Didactic transposition from scholarly knowledge of mathematics to school mathematics on sets theory

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Abstract. A study was conducted to analyze the didactic transposition process on sets theory. Specifically, this study aimed to analyze the transposition of knowledge on sets theory from scholarly mathematics to school mathematics. This study was a phenomenology study. Abstract algebra textbooks, the 2013 curriculum and school mathematics textbooks were sources of data in this study. The analysis of didactic transposition process used descriptive analysis. The results of this study show that there was a transposition of knowledge in the concept of the set from scholarly mathematics to school mathematics. The transposition of knowledge occurs in the structure and form, and context, but not in sequences. In context, the transposition of knowledge to the concept of the set can be seen from how the concept of the set is built. In scholarly mathematics, the concept of sets is built from abstract formulas. Whereas in school mathematics, the concept of a set is built from concrete formulas, to later obtain semi-concrete and abstract formulas. Structurally and formally, the transposition of knowledge can be seen from the presentation of concepts. Whereas in sequence, there was no transposition of knowledge to the concept of sets.

1. Introduction
The idea of didactic transposition highlights the fact that the knowledge taught in schools comes from other institutions, where the knowledge taught generally comes from universities or other scholarly institutions [1]. This statement shows that the knowledge of mathematics taught in schools has actually been produced outside the school (scholarly institutions) and it’s transferred to schools with a series of adaptation processes [2]. The adaptation process is a process of transposition of knowledge from the knowledge produced (scholarly knowledge) into knowledge to be taught, and became knowledge taught and learnt knowledge in school. By Chevallard, this activity is called the didactic transposition [2]. Didactic transposition process can be seen in Figure 1.
According to Figure 1, the knowledge in this research refer to the concepts of sets theory. The scholarly knowledge refer to the concept of sets theory, which is produced by mathematicians. Knowledge to be taught is knowledge that has been selected from scholarly knowledge and reorganized by noosphere (curriculum designer and author of school mathematics textbooks). In this case, the noosphere rearranges the concept of sets theory from the context of scholarly mathematics to the context of mathematics education (school mathematics). Furthermore, the taught knowledge is the concept of sets theory taught in school by the teacher. The concept of sets theory taught to students has also been rearranged by the teacher and adjusted the conditions of the learners (students). And the last, the learnt knowledge is the concept of sets theory learned by students.

The didactic transposition process is a very important process in mathematics education. This is important because it is impossible to interpret school mathematics without considering phenomena related to the reconstruction of school mathematics knowledge from scholarly knowledge [3]. In addition, a didactic transposition process that is carried out well will have an impact on obtaining a comprehensive translation of the mathematics education curriculum, school mathematics textbooks that are capable of being a means of delivering good knowledge, and the formation of appropriate learning situations. Thus, learning obstacles can be anticipated or minimized. Because of that, in this research, researchers describes the didactic transposition process on sets theory that has been done. But, the didactic transposition process studied is only limited to the didactic transposition process from scholarly mathematics to mathematics to be taught. This research was conducted with the aim to describe the transitional form of sets theory concepts from the context of scholarly mathematics to the context of mathematics education.

2. Methods
This research is a phenomenology study. Grbich [4] mentions that phenomenology is an approach to understanding hidden meaning and essence with regard to human experience or phenomena that occur in humans. Giorgi & Moustakas said that phenomenology research is a research design derived from philosophy and psychology in which researchers describe the experience of human life about a particular phenomenon as explained by participants [5]. In this research, researchers describe phenomena related to the transposition knowledge in concepts of sets theory from scholarly mathematics to school mathematics. In more detail, the observed phenomenon is the transposition of concepts which includes the transition of structure and form from sets theory, and the transition of context from scholarly mathematics to school mathematics.

Data sources used to obtain this information are scholarly mathematics textbooks, that is abstract algebra textbooks consisting of Graduate Text in Mathematics, Fundamental Concepts of Abstract Algebra, and Modern Algebra, and textbooks of school mathematics consisting of student books and teacher's book. Data were analyzed using descriptive analysis. Data analysis procedures performed refer to the four basic stages in the analysis of qualitative research data, that is: data managing, reading-memoing, describing-classifying-interpreting, and representing-visualizing [6]. This analysis includes a description of the concepts of sets theory in scholarly mathematics and school mathematics, and the transposition of knowledge from scholarly mathematics to school mathematics.
3. Result and discussion

3.1. Sets theory in scholarly mathematics

In scholarly mathematics, this theory is presented as a fundamental concept for understand other concepts, including abstract algebra. Graduate Text in Mathematics, Fundamental Concepts of Abstract Algebra, and Modern Algebra are the book sources used for get the information about the structure and form of the concept of sets theory, and the context of its use in scholarly mathematics.

Based on the result of analysis of book sources, a summary of the structure and form of the concept of sets theory is obtained. The summary of the results of the analysis can be seen in Table 1.

Table 1. Summary of analysis of abstract algebra textbooks

| Sequence of Content | Structure and Form of Content |
|---------------------|-------------------------------|
| Definition of sets  | In the Fundamental Concept of Abstract Algebra and Modern Algebra, the definition of the sets is not explained axiomatically but informally. “sets is a collection of objects”. [7],[8] While, in the Graduate Text in Mathematics book, definition of the sets is explained axiomatically but informally. “A class to be a collection A of objects (elements) such that given any object x it is possible to determine whether or not x is a member (or element) of A” [9] Furthermore, the sets in this case is defined as a certain type of class. "Class A is defined as a sets if and only if there is class B so that A ∈ B. [9] |
| Element or member of sets | In all book, the member of sets is defined by: “a. x ∈ A to signify that x element of A. x ∉ A to signify that x not element of A”. [7], [8], [9] This part also discuss about types of sets: “A sets consisting of just one element is called a singleton. A sets consisting just two element is called a pair”. [7] |
| Equality of two sets | In the Fundamental Concept of Abstract Algebra book, equality of two sets is defined by: “two sets are equal if they consist of the same element”. [7] While, in graduate text in Mathematics book, equality of two sets is defined by: “Equality is assumed to have the following properties for all classes A, B, C; A = A; A = B ⇒ B = A; A = B dan B = C ⇒ A = C; A = B dan x ∈ A ⇒ x ∈ B”[9] “The axioms of extensionality and the properties of equality: A = B ⇔ A ⊆ B dan B ⊆ A.”[9] |
| Method to describe a sets | In Modern Algebra book mentioned two methods for describe the sets, that is: 1. the tabulation method “the one in which the elements of the sets are listed within braces” [8] 2. known as sets builder method or description method “describing the sets by the property or properties common to all the elements of the sets” [8] |
| Empty sets | The empty sets is defined by: “empty sets is a sets without element and it’s denoted ∅”.[7], [8], [9] |
| Subsets | In the Fundamental Concept of Abstract Algebra and Modern Algebra, the subsets is defined by: “If A and B are sets, then B is subsets of A if every element of B is an element of A. It’s denoted B ⊆ A “.[7], [8] Furthermore, the subsets in this case is defined as a certain type of class. “A class A is a subclass of a class B (written A ⊆ B provided: for all x ∈ A, x ∈ A ⇒ x ∈ B”. [9] Next, the subsets is defined by: “A subclass A of a class B that is itself a sets is called a subsets of B”. [9] |
| Power sets | The power sets is defined by: |
3.1.1. Sets theory in school mathematics

The sets theory is one of fundamental concepts in the school mathematics. Based on the Regulation of the Minister of Education and Culture of the Republic of Indonesia Number 37 of 2018, the material of the sets is one of the main subject matter of mathematics taught at the junior high school. The sets theory is taught in 7th grade and it’s taught after the concepts of number [10]. Textbook of Mathematics for the 7th Grade, that consist of student book and teacher book, are the book sources used for get the information about the structure and form of the concept of sets theory, and the context of its use in school mathematics.

Based on the result of analysis of book sources, a summary of the structure and form of the concept of sets theory is obtained. The summary of the results of the analysis can be seen in Table 2.

| Sequence of Content | Structure and Form of Content |
|---------------------|-------------------------------|
| “Every sets A has a power sets: the sets whose elements are the subsets of A”. [7], [8], [9] | |
| Operation on sets | In this part, the discussion about sets, consist of: union of sets, intersection of sets, difference of sets, and complement of a sets.
| “the union of two sets A and B denoted by A ∪ B is the sets consisting of all those elements each ones of which is containing in A or in B or both A and B” [7], [8] | |
| “the intersection of two sets A and B denoted by A ∩ B is the sets consisting of all those elements which are common to both A and B” [7], [8] | |
| “If A and B are two sets then their difference denoted by A-B is defined by A - B = {x: x ∈ A ∧ x ∉ B} ” [7], [8] | |
| “If A is a subsets of a sets X, then the complement of A in X written as X-A is defined by X - A = {x ∈ X: x ∉ A}” [7], [8] | |
| Law of operations | In this part, the discussion about, low of operations consist of: idempotent laws, identity laws, commutative laws, associative laws, distributive laws, and De’Morgan laws [8] |
| Cartesian Product | Cartesian product is defined by:
| “if A and B are two sets then the set of all ordered pairs (x,y) such as x ∈ A and y ∈ B ” [7],[8],[9] | |
| Cartesian product is denoted by: | “A × B = {(x, y): x ∈ A ∧ y ∈ B}” [7],[8],[9] |

### Table 2. Summary of results of analysis of junior high school mathematics text books

| Sequence of Content | Structure and Form of Content |
|---------------------|-------------------------------|
| 1. Concept of Sets | This section discuss about; |
| a. Method of describing a sets | (1) the sets and members of sets. It is presented by examples and non-examples. These examples are presented in the form of contextual problems. But in this part, it does not contain the definition of the sets either informally or axiomatically, does not contain an explanation of the rules in writing a sets and members of the sets in symbols. |
| b. Empty sets and universe sets | (2) Method of describing a sets. in this section discusses the 3 forms of sets presentations, i.e. write the members of the sets, write them with words, and write the notation forming their sets. |
| c. Venn Diagram | (3) The empty sets and universe sets. Both are presented in the form of examples and non-examples, but does not contain definitions and explanations of how to write empty sets and sets of universes in the form of symbols. |
| | (4) Venn diagram. this part discusses how to draw a Venn diagram if the sets is known and how to determine the sets member if the Venn diagram is known [10], [11]. |
2. Properties of Sets
   a. Cardinality of Sets
   b. Subsets
   c. Power Sets
   d. Equality of two sets
      (1) definitions and examples of the cardinality of a sets;
      (2) subsets. But, the concepts of subsets is presented only by giving
      examples without being equipped with definitions;
      (3) the definition and examples of power sets;
      (4) the definitions of two sets are the same and two sets are equivalent.

3. Operation on Sets
   a. Union
   b. Intersection
   c. Complement
   d. Difference
   e. Properties of operation on sets
      (1) sets operations,
      (2) the definitions and examples of intersection, union, complements,
      and differences between the two sets are discussed.
      (3) the properties of set operations, which consist of: idempotent, identity, commutative, associative, and distributive.

3.2 Discussion
3.2.1. Didactic transposition from scholarly knowledge of mathematics to school mathematics on sets theory
School mathematics basically evolved from scholarly mathematics produced by mathematicians [12]. This evolutionary process has gone through a series of adaptations in the form of rearranging scholarly mathematics into mathematics to be taught in school (school mathematics). This adaptation process allows changes in the sequence, structure and form and context of a knowledge.

   Based on the results of the research presented in Tables 1 and 2, it was found that there was a transposition of knowledge in the set concept from scholarly mathematics to the set concept in school mathematics. The transposition to the concept of the set occurs in the structure and form, and context. While the transition of concepts does not occur in sequence. The transfer of knowledge on the concept of the sets from scholarly mathematics to school mathematics in context is seen from how the concept of the set is built. In scientific mathematics, the concept of sets is built from abstract formulas. Whereas in school mathematics, the concept of a set is built from concrete formulas, to later obtain semi-concrete and abstract formulas. In school mathematics, the concept of the sets is built from examples and contextual problems. Through examples and contextual problems, students are expected to be able to build their knowledge. The rearrangement of concepts taught in schools is carried out by Noosphere (curriculum designers and school mathematics textbook writer) with the aim that students, at the level of cognitive development at their age, are able to develop conceptual concepts. This is in accordance with Piaget's Theory which states that students at the age of 11 years students have entered a formal operation [13].

   The transpositional form in context is also seen in the school curriculum documents used, the 2013 curriculum. Based on the Amendment to Minister of Education and Culture Regulation number 24 of 2016 concerning core competencies and basic competencies for learning in the 2013 curriculum in basic education and secondary education, it is stated that core competencies that must be possessed by Grade VII junior high school students in sets material are 1) understanding knowledge, conceptual, and procedural) based on the curiosity about science, technology, art, culture related to the phenomena and occurrences of visible eyes; and 2) trying, processing, and presenting in concrete realms (using, parsing, arranging, modifying, and making) and abstract domains (writing, reading, counting, drawing, and making) in accordance with what is learned in school and other sources the same in a theoretical perspective. Meanwhile, the basic competencies that must be possessed are 1) explaining the sets, subsets, the sets of universes, the blank sets, the complementary sets, and performing binary operations on the sets using contextual problems; and 2) solving contextual problems related to sets, subsets, sets of universes, empty sets, complement sets, and binary operations on sets [10].

   The form of the transposition of knowledge to the set concepts from scholarly mathematics to school mathematics in the order is not very significant. In order, the concept of the set which is arranged to be
taught in schools is not much different from the order in scholarly mathematics. It's just that, in school mathematics, the textbook writer presents set theory by making 3 main subjects, that are (1) the concept of the set, which contains the concepts of the set and members of the set, presentation of the set, empty set and universal set, and Venn diagram; (2) the characteristics of the set which contain the cardinality of the set, the subset, the power set, and the similarity of the two sets; (3) set operation which contains slices of two sets, a combination of two sets, complement, difference of two sets, and set operation properties.

Furthermore, the form of the transposition of knowledge on the concept of sets from scholarly mathematics to school mathematics is structurally and form seen from the presentation of concepts. The concept of association in scholarly mathematics is presented in the form of definitions, theorems and examples that are abstract. While the concept of the set in school mathematics is presented with examples and contextual problems. Some set concepts that are taught do not contain definitions, such as definition of sets, definition of member of sets, definitions empty set. In addition, there are concepts in the set contained in school mathematics but not contained in scientific mathematics, that is the Venn diagram. Instead, there are concepts contained in scholarly mathematics, but not contained in school mathematics, that is Cartesian products. Whereas Cartesian products are one of the concepts that serve as a bridge between the set and the relation of functions.

4. Conclusion
Based on the results and discussion, it can be concluded that there is a transposition of knowledge in the concept of the set from scholarly mathematics to school mathematics. The transfer of knowledge occurs in the structure and form, and context, but not in order. In context, the transfer of knowledge to the concept of the set can be seen from how the concept of the set is built. In scientific mathematics, the concept of sets is built from abstract formulas. Whereas in school mathematics, the concept of a set is built from concrete formulas, to later obtain semi-concrete and abstract formulas. Structurally and formally, the transposition of knowledge can be seen from the presentation of concepts. Whereas in sequence, there is no transposition of knowledge to the concept of sets.

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