Measuring $\Omega_0$ using cluster evolution

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ABSTRACT

The evolution of the abundance of galaxy clusters depends sensitively on the value of the cosmological density parameter, $\Omega_0$. Recent ASCA data are used to quantify this evolution as measured by the cluster X-ray temperature function. A $\chi^2$ minimisation fit to the cumulative temperature function, as well as a maximum likelihood estimate (which requires additional assumptions about cluster luminosities), lead to the estimate $\Omega_0 \approx 0.45 \pm 0.2$ (1-$\sigma$ statistical error). Various systematic uncertainties are considered, none of which enhance significantly the probability that $\Omega_0 = 1$. These conclusions hold for models with or without a cosmological constant, i.e. with $\Lambda_0 = 0$ or $1 - \Omega_0$. The statistical uncertainties are at least as large as any of the individual systematic errors that have been considered here, suggesting that additional temperature measurements of distant clusters will allow an improvement in this estimate. An alternative method that uses the highest redshift clusters to place an upper limit on $\Omega_0$ is also presented and tentatively applied, with the result that $\Omega_0 = 1$ can be ruled out at the 98 per cent confidence level. Whilst this method does not require a well-defined statistical sample of distant clusters, there are still modelling uncertainties that preclude a firmer conclusion at this time.

Key words: galaxies: clusters – cosmology: theory.

1 INTRODUCTION

Galaxy clusters are the largest virialised objects in the universe and, as such, provide useful cosmological probes. For example, the evolution in the abundance of clusters is strongly dependent on, and is therefore a sensitive probe of, the cosmological density parameter, $\Omega_0$, (e.g. Evrard 1989; Frenk et al. 1990; Lilje 1992; Oukbir & Blanchard 1992).

The amount of observational data concerning high-redshift cluster properties has increased significantly in recent years. EMSS (Henry et al. 1992; Gioia & Luppino 1994), ASCA (e.g. Donahue 1996; Mushotzky & Scharf 1997; Henry 1997) and ROSAT (Ebeling et al. 1997, Burke et al. 1997, Rosati et al. 1998) measurements of the X-ray emitting intracluster plasma now complement the low-redshift studies carried out by Edge et al. (1990) and David et al. (1993). In addition, the CNOC survey provides galaxy velocity distributions for a well-defined sample of high-redshift clusters (Carlberg et al. 1996) and further information on mass distributions is coming from weak gravitational lensing measurements (e.g. Luppino & Kaiser 1997; Smail et al. 1997).

The modelling of the cluster population has relied heavily upon the expression provided by Press & Schechter (1974) for the mass function of haloes. This has received support from dark matter simulations of structure formation (e.g. Efstathiou et al. 1988; White, Efstathiou & Frenk 1993, Lacey & Cole 1994). More recently the treatment of a baryonic component in simulations has led to a closer link with observational measurements (Evrard 1990; Cen & Ostriker 1994; Metzler & Evrard 1994; Bryan et al. 1994; Navarro, Frenk & White 1995, hereafter NFW). In addition, analytical work investigating the evolution of properties of the cluster population as a whole has been performed by Kaiser (1986, 1991), Evrard & Henry (1991) and Bower (1997).

There is now sufficient overlap between these strands of research to allow useful conclusions concerning cosmological parameters to be drawn from the available data (see however Colafrancesco, Mazzotta & Vittorio 1997). Several groups have already performed this operation (Henry 1997; Carlberg et al. 1997b; Bahcall, Fan & Cen 1997; Sadat, Blanchard & Oukbir 1998; Blanchard & Bartlett 1997; Reichart et al. 1998), although the lack of consensus on the measured value for $\Omega_0$ casts doubt on the reliability of at least some of these studies. The purpose of this paper is to investigate the systematic uncertainties in the estimation of $\Omega_0$ using cluster evolution, and to understand why the various studies differ.

In Section 2 the cosmology-dependent model for de-
scribing the evolution of the cluster population is described. The procedure for and results from fitting the models to the observed temperature functions are presented in Section 3. Section 4 contains a description of a maximum likelihood method used to estimate cosmological parameters using the cluster redshifts and temperatures. The results from these two methods are compared with other studies in Section 5. In Section 6 an alternative method is explored for using high-redshift cluster data to constrain \( \Omega_0 \). A discussion of the different techniques is presented in Section 7. The implications of these results for \( \Omega_0 = 1 \) scenarios are discussed in Section 8 and conclusions are summarized in Section 9.

2 MODEL

The model of the population of hot galaxy clusters employed here is the same one described by Eke, Cole & Frenk (1996). This is based upon the analytical expression for the comoving number density of dark matter haloes first derived by Press & Schechter (1978):

\[
\frac{dn}{dM_{\text{vir}}} = \left( \frac{2}{\pi} \right)^{\frac{3}{2}} \frac{\bar{\rho}}{M_{\text{vir}}^2} \frac{\delta_c(z)}{\sigma} \frac{d \ln \sigma}{d \ln M_{\text{vir}}} \exp \left( -\frac{\delta_c(z)^2}{2\sigma^2} \right).
\]

Here \( M_{\text{vir}} \) denotes the halo virial mass, defined such that a halo has a mean density \( \Delta_c \) times the critical density at redshift \( z \), where \( \Delta_c \) is evaluated using the spherical collapse model; \( \bar{\rho} \) represents the present-day mean density of the universe and \( \sigma(M_{\text{vir}}) \) the present-day, linear theory rms density fluctuation in spheres containing a mean mass \( M_{\text{vir}} \). Throughout this paper, we use the fit to the cold dark matter power spectrum of Bardeen et al. (1986). The shape of the power spectrum and its amplitude at \( 8h^{-1}\) Mpc are described by \( \sigma_8 \) and \( \Gamma \) respectively (Efstathiou, Bond & White 1992). The spherical collapse threshold, \( \delta_c \), is employed for most of the analysis in the rest of this paper. Bryan & Norman (1998) report that their work, combined with the majority of published results, suggests that a slightly (<10 per cent) higher value provides a better fit to mass functions from dark matter simulations. However, the recent simulations of Tozzi & Governato (1997) appear to support a lower value. This will be considered further in Section 3. Monaco (1998) has provided a review of the present understanding of the mass function.

An additional crucial ingredient in the model is the conversion from virial mass to emission-weighted X-ray temperature:

\[
kT_{\text{gas}} = \frac{7.75}{\beta_{TM}} \frac{6.8}{5X + 3} \left( \frac{M_{\text{vir}}}{10^{15} h^{-1} M_\odot} \right)^{\frac{3}{2}} \times (1 + z) \left( \frac{\Omega_0}{\Omega(z)} \right)^{\frac{1}{2}} \left( \frac{\Delta_c}{178} \right)^{\frac{1}{2}} \text{keV.}
\]

In this equation, \( \beta_{TM} \) is a conversion factor, the numerical value of which will be discussed shortly, and \( X \) is the hydrogen mass fraction which is taken to be \( X = 0.76 \). If the specific galaxy kinetic energy equals the specific gas thermal energy within the virial radius, then a value of \( \beta_{TM} = 1 \) would be appropriate for an isothermal gas. Alternatively, this value follows from applying hydrostatic equilibrium at the virial radius and assuming both that the sum of the logarithmic derivatives of the gas density and temperature profiles at this radius is ~2 and that the gas temperature at this radius equals the overall emission-weighted value. Neither of these reasons is particularly persuasive. Firstly, there is likely to be some residual kinetic energy in the gas as a result of incomplete thermalisation (Evrard 1989). Secondly, numerical simulations show that the gas density is dropping off faster than \( r^{-2} \) at the virial radius and that the temperature is also decreasing rather than remaining constant (e.g. NFW). The value of \( \beta_{TM} \) chosen here is based solely upon the results of hydrodynamical cluster simulations. These usually include shock heating but no radiative cooling or heating from any source. Such experiments (NFW; Evrard, Metzler & Navarro 1996; Eke, Navarro & Frenk 1998; Bryan & Norman 1998; Frenk et al. 1998) suggest that \( \beta_{TM} = 1.0 \) is a reasonable choice, and it is adopted as the default value in this paper, although the dependence of the results on this assumption will be investigated in Section 6. Values returned from different types of hydrodynamical simulations for a variety of cosmologies range between 1 and 1.3 (see Bryan & Norman 1998), with a slight preference for larger values of \( \beta_{TM} \) when \( \Omega_0 \) is larger. No significant mass dependence of \( \beta_{TM} \) is found.

The redshift dependence of the mean value of \( \beta_{TM} \) for the 10 clusters simulated by Eke et al. (1998), and the scatter among the individual cluster values are shown in Fig. 1. Most importantly for estimating \( \Omega_0 \) from cluster evolution out to \( z \sim 0.4 \), there is no significant evolution in either the mean \( \beta_{TM} \) or in the magnitude of the scatter about this value. As the temperature function has such a steep dependence on cluster temperature, uncertainties and possible biases in the mass-temperature conversion are very important (e.g. Pen 1998). A Gaussian scatter of 20 per cent in the temperature at a given mass has been included, consistent with Fig. 1 when calculating the model predictions. The neglect of physical processes associated with galaxy formation in hydrodynamical simulations of clusters is a concern, but the work of Metzler & Evrard (1994) suggests that even extreme feedback from stellar evolutionary effects cannot heat the intracluster medium of a rich cluster by more than about 15 per cent. Similarly, NFW found that \( \beta_{TM} \) was decreased by about 20 per cent if preheating was included in rich cluster simulations in order to match the slope of the observed luminosity-temperature relation. The study by White (1991) comparing observed velocity dispersions with temperatures is also suggestive that non-gravitational processes have a relatively small effect on rich clusters. In Section 3 the possible extent to which the results presented here might depend upon physics missing from the numerical simulations will be considered further.

3 EVOLUTION OF THE CLUSTER TEMPERATURE FUNCTION

3.1 Estimating the cumulative temperature functions

The cluster X-ray emission-weighted temperature measurements that are used here come from two different sources. At low redshift, an updated version of the flux-limited
The top panel shows the evolution with redshift of the parameter $\beta_{TM}$, relating the emission weighted X-ray temperature to the halo mass, as measured from an ensemble of 10 high-resolution hydrodynamical simulations of an $\Omega_0 = 0.3$, $\Lambda_0 = 0.7$ cosmology (Eke et al. 1998). The lower panel shows the standard deviation of the individual cluster $\beta_{TM}$ values about this mean value as a function of redshift.

\[ (f(2-10\text{ keV}) > 3 \times 10^{-14}\text{Wm}^{-2}\text{sr}^{-1}) \] sample of 25 clusters compiled by Henry & Arnaud (1991, hereafter HA91) is employed (see Henry 1998 for details). These authors report that this sample is “at least 90 per cent complete”. In this Section, a completeness of 100 per cent will be assumed, but the effect of this uncertainty will be investigated further in Section 6. The average 1σ measurement uncertainty in these temperatures is 7 per cent. For the high redshift data, the 10 EMSS $0.3 < z < 0.4$ cluster temperatures obtained by Henry (1997) are employed. These have an average 1σ uncertainty of 14 per cent. No systematic differences are found between these results and those of Mushotzky & Scharf (1997) for the 5 cases where they studied the same clusters.

In each case the cumulative cluster temperature function is estimated using

\[ N(> kT) = \sum_{kT > kT} 1/V_{\text{max}, i}, \tag{3.1} \]

where $V_{\text{max}, i}$ is the maximum volume in which cluster $i$ could be detected given the survey selection criteria, and is calculated using the cluster fluxes and redshifts. A complication for the high-z sample is that the area of sky surveyed was a function of the sensitivity, i.e. only a small fraction of the total survey area was observed at the lowest flux levels. Using the data in table 3 of Henry et al. (1992), the following expression has been found to give the area of sky, $A$, in which a cluster providing a detect cell flux, $f_{\text{det}}$, would have been detectable by the EMSS:

\[ \frac{A(f_{\text{det}})}{A_{\text{tot}}} = 1 - 3.05 e^{-0.43f_{\text{det}}} + 2.30 e^{-0.77f_{\text{det}}}. \tag{3.2} \]

Here, $f_{\text{det}}$ is in units of $10^{-16}$W m$^{-2}$ and $A_{\text{tot}}$ is the maximum area covered by the EMSS cluster survey, 735 square degrees. This fit is accurate to within 10 per cent at all detect cell fluxes above the Henry (1997) catalogue limit of $2.5 \times 10^{-16}$W m$^{-2}$. Using the cluster detect cell fluxes and redshifts provided in table 1 of Henry (1997), the cluster flux can be calculated as a function of redshift. $V_{\text{max}, i}$ is then evaluated by integrating the product of $dV/dz$ and survey area at a particular redshift from $z=0.3$ to $z=0.4$.

Before this estimate of $N(> kT)$ can be compared with the model, a correction for incompleteness in the measured high-redshift temperature function is required. This arises because of the non-zero scatter in the luminosity-temperature relation and the fact that the high-redshift sample has a non-zero lower redshift limit. As a consequence, at the low redshift limit of the sample, the flux limit of $2.5 \times 10^{-16}$W m$^{-2}$ corresponds to a non-zero luminosity limit such that for any given temperature, only clusters with luminosities higher than this limit will be included. The incompleteness will be greater for cooler clusters because, on average, they have lower luminosities. To calculate the amplitude of this effect, the data of HA91 for low-redshift clusters were used to provide both a mean luminosity as a function of cluster temperature and a scatter about this mean. What observational evidence there is suggests that the luminosity-temperature relation evolves very little out to redshifts of 0.4 (Tsura et al. 1996; Mushotzky & Scharf 1997; Henry 1997). This gave

\[ \log_{10} L_{\text{lim}}^{0.3-3.5} = 3.54 \log_{10}(kT) - 2.53, \tag{3.3} \]

where $L_{\text{lim}}^{0.3-3.5}$ is the 0.3 – 3.5 keV luminosity measured in $10^{37}h^{-2}$ W and $kT$ is in units of keV, and a Gaussian scatter in $\log_{10} L_{\text{lim}}^{0.3-3.5}$ of

\[ \sigma = 0.69 - 0.47 \log_{10}(kT). \tag{3.4} \]

At any given temperature, the fraction of clusters that are actually selected is then just given by

\[ F(T, z_{\text{lim}}) = 0.5 \text{erf}(\sqrt{2} \sigma) \]

where erf represents the complementary error function, $\sigma_{\text{lim}} = (\log_{10} L_{\text{lim}} - \log_{10} L)/\sqrt{2} \sigma$ and $L_{\text{lim}}$ is the (cosmology-dependent) luminosity of a cluster at $z = 0.3$ having a flux equal to $f_{\text{lim}}$. To allow for the fact that the catalogue flux limit refers to the EMSS detect cell whilst the clusters are typically more extended, a factor of 2.1 has been included to account for the conversion between these two fluxes (see figure 1 of Henry et al. 1992). The product of this factor and the differential number density at the same temperature can then be integrated with respect to temperature to give the correction to the cumulative temperature.

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* For the benefit of older readers, 1 W $\equiv 10^7$ erg s$^{-1}$. 

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function. Even for temperatures as low as 4 keV the incompleteness is no more than about 20 per cent. This can be seen in Figure 2, where the low redshift cumulative cluster temperature function is shown as a stepped line along with that from the high-z sample, calculated assuming that $\Omega_0 = 1$ and ignoring the incompleteness correction. The points represent the data used for the quantitative comparison with the models, and they include the correction factor which only has any significant effect on the lowest temperature measurement. The 1σ uncertainties on the number densities come from a bootstrap resampling procedure using $10^4$ catalogues.

### 3.2 Comparison with model predictions

Given that the cumulative cluster number density varies rapidly with temperature, uncertainties in either the mass-to-temperature conversion or in the observed temperatures of individual clusters will impact more on the results than any minor incompleteness. These effects can both be modelled by smoothing the model temperature function before it is compared with observational data. The width of the smoothing function, which has been taken to be a Gaussian, has been estimated by adding the fractional uncertainties from the two sources in quadrature. From the results in Fig. 1, the scatter in individual cluster $\beta_{TM}$ values is slightly less than 20 per cent and it appears to be independent of redshift for $z < 0.4$. While these simulations were of an $\Omega_0 = 0.3$, $\Lambda_0 = 0.7$ model, in the absence of information to the contrary, this scatter is assumed to apply to other cosmological models as well. For both observational redshift ranges, the temperature uncertainties are sufficiently small compared to the scatter in $\beta_{TM}$, that neglecting them will not affect the final results significantly. A Gaussian smoothing width of $0.2T$ was therefore used. From Fig. 3 it is apparent that this correction is not negligible compared with the amount of evolution that the $\Omega_0 = 1$ model predicts. However, if the width of the smoothing Gaussian is similar for both the high- and low-redshift samples, then the amount of evolution is largely unaffected because the cumulative temperature function is almost a power law. The estimate of $\sigma_8$ will be decreased slightly by this effect. Compared with the rapid evolution in the number of clusters at a particular temperature, Fig. 3 illustrates how little the temperature at a fixed number density evolves, and how important any evolution in the mass-temperature relation is over this range of redshifts.

The best-fitting parameter set ($\Omega_0$, $\Gamma$, $\sigma_8$) was found by minimising $\chi^2$ for the models calculated at five and three temperatures in the low- and high-redshift bins respectively. Bin temperatures were chosen to be far apart, in order to reduce the correlations between the different points. Nevertheless, the full covariance matrix was employed when calculating the $\chi^2$ values. The number of bins used was the minimum for which the results of the model fitting were found to be robust to the exact positioning of the bins. If any fewer bins are taken, then the results depend upon their chosen temperatures. This is particularly so for the low-redshift data where the observed function is less smooth than in the high-redshift case. The statistical uncertainties in the best-fitting parameters were quantified using both the change in $\chi^2$ and
the distribution of best-fitting values from the $10^4$ bootstrap catalogues that were created from the observational data.

### 3.3 Results

The $\chi^2$ minimisation gave rise to the following best-fitting parameter values: $\Omega_0 = 0.35$, $\Gamma = 0.07$ and $\sigma_8 = 0.71$ for $\Lambda_0 = 0$ and $\Omega_0 = 0.31$, $\Gamma = 0.07$ and $\sigma_8 = 0.78$ for $\Lambda_0 = 1 - \Omega_0$. In the latter case, two different effects act in opposite directions on the most likely value of $\Omega_0$ compared to the $\Lambda_0 = 0$ case. If $\Lambda_0$ is non-zero, then the expected amount of evolution at a particular $\Omega_0$ increases because of the more rapidly changing growth factor. However, the volume surveyed between redshifts 0.3 and 0.4 also increases if a $\Lambda_0$ term is included, and this has the effect of decreasing the observed cluster number density at high-redshift relative to the low-redshift measurement. Given that the most likely $\Omega_0$ decreases, albeit only slightly, when $\Lambda_0$ is assumed to be non-zero, the former effect is the more important. The best-fitting curves for $\Lambda_0 = 0$, together with the observational data points, are shown in Fig. 4.

In order to assess the statistical uncertainties in the estimated parameters, one can consider both contours of $\Delta \chi^2$ and the distributions of the best-fitting values of the bootstrap catalogues. It is interesting to look at both of these because the $\chi^2$ method formally assumes that the uncertainty in the estimated temperature function is Gaussian distributed, which is not fully justified given the small number of clusters in the two samples. Fig. 5 shows the distribution of best-fitting bootstrap catalogue parameters. The open and flat models are represented by solid and dotted lines respectively. Figures 6 and 7 show slices through the three-dimensional parameter space at different values of $\Gamma$ and $\Omega_0$ respectively. The contour levels represent 1, 2 and $3\sigma$ contours for two degrees of freedom, i.e. not marginalised with respect to the parameter that is fixed. All panels are for open models with $\Lambda_0 = 0$. Very little changes with the inclusion of a non-zero $\Lambda_0$. The minimum values of $\chi^2$ are given for each of the slices. Estimates of the uncertainties have been obtained by looking at the cumulative distributions of the bootstrap catalogue best-fitting parameters. The following ranges contain 68 per cent of these values:

$$
\Omega_0 = [0.24, 0.49], \quad \Gamma = [0.02, 0.16], \quad \sigma_8 = [0.64, 0.76] \quad (3.6)
$$

if $\Lambda_0 = 0$, and

$$
\Omega_0 = [0.20, 0.49], \quad \Gamma = [0.02, 0.16], \quad \sigma_8 = [0.67, 0.88] \quad (3.7)
$$

if $\Lambda_0 = 1 - \Omega_0$. When the individual bootstrap catalogue best-fitting parameters were overplotted on the $\Delta \chi^2$ contours shown in Figs. 6 and 7, they were found to trace very similar patterns. Only a few lay outside the $3\sigma$ contours, having $\Omega_0$ values slightly larger than would be expected given these two figures. However, as is evident from Fig. 5, there are nevertheless very few catalogues favouring $\Omega_0 > 0.9$. This provides some reassurance that the $\Delta \chi^2$ contours are not providing a misleading impression of the statistical uncertainties in the various parameters, despite the low number of clusters per bin.
Figure 6. Contours of constant $\Delta \chi^2$ for slices in parameter space at fixed $\Gamma$, as given in the individual panels. The $\Delta \chi^2$ values shown are 2.30, 6.17 and 11.8 and, for two interesting parameters, these correspond to enclosed probabilities of 1, 2 and 3$\sigma$. All panels are for the $\Lambda_0 = 0$ models. In each case, the minimum $\chi^2$ from all of the models with the chosen $\Gamma$ is also given.

Figure 7. Contours of constant $\Delta \chi^2$ for slices in parameter space at fixed $\Omega_0$, as given in the individual panels. The $\Delta \chi^2$ values shown are 2.30, 6.17 and 11.8 and, for two interesting parameters, these correspond to enclosed probabilities of 1, 2 and 3$\sigma$. All panels are for the $\Lambda_0 = 0$ models. In each case, the minimum $\chi^2$ from all of the models with the chosen $\Omega_0$ is also given.

4 MAXIMUM LIKELIHOOD PARAMETER ESTIMATION

One drawback of the method presented in the previous Section is that the precise cluster redshifts are not employed and so the modelling ignores any evolution within the redshift ranges spanned by the two data sets. In this Section, details are given of a maximum likelihood technique that has been applied to determine the most likely model for producing the measured values of cluster temperatures and redshifts. This has the benefit over the previous method that no particular binning in temperature is required and that the individual cluster redshifts are used. The disadvantage is that the measured cluster fluxes are not included in the scheme, and additional assumptions about the cluster luminosities need to be made.

If $a(z,T)dzdT$ is the model probability of observing a cluster with temperature $T$ at redshift $z$, then the likelihood of the observed distribution of cluster temperatures and redshifts can be written as

$$L = \Pi_{i=1}^{N} a_i dz dT e^{-a_i dz dT} \Pi_{j=N+1}^{N} e^{-a_j dz dT}.$$  \hspace{1cm} (4.1)

Here $j$ is an index running over all those of the $N_{\text{Tot}}$ total bins in redshift and temperature that are empty, and $i$ runs over just the $N$ bins occupied by the observed clusters. This equation follows from assuming that the cluster redshifts and temperatures are independent, and taking bins such that $a_i \ll 1 \forall i$. Neglecting the terms that do not depend upon the cosmological parameters, equation (4.1) can be rewritten as

$$\ln L = \Sigma_{i=1}^{N} a_i - \int \int adz dT.$$  \hspace{1cm} (4.2)

The function $a(z,T)$ can be defined as

$$a(z,T) = n(z,T,p) \frac{dV(z,p)}{dz} \zeta(z,T,p)$$  \hspace{1cm} (4.3)

where $p$ represents the three model parameters being investigated, $n$ is the comoving cluster number density given by the Press-Schechter-based model including a correction for the 20 per cent uncertainty in the mass-temperature conversion, $V$ is the comoving volume and $\zeta$ is the product of the fraction of sky surveyed and the estimated completeness of the survey. For the high redshift catalogue, the fraction of sky surveyed has been calculated by integrating over all possible fluxes at a given $(z,T,p)$, thereby accounting for the scatter in the luminosity-temperature relation and the variation of sky covered as a function of flux in the EMSS. A single temperature bremsstrahlung model has been used to calculate the ‘K’-corrections to the cluster fluxes. The default temperature to luminosity conversions that have been employed are given by equation (4.3) for the $0.3 - 3.5$ keV band and, again using the HA91 clusters,

$$\log_{10} \frac{F_{0.3-10}}{\text{ergs} \text{s}^{-1} \text{cm}^{-2} \text{arcsec}^{-2}} = 3.93 \log(kT/\text{keV}) - 2.92$$  \hspace{1cm} (4.4)

for the $2 - 10$ keV band. The scatter in $\log_{10}(\text{luminosity})$ about these relations is described by equation (4.4) and by

$$\sigma = 0.70 - 0.52 \log_{10}(kT/\text{keV})$$  \hspace{1cm} (4.5)

respectively. To estimate how the choice of $L-T$ conversion affects the results, the David et al. (1993) sample has also been considered, giving

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The dependence of the best-fitting parameters on the following factors has been investigated: the assumed completeness of the low-redshift sample, the assumed conversion between cluster flux and EMSS detect cell flux, the luminosity-temperature conversions, and the amplitudes of the scatter about these mean relations. The default choice is to assume that both high and low-redshift samples are 100 per cent complete. Note that a lower completeness for the high-redshift sample would give rise to lower estimates for Ω₀. Unless stated otherwise, a cluster at redshift z is taken to have 2.4 − z times as much flux as would fall into the EMSS detect cell (see figure 1 of Henry et al. 1992). The redshift and temperature limits for the binning are chosen so as just to encompass all of the data points. For the low-redshift sample this means 2.5 <kT/keV< 10 and 0 < z < 0.1, and for the high-redshift bins 3.7 <kT/keV< 10 and 0.3 < z < 0.4. The region of parameter space that has been searched for the most likely parameters is 0.01 ≤ Ω₀ ≤ 1 (with Λ₀ = 0 or 1 − Ω₀), 0.01 ≤ Γ ≤ 0.25 and 0.5 ≤ σ₈ ≤ 1.2.

4.1 Results

With the default assumptions described in the preceding section, the most likely parameter values are Ω₀ = 0.52, Γ = 0.08 and σ₈ = 0.63 for the Λ₀ = 0 model. The statistical uncertainties on these parameters can be determined by considering the marginalised likelihoods. These one-dimensional likelihoods are shown for all three of the fitted parameters in Fig. 3. In each case the maximum likelihood is found at each value of the chosen parameter, independently of the other two parameters. Confidence levels are therefore taken for a single interesting parameter, leading to 1σ uncertainties of Ω₀ = 0.52±0.17, Γ = 0.08±0.04 and σ₈ = 0.63±0.08. The model predictions for the marginalised number of clusters as a function of either redshift or temperature are shown in Fig. 4 for the most likely case. It is evident that the high-redshift sample cluster distribution is not very well matched by the model. In fact a Kolmogorov-Smirnov test indicates that the probability that this data set was produced from the model distribution is of order 10⁻³. A comparison of CNOC (Carlberg, Yee & Ellingson 1997a) with EMSS (Henry et al. 1992) redshifts suggests that the low probability cannot be accounted for by redshift measurement errors. If the maximum redshift limit of the Henry (1997) cluster sample is decreased from 0.4 to 0.33 and the likelihoods are recomputed then the resulting Ω₀ becomes ≤ 0.13 (the best-fitting σ₈ is at least 1.2, which is the largest value considered on the grid of likelihoods). This suppression of the high-z survey cluster distribution with the corresponding removal of only one of the 10 clusters has the effect of reducing the apparent evolution and, likewise Ω₀. However, if the redshift distribution of the z > 0.4 EMSS clusters is also considered, then models with so little evolution will overproduce distant objects so vastly that even the additional uncertainties in such predictions are unlikely to account for the results. The redshift distribution of the 0.3 < z < 0.4 clusters is therefore rather difficult to explain.

Fig. 4 shows the effect on the most likely parameters produced by altering the model assumptions. From top to bottom, the rows show the variation with 1) βTM, 2) survey incompleteness and uncertain f_clus/f_int, 3) L−T treatment and 4) the inclusion of a non-zero Λ₀ term. Table 2 contains details of the most likely parameters in the different cases.

Increasing βTM has the effect of boosting σ₈. This suppresses the amount of evolution at any given Ω₀, thus leading
The second row in Fig. 11 shows the relative insensitivity of the results to the assumed conversion between total cluster flux and EMSS detect cell flux, $\Delta \Omega_0 = \pm 0.08$ for extreme choices (see Henry et al. 1992 figure 1). When the ratio $f_{\text{clus}}/f_{\text{det}}$ is taken to be larger, this corresponds to a need for more luminous clusters in the high-redshift sample and hence a lower value for $\Omega_0$. A 10 per cent incompleteness in the low-redshift cluster sample enhances the evolution enough to create $\Omega_0 = 0.58$. However, if the high-redshift sample was less than 100 per cent complete then the best-fitting value would decrease accordingly (see Section 8).

The scatter in the luminosity-temperature relations has varied to see how much the most likely density parameter can be increased. In the third row of Fig. 11, it is apparent that very little impact is made with these alterations. More significant is the reduction in $\Omega_0$ that comes from exchanging the $L - T$ equations (4.6) and (4.7) for the default choices (3.3) and (4.3). This yields $\Omega_0 = 0.43$. The amplification of the high-redshift luminosity at a given temperature by 50 per cent, which is more than is observed (Tsuru et al. 1996; Mushotzky & Scharf 1997; Henry 1997), increases the number of clusters that should be seen further away. Consequently, to fit the observed abundance, it is necessary to increase the amount of evolution to compensate, giving a best-fitting $\Omega_0 = 0.64$.

As is evident from this figure, the best-fitting value of $\Omega_0$ is difficult to change by more than ~ 0.1 with any single one of these systematic changes. Additional reasons why these results might be misleading will be considered in Section 8, with particular emphasis on what can be said about the health of the critical density model in the light of cluster evolution.

The final row in Fig. 11 shows how the most likely parameters are affected by the inclusion of a $\Lambda_0 = 1 - \Omega_0$ term. In this case, the most likely parameters and $1\sigma$ marginalised uncertainties are $\Omega_0 = 0.45^{+0.18}_{-0.16}$, $\Gamma = 10^{+0.08}_{-0.05}$, and $\sigma_{B} = 0.69^{+0.13}_{-0.08}$.

Only the choice of upper redshift limit for the Henry (1997) sample and the excessive change in $\beta_{TM}$ of the systematic effects investigated above, manage to alter the most likely $\Omega_0$ by much more than 0.15. However, the statistical uncertainties are sufficiently large that in none of the above cases is $\Omega_0 = 1$ ruled out with a significance of 3$\sigma$ or greater.

## 5 COMPARISON WITH OTHER STUDIES

The analysis presented here produces results in good agreement with those of Henry (1997). This is not surprising, given that very nearly identical data have been employed, albeit within different modelling procedures. These results fit happily with the ROSAT logN-logS analysis by Mathiesen & Evrard (1998) as long as the luminosity evolves like $L \propto (1 + z)^{-\gamma}$. This is approximately what is implied by equations (1.6) and (5.5), although it should be noted that the results in Section 3 are almost completely insensitive to the assumed form of the L-T relation, unlike those in Section 5. Kitayama & Suto (1997) have also used the logN-logS relation of X-ray clusters to arrive at the conclusion that $\Omega_0 = 0.45$ is favoured if $\Lambda_0 = 0$.

Of the other results in this field, there appear to be two more distinct camps, one on either side of the above settle-
Figure 10. $\Delta(-\ln \text{likelihood})$s for $\Omega_0$ (left panels), $\Gamma$ (middle) and $\sigma_8$ (right), marginalised over the other two parameters in each case. Dashed lines at 1 and 2$\sigma$ significance are shown, and the top of each panel corresponds to 3$\sigma$ for one interesting parameter. In all rows, the default curves shown in Fig. 8 are reproduced with solid lines. The top row shows curves for $\beta_{TM} = 1.2, 1.5$ and 2 with dotted, short dashed and long dashed lines respectively. In the second row, the results of a cluster to EMISS detect cell flux ratio of $2 - z$ (dotted) and $2.8 - z$ (short dashed) are given along with results obtained by assuming that the low-redshift data are 10 per cent incomplete (long dashed). Illustrated in the third row are curves resulting from decreasing the $L - T$ scatter for the low-z sample by 30 per cent (dotted), increasing the scatter by 30 per cent for the high-z sample (short dashed), using the different mean $L - T$ relations described in equations (4.7) and (4.6) (long dashed) or simply increasing the high-z luminosity at a given temperature by 50 per cent (dot-dashed). The final row shows the likelihoods found after assuming that $\Lambda_0 = 1 - \Omega_0$ (dotted).
ment in the middle of $\Omega_0$-space. On the lower-side, although not by much, favouring $\Omega_0 \approx 0.40 \pm 0.12$ and $0.30 \pm 0.10$ are Carlb erg et al. (1997b) and Bahcall et al. (1997) respectively. These results are consistent with those found here, although the small quoted uncertainties strongly imply that $\Omega_0 < 1$. The 'high-ground', with $\Omega_0 = 0.8 - 1$, is occupied by Sadat et al. (1998) and Blanchard & Bartlett (1997). At this point, the reader might wonder why any of these conclusions concerning $\Omega_0$ are worth taking seriously, given that they appear to be inconsistent and span the whole range of interesting values. That such disparate results are obtained is presumably a suggestion that unaccounted-for systematic errors are lurking beneath the surface in at least one of these analyses. As has been stressed in this paper, and was also discussed by Pen (1998), the cluster number density evolution is rapid if considered at a particular mass. However, at a fixed number density the mass varies relatively little over the range of redshifts probed by the presently available data. For instance, in an $\Omega_0 = 1$ model normalised to match the $z = 0$ cluster abundance, the evolution between $z = 0.05$ and $z = 0.33$ of the mass at a constant comoving cluster number density is $\sim 45$ per cent. Any unaccounted-for relative systematic temperature errors (i.e. affecting the low- and high-redshift samples differently) must therefore be smaller than $\sim 30$ per cent, or else the comparison between the models and observations will become completely unreliable. The quantification of any potential systematic uncertainties in mass estimation is thus crucial in determining the size of the error bar on the derived $\Omega_0$.

For the studies producing lower-$\Omega_0$ determinations, velocity dispersions obtained using different procedures are used to estimate masses. Frenk et al. (1996) and van Haarlem, Frenk & White (1997) investigated the biases in mass estimation using velocity dispersions, and more recently Borgani et al. (1997) considered the model dependence of these biases. These studies suggest that only after very careful modelling of the velocity dispersion calculation procedure, should this method be applied with any confidence. From the simple scaling laws $M \propto \sigma_8^3$ and $M \propto T^{1.5}$, it is immediately apparent that fractional uncertainties in velocity dispersion would need to be half those in the X-ray temperature in order for the virial mass to be similarly uncertain. The importance of such conversions is shown by figures 1 and 2 of Carlb erg et al. (1997b) where the higher redshift CNOC number density determination lies at a greater mass than the other CNOC point. This is slightly surprising because the virial theorem suggests that the same velocity dispersion should be produced by equal masses out to a given physical radius, independent of redshift. Under the assumption that both CNOC redshift ranges have the same cluster mass limit, which was the choice made by Bahcall et al. (1997), the higher redshift CNOC point can be made to lie rather happily on the $\Omega_0 = 1$ model prediction shown in Carlberg et al.'s figure 2, as long as their high-redshift point is shifted to the mass limit of their low-redshift sample. While it is not clear that this is the correct mass to choose, and Bahcall et al. took an intermediate value, this demonstrates how sensitively the conclusions about $\Omega_0$ depend on such details. In addition to these difficulties, the CNOC sample, which is almost a subsample of EMSS clusters and contains 5 from the high-redshift set studied by Henry (1997), has both a luminosity and a velocity dispersion cut. Consequently, when converting to an observed number density of clusters above a particular mass, a correction for scatter in the correlations between these properties is required. As Carlb erg et al. (1997b) showed, this is uncertain by $\sim 50$ per cent.

Bahcall et al. also consider the richnesses in the Palomar Distant Cluster Survey. This is suspect because of the well-known difficulties associated with trying to calibrate the richness-mass correlation and, more importantly, any possible evolution in it (e.g. van Haarlem, Frenk & White 1997).

Moving on to the large-$\Omega_0$ camp, Blanchard & Bartlett claim that their result "in conjunction with the absence of any negative evolution in the luminosity-temperature relation, provides robust evidence in favour of a critical density universe". The data used to draw this conclusion, come from a variety of sources: CNOC velocity dispersions, EMSS luminosities and ROSAT luminosities. As these authors point out, there is considerable uncertainty in the number density of clusters above 6 keV at redshift zero. A consistent value of $\sim 30 \times 10^{-9} (h^{-1} \text{Mpc})^{-3}$ is obtained from both the samples of Edge et al. (1990) and HA91. Integrating the ROSAT Brightest Cluster Sample (BCS) (Ebeling et al. 1997) best-fitting Schechter function for the EMSS passband (0.3-3.5 keV) from infinity to the luminosity corresponding to 6 keV ($1.25 \times 10^{37} h^{-2} W$) suggests a number density of clusters of $\sim 40 \times 10^{-8} (h^{-1} \text{Mpc})^{-3}$. The value calculated by Blanchard & Bartlett is twice as large as this. They derived their estimate by integrating in the ROSAT passband to a limit determined from the L-T relation of Arnaud & Evrard (in preparation). The consequence of having a larger redshift zero abundance is to increase the apparent evolution and hence increase the derived $\Omega_0$.

Sadat et al. (1998) used the lack of evolution of the cluster luminosity-temperature relation to give a "tentative" measurement of $\Omega_0 = 0.85 \pm 0.2$. This determination was based on an application of the method proposed by Oukbir & Blanchard (1997). They showed how, by using the redshift zero cluster temperature function normalisation and the redshift distribution of EMSS clusters, it was possible to relate $\Omega_0$ to the evolution of the cluster luminosity-temperature relationship. However, as Sadat et al. note, Oukbir & Blanchard (1997) used the uncorrected (see Eke et al. 1996) HA91 temperature function, which gives an overabundance of present-day clusters. This will feed through into the $\Omega_0$ determination (equation 7 of Sadat et al.) and produce an overestimate, although it is not clear by how much. The measurement of the L-T evolution found by Sadat et al. relies upon the correct choice of the slope for this relation. It is important because the more distant observed clusters are intrinsically hotter and more luminous than the nearby ones, and if the slope of the $L - T$ relation (they choose $L \propto T^3$) is not accurate then some spurious evolution (or lack of evolution) would be introduced because of the different types of clusters being probed at high and low redshifts. For the low-redshift clusters contained in their table 1 the mean temperature is 6.05 keV and the mean luminosity is $15.9 \times 10^{37} W$. From table 2 in their paper, the mean temperature and luminosity for clusters satisfying $0 < z < 0.6$ are 7.13 keV and $55 \times 10^{37} W$. If the $L - T$ relation is non-evolving then this suggests that $L \propto T^{-7}$ (This exponent is still $> 6$ if the one high-redshift cooling flow cluster RXJ1437.5-1145 is removed from this calculation.), so there is an apparent inconsistency in the method that is likely to have an impact.
on the derived value of $\Omega_0$. Their figure 3 and equation 5 suggest that, if the high-z clusters are biased to have large luminosities, then this would cause $\Omega_0$ to be overestimated.

## 6 ORDER STATISTICS AND THE EMSS CLUSTER SAMPLE

A number of recent papers have reported the existence of large mass concentrations at redshifts above 0.5 (Luppino & Gioia 1995, Donahue 1996, Luppino & Kaiser 1997, Donahue et al. 1998). Very often, these findings are accompanied by a claim that the existence of such objects proves troublesome for hierarchically clustering models of structure formation, particularly if $\Omega_0 = 1$. However, the magnitude of this difficulty is not quantified. The purpose of this Section is to propose and then apply a method by which the extent of the problem can be gauged. To this end, the question: ‘What is the expected redshift of the nth furthest galaxy cluster with a temperature above $kT$?, will be addressed. The answer will depend sensitively upon $\Omega_0$ because of the dependence of the growth rate on $\Omega_0$. Thus, one should be able to use the observations of hot, distant clusters to discriminate between different values of $\Omega_0$. More specifically, given that any observational survey is likely to be less than 100 per cent complete, the presence of a hot, massive cluster can be used to set an upper limit on $\Omega_0$.

### 6.1 Method and application

The Press-Schechter expression for the abundance of haloes as a function of mass has been tested using large N-body simulations out to about $z = 0.5$ for haloes having masses corresponding to $kT \sim 5$ keV in an $\Omega_0 = 1$ universe (e.g. Eke et al. 1996). It appears to provide a good description of the cluster mass function over this range of redshifts when the spherical collapse threshold density is used. The more recent, and larger, simulation of Tozzi & Governato (1997) suggests that the Press-Schechter equation overestimates the observed evolution in very massive haloes. However, it is not yet clear how much of this apparent discrepancy results from their particular choice of groupfinder. Whilst the theoretical expectation is still somewhat uncertain for very rare objects at $z \gtrsim 0.5$, this is an issue that can be addressed in the near future with very large-volume dark matter simulations (e.g. Evrard et al. in preparation), so this need not be an insurmountable barrier to the usefulness of the approach described here.

As was described in Section 2, the simple spherical collapse model will be used in conjunction with the Press-Schechter expression in order to provide a description of the halo mass function. Additionally, $\beta_{FM} = 1$ will be assumed to hold at all redshifts, along with a 20 per cent scatter about this mean value. Using the results of Eke et al. (1996), the normalisation of the mass fluctuation spectrum will be taken as $0.50 \pm 0.04$ for the $\Omega_0 = 1$ model considered in this Section. (Note that this is slightly different from the best-fitting value of 0.52 found by Eke et al. (1996).) This new estimate takes into account both the systematic over-estimation of $\sigma_8$ resulting from the measured temperature uncertainties (see table 1 of Eke et al.) and another, slightly larger, term (neglected in their table) arising from scatter in the mass-temperature relation.

From these stipulations, it is possible to calculate the redshift distribution of all clusters hotter than a specified value. The probability that, from the population of clusters hotter than a particular temperature, a cluster is situated at a redshift no greater than $z$ will be denoted by $P(z \leq \delta) \equiv P(z \leq \delta | kT > kT_{min})$. This quantity is calculable from the model for any given temperature. For brevity and clarity the temperature dependence is suppressed in the following expressions. Using this notation the probability that the furthest cluster (at $z_1$) has a redshift greater than $z$ can be written

$$P(z_1 > z) = 1 - P(z_1 \leq z) = 1 - (P(z \leq z))^N, \quad (6.1)$$

where $N$ is the total number of clusters above the chosen temperature limit in the whole volume, given by the Press-Schechter formula. For a particular redshift distribution, larger $N$ will probe further into the tail of the distribution and $z_1$ will be expected to be larger. This procedure can be extended to give the redshift distributions of the kth furthest cluster in the following manner. The probability that the kth furthest cluster (hotter than $kT_{min}$) has a redshift greater than a particular value is given by

$$P(z_k > z) = 1 - P(z_k \leq z), \quad (6.2)$$

To calculate the last term in this equation, it is necessary to consider the various combinations of clusters that would give rise to $z_k \leq z$. This probability can then be written, using the combination function $\binom{N}{k}$ as

$$P(z_k \leq z) = \sum_{j=0}^{k-1} \binom{N}{j} C_j^k P(z \leq z)^N - j(1 - P(z \leq z))^j \quad (6.3)$$

and this, together with equation (6.2), gives $P(z_k > k)$. This probability will be very sensitive to both the model normalisation $\sigma_8$ and the error in the measured cluster temperatures. Consequently, a marginalisation technique allowing for Gaussian errors on both of these quantities has been employed. This involves integrating over all normalisations and temperatures, weighting with the probabilities that these are in fact the actual values of $\sigma_8$ and the lowest temperature of the clusters under consideration.

The three clusters that will be used in this analysis are $MS0015.9 + 1609$ at $z = 0.54$, $MS0451.6 - 0305$ at $z = 0.54$ and $MS1054.5 - 0321$ at $z = 0.83$. ASCA temperatures of $8.0 \pm 0.6$ keV (Mushotzky & Scharf 1997), $10.4 \pm 1.2$ keV (Donahue 1996) and $14.7 \pm 2.1$ keV (Donahue et al. 1998) respectively have been found for these objects. Therefore in the EMSS cluster sample, the third furthest cluster having a temperature greater than or equal to $8.0 \pm 0.6$ keV has a redshift larger than 0.53. Given that these clusters were all found during a search containing only 735 square degrees of sky, this value, rather than the entire 4r steradians will be used for calculating the model predictions.

$\uparrow$ The combination function is $\binom{N}{j} = N!/(j!(N - j)!)$.
it would be premature to consider the existence of these redshifts larger than 0.8
EMSS if an extra fitting of the halo mass function was badly awry, or if the best-
sult would vary somewhat if the Press-Schechter treatment of the factor relating mass to
temperature, $\beta z$, larger than 0.8. There is less than a 1 in 7 chance that three such objects will exist in an EMSS cluster survey area of sky, and only a probability of $\sim 0.02$ that the third furthest hot cluster will have a redshift greater than 0.53.

6.2 Results

As can be seen in Figure 11, the probability that three clusters with $kT \geq 8.0 \pm 0.6$ keV exist in an $\Omega_0 = 1$, $\sigma_8 = 0.50 \pm 0.04$, $\Gamma = 0.25$ model is less than 0.14. Furthermore, it is another factor of $\sim 6$ less likely that the third most distant of these has a redshift of more than 0.53, as is observed. The conclusion from this exercise would appear to be that such a model can be ruled out at the 98 per cent confidence level. This would improve to $\sim 99$ per cent if an extra EMSS cluster hotter than 8 keV were found at a redshift larger than 0.53. It is worth reiterating that this result would vary somewhat if the Press-Schechter treatment of the halo mass function was badly awry, or if the best-fitting $\sigma_8$ were underestimated because of a poor choice of the factor relating mass to temperature, $\beta_T M$. Therefore, it would be premature to consider the existence of these hot clusters at high redshifts as being sufficient to rule out $\Omega_0 = 1$ cosmologies. Performing the above analysis for an $\Omega_0 = 0.52$ and $\sigma_8 = 0.63 \pm 0.06$ model (keeping $\Gamma$ fixed at 0.25) yielded a probability of 0.38 for the third furthest of the EMSS $kT \geq 8.0 \pm 0.6$ keV clusters having $z > 0.53$. 

7 OVERVIEW OF THE DIFFERENT TECHNIQUES

Having analysed cluster data in three distinct ways, it is worth discussing the strengths and weaknesses of the different methods that were employed in Sections 3, 4 and 6. While all of these techniques use similar input data in an attempt to determine $\Omega_0$, they are sufficiently diverse to merit some discussion. Table 1 contains a summary of the main pros and cons of these approaches.

The main drawback with the first two methods, which is not so important for the order statistics approach, is the need for a statistically well-defined sample of data with well-understood selection effects. Given such observational catalogues, the particular data that are used by the $\chi^2$ and maximum likelihood (M.L.) techniques differ somewhat. In the former case, only the cluster temperatures and fluxes are important. The $1/V_{\text{max}}$ estimator enables this method to sidestep the need for any cluster luminosity-temperature relation, apart from a minor incompleteness correction. However, the redundancy of the individual redshift measurements, other than to determine if a cluster should be included in the 'high-' or 'low-'z temperature function, leaves a certain freedom to choose what redshifts to assign to the fits. The choice of temperature bins is also something that can alter the resulting $\Omega_0$ determination to a small extent. In contrast, the M.L. method uses all the cluster temperatures and redshifts and avoids awkward binning problems. However, the limits of integration in equation (4.2), coming from the adopted ranges in $z$ and $T$, do create some freedom in the most likely parameters. This comes as a result both of the peculiar redshift distribution of the high-z clusters, and the uncertainties in the selection functions at the extreme temperatures and redshifts. Additionally, the price paid for not using the measured flux values is that one must adopt a luminosity-temperature relation (at all relevant redshifts) instead of using cluster evolution.

The difference between the derived best-fitting parameters from the $\chi^2$ and M.L. techniques has non-negligible contributions from the systematic uncertainties mentioned in the previous paragraph. Looking at the situation more positively, it is perhaps reassuring that, despite the various

| Method                  | Strengths                                      | Weaknesses                                      |
|-------------------------|------------------------------------------------|-------------------------------------------------|
| $\chi^2$ (Section 3)    | Utilises the cluster flux measurements.        | Requires carefully selected data.               |
|                         | Almost independent of $L - T$ relation.        | Results depend slightly on $z$ and $T$ bin choice. |
| M.L. (Section 3)        | All cluster redshifts are used.                | Requires carefully selected data.               |
|                         | No $z$ or $T$ bin choices are necessary.       | Uncertainty introduced with $L - T$ relation.   |
| Order statistics (Section 6) | No need for a statistically well-defined data sample. | Based on a model extrapolation.                |
|                         | Independent of $L - T$ relation.               | Results are very sensitive to the assumed $\sigma_8$. |

Figure 11. The probability that the $k$th furthest of all EMSS clusters with a temperature exceeding $8.0 \pm 0.6$ keV has a redshift larger than $z$, assuming a cosmological model with $\Omega_0 = 1$, $\sigma_8 = 0.50 \pm 0.04$ and $\Gamma = 0.25$. There is less than a 1 in 7 chance that three such objects will exist in an EMSS cluster survey area of sky, and only a probability of $\sim 0.02$ that the third furthest hot cluster will have a redshift greater than 0.53.
differences between the two methods, such similar results were returned.

An opportunity to win the jackpot is offered by the order statistics method; namely the chance to rule out \( \Omega_0 = 1 \) with observations of only a few very hot and distant clusters. However, the size of the payout for an apparently successful data set is not yet well-defined because of the uncertainties in both the cluster mass function evolution and the mass-temperature conversion for such rare objects at such high redshifts. Nevertheless, these are both eminently measurable using numerical simulations, and the Hubble volume simulation performed by the Virgo consortium (Evrard et al. in preparation) should address the former of these issues in the near future. It is worth noting that the potential non-virialisation of large mass concentrations at high redshifts, or the boosting in emission-weighted temperature coming from a recent merger event should not cause any difficulties for this method. This is because such effects are included in hydrodynamical simulations and would be calibrated through the mean \( \beta_{\text{TM}} \) and the scatter around this value.

8 WHAT WENT WRONG?

Given that all ‘right-minded’ cosmologists know that \( \Omega_0 = 1 \), a pertinent question to ask at this point, on their behalf, is “What has gone wrong?”, i.e. what would it need for an \( \Omega_0 = 1 \) model to be more favoured by cluster evolution. In this Section, consideration is given to the magnitudes of the errors that are required in some of the preceding assumptions.

If \( \Omega_0 \) is in fact unity, then it is very probable that at least one of the observations or the model used above is wrong. The first of these possibilities seems unlikely. An \( \Omega_0 = 1 \) model would require a systematic overestimation of the high-redshift cluster temperatures by ASCA of about 25 per cent, or a corresponding underestimation of the low-redshift temperatures by the same amount. This corresponds to a shift in all of the cluster temperatures that is typically twice the quoted uncertainty and, as such, seems highly unlikely. Having said that, the analysis of ASCA data by Markevitch et al. (1998) showed, for single temperature fits to 15 of the original 25 HA91 clusters, an average 6 per cent increase in the measured temperatures. While this is not sufficiently large, if such a systematic shift were applied to the entire low-redshift temperature function, then the most likely \( \Omega_0 \) would increase by \( \sim 20 \) per cent.

Contamination of the X-ray emission by active galactic nuclei (AGN) or cooling flows would alter the mass-temperature conversion. The effect of AGN contamination is to increase the emission-weighted temperature. Cooling flows have the opposite effect. These potential complications are better addressed from the point of view of decontaminating the data, rather than adding extra processes into the numerical simulations, so will be considered as ‘observational’ problems. The likelihood of any interloping AGN in the high-redshift sample is very low, and high-resolution images show no evidence for such contamination. More important is the effect of cooling flows on the two different redshift samples. If cooling flows were more prevalent in the low-redshift sample than the high-redshift one, then their removal would result in an increased separation of the two temperature functions, thus favouring higher \( \Omega_0 \). By fitting both single-temperature and two isothermal models to ASCA data, Markevitch et al. (1998) showed that, for the 15 low-z clusters in common with HA91, the decontaminated emission-weighted temperature is, on average, 6 per cent larger than the single-temperature value. For the most discrepant single cluster, the effect was 5 times as great. Allen (1998) performs a similar analysis including two of the Henry (1997) (i.e. high-z) clusters, and finds the temperatures increase by 15 and 24 per cent when a cool component is removed. There are two simple reasons why one might expect cooling flows to be more prominent in the EMSS high-z sample rather than the HA91 catalogue. Firstly the densities are larger and hence cooling times are shorter in the higher redshift range. Secondly, the EMSS selection algorithm tends to favour clusters that have centrally peaked emission, with only those exceeding a flux threshold in a 2.4 \( \times \) 2.4 detect cell being included in the catalogue. If the results from Allen (1998) are typical of the whole Henry (1997) sample, then the effect of cooling flow decontamination would be to increase the high-redshift cluster temperatures by more than the low-redshift ones. This would somewhat overcompensate for the systematic temperature shift described in the previous paragraph, leading to an overall estimate of \( \Omega_0 \) that is about 20 per cent lower than the value found in Section 4, where these considerations were neglected.

As was discussed in Section 4, a 10 per cent incompleteness in the low-redshift sample only increases \( \Omega_0 \) by 0.06, so this is not a sufficiently large effect to help the critical density model. Independent checks are available on the sample completenesses. These come from the comparison of luminosity functions obtained from the cluster catalogues used here with those from the larger ROSAT cluster samples at low (Ebeling et al. 1997) and high (Rosati et al. 1998) redshifts. ROSAT does not detect enough very luminous clusters to compare with the majority of the luminosity range of the EMSS high-z sample, because of the smaller fraction of sky it surveyed. Nevertheless, where an overlap in luminosity exists, there is agreement between the cluster abundances in the different surveys. These comparisons have enough statistical uncertainty to make them reassuring rather than highly restrictive. There could still plausibly be \( \sim 40 \) per cent difference between the various samples. In order to favour \( \Omega_0 = 1 \) though, there would need to be a significantly greater incompleteness in the low-redshift sample than exists in the more distant one, and this seems unlikely.

The considerations in the preceding few paragraphs suggest that if anything, relative to the results found in Sections 2 and 3, lower rather than higher values of \( \Omega_0 \) would be favoured by the potential ‘observational’ systematic effects. Thus, if the data used in this paper are the product of an \( \Omega_0 = 1 \) universe then it seems that the finger of suspicion should point firmly in the direction of the modelling of cluster evolution.

While the statement that the Press-Schechter expression provides a good description of the mass function of galaxy-cluster-sized objects had stood up to much scrutiny, the recent report by Tozzi & Governato (1997) suggests that it overestimates the actual evolution for \( \Omega_0 = 1 \). Their simulation is of a cube with side 500h\(^{-1}\) Mpc and thus allows the evolution of very rare objects to be traced back to higher redshifts than was possible with the smaller vol-
Nevertheless, it is worthwhile to investigate the possibility that an evolving threshold is required. Assuming that \( \delta \) is the form of the spherical collapse threshold used in previous studies. These authors suggest that the spherical collapse threshold \( \delta_c \) should be replaced by \( \delta_{\text{eff}} = 1.48(1+z)^{-0.06} \). This result appears to be slightly discrepant with previous findings where comparison is possible, but it may be that this is just an artefact of the groupfinder that was employed (see Cole & Lacey 1996 for more details). Nevertheless, it is worthwhile to investigate the possibility that an evolving threshold is required. Assuming that \( \delta_{\text{eff}} \) is the form of \( \delta_c \) that is applicable to all \( \Lambda_0 = 0 \) models (similar redshift evolution is found in a large \( \Omega_0 = 0.3 \) CDM simulation; Governato, private communication), the effect of such a varying threshold on the most likely value of \( \Omega_0 \) can be investigated using the likelihood procedure described in Section 4. The resulting marginalised likelihoods are shown in Fig. 12. It is apparent that, whilst the most likely value of \( \Omega_0 \) has only increased to 0.58, the \( \Delta(-\ln L) \) required to reach \( \Omega_0 = 1 \) is now just 2.2, i.e. only \( \sim 2\sigma \). Whilst this gives the impression that a small hit has been scored on the good ship \( \Omega_0 < 1 \), the captain might retaliate by pointing out that the EMSS survey found clusters with \( z > 0.4 \) and, given that the Tozzi & Governato \( \delta_{\text{eff}} \) fitted their simulated mass function out to a redshift of one, these distant objects can now be included in the likelihood calculation. Temperatures have yet to be measured for all of these clusters, but they do have known redshifts, so an additional contribution to the likelihood is computed by replacing the \( a \) in equation (4.2) by the integral of \( a \) over all cluster temperatures (2 – 20 keV in practice) and just comparing the probability of finding clusters at the observed redshifts. Additionally, the value of \( f_{\text{clus}}/f_{\text{det}} \) is taken to be 1.75, independent of redshift for the \( z > 0.4 \) EMSS clusters. The identified EMSS clusters with \( z > 0.4 \) and their corresponding redshifts are taken from Gioia & Luppino (1994) and Luppino & Gioia (1995), and the marginalised likelihoods resulting from their inclusion are shown in Fig. 13. It should be noted that there are also 9 unidentified EMSS sources (Gioia & Luppino 1994), some of which might contribute to this list, and consequently reduce the estimated value of \( \Omega_0 \) from that found here. The most recent HR1 data show that MS1610.4 + 6616 is a point source (see Henry 1998) and it is not included.

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Figure 12. \( \Delta(-\ln L) \) for \( \Omega_0 \) (left panel), \( \Gamma \) (middle) and \( \sigma_8 \) (right), marginalised over the other two parameters in each case. Dashed lines at 1 and 2\( \sigma \) significance are shown, and the top of each panel corresponds to 3\( \sigma \) for one interesting parameter. The curves are calculated assuming that the collapse threshold is given by \( \delta_{\text{eff}} = 1.48(1+z)^{-0.06} \) rather than the spherical collapse \( \delta_c \).

Figure 13. As for Fig. 12 but with the redshift information for the 9 \( z > 0.4 \) EMSS clusters included in the likelihood calculation.

likely parameters change only slightly, becoming \( \Omega_0 = 0.62 \), \( \Gamma = 0.06 \) and \( \sigma_8 = 0.53 \). Furthermore, the inclusion of the more distant clusters significantly decreases the sizes of the statistical uncertainties on the most likely parameters, such that \( \Omega_0 = 1 \) is now excluded at a higher confidence level than was the case with the original default assumptions, despite the favourably evolving \( \delta_{\text{eff}} \). Distributions of the numbers of clusters marginalised with respect to either redshift or temperature are shown in Fig. 14. As the temperatures have yet to be measured for the highest redshift clusters, this model ‘prediction’ is shown as a dotted line.

In addition to any uncertainties in the accuracy of the Press-Schechter description of the mass function, there is some leeway in the conversion from mass to temperature that may effect the most likely \( \Omega_0 \). There are at least 5 options for tinkering in an attempt to salvage \( \Omega_0 = 1 \):

1. Evolution of the scatter in both the cluster mass-temperature conversion and the observational temperatures would give rise to a systematic error in the estimation of \( \Omega_0 \). It has been assumed that the intrinsic scatter is unchanging and the low- and high-redshift temperature errors are similar (which the quoted values suggest they are). If these scatters at high redshift implied that a Gaussian smoothing width of 0.4\( T \) rather than 0.2\( T \) was appropriate, then even the \( \Omega_0 = 1 \) model cluster temperature function would appear not to evolve.

However, for the mass-temperature relation, the low-\( \Omega_0 \) simulations of Eke et al. (1998) show no significant evolution in the scatter over this range of redshifts (see Fig. 1). The observational uncertainties are relatively small and, only slightly larger for the high-\( z \) sample. The conclusion is therefore that such an escape for \( \Omega_0 = 1 \) is not easily arranged.

2. Evolution of \( \mu \), the mean molecular weight of the intracluster gas, has been ignored in the modelling. If it were 25 per cent larger at \( z = 0.33 \) than \( z = 0 \) then the model predictions for \( \Omega_0 = 1 \) would show no evolution over this redshift range. This change in \( \mu \) is both extreme and in a counter-intuitive direction. Mushotzky & Loewenstein (1997) recently used ASCA data to show that the iron abundance in the intracluster medium (ICM) does not evolve over the range \( 0 < z \leq 0.3 \). Also, Kauffmann & Charlot (1998) adopted a semi-analytical approach to the formation and evolution of elliptical galaxies, concluding that the metallicity of the ICM is approximately constant.
Figure 14. Marginalised distributions of clusters. Calculated assuming that the threshold varies with redshift according to $\delta_{\text{eff}}$, and using the 9 $z > 0.4$ extra EMSS clusters. Smooth curves are shown for the most likely model, namely $\Omega_0 = 0.62$, $\Gamma = 0.06$ and $\sigma_8 = 0.53$. The dotted curve in the temperature panel corresponds to the model prediction for the distribution of the temperatures that are yet to be measured. The stepped curves represent the observed cluster distributions.

out to $z > 1$. This approach for saving $\Omega_0 = 1$ can be ruled out.

3. Evolution of $\beta_{TM}$ to the extent described for $\mu$ in point 2, would also make $\Omega_0 = 1$ compatible with the observations. The definition of $\beta_{TM}$ is such that it would need to be 25 per cent smaller at the larger redshift. As shown by the simulations of Eke et al. (1998) (see Fig. 4) this amount of evolution is difficult to justify and, if anything, the $\beta_{TM}$ at $z = 0.33$ is a few per cent larger than that at low redshift. For the various cosmologies studied numerically by Bryan & Norman (1998), a similar lack of evolution was also found (see their figure 4). This suggests that such a route will not assist $\Omega_0 = 1$ models either, and that the most likely value for $\Omega_0$ quoted here may be slightly biased to a value that is too large.

4. One clear deficiency of the hydrodynamical simulations that have been used to relate virial mass to emission-weighted X-ray temperature is the lack of any heating of the intracluster gas by supernovae. That this is a problem becomes apparent when the simulated cluster luminosity-temperature relation is compared with the observed one.Whilst the models and scaling laws show $L \propto T^2$, the observations suggest a steeper dependence of luminosity on temperature (see equation (3.3)). Allen & Fabian (1998) find that decontaminating the cooling flow contribution to the X-ray emission can leave $L \propto T^{-2}$. However, a similar study by Markevitch (1998) implies that, in order to produce this shallow slope, some feedback is still necessary after the cooling flow correction has been applied. NFW showed that by preheating gas in the manner suggested by Evrard & Henry (1991), the low-temperature simulated clusters could be warmed up enough, along with the corresponding decrease in luminosity as a result of the less dense cores, to produce a sufficiently steep luminosity-temperature relation to match observations. To obtain a very crude idea of the maximum extent to which such a change will affect the evolution of the $\Omega_0 = 1$ temperature functions, a simple mapping of temperature has been applied. NFW showed that by preheating gas in the manner suggested by Evrard & Henry (1991), the low-temperature simulated clusters could be warmed up enough, along with the corresponding decrease in luminosity as a result of the less dense cores, to produce a sufficiently steep luminosity-temperature relation to match observations. To obtain a very crude idea of the maximum extent to which such a change will affect the evolution of the $\Omega_0 = 1$ temperature functions, a simple mapping of temperature has been applied.

This route for converting the simulated clusters to lie on the observed luminosity-temperature relation is extreme in its effect on the cluster temperatures. In practice, one would expect any energy injection to heat the gas and decrease the central density. As a result, the simulated clusters would fall onto the observed $L_x - T_x$ relation more as a result of decreasing luminosity than increasing temperature as is assumed for the mapping above.

Figure 15. The temperature functions for an $\Omega_0 = 1$, $\sigma_8 = 0.52$ and $\Gamma = 0.25$ model are shown with solid and dashed lines for $z = 0.05$ and $z = 0.33$ respectively. In each case, the curves resulting from applying the temperature mapping of equation (8.1) are also shown. These are steeper than the original predictions. Stepped curves show the observationally determined temperature functions.
Figure 13 shows the predictions before and after ‘feedback’ when the same distortion is applied to both high- and low-redshift models. With the ‘feedback’ essentially just heating the cooler clusters, it produces steeper cumulative temperature functions, thus favouring lower \( \Gamma \) values. (It is worth noting that, if the lower temperature clusters were preferentially contaminated by cooling flows then the estimated \( \Gamma \) would be biased low, but Markovitch (1998) reports that slightly the reverse is seen in his sample of nearby clusters.) Interestingly, the extreme feedback simulations of Metzler & Evrard (1994) increased the gas temperature of a 5.6 keV cluster by \( \sim 15 \) per cent. This size of fractional change is only reached with the above mapping for clusters cooler than \( \sim 4.5 \) keV. Despite all of this, the predicted amount of evolution still exceeds that observed. Only if such a prescription were proscribed for the high-redshift sample alone would feedback be effective in reducing the amount of evolution seen in \( \Omega_0 = 1 \) models.

5. The effects of having a two-phase intracluster gas medium or magnetic fields are also lacking from the hydrodynamical simulations used to normalise the mass-temperature relation. While it seems unlikely that they should have a different effect on the two redshift ranges considered here, they could still bias the best-fitting parameters as was illustrated in Section 4, by shifting a different part of the mass function into the observed range of temperatures. The amount by which \( \beta_{TM} \) can be affected is roughly constrained by comparisons between weak lensing masses and X-ray masses. Present results (Smail et al. 1997; Allen 1998) appear to imply that this additional physics is not sufficiently misleading to allow \( \Omega_0 = 1 \) to be consistent with the lack of evolution of the cluster number density.

9 CONCLUSIONS

In this paper, the value of the density parameter \( \Omega_0 \) has been constrained using three different, yet similar, methods. The results are entirely consistent with those presented by Henry (1997). Combining the results from Sections 3 and 4 gives the most likely parameters as \( \Omega_0 \approx 0.44 \pm 0.2, \Gamma \approx 0.08 \pm 0.07 \) and \( \sigma_8 \approx 0.97 \pm 0.1 \) if \( \Lambda_0 = 0 \). The inclusion of a non-zero cosmological constant term has very little effect, leading to \( \Omega_0 \approx 0.38 \pm 0.2, \Gamma \approx 0.09 \pm 0.08 \) and \( \sigma_8 \approx 0.74 \pm 0.1 \).

Apart from the difficulty arising because of the redshift distribution of the Henry (1997) sample, none of the systematic effects that have been considered alter the best-fitting \( \Omega_0 \) by more than one of the statistical standard deviations quoted above. Unless an important systematic error or uncertainty has been overlooked in the analysis, these results would appear to be limited by a lack of data, so, while this preliminary study suggests that the \( \Omega_0 = 1 \) model is still bobbing about unhappily in the water at a rejection level of \( 2 - 3 \sigma \), a large sample of clusters would offer the opportunity, with this method alone, to sink it properly. Enough ammunition should be provided by the upcoming X-ray telescope missions XMM and AXAF.

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