LOW LYING SCALAR RESONANCES AND CHIRAL SYMMETRY

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Current theoretical studies on the \( \sigma \) and \( \kappa \) resonances are reviewed. It is emphasized that all evidences accumulated so far are consistent with the picture that the \( \sigma \) meson is the chiral partner of the Nambu–Goldstone bosons in a linear realization of chiral symmetry.

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1. The Renaissance of the \( \sigma \) Meson

1.1. Early studies on the physics related to the \( \sigma \) meson

The \( \sigma \) particle was firstly introduced by Gell-Mann and Levy in association with the linear \( \sigma \) model. In such a model, the \( \sigma \) meson develops a non-vanishing vacuum expectation value which triggers the spontaneous breaking of chiral symmetry. Pions act as the (pseudo-)goldstone bosons associated with the spontaneous chiral symmetry breaking (S\( \chi \)SB). In the linear \( \sigma \) model the \( \sigma \) meson fills in the linear chiral multiplet together with pions. The \( \sigma \) and the pion fields transform and mix with each other under chiral rotations, and before S\( \chi \)SB the \( \sigma \) field and the \( \pi \) field were essentially the same.

What is described above only repeats standard textbook content. The question is then whether the theory can accommodate for experimental data. Early studies on nuclear physics require \( \sigma \) in order to cancel the large \( \pi N \) scattering lengths caused by the \( \pi \) nucleon Born term. However these studies are based on models with linearly realized chiral symmetry. In such models, there must exist a large cancelation between the \( \sigma \) contribution and the \( \pi \) contribution at low energies in order to obey various constraints from soft pion theorems. One example of the latter is the Adler zero condition.

Non-linear realization of chiral symmetry was later discovered, which satisfies all low energy theorems induced by PCAC. Hence it is suggested that the \( \sigma \) is not necessary for chiral symmetry breaking. Chiral perturbation theory (\( \chi \)PT) is established based on the non-linear realization of chiral symmetry. It is a model

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independent approach and successfully describes strong interaction physics at (very) low energies. Furthermore it was shown that the (renormalizable) linear $\sigma$ model is not QCD at low energies.\cite{6}

On the other side, remarkable efforts have been made by physicists who insist on the existence of $\sigma$ meson, which led to the return of the $\sigma$ meson in PDG after disappearing for more than 30 years.\cite{7} Most of these studies and hence conclusions are model dependent, but are however important in keeping the thoughts on the right trajectory. On experimental side, it is also very difficult to claim the discovery of the $\sigma$ pole from experimental data, since the $\sigma$ meson, if exists, has to be a very broad resonance. In other words, $s$ wave $I=0$ channel $\pi\pi$ interaction at low energies is of highly non-perturbative, strong interaction nature. It is not easy to extract the $\sigma$ pole from the background contributions.\cite{7} As a consequence, previous conclusions in supporting the existence of the $\sigma$ meson are not very convincing.

1.2. The $\sigma$ meson must exist to adjust $\chi$PT to experiments

The situation still seemed to be rather confusing: chiral perturbation theory does not seem to support the existence of the $\sigma$ meson. On the other hand, see Fig. 1, the steady rise of the $I,J=0,0$ channel $\pi\pi$ scattering phase shift data below 1GeV, provided by the old CERN-Munich collaboration, left a big puzzle for imagination.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1.png}
\caption{IJ=00 channel $\pi\pi$ scattering phase shift data from Ref.\cite{9} and Ref.\cite{10}}
\end{figure}

In my personal opinion, the clearest way to reveal the very existence of the $\sigma$ meson hidden behind the phase shift data in Fig. 1 may be to write down a

\begin{itemize}
\item[$\ast$] The major difficulties in accepting the $\sigma$ resonance, and how can they be overcome, have been reviewed in Ref\cite{8}
\end{itemize}
dispersion relation for the $\sin(2\delta_0)$. Define

$$F(s) \equiv \frac{1}{2i\rho} \left( S(s) - \frac{1}{S(s)} \right),$$

it is easy to understand that $F(s)$ is analytic across the elastic cut of $\pi\pi$ elastic scattering and

$$\sin(2\delta_\pi) = \rho F,$$

where $\rho = \sqrt{\frac{s-4m^2}{s}}$. One assumes a high energy Regge behavior for the partial wave amplitude, hence $F(s)$ satisfies a once-subtracted dispersion relation,

$$F(s) = \alpha + \sum \text{poles} + \frac{1}{\pi} \int_L \text{Im} \frac{F(s')}{s'-s} ds' + \frac{1}{\pi} \int_R \text{Im} \frac{F(s')}{s'-s} ds',$$

where $\alpha$ is a subtraction constant and the rest contribution to $F(s)$ come either from poles or cuts. For $\pi\pi$ scatterings $L = (-\infty, 0]$ and $R$ starts from first inelastic threshold, practically $R = [4m^2, \infty)$.

The right hand cut can be estimated using experimental data and it is found that its contribution is not important at low energies. The left hand cut is estimated using $\chi$PT. As seen in Fig. 2, the left hand cut contribution to the phase shift is negative and concave whereas the experimental data curve on $\sin(2\delta^0_0)$ is positive and convex. Comparing with Fig. 1, Fig. 2 clearly demonstrates that it is necessary to include the $\sigma$ meson to adjust chiral perturbation theory to experiments, according to Eq. (3).
2. Dispersive Analyses to the $\sigma$ Meson

Because the physics related to the $\sigma$ meson and the $\kappa$ meson in $s$ wave $I=1/2$ channel $\pi K$ scatterings are of highly non-perturbative nature, a dispersive analysis is hence needed to extract the physical information in a model independent way. In the next we briefly review a new dispersive approach we have developed which is particularly suitable to study low lying resonance structure.

2.1. Factorized $S$ matrix element and separable singularities – the PKU parametrization form

One constructs, using analyticity, unitarity, and partial wave dispersion relations, a factorized form for elastic scattering $S$ matrix element:

$$ S^\text{phys.} = \prod_i S^{R_i} \cdot S^\text{cut}, $$

where $S^{R_i}$ denotes the $i$-th second sheet pole contribution and $S^\text{cut}$ denotes the contribution from cuts or background. The information from higher sheet poles is hidden in the right hand integral which consists of one part of the total background contribution,

$$ S^\text{cut} = e^{2i\rho f(s)}, $$

$$ f(s) = \frac{s}{\pi} \int_L \frac{\text{Im} f(s')}{s'(s' - s)} + \frac{s}{\pi} \int_R \frac{\text{Im} f(s')}{s'(s' - s)}. $$

The ‘left hand’ cut $L = (-\infty, 0]$ for equal mass scatterings and may contain a rather complicated structure for unequal mass scatterings. The right hand cut starts from first inelastic threshold to positive infinity. It can be demonstrated that the dispersive representation for $f$ is free from the subtraction constant. Again the right hand cut can be estimated from experimental input. Nearby left hand cut ($s > -32m^2 \pi^2$ for $\pi\pi$ scatterings if assuming Mandelstam representation) can in principle be estimated from experimental data as well, using Froissart–Gribov projection formula. Nevertheless, one expects such an estimate gives more or less the same effect as using $\chi$PT results when estimating the nearby left cut. There is no reliable way to estimate further left hand cut effects, nevertheless physics around physical threshold should not be strongly affected by what might happen far away in the complex $s$ plane. Moreover, further left cut contribution as defined by Eq. (5) is very mild which is understood by the fact that the integrand appeared in Eq. (5) is of logarithmic form. Numerical evaluation justifies this argument.

Estimates in various channels of $\pi\pi$ and $\pi K$ scatterings reveal a common feature: all the background contributions as defined in Eq. (5) are numerically found to be negative! This fact is actually crucial to establish the existence of the $\sigma$ and $\kappa$ pole in the present approach and also helps greatly in stabilizing the pole location in the data fit. It is interesting to notice that, there actually exists a correspondence of
Eq. (4) in quantum mechanical scattering theory, obtained sixty years ago:

\[ S(k) = e^{-2ikR} \prod_{1}^{\infty} \frac{k_{n} + k}{k_{n} - k}, \]  

where \( k \) is the (single) channel momentum and \( k_{n} \) pole locations in the complex \( k \) plane. The above formula is written down for any finite range potential, in s wave. It amazing to notice that the Eq. (6) automatically predicts a negative background contribution!

2.2. The \( \sigma \) and \( \kappa \) pole locations

The Eq. (4) can actually be understood as a simple combination of single channel unitarity and the partial wave dispersion relation. In the data fit it is found that the parametrization form is sensitive to \( S \) matrix poles not too far away from physical threshold, hence providing a useful tool to explore the broad resonance \( \sigma \) and \( \kappa \).

In the data fit it is found that crossing symmetry also plays an important role in fixing the \( \sigma \) pole location. Taking this fact into account it gives the \( \sigma \) pole location at \( M_{\sigma} = 470 \pm 50 \text{MeV} \), \( \Gamma_{\sigma} = 570 \pm 50 \text{MeV} \), in good agreement with the determination using more sophisticated Roy equation analysis. The application of Eq. (4) to LASS data also unambiguously establish the existence of the \( \kappa \) meson with the pole location \( M_{\kappa} = 694 \pm 53 \text{MeV} \), \( \Gamma_{\kappa} = 606 \pm 59 \text{MeV} \), which are also in agreement with the later determination on \( \kappa \) pole parameters using Roy–Steiner equations.

3. The physical properties of \( \sigma \) and \( \kappa \)

Though the existence of the broad \( \sigma \) and \( \kappa \) resonances are firmly established, its nature remains somewhat mysterious. Especially the question remains completely open on how to understand it from the underlinging theory, QCD. Though it is natural to attempt to relate \( f_{0}(600) \) or the \( \sigma \) meson to the quantum excitation of the order parameter \( \langle \bar{\psi} \psi \rangle \), a proof at the fundamental level is still missing. The first thing to notice is that the similarity between the \( \sigma \) and \( \kappa \) excludes very likely the possibility that the \( \sigma \) contains a large gluball component. Efforts have been made in trying to understand the \( \sigma \) meson using lattice QCD technique, but the approach is still at the stage of prematurity.

Most papers devoted to the study of physical interpretation of the \( \sigma \) or \( f_{0}(600) \) meson are at phenomenological level, using for example, linear sigma model at hadron level, or quark level, or the ENJL model. Also the \( \sigma \) meson are considered as a tetra quark state as a dynamically generated resonance from chiral perturbation theory lagrangian or as a resonance generated from 3P\(_{0}\) potential model.
It is of course very helpful, when trying to understand $\sigma$, to investigate the $\kappa$, $f_0(980)$, $a_0(980)$ simultaneously. One of the most challenging problems is to understand not only the mass spectrum but also the widely spread widths between these lightest scalars in different channels. This is recently reinvestigated using the ENJL model.\textsuperscript{30} It is found that the masses and widths of these lightest scalars, except the $f_0(980)$, can be understood simultaneously, taking them as the chiral partners of the $SU(3)$ pseudo-goldstone bosons, within linearly realized chiral symmetry.\textsuperscript{30}

The estimate is very crude because it is based upon a $K$-matrix unitarization approach. But one may hope the study may be of help to grasp the major physics and the qualitative picture presented by these lightest scalars.

An examination to the unitarized chiral perturbative amplitudes finds a light and broad pole on the complex $s$ plane, in the $I,J=0,0$ channel of $\pi\pi$ scatterings. It is also found that the $N_c$ trajectory of the $\sigma$ meson has a non-typical behavior as comparing with that of a normal resonance, e.g., a $\rho$ pole. Hence it is argued that the $\sigma$ is a dynamically generated resonance from a lagrangian without the $\sigma$ degree of freedom.\textsuperscript{28} This idea has been carefully examined\textsuperscript{31,32}, and it is found that the $[1,1]$ Padé approximation leads to a ‘$\sigma$’ pole falling back to the real $s$ axis in the large $N_c$ limit. A correct understanding to what does the $[1,1]$ Padé approximant mean is obtained through these studies. It is also pointed out that the bent structure of the $\sigma$ pole trajectory with respect to $N_c$ found in $[1,1]$ Padé approximant is in qualitative agreement with what one finds in $O(N)$ $\sigma$ model, hence suggesting a fundamental role of the light and broad resonance pole played at lagrangian level, even though it can be generated from certain dynamical approximations.\textsuperscript{32}

On the other hand, both lattice simulation\textsuperscript{23} or QCD sum rule (SR) approach\textsuperscript{33} seem to suggest that the ‘$\sigma$’ pole couples strongly to quark quadru-linear currents. Based on this observation, one may argue that the $\sigma$ is a tetra quark state. Nevertheless at low energies quark and gluons are not the appropriate degrees of freedom to interpret QCD. To understand this, let us ask a question whether a pion is a $\bar{q}q$ state. We know that $\pi$ is a pseudo-goldstone boson, i.e., a collective excitation of QCD. Hence in the chiral limit, the pion wave function in the Fock space expansion contains equally important $\bar{q}q$, $\bar{q}^2q^2$, $\bar{q}^3q^3$, \ldots components. Then can one conclude that $\pi$ is not a $\bar{q}q$ state, even if it contains $\bar{q}q$ as the leading component providing the quantum number? If the $\sigma$ meson is the chiral partner of the $\pi$ field in a lagrangian with a linear realization of chiral symmetry, it must share many properties of the pion field, i.e, being also a collective excitation. The lattice or QCD SR results, in my opinion, seem to support this simple picture that the $\sigma$ is indeed the chiral partner of the $\pi$ fields. The leading component in the $\sigma$ wave function is still $\bar{q}q$ with vacuum quantum number. Actually the clearest way to see the $\bar{q}q$ component is in the large $N_c$ limit, where $\bar{q}q$ becomes dominant, otherwise the amplitude will have difficulty to fulfil the requirements of crossing symmetry and analyticity.\textsuperscript{31,19}

Therefore a calculation in lattice or QCD SR with a $\bar{q}q$ component would be more realistic and welcome.\textsuperscript{34}

Furthermore, a quark quadru-linear current is actually not distinguishable from
a product of two quark bi-linear current. In this sense it is also ambiguous to state that the σ meson is a tetra quark state even if there is a large $\bar{q}q^2$ component inside the σ. The only unambiguous way to explore the tetra quark state is through searching for states with exotic quantum numbers. However, it is shown that such states in general are at least not favorable to be formed.

To summarize, the results obtained so far from theoretical studies to the σ meson are well consistent with the picture that it is the chiral partner of the pseudogoldstone bosons of QCD. The σ meson contains a $\bar{q}q$ seed, but is heavily renormalized by $\pi\pi$ continuum. The latter shifts its pole from near real axis to a place deep in the s plane.

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