Hadron Modifications in a Dense Baryonic Matter

G. Musulmanbekov*
Joint Institute for Nuclear Research, Dubna, 141980 Russia
*e-mail: genis@jinr.ru

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Abstract—Starting with the Strongly Correlated Quark Model (SCQM) of a hadron structure, SCQM, we propose a possible scenario of modification of the properties of mesons and baryons in a dense nuclear environment. These in-medium modifications can lead to the observable effects in heavy ion collisions, such as enhancement of strangeness and dropping/broadening vector meson masses.

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INTRODUCTION

In head-on heavy ion collision (HIC) at high energies the energy and baryonic density increases multiply. There are some conceptual questions arising from such large densities: How much of the nuclear collision energy is converted into compression? Can hadrons exist as a free species in such a dense environment and how this environment changes their properties? As demonstrated by experiments the large baryon density has a strong impact on the characteristic features of strangeness production in HICs. Mainly, theoretical and experimental efforts have been dedicated to the physics of relativistic HICs looking for signatures of quark gluon plasma (QGP). Restoration of chiral symmetry and deconfinement are expected to occur at high density and/or temperature of a hadronic matter. When the energy density exceeds some typical hadronic value (~1 GeV/fm³), matter, as assumed, no longer consists of separate hadrons (protons, neutrons, etc.), but as their fundamental constituents, quark and gluons.

The other issue relates to the fact that in the most theoretical descriptions nucleons in nuclear matter are treated as point particles. However, in a compressed nuclear matter the assumption of point like nucleons is difficult to accept because, as known, at the normal nuclear density nucleons possessing a finite volume $V_N$ occupy considerable part of the nuclear volume $AV_N$, therefore the available space for nucleon motion is smaller. So theoretical treatment of nucleons in the nuclear matter should involve corrections to nucleon energies dependent on baryonic density with parameters — pressure, nucleon radius and nucleon mass.

Current and future experiments devoted to heavy ion collisions focus on observables which are sensitive to QGP phase transition, especially to the range of the phase diagram which close the critical point. In this way the study of the strange particle production in HICs is promising as they could serve a good diagnostic tool to investigate the properties of nuclear matter under extreme conditions. Strange particle yields have been thoroughly measured by many experiments with various colliding nuclei at different energies starting from kaon subthreshold production up to energies of SPS and RHIC. An enhanced yield of $K^+$ mesons below threshold in HIC has been observed by the KaoS Collaboration [1]. The systematic study of hadron production in central Pb + Pb collisions at SPS performed by NA49 collaboration revealed a sharp structure in energy dependence of positive kaon to pion multiplicity ratio, $K^+/\pi^-$ [2]. That peculiarity, called “horn”-effect, was later confirmed by Beam Energy Scan (BES) program of STAR collaboration at RHIC [3] (Fig. 1). At the same time there were no any peculiarities observed in the energetic behaviour of $K^-/\pi^-$ ratio. The idea that strangeness is a good signal of deconfinement was put forward by J. Rafelski in 1982 [4]. The argument was the following: it is energetically favourable to produce $s\bar{s}$ -pairs in deconfined medium than a pairs strange hadrons in hadron gas. Interpretation of the non–monotonic structure of $K^+/\pi^+$ has initiated intense theoretical activity. Authors attempted to reproduce the horn structure employing approaches either with phase transition to QGP or without it. “Horn”-like structure has been predicted in [5], as a manifestation of phase transition between thermalized hadronic and partonic phases. Albeit a variety of models, statistical [6–9] and kinetic [10, 11] (with or without deconfinement) have been proposed for interpretation of “horn” structure its satisfactory understanding is still not complete.

Another promising observable is a yield of dileptons. Dileptons are an ideal probe to study the proper-
ties of hot and dense nuclear matter, since they are emitted by hadronic resonances at different stages of reaction and escape the medium nearly unperturbed. They allow unique access to the properties both of the medium and resonances that decay within a strongly interacting medium. Measurements of emission of dielectrons in different nuclear reactions at wide range of collision energy revealed an enhancement of invariant mass spectra of dileptons yield in the interval 0.2–0.6 GeV [12–14] (Fig. 2). This enhancement was interpreted as in-medium modifications of hadrons at high temperature and density resulting in strong broadening of the $\pi^0$-meson and/or its “mass–dropping” [15–17].

In this paper we give our interpretation of the observed phenomena without QGP involvement and using for this purpose the Strongly Correlated Quark Model, SCQM, which takes into account the finite dimension of hadrons [18].

1. THE MODEL, SCQM

1.1. The Structure of Nucleon

The physical vacuum, whose energy is below the “empty” perturbative vacuum, is populated by gluon and quark–antiquark condensates. The QCD vacuum is the Bose–Einstein condensate of quark–antiquark pairs $\langle \psi \bar{\psi} \rangle = -(250 \text{ MeV})^3$. Suppose hypothetically a single quark of a certain color embedded in the physical vacuum. The color field of the quark polarizes the surrounding vacuum creating a condensate. At the same time it experiences the pressure of the vacuum, as a reaction on the ordering, because of the presence of quantum fluctuations of quark–antiquark condensate, or zero point radiation field in a classical sense. Suppose we place a corresponding antiquark in the vicinity of the first quark. Owing to their opposite signs, color polarization fields of the quark and antiquark interfere destructively in the overlap regions eliminating each other maximally at the midpoint between them. This effect leads to a decreasing value of the condensate density in that region and overbalancing of the isotropic vacuum pressure acting on both quarks (Fig. 3). As a result, an attractive force between the quark and antiquark emerges and the quark and antiquark start to move towards each other. We can relate this mechanism of quark–antiquark attraction to gluon exchange between them in QCD. The density of the resulting condensate around the quark (antiquark) is identified with the hadronic matter distribution which is associated with a dynamical mass of the quark. At maximum displacement in the system corresponding to small overlap of color fields, hadronic matter distributions have maximum extent and densities. The quark (antiquark) in this state possesses a constituent mass. The closer they come each other, the larger is the destructive interference effect and the smaller hadronic matter distributions around quarks and the larger their kinetic energies. At a minimal displacement the quark (antiquark) becomes a relativistic one with a current mass. So, the quark and antiquark start to oscillate back and forth around their midpoint. Suppose that the profile of color field and density of condensate around the quark (antiquark) has a form close to gaussian. Then oscillation of the quark–antiquark system looks like as shown.
For such interacting $q\bar{q}$ pair located from each other on a distance $2x$, the total Hamiltonian is

$$H = \frac{m_q}{\sqrt{1-\beta^2}} + \frac{m_{\bar{q}}}{\sqrt{1-\beta^2}} + V_{q\bar{q}}(2x), \quad (1)$$

where $m_q$, $m_{\bar{q}}$ are the current masses of the valence quark and antiquark, $\beta = \beta(x)$ is a velocity depending on their displacement from each other, and $V_{q\bar{q}}$ is the quark–antiquark potential energy with separation $2x$. It can be rewritten as

$$H = \left[ \frac{m_q}{\sqrt{1-\beta^2}} + U(x) \right] + \left[ \frac{m_{\bar{q}}}{\sqrt{1-\beta^2}} + U(x) \right] = H_q + H_{\bar{q}}, \quad (2)$$

where $U(x) = \frac{1}{2} V_{q\bar{q}}(2x)$ is the potential energy of the quark or antiquark. We postulate that the potential energy of quark is equal to its dynamical mass:

$$2U(x) = \int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dx' \rho(x, r') \approx 2M_q(x) \quad (3)$$

with

$$\rho(x, r') = c \left| \phi(x, r') \right| = c \left| \phi_0(x' + x, y', z') - \phi_{\bar{q}}(x' - x, y', z') \right|, \quad (4)$$

where $\rho$ is the resulting density of hadronic matter (quark–antiquark condensate) formed by color fields $\phi_0$ and $\phi_{\bar{q}}$ of the quark and antiquark, respectively. $c$ is a normalization constant. The resulting function $\phi(d, x)$ shown on Fig. 4 represents a potential energy density profile, and its integral (3) corresponds to dynamical mass of quarks. The structure and shape of vacuum polarization around the color quark/antiquark which could give us the information about the confining potential is not known.

It turned out that the mentioned above quark–antiquark system behaves similarly to a so-called breather solution of one–dimensional Sine-Gordon equation [19] which in scaled form reads

$$\square \phi(x, t) + \sin \phi(x, t) = 0, \quad (5)$$
where $\phi(x,t)$ is a scalar function and $x$ and $t$ are dimensionless. The *breather* solution
\[
\phi(x,t)_{br} = 4 \arctan \left[ \sqrt{\frac{T}{2\pi}}t - \frac{\sin(2\pi t/T)}{\cosh(2\pi x/T)} \right]
\]  
(6)
describes the periodic soliton–antisoliton system oscillating with a period $T$ (upper row on Fig. 5). The energy density profile of the soliton–antisoliton system
\[
\varphi(x,t)_{br} = \frac{d\phi(x,t)_{br}}{dx}
\]
(7)
oscillates (bottom row on Fig. 5) the same way as our quark–antiquark system (Fig. 4). At maximal displacements ($t = 0, (1/2)T$) the soliton and antisoliton energy density profile, $\phi(x,t)$, is maximal and at minimum displacement ($t = (1/4)T, (3/4)T$) they “annihilate”. It is not surprising, because our quark–antiquark system is built in close analogy with the model of dislocation–antidislocation [20], which in its continuous limit is described by breather solution (6) of SG equation. W. Troost [21] demonstrated that the Hamiltonian (2) corresponds to the dynamics of the coupled soliton–antisoliton pair. He derived the effective potential $U(x)$ for (2):
\[
U(x) = M \tanh^2(\alpha x),
\]  
(8)
where $M$ is a mass of soliton/antisoliton and $\alpha$ is an adjusting parameter. Hence, we can identify our potential of quark–antiquark interaction in hamiltonian (2) with the potential of soliton–antisoliton interaction.

Since quarks are the members of the fundamental color triplet, generalization to the 3-quark system (baryons, composed of Red, Green and Blue quarks) is performed according to $SU(3)_{\text{color}}$ symmetry: a pair of quarks has coupled representations $3 \otimes 3 = 6 \oplus \bar{3}$ and for quarks within the same baryon only the $\bar{3}$ (antisymmetric) representation is realized. Hence, an antiquark can be replaced by two correspondingly colored quarks to get a color singlet baryon; destructive interference takes place between color fields of three valence quarks (Fig. 6). The area of overlapping color fields of pair of quarks possesses the anti-color of the third quark. So, overlap of green (G) and red (R) fields gives yellow (Y) color which is the anti-color for the blue color of third quark and so forth. Overlap of quark color fields can be associated with color gluon exchange in QCD. In QCD eight gluons are combined from nine two-color states: $R\bar{R}, RG, RB, GR, GG, GB, BR, BG, BB$ and gluon exchange between color quarks will be of the form
\[
H_{12} \sim \mathbf{F}_i \cdot \mathbf{F}_j,
\]  
(9)
where $\mathbf{F} = \frac{1}{2} \mathbf{\lambda}$ are $3 \times 3$-matrices. For 3 body system the interaction between color quarks has the form
\[
H_{3q} \sim \sum_{ij} \mathbf{F}_i \cdot \mathbf{F}_j.
\]  
(10)

It is interesting that three gluon self-interaction corresponds to destructive interference of $R, G, B$ color fields of three quarks at the midpoint of nucleon (white area at the midpoint on Fig. 6) where all three colors are overlapped.

Putting aside the mass and charge differences of valence quarks one can consider three quarks oscillating synchronously along the bisectors of equilateral triangle turning from the constituent to current state.
and inversely. Therefore, the model unifies the features of bag models and constituent models. At a maximal displacement quark becomes nonrelativistic with constituent mass corresponding to the maximal value of condensate surrounding it. Further, owing to the prevailing condensate pressure from the outside, it moves under influence of the potential (8) (see Fig. 7a) towards two other quarks, and at the origin of oscillation it becomes relativistic with the current mass. Thus, during oscillation quarks transit from constituent states to current states and inversely that corresponds to dynamical restoration/braking of chiral symmetry. A nucleon composed of three oscillating quarks runs over the states corresponding to terms of the infinite series of Fock space:

\[ |\mathcal{B} \rangle = c_1 |q_1 q_2 q_3 \rangle + c_2 |q_1 q_2 \bar{q} q_3 \rangle + \ldots \]  

(11)

Important feature of the model is that there is no a confining potential/force inside a nucleon. During oscillations (putting aside Coulomb and spin interactions) the interaction force between quarks \( F(x) = -dU/dx \) vanishes both at the origin of oscillation and asymptotic displacement (Fig. 7b). It becomes maximal in between the origin and maximal displacement. Thus, at the origin of oscillations, where the quark and antiquark in mesons and three quarks in baryons being as relativistic ones do not interact, the state of asymptotic freedom is realized (Fig. 6). As to real confining potential, it should act at distances exceeding hadronic radii. Apparently, “imprisonment” of quarks is a consequence of the topological nature of hadrons. Hereinafter we assume that the quark–antiquark describing mesons and three quark systems describing baryons are topological solitons. Topological solitons are characterized by the conserving numbers, so-called, winding numbers. For baryons the winding number is identified with the baryonic number. It means that at any temperature and density of nuclear environment baryon conserves its identity, i.e. baryonic number. The model meets the local gauge invariance. Indeed, suppose \( \psi_{\text{color}} \) is a wave function of a single quark in color space where index color corresponds to one of the values Red, Green, Blue. Interactions of R, G, and B quarks in a nucleon which result in their oscillations can be reduced to the phase rotation the wave function \( \psi_{\text{color}} \) of each quark in its color space

\[ \psi_{\text{color}}(x) \rightarrow e^{i\theta(x)} \psi_{\text{color}}(x). \]  

(12)

This phase rotation results in dressing (undressing) of the quark by quark–antiquark condensate that can be linked with transformation of the gauge field \( A^\mu \) :

\[ A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu \theta(x). \]  

(13)

Such consideration comes from correlated dynamics of quark–antiquark and three quark interactions in mesons and baryons accordingly. As 3-quark system (baryon) has been constructed on the basis on that fact that any pair of quarks is a member of antitriplet 3 we
can limit our consideration by the quark–antiquark system which is a color singlet
\[
\sqrt[3]{\frac{1}{3}} (R\vec{R} + G\vec{G} + B\vec{B}).
\]  

(14)

Strong correlation of quarks coming from destructive interference of opposite color fields of quark and antiquark allows us to take into consideration only a single colored quark (one of \(R,G,B\)) described by a single component function (12). At maximal displacement, where the phase \(\theta = 0, \pi, 2\pi\), the color field of quark (Fig. 4) and the wave function \(\psi_{\text{color}}\) have a maximal value; at the midpoint of oscillation, where the phase \(\theta = (1/2)\pi, (3/2)\pi\), the quark color field and \(\psi_{\text{color}}\) go to zero (due to distractive interference). Therefore, the strong interaction between quarks (Eqs. (9) and (10)), described in our model by (2), is effectively “reduced” to the phase rotation of the quark wave function in a single color space that, in turn, allows one to drop the color indices of a gauge field: \(A^\mu_\alpha(x) \rightarrow A^\mu(x)\). Hence interaction of color quarks via non–Abelian fields of QCD in our model is “reduced” to its electrodynamical analog
\[
F^{\mu\nu}_\alpha = \partial^\mu A^\nu_\alpha - \partial^\nu A^\mu_\alpha - \lambda^{abc} A^\mu_\beta A^\nu_\gamma A^\nu_\alpha A^\nu_\beta A^\mu_\gamma.
\]

\[
(15)
\]

The parameters of the model are the maximum displacement of valence quark and antiquark in mesons and 3 quarks in baryons, \(x_{\text{max}}\), and the parameters of the hadronic matter distribution formed by quark–antiquark condensate around them. In the absence of knowledge about the shape of quark–antiquark condensate around valence quarks, or the form of hadronic matter in a constituent quark \(\varphi_{0(\vec{0})}\), we take it in a gaussian form
\[
\varphi_{0(\vec{0})}(x, y, z) = \varphi_{0(\vec{0})}(x_1, x_2, x_3)
\]
\[
= \left(\frac{\det \hat{A}}{(\pi)^{3/2}}\right)^{1/2} \exp \left(-X^T \hat{A} X\right),
\]

where the exponent is written in a quadratic form. The value of the maximal quark (antiquark) displacement, and parameters of the gaussian function for hadronic matter distribution around VQ are chosen to be \(x_{\text{max}} = 0.64\ \text{fm},\ \sigma_{x,y} = 0.24\ \text{fm},\ \sigma_{z} = 0.12\ \text{fm}\). They are adjusted by comparison of calculated and experimental values of the total, inelastic and differential cross sections for \(pp\) and \(\bar{p}p\) collisions [22]. The mass of the constituent quark at maximum displacement is taken as \(M_{0(\vec{0})}(x_{\text{max}}) = \frac{1}{3}\left(m_\Delta + m_N\right) = 360\ \text{MeV}\), where \(m_\Delta\) and \(m_N\) are masses of the delta isobar and nucleon correspondingly. The current mass of the valence quark is taken to be 5 MeV.

1.2. The Structure of Nuclei

As shown in the previous section interaction between quarks within the nucleon arises owing to overlapping of their colour fields. The same overlapping mechanism of quark–quark interactions is responsible for nucleon–nucleon binding in nuclei. In this case different color fields of adjacent quarks, i.e. quarks belonging to the neighbour nucleons being overlapped create additional minima of the potentials at the maximal quark displacements in each nucleon with a small (~8 MeV) well depth. Thus, due to this minima the amplitude of oscillations is reduced up to dimensions of this potential well. We start with light nuclei \(d, ^3H, ^3He\) and \(^4He\), which consist of 6, 9, 9,
and 12 quarks correspondingly. The demand for the multibaryonic states to be singlets of colour together with the Pauli principle leads to the possibility of assigning these states to completely antisymmetric representation of the $SU(12)$ group which contains the direct product

$$SU(2)_{\text{isospin}} \otimes SU(2)_{\text{spin}} \otimes SU(3)_{\text{color}}.$$  

(17)

Up to 12 quarks can occupy the $s$-state. The most of configurations built according to the group representations correspond to, so-called, “hidden color” states (color baryons) as these can not be represented in terms of the free (color-singlet) nucleons [23]. We restrict the multiquark configurations only by the color-singlet clusters—nucleons forming nuclei. In order to form bonds between nucleons we have to specify quantum numbers of adjacent quarks. Obviously, pairs of adjacent quarks have to possess different colors and isospins ($u$, $d$), because in this case the interaction between them is attractive. Their spin alignments are specified by using the fact that the $s$-state in $^4$He is totally occupied by 12 quarks. Assuming that all 3 quarks of any nucleon in $^4$He are bound with quarks of other nucleons and performing combinatoric calculations we unambiguously determine that adjacent quarks are spin-symmetric. In this case the spins of 2 protons and 2 neutrons are arranged in a right way. Therefore, using the following rules for quantum numbers of adjacent quarks

1. $SU(3)_{\text{color}}$—different colors;
2. $SU(2)_{\text{isospin}}$—antisymmetric;
3. $SU(2)_{\text{spin}}$—symmetric;

we can construct 2- and 3-nucleon nuclei. The three-nucleon system is built by the linkage of two quarks of each nucleon with quarks of two other nucleons according to the above rules. Three-nucleon nuclei, namely $^3$H and $^3$He, represent triangular configurations with a quark loop and three quarks at free ends (Fig. 9). Completion to four-nucleon system, $^4$He, from three-nucleon one, is performed by binding the free quark ends in $^3$H ($^3$He) with the three quarks of an additional proton (neutron) again in accordance with the above rules. This arrangement of 2 protons and 2 neutrons corresponds to octahedral symmetry. Construction of more complex nuclei leads to arrangement of nucleons in alternating spin—isospin layers. It turned out that nucleons occupy the nodes of face-centered-cubic (FCC) lattice [24]. In magic nuclei nucleons form closure shells having shapes of octahedron ($s$, $p$, $d$, ... — shells). The model allows transparent explanation of pairing effect, symmetry energy, nuclear deformation, to make prediction of nuclear drip-line.

2. HADRON MODIFICATIONS IN HEAVY ION COLLISIONS

In head-on collisions of two heavy nuclei the baryon and energy densities in the overlap zone increase drastically that results in perturbation of a nuclear structure and breaking of binding of nucleons. The time of “crossing” two symmetric nuclei through each other when they cease to overlap is $t_{\text{cross}} = 2R/\gamma$, where $R$ is the rest-frame radius of the nucleus. Particles created in the overlap zone can be considered “formed” at some proper time $\tau_{\text{form}}$ which is $\sim 1$ fm/c. At low and moderate collision energies where $\tau_{\text{form}} < t_{\text{cross}}$ particle production and their interactions take place mainly in the overlap zone with high baryonic density. At very high collision energies the remnants of Lorenz-contracted disks leave behind the interaction zone which is occupied by a hot and dense “fireball” of interacting secondaries characterized by low baryonic density. Obviously, that the crossing time depends on the additional factor, so-called “stopping
power” caused by nucleon-nucleon interactions. This factor prolongates the crossing time, $t_{\text{cross}}$.

As mentioned in Introduction, most theoretical models describe nuclear matter as composed of Fermi gas of point nucleons. For quantities characterizing nuclear matter at zero temperature one refers to energy density $\epsilon = E_A/A$, baryonic density $\rho = A/V$, pressure $p = \partial E_A/\partial V$. In nuclear potential these point nucleons move with Fermi energy connected with these quantities as $E_f = \epsilon + p/\rho$. When nucleons are extended objects the available space for nucleon motion becomes smaller: $V_{\text{av}} = V - AV_N$. Here $V_N$ is the nucleon volume. Therefore, the smaller available volume causes a bigger density $\rho$ and effectively a bigger pressure

$$
\rho_{\text{av}} = \partial E_A/\partial V_{\text{av}} = (\partial E_A/\partial V)(V/V_{\text{av}}) = \rho(1 - \rho V_N).
$$

(18)

If the volume of nucleon $V_N$ is fixed, then the increasing nuclear density up to the close-packing limit, $\rho \to 1/V_N$, can lead to infinite pressure $p_{\text{av}} \to \infty$. Thus inclusion of nucleon dimension into consideration especially in heavy ion collision cause some difficulties. With aim to describe the enhancement of strangeness production in HIC some authors include in their theoretical models hard cores of nucleons justifying that by repulsion of nucleons at small distances [25].

Exploiting the above model of nucleon structure, SCQM, we offer the other than QGP scenario in heavy ion collisions. It is based on the idea that in a dense nuclear environment the physical vacuum is essentially destroyed i.e. quark condensate is reduced. It has been shown in [26, 27] that in the interior of two overlapped nuclei, even with no interactions between nucleons, quark condensate is reduced up to $\frac{1}{3}$ of its vacuum value. Let us start with a static situation when two heavy nuclei overlap, like in the interior of neutron star with 2 times of normal nuclear density. Since the volume occupied by nucleons decreases twice, the accessible volume occupied by each nucleon composed of light quarks is reduced, at least, twice. As a consequence of available volume reduction, the vacuum condensate around the valence quarks decreases that, in turn, results in decrease of a dynamical mass of quarks and amplitude of their oscillation. Further compression of nuclear matter, that is typical in the interior of neutron stars, would force nucleons to collapse. Since nucleons are topological solitons, it could result in violation of the winding (topological) number conservation. To avoid collapsing there should be pressure arising from nucleon’s interior and balancing external pressure. This internal pressure arises if spin of one of three quarks in a nucleon flips transforming thus it to delta–isobar: $p_{\text{av}} \to \Delta S$, and their higher states. The $\Delta$ and nucleon $N$ are made of three quarks in the overall S-wave with spins $\frac{3}{2}$ and $\frac{1}{2}$ respectively.

The mass separation of $\Delta-N$ states is manifestation of QCD hyperfine splitting which is proportional to the product of quarks’ color-magnetic moments defined in analogy to their electromagnetic moments:

$$
H_{3q} = -c^2 \sum_{j<k} \langle F_j, S_j, S_k | \epsilon, \epsilon_k \rangle,
$$

(19)

where $F, S$ are SU(3) color (9) and SU(2) spin matrices respectively, and $\epsilon$ is effective mass of quark. $c^2$ is a parameter with dimensions (mass)$^3$, related to quarks’ fields overlap. Hence color-magnetic interaction results in the $\Delta-N$ mass separation between $\frac{3}{2}$ and $\frac{1}{2}$ spin states:

$$
m_{\Delta-N} = \langle H_{3} | \frac{1}{2} \frac{1}{2} \rangle \simeq 300 \text{ MeV}.
$$

(20)

Due to color-magnetic interaction (19) the internal pressure inside the nucleon transformed to $\Delta$-isobar arises to balance the external pressure from compressed nuclear environment. Denoting the modified nucleon by $N'$, we assume that in a compressed nuclear matter the size of the $N'$ decreases but its mass remains constant, equal mass of a nucleon. Reduction of the constituent mass of quarks at decreasing of the $N'$ dimensions is compensated by the energy of color-magnetic interactions (19). Thus, the balance between the external pressure acting on surface of modified nucleon $N'$ and the internal pressure arising from color-magnetic interaction (20) takes the form

$$
m_{\Delta-N} = (4/3)\pi R_{N'}^3 p,
$$

(21)

where $p$ is the external pressure and $R_{N'}$ is the radius of modified nucleon. Then the resulting mass of the $N'$ remaining constant (nucleon mass) relates the value of external pressure to the dimensions of $N'$ as

$$
M_{N} = 3m_0(r_q) + (4/3)\pi R_{N'}^3 p,
$$

(22)

where $r_q$ is the maximal displacement of quark depending in this case on the external pressure. Taking into account that the size of the nucleon, $R_N$, is composed of the maximal displacement of quark and dimensions of color condensate around it we make substitution $r_q = 0.57R_N$. The assumption that the mass of modified nucleon remains constant corresponds to the condition

$$
\partial M_{N'}/\partial R_{N'} = 0.
$$

(23)
Hence we can determine the relationship between pressure and the radius of modified nucleon $N'$

$$4\pi R_N^3 \rho = 1.14c\alpha (\tanh^2 0.57 R_N \alpha - 1) \times \tanh 0.57 R_N \alpha = 0$$  \hspace{1cm} (24)

and

$$p = \frac{c\alpha}{4\pi R_N} (\tanh^2 0.57 R_N \alpha - 1) \times \tanh 0.57 R_N \alpha.$$  \hspace{1cm} (25)

Therefore, there should be a limit of accessible volume reduction which can be specified as a “hard-core” of delta isobars and their excited states. However, at higher compression this mechanism is not sufficient because the cores of light quarks need to occupy relatively large volume. Moreover, at higher compression when the pressure (25) and energy density in the interior of a neutron star increase, it is preferable for nucleons to be converted to hyperons and their resonances, as dimensions/cores of strange quarks are small compared with those ones of light quarks. This transition of nucleons to hyperons can be described in the framework of $^0 P_1$-model of vacuum. In a compression zone the production of $s\bar{s}$-pairs should be dominating in the content of condensate. $s$-quarks of the pairs replace one or more of $d\bar{u}$-pairs of the nucleons forming hyperons and their resonances, and $\Xi$-quarks form with those replaced quarks strange mesons:

$$p(uud) \to \Sigma^+(uus) + K^0(d\bar{s}),$$
$$\to \Sigma^0(uds) + K^+(u\bar{s}),$$
$$\to \Lambda^0(uds) + K^0(u\bar{s}),$$
$$\to \Xi^0(dss) + 2K^+(u\bar{s}),$$
$$\to \Xi^0(uss) + K^+(u\bar{s}) + K^0(d\bar{s}),$$
$$\to \Omega^0(sss) + 2K^+(u\bar{s}) + K^0(d\bar{s}),$$

$$n(udd) \to \Sigma^-(dds) + K^+(u\bar{s}),$$
$$\to \Sigma^0(dds) + K^0(d\bar{s}),$$
$$\to \Lambda^0(dds) + K^0(d\bar{s}),$$
$$\to \Xi^- (dss) + K^+(u\bar{s}) + K^0(d\bar{s}),$$
$$\to \Omega^- (sss) + 2K^0(d\bar{s}) + K^+(u\bar{s}).$$

In these transition channels the $K^-$s and $K^0$s are produced only, but not $K^-$s. At higher compression the production of heavier resonances with all quark spins aligned parallel should be dominating.

However, nucleus-nucleus collision is not a static process, but dynamical one starting with compression, followed by interaction of nucleons with particle production and formation of heated and expanding ’fireball’. The transition mechanism works if the “crossing” time, $t_{\text{cross}}$, is larger than formation time, $\tau_{\text{form}} \sim 1$ fm/c. With increasing collision energy $t_{\text{cross}}$ becomes very short, and the Lorenz-contracted disks with excited baryons penetrate each other leaving behind a hot and dense fireball with a low baryonic chemical potential. Hence, at $t_{\text{cross}} < \tau_{\text{form}}$ the transition mechanism ceases.

In our approach the “horn”-effect arises from interplay between the energy/baryon density in the overlap zone, the formation time, $t_{\text{form}}$, and the crossing time, $t_{\text{cross}}$. Obviously, there is an additional mechanism affecting the effect, namely, baryon-baryon interactions, while nuclei cross each other. First, these interactions, due to stretching of quark-gluon strings, prolongate the crossing time, and second, particles produced at breaking of these strings increase the pressure (25) and energy density in the overlap zone. Among produced particles, according to the above arguments, the production of baryon resonances and vector mesons should be dominating. Because of high compression in the overlap zone decay of resonances on particles consisting of light quarks are suppressed. At higher energy of a collision both mechanisms lead to increasing production of strange particles: hyperons and kaons and their resonances. Again, production of $K^-$s prevails over $K^+$s. This enhanced production of hyperons and positive kaons in the overlap zone goes up to the moment when the crossing time, $t_{\text{cross}}$, becomes shorter than $\tau_{\text{form}}$ that leads to a drastic reduction of baryonic density in the interaction zone. Figure 10 qualitatively demonstrates the appearance of the “horn”-effect. Increasing collision energy leads to increase of baryon/energy density in the overlap zone (left plot) and simultaneously to reduction of the crossing time (middle plot). As the “horn” takes place at $\sqrt{s} = 7-9$ GeV (Fig. 1), that corresponds the crossing time interval $3-4$ fm/c. Therefore, the ratio of the yield of strange particles to non-strange ones rising up to $\sqrt{s} = 7-9$ GeV then drops above this energies (right plot).

Another effect, which is a consequence of modification of hadrons properties produced in a dense/hot hadronic medium, is enhancement of the invariant mass spectra of dileptons. In the literature this enhancement is interpreted as in-medium modifications of resonances at high temperature and density resulting in strong broadening of the $\rho$-meson and/or its “mass-dropping”.

In a compression zone the production of heavier resonances with all quark spins aligned parallel should be dominating. Because of high compression in the overlap zone decay of resonances on particles consisting of light quarks are suppressed. At higher energy of a collision both mechanisms lead to increasing production of strange particles: hyperons and kaons and their resonances. Again, production of $K^-$s prevails over $K^+$s. This enhanced production of hyperons and positive kaons in the overlap zone goes up to the moment when the crossing time, $t_{\text{cross}}$, becomes shorter than $\tau_{\text{form}}$ that leads to a drastic reduction of baryonic density in the interaction zone. Figure 10 qualitatively demonstrates the appearance of the “horn”-effect. Increasing collision energy leads to increase of baryon/energy density in the overlap zone (left plot) and simultaneously to reduction of the crossing time (middle plot). As the “horn” takes place at $\sqrt{s} = 7-9$ GeV (Fig. 1), that corresponds the crossing time interval $3-4$ fm/c. Therefore, the ratio of the yield of strange particles to non-strange ones rising up to $\sqrt{s} = 7-9$ GeV then drops above this energies (right plot).

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As have been shown above, production of particles in a dense baryonic matter, according our model, is predominated by baryonic and vector meson resonances. Because of the reduced value of condensate in a dense nuclear environment the mass of $\rho$-meson can drop. Moreover, in a dense matter the decay of vector mesons on pions should be suppressed. This suppression leads to more rescattering $\rho$-mesons that, in turn, to broadening of decay widths of these mesons. At essentially high energies excited remnants of colliding nuclei leave the interaction zone of a dense and hot hadronic matter, “fireball”. Evolution of the fireball starting with multiparticle production goes through heating, thermalization and cooling during its expansion. Again, since the nuclear matter inside the fireball is highly compressed the production of pions composed of light $u$ and $d$ quarks in pseudoscalar state is suppressed, and vector mesons, $\rho, \omega, \phi$, and heavier mass resonances will be dominating in the composition of fireball. Obviously, there should the same modifications of the features of $\rho$-mesons like in a dense baryonic matter. In hadronic channels the vector mesons can decay down to the threshold, $2m_\pi$. In dilepton decay mode the threshold goes down to $2m_\pi$. Therefore, in the framework of our approach, the enhancement of spectral functions of vector mesons (Fig. 2) can be explained by domination of their production and decay width broadening and mass dropping.

### 3. CONCLUSIONS

The model of hadron structure based on soliton–antisoliton (breather) solution of Sine-Gordon equation, SCQM, is used to analyze the non-monotonic behaviour of the ratio $K^+ / \pi^+$ and enhancement of the invariant mass spectra of di-electrons in heavy ion collisions. According to the model composing the features of both current and constituent quark models, baryons and mesons are extended objects, and their finite sizes play a key role in modification of their features in a dense/hot hadronic matter. In a dense and hot hadronic matter (i) nucleons of colliding nuclei preferably convert into delta-isobars and hyperons, (ii) among produced particles hadronic resonances are dominating. Analyzing the “horn”-effect in heavy ion collisions we qualitatively demonstrate that it arises from the interplay between energy/baryon density in the overlap zone of collision, the formation time ($t_{\text{form}} \approx 1 \text{ fm/c}$) and the crossing time, $t_{\text{cross}}$. It is also shown that dominating production of vector mesons, particularly $\rho$-mesons with the broadening decay width and dropping mass can qualitatively describe the enhancement of dilepton spectra.

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**Fig. 10.** “Horn”-effect as interplay between energy/baryon density in the overlap zone, the formation time ($t_{\text{form}} \approx 1 \text{ fm/c}$) and the crossing time, $t_{\text{cross}}$. Dashed lines show intervals of collision energy and crossing time corresponding to the “horn”.

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**TABLE 1.**

| Baryon/Energy density | Crossing time | Strangeness/Entropy |
|-----------------------|---------------|---------------------|
| $\sqrt{s} \approx 7–9 \text{ GeV}$ | $t_{\text{cross}} \approx 3–4 \text{ fm/c}$ | Collision energy |
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