Will at least one of the Higgs bosons of the next-to-minimal supersymmetric extension of the Standard Model be observable at LEP2 or the LHC?

John F. Gunion  
_Davis Institute for High Energy Physics, University of California, Davis, California 95616_  
Howard E. Haber  
_Santa Cruz Institute for Particle Physics, University of California, Santa Cruz, California 95064_  
Takeo Moroi  
_Lawrence Berkeley National Laboratory, Berkeley, California 94720_

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Will at least one of the Higgs bosons of the next-to-minimal supersymmetric extension of the Standard Model be observable at LEP2 or the LHC?*

John F. Gunion

Davis Institute for High Energy Physics, University of California, Davis, California 95616

Howard E. Haber

Santa Cruz Institute for Particle Physics, University of California, Santa Cruz, California 95064

Takeo Moroi

Lawrence Berkeley National Laboratory, Berkeley, California 94720

ABSTRACT

We demonstrate that there are regions of parameter space in the next-to-minimal (i.e., two-Higgs-doublet, one-Higgs-singlet superfield) supersymmetric extension of the SM for which none of the Higgs bosons are observable either at LEP2 with $\sqrt{s} = 192$ GeV and an integrated luminosity of $L = 1000 \text{ pb}^{-1}$ or at the LHC with $L = 600 \text{ fb}^{-1}$.

I. Introduction

It has been demonstrated that detection of at least one of the Higgs bosons of the minimal supersymmetric standard model (MSSM) is possible either at LEP2 or at the LHC throughout all of the standard $(m_A, \tan \beta)$ parameter space (for a recent review, see Ref. [1]). Here, we reconsider this issue in the context of the next-to-minimal supersymmetric standard model (NMSSM) [2] in which there is one Higgs singlet superfield in addition to the two Higgs doublet superfields of the MSSM. (The NMSSM Higgs sector is taken to be CP-conserving.) We will demonstrate that there are regions of parameter space for which none of the NMSSM Higgs bosons can be detected at either LEP2 or the LHC. This result should be contrasted with the NLC no-lose theorem [3], according to which at least one of the CP-even Higgs bosons of the NMSSM will be observable in the $Z^* \rightarrow Zh$ production mode. However, we do find that the parameter regions for which Higgs boson observability is not possible at LEP2 or the LHC represent a small percentage of the total possible parameter space.

Many detection modes are involved in establishing the LHC no-lose theorem for the MSSM. A more than adequate set is: 1) $Z^* \rightarrow Zh$ at LEP2; 2) $Z^* \rightarrow ha$ at LEP2; 3) $gg \rightarrow h \rightarrow \gamma \gamma$ at LHC; 4) $gg \rightarrow h \rightarrow ZZ^*$ or $ZZ \rightarrow 4\ell$ at LHC; 5) $t \rightarrow H^+ b$ at LHC; 6) $gg \rightarrow b\overline{b}h, b\overline{b}a \rightarrow b\overline{b}\tau^+ \tau^−$ at LHC; 7) $gg \rightarrow h, a \rightarrow \tau^+ \tau^−$ at LHC. Additional LHC modes that have been considered include: a) $a \rightarrow Zh$; b) $h \rightarrow aa$; c) $h_j \rightarrow h_i h_i$; d) $a, h \rightarrow t\overline{t}$. Because of the more complicated Higgs self interactions, b) and c) cannot be reliably computed in the NMSSM without additional assumptions. The Higgs mass values for which mode a) is kinematically allowed can be quite different than those relevant to the MSSM and thus there are uncertainties in translating ATLAS and CMS results for the MSSM into the present more general context. Finally, mode d) is currently of very uncertain status and might turn out to be either more effective or less effective than current estimates. Thus, to be conservative, we excluded from our considerations any choice of NMSSM parameters for which the modes a)-d) might be relevant. Even over this restricted region of parameter space, we shall demonstrate that NMSSM parameter choices can be found such that there are no observable Higgs signatures at either LEP2 or the LHC.

II. Parameters and Scanning Procedure

In order to specify a point in NMSSM parameter space, we have adopted the following procedure.

- Employ a basis in which only the first neutral Higgs field has a vev: $\langle \phi_1 \rangle = v = 246 \text{ GeV}$. In this basis, the $(11, 12, 21, 22)$ elements of the Higgs mass-squared matrix (denoted $M^2$ below) take the simple form

$$
\begin{pmatrix}
m_Z^2 + m_{Z\lambda}^2 s_{2\beta}^2 + \delta_{11} \\
m_{Z\lambda}^2 s_{2\beta} c_{2\beta} + \delta_{12} \\
m_p^2 + m_{Z\lambda}^2 s_{2\beta}^2 + \delta_{22}
\end{pmatrix}
$$

(1)
where \( \lambda \) appears in the superpotential in the term \( W \equiv \lambda H_1 H_2 N \), \( m^2_{\lambda} = \frac{1}{2} \lambda^2 v^2 - m^2_{\tilde{g}} \), and \( \delta_{11} (1,2,2) \) are the radiative corrections\(^2\) (which are independent of \( \lambda \) and \( m_{pp} \), but depend on \( \tan \beta \) and \( m_t \) — we take \( m_t = 175 \, \text{GeV} \)). We note that there are enough parameters in the NMSSM model superpotential and soft-supersymmetry-breaking terms that the \( M_{11,12,23,33} \) entries can have arbitrary values. (Specific Planck scale boundary conditions could restrict these latter \( \mathcal{M}^2 \) entries and thereby impose restrictions on the allowed parameter space beyond those described below; such boundary conditions will not be imposed here.)

- Pick a value for \( \tan \beta \) and a value for \( m_{h_1} \leq m_{h_1}^{\text{max}} \), where \( m_{h_1}^{\text{max}} = M_{11} (\lambda = \lambda_{\text{max}}) \). The crucial ingredient in limiting the scan is the upper limit of \( \lambda_{\text{max}} = 0.7 \)\(^3\) obtained by requiring that \( \lambda \) remain perturbative during evolution from scale \( m_Z \) to the Planck scale.

- Pick values for the angles \( -\pi/2 \leq \alpha_1 \leq +\pi/2, 0 \leq \alpha_2 \leq 2\pi \), and \( 0 \leq \alpha_3 \leq \pi/2 \) that appear in the matrix \( V \) which diagonalizes the CP-even Higgs mass-squared matrix via \( V^\dagger \mathcal{M}^2 V = \text{diag}(m_{h_1}^2, m_{h_2}^2, m_{h_3}^2) \):

\[
V = \begin{pmatrix}
  c_1 & -s_1 c_3 & -s_1 s_3 \\
  s_1 c_2 & c_1 c_2 c_3 + s_2 s_3 & c_1 c_2 s_3 + c_2 c_3 \\
  s_1 s_2 & c_1 s_2 c_3 + s_2 c_3 & c_1 s_2 s_3 - c_2 s_3
\end{pmatrix}
\]

(2)

where \( c_1 = \cos \alpha_1 \), and so forth. It is useful to note that

\[
m_{h_2}^2 \leq \frac{\left[ m_{h_1}^{\text{max}} \right]^2 - V_{11}^2 m_{h_1}^2}{1 - V_{11}^2}
\]

(3)

\[
m_{h_3}^2 \leq \frac{\left[ m_{h_1}^{\text{max}} \right]^2 - V_{11}^2 m_{h_3}^2 - V_{22}^2 m_{h_2}^2}{1 - V_{11}^2 - V_{12}^2}
\]

(4)

- Pick a value \( \lambda_{\text{min}} \leq \lambda \leq \lambda_{\text{max}} \), and compute

\[
m_{h_2}^2 = \frac{V_{13} M_{12} - V_{12} M_{13} - m_{h_3}^2}{v^2 (V_{12} V_{13} + V_{12} V_{13})},
\]

\[
m_{h_3}^2 = \frac{V_{12} M_{13} - V_{13} M_{12} - m_{h_3}^2}{v^2 (V_{12} V_{13} + V_{12} V_{13})},
\]

\[
m^2_{pp} = \sum_{i=1,2,3} V_{ii}^2 m_{h_i}^2 + m_{Z}^2 s_2^2 - \delta_{22},
\]

\[
m^2_{H^+} = m^2_{pp} - m^2_{Z} \lambda.
\]

The lower limit on \( \lambda \) is given by

\[
\lambda_{\text{min}}^2 v^2 = \frac{m_{h_2}^2 - \delta_{11} - m_{Z}^2}{s_2^2} + m_{Z}^2,
\]

(5)

which is obtained by noting that \( m_{h_1}^2 \approx M_{11}^2 \). If \( \lambda_{\text{min}} < 0 \) then use \( \lambda_{\text{min}} = 0 \). It is consistent to consider only those \( \alpha_1, \lambda \) values such that \( m_{h_2}^2 \geq m_{h_1}^2 \geq m_{h_3}^2 \). Further restrictions are imposed on the \( m_{h_2}^2 \) as follows. First, we require that \( m_{h_3} \leq 2m_{h_1} \), in which case the decays \( h_2 \rightarrow h_1 h_1, h_3 \rightarrow h_1 h_2, h_2 \rightarrow h_2 h_2 \) are all kinematically disallowed. (If kinematically allowed, such decays are model dependent and could be dominant; their experimental accessibility would have to be evaluated.) Second, we require that \( m_{h_3} \leq 2m_t \) so that the decays \( h_{1,2,3} \rightarrow t \bar{t} \) are forbidden.

- The CP-odd mass-squared matrix takes the form

\[
\mathcal{N}^2 = \begin{pmatrix}
  m^2_{pp} & \cdot & \cdot \\
  \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot
\end{pmatrix},
\]

(6)

where the unspecified entries may take on any value given the parameter freedom of the model. For simplicity, we assume that only one CP-odd scalar, the \( a \) (which must have \( m_a^2 \leq m_{pp}^2 \)), is possibly light and that the other is heavy and, therefore, unobservable. In principle, we could scan \( 0 \leq m_a \leq m_{pp} \). However, we impose three additional restrictions on \( m_a \) as follows. In order to avoid the presence of the model-dependent, possibly dominant \( h_{1,2,3} \rightarrow aa \) decays, we restrict the scan to \( m_a \geq m_{h_3}/2 \). In particular, this implies that no \( m_a \) scan is possible if \( m_{pp} \leq m_{h_3}/2 \). We also impose the restrictions: \( m_a \leq 2m_t \), so that \( a \rightarrow t \bar{t} \) decays are forbidden; and \( m_a \leq m_{Z} + m_{h_1} \), which implies that the model-dependent decays \( a \rightarrow Z h_{1,2,3} \) are absent.

We emphasize that there may be parameter choices, for which no Higgs bosons of the NMSSM are observable, that lie outside the restricted portion of parameter space that we search. Our goal here is not to fully delineate all problematical parameter choices, but rather to demonstrate the existence of parameters for which it is guaranteed that no NMSSM Higgs boson can be found without increased LEP2 energy and/or luminosity, or increased LHC luminosity or LHC detector improvements.

### III. Detection Modes

In order to assess the observability of the \( h_{1,2,3} \) and \( a \). Those required are:

\[
ZZh_i, WWh_i : \left[ \frac{g m_Z}{c_W}, \frac{g m_W}{c_W} \right] V_{ii}
\]

(7)

\[
Zh_1 a : \frac{g}{2c_W} V_{2i}
\]

(8)

\[
\tilde{t} h_i : \frac{g m_t}{2m_W} (V_{ii} + V_{2i} \cot \beta)
\]

(9)

\[
b \bar{b} h_i : \frac{g m_b}{2m_W} (V_{ii} - V_{2i} \tan \beta)
\]

(10)

\[
t \bar{t} a, b \bar{b} a : \frac{g m_t}{2m_W} \cot \beta, \frac{g m_b}{2m_W} \tan \beta
\]

(11)

As already noted, we do not search parameter regions in which the very model-dependent Higgs self-couplings would be needed.

Within the domain of parameter space that we search, we evaluate the potential of modes 1)-7) as follows. For the LEP2 modes 1) and 2), we require 30 and 50 events, respectively, for \( L = 1000 \, \text{pb}^{-1} \), before any cuts, branching ratios, or efficiency factors. For the LHC modes 3)-7), we require \( 5\sigma \) statistical significance for \( L = 600 \, \text{fb}^{-1} \). The individual mode treatments are as follows.
• For the $h_i \rightarrow \gamma\gamma$ and $h_i \rightarrow ZZ^*$, $ZZ \rightarrow 4\ell$ modes, 3) and 4), we compute the number of events as compared to predictions for the SM Higgs boson, and then compute the resulting statistical significance assuming scaling proportional to the signal event rate. The most optimistic SM Higgs statistical significances for the $\gamma\gamma$ and $4\ell$ channels as a function of Higgs mass are those from CMS, Fig. 4 ($\gamma\gamma$) and Fig. 8 ($ZZ^*$), and Tables 35 and 36 ($ZZ$) of Ref. [8]. We increase these $L = 100 \text{ fb}^{-1}$ statistical significances by a factor of $\sqrt{6}$ for $L = 600 \text{ fb}^{-1}$ and then apply the NMSSM corrections.

• For the $t \rightarrow H^+b$ detection mode 5) we employ the $L = 600 \text{ fb}^{-1}$ contours, Fig. 76, of Ref. [8]. We note that when $t \rightarrow H^+b$ is kinematically allowed, the $H^+ \rightarrow W^+ h_{1,2,3}$ decays are forbidden for the $m_{h_{1}}$ values we consider here. Thus, the $H^+$ decays are exactly as in the MSSM and the MSSM results can be employed ‘as is’ when the $5\sigma$ contour is specified as a function of $m_{H^+}$ and $\tan \beta$.

• For the $b\bar{b}h$ and $b\bar{b}a$ final states we refer to the $L = 100 \text{ fb}^{-1}$ statistical significances quoted for the MSSM model $b\bar{b}A^0$ process at $\tan \beta = 10$ in Table 34 of Ref. [8] and the input $B(A^0 \rightarrow \tau^+\tau^-)$ from Fig. 22 ($\tan \beta = 10$ results) of Ref. [8]. From these results we compute a standard statistical significance for $\tan \beta = 1, B(a \rightarrow \tau^+\tau^-) = 1$, and $L = 600 \text{ fb}^{-1}$. Statistical significances in the NMSSM model are obtained for the $h_1$ and $a$ by multiplying these standard statistical significances by the appropriate ($b\bar{b}h_1$)$^2$ enhancement factor or by ($b\bar{b}a$)$^2 = \tan^2 \beta$ and by the computed $\tau^+\tau^-$ branching ratio of the Higgs boson in question. Recall that we do not search parameter regions for which the $\tau^+\tau^-$ branching ratios would be uncertain due to Higgs pair decay channels being kinematically allowed.

• Finally, we assume that mode 7) is only relevant for the $a$ (as in the MSSM). However, we cannot directly use the discovery region shown for $L = 300 \text{ fb}^{-1}$ in Fig. 53 of Ref. [8] since $A^0 \rightarrow Zh^0$ decays deplete the $\tau^+\tau^-$ branching ratio for $m_{A^0} \sim 190 \text{ GeV}$. Thus, we use an optimistic limit for this mode’s $L = 600 \text{ fb}^{-1}$ region of viability: $\geq 5\sigma$ is assumed to be achieved in this mode for $\tan \beta \leq 4$ if $100 \leq m_a \leq 350 \text{ GeV}$.

If none of the Higgs bosons $h_{1,2,3}$, $a$ or $H^\pm$ are observable as defined above we declare a parameter point in our search to be a “point of unobservability” or a “bad point”.

IV. Results

We now summarize our results. We find that if $\tan \beta \lesssim 1.5$ then all parameter points that are included in our search are observable for $m_{h_1}$ values up to the maximum allowed ($m_{h_1}^\text{max} \sim 137 \text{ GeV}$ for $\lambda_{\text{max}} = 0.7$, after including radiative corrections). For such low $\tan \beta$, the LHC $\gamma\gamma$ and $4\ell$ modes allow detection if LEP2 does not. For high $\tan \beta \gtrsim 10$, the parameter regions where points of unobservability are found are also of very limited extent, disappearing as the $b\bar{b}h_{1,2,3}$ and/or $b\bar{b}a$ LHC modes allow detection where LEP2 does not. However, significant portions of searched parameter space contain points of unobservability for moderate $\tan \beta$ values. That such $\tan \beta$ values should be the most ‘dangerous’ can be anticipated from the MSSM results. It is well-known (see, for example, Ref. [9]) that there is a wedge of MSSM parameter space at moderate $\tan \beta$ and with $H^0$ and $A^0$ masses above about $200 \text{ GeV}$ for which the only observable MSSM Higgs boson is the light SM-like $h^0$, and that it can only be seen in the $\gamma\gamma$ mode at the LHC ($m_{h^0} + m_{Z, A^0} > \sqrt{s}$ at LEP2). By choosing $m_{h_{1}}$ and $m_a$ in the NMSSM so that $m_{h_{1}} + m_Z$ and $m_{h_{1}} + m_a$ are close to or above the $\sqrt{s}$ of LEP2, then, by analogy, at moderate $\tan \beta$ we would need to rely on the $h_{1,2,3} \rightarrow \gamma\gamma$ modes. However, in the NMSSM, parameter choices are possible for which all the $WWh_{1,2,3}$ couplings are reduced relative to SM strength. This reduction will suppress the $\gamma\gamma$ couplings coming from the $W$-boson loop. All the $h_i \rightarrow \gamma\gamma$ widths can be sufficiently smaller than the somewhat enhanced $b\bar{b}$ widths so that the $\gamma\gamma$ branching ratios are all no longer of useful size.

Figure 1: For $\tan \beta = 5$ and $m_{h_{1}} = 105 \text{ GeV}$, we display in three dimensional ($\alpha_1, \alpha_2, \alpha_3$) parameter space the parameter regions searched (which lie within the surfaces shown), and the regions therein for which the remaining model parameters can be chosen so that no Higgs boson is observable (interior to the surfaces shown).

To illustrate, we shall discuss results for $\tan \beta = 3$, $\tan \beta = 5$ and $\tan \beta = 10$ (for which $m_{h_{1}}^\text{max} \sim 124 \text{ GeV}$, $118 \text{ GeV}$ and $114 \text{ GeV}$, respectively) and $m_{h_{1}} = 105 \text{ GeV}$.

• In Fig. 4 we display for $\tan \beta = 5$ both the portions of ($\alpha_1, \alpha_2, \alpha_3$) parameter space that satisfy our search restrictions, and the regions (termed “regions of unobservability”) within the searched parameter space such that, for some choice of the remaining parameters ($\lambda$ and $m_a$), no
Higgs boson will be detected using any of the techniques discussed earlier. \[ ^3 \] Relatively large regions of unobservability within the searched parameter space are present.

- At \( \tan \beta = 3 \), a similar picture emerges. The search region that satisfies our criteria is nearly the same; the regions of unobservability lie mostly within those found for \( \tan \beta = 5 \), and are about 50% smaller.

- For \( \tan \beta = 10 \), the regions of unobservability comprise only a very small portion of those found for \( \tan \beta = 5 \). This reduction is due to the increased \( b \bar{b} \) couplings of the \( h_i \) and \( a \), which imply increased \( b \bar{b} h_i, b \bar{b} a \) production cross sections. As these cross sections become large, detection of at least one of the \( h_i, a \) in the \( b \bar{b} \tau^+ \tau^- \) final state becomes increasingly difficult to avoid. For values of \( \tan \beta \gtrsim 10 \), we find that one or more of the \( h_i, a \) should be observable regardless of location in \((\alpha_1, \alpha_2, \alpha_3, \lambda, m_a)\) parameter space (within the somewhat restricted search region that we explore).

Another perspective on the parameter space and the location of points of unobservability is provided in Fig. \( \circled{3} \). There, we display for \( \tan \beta = 5 \) and \( m_{h_1} = 105 \text{ GeV} \) the regions searched in the \((V_{11}^2, m_{h_2})\), \((V_{12}^2, V_{13}^2)\) and \((m_{h_3}, m_{h_2})\) parameter spaces, and the portion thereof in which the remaining model parameters can be chosen such that no Higgs boson is observable. The \((V_{11}^2, m_{h_2})\) plot shows that Higgs boson unobservability is possible for any value of \( V_{11}^2 \) and for all values of \( m_{h_2} \) up to the bound of Eq. \( \circled{3} \), so long as \( V_{11}^2 \lesssim 0.5 \). For \( V_{11}^2 \gtrsim 0.5 \), the region of \( m_{h_2} \) for which Higgs boson unobservability is possible does not include the highest \( m_{h_2} \) values.

The \((V_{11}^2, V_{12}^2)\) plot shows that unobservability is possible only if \( V_{11}^2 + V_{12}^2 \gtrsim 0.7 \), i.e. the \( ZZh_i \) coupling is reduced relative to SM strength by \( V_{12}^2 \lesssim 0.3 \), implying that \( h_3 \) is difficult to detect in the \( ZZ \rightarrow 4\ell \) mode. The \((m_{h_3}, m_{h_2})\) plot shows that unobservability is possible for almost all \( m_{h_3} \) values as long as \( m_{h_2} \lesssim 2m_Z \). For \( m_{h_2} \gtrsim 2m_Z \), the \( h_2 \) must be detected in the relatively weak \( h_2 \rightarrow ZZ^* \) or \( \gamma \gamma \) modes; both are typically somewhat suppressed at moderate (or large) \( \tan \beta \) by a \( gg h_2 \) coupling that is smaller than SM-strength and by an enhanced \( b \bar{b} \) decay width that diminishes the \( ZZ^*, \gamma \gamma \) branching fractions. Throughout the regions displayed in Fig. \( \circled{3} \) where choices for the remaining model parameters can make observation of any of the Higgs bosons impossible, there are other choices for the remaining parameters such that at least one Higgs boson is observable.

The mass \( m_{h_1} = 105 \text{ GeV} \) is typical of the ‘intermediate’ values that yield the largest regions of unobservability. If \( m_{h_1} \lesssim 85 \text{ GeV} \), then discovery of one of the \( h_i \) at LEP2 is almost certain. As \( m_{h_1} \rightarrow m_{h_1}^{\text{max}} \), then discovery of at least one Higgs boson at the LHC becomes possible over most of parameter space, as we now describe. As \( m_{h_1} \rightarrow m_{h_1}^{\text{max}} \),

\[ \text{Figure 2: For } \tan \beta = 5 \text{ and } m_{h_1} = 105 \text{ GeV}, \text{ we display the regions of the } (V_{11}^2, m_{h_2}), (V_{12}^2, V_{13}^2) \text{ and } (m_{h_3}, m_{h_2}) \text{ parameter spaces that were searched and the regions therein (labeled “bad points found”) for which there is some choice for the remaining NMSSM parameters such that no Higgs boson is observable.} \]

\[ \text{For } V_{13}^2 \rightarrow 0. \] Since \( V_{13} = -s_1 s_3 \), this means either \( \alpha_1 \sim 0 \) or \( \alpha_3 \sim 0 \). However, only if \( \alpha_3 \sim 0 \) can all the Higgs bosons be observable. If \( \alpha_3 \) is not near 0, \( \alpha_1 \) must be, in which case \( V_{21} \sim 0 \) and \( V_{11} \sim 1 \) and the \( h_1 \) has completely SM-like couplings.

\[ ^{5} \text{If } V_{13} \neq 0, \text{ then Eqs. } \circled{4} \text{ and } \circled{6} \text{ imply that } m_{h_3} \sim m_{h_2} \sim m_{h_1} \text{ as } m_{h_1} \rightarrow m_{h_1}^{\text{max}}. \text{ In this limit we have } M_{12}^2 = \sum_{i=1,2,3} V_{i1} V_{2i} m_{h_i}^2 \rightarrow m_{h_1}^2 \sum_{i=1,2,3} V_{i1} V_{2i} = 0 \text{ by orthogonality of } V. \text{ Unless } M_{12}^2 = 0, \text{ there is an inconsistency which can only be avoided by simultaneously taking } V_{13} \rightarrow 0. \]
plings [see Eqs. (7)-(11)], and for \(m_{h_1} \sim m_{h_1}^{\text{max}} \sim 118 \text{ GeV} \) at \(\tan \beta = 5\) \(h_1\) will be detectable in the \(\gamma\gamma\) final state. If \(\alpha_3 \sim 0\), then any value of \(\alpha_1\) is possible, but (again) \(\alpha_1 \sim 0\) would make \(h_1\) SM-like and observable; in addition, \(\alpha_1 \sim \pm \pi/2\) (i.e. \(s_1 \sim \pm 1, c_1 \sim 0\)) yields \(V_{22} \sim 0\) and \([V_{21}] \sim 1\) implying that \(h_2\) would be SM-like and observable (in the \(\gamma\gamma\) or \(ZZ^*, ZZ\) modes). Thus, the only ‘dangerous’ region is \(\alpha_3 \sim 0\) and \(\alpha_1 \neq 0, \pm \pi/2\), for which, Eq. (8) implies \(m_{h_2} \sim m_{h_1}\) so that both \(h_2\) and \(h_1\) would have to be found in the \(\gamma\gamma\) mode\(] If the value of \(o_2\) is such that neither \(s_2\) nor \(o_2\) is small, then both \(V_{21}\) and \(V_{22}\) can be substantial, and the \(\gamma\gamma\) mode can be suppressed for both \(h = h_1\) and \(h = h_2\) by a combination of \(t\bar{t}h\) coupling suppression (to diminish \(g\gamma \rightarrow h\) production) and \(b\bar{b}h\) coupling enhancement (as natural for moderate or large \(\tan \beta\)). The latter enhances the \(b\bar{b}\) partial width and diminishes the \(h \rightarrow \gamma\gamma\) branching ratio. The moderate \(\tan \beta \sim 5\) value makes it possible to have the required \(b\bar{b}h\) coupling enhancement without its being so large as to make the \(h \rightarrow \tau^+\tau^-\) mode observable in \(b\bar{b}h\) production.

It is useful to present details on what goes wrong at a typical point of unobservability. For \(\tan \beta = 5\) and \(m_{h_1} = 105 \text{ GeV}\), no Higgs boson can be observed for \(m_a = 103 \text{ GeV} \) if \(\alpha_1 = -0.470\), \(\alpha_2 = 0.911\), \(\alpha_3 = 0.165\), and \(\lambda = 0.294\) (for which \(m_{h_2} = 124 \text{ GeV}\), \(m_{h_3} = 206 \text{ GeV}\), \(m_{H^+} = 201 \text{ GeV}\), and \(m_{h^0} = 186 \text{ GeV}\)). The corresponding \(V\) matrix entries are:

\[
V = \begin{pmatrix}
0.887 & 0.455 & 0.0757 \\
-0.283 & 0.407 & 0.869 \\
-0.364 & 0.792 & -0.490
\end{pmatrix}
\]

From the \(V_{ij}\) and the value of \(\tan \beta\), we compute (relative to the SM values)

\[
(VVh_1)^2 = 0.79 \quad (VVh_2)^2 = 0.21 \quad (VVh_3)^2 = 0.006
\]

\[
(b\bar{b}h_1)^2 = 5.3 \quad (b\bar{b}h_2)^2 = 2.5 \quad (b\bar{b}h_3)^2 = 18
\]

\[
(t\bar{t}h_1)^2 = 0.69 \quad (t\bar{t}h_2)^2 = 0.29 \quad (t\bar{t}h_3)^2 = 0.062
\]

where \(V = W \) or \(Z\). Note that \(h_3\) has very small couplings to \(V\).

The manner in which this point escapes discovery is now apparent. First, the minimum values required for the \((b\bar{b}h_1)^2\) values for \(h_1\) observability in the \(\tau^+\tau^-\) mode are: 53 (\(i = 1\)); 32 (\(i = 2\)); 35 (\(i = 3\)). The actual values all lie below those required. Observation of the \(a\) at \(m_a = 103 \text{ GeV}\) (without adding in the much smaller overlapping \(h_1\) signal) would require \(\tan \beta = 8\). Regarding the other discovery modes, \(h_1\) and \(h_2\) are both in the mass range for which the \(\gamma\gamma\) mode is potentially viable and the \(h_3\) is potentially detectable in the \(ZZ \rightarrow 4\ell\) channel. However, the suppressed \(t\bar{t}h_{1,2,3}\) couplings imply smallish \(gg\) production rates for \(h_{1,2,3}\). Relative to a SM Higgs of the same mass we have:

\[
\frac{(ggh_i)^2}{(ggh_{SM})^2} = 0.58 \quad (i = 1); \quad 0.43 \quad (i = 2); \quad 0.15 \quad (i = 3).
\]

(Note that these strengths are not simply the \((t\bar{t}h_i)^2\) magnitudes due to enhanced \(b\bar{b}h\) loop contributions which interfere with the \(t\bar{t}h\) loop contributions at amplitude level.) Further, the enhanced Higgs decay rate to \(b\bar{b}\) and the reduced \(W\)-loop contributions to the \(\gamma\gamma\) coupling suppress the \(\gamma\gamma\) branching ratios of \(h_1\) and \(h_2\) relative to SM expectations. We find:

\[
\frac{B(h_i \rightarrow \gamma\gamma)}{B(h_{SM} \rightarrow \gamma\gamma)} = 0.18 \quad (i = 1); \quad 0.097 \quad (i = 2); \quad (14)
\]

\(i.e.\) suppression sufficient to make \(h_1\) and \(h_2\) invisible in the \(\gamma\gamma\) mode. The suppressed \(ZZh_3\) coupling and the enhanced \(h_3 \rightarrow b\bar{b}\) decays are sufficient to suppress \(B(h_3 \rightarrow ZZ)\) much below SM expectations:

\[
\frac{B(h_3 \rightarrow ZZ)}{B(h_{SM} \rightarrow ZZ)} = 0.11 , \quad (15)
\]

\(i.e.\) such that the \(4\ell\) signal has a significance of only \(1.5\sigma\), even though a SM Higgs of this mass would yield a \(\sim 37\sigma\) signal.

In short, there is enough flexibility due to the addition of the singlet Higgs field (which has no couplings to SM fermions and vector bosons!) for all the Higgs bosons to escape detection for certain choices of model parameters, provided \(\tan \beta\) is moderate in size. Moderate \(\tan \beta\) implies that \(h \rightarrow \gamma\gamma\) decays for light Higgs are suppressed, while at the same time \(b\bar{b}h\) production is not adequately enhanced for detection of the \(h \rightarrow \tau^+\tau^-\) mode.

V. Discussion and Conclusions

The regions of NMSSM parameter space where no Higgs boson can be detected will expand if full \(L = 600 \text{ fb}^{-1}\) \((L = 1000 \text{ pb}^{-1})\) luminosity is not available at the LHC (LEP2) or efficiencies are smaller than anticipated. Conversely, these “regions of unobservability” could decrease substantially (perhaps disappear) with improved efficiency \((e.g.\) due to the expanded calorimeter option discussed in Ref. [8]) in the \(\tau\tau\) final state or higher luminosity. These issues will be pursued elsewhere.

We have explicitly neglected supersymmetric (SUSY) decay modes of the Higgs bosons in our treatment. If these decays are important, the regions of unobservability found without using the SUSY final states will increase in size. However, Higgs masses in the regions of unobservability are typically modest in size \((100 - 200 \text{ GeV})\), and as SUSY mass limits increase with LEP2 running this additional concern will become less relevant. Of course, if SUSY decays are significant, detection of the Higgs bosons in the SUSY modes might be possible, in which case the regions of unobservability might decrease in size. Assessment of this issue is dependent upon a specific model for soft SUSY breaking and will not be pursued here.

Finally, although we cannot establish a no-lose theorem for the NMSSM Higgs bosons at LEP2 and the LHC (in contrast to the no-lose theorems applicable to the NLC Higgs search with \(\sqrt{s} \gtrsim 300 \text{ GeV}\), the regions of complete Higgs boson unobservability appear to constitute a small fraction of the total model parameter space. It would be interesting to see whether or not these regions of unobservability correspond to unnatural choices for the Planck scale supersymmetry-breaking parameters.
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[5] See S.F. King and P.L. White, Ref. [3], Fig. 1; $\lambda_{\text{max}} = 0.7$ is the largest value found for $m_t = 175 \text{ GeV}$, $\alpha_s(m_Z) = 0.13$. If $\tan \beta \lesssim 2$ or $\gtrsim 40$, the bound could be much smaller.

[6] CMS Technical note, CMS TN/95-156; the same figure is repeated in the CMS Technical Proposal, CERN/LHCC 94-38, LHCC/P1 (1994).

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