Why are Orbital Currents Central to High T_c Theory?

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We explain qualitatively why the staggered flux state plays a central role in the SU(2) formulation of the $t$-$J$ model, which we use to model the pseudogap state in underdoped cuprates. This point of view is supported by studies of projected wavefunctions. In addition to staggered orbital current correlations, we present here for the first time results of correlations involving hole and spin chirality and show that the two are closely related. The staggered flux state allows us to construct cheap and fast vortices, which may hold the key to explaining the many anomalous properties of the normal state.

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It is now a widely accepted view that the problem of high T_c superconductivity is the problem of doping into a Mott insulator. By doping $x$ holes per unit cell, the Néel order is rapidly destroyed and $d$-wave superconductivity emerges. Many of us believe the physics is captured by the $t$-$J$ model, and the competition is between the kinetic energy of the hole $xt$, where $t$ is the hopping matrix element, and the exchange energy $J$. This competition leads to spectacular new physics in the underdoped region, where the pseudogap phenomenon has been well documented. An understanding of the underdoped region is prerequisite to understanding the entire phase diagram.

One view of the pseudogap phase is that it is a local superconductor with robust amplitude but strong phase fluctuations. Setting aside the question of where the strong pairing amplitude comes from in the first place, that this view is incomplete can be seen from the following argument. In two dimensions the destruction of superconducting order is via the Berezinskii-Kosterlitz-Thouless (BKT) theory of vortex unbinding. Above $T_c$ the number of vortices proliferate and the normal metallic state is reached only when the vortex density is so high that the cores overlap. (There is considerable latitude in specifying the core radius, but this does not affect the conclusion.) At lower vortex density, transport properties will resemble a superconductor in the flux flow regime. In ordinary superconductors, the BKT temperature is close to the mean field temperature, and the core energy rapidly becomes small. However, in the present case, it is postulated that the mean field temperature is high, so that a large core energy is expected. Indeed, in a conventional core, the order parameter and energy gap vanish with an energy cost of $\Delta_0^2 / E_F$ per unit area. Using a core radius of $\xi = V_F / \Delta_0$, the core energy of a conventional superconductor is $E_F$. In our case, we may replace $E_F$ by $J$. If this were the case, the proliferation of vortices will not happen until a high temperature $\sim J$ independent of $x$ is reached. Thus for the phase fluctuation scenario to work, it is essential to have “cheap” vortices, with energy cost of order $T_c$. Then the essential problem is to understand what the vortex core is made of.

In the past several years, Wen and Lee have developed an SU(2) formulation of the $t$-$J$ model and the staggered flux state has emerged as the natural candidate for the competing state which makes up the vortex core. Indeed, Lee and Wen have successfully constructed a “cheap” vortex state. Other possibilities, such as a Néel ordered state or spin density waves or dimers, have been proposed. In our view, the staggered flux phase has an advantage over other possibilities in that its excitation spectrum is similar to the $d$-wave superconductor. In any event the theory is not complete until the nature of the alternative state which constitutes the vortex core is understood. Then the pseudogap phase can be understood equally well as fluctuating superconductors with regions of the alternative state or as a fluctuating alternative state with regions of superconductivity.

The staggered flux state was first introduced as a mean field solution at half-filling and later was extended to include finite doping. It exhibits the remarkable property that fermions hopping on a square lattice penetrated by
staggered flux $\pm \Phi_o$ has an excitation spectrum identical to that of a $d$-wave superconductor at half-filling. There is a gap $\Delta_o$ at $(0, \pi)$ and nodes at $(\pm \pi/2, \pm \pi/2)$, provided that $\tan \left( \frac{2\Phi_o}{\chi_o} \right) = \frac{\Delta_o}{\chi_o}$, where $\chi_o$ is the hopping amplitude. At half-filling, due to the constraint of no double occupation, the staggered flux state corresponds to an insulating state with power law decay in the spin correlation function. It is known that upon including gauge fluctuations which enforce the constraint, the phenomenon of confinement and chiral symmetry breaking occurs, which directly corresponds to Néel ordering. The idea is that with doping, confinement is suppressed at some intermediate energy scale, and the state can be understood as fluctuating between the staggered flux state and the $d$-wave superconducting state. Finally, when the holes become phase coherent, the $d$-wave superconducting state is the stable ground state. Thus the staggered flux state may be regarded as the “mother state” which is an unstable fixed point due to gauge fluctuations. It flows to Néel ordering at half-filling and to the $d$-wave superconductor for sufficiently large $x$. Thus the staggered flux state plays a central role in this kind of theory. We should point out that the staggered flux state (called the $D$-density wave state) has recently been proposed as the ordered state in the pseudogap region. As explained elsewhere, we think that this view is not supported by experiment and we continue to favor the fluctuation picture.

The above picture finds support from studies of projected wavefunctions, where the no-double-occupation constraint is enforced by hand on a computer. This field has a long history. For example, it has been known for a long time that if one takes a spin density wave state for the fermion (i.e., introduce a staggered magnetization mean field as a variational parameter) and performs what is called the Gutzwiller projection, i.e., project out all doubly-occupied configurations, one does not obtain a very good wavefunction. On the other hand, projection of a $\pi$-flux phase without any variational parameter does surprisingly well. The best state is the combination of staggered magnetization with some flux, either $\pi$-flux or staggered-flux, and an excellent energy is achieved. With doping the best state is a projected $d$-wave state. Not surprisingly, this state has long range pairing order after projection. Recently we calculated the current-current correlation function of this state

$$c_j(k, \ell) = \langle j(k)j(\ell) \rangle$$

where $j(k)$ is the physical electron current on the bond $k$. The average current $\langle j(k) \rangle$ is obviously zero, but the correlator exhibits a staggered circulating pattern as shown in Fig.1. (Note that the pattern is shifted by $\pi$ relative to a pattern constructed from the reference bond at the origin.) Within our numerical accuracy, this correlation decays as a power law and the decay is faster with increasing doping. Such a pattern is absent in the $d$-wave BCS state before projection, and is a result of the Gutzwiller projection.
FIG. 1. (a) Current-current correlations for 2 holes in the 10 × 10 lattice. Boundary conditions are periodic in one and antiperiodic in the other directions (the data are averaged over the two orientations). The number on a link is the correlation of the current on this link and of the current on the circled link divided by hole density. The arrows point in the direction of the positive correlations of the current. (b) The same data in the form of vorticity defined as the sum of the current around a plaquette. The number on a plaquette is the vorticity correlation divided by $x$ with the crossed plaquette. (c) Same as (b) for 10 holes in 10 × 10 lattice.

We were motivated to look for the staggered pattern in the current-current correlation function because that is what we expect to find in the staggered flux phase. Consider a plaquette with a hole in the top left corner (4) and spins on the other three corners (labelled clockwise 1–3). A hole hopping around the plaquette sees a wandering spin quantization axis from site to site and will pick up a Berry’s phase $\Phi$ which is given by the solid angle subtended by the 3 spin directions. The flux in the flux phases is designed to capture this piece of physics, as a hole hopping in the presence of a gauge “magnetic flux” will also pick up an Aharonov-Bohm phase. The physical idea is that the Néel state is detrimental to hole hopping, but if the spins are not coplanar, we may achieve a better compromise between the exchange energy and the hole kinetic energy. More formally, in the slave boson representation where the electron operator is written as $c_{i\sigma} = f_{i\sigma} b_i^\dagger$, it is known that if we define $Q_{ij} = \sum_\sigma f_{i\sigma}^\dagger f_{j\sigma}$, then

$$\text{Im} \left( \prod : Q_{12} Q_{23} Q_{34} Q_{41} : \right) = \left( f_{4\sigma}^\dagger f_{4\sigma} \right) \vec{S}_1 \cdot \vec{S}_2 \times \vec{S}_3 + \text{permutations},$$

In the mean field theory, the L.H.S. of Eq.(2) is $|\chi|^4 \sin \Phi$. Thus the flux is also related to the spin chirality defined as $\chi = \vec{S}_1 \cdot \vec{S}_2 \times \vec{S}_3$. We note that in Eq.(2), $f_{4\sigma}^\dagger f_{4\sigma} = 1 - b_i^\dagger b_i = 1 - n_H$ where $n_H$ is the hole density. We thus consider

$$c_{\chi_H\chi_H}(k, \ell) = \left< n_H(4) \vec{S}_1 \cdot \vec{S}_2 \times \vec{S}_3(k) \right| n_H(4') \vec{S}_{1'} \cdot \vec{S}_{2'} \times \vec{S}_{3'}(\ell) \right> \cdot$$

where $(k, \ell)$ labels plaquettes and the spins 1,2,3 form a triangle with a fixed orientation around plaquette $k$, as described earlier.
(a)

(b)
Fig. 2. The correlation function \( c_{\chi_H \chi_H} \times \frac{1}{x} \) as a function of plaquette position. The reference plaquette is at the center. The lower number is the statistical error. Plaquettes with positive (negative) values are shown in white (black) while plaquettes whose signs are undetermined within the error bar are shown in grey. The calculation was done with periodic boundary conditions in one direction and antiperiodic boundary conditions in the other. The only symmetry is between \( r \) and \(-r\) and this symmetry was used in the computation. Fig. 2(a) is for two holes in a 16 \times 16 lattice and Fig. 2(b) is for six holes in a 10 \times 10 lattice. The correlation data is in unit of \( 10^{-5} \).

In Fig. 2 we show the results for \( c_{\chi_H \chi_H} \) for two holes in 16 \times 16 and six holes in 10 \times 10. The large number of operators in Eq. (3) makes the computation more time consuming than for \( c_j \) and the resulting error bars are larger. Setting aside results with undetermined signs within the error bar, we find a perfect staggered correlation given by the black and white checkerboard pattern. The pattern is also phase shifted from the central one, just as we found for \( c_j \). Note that the correlator on the same site is very large. This is because it only require the presence of a hole on a single site. To get a fair comparison, we should multiply the equal site correlator by a factor of \( x \). Note that the correlator decreases rather slowly with distance. This decrease is shown in Fig. 3, which plots \( c_{\chi_H \chi_H} \) vs \( R^2 \) on a log-log plot. We also plot the vorticity correlator constructed from \( c_j \) from ref. (13). We see that both are consistent with the same power law decay.

![Fig.3](image)

Fig. 3 The absolute value of the correlation function \( c_{\chi_H \chi_H} \times \frac{1}{x} \) plotted vs the square of the distance between plaquettes on a log-log plot for two holes in a 16 \times 16 lattice. Values of plaquettes with the same distance have been averaged. The correlation data is in unit of \( 10^{-5} \). We also reproduced the data on the current vorticity correlator divided by \( x \) for 2 holes in 18 \times 18 sites from Ivanov et al. We have divided the latter by 10 for clarity.

We have also computed \( c_{\chi_H \chi} \) given by

\[
c_{\chi_H \chi}(k, \ell) = \left\langle n_H(4) \vec{S}_1 \cdot \vec{S}_2 \times \vec{S}_3(k) \vec{S}_{1'} \cdot \vec{S}_{2'} \times \vec{S}_{3'}(\ell) \right\rangle.
\]

This gives information about the spin chirality generated by a hole and chirality combination some distance away.
Naively we might expect that $c_{\chi H \chi H}$ should be larger than $c_{\chi H \chi H}$ by a factor $\frac{1}{2}$, because a hole is not required to be associated with one of the chirality factors. Surprisingly we found that $c_{\chi H \chi H}$ is of similar magnitude as $c_{\chi H \chi H}$ and shows an intricate correlation. However, due to the small size of the correlation that results from delicate cancellations, the error bars are too large for us to make a detailed analysis. Our results for $c_j$, $c_{\chi H \chi H}$, and $c_{\chi H \chi}$ are consistent with exact diagonalization of two holes in 32 sites.

We conclude that the correlation of the hole with the spin chirality on the same plaquette is the key physics and that the current correlator may be viewed as the symptom, rather than the root cause of the pseudogap phenomenon. Of course, the current correlator has the advantage that it is easier (relatively speaking) to measure than the $\chi H \chi H$ correlator. The staggered current generates a staggered physical magnetic field (estimated to be 10–40 gauss), which may be detected, in principle, by neutron scattering. In practice the small signal makes this a difficult, though not impossible experiment and we are motivated to look for situations where the orbital current may become static or quasi-static. Recently, we analyzed the structure of the $\hbar c/2e$ vortex in the superconducting state within the SU(2) theory and concluded that in the vicinity of the vortex core, the orbital current becomes quasi-static, with a time scale determined by the tunnelling between two degenerate staggered flux states. It is very likely that this time is long on the neutron time scale. Thus we propose that a quasi-static peak at $(\pi, \pi)$ will appear in neutron scattering in a magnetic field, with intensity proportional to the number of vortices. The time scale may actually be long enough for the small magnetic fields generated by the orbital currents to be detectable by $\mu$-SR or Yttrium NMR. Again, the signal should be proportional to the external fields. (The NMR experiment must be carried out in 2–4–7 or 3 layer samples to avoid the cancellation between bi-layers.) We have also computed the tunnelling density of states in the vicinity of the vortex core, and predicted a rather specific kind of period doubling which should be detectable by atomic resolution STM. The recent reports of a static field of $\pm 18$ gauss in underdoped YBCO which appears in the vortex state is promising, even though muon cannot distinguish between orbital current or spin as the origin of the magnetic field. We remark that in the underdoped antiferromagnet, the local moment gives rise to a field of 340 gauss at the muon site. Thus if the 18 gauss signal is due to spin, it will correspond to roughly 1/20th of the full moment.

We remark that our analytic model of the vortex core is in full agreement with the numerical solution of unrestricted mean field $\Delta_{ij}$ and $\chi_{ij}$ by Han and Lee. Upon re-examination of their numerical solution, they also found staggered orbital current in their vortex core. This vortex solution is also interesting in that the tunnelling density of states show a gap, with no sign of the large resonance associated with Caroli-deGennes-type core levels found in the standard BCS model of the vortex. There exists a single bound state at low lying energy in agreement with STM experiments. The low density of states inside the vortex core has an important implication. In the standard Bardeen-Stephen model of flux-flow resistivity, the friction coefficient of a moving vortex is due to dissipation associated with the vortex core states. Now that the core states are absent, we can expect anomalously small friction coefficients for underdoped cuprates. The vortex moves fast transverse to the current and gives rise to large flux-flow resistivity. Indeed, the flux-flow resistivity is given by

$$\rho_{\text{flux-flow}} = \frac{B\Phi_o}{\eta e^2}.$$  

Since the total conductivity is the sum of the flux-flow conductivity and the quasiparticle conductivity, it is possible to get into a situation where the quasiparticle conductivity dominates even for $H \ll H_{c2}$. Thus the “cheap” and “fast” vortex opens the possibility of having vortex states above the nominal $T_c$ and $H_{c2}$, when the resistivity looks like that of a metal, with little sign of flux-flow contribution. From this point of view, the large Nerst effect observed by Ong and co-workers over a large region in the $H$-$T$ plane above the nominal $T_c$ and $H_{c2}$ (as determined by resistivity) may be qualitatively explained.
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