Unitarity Triangle analysis
in the Standard Model
and beyond from UTfit

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Unitarity Triangle analysis in the SM

SM UT analysis:
- provide the best determination of CKM parameters
- test the consistency of the SM ("direct" vs "indirect" determinations)
- provide predictions (from data..) for SM observables

.. and beyond

NP UT analysis:
- model-independent analysis
- provides limit on the allowed deviations from the SM
- obtain the NP scale
Other UT analyses exist, by:

CKMfitter (http://ckmfitter.in2p3.fr/),
Laiho&Lunghi&Van de Water (http://latticeaverages.org/)
Lunghi&Soni (1010.6069)

C. Alpigiani, A. Bevan, M.B., M. Ciuchini,
D. Derkach, E. Franco, V. Lubicz, G. Martinelli,
F. Parodi, M. Pierini, C. Schiavi, L. Silvestrini,
A. Stocchi, V. Sordini, C. Tarantino and V. Vagnoni
the method and the inputs:

\[ f(\bar{\rho}, \bar{\eta}, X | c_1, ..., c_m) \sim \prod_{j=1}^{m} f_j(C | \bar{\rho}, \bar{\eta}, X) * \prod_{i=1}^{N} f_i(x_i) f_0(\bar{\rho}, \bar{\eta}) \]

**Bayes Theorem**

\[ X \equiv x_1, ..., x_n = m_t, B_K, F_B, ... \]

\[ C \equiv c_1, ..., c_m = \epsilon, \Delta m_d/\Delta m_s, A_{CP}(J/\psi K_S), ... \]

\[
\begin{array}{|l|c|c|}
\hline
(b \to u)/(b \to c) & \bar{\rho}^2 + \bar{\eta}^2 & \bar{\Lambda}, \lambda_1, F(1), ... \\
\hline
\epsilon_K & \bar{\eta}[1 - \bar{\rho}] + P & B_K \\
\Delta m_d & (1 - \bar{\rho})^2 + \bar{\eta}^2 & f_B^2 B_B \\
\Delta m_d/\Delta m_s & (1 - \bar{\rho})^2 + \bar{\eta}^2 & \xi \\
A_{CP}(J/\psi K_S) & \sin 2\beta & m_t \\
\hline
\end{array}
\]

Standard Model + OPE/HQET/Lattice QCD to go from quarks to hadrons

M. Bona et al. (UTfit Collaboration)  
JHEP 0507:028,2005 hep-ph/0501199  
M. Bona et al. (UTfit Collaboration)  
JHEP 0603:080,2006 hep-ph/0509219
**V_{cb} and V_{ub}**

Average of the two FLAG Nf=2+1 averages

\[
|V_{cb}| \text{ (excl)} = (40.1 \pm 1.2) \times 10^{-3}
\]

\[
|V_{cb}| \text{ (incl)} = (42.00 \pm 0.64) \times 10^{-3}
\]

Gambino 1606.06174

\sim 1.3\sigma \text{ discrepancy}

Nf=2+1 FLAG average

\[
|V_{ub}| \text{ (excl)} = (3.62 \pm 0.14) \times 10^{-3}
\]

\[
|V_{ub}| \text{ (incl)} = (4.41 \pm 0.22) \times 10^{-3}
\]

\sim 3\sigma \text{ discrepancy}

\[
|V_{ub} / V_{cb}| \text{ (LHCb)} = (8.3 \pm 0.6) \times 10^2
\]
$V_{cb}$ and $V_{ub}$

2D average inspired by D'Agostini skeptical procedure (hep-ex/9910036) with $\sigma=1$. Very similar results obtained from a 2D a la PDG procedure.

$|V_{cb}| = (41.7 \pm 1.0) \times 10^{-3}$
uncertainty $\sim 2.4\%$

$|V_{ub}| = (3.74 \pm 0.21) \times 10^{-3}$
uncertainty $\sim 5.6\%$

$|V_{cb}| = (42.6 \pm 0.7) \times 10^{-3}$

$|V_{ub}| = (3.66 \pm 0.13) \times 10^{-3}$

UTfit predictions updated for ICHEP16
### Updates from UTfit

#### $V_{cb}$ and $V_{ub}$

- **$|V_{cb}|$ (excl)** = $(38.88 \pm 0.60) \times 10^{-3}$
- **$|V_{cb}|$ (incl)** = $(42.19 \pm 0.78) \times 10^{-3}$

New HFAG

- ~$3.3\sigma$ discrepancy

- **$|V_{ub}|$ (excl)** = $(3.65 \pm 0.14) \times 10^{-3}$
- **$|V_{ub}|$ (incl)** = $(4.50 \pm 0.20) \times 10^{-3}$

New HFAG

- ~$3.4\sigma$ discrepancy

- **$|V_{ub}| / |V_{cb}|$ (LHCb)** = $(8.0 \pm 0.6) \times 10^{-2}$

Updated value

- **UTfit average not updated!**

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**New HFAG numbers @CKM16**

- **$V_{ub}$**
  - Incl. $V_{ub}$
  - Excl. $V_{ub}$

- **$V_{cb}$**
  - Incl. $V_{cb}$
  - Excl. $V_{cb}$

- **UTfit average**

- **UTfit prediction**

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$V_{cb}$ and $V_{ub}$

some new extra results @CKM16: a lay(wo)man’s visualisation..

new BaBar result for inclusive $V_{ub}$ @Kowalewski on Tue in WG2
$V_{cb}$ and $V_{ub}$

some new extra results @CKM16: a lay(wo)man’s visualisation..

new from Fermilab Lattice collaboration MILC

new from HPQCD collaboration @Davis on Mon in Plenary
exclusives vs inclusives

only exclusive values

only inclusive values

UTfit
CKM16

exclusives only

includes only

BR($B \rightarrow \tau \nu$)

$\rho$

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exclusives vs inclusives

\[
\sin^2 \beta_{\text{exp}} = 0.745 \pm 0.050
\]

\[
\sin^2 \beta_{\text{exp}} = 0.729 \pm 0.019
\]

\[
\sin^2 \beta_{\text{exp}} = 0.796 \pm 0.026
\]

\[
\sin^2 \beta_{\text{UTfit}} = 0.725 \pm 0.030
\]
\[ \sin2\alpha \ (\phi_2) \] from charmless B decays: \( \pi\pi, \rho\rho, \pi\rho \)

\[ \pi^0\pi^0 \] from Belle at CKM14 to be updated soon (?

\[ \text{BR}(\pi^0\pi^0) = (1.17 \pm 0.13) \times 10^{-6} \]

HFAG 2014

a à la PDG average would give an inflated uncertainty of 0.41

\[ \rho^+\rho^- \] average updated including Belle arXiv:1510.01245

\[ \rho^0\rho^0 \] average updated including LHCb arXiv:1503.07770

\[ \alpha \] from \( \pi\pi, \rho\rho, \pi\rho \) decays:

combined: \( (94.2 \pm 4.5) \degree \)

UTfit prediction: \( (90.9 \pm 2.5) \degree \)
After a decade of analyses and almost 50 papers published, the world average uncertainty has decreased by a factor 3 combined: $(70.5 \pm 5.7)\degree$

UTfit prediction: $(65.4 \pm 2.1)\degree$
Unitarity Triangle analysis in the SM:

$|v_{cb}/v_{ub}|$

$\bar{\rho}^2 + \bar{\eta}^2$

$\bar{\eta}[(1 - \bar{\rho}) + P]$

$\Delta m_d$

$\Delta m_s/\Delta m_d$

$(1 - \bar{\rho})^2 + \bar{\eta}^2$

$\gamma$

$2\beta + \gamma$

$B \rightarrow \tau \nu$
Unitarity Triangle analysis in the SM:

Levels @ 95% Prob:

\[ \bar{\rho} = 0.154 \pm 0.015 \]
\[ \bar{\eta} = 0.344 \pm 0.013 \]

\( \sim 10\% \)
\( \sim 4\% \)
angles vs sides (and $\varepsilon_K$)

$\bar{\rho} = 0.147 \pm 0.022$
$\eta = 0.333 \pm 0.016$

levels @ 95% Prob

$\bar{\rho} = 0.160 \pm 0.018$
$\eta = 0.359 \pm 0.021$
compatibility plots

A way to “measure” the agreement of a single measurement with the indirect determination from the fit using all the other inputs: test for the SM description of the flavour physics

Color code: agreement between the predicted values and the measurements at better than 1, 2, ...\( \sigma \)

The cross has the coordinates \((x,y) = \text{(central value, error)}\) of the direct measurement

\[\gamma_{\exp} = (70.5 \pm 5.7) ^\circ \quad \gamma_{\text{UTfit}} = (65.4 \pm 2.1) ^\circ\]

\[\alpha_{\exp} = (94.2 \pm 4.5) ^\circ \quad \alpha_{\text{UTfit}} = (90.9 \pm 2.5) ^\circ\]
tensions? not really.. still that $V_{ub}$ inclusive

$V_{ub ~(excl)} = (3.62 \pm 0.14) \times 10^{-3}$

$V_{ub ~(incl)} = (4.41 \pm 0.22) \times 10^{-3}$

$\sin 2\beta_{exp} = 0.680 \pm 0.023$

$\sin 2\beta_{UTfit} = 0.725 \pm 0.030$

$V_{ub exp} = (3.74 \pm 0.21) \cdot 10^{-3}$

$V_{ub UTfit} = (3.66 \pm 0.11) \cdot 10^{-3}$
## Unitarity Triangle analysis in the SM:

| Observables | Measurement | Prediction | Pull (#σ) |
|-------------|-------------|------------|-----------|
| sin2β       | 0.680 ± 0.023 | 0.725 ± 0.030 | ~ 1.2     |
| γ           | 70.5 ± 5.7   | 65.4 ± 2.1  | < 1       |
| α           | 94.2 ± 4.5   | 90.9 ± 2.5  | < 1       |
| |V_{ub}| · 10^3     | 3.74 ± 0.21 | 3.66 ± 0.11 | < 1       |
| |V_{ub}| · 10^3 (incl) | 4.41 ± 0.22 | -          | ~ 2.9     |
| |V_{ub}| · 10^3 (excl) | 3.62 ± 0.14 | -          | < 1       |
| |V_{cb}| · 10^3     | 41.7 ± 1.0  | 42.6 ± 0.7 | < 1       |
| β_s         | 0.97 ± 0.94  | 1.05 ± 0.04 | < 1       |
| BR(B → τν)[10^-4] | 1.06 ± 0.20 | 0.81 ± 0.06 | ~ 1.2     |
| A_{SL}^d · 10^3  | 0.2 ± 2.0  | -0.283 ± 0.024 | < 1       |
| A_{SL}^s · 10^3  | 1.7 ± 3.0  | 0.013 ± 0.001 | < 1       |

LHCb arXiv:1605.09768

obtained excluding the given constraint from the fit
Unitarity Triangle analysis in the SM:

| Observables | Measurement     | Prediction      | Pull (#σ) |
|-------------|----------------|----------------|-----------|
| $B_K$       | $0.740 \pm 0.029$ | $0.81 \pm 0.07$ | < 1       |
| $f_{Bs}$    | $0.226 \pm 0.005$ | $0.220 \pm 0.007$ | < 1       |
| $f_{Bs}/f_{Bd}$ | $1.203 \pm 0.013$ | $1.210 \pm 0.030$ | < 1       |
| $B_{Bs}/B_{Bd}$ | $1.032 \pm 0.036$ | $1.07 \pm 0.05$ | < 1       |
| $B_{Bs}$    | $1.35 \pm 0.08$  | $1.30 \pm 0.07$  | < 1       |

in general: average the Nf=2+1+1 and Nf=2+1 FLAG averages, through eq.(28) in arXiv:1403.4504

for $B_K$, $f_{Bs}$, $f_{Bs}/f_{Bd}$:
FLAG Nf=2+1+1 (single result) and Nf=2+1 average

for $B_{Bs}$, $B_{bs}/B_{bd}$:
update w.r.t. the Nf=2+1 FLAG average (no Nf=2+1+1 results yet)
updating the FNAL/MILC result to FNAL/MILC 2016 (1602.03560)
some old plots coming back to fashion:

As NA62 and KOTO are approaching data taking:

\[ \text{BR}(K^+ \rightarrow \pi^+\nu\bar{\nu}) \]

- **E949 central value**
  - Projection: 100 events

- **2007 global fit area**
- **7 events**

- **SM central value**
  - Projection: 100 events

Including:

\[ \text{BR}(K^0 \rightarrow \pi^0\nu\bar{\nu}) \]

**SM central value**
UT analysis including new physics

fit simultaneously for the CKM and the NP parameters (generalized UT fit)
- add most general loop NP to all sectors
- use all available experimental info
- find out NP contributions to $\Delta F=2$ transitions

$B_d$ and $B_s$ mixing amplitudes
(2+2 real parameters):

$$A_q = C_{B_q} e^{2i \phi_{B_q}} A_q^{SM} e^{2i \phi_{q}^{SM}} = \left( 1 + \frac{A_q^{NP}}{A_q^{SM}} e^{2i(\phi_{q}^{NP} - \phi_{q}^{SM})} \right) A_q^{SM} e^{2i \phi_{q}^{SM}}$$

$$\Delta m_{q/K} = C_{B_q/\Delta m_K} (\Delta m_{q/K})^{SM}$$

$$A_{CP}^{B_d \rightarrow J/\psi K_s} = \sin 2(\beta + \phi_{B_d})$$

$$A_{SL}^{q} = \text{Im} \left( \Gamma_{12}^{q} / A_q \right)$$

$$\Delta m_{q/K} = C_{B_q/\Delta m_K} (\Delta m_{q/K})^{SM}$$

$$\varepsilon_K = C_{\varepsilon} \varepsilon^{SM}$$

$$A_{CP}^{B_s \rightarrow J/\psi \phi} \sim \sin 2(-\beta_s + \phi_{B_s})$$

$$\Delta \Gamma_{12}^{q} / \Delta m_q = \text{Re} \left( \Gamma_{12}^{q} / A_q \right)$$

$$\Delta \Gamma_{12}^{q} / \Delta m_q = \text{Re} \left( \Gamma_{12}^{q} / A_q \right)$$
new-physics-specific constraints

\[ A_{SL}^s = \frac{\Gamma( \bar{B}_s \to \ell^+ X) - \Gamma( B_s \to \ell^- X)}{\Gamma( \bar{B}_s \to \ell^+ X) + \Gamma( B_s \to \ell^- X)} = \text{Im} \left( \frac{\Gamma_{12}^s}{A_{full}^s} \right) \]

**semileptonic asymmetries in B^0 and B_s:** sensitive to NP effects in both size and phase. Currently using a 2D average done by LHCb in 1605.09768 (pre-ICHEP16 value).

**same-side dilepton charge asymmetry:** admixture of B_s and B_d so sensitive to NP effects in both.

\[ A_{\mu\mu} \times 10^3 = -7.9 \pm 2.0 \]

**lifetime τ^FS in flavour-specific final states:**
average lifetime is a function to the width and the width difference

\[ \tau^FS(B_s) = 1.511 \pm 0.014 \text{ ps} \]

φ_s=2β_s vs ΔΓ_s from B_s→J/ψφ
angular analysis as a function of proper time and b-tagging

BaBar, Belle, D0 + LHCb

D0 arXiv:1106.6308

HFAG

68% CL contours

Δ log L = 1.15
NP analysis results

NP fit

SM is

\[ \bar{\rho} = 0.154 \pm 0.015 \]
\[ \bar{\eta} = 0.344 \pm 0.013 \]
NP parameter results

dark: 68%
ligh: 95%
SM: red cross

\[ A_q = C_{Bq} e^{2i\phi_{Bq}} A_{q}^{SM} e^{2i\phi_{q}^{SM}} \]

- \[ C_{Bd} = 1.04 \pm 0.12 \]
  \[ \phi_{Bd} = (-1.8 \pm 1.7)^{\circ} \]

- \[ C_{Bs} = 1.07 \pm 0.09 \]
  \[ \phi_{Bs} = (0.1 \pm 1.0)^{\circ} \]

K system:
\[ C_{\epsilon K} = 1.05 \pm 0.11 \]
The ratio of NP/SM amplitudes is:

\[ A_q = \left(1 + \frac{A_q^{NP}}{A_q^{SM}} e^{2i(\phi_q^{NP} - \phi_q^{SM})}\right) A_q^{SM} e^{2i\phi_q^{SM}}. \]

The ratio of NP/SM amplitudes is:

- < 15% @68% prob. (30% @95%) in \( B_d \) mixing
- < 15% @68% prob. (25% @95%) in \( B_s \) mixing

see also Lunghi & Soni, Buras et al., Ligeti et al.
At the high scale
new physics enters according to its specific features

At the low scale
use OPE to write the most general effective Hamiltonian.
the operators have different chiralities than the SM
NP effects are in the Wilson Coefficients C

NP effects are enhanced
◉ up to a factor 10 by the values of the matrix elements especially for transitions among quarks of different chiralities
◉ up to a factor 8 by RGE

\[
H_{\text{eff}}^{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i^{bq} + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i^{bq}
\]

\[
Q_1^{q_i q_j} = \bar{q}_j^\alpha \gamma_\mu q_i^\alpha \bar{q}_j^\beta \gamma_\mu q_i^\beta,
Q_2^{q_i q_j} = \bar{q}_j^\alpha q_i^\alpha \bar{q}_j^\beta q_i^\beta,
Q_3^{q_i q_j} = \bar{q}_j^\alpha q_i^\beta \bar{q}_j^\beta q_i^\alpha,
Q_4^{q_i q_j} = \bar{q}_j^\alpha q_i^\alpha \bar{q}_j^\beta q_i^\beta,
Q_5^{q_i q_j} = \bar{q}_j^\alpha q_i^\beta \bar{q}_j^\beta q_i^\alpha.
\]
effective BSM Hamiltonian for $\Delta F=2$ transitions

The Wilson coefficients $C_i$ have in general the form

$$C_i(\Lambda) = \frac{F_i L_i}{\Lambda^2}$$

Putting bounds on the Wilson coefficients give insights into the NP scale in different NP scenarios that enter through $F_i$ and $L_i$.

- $F_i$: function of the NP flavour couplings
- $L_i$: loop factor (in NP models with no tree-level FCNC)
- $\Lambda$: NP scale (typical mass of new particles mediating $\Delta F=2$ transitions)
The dependence of $C$ on $\Lambda$ changes depending on the flavour structure. We can consider different flavour scenarios:

- **Generic**: $C(\Lambda) = \alpha/\Lambda^2$; $F_i \sim 1$, arbitrary phase
- **NMFV**: $C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2$; $F_i \sim |F_{SM}|$, arbitrary phase
- **MFV**: $C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2$; $F_1 \sim |F_{SM}|$, $F_{i \neq 1} \sim 0$, SM phase

$\alpha (L_i)$ is the coupling among NP and SM

- $\alpha \sim 1$ for strongly coupled NP
- $\alpha \sim \alpha_w (\alpha_s)$ in case of loop coupling through weak (strong) interactions

If no NP effect is seen lower bound on NP scale $\Lambda$
if NP is seen upper bound on NP scale $\Lambda$
results from the Wilson coefficients

**Generic:** \( C(\Lambda) = \alpha/\Lambda^2 \), \( F_i \sim 1 \), arbitrary phase

\[ \alpha \sim 1 \] for strongly coupled NP

To obtain the lower bound for loop-mediated contributions, one simply multiplies the bounds by \( \alpha_s (\sim 0.1) \) or by \( \alpha_w (\sim 0.03) \).

\[ \alpha \sim \alpha_w \] in case of loop coupling through **weak** interactions

NP in \( \alpha_w \) loops

\[ \Lambda > 1.5 \times 10^4 \text{ TeV} \]

Best bound from \( \varepsilon_K \)
dominated by CKM error

**CPV** in charm mixing follows,
exp error dominant

Best CP conserving from \( \Delta m_K \),
dominated by long distance

\( B_d \) and \( B_s \) behind,
errors from both CKM and B-parameters

Lower bounds on NP scale
(in TeV at 95% prob.)

Non-perturbative NP

\[ \Lambda > 5.0 \times 10^5 \text{ TeV} \]
results from the Wilson coefficients

**NMFV:** $C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2$, $F_i \sim |F_{SM}|$, arbitrary phase

$\alpha \sim 1$ for strongly coupled NP

To obtain the lower bound for loop-mediated contributions, one simply multiplies the bounds by $\alpha_s (\sim 0.1)$ or by $\alpha_W (\sim 0.03)$.

$\alpha \sim \alpha_W$ in case of loop coupling through weak interactions

NP in $\alpha_W$ loops

$\Lambda > 3.4$ TeV

If new chiral structures present, $\varepsilon_K$ still leading

$B_{(s)}$ mixing provides very stringent constraints, especially if no new chiral structures are present

Constraining power of the various sectors depends on unknown NP flavour structure.
Look at the near future

**future I scenario:** errors from
Belle II at 5/ab
+ LHCb at 10/fb

$r = 0.150 \pm 0.027$
$h = 0.363 \pm 0.025$

$ar{r} = 0.154 \pm 0.015$
$\bar{h} = 0.344 \pm 0.013$

**current sensitivity**

$ho = \pm 0.015$
$\eta = \pm 0.015$

$r = 0.154 \pm 0.015$
$h = 0.344 \pm 0.013$

$ar{r} = 0.150 \pm 0.027$
$\bar{h} = 0.363 \pm 0.025$

$r = 0.150 \pm 0.027$
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$\bar{h} = 0.344 \pm 0.013$

$r = 0.154 \pm 0.015$
$h = 0.344 \pm 0.013$
conclusions

- SM analysis displays very good overall consistency
- Still open discussion on semileptonic inclusive vs exclusive
- UTA provides determination of NP contributions to $\Delta F=2$ amplitudes. It currently leaves space for NP at the level of 25-30%
- So the scale analysis points to high scales for the generic scenario and at the limit of LHC reach for weak coupling. Indirect searches are complementary to direct searches.
- Even if we don't see relevant deviations in the down sector, we might still find them in the up sector.
Back up slides
**$V_{cb}$ and $V_{ub}$**

Average of the two FLAG Nf=2+1 averages

$V_{cb} \ (excl) = (40.1 \pm 1.2) \times 10^{-3}$

$V_{cb} \ (incl) = (42.00 \pm 0.64) \times 10^{-3}$

Gambino 1606.06174

~3.0σ discrepancy

$V_{ub} \ (excl) = (3.69 \pm 0.14) \times 10^{-3}$

$V_{ub} \ (incl) = (4.40 \pm 0.22) \times 10^{-3}$

~2.7σ discrepancy

$V_{ub} / V_{cb} \ (LHCb) = (8.3 \pm 0.6) \times 10^{-2}$

Average of the two FLAG Nf=2+1 averages:
- from B->D  => 40.85(98) $10^{-3}$
- from B->D* => 39.27(56)(49) $10^{-3}$
sin$2\alpha$ ($\phi_2$) from charmless B decays: pp, ($\rho\rho$, $\pi\rho$)

$\pi\pi$ only

$\pi^0\pi^0$ from Belle at CKM14

to be updated soon (?)

$\text{BR}(\pi^0\pi^0) = (1.17 \pm 0.13) \times 10^{-6}$

compared with a $\textit{a la PDG}$ average

giving an inflated uncertainty of 0.41

$\alpha$ from $\pi\pi$, $\rho\rho$, $\pi\rho$ decays:
combined: $(94.2 \pm 4.5)^\circ$