MSSM curvaton in the gauge-mediated SUSY breaking

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Abstract

We study the curvaton scenario using the MSSM flat directions in the gauge-mediated SUSY breaking model. We find that the fluctuations in the both radial and phase directions can be responsible for the density perturbations in the universe through the curvaton mechanism. Although it has been considered difficult to have a successful curvaton scenario with the use of those flat directions, it is overcome by taking account of the finite temperature effects, which induce a negative thermal logarithmic term in the effective potential of the flat direction.

1 Introduction

The curvaton is a scalar field whose fluctuations are isocurvature-like during inflation, but later converted to an adiabatic counterpart to explain the structures of the universe \cite{1,2,3}. The amplitude of density fluctuations of the curvaton during inflation is estimated as $H^*/\phi^*$, provided that the curvature scale is small enough, i.e., $V''(\phi_*) \ll H^2_*$, where $H_*$ and $\phi_*$ denote the Hubble parameter and the amplitude of the curvaton, respectively. The subscript '*' means that the variable is evaluated when the cosmological scale is departing the horizon during inflation.

Among realistic particle theories, the minimal supersymmetric standard model (MSSM) is the most attractive one, in which there are many scalar fields called flat directions. The potential of flat directions vanishes in the supersymmetry (SUSY) exact limit, but it is lifted by the SUSY breaking effect and some nonrenormalizable operators ($W \sim \phi^n$, $n = 4 - 9$) \cite{4}. The flat direction (FD) field could be a good candidate of a curvaton. The earlier studies revealed that the curvaton scenario is usually difficult to achieve along the context of the gravity-mediated SUSY breaking models \cite{2,3}. This is because thermal effects, especially corrections to the potential, lead to the fact that the energy density of the FD field will not dominate the universe before it decays. Even if thermal effects are suppressed as in the case of the hidden radiation \cite{2,3}, the energy density of the FD field does not dominate the universe except for the $n=9$ direction, and the amplitude of the fluctuations generated during inflation usually damps considerably in the course of the evolution after inflation. The crucial point is that the oscillation of the FD field starts rather early so that it decays well before it dominates the energy of the universe.

One may wonder if the oscillation could be ‘delayed’ by some mechanism. This is exactly achieved in the context of the gauge-mediated SUSY breaking models. For many flat directions, the sign of the two-loop thermal correction to the potential is negative, which traps the field at a large amplitude until very late epoch \cite{5}. The energy density of the FD field dominates the universe soon after the FD field oscillation starts. The field naturally deforms into Q balls, which act as a protector from thermal scatterings, and have relatively long lifetime. In this way, it is possible to have successful curvaton models with the use of the MSSM FD in the gauge-mediated SUSY breaking models. We will show how it is actually realized below. Notice that the key to the problem is the trap due to this two-loop thermal correction to the potential. If it were not there
(or if it were ineffective), the energy density of the Q balls would not be able to dominate before the big bang nucleosynthesis (BBN) and/or the decay would occur before its domination.

2 Flat direction

The scalar potential vanishes along the flat direction $\Phi = \phi e^{i\theta}/\sqrt{2}$, and it is only lifted by the gauge-mediated SUSY breaking effect and some nonrenormalizable operators. It can be written as

$$V(\Phi) = M_F^4 \log \left(1 + \frac{|\Phi|^2}{M_S^2}\right) + \lambda^2 \frac{|\Phi|^{2(n-1)}}{M_P^{2(n-3)}},$$  

(1)

where $M_S$ is the messenger scale, $M_F = (m_\phi M_S)^{1/2}$ is the SUSY breaking scale, $m_\phi \sim \text{TeV}$ is the mass scale of squarks, and $M_P = 2.4 \times 10^{18}$ GeV is the Planck mass.

For the curvaton mechanism to work, the mass of the FD field during inflation should be negligible compared with the Hubble parameter. It can be achieved in, say, supersymmetric D-term inflation models [7] or the no-scale type inflation [8], but here we just assume that there is no Hubble-induced mass term during inflation. Then the FD field slow rolls on the nonrenormalizable potential $V_{NR}$, obeying the equation of motion

$$3H^2 + V_{NR}(\phi) \simeq 0.$$  

(2)

After inflation, inflaton oscillates around the minimum of its potential, and the universe is dominated by the inflaton oscillation energy, which behaves as nonrelativistic matter. During this stage, inflaton gradually decays into light degrees of freedom, forming dilute radiation. Although the energy density of this radiation is very small compared with the total energy density, it actually affects the potential of the FD field. There are thermal effects at both one-loop and two-loop order. The former one is a thermal mass term which is effective for relatively small amplitude of the flat direction : $\phi \lesssim f^{-1}T$, where $f$ is gauge or Yukawa coupling constant. The latter one is a two-loop thermal logarithmic potential that dominates over the thermal mass term for larger $\phi$. Hereafter we concentrate on the two-loop thermal effects, which can be written as [9, 10, 5]

$$V_T = c_T f^4 T^4 \log \left(\frac{|\Phi|^2}{T^2}\right),$$  

(3)

where $c_T$ is a constant of order unity. We expect $f \sim 0.1$, since the contribution of the gauge interactions dominate over that of Yukawa interactions. We found that the sign of $c_T$ is negative for most of the flat directions, so we assume this is the case, setting $c_T = -1$. For the sake of completeness, we will discuss the positive case in App. A, in which there is no successful curvaton scenario.

Because of this negative thermal logarithmic potential, the FD field will soon be trapped at the amplitude

$$\phi_m \sim \left(\frac{f^2 T^2 M_{P}^{-3}}{\lambda}\right)^{\frac{1}{n-1}}.$$  

(4)

It is not released until the zero temperature potential overcomes the thermal counterpart, $fT < M_F$. Notice that the Hubble parameter becomes much smaller than the curvature of the potential at that time.

After the oscillation of the FD field commence, it feels spatial instabilities, and deforms into Q balls right after the oscillation begins [6, 7, 11, 12, 13, 5]. The oscillation starts at the amplitude

$$\phi_{osc} \sim \left(\frac{M_F^2 M_{P}^{-3}}{\lambda}\right)^{\frac{1}{n-1}},$$  

(5)
so that the charge of the produced Q ball becomes
\[ Q \sim \beta \left( \frac{\phi_{osc}}{M_F} \right)^4 \sim \beta \left( \frac{M_{n-3}^{n-3}}{\lambda M_{n-3}^{n-3}} \right)^{\frac{1}{n-1}}. \]  
(6)

where \( \beta \lesssim 0.1. \) \(^\dagger\) Here and hereafter we assume that \( \phi_{osc} \gtrsim M_S \sim M_F^2/m_\phi \) is satisfied.

If the universe has already become radiation-dominated when \( T \sim f^{-1} M_F \), the energy density of the Q balls the FD condensate is only \( f^4 \) times smaller than that of the radiation. Hence the energy of the Q balls will dominate the universe soon because it decreases as \( \propto a^{-3} \propto T^3 \), while the radiation density decreases as \( \propto a^{-4} \propto T^4 \). Thus, the energy density of the Q balls becomes equal to that of radiation when
\[ T = T_{eq} \sim f^3 M_F. \]  
(7)

On the other hand, if the FD field starts its oscillation during the inflaton oscillation dominated (IOD) universe, the energy density of the Q balls evolves as \( a^{-3} \), while the radiation decreases as \( a^{-3/2} \). The energy density of the Q balls will not dominate the universe until
\[ T = T_{eq} \sim f^3 \left( \frac{T_{RH}}{f^{-1} M_F} \right)^5 M_F. \]  
(8)

Notice that \( T_{RH} \lesssim f^{-1} M_F \) in this case. For the curvaton scenario to work, the temperature \( T_{eq} \) must be higher than the decay temperature \( T_d \) (see Eqs. (12) and (13)).

The Q balls can decay if the mass per unit charge is larger than the mass of the decay particle \( m_d \). This condition is expressed as
\[ M_F Q^{-\frac{1}{4}} > m_d. \]  
(9)

In the case of the Q ball with \( B \neq 0 \), nucleons must be in the decay particles, so that \( m_d = m_N \sim 1 \) GeV, and the condition becomes
\[ M_F > \left( \beta \frac{\lambda^{-1} m_N^{-1} M_P^{-3}}{\lambda M_P^{-3}} \right)^{\frac{1}{n-1}}, \]
\[ \sim \begin{cases} 3 \times 10^4 \text{ GeV} & \text{for } n = 4, \\ 9 \times 10^5 \text{ GeV} & \text{for } n = 5, \\ 5 \times 10^6 \text{ GeV} & \text{for } n = 6, \end{cases} \]  
(10)

where we set \( \beta = 0.1 \) and \( \lambda = 1 \). If the Q ball has no baryon number \( B = 0 \) but its constituent includes squarks, the decay particles are pions; \( m_N \) should replaced by \( m_\pi \sim 0.1 \) GeV. If the Q ball is non-baryonic, it can decay into neutrinos, and essentially there is no condition like Eq. (9).

The Q ball decays by loosing its charge through the surface with the rate \( [14] \)
\[ \Gamma_Q \sim \frac{M_F}{48\pi Q^2}. \]  
(11)

so that the decay temperature is given by
\[ T_d \sim (\Gamma_Q M_P)^{\frac{1}{2}} \sim \frac{\lambda^{-1} m_N^{-1}}{\sqrt{48\pi}} \beta^{-\frac{1}{2}} \left( \frac{M_F}{M_P} \right)^{\frac{3n-8}{n-8}} M_P. \]  
(12)

Imposing \( T_d \gtrsim \) MeV so that the BBN is successful, we must have \( M_P \) larger than \( 10^3, 3 \times 10^6 \) and \( 10^8 \) GeV for \( n = 4, 5, \) and 6, respectively. For larger \( n \), Q balls cannot decay before the BBN time. Notice that there is another constraint from the gauge-mediated SUSY breaking at the weak scale.

\( \dagger \) Since the Hubble parameter is much smaller than the curvature of the potential at the onset of the oscillation, we expect \( \beta \lesssim 0.1 \) instead of \( \beta \sim 10^{-4} \) \([14]\).
models; \(M_F \lesssim \sqrt{gm_{3/2}M_P}\), i.e., \(M_F \lesssim 10^8\) GeV for \(m_{3/2} \lesssim\) GeV, where \(g\) is a gauge coupling. For \(n = 4\), more stringent bound appears as \(M_F \lesssim 10^7\) GeV, which comes from the condition \(\phi_{\text{osc}} \gtrsim M_S \sim \frac{M_F^2}{m_{\phi}}\).

On the other hand, if the Q-ball charge is small enough, such that \(Q < Q_c \simeq 10^{16}\), Q-balls decay through thermal effects. In this case, the decay temperature can be written as

\[T_d \sim 10^{3.5 - 4/11} M_F \frac{Q}{Q_c}.\] (13)

Notice that this decay process is realized only for \(n = 4\) with \(M_F \gtrsim 3 \times 10^4\) GeV.

3 Dynamics of fluctuations

3.1 Fluctuations in the radial direction

The second important aspect of the curvaton mechanism is to produce enough amount of the fluctuations during inflation, i.e., \(\delta \equiv H_* / \phi_* \gtrsim 10^{-5}\). In general, the position of FD field during inflation is constrained as \(V'' \lesssim H^2\), leading to \(\phi_* \lesssim \phi_{sr} \equiv (H M_P^{-3}/\lambda)^{1/(n-2)}\). Hereafter we assume that \(\phi_*\) is of the order of \(\phi_{sr}\) for a definite discussion [13]. Thus

\[\frac{H_*}{\phi_*} \sim \lambda \left( \frac{\phi_*}{M_P} \right)^{n-3},\] (14)

leading to

\[\phi_* \sim \left( \frac{\delta}{\lambda} \right)^{1/(n-3)} M_P,\] (15)

\[H_* \sim \lambda^{-1/2} \frac{\delta_{\phi}}{\phi_{\phi}} M_P.\] (16)

As we will see shortly, however, the amplitude of the fluctuations will damp in the course of the evolution after inflation. In general, the equations of motion of the homogeneous and fluctuation modes are given by

\[\ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0,\] (17)

\[\ddot{\phi} + 3H \dot{\phi} + V'(\phi)\delta \phi = 0,\] (18)

where the wave number \(k \to 0\) for the superhorizon mode is assumed for the fluctuation. After inflation the homogeneous mode of the FD field still (marginally) slow rolls in the nonrenormalizable potential during IOD, and the damping factor is estimated as

\[\frac{\delta \phi}{\phi} \sim \left( \frac{\delta \phi}{\phi}_{\phi_{\phi}} \right)^{1/(n-2)} \frac{m_{\phi}}{M_P},\] (19)

where the subscript \(i\) denotes the initial values. This slow-roll regime will end when the field is trapped by the negative thermal logarithmic potential. Since the curvature at the minimum is

\[V''(\phi_m) \sim \left[ \frac{\lambda f(T)^{2(n-2)}}{M_P^{n-3}} \right] \frac{1}{M_P^{n-3}}\] (20)

and \(T \sim (T_{RH}^2 H M_P)^{1/3}\) during inflaton oscillation dominated universe, the trap starts when

\[H = H_{tr} \sim \left( \frac{\lambda^2 f(T)^{(2n-2)}}{M_P^{n-2}} \right)^{1/3}.\] (21)
The amplitude of the field and the temperature at that moment are

$$\phi_{tr} \sim \left( \frac{f^4 T_{RH}^2 M_P^{n-2}}{\lambda} \right)^{\frac{1}{2n}} ,$$

$$T_{tr} \sim \left( \lambda^2 f^{n-2} T_{RH}^{n-1} M_P \right)^{\frac{1}{2n}} ,$$

respectively, so that the damping factor during slow roll in the nonrenormalizable potential is estimated as

$$\xi_{\phi}^{(SR)} \equiv \frac{\langle \dot{\phi} \rangle_{tr}}{\langle \dot{\phi} \rangle_{SR}} \sim \left( \frac{\phi_{tr}}{\phi_*} \right)^{\frac{n-4}{2}} \sim \left[ \left( \frac{f^4 T_{RH}^2}{\lambda^2 M_P^2} \right)^{\frac{1}{2n}} \left( \frac{\lambda}{\lambda^*} \right)^{\frac{1}{2n}} \right]^{\frac{n-4}{2}} .$$

This result is applicable if the amplitude of the field during inflation, $\phi_*$, is larger than that of the minimum determined by the balance of nonrenormalizable and negative thermal logarithmic terms in the potential, namely

$$T_{RH} \lesssim T_{RH}^C \equiv \lambda^{-\frac{n-3}{2n}} f^{-2} \delta^{\frac{n}{n-3}} M_P .$$

In the opposite case, the field fast rolls in the negative thermal logarithmic potential, and quickly settles down to the minimum. In this course, the amplitude of $\delta \phi/\phi$ does not evolve so much. Typically, it only decreases an order of magnitude at most.

Once the field is trapped at the instantaneous minimum $\phi_m$, it is not released until $T \lesssim f^{-1} M_F$. During this stage, the amplitude of fluctuation compared to the homogeneous mode decreases as

$$\frac{\delta \phi}{\phi} \propto \begin{cases} H^{\frac{n-4}{2(n-3)}} & \text{(IOD)}, \\ H^{\frac{n-3}{4(n-1)}} & \text{(RD)}. \end{cases}$$

Thus, the additional damping factor during the trap is estimated as

$$\xi_{\phi}^{(tr)} \equiv \frac{\langle \dot{\phi} \rangle_{osc}}{\langle \dot{\phi} \rangle_{tr}} \sim \left( \frac{H_{RH}}{H_{tr}} \right)^{\frac{n-4}{2(n-3)}} \left( \frac{H_{osc}}{H_{RH}} \right)^{\frac{n-3}{2(n-1)}} ,$$

$$\sim \lambda^{-\frac{n-4}{2(n-3)}} f^{-\frac{n-6}{2(n-1)}} T_{RH}^{-\frac{n-6}{2n-11}} M_F^{\frac{n-3}{2n-11}} M_P^{\frac{n-4}{2n-11}} ,$$

where we assumed that the release of the FD field from the trap by the negative thermal logarithmic potential takes place during radiation-dominated universe, i.e., $T_{RH} > f^{-1} M_F$. Thus the amplitude of the fluctuations damps by a factor $\xi_{\phi}^{(SR)} \xi_{\phi}^{(tr)}$ during both the slow-roll and the trap periods. Since $\xi_{\phi}^{(SR)}$ has dependences on the reheating temperature as

$$\xi_{\phi}^{(SR)} \propto \begin{cases} \frac{1}{T_{RH}^{\frac{n-4}{2n}}} & \text{(TRH < T_{RH}^C)}, \\ \text{const.} & \text{(TRH > T_{RH}^C)}. \end{cases}$$

the total damping factor behaves according to

$$\xi_{\phi}^{(SR)} \xi_{\phi}^{(tr)} \propto \begin{cases} \frac{T_{RH}^{\frac{n-4}{2n}}} {T_{RH}^{\frac{8-n}{2n}}} & \text{(TRH < T_{RH}^C)}, \\ \frac{T_{RH}^{s-n}} {T_{RH}^{\frac{4}{n}}} & \text{(TRH > T_{RH}^C)}. \end{cases}$$

At a glance, higher reheating temperature than $T_{RH}^C$ seems to give less damping factor, but it will be shown shortly that it does not open parameter space for successful scenario.
To this end, let us estimate the damping factor when $T_{RH} = T_{RH}^C$. In this case we can set $\xi_{\phi}^{(SR)} \sim 1$. The amplitude of the fluctuation can be estimated as

$$\delta \equiv \left. \frac{\delta \phi}{\phi} \right|_{osc} \sim \lambda^{-\frac{3n+1}{8(n-1)(n-3)}} f^{-\frac{3}{2}} \left( \frac{M_P}{M_F} \right)^{\frac{n-3}{2(n-1)}} \delta^{-\frac{3n-4}{8(n-3)}}.$$  \hspace{1cm} (30)

Now let us look at each case of $n = 4, 5,$ and 6 when the Q-ball charge is larger than $Q_c$. For $n = 4$, $\delta \sim 10^{-5}$ can be realized if we take $\lambda \sim 1$, $f \sim 0.1$, $M_F \sim 10^8$ GeV, and $\delta \sim 2 \times 10^{-4}$. However, the reheating temperature is very high $\sim 10^{16}$ GeV, which is highly speculative. In addition, the entropy production due to the decay of Q balls is not enough to dilute the overproduced gravitinos. For $n = 6$, the parameter set of $\lambda \sim 1$, $f \sim 0.1$, $M_F \sim 10^8$ GeV, and $\delta \sim 10^{-3}$ lead to the right amount of fluctuations, i.e., $\delta \sim 10^{-5}$, but still very high reheating temperature such as $\sim 10^{16}$ GeV is needed.

To sum up, $n = 4$ direction could be a curvaton provided that there was some other entropy production. The cases of $n = 5$ and 6 are hopeless, since too high reheating temperature such as $\sim 10^{16}$ GeV is necessary, where the adiabatic fluctuations of the inflaton should dominate among others. As for the larger reheating temperature than $T_{RH}^C$, it cannot be realized in curvaton context for $n = 5$ and 6, because $T_{RH}$ exceeds $10^{16}$ GeV. For $n = 4$, $T_{RH}$ can be raised an order of magnitude at most, but it only makes 3 times less damping effect, and does not essentially change our result here.

However, the situation changes dramatically in the case of $Q < Q_c$ for $n = 4$ directions, in which the Q ball decays through thermal effects. The ideal example is QuQd direction $^5$ which does not carry any baryonic or leptonic charge, thus we do not have to care about the problematic baryonic isocurvature fluctuations. Since there is no damping effect during the slow roll regime in the case of $n = 4$, we only have to estimate the damping factor during the trap due to the negative thermal log potential. For $\lambda \simeq 10^{-2}$, $f \simeq 0.2$, $M_F = 10^7$ GeV, and $T_{RH} \simeq 10^{11}$ GeV, the charge of the Q ball becomes $\sim 10^{16}$, and the decay temperature is as high as $\sim 10^{5}$ GeV, while the energy density of the Q ball dominates the universe when $T = T_{eq} \simeq 10^5$ GeV. Thus, enough entropy will be released through the Q-ball decay (dilution factor $\sim 100$) to evade the gravitino problem. As for the amplitude of the fluctuation, we can obtain $\xi \sim 10^{-5}$ for $\phi_\ast \sim 10^{-3}M_p$ and $H_\ast \sim 5 \times 10^{-5}M_p$. Since the decay temperature is higher than the electroweak scale, the baryogenesis may take place in the course of the electroweak phase transition.

Finally we comment on the case that the FD field is released from the trap during IOD (i.e., $T_{RH} < f^{-1}M_F$). In this case it is impossible to adjust parameters to be $\phi_\ast \simeq \phi_{tr}$, hence the damping effects is unavoidable. Also, the later energy dominance by the Q balls is difficult to achieve while keeping the amplitude of the fluctuations larger than $10^{-5}$. Thus, there is no successful scenario.

$^1$Since there is no damping effect during the slow-roll regime in the case of $n = 4$, it seems possible to have lower reheating temperature $\sim 10^9$ GeV such that the gravitino problem can be evaded. However, the coupling constant $f$ should be unnaturally small $\sim 0.003$ in order to realize $T_{eq} \simeq T_d$ in this case.

$^5$The negative thermal logarithmic potential for this direction is shown in App. B.
3.2 Fluctuations in the phase direction

The FD field generally has the A-terms in the potential. For the superpotential \( W_{NR} \sim \lambda \Phi^n / M_p^{n-3} \), they are written as

\[
V_A \sim \frac{\lambda m_3/2 \Phi^n}{M_p^{n-3}} + \text{h.c.},
\]

where we assumed the vanishing (or negligible) cosmological constant in the vacuum. As is well known, such an A-term appears only if the nonrenormalizable superpotential is \( \phi^n \)-type. If it is \( \chi \phi^{n-1} \)-type, where \( \chi \) is any (super)field other than those consist of the flat direction, no A-term results. The \( \chi \phi^{n-1} \)-type superpotential appears for all \( n = 5 \) flat directions [17]. (Note that this fact does not mean that there is no A-term in the case of \( n = 5 \); \( \phi^n \) type superpotential with \( \tilde{n} > 5 \) induces the A-terms, although they are suppressed by \( \Phi (M_p)^{\tilde{n} - n} \). In fact, we have found that there is no successful scenario in the case of \( n = 5 \).) Therefore, we will consider \( n = 4 \) and 6 cases here.

Since the fluctuation of the potential becomes

\[
\Delta V(\theta) \sim \frac{\lambda m_3/2 \Phi^n}{M_p^{n-3}} \sin n\theta \delta \theta \big|_{osc},
\]

at the onset of the FD field oscillations, the density perturbation is estimated as

\[
\frac{\delta \rho}{\rho} \sim \frac{\Delta V(\theta)}{V(\phi_{osc})} \sim \lambda^{-1} \left( \frac{m_3/2}{M_P} \right) \left( \frac{M_F}{M_P} \right)^{-\frac{2(n-2)}{n-1}} \sin n\theta \delta \theta \big|_{osc},
\]

where \( V(\phi_{osc}) \sim M_F^3 \) and Eq. (31) are used.

Now we discuss the evolution of the fluctuations. Since the amplitude of the adiabatic fluctuations is determined by \( \theta \) and \( \delta \theta \), not the combination like \( \delta \theta / \theta \), we need follow the evolution of \( \theta \) and \( \delta \theta \) in the potential \( V(\eta) \sim \lambda m_3/2 \phi^{n-2} \eta^2 / M_p^{n-3} \), where \( \eta = \theta \) or \( \delta \theta \). The mass scale of the phase direction, \( m_\theta \), remains smaller than the Hubble parameter until well after the radial direction \( \phi \) is trapped at the instantaneous minimum \( \phi_m \), and the amplitudes of \( \theta \) and \( \delta \theta \) will not decrease during the slow roll. The damping takes place only at the very late stage, just before \( \phi \) is released from the trap. This happens when the slow roll condition on the phase direction is violated, \( m_\theta > H \), and hence

\[
T < T_{sr} \equiv \left\{ \begin{array}{ll}
\left( \lambda f^{2(n-2)} m_{3/2}^{-1} T_{RH}^{4(n-1)} M_p^{n+1} \right)^{\frac{1}{5(n-2)}} & \text{(IOD)}, \\
\left( \lambda f^{2(n-2)} m_{3/2}^{-1} M_p^{n+1} \right)^{\frac{1}{2n}} & \text{(RD)}. 
\end{array} \right.
\]

After the temperature drops down to this value, the amplitude \( \eta \) \((= \theta \) or \( \delta \theta \)) will decrease obeying the equation

\[
\dot{\eta} + 3H \dot{\eta} + 2\frac{\dot{\phi}}{\phi} \dot{\eta} + m_\theta^2 \eta = 0,
\]

where we assumed that \( \delta \phi \) has died out completely. It is easily derived that

\[
\eta \propto \left\{ \begin{array}{ll}
H^{\frac{2n-10}{2(n-2)}} \propto T^{\frac{2n-10}{2(n-2)}} & \text{(IOD)}, \\
H^{\frac{2n-5}{2(n-2)}} \propto T^{\frac{2n-5}{2(n-2)}} & \text{(RD)}. 
\end{array} \right.
\]

First we consider the case that the trap ends after the reheating, \( i.e. \), \( T_{RH} > T_{sr} \). Since the damping effect is very mild during the radiation-dominated era compared to inflaton-oscillation-domination, one can avoid considerable damping of \( \theta \) and \( \delta \theta \) in this case. In fact, we obtain a damping factor

\[
\xi_\theta \equiv \frac{(\theta \delta \theta)_{osc}}{(\delta \theta)_{osc}} \sim \left( \frac{T_{osc}}{T_{sr}} \right)^{\frac{2n-5}{2n}} \sim \lambda^{-\frac{2n-5}{2n}} f^{-\frac{2(2n-5)}{n}} \left( \frac{m_{3/2}}{M_p} \right)^{-\frac{2n-5}{2n}} \left( \frac{M_F}{M_p} \right)^{\frac{2n-5}{2n}},
\]
considering both $\theta$ and $\delta \theta$. Thus, the amplitude of the fluctuations can be estimated as

$$\frac{\delta \rho}{\rho} \sim \lambda^{-\frac{4n-5}{2n(2n-5)}} f^{-\frac{2(2n-5)}{n}} \left(\frac{m_{3/2}}{M_P}\right)^{\frac{1}{2n}} \left(\frac{M_F}{M_P}\right)^{-\frac{1}{2n}} \theta_\ast \frac{H_\ast}{\phi_\ast},$$

(38)

where $\theta_\ast$ is the value of $\theta$ during inflation, and $\delta \theta \sim H_\ast/\phi_\ast$ is used. It is easily seen that successful curvaton scenario is achieved only for $n = 6$. For example, the parameter set of $\lambda \sim 1$, $f \sim 0.05$, $M_F \sim 10^{8}$ GeV, $m_{3/2} \sim 1$ GeV, $\theta_\ast \sim 0.3$ and $\delta \sim 10^{-2}$ realizes the desired density perturbation, $\delta \rho/\rho \sim 10^{-5}$, where the damping factor $\xi_\theta$ is $\sim 0.3$. Notice that the disastrous baryonic isocurvature problem is naturally avoided because the baryon asymmetry vanishes due to the nonlinear dynamics of the Q-ball formation, in spite of the fact that any $n = 6$ FD field has non-zero $B$. This happens when the oscillation of the FD field starts after $H < m_\theta$ in negative thermal logarithmic potential $\delta^\ast$.

For the parameter set exemplified above, the reheating temperature is constrained as $T_{RH} > T_{sr} \sim 7 \times 10^9$ GeV. Such high reheating temperature leads to overproduction of the gravitino by a factor $\sim (T_{RH}/T_{3/2}) \sim 7$. However, the Q balls dominate the universe later, and enormous amount of entropy will be released. Since the domination by the Q balls begins soon after the oscillation of the FD fields start, the dilution factor becomes $(H_{eq}/H_d)_{1/2} \sim (f^{-1}M_F/T_d) \sim 2 \times 10^5$, where $T_{eq} \sim 600$ GeV and $T_d \sim 3$ MeV (see below Eq. (12)), which is enough to dilute the overproduced gravitinos. Notice that we just require the reheating temperature higher than $T_{sr}$ (RD). Of course, there is an upper limit in order not to have gravitino overproduction. Thus, $7 \times 10^9$ GeV $\lesssim T_{RH} \lesssim 10^{13}$ GeV is allowed.\footnote{If the messenger scale $M_S$ is less than the reheating temperature, the particles in the messenger sector give significant contributions to the gravitino production $\delta^\ast$, which make the constraint more stringent. However, in the present case, $M_S \sim M_F/m_\theta \sim 10^{13}$ GeV $> T_{RH}$, and hence the messenger contributions are neglected.}

Now let us consider the cases that the trap ends during RD. Then the damping factor becomes

$$\xi_\theta \sim \left(\frac{T_{osc}}{T_{sr}}\right)^{\frac{7n-10}{4n-10}} \sim \left[\lambda^{-1} f^{-4(n-1)} \left(\frac{T_{RH}}{f^{-1}M_F}\right)^{-4(n-1)} \frac{M_F^{n^2}}{m_{3/2}^{n+1} M_P^{n+1}}\right]^{\frac{7n-10}{2(n-1)(4n-10)}}.$$

(39)

The damping is tremendous for $n = 4$, since $\xi_\theta \lesssim 10^{-10}$ even for $M_F \sim 3 \times 10^4$ GeV, $f \sim 10^{-2}$, and $T_{RH}/(f^{-1}M_F) \lesssim 1$. For $n = 6$, one could obtain the right amount of the fluctuations. It is achieved if we set the parameters as, for example, $f \sim 0.1$, $M_F \sim 10^8$ GeV, $T_{RH} \sim 5 \times 10^8$ GeV, and $\delta \sim 10^{-2}$. In this case, $T_{eq} \sim 3 \times 10^3$ GeV and $T_d \sim 1$ MeV for $\beta \sim 0.1$. This situation is very similar to the above $n = 6$ case when the trap ends during RD, since the baryon asymmetry is not created because of the nonlinear dynamics of Q-ball formation. However, it is different in that there is no gravitino overproduction for relatively low reheating temperature. An example of $n = 6$ is LLe direction, and the derivation of the thermal potential is given in the App. B.

Finally we must mention that this mechanism of generating the density perturbation using the phase direction is independent of any damping effects on the radial direction, so that a negative Hubble-induced mass term can appear during and after inflation. Of course, one must avoid Hubble-induced A-terms in order that $\theta$ and $\delta \theta$ should not decrease considerably. They appear only if there is three-point interaction in nonrenormalizable Kähler potential, and these terms do not exist if the inflaton carries some nontrivial charge.

## 4 Conclusion

We have investigated the possibility of curvaton scenario with the use of the MSSM flat directions in the gauge-mediated SUSY breaking models, and shown that the both radial and phase directions of the flat directions can act as a curvaton. The later energy domination (before the decay) is
usually very difficult to achieve in the realistic particle physics. In addition, thermal effects such as corrections to the potential and scatterings leading to early decay (or evaporation) of the field make the scenario more difficult to be constructed. In our scenario, both of them are overcome by the thermal effects and Q balls. The FD field dominates the universe soon after the release from the rather long trap by the negative thermal logarithmic potential. Thermal scatterings do not affect so much, since the FD field deforms into Q balls soon after the oscillation starts. The Q balls in the gauge-mediated SUSY breaking have a long lifetime, and survive from too early evaporation due to thermal scatterings, if the charge is large enough. On the other hand, they can decay before BBN (and even electroweak phase transition) for small enough charge. We have utilized the intermediate region suitable for the curvaton scenario.

We have shown that the radial component of the flat direction can play a role of curvaton in the case of $n = 4$. For larger $n$, $\delta \phi / \phi$ damp considerably in the higher power potential, and we have found no successful scenario. In addition, the fluctuations in the phase direction do not damp so much and can be responsible for the right amount of the adiabatic perturbation of order $\sim 10^{-5}$ in the case of $n = 6$, provided that there are no Hubble-induced A-terms. Although such terms can appear due to nonrenormalizable three-point interactions $\sim I \Phi \Phi$ between inflaton $I$ and FD field in the Kähler potential, it can be easily forbidden with use of some symmetry principles.

Furthermore, there is another problem associated with the baryonic isocurvature perturbation. Except for three $n = 4$ directions, flat directions possess non-zero $B$ (and $B - L$), which usually leads to too much baryonic isocurvature perturbation. In the case of the phase directions as a curvaton, the vanishing baryon asymmetry is achieved through non-perturbative dynamics of producing Q balls [5], in which the negative thermal logarithmic potential plays the crucial role. However, baryogenesis may be still problematic in this case. In order to avoid large baryonic isocurvature fluctuations, the baryogenesis must take place after the Q-ball dominates the Universe. In our scenario the Q-ball dominated Universe begins at the temperature $T_{eq} \sim 6 \times 10^3$ or $3 \times 10^3$ GeV, depending on the thermal history (see Sec.3). Moreover, in order for baryon number to survive dilution by the entropy production due to the Q-ball decay, large baryon-to-entropy ratio should be generated. Unfortunately, at the moment we do not know such effective baryogenesis mechanism which works at low energy scales.

On the other hand, the $n = 4$ FD fields such as $QuLe$ and $QuQd$ possess no baryon number, so that there is no baryonic isocurvature perturbations from the first place in the case of the radial direction as curvaton. Also, the decay temperature can be higher than the electroweak scale, therefore the baryogenesis may occur through the electroweak phase transition. We can thus conclude that these directions are most promising candidate of a curvaton.

## A Positive thermal logarithmic potential

Here we comment on the case in which the two-loop thermal correction to the potential has a positive sign, and show that there is no successful scenario. We have found that the positive potential seems to appear only for some $n = 4$ directions, so we restrict our discussion only to $n = 4$, although some formulae will be given in general $n$. The FD field starts its oscillation earlier in the positive logarithmic potential, and the charge of the produced Q ball is rather small. It is estimated as

$$Q \sim \beta \left( \frac{\phi_{osc}}{T_{osc}} \right)^4 \sim \left\{ \begin{array}{ll} \beta \lambda^{4 \frac{n}{n-3}} \left( \frac{M_P}{f T_{RH}} \right)^{\frac{4(n-3)}{n}} & \text{(IOD),} \\ \beta \lambda^{-2} f^{-4(n-3)} & \text{(RD),} \end{array} \right.$$

(A.1)

where the field oscillation starts during IOD and RD at the temperature and amplitude

$$T_{osc} \sim \left\{ \begin{array}{ll} \left( \lambda^{\frac{n}{n-2}} T_{RH}^{-n-1} M_P \right)^{\frac{1}{n}}, & \text{(IOD),} \\ \lambda^{\frac{n}{n-2}} f^{n-2} M_P, & \text{(RD),} \end{array} \right.$$

(A.2)
survive from the evaporation. Then the decay temperature is

\[ T_d \sim \begin{cases} \frac{10^7 M_p}{Q} & (Q < Q_c), \\ \left( \frac{M_F M_p}{4 \pi Q} \right)^{\frac{1}{2}} & (Q > Q_c), \end{cases} \]

respectively. The decay process is determined by the evaporation (and/or diffusion) for the Q ball with small charge, while the Q ball decays at the rate \( 11 \) when the charge is large enough to survive from the evaporation. Then the decay temperature is

\[ T_{eq} \sim \begin{cases} \lambda^{-1} f^4 T_{RH}^2 M_p^{n-2} & (IOD), \\ f^2 M_p & (RD), \end{cases} \]

(A.3)

where the survival condition is \( Q > Q_c \sim 10^{16} (M_F/10^7 \text{GeV})^{-4/11} \). As the energy density of the Q ball equals to that of radiation when

\[ T_{eq} > T_d \]

it is easily seen that \( T_{eq} > T_d \) does not hold for \( n = 4 \) in any case. Thus, there is no working case for the curvaton scenario in the positive thermal logarithmic potential.

B Thermal logarithmic potential of \( LLe \) and \( QuQd \) flat directions

Here we first derive explicitly the negative thermal logarithmic potential arises taking \( n = 6 \) for example. In the end, we also show for \( n = 4 \) for \( Q_1 u_2 Q_2 d_3 \) direction. Generally, the free energy depending the gauge coupling is provided by two loop diagrams as \( V = A g_3^2(T) T^4 \), where \( A \) is a numerical factor, and \( a = 1, 2, \) and \( 3 \). Usually, \( g_a(T) \) can be determined from the renormalization group equation

\[ \frac{d}{dt} g_a = \frac{1}{16 \pi^2} b_a g_a^3, \]

(B.1)

where \( t = \ln(\mu) \), and \( b_4 = 11, 1, \) and \( -3 \), respectively for \( U(1)_Y \), \( SU(2)_L \), and \( SU(3)_C \) in MSSM. However, in the presence of the flat direction, those fields coupled directly to \( t \) acquire the mass of order \( f \), where \( f \) is gauge or Yukawa coupling. Then, the runnings of gauge couplings (values of \( b_a \)) change at the scale \( \mu = f \phi \) accordingly. We will write the new values by \( \tilde{b}_a \). Thus, the difference of the gauge coupling at the scale \( \mu = T < f \phi \) becomes

\[ \Delta g_a(T) \sim \frac{b_a - \tilde{b}_a}{32 \pi^2} g_a^3(T) \big|_{\phi = 0} \log \left( \frac{f^2 \phi^2}{T^2} \right), \]

(B.2)

where \( g_a(T) \big|_{\phi = 0} \) is the coupling without the presence of the flat direction. Since the leading term of the potential dependent on \( \phi \) is written as \( V = 2 g_a \Delta g_a(\phi) T^4 \), the thermal logarithmic potential is derived as

\[ V_{T^2}(\phi) \sim A \frac{b_a - \tilde{b}_a}{16 \pi^2} g_a^3(T) \big|_{\phi = 0} T^4 \log \left( \frac{\phi^2}{T^2} \right). \]

(B.3)

Now we have to evaluate \( A \) and \( b_a \) for the presence of the flat direction. We take an explicit example of \( L_1 L_2 e_3 \) for simplicity. Because of the flat direction, only \( Q, u, d, H_u, \) and \( SU(3) \) gauge fields are in thermal bath. Since the \( g_3 \)-running will not change, we have to see the changes of the runnings of \( g_2 \) and \( g_1 \). They are easily derived as \( \tilde{b}_2 = 5 \) and \( \tilde{b}_1 = 6 \). The only diagrams that
can contribute to the free energy of the type $g^2T^4$ are those four scalar interactions in the D-term. Counting the number of degrees of freedom, we obtain $A_2 = 5/96$ and $A_1 = 7/288$. Thus,

$$V_{T_2}(\phi) \simeq \left( \frac{35}{4608\pi^2} R - \frac{5}{384\pi^2} \right) g^4(T) T^4 \log \left( \frac{\phi^2}{T^2} \right),$$

(B.4)

$$\simeq -f_{eff}^4 T^4 \log \left( \frac{\phi^2}{T^2} \right),$$

(B.5)

where $R = (g'/g)^4$, and the last line is evaluated at $\mu = T = 10^{10}$ GeV (see Eq. (23)), so $f_{eff} \approx 0.1$. This is the ideal example for the scenario of the phase direction as a curvaton where the trap is released during IOD.

For $Q_1u_2Q_2d_3$ direction, we only show the final result:

$$V_{T_2}(\phi) \simeq -\frac{1}{16\pi^2 \cdot 576} \left( \frac{427}{2} + \frac{9}{2} R_2 - \frac{1829}{18} R_1 \right) g_4^4(T) T^4 \log \left( \frac{\phi^2}{T^2} \right),$$

(B.6)

where $R_2 = (g_2/g_3)^4$ and $R_1 = (g'/g_3)^4$. Evaluating at $\mu \approx 2 \times 10^{11}$ GeV, we obtain $f_{eff} \approx 0.2$, which gives the perfect example for the scenario of the radial direction as a curvaton when $Q < Q_c$.

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