The Ramsauer model for the total cross sections of neutron nucleus scattering

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Theoretical study of systematics of neutron scattering cross sections on various materials for neutron energies up to several hundred MeV are of practical importance. In this paper, we analysed the experimental neutron scattering total cross sections from 20 MeV to 550 MeV using Ramsauer model for nuclei ranging from Be to Pb.

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In recent times world over, there is a renewed interest in the neutron nucleus scattering data. This is owing to a new concept of controlled nuclear energy source called the accelerator driven sub-critical (ADS) system [1, 2]. In this ADS system, neutrons are produced by bombarding a heavy element target with a high energy proton beam of typically above 1.0 GeV with a current of > 10 mA [1]. The sub-critical reactor is driven critical by spallation neutrons produced by proton beam on typically a molten lead target. Such accelerator driven systems can also be used for waste incineration of the long lived radio active waste produced in reactors based on thermal neutron induced fission. Reactor physics calculations of these ADS type of systems require neutron-nucleus scattering cross sections up to 500 MeV neutron energy. Unlike the neutron energy spectrum from a thermal neutron fission, the spallation neutron energies reach up to several hundred MeV. Therefore, it is currently very important to study the systematics of neutron scattering cross sections on various nuclei for neutron energies up to several hundred MeV. In the present work, we performed an empirical study of the energy dependence of total cross sections (σtot) of the neutron-nucleus (n-N) scattering. It is well known that the total cross sections are explained by the nuclear Ramsauer model. The Ramsauer model was first proposed by Lawson [3] in the year 1953 to phenomenologically explain the energy dependence of total cross sections of neutron nucleus scattering. In order to have insight of the working of this model, it is necessary to understand the optical model (OM) description of neutron scattering. In the OM approach, complex optical model potentials (OMP) are used and the Schrodinger’s equation is solved to obtain the scattering amplitude. The real part of the OMP describes the scattering and the imaginary part results in attenuation or absorption of the incident wave. The reaction cross section is given by the absorption of the neutron flux. The scattering calculations are performed using partial wave expansion method and the phase shifts (ηe = αeiβ) are determined. These complex phase shifts are strongly angular momentum and energy dependent for a given set of potentials. In terms of the phase shifts and the wave number (λ = ℏ/√2mE), the total cross sections are given below.

$$\sigma_{\text{tot}} = 2\pi R^2 \sum_{\ell}(2\ell + 1) \left[1 - \Re \eta_e\right]$$ (1)

Extensive study of the optical model fits of scattering cross sections on various nuclei over wide energy range have been made by several groups. This is owing to the excellent data base of neutron total cross sections available in the energy range up to 600 MeV [4, 5, 6]. The most recent work by Koning and Delaroche (KD) [7] presents a very exhaustive search for OMP parameters that fit the data very well up to 200 MeV. Alternatively, the nuclear Ramsauer model [8] provides a simple means to parameterise the energy dependence of neutron nucleus total scattering cross sections. This model assumes that the scattering phase shifts are independent of angular momentum (ℓ) as given in Eq.(2) (η = αeiβ), in contrast to the predictions of the optical model given in Eq.(1). Further, it was proposed that the ℓ-independent phase shift varies slowly with energy. This model was successfully applied for neutron scattering from various nuclei by Peterson [8, 9]. There were some attempts [10, 11, 12, 13] (see references therein) to put this Ramsauer model on a sound theoretical basis. The neutron total cross sections have thus been well studied using this model, over a wide range of nuclear masses as well as neutron energies up to 500 MeV [12, 13, 14, 15, 16, 17, 18]. Deb et al. [19] have achieved simple functional forms for the total cross sections by parameterising the maximum partial wave (ℓ0 = ℓmax) values.

In our earlier work [20], we presented the Ramsauer model analysis of the results of optical model code SCAT2 [21] using KD potentials. In the present work, we performed the Ramsauer model analysis of the experimental data of neutron total cross sections for heavy and light nuclei by using Eq.(2). The quantities R(fm), α, β are functions of atomic mass number (A) and the center

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of mass energy (E).

\[ \sigma_{\text{tot}} = 2\pi (R + \chi)^2 (1 - \alpha \cos \beta) \]  
\[ \beta = \beta_x A^{\frac{1}{2}} (\sqrt{E + V} - \sqrt{E}) \]  
\[ V = V_0 + V_E\sqrt{E} \]  
\[ V_A = V_0 + V_1(N - Z)/A + V_2/A \]  
\[ \alpha = \alpha_0 + \alpha_A\sqrt{E} \]  
\[ \alpha_A = \alpha_1 \ln(A) + \alpha_2/\ln(A) \]  
\[ R = r_0 A^{\frac{1}{2}} + r_A\sqrt{E} + r_2 \]  
\[ r_A = r_10 \ln(A) + r_11/\ln(A) \]  
\[ r_0 = 1.42988, \ r_10 = -0.02298, \ r_11 = 0.10268 \]  
\[ r_2 = 0.23216, \ V_0 = 46.51099, \ V_1 = 6.73833 \]  
\[ V_2 = -117.52082, \ V_E = -3.21817, \ \beta_x = 0.5928 \]  
\[ \alpha_0 = 0.02868, \ \alpha_1 = -0.00274, \ \alpha_2 = 0.13211 \]

Figure 1 shows the Ramsauer model fits (solid lines) for \( \sigma_{\text{tot}} \) cross sections using Eqs. (2), (3), (4) and the symbols represent the experimental data. The fits are obtained with total of twelve free parameters as given in Eq. (4), over wide mass range of \(^{24}\text{Mg}\) to \(^{208}\text{Pb}\). In Fig.2, we show the Ramsauer model fits for light nuclei from \(^9\text{Be}\) to \(^{24}\text{Mg}\). These fits cover the neutron energy region \((E_{\text{cm}})\) of 20-550 MeV. Similar Ramsauer model fits to total cross sections were already shown by various groups \([13, 16, 17]\) (see the references therein). As shown in Figs. (1,2), the functional dependence on energy and mass given in Eqs. (2), (3) with twelve global parameters was able to reproduce the experimental data very well.

As shown in Fig. 2, the parameters are same as those for heavy systems, except for the radius parameter. The radius parameter for scattering from light nuclei turns out to be rather larger than for heavy nuclei. In some cases of neutron nucleus scattering, the radius parameter (only \(r_0\)) has to be specially adjusted to achieve best fits. These cases did not obey the Ramsauer model systematics for either light or heavy nuclei. In case of \(^{27}\text{Al}, \ 232\text{Th}\) and \(^{238}\text{U}\), the adjusted radius parameters (in fm) are respectively, \(r_0 = 1.3892(\text{Al}), 1.4488(\text{Th}), 1.44641(\text{U})\). These values are comparable to the general systematics for \(r_0\) values for heavy nuclei (1.42988fm) and for light nuclei (1.53091fm).

In conclusion, we performed the Ramsauer model parameterization of experimental neutron total scattering cross sections from light and heavy nuclei. The parameters have been found to be same for light and heavy nuclei, except for the radius parameters. We could reproduce the experimental neutron total cross sections by using twelve parameters in the energy range of 20MeV to 550 MeV. We proposed a new functional form for energy dependence and atomic mass dependence of the Ramsauer model parameters.

[1] C. Rubbia et. al. ” Conceptual design of a fast Neutron Operated High power Energy Amplifier”, CERN Rep. CERN/AT/95-44 (ET), Geneva, 29 Sep. 1995.
[2] IAEA-TEC-DOC-985, Accelerator Driven Systems ; Energy generation and Transmutation of Waste Status Report, Nov 1997, International Atomic Energy Agency, Vienna.
[3] J. D. Lawson, Phil. Mag., 44, 102 (1953).
[4] R. W. Finlay, W. P. Abfalterer, G. Fink et. al., Phys. Rev. C47, 237 (1993).
[5] F. S. Dietrich, W. P. Abfalterer, et. al., LANSCEN – WNR, Proc. Int. Conf. Nucl. data for Sci. and Tech., Trieste, Italy, May 19-24, 1997, Vol.59, p.402, Italian Phys. Soc. (1997).
[6] W. P. Abfalterer, F. B. Bateman, et. al., Phys. Rev C63, 044608, (2001).
[7] A. J. Koning and J. -P. Delaroche, Nucl. Phys. A713, 231 (2003).
[8] J. M. Peterson, Phys. Rev. 125, 955 (1962).
[9] A. Bohr and B. Mottelson, Nuclear Structure, Vol. 1 P.166, Benjamin, N.Y. (1969).
[10] V. Franco, Phys. Rev. B140, 1501 (1965).
[11] C. R. Gould et. al., Phys. Rev. Lett. 53, 2371 (1986).
[12] J. D. Anderson and S. M. Grimes, Phys. Rev. C41, 2904 (1990).
[13] S. M. Grimes, J. D. Anderson, R. W. Bauer and V. A. Madsen, Nucl. Sci. and Engg. 130, 340 (1998).
[14] V. A. Madsen, J. D. Anderson, S. M. Grimes, V. R. Brown and P. M. Antony, Phys. Rev. C56, 365 (1997).
[15] R. W. Bauer et. al., Nucl. Sci. and Engg. 130, 348 (1998).
[16] S. M. Grimes, J. D. Anderson, R. W. Bauer and V. A. Madsen, Nucl. Sci. and Engg. 134, 77 (2000).
[17] S. M. Grimes, J. D. Anderson and R. W. Bauer, Nucl. Sci. and Engg. 135, 296 (2000).
[18] F. S. Dietrich, J. D. Anderson and R. W. Bauer, Phys. Rev. C68, 064608 (2003).
[19] P. K. Deb and K. Amos, Phys. Rev. 67, 067602 (2003) ; ibid 69, 064608 (2004).
[20] S. S. V. Suryanarayan, Rajesh S. Gowda and S. Ganesan, arXiv: nucl-th/0409005
[21] O. Bersillon, “SCAT2” Program, Note CEA-N-2227, Centre d’Etudes Nucleaires de Bruyeres-Chatel, Service de Physique et techniques Nucleaires, France (Oct. 1981).
FIG. 1: Ramsauer model fits (solid lines) to experimental neutron total cross sections (symbols) versus $E_{\text{lab}}$, using Eq. (2). The twelve parameters required are mentioned in Eq. (4). The fits are for heavy nuclei as shown by the figure labels.

FIG. 2: Ramsauer model fits (solid lines) to light nuclei, similar to Fig. 1. The twelve parameters required are same as in Fig. 1, except that the radius parameters for light systems are different as mentioned on figure.