Prediction of the Remaining Useful Life of Lithium-ion Batteries Based on Dempster-Shafer Theory and the Support Vector Regression-particle Filter

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ABSTRACT  Lithium-ion batteries (LIBs) have been widely used in various electronic equipment. The development of an effective method for predicting the remaining useful life (RUL) of LIBs can ensure the normal operation of equipment by providing an appropriate warning before the battery fails. This study presents a method for predicting the RUL of LIBs based on Dempster-Shafer theory (DST) and the support vector regression-particle filter (SVR-PF), which improves the prediction accuracy when the available data are relatively sparse. The model of LIB RUL prediction based on DST and SVR-PF was developed and proposed based on a DST algorithm and the central limit theorem. Moreover, this study proposes an approach to update the basic probability assignment (BPA) of DST, which represents the confidence of the prediction, at each iteration during the RUL prediction. The updated BPA at each iteration will increase the importance of the high confidence prediction method in the combined results. Thus, it will provide a more accurate prediction result. The proposed method can also be used as a framework to combine the prediction results obtained from various independent data. The simulation results and the comparison with the existing LIB RUL prediction methods show that the proposed method provides more accurate and reliable prediction results.

INDEX TERMS Lithium-ion Battery, Remaining Useful Life, Dempster-Shafer Theory, Support Vector Regression-particle Filter, State of Health

NOMENCLATURE

LIB  Lithium-ion battery
RUL  Remaining useful life
PF  Particle filter
SVR  Support vector regression
DST  Dempster-Shafer theory
BPA  Basic probability assignment
SOH  State of health
R  Impedance data
R_e  Electrolyte resistance
R_{ct}  Charge-transfer resistance
\lambda_{R,k}  Impedance growth parameter at k-th charge/discharge cycle
\alpha, \beta  Impedance-based capacity fade parameters
v  System noise of RUL prediction model
n  Measurement noise of RUL prediction model
Q  Maximum prediction step
L  RUL value
C^*_{im,k+1}  Predicted capacity at k-th step using impedance data
I. INTRODUCTION

Lithium-ion batteries (LIBs) have been widely used in various electronic devices to provide the necessary electrical energy for the normal operation of equipment. They are the dominant technology for powering portable electronic devices and electric vehicles [1]. Reliable and safe operation of LIBs is therefore vital for maintaining device functionality. To this end, the remaining useful life (RUL) is a useful indicator of the condition of a LIB. The RUL of a battery is defined as the time required under a specified operating condition for the battery to decay from the current state to a state in which the battery can no longer function normally. The RUL is an important component of the condition-based maintenance of a battery [2]. Therefore, to provide relevant information regarding LIB system maintenance or replacement, it is essential to develop a reliable and accurate method for predicting the RUL of an LIB. LIB RUL prediction has been a popular research area; Stroe et al. [3], Ge et al. [4], Lei et al. [5], Cai et al. [6], and Li et al. [7] have provided thorough reviews of the recent research works in this field.

Machine learning algorithms such as neural networks [8] and deep learning algorithms such as long short-term memory networks have been used in LIB RUL prediction. Tang et al. [9] used a feed-forward migration neural network to predict the degradation trajectories of different types of LIBs. The accurately predicted degradation trajectories were useful in predicting the RUL of the LIB. Zhao et al. [10] used a fusion of an RUL prediction approach based on a deep belief network and a relevance vector machine. The data-driven approach extracted the feature of capacity degradation of LIBs and was able to provide higher RUL prediction accuracy. Liu et al. [11] established a data-driven model to estimate the battery RUL using advanced machine learning techniques; their results showed that the model that combined the long short-term memory neural network and the Gaussian process regression had the best performance. Wang et al. [12] applied a dynamic long short-term memory neural network to indirectly predict the RUL of an LIB. Mao et al. [13] developed a battery RUL prediction method using a long short-term memory network, with a sliding time window and Gaussian or sine function and Levenberg-Marquardt algorithm fusion based on ensemble empirical mode decomposition.

Although the artificial intelligence-based RUL prediction methods have their advantages, the training of these neural networks requires plenty of computational power. Moreover, to obtain a reliable neural network, large amounts of data will be needed to train it and the size of the trained network will be large. These disadvantages make the neural network approach unsuitable for the in-vehicle embedded environment. Edge/cloud [14] computing can be a solution, but the cost of infrastructure will be high, and it will not be suitable for all applications. Therefore, this study did not choose neural networks as a tool for LIB RUL prediction. Because recursive filtering algorithms can be used to analyze, estimate, and predict battery failure using measured battery data based on the state-space model of the battery, they have often been applied in the prediction of the RUL of LIBs [15]. The Kalman filter [16], extended Kalman filter [17], and unscented Kalman filter [18] [19] have been used in different research works for LIB RUL prediction. The particle filter (PF) [20], which can provide a good RUL prediction performance, has been widely used by researchers. The weight of a PF’s particle can provide the probability distribution of the predicted RUL. The resampling algorithm of the standard PF suffers from the degeneracy phenomenon. After several iterations, most weights of the particles will become zero and most of the computational power will be wasted on these particles [21]. Therefore, most of the existing research changed the resampling algorithm for the PF to improve the LIB RUL prediction performance. Zhang et al. [22] used the linear optimizing combination resampling algorithm to improve the RUL prediction accuracy of the unscented particle filter algorithm. Chen et al. [23] used a second-order central difference algorithm for the resampling of the PF and applied the PF to the LIB RUL prediction. Ma et al. [24] used a LIB RUL prediction method based on a Gauss-Hermite particle filter, in which Gauss-Hermite algorithm-based resampling was used for the PF. Jiao et al. [25] developed an
LIB RUL prediction based on a conditional variational autoencoder PF. Their resampling algorithm was a conditional variational autoencoder. Zhang et al. [26] proposed a LIB RUL prediction method based on F-distribution particle filter. The particle weight was updated by a kernel smoothing algorithm. Support vector regression (SVR) has been used for LIB RUL prediction [27] [28]. It can also be used for PF resampling [29] and be applied to the LIB RUL prediction [30][31]. Dong et al. [31] proposed an online LIB capacity degradation analysis approach and built a prediction model to predict the RUL of an LIB. Moreover, a support vector regression-particle filter (SVR-PF) algorithm is implemented in the research to improve the standard PF with respect to the degeneracy phenomenon.

However, the recursive filtering algorithms still require a large amount of data to achieve an accurate LIB RUL prediction. The algorithms do not perform well when the data are sparse. Moreover, the RUL prediction results were only based on a single algorithm. If the prediction results based on different algorithms could be combined, they could achieve more accurate prediction results and less data would be required. Dempster-Shafer theory (DST) [32] has been used in several research areas such as sensor fusion for fault detection [33] and mining shearer condition monitoring [34]. Some researchers have also used DST in LIB RUL prediction. He et al. [35] used DST for the generation of the initial particles for the PF in LIB RUL prediction. However, their research work did not take advantage of the DST algorithm by combining the prediction results from different methods.

DST can combine the prediction results that come from different LIB RUL prediction methods by associating them with a basic probability assignment (BPA), which represents the confidence of prediction for each method. Combination using this method will provide a more reliable and accurate LIB RUL prediction than a single method when the data are sparse because the results are the combination of two different methods.

Moreover, data sparsity addresses the problem of insufficient data in a data set [36]. Existing research work on data sparsity covers the sparse data problem such as lack of data on rating-based movie recommendation systems [37], and the measure for sparsity channels in wireless communication networks [38]. The focus of existing research works on sparse data were on data mining and data feature extraction [39], which are different from the work in this paper. Currently, there is no research addressing data sparsity in LIB RUL prediction.

A. Work of this paper
From the above description of past studies, it can be deduced that predicting the RUL of LIBs requires analysis of available data to estimate the state of the battery over time, followed by application of the estimation results to predict the performance decline of the battery. However, when the available data are relatively sparse, the accuracy of RUL prediction will be reduced, and an effective solution to such a problem has yet to be developed. The present study proposes a novel method based on DST and the SVR-PF for predicting the RUL of LIBs by combining the RUL prediction results derived from various independent data sources to ensure an accurate prediction when the available data are relatively sparse. The novelty of the proposed LIB RUL prediction method can be summarized as following:

- This study proposes a method that has an accurate LIB RUL prediction when the data are sparse.
- This study proposes a model of LIB RUL prediction based on DST and SVR-PF (combination of Equations (60), (61), (62), and (63)).
- This study proposes an approach to update the BPA of the DST algorithm at each prediction step based on prediction error (Equations (34), (37), (46), and (47)). The value of BPA, which represents the confidence as to whether the prediction is accurate, is different at each prediction step because the prediction error changes. The updated BPA at each prediction step will make the high confidence prediction method (which has lower prediction errors) have more importance in the combined result, thus making the combined prediction results more accurate.

The proposed method can be used as the framework to combine the results of two LIB RUL prediction methods if the measured capacity data satisfy the central limit theorem (Equation (67)).

These are the use cases that are suitable for the proposed LIB RUL prediction method:
1. During the early stage of the LIB lifecycle when the capacity and impedance data are sparse.
2. When noise and interference limit the equipment’s capability to collect the LIB data, leading to a sparse dataset.
3. When combining RUL prediction results from different prediction methods.

The remaining sections of this article are organized as follows. Section 2 describes methods for predicting the RUL of LIBs using impedance data and capacity data as a basis for DST data fusion. Section 3 describes the measured LIB data. Section 4 introduces DST. Section 5 presents the proposed method and process of predicting the RUL of LIBs based on an RUL prediction model using DST and SVR-PF. Section 6 provides simulation examples and compares the prediction performance between the method proposed in this paper and existing state-of-art methods, whereas the final two sections present the conclusions of the study and proposals for future work.

II. Introduction to RUL predication methods using impedance data and capacity data
A. RUL prediction using impedance data
LIB RUL prediction using impedance data was proposed in prior research work by the author [31]. The SVR-PF algorithm
used in this study can be found in Ref. [31]. The following is a brief introduction to this method.

The method for RUL prediction of LIBs using impedance data can be summarized by the following steps. First, an impedance growth model [31] is established to characterize the trend of impedance increase over time and used to estimate the impedance growth parameter. Second, according to the linear correlation between battery capacity and impedance [40][41], a capacity fade model is established to estimate the capacity fade parameters. Third, after completing the estimation of the two types of parameters, an RUL prediction model based on the SVR-PF is established. The model and its corresponding components are described in the following discussion.

The impedance growth model is given by the following:

\[
\begin{align*}
\lambda_{R,k} &= \lambda_{R,k-1} + v_{R,1,k} \\
\tilde{R}_k &= \tilde{R}_{k-1} \exp(\lambda_{R,k-1} \Delta k) + v_{R,2,k} \\
R_k &= \tilde{R}_k + n_{R,k}
\end{align*}
\]  

(1)

Here, the variables represent values measured with respect to the \( k \)-th charge/discharge cycle, where \( R_k \) represents the impedance data given as the electrolyte resistance \( R_e \) or the charge-transfer resistance \( R_{ct} \). \( \lambda_{R,k} \) represents the impedance growth parameter, \( v_{R,1,k} \) and \( v_{R,2,k} \) are the system noise, and the measurement noise is denoted by \( n_{R,k} \). In addition, \( \Delta k \) represents the difference of time step; in this case, \( \Delta k = k - (k-1) = 1 \). The tilde over the variable \( R \) represents the impedance value smoothed by the SVR-PF.

The impedance-based capacity fade model is given by the following equation:

\[
\begin{align*}
\alpha_k &= \alpha_{k-1} + v_{C,1,k-1} \\
\beta_k &= \beta_{k-1} + v_{C,2,k-1} \\
C_k &= \alpha_k (\tilde{R}_e,k + \tilde{R}_{ct,k}) + \beta_k + n_{C,k}
\end{align*}
\]  

(2)

In this model, \( \alpha_k \) and \( \beta_k \) are impedance-based capacity fade parameters, \( C_k \) is the battery capacity, \( \tilde{R}_{e,k} \) and \( \tilde{R}_{ct,k} \) are the impedances obtained after smoothing model (1), \( v_{C,1,k} \) and \( v_{C,2,k} \) are the system noise, and \( n_{C,k} \) is the measurement noise. It is assumed that the estimation of the impedance growth parameter \( \lambda_{R,k} \) and of the impedance-based capacity fade parameters \( \alpha_k \) and \( \beta_k \) stops when the charge/discharge cycle is \( N \) (resulting in \( \tilde{R}_{e,N}, \tilde{R}_{ct,N}, \lambda_{R,N}, \lambda_{C,N}, \alpha_N \) and \( \beta_N \)). The impedance data are first predicted based on \( \lambda_{R,k} \). The predicted impedance values are \( \tilde{R}_e \) and \( \tilde{R}_{ct} \):

\[
\begin{align*}
\tilde{R}_{e,N+q} &= \tilde{R}_{e,N} \exp(\lambda_{R,e,N} q), \\
\tilde{R}_{ct,N+q} &= \tilde{R}_{ct,N} \exp(\lambda_{R,ct,N} q)
\end{align*}
\]  

(3)

where \( q = 1, \ldots, Q \) and \( Q \geq L \) is a constant that represents the max prediction step. \( L \) represents the RUL. With respect to \( N \) charge/discharge cycles, \( \tilde{R}_{e,N} \) and \( \tilde{R}_{ct,N} \) are the smoothed impedance values, while \( \lambda_{R,e,N} \) and \( \lambda_{R,ct,N} \) are the impedance growth parameters.

Next, the predicted impedance values are used as the measurement output to establish the model for predicting the RUL of LIBs based on the SVR-PF, which is established as follows:

\[
\begin{align*}
\tilde{R}_{e,k+1}^* &= \tilde{R}_{e,k} + v_{a,k} \\
\tilde{R}_{ct,k+1}^* &= \tilde{R}_{ct,k} + v_{b,k} \\
R_{e,k+1}^* &= R_{e,k} \exp(\lambda_{R,e,k} \Delta k) + v_{c,k} \\
R_{ct,k+1}^* &= R_{ct,k} \exp(\lambda_{R,ct,k} \Delta k) + v_{d,k}
\end{align*}
\]  

(5)

In this model, variables with a superscript * denote prediction of the corresponding variables. Here, \( C_{im,k+1}^* \) is the predicted capacity value. The prediction process stops when \( C_{im,k+1}^* \) decays to the end of life (EOL) threshold of the battery, given as the capacity value when the battery reaches the EOL as a percentage of the nominal capacity.

B. RUL prediction using capacity data

The method for RUL prediction of LIBs using capacity data was proposed by the author in Ref. [42], and it can be summarized by the following steps. First, using the trend of the capacity decay over time, a capacity fade model is established to estimate the capacity fade parameters. Second, after completing parameter estimation, the RUL prediction model based on the SVR-PF is established. The model and its corresponding components are described in the following discussion.

The capacity-based capacity fade model is given by the following:

\[
\begin{align*}
F_{1,k+1} &= F_{1,k} + v_{1,k} \\
\lambda_{F_{1,k+1}} &= \lambda_{F_{1,k}} + v_{2,k} \\
F_{2,k+1} &= F_{2,k} + v_{3,k} \\
\lambda_{F_{2,k+1}} &= \lambda_{F_{2,k}} + v_{4,k} \\
C_k &= F_{1,k} \exp(\lambda_{F_{1,k}} k) + F_{2,k} \exp(\lambda_{F_{2,k}} k) + n_k
\end{align*}
\]  

(6)

Next, with respect to the \( k \)-th charge/discharge cycle, \( F_{1,k}, \lambda_{F_{1,k}}, F_{2,k}, \lambda_{F_{2,k}} \), and \( n_k \) are the capacity-based capacity fade parameters; \( C_i \) is the battery capacity; \( v_{1,k}, v_{2,k}, v_{3,k}, \) and \( v_{4,k} \) are the system noise; and \( n_k \) is the measurement noise.

As was discussed in Subsection A, it is assumed that the estimation of the capacity fade parameters stops when the number of charge/discharge cycles is \( N \). The RUL prediction model is established, and the battery capacity fade parameters \( F_{1,N}, \lambda_{F_{1,N}}, F_{2,N}, \) and \( \lambda_{F_{2,N}} \) estimated at this point are recorded. Then, the measurement output of model (6) is split into two parts, namely,

\[
\begin{align*}
F_a &= F_1 \exp(\lambda_{F_{11}}), \\
F_b &= F_2 \exp(\lambda_{F_{22}})
\end{align*}
\]  

(7)

(8)
The values of $F_a$ and $F_b$ are the capacity prediction parameters, and they are predicted according to the following:

$$F_{a,k} = F_{1,N} \exp(\lambda_{F_{1,N}} k), \; k = 1,2,\ldots, Q, Q \geq 0,$$  

$$F_{b,k} = F_{2,N} \exp(\lambda_{F_{2,N}} k), \; k = 1,2,\ldots, Q, Q \geq 0.$$  

These values are used as the measurement data to establish the prediction model. The equation of the model is given by the following:

$$\begin{align*}
\hat{F}_{1,k+1} &= \hat{F}_{1,k} + \nu_{1,k} \\
\hat{F}_{2,k+1} &= \hat{F}_{2,k} + \nu_{2,k} \\
\hat{C}_{\text{cap},k+1} &= \hat{F}_{1,k+1} \exp(\lambda_{F_{1,N}} k) + \nu_{3,k} \\
\hat{F}_{a,k} &= \hat{F}_{a,k} + \nu_{4,k} \\
\hat{F}_{b,k} &= \hat{F}_{b,k} + \nu_{5,k}
\end{align*}$$

Here, $\nu_{1,k}$, $\nu_{2,k}$, $\nu_{3,k}$, $\nu_{4,k}$, and $\nu_{5,k}$ are the system noise, and $\nu_{4,k}$ and $\nu_{5,k}$ are the measurement noise. The prediction process ends when the predicted capacity $\hat{C}_{\text{cap},k+1}$ decays to the EOL threshold.

### III. Experimental LIB impedance and capacity data

Satellite LIB impedance and capacity data were measured by the NASA Idaho National Laboratory [43] in the United States. The LIB device employed was the Gen 2 18650-size model. The experiments were conducted under constant temperature conditions at room temperature ($24^\circ$C) and at $43^\circ$C. The experiments included three cyclic processes: charging, discharging, and impedance testing.

1) Charging: First, the battery was charged at a constant current of 1.5 A until the battery voltage attained a value of 4.2 V; then, the battery was charged at a constant voltage until the charging current dropped to 20 mA, upon which charging was ceased.

2) Discharging: The battery was discharged at a constant current of 2 A until the battery voltage dropped to 2.5 V, upon which discharging was ceased.

3) Impedance testing: Electrochemical impedance spectroscopy testing was conducted over a frequency sweep from 0.1 Hz to 5 kHz to evaluate the LIB impedance.

Continual charging and discharging cycles resulted in the decline of battery performance, whereas the impedance testing provided measurements of the changing parameters characterizing battery performance. The experiment was completed when the battery reached the EOL threshold. Figure 1 and Figure 2 present measured battery data reflecting the battery capacity, $R_c$, and $R_{ct}$ of batteries 5, 6, and 7, and those of battery 32. The data collected for batteries 5, 6, and 7 were measured at room temperature, whereas those collected for battery 32 were measured at $43^\circ$C.

![Figure 1: Capacity and impedance data for batteries 5, 6, and 7.](image1)

![Figure 2: Capacity and impedance data for battery 32.](image2)
IV. Dempster-Shafer theory

DST [32] uses the basic probability assignment (BPA) function to fuse data from different sources and analyzes the degree of belief attributable to all possible propositions in a frame of discernment. Using fusion rules, the degrees of belief derived from all propositions are combined, thereby achieving the goal of data fusion.

Let \( \Omega = \{w_1, w_2, ..., w_n\} \) be the frame of discernment, where \( w_i \) is a possible proposition within \( \Omega \), such that the power set \( 2^\Omega \) represents all possible proposition combinations in \( \Omega \) (e.g., \( w_i \cup w_j \)). The BPA function is defined as a mapping from the power set \( 2^\Omega \) to the closed interval \([0, 1]\), namely,

\[
m: 2^\Omega \rightarrow [0,1].
\]  
(12)

If \( A \) is a possible proposition in \( 2^\Omega \), then its BPA function meets the following conditions:

\[
\sum_{A \subseteq 2^\Omega} m(A) = 1,
\]  
(13)

\[
m(\emptyset) = 0.
\]  
(14)

This represents the combination of all propositions that support \( A \) in the power set \( 2^\Omega \). The belief function \( Bel(A) \) and the plausibility function \( Pl(A) \) are defined as follows:

\[
Bel(A) = \sum_{B \subseteq A} m(B),
\]  
(15)

\[
Pl(A) = \sum_{B \supseteq A \neq \emptyset} m(B) = 1 - Bel(\bar{A}).
\]  
(16)

Here, \( \bar{A} \) represents the complement of \( A \), and \( Bel(A) \) represents the sum of the BPA of all the subsets of \( A \), and is the lower bound on the probability that \( A \) occurs. \( Pl(A) \) represents the sum of the BPA of all the sets that intersect with \( A \) and are not empty. The plausibility function thus represents the probability of not denying the occurrence of \( A \), and is the upper bound on the probability that \( A \) occurs. Thus, the probability of \( A \) occurring belongs to the following closed interval:

\[
[Bel(A), Pl(A)].
\]  
(17)

The core concept of DST is the fusion of two independent BPA functions for the power set \( 2^\Omega \) to obtain the posterior BPA function of an event after the fusion. In comparison to the original BPA functions individually, the BPA function after fusion combines more data sources, and, thus, potentially produces more reliable results.

To illustrate, let \( m_1 \) and \( m_2 \) be two BPA functions for the power set \( 2^\Omega \), and \( m_{1,2} \) be the BPA function after fusion. Then, DST can be expressed by the following formulas:

\[
m_{1,2}(\emptyset) = 0,
\]  
(18)

\[
m_{1,2}(A) = \frac{1}{1 - K} \sum_{B \cap C = A} m_1(B)m_2(C).
\]  
(19)

Here,

\[
K = \sum_{B \cap C = \emptyset} m_1(B)m_2(C).
\]  
(20)

Therefore, DST combines views of the same issue derived from different sources, and simultaneously eliminates all conflicting views to obtain a reliable posterior BPA function after fusion.

V. Method of RUL prediction based on DST and SVR-PF

A. Application of DST to RUL prediction

The proposed prediction of the RUL of LIBs derives from two data sources: (1) impedance data (im), and (2) capacity data (cap). The method proposed in this paper for predicting the RUL of LIBs based on DST and the SVR-PF is illustrated in Figure 3.

In the proposed method, the universal set \( \Omega \) is denoted as

\[
\Omega = \{im, cap\}.
\]  
(21)

Then, its power set \( 2^\Omega \) is

\[
2^\Omega = \{\emptyset, \{im\}, \{cap\}, \{im \cup cap\}\}.
\]  
(22)

The meanings of the propositions in the power set \( 2^\Omega \) are given as follows:

1. \( \{im\} \) The RUL obtained using the impedance data is believable;
2. \( \{cap\} \) The RUL obtained using the capacity data is believable;
3. \( \{im \cup cap\} \) The RUL obtained using either impedance or capacity data is believable.

Meanwhile, the BPA functions \( m_1 \) and \( m_2 \) for the power set \( 2^\Omega \) are defined as follows:

1. \( m_1 \) is the belief distribution of propositions in \( 2^\Omega \) when predicting the RUL using \( im \);
2. \( m_2 \) is the belief distribution of propositions in \( 2^\Omega \) when predicting the RUL using \( cap \).

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Let $a$ be the number of charge/discharge cycles when the capacity state is determined. From the central limit theorem, we know that the measurement errors of a large body of capacity data follow a normal distribution, namely,

$$R_{c,N} \sim \mathcal{N}\left(\mu_{R_{c,N}}, \sigma_{R_{c}N}^2\right).$$

$$C_{im,N} \sim \mathcal{N}\left(\mu_{C_{im,N}}, \sigma_{C_{im}}^2\right).$$

Here, $\mu_{R_{c,N}}$ and $\mu_{C_{im,N}}$ are the mean values and $\sigma_{R_{c}N}^2$ and $\sigma_{C_{im}}^2$ are the variances of $R_{c}$ and $R_{im}$, respectively. From model (5), we obtain

$$C_{im,N} = \alpha_N (R_{e,N}^* + R_{ct,N}^*) + \beta_N. \quad (30)$$

Thus, the estimation of $C_{im,N}$ follows a normal distribution, namely,

$$C_{im,N} \sim \mathcal{N}\left(\mu_{C_{im,N}}, \sigma_{C_{im}}^2\right). \quad (31)$$

where the following definitions apply.

$$\mu_{C_{im,N}} = \alpha_N (\mu_{R_{e,N}} + \mu_{R_{ct,N}}) + \beta_N = C_N, \quad (32)$$

$$\sigma_{C_{im}}^2 = \alpha_N \left(\sigma_{R_{e}}^2 + \sigma_{R_{ct}}^2\right). \quad (33)$$

From the preceding, $m_{1,N+1}(im)$, the initial value of $m_{1,i}(im)$, is

$$m_{1,N+1}(im) = \frac{1}{\sqrt{2\pi\sigma_{C_{im}}}^2} \exp\left[-\left(\frac{C_{im,N} - \mu_{C_{im}}}{2\sigma_{C_{im}}}^2\right)\right]. \quad (34)$$

Next, the initial value of the BPA function $m_{2,i}(cap)$ is determined. Similarly, from the central limit theorem, we know that the measurement errors of a large body of capacity data follow a normal distribution, namely,

$$C_{cap,N} \sim \mathcal{N}\left(\mu_{C_{cap,N}}, \sigma_{C_{cap}}^2\right). \quad (35)$$

where $\mu_{C_{cap,N}}$ is the mean of $C_{cap,N}$, and $\sigma_{C_{cap}}^2$ is its variance. We know that $\mu_{C_{cap,N}}$ satisfies the following:

$$\mu_{C_{cap,N}} = C_N. \quad (36)$$

Thus, $m_{2,N+1}(cap)$, the initial value of $m_{2,i}(cap)$, is

$$m_{2,N+1}(cap) = \frac{1}{\sqrt{2\pi\sigma_{C_{cap}}}^2} \exp\left[-\left(\frac{C_{cap,N} - \mu_{C_{cap}}}{2\sigma_{C_{cap}}}^2\right)\right]. \quad (37)$$

Based on the above formulation, the possible BPA function combinations based on DST are listed in Table I.

The posterior BPA function after fusion, $m$, is then given as follows:

$$m(iim) = \frac{1}{1 + K} \sum_{B \cap C = iim} m_i(B)m_2(C) = \frac{1}{1 + K} \cdot a. \quad (23)$$

$$m(cap) = \frac{1}{1 + K} \sum_{B \cap C = cap} m_i(B)m_2(C) = \frac{1}{1 + K} \cdot b. \quad (24)$$

Here, the following definitions are applied:

$$a = m_i(im)m_2(im) + m_i(im \cup cap)m_2(im) + m_i(im \cap cap)m_2(cap) \quad (25)$$

$$b = m_i(cap)m_2(cap) + m_i(im \cup cap)m_2(cap) + m_i(cap \cap cap)m_2(cap) \quad (26)$$

$$K = \sum_{B \cap C = \emptyset} m_i(B)m_2(C) = m_1(cap)m_2(im) + m_1(im)m_2(cap) \quad (27)$$

Thus, a method for applying DST to the prediction of the RUL of LIBs is obtained. Using the posterior fused BPA functions (23) and (24), the prediction results derived from the two data sources are combined, and a prediction result of improved accuracy can be obtained.

**B. Process of RUL prediction based on DST and SVR-PF**

In the process of predicting the RUL of LIBs based on DST and the SVR-PF, we begin with the calculation of the initial state. Let $N$ be the number of charge/discharge cycles when the prediction begins, wherein values for the measured capacity $C_N$, the estimated capacity $C_{im,N}^*$ derived from the impedance data, and the estimated capacity $C_{cap,N}^*$ derived from the capacity data have been obtained for the $N$-th charge/discharge cycle.

First, the initial value of the BPA function $m_{1,i}(im)$ is determined. From the central limit theorem, we know that the measurement errors of a large body of impedance data follow a normal distribution as follows:

$$R_{e,N} \sim \mathcal{N}\left(\mu_{R_{e,N}}^*, \sigma_{R_{e}N}^2\right). \quad (28)$$

$$R_{ct,N} \sim \mathcal{N}\left(\mu_{R_{ct,N}}^*, \sigma_{R_{ct}N}^2\right). \quad (29)$$

Thus, $m_{1,N+1}(im)$ and $m_{2,N+1}(cap)$, other BPA function values can be determined. Because the two prediction methods are not associated, we obtain...
After obtaining the initial state, the prediction of the RUL is conducted, which includes the following three steps.

**Step 1**: Calculation of the BPA function values.
Let $N_{EOL}$ be the number of charge/discharge cycles when the battery reaches EOL, and, with respect to the $k$-th charge/discharge cycle, where $N+1 \leq k \leq N_{EOL}$, let $C^*_{im,k}$ be the predicted capacity obtained after data fusion, let $C^*_im,k$ be the predicted capacity obtained using impedance data analysis, and let $C^*_cap,k$ be the predicted capacity obtained using capacity data analysis. Then, the values of the BPA functions $m_{1,k+1}(im)$ and $m_{2,k+1}(cap)$ are as follows:

$$m_{1,k+1}(im) = \frac{1}{\sqrt{2\pi} \sigma_{im}} \exp \left(-\frac{(C^*_im,k - C^*_im)^2}{2\sigma_{im}^2}\right), \quad (46)$$

$$m_{2,k+1}(cap) = \frac{1}{\sqrt{2\pi} \sigma_{cap}} \exp \left(-\frac{(C^*_cap,k - C^*_cap)^2}{2\sigma_{cap}^2}\right). \quad (47)$$

In a similar manner as above, because the two data sources are not correlated, we obtain

$$m_{1,k+1}(cap) = m_{2,k+1}(im) = 0. \quad (48)$$

According to the property of a BPA function given in Equation (13), we have the following:

$$m_{1,k+1}(im \cup cap) = 1 - m_{1,k+1}(im), \quad (49)$$

$$m_{2,k+1}(im \cup cap) = 1 - m_{2,k+1}(cap). \quad (50)$$

Thus, all the required BPA function values are obtained.

**Step 2**: Data fusion and prediction of the capacity value in the $k+1$-th charge/discharge cycle.
From Equations (23) and (24), in addition to Equations (46), (47), (48), (49), and (50), the values of the posterior fused BPA functions $m_{k+1}(im)$ and $m_{k+1}(cap)$ are given as follows:

$$m_{k+1}(im) = \frac{1}{1 - K_k} \sum_{BNC=im} m_{1,k+1}(B)m_{2,k+1}(C) \quad (51)$$

$$m_{k+1}(cap) = \frac{1}{1 - K_k} \sum_{BNC=cap} m_{1,k+1}(B)m_{2,k+1}(C). \quad (52)$$

Thus, the predicted capacity value in the $k+1$-th charge/discharge cycle based on DST is obtained as follows:

$$C_{k+1}^* = m_{k+1}(im)C_{im,k+1}^* + m_{k+1}(cap)C_{cap,k+1}^*. \quad (53)$$

After determining all BPA function values, the posterior fused BPA function values, $m$, are calculated from Equations (23) and (24) as follows:

$$m_{N+1}(im) = \frac{1}{1 - K_{N+1}} \sum_{BNC=im} m_{N+1}(B)m_{2,N+1}(C)$$

$$m_{N+1}(im \cup cap) = 1 - m_{N+1}(im) - m_{N+1}(cap)$$

$$m_{N+1}(cap) = \frac{1}{1 - K_{N+1}} \sum_{BNC=cap} m_{N+1}(B)m_{2,N+1}(C)$$

$$m_{N+1}(im \cup cap) = 1 - m_{N+1}(im) - m_{N+1}(cap). \quad (38)$$

According to the property of a BPA function given in Equation (13), we obtain the following:

$$m_{1,N+1}(im \cup cap) = 1 - m_{1,N+1}(im) - m_{1,N+1}(cap)$$

$$m_{2,N+1}(im \cup cap) = 1 - m_{2,N+1}(im) - m_{2,N+1}(cap). \quad (39)$$

Thus, all the required BPA function values are obtained.

**Step 3**: Calculation of the BPA function values.
After obtaining the posterior fused BPA function values, the capacity value in the $N+1$-th charge/discharge cycle can be predicted. For the $N+1$-th charge/discharge cycle, let $C^*_{im,N+1}$ be the predicted capacity value obtained using impedance data, $C^*_{cap,N+1}$ be the predicted capacity value obtained using the capacity data, and $C^*_N$ be the predicted capacity value based on DST analysis. Then, according to Equations (41), (42), and (43), $C^*_N$ satisfies

$$C^*_N = m_{N+1}(im)C^*_{im,N+1} + m_{N+1}(cap)C^*_{cap,N+1}. \quad (44)$$

Thus, the initial state of the RUL prediction for LIBs based on DST and the SVR-PF is obtained, which is given by the following:

Initial \begin{array}{c}
\vdots 
\vdots 
\vdots \end{array}
\begin{array}{c}
m_{1,N+1}(im) 
m_{2,N+1}(im) 
m_{1,N+1}(im \cup cap) 
m_{2,N+1}(im \cup cap) 
m_{1,N+1}(cap) 
m_{2,N+1}(cap) 
m_{N+1}(im) 
m_{N+1}(cap) 
\end{array}, \quad (45)
Part 3: The state equations based on DST. They are deduced in Section V.B.

\[
\begin{align*}
    m_{1,k+1}(im) &= \frac{1}{\sqrt{2\pi} \sigma_{im}} \exp \left( - \frac{(C_{im,k} - C_k)^2}{2\sigma_{im}^2} \right) \\
    m_{2,k+1}(cap) &= \frac{1}{\sqrt{2\pi} \sigma_{cap}} \exp \left( - \frac{(C_{cap,k} - C_k)^2}{2\sigma_{cap}^2} \right) \\
    m_{1,k+1}(im \cup cap) &= 1 - m_{1,k+1}(im) \\
    m_{2,k+1}(im \cup cap) &= 1 - m_{2,k+1}(cap) \\
    K_{k+1} &= m_{1,k+1}(im)m_{2,k+1}(cap) \\
    m_{k+1}(im) &= m_{1,k+1}(im)m_{2,k+1}(im \cup cap) \\
    m_{k+1}(cap) &= m_{1,k+1}(im \cup cap)m_{2,k+1}(cap) \\
    C^*_k &= m_k(im)c_{im,k} + m_k(cap)c_{cap,k} \\
    C^*_{k+1} &= m_{k+1}(im)c^*_{im,k+1} + m_{k+1}(cap)c^*_{cap,k+1} \\
\end{align*}
\]

By combining Equations (60), (61), and (62), we obtain the state equations of model (55). The measurement equations of model (55) can be expressed as the following:

\[
\begin{align*}
    \bar{R}_{e,k} &= R_{e,k} + n_{e,k} \\
    \bar{R}_{ct,k} &= R_{ct,k} + n_{d,k} \\
    \bar{F}_{a,k} &= F_{1,k} + n_{1,k} \\
    \bar{F}_{b,k} &= F_{2,k} + n_{2,k} \\
    \bar{C}_k &= m_k(im)c_{im,k} + m_k(cap)c_{cap,k} \\
\end{align*}
\]

A flow chart summarizing the proposed procedure for predicting the RUL of LIBs based on DST and the SVR-PF is shown in Figure 4.

**Step 3:** Determine whether the predicted capacity reaches the EOL threshold.

In this step, if the predicted capacity fails to reach the EOL threshold, return to Step 1 to continue the prediction process. However, if it does reach the EOL threshold, the predicted value of RUL, \( \hat{L} \), is calculated as

\[
\hat{L} = N_{\text{EOL}}^* - N = (k + 1) - N.
\]

In Equation (54), \( N_{\text{EOL}}^* \) is the predicted value of the EOL.

**C. RUL prediction model based on DST and SVR-PF**

Based on the above, and combining models (5) and (11), a model for predicting the RUL of LIBs based on DST and the SVR-PF is established as follows:

\[
\begin{align*}
    X_{k+1} &= f(X_k, V_k) \\
    Y_k &= g(X_k, N_k) 
\end{align*}
\]

Here, the state \( X_k \) is

\[
X_k = [X^{T}_{im,k}, X^{T}_{cap,k}, X^{T}_{DS,k}]^T. \tag{56}
\]

In Equation (56) above, \( X_{im,k} \) is the state obtained using impedance data analysis, \( X_{cap,k} \) is the state obtained using capacity data analysis, and \( X_{DS,k} \) is the state obtained using DST analysis. In addition, we have the following definitions:

\[
\begin{align*}
    X_{im,k} &= \left[ \lambda^{*}_{R_{e,k}}, \lambda^{*}_{R_{ct,k}}, R^{*}_{e,k}, R^{*}_{ct,k}, C^{*}_{im,k} \right]^T, \tag{57} \\
    X_{cap,k} &= \left[ \lambda^{*}_{R_{e,k}}, \lambda^{*}_{R_{ct,k}}, R^{*}_{e,k}, R^{*}_{ct,k}, C^{*}_{cap,k} \right]^T, \tag{58} \\
    X_{DS,k} &= [m_{1,k}(im), m_{1,k}(im \cup cap), ... \ m_{2,k}(cap), m_{2,k}(im \cup cap), ... \ m_{k}(im), m_{k}(cap), C_k]^T. \tag{59}
\end{align*}
\]

Thus, the state equations of model (55) are given by the following three parts:

**Part 1:** The state equations based on RUL prediction using impedance data. They are based on Equation (5).

\[
\begin{align*}
    \lambda^{*}_{R_{e,k+1}} &= \lambda^{*}_{R_{e,k}} + v_{a,k} \\
    \lambda^{*}_{R_{ct,k+1}} &= \lambda^{*}_{R_{ct,k}} + v_{b,k} \\
    R^{*}_{e,k+1} &= R^{*}_{e,k} \exp(\lambda^{*}_{R_{e,k}}(\Delta k)) + v_{e,k} \\
    R^{*}_{ct,k+1} &= R^{*}_{ct,k} \exp(\lambda^{*}_{R_{ct,k}}(\Delta k)) + v_{d,k} \\
    C^{*}_{im,k} &= \alpha_n(R^{*}_{e,k+1} + R^{*}_{ct,k+1}) + \beta_n + v_{e_k} 
\end{align*}
\]

**Part 2:** The state equations based on RUL prediction using capacity data. They are based on Equation (11).
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From Figure 4, we can see that the LIB RUL prediction process stops when the EOL threshold criterion is met. The RUL value is calculated based on Equation (54). Furthermore, the RUL distribution is also given by using the particle weights when the prediction stops. Equation (27) in Ref. [31] provides a predicted RUL distribution.

D. DST-based LIB RUL prediction framework

The proposed DST LIB RUL prediction method can be applied to the combination of any two methods. In definition (22), if we change \{im\} and \{cap\} to any other RUL prediction methods, for example, \{meth1\} and \{meth2\}, we modify the definition in Equation (22) to

\[
2^{Ω} = \{∅, \{meth1\}, \{meth2\}, \{meth1 \cup meth2\}\}. \tag{64}
\]

\(meth1\) represents the first method of choice, and \(meth2\) represents the second method of choice.

With the change of the power set \(2^{Ω}\), the meaning of the propositions will also change:

1. \(meth1\) The RUL obtained using the first method is believable.
2. \(meth2\) The RUL obtained using the second method is believable.
3. \(meth1 \cup meth2\) The RUL obtained using either method’s data is believable.

The BPA functions, \(m_1\) and \(m_2\), will also change:

1. \(m_1\) is the belief distribution of propositions in \(2^{Ω}\) when predicting the RUL using the first method.
2. \(m_2\) is the belief distribution of propositions in \(2^{Ω}\) when predicting the RUL using the second method.

Furthermore, if we assume the capacity data from both chosen methods satisfy the central limit theorem, we have

\[
C_{meth1} \sim \mathcal{N}(μ_{meth1}, σ^2_{meth1}), \tag{65}
\]

\[
C_{meth2} \sim \mathcal{N}(μ_{meth2}, σ^2_{meth2}). \tag{66}
\]

Therefore, using Equation (62), the combined predicted LIB capacity at each prediction step \(k\) is

\[
\begin{align*}
    m_{k+1}(meth1) &= \frac{1}{\sqrt{2πσ_{meth1}}} \exp \left( -\frac{(C_{meth1,K} - C_k^{meth1})^2}{2σ_{meth1}^2} \right) \\
    m_{k+1}(meth2) &= \frac{1}{\sqrt{2πσ_{meth2}}} \exp \left( -\frac{(C_{meth2,K} - C_k^{meth2})^2}{2σ_{meth2}^2} \right) \\
    m_{k+1}(meth1 \cup meth2) &= 1 - m_{k+1}(meth1) - m_{k+1}(meth2) \\
    K_{k+1} &= m_{k+1}(meth1)m_{k+1}(meth2) \\
    m_{k+1}(meth1) &= m_{k+1}(meth1)m_{k+1}(meth1 \cup meth2) \\
    m_{k+1}(meth2) &= m_{k+1}(meth2)m_{k+1}(meth1 \cup meth2) \\
    C_{k}^{meth1} &= m_{k+1}(meth1)C_{meth1,K+1} + m_{k+1}(meth2)C_{meth2,K+1}
\end{align*}
\]  

Using Equation (67), we can combine the LIB RUL prediction results from both methods.

VI. SIMULATION

A. Comparing the prediction performances between impedance-based prediction, capacity-based prediction, and DST-based prediction

To compare the performances of the three methods for RUL prediction using impedance data, capacity data, and the fusion of the two based on DST when the available data are relatively sparse, the data obtained for batteries 5, 6, 7, and 32 were used. RUL prediction thresholds of batteries 5, 6, 7, and 32 were respectively set to be 90\%, 85\%, 95\%, and 95\% of the nominal capacity. The RUL prediction threshold is defined as a percentage that is lower than the maximum capacity of the battery (100\%), and higher than the capacity corresponding to the EOL threshold. A high RUL prediction threshold reduces the available data that can be used for analyzing the state of health of the battery, making the conditions for the algorithm more stringent. Hence, the performance of the algorithm when the available data are relatively sparse can be tested using a high RUL prediction threshold. Moreover, Gaussian noise model was used in the simulation. The PF in this study used 100 particles. The prediction results are presented graphically in Figure 5, Figure 6, Figure 7, and Figure 8, whereas the predicted EOL and RUL values with corresponding prediction errors are listed in Tables II and III.
FIGURE 5. Prediction results for battery 5 using the three methods employing the fusion of impedance and capacity data using DST (left), impedance data (middle), and capacity data (right).

FIGURE 6. Prediction results for battery 6 using the three methods employing the fusion of impedance and capacity data using DST (left), impedance data (middle), and capacity data (right).

FIGURE 7. Prediction results for battery 7 using the three methods employing the fusion of impedance and capacity data using DST (left), impedance data (middle), and capacity data (right).
FIGURE 8. Prediction results for battery 32 using the three methods employing the fusion of impedance and capacity data using DST (left), impedance data (middle), and capacity data (right).

TABLE II. Comparison of the three methods for each battery using the impedance data, the capacity data, and the fusion of the two based on DST for RUL and EOL prediction.

| Battery | DST with SVR-PF | Impedance | Capacity | Measured value |
|---------|-----------------|-----------|----------|----------------|
|         | RUL | EOL | RUL | EOL | RUL | EOL | RUL | EOL |
| Battery 5 | 94  | 159 | 104 | 169 | 88  | 153 | 97  | 162 |
| Battery 6 | 89  | 144 | 95  | 150 | 69  | 124 | 85  | 140 |
| Battery 7 | 120 | 165 | 127 | 172 | 96  | 141 | 115 | 160 |
| Battery 32| 19  | 39  | 32  | 52  | 16  | 36  | 20  | 40  |

TABLE III. Comparison of the RUL prediction errors for each battery using methods employing the impedance data, the capacity data, and the fusion of the two based on DST.

| Battery | DST with SVR-PF | Impedance | Capacity |
|---------|-----------------|-----------|----------|
|         | RUL | EOL | RUL | EOL|
| Battery 5 | 3   | 7   |
| Battery 6 | 4   | 10  |
| Battery 7 | 5   | 12  |
| Battery 32| 1   | 12  |

From Figure 5 to Figure 8, DST demonstrates a more accurately predicted capacity. The accuracy is further verified by the comparisons of the three methods regarding the prediction results for the RUL and EOL listed in Table II and the prediction errors for the RUL listed in Table III, which show that the prediction method based on DST provides values for the RUL and the EOL that are closer to the actual measured values than those obtained by the methods employing either impedance or capacity individually. This suggests that the proposed method can effectively combine the prediction results obtained using impedance data with those obtained using capacity data, and provide a more accurate prediction when the available data are relatively sparse.

B. Comparing the prediction performance with existing LIB RUL prediction methods introduced in other literature

To compare the LIB RUL predicting performance between the proposed method in this study and existing methods introduced in other literature, research works that used the same battery data sets were compared according to their prediction performance. Because the prediction start and stop thresholds were different across the literature, relative error (RE) was used to represent the prediction performance, as defined by the following equation:

$$RE = \frac{RUL_{predicted} - RUL_{measured}}{RUL_{measured}} \times 100\%.$$ (68)

Here, $RUL_{predicted}$ is the predicted LIB RUL value, $RUL_{measured}$ is the measured LIB RUL value, and RE is the relative error.

Data for batteries 5 and 6 were most commonly used by the existing RUL prediction methods, so the data sets of these two batteries were chosen for the comparison. The comparison is shown in Tables IV and V. Reference is made to the prediction start and stop thresholds, predicted RUL, measured RUL, and error given in the corresponding literature. Relative error was calculated based on Equation (68):
### TABLE IV. Comparison among the existing LIB RUL prediction approaches and the proposed DST approach in this paper (battery 5, unit: cycle).

| Algorithm used in method                          | Prediction starts threshold | Prediction stops threshold/measured EOL | Cycles used for model training/SOH estimation | Predicted RUL | Measured RUL | Error | Relative error |
|---------------------------------------------------|----------------------------|----------------------------------------|-----------------------------------------------|---------------|--------------|-------|----------------|
| 1 Conditional variational autoencoder PF [25]     | 74                         | 124                                    | 74                                            | 55            | 50           | 5     | 10%            |
| 2 Conditional variational autoencoder PF [25]     | 99                         | 124                                    | 99                                            | 25            | 25           | 0     | 0%             |
| 3 F-distribution particle filter and kernel        | 100                        | 123                                    | 100                                           | 19            | 23           | 4     | 17.4%          |
| smoothing algorithm [26]                          |                            |                                        |                                               |               |              |       |                |
| 4 F-distribution particle filter and kernel        | 67                         | 123                                    | 67                                            | 67            | 56           | 11    | 19.6%          |
| smoothing algorithm [26]                          |                            |                                        |                                               |               |              |       |                |
| 5 Dynamic long short-term memory neural-network   | 81                         | 128                                    | 81                                            | 45            | 47           | 2     | 4.3%           |
| [12]                                              |                            |                                        |                                               |               |              |       |                |
| 6 Long short-term memory network–sliding time      | 70                         | 162                                    | 70                                            | 100           | 92           | 8     | 8.7%           |
| window and Gaussian or sine function, Levenberg-  |                            |                                        |                                               |               |              |       |                |
| Marquardt algorithm fusion [13]                   |                            |                                        |                                               |               |              |       |                |
| 7 Long short-term memory network–sliding time      | 80                         | 162                                    | 80                                            | 95            | 88           | 7     | 8.0%           |
| window and Gaussian or sine function, Levenberg-  |                            |                                        |                                               |               |              |       |                |
| Marquardt algorithm fusion [13]                   |                            |                                        |                                               |               |              |       |                |
| 8 Long short-term memory network–sliding time      | 90                         | 162                                    | 90                                            | 74            | 72           | 2     | 2.8%           |
| window and Gaussian or sine function, Levenberg-  |                            |                                        |                                               |               |              |       |                |
| Marquardt algorithm fusion [13]                   |                            |                                        |                                               |               |              |       |                |
| 9 Proposed DST method                             | 65                         | 162                                    | 65                                            | 94            | 97           | 3     | 3.1%           |

*The authors in the references did not give the actual values. So, the values were deduced based on the information provided by the other authors.*

### TABLE V. Comparison among the existing LIB RUL prediction approaches and the proposed DST approach in this paper (battery 6, unit: cycle).

| Algorithm used in method                          | Prediction starts threshold | Prediction stops threshold/measured EOL | Cycles used for model training/SOH estimation | Predicted RUL | Measured RUL | Error | Relative error |
|---------------------------------------------------|----------------------------|----------------------------------------|-----------------------------------------------|---------------|--------------|-------|----------------|
| 1 F-distribution particle filter and kernel        | 89                         | 107                                    | 89                                            | 13            | 18           | 5     | 27.8%          |
| smoothing algorithm [26]                          |                            |                                        |                                               |               |              |       |                |
| 2 F-distribution particle filter and kernel        | 54                         | 107                                    | 54                                            | 43            | 53           | 10    | 18.9%          |
| smoothing algorithm [26]                          |                            |                                        |                                               |               |              |       |                |
| 3 Long short-term memory network–sliding time      | 70                         | 101*                                   | 70                                            | 41*           | 31           | 10    | 32.3%          |
| window and Gaussian or sine function, Levenberg-  |                            |                                        |                                               |               |              |       |                |
| Marquardt algorithm fusion [13]                   |                            |                                        |                                               |               |              |       |                |
| 4 Long short-term memory network–sliding time      | 80                         | 101*                                   | 80                                            | 26*           | 21           | 5     | 23.8%          |
| window and Gaussian or sine function, Levenberg-  |                            |                                        |                                               |               |              |       |                |
| Marquardt algorithm fusion [13]                   |                            |                                        |                                               |               |              |       |                |
| 5 Long short-term memory network–sliding time      | 90                         | 101*                                   | 90                                            | 12*           | 11           | 1     | 9.1%           |
| window and Gaussian or sine function, Levenberg-  |                            |                                        |                                               |               |              |       |                |
| Marquardt algorithm fusion [13]                   |                            |                                        |                                               |               |              |       |                |
| 6 Proposed DST method                             | 55                         | 140                                    | 55                                            | 89            | 85           | 4     | 4.7%           |

*The authors in the references did not give the actual values. So, the values were deduced based on the information provided by the other authors.*
In Tables IV and V, the cycles used for model training/state of health (SOH) estimation represent the number of cycles of data that were used to train the model if the prediction methods were neural network based, or the number of cycles of data that were used for SOH estimation to determine the degradation trend of the LIB.

From the above tables we can observe:

- The relative error of the proposed DST LIB RUL prediction method can outperform most of the existing methods.
- For the methods that had lower relative errors than the proposed DST method (rows 2 and 8 in Table IV), either more cycles of data were used to train their models or LIB’s SOH estimation (65 cycles for DST vs. 99 and 90 cycles for rows 2 and 8 respectively). Moreover, the prediction stop thresholds of the proposed method were lower, which means that the proposed method had a more difficult prediction task since it had to predict more data.
- For the method that has similar prediction start and stop thresholds (row 6 in Table IV), the proposed DST method has a better relative error.
- When the available data are sparse (rows 1, 4, and 6 in Table IV and rows 2 and 3 in Table V), the proposed DST method has a better relative error. Therefore, from the analysis above, compared to other existing LIB RUL prediction methods, the proposed DST LIB RUL prediction method has better prediction performance (despite relatively sparse data) and has the lowest prediction stop thresholds.

VII. CONCLUSION

This study presents a method for predicting the RUL of LIBs based on DST and the SVR-PF. The proposed method combines RUL prediction methods that use impedance data and capacity data individually, making it possible to provide more accurate RUL predictions even when the available data are relatively sparse. First, the application of DST in predicting the RUL of LIBs was discussed. Subsequently, the process of predicting the RUL of LIBs based on DST and an SVR-PF was proposed. Finally, a novel RUL prediction model was established. The simulation results suggest that, with the proposed method, accurate RUL prediction can be achieved when the available data are relatively sparse. Moreover, the comparison with the existing LIB RUL prediction methods shows that the proposed method can achieve more accurate prediction results with relatively sparse datasets and the lowest prediction stop thresholds. Hence, the proposed method ensures the accuracy of RUL prediction.

VIII. FUTURE WORK

Future work based on these results can be summarized as follows:

- Implement this method for the design testing of LIB production, particularly those that will be used in battery electric vehicles. Collect their degradation data, and verify the proposed LIB RUL prediction method on the data.
- Using vehicle to cloud communication, design a real-time battery electric vehicle LIB data collection system using edge computing and/or cloud computing. Apply the proposed DST and SVR-PF-based LIB RUL prediction framework in Equation (67) using edge computing/cloud computing.

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