Superconducting dark energy

Shi-Dong Liang‡ and Tiberiu Harko†

1 State Key Laboratory of Optoelectronic Material and Technology, and Guangdong Province Key Laboratory of Display Material and Technology, School of Physics and Engineering, Sun Yat-Sen University, Guangzhou 510275, People’s Republic of China and
2 Department of Mathematics, University College London, Gower Street, London, WC1E 6BT, United Kingdom

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Based on the analogy with superconductor physics we consider a scalar-vector-tensor gravitational model, in which the dark energy action is described by a gauge invariant electromagnetic type functional. By assuming that the ground state of the dark energy is in a form of a condensate with the U(1) symmetry spontaneously broken, the gauge invariant electromagnetic dark energy can be described in terms of the combination of a vector and of a scalar field (corresponding to the Goldstone boson), respectively. The gravitational field equations are obtained by also assuming the possibility of a non-minimal coupling between the cosmological mass current and the superconducting dark energy. The cosmological implications of the dark energy model are investigated for a Friedmann-Robertson-Walker homogeneous and isotropic geometry for two particular choices of the electromagnetic type potential, corresponding to a pure electric type field, and to a pure magnetic field, respectively. The time evolutions of the scale factor, matter energy density and deceleration parameter are obtained for both cases, and it is shown that in the presence of the superconducting dark energy the Universe ends its evolution in an exponentially accelerating vacuum de Sitter state. By using the formalism of the irreversible thermodynamic processes for open systems we interpret the generalized conservation equations in the superconducting dark energy model as describing matter creation. The particle production rates, the creation pressure and the entropy evolution are explicitly obtained.

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I. INTRODUCTION

The Λ Cold Dark Matter (ΛCDM) model of cosmology is remarkably successful in accounting for most of the observed features of the Universe. The recent Planck satellite data from the 2.7 degree Cosmic Microwave Background full sky survey [1, 2] have generally confirmed again the present day standard cosmological paradigm. However, a number of fundamental questions at the very foundations of cosmology and gravitation still remain open, and unanswered. Perhaps the most important challenge facing modern cosmology is the understanding of the mechanism of the acceleration of the late universe, which is usually attributed to the presence of the mysterious dark energy. In fact, as fundamental candidates responsible for the cosmic expansion, the standard ΛCDM model of cosmology has favored dark energy models involving time-dependent scalar fields. Scalar fields naturally arise in many particle physics models, including string theory. On the other hand, the underlying dynamics of inflationary models, assumed to be of fundamental importance for the understanding of the early history of the Universe, also depend essentially on a single scalar field, the inflaton, rolling in some underlying potential [5]. The possibility that a single canonical scalar field φ, with a non-zero potential, called quintessence, could be responsible for the late-time cosmic acceleration, was also explored in much detail [8–7]. The well-known action for a scalar field in the presence of gravity is

\[ S_\phi = \int \left[ \frac{R}{16\pi G} - \frac{1}{2} \nabla^\alpha \phi \nabla_\alpha \phi - V(\phi) \right] \sqrt{-gd^4x}, \quad (1) \]

where \( R \) is the Ricci scalar, \( G \) is the gravitational constant, and \( V(\phi) \) is the self-interaction potential, respectively [9].

In opposition to the behavior of the cosmological constant, the quintessence equation of state changes dynamically with time [9]. In fact, many other exotic fluids have been proposed to explain the accelerated expansion of the Universe. Some of the proposed models are the so-called k—essence models, in which the late-time acceleration is driven by the kinetic energy term of the scalar field [10]. A number of coupled models, where dark energy interacts both quantitatively and qualitatively with dark matter, have also been proposed [11], as well as unified models of dark matter and dark energy [12]. For a review of the dark energy candidates see [13].

An intriguing alternative about the nature of dark energy, which was also intensively investigated in the literature, is the possibility that it could be described by a vector field, which can be at the origin of the present stage of cosmic acceleration. In its simplest formulation
the action for the vector field dark energy model is
\[
S_V = \int d^4x \sqrt{-g} \left\{ \frac{R}{16\pi G} - \sum_{a=1}^{3} \left[ \frac{1}{4} F^a_{\mu\nu} F^a{^{\mu\nu}} + V(A^2) \right] + L_m \right\},
\]
where \( F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu, A^2 = g^{\mu\nu} A^a_\mu A^a_\nu, \) and \( L_m \) is the matter Lagrangian \[14\]. This vector (or more exactly Yang-Mills) type action for the dark energy thus contains three identical components obtained by generalizing the Lagrangian of a single vector field. The term \( V(A^2) \) is a self-interaction potential that explicitly violates gauge invariance. The cosmological implications of the vector type dark energy models have been investigated in \[12\].

More general vector field dark energy models, in which the vector field is non-minimally coupled to the gravitational field, have been proposed in \[16\]. By assuming that the Universe is filled with a massive cosmological vector field, with mass \( m_A \), which is characterized by a four-potential \( A^\mu(x^\nu), \mu, \nu = 0, 1, 2, 3 \), which couples non-minimally to gravity, and by introducing, in analogy with electrodynamics, the field tensor \( C_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu \), the action for the non-minimally coupled vector dark energy theory can be written as
\[
S = -\int \left[ R + C_{\mu\nu} C^{\mu\nu} + \frac{1}{2} \mu^2 A_\mu A^\mu + \omega A_\mu A^\mu R + \eta A_\mu A^\mu R_{\mu\nu} + 16\pi G_0 L_m \right] \sqrt{-g} d^4x \Omega,
\]
where \( R_{\mu\nu} \) is the Ricci tensor and \( G_0 \) is the gravitational constant. In Eq. \(3\) \( \omega \) and \( \eta \) are dimensionless coupling parameters.

At first sight the gravitational actions given by Eqs. \[11\] and \[12\] look totally different, from both mathematical point of view, as well as from the physical interpretation point of view. However, they can be in fact interpreted and understood as the limiting cases of a single physical model, related to the spontaneous breaking of the electromagnetic U(1) symmetry. Thus an approach is used to describe superconductivity from a fundamental point of view \[17, 18\].

From a general physical point of view in the theory of superconductivity the existence of a quantum condensate (superconducting state) is described by a non-vanishing value of a gauge dependent complex order parameter \[17, 18\]. In bosonic systems superfluid behavior occurs when the expectation value of the bosonic field parameter \( \psi \) has a nonzero value, \( \langle \psi \rangle \neq 0 \). On the other hand the existence of superconductivity is also induced by a nonzero value of the expectation value of the pair field operator. Therefore, in the ground state of a superconducting system a quantum condensate \( \langle \epsilon_{\alpha\beta} \psi^\alpha \psi^\beta \rangle \) forms \[17, 18\]. Since the dimeron operator has charge \(-2e\), the important result that the quantum condensate breaks the electromagnetic U(1) symmetry is obtained. Another fundamental quantity in the model is a scalar field, \( \Phi \), which plays the role of the order parameter. Under a gauge transformation \( A_\mu \to A_\mu + \partial_\mu \Lambda \), the scalar field transforms like the condensate wave function \( \psi \to e^{ie\Lambda} \psi \Rightarrow \Phi \to e^{2ie\Lambda} \Phi \). Note that in the zero temperature superconductivity theory one also introduces the Goldstone field \( \phi \) as the phase of the field \( \Phi, \Phi = \rho e^{i\phi/\pi} \), as well as the gauge invariant Fermi fields, \( \psi = e^{-ie\phi} \).

Hence from a fundamental point of view a superconducting system can be described by a gauge invariant Lagrangian, depending on the wave function \( \psi \), and on the vector potentials \( A_\mu \) and \( \nabla_\mu \phi \). A simplified model is obtained after integrating out the Fermi fields. Thus one obtains a gauge invariant Lagrangian, depending only on \( A_\mu \) and \( \nabla_\mu \phi \), respectively. The important requirement of the gauge invariance of the theory implies that these bosonic fields must appear only in the combinations \( F^{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu \) and \( A_\mu - \nabla_\mu \phi \), respectively. Therefore the Lagrangian describing a superconductor from a fundamental physical point of view has the form \[17, 18\]
\[
L = -\frac{1}{4} \int F^{\mu\nu} F^{\mu\nu} d^4x + L_s (A_\mu - \nabla_\mu \phi),
\]
where \( L_s (A_\mu - \nabla_\mu \phi) \) is an arbitrary function of the argument \( A_\mu - \nabla_\mu \phi \). The only physical condition required on the superconductor Lagrangian \( L_s \) is that in the absence of \( A^0 \) and \( \phi \) it gives rise to a stable state of the system. In particular, this requires that the point \( A_\mu = \nabla_\mu \phi \) is a local minimum of the theory (this property can fully explain the Meissner effect in superconductivity theory \[17\]). Therefore, we require that the second derivative of the superconductor Lagrangian \( L_s \) with respect to its argument must be nonzero at the point \( A_\mu = \nabla_\mu \phi \) \[17, 18\].

It is the goal of the present paper to consider a gravitational model in which dark energy is described by a Lagrangian of the form given by Eq. \[3\], resulting from the breaking of the U(1) symmetry in the ground state dark energy condensate. By analogy with condensed matter physics we call this model the \textit{superconducting dark energy model}. The gravitational field equations of the model are derived from an action principle, and the cosmological implications are investigated in a background homogeneous and isotropic flat Friedmann-Robertson-Walker geometry. We consider two distinct classes of cosmological models, corresponding to two different choices of the electromagnetic potential \( A_\mu \) of the dark energy. In the first model \( A_\mu \) has only a non-vanishing temporal component, while in the second case we assume non-vanishing spatial (magnetic) components of the potential. In both cases we assume that the dark energy self-interaction potential is constant.

In the present dark energy model, due to the coupling between the matter current and the electromagnetic and scalar potentials of the dark energy, the matter energy-momentum tensor is not conserved. By using the formalism of the open thermodynamic systems introduced in
see also [20] for recent investigations of particle creation in cosmology), we interpret the generalized conservation equations in the superconducting dark energy model from a thermodynamic point of view as describing irreversible matter creation processes. Thus in the present model particle creation corresponds to an irreversible energy flow from the superconducting dark energy to the created matter constituents (both normal and dark). We explicitly obtain the equivalent particle number creation rates, the creation pressure and the entropy production rates. The temperature evolution laws of the newly created particles are explicitly derived. We also show that due to the superconducting dark energy - matter current coupling, during the cosmological evolution a large amount of comoving entropy could be produced.

The present paper is organized as follows. In Section II the gravitational field equations of the superconducting dark energy model, a scalar-vector-tensor theory with broken U(1) symmetry, are derived from a variational principle. The equations of motion of the scalar and vector fields are also obtained. The cosmological applications of the theory are investigated in Section III. Two distinct dark energy models are considered: an electric type, in which the vector potential has only a time component, and a magnetic type, with the vector potential having only spatial components. The cosmological properties of both models are investigated in detail. The thermodynamic interpretation of the superconducting dark energy model is considered, in the framework of the thermodynamic of open systems and irreversible processes, in Section V. We discuss and conclude our results in Section VI. In this paper we adopt the Landau-Lifshitz [20] metric conventions, and we use the natural system of units with $8\pi G = c = 1$.

II. FIELD EQUATIONS OF THE SUPERCONDUCTING DARK ENERGY MODEL

In the following we assume that the interaction of the gravitational and of the superconducting dark energy scalar-vector fields is described by a Lagrangian which is required to satisfy the following standard conditions: a) the Lagrangian density is a four-scalar b) the free-field energies are positive-definite for all the metric, scalar and vector fields c) the resulting theory is metric and d) the field equations contain no higher than second order derivatives of the fields [21]. Based on the analogy with superconductor physics we consider a gravitational scalar-vector-tensor action of the form

$$S = - \int \left[ \frac{R}{2} + \frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{2} g^{\mu\nu} \times (A_\mu - \nabla_\mu \phi) (A_\nu - \nabla_\nu \phi) + V(A^2, \phi) - \frac{\alpha}{2} g^{\mu\nu} j_\mu (A_\nu - \nabla_\nu \phi) + L_m (g_{\mu\nu}, \psi) \right] \sqrt{-g} d\Omega, \quad (5)$$

where $\lambda$ and $\alpha$ are constants, $L_m (g_{\mu\nu}, \psi)$ is the Lagrangian of the total (ordinary baryonic plus dark) matter, and $j^\mu = \rho u^\mu$ is the total mass current, where $\rho$ is the total matter density (including dark matter), and $u^\mu$ is the matter four-velocity. We assume that the baryonic and dark matter are comoving. The third term in the action Eq. (5) follows from the assumption that the superconducting dark energy is close to the minimum $A_\mu = \nabla_\mu \phi$. In this case the general superconducting Lagrangian [21] can be expanded in power series as

$$L_s (A_\mu - \nabla_\mu \phi) \approx L_0 + \frac{\delta^2 L_s}{\delta (A_\mu - \nabla_\mu \phi)^2} (A_\mu - \nabla_\mu \phi)^2 + \ldots, \quad (6)$$

where $L_0$ is a constant. Hence the superconducting type Lagrangian $L_s (A_\mu - \nabla_\mu \phi)$ gives a quadratic contribution in $A_\mu - \nabla_\mu \phi$ to the gravitational Lagrangian. We have also assumed the possibility of an interaction between the total matter flux $j^\mu$ and the superconducting dark energy gauge invariant potentials $A_\mu - \nabla_\mu \phi$. $V(A^2, \phi)$ is the self-interaction potential of the scalar and vector fields, with $A^2 = A_\mu A^\mu$, in which we have also included the constant $L_0$. When $\phi \equiv 0$, that is, the scalar field vanishes, the action (5) gives the pure vector model of the dark energy. When the electromagnetic type potential $A^\mu = 0$, we recover the standard action of the minimally coupled scalar-tensor theory. Hence the gravitational action (5) gives a unified framework for the minimal inclusion into the gravitational action of the scalar-vector interactions, under the assumption of the existence of a U(1) broken symmetry. The second and third terms in the gravitational action are also similar to the Stueckelberg Lagrangian [27].

We define the energy-momentum tensor of the matter as

$$T_{\mu\nu} = - \frac{2}{\sqrt{-g}} \left[ \frac{\partial}{\partial x^\lambda} \left( \sqrt{-g} L_m \right) \frac{\partial}{\partial g_{\mu\nu}^\lambda} - \frac{\partial}{\partial g_{\mu\nu}^\lambda} \frac{\partial}{\partial \left( \sqrt{-g} L_m \right)} \right]. \quad (7)$$

By making the important assumption that the Lagrangian density $L_m$ of the matter depends only on the metric tensor components $g_{\mu\nu}$, and not on its derivatives, we obtain the expression $T_{\mu\nu} = L_m g_{\mu\nu} - 2\partial L_m / \partial g^{\mu\nu}$. By varying the action (5) with respect to the metric tensor we obtain the gravitational field equations for the superconducting dark energy model as

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu} + \frac{4}{\pi} \left( - F_{\mu\alpha} F^\alpha_\nu + \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} g_{\mu\nu} \right) +$$

$$\lambda (A_\mu - \nabla_\mu \phi) (A_\nu - \nabla_\nu \phi) - \frac{\lambda}{2} (A^\alpha - \nabla^\alpha \phi) (A_\alpha - \nabla_\alpha \phi) \times$$

$$g_{\mu\nu} + \alpha j_\mu (A_\nu - \nabla_\nu \phi) - \frac{\alpha}{2} j^\beta (A_\beta - \nabla_\beta \phi) g_{\mu\nu} +$$

$$V(A^2, \phi) g_{\mu\nu}, \quad (8)$$

where $T_{\mu\nu}$, the energy-momentum tensor of the ordinary matter, is given by

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu - pg_{\mu\nu}, \quad (9)$$
where $p$ is the total thermodynamic pressure of the matter components (baryonic and dark). By taking the variation of the action Eq. (5) with respect to the scalar field $\phi$ we obtain

$$
\delta_{\phi}S = - \int \left[ \lambda A^{\mu} \nabla_{\mu} \delta \phi - \lambda g^{\mu\nu} \nabla_{\mu} \delta \phi \nabla_{\nu} \phi + \frac{\alpha}{2} j^{\mu} \nabla_{\mu} \delta \phi + \partial_{\phi} V \left( A^{2}, \phi \right) \delta \phi \right] \sqrt{-g} d\Omega. \tag{10}
$$

With the use of the mathematical identity

$$
\nabla_{\mu} (B^{\mu} \delta \phi) = \nabla_{\mu} B^{\mu} \delta \phi + B^{\mu} \nabla_{\mu} \delta \phi, \tag{11}
$$

it follows that if the conditions

$$
\nabla_{\mu} A^{\mu} = 0, \nabla_{\mu} j^{\mu} = 0, \tag{12}
$$

are imposed on the dark energy vector potential, and on the baryonic matter flow, the variation in the integral of the first term, containing $A^{\mu}$, and of the last term, containing $j^{\mu}$, vanish identically. Therefore we obtain the result that if the conditions given by Eqs. (12) are satisfied, then the scalar field satisfies the standard Klein-Gordon equation,

$$
\lambda g^{\mu\nu} \nabla_{\mu} \nabla_{\nu} \phi + \partial_{\phi} V \left( A^{2}, \phi \right) = 0. \tag{13}
$$

This case corresponds to the minimal coupling of the scalar and vector fields. However, in the following we will use a more general approach, in which no additional constraints are imposed on the fields or on the hydrodynamic flow. Therefore, the variation of the action Eq. (5) gives the following coupled evolution equation for the scalar and vector fields,

$$
\lambda g^{\mu\nu} \nabla_{\mu} \nabla_{\nu} \phi + \partial_{\phi} V \left( A^{2}, \phi \right) - \lambda \nabla_{\mu} A^{\mu} - \frac{\alpha}{2} \nabla_{\mu} j^{\mu} = 0. \tag{14}
$$

By varying the superconducting dark energy action Eq. (6) with respect to $A_{\mu}$, we obtain first

$$
\delta_{A_{\mu}} S = - \int \left[ - \frac{1}{4 \pi} F^{\mu\nu} \nabla_{\nu} \delta A_{\mu} - \lambda g^{\mu\nu} \left( A_{\mu} - \nabla_{\mu} \phi \right) \delta A_{\mu} - \frac{\alpha}{2} j^{\mu} A_{\mu} + 2 \partial_{A^{2}} V \left( A^{2}, \phi \right) A^{\mu} \delta A_{\mu} \right] \sqrt{-g} d\Omega = 0. \tag{15}
$$

By taking into account the identity $\nabla_{\nu} (F^{\mu\nu} \delta A_{\mu}) = \nabla_{\nu} F^{\mu\nu} \delta A_{\mu} + F^{\mu\nu} \nabla_{\nu} \delta A_{\mu}$, after partial integration and the use of Gauss’ theorem, it follows that the superconducting dark energy vector field satisfies the equation

$$
\frac{1}{4 \pi} \nabla_{\nu} F^{\mu\nu} = J^{\mu}, \tag{16}
$$

where

$$
J^{\mu} = \left[ \lambda g^{\mu\nu} \left( A_{\nu} - \nabla_{\nu} \phi \right) + \frac{\alpha}{2} j^{\mu} - 2 \partial_{A^{2}} V \left( A^{2}, \phi \right) A^{\mu} \right]. \tag{17}
$$

The divergence of the dark energy field tensor can be obtained as $\nabla_{\nu} F^{\mu\nu} = (1/\sqrt{-g}) \partial_{\nu} (\sqrt{-g} F^{\mu\nu})$.

By its definition the dark energy electromagnetic type tensor $F^{\mu\nu}$ satisfies the Bianchi identity

$$
\varepsilon^{\alpha\beta\mu\nu} \nabla_{\beta} F_{\mu\nu} = 0, \tag{18}
$$

where $\varepsilon^{\alpha\beta\mu\nu}$ is the complete antisymmetric unit tensor of rank four.

Finally, by taking the covariant derivative of the field equations Eqs. (5) we obtain the matter conservation equation in the presence of a superconducting dark energy as

$$
\nabla_{\mu} T^{\mu}_{\nu} + \frac{\alpha}{2} \nabla_{\mu} \left[ j^{\mu} \left( A_{\nu} - \nabla_{\nu} \phi \right) \right] - \frac{\alpha}{2} \nabla_{\nu} j^{\beta} \left( A_{\beta} - \nabla_{\beta} \phi \right) + \partial_{\phi} V \left( A^{2}, \phi \right) A_{\nu} + 2 \partial_{A^{2}} V \left( A^{2}, \phi \right) A^{\nu} \nabla_{\alpha} A_{\nu} = 0. \tag{19}
$$

The derivation of Eq. (19) is presented in Appendix A.

By taking into account the explicit form of the energy-momentum tensor, given by Eq. (9) we obtain

$$
\left( \nabla^{\mu} \rho + \nabla^{\mu} p \right) u_{\mu} u_{\nu} + (\rho + p) u_{\mu} \nabla^{\mu} u_{\nu} + \nabla^{\mu} \nabla^{\lambda} u_{\mu} \left( A_{\nu} - \nabla_{\nu} \phi \right) \right] - \frac{\alpha}{2} \nabla_{\nu} j^{\beta} \left( A_{\beta} - \nabla_{\beta} \phi \right) + \partial_{\phi} V \left( A^{2}, \phi \right) u^{\nu} A_{\nu} + 2 \partial_{A^{2}} V \left( A^{2}, \phi \right) u^{\nu} A^{\nu} A_{\alpha} A_{\nu} = 0, \tag{20}
$$

By multiplying Eq. (20) with $u^{\nu}$, and by taking into account the mathematical identity $u^\nu \nabla^\mu u_\mu = 0$ we obtain the energy conservation equation in the superconducting dark energy model as

$$
\dot{\rho} + 3 (\rho + p) H + \frac{\alpha}{2} u^{\nu} \nabla_{\mu} \left[ j^{\mu} \left( A_{\nu} - \nabla_{\nu} \phi \right) \right] = \frac{\alpha}{2} \frac{d}{ds} \left[ j^{\beta} \left( A_{\beta} - \nabla_{\beta} \phi \right) \right] + \partial_{\phi} V \left( A^{2}, \phi \right) u^{\nu} A_{\nu} + 2 \partial_{A^{2}} V \left( A^{2}, \phi \right) u^{\nu} A^{\nu} \nabla_{\alpha} A_{\nu} = 0, \tag{21}
$$

where we have introduced the Hubble function $H = (1/3) \nabla^{\mu} u_{\mu}$, and we have denoted $' = u^{\nu} \nabla_{\mu} = d/ds$, respectively, where $ds$ is the line element corresponding to the metric $g_{\mu\nu}$, $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$.

By multiplying Eq. (21) with the projection operator $h^\nu_{\lambda}$, defined as $h^\nu_{\lambda} = \delta^\nu_{\lambda} - u_{\lambda} u^{\nu}$, and satisfying the relation $u_{\nu} h^\nu_{\lambda} = 0$, gives the momentum balance equation for a perfect fluid in the superconducting dark energy model as

$$
\begin{align*}
\dot{\rho} + 3 (\rho + p) H &+ \frac{\alpha}{2} u^{\nu} \nabla_{\mu} \left[ j^{\mu} \left( A_{\nu} - \nabla_{\nu} \phi \right) \right] = \frac{\alpha}{2} \frac{d}{ds} \left[ j^{\beta} \left( A_{\beta} - \nabla_{\beta} \phi \right) \right] + \partial_{\phi} V \left( A^{2}, \phi \right) u^{\nu} A_{\nu} + 2 \partial_{A^{2}} V \left( A^{2}, \phi \right) u^{\nu} A^{\nu} \nabla_{\alpha} A_{\nu} \\
&= \frac{\alpha}{2} \nabla_{\nu} \left[ j^{\mu} \left( A_{\nu} - \nabla_{\nu} \phi \right) \right] + \partial_{\phi} V \left( A^{2}, \phi \right) u^{\nu} A_{\nu} + 2 \partial_{A^{2}} V \left( A^{2}, \phi \right) u^{\nu} A^{\nu} \nabla_{\alpha} A_{\nu} .
\end{align*} \tag{22}
$$

III. COSMOLOGICAL APPLICATIONS

We assume that the metric of the Universe is given by the isotropic and homogeneous Friedmann-Robertson-
Walker metric,

\[ ds^2 = dt^2 - a^2(t) \left( dx^2 + dy^2 + dz^2 \right), \]  

(23)

where \( a(t) \) is the scale factor describing the expansion of the Universe. We assume that the cosmological matter is comoving with the cosmological expansion, and therefore we choose the four velocity of the cosmological fluid as \( u^\mu = (1, 0, 0, 0) \). Hence the components of the four-current vector are \( j^\mu = (\rho, 0, 0, 0) \). In the Friedmann-Robertson-Walker geometry the Hubble function takes the form \( H = \dot{a}/a \), since \( u^\mu \nabla_\mu = \cdot = d/dt \). To describe the decelerating/accelerating nature of the cosmological expansion, we use the deceleration parameter \( q \), with the definition

\[ q = \frac{d}{dt} \frac{1}{H} - 1 = -\frac{\ddot{H}}{H^2} - 1. \]  

(24)

Moreover, from the homogeneity of the Universe it follows that the scalar and vector fields \( \phi \) and \( A_\mu \) can be only functions of the cosmological time \( t \), so that \( \phi = \phi(t) \) and \( A_\mu = A_\mu(t) = (A_0(t), A_1(t), A_2(t), A_3(t)) \), respectively. The non-zero components of the dark energy tensor \( F_{\mu \nu} \) are given by \( F_{0i}(t) = -\dot{A}_i(t) \), \( i = 1, 2, 3 \), and \( F^{00}(t) = \dot{A}_i(t)/a^2(t), \ i = 1, 2, 3 \). Hence we obtain

\[ F_{\alpha \beta} F^{\alpha \beta} = -\left(2/a^2(t)\right) \sum_{i=1}^3 \dot{A}_i(t)^2. \]  

Then, the cosmological equations corresponding to the superconducting dark energy model are given by

\[ 3H^2 = \left[1 + \frac{\alpha}{2} \left( A_0 - \dot{\phi} \right)\right] \rho + \frac{1}{8\pi a^2(t)} \left( \sum_{k=1}^3 \dot{A}_k^2 \right) + \frac{\lambda}{2} \left( A_0 - \dot{\phi} \right)^2 + \frac{\lambda}{2a^2(t)} \left( \sum_{k=1}^3 \dot{A}_k^2 \right), \]

\[ 2\dot{H} + 3H^2 = -p - \frac{1}{4\pi a^2(t)} \dot{A}_0^2 - \frac{1}{8\pi a^2(t)} \left( \sum_{k=1}^3 \dot{A}_k^2 \right) - \frac{\lambda}{2a^2(t)} \left( A_0 - \dot{\phi} \right)^2 + \frac{\lambda}{2a^2(t)} \left( \sum_{k=1}^3 \dot{A}_k^2 \right) - \frac{\alpha}{2} \left( A_0 - \dot{\phi} \right) \rho + V(A^2)\phi, \]

\[ \lambda \dot{\phi} - 3 \left[ \frac{\alpha}{2} \rho + \lambda \left( A_0 - \dot{\phi} \right) \right] H - \lambda \dot{A}_0 - \frac{\alpha}{2} \dot{\phi} + \partial_\phi V(A^2, \phi) = 0, \]

\[ \lambda \left( A_0 - \dot{\phi} \right) + \frac{\alpha}{2} \rho - 2\partial A^2 V(A^2, \phi) = 0. \]

(25)

As an independent variable we introduce, instead of the cosmological time \( t \), the redshift \( z \), defined as \( 1 + z = 1/a \). Therefore

\[ \frac{dH}{dt} = \frac{dH}{dz} \frac{dz}{dt} = -(1 + z)H \frac{dH}{dz}. \]  

(30)

As a function of the redshift the deceleration parameter is obtained as

\[ q = (1 + z) \frac{1}{H(z)} \frac{dH(z)}{dz} - 1. \]  

(31)

In the following we will explicitly investigate two distinct superconducting dark energy models.

### A. Electric dark energy models

We assume that the dark energy vector potential has the form \( A_\mu = (A_0(t), 0, 0, 0) \), that is, the dark energy vector potential has only one, electric type, component. For this choice \( F_{\mu \nu} \equiv 0 \), \( \forall \mu, \nu \in \{0, 1, 2, 3\} \). The gravitational field equations describing the cosmological dynamics in the presence of the superconducting dark energy take the form

\[ 3H^2 = \left[1 + \frac{\alpha}{2} \left( A_0 - \dot{\phi} \right)\right] \rho + \frac{\lambda}{2} \left( A_0 - \dot{\phi} \right)^2 + V(A^2, \phi), \]

\[ 2\dot{H} + 3H^2 = -p - \frac{\alpha}{2} \left( A_0 - \dot{\phi} \right)^2 - \frac{\alpha}{2} \left( A_0 - \dot{\phi} \right) \rho + V(A^2, \phi), \]

\[ \frac{d}{dt} \left( a^2 \left[ \lambda \left( A_0 - \dot{\phi} \right) + \frac{\alpha}{2} \rho \right] \right) - a^3 \partial_\phi V(A^2, \phi) = 0, \]

\[ \lambda \left( A_0 - \dot{\phi} \right) + \frac{\alpha}{2} \rho - 2\partial A^2 V(A^2, \phi) = 0. \]

(32)

(33)

(34)

(35)

In order to close the system of equations \( (32)-(35) \), the baryonic equation of state \( p = p(\rho) \) must also be provided. In the following we will restrict our analysis to the case of a constant self-interaction potential of the superconducting dark energy field, \( V(A^2, \phi) = V_0 = \text{constant} \). Then from Eqs. \( (34) \) and \( (35) \) we obtain

\[ \left( A_0 - \dot{\phi} \right) = -\frac{\alpha}{2\lambda} \rho. \]

(36)

Hence the generalized Friedmann equations of the cosmological expansion in the presence of the electric type superconducting dark energy become

\[ 3H^2 = \rho - \frac{\alpha^2}{8\lambda} \rho^2 + V_0 = \rho + \rho_{DE}, \]

\[ 2\dot{H} + 3H^2 = -p + \frac{\alpha^2}{8\lambda} \rho^2 + V_0 = -p - \rho_{DE}. \]

(37)

(38)
where we have denoted
\[ \rho_{DE} = \frac{\alpha^2}{8\lambda} \rho^2 + V_0, \tag{39} \]
and
\[ p_{DE} = -\frac{\alpha^2}{8\lambda} \rho^2 - V_0, \tag{40} \]
respectively. From Eqs. (37) and (38) we obtain
\[ 2\dot{H} = - (\rho + p) + \frac{\alpha^2}{4\lambda} \rho^2. \tag{41} \]
The energy conservation equation can be written as
\[ \frac{d}{dt} \left[ \left( \rho - \frac{\alpha^2}{8\lambda} \rho^2 \right) a^3 \right] + \left( p - \frac{\alpha^2}{8\lambda} \rho^2 \right) \frac{da}{d\tau} = 0. \tag{42} \]
The deceleration parameter can be obtained as
\[ q = \frac{(\rho + 3p) - (\alpha^2/2\lambda) \rho^2 - 2V_0}{2[\rho - (\alpha^2/\lambda) \rho^2 + V_0]}. \tag{43} \]
For a dust Universe with \( p = 0 \), the deceleration parameter takes the form
\[ q = \frac{1}{2} \frac{\rho - (\alpha^2/2\lambda) \rho^2 - 2V_0}{\rho - (\alpha^2/\lambda) \rho^2 + V_0}. \tag{44} \]
From the field equations Eqs. (37), (38), we obtain the time evolution equation of the baryon density as
\[ \dot{\rho} = -\sqrt{3} \rho \left[ \rho - (\alpha^2/8\lambda) \rho^2 + V_0 \right] \left\{ \rho + \rho \left[ 1 - (\alpha^2/4\lambda) \rho \right] \right\}. \tag{45} \]
In terms of the redshift \( z \), the density evolution equation of the baryon density in the electric type superconducting dark energy models is given by
\[ \frac{dp(z)}{dz} = \frac{3}{1+z} \left[ p(z) + \rho(z) \left[ 1 - (\alpha^2/4\lambda) \rho(z) \right] \right]. \tag{46} \]
In order to characterize the dark energy, and its evolution properties, we also introduce the dark energy equation of state parameter \( w_{DE} \), defined as
\[ w_{DE} = \frac{p_{DE}}{\rho_{DE}} = -\frac{\left( \alpha^2/8\lambda \right) \rho^2 + V_0}{\left( \alpha^2/8\lambda \right) \rho^2 - V_0}. \tag{47} \]

1. Dust Universes with electric type superconducting dark energy

In the case of the dust Universe, with \( p = 0 \), Eq. (45), describing the time dynamics of the matter density in the presence of the superconducting electric type dark energy takes the form
\[ \dot{\rho} = -\sqrt{3} \rho \sqrt{\rho - (\alpha^2/8\lambda) \rho^2 + V_0}. \tag{48} \]
By introducing a set of dimensionless variables \( (\theta, \tau, h, v_0) \), defined as \( \rho = (8\lambda/\alpha^2) \theta \), \( t = \alpha/\sqrt{24\lambda} \theta \), \( H = \left( \sqrt{8\lambda/\sqrt{3}} \right) h \), \( v_0 = (\alpha^2/8\lambda) V_0 \), Eq. (45) becomes
\[ \frac{d\theta}{d\tau} = -\theta \sqrt{\theta - \theta^2 + v_0}, \tag{49} \]
while the dimensionless Hubble parameter \( h \) is obtained as
\[ h = \sqrt{\theta - \theta^2 + v_0}. \tag{50} \]
In the dimensionless time variable \( \tau \) we have \( h(\tau) = [3/a(\tau)](da/d\tau) \). The deceleration parameter of this model is given by
\[ q = \frac{1}{2} \frac{\theta - 4\theta^2 - v_0}{\theta - \theta^2 + v_0}, \tag{51} \]
while the dark energy equation of state parameter \( w_{DE} \) can be obtained as
\[ w_{DE} = -\frac{\theta^2 + v_0}{\theta^2 + v_0}. \tag{52} \]
The time variation of the redshift \( z \) can be obtained from the equation
\[ \frac{dz}{d\tau} = -(1+z) \frac{1}{a} \frac{da}{d\tau} = - \frac{1+z}{3} \sqrt{\theta - \theta^2 + v_0}. \tag{53} \]
In terms of the redshift \( z \) the evolution of the dust electric type superconducting dark energy Universe is described by the simple relations
\[ \rho(z) = \rho_0 (1+z)^3, \tag{54} \]
\[ H(z) = \frac{1}{\sqrt{3}} \left[ \rho_0 (1+z)^3 - \frac{\alpha^2 \rho_0^2}{8\lambda} (1+z)^6 + V_0 \right]^{1/2}, \tag{55} \]
\[ q(z) = \frac{1}{2} \frac{\rho_0 (1+z)^3 - (\alpha^2 \rho_0^2/2\lambda) (1+z)^6 - 2V_0}{\rho_0 (1+z)^3 - (\alpha^2 \rho_0^2/8\lambda) (1+z)^6 + V_0}, \tag{56} \]
where \( \rho_0 \) is the matter density of the Universe at the present time \( z = 0 \).

The variations with respect to the redshift \( z \) of the Hubble function \( h \) of the Universe, of the matter energy density \( \theta \), of the deceleration parameter \( q \), and of the parameter of the dark energy equation of state are represented, for different values of \( v_0 \), in Figs. 11-14. The initial values used to numerically integrate the cosmological evolution equations are \( \theta(0) = 0.1, z(0) = 5, \) and \( a(0) = 1/6, \) respectively.

As one can see from Fig. 11, the Hubble function \( h \) of the Universe is a monotonically increasing function of the redshift (monotonically time decreasing function). In the early stages of evolution, at around \( z \approx 5, h \) is basically
FIG. 1: Variation with respect to the redshift \( z \) of the dimensionless Hubble function \( h \) of the electric type superconducting dark energy filled Universe with \( p = 0 \) for different values of \( v_0 \): \( v_0 = 0.001 \) (solid curve), \( v_0 = 0.002 \) (dotted curve), \( v_0 = 0.003 \) (short dashed curve), \( v_0 = 0.004 \) (dashed curve), and \( v_0 = 0.005 \) (long dashed curve), respectively.

FIG. 2: Variation with respect to the redshift \( z \) of the dimensionless matter energy density \( \rho \) of the electric type superconducting dark energy filled Universe with \( p = 0 \) for different values of \( v_0 \): \( v_0 = 0.001 \) (solid curve), \( v_0 = 0.002 \) (dotted curve), \( v_0 = 0.003 \) (short dashed curve), \( v_0 = 0.004 \) (dashed curve), and \( v_0 = 0.005 \) (long dashed curve), respectively.

FIG. 3: Variation with respect to the redshift \( z \) of the deceleration parameter \( q \) of the electric type superconducting dark energy filled Universe with \( p = 0 \) for different values of \( v_0 \): \( v_0 = 0.001 \) (solid curve), \( v_0 = 0.002 \) (dotted curve), \( v_0 = 0.003 \) (short dashed curve), \( v_0 = 0.004 \) (dashed curve), and \( v_0 = 0.005 \) (long dashed curve), respectively.

FIG. 4: Variation with respect to the redshift \( z \) of the parameter \( w_{DE} \) of the dark energy equation of state of the electric type superconducting dark energy filled Universe with \( p = 0 \) for different values of \( v_0 \): \( v_0 = 0.001 \) (solid curve), \( v_0 = 0.002 \) (dotted curve), \( v_0 = 0.003 \) (short dashed curve), \( v_0 = 0.004 \) (dashed curve), and \( v_0 = 0.005 \) (long dashed curve), respectively.

Independent on the values of \( v_0 \). The matter density of the Universe, presented in Fig. 2, is a monotonically increasing function of \( z \), tending in the small \( z \) limit to zero, \( \lim_{z \to 0} \rho(z) = 0 \). Its evolution is basically independent of the range if the numerical values of \( v_0 \). The redshift variation of the deceleration parameter \( q \), depicted in Fig. 3, shows that in the present model the Universe starts from a decelerating phase at around \( z \approx 5 \), with \( q \) having values of the order of \( q \approx 0.2 - 0.3 \). This initial value increases in the early stages of the cosmological evolution, showing a decelerating expansion. At \( z \approx 1 - 2 \), the Universe starts to accelerate, with the decelerating parameter slightly decreasing and taking negative values \( q < 0 \). The values of the deceleration parameter gradually decrease with decreasing \( z \), and the Universe enters into an accelerating stage, ending its evolution in a de Sitter stage, with \( q \approx -1 \) at \( z \approx 0 \). The parameter \( w_{DE} \) of the equation of state of the dark energy, presented in Fig. 4, starts with positive values, and, with decreasing \( z \), it takes negative values. In the small redshift limit it tends to the value \( w_{DE} = -1 \).

The present day numerical values of the Hubble function \( H_0 \) and of the deceleration parameter \( q_0 \) can be obtained as

\[
H_0 = \left[ \rho_0 - \frac{(\alpha^2 \rho_0^2 / 8 \lambda) + V_0}{\sqrt{3}} \right]^{1/2}, \tag{57}
\]

and

\[
q_0 = \frac{1}{2} \left[ \frac{\rho_0 - (\alpha^2 \rho_0^2 / 8 \lambda) - 2V_0}{\rho_0 - (\alpha^2 \rho_0^2 / 8 \lambda) + V_0} \right], \tag{58}
\]

respectively. Therefore the free parameters \( \alpha \) and \( \lambda \) of
the superconducting electric type dark energy model can be obtained from astronomical observations.

Eq. (19) can also be solved exactly, and thus we obtain the density as a function of time in an exact analytical form as given by

\[
\theta(\tau) = 4\theta_0 v_0 e^{\frac{\theta}{\sqrt{v_0}}} \sqrt{2} \left[ 2\sqrt{v_0} \left( -\theta_0^2 + \theta_0 + v_0 \right) e^{2\theta v_0 \sqrt{v_0}} + \left( \theta_0 + 2v_0 \right)e^{\theta v_0 \sqrt{v_0}} \right] \left\{ \frac{\theta_0^2 + 8v_0^2 - 4(\theta_0 - 2)v_0}{\theta_0^2 + \theta_0 + v_0} e^{2\theta v_0 \sqrt{v_0}} + \theta_0^2 (4v_0 + 1)e^{2\theta v_0 \sqrt{v_0}} + 4(\theta_0 + 2v_0)e^{\theta v_0 \sqrt{v_0}} \right\}^{-1},
\]

(59)

where we have used the initial condition \( \theta(\tau_0) = \theta_0 \).

2. The radiation fluid Universe with electric type superconducting dark energy

For a high density radiation fluid Universe, with matter equation of state satisfying the condition \( p = \rho/3 \), in the presence of electric type superconducting dark energy the basic evolution equation of the dimensionless matter density \( \theta \) is given by

\[
\frac{d\theta}{d\tau} = -\frac{\sqrt{\theta - \theta^2 + v_0(\theta/3 + \theta(1 - 2\theta))}}{(1 - 2\theta)}. \tag{60}
\]

The deceleration parameter of the radiation Universe can be obtained as

\[
q = \frac{2\theta - 4\theta^2 - 2v_0}{2(\theta - \theta^2 + v_0)}. \tag{61}
\]

The variations with respect to the redshift \( z \) of the Hubble function, of the energy density, of the deceleration parameter and of the parameter of the dark energy equation of state of the radiation fluid Universe in the presence of the electric type superconducting dark energy are presented in Figs. 5-8. The initial conditions used to numerically integrate the cosmological evolution equation are \( \theta(0) = 0.45 \) and \( z(0) = 25 \).

We assume that the Universe was radiation dominated in the redshift range \( 5 \leq z \leq 25 \). The dimensionless Hubble function \( h \) of the high density Universe, presented in Fig. 5 is a monotonically increasing function of the redshift (monotonically decreasing in time), while the energy density, depicted in Fig. 4 increases monotonically with the redshift \( z \) during the cosmological evolution. The deceleration parameter, shown in Fig. 7 has positive values for the redshift interval \( 9 \leq z \leq 25 \), indicating a decelerating expansion. For large values of \( v_0 \) the deceleration parameter can reach the zero value at redshifts as high as \( z \approx 9 \), \( \lim_{z \to 9} q |_{v_0 = 0.005} \approx 0 \). The time variation of the cosmological parameters \( h \) and \( \theta \) is practically independent of the adopted small values of the parameter \( v_0 \). The parameter \( w_{DE} \) of the dark energy equation of state is represented in Fig. 8. For large redshift values \( 15 \leq z \leq 25 \), \( w_{DE} \) is positive, while for \( z \approx 5 \) it approaches the value \( w_{DE} = -1 \), showing that in the present model the Universe becomes dark energy dominated at around \( z \approx 5 \).
Finally, to conclude the investigation of the electric type superconducting dark energy model, we present a unified picture of the evolution of the Universe for the redshift range $z \in [0, 25]$. The variations of the Hubble function, matter density, deceleration parameter, and the parameter of the dark energy equation of state are plotted in Figs. 9 and 10.

To study the evolution of the electric type superconducting dark energy we adopt for the redshift $z$ the range from 0 to 25. We assume that in the range $z \in [5, 25]$ the matter content of the Universe can be (at least approximately) described by a radiation type equation of state $p = \rho/3$. At $z = 5$ the Universe enters in the matter dominated era, with $p \approx 0$. In this simplified model the transition from the radiation dominated era to the matter dominated phase is smooth, with all physical and thermodynamical quantities continue at the transition point. Therefore the Hubble function and the matter density, represented in Figs. 9 and 10, are monotonically increasing functions of the redshift for the entire period. The deceleration parameter, shown in Fig. 10, has a complex behavior. The Universe starts at $z = 25$ with a deceleration parameter having a value of $q \approx 0.10$, and its early evolution is strongly decelerating, with $q$ reaching the value $q \approx 0.9$ at $z \approx 12$, for $v_0 = 0.001$. For larger values of $v_0$ the expansion of the Universe is faster, with $q$ reaching the value $q = 0.6$ at $z \approx 14$. After $q$ has reached its maximum, it starts to decrease with decreasing $z$, and, depending on the numerical value of $v_0$, reaches the value
q = 0 for z ∼ 5 − 7. Then the deceleration parameter enters the negative range, with the Universe starting to accelerate at a higher rate, and reaching the value q ≈ −1 (the de Sitter phase) at around z = 0. The parameter of the dark energy equation of state w_{DE}, represented in Fig. 12 is slowly decreasing from its maximum value 1 in the redshift range z ≈ 17−25. For z < 15, w_{DE} decreases rapidly with decreasing z, and, depending on the numerical value of v_0, reaches the limiting value w_{DE} ≈ −1 at z ≈ 7.5 − 10.

4. Electric type superconducting dark energy Universe with conserved electric field and matter current

Finally, we investigate the case in which the electric potential A^0 and the matter current satisfy the conservation equations given by Eqs. (12). The time variation of A^0 can be immediately obtained from the Lorentz gauge equation imposed on A^μ, which gives

\[ \nabla_\mu A^\mu = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} (\sqrt{-g} A^\mu) = \frac{1}{a^3} \frac{d}{dt} (a^3 A^0) = 0, \]  

and

\[ A^0(t) = \frac{C_0}{a^3}, \]  

respectively, where C_0 is an arbitrary integration constant. The continuity equation of the matter hydrodynamic flow, \( \nabla_\mu (\rho u^\mu) = 0 \) gives a similar dependence for the matter density \( \rho \),

\[ \rho(t) = \frac{\rho_0}{a^3}, \]

where \( \rho_0 \) is an arbitrary integration constant. The evolution of the scalar field \( \phi \) is decoupled from the electric and matter component, and in the presence of a constant potential, \( V(A^2, \phi) = \text{constant} \), follows a similar law as the electric and the matter fields,

\[ \dot{\phi}(t) = \frac{\phi_0}{a^3}, \]  

where \( \phi_0 \) is an arbitrary integration constant. In the large time limit, all these fields tend to zero, and the Universe enters in an exponential, de Sitter type expansionary phase. However, the presence of the electric type superconducting dark energy modifies the cosmological dynamics of the Universe before it enters in the de Sitter stage.

B. Magnetic dark energy models

As a second superconducting vector type dark energy model we consider the case in which dark energy has a magnetic type structure, with its vector potential given by \( A_\mu = (0, A_1(t), A_2(t), A_3(t)) \). In order to have an isotropic expansion the components of the superconducting magnetic vector potential must satisfy the condition \( A_1(t) = A_2(t) = A_3(t) = A(t) \). Hence for this choice of \( A_\mu \) we obtain \( F_{0i} = -\dot{A}_i \), \( F_{00} = \dot{A}_i / a^2 \), \( i = 1, 2, 3 \), \( (1/16\pi) F_{\alpha\beta} F^{\alpha\beta} = -(3/8\pi) \dot{A}_i^2 / a^2 \), \( (1/4\pi) F_{0i} F_0^i = -(3/4\pi) \dot{A}_i^2 / a^2 \), and \( (1/4\pi) F_A F_\alpha^\alpha = -(1/4\pi) \dot{A}_i^2 / a^2 \) (no summation upon the index \( i \)). Therefore the gravitational field equations describing the isotropic homogeneous Universe in the presence of superconducting dark energy take the form

\[ 3H^2 = \left( 1 - \frac{\alpha}{2\phi} \right) \rho + \frac{\lambda}{3} \dot{\phi}^2 + \frac{3}{8\pi} \frac{A^2}{a^2} + \frac{3\lambda}{2} \frac{A^2}{a^2} + V_0, \]  

where \( \alpha \) and \( \lambda \) are the magnetic and coupling constants, respectively.
\[ 2\dot{H} + 3H^2 = -p + \frac{\alpha}{2} \rho \dot{\phi} - \frac{\lambda}{2} \dot{\phi}^2 - \frac{1}{8\pi} \frac{\dot{A}^2}{a^2} + \frac{\lambda A^2}{2 a^2} + V_0, \] (67)

\[ \lambda \dot{\phi} - \frac{\alpha}{2} \rho = 0, \] (68)

\[ \frac{1}{a} \frac{d}{dt} \left( a \dot{A} \right) = -4\pi \lambda A, \] (69)

where for simplicity we have adopted a constant value \( V_0 \) for the self-interaction potential of the scalar and vector fields, \( V(A^2, \phi) = V_0 = \text{constant}. \) The energy conservation equation takes the form

\[ \frac{d}{dt} \left( a^3 \rho \right) + p \frac{d}{dt} \left( a^3 \right) = \frac{\alpha}{2} a^3 \dot{\phi} \rho + \frac{15}{8\pi} a \dot{A}^2 + \frac{7 \lambda a A^2}{3} - 3 \left( \frac{1}{2} \lambda \dot{\phi}^2 + V_0 \right) a^2 \dot{a}. \] (70)

With the use of Eq. (68) we can substitute the derivative of the scalar field in terms of the matter density \( \rho. \)

Therefore the system of gravitational field equations describing the superconducting magnetic type cosmological dark energy model takes the form

\[ \frac{3 \dot{\phi}^2}{a^2} = \rho - \frac{\alpha^2}{8\pi} \rho^2 + \frac{3 \dot{A}^2}{2 a^2} + \frac{3 \lambda A^2}{a^2} + V_0, \] (71)

\[ 2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -p + \frac{\alpha^2}{8\pi} \rho^2 - \frac{\dot{A}^2}{2 a^2} + \frac{\lambda A^2}{a^2} + V_0, \] (72)

\[ \ddot{A} + \frac{\dot{A}^2}{a} + 4\pi \lambda A = 0. \] (73)

From Eqs. (71) and (72) we obtain

\[ 2 \dot{H} = - (\rho + p) + \frac{\alpha^2}{4 \lambda} \rho^2 - \frac{1}{2} \frac{\dot{A}^2}{a^2} - \lambda \frac{A^2}{a^2}. \] (74)

The deceleration parameter of the magnetic type superconducting dark energy model can be represented as

\[ q = \frac{\rho + 3p - (\alpha^2/2\lambda) \rho^2 + (3/4\pi) \left( \dot{A}^2/a^2 \right)}{2 \left[ \rho - ((\alpha^2/2\lambda) \rho^2) + (3/8\pi) \left( \dot{A}^2/a^2 \right) + (3\lambda/2) \left( A^2/a^2 \right) + V_0 \right].} \] (75)

\[ H = \left( \sqrt{8\pi/\alpha} \right) h, V_0 = \left( 8\lambda/\alpha^2 \right) V_0 \], and by denoting \( \sigma = \pi \alpha^2/2, \) the system of Eqs. (71)-(73) can be written in a dimensionless form as

\[ 3h^2 = \theta - \dot{\theta}^2 + \frac{1}{a^2} \left( \frac{d\Sigma}{dr} \right)^2 + \sigma \frac{\Sigma^2}{a^2} + v_0, \] (76)

\[ 2 \frac{dh}{dr} = \theta - \dot{\theta}^2 - \frac{1}{3} \frac{1}{a^2} \left( \frac{d\Sigma}{dr} \right)^2 + \frac{\Sigma^2}{3 a^2} + v_0, \] (77)

In the case of a dust Universe, with \( p = 0, \) from the conservation equation Eq. (70) we obtain for the time derivative of the energy density \( \rho \) the equation

\[ \dot{\rho} = \frac{\frac{15}{8\pi} \frac{\dot{A}^2}{a^2} + \frac{7 \lambda A^2}{a^2} - \frac{3 \rho^2}{a^2} + 3 \rho - 3V_0}{(1 - \alpha^2/a^2 \rho)} H. \] (78)

1. Dust Universes with magnetic type superconducting dark energy

For the case of dust matter, with negligible thermodynamic pressure, we can take \( p = 0 \) in the gravitational field equations. Then by introducing a set of dimensionless variables \( (\theta, \Sigma, \tau, h, v_0) \) defined as \( \{ \rho = (8\lambda/\alpha^2) \theta, A = \sqrt{8\pi/3\Sigma}, t = (\alpha/\sqrt{8\lambda}) \tau, \)

\[ \frac{dz}{d\tau} = -(1 + z) \frac{1}{\sqrt{3}} \theta - \dot{\theta}^2 + \frac{1}{a^2} \left( \frac{d\Sigma}{d\tau} \right)^2 + \sigma \frac{\Sigma^2}{a^2} + v_0. \] (79)

The deceleration parameter and the parameter of the equation of state of the dark energy of the Universe filled
with magnetic type superconducting dark energy are obtained as

\[ q = \frac{\theta - 4\theta^2 + 2\Sigma^2/a^2 - 2v_0}{2(\theta - \theta^2 + \Sigma^2/a^2 + \sigma \Sigma^2/a^2 + v_0)}, \]  

(83)

\[ w_{DE} = -\frac{\theta^2 + \Sigma^2/a^2 + \sigma \Sigma^2/a^2 + v_0}{\theta^2 - \Sigma^2/3a^2 + \sigma \Sigma^2/3a^2 + v_0}, \]  

(84)

where a prime denotes the derivative with respect to the dimensionless time \( \tau \). By taking the derivative with respect to \( \tau \) of Eq. (78), and with the use of Eqs. (80) and (84), we obtain for the time variation of the matter density the equation

\[ \theta = \frac{\theta_0}{a^3}, \]  

(85)

where \( \theta_0 \) is an arbitrary constant of integration.

The variations with respect of the redshift \( z \in [0, 3] \) of the Hubble function, of the matter energy density \( \theta \), and of the deceleration parameter \( q \) of the Universe filled with magnetic type superconducting dark energy, obtained by numerically integrating Eqs. (79), (80), and (84), are presented, for a fixed value of \( \sigma = 0.0001 \), and for different values of \( v_0 \), in Figs. 7-13. The initial conditions used to integrate the system of cosmological evolution equations are \( \theta(0) = \theta_0 = 0.25 \), \( a(0) = 1 \), \( \Sigma(0) = 0.01 \), \( z(0) = 5 \), and \( \Sigma'(0) = 0.01 \), respectively.

As one can see from Fig. 13, the Hubble function of the Universe filled with magnetic type superconducting dark energy is a monotonically increasing function of \( z \) (time decreasing function), indicating an expansionary evolution. The matter energy density \( \theta \), represented in Fig. 14, monotonically increases with the redshift, and tends to zero in the limit of small \( z \). Its dynamics is basically independent on the adopted numerical values of the parameters \( \sigma \) and \( v_0 \). The dust magnetic Universe starts from a decelerating state at \( z = 5 \), with positive values of the deceleration parameter \( q > 0 \), shown in Fig. 15. The cosmological evolution is generally decelerating for \( 2 \leq z \leq 5 \), with \( q \) reaching the value zero at \( z \approx 2 \). Then the Universe begins to accelerate, with \( q < 0 \), and in the large time (small \( z \)) limit we have \( \lim_{z \to 0} q(z) = -1 \). Thus, in the final stages of evolution of the Universe filled with magnetic type superconducting dark energy the cosmological expansion is of de Sitter type, with the dark energy driving the Universe's expansion. The parameter of the dark energy equation of state, represented in Fig. 16, is smaller than zero in the entire redshift range \( 0 \leq z \leq 5 \), and it tends to -1 in the limit of small redshifts.

\[ \Theta_1 \]

\[ \Theta_2 \]

\[ \Theta_3 \]

\[ \Theta_4 \]

\[ \Theta_5 \]

\[ \Theta_6 \]

\[ \Theta_7 \]

\[ \Theta_8 \]

\[ \Theta_9 \]

\[ \Theta_{10} \]

\[ \Theta_{11} \]

\[ \Theta_{12} \]
FIG. 16: Redshift evolution of the parameter of the dark energy equation of state of the dust Universe in the presence of magnetic type superconducting dark energy for $\sigma = 0.0001$, and for different values of $v_0$: $v_0 = 0.001$ (solid curve), $v_0 = 0.002$ (dotted curve), $v_0 = 0.003$ (short dashed curve), $v_0 = 0.004$ (dashed curve), and $v_0 = 0.005$ (long dashed curve), respectively.

**IV. THERMODYNAMIC INTERPRETATION OF THE SUPERCONDUCTING DARK ENERGY MODELS**

In the present Section we analyze the physical interpretation of the superconducting dark energy model by adopting the point of view of the thermodynamics of the matter creation irreversible processes [23]-[25]. As we have already seen, the energy conservation equation of the superconducting dark energy models, Eq. (21), contain, as compared to the standard adiabatic conservation equation, an extra term, which can be interpreted thermodynamically as a matter creation rate. According to irreversible thermodynamics, matter creation represents an entropy source, generating an entropy flux, and thus modifying the temperature evolution of the considered gravitational system. On the other hand, due to our choice of the geometry of the Universe, all the non-diagonal components of the total energy–momentum tensor of the superconducting dark energy model are equal to zero, so that $T^\mu\nu_{\text{total}} = 0$, $\mu \neq \nu$. In particular, from the point of view of the thermodynamics of the irreversible processes and open systems, this condition implies the impossibility of heat transfer in the Friedmann–Robertson–Walker models of superconducting dark energy, since the condition $T^\mu\nu_{\text{di}}(\text{total}) = 0$, $i = 1, 2, 3$ must always hold.

**A. Matter creation rates and the creation pressure**

To analyze the thermodynamical implications of the superconducting dark energy models at the cosmological scale we start with an open system containing $N$ particles in a volume $V = a^3$, and characterized by an energy density $\rho$ and a thermodynamic pressure $p$. For such a system the second law of thermodynamics, in its most general form, is given by [23]

$$\frac{d}{dt} (\rho a^3) + p \frac{d}{dt} a^3 = \frac{dQ}{dt} + \frac{\rho + p}{n} \frac{d}{dt} (na^3),$$  

(86)

where $dQ$ is the heat received by the system during time $dt$, and $n = N/V$ is the particle number density, respectively. Due to our choice of the geometry of the Universe, in a homogeneous and isotropic system filled with superconducting dark energy only adiabatic transformations, defined by the condition $dQ = 0$, are possible. Therefore in the following we ignore proper heat transfer processes in the superconductor type cosmological system. However, under the assumption of adiabatic transformations, Eq. (86), representing the general formulation of the second law of thermodynamics, contains the term $((\rho + p)/n)d(\rho a^3)/dt$, which explicitly takes into account the variation of the number of particles in a given volume. Hence, in the general thermodynamic approach of open systems, even for the case of adiabatic transformations with $dQ = 0$, there is a "heat" (internal energy), received/lost by the system, which is entirely due to the change in the particle number $n$. From the cosmological perspective of the superconducting dark energy models, the change in the particle number is due to the transfer of energy from dark energy to matter. Thus in this class of cosmological models matter creation acts as a source of internal energy, as well as of entropy. For adiabatic transformations $dQ/dt = 0$, Eq. (86) can be written in an equivalent form as

$$\dot{\rho} + 3(\rho + p)H = \frac{\rho + p}{n} (\dot{n} + 3Hn).$$  

(87)

Therefore, from the point of view of the thermodynamics of open systems, Eq. (21), giving the energy conservation equation in the superconducting dark energy models, can be interpreted as describing particle creation in an homogeneous and isotropic geometry, with the time variation of the particle number obtained from the equation

$$\dot{n} + 3nH = \Gamma n,$$  

(88)

where the particle creation rate $\Gamma$ is defined as

$$\Gamma = \frac{1}{\rho + p} \left\{ - \frac{\alpha}{2} u^\sigma \nabla_\sigma \left[ j^\mu (A_\nu - \nabla_\nu \phi) \right] + \frac{\alpha}{2} \frac{d}{ds} \left[ j^\beta (A_\beta - \nabla_\beta \phi) \right] - \partial_\delta V (A^2, \phi) u^\sigma A_\nu - 2 \partial_{A^\sigma} V (A^2, \phi) u^\rho A^\sigma \nabla_\rho A_\nu \right\}.$$  

(89)

Therefore the energy conservation equation in the superconducting dark energy model can be written in the alternative form

$$\dot{\rho} + 3(\rho + p)H = (\rho + p)\Gamma.$$  

(90)
As shown initially in [23], for adiabatic transformations Eq. (80), describing irreversible particle creation in an open thermodynamic systems, can be formulated as an effective energy conservation equation of the form

$$\frac{d}{dt} \left( p a^3 \right) + (p + p_c) \frac{d}{dt} a^3 = 0,$$  \hspace{1cm} (91)

which can be written in an equivalent form as,

$$\dot{\rho} + 3 (\rho + p + p_c) H = 0,$$  \hspace{1cm} (92)

where we have introduced the term $p_c$, called the creation pressure, and which is defined as [23]

$$p_c = -\frac{\rho}{n} \frac{d}{dn} (na^3) = -\frac{\rho + p}{3nH} \left( \dot{n} + 3nH \right) = \frac{-\rho + p}{3H} \Gamma.$$  \hspace{1cm} (93)

Therefore in the superconducting dark energy model the creation pressure can be obtained as

$$p_c = \frac{1}{3H} \left\{ -\frac{\alpha}{2} u^\mu \nabla_{\mu} \left[ j^{\mu} (A_\nu - \nabla_\nu \phi) \right] + \alpha \frac{d}{ds} \left[ j^{\beta} (A_\beta - \nabla_\beta \phi) \right] - \partial_\phi V (A^2, \phi) u^\nu A_\nu - 2 \partial A^{\nu} V (A^2, \phi) u^\nu A^\alpha \nabla_\alpha A_\nu \right\}.$$  \hspace{1cm} (94)

### 1. Particle creation rates and creation pressure in the electric type superconducting dark energy model

As an example of the thermodynamic description of the superconducting dark energy models we consider the electric type superconducting dark energy case, for which the energy conservation equation is given by Eq. (42), can be formulated as

$$\dot{\rho} + 3 (\rho + p) H = \frac{\alpha^2}{4\lambda} \rho (\dot{\rho} + 3H \rho).$$  \hspace{1cm} (95)

From Eq. (95) it follows that for $p = 0$ the matter energy is conserved, $\dot{\rho} + 3(\rho + p) H = 0$, and there is no particle creation from the superconducting dark energy. However, for $p \neq 0$, matter and energy transfer processes take place in the presence of the superconducting electric type dark energy, with the particle creation rate $\Gamma$ given by

$$\Gamma = \frac{\alpha^2}{4\lambda} \rho \left( \dot{\rho} + 3H \rho \right).$$  \hspace{1cm} (96)

The creation pressure for this model can be obtained as

$$p_c = -\frac{\alpha^2}{12\lambda H} \rho (\dot{\rho} + 3H \rho).$$  \hspace{1cm} (97)

### B. Entropy and temperature evolution

In order to formulate the second law of thermodynamics for open systems, and to apply it to the superconducting dark energy model, we must decompose the entropy change in the cosmological fluid into two components: the entropy flow term $d_i S$, and the entropy creation term $d_i S$. Hence the total entropy $S$ of an open thermodynamic system can be written as [22, 23]

$$dS = d_e S + d_i S,$$  \hspace{1cm} (98)

where we assume that $d_i S > 0$. Both the entropy flow and the entropy production rate in the superconducting dark energy model can be evaluated by starting from the total differential of the entropy, given by [23],

$$Td \left( \dot{\bar{s}} a^3 \right) = d \left( \rho a^3 \right) + p da^3 - \mu d \left( na^3 \right),$$  \hspace{1cm} (99)

where $T$ is the temperature of the open thermodynamic system with superconducting particle creation, $\bar{s} = S/a^3$ is the entropy per unit volume, and $\mu$ is the chemical potential, defined in the usual way as

$$\mu n = (\rho + p) - Ts.$$  \hspace{1cm} (100)

For closed systems and adiabatic transformations $dS = 0$ and $d_i S = 0$. However, in the presence of matter creation there is a non-zero contribution to the entropy. For homogeneous systems the entropy flow term $d_i S$ cancels, so that $d_e S = 0$. But matter creation from superconducting dark energy acts as a source for entropy production, with the corresponding entropy time variation obtained as [23]

$$\frac{T d_{e} S}{dt} = \frac{T d S}{dt} = -\frac{\rho + p}{n} \frac{d}{dt} (na^3) - \mu \frac{d}{dt} (na^3) = T \frac{\dot{s}}{n} \frac{d}{dt} (na^3) \geq 0.$$  \hspace{1cm} (101)

From Eq. (101) we obtain for the time variation of the entropy the equation

$$\frac{dS}{dt} = \frac{S}{n} \left( \dot{n} + 3H n \right) = \Gamma S \geq 0,$$  \hspace{1cm} (102)

giving for the entropy increase due to particle creation the expression

$$S(t) = S_0 e^{\int_0^t \Gamma(t') dt'},$$  \hspace{1cm} (103)

where $S_0 = S(0)$ is a constant. With the use of Eq. (99), we obtain for the entropy production in the superconducting dark energy models the equation

$$\frac{1}{S} \frac{dS}{dt} = \frac{1}{\rho + p} \left\{ -\frac{\alpha}{2} u^\mu \nabla_{\mu} \left[ j^{\mu} (A_\nu - \nabla_\nu \phi) \right] + \frac{\alpha}{2} \frac{d}{ds} \left[ j^{\beta} (A_\beta - \nabla_\beta \phi) \right] - \partial_\phi V (A^2, \phi) u^\nu A_\nu - 2 \partial A^{\nu} V (A^2, \phi) u^\nu A^\alpha \nabla_\alpha A_\nu \right\} \geq 0.$$  \hspace{1cm} (104)
Equivalently, the above equation can be written as
\[
\frac{1}{S} \frac{dS}{dt} = \frac{\alpha^2}{4\lambda} \frac{\rho}{\rho + p} (\dot{\rho} + 3H\rho) \geq 0.
\] (105)

An important thermodynamic quantity, the entropy flux vector \( S^\mu \) of the particles created from the superconducting dark energy, is defined according to \[24\]
\[
S^\mu = n\sigma u^\mu,
\] (106)
where \( \sigma = S/N \) is the specific entropy per particle. The entropy flux vector \( S^\mu \) is the specific entropy per particle. The entropy flux vector \( S^\mu \) can be written in a simpler form as
\[
\frac{nTds}{dn} = d\rho - \frac{\rho + p}{n} dn.
\] (107)
and by using the definition of the chemical potential \( \mu \) of the superconducting thermodynamic system as given by
\[
\mu = \frac{\rho + p}{n} - T\sigma,
\] (108)
we obtain
\[
\nabla_\mu S^\mu = (\dot{n} + 3nH)\sigma + nu^\nu \nabla_\mu \sigma =
\frac{1}{T} (\dot{n} + 3nHn) \left( \frac{\rho + p}{n} - \mu \right),
\] (109)
where we have taken into account the important relation
\[
\frac{nTds}{dn} = \dot{\rho} - \frac{\rho + p}{n} \dot{n} = 0,
\] (110)
which follows immediately from Eq. \[87\]. With the use of Eq. \[106\], we obtain for the entropy production rate due to the particle creation processes in the superconducting dark energy model the expression
\[
\nabla_\mu S^\mu = \frac{n}{(\rho + p)T} \left\{ -\frac{\alpha}{2} u^\nu \nabla_\mu \left[ j^\mu (A_\nu - \nabla_\nu \phi) \right] + \right.
\]
\[
\frac{\alpha d}{2} ds \left[ j^\beta (A_\beta - \nabla_\beta \phi) \right] - \partial_\phi V (A^2, \phi) u^\nu A_\nu - 2\partial_{\lambda} V (A^2, \phi) u^\nu A^\alpha \nabla_\alpha A_\nu \left( \frac{\rho + p}{n} - \mu \right).
\] (111)

The entropy production rate via superconducting particle creation processes given by Eq. \[111\] can be written in a simpler form as
\[
\nabla_\mu S^\mu = \frac{\alpha^2}{4\lambda T} n \left( \frac{\rho + p}{n} - \mu \right) \frac{\rho}{\rho + p} (\dot{\rho} + 3H\rho).
\] (112)

In the general case the thermodynamic state of a perfect comoving fluid is described by only two essential thermodynamic variables, the particle number density \( n \), and the temperatures \( T \), respectively. Hence the energy density \( \rho \) and the thermodynamic pressure \( p \) can be obtained, in terms of the particle number \( n \) and temperature \( T \), by using the standard form of the equilibrium equations of state of the matter created by the superconducting dark energy,
\[
\rho = \rho(n, T), p = p(n, T).
\] (113)
Therefore the energy conservation equation Eq. \[90\] can be written in the expanded form
\[
\frac{\partial \rho}{\partial n} \dot{n} + \frac{\partial p}{\partial T} \dot{T} + 3(\rho + p)H = (\rho + p)\Gamma.
\] (114)
With the use of the general thermodynamic relation \[24\]
\[
\frac{\partial p}{\partial n} = \frac{\rho + p}{T} - \frac{T}{n} \frac{\partial p}{\partial T},
\] (115)
we obtain for the temperature evolution of the newly created particles in the superconducting dark energy model the expression
\[
\frac{\dot{T}}{T} = c_s^2 \frac{\dot{n}}{n} = c_s^2 (\Gamma - 3H).
\] (116)
where the speed of sound \( c_s \) is defined as \( c_s^2 = \partial p/\partial \rho \). If the matter newly created from the superconducting dark energy satisfies a barotropic equation of state \( p = (\gamma - 1)\rho \), \( 1 \leq \gamma \leq 2 \), the matter temperature evolution can be obtained as
\[
T = T_0 n^{\gamma - 1}.
\] (117)

V. DISCUSSIONS AND FINAL REMARKS

In the present paper we have considered an electromagnetic type dark energy model, in which the electromagnetic gauge invariance is spontaneously broken. The action for such a system must be invariant under gauge transformations, \( A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \Lambda(x) \), \( \psi_n(x) \rightarrow \exp (iq_n \Lambda(x)/\hbar) \psi_n(x) \), where \( q_n \) are the charges destroyed by the field \( \psi_n \) \[17\]. These phase changes lead to the formation of an ordered state. By writing all charged fields as a function of a scalar field \( \phi(x) \), when the matter fields are integrated out we obtain a Lagrangian that is a gauge invariant functional of the fields \( A^\mu \) and \( \phi \). Such a physical model can explain easily all the observed properties of superconductors \[17\]-\[19\]. Tentatively, we also propose it as a dark energy model with a broken electromagnetic gauge invariance.

From a physical point of view the superconducting dark energy model is a two-field model, leading to a scalar-vector-tensor cosmological theory. It can also be viewed as a unified scalar - vector field dark energy model, in which the scalar field \( \phi \) and the vector field \( A_\mu \) appear in the gauge invariant combination \( A_\mu - \nabla_\mu \phi \).
Moreover, similarly the standard electrodynamic case, we have assumed the possible existence of a generalized coupling between the matter current $j^\mu$ and the gauge invariant combination of the potentials $A_\mu - \nabla_\mu \phi$. We have investigated the cosmological implications of this model, by restricting our analysis to the case of a homogeneous and isotropic geometry. We have considered two distinct classes of models, whose main properties are determined by the form of the electromagnetic potential $A_\mu$. The first model corresponds to an electric type choice for the dark energy potential, with $A_0$ the only non-zero component. For this case the general solution of the gravitational field equations was obtained numerically for the dust and the radiation filled Universe, respectively. In both cases we have assumed that the self-interaction potential of the electromagnetic type dark energy is a constant. In both cases in the long time limit the Universe ends in a decelerating phase. A similar result is obtained for the magnetic type dark energy model, in which the electromagnetic potential is restricted to the form $A_\mu = (0, A(t), A(t), A(t))$. For this case we have investigated, by numerically solving the gravitational field equations, the zero thermodynamic pressure (dust) cosmological model, in the presence of a constant self-interaction potential. Similarly to the electric type dark energy model, the magnetic superconducting dark energy model drives the Universe, in the long time limit, in an accelerated, de Sitter type, expansionary phase.

Due to the coupling between the dark energy potentials and the matter current, the matter energy-momentum tensor is not conserved in the present approach. We have interpreted, in the framework of the thermodynamics of open systems and irreversible processes, the non-conservation of the matter-energy-momentum tensor as describing particle creation, and energy transfer from the superconducting dark energy to ordinary matter. We have explicitly obtained the particle creation rates, as well as the effective creation pressure generated by the irreversible transformation of the field energy into matter. The entropy production rate and the overall entropy increase during the creation process. The entropy being given by the exponential of the time evolution of the Universe was also obtained, with the total entropy being given by the exponential of the time integral of the particle production rate $\Gamma$.

The possible observational study in a cosmological context of the irreversible matter–creation processes in the homogeneous and isotropic flat Friedmann–Robertson–Walker geometry in the superconducting dark energy models may represent one of the possibilities of considering the viability of this dark energy model. However, in order to confirm the validity of the superconducting dark energy model developed in the present paper, it is necessary to carefully consider a much wider range of cosmological and astrophysical tests for this type of models. In particular, an essential test of the superconducting dark energy model would be the investigation of the classical macroscopic predictions of the model in large-scale structure formation with linear perturbations, and with the consideration of the Newtonian limit for small scales. Supernovae observations fitting, and the study of the effects of the matter creation on the Cosmic Microwave Background anisotropies could lead to other important tests and parameter constraints of the model. Essentially the superconducting dark energy model introduced in the present paper is a simple toy model, whose main goal is to stimulate the study of alternative or more general electromagnetic type dark energy models.

The superconducting dark energy model introduced in the present paper leads to the possibility that matter creation, associated with matter current - dark energy electromagnetic potential coupling may also happen in the present - day universe, as initially considered by Dirac. The existence of some forms of coupling between matter and dark energy are not in contradiction with the cosmological observations or with some astrophysical data. However, firm observational evidence for particle creation on a cosmological scale is still missing. Hopefully, a key ingredient of the present model, the functional form of the self-interaction potential $V(A^2, \phi)$, which essentially determines the cosmological dynamics in the superconducting dark energy model, will be provided by fundamental particle physics (or perhaps even condensed matter) models, thus permitting an in depth comparison of the predictions of the model with high precision observational cosmological and astrophysical data.

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Appendix A: The divergence of the energy-momentum tensor

In order to obtain the divergence of the matter energy-momentum tensor in the superconducting dark energy model we compute first the divergence of the electromagnetic type term,

$$T^{(em)}_{\nu} = \frac{1}{4\pi} \left( - F_{\nu\alpha} F^{\alpha\mu} + \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta\mu\nu} \right). \quad (A1)$$

Hence for the divergence of the electromagnetic component we find first

$$\nabla_\mu T^{(em)}_{\nu} = \frac{1}{4\pi} \left( - \frac{1}{2} F^{\alpha\beta\nu} F_{\alpha\beta} - \nabla_\mu F_{\nu\alpha} F^{\alpha\mu} - F_{\nu\alpha} \nabla_\mu F^{\alpha\mu} \right). \quad (A2)$$

By taking into account that \((1/4\pi) \nabla_\mu F^{\mu\nu} = J^\nu\), and \(\nabla_\mu F_{\alpha\beta} = - \nabla_\alpha F_{\beta\nu} - \nabla_\beta F_{\nu\alpha}\), it follows that

$$\nabla_\nu T^{(em)}_{\mu} = \frac{1}{4\pi} \left( - \frac{1}{2} F^{\alpha\beta} \nabla_\alpha F_{\beta\nu} - \frac{1}{2} F^{\alpha\beta} \nabla_\beta F_{\nu\alpha} -$$
\[ F^{\mu \alpha} \nabla_{\mu} F_{\nu \alpha} \) \] \[ F_{\nu \alpha} J^\alpha. \] \[ (A3) \]

The terms in the bracket vanish, and therefore we find
\[
\nabla_\mu T_{(\text{em})\nu} = F_{\nu \alpha} \left[ \lambda g^{\alpha \beta} (A_\beta - \nabla_\beta \phi) + \frac{\alpha}{2} j^\alpha - 2 \partial_A V (A^2, \phi) A^\alpha \right]. \] \[ (A4) \]

Then for the divergence of the matter energy-momentum tensor we obtain
\[
\nabla_\mu T_{\mu \nu} - F_{\nu \alpha} J^\alpha + \lambda (\nabla_\nu A_\mu - \nabla_\mu \nabla_\nu \phi) (A_\nu - \nabla_\nu \phi) + \lambda (A_\mu - \nabla_\mu \phi) F_{\mu \nu} + \alpha \nabla_\mu j^\mu (A_\nu - \nabla_\nu \phi) + \frac{\alpha}{2} j^\mu (\nabla_\mu A_\nu - \nabla_\nu \nabla_\mu \phi) + \frac{\alpha}{2} j^\nu F_{\mu \nu} - \frac{\alpha}{2} \nabla_\nu j^\beta (A_\beta - \nabla_\beta \phi) + \partial_\phi V (A^2, \phi) \nabla_\nu \phi + 2A^\alpha \partial_\phi V (A^2, \phi) \nabla_\nu A_\alpha = 0, \] \[ (A5) \]

where we have used the identities
\[
\lambda (A_\mu - \nabla_\mu \phi) (\nabla_\nu A_\nu - \nabla_\mu \nabla_\nu \phi) - \frac{\lambda}{2} (\nabla_\nu A_\alpha - \nabla_\nu \nabla_\alpha \phi) \times (A_\alpha - \nabla_\alpha \phi) - \frac{\lambda}{2} (A_\alpha - \nabla_\alpha \phi) (\nabla_\nu A_\alpha - \nabla_\nu \nabla_\alpha \phi) = \]

\[ \lambda F_{\mu \nu} (A_\mu - \nabla_\mu \phi), \] \[ (A6) \]

and
\[
\alpha j^\mu (\nabla_\mu A_\nu - \nabla_\nu \nabla_\mu \phi) - \frac{\alpha}{2} j^\beta (\nabla_\nu A_\beta - \nabla_\nu \nabla_\beta \phi) = \]

\[ \frac{\alpha}{2} j^\mu (\nabla_\mu A_\nu - \nabla_\nu \nabla_\mu \phi) + \frac{\alpha}{2} j^\nu F_{\mu \nu}, \] \[ (A7) \]

respectively.

With the use of the evolution equation of the scalar field we find the relation,
\[
\lambda (\nabla_\mu A_\mu - \nabla_\mu \nabla_\mu \phi) (A_\nu - \nabla_\nu \phi) = \]

\[ \partial_\phi V (A^2, \phi) (A_\nu - \nabla_\nu \phi) - \frac{\alpha}{2} \nabla_\mu j^\mu (A_\nu - \nabla_\nu \phi). \] \[ (A8) \]

By substituting the above relation and the expression of the divergence of the electromagnetic part of the energy-momentum tensor in Eq. \( A8 \), we finally obtain
\[
\nabla_\mu T_{\mu \nu} + \frac{\alpha}{2} \nabla_\mu [j^\mu (A_\nu - \nabla_\nu \phi)] - \frac{\alpha}{2} \nabla_\nu j^\beta (A_\beta - \nabla_\beta \phi) + \partial_\phi V (A^2, \phi) A_\nu + 2A_\alpha \partial_\phi V (A^2, \phi) A^\alpha \nabla_\alpha A_\nu = 0. \] \[ (A9) \]