I address the phenomenology of CPT violation in the neutral kaon system under the assumption that it originates from Planck scale physics. This assumption opens the door to a new set of CPT violating parameters whose phenomenology is distinct from the $\Delta$ parameter usually considered in the Hamiltonian. The origin of these parameters reflects a possible departure from a $S$-matrix evolution. Existing bounds on CPT violation are near the expected range based on naive dimensional analysis. This provides a strong incentive to pursue the quest of CPT violation in near-future kaon experiments.
In 1983, S.W. Hawking\textsuperscript{1} extrapolated on earlier developments in the quantum theory of gravity, and proposed a generalization of quantum mechanics which allows the evolution of pure states to mixed states. This generalization of quantum mechanics departs from a $S$-matrix evolution and was shown to conflict\textsuperscript{2} with $CPT$ conservation.\textsuperscript{2} It is worth noting that departure from a unitary evolution predicted by quantum mechanics had already been given an experimental scrutiny in the early 70’s,\textsuperscript{3} without reference to a particular theoretical framework.

Soon after, Ellis, Hagelin, Nanopoulos, and Srednicki\textsuperscript{4} observed that systems which possesses a high degree of quantum coherence are most appropriate to probe the violation of quantum mechanics of the type proposed by Hawking. They conveniently wrote a differential form of Hawking’s generalization of the time evolution of a density matrix $\rho_K$, namely,

$$i \frac{d}{dt} \rho_K = H \rho_K - \rho_K H^\dagger + \delta \hat{h} \rho_K.$$  \hfill (1)

The first two terms on the RHS of this equation, accounts for the quantum mechanical evolution of the system, which is described by an Hamiltonian $H$. The third term accounts for the loss of coherence in the evolution of the beam; $\delta \hat{h}$ is a linear operator and is written so to require that it does not break conservation of probability and does not decrease the entropy of the system.

One of the simplest systems exhibiting “macroscopic” quantum coherence is a beam of neutral kaons. As it so happens, a major experimental consequence of the presence of the $\delta \hat{h}$-term in the $K_0-\overline{K}_0$ system is $CPT$ violation. There is no a priori reason for $CPT$ symmetry to be exactly conserved. $CPT$ conservation, however, arises naturally in the framework of a local, Lorentz-invariant, quantum field theory—the tool of modern theoretical physics. The immense success of the latter is the root of the conjecture that $CPT$ may be exactly conserved. Previous experimental searches for $CPT$ violation in departing for that conjecture, have assumed $\delta \hat{h} = 0$. This is the reason why $CPT$ violation has been parameterized with a single (complex) parameter $\Delta$, $= (\epsilon_S - \epsilon_L)/2$, in the effective Hamiltonian $H$, reflecting a difference in mass or in decay properties between $K_0$ and $\overline{K}_0$. One should remember, however, that the motivation for the $CPT$ violating perturbation $\delta \hat{h}$ arises from considering quantum gravitational effects, i.e., quantum fluctuations of the spacetime, which, until today, have not been successfully incorporated in a quantum field theory.\textsuperscript{5} In the future, the latter might turn out to describe those fluctuations, or, perhaps more likely, a more general (yet to be developed) framework might be required to meet the challenge of dealing with a “spacetime foam”. That framework will not be required to conserve $CPT$ symmetry. In fact, preliminary theoretical considerations suggest that it might not. Some of these considerations are: (a) Quantization of matter in presence of a gravitational background; the extrapolation of which suggests the introduction of the $CPT$-violating term $\delta \hat{h}$. (b) Local, but non-Lorentz-invariant, operators induced in the low energy world by the intrinsic non-local Planckian structure of string theory;\textsuperscript{5} these operators induce a non-zero value for the $CPT$ violating parameter $\Delta$ (cf. Alan Kostelecký’s contribution to these proceedings).\textsuperscript{5}

At the light of these speculative, yet suggestive, theoretical considerations, it is appropriate to seriously consider Planck scale physics as a prime candidate for inducing $CPT$ violation and to generalize the search of $CPT$ violation in the $K_0-\overline{K}_0$ system from the single parameter $\Delta$, to the more complete set $\{\Delta, \delta \hat{h}\overline{ij}/\Delta m\}$ (I have appropriately divided $\delta \hat{h}\overline{ij}$ with the mass

\textsuperscript{2} $CPT$ conservation could be preserved in a weaker form.
\textsuperscript{3} A low energy effective field theory which successfully describes aspects of quantum gravity, cannot insure $CPT$ conservation as it does not describe short distances physics.
\textsuperscript{4} For more speculative consequences of string theory leading to the generation of a $\delta \hat{h}$ term, see John Ellis’ contribution to these proceedings and references therein.
difference $\Delta m$, to obtain a dimensionless parameter). This addition has more than an academic value since these parameters have a different experimental signature — a nice feature when it comes to extracting them from data.\footnote{Although, it has been suggested in this workshop that $n$ could assume arbitrary non-integer values, I will restrict it to near-integer values as otherwise this exercise in dimensional analysis looses its interest.}

The assumption of Planckian physics as the origin of CPT violation has also the virtue of providing a scale which allows to make an order-of-magnitude estimate and so to give experimentalists a range to shoot for. One naively expects

$$
\Delta \sim \frac{\delta \hbar}{\Delta M} \sim \frac{E}{\Delta M} \left( \frac{E}{M_{Pl}} \right)^n.
$$

Here, $E$ is the energy of the system (say, the kaon mass), $1/\Delta m$ is the typical coherence time of the system (say the oscillation length of the kaon), and $n$ is an unknown power.\footnote{as well as CP and CPT perturbations compatible with quantum mechanics.}

In the particular case of the kaon system, we estimate $\{\Delta, \delta \hbar/\Delta m\} \sim 10^{-5}$ for $n = 1$; that is, about 1% of the total CP violation observed in the kaon system. This is precisely near the range accessible by current experiments. Higher values of $n$ renders the effect nearly inobservable. These estimates, taken at faith value, are very suggestive: CPT violation is at the reach of near-future experiments, otherwise, it is unlikely to be seen at all unless it originates from physics at a scale far below $M_{Pl}$.

Let us take a closer look at a beam of kaons evolving under Eq. (1) and understand how CPT violation originates in this particular system. The remaining of this paper follows closely the analysis of Ref. \cite{6} (See Ref. \cite{7} for an alternative analysis).

## Violation of CPT in the $K_0-\bar{K}_0$ system

Any observable $\langle P \rangle$ along the kaon beam can be computed by tracing the product of the density matrix $\rho_K(\tau)$ with an appropriate operator $O_P$, as $\langle P \rangle = \text{Tr}[\rho_K O_P]$. The time evolution of $\rho_K(\tau)$ is determined by Eq. (1) and is completely characterized by an effective Hamiltonian $H_e = M - \frac{i}{2} \Gamma$, which incorporates the natural width of the system and by a linear operator $\delta \hbar$. The latter is only constrained so as not to break conservation of probability and not to decrease the entropy of the system; that makes it expressible in terms of six parameters. In order to lower this number to a more tractable one, one neglects its strangeness violating components, reducing $\delta \hbar$ to three unknown positive parameters written as $\rho_L$, $\rho_S$, and $\rho_T$. They satisfy the relation $\rho_L > \rho_S^2$, they are dimensionless and might be as large as $m_{K}^2/(\Delta m m_{Pl}) \sim 3 \times 10^{-5}$. These are the parameters which eventually shift the observed value of the CP violating parameter $\epsilon \sim 2.32 \times 10^{-3}$. The solution of Eq. (1) is generally expressible as

$$
\rho_K(\tau) = A_L \rho_L^{(\phi)} e^{-\Gamma_L \tau} + A_S \rho_S^{(\phi)} e^{-\Gamma_S \tau} + \left( A_I \rho_I^{(\phi)} e^{-\Gamma_I \tau} e^{-i \Delta m \tau} + \text{h.c.} \right)
$$

The parameters $A_{S,L,I}$ are fixed by the production mechanism of the beam. In the absence of the quantum mechanics violating perturbation $\delta \hbar$, the eigenmodes $\rho_L$, $\rho_S$ and $\rho_T$, are expressible in terms of the pure states $|K_L\rangle$ and $|K_S\rangle$ as $\rho_L^{(\phi)} = |K_L\rangle \langle K_L|$, $\rho_S^{(\phi)} = |K_S\rangle \langle K_S|$ and $\rho_T^{(\phi)} = |K_L\rangle \langle K_L|$, while $\Gamma = (\Gamma_L + \Gamma_S)/2$, $\Delta \Gamma = \Gamma_S - \Gamma_L$ and $\Delta m = m_L - m_S$. After adding the quantum mechanics violating term $\delta \hbar$, the eigenmodes are changed to, in first order in small quantities ($\Delta m + i\Delta \Gamma/2 \sim i\sqrt{2} \Delta m e^{-i \phi_{SW}}$)

\[\text{(2)}\]
\[ \rho_L = \rho^{(\otimes)} + \frac{7}{2} \rho^{(\otimes)} - \frac{\beta}{\sqrt{2}} \left( i e^{i \phi_{SW}} \rho^{(\otimes)} + \text{h.c.} \right), \quad \rho_S = \rho^{(\otimes)} - \frac{\gamma}{2} \rho^{(\otimes)} - \frac{\beta}{\sqrt{2}} \left( i e^{-i \phi_{SW}} \rho^{(\otimes)} + \text{h.c.} \right) \]

\[ \rho_I = \rho^{(\otimes)} - \frac{\beta}{\sqrt{2}} i \left( e^{-i \phi_{SW}} \rho^{(\otimes)} + e^{i \phi_{SW}} \rho^{(\otimes)} \right) - \frac{\alpha}{2} (\rho^{(\otimes)})^\dagger. \] (4)

The corresponding eigenvalues are corrected by the shifts \( \Gamma_{L,S} \rightarrow \Gamma_{L,S} + \tau \cdot \Delta m, \, \bar{\Gamma} \rightarrow \bar{\Gamma} + \pi \cdot \Delta m \) and \( \Delta m \rightarrow \Delta m \cdot (1 - (\beta/8)^2) \).

The major effect of violation of quantum mechanics is embodied in the eigenmodes \( \rho_L, \rho_S, \rho_I \). These density matrices are no longer pure density matrices in contrast to their quantum mechanical counterparts (labelled with a diamond). This loss of purity alters the decay properties of the beam. For example, the properties of the beam at large time, \( \tau \gg 1/\Gamma_S \), are dominated by the properties of \( \rho_L \). The second term on the RHS of the equation for \( \rho_I \) as given in Eq. (4) is proportional to \( \rho^{(\otimes)}_S \) and is even under CP conjugation. That results in an enhancement of the rate of decay into two pions at late time in the evolution of the beam, proportional to \( \pi/2 \).

A similar argument leads to expect an enhancement by an amount \( \propto \bar{\beta} \cos \phi_{SW} \) and \( \beta \sin \phi_{SW} \) in the intermediate time region, \( \tau \sim 1/\bar{\Gamma} \). Furthermore, these effects distinguish between \( K_0 \) and \( \bar{K}_0 \) (as seen in an appropriate basis) and, consequently, violate CPT symmetry.

A proper method of extraction of the parameters \( |\Delta|, \bar{\alpha}, \bar{\beta} \) and \( \tau \) is a method which accounts for possible correlations or accidental cancelations among these parameters. Such a method was first proposed in Ref. [4], the essence of which is as follows. (1) \( \beta \) and \( \gamma \) are extracted by comparing measurements of the 2\( \pi \) decay rates \( R_L \) and \( \eta_{+-} = |\eta_{+-}| \exp(i \phi_{+-}) \) along with the semileptonic asymmetry \( \delta_L \). In quantum mechanics, these quantities relate according to \( R_L = |\eta_{+-}|^2 \) and \( \delta_L/2 = \Re \eta_{+-} \). After allowance has been made for violation of quantum mechanics, they relate according to \( R_L \simeq |\eta_{+-}|^2 + \tau/2 + 2\sqrt{\beta} |\eta_{+-}| \) and \( \delta_L/2 = \Re \eta_{+-} - \sqrt{2} \beta \sin \phi_{SW} \). The geometry of these corrections is given in Ref. [4]. (2) A linear combination of \( \beta \) and \( |\Delta| \) is constrained from the value of \( \phi_{+-} - \phi_{SW} = 0.21 \pm 0.6 \) (PDB'96) by accounting for the dominance of the isospin-0 \( \pi\pi \) decay channel. (3) Finally, \( \bar{\alpha} \) is obtained from the shift \( \bar{\Gamma} \rightarrow \bar{\Gamma} + \pi \cdot \Delta m \) mentioned above. This yields\footnote{- These bounds depart slightly from the ones obtained in Ref. [4]. This reflects a recent adjustment of the value of \( \phi_{+-} \) given in the PDB'96. - In obtaining these bounds, we set to zero the CPT violating quantum mechanics perturbations of the decay amplitudes in the two-pion and semi-leptonic channels. For more complete formulas, see [3].} \( \bar{\alpha} \leq 10^{-2}, \bar{\beta} = (0.2 \pm 0.67) \times 10^{-4}, \, \bar{\tau} = (0.3 \pm 0.54) \times 10^{-6} \) and \( \bar{\beta}/\sqrt{2} |\Delta| = (0.08 \pm 0.24) \times 10^{-4} \). These bounds are in agreement with the ones obtained in Ref. [4] under the assumption that \( \Delta = 0 \).

To answer a question asked during this workshop, neither of these parameters contributes more than 10% of the total CP violation observed in the \( K_0 - \bar{K}_0 \) system.

We conclude this section by noting that, these bounds are of the order of the expectations presented earlier on the assumption that CPT violation arises from Planck scale physics. This is an incentive to pursue the quest of CPT violation in the kaon system.

### Tests of quantum mechanics at a \( \phi \)-factory

At a \( \phi \) factory, a spin-1 meson decays to an antisymmetric state of two kaons which propagate with opposite momenta. If the kaons are neutral, the resulting wavefunction, in the basis of CP eigenstates \( |K_1 \rangle, |K_2 \rangle \), is \( \phi \rightarrow (|K_1, p > \otimes |K_2, -p > - |K_2, p > \otimes |K_1, -p >) \sqrt{2} \). The two-kaon density matrix resulting from this decay is a 4 \( \times \) 4 matrix \( P \), which, in the context of
generalized quantum mechanics, evolves according to Eq. (I). When expressed in terms of the eigenmodes \( \rho_L, \rho_S \) and \( \rho_I \), it takes the form\(^6\)

\[
P = \frac{1}{2} \left[ \rho_S \otimes \rho_L + \rho_L \otimes \rho_S - \rho_I \otimes \rho_I^\dagger - \rho_I^\dagger \otimes \rho_I \right]
+ \left[ \frac{\beta}{\sqrt{2}} i \left( \left( e^{i\phi_{SW}} \rho_S + e^{-i\phi_{SW}} \rho_L \right) \otimes \rho_I + \rho_I \otimes \left( e^{i\phi_{SW}} \rho_S + e^{-i\phi_{SW}} \rho_L \right) \right)
+ \alpha i \rho_I \otimes \rho_I + \frac{\Gamma}{2} \left( \rho_L \otimes \rho_L - \rho_S \otimes \rho_S \right) + \text{h.c.} \right].
\]

(5)

The time dependence of each term is obtained from the substitutions \( \rho_i \otimes \rho_j \rightarrow \rho_i \otimes \rho_j e^{\left( -\lambda_i \tau_1 - \lambda_j \tau_2 \right)} \) with \( \lambda_L = \Gamma_L, \lambda_S = \Gamma_S \) and \( \lambda_I = \Gamma + i \Delta m \). The first line (inside the first pair of brackets) has the canonical form predicted by quantum mechanics after the replacement \( \rho_K^{(\otimes)} \rightarrow \rho_K \), while the remaining terms (inside the second pair of brackets) have a peculiar dependence on \( \tau_1 \) and \( \tau_2 \) and provide an unambiguous method to isolate the \( \delta f_{ij} \) parameters from the quantum mechanics CPT violating perturbations.\(^6\) The basic observables computed from \( P \) are double differential decay rates, \( P(f_1, \tau_1; f_2, \tau_2) \), the probabilities that the kaon with momentum \( p \) decays into the final state \( f_1 \) at proper time \( \tau_1 \) while the kaon with momentum \( -p \) decays to the final state \( f_2 \) at proper time \( \tau_2 \). As an illustration, let us consider the decay into two identical final states \( f_1 = f_2 = f \). In quantum mechanics, the quantity \( P(f, \tau_1; f, \tau_2) \) has only an overall dependence on the choice of the final state \( f \): its dependence on the two times \( \tau_1, \tau_2 \) is entirely fixed by the initial antisymmetry of the wave function which is preserved by the Hamiltonian evolution. This characteristic is lost when violation of quantum mechanics is incorporated as in Eq. (I). One can, for instance, interpolate the double decay rates into identical final states \( P(f, \tau_1; f, \tau_2) \) on the line of equal time \( \tau_1 = \tau_2 \). This quantity vanishes identically according to the principles of quantum mechanics and is thus of order \( \overline{\alpha}, \overline{\beta} \) and \( \overline{\gamma} \). As an illustration, the semileptonic double decay rate at equal time yields(\( \ell^\pm \equiv \pi^\mp \ell^\pm \nu \))

\[
\frac{P(\ell^+, e^\mp; \ell^-, e^\mp)}{P(\ell^+, e^\pm; \ell^-, e^\pm)} = \frac{1}{2} \left[ 1 - e^{-2(\overline{\alpha} e^\mp - \overline{\alpha} e^\pm) \Delta m \tau} (1 - \overline{\alpha} \sin 2 \Delta m \tau) \right] + \frac{\overline{\gamma}}{2} \sinh(\Delta \Gamma \tau)
\pm 2 \sqrt{2} \overline{\beta} \left[ \sin(\Delta m \tau - \phi_{SW}) e^{-\Delta \Gamma \tau/2} + \sin(\Delta m \tau + \phi_{SW}) e^{+\Delta \Gamma \tau/2} \right].
\]

(6)

The three coefficients \( \overline{\alpha}, \overline{\beta} \), and \( \overline{\gamma} \) are selected by terms with different time dependences.

**Outlook**

One might be concerned that attempts to depart from unitarity may be plagued with inconsistencies and consequently be ruled out. This is indeed a possibility. For example, the authors of Ref. \(^7\) pointed out that the spurious time-dependence which arises in the evolution of correlated kaons resulting from the decay of a \( \phi \) (Eq. (I)), and which may signal violation of quantum mechanics at a \( \phi \)-factory, is a consequence of the loss of the initial antisymmetry of the wave function, otherwise preserved by a unitary evolution. This antisymmetry was initially guaranteed by conservation laws. Do these non-unitary effects intrinsically violate conservation laws?\(^8\) This possibility has been pointed out in Ref. \(^8\) and used to argue the inconsistency of Hawking’s idea. These arguments are however not full proof as shown, for example, in Ref. \(^9\).

Two attitudes are then possible: one may await the resolution of these issues on a theoretical note.\(^9\)

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\(^6\) That is, from the parameter \( \Delta \) a well as other CPT violating parameters in the decay amplitudes.

\(^7\) It was pointed out in Ref. \(^7\) that, in the extended framework given by Eq. (I), violation of a conservation law no longer implies violation of a symmetry principle.
ground or one may go on and resolve them in a laboratory. The latter seems to be the natural path to follow in the instance where existing theories are not able to address a question whose answer is at the reach of an experiment. As an example, let us contemplate for a moment, violation of angular momentum resulting from loss of unitarity in the simplest possible system: a spin $\frac{1}{2}$ atom in a magnetic field. Evolving that system with Eq. (1) results in the relaxation, $\sim \exp(-\delta h \tau)$, into an equal mixture of spin up and spin down, explicitly violating angular momentum conservation. Authors of Ref. [10], in their experiments on atomic electric dipole moments, have observed a spin $\frac{1}{2}$ Xe 129 atom precess and relax over a period of a few hundred seconds, providing the constraints $|\delta h| < 10^{-26}$ GeV. Furthermore, Helium 3 atoms, used as a gaseous polarized target for use with a polarized electron beam at SLAC, were observed to precess during a time larger than 75 hours, bringing the bound down to $\sim 10^{-31}$ GeV much below our naive expectations, $> 10^{-19}$ GeV. One can safely conclude that whether or not it is consistent to include a term $\delta h$ in the evolution of a spin $\frac{1}{2}$-system, this term, if present, ought to be too small to give observable consequences. This is a particular instance where experimentalists do not have to wait for theorists to develop a theoretical framework in order to probe a fundamental property of Nature. Similarly, CPT violation, if it originates from Planck scale physics, may be at the reach of future kaon experiments and a genuine phenomenological parameterization in terms of $\{\Delta, \delta h ij / \Delta m\}$ may be all that is needed to discover it or demonstrate its irrelevance to low energy physics.

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