Application of the non-linear harmonic method to study the rotor-stator interaction in Francis-99 test case

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Abstract. Steady state and non-linear harmonic (NLH) flow simulations were performed within the framework of the Francis-99 project in order to assess the capacity of the NLH method to capture the main pressure fluctuations associated with the rotor-stator interactions between the distributor and the runner of the turbine. This paper focuses on the methodology developed to obtain harmonic solutions and presents preliminary results from the simulations using the flow solver NUMECA FineTURBO on intermediate grid level meshes. Comparisons of the first simulations to experimental data reveal good agreement concerning the predicted pressure amplitudes notably at high load operating condition.

1. Introduction

The process of energy extraction in hydraulic turbines requiring the presence of rotating and non-rotating hydraulic surfaces leads to pressure fluctuations that translate in unsteady hydrodynamic forces on the different parts of the machines. Those fluctuations originate from different flow phenomena like the part load vortex, the inter-blade channel vortex or the rotor-stator interaction. This last phenomenon, present for all operating conditions at fixed frequencies based on the runner rotation speed, is highly dependent on the flow rate and the geometry of the turbines. It will mainly affect the runner blades and the guide vanes through the interaction of the non-uniform pressure field associated with the flow in both rows of blade.

The prediction of the unsteady loading stemming from rotor-stator interaction is of the foremost importance to perform structural analysis that will provide life span and maintenance estimates at the design stages. Nowadays, Computational Fluid Dynamics (CFD) has deepened the general understanding of the rotor-stator interaction as well as giving what might be the only tool to evaluate it. The typical method consists of performing simulations of the complete interactions between the components in the time-domain. Although this approach can be quite precise, it does require significant computing resources and more importantly leads to long simulation time.

The nonlinear harmonic decomposition method (NLH) implemented within the CFD package FineTURBO of the NUMECA Corporation was aimed specifically at lowering significantly those computing costs by focusing only on the main frequencies associated with the rotor-stator interaction. NLH simulations have already been performed successfully on different cases using the FineTURBO solver by [5, 9, 10], where the reduction in computational costs was also highlighted. The Francis-99 test case [6], with its high head Francis turbine, is well suited for the application of the method since the rotor-stator interaction causes the dominant fluctuations within the runner. This paper focuses on preliminary NLH simulations results for two operating conditions validated through experimental
measurements performed at the Waterpower laboratory, NTNU, Norway [2]. The paper presents a brief overview of the NLH method, the methodology and an analysis of the first results.

2. Non-linear harmonic method
The non-linear harmonic method (NLH), introduced by He and Ning [4], is based on the unsteady Reynolds Averaged Navier-Stokes (URANS) formulation. For an incompressible fluid, the equations are:

\[
\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial U_i}{\partial x_j} - \tau_{ij} \right)
\]

where \(U_i\) are the mean velocity components, \(P\) the mean pressure, \(\nu\) the kinematic viscosity and \(\tau_{ij}\) the Reynolds stresses. A simple derivation of the harmonic method is done by decomposing these equations into a time-averaged flow and unsteady perturbations. The decomposition of the conservative variables into a mean value and periodic perturbations has the form:

\[
\vec{U} = \overline{\vec{U}} + \vec{U}'
\]

Substituting equation (2) in the unsteady equations (1) and time-averaging them yields the mean flow equations. Considering only one perturbation and adding a pseudo time-dependence, this gives:

\[
\frac{\partial \overline{U}_i}{\partial t} + \overline{U}_j \frac{\partial \overline{U}_i}{\partial x_j} + \frac{\partial U'_i U'_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \overline{P}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \overline{U}_i}{\partial x_j} - \tau_{ij} \right)
\]

Because of the non-linearity of the URANS equations, this operation brings new terms that must be modeled, the deterministic stresses \(\overline{U'_i U'_j}\), which are similar to the Reynolds stresses appearing when time-averaging the Navier-Stokes equations. These terms are responsible for the coupling between the perturbations and the time-mean flow, hence the nonlinearity of the method. Subtracting the time-mean flow equations (4) from the URANS equations (1) and keeping only the first order terms yields the transport equations required to solve the unsteady perturbations:

\[
\frac{\partial U'_i}{\partial x_j} = 0
\]

The complex perturbations (3) can then be substituted in the transport equations to obtain the non-linear harmonic method equations. Equation (4) is equivalent to a RANS formulation that is solved to obtain the mean flow field and equation (5) is solved simultaneously for each of the complex amplitudes \(\Phi_k\), which are space dependent only. The final solution can be constructed in time by computing the perturbations at a given time using equation (3) and adding them to the time-mean flow. This operation allows to obtain an approximation of the unsteady flow field. It should be noted
that interactions between the perturbations themselves can only be achieved through their respective interaction with the mean flow by the deterministic stresses. More details about this formulation and its implementation can be found in [4, 9].

The resolution of the perturbations equations in the frequency domain allows the usage of periodic computational domains with mixing plane interfaces, thus lowering the required memory and simulation time. Periodic boundary conditions are implemented using a simple phase shift of the complex amplitudes, which are solved for the frequencies of the perturbations coming from adjacent computational domains with different rotational speeds.

3. Francis-99 test case

3.1. Geometry

The Francis-99 test case is a model turbine installed and investigated at the Water Power Laboratory at the Norwegian University of Science and Technology (NTNU) in Norway. It is a 1:5.1 model of a high head Francis turbine in operation at the Tokke power plant containing a spiral casing, 14 stay vanes, 28 guide vanes, and a runner with 15 full length and 15 splitter blades. The model is illustrated in Figure 1. More details about the test case and its geometry can be found in [2, 6]. For the needs of this project, the complete turbine geometry was reconstructed with ProEngineer according to the model drawings and compared with the provided geometry. The process resulted in a slight decrease, below 1%, of the distributor height and adjustment of the runner meridional profile.

![Figure 1. Tokke model turbine geometry.](image)

3.2. Operating points

For the first Francis-99 workshop, three operating conditions are studied with experimental measurements data provided at part load, best efficiency and high load. These operating conditions are summarized in Table 1.

|                | Part load | BEP    | High load |
|----------------|-----------|--------|-----------|
| Net head [m]   | 12.29     | 11.91  | 11.84     |
| Guide vanes angle $\alpha$ [°] | 3.91     | 9.84   | 12.44     |
| Flow rate [m$^3$/s] | 0.071    | 0.203  | 0.221     |
| Runner frequency [Hz] | 6.77    | 5.59   | 6.16      |
| Efficiency [%]  | 71.69    | 92.61  | 90.66     |

3.3. Pressure measurements

Pressure measurements are provided at six locations illustrated in Figure 2: one in the vaneless space between the guide vanes and the runner blades (VL01), two on a blade pressure side (P42 and P71),
one on the suction side of the adjacent blade (S51) and two in the draft tube cone (DT11 and DT21). Again, more details concerning the pressure measurement campaign can be found in [2].

4. Numerical methodology
The simulations are conducted with both standard RANS method and nonlinear harmonic method using the same computational domains for the distributor and the runner. The numerical methodology and simulation parameters used are described in this section.

4.1. Computational domains and boundary conditions
The computational domains of the distributor and runner have a periodicity of 25.7° and 24°, respectively, and are illustrated in Figure 3a. The value of the periodicity angle is important since it is used to compute the perturbation frequency and harmonics by the NLH solver. The axisymmetric geometry of the distributor is obtained by the revolution of a representative spiral casing section. The inlet is located at the radius where the spiral casing height is maximal and the distributor domain includes one stay vane and two guide vanes. The runner domain includes one full length and one splitter blade, as well as the conical part of the draft tube. The inclusion of the conical diffuser in the periodic runner domain implies that one cannot expect to capture different perturbations than the NLH frequencies coming from the distributor domain.

At the inlet, the mass flow rate, flow angle and turbulent viscosity are imposed. The flow angle is set to a constant value of \( \alpha = 5^\circ \) relative to the boundary and a turbulent viscosity \( \nu_t = 1 \times 10^{-4} \text{ m}^2/\text{s} \) is imposed, both values coming from spiral casing simulations. It was verified with steady simulations that the results were not highly sensitive to these parameters. At the outlet of the conical diffuser, an average static pressure is imposed. Modelling of the rotor-stator interface between the distributor and runner is made with a mixing plane approach for the steady simulations while a non-reflecting mixing plane approach is used for the NLH simulations.

4.2. Flow solver and numerical parameters
The software used for the computations is NUMECA FineTURBO v9.0-3, a structured multigrid solver specialized in turbomachinery cases. The Spalart-Almaras turbulence model is used with an extended wall function and a finite volume second order central spatial discretization is used. A multi-stage Runge-Kutta explicit scheme is used to advance the solution in time with local time stepping and
a CFL number equal to 3. Finally, nonlinear harmonic simulations are set to compute one perturbation per domain, i.e. the perturbation coming from the domain separated by the rotor-stator interface. Simulations are conducted using two \((n = 2)\) or four \((n = 4)\) harmonics to represent the perturbation, with harmonic number two being related to the relative blade passing frequency.

4.3. Meshes
The meshes are made using NUMECA Autogrid5 9.0-3 for the distributor and runner and NUMECA IGG 9.0-3 for the diffuser. These meshers produce structured multigrid meshes readily functional with the FineTURBO solver. The simulations are conducted on two different grid levels of the meshes: an intermediate grid level denoted 111, and a fine grid level denoted 000 which represents the finer grid of a given mesh. Examples of these two grid levels are shown in Figure 3b and 3c, respectively.

Grid orthogonality, aspect ratios and expansion ratios are checked to assure a good quality according to NUMECA recommendations, and grid distribution is made to be identical on both sides of the mixing plane interface in the three spatial directions. In addition, all meshes are checked to produce \(y^+\) values above \(\sim 30\) on the finer grid level so the turbulence model with wall functions can be used.

\[ \begin{align*}
\text{Inlet} & \quad m, v_z, \alpha \\
\text{Mixing plane interface} & \\
\text{Runner + Conical diffuser} & \\
\text{Outlet} & \quad \vec{p}_s
\end{align*} \]

\textbf{Figure 3.} a) Computational domains and boundary conditions; b) Intermediate grid level of the mesh (111); c) Fine grid level of the same mesh (000).

In order to obtain solutions that are not dependent on the mesh refinement level, steady state simulations were conducted with three different meshes (coarse, medium and fine) at the two grid levels for each of them. Analysis of the results showed that pressure values are weakly affected by the grid level or mesh used. For a given grid level, differences range from 0.1% in the runner to 1.5% in the vaneless space between the three mesh refinements. For a given mesh, the differences between the grid levels 111 and 000 are around 1.5% in the vaneless space, and 0.5% in the runner.
Figure 4 shows the head loss in the different components of the simulations using the three meshes and the two grid levels, indicating that the medium mesh, whose grid levels are illustrated in Figure 3, is adequate to obtain good results. Computation of the grid convergence index (GCI) for this mesh on each component yields errors of 1.7%, 0.6% and 0.4% for the distributor, runner and conical diffuser respectively [1]. Since this refinement level also produces acceptable computation times, the medium mesh is used for the simulations presented in this paper. Its statistics and quality on the finer grid level are detailed in Table 2.

![Figure 4](image-url)  Head losses in the distributor, runner and conical diffuser for the three meshes at the two different grid levels.

| Component | Nb. cells | Min. angle ° | Max. aspect ratio | Max. exp. ratio | Max. y+ |
|-----------|-----------|--------------|-------------------|-----------------|---------|
| Distributor | 5 107 200 | 34.9 | 20.2 | 2 | 80.7 |
| Runner | 6 300 992 | 33.7 | 66.7 | 1.77 | 87.6 |
| Diffuser | 2 078 272 | 33.6 | 141 | 1.34 | 58.2 |

5. Results
In this section, preliminary results are shown from steady and NLH simulations at the best efficiency point and high load. For both operating points, steady simulations were made at grid levels 111 and 000. Non-linear harmonic simulations are presented on grid level 111 only with 2 and 4 harmonics. The residuals, the real and imaginary parts of the pressures at each measurement points for each harmonic and other performance quantities were monitored during the simulations. The simulations were considered converged when all the performance and harmonic values were stabilized.

5.1. Mean pressure values
Comparison of the steady and harmonic simulations on both grid levels showed negligible differences in the averaged pressure predictions for all average pressures at VL01, P42, P71, S51 and DT when compared to the fine grid steady simulations, with differences ranging from ~0.0% in the conical diffuser up to ~0.5% in the vaneless space for the simulation on intermediate grid level at high load. The mean flow stemming from the harmonic computations is thus in accordance with the results from the steady computations with regard to the mean pressure field prediction.
The averaged numerical pressure values from the steady simulation on the fine grid for the best efficiency condition are compared to the average of the experimental pressure signals in Figure 5. Globally, the simulations are showing good agreement with the experimental values with a maximum difference of ~5.5% in the vaneless space. The simulations do appear to overestimate slightly the average pressure in the distributor and runner. It should be noted that the computed data is corrected according to the atmospheric pressure and water level in the test rig downstream tank. The draft tube experimental value shown corresponds to the average of sensors DT11 and DT21.

![Figure 5. Mean experimental and numerical pressure values at BEP.](image)

The computed values at high load are shown and compared to the averaged experimental pressures in Figure 6. In the same way as for the BEP, the simulations overestimate the pressure in the distributor and runner, and underestimate it in the draft tube. Here, the maximum difference of ~7.5% arises at the P42 sensor on the runner blade pressure side.

![Figure 6. Mean experimental and numerical pressure values at high load.](image)

5.2. **BEP pressure amplitudes and signals**

Pressure frequency spectra are computed with the complex amplitudes from the intermediate grid harmonic simulations and compared to the Fourier spectral analysis of the experimental values at the NLH frequencies. The FFT were performed on the experimental pressure signals using a Hanning window to prevent side lobe leakage.

The spectra for the best efficiency condition are illustrated in Figure 7 and show that the simulations generally overestimate the amplitude related to the blade passing frequency \( F/F_{runner} = 28 \) or 30) in the vaneless space and underestimate the other fluctuation harmonics. Caution should be exercised when comparing amplitudes below 0.05 kPa for the VL01 sensor and 0.2 kPa in the runner.
since they fall in the measurements noise range. The computation with $n = 4$ allows a slight prediction improvement over the $n = 2$ simulations, with a decrease of $\sim 7.2\%$ of the relative blade passing frequency amplitude in the distributor and $\sim 1.2\%$ for each of the sensors on the runner blade. The relative amplitudes between the third and fourth harmonics also appear to match.

**Figure 7.** Experimental and numerical frequency spectra for the main perturbation harmonics at best efficiency point.

The reconstructed signals from the NLH simulations are compared to the experimental ones for the VL01 and P42 sensors in Figure 8 and Figure 9, respectively. The experimental pressures are averaged over ten full runner rotations and a bandstop filter was used to remove the electrical grid frequency (50 Hz) and its harmonics. The numerical VL01 signal shows similarities with the experimental data regarding the global amplitude and frequencies. The mean difference lies in the over prediction of the second harmonic relatively to the three others and in the other frequencies present experimentally.

**Figure 8.** Experimental and NLH pressure signals for the VL01 sensor at best efficiency point.
Similar observations are made when comparing the numerical and experimental signals for the P42 sensor, where the mean amplitude of the signal seems to be respected even though the fluctuations from the simulations are still slightly lower than the measured values. The important energy content from the turbulent fluctuations over a wide frequency range, absent from the simulations, can explain this behaviour.

**Figure 9.** Experimental and NLH pressure signals for the P42 sensor at best efficiency point.

**Figure 10.** Experimental and numerical frequency spectra for the main perturbation harmonics at high load.

5.3. High load pressure amplitudes and signals
The results from the two harmonics simulation at high load, illustrated in Figure 10, show a better agreement with the experimental values for the blade passing frequency, with differences between ~0.9% for the P71 sensor and ~10.5% for the P42 sensor. The first harmonic for the VL01 sensor is
also predicted with a better accuracy than the BEP case. Once again, the general tendency of the computations is to overestimate the second harmonics and underestimate the first one.

Comparison of the reconstructed signal at VL01 with the experimental measurements also shows a very good agreement. The amplitudes are, in fact, well predicted as well as the relative phase between the two simulated harmonics. This comparison also provides confidence in the NLH base assumption to represent the unsteady flow field with a superposition of periodic sinusoidal values.

![Figure 11](image-url)

**Figure 11.** Experimental and NLH pressure signals for the VL01 sensor at high load.

### 5.4. Pressure visualisations

The distribution of the second harmonic amplitude on the blade for simulations with \( n = 2 \) is illustrated with contours in Figure 12. At the best efficiency point, the maximum amplitudes are obtained near the leading edge of the blade on the suction side, as it can be seen on the top-right picture. At high load, the leading edge is characterized by a diminution of the amplitude. In both cases, the fluctuations tend to be higher on the pressure side than on the suction side, and decrease linearly until they become almost non-existent near the blade trailing edge.

Colour contours are shown at midspan in the distributor and runner for the second harmonic in Figure 13. Again, the maximum amplitudes, which are higher than 4 kPa, are obtained for the best efficiency point and are located in the runner domain near the rotor-stator interface. The amplitude distributions for both operating points follow a similar pattern, but the gradients at BEP are higher particularly between the guide vanes. At high load, the lower gradients allow the perturbation to be more spread out in the upstream part of the distributor.

### 5.5. Computation times

The computations shown in this paper were conducted on a cluster and were partitioned over 32 processors. The average computation times per iteration were checked for each simulation type on different grid levels and are summarized in Table 3. For the finer grid level, NLH simulations with \( n = 2 \) require about four times as much time as the steady simulations. The simulation of four harmonics requires about 35% more computational time as the simulation of two harmonics.

| Grid level | Nb. cells | Steady | NLH \( n = 2 \) | NLH \( n = 4 \) |
|------------|-----------|--------|----------------|----------------|
| 111        | 1,733,664 | 0.10   | 2.40           | 3.24           |
| 000        | 13,486,464| 2.54   | 10.4           | n/a            |
Figure 12. Pressure amplitude of the second harmonic on the runner blade. Top: BEP; Bottom: High load; Left: Pressure side; Right: Suction side.

Figure 13. Pressure amplitude of the second harmonic at midspan in the distributor and runner seen from above. Left: BEP; Right: High load.

6. Conclusion
This paper focused on the methodology used to apply the nonlinear harmonic method to the Francis-99 test case and on preliminary results from intermediate grid level simulations. It was shown from steady and nonlinear harmonic simulations that the grid level used and the computation method does not modify the mean pressure values. NLH simulations on intermediate grid level tend to overestimate the pressure fluctuation amplitudes corresponding to the blade passing frequency. No major differences in the predictions were found between the simulations with two or four harmonics. At the
best efficiency point, some similarities are found between the experimental and NLH pressure signals, but better agreement with experimental data is found at high load. In this particular case, simulations on the finer grid level might not be required to predict adequately the main RSI amplitude.

Further investigations are underway to assess the nonlinear harmonic method ability to approximate the unsteady flow in the Francis-99 model. These include simulations on the finest grid level for the three operating points as well as comparisons with full URANS simulations. The effect of the turbulence and boundary layer modeling is also being evaluated.

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Nomenclature

| Symbol | Definition |
|--------|------------|
| $P, P_g$ | Static pressure [kPa] |
| $U_i$ | Mean velocity components [m/s] |
| $D$ | Runner diameter [m] |
| $F$ | Frequency [Hz] |
| $F_{\text{runner}}$ | Runner frequency [Hz] |
| $\dot{m}$ | Mass flow rate [kg/s] |
| $N$ | Number of perturbations |
| $n$ | Number of simulated harmonics |
| $\alpha$ | Inlet flow angle [$^\circ$] |
| $\tau_{ij}$ | Reynolds stresses |
| $\nu, \nu_t$ | Kinematic, turbulent viscosity [m$^2$/s] |
| $\phi(\vec{x}, t)$ | Conservative variable [m/s, kPa] |
| $\phi(\vec{x})$ | Conservative variable amplitude [m/s, kPa] |
| $\phi'(\vec{x}, t)$ | Conservative variable perturbation |

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