Phase-space densities and effects of resonance decays in hydrodynamic approach to heavy ion collisions

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Abstract

A method allowing analysis of the overpopulation of phase-space in heavy ion collisions in a model independent way is proposed within the hydrodynamic approach. It makes it possible to extract a chemical potential of thermal pions at freeze out irrespective of the form of freeze-out (isothermal) hypersurface in Minkowski space and transverse flows on it. The contributions of resonance (with masses up to 2 GeV) decays to spectra, interferometry volumes and phase-space densities are calculated and discussed in detail. The estimates of average phase-space densities and chemical potentials of thermal pions are obtained for SPS and RHIC energies. They demonstrate that multibosonic phenomena at those energies might be considered as a correction factor rather than as a significant physical effect. The analysis of the evolution of the pion average phase-space density in chemically frozen hadron systems shows that it is almost constant or slightly increases with time while the particle density and phase-space density at each space point drops down rapidly during the system’s expansion. We found that, unlike the particle density, the average phase-space density has no direct link to the freeze-out criterion and final thermodynamic parameters, being connected rather to the initial phase-space density of hadronic matter formed in relativistic nucleus-nucleus collisions.

1 Introduction

The interferometry method to measure particle phase space densities averaged over the volume of a system that emits the particles was first proposed by Bertshch [1]. If systems formed in ultrarelativistic A+A collisions are thermal, then it is possible to compare the experimental value of the average phase-space density (APSD) with one associated with the Bose-Einstein [2] or Fermi-Dirac [3] thermal distributions, \( f_{BE,FD}(p) \), and extract in this way a particle chemical potential (or fugacity) at the freeze-out temperature. Such a chemical potential could have a positive value even for pions since a hadron gas evolution is chemically frozen rather than chemically equilibrated [4, 5, 6, 7, 8]. If it turns out that the particle fugacity is fairly large, then the boson density could
be around the critical one and significant multibosonic \cite{9, 10, 11, 12}, and coherent \cite{13} effects might be present at low particle momenta.

There are, however, two serious problems which have to be solved to apply the above method. It is obvious that the APSD $\langle f(p) \rangle$ can be associated directly with a thermally equilibrated distribution for static homogeneous fireballs, $f_{BE}(p)$ for bosons. For expanding inhomogeneous systems, characterized by a set of (local) equilibrium distributions which are Doppler shifted in the vicinities of each space-time point according to the collective velocity field, the phase-space density averaged over some hypersurface situated at or ”after” the freeze-out could lose, in general, the direct connection to the Bose-Einstein or Fermi-Dirac phase-space distributions. As it was demonstrated in Ref. \cite{14}, even at a uniform temperature and chemical potential at freeze-out, the intensity and profile of longitudinal and transverse flows influence very essentially the value of $\langle f(p) \rangle$, making, thus, the extraction of fugacity by comparison with $f_{BE}(p)$ ambiguous and model dependent. One of the aims of this paper is to study and substantiate the model independent possibility for an analysis of overpopulation of the phase-space in 3D expanding fireballs at freeze-out.

The other problem is resonance contributions to particle spectra and interferometry radii \cite{15}. If chemical freeze-out takes place, then about 2/3 of pions comes from decays of resonances in high energy heavy ion collisions \cite{6, 8}. Only a small part of them are long-lived resonances which give the effect of a halo \cite{16} - a suppression of correlation functions of identical pions (see also \cite{13}). The others, short-lived, enhance significantly the pion single-particle spectrum, as compare to the thermal pion one, and also modify the interferometry radii. Our analysis of pion production by some con concrete resonance species shows that even for thermal distribution of those resonance species at freeze-out the spectrum of the produced pions is far from the thermal one. Also, an effective region of such a pion emission at small momenta is not associated with the interferometry volume at the corresponding small mean momenta of pion pairs from the resonance decays. Peculiarities of pion production by resonances are important to understand the influence of resonance decays on the average phase-space densities which are currently measured in experiments at CERN SPS and BNL RHIC. We analyse quantitatively the total effect on spectra and interferometry volume of pion production by huge number of resonances with masses up to 2 GeV and dependence of the effect on the intensity of flows. As a result, we propose how to cope with the resonance contributions to study the freeze-out properties of the expanding systems.

One of the most serious challenges to the theoretical understanding of the processes of A+A collisions is multiplicity (and energy) dependence of the APSD $\langle f \rangle$ in central events. According to the Bertsch result \cite{1} $\langle f \rangle \sim \frac{dN/dy}{T_{eff}V_{int}}$. If at different energies of A+A collisions the pion interferometry volume $V_{int}$ of the created systems is proportional to the pion multiplicity $dN/dy$, and $T_{eff}$, the effective temperature of the transverse spectra of pions, changes only slightly, then approximate constancy of the APSD could be considered a the natural property of the system at freeze-out in high energy A+A collisions. It was a very likely hypothesis at relatively small energies \cite{2}, since it had been supported by the observed proportionally between $dN/dy$ and $V_{int}$ \cite{17}. However, this proportionally law starts to be violated already at SPS energies and is completely destroyed in experiments at RHIC \cite{18}. In the latter case the interferometry radii and volume turn out to be essentially smaller than was expected at the corresponding RHIC multiplicities (the HBT puzzle) \cite{19}. As a consequence, the APSD at RHIC energies is found to be the considerably higher than at the formerly achieved energies \cite{20}. Does this mean that freeze-out conditions of the system in heavy ion collisions at RHIC are essentially different from the ones at SPS? Does this signify that the freeze-out particle densities at RHIC come close to the critical value? The latter could lead to
the multibosonic effects at freeze-out which, in turn, can reduce the observed HBT radii \[9, 10\]. Are thus multibosonic effects relevant to the HBT puzzle?

To understand the above problems we study the APSD and particle density temporal evolution in a few hydrodynamic models. We observe, in particular, that the phase-space density averaged over the freeze-out volume, \( \langle f(p) \rangle \), drops rather slowly during the 3D expansion of the system, - much slower than the phase-space density \( f(t, r, p) \) at any fixed \( r \) and than the particle density. This difference in the behavior of the two groups of values becomes bigger when the intensity of the transverse flow grows. The detailed analysis of this effect as well as the model independent estimates of chemical potential for the thermal pions at freeze-out makes it possible to shed light on the reason of the relatively high APSD at RHIC and on the HBT puzzle.

2 Evolution of the average phase-space density.

2.1 Temporal behaviour of APSD \( \langle f(p) \rangle \) in analytically solved hydrodynamic models

Let us demonstrate the important features of the APSD evolution versus particle density one by one using hydrodynamic models of A+A collisions. To see the main idea we start from the exact solutions of perfect non-relativistic hydrodynamics. If the initial conditions correspond to the thermal Boltzmann distribution with uniform temperature \( T_0 \), zero initial flows, \( u(t=0, r) = 0 \), and spherically symmetric Gaussian (with radius \( R_0 \)) profile of particle density, then the hydrodynamic evolution of that system is described by the local equilibrium phase-space distribution which is an exact solution of Boltzmann equation for any cross-section of particle interactions \[21\] and has the form:

\[
\frac{N}{R_0^3} \left( \frac{1}{(2\pi)^2 m T_0} \right)^{\frac{3}{2}} \exp\left(-\frac{m(v - u(r))^2}{2T(t)} + \frac{\mu(r)}{T(t)} \right),
\]

where \( N \) is total number of particles, \( v = p/m \) are their velocities, and \( T(t) = \frac{T_0}{1 + T_0 t^2 / m R_0^2} \), \( u(r) = \frac{T_0}{m R_0^2} r t T(t) \).

One can check directly that the momentum spectrum and the interferometry radii, evaluated at any time \( t \) (at any isotherm) are identical to those calculated at the initial time \( t = 0 \). The easiest way to see it is to rewrite expression (1) in the form

\[
f^{\text{eq}}(t, r, p) = \frac{N}{R_0^3} \left( \frac{1}{(2\pi)^2 m T_0} \right)^{\frac{3}{2}} \exp\left(-\frac{p^2}{2m T_{\text{eff}}} - \frac{(r - t p/m)^2}{2R_0^2} \right),
\]

where the effective temperature of the spectra is a constant in time and \( T_{\text{eff}} = T_0 \). Thus, the maximal phase space density and Bertsch’s average phase-space density \[1\]

\[
\langle f(t, p) \rangle = \int \frac{f^{\text{eq}}(t, r, p)^2 d^3 r}{\int f^{\text{eq}}(t, r, p) d^3 r} = \frac{N}{R_0^3} \left( \frac{m}{2(2\pi)^2 T_0} \right)^{\frac{3}{2}} \exp\left(-\frac{p^2}{2m T_{\text{eff}}} \right)
\]

is the same at each \( p \) as it was at the initial time!

The space density of the particles falls down with time as \( 1/t^3 \) at large \( t \). So, the particles in the system will really stop interacting at some later stage and the system decouples, preserving its maximal PSD and average PSD values as they were at the moment when the fireball was created.
It is noteworthy that the local phase-space density \( f^{l,eq.}(t, \mathbf{r}, \mathbf{p}) \) in the system drops with time at any fixed point \( \mathbf{r} \) very rapidly, as \( \exp[-(c_1^2 t^2 + c_2 t)] \).

The perfect constancy of the APSD is caused by the peculiarities of this specific model: in particular, its spherical symmetry, as was discussed in detail in Ref. \[22\]. At the same time, as we will demonstrate, the important feature of this simple model, namely, the principal distinction in temporal behaviours of different types of densities, is preserved in realistic models of A+A collisions: the APSD decreases with time essentially slower than the particle density.

To make the next step towards clarifying the effect, let us consider one more analytical illustration of it based on the model of a 3D expanding fireball with longitudinally Bjorken-like flow \( v_z = r_z/t \). Here is the exact non-relativistic solution of the ideal hydro equations for an initially Gaussian transverse density profile which results in the following evolution of the locally equilibrated phase-space density \[8\]:

\[
f^{l,eq.}(t, \mathbf{r}, \mathbf{p}) = \left( \frac{1}{2\pi m T} \right)^{3/2} n \exp(-\frac{m}{2T}(\mathbf{v} - \mathbf{u})^2),
\]

\[
n(t, \mathbf{r}) = n_0 \frac{R_0^2 t_0}{R^2(t)} \exp(-\frac{r_z^2}{2R^2} - \frac{r_y^2}{2R^2}),
\]

\[
\mathbf{v} = \frac{p}{m}, \mathbf{u}(t, \mathbf{r}) = \left( \frac{\dot{R}_x}{R}, \frac{\dot{R}_y}{R}, \frac{1}{t} \dot{r}_z \right),
\]

(4)

where the initial particle density in the central part of system, \( r_x = r_y = 0 \), is \( n_0 = N/(2\pi)^{3/2} R_0^2 t_0 \) and

\[
T = T_0 \left( \frac{R_0^2 t_0}{R^2(t)} \right)^{2/3}, R = \frac{m}{T}.
\]

(5)

At each moment \( t \) the longitudinal spectrum in this model is \( m \frac{dN}{dp_z} = 2\pi R_0^2 t_0 n_0 = \text{const} \), similar to \( \frac{dN}{dy} = \text{const} \) in the relativistic boost-invariant model, and the momentum spectrum is

\[
\frac{d^3 N}{d^3 p} = \frac{n_0 R_0^2 t_0}{m^2 T_{eff}(t)} \exp(-\frac{p_T^2}{2m T_{eff}(t)}),
\]

(6)

where the effective temperature, \( T_{eff} \), is

\[
T_{eff} = \frac{m}{R} + T.
\]

(7)

The \( T_{eff}(t) \) does not change much since the decrease of temperature \( T \) when the system expands is accompanied by the increase of transverse flows described by \( \dot{R} \) in Eq. \[5\]. The typical behavior of the temperature and effective temperature in this model can be seen in Fig. 1.

The interferometry radii, out-, side- and long- take the forms

\[
R_O^2 = R_S^2 = \frac{T}{T_{eff}} R^2, R_L^2 = t^2 \frac{T}{m}
\]

(8)

and are similar to the ones in the relativistic case at the central rapidity point \[23\ \[24\,], if \( m \rightarrow m_T \equiv \sqrt{m^2 + p_T^2} \). The interferometry volume, taking into account \[5\], is

\[
V_{int} \equiv R_O R_S R_L = \frac{R_0^2 t_0 (T_0)^{3/2}}{T_{eff} \sqrt{m}}.
\]

(9)
and does not change significantly since it depends on $T_{\text{eff}}$ only. So, it is not surprising after all, that the APSD is, at least approximately, preserved with time

$$
\langle f(t,p) \rangle = n_0 \left( \frac{1}{4\pi m T_0} \right)^{3/2} \exp\left( -\frac{p_T^2}{2m T_{\text{eff}}(t)} \right).
$$

The average momentum phase-space density, $\langle f(t,p) \rangle$, as well as the interferometry volume $V_{\text{int}}$, will be preserved in time if and when, at some stage of the evolution, $T_{\text{eff}}(t)$ is constant in time.

Practically, the effective temperature, and thus the APSD $\langle f(t,p) \rangle$, could be slightly reduced or even increased with time during the evolution, depending on initial conditions, the evolution stage and particle mass. Behavior of the effective temperature is correlated unambiguously with the behavior of particle transverse energy per unit of rapidity (or average particle transverse momentum).

In the idealized case of the 1D longitudinal expansion of a one-component gas, the particle transverse energy per unit of rapidity can only decrease since the effective temperature of the transverse spectrum in that case is just the system’s temperature that falls down permanently. However, in a realistic case when transverse expansion is switched on, the transformation of a system’s heat energy into the transverse and longitudinal kinetic energies of particles becomes much more complicated. In the 3D expansion the diminution of the total energy of the part of the hydrodynamic tube confined within a small rapidity interval is accompanied by a faster decrease in the particle longitudinal kinetic energy, comparing to the 1D case. It is caused by faster temperature decrease, while longitudinal collective velocities are the same in that piece of fluid. As for the transverse energy, it accounts for not only a decrease of the heat energy, associated with temperature, but also an increase with time of transverse flows. Therefore, the particle transverse energy (in the relativistic case it is better to consider average transverse momentum of particles) essentially does not change during the evolution. Such a process of transformation of heat into transverse flow is accompanied by an appearance of positive correlations between the absolute value of particle transverse momentum and its distance from the centre of the system. The degree of that correlation grows with time compensating for the contribution of increasing geometrical radii to the interferometry volume $V_{\text{int}}(p_T)$ \cite{24}. Since the transverse momentum spectra do not change much, the APSD $\langle f(t,p) \rangle \sim \frac{d^n N}{dp_T^{n-1}} / V_{\text{int}}(p_T)$ becomes about constant at some stage of evolution unlike the local phase-space density $f(t,r,p)$ and particle density. The latter is just a fast monotonically decreasing function (see \ref{11}):

$$
n(t,0) = n_0 \frac{R_0^2 t_0}{R^2(t) t} \xrightarrow{t \to \infty} \frac{1}{t^3}
$$

This is one of our main points - the system can reach small densities and decay with rather high APSD reflecting the high average phase-space density at the moment when the system was formed!

Now we pass from general physical ideas to the concrete results within the realistic model of Pb+Pb collisions at CERN energies 158 AGeV \cite{8}. The model describes hydrodynamically the evolution of chemically frozen hadron-resonance gas from the hadronization stage to the thermal, or kinetic, freeze-out. As was found recently \cite{5}, if chemical freeze-out is incorporated into hydrodynamics, then the final spectra and fireball lifetimes are insensitive to the temperature at which the switch from hydrodynamics to cascade RQMD is made. This implies that the local conservation law describing the transformation of heat energy into collective flows for a chemically frozen hydrodynamic evolution results in spectra which correspond approximately to the microscopic cascade calculations. Based on an analysis of the particle number ratios, pion rapidity density at $y \approx 0$,
slopes of transverse spectra and values of interferometry radii for pions, kaons and protons, the reconstruction of the hadronization stage was done in Ref. [8] by solving backwards in time the hydrodynamic equations for the chemically frozen composition of mesons and baryons with masses up to 2 GeV. The "initial conditions" for such a solution were defined from an analysis of spectra and correlations at the thermal freeze-out stage. Supposing that the hadronization of Bjorken-like expanding quark-gluon plasma (QGP) happens at a uniform chemical freeze-out temperature $T_{\text{ch}}$ with Gaussian-like particle density transverse profile of some width $R$ and self-similar transverse flow $y_T = r \frac{R}{R}$, one can restore from the hydrodynamic solutions the hadronization proper time $\tau_{\text{ch}}$, transverse radius $R(\tau_{\text{ch}})$, transverse velocity $v_R(\tau_{\text{ch}}) = \frac{R}{T_{\text{ch}}} \frac{\partial R}{\partial \tau_{\text{ch}}}$ at the Gaussian "boundary" of the system, and then the energy and particle number densities in the center of the system, $\epsilon_{\text{ch}}$ and $n_{\text{ch}}$. The temperature $T_{\text{ch}}$ and baryonic chemical potential $\mu_{B}(\tau_{\text{ch}})$ are defined in Ref. [8] from the analysis of particle number ratios.

| $T_{\text{ch}}$ GeV | $\mu_{B,\text{ch}}$ GeV | $\tau_{\text{ch}}$ fm | $R_{\text{ch}}$ fm | $v_{R,\text{ch}}$ | $\epsilon_{\text{ch}}$ GeV fm$^{-3}$ | $n_{\text{ch}}$ fm$^{-3}$ |
|---------------------|------------------------|---------------------|---------------------|-----------------|-------------------------|---------------------|
| 0.164               | 0.224                  | 7.24                | 4.63                | 0.304           | 0.420                   | 0.421               |

The evolution of the APSD $\langle f(\tau, p) \rangle$ for the frozen number of direct pions in hadron-resonance gas with the initial conditions at chemical freeze-out as in Table 1 is demonstrated in Fig. 2.

As one can see, the APSD is changed less than 10 per cent during the evolution from the hadronization stage to the kinetic freeze-out and its values at small transverse momenta are even higher at the final than at the initial moment. At the same time, as is shown in Fig. 3, the densities of direct pions drop rather quickly in the central part of the system where soft momentum particles are produced. One can see the principal difference in the evolutions of the space- and average phase-space densities from the comparison of correspondent curves at $\tau = 15$ fm/c and $\tau = 7.24$ fm/c. While the APSD changes not more than 35 per cent and even increases at small $p_T$, the particle density drops a factor of 6! We can conclude, therefore, that the basic physical phenomenon of approximate "conservation" of the pion APSD during the system evolution that we demonstrated analytically in the non-relativistic models of the 3D hydrodynamic evolution also takes place in the general case of relativistic chemically frozen 3D expansion of a many-component hadron resonance gas.

2.2 Totally averaged phase-space density $\langle f \rangle$ as integral of motion

Let us devote the end of this Section to the new, important results concerning the evolution of the phase-space density $\langle f(t) \rangle$ that is totally averaged over momentum and space with the integral measure $d^3r d^3p$:

$$\langle f(t) \rangle = \frac{\int f^2(t, r, p) d^3r d^3p}{\int f(t, r, p) d^3r d^3p}.$$  

(12)

It is obvious that $\langle f(t) \rangle \neq \int d^3p \langle f(t, p) \rangle$. Supposing the Boltzmann non-relativistic approximation for the distribution function holds:

$$f(t, r, p) = (2\pi)^{-3} \exp \left( \frac{m(v - u(t, r))^2}{2T(t, r)} \right) \xi(t, r)$$  

(13)
where \( \mathbf{u}(t, \mathbf{r}) \) is the collective velocity field, \( \xi(t, \mathbf{r}) = \exp(\mu/T) \) is the fugacity, and \( \mu \) is the (non-relativistic) chemical potential, one can find for the finite expanding system that \( \frac{d}{dt} \langle f(t) \rangle = 0 \).

Indeed, from the non-relativistic hydrodynamic equations for the one-component Boltzmann gas (see, e.g., [25]) it follows the fugacity \( \xi \) is preserved along the current lines: \( (\partial_t + u_i \partial_{r_i}) \xi = 0 \). Then one should appeal again to the hydro equations to deduce that
\[
(\partial_t + \partial_{r_i} u_i) T^{3/2}/\xi = 0,
\]
and get, finally, using the property of finiteness of the system, that \( (k = 1, 2, \ldots) \)
\[
\frac{d}{dt} \left( \int d^3r \int f^k(t, \mathbf{r}, \mathbf{p}) d^3p \right) = \int d^3r \frac{\partial}{\partial t} \left( \int f^k(t, \mathbf{r}, \mathbf{p}) d^3p \right) = -\left( \frac{m}{2\pi k} \right)^{3/2} \int d^3r \partial_{r_i}(u_i T^{3/2}/\xi^k) = 0
\]
(15)
So the totally averaged phase-space density is conserved during the system expansion! Such models which describe initially finite systems, \( f(0, r, t) \rightarrow 0 \), belong to the class of Landau-like models [26]. Of course, the property \( \langle f(t) \rangle = \text{const} \) is satisfied in the concrete model \( \Pi \) of spherically expanding fireball.

There is, however, another, Bjorken-like class of hydrodynamic models, which are boost invariant and, so, formally infinite in the longitudinal direction. For this situation it is better to define the phase-space densities averaged over all phase-space except particle longitudinal momentum \( p_z \), or rapidity \( y \), in order to associate the APSD with the small rapidity interval near mid-rapidity, \( v_L = p_z/m \approx 0 \), or \( y \approx 0 \). We will mark the corresponding APSD as \( \langle f(t, y) \rangle \). Since the boost-invariance dictates that distribution function depends just on the difference \( v_L - u_L \) (\( u_L = r_z/t \)) for the non-relativistic case, or \( y - \eta \) (\( \eta \) is hydrodynamic rapidity) for the relativistic one, the integration over fluid longitudinal coordinates at fixed particle velocities \( v_L \approx 0 \), or \( y \approx 0 \), in [12] can be substituted with the integration over particle rapidity at fixed hydrodynamic velocities \( u_L \approx 0 \), or \( \eta \approx 0 \). Therefore, \( \langle f(t, y) \rangle \) corresponds to a totally averaged phase-space density in a small central slice \( \Delta \eta \) of the hydrodynamic tube that expands conserving entropy \( S \) and particle number \( (dN/d\eta) \Delta \eta \). Using Eq. (14) one can find, similar to (15), that
\[
\frac{d}{dt} \left( \int t d^2r \int f^k(t, \mathbf{r}, \mathbf{p}) d^3p \right) = 0,
\]
(16)
and, thus, the APSD \( \langle f(t, y) \rangle \) in this case is also preserved in time.

It is useful to explain and generalize to the relativistic case the above results without resorting to hydrodynamic equations. Let us consider the time evolution of the set of fluid elements starting from some initial time \( t_0 \). They contain particle numbers \( \Delta N_i \), entropy \( \Delta S_i \), energy in their rest frames (marked by asterisk) \( \Delta E_{i(0)}^* \) and occupy initially the small volumes \( \Delta V_{i(0)}^* \) near points \( x_{i(0)} \) with temperatures \( T_{i(0)} \) and chemical potentials \( \mu_{i(0)} \). At some later time \( t_f \) the fluid elements are characterized by the corresponding values: \( \Delta N_i \rightarrow \Delta N_i \) while preserving their entropy \( \Delta S_i \) (due to isentropic expansion) and particle number \( \Delta N_i \) (because of chemically frozen evolution). Then, according to the thermodynamic identity (\( p \) is pressure)
\[
\Delta S_i = \frac{p_{i(0)} \Delta V_{i(0)}^*}{T_{i(0)}} + \frac{\Delta E_{i(0)}^*}{T_{i(0)}} - \mu_{i(0)} \Delta N_i = \frac{p_{i(f)} \Delta V_{i(f)}^*}{T_{i(f)}} + \frac{\Delta E_{i(f)}^*}{T_{i(f)}} - \mu_{i(f)} \Delta N_i
\]
(17)
In the ideal Boltzmann gas $pV^* = NT$ and at $m/T \gg 1$ the particle mass can be neglected and $E \approx 3NT$. In the ultrarelativistic case $m/T \ll 1$ the particle mass is small and $E \approx mN + \frac{3}{2}NT$. As the result of Eq. (17), in both cases the "non-relativistic fugacities" will be conserved, \( \mu_{i(0)} = \mu_{i(f)} \), along the world lines of each fluid element. What happens to the APSD during the system evolution? In the case of relativistic expansion the averaging in (12) should be done in relativistic covariant form \( d^3r d^3p \rightarrow d\sigma \mu^\mu \frac{d^3p}{p_0} \), $f(t) \rightarrow f(\sigma)$, $f(t, v_L) \rightarrow f(\sigma, y)$ where the hypersurface $\sigma$ generalizes the hypersurface of constant time $t$. The 4-vector formed by the world lines of the $i$–fluid element crossing $\sigma$ and moving with 4-velocity $u^\mu$ is $\int dV_i^* d\sigma^\mu = \frac{\Delta V_i^* n^\mu}{u(\sigma) n_{i(0)}(\sigma)}$, where $n^\mu(x)$ is normal to $\sigma(x)$. Then ($k = 1, 2, ...$)

$$
\int dV_i^* d\sigma^\mu \int \frac{d^3p}{p_0} \exp[-k(p_\mu u^\mu - m)/T]\xi^k
$$

\( \sim T_i^{3/2} \xi^k \Delta V_i^* \) if $m/T \gg 1$ and \( \sim T_i^3 \xi^k \Delta V_i^* \) if $m/T \ll 1$. Since the "non-relativistic fugacity" $\xi = \exp[\mu - m]$, and corresponding values \( (mT_i)^{3/2} \Delta V_i^* \) are constants along the current line, then

$$
\int dV_i(0) d\sigma f_k = \int dV_i(f) d\sigma f_k
$$

and, finally, the values of phase-space densities \( \langle f(\sigma) \rangle \) and \( \langle f(\sigma, y) \rangle \) averaged over $d\sigma \mu^\mu \frac{d^3p}{p_0}$ are the same on different hypersurfaces. In other words, the totally averaged PSD of the relativistically expanding Boltzmann gas is approximately preserved during the system’s evolution if the mass of the particles is much higher or smaller than the temperature of the system.

The non-relativistic Bjorken-like model \( \mathbf{3} \) gives an example of such a conservation of the APSD, resulting in $m \langle f(t, v_L) \rangle = \frac{n_0}{8\sqrt{\pi}(mT_0)^{3/2}} = \text{const}$. The pion APSD \( \langle f(t, y) \rangle \) in the relativistic chemically frozen 3D Bjorken model \( \mathbf{3} \) with the initial conditions as in Table 1 is also roughly constant: it changes from the chemical to kinetic freeze-out stages 5 – 7 per cent only, but it increases with time even without feeddown from resonance decays! This means, that the non-relativistic fugacity of pions, $\exp[\mu - m]$, grows with time. This paradoxical behavior of the APSD is, however, typical for a mixture of heavy and light hadron gases. Indeed, let us imagine that we have a thermal mixture of two gases: massless (e.g., neutrino) with concentration $\kappa_1$ and massive (e.g., neutrons) with concentration $\kappa_2$: $\kappa_1 + \kappa_2 = 1$. Each of them will preserve its non-relativistic fugacity if they expand separately. For evolution of the mixture we have, generalizing (17), that $\kappa_1 \mu_1/T + \kappa_2 (\mu_2 - m)/T = \text{const}$ along the current line at isentropic expansion. One could suppose that the expansion is conditioned mainly by the heavy component: \( (mT_1)^{3/2} \Delta V_i^* \approx \text{const} \) during the evolution of the i-th fluid element. If then both fugacities are preserved then the entropy of the mixture will decrease: the light component needs the volume $\Delta V_i^* = \Delta V_i(0) \frac{\sigma(0)}{\sigma(f)} \approx \text{const}$ to preserve entropy at the same fugacity, but it gets only $\Delta V_i(0) \frac{\sigma(0)}{\sigma(f)} \Delta V_i(0) < \Delta V_i(0) \frac{\sigma(0)}{\sigma(f)}$ while the heavy component preserves its entropy. Thus, the fugacity of the light component should grow and, correspondingly, the fugacity of the heavy component should decrease. The expansion of the mixture then follows a law which is intermediate between $V \sim 1/T^{3/2}$ and $V \sim 1/T^3$. Such behavior was considered in detail in Ref. \( \mathbf{8} \). The mean mass of the chemically frozen hadron-resonance Boltzmann gas is significantly higher than the temperature of hadronization: $\bar{m} = \sum \kappa_j m_j = 0.662 \text{ GeV}$, where $\kappa_j$
is the concentrations of $j$-hadronic species. The aforementioned result on the increase of the pion APSD $\langle f(t, y) \rangle$ during the chemically frozen hydrodynamic evolution [8] is explained, therefore, by the fact that the pion gas is the lightest component in the mixture and so the pion non-relativistic fugacity should grow regardless of inelastic rescatterings and resonance decays.

3 Resonance influence on spectra and interferometry volume in A+A collisions

3.1 Emission and decays of resonances in quasi-classical approximation

The above analysis is based on thermal local equilibrium distributions which in hydrodynamic models describe so called direct particles and resonances. The pion spectra from resonance decays are not thermal and, even under an assumption of sharp kinetic freeze-out, are formed in some effective post freeze-out 4-volume that depends also on resonance widths. Unfortunately, one cannot ignore a priori the contributions to the pion spectra and interferometry radii by the short-lived resonances as they give more than half of total pion yields in ultrarelativistic A+A collisions. Such a big contribution arises since the chemically equilibrated composition of the hadron resonance gas, corresponding to relatively high hadronization temperature, is nearly frozen during the subsequent evolution to low temperatures. Of course, the resonance decays could take place during the evolution of the dense system and lead to some increase of number of (quasi) stable particles even before the kinetic freeze-out. On the other hand, there are back reactions that produce the resonances in the dense system. These reactions do not lead to chemical equilibrium, but just reduce the effect of resonance decays into chemically non-equilibrated media. According to the results of Ref. [8], nearly $2/3$ of all pions arise from the resonance decays after the thermal freeze-out if one totally ignores the reduction of resonance numbers during the hydro evolution. Taking into account a rather short duration of the hydrodynamic stage for the hadron gas, about $1.7 \text{ fm/c}$, which is comparable with the average life-time of the resonances, we can very roughly estimate that at least half of the pions arise from resonance decays during the post hydrodynamic stage.

The only attempt to analyse the contribution of large numbers of short-lived resonances to the spectra and interferometry volume was done in Refs. [27] within some hydrodynamic parametrization of the kinetic freeze-out. The aim of those papers was to fit the spectra and interferometry data from CERN SPS and BNL RHIC supposing that kinetic freeze-out happens just after hadronization. However, the analysis of relative resonance contribution to the interferometry radii has not been done probably since a non-physical approximation corresponding to infinite widths (zero lifetimes) of all resonances was used. But such estimates are necessary for an analysis of the phase-space and particle density properties at kinetic freeze-out. Pursuing this aim we analyse the resonance contributions to the spectra and interferometry radii based on the hydrodynamic model of Ref. [8] and take the widths $\Gamma$ of mesonic and baryonic resonances with masses up to $2 \text{ Gev}$ from Particle Data Group [28]. Because of technical reasons we ignore, however, the contributions from many-cascade decays. We hope these do not change the general picture and conclusions. As we show below, the single particle transverse pion spectra have the same main features as in the model accounting for the many-cascade decays of heavy resonances [27].

To evaluate the contribution from resonance decays to the pion spectra and interferometry radii we use the quasi-classical approximation [29, 30] for the emission function $g_i(x, p)$ of the $i$-th
resonance species:

\[ g_i(x, p) = \int_{\omega^-}^{\omega^+} d\omega^2 \varphi_i(\omega^2) \int d^3 p_i \frac{2m_i}{E_i} \delta((p_i - p)^2 - \omega^2). \]

\[ \Gamma_i \exp(-\Gamma_i \tau^*) \int d\sigma \mu(x_i) p_i^\mu \delta^{(4)}(x_i - \mu) f_{i, eq.}(x_i, p_i) \]

\[ \int_{\sigma}^\infty d\tau^* \Gamma_i \exp(-\Gamma_i \tau^*) \int d\sigma \mu(x_i) p_i^\mu \delta^{(4)}(x_i - \mu) f_{i, eq.}(x_i, p_i) \]

The emission function \( g_i(x, p) \) describes the production of pions with momentum \( p \) from the \( i \)-th resonance species, the latter being distributed according to the local equilibrium phase-space density \( f_{i, eq.}(x_i, p_i) \). The quasi-classical picture supposes that the \( i \)-th resonance is created from the hydrodynamic tube at a space-time point \( x_i^\mu \) of the kinetic freeze-out hypersurface \( \sigma \) and decays into \( \pi + X \) after some proper time with the mean value \( 1/\Gamma_i \). The probability to produce a group of particles \( X \) with an invariant mass \( \omega \) which accompany the pion at the decay of the \( i \)-th resonance is described by the covariant amplitude of the resonance decay \( \varphi_i(\omega^2) \) that is supposedly known.

The pion spectra and correlations are defined by the phase-space densities of thermal ("direct") pions and resonances at \( \sigma \). According to Ref. [8], the local Boltzmann distribution \( f_{i, eq.}(x_i, p_i) \) has a transverse Gaussian-like density profile with width \( R(\tau_{th}) \), and is attributed to the kinetic freeze-out hypersurface \( \sigma: \tau_{th} = const \) where the system is characterized by the constant temperature \( T_{th} \), boost-invariant longitudinal flow and transverse flow \( y_T = v_R r/R(\tau_{th}) \). The numerical values of correspondent parameters for Pb+Pb 158 AGeV collisions were extracted in Ref. [8] from an analysis of the spectra and interferometry radii at fairly large \( p_T \) to minimize the influence of resonance decays. They are \( T_{th} = 0.135 \) GeV, \( R(\tau_{th}) = 5.3 \) fm, \( v_R = 0.4 \). We take proper freeze-out time \( \tau_{th} = 8.2 \) fm/c for the "direct" pions [8], and use this freeze-out time also for resonances which produce pions. Note, however, that the average freeze-out time for different hadronic species was found to be \( 8.9 \) fm/c [8] which does not differ much from what we use. We do not intend in this paper to perform a detailed fitting of the spectra and radii; our aim is to analyse the dependence on flow intensity of the relative contributions of resonances to the spectra and interferometry radii within the appropriate model [8] and, thus, to estimate the influence of resonances on the phase-space densities. So, we just take the parameters which were found in [8], and use them for highest SPS energies. Also, we estimate roughly the correspondent contribution to the APSD at RHIC energies by using the same model, simply enhancing the intensity of transverse flows: \( v_R = 0.4 \rightarrow v_R = 0.6 \).

### 3.2 The results on spectra and interferometry and their interpretations

The analysis of the single pion spectra from decays of the fixed resonance species shows it is essentially non-thermal, non-exponential, at least in the transverse momentum region below \( p_0 \). One can see it at the top of Fig. 4 for the fixed heavy resonance species \( f_2(1270) \) with \( p_0 = 0.622 \) GeV. Nevertheless, the total contribution (thermal + decays) from many such resonances with different masses and different \( p_0 \) results in a slope of the transverse spectrum which, as one can see from Fig. 4, is surprisingly similar to the spectrum slope for the thermal ("direct") pions in the wide momentum region at different intensities of transverse flows. The same peculiarities of the resonance contributions to the transverse pion spectra were found also in Ref. [31] within the Landau hydrodynamic model with realistic freeze-out conditions. In contrast to these observations, the
results of Refs. [27] demonstrate a noticeable decrease of the effective temperature, or inverse of the slope of total spectra, as compared to thermal pions. The main reason for such a discrepancy is the (unrealistically) high decoupling temperature \( T \) that is taken to be equal to the hadronization, or chemical freeze-out one in Refs. [27]. It is qualitatively clear that the ratio of effective temperatures of total to thermal pion spectra depends on the relation between the mean \( p_0 \) and \( T \). We demonstrate this effect in Fig. 5 where one can see that the slopes of the total pion spectra are about the same as they are for thermal pions in the realistic interval of freeze-out temperatures 100 – 140 MeV. Above this region the pion effective temperatures for the total transverse spectra are significantly smaller than for just thermal spectra. The result corresponding to the decoupling temperature equal to the chemical freeze-out one, 163 MeV \([8]\), is close to what was found in Refs. [27] for similarly high temperatures \( T \approx 165 \) MeV. Clearly, at freeze-out temperatures below 100 – 140 MeV the transverse effective temperature of total pion production has to be noticeably larger than in the case of thermal pions only. The correspondent tendency one can perceive from the plots in Fig. 5.

Let us move to a discussion on the influence of resonance decays on Bose-Einstein correlations. Fig. 6 demonstrates that resonance decays extend the interferometry volume, especially at small transverse momenta. The ratio of the complete volume to the direct one grows when the intensity of transverse flow increases. It could be the double-effect of an increase in the resonance velocities in the c.m. of the system and correspondent Lorentz dilation for the resonance life-times; both effects increase the mean free path for outgoing resonances. Another impressive effect of the flow and resonance decays on the interferometry radii is demonstrated in Fig. 7. One can see that the "standard" hydrodynamic behavior of the ratio \( R_{out}/R_{side} \) which is typically larger than unity\(^1\) becomes essentially smaller, \( R_{out}/R_{side} < 1 \), outside the region of small \( p_T \) if the resonance decays are taken into account. The above effects are created by the rather complicated interplay between the kinematics of resonance decays and flows. A description and understanding of the correspondent effects are crucial for the correct estimates of the phase-phase densities.

Let us consider the effect of the relativistic longitudinal flow. If there is no flow and the resonance momentum spectra are, for instance, pure thermal, then all the points of the whole source volume are accessible with equal probability for resonance emission. Since the resonances have finite average life-times, they can decay inside as well as outside of the source volume, thereby, at any momenta the pion interferometry radii will be larger than the size of the source that emits the resonances. If the source expands relativistically in the longitudinal direction, then the spectra of heavy resonances are defined mainly by the flow, not the temperature. Let us suppose that pairs of identical pions with average rapidity \( y \approx 0 \) and half-sum of pion transverse momenta \( p_T \) are produced by two resonances \( 1^* \) and \( 2^* \) having, for simplicity, only two- particle modes of decay (pion plus, e.g., nucleon) with the same absolute value of pion momenta \( p_0 \) in the rest frame of each resonance. If one studies the long-projection of the correlation function, the transverse momenta of both pions is supposed to equal \( p_T \). Since only pions with close momenta interfere, the resonances should have rapidities \( y_1^* \approx y_2^* = \pm y_0 = \pm \sinh^{-1} \sqrt{\left(p_0^2 - m^2\right)/p_T^2} \) to guarantee \( y \approx 0 \), if the resonance masses are much bigger than the temperature and there are no transverse flows. Thus, the heavy resonances with \( y_1^* = y_2^* \) are emitted just from the two fluid elements with correspondent fluid rapidities \( y_i = y^*_i \) see Fig. 8. In the case \( y_1^* \approx y_2^* \) the fluid elements are overlapping. The sizes \( l \) of these fluid elements are rather small, \( l \approx \tau \sqrt{m_i} \), as compared to the system’s scales and also to the

\(^1\)Except for an (artificial) situation with a box-like density profile [32].
correspondent homogeneity lengths of direct pions. If the half-sum of pion transverse momenta is small, $p_T \ll p_0$, then the distance between the two fluid elements could be also small if $y^*_1 = y^*_2$ or fairly large, $L \gg l$, if $y^*_1 = -y^*_2$. The former configuration corresponds to situations a) and b) in Fig. 8 at the moment of resonance decays $\tau > \tau_{f.o}$. The later configuration c) increases significantly at small $p_T$ the long- interferometry radius, that corresponds to a Gaussian fit to the correlation function. It is enhanced additionally since in this case the velocities of the resonances are back-to-back directed (total momentum of pair is selected to be near zero), and they propagate for some time until they decay. At large $p_T > p_0$ the distance between the two resonances is reduced according to the kinematics, especially when transverse flow takes place, which is why the resonance contribution to the interferometry volume dies out with $p_T$ much faster than their contribution to the spectra, see Fig. 6.

Let us illustrate also the reduction of the ratio $R_{out}/R_{side}$ due to heavy resonance decays in the presence of transverse flows. For simplicity we omit the longitudinal coordinate and will keep in mind the linear with transverse radial flow and the Gaussian-like particle density profile at freeze-out. Let us consider at $p_T = p_0$ the interferometry in transverse plane for the pions coming, as we described above, from the one sort of resonances characterized by $m^*_i \gg T$ and with pion decay momentum $p_0$. In Fig. 9 we mark the four points a, b, c, d of emission of the resonances at freeze-out which are minimal and maximal in $x$ (out-) and $y$ (side-) directions at the given restriction for pion momenta. Those positions are dictated by the kinematics of the decay $(v_0 = p_0/E_\pi)$ to guarantee the pion transverse momentum to be $p_T = p_0$ and directed along the $x$ axis. As one can see from Fig. 9, the maximal sideways extension $R_{red}$ (along the $y$ axis) of the emission region for the resonances producing the pions is equal to the maximal outward extension $R_{ac}$ of the region, and the maximal relative resonance velocities in the $x$-direction $\Delta v_x = 2v_0$ is equal to the maximal relative velocities in the $y$-direction $\Delta v_y = 2v_0$. If the density were constant, the effective emission sizes would be $R_x = R_y$. But since the density decreases with the radius, the effective outward size of the emission region, accounting for the probabilities of emission, will be less, $R_x < R_y$, since point $R_c$ is maximally remote from the centre as one can see from Fig. 9. Then we have also $\Delta v_x < \Delta v_y$ effectively. It reduces the ratio of the interferometry out- to side- radii especially when mean life-time $\tau^*$ of the resonances are fairly large: $R_{out} = R_x + \tau^* \Delta v_x < R_{side} = R_y + \tau^* \Delta v_y$. The results of Fig. 7 demonstrate the total effect from the contributions of many resonance species in the 3D expanding hydrodynamic model.

Note that the above effects are typical in the hydrodynamic picture with sudden freeze-out when the fraction of pions produced by resonances is fairly big as takes place in the case the chemical freeze-out. In the opposite case, mainly direct-direct and direct-resonance pion pairs are important for interferometry radii formation; the correspondent results are well known [29] [30].

3.3 Resonance decays and phase-space densities

This detailed consideration allows us to clarify what we should measure to estimate the overpopulation of phase-space in the system at the latest stage of its evolution. First, the spectra of pions from decays of some fixed resonance species are primarily non-thermal; second, the resonances that produce the pion pair are emitted, in general, from different small elements of fluid separated by some distance that could be rather large at small transverse momenta of the pair. The pions of the pair are produced also at different (proper) times. This is completely unlike the emission of direct pions at freeze-out in the hydrodynamic picture: the pions with similar momenta $p$ are emitted from the same fluid element, the value of momenta $p$ dictates the size of that element
(the homogeneity length) and the HBT interferometry measures the correspondent size. Hence, the maximal phase-space density at each \( p \) is just the spectra \( n(p) \) divided by the interferometry volume and the APSD is less than it by a factor \( (2)^{3/2} \) if one supposes a Gaussian phase-space density profile (at fixed \( p \)!) outside of that fluid element. Such a quantity, the APSD, for the case when \( x-p \) correlations are absent and the Gaussian profile is universal for all \( p \) was analysed in the pioneer Bertsch paper [1]. For pions created by the resonances there is no such transparent physical meaning of the ratio of the spectrum to interferometry volume, even if the resonance width is set to infinity and so they are only allowed to produce pions at the freeze-out hypersurface. Indeed, if we suppose the absence of transverse flows, then at small \( p_T \) of pairs there are, as was discussed above in detail (see also Fig. 8), the two rather small, \( l \sim \tau \sqrt{T/m^*} \), and well separated by distance \( L \sim 2\tau \sinh \eta_1 \gg l \), fluid elements which emit the pions by means of resonances. Obviously, the maximal APSD here is proportional to \( n(p)/l \) and is a high value, while it will be estimated as a small value, \( \sim n(p)/L \), according to the Bertsch prescription, when the interferometry radius, extracted by a Gaussian fit to the interferometry peak, is proportional to \( L \). Of course, the correspondent correlation function in a large interval of \( qL \) is rather different than the Gaussian form; it is proportional to \( 1 + \exp(-q^2 l^2) \cos^2(qL/2) \), and this is the solution of the paradox - the simplest form, the Bertsch prescription, just fails here, but how does one distinguish such effects of the non-Gaussian contribution, when experimenters are fitting the total correlation function from many similar contributions to a Gaussian? The finite life-time of the resonances makes the situation even more complicated. We can conclude that the effective region (homogeneity volume), where the single pion spectrum from the resonance decays forms, is not associated with an interferometry volume of pion "resonance emission" that is topologically non-trivial and corresponds, in general, to the superposition of space-time separated Gaussian sources. In the next Section we propose a way to extract the actual APSD of pions in such a situation.

4 Extraction of pion fugacities in A+A collisions.

4.1 Sudden freeze-out vs continuous emission: how to treat the APSD?

Our aim is to propose a simple method to estimate the overpopulation of phase-space that does not depend on the details of hydrodynamic evolution. Before discussing it, let us make some comments on the applicability of the hydrodynamic approach to the post hadronization stage in A+A collisions. As we discussed in Sec. 2, there is approximate equivalence between the chemically frozen hydrodynamics of a hadron gas and its evolution within the cascade approach in the temperature region of applicability of hydrodynamics. Out of this region, below the temperature 0.12 GeV [5], the formation of spectra is basically continuous in time with fairly long "tails". That continuous emission is characterized by the emission function \( g(t, r, p) \) which, normally, is very far from thermal. The example of it is presented in Ref. [22] and is based on the exact analytic solution [11] of the Boltzmann equation for an expanding fireball. Unlike the very complicated structure of the emission function \( g(t, r, p) \) that was found in [22], the interferometry radii, spectra and the APSD have simple and clear analytical forms and correspond to the thermal distribution function in the fireball before it starts to decay. It looks like there is some kind of duality [34] in the description of the spectra and the interferometry data based either on the (thermal) distribution functions \( f(t, r, p) \) which characterize the system just before decay begins or on the emission function \( g(t, r, p) \) that describes the process of continuous particle emission after the system loses its simple thermal properties. The replacement of the complicated emission process by the simple
Landau [26] prescription of sudden freeze-out at the lower boundary of the region of applicability of hydrodynamics, at $T \approx 0.1 - 0.14$ GeV (depending on the system size) is, of course, a rather rough approximation. Nevertheless, as we discussed above, the momentum-energy conservation laws, particle and entropy (partial) conservation as well as some symmetry properties minimize the correspondent deviation for observables. As for the average phase-space densities, which, as we discussed in Sec. 2, are approximately frozen during the hydrodynamic evolution, there is no reason to expect that they are changed significantly during the transition to their complete freeze-out (the APSD does not change during the stage of free streaming).

Therefore, one can consider the analysis of the APSD in two aspects. The first one associates the observed APSD with the corresponding value \textit{in the thermal hadron medium} just before its decay, that allows one to obtain, based on the distribution function $f(t, r, p)$, the particle number and other densities in the medium at the pre-decay stage. On the other hand one can link the APSD with the properties of the \textit{particle continuous emission}, and thus analyse the process of particle liberation described by the emission function $g(t, r, p)$. If the decay of the medium starts at time $t_f$, then one can compare the particle density just before decay $n(t_f, r) = \int d^3 p f(t_f, r, p)$ with the density of "emission capacity" of the source at point $r$, $n_{\text{emit}}(r) = \int_{t_f}^{\infty} dt \int d^3 p g(t, r, p)$. Since the system continues to expand during the process of particle liberation, then $n(t_f, r) \geq n_{\text{emit}}(r)$ and the equality is reached only in the case of a \textit{real} sharp freeze-out when, in the non-covariant form, $g(t, r, p) = f(t, r, p) \delta(t - t_f)$. Thus the results of our analysis of the APSD based on the hydrodynamic picture with the Landau freeze-out prescription gives the possibility, on the one hand, to find the particle density \textit{in the thermal hadron medium} just before its decay and, on the other hand, to make the upper estimate of the "emission capacity" density for the continuous process of particle liberation.

It is clear, however, that the resonance decays at the post freeze-out stage destroy the above conception of duality in the APSD description since they seriously affect the post-hydrodynamic (pion) emission function $g(t, r, p)$. The attempt to attribute the decays of resonances just to the sharp freeze-out hypersurface will make an estimate of the pion density to be artificially high. Indeed, the proper life-time $1/\Gamma$ of resonances averaged over all the species is nearly $2 \text{ fm}/c$ that gives us, taking into account the Lorentz-dilation of life-times of resonances in the expanding swarm, an even larger value, say $2.5 \text{ fm}/c$, when the total number of resonances drops by a factor of $1/e$. Since at the last quasi-inertial 3D-stage of expansion the system volume grows roughly proportional to $\tau^3$, one can easily estimate, keeping in mind the typical (proper) time $10 \text{ fm}/c$ for the freeze-out in A+A collisions at SPS and RHIC, that despite the resonance decays bringing about half of the pion yields the pion densities nevertheless decrease at the post freeze-out stage! It means that at the final decoupled stage of the system evolution the maximal densities, including the APSD of pions, are reached at the thermal freeze-out, at the boundary of the applicability region of the hydrodynamic approach. Thereby, taking into account also that resonance contributions to the APSD is physically obscure and their including complicates the treatment of the APSD very much (see the discussion at the end of Sec. 3) we will exclude the resonance contributions to the spectra and interferometry radii when estimating the maximal pion APSD. Since the effective temperatures of the total pion spectra and direct pions are approximately the same until rather high $p_T$, one can just reduce the spectra by the corresponding factor, $1/3 - 1/2$, in accordance with the chemical freeze-out concept [6, 8]. As for the interferometry volume, the correction factor can be taken as 0.6 for SPS energies and 0.5 for RHIC energies as follows from Fig. 4 and will be explained below.
4.2 A model independent method extracting fugacity

Coming back to a hydrodynamic analysis of the spectra, interferometry radii and the APSD of thermal particles in Sec. 2, one can see (e.g. from Eqs. (6), (10)) the simple link between the above values in the non-relativistic hydrodynamic models

\[
\langle f(t, p) \rangle = \frac{d^3N}{d^3p}/(8\pi^3/2 V_{int})
\]  

Eq. (21) expresses the basic Bertsch idea [1] and can be generalized in covariant way for a relativistic situation [35]. If a system decays on some freeze-out hypersurface \(\sigma\), the PSD averaged over this hypersurface can be expressed through the two-particle correlation function \(C(p, q)\) of identical bosons as the following

\[
\langle f(\sigma, p) \rangle = \frac{1}{(2\pi)^2} \int \frac{d^3N}{d^3p} (C(p, q) - 1)d^3q
\]  

Thus, to estimate the APSD one should use the experimental data on spectra and correlation functions or just the interferometry radii if \(C(p, q) - 1\) is represented in standard Gaussian form, see (21). As for the left hand side of Eq. (22), it is strongly model dependent since it is determined by unknown functions in space-time such as the fugacity \(\xi = \exp(\beta\mu)\), temperature \(T \equiv 1/\beta\) and flow 4-velocities \(u^\mu\) in the local equilibrium Bose-Einstein distribution \(f(x, p) = (2\pi)^{-3}[\xi^{-1}\exp(\beta p\cdot u) - 1]\). More concretely, it was demonstrated in Ref. [14] that boost-invariant longitudinal flows in the system could bring significant error if the fugacity is extracted from \(\langle f(p) \rangle\) assuming a static source. It was shown also that the extracted value of fugacity depends strongly on transverse flows - all of which makes estimates of this value essentially model dependent. Therefore the experimental analysis (see, e.g. [20]) of the overpopulation of phase space is carried out within some concrete model and, thus, depends on many assumptions such as distribution functions on \(\sigma\) and the form of this freeze-out hypersurface in Minkowski space.

Here we propose a method that allows one to extract the fugacity in a model independent way. Let us explain the basic idea using the oversimplified example when the freeze-out happens at uniform temperature and uniform particle densities across the whole freeze-out hypersurface enclosing the space-time of the expanding hydrodynamic system. Then the phase-space density averaged over the hypersurface and momentum,

\[
\langle f \rangle = \frac{\int \frac{d^3p}{p_0} p^\mu d\sigma_\mu f^2(x, p)}{\int \frac{d^3p}{p_0} p^\mu d\sigma_\mu f(x, p)} = \frac{\int \left(\frac{d^3N}{d^3p}\right)^2 (C(p, q) - 1)d^3pd^3q}{(2\pi)^3N},
\]  

will be a suitable value which allows one to extract the fugacity at the freeze-out unambiguously, no matter to the form of the hypersurface and flow profile. Indeed, at each point \(x\) one can make the integration over \(p\) in the rest frame of the fluid element at the point \(x\) taking into account that in this comoving system the \(f(x, p)\) depends on the absolute value of \(p\) only (property of local equilibrium). Then, coming back to the common c.m. system, and exploring the Lorentz-invariant

\footnote{Note, it is erroneously claimed in Ref. [14] that such a discrepancy vanishes at high \(p_T\), while the fugacity is still a factor of \(\sqrt{2}\) larger (in the Boltzmann approximation) at high \(p_T\) than one could estimate supposing that the source is static.}
properties of the local equilibrium distribution function, one gets

\[
(2\pi)^3 \langle f(\sigma) \rangle = \frac{\int d\sigma \mu \omega(\sigma, \eta) \int d\mu \frac{d^3p}{\xi^{-1}(\nu, \eta) \exp(\beta(\nu, \eta)p_0) - 1}}{\int d\sigma \mu \omega(\sigma, \eta) \int d\mu \frac{d^3p}{\xi^{-1}(\nu, \eta) \exp(\beta(\nu, \eta)p_0) - 1}}
\]  

(24)

If, as we supposed, the temperature and particle densities are constant at freeze-out, then

\[
(2\pi)^3 \langle f(\sigma) \rangle = \frac{\int d\sigma \mu \omega(\sigma, \eta) \int d\mu \frac{d^3p}{\xi^{-1} \exp(\beta(p_0) - 1)^2}}{\int d\sigma \mu \omega(\sigma, \eta) \int d\mu \frac{d^3p}{\xi^{-1} \exp(\beta(p_0) - 1)}}
\]  

(25)

and \( \langle f(\sigma) \rangle = \langle f \rangle_{eq} \) where \( \langle f \rangle_{eq} \) is just the APSD for emission of a static thermal homogeneous source.

Despite the extreme simplicity of this result, it cannot be used practically for two reasons. The first is that the integration in Eq. (24) must be done over the whole closed freeze-out hypersurface, including, of course, butt-ends of the hydrodynamic tube. It implies the knowledge of one- and two-particle spectra in the whole momentum region \((y, p)\), including, of course, butt-ends of the hydrodynamic tube. The second important reason why the formula \( \langle f(\sigma) \rangle = \langle f \rangle_{eq} \) cannot be used is that the baryochemical potential and, hence, the temperature are not uniform across the longitudinal direction as the data on proton to antiproton ratios in different rapidity regions demonstrates \[36\]; see also the discussion about inhomogeneities of thermodynamic parameters in rapidity in Refs. \[3, 37\].

Fortunately, a result similar to (24) can be obtained for the central rapidity interval where the spectra are usually measured and the thermodynamic parameters are nearly uniform. Let us start from the well known observation \[26\] that at the latest stage of a hydrodynamic evolution the longitudinal velocity distribution corresponds to the asymptotic quasi-inertial regime, \( v_L = \frac{z}{t} \). It takes place even in the Landau model of complete stopping, and, of course, in the Bjorken model \[38\] where this quasi-inertial regime exists at all times of hydrodynamic evolution. Further, if deviations of thermodynamic values such as temperature, baryochemical potential, etc. in the longitudinal direction near central fluid rapidity \( \eta = 0 \) are negligible within the maximal rapidity length of homogeneity \[39\], \( \sqrt{\frac{T}{m_\pi}} \simeq 1 \), then all physical values at mid-rapidity can be calculated supposing boost-invariance. The latter implies that the distribution function which is applied for evaluation of the spectra at mid-rapidity near \( y = 0 \) can be considered to depend only on the difference between fluid and particle rapidities, \( \eta - y \). As a result, the APSD at fixed particle rapidity \( \langle f(\sigma, y) \rangle \) is equal to the APSD \( \langle f(\sigma, \eta) \rangle \),

\[
\langle f(\sigma, \eta) \rangle \equiv \frac{\int f^2(t, r, p) \mu \frac{d\sigma}{dn} \frac{d^3p}{p_0} \int f(t, r, p) \mu \frac{d\sigma}{dn} \frac{d^3p}{p_0}}{\int f(t, r, p) \mu \frac{d\sigma}{dn} \frac{d^3p}{p_0}}
\]  

(26)

at the fixed ratio \( r_z/t = \tanh \eta \), corresponding to the fluid rapidity \( \eta = y \). Then using the Lorentz transformations as described above one can absorb all dynamic and kinematic characteristics into common factor \( V_{eff}(\eta) = \int \frac{d\sigma}{dn} \omega(\sigma, \eta) \) for numerator and denominator in Eq. (26). Note, if the freeze-out hypersurface \( \tau(r) \) has non-space-like sectors, the measure of integration in (26) should be modified to exclude negative contributions to particle numbers at some momenta. As a result, it contains \( \theta - \) functions like \( \theta(p_\mu n^\mu(x)) \) where \( n^\mu \) is a 4-vector orthogonal to the hypersurface \( \sigma \) \[39, 40\]. If a fluid element that crosses the time-like sector of the freeze-out hypersurface decays
preserving its total particle number, then the measure of integration is modified according to the prescription in Ref. \[39\]. In that case, after integration over all momenta at each point \(x\) the factorization of the numerator and denominator in \(26\) takes place again resulting finally in the common factor \(V_{\text{eff}}(\eta)\) which cancels. Thereby, the expression \(26\) for the APSD can be used in the same form even when freeze-out hypersurface contains non-space-like sectors, if one considers the prescription from Ref. \[39\] to describe the decay of the system on the time-like parts of the hypersurface. Finally, the common effective volumes are canceled and the result is

\[
(2\pi)^3 \langle f(\sigma, y) \rangle_{y=0} = \int \frac{dp}{\xi^{-1} \exp(\beta p) - 1} \int \frac{dp}{\xi^{-1} \exp(\beta p) - 1}
\]

Here the inverse of temperature \(\beta\) and fugacity \(\xi\) characterize thermal properties of the decaying system at mid-rapidity and are supposed to be approximately constant within one unit of rapidity near \(\eta = 0\). On the other hand, the above value of the APSD is expressed through the transverse spectra and the interferometry radii at mid-rapidity similar to Eq. \(23\),

\[
(2\pi)^3 \langle f(\sigma, y) \rangle \approx \kappa \frac{2\pi^{5/2} \int_{R_O R_S R_L} \left( \frac{d^2 N}{2\pi m_T dN_T dy} \right)^2 dN_T}{dN/dy}
\]

Here we neglect interferometry cross-terms since they are usually rather small in the mid-rapidity region. The factor \(\kappa\), which will be calculated later from the results of the preceding Section, is introduced to eliminate contribution of short-lived resonances to the spectra and interferometry radii. It absorbs also the effect of the suppression of the correlation function due to long-lived resonances.

Thereby we proved, using very natural assumptions, the possibility of extracting in a model-independent way the fugacity in expanding thermal systems at the stage of their freeze-out, realizing, thus, the program which was declared by Bertsch in his pioneer paper \[1\].

4.3 Pion APSDs and chemical potentials at SPS and RHIC

Let us use our approach to find the chemical potential of direct pions at thermal freeze-out at SPS and RHIC. First we have to obtain the value of \(\kappa\) in Eq. \(28\). This coefficient includes, in particular, the factor \(\sqrt{\Lambda}\) (see, e.g., review \[41\]) arising because decays of the long-lived \((l)\) resonances such as \(\eta\)-, \(\eta'\)-mesons give the contribution \(d^3 N^{(l)}_i / d^3 p\) to the total pion spectra but not to the observed interferometry radii. The reason is that the (theoretical) correlation function due to pions produced by the long-lived resonances contains very narrow (much smaller than momentum resolution \(q_{\text{min}}\) of a detector) peak at \(q = 0\) on the top of smooth curve associated with bulk pion source. This narrow peak gives, in fact, no contribution to integral over \(q\) of \((C(p, q) - 1)\) (see Eq. \(23\)) in Bertsch’s method, and, therefore, results just in reduction of the total value of the APSD. In accordance with the estimates made in Refs. \[29\] \[42\] we choose the effective suppression factor \(\Lambda\) which measures the fraction of pion pairs containing no pions from long-lived sources,

\[
\Lambda(p) = \left(1 - \frac{d^3 N^{(l)} / d^3 p}{d^3 N / d^3 p}\right)^2 < 1,
\]

to be equal to 0.8; thereby \(\kappa\) contains a factor \(\sqrt{\Lambda} \approx 0.9\). Note, that experimental value of the suppression factor is, normally, smaller than the above theoretical value since the many effects,
except the decays of long-lived particles, could suppress the measured correlation functions. The most important among them are the single- and two-track resolution and particle misidentification. These effects are irrelevant to the physics of A+A collisions and we use, therefore, theoretical value of the suppression factor associated with long-lived resonances.

Since the slopes of the complete pion spectra and thermal pions are very similar in a wide $m_T$ region (see Fig. 4), we just reduce the experimental spectra by the ratio of the total pion number to the number of direct pions that follows from an analysis of the particle number ratios within the concept of chemical freeze-out. We use the reduction factor of 3 as is found in Ref. [8], also we estimate for comparison the fugacity supposing the correspondent reduction factor to be 2. The ratio of the complete to “direct” interferometry volumes can be fitted by a $const/\sqrt{m_T}$ function in the region shown in Fig. 4. We take into account this momentum dependence to define the effective reduction factor to observed interferometry volume which we will accumulate again in $\kappa$ in Eq. (28). The correspondent contribution is 1.65 for central Pb+Pb 158 AGeV collisions at CERN SPS and is 1.92 for Au+Au $\sqrt{s_{NN}} = 130$ GeV at BNL RHIC. Thus, $\kappa = 0.9 \cdot (1/3) \cdot (1.65 - 1.92) = 0.5$ (SPS) - 0.6 (RHIC). If one supposes the ratio of the total pion number to the direct one to be 2, it changes the values of $\kappa$ to $\kappa = 0.65$ (SPS) - 0.7 (RHIC). We use also the latter values to estimate the sensitivity of the results to this ratio.

To evaluate the APSD of direct pions [28] we utilize the following parametrization of the $\pi^-$ transverse spectra and interferometry radii.

For SPS Pb+Pb(Au) 158 AGeV:

The transverse spectrum is $\frac{dN^-}{2\pi m_T dm_T dy} = A \exp(-m_T/T_{eff})$, $T_{eff} \approx 0.180$ GeV; the midrapidity density is $\frac{dN^-}{dy} \approx 175$ (the NA49 Collaboration, [13]). The interferometry radii are $R_L = C_L/\sqrt{m_T}$, $R_S = C_{S,1}/\sqrt{1 + C_{S,2}m_T}$ and correspondent numerical parameters are taken from Ref. [44] (the CERES Collaboration). We use the approximation $R_O = R_S$ until the minimal measured $p_T$ momentum, $p_T = 0.125$ GeV, and our analytical approximation of the CERES outward interferometry radii data for $p_T > 0.125$ GeV.

For RHIC Au+Au $\sqrt{s_{NN}} = 130$ GeV:

The transverse spectrum is $\frac{dN^-}{2\pi m_T dm_T dy} = A \left(\exp(-m_T/T_{eff}) - 1\right)^{-1}$, $T_{eff} \approx 0.218$ GeV, and $\frac{dN^-}{dy} \approx 249$ (the STAR Collaboration, [15]). We use here the Bose-Einstein parameterization of the transverse spectra from Ref. [15] since the integrated rapidity density of negative pions with this fitting function is closer to value $\frac{dN^-}{dy} \approx 270$ presented recently by the PHENIX Collaboration at midrapidity [40] than what one can get from the exponential parameterization of the transverse spectra, $\frac{dN^-}{dy} \approx 229$ [15]. The phenomenological parameterization of interferometry radii, $R_L = C_L/\sqrt{m_T}$, $R_S = C_{S,1}/\sqrt{1 + C_{S,2}m_T}$, and the correspondent numerical parameters are taken from Ref. [47] of the PHENIX Collaboration. As one can see from Fig. 3 of Ref. [47], there is some discrepancy in the data on $R_S$ radii between the STAR [18] and PHENIX Collaborations [47]. To optimize uncertainties in the forthcoming estimates, we utilize for the parameter $C_{S,1}$ the value 8.75 fm that is the average between ”PHENIX data motivated” value, 8.1 fm, and ”STAR data motivated” value, 9.4 fm, [47]. We use the approximation $R_O = R_S$.

The results of our calculations of the APSD according to (28) are collected in Table 2. The values found are used then to extract the pion fugacities based on our result (27). In Table 3 we present the pion chemical potentials and densities at different typical freeze-out temperatures.

TABLE 2. The average phase-space densities (APSD) of negative pions for all particles ("raw") and for thermal ones.
\[
\left(2\pi\right)^4 \left< f(\sigma, y) \right>
\]
without corrections
\[
\kappa \quad \text{corrections for resonances}
\]
\[
\left(2\pi\right)^4 \left< f(\sigma, y) \right>
\]
for thermal (direct) pions

|               | \(\sqrt{s_{NN}} \) = 17.3 GeV | \(\sqrt{s_{NN}} \) = 130 GeV |
|---------------|--------------------------------|-----------------------------|
| SPS           | 0.210                          | 0.240                       |
| \(\sqrt{s_{NN}} \) = 17.3 GeV | 0.5 – 0.65                    | 0.6 – 0.7                   |
| RHIC          | 0.105 – 0.137                  | 0.144 – 0.168               |

TABLE 3. The chemical potentials, "raw" and for thermal pions, and thermal densities of negative pions vs critical densities extracted at two typical temperatures of the kinetic freeze-out.

All the values are related to the negative pions. The values in square brackets in the last column of Table 3 are the critical pion densities (in the rest frames of the fluid elements) at the correspondent temperatures for one (negative) component of the isospin pion triplet. As we can conclude, no exotic phenomena associated with the overpopulation of phase-space could be expected at the current energies. Some increase of the APSD takes place at RHIC; the pion chemical potentials are higher than at SPS and correspondingly the pion number density at RHIC is closer to the critical value for Bose-Einstein condensation. However the density values are still too small to expect the critical phenomena, such as serious spreading of multiplicity distributions, significant reduction of the interferometry radii, etc., which are associated with the Bose-Einstein condensation of pions in momentum space. Note, as was discussed above, our estimates of the pion densities at the end of the hydrodynamic stage of the evolution are, at the same time, the upper limits for the "emission capacity" density of the source of the direct pions.

The APSDs per pion component in the chemically equilibrated neutral pion gas are 0.07 and 0.09 for \(T_{th} = 0.100 \text{ GeV} \) and \(T_{th} = 0.120 \text{ GeV} \) respectively. As one can see from the Table 2, the estimated APSD for thermal pions are somewhat higher than in the case of chemical equilibrium. Thereby some corrections to observables conditioned by the partial overpopulation of the pion phase-space should be taken into account in advanced models of A+A collisions.

5 Conclusions

An approach which allows one to estimate the chemical potential and, thus, the overpopulation of particle phase-space at thermal freeze-out in a way that does not depend on the flow profile.
and form of the freeze-out hypersurface has been developed. It realizes Bertsch’s basic idea to find from the interferometry data the phase-space density averaged over both configuration and momentum spaces and compare it with the correspondent value for static source with equilibrium (homogeneous) Bose-Einstein thermal distribution to estimate thereby the overpopulation of the phase-space. We have proved that under the assumption of uniformity of particle density at the freeze-out hypersurface the flows and form of the hypersurface are absorbed into a factor that is canceled when the APSD is determined. It is worth to note that such a “cancellation” takes place not only for the APSD. For example, as one can find easily under the same conditions, the thermal pion entropy per particle also does not depend on the form of the hypersurface and flows.

Another problem that has been analysed is the resonance contributions to pion phase space densities. Because of chemical freeze-out an essential fraction of pions, about half, will be produced by short-lived resonances after thermal freeze-out. It is found that the decays of many heavy resonances correct significantly the interferometry radii of direct pions. It could result in up to a 50 per cent increase of the correspondent interferometry volume at small transverse momenta. Since the heavy resonances have, typically, large momentum $p_0$ of pions produced, the resonances contribute to the interferometry radii up to $p_T \sim 1 \text{ GeV}$. One of the important consequences of this is the ratio of the outward interferometry radius to the sideward one, $R_{\text{out}}/R_{\text{side}}$, that becomes less than unity due to heavy resonance decays in contrast to standard results $R_{\text{out}}/R_{\text{side}} > 1$ for direct pions in the hydrodynamic models (except for the so-called ”blast-wave” model [32], see Footnote 1). It could be a step towards understanding the HBT puzzle.

The correlation function of two identical pions arising from decays of heavy resonances of fixed species, even if one neglects their life-times, is essentially non-Gaussian, especially at small pion transverse momenta. Then, if one gets the interferometry radii from a Gaussian fit to the correlation function in the region of the interferometry peak, the longitudinal radii will be larger than the homogeneity lengths of regions where the spectra of ”resonance pions” are formed. This violates the natural physical correlation $\langle f(t,p) \rangle \sim \frac{d^3N}{dp^3}/V_{\text{int}}$ that is typical for direct particles. Accounting for the finite life-times of resonances makes the picture even more complicated: the emission is now spread out in time direction and therefore the $V_{\text{int}}$ cannot be attributed to the thermal freeze-out hypersurface. We have argued thereby that resonance contributions should be excluded when the averaged pion phase-space densities at the freeze-out stage are determined. Our estimates of the total and thermal spectra and the ratios of correspondent interferometry volumes at different intensities of transverse flow, including what are typical for SPS and RHIC energies, gives the possibility of restoring the APSD of thermal pions at thermal freeze-out. We argue that the values found are the maximal (total) pion APSD at and after the decoupling of the hydrodynamic system.

We found that the overpopulation of thermal pions at the thermal freeze-out, $100 \leq T_{\text{th}} \leq 120 \text{ MeV}$, is associated with the pion chemical potentials $15 - 51 \text{ MeV}$ for Pb+Pb(Au) 158 AGeV collisions at CERN SPS and $42 - 64 \text{ MeV}$ for Au+Au $\sqrt{s_{NN}} = 130 \text{ GeV}$ collisions at BNL RHIC. The estimates of the correspondent values in the recent paper [49] based on the concrete hydrodynamic parametrization are close to our results for SPS energies, while for RHIC energies the pions chemical potentials estimated in Ref. [49] are somewhat higher. The thermal pion densities turn out to be essentially smaller than the critical ones. The correspondent values at the SPS energies are $0.014 - 0.017 < 0.065 \text{ fm}^{-3}$ for $T_{\text{th}} = 100 \text{ MeV}$ and $0.022 - 0.027 < 0.095 \text{ fm}^{-3}$ for $T_{\text{th}} = 120 \text{ MeV}$. For the RHIC energies we found $0.018 - 0.020 < 0.065 \text{ fm}^{-3}$ for $T_{\text{th}} = 100 \text{ MeV}$ and $0.028 - 0.031 < 0.095 \text{ fm}^{-3}$ for $T_{\text{th}} = 120 \text{ MeV}$. Thereby the multibosonic effects at those energies would be considered rather as a correction factor than as an important physical phenomenon.
It looks puzzling that the averaged phase-space densities of thermal pions at RHIC are higher than at SPS, \(0.144 - 0.168 > 0.105 - 0.137\) while the particle freeze-out densities are comparable. The detailed analysis of the evolution of the APSD could explain such a behaviour. Indeed, if one imagines that at some initial time hadronic system has no significant transverse flow, then the averaged PSD is proportional to the PSD and particle density, as in a hot, dense gas in a box, and all the densities are high. However, when the system expands and transverse flow develops, the behaviors of the APSD and particle density are completely different. While the pion density is reduced dramatically, at the last stage as \(t^{-3}\), the pion APSD changes (it can even increase) rather slowly. Thereby the systems reach kinetic freeze-out at some typical particle densities that are similar for different collision energies, and, at the same time, their APSD could increase with energy since it ”memorizes” the higher initial hadronic densities at higher energies. It is found that such a ”memory” is conditioned by the conservation of entropy and particle numbers during chemically frozen hydrodynamic evolution of the mixture of hadronic gases.

The main aspect of the HBT puzzle concerns the energy dependence of the interferometry volume that turns out essentially smaller than expected at the RHIC energies. Our results could shed light on this phenomenon. Since the pion APSD and effective transverse temperature do not change essentially during the evolution, the ”interferometry volume”, \(V_{\text{int}} \sim (dN/dy)/(T_{\text{eff}}^3 \langle f(t) \rangle)\), defined formally at any time during the evolution of hadronic system, also changes rather slowly with time. The concrete mechanism of the effect is that, if the intensity of the transverse flow grows during the evolution, the contribution of an enlarged geometric radius of the expanding system to the interferometry volume is almost compensated by a stronger reduction of the interferometry radii due to the strengthening of the flow (see, e.g., [21]). The initial effective volume is, thus, partially frozen in the form of the interferometry volume. There is a rough similarity between the effective, or ”interferometry” volumes at higher (RHIC) and smaller (SPS) energies because the larger configuration volume at hadronization stage at RHIC (particle densities at the hadronization are approximately the same as at the SPS) is partially compensated by the higher intensities of flow. Therefore, the unexpectedly small increase of the interferometry volume at RHIC is caused by strengthening of transverse flows at higher energies. During the evolution of hadronic system the APSD is reduced rather slowly ("frozen") while the particle density falls down quickly until it reaches the standard conditions for system decay. That is the reason why the observed value of the APSD has no direct link to the freeze-out criteria and final thermodynamic parameters, being connected rather to the initial phase-space density of hadronic matter formed in relativistic nucleus-nucleus collisions.

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Figure 1: Evolution of the pion effective temperature $T_{eff}$ (solid line) versus the temperature of the system $T$ (dashed line) in the non-relativistic one-component Bjorken-like model with transverse expansion. The plots correspond to initial conditions: $T_0 = 0.1 \text{ GeV}$, time $t_0 = 5 \text{ fm/c}$, transverse radius $R_0 = 5 \text{ fm}$ and initial transverse flow $\dot{R}(t_0) = 0.3$. 
Figure 2: The average phase-space densities $\langle f(\tau, p) \rangle$ at typical proper times: at hadronization, $\tau = 7.24 \text{ fm/c}$, (solid line) and at kinetic freeze-out $\tau = 8.9 \text{ fm/c}$ (dashed line). The dot-dashed line corresponds to the "asymptotic" time $\tau = 15 \text{ fm/c}$ of hydrodynamic solutions continued beyond kinetic freeze out. The initial conditions of hydrodynamic expansion are taken from Table 1 and based on Ref. [8].
Figure 3: The space densities $n(r)$ taken at typical proper times: at hadronization, $\tau = 7.24$ fm/c, (solid line) and at kinetic freeze-out $\tau = 8.9$ fm/c (dashed line). The dot-dashed line corresponds to the "asymptotic" time $\tau = 15$ fm/c of hydrodynamic evolution continued beyond kinetic freeze out. The initial conditions of the hydrodynamic expansion are taken from Table 1 and based on Ref. [8].
Figure 4: The pion transverse spectra for the complete pion yields (solid lines) and for "direct" pions only (dashed line) in the cases with and without transverse flow: $v_{R} = 0, 0.4$ and $0.6$. The shape of the pion spectra from the fixed heavy resonance species $f_{2}(1270)$ is presented for illustration by dot-dashed line for $v_{R} = 0$. The overall normalization is arbitrary.
Figure 5: The pion transverse spectra for the complete pion yields (solid lines) and for "direct" pions only (dashed line) taken at a few typical temperatures without transverse flow, $v_R = 0$. The overall normalization is arbitrary.
Figure 6: The ratio of the complete interferometry volume $V_{\text{tot}}$, that includes effects of the resonance decays to pions, to the interferometry volume for "direct" pions taken at a few typical intensities of flow. The resonance contributions are calculated in accordance with the chemical freeze-out conception, when approximately $2/3$ of pions are produced by resonance decays. The dashed line corresponds to $v_R = 0$, solid line: $v_R = 0.4$ and dash-dotted line: $v_R = 0.6$. The other parameters are taken from Ref. \[8\] as described in the text.
Figure 7: The $R_{out}$ to $R_{side}$ ratio for complete interferometry radii (solid line) vs ones for "direct" pions (dashed line) at transverse flow $v_R = 0.6$ which is associated with RHIC energies.
Figure 8: The two-pion spectra formation at $p_T \simeq 0$ and $y \simeq 0$ due to decays of heavy resonances involved in the longitudinal flow before and at the freeze-out time $\tau_{f.o.}$, see the text for details.
Figure 9: The formation of the pion transverse spectra for decays of the heavy resonances involved in transverse flows as it is described in the text. The solid arrows are associated with radial flow, the dashed arrows correspond to emission of pions from the resonances.