Higher Moments of Net-Baryon Distribution as Probes of QCD Critical Point

Y. Zhou\textsuperscript{1,2\ast}, S. S. Shi\textsuperscript{1,2}, K. Xiao\textsuperscript{1,2}, K. J. Wu\textsuperscript{1,2} and F. Liu\textsuperscript{1,2}

\textsuperscript{1}Institute Of Particle Physics, HuaZhong Normal University(CCNU), Wuhan 430079, People’s Republic of China
\textsuperscript{2}Key Laboratory of Quark & Lepton Physics, HuaZhong Normal University(CCNU), Ministry of Education, Wuhan 430079, People’s Republic of China
\textsuperscript{*}zhou@iopp.ccnu.edu.cn

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It is crucially important to find an observable which is independent on the acceptance and late collision process, in order to search for the possible Critical Point predicted by QCD. By utilizing A Multi-Phase Transport (AMPT) model and Ultra Relativistic Quantum Molecular Dynamics (UrQMD) model, we study the centrality and evolution time dependence of higher moments of net-baryon distribution in Au + Au collisions at $\sqrt{s_{NN}} = 17.3$ GeV. The results suggest that Kurtosis and Skewness are less sensitive to the acceptance effect and late collision process. Thus, they should be good observables providing the information of the early stage of heavy ion collision. In addition, our study shows that the Kurtosis times $\sigma^2$ of net-proton distribution are quite different to that of net-baryon when collisions energy is lower than $\sqrt{s_{NN}} = 20$ GeV, the Monte Carlo calculations on Kurtosis-$\sigma^2$ have a deviation from the theoretical predictions.

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I. INTRODUCTION

In QCD phase diagram, the first order phase transition boundary separates the two phases: the hadron gas and the quark gluon plasma(QGP). The Critical Point(CP) which locates at the end of the transition boundary, is a distinct singular feature of QCD phase diagram\textsuperscript{[1]}. To map the components of the QCD phase diagram is one of the main goals of heavy-ion physics, and searching for the critical point has been addressed in both theoretical and experimental studies\textsuperscript{[2–7]}, all these works help us to gather information of the singularities near the CP. If the CP existence around $\mu_B \sim 300$ MeV\textsuperscript{[4]} it can be studied by heavy ion experiments, like BNL Relativistic Heavy Ion Collider (RHIC) with its Beam Energy Scan (BES) program\textsuperscript{[5]}, CERN Super Proton Synchrotron (SPS)\textsuperscript{[6]} and the future GSI Facility for Antiproton and Ion Research (FAIR)\textsuperscript{[7]}. The fluctuations of conserved quantities, like net baryon number, electric charge and strangeness, are considered to be sensitive indicators for the structure of created system\textsuperscript{[8]}. The characteristic feature of a CP is the diverge of the fluctuations. Most proposed fluctuation of observables are variations of $2^{\text{nd}}$ order moments of the distribution, such as particle ratio\textsuperscript{[8]}, charged dynamical\textsuperscript{[9]} and $\Phi$-measure\textsuperscript{[10]} . It has been proved that all these observables are proportional to approximately $\xi^2$\textsuperscript{[11]} , where $\xi$ is the correlation length which will diverges at the CP. Theoretical calculations predict that $\xi \approx 2–3$ fm for heavy ion collision\textsuperscript{[12]}, thus, it is extremely difficult to measure in experiments. However, the recent results shown that higher moments of conserved quantities distribution are more sensitive to CP due to their strong dependence on $\xi$, the $3^{\text{rd}}$ order moment Skewness$\sim \xi^{4.5}$ while the $4^{\text{th}}$ order moment Kurtosis$\sim \xi^2$\textsuperscript{[3]}. Also the product Kurtosis-$\sigma^2$ (called $K_{\text{eff}}$ in reference\textsuperscript{[13]}) which is equal to the ratio of $4^{\text{th}}$ order to $2^{\text{nd}}$ order susceptibilities shows a large deviation from unity near the CP by lattice calculations and QCD based model studies\textsuperscript{[14]}.

Studying the event-by-event fluctuations\textsuperscript{[13,16]} is an effective way to address fluctuations of a system created in a heavy ion collision\textsuperscript{[17]}. In this paper, we will present the study of higher moments as a function of evolution time, and we will discuss the acceptance effect on higher moments of net baryon distribution.

II. MONTE CARLO MODELS

Monte Carlo event generators, AMPT and UrQMD, have been used in this study. A Multi-Phase Transport(AMPT) model is made up by four main parts: the initial conditions, partonic interactions, hadronization and hadronic rescattering. The initial conditions, which include the spatial and momentum distributions of minijet partons and soft string excitations, are obtained from the Heavy Ion Jet Interaction Generator(HIJING) model\textsuperscript{[18]}. Scattered between partons are modeled by Zhang’s Parton Cascade(ZPC)\textsuperscript{[19]}, which presently includes only two-body scatterings with cross sections obtained from the pQCD with screening masses. In the default AMPT model(abbr. “AMPT Default”)\textsuperscript{[20]}, partons are recombined with their parent strings when they stop interacting, and the resulting strings are converted to hadrons using the Lund string fragmentation model\textsuperscript{[21,22]}. In the AMPT model with string melting(abbr. “AMPT StringMelting”)\textsuperscript{[23]}, the transition from the partonic matter to the hadronic matter is achieved by a simple coalescence model, which combines two quarks into mesons and three quarks into...
baryons \(^{24}\). The authors of AMPT model use hadronic cascade, which is based on the A Relativistic Transport (ART) model \(^{25}\), to describe the dynamics of the subsequent hadronic matter. Final results from the AMPT model are obtained after hadronic interactions are terminated at a cutoff time \(t_{\text{cut}}\). When setting the time-step (in fm/c) for hadron cascade to 0.2 (default value), the termination time of hadron cascade \(t_{\text{cut}} = 0.2 \times \text{NTMAX}\). Here NTMAX is the number of time-steps, the default setting is NTMAX. When the time-step is fixed, larger NTMAX means longer time of hadron rescatterings. Note NTMAX lets all resonance rapidly decay (less than 0.6fm/c) and turns off hadronic interactions effectively. In this paper, we measure the fluctuation which belongs to different processes: scenario (a) "parton" is a process between ZPC and quark coalescence, it still locates in the partonic phase, scenario (b) "w/o ART" is the time that hadronization (or quark coalescence) has finished, it is in hadronic phase but no hadron cascade happens, scenario (c) and (d) are originated from different termination time \(t_{\text{cut}}\), by setting different NTMAX value we can control the hadronic process in the simulated heavy ion collisions. It helps us to understand the hadronic effect on the observable. In order to investigate different effects in time evolution, we will study the fluctuation of higher moments in each process. Comparing the measured value from different processes, we can study whether the fluctuation of the higher moment can provide the information from the early stage of the collisions.

The Ultra Relativistic Quantum Molecular Dynamics (UrQMD) model \(^{26}\) is also been used in this paper. It is a microscopic transport theory based on the covariant propagation of all hadrons on classical trajectories in combination with stochastic binary scatterings, color string formation and resonance decay. The UrQMD model represents a Monte Carlo solution of a large set of coupled partial integro-differential equations for the time evolution of the various phase space densities \(f_i(x,p)\) of different species of particles. In the input file, one can control the time to propagate and output time-interval (in fm/c), in this paper we set the time to propagate is 40 fm/c and the time-interval is 2 fm/c. More detail descriptions can be found in Ref. \(^{26}\).

### III. RESULTS AND DISCUSSION

In experiments, neutrons can not be detected, and the reconstruction efficiency is relatively low for strange hadrons, especially for multi-strange hadrons, such as \(\Xi\) and \(\Omega\) \(^{30}\). Fortunately, theoretical calculation suggests that the proton number is a meaningful observable for the purpose of detecting the CP in heavy ion experiments \(^{27}\), its fluctuation completely reflect the singularity of the baryon number susceptibility. Thus, only if the measurements from net baryon and net proton distribution are similar, we can searching for the CP by measuring the fluctuation of various moments of net proton distribution.

Observables that can reflect real dynamics and have little influence on the finite acceptance, are worth exploring from the theoretical point of view \(^{28, 29}\). Due to the limited acceptances in experiments, it is necessary to study the acceptance effect by Monte Calor model. In the previous work \(^{31}\), we presented that the Skewness and Kurtosis have little influence on the size of rapidity windows and transverse momentum regions, they could be good candidate for searching the CP in experiments.

Firstly, we study the transverse momentum window...
Kurtosis (B)

FIG. 3: (Color online) The centrality dependence of Kurtosis of net-proton, net-cBaryon and net-baryon distribution in time evolution at √sNN = 17.3 GeV within AMPT StringMelting.

Kurtosis can provide the information of the signature of the early stage. With AMPT StringMelting, we can illustrate different effects on Kurtosis in time evolution clearly. In Fig. 3 we study the Kurtosis as a function of centralities in scenario (a) parton, (b) w/o ART, (c) NTMAX = 3, (d) NTMAX = 150 with 0 < pT < 1 (GeV/c) and |y| < 0.5 at √sNN = 17.3 GeV, respectively. From scenario (a) to (b), the system experiences the hadronization process. Comparing panel (a) and (b) of Fig. 3, we find the Kurtosis of the distributions originate from early partonic phase, the quark coalescence-like hadronization almost doesn’t affect Kurtosis. In AMPT StringMelting, the hadron cascade procedure is regarded as two processes, resonance decay and hadron rescattering. From scenario (b) w/o ART to scenario (c) NTMAX = 3, the system experiences the resonance decay process. Fig. 4 is the particles’ yield in different evolution time in central collision at √sNN = 17.3 GeV by AMPT StringMelting. We can find all resonances decay before tc = 0.6fm/c and the certain particle yield doesn’t change after NTMAX = 3. Here the yield is the ratio of a certain particle number to the total particles number. When we compare panel (b) and (c) of Fig. 3,
it seems that the resonance decay process doesn’t affect Kurtosis.

On the other hand, large hadron rescattering effect may destroy the fluctuation which originates from the early stage of heavy ion collision. This effect depends on two factors: one is the time that particles go through the collision region, the other is the density in the collision region [33]. In AMPT StringMelting, we can study the hadron rescattering effect on higher moments by controlling the termination time of hadron cascade ($t_{\text{cut}}$). NTMAX = 3 is regarded as that there is no hadron rescattering, while NTMAX = 150 is corresponding to termination time of 30 fm/c. From panel (c) and (d) of Fig. 3, it suggests the observed value of Kurtosis nearly does not change and almost keeps the same trend, even though the system experiences full hadronic rescatterings. It is reasonably to conclude that hadron rescatterings do not have clear influence on Kurtosis.

Besides Kurtosis and Skewness, the product of Kurtosis and Variance (Kurtosis-$\sigma^2$) of the net-baryon distribution has also been used to search for the CP. This observable is related to the ratio of quartic ($4^{\text{th}}$ order) and quadratic ($2^{\text{nd}}$ order) cumulants of baryon number susceptibilities. Lattice QCD and some QCD based model [14] predicted that the fluctuation of Kurtosis-$\sigma^2$ changed rapidly at the transition temperature $T_C$. Furthermore, it is predicted that baryon number susceptibilities will diverge when close to the CP, this will bring about the deviation of Kurtosis-$\sigma^2$ from being constant. All of the works argue that Kurtosis-$\sigma^2$ is a worthwhile observable. However, as we presented before [31], the Kurtosis-$\sigma^2$ is dependent on the acceptance, namely, larger acceptance lead to smaller measured value. Also, as shown in Fig. 5, results of Kurtosis-$\sigma^2$ of net-baryon distribution from two versions of AMPT model have a strong dependence on the collision energy, it is quite different from net-proton distribution. This difference may due to baryon stopping effect in low energy region. The similar result can be found in Ref. [12] by UrQMD calculation.

IV. SUMMARY AND OUTLOOK

In this paper we study fluctuation of higher moments of net-baryon distribution with AMPT and UrQMD models. Based on these results, we find the higher moments of net-baryon distribution are independent on the chosen transverse momentum window. Together with the previous results of rapidity and transverse momentum sizes dependence, we conclude that Kurtosis and Skewness are independent on acceptance.

We also study the time evolution effects and late hadronic effects on higher moments of net-baryon distribution in Au+Au collisions at $\sqrt{s_{NN}} = 17.3$ GeV with UrQMD and AMPT model. The results seem that quark coalescence hadronization process, resonances decay process and hadronic rescatterings don’t affect the trend and value of Kurtosis and Skewness as a function of centralities. Thus, they should be good observables to searching for the possible critical point predicted by QCD, non monotonic behaviors of the Kurtosis and Skewness as a function of beam energy or centrality will demonstrate the existence of a CP. The Kurtosis-$\sigma^2$ of net-proton distribution show great difference to that of net-baryon distribution when collisions energy is lower than $\sqrt{s_{NN}} = 20$ GeV, it deviates from the theoretical predictions.

In the future, the RHIC Beam Energy Scan and GSI Facility for Antiproton and Ion Research will provide the possibility to locate the critical point in experiment. Higher moments of net-baryon distribution should be one of the powerful tools.

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