Approximation of Capacity for Downlink Multi-User System with Combination of Precoding and NOMA Methods

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Abstract: Enhancing performance of downlink MU systems is an attractive and important research for future wireless systems. The non-orthogonal multiple access (NOMA) method was proposed to improve the performance of MU systems. In order to further improve the outage probability (OP) and ergodic capacity (EC) of downlink NOMA MU systems, we propose the combination of precoding and NOMA methods, and then the OP and EC of MU systems with our novel method are derived in scenarios of perfect and imperfect successive interference cancellation (SIC) scheme. Moreover, the closed-form expression of OP and EC for both scenarios is theoretically derived and compared with Monte Carlo simulations. The results show that, the analysis method is accurate, and the proposed combining precoding and NOMA can further enhance the performance of MU systems in comparing with the original orthogonal multiple access method.

Keywords: non orthogonal multiple access; successive interference cancellation; close-form expression; outage probability; ergodic capacity; combination of precoding and NOMA; perfect and imperfect SIC

1. Introduction

The fifth generation (5G) wireless networks is deployed for actualizing the full potential of the Internet of Things (IoT). The key idea of IoT is a large number of devices which connect to the internet, collect and share data together. As mentioned in [1], the big challenge in IoT implementation is to obtain reliable communication under the condition of low cost and limited spectrum. Owing to superior spectral efficiency of the non-orthogonal multiple access (NOMA), it is a promising multiple access technique for future wireless systems, especially the NOMA is considered to be applied to 5G mobile networks [2–4].

Being different to another orthogonal multiple access (OMA) method, such as code division multiple access (CDMA), frequency division multiple access (FDMA) and so on, the NOMA method uses the power domain for multiple access technique at transmitters, and it applies successive interference cancellation (SIC) method for detecting the desired signals at users site [5]. The SIC separates superimposed symbols and mitigates the inter-user interference while operating at users. In the downlink NOMA MU system, the users that has better channel conditions are provided by less transmit power, whereas the users that has worse channel conditions are allocated more transmit power [6,7]. This strategy can keep the balance between the performance of MU systems and the fairness of every user [8]. In the other hand, the NOMA MU system allocates transmit power based on priority of users, the user with higher priority is provided by more transmit power, whereas the user with lower priority is allocated less transmit power [9,10].

The previous researches on NOMA downlink MU systems are summarized as follows. In [11], Ding et al. investigated the simple NOMA downlink MU system, in which all users...
are located randomly, and the approximated expressions of outage probability (OP) and ergodic capacity (EC) of the system are derived. In [12], Yang et al. researched on downlink and uplink of MU NOMA systems. The authors proposed the dynamic power allocation based on QoS of different users was proposed. According to the obtained results, it is indicated that more whippiness to fairness performance of all users is provided based on the proposed dynamic power allocation method. The OP and EC were discussed under the condition of fairness of quality for the users in [13,14]. The authors in [15,16] analyzed the NOMA MU—MIMO systems, and shown that the performance of NOMA MU—MIMO systems is improved when MU are gathered into a cluster.

Throughout the previous works, the throughput and the bandwidth efficiency of NOMA systems are improved, furthermore, many billions electronic devices are provided an ultrahigh connectivity by the non-orthogonal property of NOMA technology. Moreover, comparing with another multiple access method such as: sparse-code multiple access, MU shared access, pattern-division multiple access, the NOMA MU systems has a low-complexity [3]. Therefore, the NOMA technology is suitable for downlink MU systems. On the other hand, the precoding in NOMA multiple input single output (MISO) systems was proposed and analyzed [17]. However, every cluster was assumed to consist of only two users, we are going to extend this work in order to propose a system that can serve more end-users. We have proposed and analyzed the dual hops MU system with precoding at the first hop and NOMA at the second hop in the short paper [18], and the combination of NOMA and beamforming for downlink MU systems in [19,20]. However, these systems were analyzed under assumption of perfect SIC, perfect channel state information (CSI), perfect beamforming, and completely decoding superposition signals. It is almost impossible in practical applications, therefore we are going to analyze the proposed system with combination of precoding and NOMA under the condition of imperfect SIC. Furthermore, the approximated equations of not only the outage probability but also the ergodic capacity are derived and compared in both scenarios: perfect and imperfect SIC. The contributions of this paper can be summarized as follows:

- The downlink NOMA system with the base station (BS) of multiple antennas and the user of single antenna is analyzed. Multiple users become an one cluster according to their location and Zero-forcing based precoding is applied at the BS to mitigate the inter-cluster interference.
- The approximated equations of OP and EC are derived for every user for evaluating the proposed method based on perfect and imperfect SIC. The proposed approximated expressions are verified by simulation results.
- The EC of proposed method is calculated and compared with the OMA method to indicate the better one via both theoretical analysis and simulation in cases of perfect and imperfect SIC.

The organization of the rest of paper is as follows. Section 2 briefly depicts the downlink NOMA MU-MISO system and its channel model. The approximation of EC and OP is proposed in Section 3. The proposed method is evaluated by numerical results that are depicted in Section 4. Finally, the conclusion of paper is given in Section 5.

**Notations:** The notations which are used in this paper are denoted as: Bold upper letters are the matrices, whereas bold lower letters indicate the column vectors, the conjugate transpose operation is denoted by $(\cdot)^H$ and $(\cdot)^\ast$. A vector norm is denoted by $\| \cdot \|$, whereas $| \cdot |$ denotes the absolute value. Furthermore, the $E\{ \cdot \}$ and $[X]^T$ denote the average operator and the transpose matrix of $X$, respectively. $f_X(\cdot)$ and $F_X(\cdot)$ respectively denote the probability density function (PDF) and the cumulative distribution function (CDF).

### 2. System Model

The system model of downlink MU system with combination of precoding and NOMA is presented in Figure 1, the BS is equipped with $M \geq 2$ antennas and zero-forcing based precoding (ZFBP) method to serve the $M$ user clusters, while every user has only a single antenna because of its limited size. All users are simply clustered by user location tracking.
algorithms or spatial direction method such as Global Positioning System (GPS) technique. The number of users in every cluster, \( N \), is assumed to be the same.

![System Model](image)

**Figure 1. System Model.**

The precoding matrix \( \mathbf{w}_m \) according to ZFBP method is designed for the \( m \)th cluster in order to remove the inter-cluster interference as follows. The \( n \)th user in the \( m \)th cluster is called the \( (m,n) \)th user, and the channel matrix between the \( (m,n) \)th user and the BS is denoted by \( \mathbf{h}_{m,n} = [h_{m,n,1}, h_{m,n,2}, \ldots, h_{m,n,M}]^T \in \mathbb{C}^{M \times 1} \) with \( m \in \{1, \cdots, M\} \) and \( n \in \{1, \cdots, N\} \). \( h_{m,n,i} \sim \mathcal{CN}(0, \Omega_{m,n}) \) denotes the channel coefficient, and \( \mathbb{E}\{|h_{m,n,i}|^2\} = \Omega_{m,n} \) is the variance of the channel gain. All channels are assumed to be quasi-static independent and identically distributed (i.i.d.) and block fading. For the purpose of maximizing the signal to noise ratio (SNR), the precoding matrix \( \mathbf{w}_m \) can be expressed as the projection of \( \mathbf{h}_{m} = [h_{m,1}, h_{m,2}, \cdots, h_{m,N}] \in \mathbb{C}^{M \times N} \) in the direction of interference user in the \( m \)th cluster. Mathematically, the precoding matrix \( \mathbf{w}_m \) is represented in Equation (1).

\[
\mathbf{w}_m = \frac{\mathbf{B}_m \mathbf{h}_m}{\|\mathbf{B}_m \mathbf{h}_m\|},
\]

where \( \mathbf{B}_m = \mathbf{I}_N - \mathbf{H}_m (\mathbf{H}_m^H \mathbf{H}_m)^{-1} \mathbf{H}_m^H \), \( \mathbf{I}_N \) denotes a \( N \times N \) identity matrix and the structure of channel matrix \( \mathbf{H}_m \) is given in detail as follows in Equation (2):

\[
\mathbf{H}_m = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_{m-1}, \mathbf{h}_{m+1}, \cdots, \mathbf{h}_M]^T.
\]

We assume that the CSI and the precoding at the BS are perfect. Hence, we have \( \mathbf{h}_{m}^H \mathbf{w}_j = 0, \forall m \neq j \). It means that, the ZFBP lets the complexity of the system be high. However, all users in the \( m \)th cluster don’t receive signals of user in the other clusters, they receive signals of all users in the same cluster. Consequently, the NOMA method should be applied to cancel the inter-user interference (IUI).

Based on the principle of NOMA method, all users are assumed to determine the order of decoding signal according to the channel gains. Without loss of generality, the distances between the BS and users are assumed to be sorted as \( d_{m,1} > d_{m,2}, \cdots, > d_{m,N} \), therefore, the channel gains of these users satisfies the following condition in Equation (3).

\[
|\mathbf{w}_m \mathbf{h}_{m,1}|^2 \leq |\mathbf{w}_m \mathbf{h}_{m,2}|^2 \leq \cdots \leq |\mathbf{w}_m \mathbf{h}_{m,N}|^2.
\]
Hence, according to the principle of NOMA, the power allocation coefficient of every user is sorted by Equation (4):

\[ a_{m,1} \geq a_{m,2} \geq \ldots \geq a_{m,N}. \]  (4)

The superposition code which consists of signals of all users in the \( m \)th cluster is defined by \( x_{S,m} = [x_{m,1}, \ldots, x_{m,N}]^T \), where \( x_{m,n} \) denotes the signal of the \((m, n)\)th user, and is normalized with zero mean unit variance. The superposition code are multiplied by the precoding matrix, \( w_m \), at the output antenna following the principle of ZF method. The transmit power of all antennas of BS is fixed, \( E\{|x_{S,m}|^2\} = P_s \), the power allocation coefficient for the \((m, n)\)th user is denoted by \( a_{m,n} \) and satisfies condition \( \sum_{n=1}^{N} a_{m,n} = 1 \). Therefore, the signal of \( m \)th cluster which is transmitted from every antenna of the BS, is described as follow in Equation (5):

\[ x_{S,m} = w_m \sum_{n=1}^{N} \sqrt{a_{m,n}} P_s x_{m,n}. \]  (5)

Let \( n_{m,n} \sim \mathcal{CN}(0, \sigma_n^2) \) be an i.i.d additive white Gaussian noise (AWGN) at the \((m, n)\)th user. In the case of perfect precoding, the received signal of the \((m, n)\)th user includes the signal of all users in the \( m \)th cluster in Equation (6):

\[
y_{m,n} = h_{m,n} w_m \sum_{i=1}^{N} \sqrt{a_{m,i}} P_s x_{m,i} + n_{m,n} = h_{m,n} w_m \sum_{i=1}^{N} \sqrt{a_{m,i}} P_s x_{m,i} + h_{m,n} w_m \sum_{i=n+1}^{N} \sqrt{a_{m,i}} P_s x_{m,i} + n_{m,n}.\]  (6)

The term \( h_{m,n} w_m \sum_{i=n+1}^{N} \sqrt{a_{m,i}} P_s x_{m,i} \) is equal to zero in the case of perfect SIC.

As mentioned above, although the ZFBP can remove the inter-cluster interference, the intra-cluster interference meaning the interference between the users which are located in the same cluster, also called IUI, is still remained. Hence, we propose to apply the SIC technique to cancel the IUI. Owing to SIC principle, at receiver site, the interference of users whose channel gain is worse is deleted. According to above assumption of order, the \((m, n)\)th user should remove the interference of the \((m, 1), \ldots, (m, n-1)\)th users. Let the signal to interference plus noise ratio (SINR) be denoted by \( \gamma \), and the SINR of \((m, n)\)th user is \( \gamma_{m,n} \), from Equation (6) we have Equation (7):

\[
\gamma_{m,n} = \frac{P_s a_{m,n} |h_{m,n} w_m|^2}{\sum_{i=n+1}^{N} P_s a_{m,i} |h_{m,n} w_m|^2 + \sigma_n^2_x}. \]  (7)

Because the \((m, N)\)th user apply the SIC processing to remove the interference signals of all users in the same cluster, the SNR of \((m, N)\)th user is represented by Equation (8):

\[
\gamma_{m,N} = \frac{P_s a_{m,N} |h_{m,N} w_m|^2}{\sigma_n^2_x}. \]  (8)

3. Performance Analysis

3.1. OP of Perfect SIC

The MU-MISO system with our proposed combination of precoding and NOMA approaches has modeled in the previous section, in this section, we start analyzing the performance of system with deriving the outage probability of \((m, n)\)th user. The data rate from the BS to every user which is denoted by \( r \) is assumed to be the same in the sense of
fairness of all users. Therefore, the outage event occurs when the instantaneous capacity between the BS and the user which is calculated by \( \log_2(1 + \gamma_{m,n}) \), is below \( r \). The OP is described as follows.

\[
\text{OP}_{m,n} = \text{Pr} \left( \gamma_{m,n} \leq \gamma_{th} \right). 
\]  

(9)

where \( \gamma_{th} = 2^r - 1 \) denotes the outage threshold.

From Equation (7), we can rewrite Equation (9) by Equation (10):

\[
\text{OP}_{m,n} = \text{Pr} \left( \sum_{i=n+1}^{N} \frac{P_S a_{m,i}|h_{m,i}w_{m}|^2}{\sum_{i=n+1}^{N} P_S a_{m,i}|h_{m,i}w_{m}|^2 + \sigma_{m,n}^2} \leq \gamma_{th} \right). 
\]  

(10)

Let

\[
b_{m,n} = \sum_{i=n+1}^{N} a_{m,i},
\]

\[
\gamma_{m,n} = |w_{m}h_{m,n}|^2,
\]

and then Equation (10) is rewritten as Equation (11):

\[
\text{OP}_{m,n} = \text{Pr} \left( \frac{\gamma_{th}}{P_S(a_{m,n} - \gamma_{th}b_{m,n})} = \theta_i \right). 
\]  

(11)

From Equation (11), we can recognize that the outage event always happens in case of \( a_{m,n} \leq \gamma_{th}b_{m,n} \), thus, more transmit power should be allocated to users whose channel gains are weak.

The ordered variable \( X_{m,n} \) of PDF function is represented by Equation (6.58) \[21\] in Equation (12):

\[
f_{X_{m,n}}(x) = \frac{N!}{(N-n)![(n-1)!]} \sum_{j=0}^{N-n} (-1)^j \binom{N-n}{j} f_{X_i}(x_k) [F_{X_i}(x_k)]^{n-j-1}, 
\]  

(12)

where Equations (13) and (14):

\[
f_{X_i}(x_k) = \frac{1}{\Omega_k} \exp \left( - \frac{x_k}{\Omega_k} \right),
\]  

(13)

\[
F_{X_i}(x_k) = 1 - \exp \left( - \frac{x_k}{\Omega_k} \right).
\]  

(14)

Replacing Equations (13) and (14) into Equation (12), and then using the approximation of binomial expansion, the PDF function for the SINR of \( (m,n) \)th user is obtained.

Owing to the definition of OP, e.g., \( \text{OP} = \int_{0}^{\theta^*} f_{X_{m,n}}(x) \) \( dx \), the approximation of OP of the \( (m,n) \)th user is depicted by Equation (15):

\[
\text{OP}_{m,n} = \frac{N!}{(N-n)![(n-1)!]} \sum_{k=0}^{N-n} \frac{(-1)^k}{n+k} \binom{N-n}{k} \left[ 1 - \exp \left( - \frac{\theta^*}{P_S \Omega_{m,k}} \right) \right]^{n+k}, 
\]  

(15)

with \( k \leq n \leq N, \theta^* = \max_{i=1:N} \{ \theta_i = \frac{\gamma_{th}}{a_{m,n} - \gamma_{th}n_{m,n}} \} \).
3.2. EC of Perfect SIC

In this section, we are going to derive the approximated expression of data rate of MU NOMA system, which is defined as the total rate of all users which are served. The instantaneous data rate based on the Claude Shannon theory of \((m,n)\)th user is represented by Equation (16):

\[
R_{m,n} = \log_2 \left( 1 + \frac{P_3s_{m,n} |w_m h_{m,n}|^2}{\sum_{i=n+1}^{N} P_3s_{m,i} |w_m h_{m,n}|^2 + \sigma_{m,n}^2} \right),
\]

\[
= \log_2 \left( 1 + |w_m h_{m,n}|^2 \rho \left( \sum_{i=n+1}^{N} a_{m,i} + a_{m,n} \right) \right) - \log_2 \left( 1 + \sum_{i=n+1}^{N} \rho a_{m,i} |w_m h_{m,n}|^2 \right), \tag{16}
\]

where \(\rho = \frac{\Omega}{\sigma_{m,n}^2}\).

From Equation (16), the EC of the \((m,n)\)th user is calculated as in Equation (17):

\[
\bar{R}_{m,n} = \mathbb{E}\{R_{m,n}\} = \mathbb{E}\left\{ \log_2 \left( 1 + |w_m h_{m,n}|^2 \rho \left( \sum_{i=n+1}^{N} a_{m,i} + a_{m,n} \right) \right) \right\} - \mathbb{E}\left\{ \log_2 \left( 1 + \sum_{i=n+1}^{N} \rho a_{m,i} |w_m h_{m,n}|^2 \right) \right\}. \tag{17}
\]

The first expectation of a random variable \(X\) is defined by Equation (18):

\[
\mathbb{E}[\log_2(1 + X)] = \int_0^{\infty} \log_2(1 + x) f_X(x) dx, \tag{18}
\]

where \(f_X(x)\) is PDF of \(X\) with \(X \in \{X,Y\}\) and

\[
X = P_3 |w_m h_{m,n}|^2,
\]

\[
Y = \sum_{i=n+1}^{N} P_3s_{m,i} |w_m h_{m,n}|^2.
\]

Using the integration-by-parts method we can rewrite Equation (17) as Equation (19):

\[
R_{m,n} = \ln 2 \int_0^{\infty} \frac{1 - F_X(x)}{1 + x} dx - \ln 2 \int_0^{\infty} \frac{1 - F_Y(y)}{1 + y} dy, \tag{19}
\]

where the \(F_X(x)\) and \(F_Y(y)\) are the CDFs of the random variables \(X\) and \(Y\), respectively. From Equation (19) and above suppositions, the EC depicted as Equation (20):

\[
\bar{R}_{m,n} = \sum_{n=1}^{N} \left( \frac{N}{n} \right) (-1)^{n-1} \ln 2 \int_0^{\infty} \frac{1}{1 + y} \exp \left( - \frac{y}{\sum_{i=n+1}^{N} \rho a_{m,i} \Omega_{m,n}} \right) dy, \tag{20}
\]

where Equation (21):

\[
\mathcal{J}_1(x) = \int_0^{\infty} \frac{1}{1 + x} \exp \left( - \frac{x}{\rho \left( \sum_{i=n+1}^{N} a_{m,i} + a_{m,n} \right) \Omega_{m,n}} \right) dx. \tag{21}
\]
To solve Equation (20), the following equation is applied Equation (3.352.4) [22] in Equation (22):

\[
\int_0^\infty \frac{1}{x+\alpha} \exp(-\mu x) dx = -\exp(-\alpha \mu) \text{Ei}(-\mu \beta),
\]

(22)

with \( \alpha = \beta = 1 \) and \( \text{Ei}(x) = \int_0^\infty \frac{\exp(t)}{t} dt \) denotes the exponential integral function [22].

And then, the approximated expression of the EC of \((m,n)\)th user is given in Equation (23).

\[
\hat{R}_{m,n} = \sum_{n=1}^{N} \binom{N}{n} \left( \frac{-1}{\ln 2} \right)^{n-1} \exp \left( \frac{1}{\rho \left( \sum_{i=n+1}^{N} a_{m,i} + a_{m,n} \right) \Omega_{mn}} \right) \text{Ei} \left( - \frac{1}{\rho \left( \sum_{i=n+1}^{N} a_{m,i} + a_{m,n} \right) \Omega_{mn}} \right) - \sum_{n=1}^{N} \binom{N}{n} \left( \frac{-1}{\ln 2} \right)^{n-1} \exp \left( \frac{1}{\rho \left( \sum_{i=n+1}^{N} \rho a_{m,i} \Omega_{mn}} \right) \text{Ei} \left( - \frac{1}{\rho \left( \sum_{i=n+1}^{N} \rho a_{m,i} \Omega_{mn}} \right) .
\]

(23)

Finally, the EC of the \(m\)th cluster is the sum of EC of all users in this cluster Equation (24):

\[
\bar{R}_m = \sum_{n=1}^{N} \hat{R}_{m,n}.
\]

(24)

### 3.3. OP of Imperfect SIC

Up to now, the MU MISO system with the proposed method has been analyzed based on the assumption of perfect SIC, however, in practice, it is very difficult to achieve the ideal SIC. The imperfect SIC causes the error in decoding the received superposition code at users. Investigation of imperfect SIC concerns in designing NOMA system, therefore we analyze the proposed method in case of imperfect SIC hereafter.

The term of \(h_{m,n}w_m \sum_{k=1}^{n-1} \sqrt{a_k P_S x_{m,k-1}}\) in Equation (6) depends on the quality of SIC operation, and it is unequal to zero in the case of imperfect SIC. Thus, the instantaneous SINR and the OP of the \((m,n)\)th user hereafter.

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\[
\gamma_{mn}^{impSIC} = \frac{P_S a_{m,n} |h_{m,n}w_m|^2}{\sum_{i=n+1}^{N} P_S a_{m,i} |h_{m,n}w_m|^2 + |h_{m,n}w_m|^2 \sum_{k=1}^{\xi-1} a_k P_S + \sigma_{m,n}^2},
\]

(25)

\[
\text{OP}_{mn}^{impSIC} = \text{Pr} \left( \gamma_{mn}^{impSIC} \leq \gamma_{th} \right) = \text{Pr} \left( \sum_{i=n+1}^{N} P_S a_{m,i} |h_{m,n}w_m|^2 \left| h_{m,n}w_m \right|^2 + \Delta_k \right) \left| h_{m,n}w_m \right|^2 + \sigma_{m,n}^2 \leq \gamma_{th} \right),
\]

(26)

Based on the order of channel gains of the users which is represented as in Equation (3) and from Equation (26) we can rewrite the OP of the \((m,n)\)th user as in Equation (27):

\[
\text{OP}_{mn}^{impSIC} = 1 - \text{Pr} \left( \frac{X_{mn}}{N_{1:n}} > \frac{\gamma_{th}}{P_S a_{m,n} - \gamma_{th} b_k} = a_k \right),
\]

(27)
where \( b_k = \sum_{i=n+1}^{N} P_S a_{m,i} + \Delta_k, \forall k \in \{ n : 1 \div N \} \), and

\[
\alpha_k = \max_{n \div N} \left\{ \frac{\gamma_n}{\frac{\gamma_n}{P_S a_{m,n} - \gamma_n b_k}}, \forall k \in \{ n < k < N \} \right\}.
\]

As shown in Equation (27), the outage always occurs if \( P_S a_{m,n} < \gamma_n b_k \); it means that the transmit power which is allocated to the \((m, n)\)th user, should be large enough. Furthermore, the PDF function of \( X_{m,n} \) is depicted by Equation (6.58) [21] in Equation (28):

\[
f_{x_{m,n}}(x) = \frac{N!}{(N-n)! (n-1)!} f_{X_M}(x) \left[ F_{X_M}(x) \right]^{n-1} \left[ 1 - F_{X_M}(x) \right]^{N-n} \sum_{i=0}^{N-n} \left( \frac{-1}{n+k} \right)^{N-n} f_{X_M}(x) \left[ F_{X_M}(x) \right]^{n+k-1}.
\]

Using Equation (28), we can rewrite the OP in the case of imperfect SIC as follows in Equation (29):

\[
\text{OP}_{m,n}^{\text{impSIC}} = \frac{N!}{(N-n)! (n-1)!} \sum_{k=0}^{N-n} \left( \frac{-1}{n+k} \right)^{N-n} f_{X_M}(x) \left[ F_{X_M}(x) \right]^{n+k} \left[ 1 - \exp \left( -\frac{\alpha_k}{P_S \Omega_{m,n}} \right) \right]^{n+k}.
\]

where \( n \leq k \leq N \).

### 3.4. EC of Imperfect SIC

From the instantaneous SINR given in Equation (25), we have the instantaneous capacity of the \((m, n)\)th user is represented by Equation (30):

\[
R_{m,n}^{\text{impSIC}} = \log_2 \left( 1 + \gamma_{m,n}^{\text{impSIC}} \right) = \log_2 \left( 1 + \frac{P_S a_{m,n} |h_{m,n} w_m|^2}{\left( \sum_{i=n+1}^{N} P_S a_{m,i} + \Delta_k \right) |h_{m,n} w_m|^2 + \sigma_{m,n}^2} \right),
\]

\[
= \log_2 \left( \frac{\left( \sum_{i=n+1}^{N} P_S a_{m,i} + \Delta_k \right) |h_{m,n} w_m|^2 + \sigma_{m,n}^2}{\left( \sum_{i=n+1}^{N} P_S a_{m,i} + \Delta_k \right)|h_{m,n} w_m|^2 + \sigma_{m,n}^2} \right),
\]

where \( \Delta_k = \xi \sum_{k=1}^{n-1} a_k P_S \). From Equation (30), we can rewrite the EC as Equation (31).

\[
R_{m,n}^{\text{impSIC}} = E \left\{ \log_2 \left( \left( \sum_{i=n+1}^{N} \rho a_{m,i} + \frac{\Delta_k}{\sigma_{m,n}^2} \right) |h_{m,n} w_m|^2 + 1 \right) \right\}

- E \left\{ \log_2 \left( \left( \sum_{i=n+1}^{N} \rho a_{m,i} + \frac{\Delta_k}{\sigma_{m,n}^2} \right) |h_{m,n} w_m|^2 + 1 \right) \right\}.
\]

Let

\[
Z_1 = \left( \sum_{i=n+1}^{N} \rho a_{m,i} + \frac{\Delta_k}{\sigma_{m,n}^2} \right) |h_{m,n} w_m|^2,
\]

\[
Z_2 = \left( \sum_{i=n+1}^{N} \rho a_{m,i} + \frac{\Delta_k}{\sigma_{m,n}^2} \right) |h_{m,n} w_m|^2.
\]
using integration-by-parts method and Equation (18), the $R_{impSIC}^{m,n}$ is rewritten as follows in Equation (32):

$$R_{impSIC}^{m,n} = \frac{1}{\ln 2} \left( \frac{1}{\infty} \int_0^\infty \frac{1 - F_{Z_1}(z_1)}{1 + z_1} dz_1 - \frac{1}{\ln 2} \left( \frac{1}{\infty} \int_0^\infty \frac{1 - F_{Z_2}(z_2)}{1 + z_2} dz_2 \right) \right).$$

Note that, according to the Equations (7)–(14) [23] the CDFs of $Z_1$ and $Z_2$ can be given as Equation (33):

$$F_{Z_u}(z) = 1 - \sum_{n=1}^N \left( \frac{N}{n} \right) (-1)^{n-1} \exp\left(-\frac{n z_u}{\Omega_{m,n}}\right),$$

with $u \in \{1, 2\}$.

The functions $Q_1$ and $Q_2$ are calculated by replacing the CDF in Equation (33) into Equation (32). In Equations (34) and (35):

$$Q_1 = \sum_{n=1}^N \left( \frac{N}{n} \right) (-1)^{n-1} \exp\left(-\frac{z_1}{\Xi_1 \Omega_{m,n}}\right),$$

where $\Xi_1 = \left( \sum_{i=n+1}^N \frac{\rho \alpha_{m,i}}{r_{m,n,i}} \right) + \rho \alpha_{m,n}$.

$$Q_2 = \sum_{n=1}^N \left( \frac{N}{n} \right) (-1)^{n-1} \exp\left(-\frac{z_2}{\Xi_2 \Omega_{m,n}}\right),$$

where $\Xi_2 = \left( \sum_{i=n+1}^N \frac{\rho \alpha_{m,i}}{r_{m,n,i}} \right)$. Using Equation (22), we can obtain $Q_1$ and $Q_2$ as in Equations (36) and (37):

$$Q_1 = \sum_{n=1}^N \left( \frac{N}{n} \right) (-1)^{n-1} \exp\left(-\frac{1}{\Xi_1 \Omega_{m,n}}\right) Ei\left(-\frac{1}{\Xi_1 \Omega_{m,n}}\right),$$

$$Q_2 = \sum_{n=1}^N \left( \frac{N}{n} \right) (-1)^{n-1} \exp\left(-\frac{1}{\Xi_2 \Omega_{m,n}}\right) Ei\left(-\frac{1}{\Xi_2 \Omega_{m,n}}\right).$$

Finally, replace $Q_1$ and $Q_2$ into Equation (32), we can obtain the EC of the $(m, n)$th user, and then the EC of $m$th cluster in case of imperfect SIC.

The result of our paper investigated the performance of the considered NOMA system in derivate the closed-form expressions of outage probability and ergodic capacity of each cluster. Based on the total of users, the number of users in each cluster $N$, and the number of clusters, $M$ of the system can be varied. The derived expressions can not apply to another system.

4. Numerical Results

This section provide numerical results of OP and EC of the proposed combining precoding and NOMA approaches for downlink MU MISO systems. The result of proposed method is compared to the conventional method to validate the derived theoretical analysis in both perfect and imperfect SICs. In most of published works, they indicated that the performance of system is deteriorated when the number of users is more than three because of remained IUI. Configurations and parameters of the system are explained as follows. The channel between the BS and each user follows the Rayleigh distribution. We perform
10^6 independent trials for each Monte-Carlo simulation. In this system, we assume that the terminals are stationary, thus only small-scale fading impacts on the system performance. The average channel gains are modeled as \( \Omega_{m,1} = \mathbb{E}\{|h_{m,1}|^2\} \), \( \Omega_{m,2} = \mathbb{E}\{|h_{m,2}|^2\} \) and \( \Omega_{m,3} = \mathbb{E}\{|h_{m,3}|^2\} \). And, the average channel gains of users are chosen as \( \Omega_{m,1} = 1 \), \( \Omega_{m,2} = 2 \), \( \Omega_{m,3} = 3 \). Furthermore, unless otherwise stated, the data rate threshold is set at 1 bit/s/Hz. Consequently, we assume that the number of user in every cluster is three, and the precoding at the BS for each cluster is perfect. The power allocation coefficient is calculated by \( a_n = \frac{N-n+1}{\psi} \), with \( \psi = \frac{N(N+1)}{2} \) to make sure that \( \sum_{n=1}^{N} \sqrt{a_n} = 1 \).

In Figure 2, we investigated the total EC under the impact of the power allocation coefficient of User 1, \( a_1 \), with different values of SNR. We can recognize that, the total EC reaches the maximal value when \( a_1 = 0.7 \) in all cases of considered SNR. The User 1 is the farthest from the BS, thus the largest transmit power should be assigned to the User 1 for ensuring the fairness. From the result of this figure, \( a_1 = 0.7 \), \( a_2 = 0.2 \) and \( a_3 = 0.1 \) are utilized for further analyzing.

In Figure 3, we present the theoretical and simulation results of OP versus SNR for two cases perfect and imperfect SIC with \( a_3 = 0.1 \), \( a_2 = 0.2 \) and \( a_1 = 0.7 \). Although the transmit power of the third user is the lowest, its OP is the lowest. It is explained as, the third user is the nearest from the BS, hence its channel gain is the largest. In addition, the third user applies the SIC processing to cancel interference of all other users, whereas, the first user detects its own signal without SIC operation, and the second user performs the SIC operation to remove the signal of user 1 only and considers the signal of third user as interference. Therefore, its received SNR is much higher than the received SINR of the others. The OP of the first user is the same in two cases because the first user does not employ SIC operation. Moreover, the accuracy between theoretical and simulation results verifies the proposed theoretical analysis in both perfect and imperfect SIC schemes.

Figure 4 represents the OP of the first user when the required data rate is changed, i.e., \( r = 1, 2, \) and 3 bit per symbol, these required data rates respectively correspond to BPSK, QPSK and 8-PSK modulations. The OP of the first user is represented, the case of others is similar. As shown in Figure 4, the OP is increased when the required data rate increases, and the Monte Carlo simulation guarantees the correctness of the theoretical results. For the same value of OP, the difference of SNR of BPSK modulation from QPSK and 8-PSK modulations is respectively 3 dB and 5 dB. It means that, increasing the number of information bits in every symbol lets the potential detection of information bit of users reduce. Thus, the NOMA system should to select the reasonable modulations scheme according to the system model.
Figure 3. OP in term of perfect and imperfect SIC with residual of coefficient $\xi = 0.05$.

Figure 4. OP of the first user with different data rates.

Figure 5 illustrates the EC of the first and the third users in term of perfect and imperfect SIC to investigate an impact of quality of SIC process on the performance of users. We can recognize that the EC of the third user in the case of perfect SIC outperforms its capacity of imperfect SIC, whereas the EC of the first user is fixed for both cases. The reason is the same as explained above, the first user does not apply the SIC operation to cancel the interference, it considers the signal of other users as the interference signal. Moreover, the EC of the third user in perfect SIC is much higher than the EC of the first user, however it is rapidly decreased when the $\xi$ increases, and it trends to be lower than that of the first user when the $\xi$ becomes higher. It can be said that, the impact of imperfect SIC on the user close to the BS is high, the EC of the nearer user decreases more rapidly in the case of imperfect SIC. Because the quality of SIC significantly affects the performance of users, designing high quality of SIC operation at the receiver is important for NOMA systems.
The EC of the first and the second users in terms of perfect and imperfect SIC is demonstrated in Figure 6. The EC of the first user is higher than that of the second user in both perfect and imperfect SIC. Especially in the case of imperfect SIC, the EC of the second user decreases significantly whereas the EC of the first user is unchanged. This result also demonstrates the saturation of EC in the high SNR region. The reason is, in the high SNR region, the power level of designed signal is high, however the power level of remained interference signal after imperfect SIC operation is also higher. Therefore, the SINR as well as the EC is saturated.

Our proposed method is compared with OMA method in Figure 7 for the both cases perfect and imperfect SIC with $\xi = 0.01$ and BPSK modulation. Similar to our proposed
method, the conventional OMA method is also combined with the beamforming method to cancel the infer-beam interference. As shown in Figure 7, in the case of perfect SIC, the proposed method outperforms the conventional OMA method because of higher spectrum efficiency of NOMA method [24]. However, the proposed method is significantly affected by the remained interference signal due to the imperfect SIC, the total EC of proposed method therefore becomes lower than that of conventional OMA method when the SNR is over 20 dB, it is the trade-off of our proposed method.

![Graph comparing total EC of 3 users of proposed method with OMA method in the case of BPSK modulation.]

Figure 7. Comparing total EC of 3 users of proposed method with OMA method in the case of BPSK modulation.

5. Conclusions

We have analyzed a MU MISO downlink systems with combining precoding and NOMA approaches, and proposed the OP as well as EC calculation method for all users. Furthermore, the approximation of OP and EC is derived in both perfect and imperfect SIC schemes. The proposed method was compared with the conventional OMA method, and the numerical result indicated that the combining precoding and NOMA approach outperforms the conventional OMA method in case of perfect SIC. Whereas in case of imperfect SIC, the total EC of proposed method decreases and becomes lower than that of conventional OMA method when the SNR is high. Moreover, the theoretical analysis was verified by simulation in both cases of perfect and imperfect SIC.

The ZFBP method was equipped to remove the inter-cluster interference, however, the performance of system was analyzed under the assumption of perfect implementation of ZFBP, and thus the inter-cluster interference was removed clearly. The analysis of system performance under the assumption of imperfect ZFBP will be proposed in our future works. Moreover, another precoding method will be taken into consideration to further improve the performance of downlink MU systems.

Author Contributions: Conceptualization, P.T.H.; methodology, V.V.S.; software, T.M.H.; validation, V.V.S. and T.M.H.; formal analysis, N.T.P. and N.L.C.; investigation, T.M.H.; resources, N.T.P.; data curation, N.T.P.; writing—original draft preparation, V.V.S.; writing—review and editing, N.T.P. and N.L.C.; visualization, V.V.S.; supervision, T.M.H.; project administration, P.T.H. All authors have read and agreed to the published version of the manuscript.

Funding: This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 102.04-2017.311.
Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare that there is no conflict of interest regarding the publication of this paper.

Abbreviations

The following abbreviations are used in this manuscript:

- BS: Base station
- IUI: Inter-user interference
- MU: Multiple users
- MIMO: Multiple input multiple output
- MISO: Multiple input single output
- NOMA: Non-orthogonal multiple access
- OMA: Orthogonal multiple access
- SIC: Successive interference cancellation
- SINR: Signal to interference plus noise ratio
- SNR: Signal to noise ratio
- ZFBF: Zero forcing beamforming

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