Gauss polynomials based modelling of grid-connected PV system output power in high-altitude area

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Abstract. Complicated ordering management and improper providing electricity are a problem encountered during the PV system generate electricity. Accurately predict the output power is an effective approach for minimizing these problems. In this paper, electric power exchange relationship of each link is analysed, according to the topology of the photovoltaic (PV) power stations. We propose the under prediction model of output power, based on the theory model of output PV current in the high-altitude environment. The composite error could be calculated by the difference between the under prediction power and the measure power. Comprehensive error expression is obtained by using the high-order polynomial fitting method. An improved prediction model, which adapts to the high-altitude environment, is established, while compensate the original model with the composite error. Compared with the actual output power of the PV power station, the results show that the curves of the two are highly consistent, the correlation between the two is more than 0.8 and the relative error between the predicted and measured power is less than that of the under-predicted power model. The results show that the improved predictive power model has a good tracking performance.

1. Introduction
Photovoltaic technology can provide a more environmentally friendly alternative for fossil energy power generation. In the last two decades, the rate of compound of PV system is more than 15% per annum [1, 2]. However, renewable energy needs more maintenance than traditional energy sources. Despite the development of new high-performance materials, the efficiency of the PV system is still limited, ranging from 12% to 20% [3]. The output power of PV system is also different due to the different total radiation in different regions. In addition to radiation, ambient temperature, battery temperature and weather conditions will also affect output power [4].

Predicting the output power of grid-connect PV system has become a major trend in recent years. As the output power of PV system is random and intermittent fluctuation [5, 6], when large-scale PV power station merge into the power grid, it will impulse the power grid company in dispatching management, power balance and the safety and stability of electricity generation, so the grid company should strictly monitor and record the output power quality of photovoltaic power station. Therefore, accurate prediction of grid connected output power becomes a problem to be solved urgently.

Many scholars have studied the output power of grid-connected PV system. Ayompe had compared the capture loss, the component temperature loss, the component conversion efficiency and the system power efficiency of PV array in different places [7]. Li studied the performance of PV system from the view of the economic and environmental, which provides a new direction for the research of PV system [8]. Typical power forecasting models are mainly divided into three categories: physical model,
time series model and neural network model [9]. The first type is based on the mathematical model of PV modules and the conversion efficiency of each device in the PV power station, and predicts the output power of the grid-connected, but there is no error compensation [10, 11]. Su proposed power forecasting model based on time series, because the power output curve shows a normal distribution in sunny weather, so the Gaussian formula is used to approximate the output power, but a large amount of historical data is needed to determine the parameters of the Gaussian formula [9]. The neural network model is complex and requires irradiance, ambient temperature and component temperature as input [12-14]. Moreover, the design of the model is subject to historical experience. Since the pioneering work of Goh and Tan [15], time series models have been well established for statistical prediction of solar data. Therefore, the purpose of this paper is to propose a simple but accurate power prediction model.

Firstly, the topological structure of PV power station and the relationship between power and electricity in each link are analyzed, owing to the parameter of each link are difficult to obtain or measure, the power model that is not included in these parameters is defined as an under prediction power model. Then the daily under-predicted output power is calculated by using irradiance and ambient temperature, the predicted power subtracts the actual power, the difference between the two is defined as a composite error, the results show that the daily composite error curve shows a Gaussian distribution, thus the Gaussian polynomial is used to approximate the monthly composite error. Finally, the composite error polynomial is compensated for the under prediction power model, and the prediction model of the grid-connected output power in the high-altitude desert area is obtained. The results show that the grid-connected output power is predicted by the proposed model can track the actual output power well.

2. Grid-connected power prediction model for PV system

2.1. Output current model of silicon battery board in high altitude area
The choice of the output current model for silicon panels requires a strong theoretical foundation. At the same time, its static parameters must be within the date range provide by the panel manufacturer, so the model is selected as follows [16]:

\[
I = I_s \left\{ 1 - \exp \left[ \left( U - U_{oc} \right) \ln \left( 1 - \frac{I_{mpp}}{I_{sc}} \left( U_{mpp} - U_{oc} \right)^{-1} \right) \right] \right\} \tag{1}
\]

Where, \( I \) is output current for a single cell board; \( U \) is the terminal voltage of a single battery board; \( I_{sc} \) is short circuit current; \( U_{oc} \) is open circuit voltage; \( I_{mpp} \) is the maximum current; \( U_{mpp} \) is the maximum voltage.

Considering the changes of environmental temperature and irradiance in high-altitude desert area, the temperature of PV module is:

\[
T = T_{air} + \delta \tag{2}
\]

Where, \( T_{air} \) is the ambient temperature, \( \delta \) is the temperature coefficient of battery board, and \( S \) is the irradiance.

The temperature difference and irradiance difference are as follows:

\[
\Delta T = T + T_{ref} \tag{3}
\]

\[
\Delta S = \frac{S}{S_{ref}} - 1 \tag{4}
\]

Where, \( T_{ref} \) and \( S_{ref} \) are ambient temperature and irradiance under standard conditions, respectively.
(usually $T_{ref} = 25 \text{ and } S_{ref} = 1000 W / m^2$).

Therefore, the open circuit voltage, short circuit current, maximum current and maximum voltage under the general operating conditions are as follows:

\[ U_{oc} = U_{oc} (1- \gamma \Delta T) \ln(e + \beta \Delta S) \]  
(5)

\[ I_{oc} = I_{oc} \frac{S}{S_{ref}} \left[1 + \alpha(\Delta T + 273.15)\right]^{-1} \]  
(6)

\[ I_{mpp} = I_{mpp} \frac{S}{S_{ref}} \left[1 + \alpha(\Delta T + 273.15)\right]^{-1} \]  
(7)

\[ U'_{mpp} = U'_{mpp} (1- \gamma \Delta T) \ln(e + \beta \Delta S) \]  
(8)

where $U_{oc}$, $I_{oc}$, $I_{mpp}$, $U_{mpp}$ are the open circuit voltage, short circuit current, maximum current and maximum voltage under the standard conditions, $\alpha$ and $\gamma$ are temperature coefficient of current variation and voltage variation under reference illumination conditions, usually $\beta = 0.5$.

### 2.2. Grid-connected power model and under predictive power model of PV system in high-altitude desert area

The PV array has $m$ strand and each strand battery board is connected in parallel, and each strand is connected by the same type of battery plate in series with $n$ blocks. The total output current and the total terminal voltage can be expressed as follows:

\[ I_z = ml \]  
(9)

\[ U_z = nU \]  
(10)

Where $I_z$ is the output current of single solar panels, $U_z$ is the terminal voltage of single solar panel.

And the total power becomes:

\[ P_z = \eta_z I_z U_z = \eta_z mnIU \]  
(11)

Where $\eta_z$ is the comprehensive efficiency of the PV array.

The conversion relationship between the total power of PV array ($P_z$) and the effective value of the output power ($P_n$) is $L_z$, and the PV array is connected to a transformer and inverter. Therefore, the effective value of the output of the output grid power is shown as follows:

\[ P_n = \eta_z\eta_t LP_z = \eta_{Lz}\eta_t\eta_z LmnIU \]  
(12)

Where $\eta_{Lz}$ is the efficiency of the transformer, $\eta_t$ is the efficiency of inverter.

The nonlinear curve of $\eta_{Lz}$ and $\eta_t$, are usually provided by manufacturers. And the comprehensive efficiency of PV engineering is not easy to obtain need field measurement. In order to reduce the amount of engineering and convenient for calculation, $\eta_z$, $\eta_{Lz}$ and $\eta_t$ will bring about deviations, and make the deviations incorporate into the comprehensive error ($\varepsilon_p$), therefore the prediction power of grid-connected PV system is:

\[ P = LmnIU + \varepsilon_p = P_n + \varepsilon_p \]  
(13)
Where $\epsilon_p$ is the comprehensive error polynomial. And the under prediction power of grid-connected PV system is:

$$P_n = LnnIU$$  \hspace{1cm} (14)

3. Comprehensive error

The PV power station in Qinghai University has record the actual grid-power, irradiance and ambient temperature since 2016 June. By calculating the daily predictive power and actual power from June to December, the actual comprehensive error curve was obtained. In order to observe the change regulation of the actual comprehensive error curve, the different forms of the actual comprehensive error curve in the 6 months were classified, which are shown in figures 1 and 2.

The actual comprehensive error curves in 6 months are shown in figures 1 and 2. For the weather changes in a month, the comprehensive error also varies with the weather. We summarized the following two kinds of comprehensive error and both of them are piecewise Gaussian distribution, the two curves, as shown in figure 3.

![Figure 1. Actual comprehensive error curve 1.](image-url)
As can be obtained from figure 3, the comprehensive error power varies with time $t$, and approximates the Gauss curve. Because of the nonlinearity of the curve, to reduce the computational complexity, the Gaussian polynomial is used to define the comprehensive error relation formula.

The comprehensive error function is established as follows:

$$\varepsilon_p(t) = a + bt + ct^2 + dt^3 + et^4 + \ldots$$ \hspace{1cm} (15)

The composite error of each moment is determined by the time $t$. $a - e$ are the weight coefficient, and can be changed according to different ambient requirements.

4. Actual cases and model validation

4.1. Actual case description
Take the photovoltaic power station, which is now operating in Qinghai University, as an example:

- This article uses the STS-156P-255W polysilicon battery board, factory parameters under standard conditions are provided by the manufacturer are: short-circuit current $I_{sc} = 8.95$ A,
open-circuit voltage $U_{oc}=37.8$ V, maximum current $I_{mpp}=8.32$ A and maximum voltage $U_{mpp}=30.65$ V.

- The topology of PV power station. A total of 12 pieces of battery boards are connected in series. After an inverter is connected to the grid, no transformer is connected.
- Acquisition and recording data. The irradiance and ambient temperature are collected every 1 minute with a light-wet-temperature recorder, and the sampling period was December 2016. The output power of the PV power generation system is recorded every 1 minute with the power recorder, which is the actual power. The acquisition point of the actual power value is the inverter outlet side, and the power prediction is also carried out at the point.

![Figure 4](image)

**Figure 4.** The curves of under-predicted power, measured power and comprehensive error. (a) The curves of under-predicted power, measured power and comprehensive error in Dec. 14th, (b) The curves of under-predicted power, measured power and comprehensive error in Dec. 16th, (c) The curves of under-predicted power, measured power and comprehensive error in Dec. 18th and (d) The curves of under-predicted power, measured power and comprehensive error in Dec. 20th.

First, the daily irradiance and ambient temperature in December were put in equations (2)-(9) and the relation between power and voltage and current ($P=IU$), so the under-predicted power value per minute in the month is calculated. Then the daily under predictice power curve, the actual power curve and the comprehensive error curve are drawn by MATLAB programming. Because of the large amount of data, select four of them, as shown in figure 4, figure 4(a) to 4(d) are the under-predicted power curves, the actual power curves and the composite error curves of the 14th, 16th, 18th and 20th in December.
The comprehensive error of under-predicted power and measured power is shown in figure 4, and the curve belongs to the first form in figure 3. The comprehensive error polynomial of four days in December is listed in table 1.

Table 1. Comprehensive error polynomial.

| Date   | Comprehensive error polynomial                                                                 |
|--------|-----------------------------------------------------------------------------------------------|
| 1214   | \( \varepsilon_p(t) = -29917.5616 + 9235.8126t - 1032.0415t^2 + 49.5424t^3 - 0.8648t^4 \)   |
| 1216   | \( \varepsilon_p(t) = -47605.0828 + 15630.9906t - 1863.1604t^2 + 95.3749t^3 - 1.7741t^4 \)   |
| 1218   | \( \varepsilon_p(t) = -41050.7220 + 13190.1787t - 1529.2654t^2 + 75.6021t^3 - 1.3501t^4 \)   |
| 1220   | \( \varepsilon_p(t) = -41574.4233 + 13139.3314t - 1501.5260t^2 + 73.4516t^3 - 1.3025t^4 \)   |

As can be seen from table 1, the positive and negative laws of the corresponding coefficients of each comprehensive error polynomial are consistent, but the corresponding coefficient values are different. The main reason is that the weather conditions are different every day, if get rid of mostly sunny, continuous cloudy, rain, snow, dust storm and other weather, the corresponding coefficients of the comprehensive error polynomial of this month are all on the order of magnitude. Therefore, the comprehensive error expression of the first 20 days in December can be obtained by means of the mean value method, and the output power of the last 10 days in this month could be predicted.

By using the comprehensive error polynomial of the first 20 days in December, the average of corresponding coefficients of the comprehensive error polynomial in these 20 days is obtained, where \( a = -40036.9474 \), \( b = 12799.0783 \), \( c = -1481.4984 \), \( d = 73.4927 \), \( e = -1.3229 \). Put them in equation (1):

\[
\varepsilon_p(t) = -40036.9474 + 12799.0783t - 1481.4984t^2 + 73.4927t^3 - 1.3229t^4 + \ldots
\]

Equation (16) is the comprehensive error polynomial in December. Compensate equation (16) to the under predictive power model (equation (15)), a power prediction model for the PV generation system of the month is obtained, as in equation (17).

The voltage \( U \) and current \( I \) in equation (17) are related to the irradiance and environment temperature, and can be obtained by equations (2)-(9).

So the output power of grid-connected PV generation system in the last 10 days of December is predicted by using equation (17) and the 10 days in late December of irradiance and ambient temperature. Plotting the curves of the predictive power, the actual power and the under-predicted power, the accuracy of the model is verified by comparing the curves of the three. Due to the limited space, we chose the four groups show in figure 5. Figure 5(a) to 5(d) are total predict power, actual power and under predictive power curves of the PV generation system in December 22nd, 25th, 28th and 31st, respectively.
Figure 5. The curves of under-predicted power, measured power and comprehensive error. (a) The predicted power, actual power and under predicted power in Dec. 14th, (b) The predicted power, actual power and under predicted power in Dec. 16th, (c) The predicted power, actual power and under predicted power in Dec. 18th and (d) The predicted power, actual power and under predicted power in Dec. 20th.

As is shown in figure 5, the compensated total prediction power of PV system is closer to the actual power compared with the under-predicted power. It is shown that the comprehensive error polynomial is compensated for the under predictive power model after being compared with the no compensation, it can better track the actual power curve.

\[ P_n(t) = LmnIU + \varepsilon_n(t) \]
\[ = LmnIU - 1.3229t^4 + 73.4927t^3 - 1481.4984t^2 + 12799.0783t - 40036.9474 \]  

(17)

4.2. Model validation

In order to quantitatively observe the accuracy of the prediction model of photovoltaic power generation system, correlation degree and relative error are introduced to evaluate the accuracy of the model.

\[ \gamma = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\left(\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2\right)^{1/2}} \]  

(18)

\[ e_{ss} = \frac{\sum x_i - \sum y_i}{\sum y_i} \times 100\% \]  

(19)

Where, \( \gamma \) correlation degree; \( e_{ss} \) is relative error; \( x_i \) is the power data calculated from the power prediction model of the PV generation system; \( y_i \) is actual power data; (Tex translation failed) is the average power of \( x_i \); \( \bar{y} \) is the average power of \( y_i \).

The irradiance, environment temperature and corresponding time are replaced by equation (2)-(9) and equation (17), the predicted power data of the 10 days in late December ( \( x_i \) ) was calculated, then the average predicted power of each day (\( \bar{y} \)) in the last 10 days is calculated. The average actual power of each day (\( \bar{y} \)) in the last 10 days is also calculated. These values are computed by substituting equation (18) and equation (19). The correlation between the predicted power and the actual power, and between the under-predicted power and the actual power, the relative
error between the predicted power and the actual power, and between the under-predicted power and the actual power of the PV generation system in these 10 days is calculated respectively. The correlations between the predicted power and the actual power, and between the under-predicted power and the actual power of the 4 plots in figure 4 are given in table 2. And the relative errors between the predicted power and the actual power, and between the under-predicted power and the actual power of the 4 plots in figure 4 are given in table 3.

### Table 2. Correlation degree parameters.

| Time       | Dec. 22 | Dec. 25 | Dec. 28 | Dec. 31 |
|------------|---------|---------|---------|---------|
| Predicted power model (%) | 92.0478 | 86.0918 | 90.0448 | 91.0712 |
| Under predicted power model (%) | 87.6345 | 79.9899 | 82.9967 | 89.8718 |

### Table 3. Relative error parameters.

| Time       | Dec. 22 | Dec. 25 | Dec. 28 | Dec. 31 |
|------------|---------|---------|---------|---------|
| Predicted power model (%) | 39.1012 | 17.3164 | 59.3277 | 28.0054 |
| Under predicted power model (%) | 99.2426 | 40.5303 | 124.3082 | 86.1631 |

As shown in table 2, the correlation between the predicted power and the actual power is better than the correlation between the under-predicted power and the actual power, it shows that the compensation model is better than the under predictive model, the accuracy of the comprehensive error polynomial and the feasibility of the prediction method are also illustrated. The results show that the grid-connected prediction model of PV power generation system is applicable to the climate characteristics of altitude desert area.

As shown in table 3, the relative error between the predicted power and the actual power is less than the relative error between the under-predicted power and the actual power, it shows that the compensation model is better than the under predictive model, the accuracy of the comprehensive error polynomial and the feasibility of the prediction method are also illustrated. The results show that the grid-connected prediction model of PV power generation system is applicable to the climate characteristics of high altitude desert area.

In view of the difference between the predicted total power and the actual power curve in figure 5, the main reason is limited by the number of years’ study in this subject, that is, the sunshine intensity, environment temperature and actual power data cannot be collected within a few years. Therefore, the establishment of the comprehensive error polynomial is based on the data of the first 20 days in the month and predicts the output power for the last 10 days in this month, instead of predicting the output of the same month or the same day of next year based on the annual corresponding month or the data corresponding to each day. But as figure 5, tables 2 and 3 shown, the comprehensive error polynomial compensates for the predicted power, it can still reduce the error between the grid-connected prediction power and the actual power, it shows that the method of obtaining comprehensive error polynomial is feasibility. As a result, the relative error between the predicted power and the actual power remains the same, but the results are still scientific.

### 5. Conclusion and feature work

In this paper, the output current model of silicon battery board in high-altitude desert area is selected, the grid-connected power prediction model of PV system is studied, draw the following conclusions:

- The actual comprehensive error curve is analyzed, two different forms of the comprehensive error curve are summarized, the results show that the comprehensive error is Gaussian distribution, and the Gaussian polynomial is used to approximate the comprehensive error.
- The topological structure of PV system is analyzed, combining the prediction model of the output current of single silicon battery board in high-altitude desert area, the under predictive power model is defined. By using the method of error compensation, the comprehensive error
polynomial is compensated to the under predictive power model, and the grid-connected power prediction model of high-altitude PV system is proposed.

- Taking the actual PV power station as an example, the comprehensive error polynomial in December is calculated. The comprehensive error polynomial of December is compensated to the predictive power model, the daily forecast power curve of December is obtained, compared with the actual power, the results show that the trend of the two curves are consistent. The correlation degree and relative error of both are calculated, and the correlation degree is greater than 0.8, the relative error between the predicted power and the actual power is less than the relative error between the under-predicted power and the actual power. The results show that the power predicted model after error compensation can track the actual power of each day very well.

The annual measured power data are recorded, and the comprehensive error polynomial of each month is obtained. Then, the polynomial coefficients of the comprehensive errors in each corresponding month (such January 2016, January 2017, January 2018, etc.) are averaged, annual mean comprehensive error polynomial is obtained, it is used to predict the output power of PV systems in the corresponding month of next year (such January 2019). The accumulation of large data to obtain predictive power will make the prediction closer to the actual value, and make the prediction model more accurate.

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