A Case Study on Optimal Allocation of Call Center Staff — With Un-used Phones and Repeated Calls Situation Considered —

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Abstract: In this paper, we consider the customer call center operation which is one of the personal computer end-user services. We deal with the problem as a case study where the optimal allocation of the call center staff which is called agents is determined. This previous proposed model was to provide the optimal agents’ allocation for the call center service based on queuing (Erlang loss formula) [18, 19] and linear programming theory [20]. Here, besides it, we’re going to address or examine the mode, considering the queuing space of not-using phones or the phones that no agents are using and the repeated calls that the customers would call when they couldn’t reach any agents directly or immediately, and they couldn’t even get into the queuing space of not-using phones. Then we show the effectiveness or the extent of the effectiveness of the model.

Key Words: Call center, Poisson distribution, Erlang distribution, Erlang loss formula, Linear programming

1. Introduction

In one of the Outsourcing Businesses of IT industry, there is the call center service for the end-users of the customers. It provides the customers with a single point of contact for the solution support of various IT problems, especially of the most familiar product of PCs’. You could say that the call center is also the first contact point for the customers; it is, for that reason, one of the most important services to establish or enhance the product brand and to improve the customer satisfaction.

In this study, we’re considering the model with the un-used phones and the repeated calls based on the previous model, where the optimal agents are fixed according to the arrival rate each hour.

There have been many other studies on the agents’ scheduling problem of the call center so far. As some of the latest papers on it, they addressed the enhanced model itself [1, 2, 4, 6, 7-9, 13], the new solution approaches [3, 5, 11, 12, 14, 15], and then tried practically to make the call center more effective [10, 17], so on. In those papers, they considered the agents’ cyclic work shift and their detailed skills levels, or they basically touched on the agents’ work-force allocation from the points of a broad scale and long time range or long term. It is often difficult to estimate the arrival calls for a long time span.

While we take it into account that the arrival calls are fluctuating and depend on the types of the PCs that the customers use or the timing of the software up-grades and so on. So we focus on doing the arrival calls’ forecast for quite a short time period or the unit time of one hour at each service day. Then, we considered making a model of the agents’ scheduling dynamically and flexibly based on it [21]. We’re now going over or exploring the above situation, plus the un-used phones available (as the waiting space) and repeated calls considered. And we’re going to get the result that we’d find the previous model suited or applicable enough, in the actual call center situation except such special or typical cases as system programs’ changes or updates, etc.

2. Arrival Rate

The table 1 shows the average call arrival rate each hour over actual 84 months’ period. We’re going to focus on addressing the most frequent call arrival time of 10 a.m.-11 a.m. to explain the core of this study. And we show that the actual call arrival number’s frequency and its theoretical Poisson arrival in the figure 1.

The expected values in the figure 1 are the theoretical values of Poisson arrival based on the actual arrival calls. We used the Chi-squared test to show how well the Poisson arrival fits for the actual arrival rate in the table 2. We got the result, good enough within the rejection region of 5% and its average of 14.40, standard deviation of 3.07. And the actual maximum call arrival number is 25.38 and its arrival possibility is 0.33%, theoretically. And 25.38 is also the highest call arrival number over 84 months’ data-tracking period of all the service time of 8 a.m. to 20 p.m.

3. Service Rate

In the same manner as arrival rate, we got the service time distribution as we measured the actual service time over the past
84 months’ period, and below was depicted the figure 2 with the vertical line of Frequency to the service time range of every 50 second. The actual average service time is 428.57 seconds. While the theoretical expected service time is expressed as Erlang distribution, and its fitness is shown in the table 3 with Chi-squared test.

You would find the fitness quite good from the table 3. Here, we assume that the service time doesn’t depend on the time itself or each time. Now we’re going to address or look into the call center model with the repeated calls and the waiting space involved, based on these arrival rate and service time.

4. State Transition Model

We’re going to set the probability variable of \( i, j, \) and \( l \) as the number of agents working each hour, the number of users in the waiting space, and the number of users trying to re-call for the services, respectively, and to define the notation or the probability as \( (i,j,l) \) and \( P_{(i,j,l)} \). Also, we set the number of the physical existing phones available as \( M \). Here, we’re going to address or think about the probability of the steady-state system where one situation is moving to another during a very tiny time period of \( \Delta t \) or the state transition probability. Whenever the changes occur, we have such situations as a call arrival, a service completion, a call dropping-away from the waiting space, or a repeated call occurring (including a giving-up repeated call).

Here, the probability of one call’s arrival in tiny bits of time \( \Delta t \) is always, regardless of the system situation, \( \lambda \Delta t + o(\Delta t) \), according to the Poisson arrival assumption. We’re going to discuss, using the actual service time as a service time parameter \( \mu \), which we found as Erlang distribution. The probability that the number of \( i \) agents \((i \leq 1)\) are servicing in tiny bits of time is \( i\mu\Delta t + o(\Delta t) \). And we also assume that the abandon rate (or the dropping-away rate) is \( \beta\Delta t + o(\Delta t) \), where \( \beta \) is the dropping-away rate in the waiting space, and the probability of the repeated calls in the tiny bits of time similarly is \( f\Delta t + o(\Delta t) \), where \( f(l \leq 1) \) is the number of users who would be going to re-call for the services and \( \gamma \) is the interval time of recalling, that is the time until when the users could wait and try to re-call for the services. And the users have the options that they re-call or not, with the probability of \( \alpha \) for re-calling and \((1 - \alpha)\) for NOT re-calling. And \( o(\Delta t) \) means that \( \lim o(\Delta t)/\Delta t = 0 \)(lim : as \( \Delta t \) goes to 0).

5. State Transition Probability

5.1 State Transition

In the call center operation, we have been taking the approach that a minimum number of agents \((i = c)\) is assigned or fixed each hour according to each hour’s call arrival rate. From here, we’re going to take more realistic situation where the minimal number of agents \((i = c)\) is already set, there is the number of unused phones available \((M - c)\) as the waiting space and repeated calls are considered. Here, \( M \) is the total number of phones existed. The table 4 shows each state transition status according to the state transition diagram (the figure 4). And potential re-calling users \((l \leq 1)\) are trying to re-call with the
probability of $l_y$, but there are no agents who can take the calls and provide the services. Therefore, they cannot receive the services, or they are back to the current state of $(c, M - c, l)$. $l_y$ does not give any changes to the state of $(c, M - c, l)$ so that we exclude it in the table 4. The figure 3 shows this state transition. We’re thinking about the system or the situation where the number of agents($i = c$) is already set or fixed each hour, and up to the number of calls of $c$, that is, the agents assigned ($i = c$) can take the calls of up to calls of $c$ and give the services immediately. This means that the row of ($i - 3$)in the figure 4, for example, has no waiting queues or no users in the waiting space (the horizontal cells of $j$). Once agents ($i = c$) can not take care of calls ($i > c$), the state transition moves to the horizontal columns of the waiting space. Therefore, we’ll address or solve the state transition equation, only focusing on the horizontal cells of $(i, j, l)$ in the figure 4. In the notation of $(i, j, l)$ of the figure 4, if $l = 0$, then we simply set $(i, j)$ and the figure 4 specifically shows the case of $i = 5$.

![Fig. 4 State Transition](image)

5.2 State Equation

We could generally notify the state transition as the following equation.

When

$$
1 \leq i \leq c \\
0 \leq j \leq (M - i) \\
0 \leq l \leq k,
$$

$\lambda l_0 = \mu \Pi_{1,0}$

$\lambda (\mu)\Pi_{1,0} = \lambda \Pi_{0,0} + 2\mu \Pi_{2,0}$
(\lambda + 5\mu + \beta)\Pi_{5,1} = \lambda \Pi_{5,0} + (5\mu + 2\beta)\Pi_{5,2}

(\lambda + 5\mu + 2\beta)\Pi_{5,2} = \lambda \Pi_{5,1} + (5\mu + 3\beta)\Pi_{5,3}

(\lambda + 5\mu + 3\beta)\Pi_{5,3} = \lambda \Pi_{5,2} + (5\mu + 4\beta)\Pi_{5,4}

(\lambda + 5\mu + 4\beta)\Pi_{5,4} = \lambda \Pi_{5,3} + (5\mu + 5\beta)\Pi_{5,5}

(\lambda + 5\mu + 5\beta)\Pi_{5,5} = \lambda \Pi_{5,4} + \gamma(1 - \alpha)\Pi_{5,5,1}

(\lambda + 2\gamma(1 - \alpha))\Pi_{5,5,2} = \lambda \alpha \Pi_{5,5,1} + 3\gamma(1 - \alpha)\Pi_{5,5,2}

(\lambda + 2\gamma(1 - \alpha))\Pi_{5,5,3} = \lambda \alpha \Pi_{5,5,2} + 3(1 - \alpha)\Pi_{5,5,3}

(\lambda + 3\gamma(1 - \alpha))\Pi_{5,5,4} = \lambda \alpha \Pi_{5,5,3} + 5\gamma(1 - \alpha)\Pi_{5,5,4}

5\gamma(1 - \alpha)\Pi_{5,5,5} = \lambda \alpha \Pi_{5,5,4}

Focusing on the equations of \(\square\) and \(\text{underlined}\), we’re starting to solve sequentially from the equation of \(\square\) in the descending order. Then, we got the following simpler forms of equations.

\(\mu \Pi_{1,0} = \lambda \Pi_{0,0}\)

\(2\mu \Pi_{2,0} = \lambda \Pi_{1,0}\)

\(3\mu \Pi_{3,0} = \lambda \Pi_{2,0}\)

\(4\mu \Pi_{4,0} = \lambda \Pi_{3,0}\)

\(5\mu \Pi_{5,0} = \lambda \Pi_{4,0}\)

\((5\mu + \beta)\Pi_{5,1} = \lambda \Pi_{5,0}\)

\((5\mu + 2\beta)\Pi_{5,2} = \lambda \Pi_{5,1}\)

\((5\mu + 3\beta)\Pi_{5,3} = \lambda \Pi_{5,2}\)

\((5\mu + 4\beta)\Pi_{5,4} = \lambda \Pi_{5,3}\)

\((5\mu + 5\beta)\Pi_{5,5} = \lambda \Pi_{5,4}\)

\(\gamma(1 - \alpha)\Pi_{5,5,1} = \lambda \alpha \Pi_{5,5,1}\)

\(2\gamma(1 - \alpha)\Pi_{5,5,2} = \lambda \alpha \Pi_{5,5,1}\)

\(3\gamma(1 - \alpha)\Pi_{5,5,3} = \lambda \alpha \Pi_{5,5,2}\)

\(4\gamma(1 - \alpha)\Pi_{5,5,4} = \lambda \alpha \Pi_{5,5,3}\)

\(5\gamma(1 - \alpha)\Pi_{5,5,5} = \lambda \alpha \Pi_{5,5,4}\)

These 15 equations are showing the relationships of a neighboring variable of \(\Pi_{i,j}\). Hence, with these equations and the total sum of each state transition probability to 1, we can notify each state transition probability as the form of the probability of \(\Pi_{0,0}\).

### Table 5 Solution of Steady-state transition equation

| State Transition | Probability | User in Queue |
|------------------|-------------|---------------|
| \(\Pi_{0,0}\)    | 17.93%      | 0.000         |
| \(\Pi_{1,0}\)    | 30.73%      | 0.000         |
| \(\Pi_{2,0}\)    | 26.34%      | 0.000         |
| \(\Pi_{3,0}\)    | 13.05%      | 0.000         |
| \(\Pi_{4,0}\)    | 6.45%       | 0.000         |
| \(\Pi_{5,0}\)    | 2.21%       | 0.000         |
| \(\Pi_{5,1}\)    | 0.75%       | 0.007         |
| \(\Pi_{5,2}\)    | 0.25%       | 0.005         |
| \(\Pi_{5,3}\)    | 0.08%       | 0.002         |
| \(\Pi_{5,4}\)    | 0.03%       | 0.001         |
| \(\Pi_{5,5}\)    | 0.01%       | 0.000         |
| \(\Pi_{5,5,1}\)  | 0.04%       | 0.002         |
| \(\Pi_{5,5,2}\)  | 0.05%       | 0.003         |
| \(\Pi_{5,5,3}\)  | 0.04%       | 0.003         |
| \(\Pi_{5,5,4}\)  | 0.02%       | 0.002         |
| \(\Pi_{5,5,5}\)  | 0.01%       | 0.001         |
| Total            | 1           | 0.029         |

### Fig. 5 State Transition Probability (\(\lambda = 14.40\))

6. Steady-state equation and its solution

The table 5 shows the result that we’ve solved the steady-state equation of the specific case of 5.3 to get each state transition probability. Similarly, we could get the probability of the general steady-state equation of 5.2, given the parameters fixed or set. The parameters of the case of 5.3 are shown in the table 5. The figure 5 shows the probability of each state transition. The agents of 5 could take care of 98.72% calls. Then, they could take care of 99.84% calls with the waiting space provided. In the total system, there are 0.029 users waiting, with the waiting space and re-calling users included.

In the same way, at \(\lambda\) of 20.57 which is the point of the right hand of \(+2\sigma\) from the center of arrival rate of 14.40, we get the solution of the steady-state equation in the table 6 and their probabilities’ status in the figure 6. The agents of 5 could take care of 87.99% calls. Furthermore, they could take care of 93.17% calls with the waiting space. And there are 0.0637 users waiting totally. We would say that the agents of 5 could do the services with almost no impact to the services or users.

From the figure 6, we guess that we would have more impacts to the system or the services if the arrival rate is increas-
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Table 6 Solution of Steady-state transition equation $(\lambda = 20.57)$

| State Transition | Probability | User in Queue |
|------------------|-------------|---------------|
| $\pi_{0;0}$      | 7.91%       | 0.000         |
| $\pi_{0;1}$      | 19.36%      | 0.000         |
| $\pi_{0;2}$      | 23.71%      | 0.000         |
| $\pi_{0;3}$      | 19.35%      | 0.000         |
| $\pi_{0;4}$      | 11.85%      | 0.000         |

Table 7 Solution of Steady-state transition equation $(\lambda = 25.38)$

| State Transition | Probability | User in Queue |
|------------------|-------------|---------------|
| $\pi_{0;0}$      | 2.83%       | 0.000         |
| $\pi_{1;0}$      | 8.54%       | 0.000         |
| $\pi_{1;1}$      | 12.90%      | 0.000         |
| $\pi_{1;2}$      | 12.99%      | 0.000         |
| $\pi_{1;3}$      | 9.81%       | 0.000         |

Fig. 6 State Transition Probability $(\lambda = 20.57)$

Fig. 7 State Transition Probability $(\lambda = 25.38)$

Fig. 8 State Transition Probability $(\lambda = 25.38, Agents of 7)$

...ing with the current service situation of 5 agents and service rate of 8.40 calls/hour. The theoretical probability of the arrival rate of 25.38 is 0.33%. But, once the agents of 5 would get this arrival rate of 25.38, the re-calling status would suddenly increase, as shown in the table 7 or the figure 7. In the month of having the arrival rate of 25.38, we had actually known the fact or the event hat we had the system program’s update. And we made ready for it and assigned 7 agents to do the services well enough, as shown in the table 8 or the figure 8.

Here, the table 9 shows the number of waiting users according to the arrival rate $\lambda$ with the agents of 5. Supposed the abandon rate is less that or equal to 5%, we would do the services well enough with the arrival rate of 17.5 (waiting users; 0.145).

From the standpoint of the daily abandon rate in the previous study, we could do the services with the service rate of 20.56 (waiting users; 0.637).

We have been doing the agent scheduling, based on the past call arrival rate and any IT changes, such as software changes or updates. We would say from this study that we could provide the services with our previous proposed model if we have the call arrival rate range of up to +2σ from the average call arrival rate of 10 to 11 a.m. in the table 1.

If we have an arrival rate increasing by the impact of re-calling in a unit hour, we would have the higher arrival rate in the next hour. But, take, for example, the arrival rate of 11 a.m. ~ 12 noon in the table 1, we have lower arrival rate of
than the arrival rate of 10 a.m. — 11 a.m., or 14.40. Moreover, the table 10 shows the arrival rate each hour of the month in which we had the highest arrival rate of 25.38 during 10 — 11 a.m. In the next service hour of 11 a.m. — 12 noon, we had the lower arrival rate of 20.38, which is not affected, impacted, or not being increased. Therefore, we think that it is meaningful or effective enough to use our optimal agents’ scheduling with the abandon rate. Here, the table 11 shows the minimal number of agents each hour, and the table 12 shows the optimal agents’ scheduling using Linear Programming based on the minimal agents each hour as constraints.

### 7. Conclusion

In the previously proposed paper on the optimal agents’ scheduling for the call center, we got the solution by solving the Linear Programming based on the minimal number of agents each hour, meeting the service level (of the abandon rate) as constraints. But in this paper, we further studied, or made the model much closer to the real call center situation, being focusing on the users who can not receive the services at their first call because of all agents handling the calls. And some of them might be going to wait for the services available in the waiting space or others might be going to re-call for the services.

In this situation, we went over or explored the extent to the system which or how much the re-calling calls affect, according to such assumptions as the theoretical calls’ arrival distribution based on the actual calls’ arrival rate and the theoretical service distribution based on the actual service time, and there are un-used phones available as the waiting space, from which the automatic guiding message of asking to wait is running. There are also another automatic guideline message when the waiting space is full, saying that please try to call later again. These messages actually work for not re-calling or mitigating re-calling immediately.

As a result of this model with 5 agents and the waiting space for up to 5 users available, our call center could provide the services with 93.22% calls’ handling coverage against the potential arrival calls’ moving range of $+2\sigma$ (standard deviation) at the center of 14.40, the average calls arrival rate. And we believe that our previous proposed optimal scheduling model is enough good to use.

### References

1. E. Chassioti and D. J. Worthington. 2004. A New Model for Call Centre Queue Management, The Journal of the Operational Research Society, Vol.55, No.12, pp.1352-1357.

2. Thomas, R. Robbins and Terry, P. Harrison. 2010. A Stochastic Programming Model for Scheduling Call Centers with Global Service Level Agreements, Vol.207, No.3, pp.1608-1619.
[3] Linda, V. Green Peter J. Kolesar and João, oares. 2003. An Improved Heuristic for Staffing Telephone Centers with Limited Operating Hours, Production and Operations Management, Vol.12, No.1, pp.46-61.

[4] Tolga Cezik, Oktay Gunluk, and Hanan, Luss. 2001. An Integer Programming Model for the Weekly Tour Scheduling Problem, Naval Research Logistics, Vol.48, No.7.

[5] Jólius, Atlason and Marina, A. Epelman. 2004. Call Center Staffing with Simulation and Cutting Plane Methods, Annals of Operations Research, Vol.127, No. 1, pp.333-358.

[6] Athanassions, N. Avramidis, Alexandre Deslauriers, and Pierre, L’ecuyer. 2004. Moldeing Daily Arrivals to a Telephone Call Center, Management Science, Vol.50, No.7, pp.896-908.

[7] Michael, J. Brusco and Larry, W. Jacobs. 2000. Optimal Models for Meal-Break and Start-Time Flexibility in Continuous Tour Scheduling, Management Science, Vol.46, No.12, pp.1630-1641.

[8] Ger, Koole and Erik, van der Sluis 2003. Optimal Shift Scheduling with a Global Service Level constraint, IIE Transactions, Vol.35, No.11.

[9] Athanassions, N. Avramidis, Wyean Chan, Michel, Gendreau, Pierre L’ecuyer, and Omella, Pisacane. 2010. Optimizing Daily Agent Scheduling in a Multiskill Call Center, European Journal of Operational Research, Vol.200, No.3, pp.822-832.

[10] Dennis, C. Dietz. 2011. Practical Scheduling for Call Center Operations, Omega, Vol.39, No.5, pp.550-557.

[11] Alex, Fukunaga, Ed, Hamilton, Jason, Fama., David, Andre. Ofer, Matan and Illah, Nourbakhsh. 2002. Staff Scheduling in Inbound Call Centers and Customer Contact Centers, AI Magazine, Vol.23, No.4, pp.30-40

[12] Lawrence, Brown. Noah. Gans, Avishai, Mandelbaum., Anat, Sakov. Haipeng, Shen. Sergey, Zeltyn. and Linda, Zhao. 2005. Statistical Analysis of a Telephone Call Center: A Queuing-Science Perspective, Journal of the American Statistical Association, Vol.100, No.469, pp.36-50

[13] Ignacio, Castillo. Tarja, Joro. and Yong, Yue Li. 2009. Workforce Scheduling with Multiple Objectives, European Journal of Operational Research, Vol.196, No.1, pp.162-170

[14] Vijay, Mehtrota and Jason, Fama. Fama. Call Center Simulation Modeling: Methods, Challenges, and Opportunities, Proceedings of the 2003 Winter Simulation Conference.

[15] Jólius Atlason. Marina A. Epelman, and Shane, G. Henderson. 2008. Optimizing Call Center Staffing using Simulation and Analytic Center Cutting-Plane Methods, Management Science, Vol.54, No.2, pp.295-309.

[16] Sandjai, Bhalai. Ger. Koole, and Auko, Pot. Simple Methods for Shift Scheduling in Multiskill Call Centers, Manufacturing & Service Operations Management, Vol.10, No.3, pp.411-420.

[17] Hidenori Morimura, Yoshitsugu Ohmae: Applied Queueing Theory,NITSUKAGIREN,ISBN4-8171-5313-X(1996).

[18] Masao Mori, Tomomi Matsui: Operations Research, ASAKURASHOTEN, ISBN4-254-27538-2(2004).

[19] Akio Koyama: Linear Programming Introduction, Japan Economy News Paper Publication ,ISBN4-532-0107.

[20] Masahiko Tanaka, Bong-Sung Chu, Shuhei Inada, and Hiroaki Matsukawa: Case Study on Optimal Allocation of Call Center Staff for IT Equipment Products, SRERM20.