Mathematical modeling of polymer flooding using the unstructured Voronoi grid

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Abstract. Effective recovery of unconventional oil reserves necessitates development of enhanced oil recovery techniques such as polymer flooding. The study investigated the model of polymer flooding with effects of adsorption and water salinity. The model takes into account six components that include elements of the classic black oil model. These components are polymer, salt, water, dead oil, dry gas and dissolved gas. Solution of the problem is obtained by finite volume method on unstructured Voronoi grid using fully implicit scheme and the Newton’s method. To compare several different grid configurations numerical simulation of polymer flooding is performed. The oil rates obtained by a hexagonal locally refined Voronoi grid are shown to be more accurate than the oil rates obtained by a rectangular grid with the same number of cells. The latter effect is caused by high solution accuracy near the wells due to the local grid refinement. Minimization of the grid orientation effect caused by the hexagonal pattern is also demonstrated. However, in the inter-well regions with large Voronoi cells flood front tends to flatten and the water breakthrough moment is smoothed.

1. Introduction

While the polymer flooding, the aqueous solution of high molecular weight polymer (about 1 million g/mol for partially hydrolyzed polyacrylamide) is injected into a reservoir and its mobility is several times lower than water mobility, providing the increase in oil recovery. This method, as well as most reservoir development technologies, requires a numerical modeling.

The software products known in the oil and gas industry include only the preselected sets of mathematical models and calculation tools. For a user it is not possible to make changes in these models. In addition, such software is expensive and for small companies it is more practical to develop the in-house software and make calculations by themselves.

The studies of polymer flooding modeling began in the late 1960s and are actively continued at present time [1, 2].

In the late 1980s, for the first time the unstructured grid was applied for the oil reservoir modeling [3-5]. These grids provide a number of advantages over classical rectangular grids: they allow better description of reservoir heterogeneity and more accurate calculation of the problem solution in the near-well area, reduction of the grid orientation effect and/ in some cases, the calculating speed increase from reducing the number of grid cells.

In the presented work we review a model of polymer flooding with the reservoir water salinity consideration, which was generated from the classical black-oil model. The novelty of the work is in the
application of the unstructured Voronoi grid for space representation. The flow under consideration in a porous medium consists of six-components: water, oil, free gas, dissolved gas, polymer and salt brine. The viscosity of water phase increases with the polymer concentration increase and the polymer adsorption on the rock surface as defined by the adsorption isotherms with subsequent decrease of rock permeability to water.

2. Basic equations
The equations of polymer flooding model are:

\[
\frac{\partial}{\partial t} \left[ \phi \left( \frac{S_w}{B_w} \right) \right] + \nabla \cdot \left( \frac{u_w}{B_w} \right) + q_w \delta_{\text{well}} = 0, \tag{1}
\]

\[
\frac{\partial}{\partial t} \left[ \phi \left( \frac{S_o}{B_o} \right) \right] + \nabla \cdot \left( \frac{u_o}{B_o} \right) + q_o \delta_{\text{well}} = 0, \tag{2}
\]

\[
\frac{\partial}{\partial t} \left[ \phi \left( \frac{R_s S_o + S_g}{B_o} \right) \right] + \nabla \cdot \left( \frac{R_s u_o + u_g}{B_o} \right) + q_g \delta_{\text{well}} = 0, \tag{3}
\]

\[
\frac{\partial}{\partial t} \left[ \phi \left( \frac{S_w C_p}{B_w} \right) \right] + \frac{\partial}{\partial t} (1 - \phi) \rho_C C_a + \nabla \cdot \left( \frac{u_w C_p}{B_w} \right) + q_w C_p \delta_{\text{well}} = 0, \tag{4}
\]

\[
\frac{\partial}{\partial t} \left[ \phi \left( \frac{S_w C_a}{B_w} \right) \right] + \nabla \cdot \left( \frac{u_w C_a}{B_w} \right) + q_w C_a \delta_{\text{well}} = 0, \tag{5}
\]

\[
S_w + S_o + S_g = 1. \tag{6}
\]

where: \( \phi \) = rock porosity; \( S_w \) = water saturation; \( S_o \) = oil saturation; \( S_g \) = gas saturation; \( B_w, B_o, B_g \) = water, oil and free gas volume factors, respectively; \( R_s \) = solution gas-oil ratio; \( p_b \) = bubble point pressure, which is constant at \( S_g = 0 \) and equal to reservoir pressure at \( S_g > 0 \); \( u_w, u_o, u_g \) = water, oil and free gas flow rates, respectively; \( C_p \) = mass of polymer dissolved in a unit volume of water; \( C_s \) = mass of salt dissolved in a unit volume of water; \( C_a \) = mass of polymer adsorbed on the surface of unit rock mass (adsorption isotherm); \( \rho_C \) = rock density; \( q_w, q_o, q_g \) = the flow rate of water, oil and free gas, respectively, produced from a reservoir unit volume per unit time; \( \delta_{\text{well}} \) = Dirac delta function localized for the well location.

The first three equations (1) - (3) describe the classical black-oil model. They are the equations of mass conservation for the water, oil and gas phases, respectively. Equations (4) and (5) reflect the mass conservation laws for polymer and salt components, respectively. The velocities of fluids flowing in a porous media are described by the Darcy’s law (7) - (9):

\[
\bar{u}_w = \frac{k k_{nw}}{\mu_{w,\text{eff}}} \nabla p, \tag{7}
\]

\[
\bar{u}_o = \frac{k k_{ro}}{\mu_o} \nabla p, \tag{8}
\]

\[
\bar{u}_g = \frac{k k_{rg}}{\mu_g} \nabla p. \tag{9}
\]

The sources simulating the wells are described by equations (10) – (12):
The reservoir boundary is \( \partial \Omega = \partial \Omega \). The reservoir thickness, rock density, residual resistance factor, reflecting the rock permeability reduction by the polymer adsorption; \( h \) = reservoir thickness; \( r_{eq} \) = equivalent wellbore radius [6]; \( r_w \) = wellbore radius; \( p_{bh} \) = bottomhole pressure. The resistance factor is calculated as:

\[
R = 1 + (R_{res} - 1) \frac{C_a}{C_{a,max}},
\]

where \( R_{res} \) = residual resistance factor, \( C_{a,max} \) = maximum adsorption isotherm.

The basic assumptions of the model are:

- the fluids temperature remains constant;
- water does not mix and does not exchange masses with oil and gas phases;
- water, polymer and salt are completely mixed into a homogeneous solution;
- the gas phase contains free gas and oil-dissolved gas;
- volume factors of water, oil and gas do not depend on polymer and salt concentrations;
- porosity and volume factors are the functions of reservoir pressure;
- solution gas-oil ratio depends on reservoir pressure and bubble point;
- relative permeability is a function of water saturation and gas saturation;
- effective water viscosity is a two-dimensional function of polymer and salt concentrations (equal to pure water viscosity in the absence of polymer and salt admixtures);
- the adsorption isotherm is a function of polymer concentration.

3. The problem statement

Let us assume that the fluid flow occurs in a thin reservoir, which properties are weakly varied along a vertical axis. Neglecting gravitational effects and variations of all quantities along the vertical axis, a planar two-dimensional reservoir representation can be used \( \Omega \subset \mathbb{R}^2 \). The reservoir boundary is described by a closed contour \( \Gamma = \partial \Omega \). The reservoir is penetrated by several vertical wells, which operate at the given constant bottomhole pressure. The rock permeability is given as non-uniform and isotropic. The reservoir thickness, rock density, residual resistance factor, and viscosity of pure water, oil and gas are known and constant. The functions \( \phi(p), B_w(p), B_o(p), B_g(p), k_{ro}(S_w, S_g), k_{rg}(S_g), R_s(p, p_b), \mu_{w\_eff}(C_p, C_s), C_a(C_p) \) are given and the values of required functions are known at the reference time.

The reservoir boundary is considered as a non-flow (barrier) boundary:

\[
\frac{\partial p}{\partial n} \bigg|_{\Gamma} = 0.
\]
Thus, the values of \( p, S_w, S_g, p_b, C_p, C_s, q_w, q_o, q_g \) should be calculated for the subsequent time period \( t > 0 \).

4. Grid generation
A two-dimensional Voronoi grid is generated from a given set of points on the plane called ‘grid nodes’. This is such partition of the plane where each grid cell forms a set of points being closer to one particular node than to any other node (Figure 1). Let’s denote the grid nodes as \( P_i, i = 1...N \). Then, the corresponding cells of the Voronoi grid \( G_i \) can be defined as:

\[
G_i = \{ x \in \Omega \mid |x, P_i| \leq |x, P_j|, \ j = 1...N, \ i \neq j \}, \ i = 1...N.
\]

For representing the reservoir as a grid, the grid nodes should be arranged on contour \( \Gamma \) and as a certain pattern within the contour. First of all, the entire \( \mathbb{R}^2 \) plane is covered by the Voronoi segmentation for using the sweeping line algorithm [3]. After that, the algorithm [5] cuts off the parts of the cells beyond the contour \( \Gamma \) (Figure 1). Around the well locations we can increase the density of grid nodes and arrange them radially from the center of the well. This approach allows reflecting the radial nature of reservoir pressure distribution in the wellbore vicinity. If in any area to increase a node density, it defines a refined grid ensuring more accurate solution of the problem. Fine grids should be used for the reservoir regions with high pressure gradient and strong permeability variation. The classical rectangular grid, used for local grid refinement, will require the modification of mass conservation equations.

We have to point to one important property of the Voronoi grid: a segment connecting the nodes of any two neighboring cells is perpendicular to a face between these cells (Figure 2). And thus, the reservoir pressure gradient in the normal direction to the face between two cells can be approximated by the finite difference of pressure values at the nodes of these cells – in the same way as in the rectangular grid case. This approach is called ‘two-point flux approximation’.

Figure 1. Reservoir representation as a Voronoi grid with a hexagonal pattern and local grid refinement in the well vicinity.

5. Solution of the problem
Though the computational grid is two-dimensional, the cells were considered as three-dimensional of height \( h \) equal to the reservoir thickness. Let’s make discretization of equation (4). Integrating equation (4) by the cell volume \( i \), we obtain:

\[
\int_{V_i} \left[ \frac{\partial}{\partial t} \left( \phi \left( \frac{S_w C_p}{B_w} \right) \right) \right] dV + \int_{V_i} \nabla \cdot \left( \frac{\bar{u}_w C_p}{B_w} \right) dV + \int_{V_i} q_w C_p \delta_{well} dV = 0.
\]
Let subscript $i$ to determine the average value inside cell $i$. Replacing the integrands by the means and applying the Gauss-Ostrogradsky formula, the resulted equation is:

$$V_i \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_w C_p}{B_w} \right) + (1-\phi) \rho_f C_a \right]_i + \int_{S_i} \frac{C_p}{B_w} \vec{n} \cdot \vec{n} dS + q_{w,i} C_{p,i} = 0.$$  \hspace{1cm} (1)

The integral over the surface $S_i$ of cell $i$ in the last equation can be substituted by a sum of the surface integrals over each face of this cell:

$$V_i \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_w C_p}{B_w} \right) + (1-\phi) \rho_f C_a \right]_i + \sum_{j \in \Psi(i)} \frac{C_{p,ij}}{B_{w,ij}} \int_{S_{ij}} \vec{n} \cdot \vec{n} dS + q_{w,i} C_{p,i} = 0,$$

$$V_i \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_w C_p}{B_w} \right) + (1-\phi) \rho_f C_a \right]_i + \sum_{j \in \Psi(i)} \frac{C_{p,ij} s_{ij}}{B_{w,ij}} \vec{n}_{ij} \cdot \vec{n}_{ij} + q_{w,i} C_{p,i} = 0.$$  \hspace{1cm} (2)

Where $\Psi(i) = \text{set of cells adjacent to } i\text{-th; } S_{ij} = \text{face between } i\text{-th and } j\text{-th cells, } s_{ij} = \text{area of this face; } \vec{n}_{ij} = \text{normal to face } S_{ij}, \text{directed toward the cell } j$ (Figure 2); $d_{ij} = \text{distance between nodes } P_i \text{ and } P_j$. The subscript $ij$ for all other parameters reflects the mean value of the parameter on face $S_{ij}$.

**Figure 2.** Computation of flow between grid cells.

$p_i$, $p_j$ are the reservoir pressure values at nodes $P_i$ and $P_j$, respectively. The fluid flow velocity (7) normal to a face is described by two-point approximation:

$$V_i \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_w C_p}{B_w} \right) + (1-\phi) \rho_f C_a \right]_i + \sum_{j \in \Psi(i)} \frac{C_{p,ij} s_{ij}}{B_{w,ij}} \frac{k_{ij} k_{rw,ij}}{\mu_{w,eff} d_{ij} B_{w,ij} R_{ij}} \left( p_i - p_j \right) + q_{w,i} C_{p,i} = 0.$$  \hspace{1cm} (3)

The time derivative is described by a finite-difference approximation:

$$\frac{V_i}{\Delta t} \left[ \phi \left( \frac{S_w C_p}{B_w} \right) + (1-\phi) \rho_f C_a \right]_i^{n+1} - \left[ \phi \left( \frac{S_w C_p}{B_w} \right) + (1-\phi) \rho_f C_a \right]_i^n +$$

$$\sum_{j \in \Psi(i)} \left[ \frac{s}{d_{ij} \mu_{w,eff} B_{w,ij} R_{ij}} \right]^{n+1} \left( p_i^{n+1} - p_j^{n+1} \right) + q_{w,i}^{n+1} C_{p,i}^{n+1} = 0.$$  \hspace{1cm} (4)

The value of rock permeability on the cell face $k_{ij}$ is calculated as the cell-average harmonic permeability.
The values of relative permeability, volume factor, viscosity, resistance factor and polymer concentration are calculated for the face in the upstream direction. The equivalent radius of the well grid cell is calculated as:

\[
    r_{eq} = \left( e^{-2 \pi kh} \prod_{j \in \Psi(i)} d_{ij}^{T_{ij}} \right)^{1/U} \sum_{j \in \Psi(i)} T_{ij},
\]

\[
    T_{ij} = \frac{k_{ij} \sigma_{ij}}{d_{ij}}.
\]

After equations (1) - (3) discretization and a similar procedure for (5) gives a system of five nonlinear equations for the \( i \)-th grid cell.

For the grid with \( N \) cells the number of equations equal to \( 5N \). The unknown variables are \( p_i \), \( S_{w,i} \), 'gas variable in the \( i \)-th cell', \( C_{p,i} \) and \( C_{s,i} \) for \( i = 1 \ldots N \). If the oil in the cell is saturated with gas (\( S_{g,i} > 0 \)), then the ‘gas variable’ equals to gas saturation \( S_{g,i} \), in the contrary case – to gas-in-oil solubility factor \( R_{s,i} \). Oil saturation \( S_{o,i} \) is a known quantity as per equation (6).

If all \( 5N \) variables are known for a time layer \( n \), then the system of equations, becomes a closed system. Its solution will be the values of unknown variables for the time layer \( n+1 \). This system of equations in vector form is expressed as:

\[
    \vec{F}(\chi^{n+1}) = 0.
\]  

(13)

The no-flow boundary condition is valid providing that in the mass conservation equations the flows across the cell boundary are summed only by faces of two neighboring cells. The system is solved by Newton's iteration method:

\[
    \vec{x}^{n+1} = \vec{x}^{n} - J^{-1}(\vec{x}^{n}) \cdot \vec{F}(\vec{x}^{n}), \quad J(\vec{x}^{n}) = \frac{d\vec{F}(\vec{x}^{n})}{d\vec{x}^{n}}.
\]

The largest part of the computational time is related to finding the inverse Jacobian matrix \( J^{-1} \), which order linearly depends on the number of cells in the computational grid.

6. Numerical experiment

The relative permeability curves to water and to gas were built using method [7], and relative permeability to oil was calculated from model [8].

The correlation between the viscosity of polymer solution and local concentrations of polymer and salt were assumed as linear. Volumetric coefficients of water and degassed oil and a linear isotherm of polymer adsorption are also linear. The reservoir porosity and permeability are constant.

The injection of polymer solution was simulated for 1750 days (57 months). The polymer and salt concentrations in the injected solution are 2 kg/m³ and 0.01 kg/m³, respectively. After the polymer, pure water was injected during 2700 days (90 months). For numerical simulation 6 different grid configurations were used: a hexagonal locally refined Voronoi grid with 121, 444 and 2678 grid cells and a rectangular grid with 121, 441 and 2601 grid cells (Figure 3).

Throughout the entire period of polymer rim injection, an oil production rate shows the decline (Figure 5), caused by the increased high viscosity areas in the reservoir. After the beginning of pure water injection, in 1750 days the size of these areas decreases due to low viscosity of pure water and partial polymer adsorption by rock surfaces that leads to a sharp increase in the oil production rate. After 2400 days the injected polymer is entirely adsorbed, the oil production profile achieves a plateau.
3000 days, the injected water reaches a production well causing the oil production rate decrease and water production rate increase.

Figure 3. Grid configuration and reservoir pressure after 60 months of waterflooding. The hexagonal locally refined Voronoi grid is shown in (a), (b), (c). A rectangular grid is shown in (d), (e), (f).

Figure 4. Oil flow rate calculated on the rectangular grid.

Figure 5. Oil flow rate calculated on the hexagonal locally refined Voronoi grid.

Figure 6. Water saturation after 80 month of waterflooding.
Figure 3 demonstrates that the Voronoi grid gives more realistic picture of the reservoir pressure distribution in the well vicinity than a rectangular grid with the same number of grid cells. From analysis of Figures 4 and 5 the following conclusion is made: unlike the rectangular grid, changes in the grid cell number the Voronoi grid yields smaller dispersion in oil production. Using this grid, the production rates calculated on a small number of grid cells are close to the rates for a large number of grid cells. Thus, the Voronoi grid keeps the calculation accuracy for well indicators giving a significant reduction in the number of grid cells. This can be explained by the fact that in the regions with high pressure gradient the solution quality does not change much, since the grid cells in these regions always remain fine and the cells with wells do not change their location. However, with a lower number of grid cells the fluid front is worse described at a distance from the wells, which is expressed in the oil production profile smoothing at water breakthrough into the production well (Figure 5).

Figure 6 illustrates the grid orientation effect, causing the distortion of water front shape. Compared with the rectangular grid, the Voronoi grid gives more realistic picture of the water front movement when using the hexagonal pattern; however, it required longer computational time due to the Jacobian matrix structure complication.

7. Conclusion
A tool was developed for evaluating the polymer flooding performance, which can be used for defining optimal polymer properties and injection volumes.

The Voronoi grid with a given number of cells allows achieving solution accuracy by redistributing the grid cell density through the reservoir regions. With local grid refinement near the wells, the computation of well technological parameters is characterized by higher accuracy, but with coarse estimation of the fluid front movement. The grid orientation effect can be reduced from using different grid pattern (rectangular, hexagonal or octagonal). The Voronoi grid describes much more accurately the structure of complex reservoirs, horizontal well geometry and wells with hydraulic fracture than the classical rectangular one.

Summarizing, we can state that the Voronoi grid provides more detail solution to the problem as well as flexibly in balancing between accuracy and computational time.

8. Acknowledgments
This study was supported by the Russian Foundation for Basic Research (Project 17-41-020226 r_a) and UNTC LLC.

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