JOULE HEATING AND THE THERMAL EVOLUTION OF OLD NEUTRON STARS
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ABSTRACT

We consider Joule heating caused by dissipation of the magnetic field in the neutron star crust. This mechanism may be efficient in maintaining a relatively high surface temperature in very old neutron stars. Calculations of the thermal evolution show that, at the late evolutionary stage \((t \geq 10\text{ Myr})\), the luminosity of the neutron star is approximately equal to the energy released due to the field dissipation and is practically independent of the atmosphere models. At this stage, the surface temperature can be of the order of \(3 \times 10^4 – 10^5\) K. Joule heating can maintain this high temperature for an extremely long time \((\geq 100\text{ Myr})\), comparable with the decay time of the magnetic field.

Subject headings: MHD — pulsars: general — stars: magnetic fields — stars: neutron

1. INTRODUCTION

Neutron stars are very hot at birth, with temperatures well above \(10^{10}\) K. This heat is radiated away mainly by neutrinos from the inner layers during the first million years or so (the neutrino cooling era) and, later on, the emission of photons from the surface dominates the cooling of the star. This photon luminosity and its change with time depend on the properties of matter inside the neutron star and its magnetic field. Observations of this radiation can thus provide important information about the state of matter above and below nuclear density as well as about the magnetic field.

The magnetic field can influence the thermal evolution of neutron stars in different ways. This influence is probably less appreciable during the neutrino cooling era despite the fact that the neutrino emissivities for some mechanisms can essentially alter in strong magnetic fields \(\sim 10^{13}\). The effect of the magnetic field may be of particular importance for neutrino processes in the inner crust where, at some conditions, the synchrotron emission may dominate the rate of neutrino cooling (Vidauuere et al. 1995).

The influence of the magnetic field on the cooling history is much more important during the photon cooling era. In the presence of a strong field \(\sim 10^{12} – 10^{13}\) G the transport properties of plasma are different compared to those in non-magnetic neutron stars. Both the electron and radiative thermal conductivities can be affected by the field. Generally speaking, the cooling efficiency longitudinal to the field lines exceeds that in the transverse direction. This change in the thermal conductivity can have an appreciable effect on the thermal evolution (see Van Riper 1991; Nomoto & Tsuruta 1987). Besides, an anisotropic heat transport will result in a characteristic temperature difference between the hot magnetic poles of the neutron star and the relatively cold magnetic equator (see, e.g., Schauf 1990).

Additional heating associated with the ohmic dissipation of currents may be one more important effect caused by the magnetic field. The rate of Joule heating depends on both the geometry of the magnetic field and conductive properties of plasma and may be rather high for some magnetic configurations. The total magnetic energy of the neutron star can probably reach \(10^{43} – 10^{44}\) ergs. Observational data on the spin and magnetic evolution of isolated pulsars provide some evidence that the field decay is rather slow in isolated pulsars. Thus, according to Narayan & Ostriker (1990) the decay timescale is about 20 Myr. Bhattacharya et al. (1992) inferred even a longer decay time \((\geq 30 – 100\text{ Myr})\) from the same observational data. Nevertheless, even for such a slow decay the rate of Joule heating (if the decay is caused by ohmic dissipation) may be as large as \(10^{28} – 10^{30}\) ergs s\(^{-1}\). Clearly, Joule heating cannot substantially change the early thermal evolution when the neutron star is relatively hot and its luminosity exceeds this value. However, the ohmic dissipation produces enough heat to completely change the thermal history of old neutron stars. Assuming that all heat released due to dissipation of the magnetic field is transferred to the surface (that is true during the photon cooling era) and emitted with the blackbody spectrum, one can obtain an estimate of the surface temperature \(T_s\) required to maintain the neutron star under the thermal equilibrium. This temperature may be as high as \(3 \times 10^5 – 10^6\) K, and the neutron star can maintain this temperature for a long time, comparable with the decay time of the magnetic field, \(\sim 30 – 100\text{ Myr}\), whereas \(T_s\) has to fall down to the value below \(10^5\) K after \(\sim 1 – 3\) Myr in accordance with the so-called standard cooling scenario. Therefore, an observational and theoretical study of the late thermal evolution of neutron stars may be a powerful diagnostic of their magnetic fields and can provide important information about the magnetic configuration and mechanisms of its decay.

Detection of the thermal radiation from old and close neutron stars with the surface temperature \(\leq 10^5\) K has become possible only in recent years. Thus, Becker & Trümper (1997) detected several middle-aged and old neutron stars in the soft X-ray band. The Hubble Space Telescope also detected the optical and UV thermal emission from few relatively old pulsars (Pavlov, Stringfellow, & Cordova 1996; Mignani, Caraveo, & Bignami 1997). The corresponding surface temperatures turn out surprisingly high compared to predictions of the standard cooling model. Such high temperatures can be understood only if some mechanisms of additional heating operate in relatively old neutron stars.

One possible source of heating can be caused by the frictional interaction of neutron superfluid with the normal matter in the inner crust (Shibazaki & Lamb 1989; Umeda

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et al. 1993). If the inner crust of neutron stars contains superfluid rotating faster than the rest of the star, the differential rotation causes the frictional heat generation. The authors found that the rate of frictional heat generation does not depend on specific models for the superfluid-crust interaction. Note that the frictional heating is independent of the magnetic field, whereas the Joule heating is strongly sensitive to the field strength. This difference provides hope that it may be possible to discriminate between these heating mechanisms from observational data.

In the present paper, we consider the thermal evolution of a neutron star assuming the crustal origin of its magnetic field. The models with a crustal magnetic field turn out to be quite suitable to account for a wide variety of observational data on the magnetic and spin evolution of both isolated and entering binary systems neutron stars (Urpin & Konenkov 1997; Urpin, Geppert, & Konenkov 1998). We calculate the rate of Joule heating caused by dissipation of currents in the neutron star crust. Incorporating the expression for Joule heating into the numerical codes of thermal evolution, we examine the effect of this heating on the cooling history. In § 2 the physical model adopted for our calculations is described. The results of calculations of the thermal evolution with Joule heating are presented in § 3. In § 4 we briefly summarize the results of our study.

2. BASIC EQUATIONS

We assume that the magnetic field has been generated in the neutron star crust by some unspecified mechanism during or shortly after neutron star formation. The evolution of such a field is controlled by the conductive properties of the crust. Shortly after collapse the main fraction of the crustal material solidifies, and the evolution of the crustal field is governed by the induction equation without the convective term,

$$\frac{\partial B}{\partial t} = -\frac{c^2}{4\pi} \mathbf{V} \times \left( \frac{1}{\sigma} \mathbf{V} \times \mathbf{B} \right),$$

where $\sigma$ is the conductivity. We restrict our consideration to a dipolar field that can be described by the vector potential $A = (0, 0, A_r)$, $A_r = S(r, t) \sin \theta/r$, where $r$ and $\theta$ are the spherical radius and polar angle, respectively. Then the function $S(r, t)$ obeys the equation (see, e.g., Sang & Chantmugam 1987)

$$\frac{\partial^2 S}{\partial r^2} - \frac{2S}{r^2} = \frac{4\pi \sigma}{c^2} \frac{\partial S}{\partial t},$$

with the boundary condition

$$\frac{\partial S}{\partial r} + \frac{S}{R} = 0$$

at the stellar surface $r = R$. For a field confined to the crust, $S(r, t)$ should vanish in the deep layers. The field components in the interior of the star are

$$B_r = \frac{2S}{r^2} \cos \theta, \quad B_\theta = -\frac{\sin \theta}{r} \frac{\partial S}{\partial r}.$$  

The $\phi$ component of the electric current maintaining the dipolar magnetic configuration is given by

$$j_\phi = -\frac{c}{4\pi} \frac{\sin \theta}{r} \left( \frac{\partial^2 S}{\partial r^2} - \frac{2S}{r^2} \right).$$

Then for the rate of Joule heating one finds

$$\dot{q} = \frac{c^2}{16\pi^2 \sigma} \sin^2 \theta \left( \frac{\partial^2 S}{\partial r^2} - \frac{2S}{r^2} \right)^2.$$  

For the sake of simplicity, we will neglect nonsphericity in cooling calculations. Therefore, instead of equation (6), we will use the polar-averaged expression for Joule heating,

$$\dot{q} = \frac{c^2}{24\pi^2 r^2} \left( \frac{\partial^2 S}{\partial r^2} - \frac{2S}{r^2} \right)^2.$$  

It is convenient to normalize the function $S(r, t)$ to its initial value at the surface, $S(R, 0)$, which in turn can be related to the initial field strength at the magnetic equator $B_i$ by $S(R, 0) = R^2 B_i$. Finally, we obtain the expression for the rate of Joule heating in the form

$$\dot{q} = \frac{c^2 R^4 B_i^2}{24\pi^2 r^2} \left( \frac{\partial^2 S}{\partial r^2} - \frac{2S}{r^2} \right)^2,$$

where $S(r, t) = S(r, t)/S(R, 0)$.

The evolution of the magnetic field, as well as the rate of heat production, is strongly sensitive to the conductivity. In the crust, the conductivity is determined by electron scattering with phonons and lattice impurities; which mechanism dominates depends on the density $\rho$ and the temperature $T$. Electron-phonon scattering gives the main contribution to the conductivity at a relatively low density. For this mechanism, $\sigma \approx T^{-1}$ when $T$ is above the Debye temperature and $\sigma \approx T^{-2}$ for lower $T$. Electron-impurity scattering becomes more important with increasing density and decreasing $T$. When it dominates, $\sigma$ is nearly independent of $T$, and its magnitude depends on the impurity parameter

$$\xi = \frac{1}{n_i Z_a^2} \sum_i n_i (Z - Z_a)^2,$$

where $n_i$ and $Z$ are the number density and charge number of the dominant background ion species, respectively, and $n_a$ is the number density of an interloper species of charge $Z_a$; the summation is over all species of impurities. In our calculations, we use the numerical data on the phonon conductivity obtained by Itoh, Hayashi, & Kohyama (1993) and the analytical expression for the impurity conductivity derived by Yakovlev & Urpin (1980).

Our cooling calculations, which take into account the additional heating caused by the ohmic dissipation, have been done within the isothermal approximation that follows from assuming constant temperature (corrected by the redshift factor) in the interior of the star ($\rho > 10^{10} \text{ g cm}^{-3}$) and a temperature drop at the surface given by the atmosphere model that relates the interior temperature with the effective temperature (outer boundary condition) and allows calculation of the photon luminosity of the star. Although the calculations presented here correspond to the isothermal approximation, we have checked for isothermality running some models where this restriction is not enforced. At the time when Joule heating becomes important, the thermal conductivity of the crust is so high that the isothermal condition is very well satisfied and we do not observe significant differences in the thermal evolution with respect to the isothermal approximation.

We use the atmosphere models calculated by Van Riper (1988) to impose the outer boundary condition to the
cooling calculations. These atmosphere models are obtained by solving the hydrostatic and radiative equilibrium equations for a pure $^{56}$Fe composition (see Van Riper 1988 for details).

Calculations presented here are based on the so-called standard cooling scenarios, which correspond to a star with standard neutrino emissivities. We consider the thermal history for 1.4 $M_\odot$ models constructed with the equations of state of Friedman & Pandharipande (1981, hereafter FP) and Pandharipande & Smith (1975, hereafter PS). The PS model is representative of stiff equations of state with a low central density and a massive crust; the FP model represents intermediate equations of state. The stiffer the equation of state, the larger the radius and crustal thickness for a given neutron star mass. For the FP and PS models, the radii are 10.61 km and 15.98 km, respectively; the corresponding crustal thicknesses are $\approx 980$ and 4200 m; the crust bottom is located at the density $2 \times 10^{14}$ g cm$^{-3}$. Our choice is compelled by the fact that only models with the equation of state stiffer than that of Friedman & Pandharipande (1981) seem to be suitable to account for the available observational data on the magnetic evolution of pulsars (Urpin & Konenkov 1997).

3. NUMERICAL RESULTS

In our calculations, we assume the initial magnetic field to be confined to the outer layers of the crust with density $\rho \leq \rho_0$. The calculations have been performed for a wide range of $\rho_0$, $10^{14} \leq \rho_0 \leq 10^{12}$ g cm$^{-3}$. In the present work, we choose the initial function $s(r, 0)$ in the form

$$s(r, 0) = (1 - r^2/r_0^2)/(1 - R^2/r_0^2), \quad r \geq r_0,$$

$$s(r, 0) = 0, \quad r < r_0,$$

where $r_0$ is the boundary radius of the region originally occupied by the magnetic field, $\rho_0 = \rho(r_0)$. Note that the field decay and Joule heating are sensitive to the initial depth penetrated by the field and, hence, to the value $\rho_0$. However, both these quantities are much less affected by the particular form of the original field distribution.

The impurity parameter, $\xi$, is taken within the range $0.1 \leq \xi \leq 0.001$, and it is assumed to be constant throughout the crust. This parameter plays a key role in any model of the magnetic evolution of neutron stars, but unfortunately there are no reliable theoretical estimates of $\xi$. Calculations of the impurity charge and concentration in the crust meet troubles because of large uncertainties in the nonequilibrium processes during the very early evolution of a neutron star. Flowers & Ruderman (1977) made an attempt to estimate the final crustal composition taking into account slow neutrino reactions at $T < 10^{10}$ K (but above the melting point). They calculated small deviations from the equilibrium composition predicted by energy-minimization criteria and estimated $\xi \approx 0.004$. However, this estimate contains the binding energies of nuclei in the exponential factor; thus, the obtained value is rather uncertain. The properties of the crust can also be influenced by accretion during the early phase after the supernova explosion (see, e.g., Chevalier 1989). During this phase, the unbind fraction of matter falls back onto the neutron star surface and the total amount of accreted mass may be as large as 0.1 $M_\odot$. Evidently, this material can substantially change the crustal composition and increase the impurity parameter. Besides, $\xi$ can generally be nonuniform within the crust (De Blasio 1998) because mixing between the layers with different composition may be an important mechanism of the impurity production. Note also that calculations of the magnetic evolution of neutron stars with the crustal magnetic field (Urpin & Konenkov 1997) give a better fit to observational data if $\xi \sim 0.1$–0.01.

Figure 1 plots the evolution of the rate of Joule heating integrated over the neutron star volume,

$$Q = 4\pi \int_0^R \hat{q} r^2 dr,$$

for the FP and PS models. Calculations have been performed for several values of $\rho_0$ and $\xi$. At $t > 1$ Myr, the efficiency of Joule heating at given $\rho_0$ turns out to be essentially different for the FP and PS models with a much lower rate of heating for the FP model. This difference is evident because the thickness of the crust is much smaller for the FP model and, hence, the field strength decreases faster. Since the rate of Joule heating is proportional to $\hat{j}^2$, it is much greater during the late evolution for the PS model that experiences a lower field decay. The rate of heating is very sensitive to the initial depth penetrated by the field. If the field is initially confined to the layers with density $\rho < 10^{14}$ g cm$^{-3}$ for the FP model and $\rho < 10^{12}$ g cm$^{-3}$ for the PS model then, after $\sim$1 Myr, the rate of heating is likely too low to heat the neutron star to a sufficiently high surface temperature even if the initial magnetic field is of the order of the maximal field observed in neutron stars, $\sim 4 \times 10^{13}$ G. Note that at $t < 1$ Myr the effect of Joule heating on the thermal evolution is negligible (see below). The rate of heating is also sensitive to the impurity parameter $\xi$. For a given $\rho_0$ and $B_c$, $Q$ is initially higher for larger values of $\xi$ since the conductivity is lower for such $\xi$. However, a lower conductivity leads to a faster decrease of the field strength and, hence, $Q$. At some age (which generally depends on $\rho_0$ and $\xi$), the heat production becomes larger for a smaller $\xi$. Note that the ohmic dissipation can maintain the rate of heating at approximately the same level during extremely long time. Thus, for a very pure crust with $\xi = 0.001$, the neutron star may have a practically constant surface temperature during $\approx 1000$ Myr after the initial cooling stage ($t < 1$–3 Myr). For more polluted crust with $\xi = 0.01$, this phase of almost constant surface temperature can last $\sim 100$ Myr.

Figure 2 shows the thermal evolution of the neutron star with Joule heating and with the FP equation of state. We plot the cooling curves only for the age $t > 2 \times 10^6$ yr, since the earlier evolution is not affected practically by Joule heating. For a comparison, the cooling history of a nonmagnetized neutron star is also shown. All the cooling curves presented in this paper are obtained considering a nonmagnetized atmosphere. This is done so for the sake of simplicity in the comparison of the different models. Had we used magnetic atmospheres we would have obtained significant differences in the evolution of the surface temperature with respect to the nonmagnetized atmospheres only before the Joule heating drives the evolution but not after this time, as we will explain below.

It turns out that Joule heating may have an appreciable influence on the thermal evolution of the FP model only if the magnetic field initially occupies a significant fraction of the crust volume and if the field is initially very strong. Therefore, calculations presented here have been performed for $B_c \geq 5 \times 10^{12}$ G and $\rho_0 = 10^{13}$ and $10^{14}$ g cm$^{-3}$. Note
that the value \( \rho_0 = 10^{14} \text{ g cm}^{-3} \) corresponds to a depth from the surface of \( \approx 660 \text{ m} \); thus, about 60% of the crust volume has to be occupied initially by the field. Except for a short initial phase (\( \sim 3-10 \text{ Myr} \)) when the surface temperature is high, the effect of Joule heating on neutron star cooling turns out to be surprisingly simple: approximately all heat released due to the field dissipation has to be emitted from the surface; thus, the surface temperature \( T_s \) obeys with a high accuracy the equation
\[
\dot{Q} \approx 4\pi R^2 \sigma_{SB} T_s^4,
\]
where \( \sigma_{SB} \) is the Stefan-Boltzmann constant. The heat flux to the interior is negligible for all considered models and, therefore, the luminosity of old neutron stars has to be completely determined by the field strength and geometry and the conductive properties of the crust. Note that equation (9) is also valid for the PS model (see below).

If the initial field of a neutron star is of the order of the maximal pulsar field, \( B_e \sim (3-4) \times 10^{13} \text{ G} \), Joule heating can maintain a relatively high surface temperature \( \sim 5 \times 10^4-10^5 \text{ K} \) in relatively old neutron stars with \( t \geq 10 \text{ Myr} \). Evidently, this temperature is well above the surface temperature predicted by the standard cooling models without additional heating mechanisms. For the FP model, the neutron star can stay in such a “warm” state for a rather long time, \( \sim 30-60 \text{ Myr} \). The efficiency of heating is very sensitive to \( \rho_0 \); if the field is initially anchored in the layers with \( \rho \leq 10^{13} \text{ g cm}^{-3} \) the surface temperature at \( t > 10 \text{ Myr} \) turns out to be lower than \( 3 \times 10^4 \text{ K} \) even for the maximal pulsar field.

In Figure 3, we plot the thermal evolution of the PS model with and without Joule heating. The cooling curves are shown only for \( t > 2 \text{ Myr} \) since for the earlier age the influence of Joule heating is unimportant. As in the case of the FP model, at the late evolutionary stage (\( t > 10 \text{ Myr} \)) the surface temperature of the PS model is determined by balancing the Joule heating with the photon luminosity (eq. [9]). However, for the PS model the effect of additional

**Fig. 1.**—Time dependence of the rate of Joule heating integrated over the star volume and normalized to \( B_{e 13} = B_e /10^{13} \text{ G} \). Numbers near the curves indicate the logarithm of \( \rho_0 \). Different types of lines correspond to different values of \( \zeta : 0.1 \) (solid line), 0.01 (dashed line), 0.001 (dotted line).

**Fig. 2.**—Thermal evolution of the FP model with and without Joule heating. Curve 1: \( B_e = 1.5 \times 10^{13} \text{ G}, \rho_0 = 10^{14} \text{ g cm}^{-3}, \zeta = 0.1 \). Curve 2: \( B_e = 1.5 \times 10^{13} \text{ G}, \rho_0 = 10^{13} \text{ g cm}^{-3}, \zeta = 0.1 \). Curve 3: \( B_e = 1.5 \times 10^{13} \text{ G}, \rho_0 = 10^{14} \text{ g cm}^{-3}, \zeta = 0.01 \). Curve 4: \( B_e = 5 \times 10^{12} \text{ G}, \rho_0 = 10^{14} \text{ g cm}^{-3}, \zeta = 0.1 \). Curve 5: without Joule heating.
heating is much more pronounced because the field decay is substantially slower and, after 10 Myr of evolution, the field is stronger for this model. The surface temperature caused by Joule heating may be as high as \(3 \times 10^4 - 10^5\) K even if the field occupied initially a relatively small fraction of the crust volume. It turns out that \(T_s\) is strongly sensitive to all parameters determining the magnetic evolution \((\xi, \rho_0, B_0)\).

At \(t > 10\) Myr, a relatively high temperature \((T_s \geq 5 \times 10^4\) K) can only be reached for strongly magnetized neutron stars with the initial field \(B_0 \geq 5 \times 10^{12}\) G. The surface temperature is rather low if the magnetic field is initially confined to the layers with a small density, \(\rho \leq 10^{12} \text{g cm}^{-3}\). This is because the crustal field anchored initially in not very deep layers experiences a fast decay during the very early evolutionary stage \((t < 10^5\) yr) when the neutron star is hot and the crustal conductivity is low. For such initial magnetic configurations, the field strength at \(t > 10\) Myr is too weak to produce an appreciable Joule heating.

The most remarkable point is that all considered models with Joule heating can maintain a sufficiently high temperature for an extremely long time. Thus, our calculations show that for the “polluted” crust \((\xi = 0.1)\) \(T_s\) decreases only by a factor \(\sim 3\) when \(t\) runs from 10 to 100 Myr. For \(\xi = 0.01\), the temperature is practically unchanged during the same period. Evidently (see eq. [9]), the characteristic cooling time in our model is determined by the ohmic decay time of the magnetic field and, therefore, should be very long.

Note that a simple estimate of the surface temperature caused by Joule heating can be obtained directly from equation (9). The rate of heat production is equal to the rate of decrease of the magnetic energy. In the main fraction of the crust volume (except surface layers and a region near the magnetic pole), the \(\theta\) component of the crustal field is stronger than the radial one, which in its turn is of the order of the current surface field, \(B(t)\). We have \(B_\theta \sim B(t)(R/\ell) \sim B(t)(R/\ell)\) (see eq. [4]) where \(\ell\) is the radial length scale of the magnetic field; this length scale depends on time, since the field diffuses inward. Therefore, the energy of the crustal field can be estimated as \(E_m \sim 4\pi R^2(B_\ell^2/8\pi)\) and, correspondingly, \(Q \sim E_m/t\). Substituting this expression into equation (9), we obtain the estimate of the surface temperature,

\[
T_s \sim \frac{R^2 B^2(t)^{1/4}}{8\pi \sigma_{\text{SS}} t},
\]

or

\[
T_s \sim 4 \times 10^4 B_1^{-1/2} R_{10}^{1/2} t_s^{-1/4} \ell_5^{-1/4} K,
\]

where \(B_1(t) = B(t)/10^{12}\) G, \(R_{10} = R/10^6\) cm, \(\ell_5 = \ell/10^5\) cm, and \(t_s = t/10^8\) yr. In this equation, both the current field strength \(B(t)\) and the depth penetrated by the field \(\ell\) depend generally on the impurity parameter \(\xi\), and these dependences are rather complex because of a nonuniform chemical composition of the crust. Nevertheless, sometimes the estimate (eq. [10]) may be useful because the dependence of \(T_s\) on \(\ell\) is weak, and \(\ell\) varies within a relatively narrow range. For example, for the models presented here, \(\ell_5\) ranges from 0.5 to 1 for the FP model and from 1 to 4 for the PS model. Of course, this simplified estimation is only valid soon after the Joule heating becomes important, \(t \sim 10–30\) Myr, until the magnetic field reaches the crust-core boundary \((t \sim 1000\) Myr for the PS model).

Obviously, in the suggested mechanism, the rate of Joule heating and the surface temperature depend strongly on the current magnetic field strength at the equator \(B\). In Figure 4, we plot the dependence of the surface temperature on \(B\) for the PS neutron star model. The chosen range of the field strength corresponds to the age from \(\sim 3\) Myr to \(\sim 1000\) Myr (see Fig. 3). Note that the magnetic field strength, plotted in Figure 4, is different from what is usually calculated by use of observational data on the pulsar period \(P\) and its derivative \(\dot{P}\), and assuming magnetodipole braking. The standard estimate (Ostriker & Gunn 1969) gives the field strength at the magnetic equator,

\[
B_{\text{obs}} = (3Ic^3 \dot{P})/8\pi^2 R^2),
\]

where \(I\) is the moment of inertia. However, this equation gives an estimate of the equatorial field produced by the component of the magnetic dipole perpendicular to the spin axis because the parallel component does not contribute to braking. The rate of Joule heating is determined by the true magnetic field produced by both components. Therefore, the equatorial field strength entering our calculations is by a factor \(1/\sin \alpha\) larger than the observable pulsar magnetic

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**Fig. 3**—Thermal evolution of the PS model with and without Joule heating. Curve 1: \(B_0 = 1.5 \times 10^{13}\) G, \(\rho_0 = 10^{13} \text{g cm}^{-3}\), \(\xi = 0.1\). Curve 2: \(B_0 = 1.5 \times 10^{13}\) G, \(\rho_0 = 5 \times 10^{12} \text{g cm}^{-3}\), \(\xi = 0.1\). Curve 3: \(B_0 = 1.5 \times 10^{13}\) G, \(\rho_0 = 10^{13} \text{g cm}^{-3}\), \(\xi = 0.01\). Curve 4: \(B_0 = 5 \times 10^{12}\) G, \(\rho_0 = 10^{13} \text{g cm}^{-3}\), \(\xi = 0.1\). Curve 5: without Joule heating.

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**Fig. 4**—Dependence of the surface temperature \(T_s\) on the current equatorial magnetic field \(B\) for the PS model. Numbers near the curves correspond to the same values of parameters as in Fig. 3.
field $B_{\text{obs}}$, where $\alpha$ is the angle between magnetic and spin axes.

To plot $T_s(B)$, we simply eliminate the $t$ dependence from the cooling curves $T_s(t)$ and the magnetic decay curves $B(t)$ calculated for the same values of initial parameters (for more details concerning magnetic decay curves see Urpin & Konenkov 1997). Obviously, $T_s$ decreases with the magnetic field decay. During the first several Myr while Joule heating is negligible, the magnetic evolution goes much slower than the thermal one because neutrino emissivity and photon luminosity make the characteristic cooling time much shorter than the characteristic time for ohmic decay. This phase of evolution (represented by nearly vertical pieces of curves) lasts while the total rate of Joule heating $\dot{Q}$ is smaller than the luminosity. At $B \lesssim (1-3) \times 10^{12}$ G (depending on the parameters) when Joule heating plays a dominating role in the thermal evolution, the characteristic cooling time becomes comparable with the decay time of the magnetic field as it follows from equation (9).

Figure 5 shows the dependence of the surface temperature on the spin-down age of neutron stars with the PS equation of state. Our knowledge of the field strength and its behavior with time comes mainly from radio pulsars with measured spin-down rates. For most pulsars, the true age is unknown and observations provide information only on the so-called spin-down age, $\tau = P/2\dot{P}$. Therefore, for a comparison with observational data, it is convenient to analyze the dependence of $T_s$ on $\tau$ rather than on $t$. Assuming magnetodipole braking and integrating equation (11), we can calculate the spin-down age as a function of time, $\tau = \tau(t)$. Eliminating $t$ from the couple of functions $T_s(t)$ and $\tau(t)$, we obtain the dependences $T_s(\tau)$ shown in Figure 5.

The $\tau$ dependence of cooling curves is qualitatively similar to their $t$ dependence. The only difference is a bit slower decrease of $T_s$ in terms of the spin-down age. This difference is clear because $\tau(t) > t$ for a decaying magnetic field. In Figure 5, we also plot the data for three middle-aged and old pulsars: B0823 + 26 ($\tau = 4.9$ Myr), B1929 + 10 ($\tau = 3$ Myr), and B0950 + 08 ($\tau = 17.4$ Myr). The surface temperatures of these pulsars are $\sim 1.6 \times 10^5$ (this estimate has been obtained from the luminosity given by Becker & Trümper 1997, assuming the blackbody spectrum), $(1-3) \times 10^5$ and $(7 \pm 1) \times 10^4$ K (Pavlov et al. 1996), respectively. Of course, these observational data are too poor to infer categorically, nevertheless it seems that the ohmic dissipation can produce enough heat to maintain the observed surface temperatures of middle-aged and old pulsars.

4. SUMMARY

We considered Joule heating caused by the decay of the crustal magnetic field in neutron stars. Calculations of the thermal evolution of neutron stars show that the heat released in the crust due to the field decay diffuses mainly outward; thus, practically all the Joule heat has to be radiated from the surface. Because of this, the surface temperature at the late evolutionary stage ($t > 10$ Myr) turns out to be independent of the atmosphere models and is determined by balancing between the rate of Joule heating integrated over the neutron star volume and the luminosity (see eq. [9]). Being independent of the atmosphere models, $T_s$ is, however, strongly dependent on parameters of the magnetic configuration and the conductive properties of the crust. Therefore, the observational study of the late thermal history of neutron stars could be a useful diagnostic of their internal magnetic fields and properties of the crust.

The decay of the crustal magnetic field can produce enough heat to maintain a sufficiently high surface temperature $\sim 3 \times 10^4-10^5$ K. Our calculations predict that Joule heating becomes important after a relatively short ($\sim 3-10$ Myr depending on the model) initial phase when the neutron star cools down to $T_s \sim 3 \times 10^4-10^5$ K. The further thermal evolution slows down substantially: a characteristic cooling time becomes comparable with the decay time of the magnetic field. Since the field decay in the crust is very slow, the neutron star can maintain a practically unchanged surface temperature for an extremely long time, $t \geq 100$ Myr.

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