A nonlinear optimal control approach for the pulping process of paper mills

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Abstract
The mechanical pulping process is non-linear and multivariable. To solve the related control problem, the dynamic model of the pulping process undergoes first approximate linearization around a temporary operating point which is updated at each iteration of the control algorithm. The linearization process relies on first-order Taylor series expansion and on the computation of the Jacobian matrices of the state-space model of the pulping process. For the approximately linearized description of the pulping process, a stabilizing H-infinity feedback controller is designed. To compute the controller's feedback gains, an algebraic Riccati equation is solved at each time-step of the control method. The stability properties of the control scheme are proven through Lyapunov analysis.

1 | INTRODUCTION

The mechanical pulping process is among the most energy intensive processes in industry [1–5]. Paper is an important commodity, and the demand for paper of various types and quantity grows steadily [6–10]. The annual revenues of the paper industry are at the level of 500 billion USD, while the total produced quantity reaches the amount of 300 million tons. A pulp and paper mill with a production capacity of 300,000 tons per year has a construction cost of about 1 billion USD [6, 7]. Challenging objectives for the paper making industry are: (i) to minimize energy consumption, (ii) to reach specific quality features for the produced paper despite variability of the raw material, (iii) to minimize hazardous emissions and environmental risks [11–14]. Owing to the strongly non-linear and multivariable structure of the state-space model of the pulping process, control and stabilization of this process at setpoints that optimize the paper mills’ functioning is a non-trivial task [15–18]. So far, several approaches have been proposed for the control of the mechanical pulping process in the paper industry with the majority of them to consider linear models of the process. Often, there is need for further elaboration on the global stability properties of the related control schemes [19–22]. Energy efficiency and energy cost minimization in the paper industry remains a challenge which is directly related to the solution of the control problem for the mechanical pulping process in paper mills [23–26].

A non-linear optimal (H-infinity) control method is developed for the dynamic model of the mechanical pulping process in the paper industry [27–29]. First, the dynamic model of the pulping process undergoes approximate linearization around a temporary operating point which is updated at each iteration of the control algorithm. This operating point is defined by the present value of the process’s state vector and by the last value of the control inputs vector that was applied to the process. The linearization relies on first-order Taylor series expansion and on the computation of the related Jacobian matrices [30–32]. For the approximately linearized model of the pulping process an optimal (H-infinity) stabilizing feedback controller is defined. The modelling error which is due to the truncation of higher-order terms in the Taylor series expansion, is considered to be a
perturbation which is asymptotically compensated by the robustness of the control algorithm.

The proposed H-infinity controller provides a solution to the optimal control problem of the pulping process under model uncertainty and external perturbations. Actually, it represents the solution to a min-max differential game in which the controller tries to minimize a cost function that comprises a quadratic term of the state vector's tracking error, whereas the model uncertainty and external perturbation terms try to minimize this cost function. To compute the controller's feedback gains, an algebraic Riccati equation is solved at each time-step of the control method [33–35]. The stability properties of the control scheme are proven through Lyapunov analysis. First, it is shown that the control loop satisfies the H-infinity tracking performance criterion, which signifies elevated robustness against model uncertainty and external perturbations [27, 36, 37]. Moreover, under moderate conditions, the global asymptotic stability properties of the control method are proven. To perform feedback control without the need of measuring the entire state vector of the process, the H-infinity Kalman Filter is proposed as reliable state estimator. The proposed optimal (H-infinity) control method retains the advantages of typical linear optimal control, that is fast and accurate tracking of the reference setpoints, under moderate variations of the control inputs.

The article's control approach differs from all control methods that one could have considered for the non-linear model of the mechanical pulping process in paper mills. A large part of the competitive control methods is of known suboptimal performance (e.g. global linearization-based control methods, sliding-mode control, back-stepping control, and PID control) while the rest of the control methods are not of proven global stability (MPC, NMPC, or PID control). Besides, multiple models-based optimal control approaches require the definition of multiple operating points, and the solution of multiple Riccati equations or LMIs and this incurs a significant computation effort comparing to the article's non-linear optimal control approach. Finally, about LQR control, it can be stated that this method is completely unsuitable for use in the case of the non-linear mechanical pulping process of paper mills because of not performing compensation of the modelling errors of the linearization procedure and consequently because of lacking a global stability proof.

Proceeding results on the use of H-infinity control to non-linear dynamical systems were limited to the case of affine-in-the-input systems with drift-only dynamics and by considering that the control inputs gain matrix is not dependent of the values of the system's state vector. Moreover, in these approaches the linearization was performed around points of the desirable trajectory whereas in the present article's control method the linearization points are related with the value of the state vector at each sampling instant as well as with the last sampled value of the control inputs vector. The Riccati equation which has been proposed for computing the feedback gains of the controller is novel, so is the presented global stability proof through Lyapunov analysis. For the reasons that have already been explained, the presented non-linear optimal control method has improved performance when compared against other non-linear control schemes that one can consider for the dynamic model of the mechanical pulping process in paper mills (such as Lie algebra-based control, differential flatness theory-based control, Model-based Predictive Control, Non-linear Model-based Predictive Control, Sliding-mode control, Backstepping control, PID control and multiple linear models-based feedback control).

The structure of the article is as follows: in Section 2 the dynamic model of the mechanical pulping process in papermaking mills is formulated. In Section 3, approximate linearization is performed for the model of the pulping process and a stabilizing H-infinity feedback controller is developed. In Section 4, the global stability properties of the control scheme are proven through Lyapunov analysis. In Section 5, the H-infinity Kalman filter is introduced as a robust state estimator which allows for performing state-estimation-based control. In Section 6, the performance of the control method is further confirmed through simulation experiments. Finally, in Section 7 concluding remarks are stated.

2 | DYNAMIC MODEL OF THE MECHANICAL PULPING PROCESS IN PAPER MILLS

2.1 The state-space model of the pulping process

The multi-stage mechanical pulping process comprises the following stages [1–5]: (i) wood chip pre-treatment, (ii) wood chip refining, and (iii) pulp refining. In the wood chip pretreatment stage, wood chips are screened to remove over and under-sized particles and next they are heated and steamed at a temperature of about 100°C. In the wood chip refining stage, there are typically two high consistency refiners, the primary and the secondary one, where the refined wood material is exposed again to steam flow.

After pre-treatment, the wood chips are introduced into the inlet of the primary High Consistency (HC) refiner by the cylindrical chip transfer screw feeder. Dilution water is usually fed into the inlet of the refiner to control the consistencies of the refining zone. Finally, the wood chips end at being broken down into fibres as they pass through the two rotating discs of the refiners. The pressure exerted by the discs that rotate in opposite directions frees fibres from lignin, thus releasing the mass that will be turned into paper pulp.

The diagram of the mechanical pulping process in paper mills is shown in Figure 1. The following state variables are defined: $x_1$ is the production rate (tons/day), $x_2$ is the primary motor load (MW), $x_3$ is the primary consistency indicator (%), $x_4$ is the secondary motor load (MW), and $x_5$ is the secondary consistency (%). The control inputs of the pulping process are defined as follows: $u_1$ is the chip transfer screw speed (rpm), $x_2$ is the primary refiner plane gap (mm), $u_3$ is the secondary refiner plane gap (mm), and $u_5$ is the secondary dilution flow rate (kg/s).
The significance of the aforementioned state variables and control inputs is as follows [1–5]:

\( x_1 \): the production rate. The production rate is related with the wood chips feed-in rate. By changing the chip transfer screw speed, the production rate can be also modified. The production rate is also affected by parameters such as the quality of the wood species, the chip density and the moisture content. The production rate also affects the specific energy of the process and the pulp's quality.

\( x_2 \), \( x_3 \): Primary (secondary) electric motor's load. The motor's load is a parameter that significantly affects the quality of the produced paper's pulp. As noted, the motor's load is modified by changing the plate's gap. Actually, the motor's load exhibits high sensitivity to changes of the plate's distance.

\( u_1 \): the chip transfer screw speed. The chip transfer screw for the primary refiner is the main manipulated variable that controls the flow of chips from the preheater at the pretreatment stage to the inlet of the primary refiner. Changes to the screw speed affect the flow of dry fibres to the refiner. The variable is also used to modify the pulp's production rate.

\( u_2 \), \( u_3 \): primary (secondary) refiner plate gap. The plate gap is the distance between the plates of a refiner, that is of the two discs that squeeze wood chips while rotating in opposite directions. The plate gap is controlled by a mechanical load system. Variations in gap size affect the mechanical force exerted by the plates on the wood chips and thus they also affect the loads of the two electric motors.

\( u_4 \), \( u_5 \): primary (secondary) dilution flow rate. The consistency of the refining zone affects the pulp's properties. The added water modifies this consistency and finally changes the pulp's quality. Large variations in the dilution matter flow rate can even result into an unstable refining process. Thus the incoming water's flow at the refiners is among the control variables that determine the quality of the paper's pulp.

A description of the mechanical pulping process has been first formulated in discrete-time form [1, 21]:

\[
\begin{align*}
  x_1(k+1) &= a_1 x_1(k) + b_1 k_p x_5 u_1(k) \\
  x_2(k+1) &= a_2 x_2(k) + b_2 \frac{k_m x_1(k)}{u_3(k)} (1 - e^{-10 u_1(k)}) (c_1 - e_1 u_3(k)) \\
  x_3(k+1) &= a_3 x_3(k) + b_3 \frac{100 x_1(k)}{x_1(k) + k_x u_3(k) - k_c x_3(k)} \\
  x_4(k+1) &= a_4 x_4(k) + b_4 \frac{k_m x_1(k)}{u_3(k)} (1 - e^{-10 u_1(k)}) (c_2 - e_2 u_4(k)) \\
  x_5(k+1) &= a_5 x_5(k) + b_5 \frac{x_1(k)}{0.01 x_3(k)} + k_d u_5(k) - k_c x_4(k)
\end{align*}
\]

By using the first-order Euler approximation to describe the time-derivative and by defining the following normalized coefficients \( \tilde{a}_1 = \frac{a_1 - 1}{T_s} \), \( \tilde{b}_1 = \frac{b_1}{T_s} \), \( \tilde{a}_2 = \frac{a_2 - 1}{T_s} \), \( \tilde{b}_2 = \frac{b_2}{T_s} \), \( \tilde{a}_3 = \frac{a_3 - 1}{T_s} \), \( \tilde{b}_3 = \frac{b_3}{T_s} \), \( \tilde{a}_4 = \frac{a_4 - 1}{T_s} \), \( \tilde{b}_4 = \frac{b_4}{T_s} \), and \( \tilde{a}_5 = \frac{a_5 - 1}{T_s} \), \( \tilde{b}_5 = \frac{b_5}{T_s} \), one can obtain a continuous-time state-space description of the dynamics of the mechanical pulping process in paper mills:

\[
\begin{align*}
  x_1(t) &= \tilde{a}_1 x_1(t) + \tilde{b}_1 k_p x_5 u_1(t) \\
  x_2(t) &= \tilde{a}_2 x_2(t) + \tilde{b}_2 \frac{k_m x_1(t)}{u_3(t)} (1 - e^{-10 u_1(t)}) (c_1 - e_1 u_3(t)) \\
  x_3(t) &= \tilde{a}_3 x_3(t) + \tilde{b}_3 \frac{100 x_1(t)}{x_1(t) + k_x u_3(t) - k_c x_3(t)} \\
  x_4(t) &= \tilde{a}_4 x_4(t) + \tilde{b}_4 \frac{k_m x_1(t)}{u_3(t)} (1 - e^{-10 u_1(t)}) (c_2 - e_2 u_4(t)) \\
  x_5(t) &= \tilde{a}_5 x_5(t) + \tilde{b}_5 \frac{x_1(t)}{0.01 x_3(t)} + k_d u_5(t) - k_c x_4(t)
\end{align*}
\]

It is assumed that the control inputs \( u_1 \) (primary dilution flow rate) and \( u_5 \) (secondary dilution flow rate) are kept constant. Moreover, the following parameters and control inputs variables are considered in the individual rows of the state-space model: In the first row one defines \( \tilde{b}_1 k_p x_5 u_1 = \tilde{b}_1, \tilde{a}_1 = \tilde{a}_1 \) and \( u_1 = v_1 \). In the second row of the state-space model one defines \( \tilde{b}_2 k_m x_1 u_3 = \tilde{b}_2, e^{-10 u_1} = v_2, \) and \( \tilde{a}_2 = \tilde{a}_2 \). In the third row of the state-space model one defines \( \tilde{b}_3 x_1 u_3 = \tilde{b}_3, k_x u_3 = q_1 \) (constant) and \( \tilde{a}_3 = \tilde{a}_3 \). In the fourth row of the state-space model one defines \( \tilde{b}_4 k_m x_1 u_3 = \tilde{b}_4, (1 - e^{-10 u_1}) (c_2 - e_2 u_4) = \tilde{v}_3 \) where \( v_3 = c_2 e^{-10 u_1} - e^{-10 u_1} e_2 u_4, \) and \( \tilde{a}_4 = \tilde{a}_4 \). Finally, in the fifth row of the state-space model one defines \( \tilde{b}_5 = \tilde{b}_5 \),
\( q_3 = 0.01 k_5 \mu_5 \) (constant), \( q_1 = 0.01 \) and \( \bar{a}_5 = \bar{a}_s \). Using the previous definitions, the state-space model of the pulping process can be written as

\[
\begin{aligned}
\dot{x}_1 &= \bar{a}_1 x_1 + \bar{b}_1 v_1 \\
\dot{x}_2 &= \bar{a}_2 x_2 + \bar{b}_2 (1 - v_2) \\
\dot{x}_3 &= \bar{a}_3 x_3 + \bar{b}_3 (x_1 + q_1 - k_p x_2) \\
\dot{x}_4 &= \bar{a}_4 x_4 + \bar{b}_4 (c_2 - v_3) \\
\dot{x}_5 &= \bar{a}_5 x_5 + \bar{b}_5 (x_1 + q_2 x_3 - q_3 k_c x_3 x_4)
\end{aligned}
\]

(3)

By defining \( x \in R^{5 \times 1}, u = [v_1, v_2, v_3]^T \in R^{3 \times 1} f(x) \in R^{5 \times 1} \) and \( g(x) \in R^{5 \times 3} \) the dynamic model of the process is written in the following concise form:

\[
\dot{x} = f(x) + g(x)u
\]

(4)

where vector \( f(x) \) and matrix \( g(x) \) are given by

\[
f(x) = \begin{pmatrix}
\bar{a}_1 x_1 \\
\bar{a}_2 x_2 + \bar{b}_2 x_1 \\
\bar{a}_3 x_3 + \bar{b}_3 (x_1 + q_1 - k_p x_2) \\
\bar{a}_4 x_4 + \bar{b}_4 x_1 c_2 \\
\bar{a}_5 x_5 + \bar{b}_5 (x_1 + q_2 x_3 - q_3 k_c x_3 x_4)
\end{pmatrix}
\]

\[
g(x) = \begin{pmatrix}
b_1 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{pmatrix}
\]

(5)

2.2 Parameters of the mechanical pulping process

Next, main parameters of the mechanical pulping process are overviewed [1–5].

(a) The production rate \( P \) is measured in tons/day and is given by

\[
P = k_p s_c d_c R
\]

(6)

where \( k_p \) and \( k_s \) (m³/rpm) are constant parameters with values depending on the specific production units. Besides, \( s_c(\%) \) is the chip solid content, \( d_c \) (kg/m³) is the chip bulk density and \( R \) (rpm) is the chip transfer screw speed.

(b) The motor load \( (\text{MW}) \) is defined as

\[
M = \frac{k_m P}{D_e} (1 - e^{-10G_i})(c_i - e_i G_i)
\]

(7)

where \( i = \{p, s\} \) with \( p \) to stand for primary and \( s \) to stand for secondary. It holds that \( M(MW) \) is the motor's load at the \( i \)-th refiner, \( D \) (l/min) is the dilution mater flow rate, \( G_i \) (mm) is the gap distance, and \( c_i, e_i \) and \( k_m \) are parameters of each refiner.

(c) The consistency of the refiners is defined as

\[
C_p = \frac{100P}{k_4 D_p - k_5 M_p}
\]

(8)

\[
C_S = \frac{100P}{0.01C_p + k_4 D_s - k_6 M_s}
\]

(9)

where \( C_p \) and \( C_s \) denote the consistency for the primary and the secondary refiner, and \( k_4, k_5 \) and \( k_6 \) are the refiner parameters.

(d) The specific energy \( (\text{MW/tons/day}) \) is the energy consumed per ton of dry pulp, which is defined as the ratio between the motor load and the production rate. For the \( i \)-th refiner and motor the specific energy (SE) is defined as

\[
SE = \frac{\text{Motor load } i}{\text{Production rate}}
\]

One can also define the total specific energy (TSE)

\[
TSE = \frac{\text{Total Motor load}}{\text{Production rate}}
\]

(11)

2.3 Quality indicators in the mechanical pulping process

At present, the widely used technological indicators for evaluating pulp quality are the Canadian Standard Freeness (CSF), the long fibre content and the shive content. CSF is closely associated with the energy consumption of the pulping process. Another quality indicator is the pulp consistency which stands for the ratio of the mass of dry fibres over the sum of the total mass of dry fibres and of the water suspension. When this ratio is between 20% and 50% the pulp is considered to be of high consistency, while when it ranges between 3% and 5% the pulp is considered to be low consistency [1–5].

In general, wood consists of cellulose ~42%, hemicellulose ~27%, lignin ~28%, and extractives ~3%. Lignin is a glue-like substance binding cellulose and semi-cellulose fibres throughout the wood’s cellular structure. Pulp’s quality is also determined by the so-called kappa number \( \kappa \) which indicates the residual lignin content in the wood pulp. The purpose of
3 | APPROXIMATE LINEARIZATION OF THE STATE-SPACE MODEL OF THE PULPING PROCESS

3.1 | Approximately linearized model

The dynamic model of the pulping process of Equation (4) undergoes approximate linearization around the temporary operating point (equilibrium) \( (x^*, u^*) \), where \( x^* \) is the present value of the process's state vector and \( u^* \) is the last value of the control inputs vector that was applied to the process. The linearization procedure gives

\[
\dot{x} = Ax + Bu + \tilde{d}
\]  

(12)

where \( \tilde{d} \) is the cumulative disturbances vector which comprises (i) the modeling error due to the truncation of higher order terms in the Taylor series expansion, (ii) exogenous perturbations, and (iii) sensors noise of any distribution. After linearization around its current operating point, the dynamic model of the mechanical pulping process is written as

\[
\dot{x} = Ax + Bu + d_1
\]  

(15)

Parameter \( d_1 \) stands for the linearization error in the pulping process's dynamic model appearing previously in Equation (15). The reference setpoints for the pulping process's state vector are denoted by \( x_a = [x_1^d, ..., x_5^d] \). Tracking of this trajectory is achieved after applying the control input \( u^* \). At every time instant the control input \( u^* \) is assumed to differ from the control input \( u \) appearing in Equation (15) by an amount equal to \( \Delta u \), that is \( u^* = u + \Delta u \)

\[
\dot{x}_d = Ax_d + Bu^* + d_2
\]  

(16)

The dynamics of the controlled system described in Equation (15) can be also written as

\[
\dot{x} = Ax + Bu + Bu^* - Bu^* + d_1
\]  

(17)

and by denoting \( d_3 = -Bu^* + d_1 \) as an aggregate disturbance term one obtains

\[
\dot{x} = Ax + Bu + Bu^* + d_3
\]  

(18)

By subtracting Equation (16) from Equation (18) one has

\[
\dot{x} - \dot{x}_d = A(x - x_d) + Bu + d_3 - d_2
\]  

(19)

By denoting the tracking error as \( e = x - x_d \) and the aggregate disturbance term as \( d = d_3 - d_2 \), the tracking error dynamics becomes

\[
\dot{e} = Ae + Bu + \tilde{d}
\]  

(20)
For the approximately linearized model of the system a stabilizing feedback controller is developed. The controller has the form

\[ u(t) = -K e(t) \]  

(21)

with \( K = \frac{1}{2} B^T P \) where \( P \) is a positive definite symmetric matrix which is obtained from the solution of the Riccati equation [27].

\[ A^T P + PA + Q - P \left( \frac{2}{r} BB^T - \frac{1}{\rho^2} LL^T \right) P = 0 \]  

(22)

where \( Q \) is a positive semi-definite symmetric matrix. The diagram of the considered control loop is depicted in Figure 2.

4 | LYAPUNOV STABILITY ANALYSIS

Through Lyapunov stability analysis it will be shown that the proposed non-linear control scheme assures \( H_\infty \) tracking performance for the pulping process, and that under moderate conditions about the disturbance terms, asymptotic convergence to the reference setpoints (global stability) is achieved. As shown before, the tracking error dynamics for the pulping process is written in the form

\[ \dot{e} = Ae + Bu + \tilde{L}d \]  

(23)

where in the pulping process case \( L = I \in \mathbb{R}^{5 \times 5} \) with \( I \) being the identity matrix. Variable \( d \) denotes model uncertainties and external disturbances of the pulping process’s model. The following Lyapunov equation is considered

\[ V = \frac{1}{2} e^T Pe \]  

(24)

where \( e = x - x_d \) is the tracking error. By differentiating with respect to time one obtains

\[ \dot{V} = \frac{1}{2} e^T P \dot{e} + \frac{1}{2} e \dot{e} \Rightarrow \]

\[ \dot{V} = \frac{1}{2} [Ae + Bu + \tilde{L}d]^T P + \frac{1}{2} e^T P[Ae + Bu + \tilde{L}d] \Rightarrow \]

\[ \dot{V} = \frac{1}{2} [e^T A^T + u^T B^T + \tilde{d}^T L^T] P e + \frac{1}{2} e^T P[Ae + Bu + \tilde{L}d] \Rightarrow \]

\[ \dot{V} = \frac{1}{2} e^T A^T Pe + \frac{1}{2} u^T B^T Pe + \frac{1}{2} \tilde{d}^T L^T Pe + \frac{1}{2} e^T PAe + \frac{1}{2} e^T PBu + \frac{1}{2} e^T P\tilde{L}d \]  

(27)

The previous equation is rewritten as

\[ \dot{V} = \frac{1}{2} e^T (A^T P + PA)e + \left( \frac{1}{2} u^T B^T Pe + \frac{1}{2} e^T PBu \right) + \left( \frac{1}{2} \tilde{d}^T L^T Pe + \frac{1}{2} e^T P\tilde{L}d \right) \]

(28)

\[ A^T P + PA = -Q + P \left( \frac{2}{r} BB^T - \frac{1}{\rho^2} LL^T \right) P \]  

(29)

**Assumption 1** For given positive definite matrix \( Q \) and coefficients \( r \) and \( \rho \) there exists a positive definite matrix \( P \), which is the solution of the following matrix equation:

Moreover, the following feedback control law is applied to the system

\[ u = -\frac{1}{r} B^T Pe \]  

(30)

By substituting Equations (29) and (30), one obtains

\[ \dot{V} = \frac{1}{2} e^T \left[ -Q + P \left( \frac{2}{r} BB^T - \frac{1}{\rho^2} LL^T \right) P \right] e + e^T PB \left( \frac{1}{r} B^T Pe \right) + e^T P\tilde{L}d \Rightarrow \]

\[ \dot{V} = \frac{1}{2} e^T \left( \frac{1}{r} e^T PBB^T Pe - \frac{1}{2\rho^2} e^T PLLL^T Pe \right) - \frac{1}{r} e^T PBB^T Pe + e^T P\tilde{L}d \]  

(32)

which after intermediate operations gives

\[ \dot{V} = -\frac{1}{2} e^T Qe - \frac{1}{2\rho^2} e^T PLLL^T Pe + e^T P\tilde{L}d \]  

(33)
or, equivalently
\[
\dot{V} = -\frac{1}{2} e^T Q e - \frac{1}{2\rho^2} e^T P L L^T P e +\]
\[+ \frac{1}{2} e^T P L \tilde{d} + \frac{1}{2} \tilde{d}^T L^T P e \tag{34}
\]

**Lemma** The following inequality holds:
\[
\frac{1}{2} e^T P L \tilde{d} + \frac{1}{2} \tilde{d}^T L^T P e - \frac{1}{2\rho^2} e^T P L L^T P e \leq \frac{1}{2} \rho^2 \tilde{d}^T \tilde{d} \tag{35}
\]

**Proof** The binomial \((\rho \alpha - \frac{1}{\rho} b)^2\) is considered. Expanding the left part of the above inequality one gets:
\[
\rho^2 \alpha^2 + \frac{1}{\rho^2} b^2 - 2 ab \geq 0 \Rightarrow \frac{1}{2} \rho^2 \alpha^2 + \frac{1}{2\rho^2} b^2 - ab \geq 0 \Rightarrow \]
\[
ab - \frac{1}{2\rho^2} b^2 \leq \frac{1}{2} \rho^2 \alpha^2 \Rightarrow \frac{1}{2} ab + \frac{1}{2\rho^2} b^2 \leq \frac{1}{2} \rho^2 \alpha^2 \tag{36}
\]

The following substitutions are carried out: \(a = \tilde{d}\) and \(b = e^T P L\) and the previous relation becomes
\[
\frac{1}{2} \tilde{d}^T L^T P e + \frac{1}{2} e^T P L \tilde{d} - \frac{1}{2\rho^2} e^T P L L^T P e \leq \frac{1}{2} \rho^2 \tilde{d}^T \tilde{d} \tag{37}
\]

Equation (37) is substituted in Equation (34) and the inequality is enforced, thus giving
\[
\dot{V} \leq -\frac{1}{2} e^T Q e + \frac{1}{2} \rho^2 \tilde{d}^T \tilde{d} \tag{38}
\]

Equation (38) shows that the \(H_\infty\) tracking performance criterion is satisfied. The integration of \(V\) from 0 to \(T\) gives
\[
\int_0^T \dot{V}(t) dt \leq -\frac{1}{2} \int_0^T \| e \|_Q^2 dt + \frac{1}{2} \rho^2 \int_0^T \| \tilde{d} \|^2 dt \Rightarrow \]
\[
2V(T) + \int_0^T \| e \|_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T \| \tilde{d} \|^2 dt \tag{39}
\]

Moreover, if there exists a positive constant \(M_d > 0\) such that
\[
\int_0^\infty \| \tilde{d} \|^2 dt \leq M_d \tag{40}
\]
then one gets
\[
\int_0^\infty \| e \|_Q^2 dt \leq 2V(0) + \rho^2 M_d \tag{41}
\]

Thus, the integral \(\int_0^\infty \| e \|_Q^2 dt\) is bounded. Moreover, \(V(T)\) is bounded and from the definition of the Lyapunov function \(V\) in Equation (24), it becomes clear that \(e(t)\) will be also bounded since \(e(t) \in \Omega_e = \{ e | e^T P e \leq 2V(0) + \rho^2 M_d \} \).

According to the above and with the use of Barbalat's Lemma, one obtains \(\lim_{t \to \infty} e(t) = 0\).

The outline of the global stability proof is that at each iteration of the control algorithm the state vector of the pulping process converges towards the temporary equilibrium and the temporary equilibrium in turn converges towards the reference trajectory [27]. Thus, the control scheme exhibits global asymptotic stability properties and not local stability. Assume the i-th iteration of the control algorithm and the i-th time interval about which a positive definite symmetric matrix \(P\) is obtained from the solution of the Riccati equation appearing in Equation (29). By following the stages of the stability proof one arrives at Equation (38) which shows that the \(H\)-infinity tracking performance criterion holds. By selecting the attenuation coefficient \(\rho\) to be sufficiently small and in particular to satisfy \(\rho^2 < \| e \|_Q / \| \tilde{d} \|^2\) one has that the first derivative of the Lyapunov function is upper bounded by 0. Therefore, for the i-th time interval it is proven that the Lyapunov function defined in Equation (24) is a decreasing one. This signifies that between the beginning and the end of the i-th time interval there will be a drop of the value of the Lyapunov function and since matrix \(P\) is a positive definite one, the only way for this to happen is the Euclidean norm of the state vector error \(e\) to be decreasing. This means that comparing to the beginning of each time interval, the distance of the state vector error from zero at the end of the time interval has diminished. Consequently as the iterations of the control algorithm advance, the tracking error will approach zero, and this is a global asymptotic stability condition.

## 5 | ROBUST STATE ESTIMATION WITH THE USE OF THE \(H_\infty\) KALMAN FILTER

The control loop has to be implemented with the use of information provided by a small number of sensors and by processing only a small number of state variables. To reconstruct the missing information about the state vector of the mechanical pulping process in paper mills it is proposed to use a filtering scheme and based on it to apply state estimation-based control [27, 34]. The recursion of the \(H_\infty\) Kalman Filter, for the model of the pulping process, can be formulated in terms of a measurement update and a time update part.

**Measurement update:**
\[
D(k) = [I - \theta W(k) P^-(k) + C^T(k) R(k)^{-1} C(k) P^-(k)]^{-1}
\]
\[
K(k) = P^-(k) D(k) C^T(k) R(k)^{-1}
\]
\[
\hat{x}(k) = \hat{x}^-(k) + K(k) [y(k) - C \hat{x}^-(k)]
\]

\[
\hat{x}^-(k+1) = A(k) \hat{x}(k) + B(k) u(k)
\]
\[
P^-(k+1) = A(k) P^-(k) A^T(k) + Q(k)
\]

where it is assumed that parameter \(\theta\) is sufficiently small to assure that the covariance matrix \(P^-(k)^{-1} - \theta W(k) + C^T(k) R(k)^{-1} C(k)\) will be positive definite. When \(\theta = 0\) the \(H_\infty\) Kalman Filter becomes equivalent to the standard Kalman
Filter. One can measure only a part of the state vector of the pulping process (state variables \(x_1, x_2, x_3\)), and can estimate through filtering the rest of the state vector elements (state variables \(x_4, x_5\)). Moreover, the proposed Kalman filtering method can be used for sensor fusion purposes.

To elaborate on the matrices which appear in the Measurement update part and in the Time update part of the H-infinity Kalman Filter, the following can be noted: Matrix \(R(k) \in R^{3 \times 3}\) is the measurement noise covariance matrix, that is the covariance matrix of the measurement error vector of the system. Matrix \(P^*(k) \in R^{5 \times 5}\) is a priori state vector estimation error covariance matrix of the system, that is the covariance matrix of the state vector estimation error prior to receiving the updated measurement of the system’s outputs. Matrix \(W(k) \in R^{5 \times 5}\) is a weight matrix which defines the significance to be attributed by the H-infinity Kalman Filter in minimizing the state vector’s estimation error, relatively to the effects that the noise affecting the system may have. Actually, \(W\) is a diagonal matrix and the elements on its diagonal are given the value \(10^{-3}\). Finally, matrix \(D(k) \in R^{5 \times 5}\) stands for a modified a posteriori state vector estimation error covariance matrix, that is the covariance matrix of the state vector estimation error after receiving the updated measurement of the system’s outputs. Besides, about the process noise covariance matrix \(Q \in R^{5 \times 5}\) that appears in the filter’s Time-Update part it holds that this is also a diagonal matrix and the elements on its diagonal are given the value \(10^{-4}\). Conclusively, the H-infinity Kalman Filter retains the structure of the typical Kalman Filter, that is a recursion in discrete time comprising a Time update part (computation of variables prior to receiving measurements) and a Measurement update part (computation of variables after measurements have been received). There is a modified a posteriori state vector estimation error covariance matrix, which in turn takes into account a weight matrix that defines the accuracy of the state estimation under the effects of elevated noise.

6 | SIMULATION TESTS

The performance of the proposed non-linear optimal control method for the mechanical pulping process in paper industry is tested through simulation experiments. To compute the feedback gains of the controller, the algebraic Riccati equation appearing in Equation (29) is iteratively solved at each time-step of the control method. The parameters of the state-space model of the pulping process were according to Reference [1, 21]. Considering a sampling period of \(T_s = 0.01s\), indicative values for these parameters have been \(\bar{a}_1 = \frac{a_{11}}{T_s} = -0.6, \bar{b}_1 = \frac{b_{11}}{T_s} = 0.1, \bar{a}_2 = \frac{a_{21}}{T_s} = -0.6, \bar{b}_2 = \frac{b_{21}}{T_s} = 0.1, \bar{a}_3 = \frac{a_{31}}{T_s} = -0.6, \bar{b}_3 = \frac{b_{31}}{T_s} = 0.1, \bar{a}_4 = \frac{a_{41}}{T_s} = -0.6, \bar{b}_4 = \frac{b_{41}}{T_s} = 0.1, \bar{a}_5 = \frac{a_{51}}{T_s} = -0.6, \bar{b}_5 = \frac{b_{51}}{T_s} = 0.1\).

In the present simulation test the following values were given to the parameters \(r, \rho\) and \(Q\) which appear in the Riccati equation of the control method: \(r = 0.001, \rho = 0.1\) and \(Q = 40I_{5 \times 5}\) while \(L_m = 10^{-7}I_{5 \times 5}\). There is a rationale in the selection of the values of these parameters. At each sampling period one has to confirm the existence of a solution for the algebraic Riccati equation of the control method, and to assure that the selection of parameters \(r, \rho\) and \(Q\) enables a positive definite matrix \(P\) to be computed from this Riccati equation. The attenuation coefficient \(\rho\) can be given a small value up to the point that the method’s algebraic Riccati equation still returns a valid solution in the form of the positive definite and symmetric matrix \(P\). According to the relation about the worst case disturbance that the non-linear optimal control loop can sustain one has \(d = \frac{1}{\rho}L^TPe\); therefore, the smallest value of \(\rho\)
for which a valid solution of the method's algebraic Riccati equation can be obtained (in the form of a positive definite and symmetric matrix $P$) is the one that provides maximum robustness to the control loop. On the other side, relatively small values of parameter $r$ result into elimination of the state vector's tracking error, while relatively large values of the diagonal elements of matrix $Q$ enable fast convergence of the system's state variables to their reference setpoints.

The obtained results are shown in Figures 3–8. The real values of the state variables of the process are printed in blue,
the estimated values which are provided by the H-infinity Kalman Filter are plotted in green, while the related setpoints are plotted in red. The values of the control inputs are shown in Figures 9–11. It can be noticed, that under the proposed non-linear optimal control method, precise tracking of the reference setpoints was achieved. The state variables of the system converged fast to the targeted values while the variations of the control inputs remained moderate. Since the control algorithm provides the values of the control variables $u_i$, $i = 1, 2, 3$ one has to solve also a set of three algebraic equations given in Section 2 to find finally the initial control variables $u_{i0}, i = 1, 2, 3$. By using the H-infinity Kalman Filter as a state estimator it was possible to implement state estimation-based control through the processing of the
measurable state variable, that is the production rate $x_1$, the motor's load at the first refiner $x_2$ and the motor's load at the second refiner $x_4$.

To elaborate on the results of the proposed non-linear optimal control method, three new tables are given next: (i) Table 1 which shows the accuracy of tracking of setpoints by the state variables of the mechanical pulping process in paper mills under exact parameters of the state-space model of the process, (ii) Table 2 which shows the accuracy of tracking of setpoints by the state variables of the mechanical pulping process in paper mills under parametric uncertainty in the state-space model of process (actually a $\Delta a\%$ change has been considered about parameter $\tilde{a}_1$ in the state-space model of process).
the system), and (iii) Table 3 which shows the accuracy of state estimation which is achieved by the H-infinity Kalman Filter.

The following points outline the advantages of the proposed non-linear optimal control method against other control schemes that one could have considered for the mechanical pulping process in paper mills: (1) unlike global linearization-based control approaches, such as Lie algebra-based control and differential flatness theory-based control, the optimal control approach does not rely on complicated transformations (diffeomorphisms) of the system's state variables. Besides, the computed control inputs are applied directly on the initial non-linear model of the process and not on its linearized equivalent. The inverse transformations which are met in global linearization-based control are avoided and consequently one does not come against the related singularity

**FIGURE 10** (a) Non-linear optimal control of the mechanical pulping process in the paper industry: variations of the control inputs $v_1$ to $v_3$ when tracking setpoint 3, (b) variations of the control inputs $v_1$ to $v_3$ when tracking setpoint 4

**FIGURE 11** (a) Non-linear optimal control of the mechanical pulping process in the paper industry: variations of the control inputs $v_1$ to $v_3$ when tracking setpoint 5, (b) variations of the control inputs $v_1$ to $v_3$ when tracking setpoint 6
sliding mode control can be an intuitive procedure. About
found in the input control it is known that when the controlled system is not
the system to be found in a specific form. About sliding
control method does not require the state
mode control and backstepping control the proposed optimal
values selection and consequently the global stability of this
search for an optimum depends on initialization and parameter
will be lost. Besides, in NMPC the convergence of its iterative
dynamics of the pulping process the stability of the control loop
linear control approach that if applied to the non-linear
models-based control the non-linear optimal control method uses
only one linearization point and needs the solution of only
one Riccati equation so as to compute the stabilizing feedback
gains of the controller. Consequently, in terms of computation
to the proposed control method is much more efficient.

### TABLE 1 RMSE × 10⁻³ of the pulping process in the disturbance-free case

| Test # | RMSE₁₀⁻³ | RMSE₂₀⁻³ | RMSE₃₀⁻³ | RMSE₄₀⁻³ | RMSE₅₀⁻³ |
|--------|-----------|-----------|-----------|-----------|-----------|
| No 1   | 0.6677    | 0.3778    | 0.2990    | 0.3129    | 0.0316    |
| No 2   | 0.8004    | 0.3912    | 0.2208    | 0.3369    | 0.0166    |
| No 3   | 0.0029    | 0.0017    | 0.0002    | 0.0014    | 0.0001    |
| No 4   | 0.5993    | 0.3389    | 0.6586    | 0.2600    | 0.0255    |
| No 5   | 0.6052    | 0.3283    | 0.1087    | 0.2904    | 0.1115    |
| No 6   | 0.0230    | 0.0100    | 0.0013    | 0.0007    | 0.0002    |
| No 7   | 0.4223    | 0.2964    | 0.3265    | 0.2331    | 0.0230    |

### TABLE 2 RMSE × 10⁻³ of the pulping process under disturbances

| Δa%   | RMSE₁₀⁻³ | RMSE₂₀⁻³ | RMSE₃₀⁻³ | RMSE₄₀⁻³ | RMSE₅₀⁻³ |
|-------|-----------|-----------|-----------|-----------|-----------|
| 0%    | 0.5991    | 0.3389    | 0.6585    | 0.2600    | 0.0235    |
| 10%   | 0.6576    | 0.3410    | 0.6648    | 0.2602    | 0.0240    |
| 20%   | 0.7160    | 0.3410    | 0.6711    | 0.2604    | 0.0245    |
| 30%   | 0.7743    | 0.3451    | 0.6774    | 0.2605    | 0.0250    |
| 40%   | 0.8324    | 0.3476    | 0.6836    | 0.2607    | 0.0256    |
| 50%   | 0.8904    | 0.3492    | 0.6908    | 0.2609    | 0.0262    |
| 60%   | 0.9482    | 0.3512    | 0.7048    | 0.2609    | 0.0847    |

### TABLE 3 RMSE × 10⁻³ of the Kalman Filter's estimation error

| Test # | RMSE₁₀⁻³ | RMSE₂₀⁻³ | RMSE₃₀⁻³ | RMSE₄₀⁻³ | RMSE₅₀⁻³ |
|--------|-----------|-----------|-----------|-----------|-----------|
| No 1   | 0.0003    | 0.0003    | 0.3330    | 0.0003    | 0.0528    |
| No 2   | 0.0003    | 0.0003    | 0.2382    | 0.0003    | 0.0338    |
| No 3   | 0.0003    | 0.0003    | 0.3748    | 0.0003    | 0.0584    |
| No 4   | 0.0003    | 0.0003    | 0.6441    | 0.0003    | 0.0889    |
| No 6   | 0.0003    | 0.0003    | 0.2516    | 0.0003    | 0.0582    |
| No 6   | 0.0003    | 0.0003    | 0.9219    | 0.0003    | 0.0106    |
| No 7   | 0.0003    | 0.0003    | 0.2522    | 0.0003    | 0.0356    |

problems. (2) unlike Model Predictive control (MPC) and Non-linear Model Predictive control (NMPC), the proposed control method is of proven global stability. It is known that MPC is a linear control approach that if applied to the non-linear dynamics of the pulping process the stability of the control loop will be lost. Besides, in NMPC the convergence of its iterative search for an optimum depends on initialization and parameter values selection and consequently the global stability of this control method cannot be always assured. (3) unlike sliding mode control and backstepping control the proposed optimal control method does not require the state-space description of the system to be found in a specific form. About sliding-mode control it is known that when the controlled system is not found in the input-output linearized form the definition of the sliding mode control can be an intuitive procedure. About backstepping control it is known that it cannot be directly applied to a dynamical system if the related state-space model is not found in the triangular (backstepping integral) form (4) unlike PID control, the proposed non-linear optimal control method is of proven global stability, the selection of the controller’s parameters does not rely on a heuristic tuning procedure, and the stability of the control loop is assured in the case of changes of operating points, (5) unlike multiple local models-based control the non-linear optimal control method uses only one linearization point and needs the solution of only one Riccati equation so as to compute the stabilizing feedback gains of the controller. Consequently, in terms of computation load the proposed control method is much more efficient.

### 7 CONCLUSIONS

Owing to the non-linear and multivariable structure of the state-space model of the mechanical pulping process in the paper industry, the solution to the related control problem is a non-trivial task. A new non-linear optimal (H-infinity) control method is proposed for this industrial process. To this end, the dynamic model of the process undergoes approximate linearization around a temporary operating point which is recomputed at each time-step of the control algorithm. The linearization procedure makes use of the first-order Taylor series expansion and of the computation of the related Jacobian matrices.

For the approximately linearized model of the system, a stabilizing H-infinity feedback controller is designed. To compute the feedback gains of the controller, an algebraic Riccati equation is solved at each iteration of the control method. The global stability properties of the control scheme are proven through the Lyapunov analysis. The use of the H-infinity Kalman Filter allows also to implement state estimation-based control without the need to measure the entire state vector of the pulping process. The control method retains the advantages of the linear optimal control, that is fast and accurate tracking of the reference setpoints under moderate variations of the control inputs.

The article has demonstrated the advantages of the proposed non-linear optimal control method when compared against other control approaches that one could have considered for the model of the mechanical pulping process in paper mills. The proposed method exhibits the following advantages: (1) it is computationally efficient because it avoids the complicated state variables transformations which are met in global linearization-based control methods, while also avoiding singularity problems; (2) it is of proven global stability and its convergence to optimum does not depend on initialization and parameters value selection and is not affected by the change of operating points and (3) it is energy efficient because it minimizes the amount of energy that is consumed for the completion of the mechanical pulping process in the paper mills.

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