A LiFo Dynamic Dictionary

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Abstract

Data structures that realize a dictionary are characterized by three basic instructions: (1) Insert a new entry (key and value). (2) Search by a key, returning the associated value. (3) Delete an entry.

Known realizations are hashing schemes and various types of search trees. Time complexity of the fundamental operations is measured as a function of the number of entries \( n \), the (binary) size of a key \( s \) is usually not considered. For search trees the expected time as well as the upper limit of time is \( O(\log n) \). LiFo dictionary is a new implementation, the time limits for Insert and for Search both are a linear function of the length \( s \) of the used key, that is \( O(s) \). The LiFo dictionary furthermore provides two additional basic operations: (4) open environment and (5) close environment with a constant time for performing. Close environment needs only one assignment by which it restores exactly the same internal situation as before the last call of open environment. This feature cannot be realized by any of the data structures mentioned above, therefore the prefix LiFo (= last in first out ), another name for stack. This ability is highly suitable for software applications that frequently perform local symbol to value binding throughout multiple levels of environments. E.g. Lisp-Interpreter or D. Knuth’s TeXprogramm are such applications.

Keywords: Datastructure Dictionary, LiFo-Facility, Computational Complexity.

1 Introduction

To illustrate the principle of this data structure in a simplified fashion, a game model, which is demonstrated by an example, seems to be suitable. Assume that the president of some association keeps a dictionary of the members. Keys of the dictionary are the member’s names, information to be accessed as value assigned to a key is the year of birth. Assume the current state of the dictionary are three entries: ALFRED (1940), ALBERT (1955), PETRA (1960). This current
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state in the game is modelled by a route pattern of square shaped cards, each with one letter written on it, as the following figure shows.

![Figure 1](image)

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(the author will send a sheet of 3 Figures on request).

We label card \(A\) by \(\triangleright\) as starting square and mark the ends with the value encircled (the birth year). Using another set of these cards beside we place a so called argument line, whose letters form the word to be searched (the input). The search is simulated by the following procedure: First we put a piece \(a\) onto the starting square of the above route pattern and a piece \(b\) onto the first square of the argument line. This is the starting configuration. A configuration of the game in process is characterized by the square locations of the two pieces, we shall refer to it as the current squares in the dictionary and in the argument line. A successful final configuration is reached, if \(a\) covers the last square of the line, \(b\) is upon a square with circled value attached and the letters of both squares agree. The answer of the search then is the circled value next to \(b\). The change from one configuration to the next is described as follows:

If the letters of the current fields are the same, the pieces move one square in straight direction.

In the other case, if on the left hand of piece \(b\) a square is adjacent, then \(b\) moves left to this square and \(a\) keeps its position; if there is no square lefthand, however, the game fails, that is the argument does not appear in the dictionary.

If instead of searching an extension of the dictionary is to be performed by inserting the input as a key, then we continue in the last case as follows. We place a card with the same letter as that below piece \(a\) so that it adjoins lefthand to the square where piece \(b\) resides. We twist the card a small angle to the left, then we append a copy of the part of the argument line in front of piece \(b\) as a new branch. To finish inserting, a further argument, carrying the value to be associated with the key is required. (The value is the encircled year in the example above)

If we extend our example by inserting an entry for PETER (1965) we obtain:
This game should illustrate only how the mechanism of the data structure works in principle. We observe the following drawback in this model: From a certain size upwards considerable multiple branching will arise so that search becomes inefficient. A simple rule helps to overcome this difficulty: rely on a binary coding of the letters and play the game with the alphabet of 0 and 1! Let us record that access by a matching chain of bits (input strip) is realized by a sequence of moves in the game model. Each move comprises testing agreement of the two current (the focused) bits of the input strip and the search tree respectively; if testing results to false, the focused bit migrates to the leftside branch (which exists for a matching input) and testing agreement is repeated; if finally testing results in true, both focuses move one node ahead (to the next bit). For a correct input the focus migrates through the branch in the search tree that carries the same sequence of bits as the input strip. It is significant to record that each entry except the first (the oldest one) is mapped to exactly one branching node in the search tree. Now let us design an adequate data structure.

2 Data Structure of Dictionary

An entry key + value is recorded as a data type Entry with field components. The dictionary as a whole is an organized collection of objects of type Entry together with a pool for keys, that is a compound of objects, which are interconnected by references, sometimes called pointers. One component to be considered is the bit-string of the key of an entry. With exception of the first entry the right part, that is the continuation after a branching off from the key of an elder entry, is sufficient for the searching algorithm. We shall record this bit string as a component of ENTRY referred to by selector name remkey. Now let us imagine further branching off nodes located in certain places of remkey. Remember the fact that each entry is mapped to exactly one branching node in the search tree, namely the one which leads into the entry. Hence it seems to be advisable to record only the first branching off by a component attached to the structure of ENTRY and to delegate further branching offs to the entry reachable from the first one. This is managed by introducing three further com-
ponents for ENTRY: \texttt{dist} as the place of first branching in bit string \texttt{remkey}; \texttt{branch} as pointer to the entry (object of type ENTRY) that is reachable by the first branching off; and finally \texttt{link} as pointer to record further branching offs, which belong to the bit string \texttt{remkey} of the entry that was the focus (the current ENTRY) before the last assignment \texttt{focus} $\leftarrow\texttt{focus} \uparrow \texttt{branch}$.

A pointer or reference to a certain object is the address or position of the object within the digital storage. A declaration $\uparrow\texttt{ENTRY}$ \texttt{lexpos} stipulates \texttt{lexpos} as a pointer to an object of type \texttt{ENTRY}, the object is denoted by \texttt{lexpos}.$\uparrow$.

The meaning of the components \texttt{branch} and \texttt{link} is illustrated by the following sketch that enters into our game model. Assume the dictionary contains 4 entries, the addresses of these are denoted by $L_1, L_2, L_3, L_4$ and the corresponding route pattern is:

\begin{center}

text{Sorry, Figure 3 cannot be presented!}
\end{center}

Then we have $L_2 = L_1 \uparrow \texttt{branch}$, $L_3 = (L_1 \uparrow \texttt{branch}) \uparrow \texttt{link}$, and $L_4 = ((L_1 \uparrow \texttt{branch}) \uparrow \texttt{link}) \uparrow \texttt{link}$. If for an object $X$ of type ENTRY there are $n+1$ entries corresponding to the branching offs on bit string $X.\texttt{remkey}$ according to the above model, that is each key associated with such an entry (component \texttt{remkey}) exhibit the first difference to $X.\texttt{remkey}$ in the branching locations, then the entries are $X.\texttt{branch} \uparrow, X.\texttt{branch} \uparrow \texttt{link} \uparrow, \ldots, X.\texttt{branch} \uparrow \texttt{(link} \uparrow \texttt{)}^n$. The branching positions can be read out of these objects: component $Y.\texttt{dist}$ for each $Y = X.\texttt{branch} \uparrow \texttt{(link} \uparrow \texttt{)}^j$ $(j \leq n)$ is the information that the leftmost difference of the key of $X$ and that of $Y$ is the bit at position $Y.\texttt{dist}$ in $X.\texttt{remkey}$ (although \texttt{remkey} in general is only a right piece of the real key).

\section{Preliminary Data Types}

\begin{verbatim}
define type Bit = \{0, 1\};
define type Bool = \{false, true\};
define type BitString = array [\ast] of Bit ;
/\ast BitString \texttt{s} means \texttt{s} = \langle s[i] \rangle_{i<\text{length}(\texttt{s})} \ast/
define type NatN = Elements of N;
define type ResultType = any type you may need below;
define type Location = position of storage referring to some object;
\end{verbatim}

\section{Data Types and References}

For each data type \texttt{T} we consider the type $\uparrow\texttt{T}$ of reference to an object of type \texttt{T} (location of the object within storage). If a variable \texttt{p} is declared to be of type \texttt{T} (usually by writing "\texttt{T p}") then \texttt{p}.$\uparrow$ denotes the object referred to
by \( p \). The relationship between \( \uparrow T \) and Location is that under declarations “\( T p, q \); Location \( r \)” the commands “\( r \leftarrow p; q \leftarrow r \)” yield \( p = q \).

### 2.3 Procedures of Storage-Allocation

\[ \uparrow T \text{ valued} /* T may be any type */ \]

\begin{verbatim}
create(T, var Location freeloc) ≡
/* freeloc is considered as the top of a stack */
begin
  \( \uparrow T \) result \( \leftarrow \) freeloc;
  Increase freeloc by the size of \( T \);
  return(result);
end.
\end{verbatim}

\[ \uparrow T \text{ valued} /* T may be any type */ \]

\begin{verbatim}
create_copy(T v, var Location freeloc) ≡
begin
  \( \uparrow T \) result \( \leftarrow \) freeloc;
  Increase freeloc by the size of \( T \);
  result\( \uparrow \leftarrow v; \)
  return(result);
end.
\end{verbatim}

First we study a simpler version of the data structure Entry , which does not yet realize the LiFo-property but differs from the final version essentially only with regard to the operation of adding a new entry by procedure insert. The logical connectives \( \land \) and \( \lor \) are used with semantics that differ from the one used in logic, as they will be interpreted as a sequential and and or respectively. That is, if the first operand of \( \lor \) yields \text{true} within an evaluation, then evaluation of the expression built upon \( \lor \) ceases with value \text{true} without any computation of the second operand. In a similar way this applies to \( \land \) with \text{false} in place of \text{true}. This can prevent e.g. for a conditional instruction as \( \text{if} (\text{findpos} \neq \text{nil} \land d > \text{findpos}\uparrow.d) \ldots \) the computation of an undefined expression \( \text{findpos}\uparrow.d \) in case of \( \text{findpos} = \text{nil} \).

### 2.4 Data Structure Dictionary /* simple (non-lifo) Version */

\begin{verbatim}
define type Entry = structure
  \uparrow BitString remkey;
  NatN dist;
  \uparrow Entry branch,link;
  ResultType val;
end

define type Dictionary =
structure begin \uparrow Entry gate; Location freeloc; end;
\end{verbatim}
DATA STRUCTURE OF DICTIONARY

/* Procedure search computes the ↑Dictionary position findpos for a bitstring x that corresponds to an entry in the dictionary L, the key of which is x, provided that this key occurs in L. The proper answer after a successful search then is findpos ↑.val. The answer whether the search was successful is the return value of procedure search. Procedure insert relies on search. If search(...) returns value true, then findpos ↑.val needs only to be rewritten by the new associated value (a parameter of insert). In the other case L will be enlarged by one element (type Entry). Particularly for this purpose search is provided with output parameters insertpos and d. */

Bool valued

search(Dictionary L, BitString x, var ↑Entry findpos, insertpos, var NatN d) ≡
begin
findpos ← L.gate;
if (findpos = nil) then return(false); /* empty dictionary */
while (true) do /* terminated by return */
begin
if (findpos ↑.remkey ↑ = x) then return(true);
if (one of x and findpos ↑.remkey ↑ is a left substring of the other) then
  d ← min(length(x), length(findpos ↑.remkey ↑)) + 1;
else let d be the smallest d, such that x[d] ≠ (findpos ↑.remkey ↑)[d];
insertpos ← findpos; /* ≠ nil (on account of 20 and 35) */
findpos ← findpos ↑.branch;
while (findpos ≠ nil ∧ d > findpos ↑.dist) do
begin
  insertpos ← findpos;
  findpos ← findpos ↑.link;
end;
if (findpos = nil ∨ d ≠ findpos ↑.dist) then return(false); /* x is not a valid key */
x ← ⟨x[d + j]|j<length(x)−d⟩; /* ≡ shift_right(x, d) */
/* search continues at the entry the branch off leads to, i.e. after position d of remkey ↑. */
end; /* search */

insert(Dictionary L, BitString x, ResultType v) ≡
begin
  Entry findpos, insertpos; NatN d;
if (search(L, x, findpos, insertpos, d))
  then findpos ↑.val ← v
else
begin
newpos ← create(Entry, L.freeloc);
newpos↑.dist ← d;
if (insertpos = nil) then L.gate ← newpos;
/* note that insertpos = nil iff L.gate = nil */
if (insertpos↑.branch = findpos)
then insertpos↑.branch ← newpos
else insertpos↑.link ← newpos;
newpos↑.branch ← nil;
newpos↑.link ← findpos;
newpos↑.remkey ← create_copy(⟨x[d + j]⟩j length(x)−d, L.freeloc);
newpos↑.val ← v;
end
end /* insert */

2.5 Observation The worst case time complexity of procedure search of 2.4 is $O(s)$ where $s = \text{length of } x$.

Proof: The instructions within the while loop at lines 21-40 are to be considered as follows: Assume the loop is passed $r$ times, the associated values of $d$ being $⟨d_j⟩_{j<r}$. Looking at lines 25,26 and 38 we have $d_0 + d_1 + \cdots + d_{r−1} \leq s = \text{length}(x)$. More than two subsequent $d_i$'s cannot be equal to 0 ($d_i = d_{i+1} = 0$ is only if keys $u, u0v, u1w$ occur), therefore $r \leq 2s$. As to the inner loop of lines 30-34, the construction of the link components according to insert ensures that an ypos↑.link" ↑.dist is a strictly increasing sequence limited by $d$. In this way the inner loop is repeated $p_j$ times, if

$$0 \leq \text{findpos}↑.\text{dist} < \text{findpos}↑.\text{link}↑.\text{dist} < \cdots < \text{findpos}↑.\text{link}^{p_j−1}↑.\text{dist} \leq d_j$$

If we take $d_j k = \text{findpos}↑.\text{link}^{k+1}↑.\text{dist} − \text{findpos}↑.\text{link}^k↑.\text{dist}$ then each $d_j k \geq 0$ and $d_j = d_j 0 + \cdots + d_j p_j−1$. If we concatenate the tuples $⟨d_j k⟩_{k<p_j}$ and let $⟨a_h⟩_{h<k}$ be the result of the concatenation, we conclude, as above for the sequence of the $d_j$'s, that $h \leq s$, where $h$ is the sum of the number of repetitions of the small loop for each traverse of the big loop. Hence the amount of computation time due to the small loop is $O(s)$, the number of repetitions of the big loop is $O(s)$, the instructions outside the small loop either require constant time or are limited by $d_j$ and, as the sum of the $d_j$'s is not greater then $s$, the whole algorithm of search has worst case time complexity $O(s)$.

2.6 Data Structure LiFo-Dictionary

Each component of the preceding data-structure is included, partly modified. Two additional procedures open_environment and close_environment are introduced. Data-type DICTIONARY gets the new component pop of type ↑DICTIONARY, so that a stack of DICTIONARY-frames is maintained.
**DATA STRUCTURE OF DICTIONARY**

1. **define type** Entry = as in 2.4 Dictionary;
2. **define type** Dictionary =
   **structure** begin
   ↑Entry gate;
   Location freeloc;
   ↑Dictionary pop
   **end:**
3. **open_environment** (var ↑Dictionary DictPtr) ≡
   **begin**
   ↑Dictionary OldDictPtr ← DictPtr;
   Location NewFreeLoc ← DictPtr↑.freeloc;
   DictPtr ← create(Dictionary, NewFreeLoc);
   DictPtr↑.gate ← create_copy(OldDictPtr↑.gate↑, NewFreeLoc);
   DictPtr↑.pop ← OldDictPtr;
   DictPtr↑.freeloc ← NewFreeLoc
   **end**
4. **close_environment** (var ↑Dictionary DictPtr) ≡
   DictPtr ← DictPtr↑.pop
5. /* Note, that we associate our dictionary with a variable ↑Dictionary DictPtr. The LiFo-storage for the whole compound is characterized by its top-position DictPtr↑.freeloc. Thus the single assignment of close_environment suffices to restore the state before the last call of open_environment. */
6. Bool valued
7. **search** (↑Dictionary DictPtr, BitString x) ≡
   **begin**
   ↑Entry findpos, insertpos: NatN d;
   /* these are no longer required for insert, */
   /* we use search of 2.4 */
   **return** Dictionary.search(DictPtr↑, x, findpos, insertpos);
   **end**
8. **insert** (↑Dictionary DictPtr, BitString x, ResultType v) ≡
   **begin**
   ↑Entry findpos, insertpos, newpos;
   NatN d; Bool key_exists;
   findpos ← DictPtr↑.gate;
   while (findpos ≠ nil) do
   **begin**
   if (findpos < DictPtr)
DATA STRUCTURE OF DICTIONARY

```plaintext
42 then findpos ← create_copy(findpos↑, DictPtr↑.freeloc);
43 key_exists ← (findpos↑.remkey↑ = x);
44 if (key_exists) then breakloop;
45 if (one of x and findpos↑.remkey↑ is a left substring of the other)
46 then d ← min(length(x), length(findpos↑.remkey↑)) + 1;
47 else let d be the smallest d, such that x[d] ≠ (findpos↑.remkey↑)[d];
48 /* Note that findpos < DictPtr iff findpos has been created before last
49 call of open_environment, which caused the creation of DictPtr. If
50 this is true, then findpos shall be replaced by a copy of it which is
51 located atop of DictPtr. This copy will be removed at next call of
52 close_environment. */
53 insertpos ← findpos;
54 findpos ← findpos↑.branch;
55 if (findpos ≠ nil ∧ findpos < DictPtr) then begin
56 newpos ← create_copy(findpos↑, DictPtr↑.freeloc);
57 insertpos↑.branch ← newpos;
58 findpos ← newpos;
59 end;
60 while (findpos ≠ nil ∧ d > findpos↑.dist) do begin
61 insertpos ← findpos;
62 findpos ← findpos↑.link;
63 if (findpos ≠ nil ∧ findpos < DictPtr) then begin
64 newpos ← create_copy(findpos↑, DictPtr↑.freeloc);
65 insertpos↑.link ← newpos;
66 findpos ← newpos;
67 end;
68 end;
69 if (findpos = nil ∨ d ≠ findpos↑.dist) then breakloop;
70 /* with key_exists=false, i.e. x is not a valid key */
71 x ← ⟨x[d + j]⟩j<length(x)−d; /* ≡ shift_right(x, d) */
72 /* searching continues in branching off (after position d of remkey↑) */
73 end;
74 if (key_exists)
75 then findpos↑.val ← v
76 else begin
77 newpos ← create(ENTRY, DictPtr↑.freeloc);
78 newpos↑.dist ← d;
79 if (insertpos = nil) then DictPtr↑.gate ← newpos;
80 /* note that insertpos = nil iff DictPtr↑.gate = nil */
```

There is a stack of items of type Dictionary with top DictPtr. These items are connected by the downward link pop. Next segment below DictPtr↑ is DictPtr↑.pop↑. Each element (of type Entry or BitString) we can access from DictPtr↑.pop↑ was created before DictPtr↑. By Definition of create and create_copy an element p↑ is older (created later) than another element q↑ if and only if p < q. Hence the address value of each element accessible from DictPtr↑.pop↑ (by iterated application of .branch↑, .link↑) is less than DictPtr.

Part 39-73 of procedure insert is a modification of search from 2.4, which additionally copies the traversed entries findpos↑ and chain the copies parallel to the originals by branch and link. The same considerations as for search yield that the worst case time complexity of procedure insert also is $O(s)$. 