Fault-tolerant Control of a Class of Uncertain Systems with Actuator Failure

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Abstract. The fault-tolerant control of uncertain networked control systems with actuator failure and packet loss is studied. In the event of a failure of the actuator component, there is a problem of packet loss. The uncertain control system with actuator failure and packet loss is modeled as a Markov jump linear system based on the different locations of packet loss during execution. Then, analysing the stability of the fault system by using Lyapunov stability theory and linear matrix inequalities technology, the adequacy of the random stability of the closed-loop fault system is proved. At the same time, the mature cone repair linearization method is adopted for the controller design. Finally, the feasibility of the design method was verified by using the example of a double rotor helicopter simulation model.

1. Introduction
With the rapid development of computer technology, control technology and communication technology, network control systems have received great attention in modern industry due to their advantages of resource sharing, less cabling, low cost. The network control system is a closed-loop control system composed of sensors, controllers and actuators as well as mutual networks. The network control system inevitably causes delay, packet loss, and timing inconsistency. Fault-tolerant control has become a research hotspot in the control field and has achieved great results. [1-3]

Packet loss is a common problem after the control system is introduced into the network. The upper and lower limits of the signal or control action scale factor [4]. An active fault-tolerant control system based on sliding mode control theory is studied in [5]. But the previous researches were mostly based on the time-delay network control system. The system components were faulted and the delay was considered for the component failure.

Loss of different situations for analysis, in the event of failure of the actuator parts based on the different locations of packet loss will be divided into four subsystems of uncertain control system, the four faulty control system is modeled as four Markov jumps Linear system, then use Lyapunov stability theory and matrix inequality technology to analyze the stability of the system, use Schur's complement lemma to design a suitable controller. Finally, simulation to prove the design method are given in this paper.

2. Problem Description
Figure 1 is a block diagram of the network system studied in this paper.
Network control system controlled objects:

\[ x(k + 1) = Ax(k) + Bu(k) \]  \hspace{1cm} (1)

In the actual control system, due to the disturbance of the external environment, changes in system parameters, and disturbances of unknown factors, the uncertainty in the control system makes it difficult to obtain the ideal system model. Considering the uncertainty, the controlled object model is:

\[ x(k + 1) = (A + \Delta A)x(k) + (B + \Delta B)u(k) \]  \hspace{1cm} (2)

\[ \Delta A, \Delta B \] are unknown matrices and there are uncertainties in the system. Assume that \( \Delta A \) and \( \Delta B \) satisfy the following equations:

\[ \Delta A(k) = D_aF_a(k)E_a, \Delta B(k) = D_bF_b(k)E_b \]  \hspace{1cm} (3)

Where \( D_a, D_b, E_a, E_b \) are known constant matrices, \( F_a(k), F_b(k) \) are unknown matrices, and \( F_a(k), F_b(k) \) satisfy:

\[ F_a^T(k)F_a(k) \leq I, \quad F_b^T(k)F_b(k) \leq I \]  \hspace{1cm} (4)

The uncertainty parameters \( \Delta A(k), \Delta B(k) \) existing in the system are related to the unknown matrices \( F_a(k), F_b(k) \), which are different from the previously used exact matching uncertainties \( [\Delta A(k), \Delta B(k)] = DF(k)[E_a, E_b] \) [6]. For possible actuator failures, it must be studied to define the actuator form and actuator failure model:

\[ u(k) = Kx(k), \quad u^F(x) = Mu(k) \]  \hspace{1cm} (5)

When \( m_i = 0 \), the i-th channel of the actuator completely fails; when \( m_i = 1 \), the i-th channel can work normally; when \( 0 \leq m_{ii} \leq m_i \leq m_{iu}, 1 > m_{ii}, m_{ii} \geq 1 \) and \( m_i \neq 1 \) indicates that the i-th channel has not completely failed. In order to represent the actuator fault matrix, a partial matrix is now introduced:

\[ M_0 = \text{diag}[m_{01}, m_{02}, ..., m_{0p}], L = \text{diag}[l_1, l_2, ..., l_p], H = \text{diag}[h_1, h_2, ..., h_p], |L| = \text{diag}[|l_1|, |l_2|, ..., |l_p|], m_{0i} = \frac{m_{ii} + m_{iu}}{2}, l_i = \frac{m_{ii} - m_{iu}}{m_{iu} + m_{ii}}, h_i = \frac{m_{ii} - m_{iu}}{m_{iu} + m_{ii}}, i = 1, 2, ..., p \]  \hspace{1cm} (6)

From \( m_{0i}, l_i, h_i \) you can get:

\[ M = M_0(1 + L), \quad |L| \leq H \leq 1 \]  \hspace{1cm} (7)

It can be concluded that the controlled equation with an uncertain system of actuator failure is

\[ x(k + 1) = \tilde{A}x(k) + \tilde{B}Mu(k) \]  \hspace{1cm} (8)

\( \tilde{A}, \tilde{B} \) represent the status matrix parameters and control input parameters, respectively:
When the switch is closed, the sensor data can be directly transmitted to the controller, the sensor output $x(k)$. The data that the controller needs to process is the control signal $\tilde{x}(k)$ after the switch. If the signal received by the controller is the switch off state, then the data packet is lost. The signal that the controller can receive is actually a switch. Keep a signal before the output, that is $\tilde{x}(k-1)$, at this time we represent the model of the switch $S_1$:

$$\tilde{x}(k) = \begin{cases} 
  x(k) & S_1 \text{ closed} \\
  \tilde{x}(k-1) & S_1 \text{ opened}
\end{cases}$$

(10)

In the same way, calculate the model of $S_2$:

$$u(k) = \begin{cases} 
  \tilde{u}(k) & S_2 \text{ closed} \\
  u(k-1) & S_2 \text{ opened}
\end{cases}$$

(11)

When analyzing the position of the packet loss, let the dimension vector be

$$\xi(k) = [x^T(k)\tilde{x}^T(k-1), u^T(k-1)]^T$$

(12)

1. No packet loss occurred
2. Switch $S_1$ opened, packet loss is only between sensor and controller.
3. Switch $S_2$ opened, packet loss is only between controller and actuator.
4. Switch $S_1$, $S_2$ opened, packet loss occurs at the same time between sensor and controller and controller and actuator.

The packet loss situation is divided into four categories. Now the four situations are handled in a unified manner. The closed-loop fault network control system model:

$$\xi(k + 1) = \Phi_{\sigma(k)}\xi(k)$$

(13)

Definition 1: For the initial condition $(\xi_0, \sigma_0)$, that has been given, if the constant $\Psi(\xi_0, \sigma_0)$ exists and the solution of the control system (13) satisfies $\lim_{t \to \infty} E[\sum_{k=0}^{T} ||\xi(k)||^2] \leq \Psi(\xi_0, \sigma_0)$, the uncertain network control system, which can indicate actuator failure, is stable.

3. Stability analysis

If there is an actuator failure, there is a known matrix $P_i > 0$, controller parameter $K$, $\alpha_{ij}, \beta_{ij}, y_{ij}, \kappa_{ij}$, scalar for the control system (13), such that if equation (14) holds at any $i, j \in S$, the control system (13) is stable.

$$\begin{bmatrix}
-P_{11} & * & * & * & * & * & * & * \\
\bar{\xi}_{11} & Z_{11} & * & * & * & * & * & * \\
\bar{\xi}_{12} & 0 & Z_{12} & * & * & * & * & * \\
\bar{\xi}_{13} & 0 & 0 & \bar{Z}_{13} & * & * & * & * \\
\bar{\xi}_{14} & 0 & 0 & 0 & \bar{Z}_{14} & * & * & * \\
\bar{\Psi}_{11} & 0 & 0 & 0 & 0 & \bar{\Xi}_{11} & * & * \\
\bar{\Psi}_{12} & 0 & 0 & 0 & 0 & \bar{\Xi}_{12} & * & * \\
\bar{\Psi}_{13} & 0 & 0 & 0 & 0 & 0 & \bar{\Xi}_{13} & * \\
\bar{\Psi}_{14} & 0 & 0 & 0 & 0 & 0 & 0 & \bar{\Xi}_{14}
\end{bmatrix} < 0$$

(14)

Lemma 1: Known symmetric matrix $M$, $D$, $E$ matrix of a certain dimension, when the matrix $F(k)$ is satisfied $M + DF(k)E + E^T F^T(k)D^T < 0$, And if and only positive scalar $\varepsilon$ exists, $M + \varepsilon DD^T + \varepsilon^{-1}E^TED < 0$ is established.

Lemma 2: If $R_1, R_2, Y$ is a symmetric real matrix of a certain dimension, $L$ is a diagonal matrix of a certain dimension, $H$ is a positive definite diagonal matrix of a certain dimension, and $|L| \leq H$, If there is a normal number $\beta > 0$ such that $Y + \beta R_1 HR_1^T + \beta^{-1} R_2^T H R_2 < 0$ holds, then $R_1 L R_2 + \beta R_1 HR_1^T + \beta^{-1} R_2^T H R_2 < 0$ holds.
Lemma 3 (Schur Theorem): For symmetric matrix $A$, matrix $B$ and symmetrical positive definite matrix $C$, then $A + B^TCB < 0$ can be written as $\begin{bmatrix} A & B^T \\ B & -C^{-1} \end{bmatrix} < 0$.

Proof: Construct a Lyapunov function,
\[ V(k) = \xi^T(k)P_\sigma(k)\xi(k) \] (15)

$P_\sigma(k)$ is a positive definite matrix.

Assumptions
\begin{equation}
\beta = \min_{i \in \mathbb{S}} \lambda_{\min} (-\Omega_{li}), \lim_{t \to \infty} E\{\sum_{k=0}^{T} \|\xi(k)\|^2\} \leq \frac{\beta}{\rho} E\{V(0)\} = \frac{1}{\rho} \xi^T(k)P_{\sigma 0}\xi(0) \tag{16}
\end{equation}

If there is a constant $\Psi(\xi_0, \sigma_0)$ such that $\Psi(\xi_0, \sigma_0) = \frac{1}{\rho} \xi^T(k)P_{\sigma 0}\xi(0)$, then according to definition 1 it can be concluded that the control system (13) is stable.

From Lemma 3, $\Omega_{li} \equiv \sum_{j=1}^{\mathcal{T}} \pi_{ij}\Phi_j^T P_l \Phi_l - P_l < 0$ can be expressed as
\[ \begin{bmatrix}
-P_i & * & * & * & * \\
\sqrt{\pi_{i1}}\Phi_1 & -P_i^{-1} & * & * & * \\
\sqrt{\pi_{i2}}\Phi_2 & 0 & -P_i^{-1} & * & * \\
\sqrt{\pi_{i3}}\Phi_3 & 0 & 0 & -P_i^{-1} & * \\
\sqrt{\pi_{i4}}\Phi_4 & 0 & 0 & 0 & -P_i^{-1} 
\end{bmatrix} < 0 \tag{17}
\]

Applying Lemma 1, equation (17) can be expressed as:
\[ \begin{bmatrix}
-P_i + Y_{11} + Y_{12} & * & * & * & * \\
\xi_{i1} & Z_{i1} & * & * & * \\
\xi_{i2} & 0 & Z_{i2} & * & * \\
\xi_{i3} & 0 & 0 & Z_{i3} & * \\
\xi_{i4} & 0 & 0 & 0 & Z_{i4} 
\end{bmatrix} < 0 \tag{18}
\]

Equation (18) is equivalent to Lemma 3:
\[ \begin{bmatrix}
-P_i & * & * & * & * & * \\
\xi_{i1} & Z_{i1} & * & * & * & * \\
\xi_{i2} & 0 & Z_{i2} & * & * & * \\
\xi_{i3} & 0 & 0 & Z_{i3} & * & * \\
\xi_{i4} & 0 & 0 & 0 & Z_{i4} & * \\
\psi_{i1} & 0 & 0 & 0 & 0 & \Sigma_{i4} \\
\psi_{i2} & 0 & 0 & 0 & 0 & \Sigma_{i2} \end{bmatrix} < 0 \tag{19}
\]

Using Lemma 3 again, we can conclude that Eq. (19) is equivalent to Eq. (14). $-P_i$ is nonlinear item, cannot be directly solved, so new variables $Q_i$ need to be introduced. $Q_1 = P_1^{-1}$, $Q_2 = P_2^{-1}$, $Q_3 = P_3^{-1}$, $Q_4 = P_4^{-1}$. Substitute (14), available
\[ \begin{bmatrix}
-P_i & * & * & * & * & * & * \\
\xi_{i1} & X_{i1} & * & * & * & * & * \\
\xi_{i2} & 0 & X_{i2} & * & * & * & * \\
\xi_{i3} & 0 & 0 & X_{i3} & * & * & * \\
\xi_{i4} & 0 & 0 & 0 & X_{i4} & * & * \\
\psi_{i1} & 0 & 0 & 0 & 0 & \Sigma_{i1} & * \\
\psi_{i2} & 0 & 0 & 0 & 0 & \Sigma_{i2} & * \\
\psi_{i3} & 0 & 0 & 0 & 0 & 0 & \Sigma_{i3} \\
\psi_{i4} & 0 & 0 & 0 & 0 & 0 & \Sigma_{i4} \end{bmatrix} < 0 \tag{20}
\]
Using the cone complementarity linearization method proposed in Article 7, the nonlinear matrix inequality of Theorem 1 can be solved by solving the nonlinear minimization problem (13):

$$\min \text{tr}\left(\sum_{i=1}^{n} P_i Q_i\right), \quad \text{s.t.} (20) \text{ and } (22)$$

From equation (22), $P_i Q_i \geq I$ can be obtained, and $\min \text{tr}\left(\sum_{i=1}^{n} P_i Q_i\right) \geq 4(2n + p)$ can be deduced. Then, $\text{tr}(\sum_{i=1}^{n} P_i Q_i) = 4(2n + p)$, then $P_i Q_i = I$, the matrix inequality (10) is established. The iterative algorithm of the fault-tolerant controller is now designed to solve the minimization problem.

1. Solve the feasible solutions $(P_i^0, Q_i^0, K_i, \alpha_i, \beta_i, \gamma_i, \kappa_i)$ and $k = 0$ for (20) and (22).
2. Solve the optimization problem with $(P_i, Q_i, K_i, \alpha_i, \beta_i, \gamma_i, \kappa_i)$ as variables.
3. Substituting $(P_i, K_i)$ into the formula (13), if the formula (13) is true, then the algorithm terminates and is no longer executed. If it is not established, then $k + 1$ returns to step 2 to continue execution.

4. Simulation example
Consider the system model of a horizontal-rotation double rotor helicopter [7]:

$$\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}$$

And $A = \begin{bmatrix} -0.02 & 0.005 & 2.4 & -32 \\ -0.14 & 0.44 & -1.3 & -30 \\ 0 & 0.018 & -1.6 & 1.2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0.14 & -0.12 \\ 0.36 & -8.6 \\ 0.35 & 0.009 \\ 0 & 0 & 57.3 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

The input is the combined propeller thrust and the differential collecting propeller thrust, which are denoted by $u_1, u_2$, respectively. The output is the vertical speed and the inclination angle, which are respectively expressed by $y_1, y_2$. With a radius of 2, the center of the circle $(−3,0)$ represents a stable area, the fluctuation range of the fault value are $0.6 \leq m_i \leq 1.25, 0.4 \leq m_i \leq 1.45$, then the pole $(-2.23,0.07,0.49 + 0.42 i + 0.49 - 0.42 i)$ of the dual-rotor helicopter system (23) is not within the stable area.

$$K_{\text{nomal}} = \begin{bmatrix} 2.1161 & -0.0981 & -21.059 & -81.282 \\ 0.076 & 0.3958 & -1.0262 & -6.9184 \end{bmatrix}$$

$$K_{\text{reflable}} = \begin{bmatrix} 0.7861 & -0.0822 & -15.2475 & -46.3392 \\ -0.1947 & 0.428 & -1.0237 & -3.7203 \end{bmatrix}$$

In the absence of actuator failure, the pole sets of the normal controller and the fault-tolerant controller system are $\{-1.17, -2.23, -3.46, -3.24\}, \{-2.52, -3.09 + 0.18 i, -3.09 - 0.18 i, -2.99\}$. It can be concluded that the system is stable because the collection is in a stable circular area.

From Figure 2, we can see that in the event of system failure while considering about the phenomenon of packet loss, the system under the control of the original normal controller can’t stabilize the system, not in the stable area, the designed fault-tolerant controller pole is still within the stable area, indicating that the designed controller has a role.
Figure 2. Fault distribution of unstable system

5. Conclusion
The uncertain control system is divided into four sub-systems based on the different positions of the lost parts when the actuator parts fail, and these four faulty control systems are modeled as four Markov jump linear systems. Then use Lyapunov stability theory and matrix inequality technology to analyze the stability of the system, prove the stability of the system, and finally use Schur's complement lemma to design a suitable controller. The example of a dual-rotor aircraft is used to prove the effectiveness of the designed controller.

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