THE AUTOMORPHISM AND ISOMETRY GROUPS OF \( \ell_\infty(\mathbb{N}, B(\mathcal{H})) \) ARE TOPOLOGICALLY REFLEXIVE

LAJOS MOLNÁR

Abstract. The aim of this paper is to show that the automorphism and isometry groups of the C*-algebra \( \ell_\infty(\mathbb{N}, B(\mathcal{H})) \) are topologically reflexive which, as one of our former results shows, is not the case with the "scalar algebra" \( \ell_\infty \).

1. Introduction and Statement of the Results

Let \( \mathcal{A} \) be a C*-algebra. A subset \( \mathfrak{T} \) of the algebra \( B(\mathcal{A}) \) of all bounded linear transformations on \( \mathcal{A} \) is called topologically reflexive if for every \( \Phi \in B(\mathcal{A}) \), the condition that \( \Phi(A) \in \overline{\mathfrak{T}(A)} \) (the norm-closure of \( \mathfrak{T}(A) \)) holds true for every \( A \in \mathcal{A} \) implies \( \Phi \in \mathfrak{T} \). Similarly, we say that \( \mathfrak{T} \) is algebraically reflexive if we have \( \Phi \in \mathfrak{T} \) for any \( \Phi \in B(\mathcal{A}) \) with \( \Phi(A) \in \mathfrak{T}(A) \) (\( A \in \mathcal{A} \)). Roughly speaking, reflexivity means that the elements of \( \mathfrak{T} \) are, in some sense, completely determined by their local actions.

In what follows, let \( \mathcal{H} \) stand for a complex separable infinite dimensional Hilbert space. Reflexivity problems concerning subspaces of \( B(\mathcal{H}) \) represent one of the most active and important research areas in operator theory. The question of reflexivity in our sense described above was first considered by Kadison [Kad] and Larson and Sourour [LaSo] in the case of derivation algebras of operator algebras. In [BrSe] Brešar and Šemrl proved the algebraic reflexivity of the automorphism group of \( B(\mathcal{H}) \). We emphasize that by an automorphism we mean a multiplicative linear bijection, so we do not assume the *-preserving property. As for topological reflexivity, the first result was obtained by Shul’man in [Shu] concerning the

\[ \text{Date: June 11, 1997.} \]

1991 Mathematics Subject Classification. Primary: 47B49, 46L40, 47D25; Secondary: 54D35.

Key words and phrases. Reflexivity, automorphism group, isometry group, operator algebra, Stone-Čech compactification.

This paper was written when the author, holding a scholarship of the Volkswagen-Stiftung of the Konferenz der Deutschen Akademien der Wissenschaften, was a visitor at the University of Paderborn, Germany. He is very grateful to Prof. K.-H. Indlekofer for his kind hospitality. This research was partially supported also by the Hungarian National Foundation for Scientific Research (OTKA), Grant No. T–016846 F–019322.
derivation algebra of any $C^*$-algebra. In our papers [BaMo, Mol1, Mol2] we investigated the topological reflexivity of the groups of all *-automorphisms, respectively automorphisms, respectively surjective isometries of some operator algebras. The results which are in the closest relation to our present investigation are the following. In [Mol1] we showed that the automorphism and isometry groups of $\mathcal{B}(\mathcal{H})$ are topologically reflexive. In [BaMo] we obtained that in the case of the commutative algebra $\ell_\infty$, these groups are topologically nonreflexive. The main result of this paper is that the "mixed" $C^*$-algebra

$$\ell_\infty(\mathbb{N}, \mathcal{B}(\mathcal{H})) = \{(A_n) : A_n \in \mathcal{B}(\mathcal{H}) (n \in \mathbb{N}), \sup_n \|A_n\| < \infty\}$$

has topologically reflexive automorphism and isometry groups. This could be surprising, since these groups in question "cannot be more reflexive" than they are in the case of $\mathbb{C}$, the role of which is played by the operator algebra $\mathcal{B}(\mathcal{H})$ in $\ell_\infty(\mathbb{N}, \mathcal{B}(\mathcal{H}))$. On the way to the proof of this assertion we present some hopefully interesting results concerning the tensor product of $\ell_\infty$ and $\mathcal{B}(\mathcal{H})$ as well as the Stone-Čech compactification $\beta\mathbb{N}$ of $\mathbb{N}$.

In what follows we need the concept of Jordan homomorphisms. If $\mathcal{R}$ and $\mathcal{R}'$ are algebras, then a linear map $\phi : \mathcal{R} \to \mathcal{R}'$ is called a Jordan homomorphism if

$$\phi(A)^2 = \phi(A^2) \quad (A \in \mathcal{R}).$$

The following equations are well-known to be fulfilled by any Jordan homomorphism

1a) \[ \phi(A)\phi(B) + \phi(B)\phi(A) = \phi(AB + BA) \]

1b) \[ \phi(A)\phi(B)\phi(A) = \phi(ABA) \]

1c) \[ \phi(A)\phi(B)\phi(C) + \phi(C)\phi(B)\phi(A) = \phi(ABC + CBA) \]

where $A, B, C \in \mathcal{R}$ (e.g. [Pal, 6.3.2 Lemma]).

It is well-known that the $C^*$-algebras $\ell_\infty$ and $C(\beta\mathbb{N})$ (the algebra of all continuous valued functions on $\beta\mathbb{N}$) are isomorphic. In fact, this follows from the property of the Stone-Čech compactification that every continuous function from a completely regular space $X$ into a compact Hausdorff space $Y$ can be uniquely extended to a continuous function defined on $\beta X$. The map which sends every element of $\ell_\infty$ to its unique extension to an element of $C(\beta\mathbb{N})$ gives the desired isomorphism. Due to the topological properties of $\beta\mathbb{N}$, we have "singular" characters of the commutative algebra $\ell_\infty$ by simply considering any one-point evaluation functional on $C(\beta\mathbb{N})$ which corresponds to a point in $\beta\mathbb{N}\setminus\mathbb{N}$. The word "singular" means here that this character annihilates the cofinite sequences in $\ell_\infty$ (and hence
it is not $w^*$-continuous). In our first theorem we show that this is not the case if we replace the set $C$ of values by the operator algebra $B(H)$.

**Theorem 1.** There is no nonzero Jordan homomorphism $\Phi : \ell_\infty(N, B(H)) \to B(H)$ which vanishes on the set of all cofinite sequences.

This result has the corollary that the kernels of irreducible Jordan homomorphisms of the above type correspond to elements of $N$. Comparing this with the case of the characters of $\ell_\infty$, one can say that the operator algebra $B(H)$ cleans up the Stone-Čech compactification of $N$.

**Corollary 1.** Let $\Phi : \ell_\infty(N, B(H)) \to B(H)$ be an irreducible Jordan homomorphism (i.e. a Jordan homomorphism whose range has only trivial invariant subspaces). Then there is a positive integer $n$ so that

$$\ker \Phi = \{ (A_k)_k \in \ell_\infty(N, B(H)) : A_n = 0 \}.$$ 

It is well-known in the theory of tensor products that for any compact Hausdorff space $X$ and for any $C^*$-algebra $A$, the $C^*$-algebras $C(X) \otimes A$ and $C(X, A)$ (the space of all continuous functions from $X$ into $A$) are isomorphic. The following corollary shows that a similar assertion does not hold true if one considers $\ell_\infty$ instead of $C(X)$.

**Corollary 2.** The $C^*$-algebras $\ell_\infty \otimes B(H)$ and $\ell_\infty(N, B(H))$ are nonisomorphic even as Jordan algebras.

This statement can be reformulated in the following way.

**Corollary 3.** The $C^*$-algebras $\ell_\infty(N, B(H))$ and $C(\beta N, B(H))$ are nonisomorphic even as Jordan algebras.

One short remark should be added here. Namely, for any finite dimensional Hilbert space $K$ we do have an isomorphism between the $C^*$-algebras $\ell_\infty(N, B(K))$ and $C(\beta N, B(K))$. In fact, since in that case the functions in $\ell_\infty(N, B(K))$ have precompact ranges, one can use the function extension property of the Stone-Čech compactification. Nevertheless, Corollary 3 shows that not only this extension stuff breaks down in the infinite dimensional case, but there does not exist any isomorphism between $\ell_\infty(N, B(H))$ and $C(\beta N, B(H))$.

In the next theorem which can also be consireded as a corollary of Theorem 1, we describe the automorphism and isometry groups of $\ell_\infty(N, B(H))$.

**Theorem 2.** Let $\Phi : \ell_\infty(N, B(H)) \to \ell_\infty(N, B(H))$ be an automorphism. Then there are automorphisms $\phi_n$ ($n \in N$) of $B(H)$ and a bijection $\varphi : N \to N$ so that
Φ is of the form

$$Φ((A_k)_k) = (ϕ_n(A_{ϕ(n)})) \quad ((A_k)_k ∈ ℓ_∞(N, B(ℋ))).$$

(2)

An analogue statement holds true for the surjective isometries of $ℓ_∞(N, B(ℋ))$ as well.

To be more specific with (2), we recall the well-known folk results that every automorphism of $B(ℋ)$ is of the form

$$A ↦ TAT^{-1}$$

with some invertible operator $T ∈ B(ℋ)$ and that every surjective isometry of $B(ℋ)$ is either of the form

$$A ↦ UAU^*$$

or of the form

$$A ↦ UA^*U^*$$

with some unitary operator $U ∈ B(ℋ)$, where $^*$ denotes the transpose operation with respect to an arbitrary but fixed complete orthonormal system in $ℋ$.

After these preliminary results we shall be able to prove the main result of the paper which follows.

**Main Theorem.** The automorphism and isometry groups of $ℓ_∞(N, B(ℋ))$ are topologically reflexive.

2. Proofs

We begin with a lemma which we shall use repeatedly throughout.

**Lemma 1.** Let $ϕ : B(ℋ) → B(ℋ)$ be a Jordan homomorphism. Then $ϕ$ is either injective or we have $ϕ = 0$.

**Proof.** First observe that by (1a) every Jordan homomorphism preserves the idempotents as well as the orthogonality between them (the idempotents $P, Q$ are called orthogonal if $PQ = QP = 0$). Now, since every Jordan homomorphism of $B(ℋ)$ is continuous (see [Mol1, Lemma 1]), the kernel of $ϕ$ is a closed Jordan ideal of a $C^*$-algebra and hence it is an associative ideal as well (see [CiYo]). Therefore, ker $ϕ$ is either $\{0\}, C(ℋ)$ (the ideal of all compact operators on $ℋ$) or $B(ℋ)$. Since the Calkin algebra $B(ℋ)/C(ℋ)$ has uncountably many pairwise orthogonal nonzero idempotents which does not hold true for $B(ℋ)$, we have the assertion. □
Proof of Theorem 1. Suppose that $\Phi : \ell_\infty(N, B(H)) \to B(H)$ is a Jordan homomorphism which vanishes on the cofinite sequences. Define $\phi : B(H) \to B(H)$ by

$$\phi(A) = \Phi(A, A, \ldots) \quad (A \in B(H)).$$

Obviously, $\phi$ is a Jordan homomorphism. Let $(e_n)$ be a complete orthonormal sequence in $H$ and denote by $S \in B(H)$ the corresponding unilateral shift. For any $n \in \mathbb{N}$, let $P_n$ be the orthogonal projection onto the subspace generated by $e_1, \ldots, e_n$. If $n, m \in \mathbb{N}$, since $\Phi$ vanishes on the cofinite sequences, by (1c) we can compute

$$\Phi(P_nSP_m + P_mSP_n, P_nS^2P_m + P_mS^2P_n, P_nS^3P_m + P_mS^3P_n, \ldots) = 0.$$

Since $\phi : B(H) \to B(H)$ is a Jordan homomorphism, it follows from [Mol1, Lemma 2] that the sequence $(\phi(P_n))$ converges strongly to an idempotent $E \in B(H)$ which does not depend on the particular choice of $(e_n)$ and $E$ commutes with the range of $\phi$. Then the map $\phi' : B(H) \to B(H)$ defined by $\phi'(A) = \phi(A)(I - E)$ is a Jordan homomorphism which vanishes on the finite rank operators. By Lemma 1, we have $\phi' = 0$ and this gives us that $\phi(.) = \phi(.)E = E\phi(.)$. Now, using (1b), from (3) we obtain

$$0 = \phi(I)E\Phi(S, S^2, S^3, \ldots)E\phi(I) = \phi(I)\Phi(S, S^2, S^3, \ldots)\phi(I) = \Phi(S, S^2, S^3, \ldots).$$

By (1a) this further implies that

$$0 = \Phi(S, S^2, S^3, \ldots)\Phi(S^*, S^{*2}, S^{*3}, \ldots) + \Phi(S^{*}, S^{*2}, S^{*3}, \ldots)\Phi(S, S^2, S^3, \ldots) = \Phi((I - P_1, I - P_2, I - P_3, \ldots) + (I, I, I, \ldots))$$

which results in

$$2\Phi(I, I, I, \ldots) = \Phi(P_1, P_2, P_3, \ldots).$$

Since the operators $\Phi(I, I, I, \ldots)$ and $\Phi(P_1, P_2, P_3, \ldots)$ are idempotents, we deduce from (4) that $\Phi(I, I, I, \ldots) = 0$. By (1a) we clearly have $\Phi = 0$. This completes the proof.

Remark 1. Observe that an inspection of the previous proof (see (3)) shows that the conclusion in Theorem 1 remains true if we assume that $\Phi : \ell_\infty(N, B(H)) \to B(H)$ vanishes only on all cofinite sequences with finite-rank coordinates.
Proof of Corollary 1. First, by Theorem 1, we obtain that \( \Phi \) takes a nonzero value on a cofinite sequence, say \((A_1, \ldots, A_n, 0, \ldots)\). Therefore, in the equation

\[
0 \neq \Phi(A_1, \ldots, A_n, 0, \ldots) = \Phi(A_1, 0, \ldots) + \cdots + \Phi(0, \ldots, 0, A_n, 0, \ldots)
\]

one term on the right hand side, say the last one, must be nonzero. Thus, the map

\[
\Phi_n : A \mapsto \Phi(0, \ldots, 0, A_n, 0, \ldots)
\]

is a nonzero Jordan homomorphism of \( \mathcal{B}(\mathcal{H}) \). Now, by (1a) it is easy to see that the idempotent \( \Phi_n(I) \) is necessarily nonzero. Furthermore, using (1b), for any \((A_k)_k \in \ell_\infty(\mathbb{N}, \mathcal{B}(\mathcal{H}))\) we infer

\[
2\Phi_n(I)\Phi((A_k)_k)\Phi_n(I) = 2\Phi_n(A_n) = \Phi((A_k)_k)\Phi_n(I) + \Phi_n(I)\Phi((A_k)_k).
\]

Multiplying this equation by \( \Phi_n(I) \) from the right, we obtain that

\[
\Phi_n(I)\Phi((A_k)_k)\Phi_n(I) = \Phi((A_k)_k)\Phi_n(I)
\]

which means that the range of \( \Phi_n(I) \) is an invariant subspace of the range of \( \Phi \). Therefore, we have \( \Phi_n(I) = I \). Now, we can compute

\[
2\Phi((A_k)_k) = \Phi((A_k)_k)\Phi_n(I) + \Phi_n(I)\Phi((A_k)_k) = 2\Phi_n(A_n).
\]

Clearly, it remains to prove that \( \Phi_n \) is injective. Since \( \Phi_n \neq 0 \), by Lemma 1 we have \( \ker \Phi_n = \{0\} \).

Proof of Corollary 3. Let us suppose on the contrary that there exists a Jordan isomorphism \( \Phi : \ell_\infty(\mathbb{N}, \mathcal{B}(\mathcal{H})) \to C(\beta\mathbb{N}, \mathcal{B}(\mathcal{H})) \). Plainly, \( \Phi \) preserves the nonzero minimal idempotents in these algebras. Any idempotent \( f \in C(\beta\mathbb{N}, \mathcal{B}(\mathcal{H})) \) is a continuous function whose values are idempotents in \( \mathcal{B}(\mathcal{H}) \). Obviously, if we multiply \( f \) by the characteristic function of any point in \( \mathbb{N} \subset \beta\mathbb{N} \), we obtain an idempotent in \( C(\beta\mathbb{N}, \mathcal{B}(\mathcal{H})) \). On the other hand, if a function \( f \in C(\beta\mathbb{N}, \mathcal{B}(\mathcal{H})) \) vanishes on \( \mathbb{N} \), then by \( \|f(.)\| \in C(\beta\mathbb{N}) \) we have \( f = 0 \). Putting these together, we find that the nonzero minimal idempotents in \( C(\beta\mathbb{N}, \mathcal{B}(\mathcal{H})) \) are those functions on \( \beta\mathbb{N} \) which take only one nonzero value, they take it at some point in \( \mathbb{N} \) and this value is a rank-one idempotent in \( \mathcal{B}(\mathcal{H}) \). Now, let \( p \in \beta\mathbb{N} \setminus \mathbb{N} \) and consider the homomorphism \( \Psi : C(\beta\mathbb{N}, \mathcal{B}(\mathcal{H})) \to \mathcal{B}(\mathcal{H}) \) defined by \( \Phi(f) = f(p) \). Clearly, \( f(p) = 0 \) for every minimal idempotent in \( C(\beta\mathbb{N}, \mathcal{B}(\mathcal{H})) \). This gives us that the nonzero Jordan homomorphism \( \Psi \circ \Phi : \ell_\infty(\mathbb{N}, \mathcal{B}(\mathcal{H})) \to \mathcal{B}(\mathcal{H}) \) vanishes on every cofinite sequence having finite rank coordinates. But this is a contradiction by Remark 1. \( \square \)
Proof of Corollary 2. Since $\ell_\infty$ and $C(\beta\mathbb{N})$ are isomorphic, we obtain that $\ell_\infty \otimes B(\mathcal{H})$ and $C(\beta\mathbb{N}) \otimes B(\mathcal{H}) \simeq C(\beta\mathbb{N}, B(\mathcal{H}))$ are isomorphic as $C^*$-algebras. Now, the statement follows from Corollary 3.

Proof of Theorem 2. First let $\Phi : \ell_\infty(\mathbb{N}, B(\mathcal{H})) \to \ell_\infty(\mathbb{N}, B(\mathcal{H}))$ be a Jordan automorphism. Plainly, every coordinate function $\Phi^m$ of $\Phi$ satisfies the condition in Corollary 1. Hence, to every $m \in \mathbb{N}$ there corresponds a positive integer $\varphi(m)$ such that

$$\Phi^m((A_k)) = \Phi^m(0, \ldots, 0, A_{\varphi(m)}, 0, \ldots) = \Phi^m_{\varphi(m)}(A_{\varphi(m)}),$$

where $\Phi^m_{\varphi(m)}$ is a Jordan automorphism of $B(\mathcal{H})$ (see the proof of Corollary 1). Let us show that $\varphi : \mathbb{N} \to \mathbb{N}$ is a bijection. Since $\Phi$ preserves the nonzero minimal idempotents in $\ell_\infty(\mathbb{N}, B(\mathcal{H}))$, thus for any rank-one idempotent $P \in B(\mathcal{H})$, two different coordinates of $\Phi(0, \ldots, 0, P, 0, \ldots)$ cannot be nonzero. This readily implies that $\varphi$ is injective. Assume now that $\varphi$ is not surjective. Then it is easy to see that there is at least one coordinate, say the first one, for which $\Phi(A, 0, \ldots) = 0$ ($A \in B(\mathcal{H})$). But this contradicts the injectivity of $\Phi$. Consequently, we obtain that $\varphi$ is bijective. Since we apparently have

$$\Phi((A_k)_k) = (\Phi^m_{\varphi(m)}(A_{\varphi(m)})) = ((A_k)_k \in \ell_\infty(\mathbb{N}, B(\mathcal{H}))),$$

the first statement of the theorem follows. As for the second one, we refer to a well-known theorem of Kadison stating that the surjective isometries of a $C^*$-algebra are exactly those maps which can be written as a Jordan $*$-automorphism multiplied by a fixed unitary element (see [KaRi, 7.6.16, 7.6.17]).

Proof of the Main Theorem. Let first $\Phi : \ell_\infty(\mathbb{N}, B(\mathcal{H})) \to \ell_\infty(\mathbb{N}, B(\mathcal{H}))$ be a continuous linear map which is an approximately local automorphism, i.e. suppose that $\Phi$ can be approximated at every element $(A_k)_k \in \ell_\infty(\mathbb{N}, B(\mathcal{H}))$ by the values of a sequence of automorphisms at $(A_k)_k$. Using Theorem 2, one can readily verify that the map

$$A \mapsto \Phi^m(A, A, \ldots)$$

is an approximately local automorphism of $B(\mathcal{H})$ and then, by [Mol1, Theorem 2], it is an automorphism. In particular, $\Phi^m$ is surjective and using Corollary 1 as well as its proof, we obtain that for every $m \in \mathbb{N}$ there is a positive integer $\varphi(m)$ so that $\Phi^m_{\varphi(m)}$ is an automorphism and we have

$$\Phi((A_k)_k) = (\Phi^m((A_k)_k)) = (\Phi^m_{\varphi(m)}(A_{\varphi(m)})) = ((A_k)_k \in \ell_\infty(\mathbb{N}, B(\mathcal{H}))).$$

We show that the function $\varphi : \mathbb{N} \to \mathbb{N}$ is a bijection. Like in the proof of Theorem 2, we obtain immediately that $\varphi$ is injective. The surjectivity is also easy to see.
Indeed, if \( \varphi \) were not surjective, then we would have such a coordinate, say the first one for which \( \Phi(A,0,\ldots) = 0 \) holds true for every \( A \in \mathcal{B}(\mathcal{H}) \). But this is a contradiction, since, for example, the norm of the image of \( (I,0,0,\ldots) \) under any automorphism of \( \ell_\infty(\mathbb{N},\mathcal{B}(\mathcal{H})) \) is 1. For every \( m \in \mathbb{N} \) let \( T_m \in \mathcal{B}(\mathcal{H}) \) be an invertible operator such that

\[
\Phi^m_{\varphi(m)}(A) = T_mA \Phi^m(A) T_m^{-1} \quad (A \in \mathcal{B}(\mathcal{H})).
\]

Since \( \Phi \) is of the form

\[
\Phi((A_k)_k) = (T_m A_{\varphi(m)} T_m^{-1}) \quad ((A_k)_k \in \ell_\infty(\mathbb{N},\mathcal{B}(\mathcal{H})))
\]

and \( \Phi \) is bounded, considering sequences of the form \( (x_k \otimes y_k)_k \), one can easily verify that \( \sup_k \|T_k\| \|T_k^{-1}\| < \infty \). This immediately gives us the bijectivity of \( \Phi \), concluding the proof of our statement in the case of automorphisms.

Let next \( \Phi \) be an approximately local surjective isometry of \( \ell_\infty(\mathbb{N},\mathcal{B}(\mathcal{H})) \). Since \( \Phi \) clearly preserves the unitaries in the \( C^* \)-algebra \( \ell_\infty(\mathbb{N},\mathcal{B}(\mathcal{H})) \), it follows from a well-known theorem of Russo and Dye [RuDy, Corollary 2] that \( \Phi \) is a Jordan *-homomorphism multiplied by a fixed unitary element of \( \ell_\infty(\mathbb{N},\mathcal{B}(\mathcal{H})) \). Consequently, we can assume that our approximately local surjective isometry \( \Phi \) is a unital Jordan *-homomorphism. Now, the proof can be completed similarly to the case of automorphisms, using [Mol1, Theorem 3] in the place of [Mol1, Theorem 2].

To conclude the paper, we note that we feel it to be an interesting algebraic-topological question to investigate for which completely regular spaces \( X \) holds it true that the operator algebra \( \mathcal{B}(\mathcal{H}) \) ”cleans up” the Stone-Čech compactification of \( X \) in the sense that we have seen above.

References

[BaMo] C.J.K. Batty and L. Molnár, *On topological reflexivity of the groups of *-automorphisms and surjective isometries of \( B(H) \)*, Arch. Math. 67 (1996), 415–421.

[BrSe] M. Brešar and P. Šemrl, *On local automorphisms and mappings that preserve idempotents*, Studia Math. 113 (1995), 101–108.

[CiYo] P. Civin and B. Yood, *Lie and Jordan structures in Banach algebras*, Pacific J. Math. 15 (1965), 775–797.

[KaRi] R.V. Kadison and J.R. Ringrose, *Fundamentals of the Theory of Operator Algebras, Vol. II.*, Academic Press, 1986.

[LaSo] D.R. Larson and A.R. Sourour, *Local derivations and local automorphisms of \( B(X) \)*, Proc. Sympos. Pure Math. 51, Providence, Rhode Island 1990, Part 2, 187–194.

[Mol1] L. Molnár, *The set of automorphisms of \( B(H) \) is topologically reflexive in \( \mathcal{B}(\mathcal{B}(H)) \)*, Studia Math. 122 (1997), 183–193.
[Mol2] Reflexivity of the automorphism and isometry groups of some standard operator algebras, (submitted for publication).

[Pal] T.W. Palmer, Banach Algebras and The General Theory of *-Algebras, Vol. I., Encyclopedia Math. Appl. 49, Cambridge University Press, 1994.

[RuDy] B. Russo and H.A. Dye, A note on unitary operators in $C^*$-algebras, Duke Math. J. 33 (1966), 413–416.

[Shu] V.S. Shul’mann, Operators preserving ideals in $C^*$-algebras, Studia Math. 109 (1994), 67–72.

Institute of Mathematics, Lajos Kossuth University, 4010 Debrecen, P.O.Box 12, Hungary

E-mail address: molnari@math.klte.hu