ABSTRACT
This paper extends previous work of the authors to reconsider the capital-asset pricing model (CAPM) in South Africa in real terms. Conventional and index-linked bonds were included both in the composition of the market portfolio and in tests of the securities market line. For the investigation, quarterly total returns from the FTSE/JSE All-Share Index listed on the JSE Securities Exchange from 30 September 1964 to 31 December 2010 were used, together with yields on government bonds and consumer price indices over the same period. As expressed in the securities market line, the CAPM suggests that higher risk, as measured by beta, is associated with higher expected returns. In addition, the theoretical underpinnings of the CAPM are that it explains expected excess return, and that the relationship between expected return and beta is linear. In this investigation the above-mentioned predictions of the CAPM were tested. Direct tests of the securities market line were made, using both prior betas and in-period betas. A nonparametric test was also made, as well as a regression analysis. These tests were made for individual periods as well as for all periods combined.

KEYWORDS
Beta, bonds, capital-asset pricing model; excess return; South Africa

THE CAPITAL-ASSET PRICING MODEL RECONSIDERED: TESTS IN REAL TERMS ON A SOUTH AFRICAN MARKET PORTFOLIO COMPRISING EQUITIES AND BONDS

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The wrong sort of bees would make the wrong sort of honey.
Winnie-the-Pooh

1. INTRODUCTION
1.1 The capital-asset pricing model's simplicity has made it the central equilibrium model of financial economics (Ross, 1978). The attraction of the CAPM is that it offers powerful and intuitively pleasing predictions about how to explain the relationship between expected return and risk (Fama & French, 1992) and the variations in risk differentials on different risky assets (Friend et al., 1978). Ross (op. cit.) describes the CAPM as a paradigm, precisely because it is cast in terms of variables that are, at least in principle and with the usual exception of the ex-ante–ex-post distinction, empirically observable and statistically testable.

1.2 As argued by Ross (op. cit.), expected returns are linear in beta if and only if the market portfolio is mean–variance efficient.

1.3. As observed in the previous study (Reddy & Thomson, unpublished), the market portfolio, which is central to testing, is unobservable in practice. In that study the FTSE/JSE All Share Index was used as a market proxy. Roll (1977) and Bowie & Bradfield (1998) warned that the choice of a wrong proxy would reduce the predictive ability of the CAPM. It is possible that the results obtained in the previous study were misstated. In this study, in order to obtain a more realistic market portfolio, bonds were included; both in the composition of the market portfolio and in tests of the securities market line.

1.4 Furthermore, this paper extends to reconsider the CAPM is real terms. Although most tests of the CAPM are applied in nominal terms, it is preferable to measure returns in real terms. The reason for this is that investors’ preferences must ultimately be expressed in terms of consumption of goods and services, not merely in terms of currency. In short-term applications the difference may not be material. Furthermore, since consumer price indices are calculated monthly in arrears, there are no short-term inflation-protected money-market instruments. However, in longer-term applications such as those typically used in actuarial modelling, the difference may be material and index-linked instruments exist.
1.5 The main question this study aimed to answer remains: Is the CAPM valid in the South African market? And again, in particular, does the CAPM explain expected excess return and is the relationship between return and beta linear? But in addressing these questions, the authors have extended the analysis in the previous study by including bonds in the market portfolio and by expressing returns in real terms. (Again, ‘excess return’ is the excess of the return over the risk-free rate; it is defined more formally in §4.4.2 below.)

1.6 The rest of the paper is organised as follows. In Section 2, some literature on the inclusion of bonds in the CAPM market portfolio and on the expression of returns in real terms is briefly reviewed. In Section 3, a description of the data used for this study is provided. The method used is described and explained in Section 4. In Section 5 the results of the empirical tests are presented and discussed. The results are summarised, and concluding comments are made, in Section 6.

2. LITERATURE REVIEW

2.1 This section extends the literature of the previous study, dealing only with the literature on the inclusion of bonds in the CAPM market portfolio and on the expression of returns in real terms. For more general literature relevant to this paper, the reader is referred to the literature review in that study.

2.2 Although the market portfolio is unobservable, the closer the chosen proxy is to the market portfolio, the more reliable will be the predictions of the CAPM. Actuarial applications will generally require at least the inclusion of bonds in the market portfolio. Friend et al. (op. cit.) appear to have been the first to have included bonds in the market portfolio. They studied the USA market during the period from 1964 to 1973. The risk–return relationship obtained in their study was significantly different from that using a market portfolio consisting of only equity. In particular, during the period from 1964 to 1968 the zero-beta returns on bonds and on equity were significantly lower than the risk-free rates. During the period from 1968 to 1973, the zero-beta return on bonds was lower than that on equity. They suggested that other periods needed to be analysed before any conclusion could be drawn. They also suggested that their findings might imply some segmentation between bond and equity markets.

2.3 In an attempt to quantify the effects of co-skewness on quarterly ex-post expected returns in a market portfolio comprising equities and conventional bonds in the USA over the period 1952 to 1976, Friend & Westerfield (1980) found that estimates of expected returns on the zero-beta portfolio were significantly higher than actual risk-free returns.
2.4 Bollerslev et al. (1988) tested the use of time-varying conditional covariances of real returns in the USA over the period 1959 to 1984. They found that the conditional covariance matrix varied significantly over the time interval considered.

2.5 Other literature of relevance is reviewed in Reddy & Thomson (unpublished); that review is not repeated here.

3. DATA AND PERIODS CONSIDERED

3.1 Data
Data for this study were obtained as explained in Appendix A. The time intervals used were calendar quarters. Particularly for earlier periods, the data were not generally available in the form required. The approximations and assumptions made are explained in that appendix, as well as the formulae used to obtain the values required.

3.2 Periods Considered
3.2.1 As explained in Appendix A, data were available from 30/9/1964 to 31/12/2011 (i.e. for quarters \( t \in [1, \ldots, 185] \)). However, this period has seen a number of changes, which may have affected the underlying assumptions of the CAPM or its testability.

3.2.2 The onset of high inflation in the 1970s arguably created a discontinuity in expected returns on equities relative to bonds, as evidenced by the emergence of a reverse yield gap. The first sub-period considered was therefore from 30/9/1964 to 31/12/1972 (i.e. for quarters \( t \in [1, \ldots, 33] \)).

3.2.3 Until 31/12/1985 the yield curve comprised only three points, which effectively represented yields on the primary market. The secondary market was not well developed and it was therefore not possible to determine a descriptive yield curve based on trades between investors. The yield curve reflected the yields at which the most recent issues had been made. Tests of the CAPM including bonds as well as equities prior to that date may be affected by the inefficiency of the bond market. The second sub-period considered was therefore from 31/12/1972 to 31/12/1985 (i.e. for quarters \( t \in [34, \ldots, 85] \)).

3.2.4 Before 30/9/1989, the South African government imposed prescribed asset requirements on life offices, and even more exacting requirements on pension funds. This arguably created a disequilibrium in bonds relative to equities. Whilst a cursory examination of the data before and after the abolition of prescribed assets reveals no obvious discontinuity either in the yields on or in the market capitalisation of government bonds, it was decided to treat this change as a discontinuity for the purposes of testing the CAPM. The third sub-period considered was therefore from 31/12/1985 to 30/9/1989 (i.e. for quarters \( t \in [86, \ldots, 100] \)).
3.2.5 In South Africa inflation-linked bonds were first issued in 2000. Figure 1 shows the value of $z_{t,40}$ for all times $t$ at which there were inflation-linked bonds in issue. As shown there, during the early years the yields were unsustainably high; investors were unfamiliar with these bonds and were reluctant to buy them. During the period from 2000 to 2002 (i.e. for quarters $t \in [143, \ldots, 152]$) yields gradually decreased from unreasonably high yields to more reasonable levels. Under these circumstances it is not reasonable to suppose that the inflation-linked bond market was in equilibrium with the rest of the market. Furthermore, until 2002 there were less than four bonds in issue. While the approximations suggested in the preceding paragraph may be adequate for the purpose of the analysis illustrated in Figure 1, they are not ideal for the purposes of this paper. It was therefore decided to ignore inflation-linked bonds before 31/12/2002 (i.e. before $t = 153$). It was therefore not possible to include such bonds in prior periods. The fourth and fifth periods were therefore from 30/9/1989 to 31/12/2002 (i.e. for quarters $t \in [101, \ldots, 152]$) and from 31/12/2002 to 31/12/2011 (i.e. for quarters $t \in [153, \ldots, 185]$) respectively.

4. Method

4.1 Real Returns
As discussed in §1.4, it is preferable to measure returns in real terms. Most tests of the CAPM, being more concerned about applications to trading, use relatively short
intervals. Because of the difficulty of measuring real returns over such intervals—and because of the relative certainty of the level of inflation over such intervals—tests are usually applied to nominal returns. Those problems do not apply to long-term modelling. Conversely, the longer the term of the model, the greater is the uncertainty about levels of inflation. For the purposes of this research, real returns were therefore used.

4.2 The Risk-Free Rate
Because this study considered the CAPM in real terms, the risk-free return used was the real risk-free return, that is the spot rate for an inflation-linked bond maturing one quarter hence. As mentioned in ¶3.2.5, inflation-linked bonds were first issued in 2000 and it was decided to ignore inflation-linked bonds before 31/12/2002. For earlier periods real risk-free returns were therefore not available. For those periods it was not possible apply tests of the CAPM based on the risk-free return. However, tests based on zero-beta returns were made for each sub-period considered and for all periods combined.

4.3 Constituents of the Market Portfolio
As explained in Appendix A, the market portfolio was assumed to comprise:
— listed equities included in the all-share index on the JSE Securities Exchange;
— zero-coupon conventional bonds with maturities of 3, 10 and 20 years; and
— with effect from 31/12/2002, zero-coupon inflation-linked bonds with the same maturities.

As further explained in that appendix, almost all the variability in the yields on bonds may be explained by the first three principal components of the yield curve, and therefore by those on three well-dispersed zero-coupon bonds. The reduction of the bond portfolio to maturities of 3, 10 and 20 years therefore results in no material loss in generality.

4.4 Variables
4.4.1 From the data, the value of \( R_i \), being the return on component \( i \) of the market portfolio during quarter \( t \) was determined as explained in Appendix A for \( i \in I^- \), \( t \in [1,\ldots,152] \) and for \( i \in I^+ \), \( t \in [153,\ldots,185] \); where \( I^- \) comprises:
— listed equities included in the all-share index on the JSE Securities Exchange; and
— zero-coupon conventional bonds with maturities of 3, 10 and 20 years;
and \( I^+ \) comprises:
— listed equities as above;
— zero-coupon conventional bonds with maturities as above; and
— zero-coupon inflation-linked bonds with the same maturities.
4.4.2 For \( t \in [153, \ldots, 185] \) the excess return on component \( i \) during quarter \( t \) was determined as:

\[
r_t = R_t - R_{Ft},
\]

where:

\( R_{Ft} \) is the return during quarter \( t \) on an inflation-linked bond maturing at the end of that quarter.

4.4.3 The market capitalisation \( m^*_t \) of component \( i \) of the market portfolio at time \( t \) was determined as explained in Appendix A. From these values the return on the market portfolio during quarter \( t \) was determined as:

\[
R_{Mt} = \begin{cases} 
\frac{\sum_{i \in I} m^*_{i,t-1} R_t}{\sum_{i \in I} m^*_{i,t-1}} & \text{for } t \in [1, \ldots, 152]; \\
\frac{\sum_{i \in I} m^*_{i,t-1} R_t}{\sum_{i \in I} m^*_{i,t-1}} & \text{for } t \in [153, \ldots, 185].
\end{cases}
\]

The excess return on the market portfolio during quarter \( t \) was determined as:

\[
r_{Mt} = R_{Mt} - R_{Ft}.
\]

### 4.5 Exogenous Variables

4.5.1 While the variances and covariances of the variables to be modelled are endogenous to the requirements of long-term actuarial modelling, other explanatory variables would be exogenous, and would require separate modelling. As observed in Reddy & Thomson (unpublished), many authors have investigated the effects of exogenous variables. However, as observed there, these effects add little value to long-term modelling and they are diluted by the aggregation of equities into sectors and portfolios.

4.5.2 As in Reddy & Thomson (op. cit.) the effects of exogenous variables have been ignored. As explained below, however, (cf. sections 5.1.2, 5.1.4, 5.2.2 and 5.2.4) the effects of nonlinearity were tested.

### 4.6 The Zero-Beta Version of the CAPM

In terms of the zero-beta version of the CAPM, it is not necessary to refer to the risk-free asset. In order to test the zero-beta version of the CAPM, it is necessary to translate the \textit{ex-ante} parameters of an equilibrium model into \textit{ex-post} realisations. For that purpose it is necessary to assume the validity of some return-generating function. For any \textit{ex-ante} model and \textit{ex-post} realisations there is almost certainly some generating function that will link those realisations with the model (Blume & Husic, 1973). The following return-generating process for component \( i \) in quarter \( t \) may be used to test the zero-beta version of the CAPM.
\[ R_{it} = \gamma_{0t} + \beta_{it} y_{1t} + \varepsilon_{it}; \]  

where:

\[ \beta_{it} = \frac{\sigma_{Mrt}}{\sigma_{MMt}}; \]

\[ \varepsilon_{it} \sim N(0, \sigma^2_{\varepsilon_{it}}); \]

\[ \sigma_{ijt} = \text{cov}(R_{it}, R_{jt}); \]

\[ \sigma^2_{\varepsilon_{it}} = \text{var}(\varepsilon_{it}); \text{ and} \]

\[ \text{cov}(\varepsilon_{it}, \varepsilon_{ut}) = 0 \text{ for } t \neq u. \]

4.6.1 Prior Betas

The investigation was first carried out by calculating a prior beta for each component \( i \) for each quarter \( t = 21, \ldots, 185 \). For this purpose, five years of quarterly prior real returns were used to give the prior beta as:

\[ \hat{\beta}_{it} = \frac{\hat{\sigma}_{Mrt}}{\hat{\sigma}_{MMt}}; \]  

where:

\[ \hat{\sigma}_{ijt} = \frac{1}{19} \sum_{v=t-20}^{t-1} (R_{iv} - \bar{R}_u)(R_{jv} - \bar{R}_u); \text{ and} \]

\[ \bar{R}_u = \frac{1}{20} \sum_{v=20}^{t-1} R_{uv} \text{ for } u = i, j. \]

4.6.2 In-Period Betas

4.6.2.1 Another investigation was carried out using in-period betas, which, for each component \( i \), were estimated for each calendar year \( Y = 1965, \ldots, 2010 \) as:

\[ \hat{\beta}_{i[Y]} = \frac{\hat{\sigma}_{i[Y]}}{\hat{\sigma}_{MM[Y]}}; \]  

where:

\[ \hat{\sigma}_{i[Y]} = \frac{1}{3} \sum_{t \in Y} (R_{it} - \bar{R}_{i[Y]})(R_{jt} - \bar{R}_{j[Y]}); \text{ and} \]

\[ \bar{R}_{u[Y]} = \frac{1}{4} \sum_{t \in Y} R_{ut} \text{ for } u = i, j. \]
The in-period return for each calendar year $Y$ was calculated as:

$$ R_{i[Y]} = \sum_{t \in [Y]} R_{it}. $$ (8)

4.6.2.2 A further investigation was carried out using in-period betas for each sub-period $[p] \in \{[1,33],[34,85],[86,100],[101,152],[153,185]\}$; where $[a,b]$ denotes the sub-period from quarter $a$ to quarter $b$. The in-period beta for each component $i$ in sub-period $[p]$ was estimated as:

$$ \hat{\beta}_{i[p]} = \frac{\hat{\sigma}_{i[p]}}{\hat{\sigma}_{MM[p]}}; $$ (9)

where:

$$ \hat{\sigma}_{i[p]} = \frac{1}{q_{[p]} - 1} \sum_{t \in [p]} \left( R_{it} - \bar{R}_{i[p]} \right) \left( R_{jt} - \bar{R}_{j[p]} \right); $$ and

$$ \bar{R}_{u[p]} = \frac{1}{q_{[p]} \sum_{t \in [p]} R_{ut}} \text{ for } u = i, j; $$ and

$$ q_{[p]} \text{ is the number of quarters in } [p]. $$ (10)

The in-period return for each sub-period $p$ was calculated as:

$$ R_{i[p]} = \frac{1}{q_{[p]} \sum_{t \in [p]} R_{it}}. $$ (12)

4.6.2.3 If the rational-expectations hypothesis (REH) holds, then it is not necessary to use prior betas. As explained by Blume & Husic (op. cit., if ex-ante values of beta differ from ex-post values at the start of a period, then ex-post estimates derived from values during the period “may more accurately mirror investors’ ex-ante expectations”. If both ex-ante betas and ex-post in-period sample betas are unbiased estimates of population betas for the respective indices, then the in-period sample betas are unbiased estimates of the ex-ante betas. The in-period sample betas may then be used to test the joint hypothesis that both the CAPM and the REH hold. Such a test is useless as an operational test: it does not test whether the CAPM works, because in-period sample betas are not available ex ante. However, for the purpose of testing whether the CAPM can be used in long-term models it is relevant, since such a model can generate unbiased ex-ante betas.

4.6.2.4 A further investigation was carried out using all sub-periods combined. For this purpose, for each component $i$, the in-period beta for all sub-periods combined was estimated as:

$$ \hat{\beta}_i = \frac{\hat{\sigma}_{iM}}{\hat{\sigma}_{MM}}; $$ (13)
where:
\[ \hat{\sigma}_{ij} = \frac{1}{184} \sum_{t=1}^{185} (R_{it} - \bar{R}_i)(R_{jt} - \bar{R}_j); \] and

\[ \bar{R}_u = \frac{1}{185} \sum_{t=1}^{185} R_{ut} \text{ for } u = i, j. \]

The in-period return for all sub-periods combined was calculated as:
\[ R_i = \frac{1}{185} \sum_{t=1}^{185} R_{it}. \]

4.7 The Standard Version of the CAPM

As with the zero-beta version of the CAPM, in order to test the standard version of the CAPM it is necessary to assume the validity of some return-generating process. With the standard version of the CAPM it is necessary to refer to the risk-free asset. The following return-generating process may be used to test the zero-beta version of the CAPM for component \( i \) in quarter \( t \):
\[ r_{it} = \beta_{it} r_{Mt} + e_{it}; \]

where:
\[ \beta_{it} = \frac{\sigma_{Mi}}{\sigma_{MM}}; \]
\[ e_{it} \sim N(0, \sigma_{e}^2); \]
\[ \sigma_{ij} = \text{cov}(R_{it}, R_{jt}); \]
\[ \sigma_{e}^2 = \text{var}(e_{it}); \] and
\[ \text{cov}(e_{it}, e_{iu}) = 0 \text{ for } t \neq u. \]

4.7.1 Prior Betas

As for the zero-beta version, the investigation was first carried out by calculating a prior beta for each component \( i \) for each quarter \( t = 173, \ldots, 185 \). Again, five years of quarterly prior returns were used to give the prior beta as in equation (2).

4.7.2 In-Period Betas

4.7.2.1 Another investigation was again carried out using in-period betas which, for each component \( i \), were estimated for each calendar year \( Y = 2003, \ldots, 2010 \) as in equation (5). The in-period return for each calendar year \( Y \) was calculated as:
\[ r_{i[Y]} = \sum_{t \in Y} r_{it}. \]

4.7.2.2 A further investigation was carried out using all quarters combined. For this purpose, for each asset class \( i \), in-period beta for all quarters \( t = 153, \ldots, 185 \) was
estimated as in equation (13). The in-period return for all quarters combined was calculated as:
\[ r_i = \frac{1}{33} \sum_{\forall t} r_{it}. \]  
(19)

5. **EMPIRICAL TESTS**

5.1 **The Zero-Beta Version of the CAPM**

5.1.1 **A Test of the Explanatory Power of the CAPM Using Prior Betas**

5.1.1.1 To reduce the confounding effect of the REH and to allow extension to the zero-beta version of the CAPM, a less explicitly expressed statement may be tested, viz. that:
\[ E \{ R_i \} = \gamma_0 + \gamma_1 \beta_i. \]  
(20)

The null hypothesis for the zero-beta version of the CAPM, for a given quarter \( t \) may be expressed in terms of the equation:
\[ R_t = \gamma_0 + \gamma_1 \beta_t + \epsilon_t; \]  
(21)

In the zero-beta version of the CAPM, equation (21) implies that:
\[ \gamma_0 = R_{Z_t}; \]  
and
\[ \gamma_1 = E \{ R_{M_t} - R_{Z_t} \}; \]

where \( R_{Z_t} \) is the return on the zero-beta portfolio \( Z \) for a given quarter \( t \).

5.1.1.2 A major advantage of such tests is that they do not use ex-post expected values of \( R_{M_t} \).

5.1.1.3 Using the above test to investigate whether the CAPM explains rates of return, the return for each component \( i \) of the market portfolio was regressed, for each quarter \( t \in [Y] \) where \( t = 21, \ldots, 185 \) and \( Y = 1969, \ldots, 2010 \), against the corresponding prior beta estimate. The relationship examined is equation (21), expressed in terms of estimated prior betas as:
\[ R_t = \gamma_{0(Y)} + \gamma_{1(Y)} \hat{\beta}_t + \epsilon_t. \]  
(22)

The results of the analysis for each calendar year \( Y \) are given in Table B.1 of Appendix B.

5.1.1.4 A similar regression analysis was applied to each sub-period \([p]\). The return for each component \( i \) was regressed, for each sub-period \([p] \in \{[21, 33],[34, 85],[86, 100],[101, 152],[153, 185]\} \), against the corresponding prior beta estimate. The relationship examined is again equation (21), expressed in terms of estimated prior betas as in equation (22).
5.1.1.5 A further investigation involved a regression of the return on each component \( i \) for each quarter \( t \) for \( t = 21, \ldots, 185 \), against the corresponding prior beta estimate, which represents a regression analysis across all quarters. The relationship examined is again equation (21), expressed in terms of estimated prior betas as:

\[
R_t = \gamma_0 + \gamma_1 \beta_t + \epsilon_t
\]

5.1.1.6 The results of the analysis for each sub-period \( p \) and for all sub-periods combined are given in Table 1.

| Table 1 Summary of regression analysis for the test of the explanatory power of the CAPM using prior betas |
|---|---|---|---|---|
| Period | \( R^2 \) | Estimate | Value | \( t \)-value | \( p \)-value |
| [21,33] | 0.004 | \( \gamma_{0(p)} \) | -0.014 | -0.841 | 0.404 |
| | | \( \gamma_{1(p)} \) | 0.010 | 0.442 | 0.660 |
| [34,85] | 0.013 | \( \gamma_{0(p)} \) | -0.025 | -2.254 | 0.025 |
| | | \( \gamma_{1(p)} \) | 0.023 | 1.653 | 0.100 |
| [86,100] | 0.014 | \( \gamma_{0(p)} \) | -0.012 | -0.319 | 0.751 |
| | | \( \gamma_{1(p)} \) | 0.032 | 0.898 | 0.373 |
| [101,152] | 0.000 | \( \gamma_{0(p)} \) | 0.024 | 1.731 | 0.085 |
| | | \( \gamma_{1(p)} \) | -0.004 | -0.283 | 0.778 |
| [153,185] | 0.017 | \( \gamma_{0(p)} \) | 0.010 | 1.200 | 0.232 |
| | | \( \gamma_{1(p)} \) | 0.023 | 1.715 | 0.088 |
| All | 0.005 | \( \gamma_0 \) | -0.001 | -0.141 | 0.888 |
| | | \( \gamma_1 \) | 0.015 | 1.947 | 0.052 |

5.1.1.7 First, as indicated in Table 1, the values of \( R^2 \) for the tests for each sub-period and for all sub-periods combined are low, indicating that most of the risk is unsystematic. The zero-beta version of the CAPM predicts that \( \gamma_0 > 0 \). For this version of the CAPM the null hypothesis is that \( \gamma_0 = 0 \) and the alternative hypothesis is that \( \gamma_0 < 0 \). Again, the CAPM is rejected if the null hypothesis is rejected. The \( p \)-value for each sub-period other than sub-period [34,85] and for all sub-periods combined with negative values of \( \gamma_0 \) are greater than 5% (using a one-tailed test). The null hypothesis is rejected only for sub-period [34,85].

5.1.1.8 The zero-beta version of the CAPM predicts that \( \gamma_1 > 0 \). The null hypothesis is that \( \gamma_1 = 0 \) and the alternative hypothesis is that \( \gamma_1 < 0 \). The CAPM is rejected if the
null hypothesis is rejected. The null hypothesis is not rejected for any of the sub-periods nor for all sub-periods combined. The zero-beta form of the CAPM also predicts that $\gamma_1 > \gamma_0$. This prediction is not tested here, though by inspection it appears that this may hold true for most sub-periods and sub-periods combined.

5.1.1.9 One would not necessarily expect all sub-periods to be insignificant at the 5% level as the probability of that is $0.95^4 = 0.77$. The results reported in Table 1 do therefore not constitute grounds for rejection of the CAPM. However, it may be argued on the basis of that table that the CAPM did not necessarily apply in every sub-period. From Table B.1 it may be seen that, of the 42 years considered, there were six years during which one or both of the parameters were outside of their 95% confidence limits. The probability of having at least six outside of their 95% confidence limits is about 1.7%, which is significant. The CAPM must therefore be rejected on the grounds of these tests.

5.1.2 A Test for Nonlinearity Using Prior Betas

5.1.2.1 In order to test for nonlinearity between the return for each component $i$, for each quarter $t \in [Y]$ where $Y = 1969, \ldots, 2010$, and the corresponding prior beta estimate, the relationship examined, for each calendar year $Y$, expressed in terms of estimated prior betas is:

$$R_t = \gamma_0[Y] + \gamma_1[Y]\hat{\beta}_t + \gamma_2[Y]\hat{\beta}_t^2 + \epsilon_t.$$

The results of the analysis for each calendar year $Y$ are given in Table B.2 in Appendix B. The estimate values, $t$-values and $p$-values are those for $\gamma_{2[Y]}$ only; other coefficients are not relevant to this test.

5.1.2.2 Similarly, in order to test for nonlinearity between return for each component $i$ for each quarter $t \in [p]$ where $[p] \in \{[1,33],[34,85],[86,100],[101,152],[153,185]\}$, and the corresponding prior beta estimate. The relationship examined, for each sub-period $p$, expressed in terms of estimated prior betas is:

$$R_t = \gamma_0[p] + \gamma_1[p]\hat{\beta}_t + \gamma_2[p]\hat{\beta}_t^2 + \epsilon_t.$$

5.1.2.3 Furthermore, in order to test for nonlinearity between the return on each component $i$ for all periods combined, and the corresponding prior beta estimate, the relationship examined expressed in terms of estimated prior betas is:

$$R_t = \gamma_0 + \gamma_1\hat{\beta}_t + \gamma_2\hat{\beta}_t^2 + \epsilon_t.$$

5.1.2.4 The results of the analysis for each sub-period $[p]$ and for all sub-periods combined are given in Table 2.

5.1.2.5 For the relationship between the return and the beta estimate to be linear, one would expect that $\gamma_2 = 0$. This is the null hypothesis; the alternative hypothesis is
that $\gamma_2 \neq 0$. As indicated in Table 2, $\gamma_2$ is not significantly different from zero for any sub-period nor for all sub-periods combined.

**Table 2** Summary of the regression analysis for the test for nonlinearity using prior betas

| Period     | $R^2$ | Estimate $\gamma_2$ | Value | $t$-value | $p$-value |
|------------|-------|----------------------|-------|-----------|-----------|
| [21,33]    | 0.005 | $\gamma_2(p)$       | 0.058 | 0.197     | 0.845     |
| [34,85]    | 0.022 | $\gamma_2(p)$       | 0.068 | 1.365     | 0.174     |
| [86,100]   | 0.014 | $\gamma_2(p)$       | -0.008| -0.124    | 0.902     |
| [101,152]  | 0.005 | $\gamma_2(p)$       | -0.030| -0.976    | 0.330     |
| [153,185]  | 0.017 | $\gamma_2(p)$       | -0.001| -0.029    | 0.977     |
| All        | 0.005 | $\gamma_2$          | -0.003| -0.184    | 0.854     |

5.1.2.6 However, from Table B.2 it may be seen that, of the 42 years considered, there were five years during which $\gamma_2(Y)$ was outside of its confidence limits. The probability of at least six such occurrences is 5.7%, which is not significant. It may be concluded that, considered annually, the relationship is linear. However, this does not detract from the rejection of the CAPM itself in terms of the tests described in section 5.1.1.

5.1.3 **A Test of the Explanatory Power of the CAPM Using In-Period Betas**

5.1.3.1 A similar test used in section 5.1.1 was used to investigate the explanatory power of the CAPM. In this case, the in-period return for each component $i$ was regressed, for each calendar year $Y$ where $Y = 1965, \ldots, 2010$, against the corresponding in-period beta estimate for each component $i$ for each calendar year $Y$. The relationship examined is equation (21), expressed in terms of estimated in-period betas as:

$$ R_{i[Y]} = \gamma_{0[Y]} + \gamma_{1[Y]} \beta_{i[Y]} + \epsilon_{i[Y]} $$

The results of the analysis for each calendar year $Y$ are given in Table B.3 of Appendix B.

5.1.3.2 A similar regression analysis was applied to each sub-period $p$. The in-period return for each component $i$ was regressed, for each sub-period $p$ where $[p] \in \{[1,33],[34,85],[86,100],[101,152],[153,185]\}$, against the corresponding in-period beta estimate for each component $i$ for each sub-period $p$. The relationship examined is equation (21), expressed in terms of estimated in-period betas as:
5.1.3.3 A further investigation involved a regression of the in-period return on each component $i$ for the entire period from $t = 1, ..., 185$, against the corresponding in-period beta estimate for each component $i$. The relationship examined is equation (21), expressed in terms of estimated betas as:

$$R_i = \gamma_0 + \gamma_1 \tilde{\beta}_i + \epsilon_i.$$

5.1.3.4 The results of the analysis for each sub-period $p$ and for all sub-period combined are given in Table 3.

Table 3 Summary of regression analysis for the test of the explanatory power of the CAPM using in-period betas

| Period  | $R^2$ | Estimate | Value | $t$-value | $p$-value |
|---------|-------|----------|-------|-----------|-----------|
| [1,33]  | 0.886 | $\gamma_{0,p}$ | -0.006 | -1.715    | 0.228     |
|         |       | $\gamma_{1,p}$ | 0.019 | 3.948    | 0.059     |
| [34,85] | 0.735 | $\gamma_{0,p}$ | -0.034 | -2.957    | 0.098     |
|         |       | $\gamma_{1,p}$ | 0.033 | 2.356    | 0.143     |
| [86,100]| 0.422 | $\gamma_{0,p}$ | 0.004 | 0.275    | 0.809     |
|         |       | $\gamma_{1,p}$ | 0.021 | 1.208    | 0.351     |
| [101,152]| 0.044| $\gamma_{0,p}$ | 0.024 | 1.736    | 0.225     |
|         |       | $\gamma_{1,p}$ | -0.005 | -0.303    | 0.791     |
| [153,185]| 0.763| $\gamma_{0,p}$ | 0.013 | 7.943    | 0.001     |
|         |       | $\gamma_{1,p}$ | 0.015 | 4.015    | 0.010     |
| All     | 0.368 | $\gamma_0$    | 0.006 | 2.107    | 0.089     |
|         |       | $\gamma_1$    | 0.009 | 1.705    | 0.149     |

5.1.3.5 First, as indicated in Table 3, the values of $R^2$ for the tests of the sub-periods and all sub-periods combined have increased. The zero-beta version of the CAPM predicts that $\gamma_0 > 0$. For this version of the CAPM the null hypothesis is that $\gamma_0 = 0$ and the alternative hypothesis is that $\gamma_0 < 0$. Again, the CAPM is rejected if the null hypothesis is rejected. Since the $p$-values for each sub-period and for all sub-periods combined with positive values of $\gamma_0$ are all greater than 5% (using a one-tailed test), with the exception of sub-period [153,185], the null hypothesis is not rejected for any of the periods, except that sub-period.
5.1.3.6 The zero-beta version of the CAPM predicts that $\gamma_1 > 0$. The null hypothesis is that $\gamma_1 = 0$ and the alternative hypothesis is that $\gamma_1 < 0$. The CAPM is rejected if the null hypothesis is rejected. The null hypothesis is not rejected for any sub-period except sub-period [153,185], nor for all periods combined. The zero-beta form of the CAPM also predicts that $\gamma_1 > \gamma_0$. This prediction is not tested here, though by inspection it appears that it may not hold true for the last two sub-periods nor for all sub-periods combined.

5.1.3.7 Again one would not necessarily expect all sub-periods to be insignificant at the 5% level. The results reported in Table 1 do therefore not constitute grounds for rejection of the CAPM. However, it may be argued on the basis of that table that the CAPM did not necessarily apply in every sub-period. Of particular concern here is the fact that it is the last sub-period, for which inflation-linked bonds are included, that fails the test. From Table B.3 it may be seen that, of the 46 years considered, there were 18 years during which one or both of the parameters were outside of their 95% confidence limits. The probability of having at least 18 outside of their 95% confidence limits is minimal. Considered annually, the CAPM is rejected on the grounds of these tests.

5.1.4 A Test for Nonlinearity using In-Period Betas
5.1.4.1 In order to test for nonlinearity between return for each component $i$, for each calendar year $Y$ where $Y = 1965, ..., 2010$, and the corresponding in-period beta estimate for each component $i$ for each calendar year $Y$. The relationship examined, for each calendar year $Y$, expressed in terms of estimated in-period betas is:

$$R_{i(Y)} = \gamma_0(Y) + \gamma_1(Y) \hat{\beta}_{i(Y)} + \gamma_2(Y) \hat{\beta}_{i(Y)}^2 + \varepsilon_{i(Y)}.$$ 

The results of the analysis for each calendar year $Y$ are given in Table B.4 in Appendix B. The estimate values, $t$-values and $p$-values are those for $\gamma_2(Y)$ only; other coefficients are not relevant to this test.

5.1.4.2 Similarly, in order to test for nonlinearity between return for each component $i$ for each sub-period $p$ where $[p] \in \{[1,33],[34,85],[86,100],[101,152],[153,185]\}$, and the corresponding in-period beta estimate for each component $i$ for each sub-period $p$. The relationship examined, for each sub-period $p$, expressed in terms of estimated in-period betas is:

$$R_{i([p])} = \gamma_0([p]) + \gamma_1([p]) \hat{\beta}_{i([p])} + \gamma_2([p]) \hat{\beta}_{i([p])}^2 + \varepsilon_{i([p])}.$$ 

5.1.4.3 Furthermore, in order to test for nonlinearity between the in-period return on each component $i$ for the entire period from $t = 1, ..., 185$, and the corresponding in-period beta estimate for each component $i$. The relationship examined expressed in terms of estimated in-period betas is:
\[ R_t = \gamma_0 + \gamma_1 \beta_t + \gamma_2 \beta_t^2 + \epsilon_t. \]

5.1.4.4 The results of the analysis for each sub-period \( p \) and for all sub-periods combined are given in Table 4.

| Period          | \( R^2 \) | Estimate | Value | \( t \)-value | \( p \)-value |
|-----------------|-----------|----------|-------|---------------|--------------|
| [1,33]          | 0,996     | \( \gamma_{2[p]} \) | 0,074 | 5,609         | 0,112        |
| [34,85]         | 0,967     | \( \gamma_{2[p]} \) | 0,057 | 2,658         | 0,229        |
| [86,100]        | 1,000     | \( \gamma_{2[p]} \) | -0,080| -44,592       | 0,014        |
| [101,152]       | 0,990     | \( \gamma_{2[p]} \) | -0,084| -9,548        | 0,066        |
| [153,185]       | 0,875     | \( \gamma_{2[p]} \) | 0,041 | 1,894         | 0,131        |
| All             | 0,545     | \( \gamma_2 \)     | 0,020 | 1,250         | 0,279        |

5.1.4.5 For the relationship between the return and the beta estimate to be linear, one would expect that \( \gamma_2 = 0 \). This is the null hypothesis; the alternative hypothesis is that \( \gamma_2 \neq 0 \). As indicated in Table 4, \( \gamma_2 \) is significantly non-zero in sub-period [86,100].

5.1.4.6 Again one would not necessarily expect all sub-periods to be insignificant at the 5% level and the results reported in Table 4 do therefore not constitute grounds for rejection of the CAPM. However, it may again be argued on the basis of that table that the CAPM did not necessarily apply in every sub-period. From Table B.4 it may be seen that, of the 46 years considered, there were four years during which one or both of the parameters were outside of their 95% confidence limits. The probability of having at least four outside of their 95% confidence limits is about 19,7%, which is not significant. On the basis of these tests the linearity of the CAPM cannot be rejected. However, these results are secondary to those of §5.1.3.7 above, on the strength of which the CAPM must be rejected.

5.2 THE STANDARD VERSION OF THE CAPM

5.2.1 A Test of the Explanatory Power of the CAPM Using Prior Betas

5.2.1.1 As in equation (21) the null hypothesis for the standard version of the CAPM, for a given quarter \( t \) may be expressed in terms of the equation:

\[ r_{it} = \gamma_0 + \gamma_1 \beta_{it} + \epsilon_{it}. \] (23)
In the standard form of the CAPM, equation (29) implies that:

\[ \gamma_0 = 0 \; ; \; \text{and} \]

\[ \gamma_1 = E \{ r_{mt} \} \]

5.2.1.2 Using the above test to investigate whether the CAPM explains excess rates of return, the excess return for each component \( i \) was regressed, for each quarter \( t \in Y \) where \( t = 173, \ldots, 185 \) and \( Y = 2007, \ldots, 2010 \), against the corresponding prior beta estimate for each component \( i \) for each quarter \( t \in Y \). The relationship examined is equation (23), expressed in terms of estimated prior betas as:

\[ r_{it} = \gamma_{0(Y)} + \gamma_{1(Y)} \hat{\beta}_{it} + \epsilon_{it}. \]

The results of the analysis for each calendar year \( Y \) are given in Table B.5 of Appendix B.

5.2.1.3 A further investigation involved a regression of the return on each component \( i \) for \( \forall t \) where \( t = 173, \ldots, 185 \), against the corresponding prior beta estimate for each component \( i \) for \( \forall t \). The relationship examined is equation (23), expressed in terms of estimated betas as:

\[ r_{it} = \gamma_0 + \gamma_1 \hat{\beta}_{it} + \epsilon_{it}. \]

5.2.1.4 The results of the analysis for period \([153,185]\) are given in Table 5.

Table 5 Summary of regression analysis for the test of the explanatory power of the CAPM using prior betas

| Period  | \( R^2 \) | Estimate | Value | \( t \)-value | \( p \)-value |
|---------|-----------|----------|-------|---------------|-------------|
| \([153,185]\) | 1.9E-04 | \( \gamma_0 \) | -0.001 | -0.072 | 0.942 |
|         |          | \( \gamma_1 \) | 0.003 | 0.130 | 0.897 |

5.2.1.5 First, as indicated in Table 5, the value of \( R^2 \) for the test for all quarters combined is low, indicating that most of the risk is unsystematic. The standard version of the CAPM predicts that \( \gamma_0 = 0 \). For this version of the CAPM the null hypothesis is that \( \gamma_0 = 0 \) and the alternative hypothesis is that \( \gamma_0 \neq 0 \). Again, the CAPM is rejected if the null hypothesis is rejected. Since the \( p \)-value for all quarters combined is greater than 2.5% (using a two-tailed test), the null hypothesis is not rejected.

5.2.1.6 The standard version of the CAPM also predicts that \( \gamma_1 > 0 \). The null hypothesis is that \( \gamma_1 = 0 \) and the alternative hypothesis is that \( \gamma_1 < 0 \). The CAPM is rejected if the null hypothesis is rejected. The null hypothesis is not rejected.
5.2.1.7 Table B.5 shows that, of the four years considered, only one showed parameters outside of their 95% confidence limits. The probability of one or more parameters outside of their 95% confidence limits is 18.6%, which is not significant.

5.2.2 A Test for Nonlinearity using Prior Betas

5.2.2.1 In order to test for nonlinearity between return for each component \( i \), for each quarter \( t \in Y \) where \( t = 173, \ldots, 185 \) and \( Y = 2007, \ldots, 2010 \), and the corresponding prior beta estimate for each component \( i \) for each quarter \( t \in Y \). The relationship examined, for each calendar year \( Y \), expressed in terms of estimated prior betas is:

\[
 r_{it} = \gamma_{0[Y]} + \gamma_{1[Y]} \hat{\beta}_{it} + \gamma_{2[Y]} \hat{\beta}_{it}^2 + \varepsilon_{it}.
\]

The results of the analysis for each calendar year \( Y \) are given in Table B.6 in Appendix B. The estimate values, \( t \)-values and \( p \)-values are those for \( \gamma_{2[Y]} \) only; other coefficients are not relevant to this test.

5.2.2.2 Furthermore, in order to test for nonlinearity between on each component \( i \) for \( \forall t \) where \( t = 173, \ldots, 185 \), and the corresponding prior beta estimate for each component \( i \) for \( \forall t \). The relationship examined expressed in terms of estimated betas as:

\[
 r_{it} = \gamma_{0} + \gamma_{1} \hat{\beta}_{it} + \gamma_{2} \hat{\beta}_{it}^2 + \varepsilon_{it}.
\]

5.2.2.3 The results of the analysis for period \([153, 185]\) are given in Table 6.

**Table 6** The results of the regression for the test for nonlinearity using prior betas

| Period     | \( R^2 \) | Estimate | Value | \( t \)-value | \( p \)-value |
|------------|-----------|----------|-------|---------------|--------------|
| \([153, 185]\) | 0.029     | \( \gamma_2 \)     | 0.003 | 0.522         | 0.603        |

5.2.2.4 For the relationship between the return and the beta estimate to be linear, one would expect that \( \gamma_2 = 0 \). This is the null hypothesis; the alternative hypothesis is that \( \gamma_2 \neq 0 \). As indicated in Table 6, the null hypothesis is not rejected.

5.2.2.5 Table B.6 shows one year in which \( \gamma_{2[Y]} \) was outside its 95% confidence limits. The probability that, in four years, one or more is outside its confidence limits is 18.6%, which is not significant.

5.2.3 A Test of the Explanatory Power of the CAPM Using In-Period Betas

5.2.3.1 A similar test used in section 5.1.3 was used to investigate the explanatory power of the CAPM. In this case, the in-period return for each component \( i \)
was regressed, for each calendar year $Y$ where $Y = 2003,\ldots,2010$, against the corresponding in-period beta estimate for each component $i$ for each calendar year $Y$. The relationship examined is equation (23), expressed in terms of estimated in-period betas as:

$$r_{i(Y)} = \gamma_{0(Y)} + \gamma_{i(Y)} \hat{\beta}_{i(Y)} + \varepsilon_{i(Y)}.$$ 

The results of the analysis for each calendar year $Y$ are given in Table B.7 of Appendix B.

5.2.3.2 A further investigation involved a regression of the in-period return on each component $i$ for the entire period from $t = 153,\ldots,185$, against the corresponding in-period beta estimate for each component $i$. The relationship examined is equation (23), expressed in terms of estimated in-period betas as:

$$r_i = \gamma_0 + \gamma_1 \hat{\beta}_i + \varepsilon_i.$$ 

5.2.3.3 The results of the analysis for period $[153,185]$ are given in Table 7.

**Table 7** Summary of regression analysis for the test of the explanatory power of the CAPM using in-period betas

| Period | $R^2$ | Estimate | Value | $t$-value | $p$-value |
|--------|-------|----------|-------|-----------|-----------|
| All    | 7,7E-01 | $\gamma_0$ | 0,007 | 4,207 | 0,008 |
|        |        | $\gamma_1$ | 0,015 | 4,134 | 0,009 |

5.2.3.4 The value of $R^2$ is low indicating that most of the risk is unsystematic. The standard version of the CAPM predicts that $\gamma_0 = 0$. For this version of the CAPM the null hypothesis is that $\gamma_0 = 0$ and the alternative hypothesis is that $\gamma_0 \neq 0$. Again, the CAPM is rejected if the null hypothesis is rejected. Since the $p$-value for all quarters combined is less than 2.5% (using a two-tailed test), the null hypothesis is rejected.

5.2.3.5 The standard version of the CAPM also predicts that $\gamma_1 > 0$. The null hypothesis is that $\gamma_1 = 0$ and the alternative hypothesis is that $\gamma_1 < 0$. The CAPM is rejected if the null hypothesis is rejected. The null hypothesis is rejected.

5.2.3.6 Table B.7 shows that the parameters were outside of their 95% confidence limits in four of the eight years considered. The probability that the parameters are outside of their 95% confidence limits at least four years is 0,04%, which is significant. On the basis of these tests the CAPM must be rejected.
5.2.4 A Test for Nonlinearity Using In-Period Betas

5.2.4.1 In order to test for nonlinearity between return for each component $i$, for each calendar year $Y$ where $Y = 2003,...,2010$, and the corresponding in-period beta estimate for each component $i$ for each calendar year $Y$. The relationship examined, for each calendar year $Y$, expressed in terms of estimated in-period betas is:

$$r_{i[Y]} = \gamma_0[Y] + \gamma_1[Y] \beta_{i[Y]} + \gamma_2[Y] \beta_{i[Y]}^2 + \epsilon_{i[Y]}$$

The results of the analysis for each calendar year $Y$ are given in Table B.8 in Appendix B. The estimate values, $t$-values and $p$-values are those for $\gamma_2[Y]$ only; other coefficients are not relevant to this test.

5.2.4.2 Furthermore, in order to test for nonlinearity between the in-period return on each component $i$ for the entire period from $t = 153,...,185$, and the corresponding in-period beta estimate for each component $i$. The relationship examined expressed in terms of estimated in-period betas as:

$$r_i = \gamma_0 + \gamma_1 \beta_i + \gamma_2 \beta_i^2 + \epsilon_i$$

5.2.4.3 The results of the analysis for period $[153,185]$ are given in Table 8.

| Period     | $R^2$ | Estimate | Value | $t$-value | $p$-value |
|------------|-------|----------|-------|-----------|-----------|
| [153,185]  | 0.862 | $\gamma_2$ | 0.040 | 1.606     | 0.184     |

5.2.4.4 For the relationship between the return and the beta estimate to be linear, one would expect that $\gamma_2 = 0$. This is the null hypothesis; the alternative hypothesis is that $\gamma_2 \neq 0$. As indicated in Table 8, the null hypothesis is not rejected.

5.2.4.5 Table B.8 shows one year in which $\gamma_{2[Y]}$ was outside its 95% confidence limits. The probability that, in eight years, one or more is outside its confidence limits is 33.7%, which is not significant. However, these results are secondary to those of §5.2.3.6 above, on the strength of which the CAPM must be rejected.

5.3 A Parametric Test of the Securities Market Line using In-Period Betas

5.3.1 Let us suppose that both ex-ante assumptions at the start of a period and estimates based on ex-post observations during that period are unbiased estimates of the underlying values during that period, and therefore that the latter are unbiased estimates of the former. While this does not imply perfect foresight, it does imply
greater correspondence than might reasonably be expected. Nevertheless, as explained in the previous study, for the purpose of testing whether the CAPM can be used in long-term models it is relevant, since such a model can generate unbiased ex-ante betas.

5.3.2 The method used for this test follows Shanken (1985). Because the covariances and betas are estimated in-period, the quadratic form does not follow a multivariate $\chi^2$ distribution. Instead, with Shanken’s (op. cit.) notation, the regression statistic

$$Q_c = T\hat{e}'\hat{\Sigma}^{-1}e$$

follows a Hotelling’s $T^2$ with $N–2$ and $T–2$ degrees of freedom, where:

- $N$ is the number of asset classes and $n = N–2$
- $T$ is the length of the time series and $m = T–2$

$$e = \hat{\mu} - \hat{X}'\hat{\Gamma}_c;$$

5.3.3 For the zero-beta version of the CAPM this test was performed on each sub-period $p$ for which $m > n$. It was not possible to apply this test to the sub-period [153,185] either for the zero-beta version or for the standard CAPM, as the number of components in the market portfolio was too great in comparison with the length of the time series. It was also not possible to apply the test to the sub-period [34,85] as the covariance matrix was virtually singular.

5.3.4 Therefore for sub-periods $[p] \in \{[1,33],[101,152]\}$ the test was constructed as follows:

$$\hat{\mu} = \begin{bmatrix} \hat{\mu}_1 \\ \vdots \\ \hat{\mu}_n \end{bmatrix};$$

$$\hat{\mu}_i = R_{i(p)};$$

$$\hat{X} = \begin{bmatrix} 1 & \hat{\beta}_1 \\ 1 & \vdots \\ 1 & \hat{\beta}_4 \end{bmatrix};$$

$$\hat{\beta}_i = \beta_{i(p)};$$

$$\hat{\Gamma}_c = \begin{bmatrix} \hat{\gamma}_0 \\ \hat{\gamma}_1 \end{bmatrix} = (\hat{X}'\hat{\Sigma}^{-1}\hat{X})^{-1}\hat{X}'\hat{\Sigma}^{-1}\hat{\mu};$$
\[ \hat{\Sigma} = \begin{pmatrix} \hat{\sigma}_{e11} & \ldots & \hat{\sigma}_{e14} \\ \vdots & \ddots & \vdots \\ \hat{\sigma}_{e41} & \ldots & \hat{\sigma}_{e44} \end{pmatrix} ; \]

\[ \hat{\sigma}_{eij} = \frac{1}{Y} \sum_{Y \in p} (\varepsilon_{i(Y)} - \bar{\varepsilon}_i)(\varepsilon_{j(Y)} - \bar{\varepsilon}_j) ; \]

\[ \bar{\varepsilon}_u = \frac{1}{Y} \sum_{Y \in p} \varepsilon_{u(Y)} ; u = i, j ; \]

\[ Q_c^1 = \frac{Q_c}{1 + \frac{\hat{\sigma}^2_{MM}}{\hat{\sigma}_{MM}}} . \]

\[ F = T^2 \frac{m - n + 1}{mn} \sim F_{n,m-n+1} \]

5.3.5 For the zero-beta version of the CAPM the test for all sub-periods combined was constructed as above where:

\[ \hat{\mu} = \begin{pmatrix} \hat{\mu}_1 \\ \vdots \\ \hat{\mu}_7 \end{pmatrix} ; \]

\[ \hat{\mu}_i = R_i ; \]

\[ \hat{\beta}_i = \beta_i ; \]

\[ \hat{\Sigma} = \begin{pmatrix} \hat{\sigma}_{e11} & \ldots & \hat{\sigma}_{e17} \\ \vdots & \ddots & \vdots \\ \hat{\sigma}_{e71} & \ldots & \hat{\sigma}_{e77} \end{pmatrix} ; \]

\[ \hat{\sigma}_{eij} = \frac{1}{Y - [p] - 1} \sum_{Y \subseteq [p]} (\varepsilon_{i(Y)} - \bar{\varepsilon}_{i[p]})(\varepsilon_{j(Y)} - \bar{\varepsilon}_{j[p]}) . \]
For the zero-beta version of the CAPM $\varepsilon_{i(Y)}$ is defined as in §5.1.3.1 and for the standard version of the CAPM $\varepsilon_{i(Y)}$ is defined as:

$$r_{i(Y)} = \beta_{i(Y)} r_{M[Y]} + \varepsilon_{i(Y)}.$$ 

5.3.7 The results of these tests are summarised in Table 9.

### Table 9 Summary of results for Hotelling’s test

| Period     | $Q_C$  | $Q_C^d$ | $F$     | $p$-value |
|------------|--------|---------|---------|-----------|
| [1,33]     | 0,0748 | 0,0742  | 16,000  | 0,007     |
| [101,152]  | 0,027  | 0,027   | 61,455  | 0,000     |
| All        | 0,452  | 0,450   | 365,491 | 0,000     |

5.3.8 For each of the sub-periods tested the zero-beta version of the CAPM is rejected.

### 5.4 A Non-Parametric Test of the Securities Market Line

The above test assumes that $\varepsilon_{i(Y)}$ is normally distributed. No such assumption is made in the statement of the CAPM.

#### 5.4.1 The Zero-Beta Version of the CAPM

5.4.1.1 Of the 208 values of $\varepsilon_{i(Y)}$, 98 are negative and 110 positive. Also, the median absolute value is 0,0453. The observed values of $\varepsilon_{i(Y)}$ may be grouped according to whether $\varepsilon_{i(Y)} \leq 0$ and according to whether $|\varepsilon_{i(Y)}| \leq 0,0453$. If

$$E\{\varepsilon_{i(Y)}\} = 0$$

and the distribution of $\varepsilon_{i(Y)} \leq 0$ is symmetric, then the median will be 0 and, for both non-positive and positive values, the expected numbers of absolute values greater than the median absolute value ($e_{\rightarrow}$ and $e_{+\rightarrow}$ respectively) will be equal to the expected number of absolute values less than or equal to the median absolute value ($e_{\rightarrow\leftarrow}$ and $e_{+\leftarrow}$ respectively). Table 10 shows the observed numbers $o_j$ of values in each group $j \in \{-, \leq, -, >, +, \leq, +, >\}$, with the corresponding expected numbers $e_j$ in brackets.
Table 10 Grouping of $\varepsilon_{i(Y)}$ relative to 0 and of $|\varepsilon_{i(Y)}|$ relative to its median value

|          | $\varepsilon_{i(Y)} \leq 0$ | $\varepsilon_{i(Y)} > 0$ | Total |
|----------|-----------------------------|---------------------------|-------|
| $|\varepsilon_{i(Y)}| \leq 0,0453$ | 48 (49)                   | 56 (55)                   | 104   |
| $|\varepsilon_{i(Y)}| > 0,0453$ | 50 (49)                   | 54 (55)                   | 104   |
| **Total**| 98                          | 110                       | 208   |

The statistic:

$$G = \sum_j \left( \frac{o_j - e_j}{e_j} \right)^2$$

has a $\chi^2$ distribution with 1 degree of freedom. The value of $G$ is 0.0772. This is not significant at the 95% level, and the CAPM cannot be rejected on the basis of this test.

5.4.2 The Standard Version of the CAPM

5.4.2.1 Of the 56 values of $\varepsilon_{i(Y)}$, 23 are negative and 33 positive. Also, the median absolute value is 0.0537. The observed values of $\varepsilon_{i(Y)}$ may be grouped according to whether $\varepsilon_{i(Y)} \leq 0$ and according to whether $|\varepsilon_{i(Y)}| \geq 0.0537$. If

$$E\{\varepsilon_{i(Y)}\} = 0$$

and the distribution of $\varepsilon_{i(Y)} \leq 0$ is symmetric, then the median will be 0 and, for both non-positive and positive values, the expected numbers of absolute values greater than the median absolute value ($e_{-,>}$ and $e_{+,>}$ respectively) will be equal to the expected number of absolute values less than or equal to the median absolute value ($e_{-,\leq}$ and $e_{+,\leq}$ respectively). Table 11 shows the observed numbers $o_j$ of values in each group $j \in \{-,\leq-,>,+,\leq+,>\}$, with the corresponding expected numbers $e_j$ in brackets.

Table 11 Grouping of $\varepsilon_{i(Y)}$ relative to 0 and of $|\varepsilon_{i(Y)}|$ relative to its median value

|          | $\varepsilon_{i(Y)} \leq 0$ | $\varepsilon_{i(Y)} > 0$ | Total |
|----------|-----------------------------|---------------------------|-------|
| $|\varepsilon_{i(Y)}| \leq 0.0537$ | 10 (11,5)                 | 18 (16,5)                 | 28    |
| $|\varepsilon_{i(Y)}| > 0.0537$ | 13 (11,5)                 | 15 (16,5)                 | 28    |
| **Total**| 23                          | 33                        | 56    |

5.4.2.2 The statistic:

$$G = \sum_j \left( \frac{o_j - e_j}{e_j} \right)^2$$
has a $\chi^2$ distribution with 1 degree of freedom. At the 5% level, the critical value of $\chi^2$ is 5.991. The value of $G$ is 0.664. The hypothesis can therefore not be rejected. On the basis of this test the standard version of the CAPM is not rejected.

6. SUMMARY AND CONCLUSION

6.1 In section 5.1 it was shown using prior betas that the zero-beta version of the CAPM must be rejected.

6.2 Using in-period betas it was found that the explanatory power of the CAPM was higher. Again, one period—in this case [153,185] (a sub-period of particular concern, being that for which inflation-linked bonds are included)—showed significant results, but again on the basis of the overall results it was not possible to reject the zero-beta version of the CAPM. However, it appeared that, for some periods, $\gamma_1 < \gamma_0$, which is contrary to the prediction of the CAPM. Furthermore, the results for individual years showed that for many years the parameters were outside of their 95% confidence limits and on the basis of this result the CAPM must be rejected.

6.3 In section 5.2 it was shown using prior betas that the standard version of the CAPM could not be rejected. Tests of its linearity could also not be rejected. However, the explanatory power of the CAPM was low.

6.4 Using in-period betas it was found that the standard CAPM must be rejected for the period [153,185] not only on the grounds of tests for the parameters $\gamma_0$ and $\gamma_1$ for the sub-period as a whole, but also because of tests of individual years.

6.5 Using in-period betas, Hotelling’s test for the residuals of the zero-beta version of the CAPM was applied to those periods for which it was possible to do so. This comprised the sub-periods [1,33] and [101,152], as well as all sub-periods combined. For each of the sub-periods tested the zero-beta version of the CAPM is rejected.

6.6 The above tests are all parametric, assuming the normality of the residuals. This assumption is not necessary for the CAPM, and non-parametric tests of the residuals were therefore made. These tests were applied both to the zero-beta version of the CAPM over all sub-periods combined and to the standard version over the sub-period [153,185]. Neither of these tests rejected the CAPM.

6.7 In summary, parametric tests show that both the zero-beta and standard versions of the CAPM must be rejected for in-period betas and that the zero-beta version must also be rejected for prior betas. However, for prior betas the standard version of the CAPM could not be rejected. It is counter-intuitive that the tests should be failed for in-period betas and yet passed for prior betas: one would expect that prior knowledge of in-period betas would enhance the effectiveness of the CAPM.
6.8 On the other hand, non-parametric tests of the residuals could not be rejected, suggesting that the assumption of normality needs to be relaxed in tests of the CAPM in this market. Conversely, however, if the CAPM is used for long-term modelling, care needs to be taken about the choice of distribution: a normal distribution may be inappropriate. Whilst the CAPM does not necessarily imply a normal distribution, it does necessitate an elliptically symmetric distribution. Some other elliptically symmetric distribution than the normal may be required.

6.9 It may be, though, that the non-parametric tests used in this paper were not sufficiently powerful to reject the CAPM. Further research is required to test it more rigorously.

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APPENDIX A

CALCULATION OF RETURNS AND THE EFFECTIVE MARKET PORTFOLIO

In this appendix the calculation of the quarterly returns on equities and bonds is explained, as well as the determination of the effective market portfolio at quarterly intervals, during the period from 30/9/1964 to 185 at 31/12/2010. The meaning of ‘effective’ becomes clear below. We let t denote the quarter-end, or if the context so requires, the quarter, from 0 at 30/9/1964 to 185 at 31/12/2010.

A.1 Inflation

For \( t = 101, \ldots, 185 \) the data for inflation comprised values of the consumer price index.\(^1\) For earlier quarters these values comprised only two significant digits, and over some periods these did not change for up to three quarters. To avoid the resulting errors, earlier consumer price indices\(^2\) were used, after rebasing them to correspond to the current series. The force of inflation during quarter \( t \) was calculated as:

\[
i_t = \ln \left( \frac{p_t}{p_{t-4}} \right);
\]

where \( p_t \) is the consumer price index, recalculated as explained above, at time \( t \).

A.2 Equities

A.2.1 Data

The data for equities comprised the following:

- quarterly from 30/9/1964 to 31/12/2001: the JSE-Actuaries all-share price index and the dividend yield on that index;\(^3\)
- quarterly from 31/3/1970 to 31/12/1981\(^4\) and from 31/3/1989 to 31/12/2001: \(^5\) the total market capitalisation of equities included in the JSE-Actuaries all-share price index; and
- quarterly from 31/12/2001 to 31/12/2010: the FTSE-JSE all-share price index, the dividend yield on that index, the corresponding total-return index and the market capitalisation of equities included in that index.\(^6\)

\(^1\) Source: INet Bridge; code ECPI

\(^2\) Source: South African Reserve Bank. Quarterly reports

\(^3\) Source: INet Bridge; codes CI01[CL] and CI01[DY]

\(^4\) Source: Actuarial Society of South Africa & Johannesburg Stock Exchange (1982). *The JSE Actuaries Index*. Old Mutual, Cape Town

\(^5\) Source: INet Bridge, code CI01[MC]

\(^6\) Source: INet Bridge; code J203[CL], J203[DY] and J203[MC]
A.2.2 Returns

A.2.2.1 Let $S_t$, $D_t$ and $T_t$ denote the all-share price index, the dividend yield on that index and the corresponding total-return index respectively. We identify the JSE–Actuaries series and the FTSE–JSE series by means of the superscripts $^A$ and $^F$ respectively. For the period from 1/1/2002 ($t = 150,\ldots,185$) returns on equities were calculated as:

$$R_t = \ln \left( \frac{T_{t-1}^F}{T_t^F} \right) - i_t.$$

For earlier periods it was necessary to calculate returns from price indices and dividend yields. For this purpose it was assumed that:

$$S_{t-1}^A (1 + h_t) = S_t^A (1 + \frac{1}{4} h_t);$$

where:

$$h_t = e^{R_{t-1}^A} - 1;$$

and $d_t$ is the amount of dividends paid on the all-share index. Since the dividend yield $D_t$ is annually retrospective, we assume the formula:

$$d_t = \frac{1}{16} \left( S_t^A D_t^A + S_{t+1}^A D_{t+1}^A + S_{t+2}^A D_{t+2}^A + S_{t+3}^A D_{t+3}^A \right);$$

where each of the terms in parentheses reflects one-quarter of the annual dividends that became payable in quarter $t$, together with noise from various other quarters reduced by averaging. The formula used was therefore:

$$R_t = \ln \left( \frac{S_t^A + \frac{1}{2} D_t^A}{S_{t-1}^A - \frac{1}{2} D_{t-1}^A} \right) - i_t.$$

A.2.3 Market capitalisation

A.2.3.1 For the periods for which values of the market capitalisation of the all-share index were not available, these values were estimated as follows. The ratio of market capitalisation to the all-share index was calculated for the period for which market-capitalisation values were available and these ratios were extrapolated or interpolated for the remaining periods. The resulting ratios were then applied to the all-share index values for those periods.

A.3 Conventional Bonds

A.3.1 Data

The data for conventional bonds, on which coupons are payable half-yearly, comprised the following:

- quarterly from 30/9/1964 to 31/12/1964: the yield to redemption on government bonds for terms to redemption of 3 years and 20 years;
- quarterly from 31/3/1965 to 31/12/1985: the yield to redemption on government bonds for terms to redemption of 3 years, 10 years and 20 years;
- quarterly from 31/3/1985 to 31/12/2010: the yield to redemption on government bonds for terms to redemption of 1 year to 25 years at annual maturity intervals;
— on 30/6/1986 and 30/9/1986: the total loan debt of national government;
— quarterly from 30/9/1964 to 30/9/2010: the loan debt of national government by
  term to redemption for maturity intervals not exceeding 1 year, exceeding 1 but
  not 3 years, exceeding 3 but not 10 years and exceeding 10 years;
— quarterly or annually over various ranges from 31/3/1980 to 30/9/1990: the total
  nominal value of domestic marketable bonds issued by public-sector bodies
  other than national government; and
— quarterly from 30/9/1990 to 30/9/2010: the total nominal value of domestic
  marketable bonds issued by the public sector.

Yields to redemption are annual yields convertible half-yearly. Loan debt is the
nominal amount in issue. From the data available for the period from 31/3/1980 to
30/9/1990 it was possible to estimate the total nominal value of domestic marketable
bonds issued by the public sector.

### A.3.2 Returns

#### A.3.2.1

Let $y_{tq}$ denote the yield to redemption at time $t$ for $q$ quarters to redemption.
First we need to interpolate these yields between the maturity intervals available
to give yields to redemption for half-yearly intervals. For $t = 86, 87, ..., 185$ we have
$y_{1.4}, y_{1.8}, ..., y_{1.100}$ from which to interpolate. For this purpose the third-order difference
formula:

$$y_{tq_0 + d} = y_{q_0} + \left(\frac{d}{4}\right) \Delta y_{q_0} + \frac{1}{2} \left(\frac{d}{4}\right) \left(\frac{d}{4} - 1\right) \Delta^2 y_{q_0} + \frac{1}{6} \left(\frac{d}{4}\right) \left(\frac{d}{4} - 1\right) \left(\frac{d}{4} - 2\right) \Delta^3 y_{q_0}$$

was used. This formula uses the range $q_0, q_0 + 4, q_0 + 8, q_0 + 12$. Where possible, for the
purpose of calculating $y_{tq}$, the range of differencing was selected so as to straddle $q$;
i.e. $q_0 = q - 6$, so that $d = 6$. At the extremes of maturity, the available data were used,
including $d = -2, 2$ at the lower extreme and $d = 10$ at the upper.

#### A.3.2.2

For $t = 2, 3, ..., 85$ (i.e. from 31/3/1965 to 31/12/1985) we have just three
values from which to interpolate: $y_{2.12}, y_{3.40}$ and $y_{5.80}$. We assume for the purpose
of interpolation that the yield curve follows:

$$y_{tq} = a_t + b_t \exp\left\{-\left(\frac{q - c_t}{100}\right)^2\right\};$$  \hspace{1cm} (A.1)

where $a_t > 0$ and $a_t + b_t > 0$. Unlike most descriptive yield-curve formulae—apart
from the Ayres–Barry yield-curve formula—this formula comprises only three
parameters. Unlike the Ayres–Barry yield-curve formula, however, this formula allows
for humped yield curves if $c_t > 0$.

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7 Source: INet Bridge; codes JAYC01 to JAYC20
8 Source: South African Reserve Bank, www.sarb.gov.za; codes KBP 4086, 4140–3 and 4564
A.3.2.3 Equation (A.1) may be solved by means of Newton’s formula. Suppressing the subscripts \( t \) and \( q \), and instead denoting the \( j \)th values of \( q \) and \( y_{tq} \) as \( q_j \) and \( y_j \) respectively for \( j = 1, 2, 3 \), we obtain:

\[
\begin{align*}
c &= \lim_{n \to \infty} c_n; \\
b &= \lim_{n \to \infty} b_n; \quad \text{and} \\
a &= \lim_{n \to \infty} a_n;
\end{align*}
\]

where:

\[
\begin{align*}
c_n &= c_{n-1} - \frac{w_{n-1}}{w'_{n-1}}, \\
b_n &= \frac{y_1 - y_3}{r_n - r_{3n}}, \\
a_n &= y_1 - br_{1n}; \\
w_n &= (y_1 - y_3)(r_{3n} - r_{2n}) - (y_3 - y_2)(r_n - r_{3n}); \\
w'_{n} &= \frac{2}{d^2} \left\{ (y_1 - y_3)(s_{3n} - s_{2n}) - (y_3 - y_2)(s_n - s_{3n}) \right\}; \\
r_{jn} &= \exp \left\{ -\left( \frac{q_j - c_n}{d} \right)^2 \right\}; \quad \text{and} \\
s_{jn} &= (q_j - c_n)r_{jn}.
\end{align*}
\]

For the purposes of this paper, it was assumed that:

\[
d = 100.
\]

A.3.2.4 A problem arises where \( y_{t,40} = y_{t,80} \). This is clearly intended to be an approximation due to scarcity of data. An exact fit would generally place a hump between \( y_{t,40} \) and \( y_{t,80} \), which would not reflect the intention. It is therefore appropriate to assume that \( c_t = c_{t+1} \) and hence find \( a_t \) and \( b_t \) from equations (A.2) and (A.3) respectively. For \( t = 0, 1 \) (i.e. for 30/9/1964 and 31/12/1964) we have just two values from which to interpolate: \( y_{t,12} \) and \( y_{t,80} \). At these dates we again assume that \( c_t = c_{t+1} \) and hence find \( a_t \) and \( b_t \) from equations (A.2) and (A.3).

A.3.2.5 We now have \( y_{tq} \) for \( t = 0, \ldots, 185, \ q = 2, 4, \ldots, 100 \). Next we need to calculate continuously compounded quarterly spot yields \( z_{tq} \) for the same ranges of \( t \) and \( q \). For this purpose we assume, for the sake of simplicity, that the yields to redemption \( y_{th} \) are for bonds at par, so that:

\[
\frac{1}{2} y_{tq} e^{-2z_{tq}} + \frac{1}{2} y_{tq} e^{-4z_{tq}} + \ldots + \left(1 + \frac{1}{2} y_{tq}\right)^{-q_{tq} A_t} = 1;
\]
giving:

\[ z_{tq} = \begin{cases} 
\frac{1}{2} \ln \left(1 + \frac{1}{2} y_{t2}^{q} \right) & \text{for } q = 2; \\
\frac{1}{q} \ln \left( \frac{1 + \frac{1}{2} y_{tq}^{q}}{1 - \frac{1}{2} y_{tq}^{q} \sum_{n=1}^{q-1} \exp(-2n z_{t,2n})} \right) & \text{for } q = 4, 6, \ldots, 100.
\end{cases} \]

A.3.2.6 Now, for each \( t \), we need to interpolate between \( z_{t2}, z_{t4}, \ldots, z_{t100} \) to find \( z_{t1}, z_{t3}, \ldots, z_{t99} \). Here we use the interpolation formula:

\[ z_{t, q_0 + d} = z_{q_0} + \left( \frac{d}{2} \right) \Delta z_{q_0} + \frac{1}{2} \left( \frac{d}{2} \right) \left( \frac{d}{2} - 1 \right) \Delta^2 z_{q_0} + \frac{1}{6} \left( \frac{d}{2} \right) \left( \frac{d}{2} - 1 \right) \left( \frac{d}{2} - 2 \right) \Delta^3 z_{q_0}. \]

This gives us \( z_{tq} \) for \( t = 0, \ldots, 185, q = 1, 2, \ldots, 100 \).

A.3.2.7 As shown by Maitland (2001), the first three principal components of the JSE–Actuaries yield curve over the period from February 1986 to May 200 were sufficient to explain 98.45% of its variability. For the purposes of this paper it was therefore assumed that the conventional bonds market consisted in a portfolio comprising three zero-coupon bonds. The terms chosen for this purpose were 20, 40 and 80 quarters. For the purpose of modelling risk-free short-term interest rates, a one-quarter bond was also modelled, but this was not included in the market capitalisation.

A.3.2.8 Finally, then, we calculate the real return on conventional bonds with terms to redemption of \( \frac{1}{4}, 5, 10 \) and 20 years as:

\[ R_{t,q} = q z_{t-1,q} - (q - 1) z_{t,q-1} - i, \quad \text{for } t = 1, \ldots, 185, q = 1, 20, 40, 80. \]

A.3.3 Effective market capitalisation of zero-coupon bonds by term to redemption

A.3.3.1 Let \( m_{t} \) denote the total nominal value of conventional domestic marketable bonds issued by the public sector at time \( t \). This is obtained by deducting from the total nominal value of domestic marketable bonds issued by the public sector the total nominal value of inflation-linked domestic marketable bonds issued as specified in section A.4.1 below. Then we estimate the total nominal value of conventional domestic marketable bonds issued by the public sector at time \( t \) for maturity group \( g \) as:

\[ m_{t[g]} = \frac{m_{t[g]}^{N}}{\sum_{g=1}^{4} m_{t[g]}^{N}} m_{t}; \]

where \( m_{t[g]}^{N} \) is the loan debt of national government for maturity group \( g \); and:
1 for maturity intervals not exceeding 4 quarters;
2 for maturity intervals exceeding 4 but not 12 quarters;
3 for maturity intervals exceeding 12 but not 40 quarters;
4 for maturity intervals exceeding 40 quarters.

A.3.3.2 We now assume that, in each group, the bonds are at par with terms to redemption uniformly distributed across its maturity interval. We also assume that the upper limit of group 4 is 100 quarters. At time \( t \) the amounts payable during each subsequent maturity interval may then be estimated as:

\[
\pi_{tg} = \begin{cases} 
(1 + 0.5y_{t2}^1)m_{t1} + (y_{t18}^1m_{t2} + y_{t26}^1m_{t3} + y_{t170}^1m_{t4}) & \text{for } g = 1; \\
(1 + y_{t18}^1)m_{t2} + 2(y_{t26}^1m_{t3} + y_{t170}^1m_{t4}) & \text{for } g = 2; \\
(1 + 3.5y_{t26}^1)m_{t3} + 7(y_{t170}^1m_{t4}) & \text{for } g = 3; \\
(1 + 7.5y_{t170}^1m_{t4}) & \text{for } g = 4.
\end{cases}
\]

A.3.3.3 We now need the market portfolio of conventional bonds of the selected maturities. It would be possible to calculate this using Maitland (2002). However, it was decided to adopt a more heuristic approach, particularly since Maitland’s method relates to yields to redemption rather than spot yields. Let \( k_{ij} \) denote the market value at maturity \( j \).

A.3.3.4 We denote the price of a zero-coupon bond with term to redemption \( q \) at time \( t \) as:

\[
P(t, q) = \exp\left(-qz_{tq}\right).
\]

Let:

\[
k_i^{(p)} = \sum_{g=1}^{4} \pi_{tg} \left[ \frac{1}{2} f_g^p P(t, f_{g-1}) + (f_{g-1} + 1)^p P(t, f_{g-1} + 1) + \ldots \right]
\]

for \( p = 0, 1, 2; \)

where \( f_0 = 0; f_1 = 4; f_2 = 12; f_3 = 40 \) and \( f_4 = 100; \)

so that, in particular, \( k_i^{(0)} \) is the market capitalisation of bonds at time \( t \). Let \( m_{ij}^* \) be the exposure of the market at time \( t \) to a zero-coupon bond with term to redemption \( q_j \) such that:

\[
q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \\ 20 \end{pmatrix}.
\]
In other words,

\[ m_t^* = \begin{pmatrix} m_{t_1}^* \\ m_{t_2}^* \\ m_{t_3}^* \end{pmatrix} \]

gives the effective market capitalisations at that time of zero-coupon bonds of the selected terms to redemption.

A.3.3.5 Then:

\[ Qm_t^* = \kappa_t \]

where:

\[ Q = \begin{pmatrix} 1 & 1 & 1 \\ q_1 & q_2 & q_3 \\ q_1^2 & q_2^2 & q_3^2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 12 & 40 & 80 \\ 144 & 1600 & 6400 \end{pmatrix} \]

\[ \kappa_t = \begin{pmatrix} \kappa_t^{(0)} \\ \kappa_t^{(1)} \\ \kappa_t^{(2)} \end{pmatrix} \]

This gives:

\[ m_t^* = Q^{-1} \kappa_t \]

A.4 Inflation-Linked Bonds

A.4.1 Data
The data for inflation-linked bonds comprised:

— for each bond issued: the date of issue, the amount originally issued, the coupon and the redemption date; and

— at quarterly intervals from 30/6/2000 (when the first bond was issued), for each bond issued, the yield to redemption;

— at quarterly intervals from 30/6/2009 for each bond issued, the cumulative amount issued and the total market capitalisation.

A.4.2 Returns
A.4.2.1 The price of a zero-coupon bond, per unit of the amount issued, inflated to the date of calculation, is taken to be:

\[ P = \frac{1}{2} \left[ 1 - \left(1 + \frac{y}{2}\right)^{-\frac{q}{2}} \right] \left(1 + \frac{y}{2}\right)^{-\frac{q}{2} + \frac{y}{2}} + \left(1 + \frac{y}{2}\right)^{-\frac{q}{2}} ; \]
where:
- $q$ is the term to redemption in quarters;
- $c$ is the half-yearly coupon;
- $y$ is the annual yield to redemption, convertible half-yearly; and
- $[x]$ is the integral portion of $x$.

A.4.2.2 This gives:

$$
P = \frac{c}{y} \left( \left( 1 + \frac{y}{2} \right)^{-q/2} + \left( 1 + \frac{y}{2} \right)^{-q/2} \right) - \left( 1 + \frac{y}{2} \right)^{-q/2} + \left( 1 + \frac{y}{2} \right)^{-q/2}
$$

(A.4)

where:
- $g = \left( 1 + \frac{y}{2} \right)$; and
- $h = \left\lfloor \frac{q}{2} \right\rfloor$.

A.4.2.3 The duration of a zero-coupon bond in quarters is defined as:

$$
D = -\frac{1}{P} \frac{\partial P}{\partial z} ; \quad (A.6)
$$

where $z$ is the quarterly continuously compounded yield, such that:

$$
z = \frac{1}{2} \ln g .
$$

Now:

$$
\frac{\partial z}{\partial y} = \frac{\partial z}{\partial g} \frac{\partial g}{\partial y} = \frac{1}{2g} \frac{\partial g}{\partial y};
$$

and from equation (A.5):

$$
\frac{\partial g}{\partial y} = \frac{1}{2} . \quad (A.7)
$$

Thus:

$$
\frac{\partial z}{\partial y} = \frac{1}{4g} . \quad (A.8)
$$

It follows from equation (A.6) that:

$$
D = -\frac{1}{P} \frac{\partial P}{\partial y} \frac{\partial y}{\partial z} = -\frac{4g}{P} \frac{\partial P}{\partial y} .
$$
Also, from equation (A.4):
\[
\frac{\partial P}{\partial y} = -\frac{q}{2} g^{-q/2-1} \frac{\partial g}{\partial y} \left\{ 1 - \frac{c}{y} (1 - g_y) \right\} + g^{-q/2} \left\{ \frac{c}{y^2} (1 - g_y^2) - \frac{c}{y} \left( -hg_y^{-1} \frac{\partial g}{\partial y} \right) \right\};
\]
and hence from equation (A.7):
\[
\frac{\partial P}{\partial y} = -\frac{P q}{4g} + \frac{cg^{-q/2}}{y} \left\{ \frac{1}{y} (1 - g_y) + \frac{1}{2} \left( hg_y^{-1} \right) \right\}.
\]
Thus:
\[
D = q - \frac{2cg^{-q/2}}{Py} \left\{ \frac{2g}{y} (1 - g_y) + hg_y \right\}.
\]
(A.9)

A.4.2.4 For a zero-coupon bond we have \( c = 0 \), so that the yield to redemption for a term to redemption of \( D \) quarters may be taken as that of a coupon-paying bond with \( q \) quarters to redemption. In other words, we may plot the yield to redemption as a function of \( D \) and read the spot yield off that curve. In order to obtain spot yields at integral values of \( q \) we may use divided differences to give:
\[
z_q = z_a + (D - a) \Delta_{a,b} + (D - a)(D - b) \Delta_{a,b,c}^2 + (D - a)(D - b)(D - c) \Delta_{a,b,c,d}^3
\]
\[
= z_a + (D - a) \left[ \Delta_{a,b} + (D - b) \left( \Delta_{a,b,c}^2 + (D - c) \Delta_{a,b,c,d}^3 \right) \right];
\]
(A.10)
where:
\[
\Delta_{a,b} = \frac{z_b - z_a}{b - a} \quad \text{etc.}; \quad \Delta_{a,b,c}^2 = \frac{\Delta_{b,c} - \Delta_{a,b}}{c - b} \quad \text{etc.}; \quad \Delta_{a,b,c,d}^3 = \frac{\Delta_{b,c,d} - \Delta_{a,b,c}}{d - a};
\]
and \( z_a, z_b, z_c \) and \( z_d \) are quarterly continuously compounded yields to redemption for durations \( a, b, c \) and \( d \) respectively. Where possible (i.e. except at the extremes), the bonds were chosen so that \( a < b < D \leq c < d \). Where there were fewer than four bonds, the higher-order differences were ignored.

A.4.2.5 In some cases the method explained in the preceding paragraph resulted in extrapolation to unacceptable levels at the extremes. In these cases reasonable values were assumed for \( D = 0 \) and \( D = 100 \).

A.4.2.6 Finally we calculate the real return on inflation-linked bonds with terms to redemption of \( \frac{1}{4}, 5, 10 \) and \( 20 \) years as:
\[
R_q = qz_{t-1,q} - (q-1)z_{t,q-1} \quad \text{for} \ t = 144, \ldots, 185, \ q = 1, 20, 40, 80.
\]
Here it is not necessary to subtract the force of inflation as the return itself is expressed in real terms.
A.4.3 EFFECTIVE MARKET CAPITALISATION OF ZERO-COUPON BONDS BY TERM TO REDEMPTION

A.4.3.1 For the purpose of calculating the total nominal value of conventional domestic marketable bonds issued by the public sector at time \( t \) (\( \Pi \), A.3.3.1), the amount of inflation-linked domestic marketable bonds so issued must be deducted from the total nominal value of domestic marketable bonds so issued. From 30/6/2009 onwards the amounts of inflation-linked domestic marketable bonds so issued were available. At earlier times it was necessary to estimate them. For this purpose, for each bond issued before 30/6/2009, the amount issued at times between the date of issue and that date was interpolated between the amounts issued at those dates.

A.4.3.2 In order to calculate the effective market capitalisation of zero-coupon bonds by term to redemption \( q \) at a particular date \( t \), the payment during each subsequent quarter was calculated and discounted at the mean spot rate for that quarter, to give the total payment during quarter \( q \):

\[
\pi_{tq} = \sum_{j \in J} \frac{m'_{jt}}{P_{jt}} \left( \frac{1}{2} C_j \delta_{t + q - q_j} + \delta_{t + q - q_j} \right); 
\]

where:

\[
P_{jt} = \sum_{q \in Q_j} \left( \frac{1}{2} C_j + \delta_{q - q_j} \right) \exp \left( -q \kappa_{tq} \right); 
\]

\[
\delta_B = \begin{cases} 
1 & \text{if } B; \\
0 & \text{otherwise}; 
\end{cases} 
\]

\( J \) is the set of inflation-linked bonds;

\( C_j \) is the annual coupon on bond \( j \);

\( Q_j \) is the set of quarters in which coupons are payable on bond \( j \);

\( q_j^* \) is the quarter in which bond \( j \) is redeemable; and

\( m'_{jt} \) is the market capitalisation of bond \( j \) at time \( t \).

A.4.3.3 Following \( \Pi \), A.3.3.4, let:

\[
\kappa^{(p)}_t = \sum_{q=1}^{100} \pi_{tq} q^p P(t, q) \quad \text{for } p = 0, 1, 2; 
\]

where:

\[
P(t, q) = \exp \left( -q \kappa_{tq} \right); 
\]

so that, in particular, \( \kappa^{(0)}_t \) is the market capitalisation of bonds at time \( t \). As before:

\[
m^*_t = Q^{-1} \kappa_t. 
\]
### APPENDIX B

**SUMMARY OF REGRESSION ANALYSIS PER CALENDAR YEAR**

Table B.1 Summary of the regression analysis for the test of the explanatory power of the CAPM using prior betas for the zero-beta version of the CAPM

| Year | $R^2$ | Estimate 1 | Value 1 | $t$-value 1 | $p$-value 1 | Estimate 2 | Value 2 | $t$-value 2 | $p$-value 2 |
|------|-------|------------|--------|-------------|------------|------------|--------|-------------|------------|
| 1969 | 0.998 | $\gamma_{0[Y]}$ | -0.002 | -1.141 | 0.372 | $\gamma_{1[Y]}$ | -0.096 | -31.855 | 0.001 |
| 1970 | 0.148 | $\gamma_{0[Y]}$ | -0.020 | -0.917 | 0.374 | $\gamma_{1[Y]}$ | -0.048 | -1.559 | 0.141 |
| 1971 | 0.008 | $\gamma_{0[Y]}$ | -0.028 | -0.680 | 0.508 | $\gamma_{1[Y]}$ | 0.020 | 0.336 | 0.742 |
| 1972 | 0.630 | $\gamma_{0[Y]}$ | 0.003 | 0.252 | 0.805 | $\gamma_{1[Y]}$ | 0.079 | 4.878 | 0.000 |
| 1973 | 0.004 | $\gamma_{0[Y]}$ | -0.019 | -0.930 | 0.368 | $\gamma_{1[Y]}$ | 0.007 | 0.246 | 0.809 |
| 1974 | 0.023 | $\gamma_{0[Y]}$ | -0.065 | -1.589 | 0.134 | $\gamma_{1[Y]}$ | 0.032 | 0.575 | 0.575 |
| 1975 | 0.004 | $\gamma_{0[Y]}$ | -0.026 | -1.056 | 0.309 | $\gamma_{1[Y]}$ | 0.009 | 0.250 | 0.806 |
| 1976 | 0.010 | $\gamma_{0[Y]}$ | -0.016 | -0.682 | 0.506 | $\gamma_{1[Y]}$ | 0.012 | -0.367 | 0.719 |
| 1977 | 0.139 | $\gamma_{0[Y]}$ | 0.001 | 0.063 | 0.951 | $\gamma_{1[Y]}$ | 0.029 | 1.501 | 0.156 |
| 1978 | 0.031 | $\gamma_{0[Y]}$ | 0.028 | 1.416 | 0.179 | $\gamma_{1[Y]}$ | 0.018 | 0.670 | 0.514 |
| 1979 | 0.449 | $\gamma_{0[Y]}$ | -0.023 | -0.909 | 0.379 | $\gamma_{1[Y]}$ | 0.102 | 3.375 | 0.005 |
| 1980 | 0.281 | $\gamma_{0[Y]}$ | -0.091 | -2.905 | 0.012 | $\gamma_{1[Y]}$ | 0.089 | 2.339 | 0.035 |
| Year | \( Y_t \) | \( \gamma_{ty} \) | \( \gamma_{ty} \) | \( \gamma_{ty} \) | \( \gamma_{ty} \) |
|------|---------|----------------|----------------|----------------|----------------|
| 1981 | 0.000   | -0.041         | -1.509         | 0.154          | 0.989          |
| 1982 | 0.004   | 0.044          | 0.478          | 0.640          |
| 1983 | 0.005   | -0.050         | -0.903         | 0.382          |
| 1984 | 0.015   | -0.044         | -0.741         | 0.471          |
| 1985 | 0.046   | -0.045         | -0.754         | 0.463          |
| 1986 | 0.030   | 0.079          | 0.655          | 0.523          |
| 1987 | 0.006   | -0.024         | -0.371         | 0.716          |
| 1988 | 0.025   | -0.017         | -0.477         | 0.641          |
| 1989 | 0.014   | 0.015          | 0.298          | 0.770          |
| 1990 | 0.054   | -0.048         | -0.891         | 0.388          |
| 1991 | 0.000   | 0.005          | 0.101          | 0.921          |
| 1992 | 0.053   | 0.027          | 0.681          | 0.507          |
| 1993 | 0.071   | 0.034          | 1.031          | 0.320          |
| 1994 | 0.008   | -0.077         | -1.479         | 0.161          |
| 1995 | 0.012   | 0.064          | 1.634          | 0.124          |
| Year | \( \gamma \) | \( \gamma_{\text{df}} \) | \( \gamma_{\text{df}} \) | \( \gamma_{\text{df}} \) | \( \gamma_{\text{df}} \) |
|------|-----|-----|-----|-----|-----|
| 1996 | 0.026 | -0.013 | -0.418 | 0.682 | 0.554 |
| 1997 | 0.009 | 0.075 | 1.675 | 0.116 | 0.724 |
| 1998 | 0.002 | -0.020 | -0.214 | 0.833 | 0.861 |
| 1999 | 0.054 | 0.037 | 0.583 | 0.569 | 0.385 |
| 2000 | 0.017 | 0.047 | 1.030 | 0.321 | 0.633 |
| 2001 | 0.024 | 0.011 | 0.147 | 0.885 | 0.569 |
| 2002 | 0.045 | 0.062 | 0.871 | 0.398 | 0.433 |
| 2003 | 0.009 | 0.061 | 1.671 | 0.117 | 0.733 |
| 2004 | 0.004 | 0.041 | 1.402 | 0.183 | 0.809 |
| 2005 | 0.139 | 0.024 | 1.286 | 0.219 | 0.155 |
| 2006 | 0.107 | 0.001 | 0.025 | 0.980 | 0.215 |
| 2007 | 0.110 | -0.011 | -0.800 | 0.435 | 0.165 |
| 2008 | 0.026 | 0.019 | 0.547 | 0.589 | 0.410 |
| 2009 | 0.213 | -0.029 | -1.834 | 0.078 | 0.013 |
| 2010 | 0.000 | 0.025 | 1.975 | 0.059 | 0.931 |
Table B.2 Summary of the regression analysis for the test for nonlinearity using prior betas for the zero-beta version of the CAPM

| Year | $R^2$ | Estimate | Value | t-value | p-value |
|------|-------|----------|-------|---------|---------|
| 1969 | 0,999 | $\gamma_{0[Y]}$ | -0,003 | -1,315 | 0,414   |
|      |       | $\gamma_{1[Y]}$ | 0,052 | 0,365  | 0,777   |
|      |       | $\gamma_{2[Y]}$ | -0,106 | -1,033 | 0,490   |
| 1970 | 0,250 | $\gamma_{0[Y]}$ | 0,012  | 0,377  | 0,712   |
|      |       | $\gamma_{1[Y]}$ | -1,368 | -1,376 | 0,192   |
|      |       | $\gamma_{2[Y]}$ | 0,928  | 1,329  | 0,207   |
| 1971 | 0,064 | $\gamma_{0[Y]}$ | -0,074 | -1,112 | 0,286   |
|      |       | $\gamma_{1[Y]}$ | 1,351  | 0,897  | 0,386   |
|      |       | $\gamma_{2[Y]}$ | -0,914 | -0,885 | 0,392   |
| 1972 | 0,643 | $\gamma_{0[Y]}$ | -0,008 | -0,390 | 0,703   |
|      |       | $\gamma_{1[Y]}$ | 0,259  | 0,986  | 0,342   |
|      |       | $\gamma_{2[Y]}$ | -0,120 | -0,686 | 0,504   |
| 1973 | 0,004 | $\gamma_{0[Y]}$ | -0,018 | -0,604 | 0,556   |
|      |       | $\gamma_{1[Y]}$ | -0,009 | -0,025 | 0,981   |
|      |       | $\gamma_{2[Y]}$ | 0,011  | 0,043  | 0,966   |
| 1974 | 0,104 | $\gamma_{0[Y]}$ | -0,071 | -1,744 | 0,105   |
|      |       | $\gamma_{1[Y]}$ | 0,768  | 1,126  | 0,280   |
|      |       | $\gamma_{2[Y]}$ | -0,498 | -1,084 | 0,298   |
| 1975 | 0,006 | $\gamma_{0[Y]}$ | -0,020 | -0,397 | 0,698   |
|      |       | $\gamma_{1[Y]}$ | -0,116 | -0,165 | 0,871   |
|      |       | $\gamma_{2[Y]}$ | 0,071  | 0,153  | 0,881   |
| 1976 | 0,059 | $\gamma_{0[Y]}$ | -0,055 | -1,037 | 0,318   |
|      |       | $\gamma_{1[Y]}$ | 0,433  | 0,801  | 0,438   |
|      |       | $\gamma_{2[Y]}$ | -0,290 | -0,824 | 0,425   |
| 1977 | 0,150 | $\gamma_{0[Y]}$ | 0,008  | 0,350  | 0,732   |
|      |       | $\gamma_{1[Y]}$ | -0,061 | -0,278 | 0,785   |
|      |       | $\gamma_{2[Y]}$ | 0,060  | 0,414  | 0,685   |
| Year | $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | $\gamma_4$ |
|------|-----------|-----------|-----------|-----------|-----------|
| 1978 | 0.103     | 0.051     | 1.695     | 0.357     |
|      |           | -0.282    | -0.956    |           |
|      |           | 0.194     | 1.020     | 0.327     |
| 1979 | 0.581     | 0.035     | 0.968     | 0.351     |
|      |           | -0.180    | -1.270    | 0.226     |
|      |           | 0.169     | 2.031     | 0.063     |
| 1980 | 0.282     | -0.088    | -2.176    | 0.049     |
|      |           | 0.071     | 0.365     | 0.721     |
|      |           | 0.011     | 0.096     | 0.925     |
| 1981 | 0.037     | -0.018    | -0.419    | 0.682     |
|      |           | -0.130    | -0.697    | 0.498     |
|      |           | 0.081     | 0.707     | 0.492     |
| 1982 | 0.039     | -0.020    | -0.153    | 0.881     |
|      |           | 0.440     | 0.719     | 0.485     |
|      |           | -0.272    | -0.691    | 0.502     |
| 1983 | 0.049     | 0.025     | 0.225     | 0.826     |
|      |           | -0.314    | -0.732    | 0.477     |
|      |           | 0.211     | 0.776     | 0.452     |
| 1984 | 0.383     | 0.164     | 1.845     | 0.088     |
|      |           | -0.948    | -2.838    | 0.014     |
|      |           | 0.591     | 2.783     | 0.016     |
| 1985 | 0.078     | 0.038     | 0.272     | 0.790     |
|      |           | -0.278    | -0.554    | 0.589     |
|      |           | 0.207     | 0.665     | 0.517     |
| 1986 | 0.121     | 0.220     | 0.908     | 0.380     |
|      |           | -0.799    | -1.042    | 0.316     |
|      |           | 0.502     | 1.160     | 0.267     |
| 1987 | 0.007     | -0.033    | -0.325    | 0.750     |
|      |           | 0.038     | 0.188     | 0.854     |
|      |           | -0.010    | -0.117    | 0.908     |
| Year | Value | $\gamma_{0\gamma}$ | $\gamma_{1\gamma}$ | $\gamma_{2\gamma}$ |
|------|-------|------------------|------------------|------------------|
| 1988 | 0.068 | -0.045           | 0.178            | -0.104           |
|      |       |                  | 0.888            | -0.775           |
|      |       |                  | 0.392            | 0.452            |
| 1989 | 0.015 | 0.024            | -0.009           | 0.026            |
|      |       |                  | 0.295            | 0.120            |
|      |       |                  | 0.773            | 0.906            |
| 1990 | 0.060 | 0.030            | -0.126           | 0.052            |
|      |       |                  | 0.481            | 0.304            |
|      |       |                  | 0.639            | 0.766            |
| 1991 | 0.000 | 0.004            | 0.013            | -0.007           |
|      |       |                  | 0.064            | -0.032           |
|      |       |                  | 0.950            | 0.975            |
| 1992 | 0.081 | 0.045            | -0.212           | 0.113            |
|      |       |                  | 0.907            | 0.624            |
|      |       |                  | 0.381            | 0.543            |
| 1993 | 0.090 | 0.097            | 0.071            | -0.036           |
|      |       |                  | 2.118            | -0.527           |
|      |       |                  | 0.054            | 0.607            |
| 1994 | 0.020 | -0.079           | -0.155           | 0.093            |
|      |       |                  | -1.468           | 0.403            |
|      |       |                  | 0.166            | 0.693            |
| 1995 | 0.140 | 0.049            | 0.349            | -0.231           |
|      |       |                  | 1.244            | -1.395           |
|      |       |                  | 0.235            | 0.186            |
| 1996 | 0.185 | 0.054            | -0.353           | 0.214            |
|      |       |                  | 1.066            | 1.595            |
|      |       |                  | 0.306            | 0.022            |
| 1997 | 0.348 | -0.087           | 0.621            | -0.399           |
|      |       |                  | -1.193           | -2.599           |
|      |       |                  | 0.254            | 0.027            |
| Year | $\gamma_{0}$ | $\gamma_{1}$ | $\gamma_{2}$ |
|------|--------------|--------------|--------------|
| 1998 | 0.006        | -0.146       | 0.081        |
|      |              | -0.015       | -0.095       |
|      |              | 0.222        | 0.512        |
|      |              | -0.096       | -0.361       |
| 1999 | 0.064        | -0.093       | -0.282       |
|      |              | 0.409        | 1.537        |
|      |              | -0.137       | -1.667       |
| 2000 | 0.190        | -0.055       | -0.333       |
|      |              | 0.259        | 0.568        |
|      |              | -0.137       | -0.469       |
| 2001 | 0.040        | -0.090       | -0.333       |
|      |              | 0.442        | 0.904        |
|      |              | -0.037       | -1.077       |
| 2002 | 0.123        | -0.243       | -1.619       |
|      |              | 2.313        | 4.028        |
|      |              | -0.015       | -0.110       |
| 2003 | 0.009        | 0.066        | 0.013        |
|      |              | -0.037       | -0.115       |
|      |              | 0.013        | 0.060        |
| 2004 | 0.555        | 0.064        | 0.663        |
|      |              | -0.836       | 3.742        |
|      |              | -0.021       | 3.789        |
| 2005 | 0.591        | -0.145       | 0.045        |
|      |              | -0.180       | 1.262        |
|      |              | 0.079        | 0.493        |
| 2006 | 0.174        | -0.015       | -0.810       |
|      |              | 0.066        | -0.835       |
|      |              | 0.079        | 0.493        |
| 2007 | 0.258        | -0.006       | 1.262        |
| Year | \( r_{0,1} \) | \( r_{1,1} \) | \( r_{2,1} \) | \( \gamma_{0,1} \) | \( \gamma_{1,1} \) | \( \gamma_{2,1} \) |
|------|----------------|----------------|----------------|----------------|----------------|----------------|
| 2008 | 0.227          | 0.063          | 1.766          | -0.556         | -2.694         | 0.012          |
|      |                |                |                | 0.417          | 2.546          | 0.017          |
| 2009 | 0.331          | -0.021         | -1.417         | 0.239          | 3.057          | 0.005          |
|      |                |                |                | -0.153         | -2.103         | 0.046          |
| 2010 | 0.022          | 0.020          | 1.366          | -0.042         | -0.636         | 0.184          |
|      |                |                |                | 0.048          | 0.738          | 0.467          |
### Table B.3 Summary of the regression analysis for the test of the explanatory power of the CAPM using in-period betas for the zero-beta version of the CAPM

| Year | \( R^2 \) | Estimate | Value | \( t \)-value | \( p \)-value |
|------|-----------|----------|-------|---------------|--------------|
| 1965 | 0.076     | \( \gamma_0 \) | -0.041| -0.642        | 0.586        |
|      |           | \( \gamma_1 \) | 0.034 | 0.406         | 0.724        |
| 1966 | 0.479     | \( \gamma_0 \) | 0.048 | 1.211         | 0.349        |
|      |           | \( \gamma_1 \) | 0.018 | 1.357         | 0.308        |
| 1967 | 1.000     | \( \gamma_0 \) | 0.007 | 5.917         | 0.027        |
|      |           | \( \gamma_1 \) | 0.108 | 71.456        | 0.000        |
| 1968 | 0.970     | \( \gamma_0 \) | 0.066 | 3.314         | 0.080        |
|      |           | \( \gamma_1 \) | 0.207 | 8.081         | 0.015        |
| 1969 | 0.988     | \( \gamma_0 \) | 0.010 | 1.620         | 0.247        |
|      |           | \( \gamma_1 \) | -0.118| -12.918       | 0.006        |
| 1970 | 0.731     | \( \gamma_0 \) | -0.107| -2.201        | 0.159        |
|      |           | \( \gamma_1 \) | -0.143| -2.330        | 0.145        |
| 1971 | 0.091     | \( \gamma_0 \) | -0.098| -1.877        | 0.201        |
|      |           | \( \gamma_1 \) | 0.026 | 0.448         | 0.698        |
| 1972 | 0.938     | \( \gamma_0 \) | 0.029 | 0.748         | 0.532        |
|      |           | \( \gamma_1 \) | 0.262 | 5.514         | 0.031        |
| 1973 | 0.442     | \( \gamma_0 \) | -0.081| -4.417        | 0.048        |
|      |           | \( \gamma_1 \) | 0.032 | 1.258         | 0.335        |
| 1974 | 0.731     | \( \gamma_0 \) | -0.251| -5.924        | 0.027        |
|      |           | \( \gamma_1 \) | 0.134 | 2.331         | 0.145        |
| 1975 | 0.685     | \( \gamma_0 \) | -0.098| -6.537        | 0.023        |
|      |           | \( \gamma_1 \) | -0.041| -2.084        | 0.173        |
| 1976 | 0.902     | \( \gamma_0 \) | -0.048| -3.260        | 0.083        |
|      |           | \( \gamma_1 \) | -0.054| -4.292        | 0.050        |
| 1977 | 0.941     | \( \gamma_0 \) | -0.006| -0.332        | 0.772        |
|      |           | \( \gamma_1 \) | 0.098 | 5.644         | 0.030        |
| Year | \( \gamma_0 \) | \( \gamma_1 \) | \( \gamma_2 \) | \( \gamma_3 \) |
|------|--------------------|--------------------|--------------------|--------------------|
| 1978 | 0.596              | 0.044              | 0.548              | 0.639              |
| 1979 | 0.871              | -0.030             | -0.354             | 0.757              |
| 1980 | 0.867              | -0.357             | -4.560             | 0.045              |
| 1981 | 0.167              | -0.183             | -3.049             | 0.093              |
| 1982 | 0.532              | 0.094              | 0.849              | 0.485              |
| 1983 | 0.640              | -0.140             | -2.453             | 0.134              |
| 1984 | 0.348              | 0.068              | 0.185              | 0.870              |
| 1985 | 0.328              | 0.156              | 0.766              | 0.524              |
| 1986 | 0.587              | 0.131              | 2.372              | 0.141              |
| 1987 | 0.418              | 0.029              | 0.346              | 0.762              |
| 1988 | 0.846              | -0.126             | -2.863             | 0.103              |
| 1989 | 0.647              | 0.139              | 3.348              | 0.079              |
| 1990 | 0.749              | 0.072              | 1.916              | 0.196              |
| 1991 | 0.913              | -0.078             | -2.440             | 0.135              |
| 1992 | 0.479              | -0.053             | -1.994             | 0.044              |
| Year | \( \gamma \) | \( \gamma \) | \( \gamma \) | \( \gamma \) |
|------|-------------|-------------|-------------|-------------|
| 1993 | 0.060       | 0.420       | -0.094      | 0.174       |
|      |             |             |             |             |
| 1994 | 0.894       | 0.211       | -0.309      | 0.054       |
|      |             |             |             |             |
| 1995 | 0.055       | 0.448       | -0.189      | 0.442       |
|      |             |             |             |             |
| 1996 | 0.923       | -0.101      | 0.102       | 0.039       |
|      |             |             |             |             |
| 1997 | 0.014       | 0.201       | 0.055       | 0.588       |
|      |             |             |             |             |
| 1998 | 0.377       | 0.012       | -0.195      | 0.942       |
|      |             |             |             |             |
| 1999 | 0.920       | 0.142       | 0.277       | 0.041       |
|      |             |             |             |             |
| 2000 | 0.153       | -0.006      | 0.110       | 0.978       |
|      |             |             |             |             |
| 2001 | 0.016       | 0.196       | 0.017       | 0.094       |
|      |             |             |             |             |
| 2002 | 0.848       | -0.003      | -0.116      | 0.961       |
|      |             |             |             |             |
| 2003 | 0.007       | 0.131       | -0.019      | 0.054       |
|      |             |             |             |             |
| 2004 | 0.917       | 0.041       | 0.157       | 0.001       |
|      |             |             |             |             |
| 2005 | 0.871       | 0.103       | 0.189       | 0.002       |
|      |             |             |             |             |
| 2006 | 0.034       | 0.091       | -0.020      | 0.239       |
|      |             |             |             |             |
| 2007 | 0.058       | -0.010      | 0.030       | 0.603       |
| Year | Y | Y | Y | Y |
|------|---|---|---|---|
| 2008 | 0.605 | -0.031 | -0.665 | 0.536 |
| 2009 | 0.147 | -0.021 | -0.274 | 0.795 |
| 2010 | 0.418 | 0.088 | 6.459 | 0.001 |

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Table B.4 Summary of the regression analysis for the test for nonlinearity using in-period betas for the zero-beta version of the CAPM

| Year | $R^2$ | Estimate | Value | $t$-value | $p$-value |
|------|-------|----------|-------|-----------|-----------|
| 1965 | 0.953 | $\gamma_{0F1}$ | -0.050 | -2.436 | 0.248 |
|      |       | $\gamma_{1F1}$ | -0.423 | -3.857 | 0.162 |
|      |       | $\gamma_{2F1}$ | 0.330  | 4.301  | 0.145 |
| 1966 | 1.000 | $\gamma_{0F1}$ | 0.014  | 8.016  | 0.079 |
|      |       | $\gamma_{1F1}$ | 0.056  | 48.906 | 0.013 |
|      |       | $\gamma_{2F1}$ | 0.009  | 37.082 | 0.017 |
| 1967 | 1.000 | $\gamma_{0F1}$ | 0.012  | 0.697  | 0.613 |
|      |       | $\gamma_{1F1}$ | 0.054  | 0.325  | 0.800 |
|      |       | $\gamma_{2F1}$ | 0.035  | 0.330  | 0.797 |
| 1968 | 1.000 | $\gamma_{0F1}$ | 0.020  | 9.678  | 0.066 |
|      |       | $\gamma_{1F1}$ | -0.117 | -9.568 | 0.066 |
|      |       | $\gamma_{2F1}$ | 0.233  | 26.669 | 0.024 |
| 1969 | 0.999 | $\gamma_{0F1}$ | 1.111  | 3.852  | 0.162 |
|      |       | $\gamma_{1F1}$ | 106.478| 3.813  | 0.163 |
|      |       | $\gamma_{2F1}$ | -79.508| -3.817 | 0.163 |
| 1970 | 1.000 | $\gamma_{0F1}$ | -0.003 | -1.205 | 0.441 |
|      |       | $\gamma_{1F1}$ | 0.507  | 33.655 | 0.019 |
|      |       | $\gamma_{2F1}$ | -0.464 | -43.536| 0.015 |
| 1971 | 0.913 | $\gamma_{0F1}$ | -0.091 | -3.946 | 0.158 |
|      |       | $\gamma_{1F1}$ | -0.243 | -2.664 | 0.229 |
|      |       | $\gamma_{2F1}$ | 0.178  | 3.073  | 0.200 |
| 1972 | 0.999 | $\gamma_{0F1}$ | 0.041  | 7.277  | 0.087 |
|      |       | $\gamma_{1F1}$ | -0.534 | -6.681 | 0.095 |
|      |       | $\gamma_{2F1}$ | 0.482  | 9.996  | 0.063 |
| 1973 | 0.466 | $\gamma_{0F1}$ | -0.069 | -1.123 | 0.463 |
|      |       | $\gamma_{1F1}$ | -0.040 | -0.118 | 0.925 |
|      |       | $\gamma_{2F1}$ | 0.047  | 0.214  | 0.866 |
| Year | 1974 | 0.829 | $\gamma_0^Y$ | -0.227 | -3.974 | 0.157 | $\gamma_1^Y$ | 0.318 | 1.258 | 0.428 | $\gamma_2^Y$ | -0.145 | -0.754 | 0.589 |
|------|------|-------|------------|--------|--------|------|-----------|-------|--------|-------|-----------|--------|--------|------|
|      | 1975 | 0.695 | $\gamma_0^Y$ | -0.092 | -2.435 | 0.248 | $\gamma_1^Y$ | -0.074 | -0.406 | 0.755 | $\gamma_2^Y$ | 0.021  | 0.183  | 0.885 |
|      | 1976 | 0.978 | $\gamma_0^Y$ | -0.066 | -4.785 | 0.131 | $\gamma_1^Y$ | -0.093 | -4.167 | 0.150 | $\gamma_2^Y$ | 0.033  | 1.877  | 0.312 |
|      | 1977 | 0.966 | $\gamma_0^Y$ | -0.023 | -0.857 | 0.549 | $\gamma_1^Y$ | 0.221  | 1.549  | 0.365 | $\gamma_2^Y$ | -0.061 | -0.869 | 0.544 |
|      | 1978 | 0.992 | $\gamma_0^Y$ | -0.025 | -1.347 | 0.406 | $\gamma_1^Y$ | 0.585  | 8.861  | 0.072 | $\gamma_2^Y$ | -0.249 | -6.998 | 0.090 |
|      | 1979 | 0.873 | $\gamma_0^Y$ | -0.038 | -0.279 | 0.827 | $\gamma_1^Y$ | 0.469  | 0.631  | 0.642 | $\gamma_2^Y$ | -0.055 | -0.123 | 0.922 |
|      | 1980 | 0.944 | $\gamma_0^Y$ | -0.317 | -3.978 | 0.157 | $\gamma_1^Y$ | -0.030 | -0.077 | 0.951 | $\gamma_2^Y$ | 0.338  | 1.180  | 0.448 |
|      | 1981 | 0.734 | $\gamma_0^Y$ | -0.177 | -3.676 | 0.169 | $\gamma_1^Y$ | 0.542  | 1.582  | 0.359 | $\gamma_2^Y$ | -0.345 | -1.460 | 0.382 |
|      | 1982 | 0.579 | $\gamma_0^Y$ | 0.252  | 0.507  | 0.701 | $\gamma_1^Y$ | -0.438 | -0.244 | 0.848 | $\gamma_2^Y$ | 0.336  | 0.333  | 0.795 |
|      | 1983 | 0.716 | $\gamma_0^Y$ | -0.085 | -0.656 | 0.630 | $\gamma_1^Y$ | 0.027  | 0.587  | 0.662 | $\gamma_2^Y$ | -0.011 | -0.519 | 0.695 |
| Year | γ₀ | γ₁ | γ₂ |
|------|----|----|----|
| 1984 | 0.403 | -0.201 | -0.198 | 0.876 |
|      |     | 0.559 | 0.182 | 0.885 |
|      |     | -0.527 | -0.303 | 0.813 |
| 1985 | 0.482 | 0.313 | 0.816 | 0.564 |
|      |     | -0.300 | -0.671 | 0.624 |
|      |     | 0.046 | 0.544 | 0.683 |
| 1986 | 0.954 | 0.102 | 3.641 | 0.171 |
|      |     | 0.157 | 3.564 | 0.174 |
|      |     | -0.025 | -2.821 | 0.217 |
| 1987 | 0.675 | -0.061 | -0.456 | 0.728 |
|      |     | 1.006 | 0.775 | 0.580 |
|      |     | -0.789 | -0.889 | 0.537 |
| 1988 | 0.870 | -0.088 | -0.845 | 0.553 |
|      |     | 0.028 | 0.134 | 0.915 |
|      |     | 0.037 | 0.436 | 0.738 |
| 1989 | 0.844 | 0.087 | 1.431 | 0.388 |
|      |     | 0.039 | 0.832 | 0.558 |
|      |     | 0.044 | 1.124 | 0.463 |
| 1990 | 0.920 | -0.003 | -0.058 | 0.963 |
|      |     | -0.161 | -2.583 | 0.235 |
|      |     | 0.038 | 1.461 | 0.382 |
| 1991 | 0.999 | -0.029 | -7.206 | 0.088 |
|      |     | 0.309 | 16.981 | 0.037 |
|      |     | -0.141 | -10.963 | 0.058 |
| 1992 | 0.561 | -0.060 | -0.391 | 0.763 |
|      |     | -0.073 | -1.076 | 0.477 |
|      |     | 0.024 | 0.433 | 0.740 |
| 1993 | 0.964 | 0.322 | 5.404 | 0.116 |
|      |     | -2.069 | -5.148 | 0.122 |
|      |     | 1.366 | 4.997 | 0.126 |
| Year | Value | $\gamma_0$ | $\gamma_1$ | $\gamma_2$ |
|------|-------|-------------|-------------|-------------|
| 1994 | 0.905 | 0.489       | -0.642      | 0.067       |
|      |       |             | -0.650      | 0.339       |
|      |       |             |             | 0.792       |
| 1995 | 0.709 | -0.699      | 3.135       | -2.000      |
|      |       |             | 1.389       | -1.501      |
|      |       |             |             | 0.374       |
| 1996 | 0.938 | -0.082      | 0.087       | -0.013      |
|      |       |             | 2.149       | -0.476      |
|      |       |             |             | 0.717       |
| 1997 | 0.014 | 0.205       | 0.033       | 0.014       |
|      |       |             | 0.009       | 0.006       |
|      |       |             |             | 0.996       |
| 1998 | 0.635 | 0.361       | -1.194      | 0.622       |
|      |       |             | -0.992      | 0.841       |
|      |       |             |             | 0.555       |
| 1999 | 0.920 | 0.143       | 0.272       | 0.003       |
|      |       |             | 1.180       | 0.012       |
|      |       |             |             | 0.992       |
| 2000 | 0.470 | 0.265       | -0.779      | 0.520       |
|      |       |             | -0.668      | 0.775       |
|      |       |             |             | 0.580       |
| 2001 | 0.019 | 0.207       | 0.072       | -0.046      |
|      |       |             | 0.964       | -0.055      |
|      |       |             |             | 0.965       |
| 2002 | 0.997 | 0.067       | -0.189      | -0.043      |
|      |       |             | 4.764       | -7.289      |
|      |       |             |             | 0.132       |
|      |       |             | -15.708     | 0.040       |
|      |       |             |             | 0.087       |
| 2003 | 0.511 | 0.136       | -0.752      | 0.592       |
|      |       |             | 3.321       | 2.032       |
|      |       |             |             | 0.029       |
| Year | R_0 | R_1 | R_2 | R_3 |
|------|-----|-----|-----|-----|
| 2004 | 0.981 | 0.058 | 5.869 | 0.004 |
| 2005 | 0.894 | 0.102 | 5.808 | 0.004 |
| 2006 | 0.280 | 0.153 | 0.989 | 0.378 |
| 2007 | 0.556 | 0.020 | 0.227 | 0.832 |
| 2008 | 0.788 | 0.135 | 2.119 | 0.307 |
| 2009 | 0.500 | 0.023 | 0.491 | 0.649 |
| 2010 | 0.517 | 0.086 | 6.147 | 0.004 |

Source: 2011 Convention, Johannesburg, 8–9 November 2011
Table B.5 Summary of the regression analysis for the test of the explanatory power of the CAPM using prior betas for the standard version of the CAPM

| Year | $R^2$ | Estimate | Value | $t$-value | $p$-value |
|------|-------|----------|-------|-----------|-----------|
| 2007 | 0.339 | $\gamma_{q[1]}$ | 0.019 | 1.043 | 0.345 |
|      |       | $\gamma_{i[1]}$ | -0.057 | -1.601 | 0.170 |
| 2008 | 0.026 | $\gamma_{q[1]}$ | 0.013 | 0.369 | 0.715 |
|      |       | $\gamma_{i[1]}$ | -0.046 | -0.826 | 0.416 |
| 2009 | 0.208 | $\gamma_{q[1]}$ | -0.033 | -2.094 | 0.046 |
|      |       | $\gamma_{i[1]}$ | 0.088 | 2.610 | 0.015 |
| 2010 | 0.000 | $\gamma_{q[1]}$ | 0.023 | 1.766 | 0.089 |
|      |       | $\gamma_{i[1]}$ | 0.002 | 0.081 | 0.936 |

Table B.6 Summary of the regression analysis for the test for nonlinearity using prior betas for the standard version of the CAPM

| Year | $R^2$ | Estimate | Value | $t$-value | $p$-value |
|------|-------|----------|-------|-----------|-----------|
| 2007 | 0.402 | $\gamma_{q[1]}$ | 0.004 | 1.026 | 0.301 |
|      |       | $\gamma_{i[1]}$ | 0.051 | 3.030 | 0.006 |
|      |       | $\gamma_{2[i]}$ | -0.081 | -2.094 | 0.043 |
| 2008 | 0.227 | $\gamma_{q[1]}$ | 0.061 | 1.658 | 0.110 |
|      |       | $\gamma_{i[1]}$ | -0.573 | -2.692 | 0.012 |
|      |       | $\gamma_{2[i]}$ | 0.429 | 2.549 | 0.017 |
| 2009 | 0.329 | $\gamma_{q[1]}$ | -0.026 | -1.717 | 0.098 |
|      |       | $\gamma_{i[1]}$ | 0.250 | 3.030 | 0.006 |
|      |       | $\gamma_{2[i]}$ | -0.163 | -2.129 | 0.043 |
| 2010 | 0.024 | $\gamma_{q[1]}$ | 0.017 | 1.182 | 0.248 |
|      |       | $\gamma_{i[1]}$ | -0.049 | -0.693 | 0.495 |
|      |       | $\gamma_{2[i]}$ | 0.054 | 0.787 | 0.439 |
### Table B7: Summary of the regression analysis for the test of the explanatory power of the CAPM using in-period betas for the standard version of the CAPM

| Year | $R^2$ | Estimate Variable | Value | t-value | p-value |
|------|-------|-------------------|-------|---------|---------|
| 2003 | 0.007 | $\gamma_{0,1}^Y$ | 0.097 | 1.867   | 0.121   |
|      |       | $\gamma_{1,1}^Y$ | -0.019| -0.184  | 0.861   |
| 2004 | 0.918 | $\gamma_{0,1}^Y$ | 0.006 | 0.375   | 0.723   |
|      |       | $\gamma_{1,1}^Y$ | 0.157 | 7.488   | 0.001   |
| 2005 | 0.887 | $\gamma_{0,1}^Y$ | 0.070 | 4.228   | 0.008   |
|      |       | $\gamma_{1,1}^Y$ | 0.198 | 6.260   | 0.002   |
| 2006 | 0.034 | $\gamma_{0,1}^Y$ | 0.068 | 0.998   | 0.364   |
|      |       | $\gamma_{1,1}^Y$ | -0.020| -0.420  | 0.692   |
| 2007 | 0.053 | $\gamma_{0,1}^Y$ | -0.041| -1.476  | 0.200   |
|      |       | $\gamma_{1,1}^Y$ | 0.028 | 0.527   | 0.621   |
| 2008 | 0.606 | $\gamma_{0,1}^Y$ | -0.057| -1.242  | 0.269   |
|      |       | $\gamma_{1,1}^Y$ | -0.097| -2.771  | 0.039   |
| 2009 | 0.147 | $\gamma_{0,1}^Y$ | -0.037| -0.490  | 0.645   |
|      |       | $\gamma_{1,1}^Y$ | -0.067| -0.930  | 0.395   |
| 2010 | 0.418 | $\gamma_{0,1}^Y$ | 0.077 | 5.660   | 0.002   |
|      |       | $\gamma_{1,1}^Y$ | 0.043 | 1.896   | 0.117   |
Table B8 Summary of the regression analysis for the test for nonlinearity using in-period betas for the standard version of the CAPM

| Year | $R^2$ | Estimate | Value | $t$-value | $p$-value |
|------|-------|----------|-------|-----------|-----------|
| 2003 | 0.511 | $\gamma_{0[y]}$ | 0.099 | 2.438 | 0.071 |
|      |       | $\gamma_{1[y]}$ | -0.748 | -2.033 | 0.112 |
|      |       | $\gamma_{2[y]}$ | 0.589 | 2.032 | 0.112 |
| 2004 | 0.983 | $\gamma_{0[y]}$ | 0.024 | 2.484 | 0.068 |
|      |       | $\gamma_{1[y]}$ | -0.023 | -0.479 | 0.657 |
|      |       | $\gamma_{2[y]}$ | 0.134 | 3.867 | 0.018 |
| 2005 | 0.899 | $\gamma_{0[y]}$ | 0.072 | 4.056 | 0.015 |
|      |       | $\gamma_{1[y]}$ | 0.126 | 1.145 | 0.316 |
|      |       | $\gamma_{2[y]}$ | 0.066 | 0.691 | 0.528 |
| 2006 | 0.278 | $\gamma_{0[y]}$ | -0.003 | -0.032 | 0.976 |
|      |       | $\gamma_{1[y]}$ | 0.153 | 0.984 | 0.381 |
|      |       | $\gamma_{2[y]}$ | -0.055 | -1.164 | 0.309 |
| 2007 | 0.558 | $\gamma_{0[y]}$ | -0.072 | -2.801 | 0.049 |
|      |       | $\gamma_{1[y]}$ | -0.041 | -0.791 | 0.473 |
|      |       | $\gamma_{2[y]}$ | 0.136 | 2.140 | 0.099 |
| 2008 | 0.789 | $\gamma_{0[y]}$ | -0.005 | -0.115 | 0.914 |
|      |       | $\gamma_{1[y]}$ | -0.175 | -3.454 | 0.026 |
|      |       | $\gamma_{2[y]}$ | -0.045 | -1.868 | 0.135 |
| 2009 | 0.499 | $\gamma_{0[y]}$ | 0.008 | 0.119 | 0.911 |
|      |       | $\gamma_{1[y]}$ | 0.109 | 0.899 | 0.419 |
|      |       | $\gamma_{2[y]}$ | -0.116 | -1.677 | 0.169 |
| 2010 | 0.516 | $\gamma_{0[y]}$ | 0.075 | 5.399 | 0.006 |
|      |       | $\gamma_{1[y]}$ | -0.017 | -0.242 | 0.821 |
|      |       | $\gamma_{2[y]}$ | 0.062 | 0.901 | 0.418 |