Abstract: Due to the curve of the LEO synthetic aperture radar (SAR) orbit and the rotation of the earth, the high resolution imaging for LEO SAR is always facing the problem of obtaining accurate range history. Meanwhile, with the increase of squint angle, it will take longer azimuth time to meet the same high resolution. In this paper, an improved hyperbolic range equation (IHRE) is presented to model the accurate range history for high squint LEO SAR. First, the state vectors of satellite are used to estimate the Doppler parameters. Then, the improved hyperbolic range equation is recovered based on the estimation of the Doppler parameters. Compared with other existing methods, the superiority of IHRE is more precise in the condition of high squint LEO SAR which can provide sufficient azimuth time for high resolution imaging. The simulation and analysis validate the precision of the proposed range history model.

1 Introduction

Synthetic aperture radar (SAR) has been widely used in many fields due to its high-resolution imaging capabilities regardless of weather conditions. Since the SEASAT that is the first LEO SAR was launched into space, the LEO SAR has developed rapidly. The high squint LEO SAR has received great attention in recent years, because it can provide more diversity and flexibility for SAR tasks. However, the satellite orbit is not a straight line and the target is rotating with the earth, the range model of airborne is no longer applicable. Thus, the adaptive range history model becomes the most important challenge in spaceborne SAR.

There are several existing range history model up to now, including conventional hyperbolic range equation (CHRE) [1–3], advanced hyperbolic range equation (AHRE) [4] and the four-order Doppler range model (DRM4) [5]. For CHRE, although it can provide integration time <10 s in side-looking, its performance will decrease rapidly with the increase of squint angle, which is unable to meet high resolution requirements. In [4], AHRE was proposed to discuss the range history of medium earth orbit (MEO) SAR and achieve a longer integration time, but it did not consider the changes with the increase of squint angle. DRM4 was first presented in [5] which was more accurate than the two methods mentioned earlier, but it was more complicated and needed to estimate more Doppler parameters than other models.

In this paper, an IMPROVED range model for high squint LEO SAR is proposed. It considers the local orbit curve of the LEO SAR and introduces an additional quadratic component to fitting the local orbit curve. The improved range mode uses the state vectors between target and the satellite to estimate Doppler parameters and calculate the coefficient of the improved range mode according to the relationship between the azimuth phase and range history. The simulation results with different squint angle demonstrate the superior precision of the proposed range history model.

2 High squint LEO SAR range history model

2.1 Improved hyperbolic range equation

As shown in Fig. 1, the solid line is the real satellite orbit and the dashed is the equivalent model with straight line. Compared with the two paths, it can be considered that the real satellite orbit is a circle arc in local curve orbit and the range error between the two paths is a quadratic component. Thus, an extra quadratic component is introduced into CHRE which can approximate the relative Satellite–Earth motion based on the Doppler parameter estimation for high squint LEO SAR. The expression of proposed range mode is as follow:

\[
R(t) = R_0 - V_i \sin(\theta_{sq}) \cdot t + \left\{ \frac{V_i \cos^2(\theta_{sq})}{2R_0} + a \right\} t + \left\{ \frac{V_i \sin(\theta_{sq}) \cos(\theta_{sq})}{2R_0^2} \right\} t^2 + \left\{ \frac{V_i \cos^2(\theta_{sq}) \cdot (5 \sin^2(\theta_{sq}) - 1)}{8R_0^3} \right\} t^3 + \ldots .
\]

Using (1), (2), the Doppler frequency is defined as

\[
f_d(t) = -\frac{2}{\lambda} \frac{dR}{dt} = \frac{2V_i \sin(\theta_{sq})}{\lambda} + \left\{ -\frac{2V_i \cos^2(\theta_{sq})}{R_0} \right\} t + \left\{ \frac{-6V_i \sin(\theta_{sq}) \cos(\theta_{sq})}{R_0^2} \right\} t^2 + \left\{ \frac{-V_i \cos^2(\theta_{sq}) \cdot (5 \sin^2(\theta_{sq}) - 1)}{R_0^3} \right\} t^3 + \ldots .
\]

Compared to the characteristics of the Doppler frequency

\[
f_d(t) = f_{dc} + f_{d1} t + f_{d2} t^2 + f_{d3} t^3 + \ldots .
\]
where $f_{dc}$ is the Doppler centroid frequency, $f_{at}$ denotes the nth-order Doppler FM rate. Then combine (3) and (4), the variables in (1) could be solved with the following equations

$$\begin{equation}
\begin{aligned}
2V_s\sin(\theta_{sq}) - f_{at} &= 0 \\
-2V_s\cos(\theta_{sq}) + \frac{4a}{\lambda} - f_u &= 0 \\
-6V_s\sin(\theta_{sq})\cos(\theta_{sq}) + \frac{\lambda R_0}{2} - f_{w} &= 0.
\end{aligned}
\end{equation}$$

Finally, the result of the variables could be calculated as follows:

$$a = \frac{Rf_{at}}{6f_{dc}} - \frac{\lambda f_u}{4},$$

$$V_z = \sqrt{\left(\frac{\lambda f_{dc}}{2}\right)^2 - 2R_0(f_{at} + (4a/\lambda))} = \sqrt{\left(\frac{\lambda f_{dc}}{2}\right)^2 - 3Rf_{at}},$$

$$\theta_{sq} = \arcsin\left(\frac{\lambda f_{dc}}{2V_s}\right).$$

The estimation of Doppler parameters is the key which utilises the state vectors of the satellite to treat the relative motion between satellite and target. In [7–9], detailed calculation method of the Doppler parameters was derived as follows

$$f_{at} = -\frac{2R'}{\lambda} - \frac{2}{\lambda}(r_s - r_v)(r_s - r_v),$$

$$f_u = -\frac{2R'\lambda}{\lambda},$$

$$f_{w} = -\frac{2R^0\lambda}{\lambda},$$

$$f_u = -\frac{2R'\lambda}{\lambda},$$

$$f_{w} = -\frac{2R^0\lambda}{\lambda},$$

where $R$ is the instantaneous range between target and the satellite. In the Earth Centred Inertial (ECI) coordinates, $r$ is the three dimension position vector, $v$ is the three dimension velocity vector, $A$, $A'$ and $A''$ are the nth-order three dimension acceleration vector. The subscript of state vectors represents target and the satellite, respectively.

### 2.2 Model parameter analysis

The expression of IHRE introduced an additional quadratic component which will influence the Taylor series expression of CHRE seen in (2). Compared the linear and quadratic term between CHRE and IHRE as follows

$$\begin{equation}
\begin{aligned}
\left\{V_s\sin(\theta_{sq})\right\}_{CHRE} &= \left\{V_s\sin(\theta_{sq})\right\}_{IHRE} \\
\left\{V_s\cos(\theta_{sq})\right\}_{CHRE} &= \left\{V_s\cos(\theta_{sq})\right\}_{IHRE} + a
\end{aligned}
\end{equation}$$

From (11), it can be found that the expression of quadratic term are different because of the existence of the coefficient $a$. However, according to the expression of CHRE, the Taylor series expansion of CHRE can be accurately fitted to linear term of the Doppler history, namely, the two order component of the actual range process. Thus, the true value of quadratic term is identical that means the coefficient $V_z$ and $\theta_{sq}$ of CHRE are different from these of IHRE.

Contrary to (11), the true value of cubic term between CHRE and IHRE will be different as follows

$$\left\{V_s\sin(\theta_{sq})\cos(\theta_{sq})\right\}_{CHRE} \neq \left\{V_s\sin(\theta_{sq})\cos(\theta_{sq})\right\}_{IHRE}.$$  

The cubic term of IHRE is recovered from the estimated Doppler parameter which is accurate, and the cubic term of CHRE is calculated from the expression of Taylor series. Thus, the range error of IHRE must be better than that of CHRE and the main range error comes from the cubic term fitting error.

### 3 Simulation and analysis

In order to further demonstrate the accuracy of the proposed hyperbolic range equation, there are several simulations for evaluating the accuracy among CHRE, AHRE and IHRE with high squint LEO parameters. The simulation parameters are listed in Table 1.

Fig. 2a shows that the range error of CHRE will increase sharply with the increasing of squint angle and the order of magnitude of the range error can reach a few meters. However, as seen in Figs. 2b and c, the accuracy of AHRE and IHRE are better and the range error of them can reduce an order of magnitude, which will greatly increase the available azimuth time.
As described in Section 2.2, the result of Fig. 3 proves that the main range error between CHRE and IHRE comes from the cubic term fitting error. In addition, with the increase of squint angle and azimuth time, cubic term fitting error will affect the range accuracy seriously.

Compared to AHRE, the expression of IHRE is similar to that of AHRE, only replacing the linear term with quadratic term. Due to the accurate estimation of quadratic term of the Doppler history, they do not have three order fitting error and face the same problem, that is, the main fitting error is four order term. However, AHRE was initially proposed to solve the range process of MEO SAR orbit and did not take into account the change of squint angle. Fig. 4 indicates that with the increasing of squint angle, the ratio of four order fitting error of IHRE is less than that of AHRE and the simulation results Figs. 2b and c also verify the conclusion.

Fig. 5 illustrates the phase errors respect to the 0°, 15°, 30° and 45° squint angle among CHRE, AHRE and IHRE. The phase errors can be calculated by

\[\phi_{err}(t) = -\frac{4\pi}{\lambda} \Delta R_{err}(t),\]  

where \(\Delta R_{err}(t)\) denotes the range error during azimuth time. While the phase error during azimuth time is \(<\pi/4\), the influence on imaging can be ignored. Fig. 5 shows that the accuracy of IHRE is best with the increasing squint angle and the performance of all the range models will degrade due to the increasing azimuth time.

Finally, the detailed azimuth time available under different squint angle are shown in Table 2. From Table 2, the available azimuth time of CHRE will decrease rapidly while the squint angle increases. Although the available azimuth time of AHRE shows fewer changes in the condition of the increasing of squint angle, the LEO SAR flies with L-band can only provide the focusing ability of an azimuth resolution around 3 m at high squint angle. Contrary to AHRE, IHRE can achieve longer available azimuth time which is suitable for ultrahigh resolution case.
Conclusions

This paper has presented an improved hyperbolic range equation model for high squint LEO SAR which fits well for high resolution case. An additional quadratic component introduced into CHRE achieves better range accuracy and longer available azimuth time than AHRE, which is similar to the equation of proposed range model. All simulations and analyses illustrate the effectiveness of the proposed range model.

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Fig. 5 Phase error analysis of three models with different squint angles
(a) Phase error with 0° squint angle, (b) Phase error with 15° squint angle, (c) Phase error with 30° squint angle, (d) Phase error with 45° squint angle

Table 2 Azimuth time available under different squint angle

| Squint angle | CHRE, s | AHRE, s | IHRE, s |
|--------------|---------|---------|---------|
| 0°           | 8.9680  | 8.9680  | 7.5640  |
| 15°          | 3.3340  | 8.4980  | 9.3080  |
| 30°          | 2.6760  | 7.7760  | 11.1060 |
| 45°          | 2.3840  | 7.4360  | 20.5540 |

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