Polarizations of two vector mesons in $B$ decays

Chuan-Hung Chen$^{1,2,*}$

$^1$Department of Physics, National Cheng-Kung University, Tainan, 701 Taiwan

$^2$National Center for Theoretical Sciences, Taiwan

(Dated: July 6, 2021)

Abstract

Inspired by the small longitudinal polarizations (LPs) of $B \to K^*\phi$ decays observed by BELLE and BABAR, we revise the theoretical uncertainties of perturbative QCD approach for determining hard scales of $B$ decays, we find that the LPs of $B \to K^*\phi$ could approach to 60% while the branching ratios (BRs) could be around $9 \times 10^{-6}$. In addition, we also study the BRs and polarization fractions of $B \to \rho(\omega)\rho(\omega)$ and $B \to \rho(\omega)K^*$ decays. For those tree dominant and color-allowed processes in $B \to \rho(\omega)\rho(\omega)$ decays, we get that the BRs of $(\rho^+\rho^-, \rho^0\rho^+, \omega\rho^-)$ are $(23.06, 11.99, 14.78) \times 10^{-6}$ while their LPs are close to unity. Interestingly, due to significant tree contributions, we find that the BR(LP) of $\rho^-K^{*+}$ could be around $10.13 \times 10^{-6}(60\%)$; and due to the tree and electroweak penguin, the BR(LP) of $\omega K^{*+}$ could be around $5.67 \times 10^{-6}(61\%)$.

* Email: phychen@mail.ncku.edu.tw
Since the transverse polarizations (TPs) of vector mesons are associated with their masses, by naive estimations, we can easily obtain that the longitudinal polarization (LP) of the two light vector mesons produced by $B$ decay is approaching to unity. The expectation is confirmed by BELLE [1] and BABAR [2, 3] in $B \to \rho(\omega)\rho$ decays, in which the longitudinal parts occupy over 88%. Furthermore, TP(LP) could be large(small) while the final states include heavy vector mesons. The conjecture is verified in $B \to J/\Psi K^*$ decays [4, 5], in which the longitudinal contribution is only about 60%. However, the rule for small LP seems to be broken in $B \to \phi K^*$ decays. From the recent measurements of BELLE [6] and BABAR [2, 7], summarized in the Table I, it is quite clear that the LPs of $B \to K^*\phi$ are only around 50%. According to the observations, many mechanisms are proposed to solve the puzzle, where the methods include not only new QCD effects [8] but also the effects of the extension of the standard model (SM) [9, 10].

### TABLE I: The branching ratios (in units of $10^{-6}$), polarization fractions and relative phases for $B \to \phi K^*$.

| Mode   | Observation | BELLE               | BABAR               |
|--------|-------------|---------------------|---------------------|
| $K^{*+}\phi$ | BR  | $10.0^{+1.6+0.7}_{-1.5-0.8}$ | $12.7^{+2.2}_{-2.0} \pm 1.1$ |
|        | $R_L$       | $0.52 \pm 0.08 \pm 0.03$           | $0.46 \pm 0.12 \pm 0.03$           |
|        | $R_\perp$   | $0.19 \pm 0.08 \pm 0.02$           |                           |
|        | $\phi_\parallel (rad)$ | $2.10 \pm 0.28 \pm 0.04$ |                         |
|        | $\phi_\perp (rad)$  | $2.31 \pm 0.20 \pm 0.07$           |                           |
| $K^{*0}\phi$  | BR  | $6.7^{+2.1+0.7}_{-1.9-1.0}$       | $9.2 \pm 0.9 \pm 0.5$       |
|        | $R_L$       | $0.45 \pm 0.05 \pm 0.02$           | $0.52 \pm 0.05 \pm 0.02$           |
|        | $R_\perp$   | $0.30 \pm 0.06 \pm 0.02$           | $0.22 \pm 0.05 \pm 0.02$           |
|        | $\phi_\parallel (rad)$ | $2.39 \pm 0.24 \pm 0.04$ | $2.34^{+0.23}_{-0.20} \pm 0.05$ |
|        | $\phi_\perp (rad)$  | $2.51 \pm 0.23 \pm 0.04$           | $2.47 \pm 0.25 \pm 0.05$           |

It is known that most proposals to solve the anomalous polarizations only concentrate on how to make the LPs of $B \to K^*\phi$ be small. It is few to analyze the problem by combing other decays such as the decays $B \to \rho(\omega)\rho(\omega)$ and $B \to \rho(\omega)K^*$ etc. That is, maybe we can invent a way to solve the anomalies in $K^*\phi$, however, we still don’t have the definite reason to say why the considering effects cannot contribute to $\rho(\omega)\rho(\omega)$ or
\[ \rho(\omega) K^* \] significantly. By this viewpoint, in this paper, we are going to reanalyze the decays \( B \to K^* \phi \) in terms of perturbative QCD(PQCD) approach in the SM. By revising the theoretical uncertainties of PQCD, which come from the man-made chosen conditions for hard scales of \( B \) decays, we will show how well we can predict and how close we can reach in theoretical calculations, while the processes of light mesons production are assumed to be dominated by the short-distance effects. We note that the wave functions of mesons, representing the nonpertubative QCD effects, are assumed to be known and obtained by the QCD sum rules. Moreover, according to the improving conditions, we also make the predictions on the decays \( B \to \rho(\omega) \rho(\omega) \) and \( B \to \rho(\omega) K^* \).

Although the effective interactions, governing the transition decays \( b \to s(d) \) at the quark level, are well known, to be more clear for explanation, we still write them out to be

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_q \left[ C_1(\mu)O_1^{(q)}(\mu) + C_2(\mu)O_2^{(q)}(\mu) + \sum_{i=3}^{10} C_i(\mu)O_i(\mu) \right],
\]

where \( V_q = V_{q\bar{q}}^* V_{q\bar{q}} \) are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, the subscript \( q' \) could be \( s \) or \( d \) quark and the operators \( O_1-O_{10} \) are defined as

\[
\begin{align*}
O_1^{(q)} &= (\bar{q}'_a q_\beta)_V A (\bar{q}_\beta b_\alpha)_V A , \\
O_3 &= (\bar{q}'_a b_\alpha)_V A \sum_q (\bar{q}_\beta q_\beta)_V A , \\
O_5 &= (\bar{q}'_a b_\alpha)_V A \sum_q (\bar{q}_\beta q_\beta)_V A , \\
O_7 &= \frac{3}{2} (\bar{q}'_a b_\alpha)_V A \sum_q e_q (\bar{q}_\beta q_\beta)_V A , \\
O_9 &= \frac{3}{2} (\bar{q}'_a b_\alpha)_V A \sum_q e_q (\bar{q}_\beta q_\beta)_V A , \\
O_2^{(q)} &= (\bar{q}'_a q_\alpha)_V A (\bar{q}_\beta b_\beta)_V A , \\
O_4 &= (\bar{q}'_a b_\beta)_V A \sum_q (\bar{q}_\alpha q_\alpha)_V A , \\
O_6 &= (\bar{q}'_a b_\beta)_V A \sum_q (\bar{q}_\alpha q_\alpha)_V A , \\
O_8 &= \frac{3}{2} (\bar{q}'_a b_\beta)_V A \sum_q e_q (\bar{q}_\alpha q_\alpha)_V A , \\
O_{10} &= \frac{3}{2} (\bar{q}'_a b_\beta)_V A \sum_q e_q (\bar{q}_\alpha q_\alpha)_V A ,
\end{align*}
\]

with \( \alpha \) and \( \beta \) being the color indices. In Eq. (1), \( O_1-O_2 \) are from the tree level of weak interactions, \( O_3-O_6 \) are the so-called gluon penguin operators and \( O_7-O_{10} \) are the electroweak penguin operators, while \( C_1-C_{10} \) are the corresponding WCs. Using the unitarity condition, the CKM matrix elements for the penguin operators \( O_3-O_{10} \) can also be expressed as \( V_u + V_c = -V_t \). To describe the decay amplitudes for \( B \) decays, we have to know not only the relevant effective weak interactions but also all possible topologies for the specific process. In terms of penguin operators, we display the general involving flavor diagrams for \( b \to q' q\bar{q} \) in Fig. (1) where (a) and (b) denote the emission topologies while (c) is the annihilation
The flavor $q$ in Fig. 1(a) and (b) is produced by gauge bosons and could be $u$, or $d$ or $s$ quark if the final states are the light mesons; however, $q''$ stands for the spectator quark and could only be $u$ or $d$ quark, depending the $B$ meson being charged or neutral one. However, the role of $q$ and $q''$ in Fig. 1(c) is reversed so that $q = u$, or $d$, or $s$ is the spectator quark while $q'' = u$ or $d$ is dictated by gauge interactions. Since the matrix elements obtained by the Fierz transformation of $O_{3,4}$ are the same as those of $O_{1,2}$, we don’t further consider the flavor diagrams for tree contributions.

In the beginning, we first pay attention to $B \rightarrow K^* \phi$ decays. Although there are charged and neutral modes in $B \rightarrow K^* \phi$ decays, because the differences in charged and neutral modes are only the parts of small tree annihilation, for simplicity our discussions will concentrate on the decay $B_d \rightarrow K^{*0} \phi$. As known that at quark level, the decay corresponds to $b \rightarrow ss\bar{s}$; thus, by the flavor diagrams, we have $q = q' = s$ and $q'' = d$. According to our previous results [17], the helicity amplitude could be expressed by

$$\mathcal{M}^{(h)} = m_B^2 \mathcal{M}_L + m_B^2 \mathcal{M}_N \epsilon^*_1(t) \cdot \epsilon^*_2(t) + i \mathcal{M}_T \epsilon^{\alpha\beta\gamma\rho} \epsilon^*_1(t) \epsilon^*_2(t) P_{1\gamma} P_{2\rho}$$

with the convention $\epsilon^{0123} = 1$, where the superscript $h$ is the helicity, $\mathcal{M}_h$ is the amplitude with helicity $h$ and it’s explicit expression could be found in Ref. [17], the subscript $L$ stands for $h = 0$ component while $N$ and $T$ express another two $h = \pm 1$ components, $P_{1(2)}$ denote the four momenta of vector mesons, and $\epsilon^*_1(t) \cdot \epsilon^*_2(t) = 1$ with $t = \pm 1$. Hence, each helicity
amplitude could be written as \[17\]

\[ H_0 = m_B^2 M_L , \]
\[ H_\pm = m_B^2 M_N \mp m_{V_1} m_{V_2} \sqrt{r^2 - 1} M_T , \]  

(4)

and \( r = P_1 \cdot P_2 / (m_{V_1} m_{V_2}) \) in which \( m_{V_1(2)} \) are the masses of vector mesons. Moreover, we can also write the amplitudes in terms of polarizations as

\[ A_L = H_0, \quad A_{\parallel(\perp)} = \frac{1}{\sqrt{2}} (H_- \pm H_+) . \]  

(5)

The relative phases are defined as \( \phi_{\parallel(\perp)} = \text{Arg}(A_{\parallel(\perp)}/A_0) \). Accordingly, the polarization fractions (PFs) can be defined as

\[ R_i = \frac{|A_i|^2}{|A_L|^2 + |A_\parallel|^2 + |A_{\perp}|^2} \quad (i = L, \parallel, \perp) . \]  

(6)

Since we have derived the formalisms for the decay amplitudes \( M_{L,N,T} \) by PQCD approach in Ref. \[17\], in our following discussions, we only concentrate on the theoretical uncertainties of PQCD.

It is known that by PQCD the transition amplitude is factorized into the convolution of hadron wave functions and the hard amplitude of the valence quarks, in which the wave functions absorb the infrared divergences and represent the effects of nonperturbative QCD. With including the transverse momentum of valence quark, \( k_T \), the factorization formula for the decay of \( B \) meson could be briefly described as \[12\]

\[ H_r(m_W, \mu) H(t, \mu) \Phi(x, P, b, \mu) = c(t) H(t, t) \Phi(x, b, 1/b) \]
\[ \times \exp \left[ -s(P, b) - \int_{1/b}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_\Phi(\alpha_s(\bar{\mu})) \right] \]  

(7)

where \( H_r(m_W, \mu) \) and \( H(t, \mu) \) denote the renormalized hard parts which the running scale starts from \( m_W \) and typical hard scale \( t \), respectively, \( \Phi(x, P, b, \mu) \) is the wave function of meson, \( c(t) \) is the effective Wilson coefficient, \( b \) is the conjugate variable of \( k_T \), \( s(P, b) \) is Sudakov factor for suppressing the radiative corrections at large \( b \) parameter, and \( \gamma_\Phi \) stands for the anomalous dimension of valence quark. Clearly, for calculating the decay amplitudes of \( B \) decays, we have to determine the typical scale which dictates the decaying scale of \( B \) meson. To illustrate the chosen hard scale in conventional PQCD, we take the transition matrix element \( \langle M(P_2) | \bar{b} \gamma_\mu q | B(P_1) \rangle \) as the example. As usual, the condition for the hard
scale is set to be
\[ t = \max \left( \sqrt{x_1 m_B^2}, \sqrt{x_2 m_B^2}, 1/b_1, 1/b_2 \right), \] (8)
where \( x_1(2) \) are the momentum fraction carried by the quark of \( B(M) \) meson. Since the allowed range of momentum fraction is between 0 and 1, therefore the value of hard scale could be less than 1 GeV. However, the wave functions such as twist-2 wave function expressed by
\[ \Phi(x, \mu^2) = 6x (1 - x) \left( 1 + \sum_{n=1}^{\infty} a_n (\mu^2) C_n^{3/2} (2x - 1) \right), \] (9)
are expanded by the Gegenbauer polynomials; and the scale-dependent coefficients are usually estimated at \( \mu = 1 \text{ GeV} \). That is, the physics below 1 GeV belongs to nonperturbative region and hard scale should end up at this scale. Consequently, we regard that the condition of Eq. (8) should be revised to be
\[ t = \max \left( \sqrt{x_1 m_B^2}, \sqrt{x_2 m_B^2}, 1/b_1, 1/b_2, \bar{\Lambda} \right) \] (10)
where \( \bar{\Lambda} \) indicates the cutoff for distinguishing the region of perturbation and nonperturbation, i.e. below \( \bar{\Lambda} \) the physics is dominated by nonperturbative effects. Roughly, the order of magnitude of the hard scale could be estimated by the momentum of exchanged hard gluon as \( t \sim \sqrt{x_1 x_2 m_B^2} \). It is known that \( x_1 \sim (m_B - m_b)/m_B \) and \( x_2 \sim O(1) \). By taking \( x_1 = 0.16, x_2 = 0.5 \) and \( m_B = 5.28 \text{ GeV} \), the average value of hard scale could be estimated to be around \( \bar{t} \sim 1.5 \text{ GeV} \). Besides the chosen condition for hard scale and wave functions of light mesons, the remaining uncertainties of PQCD are the shape parameter \( \omega_B \) of the \( B \) meson wave function and the parametrization of threshold resummation, denoted by \( S_t(x) = 2^{1+2c}\Gamma(1+2c)[x(1-x)]^c/\sqrt{\pi}(1+c) \) [17]. In our following numerical estimations, we will set \( \omega_B = c = 0.4 \). Hence, according to the wave functions derived by QCD sum rules [13] and using \( f_{K^*}^{(T)} = 210(170) \text{ MeV} \), the values of \( B \to K^* \) form factors, defined by [18]
\[ \langle M(P_2, \epsilon)|\bar{b}\gamma_\mu q|B(P_1)\rangle = i \frac{V(q^2)}{m_B + m_M} \varepsilon_{\mu\alpha \beta \rho} \varepsilon^*_{\alpha\beta} P^\beta q^\rho, \]
\[ \langle M(P_2, \epsilon)|\bar{b}\gamma_\mu \gamma_5 q|B(P_1)\rangle = 2m_M A_0(q^2) \frac{\varepsilon^* \cdot q}{q^2} q_\mu + (m_B + m_M) A_1(q^2) \left( \varepsilon^*_\mu - \frac{\varepsilon^* \cdot q}{q^2} q_\mu \right) - A_2(q^2) \frac{\varepsilon^* \cdot q}{m_B + m_M} \left( P_\mu - \frac{P \cdot q}{q^2} q_\mu \right), \] (11)
are given in Table [11] where \( M \) and \( m_M \) denote the vector meson and it’s mass, \( P = P_1 + P_2 \) and \( q = P_1 - P_2 \). In the table, for comparison, we also show the results of quark model
TABLE II: Form factors for $B \to K^*$ at $q^2 = 0$ in various QCD models.

| Model     | $V(0)$ | $A_0(0)$ | $A_1(0)$ | $A_2(0)$ |
|-----------|--------|----------|----------|----------|
| QM [19]   | 0.44   | 0.45     | 0.36     | 0.32     |
| LCSR [14] | 0.41   | 0.37     | 0.29     | 0.26     |
| LFQM [20] | 0.31   | 0.31     | 0.26     | 0.24     |
| PQCD [18] | 0.34   | 0.37     | 0.23     | 0.22     |

(QM) [19], light-cone sum rules (LCSR) [14], and light-front quark model (LFQM) [20]. In terms of the formulas, which are derived in Ref. [17] and have included nonfactorizable and annihilation effects, and by taking $V_{us}^* V_{ub} = A \lambda^3 R_b e^{-i \phi_3}$ and $V_{tb} V_{ts}^* = -A \lambda^2$ with $A = 0.82$, $\lambda = 0.224$, $R_b = 0.38$ and $\phi_3 = 63^\circ$, the calculated BR, PFs, and $\phi_{\parallel (\perp)}$ of $B_d \to K^{*0} \phi$ with different values of $\bar{\Lambda}$ are presented in Table III. Although there exist other chosen conditions for nonfactorized and annihilated parts, since the conditions are similar to Eq. (8), we neglected showing them. The details could be referred to Ref. [17]. In the table, we have

TABLE III: BR (in units of $10^{-6}$), PFs and relative phases of $B_d \to K^{*0} \phi$ for $\bar{\Lambda} = 0, 1.0, 1.3$ and $1.6$ GeV.

| $\bar{\Lambda}$ | $BR$ | $R_L$ | $R_{\parallel}$ | $R_{\perp}$ | $\phi_{\parallel}(rad)$ | $\phi_{\perp}(rad)$ |
|-----------------|------|-------|-----------------|-------------|-------------------------|---------------------|
| 0               | 14.54| 0.71  | 0.16            | 0.13        | 2.48                    | 2.47                |
| 1.0             | 10.32| 0.65  | 0.19            | 0.16        | 2.33                    | 2.32                |
| 1.3             | 8.91 | 0.63  | 0.20            | 0.17        | 2.27                    | 2.26                |
| 1.6             | 7.69 | 0.61  | 0.21            | 0.18        | 2.22                    | 2.21                |

set $\bar{\Lambda} = 0$ as the old chosen conditions for the hard scales. From the table, we clearly see that the BR and $R_L$ are decreasing while $\bar{\Lambda}$ is increasing. If we regard $\bar{t} \sim \bar{\Lambda} \sim 1.5$ GeV, we obtain that the $R_L$ of $B \to K^{*0} \phi$ could be around 62% while the BR could be $8 \times 10^{-6}$. Since the errors of neutral $B$ decay are still big, if we use the observed world averages of charged mode, which they are $BR = (9.7 \pm 1.5) \times 10^{-6}$ and $R_L = 0.50 \pm 0.07$ [22], as the illustration, we find that our $R_L$ has approached to the upper bound of world average of $B_u \to K^{*+} \phi$ while the BR is close to the lower bound. Clearly, by using Eq. (10), we can improve our results to be more close to the indications of data. Furthermore, in
order to understand the influence of nonfactorizable and annihilation effects, we present the results without either and both contributions in Table IV. By the results, we could see nonfactorizable and annihilation contributions play important role on the PFs, especially, the annihilation effects. The brief reason is that the penguin dominant processes involve $O_{6,8}$ operators which the chiral structures are $(V - A) \otimes (V + A)$. The detailed interpretation could be referred to Refs. [21, 23].

TABLE IV: BR (in units of $10^{-6}$), PFs and relative phases for $B_d \to K^{*0} \phi$ without nonfactorization or/and annihilation.

| topology | BR | $|A_0|^2$ | $|A_1|^2$ | $|A_{1\parallel}|^2$ | $\phi_{\parallel}(\text{rad})$ | $\phi_{\perp}(\text{rad})$ |
|----------|----|---------|---------|-----------------|-----------------|-----------------|
| no nonfac. | 12.05 | 0.78 | 0.12 | 0.10 | 2.15 | 2.12 |
| no anni. | 8.42 | 0.83 | 0.09 | 0.08 | 3.30 | 3.32 |
| no both | 9.41 | 0.92 | 0.04 | 0.04 | $\pi$ | $\pi$ |

Next, we discuss the tree dominant processes $B \to \rho(\omega)\rho(\omega)$ in which at quark level the decays are governed by $b \to d\bar{q}q$. Since for those color-allowed decays, penguin contributions are small, according to the analysis of Ref. [23], it is expected that the annihilation effects are negligible. In addition, since the nonfactorizable effects are associated with $C_1/N_c$ in which $N_c$ is the number of color and $C_1$ is roughly less than $C_2$ by a $N_c$ factor, thus, we conjecture that the nonfactorizable contributions for color-allowed processes are also negligible. Consequently, we conclude that the PFs should be the same as the naive estimations, i.e. $R_L \approx 1 - m_{M}^2/m_B^2$. By using the decay constants $f_{\rho} = f_{\omega} = 200$ MeV, $f_{\rho}^T = f_{\omega}^T = 160$ MeV and the same taken values of parameters for $B \to K^* \phi$, the values of $B \to \rho$ form factors, defined by Eq. (11), in various QCD models are given in Table V. Again, in terms of the

TABLE V: Form factors for $B \to \rho$ at $q^2 = 0$ in various QCD models.

| Model     | $V(0)$ | $A_0(0)$ | $A_1(0)$ | $A_2(0)$ |
|-----------|--------|----------|----------|----------|
| QM [19]   | 0.31   | 0.30     | 0.26     | 0.24     |
| LCSR [14] | 0.32   | 0.30     | 0.24     | 0.22     |
| LFQM [20] | 0.27   | 0.28     | 0.22     | 0.20     |
| PQCD      | 0.26   | 0.29     | 0.22     | 0.21     |
formulas derived by Ref. [17], by setting $\bar{\Lambda} = 1$ GeV and by using the wave functions of $\rho$ and $\omega$ instead of those of $\phi$ and $K^*$, the BRs, PFs and $\phi_{\parallel(\perp)}$ of $B \to \rho(\omega)\rho(\omega)$ are shown in Table VII. The results with conventional chosen conditions could be referred to Ref. [24].

Compare to the data displayed in Table VII, we find that the BR of $B_d(\to \rho^-\rho^+)$ is consistent with the observation of BELLE(BABAR). Although the result of $B_u \to \rho^0\rho^+$ doesn’t fit well with current data, since the errors of data are still large, more accumulated data are needed to further confirm. On the other hand, in the theoretical viewpoint, the BR of $B_u \to \rho^0\rho^+$ should be similar to that of $B_u \to \omega\rho^+$. Without any anomalous effects, we still expect $BR(B_u \to \rho^0\rho^+) \sim BR(B_u \to \omega\rho^+)$. As for the polarizations, like our expectation, the data show that nonfactorization and annihilation are not important in color-allowed processes of $B \to \rho(\omega)\rho(\omega)$. We note that for those color-suppressed decays, since the penguin effects are not small anymore, therefore, the nonfactorizable and annihilation effects may become important. This is the reason why we get a very small $R_L$ in $B_d \to \rho^0\rho^0$ decay. It is worth mentioning that the CP asymmetry (CPA), defined by $A_{CP} = [\Gamma(B \to f) - \Gamma(B \to \bar{f})]/[\Gamma(B \to f) + \Gamma(B \to \bar{f})]$ with $f$ being any final state, for $B_d \to \rho^\pm\rho^\mp$ has only few percent. That is, the penguin pollution in this decay is small. Thus, we speculate that the observed time-dependent CPA could directly indicate the bound on the angle $\phi_2$ of CKM.

| Mode       | BR   | $R_L$ | $R_{\parallel}$ | $R_{\perp}$ | $\phi_{\parallel}(rad)$ | $\phi_{\perp}(rad)$ | $A_{CP}$ |
|------------|------|------|-----------------|-------------|-------------------------|---------------------|---------|
| $B^0 \to \rho^-\rho^+$ | 23.06 | 0.95 | 0.03            | 0.02        | $\approx \pi$           | $\approx 0$         | -2.96   |
| $B^0 \to \rho^0\rho^0$   | 0.12  | 0.07 | 0.43            | 0.50        | 3.46                    | 3.63                | 83.21   |
| $B^0 \to \rho^0\omega$   | 0.38  | 0.93 | 0.04            | 0.03        | 4.03                    | 3.93                | 55.29   |
| $B^0 \to \omega\omega$   | 0.35  | 0.76 | 0.12            | 0.12        | 1.70                    | 1.69                | -92.72  |
| $B^+ \to \rho^0\rho^+$   | 11.99 | 0.98 | 0.01            | 0.01        | $\approx \pi$           | $\approx 0$         | $\approx 0$ |
| $B^+ \to \omega\rho^+$   | 14.78 | $\approx 1$ | $\approx 0$   | $\approx 0$ | $\approx \pi$           | 3.36                | -11.11  |

Based on the previous analyses, we have learnt that by the assumption of short-distant dominance in the $B$ decays, the nonfactorization and annihilation are unimportant and negligible for the tree amplitude; however, when penguin contributions are dominant, their effects become essential on PFs. For more comparisons with the experiments, we also cal-
TABLE VII: The experimental data on BRs (in units of $10^{-6}$) and PFs of $B \to \rho(\omega)\rho$ [1, 2, 3].

| Mode          | Observation | BELLE       | BABAR       |
|---------------|-------------|-------------|-------------|
| $\rho^-\rho^+$| BR          | $24.4 \pm 2.2^{+3.8}_{-4.1}$ | $30 \pm 4 \pm 5$ |
|               | $|A_0|^2$    | $0.951^{+0.033+0.029}_{-0.039-0.031}$ | $0.99 \pm 0.03^{+0.04}_{-0.03}$ |
| $\rho^{0}\rho^+$ | BR          | $31.7 \pm 7.1^{+3.8}_{-6.7}$ | $22.5^{+5.7}_{-5.4} \pm 5.8$ |
|               | $|A_0|^2$    | $0.95 \pm 0.11 \pm 0.02$ | $0.97^{+0.03}_{-0.07} \pm 0.04$ |
| $\omega \rho^+$ | BR          | --          | $12.6^{+3.7}_{-3.3} \pm 1.6$ |
|               | $|A_0|^2$    | --          | $0.88^{+0.12}_{-0.15} \pm 0.03$ |

TABLE VIII: The BRs (in units of $10^{-6}$), PFs and relative phases for $B \to \rho(\omega)K^*$. 

| Mode          | BR     | $R_L$ | $R_{\parallel}$ | $R_{\perp}$ | $\phi_{\parallel}(\text{rad})$ | $\phi_{\perp}(\text{rad})$ | $A_{CP}$ |
|---------------|--------|-------|-----------------|-------------|-------------------------------|--------------------------|----------|
| $B^0 \to \rho^- K^{*+}$ | 10.13  | 0.60  | 0.21            | 0.19        | 1.60                          | 1.59                     | -19.17   |
| $B^0 \to \rho^0 K^{*0}$  | 4.15   | 0.70  | 0.16            | 0.14        | 1.17                          | 1.17                     | 9.38     |
| $B^0 \to \omega K^{*0}$  | 6.75   | 0.75  | 0.13            | 0.12        | 1.79                          | 1.82                     | -7.93    |
| $B^+ \to \rho^+ K^{*0}$  | 11.99  | 0.78  | 0.12            | 0.10        | 1.45                          | 1.46                     | 0.79     |
| $B^+ \to \rho^0 K^{*+}$  | 7.53   | 0.72  | 0.15            | 0.13        | 1.82                          | 1.81                     | -19.74   |
| $B^+ \to \omega K^{*+}$  | 5.67   | 0.61  | 0.21            | 0.18        | 2.03                          | 2.06                     | -14.31   |

TABLE IX: The experimental data on BRs (in units of $10^{-6}$) and PFs of $B \to \rho K^*$ [2, 26, 27].

| Mode          | Observation | BELLE       | BABAR       |
|---------------|-------------|-------------|-------------|
| $\rho^+ K^{*0}$ | BR          | $8.9 \pm 1.7 \pm 1.2$ | $17.0 \pm 2.9^{+2.0}_{-2.8}$ |
|               | $|A_0|^2$    | $0.43 \pm 0.11^{+0.05}_{-0.02}$ | $0.79 \pm 0.08 \pm 0.04$ |
| $\rho^{0} K^{*+}$ | BR          | --          | $10.6^{+3.0}_{-2.6} \pm 2.4$ |
|               | $|A_0|^2$    | --          | $0.96^{+0.04}_{-0.15} \pm 0.04$ |

summarize the main findings as follows.
• Although the decay constants $f_{\rho(K^*)}$ are larger than $f_{\pi(K)}$, the BRs of $B \to \rho(\omega)K^*$ all are smaller than those of $B \to \pi K$ in which the corresponding flavor diagrams for $\pi K$ and $\rho(\omega)K^*$ in Fig. 1 are the same. The reason is that the factorizable contributions of $O_{6,8}$ operators are vanished in vector-vector modes, i.e. $\langle V_1 V_2 | (V - A) \otimes (V + A) | B \rangle \sim -2 \langle V_1 | S - P | 0 \rangle \langle V_2 | S + P | B \rangle = 0$ due to $\langle V_1 | S | 0 \rangle = m_{V_1} f_{V_1} \epsilon_1 \cdot P_{V_1} = 0$ where $S(P)$ denotes the scalar(pseudoscalar) current. As a result, the decays, which the tree amplitudes are color-allowed such as $\rho^{\mp} K^{*\pm}$ and $\rho^0(\omega)K^{*\pm}$, have larger CPAs.

• The $R_L$ of $B_d \to \rho^- K^{*+}$ could be as small as 60%. The result could be understood as follows: since the involving tree contributions are color-allowed, as mentioned in the decays $B \to \rho(\omega)\rho(\omega)$, we know that the nonfactorizable effects are negligible and transverse parts are small. Moreover, the amplitude of penguin is opposite in sign to that of tree. Therefore, the longitudinal part gets a large cancelation in tree and penguin such that the $R_L$ is reduced. And also, the magnitude of CPA is enhanced to be around 20%.

• Although the decays $B_u \to \rho^0(\omega)K^{*+}$ possess sizable tree contributions, however besides the diagrams Fig. (a) and (c), Fig. (b), representing the effects of electroweak penguin mainly, also has the contributions. And also, due to different flavor wave functions in $\rho$ and $\omega$, respectively denoted by $(u\bar{u} \mp d\bar{d})/\sqrt{2}$, interestingly we find that the $R_L$ of $B_u \to \rho^0 K^{*+}$ is around 72% but the $R_L$ of $B_u \to \omega K^{*+}$ could be around 61% which is similar to the value of $B_d \to \rho^- K^{*+}$.

• By naive analysis, one could expect that by neglecting the small tree contributions which are arisen from annihilation topologies, the obtained $R_L$ of $B_u \to \rho^+ K^{*0}$ should be similar to the value of $B_d \to K^{*0}\phi$. However, the calculated results shown in the Tables III and VIII are contrary to the expectation. The main reason is that the sign of real part of annihilated amplitude for $B_u \to \rho^+ K^{*0}$ decay is opposite to that for $B_d \to K^{*0}\phi$ decay. In other words, the annihilation is constructive effect in $R_L$ of $\rho^+ K^{*0}$ while it is destructive in $K^{*0}\phi$. We find that the differences are ascribed to the wave functions of mesons. In sum, the calculations of PQCD in some physical quantities, such as PFs, strongly depend on the detailed shapes of wave functions. Due to the sign difference in the real part of annihilation, we predict that LPs in most $\rho(\omega)K^*$ modes are much larger than those in $B \to K^{*}\phi$. We note that the conclusion is not suitable for those tree color-allowed processes, such as $\rho^+ K^{*+}$ and $B_u \to \rho^0(\omega)K^{*+}$, because according to previous discussions, the tree and/or electroweak penguin amplitudes have significant contributions so that the effective factors become more
complicated, i.e. tree, electroweak and annihilation all are important in these decays.

In summary, we have reanalyzed the BRs and PFs of $B \to K^\ast \phi$ in the framework of PQCD. In terms of the revised conditions for the hard scales of $B$ decays, we find the LPs of $B \to K^\ast \phi$ could approach to around 60% while the BRs are around $9 \times 10^{-6}$. It is confirmed that the annihilation and nonfactorizable contributions have no effects on PFs of $B \to \rho(\omega)\rho(\omega)$ decays so that the LPs are all close to unity; and also, we find that the BR of $B_{d(u)} \to \rho^- (\omega)\rho^+$ is consistent with the observation of BELLE(BABAR). By the calculations, we obtain that the penguin pollution in $B_d \to \rho^\pm \rho^\mp$ decays is very small so that the observed time-dependent CPA could directly indicate the bound on the angle $\phi_2$ of CKM. In addition, we also find that due to significant tree contributions, the BR(LP) of $\rho^- K^{*+}$ could be around $10.13 \times 10^{-6}$ (60%); and due to the tree and electroweak penguin, the BR(LP) of $\omega K^{*+}$ could be around $5.67 \times 10^{-6}$ (61%).

Acknowledgments

The author would like to thank Hai-Yang Cheng, Darwin Chang, Chao-Qiang Geng, Hsiang-Nan Li, Kingman Cheung, Chu-Khiang Chua, Cheng-Wei Chiang and We-Fu Chang for useful discussions. This work is supported in part by the National Science Council of R.O.C. under Grant #s:NSC-94-2112-M-006-009.

[1] BELLE Collaboration, J. Zhang et al., Phys. Rev. Lett. 91, 221801 (2003); K. Abe et al., arXiv:hep-ex/0507039.
[2] BABAR Collaboration, B. Aubert et al., Phys. Rev. Lett. 91, 171802 (2003); A. Gritsan, arXiv:hep-ex/0409059.
[3] BABAR Collaboration, B. Aubert et al., Phys. Rev. D69, 031102 (2004); Phys. Rev. Lett. 93, 231801 (2004); B. Aubert et al., Phys. Rev. D71, 031103 (2005).
[4] BELLE Collaboration, K. Abe et al., Phys. Lett. B538, 11 (2002); arXiv:hep-ex/0408104.
[5] BABAR Collaboration, B. Aubert et al., Phys. Rev. Lett. 87, 241801 (2001).
[6] BELLE Collaboration, K. F. Chen, et al., Phys. Rev. Lett. 94, 221804 (2005).
[7] BABAR Collaboration, B. Aubert et al., arXiv:hep-ex/0303020 B. Aubert et al., Phys. Rev.
Lett. 93, 231804 (2004).

[8] A. Kagan, Phys. Lett. B601, 151 (2004); W.S. Hou and M. Nagashima, hep-ph/0408007 P. Colangelo, F. De Fazio and T.N. Pham, Phys. Lett. B597, 291 (2004); M. Ladisa et al., Phys. Rev. D70, 114025 (2004); H.Y. Cheng, C.K. Chua and A. Soni, Phys. Rev. D71, 014030 (2005); H.N. Li, Phys. Lett. B622, 63 (2005).

[9] A. Kagan, hep-ph/0407076 E. Alvarez et al, Phys. Rev. D70, 115014 (2004); Y.D. Yang, R.M. Wang and G.R. Lu, Phys. Rev. D72, 015009 (2005); A.K. Giri and R. Mohanta, hep-ph/0412107 P.K. Das and K.C. Yang, Phys. Rev. D71, 094002 (2005); C.S. Kim and Y.D. Yang, arXiv:hep-ph/0412364 C.S. Hung et al., arXiv:hep-ph/0511129 S. Nandi and A. Kundu, arXiv:hep-ph/0510245 S. Baek et al., Phys. Rev. D72, 094008 (2005).

[10] C.H. Chen and C.Q. Geng, Phys. Rev. D71, 115004 (2005).

[11] G.P. Lepage and S.J. Brodsky, Phys. Lett. B87, 359 (1979); Phys. Rev. D22, 2157 (1980); H.N. Li and G. Sterman, Nucl. Phys. B381, 129 (1992); G. Sterman, Phys. Lett. B179, 281 (1986); Nucl. Phys. B281, 310 (1987); S. Catani and L. Trentadue, Nucl. Phys. B327, 323 (1989); Nucl. Phys. B353, 183 (1991); Y.Y. Keum, H.N. Li and A.I. Sanda, Phys. Rev. D63, 054008 (2001); H.N. Li, Phys. Rev. D64, 014019 (2001); H.N. Li, Phys. Rev. D66, 094010 (2002).

[12] T.W. Yeh and H.N. Li, Phys. Rev. D56, 1615 (1997).

[13] P. Ball et al., Nucl. Phys. B529, 323 (1998); P. Ball and R. Zwicky, Phys. Rev. D71, 014015 (2005).

[14] P. Ball and R. Zwicky, Phys. Rev. D71, 014029 (2005).

[15] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).

[16] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[17] C.H. Chen, Y.Y. Keum and H.N. Li, Phys. Rev. D66, 054013 (2002).

[18] C.H. Chen and C.Q. Geng, Nucl. Phys. B636, 338 (2002).

[19] D. Melikhov and B. Stech, Phys. Rev. D62, 014006 (2000).

[20] H.Y. Cheng et al., Phys. Rev. D69, 0774025 (2004).

[21] C.H. Chen, Y.Y. Keum and H.N. Li, Phys. Rev. D64, 112002 (2001).

[22] Heavy Flavor Averaging Group, K. Anikeev et al., arXiv:hep-ex/0505100

[23] C.H. Chen et al., Phys. Rev. D72, 054011 (2005).
[24] Y. Li and Cai-Dian Lu, arXiv:hep-ph/0508032

[25] H.W. Hunag et al., arXiv:hep-ph/0508080

[26] BABAR Collaboration, B. Aubert et al., arXiv:hep-ex/0408063, arXiv:hep-ex/0408093

[27] BELLE Collaboration, J. Zhang et al., arXiv:hep-ex/0505039