Unpolarized transverse momentum-dependent densities based on the modified chiral quark model

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Abstract. We investigate the transverse momentum-dependent (TMD) quark and gluon distribution functions in the modified chiral quark model (χQM). Calculations of the TMD quark and gluon densities, using the modified χQM, are carried out for the first time in this article. For this propose, we first formulate the TMD interactions that occur in the χQM at low $Q^2$ scale ($Q^2 = 0.35 \text{ GeV}^2$) and then obtain the TMD parton distributions inside the proton, considering these interactions. To this end, we need to compute the TMD bare quark distributions. These TMD bare densities are calculated using the solution of the Dirac equation with a squared radial symmetry potential. It is shown that our results present appropriate behavior, which is expected for the TMD parton distributions.

1 Introduction

To extend our knowledge of the nucleon structure far beyond what we know from parton distribution functions (PDFs) about longitudinal momentum distributions, we need a generalization of PDFs which are known as transverse momentum-dependent parton distribution functions (TMDs). These functions contain also some information on transverse momenta of partons as well as spin-orbit correlations.

TMDs are important since they play essential roles in the theoretical description of some experimental quantities like single spin asymmetries, which exist in various hard processes, including semi-inclusive deep inelastic scattering (SIDIS), Drell-Yan processes, etc. [1–3]. These functions can also be called the unintegrated parton distributions [4].

It is expected that TMDs give us a three-dimensional view of the parton distributions in momentum space. Therefore, they provide sufficient information in addition to what can be learned from the generalized parton distributions [5–8].

In comparison with the integrated parton densities, our knowledge about the TMD parton distributions is not rich. In contrast to the integrated parton distributions, for which there are many model calculations, for the TMD distributions there are not enough.

The purpose of this work is to extract the unpolarized TMD quark and gluon distributions using the chiral quark model.

The main property of the χQM is its application at low $Q^2$ scales [9]. This property is due to breaking the chiral symmetry at low energy scales. In these scales, the light quark masses are not ignorable in comparison with the nucleon mass, and chiral symmetry is broken.

In the χQM, the bare quarks inside the nucleon are surrounded by the clouds of Goldstone (GS) bosons and gluons [10,11]. Considering the interactions which occur in this bounded system at the first approximation and also the relations between the transverse momentum of the bare quarks and GS bosons (gluons) in these interactions, we can obtain the TMD quark and gluon distributions inside the proton. In order to do these calculations, we need to know the proton TMD bare quark distributions. Using the solution of the Dirac equation by considering the harmonic oscillator potential, these TMD bare distributions can be calculated.

The plan of this paper is as follows. Applying the χQM, the TMD quark and gluon densities are calculated in sect. 2. For this purpose, we first investigate the interactions, which occur at the vertex of bare quark-GS bosons and also the bare quark-gluon vertex in the χQM in subsects. 2.1 and 2.2. In subsect. 2.3 the required TMD bare quark distributions are computed. We give our results in sect. 3 and, finally, render our conclusions in sect. 4.

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2 TMD quark and gluon distributions in the chiral quark model

In this section, we calculate the transverse momentum-dependence of unpolarized quark and gluon distribution functions applying the chiral quark model. In this model which is used at low $Q^2$ scales, the important degrees of freedom are expressed in terms of quarks, gluons and GS bosons. In the $\chi$QM, the nucleon consists of the bare up and down quarks ($u_0, d_0$), which are surrounded by the clouds of GS bosons and gluons (fig. 1). We first investigate, in subsects. 2.1 and 2.2, the different types of interactions, which can be occurred at the vertexes of fig. 1.

2.1 The bare quark-GS boson vertex

At the first approximation, one basic process, which can be occurred at the vertex of bare quark-GS boson in fig. 1, is considered. In this process, the bare quark $q_i$ fluctuates into an intermediate quark $q_j$ and a GS boson $\mathcal{B}$, which is represented in fig. 2(a). This fluctuation is given by

$$ q_j(x, p_T) = \int_0^1 \frac{dy}{y} P_{j\mathcal{B}/i}(y, p_T) \left( \frac{x}{y}, p_T \right). $$  (1)

In eq. (1), $P_{j\mathcal{B}/i}$ is the splitting function that expresses the probability of finding the struck quark $q_j$ with longitudinal momentum fraction $y$ and the transverse momentum $p_{jT}$ and also the GS boson, which carries the longitudinal momentum fraction $1 - y$ of the parent quark's momentum and the transverse momentum $p_{BT}$.

We can write the transverse momenta of the quark $q_i$ and the GS boson in terms of the transverse momentum of the bare quark $q_i$, $p_T$, and the intrinsic variables [12]:

$$ p_{jT} = k_T + y p_T, \quad p_{BT} = -k_T + (1 - y) p_T. $$  (2)

Based on Sullivan processes [13] we can suggest the TMD splitting function $P_{j\mathcal{B}/i}$ as follows:

$$ P_{j\mathcal{B}/i}(y, p_T) = \frac{1}{8\pi^2} \left( \frac{g_A m}{f} \right)^2 (1 - y) \int_{A_i^2}^{\min} \frac{[(m_i - m_j)^2 - t]}{(t - m_B^2)^2} dt. $$  (3)

where $A_i$ is the cut-off parameter, $m_i$, $m_j$ and $m_B$ are the mass of quarks $q_i, q_j$ and the GS boson, respectively. The parameter $t$ is defined as

$$ t = \frac{-[p_{jT}^2 + (1 - y)[m_j^2 - y m_i^2]]}{y} = -[(k_T + y p_T)^2 + (1 - y)[m_j^2 - y m_i^2]]. $$  (4)

Substituting the $t$ parameter in eq. (3), we will arrive at

$$ P_{j\mathcal{B}/i}(y, p_T) = \left( \frac{2(k_T + y p_T)}{y^2(1 - y)(m_j^2 - M_{j\mathcal{B}}^2)^2} \right)^2 \left( (m_j - m_i y)^2 + (k_T + y p_T)^2 \right) \times \frac{1}{8\pi^2} \left( \frac{g_A m}{f} \right)^2. $$  (5)

In eq. (5), $g_A$ and $f$ denote the axial vector constant and the pseudo-scalar decay constant, respectively, $\bar{m}$ is the average mass of $q_i$ and $q_j$ and $M_{j\mathcal{B}}^2$ is the square invariant mass of the final state,

$$ M_{j\mathcal{B}}^2 = \frac{m_j^2 + (k_T + y p_T)^2}{y} + \frac{m_i^2 + (k_T + y p_T)^2}{1 - y}. $$  (6)

It is obvious that $t_{\min}$ in eq. (3) can be obtained from eq. (4) by substituting $k_T = 0$ in this equation [14]. It is found that if we put $p_T = 0$ in eq. (4), this equation will be casted in its usual form, while there is not, finally, any transverse momentum dependence in the $\chi$QM [15,16]. In this case the splitting function $P_{j\mathcal{B}/i}$ and also the $q_j$ distribution are given by expressions which are used in refs. [10,17–19].
Fig. 2. (a) The fluctuation of a bare quark $q_i$ into a GS boson $B$ plus a struck quark $q_j$. (b) A bare quark $q_i$ emits a gluon and transforms to a constituent quark $q_j$.

2.2 The vertex of the bare quark-gluon

A similar process can be occurred in the vertex of the bare quark gluon, at the first approximation (fig. 2(b)). In this process the bare quark $q_i$ emits a gluon and appears as a recoiled quark $q_j$. The TMD $q_j$ distribution has the following form:

$$q_j(x, p_T) = \int_1^x \frac{dy}{y} P_{jg/i}(y, p_T) q_{0,i} \left( \frac{x}{y}, p_T \right).$$

(7)

Here the related TMD splitting function $P_{jg/i}$ is

$$P_{jg/i}(y, p_T) = \int dk_T G_{jg/i}^2 \left( \frac{2(k_T + y p_T)}{y(1 - y)(m_i^2 - M_{jg}^2)^2} \right) \times \left[ \frac{(m_j - m_i y)^2 + (k_T + y p_T)^2}{1 - y} \right] \times C_f \frac{\alpha_s(Q^2)}{4\pi},$$

(8)

where $C_f$ is the color factor and the square invariant mass of the final quark-gluon state, $M_{jg}^2$, is written as

$$M_{jg}^2 = \frac{m_j^2 + (k_T + y p_T)^2}{y} + \frac{m_i^2 + (k_T + y p_T)^2}{1 - y}.$$

(9)

In this equation $m_g$ denotes the gluon mass.

In eq. (8), the vertex function $G_{jg/i}$ is defined as

$$G_{jg/i} = \exp \left( \frac{m_i^2 - M_{jg}^2}{2\Lambda_ch} \right).$$

(10)

The TMD gluon distribution is also calculated via the interaction of fig. 2(b).

We use the following notation for the convolution integrals in eqs. (1) and (7):

$$P_{jB/i} \otimes q_0 = \int_x^1 \frac{dy}{y} P_{jB/i}(y, p_T) q_0 \left( \frac{x}{y}, p_T \right),$$

$$P_{jg/i} \otimes q_0 = \int_x^1 \frac{dy}{y} P_{jg/i}(y, p_T) q_0 \left( \frac{x}{y}, p_T \right).$$

(11)

2.3 TMD bare quark distribution functions

In order to calculate the TMD distribution functions inside the proton using eqs. (1) and (7), we should first compute the TMD bare quark distributions. For this purpose, we use the solution of the Dirac equation under harmonic oscillator potential [10,20–22]. Applying this approach, the ground state wave function of the bare quark in the momentum space in terms of two parameters $\rho$ and $R$ is obtained as [10]

$$\phi_0(p) = -\pi^{-\frac{1}{2}} R^\frac{3}{2} \left( 1 + \frac{3\rho^2}{2} \right)^{-\frac{1}{2}} e^{-\frac{\rho^2}{2}} \chi_s \chi_f \chi_c,$$

(12)

where $\chi_s$, $\chi_f$ and $\chi_c$ are the related spin, flavor and color parts of the wave function.

We consider the probability density as $\varrho = \phi_0^*(p) \phi_0(p)$. 


The TMD bare quark distribution, $f(x, p_T)$, satisfies the following relation \[10,23\]:

$$
\int d\phi \left( p^0 - \sqrt{(p^3)^2 + (p_T)^2 + m^2} \right) dp^0 dp^3 d^2 p_T = \int f(x, p_T) d^2 p_T dx.
$$

(13)

In the above equation $p^0, p = (p^1, p^2, p^3)$ and $m$ are the bare quark energy, 3-momentum and mass, respectively. In eq. (13), we use the relations which exist between the components of the momentum four vector in standard coordinates and light cone coordinates, so we can write \[10,23\]

$$
dp^0 dp^3 d^2 p_T = \frac{1}{2} M_t dp^- dx d^2 p_T.
$$

(14)

Finally, by comparing both sides of eq. (13), the TMD bare quark distribution is determined as \[10\]

$$
f(x, p_T) = \frac{1}{2} M_t R^3 \pi^- \left( 1 + \frac{3p_T^2}{2} \right)^{-1} \left[ 1 + \frac{(p_T)^2 + m^2}{(M_t x)^2} \right] e^{-R^2(p_T)^2} e^{-\frac{M_t x (p_T)^2 + m^2}{M_t x}}.
$$

(15)

According to the above discussion, the unpolarized TMD quark distributions are given as follows:

$$
u(x, p_T) = Zu_0(x, p_T) + Pu_{\pi^-/d} \otimes d_0 + \frac{1}{2} Pu_{\pi^+/u} \otimes u_0 + Pu_{ug/u} \otimes u_0,
$$

(16)

$$
d(x, p_T) = Zd_0(x, p_T) + Pd_{\pi^+/u} \otimes u_0 + \frac{1}{2} Pd_{\pi^-/d} \otimes d_0 + Pd_{dg/d} \otimes d_0,
$$

(17)

$$
s(x, p_T) = Ps_{\kappa^+/u} \otimes u_0 + Ps_{\kappa^-/d} \otimes d_0.
$$

(18)

$Z_u$ and $Z_d$ are the renormalization constants of the $u$ and $d$ bare quark distributions.

Finally, the TMD gluon distribution function is written as

$$
g(x, p_T) = Pu_{ug/u} \otimes u_0 + Pd_{dg/d} \otimes d_0.
$$

(19)

Now we are able to calculate the TMD quark and gluon densities using the above relations.

3 Results

We calculate the unpolarized TMD quark and gluon distribution functions inside the proton using the effective chiral quark model at $Q^2 = 0.35$ GeV$^2$. To this end, we first compute the TMD splitting functions and also bare quark densities, which were discussed in previous section.

In fig. 3, we have displayed the three-dimensional representation of the TMD splitting functions $Pu_{\pi^-/d}, Pu_{\pi^+/u}$ and $Ps_{\kappa^+/d}$ with respect to $y$ and $p_T$.

We have depicted the TMD $u, d$ and $s$ quark and also gluon distributions with respect to $x$ at three values of $p_T$ ($p_T = 0.1, 0.2$ and $0.3$ GeV) in fig. 4.
It is shown that, as have been expected, by increasing the $p_T$ value, the TMD densities falloff down. In fact, the probability of finding the partons at larger values of $p_T$ is lower. This behavior of the TMD distributions is also seen in refs. [24–27].

In figs. 5 and 6 we have plotted the TMD quark and gluon densities with respect to $p_T$ at different values of $x$. It is found that these $p_T$ distributions have the forms very close to the Gaussian distributions [27] and also the width of these densities is $x$ dependent [27].

It should be pointed out that we have displayed each of $p_T$ densities in two sets of $x$ values. In the first set, which contains the small values of $x$, the TMD distributions grow by increasing the $x$ value in spite of the behavior of the second set, in which the $p_T$ densities decrease by increasing the amount of the $x$ variable.

Our results are arising out completely from a theoretical framework. As can be seen in figs. 4–6, these results yield us an appropriate behavior in comparison with the results of refs. [24–27]. Furthermore, we have calculated the TMD strange ($s$) quark and gluon distribution functions, using our theoretical model. Based on the current quark models, our investigations indicate that so far computations of the TMD gluon and $s$-quark distributions have not been done.
4 Conclusion

A good understanding of parton densities can be obtained by studying the deep-inelastic lepton nucleon scattering. These studies provide us how the momenta of partons are distributed parallel to the nucleon momentum. To go beyond the one-dimensional consideration of the nucleon substructure, we need to investigate the transverse momentum dependence of the parton densities. This can be done by taking into account the transverse momenta of produced hadrons via processes, for instance, semi-inclusive DIS or dileptons resulted from the Drell-Yan process.

In this article we have used for the first time the modified $\chi$QM to achieve the transverse momentum-dependent parton densities in unpolarized case. For this propose, we have applied the Sullivan processes and suggested the TMD splitting functions which are being used in the $\chi$QM. Another gradient key in our calculations, is obtaining the TMD bare quarks. This would be possible if we convert properly the measure of related integration to the light cone coordinates. What we got for the TMD parton densities, is indicating an acceptable behavior with respect to the variation of transverse momentum and the $x$-Bjorken variable. Extending the calculations to the polarized case, using the modified $\chi$QM, is possible, which we hope to report in the future.

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