QUARK COLOR SUPERCONDUCTIVITY AND THE COOLING OF COMPACT STARS

I. A. SHOVKOY AND P. J. ELLIS

School of Physics and Astronomy,
University of Minnesota,
Minneapolis, MN 55455, USA

The thermal conductivity of the color-flavor locked phase of dense quark matter is calculated. The dominant contribution to the conductivity comes from photons and Nambu-Goldstone bosons associated with the breaking of baryon number, both of which are trapped in the quark core. Because of their very large mean free path the conductivity is also very large. The cooling of the quark core arises mostly from the heat flux across the surface of direct contact with the nuclear matter. As the thermal conductivity of the neighboring layer is also high, the whole interior of the star should be nearly isothermal. Our results imply that the cooling time of compact stars with color-flavor locked quark cores is similar to that of ordinary neutron stars.

1. Introduction

At sufficiently high baryon density the nucleons in nuclear matter should melt into quarks so that the system becomes a quark liquid. It should be weakly interacting due to asymptotic freedom [1], however, it cannot be described as a simple Fermi liquid. This is due to the nonvanishing attractive interaction in the color antitriplet quark-quark channel, provided by one-gluon exchange, which renders a highly degenerate Fermi surface unstable with respect to Cooper pairing. As a result the true ground state of dense quark matter is, in fact, a color superconductor [2].

Recent phenomenological [3] and microscopic studies [4,5,6,7] have confirmed that quark matter at a sufficiently high density undergoes a phase transition into a color superconducting state. Phenomenological studies are expected to be appropriate to intermediate baryon densities, while microscopic approaches are strictly applicable at asymptotic densities where perturbation theory can be used. It is remarkable that both approaches concur that the superconducting order parameter (which determines the gap \( \Delta \) in the quark spectrum) lies between 10 and 100 MeV for baryon densities existing in the cores of compact stars.
At realistic baryon densities only the three lightest quarks can participate in the pairing dynamics. The masses of the quarks are much smaller than the baryon chemical potential, thus, to a good approximation, all three flavors participate equally in the color condensation. The ground state is then the so-called color flavor locked (CFL) phase [8]. The original gauge symmetry $SU(3)_c$ and the global chiral symmetry $SU(3)_L \times SU(3)_R$ break down to a global diagonal “locked” $SU(3)_{c+L+R}$ subgroup. Because of the Higgs mechanism the gluons become massive and decouple from the infrared dynamics. The quarks also decouple because large gaps develop in their energy spectra. The breaking of the chiral symmetry leads to the appearance of an octet of pseudo-Nambu-Goldstone (NG) bosons ($\pi^0$, $\pi^\pm$, $K^\pm$, $K^0$, $\bar{K}^0$, $\eta$). In addition an extra NG boson $\phi$ and a pseudo-NG boson $\eta'$ appear in the low energy spectrum as a result of the breaking of global baryon number symmetry and approximate $U(1)_A$ symmetry, respectively.

The low energy action for the NG bosons in the limit of asymptotically large densities was derived in Refs. [9,10]. By making use of an auxiliary “gauge” symmetry, it was suggested in Ref. [11] that the low energy action of Refs. [9,10] should be modified by adding a time-like covariant derivative to the action of the composite field. Under a favorable choice of parameters, the modified action predicted kaon condensation in the CFL phase. Some unusual properties of such a condensate were discussed in Ref. [12].

While the general structure of the low energy action in the CFL phase can be established by symmetry arguments alone [9], the values of the parameters in such an action can be rigorously derived only at asymptotically large baryon densities [10,11]. Thus, in the most interesting case of intermediate densities existing in the cores of compact stars, the details of the action are not well known. For the purposes of the present paper, however, it suffices to know that there are 9 massive pseudo-NG bosons and one massless NG boson $\phi$ in the low energy spectrum. If kaons condense [11] an additional NG boson should appear. These NG bosons should be relevant for the kinetic properties of dense quark matter.

It has been found [13] that neutrino and photon emission rates for the CFL phase are very small so that they would be inefficient in cooling the core of a neutron star. The purpose of the present investigation is to determine quantitatively the thermal conductivity of the CFL phase of dense quark matter in order to see whether it can significantly impact the cooling rate. We shall argue that the temperature of the CFL core, as well as the neighboring neutron layer which is in contact with the core, falls quickly due to the very high thermal conductivities on both sides of the interface. In fact, to a good approximation, the interior of the star
is isothermal. A noticeable gradient of the temperature appears only in a relatively thin surface layer of the star where a finite flux of energy is carried outwards by photon diffusion. A more complete account of this work is to be found in Ref. [14].

2. Thermal conductivity

A detailed understanding of the cooling mechanism of a compact star with a quark core is not complete without a study of thermal conductivity effects in the color superconducting quark core. The conductivity, as well as the other kinetic properties of quark matter in the CFL phase, is dominated by the low energy degrees of freedom. It is clear then that at all temperatures of interest to us, \( T \ll \Delta \), it is crucial to consider the contributions of the NG bosons. In addition, there may be an equally important contribution due to photons; this is discussed in Sec. 4. Note that, at such small temperatures, the gluon and the quark quasiparticles become completely irrelevant. For example, a typical quark contribution to a transport coefficient would be exponentially suppressed by the factor \( \exp(-\Delta/T) \).

Let us start from the general definition of the thermal conductivity as a characteristic of a system which is forced out of equilibrium by a temperature gradient. In response to such a gradient transport of heat is induced. Formally this is described by the following relation:

\[
    u_i = -\kappa \partial_i T, \tag{1}
\]

where \( u_i \) is the heat current, and \( \kappa \) is the heat conductivity. As is clear from this relation, the heat flow would persist until a state of uniform temperature is reached. The higher the conductivity, the shorter the time for this relaxation.

In the linear response approximation, the thermal conductivity is given in terms of the heat current correlator by a Kubo-type formula. We derive the expression for the heat (energy) current carried by a single (pseudo-) NG boson field \( \varphi \). The corresponding Lagrangian density reads

\[
    L = \frac{1}{2} \left( \partial_0 \varphi \partial_0 \varphi - v^2 \partial_i \varphi \partial_i \varphi - m^2 \varphi^2 \right) + \ldots, \tag{2}
\]

where the ellipsis stand for the self-interaction terms as well as interactions with other fields. Notice that we introduced explicitly the velocity parameter \( v \). In microscopic studies of color superconducting phases, which are valid at very large densities, this velocity is equal to \( 1/\sqrt{3} \) for all (pseudo-) NG bosons. It is smaller than 1 because Lorentz symmetry is broken due to the finite value of the quark chemical potential. By making use of the
above Lagrangian density, we derive the following expression for the heat current:

\[ u_i = \frac{\partial L}{\partial (\partial^\mu \varphi)} \partial_\mu \varphi = v^2 \partial_i \varphi \partial_0 \varphi. \] (3)

This definition leads to an expression for the heat conductivity in terms of the corresponding correlator [15]:

\[ \kappa_{ij} = -\frac{i}{2T} \lim_{\Omega \to 0} \frac{1}{\Omega} \left[ \Pi^R_{ij}(\Omega + i\epsilon) - \Pi^A_{ij}(\Omega - i\epsilon) \right], \] (4)

where, in the Matsubara formalism,

\[ \Pi_{ij}(i\Omega_m) = v^4 T \sum_n \int \frac{d^3 k}{(2\pi)^3} k_i k_j i\Omega_n (i\Omega_n + i\Omega_{n-m}) \times S(i\Omega_n, \vec{k}) S(i\Omega_{n-m}, \vec{k}). \] (5)

Here \( \Omega_n \equiv 2\pi n T \) is the bosonic Matsubara frequency, and \( S(i\Omega_n, \vec{k}) \) is the propagator of the (pseudo-) NG boson. In general, the propagator should have the following form:

\[ S(\omega, \vec{k}) = \frac{1}{(\omega + i\Gamma/2)^2 - v^2 \vec{k}^2 - m^2}, \] (6)

where the width parameter \( \Gamma(\omega, \vec{k}) \) is related to the inverse lifetime (as well as the mean free path) of the boson. In our calculation, it is very convenient to utilize the spectral representation of the propagator,

\[ S(i\Omega_n, \vec{k}) = \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega A(\omega, \vec{k}) i\Omega_n - \omega. \] (7)

Then the conductivity is expressed through the spectral function \( A(\omega, \vec{k}) \) as follows:

\[ \kappa_{ij} = \frac{v^4}{2\pi T^2} \int_{-\infty}^{\infty} \frac{\omega^2 d\omega}{\sinh^2 \frac{\omega}{2T}} \int \frac{d^3 k}{(2\pi)^3} k_i k_j A^2(\omega, \vec{k}). \] (8)

By making use of the explicit form of the propagator in Eq. (6), we see that the spectral function of the (pseudo-) NG boson is

\[ A(\omega, \vec{k}) = \frac{\omega \Gamma}{(\omega^2 - e_k^2 - \Gamma^2/4)^2 + \omega^2 \Gamma^2}, \] (9)

where \( e_k \equiv \sqrt{v^2 k^2 + m^2} \). Because of the rotational symmetry of the system, the conductivity is characterized by a single scalar quantity \( \kappa \) which is introduced as follows: \( \kappa_{ij} = \kappa \delta_{ij} \). The explicit expression for this scalar function reads

\[ \kappa = \frac{1}{48 \sqrt{2\pi^2 \epsilon} \Gamma T^2} \int_{0}^{\infty} \frac{\omega d\omega}{\sinh^2 \frac{\omega}{2T}} \left( \sqrt{X^2 + \omega^2 \Gamma^2} + X \right)^{3/2}, \] (10)
where we introduced the notation $X \equiv \omega^2 - m^2 - \Gamma^2/4$. For our purposes it will be sufficient to consider the conductivity in the limit of a small width, $\Gamma \to 0$. This is because the (pseudo-) NG bosons in the CFL quark matter are weakly interacting. Thus, we derive the following approximate relation:

$$\kappa = \frac{1}{24\pi^2 v T^2 \Gamma} \int_{m}^{\infty} \frac{d\omega \omega}{\sinh^2 \frac{\omega}{2T}} (\omega^2 - m^2)^{3/2}. \quad (11)$$

At small temperature, $T \ll m$, this result is further approximated by

$$\kappa \simeq \frac{m^{5/2} \sqrt{T}}{2\sqrt{2\pi^3/2} v} e^{-m/T}. \quad (12)$$

This demonstrates clearly that the contributions of heavy pseudo-NG bosons to the thermal conductivity are suppressed. The largest contribution comes from the massless NG boson $\phi$ for which the thermal conductivity is

$$\kappa_\phi = \frac{4T^3}{3\pi^2 v \Gamma_\phi} \int_{0}^{\infty} \frac{x^4 dx}{\sinh^2 x} = \frac{2\pi^2 T^3}{45 v \Gamma_\phi}. \quad (13)$$

In order to calculate $\kappa_\phi$ the width $\Gamma_\phi$ is required, or equivalently the mean free path $\ell_\phi$ since $\ell_\phi \equiv \bar{v}/\Gamma_\phi$, where $\bar{v}$ is the average thermal velocity of the NG bosons. This will be discussed in the next section.

3. Mean free path of the NG boson

As we have remarked, the contribution of massive pseudo-NG bosons to the thermal conductivity is suppressed. In the CFL phase of quark matter, however, there is one truly massless NG boson $\phi$ which should therefore give the dominant contribution to the heat conductivity. The interactions of $\phi$ with the CFL matter leads to a finite value for its mean free path. Since this boson is a composite particle there is always a non-zero probability at finite temperature for its decay into a pair of quark quasiparticles. It is natural to expect that such a process is strongly suppressed at small temperatures, $T \ll \Delta$. This is confirmed by a direct microscopic calculation in the region of asymptotic densities which yields a decay width [16]:

$$\Gamma_{\phi \to qq}(k) \simeq \frac{5\sqrt{2\pi} v k}{4(21 - 8 \ln 2)} \exp \left( -\sqrt{\frac{3}{2} \frac{\Delta}{T}} \right). \quad (14)$$

If this were the only contribution, then the order of magnitude of the mean free path of the NG boson would be

$$\ell_{\phi \to qq} \sim \frac{v}{T} \exp \left( \sqrt{\frac{3}{2} \frac{\Delta}{T}} \right). \quad (15)$$
This grows exponentially with decreasing temperature. For example, if $\Delta \simeq 50$ MeV and $T \lesssim 1.5$ MeV, the mean free path is 30 km or more. This scale is a few times larger than the typical size of a compact star.

The decay channel of the NG bosons into quarks is not the only contribution to the mean free path. They can also scatter on one another. The corresponding amplitude is of order $k^4/\mu^4$ [17] which gives a cross section of $\sigma_{\phi\phi} \simeq T^6/\mu^8$, yielding the following contribution to the width:

$$
\Gamma_{\phi\phi} = v \sigma_{\phi\phi} n_{\phi} \sim \frac{T^9}{\mu^8},
$$

where $n_{\phi}$ is the equilibrium number density of the NG bosons [14]. At small temperatures the scattering contribution in Eq. (16) dominates the width. This leads to a mean free path

$$
\ell_{\phi\phi} \sim \frac{\mu^8}{T^9} \approx 8 \times 10^5 \frac{\mu_{500}^8}{T_{\text{MeV}}^9} \text{ km.}
$$

Here we defined the following dimensionless quantities: $\mu_{500} \equiv \mu/(500 \text{ MeV})$ and $T_{\text{MeV}} \equiv T/(1 \text{ MeV})$. Both $\ell_{\phi\phi}$ and $\ell_{\phi\rightarrow q\bar{q}}$ depend very strongly on temperature, however the salient point is that they are both larger than the size of a compact star for temperatures $T_{\text{MeV}}$ of order 1.

We define $\tilde{T}$ to be the temperature at which the massive NG bosons decouple from the system. This is determined by the mass of the lightest pseudo-NG boson for which it is not presently possible to give a reliable value. Different model calculations [10,11,18] produce different values which can range as low as 10 MeV. Thus, conservatively, we choose $\tilde{T} \simeq 1$ MeV. Then the mean free path of the NG boson is comparable to or even larger than the size of a star for essentially all temperatures $T \lesssim \tilde{T}$. It is also important to note that the mean free path is very sensitive to temperature changes. In particular, at temperatures just a few times higher than $\tilde{T}$ the value of $\ell$ may already become much smaller than the star size. This suggests that, during the first few seconds after the supernova explosion when the temperatures remain considerably higher than $\tilde{T}$, a noticeable temperature gradient may exist in the quark core. This should relax very quickly because of the combined effect of cooling (which is very efficient at $T \gg \tilde{T}$) and diffusion. After that almost the whole interior of the star would become isothermal.

Before concluding this section, we point out that the geometrical size of the quark phase limits the mean free path of the NG boson since the scattering with the boundary should also be taken into account. It is clear from simple geometry that $\ell \sim R_0$, where $R_0$ is the radius of the quark core.
4. Photon contributions

Now, let us discuss the role of photons in the CFL quark core. It was argued in Ref. [13] that the mean free path of photons is larger than the typical size of a compact star at all temperatures $T \lesssim \tilde{T}$. One might conclude therefore that all photons would leave the stellar core shortly after the core becomes transparent. If this were so the photons would be able to contribute neither to the thermodynamic nor to the kinetic properties of the quark core. However the neighboring neutron matter has very good metallic properties due to the presence of a considerable number of electrons. As is known from plasma physics, low frequency electromagnetic waves cannot propagate inside a plasma. Moreover, an incoming electromagnetic wave is reflected from the surface of such a plasma [19]. In particular, if $\Omega_p$ is the value of the plasma frequency of the nuclear matter, then all photons with frequencies $\omega < \Omega_p$ are reflected from the boundary. This effect is similar to the well known reflection of radio waves from the Earth’s ionosphere.

The plasma frequency is known to be proportional to the square root of the density of charge carriers and inversely proportional to the square root of their mass. It is clear therefore that the electrons, rather than the more massive protons, will lead to the largest value of the plasma frequency in nuclear matter. Our estimate for the value of this frequency is

$$
\Omega_p = \sqrt{\frac{4\pi e^2 Y_e \rho}{m_e m_p}} \simeq 4.7 \times 10^2 \sqrt{\frac{\rho Y_e}{\rho_0}} \text{ MeV},
$$

(18)

where the electron density $n_e = Y_e \rho / m_p$ is given in terms of the nuclear matter density $\rho$ and the proton mass $m_p$. Also $m_e$ denotes the electron mass, $Y_e \simeq 0.1$ is the number of electrons per baryon, and $\rho_0 \approx 2.8 \times 10^{14}$ g cm$^{-3}$ is equilibrium nuclear matter density.

Since $\Omega_p$ is more than 100 MeV, essentially all thermally populated electromagnetic waves at $T \lesssim \tilde{T}$ will be reflected back into the core region. In a way the boundary of the core looks like a good quality mirror with some leakage which will allow a thermal photon distribution to build up and stay. Thus photons will be trapped in such a core surrounded by a nuclear layer. Notice that the transparency of the core is reached only after the temperature drops substantially below $\tilde{T} \simeq 2m_e$, i.e., when the density of thermally excited electron-positron pairs becomes very small.

Now, since photons are massless they also give a sizable contribution to the thermal conductivity of the CFL phase. The corresponding contribution $\kappa_\gamma$ will be similar to the contribution of massless NG bosons in Eq. (13). Since the photons move at approximately the speed of light [20] ($v \simeq 1$ at
the densities of interest) and they have two polarization states, we obtain
\[ \kappa_\gamma = \frac{4\pi^2 T^3}{45 \Gamma_\gamma}. \]  
(19)

Since the thermal conductivity is additive the total conductivity of dense quark matter in the CFL phase is given by the sum of the two contributions:
\[ \kappa_{CFL} = \kappa_\phi + \kappa_\gamma \simeq \frac{2\pi^2}{9} T^3 R_0, \]  
(20)

where for both a photon and a NG boson the mean free path \( \ell \sim R_0 \). This yields the value
\[ \kappa_{CFL} \simeq 1.2 \times 10^{32} T_{\text{MeV}}^3 R_{0,\text{km}} \text{ erg cm}^{-1} \text{ sec}^{-1} \text{ K}^{-1}, \]  
(21)

where \( R_{0,\text{km}} \) is the quark core radius measured in kilometers. The value of \( \kappa_{CFL} \) is many orders of magnitude larger than the thermal conductivity of regular nuclear matter in a neutron star [21].

5. Stellar cooling

In discussing the cooling mechanism for a compact star we have to make some general assumptions about the structure of the star. We accept without proof that a quark core exists at the center of the star. This core stays in direct contact with the neighboring nuclear matter. From this nuclear layer outwards the structure of the star is essentially the same as an ordinary neutron star. The radius of the core is denoted by \( R_0 \), while the radius of the whole star is denoted by \( R \).

A detailed analysis of the interface between the quark core and the nuclear matter was made in Ref. [22]. A similar analysis might also be very useful for understanding the mechanism of heat transfer from one phase to the other. Here we assume that direct contact between the phases is possible, and that the temperature is slowly varying across the interface.

Now, let us consider the physics that governs stellar cooling. We start from the moment when the star is formed in a supernova explosion. Immediately after the explosion many high-energy neutrinos are trapped inside the star. After about 10 to 15 seconds most of them escape from the star by diffusion. The presence of the CFL quark core could slightly modify the rate of such diffusion [23,24,25]. By the end of the deleptonization process, the temperature of the star will have risen to a few tens of MeV. Then, the star cools down relatively quickly to about \( \tilde{T} \) by the efficient process of neutrino emission. It is unlikely that the quark core would greatly affect the time scale for this initial cooling stage. An ordinary neutron star would
continue to cool by neutrino emission for quite a long time even after that [26]. Here we discuss how the presence of the CFL quark core affects the cooling process of the star after the temperature drops below $\tilde{T}$.

Our result for the mean free path of the NG boson demonstrates that the heat conductivity of dense quark matter in the CFL phase is very high. For example, a temperature gradient of 1 MeV across a core of 1 km in size is washed away by heat conduction in a very short time interval of order $R_{0, \text{km}}/v\ell(T) \simeq 6 \times 10^{-4}$ sec. In deriving this estimate, we took into account the fact that the specific heat and the heat conductivity in the CFL phase are dominated by photons and massless NG bosons and that $\ell \sim R_0$. In addition, we used the classical relation $\kappa = \tilde{v}c_\nu/3$, where $c_\nu$ is the specific heat; this can be shown to hold in the present context [14]. Since heat conduction removes a temperature gradient in such a short time interval, it is clear that, to a good approximation, the quark core is isothermal at all temperatures $T \lesssim \tilde{T}$.

The heat conductivity of the neighboring nuclear matter is also known to be very high because of the large contribution from degenerate electrons which have a very long mean free path. It is clear, then, that both the quark and the nuclear layers should be nearly isothermal with equal values of the temperature. When one of the layers cools down by any mechanism, the temperature of the other will adjust almost immediately due to the very efficient heat transfer on both sides of the interface.

Now consider the order of magnitude of the cooling time for a star with a CFL quark core. One of the most important components of the calculation of the cooling time is the thermal energy of the star which is the amount of energy that is lost in cooling. There are contributions to the total thermal energy from both the quark and the nuclear parts of the star. The dominant amount of thermal energy in the CFL quark matter is stored in photons and massless NG bosons. Numerically, its value is [14]

$$E_{CFL}(T) \simeq 2.1 \times 10^{42} R_{0, \text{km}}^3 T_{\text{MeV}}^4 \text{ erg}. \quad (22)$$

The thermal energy of the outer nuclear layer is provided mostly by degenerate neutrons. The corresponding numerical estimate is [27]:

$$E_{NM}(T) \simeq 8.1 \times 10^{49} \frac{M - M_0}{M_\odot} \left(\frac{\rho_0}{\rho}\right)^{2/3} T_{\text{MeV}}^2 \text{ erg}, \quad (23)$$

where $M$ is the mass of the star, $M_0$ is the mass of the quark core and $M_\odot$ is the mass of the Sun. It is crucial to note that the thermal energy of the quark core is negligible in comparison to that of the nuclear layer.

The second important component that determines stellar cooling is the luminosity which describes the rate of energy loss due to neutrino and
photon emission. Typically, the neutrino luminosity dominates the cooling of young stars when the temperatures are still higher than about 10 keV and after that the photon diffusion mechanism starts to dominate. Photon and neutrino emission from the CFL quark phase is strongly suppressed at low temperatures [13]. The neighboring nuclear layer, on the other hand, emits neutrinos quite efficiently. As a result, it cools relatively fast in the same way as an ordinary neutron star. The nuclear layer should be able to emit not only its own thermal energy, but also that of the quark core which constantly arrives by the very efficient heat transfer process. The analysis of the cooling mechanism is greatly simplified by the fact that the thermal energy of the quark core is negligible compared to the energy stored in the nuclear matter. By taking this into account, we conclude that the cooling time of a star with a quark core is essentially the same as for an ordinary neutron star provided that the nuclear layer is not extremely thin.

6. Conclusions

Our analysis shows that the thermal conductivity of CFL color superconducting dense quark matter is very high for typical values of the temperature found in a newborn compact star. This is a direct consequence of the existence of the photon and the massless NG boson $\phi$ whose mean free paths are very large. Note that the photons are trapped in the core because of reflection by the electron plasma in the neighboring nuclear matter. The NG bosons are also confined to the core since they can only exist in the CFL phase.

It is appropriate to mention that the (pseudo-) NG bosons and photons should also dominate other kinetic properties of dense quark matter in the CFL phase. For example, the shear viscosity should be mostly due to photons and the same massless NG bosons associated with the breaking of baryon number. The electrical conductivity, on the other hand, would be mostly due to the lightest charged pseudo-NG boson, i.e., the $K^+$. Thus, in the limit of small temperatures, $T \to 0$, the electrical conductivity will be suppressed by a factor $\exp(-m_{K^+/T})$.

Since the neutrino emissivity of the CFL core is strongly suppressed, the heat is transferred to the outer nuclear layer only through direct surface contact. While both the core and the outer layer contribute to the heat capacity of the star, it is only the outer layer which is capable of emitting this heat energy efficiently in the form of neutrinos. The combination of these two factors tends to extend the cooling time of a star. However, because of the very small thermal energy of the quark core, the time scale for cooling could be noticeably longer than that for an ordinary neutron
star only if the outer nuclear layer was very thin. (Note that, while little is known about the properties of thin boundary layers outside CFL matter, it is possible that photon emission from this layer might quickly drain the relatively small amount of CFL thermal energy.) Thus it appears that the cooling of stars with not too large CFL quark cores will differ little from the cooling of typical neutron stars. A similar conclusion has been reached for stars with regular, non-CFL quark interiors [28].

In passing it is interesting to speculate about the possibility that a bare CFL quark star made entirely of dense quark matter could exist. If it were possible, it would look like a transparent dielectric [29]. Our present study suggests such a star would also have very unusual thermal properties. Indeed, if the star has a finite temperature $T \lesssim \tilde{T}$ after it was created, almost all of its thermal energy would be stored in the NG bosons. Notice that all the photons would leave the star very soon after transparency set in because the star is assumed to have no nuclear matter layer. The local interaction as well as the self-interaction of the NG bosons is very weak so that we argued in Sec. 3 that their mean free path would be limited only by the geometrical size of the star. This suggests that, since photon and neutrino emission inside the CFL phase is strongly suppressed [13], the only potential source of energy loss in the bare CFL star would be the interaction of the NG bosons at the stellar boundary and photon emission there. It is likely, therefore, that such stars might be very dim and might even be good candidates for some of the baryonic dark matter in the Universe.

Acknowledgments

This work was supported by the U.S. Department of Energy Grant No. DE-FG02-87ER40328.

References

1. J.C. Collins and M.J. Perry, *Phys. Rev. Lett.* **34**, 1353 (1975).
2. B. C. Barrois, *Nucl. Phys.* **B129**, 390 (1977); S. C. Frautschi, in *Hadronic Matter at Extreme Energy Density*, edited by N. Cabibbo and L. Sertorio (Plenum, New York, 1980); D. Bailin and A. Love, *Phys. Rep.* **107**, 325 (1984).
3. M. G. Alford, K. Rajagopal and F. Wilczek, *Phys. Lett.* **B422**, 247 (1998); R. Rapp, T. Schäfer, E. V. Shuryak and M. Velkovsky, *Phys. Rev. Lett.* **81**, 53 (1998).
4. D. T. Son, *Phys. Rev.* **D59**, 094019 (1999); R. D. Pisarski and D. H. Rischke, *Phys. Rev. Lett.* **83**, 37 (1999).
5. T. Schafer and F. Wilczek, *Phys. Rev.* **D60**, 114033 (1999); D. K. Hong, V. A. Miransky, I. A. Shovkovy and L. C. R. Wijewardhana, *ibid.* **D61**, 056001
(2000); D62, 059903(E) (2000); R. D. Pisarski and D. H. Rischke, ibid. D61, 051501 (2000).
6. S. D. Hsu and M. Schwetz, Nucl. Phys. B572, 211 (2000); W. E. Brown, J. T. Liu and H. C. Ren, Phys. Rev. D61, 114012 (2000).
7. I. A. Shovkovy and L. C. R. Wijewardhana, Phys. Lett. B470, 189 (1999); T. Schäfer, Nucl. Phys. B575, 269 (2000).
8. M. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. B537, 443 (1999); note that the 2SC phase involving just two quark flavors is thought to be absent in compact stars, see M. Alford and K. Rajagopal, JHEP 0206, 031 (2002).
9. R. Casalbuoni and R. Gatto, Phys. Lett. B464, 111 (1999).
10. D. T. Son and M. A. Stephanov, Phys. Rev. D61, 074012 (2000); D62, 059902(E) (2000).
11. P. F. Bedaque and T. Schäfer, Nucl. Phys. A697, 802 (2002); D. B. Kaplan and S. Reddy, Phys. Rev. D65, 054042 (2002).
12. V. A. Miransky and I. A. Shovkovy, Phys. Rev. Lett. 88, 111601 (2002); T. Schäfer, D. T. Son, M. A. Stephanov, D. Toublan and J. J. Verbaarschot, Phys. Lett. B522, 67 (2001).
13. P. Jaikumar, M. Prakash and T. Schäfer, astro-ph/0203088.
14. I. A. Shovkovy and P. J. Ellis, Phys. Rev. C66, 015802 (2002).
15. E. J. Ferrer, V. P. Gusynin and V. de la Incera, cond-matt/0203217.
16. V. P. Gusynin and I. A. Shovkovy, Nucl. Phys. A700, 577 (2002).
17. D. T. Son, hep-ph/0204199.
18. T. Schäfer, hep-ph/0201189.
19. P. A. Sturrock, Plasma Physics (Cambridge University Press, Cambridge, 1994).
20. D. F. Litim and C. Manuel, Phys. Rev. D64, 094013 (2001).
21. J. M. Lattimer, K. A. Van Riper, M. Prakash and M. Prakash, Astrophys. J. 425, 802 (1994).
22. M. G. Alford, K. Rajagopal, S. Reddy and F. Wilczek, Phys. Rev. D64, 074017 (2001).
23. G. W. Carter and S. Reddy, Phys. Rev. D62, 103002 (2000).
24. A. W. Steiner, M. Prakash and J. M. Lattimer, Phys. Lett. B509, 10 (2001).
25. S. Reddy, M. Sadzikowski and M. Tachibana, nucl-th/0203011.
26. M. Prakash, J. M. Lattimer, J. A. Pons, A. W. Steiner and S. Reddy, Lect. Notes Phys. 578, 364 (2001).
27. S. L. Shapiro and S. A. Teukolsky, Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects (Wiley, New York, 1983).
28. D. Page, M. Prakash, J. M. Lattimer and A. W. Steiner, Phys. Rev. Lett. 85, 2048 (2000).
29. K. Rajagopal and F. Wilczek, Phys. Rev. Lett. 86, 3492 (2001).