Anomalous Couplings in $e^+e^- \rightarrow W^+W^-\gamma$ at LEP2 and NLC

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Abstract

We present sensitivity limits on the coefficients of a dimension–6 effective Lagrangian that parametrizes the possible effects of new physics beyond the Standard Model. Our results are based on the study of the process $e^+e^- \rightarrow W^+W^-\gamma$ at LEP2 and NLC energies. In our calculations, we include all the new anomalous interactions, involving vector and Higgs bosons, and take into account the Standard Model irreducible background. We analyse the impact of these new interactions on the total cross section, including the effects of the initial electron and final $W$ polarizations. We then focus on the operators that will not be constrained by the $e^+e^- \rightarrow W^+W^-$ process, obtaining limits based on the photon energy distribution.

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I. INTRODUCTION

One of the main physics goals of LEP2 and future $e^+e^-$ colliders is to directly test the gauge nature of couplings among the electroweak gauge bosons. The process with largest cross section at LEP2 involving these couplings is the $W$-pair production, $e^+e^- \rightarrow W^+W^-$, which is sensitive to the trilinear $WW\gamma$ and $WWZ$ couplings. The measurement of these couplings and the sensitivity to possible deviations from the Standard Model (SM) predictions have been extensively studied in the recent years [1].

The most general phenomenological parametrization for these couplings [2] can be achieved by means of an effective Lagrangian [3] that involves operators with dimension higher than four, containing the relevant fields at low energies and respecting the symmetries of the Standard Model. The effective Lagrangian approach is a model-independent way to describe new physics that can occur at an energy scale $\Lambda$ much larger than the scale where the experiments are performed.

The effective Lagrangian depends on the particle content at low energies and since the Higgs boson has not yet been found, there are two logical possibilities to describe the new physics effect at low energies. In one of them, the Higgs boson can be light, being present in the higher dimensional operators, in addition to the electroweak gauge bosons, and the SM symmetries are linearly realized [4,5]. Alternatively, the Higgs boson can be very heavy and it must be integrated out at low energies. In this case, the relevant fields at low energies are only electroweak gauge bosons and the SM symmetries are realized non-linearly [6]. Here we focus on a linearly realized $SU_L(2) \times U_Y(1)$ invariant effective Lagrangian to describe the bosonic sector of the Standard Model, keeping the fermionic couplings unchanged.

The same effective Lagrangian used to describe anomalous trilinear gauge couplings can, in general, lead to anomalous quartic interaction among gauge bosons and also to anomalous couplings of these particles with the Higgs field. All these interactions should also be investigated at LEP2 and at the Next Linear Colliders (NLC) in order to search for hints about the nature of the new physics described by these higher dimensional operators.
New quartic gauge boson couplings have been studied before in many different processes at future $e^+e^-$, $e\gamma$, $\gamma\gamma$, $e^-e^-$ and $pp$ colliders [7]. However, most of these previous works have focused on the so-called genuinely quartic operators, i.e. operators that give rise only to quartic gauge boson interactions without altering the trilinear couplings [8]. Since these operators do not appear in a dimension–6 linearly realized $SU(2)_L \times U_Y(1)$ invariant effective Lagrangian [9], they will not be considered here. Anomalous Higgs boson couplings have also been studied before in Higgs and Z boson decays [10], in $e^+e^- [11,12]$ and $\gamma\gamma$ colliders [13].

The process with largest cross section in $e^+e^-$ colliders that also involves quartic couplings, and possibly anomalous Higgs couplings, besides the trilinear couplings, is $e^+e^- \rightarrow W^+W^-\gamma$. Therefore, it is the most promising channel to look for possible deviations from the Standard Model predictions. This process has been considered by Bélanger and Boudjema [8] and by Leil and Stirling [14] in the context of genuinely quartic operators, where the Higgs and trilinear couplings were set to the Standard Model values and $3\sigma$ deviations in the total cross section were used to determine the reach of this reaction. Grosse-Knetter and Schildknecht [15] have considered the effect of a single higher dimensional operator usually denoted by $O_W$ (see below) in the above process, taking into account modifications on both trilinear and quartic couplings. However, they assumed that the Higgs boson mass lies above the energy region to be investigated and therefore they disregarded its contribution.

The purpose of this work is to study the sensitivity to these anomalous couplings of the process $e^+e^- \rightarrow W^+W^-\gamma$ at LEP2 and the NLC. We consistently include in our calculations all new couplings introduced by the effective Lagrangian that has become widely adopted to describe new physics beyond the Standard Model. In particular, this process is sensitive to operators related to anomalous Higgs boson couplings that do not affect the self-coupling of gauge bosons and hence are not constrained by the LEP2 measurements of $e^+e^- \rightarrow W^+W^-$. Therefore, the process $e^+e^- \rightarrow W^+W^-\gamma$ may provide important information about these operators at the NLC.
This paper is organized as follows. In Section II, we review the framework of effective Lagrangians that we use to parametrize anomalous couplings and explain the methodology used to study the $W^+W^-\gamma$ production. In Section III, we analyze the sensitivity at LEP2 based on the total cross section. In Section IV, we study the improvements arising from going to NLC energies, the effects of having a polarized electron beam, and the impact of being able to measure the $W$ boson polarization. We then concentrate on the analysis of operators which will not be probed by the $e^+e^-\rightarrow W^+W^-$ process, obtaining limits based on the photon energy spectrum. We present our conclusions in Section V.

II. EFFECTIVE LAGRANGIAN AND THE PROCESS $e^+ e^- \rightarrow W^+W^-\gamma$

In order to write down the most general dimension–6 effective Lagrangian containing all SM bosonic fields, i.e. $\gamma, W^\pm, Z^0,$ and $H,$ we adopt the notation of Hagiwara et al. [5]. This Lagrangian has eleven independent operators in the linear representation that are locally $SU_L(2) \times U_Y(1)$ invariant, $C$ and $P$ even. We discard the four operators which affect the gauge boson two–point functions at tree–level and therefore are strongly constrained by LEP1 measurements. We also do not consider the two operators that modify only the Higgs boson self–interactions, since they are not relevant for our calculations. We are then left with five independent operators, and the Lagrangian is written as,

$$L_{\text{eff}} = L_{\text{SM}} + \frac{1}{\Lambda^2} \left( f_{WWW}O_{WWW} + f_{WW}O_{WW} + f_{BB}O_{BB} + f_WO_W + f_BO_B \right), \quad (1)$$

with each operator $O_i$ defined as,

$$O_{WWW} = \text{Tr} \left[ \hat{W}_{\mu\nu} \hat{W}^{\nu\rho} \hat{W}^\mu_{\rho} \right] \quad (2)$$

$$O_{WW} = \Phi^\dagger \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \Phi \quad (3)$$

$$O_{BB} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \Phi \quad (4)$$

$$O_W = (D_\mu \Phi)^\dagger \hat{W}^{\nu\mu} (D_\nu \Phi) \quad (5)$$

$$O_B = (D_\mu \Phi)^\dagger \hat{B}^{\nu\mu} (D_\nu \Phi) \quad (6)$$
where $\Phi$ is the Higgs field doublet, which in the unitary gauge assumes the form,

$$
\Phi = \begin{pmatrix}
0 \\
(v + H)/\sqrt{2}
\end{pmatrix},
$$

and

$$
\hat{B}_{\mu\nu} = ig' \hat{B}_{\mu\nu}, \quad \hat{W}_{\mu\nu} = ig^2 \sigma^a W^a_{\mu\nu},
$$

with $B_{\mu\nu}$ and $W^a_{\mu\nu}$ being the field strength tensors of the $U(1)$ and $SU(2)$ gauge fields respectively.

The operator $O_{WWW}$ contributes only to anomalous gauge couplings, $O_{WW}$ and $O_{BB}$ contribute only to anomalous Higgs couplings, $HZZ$ and $HZ\gamma$, whereas $O_{W}$ and $O_{B}$ give rise to both types of new couplings. Therefore, the existence of anomalous trilinear gauge couplings could be related to the anomalous quartic gauge couplings and Higgs interaction, which are the subject of our investigation.

Studies of anomalous trilinear gauge boson couplings from $W$–pair production will significantly constrain combinations of the parameters $f_{WWW}$, $f_W$ and $f_B$. However they are "blind" with respect to $f_{WW}$ and $f_{BB}$. We chose to study the reaction $e^+e^- \rightarrow W^+W^-\gamma$ since it is the process with the largest cross section involving triple, quartic gauge boson couplings and also anomalous Higgs–gauge boson couplings. Therefore, it is also sensitive to $f_{WW}$ and $f_{BB}$, offering an excellent possibility for a detailed study of these couplings.

The Standard Model cross section for the process $e^+e^- \rightarrow W^+W^-\gamma$ was evaluated in Ref. [16]. When we neglect the electron mass, Higgs contributions for this reaction do not appear at tree level since the couplings $H\gamma\gamma$ and the $HZ\gamma$ are generated only at one loop [17,18]. Taking into account these contributions, there are 16 Feynman diagrams involved in the reaction $e^+e^- \rightarrow W^+W^-\gamma$, which are represented in Fig. 1 (the crossed diagrams are not shown) which yields,

$$
\sigma^{SM}_{WWW\gamma} = 46\ (418) \text{ fb}, \quad \text{with } E_\gamma > 20\ (5) \text{ GeV}, \quad \text{at } \sqrt{s} = 190 \text{ GeV}
$$

$$
\sigma^{SM}_{WWW\gamma} = 144 \text{ fb}, \quad \text{with } E_\gamma > 20 \text{ GeV}, \quad \text{at } \sqrt{s} = 500 \text{ GeV}
$$

(8)
where we have required that the angle between any two particles is larger than 15°. The cross section peaks at roughly $\sqrt{s} = 300$ GeV and is typically two orders of magnitude smaller than the two–body process $e^+e^- \rightarrow W^+W^-$, used to constrain anomalous trilinear couplings.

In order to compute the contribution from all possible anomalous couplings, we have developed a Mathematica code to automatically generate the Feynman rules for the Lagrangian (1) that were then incorporated in Helas–type [19] subroutines. These new subroutines were used to extend a Madgraph [20] generated code to include all the anomalous contributions and to numerically evaluate the helicity amplitudes and the squared matrix element. In our calculations, we have taken into account the standard loop Higgs contributions besides all the relevant anomalous couplings, which give rise to the 42 contributions shown in Fig. 1, 2, and 3. We have checked that our code passed the non–trivial test of electromagnetic gauge invariance. We employed Vegas [21] to perform the Monte Carlo phase space integration with the appropriate cuts to obtain the differential and total cross sections. Moreover, we have studied the angular variables in order to find optimal cuts to improve the anomalous contribution over the SM signal.

III. $WW\gamma$ PRODUCTION AT LEP2

We studied the reaction $e^+e^- \rightarrow W^+W^-\gamma$ at LEP2 assuming a center–of–mass energy of $\sqrt{s} = 190$ GeV and an integrated luminosity of $L = 0.5$ fb$^{-1}$. We applied a cut in the photon energy ($E_\gamma > 5$ GeV), and we required the angle between any two particles to be larger than $\theta_{ij} > 15^\circ$.

Our results for the sensitivity of LEP2 to the operators appearing in the effective Lagrangian (1), from an analyses of the total cross section, are summarized in Fig. 4 for a fixed value of the Higgs boson mass, $M_H = 170$ GeV. We plot the contributions of the 5 different operators separately, assuming that only one operator contributes each time. We also show the result for an extension of the so–called HISZ scenario [3], where all the coefficient are
considered equal, i.e. $f_{WWW} = f_{WW} = f_{BB} = f_W = f_B = f$, in order to reduce the number of free parameters to only one ($f$). The Standard Model cross section and its value with 1, 2 and 3 $\sigma$ deviations are depicted as horizontal lines.

The most sensitive contribution comes from $O_{WWW}$, $O_W$, and $O_B$. A 1$\sigma$ deviation in the total cross section would be observed for the following ranges of the coefficients of these operators, for $\Lambda = 1$ TeV,

\[-75 < f_{WWW} < 178, \ -48 < f_W < 192, \ -188 < f_B < 550, \ -253 < f_{WWW} < 110;\]

(9)

whereas for the extended HISZ scenario, we have,

\[-33 < f < 119.\]

(10)

Of course, the operators that also give rise to changes in the triple vector boson couplings can also be constrained at LEP2 via the reaction $e^+e^- \rightarrow W^+W^-$. A recent analyses of $W$–boson pair production based on a log–likelihood fit to a five–fold differential cross section obtained the 1$\sigma$ limits [22], $|f_{WWW}| < 10$, $|f_W| < 7.1$, and $|f_B| < 46$. However, one should keep in mind that this reaction is insensitive to $f_{WWW}$ and $f_{BB}$, and therefore the study of the process $e^+e^- \rightarrow W^+W^-\gamma$ can provide further information on these operators, as we show in this paper.

The contribution of the anomalous couplings involving only the Higgs boson, i.e. $f_{WW}$ and $f_{BB}$ (see Fig. 3), is dominated by on–mass–shell Higgs production with the subsequent $H \rightarrow W^+W^-$ decay,

$$\sigma(e^+e^- \rightarrow W^+W^-\gamma) \propto \sigma(e^+e^- \rightarrow H\gamma) \times \frac{\Gamma(H \rightarrow W^+W^-)}{\Gamma(H \rightarrow \text{all})}.\tag{11}$$

For large values of the operator coefficients, the total Higgs width is dominated by the anomalous decay $H \rightarrow \gamma\gamma$ [10], which is also proportional to $f_{WWW}$ and $f_{BB}$. On the other hand, the anomalous width $\Gamma(H \rightarrow W^+W^-)$ depends only on $f_{WWW}$. Therefore, the contribution from the anomalous coupling $f_{BB}$ is much less sensitive than the contributions from
the other operators since $\sigma(e^+e^- \rightarrow W^+W^-\gamma)$ becomes almost independent of this coefficient. Fortunately, this is not the case if one is sensitive to small values of the coefficients, as will occur at the NLC study in the next Section.

We have investigated various distributions to try to improve the LEP2 sensitivity. The most promising distribution is the angular distribution of the $W$ bosons with respect to the beam direction (see Fig. 5). We computed the total cross section with the extra cut $\cos \theta_{W^+e^+} > 0$, as suggested by this distribution, and found an increase in sensitivity from 2$\sigma$ to 2.8$\sigma$. However, due to the small deviations in the shape of the kinematical distributions and small statistics, no further improvement seems to be possible.

**IV. WW$\gamma$ PRODUCTION AT NLC**

The effect of the anomalous operators becomes more evident with the increase of energy, and we are able to put tighter constraints on the coefficients by studying their contribution to different processes at the Next Linear Collider. We studied the sensitivity of NLC to the process $e^+e^- \rightarrow W^+W^-\gamma$ assuming $\sqrt{s} = 500$ GeV and an integrated luminosity $\mathcal{L} = 50$ fb$^{-1}$. We adopted a cut in the photon energy of $E_\gamma > 20$ GeV and required the angle between any two particles to be larger than 15°. We have analyzed this process for different values of the Higgs boson mass.

In Fig. 6, we show the results for the total cross section, for $M_H = 170$ GeV, including the effects of the anomalous operators. The values of the coefficients $f$’s for which a 2$\sigma$ deviation is obtained are shown in Table 1, being typically of the order of $1 - 10$ TeV$^{-2}$. As we could expect, the $W$–pair production at NLC is able to put a limit that is one order of magnitude better for the coefficients $f_{B,W,WW}$ [22]. However this latter reaction is not able to constraint $f_{BB,WW}$.

In an attempt to increase the sensitivity, we looked at the effects of a 90% polarized electron beam in order to reduce the SM background, mainly the one coming from diagrams of Fig. 1a and 1b were just left–handed electrons are present. We have considered both left–
handed (LH) and right–handed (RH) polarizations, expecting a larger anomalous sensitivity for RH electrons.

In Fig. 7, we show the results for the total cross section, for a 90% right–handed (RH) polarized electron, for $M_H = 170$ GeV. Comparing Fig. 6 and Fig. 7, we notice that the effect of the anomalous contributions in the total cross section are larger for the polarized case. However, the small absolute value of the cross section for the polarized case reduces the statistics and leads to no improvement in the established limits, as shown in Table II.

Since we expect the new interactions to involve mainly longitudinally polarized gauge bosons, we studied the sensitivity for different combinations of the polarizations of the $W$–pair. In Fig. 8, we show the analogous of Fig. 6 for the $W_L W_L$ case. Again, the effect of the anomalous contributions to the total cross section is increased, but no further improvements are found due to the small statistics. The results for the bounds on the anomalous coefficients for the $W_L W_L, W_T W_T$, and $(W_L W_T + W_T W_L)$ cases can be seen in Table III. These bounds were obtained requiring a $2\sigma$ effect on the total cross section.

It is important to notice that the kinematical distributions of the longitudinally polarized $W$’s are quite different from the SM results. As we could expect, the new physics effects becomes more evident for longitudinal $W$’s since the decay $H \to W^+ W^-$ is dominated by this state of polarization. In Fig. 9, we present the angular distribution of the longitudinal $W^+$ boson with the initial positron and with the final photon, the energy and the transverse momentum distributions. We can see, for instance, that the $W$ energy distribution is very different from the SM prediction. Its characteristic behavior for $100 < E_W < 175$ GeV is due to the presence of the Higgs boson, which decays into the $W$ pair giving rise, at the same time, to a monochromatic photon.

We present in Fig. 10 the percent deviation of the SM prediction in the photon transverse momentum distribution, i.e.,

$$\Delta = \left( \frac{d\sigma_{\text{ANO}}}{d\mathbf{p}_T} / \frac{d\sigma_{\text{SM}}}{d\mathbf{p}_T} - 1 \right) \times 100\% ,$$

for the different polarization of the $W$’s. Once again the relevance of the $W_L W_L$ case is
evident: $\Delta > 100\%$ for $p_{T_\gamma} > 120\text{ GeV}$. When a cut of $p_{T_\gamma} > 100\text{ GeV}$ is implemented, the background is drastically reduced and the ratio of anomalous over SM events per year goes from 576/442 to 424/74, for $f_{\text{all}} = 15\text{ TeV}^{-2}$.

Using the reaction $e^+e^- \rightarrow W^+W^-\gamma$, we are also able to establish bounds on the values of the coefficients $f_{WW}$ and $f_{BB}$, for which the $W$–pair process is insensitive, since they only affect the Higgs boson couplings. In Fig. 11, we present the results of a combined sensitivity analysis in the form of a contour plot for the two free parameter, $f_{BB}$ and $f_{WW}$, for $M_H = 170\text{ GeV}$. These are the most relevant coefficients for the anomalous Higgs boson phenomenology and they are not constrained by the $W$–pair production. We should keep in mind that the $WW\gamma$ production at LEP2 can put a $1\sigma$ bound on $f_{WW}$ while it is not possible to impose a limit on $f_{BB}$ since the cross section is quite insensitive to this coefficient.

If the Higgs boson is found with a mass in the range from 170 to 300 GeV, one would have a large sensitivity for the anomalous Higgs couplings $f_{WW}$ and $f_{BB}$ in the photon energy distribution of the process $e^+e^- \rightarrow W^+W^-\gamma$. This increased sensitivity comes about because the existence of a peak in the photon energy spectrum due to the 2–body nature of the dominant contribution, i.e. $e^+e^- \rightarrow H\gamma$ followed by the subsequent decay $H \rightarrow W^+W^-$ (see Fig. 3). In Fig. 12, we illustrate this effect with a typical photon energy distribution, for $f_{WW}/\Lambda^2 = f_{BB}/\Lambda^2 = 5\text{ TeV}^{-2}$ and $M_H = 170\text{ GeV}$, where the Higgs peak appears very clearly in the photon spectrum of the anomalous contribution.

In order to analyse the significance of the signal based on the photon energy spectrum, we took different energy bins of 1, 3 and 5 GeV. The reason is to roughly mimic the effects of a realistic simulation including the finite energy resolution of the detector and the small spread in the real center–of–mass energy due to initial state radiation. We have not considered the experimental efficiency, $\epsilon_{\text{eff}}$, for $W$ reconstruction. It can be easily incorporated by multiplying the obtained significances by $\sqrt{\epsilon_{\text{eff}}}$. Table V shows the improvement on the sensitivity compared to the total cross section analysis for the $f_{WW}/\Lambda^2 = f_{BB}/\Lambda^2 = 5\text{ TeV}^{-2}$ and $M_H = 170\text{ GeV}$ case.

In Table V, we present our results for the sensitivity on $f_{BB}/\Lambda^2$ and $f_{WW}/\Lambda^2$, assuming
$f_{BB} = f_{WW}$, for the three energy bins above. We obtained a sensitivity of the order of a TeV$^{-2}$ for $M_H = 170$ GeV, decreasing by a factor of roughly four for $M_H = 300$ GeV, which does not depend in a significant way of the bin size. For larger Higgs boson masses, the cross section is reduced due to phase space suppression. For smaller Higgs boson masses, the cross section is reduced since the Higgs boson is off–mass–shell, and in this case it would be better to study processes like $e^+e^- \rightarrow b\bar{b}\gamma$ or $e^+e^- \rightarrow \gamma\gamma\gamma$ [12].

V. CONCLUSION

The search for the effect of higher dimensional operators that give rise to anomalous bosonic couplings should be pursued in all possible processes since the results may provide important information on physics beyond the Standard Model. We have studied here the production of a $W$–pair plus a photon in $e^+e^-$ colliders in order to analyse the contributions of anomalous couplings arising from dimension–6 operators of a linearly realized $SU_L(2) \times U_Y(1)$ invariant effective Lagrangian. We have included all the anomalous trilinear and quartic gauge couplings, as well as the anomalous Higgs couplings with gauge bosons.

We present the limits attainable at LEP2 and at NLC, including the Standard Model irreducible background. Polarization of the electron beam and of the $W$–pair are found to be insufficient to improve the limits obtained from the total cross section.

We also focused on the operators $O_{WW}$ and $O_{BB}$, which cannot be tested in the $W$–pair production process. We showed, in particular, that for Higgs boson masses in the range $M_H = 170–300$ GeV, the photon energy spectrum provides a sensitive signature for the anomalous Higgs couplings. Typical sensitivities of a few TeV$^{-2}$ at the NLC are obtained for these coefficients, providing complementary information on different higher dimensional operators.
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FIGURES

FIG. 1. Feynman diagrams for the Standard Model process $e^+e^- \rightarrow W^+W^-\gamma$. Crossed diagrams are not shown.

FIG. 2. The vector bosons anomalous contributions to $e^+e^- \rightarrow W^+W^-\gamma$. Crossed diagrams are not shown.

FIG. 3. The Higgs boson anomalous contributions to $e^+e^- \rightarrow W^+W^-\gamma$.

FIG. 4. Total cross section (SM + Anomalous) for the process $e^+e^- \rightarrow W^+W^-\gamma$, at LEP2 as a function of different anomalous coefficients and also for the HISZ scenario ($f_{all}$). We assumed $m_H = 170$ GeV, and $L = 0.5$ fb$^{-1}$. The results for the SM and for 1, 2, and 3$\sigma$ deviations are displayed (see text for energy and angular cuts).

FIG. 5. Normalized $W^+ - e^+$ angular distribution. The solid (dashed) line represents the SM (SM + Anomalous) contribution for $f_{all}/\Lambda^2 = 150$ TeV$^{-2}$ and $m_H = 170$ GeV.

FIG. 6. The same as Fig. 4 for NLC, with $\sqrt{s} = 500$ GeV and $L = 50$ fb$^{-1}$.

FIG. 7. The same as Fig. 6 for a 90% right–handed polarized electron, with $\sqrt{s} = 500$ GeV and $L = 50$ fb$^{-1}$.

FIG. 8. The same as Fig. 6 for longitudinal $W$ bosons ($W_LW_L$), with $\sqrt{s} = 500$ GeV and $L = 50$ fb$^{-1}$.

FIG. 9. Kinematical distributions of the longitudinally polarized $W^+$ vector boson for the SM (solid histogram) and for the anomalous contribution (dotted histogram).

FIG. 10. Plot of deviation ($\Delta$) in the photon $P_T^\gamma$ distribution for the cases of $W_LW_L$ (solid line), $W_LW_T + W_TW_L$ (dashed line) and $W_TW_T$(dotted line).
FIG. 11. Contour plot of $f_{BB} \times f_{WW}$, for $M_H = 170$ GeV. The curves show the one, two, and three sigma deviations from the Standard Model value of the total cross section.

FIG. 12. Photon energy distribution for the SM (solid line) and for the SM + Anomalous (dashed line), for $f_{WW}/\Lambda^2 = f_{BB}/\Lambda^2 = 5$ TeV$^{-2}$, and $M_H = 170$ GeV, with a 5 GeV bin.
### TABLE I
The minimum and maximum values (min, max) of the coefficients $f_i/\Lambda^2$ in units of $\text{TeV}^{-2}$ for a $2\sigma$ deviation of the unpolarized total cross section.

| Anomalous Couplings | $f_{\text{all}}/\Lambda^2$ | $f_{B}/\Lambda^2$ | $f_{BB}/\Lambda^2$ | $f_{W}/\Lambda^2$ | $f_{WW}/\Lambda^2$ | $f_{WWW}/\Lambda^2$ |
|---------------------|-----------------------------|-------------------|-------------------|------------------|------------------|------------------|
|                     | $(-2, 5)$                   | $(-5, 31)$        | $(-11, 7)$        | $(-3, 23)$       | $(-8, 4)$        | $(-5, 5)$        |

### TABLE II
The minimum and maximum values (min, max) of the coefficients $f_i/\Lambda^2$ in units of $\text{TeV}^{-2}$ for a $2\sigma$ deviation of the total cross section with 90% polarized LH and RH electrons for a reduced luminosity of $\mathcal{L} = 25 \text{ fb}^{-1}$.

| Anomalous Couplings | $e_{\text{LH}}$ | $e_{\text{RH}}$ |
|---------------------|-----------------|-----------------|
| $f_{\text{all}}/\Lambda^2$ | $(-2, 5)$ | $(-3, 5)$ |
| $f_{B}/\Lambda^2$ | $(-11, 42)$ | $(-2, 26)$ |
| $f_{BB}/\Lambda^2$ | $(-19, 17)$ | $(-8, 3)$ |
| $f_{W}/\Lambda^2$ | $(-2, 26)$ | $(-17, 9)$ |
| $f_{WW}/\Lambda^2$ | $(-7, 5)$ | $(-15, 11)$ |
| $f_{WWW}/\Lambda^2$ | $(-5, 5)$ | $(-12, 11)$ |
### TABLE III. The minimum and maximum values (min, max) of the coefficients \( f_i/\Lambda^2 \) in units of TeV\(^{-2} \) for a 2σ deviation of the total cross section for different combinations of the final state W–pair polarization.

| Anomalous Couplings | \( W_L W_L \) | \( W_T W_T \) | \( (W_L W_T + W_T W_L) \) |
|---------------------|---------------|---------------|----------------------------|
| \( f_{all}/\Lambda^2 \) | (−1, 14) | (−5, 3) | (−2, 7) |
| \( f_B/\Lambda^2 \) | (−2, 35) | (−29, 29) | (−9, 36) |
| \( f_{BB}/\Lambda^2 \) | (−12, 9) | (−14, 10) | (−13, 8) |
| \( f_W/\Lambda^2 \) | (−1, 25) | (−36, 22) | (−4, 23) |
| \( f_{WW}/\Lambda^2 \) | (−8, 6) | (−9, 6) | (−8, 5) |
| \( f_{WWW}/\Lambda^2 \) | (−51, 96) | (−6, 4) | (−5, 25) |

### TABLE IV. Number of standard deviations \( \sigma \) from the Standard Model from a sensitivity analysis based on the total cross section compared to a sensitivity analysis based on the peak of the photon energy distribution, considering a 1, 3 and 5GeV bin for different values of the Higgs mass. We fixed \( f_{WW}/\Lambda^2 = f_{BB}/\Lambda^2 = 5 \text{ TeV}^{-2} \).

| \( M_H \) (GeV) | Total Cross Section | 1 GeV bin | 3 GeV bin | 5 GeV bin |
|-----------------|-------------------|-----------|-----------|-----------|
| 170             | 4.2               | 52.2      | 43.1      | 35.8      |
| 200             | 2.8               | 17.8      | 20.9      | 17.8      |
| 250             | 1.8               | 8.3       | 10.7      | 9.9       |
| 300             | 1.0               | 2.5       | 3.8       | 4.3       |
| $M_H$ (GeV) | Total Cross Section | 1 GeV bin | 3 GeV bin | 5 GeV bin |
|-------------|---------------------|----------|----------|----------|
| 170         | $(−5.9, 2.8)$       | $(−3.9, 0.3)$ | $(−3.9, 0.4)$ | $(−3.9, 0.5)$ |
| 200         | $(−6.4, 3.6)$       | $(−4.4, 0.9)$ | $(−4.1, 0.8)$ | $(−4.2, 0.9)$ |
| 250         | $(−7.0, 4.9)$       | $(−4.2, 1.8)$ | $(−3.9, 1.6)$ | $(−4.0, 1.6)$ |
| 300         | $(−8.3, 6.9)$       | $(−6.2, 4.3)$ | $(−5.1, 3.2)$ | $(−4.9, 3.0)$ |

TABLE V. The minimum and maximum values (min, max) of the coefficients $f_i/\Lambda^2$ (for $f_{BB} = f_{WW} = f$) in units of TeV$^{-2}$ that generate a 95% C.L. signal for the total cross section analysis and for the photon energy spectrum analysis with 1, 3 and 5 GeV energy bins for different values of the Higgs mass.
Fig. 1
Fig. 2
Fig. 3
\( (1/\sigma) d\sigma / d\cos\theta_{We} \)
\[
\frac{1}{\sigma} \frac{d\sigma}{d\cos \theta_{W\gamma}} \quad \frac{1}{\sigma} \frac{d\sigma}{d\cos \theta_{W e}}
\]

Fig. 9
\(\frac{d\sigma}{dE_\gamma}(\text{fb/GeV})\)