Dark SU(2) Antecedents of the U(1) Higgs Model

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Abstract

The original spontaneously broken $U(1)$ gauge model with one complex Higgs scalar field has been known in recent years as a possible prototype dark-matter model. Its antecedents in the context of $SU(2)$ are discussed. Three specific examples are described, with one dubbed “quantum scotodynamics”.
Introduction: Consider the addition of the $U(1)_D$ Higgs model [11] to the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge model (SM) of quarks and leptons. The former may be used for dark matter [2, 3, 4, 5, 6, 7] because it has the built-in $Z_2$ symmetry where the massive gauge boson $Z_D$ after spontaneous symmetry breaking is odd and the one physical real scalar boson $h_D$ is even. However, $U(1)_D$ may mix kinetically [8] with $U(1)_Y$, in which case the above $Z_2$ symmetry would be violated. To avoid this problem, it is suggested here that $U(1)_D$ be replaced with an $SU(2)$ antecedent, with an enriched dark-matter sector. Three explicit examples will be discussed. Note that this version of dark $SU(2)$ requires that it be broken to $U(1)$, in contrast to the case where a local or global $SU(2)$ dark symmetry remains [9].

$SU(2)_D$ with Scalar Doublet and Triplet: To break $SU(2)_D$ to $U(1)_D$, the simplest choice is a real scalar triplet

$$\chi = (\chi_1, \chi_2, \chi_3)$$

with $\langle \chi_3 \rangle = v_3$. In that case, the vector gauge bosons

$$W_D^\pm = D_1 \pm iD_2$$

acquire mass given by $m_{W_D}^2 = 2g_D^2v_3^2$. Note that the superscript $\pm$ refers to dark charge, the details of which will be discussed later.

To break $U(1)_D$ in the context of $SU(2)_D$ so that $D_3 = Z_D$ acquires mass, a complex scalar doublet

$$\Phi = \left( \phi_1, \phi_2 \right)$$

is used. Moreover, a global $U(1)_\Phi$ symmetry is imposed, i.e.

$$\Phi \rightarrow e^{i\theta} \Phi,$$

which prevents the coupling of $\vec{\chi}$ to the triplet $\phi_i \epsilon_{ij} \vec{\sigma}_{jk} \phi_k$. The scalar potential consisting of $\chi$ and $\Phi$ is then given by

$$V = m_2^2 \Phi^\dagger \Phi + \frac{1}{2} m_3^2 \langle \vec{\chi} \cdot \vec{\chi} \rangle + \mu_0 \Phi^\dagger (\vec{\sigma} \cdot \vec{\chi}) \Phi$$
\[ + \frac{1}{2} \lambda_2 (\Phi^i \Phi)^2 + \frac{1}{2} \lambda_3 (\vec{\chi} \cdot \vec{\chi})^2 + \lambda_4 (\Phi^i \Phi)(\vec{\chi} \cdot \vec{\chi}). \] (5)

Note that the triplet combination of two identical real scalar triplets is zero. The minimum of \(V\) admits a solution

\[ \langle \chi_{1,2} \rangle = 0, \quad \langle \chi_3 \rangle = v_3, \quad \langle \phi_1 \rangle = 0, \quad \langle \phi_2 \rangle = v_2 / \sqrt{2}, \] (6)

where \(v_2\) is assumed real without any loss of generality, and

\[ 0 = v_3 [m_3^2 + 2 \lambda_3 v_3^2 + \lambda_4 v_2^2] - \mu_0 v_2^2 / 2, \] (7)

\[ 0 = v_2 [m_2^2 + \lambda_2 v_2^2 / 2 + \lambda_4 v_3^2 - \mu_0 v_3], \] (8)

provided that

\[ m_2^2 + \lambda_4 v_3^2 + \mu_0 v_3 > 0, \] (9)

\[ m_2^2 + \lambda_4 v_3^2 - \mu_0 v_3 < 0. \] (10)

As a result

\[ m_{W_D}^2 = 2g_D^2 v_3^2 + \frac{1}{4} g_D^2 v_2^2, \quad m_{Z_D}^2 = \frac{1}{4} g_D^2 v_2^2, \quad m_{\phi_1}^2 = 2 \mu_0 v_3, \] (11)

and the \(2 \times 2\) mass-squared matrix spanning \(h_D = \sqrt{2} Re(\phi_2) - v_2\) and \(H_D = \chi_3 - v_3\) is given by

\[ \mathcal{M}_{h_D,H_D}^2 = \begin{pmatrix} \lambda_2 v_2^2 & v_2 (2 \lambda_4 v_3 - \mu_0) \\ v_2 (2 \lambda_4 v_3 - \mu_0) & 4 \lambda_3 v_3^2 + \mu_0 v_2^2 / 2 v_3 \end{pmatrix}. \] (12)

A global residual symmetry remains, under which

\[ W_{D,+}^+, \phi_1 \sim +1, \quad W_{D,-}^-, \phi_1^* \sim -1, \quad Z_D, h_D, H_D \sim 0. \] (13)

This comes from \(I_{3D} + S_\Phi\), where \(S_\Phi = 1/2\) for \(\Phi\) and zero for all other fields. It is possible because of the imposed global \(U(1)_\Phi\) symmetry. Whereas \(\langle \phi_2 \rangle = v_2 / \sqrt{2}\) breaks both \(I_{3D}\) and \(S_\Phi\), the linear combination \(I_{3D} + S_\Phi\) is zero for \(\phi_2\), so it remains as a residual dark symmetry.
An important consequence of this structure is the emergence of a dark charge conjugation symmetry as in the original Higgs model \[1\], i.e.

\[ W_D^+ \leftrightarrow W_D^- \quad (D_2 \leftrightarrow -D_2), \quad \phi_1 \leftrightarrow \phi_1^*, \quad Z_D(D_3) \leftrightarrow -Z_D(D_3). \]  

(14)

This comes from the gauge-invariant terms

\[ -\frac{1}{4}(\partial_\mu \vec{D}_\nu - \partial_\nu \vec{D}_\mu + g_D \vec{D}_\mu \times \vec{D}_\nu)^2 + |\partial_\mu \Phi - \frac{ig_D}{2} \vec{D}_\mu | \Phi|^2. \]  

(15)

It means that $Z_D$ is stable if its mass is less than twice that of $\phi_1$, in complete analogy to the $U(1)_D$ model of Ref. \[7\]. This makes it possible in principle to implement the inception of self-interacting dark matter, i.e. $\phi_1$ or $W_D$ of order 100 GeV with $Z_D$ as the light stable mediator of order 10 to 100 MeV, to explain \[10\] the observed core-cusp anomaly in dwarf galaxies \[11\]. If $Z_D$ is unstable and decays to SM particles, as is the case for the light mediator proposed in most models, then very strong constraints exist \[12\] from the cosmic microwave background (CMB) which basically rule out \[13\] this scenario. On the other hand, $h_D$ must also be light and decay quickly through its mixing with the SM Higgs boson $h$ before big bang nucleosynthesis (BBN). In that case, the elastic scattering of $W_D$ or $\phi_1$ off nuclei through $h_D$ exchange is much too large to be acceptable with present data. In Ref. \[7\], this is not a problem because the dark matter is a Dirac fermion which couples to $Z_D$ but not $h_D$.

As it is, this specific $SU(2)_D$ antecedent of the $U(1)$ Higgs model may still be a model of dark matter without addressing the core-cusp anomaly in dwarf galaxies. Assuming that $W_D$ is heavy enough to decay into $\phi_1 h_D$ and $Z_D$ heavy enough to decay into $\phi_1 \phi_1^*$, then the complex scalar $\phi_1$ may be considered dark matter. Assuming that $h_D$ is lighter than $\phi_1$, the annihilation cross section of $\phi_1 \phi_1^*$ at rest $\times$ relative velocity is given by

\[ \sigma(\phi_1 \phi_1^* \rightarrow h_D h_D) \ v_{rel} = \frac{\lambda_2^2 \sqrt{1 - r_1}}{64 \pi m_{\phi_1}^2} \left[ 1 + \frac{r_1(2 + r_1)}{(2 - r_1)(4 - r_1)} \right]^2, \]  

(16)
where \( r_1 = m_{h_D}^2/m_{\psi_1}^2 \). Assuming as an example \( m_{\phi_1} = 150 \text{ GeV} \) and \( m_{h_D} = 100 \text{ GeV} \), the above may be set equal to \( 4.4 \times 10^{-26} \text{ cm}^3/\text{s} \) for \( \lambda_2 = 0.126 \).

There is always the allowed quartic \( \lambda_2 h \) coupling between the \( SU(2)_D \) Higgs doublet and the \( SU(2)_L \times U(1)_Y \) Higgs doublet of the SM, so that \( \phi_1 \) interacts with quarks through the SM Higgs boson \( h \) in direct-search experiments. Using present data \([14]\), it has been shown \([15]\) that \( \lambda_2 h < 4.4 \times 10^{-4} \). This is also the mixing between \( h_D \) and \( h \). Even with this limit on \( \lambda_2 h \), it can still be large enough so that \( h_D \) decays promptly to \( b \bar{b} \) in the early Universe. This interaction \([7]\) also keeps \( h_D \) in thermal equilibrium with the particles of the SM.

**\( SU(2)_D \) with Fermion Doublet:** Consider the addition of a fermion doublet

\[
\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}_L
\]  

(17)

to the \( SU(2)_D \) model discussed in the previous section. It has the allowed interactions

\[
i\bar{\Psi} \gamma^\mu (\partial_\mu - ig_D/2 \vec{\sigma} \cdot \vec{D}_\mu) \Psi + [f \bar{\Psi} (\vec{\sigma} \cdot \vec{\chi}) \Psi + H.c.],
\]  

(18)

where \( \Psi = (\psi_2, -\psi_1)_L \). Since \( \langle \chi_3 \rangle = v_3 \), this shows that \( \psi_{1,2} \) combine to form a Dirac fermion of mass \( f v_3 \). To be specific, let \( \psi_{1L} \) be the left-handed component of the Dirac fermion \( \psi \), and \( \psi_{2L} \) redefined as the conjugate of its right-handed component, i.e. \( \psi_{2L} \sim \bar{\psi}_R \). Now \( \psi_1 \) has dark charge \( 1/2 \) and \( \psi_2 \) has dark charge \( -1/2 \). Together they form a Dirac fermion \( \psi \) of charge \( 1/2 \), which interacts vectorially with \( D_3 = Z_D \). Note that \( \bar{\psi}_R \gamma_\mu \psi \) is odd under dark charge conjugation as expected. Note also that \( \psi \) has no direct coupling to \( h_D \) because of \( SU(2)_D \) gauge invariance. This allows the inception of self-interacting dark matter as described below.

Consider the elastic scattering of \( \psi \) with \( \bar{\psi} \) through the exchange of the light mediator \( Z_D \). Its cross section in the limit of zero momentum is

\[
\sigma(\psi \bar{\psi} \rightarrow \psi \bar{\psi}) = \frac{g_D^4 m_{\psi_1}^2}{64\pi m_{Z_D}^4} = \frac{m_{\psi}^2}{4\pi v^4}.
\]  

(19)
For the benchmark value of $\sigma/m_{\phi_1} \sim 1 \text{ cm}^2/g$ for self-interacting matter, this is satisfied for example with

$$m_{\psi} = 100 \text{ GeV}, \quad v_2 = 200 \text{ MeV}. \quad (20)$$

This low-energy effective theory consisting of $\psi$, $Z_D$ and $h_D$ may be dubbed quantum scotodynamics, from the Greek ‘scotos’ meaning darkness.

Consider now the annihilation of $\psi \bar{\psi} \rightarrow Z_D Z_D$. Since $Z_D$ is much lighter than $\psi$, this cross section $\times$ relative velocity is given by

$$\sigma(\psi \bar{\psi} \rightarrow Z_D Z_D) \, v_{rel} = \frac{g_D^4}{256\pi m_{\psi}^2}. \quad (21)$$

For $m_{\psi} = 100 \text{ GeV}$, and setting $\sigma v_{rel} = 4.4 \times 10^{-26} \text{ cm}^3/s$,

$$g_D = 0.42 \quad (22)$$

is obtained, which implies from Eq. (11) that

$$m_{Z_D} = 42 \text{ MeV}. \quad (23)$$

As shown in Ref. [7], the light mediator $Z_D$ is stable but annihilates quickly to $h_D$ which decays. The cross section $\times$ relative velocity is given by

$$\sigma(Z_D Z_D \rightarrow h_D h_D) \, v_{rel} = \frac{g_D^4 \sqrt{1-r}}{64\pi m_{Z_D}^2} \left[ \frac{4[r^2 + 4(2-r)^2]}{(4-r)^2} - \frac{24r(2+r)}{9(2-r)(4-r)} + \frac{8(2+r)^2}{9(2-r)^2} \right]. \quad (24)$$

where $r = m_{h_D}^2/m_{Z_D}^2$. Assuming $m_{h_D} = 21 \text{ MeV}$ as an example so that $r = 0.25$, the above is equal to $2 \times 10^{-18} \text{ cm}^3/s$, which is orders of magnitude greater than what is required for $Z_D$ to be a significant component of dark matter. It may re-emerge at late times by $\phi_1 \phi_1^*$ annihilation through Sommerfeld enhancement, but its fraction as dark matter remains negligible. Since $Z_D$ is stable, it would also not disturb [12, 13] the cosmic microwave background (CMB).
As for $h_D$, it is allowed to mix with the SM Higgs boson $h$ in the $2 \times 2$ mass-squared matrix

$$M^2_{h,D,h} = \begin{pmatrix} \lambda_2 v_2^2 & \lambda_{2h} v_2 v_h \\ \lambda_{2h} v_2 v_h & m_h^2 \end{pmatrix},$$  \hspace{1cm} (25)$$

where $v_h = 246$ GeV and $m_h = 125$ GeV. For $m_{h_D} \ll m_h$, the $h_D - h$ mixing is $\theta_{2h} = \lambda_{2h} v_2 v_h / m_h^2$. Assuming $\lambda_{2h} = 0.01$, \hspace{1cm} (26)

then $\theta_{2h} = 3.8 \times 10^{-5}$ and the $h_D$ lifetime for $e^- e^+$ decay is given by

$$\Gamma^{-1}(h_D \rightarrow e^- e^+) = \frac{8\pi v_h^2}{m_{h_D} m_h^2 \theta_{2h}^2} = 0.13 \text{ s},$$ \hspace{1cm} (27)$$

which is short enough not to affect big bang nucleosynthesis (BBN). The decay of the SM Higgs boson to $h_D h_D$ is given by

$$\Gamma(h \rightarrow h_D h_D) = \frac{\lambda_{2h}^2 v_h^2}{16\pi m_h} = 0.96 \text{ MeV},$$ \hspace{1cm} (28)$$

which is less than 25% of the SM width of 4.12 MeV and allowed by present data. Note that $\lambda_2 = 0.008$ in Eq. (25) for $m_{h_D} = 21$ MeV. Note also the important fact that $\psi$ does not couple directly to $h_D$, otherwise Eq. (26) would be impossible, as discussed in the previous section.

In summary, a successful description of self-interacting fermion dark matter ($\psi$ with $m_\psi = 100$ GeV) through a stable light vector gauge boson ($Z_D$ with $m_{Z_D} = 42$ MeV) in an $SU(2)_D$ gauge model has been rendered. The Higgs scalar $h_D$ associated with $Z_D$ is also light (21 MeV), but it decays away quickly before the onset of BBN. Other heavier particles in the dark sector are $W^\pm_D$ (which decays to $\psi_1 \psi_1 / \psi_2 \psi_2$), $\phi_1$ (which decays to $W^+_D h_D$), and $H_D$ which mixes slightly with $h$ and $h_D$.

**SU(2)$_D$ with Scalar Doublet and Quintet**: In the previous two examples, an imposed symmetry of the $SU(2)_D$ scalar doublet $\Phi$, i.e. Eq. (4), is necessary for obtaining a dark symmetry. Hence the latter is not predestined \cite{16}, i.e. not the automatic consequence of gauge
symmetry and particle content. To have a predestined dark $Z_2$ symmetry, the simpler scalar triplet is now replaced with a scalar quintet. This is analogous to having a fermion quintet [17] in the SM for minimal dark matter.

Consider thereby the real scalar quintet

$$\zeta = (\zeta^{++}, \zeta^+, \zeta^0, \zeta^-, \zeta^{--})$$

(29)

with $\langle \zeta^0 \rangle = v_5$, then $W_D^\pm$ obtains a mass given by $m_{W_D}^2 = 6g_D^2v_5^2$ from absorbing $\zeta^\pm$. This leaves $\zeta^{\pm\pm}$ as physical scalar bosons with two units of dark charge, interacting with $Z_D$. The scalar potential consisting of $\zeta$ and $\Phi$ is then given by

$$V = m_\zeta^2 \Phi^\dagger \Phi + \frac{1}{2}m_\zeta^2\zeta^\dagger \zeta + \frac{1}{2}\lambda_2(\Phi^\dagger \Phi)^2 + \lambda_5(\Phi^\dagger \Phi)(\zeta^\dagger \zeta)^2 + V_3 + V_4,$$

(30)

where $V_3$ contains the one cubic invariant formed out of 3 scalar quintets and $V_4$ contains two quartic invariants. Note that the triplet combination of two identical real scalar quintets is zero. As a result, this scalar potential automatically has an extra $U(1)_\Phi$ symmetry, so that $I_{3D} + S_\Phi$ remains unbroken as $\phi_2$ acquires a vacuum expectation value $v_2/\sqrt{2}$ as explained previously.

Assuming that

$$m_\zeta < 2m_{\phi_1} < m_{Z_D} < m_{W_D},$$

(31)

then $W_D^+$ decays to $\phi_1 h_D$, $Z_D$ decays to $\phi_1 \phi_1^*$, but both $\phi_1$ and $\zeta$ are stable. Hence this is an explicit example of two-component dark matter under one dark $U(1)$ symmetry. Let

$$m_\zeta = 200 \text{ GeV}, \quad m_{\phi_1} = 150 \text{ GeV}, \quad m_{h_D} = 100 \text{ GeV},$$

(32)

then using Eq. (16) for $\sigma_1(\phi_1 \phi_1^* \rightarrow h_D h_D) v_{rel}$ and the analogous

$$\sigma_2(\zeta \zeta^* \rightarrow h_D h_D, \phi_1 \phi_1^*) v_{rel} = \frac{\lambda_5^2 \sqrt{1 - r^2}}{64\pi m_\zeta^2} \left[ 1 + \frac{2(\lambda_5/\lambda_2)r_2}{2 - r_2} - \frac{3r_2}{4 - r_2} \right]^2$$

$$+ \frac{\lambda_5^2 \sqrt{1 - r_3^2}}{32\pi m_\zeta^2} \left[ 1 - \frac{r_2}{4 - r_2} \right]^2,$$

(33)

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where \( r_2 = m_{h_D}^2 / m_\zeta^2 \) and \( r_3 = m_{\phi_1}^2 / m_\zeta^2 \), the condition for the correct relic abundance is roughly given by

\[
\langle \sigma_1 v_{\text{rel}} \rangle^{-1} + \langle \sigma_2 v_{\text{rel}} \rangle^{-1} = (4.4 \times 10^{-26} \text{ cm}^3/\text{s})^{-1}.
\] (34)

It has for example the reasonable solution \( \lambda_5 = \lambda_2 = 0.173 \), in which case \( \phi_1 \) is 53% and \( \zeta \) 47% of dark matter. Again the mixing of \( \zeta \) with the SM Higgs boson \( h \) must be small as it is for \( \phi_1 \) to satisfy direct-search limits as discussed previously.

In this scenario, the addition of the fermion doublet of Eq. (17) could also provide a low-energy effective theory of quantum scotodynamics with light \( Z_D \) and \( h_D \). In that case, \( \zeta^{\pm \pm} \) would decay into \( W_D^\pm W_D^\pm \), \( \phi_1 \) would decay into \( W_D^+ h_D \), and \( W_D^\pm \) would decay into \( \psi_1 \psi_1 / \psi_2 \psi_2 \).

**Concluding Remarks:** Exploring the possible \( SU(2) \) antecedents of the famous \( U(1) \) Higgs model for a nontrivial application to dark matter, three interesting examples have been identified and discussed. The minimal version with one real scalar triplet \( \chi \) and one complex scalar doublet \( \Phi \) admits \( \phi_1 \) as dark matter, but a global \( U(1) \) symmetry has to be imposed. With the addition of a fermion doublet \( \psi \), the inception of self-interacting dark matter may be implemented successfully, avoiding all potential astrophysical and laboratory constraints. A third example replaces \( \chi \) with the real scalar quintet \( \zeta \), in which case the dark \( U(1) \) symmetry becomes predestined, i.e. automatic from the gauge symmetry and particle content.

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### References

[1] P. W. Higgs, Phys. Rev. Lett. **13**, 508 (1964).

[2] O. Lebedev, H. M. Lee, and Y. Mambrini, Phys. Lett. **B707**, 570 (2012).
[3] Y. Farzan and A. Rezaei Akbarrieh, JCAP **1210**, 026 (2012).

[4] S. Baek, P. Ko, W.-I. Park, and E. Senaha, JHEP **1305**, 036 (2013).

[5] A. DiFranzo, P. J. Fox, and T. M. P. Tait, JHEP **1604**, 135 (2016).

[6] A. DiFranzo and G. Mohlabeng, JHEP **1701**, 080 (2017).

[7] E. Ma, Phys. Lett. **B772**, 442 (2017).

[8] B. Holdom, Phys. Lett. **B166**, 196 (1986).

[9] T. Hambye, JHEP **0901**, 028 (2009).

[10] See for example J. L. Feng, M. Kaplinghat, H. Tu, and H.-B. Yu, JCAP **0907**, 004 (2009).

[11] See for example F. Donato, G. Gentile, P. Salucci, C. Frigerio Martins, M. I. Wilkinson, G. Gilmore, E. K. Grebel, A. Koch, and R. Wyse, MNRAS **397**, 1169 (2009).

[12] S. Galli, F. Iocco, G. Bertone, and A. Melchiorri, Phys. Rev. **D80**, 023505 (2009).

[13] T. Bringmann, F. Kahlhoefer, K. Schmidt-Hoberg, and P. Walia, Phys. Rev. Lett. **118**, 141802 (2017).

[14] E. Aprile, *et al.*, XENON Collaboration, Phys. Rev. Lett. **119**, 181301 (2017).

[15] C. Kownacki, E. Ma, N. Pollard, O. Popov, and M. Zakeri, Eur. Phys. J. **C78**, 148 (2018).

[16] E. Ma, [arXiv:1803.03891](https://arxiv.org/abs/1803.03891) [hep-ph].

[17] M. Cirelli, N. Fornengo, and A. Strumia, Nucl. Phys. **B753**, 178 (2006).