AdS/CFT and the Little Hierarchy Problem

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Abstract

The AdS/CFT correspondence is applied to a close analogue of the little hierarchy problem in $AdS_{d+1}$, $d \geq 3$. The new mechanism requires a Maxwell field that gauges a $U(1)_{R}$ symmetry in a bulk supergravity theory with a negative cosmological constant. Supersymmetry is explicitly broken by a non-local boundary term with dimensionless coupling $h$. Non-locality appears to cause no pathology, and the SUSY breaking deformation engendered is exactly marginal. SUSY breaking effects in the bulk are computed using the $U(1)$ Ward identity. Conformal dimensions and thus masses of scalar and spinor partners are split simply because they carry different $R$-charges. However SUSY breaking effects cancel to all orders for $R$-neutral fields, even in diagrams with internal $R$-charged loops. SUSY breaking corrections can be summed to all orders in $h$. Diagrams involving graviton loops do not introduce any further SUSY breaking corrections. A possible scenario for a flat spacetime limit is outlined.

1 Introduction

In a quantum field theory, scalar masses are naturally heavy due to quantum corrections if they are not protected by a symmetry. Global supersymmetry eliminates the most acute quadratic sensitivity to unknown UV physics. But supersymmetry is usually assumed to be spontaneously broken and scalar masses generically rise to the SUSY breaking scale. LHC results indicate that this scale is larger than a few TeV. The separation between the SUSY breaking scale and the Higgs mass is the little hierarchy problem. A mechanism is then needed to protect the Higgs mass.

There has already been much effort in model building to address the little hierarchy problem. For more detail, see [1, 2, 3] and references therein. An especially interesting cosmological relaxation mechanism [4] has recently been proposed. This may offer a new direction to approach the little hierarchy problem.

In this paper, we propose a mechanism to solve this problem in $AdS_{d+1}$, $d \geq 3$ based on the AdS/CFT correspondence. It generalizes the $AdS_{3}$ model discussed in [5]. In that model we considered Chern-Simons gauge fields $A_{\mu}$, $\tilde{A}_{\mu}$ which gauge a $U(1)_{R} \times \tilde{U}(1)_{R}$ symmetry of a $D = 3$ supergravity theory. The gauge fields are bulk duals of conserved currents $J_{i}, \tilde{J}_{i}$ in the boundary CFT. Supersymmetry is broken explicitly by a local boundary term quadratic in the gauge fields, and these fields propagate into the bulk and couple to $R$-charged matter fields. The Chern-Simons equation of motion is $F_{\mu\nu} = 0$, so the bulk-to-boundary propagator is a total bulk derivative, i.e. $K_{\mu}(x, \bar{w}) = \partial_{\mu} A_{i}(x, \bar{w})$. Bulk integrals induced by insertion of the SUSY breaking operator can then be evaluated by partial integration and the $U(1)$ Ward identity. The
SUSY breaking correction to any Witten diagram is then expressed by a quite simple boundary integral. The major results are:

i. The scaling dimensions of CFT operators $O_c, \Psi_c$ which are superpartners are split because superpartners carry different R-charges. AdS/CFT then implies that the masses of their dual bulk fields $\phi_c, \psi_c$ are also split.

ii. SUSY breaking corrections cancel completely for moduli fields which are $R$-neutral and massless in $AdS_3$, so moduli remain massless to all orders.

iii. Coupling constant relations required by SUSY are also corrected by this breaking mechanism.

Properties of the $AdS_3$ model are reviewed in Secs. 2 and 3 below, and we emphasize several features that are modified in the higher dimensional generalization discussed in Sec. 4. It turns out that a successful extension can be constructed with a single Maxwell field $A_\mu$ which gauges a $U(1)_R$ symmetry of bulk supergravity. The bulk-to-boundary propagator of a Maxwell field in $AdS_d+1$ for $d \geq 3$ is not a total derivative. But we devise a SUSY breaking boundary term that converts the bulk-to-boundary propagator to a total derivative in the Witten diagrams that describe SUSY breaking corrections. The SUSY breaking term is non-local but this appears to cause no pathology. The features i.-iii. above hold in higher dimensions. The possibility of a flat spacetime limit in which SUSY breaking effects survive is discussed in Sec. 5. Section 6 contains a summary and future outlook for theories of this type. An appendix is devoted to the question of contact terms in the the divergence of the current-current correlator.$^1$

2 Review of $AdS_3$

In this section, we review the features of the $AdS_3$ model in [5]. This will serve to introduce the underlying ideas and to motivate the changes needed for a viable model in $AdS_d+1$.

Let us consider a supergravity theory in $D = 3$ dimensions with gauged R-symmetry group $U(1) \times \tilde{U}(1)$. The R-gauge bosons $A$ and $\tilde{A}$ have Chern-Simons dynamics. Gauging requires a negative cosmological constant, so the natural classical solution is $AdS_3$. Pure supergravity models of this type were first constructed by Achucarro and Townsend in 1986 [6] and general matter field couplings studied in [7, 8, 9].

The bulk Chern-Simons action together with boundary terms that ensure a consistent variational problem is written as

$$S_{CS} = \int_{bulk} \left[ \frac{k}{8\pi} A \wedge dA - \frac{k}{8\pi} \tilde{A} \wedge d\tilde{A} \right] - \int_{bdy} \left[ \frac{i k}{16\pi} A \wedge *A + \frac{i k}{16\pi} \tilde{A} \wedge *\tilde{A} \right]. \quad (2.1)$$

The boundary action chosen enforces that the anti-holomorphic component of $A$ and the holomorphic component of $\tilde{A}$ vanish on the boundary.

Now let us add supersymmetric matter fields to the theory. First, we introduce the chiral multiplet $\Phi_m = (\phi_m, \psi_m, \tilde{\psi}_m, \ldots)$. The R-charges of the scalar $\phi_m$ vanish, while its spinor partners $\psi_m$ and $\tilde{\psi}_m$ have R-charges $(-1, 0)$ and $(0, -1)$ respectively. The subscript $m$ stands for “modulus” which means a scalar field with zero mass and vanishing potential interactions. We also add the chiral multiplet $\Phi_c = (\phi_c, \psi_c, \tilde{\psi}_c, \ldots)$ whose scalar and spinor components have non-vanishing R-charges $(q, \bar{q})$ and $(q - 1, \bar{q})$. The subscript $c$ stands for “charged”. The basic gauge invariant kinetic Lagrangian for $\phi_c$ is

$$\mathcal{L}_{kin} = \sqrt{g} D^\mu \phi_c^\dagger D_\mu \phi_c = \sqrt{g} g^{\mu\nu} (\partial_\mu - iqA_\mu - i\bar{q}\tilde{A}_\mu) \phi_c^\dagger (\partial_\nu + iqA_\nu + i\bar{q}\tilde{A}_\nu) \phi_c. \quad (2.2)$$

Non-trivial interactions between $\Phi_m$ and $\Phi_c$ can be included. For example, the two-derivative cubic coupling

$$S_{cubic} = \int d^4x \sqrt{g} (\lambda \phi_m D^\mu \phi_c^\dagger D_\mu \phi_c + h.c.) \quad (2.3)$$

arises from the Kähler $\sigma$-model of the theory.$^2$ Since $\phi_m$ is R-neutral, it does not couple to the R-gauge bosons directly. However, it communicates to $A$ and $\tilde{A}$ through loop diagrams involving R-charged particles such as $\phi_c$.

$^1$Some readers may be able to bypass the systematic review of the $AdS_3$ model and start instead in Sec. 3 or Sec. 4.

$^2$A non-derivative cubic coupling is forbidden because the superpotential from which it arises does not conserve R-charge. Results in the following sections remain valid for couplings with any even number of gauge covariant derivatives.
In this model, we introduce a special SUSY breaking mechanism. Explicit SUSY breaking is generated by the \( AdS_3 \) boundary term

\[
S_{SB} = \frac{h}{2} \int_{bdy} A \wedge \tilde{A}.
\]  

(2.4)

One can show that this SUSY breaking term is exactly marginal. As we review below, this term breaks the SUSY relation between boson and fermion mass spectra of R-charged multiplets as well as relations among coupling constants. However, the masses of moduli fields vanish to all orders in the coupling \( h \).

Since SUSY breaking term is exactly marginal, the \( AdS/CFT \) dual boundary theory is a \( CFT_2 \). Indeed the dual of the SUSY breaking term \((2.4)\) is

\[
S_{SB,CFT} = h \int d^2z J(z) \bar{J}(\bar{z}),
\]  

(2.5)

where \( J(z) \) and \( \bar{J}(\bar{z}) \) are holomorphic and anti-holomorphic currents dual to \( A \) and \( \tilde{A} \) respectively. Therefore we can verify bulk calculations of SUSY breaking effects using well-known CFT methods. For example, one can bosonize the currents as \( J(z) \sim \partial_z \eta(z) \) and \( \bar{J}(\bar{z}) \sim \partial_{\bar{z}} \bar{\eta}(\bar{z}) \). Exact marginality also follows from CFT arguments [10].

Here we point out an important subtlety in order to avoid potential confusions when we present the higher-dimensional generalization in later sections. If the bosonized expressions for \( J \) and \( \bar{J} \) are inserted into \((2.5)\), it appears that one can integrate by parts and use current conservation to remove the SUSY breaking term. However, currents are conserved only when equations of motion are satisfied, and we cannot naively use equations of motion in a CFT Lagrangian. The observables in a CFT are correlation functions, not S-matrices, as there is no well-defined asymptotic free particle state. Correlation functions, unlike S-matrix elements, can be sensitive to field redefinitions. Thus the SUSY breaking term \((2.5)\) cannot be trivialized or removed using equations of motion. The generalization to higher-dimensional \( AdS \) space has a similar feature. For both \( AdS_3 \) (Sec. 4 of [5]) and \( AdS_{d+1} \) (Sec. 4.4.2 below) we have used conformal perturbation theory to calculate the shift of scaling dimension. This shows that the deformation \((2.5)\) and its higher-dimensional generalization \((4.6)\) are nontrivial.

### 2.1 Mass Spectra

In this section, we explicitly show how the mass spectra are affected by turning on the SUSY breaking term in Eq. \((2.4)\). Using the \( AdS/CFT \) dictionary, studying the change of the mass of a particle is equivalent to studying the shift of conformal dimension of the dual CFT operator. In \( AdS_3 \), they are related by

\[
\begin{align*}
\Delta_B &= 1 + \sqrt{1 + (m_B L)^2}, \\
\Delta_F &= 1 + |m_F L|.
\end{align*}
\]  

(2.6)

When SUSY is a good symmetry, one has

\[
\Delta_F = \Delta_B + \frac{1}{2}.
\]  

(2.7)

A violation of this relation is a definite signal of SUSY breaking. In this section, we show that after we turn on the SUSY breaking term, i.e. Eq. \((2.4)\), the relation \((2.7)\) is no longer preserved.

In Sec. 2.1.1, we show that the mass spectra are changed in a non-supersymmetric way. In Sec. 2.1.2, we show that the mass, or even the full self-potential, of the moduli field remains zero at all loop order.

#### 2.1.1 R-charged particle

Let us start with the leading order mass correction for a bulk scalar \( \phi_c \) whose R-charges are \((q, \tilde{q})\). Because it is charged, this field couples directly to \( A_\mu, \tilde{A}_\mu \) through the gauge vertices in \((2.2)\). To first order in \( h \), the shift of mass or conformal dimension can be calculated by studying the insertion of \( S_{SB} \) in the two-point correlation function \( \langle O_c(\vec{y})\bar{O}_c(\vec{z}) \rangle \), where \( O_c \) is the CFT dual to \( \phi_c \). The relevant Witten diagrams are shown in Fig. 1.
Figure 1: The relevant diagrams for the leading order mass deformation of a charged scalar field.

The essential ingredients in the calculation of these diagrams are the bulk-to-boundary propagator of the Chern-Simons gauge field and the $U(1)$ Ward identity of elementary quantum field theory. The Chern-Simons equation of motion is $F_{\mu\nu} = 0$, so the bulk-to-boundary propagator is a total derivative with respect to the bulk coordinates. The general structure is $K_{\mu,i}(x, \vec{w}) = \partial_\mu \Lambda_i(x, \vec{w})$ where $x$ and $\vec{w}$ denote bulk and boundary points respectively. The specific form found in [5] is

$$K_{\mu i}(x, \vec{z}) = \frac{\partial}{\partial x^\mu} \Lambda_i(x, \vec{z}) = \frac{\partial}{\partial x^0} \Lambda_i(x, \vec{z}) = \frac{1}{\sqrt{2}} \epsilon_{ij} (\vec{x} - \vec{z})^j x^0 + \frac{1}{2} (\vec{x} - \vec{z})^2.$$  

(2.8)

This form must be modified by multiplication on the boundary index by the projection operators $(\delta_{ij} \pm i\epsilon_{ij})/2$ to enforce the (anti-)self-duality property of $A_\mu$, $(\tilde{A}_\mu)$.

Because the bulk-to-boundary propagator is a total derivative one can integrate by parts in the $x$ and $x'$ integrals in Fig. 1. For each integral one finds a boundary term that must be carefully studied plus a bulk term to which the Ward identity (aka Green’s theorem) applies. A detailed calculation shows that only boundary terms from partial integration contribute to the final answer; all bulk terms cancel among the contributing diagrams. In the boundary terms that remain, the $\Lambda_i$ and $\tilde{\Lambda}_i$ factors are pinned at the end-points of the $\phi_c$ line as shown in Fig. 2.

Figure 2: Two of diagrams that depict the $\Lambda$-factors of the photon pinned at the boundary after bulk integrals are performed. There are also diagrams with $\vec{y}$ and $\vec{z}$ switched.

The $d^2w$ boundary integral then produces the final result

$$\delta_h \langle \tilde{O}_c(\vec{y})O_c(\vec{z}) \rangle = (2\pi h \tilde{q} \log \frac{|y - z|^2}{|a|^2}) \langle \tilde{O}_c(\vec{y})O_c(\vec{z}) \rangle_a ,$$  

(2.9)

where $a$ is the short-distance regulator. This indicates that the conformal dimension of $O_c$ changes by

$$\delta_h \Delta_{O_c} = -2\pi h \tilde{q} .$$  

(2.10)

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3 Throughout this paper we use the Poincaré patch metric $ds^2 = (L^2/x_0^2)(dx_0^2 + d\vec{x} \cdot d\vec{x})$.

4 A complete calculation in a simpler example is given in Sec. 3 below. In the main, however, we refer readers to [5] for details of these straightforward but tedious calculations.
A similar result is obtained for the change of scaling dimension of the spinor operator $\Psi_c$, namely

$$\delta_h \Delta \Psi_c = -2\pi h(q - 1)\tilde{q}.$$ (2.11)

This example illustrates a basic feature of our results. We find SUSY breaking shifts of scaling dimension and bulk mass simply because the R-charges of boson and fermion components of a supermultiplet are different.

### 2.1.2 R-neutral particle

In this section, we outline the calculations which show that the mass of an R-neutral field such as the modulus $\phi_m$ is not shifted by the SUSY breaking term (2.4).

Unlike R-charged particles, $\phi_m$ is R-neutral and thus does not couple directly to R-gauge bosons. However, $\phi_m$ does couple to $\phi_c\phi_c^*$ pairs via the cubic coupling in Eq. (2.3). Thus SUSY breaking effects might propagate to $\phi_m$ through loops. In the MSSM this effect is a major contributor to the shift in the Higgs boson mass. In our theory the $U(1)$ Ward identity saves the day, and $\phi_m$ remains massless to all perturbative orders.

![Figure 3: Two of several diagrams that contribute to the leading order deformation of $\langle O_m O_m \rangle$. The quartic vertex in the second diagram comes from the covariant derivative in (2.3).](image)

Two of several contributing one-loop Witten diagrams are shown in Fig. 3. As in the previous section one can use integration by parts and the Ward identity to evaluate bulk integrals. Since $\phi_m$ does not couple to $A$ or $\tilde{A}$, the $\Lambda_i$ factors cannot reach the endpoints on the AdS boundary. Instead they simply cancel among all diagrams. Indeed a careful calculation in [5] shows that the sum of all diagrams vanishes, so that $\phi_m$ remains massless.

The same cancellation can be proven diagrammatically at any loop order. Again it is a consequence of the structure $K_{\mu i} = \partial_{\mu} \Lambda_i$ and the Ward identity. Since there are no external $\phi_c$ lines, the boundary terms from partial integration at each vertex vanish (provided that the unitarity bound $\Delta_c \geq 0$ for AdS$_3$ is satisfied). The $\Lambda_i$ factor from any given bulk vertex is pinned at the adjacent vertices and these terms cancel among all diagrams. This is just the $U(1)$ charge conservation in the action!

Analogous cancellations occur for n-particle correlation functions of $\phi_m$, which indicates the self-potential of $\phi_m$ remains exactly flat.

### 2.2 Coupling Constants

Supersymmetry not only requires mass relations between superpartners but also specific relations among coupling constants. In this section, we calculate the change of coupling constants induced by the SUSY breaking term in Eq. (2.4). We will show that the coupling constants are shifted in a non-supersymmetric way. To simplify the calculation, let us focus on the cubic coupling in Eq. (2.3). See Sec. 6 of [5] for more detail.

From AdS/CFT, the cubic coupling is related to the three point correlation function $\langle O_i^* O_c O_m \rangle$. By conformal invariance, it can written as

$$\langle O_i^*(\vec{y}) O_c(\vec{z}) O_m(\vec{w}) \rangle = \frac{c_3}{|\vec{y} - \vec{z}|^{2\Delta_c - \Delta_m} |\vec{y} - \vec{w}|^{\Delta_c} |\vec{z} - \vec{w}|^{\Delta_m}}.$$ (2.12)
When we turn on the SUSY breaking term in Eq. (2.4), both \( c_3 \) and \( \Delta_c \) are modified. After properly normalizing \( O_c \), the change of \( c_3 \) can be written as
\[
\delta_h c_3 = -\frac{4\pi q\bar{q}h}{\Delta_c - 1} c_3.
\] (2.13)

If the coupling (2.3) is the only cubic term in the bulk action, then the Witten diagram which determines \( \Delta_c \) normalizing to the 3-pt correlation function is given by
\[
\langle O^1(y) O_c(z) O_m(w) \rangle = -\lambda \int \frac{d^3 x}{x_0^3} \partial_\mu K_{\Delta_c}(x, \bar{y}) \partial^\mu K_{\Delta_c}(x, \bar{z}) K_{\Delta_m}(x, \bar{w}) .
\] (2.14)

It is straightforward to check that the leading order correction to \( K_{\Delta_c} \) can be simply accounted by shifting \( \Delta_c \) to \( (\Delta_c - 2\pi q\bar{q}h) \), i.e.
\[
K_{\Delta_c,h}(x,x') = K_{\Delta_c-2\pi q\bar{q}h}(x,x').
\] (2.15)

After a careful calculation, we find
\[
\frac{\delta \lambda}{\lambda} = \frac{2\pi q\bar{q}h}{\Delta_c} \frac{\partial}{\partial \Delta_c} \log \frac{\Gamma(\Delta_c - \frac{\Delta_m}{2})\Gamma(\Delta_c + \frac{\Delta_m}{2} - 1)}{\Gamma(\Delta_c)^2} (\Delta_c^2 - 2\Delta_c - \frac{1}{2} \Delta_m^2 + \Delta_m).\]
(2.16)

Thus we see that the shifts of coupling constants depend on R-charges. This is particularly interesting because it indicates that supersymmetric relations between coupling constants are also broken when the SUSY breaking deformation is turned on.

However, there is one subtlety that we need to address at the end of this section. In the calculation above we make the assumption that the cubic vertex in Eq. (2.3) is the only interaction vertex contributing to the 3-pt correlation function after SUSY is broken. However, it is possible that higher derivative bulk couplings, such as \( \lambda' (\nabla^\mu \nabla^\nu \phi_1)(\nabla^\mu \nabla^\nu \phi_2)\phi_{km} \), can be induced when \( h \) is turned on. These vertices will also contribute to the 3-pt correlation function. We expect a careful calculation of 4-pt functions may resolve this ambiguity. However that is beyond the scope of this paper. It is sufficient for our purpose here to show that the SUSY relation among coupling constants is explicitly violated.

### 2.3 Singly charged particles

In this section, we consider SUSY breaking effects on particles that are charged under only one of the R-symmetry groups. We choose bulk fields with charge \((q, 0)\). This simplification is especially helpful since it provides important intuition for the extension of our mechanism to higher dimensions.

The order \( h \) correction discussed in Sec. 2.1.1 vanishes for a singly charged particle, but there is an order \( h^2 \) shift of conformal dimension which was calculated by CFT methods in [5] with the result
\[
\delta_{h^2} \Delta = \pi^2 k h^2 q^2 / 2.
\] (2.17)

Thus SUSY breaking effects also occur for particles with only one non-vanishing R-charge.

We now assume that all charged fields in the theory couple only to \( A_\mu \). The gauge field \( \tilde{A}_\mu \) then appears only in the Chern-Simons term (2.1) and in the SUSY breaking term (2.4). One can then integrate out \( \tilde{A} \) and obtain an action with the new non-local SUSY breaking boundary term
\[
S_{SB}' = \frac{h^2 k}{8} \int_{bdy} d^2 w_1 d^2 w_2 A_\mu(w_1) J^{\mu}(w_1,w_2) A_\mu(w_2).
\] (2.18)

One can now apply the basic partial integration plus Ward identity methods to the Witten diagrams that describe the correction of this non-local interaction to the correlator \( \langle O_c O_c^* \rangle \). This calculation reproduces the result (2.17) exactly.

Integrating out a light field always produces a non-local effective action, so there is no fundamental problem with \( S_{SB}' \). However it is worthwhile to discuss how the common concern that a non-local operator violates causality is avoided operationally in our framework. As we have seen in earlier sections, the net result of the calculations is that the \( \Lambda_i \) factors of the gauge field propagators are pinned at the boundary points of R-charged fields. The same feature occurs for insertions of \( S_{SB}' \). This means that the original
correlation function appears as a factor in the final amplitude. Thus if two points are causally disconnected in the original theory, i.e. the commutator at these two points vanishes, these two points are still causally disconnected in the deformed theory.

### 2.4 All-order sum of SUSY breaking corrections

Since SUSY is broken by a bilinear operator, one might expect to obtain an exact solution for SUSY breaking corrections which supersedes the order $h$ and $h^2$ contributions discussed so far. We obtained the exact solution by summing the diagrams shown in Fig. 4. The result for the shifted scaling dimension of an operator with general R-charges $(q, \tilde{q})$ is presented in [5]. For the slightly simpler case of charge $(q, 0)$, the shifted scaling dimension is

$$
\Delta = \Delta_0 + \frac{1}{2} \frac{\pi^2 h^2 k q^2}{1 - \pi^2 h^2 k^2/4}.
$$

(2.19)

As the coupling $h$ is increased toward the root of the denominator the difference in conformal dimensions between $\phi_c$ (with $q^2$ in the numerator) and $\psi_c$ (with $(q-1)^2$) becomes arbitrarily large.

The diagrams of Fig. 4 were summed by deriving a recursion relation which reduces higher $h$ order corrections to the lowest order. This recursion relation requires the identity

$$
\partial_i \langle J^i(\vec{x}) J^j(\vec{y}) \rangle = -\pi \frac{\partial}{\partial y_j} \delta^{(2)}(\vec{x} - \vec{y}).
$$

(2.20)

This identity is special to $d = 2$. In higher dimensions, $\partial_i \langle J^i(\vec{x}) J^j(\vec{y}) \rangle$ is precisely zero, and we will obtain a very different all-order formula in later sections.

The two diagrams in Fig. 4 are the lowest terms of a series of diagrams of order $h^{2n} q^2$. The summed series may be thought of as a "necklace" that determines the all-order shift of the conformal dimension (2.19). Fig. 5 depicts the insertion of two independent necklaces along the charged line. As discussed in Sec. 5 of [5], the partial integration and Ward identity arguments apply to each necklace independently. This produces an exponential series so that the fully corrected correlator $\langle O_c^† O_c \rangle$ acquires the shifted power
law required by conformal symmetry:

\[ \langle O_c^\dagger(\bar{y})O_c(\bar{z}) \rangle \sim \frac{1}{(\bar{y} - \bar{z})^{2\Delta}}. \]  

(2.21)

3 The undeformed correlator \( \langle J^i(\bar{z})O_c^\dagger(\bar{y})O_c(\bar{w}) \rangle \)

This is a transitional section from \( AdS_3 \) to \( AdS_{d+1} \). We will study the 3-point function \( \langle J^i(\bar{z})O_c^\dagger(\bar{y})O_c(\bar{w}) \rangle \) in the undeformed theory. This will illustrate the basic partial integration and Ward identity technique in a simple example that will provide a very important clue to the higher dimensional extension of the \( AdS_3 \) model.

The previous discussion has made clear the importance of the total derivative structure of the propagator for the bulk scalar \( \phi \). This suggests that one extension to higher dimensions might be based on a \( BF \) theory with Lagrangian

\[ L \sim B \wedge F. \]  

(3.1)

For \( AdS_{d+1} \), the field \( B \) is a \((d - 1)\)-form and the field strength \( F = dA \) with \( A \) a 1-form. However this suggestion is not viable because the 3-point function to which it leads agrees with the unique conformal invariant form (see (23.54) of [12])

\[ \langle J_i(\bar{z})O_c^\dagger(\bar{y})O_c(\bar{w}) \rangle \sim \frac{1}{(\bar{y} - \bar{w})^{2\Delta_{d-2}}(\bar{w} - \bar{z})^{2\Delta - d}} \frac{(\bar{y} - \bar{z})}{(\bar{w} - \bar{z})^2} \]  

(3.2)

only for \( d = 2 \).

To demonstrate this let us evaluate the Witten diagram of Fig. 6 for general boundary dimension \( d \). We assume that the bulk-boundary propagator of the gauge field dual to the conserved current \( J^i(\bar{w}) \) has the total derivative form \( K_{\mu i}(\bar{w}) = \partial_\mu \Lambda_i(x, \bar{w}) \), with \( \Lambda_i \) to be specified later. We also need the bulk-boundary propagator for the bulk scalar \( \phi_c(x) \), namely

\[ K_\Delta(x_0, \bar{x}) = c_\Delta \left( \frac{x_0}{x_0^2 + \bar{x}^2} \right)^\Delta, \quad (\Box - m^2)K_\Delta(x_0, \bar{x}) = 0, \]  

(3.3)

\[ \lim_{x_0 \to 0} K_\Delta(x_0, \bar{x}) = x_0^{d-\Delta} \delta^{(d)}(\bar{x}), \quad C_\Delta = \frac{\Gamma(\Delta)}{\pi^{d/2} \Gamma(\Delta - d/2)}. \]  

(3.4)

The value of the Witten diagram is given by the bulk integral

\[ \langle J^i(\bar{w})O_c^\dagger(\bar{y})O_c(\bar{z}) \rangle = \int d^{d+1}x \sqrt{g} \partial_i \Lambda_i(x, \bar{w}) K_\Delta(x_0, \bar{x} - \bar{y}) \delta^{+\mu} K_\Delta(x_0, \bar{x} - \bar{z}) \]  

(3.5)

\[ = -\int d^{d+1}x \sqrt{g} \Lambda_i(x, \bar{w}) K_\Delta(x_0, \bar{x} - \bar{y}) \delta^{\mu} K_\Delta(x_0, \bar{x} - \bar{z}) \]  

(3.6)

\[ - \lim_{x_0 \to 0} \int d^d x \sqrt{g} K_\Delta(x_0, \bar{x} - \bar{y}) \delta^{+\mu} K_\Delta(x_0, \bar{x} - \bar{z}). \]  

(3.7)
After partial integration and use of Green’s theorem we find the bulk integral in the second line (which vanishes by the equation of motion for \(K_\Delta\)) plus the boundary term in the third line. The limit \(x_0 \to 0\) of the boundary term vanishes due to the \(x_0\) factors in \(K_\Delta(x_0, \vec{x})\) except when \(\vec{x} \approx \vec{y}\) and \(\vec{x} \approx \vec{z}\). The contribution from these regions is determined by the limit of \(K_\Delta\) given in (3.3) above. We find

\[
\langle J^i(\vec{w})O_i^\dagger(\vec{y})O_i(\vec{z})\rangle = (\Delta - d)[\Lambda_i(\vec{z} - \vec{w}) - \Lambda_i(\vec{y} - \vec{w})] \frac{1}{|y - z|^{2\Delta}}. \tag{3.8}
\]

We can now compare this result with the required form (3.2). In our \(AdS_3\) example, the R-gauge boson propagator is a total derivative as Eq. (2.8), and we find agreement with (3.2) when its \(\Lambda_i\) factor is inserted in (3.8). For \(d \geq 3\), however, no choice of \(\Lambda_i(\vec{z} - \vec{w})\) can bring (3.8) and (3.2) into agreement because it fails to produce the correct parametric coordinate dependence.

Thus we pass to the next section with an acute puzzle. We cannot use a gauge field of the form \(K_{\mu i} = \partial_\mu \Lambda_i\) in the undeformed current-matter correlator, yet we need precisely such a form to use Ward identity methods to evaluate SUSY breaking corrections. Please read on to see how this dilemma is resolved.

4 Generalization to \(AdS_{d+1}\)

The puzzle in Sec. 3 motivates us to proceed as follows towards the desired higher dimensional model:

a. We use a gauge field \(A_\mu\) with Maxwell dynamics which is dual to a conserved \(U(1)_R\) current \(J_i\) with correct undeformed correlation functions. A single gauge field suffices.

b. Its bulk-boundary propagator \(K_{\mu i}\) is not a total \(\partial_\mu\) derivative, but we find a simple identity which shows that \(\partial_\mu K_{\mu i}\) is a total \(\partial_\mu\) derivative.

c. We employ a SUSY breaking boundary term which always produces \(\partial_\mu K_{\mu i}\) when inserted in a Witten amplitude. Thus the same powerful Ward identity methods used in the \(AdS_3\) model are also valid in higher dimensions.

d. The SUSY breaking term is non-local, but its structure is similar to that of Sec. 2.3 and has conventional causal properties. It is also exactly marginal.

4.1 Maxwell theory in \(AdS_{d+1}\) space

The well-known Maxwell action is

\[
S_{Maxwell} = \int d^{d+1}x \sqrt{g} \frac{1}{4} F_{\mu \nu} F^{\mu \nu}. \tag{4.1}
\]

A solution of the classical equation of motion in \(AdS_{d+1}\) has the near-boundary expansion

\[
A_i(x) = \alpha_i(\vec{x})(1 + \ldots) + \beta_i(\vec{x})x_0^{d-2} + \ldots. \tag{4.2}
\]

We use standard quantization in which \(\alpha_i(\vec{x})\) is fixed as the boundary value of the bulk field and \(\beta_i(\vec{x})\) is the response, which can be thought of as the current operator \(J_i(\vec{x})\) in the dual CFT\(_d\). Note that \(\beta_i(\vec{x})\) has scaling dimension \(d - 1\) and that \(\partial_\mu \beta^i = 0\) as a consequence of the Maxwell equation. The bulk-to-boundary propagator of the Maxwell field in \(AdS_{d+1}\) satisfies the sourceless Maxwell equation. It is gauge dependent, and we use the conformal covariant form (see (48) of [11])

\[
K_{\mu i} = \left[\frac{\Gamma(d)}{2\pi^{d/2} \Gamma(d/2)}\right] \frac{J_{\mu i}(\vec{x} - \vec{z})x_0^{d-2}}{x_0^2 + (\vec{x} - \vec{z})^2} x_0^{d-4}. \tag{4.3}
\]

Standard quantization of Maxwell theory is well defined for \(d > 2\). It is important to realize that this bulk-to-boundary propagator cannot be written as a total derivative. This is as expected because \(K_{\mu i}\) satisfies the Maxwell equation \(\nabla^\mu F_{\mu \nu} = 0\), which does not imply \(F_{\mu \nu} = 0\).

4.2 The identity

In order to convert this bulk-to-boundary propagator to an object useful for our purposes, we need an identity. We first present it as a general property of bulk-to-boundary propagators for \(n\)-form gauge fields.
with Maxwell dynamics in $AdS_{d+1}$, namely
\begin{equation}
K_{\mu_1 \ldots \mu_n}^{i_1 \ldots i_n}(x, z) \equiv \frac{\Gamma(d)}{2\pi^{d/2}\Gamma(d/2)} \frac{J_{\mu_1 \ldots J_{\mu_n}}^{[i_1 \ldots J_{i_n}]}}{(x_0^2 + (\vec{x} - \vec{z})^2)^{(d-n)/2}},
\end{equation}
\begin{equation}
\partial_1 K_{\mu_1 \ldots \mu_n}^{i_1 \ldots i_n} = \frac{2d - 2n + 2}{d - 2n + 2} \partial_{\mu_1} K_{\mu_2 \ldots \mu_n}^{i_2 \ldots i_n}.
\end{equation}

In this paper we need only the case $n = 1$. This gives us a very useful identity, because it converts the bulk-to-boundary propagator in Maxwell theory to a total derivative respect to bulk coordinates, i.e.
\begin{equation}
\partial_1 K_{\mu}^{i}(x, z) = \frac{\Gamma(d)}{\pi^{d/2}\Gamma(d/2)} \partial_\mu \frac{x_0^d}{(x_0^2 + (\vec{x} - \vec{z})^2)^{d/2}} = \partial_\mu K(x, z),
\end{equation}
where $K$ is the bulk-boundary propagator of a massless scalar. This identity will be a critical tool in the evaluation of SUSY breaking corrections in higher dimension $d \geq 3$.

Here is a qualitative interpretation of the identity. The boundary integral of $K_{\mu}$ computes the bulk field $A_{\mu}(x)$ with boundary value $\alpha_i$. In general there is a non-vanishing $F_{\mu\nu}$. The $\partial_1$ in the identity means that $\alpha_i$ is purely longitudinal. In this case one expects that $F_{\mu\nu}$ vanishes and that $A_{\mu}$ is a total derivative.

### 4.3 SUSY breaking term in $AdS_{d+1}$

The generalization of our SUSY breaking mechanism to higher dimensional $AdS$ space is motivated by the second-order SUSY breaking action in $AdS_3$, given in (2.18). We propose that the general SUSY breaking action in the CFT language is
\begin{equation}
S_{\text{CFT},d} = \hbar \int d^d \vec{w} \, d^d \vec{w} J_i(\vec{w}) \frac{J_{ij}(\vec{w}_1 - \vec{w}_2)}{\vec{w}_1 - \vec{w}_2^2} J_j(\vec{w}_2),
\end{equation}
\begin{equation}
J_{ij}(\vec{w}) = \delta_{ij} - 2\frac{w_i w_j}{\vec{w}^2}.
\end{equation}

Using the BDHM dictionary [13], we may write the CFT current $J_i(\vec{w})$ as the limit of the bulk operator $ciw_{i0}^2 A_i(\vec{w})$ as $w_{00}$ goes to zero, where $c$ is a constant. The above deformation can therefore be written in the bulk language as
\begin{equation}
S_{\text{AdS}_{d+1}} = \lim_{w_{10}, w_{20} \to 0} c^2 \hbar \int d^d \vec{w}_1 d^d \vec{w}_2 (w_{10}^{2-d})(w_{20}^{2-d}) A_i(\vec{w}_1) \frac{J_{ij}(\vec{w}_1 - \vec{w}_2)}{\vec{w}_1 - \vec{w}_2^2} A_j(\vec{w}_2).
\end{equation}

This action is written on the constant radial coordinate slice $w_{10} = w_{20} = \epsilon$, and the boundary limit is taken after its insertion in a correlation function. For future use, $\frac{J_{ij}(\vec{w})}{\vec{w}^2}$ can be written in the more convenient form
\begin{equation}
\frac{J_{ij}(\vec{w})}{\vec{w}^2} = \frac{1}{2} \partial_1 \partial_1 \log(\vec{w}^2).
\end{equation}

In later calculations, partial integration of $\partial_1$ will convert the bulk-to-boundary propagator $K_{\mu i}$ to a total derivative. Naively, if one substitutes Eq. (4.9) to Eq. (4.6) and integrates by parts, it seems that the SUSY breaking term vanishes due to current conservation. We emphasize again that unlike ordinary QFT in flat spacetime, one should not use current conservation or the equation of motion to remove a term in the CFT Lagrangian or in a theory living in AdS space. This subtlety already appears in the SUSY breaking term that we introduced in our $AdS_3$ model, as discussed in Sec. 2. In the following discussion, we will present the detailed calculation to show that this SUSY breaking term generates non-trivial physical effects, and its non-triviality is consistent with the Ward identity from the CFT point of view.

It is worthwhile to understand the proposal (4.8) better before applying it to calculate SUSY breaking effects. First we note that $S_{\text{AdS}_{d+1}}$ is invariant under dilatations $w_{i\mu} \to \lambda w_{\mu}$ and $A_i(w_1) \to \lambda^{-1} A_i(\lambda w_1)$. This means that it acts as a marginal deformation of the boundary CFT$_d$. Indeed we will verify from its effect on correlation functions that it is exactly marginal.

We will compare the SUSY breaking deformation of a correlation function from both the insertion of $S_{\text{AdS}_{d+1}}$ and of $S_{\text{CFT},d}$. In the first case we compute the bulk and boundary integrals using Ward identity methods. In the second case we use conformal perturbation theory. The results agree.
The SUSY breaking actions are non-local. They have the same structure as the term (2.18) used in $AdS_4$. However, as far as we know, there is no way to rewrite them as the result of integrating out massless degrees of freedom in a larger theory. Nevertheless, as we discussed in Sec. 2.3, we can check by explicit computation of SUSY breaking deformations of correlation functions that no pathology occurs, i.e. the casuality properties are exactly the same as undeformed theory.

### 4.4 SUSY breaking corrections

In this section, we examine physical effects of the SUSY breaking term of Sec. 4.3. First we show that the 2-pt correlation functions of R-charged particles receive non-trivial corrections. Thus there are explicit SUSY breaking effects in the particle mass spectrum. Next we will show that the masses of R-neutral particles do not receive any SUSY breaking corrections. Then we return to charged particles and show that SUSY breaking effects vanish beyond leading order $h$. Finally we consider Witten diagrams containing bulk gravitons and show that SUSY breaking corrections vanish.

#### 4.4.1 Mass corrections for R-charged and R-neutral particles

In this section, we calculate the leading order SUSY breaking correction to the 2-pt correlation function of R-charged particle. The Witten diagram to be computed is shown in Fig. 7. The effect of another diagram with the $A_{\mu}A^{\mu}\phi_{c}\phi_{c}^{r}$ seagull vertex is included in the final result.

![Figure 7: A Witten diagram that gives an order h SUSY breaking correction to the 2-pt correlation function of R-charged particle.](image)

The diagram can be calculated as follows:

$$
\delta_h \langle O_c(x)O_c(z) \rangle = \frac{\hbar q^2}{2} \int d^4w_1d^4w_2 \left. d^{d+1}x \right| d^{d+1}x' \sqrt{g(x)} \sqrt{g(x')} \frac{J_{\mu}(w_1-w_2)}{(w_1-w_2)^2} K_{\nu j}(x', \bar{w}_2) \times \right.
$$

\begin{align*}
&\times \left[ K_\Delta(x, \bar{y}) \frac{\partial \phi_{c}}{\partial \mu} \left( G(x, x') \frac{\partial \phi_{c}}{\partial \nu} \Delta_{\nu j}(x', \bar{z}) \right) \right] + (\bar{y} \leftrightarrow \bar{z}) \\
&= \frac{\hbar q^2}{4} \int d^4w_1d^4w_2 \left. d^{d+1}x \right| d^{d+1}x' \sqrt{g(x)} \sqrt{g(x')} \frac{J_{\mu}(w_1-w_2)}{(w_1-w_2)^2} K_{\nu j}(x', \bar{w}_2) \times \right.
\end{align*}

\begin{align*}
&\times \left[ K_\Delta(x, \bar{y}) \frac{\partial \phi_{c}}{\partial \mu} \left( G(x, x') \frac{\partial \phi_{c}}{\partial \nu} \Delta_{\nu j}(x', \bar{z}) \right) \right] + (\bar{y} \leftrightarrow \bar{z}) \\
&= \frac{\hbar q^2}{4} \left( \frac{\Gamma(d)}{\pi^{d/2}} \right)^2 \int d^4w_1d^4w_2 \left. d^{d+1}x \right| d^{d+1}x' \sqrt{g(x)} \sqrt{g(x')} \times \right.
\end{align*}

\begin{align*}
&\times \partial_{\nu j} \log \left( \frac{(w_1-w_2)^2}{(w_1-w_2)^2} \right) \times \left[ K_\Delta(x, \bar{y}) \frac{\partial \phi_{c}}{\partial \mu} \left( G(x, x') \frac{\partial \phi_{c}}{\partial \nu} \Delta_{\nu j}(x', \bar{z}) \right) \right] + (\bar{y} \leftrightarrow \bar{z}) \\
&\text{(4.10)}
\end{align*}

Here in the second step, we use Eq. (4.9) and integrate by parts to move $\partial_{\mu}$ to $K_{\mu i}(x, \bar{w}_1)$ and $\partial_{j}$ to $K_{\nu j}(x', \bar{w}_2)$. The extra minus sign is because $\partial_{\nu j}$ now acts on $\bar{w}_2$ instead of $\bar{w}_1$. In the last step, we use Eq. (4.5) to convert $\partial_i K_{\mu i}$ to a total derivative respect to $x_{\mu}$, similar for $\partial_{\nu j}$. One can integrate by parts on $x$ and $x'$ integrals to simplify the calculation. The remaining calculation is very similar to the one carried
out for $AdS_3$, and the result is the following:

$$
\delta_h (O^1_c (\vec{y}) O_c (\vec{z})) = \frac{h q^2}{4} \frac{\Gamma(d)}{\pi^{d/2} \Gamma(d/2)} \lim_{x_0 \to 0} \int d^d w_1 d^d w_2 d^d x' \frac{x_0^d}{(x_0^d + (\vec{y} - \vec{w}_1)^2)^d} \frac{x_0^d}{(x_0^d + (\vec{z} - \vec{w}_2)^2)^d} \log[(w_1 - w_2)^2] \times 
\times 2 \Delta \frac{\delta^d(\vec{x}', \vec{y}) - \delta^d(\vec{x}', \vec{z})}{|\vec{x}' - \vec{y}|^{2\Delta}} + (\vec{y} \leftrightarrow \vec{z})
= - \frac{\Gamma(\Delta)}{\Gamma(\Delta - d/2)} \frac{(\Delta - d/2)}{\pi^{d/2}} h q^2 \frac{1}{a^2} \log[(\vec{y} - \vec{z})^2]
= - h q^2 \log[(\vec{y} - \vec{z})^2] \langle O^1_c(O^1_c(O_c(z))) \rangle_0.
$$

(4.11)

Here $a$ is the UV cut-off. In the second step, we used

$$
\lim_{x_0 \to 0} \left( \frac{x_0^d}{x_0^d + (\vec{x} - \vec{z})^2} \right)^d = \frac{\pi^{d/2} \Gamma(d/2)}{\Gamma(d)} \delta^d(\vec{x} - \vec{z}).
$$

(4.12)

This logarithmic factor implies a shift of conformal dimension of the dual operator,

$$
\delta \Delta_{O_c} = h q^2, \quad \delta \Delta_{\Phi_c} = h (q - 1)^2.
$$

(4.13)

We have included the shift for the fermion partner of $O_c$ which is computed by similar steps. We emphasize again that shifts of conformal dimension of bosonic and fermionic components of a supermultiplet differ because they have different R-charges. The shifts of conformal dimension and the consequent mass shifts definitely break SUSY!

It was crucial in this calculation that the initial bulk-to-boundary propagator is converted to a bulk derivative $\partial_{\vec{x}} K$ by partial integration of the $\partial_{\vec{i}}$ derivatives in the SUSY breaking action (4.8). Then $\partial_{\vec{x}}$ can be partially integrated and acts via the Ward identity on the charged field line as in the $AdS_3$ calculations in Sec. 2.1.1. The final answer comes from boundary terms in which the gauge fields are pinned at the location of the external R-charged particles.

The calculation of the deformation of the 2-point function $\langle O^i_m O^m_0 \rangle$ for R-neutral particles proceeds in a similar fashion to that in Sec. 2.1.2. The $U(1)$ Ward identity guarantees that the contribution from each loop containing R-charged particles cancels exactly, and there are no residual boundary terms. The $U(1)$ Ward identity is a general and exact principle, so there are no mass corrections for R-neutral particles at any loop order.

### 4.4.2 The same calculation in the CFT

It is instructive (and even easier!) to show that the same result for the mass shift can be obtained using conformal perturbation theory and the $U(1)$ Ward identity

$$
\partial \langle ... J^i (\vec{w}) O_c (\vec{z}) ... \rangle \approx_{\vec{w} = \vec{z}} q \delta^{(d)}(\vec{w} - \vec{z}) \langle ... O_c (\vec{z}) ... \rangle.
$$

(4.14)

To obtain the leading order $h$ contribution, we insert the CFT dual SUSY breaking operator in $\langle O^1_c (\vec{y}) O_c (\vec{z}) \rangle$ and proceed as follows:

$$
\delta (O^1_c (\vec{y}) O_c (\vec{z})) \sim h \int d^d w_1 d^d w_2 \int d^d x' \langle O^1_c (\vec{y}) J_i (\vec{w}_1) J_j (\vec{w}_2) O_c (\vec{z}) \rangle 
= h \int d^d w_1 d^d w_2 \partial_{\vec{x}} \partial_{\vec{y}} \log[(\vec{w}_1 - \vec{w}_2)^2] \langle O^1_c (\vec{y}) J_i (\vec{w}_1) J_j (\vec{w}_2) O_c (\vec{z}) \rangle.
$$

(4.15)

We may integrate by parts and use the Ward identity

$$
\partial_{\vec{x}} \partial_{\vec{y}} \langle O^1_c (\vec{y}) J_i (\vec{w}_1) J_j (\vec{w}_2) O_c (\vec{z}) \rangle \sim h q^2 \left[ \delta^d(\vec{y} - \vec{w}_1) - \delta^d(\vec{z} - \vec{w}_1) \right] \left[ \delta^d(\vec{y} - \vec{w}_2) - \delta^d(\vec{z} - \vec{w}_2) \right] \langle O^1_c (\vec{y}) O_c (\vec{z}) \rangle.
$$

(4.16)

The integrals over $\vec{w}_1$ and $\vec{w}_2$ become trivial with the same result as (4.11), which induces the shift of conformal dimension (4.13).
4.4.3 The order $h$ mass shift is exact

Possible higher order contributions come from the insertion of higher powers of $S_{\text{AdS}_{d+1}}$ or $S_{\text{CFT}_d}$ in correlation functions. To study these we carry out the explicit calculation at order $h^2$ order from the CFT viewpoint. The $h^2$ order correction to $\delta \langle O_c^1(\vec{y})O_c(\vec{z}) \rangle$ can be written as

$$
\delta_{h^2} \langle O_c^1(\vec{y})O_c(\vec{z}) \rangle \sim h^2 \int d^d\vec{w}_1 \ldots d^d\vec{w}_4 \langle O_c^1(\vec{y})J_1(\vec{w}_1) J_2(\vec{w}_2) J_3(\vec{w}_3) J_4(\vec{w}_4) \rangle \langle O_c(\vec{z}) \rangle .
$$

Applying conformal perturbation theory, one sees that there are two inequivalent sets of connected contributions which play rather different roles:

i. order $q^2$, with a Wick contraction of any pair of currents such as

$$
\langle \ldots J_i(\vec{w}_2)J_k(\vec{w}_3) \ldots \rangle \sim \langle \ldots \frac{J_i(\vec{w}_2 - \vec{w}_3)}{(\vec{w}_2 - \vec{w}_3)^2} \rangle .
$$

ii. order $q^4$, in which one uses (4.9) and partial integration of $\partial_i \partial_j$ followed by the $\partial_i J^i(\vec{w}_1)O_c^1(\vec{y})$ and $\partial_j J^j(\vec{w}_2)O_c(\vec{z})$ Ward identities, and then similar manipulations on the currents $J_k(\vec{w}_3)$ and $J_l(\vec{w}_4)$.

Order $q^2$: Partial integration of $\partial_i$ leads to $\langle \ldots \partial_i J_i(\vec{w}_2)J_k(\vec{w}_3) \ldots \rangle$. The current is conserved, but the question is whether there is a contact term in the OPE when $\vec{w}_2 \approx \vec{w}_3$. A regulated calculation$^5$ shows that

$$
\partial_i \langle J_i(\vec{w}_2)J_k(\vec{w}_3) \rangle = -\pi \partial_i \delta^2(\vec{w}_2 - \vec{w}_3), \quad d = 2
$$

$$
\partial_j \langle J_j(\vec{w}_2)J_l(\vec{w}_3) \rangle = 0. \quad d > 2.
$$

There is a natural contact term in $d = 2$, but none in higher dimensions. Therefore the shift $\delta_{h^2} q^2 \Delta O_c$ (and higher order shifts) vanishes for $d \geq 3$.

Order $q^4$: The operations described above together with similar operations with permutations of the four currents lead to the result

$$
\delta_{h^2} \langle O_c^1(\vec{y})O_c(\vec{z}) \rangle = -\frac{1}{2} h^2 q^4 \log^2 \frac{(\vec{y} - \vec{z})^2}{a^2} \frac{1}{(\vec{y} - \vec{z})^{2\Delta}} .
$$

When added to the order $h^0$ and $h$ terms, we find the beginning of the exponential series that expresses the fully corrected $\langle O_c^1\rangle$ correlation in the power law form

$$
\langle O_c^1(\vec{y})O_c(\vec{z}) \rangle_{\text{exact}} = c(\Delta + hq^2) \frac{1}{(\vec{y} - \vec{z})^{2(\Delta + hq^2)}}
$$

required by conformal symmetry. To see the full exponential series one must consider $n$ insertions of $S_{\text{CFT}_d}$. For $d \geq 3$ the only non-vanishing contributions are of order $h^n q^{2n}$ and come from $n$ applications of (4.9) with subsequent partial integration and use of (4.14). The combinatoric structure is the same as that of $\text{AdS}_3$ scenario [5] and leads to exponentiation.

4.4.4 Exact marginality

As stated in Sec. 4.3, insertions of $S_{\text{AdS}_{d+1}}$ in any correlator are expected to preserve its conformal structure. Thus $S_{\text{AdS}_{d+1}}$ is effectively exactly marginal. The result (4.21) above is one indication that this is true.

We may also verify this by calculating the correction to $\langle J_i(\vec{y})J_j(\vec{z}) \rangle$ which at the leading order is given by

$$
\delta_h \langle J_i(\vec{y})J_j(\vec{z}) \rangle \sim h \int d^d\vec{w}_1 d^d\vec{w}_2 \langle J_i(\vec{y})J_i(\vec{w}_1) \frac{J_j(\vec{w}_1 - \vec{w}_2)}{\vec{w}_1 - \vec{w}_2^2} J_j(\vec{w}_2)J_i(\vec{z}) \rangle
$$

$$
\sim h \int d^d\vec{w}_1 d^d\vec{w}_2 \langle J_i(\vec{y})J_i(\vec{w}_1)\partial_i \partial_j \text{Log}[(\vec{w}_1 - \vec{w}_2)^2] J_j(\vec{w}_2)J_i(\vec{z}) \rangle .
$$

---

$^5$One natural method, based on differential regularization, is used in Appendix A.
Using integration by parts and (4.19), we find $\delta_h(J_a(\vec{y})J_b(\vec{z})) = 0$ for $d > 2$. Higher order corrections also vanish, and the conformal properties are preserved.\(^6\)

### 4.4.5 Gravity mediated SUSY breaking effects vanish

As discussed in the previous section, SUSY is explicitly broken in our theory by an effectively marginal deformation. Unlike common SUSY breaking mechanisms, especially spontaneous breaking, there is no energy scale above which SUSY is restored. Then one concern in our SUSY breaking mechanism is whether the diagrams involving graviton loops can introduce additional SUSY breaking corrections to R-charged particles. More seriously, if SUSY breaking effects can propagate to R-neutral particles through graviton loops, the SUSY breaking effects may be huge due to the absence of SUSY restoring scale. These concerns can be explored by studying the Witten diagrams shown in Fig. 8.

![Witten diagrams](image)

**Figure 8:** Witten diagrams which involve gravitons. The double lines indicate gravitons. The first diagram describes the SUSY breaking corrections, mediated by the graviton, to R-charged particles. The horizontal line in the second and third diagrams can be charged or neutral. In the third diagram a graviton couples to a charged loop.

There are two categories of diagrams involving the bulk graviton.

1. The graviton line ends on an R-gauge photon emitted directly from the SUSY breaking vertex.

2. The graviton line ends on an internal loop of an R-charged matter field that also couples to an R-gauge field from the SUSY breaking vertex.

The main new observation needed to show that the first class of diagrams vanish is that the graviton couples to $A_\mu$ through the field strength $F^{\mu\nu}$, i.e.

$$L \sim \delta g_{\mu\nu} T^{\mu\nu} \sim \delta g_{\mu\nu} (F^{\mu\rho} F_{\rho\nu} - \frac{1}{4} g^{\mu\nu} F^2).$$

(4.23)

If the R-gauge boson comes directly from the SUSY breaking vertex, its bulk boundary propagator couples to $F_{\mu\nu}$ as $\partial_{[\mu} K_{\nu]}$. This vanishes after partial integration on the boundary which converts the photon propagator to a bulk total derivative, i.e. $\partial_i \partial_{[\mu} K_{\nu]} = \partial_{[\mu} \partial_{\nu]} K = 0$.

Diagrams containing charged loops coupled both to a graviton and one or more gauge fields from the SUSY breaking vertex vanish by the same arguments used in Sec. 4.4.1. The presence of a graviton does not affect the validity of the $U(1)$ Ward identity.

### 5 Flat spacetime limit?

The approach to the little hierarchy problem described in this paper works in anti-de Sitter spacetime, but it would be far more interesting if SUSY breaking effects survive in the limit $L \to \infty$ to Minkowski space. We give a preliminary discussion of this limit; details are under investigation.

At large $L$, the AdS/CFT mass formulas for the superpartners $\phi_c$ and $\Psi_c$ (which we now denote by B

---

\(^6\)For $d = 2$ the right hand side of (4.19) does not vanish, and there is a residual correction proportional to the undeformed correlator $1/(\vec{y} - \vec{z})^2$. The conformal structure is preserved in a less trivial fashion.
and F, respectively) behave as

\[ \Delta_B = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m_B^2 L^2} \rightarrow m_B L, \]

(5.1)

\[ \Delta_F = \frac{d}{2} + |m_FL| \rightarrow m_FL. \]

(5.2)

The SUSY breaking shifts of (4.13) are exact for any value of the coupling h. For large h the boson-fermion mass splitting is

\[ m_B - m_F = \frac{\Delta_B - \Delta_F}{L} = \frac{h(2q - 1)}{L}. \]

(5.3)

The mass splitting survives in the flat spacetime limit if we write h = m_0 L, where m_0 is a chosen mass scale, and scale L → ∞. In the limit we find

\[ m_B - m_F = (2q - 1)m_0. \]

(5.4)

We now bring into the discussion the Mellin representation of AdS/CFT correlators proposed by Mack [14, 15] and further developed by Penedones, Fitzpatrick, Kaplan, Raju, and van Rees [16, 17, 18]. The current status of this interesting research program was discussed by Penedones at the Strings 2015 conference [19]. In the limit L → ∞, the Mellin representation yields a quite direct relation between n-point correlation functions computed in a given bulk theory in AdS\(_{d+1}\) and the flat space S-matrix of that theory. It may be the case that the relation can be extended to include the SUSY breaking effects generated by the boundary interaction of Sec. 4.3 above.

6 Discussion

In this paper, we propose a new SUSY breaking mechanism in AdS\(_{d+1}\), d ≥ 3, in which breaking effects are generated by a boundary action. This is a generalization of the mechanism studied in AdS\(_3\). The most appealing fact of this new SUSY breaking mechanism based on gauged U(1)\(_R\) symmetry is that breaking effects of arbitrary strength can be evaluated exactly. One can thereby achieve large mass splitting between boson and fermion partners in a supermultiplet, while R-neutral fields do not receive SUSY breaking corrections to any loop order. Another important feature of our model is that diagrams involving graviton loops do not introduce any further SUSY breaking corrections.

The SUSY breaking action preserves the conformal properties of AdS/CFT correlation functions and is therefore effectively exactly marginal. Most effects can be calculated by two methods.

i. We can evaluate SUSY breaking in Witten diagrams exactly because the bulk-to-boundary propagator of the R-gauge boson is converted into a total derivative at the bulk point where it couples to R-charged fields. Partial integration and use of the U(1) Ward identity permit straightforward integration over each bulk insertion point, and the R-gauge bosons become pinned at the boundary. The remaining boundary integrals can also be done.

ii. We can evaluate the same effect in the boundary CFT using conformal perturbation theory and results always agree.

So far our mechanism operates in AdS spacetime, but there are several directions to pursue toward possible phenomenological models in flat spacetime. As suggested in Sec. 5, it may be possible to extend the recently developed connection between the Mellin transform of AdS/CFT correlators and the flat space S-matrix to include our SUSY breaking mechanism. This can be pursued in the flat space limit of AdS\(_4\). Alternatively in AdS\(_5\) one can consider the implementation of boundary SUSY breaking in the supersymmetric Randall-Sundrum model [21, 22, 23]. Or equivalently, one can study the phenomenology of new physics generated by deformations of a four-dimensional strongly coupled SCFT. We leave these interesting topics to future work.

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A  Differential Regularization of \( \langle J_i(\vec{w}) J_j(\vec{w}') \rangle \rangle \sim \frac{J_{ij}(\vec{w} - \vec{w}')}{(\vec{w} - \vec{w}')^{2(\Delta - 1)}} \)

We apply differential regularization [24] to argue that there is no contact term on the right side of (4.19) for boundary dimension \( d \geq 3 \). The basic idea of this method is quite simple. A singular function such as \( 1/(\vec{w} - \vec{w}')^{2p} \) in an integrated expression can usually be expressed as the total derivative of a less singular function. For example, one can use the identity (for \( \vec{w} - \vec{w}' \neq 0 \) and \( 2p - d > 0 \))

\[
\frac{1}{(\vec{w} - \vec{w})^{2p}} = \frac{1}{2(p - 1)(2p - d)} \left( \frac{1}{\vec{w} - \vec{w}'} \right)^{2(p - 1)}.
\]  

(A.1)

If the derivatives in \( \square \) are integrated by parts and applied to other factors in the amplitude under study, the degree of divergence of the initial integral is decreased. In the special case \( 2p = d \) of a log divergence we use instead

\[
\frac{1}{|\vec{w}|^d} = -\frac{\ln(\vec{w}^2 M^2)}{|\vec{w}|^{d-2}}.
\]  

(A.2)

The integration constant \( M^2 \) can often be interpreted as the renormalization group scale.

In our application to the tensor \( \frac{J_{ij}(\vec{w})}{|\vec{w}|^{2(d-1)}} \), we need another identity

\[
\frac{w_i w_j}{|\vec{w}|^{2d}} = \frac{1}{4(d - 1)(d - 2)} \left[ \frac{1}{|\vec{w}|^{2(d - 2)}} \right].
\]  

(A.3)

We used (A.1) with \( 2p = 2(d - 1) \) above and now again to obtain

\[
\frac{J_{ij}(\vec{w})}{|\vec{w}|^{2(d-1)}} = \frac{1}{2(d - 1)(d - 2)} \left[ \delta_{ij} \square - \partial_i \partial_j \right] \left[ \frac{1}{|\vec{w}|^{2(d - 2)}} \right].
\]  

(A.4)

It is no surprise that the result is proportional to the transverse projector \( \delta_{ij} \square - \partial_i \partial_j \) since the \( \langle J_i J_j \rangle \) correlator is formally conserved. This result is valid for \( d \geq 3 \).

For \( d = 2 \), current correlator is not manifestly conserved. However, the identity (4.9) provides a regularization and gives the well known contact term already noted in (2.20). The correlator is conserved for separated points.

For \( d = 3 \), the identity (A.4) provides a regularization, since the final power \( 1/|\vec{w}|^2 \) is integrable. To see how the identity is used, we return to (4.17) and substitute the regulated expression for the operator product \( J_j(\vec{w}_2) J_k(\vec{w}_3) \) and substitute (4.9) for the factor \( J_{ij}(\vec{w}_1 - \vec{w}_2)/(\vec{w}_1 - \vec{w}_2)^2 \). We then partially integrate the projector to obtain, in an entirely finite manner,

\[
(\delta_{ijk} \square - \partial_i \partial_k) \partial_i \partial_j \ln((\vec{w}_1 - \vec{w}_2)^2) = 0.
\]  

(A.5)

For \( d = 4 \), we need to use (A.2) to regulate the operator product completely. After further partial integration we still find (A.5). This procedure may be applied for \( d \geq 5 \) after further use of (A.1) and/or (A.2) to regulate the singular factor \( 1/(\vec{w}_2 - \vec{w}_3)^{3(d-2)} \) in (A.4). In this way we see that there are no contact terms in (4.19) for \( d \geq 3 \).

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