Atmospheric Neutrino Tests of Neutrino Oscillation Mechanisms

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Abstract

Recent Super-Kamiokande data on the atmospheric neutrino anomaly are used to test various mechanisms for neutrino oscillations. It is found that the current atmospheric neutrino data alone cannot rule out any particular mechanism. Future long-baseline experiments should play an important role in identifying the underlying neutrino oscillation mechanism.
The atmospheric neutrino anomaly [1] has been confirmed by the Super-Kamiokande experiment [2]. The observed up-down asymmetries of the detected muons indicate that, with the three known neutrino flavours, the most likely solution to this anomaly is \( \nu_\mu \rightarrow \nu_\tau \) oscillations, although significant mixing with \( \nu_e \) cannot yet be excluded [3].

The phenomenon of neutrino flavour oscillations was first proposed as a consequence of nondegenerate neutrino masses [4]. Although this mass mixing of weak eigenstates is the most likely mechanism for the observed atmospheric neutrino oscillations, other possible mechanisms have been proposed and, as stressed in Ref. [5], it is important to let experiments rather than our theoretical prejudice determine which is the correct mechanism. It is in this spirit that we undertake the present study.

The alternative neutrino oscillation mechanisms considered below share a common feature: each requires the existence of an interaction (other than the neutrino mass terms) that can mix neutrino flavours. In this paper we shall focus on such interactions which are mediated by a scalar \((J = 0)\), a vector \((J = 1)\), or a tensor \((J = 2)\) field. Assuming a two-neutrino mixing scheme, the \( \nu_\mu \rightarrow \nu_\tau \) transition probability for each of these possibilities may be parametrized as [6]

\[
P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta_J \sin^2(E_\nu^{J-1} L \delta_J),
\]

where \( E_\nu \) is the neutrino’s energy, \( L \) is the neutrino’s path length (i.e., the distance from where the neutrino is produced to the detector), \( \delta_J \) is a parameter specific to the neutrino oscillation mechanism, and \( \theta_J \) is the corresponding mixing angle. The subscript \( J \) in \( \theta_J \) and \( \delta_J \) is simply a label for the different mechanisms, and does not imply that the values of these parameters depend on the value of \( J \). Note the distinct energy dependence of the oscillation probability for each value of \( J \). This is the key for determining which neutrino oscillation mechanism is at work.

Proposed mechanisms for the \( J = 0 \) case include the mass mixing mechanism [1] for which the parameter \( \theta_0 \) in Eq.(1) represents the mixing angle between the neutrino flavour eigenstates and the mass eigenstates, and the parameter \( \delta_0 \) is given by
\[ \delta_0(\text{mass}) = \frac{\Delta m^2}{4} \equiv \frac{m_2^2 - m_1^2}{4}, \] (2)

where \( m_i \) are the masses of the neutrino mass eigenstates. Another possibility for the \( J = 0 \) case is neutrino oscillations induced by nonuniversal couplings of the neutrinos to a massless string dilaton for which \( \delta_0 \) is given by [7]

\[ \delta_0(\text{dilaton}) = \frac{1}{4} \left[ \Delta m^2 - 2\phi(\alpha_2 m_2^2 - \alpha_1 m_1^2) \right]. \] (3)

Here \( \alpha_i \) are the coupling strengths of different neutrino gravitational eigenstates to the dilaton field and it has been assumed that the neutrino mass eigenstates and the gravitational eigenstates are the same. The angle \( \theta_0 \) for this case is therefore the mixing angle between the flavour eigenstates and the gravitational eigenstates. Note that the nonuniversal neutrino-dilaton couplings constitute a violation of the principle of equivalence. The parameter \( \phi \) in Eq. (3) denotes the local Newtonian gravitational potential. It is assumed to be constant over the neutrino path length since the dominant contribution to \( \phi \) appears to come from the great attractor [8] which has been estimated to be [9]: \( |\phi| \sim 3 \times 10^{-5} \). With the constant \( \phi \) approximation, the energy dependence in the oscillation probability is the same for this mechanism as it is for the mass mixing mechanism. Neutrino oscillation experiments alone will not be able to distinguish these two possibilities. For the purposes of this paper, we shall refer to them collectively as the scalar mechanism. Note, however, that the mass mixing mechanism requires the neutrino masses to be nondegenerate whereas the dilaton induced oscillations can take place even if the masses are degenerate. We also observe that, if the neutrino-dilaton couplings are universal, i.e., if \( \alpha_1 = \alpha_2 \), and if \( \delta_0(\text{mass}) \neq 0 \), \( \delta_0(\text{dilaton}) \) and \( \delta_0(\text{mass}) \) differ only by a multiplicative constant.

As an example of the \( J = 2 \) case, consider neutrino oscillations induced by the equivalence principle violation that results from nonuniversal couplings of neutrinos to gravity [10]. In this case \( \delta_2 \) is given by [8]

\[ \delta_2(\text{VEP}) = |\phi|\Delta \gamma, \] (4)
where $\Delta \gamma$ measures the degree of violation of the equivalence principle and, as in Eq. (3), $\phi$ denotes the essentially constant local gravitational potential. With a constant $\phi$, this mechanism is phenomenologically identical \[11\] to the velocity oscillation mechanism that arises from a breakdown of Lorentz invariance \[12\] for which $\delta_2$ assumes the form

$$\delta_2(\text{velocity}) = \frac{\Delta v}{2},$$

(5)

where $\Delta v = v_2 - v_1$ is the difference between the velocities of two neutrino velocity eigenstates. We shall refer to these two mechanisms collectively as the tensor mechanism. In the former case, the angle $\theta_2$ corresponds to the mixing angle between the flavour eigenstates and the gravitational eigenstates, whereas in the latter case $\theta_2$ corresponds to the mixing angle between the flavour eigenstates and the velocity eigenstates. It should be emphasized that the tensor mechanism can lead to neutrino oscillations even if neutrinos are massless (or degenerate). These mechanisms were proposed not so much as a competing mechanism for the mass mixing mechanism, but to point out that neutrino oscillation experiments could be used as high-precision tests of the symmetry principles fundamental to the theories of general and special relativity.

We include the vector mechanism (the $J = 1$ case in Eq. (1)) in our phenomenological study. Although there are models which can yield energy-independent neutrino oscillations in matter \[13\] \[15\], we are not aware of any such model that can explain the atmospheric neutrino anomaly. The aim of our study is not to test any specific model, but rather to check if an energy-independent oscillation mechanism is compatible with the atmospheric neutrino data.

To test the three classes of neutrino oscillation mechanisms, we compare the measured values of

$$R \equiv \frac{(N_\mu/N_e)_{\text{data}}}{(N_\mu/N_e)_{\text{MC}}}$$

and the up-down asymmetry parameters

$$Y_\alpha^\eta \equiv \frac{(N_{\alpha^-}/N_{\alpha^+})_{\text{data}}}{(N_{\alpha^-}/N_{\alpha^+})_{\text{MC}}}, \quad \alpha = e, \mu$$

(7)
with the corresponding predictions of these mechanisms, assuming $\nu_\mu \rightarrow \nu_\tau$ transitions. Here $N_e$ and $N_\mu$ are the number of $e$-like and $\mu$-like events, respectively. $N_\alpha^\eta$ and $N_\alpha^{+\eta}$ are the number of $\alpha$-like events produced in the detector with zenith angle $\cos \Theta < -\eta$ and $\cos \Theta > \eta$, respectively. Note that $\Theta$ is defined to be negative for upward going directions and we have chosen $\eta = 0.2$ in our analysis. The calculational method is identical to that described in [16] and will not be reproduced here for environmental reasons.

Our results for the $R$’s and $Y$’s are displayed in Figures 1 and 2 together with the Super-Kamiokande results [2],

$$
R_{\text{sub} - \text{GeV}} = 0.63 \pm 0.03 \pm 0.05,
$$

$$
R_{\text{multi} - \text{GeV}} = 0.65 \pm 0.05 \pm 0.08,
$$

$$
Y_\mu^{0.2}_{\text{sub} - \text{GeV}} = 0.79 \pm 0.05,
$$

$$
Y_\mu^{0.2}_{\text{multi} - \text{GeV}} = 0.56 \pm 0.06.
$$

(8)

The experimental results we use correspond to 535 live days of running. For completeness we mention that the Super-Kamiokande results for the $e$-like up-down asymmetries are

$$
Y_e^{0.2}_{\text{sub} - \text{GeV}} = 1.14 \pm 0.07,
$$

$$
Y_e^{0.2}_{\text{multi} - \text{GeV}} = 0.92 \pm 0.12.
$$

(9)

Finally, note that only statistical errors are given for the up-down asymmetries since they should be much larger than possible systematic errors at the moment. We have not shown any figure for $Y_e$ - it is expected to be about 1 because $\nu_\mu \rightarrow \nu_\tau$ oscillations have almost no effect on the expected number of electron events.

From Figures 1 and 2 we see that all three mechanisms can fit the data for a range of the parameter $\delta J$. The combined $\chi^2$ fit to $Y_\mu, Y_e$ and $R$ yield the allowed regions for the neutrino oscillation parameters shown in Figure 3. The $\chi^2$ function is defined to be

$$
\chi^2_{\text{atm}} \equiv \chi^2(R) + \chi^2(Y),
$$

(10)

where
\[
\chi^2(R) \equiv \sum_E \left[ \left( \frac{R^{SK} - R^{th}}{\delta R^{SK}} \right)^2 \right]
\]  \hspace{1cm} (11)

and

\[
\chi^2(Y) \equiv \sum_E \left[ \left( \frac{Y^{SK}_{\mu} - Y^{th}_{\mu}}{\delta Y^{SK}_{\mu}} \right)^2 + \left( \frac{Y^{SK}_e - Y^{th}_e}{\delta Y^{SK}_e} \right)^2 \right].
\]  \hspace{1cm} (12)

The sum is over the sub-GeV and multi-GeV data samples. The measured Super-Kamiokande values and errors are denoted by the superscript “SK” and the theoretical predictions for the corresponding quantities are labelled by the superscript “th”. The minimal \( \chi^2 \) for the scalar, vector, and tensor mechanisms is 5.4, 5.7, and 10.4, respectively, for 4 degrees of freedom. All three mechanisms are consistent with the data, but the tensor mechanism has the worst \( \chi^2 \).

For check we have performed a second \( \chi^2 \) analysis using the SK \( \chi^2 \) function,

\[
\chi^2 = \frac{\alpha^2}{\sigma^2_{\alpha}} + \frac{\beta^2}{\sigma^2_{\beta}} + \chi^2_{\text{sub-GeV}} + \chi^2_{\text{multi-GeV}},
\]  \hspace{1cm} (13)

where

\[
\chi^2_{\text{sub-GeV}} = \sum_{\alpha=e,\mu} \sum_{a=1}^5 \frac{(y^\alpha_a - n^\alpha_a)^2}{n^\alpha_a}
\]  \hspace{1cm} (14)

and similarly for \( \chi^2_{\text{multi-GeV}} \). Here \( \alpha \) and \( \beta \) are the absolute and relative normalizations of the neutrino flux, \( y^\alpha_a \) are theoretical predictions for each zenith angle bin, \( n^\alpha_a \) are the experimental data, and we assume the uncertainties are given by \( \sigma_{\alpha} = \infty \) and \( \sigma_{\beta} = 0.12 \) (we assumed the larger value of \( \sigma_{\beta} \) given in the third reference of [2]). It is understood that \( y^\mu_a \) (\( \mu \)-like events) are multiplied by \((1 + \alpha)(1 + \beta/2)\) while \( y^e_a \) (\( e \)-like events) are multiplied by \((1 + \alpha)(1 - \beta/2)\), and \( \chi^2 \) is optimized with respect to \( \alpha \) and \( \beta \). With this alternative \( \chi^2 \), we also found that all three cases \( (J = 0, 1, 2) \) gave an acceptable fit to the data.

The basic reason that the Super-Kamiokande atmospheric neutrino data cannot distinguish these rather different oscillation probabilities is geometrical. Neutrinos from above typically travel quite short distances \((\lesssim 50 \text{ km})\) whereas neutrinos going up through the Earth travel quite long distances \((\gtrsim 5000 \text{ km})\). The atmospheric neutrino data can be
explained by assuming that neutrinos from above do not have time to oscillate while neutrinos travelling through the Earth experience averaged oscillations. In fact the data suggest that this occurs for both the sub-GeV and multi-GeV energy range (which is, very roughly, $0.3 \text{ GeV} \lesssim E_\nu \lesssim 6 \text{ GeV}$). Thus, while atmospheric neutrino experiments can sensitively test for oscillations through the zenith angle dependence (or up-down asymmetry) they are really only sensitive to averaged oscillations and are consequently not sensitive to the oscillation mechanism. The explicit energy dependence of the oscillation probability is only important in the region where the oscillations are significant and are not averaged. The proposed long-baseline experiments should be much more sensitive to the oscillation mechanism. The reason is that the neutrino path length is fixed and at a moderate distance (typically about 250 - 600 km). Provided that not all of the oscillations are averaged, and provided that at least some of the neutrinos have time to oscillate, then a comparison of the detected energy distribution with the expected energy distribution should distinguish the oscillation mechanisms.

In summary, we have examined three distinct oscillation mechanisms ($J = 0, 1, 2$ in Eq. (1)) and compared these with the current Super-Kamiokande atmospheric neutrino data. We have found that the current data are quite insensitive to the oscillation mechanism responsible for the atmospheric neutrino anomaly. To ultimately determine the underlying neutrino oscillation mechanism, it is important to do a more controlled experiment. This should be possible in the near future with the advent of long-baseline experiments.
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FIGURE CAPTIONS

Figure 1: \( R \) as a function of \( \delta_J \) (see Eq.(1)) for the tensor (bold solid line), vector (solid line), and scalar (dashed line) mechanisms. Maximal mixing has been assumed in each case. The dimensionless variable \( X \) is equal to \( \delta_2 \) GeV-km, \( \delta_1 \) km, and \( \frac{\delta_0}{1.27} \) GeV\(^{-1}\)-km, respectively for the tensor, vector, and scalar mechanisms (the choice of \( X \) for the mass mixing mechanism corresponds to the familiar \( \Delta m^2/eV^2 \)). The horizontal dashed lines correspond to Super-Kamiokande’s measured \( R \) value \( \pm 1\sigma \) error. Figure 1a is for the sub-GeV neutrinos and Figure 1b is for the multi-Gev neutrinos.

Figure 2: Same as Figure 1 except \( Y_\mu \) (see text) is plotted instead of \( R \).

Figure 3: The allowed region for the mixing parameter \( \theta_J \) and the oscillation parameter \( \delta_J \) obtained with the \( \chi^2 \) function defined in Eq.(10). Figure 3a, 3b, and 3c are for the scalar, vector, and tensor mechanisms, respectively.
Figure 1a

Graph showing the variation of $R_{\text{sub-GeV}}$ with $X$, where $R_{\text{sub-GeV}}$ is the survival probability for particles with energies below 1 GeV.
Figure 1b

Graph showing the relationship between $R$ (multi-GeV) and $X$. The graph displays multiple curves, each representing different data sets or conditions. The x-axis represents the range $10^{-5}$ to $10^{1}$, and the y-axis represents $R$ ranging from 0.40 to 1.00.
Figure 2a
Figure 2b
$\nu_{\mu} \leftrightarrow \nu_{\tau}$ with $R+Y$

Fig. 3a
$\nu_\mu \leftrightarrow \nu_\tau$ with $R+Y$

Fig. 3b
\( \nu_\mu \leftrightarrow \nu_\tau \) with \( R+Y \)

**Fig. 3c**