Chapter 1

Testing Lorentz Invariance Violation in Quantum Gravity Theories

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Abstract

Much research has been done in the latter years on the subject of Lorentz violation induced by Quantum Gravity effects. On the theoretical side it has been shown that both Loop Quantum Gravity and String Theory predict that Lorentz violation can be induced at an energy near to the Planck scale. On the other hand, most of the experimental results in the latter years, have confirmed that the laws of physics are Lorentz invariant at low energy with very high accuracy.

The inclusion of one- and two-loop contributions from a Lorentz violating Lagrangian dramatically change the above picture: the loop momenta run into the Planck scale and above and from the ”divergent” terms finite Lorentz violating contributions of order one arise. These can be suppressed through suitable counterterms in the Lagrangian, originating a strong fine tuning problem.

A brief discussion of these issues and their possible influence in future research follows.

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1.1 Introduction

For a long time, the search of experimental clues on the nature of quantum gravity, was dismissed as impractical, based on the simplistic argument that those effects should appear only at energies of the order of Planck scale \( \left( E_P = 1.2 \times 10^{19} \text{GeV} \right) \), far beyond present day experimental possibilities. But recently there was a revolutionary change in this attitude, originated in references \([1, 2, 3, 4, 5]\). Here, a spontaneous breakdown of Lorentz symmetry at the Planck scale motivated in string theory was proposed, breaking CPT symmetry and consequently of Lorentz invariance. Later, an extension of the standard model was developed by the same group, including all possible CPT and Lorentz violating interactions \([6, 7]\), which has been used to develop sensitive tests of Lorentz symmetry.

A different approach \([8, 9, 10, 11]\). (See also \([12]\)) was based on the possibility that Quantum Gravity effects would modify the dispersion relations for particle propagation, such as photons. These modifications in turn would change the propagation velocity of photons, introducing delays for particles of different energies which could be detected if these particles would travel cosmological distances.

Such modifications of the dispersion relations have been found in the two most popular approaches to Quantum Gravity: Loop Quantum Gravity \([13, 14, 15, 16, 17]\) and String theory \([18, 19, 20, 21]\). These theories predict corrections to the dispersion relations which depend on energy in the form \((E\ell_P)^n\), with \(\ell_P = 1.6 \times 10^{-33} \text{cm}\) the Planck length scale.

String theory has suggested another form of Lorentz violation: non commutative field theory \([22, 23]\), where it is assumed that the coordinate themselves are functions from the differential variety to a noncommutative algebra. This was originally found in \([22]\) where it was shown that the low energy behavior of a bosonic string propagating in a Neveu-Schwartz condensate can be represented by a noncommutative field theory.\(^1\) Noncommutativity induces of course Lorentz violation and and it constitutes one of the most interesting sources of it.

More recently, a new approach to the testing of Quantum Gravity effects was introduced \([25, 26]\), based on the fact that if the dispersion relations of photons in the vacuum are modified in the form \(v(E) \neq c\), this implies a breakdown of Lorentz invariance since such a statement can be valid at most in a single reference frame. In this way a privileged reference frame (the \textit{New Aether}) is introduced in the theory where a particular simple form of the equations of motion is valid. Since Lorentz invariance must be broken, the motion of the laboratory with respect to the privileged frame should be detected in suitable experiments. Moreover, modern cosmology suggests a single candidate for the New Aether, namely, the reference system at rest with respect to the Cosmic Microwave Background. In this way, the possibility of accurate tests of Quantum Gravity induced Lorentz breakdown opens.

The rest of the paper is organized as follows: Section 1.2 is devoted to a brief

\(^1\)A didactic introduction to noncommutative field theory can be found in \([24]\).
discussion of the main differences between Lorentz abiding and Lorentz violating theories. Section 1.3 discusses several of the test theories that have been used to design and interpret tests Lorentz invariance. In section 1.4 the origin of Lorentz invariance violation in Quantum Gravity is discussed, centering on the predictions of Loop Quantum Gravity, and in Section 1.5 some of the tests carried on the theory at the tree-level are discussed. The dramatic consequences of the inclusion of radiative correction effects are discussed in section 1.6. Finally, in section 1.7 we state our conclusions.

1.2 Issues of Lorentz violation

Lorentz invariant theories have well defined properties, such as the nonexistence of privileged reference frames. Many of these properties are lost if Lorentz invariance does not hold and new phenomena may arise in these conditions. In this section we discuss some of these issues since they may be used to check the validity of Lorentz symmetry.

1.2.1 Privileged reference systems

A characteristic fact of Lorentz violating theories is that most of them predict the existence of privileged reference systems, where the equations of motion take their simplest form. This is akin to the old notion of “luminiferous aether”, before the formulation of special relativity one century ago, and we shall call sometimes these privileged reference systems “the new aether”.

That Lorentz violating theories should generically predict the existence of privileged reference systems can be easily inferred if propagating particles have general functions of energy as dispersion relations. Consider for instance a photon: if its dispersion relation does not have the Lorentz covariant form $\omega = ck$, but propagate with an energy dependent velocity $v(E)$, such statement can be at best valid in one specific inertial frame. This selects a preferred frame of reference, where a particular form of the equations of motion is valid, and one should then be able to detect the laboratory velocity with respect to that frame [25]. It is this fact that opens the possibility of detecting tiny violations of Lorentz symmetry.

Fortunately, we have today, in contrast with the situation at the end of the 19th century, a rather unique choice for that “preferred inertial frame”: the frame where the Cosmic Microwave Background (CMB) looks isotropic. Our velocity $w$ with respect to that frame has already been determined to be $w/c \approx 1.23 \times 10^{-3}$ by the measurement of the dipole term in the CMB by COBE, for instance [27]. We shall usually refer tests of Lorentz invariance to the CMB reference system.
1.2.2 “Particle” and “Observer” Lorentz transformations

As it is well known, there are two possible formulations of Lorentz-Poincaré transformations: passive or observer transformations, where coordinates are transformed in the form

$$x' = \Lambda^{-1}(x - a)$$ (1.1)

and active or particle transformations, where fields and states are transformed

$$U(\Lambda, a)\phi(x)U^\dagger(\Lambda, a) = \phi(\Lambda x + a) \quad U(\Lambda, a)|0\rangle = |0\rangle$$ (1.2)

These two forms of the Lorentz transformation are equivalent in Lorentz-invariant theories, but this will not be so in Lorentz-violating theories. In particular, particle transformations may be limited to a given subset of Lorentz transformations, while observer transformations, in principle, are unrestricted. These will be interpreted as the transformations related to the laboratory (L) frame, and these are important in the design and interpretation of experiments.

1.2.3 Discrete Lorentz transformations

Among the testable forms of Lorentz violations, the breakdown of CPT symmetry is paramount, since it signals the breakdown of Lorentz symmetry altogether. Indeed, the CPT Theorem, i.e., the validity of CPT symmetry, can be proved from very weak assumptions in axiomatic field theory [28, 29]. The converse proposition has been proved recently [30].

The consequences of CPT invariance are well known: masses, magnetic moments and charge of particles and antiparticles must be equal in absolute value, as well as cross sections and decay rates. Any of these properties may be used to check the validity of CPT invariance and some of the most sensitive tests of Lorentz Invariance are based on them.

1.3 Test theories for Lorentz Invariance Violation

Testing symmetries of nature such as Lorentz invariance is better made through the development of test theories: theories that suitably generalize the symmetry being tested through the introduction of a set of well defined parameters $C_i$. These are chosen in such a way that the symmetry being tested is recovered for particular values of the parameters $C_i^0$, while the other values represent a breakdown of the symmetry. Let us examine a few of the proposed test theories of Lorentz invariance.

1.3.1 The Robertson model

The simplest of the proposed test theories are the Robertson model [31] and its generalization by Mansouri and Sexl [32, 33, 34]. The connection between both
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references has been discussed in reference [35]. These are purely kinematic test theories, where the parameters are introduced through suitable generalizations of the Lorentz transformation.

Consider the privileged reference system $S_0$ chosen in such a way that Maxwell’s equations are there valid. Let us introduce the metric, valid in this rest frame

$$ds^2 = dx_0^2 - (dx^2 + dy^2 + dz^2)$$ (1.3)

This quantity is important, since it has the following properties

1. A time interval at a given point $x$ is expressed as

$$dt = \frac{ds}{c} \quad (dr = 0)$$ (1.4)

2. The distance between two points $x_1, x_2$ is expressed as

$$dl = +\sqrt{-ds^2} \quad (dt = 0)$$ (1.5)

3. Light rays are along the normal to the spheres

$$ds^2 = 0$$ (1.6)

Now consider an inertial reference system $S'$, moving with respect to $S$ with velocity $w$. The Robertson transformation between both reference systems, with a suitable choice of axis, is defined as

$$x = Ax' + Bx'_0$$
$$x_0 = B'x'_0 + A'x'$$
$$y = Cy'$$
$$z = Cz'$$ (1.7)

with $A, B, C$ arbitrary functions of the relative velocity $w$. These are, of course, “observer” transformations.

Imposing Einstein’s synchronization on $S'$ the conditions

$$A' = \frac{w}{c}A \quad B' = \frac{w}{c}B$$ (1.8)

are obtained. Finally, the metric in $S'$ results

$$ds^2 = g_0^2dx_0^2 - g_1^2dx'^2 - g_2^2(dy'^2 + dz'^2)$$ (1.9)

where

$$g_0(w) = \sqrt{1 - \frac{w^2}{c^2}B}$$
$$g_1(w) = \sqrt{1 - \frac{w^2}{c^2}A}$$
$$g_2(w) = C$$ (1.10)
Thus, these tests theories are equivalent to the proposal of a metric of the form
\[ ds^2 = g_0^2 dx_0^2 - g_1^2 dx^2 - g_2^2 (dy^2 + dz^2) \] (1.11)
for space-time, where \( g_i(V) \) are arbitrary functions of the velocity of the local reference system with respect to a privileged inertial frame. Lorentz symmetry is recovered for the particular case \( g_i(V) = 1 \). For low velocity experiments, it is usual to parametrize these functions in terms of the Mansouri-Sexl parameters \[g_0(V) = 1 + \left(1 - \frac{\alpha}{2}\right) \frac{V^2}{c^2} + \ldots \] (1.12)
\[ g_1 \equiv g_0 \] (1.13)
\[ g_2 \equiv g_1 \] (1.14)
which have the values
\[ \alpha = \frac{1}{2}, \quad \beta = \frac{1}{2}, \quad \delta = 0 \] (1.15)
if Lorentz invariance is valid.

Robertson \[31\] showed that the classical experiments of Michelson-Morley \[36\], Kennedy-Thorndike \[37\] and Yves-Stillwell \[38\] restricted the functions \( g_i \) to a small neighborhood of 1. We shall discuss modern limits on \( g_i \) later on.

### 1.3.2 The Dispersion Relations models

A set of simple tests theories for testing Lorentz violation has been proposed by Amelino-Camelia \[39\] (See also \[40\]). These are kinematic models, but applied to the dynamical dispersion relations they introduce a bridge between kinematics and dynamics.

The dispersion relation for a low-energy particle (\( i.e. \) small with respect to the Planck scale \( E_P = 1.2 \times 10^{19} \text{GeV} \)) in a Lorentz violating theory, can be written in the form
\[ m^2 \simeq E^2 - p^2 + \eta p^2 \left( \frac{E}{E_P} \right)^n + O \left( \frac{E}{E_P} \right)^{n+3} \] (1.16)
where \( \eta \) and \( n \) are free parameters. The exponent \( n \) is characteristic of the magnitude of the effects expected. On the other hand, for Lorentz violating effect generated in quantum gravity, one expects \( \eta \sim O(1) \). The case \( n = 1 \) is specially interesting, since it arises in many contexts such as low energy limits of Loop Quantum Gravity.

It should be noted, however, that a photon dispersion relation of the form (1.16) with \( n \) odd is forbidden by causality. The detection of such a correction would signal not only the breakdown of Lorentz invariance but of causality relations.

To close the test theory, two prescriptions must be added to the above dispersion relation
1. The law of energy-momentum conservation is valid

\[ E_1 + E_2 = E'_1 + E'_2 \]
\[ p_1 + p_2 = p'_1 + p'_2 \quad (1.17) \]

2. The velocity of the particle is computed from the Lorentzian expression

\[ v = \frac{dE}{dp} \quad (1.18) \]

These simple prescriptions suffice for analyzing many Lorentz-violating phenomena, specially those related to threshold modifications by Quantum Gravity effects. The set of equations (1.16), (1.17) and (1.18) are the basis of threshold analysis: the very powerful tool to discuss many astrophysical phenomena in the presence of Lorentz violation [41, 42, 39, 11].

A variation of the above dispersion relation is obtained assuming that different polarization states of the particle have different propagation velocities

\[ m^2 = E^2 - p^2 \pm \theta_i p^2 \left( \frac{E}{E_P} \right) \quad (1.19) \]

where the sign depends on the helicity of the particle. This type of test theories should be analyzed in the context of low energy field theories.

### 1.3.3 The TH$\epsilon \mu$ model

A dynamical test theory for local Lorentz invariance was introduced in reference [43] as a test theory for the validity of Einstein’s Equivalence Principle. In this model, the breakdown of Local Lorentz Invariance comes from the structure of the Lagrangian of the system.

Let us consider a system of classical particles interacting through the electromagnetic field in a background spherically symmetric gravitational field $U$. The action of the system is

\[ S_{TH\epsilon\mu} = -\sum_a m_a \int \sqrt{T - H v_a^2} dt \]
\[ + \sum_a e_a \int A_\mu (x_a^\nu) v_a^\mu dt \]
\[ + \frac{1}{8\pi} \int \left( \epsilon E^2 - \frac{B^2}{\mu} \right) \quad (1.20) \]

where $\hbar = c = 1$, $m_a, e_a$ are the mass and charge of the $a$-th particle, $x_a^\mu, v_a^\mu$ its world-line and velocity and $E, B$ the electric and magnetic fields. $T, H, \epsilon, \mu$ are arbitrary functions of the spherically symmetrical gravitational field $U = GM/r$. 

To see how a Lorentz invariance violation is generated in this test theory, let us pass to a freely falling reference frame. Consider a given point in space-time $x_0^\mu = (t_0, r_0)$, and a local reference frame with origin in $x_0^\mu$. If $g = \nabla U$ is the local acceleration of gravity, the transformation equations to the freely falling reference frame, correct to first order in the small quantities $g_t$ and $g \cdot x$ are

$$\hat{t} = \sqrt{T_0 t} \left( 1 + \frac{T_0'}{2T_0} g \cdot x \right) \quad (1.21)$$

$$\hat{x} = \sqrt{H_0} \left[ x + \frac{T_0'}{4H_0} g t^2 + \frac{H_0'}{4H_0} (2x(g \cdot x) - g x^2) \right] \quad (1.22)$$

The transformed action, keeping only the lowest order terms, is

$$\hat{S}_{TH\epsilon\mu} = -\sum_a m_0 a \int \sqrt{1 - \hat{v}^2} d\hat{t} + \sum_a e_a \int \hat{A}_\mu \hat{v}_a^\mu d\hat{t} + \frac{1}{8\pi} \epsilon_0 \sqrt{\frac{T_0}{H_0}} \int \left( \hat{E}^2 - \frac{H_0}{T_0 \epsilon_0 \mu_0} \hat{B}^2 \right) d^4\hat{x} \quad (1.23)$$

where the “hatted” quantities are referred to the local reference system. The first two terms in (1.23) are locally Lorentz invariant, but the third one breaks the symmetry unless

$$\frac{H_0}{T_0 \epsilon_0 \mu_0} = 1 \quad \epsilon_0 \sqrt{\frac{T_0}{H_0}} = 1 \quad (1.24)$$

which are the conditions for the validity of Local Lorentz Invariance. The first one implies that the limiting speed for massive particles is equal to the propagation speed for electromagnetic signals

$$c_L^2 = \frac{T_0}{H_0} = \frac{1}{\epsilon_0 \mu_0} = c_{em}^2 \quad (1.25)$$

The second one, which is related to the validity of the Weak Equivalence Principle, can be recast as position invariance of the electric charge.

The $TH\epsilon\mu$ model has been generalized to the Standard Model [44] where a set of conditions similar to (1.24) should hold for the validity of Lorentz symmetry.

### 1.3.4 The Kostelecký Model

The most general treatment of Lorentz Invariance violations within the Standard Model is the model developed by Kostelecký and coworkers [6, 7, 45, 46], which we shall simply call the Kostelecký Model (KM).
The KM Lagrangian density is based on a careful inclusion of all terms of dimension 4 that break Lorentz invariance for the particles in the Standard Model. Thus, the Dirac particle modified Lagrangian takes the form

\[ \mathcal{L}_D = \frac{1}{2} i \bar{\psi} \Gamma^\mu \partial_\mu \psi - \bar{\psi} M \psi \]  

(1.26)

where the \( \Gamma \) and \( M \) operators are

\[ \Gamma^\mu = \gamma^\mu + c^{\mu\nu} \gamma_\nu + d^{\mu\nu} \gamma_\nu \gamma_5 + e^\mu + i f^\mu \gamma_5 + \frac{1}{2} i g^{\lambda\mu} \sigma_{\lambda\nu} \]  

(1.27)

\[ M = m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \frac{1}{2} H^{\mu\nu} \sigma_{\mu\nu} \]  

(1.28)

These operators have well defined transformation properties under CPT symmetry. In particular, the \( a_\mu \) and \( b_\mu \) terms are CPT-odd, making the masses of particle and antiparticle different.

There are corresponding modifications for the gauge field and Higgs Lagrangians. For the particular case of the photon Lagrangian, these modifications take the form

\[ \mathcal{L}_{em} = -\frac{1}{4} (k_F)_{\kappa\lambda\mu\nu} F^{\kappa\lambda} F^{\mu\nu} + \frac{1}{2} (k_{AF})^{\kappa} \epsilon_{\kappa\lambda\mu\nu} A^\lambda F^{\mu\nu} \]  

(1.29)

where the first term is CPT-even while the second is CPT-odd.

The CPT-even tensor \( k_F \) has 19 independent components, which can be grouped into a set of 10 P-odd components (responsible for vacuum birefringence phenomena) and a set of 9 P-even components, which describe breakdown of Lorentz boost invariance [47, 48, 49].

KM is an extremely general test theory for Lorentz violation: it includes all dimension 4- terms and thus it is “closed” under Lorentz transformations. The physical properties of the model have been extensively studied [6, 50, 51, 52, 46, 53, 54] and it has been successfully used to design and analyze [55, 56, 57, 58, 59, 60, 61, 62] experiments on Lorentz violation.

The nonrelativistic limit of the generalized Dirac operator is specially interesting since same of the most accurate test of Lorentz invariance have been carried in that régime. Although the Lorentz violating terms are small and can be treated perturbatively, it is not straightforward to carry out the nonrelativistic limit because several of the small terms have nontrivial time derivatives. A generalization of the Foldy-Wouthuysen transformation developed in reference [50] can be used to obtain
it. Neglecting terms of order \((P/m)^3\) the following Hamiltonian is obtained

\[
H_{NR} = m + \frac{p^2}{2m} + \left[a_0 + m(c_{00} + e_0)\right] + \left[-b_j + \frac{1}{2} \epsilon_{jkl} H_{kl} + m(d_{j0} - \epsilon_{jkl} g_{kl0})\right] \sigma^j
+ \left[-a_j + m(c_{j0} + e_{j0} + e_j)\right] \frac{p^j}{m}
+ \left[b_0 \delta_{kj} - \epsilon_{jkl} H_{l0} - m(d_{kJ} + d_{J0} \delta_{kJ} + \frac{1}{2} \epsilon_{klm} g_{mlj} - \epsilon_{jkm} g_{m00})\right] \frac{p^j}{m} \sigma^k
\]

The first line of (1.30) corresponds to the usual NR Hamiltonian for a free particle. The rest of the terms describe Lorentz violating phenomena of different types. We shall discuss some of these phenomena in the following pages.

1.3.5 Other models of Lorentz violation

As a last example of test theories for Lorentz violation, let us mention the Myers-Pospelov model \[63, 64\]. In this model, terms of dimension five are added to the Lagrangian of the standard model satisfying the following criteria

1. The terms must be quadratic in the same field.
2. They should have one more derivative than the kinetic energy term.
3. They must respect gauge invariance.
4. They must be Lorentz invariant except for the appearance of a unit vector \(n \cdot n = 1\) parameterizing a privileged direction in spacetime.
5. The terms should not reduce to lower dimension terms through the equations of motion.
6. They should not reduce to a total derivative.

The simplest correction satisfying the above criteria are

\[
\delta L_S = i\kappa \ell P \phi (n \cdot \partial)^3 \phi
\]

\[
\delta L_\phi = \xi \ell P \nu F_{\mu\alpha} (n \cdot \partial) n_\nu F^{\nu\alpha}
\]

\[
\delta L_f = \ell P \tilde{\psi} (\eta_1 \ t + \eta_2 \ \gamma_5 (n \cdot \partial)^2 \psi
\]
In these equations $\xi, \eta_1, \eta_2$ are free parameters and

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$$

is the dual of the electromagnetic field tensor.

These terms are CPT-odd and thus induce a host of Lorentz breaking phenomena. The model can be applied to the tree level, but radiative corrections will generate terms of dimension 2 or 3 badly breaking Lorentz invariance [65, 66]. These issues will be treated later on.

Last but not least I would like to mention double special relativity as a possible test theory, not of Lorentz violation but of new physics near the Planck scale [67, 68]. Rather than a violation of Lorentz invariance, double special relativity is a nonlinear realization of the Lorentz group, that exhibits both a limiting velocity (the speed of light $c$) and a limiting energy (the Planck energy $E_P$). This beautiful theory falls beyond the scope of this article, which is concerned with “true” violations of Lorentz invariance. For a discussion on double special relativity, see for instance [69].

### 1.4 Quantum Gravity as a source of Lorentz violation

Quantum Gravity has been proposed as a source of Lorentz violations [1, 8, 13, 70, 71] by many authors. The rationale behind the proposal is that quantum gravity effects will make space-time granular at a scale near the Planck scale $\ell_P$. This in turn will change the dispersion relations of elementary particles, breaking in general Lorentz invariance.

As it has been mentions, such modifications have been found within the two currently most popular approaches to quantum gravity, namely Loop Quantum Gravity and String theory. Let us discuss briefly how these phenomena arise, taking Loop Quantum Gravity as an example. The analysis of the resulting Lorentz violations can be carried out in the framework of the test theory developed by Kostelecký and coworkers.

#### 1.4.1 A reminder of Loop Quantum Gravity

Loop Quantum Gravity is a theory of canonical quantization of the gravitational field. References [72, 73, 74] are introductions in different levels, while [75] is a critical review on the subject.

The details of the theory are complicated, so in this note we shall only give some qualitative description.

As it is well-known, gravitation has gauge invariance under diffeomorphisms and thus physical states of the gravitational field must satisfy the momentum constraint and the Hamiltonian constraint. The strategy of Loop Quantum Gravity is to find a suitable way of dealing with these constraints.
1. In order to perform the quantization, a suitable set of gravitational canonical variables must be introduced. These are cunningly contrived modifications of the Ashtekar variables [76, 77].

2. A regularization is introduced in the theory by breaking space-time in a lattice-like structure, at a scale of the order of $\ell_P$ (Figure 1.1). With this regularization, all physical quantities became well defined, although gauge dependent.

3. In terms of these variables, a suitable auxiliary Hilbert space $\mathcal{H}_{\text{aux}}$ is built [72]. However, because of the invariance of the theory under diffeomorphisms, not all of the states in $\mathcal{H}_{\text{aux}}$ represent physical states of the gravitational field.

4. The physical states are chosen so that the Wilson loops built from the canonical variables are well defined. This is the crucial step, from which the theory can be developed. This representation, originally introduced for the electromagnetic field [78, 79, 80] allows the definition of well-defined physical quantities.

5. In principle, the regularization introduced in step 2 can be eliminated shrinking the scale of the lattice to zero. However, it can be shown that the volume element operator acts like a regulator, yielding well-defined quantities in this limit.
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Although the technical details are very complex, the above steps have been carried to success (see, however, Ref. [75]). Among the most interesting results obtained, let us mention

1. Geometrical quantities, such as areas and volumes, are quantized s multiples of $\ell_P^2$ (or $\ell_P^3$), even in the limit of the auxiliary lattice size going to zero [72, 73].

2. Other field theories can be included in the scheme. The lattice-like discretization provides a suitable regularization of the field quantities [72].

3. Solutions for highly symmetrical states, such as spherically symmetrical or cosmological ones, can be found by solving additional constraints. For instance, a beautiful solution for a Robertson-Walker universe has been found, free of singularities (For a review, see [81, 82]).

1.4.2 Lorentz Invariance Violation in Loop Quantum Gravity

Let us examine the origin of Lorentz invariance violation in Loop Quantum Gravity with an example: the lowest order corrections to the Maxwell Hamiltonian [13]. The latter can be written in the form

$$H_M = \frac{1}{2\alpha} \int d^3x \frac{g_{ab}}{g} \left( \tilde{E}^a \tilde{E}^b + \tilde{B}^a \tilde{B}^b \right)$$

(1.35)

where the tildes denote that the fields are tensor densities in the canonical framework, $g$ is the determinant of the metric and $\alpha$ is the fine structure constant. We wish to average (1.35) over a semiclassical state $|g, \mathcal{L}\rangle$. Indexes $a, b$ are purely spatial.

There is currently no clear characterization in Loop Quantum Gravity of a semiclassical state. Following [13] we shall use a weave: this is a state characterized by a semiclassical length scale $L$ such that

$$\ell_P \ll L \ll \lambda$$

(1.36)

Intuitively, a weave can be thought as a big carpet, of size $\mathcal{L}$, weaved of tiny pieces of scale $\ell_P$ (Fig. 1.2). The characteristic size $\mathcal{L}$ should be smaller than any “macroscopic” characteristic length $\lambda$ such that spacetime can be considered continuous. Thus, weaves are states where whose scale $\mathcal{L}$ marks the transition from quantum (discrete) spacetime to classical one.

In the following, we shall assume that the weave state $|g, \mathcal{L}\rangle$ correctly describes the privileged reference frame $S$.

Consider the averages of the Hamiltonian (1.35) in the weave state $|g, \mathcal{L}\rangle$

$$H_{ef} = \langle g, \mathcal{L} | H_M | g, \mathcal{L} \rangle$$

(1.37)

The average is performed for fixed values of the electromagnetic field operators $E, B$ which, for all practical purposes, can be considered classical fields. That is,
the average is made on the fluctuations of the quantum gravitational field. We shall also assume that the classical geometry in the privileged reference frame is flat euclidean and so we expect

$$\langle g, L \, \big| \, g, L \rangle = \delta_{ab} + O\left(\frac{\ell_P}{L}\right)$$

(1.38a)

$$\langle g, L \, | A_{ia} \, | g, L \rangle = 0 + \frac{1}{L} \left(\frac{\ell_P}{L}\right)^{\Upsilon}$$

(1.38b)

where $A$ is the Ashtekar variable operator and $\Upsilon$ is a real number parameter.

Let us now introduce the Thiemann factorization of the metric operator

$$g_{ab}^{-1} = \hat{w}_a \hat{w}_b$$

(1.39)

where the operators $\hat{w}$ are finite and have nonzero contributions only at the vertexes $v_i$. Introducing a point splitting regulator $\Delta(x,y) \rightarrow \delta(x-y)$ when the triangulation scale goes to zero, the Maxwell Hamiltonian can be written as a sum over the vertexes in a weave

$$H_{ef} = \frac{1}{2\alpha} \sum_{v_i,v_j} \langle g, L \, | \hat{w}(v_i)\hat{w}(v_j) \rangle \, | g, L \rangle \left[ E^a(v_i)E^b(v_j) + B^a(v_i)B^b(v_j) \right]$$

(1.40)

The electromagnetic field, being quasiclassical, is slowly varying across the weave and can be expanded in a Taylor series about some point $x_0$

$$E^a(v_i) = E^a(x_0) + (v_i - x_0)_a \partial^a E^a(x_0)$$

(1.41)

and after a simple calculation the low energy Maxwell Hamiltonian takes the form

$$H_{ef} = \frac{1}{2\alpha} \int d^3x \left[ (E^2 + B^2) + 2t_{abc} \left( E^a \partial^b + B^a \partial^b \right) \right]$$

(1.42)
where the 3-tensor
\[ t_{abc} = \frac{1}{2} \sum_{v_i, v_j} \langle g, \mathcal{L} | \hat{w}(v_i) \hat{w}(v_j) | g, \mathcal{L} \rangle (v_i - x_0)_c \] (1.43)

Now, let us consider the electromagnetic field in the privileged reference frame \( S \). We expect that the electromagnetic field will behave isotropically in this particular reference frame and so we shall write the averages as isotropic tensors in \( S \); that is, tensors built from the metric tensor \( \delta_{ab} \) or the Levi-Civita tensor \( \varepsilon_{abc} \). Thus the 3-tensor \( t_{abc} \) must have the form
\[ t_{abc} = \theta_{GP} \ell_P \varepsilon_{abc} \] (1.44)
the last factor coming from the fact that a nonzero value of \( t_{abc} \) can be expected only at scales near the Planck length, where the distributions of vertexes is very anisotropic.

As a final result, we get the \textit{Gambini-Pullin Hamiltonian}
\[ H_{GP} = \frac{1}{2\alpha} \int d^3x \left[ (E^2 + B^2) + 2 \theta_{GP} \ell_P (E \cdot \nabla \times E + B \cdot \nabla \times B) \right] \] (1.45)

The second term is parity odd and originates birefringence in vacuum propagation. This happens because we have implicitly assumed that weaves can break parity, although in the privileged system rotational invariance is preserved.

The above informal method was developed in a much more rigorous way in reference [15], where a detailed exposition of the used techniques is given, and higher order corrections in the small parameter \( \ell_P / \mathcal{L} \) were found. These higher order corrections depend on the \( \Upsilon \) and \( \mathcal{L} \) parameters and have a rather complex structure.

The method was also applied to the Dirac Hamiltonian [14, 16]. These latter corrections are particularly important, since they can be submitted to strict experimental tests.

The modified Dirac equation, in the privileged reference frame, takes the form
\[ [i\gamma^\mu \partial_\mu + \Theta_1 m \ell_P i\gamma \cdot \nabla - \mathcal{K} / 2 \gamma_5 \gamma^0 - m (\alpha - i \Theta_2 \ell_P \Sigma \cdot \nabla)] \Psi = 0 \] (1.46)
where the spin operator is \( \Sigma^k = (i/2)\epsilon_{klm} \gamma^l \gamma^m \). Here
\[ K = m \Theta_4 m \ell_P, \quad \alpha = (1 + \Theta_3 m \ell_P) \] (1.47)
and \( \Theta_1, \Theta_2, \Theta_3, \Theta_4 \) are constants of order one. Besides we have assumed \( \mathcal{L} = 1/m \), where \( m \) is typically the particle mass.

The term \( m (1 + \Theta_3 m \ell_P) \) can be interpreted as a renormalization of the mass whose physical value is taken to be \( M = m (1 + \Theta_3 m \ell_P) \). The other terms, however cannot be eliminated by a simple transformation, since they break discrete symmetries. Indeed, the first two correction terms are CPT odd, while the last one is CP odd. Thus, they represent new physical effects induced by Quantum Gravity.
1.4.3 Transforming into the Laboratory System

Although many phenomena can be conveniently analyzed in the Privileged reference system $S$, many others are manifest in the Laboratory reference system $S'$, moving with four velocity $W^\mu$ with respect to $S$. This transformation can be accomplished in a very elegant way using the Lagrangian density, which should be a scalar density \[25\]. As an example, consider transforming (1.46) to the lab system.

We shall write (1.46) in a formally covariant form using $W^\mu$ and projectors constructed from it. In this way, we find

\[
L = \frac{1}{2} \bar{\Psi} \gamma^\mu \partial_\mu \Psi - \frac{1}{2} \bar{\Psi} M \Psi \\
+ \frac{1}{2} i(\Theta_1 M \ell_P) \bar{\Psi} \gamma_\mu (g^{\mu\nu} - W^\mu W^\nu) \partial_\nu \Psi \\
+ \frac{1}{4} (\Theta_2 M \ell_P) \bar{\Psi} \epsilon_{\mu\nu\alpha\beta} W^\mu \gamma^\nu \gamma^\alpha \partial^\beta \Psi \\
- \frac{1}{4} (\Theta_4 M \ell_P) MW^\mu \bar{\Psi} \gamma_5 \gamma_\mu \Psi \tag{1.48}
\]

In this way, the Dirac particle Lagrangian takes the form of the Kostelecký Lagrangian, with

\[
a_\mu = H_{\mu\nu} = d_{\mu\nu} = c_\mu = f_\mu = 0 \tag{1.49}
\]
\[
c_{\mu\nu} = \Theta_1 M \ell_P (g^{\mu\nu} - W^\mu W^\nu) \tag{1.50}
\]
\[
g_{\alpha\beta\gamma} = \Theta_2 M \ell_P \epsilon_{\mu\alpha\beta\gamma} W^\mu \tag{1.51}
\]
\[
b_\mu = \frac{1}{2} \Theta_4 M^2 \ell_P W^\mu \tag{1.52}
\]

and from these expressions the low energy Hamiltonian for the fermion can be found in the form

\[
\tilde{H} = Mc^2 (1 + \Theta_1 M \ell_P (w/c)^2) \\
+ \left(1 + 2 \Theta_1 M \ell_P \left(1 + \frac{5}{6} (w/c)^2\right) \right) \left( \frac{p^2}{2M} + g \mu \mathbf{s} \cdot \mathbf{B} \right) + \\
+ \left( \Theta_2 + \frac{1}{2} \Theta_4 \right) M \ell_P \left[ \left(2M^2c^2 - \frac{2p^2}{3M}\right) s \cdot \frac{w}{c} + \frac{1}{M} s \cdot \mathbf{Q}_P \cdot \frac{w}{c} \right] \\
+ \Theta_1 M \ell_P \left[ \frac{w \cdot \mathbf{Q}_P \cdot w}{Mc^2} \right], \tag{1.53}
\]

where $g$ is the standard gyromagnetic factor and $\mathbf{Q}_P$ represents the momentum quadrupole tensor, with components $Q_{Pij} = \langle p_i p_j - 1/3 p^2 \delta_{ij}\rangle$.

The first two lines of (1.53) represent the Schrödinger-Pauli Hamiltonian, with corrections to the rest and inertial mass, coming from the motion with respect to the “New Aether”. The third and fourth lines represent an “Aether wind” blowing on the spin and an anisotropy of the inertial mass of the particle, which could be tested in Hughes-Drever experiments.
1.4.4 Other models of Lorentz violation by Quantum Gravity

A similar treatment can be applied to other models of Lorentz violation generated by Quantum Gravity. As an example let us briefly discuss the so called Liouville approach to Non-critical String Theory. In this scheme our universe is identified with a 4 dimensional D-brane, which will naturally contains a type of topological defect called a D-particle. These will interact with ordinary particles, represented by strings in the corresponding mode, by elastic scattering, which would produce a recoil of the D-particle. This recoil will produce a local disturbance of the space-time geometry which will in turn affect the propagation of the particle [18, 19, 20].

We take this part of the analysis directly from Ref. [83], starting with their modified Dirac equation

\[
[\gamma^\mu (i\partial_\mu - eA_\mu) - v^i \gamma^0 (i\partial_i - eA_i) - m] \Psi = 0,
\]

(1.54)

Here \(\gamma^\mu\) are the standard flat-space gamma matrices and \(v^i\) is the D-particle recoil velocity in the CMB frame, where it is initially at rest. The above modified Dirac equation preserves gauge invariance due to the standard minimal coupling. In the following we set \(A_\mu = 0\). We will be concerned with a perturbative expansion in the small parameter \(v^i\).

The above equation can be interpreted as describing the motion of a fermion in an effective metric

\[
p^\mu G_{\mu\nu} p^\nu = E^2 - 2E v \cdot p - p^2, \quad p^\mu = (E, p), \quad |v| \approx \frac{|p|}{M}.
\]

(1.55)

The analysis of the above equation requires some care to define correctly the D-particle recoil velocity in the nonrelativistic limit [26]. Passing to the laboratory frame the following form of the equation is found

\[
[i\gamma^\mu \partial_\mu - i\gamma^0 (W_\mu V^\nu) \partial_\nu - m] \Psi = 0,
\]

(1.56)

which can be put in the standard form of [50]. The identification of the corresponding Kostelecký parameters leads to

\[
a_\mu = b_\mu = 0 = H_{\mu\nu}, \quad g_{\lambda\mu\nu} = 0 = d_{\mu\nu}, \quad c_{\mu\nu} = -W_\mu V_\nu.
\]

(1.57)

Let us note that \(c_{\mu\nu}\) is not symmetrical in this case.

Non commutative field theory is another important model of Lorentz violating field theory. We shall discuss issues connected with these theories in Section 1.6.

1.5 Testing Lorentz Invariance Violation at the tree level

The results collected in Section 1.4 can be tested with sensitive well defined experiments if one neglects the effects of radiative corrections. In this section we shall
examine these tests at the “tree level”, leaving a discussion of radiative corrections for the next one.

There is a host of phenomena which can be used to detect Lorentz Invariance violation. Among them, let us mention

1. Breakdown of local rotational symmetry (“Aether wind effects”).
2. Breakdown of Lorentz Boost Invariance (“Kennedy-Thorndike experiments”).
3. Occurrence of “forbidden processes” (such as photon decay in vacuum).
4. Dispersive processes in vacuum (such as energy dependent velocities or birefringence).
5. Breakdown of discrete symmetries, principally violation of the CPT theorem.

There are two main group of observations with high enough sensitivity to probe the Quantum Gravity regime: Cosmological or astrophysical tests of Lorentz invariance (“Threshold analysis type”), based on the modification of the dispersion relations of particles, and Clock comparison experiments (“Hughes-Drever type”). The sensitivity requirements are indeed very strict; for instance, for terms linear in the Planck length $\ell_P$, one should have an accuracy better than

$$\ell_P m \sim 10^{-19}$$

for a mass of the order of the nucleon mass.

In the following we shall consider some of the most interesting tests of Lorentz invariance. Our selection is motivated by their effectiveness in testing Quantum Gravity induced Lorentz violation. More complete covering of the phenomenology can be found in references [59, 84, 40, 60, 62, 85, 86, 87].

### 1.5.1 Kinematic tests of Lorentz invariance

These are the classical Michelson-Morley [36] and Kennedy-Thorndike [37] experiments, which test isotropy and boost independence of the local velocity of light. Using the Robertson metric (1.11) it is easy to show that the light velocity is given by

$$\frac{c(\theta, V)}{c_0} = 1 + (\alpha + \beta - 1) \frac{V^2}{c^2} + \left(\beta - \delta - \frac{1}{2}\right) \frac{V^2}{c^2}$$

(1.59)

The last term is a local anisotropy in the velocity of light, while the second one represents a boost dependence. Several modern versions of both experiments have been carried in the latter years [88, 89, 90, 49], with strict limits on the Mansouri-Sexl parameters

$$|\alpha + \beta - 1| < 6.9 \times 10^{-7}$$

(1.60a)

$$|\delta - \beta + \frac{1}{2}| < 4.5 \times 10^{-9}$$

(1.60b)
The result of the new version Michelson-Morley experiment\cite{49} were also interpreted also in terms of the Kostelecký $k_F$ parameter tensor. It tests only the P-even part of the tensor and it imposes limits of

$$|k_{Fe^-}| < 10^{-14} \quad |k_{Fo^-}| < 10^{-10}$$

(1.61)

where the limits refer to the P-even and P-odd decomposition of $k_F$\cite{48,49}.

$\alpha$ is an important parameter (or rather, the function $g_0(V)$ is) since its value is related to the dispersion relation for the particle. Indeed, it can be shown\cite{35} that the energy of a moving particle is related to its velocity in the form

$$E = \frac{mc^2}{g_0(V)\sqrt{1 - \frac{V^2}{c^2}}}$$

(1.62)

which provides an interesting connection between the Robertson-Mansouri-Sexl test theory and some dispersion relation ones. But (1.62) defines also the transverse Doppler effect, from which $\alpha$ can be measured. The most accurate value obtained is\cite{91}

$$\left|\alpha + \frac{1}{2}\right| < 8 \times 10^{-7}$$

(1.63)

In spite of these impressive results, the accuracy of the above experiments is still below the requirement for testing Quantum Gravity at the tree level.

### 1.5.2 “Forbidden” processes

If the limiting velocity of a high energy particle is different from the light velocity, several processes forbidden in a Lorentz invariant context became possible\cite{92,93}. As an example, consider the decay of a photon in an electron-positron pair, a process which is usually forbidden by energy-momentum conservation (Fig. 1.3)

$$\gamma \rightarrow e^+ + e^-$$

(1.64)

But if the speed of photons is greater than the limiting speed of electrons this is no more so. Indeed, energy-momentum conservation implies

$$q = p_+ + p_-$$

(1.65)

Let us examine this process in the context of the $TH\epsilon\mu$ formalism\cite{128}. There is a threshold for this reaction: the photon energy should be enough to produce an electron-positron pair at rest, so that

$$E_\gamma > \frac{2m}{\sqrt{\left(\frac{c_0}{c\gamma}\right)^2 - 1}} = E_0$$

(1.66)
Figure 1.3: The forbidden process $\gamma \rightarrow e^+ + e^-$. 

Any photon with energy $E > E_0$ will rapidly decay into electron-positron pairs so that they should never arrive to earth. However, photons of energy $E_\gamma > 20$ TeV have been observed so that an upper limit is obtained \[92\]

$$\frac{c_\gamma}{c_e} - 1 < 1.5 \times 10^{-15} \quad (1.67)$$

On the other hand, if the limiting speed is greater than the photon speed, the energy of charged particles is limited by vacuum Cerenkov process

$$p \rightarrow p + \gamma \quad (1.68)$$

whose threshold is

$$E_0' = \frac{m_p}{\sqrt{(1 - \frac{c_\gamma}{c_e})^2}} \quad (1.69)$$

From the fact that the upper limit of cosmic ray spectrum is $E_u \sim 10^{20}$ eV a lower bound of

$$\frac{c_\gamma}{c_e} - 1 > -5 \times 10^{-23} \quad (1.70)$$

was obtained \[92\].

Of course, the same phenomena can be analyzed in the context of any dispersion relation model, once the effective limit- and light- speeds can be determined. For
instance, much more stringent limits have been obtained from the γ-ray spectrum of the Crab nebula [94]. These limits have been obtained within the Myers-Pospelov test theory.

1.5.3 Propagation phenomena: delays and polarization

The propagation of high energy particles, specially photons, has been one of the most important tests of Lorentz Invariance since the seminal paper of Amelino-Camelia et. al. [8]. These are based on the simple fact that particles of different energy, with dispersion relations of the form (1.16), will have different velocities and so different travel times from the same sources.

For a pair of photons with energies $E_1, E_2$ and dispersion relation of the form (1.16) with $n = 1$ the difference in arrival time would be

$$\frac{\Delta t}{T} \simeq \eta \frac{\Delta E}{E_P}$$

(1.71)

where $T \simeq D/c$ is the travel time of the photon. For a photon energy difference of 1 GeV and a travel time of $10^{17}$ s a time delay of the order of a second results, depending on the value of $\eta$. A typical bound from TeV flares of Makarian 421 is [95]

$$\frac{E_P}{\eta} \geq 4 \times 10^{16} \text{GeV}$$

(1.72)

implying

$$\eta > 300$$

(1.73)

This limit is still below the accuracy requirements (1.58) for testing Quantum gravity.

On the other hand, very interesting results have been found studying the polarization behavior of the traveling radiation. As an example, consider the modified Maxwell equations derived from the Hamiltonian (1.45) [13, 96]

$$\dot{E} = -\nabla \times B + 2\theta_{GP}\ell_P\nabla^2 B$$

(1.74a)

$$\dot{B} = \nabla \times E - 2\theta_{GP}\ell_P\nabla^2 B$$

(1.74b)

These modified Maxwell equations (written in the privileged reference system) have a solution [13, 96]

$$E_{\pm} = (\hat{e}_1 \pm \hat{e}_2)e^{i(\Omega_{\pm}t - k \cdot x)}$$

(1.75a)

and a similar expression for $B$. Here $\hat{e}_{1,2}$ denote the polarization unit vectors and

$$\Omega_{\pm} = k (1 \mp 2\theta_{GP}\ell_P k)$$

(1.75b)

is the dispersion relation for photon propagation in vacuum.
The latter equation predicts birefringence of the vacuum. Indeed, for a wave packet traveling along the $x$ axis, formed with a superposition of plane waves with central frequency $\Omega_0$ \[(1.75a),\] one finds

$$E \simeq e^{i\Omega_0(t-x)} \left[ A(x-v_+ t)e^{-i\theta_{GP}\ell \Omega_0^2 z} \hat{e}_+ + A(x-v_- t)e^{i\theta_{GP}\ell \Omega_0^2 z} \hat{e}_- \right]$$ \[(1.76)\]

with $A(x)$ the initial wave packet, $\hat{e}_\pm = \hat{e}_1 \pm \hat{e}_2$ and the group velocities are given by

$$v_\pm = 1 \mp 4\theta_{GP}\ell \Omega_0^2 k$$ \[(1.77)\]

For short enough distances, so that

$$8\theta_{GP}\ell \Omega_0 \delta \Omega \ll 1$$ \[(1.78)\]

where $\delta \Omega$ is the frequency width of the wave packet, equation \[(1.76)\] can be recast in the form

$$E \sim B(t-x) \left[ \cos (\theta_{GP}\ell \Omega_0^2 z) \hat{e}_1 + \sin (\theta_{GP}\ell \Omega_0^2 z) \hat{e}_2 \right]$$ \[(1.79)\]

showing explicitly a rotation of the polarization vector.

From the fact that the optical polarization spectrum of many distant galaxies is flat, the following bound was found for the Gambini-Pullin parameter \[(96)\]

$$|\theta_{GP}| < 10^{-4}$$ \[(1.80)\]

This is a very strict bound on a parameter directly connected to Quantum Gravity.

Even more strict bounds were obtained for the P-odd part of the $k_F$ tensor in the Lorentz violating extension of the Standard Model \[(47, 48)\]. From an analysis of a carefully selected set of cosmologically distant sources, whose polarization had been measured. From a careful analysis of these sources, a limit

$$|\kappa_F^\gamma| < 10^{-31}$$ \[(1.81)\]

was found.

The bounds \[(1.80)\] and \[(1.81)\] are in a certain sense complementary, since the Lorentz violating extension of the Standard Model does not include terms of dimension 5, such as those in the Gambini-Pullin Hamiltonian.

### 1.5.4 Laboratory experiments

There has been a great number of laboratory experiments designed to detect tiny violations of Lorentz invariance. Among them, the Hughes-Drever-like experiments \[(97, 98)\] designed to detect tiny anisotropies in the laboratory reference system, are the most sensitive ones.
In a Hughes-Drever experiment the spin of an atomic or nuclear system is oriented in a magnetic field in a reference system rotating with angular velocity $\Omega$ and moving with velocity $w$ with respect of the privileged one (fig. 1.4). The motion with respect to the privileged system generates local anisotropies that in turn generate a signal of frequency $n\Omega$.

In the latter years many new versions of the original Hughes-Drever experiment have been devised, with ever increasing accuracy \cite{99, 100, 101, 102, 103, 104, 105}. As an example, let us consider bounds on the Dirac Hamiltonian parameters \cite{25}.

As we have already said, the last line in (1.53) represent a coupling of the spin to the velocity with respect to the privileged frame $w$. Keeping only the dominant terms and passing to the usual units we find the dominant perturbation

$$\delta H_S = \left( \Theta_1 + \frac{1}{2} \Theta_4 \right) M\ell_p (2Mc^2) \left[ 1 + O \left( \frac{p^2}{2Mc^2} \right) \right] s \cdot \frac{w}{c} \tag{1.82}$$

The third line in (1.53) represents an anisotropy of the inertial mass, to test which the Hughes-Drever experiments were designed. With the approximation

$$Q_P \simeq -\frac{5}{3} \left( \frac{p^2}{4M^2} \right) Q \frac{R^2}{R^2} \tag{1.83}$$

for the momentum quadrupole moment, with $Q$ the electric quadrupole moment and $R$ the nuclear radius, we obtain

$$\delta H_Q = -\Theta_1 M\ell_p \frac{5}{3} \left( \frac{p^2}{2M} \right) Q \frac{R^2}{R^2} \left( \frac{w}{c} \right)^2 P_2(\cos \theta) \tag{1.84}$$
for the quadrupole moment perturbation, where $\theta$ is the angle between the quantization axis and $w$, $\langle p^2/2M \rangle \sim 40$ MeV for the kinetic energy of the last shell of a typical heavy nucleus. From the results of references [101, 104] we find \[ \left| \Theta_1 + \frac{1}{2} \Theta_4 \right| < 2 \times 10^{-9} \quad \left| \Theta_1 \right| < 3 \times 10^{-5} \] (1.85)

This set of experiments can be interpreted in different test theories. For instance, the above results have been used to set bounds for the Myers-Pospelov parameters [63].

Besides, the parameter $c - 1$ in the $TH\epsilon\mu$ model has been constrained from the results of references [99, 100, 101] \[ |c - 1| < 10^{-21} \] (1.86)
the result being less sensitive than (1.85) because of several suppressing nuclear factors.

Much more interesting is the possibility of constraining the mass of the D-particle from the above results. Indeed, from equation (1.56) one obtains the low-energy perturbation Hamiltonian \[ H'_Q = -\frac{4m_N}{M_D} \frac{w \cdot Q_P \cdot w}{Mc^2} \] (1.87)
and comparison with (1.53) shows that \[ M_D = \frac{4}{\Theta_1 \ell_P} > 1.2 \times 10^5 M_P \] (1.88)

This is the strongest bound found so far for the mass of the D-particle. The corresponding recoil velocity \[ v < 2 \times 10^{-27}c \] (1.89)
is extremely small, even by the standards of everyday experience (e.g. the speed of a crawling snail is $10^{-11}c$), that it seems quite unlikely that it will be detected in more direct experiments, such as time delays.

1.5.5 The GZK cutoff

The spectrum of high energy cosmic rays should show a cutoff at energies about 50 EeV, called the GZK cutoff [105, 107]. The origin of this cutoff comes from several processes that eat up the energy of primary cosmic rays, such as inverse Compton effect by the Cosmic Microwave Background photons. A modern evaluation of the ultrahigh energy spectrum of cosmic rays can be found in [108] and a complete review of the problem in [109].
In spite of the theoretical prediction, quite a few cosmic rays of energies greater than the GZK cutoff have been found in several cosmic ray observatories, mainly AGASA [110, 111]. This is usually called the GZK anomaly.

Alfaro and Palma [112, 111] have shown that an explanation of the GZK anomaly can be found within the Loop Quantum gravity model, and besides, that this can be used to test quadratic terms in the Planck scale.

To see how this can be done, observe that the dispersion relations for particles in the Loop Quantum Gravity of section 1.4.2 take the form

\[ E^2_{\pm} \simeq 1 + 2\kappa_\alpha \left( \frac{\ell_P}{\mathcal{L}} \right)^2 p^2 + \kappa_\eta \ell_P p^4 \pm \kappa_\lambda \frac{\ell_P}{2\mathcal{L}^2} p + m^2 \]  
\[ \omega_{\pm} \simeq k \left[ 1 + \kappa_\gamma \left( \frac{\ell_P}{\mathcal{L}} \right)^{2+2\Upsilon} - \theta_3 (\ell_P k)^2 \pm \theta_{GP} \ell_P k \right] \]  

for massive fermions and photons respectively. In these equations the \( \kappa_i \) and \( \theta \) parameters are numbers of order unity. The explicit dependence with the \( \mathcal{L} \) and \( \Upsilon \) parameters has been left explicit.

With these equations, a careful analysis of the main reactions in the presence of Lorentz violations was carried with the result that the GZK anomaly could be explained. A fit to the data is possible assuming that \( \mathcal{L} \) is an universal constant with a value

\[ 2.6 \times 10^{-18} \text{ eV}^{-1} \lesssim \mathcal{L} \lesssim 1.6 \times 10^{-17} \text{ eV}^{-1} \]  

The good fit shown in Figure 1.5 shows the importance of threshold analysis for the test of even quadratic order in the Planck scale.
1.5.6 Other tests of Lorentz invariance

There are many other tests of Lorentz invariance, but few of them satisfy our accuracy requirement (1.58). The most important of them are the laboratory test for CPT symmetry [113, 114, 115].

These experiments test the CPT symmetry of an electron (proton) in a Penning trap. Although potentially they should be very sensitive, they have not yet our required accuracy (1.58). Their impressive result

$$\frac{\Delta a}{2m_e c^2} = (3 \pm 12) \times 10^{-22}$$

(1.92)

translates into the weak constraint [84]

$$\Theta_2 + \frac{1}{2} \Theta_4 \lesssim 1$$

(1.93)

Another important group of experiments are made with spin-polarized matter [116, 84]. In these experiments a torsion pendulum with a total spin $S \sim 10^{22}$ can be used to extract a signal coupling spin and velocity with respect to a privileged frame. Their result translates into the mild bound

$$\Theta_2 + \frac{1}{2} \Theta_4 \lesssim 0.002$$

(1.94)

Although the increasing accuracy of these experiments will improve very much the above bound, they are still far from the results of Hughes-Drever experiments. Their main interest is that the parameters $\Theta_i$ are estimated from electron instead of proton, yielding independent tests of Lorentz invariance.
1.6 The influence of radiative corrections

Radiative corrections, whether at one-loop or higher order, open the possibility of testing energies as high as Planck’s in nowadays existing laboratory experiments. Indeed, consider the diagram in Figure 1.7: the loop variable $k$ runs over all energies, probing thus near Planck scale.

Although quite a lot of work has been done on radiative corrections in Lorentz non-invariant theories [52, 117, 118, 65], only quite recently it was shown that they impose very stringent constraints on Lorentz violation phenomena [66]. In the context of renormalization theory, this originates a new fine tuning problem.

The main idea is that Planck scale physics should cutoff the range of the loop variable at a given scale $\Lambda \sim E_P$. If this cutoff is anisotropic, this anisotropy will be “dredged up” to a low energy scale. This would include corrections to the dispersion relations of the order of the coupling constants of the standard model, rather than $O(\ell P)$. In the following sections, we shall discuss first a couple of examples and then discuss the general case.

1.6.1 A couple of examples

Let us consider first the electron self-energy, introducing a cutoff function $C(k^2)$ to model Planck scale effects

$$\Sigma^{(2)}(p, \Lambda, \xi) = \frac{e^2}{4\pi i} \int d^4 k \gamma^\mu \left( \frac{p \cdot (p-k)}{(p-k)^2 - m^2} \gamma^\mu k^2 + i \epsilon \right) C(k^2 + \xi (W \cdot k)^2) \quad (1.95)$$

where $\xi$ parametrizes an anisotropy in the laboratory system. Consider the particular case

$$C(k^2) = \frac{\Lambda^2}{\Lambda^2 - k^2} \quad (1.96)$$

with $\Lambda \sim E_P$ and $\xi \ll 1$. Then we can expand the self energy in powers of $\xi$ and keep the lowest order term

$$\Sigma^{(2)}(p, \Lambda, \xi) = \Sigma^{(2)}(p, \Lambda, 0) + \delta \Sigma \quad (1.97)$$
with
\[
\delta \Sigma(p, \Lambda, \xi) = -i\alpha \int d^4 k \gamma^\mu \frac{\gamma \cdot (p - k) + m}{(p - k)^2 - m^2} \frac{1}{k^2 + i\epsilon} \frac{\Lambda^2}{\Lambda^2 - k^2} \frac{\xi(W \cdot k)^2}{\Lambda^2 - k^2} \quad (1.98)
\]

After the usual procedure of combining denominators and shifting the loop variable one is lead to the on-shell expression
\[
\delta \Sigma(\not p = m, \Lambda, \xi) = 12\alpha \xi \int_0^1 zdz \int_0^1 y^2 dy \int d^4 q \frac{P(q)\Lambda^2}{[q^2 - m^2(1 - y)^2 - \Lambda^2 xy]} \quad (1.99)
\]
with
\[
P(q) = \frac{1}{4} [(4 + 2y)mW^2 - 4(\gamma \cdot W)(W \cdot p)(1 - y)] q^2 + O(q^0) \quad (1.100)
\]

To compute this integral one usually performs a Wick rotation, but it is not clear that this can be done in the highly fractal structure of spacetime at the Planck scale. Indeed, one expects that the physical cutoff induced by discreteness will be in real spacetime. But this can be done in this particular model with the result
\[
\delta \Sigma(\not p = m, \Lambda, \xi) = \pi^2 \alpha \xi \int_0^1 zdz \int_0^1 y^2 dy \tilde{P}(y) \frac{\Lambda^2}{m^2(1 - y)^2 + \Lambda^2 xy} \quad (1.101)
\]
with
\[
\tilde{P}(y) = [(4 + 2y)mW^2 - 2(\gamma \cdot W)(W \cdot p)(1 - y)] \quad (1.102)
\]

The integral is finite in the limit \( \Lambda \to \infty \) and we find thus a correction term in the nonrelativistic limit
\[
\delta \Sigma \sim \xi \alpha m (s \cdot w) \quad (1.103)
\]
that contradicts experiment unless \( \xi \ll 10^{-28} \).

As a second example, let us consider a non-commutative version of QED, with
\[
[x^\mu, x^\nu] = i\theta^{\mu\nu} \quad (1.104)
\]
with \( \theta^{\mu\nu} \) a c-number.

This case has been analyzed in reference [119] with the result
\[
\delta L_{\text{ct}} = \left\{ \begin{array}{ll}
\frac{3}{4} m\Lambda^2 \alpha^2 \frac{\psi \theta^{\mu\nu} \sigma_{\mu\nu} \psi}{2\pi m\alpha^2} & , \quad \theta \Lambda^2 \ll 1 \\
2\pi m\alpha^2 \frac{\psi \theta^{\mu\nu} \sigma_{\mu\nu} \psi}{\text{Tr} \theta^2} & , \quad \theta \Lambda^2 \gg 1
\end{array} \right. \quad (1.105)
\]
where \( \theta \) is a typical scale of non-commutativity, such as \( \theta = \sqrt{\theta^{\mu\nu} \theta_{\mu\nu}} \).

The second result predicts an anisotropy of order \( \alpha^2 \), independent of both the scale of non-commutativity and cutoff and thus is in strong disagreement with experiment, while the first can only be consistent in the strange situation where the cutoff \( \Lambda \) is much smaller than the interesting non-commutative scale. This means,
of course that the non-commutative field theory ceases to be valid at energy scales much smaller than the non-commutativity scale.

For our last example, consider the second order self energy of a scalar particle in a Yukawa theory $\Pi^{(2)}(p)$ (Figure 1.8). A measure of Lorentz violating in this graph is given by the quantity [66]

$$\xi = \frac{\partial^2 \Pi^{(2)}(p)}{\partial p_0^2} + \frac{\partial^2 \Pi^{(2)}(p)}{\partial p_1^2} \bigg|_{p=0}$$ (1.106)

Without Planck scale modifications one would obtain

$$\xi = -\frac{ig^2}{\pi^4} \int d^4k \frac{(k_0^2 + k_1^2)(k^2 + 3M^2)}{(k^2 - M^2 + i\epsilon)^4}$$ (1.107)

This integral could be shown to be zero by a Wick rotation, after a suitable regularization to avoid the logarithmic divergence. However, if Planck scale physics introduces a cutoff, things are different. For instance, assume that the fermion propagator has a cutoff in the form

$$S'(k) = f(k)\frac{\gamma \cdot k + M}{k^2 - M^2} \quad f(0) = 1 \quad f(\infty) = 0$$ (1.108)

then the integral can be computed and the corresponding Lorentz violation is

$$\xi = \frac{g^2}{6\pi^2} \left[ 1 + \int_0^\infty dx x f'(x)^2 \right]$$ (1.109)

This shows again that a Lorentz violation at the Planck scale is translated into a Lorentz violating term at low energy, of the order of the coupling constant squared.

1.6.2 A general argument

Let us now show that the above examples are really parts of a general property of radiative corrections. To fix our ideas, consider the self energy of a scalar particle
in a Yukawa theory. The dispersion relation for the particle is obtained by solving

$$E^2 - p^2 - m^2 - \Pi(E, p) = 0$$  \hbox{(1.110)}

with $\Pi$ the sum of all self-energy graphs, to which we have added any small Lorentz-violating corrections that stem from tree-level theory.

Now we reason as follows: without a cutoff the graphs have divergences from large momenta. In the Lagrangian defining the theory the divergences correspond to terms of dimension 4 (or less) that obey the symmetries of the microscopic theory. In a Lorentz invariant case these divergences are removed by renormalization of the parameters of the theory. But Planck scale physics can both cutoff the divergences and modify the expressions that define the graph in the neighborhood of the Planck energy. The same power counting that determines the divergences also determines the natural size of the contributions from Planck scale momenta: the dominant contributions correspond to operators of dimension 4 (or less) in the Lagrangian. If the microscopic theory violates Lorentz invariance at the Planck scale, then generically we get Lorentz violating terms at low energy, without any suppression by powers of $E/E_P$.

Of course, the above Lorentz violating terms can be removed by explicitly including Lorentz violating counterterms of dimension 4 in the Lagrangian which are fine-tuned to give the observed low-energy Lorentz invariance. Such fine-tuning is unacceptable \cite{120, 121, 122} in a fundamental theory.

In the models considered in Section 1.3 and 1.5 Lorentz violating corrections to the dispersion relations, suppressed by some power of $E/E_P$, were found by considering the propagation of free particles in a granular spacetime background. The above reasoning shows that there are effects that are only suppressed by two powers of the standard model coupling constants. These effects change the effective value of $c$ in the usual dispersion relation

$$E^2 = c^2 p^2 + m^2 c^4$$  \hbox{(1.111)}

The well-known structure of the standard model shows that \hbox{(1.111)} predicts differences up to 10% between different particles, in violent disagreement with experiment that shows that the fractional differences in $c$ is below $10^{-20}$.

The above result is a consequence of well-known properties of quantum field theories, of which the standard model is an example. Indeed the actual technical result is related to more or less explicit statements that can be found in several references \cite{123, 6, 93, 7, 118, 63} although never fully recognized except in the recent \cite{64}. For a criticism of these results see \cite{65, 66}.

\section{1.7 Conclusion}

In spite of the difficult nature of the research, a group of delicate experiments has been devised to test Lorentz invariance violations induced by Quantum Gravity. Up
to now, no clear signal of Lorentz violation has been found (See, however, Section 1.5.5).

Tree-level tests of Quantum Gravity modifications include attempts to measure the parameters of the effective Lagrangian’s describing these modifications. Several of the above mentioned experiments are accurate enough to measure the parameters of corrected dispersion relations with high precision. Even better results can be found from Hughes-Drever experiments. Indeed, the bounds on the parameters are so small that they call into question the full scheme of Lorentz invariance violations induced by Quantum Gravity, at least with respect to terms linear in the Planck length [25, 26].

Even more strict results stem from the analysis of radiative corrections. We have found that the loop variables can dredge up Lorentz violations at the Planck scale up to the low-energy scale. This would violently contradict experiment, unless extremely fine tuning counterterms are introduced in the Lorentz violating Lagrangian. This shows that Lorentz invariance should be added to the well-known list of fine-tuning problems; namely, the cosmological constant, the Higgs bare mass and mass hierarchies.

The implications of the above argument for both experiment and theory are quite profound. First, the present unsuccessful searches for Lorentz violations suffice by many orders of magnitude. Of course, since it is correct science to question and test accepted principles, tests of Lorentz invariance are worthwhile. However, many searches that were started by estimates of specific orders of magnitude may be misled and should be revised.

As to the theory, the critical task concerns any proposal in which Lorentz symmetry is substantially broken at the Planck scale: to find and implement a mechanism to give automatic Lorentz invariance at low energy despite a violation at Planck scale. We assume here that the treatment involves real time, not an analytic continuation to imaginary time, as is common in quantum field theory in flat spacetime (See, however, Section 1.6.1). One mechanism is to have a custodial symmetry that is sufficient to prohibit Lorentz-violating symmetry without being itself the full Lorentz group. Such a symmetry does not appear to be known and the Coleman-Mandula theorem [124] suggests it does not exists.

It should be noted that there is not necessarily a conflict between discreteness and the absence of a preferred frame. For instance in reference 125 a Lorentz invariant macroscopic space is constructed by the use of a random causal set of points. On the other hand, the authors of 126 argue that the existence of a minimum length does not imply local Lorentz invariance violation, anymore than the discreteness of angular momentum eigenvalues signal a violation of rotational invariance.

An optimistic point of view should be stressed: a branch of theoretical physics long considered to suffer from detachment from experimental guidance is now in the opposite situation. The subtle interplay of cosmology, atomic and nuclear physics is shedding light on such a recondite subject as quantum gravity.
Our results show that Lorentz invariance still imposes stringent requirements on the mathematical theories that attempt to describe experimental results.

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