Radiative Decays of $q\bar{q}$ Chiral States in the $\tilde{U}(12)$-Scheme

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The radiative transitions between ground-states (GS) of light $q\bar{q}$ mesons are investigated in the $\tilde{U}(12)$-scheme. In this scheme the rich decay-spectra are offered even in transition between GS due to the appearance of chiral states. As a result the radiative decay widths of the ordinary $V \rightarrow P \gamma$ process are well reproduced, and furthermore, the predicted width of $b_1(1235) \rightarrow \pi\gamma$ process ($\Gamma_{\text{theor}} = 229$ keV), by assigning $b_1(1235)$ as a member of the $^3S_1$ chiral states $A(\tilde{E})$, is in good agreement with experimental one ($\Gamma_{\text{exp}} = 230 \pm 60$ keV). From this results it is indicated that $b_1(1235)$ meson, classified as $^1P_1$ state in conventional non-relativistic (NR) classification scheme, is a good candidate of chiral state in the $\tilde{U}(12)$-scheme. The other radiative transition widths of some chiral states are also predicted. Accordingly the predicted values given here, will provide useful insights in their search at BES.

§1. Introduction

($\tilde{U}(12)$-scheme and $q\bar{q}$ chiral states)

The $\tilde{U}(12)$-scheme, which has been proposed several years ago, is a Lorentz covariant framework for describing the composite hadron system based on “static” $U(12)_{SF}$-symmetry, embedded in $\tilde{U}_{SF}(12) \otimes O(3,1)_L$-space. The wave function (WF) of composite hadron is generally described as irreducible tensors of $U(12)_{SF}$-group at their rest frame. The $U(12)_{SF}$-group includes, in addition to the conventional non-relativistic $SU(6)_{SF}$-group, the new symmetry $SU(2)_\rho$ which is naturally introduced in connection with the covariant treatment of the constituent confined quarks. By inclusion of this extra $SU(2)$ spin freedom, it leads to the possible existence of many new states, called chiral states or chirals. Chiral states are described with at least one Dirac spinors with negative $\rho_3$ (the third component of the $\rho$-spin)- eigen-value, and never appear in the non-relativistic quark model (NRQM). On the other hand, the conventional states appearing in NRQM (called Pauli states or Paulons) are described with Dirac spinors with all positive $\rho_3$-eigen-values.

In the $\tilde{U}(12)$-scheme the GS of light $q\bar{q}$ meson system is assigned as $12 \times 12^* = 144$-representation of the static $U(12)_{SF}$ group and they are classified into the eight flavor nonets with $J^{PC}$ (see, Table I). As is shown in this table, the 144-plet contains two pseudo-scalar (labeled $P_s^{(N)}$ and $P_s^{(E)}$) nonets, two vector (labeled $V_\mu^{(N)}$)

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$^*$) The word “static” implies that the symmetry is imposed only at the rest frame of hadrons. For more detail, see Ref. 5.

$^{**}$) The new degree of freedom corresponding to the $SU(2)_\rho$-symmetry is called the $\rho$-spin, after the well-known $\rho \otimes \sigma$-decomposition of Dirac matrices.
and \( V_\mu^{(E)} \) nonets, two scalar (labeled \( S^{(N)} \) and \( S^{(E)} \)) nonets, and two axial-vector (labeled \( A_\mu^{(N)} \) and \( A_\mu^{(E)} \)) nonets. Here we emphasize that, in the \( \tilde{U}(12) \)-scheme, the scalar \( 0^{++} \) and axial-vector \( 1^{++}/1^{+} \) nonets appear not only in excited P-wave state but also in the S-wave GS. These GS chiralons are expected to have the lower mass than the P-wave states. The \( \sigma \)-nonet are regarded as an appropriate candidate of the \( 0^{++} \) GS chiralon, while it will be shown that \( b_1(1235) \) meson is also another good candidates of \( 1^{++} \) GS chiralon through this analysis. On the other hand, it is shown that experimentally well known \( a_1(1260) \)-meson, decaying with partial width \( \Gamma_{\exp}(a_1(1260) \to \pi\gamma) = 640 \pm 240 \text{ keV} \) is not a pure GS chiralon.

(\textit{The important features of our scheme and radiative decays})

In the radiative transition between light-quark meson system the final mesons generally move relativistically. In addition to this, considering that all actual physical observations are made through not quarks but hadrons, covariant treatment for the center of mass (CM) motion of hadron is absolutely necessary. Although the NRQM may incorporate the effect of relativistic motion of the constituent quarks, but it is unable to give the conserved current concerning the composite hadron. On the other hand, the \( \tilde{U}(12) \)-scheme has remarkable features that hadrons are treated in manifestly covariant way and the conserved effective hadron currents are explicitly given in terms of hadron variable themselves.

(Radiative decays and our previous work)

Here it will be worth mentioning the relation between the \( \tilde{U}(12) \)-scheme and our previous scheme for the relevant problem. Before possible existence of chiralons being noticed, we had been investigated systematically radiative decays of light through heavy quark meson systems in framework of the covariant oscillator quark model (COQM)\(^{\text{\*\*}}\). As the results, it had been shown that qualitative features of experimental data are well reproduced except for some cases. The COQM is also based on the \( \tilde{U}(12) \otimes O(3,1) \)-scheme, but with static SU(6)\(_{SF}\)-symmetry. Thus, the coverage of the COQM is limited to the reaction of Pauli states. The new \( \tilde{U}(12) \)-scheme with static \( U(12)_{SF} \)-symmetry is extended, keeping the above mentioned covariant properties, to be able to describe the chiral states in addition to Pauli states.

(Purpose of this work)

In this report, now taking into account the existence of chiral states, we shall examine the radiative transitions among GS mesons in the \( \tilde{U}(12) \)-scheme, leading to some useful guidance for searching the chiral particle in BES experiment.

\section{Ground State \( q\bar{q} \) Chiral Mesons and their Wave Functions}

(The \( q\bar{q} \)-meson wave functions in \( \tilde{U}(12) \)-scheme)

First we recapitulate briefly the framework of \( \tilde{U}(12) \)-scheme so far as required.

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\(^{\text{\*\*}}\) The COQM has a long history to development. In Ref.\(^{\text{[12]}}\) it is applied to investigate the light-quark meson spectra, including the one-gluon-exchange effect, with considerable success.
for the relevant application. In our scheme, mesons are unifiedly described by the bi-local Klein-Gordon field with one each of lower and upper indices in the boosted $U(12)_S F \otimes O(3)_L$ space as

$$
\Phi(x_q, x_q)_A^B = \int \frac{d^3 P}{\sqrt{(2\pi)^3 2 P_0}} (e^{+i P X} \Phi(x, P)_A^{(+)B} + e^{-i P X} \Phi(x, P)_A^{(-)B}),
$$

(2.1)

where $A = (\alpha, a) (B = (\beta, b))$ denotes Dirac spinor and flavor indices respectively, $x_q (x_q)$ represents a space-time coordinate of quark (anti-quark) which is related to the CM (relative) coordinate for composite meson as $X = (m_q x_q + m_q x_q)/(m_q + m_q)$ ($x = x_q - x_q$) ($m_q$ ($m_q$) being the quark (anti-quark) mass parameter). The $P_\mu$ denotes CM 4-momentum of the relevant meson. The above bi-local field is supposed to satisfy the Klein-Gordon type equation

$$
\left[ \frac{\partial^2}{\partial X^2} - M^2(x, \frac{\partial}{\partial x}) \right] \Phi(X, x)_A^B = 0,
$$

(2.2)

where $M^2(x, \frac{\partial}{\partial x})$ is the $U(4) (\otimes SU(2)_\sigma \otimes SU(2)_\rho)$-spin independent squared mass operator including only central confining-force potential. As the basis to expand the above internal wave function $\Phi(x, P)_A^B$, we use the “LS-coupling” type direct products,

$$
\Phi(x, P)_A^{(\pm)B} \approx f(x, P) W_A^{(\pm)B}(P),
$$

(2.3)

where the definite-metric type 4-dimensional oscillator function $f(x, P)$ and the extended Bargmann-Wigner (BW) spinor WF $W_A^{(\pm)B}(P)$ are taken for the respective parts. Decomposing the $W_A^{(+)B}(P)$ into irreducible definite $J^{PC}$-components, their detailed forms for relevant meson system are given as

$$
W_A^{(+)}(P) = (1)^{\alpha}_a(S^{(N)}(P))_a^b + (i\gamma_5)^{\alpha}_a(P^{(N)}(P))_a^b
+ (-v_\gamma)^{\beta}_a(S^{(E)}(P))_a^b + (-\gamma_5 v_\gamma)^{\beta}_a(P^{(E)}(P))_a^b
+ (i\gamma_5 v_\gamma)^{\beta}_a(A^{(N)}_a^b(P))_a^b
+ (-i\sigma_\mu_v_\gamma)^{\beta}_a(V^{(N)}_a^b(P))_a^b + (\gamma_5 \sigma_\mu v_\gamma)^{\beta}_a(A^{(E)}_a^b(P))_a^b,
$$

(2.4)

where the $v_\mu = P_\mu/M$ denotes 4-velocity of the relevant meson. Note that the expansion basis of the BW spin function consists of totally 16 members in the $\tilde{U}(4)$-space.

(Chiral representation for two kinds of Vector / Pseudo-scalar nonets)

* For simplicity, only expressions for the GS are shown in the following.

** In the present application, we neglect the form factor coming from the overlapping of the space-time internal WF, since $\int d^4 x f(x, P') f(x, P) e^{i \frac{P_{\mu}}{M} x_\mu} \approx 1$ is expected for transition between the GS.

*** The negative frequency (creation) part of the internal spin WF, $W^{(-)}(P)$ can be obtained by substituting the CM momentum by $P_\mu \to -P_\mu$ in the relevant positive frequency (annihilation) part $W^{(+)P}$ owing to the crossing relation for the meson field.
Eq. (2.5) is deduced from the basic structure of our classification scheme. See, for example, Ref. 7). Here it should be noted that the definition of chirality transformation the polarization vector and the flavor WF of the relevant vector meson, respectively. This WF is the eigen-function of chirality operation defined as Eq. (2.5), with the χ-ef (3)

Table I. Ground-state light $q\bar{q}$ meson nonets in $\bar{U}(12)$-scheme. Their spin WFs and quantum numbers of the relevant mesons are shown. The symbol $\chi$ denotes the chirality eigen-value of the relevant meson WF. Its definition and physical meaning are discussed in the text.

| Meson Nonets: | $P^{(N)}$ | $P^{(E)}$ | $V^{(N)}$ | $V^{(E)}$ | $A^{(N)}$ | $A^{(E)}$ | $S^{(N)}$ | $S^{(E)}$ |
|--------------|------------|------------|------------|------------|------------|------------|------------|------------|
| $J^{PC} \times$ | 0$^+$ | 0$^+$ | 1$^-$ | 1$^+$ | 1$^-$ | 1$^+$ | 0$^+$ | 0$^+$ |
| $W(P)^{(+)B}$ | $\frac{\lambda^a}{\sqrt{2}}$ | $\frac{\lambda^a}{\sqrt{2}}$ | $\frac{\lambda^a}{\sqrt{2}}$ | $\frac{\lambda^a}{\sqrt{2}}$ | $\frac{\lambda^a}{\sqrt{2}}$ | $\frac{\lambda^a}{\sqrt{2}}$ | $\frac{\lambda^a}{\sqrt{2}}$ | $\frac{\lambda^a}{\sqrt{2}}$ |

As was explained in the previous section, in our scheme, two kinds of vector and pseudo-scalar meson nonets appear in GS, respectively. To make a discrimination of respective meson, it is useful to see the eigen-value of the chirality operator of the relevant bi-spinor WF. The chirality transformation$^6$ for the bi-spinor WF is defined as

$$(\gamma_5)\alpha^a W(P)_\alpha^\beta (-\gamma_5)\beta^{\prime} = \chi W(P)_\alpha^{\beta^\prime}, \quad (2.5)$$

where $\chi$, denotes the eigen value of chirality operator for the composite meson system. In case that the relevant composite meson consists of quarks with same chiralities (i.e. Left×Left or Right×Right combination), then the $\chi$ becomes $\chi = +1$, and vice versa.

The $V(N)$-type vector meson WF is given by

$$W(P)^{(+)B}_A = \left(\frac{i}{2} \gamma_\mu \epsilon(P)_{\mu} V^{(N)}_a A^B \right)$$

$$= \frac{1}{\sqrt{2}} \gamma_\mu \epsilon(P)_{\mu} V^{(N)}_a A^B$$

with $\gamma_\mu = \gamma_{\mu} + (\gamma \cdot v)v_\mu$ satisfying $\gamma_\mu v_\mu = 0$, where $\epsilon_\mu$ and $V^{(N)}_a (a = 0 \sim 8)$ represent the polarization vector and the flavor WF of the relevant vector meson, respectively.

In the third expression of Eq. (2.6), the WF is written in $SU(6)_{SF} \otimes SU(2)_\mu$ form. This WF is the eigen-function of chirality operation defined as Eq. (2.5), with the eigen-value $\chi = +1$ reflecting its chiral $SU(3)_L \otimes SU(3)_R$-representation, $(1_L, 1_R) \oplus (1_L, \bar{1}_R)/(8_L, 1_R) \oplus (1_L, 8_R)$. Here it may be notable that, as shown in the last line of Eq. (2.6), the $V(N)$-WF is a superposition pure-Pauli states and pure-chiral states.

On the other hand, another type of vector meson WF, $V(E)$ as given by

$$W(P)^{(+)B}_A = \left(\frac{-i}{2} \sigma_{\mu\nu} v_\nu \epsilon(P)_{\mu} V^{(E)}_a A^B \right)$$

with $\gamma_\mu = \gamma_\mu + (\gamma \cdot v)v_\mu$ satisfying $\gamma_\mu v_\mu = 0$.

$^6$ This type representation is well known and the common expression is used to various models. See, for example, Ref. [7]. Here it should be noted that the definition of chirality transformation Eq. (2.5) is deduced from the basic structure of our classification scheme.
is also contained in the GS of the $\tilde{U}(12)$-scheme, which is corresponding to the representation $(3_L, 3_R) \oplus (\bar{3}_L, \bar{3}_R)$ of the chiral $SU(3)_L \otimes SU(3)_R$. Thus, the eigenvalue of chirality is $\chi = -1$. This $V^{(E)}$-WF is the orthogonal superposition (to the $V^{(N)}$-WF) of Pauli- and chiral-states. Here it should be noted that generally the spin WF of physical $\rho$-nonets is given as a superposition of the $(N)$ and $(E)$ components, since they have the same $J^{PC}$. Being based on its success with $SU(6)_{SF}$-description for $\rho$-nonet, it seems that its WF should be taken as the same form as in our previous COQM, containing only Pauli-states. This is obtained by taking the following two linear combinations of $(N)$ and $(E)$, $V$ and $V'$ with equal weight,

$$V \equiv (V^{(N)} + V^{(E)})/\sqrt{2}, \quad V' \equiv (V^{(N)} - V^{(E)})/\sqrt{2}.$$  \hspace{1cm} (2.8)

Here the $V$ represents $\rho$-nonet as pure Pauli state, while the $V'$ represents the other nonets as “pure chiral state”, which is newly appeared in $\tilde{U}(12)$-scheme. They are mutually orthogonal tensors in static $SU(2)_\rho$-space. The resultant WF of physical $\rho$-nonet is described as

$$W(P)^{(+)B}_A = \left( \frac{1}{2\sqrt{2}} i \tilde{\gamma}_\mu (1 + iv\gamma) \epsilon(P)_\mu V_a \frac{\lambda^a}{\sqrt{2}} \right)^B_A$$

$$\text{with} \quad P = (\mathbf{0}, \mathbf{0}) \quad \Rightarrow \quad \left[ \frac{\sigma \cdot \epsilon(P = 0)}{\sqrt{2}} V_a^{(N)} \frac{\lambda^a}{\sqrt{2}} (|+\rangle_\rho |+\rangle_\beta) \right]^B_A.$$ \hspace{1cm} (2.9)

It should be noted that the above WF no longer has definite chirality, and instead it includes only the pure eigen-states with $(\rho_3, \bar{\rho}_3) = (+1, +1)$.

The above statement also holds for the two-type pseudo-scalar WFs. However, in contrast to the case of vectors, the WF of $\pi$-nonet, which is denoted by $P$, should belong to the same chiral representation as $\sigma$-nonet from the consideration on linear realization of chiral symmetry. The resultant WF of $\pi$-nonet is described as

$$W(P)^{(+)B}_A = \left( \frac{1}{2} i \gamma_5 P_a \frac{\lambda^a}{\sqrt{2}} \right)^B_A, \quad P \equiv P^{(N)}.$$ \hspace{1cm} (2.10)

(Effective Meson Currents )

In order to treat the interaction of $q\bar{q}$-mesons with the electro-magnetic (EM) field $A_\mu(X)$, we start from the free action of the meson system,

$$S_0 = \int d^4x_q \, d^4x_\bar{q} \langle \mathcal{L}_0(x_q, x_\bar{q}) \rangle_{S,F} = \int d^4x \, d^4x \langle \mathcal{L}_0(x, X) \rangle_{S,F} \equiv \int d^4X \mathcal{L}_0(X),$$

$$\mathcal{L}_0(X) = \int d^4x \langle \Phi(x, X) \left( \frac{\partial^2}{\partial X^2_\mu} - \mathcal{M}(x)^2 \right) \Phi(x, X) \rangle_{S,F}.$$ \hspace{1cm} (2.11)

Here, $\langle \cdots \rangle_{S,F}$ denotes taking the trace over Dirac spinor and flavor indices. The Pauli-conjugate of the bi-spinor function is defined as $\Phi^\dagger_\alpha \beta \equiv (\gamma_4)_{\alpha}^\dagger (\Phi^\dagger_\alpha)_{\beta} (\gamma_4)_{\beta}^\dagger$. Note that the action (2.11), which is the simplest form to lead to our basic Eq. (2.2), has the $\tilde{U}(12)_{SF}$-symmetry. Then it is necessary to implement the following modification. We shall explain this procedure by citing a concrete descriptions in the
The action \( A_0 \) yields the amplitude in momentum representation,

\[
\tilde{A}_0 = \frac{2\pi}{\sqrt{4p_0p_0'}} \delta^4(P - P') \left( p_0^2 - M_0^2 \right) \{ (\tilde{W}_H^{(-)}(P') W_H^{(+)}(P) )_S \right. \\
+ \left. (\tilde{W}_H^{(+)}(P') W_H^{(-)}(P) )_S \right\}. \quad (2.12)
\]

In Eq. \( (2.12) \), \( \langle \cdots \rangle_S \) represents taking the trace over the only Dirac spinor indices, while the flavor indices are specified here. The first (second) term of Eq. \( (2.12) \) describes annihilation of the meson \( H \) (anti-meson \( \bar{H} \)) with momentum \( P \) and creation of the meson \( H \) (anti-meson \( \bar{H} \)) with momentum \( P' \). In order to guarantee the static \( U(12)_{SF} \)-invariance of the amplitude \( (2.12) \) embedded in the \( \tilde{U}(12)_{SF} \)-space, we introduce the vertex factor,

\[
F_U(v) \equiv -iev \cdot \gamma, \quad (2.13)
\]

called unitarizer. Inserting the unitarizer in appropriate places of the above amplitude makes it static \( U_{SF}(12) \)-invariant and leads to the correct sign of mass term for the chiral states as well as the Pauli states. By using this prescription, Eq. \( (2.12) \) is replaced by

\[
\tilde{A}_0 \to A_0 = \frac{2\pi}{\sqrt{4p_0p_0'}} \delta^4(P - P') \left( p_0^2 - M_0^2 \right) \{ (\tilde{W}_H^{(-)}(P') F_U(v') W_H^{(+)}(P) F_U(v) )_S \right. \\
+ \left. (\tilde{W}_H^{(+)}(P') F_U(v') W_H^{(-)}(P) F_U(v) )_S \right\}. \quad (2.14)
\]

Next we consider the coupling of a photon with the general \( q\bar{q} \) mesons. By applying the conventional minimal substitution method to the free action \( (2.11) \),

\[
\partial_q,\mu \to \partial_q,\mu - iQeA(x_q)_{\mu}, \quad (2.15)
\]

\( Q = \text{diag}(2/3, -1/3, -1/3) \) being quark charge matrix), we obtain the action for the relevant EM interaction, up to lowest order of coupling constant \( e \),

\[
S_{EM}^I = \int d^4 x d^4 x_q \sum_i j_{i,\mu}(x_q, x) A_{\mu}(x_i) \\
= \int d^4 X J_{\mu}(X) A_{\mu}(X) \quad (i = q, \bar{q}). \quad (2.16)
\]

Writing only the terms related to our relevant transition process, \( (2.16) \) yields

\[
j_{i,\mu}(x_q, x) = -ie \frac{m_q + m_{\bar{q}}}{m_i} \langle \Phi(x, X)Q(g^{(i)}_M \sigma_{\mu\nu}( \vec{\partial}_i, \nu + \vec{\partial}_i, \nu))\Phi(x, X) \rangle_{S, F}. \quad (2.17)
\]

Here we have introduced the parameter \( g^{(i)}_M \) concerning intrinsic magnetic moment of their constituents. Integrating on the relative space-time coordinate for the GS-WF, we obtain the effective spin current of meson in momentum representation,

\[
J_{\mu}(P, P') = J_{q,\mu}(P, P') + J_{\bar{q},\mu}(P, P'), \quad (2.18)
\]
\[ J_{q,\mu}(P, P') = e\langle \bar{W}^{(-)}(P)Q\frac{m_q + m_{\bar{q}}}{m_q}g_M^{(q)}i\sigma_{\mu\nu}q_\nu\rangle W^{(+)}(P) \rangle_{S,F}, \]  
\[ J_{\bar{q},\mu}(P, P') = e\langle W^{(+)}(P)Q\frac{m_q + m_{\bar{q}}}{m_q}g_M^{(q)}i\sigma_{\mu\nu}q_\nu\rangle \bar{W}^{(-)}(P') \rangle_{S,F}, \]

where \( q_\mu \equiv P_\mu - P'_\mu \) is the four-momentum of photon, \( M(M') \) represents the mass of initial (final) meson. Here we used the fact referred in the footnote below Eq.(2.4). The above framework to treat the EM interaction, leading to conserved EM current of hadrons, is the same as in our previous \( \tilde{U}(12) \)-scheme, COQM.\textsuperscript{8)}

(Intrinsic electric dipole transition)

In our scheme, the relativistic covariance of the spin current play an important role in some radiative transition processes, due to the inclusion of Dirac spinor with negative \( \rho_3 \)-value. To clarify this point, we rewrite the spin current vertex operator as

\[ \sigma_{\mu\nu}iq_\nu A_\mu = \sigma_{\mu\nu}F_{\mu\nu} = \boldsymbol{\sigma} \cdot \mathbf{B} - i\rho_3 \boldsymbol{\sigma} \cdot \mathbf{E}. \]  

In the cases of transition between both positive (negative) \( \rho_3 \) Dirac spinors, as is well known, a principal contribution comes from the magnetic interaction,

\[ \bar{u}_{\rho_3 = \pm}(q)(\sigma_{\mu\nu}iq_\nu A_\mu) u_{\rho_3 = \mp}(0) \approx \chi^\dagger(\pm \boldsymbol{\sigma} \cdot \mathbf{B}) \chi \quad \text{for} \quad |q| \approx 0, \]  

where \( \chi \) represents two component \( SU(2)_\sigma \)-spinor. In this case, a contribution of the electric interaction, coming from the \( \sigma_{i4}iq_i A_4 \)-term, is appeared as a relativistic correction caused by the recoil effect. On the other hand, in transitions between Dirac spinors with positive and negative \( \rho_3 \)-values, the electric interaction becomes a principal contribution,

\[ \bar{u}_{\rho_3 = \pm}(q)(\sigma_{\mu\nu}iq_\nu A_\mu) u_{\rho_3 = \mp}(0) \approx \chi^\dagger(\mp i\boldsymbol{\sigma} \cdot \mathbf{E}) \chi \quad \text{for} \quad |q| \approx 0. \]

Accordingly, this “intrinsic electric dipole” transition describes the transition between GS mesons accompanied by their parity change.

(Chirality conservation in radiative transition process)

By using the current stated above, a very useful selection rule is realized in the radiative transition processes. That is, the chirality eigen value of the relevant meson is conserved. For an example,

\[ A^{(E)}(\chi = -) \rightarrow P(\chi = -) \quad \gamma \quad \text{; allowed} \]  
\[ A^{(N)}(\chi = +) \rightarrow P(\chi = -) \quad \gamma \quad \text{; forbidden} \]  
\[ V(\chi = + \pm -) \rightarrow P(\chi = -) \quad \gamma \quad \text{; allowed}. \]

\( \S 3. \) Numerical Results

(Fixing the parameters)
From the expression of currents, we can derive the relevant formulas of radiative decay width, which are given in the following Tables II and III.

In this work, we take the following values of parameters in our scheme.

- \( M, M' \): take the physical meson mass (PDG value), or the predicted values (see, K. Yamada’s talk),
- \( g_M^{(n)} = 1.196 \) (determined from the \( \rho^\pm \to \pi^\pm \gamma \)),
- The ratio of constituent quark mass: \( x \equiv \frac{m_{q'}}{m_q} = 0.82, \ g_M^{(s)} = 1.184 \) (determined from the \( K^*(K^00) \to K^*(K^0)\gamma \)),
- flavor mixing angle: \( \phi_P = -45.3^\circ, \ \phi_V = 3.4^\circ, \ \phi_{\text{others}} = 0^\circ \).

### Table II. Effective transition currents for radiative decays between the GS mesons.

Here, \( P \equiv P_s^{(N)} \), \( S \equiv S^{(N)} \), \( V \equiv (V^{(N)} + V^{(E)})/\sqrt{2} \), \( V' \equiv (V^{(N)} - V^{(E)})/\sqrt{2} \), and \( d \equiv 2(m_q + m_{q'}) \).

| process          | current              | coupling parameter                           |
|------------------|----------------------|---------------------------------------------|
| \( V \to P\gamma \) | \( J_\mu = ie\bar{\mu}_\nu \gamma \nu \bar{q}_\rho P_\alpha \) | \( \mu_0 = (\frac{d_m}{2m_q}g_M^{(q)}(PQV) + \frac{d_m}{2m_q}g_M^{(q)}(PVQ)) \frac{\alpha}{\sqrt{2M}} \) |
| \( V \to P\gamma \) | \( J_\mu = ie\bar{\mu}_\nu \gamma \nu \bar{q}_\rho P_\alpha \) | \( \mu_0 = (\frac{d_m}{2m_q}g_M^{(q)}(PQV) + \frac{d_m}{2m_q}g_M^{(q)}(PVQ)) \frac{\alpha}{\sqrt{2M}} \) |
| \( A^{(N)} \to P\gamma \) | \( J_\mu = ie\bar{\mu}_\nu \gamma \nu \bar{q}_\rho P_\alpha \) | \( \epsilon_{A(E)}(\bar{q}VQ) = (\frac{d_m}{2m_q}g_M^{(q)}(PQA^{(E)}) + \frac{d_m}{2m_q}g_M^{(q)}(PA^{(E)}Q))(qv) \frac{\alpha}{\sqrt{2}} \) |
| \( S \to V\gamma \) | \( J_\mu = -ie\bar{\mu}_\nu \gamma \nu \bar{q}_\rho P_\alpha \) | \( \xi_0 = (\frac{d_m}{2m_q}g_M^{(q)}(VQA^{(N)}) + \frac{d_m}{2m_q}g_M^{(q)}(VA^{(N)}Q)) \frac{\alpha}{\sqrt{2}} \) |
| \( A^{(N)} \to V\gamma \) | \( J_\mu = ie\bar{\mu}_\nu \gamma \nu \bar{q}_\rho P_\alpha \) | \( \xi_0 = (\frac{d_m}{2m_q}g_M^{(q)}(VQA^{(N)}) + \frac{d_m}{2m_q}g_M^{(q)}(VA^{(N)}Q)) \frac{\alpha}{\sqrt{2}} \) |

### Table III. Formulae for radiative dacay width. Here, the \( \omega \) and \( \omega_3 \) are defined as \( \omega \equiv -v \cdot v' \) and \( \omega_3 \equiv \sqrt{\omega^2 - 1} \), respectively. The \( \alpha \) denotes the fine structure constant, \( \alpha = 1/137 \).

| process         | \( \Gamma \) |
|-----------------|-------------|
| \( V(V') \to P\gamma \) | \( \frac{\alpha}{\sqrt{2}} \frac{\mu_0^{(1)} [q]^3}{2} \) |
| \( A^{(N)} \to P\gamma \) | \( \frac{\alpha}{\sqrt{2}} \frac{\epsilon_{A(E)}^{(2)} [q]}{2} \) |
| \( S \to V\gamma \) | \( \frac{\alpha}{\sqrt{2}} \frac{\xi_0^{(2)} [q]}{2} \) |
| \( A^{(N)} \to V\gamma \) | \( \frac{\alpha}{\sqrt{2}} \frac{\xi_0^{(2)} [q]}{2} \) |

(1. Radiative decays of \( V \)-meson)

As was discussed in §2, we use the \( V \)- and \( P \)-type BW functions, Eq.(2.8) and Eq.(2.10), as the spin WFs of the physical \( p \)-nonets and \( \pi \)-nonets, respectively. A possible decay mode of \( V \)-meson is only \( P_\gamma \), as well as conventional scheme. The results in comparison with experiments are shown in Table IV. Our estimated widths seems to be consistent with the experimental data. Here we emphasize that it comes from our choice of the \( SU(2)_\rho \) WF. the \( \pi \)-on spin WF as \( i\gamma_5/2 \) and the \( \rho \)-meson spin WF as \( \pi \) as possible with equal weight superposition of \( V^{(N)} \) and \( V^{(E)} \).

(2. Radiative decays of \( V' \)-meson)

In our scheme, the existence of “extra-vector” \( V' \)-nonet as “pure chiral state” is required as the \( q\bar{q} \) GS. The plausible candidates of extra-vector meson are pointed out
Table IV. Predicted radiative decay widths of pure Pauli-vector meson $V \rightarrow P \gamma$ process in comparison with experiments. The input values are underlined.

| $V \rightarrow P \gamma$ process | $\Gamma_{\text{theor}}$ (keV) | $\Gamma_{\text{exp}}$ (keV) |
|---------------------------------|-------------------------------|----------------------------|
| $\rho^0 \rightarrow \pi^+ \gamma$ | 68                           | 68 ± 7                     |
| $\rho^0 \rightarrow \eta \gamma$ | 45.2                         | 45.09                      |
| $K^{*\pm} \rightarrow K^\pm \gamma$ | 50                           | 50 ± 5                     |
| $K^{*0} \rightarrow K^0 \gamma$ | 116                          | 116 ± 10                   |
| $\omega \rightarrow \pi^0 \gamma$ | 620                          | 757.3                      |
| $\omega \rightarrow \eta \gamma$ | 4.20                         | 4.16                       |
| $\phi \rightarrow \pi^0 \gamma$ | 2.96                         | 5.24                       |
| $\phi \rightarrow \eta \gamma$ | 70.7                         | 55.2                       |
| $\phi \rightarrow \eta' \gamma$ | 0.327                        | 0.264                      |
| $\eta \rightarrow \rho^0 \gamma$ | 72.7                         | 59.59                      |
| $\eta \rightarrow \omega \gamma$ | 9.06                         | 6.12                       |

Table V. Predicted radiative decay widths of “extra-” pure chiral-vector meson $V' \rightarrow P \gamma$ process.

| $V' \rightarrow P \gamma$ process | $\Gamma_{\text{theor}}$ (keV) | $\Gamma_{\text{exp}}$ (keV) |
|-----------------------------------|-------------------------------|----------------------------|
| $\rho^0(1290) \rightarrow \pi^\pm \gamma$ | 120                          |                            |
| $\rho^0(1290) \rightarrow \eta \gamma$ | 329                          |                            |
| $\rho^0(1290) \rightarrow \eta' \gamma$ | 47.6                         |                            |
| $K^{*\pm}(1410) \rightarrow K^\pm \gamma$ | 161                          |                            |
| $K^{*0}(1410) \rightarrow K^0 \gamma$ | 372                          | $< 52.9^{14}$              |
| $\omega'(1290) \rightarrow \pi^0 \gamma$ | 894                          |                            |
| $\omega'(1290) \rightarrow \eta \gamma$ | 36.5                         |                            |
| $\phi'(1540) \rightarrow \pi^\pm \gamma$ | 0                            |                            |
| $\phi'(1540) \rightarrow \eta \gamma$ | 186                          |                            |
| $\phi'(1540) \rightarrow \eta' \gamma$ | 73.0                         |                            |
| $\omega'(1290) \rightarrow \eta' \gamma$ | 5.28                         |                            |

in nearly 1.3 GeV region. (For more detail, see, K. Yamada\(^{10}\) and T. Komada\(^{11}\)’s talks.) Possible decay modes of $V'$-meson are (a) $V' \rightarrow P \gamma$, (b) $V' \rightarrow S^{(N)} \gamma$, (c) $V' \rightarrow S^{(E)} \gamma$, (d) $V' \rightarrow V \gamma$, (e) $V' \rightarrow A^{(N)} \gamma$, (f) $V' \rightarrow A^{(E)} \gamma$. Here we show the result only for the case of (a) in Table VI. There is few experimental data be compared with these estimated numerical values. Taking naively the well-known $K^*(1410)$ meson as a $I = 1/2$ member of $V'$-nonet, then it seems that the predicted width is much larger than experimental one. (The only one reported experimental value\(^{14}\) for the relevant process is $\Gamma_{\text{exp}} < 52.9$keV). This problem may be possibly resolved by considering the mixing effect of the GS $V'$ nonet with $^3P_1/1P_1$ vector nonet in our scheme.

3. Radiative decays of $S^{(N)}$-meson

One of most important chiral multiplet in the $q\bar{q}$ GS is the scalar $\sigma$-nonet. All members have now established experimentally. Possible decay modes of $S^{(N)}$-meson is only $V \gamma$, as well as in the conventional scheme. The results in comparison with experiments and with the other models are shown in Table VI. Note that it is
Table VI. Predicted radiative decay widths of $S^{(N)} \rightarrow V \gamma$ process in comparison with other model and experiments.

| $S^{(N)} \rightarrow V \gamma$ process | $\Gamma_{\text{theor}}$(keV) | $\Gamma_{\text{exp}}$(keV) | $\Gamma_{3\rho_0}$(keV) in NRQM | $\Gamma_{4\rho_0}$(keV) with VMD model |
|-------------------------------------|-----------------|-----------------|-----------------|-----------------|
| $a_0(980)^{\pm} \rightarrow \rho^\pm \gamma$ | 25.1 | - | 14 | 3.0 |
| $a_0(980)^0 \rightarrow \omega \gamma$ | 203 | - | 125 | 641 |
| $\kappa_4^{\pm}(1105) \rightarrow K^{*\pm}(892)\gamma$ | 36.5 | - | - | - |
| $\kappa_4^0(1105) \rightarrow K^{*0}(896)\gamma$ | 79.1 | - | - | - |
| $\rho^0 \rightarrow \sigma (600)\gamma$ | 73.3 | - | - | 0.23/17 |
| $\omega \rightarrow \sigma \gamma$ | 8.99 | - | - | 16/33 |
| $\phi \rightarrow \sigma \gamma$ | 0.280 | - | - | 137/33 |
| $f_0(980) \rightarrow \rho^0 \gamma$ | 0 | - | 0 | 19/33 |
| $f_0(980) \rightarrow \omega \gamma$ | 0.292 | - | 0.109 | 126/88 |
| $\phi \rightarrow f_0(980)\gamma$ | 0.182 | 1.87 ± 0.11 | 0.18 | - |
| $\phi \rightarrow a_0(980)\gamma$ | 0.000987 | 0.32 ± 0.026 | 0.0013 | - |

not considered the finite width effect of $\kappa$- and $\sigma$-mesons and the coupled channel ($K^+K^-\text{-loop}$) effect which may play essential role in this process.\(^{[15]}\) in this calculation.

4. Radiative decays of $A^{(E)}$-meson

The $1^{-+}$-nonet are classified as the excited $^1P_1$ state in the conventional classification scheme. However, there is another $1^{-+}$-nonet as $A^{(E)}$-type GS chiralon in our scheme. Its possible radiative decay mode are (a) $A^{(E)} \rightarrow P \gamma$, (b)$A^{(E)} \rightarrow S_A \gamma$, (c)$A^{(E)} \rightarrow V \gamma$. Their estimated decay widths are shown in the Table VII only the case of (a).

Table VII. Predicted radiative decay widths of $A^{(E)} \rightarrow P \gamma$ process in comparison with experiments.

| $A^{(E)}$ process | $\Gamma_{\text{theor}}$(keV) | $\Gamma_{\text{exp}}$(keV) |
|-------------------|-----------------|-----------------|
| $b_1(1235)^\pm \rightarrow \pi^\pm \gamma$ | 229 | 230 ± 60 |
| $b_1(1235)^0 \rightarrow \eta \gamma$ | 586 | - |
| $b_1(1235)^0 \rightarrow \eta^0 \gamma$ | 61.0 | - |
| $h_1(1170) \rightarrow \pi^0 \gamma$ | 1957 | - |
| $h_1(1170) \rightarrow \eta \gamma$ | 57.3 | - |
| $h_1(1170) \rightarrow \eta^0 \gamma$ | 3.80 | - |
| $K_{1B}(1350)^\pm \rightarrow K^{\pm} \gamma$ | 297 | - |
| $K_{1B}(1350)^0 \rightarrow K^{*0} \gamma$ | 682 | - |
| $h_1(1380) \rightarrow \pi^0 \gamma$ | 0 | - |
| $h_1(1380) \rightarrow \eta \gamma$ | 299 | - |
| $h_1(1380) \rightarrow \eta^0 \gamma$ | 79.0 | - |

Here the result on the $b_1(1235)$-decay is specially to be noted : In the previous scheme, our predicted width for $b_1(1^{P_1}) \rightarrow \pi(1S_0)$ ($\Gamma_{\text{theor}} = 66 \text{ keV}$) was much smaller than experimental one ($\Gamma_{\text{exp}} = 230 \pm 60 \text{ keV}$). We had considered at that time that this discrepancy is quite serious for the scheme, since to this process contributes only the convection current, whose form reflects the conservation of EM current. However, it has been shown now that the experiment can be reproduced...
Table VIII. Predicted radiative decay widths of $A^{(N)} \rightarrow V\gamma$ process.

| $A^{(N)} \rightarrow V\gamma$ process | $\Gamma_{\text{theo}}\text{(keV)}$ |
|----------------------------------------|-------------------------------|
| $a_1(1210)^0 \rightarrow \rho^0\gamma$ | 82.1                          |
| $a_1(1210)^0 \rightarrow \omega\gamma$ | 701                           |
| $a_1(1210)^0 \rightarrow \phi\gamma$ | 0.218                         |
| $f_1(1420)^0 \rightarrow \omega\gamma$ | 2.74                          |
| $f_1(1420)^0 \rightarrow \rho^0\gamma$ | 0                             |
| $f_1(1420)^0 \rightarrow \phi\gamma$ | 179                           |
| $K_{1A}(1328)^0 \rightarrow K^*\gamma$ | 268                           |
| $K_{1A}(1328)^\pm \rightarrow K^{*\pm}\gamma$ | 120                           |
| $f_1(1210)^0 \rightarrow \omega\gamma$ | 77.8                          |
| $f_1(1210)^0 \rightarrow \phi\gamma$ | 0.024                         |
| $f_1(1210)^0 \rightarrow \rho^0\gamma$ | 739                           |

by assigning $b_1(1235)$ as the GS chiralon. We give the other results concerning the $A^{(E)} \rightarrow P\gamma$.

Remarks on $K_1(1270)$ and $K_1(1400)$: Concerning the radiative decays of $K_1(1270)$- and $K_1(1400)$-meson, the experimental data \(^{13}\) $\Gamma(K_1(1270) \rightarrow K^0\gamma) = 73.2\text{ keV}$, $\Gamma(K_1(1400) \rightarrow \phi\gamma) = 280.8\text{ keV}$ are reported, respectively. We examined this results by assuming these states being mixed states of $K_1^{(N)}$ and $K_1^{(E)}$, finding there is no solution of $(N)$-$(E)$ mixing angle parameter consistent with the both above. Thus, at least, the one of them is not pure chiral state.

(5. Radiative decays of $A^{(N)}$-meson)

As shown in the previous section, the $A^{(N)}$-nonet, which is the GS axial-vector chiral state, does not have the decay mode $A^{(N)} \rightarrow P\gamma$ due to chirality conservation rule. This fact indicates that well known $a_1(1260)$ meson decaying to $\pi\gamma$ is not a pure GS chiralon at least. This is consistent to our previous result by COQM that $a_1(1260) \rightarrow \pi\gamma$ could be explained as the decay of the $^3P_1$ Pauli-state. We predict the another $A^{(N)}$-nonet, being different from the $^3P_1$-state, in the lower mass region compared to 1260 MeV. Possible radiative decay modes are (a) $A^{(N)} \rightarrow V\gamma$ and (b) $A^{(N)} \rightarrow S^{(E)}\gamma$. Their estimated widths are shown in Table VIII only the case of (a).

Remarks on $f_1(1285)$: From the experimental data, we have $\Gamma(f_1(1285) \rightarrow \rho^0\gamma) = 1325\text{ keV}^{12} / 674.8\text{ keV}^{13}$. We examined this results by assuming $f_1(1285)$ being $f_1^{(N)}$ chiralon, finding there is no solution of flavor octet-singlet mixing angle parameter consistent with the both above. Thus, $f_1(1285)$ is not pure chiral state as well as $a_1(1260)$.

§4. Concluding Remarks

We have reinvestigated the radiative decay of the GS light quark mesons in our new scheme. As a result, it is shown that; (1) Experimental data on the ordinary $V \rightarrow P\gamma$ process are well reproduced in the $\tilde{U}(12)$-scheme, as well as our previous COQM results. The essential points are that to take the $\pi$-on spin WF as $i\gamma_5/2$, reflecting its Goldston-boson nature, and to take the $\rho$-meson spin WF as being a pure-Pauli state.
(2) The $b_1(1235)$, has been classified as the $P$-wave excitation in the conventional classification scheme, is a promising candidate of the chiral states. (3) It is also shown that $K_1(1270)$ and $K_1(1400)$ are not both pure chiral states, and also $K^*(1410)$, $f_1(1285)$ is not pure chiral state. So, we have to consider the possibility of mixing them with the P-wave states. (4) The radiative decay widths of $V^+$, $S^{(N)}$, $A^{(N)}$, and $A^{(E)}$-mesons are predicted. They should be checked experimentally.

Acknowledgements

We are grateful to Kunio Takamatsu and other member of the sigma group for useful discussions.

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