A method of diagonalization for sfermion mass matrices

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Abstract: We present a method of diagonalization for the sfermion mass matrices of the minimal supersymmetric standard model (MSSM). It provides analytical expressions for the masses and mixing angles of rather general hermitian sfermion mass matrices, and allows the study of scenarios that extend the usual constrained - MSSM. Three signature cases are presented explicitly and a general study of flavor changing neutral current processes is outlined in the discussion.

Keywords: FCNC, sfermion mass matrix, A-terms.

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1. Introduction

The Minimal Supersymmetric Standard Model (MSSM) is one of the main extensions of the Standard Model (SM). Its main motivation is its natural resolution to the hierarchy problem. Its basic structure is obtained by assuming that there is (low-scale) supersymmetry which then immediately predicts the existence of a so-called super-partner for each of the SM particles. Additionally it requires an extension of the usual (minimal) scalar sector of the SM to two Higgs doublets \([1, 2]\).

Supersymmetry, if at all present, must be broken and the MSSM must contain this information \([3]\). Since the actual mechanism of supersymmetry breaking that would lead to the MSSM is still an open problem, the best one can do is to assume that supersymmetry is broken softly (in order not to spoil the supersymmetry based solution to the hierarchy problem). The result is that all super-partners are assumed to have soft masses that in turn become unknown free parameters of the model. One can then try to use physically sensible assumptions as to their magnitudes and hierarchies and perform phenomenological studies that eventually lead to a constrained parameter space.

Of particular interest are the sfermions mass matrices. These matrices receive contributions from the so-called trilinear terms, also called A-terms, which can be strong sources of flavor changing neutral currents (FCNC) \([4]\). The sfermions mass matrices are \(6 \times 6\) matrices usually written in the basis (for the up-type squarks, for example) \(\{\tilde{u}_L, \tilde{c}_L, \tilde{t}_L, \tilde{u}_R, \tilde{c}_R, \tilde{t}_R\}\). The entries in these matrices are arbitrary and will be determined by the supersymmetry breaking mechanism that leads to the MSSM.

One can start reducing the arbitrariness in the parameter space by assuming, for example, that the matrices are hermitian, or by imposing certain relations among the entries
through the use of flavor symmetries. Also, considering some specific scenarios of super-symmetry breaking (gauge or gravity mediated) one can impose degeneracies (universality) and or hierarchies among the different parameters [5]. In practice, however, most of the phenomenological studies have been done under the physically sound assumption that only the third generation sfermions contribute significantly to the A-terms. Besides being somehow natural (due to the contribution of the Yukawa couplings to the A-terms), this assumption is also convenient if one is interested in obtaining analytical expressions. The diagonalization of the sfermions mass matrices in this case can be easily done while more general cases are done numerically. Other scenarios exist where some of the universality is broken with some specific non-zero entries in the A-terms [6–8].

Motivated by this situation we present a method of diagonalization for the 6×6 matrices that gives analytical expressions and that can be used under more general cases than those usually studied. In Section 2 we present the method in detail and consider three different cases. We explicitly express the physical sfermion (squared) masses and give expressions for the mixing angles. This is followed by a discussion in Section 3 where some comments are given as to the relevance that this method can have in model-independent analysis of certain FCNC processes. Finally we present our conclusions.

2. The method

Consider the up-type squarks (6 × 6) mass matrix

$$\tilde{M}_u^2 = \begin{pmatrix} M_{LL}^2 & M_{LR}^2 \\ M_{LR}^2 & M_{RR}^2 \end{pmatrix},$$

(2.1)

where $M_{LL}^2$, $M_{RR}^2$, and $M_{LR}^2$ are 3 × 3 matrices given by

$$M_{LL}^2 = M_{\tilde{Q}}^2 + M_u^2 + \frac{1}{6} \cos 2\beta (4m_W^2 - m_Z^2),$$

(2.2)

$$M_{RR}^2 = M_{\tilde{u}}^2 + M_u^2 + \frac{2}{3} \cos 2\beta \sin^2 \theta_w m_Z^2,$$

(2.3)

$$M_{LR}^2 = A_u v \sin \beta - M_{\tilde{u}}^2 \mu \cot \beta,$$

(2.4)

where $v^2 = v_u^2 + v_d^2$, with $v_{u(d)} = \langle H_{u(d)} \rangle$ and $\tan \beta = v_u/v_d$. Note that these matrices receive contributions from soft-breaking terms ($M_{\tilde{Q}}^2$, $M_{\tilde{u}}^2$, $A_u$, and $\mu$), from the breaking of electroweak symmetry ($M_u^2$), and from the D-terms in the lagrangian.

The contributions involving soft-breaking terms constitute free parameters and there is no precise way to fix them. As a result, the mass matrix Eq. (2.1) is completely arbitrary. Furthermore, the so-called A-terms can contribute to flavor changing neutral current (FCNC) processes and must be handled with care. From now on we work under the assumption that $\tilde{M}_u^2$ is a hermitian matrix.

Given these considerations we proceed to the diagonalization of the mass matrix $\tilde{M}_u^2$ by constructing a matrix $V_\tilde{u}$ such that

$$\tilde{M}_{uD}^2 = V_\tilde{u}^\dagger \tilde{M}_u^2 V_\tilde{u}.$$
We reparametrize the mass matrix Eq. (2.1) as

\[ \tilde{M}^2_u = \begin{pmatrix} A & B \\ B^\dagger & C \end{pmatrix}, \tag{2.6} \]

and introduce a unitary \((6 \times 6)\) matrix \(U\) of the form

\[ U = \begin{pmatrix} U_L & 0 \\ 0 & U_R \end{pmatrix}, \tag{2.7} \]

with \(U_{L(R)}\) unitary \(3 \times 3\) matrices. Then, consider the following expression:

\[ \tilde{M}^2_{u*} = U^\dagger \tilde{M}^2_u U = \begin{pmatrix} U^\dagger_L A U_L & U^\dagger_R B U_R \\ (U^\dagger_L B U_R)^\dagger & U^\dagger_R C U_R \end{pmatrix}. \tag{2.8} \]

Since \(B\) is hermitian (by assumption), it is clear that we can use the matrices \(U_L\) and \(U_R\) to diagonalize the matrix \(B\) (with eigenvalues \(b_1, b_2,\) and \(b_3\)). On the other hand, the matrices \(A\) and \(C\) will not in general be diagonalized by these matrices, see Appendix A. Let us continue by assuming for the moment that the matrices \(A\) and \(B\) satisfy this condition (we present explicit examples below), we then obtain a matrix with form

\[ \tilde{M}^2_{u*} = U^\dagger \tilde{M}^2_u U = \begin{pmatrix} a_1 & 0 & 0 & b_1 & 0 & 0 \\ 0 & a_2 & 0 & 0 & b_2 & 0 \\ 0 & 0 & a_3 & 0 & 0 & b_3 \\ b_1 & 0 & 0 & c_1 & 0 & 0 \\ 0 & b_2 & 0 & 0 & c_2 & 0 \\ 0 & 0 & b_3 & 0 & 0 & c_3 \end{pmatrix}, \tag{2.9} \]

where \(a_1, a_2, a_3\) are the eigenvalues of \(A\) and similarly \(c_1, c_2, c_3\) the ones for \(C\).

Going back to the original mass matrix \(\tilde{M}^2_u\) we note that it is written in the basis \(\hat{u}_{LLLRRR} \equiv \{\hat{u}_L, \hat{c}_L, \hat{t}_L, \hat{u}_R, \hat{c}_R, \hat{t}_R\}\). The next step in the procedure is to express the mass matrix in the different basis \(\hat{u}_{LLRLLL} \equiv \{\hat{u}_L, \hat{u}_R, \hat{c}_L, \hat{c}_R, \hat{t}_L, \hat{t}_R\}\). This can be easily accomplished using the matrix \(T\) defined by

\[ T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{2.10} \]

and we obtain the matrix given by

\[ \tilde{M}^2_{uBD} = T \tilde{M}^2_u T^\dagger = \begin{pmatrix} a_1 & b_1 & 0 & 0 & 0 & 0 \\ b_1 & c_1 & 0 & 0 & 0 & 0 \\ 0 & a_2 & b_2 & 0 & 0 & 0 \\ 0 & 0 & a_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & a_3 & b_3 & 0 \\ 0 & 0 & 0 & 0 & b_3 & c_3 \end{pmatrix} = \begin{pmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{pmatrix}. \tag{2.11} \]
Note that since all the $3 \times 3$ blocks in the matrix in Eq. (2.8) are hermitian matrices, then their eigenvalues are real numbers (see Eq. (2.9)). Furthermore, since the off-diagonal blocks are the same ($B^\dagger = B$), the off-diagonal entries in each of the $G_x$ ($x = 1, 2, 3$) are equal and thus the $G_x$ are $2 \times 2$ real symmetric matrices. As such they can be diagonalized by orthogonal matrices $R_x$ in the following way:

$$G_{1D} = R_1^\dagger G_1 R_1, \quad G_{2D} = R_2^\dagger G_2 R_2, \quad G_{3D} = R_3^\dagger G_3 R_3,$$

with eigenvalues of $G_x$ denoted by $\lambda_{x,1}$ and $\lambda_{x,2}$ and given by

$$\lambda_{x,1} = \frac{1}{2} (a_x + c_x - \Delta_x), \quad \lambda_{x,2} = \frac{1}{2} (a_x + c_x + \Delta_x),$$

with $\Delta_x = \sqrt{(a_x - c_x)^2 + 4b_x^2}$, and where the matrices $R_x$ can be parametrized in terms of mixing angles $\theta_x$ in the usual way:

$$R_x = \begin{pmatrix} \cos \theta_x & \sin \theta_x \\ -\sin \theta_x & \cos \theta_x \end{pmatrix},$$

with

$$\cos \theta_x = \frac{1}{\sqrt{2}} \left(1 - \frac{a_x - c_x}{\Delta_x}\right)^{1/2}, \quad \sin \theta_x = \frac{1}{\sqrt{2}} \left(1 + \frac{a_x - c_x}{\Delta_x}\right)^{1/2}.$$

Finally, defining the $6 \times 6$ matrix $\tilde{R}_u = \text{diag}(R_1, R_2, R_3)$, we express the diagonal matrix $\tilde{M}_u^2$ as

$$\tilde{M}_u^2 = V_\tilde{u}^\dagger \tilde{M}_u^2 V_\tilde{u} = \tilde{R}_u^\dagger T U^\dagger \tilde{M}_u^2 U T^\dagger \tilde{R}_u,$$

and thus the matrix $V_\tilde{u} = U T^\dagger \tilde{R}_u$ diagonalizes the (up-type) squark mass matrix $\tilde{M}_u^2$. Note that the same procedure is easily extended to the down-type squark mass matrix as well as to the slepton mass matrix where one obtains (in obvious notation)

$$\tilde{M}_d^2 = V_d^\dagger \tilde{M}_d^2 V_d = \tilde{R}_d^\dagger T D^\dagger \tilde{M}_d^2 D T^\dagger \tilde{R}_d,$$

$$\tilde{M}_l^2 = V_l^\dagger \tilde{M}_l^2 V_l = \tilde{R}_l^\dagger T L^\dagger \tilde{M}_l^2 L T^\dagger \tilde{R}_l.$$

As discussed after Eq. (2.8), these results apply only to those cases in which the matrices $M_{LL}^2$ and $M_{RR}^2$ are related to the matrix $M_{LR}^2$ in such a way as to be diagonalized once $M_{LR}^2$ is (see Appendix A).

We now proceed to show the application of this method to three different cases.

### 2.1 Case a

As a first example of the application of the method described above, we choose a scenario where both $M_{LL}^2$ and $M_{RR}^2$ are proportional to the identity $I_3$ and $M_{LR}^2$ is an arbitrary hermitian matrix (we show the analysis for the up-type squark mass matrix): 

$$M_{LL}^2 = a I_3, \quad M_{RR}^2 = c I_3, \quad M_{LR}^2 = (M_{LR}^2)^\dagger.$$
Then, the mass matrix becomes (see Eq. (2.1))
\[ \tilde{M}_u^2 = \begin{pmatrix} a \mathbb{I}_3 & M_{LR}^2 \\ M_{LR}^2 & c \mathbb{I}_3 \end{pmatrix}, \] (2.20)
and using Eq. (2.8) we obtain
\[ \tilde{M}_{us}^2 = U_{L}^\dagger \tilde{M}_u^2 U_{R} = \begin{pmatrix} a & U_{L}^\dagger \mathbb{I}_3 U_{L} & U_{L}^\dagger M_{LR}^2 U_{R} \\ U_{L}^\dagger M_{LR}^2 U_{R} & c \end{pmatrix}, \] (2.21)
where the unitary matrices \( U_L \) and \( U_R \) diagonalize \( M_{LR}^2 \) with eigenvalues \( b_x \). Since all matrices are hermitian we have \( a, c, b_x \in \mathbb{R} \).

The next step is to rotate this matrix to the basis \( u_{LRRLR} \) using the matrix \( T \) in Eq. (2.10):
\[ \tilde{M}_{u_{BD}}^2 = T \tilde{M}_{us}^2 T^\dagger = \begin{pmatrix} a & b_1 & 0 & 0 & 0 & 0 \\ b_1 & c & 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 & 0 \\ 0 & 0 & b_2 & c & 0 & 0 \\ 0 & 0 & 0 & 0 & a & b_3 \\ 0 & 0 & 0 & 0 & b_3 & c \end{pmatrix} = \begin{pmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{pmatrix}. \] (2.22)

From Eq. (2.13) we immediately obtain:
\[ \lambda_{\tilde{U}_1} = \frac{1}{2} (a + c - \Delta_{\tilde{U}}), \quad \lambda_{\tilde{U}_2} = \frac{1}{2} (a + c + \Delta_{\tilde{U}}), \] (2.23)
where we have renamed \( x = 1, 2, 3 \) by \( \tilde{U} = \tilde{u}, \tilde{c}, \tilde{t} \), and where \( \Delta_{\tilde{U}} = \sqrt{(a - c)^2 + 4b_x^2} \).

Similarly, for the mixing angles in \( R_{\tilde{U}} \) we obtain
\[ \sin \theta_{\tilde{U}} = \frac{1}{\sqrt{2}} \left( 1 + \frac{a - c}{\Delta_{\tilde{U}}} \right)^{1/2}, \quad \cos \theta_{\tilde{U}} = \frac{1}{\sqrt{2}} \left( 1 - \frac{a - c}{\Delta_{\tilde{U}}} \right)^{1/2}, \] (2.24)
and so we finally arrive at the desired up-type squark mass matrix
\[ \tilde{M}_{uD}^2 = V^\dagger \tilde{M}_{u_{BD}}^2 V = \text{diag}(\lambda_{\tilde{u}_1}, \lambda_{\tilde{u}_2}, \lambda_{\tilde{c}_1}, \lambda_{\tilde{c}_2}, \lambda_{\tilde{t}_1}, \lambda_{\tilde{t}_2}). \] (2.25)

We can now make some comments regarding the spectrum. If we consider a hierarchy for the \( M_{LR}^2 \) eigenvalues of the form \( |b_{\tilde{u}}| < |b_{\tilde{c}}| < |b_{\tilde{t}}| \), then this implies that \( \Delta_{\tilde{u}} < \Delta_{\tilde{c}} < \Delta_{\tilde{t}} \) which then leads to
\[ \lambda_{\tilde{t}_1} < \lambda_{\tilde{t}_2} < \lambda_{\tilde{u}_1} < \lambda_{\tilde{u}_2} < \lambda_{\tilde{c}_2} < \lambda_{\tilde{c}_1}. \] (2.26)

We see that in this case the lightest u-type squark has squared mass \( \lambda_{\tilde{u}_1} \), and the heaviest has a squared mass \( \lambda_{\tilde{t}_2} \).\footnote{These relations for the spectrum are valid at the supersymmetry breaking scale and can be affected by their running down to the electroweak scale.}
2.2 Case b

Another interesting scenario consists of having an arbitrary (hermitian) \( M_{LL}^2 \) and both \( M_{RR}^2 \) and \( M_{LR}^2 \) proportional to the identity \( I_3 \) matrix: (again, we show the analysis for the up-type squark mass matrix):

\[
M_{LL}^2 = (M_{LL}^2)^\dagger, \quad M_{RR}^2 = c I_3, \quad M_{LR}^2 = b I_3.
\]  (2.27)

Then, the mass matrix becomes (see Eq. (2.1))

\[
\tilde{M}_u^2 = \begin{pmatrix} M_{LL}^2 b I_3 \\ b I_3 c I_3 \end{pmatrix},
\]  (2.28)

and using Eq. (2.8) we obtain

\[
\tilde{M}_{u*}^2 = U^\dagger \tilde{M}_u^2 U = \begin{pmatrix} U_L^\dagger M_{LL}^2 U_L & b U_L^\dagger I_3 U_L \\ b U_L^\dagger I_3 U_L & c U_L^\dagger I_3 U_L \end{pmatrix} = \begin{pmatrix} a_1 & 0 & 0 & b & 0 & 0 \\ 0 & a_2 & 0 & 0 & b & 0 \\ 0 & 0 & a_3 & 0 & 0 & b \\ b & 0 & 0 & c & 0 & 0 \\ 0 & b & 0 & 0 & c & 0 \\ 0 & 0 & b & 0 & 0 & c \end{pmatrix},
\]  (2.29)

where \( a_x, b, c \in \mathbb{R} \) (since all matrices are hermitian). Note that in order to obtain diagonal matrices in the 1–2 and 2–1 sub-blocks we require (wlog) \( U_R = U_L \).

Applying the method we obtain:

\[
\lambda_{\tilde{U}1} = \frac{1}{2} \left( a_{\tilde{U}} + c - \Delta_{\tilde{U}} \right), \quad \lambda_{\tilde{U}2} = \frac{1}{2} \left( a_{\tilde{U}} + c + \Delta_{\tilde{U}} \right),
\]  (2.30)

where again we have renamed \( x = 1, 2, 3 \) by \( \tilde{U} = \tilde{u}, \tilde{c}, \tilde{t} \), and \( \Delta_{\tilde{U}} = +\sqrt{(a_{\tilde{U}} - c)^2 + 4b^2} \).

The mixing angles for \( R_{\tilde{U}} \) become

\[
\sin \theta_{\tilde{U}} = \frac{1}{\sqrt{2}} \left( 1 + \frac{a_{\tilde{U}} - c}{\Delta_{\tilde{U}}} \right)^{1/2}, \quad \cos \theta_{\tilde{U}} = \frac{1}{\sqrt{2}} \left( 1 - \frac{a_{\tilde{U}} - c}{\Delta_{\tilde{U}}} \right)^{1/2},
\]  (2.31)

and so we finally arrive at the desired up-type squark mass matrix

\[
\tilde{M}_{uD}^2 = V^\dagger \tilde{M}_u^2 V = \text{diag} (\lambda_{\tilde{u}_1}, \lambda_{\tilde{u}_2}, \lambda_{\tilde{c}_1}, \lambda_{\tilde{c}_2}, \lambda_{\tilde{t}_1}, \lambda_{\tilde{t}_2} ).
\]  (2.32)

If we now consider a hierarchy for the \( M_{LL}^2 \) eigenvalues of the form \( 0 < a_{\tilde{u}} < a_{\tilde{c}} < a_{\tilde{t}} < c \), then this implies that \( \Delta_{\tilde{u}} > \Delta_{\tilde{c}} > \Delta_{\tilde{t}} \) which then leads to

\[
\lambda_{\tilde{u}_2} > \lambda_{\tilde{u}_1}, \quad \lambda_{\tilde{c}_2} > \lambda_{\tilde{c}_1}, \quad \lambda_{\tilde{t}_2} > \lambda_{\tilde{t}_1}, \quad \lambda_{\tilde{f}_1} > \lambda_{\tilde{f}_1} > \lambda_{\tilde{u}_1} .
\]  (2.33)

In this case the lightest u-type squark has squared mass \( \lambda_{\tilde{u}_1} \).
2.3 Case c

Finally we consider a scenario where both $M^2_{LL}$ and $M^2_{RR}$ are general hermitian matrices while $M^2_{LR}$ proportional to the identity $I_3$ matrix: (again, we show the analysis for the up-type squark mass matrix):

\[ M^2_{LL} = (M^2_{LL})^\dagger, \quad M^2_{RR} = (M^2_{RR})^\dagger, \quad M^2_{LR} = bI_3. \]  \hfill (2.34)

Then, the mass matrix becomes (see Eq. (2.1))

\[ \tilde{M}^2_u = \begin{pmatrix} bI_3 \end{pmatrix}, \]  \hfill (2.35)

and using Eq. (2.8) we immediately obtain

\[ \tilde{M}^2_{u} = U^\dagger \tilde{M}^2_{u} U = \begin{pmatrix} a_1 & 0 & 0 & b & 0 & 0 \\ 0 & a_2 & 0 & 0 & b & 0 \\ 0 & 0 & a_3 & 0 & 0 & b \\ b & 0 & 0 & ka_1 & 0 & 0 \\ 0 & b & 0 & 0 & ka_2 & 0 \\ 0 & 0 & b & 0 & 0 & ka_3 \end{pmatrix}, \]  \hfill (2.36)

where in order to obtain the identity in the off-diagonal sub-blocks it is necessary to require $U_R = U_L$. This in turn requires $M^2_{LL}$ and $M^2_{RR}$ to be diagonalized by the same unitary matrix and we have chosen the simplest case where they are proportional, i.e. $M^2_{RR} = kM^2_{LL}$. Again $a, b, k \in \mathbb{R}$ (since all matrices are hermitian).

Applying the method we obtain

\[ \lambda_{\tilde{u}_1} = \frac{1}{2} (a_{\tilde{u}} (k+1) - \Delta_{\tilde{u}}), \quad \lambda_{\tilde{u}_2} = \frac{1}{2} (a_{\tilde{e}} (k+1) + \Delta_{\tilde{e}}), \]  \hfill (2.37)

where again we have renamed $x = 1, 2, 3$ by $\tilde{U} = \tilde{u}, \tilde{c}, \tilde{t}$, and $\Delta_{\tilde{e}} = + \sqrt{a_{\tilde{e}}^2 (k-1)^2 + 4b^2}$, and

\[ \sin \theta_{\tilde{e}} = \frac{1}{\sqrt{2}} \left( 1 + \frac{a_{\tilde{e}} (1-k)}{\Delta_{\tilde{e}}} \right)^{1/2}, \quad \cos \theta_{\tilde{e}} = \frac{1}{\sqrt{2}} \left( 1 - \frac{a_{\tilde{e}} (1-k)}{\Delta_{\tilde{e}}} \right)^{1/2}. \]  \hfill (2.38)

In this case the up-type physical squark mass matrix is given by

\[ \tilde{M}^2_{uD} = V^\dagger \tilde{M}^2_u V = \text{diag} \left( \lambda_{\tilde{u}_1}, \lambda_{\tilde{u}_2}, \lambda_{\tilde{c}_1}, \lambda_{\tilde{c}_2}, \lambda_{\tilde{t}_1}, \lambda_{\tilde{t}_2} \right). \]  \hfill (2.39)

Taking $k > 0$ and the $M^2_{LL}$ eigenvalues with hierarchy $0 < a_\tilde{u} < a_\tilde{c} < a_\tilde{t}$ implies that $\Delta_\tilde{u} < \Delta_\tilde{c} < \Delta_\tilde{t}$ which then leads to

\[ \lambda_{\tilde{u}_2} > \lambda_{\tilde{u}_1}, \quad \lambda_{\tilde{c}_2} > \lambda_{\tilde{c}_1}, \quad \lambda_{\tilde{t}_2} > \lambda_{\tilde{t}_1}, \quad \lambda_{\tilde{t}_2} > \lambda_{\tilde{c}_2} > \lambda_{\tilde{u}_2}. \]  \hfill (2.40)

We see that in this case the heaviest $u$-type squark has squared mass $\lambda_{\tilde{t}_2}$. 

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3. Discussion of FCNC

The study of FCNC is crucial to determine the viability of any given parametrization of the sfermions mass matrices [4]. Generally, large off-diagonal terms in the squark mass matrices are strongly constrained by \(K^0 - \bar{K}^0\), \(D - \bar{D}\), and \(B - \bar{B}\) mixing, as well as by the processes \(b \to s\gamma\), \(b \to sll\), and \(K^0 \to \mu^+\mu^-\) decays (large off-diagonal terms in the slepton mass matrix are restricted by \(\mu \to e\gamma\)). This is one of the reasons why in the simplified parameter space of the MSSM these off-diagonal terms are simply put to zero. However, it is important to keep in mind that the actual form of these mass matrices is unknown and that attempts to build them from more general contexts might in fact lead to more interesting matrices with richer phenomenology. We note that general formulae exist for the analysis of FCNC processes and they have been obtained either with a general diagonalization or through the use of the mass insertion method [9, 10], however, a study considering specific textures for the sfermion mass matrices has only been done in [6].

As mentioned above, some interesting modifications to the usual scenarios discussed in the literature can be analyzed easily within the framework of our method. Take for instance the following assumptions:

\[
m_{Qij}^2 = m_Q^2 \delta_{ij}, \quad m_{Uij}^2 = m_U^2 \delta_{ij}, \quad m_{Dij}^2 = m_D^2 \delta_{ij}, \quad (3.1)
\]

\[
m_{Lij}^2 = m_L^2 \delta_{ij}, \quad m_{Eij}^2 = m_E^2 \delta_{ij}, \quad (3.2)
\]

and

\[
A_{uij} = A_u Y_{uij}, \quad A_{dij} = A_d Y_{dij}, \quad A_{eij} = A_e Y_{eij}, \quad (3.3)
\]

where \(m_Q^2, m_U^2, m_D^2, m_L^2\) and \(m_E^2\) are such that \(M^2_{LL}\) and \(M^2_{RR}\) are proportional to the identity matrix. This setting corresponds to the case a described in Section 2 and thus can be analyzed immediately. The spectrum, for instance, is already given by Eq. (2.26). This corresponds to a modification of the Minimal Flavor Violation (MFV) scenario discussed in [3].

We stress that the method described in this paper can be used to extract general expressions for the main FCNC processes listed above in terms of the parametrizations discussed in the previous section. One important observation is that by analyzing the general expressions for the sfermion mass matrices within this method one can then keep all the sub-leading terms in the mass matrices and mix the exact diagonalization with the mass insertion method. We believe this will be helpful in exploring a richer set of extensions and/or reparametrizations of the MSSM and the work is currently under preparation [11]).

4. Conclusions

A method of diagonalization for the sfermion mass matrices has been presented. It assumes hermiticity of the sfermion mass matrices and works for matrices \(M^2_{LL}, M^2_{RR},\) and \(M^2_{LR}\) such that

\[
U^\dagger_L M^2_{LL} U_L = M^2_{LDD}, \quad U^\dagger_R M^2_{RR} U_R = M^2_{RRD}, \quad U^\dagger_L M^2_{LR} U_R = M^2_{LDR}, \quad (4.1)
\]
where $M^2_{LLD}$, $M^2_{RRD}$, and $M^2_{LRD}$ are diagonal matrices. Three specific cases have been presented in the paper that represent extensions of the constrained MSSM scenario where universality of sfermion masses is assumed and all the off-diagonal A-terms contributions are set to zero. The spectrum is presented for each case. A model independent study of FCNC processes is underway and will be presented elsewhere.

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A. General conditions on the mass matrices

The method described in this paper works for hermitian matrices $M^2_{LL}$, $M^2_{RR}$, and $M^2_{LR}$ such that

$$U_L^\dagger M^2_{LL} U_L = M^2_{LLD}, \quad U_R^\dagger M^2_{RR} U_R = M^2_{RRD}, \quad U_L^\dagger M^2_{LR} U_R = M^2_{LRD},$$

(A.1)

where $M^2_{LLD}$, $M^2_{RRD}$, and $M^2_{LRD}$ are diagonal matrices.

These relations impose the following conditions on the three matrices: let $u^L_i$ denote the eigenvectors of $M^2_{LL}$ (i.e. the columns of $U_L$) and $u^R_i$ those of $M^2_{RR}$, then since $\langle u^R_j, u^R_k \rangle = \delta_{jk}$, we obtain the desired relations provided

$$M^2_{LR} u^R_i = \lambda_i u^L_i, \quad M^2_{LR} u^L_i = \lambda_i u^R_i. \quad \text{(A.2)}$$

Thus, $M^2_{LR}$ mixes the eigenvectors of matrices $U_L$ and $U_R$. Note that the matrices in Eqs. (2.19), (2.27), and (2.34) trivially satisfy these conditions.

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