Vertex rough graphs

Bibin Mathew¹ · Sunil Jacob John¹ · Harish Garg²

Received: 21 August 2019 / Accepted: 11 February 2020 © The Author(s) 2020

Abstract
This article introduces the notion of vertex rough graph and discusses certain basic graph theoretic definitions and examples. Adjacency of vertices is used to create a matrix corresponding to a vertex rough graph. Also, the membership function of a vertex rough graph is introduced with the help of Pawlak’s Rough set theory, and using this certain results are obtained. The concepts of rough precision and rough similarity degree are extended to vertex rough graphs.

Keywords Rough set · Edge rough graph · Vertex rough graph · Rough membership function

Introduction
Uncertainty and imprecision occurring in the form of vagueness and ambiguity make many of the naturally occurring situations complex and complicated. Classical mathematical techniques often fail to prosper in situations like this. Further, most of these techniques are crisp, precise and deterministic. The classical technique of probability theory has the limitation that the happening of an event is strictly determined by chance. Zadeh [1] has defined fuzzy sets which can mathematically model situations which are imprecise and vague. Pawlak [2] introduced the concept of rough sets which is an excellent mathematical tool to handle ambiguity and equivocalness associated with the given information. The main advantage of rough set theory is that it does not need any additional information about the data, like membership values in fuzzy sets. In classical set theory, Crisp sets are defined by a membership function, but in rough set theory, the primary concept to define a rough set is an indiscernibility relation. It employs indiscernibility relations to evaluate to what extent two objects were similar. Using this indiscernibility relation, one can construct lower and upper approximations of a set. Lower approximation consists of all instances which surely belongs to the concept, and upper approximation consists of all cases which possibly belongs to the concepts. One benefit of the rough set theory is that it does not require any additional parameter to extract information. Rough set theory has found main applications [3] in many branches like rough classification and logic [4,5], decision making [6,7], machine learning [8], data mining [9,10], banking [11], medicine [12], etc.

A Graph is a symmetric binary relation on a set. It is a fundamental tool in mathematical modelling and has applications in almost all branches of Science and Engineering. Many of the real life problems were solved through mathematical modelling with the help of graph theory. The theory of rough graphs is an attempt to unify rough set theory and graph theory. Graph theory, where objects are represented by vertices and relations by edges, is a convenient way of representing information involving relationship between objects. When there is ambiguity in the description of the objects or in its relationships or in both, it is quite natural that we need to design a structure supporting it, which is called a Rough Graph.

With the advent of World Wide Web, the amount of data need to be collected and stored has increased exponentially and a major part of this data can be represented as graphs which includes page link structures, social, professional and academic networks such as Facebook, Linkedin, DBLP, etc. Most of the times, the patterns of connection between entities in these, which represents non trivial topological features, which are neither purely crisp nor completely random, is called a Complex Network [13]. A major challenge nowa-
days is to mine these complex networks and the abundance
of data in these motivated a new area, called Graph mining,
which focus on investigate, propose and develop new algo-
rithms designed to mine complex networks. As ambiguity is
naturally inherited in these networks, a suitable modelling
can be achieved by utilizing the concept of Rough Graphs.

The notion of edge rough graph was introduced by He and
Shi [14]. They have established the concept using a partition
on the edge set of a graph. He et al. [15] extended this concept
to weighted rough graph by enduing the edges of rough graph
with weight attribute, and gave the algorithm of exploring the
class optimal tree in weighted rough graph, which generalizes
the classical Kruskal algorithm of exploring the optimal tree
and presented an application in relationship analysis. Another
application of Weighted Rough graph was discussed in [16].
Combining the edge rough graphs and cayley graphs, Liang
et al. [17] studied an application of rough graph in data min-
ing. Tong He introduced further rough theoretic properties of
rough graphs [18] and representation forms of rough graphs
[19]. Some other hybrid structures of rough graphs like soft
rough graphs, neutrosophic soft rough graphs, intuitionistic
fuzzy rough graphs are also introduced in [20–22].

In edge rough graph, there is no significance for vertex
set. It is not possible to compare any two arbitrary rough
graphs. They can be compared only if their vertex sets are
same. If such a comparison is possible, then the real life
applications of rough graph will have more flexibility. The
main objective of this paper was to introduce the concept
of vertex rough graph which is a more general concept than
the edge rough graph. The vertex rough graph is constructed
using a partition on the vertex set. Using a partition of vertex
set, we define lower approximation and upper approximation
of a graph. Hence, this paper is an introduction to the theory
of vertex rough graph.

In this paper, the basic idea of edge rough graph is
extended to vertex rough graph. Section 2 discusses some
basic definitions of graph theory, rough set and edge rough
graph. In Sect. 3, the notion of vertex rough graph is intro-
duced and some examples are given. Basic graph theoretic
definitions of vertex rough graphs are defined and a counter
example for a connected graph which is not surely connected
is provided. Later, adjacency matrix of vertex rough graph is
deﬁned and some of its properties are discussed. In the last
section, some rough theoretic ideas like membership func-
tions and precisions of a vertex rough graphs are deﬁned and
related properties are derived.

Preliminaries

Some basic definitions from graph theory, Rough set theory
and edge rough graph are given:

Definition 2.1 [23] A graph G is an ordered triple
(V(G), E(G), ψG) consisting of a non-empty set V(G) of
vertices, a set E(G), disjoint from V(G), of edges, and an
incidence function ψG that associates with each edge of G
an unordered pair of (not necessarily distinct) vertices of G.
If e is an edge and u and v are vertices such that ψG(e) = uv,
then e is said to join u and v; the vertices u and v are called
the ends of e. Two graphs G and H are identical (written
G = H) if V (G) = V (H), E(G) = E(H), and ψG = ψH.
Two graphs G and H are said to be isomorphic (written
G ≃ H) if there are bijections θ : V (G) → V (H) and
φ : E(G) → E(H) such that ψG(e) = uv if and only if
ψH(φ(e)) = θ(u)θ(v); such a pair (θ, φ) of mappings is
called an isomorphism between G and H.

Definition 2.2 [2] Suppose we are given a set of objects
U called the universe and an indiscernibility relation R ⊆ U ×
U, representing our lack of knowledge about elements of U.
For the sake of simplicity we assume that R is an equivalence
relation. Let X be a subset of U. We want to characterize the
set X with respect to R:

- R-lower approximation of X

\[ R(x)_R = \bigcup_{x \in X} \{ R(x) : R(x) \subseteq X \} \]

- R-upper approximation of X

\[ R(x)^*_R = \bigcup_{x \in X} \{ R(x) : R(x) \cap X \neq \emptyset \} \]

- R-boundary region of X

\[ RN_R(x) = R(x)^*_R - R(x)_R \]

The pair (R(x)_R, R(x)^*_R) is called Rough Set. X is crisp (exact
with respect to R), if the boundary region of X is empty. Set
X is rough (inexact with respect to R), if the boundary region
of X is non-empty.

Definition 2.3 [14] Given universe of discourse U, V =
\{v_1, v_2, \ldots, v_{|V|}\}, P = \{r_1, r_2, \ldots, r_{|P|}\} is attributes set on
U, and P contains vertex attribute (v_i, v_j), where v_i ∈ V,
v_j ∈ V. Let E = \bigcup_{r \in P} (v_i, v_j) is edge set on U, graph
U = (V, E) is called universe graph. For any attribute set
R ⊆ P on E, the elements (or be called edges) in E can
be classified into different equivalence classes [e]_R. For any
subgraph T = (W, X), where W ⊆ V, X ⊆ E, graph T is
called R-definable graph or R-exact graph if X is the union
of some [e]_R. Conversely, graph T is called R-undefinable
graph or R-rough graph. For R-rough graph, two exact graphs
R(T) = (W, R(X)_R) and R(T)^*_R = (W, R(X)^*_R) can be
used to define it approximately, where
Let a indiscernability relation on vertex set $V$ is presented. In this section, vertex rough graph of a graph with respect to approximations of vertex set and edge set as (Fig. 2):

The graphs $R(T)_s$ and $R(T)^s$ are called $R$-lower and $R$-upper approximate graphs of $T$. The pair of graph $(R(T)_s, R(T)^s)$ is called $R$-rough graph. The set $bn_R(X) = R(X)_s - R(X)^s$ is called the $R$-boundary of edges set $X$ of $T$.

**Vertex rough graph**

In this section, Vertex rough graph of a graph with respect to a indiscernability relation on vertex set $V$ is presented.

**Definition 3.1** Let $G = (V, E)$ be a universe graph with $V = \{v_1, v_2, \ldots, v_n\}$ and $E = \{e_1, e_2, \ldots, e_m\}$. Let $R$ be an equivalence relation defined on $V$. Then the elements in $V$ can be divided into different equivalence classes $[v]_R$.

**Definition 3.2** Let $T(W, X)$ be a subgraph of $G(V, E)$ where $W \subseteq V, X \subseteq E$, graph $T$ is called $R$-definable graph or $R$-exact graph if $W$ is the union of some $[v]_R$. Otherwise, the graph $T$ is called $R$-undefinable graph or $R$-rough graph.

**Definition 3.3** $R$-vertex rough graph is defined in terms of two exact graphs $R_s(T) = (R_s(W), R_s(X))$ and $R^s(T) = (R^s(W), R^s(X))$, where

$$
R_s(W) = \{v \in V : [v]_R \subseteq W\}
$$

$$
R^s(W) = \{v \in V : [v]_R \cap W \neq \emptyset\}
$$

$$
R_s(X) = \{(v_i, v_j) \in X : v_i, v_j \in [v]_R \text{ for some } v \in R_s(W)\}
$$

$$
R^s(X) = \{(v_i, v_j) \in X : v_i \in [v]_R \text{ & } [v]_R \cap X \neq \emptyset \text{ and } v_j \in [v]_R \text{ & } [v]_R \cap X \neq \emptyset\}
$$

The graphs $R_s(T)$ and $R^s(T)$ are called $R$-lower approximate graph of $T$ and $R$-upper approximate graph of $T$. The pair of graph $(R_s(T), R^s(T))$ is called $R$-vertex rough graph.

**Example 3.1** Consider $G(V, E) = \{v_1, v_2, v_3, v_4, v_5\}$

$$
V / R = \{\{v_3, v_4, v_5\}, \{v_1, v_2\}\}
$$

Consider $T = (W, X)$ be a subgraph of $G(V, E)$ (Fig. 1).

By using definition 3.3, we get the lower and upper approximations of vertex set and edge set as (Fig. 2):

$$
R_s(W) = \{v_3, v_4, v_5\} \quad R^s(W) = \{v_1, v_2, v_3, v_4, v_5\}
$$

$$
R_s(X) = \{e_3, e_4\} \quad R^s(X) = \{e_1, e_2, e_3, e_4, e_5, e_6\}
$$

$$
R_s(T) = (R_s(W), R_s(X)) \quad R^s(T) = (R^s(W), R^s(X))
$$

**Proposition 3.1** Lower and upper approximations of a graph have the following properties:

For all $T, T_1, T_2 \subseteq G$.

1. $R_s(T) \subseteq T \subseteq R^s(T)$.
2. $R_s(K^c) = R^s(K^c) = K^c$, $R_s(G) = R^s(G) = G$ where $K$ is the Complete graph.
3. $R_s(T_1 \cap T_2) = R_s(T_1) \cap R_s(T_2)$.
4. $R^s(T_1 \cap T_2) = R^s(T_1) \cup R^s(T_2)$.
5. $R_s(T_1 \cup T_2) \subseteq R_s(T_1) \cup R_s(T_2)$.
6. $R^s(T_1 \cup T_2) \subseteq R^s(T_1) \cup R^s(T_2)$.
7. $T_1 \subseteq T_2 \Rightarrow R_s(T_1) \subseteq R_s(T_2)$ & $R^s(T_1) \subseteq R^s(T_2)$.
8. $R_s(R_s(T)) = R^s(T)$.
9. $R^s(R^s(T)) = R^s(T)$.

**Definition 3.4** Let $T(W_1, X)$ and $S(W_2, Y)$ be subgraphs of $G(V, E)$ where $W_1 \subseteq V, W_2 \subseteq V, X \subseteq E, Y \subseteq E$, $T = (R_s(T), R^s(T))$ and $S = (R_s(S), R^s(S))$ be its rough graphs. $S$ is said to be surely subgraph of $T$ if $R_s(S) \subseteq R_s(T)$. Also $S$ is said to be possibly subgraph of $T$ if $R^s(S) \subseteq R^s(T)$ if $S$ is both surely subgraph and possibly subgraph of $T$, then $S$ is a rough subgraph of $T$.

**Definition 3.5** A set of two or more edges of a rough graph $T$ is said to be multiple or parallel edges if they have the same end vertices. An edge for which two ends are the same is called a loop at the common vertex. A rough graph $T = (R_s(T), R^s(T))$ is said to be surely simple if $R_s(T)$ contains no loops and parallel edges. A rough graph $T = (R_s(T), R^s(T))$ is said to be possibly simple if $R^s(T)$ contains no loops and parallel edges. A rough graph $T = (R_s(T), R^s(T))$ is said to be simple if it is both surely and possibly simple graphs.

**Definition 3.6** Two rough graphs $T = (R_s(T), R^s(T))$ and $S = (R_s(S), R^s(S))$ are said to be surely isomorphic if there is a graph isomorphism between $R_s(T)$ and $R_s(S)$. Also it is said to be possibly isomorphic if there is a graph isomorphism between $R^s(T)$ and $R^s(S)$. Two rough graphs $T = (R_s(T), R^s(T))$ and $S = (R_s(S), R^s(S))$ are said to be isomorphic if they are both surely and possibly isomorphic.

**Definition 3.7** Let $T = (R_s(T), R^s(T))$ be a Rough graph. The Complement $T^c$ of $T$ with respect to $G$ is defined by taking $V(T^c) = V(T)$ and $T^e = (R_s(T)^c, R^s(T)^c)$ where adjacency of $R_s(T)^c$ is defined as two vertices of $u$ and $v$ are
adjacent if and only if they are non adjacent in $R_*(T)$. Also adjacency of $R^*(T)^c$ is defined as two vertices $u$ and $v$ are adjacent if and only if they are non adjacent in $R^*(T)$.

**Remark 3.1** The connectedness of vertex rough graph is the same as the connectedness of edge rough graph.

**Result 3.1** If $T(W, X)$ is connected then it need not be surely connected. Similarly $T$ is a tree then it need not be a sure tree.

**Example 3.2** Consider $G(V, E)$

$V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$,

$V/R = \{(v_7)\{v_4, v_5\}, \{v_1, v_2\}, \{v_3, v_6\}\}$

Consider $T = (W, X)$ be a subgraph (Fig. 3) of $G(V, E)$

Then, we get the lower approximation of vertex set and edge set as (Fig. 4)

$R_*(W) = \{v_3, v_4, v_6\} R(X)_* = \{e_4, e_5\}$

$R_*(T) = (R_*(W), R_*(X))$

Here $R_*(T)$ is disconnected, but $T = (W, X)$ is connected. Also $T$ is a tree but $R_*(T)$ is not a tree. It is a forest.

**Matrix corresponding to a rough graph**

Let $G=(V, E)$ be a universe graph with $V=\{v_1, v_2, \ldots, v_n\}$, $R$ be an equivalent relation defined on $V$. Let $T(W, X)$ be a subgraph of $G$. $T = (R_*(T), R^*(T))$ be the corresponding rough graph. Then we can define a nonzero ternary matrix $A_R(T)$ of $T$ by

$$A_R(T) = (a_{ij}) = \begin{cases} 0 & \text{if } (v_i, v_j) \notin R^*(X) \\ 1 & \text{if } (v_i, v_j) \in R^*(X) \& (v_i, v_j) \notin R_*(X) \\ 2 & \text{if } (v_i, v_j) \in R_*(X) \end{cases}$$

**Example 3.3** Matrix corresponding to the rough graph in Example 3.1 is

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}$$
Definition 4.1 The rough vertex membership function of a rough graph $T = (R_a(T), R^*(T))$ is a function $\mu^R_W : V \rightarrow [0, 1]$ is defined as $\mu^R_W(v) = \frac{|W \cap [v]_R|}{|[v]_R|}$.

Also, the rough edge membership function of a rough graph $T = (R_a(T), R^*(T))$ is a function $\delta^R_W(v_i, v_j)$ is defined as $\delta^R_W(v_i, v_j) = min\{\mu^R_W(v_i), \mu^R_W(v_j)\}$.

Remark 3.2 Let $T$ be a rough graph and $A_R(T)$ be its corresponding matrix. Then

1. $T$ is exact if all entries of $A_R(T)$ are 0 & 2.
2. $T$ is Rough if atleast one entry of $A_R(T)$ is 1.

Rough properties of rough graph

In the same way of rough set theory, rough graph can be also defined employing instead of approximation, rough membership function.

**Proposition 4.1** The membership function has the following properties:

1. $\mu^R_W(v) = 1$ iff $v \in R_a(W)$ and $\delta^R_W(v_i, v_j) = 1$ iff $(v_i, v_j) \in R_a(X)$
2. $\mu^R_W(v) = 0$ iff $v \in V - R^*(W)$ and $\delta^R_W(v_i, v_j) = 0$ iff $(v_i, v_j) \in E - R^*(X)$
3. $\mu^R_W(v) = 1 - \mu^R_W(v)$ and $\delta^R_W(v_i, v_j) \leq 1 - \delta^R_W(v_i, v_j)$
4. $\mu^R_{W_1\cup W_2}(v) = \mu^R_{W_1}(v) + \mu^R_{W_2}(v) - \mu^R_{W_1\cap W_2}(v)$

Proof

$\mu^R_W(v) = 1 \iff \frac{|W \cap [v]_R|}{|[v]_R|} = 1$

$\delta^R_W(v_i, v_j) = 1 \iff min\{\mu^R_W(v_i), \mu^R_W(v_j)\} = 1$

$\mu^R_W(v) = 0 \iff \frac{|W \cap [v]_R|}{|[v]_R|} = 0$

$\delta^R_W(v_i, v_j) = 0 \iff min\{\mu^R_W(v_i), \mu^R_W(v_j)\} = 0$

$\mu^R_W(v) = 1 - \mu^R_W(v)$

Next we extend the definition of Edge precision $\alpha_R(T)$ [18] of a edge rough graph to vertex rough graph:
Definition 4.3 A vertex rough graph $T = (R_s(T), R^*(T))$ where $R^*(T) = (R^*(W), R^*(X))$ and $R_s(T) = (R_s(W), R_s(X))$. $\alpha _R(T)$ is the R-vertex precision of T and $\beta _R(T)$ is the R-edge precision of T defined by $\alpha _R(T) = \frac{|R_s(W)|}{|R^*(W)|}$ and $\beta _R(T) = \frac{|R_s(X)|}{|R^*(X)|}$ where $W \neq \emptyset$, $X \neq \emptyset$.

Result 4.1 Let M be the set of all vertex rough graphs. For any vertex attribute set W and edge attribute set X and $t \subseteq M$ then, $0 \leq \alpha _R(T) \leq 1$ & $0 \leq \beta _R(T) \leq 1$. If T is exact iff $\alpha _R(T) = 1$ & $\beta _R(T) = 1$.

Proof Since $R_s(W) \subseteq R^*(W)$ and $R_s(X) \subseteq R^*(X)$.

Therefore, $0 \leq \alpha _R(T) \leq 1$ & $0 \leq \beta _R(T) \leq 1$.

If T is exact $\iff R_s(W) = R^*(W)$ & $R_s(X) = R^*(X)$ $\iff \alpha _R(T) = 1$ & $\beta _R(T) = 1$

Result 4.2 If T and S are two vertex rough graphs, where $T = (W_1, X_1)$ and $S = (W_2, X_2)$. S is a vertex rough sub-graph of T, then $\alpha _S(S) \leq \alpha _R(T)$ and $\beta _S(S) \leq \beta _R(T)$.

To compare two rough graphs, rough similarity degree [18] is an important measure. We can extend it to vertex rough graph.

Definition 4.4 Given vertex rough graph set $M$, attribute set $R \in (M, R)$ is a knowledge system. Let $H, J \subseteq M$ where $H = (W_1, X_1), J = (W_2, Y)$ and

\[R_s(H) = (R_s(W_1), R_s(X_1)), R^*(H) = (R^*(W_1), R^*(X_1))\]

\[R_s(J) = (R_s(W_2), R_s(Y)), R^*(J) = (R^*(W_2), R^*(Y))\].

1. Rough vertex similarity degree ($\langle H, J \rangle _R$) and rough edge similarity degree ($\langle H, J \rangle _R$) between $H$ and $J$ are defined by

\[\langle H, J \rangle _R = \min \left\{ \frac{|R_s(W_1) \cap R_s(W_2)|}{|R_s(W_1) \cup R_s(W_2)|}, \frac{|R^*(W_1) \cap R^*(W_2)|}{|R^*(W_1) \cup R^*(W_2)|} \right\} \]

\[\langle H, J \rangle _R = \min \left\{ \frac{|R_s(X) \cap R_s(Y)|}{|R_s(X) \cup R_s(Y)|}, \frac{|R^*(X) \cap R^*(Y)|}{|R^*(X) \cup R^*(Y)|} \right\} \]

2. Lower rough vertex similarity degree ($\langle H, J \rangle _{R^-}$) and lower rough edge similarity degree ($\langle H, J \rangle _{R^-}$) between $H$ and $J$ are defined by

\[\langle H, J \rangle _{R^-} = \left\{ \frac{|R_s(W_1) \cap R_s(W_2)|}{|R_s(W_1) \cup R_s(W_2)|}, \frac{|R^*(W_1) \cap R^*(W_2)|}{|R^*(W_1) \cup R^*(W_2)|} \right\} \]

\[\langle H, J \rangle _{R^-} = \left\{ \frac{|R_s(X) \cap R_s(Y)|}{|R_s(X) \cup R_s(Y)|}, \frac{|R^*(X) \cap R^*(Y)|}{|R^*(X) \cup R^*(Y)|} \right\} \]

3. Upper rough vertex similarity degree ($\langle H, J \rangle _{R^+}$) and upper rough edge similarity degree ($\langle H, J \rangle _{R^+}$) between $H$ and $J$ are defined by

\[\langle H, J \rangle _{R^+} = \left\{ \frac{|R^*(W_1) \cap R^*(W_2)|}{|R^*(W_1) \cup R^*(W_2)|}, \frac{|R^*(X) \cap R^*(Y)|}{|R^*(X) \cup R^*(Y)|} \right\} \]

\[\langle H, J \rangle _{R^+} = \left\{ \frac{|R^*(X) \cap R^*(Y)|}{|R^*(X) \cup R^*(Y)|}, \frac{|R^*(X) \cap R^*(Y)|}{|R^*(X) \cup R^*(Y)|} \right\} \]

Proposition 4.2 Given vertex rough graph set $M$, R to be the attribute set. $K = (M, R)$ to be the knowledge system. $H, J \subseteq M$. Then

1. $H$ and $J$ are R-rough equal iff $\langle H, J \rangle _R = [H, J]_R = 1$

2. $H$ and $J$ are R-lower rough equal iff $\langle H, J \rangle _{R^-} = [H, J]_{R^-} = 1$

3. $H$ and $J$ are R-upper rough equal iff $\langle H, J \rangle _{R^+} = [H, J]_{R^+} = 1$

Proof 1.

$\langle H, J \rangle _R = [H, J]_R = 1$

\[\iff \min \left\{ \frac{|R_s(W_1) \cap R_s(W_2)|}{|R_s(W_1) \cup R_s(W_2)|}, \frac{|R^*(W_1) \cap R^*(W_2)|}{|R^*(W_1) \cup R^*(W_2)|} \right\} = 1, \]

\[\frac{|R_s(X) \cap R_s(Y)|}{|R_s(X) \cup R_s(Y)|}, \frac{|R^*(X) \cap R^*(Y)|}{|R^*(X) \cup R^*(Y)|} \right\} = 1, \]

\[\iff [H_s(W_1) \cap H_s(W_2)] \cup R_s(W_2)] = [R_s(W_1) \cup R_s(W_2)] \]

$[H_s(X) \cap H_s(Y)] \cup R_s(Y)] \right\} = 1 \]

$[H_s(X) \cap H_s(Y)] \cup R_s(Y)] \right\} = 1 \]

$H$ and $J$ are R-rough equal.
Conclusion

Both the Rough set theory and the graph theory have a variety of applications across different fields. This paper introduced the concept of vertex rough graph which combines rough set theory and the graph theory. Similar to rough set theory, the notion of vertex and edge rough membership function is introduced and using this membership functions, an alternative definition of vertex rough graph has been developed. Later, vertex precision and edge precision are defined and some properties are discussed. Since edge rough graph has lot of applications in various fields, like relationship analysis, data mining, etc., the vertex rough graphs also will have applications in these fields as well as many other fields. In future we will find out further rough properties and applications of vertex rough graphs.

Acknowledgements The first author acknowledges the financial assistance given by University Grants Commission (UGC), Government of India throughout the preparation of this paper. Authors are very much thankful to the reviewers for the valuable suggestions which helped greatly in improving the quality of the paper.

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