The basic principles of quantum physics such as quantum superpositions and entanglement have already had a major impact on the scientific world view. These phenomena are typically far removed from our everyday experience. It is of interest to explore various ways of bringing quantum phenomena closer to the macroscopic level, and to everyday life. One possible approach is to ask whether it might be possible to perform quantum optics experiments with human-eye detectors [1]. Quantum cloning of single photon states via stimulated emission has recently allowed the experimental creation of tens of thousands of clones starting from a single photon [2]. Here we show that cloning by stimulated emission is a very promising approach for the realization of quantum experiments with human-eye detectors.

The photon detection characteristics of the human eye have been studied in significant detail starting with Ref. [3]. Our results are based on the following theoretical model which describes the experimental evidence very well [3]. The eye is modeled as an ideal threshold detector preceded by very significant losses [9]. The eye is modeled as an ideal threshold detector preceded by very significant losses [9]. The eye is modeled as an ideal threshold detector preceded by very significant losses [9]. The eye is modeled as an ideal threshold detector preceded by very significant losses [9].

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Cloning by stimulated emission was originally introduced in the context of universal cloning [11], i.e. in a setting where all input states are treated equally. Here we focus instead on phase-covariant cloning by stimulated emission [6], in order to stay close to the experiments of Refs. [6, 12]. A phase-covariant cloner makes good copies only of input states that lie on a great circle of the Bloch sphere, e.g. the equator. Considering qubits realized by the polarization states of single photons in a spatial mode $a$, a phase-covariant cloner can be realized based on stimulated collinear type-II parametric down-conversion [6], where the appropriate Hamiltonian for the down-conversion process is $H = i\chi a_H^\dagger a_L^\dagger a_v + h.c.$, where $\chi$ is proportional to the non-linear susceptibility of the crystal and to the pump power, and $a_H$ and $a_V$ are the horizontal and vertical polarization modes corresponding to the spatial mode $a$. Identifying $a_H$ and $a_V$ with the north and south poles of the Bloch sphere, one can introduce a basis of “equatorial” modes $a_\phi$ and $a_{\phi\perp}$ via the relations $a_H = \frac{1}{\sqrt{2}}e^{i\phi}(a_{\phi} + i a_{\phi\perp})$, $a_V = \frac{1}{\sqrt{2}}e^{-i\phi}(a_{\phi} - i a_{\phi\perp})$. Different choices of the phase $\phi$ correspond to different bases. Rewriting $H$ in terms of $a_\phi$ and $a_{\phi\perp}$ gives $H = \frac{1}{\sqrt{2}}(a_\phi^{+\dagger} + a_{\phi\perp}^{+\dagger}) + h.c.$: one can see that $H$ has the same form for any choice of equatorial basis. This is why...
the cloning process is phase covariant. We will assume that a choice of basis has been made and denote the corresponding equatorial modes by $a$ and $a_\perp$ for compactness of notation.

We now show that the multi-photon states obtained by cloning single-photon qubits via stimulated emission can be distinguished with the naked eye with a high probability for a conclusive result and high fidelity. Consider the two orthogonal single-photon qubit states $a^\dagger|0,0\rangle = |1,0\rangle$ and $a_\perp^\dagger|0,0\rangle = |0,1\rangle$. The time evolution operator for the cloning process is $e^{-iHt} = UU_\perp$ with $U = e^{g(a_\perp^* a_\perp)}$, $U_\perp = e^{g(a_\perp^* a_\perp)}$, where we have defined the amplification gain $g = \chi t$, with $t$ the interaction time for the down conversion process. After the amplification, the qubit states become

\begin{align}
|\Phi\rangle &= UU_\perp|1,0\rangle = |A_1\rangle|A_0\rangle_\perp,
|\Phi_\perp\rangle &= UU_\perp|0,1\rangle = |A_0\rangle|A_1\rangle_\perp,
\end{align}

where we have introduced the notation $|A_1\rangle = U|1\rangle$, $|A_0\rangle = U|0\rangle$, and analogously for the perpendicular modes. It is easy to show, e.g. by integrating the equations of motion in the Heisenberg picture, that $U^*a^\dagger U = \cosh(g)a^\dagger + \sinh(g)a$, which allows one to calculate the mean photon numbers in the two states $|A_0\rangle$ and $|A_1\rangle$. $\langle A_1|a^\dagger a_\perp|A_1\rangle = 3\sinh^2(g) + 1$, and $\langle A_0|a^\dagger a_\perp|A_0\rangle = \sinh^2(g)$. This shows that stimulating the down-conversion process with a single photon leads to an approximate tripling of the resulting output photon number compared to a vacuum input (for large $g$).

Our proposal for distinguishing $|\Phi\rangle$ and $|\Phi_\perp\rangle$ using human eyes as detectors, which is illustrated in Fig. 1, is based on this significant difference in typical photon numbers between the states $|A_1\rangle$ and $|A_0\rangle$, in combination with the fact that the eye is a (smooth) threshold detector. The amplification gain $g$ can be adjusted in such a way that $|A_1\rangle$ will give a detection by the eye with high probability (i.e. it is “above the threshold”), whereas $|A_0\rangle$ will not (it is “below the threshold”). Under these conditions, separating the two modes $a$ and $a_\perp$ and directing each of them to one eye $|1\rangle$, $|\Phi\rangle$ will mostly give rise to detections in the eye exposed to mode $a$, whereas $|\Phi_\perp\rangle$ will mostly give rise to detections in the eye exposed to mode $a_\perp$.

Since the eye is not a perfect threshold detector, and since the photon number distributions in the two states $|A_0\rangle$ and $|A_1\rangle$ have large variances [13], there will be also events where both eyes detect something, where none of the eyes detect anything, or even where only the “wrong” eye responds. Introducing the notation $p(y, n|\Phi)$ for the probability of a detection (“yes”) in mode $a$ and no detection (“no”) in mode $a_\perp$, given the state $|\Phi\rangle$, and analogously for the other cases, one can then define the probability for a conclusive measurement, corresponding to a detection in only one eye, as

$$\varepsilon = p(y, n|\Phi) + p(n, y|\Phi) = p(y, n|\Phi_\perp) + p(n, y|\Phi_\perp),$$

where the equality follows from Eq. (1) (and $\varepsilon$ stands for “efficiency”). The accuracy of the measurement can be quantified via the visibility $V$, defined as

$$V = \frac{p(y, n|\Phi) - p(n, y|\Phi)}{p(y, n|\Phi) + p(n, y|\Phi)}.$$ 

Based on the above model of the eye as a photon detector, the probabilities can be expressed as $p(y, n|\Phi) = \langle A_0|\tilde{E}_n|A_1\rangle \langle A_0|\tilde{E}_y|A_0\rangle$, and analogously for the other probabilities. In order to evaluate the expectation values of $\tilde{E}_n$ and $\tilde{E}_y$, one has to evaluate general terms of the form $P_{mn}^{(\Phi)} = \langle A_0|C_L^m|m\rangle|C_L|A_0\rangle$ and $P_{mn}^{(\Phi)} = \langle A_1|C_L^m|m\rangle|C_L|A_1\rangle$. The projector on a Fock state $|m\rangle$ can be written as $|m\rangle|m\rangle = \delta_{m,a,m} \frac{1}{z_0} \int_{\bar{z}_0} \frac{2\pi}{dz} e^{-ik(a^\dagger a - m)}$. The above expressions can be evaluated using operator ordering techniques that follow Ref. [14]. As a first step one can show that $U = e^{\frac{1}{2} \tanh g a^\dagger e^{-\ln(\cosh g)}(a^\dagger a + \frac{i}{2}) e^{-\frac{1}{2} \tanh g a^2}}$ and $C_L = e^{\frac{1}{2} \tan g a^\dagger e^{\ln(\cosh g)} a^\dagger a [0]}$. Furthermore $C_L e^{-ika^\dagger a} C_L = e^{\ln(1 - \eta - e^{-ik}) a^\dagger a}$, where we have introduced the expression $X_0 = (1 - \eta + ne^{-ik})^{-1}$, which allows us to evaluate $U^*C_L^2 e^{-ik a^\dagger a} C_L U = U^* e^{-\ln(X_0) a^\dagger a} U = \frac{1}{z_0} e^{-\ln(z_0) a^\dagger a + \frac{i}{2} \tan g a^\dagger a [0]} e^{-\frac{1}{2} \tanh g a^2}$, with $X = X_0 \cosh^2 g - \frac{\sinh^2 g}{z_0}$, and $Z = \frac{1}{z_0} \partial_y X$. This gives $\langle A_0|C_L^m e^{-ika^\dagger a} C_L|A_0\rangle = \frac{1}{z_0} e^{-\ln(X_0) a^\dagger a} \langle A_0|C_L^m e^{-ik} a^\dagger a C_L|A_0\rangle = \frac{1}{z_0} e^{-\ln(X_0) a^\dagger a} \langle A_1|C_L^m e^{-ik} a^\dagger a C_L|A_1\rangle = \frac{1}{z_0} e^{-\ln(X_0) a^\dagger a} \langle A_1|C_L^m e^{-ik} a^\dagger a C_L|A_1\rangle = \frac{1}{z_0} e^{-\ln(X_0) a^\dagger a} \langle A_1|C_L^m e^{-ik} a^\dagger a C_L|A_1\rangle$.

These results make it possible to calculate the de-
FIG. 2: Efficiency ε and visibility V, defined in Eqs. (2) and (3), of the human-eye detection method for amplified single-photon qubits, as a function of the mean photon number after amplification \( \langle N_a \rangle \) (thick lines). The efficiency has a maximum of \( \varepsilon = 0.61 \) for \( \langle N_a \rangle = 288 \). The visibility never drops below \( \frac{1}{\sqrt{2}} \), which is relevant for Bell experiments in the micro-macro setting of Refs. [7, 13], cf. text and Fig. 3. We also show \( V \) and \( \varepsilon \) for the case of additional losses after the amplification, corresponding to overall transmission factors \( \frac{1}{2} \) (thin lines) and \( \frac{1}{4} \) (dashed lines).

Let us now apply these results to one particular interesting experimental situation, namely the micro-macro scenario of Refs. [7, 13], see also Fig. 3. In these experiments, a first low-gain down-conversion process creates an entangled photon pair into the two distinct spatial modes \( a \) and \( b \) in a polarization singlet state, \( |\psi_-\rangle = \frac{1}{\sqrt{2}}(a_H^1b_V^1 - a_V^1b_H^1)|0, 0, 0, 0\rangle \), where \( |0, 0, 0, 0\rangle \) denotes the vacuum for all participating modes. Thanks to the rotational invariance of the singlet, this can be rewritten in an equatorial mode basis as \( |\psi_-\rangle = \frac{1}{\sqrt{2}}(a^1_1b^1_\perp - a^1_\perp b^1_1)|0, 0, 0, 0\rangle \). The photon in the \( b \) spatial mode is detected directly, whereas the photon in the \( a \) mode is greatly amplified with the phase-covariant cloning process described above, leading to a micro-macro entangled state

\[
|\Psi_-\rangle = \frac{1}{\sqrt{2}}(|\Phi_a\rangle_0, 1)_b - |\Phi_\perp\rangle_a|1, 0\rangle_b)
\]

still written in the equatorial basis for both spatial modes). The capability of human-eye detectors to distinguish the two states \( |\Phi\rangle \) and \( |\Phi_\perp\rangle \) with high visibility implies the possibility of observing a violation of the CHSH Bell inequality with the same visibility for this entangled state, provided that the detection of the unamplified photon in mode \( b \) does not introduce any errors. Note that measurements in two different equatorial bases for both systems \( a \) and \( b \) are sufficient for testing the CHSH inequality. The detection of the unamplified photon also serves as a trigger, signaling that a pair has indeed been produced in the low-gain down-conversion.

It is worth noting that the proposed measurement by human eye, although clearly motivated by conceptual rather than practical considerations, can in fact be orders of magnitude more efficient than the “orthogonality filter” technique used in Refs. [7, 13], for which the success probability is of order \( 10^{-4} \). This is because the eye as a threshold detector is extremely well suited to the task of discriminating the states \( |\Phi\rangle \) and \( |\Phi_\perp\rangle \).

The robustness of the visibility with respect to losses shown in Fig. 2 means that a strong Bell inequality violation could be achieved for arbitrarily high losses, provided that the amplification is sufficiently strong. This is paradoxical at first sight, since losses are clearly going to affect the micro-macro entanglement, as information about the macro-state (\( |\Phi\rangle \) or \( |\Phi_\perp\rangle \)) leaks into the environment. Even in the case where there are only the losses intrinsic to the eye, i.e. for \( \eta = 0.08 \), most of the photons are lost, such that the environment contains almost all the available information, which means that the remaining micro-macro entanglement must be quite small. So how can the visibility of the Bell violation remain so high?

This apparent paradox can be resolved by realizing that, while the efficiency of the proposed detection method is quite high, it is always significantly smaller than one, such that the measurement is nevertheless post-selective. Moreover, whereas in the lossless case the macro-system lives in a two-dimensional Hilbert space spanned by \( |\Phi\rangle \) and \( |\Phi_\perp\rangle \), in the presence of losses it lives in a much larger (in fact, in principle infinite-dimensional) space. Together these two facts open up
an important "loophole". Conclusive (i.e. \( (y,n) \) or \( (n,y) \)) results for different equatorial bases correspond to different, almost orthogonal, subspaces of the high-dimensional Hilbert space. It is not difficult to construct separable multi-photon states that exploit this loophole to achieve the same visibility as in Fig. 2. The experimental observation of such a visibility by itself therefore allows no conclusion about the existence of micro-macro entanglement.

Nevertheless, the same measurements do allow one to prove the entanglement of the original entangled pair before amplification. From this perspective, the amplification and losses can be simply seen as part of the detection process for the original single photon. The Hilbert space of the original photon is only two-dimensional, so there is no risk of different subspaces being detected for different choices of measurement basis. Moreover, the detection efficiency is independent of the choice of equatorial basis thanks to the phase covariance of the amplification. For proving non-locality (as opposed to just entanglement), there is still the usual detection loophole due to the limited measurement efficiency. However, it is no more severe than for any other detection method that has comparable efficiency. Let us note that the amplification-based detection is different from conventional photon detection in one interesting way. Namely, in the present scenario the choice of detection basis can be made after the amplification process. This is quite different compared to conventional discussions of the measurement process, where the amplification only occurs after the choice of basis.

Briefly relaxing our focus on human eyes as detectors, we now show that proving genuine micro-macro entanglement in the presence of losses is possible using measurements that are not post-selective. Using the same methods as in Ref. 13, one can derive the following condition, which has to be fulfilled for all separable states: \( \langle J_a \cdot J_b \rangle \leq \langle N_a N_b \rangle \). As in Ref. 13, \( J_a \) and \( J_b \) are the Stokes (polarization) vectors corresponding to two different spatial modes of the light field, and \( N_a \) and \( N_b \) are the corresponding photon number operators. In particular, one can choose a convention where \( J_a = a^\dagger a H - a^\dagger a V \) and \( J_x = a^\dagger a - a^\dagger a \), i.e. the \( x \) direction is identified with the arbitrary phase choice \( \phi \) that was used to define the modes \( a \) and \( a^\dagger \) above. Let us emphasize that the dynamics of \( J_y \) (in fact, of any Stokes vector component in the \( x-y \) plane) will be exactly the same as that of \( J_x \). For our micro-macro scenario, the state of \( b \) is a single-photon state, leading to the simplified criterion \( |J_a \cdot \sigma_b| \leq \langle N_b \rangle \), where \( \sigma_b \) is the vector of Pauli spin matrices.

This means that we have to evaluate in particular \( \langle J_a \cdot \sigma_b \rangle = \langle \Psi_- | C^\dagger_{La} J_k a^\dagger \sigma_b C_{La} | \Psi_- \rangle \) for \( |\Psi_- \rangle \) from Eq. (1). One can show quite easily that \( \langle \Psi_- | C^\dagger_{La} J_a a^\dagger \sigma_b C_{La} | \Psi_- \rangle = \eta \), whereas for the equatorial components \( \langle \Psi_- | C^\dagger_{La} J_x a^\dagger \sigma_b C_{La} | \Psi_- \rangle = \eta \langle a^\dagger a | a^\dagger a \rangle - \langle a^\dagger a | a^\dagger a \rangle = \eta (2 \sinh^2 g + 1) \). On the other hand, \( \langle N_a \rangle = \langle \Psi_- | C^\dagger_{La} a^\dagger a + a^\dagger a \rangle C_{La} | \Psi_- \rangle = \eta (\langle a^\dagger a | a^\dagger a \rangle + \langle a^\dagger a | a^\dagger a \rangle) = \eta (4 \sinh^2 g + 1) \), which finally yields \( \langle J_a \cdot \sigma_b \rangle - \langle N_a \rangle = 2 \eta \). One can see that the violation of this genuine micro-macro entanglement criterion is sensitive to photon loss as expected. However, some micro-macro entanglement persists even for high loss. Note that experimentally demonstrating micro-macro entanglement in this way would require counting large photon numbers with single-photon accuracy. As a consequence of the above-mentioned "loophole", we are not aware of a way of demonstrating genuine micro-macro entanglement in the presence of losses with human eye detectors, or with the orthogonality filter technique of Refs. 12, 13.

We have shown that quantum experiments with human eyes as detectors appear possible, based on a realistic model of the eye as a photon detector. We note that these results remain valid even if losses not only after, but also before and during the amplification process, are taken into account, which is possible using similar techniques as in the present paper 10. Motivated by recent experiments 12, 13, we focused on a micro-macro scenario, but other experiments, such as bunching of amplified single photons, or macro-macro experiments where both photons from an original pair are amplified, can also be considered. The latter scenario requires a heralded source of photon pairs, since otherwise the amplified vacuum, which is invariant under equatorial rotations, will dominate all detections. Moreover universal cloning can be considered instead of phase-covariant cloning. The proposed experiments will require quantum amplifiers that operate at visible wavelengths, and pulse durations that are adapted to the timescales of the human eye. We intend to address all these points in more detail in a future publication 11.

This work was supported by the Swiss NCCR Quantum Photonics and by the EU Integrated Project Qubit Applications. We thank M.J. Collett, F. De Martini, S. Gonzalez-Andino and A. Lvovsky for helpful comments and useful discussions.

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In fact, one can show that the achievable visibility of the correlations for this approach is only of order 0.03, whereas a visibility of $\sqrt{2} = 0.71$ is required for violating the CHSH Bell inequality. This will be shown in more detail in Ref. [19].

Note that the eyes should not belong to the same person, since a single observer is usually not aware which of his eyes sees a particular light signal.

For example, the following separable $N + 1$ photon state achieves a violation of the Bell inequality with post-selection: $\rho = \frac{1}{\pi} \int_0^{2\pi} d\phi U(\phi) |N,0\rangle |0,1\rangle |N,0\rangle |0,1\rangle |U(\phi)^\dagger,$ where $U(\phi)$ is a rotation of the whole system around the z axis by an angle $\phi$. The same model also achieves high visibility for the orthogonality filter measurements of Refs. [7, 13]. These points will be discussed in more detail in Ref. [19].