Optimization of the interplanetary flight to Mars with three-pulse approach to Phobos based on Lagrange principle

A.S. Samokhin\textsuperscript{1} and M.A. Samokhina\textsuperscript{2}

\textsuperscript{1} Assistant, Department of Mechanics and Mathematics, MSU, Moscow, Russia; researcher, V.A. Trapeznikov Institute of Control Sciences of RAS, Moscow, Russia

\textsuperscript{2} Researcher, V.A. Trapeznikov Institute of Control Sciences of RAS, Moscow, Russia

E-mail: samokhin@ipu.ru

Abstract. The problem of optimal control of the spacecraft interplanetary flight from the Earth to Mars with a three-pulse approach to the Phobos tracking orbit is considered. The departure of the spacecraft from the Earth is also approximated by pulses. The attraction of the Earth and Mars is taken into account. The problem is solved numerically on the basis of Lagrange principle. The results are compared with the scheme of a full turn free approach to Phobos. The initial approximation is carried out based on a combination of Lambert problems solutions. The software package is developed. Ephemerides are taken into account in the calculations, and the NASA SPICE package is used.

1. Introduction

The paper considers one of the actual problems of optimal control of the spacecraft (SC) interplanetary flight to Phobos—the natural satellite of Mars [1]. A lot of studies are devoted to this topic (see [2] and its bibliography). Many scientists consider that Phobos may contain the relict matter closest to the Earth. There is also currently a project to create an inhabited base on Phobos.

This paper deals only with the trajectory part of the mission. At present there is no complete mathematical study of the problem with construction of an end-to-end optimal extremum, while the Russian Federation plans to carry out a mission to Phobos in order to collect soil samples in the near future.

The problem of optimizing the interplanetary spacecraft flight from the Earth to Phobos in pulse statement is considered. On the one hand, such research is necessary for solving problems in more complex continuous formulations, for example, on the basis of the L.S. Pontryagin maximum principle as a good initial approximation. On the other hand, such research is of independent interest. In this paper, the flight trajectory is constructed on the basis of the Lagrange principle.

The initial approximation for solving the boundary-value problem is taken from considering the trajectory as a combination of Lambert problems solutions. The boundary Lagrange and Lambert problems are solved numerically on a computer, taking the ephemeris into account. The combination of Lambert problems is optimized by external gradient methods.

The paper is devoted to a description of the mathematical formulation of the problem, the methods used to solve the problem numerically, the results obtained by numerical modeling. The results of the
flight along the trajectory with a three-pulse approach to Phobos and the flight with a direct one pulse approach scheme are compared.

2. The problem statement

The expedition scheme is shown in Fig. 1. At the initial moment at time $t_0$ the spacecraft is on a circular orbit of the artificial Earth satellite (AESO) at an altitude of 200 km and inclination 51.6° to the equator, corresponding to the spacecraft launching from Baikonur. At the final time $t_3$ the spacecraft is approaching to the orbit of the artificial Mars satellite (AMSO), the circular Phobos tracking orbit, corresponding to its mean orbit from the MAR097 ephemeris, and the angular positions of the spacecraft and Phobos on the final orbit may be different.

In the paper, the spacecraft and Phobos are non-attractive mass points, since their masses are small. Gravitational fields of the Sun, the Earth, and Mars are considered central Newtonian ones and are taken into account during the entire flight. Positions of the Earth’s and Mars’ centers of mass correspond to the DE424 ephemeris. The launch date $t_0$ is chosen from 2020 to 2030, the total duration of the mission $t_3-t_0$ is limited to 1300 days. Due to the effect of accuracy loss, the spacecraft motion is described in several coordinate systems.

The following scheme, shown in Fig. 2, is used to reach the AMSO.

In the paper, the spacecraft and Phobos are non-attractive mass points, since their masses are small. Gravitational fields of the Sun, the Earth, and Mars are considered central Newtonian ones and are taken into account during the entire flight. Positions of the Earth’s and Mars’ centers of mass correspond to the DE424 ephemeris. The launch date $t_0$ is chosen from 2020 to 2030, the total duration of the mission $t_3-t_0$ is limited to 1300 days. Due to the effect of accuracy loss, the spacecraft motion is described in several coordinate systems.

The following scheme, shown in Fig. 2, is used to reach the AMSO.

In the paper, the spacecraft and Phobos are non-attractive mass points, since their masses are small. Gravitational fields of the Sun, the Earth, and Mars are considered central Newtonian ones and are taken into account during the entire flight. Positions of the Earth’s and Mars’ centers of mass correspond to the DE424 ephemeris. The launch date $t_0$ is chosen from 2020 to 2030, the total duration of the mission $t_3-t_0$ is limited to 1300 days. Due to the effect of accuracy loss, the spacecraft motion is described in several coordinate systems.

The following scheme, shown in Fig. 2, is used to reach the AMSO.
After the spacecraft crosses the Mars Hill sphere at point 1, it moves along a hyperbolic approach trajectory. At time $t_1$ at point 2, which is located at the trajectory pericenter at a distance of 50 km from the Mars surface in the plane of the final Phobos tracking orbit, the braking impulse, necessary to lift the spacecraft orbit to the AMSO plane is generated. At time $t_3$ at point 3 the last braking impulse is generated, which is necessary to equalize the spacecraft velocity with the speed of motion along the final circular orbit. Potentially necessary condition of phasing is not taken into account. It is believed that hitting Phobos at the final moment can be provided by a slight decrease in the height of point 3, which will insignificantly affect the entire trajectory. This assumption is confirmed by calculations.

The spacecraft flights from point 2 to 3 and from point 3 to 4 are considered to be Hohmann flights; their corresponding impulses are calculated analytically and added to the problem functional. Thus, the numerical integration can be completed at time $t_1$ at point 2. Operation of high-thrust engines for acceleration and deceleration of the spacecraft near the Earth and Mars is approximated by impulse actions. Control is performed by the direction and magnitude of four pulses at times $t_0, t_1, t_2, t_3$.

The ascending node longitude for the initial AESO, the spacecraft position on it, time moments $t_0, t_1, t_2, t_3$, directions and magnitudes of pulses are optimized. The sum of values of four pulses of the problem – that is an analogue of mass cost – is minimized.

3. Solution method
The problem under consideration is formalized as a Lagrange problem of optimal control over an aggregate of dynamic systems [3], and on the basis of the Lagrange principle [4–5] its solution is reduced to the solution of the 36-order boundary-value problem.

The system of differential equations of this boundary-value problem consists of the equations from the initial statement of the problem and the conjugate system of equations:

$$
\begin{align*}
\dot{x}_i &= u_i, \quad y_i = v_i, \quad \dot{z}_i = w_i, \\
\dot{u}_i &= -g_i - \sum_{B=(E,M)} \mu_B \frac{x_{Bi}}{r_{Bi}^3}, \\
\dot{v}_i &= -g_i - \sum_{B=(E,M)} \mu_B \frac{y_{Bi}}{r_{Bi}^3}, \\
\dot{w}_i &= -g_i - \sum_{B=(E,M)} \mu_B \frac{z_{Bi}}{r_{Bi}^3}, \\
\dot{p}_{xi} &= \sum_{B=(E,M)} \left( -\frac{\mu_B}{r_{Bi}^5} \left( p_{ui}(r_{Bi}^2 - 3x_{Bi}^2) - 3(p_{vi}x_{Bi}z_{Bi} + p_{wi}y_{Bi}y_{Bi}) \right) \right), \\
\dot{p}_{yi} &= \sum_{B=(E,M)} \left( -\frac{\mu_B}{r_{Bi}^5} \left( p_{ui}(r_{Bi}^2 - 3y_{Bi}^2) - 3(p_{vi}y_{Bi}z_{Bi} + p_{wi}y_{Bi}x_{Bi}) \right) \right), \\
\dot{p}_{zi} &= \sum_{B=(E,M)} \left( -\frac{\mu_B}{r_{Bi}^5} \left( p_{ui}(r_{Bi}^2 - 3z_{Bi}^2) - 3(p_{vi}z_{Bi}y_{Bi} + p_{wi}z_{Bi}x_{Bi}) \right) \right), \\
\dot{p}_{ui} &= -p_{xi}, \quad \dot{p}_{vi} = -p_{yi}, \quad \dot{p}_{wi} = -p_{zi},
\end{align*}
$$

where $x_i, y_i, z_i$ and $u_i, v_i, w_i$ are components of the spacecraft position and velocity vectors, respectively, in each of the three trajectory sections, $x_{SE}, y_{SE}, z_{SE}$ and $x_{SM}, y_{SM}, z_{SM}$ are components of the Earth and Mars position vectors in heliocentric coordinate system, $x_{ME}, y_{ME}, z_{ME}$ are components of the difference of these vectors, $x_{E1} = x_1, y_{E1} = y_1, z_{E1} = z_1, x_{S1} = x_1 + x_{SE}, y_{S1} = y_1 + y_{SE}, z_{S1} = z_1 + z_{SE}, x_{M1} = x_1 + x_{ME}, y_{M1} = y_1 + y_{ME}, z_{M1} = z_1 + z_{ME}, x_{Ej} = x_j - x_{SE}, y_{Ej} = y_j - y_{SE}$.
The boundary-value problem is nonlinear and is solved numerically in this paper by the multipoint shooting method [6]. The authors have implemented a software package in C using the SPICE package for consideration of ephemeris, the peculiarities of working with the package are given in [7]. A series of Cauchy problems are solved numerically by the explicit Runge-Kutta method of order 8, based on the Dorman-Prince 8(7) calculation formulas with automatic step selection [8]. The system of nonlinear algebraic equations is solved by the modified Newton method using Fedorenko normalization in the convergence condition.

The initial approximation was constructed by solving a combination of Lambert problems. The methodology for solving the Lambert problems, based on the universal Kepler equation used by the authors, is given in [7], [9]. The construction of the initial approximation on its basis, as applied to the problem of a 3-pulse approach to Phobos, is described in [10].

4. Results
The boundary problem was solved numerically. The launch windows to Mars open every 2 years, and the dependence of the functional on the launch date was constructed. The optimal time of the spacecraft launch is November 2026. The spacecraft has a 30-day launch window, at which the loss in functionality in comparison with a direct arrival of the spacecraft in the Mars activity sphere from the dependence of the functional on the launch date was constructed. The optimal time of the spacecraft launch is November 2026. The spacecraft has a 30-day launch window, at which the loss in functionality is no more than 5%. On the best trajectory to Mars, the spacecraft flies for 312 days, then sits on Phobos for 365 days, then returns to the Earth in 340 days.

All of the intermediate impulses on the activity spheres of planets and the Hill sphere at point 1 (Fig. 2) on the local-optimal trajectories are absent. With a three-pulse approach to Phobos, the flight time on Hohmann trajectories from point 2 to point 3 and from point 3 to point 4 is 63 days. The gain in functionality in comparison with a direct arrival of the spacecraft in the Mars activity sphere from point 1 to point 4 is more than 400 m/s, which allows us to estimate the feasibility of implementing the 3-pulse scheme for the flight to Phobos.

The numerical data of the solved boundary-value problem, necessary to repeat the calculations given in the paper, are as follows. At \( t_0 = 9801.557755374460920785752 \) days from Jan 01 2000 12:00 TT in heliocentric coordinate system: \( x_1 = 0.0039199070702828109 \), \( y_1 = -0.001915466315535246 \), \( z_1 = -0.00054857442148598 \), \( u_1 = 0.238887437792377964 \), \( \psi_1 = 62.145923755019772727 \), \( \phi_1 = -0.369116096596407317 \), \( \phi_2 = -8.695881691784672185 \), \( p_{u1} = 0.362291387006706433 \), \( p_{\psi1} = 0.519845180611838709 \), \( p_{\phi1} = 0.773631655954823083 \).

At point 2 at time \( t_1 = 10110.85325468723778496 \) days from Jan 01 2000 12:00 TT in Mars-centric coordinate system: \( x_2 = 0.00065003685343234 \), \( y_2 = 0.001179785560286538 \), \( z_2 = 0.00128952123226607786 \), \( u_2 = 0.30964985849882428 \), \( \psi_2 = -0.369116095696407317 \), \( \phi_2 = -8.695881691784672185 \), \( p_{u2} = -0.02911672716688004 \), \( p_{\psi2} = 15.646498975507576290 \), \( p_{\psi3} = -43.321527117836822640 \), \( p_{\phi3} = -31.03890580358612834 \), \( p_{u3} = 0.955565947545433558 \), \( p_{\psi3} = 0.280748857287123788 \), \( p_{\phi3} = 0.08985432113379845 \).

5. Conclusion
The problem of space dynamics was considered in the paper. Based on the Lagrange principle, its solution was reduced to the boundary-value problem. The boundary-value problem was solved numerically by the multipoint shooting method. The calculation results can be used as an initial approximation for studying the problem in a more precise statement with a limited value of thrust, with a possible neat consideration of the phasing conditions for the exact hit in Phobos [11].

As a result of the problem solution, the trajectory of the flight to Phobos was numerically built using the software package implemented by the authors on a computer in C language. Additionally, its optimality in the case of a turn-free approach to Phobos was checked on the basis of the Pontryagin maximum principle in a more complex formulation [12]. The results obtained make it possible to judge the appropriateness of the three-pulse maneuver during the approach to Phobos.

The paper gives the solution of the resulting boundary problem in a form that allows other researchers to repeat the numerical calculations performed by the authors. The main calculation units were au/100 = 1495978.70691 km and days, E.D. = 86400 s. If other values are used, then the resulted values of the conjugate variables must be recalculated [13].

Further studies will be aimed at solving the problem with a similar maneuver in more complex formulations on the basis of L.S. Pontryagin maximum principle with the piecewise continuous bounded thrust. The obtained trajectory of interplanetary flight can be used to construct the trajectory of the expedition to Phobos with a return to the Earth [14].

References

[1] Eneev T M 2005 Pressing issues of the day in studying deep space Cosmic research 43:6 pp 383–387. Doi: 10.1007/s10604-005-0061-1
[2] 2011 Phobos-Grunt. The space mission project. Scientific publication in two volumes. Vol. 1, 2 ed V S Kornilenco (Moscow: Federal State Unitary Enterprise NPO SA Lavochkin Roscosmos, the Russian Academy of Sciences Space Research Institute) p 520.
[3] Grigoriev K G and Grigoriev I S 2003 Conditions of the maximum principle in the problem of optimal control over an aggregate of dynamic systems and their application to solution of the problems of optimal control of spacecraft motion Cosmic Research 41:3 pp 285–309.
[4] Pontryagin L S, Boltyansky V G, Gamkrelidze R V and Mishchenko E F 1983 Mathematical theory of optimal processes (Moscow: Nauka) p 393
[5] Grigoriev K G and Grigoriev I S 2002 Solving optimization problems for the flight trajectories of a spacecraft with a high-thrust jet engine in pulse formulation for an arbitrary gravitational field in a vacuum Cosmic Research 40:1 pp 81-103.
[6] Grigoriev I S 2005 A methodical manual on numerical methods for solving boundary value problems of the maximum principle in problems of the optimal control. (Moscow: The Center of Applied Research at the Mechanical and Mathematical Faculty of Moscow State University) p 159.
[7] Samokhin A and Samokhina M 2018 Verification of the second-order optimality conditions in the modeling of the SC expedition with the returning to the Earth based on two Lambert’s problems solving Advances in the Astronautical sciences 161 pp 843–862.
[8] Hairer E, Nørsett S P and Wanner G 1987 Solving Ordinary Differential Equations I. Nonstiff Problems (Berlin: Springer-Verlag Publ.) p 528.
[9] Samokhin A S 2014 Optimization of expedition to Phobos using the impulse control and solution to Lambert problems taking into account attraction of the Earth and Mars Moscow University Mathematics Bulletin 69:2 pp 84–87. Doi: 10.3103/S0027132214020089.
[10] Samokhin A and Samokhina M 2020 Construction of a three-pulse approach to phobos trajectories with access to the Mars Hill sphere based on the solution of a series of Lambert's problems 27th Saint Petersburg International Conference on Integrated Navigation Systems (ICINS) pp 1–3. Doi: 10.23919/ICINS43215.2020.9133816.
[11] Samokhina M A, Samokhin A S, Zapletin M P and Grigoryev I S 2018 Method of optimal trajectories design for a spacecraft with a jet engine of a large limited thrust in problems with
the phasing condition *Advances in the Astronautical Sciences* 161 pp 711–730.

[12] Samokhin A S, Samokhina M A, Grigoryev I S and Zapletin M P 2020 The optimization of interplanetary flight to Phobos with a jet engine of combined low and high limited thrust *Advances in the Astronautical Sciences* 170 pp 213–227.

[13] Grigoriev I S and Zapletin M P 2009 On a problem of optimization of the SC trajectories of an asteroids cluster visit *Cosmic research* 47:5 pp 460–470.

[14] Grigoryev I S, Zapletin M P, Samokhin A S and Samokhina M A 2017 Optimization of Phobos mission with hybrid propulsion returning to the Earth *Engineering Journal: Science and Innovation* 7 p 24. Doi: 10.18698/2308-6033-2017-7.