Research Article

Hybrid Random Regret Minimization and Random Utility Maximization in the Context of Schedule-Based Urban Rail Transit Assignment

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1. Introduction

Analysis of travelers’ route choice behavior is very important for daily urban rail transit operation. However, for a complex urban rail transit network, passengers’ route cannot usually be obtained directly, because only the departure and destination station are recorded in smart card data records in most urban rail transit networks. In this situation, passengers travel route choice can be estimated through a transit assignment model given OD demand data and train timetable. The accuracy of the estimation is highly dependent on the extent to which the model can reflect the realized passenger behaviors.

The majority of existing studies of traveler’s route choice behavior are based on the random utility maximization (RUM) [1]. These RUM models assume that when faced with a number of travel choice options, a traveler is rational enough and he or she will choose the one that has the highest utility value according to the information which was obtained by the traveler. However, it is difficult to fully get the accurate traffic information for the travelers. Moreover, the travelers’ route choices are affected by their preferences and attitudes, and the practical behavior of travelers’ route choice does not fully respect the axiomatic system of expected utility theory. So many scholars try to find a more realistic theory than RUM theory to explain and describe travelers’ route choice behavior. Among them, Loomes and Sudgen in 1982 and Bell in [2] independently proposed a regret theory, and they pointed out that the single factor’s utility function cannot explain the behavior of nonrational decision
well. People would compare the actual situation and possible situations according to their decision-making factors. If they find the chosen one can get better results than other options, they tend to rejoice. Otherwise, they would feel regret. Based on the regret theory, Casper proposed a random regret minimization (RRM) model. RRM supposed that the satisfaction degree of a travel route depends not only on the utility of selected travel route, but also on the regret of other options [3].

Sociologists found that human behavior is far more complicated than a single standard [4]. Some people tend to maximize their utility while others would minimize their regret. This is also obvious from the travelers’ behavior in route choice. Passengers may not choose a faster but unfamiliar route, because they try to avoid regret from choosing the route. Moreover, for different user class, the degree of maximizing utility and minimizing the regret may be also different. Any single standard behavior assumption may not fully explain the complex route choice behavior of passengers. Based on this, this study tries to answer the research questions as follows: What is the outcome of hybrid of utility maximization and regret minimization? Which behavior assumption reflects the route choice behavior more accurately in urban rail transit?

In this study, the hybrid route choice behaviors of random utility maximization and random regret minimization are proposed and formulated to analyze the travelers’ route choice for an urban rail transit network. Although the regret theory has been applied to many fields, including road traffic assignment [5, 6], to the best of the authors’ knowledge, this is the first research which applies the hybrid of utility theory and regret theory in the context of an urban rail transit. Different from cars on a road network where many roads can be selected to reach the same destination, in an urban rail transit network have less choices. Besides, the behaviors of drivers and passengers are different. Therefore, it is worth testing whether the hybrid of two kinds of behavioral assumptions is effective on an urban rail transit. The hybrid route choice was applied to a space-time expanded network. Till now, most transit assignment models considering the regret are based on the frequency and a physical network, which belongs to the frequency-based assignment. This research applies the regret to a schedule-based transit assignment by constructing a space-time expanded network from smart card data. Moreover, passengers’ heterogeneous choice behavior (e.g., regret, disappointment) is neglected in most previous studies. This paper incorporates regret minimization and utility maximization into a transit assignment model to characterize travelers’ route choice behavior. Two types of hybrid rules are considered, namely, attributes level hybrid rule and decision level hybrid rule. The effects of both of the hybrid rules are discussed. The model is applied to a real world case study in Beijing urban rail transit network. The effectiveness of the model is validated by the travel time estimated from smart card data. Compared with existing validation data relying on empirical investigation, using smart card data is more objective.

The remainder of the paper is organized as follows. Section 2 is the literature review. Section 3 proposes some basic assumptions and terminologies; Section 4 proposes the methodology. After that, Section 5 explains the MSA algorithm for solving the model. Furthermore, Section 6 applies the method to Beijing metro network, showing the effect of attributes level hybrid rule and decision level hybrid rule. Finally, Section 7 draws the conclusion.

2. Literature Review

The existing transit assignment models can be classified into frequency-based assignment models [7] which are known as “line-oriented” and schedule-based model [8] which are known as “run or vehicle-oriented” [9]. The former assigns the traffic to a service transit lines, while the latter assigns the traffic to service runs. Due to strong capability of assignment of the dynamic assignment model, we are in particular interested in the latter one.

On frequency-based transit network, each train on transit lines is supposed to run with a constant headway, and the network is represented in a static manner. Generally the procedures of the assignments include 3 steps: path searching, path choice probability estimation, and traffic assignment. The first step searches effective and possible path between all O-D pairs of the network. In the second step, we compare each path in the effect and possible path set by the generalized cost function or utility function individually. With the cost distribution function, the path choice probability can be estimated. Finally we assign the demand to paths with the path probability by O-D demand matrices. Frequency-based transit assignment models always consider a constant demand and minimize the path cost [10] or optimize the strategy choice [11] of individual passengers, which is similar to user equilibrium assignment on network. Schmöcker et al. [12] proposed an assignment model based on frequency of departure. The model considered the probability of passengers finding seats in their perception of route cost. They introduced the probability of “fail-to-sit” at boarding points to calculate the travel cost and distribute the passenger flow. Zhang et al. proposed an assignment model based on frequency considering day-to-day evolution under oversaturated conditions and studied the impact of passenger comfort on the overload conditions and frequency of departure on passenger route selection [13]. Leurent et al. considered the capacity and provided a static and macroscopic traffic assignment model from the line submodel and the network [14]. As the frequency-based assignment model usually assigns the passenger based on a physical topology network, another important concept on frequency-based transit model is based on hyperpaths approaches. Hyperpaths form a directed acyclic graph with a flow distribution rule in network representation. Wu et al. [15] firstly proposed the hyperpath concept with strategy-based transit link cost function. Kurauchi et al. [16] also introduced traffic split in hyperpaths of transit network. For the majority of these models, schedule of transit system is assumed to be sufficiently reliable. Therefore the headway is calculated by the average frequencies of transit line in frequency-based network. And the waiting time and transfer time are implicitly estimated based on headway. Since the time dimension is not considered in frequency-based transit
model, the assignment results in frequency-based models are the average value in the specified time period (e.g., the rush hour).

Unlike frequency-based type model, schedule-based models generally take into account explicitly timetable or schedule of the transit system, which means that the detailed departure or arrival times of vehicles in each transit line are used in assignment procedures. On the other hand, schedule-based assignment model considers the temporal and spatial structure of travel demand on the network; therefore the assignment results would estimate explicitly the accurate number of passengers to each vehicle of the schedule. At present, this approach becomes hot spot to researchers. For an overview, readers are referred to [17, 18]. In a schedule-based network, passengers choose not only the optimal hyperpaths for their trip, but also the departure/arrival time and vehicles of different crowded levels by minimizing their generalized travel costs. In order to combine the time-dependent choice, a time-dependent transit network should be presented according to different schedule-based transit assignment model, which can be classified into these types: (1) diachronic graph consisting of service subgraph, demand subgraph, and access/egress subgraph proposed by Nuzzolo et al. [19]; (2) dual graph proposed by Moller-Pedersen [20]; (3) time-dependent graph formulation with line schedule information by Tong and Wong [8]; (4) discrete space-time graph formulation with space-time nodes and space-time arcs proposed by Nguyen et al. [21], which is improved by Hamdouch et al. [22] to time-expanded network representation.

Modeling formulations of transit assignment constitute one of the schedule-based problems. Poon et al. [23] put forward a dynamic user equilibrium model considering the stochastic user equilibrium model by implementing time-dependent demand distribution and taking passengers’ microbehavior on boarding link into account. With the numerical example in the study, dynamic user equilibrium mechanism is reasonably expressed with the factors of queueing at station due to congestion. Nieson [24] proposed a stochastic transit assignment model considering differences properties in passengers’ utility function as optimized problem. Tian et al. [25] improved the model proposed by Alfa and Chen [26]. The model considers the in-vehicle crowding and schedule delay in generalized cost function with departure time decisions of passengers and presents the theoretical properties of equilibrium status.

With considering the capacity in the transit assignment model, some factors related to the capacity are concerned with the RUM model in recent years. Hamdouch et al. considered the uncertainty of vehicle capacity. Passengers used a strategy to travel under the uncertainty of capacity. Using specific examples to analyze the impact of uncertainty on passengers’ travel strategies and departure time [22], Sumalee et al. considered one of the critical factors of capacity: sitting and standing capacities, and the treatment of seat allocation is considered as a random probability to get a seat or not [27]. Nuzzolo et al. presented a joint choice model by formulating departure/arrival time and train of different crowded level for maximizing the utility function. To solve the assignment model, a simulation procedure was put forward, taking congestion into account through explicit vehicle capacity. [28]. Han B et al. [29] proposed a stochastic user equilibrium model to solve transit assignment problem. This model was based on rail transit network schedule considering the travelers’ behavior assumption of train’s overload delay. The model was transformed into a dynamic schedule-based assignment model with splitting the origin-destination demands into the schedule-based network with time-space routes, using the Beijing urban rail transit (BURT) network as a case to verify the rationality of the model [29].

In terms of different behavior assumptions, the transit assignment model can be classified into the random utility maximization model and random regret minimization model.

The majority of the transit assignment models have used the random utility maximization (RUM) rooted in discrete choice analysis [1, 30]. The RUM model assumes that when a traveler is faced with a number of travel choices, he or she will choose the one with the highest utility. Poon et al. [23] assumed that a traveler can get all travel information including travel time and transfer time and constructed a generalized travel cost function. This function included in-vehicle travel cost, waiting cost, transfer cost, and transfer penalty and in-vehicle crowds. The transit assignment model was constructed based on the train schedule, and computer simulation is used to solve the model. Nuzzolo et al. [29] expressed the traffic network using diachronic graph and proposed a schedule-based assignment model considering the vehicle capacity limit. Considering the dynamics of passenger demand, Tong and Wong [8] established a stochastic transit assignment model, in which waiting time and walking time are a part of the generalized cost function. Nieson [24] proposed a stochastic transit assignment model considering differences preferences in passengers’ utility functions as optimized problem, which presents a framework for transit assignment based on a basic probit model.

Regret theory was presented decades ago [31], and, similar to prospect theory (PT), it originally assumed a decision-making process under uncertainty. RRM constitutes an alternative to both utility theory (UT) and PT. When an individual’s awareness perceives the product of the nonchosen alternative to be better than the result of the chosen alternative, it will build an emotion called regret [32]. The concept of regret as a determinant of decisions is often employed in areas such as psychology [33, 34], marketing [35], and finance [36]. The RRM model develops from the angle of bounded rationality and captures the scheme between multiple attribute trades-offs to the traveler’s choice of psychological and traffic behavior based on minimization of the perceived regret decision criteria [3]. Recently, regret-based choice models have gained in popularity in travel behavior research, as an alternative approach to modeling choice behavior, under conditions of both certainty and uncertainty [32, 37, 38]. The RRM model has been used to analyze and predict a wide variety of choices, such as departure time choices, route choices, mode-destination choices, activity choices, on-line dating choices,
health-related choices, and policy choices [39, 40]. Chorus shows that RUM can be extended to the case of risky travel choice [32, 37]. Recently, he compared RUM with RRM in terms of theories and equations and showed their respective benefits of the scope of application [38]. Traditionally, the RUM model has dominated the travel choice behavior since it was accepted [3]. Studies to date suggest that the RRM is just as parsimonious as the standard RUM model, and it is unlike other models of contextual effects, which typically require the estimation of additional parameters [3].

Compared with the RUM, the RRM has two advantages:

1. (1) The RRM features logit choice probabilities and is easily estimated by using conventional discrete choice software packages in the research field [41, 42]. Therefore, RRM can be applied more widely.

(2) The arrival time of a passenger refers to the moment when a passenger arrives at the platform and is ready to board. If the arrival time of a passenger is earlier than the arrival time of the train, the passenger can get on the train; otherwise, they cannot board the train.

2. (2) The RRM model does not exhibit the property of the IID error terms (Hensher et al. 2016). Therefore, RRM can be extended to the case of risky travel choosing a risky traveler. RRM can accommodate the assumption of independent and identically distributed (IID) error terms (Hensher et al. 2016). Therefore, RRM can be applied more widely.

Table 1 lists the typical literature of the transit assignment problem, and a number of characteristics of this literature are shown, including the research subject, the assumption of choice behavior, whether it is scheduled based, whether it considers the capacity, and whether it uses the smart card data.

Table 1 shows that most of the models are based on the random utility maximization behavioral assumption. Although some assignment models have introduced the RRM, there is no adequate research on the RRM in an urban rail transit assignment. With regard to the hybrid model, attribute level hybrid and decision level hybrid are independently adopted in different literature. Moreover, there is no study on the effect of the two types of hybrid behavior rules in the rail transit assignment. This paper extends the previous literature to bridge the above-mentioned gaps.

3. Problem Statement

3.1. Assumptions. In the process of analyzing and modeling the passengers’ route choice behavior in urban rail transit network, the following assumptions are made:

(1) Trains run strictly according to timetable. The unexpected incidents are not considered.

(2) The arrival time of a passenger refers to the moment when a passenger arrives at the platform and is ready to board. If the arrival time of a passenger is earlier than the arrival time of the train, the passenger can get on the train; otherwise, they cannot board the train.

3.2. Representation of Transit Supply and Demand. This study is based on schedule-based approach [19]. It requires the representation of supply and demand in a detailed level. On the supply side, a space-time expanded network is developed based on the train timetable; on the demand side, the time independent OD matrix is used.

3.2.1. Space-Time Expanded Network Representation. A space-time expanded network is represented by a directed graph $G_T(N_E, A_E)$, where $N_E$ represents the nodes and $A_E$ represents the arcs. The nodes include a series of events associated with trains and passengers, e.g., passenger arrival node, train arrival node, passenger boarding node, passenger alighting node, and train departure node. The arcs include train running links, train stop arcs, passenger access arc, passenger egress arc, and passenger transfer arcs. $o$ and $d$ represent the starting and destination stations of the physical route. $S$ represents the set of stations, $L$ represents the sets of trains on all lines in the network, $l_i$ represents the $i$-th line in the network, $L_i \subset L$ represents the set of train numbers on the $i$-th line, and $l_{ij}$ expresses the $j$-th train on the route of the $i$-th line. $T$ indicates the train timetables, which mainly include the train arrival and departure times at all stations, $T = \{a_{t,i}^{hy}, d_{t,j}^{hy} \mid \forall s \in S, l_{ij} \in L_i \subset L\}$. The components of urban rail transit time-expanded network are as follows.

**Table 1: Typical researches on transit assignment problem.**

| Literature | Research subject | RRM/RUM/Hybrid | Schedule based | Capacity constraint | Smart card data |
|------------|------------------|----------------|----------------|---------------------|-----------------|
| Schmöcker et al. [12] | underground network | RUM | × | ✓ | ✓ |
| Leurent et al. [14] | bus transit network | RUM | × | ✓ | × |
| Zhang S et al. [13] | feeder bus service | RUM | × | ✓ | × |
| Tong et al. (1998) | urban rail transit | RUM | ✓ | × | ✓ |
| Poon et al. [23] | bus transit network | RUM | ✓ | ✓ | ✓ |
| Hamdouch et al. [22] | bus transit system | RUM | ✓ | ✓ | ✓ |
| Nuzzolo et al. [28] | bus transit system | RUM | ✓ | ✓ | ✓ |
| Han B et al. [29] | urban rail transit | RUM | ✓ | ✓ | ✓ |
| Chorus et al. [32, 37] | bus transit system | RUM | ✓ | ✓ | ✓ |
| Chorus et al. [38] | bus transit system | Attributes level hybrid | ✓ | × | × |
| this paper | urban rail transit | Attributes level and decision level hybrid | ✓ | ✓ | ✓ |

Note. RUM: random utility maximization; RRM: random regret minimization.
(i) Train Arrival and Departure Node. The train arrival and departure nodes are based on the train timetable, by expanding the station nodes of a physical topology in the time dimension. If there are \( n \) trains to the station corresponding to the physical node in the route forward direction, the node can be expanded to \( 2 \times n \) trains event node according to the arrival time of each train at the node. This kind of time extension node is expressed as \( n_{t}^{i,j} \in N_{t} \subset N_{E} \), in which \( s \in S, l_{ij} \in L_{j} \subset L, t \in T \), the \( j \) train on the line \( l \) on the station \( s \) at the time \( t \) to the time extension node; it can be seen that this kind of node has three properties of location \( \text{Station}(n_{t}^{i,j}) = s \), time \( \text{Time}(n_{t}^{i,j}) = t \), and train number \( \text{Train}(n_{t}^{i,j}) = l_{ij} \). The arrival time is expressed as \( at \) and \( dt \), respectively, and the node can be further represented as \( n_{at}^{i,j}, n_{dt}^{i,j} \).

(ii) Passenger Alighting and Boarding Node. The passenger boarding node set \( s_{b} \) includes the node of trip starting and the node before transfer; passenger alighting node set \( s_{a} \) includes the node of trip ending and the node after transfer. This kind of node is expressed as \( n_{s}^{i,j} \in n_{o} \subset N_{E} \), which represents the events of passenger boarding or alighting the \( j \) train at station \( s \) at the time \( a \). It also has three attributes: location \( \text{Station}(n_{s}^{i,j}) \), time \( \text{Time}(n_{s}^{i,j}) \), and train \( \text{Train}(n_{s}^{i,j}) \). Among them, the boarding node satisfies the following condition.

\[
s \in s_{b} \quad l_{ij} \in L_{j} \subset L \quad a \in \{ \text{Time}(n_{at}) \mid n_{at} \in n_{t} \}
\]

The alighting node satisfies the following condition.

\[
s \in s_{a} \quad l_{ij} \in L_{j} \subset L \quad a \in \{ \text{Time}(n_{at}) \mid n_{at} \in n_{t} \}
\]

(iii) Passenger Arrival and Left Node. A passenger arrival time extension node is expanded by the gate entry time recorded by the automatic fare collection systems when a passenger swipes his or her card. \( n_{w}^{i} \in n_{w} \subset N_{E} \) is used to represent this kind of node. In which \( s \) represents the station corresponding to the starting and destination points, \( w \) represents the time when a passenger swipes his card. For passenger arrival node, it satisfies

\[
s = o \quad w = t_{\text{entry}}
\]

where \( o \) is the enter station which is recorded by the smart card. \( t_{\text{entry}} \) indicates the entry time and the card number is \( i \). This kind of node has two attributes of location:

\[
\text{Station}(n_{w}^{i}) = s \quad \text{and Time}(n_{w}^{i}) = w.
\]

(2) Time Extension Arc. The time-expanded arc set \( A_{T} \) includes passenger access and egress arcs \( a_{\text{walkin}} \subset A_{E} \) and \( a_{\text{walkout}} \subset A_{E} \), train running arc \( a_{\text{operation}} \subset A_{E} \), train stop arc \( a_{\text{stop}} \subset A_{E} \), and passenger transfer arc \( a_{\text{transfer}} \subset A_{E} \).

(i) Passenger Access and Egress Arcs. The passenger access arc connects the passenger arrival node and boarding node in space-time expanded network. \( a_{\text{walkin}}(n_{w}^{i}, n_{b}^{i,j}) \) is used to represent the access arc between the extension nodes from the passenger arrival time \( t_{a} \) at the physical node \( s_{a} \) to the node of the passenger boarding time \( t_{b} \) of the \( j \) train on the line \( l \).

The weight \( w_{\text{walkin}}(n_{w}^{i}, n_{b}^{i,j}) \) of the access arc is represented by the sum of passenger walking time and waiting time:

\[
w_{\text{walkin}}(n_{w}^{i}, n_{b}^{i,j}) = t_{\text{walkin}} + t_{\text{wait}} = t_{b} - t_{a}
\]

\( t_{\text{walkin}} \) indicates passenger walking time in station \( s_{a} \) from ticket gate to platform, \( t_{\text{wait}} \) indicates passenger waiting time on \( s_{b} \) platform, \( s_{o} \in S, l_{ij} \in L_{j} \subset L, t \in T \).

The egress arc is the connection between the passenger alighting node and the left node. \( a_{\text{walkout}}(n_{s}^{i,j}, n_{w}^{i}) \) is used to represent the egress arc between the extension nodes from the alighting time \( t_{w} \) of the \( j \) train at physical node \( s_{d} \) on line \( l \) to the passenger left time \( t_{o} \). The weight \( w_{\text{walkout}}(n_{s}^{i,j}, n_{w}^{i}) \) of egress arc is represented by passenger departure time as

\[
w_{\text{walkout}}(n_{s}^{i,j}, n_{w}^{i}) = t_{d} - t_{o}
\]

(3) Passenger Transfer Arc. The passenger transfer arc connects the expanded node of passenger alighting node and boarding node between different lines. \( a_{\text{transfer}}(n_{w}^{i,j}, n_{w}^{p,k}) \) represents the passenger transfer arc which is from the passenger alighting time from the \( j \)-th train at station \( s_{m} \) on line \( l_{j} \) to the boarding time to the \( k \)-th train at station \( s_{n} \) on line \( l_{k} \).

The weight of passenger transfer arcs \( w_{\text{transfer}}(n_{w}^{i,j}, n_{w}^{p,k}) \) is the sum of passenger walking time and waiting time:

\[
w_{\text{transfer}}(n_{w}^{i,j}, n_{w}^{p,k}) = t_{\text{transfer}} + t_{\text{wait}} = d_{t} - d_{a}
\]

where \( t_{\text{wait}} \) indicates the passenger’s waiting time from \( s_{m} \) to \( s_{n} \), \( t_{\text{transfer}} \) indicates the passenger’s walking time from \( s_{m} \) to \( s_{n} \).
Step 4. According to the passenger arrival and left time from the AFC data, the expanded physical nodes of the origin station \( n_0 \) and the destination \( n_d \) are further expanded in the time dimension, and the time label is added to form the expanded node to express the time of the passengers' arrival and departure, as shown in Figure 1(d).

Step 5. The train arrival expansion node \( n_o \) corresponding to the origin station time is connected to the passenger arrival expansion node. The arrival time should meet the access time constraint (4) and egress time constraint (5) at the station, as shown in Figure 1(e).

Taking the Beijing subway as an example, a passenger's AFC record shows that the passenger arrived at 7:10:28 from the CHEGONZHUANG station and 7:28:18 from the TIAN'ANMEN West. The passenger's physical route is only "CHEGONZHUANG-FUCHENGMEN-FUXINGMEN (transfer station)-XIDAN-TIAN'ANMEN West", but in the time-expanded network, there are three time-expanded routes satisfying the condition of passenger arrival time and passenger departure time, which are shown in Figure 2. This could happen when the trains are too crowded during peak hours so that passengers have to choose the next train.

3.2.3. Demand Representation. On the demand side, the proposed assignment model is specified considering a reference period (e.g., a day) divided into elementary time intervals of width \( h \) (e.g., \( h = 2 \) min), for which the generic time slice covers time interval \([t, t + h]\).

We consider an urban rail transit network, in which the AFC system is a tap in and tap out system, where both passengers' swipe card information is accurately recorded. In the AFC system, there are two records for one trip. One is created when the passenger taps in, and the other is created when the passenger taps out. Trips with only one record are considered as incomplete trips and are neglected in this study.

3.3. Generalized Travel Cost of Passengers. A passenger would choose his or her travel route based on the generalized travel cost of the corresponding route on a space-time expansion network. In this study, the passengers' generalized travel cost is represented by a weighted sum of a number of independent variables, such as access time, egress time, in-vehicle time, in-vehicle crowd, and transfer time.

(1) Access Time. The access time of passenger is the cost on the access arc. In Section 3.2.1, the access time includes the walking time and waiting time. This time can be obtained from the space-time expanded network. Passengers’ arrival time is different and the train is different, so the arrival time is different. The access time is represented by \( ET \). Based on the space-time expanded network, the formula of access’ time is

\[
ET = t^s_{\text{walkin}} + t^s_{\text{wait}} = t_b - t_o
\]
where $t_{\text{walk}}^s$ indicates passengers’ walking time from the tap in gate to the platform at station $s_o$ and $t_{\text{wait}}^s$ indicates passenger waiting time on the platform at station $s_o$.

(2) Train Running Time. Train running time is the cost of the train running arc. In the space-time expanded network, the train running time includes train dwell time and train stop time. In the urban rail transit system, the train operation strictly follows the train timetable, so the cost of the train running arc $e_j$ is known and fixed, in other words, the running time of the train between two stations. When $VT$ is used to indicate the running time of trains, the formula of train running time is as follows.

$$VT = w_{\text{operation}}\left(n_{at}^{i,j}, n_{at}^{i+1,j}\right) = at - dt$$ (10)
(3) Crowd Cost. In the train operation, the more passengers on the train, the more uncomfortableness perceived by the passengers. When the number of passengers in the train is large, the capacity limit of the train will cause the crowding between passengers. If the number of passengers in the train does not reach the number of train seats, there will be no congestion. The degree of crowding caused by the number of passengers on trains is expressed by the crowd coefficient $\rho$, which is estimated by

$$\rho(t) = \begin{cases} 0 & x_e(t) < Z_l \\ \alpha (x_e(t) - Z_l) / Z_l & Z_l < x_e(t) < N_l \\ \alpha (x_e(t) - Z_l) / Z_l + \beta (x_e(t) - N_l) / N_l & x_e(t) > N_l \end{cases}$$  

where $\rho(t)$ is the crowd coefficient based on passengers number on train running arcs during $t$ interval, $x_e(t)$ is the number of passengers on the running arc $e$ during $t$ interval, $Z_l$ is the number of seats on the train running arc $e$ on the line $l$, $N_l$ is the maximum capacity of train running arc $e$ on line $l$, and $\alpha$, $\beta$ are corresponding coefficients, which are derived from specific statistical data.

When considering the impact of crowd, the train running time on the running arcs is magnified, which is represented by $CT$. It is calculated as follows:

$$CT = V_T \cdot \rho_{t_1}(t)$$  

(12)

(4) Transfer Time. Transfer time includes transfer walking time and transfer waiting time. In the space-time expanded network, the transfer time is the cost on the transfer arc. The cost is composed of two parts, which are the transfer time and the number of transfers, respectively. The transfer time on the transfer arc can be expressed as

$$t_{l_1,l_2}^{t_1,t_2} = t_{\text{transfer}}^{t_1,t_2} + t_{\text{wait}}^{t_1,t_2} = dt_2 - at_1$$  

(13)

where $t_2$ and $t_1$ represent the starting and ending times of transfer arcs, respectively.

The perceived cost of transfer time will be greater than in-vehicle time and waiting time. This is because the passengers need to walk during transfer and pay much physical effort. Therefore, the transfer time perception cost on the transfer arc is magnified and expressed as

$$\tilde{t}_{l_1,l_2}^{t_1,t_2} = \theta t_{l_1,l_2}^{t_1,t_2}$$  

(14)

where $\theta > 0$ and $\theta$ can be estimated by fitting the survey data.

When the number of transfers is more than two times, the passenger will increase the extra psychological cost for each additional transfer. The additional cost is expressed as $\kappa$. The comprehensive cost on the transfer arc is expressed by $TT$, which can be expressed as follows.

$$TT = \tilde{t}_{l_1,l_2}^{t_1,t_2} + \kappa = \theta (t_2 - t_1) + \kappa$$  

(15)
In summary, the generalized travel cost of passengers is represented as the sum of the four parts. It is formulated as follows.

\[
C = \beta_E ET + \beta_V VT + \beta_CT CT + \beta_{TT} TT
\]  

(16)

Substituting the formulation of variables to (16), the generalized cost can be represented as follows.

\[
C = \beta_E (t_b - t_o) + \beta_V (t_2 - t_1) + \beta_CT \left( t_o \cdot \rho(t) \right) \\
+ \beta_{TT} \left( \theta (t_2 - t_1) + \kappa \right)
\]  

(17)

4. Methodology

4.1. Passenger Route Choice Behavior. At present, there are two basic types of the choice behavior assumptions: one is that the choice behavior of passengers is fully rational, which is mainly based on the theory of utility maximization, and the other is that the choice behavior of passengers is finite rational, which is mainly based on the theory of regret minimization.

We would argue that neither RUM nor RRM could reflect the complexity of the travelers’ route choice behavior. Therefore, this paper uses the hybrid of utility maximization rule and the regret minimization rule to model the passenger route choice behavior. Based on the different route choice rules, the random utility maximization (RUM) route choice model, the random regret minimization (RRM) route choice model, and the hybrid utility-regret (RUMR) route choice model are established, respectively.

4.1.1. Random Utility Maximization (RUM) Formulation. The random utility maximization (RUM) is aiming to choose the least generalized travel cost in the process of the passenger route choice and choose the route with the maximum utility value.

Under the rule of utility maximization, it is assumed that the passengers’ choice is completely rational; that is, the passengers always compare the utility of different routes and choose a route with the maximum travel utility. The travel utility of a route consists of two parts: one is determined cost, which can be calculated through the generalized travel cost of passengers. The other part is random cost, which is caused by some uncertain factors such as the different passengers’ attributes or the estimated deviation of passengers. Determined cost and random cost together constitute the travel cost of passengers, and the reverse number of travel costs is the utility function.

Assuming that there are J routes between a certain OD pair, the random utility function is composed of two parts: a deterministic trip cost which reflects the impact of observed variables and a random error term which represents unobserved factors. The two parts of the cost reflect the average perception of the passenger’s route utility and the perception error, respectively. The random utility function of the route i can be expressed as follows.

\[
RU_i = C_i + \varepsilon_i
\]  

(18)

Here, \( RU_i \) is the random utility value of the route i and \( C_i \) is the determined utility value of the route i, which can be determined by the analyst. \( \varepsilon_i \) is the random error.

If the distribution of random error \( \varepsilon_i \) is known, it is possible to calculate the probability of different routes being selected. If the route i is selected, it needs to satisfy the following equation.

\[
P_U(i) = \text{prob} (RU_i > RU_j, \forall i \neq j)
\]  

(19)

When the random error obeys the independent identical distribution with Generalized Extreme Value Distribution type 1 (also known as Gumbel distribution), the probability of passenger choosing route i can be formulated as follows.

\[
P_U(i) = \frac{\exp (C_i)}{\sum_{j=1}^{J} \exp (C_j)}
\]  

(20)

4.1.2. Random Regret Minimization (RRM) Formulation. The random regret minimization rule assumes that when there are multiple routes corresponding to one travel OD pair, passengers can compare the attributes of different routes.

After choosing a route, if passengers find that other uns-  

elected routes’ travel costs of a certain attribute are less than their chosen routes, regret will arise. Under the regret minimization rule, passengers will choose a route with the least regret.

The regret value of choosing a route can be quantified by a regret function. The regret function is a function of the attributes of the two or more alternatives. Taking two alternatives as an example, the regret function is \( R_j(s) = \phi(x_i(s), x_j(s)) \). It is assumed that \( R_j(s) = -R_i(s) \). Consider a traveler n who faces a route choice among alternatives i, j, and k, and the alternatives are fully defined in terms of the attributes x, y, and z; the regret associated with alternative i can be defined as follows.

\[
R_{ij} = \phi_x \left( x_i, x_j \right) + \phi_y \left( y_i, y_j \right) + \phi_z \left( z_i, z_j \right)
\]  

(21)

In this study, the regret function of route choice is defined as a logarithm function as in the following equation.

\[
\phi_x \left( x_i, x_j \right) = \ln \left( 1 + \exp \left( \frac{y_i - x_j}{x_i} \right) \right)
\]  

(22)

Similar to the composition of random utility functions, the random regret function is also composed of two parts of the deterministic regret term and the random regret term. The deterministic term indicates the
perception of the travelers’ average regrets between different routes, and the random regret term represents the perception of the passengers’ regrets caused by random errors.

For route $i$, the regret value generated by passengers choosing this route can be calculated by the random regret function of

$$RR_i = \sum_{j \neq i} \left( \ln \left( 1 + \exp \left( \gamma_{ET} \cdot \frac{(ET_j - ET_i)}{ET_i} \right) \right) + \ln \left( 1 + \exp \left( \gamma_{VT} \cdot \frac{(VT_j - VT_i)}{VT_i} \right) \right) + \ln \left( 1 + \exp \left( \gamma_{CT} \cdot \frac{(CT_j - CT_i)}{CT_i} \right) \right) + \ln \left( 1 + \exp \left( \gamma_{TT} \cdot \frac{(TT_j - TT_i)}{TT_i} \right) \right) + \epsilon_i \right)$$  (23)

where $RR_i$ is the random regret value of the route $i$, $RR_j$ is the determined regret value of the route $j$, $\gamma_{ET}$ is the coefficient of the entry time, $\gamma_{VT}$ is the coefficient of the train running time, $\gamma_{CT}$ is the coefficient of the crowded cost, and $\gamma_{TT}$ is the coefficient of the transfer time, which is between 0 and 1.

The probability of passengers choosing route $i$ in the following formula is

$$P_R(i) = \text{prob} (KR_i < KR_j, \forall j \neq i) = \frac{\exp(-RR_i)}{\sum_{j=1 \to J} \exp(-RR_j)}$$  (24)

4.1.3. Hybrid of Utility Maximization and Regret Minimization (RURM). In the above, the corresponding choice models of the random utility maximization and the random regret minimization are established, respectively. However, passengers’ behavior can be a hybrid of both. In this study two types of hybrid rules are applied, namely, attribute hybrid rule and decision level hybrid rule.

Attribute level hybrid rule reflects the different behavioral tendencies (utility or regret) for different decision attributes. In the actual travel process, passengers may have different behaviors when facing different route attributes such as time and crowd. For some attributes, passengers tend to maximize the utility, while for others, passengers tend to minimize the regret. Chorus [5, 6] introduced regret-weights to model such kind of hybrid. As a result, the hybrid utility and regret function can be formulated as

$$HUR_i = \rho \cdot RU_i - (1 - \rho) \cdot RR_i$$

$$= \rho \cdot (\beta_{ET}ET_i + \beta_{VT}VT_i + \beta_{CT}CT_i + \beta_{TT}TT_i) - (1 - \rho)$$

$$\left( \sum_{j \neq i} \left( 1 + \exp \left( \gamma_{ET} \cdot \frac{(ET_j - ET_i)}{ET_i} \right) \right) + \ln \left( 1 + \exp \left( \gamma_{VT} \cdot \frac{(VT_j - VT_i)}{VT_i} \right) \right) + \ln \left( 1 + \exp \left( \gamma_{CT} \cdot \frac{(CT_j - CT_i)}{CT_i} \right) \right) + \ln \left( 1 + \exp \left( \gamma_{TT} \cdot \frac{(TT_j - TT_i)}{TT_i} \right) \right) + \epsilon_i \right)$$  (26)

where $\alpha_{ET}, \alpha_{VT}, \alpha_{CT},$ and $\alpha_{TT}$ are attributes level hybrid parameters (since this parameter is only one kind of hybrid rule in terms of attributes, we use the term “attributes level hybrid parameter” instead of regret-weight) for access time, in-vehicle time, crowd cost, and transfer time, respectively. These parameters represent the behavioral tendencies of utility maximization or regret minimization. When the value of the attributes level hybrid parameter is zero for observed variables, the decision tends to be close to RUM.

Decision level hybrid rule refers to the coincident traits of passengers when making decisions. For instance, some individuals are more risk averse than others considering the heterogeneity of passengers. These traits are unobservable and related to some observable characteristics such as gender (Hess 2013). Without loss of generality, the possibility of a passenger choosing RUM or RRM can be considered as a latent variable. As a result, the method of developing a hybrid of utility and regret can be represented by a latent class model.
4.1.4. Illustrative Example. In order to compare and analyze the effect of hybrid of RUM and RRM on route choice behavior, a small numerical experiment is selected, which is shown in Figure 3. There are three routes: \( k_1, k_2, \) and \( k_3 \); the in-vehicle times of them are 15min, 20min, and 25min, and the transfer times are 5min, 4min, and 2min, respectively. To simplify the case, other attributes are neglected. The parameters of the in-vehicle time \( \beta_{VT} \) and \( \gamma_{VT} \) are -1/min, and the parameters of the transfer time \( \beta_{TT} \) and \( \gamma_{TT} \) are -2/min. Both attribute level hybrid rule and decision level hybrid rule are applied.

(1) Effects of Attribute Level Hybrid on Passengers’ Route Choice Behavior. Assuming \( \omega_1 \) and \( \omega_2 \) are attributes level hybrid parameters for in-vehicle time and transfer time, respectively, in order to study the effect of the attributes level hybrid parameters on the route selection probability, \( \omega_1 \) and \( \omega_2 \) are chosen from values range \([0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1]\), respectively. The closer the attributes level hybrid parameters to 1, the greater the possibility of using the regret minimization model for the route; otherwise, the utility maximization model will be used. The selection probabilities of the three routes of the above cases are illustrated in Figures 4–6.

As can be seen from Figures 4, 5, and 6, when attributes level hybrid parameters \( \omega_1 \) for in-vehicle time keep a constant, with the increase of the attributes level hybrid parameters \( \omega_2 \) for transfer time, the selection probability of route 1 and route 2 increases and the selection probability of route 3 decreases rapidly. This implies that the route selection is very sensitive to the regret of transfer time. The transfer time of route 1 and route 2 is longer than that of the route 3. Passengers prefer to choose route 3 under RUM condition. Increasing the value of decision regret parameter \( \omega_2 \) leads to more regret of this decision, and thus the probability of selecting route 3 decreased. Then \( \omega_2 \) keeps a constant, and with the increase of \( \omega_1 \), the selection probability of route 3 increases and the selection probability of route 1 and route 2 decreases gradually. This is because the in-vehicle times of the route 1 and the route 2 are less than the in-vehicle time of the route 3. Increasing the value of attribute level parameter \( \omega_1 \) would generate regret of choosing route 1 and route 2, and thus the probability of selecting route 1 and route 2 decreases.
The influence of the value of decision level hybrid rule parameter on the route probability

(2) Effects of Decision Level Hybrid on Passengers’ Route Choice Behavior. In order to study the effect of the decision level hybrid rule parameter on the route selection probability, the decision level hybrid rule parameter \( \rho \) is chosen from values range \([0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1]\). The closer the decision level hybrid rule parameter \( \rho \) to one, the greater the possibility of using the utility maximization model for the route; otherwise, the regret minimization model will be used. The route selection probabilities of the above cases are listed in Figure 7.

As can be seen from Figure 7, under regret minimization decision rule (\( \rho = 0 \)), most passengers would choose the route 3 to reduce their regret. With the increase of the decision level hybrid rule parameter \( \rho \) (meaning passengers tend to make decision by utility maximization), the selection probability of route 1 increases and the selection probability of route 3 decreases. In this particular case, the selection results of RUM and RRM are significantly different.

4.2. Transit Assignment Model. It is assumed that the passenger flow of the urban rail transit network meets the user equilibrium distribution, which means passengers can obtain accurate travel information. Based on the user equilibrium theory, the dynamic transit assignment model of urban rail transit can be developed based on RURM. The dynamic transit assignment model based on the space-time expanded network is

\[
\min \quad Z = \sum_k [(1 - \rho) \cdot RR_k - \rho \cdot RU_k] \tag{27}
\]

subject to

\[
q_{rs} = \sum_k f_{rs}^k, \quad \forall r, s \tag{28}
\]

\[
f_k^r > 0 \tag{29}
\]

\[
x_{mn} = \sum_{rs} \sum_k f_{rs}^k \cdot \delta_{mn,k} \tag{30}
\]

where \( x_{mn} \) represents the traffic on the space-time arc \( m \rightarrow n \); \( f_{rs}^k \) represents the flow of OD pair \( rs \) on route \( k \); \( \delta_{mn,k} \) represents the relationship between route \( k \) and arc \( m \rightarrow n \);\( q_{rs} \) represents passenger demand of OD pair \( rs \).

Formula (29) represents the conservation of OD demand and route traffic flow. Formula (30) indicates the conservation of route traffic flow and space-time arc’s traffic flow.

When the network reaches an equilibrium, it satisfies

\[
v_{rs} - f_{rs} = \begin{cases} 0 & f_{rs} > 0, \\ \leq 0 & f_{rs} = 0, \end{cases} \tag{31}
\]

where \( v_{rs} \) is the traffic flow on the minimum hybrid RURM route.

5. Algorithm

The method of successive algorithm (MSA) is applied to solve the transit assignment model. The steps of the algorithm are shown in Figure 8.

Step 1 (initialization). Firstly, set the number of iterations \( n = 0 \). At this time, all space-time arc flows in the space-time expanded network are set to 0, and each space-time arc has an initial cost. According to the shortest route search algorithm, the route corresponding to the minimum hybrid RUM and RRM value (RURM) of each OD pair in the space-time network can be obtained. Then, all the OD demands that meet the train departure time are assigned to the shortest...
space-time route, and the space-time route flow is $f_{k}^{r,s}(0)$. According to the corresponding relationship between the space-time route and the space-time arc, the flow of space-time arc for the space-time route is $x_{mn}^{(0)}$. At this point, the number of iterations is $i = 1$.

Step 2 (update the hybrid RUM and RRM value of the space-time arc). According to flow of the space-time arcs which is obtained in Step 1, the space-time arcs’ costs under the utility maximization rules and the regret minimization rule are calculated.

Step 3 (calculate the additional flow of space-time arc). After calculating the cost of each space-time arc by Step 2, the cost of space-time route is calculated under the utility maximization rule for formula (17), and the cost of space-time route is calculated under the regret minimization rule for formula (20).

According to the shortest route algorithm, the new route of the minimum travel cost or minimum regret value between each OD pair in the space-time network is obtained. Then, all the OD demands that meet the train departure time are assigned to the new space-time route. At this time, according to the relationship between space-time arc and space-time route and the all-or-none assignment model, the space-time arc flow $y_{mn}^{(i)}$ on the space-time network is obtained, and the additional space-time arc flow is $y_{mn}^{(i)}$.

Step 4 (update the time and space arc flow). The updated space-time arcs flows are determined by the following formula.

$$x_{mn}^{(i+1)} = x_{mn}^{(i)} + \frac{1}{n} (y_{mn}^{(i)} - x_{mn}^{(i)})$$  (32)

Step 5 (convergence judgment). If there is no significant difference between the flows of space-time arcs obtained in two successive times, that is to say, the following formula can be satisfied, then the flow assignment ends; otherwise, it will return to Step 2. Here, $\varepsilon$ is a very small number, which will be taken as 0.5.

$$\max \{|x_{mn}^{(i+1)} - x_{mn}^{(i)}|\} \leq \varepsilon$$  (33)

6. Case Study

6.1. Case Information

6.1.1. Case Description. In this study, part of the Beijing subway network is selected to test the models. Beijing subway is the second largest metro network in China, with 22 operation lines and 370 stations in the network. The scale of the Beijing subway network is 608 kilometers. This paper selects four busiest lines in the Beijing subway network in 2017. The four lines are line 2, line 4, line 5, and line 6, respectively. All transfer stations are retained. For convenience, some intermediate stations are neglected, which does not affect the estimation result. The physical topology of the test network is shown in Figure 9.

In the physical topology in Figure 9, the stations marked by the boxes represent intermediate stations, and the stations marked by the circle represent the transfer stations. The physical topology network is a directed graph, so the two running directions of the same line should be represented.

Figure 9: Subway network physical topology.
Table 2: Line direction and number.

| Line direction | Real line direction |
|----------------|---------------------|
| Line direction 1 | The clockwise direction of inner ring of line 2 |
| Line direction 2 | The counter clockwise direction of the outer ring of line 2 |
| Line direction 3 | The direction of the TIANTONGYUAN on line 4 |
| Line direction 4 | The direction of the ANHEQIAO North on line 4 |
| Line direction 5 | The direction of the TIANTONGYUAN North on line 5 |
| Line direction 6 | The direction of the SONGJIAZHUANG on line 5 |
| Line direction 7 | The direction of the LUCHENG on line 6 |
| Line direction 8 | The direction of the HUIDIAN WULUJU on line 6 |

Table 3: Station number.

| Station number | Real station     | Station number | Real station     |
|----------------|------------------|----------------|------------------|
| 1              | CHEGONGZHUANG West | 12             | CHONGWENMEN     |
| 2              | CHEGONGZHUANG    | 13             | HEPINGMEN       |
| 3              | PINGANLI         | 14             | XUANWUMEN        |
| 4              | NANLUOGUXIANG    | 15             | CHANGCHUNJIE     |
| 5              | DONGSI           | 16             | National Library |
| 6              | CHAOYANGMEN      | 17             | LINGJING Hutong  |
| 7              | HUJIALOU         | 18             | Beijing South Railway Station |
| 8              | XIZHIMEN         | 19             | HEPINGXIAO       |
| 9              | GULOUDAJIE       | 20             | BEIXINQIAO       |
| 10             | YONGHEGONG       | 21             | DONGDAN          |
| 11             | Beijing Railway Station | 22       | TIANTANDONGMEN   |

Table 4: Passenger demand.

| Time   | OD pair       | 7-16 | 16-7 | 16-13 | 1-20 | 16-22 | 1-22 | 18-19 | 1-7 | 19-1 | 22-1 | 7-18 |
|--------|---------------|------|------|-------|------|-------|------|-------|-----|------|------|------|
| 9:02   | 65            | 50   | 32   | 69    | 65   | 43    | 45   | 44    | 52  | 55   | 53   |
| 9:04   | 50            | 79   | 68   | 47    | 84   | 59    | 51   | 60    | 56  | 54   | 54   |
| 9:06   | 48            | 52   | 42   | 52    | 30   | 48    | 49   | 47    | 39  | 47   | 55   |
| 9:08   | 37            | 52   | 49   | 49    | 69   | 50    | 45   | 49    | 49  | 41   | 44   |
| 9:10   | 73            | 47   | 69   | 30    | 58   | 57    | 56   | 53    | 57  | 50   | 48   |
| 9:12   | 62            | 40   | 78   | 79    | 39   | 62    | 50   | 60    | 62  | 45   | 54   |
| 9:14   | 56            | 55   | 40   | 58    | 49   | 58    | 50   | 57    | 42  | 45   | 51   |
| 9:16   | 68            | 61   | 82   | 49    | 63   | 65    | 57   | 64    | 59  | 54   | 61   |
| 9:18   | 45            | 80   | 69   | 74    | 52   | 68    | 50   | 67    | 69  | 48   | 64   |
| 9:20   | 58            | 54   | 59   | 40    | 68   | 52    | 51   | 50    | 58  | 58   | 48   |
| 9:22   | 61            | 45   | 60   | 60    | 50   | 55    | 53   | 53    | 54  | 50   | 51   |
| 9:24   | 59            | 82   | 54   | 59    | 72   | 53    | 61   | 52    | 65  | 58   | 49   |
| 9:26   | 60            | 75   | 52   | 62    | 58   | 68    | 63   | 62    | 60  | 60   | 59   |
| 9:28   | 49            | 62   | 62   | 59    | 60   | 65    | 52   | 65    | 56  | 49   | 62   |
| 9:30   | 59            | 42   | 59   | 52    | 51   | 46    | 49   | 44    | 46  | 47   | 42   |

separately. For the convenience of the following research, the different directions of each line are numbered separately, as shown in Table 2. In addition, the stations’ numbers are shown in Table 3.

Taking 9:00 to 9:30 as the study period, each 2 min is a time step, so there are 15 periods in total. 4 OD pairs (10 routes) in Figure 9 are selected and the passenger demand data are obtained from smart card records at each period in Table 4. The train timetables of all lines are shown in the appendix.

6.1.2. Route Selection Survey and Parameter Estimation. In order to verify the applicability of the improved generalized random regret model in the route selection, the parameter estimation in the model was carried out based on
the data derived from a survey on passenger route choice behavior.

Route choice data were collected based on a face to face SP survey [43]. The survey was conducted between 8:00 and 17:00 on Sept. 12 and Sept. 19 in trains and at stations. A questionnaire is designed to obtain the route choice result of passengers who use Beijing urban rail transit. Responders are asked to make a choice among a number of possible routes given the associated values of attributes on the subnetwork. A total number of 1030 effective samples are collected in the survey. The parameters of utility maximization model and regret minimization model are estimated by Biogeme software [44]. The results are shown in Table 5.

Three experiments are designed: (1) random utility maximization; (2) hybrid of random utility maximization and random regret minimization with attributes level; (3) hybrid of random utility maximization and random regret minimization with decision level.

### 6.2. Result with Attributes Level Hybrid Rule

According to the physical topology network, train timetable, and the smart card data, the space-time extended network is constructed, passenger demand and the parameter value are put into the algorithm, and the results can be obtained by using the random utility maximization and the random regret minimization model with different attributes level hybrid parameters, which are shown in Table 6.

| parameter | values | parameters | values |
|-----------|--------|------------|--------|
| $\theta$  | 1.3    | $Z_i$      | 256    |
| $\kappa$ | 2      | $N_j$      | 1428   |
| $\gamma_{ET}$ | -1  | $\gamma_{TT}$ | -3    |
| $\gamma_{VT}$ | -1  | $\omega_{VT}$ | 0.5   |
| $\gamma_{CT}$ | -2.5 | $\omega_{CT}$ | 0.5   |
| $\alpha$ | 0.5    | $\omega_{TT}$ | 0.9   |
| $\beta$  | 0.8    |            | 0.8    |

Note: The number of seats $Z_i$ and the capacity of train $N_j$ are directly obtained from rolling stock profile.

### 6.2.2. Flow Assignment

The transit assignment results of RUM and RRM with attributes level hybrid parameters are shown in Figures 10 and 11. In the figures, the number represents the passenger flow on each section, and the time for each train stopping at the station is indicated beside the station.

It can be observed from Figures 10-11 that the flows assigned to the links are different with different rules. The results show that most of the passengers would choose the fastest route under RUM conditions, while under RRM with different attributes level hybrid parameters, more passengers choose a moderate route.

Between the same OD, the route traffic under RUM is different from the route traffic under RRM. For example, there are two routes of National Library-HEPINGMEN; the first route is National Library- XUANWUMEN -HEPINGMEN, and the second route is National Library-XIZHIMEN-HEPINGMEN. They have the same departure time and number of transfer times, but the transfer station is different, so the transfer time is different. Under the RUM, the flow of the first route is less than the flow of second route, but under the RRM, the flow of the first route decreases and the flow of the second route increases. The reason is that the transfer time of the first route in XUANWUMEN is larger than the transfer time of the second route in XIZHIMEN. In addition, the attribute hybrid parameter of the transfer time is larger under the regret rule, which indicates that the passenger's
Table 6: Route choice result under utility maximization and regret minimization.

| O-D                  | Station sequence/Time sequence on the route | Line direction | Transfer station | Utility value | Assignment | Regret value | Assignment |
|----------------------|---------------------------------------------|----------------|------------------|--------------|------------|--------------|------------|
| National Library-HEPINGMEN (16-13) | National Library9:08-XIZHIMEN9:13-PINGANLI9:18-LINGJING Hutong9:21-XUANWUMEN9:25(9:30)-HEPINGMEN9:32 National Library9:08-XIZHIMEN9:13(9:16)-CHEGONGZHUANG9:18-CHEGONGZHUANG9:25-XUANWUMEN9:27-HEPINGMEN9:29 | 3-2          | XUANWUMEN       | 34.4         | 38         | 3.26         | 33         |
|                      | CHEGONGZHUANG Hutong9:14-CHEGONGZHUANG9:16(9:18)-CHEGONGZHUANG9:25-XUANWUMEN9:27-HEPINGMEN9:29-CHEGONGZHUANG9:34(9:36)-TIANTANDONGMEN9:38 CHEGONGZHUANG Hutong9:14-CHEGONGZHUANG9:16-PINGANLI9:19-NANLUOGUXIANG9:24-DONGSI9:27(9:34)-DONGDAN9:38-CHEGONGZHUANG9:40-TIANTANDONGMEN9:44 | 7-2-6        | CHEGONGZHUANG   | 29.9         | 115        | 2.64         | 108        |
|                      | Beijing South Railway Station9:06-XUANWUMEN9:14(9:18)-HEPINGMEN9:28-CHEGONGZHUANG9:25(9:28)-DONGDAN9:30-DONGSI9:34-BEIXINQIAO9:38-YONGHEGONG9:40-HEPINGXIQIAO9:45 Beijing South Railway Station9:06-XUANWUMEN9:14-LINGJING Hutong9:18-PINGANLI9:22(9:29)-NANLUOGUXIANG9:34-DONGSI9:37(9:42)-BEIXINQIAO9:46-YONGHEGONG9:48-HEPINGXIQIAO9:53 | 4-2-5        | XUANWUMEN       | 48.3         | 50         | 1.06         | 57         |
|                      | National Library9:04-XIZHIMEN9:09-PINGANLI9:14-LINGJING Hutong9:17-XUANWUMEN9:21(9:24)-HEPINGMEN9:26-CHEGONGZHUANG9:15-CHEGONGZHUANG9:22-XUANWUMEN9:24-HEPINGMEN9:26-CHEGONGZHUANG9:31(9:36)-TIANTANDONGMEN9:40 National Library9:04-XIZHIMEN9:09(9:13)-CHEGONGZHUANG9:15-CHEGONGZHUANG9:22-XUANWUMEN9:24-HEPINGMEN9:26-CHEGONGZHUANG9:31(9:36)-TIANTANDONGMEN9:40 National Library9:04-XIZHIMEN9:09-PINGANLI9:14(9:19)-NANLUOGUXIANG9:24-DONGSI9:27(9:34)-DONGDAN9:38-CHEGONGZHUANG9:40-TIANTANDONGMEN9:44 National Library9:04-XIZHIMEN9:09(9:12)-GULOUDAJIE9:18-YONGHEGONG9:22(9:24)-BEIXINQIAO9:26-DONGSI9:30-DONGDAN9:34-CHEGONGZHUANG9:36-TIANTANDONGMEN9:40 | 3-2-6        | XIZHIMEN        | 44.5         | 14         | 7.69         | 5          |
|                      | National Library9:04-XIZHIMEN9:09-PINGANLI9:14-LINGJING Hutong9:17-XUANWUMEN9:21(9:24)-HEPINGMEN9:26-CHEGONGZHUANG9:15-CHEGONGZHUANG9:22-XUANWUMEN9:24-HEPINGMEN9:26-CHEGONGZHUANG9:31(9:36)-TIANTANDONGMEN9:40 National Library9:04-XIZHIMEN9:09(9:13)-CHEGONGZHUANG9:15-CHEGONGZHUANG9:22-XUANWUMEN9:24-HEPINGMEN9:26-CHEGONGZHUANG9:31(9:36)-TIANTANDONGMEN9:40 National Library9:04-XIZHIMEN9:09-PINGANLI9:14(9:19)-NANLUOGUXIANG9:24-DONGSI9:27(9:34)-DONGDAN9:38-CHEGONGZHUANG9:40-TIANTANDONGMEN9:44 National Library9:04-XIZHIMEN9:09(9:12)-GULOUDAJIE9:18-YONGHEGONG9:22(9:24)-BEIXINQIAO9:26-DONGSI9:30-DONGDAN9:34-CHEGONGZHUANG9:36-TIANTANDONGMEN9:40 | 3-7-6        | PINGANLI       | 45.6         | 8          | 5.79         | 16         |
|                      | National Library9:04-XIZHIMEN9:09-PINGANLI9:14-LINGJING Hutong9:17-XUANWUMEN9:21(9:24)-HEPINGMEN9:26-CHEGONGZHUANG9:15-CHEGONGZHUANG9:22-XUANWUMEN9:24-HEPINGMEN9:26-CHEGONGZHUANG9:31(9:36)-TIANTANDONGMEN9:40 National Library9:04-XIZHIMEN9:09(9:13)-CHEGONGZHUANG9:15-CHEGONGZHUANG9:22-XUANWUMEN9:24-HEPINGMEN9:26-CHEGONGZHUANG9:31(9:36)-TIANTANDONGMEN9:40 National Library9:04-XIZHIMEN9:09-PINGANLI9:14(9:19)-NANLUOGUXIANG9:24-DONGSI9:27(9:34)-DONGDAN9:38-CHEGONGZHUANG9:40-TIANTANDONGMEN9:44 National Library9:04-XIZHIMEN9:09(9:12)-GULOUDAJIE9:18-YONGHEGONG9:22(9:24)-BEIXINQIAO9:26-DONGSI9:30-DONGDAN9:34-CHEGONGZHUANG9:36-TIANTANDONGMEN9:40 | 3-1-6        | XIZHIMEN       | 43.3         | 28         | 5.23         | 29         |
Figure 10: The assignment results based on RUM.

Figure 11: The assignment results based on RRM.
avoidance of the transfer time is higher, so the flow rate of the second route will increase.

6.2.3. Train Occupancies. Based on the route choice result on the space-time expanded network, the number of passengers is assigned to different trains. So the train occupancies can be obtained under different behavioral assumptions.

(1) Based on Utility Maximization. The number of passengers in each train at each section of each line based on the utility maximization model is shown in Figure 12.

In the above assignment results, the number of people in the train departing at each station is different depending on trains. That is, the number of people in the train changes dynamically with time. In the results of the above assignment, the number of people on the train in the station at the time of departure is 0, or the number of people in a train on a certain line is all 0, because in this case, only the OD demand within 30 minutes is selected. Moreover, due to limited data acquisition, the OD passenger flow demand is not between the two OD, so that some stations or trains appear with 0 people in the train. If we study the assignment results of a longer period of time and can get the passenger flow demand between all ODs, we can get more detailed assignment results. It can be seen from the above assignment results that each train shows the change of passenger flow only when passengers get on the train at the starting point of the OD and transfer stations, which is consistent with the actual urban rail transit operation.

(2) Based on Regret Minimization. The number of passengers in each train at each section of each line based on regret minimization model is shown in Figure 13.

In the assignment results under the regret minimization rule, the number of people in the train at the time of departure at each station is different from the utility maximization rule. Because under the regret rule, the regrets of passengers are different for various times in the train, transfer times, etc., the number of people in the train is different from the number under the utility rule.

6.2.4. Validation. The result is validated by the travel times of passengers derived from smart card data. At first, the travel time of each route is calculated, and then the OD travel time is aggregated according to the passenger flow distribution in Section 6.2. After that, the accuracy of the result is verified by comparing it with the actual travel time obtained from the AFC. The results are shown in Table 7.

The estimation errors are calculated by formula (34), and the errors with different models are listed in Table 8.

$$\text{OD travel time error} = \left( \frac{\text{Calculated time} - \text{actual time}}{\text{actual time}} \right) \times 100\% \quad (34)$$

The validation result shows that, from the aspect of travel time, RRM is slightly better than RUM for some of the OD pairs. For some OD pairs, the RRM performs worse than RUM. This is different from previous research findings in road traffic. This can be explained by the fact that passengers’ behavior in urban rail transit is different from that of road travelers. For different types of OD pairs, passenger behaviors can be different. In urban rail transit, some people tend to maximize their utility while others would minimize their regret. Some passengers may choose a faster route to maximize their utility and ignore the risk of crowd regret, but some passengers also may not choose a faster but crowd route, because they try to avoid crowd regret from choosing the route. Any single standard behavior assumption may not fully explain the complex route choice behavior of humans.
From this point of view, we only assign transit flow based on the different rules and compare the differences. Moreover, considering the land use of the station in different OD pairs, it is observed that the crowd would influence the regret value of the OD pairs with more commuters, e.g., 16-13 and 1-22. The crowd does not influence the route choice of the OD pairs with more tourists, e.g., 18-19 and 16-22.

6.3. Result with Decision Level Hybrid Rule. In order to study the different effects of utility and regret on passenger route choice, the different values of decision level hybrid parameter are also tested. The value of the decision level hybrid parameter $\rho$ is taken from 0 to 1, and the route choice and flow assignment results are obtained.

6.3.1. Flow Assignment. The results are shown in Figures 14–22.

Based on the hybrid utility and regret rule, the route flow of the same OD is different in different decision level hybrid parameters. With the increase of $\rho$, the route flow gradually tends to the flow based on the utility maximization rule, which means the proportion of passengers choosing utility maximization rule increases.

From Figures 14–22, we can find that, with the increase of $\rho$, there are eight sections whose flows remain unchanged,
and nine sections whose flows have changed. The distribution of passenger flow in four sections has increased, which are LINGJING Hutong-XUANWUMEN, XIZHIMEN-CHEGONGZHUANG, CHEGONGZHUANG-CHANGCHUNJIE, and XUANWUMEN-HEPINGMEN. The passenger flow in five sections has decreased, which are XIZHIMEN-PINGANLI, CHEGONGZHUANG-PINGANLI, NANLUOGUOXIANG-DONGSI, CHONGWENMEN-Beijing Railway Station, and DONGDAN-CHONGWENMEN.

The distribution of passenger flow in the vicinity of transfer station changed greatly. In addition, when ρ is reduced, the distribution of passenger flow is more uniform, especially in the vicinity of XIZHIMEN.

6.3.2. Validation. Figure 23 shows that, from the aspect of travel time, with the increase of ρ, some OD travel time errors with different hybrid parameter perform slightly better than RRM and some OD perform slightly worse. For National Library-HEPINGMEN and CHEGONGZHUANG West-TIANTANDONGMEN, RRM is slightly better than RUM. For Beijing South Railway Station-HEPINGXIQIAO and National Library-TIANTANDONGMEN, RUM is slightly better than RRM.

Taking CHEGONGZHUANG West-TIANTANDONGMEN as an example, RRM is slightly better than RUM. The study time is after 9 a.m., and the route travel cost is smaller for route 1 (transfer at XUANWUMEN and CHONGWENMEN-Beijing Railway Station, and DONGDAN-CHONGWENMEN). The distribution of passenger flow in the vicinity of transfer station changed greatly. In addition, when ρ is reduced, the distribution of passenger flow is more uniform, especially in the vicinity of XIZHIMEN.

Table 8: Travel time error of each OD.

| OD pair                                      | OD travel time error |
|----------------------------------------------|----------------------|
| National Library-HEPINGMEN (16-13)          | RUM: 4.36% RRM: 2.84% |
| CHEGONGZHUANG West-TIANTANDONGMEN (1-22)    | RUM: 29.93% RRM: 27.83% |
| Beijing South Railway Station-HEPINGXIQIAO  | RUM: 0.70% RRM: 4.02%  |
| National Library – TIANTANDONGMEN (16-22)   | RUM: 16.58% RRM: 18.85% |

Figure 14: Flow distribution based on hybrid of RUM and RRM (ρ=0.1).
Figure 15: Flow distribution based on hybrid of RUM and RRM ($\rho=0.2$).

Figure 16: Flow distribution based on hybrid of RUM and RRM ($\rho=0.3$).
Figure 17: Flow distribution based on hybrid of RUM and RRM ($\rho=0.4$).

Figure 18: Flow distribution based on hybrid of RUM and RRM ($\rho=0.5$).
Figure 19: Flow distribution based on hybrid of RUM and RRM ($\rho=0.6$).

Figure 20: Flow distribution based on hybrid of RUM and RRM ($\rho=0.7$).
Figure 21: Flow distribution based on hybrid of RUM and RRM ($\rho=0.8$).

Figure 22: Flow distribution based on hybrid of RUM and RRM ($\rho=0.9$).
Table 9: Timetable of line 1.

| Station           | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  |
|-------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| XIZHIMEN          | 9:00| 9:03| 9:06| 9:09| 9:12| 9:15| 9:18| 9:21| 9:24| 9:27| 9:30|
| GULOUDAJIE        | 9:06| 9:09| 9:12| 9:15| 9:18| 9:21| 9:24| 9:27| 9:30| 9:33| 9:36|
| YONGHEGONG        | 9:10| 9:13| 9:16| 9:19| 9:22| 9:25| 9:28| 9:31| 9:34| 9:37| 9:40|
| CHAOYANGMEN       | 9:19| 9:22| 9:25| 9:28| 9:31| 9:34| 9:37| 9:40| 9:43| 9:46| 9:49|
| Beijing Railway Station | 9:23| 9:26| 9:29| 9:32| 9:35| 9:38| 9:41| 9:44| 9:47| 9:50| 9:53|
| CHONGWENMEN       | 9:26| 9:29| 9:32| 9:35| 9:38| 9:41| 9:44| 9:47| 9:50| 9:53| 9:56|
| HEPINGMEN         | 9:31| 9:34| 9:37| 9:40| 9:43| 9:46| 9:49| 9:52| 9:55| 9:58| 10:01|
| XUANWUMEN         | 9:33| 9:36| 9:39| 9:42| 9:45| 9:48| 9:51| 9:54| 9:57| 10:00| 10:03|
| CHANGCHUNJIE      | 9:35| 9:38| 9:41| 9:44| 9:47| 9:50| 9:53| 9:56| 9:59| 10:02| 10:05|
| CHEGONGZHUANG     | 9:42| 9:45| 9:48| 9:51| 9:54| 9:57| 10:00| 10:03| 10:06| 10:09| 10:12|

Figure 23: Travel time errors with different hybrid parameter.

7. Conclusions

This paper proposed a hybrid model of passenger route choice in urban rail transit network. The utility maximization rule and regret minimization rule are combined to analyze the passengers’ route choice behavior. Two types of hybrid rules are introduced, namely, attributes level hybrid rule and decision level hybrid rule. Based on the train timetable, the space-time extension network is constructed based on the smart card data. The utility function and regret function are formulated by calculating the utility value and the regret value of different space-time arcs, respectively. After that, the MSA algorithm is used to solve the model. Finally, it took some OD pairs in Beijing urban rail transit network as an example to validate the hybrid model.

Three conclusions can be drawn from this study:

1. Passengers’ route choice behavior in urban rail transit does not obey a single criteria, e.g., utility maximization or regret minimization. For some OD pairs, regret minimization model performs slightly better than utility maximization, while for others, the results are opposite. For those in which RRM performs better, this implies that the utility of the chosen route is not only related to its own utility, but also related to the utility value of other unselected routes, which provide the multiangle method for transit assignment of urban rail transit.

2. For OD pairs with only two alternative routes, the optimal route with RRM is close to RUM. In contrast, for those with more than two alternative routes, the route choice with RRM and RUM is different.

3. Case studies show that crowd would only influence the regret value of the OD pair with more commuters. The crowd does not influence the route choice of OD pairs with more tourists. So it is necessary to use different attributes level hybrid parameters for different observable variables considering the types of passengers traveling between two stations.

So far, it is not clear for what kind of OD pairs, RRM is better. So an alternative choice is to use a hybrid transit assignment model which can reduce the risk of average estimation error.

This research is only a pilot study of hybrid route choice model in an urban rail transit network. The future research would focus on the key problem of which is the main influencing factor on RUM or RRM in an urban rail transit network. It is also necessary to validate the result in a more practical manner for a larger network scale. The advantages and disadvantages of passenger route choice behavior based on the utility maximization and regret minimization would be verified from the actual situation.

Appendix
See Tables 9, 10, 11, 12, 13, 14, 15, and 16.

Data Availability

The data are available from Beijiao SRAIL Technology (Beijing) Co., Ltd, for researchers who meet the criteria for access to confidential data. Access to data used in this paper is granted through a formal application process that requires
Table 10: Timetable of line 2.

| Station          | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 |
|------------------|----|----|----|----|----|----|----|----|----|----|----|
| XIZHIMEN         | 9:01 | 9:04 | 9:07 | 9:10 | 9:13 | 9:16 | 9:19 | 9:22 | 9:25 | 9:28 | 9:31 |
| CHEGONGZHUANG    | 9:03 | 9:06 | 9:09 | 9:12 | 9:15 | 9:18 | 9:21 | 9:24 | 9:27 | 9:30 | 9:33 |
| CHANGCHUNJIE     | 9:10 | 9:13 | 9:16 | 9:19 | 9:22 | 9:25 | 9:28 | 9:31 | 9:34 | 9:37 | 9:40 |
| XUANWUMEN        | 9:12 | 9:15 | 9:18 | 9:21 | 9:24 | 9:27 | 9:30 | 9:33 | 9:36 | 9:39 | 9:42 |
| HEPINGMEN        | 9:14 | 9:17 | 9:20 | 9:23 | 9:26 | 9:29 | 9:32 | 9:35 | 9:38 | 9:41 | 9:44 |
| CHONGWENMEN      | 9:19 | 9:22 | 9:25 | 9:28 | 9:31 | 9:34 | 9:37 | 9:40 | 9:43 | 9:46 | 9:49 |
| Beijing Railway Station | 9:22 | 9:25 | 9:28 | 9:31 | 9:34 | 9:37 | 9:40 | 9:43 | 9:46 | 9:49 | 9:52 |
| CHAOYANGMEN      | 9:26 | 9:29 | 9:32 | 9:35 | 9:38 | 9:41 | 9:44 | 9:47 | 9:50 | 9:53 | 9:56 |
| YONGHEGONG       | 9:35 | 9:38 | 9:41 | 9:44 | 9:47 | 9:50 | 9:53 | 9:56 | 9:59 | 10:02 | 10:05 |
| GULOUDAJIE       | 9:39 | 9:42 | 9:45 | 9:48 | 9:51 | 9:54 | 9:57 | 10:00 | 10:03 | 10:06 | 10:09 |
| XIZHIMEN         | 9:45 | 9:48 | 9:51 | 9:54 | 9:57 | 10:00 | 10:03 | 10:06 | 10:09 | 10:12 | 10:15 |

Table 11: Timetable of line 3.

| Station          | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|------------------|----|----|----|----|----|----|----|----|----|
| National Library | 9:00 | 9:04 | 9:08 | 9:12 | 9:16 | 9:20 | 9:24 | 9:28 | 9:32 |
| XIZHIMEN         | 9:05 | 9:09 | 9:13 | 9:17 | 9:21 | 9:25 | 9:29 | 9:33 | 9:37 |
| PINGANLI         | 9:10 | 9:14 | 9:18 | 9:22 | 9:26 | 9:30 | 9:34 | 9:38 | 9:42 |
| LINGJING Hutong  | 9:13 | 9:17 | 9:21 | 9:25 | 9:29 | 9:33 | 9:37 | 9:41 | 9:45 |
| XUANWUMEN        | 9:17 | 9:21 | 9:25 | 9:29 | 9:33 | 9:37 | 9:41 | 9:45 | 9:49 |
| Beijing South Railway Station | 9:25 | 9:29 | 9:33 | 9:37 | 9:41 | 9:45 | 9:49 | 9:53 | 9:57 |

Table 12: Timetable of line 4.

| Station          | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|------------------|----|----|----|----|----|----|----|----|----|
| Beijing South Railway Station | 9:01 | 9:06 | 9:11 | 9:16 | 9:21 | 9:26 | 9:31 | 9:36 | 9:41 |
| XUANWUMEN        | 9:09 | 9:14 | 9:19 | 9:24 | 9:29 | 9:34 | 9:39 | 9:44 | 9:49 |
| LINGJING Hutong  | 9:13 | 9:18 | 9:23 | 9:28 | 9:33 | 9:38 | 9:43 | 9:48 | 9:53 |
| PINGANLI         | 9:17 | 9:22 | 9:27 | 9:32 | 9:37 | 9:42 | 9:47 | 9:52 | 9:57 |
| XIZHIMEN         | 9:22 | 9:27 | 9:32 | 9:37 | 9:42 | 9:47 | 9:52 | 9:57 | 10:02 |
| National Library | 9:27 | 9:32 | 9:37 | 9:42 | 9:47 | 9:52 | 9:57 | 10:02 | 10:07 |

Table 13: Timetable of line 5.

| Station          | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|------------------|----|----|----|----|----|----|----|----|----|
| TIANTANDONGMEN   | 9:00 | 9:04 | 9:08 | 9:12 | 9:16 | 9:20 | 9:24 | 9:28 | 9:32 |
| CHONGWENMEN      | 9:04 | 9:08 | 9:12 | 9:16 | 9:20 | 9:24 | 9:28 | 9:32 | 9:36 |
| DONGDAN          | 9:06 | 9:10 | 9:14 | 9:18 | 9:22 | 9:26 | 9:30 | 9:34 | 9:38 |
| DONGSI           | 9:10 | 9:14 | 9:18 | 9:22 | 9:26 | 9:30 | 9:34 | 9:38 | 9:42 |
| BEIXINQIAO       | 9:14 | 9:18 | 9:22 | 9:26 | 9:30 | 9:34 | 9:38 | 9:42 | 9:46 |
| YONGHEGONG       | 9:16 | 9:20 | 9:24 | 9:28 | 9:32 | 9:36 | 9:40 | 9:44 | 9:48 |
| HEPINGXIQIAO     | 9:21 | 9:25 | 9:29 | 9:33 | 9:37 | 9:41 | 9:45 | 9:49 | 9:53 |

peer review. Further information can be obtained by readers from the company manager Dr. Lu (srail_beijing@163.com).

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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Table 14: Timetable of line 6.

| Station        | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
|----------------|------|------|------|------|------|------|------|------|------|
| HEPINGXIQIAO   | 8:59 | 9:03 | 9:07 | 9:11 | 9:15 | 9:19 | 9:23 | 9:27 | 9:31 |
| YONGHEGONG     | 9:04 | 9:08 | 9:12 | 9:16 | 9:20 | 9:24 | 9:28 | 9:32 | 9:36 |
| BEIXINQIAO     | 9:06 | 9:10 | 9:14 | 9:18 | 9:22 | 9:26 | 9:30 | 9:34 | 9:38 |
| DONGSI         | 9:10 | 9:14 | 9:18 | 9:22 | 9:26 | 9:30 | 9:34 | 9:38 | 9:42 |
| DONGDAN        | 9:14 | 9:18 | 9:22 | 9:26 | 9:30 | 9:34 | 9:38 | 9:42 | 9:46 |
| CHONGWENMEN    | 9:16 | 9:20 | 9:24 | 9:28 | 9:32 | 9:36 | 9:40 | 9:44 | 9:48 |
| TIANTANDONGMEN | 9:20 | 9:24 | 9:28 | 9:32 | 9:36 | 9:40 | 9:44 | 9:48 | 9:52 |

Table 15: Timetable of line 7.

| Station               | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|-----------------------|------|------|------|------|------|------|------|
| CHEGONGZHUANG West   | 8:59 | 9:04 | 9:09 | 9:14 | 9:19 | 9:24 | 9:29 |
| CHEGONGZHUANG        | 9:01 | 9:06 | 9:11 | 9:16 | 9:21 | 9:26 | 9:31 |
| PINGANLI             | 9:04 | 9:09 | 9:14 | 9:19 | 9:24 | 9:29 | 9:34 |
| NALNUOGUXIANG        | 9:09 | 9:14 | 9:19 | 9:24 | 9:29 | 9:34 | 9:39 |
| DONGSI               | 9:12 | 9:17 | 9:22 | 9:27 | 9:32 | 9:37 | 9:42 |
| CHAOYANGMEN          | 9:14 | 9:19 | 9:24 | 9:29 | 9:34 | 9:39 | 9:44 |
| HUJIALOU             | 9:19 | 9:24 | 9:29 | 9:34 | 9:39 | 9:44 | 9:49 |

Table 16: Timetable of line 8.

| Station       | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|---------------|------|------|------|------|------|------|------|
| HUJIALOU      | 9:00 | 9:05 | 9:10 | 9:15 | 9:20 | 9:25 | 9:30 |
| CHAOYANGMEN   | 9:05 | 9:10 | 9:15 | 9:20 | 9:25 | 9:30 | 9:35 |
| DONGSI        | 9:07 | 9:12 | 9:17 | 9:22 | 9:27 | 9:32 | 9:37 |
| NALNUOGUXIANG | 9:10 | 9:15 | 9:20 | 9:25 | 9:30 | 9:35 | 9:40 |
| PINGANLI      | 9:15 | 9:20 | 9:25 | 9:30 | 9:35 | 9:40 | 9:45 |
| CHEGONGZHUANG | 9:18 | 9:23 | 9:28 | 9:33 | 9:38 | 9:43 | 9:48 |
| CHEGONGZHUANG West | 9:20 | 9:25 | 9:30 | 9:35 | 9:40 | 9:45 | 9:50 |

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