Retraction

Retraction: Mixture Weibull probabilistic model in Wind Turbine Power Analysis (J. Phys.: Conf. Ser. 1916 012109)

Published 23 February 2022

This article (and all articles in the proceedings volume relating to the same conference) has been retracted by IOP Publishing following an extensive investigation in line with the COPE guidelines. This investigation has uncovered evidence of systematic manipulation of the publication process and considerable citation manipulation.

IOP Publishing respectfully requests that readers consider all work within this volume potentially unreliable, as the volume has not been through a credible peer review process.

IOP Publishing regrets that our usual quality checks did not identify these issues before publication, and have since put additional measures in place to try to prevent these issues from reoccurring. IOP Publishing wishes to credit anonymous whistleblowers and the Problematic Paper Screener [1] for bringing some of the above issues to our attention, prompting us to investigate further.

[1] Cabanac G, Labbé C and Magazinov A 2021 arXiv:2107.06751v1

Retraction published: 23 February 2022
Mixture Weibull probabilistic model in Wind Turbine Power Analysis

Indhumathy D1, Narmatha D1, Meenambika K2
1Assistant Professor, Department of Mathematics, Sri Ramakrishna Engineering College, Coimbatore, India.
2Assistant Professor, Department of Mathematics, Sri Shanmugha College of Engineering & Technology, Sankari, India
indhu0313@gmail.com, narmathakrishna@gmail.com, meenabalaji08@gmail.com

Abstract. Wind energy is renewable and environment friendly. It is pollution free source of energy when compared with fossil fuels that pollute the lower layer of atmosphere. A new method depending on the parameters of three parameter mixture Weibull distribution is presented in this paper for calculating the output energy. Parameter estimation is much critical for the application of statistical model and is a challenging problem precisely for a Weibull distribution with more than two parameters. A study on estimation of parameters using maximum likelihood method is discussed. An expression to estimate the Capacity factor is formulated and the average power produced from a turbine is computed using the three parameter mixture Weibull distribution.

Keywords: Three parameter mixture Weibull distribution, Maximum likelihood method, Capacity Factor, Wind Power

1. Introduction
An accurate determination of probability distribution in wind speed analysis plays a vital role in evaluating wind speed energy potential in a particular region. Waloddi Weibull is one of the scientist who promoted the distribution both internationally and interdisciplinary[2]. His discovery lead the Weibull distribution to be the most productive in engineering practice, statistical modeling and probability theory. Probability density function of wind speed is not always statistically accepted as two parameter Weibull probability density function. Since it has some drawbacks. So a new improved model with a combination of two component is used (mixture of three parameter). Carta has discussed about the mixture bimodal Weibull distribution in their study about wind speed Analysis [3-6]. The location parameter included in the proposed model is closely associated with the null wind speed. So the bimodal Weibull distribution which is nothing but a two component mixture of Two parameter Weibull distribution is now extended to mixture of three parameter Weibull distribution with two components $MWb(3,3)$ [14-16].

In section two we have the discussion about three parameter and mixture three parameter Weibull distribution. In the third section the estimation of seven parameters of the mixture three parameter Weibull distribution using the maximum likelihood technique is discussed. In the next section we have a study on the average wind power that can be generated from a turbine and the capacity factor. A new mathematical model for computing wind power and capacity factor is the main objective of the study.
2. Weibull Distribution

2.1 Three parameter mixture Weibull distribution

The general form of the three parameter Weibull probability density function is defined by Equations (1-26)

\[ f(v; \alpha, \beta, \gamma) = \frac{\alpha}{\beta} \left(\frac{v - \gamma}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{v - \gamma}{\beta}\right)\right] \quad \text{for } v > 0, \]  

(1)

Where \( v \) is the wind speed data, \( \beta \) is the shape parameter, \( \alpha \) is the shape parameter that indicates the spread of the distribution of the observed data and \( \gamma \) is the location parameter [1].

The cumulative distribution function is mathematically given as

\[ F(v) = 1 - \exp\left[-\left(\frac{v - \gamma}{\beta}\right)\right] \]  

(2)

If \( \gamma = 0 \) the distribution reduces to two parameter Weibull distribution. If \( \gamma = 1 \), the distribution is exponential and if \( \beta = 2 \) it reduces to Rayleigh distribution [8].

2.2 Three parameter mixture Weibull distribution

A mixture probability density function is a two component function with a combination of two or more than two Weibull distribution [7]. The mixture three parameter Weibull distribution is a linear combination of more than one three parameter Weibull distribution\( MWbl(3,3) \).

\[ f_{f(3,3)}(v; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega, \gamma_1, \gamma_2) = \omega f(v; \alpha_1, \beta_1, \gamma_1) + (1 - \omega) f(v; \alpha_2, \beta_2, \gamma_2) \]  

(3)

\[ = \omega \left(\frac{\alpha_1}{\beta_1} \left(\frac{v - \gamma_1}{\beta_1}\right)^{\alpha_1-1} \exp\left[-\left(\frac{v - \gamma_1}{\beta_1}\right)\right] + (1 - \omega) \left(\frac{\alpha_2}{\beta_2} \left(\frac{v - \gamma_2}{\beta_2}\right)^{\alpha_2-1} \exp\left[-\left(\frac{v - \gamma_2}{\beta_2}\right)\right] \right) \right), \quad v \geq 0 \]

where \( v > 0 \), \( \alpha_1, \alpha_2 \) are shape parameters, \( \beta_1, \beta_2 \) are scale parameters and \( \gamma_1, \gamma_2 \) are locations parameters.

The cumulative distribution function of \( MWbl(3,3) \) is defined by

\[ F_{F(3,3)}(v; \alpha_1, \beta_1, \alpha_2, \beta_2, \gamma_1, \gamma_2, \omega) = P(V \leq v) = \omega F(v; \alpha_1, \beta_1, \gamma_1) + (1 - \omega) F(v; \alpha_2, \beta_2, \gamma_2) \]  

(4)

\[ = \omega \left(1 - \exp\left[-\left(\frac{v - \gamma_1}{\beta_1}\right)^{\alpha_1} + (1 - \omega) \left(1 - \exp\left[-\left(\frac{v - \gamma_2}{\beta_2}\right)^{\alpha_2}\right]\right)\right]\right) \]

If \( \gamma_1 = 0 \) the distribution reduces to improved mixture Weibull distribution and if \( \gamma_1 = 0, \gamma_2 = 0 \) the distribution reduces to Bimodal Weibull distribution.

The first order moment of the \( MWbl(3,3) \) is given by

\[ Mean = \omega \beta_1 \Gamma\left(1 + \frac{1}{\alpha_1}\right) + (1 - \omega) \beta_2 \Gamma\left(1 + \frac{1}{\alpha_2}\right) + \omega \gamma_1 + (1 - \omega) \gamma_2 \]  

(5)

3. Estimation of Parameters

3.1 Maximum Likelihood Method

Fisher popularized the MLE techniques in 1920s. Since 1920 this technique was used by several
authors in literature to estimate the parameters. The method of Maximum likelihood method is a
commonly followed procedure because of its much desirable property [10].

3.2 Definition of MLE
Let \( v_1, v_2, \ldots, v_n \) be a sample of size \( n \) wind speed observations drawn from a pdf\( f(x; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega, \gamma_1, \gamma_2) \), the parameters except \( v \) are unknown.

\[ L = \prod_{i=1}^{n} f(v_i) \]
where \( f(v_i) \) is the probability density function associated with the observation of the wind speed data \( v_i \).

3.3 Computation of parameters
The ML estimator say \( \hat{\alpha}_1, \hat{\alpha}_2, \hat{\beta}_1, \hat{\beta}_2, \hat{\gamma}_1, \hat{\gamma}_2 \) are the values of the unknown parameter that maximize \( L \) or equivalently the logarithm of \( L \). Often MLE of the unknown parameter has the solution as the derivative of the logarithmic of \( L \) with respect to the parameters is equated to zero.

The log likelihood function of the improved mixture Weibull distribution is given by

\[ L(\alpha_1, \beta_1, \alpha_2, \beta_2, \gamma_1, \gamma_2, \omega) = \prod_{i=1}^{n} f(v_i; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega, \gamma_1, \gamma_2) \]

\[ = \prod_{i=1}^{n} \omega \left( \frac{\beta_1}{\beta_2} \right)^{\alpha_1} \exp \left( - \left( \frac{v_i - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) + (1 - \omega) \left( \frac{\beta_1}{\beta_2} \right)^{\alpha_2} \exp \left( - \left( \frac{v_i - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) \]

Taking the natural log on both the sides, we obtain that the log-likelihood function is

\[ \log L(v_i; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega, \gamma_1, \gamma_2) = \]

\[ n \log \alpha_1 - n \log \beta_1 - \frac{1}{\beta_1^{\alpha_1}} \sum_{i=1}^{n} (v_i - \gamma_1)^{\alpha_1} + \sum_{i=1}^{n} (\alpha_1 - 1) \log(v_i - \gamma_1) \]

\[ + n \log \alpha_2 - n \log \beta_2 - \frac{1}{\beta_2^{\alpha_2}} \sum_{i=1}^{n} (v_i - \gamma_2)^{\alpha_2} + \sum_{i=1}^{n} (\alpha_2 - 1) \log(v_i - \gamma_2) \]

Consider the partial derivatives w.r.t \( \alpha_1, \beta_1, \alpha_2, \beta_2, \gamma_1, \gamma_2 \) we get

\[ \frac{\partial}{\partial \beta_1} \left[ \log L(v_i; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega, \gamma_1, \gamma_2) \right] = -n \alpha_1 + \frac{\alpha_1}{\beta_1^{\alpha_1+1}} \sum_{i=1}^{n} (v_i - \gamma_1)^{\alpha_1} \]

\[ \frac{\partial}{\partial \beta_2} \left[ \log L(v_i; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega, \gamma_1, \gamma_2) \right] = -n \alpha_2 + \frac{\alpha_2}{\beta_2^{\alpha_2+1}} \sum_{i=1}^{n} (v_i - \gamma_2)^{\alpha_2} \]

\[ \frac{\partial}{\partial \alpha_1} \left[ \log L(v_i; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega, \gamma_1, \gamma_2) \right] = -n \frac{\alpha_1}{\beta_1^{\alpha_1+1}} + \frac{\log \beta_1}{\beta_1^{\alpha_1}} \sum_{i=1}^{n} (v_i - \gamma_1)^{\alpha_1} \]

\[ \frac{\partial}{\partial \alpha_2} \left[ \log L(v_i; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega, \gamma_1, \gamma_2) \right] = -n \frac{\alpha_2}{\beta_2^{\alpha_2+1}} + \frac{\log \beta_2}{\beta_2^{\alpha_2}} \sum_{i=1}^{n} (v_i - \gamma_2)^{\alpha_2} \]
Wind Power distribution

4.1 Maximum Likelihood Method

Wind is the moving mass of air which is such as the kinetic energy. A part of the kinetic energy is exploited to drive a wind turbine. The energy available in the wind is given by [7].

\[ P = \frac{1}{2} \rho Av^3 \]

where \( \rho \) is the air density, 1.225 kg/m^3, \( A \) is the cross section of the area in m^2, \( v \) is the velocity in m/s.

It is definitely not possible to extract all the energy available in the wind because of its movement away from the disc of the turbine and should be replaced by the incoming mass. Therefore the above stated equation can be replaced as
\[ P_e = \frac{1}{2} C_p \rho A v^3 \] 

where \( P_e \) is the extractable energy, \( C_p \) is the coefficient of power = 0.59, \( \eta \) is the Betz limit.

4.2 Power probability distribution

The cumulative distribution function of power is defined by [11]

\[ FF_{p(3,3)}(v) = \omega \left\{ 1 - \exp \left[ -\frac{1}{\beta_1} \left( \frac{2(v - \gamma_1)}{\rho A} \right)^{\frac{\alpha_1}{3}} \right] \right\} + (1 - \omega) \left\{ 1 - \exp \left[ -\frac{1}{\beta_2} \left( \frac{2(v - \gamma_2)}{\rho A} \right)^{\frac{\alpha_2}{3}} \right] \right\} \] 

And the probability density function corresponding to the cdf is

\[ f_{f(3,3)}(v) = \omega \left\{ \exp \left[ -\frac{1}{\beta_1} \left( \frac{2(v - \gamma_1)}{\rho A} \right)^{\frac{\alpha_1}{3}} \right] \right\} \frac{\alpha_1}{3} \left( \frac{2}{\rho A} \right)^{\frac{\alpha_1}{3}} \] 

\[ + (1 - \omega) \left\{ 1 - \exp \left[ -\frac{1}{\beta_2} \left( \frac{2(v - \gamma_2)}{\rho A} \right)^{\frac{\alpha_2}{3}} \right] \right\} \frac{1}{\beta_2^{\frac{\alpha_2}{3}}} \alpha_2 \left( \frac{2}{\rho A} \right)^{\frac{\alpha_2}{3}} \] 

The CDF of power is defined by

\[ FF_{p(2,3)}(v) = \omega \left\{ 1 - \exp \left[ -\frac{1}{\beta_1} \left( \frac{2v}{\rho A} \right)^{\frac{\alpha_1}{3}} \right] \right\} + (1 - \omega) \left\{ 1 - \exp \left[ -\frac{1}{\beta_2} \left( \frac{2(v - \gamma_0)}{\rho A} \right)^{\frac{\alpha_2}{3}} \right] \right\} \] 

And PDF is

\[ f_{f(2,3)}(v) = \omega \left\{ \exp \left[ -\frac{1}{\beta_1} \left( \frac{2v}{\rho A} \right)^{\frac{\alpha_1}{3}} \right] \right\} \frac{\alpha_1}{3} \left( \frac{2}{\rho A} \right)^{\frac{\alpha_1}{3}} \] 

\[ + (1 - \omega) \left\{ 1 - \exp \left[ -\frac{1}{\beta_2} \left( \frac{2(v - \gamma_0)}{\rho A} \right)^{\frac{\alpha_2}{3}} \right] \right\} \frac{1}{\beta_2^{\frac{\alpha_2}{3}}} \alpha_2 \left( \frac{2}{\rho A} \right)^{\frac{\alpha_2}{3}} \] 

Proof:

\[ FF_{p(2,3)}(v) = P(P \leq v) = P \left\{ V \leq \left( \frac{2v}{\rho A} \right)^{\frac{1}{3}} \right\} \]

\[ = \omega \left\{ 1 - \exp \left[ -\frac{1}{\beta_1} \left( \frac{2v}{\rho A} \right)^{\frac{\alpha_1}{3}} \right] \right\} + (1 - \omega) \left\{ 1 - \exp \left[ -\frac{1}{\beta_2} \left( \frac{2(v - \gamma_0)}{\rho A} \right)^{\frac{\alpha_2}{3}} \right] \right\} \] 

Diff w.r.t \( v \),

\[ f_{f(2,3)}(v) = \omega \left\{ \exp \left[ -\frac{1}{\beta_1} \left( \frac{2v}{\rho A} \right)^{\frac{\alpha_1}{3}} \right] \right\} \frac{\alpha_1}{3} \left( \frac{2}{\rho A} \right)^{\frac{\alpha_1}{3}} \] 

\[ + \frac{1}{\beta_2^{\frac{\alpha_2}{3}}} \alpha_2 \left( \frac{2}{\rho A} \right)^{\frac{\alpha_2}{3}} + (1 - \omega) \left\{ 1 - \exp \left[ -\frac{1}{\beta_2} \left( \frac{2(v - \gamma_0)}{\rho A} \right)^{\frac{\alpha_2}{3}} \right] \right\} \frac{1}{\beta_2^{\frac{\alpha_2}{3}}} \alpha_2 \left( \frac{2}{\rho A} \right)^{\frac{\alpha_2}{3}} - \left( \frac{1}{\beta_2^{\frac{\alpha_2}{3}}} \alpha_2 \left( \frac{2}{\rho A} \right)^{\frac{\alpha_2}{3}} \right) \]
is the wind speed at which the turbine starts operating and

\[ \alpha (v - \gamma_0) \frac{2}{\beta_2} \left( \frac{2(v - \gamma_0)}{\rho A} \right)^{\gamma - 1} \]

\[ f_{\mu,v}(v) = \omega \left[ 1 - \exp \left[ - \frac{1}{\beta_2} \left( \frac{2(v - \gamma_0)}{\rho A} \right) \right] \right] \frac{1}{\beta_2} \frac{\alpha_2}{3} \left( \frac{2(v - \gamma_0)}{\rho A} \right)^{\gamma - 1} + (1 - \omega) \left[ 1 - \exp \left[ - \frac{1}{\beta_2} \left( \frac{2(v - \gamma_0)}{\rho A} \right) \right] \right] \frac{1}{\beta_2} \frac{\alpha_2}{3} \left( \frac{2(v - \gamma_0)}{\rho A} \right)^{\gamma - 1} \]

(14)

### 4.3 Power Distribution

The wind speed experienced by the turbine is given by

\[ V_{\text{turbine}} = \begin{cases} 0 & v_{\text{cut-in}} < V \\ v_{\text{cut-in}} \leq V \leq v_{\text{rated}} & v_{\text{rated}} < V < v_{\text{cut-off}} \\ 0 & V \geq v_{\text{cut-off}} \end{cases} \]

(15)

Where \( v_{\text{cut-in}} \) is the wind speed at which the turbine starts operating and \( v_{\text{cut-off}} \) is the highest wind speed at which the turbine shuts down [12].

\[ P_{\text{turbine}} = \begin{cases} \frac{1}{2} \rho C_p \eta A V^3 & v_{\text{turbine}} < v_{\text{cut-in}} \\ \frac{1}{2} \rho C_p \eta A v_{\text{rated}}^3 & v_{\text{cut-in}} \leq V \leq v_{\text{rated}} \\ 0 & v_{\text{rated}} < V < v_{\text{cut-off}} \\ V \geq v_{\text{cut-off}} \end{cases} \]

(16)

### 4.4 Proposition

\( P_{\text{turbine}} \) has CDF given by

\[ F_{F_{\nu_{\text{off}},\beta,3}}(v) = \begin{cases} 0 & -\infty < v < 0 \\ 1 & 0 \leq v \leq \frac{1}{2} \rho C_p \eta A v_{\text{cut-off}} \\ \frac{1}{2} \rho C_p \eta A v_{\text{rated}} & v_{\text{cut-off}} < v < \frac{1}{2} \rho C_p \eta A v_{\text{cut-off}} \\ 1 & \frac{1}{2} \rho C_p \eta A v_{\text{rated}} \leq v < \infty \end{cases} \]

(17)

Proof: \( P(P_{\text{turbine}} \leq 0) \) is

\[ P\left[ V \leq v_{\text{cut-in}} \right] \cup \left[ V \leq v_{\text{cut-off}} \right] = F_{\nu_{\text{off}},\beta,3}(v_{\text{cut-in}}) + \left(1 - F_{\nu_{\text{off}},\beta,3}(v_{\text{cut-off}})\right) \]

\[ = \omega \left[ 1 - \exp \left( -\frac{(v_{\text{cut-in}} - \gamma_0)^{\alpha_1}}{\beta_2} \right) \right] + (1 - \omega) \left[ 1 - \exp \left( -\frac{(v_{\text{cut-off}} - \gamma_2)^{\alpha_2}}{\beta_2} \right) \right] \]

\[ + \omega \left[ 1 - \exp \left( -\frac{(v_{\text{cut-off}} - \gamma_0)^{\alpha_1}}{\beta_2} \right) \right] + (1 - \omega) \left[ 1 - \exp \left( -\frac{(v_{\text{cut-off}} - \gamma_2)^{\alpha_2}}{\beta_2} \right) \right] \]

\[ = 1 - \left( 1 - \omega \right) e^{-\frac{(v_{\text{cut-in}} - \gamma_0)^{\alpha_1}}{\beta_2}} - e^{-\frac{(v_{\text{cut-off}} - \gamma_0)^{\alpha_1}}{\beta_2}} - \omega e^{-\frac{(v_{\text{cut-off}} - \gamma_2)^{\alpha_2}}{\beta_2}} - e^{-\frac{(v_{\text{cut-off}} - \gamma_2)^{\alpha_2}}{\beta_2}} \]

(18)
For \( 0 < v = \frac{1}{2} \rho C_p \eta \Delta v^3 \leq \frac{1}{2} \rho C_p \eta \Delta v_\text{r}^3 \), note that \( v \leq \frac{1}{2} \rho C_p \eta \Delta v_\text{r}^3 \Leftrightarrow v \leq v_\gamma \), and \( v - v_\gamma \leq v_r \),

\[
\nu - v_\gamma \leq v_r
\]

For \( \rightarrow FF_{\text{turbine}}(3,3) \) \( (v) = P(0 < P_{\text{turbine}} \leq v) + P(P_{\text{turbine}} = 0) \)

\[
= P \left\{ \frac{1}{2} \rho C_p \eta \Delta v_{\text{cut-in}}^3 \leq P_{\text{turbine}} \leq v \right\}
+ 1 - (1 - \omega) \left\{ e^{-\frac{\left( v_{\text{cut-in}} - v_\gamma \right)^2}{\beta_1^2}} - e^{-\frac{\left( v_{\text{cut-in}} - v_\gamma \right)^2}{\beta_2^2}} - \omega \left\{ e^{-\frac{\left( \nu_{\text{cutoff}} - v_\gamma \right)^2}{\beta_1^2}} - e^{-\frac{\left( \nu_{\text{cutoff}} - v_\gamma \right)^2}{\beta_2^2}} \right\} \right\}
\]

\[
v = \frac{1}{2} C_p \eta \Delta v_{\text{cut-in}}^3 \Rightarrow v_{\text{cut-in}} = \frac{2v}{C_p \eta \Delta A} \Rightarrow v_{\text{cut-in}} = \frac{2v}{C_p \eta \Delta A}
\]

\[
= 1 - (1 - \omega) \left\{ e^{-\frac{1}{\beta_1^2} \left( 2(v - v_\gamma) \right)^2} - \omega \left\{ e^{-\frac{1}{\beta_1^2} \left( 2(v - v_\gamma) \right)^2} \right\} \right\} + \omega \left\{ e^{-\frac{1}{\beta_1^2} \left( 2(v - v_\gamma) \right)^2} \right\} + (1 - \omega) e^{-\frac{\left( v_{\text{cut-in}} - v_\gamma \right)^2}{\beta_2^2}}
\]  \hspace{1cm} (19)

\[
\frac{1}{2} \rho C_p \eta \Delta v_{\text{rated}}^3 \leq v < \infty \Rightarrow FF_{\text{turbine}}(3,3) \) \( (v) = 1 \). \hspace{1cm} (20)

Equation (17) is combination of (18),(19) and (20).

4.5 Proposition

\[
E[P_{\text{turbine}}]\left[ \frac{1}{2} \rho C_p \eta \Delta A \right] = \left\{ \frac{1}{2} \rho C_p \eta \Delta A \right\} \left( 1 + \frac{3}{\alpha_1} \left( \frac{v_{\text{rated}} - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) \left( 1 + \frac{3}{\alpha_1} \left( \frac{v_{\text{cut-in}} - \gamma_1}{\beta_1} \right)^{\alpha_1} \right)
\]

\[
+ \frac{1}{2} \rho C_p \eta \Delta A \left\{ 3 \beta_1^2 \gamma_1 \left( 1 + \frac{2}{\alpha_1} \left( \frac{v_{\text{rated}} - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) - \left( 1 + \frac{2}{\alpha_1} \left( \frac{v_{\text{cut-in}} - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) \right\}
\]

\[
+ \frac{1}{2} \rho C_p \eta \Delta A \left\{ 3 \beta_1^2 \gamma_1 \left( 1 + \frac{1}{\alpha_1} \left( \frac{v_{\text{cut-in}} - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) - \left( 1 + \frac{1}{\alpha_1} \left( \frac{v_{\text{cut-in}} - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) \right\}
\]

\[
+ \frac{1}{2} \rho C_p \eta \Delta A \left\{ 3 \beta_1^2 \gamma_1 \left( 1 + \frac{3}{\alpha_2} \left( \frac{v_{\text{rated}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) - \left( 1 + \frac{3}{\alpha_2} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) \right\}
\]

\[
+ \frac{1}{2} \rho C_p \eta \Delta A \left\{ 3 \beta_1^2 \gamma_1 \left( 1 + \frac{2}{\alpha_2} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) - \left( 1 + \frac{2}{\alpha_2} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) \right\}
\]

\[
+ \frac{1}{2} \rho C_p \eta \Delta A \left\{ 3 \beta_1^2 \gamma_1 \left( 1 + \frac{1}{\alpha_2} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) - \left( 1 + \frac{1}{\alpha_2} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) \right\}
\]

\[
+ \frac{1}{2} \rho C_p \eta \Delta A \left\{ 3 \beta_1^2 \gamma_1 \left( 1 + \frac{3}{\alpha_1} \left( \frac{v_{\text{rated}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) - \left( 1 + \frac{3}{\alpha_1} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) \right\}
\]

\[
+ \frac{1}{2} \rho C_p \eta \Delta A \left\{ 3 \beta_1^2 \gamma_1 \left( 1 + \frac{2}{\alpha_1} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) - \left( 1 + \frac{2}{\alpha_1} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) \right\}
\]

\[
+ \frac{1}{2} \rho C_p \eta \Delta A \left\{ 3 \beta_1^2 \gamma_1 \left( 1 + \frac{1}{\alpha_2} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) - \left( 1 + \frac{1}{\alpha_2} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) \right\}
\]

\[
+ \frac{1}{2} \rho C_p \eta \Delta A \left\{ 3 \beta_1^2 \gamma_1 \left( 1 + \frac{3}{\alpha_1} \left( \frac{v_{\text{rated}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) - \left( 1 + \frac{3}{\alpha_1} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) \right\}
\]

\[
+ \frac{1}{2} \rho C_p \eta \Delta A \left\{ 3 \beta_1^2 \gamma_1 \left( 1 + \frac{2}{\alpha_1} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) - \left( 1 + \frac{2}{\alpha_1} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) \right\}
\]

\[
+ \frac{1}{2} \rho C_p \eta \Delta A \left\{ 3 \beta_1^2 \gamma_1 \left( 1 + \frac{1}{\alpha_2} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) - \left( 1 + \frac{1}{\alpha_2} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) \right\}
\]
This implies

\[
\Gamma = \int_0^\infty e^{-\gamma} \gamma^{n-1} dy. \]  

This implies
\[ E[P_{\text{turbine}}] = \left( \frac{1}{2} C_p \rho \eta A \right) \left[ \beta_1 \left( \Gamma \left( 1 + \frac{3}{\alpha_1} \left( \frac{v_{\text{rated}} - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) - \Gamma \left( 1 + \frac{3}{\alpha_1} \left( \frac{v_{\text{cut-in}} - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) \right) \right] \]

\[ + \left( \frac{1}{2} C_p \rho \eta A \right) \left[ 3 \beta_1^2 \gamma_1 \left( \Gamma \left( 1 + \frac{2}{\alpha_1} \left( \frac{v_{\text{rated}} - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) - \Gamma \left( 1 + \frac{2}{\alpha_1} \left( \frac{v_{\text{cut-in}} - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) \right) \right] \]

\[ + \left( \frac{1}{2} C_p \rho \eta A \right) \left[ \beta_2 \left( \Gamma \left( 1 + \frac{3}{\alpha_2} \left( \frac{v_{\text{rated}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) - \Gamma \left( 1 + \frac{3}{\alpha_2} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) \right) \right] \]

\[ + \left( \frac{1}{2} C_p \rho \eta A \right) \left[ 3 \beta_2^2 \gamma_2 \left( \Gamma \left( 1 + \frac{2}{\alpha_2} \left( \frac{v_{\text{rated}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) - \Gamma \left( 1 + \frac{2}{\alpha_2} \left( \frac{v_{\text{cut-in}} - \gamma_2}{\beta_2} \right)^{\alpha_2} \right) \right) \right] \]

\[ + \left( \frac{1}{2} C_p \rho \eta A \right) \left[ 1 - \omega \right] \left[ \gamma_1^3 \left( e^{\frac{v_{\text{rated}} - \gamma_1}{\beta_1}} - e^{\frac{v_{\text{cut-in}} - \gamma_1}{\beta_1}} \right) + v_{\text{rated}}^3 \left( e^{\frac{v_{\text{rated}} - \gamma_1}{\beta_1}} - e^{\frac{v_{\text{cut-in}} - \gamma_1}{\beta_1}} \right) \right] \]

\[ + \left( \frac{1}{2} C_p \rho \eta A \right) \left[ 1 - \omega \right] \left[ \gamma_2^3 \left( e^{\frac{v_{\text{rated}} - \gamma_2}{\beta_2}} - e^{\frac{v_{\text{cut-in}} - \gamma_2}{\beta_2}} \right) + v_{\text{rated}}^3 \left( e^{\frac{v_{\text{rated}} - \gamma_2}{\beta_2}} - e^{\frac{v_{\text{cut-in}} - \gamma_2}{\beta_2}} \right) \right] \]  

(21)

### 4.6 Capacity Factor

The production of energy from wind turbines have to compete with enormous energy sources [13]. There are several methods available for calculating the output energy from a wind turbine [15]. Here we have used the mixture three parameter Weibull distribution \( MWbl(3,3) \).

The capacity factor depends on several parameters like cut-in speed, rated and cut off speed, turbine rated power \( P \), the shape parameters \( \alpha \), the scale parameters \( \beta \) and the location parameters \( \gamma \).\[ CF = \frac{E[P_{\text{turbine}}]}{\frac{1}{2} C_p \rho \eta A v_{\text{rated}}^3} \]

(22)

\[ CF = \left( \frac{\beta_1}{v_{\text{rated}}} \right)^3 \left[ \Gamma \left( 1 + \frac{3}{\alpha_1} \left( \frac{v_{\text{rated}} - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) - \Gamma \left( 1 + \frac{3}{\alpha_1} \left( \frac{v_{\text{cut-in}} - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) \right] \]
Consider a location whose wind speed data follow $\mathcal{W}bl(2,3)\left(v; \alpha_1, \beta_1, \alpha_2, \beta_2, \alpha, \gamma_1\right)$ distribution and $v_r \in \left[v_{\text{cut-in}}; v_{\text{cut-off}}\right]$ then we define the capacity factor can be defined by [11]

$$
\text{CF} = \left(\frac{\beta_3}{v}\right)^3 \left\{ \left[1 + \frac{3}{\alpha_1} \left(\frac{v - \gamma_1}{\beta_1}\right)^{\alpha_1}\right] - \left[1 + \frac{3}{\alpha_1} \left(\frac{v_{\text{cut-in}} - \gamma_1}{\beta_1}\right)^{\alpha_1}\right] \right\}
+ \left(\frac{3 \beta_1^2 \gamma_1}{v^3}\right) \left\{ \left[1 + \frac{2}{\alpha_1} \left(\frac{v - \gamma_1}{\beta_1}\right)^{\alpha_1}\right] - \left[1 + \frac{2}{\alpha_1} \left(\frac{v_{\text{cut-in}} - \gamma_1}{\beta_1}\right)^{\alpha_1}\right] \right\}
+ \left(\frac{3 \beta_1^2 \gamma_1}{v^3}\right) \left\{ \left[1 + \frac{1}{\alpha_1} \left(\frac{v - \gamma_1}{\beta_1}\right)^{\alpha_1}\right] - \left[1 + \frac{1}{\alpha_1} \left(\frac{v_{\text{cut-in}} - \gamma_1}{\beta_1}\right)^{\alpha_1}\right] \right\}
+ \left(\frac{\beta_2}{v}\right)^3 \left\{ \left[1 + \frac{3}{\alpha_2} \left(\frac{v - \gamma_2}{\beta_2}\right)^{\alpha_2}\right] - \left[1 + \frac{3}{\alpha_2} \left(\frac{v_{\text{cut-in}} - \gamma_2}{\beta_2}\right)^{\alpha_2}\right] \right\}
+ \left(\frac{\beta_2}{v}\right)^3 \left\{ \left[1 + \frac{2}{\alpha_2} \left(\frac{v - \gamma_2}{\beta_2}\right)^{\alpha_2}\right] - \left[1 + \frac{2}{\alpha_2} \left(\frac{v_{\text{cut-in}} - \gamma_2}{\beta_2}\right)^{\alpha_2}\right] \right\}
+ \left(\frac{\beta_2}{v}\right)^3 \left\{ \left[1 + \frac{1}{\alpha_2} \left(\frac{v - \gamma_2}{\beta_2}\right)^{\alpha_2}\right] - \left[1 + \frac{1}{\alpha_2} \left(\frac{v_{\text{cut-in}} - \gamma_2}{\beta_2}\right)^{\alpha_2}\right] \right\}
(23)
$$

### 4.7 Turbine - Location

To account for the fact that the wind speed data follow $\mathcal{W}bl(2,3)\left(v; \alpha_1, \beta_1, \alpha_2, \beta_2, \alpha, \gamma_1\right)$ distribution and $v_r \in \left[v_{\text{cut-in}}; v_{\text{cut-off}}\right]$ then the capacity factor can be defined by [11]

$$
\text{CF} = \left(\frac{\beta_3}{v}\right)^3 \left\{ \left[1 + \frac{3}{\alpha_1} \left(\frac{v - \gamma_1}{\beta_1}\right)^{\alpha_1}\right] - \left[1 + \frac{3}{\alpha_1} \left(\frac{v_{\text{cut-in}} - \gamma_1}{\beta_1}\right)^{\alpha_1}\right] \right\}
+ \left(\frac{3 \beta_1^2 \gamma_1}{v^3}\right) \left\{ \left[1 + \frac{2}{\alpha_1} \left(\frac{v - \gamma_1}{\beta_1}\right)^{\alpha_1}\right] - \left[1 + \frac{2}{\alpha_1} \left(\frac{v_{\text{cut-in}} - \gamma_1}{\beta_1}\right)^{\alpha_1}\right] \right\}
+ \left(\frac{3 \beta_1^2 \gamma_1}{v^3}\right) \left\{ \left[1 + \frac{1}{\alpha_1} \left(\frac{v - \gamma_1}{\beta_1}\right)^{\alpha_1}\right] - \left[1 + \frac{1}{\alpha_1} \left(\frac{v_{\text{cut-in}} - \gamma_1}{\beta_1}\right)^{\alpha_1}\right] \right\}
+ \left(\frac{\beta_2}{v}\right)^3 \left\{ \left[1 + \frac{3}{\alpha_2} \left(\frac{v - \gamma_2}{\beta_2}\right)^{\alpha_2}\right] - \left[1 + \frac{3}{\alpha_2} \left(\frac{v_{\text{cut-in}} - \gamma_2}{\beta_2}\right)^{\alpha_2}\right] \right\}
+ \left(\frac{\beta_2}{v}\right)^3 \left\{ \left[1 + \frac{2}{\alpha_2} \left(\frac{v - \gamma_2}{\beta_2}\right)^{\alpha_2}\right] - \left[1 + \frac{2}{\alpha_2} \left(\frac{v_{\text{cut-in}} - \gamma_2}{\beta_2}\right)^{\alpha_2}\right] \right\}
+ \left(\frac{\beta_2}{v}\right)^3 \left\{ \left[1 + \frac{1}{\alpha_2} \left(\frac{v - \gamma_2}{\beta_2}\right)^{\alpha_2}\right] - \left[1 + \frac{1}{\alpha_2} \left(\frac{v_{\text{cut-in}} - \gamma_2}{\beta_2}\right)^{\alpha_2}\right] \right\}
(23)
$$
\[
+ \left( \frac{3\beta_2\gamma_2}{v^3} \right) \left( \frac{1 + \frac{2}{\alpha_1} \left( \frac{v - \gamma_2}{\beta_2} \right)^{\alpha_1}}{1 + \frac{2}{\alpha_1} \left( \frac{v_{cut-in} - \gamma_2}{\beta_2} \right)^{\alpha_2}} \right) \left( 1 + \frac{1}{\alpha_2} \left( \frac{v_{cut-off}}{\beta_2} \right)^{\alpha_2} \right) \right] \\
+ \left( \frac{3\beta_2\gamma_2}{v^3} \right) \left( \frac{1 + \frac{1}{\alpha_2} \left( \frac{v - \gamma_2}{\beta_2} \right)^{\alpha_1}}{1 + \frac{1}{\alpha_2} \left( \frac{v_{cut-in} - \gamma_2}{\beta_2} \right)^{\alpha_2}} \right) \left( 1 + \frac{1}{\alpha_2} \left( \frac{v_{cut-off}}{\beta_2} \right)^{\alpha_2} \right) \right] \\
+ (1 - \omega) \left[ \frac{\gamma_3}{v^3} \left( e^{\left( \frac{v - \gamma_2}{\beta_2} \right)^{\alpha_2}} - e^{\left( \frac{v_{cut-off} - \gamma_2}{\beta_2} \right)^{\alpha_2}} \right) \right] \\
+ (1 - \omega) \left[ \frac{\gamma_3}{v^3} \left( e^{\left( \frac{v - \gamma_2}{\beta_2} \right)^{\alpha_2}} - e^{\left( \frac{v_{cut-off} - \gamma_2}{\beta_2} \right)^{\alpha_2}} \right) \right] \\
+ (1 - \omega) \left[ \frac{\gamma_3}{v^3} \left( e^{\left( \frac{v - \gamma_2}{\beta_2} \right)^{\alpha_2}} - e^{\left( \frac{v_{cut-off} - \gamma_2}{\beta_2} \right)^{\alpha_2}} \right) \right] \\
\right] \\
\right]
\right] \\
(24)
\]

4.8 Maximized Power:
Wind power that can be produced by a wind turbine depends on the operation of wind turbine and the wind speed [5]. The kinetic energy available in the wind is converted into mechanical energy and then the power output. The estimation of the rated wind speed which can maximize the average output power is computed as follows [12].

Let us define \( P_{avg}(v) : v_{cut-in, v_{cut-off}} \rightarrow R \) then

\[
P_{avg}(v) = \left( \frac{1}{2} C_p \rho A \right) \left[ \frac{1}{\beta_1} \left( 1 + \frac{3}{\alpha_1} \left( \frac{v - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) \right] \\
+ \left( \frac{1}{2} C_p \rho A \right) \left[ \frac{3\beta_1\gamma_1}{v^3} \left( 1 + \frac{3}{\alpha_1} \left( \frac{v - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) \right] \\
+ \left( \frac{1}{2} C_p \rho A \right) \left[ \frac{3\beta_1\gamma_1}{v^3} \left( 1 + \frac{3}{\alpha_1} \left( \frac{v - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) \right] \\
+ \left( \frac{1}{2} C_p \rho A \right) \left[ \frac{3\beta_1\gamma_1}{v^3} \left( 1 + \frac{3}{\alpha_1} \left( \frac{v - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) \right] \\
+ \left( \frac{1}{2} C_p \rho A \right) \left[ \frac{3\beta_1\gamma_1}{v^3} \left( 1 + \frac{3}{\alpha_1} \left( \frac{v - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) \right] \\
+ \left( \frac{1}{2} C_p \rho A \right) \left[ \frac{3\beta_1\gamma_1}{v^3} \left( 1 + \frac{3}{\alpha_1} \left( \frac{v - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) \right] \\
+ \left( \frac{1}{2} C_p \rho A \right) \left[ \frac{3\beta_1\gamma_1}{v^3} \left( 1 + \frac{3}{\alpha_1} \left( \frac{v - \gamma_1}{\beta_1} \right)^{\alpha_1} \right) \right] \\
\right]
\[ + \left( \frac{1}{2} C_p \eta A \right) \left( 1 - \omega \right) \left\{ \gamma_1^3 \left( \frac{v_{in} - v_{avg}}{\beta_1} \right)^{\alpha_2} - e^{-\left( \frac{v_{in} - v_{avg}}{\beta_1} \right)^{\alpha_1}} \right\} + \left( \frac{1}{2} C_p \eta A \right) \left( 1 - \omega \right) \left\{ \gamma_2^3 \left( \frac{v_{in} - v_{avg}}{\beta_2} \right)^{\alpha_2} - e^{-\left( \frac{v_{in} - v_{avg}}{\beta_2} \right)^{\alpha_1}} \right\} + \left( \frac{1}{2} C_p \eta A \right) \left( 1 - \omega \right) \left\{ \gamma_3^3 \left( \frac{v_{in} - v_{avg}}{\beta_3} \right)^{\alpha_2} - e^{-\left( \frac{v_{in} - v_{avg}}{\beta_3} \right)^{\alpha_1}} \right\} \]

Differentiating the above equation and equating to zero we can see that the power output is maximized when \( P_{avg} (v) = \frac{1}{2} C_p \rho A v_{1\text{st}-off}^3 \). \( \text{(26)} \)

5. Conclusion

Two parameter or three parameter Weibull distribution cannot be used in all general cases where a location experiences calm wind speed. The wind energy analysis at these regions can be done through our proposed model. The mixture three parameter Weibull distribution gives a drastic change of the computation because of the influencing of location parameters.

References

[1] Akdag S.A and Ali D 2009 A new method to estimate Weibull parameters for wind energy Applications Energy Conversion Management 50 p 1761-1766.
[2] Bhattacharya P 2000 A study on Weibull distribution for estimating the parameters Journal of Applied Quantitative Measures 5 p 234-241.
[3] Carta J A, Ramirez P 2007 Analysis of two-component mixture Weibull statistics for estimation of wind speed distributions Renewable Energy 32 p 518-31.
[4] Cheng S.K and Chen Z 2009 Benefit evaluation of wind turbine generators in Wind and Structures 23 p 351-366.
[5] Ditkovich Y, Kuperman A, Yahalom and Byalsky M 2012 A generalized approach to estimating capacity factor of fixed speed wind turbines IEEE Transaction Sustainable Energy 3 p 607 – 608.
[6] H. Anandakumar and K., Umamaheswari, Supervised machine learning techniques in cognitive radio networks during cooperative spectrum handovers, Cluster Computing, vol. 20, no. 2, pp. 1505–1515, Mar. 2017.
[7] H. Anandakumar and K., Umamaheswari, A bio-inspired swarm intelligence technique for social aware cognitive radio handovers, Computers & Electrical Engineering, vol. 71, pp. 925–937, Oct. 2018. doi:10.1016/j.compeleceng.2017.09.016
[8] Ravindra K Srinivesa R, R Narasimham S V L and Krishna M P 2012 Mixture probability distribution functions to model wind speed distributions International journal of Energy and Environmental Engineering 3 p 1-10.
[9] Salameh Z M and Safari I 1995 The effect of the windmill's parameters on the capacity factor IEEE Transaction Energy Conversion 10 p 747 -751.
[10] Sedghi M, Slaunik K H and Mehrdad B 2015 Estimation of Weibull parameters for wind energy Application in Iran Wind and Structures 21 p 203-221.
[11] Seshiah C V and Indhumathy D 2016 Mathematical Modelling of wind power Estimation using Multiple parameter Weibull Distribution Wind and Structures 23 p 351-366.
[12] Seshiah C V, Indhumathy D 2019 A Bimodal Weibull Distribution – Capacity Factor For Different Heights At Sulur Wind and Structures 28 p. 63-70.
[13] Wang L, Yeh T H, Lee W J and Chen Z 2009 Benefit evaluation of wind turbine generators in wind farms using capacity-factor analysis and economic-cost methods IEEE Transaction Power System 24 p 692 – 704.
[14] Xu Qin, Jiang-she Zhang and Xiao-dong Yan 2012 Two improved mixture Weibull models for the analysis of wind speed data *American Mathematical society* 51 p 1321-1332.

[15] Yeh T.H and Wang L 2008 A study on generator capacity for wind turbines under various tower heights and rated wind speeds using Weibull distribution *IEEE Transaction Energy Conversion* 23 p 592 – 602.

[16] Weisser D A Wind energy analysis of Grenada: an estimation using the Weibull density Function *Renewable Energy* 28 p 1803-12.