A random interaction matrix model is used to study the statistics of conductance peak heights in Coulomb blockade quantum dots. When the single-particle dynamics conserves time-reversal symmetry, the peak height statistics is insensitive to the interaction strength. But when the single-particle dynamics breaks time-reversal symmetry, the peak height statistics exhibits a crossover from unitary to orthogonal symmetry as the interaction strength increases. This crossover is driven by the time-reversal symmetry of the interaction. Our random interaction matrix model describes features of both the measured peak height and peak spacing statistics.

RMT is not limited to non-interacting systems. Indeed it was first introduced to explain the neutron resonance data in the compound nucleus. But the neutron resonances are measured at finite excitations, while the linear conductance experiments in quantum dots probe the ground state of the system with a varying number of electrons. To understand the statistics that emerges from the interplay between one-body chaos and interactions in these dots, it is necessary to construct a random matrix model that includes interactions explicitly and reduces to one-body RMT in the absence of interactions. We can break the time-reversal symmetry of the one-body Hamiltonian (e.g., with a magnetic field), but the time-reversal symmetry of the two-body interaction should be preserved. An important question is whether such a random matrix model can reproduce both the measured peak height and peak spacing statistics.

Random matrix theory (RMT) provides a useful tool for describing the universal statistical fluctuations of the spectrum and eigenfunctions of a quantum system whose associated classical dynamics is chaotic. RMT has been successfully applied to the study of mesoscopic phenomena in quantum dots—submicron 2D devices where electrons are confined by electrostatic potentials. In dots with irregular shapes the single-electron dynamics is mostly chaotic, and the use of RMT is justified within a single-particle framework. In dots that are strongly coupled to leads, or open dots, the approximation of non-interacting quasi-particles is reasonable, and interactions can be considered indirectly through their effect on the electron coherence time. When a finite dephasing time is included, RMT can describe quantitatively the observed conductance statistics in open dots.

As the dot-leads coupling is made weaker, the charge on the dot becomes quantized, and electron-electron interactions cannot be ignored. In such almost closed dots, the conductance displays sharp peaks versus the gate voltage. The measured peak height distributions were found to agree with RMT predictions. However, it is not clear why RMT should describe the peak height statistics in dots with strong electron-electron interactions. For example, the spacings between peaks were observed to have a Gaussian-like distribution and not the Wigner-Dyson distribution that is expected in the constant interaction model plus single-particle RMT. Numerical simulations of an Anderson model of a small disordered dot (≲10 electrons) with Coulomb interactions also found Gaussian peak spacing distributions, while the peak height distributions showed only a weak dependence on interactions.

The n-th conductance peak height $G_n$ corresponds to the tunneling of an electron into a dot with $n−1$ electrons...
to form a dot with n electrons. At low temperatures, $G_n = (e^2/h)(\pi T/4kT)g_n$, where
\begin{equation}
G_n = \frac{1}{\Gamma_n^r \Gamma_n^l + \Gamma_n^r}.
\end{equation}

$\Gamma_n^{l(r)}$ is the partial width of the ground state of the n-electron dot to decay into an electron in the left (right) lead and the ground state of the dot with $n-1$ electrons:
\begin{equation}
\Gamma_n \propto \left| \langle \Phi_{G,S}(n) | \psi^l(r) | \Phi_{G,S}(n-1) \rangle \right|^2.
\end{equation}

$\psi^l(r)$ is the creation operator of an electron at the point $r$ \textbf{r} \textit{r}(r) for the left (right) point contact, and $\Phi_{G,S}(n)$ is the ground state wavefunction of the n-electron dot. For non-interacting electrons, $\Phi_{G,S}(n)$ is a Slater determinant of the n lowest single-particle eigenfunctions in the dot, and Eq. 2 reduces to $\Gamma_n \propto |\phi_n(r)|^2$, where $\phi_n$ is the n-th single-particle wavefunction. If the single-particle dynamics is chaotic, $|\phi_n(r)|^2$ satisfies Porter-Thomas statistics, leading to universal distributions of the conductance peak heights \textbf{f} that are sensitive only to the underlying symmetry class \textbf{f}.

To determine how interactions might modify the Porter-Thomas statistics of the partial widths, we use the RIMM of spinless interacting fermions \textbf{f}
\begin{equation}
H = \sum_{ij} h_{ij} a_i^\dagger a_j + \frac{1}{4} \sum_{ijkl} \bar{u}_{ijkl} a_i^\dagger a_j^\dagger a_k a_l.
\end{equation}

The one-body matrix elements $h_{ij}$ are chosen from the appropriate Gaussian random matrix ensemble, while the anti-symmetrized two-body matrix elements $\bar{u}_{ijkl} = u_{ij;kl} - u_{ij;lk}$ form a GOE in the two-particle space \textbf{f}
\begin{equation}
P(h) \propto e^{-\frac{2}{N} \text{Tr} h^2}; \quad P(\bar{u}) \propto e^{-\text{Tr} \bar{u}^2/2u^2}.
\end{equation}

The states $|i\rangle = a_i^\dagger |0\rangle$ describe a fixed basis of m single-particle states. $h$ is an $m \times m$ GOE (GUE) matrix when the single-particle dynamics conserves (breaks) time-reversal symmetry. The two-body interaction is assumed to preserve time-reversal symmetry and forms a GOE, irrespective of the symmetry of the one-body Hamiltonian.

To calculate the statistics of the partial widths $\Gamma_n$, we expand $\psi^l(r) = \sum_i \psi_i(r)a_i^\dagger$, where $\psi_i(r) \equiv \langle r|i \rangle$ is the wavefunction of the fixed state $|i\rangle$. It follows from the orthogonal invariance of the ensemble \textbf{f} that the statistics of $\Gamma_n$ is identical to the statistics of
\begin{equation}
\Gamma_n^i \propto \left| \langle \Phi_{G,S}(n) | a_i^\dagger \Phi_{G,S}(n-1) \rangle \right|^2
\end{equation}
for any i.

For each realization $H$ of the ensemble \textbf{f}, we calculate the ground states for $n-1$ and $n$ electrons, and compute $\Gamma_n^i$ using (5). The distributions of the normalized width $\Gamma = \Gamma/\bar{\Gamma}$ for a GOE single-particle statistics are shown in Fig. \textbf{f} for several values of $U/\Delta$. We show $P(\text{ln} \bar{\Gamma})$ rather than $P(\bar{\Gamma})$ in order to display more clearly the small values of $\bar{\Gamma}$. The distributions are independent of $U/\Delta$ and are well described by the GOE Porter-Thomas distribution $P_{\text{GOE}}(\text{ln} \bar{\Gamma}) = (\bar{\Gamma}/2)^{1/2} \text{exp}(-\bar{\Gamma}/2)$ (solid line). As a reference we also show the GUE Porter-Thomas distribution $P_{\text{GUE}}(\text{ln} \bar{\Gamma}) = \text{exp}(-\bar{\Gamma})$ (dashed line). The conductance peak heights are calculated from \textbf{f} using two uncorrelated “sites” i and j for the left and right leads. The peak height distributions $P(g)$ for a GOE h are shown in the insets of Fig. \textbf{f}. They are all in good agreement with the GOE Porter-Thomas distribution $P_{\text{GOE}}(g) = \sqrt{2/\pi} \text{e}^{-2g}$ (solid lines) irrespective of $U/\Delta$. The dashed lines describe the GUE distribution $P(g) = 4g[K_0(2g) + K_1(2g)]\text{e}^{-2g}$ ($K_0$ and $K_1$ are Bessel functions).

The results for a GOE one-body statistics are shown in Fig. \textbf{f} for the same values of $U/\Delta$ as in Fig. \textbf{f}. When the interaction strength increases, the width and peak height distributions make a crossover from the corresponding GUE distribution (at $U = 0$) to the GOE distribution (at large $U$). Equivalently, the transition from GOE to GUE statistics due to a time-reversal symmetry-breaking one-body field is not complete because of the competing GOE symmetry of the two-body interaction. The crossover distributions are compared with distributions obtained from an RMT ensemble that describes the crossover between the orthogonal and unitary symmetries. This ensemble is $H = S + i\alpha A$, where $S$ and $A$ are $N \times N$ symmetric and antisymmetric uncorrelated Gaussian matrices and $\alpha$ is a parameter \textbf{f}. Its wavefunction statistics depends on the parameter $\lambda \equiv \alpha\sqrt{N}/\pi$. In particular, the width distribution is given in closed form by \textbf{f}.

FIG. 1. Width and conductance peak height distributions in the RIMM with GOE one-body statistics. $P(\text{ln} \bar{\Gamma})$ us shown versus $\text{ln} \bar{\Gamma}$ ($\bar{\Gamma}$ is the normalized width \textbf{f}) for $m = 12$, $n = 4$ and $U/\Delta = 0$ (circles), 2.4 (squares), 4 (diamonds) and 8 (triangles). The solid and dashed lines are the GOE and GUE Porter-Thomas distributions, respectively. Insets: the peak height distributions $P(g)$ for a log-log scale (left inset) and in a linear scale (right inset) for the same cases shown in the main figure. The solid and dashed lines are the GOE and GUE peak height distributions, respectively.
\[ P_\lambda(\Gamma) = \int_0^1 dt \, \frac{1 + t^2}{t} e^{-\left(\frac{t^2}{2}\right)^2} \Gamma_0 \left(1 - \frac{t^4}{t^2 - \Gamma}\right), \quad (6) \]

where \( \Gamma_0 \) is the modified Bessel function of order zero. The short-dashed lines are fits to the theoretical distribution (1) describing the RMT crossover from GUE \((\lambda = \infty)\), dashed line) to GOE \((\lambda = 0)\), solid line) with \( \lambda = 0.28, 0.17 \) and \( 0.08 \), respectively. Insets: \( P(g) \) in a log-log scale (left inset) and in a linear scale (right inset) for the same cases shown in the main figure. The short-dashed lines are the analytic peak height distributions in the crossover regime \( (\lambda) \) between the GUE (dashed) and GOE (solid) distributions.

For each \( U/\Delta \), we find \( \lambda \) by fitting the distribution (3) to the computed width distribution. The distributions (3), shown by the short-dashed lines in Fig. 3, accurately describe the width distributions of the RIMM with a GUE \( h \). Good agreement with closed RMT expressions (4) is also obtained for the peak height distributions \( P_\lambda(g) \) shown in the insets of Fig. 2.

An important issue is the universality associated with the RIMM (8) and (10). The model depends on three parameters: \( m, n, \) and \( U/\Delta \). The top panel of Fig. 3 shows, for a GUE one-body \( h \), the crossover parameter \( \lambda \) as a function of \( U/\Delta \) for \( m = 10 \) and \( n = 4 \), \( 5 \), \( 6 \), and \( 7 \) (symbols), and for a “reference” case \( m = 12 \) and \( n = 4 \) (solid line). The curves depend on both \( m \) and \( n \), but can all be scaled on the reference curve after scaling the interaction strength by a constant, \( U_{\text{eff}} \equiv f(m, n) U/\Delta \).

In the range \( U_{\text{eff}} \lesssim 1, \lambda \gtrsim 1 \) and the statistics is essentially GUE. For \( U_{\text{eff}} \sim 1 - 1.5 \), the peak height statistics is close to GUE while the peak spacings already follow a Gaussian distribution. This explains the measured RMT-like peak height distributions (8) and the Gaussian-like shape of the peak spacing distributions (10) within a single random matrix model. We remark that the small deviations from GUE statistics observed in the experiment of Ref. (8) and in the calculations of Ref. (10) in the presence of a magnetic field are consistent with a crossover from GUE to GOE.

The average width \( \bar{\Gamma} \) is a monotonically decreasing
function of $U/\Delta$ and saturates at large values of $U$. The top inset of Fig. 3 shows $\Gamma(U)/\Gamma(0)$ as a function of $U/\Delta$ for several $m$ and $n$. This dependence is well described by 

$$\Gamma(U) - \Gamma(\infty) = (\Gamma(0) - \Gamma(\infty))/(1 + bU^2/\Delta^2).$$

The parameter $\Gamma(\infty)$ depends on $m$ and $n$, while $b$ is found to be independent of $m$ and to depend only weakly on $n$.

Next we compare the predictions of the RIMM with those of a model of a quantum dot. We studied a 2D Anderson model with on-site disorder parameter $W$ and hopping matrix element $V = 1$. The electrons are interacting with a Coulomb interaction whose strength over those of a model of a quantum dot. We studied a 2D

The left panel of Fig. 4 shows $P(\ln \Gamma)$ in the absence of a magnetic field and for several values of the Coulomb interaction strength: $U_c = 0$ (circles), 2 (squares), 4 (diamonds) and 6 (triangles). The disorder parameter is $W = 5$. The solid (dashed) line is the GOE (GUE) Porter-Thomas distribution.

Right: $P(\ln \Gamma)$ for $U_c = 0$ (circles) and 12 (triangles) in the presence of time-reversal symmetry-breaking magnetic flux $\Phi = 0.14\Phi_0$ and for a disorder strength of $W = 3$. The short-dashed line is the distribution $\Gamma(U) / \Gamma(0)$ with $\lambda = 0.17$.

The left panel of Fig. 4 shows the width statistics in the absence of magnetic flux for a $4 \times 5$ Anderson model with disorder parameter $W = 5$ and for $n = 4$ electrons. The distributions are approximately described by the GOE Porter-Thomas distribution (solid line) irrespective of the value of $U_c$ and in agreement with the RIMM. The dependence of the average width $\Gamma$ on the interaction strength (not shown) is similar to that observed in the RIMM. The right panel shows width distributions in the presence of magnetic flux $\Phi = 0.14\Phi_0$ and for $W = 3$. The $U_c = 0$ distribution (circles) agrees with the GUE Porter-Thomas distribution (dashed line), while the $U_c = 12$ distribution (triangles) is described by $\Gamma(U) / \Gamma(0)$ with $\lambda = 0.17$. This is the distribution obtained in the RIMM for $U_{\text{eff}} \approx 4$. For weaker interactions the calculated width distributions exhibit some deviations from $\Gamma(U) / \Gamma(0)$. We note however that, for the small lattices used, it is difficult to find a disorder strength for which the model is in the metallic diffusive regime and displays universal RMT statistics. In particular, in the presence of a magnetic flux we could not find values of $W$ for which both the spectral and wavefunction statistics are GUE.

In conclusion, we have investigated the width and peak height statistics in Coulomb-blockade quantum dots using a random interaction matrix model with interactions that preserve time-reversal symmetry. For a GOE one-body symmetry the statistics is insensitive to the interaction strength. However, at strong interactions, a time-reversal symmetry-breaking field leads only to a partial crossover from GOE to GUE statistics. Our random interaction matrix model can reproduce both the observed Gaussian-like shape of the peak spacing distribution and the RMT statistics of the peak height distributions.

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