Monte Carlo Method of Shrinking Direction on Rectangular Slab of Fixed Boundary Condition

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Abstract-

The study of heat transfer is crucial for its proper implementation in engineering design process. The Shrinking Boundary Monte Carlo Method was applied to evaluate temperature distribution in a Spiral, vertically up and vertically down running direction. The outcome was equated to normal Monte Carlo Method. The results revealed that increasing the size of the rectangular slab will increase the running time to compute temperature distribution. In a given direction of running, it has different temperature distribution for the different sizes of the slab. The computational run time for a rectangular slab in a portrait shape is greater than rectangular slab in a landscape pattern. The study proved that the average of the run time for various direction of movement indicates that spiral is shorter. It is the best ways of computing temperature dispersal in a rectangular slab of fixed boundary condition. The utilization of the shrinking boundary to analyse heat transfer was successful.

Key words: Shrinking boundary, Heat transfer, Spiral, rectangular slab

1. Introduction

Thermodynamics study helps in the methods of heat transfer and calculation of transfer rates. The aim is to determine the mode of interaction and the time rates at which it occurs. Heat transfer evaluates the rate at which heat is transferred and the temperature dispersal of the system during the process [1, 2]. The quest for application of modern method prompts engineers to suggest ways to enhance the performance of heat transfer procedures. Over the decades, numerous machines that depend on heat transfer rates has been formulated and improved upon. Neumann or Dirichlet is recognized procedure to compute the temperatures of finite element that has wide availability and utilization [3, 4, 5]. Monte Carlo experiment is to discern random numbers selected in a pattern that simulate the physical haphazard process of the real life problems. Monte Carlo Methods are applied to the solution of problems by linear differential equations and linear boundary conditions. It is a method for simulating the equilibrium properties of many body system based on properties random walks. The scope of this work is to obtain suitable correlation for the description of the temperature distribution and heat behaviour. The body surface temperature $T_s$ is usually fixed which is called Dirichlet or boundary condition. Majority of the real life situations are based on heat transfer between the body and the surrounding. We assume that the heat transfer is severe when the convective coefficient $\alpha$ is reasonably high but has lower thermal conductivity and bigger heat conduction body. In these conditions $T_x$ and the temperature of the surface are constant [6].

\[ T(x = 0, t) = T_s \] (1)

2. Methodology
2.1 The Shrinking Boundary Method
The shrinking boundary method is used in this work because of the short computational time needed for random walk to be completed. It involves the same concept except that once an internal point has been computed it can be treated as a boundary point that can absorb other particles. This will lower the computation time for the next point since the walkers will have an additional point for terminating their walk. The savings in calculation time become even greater with subsequent calculations.

2.2 Finite difference method (FDM)
This is when the object being analyzed is divided into number of lumps. The differential equation is replaced by an approximate algebraic expression. It consists of backward difference, forward difference and central difference. To solve a conduction problem by the FDM, the relevant partial differential equation is replaced using Finite difference method. The space and time dimension are divided into a number of increments of finite size. The approximate expression which replaces the differential equation applies to every point in grids of points or nodes and separate equations are derived for boundary conditions. The relevant differential equation is effectively replaced by a large number of identical algebraic expressions for the temperature at each point in the space at a time. Finite difference expressions must be chosen such that the computer solution converges towards the exact solution [7, 8].

2.3 The finite element methods (FEM)
The finite element methods normally divide the system into a large number of finite elements. These elements may be one, two or three dimensional. The mathematical type of problem involves the choice of the finite element formulation. The formulation and the computer program develop for one problem is normally applicable to another problem of the same mathematical type [9,10]. A Monte Carlo Method involves using random numbers and probability to solve problems. Its application to conduction is the process of using random walk to determine temperature distribution in a system. Oduntan (2005) stated that the rapid progress in information technology resulted in the increasing popularity of the Monte Carlo Methods. It has a widely used class of approaches which follow a specific process [12].
   i. Definition of a domain of possible inputs.
   ii. Generates inputs haphazardly from the domain and perform a deterministic calculation on the result.
   iii. Summation of the results of the individual calculations into the final result.
Given any domain, the solution of a point in it is obtained by commencing many random walks from the points of interest. Such walks are determined whenever an absorbing boundary is encountered.

3. MATERIALS AND METHOD
3.1 Experimental procedure:
Heat conduction complications like unbalanced geometry and complex boundary conditions or both can be solved approximately by a technique called numerical analysis. Here, Fortran program was formulated for analyzing engineering problems using Shrinking Boundary Monte Carlo Method. The simplest boundary condition is that of known surface temperature in which the boundary nodes are specified and held constant during the iteration, which takes place only
over interior nodes. Assuming the region to be rectangular with boundary temperature known to be constant, a fortran program using Monte Carlo simulation methods to solve for the interior temperatures is formulated [12, 13]. The boundary conditions of the rectangular region are also dirichlet, that is, temperature is specified all along the boundary. Placing nodes within the slab that are spaced uniformy, with uniformly spaced nodes, the values of ΔX and ΔY is the same. Monte Carlo simulation to find temperature distribution by using equation 1. Figure 1 is the schematic diagram of arbitrary shaped, two-dimensional body

\[ T_{m,n} = \frac{1}{4} (T_{m,n+1} + T_{m+1,n} + T_{m,n-1} + T_{m-1,n}) \]  

**Figure 1:** Schematic diagram of arbitrary shaped, two-dimensional body

4. **Result and discussions**

The time it takes to do computational run for the model of steady state heat conduction in a rectangular slab using shrinking boundary Monte Carlo Method is carried out first. Secondly, computational run time for the calculation of temperature is performed using the boundary rectangular slab spiral in. The next step is to determine the computation run time from the vertical upward movement in the rectangular slab domain. Thereafter computation run time from the vertical downward movement is carried out. The rectangular domain consists of twenty-one nodal points. It is made up of two-dimensional rectangular slab with boundary conditions with constant thermal conductivity.

**Computational run time from the vertical upward iteration in the rectangular slab domain**

This is the starting of iteration from the bottom of the rectangular slab represented as downy in the program. Figure 2 is the diagram of Vertical Up shrinking direction of movement; All the values in table 1 are applicable
Computational run time from the vertical downward iteration in the rectangular slab domain.
It involves starting calculation of temperature distribution from the top of the rectangular slab. Two-dimensional steady state condition with no heat generation and constant thermal conductivity and is represented in the program as Up Y. Figure 3 is the diagram of vertical down shrinking direction of movement,

Figure 3: Vertical down shrinking direction of movement

Computational run time for spiral movement
This is the starting of counting from down X to up X, move up to the next node point and then move left to down X until it reaches the last point. The same values of table 1 are used to find temperature distribution in the slab. Figure 4 is the diagram of Spiral shrinking direction of movement
Table 1: Summary of various shrinking direction of movement and run time in the various sizes of rectangular slab

| Size of slab  | Spiral (secs) | Vertical up (secs) | Vertical down (secs) |
|--------------|--------------|--------------------|---------------------|
| y = 10, x = 10 | 3.645        | 3.545              | 3.555               |
| y = 15, x = 10 | 3.555        | 3.575              | 3.525               |
| y = 20, x = 10 | 3.445        | 3.585              | 3.685               |
| y = 15, x = 20 | 3.635        | 3.645              | 3.655               |
| y = 20, x = 15 | 3.675        | 3.655              | 3.735               |
| y = 30, x = 30 | 3.785        | 3.665              | 3.826               |
| y = 15, x = 30 | 3.655        | 3.575              | 3.715               |
| y = 50, x = 25 | 3.695        | 3.836              | 3.866               |
| y = 25, x = 50 | 3.815        | 3.846              | 3.795               |
| y = 50, x = 50 | 3.896        | 3.926              | 3.956               |

5. Conclusion
In conclusion, as the size of the rectangular slab increases, the higher the run time to calculate temperature distribution. The size of a given rectangular slab has equal temperature distribution irrespective of the direction of counting and run to calculate temperature distribution. Also it has equal surface plot graph for the same size of a given rectangular slab. In a given direction of running/counting, it has different temperature distribution for the different sizes of the slab. The computational run time for a rectangular slab in a portrait shape is greater than rectangular slab in a landscape pattern. Observation shows that the average of the run time for various sizes of the slab indicates that spiral is shorter. It is the best ways of calculating temperature distribution in a rectangular slab of fixed boundary temperature. Application of the shrinking boundary Monte Carlo Methods to study heat conduction in rectangular slabs generated good results. The obtained results are close to those obtained by finite element technique.
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