The conical flow from quenched jets in sQGP

E.Shuryak

Department of Physics and Astronomy, University at Stony Brook,
Stony Brook NY 11794 USA

Abstract

Starting with a reminder of what is strongly coupled Quark-Gluon Plasma (sQGP), we proceed to recent advances in jet quenching and heavy quark diffusion, with a brief summary of various results based on AdS/CFT correspondence. The conical flow is a hydrodynamical phenomenon created by energy and entropy deposited by high energy jets propagating in matter, similar in nature to well known sonic boom from the supersonic planes. After a brief review, we discuss excitations of two hydro modes – sound and “diffuson” – which can be excited in this way. We also study expanding matter case, with a variable speed of sound, and use adiabatic invariants to show that the parameter $v/T$ ($v$ velocity in the wave, $T$ temperature) is increasing, up to a factor 3, during expansion. At the end we discuss recent results of the Princeton group which derived conical flow from AdS/CFT.

Key words:

PACS:

1 Why do we think that QGP is strongly coupled?

A realization [1,2,3] that QGP at RHIC is a strongly coupled liquid has lead to a paradigm shift in the field. It was extensively debated at the “discovery” BNL workshop in 2004 [4] (at which the abbreviation sQGP was established) and multiple other meetings since. In the intervening three years we all had to learn a lot, in part from the branches of physics which happened to have experience with strongly coupled systems. Those range from quantum gases to classical plasmas to string theory. In short, there seem to be not one but actually two difficult issues we are facing. One is to understand why QGP at $T \sim 2T_c$ is strongly coupled, and what exactly it means. The second large problem is to understand what happens at the deconfinement, at $|T-T_c| \ll T_c$, which may be a key to the famous confinement problem.
As usual, progress proceeds from catching/formulating the main concepts and qualitative pictures, to mastering technical tools, to final quantitative predictions: and now we are somewhere in the middle of this process. The work is going on at many fronts. At classical level, first studies of the transport properties of strongly coupled non-Abelian plasmas have been made. Quantum-mechanical studies of the bound states above $T_c$ have revealed a lot of unusual states, including “polymeric chains”. At the quantum field/string theory front, a surprisingly detailed uses of AdS/CFT correspondence has been made (see below).

The list of arguments why we think QGP is strongly coupled at $T = (1-2)T_c$ is long and growing. Let me start with its short version, as I see them today.

1. Collective phenomena observed at RHIC lead hydro practitioners to a conclusion that QGP as a “near perfect liquid”, with small viscosity-to-entropy ratio $\eta/s = .1 - .2 << 1$ [5]. Not only soft partons rescatter a lot, but even high energy jets, including the charmed ones, are strongly quenched. Even in this case one needs hydrodynamics to tell what is happening (conical flow).

2. Combining lattice data on quasiparticle masses and inter-particle potentials, one finds a lot of quasiparticle bound states [7]. This approach explains why $\eta_c, J/\psi$ remains bound till $T < (2-3)T_c$, as was directly shown on the lattice [8] and perhaps experimentally at RHIC. The bound states and resonances enhance transport cross sections [2,9] which helps to explain liquid-like behavior. Similar phenomena are known for ultracold trapped atoms, which can be put in a liquid form via Feshbach resonances at which the scattering length $a \to \infty$.

3. Classical interaction parameter $\Gamma \sim <\text{potential energy}> / <\text{kinetic energy}>$ in sQGP is not small at all. Classical e/m plasmas at the comparable coupling $\Gamma \sim 1 - 10$ are known to be good liquids. Our results [16] show it to be also true for non-Abelian plasma as well.

4. Correspondence between the conformal (CFT) $\mathcal{N}=4$ supersymmetric Yang-Mills theory at strong coupling and string theory in Anti-de-Sitter space (AdS) in classical SUGRA regime was conjectured by Maldacena [11]. The results obtained this way on the $g^2 N_c \to \infty$ regime of the CFT plasma are all close to what we know about sQGP (see below).

5. The $\mathcal{N}=2$ SUSY YM (“Seiberg-Witten” theory) is a working example of confinement due to condensed monopoles[12]. If it is also true for QCD, at $T \to T_c$ magnetic monopoles become light and weakly interacting at large distances due to $U(1)$ beta function. Then the Dirac condition forces electric coupling $g$ to become large (in IR).

This list is full of fascinating issues, but most of them are not what the HP06 organizers asked me to speak about, which is a focus on “hard probe” part of the story, to which I now proceed.
2 Jet quenching and heavy quark diffusion

A decade ago, at the formation time of RHIC program, the “hard probe” community clearly looked from above at the “soft probes”. Indeed, the letter spoke about the phase diagram, temperature and entropy, flow and viscosity and other macroscopic quantities which was quite difficult to justify for not-too-large systems we have. “Hard” physics was much simpler conceptually, and had a well calibrated tool – pQCD and a parton model PDFs. “Calibration” was the key word, and many small effects (like gluon shadowing) got a lot of attention.

And yet, what happened at RHIC was quite contrary to these expectations: hydrodynamics turned out to be the only theory which actually worked at RHIC! It provided quantitative description of spectra of most secondaries\(^1\). “Parton cascades” had spectacularly flopped, unable to generate any collective flow, unless their pQCD cross sections are grossly enhanced.

Then came the jet quenching \(R_{AA}\) data, with a shocking conclusion that we can only see only small fraction of all jets, perhaps originating in a dilute corona. With recent data on charm (single electrons) hopes to describe it by the gluon radiation via pQCD are fading. Although these issues are discussed at HP06 extensively, let me still mention few main reasons for this conclusion.

(i) The quenching of charm quarks turned out to be very similar to that of the usual jets without charm. First of all, this contradicts the Casimir scaling of any perturbative diagram, which would demand that a (charmed) quark has a smaller charge than a gluon (making most of the usual jets at RHIC.

(ii) Radiative jet quenching should be reduced for heavy quarks: thus now a discussion of collisional loss restarted, perhaps enhanced by heavy-light resonances \([9]\). Zahed and myself\([10]\) have also calculated “ionization” energy loss due to binary bound states.

(iii) Last but not least: secondaries from the quenched jets are not at all emitted in the direction of a jet but rather at very large angle \(\sim 70^\circ\) to it. It is hard to see how a gluon radiation may show such pattern: but it is exactly what the hydrodynamical theory of sound emission is predicting.

Charm diffusion and drag can be described by a single constant, \(D_c\), which can be extracted from comparison to charm \(R_{AA}\) and charm elliptic flow, see e.g. Moore and Teaney \([6]\). The resulting phenomenological value from RHIC data is in the range

\[
D_c \ast (2\pi T) = (1 - 2)
\]  

\(^1\) Well, only about 99 percent, for \(p_t < 2 GeV\).
while the pQCD result is
\[ D_c^{pQCD} \ast (2\pi T) = 1.5/\alpha_s^2 \] (2)

Assuming that perturbative domain is\(^2\) somewhere at \( \alpha_s < 1/3 \) one concludes that empirical value (1) is an order of magnitude smaller than the perturbative one.

3 AdS/CFT and strongly coupled plasma: brief summary

With weak coupling methods failing, one gets interested in the strong coupling limit. AdS/CFT correspondence is one of the directions (but not the only one!) which allows to address it, so far not for QCD but for its distant cousin \( \mathcal{N}=4 \) SYM theory.

**Thermodynamics** of the CFT plasma was studied started from the early work[25], its result is that the free energy (pressure) of a plasma is

\[ F(g, N_c, T)/F(g = 0, N_c, T) = [(3/4) + O((g^2 N_c)^{-3/2})] \] (3)

which compares well with the lattice value\(^3\) of about 0.8.

**Heavy-quark potentials** in vacuum and then at finite \( T \) [26] were calculated by calculating the configuration of the static string, deformed by gravity into the 5-th dimension. Let me write the result schematically as

\[ V(T, r, g) \sim \frac{\sqrt{g^2 N_c}}{r} \exp(-\pi T r) \] (4)

The Debye radius, unlike in pQCD, has no coupling constant: but it is not far from lattice value at \( \sim 2T_c \). Although potential depends on distance \( r \) still as in the Coulomb law, \( 1/r \) (at \( T = 0 \) it is due to conformity), it is has a notorious square root of the coupling. Semenoff and Zarembo [27] noticed that summing ladder diagrams one can explain \( \sqrt{g^2 N_c} \), although not a numerical constant. Zahed and myself [3] pointed out that both static charges are color correlated during a parametrically small time \( \delta t \sim r/(g^2 N_c)^{1/4} \); this explains [28] why

\(^2\) Recall that at \( 4/3\alpha_s = 1/2 \) two scalar quarks should fall at each other, according to Klein-Gordon eqn: so this is clearly not a perturbative region.

\(^3\) Not too close to \( T_c \), of course, but in the “conformal domain” of \( T = few T_c \), in which \( p/T^4 \) and \( \epsilon/T^4 \) are constant.
a field of the dipole is \(1/r^7\) at large distance\[29\], not \(1/r^6\). Debye screening range can also be explained by resummation of thermal polarizations \[3\].

Zahed and myself \[30\] had also discussed the velocity-dependent forces, as well as spin-spin and spin-orbit ones, at strong coupling. Using ladder resummation for non-parallel Wilson lines with spin they concluded that all of them join into one common square root

\[
V(T, r, g) \sim \sqrt{(g^2 N_c)[1 - \vec{v}_1 \cdot \vec{v}_2 + (\text{spin - spin}) + (\text{spin - orbit})]/r} \tag{5}
\]

Here \(\vec{v}_1, \vec{v}_2\) are velocities of the quarks: and the corresponding term is a strong coupling version of Ampere’s interaction between two currents\[^4\]. No results on that are known from a gravity side, to my knowledge.

**Bound states:** Zahed and myself \[3\] looked for heavy quarks bound states, using a Coulombic potential with Maldacena’s \(\sqrt{g^2 N_c}\) and Klein-Gordon/Dirac eqns. There is no problem with states at large orbital momentum \(J >> \sqrt{g^2 N_c}\), otherwise one has the famous “falling on a center” solutions\[^5\]: we argued that a significant density of bound states develops, at all energies, from zero to \(2M_{HQ}\). And yet, a study of the gravity side \[31\] found that there is no falling. In more detail, the Coulombic states at large \(J\) are supplemented by two more families: Regge ones with the mass \(\sim M_{HQ}/(g^2 N_c)^{1/4}\) and the lowest \(s\)-wave states (one may call \(\eta_c, J/\psi\)) with even smaller masses \(\sim M_{HQ}/\sqrt{g^2 N_c}\).

The issue of “falling” was further discussed by Klebanov, Maldacena and Thorn \[28\] for a pair of static quarks: they calculated the spectral density of states via a semiclassical quantization of string vibrations. They argued that their corresponding density of states should appear at exactly the same critical coupling as the famous “falling” in the Klein-Gordon eqn.

AdS/CFT also has multi-body states similar to “polymeric chains” \(\bar{q}gq\ldots q\) discussed above. For the endpoints being static quarks and the intermediate gluons conveniently replaced by adjoint scalars, Hong, Yoon and Strassler \[32\] have studied such states and even their formfactors.

**Transport properties** of the CFT plasma have been pioneered by Polikastro, Son and Starinets \[33\] who have calculated viscosity (at infinite coupling). Their famous result \(\eta/s \Rightarrow 1/4\pi\) is again in the ballpark of the empirical RHIC value. Thus gravitons in the bulk are dual to sound on the brane. Dual to (viscous) sound absorption is interception of gravitons by the black hole.

\[^4\] Note that in a quarkonium their scalar product is negative, increasing attraction.

\[^5\] Note that all relativistic corrections mentioned above cannot prevent it from happening.
Heavy quark diffusion constant has been calculated by Casalderrey and Teaney [34]; their result

\[ D_{HQ} = \frac{2}{\pi T \sqrt{g^2 N_c}} \]  

(6)

is even parametrically smaller than the expression for momentum diffusion \( D_p = \eta/(\epsilon + p) \sim 1/4\pi T \). If one plug in numbers, one can get \( D_p \sim (2\pi T) \sim 1 \), in the RHIC ballpark again.

Note that this important work is methodically quite different from others in that Kruskal coordinates are used, which allows to consider the inside of the black hole and two Universes (with opposite time directions) simultaneously, see Fig.1a. This is indeed necessary in any problems when a probability is evaluated, because that contains both an amplitude and a conjugated amplitude at the same time.

Heavy quark quenching [36,37,38,39,40] for quarks heavy enough \( M > M_{eff} \sim \sqrt{g^2 N_c T} \) is obtained in a stationary setting, in which a quark is dragged with constant by “an invisible hand” via some rope through QGP, resulting in constant production of a string length per time, see Fig.1 (b). The resulted drag force

\[ \frac{dP}{dt} = -\frac{\pi T^2 \sqrt{g^2 N_c v}}{2\sqrt{1-v^2}} \]  

(7)

remarkably satisfies the Einstein relation which relates it the heavy quark diffusion constant (given above), in spite of quite different gravity settings.

4 Brief history of the conical flow

In 1980’s Greiner and collaborators discussed possible shock wave formation and Mach cone emission in light-on-heavy collisions at BEVALAC. It did not work, because nuclear matter is not a good liquid.

Since we now known sQGP is a very good liquid, Casalderrey, Teaney and

\[ \text{One such problem is evaluation of the so called } \hat{q} \text{ parameter: two lines of the loop should also belong to two different Universes, not one as assumed in [35]. It remains unknown whether similar calculation in Kruskal geometry would produce the same result or not.} \]
myself suggested\textsuperscript{7} [13] that \textit{conical} flow must be induced by jets. We argued that as jet dumped energy into the medium locally, it should partially be transformed into coherent radiation of sound waves. Unlike gluons\textsuperscript{8}, they are much less absorbed and can propagate till the end (freezeout) and be detected. (In fact we will argue below that their amplitude should even grow with time.)

Fig. 2 explains a view of the process, in a plane transverse to the beam. Two oppositely moving jets originate from the hard collision point B. Due to strong quenching, the survival of the trigger jet biases it to be produced close to the surface and to move outward. This forces its companion to move inward through matter and to be maximally quenched. The energy deposition starts at point B, thus a spherical sound wave appears (the dashed circle in Fig.2a). Further energy deposition is along the jet line, and is propagating with a

\textsuperscript{7} To be fair, this idea only came to our mind \textit{after} we have seen the first STAR and PHENIX data on angular correlations.

\textsuperscript{8} This argument is one of several good reasons to discard Cerenkov gluon radiation as a source of conical distributions.
speed \( v \) of the jet till the leading parton is found at point A at the moment of the snapshot. The non-hydrodynamical core (solid region) serves as a source for the hydrodynamic fields.

The main prediction is that the shape of the jet passing through sQGP drastically changes: most of associated secondaries fly preferentially to a very large angle with jet direction, \( \approx 70 \) degrees consistent with the Mach angle 
\[
\cos \theta_M = \frac{v}{c_s}
\]
with a (time-averaged) speed of sound

\[
\bar{c}_{s_{RHIC}} = \frac{1}{\tau} \int_0^\tau dt c_s(t) \approx 0.33
\]  

(8)

Antinori and myself [14] suggested to test it further using b-quark jets, which can be tagged experimentally. As they get less relativistic the Mach angle should shrink, till it vanishes at the critical velocity \( v = c_s = 1/\sqrt{3} \). Note that such behavior of the cone is opposite to what happens for gluon radiation, which predicts a wider distribution with decreasing \( v \).

Casalderrey and myself [15] have shown, using conservation of adiabatic invariants, that fireball expansion should greatly enhance the sonic boom. The reason is similar to enhancement of a sea wave such tsunami as it goes onshore: see below.

Chaudhuri and Heinz [46] have numerically solved (in rapidity-independent 2+1 hydro) hydrodynamical equations with the energy deposited by a jet. Although they see a shock in coordinate space, they failed to see any peaks at the Mach angle in spectra. See more on it in the next section and Heinz’ talk. On the other hand, partonic cascades (with large cross sections appropriate for sQGP) have actually found quite clear signature of the conical flow, see Ma et al. [45].

It turns out \(^9\) that observations of the Mach cone is routinely used experimentally in studies of strongly coupled “dusty” electromagnetic plasmas: see e.g. [47], where one can see double Mach cones, corresponding to both sound and “shear” modes.

5 The conical flow: excitation of two modes

Let me start by explaining why the hydrodynamical approach is able to predict only the \( \text{shape} \) of the correlation function induced by conical flow, not its

\(^9\) It was pointed out to me by B.Jacak recently.
amplitude. The reason is simple: in a region close to the jet the variation of energy density is so rapid that hydrodynamics cannot be applied there. This preclude us from predicting the amount of the produced entropy $dS/dx$ by the jet, as well as the fraction of energy going into sound and “diffuson” modes\(^\text{10}\).

The jet energy deposited in a “viscous volume”

$$\frac{E_{\text{lost}}}{E_{\text{fluid}}} \approx \frac{dE}{dx} \times \frac{\Gamma_s}{e \times \Gamma_s^3} \approx 36 - 100 \gg 1.$$ (9)

is numerically large, and so a jet is surrounded by its own small “fireball” (of size $\Gamma_s$ or more) where variation of the thermodynamic quantities is too large for hydrodynamics to be applicable. Outside of this region, there is a domain where gradients are small enough that viscous hydrodynamics can in principle be used, but the behavior of the fluid is non-linear, dissipative, and possibly turbulent. We will not discuss these complex regions and proceed to large distances where linearized hydro should work.

According to the axial symmetry of the problem, the most general expression for the initial disturbance of the energy density and momentum in medium are

$$\epsilon_{dt_0}(t = t_0, \mathbf{x}) = e_0(x, r)$$

$$g_{dt_0}(t = t_0, \mathbf{x}) = g_0(x, r)\delta^{ix} + \nabla g_1(x, r)$$ (10)

The source functions $e_0(x, r)$ and $g_1(x, r)$ excite only the sound mode, while the remaining function $g_0(x, r)$ excites the diffuson mode. The particular value of these functions depends on the interaction of the jet and the fluid in the near region.

The reader should consult the rather long paper [13] for many details about calculated angular distributions of secondaries, and its dependence on parameters involved. Like for elliptic flow, the effect of conical flow grows with $p_t$ of the secondaries and seem to get maximal at $p_t \sim 1 - 2 \text{ GeV}$. It is also strongly dependent on jet quenching deposition $dE/dx$, as demonstrated in Fig.3(a). But mostly it is sensitive to viscosity: since the conical flow has larger gradients than elliptic one, one naturally may expect that eventually this phenomenon will provide its best estimates.

\(^{10}\)It does not matter whether hydro is or is nor linearized: thus Chaudhuri and Heinz[46] could not possibly determine it as well. The only approach which does not have this problem is AdS/CFT to be discussed below.
Fig. 3. Left: Associate yield dependence on associate $p_T$ for fixed source size $\sigma = 0.75/T$, viscosity $\Gamma_s = 0.1/T$, $t_j = 8/T$, $t_f = 10/T$, and energy loss, $dE/dx = 10T^2$ (top) and $dE/dx = 63T^2$ (bottom). The label values for $dE/dx$ correspond to $T = 200$ MeV. The three curves are for $1T < p_t < 5T$ (solid), $5T < p_t < 10T$ (dotted), $(3 \times) 10T < p_t < 15T$ (dashed), $(10 \times) 15T < p_t < 20T$ (dashed-dotted). (In the upper panel all the curves are rescaled further up by a factor 10). No large angle correlation is observed for $dE/dx = 10T^2$. For $dE/dx = 63T^2$ the position of the peak shifts toward $\pi$ for lower $p_T$. Right: Experimental dihadron azimuthal distributions from STAR (top) [18] and PHENIX (bottom) [17]

6 Expanding fireball and variable speed of sound

The speed of sound is defined by $c_s^2 = dp/d\epsilon$ via the thermodynamical variables, and it is expected to change in wide range during the process of heavy ion collisions. At early stages at RHIC the matter is believed to be in form of quark-gluon plasma (QGP), and thus it is $c^2_{QGP} = 1/3$. The next stage is the so called “mixed phase” in which the energy density is increasing much more rapidly than pressure, so that $c^2$ decreases to rather small values at the so called “softest point”, and then rise again to $c^2_{RG} = 0.2$ in the hadronic “resonance gas”.

Casalderrey and myself [15] discuss issues related with it. The main result is that expansion and a decrease of the sound speed can significantly increase the amplitude of the sound wave. The second issue is what will happen if the
QCD phase transition is of the 1-st order, so that there is truly mixed phase and the minimal value of $c^2$ is zero.

A motion of any waves in weakly inhomogeneous medium can be described via geometric optics and eikonal eqns for the phase of the wave. Its amplitude is much more tricky to get, which can sometimes be addressed with the help of adiabatic invariants $I = \oint pdq$. Let me remind the reader what conservation of $I$ means for a basic example, a harmonic oscillator with a slowly variable energy and frequency $(d\log \omega(t)/d\log t \ll 1$ etc). The typical momentum and amplitude of oscillations scale as

$$p \sim \sqrt{E(t)}, \quad q \sim \sqrt{E(t)/\omega^2(t)}$$

while their product – the adiabatic invariant $I \sim E(t)/\omega(t) = \text{const}$ – remains constant.

Hydrodynamical equations for sound waves, in momentum representation in coordinates, also form an oscillator, and therefore we will use the adiabatic invariant to study the changes on the velocity field due to the expansion and variable speed of sound. Unfortunately, a simple substitution of a variable $c_s(t)$ into the equations of motion for perturbations of a static background is inconsistent. One should instead find a correct nonstatic solution of the hydrodynamical equations and only then, using this solution as zeroth order, study first order perturbations such as sound propagation. The numerical solutions for hydrodynamical equations have been done by a number of authors: but in all of them the flow and matter properties depend on several variables and is too difficult to implement.

Therefore, in order to study the effects of the variable speed of sound we have looked for the simplest example possible, in which there is a nontrivial time-dependent expansion but still no spatial coordinates are involved, keeping the problem homogeneous in space. The only way these goals can be achieved is by a Big-Bang gravitational process, in which the space is created dynamically by gravity. With such space available, the matter can cool and expand at all spatial points in the same way. For definiteness, we considered a liquid in Robertson-Walker metric of expanding Universe with time-dependent radius $R(t)$. The sound wave eqns looks then as

$$\partial^2_{\eta} \epsilon - c_s^2 \nabla \epsilon + c\partial_{\eta} \left((3c_s^2 - 1) \frac{R'}{R}\right) + (3c_s^2 - 1) \frac{R'}{R} \partial_{\eta} \epsilon = 0.$$  

in proper time $\eta$. Note that both corrections to the wave equations vanish for $c_s^2 = 1/3$ (the QGP value) and the rescaling factor actually produces an amplitude growth, if the speed of sound reduces below the ideal value $c_s^2 < 1/3$. 

11
Fig. 4. Azimuthal dihadron distributions normalized per trigger particle measured by PHENIX [17] for two different centralities $(2.5 < p_T^{\text{trigger}} < 4\text{GeV}, 1 < p_T^{\text{associated}} < 2.5\text{GeV})$. The filled arrow indicates the position of the Mach cone. The empty arrow our estimate for the position where the cone from the reflected wave should appear.

The relevant quantity for the final production of particles (the exponent of the Cooper-Fry integral) is the velocity to temperature ratio $v^i/T$: according to our calculations this ratio grows by about factor 3 by the time of kinetic freezeout.

Another important point of this paper is a study of what would happen if the QCD phase transition would be the 1st order and the speed of sound in the mixed phase would vanish. We studied numerically and analytically what happens in this case, and found rather spectacular phenomenon: the “frozen” wave after the mixed phase splits into two halves, moving in opposite directions.

So, if the QCD phase transition is of the first order, the original wave gets split into direct and reflected waves. At freezeout one would find the reflected wave on its way to the origin, the opposite to the Mach direction of the direct wave. In terms of two particle azimuthal distribution it means the appearance of a second peak at some angle $\Delta\phi < \pi/2$. We return to it at the end of the next section.
7 Experimental issues

The first experimental observations made at RHIC, first by the STAR collaboration [18] and then by PHENIX [17], are 2-particle correlation functions, for a trigger hard particle and “companion”. Their relative distributions in azimuthal angle shown above (Fig.3b and Fig.4) do indicate a depletion of correlated particles in the direction of the quenched jet. A peak observed agrees with an angular position and shape in agreement with hydro predictions.

The first experimental issue was whether this effect is real, as it only gets observed after elliptic flow subtraction. It seems by now being resolved: see e.g. a talk by B.Cole [19], who demonstrated that the shape of the correlation function is quite independent of the direction of the jet in respect to the flow. Additionally one may use a subset of the data, taken at the particular angle at which the contribution of the elliptic flow vanishes, and still see a clear minimum at $\delta \phi = \pi$ and a large-angle peak.

The next issue is whether one indeed see a conical picture in geometrical sense, not just its projection onto the azimuthal angle. The next observable after two-body correlation function is naturally tree-particle correlations. Although statistically it is very hard to do, PHENIX and STAR both have emerging data on this, which were discussed at HP06 a lot. To my eye, those distributions look convincing, but perhaps more work and further scrutiny in collaborations is required to reach the final conclusion on this.

Let me at the end show Fig. 4 with samples of experimental correlation function as obtained by PHENIX [17]. The peak around 1.9 rad (indicated by the filled arrow) corresponds to the Mach direction; this is the main effect attributed to a conical flow. The reflected wave should appear in the region $\Delta \phi < \pi/2$, and the empty arrow shows the estimated place where the corresponding reflected peak should appear. Fig 4 a) shows the most central collisions, does not show any nonzero signal at that angle. Fig 4 b), at higher centrality, has a nonzero correlation function there, but it seems likely to be just a slope of a much broader peak. As argued above, we see no indications for enhanced correlations at the expected angle of the reflected wave, and thus the deconfining phase transition cannot be of the first order.

8 Conical flow is found via AdS/CFT

This section is about a development which was actually discussed in my talk as a suggestion, but concluded after HP06. Being an optimist, I said then that “...in the AdS/CFT framework one soon be able to derive from gravi-
ton propagator what exactly is the flow pattern induced by a heavy quark jet in a strongly coupled plasma”. But even for an optimist this prediction was confirmed surprisingly soon: a month later the Princeton group [48] have completed such (technically nontrivial) calculation. It involves a solution of the (linearized) Einstein equation, which tells what is the metric correction $h_{\mu\nu}$ due to gravity of the string which is being dragged behind the jet (indicated by the graviton line in Fig.1(b)).

Quite remarkably, when these authors analyzed harmonics of the stress tensor at small momenta $k \ll T$ (shown at the lower part In Fig.5) they have seen the narrow cone of emission along the Mach angle, which is just the “conical flow”! And, as one can see from two upper plots on this figure, for “subsonic” velocities $v = 0.4, 0.5 < 1/\sqrt{3}$, that pattern of emission is completely changed and the Mach cone disappears, exactly as argued by Antoniriri and myself [14].
As I already mentioned above, the duality between gravity in the bulk and sound on the brane has been pointed out already by Polikastro et al. So it was clear this would work: the nontrivial output is the absolute normalization of the conical flow. And yet, a dynamical derivation from first principles of a complex hydro flow is a remarkable achievement.

Perhaps we are now ready to proceed from discussion of sound (linearized eqns) to a full-blooded hydro, by going from gravitons (linearized Einstein eqns) to a study of a full-blooded general relativity. The ultimate goal would be to work out a **complete gravity dual** to the whole RHIC collision process, in which one should be able to derive thermalization and subsequent hydrodynamical evolution from first principles, following dynamically the gravity field of a produced black hole [42]. Sin, Zahed and myself [43] further argued that exploding/cooling fireball on the brane is dual to *departing* black hole, falling into the AdS center. Janik and Peschanski [44] indeed found that gravity dual to the Bjorken hydro has a metric with a departing horizon. (However they so far solved only vacuum Einstein eqns without any matter.)

**Acknowledgments** My collaborators, Derek Teaney and especially Jorge Casalderrey, worked out most of the results I am presenting. The work is partially supported by the grant of US DOE.

**References**

[1] E.V.Shuryak, Prog. Part. Nucl. Phys. **53**, 273 (2004) [hep-ph/0312227].
[2] E.V.Shuryak and I. Zahed, hep-ph/0307267, Phys. Rev. C **70**, 021901 (2004)
[3] E.V.Shuryak and I. Zahed, Phys. Rev. **D69** (2004) 014011. [hep-th/0308073].
[4] M. Gyulassy and L. McLerran, Nucl. Phys. A **750**, 30 (2005) [nucl-th/0405013]. E. V. Shuryak.Prog.Part.Nucl.Phys.53:273-303,2004, hep-ph/0312227 Nucl. Phys. A **750**, 64 (2005).
[5] D. Teaney, Phys. Rev. C **68**, 034913 (2003) arXiv:nucl-th/0301099.
[6] G. D. Moore and D. Teaney, hep-ph/0412346
[7] E. V. Shuryak and I. Zahed, Phys. Rev. D **70**, 054507 (2004), hep-ph/0403127
[8] S. Datta, F. Karsch, P. Petreczky and I. Wetzorke, hep-lat/0208012. M. Asakawa and T. Hatsuda, Nucl. Phys. A **715** (2003) 863c; hep-lat/0308034.
[9] H. van Hees, V. Greco and R. Rapp, arXiv:hep-ph/0601166
[10] E. V. Shuryak and I. Zahed, arXiv:hep-ph/0406100
[11] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] arXiv:hep-th/9711200.
[12] N. Seiberg and E. Witten, Nucl. Phys. B 426, 19 (1994) [Erratum-ibid. B 430, 485 (1994)] arXiv:hep-th/9407087.

[13] J. Casalderrey-Solana, E. V. Shuryak and D. Teaney, hep-ph/0411315 arXiv:hep-ph/0602183

[14] F. Antinori and E. V. Shuryak, nucl-th/0507046

[15] J. Casalderrey-Solana and E. V. Shuryak, arXiv:hep-ph/0511263

[16] B. A. Gelman, E. V. Shuryak and I. Zahed, Phys.Rev.C, in press, arXiv:nucl-th/0601029, nucl-th/0605046

[17] S. S. Adler et al. (PHENIX Collaboration), arXiv:nucl-ex/0507004

[18] J . Adams et al. (STAR Collaboration), Phys. Rev. Lett. 95 (2005) 152301

[19] PHENIX (B.A. Cole) Ericeira, Portugal, 4-10 Nov 2004. Eur.Phys.J.C43:271-280,2005

[20] E. V. Shuryak, Zh. Eksp. Teor. Fiz. 74, 408 (1978); Sov. Phys. JETP 47, 212 (1978), Phys. Rept. 61, 71 (1980).

[21] P. Petreczky, F. Karsch, E. Laermann, S. Stickan, I. Wetzorke, Nucl. Phys. Proc. Suppl. 106 (2002) 513.

[22] J. Liao and E. V. Shuryak, Nucl.Phys.A, in press, arXiv:hep-ph/0508035 Phys. Rev. D 73, 014509 (2006) arXiv:hep-ph/0510110.

[23] B. A. Gelman, E. V. Shuryak and I. Zahed, Phys.Rev.A, in press, arXiv:nucl-th/0410067

[24] E. V. Shuryak, arXiv:nucl-th/0606046

[25] S.S.Gubser, I.R.Klebanov and A.A. Tseytlin, Nucl. Phys. B534 (1998) 202

[26] J. M. Maldacena, Phys. Rev. Lett. 80, 4859 (1998) arXiv:hep-th/9803002. S. J. Rey and J. T. Yee, Eur. Phys. J. C 22, 379 (2001) arXiv:hep-th/9803001.

[27] G. W. Semenoff and K. Zarembo, Nucl. Phys. Proc. Suppl. 108, 106 (2002) arXiv:hep-th/0202156.

[28] I. R. Klebanov, J. M. Maldacena and C. B. Thorn, JHEP 0604, 024 (2006) arXiv:hep-th/0602255.

[29] C. G. Callan and A. Guijosa, Nucl. Phys. B 565, 157 (2000) arXiv:hep-th/9906153.

[30] E. V. Shuryak and I. Zahed, Phys. Lett. B 608, 258 (2005) arXiv:hep-th/0310031.

[31] M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, JHEP 0307, 049 (2003) arXiv:hep-th/0304032.

[32] S. Hong, S. Yoon and M. J. Strassler, JHEP 0603, 012 (2006) arXiv:hep-th/0410080.
[33] G. Policastro, D. T. Son and A. O. Starinets, Phys. Rev. Lett. 87 (2001) 081601.

[34] J. Casalderrey-Solana and D. Teaney, hep-ph/0605199.

[35] H. Liu, K. Rajagopal, and U. A. Wiedemann, hep-ph/0605178.

[36] S.-J. Sin and I. Zahed, Phys. Lett. B608 (2005) 265–273, hep-th/0407215.

[37] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, and L. G. Yaffe, hep-th/0605158.

[38] S. S. Gubser, hep-th/0605182.

[39] A. Buchel, hep-th/0605178.

[40] S.-J. Sin and I. Zahed, hep-ph/0606049.

[41] J. J. Friess, S. S. Gubser, and G. Michalogiorgakis, hep-th/0605292.

[42] H. Nastase, hep-th/0501068.

[43] E. Shuryak, S. J. Sin and I. Zahed, arXiv:hep-th/0511199.

[44] R. A. Janik and R. Peschanski, Phys. Rev. D 73, 045013 (2006) arXiv:hep-th/0512116, arXiv:hep-th/0606149.

[45] G. L. Ma et al., arXiv:nucl-th/0601012.

[46] A.K. Chaudhuri and U. Heinz, Effect of jet quenching on the hydrodynamical evolution of QGP, nucl-th/0503028.

[47] V. Nosenko et al, Phys.Rev.E 68 056409 (2003).

[48] J. J. Friess, S. S. Gubser, G. Michalogiorgakis and S. S. Pufu, arXiv:hep-th/0607022.