Evaporative depolarization and spin transport in a unitary trapped Fermi gas

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We consider a partially spin-polarized atomic Fermi gas in a high-aspect-ratio trap, with a flux of predominantly spin-up atoms exiting the center of the trap. We argue that such a scenario can be produced by evaporative cooling, and we find that it can result in a substantially non-equilibrium polarization pattern for typical experimental parameters. We offer this as a possible explanation for the quantitative discrepancies in recent experiments on spin-imbalanced unitary Fermi gases.

I. INTRODUCTION

Two-component atomic Fermi gases provide an ideal experimental system in which to investigate fermion pairing and superfluidity in a controllable manner. For example, one can use a magnetically-tunable Feshbach resonance to access the unitary regime, where the scattering length diverges and one has a strongly-interacting fermionic superfluid that is ‘universal’. Of particular interest is the case where there is a spin imbalance that frustrates pairing between fermion species, because this is a situation that arises in many fields of physics, ranging from QCD to superconductivity. Here, a central question has been: what is the critical spin polarization $\delta_c$ at which pairing and superfluidity are destroyed for a unitary trapped Fermi gas at equilibrium? However, current experiments produce different answers. The experimental group at MIT finds that $\delta_c \simeq 77\%$ while experiments on highly-elongated trapped gases at Rice University suggest that $\delta_c$ is at least 90%. Moreover, even though the critical polarization is known to be a strong function of temperature, where $\delta_c$ decreases with increasing temperature, the different $\delta_c$'s observed in experiment are unlikely to be caused by differences in temperature since both experimental groups have claimed that their temperatures are low enough to yield $\delta_c$'s that are close to the zero-temperature result. Thus, there is a real discrepancy in the measured $\delta_c$ and a resolution of this problem has potentially important implications for the nature of the paired superfluid phase in a finite-sized system. Here we propose that the high $\delta_c$ observed in the Rice experiment was due to their trapped spin-imbalanced gas being out of equilibrium. We show that a combination of the trap geometry and the evaporative cooling scheme implemented in the Rice experiment can induce a spin current along the trap axis, which in turn creates a substantially non-equilibrium polarization pattern that favors a superfluid at the trap center.

To understand how such a spin current can be generated, one must first examine the evaporative cooling process. Here, the temperature, and entropy per atom, of a trapped gas is lowered when the most energetic atoms escape over the “lip” of the trap — the route of escape with the lowest potential barrier to be surmounted. For a partially-polarized Fermi gas at temperature $T$, the rate of thermal activation over this barrier is larger for the majority species by a factor of $\exp[(\mu_1 - \mu_\uparrow)/k_BT]$, assuming the two species are subject to the same trapping potential, where $\mu_1, \mu_\uparrow$ are the chemical potentials for the majority and minority species, respectively. Thus, at low $T$, the flux of evaporating atoms passing over the lip is essentially fully polarized, and we have evaporative depolarization in addition to evaporative cooling (as stated in Ref. 4). The Rice experiments we are considering had both a long, thin, high-aspect-ratio optical trap and a nonuniform magnetic field that contributed to the axial confinement of the gas. As a result, the lowest barrier for the atoms to escape from the trap was at the axial center, with the atoms escaping over this “lip” in the radial direction (and downwards, due to gravity; see Fig. 1(a)). Elsewhere in the trap the barrier to escape was significantly higher, so at their lowest temperatures, essentially all of the evaporating atoms escaped radially at the axial center of their high-aspect-ratio trap.

To achieve low temperatures in the Rice experiment, the height of this barrier was lowered by reducing the in-

FIG. 1: (Color online) Schematic diagram of atom transport during the evaporative cooling process in trapped, spin-imbalanced, highly-degenerate Fermi gases in the quasi-1D (a) and 3D (b) regimes. The flux of spin-up atoms $j_0$ is exiting the gas over the lip of the trap that is located at the axial center ($z = 0$). This flux must be drawn from the fully-polarized (FP) normal region. Thus, for a quasi-1D gas (a), a spin current must flow through the intervening partially-polarized (PP) normal region and across the normal/superfluid (SF) interface. In the 3D regime (b), the atoms can evaporate directly from the surrounding fully-polarized layer.
tensity of the optical trap. It was at the lowest temperatures that the unusually large $\delta_\parallel$ was observed, along with strong deviations from the equilibrium local density approximation (LDA) in the shapes of the regions occupied by the superfluid and partially-polarized normal phases. What we propose happened here is that the evaporation, with the $\uparrow$ atoms rapidly escaping radially over the trap lip, greatly depleted any excess unpaired $\uparrow$ atoms from the axially central region of the cloud occupied by the paired superfluid phase. This depletion, which is apparent in the $in situ$ density measurements, substantially suppressed $(\mu_\uparrow - \mu_\downarrow)$ in that region (evaporative depolarization). The flux of evaporating $\uparrow$ atoms over the lip then had to come from the fully-polarized normal regions at the axial ends of the cloud and be driven through the partially-polarized region and across the normal/superfluid interface by a substantial axial gradient of $(\mu_\uparrow - \mu_\downarrow)$. This resulted in the partially-polarized region of the cloud being much smaller in axial extent than it would be at equilibrium, which is the strong deviation from equilibrium LDA that we will focus on in this paper. There was another important deviation from equilibrium LDA: the aspect ratio of the central superfluid region was substantially reduced from that of the cloud as a whole; it is this latter feature that was emphasized in Ref. 3.

We emphasize that the non-equilibrium scenario described above assumes that the atom transport is effectively one-dimensional (1D) along the axial ($z$) direction, as illustrated in Fig. 1(a). Thus, it is only appropriate for high-aspect-ratio gas clouds with sufficiently low particle number, like those in the Rice experiments (aspect ratio $\gtrsim 30:1$) at the lowest temperatures. For higher temperatures (less evaporative cooling), where the particle numbers are larger, there is a fully-polarized layer of $\uparrow$ atoms fully surrounding the cloud (Fig. 1(b)), as can be seen in $in situ$ density measurements. In this case, the evaporation will simply draw $\uparrow$ atoms from this fully-polarized layer rather than driving a spin current through the superfluid and partially-polarized normal regions, so a strong chemical potential gradient is not produced. The MIT experiments, on the other hand, used trapped gases with a much lower aspect ratio ($\sim 5:1$) and at least an order of magnitude more atoms in the cloud. Thus, the 3D scenario for highly-polarized gases depicted in Fig. 1(b) remains correct for these experiments, even at the lowest temperatures, and so their observed $\delta_\parallel$ agrees with the equilibrium result from quantum Monte Carlo (QMC) calculations.

II. MODEL OF EVAPORATION

To model the spin transport in the Rice experiment at the lowest temperatures, we will assume that atoms are only removed from the gas at $z = 0$, the position of the trap lip, and that only $\uparrow$ atoms are evaporating (Fig. 1(a)). Also, we approximate any nonuniformities in the local chemical potentials as 1D, so that we only consider non-equilibrium axial gradients in $\mu_\uparrow$, $\mu_\downarrow$. Since we are primarily interested in the non-equilibrium polarization pattern within a trap, and this is not strongly dependent on the local temperature, we will ignore any gradients in temperature. We further assume that the atomic gas cloud is in mechanical equilibrium throughout the evaporation process, since the time required to equalize a pressure difference in the cloud is much smaller than both the duration of the evaporation and the time for spin diffusion along the length of the cloud. This implies that the local pressure $P$ always satisfies:

$$\frac{\partial P}{\partial z} = -n\frac{\partial V}{\partial z},$$

where $n \equiv n_\uparrow + n_\downarrow$ is the local total density and $V(z)$ is the axial trapping potential.

III. SPIN TRANSPORT IN A UNITARY NORMAL GAS

We can determine $\delta_\parallel$ of a spin-imbalanced Fermi gas in the presence of a spin current by focussing on the unitary normal gas at low temperatures. Here, the equation of state for the pressure at $T = 0$ is accurately known from QMC calculations and is given by:

$$P_N = \frac{2}{5}n_\uparrow \varepsilon_{F\uparrow} \left[ 1 - A \frac{n_\downarrow}{n_\uparrow} + \frac{m^*}{m} \left( \frac{n_\downarrow}{n_\uparrow} \right)^{5/3} + F \left( \frac{n_\downarrow}{n_\uparrow} \right)^2 \right]$$

where $m$ is the atomic mass, $m^* \approx 1.09m \simeq m$, $A \approx 0.99$ and $F \approx 0.14$. For the spin transport that we are interested in, we also expect Eq. (2) to provide a reasonable approximation for the pressure at low temperatures within the degenerate regime $T < \varepsilon_{F\uparrow}/k_B \equiv T_{F\uparrow}$, where $\varepsilon_{F\sigma} = \hbar^2(6\pi^2 n_\sigma)^{2/3}/2m$. In addition to Eq. (1), we require equations for the spin density $n_\sigma = n_\uparrow - n_\downarrow$ and spin current density $j_\sigma = j_\uparrow - j_\downarrow$. We will assume that the “DC” spin transport in the partially-polarized normal Fermi gas is diffusive, with the dissipation being driven by the interspecies interactions. Indeed, this situation may be regarded as the cold-atom analogue of spin Coulomb drag in electron systems. By transforming to the inertial reference frame where locally $j_\uparrow' + j_\downarrow' = 0$, i.e. where there is no net transport of the total mass density and the motion is purely diffusive, we obtain current densities:

$$j_\sigma' = -D_{\sigma} \left( \frac{\partial n_\sigma}{\partial z} - \frac{\partial n_\sigma q}{\partial z} \right),$$

where $\frac{\partial n_\sigma q}{\partial z}$ is the local equilibrium density gradient and $D_{\sigma}$ is the diffusion constant for each spin. One can determine $\frac{\partial n_\sigma q}{\partial z}$ as a function of the local densities using the LDA equilibrium condition for the chemical potentials of each spin:

$$\frac{\partial \mu_\sigma}{\partial z} = -\frac{\partial V}{\partial z}.$$
Note that this automatically implies that there are no gradients in the chemical potential difference ($\mu_1 - \mu_1$) at equilibrium. Combining Eq. (3) with the expression for the spin current density in this frame

$$j'_s = j'_\uparrow - j'_\downarrow = -D_s \left( \frac{\partial n_{\uparrow}^s}{\partial z} - \frac{\partial n_{\downarrow}^q}{\partial z} \right),$$

yields an expression for the spin diffusion constant:

$$\frac{1}{D_s} = \frac{1}{2} \left( \frac{1}{D_1} + \frac{1}{D_\uparrow} \right).$$

Transforming back to the lab reference frame then gives us the spin current density:

$$j_s = -D_s \left( \frac{\partial n_{\uparrow}}{\partial z} - \frac{\partial n_{\downarrow}^s}{\partial z} \right) + n_s v,$$

where the average net velocity $v = j/n$.

For a dilute gas with $s$-wave scattering, we have diffusion constants $D_s = \frac{1}{4} v_{\sigma}^2 \tau_\sigma$, where $v_{\sigma}$ is the average velocity of the random particle motion and $1/\tau_\sigma$ is the scattering rate of each species. In the regime $T \lesssim T_{F\uparrow} < T_{F\downarrow}$, the scattering rate for the $\uparrow$ scattering rate since the mean free paths $l_{\uparrow} = v_{\uparrow} \tau_\sigma$ are simply related by the densities: $l_{\uparrow} = l_{\downarrow}(n_{\uparrow}/n_{\downarrow})$. One can see this from the fact that $l_{\sigma} = 1/\sigma_{\sigma \sigma} n_{\sigma}$ in a dilute gas, where the scattering cross section $\sigma_{\sigma \sigma}$ is a universal function of the density at unitarity. Also, we can approximate the velocities as $v_{\sigma} \simeq \hbar k_{F\sigma}/m$. Thus, we obtain the ‘universal’ spin diffusion constant:

$$D_s \simeq \frac{4 \pi^3 A^2}{m^2} \frac{v_{\sigma}^2}{\hbar} \left[ 1 + \left( \frac{n_{\uparrow}}{n_{\downarrow}} \right)^{4/3} \right]^{-1}.$$

Note that the dependence of $D_s$ on temperature is that of a Fermi liquid, as expected. Experiments to explicitly measure this ‘universal’ transport constant would be welcome.

Referring to Fig. 1(a), if the fully-polarized normal regions at the axial ends are sufficiently large (i.e. if the gas has a sufficiently high global polarization), then we can treat them as stationary spin current sources and approximate the spin current through the partially-polarized region as steady-state:

$$\frac{\partial n_{\sigma}}{\partial t} = - \frac{\partial j_s}{\partial z} \simeq 0.$$

In this case, by solving Eq. (7) with $j_s = j = j_0$, together with Eq. (1), we find that the deviation of the total polarization from equilibrium for a given local polarization at the trap center is determined by the simple dimensionless quantity:

$$Q \equiv \frac{|j_0| L}{n_{\downarrow}(z = 0)} \left( \frac{T}{T_{F\uparrow}(z = 0)} \right)^{1/2} \frac{m}{\hbar},$$

where $L$ is the total axial length of the cloud. The critical total polarization $\delta_c$ for a trapped cloud is obtained by setting the local densities $n_{\uparrow}/n_{\downarrow} \cong 0.44$ at the trap center, which is its value at the superfluid-normal transition in the uniform system. This corresponds to the situation where a trapped normal gas is on the verge of forming a superfluid core. To obtain a quantitative estimate of $\delta_c$ as a function of $Q$ in a high-aspect-ratio harmonic trap, we determine the chemical potentials as a function of $z$ and then use LDA in the radial direction $r$ to now include the radial trapping potential $V_{\perp}(r)$ and obtain the densities of each spin:

$$n_{\sigma}(z, r) = n_\sigma \left[ \mu_\sigma(z) - V_\perp(r), \mu_\sigma(z) - V_\perp(r) \right].$$

Integrating these densities numerically then yields the total polarization $\delta \equiv (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$. As depicted in Fig. 2, the critical polarization $\delta_c$ corresponds to the equilibrium QMC result when $Q = 0$, but it dramatically increases with increasing $Q$ and asymptotically approaches 100% polarization as $Q \to \infty$. This increase in $\delta_c$ is due to the partially-polarized region being compressed along the axial direction relative to its extent in equilibrium LDA.
(inset of Fig. 2), an effect which has been observed in the Rice experiments. To obtain $\delta_c \simeq 90\%$, we require an evaporation rate of order $10^6$ atoms/sec if we use typical parameter values in the Rice experiment towards the end of evaporation: $T/T_F \simeq 0.05$, $L \simeq 1$ mm, $n_0 \simeq 10^{12}$ cm$^{-3}$, $\hbar/m \simeq 10^4$ pm$^2$/s, and a trap radius of 10 $\mu$m. This appears consistent with the data in Ref. 3 since the actual evaporation rate is estimated to be roughly $10^6$ atoms/sec at the time when the optical absorption images are taken.\textsuperscript{14}

IV. SUPERFLUID/NORMAL INTERFACE

For completeness, we now examine the effect of a superfluid core on the spin current in the trapped gas when $\delta < \delta_c$. For this purpose, we consider spin transport through the superfluid/normal interface. We approximate the interface to be a 1D sharp step-function in the local superfluid gap $\Delta(z)$ and densities (Fig. 3), which is reasonable when $T$ is well below the temperature at the tricritical point.\textsuperscript{15} Clearly, our assumption of mechanical equilibrium implies that the pressures in each phase are equal at the interface: $P_N = P_{SF}$. Since we are focusing on low $T$, we will use the $T = 0$ expression for the superfluid pressure:

$$P_{SF} = \frac{2}{15\pi^2} \left( \frac{2m}{\xi \hbar^2} \right)^{3/2} \mu_{SF}^{5/2},$$  

where $\xi \simeq 0.42$ according to QMC calculations\textsuperscript{12} and the superfluid chemical potential $2\mu_{SF} \equiv \mu_{SF}^{(SF)} + \mu_{SF}^{(N)}$. We further assume that any quasiparticles in the superfluid region are rapidly evaporated so that we can neglect any quasiparticles incident on the interface from the superfluid side. Effectively, this amounts to assuming a strong drop in the local temperature of the quasiparticles as one crosses the interface from the normal to the superfluid side. While this is not essential to our picture, this gives a simplification and likely captures what happens when the lip of the trap is held very low.

The scattering problem at the superfluid/normal interface has already been examined in Ref.\textsuperscript{10}. However, they only focussed on thermal transport via quasiparticle transmission through the interface. In our case, we must consider both mass and spin transport across the interface, since they can both change the local polarization. We must therefore take account of any mass transported via Andreev reflection. For example, if the chemical potentials $\mu_{SF}^{(N)}$, $\mu_{SF}^{(N)}$ in the normal phase lie within the gapped region of the superfluid, like in Fig. 3 the Andreev process results in a flux of pairs (mass) into the superfluid when $2\mu_{SF} < \mu_{SF}^{(N)} + \mu_{SF}^{(N)}$ and a flux out of the superfluid when $2\mu_{SF} > \mu_{SF}^{(N)} + \mu_{SF}^{(N)}$, even at $T = 0$.

Following the approach of Ref.\textsuperscript{20}, we can write the current density of $\uparrow$ atoms flowing into the superfluid as:

$$j_\uparrow = \frac{1}{(2\pi)^3 \hbar} \int dE \int dk \left[ k_\uparrow (1 - B_\uparrow) f(E - \mu_{SF}^{(N)}) - k_\downarrow A_\downarrow f(E - 2\mu_{SF} + \mu_{SF}^{(N)}) \right]$$

where $k_\uparrow (k_\downarrow)$ is the momentum normal to the interface of the incident $\uparrow$ atom (reflected $\downarrow$ hole), $E$ is the energy and $f(x)$ is the Fermi function. The coefficients $A_\downarrow$ and $B_\uparrow$ correspond to the probability of Andreev reflection and ordinary reflection, respectively. Clearly, when $|E - \mu_{SF}| < \Delta$, there is no quasiparticle transmission and we must have $k_\uparrow = k_\downarrow B_\downarrow + k_\uparrow A_\uparrow$. Also, for $E < \Delta + \mu_{SF}$ (appropriate for low $T$), the current density of $\downarrow$ holes crossing the interface into the superfluid has the same expression as in Eq. (14), but with $\uparrow$ and $\downarrow$ interchanged, and $f(x) \rightarrow f(-x)$. Note that although our calculation of reflection probabilities relies on the mean-field Bogoliubov-de Gennes equations, the chemical potentials, pressure and superfluid gap that we use are from $T = 0$ QMC results\textsuperscript{12,17,18}.

Referring to Fig. 3 if we consider chemical equilibrium, where $2\mu_{SF} = \mu_{SF}^{(N)} + \mu_{SF}^{(N)}$, then at finite $T$ we find that we have both a flux of $\uparrow$ atoms entering the superfluid region and a flux of $\downarrow$ atoms leaving the superfluid region (or a flux of $\downarrow$ holes flowing in the opposite direction). However, atoms, not holes, are removed during evaporative cooling and thus we require the flux $j_\downarrow = 0$ across the interface. This is achieved by setting $2\mu_{SF} \gtrsim \mu_{SF}^{(N)} + \mu_{SF}^{(N)}$ such that the concomitant mass current flowing into the superfluid cancels the $\downarrow$ hole current, so there is only a net motion of $\uparrow$ atoms. Thus, the superfluid core ensures that the flux of evaporating atoms remains spin-polarized even when ($\mu_\uparrow - \mu_\downarrow$) is suppressed at the trap center due to local evaporative depolarization.
V. CONCLUDING REMARKS

To summarize, we have shown that evaporative cooling of a trapped, partially-polarized, high-aspect-ratio atomic Fermi gas can create a spin current which in turn can produce the large $\delta_c$ seen in the Rice experiment. Moreover, when this low-temperature gas cloud develops a superfluid core, we find that the atom flux across the superfluid-normal interface remains spin-polarized, with a small drop in the average chemical potential across the interface. An alternative equilibrium explanation for the large $\delta_c$ in the Rice experiment is that finite-size effects in a high-aspect-ratio trapped gas are responsible. However, microscopic studies of the surface tension at the superfluid/normal interface in a trapped gas suggest that this scenario is not quantitatively consistent with experiments. Two possible avenues for the experiments to differentiate between equilibrium and non-equilibrium scenarios are: (1) To explicitly look for relaxation (on time scales that we estimate to be $\lesssim 0.1$ sec) towards equilibrium after deepening the optical trap and thus slowing or stopping the evaporative depolarization; and (2) To move the trap lip off-center with respect to the optical trap minimum by moving the minimum of the magnetic potential. If our non-equilibrium explanation is correct, at the lowest temperatures the superfluid core should form where the local evaporative depolarization is occurring at the trap lip: at equilibrium the superfluid will instead form at the minimum of the overall trapping potential where the density is highest. Indeed, a very recent experiment by Salomon’s group at the ENS already casts some doubt on the alternative equilibrium explanation: their trap aspect ratio ($\sim 22:1$) approaches that used in the Rice experiment, yet they observe a $\delta_c$ that agrees with the MIT result. Whilst it is still possible that this lower $\delta_c$ may be because the temperature in the ENS experiment is higher, another key difference between the Rice and ENS experiments is that the time scale for evaporation in the ENS experiment is longer, a fact which appears to be consistent with our non-equilibrium proposal.

More generally, the non-equilibrium scenario we have analyzed here suggests a route towards the realization and investigation of spin currents in a strongly-interacting Fermi gas. Moreover, it illustrates the importance of non-equilibrium phenomena in cold-atom systems. In typical experiments with ultracold atoms, the dynamic range between the microscopic time scales for atom motion, atom-atom interactions and the total duration of the experiments is not so large, often 5 orders of magnitude or less. This means that full equilibration of a large cloud of atoms might not be possible under some circumstances. Here we have discussed a trapped unitary Fermi gas, where the microscopic collision rate at low temperature is roughly $\varepsilon_F/\hbar$. When one instead considers atoms in an optical lattice, the microscopic rates for atom hopping or for superexchange can be substantially slower than this, making it even easier to drive the system out of equilibrium.

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