in the derivatives (Fig. 2c) and therefore also in the deduced mean lives (Fig. 2d). Neglecting the dependence of $\bar{v}$ on the distance $x$ would therefore result in a systematic error on the deduced mean life. To overcome this problem, computer codes have been developed to take into account, by a Monte-Carlo procedure [4], the process of energy loss and scattering in the target material of nuclei produced at different depths. In this way, one can exploit at best the information contained in the experimental results, but at the expense of systematic uncertainties related to the model used to describe the slowing-down process.

**Figure 2.** Numerical simulation of the analysis of RDM data with a relatively thick target (arbitrary units). The velocity distribution of the excited nuclei emerging from the target is assumed to be constant in the interval $\frac{1}{2}v_0 < v_x < \frac{3}{2}v_0$ and zero outside. The assumed distribution of the feeding time is $f(t) = [\exp(-t/\tau_1) - \exp(-t/\tau_2)]/(\tau_1 - \tau_2)$, with $\tau_1 = \frac{2}{3}\tau$, $\tau_2 = \frac{5}{6}\tau$. Solid lines show the results obtained assuming a constant $\bar{v}_x = v_0$, dotted lines take into account the correct dependence of $\bar{v}_x$ on $x$, which is depicted with a dashed line in part a. The product $\bar{v}_xI_s$ is shown in part a, and its derivative with respect to $x$ in part c. Part b shows the unshifted intensity $I_u$, and the estimated values of the mean life $\tau$ (ratios of values of part b and part c) are given in part d.
4. The optimal measurement

The proposed method, in its integral form (Eq. 8), provides a simple and model independent alternative, particularly useful when one has to deal with limited statistics. In such a case, it is unlikely that lifetime measurements can be performed for a large number of nuclear levels. Most probably, they will concern one or few levels, and the measurement conditions can be optimized for each of them.

The ideal experiment will consist of a long measurement at the largest distance $x_2$, to obtain the values of $I_s$ and $\bar{v}$ with sufficient accuracy, and of a series of shorter measurements at many points in the interval from $x_1$ to $x_2$, whose sum (or weighted sum) will give the integral of $I_u$ with comparable accuracy. The value of $I_s$ at the lower distance $x_1$ is usually rather small, and its measurement is not very significant for the result.

In this way, there is no need for a detailed Monte-Carlo simulation of the slowing-down process in the target. A detailed investigation of the slowing-down process in the stopper would still be necessary only for very short lifetimes approaching the stopping time in the stopper material.

**Figure 3.** Relevant quantities for the evaluation of the mean life by means of the Integral Decay Curve Method. The integral of the $I_u$ curve (part $b$), indicated by the shadowed area, must be compared with the difference of products $\bar{v}I_s$ at the two extremes of the interval (part $a$), $\bar{v}$ being the average velocity of nuclei decaying in flight (depicted as a dashed line).

**References**

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