The Turbulent Story of X-ray Bursts: Effects of Shear Mixing on Accreting Neutron Stars

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Abstract. During accretion, a neutron star (NS) is spun up as angular momentum is transported through its liquid surface layers. We study the resulting differentially rotating profile, focusing on the impact this has for type I X-ray bursts. The viscous heating is found to be negligible, but turbulent mixing can be activated. Mixing has the greatest impact when the buoyancy at the compositional discontinuity between accreted matter and ashes is overcome. This occurs preferentially at high accretion rates or low spin frequencies and may depend on the ash composition from the previous burst. We then find two new regimes of burning. The first is ignition in a layer containing a mixture of heavier elements with recurrence times as short as ≈ 5 – 30 minutes, similar to short recurrence time bursts. When mixing is sufficiently strong, a second regime is found where accreted helium mixes deep enough to burn stably, quenching X-ray bursts altogether. The carbon-rich material produced by stable helium burning would be important for triggering and fueling superbursts.

Keywords: accretion, accretion disks; magnetic fields; neutron stars; X-ray binaries; X-ray bursts

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INTRODUCTION

As neutron stars (NSs) in low mass X-ray binaries accrete material from their companions they are expected to be spun up by this addition of angular momentum, possibly becoming millisecond pulsars [1]. This suspicion has received support by the discovery of accretion driven millisecond pulsars [2], as well as the millisecond oscillations seen during type I X-ray bursts [3], the unstable ignition of the accumulating fuel [4, 5, 6, 7]. Angular momentum must be transported into the NS interior if it is to be spun up, which implies a non-zero, albeit small, shear throughout the outer liquid parts of the NS.

Such shearing may lead to viscous heating and chemical mixing at depths far below the low density boundary layer where the majority of the shearing occurs. This has motivated us to assess the importance of angular momentum transport in NS surface layers. In the following we summarize the results of our study [11].

ANGULAR MOMENTUM TRANSPORT

Material accreted at a rate \( \dot{M} \) reaches the NS surface with a nearly Keplerian spin frequency of \( \Omega_K = (GM/R^3)^{1/2} = 1.4 \times 10^4 \text{ s}^{-1} M_{1.4}^{1/2} R_6^{-3/2} \), where \( M_{1.4} \equiv M/1.4M_\odot \) and \( R_6 \equiv R/10^6 \text{ cm} \), which has a kinetic energy per nucleon of \( \approx 200 \text{ MeV nuc}^{-1} \). The majority of this energy is dissipated in a boundary layer of thickness \( H_{BL} \ll R \) and never reaches far into the NS surface [9]. Nevertheless, angular momentum is added at a rate of \( M\Omega_K^2 \), so that a torque of this magnitude must be communicated into the NS. This implies a non-zero shear rate in the interior liquid layers, down to the solid crust. In the present work we are interested in the shear at the depths where X-ray bursts ignite, near \( \rho \approx 10^6 \text{ g cm}^{-3} \), which is well below the boundary layer.

The viscous timescale, \( \tau_{\text{visc}} = H^2/\nu \), where \( H \) is the pressure scaleheight and \( \nu \) is the viscosity, is always much shorter than the accretion timescale, \( \tau_{\text{acc}} = y/m \), where \( y \) is the column depth and \( m \) is the mass accretion rate per unit area. Therefore we assume that angular momentum is transported in steady-state. In the limit \( \Omega \ll \Omega_K \) and \( H \ll R \) this results in [7]

\[
4\pi R^3 \rho \nu q \Omega = M R^2 \Omega_K, \quad (1)
\]

where \( q \equiv d\log \Omega/d\log r \) is the shear. This equation shows that \( q > 0 \) when angular momentum is transported inward. In general, we find that the shear is rather small (\( q \ll 1 \)) at the depths of interest. Nevertheless \( q \) is large enough to activate instabilities that help to transport angular momentum, as well as mix material.

TURBULENT MIXING

We use equation (1) to evaluate the shear for various turbulent viscosities and find that purely hydrodynamic
instabilities [7] are insufficient to prevent strong shearing of magnetic fields. This leads to generation of the Tayler-Spruit dynamo [12], where toroidal field growth is balanced by Tayler instabilities to create a steady-state magnetic field, which provides a torque on the shearing layers. The result is nearly uniform rotation and little viscous heating, too little to affect either X-ray bursts or superbursts, thermonuclear ignition of ashes from previous X-ray bursts [5,14]. Turbulent mixing is found to be non-negligible and in some cases may mix fresh material with the ashes of previous bursts, therefore we focus on this for the majority of our study.

The Buoyancy Barrier

A key point for whether mixing can occur is whether the strong buoyancy due to the larger density of the ashes below can be overcome. Analytic analysis shows that for this to occur the accretion rate must exceed

\[ \dot{m}_{\text{crit},1} \approx 5 \times 10^{-2} \dot{m}_{\text{Edd}} \alpha_{\text{TS}}^{-2} \left( \frac{\Omega}{0.1 \Omega_K} \right)^3 \left( \ln \mu - 0.44 \right), \]  

(2)

where \( \dot{m}_{\text{Edd}} = 1.5 \times 10^5 \text{ g cm}^{-2} \text{ s}^{-1} \text{ R}_6^{-1} \), is the local Eddington limit for helium-rich accretion, \( \alpha_{\text{TS}} \) is a dimensionless constant (of order unity) that adjusts the strength of mixing, and \( \ln \mu \) is the fractional change in mean molecular weight across the boundary (\( \approx 0.44 \) for helium-rich material above an iron-rich ocean). This critical \( \dot{m} \) depends on the composition of the ashes from previous bursts (which sets \( \ln \mu \)) so that the strength of mixing could very well change from burst to burst.

Mixed Unstable Ignition

We next assess the affect on the burst properties when mixing occurs. Figure [1] shows an example calculation of material accumulating and mixing on the NS surface causing premature ignition of the helium, for the parameters \( \dot{m} = 0.1 \dot{m}_{\text{Edd}} \) and \( \Omega = 0.1 \Omega_K \). The shear profile is evaluated using equation (1), and we assume that mixing occurs completely down to a depth \( y_{\text{mix}} \) at which \( t_{\text{acc}} = t_{\text{mix}} \) (the right end of each thin solid line), where \( t_{\text{mix}} = H^2 / D \) and \( D \) is the turbulent mixing diffusivity of the Tayler-Spruit dynamo [12]. This is significantly deeper than the amount of material accreted, \( y_{\text{acc}} \) (shown by the filled circles). This means that helium is significantly depleted at the time of ignition by an amount \( Y_{\text{mix}} = y_{\text{acc}} / y_{\text{mix}} \), where we use a helium-rich composition for the accreted material.

In Figure [2] we plot the recurrence times for mixed bursts (once again assuming that eq. [2] is satisfied). These timescales are similar to those found for short recurrence time bursts [2]. To explain the energetics of these bursts (i.e. their low \( \alpha \)-value [2]) still requires incomplete burning from the previous burst. This is in fact preferential for strong mixing since incomplete burning also leads to smaller compositional gradients. If the mixing is too strong we instead find stable helium burning (noted in Fig. [2]). This is analytically estimated to occur at

\[ \dot{m}_{\text{crit},2} \approx \dot{m}_{\text{Edd}} \alpha_{\text{TS}}^{-0.83} \left( \frac{\Omega}{\Omega_K} \right)^{0.62}. \]  

(3)

Above this \( \dot{m} \) we expect X-ray bursts to cease altogether.

CONCLUSIONS

Turbulent mixing is sufficiently large to have important consequences for X-ray bursts. We constructed simple models, both analytic and numerical, to explore mixing for pure helium accretion [11]. From these models we can make a few conclusions that are likely general enough to apply to most viscous mechanisms. As a guide, we show the different burning regimes we find in Figure [3]. These can be summarized as follows:

- Mixing has trouble overcoming the buoyancy barrier at chemical discontinuities. Incomplete burning results in small compositional gradients so that mixing is important in subsequent bursts.
FIGURE 2. The recurrence time for mixed-ignition models as a function of $\dot{m}$. The symbols denoted different spins, as shown in the key. Models that are at sufficiently high $\dot{m}$ or low $\Omega$ do not ignite unstably, and thus are not plotted.

FIGURE 3. The three regimes of burning found for models including mixing. The boundaries between each regime are shaded to emphasize uncertainty in the strength of mixing ($\sigma_{TS} = 0.7 - 1.5$). The light shaded region divides between stable and unstable mixed burning ($m_{\text{crit,2}}$, eq [3]). The heavy shaded region corresponds to the compositional barrier, which prevents mixing ($m_{\text{crit,1}}$, eq [2]). Its position can move significantly depending on the composition of material from the previous burst.

• Mixing is strongest at large $\dot{m}$ (when angular momentum is being added at greater rates) and small $\Omega$ (which gives a larger relative angular momentum between the NS and accreted material).
• Mixing can lead to two new effects. First, the layer may ignite, but in a mixed environment with a shorter recurrence time. Second, for strong mixing accreted helium can mix and burn in steady-state, quenching bursts. Both regimes have observed analogs, namely the short recurrence time bursts [2] and the stabilization of bursting seen near one-tenth the Eddington rate [4].

As a future extension of this work, realistic ash compositions from previous bursts should be incorporated into simulations, so that one can test whether mixing is a suitable explanation for short recurrence time bursts. Mixed ignition can be studied using current numerical experiments [16] by just artificially accreting fuel with a mixed composition of $Y_{\text{mix}} \approx 0.1 - 0.6$. These calculations are simplified by our conclusion that shearing and heating can be ignored, at least for initial studies. Another important extension would be to use different compositions for the accreted material. The current models assume it is pure-helium. Including hydrogen would allow comparison to a wider range of systems. Finally, the carbon-rich ashes from periodic stable burning may be important for explaining the recurrence times of superbursts [5, 14, 6], which also deserves further investigation.

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