Numerical study of space charge field inside Resistive Plate Chamber

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ABSTRACT: Resistive plate chamber (RPC) is one of the state-of-the-art particle detection technology for the High Energy Physics (HEP) experiments. The basic operating mechanism of an RPC involves ionization of gas due to the passage of charged particles followed by electron transport, avalanche, and subsequent electromagnetic induction on readout strips due to the movement of the electrons and ions. Especially during streamer mode of operation, the electric field applied to the RPC can get significantly modified due to the presence of large number of electrons and ions. In this study, we have worked on dominant issues related to the estimation of electric field due to the space charge arising out of the presence of electrons, ions within an RPC. For this purpose we have considered two approaches: representation of the space charge cloud as (a) a collection of ring charges, and (b) as a collection of line charges. The results from these different methods have been compared with results available in the literature.

KEYWORDS: Resistive-plate chambers; Detector modelling and simulations II (electric fields, charge transport, multiplication and induction, pulse formation, electron emission, etc); Gaseous imaging and tracking detectors; Charge induction; Avalanche-induced secondary effects

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1 Introduction

Resistive plate chamber (RPC)[1, 2] is a kind of gaseous detector to detect charged particles. It is made of two parallel resistive bakelite or glass plates. The space between the electrodes has been filled with a particular gas mixture and then a high-voltage (say ±5kV) was applied at each plate. When high energy charged particle pass through the RPC, it can knock out some primary electrons from gas molecules along its path. As an effect of the high electric field inside the gas gap of RPC, those primaries are accelerated towards the electrodes and produce secondaries by the ionization process. This process continues and develops an avalanche of numerous electrons and ions. An EM wave is generated due to the movement of electrons and ions, which induces a signal pulse in a pick-up strip placed on the detector [3]. So It is clear that to investigate the detector physics of RPC, we need to simulate that avalanche process precisely. We know that space charge plays a crucial role while an avalanche is developing. So for simulation of an avalanche, the electric field due to space charge needs dynamically.

In this paper, we intend to discuss three different methods (A, B, C) to calculate the electric field in the presence of the space charge. The method-A and C (see section 3.1,3.3) contains modeling of the space charge region as several co-centric charged rings of gradually increasing radius as in [4]. Again, a ring can be thought of as a collection of charged straight lines (see section 3.2) of equal length S, which is our method-B. Now two case needs to be discussed:
Case-1: the linear charge density ($\lambda$) of a ring is kept constant. The lines corresponding to a ring have been shared the equal amount of charge ($\lambda S$) to the method-B.

Case-2: the condition for the rings is remaining identical, as in case-1. However, the charges of lines have been calculated separately. We then compare the electric fields for these lines and rings in two cases.

2 Calculation of positions of electron and ion cluster

An avalanche has been simulated inside an RPC of dimension 30 cm $\times$ 30 and a 2 mm gas gap, from an electron created at the origin (0,0,0) using the Garfield++ simulation tool [5]. The gas mixture containing 97% of $C_2H_2F_4$, 2.5% of $i-C_4H_{10}$ and 0.5% of $SF_6$ has been selected. A uniform electric field of 50 kV/cm is applied perpendicularly to the parallel plates of the detector (which is considered here as z-direction) to perform this simulation. The operating pressure of the gas has been kept equal to one atmospheric pressure at temperature 293.15K. The Garfield++ can keep track of the drifting position and time of each primary and secondary electrons and ion generated during the avalanche. A table of position and corresponding time of those electrons and ions has been formed to calculate interpolated positions of electrons at a certain instant of time. The interpolated positions of space charge cloud at 18 ns, has been shown in figure 1a.

3 Calculation of the electric field due to the space charge distribution

3.1 Ring approximation (method A)

It is assumed that the avalanche charge distribution has a rotational symmetry about the z-axis as apparent from figure 1a. Along the z-direction, the gas gap $g$ can be divided into $N_z$ steps with the step size $\delta z = \frac{g}{N_z}$. The space charge region along the X-Y plane can also be divided into a number ($N_r$) of concentric charged rings centered at z-axis and of gradually increasing radius $r$. The size of the ring $\delta r = \frac{r_{max}}{N_r}$ and $\delta z$ have been chosen according to the transverse and longitudinal spread of the avalanche, e.g. $\delta r = 0.001$ cm and $r_{max} = 0.045$ cm. Now, starting from a height $z = \bar{z}$ ($\bar{z} < g$), if any charge is present then the electric field due to all $N_r$ rings has been calculated by doing numerical integration of the equation (3.1) and (3.3) and then the value of $z$ set to $z + \Delta z$. This process continues iteratively till $z_{max} = g$. In this method, the whole region of space charge can be covered to calculate the electric field. The calculated components of the field are summed to get the total electric field at any point.

3.1.1 Components of Electric field vector due to ring

The X,Y and Z components of electric field at any position (x,y,z) due to a ring of uniform charged density $\lambda$ (see figure 1c), centered at z-axis can be written as follows,

\begin{align}
E^A_x &= \frac{\lambda r}{4\pi \varepsilon_0} \int_0^{2\pi} \frac{(x-r \cos(\phi))}{\Delta} d\phi,
E^A_y &= \frac{\lambda r}{4\pi \varepsilon_0} \int_0^{2\pi} \frac{(x-r \sin(\phi))}{\Delta} d\phi,
E^A_z &= \frac{\lambda r}{4\pi \varepsilon_0} \int_0^{2\pi} \frac{(z-\bar{z})}{\Delta} d\phi
\end{align}

(3.1)
Figure 1: (a) The position of electron clouds at time 18 ns where the total number of electron is 510979. (b) picture of a ring of width $\delta r$ (c) Computation of electric field due to charged ring. (d) Components of the electric field due to line charge.

Where, $\Delta = [(x - r \cos(\phi))^2 + (y - r \sin(\phi))^2 + (z - \bar{z})]^2$, and $\bar{z}$ is the position of center of that ring along z-axis. If $Q_{ring}$ is total charge of that ring then $\lambda = \frac{Q_{ring}}{2\pi r}$.

3.2 Straight-line approximation with uniform charge density (method B)

A ring can be equally segmented into a number of straight-lines. If $r$ and $\delta r$ are the radius and thickness of that ring respectively, then the length of an arc of any segmented element of that ring is $S = r \delta \phi$, where $\delta \phi$ is the angle in radian, subtended to the center of the circle (see figure 1b). $S$ can be approximated as a straight-line of length $S$ and thickness $\delta r$, when $\delta \phi$ and is very small i.e. $S/r \leq 1$. It is discussed in section 3.1 that the space charge region can be divided into a number of rings. As an extension of this algorithm, those rings are also split into a number of straight lines, where $\delta \phi$ is chosen by the user. Therefore, the electric field can be calculated for each charged straight-line (for case-1 and case-2 in section 4) corresponding to a ring using the equations (3.2). Thus, after the formation of a ring by a number of straight lines the same iteration process discussed in section 3.1 can be followed to get electric fields for all charges. The components of fields of each charged line are added iteratively to get the total electric field. Thus we can reproduce the results of the electric field of charged rings using line charge approximation.
3.2.1 Components of Electric field vector due to a charged line

Let us consider a straight line of constant charged density \( \bar{\lambda} \) and of length \( S \), aligned parallel to the y-axis at \( x = \bar{r} \), and \( z = \bar{z} \). The X, Y, Z components of the electric field at any position (x,y,z) due to this straight line can be written as follows (see figure 1d).

\[
E_x^B = \frac{\bar{\lambda}(x-\bar{r})}{4\pi\varepsilon_0 P^2} \left[ \frac{(y+\bar{z})}{\sqrt{(y+\bar{z})^2 + p^2}} - \frac{(y-\bar{z})}{\sqrt{(y-\bar{z})^2 + p^2}} \right], \quad E_y^B = -\frac{\bar{\lambda}}{4\pi\varepsilon_0} \left[ \frac{1}{\sqrt{(y+\bar{z})^2 + p^2}} - \frac{1}{\sqrt{(y-\bar{z})^2 + p^2}} \right], \quad E_z^B = \frac{\bar{\lambda}(z-\bar{z})}{4\pi\varepsilon_0 P^2} \left[ \frac{(y+\bar{z})}{\sqrt{(y+\bar{z})^2 + p^2}} - \frac{(y-\bar{z})}{\sqrt{(y-\bar{z})^2 + p^2}} \right]
\]

(3.2)

Where, \( P = \sqrt{(z-\bar{z})^2 + (x-\bar{r})^2} \), and if \( Q_{st} \) is the total charge of this straight line then, \( \bar{\lambda} = \frac{Q_{st}}{S} \).

For a chosen value of \( \delta\phi \) it can be said that the total number of straight-line needs to form a ring is \( N_{st} = \frac{360}{\delta\phi} \). So \( \delta\phi \) is the minimum angle that has to rotate to reach from one segment to another nearest segment. Necessary coordinate transformations have been carried out to evaluate electric field in a consistent frame of reference.

3.3 Method available in literature (method C)

The equation of the electric field due to a ring of uniform charged density \( \lambda \) and radius a centered at a z-axis, can also be found in [6](v. 1 pp. 176), which is represented in cylindrical co-ordinate system as follows (see figure 1c),

\[
E_{ring}^C(\rho, z, a) = \frac{\lambda a}{\pi\varepsilon_0} \left[ \frac{1}{2\bar{r}_1 \rho} \left( K_1(u) - \frac{(a^2 - \rho^2 + z^2)K_2(u)}{\bar{r}_1^2(1-u^2)} \right) \hat{\rho} + \frac{zK_2(u)}{\bar{r}_1^2(1-u^2)} \hat{z} \right] = E_r^C \hat{\rho} + E_z^C \hat{z}
\]

(3.3)

Where \( E_r^C, E_z^C \) are the radial and z- components of electric field and \( K_1(u) \) and \( K_2(u) \) are the complete elliptic integrals of first and second kinds.

The components of fields calculated in Cartesian co-ordinates (for method A and B) is being converted into cylindrical co-ordinates by using “Jacobi transformation” to compare with the results of method C. The required Jacobi matrix for this transformations is given below,

\[
\begin{pmatrix}
E_r^i \\
E_\phi^i \\
E_z^i
\end{pmatrix} = \begin{pmatrix}
\cos(\phi) & \sin(\phi) & 0 \\
-\sin(\phi) & \cos(\phi) & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
E_r^i \\
E_\phi^i \\
E_z^i
\end{pmatrix}, \quad \phi = \tan^{-1}(\frac{y}{x})
\]

(3.4)

\( E_r^i, E_\phi^i, E_z^i \) are the radial, \( \phi \) and z directional components of electric fields of any charged ring or line. Where, \( i = A, B \) for method-A and method-B. These cylindrical form of the components for different methods can be represented together as \( E_r^{A,B,C}, E_\phi^{A,B}, E_z^{A,B,C} \).
4 Results and discussions

The integrations of equations (3.1) and (3.3) have been solved numerically by using standard GSL-Integrator from GSL-library available in root 6.18/04 [7]. The absolute values of calculated electric field components are set to zero when it goes below a minimum number $\epsilon = 10^{-8}$, because at this range the field due to the space charge is negligible in comparison to the applied field. The results can be divided into two cases:

**case-1:** The charge density $\lambda$ has been considered constant in equations (3.1) and (3.3) of methods A,C. In method-B the charge $Q_{\text{ring}}$ of a ring is equally shared between segmented lines from that ring, which is of amount $Q_{\text{st}} = \lambda S = \frac{Q_{\text{ring}} S}{2\pi r}$ for a line.

**case-2:** In this case, the conditions for equations (3.1) and (3.3) remain the same as in the above case-1. However, in the actual scenario the angular distribution of the total $Q_{\text{ring}}$ charge may not be uniform over the ring. Therefore, the amount of charge $Q_{\text{st}}$ will be different for different $N_{\phi}$ line of that ring. So charges are calculated separately for each line. Distribution of the charges reside on each line at different angles for all radius and height, have been shown in figure 4.

4.1 Results of case-1

It is well known to us that the electric field component $E_z$ is dominating over $E_r$ and $E_\phi$ on the z-axis of a uniformly charged ring. Because due to the axial symmetry, the components $E_r$ and $E_\phi$ cancel out each other and become zero. It can also be verified from the figures 2a, 2b and 2c, where the components of electric field $E_r^{A,B,C}, E_\phi^{A,B,C}, E_z^{A,B,C}$ have been plotted for three different methods A,B,C. Again, from the same figure 2 it is clear that the ratios $E_z^C/E_z^A = E_z^C/E_z^B = 1$ on the z-axis. However, The values of $E_r^{A,B,C}, E_\phi^{A,B,C}$ are always zero on the z-axis, so the calculation of their ratios is not possible. Hence, the ratio plot is not shown in the figures 2b and 2c.
4.2 Results of case-2

The magnitudes of $E_z$ components for three methods A,B,C ($E_{z}^{A,B,C}$) are the same which can be verified from the figure 3a. But there are discrepancies between the radial and $\phi$ components calculated in method-B ($E_{r}^{B}$, $E_{\phi}^{B}$) and method-A,C ( $E_{r}^{A,C}$, $E_{\phi}^{A,C}$) which is shown in the figures 3b and 3c. The components $E_{r}^{A,C}$, $E_{\phi}^{A,C}$ still gives the same result zero over z-axis as in case-1 for method-A and C. However, the components $E_{r}^{B}$, $E_{\phi}^{B}$ is showing a non zero value. Especially near the charge distribution, the value is much higher than zero. These discrepancies can be explained from the angular distribution of charges depicted in figure 4. It is clear from the same figure 4 that the nature of this angular distribution is not uniform; instead, it is observed from the computed avalanche data that most of the charges are like to reside within the angular range from 0 to 150 degrees.
5 Conclusions

Initially, it was assumed that the nature of the electron cloud has some axial symmetry about the z-axis. So due to this symmetry, one can easily neglect the $E_\phi$ component. But from the in-depth analysis of the angular charge distribution, it is found that the distribution may not be uniform, so the component $E_\phi$ plays a significant role while avalanche is developing. Hence, it can’t be ignored anymore. But it is also true that in this case, the avalanche generated here by using a single primary electron so this non-uniformity of angular charge distribution may come from the biasing in the diffusion of electrons. However, in an actual event, the avalanche can be formed from several primaries, so for each primary electron, the biasing may be in different directions. So the results for that case need to be found, which is also our future interest.

The merit of using straight line-approximation is that it produces similar results as uniformly charged rings as well as it can be easily used when the charged density is nonuniform over the ring. Again the most remarkable feature of the same is the components of field equations do not contain any elliptical integrals. So we do not need to worry about the numerical integrations.

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