The Rotational Mechanic Equipment Fault Diagnosis Based on the Wavelet Packet Analysis

S Q Zhang\textsuperscript{1}, L G Zhang\textsuperscript{1}, Z P Gu\textsuperscript{2}, J T Lv\textsuperscript{3} and T Huang\textsuperscript{2}

\textsuperscript{1} Electrical Engineering Institute, Yanshan University, Qinhuangdao, China, 066004
\textsuperscript{2} Hebei University of Science and Technology
\textsuperscript{3} Northeast University at Qinhuangdao, HeBei, China

E-mail: siwenren-yd@163.com

Abstract. The gears is a main driving device in the modern rotational machine equipments at the present time. In this paper, the Mallat algorithm of wavelet and wavelet packet is studied and is used in the fault diagnosis of the rotational machine equipment. The complicated mechanical vibration signals are analyzed by it. The weak signal is extracted effectively and the signal of the fault symptom is located in the time domain. The testing results proved the value of the analytic method.

1. Introduction

The gears is the most conventional parts and easy to be damaged in the modern rotational machine equipments. The working information of the gears is very complicated because of the complication of damaged forms and the effect of the equipment structure. The energy reflecting the early damaged state is very weak and often buried in the other signals. In this paper, the signals composed of all kinds of running and fault signals are decomposed to the signals in different frequency bands by the wavelet packet. The causation of all kinds of the faults will to be found according to the correlations of the fault and the frequency band. So, different kinds of the rotational mechanical equipments faults can be diagnosed effectively.

2. The mallat algorithm of the wavelet

To the optional signal $f(t) \in L^2(\mathbb{R})$, The sign is induced as (1)

$$
\begin{align*}
  c_{j,k} &= \int_{\mathbb{R}} f(t) \varphi_{j,k}(t) \, dt \\
  d_{j,k} &= \int_{\mathbb{R}} f(t) \psi_{j,k}(t) \, dt
\end{align*}
$$

They are the scale factor and the wavelet factor of the $f(t)$. At the same time, The orthogonal projection of the $f(t)$ in the closed subspace $V_j$ and $W_j$ is named $A_j f(t)$ and $D_j f(t)$ separately, so

$$
\begin{align*}
  A_j f(t) &= \sum_{k \in \mathbb{Z}} c_{j,k} \varphi_{j,k}(t) \\
  D_j f(t) &= \sum_{k \in \mathbb{Z}} d_{j,k} \psi_{j,k}(t)
\end{align*}
$$

According the space orthogonal direct sum resolution relation

$$
V_{j+1} = V_j \oplus W_j
$$

We can get the (4)

$$
A_{j+1} f(t) = A_j f(t) + D_j f(t)
$$
The relation between the scale factor and the wavelet factor of the signal is

\[ \sum_{k \in Z} c_{j,k} \varphi_{j,k}(t) = \sum_{k \in Z} d_{j,k} \psi_{j,k}(t) \quad (5) \]

To get the \( \{c_{j,n}; n \in \mathbb{Z}\} \) and \( \{d_{j,n}; n \in \mathbb{Z}\} \) according \( \{c_{m,n}; m \in \mathbb{Z}\} \), we use \( \varphi_{j,n}(t) \) and \( \psi_{j,n}(t) \) multiply the both end of the (4) and then get the integration, we can use the factor formula of the scale equation and the constitution equation

\[
\begin{align*}
    h_l &= \int_{-\infty}^{\infty} \varphi(t) \varphi_{l,J}(t) \, dt \\
    g_l &= \int_{-\infty}^{\infty} \psi(t) \varphi_{l,J}(t) \, dt 
\end{align*}
\]

(6)

to get the Mallat resolution formula

\[
\begin{align*}
    c_{j,n} &= \sum_{m \in \mathbb{Z}} \tilde{h}_{m-2n} c_{j+1,m} \\
    d_{j,n} &= \sum_{m \in \mathbb{Z}} \tilde{g}_{m-2n} c_{j+1,m} 
\end{align*}
\]

(7)

3. The Mallat algorithm of the wavelet packet

When \( l = 0, 1, 2, \ldots \), The sign is induced as \( (8) \)

\[ d_{j,n}^{(l)} = \int_{-\infty}^{\infty} f(t) \varphi_{l,j,n}(t) \, dt \]

(8)

So, The signal \( f(t) \) in the wavelet packet

\[ U_j^l = \text{Closspan} \{ \mu_{l,j,n}(t) = 2^{j/2} \mu_l(2^j t - n); n \in \mathbb{Z} \} \]

(9)

can be projected to

\[ D_j^l f(t) = \sum_{m \in \mathbb{Z}} d_{j,n}^{(l)} \mu_{l,j,n}(t) \]

(10)

The Mallat algorithm formular of wavelet packet discompose are as follows:

\[
\begin{align*}
    d_{j,n}^{(2l)} &= \sum_{m \in \mathbb{Z}} \tilde{h}_{m-2n} d_{j+1,m}^{(l)} \\
    d_{j,n}^{(2l+1)} &= \sum_{m \in \mathbb{Z}} \tilde{g}_{m-2n} d_{j+1,m}^{(l)} 
\end{align*}
\]

(11)

In fact, if the orthogonal projection of the primordial signal \( f(t) \) in the wavelet space of

\[ U_j^{2l} = \text{Closspan} \{ \mu_{2l,j,n}(t) = 2^{l/2} \mu_{2l}(2^l t - n); n \in \mathbb{Z} \} \]

(12)

\[ U_j^{2l+1} = \text{Closspan} \{ \mu_{2l+1,j,n}(t) = 2^{l/2} \mu_{2l+1}(2^l t - n); n \in \mathbb{Z} \} \]

(13)

are named by \( D_j^{(2l)} f(t) \) and \( D_j^{(2l+1)} f(t) \) separately, then the factor of \( D_j^{(2l)} f(t) \) span in the wavelet packet base \( \{ \mu_{2l,j,n}(t); n \in \mathbb{Z} \} \) is \( \{ d_{j,n}^{(2l)}; n \in \mathbb{Z} \} \) exactly; But the factor of \( D_j^{(2l+1)} f(t) \) span in the wavelet packet base \( \{ \mu_{2l+1,j,n}(t); n \in \mathbb{Z} \} \) is \( \{ d_{j,n}^{(2l+1)}; n \in \mathbb{Z} \} \) exactly.

4. The wavelet packet analysis to the signals

The signals can be analyzed very delicately by the wavelet packet. The frequency band can be separated to several depths by it. The lower and higher frequency parts of the signal can be decomposed further. So the resolution of the signal can be improved. For example: The three-layer wavelet packet analysis tree of the signal with cut-off frequency of 1 kHz is illustrated in figure 1.

The relation of the separated signals is:

\[ S = S_{111} + S_{112} + S_{121} + S_{122} + S_{211} + S_{212} + S_{221} + S_{222} \]

(14)

In the paper:

\[
\begin{align*}
    S_{111} &= (0-0.125K), S_{112} = (0.125K-0.25K), S_{121} = (0.25K-0.375K), S_{122} = (0.373-0.5K), \\
    S_{211} &= (0.5K-0.625K), S_{212} = (0.625K-0.75K), S_{221} = (0.75-0.875K), S_{222} = (0.875-1K).
\end{align*}
\]
5. The wavelet Packet detection experiment of the rotational mechanical equipment fault

5.1. The parameter of the gear-driven system and the fault analysis
In the experiment, the gear parameter as Table.1. The characteristic parameter of the fault frequency of this system can be obtained by the vibration fault frequency formula as table.2.

| modulus (mm) | The tooth width | The contour of the tooth | Speed |
|-------------|-----------------|-------------------------|-------|
| 30          | 35              | 230                     | 500   |
| 30          | 36              | 230                     | 485   |
| 25          | 19              | 300                     | 485   |
| 25          | 66              | 300                     | 139.9 |
| 40          | 18              | 500                     | 139.9 |
| 40          | 50              | 500                     | 50.3  |

5.2. The wavelet separation of the signal and the fault diagnosis
The sampling frequency of the system is 3kHz. So the frequency component contained in the signal is from 0 to 1kHz. The cut-of frequency of the signal is 1 kHz after the smoothing of the filter. The frequency band is 0-1 kHz. The Figure.4 is the result of the wavelet analysis. We can get the frequency extent after 8 layer wavelet resolution: The low frequency approximating signal scale A10: 0-0.0039Hz. The high frequency detail signal scale are: A8(0.5-0.5)Hz , A7(0.25-0.5)Hz , A6(0.125-0.25)Hz , A5(0.0625-0.125)Hz , A4(0.0313-0.0625)Hz , A3(0.0156-0.0313)Hz , A2(0.0078-0.0156)Hz , A1(0.0039-0.0078)Hz. The fault frequency of the signal can be obtained from the figure. So the fault characteristic frequency of the Tab.4 can be analyzed in different space.
6. Conclusion
The wavelet packet can decompose both the lower and higher parts of the signal. The resolution of the signal analysis can be improved greatly and the diagnosis locations of the mechanic fault can be detected precisely. The application prospect of the wavelet packet in the fault diagnosis is very spacious.

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