About one method of modeling high-gradient temperature fields in the welding of shell structures made of carbon and high-alloy structural steels

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Abstract. A method is proposed for modeling high-gradient temperature fields in welding processes of shell structures made of carbon and high-alloy structural steels based on the graph-analytical method for studying the behavior of a function determined by the differential heat equation. The method consists in approximating a family of interpolating cubic splines constructed from temperature measurements at the reference points of the welded contour during the welding process. The analytical dependences of the distribution of temperature fields due to the design features of the welded joint and the welding process are obtained.

1. Introduction

Many works have been devoted to theoretical research and mathematical modeling of thermal processes adapted to the welding conditions of spatial structures [1, 2]. Here, the primary task is to set a temperature problem and obtain a solution that allows determining temperature fields and stresses in the zones of technological influence. Confirmation of the obtained solutions by experimental methods is quite difficult, since the welding process takes place at high temperatures. This requires the use of special measuring devices that allow temperature measurements at selected points, sufficient in number to process the measured results and obtain reliable data.

This article offers a method for modeling high-gradient temperature fields caused by welding of structural elements, based on the results of experimental data obtained. A partial description of the experiment, the results of which formed the basis of the proposed modeling method, is given in [3].

Let's briefly describe the method of conducting the experiment.

A study of thermal and deformation processes during welding of the "shell – plate" joint was carried out. This connection was conventionally considered as the connection of a cylinder with a flat shell. The material of the plate is low-carbon steel 20, the material of the shell (pipe) is high-alloy austenitic steel. The technological properties of these steels for weldability are good. Type of connection-angular, non-rotating, welding method-manual arc welding with coated electrodes.

Let's highlight some features of the welding process. Multi-pass welding is performed with the expansion of the weld. Forming the weld in this way allows you to completely cover the gaps and ensure the integrity of the structure. Seams are made manually with the breakdown of the joint contour into several sections.
The purpose of the experiments was to study the effect of heterogeneity of the geometric parameters of the welded joint elements on the thermally stressed state of the structure due to heterogeneous high-gradient local temperature influence in the zone of technological influence.

During the application of interlayering during the welding of elements, the welding modes were changed, i.e. welding was performed at "slow" and "fast currents". In reference points of the contour of the weld in the direction of arc movement after each welding pass, the temperature was measured by a contact thermometer TK-5 at certain intervals.

Fig. 1 and Fig. 2 shows the schemes of the interlayering overlay and one of the graphs of temperature changes along the height of the tube under the specified welding modes.

From the graph (Fig. 2), it is extremely difficult to judge the distribution of the temperature field over the height of the tube, since the dependence of temperature change is constructed primitively, i.e. in a linear setting. It should be noted that the temperature measurements were carried out in accordance with the developed methodology. The research was conducted, the results were obtained and analyzed.

2. Statement of the problem

The proposed modeling method is based on the study of the behavior of a function defined by a differential equation and set by certain points based on known results obtained during the measurement of parameters.

According to Poicare's reasoning [4], a complete study of any function consists of qualitative and quantitative parts of this process, which include:

- Geometric study of the curve defined by this function.
- Determination of the numerical values of the function under study.

The problem is adapted to the study of temperature fields caused by the welding process of structural elements, based on the obtained temperature measurements, taking into account the features of the welded structure and the ongoing physical processes.
The features of the physical welding process include:

- Highly concentrated local heating of the welded metal edges to the melting point, provided by a moving heat source.
- High-intensity heat removal to a cold mass of metal due to its thermal conductivity.
- The assumption that during manual welding, a mobile heat source moves along the metal at a high speed [5].

The features of a welded structure include a joint formed by the intersection of two shell structures with different wall thickness.

It is quite clear that it is not possible to provide uniform temperature measurement in the welded circuit during welding. In this case, the cooling process of the seam and the near-seam zone can be calculated with acceptable accuracy in engineering practice [5].

The study of the differential equation is based on an approximating solution obtained by graphoanalytic method in the form of constructing a cubic spline passing through specified points of a simplified function represented graphically by the results of the experiment.

3. The basic approach and the description of the method

Let’s introduce the following basic concepts, which we will use in the future.

A simplified function, represented graphically by the results of the experiment, we will call the witness function.

An auxiliary graphical function that interpolates the witness function we will call the interpolating function.

The analytical function obtained from the results of approximating the interpolating function we will call the approximating function.

Many works, for example [2, 5], are devoted to the research of heat propagation processes by an instantaneous precision source. They are based on the consideration of the differential equation of thermal conductivity. For example, for two-dimensional heat propagation in a body, it has the form [6]

\[
\frac{\partial^2 T(x, y, \tau)}{\partial x^2} + \frac{\partial^2 T(x, y, \tau)}{\partial y^2} - \frac{1}{a} \frac{\partial T(x, y, \tau)}{\partial \tau} = 0 .
\]  
(1)

Here $T$ is temperature; $x, y$ are coordinates of points of the body; $a$ is coefficient of thermal conductivity; $\tau$ is time.

For simple problems, the analytical solutions obtained are well known. For more complex problems, numerical methods and algorithms for solving equation (1) have been developed.

Taking into account the above features of the process under consideration, we will search for an approximate solution of equation (1) by approximating an interpolating cubic spline with given boundary conditions, following the work [7].

The theoretical aspect of the method used is as follows. Let the cubic spline $S(x)$ be given on the grid $\Delta$: $a = x_1 < \ldots < x_n = b$. And

\[
S(x_i) = y_i, \quad x_i = (b-a) i / n, \quad i = 0, 1, \ldots, n.
\]  
(2)

Let's assume that

\[
S(x_{1/2}) = y_{1/2}; \quad S(x_{n-1/2}) = y_{n-1/2},
\]  
(3)

where

\[
x_{1/2} = (b-a) / 2n; \quad x_{n-1/2} = (b-a)(2n-1) / 2n .
\]  
(4)

The approximate solution of the boundary value problem is determined by the relations [7]
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\[ y'' + p(x) y' + q(y) S(y) = r(x); \quad y(a) = y(b) = 0. \quad (5) \]

Boundary and additional conditions have the following form

\[ S(a) = y_0 = S(b) = y_n = 0; \quad S''(y) + pS'(y) + q(y) S(y) = r(y), \quad (6) \]

where \( y = x_{1/2}, x_1, \ldots, x_{n-1}, x_{n-1/2} \).

Thus, we consider a system of \( n + 1 \) equations with \( n + 1 \) unknowns \( y_{1/2}, y_1, y_2, \ldots, y_{n-1}, y_{n-1/2} \).

However, this method allows you to get the desired result if the grid points are evenly distributed. If the intervals of measurement points are large enough, and the coordinates of these points cannot coincide with the nodes of the selected grid, then it is difficult to choose the boundary conditions.

To study the differential equation (1), consider the inverse problem by drawing an interpolating function from the specified points of the witness function. As an interpolating function, we take a cubic spline, which we take as a family of splines

\[ S = \{ S_0, S_1, \ldots, S_i \}; \quad i = 0,1, \ldots, n. \quad (7) \]

Some difficulties in drawing a family of curves from points can be eliminated by replacing the coordinates of the points of the witness function.

The family of cubic splines belongs to the class of natural cubic splines. Thus, each cubic spline from this family will be a polynomial of the third degree. In our case, this polynomial is the approximating function. The analytical formula of the polynomial has the form

\[ y = ax^3 + bx^2 + cx + d. \quad (8) \]

The first and second derivatives are defined by the equations

\[ y' = 3ax^2 + 2bx + c; \quad y'' = 6ax + 2b. \quad (9) \]

Here \( a, b, c, d \) are the coefficients of the polynomial.

To determine the coefficients of the desired polynomial, you must set at least four conditions.

The interpolating spline is a continuous function and on the segment \([x_0, x_n]\) has continuous first and second derivatives and satisfies the condition

\[ S(x_i) = y_i = f(x_i); \quad i = 0,1, \ldots, n. \quad (10) \]

The other three conditions are determined graphically using the interpolating cubic spline curve using CAD software.

The calculations will be performed in the \( XOY \) coordinate plane, using the coordinates of points in the accepted scale, Fig. 3, a. The Characteristic points of the witness function are points \( A(0; 35.2), B(20; 64.6), C(80; 3.2) \).

Point \( B \) (Fig. 2) divides the graph of the witness function into two lines, one of which characterizes its increasing \( (AB) \), the second-decreasing \( (BC) \). The interpolating spline function undergoes an extremum in the vicinity of a point \( B \), the coordinates of which are defined in a standard way.

Finally, write down all the conditions of the cubic spline family

\[ S_i(x_i) = y_i = a_i x_i^3 + b_i x_i^2 + c_i x_i + d_i; \quad (11) \]

\[ S'_i(x_i) = y'_i = 3a_i x_i^2 + 2b_i x_i + c_i = k_{1i}; \quad k_{1i} = \frac{y_i}{y_{i+1}}. \quad (12) \]

\[ S''_i(x_i) = y''_i = 6a_i x_i + 2b_i = k_{2i}. \quad (13) \]

Here \( \alpha_i \) is the angle of the tangent to the curve at the point \( x_i \) under consideration (given graphically).
We find additional conditions from the boundary conditions, when the relation is valid at points \( x_i \)

\[ S_i(y_i) = S_{i-1}(y_{i-1}). \]  

(14)

Then we take the coefficients of the polynomial \( a_i \) and \( b_i \) as

\[ a_i = a_{i-1} ; \quad b_i = b_{i-1}. \]  

(15)

The coefficients \( c_i \) and \( d_i \) will be determined from conditions (12) and (11).

We show the implementation of this approach by the example of calculating the coefficients of the desired polynomials approximated by the spline functions \( S_1(y_{M1}) \) and \( S_2(y_{M2}) \) Fig. 3, b. the calculation data are summarized in table 1.

![Figure 3](image)

**Figure 3.** Graphs: a is graphical representation of the interpolating function and the witness function; b is calculation of the coefficients of the approximating function

From condition (14), each subsequent interpolating spline is defined by the relations

\[ S_0 = f(y_0), \quad y_0 = f(x_0); \]
\[ S_1 = f(\Delta y_1), \quad \Delta y_1 = y_1 - y_0; \quad \Delta y_1 = f(x_1 - x_0); \]
\[ S_1 = f(\Delta y_1), \quad \Delta y_1 = y_1 - y_{i-1}; \quad \Delta y_1 = f(x_i - x_{i-1}). \]  

(16)

To simplify calculations we accept coefficients

\[ a_0 = b_0 = 1. \]  

(17)

| \( S_i \) | \( \Delta y_i \) | \( \Delta x_i \) | \( k_{i1} \) | \( a \) | \( b \) | \( c \) | \( d \) |
|---|---|---|---|---|---|---|---|
| \( S_0 \) | 35,2 | 0 | 1,47 | 1 | 1 | 1,47 | 35,2 |
| \( S_1 \) | 3,45 | 1,78 | 1,928 | 1 | 1 | -11,1372 | 6,507 |
| \( S_2 \) | 19,0 | 11,14 | 1,331 | 1 | 1 | -393,25 | 2823,24 |

Table 1. Calculated data.

Approximating functions have the following form

\[ S_0 = \Delta y_0 = x^3 + x^2 + 1,47x + 35,2 = 35,2; \]
\[ S_1 = \Delta y_1 = x^3 + x^2 - 11,14x + 6,507 = 3,45; \]
\[ S_2 = \Delta y_2 = x^3 + x^2 - 393,25x + 2823,24 = 19,0 \]

4. **Discussion of results**

The resulting family of polynomials is approximated on a family of interpolation splines with given conditions (11)-(15) at a given scale and accepted coefficients (16).
The convergence of the results is provided by the geometric meaning of the first derivative, namely, the specific value of the tangent of the angle of inclination of the tangent to the curve at the point under consideration (12).

If we accept other values of coefficients (16), we get a different family of polynomials. This raises the question of the uniqueness of the solution of the problem under consideration. However, this circumstance can be circumvented by formalizing the features of the processes under consideration.

Changing the scale does not affect the final results, because the final results are parameters of the process, and it is the essence of replacing the selected coordinate system with the original one.

5. Conclusions
The proposed approach and method allows us to obtain an analytical formula for a family of approximating functions and to simulate the process of forming temperature fields caused by metal welding with sufficient accuracy in engineering practice. The obtained formulas allow calculating a fairly wide range of dynamic tasks adapted to the technological processes of manufacturing complex structures using computers.

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