Dark energy models with time-dependent gravitational constant

Saibal Ray\textsuperscript{1,2} & Utpal Mukhopadhyay\textsuperscript{3}
\textsuperscript{1}Department of Physics, Barasat Government College, North 24 Parganas, Kolkata 700 124, West Bengal, India
\textsuperscript{2}Inter-University Centre for Astronomy and Astrophysics, PO Box 4, Pune 411 007, India; e-mail: saibal@iucaa.ernet.in
\textsuperscript{3}Satyabharti Vidyaipith, North 24 Parganas, Kolkata 700 126, West Bengal, India.

\textbf{ABSTRACT}

Two phenomenological models of $\Lambda$, viz. $\Lambda \sim (\ddot{a}/a)^2$ and $\Lambda \sim \ddot{a}/a$ are studied under the assumption that $G$ is a time-variable parameter. Both models show that $G$ is inversely proportional to time as suggested earlier by others including Dirac. The models considered here can be matched with observational results by properly tuning the parameters of the models. Our analysis shows that $\Lambda \sim \ddot{a}/a$ model corresponds to a repulsive situation and hence correlates with the present status of the accelerating Universe. The other model $\Lambda \sim (\ddot{a}/a)^2$ is, in general, attractive in nature. Moreover, it is seen that due to the combined effect of time-variable $\Lambda$ and $G$ the Universe evolved with acceleration as well as deceleration. This later one indicates a Big Crunch.

\textbf{Key words:} gravitation – relativity – cosmology – dark energy.

1 INTRODUCTION

The idea of variability of $G$ originated with the work of Dirac (1937) who for the first time drew the attention of the scientific community towards the possibility of a time-dependent gravitational constant in the context of a cosmological model. Afterwards, cosmological theories like Brans-Dicke theory (1961), Hoyle-Narlikar theory (1972) and the theory of Dirac (1973) himself supported the idea of a time-decreasing gravitational constant.

Since, in the classical form of the general relativity theory $G$ should remain constant, then the theories concerned with the variation of $G$ must be, to some extent, consistent with relativity theory. The last three theories mentioned above are reconcilable with the theory of General Relativity from the viewpoint of the perihelion advance of Mercury and the bending of star light. The theory of a expanding Universe also supports the idea of a time-dependent ($G/G = \sigma H_0$, where $H_0$ is the Hubble parameter and $\sigma$ is a dimensionless parameter depending on the gravitational theory as well as on the particular cosmological model) gravitational constant (Will, 1987).

After the emergence of superstring theory, in which $G$ is considered as a dynamical quantity (Marciano 1984), a resurgence of the idea of an evolving $G$ occurred. It has been shown that a scale-dependent $G$ can represent the dark matter (Goldman 1992). Also, there remain some scale-wise discrepancies in the value of the Hubble parameter. These discrepancies can be removed if $G$ is considered as a variable quantity (Bertolami et al. 1993). Recently, Štefančič (2004) has considered a phantom energy model with time-varying $G$ in which exchange of energy and momentum between vacuum and non-relativistic matter (or radiation) occur in such a way that the energy-momentum tensors $T^\Lambda_{\mu\nu}$ and $T^m_{\mu\nu}$ are not separately conserved, but as a whole $T^{\mu\nu} = T^\Lambda_{\mu\nu} + T^m_{\mu\nu}$ is conserved. Variability of $G$ is also supported by observational results coming up from Lunar Laser Ranging (Turyshev et al. 2003), spinning rate of pulsars (Arzoumanian 1995; Kaspi, Taylor & Ryba 1994; Stairs 2003) Viking Lander (Hellings 1987; Reasenberg 1983), distant Type Ia supernova observation (Gaztanaga et al. 2002), Helioseismological data (Guenther et al. 1998), white dwarf G117-B15A (Biesiada & Malec 2004; Benvenuto et al. 2004) etc.

On the other hand, instead of a cosmological constant $\Lambda$, the recent trend of searching the nature of dark energy, the driving force for accelerating the Universe, is to select a $\Lambda$-model of phenomenological character. Recently, the equivalent relationship of three kinematical models of $\Lambda$, viz. $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \dot{\rho}$ have been shown by us (Ray & Mukhopadhyay 2004) within the framework of constant $G$. Since the idea of time-dependent $G$ is supported by various theories and observations, so it is quite natural to investigate the behaviour of some of the $\Lambda$-models mentioned above when $G$ is a function of time. This is the motivation behind the present work.

However, it can be stated that invariant property of $\Lambda$ under Lorentz transformation is not satisfied for arbitrary systems, e.g., material systems and radiation. In this connection it have shown (Gliner 1965; Majernik 2001) that the energy density of vacuum represents a scalar function of the four-dimensional space-time coordinates so that it satisfies the Lorentz symmetry. We would also like to mention here that Vishwakarma (2001) has considered a particular Ricci-symmetry under the framework of general relativity which is the contracted Ricci-collineation along the fluid flow vector and shows that this symmetry does demand Lambda to be a function of time (and space, in general).

In favour of Lambda-decay scenario, irrespective of whether they come from extended theories of gravity or phenomenological considerations, it is argued that (i) they have been shown to address a number of pressing problems in cosmology; (ii) many
are independently motivated, e.g., by dimensional arguments, or as limiting cases of more complicated theories; (iii) most are simple enough that meaningful conclusions can be drawn about their viability and (iv) successful implementation would point toward the eventual Lagrangian formulation or a more complete theory (Overduin & Cooperstock 1998).

Under this background the paper is organized as follows: Sec. 2 deals with the solution of the field equations for two different models (viz. \( \Lambda \sim (\dot{a}/a)^2 \), \( \Lambda \sim \dot{a}/a \)) of \( \Lambda \) while amount of variations of \( G \), calculated on the basis of different theories as well as observations are scrutinized in Sec. 3. Comparison of the present models with various observational and theoretical results are done in Sec. 4 and conclusions are summarized in Sec. 5.

## 2 FIELD EQUATIONS AND THEIR SOLUTIONS

The Einstein field equations

\[ R^{ij} - \frac{1}{2} R g^{ij} = -8\pi G \left( T^{ij} - \frac{\Lambda}{8\pi G} g^{ij} \right) \]  

(1)

(where \( \Lambda \) is the time-dependent cosmological term and the velocity of light in vacuum, \( c = 1 \) in relativistic units) for the spherically symmetric Friedmann-Lemaître-Robertson-Walker (FLRW) metric

\[ ds^2 = -dt^2 + a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right) \]  

(2)
yield respectively the Friedmann and Raychaudhuri equations in the following forms

\[ 3 \left( \frac{\dot{a}}{a} \right)^2 = 8\pi G \rho + \Lambda, \]  

(3)

\[ 3 \frac{\ddot{a}}{a} = -4\pi G \left( \rho + 3p \right) + \Lambda. \]  

(4)

where \( a = a(t) \) is the scale factor of the Universe and \( k \), the curvature constant is taken to be zero for the flat Universe.

The energy conservation law, as usual, is given by

\[ \dot{\rho} + 3(p + \rho) \frac{\dot{a}}{a} = 0 \]  

(5)

whereas the time variable-\( \Lambda \) and \( G \) are governed by the dynamical condition \( \Lambda = -8\pi G \rho. \)

The barotropic equation of state is

\[ p = \omega \rho \]  

(6)

where \( \omega \) is the equation of state parameter which is, in general, a function of time, scale factor or redshift. However, sometimes it is convenient to consider \( \omega \) as a constant quantity because current observational data has limited power to distinguish between a time varying and constant equation of state (Kujat et al. 2002; Bartelmann et al. 2005). Some useful limits on \( \omega \) came from SNIa data, \(-1.67 < \omega < -0.62 \) (Knop et al. 2003) whereas refined values were indicated by the combined SNIa data with CMB anisotropy and galaxy clustering statistics which is \(-1.33 < \omega < -0.79 \) (Tegmark et al. 2004). Since, \( \dot{a}/a = H \) then from equation (3) we get

\[ 4\pi G \rho = \frac{1}{2} (3H^2 - \Lambda). \]  

(7)

### 2.1 Model with \( \Lambda \sim (\dot{a}/a)^2 \)

By the use of \( \Lambda = 3\alpha \left( \frac{\dot{a}}{a} \right)^2 = 3\alpha H^2 \), where \( \alpha \) is a free parameter, we get from equation (7)

\[ 4\pi G \rho = \frac{3(1 - \alpha)}{2} H^2. \]  

(8)

From equation (4) using (6), one can obtain

\[ 3(H^2 + \dot{H}) = -4\pi G \rho (1 + 3\omega) + 3\alpha H^2 \]  

(9)

which by the use of equation (8) takes the form

\[ \dot{H} = -\frac{3(1 - \alpha)(1 + \omega)}{2} H^2. \]  

(10)

Integrating the above equation we have

\[ \frac{\dot{a}}{a} = H = \frac{2}{3(1 - \alpha)(1 + \omega)} t. \]  

(11)

Integrating it further one gets

\[ a(t) = C_2 e^{\frac{t}{3(1 - \alpha)(1 + \omega)}}. \]  

(12)

where \( C_2 \) is integration constant.

It is interesting to note that equation (12) is the same expression for \( a(t) \) as obtained by us (Ray & Mukhopadhyay 2004) for the same \( \Lambda \)-model with constant \( G \).

Again, using equation (12) we get the solution set for the matter-energy density, cosmological parameter and gravitational parameter respectively as

\[ \rho(t) = C_4 t^{-2(1 - \alpha)}, \]  

(13)

\[ \Lambda(t) = \frac{4\alpha}{3(1 - \alpha)(1 + \omega)^2} t^{2\alpha/(1 - \alpha)} \]  

(14)

\[ G(t) = \frac{6C_4 \pi (1 - \alpha)^2 (1 + \omega)^2}{2\alpha(1 - \alpha)} t^{2\alpha/(1 - \alpha)} \]  

(15)

where \( C_4 \) is the another constant of integration and is given by

\[ C_4 = C_3 C_2^{-31 + \alpha}. \]  

(16)

Again, we observe that equation (14) is also represents exactly the same expression as in the case of our previous work (Ray & Mukhopadhyay 2004). This means that time variation of \( G \) does not affect the scale factor as well as the cosmological parameter. From the equation (15) it is easy to obtain

\[ \frac{\dot{G}}{G} = \frac{2\alpha}{1 - \alpha} t^{-1}. \]  

(17)

### 2.2 Model with \( \Lambda \sim \frac{\dot{a}}{a} \)

Let us put \( \Lambda = \beta \frac{\dot{a}}{a} = \beta (H^2 + \dot{H}) \), where \( \beta \) is a constant. Then, equation (7) becomes

\[ 4\pi G \rho = \frac{3 - \beta}{2} H^2 - \frac{\beta}{2} \dot{H}. \]  

(18)

Using equation (6) we get from equation (4)

\[ (3 - \beta) \dot{H} + (3 - \beta) H^2 = -4\pi G \rho (1 + 3\omega). \]  

(19)

Then by use of the equation (18) the equation (19) transforms to

\[ \dot{H} = -\frac{(3 - \beta)(1 + \omega)}{2 - \beta - \beta\omega} H^2. \]  

(20)
Integrating we get
\[ \frac{\dot{a}}{a} = H = \frac{\beta \omega + \beta - 2}{(\beta - 3)(1 + \omega)} t. \] (21)

Integrating again we get
\[ a(t) = C_6 t^{\frac{\beta \omega + \beta - 2}{(\beta - 3)(1 + \omega)}}, \] (22)

where \( C_6 \) is the integration constant.

Using equation (22) we get the general solutions for different physical parameters as follows:
\[ \rho(t) = C_6 t^{-3(\beta \omega + \beta - 2)}, \] (23)
\[ \Lambda(t) = \frac{\beta(1 + 3\omega)(\beta \omega + \beta - 2)}{(\beta - 3)^2(1 + \omega)^2} t^{-2}, \] (24)
\[ G(t) = \frac{\beta \omega + \beta - 2}{4\pi C_6(\beta - 3)(1 + \omega)^2} t^{\frac{\beta \omega + \beta + \omega}{(\beta - 3)(1 + \omega)}}, \] (25)

where \( C_6 \) is a constant of integration.

Equations (22) and (24) are the same expressions for \( a(t) \) and \( \Lambda(t) \) as obtained by us (Ray & Mukhopadhyay 2004). So, for this model also the scale factor and the cosmological parameter remain unaffected by time variation of \( G \).

### 2.3 Comparison with other models

From equation (25), in a similar way as in the previous case, we have
\[ \frac{\dot{G}}{G} = \frac{\beta \omega + \beta + 6\omega}{(\beta \omega + \beta - 2)} t^{-1}. \] (26)

Now, using the expression for \( H \) from equation (11), equation (17) can be written as
\[ \frac{\dot{G}}{G} = 3\alpha (1 + \omega) H. \] (27)

Similarly, putting the expression for \( H \) from equation (21), equation (26) can be written as
\[ \frac{\dot{G}}{G} = \frac{\beta \omega + \beta + 6\omega}{(\beta \omega + \beta - 2)} H. \] (28)

Recently, considering a time-dependent growing cosmological energy density of the form
\[ \rho_\Lambda = \rho_\Lambda_0 \left( \frac{a}{a_0} \right)^{-3(1+\eta)} \] (29)

Štefančič (2004) has arrived at the expression for \( \dot{G}/G \) at the present era as
\[ \frac{\dot{G}}{G} = 3(1 + \eta) \Omega^\Lambda_0 H_0. \] (30)

If we compare equation (27) with equation (21) of Štefančič (2004), then remembering that for the present era \( \Omega^\Lambda_0 = 2/3 \), we get for dust
\[ \eta = \frac{3\alpha - 2}{2}. \] (31)

Similarly, comparing equation (28) with equation (21) of Štefančič (2004), for dust case, we get
\[ \eta = \frac{4 - \beta}{2(\beta - 2)}. \] (32)

From equations (31) and (32) we readily arrive at the relation
\[ 3\alpha - \frac{\beta}{2 - \beta} = 2(\eta + 1). \] (33)

Equation (33) is interesting, since it inter-relates the parameter \( \eta \) of a phantom energy \( (\omega < -1) \) model with \( \alpha \) and \( \beta \), the parameters of our model with dust \( (\omega = 0) \) case. Here, for a repulsive \( \Lambda \) the constraint on \( \alpha \) is \( \alpha > 0 \).

### 3 PRESENT STATUS FOR THE VARIABLE-\( G \) MODELS

The Large Number Hypothesis (LNH) of Dirac prompted him not to admit the variability of the fundamental constants involved in atomic physics. Instead he thought of a possible change in \( G \) which, in turn, led him to the differential equation (Cetto, Peña & Santos 1986)
\[ G(t) = k_1 H(t) = k_2[H(t)^{3/2} \rho(t)]^{-1/2} \] (34)

where \( k_1 \) and \( k_2 \) are constants.

From the above equation Dirac obtained, \( G(t) \sim t^{-1} \). All the three variants (early Dirac, additive creation and multiplicative creation) tells us that \( G / G \) should be inversely proportional to \( t \).

According to Brans-Dicke theory (1961), \( G \) should vary inversely with time since according to that theory the scalar field \( \phi(t) \) is time increasing and \( G(t) \propto \phi(t)^{-1} \). Dyson (1972,1978) proposed that variation of \( G \) should be of the order of \( H \), the Hubble parameter. Since, \( H \propto t^{-1} \), then it is clear that \( G \) should decrease as \( t^{-1} \).

Coming to the amount of variation of \( G \), we find that, relying on the data provided by three distant quasars of red shift, \( z \sim 3.5 \) in favour of an increasing fine-structure constant \( \alpha \) (Murphy et al. 2002; Webb et al. 2001) and taking the present age of the Universe as 14 Gyr, it has been estimated (Lorèn-Aguilar et al. 2003) that \( G / G \sim +10^{-15} \text{yr}^{-1} \) for Kaluza-Klein and Einstein-Yang-Mills theories whereas it is of the order of \( 10^{-13} \text{yr}^{-1} \) for Randall-Sundrum theory. The data provided by the binary pulsar PSR 1913+16, is a very reliable upper bound (Damour et al. 1988), viz.,
\[ -1.10 \pm 1.07 \times 10^{-11} \text{yr}^{-1} \leq \frac{\dot{G}}{G} \leq 0 \] (35)

The spinning-down rates of pulsars PSR B0655 + 64 (Goldman 1992) and PSR J2019 + 2425 (Arzoumanian 1995; Stairs 2003) provide respectively the constraint
\[ \left| \frac{\dot{G}}{G} \right| \leq (2.2 - 5.5) \times 10^{-11} \text{yr}^{-1} \] (36)
and
\[ \left| \frac{\dot{G}}{G} \right| \leq (1.4 - 3.2) \times 10^{-11} \text{yr}^{-1}. \] (37)

The range of \( \dot{G}/G \) provided by the Helioseismological data (Guenther et al. 1998) is considered as the best upper bound and is given by
\[ -1.60 \times 10^{-12} \text{yr}^{-1} \leq \frac{\dot{G}}{G} < 0. \] (38)

Data provided by observations of Type Ia supernova (Riess 1998; Perlmutter 1999) gives the best upper bound of the variation of \( G \) at cosmological ranges as (Gaztanaga et al. 2002)
\[ -10^{-11} \text{yr}^{-1} \leq \frac{\dot{G}}{G} < 0 \text{ at } z \simeq 0.5. \] (39)
The present cosmological scenario tells us that we are living in an expanding, flat and accelerating Universe dominated by dark energy while the remaining one-third is contributed by matter. So, if we choose \( \Omega_m = 0.3 \) and \( \Omega_\Lambda = 0.7 \), then the estimated range of variation of \( \dot{G}/G \) comes out as (Lorèn-Aguilar 2003)

\[
-1.40 \times 10^{-11} \text{yr}^{-1} < \frac{\dot{G}}{G} < +2.6 \times 10^{-11} \text{yr}^{-1}.
\]

Very recently, using the data provided by the pulsating white dwarf star G117-B15A the asterooseismological bound on \( \dot{G}/G \) is found (Benvenuto et al. 2004) to be

\[
-2.50 \times 10^{-10} \text{yr}^{-1} < \frac{\dot{G}}{G} < +4.0 \times 10^{-10} \text{yr}^{-1}
\]

while using the same star Biesiada and Malec (2004) has inferred that

\[
\left| \frac{\dot{G}}{G} \right| < +4.10 \times 10^{-11} \text{yr}^{-1}.
\]

On the other hand, using Big Bang Nucleosynthesis another recent estimate of variation of \( G \) has been obtained (Copi, Davies & Krauss 2004) as

\[
-4.0 \times 10^{-13} \text{yr}^{-1} < \frac{\dot{G}}{G} < +3.0 \times 10^{-13} \text{yr}^{-1}.
\]

Some other estimates of the probable range of variation of \( G \) can be obtained from various other sources such as Lunar Laser Ranging (Turyshev et al. 2003), Viking Lander (Hellings 1987; Resemberg 1983), lunar occultation (Van Flandern 1975), lunar tidal acceleration (Van Flandern 1975) etc. Some theoretical models (Blake 1978; Faulkner 1976) also provide estimates of \( \dot{G}/G \). Various ranges of \( \dot{G}/G \) are listed in Table 1 and 2.

4 COMPARISON OF PRESENT MODELS WITH OBSERVATIONS

Equations (17) and (26) provide us an opportunity for fitting the \( \Lambda \sim \dot{a}/a^2 \) and \( \Lambda \sim \ddot{a}/a \) models respectively with the values of \( \dot{G}/G \) obtained from various sources by proper tuning of \( \alpha \) and \( \beta \), the parameters of the two models mentioned above. Assuming the present age of the Universe as 14 Gyr, values of \( \alpha \) and \( \beta \) corresponding to different values of \( \dot{G}/G \) are listed in Table 1. From this Table 1 it is evident that most of the values of \( \alpha \) are negative while those of \( \beta \) are positive. Now, a negative \( \alpha \) means an attractive \( \Lambda \) which does not match with the present status of this cosmological parameter. However, equation (17) shows that values of \( \dot{G}/G \sim 10^{-11} \text{yr}^{-1} \) can be obtained if we choose any value of \( \alpha > 1 \). For instance, if we set \( \alpha = 15 \), then assuming \( t_0 = 14 \) Gyr, we get the value of \( \dot{G}/G \) as \(-15 \times 10^{-11} \text{yr}^{-1}\) which fits well with the value of \( \dot{G}/G \) for Early Dirac theory (Blake 1978). Coming to the case of \( \Lambda \sim \dot{a}/a \) model we find that most of the tuned values of \( \beta \) are positive which corresponds to a repulsive \( \Lambda \). Hence, \( \Lambda \sim \ddot{a}/a \) model can be fitted more easily with various ranges of \( \dot{G}/G \) than that of \( \Lambda \sim \dot{a}/a^2 \) model.

Also, Table 1 shows that although majority of the values of \( \dot{G}/G \) are negative, but in some cases values of \( \dot{G}/G \) can be found positive as well. Now, a negative \( \dot{G}/G \) implies a time-decreasing \( G \). This means that by combined effect of a decreasing \( G \) and repulsive \( \Lambda \) the Universe will go on accelerating for ever. On the other hand, a positive \( \dot{G}/G \) means \( G \) is growing with time. Since, \( \Lambda \) is a time-decreasing parameter so in future a time-increasing \( G \) may overcome the effect of repulsive \( \Lambda \). In that case, the possibility of a ‘Big Crunch’ cannot be ruled out.

5 CONCLUSIONS

In the present work selecting two different kinematical models of dark energy and considering the gravitational constant \( G \) as a function of time it has been possible to solve Friedmann and Raychaudhuri equations for \( a(t), \rho(t), \Lambda(t) \) and \( G(t) \). Comparing the expressions for \( \dot{G}/G \) for both the models with those of various ranges of \( \dot{G}/G \) obtained from observations as well as from theoretical consideration, it is shown that the parameters of the two \( \Lambda \)-models presented here can be tuned in most of the cases to match with the values of \( \dot{G}/G \) obtained from various sources. It is worthwhile to mention here that all the values of \( \dot{G}/G \) listed in the Table 1 come from the consideration that \( G \sim t^{-1} \). But in the work of Milne (1935) \( G \) directly varies with \( t \). Recently, Belinchon (2002), starting from Dirac’s LNH, through dimensional analysis has arrived at the same result of Milne (1935), viz., \( G \sim t \) which obviously is in contradiction to Dirac’s result. It should be mentioned that in the present investigation also \( G \) does not vary inversely with \( t \) as far as the model \( \Lambda \sim \rho \) is concerned. It has been observed by the present authors that for \( \Lambda \sim \rho \) model, \( G \) varies as \( t^2 \). This time variation of \( G \) is different from that of Dirac (1937) and Milne (1935), and therefore needs further investigation. Finally, it is to be noted that the expressions for \( a(t) \) and \( \Lambda(t) \) for both the models considered here maintain the same form irrespective of the constancy or variability of \( G \).

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Table 1. Values of $\alpha$ and $\beta$ for average $\dot{G}/G$ when $t_0 = 14$ Gyr, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $z \simeq 0$

| Ranges of $\dot{G}/G \text{ yr}^{-1}$ | Sources | $\alpha$ | $\beta$ |
|----------------------------------|---------|--------|--------|
| $-(1.10 \pm 1.07) \times 10^{-11} < \frac{\dot{G}}{G} < 0$ | PSR 1913 + 16 ([Damour et al. 1988]) | -0.0852 | 0.4074 |
| $-1.60 \times 10^{-12} < \frac{\dot{G}}{G} < 0$ | Helioseismological data (Guenther et al. 1998) | -0.0115 | 0.0670 |
| $(-1.30 \pm 2.70) \times 10^{-11}$ | PSR B1855+09 (Arzoumanian 1995; Kaspi, Taylor & Ryba 1994) | -0.1023 | 0.4698 |
| $(-8 \pm 5) \times 10^{-11}$ | Lunar occultation (Van Flandern 1975) | -1.333 | 1.6 |
| $(-6.4 \pm 2.2) \times 10^{-11}$ | Lunar tidal acceleration (Van Flandern 1975) | -0.8421 | 1.4328 |
| $-15.30 \times 10^{-11}$ | Early Dirac theory (Blake 1978) | 11.7692 | 2.0582 |
| $-5.1 \times 10^{-11}$ | Additive creation theory (Blake 1978) | -0.5730 | 1.2644 |
| $(-16 \pm 11) \times 10^{-11}$ | Multiplication creation theory (Faulkner 1976) | 8.00 | 2.0869 |
| $-2.5 \times 10^{-10} \leq \frac{\dot{G}}{G} \leq +4.0 \times 10^{-11}$ | WDG 117-B15A (Benvenuto et al. 2004) | -3.0 | 1.8 |
| $\frac{\dot{G}}{G} \leq +4.10 \times 10^{-10}$ | WDG 117-B15A [18] | 1.1319 | 3.09 |
| $-(0.6 \pm 4.2) \times 10^{-12}$ | Double-neutron star binaries (Thorsett 1996) | -0.0043 | 0.0254 |
| $(0.46 \pm 1.0) \times 10^{-12}$ | Lunar Laser Ranging (Turyshev et al. 2003) | 0.0318 | -0.2110 |
| $1 \times 10^{-11} \pm 1$ | Wu and Wang (1986) | 0.0666 | -0.5 |

Table 2. Values of $\alpha$ and $\beta$ for average $\dot{G}/G$ when $t_0 = 6.57$ Gyr, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $z \simeq 0.5$

| Ranges of $\dot{G}/G \text{ yr}^{-1}$ | Sources | $\alpha$ | $\beta$ |
|----------------------------------|---------|--------|--------|
| $-1.40 \times 10^{-11} < \frac{\dot{G}}{G} < +2.60 \times 10^{-11}$ | Lorén-Aguilar et al. (2003) | 0.0196 | -0.125 |
| $-10^{-11} \leq \frac{\dot{G}}{G} < 0$ | Supernova Type Ia (Gaztanaga et al. 2002) | -0.0169 | 0.0967 |
| $-4.0 \times 10^{-13} < \frac{\dot{G}}{G} < +3.0 \times 10^{-13}$ | Big Bang Nucleosynthesis (Copi, Davies & Krauss 2004) | -0.0001 | 0.0009 |