Cooper-pair qubit and Cooper-pair electrometer in one device

A. B. Zorin

Physikalisch-Technische Bundesanstalt, Bundesallee 100, D-38116 Braunschweig, Germany

An all-superconductor charge qubit enabling a radio-frequency readout of its quantum state is described. The core element of the setup is a superconducting loop which includes the single-Cooper-pair (Bloch) transistor. This circuit has two functions: First, it operates as a charge qubit with magnetic control of Josephson coupling and electrostatic control of the charge on the transistor island. Secondly, it acts as the transducer of the rf electrometer, which probes the qubit state by measuring the Josephson inductance of the transistor. The evaluation of the basic parameters of this device shows its superioriority over the rf-SET-based qubit setup.

PACS numbers: 74.50.+r, 85.25.Na, 03.67.Lx

Superconducting structures with small Josephson tunnel junctions serve as a basis for electronic devices operating on single Cooper pairs and possessing remarkable characteristics. The paper Ref. [1] has demonstrated the potential of the single-Cooper-pair box circuit [2] as a charge qubit and thus has attracted renewed attention to this field. The practical realization of the Cooper pair qubit is not, however, simple and the main problems here are the achievement of a reliable readout of the quantum state and an elongation of the decoherence time. For the most part, these two issues are interrelated because a charge detector coupled to the qubit presents the principal source of quantum decoherence.

The qubit setup consisting of a Cooper pair box and a capacitively coupled single electron transistor (SET, including the rf-SET [3]) has been extensively explored [4, 5, 6]. Although a preliminary analysis shows that qubit states can, in principle, be measured in the shot regime [6], the ‘mismatch’ of the charge carriers in the setup components (namely, the incoherent nature of charges in the SET) might lead to an unaccounted enhancement of decoherence in the system. Alternatively, the generic type of Cooper-pair (Bloch-transistor [7]) electrometers made from superconductors [4, 5] seems to be quite promising as regards matching with the Cooper pair box. Furthermore, similar to dc-SQUIDs [10] and in contrast to SETs, the resistively shunted Cooper-pair electrometer belongs to the category of perfect (quantum-limited) linear detectors [11, 12] and, therefore, can perform continuous measurements of a quantum object [13].

Recently, we have proposed an rf-driven single-Cooper-pair electrometer [4] whose energy-resolution figure \( \varepsilon \) can approach the standard quantum limit of \( \hbar/2 \). The transducer of this electrometer is a Bloch transistor inserted into a superconducting loop. The magnitude of the supercurrent circulating in the loop depends on the polarization charge (quasicharge) on the transistor island induced via a coupling capacitance by the charge source, e.g., the qubit.

In this paper we present a circuit in which the electrometer’s transducer takes over the function of the Cooper pair box (qubit) as well. The device’s core element is a superconducting loop including a mesoscopic double Josephson junction with a capacitive gate (transistor); it is shown in Fig. 1a. The individual junctions are characterized by coupling energies \( E_{J1} \) and \( E_{J2} \), which do not differ significantly,

\[
k_B T \ll \delta E_J \equiv |E_{J1} - E_{J2}| \ll E_J \approx E_c, \tag{1}
\]

where \( E_J \equiv (E_{J1} + E_{J2})/2 \) is the average Josephson coupling energy and \( E_c \equiv e^2/2C \) is the total charging energy of the island (center electrode) [14]. The island’s total capacitance \( C \) is much lower than \( C_L \) (the capacitance of the ‘macroscopic’ loop with respect to ground).
thereby to localize the region, coupling $C$ of the island’s charge states $|\phi\rangle$. The resultant Josephson coupling of the Cooper pair box determines the energy gap in the degeneracy point $q = e$ and varies from $2E_j$ (at $f = 0$) to $\delta E_1$ (at $f = 0.5$). The circuit parameters are: $E_j = E_c$ and $\delta E_1 = 0.1 E_j$.

The quasicharge $q$ of the system as a Cooper-pair box (qubit), $J_{s} = (a$ good quantum variable) and the band index $n = 1$ respectively. The resultant Josephson coupling of the Cooper pair box to such electrometer via a small capacitance.

The present case of ‘100% coupling’ between box and electrometer, the amplitude of driving signal $V_{sh}$ can be reduced so that $a \ll 1$. In this regime, the impedance of the low-$\beta_M$ loop with the Bloch transistor is determined by the Josephson inductance $L_j$ whose reverse value is equal to

$$L_j^{-1}(q, n) = \frac{2\pi}{\Phi_0} \frac{\partial I_s(\varphi, q, n)}{\partial \varphi}. \tag{3}$$

Due to coupling to the loop the effective inductance of the tank is changed,

$$L_T \to L_T - M^2L_j^{-1}(q, n), \tag{4}$$

and this leads to the shift of the resonance frequency, $\delta\omega_0/\omega_0 \approx k^2L_j^{-1}/2$. To evaluate this shift we compute the expectation value of the supercurrent operator $\hat{I}_s$ using the Bloch eigenfunctions $|q, n\rangle$ as follows,

$$I_s(\varphi, q, n) = \langle q, n | \hat{I}_s | q, n \rangle = \langle q, n | I_{s1} \sin \varphi_1 | q, n \rangle = \langle q, n | I_{s2} \sin \varphi_2 | q, n \rangle. \tag{5}$$

The result of numerical calculation for the case of almost symmetric transistor with $E_j = E_c$ is presented in Fig. 3. As can be seen from these plots, not only the maximum of Josephson supercurrent, i.e. the critical current value, depends on the quasicharge $q$ and the band index $n$ [19], but also its phase dependence for $n = 0$ and $n = 1$ is different. In the ground state the current-phase dependence has a shape typical of the Josephson weak links (see, e.g., the review paper [20] and references therein). In the upper state, for all $q$ not close to $0$ mod$(2\pi)$ and not very large values of the ratio $E_j/E_c$ the dependence has a phase shift of $\pi$, which is typical of the Josephson junctions with ferromagnetic interlayer [21].

This behavior of the supercurrent results in the strong dependence of the Josephson inductance (see Eq. (3)) on $q$ and $n$ at $\varphi_0 \approx \pi$, viz., $L_j^{-1}$ is negative in the ground
state and positive in the upper state, while its absolute values \( \sim (I_{00}/\Phi_0) \). Therefore, when the band number \( n \) is changing, \( 0 \Rightarrow 1 \), the relative shift of the resonance frequency is substantial, i.e.,

\[
\delta \omega(\pi/2) \sim k^2 \beta_L \sim Q^{-1}.
\]

Near the resonance this shift leads to a dramatic change of the amplitude of ac current in the tank, \( \delta I_n \sim I_n = (\Phi_0/2\pi M) a \), that is reliably measured at \( a \gtrsim (\Theta_A B/E_1 \omega)^{1/2} \), assuming the output bandwidth \( B \lesssim \omega/Q \ll \omega \) and dominating role of the noise of the amplifier whose noise temperature \( T_A \equiv \Theta_A/k_B \gg T \) and the input impedance \( R_A \gg R_T \).

The great advantage of the superconducting loop including the Bloch transistor consists in a negligibly low dissipation and, therefore, a minor role of intrinsic sources of qubit decoherence. The processes associated with dissipation are the quasiparticle and the pair-quasiparticle interference components of the tunneling. In our case, however, both the frequency and the amplitude of the voltage across the transistor \( V_{tr} \) are small, namely \( 2eV_{tr} = a\hbar \omega \ll \hbar \omega \lesssim k_B T \ll E_c < \Delta \). This relation ensures huge suppression (by a factor \( \eta = \exp(-\Delta/k_B T) \ll 1 \)) of all dissipative processes. Moreover, since the parity effect blocks sequential quasiparticle tunneling, dissipation can only occur due to the co-tunneling effect. The effective leakage resistance of the transistor (with normal resistances of the junctions \( R_1 \) and \( R_2 \) therefore is of the order of \( (R_1 R_2/R_{q}) (\Delta/q \eta \hbar \omega)^2 \gg R_{1,2} \), where \( R_q = \hbar/e^2 \) is the resistance quantum, and can, therefore, be neglected.

The main source of decoherence therefore is the external electromagnetic circuit: it causes fluctuations of the island’s electric potential \( \tilde{u} \). The fluctuation sources, labeled as \( v_g \), \( i_b \), \( v_T \) and \( v_A \) in Fig. 1, are associated with dissipative components of the circuit, \( R_g \), \( R_b \), \( R_T \) (being presumably at the equilibrium temperature \( T \) and \( R_A \) (characterized by \( T_A \)). The rates of dephasing caused by the gate and dc-flux bias, i.e. by the sources \( v_g \) and \( i_b \), respectively, were earlier evaluated by Makhlkin, et al. They showed that reduction of the coupling strengths, i.e., \( C_g \) and \( M_b \), can make these rates small enough to allow many single-bit manipulations to be performed within the dephasing time. If \( V_{tr} = 0 \), similar reasoning can be applied to the resonance tank circuit containing the sources \( v_T \) and \( v_A \).

When the rf-drive is on, it leads to an enhancement of the fluctuations \( \tilde{u} \) at low frequencies (\( \ll \omega \)) due to parametric down-conversion of noise at frequencies around \( \omega \).

The spectral density of these fluctuations is equivalent to that of the resistance \( R_{eff} = a^2(E_1/\hbar \omega)R_q \) at temperature \( T_A \). At \( T_A \approx \hbar \omega/k_B \), the resistance \( R_{eff} \approx R_q/Q \), which allows to \( N = R_q/R_{eff} \sim Q \) (say, \( \sim 10^3 \)) single-qubit manipulations to be performed in the degeneracy point, \( q = e \), during the entanglement time \( \tau_{ent} \sim Q/\hbar \delta E_1 \). In contrast to the rf-SET setup \([\text{I}]\), there is, in principle, no need for switching the electrometer off in the degeneracy point \( q = e \).

During the measurement (away from the point \( q = e \)) the energy gap between the charge states is large, \( \Delta E \equiv \hbar \Omega \gtrsim E_c > \delta E_1 \). Due to the high impedance of the tank at the transition frequency \( \Omega \gg \omega_M \), the spectral density \( S_u(\Omega) \) is remarkably low, viz. \( \approx \hbar \omega/\pi Q \Omega L_T \), which yields a long mixing time \( \tau_{mix} = (\hbar/e)^2(\Delta E/E_1)^2 S_u^{-2}(\Omega) \). The latter value allows a measurement with a high signal-to-noise ratio,

\[
S/N = (B\tau_{mix})^{1/2} \sim (\hbar \Omega/k_B T_A)^{1/2} \gg 1,
\]

to be performed even if a cooled semiconductor amplifier with the fairly good noise temperature of \( T_A \sim 10 \) K is exploited (cf. \( S/N \approx 4 \) achievable using an Al rf-SET with extremely low noise figure \([\text{I}]\)).

I am grateful to M. Götz, M. Mück, and J. Niemeyer for valuable discussions.
[1] Y. Nakamura, Y. A. Pashkin, and J. S. Tsai, Nature 398 (1999) 768.
[2] V. Bouchiat, D. Vion, P. Joyez, D. Esteve, and M. H. Devoret, Phys. Scr. T76 (1998) 165.
[3] R. J. Schoelkopf, P. Wahlgren, A. A. Kozhevnikov, P. Delsing, and D. E. Prober, Science 280 (1998) 1238.
[4] Yu. Makhlkin, G. Schön, and A. Shnirman, Rev. Mod. Phys. 73 (2001) 357.
[5] M. H. Devoret and R. J. Schoelkopf, Nature 406 (2000) 1039.
[6] A. Aassime, G. Johansson, G. Wendin, R. J. Schoelkopf, and P. Delsing, Phys. Rev. Lett. 86 (2001) 3376.
[7] K. K. Likharev, Moscow State University Preprint No.29 (1986); D. V. Averin and K. K. Likharev, in: Mesoscopic Phenomena in Solids, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991) p. 213.
[8] S. V. Lotkhov, H. Zangerle, A. B. Zorin, T. Weimann, H. Scherer, and J. Niemeyer, IEEE Trans. Supercond. 9 (1999) 3664; A. B. Zorin, S. V. Lotkhov, Yu. A. Pashkin, H. Zangerle, V. A. Krupenin, T. Weimann, H. Scherer, and J. Niemeyer, J. Supercond. 12 (1999) 747.
[9] A. Cottet, A. Steinbach, P. Joyez, D. Vion, H. Pothier, D. Esteve, and M. E. Huber, in: Macroscopic Quantum Tunneling and Quantum Computing, edited by D. V. Averin, B. Ruggiero, and P. Silvestrini (Kluwer, New York, 2001), p. 111.
[10] V. V. Danilov, K. K. Likharev, and A. B. Zorin, IEEE Trans. Magn. 19 (1983) 572.
[11] A. B. Zorin, Phys. Rev. Lett. 76 (1996) 4408.
[12] A. B. Zorin, IEEE Trans. Instrum. and Meas. 46 (1997) 299.
[13] A. N. Korotkov, Phys. Rev. B 63 (2001) 085312; D. V. Averin, cond-mat/0004364.
[14] A. B. Zorin, Phys. Rev. Lett. 86 (2001) 3388.
[15] We assume that the superconductor energy gap $\Delta$ exceeds $E_c$, so the quasiparticle tunneling is suppressed due to the parity effect (see M. T. Tuominen, J. M. Hergenrother, T. S. Tighe, and M. Tinkham, Phys. Rev. Lett. 69 (1992) 1997). In the case of Al electrodes $\Delta_{Al} \approx 200 \mu eV$, so the energies $E_c$ and $E_J$ can be of the order of 100 $\mu eV$ and temperature $T < 100 \ mK$.
[16] K. K. Likharev and A. B. Zorin, J. Low Temp. Phys. 59 (1985) 347; D. V. Averin, A. B. Zorin, and K. K. Likharev, Sov. Phys. JETP 88 (1985) 697.
[17] P. K. Hansma, J. Appl. Phys. 44 (1973) 4191.
[18] A similar measurement scheme in a traditional single-junction rf-SQUID was proposed by R. Rifkin and B. S. Deaver, Jr., Phys. Rev. B 13 (1976) 3894. Lately, the efficiency of this method (in somewhat modified version) was demonstrated in measurement of the phase dependence of a tens-of-nanoampere supercurrent in small Nb double junctions ($E_J \approx 300 \mu eV$) at temperature $T = 4.2 \ K$ by E. Il’ichev, V. Zakosarenko, L. Fritzsche, R. Stolz, H. E. Hoening, H.-G. Meyer, M. Götz, A. B. Zorin, V. V. Khanin, A. B. Pavolotsky, and J. Niemeyer, Rev. Sci. Instrum. 72 (2001) 1882.
[19] D. J. Flees, S. Han, and J. E. Lukens, Phys. Rev. Lett. 78 (1997) 4817.
[20] K. K. Likharev, Rev. Mod. Phys. 51 (1979) 101.
[21] V. V. Ryazanov, V. A. Oboznov, A. Yu. Rusanov, A. V. Verevkin, A. A. Golubov, and J. Aarts, Phys. Rev. Lett. 86 (2001) 2427.
[22] In the state-of-the-art dc-SQUID amplifiers for the frequency band 100 MHz–1 GHz the noise figure $\Theta_A$ is of the order of $\hbar \omega$, see M. Mück, J. B. Kycia, and J. Clarke, Appl. Phys. Lett. 78 (2001) 967 and references therein.
[23] We do not address here the effect of background charge fluctuations on the qubit island, whose strength strongly depends on the sample design and operating regime (see V. A. Krupenin, D. E. Presnov, M. N. Savvateev, H. Scherer, A. B. Zorin, and J. Niemeyer, J. Appl. Phys. 48 (1998) 3212) and may present a problem in experiment (see, e.g., the recent paper on probing the box-transistor states by single electron tunneling: Y. Nakamura, Yu. A. Pashkin, T. Yamamoto, and J. S. Tsai, Preprint (2001)).
[24] D. N. Langenberg, Rev. Phys. Appl. 9 (1974) 35.