Considering Grid Division Optimization of Free-Form Surface Roof Structure

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Abstract. The free-form surface roof structure is widely used in large-span public buildings due to its beautiful shape. Its grid division form is diverse, and the size and shape of the structural grid directly affect its bearing performance. In this paper, the grid optimization research is carried out for the free-form surface single-layer reticulated shell roof, based on the Rhino platform and the grasshopper module for parametric modeling. The karamba plug-in is used for structural analysis, and the roof structure of different grid divisions under various load conditions is analyzed, computing its response and its carrying capacity. Genetic algorithm is used to optimize free-form surface roof structure, with grid density, member bar length and section size as parameters. Comprehensive consideration of structural self-weight and maximum displacement and other design goals are considered. It provides reference for the design of free-form roof structure.

Keywords. Free-form surface roof, parametrized grid division, grid structure optimization.

1. Introduction

With the continuous progress of construction technology, the form design of the large-span spatial grid structure gradually introduces the architectural elements of free-form surfaces, bringing new visual enjoyment to the public. The so-called free-form surface refers to a surface that cannot be composed of elementary analytical surfaces, but changes freely in a complex manner [1]. At present, the free-form surface space grid structure has been applied in more and more landmark buildings foreign and domestic due to its many advantages such as high structural efficiency, spatial flexibility, and rich aesthetics. The free-form surface as the roof structure is more and more popular. For free-form surfaces, it is necessary to make reasonable mesh division and optimize so that the mesh distribution is uniform, the rod length is consistent, and the lines are smooth, with the same time to meet the mechanical performance requirements of the structure and also meet the aesthetic requirements and visual effects of the building.

The domestic and foreign scholars have done a lot of research on this, and have produced many different algorithms in meshing, including mapping method, Delaunay method, grid method, paving method, triangle transformation method and combination method, etc. Li Na [2] proposed a grid mapping method based on quadratic difference to solve the problem of grid distortion caused by uneven surface curvature, and gave the evaluation standard of grid quality. Wei dajie [3] expanded free-form surface according to the principle of equal area, and then Delaunay triangular meshing is performed on the expanded two-dimensional plane, and finally the planar mesh is mapped back to the space surface to obtain a good meshing result. Sometimes only applying a certain method cannot solve
the meshing of free-form surfaces. In order to achieve a better meshing effect, it is often necessary to combine two or more of these methods [4]. Zheleznyakova [5] proposed a new molecular dynamics-based triangular meshing method, which treats the mesh nodes as interacting particles, and determines the position of the final node through the simulation of molecular dynamics, and finally uses Delaunay triangulation method connects to generate mesh.

Most of the time, it is difficult to meet the requirements of the architectural grid for the uniformity and fluency of the grid, and it is hard to directly apply to actual projects. According to the different optimization purposes, grid optimization methods can be divided into two categories, optimization based on geometric goals and optimization based on mechanical properties [6]. In the “Sunshine Valley” project design of the Expo axis, Li Chengming and Lu Dan [7] first manually subdivided the parameter domain to obtain a plane grid, and then mapped the plane grid back to a three-dimensional space surface, using grid shapes and unit the length standard, intelligent adjustment and optimization design of the initial grid. Li Na and Lu Jinyu [8] introduce the smoothing criterion of the surface energy method, and use the deviation tolerance of the mesh node as the smoothing constraint to perform the smoothing control of the free-form surface mesh structure. Su Liang, Zhu Shunlai [9] used the principal stress trajectory to represent the "force flow" path, and used the wavefront propulsion method to automatically generate a grid structure along the principal stress trajectory. Cui Changyu [10] uses strain energy as the target to optimize the node coordinates, and finally get a reasonable structure with the smallest structural strain energy. Winslow uses a multi-objective genetic algorithm and sets the structural response under various loads as the objective function to generate the mesh [11].

2. Spatial Bubble Method Meshing

In order to solve the problem of mapping distortion in the mapping method, Wang Qisheng, etc. [12] proposed the spatial bubble method. The main idea is to introduce the strong adsorption force of the curved surface to the bubble, then on the basis of the interaction force between the bubbles, and constant iteratively solve the problem in space. The equilibrium position of the bubble is obtained a point set \(\{P\}\) uniformly distributed on the surface, and then a triangular mesh based on the point set \(\{P\}\) is generated by the Delaunay method. The space bubble method can be divided into three stages: initial point layout, dynamic simulation and grid generation.

In the initial point layout stage, \(n\) points \(\{P\}\) are randomly arranged directly on the surface. In the dynamic simulation stage, the elastic bubble centered on the point set \(\{P\}\) on the surface is established, and the corresponding mechanical model is established. The optimization of the grid point position is regarded as the motion simulation solution of the bubble equilibrium state, and the three forces are introduced as the adsorption force \(Q_i\), repulsive force \(T_{ij}\) and movement resistance \(F_i\). The adsorption force keeps the center of the bubble basically on the curved surface, and the repulsive force keeps the bubbles at a certain distance, and repels each other until the bubbles are evenly distributed on the entire free-form surface. In order to make the bubbles finally reach a balanced state and avoid vibrations, the energy of the motion system must be dissipated to introduce motion resistance. Finally the force of the bubble is synthesized, and the resultant force of the i-th bubble is

\[
F_i = \sum_{j=1}^{n} T_{ij} + Q_i + f_i
\]

The force of the bubble will change with the change of position, but in a small time step, it can be assumed that the force state of the bubble remains unchanged. Therefore, according to the force equation of the bubble and Newton's law of motion, the Velocity-Verlet algorithm is used to iteratively solve the equilibrium position of the bubble [13]. In the mesh generation stage, a triangular mesh is generated by Delaunay rule based on the bubble center point set \(\{P_0\}\) after solving the equilibrium.
3. Analysis of Free-Form Surface Roof Structure

3.1. Basic Parameters of Free-Form Surface Roof

The model is a spatial free-form surface single-layer reticulated shell structure with three peaks and two valleys. The specific shape parameters are shown in figure 2. The maximum span is 73 m and the length is 135 m. Steel rods are round steel pipes, the specifications are shown in table 1, the material is
Q235 steel, and the elastic modulus \( E = 2.06 \times 10^{11} \text{ N/m}^2 \). It is assumed that the cross-sections of all the rods in the spatial grid structure are consistent, that is, one type of rod which is selected from the common specifications to build the structure. The joint form is a rigid joint, and the support adopts an ideal hinged support with four sides supported. The load conditions are applied: the structural self-weight, a uniform constant load of 0.3 \( \text{KN/m}^2 \) and a live load of 0.5 \( \text{KN/m}^2 \). The self-weight load act on the center of gravity of the bar, the constant load is converted into linear load on the bar, and the live load is transmitted to the nodes through the roof panels, so the live load processed to point load is applied to the grid nodes.

| Diameter (mm) | 48  | 60  | 75.5 | 88.5 | 114 | 159 | 180 | 219 | 245 | 273 | 325 |
|---------------|-----|-----|------|------|-----|-----|-----|-----|-----|-----|-----|
| Wall thickness (mm) | 3.5 | 3.5 | 3.75 | 4.0 | 4.0 | 6.0 | 8.0 | 14.0 | 14.0 | 14.0 | 16.0 |
|               | 8.0 | 10.0 | 16.0 | 16.0 | 16.0 | 20.0 |
|               | 10.0 | 12.0 |      |      |      |
|               | 12.0 | 14.0 |      |      |      |

The mesh obtained by the space bubble method is given to the bar properties, including the diameter and wall thickness of the section parameters of the round steel tube, the material parameters of the elastic modulus and density of the steel, and the four-sided hinge support constraints of the roof, which are in the whole roof gravity is applied to the grid, and the processed load is applied to the member bars and nodes of the grid. Use karamba plug-in to perform structural static analysis to calculate the maximum displacement, structural weight, and strain energy of the roof structure.

3.2. Mathematical Model of Free-Form Surface Roof Structure Optimization

The structural weight which directly affects the project cost is a very important indicator, and how to comprehensively consider the structural mechanical properties and structural weight to make a quantitative evaluation of the structural rationality is an urgent problem to be solved [14]. It is reasonable that the grid structure with the lightest structural weight is the best under the conditions that satisfy the constraints in the design specification of the spatial grid structure. This forms an optimized mathematical model, in which the objective function is the weight of the structure, the design variable parameters are the number of nodes and the cross-section of the member bars. The specific constraints follows.

1) Slenderness ratio conditions:

\[
\lambda = \frac{l_i}{i} \leq [\lambda]
\]

Where: \( l_i \) are the calculated length and \( i \) are radius of gyration of the \( i \)-th member bars. \([\lambda]\) is the allowable slenderness ratio. The slenderness ratio of the single-layer reticulated shell members under compression and bending shall be taken as \([\lambda] \leq 150\) in accordance with the relevant regulations of the current national standard "Code for Design of Steel Structures" GB 50017.

2) Strength conditions:

\[
\frac{N}{A_n} + \frac{M}{\gamma_m W_n} \leq f
\]

Where: \( N \) and \( M \) are the axial force and bending moment of the \( i \)-th member respectively, \( A_n \) is the cross-sectional area of the member, \( W_n \) is the bending modulus of the member's section, and \( \gamma_m \) is the plastic development of the section the coefficient is 1.0, and \( f \) is the material strength design value of 215N/mm\(^2\).
3) Stable conditions:

$$\frac{N}{\varphi A_n f} + \frac{\beta_m M}{\gamma_m W_n (1 - 0.8 N/N_E^l) f} \leq 1.0$$  \hspace{1cm} (4)$$

$$N_E^l = \frac{\pi^2 EA}{1.1L^2}$$  \hspace{1cm} (5)$$

In the equation: $\varphi$ is the member stability coefficient, $\beta_m$ is the equivalent bending moment coefficient.

4) Maximum structural displacement requirements:

$$\delta \leq \frac{B}{400} = 0.15m$$  \hspace{1cm} (6)$$

In the equation: $\delta$ is the maximum displacement of the structure, and $B$ is the maximum span of the roof in the short span direction.

5) Rod structure requirements:

$$d \geq d_{min} = 45mm \hspace{1cm} t \geq t_{min} = 3mm$$  \hspace{1cm} (7)$$

where: $d$ is the diameter of the rod; $t$ is the wall thickness of the rod.

The above constraints are added to the objective function in the form of a penalty function, the constrained optimization problem is transformed into an unconstrained optimization problem, and the genetic algorithm is used for optimization calculation. After continuous iteration, the optimal solution can be obtained by eliminating infeasible samples [15]. That is to find a set of parameters for the number of grid nodes and cross-sectional dimensions, and the grid structure with the lightest structure under the conditions of meeting the above constraints. The optimization flow chart is shown in figure 3.

Figure 3. Optimization flow chart.
3.3. The Influence of The Number of Nodes and the Cross-Section of Members on the Mechanical Properties of Roof Structures

Control the number of grid nodes respectively 600, 700, 800, 900, 1000, and use three specifications of φ219×14, φ219×16, and φ245×14 for the member bars, and calculate the maximum displacement, structure weight, and maximum stress of the spatial grid structure under different number of nodes and different sections of the member, and check whether it satisfies the constraint conditions such as the slenderness ratio and stability of the member. The results are listed in Table 2.

Table 2. Table of maximum displacement, structural weight, and constraint conditions of different cross-sections with different numbers of nodes.

| Number of nodes | Section of rod | Maximum displacement (m) | Structure weight (KN) | Maximum stress (Mpa) | Slenderness ratio constraint | Stability constraints |
|-----------------|----------------|--------------------------|-----------------------|----------------------|----------------------------|----------------------|
| 600             | φ219×14        | 0.079                    | 5373                  | 216.8                | ✓                          | ✓                    |
|                 | φ219×16        | 0.074                    | 6081                  | 203.1                | ✓                          | ✓                    |
|                 | φ245×14        | 0.072                    | 6055                  | 203.7                | ✓                          | ✓                    |
|                 | φ245×14        | 0.083                    | 5808                  | 204.2                | ✓                          | ✓                    |
| 700             | φ219×16        | 0.077                    | 6572                  | 191.9                | ✓                          | ✓                    |
|                 | φ245×14        | 0.076                    | 6544                  | 190.5                | ✓                          | ✓                    |
|                 | φ219×14        | 0.074                    | 6204                  | 213.7                | ✓                          | ✓                    |
| 800             | φ219×16        | 0.069                    | 7021                  | 201.3                | ✓                          | ✓                    |
|                 | φ245×14        | 0.068                    | 6991                  | 200.6                | ✓                          | ✓                    |
|                 | φ219×14        | 0.072                    | 6576                  | 206.5                | ✓                          | ✓                    |
| 900             | φ219×16        | 0.068                    | 7442                  | 194.8                | ✓                          | ✓                    |
|                 | φ245×14        | 0.066                    | 7410                  | 192.2                | ✓                          | ✓                    |
|                 | φ219×14        | 0.075                    | 6946                  | 192.7                | ✓                          | ✓                    |
| 1000            | φ219×16        | 0.071                    | 7861                  | 181.9                | ✓                          | ✓                    |
|                 | φ245×14        | 0.070                    | 7827                  | 182.7                | ✓                          | ✓                    |

Analyzing Table 2, when the number of nodes is the same, the cross section of the bar is increased, the maximum displacement of the grid structure is reduced, the maximum stress is reduced, and the structure weight is increased. Therefore, the increase of the cross section of bars can increase the stiffness of grid structure, and at the same time the increase in the cross section of rods increases the weight of structure. On the other hand, when the cross-section of member is same, the increase in the number of nodes makes the maximum displacement of the grid structure first increase and then decrease, and the structure weight increases, so the number of nodes has an impact on the stiffness of the grid structure. And with the number of nodes increases, the total length of rods increases and the structural weight increases. Therefore, for the free-form surface space grid structure, there is an appropriate number of nodes and member cross-sections so that the grid structure has the smallest structural weight under the conditions of satisfying various constraints.

4. Adjust the Mesh Density Based on the Member Stress

Through the optimization calculation of 20 samples per generation of 50 generations of genetic algorithm as is shown in figure 4, a set of parameters that meet the requirements of displacement, slenderness ratio and structural stability of the structure with the lightest weight are obtained. At this time, the number of nodes is 403, and the cross-sectional dimensions of the members are noted below. The diameter is 219 mm, the wall thickness is 14mm, the maximum structural displacement is 0.098 m≤0.15 m, the maximum stress is 218.5 Mpa≤235 Mpa, the strain energy is 135.8 kNm, the structural weight is 4379.7 kN, the average length of the rod is 5.5 m, and the rod length variance is 0.55. The displacement is shown in figure 5.
Although the grid divided by the space bubble method is relatively uniform in appearance, the stress of each member is not uniform, and the stress around the wave peak is large and the stress in the valley is small. The basic idea of the spring analogy method is to regard the connection between the grid nodes as a spring, and the position of the node is moved by the elastic force generated by the spring deformation, so as to optimize the density of the grid [16]. Assuming that the stiffness of each spring is the same, the specific equation for changing the length based on the stress of the rod is as follows:

$$ l = l_i \left[ 1 - \alpha \left( \frac{\sigma_i - \sigma_{\text{min}}}{\sigma_{\text{max}} - \sigma_{\text{min}}} \frac{\bar{\sigma} - \sigma_{\text{min}}}{\sigma_{\text{max}} - \sigma_{\text{min}}} \right) \right] $$  \hspace{1cm} (8)

In the equation: $l_i$ is the length of the member before adjustment, $\sigma_i$ is the stress value of the member, $\sigma_{\text{min}}$ is the smallest stress value among all members, and $\sigma_{\text{max}}$ is all members maximum stress value, $\bar{\sigma}$ is the average value of the stress of all the members and $\alpha$ is adjustment coefficient taken as (0,1). When $\alpha$ is 0 and 0.8, the grid structure is as follows respectively figure 6 and figure 7.

Adjust the rod length according to the above equation, so that the rod length with stress greater than the average value is shortened, and the rod length with the stress less than the average value is lengthened. The grid density is further optimized by genetic algorithm, with $\alpha$ as the variable and strain energy as the objective function. Optimization results are shown in table 3. When considering the stability constraints of the bar, when $\alpha$ is set to 0.26, the minimum strain energy is 146.4 KNm, which is 3.4% smaller than the uniform grid; the maximum displacement is 0.089m, which is 9.2% less than the uniform grid. Without considering the constraint of member stability, when $\alpha$ is 0.5, the minimum strain energy is 143.1 KNm, which is 5.5% smaller than the uniform grid; the maximum displacement is 0.084m, which is 14.3% smaller than the uniform grid.
Table 3. The maximum displacement, structure weight, and constraint conditions of the structure after adjusting the mesh density of 403 node number φ219×14.

| Number of nodes | α    | Maximum displacement (m) | Structure weight (KN) | Strain energy (KNm) | Maximum stress (Mpa) | Slenderness ratio constraint | Stability constraints |
|-----------------|------|--------------------------|-----------------------|---------------------|----------------------|-----------------------------|------------------------|
|                 | 0    | 0.098                    | 4380                  | 151.5               | 218.5                | √                           | √                      |
|                 | 0.26 | 0.089                    | 4364                  | 146.4               | 226.0                | √                           | √                      |
|                 | 0.5  | 0.084                    | 4370                  | 143.1               | 234.7                | √                           | ×                      |

5. Conclusion

For the free-form surface with three waves and two valleys, the spatial grid structure with relatively uniform grid appearance can be obtained by dividing the grid by the space bubble method. The genetic algorithm optimizes the number of nodes and the cross-section of the members of the grid structure, and can obtain the smallest grid structural weight with meeting the requirements of the maximum displacement conditions, maximum stress conditions and stability requirements. On this basis, the length of the member is further adjusted according to the stress of the member. The area with high stress is reduced and the mesh is crowded, and the area with low stress is increased and the mesh is sparse. The optimized grid structure can further reduce the maximum displacement and strain energy and improve the structural performance. It provides a method from free-form surface to space grid structure for the design of free-form surface roof structure, and finally obtains the beautiful appearance of the grid and good mechanical properties.

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