Strain transfer analysis of substrate fiber Bragg grating sensor in engineering structure health monitoring

Chuanyun Yue

1 Department of civil engineering, Shenyang Urban Construction University, Shenyang, Liaoning, 110000, China

Abstract. Fiber Bragg grating sensors are usually buried in the matrix or pasted on the surface of the matrix in practical engineering applications. Due to the different physical and mechanical properties of the fiber, the protective layer, the adhesive layer and the matrix material, the measured strain of fiber Bragg grating sensors is not equal to the actual strain of the matrix structure, so it is necessary to correct the measured strain. Based on substrate optical fiber gratings as the research object, established the "matrix structure, colloid, substrate, colloid, and fiber Bragg grating sensor of strain transfer model, deduce the substrate optical fiber grating sensor measurement of strain and matrix structure actual strain between the strain transfer calculation formula, and through the finite element analysis and experiment validation of theoretical derivation. Finite element analysis and experiments verify the correctness of theoretical derivation, and the strain transfer formula obtained can be applied to practical engineering.

1. Introduction

It has become an inevitable requirement of civil engineering and one of the important research fields of civil engineering to monitor the health of structures and discover the damage of structures in time so as to realize the safety evaluation of structures. Fiber Bragg grating sensing uses light wave to transmit information, which is safe and reliable without electromagnetic interference. With small volume, high sensitivity and corrosion resistance, fiber Bragg grating sensors can be used for distributed or quasi-distributed measurement and remote monitoring and transmission. Therefore, the use of fiber Bragg grating sensors for structural health monitoring has become one of the important development directions in the field of monitoring [1].

The main component of the optical fiber is silica, whose outer diameter is about 125 μm. The shear resistance is poor and it is easy to be brittle. Therefore, it is necessary to package and protect the bare optical fiber in the application. Fiber Bragg grating strain sensors are usually encapsulated in embedded [2,3] and surface [4,5,6]. Embedded in [7,8,9] is the fiber Bragg grating encapsulated in metal or other materials and directly embedded in the matrix. The surface is the fiber Bragg grating pasted on a certain substrate and then pasted on the surface of the substrate under test for monitoring. Because the elastic modulus of the materials used for encapsulating the fiber grating, the elastic modulus of the fiber and the elastic modulus of the matrix are not consistent, the strain transfer loss will be caused in the process of strain transfer, so the strain measured by the fiber grating is not the actual strain of the matrix.
2. Theoretical analysis of strain transfer

The packaging method of substrate fiber Bragg grating sensor is to encapsulate the fiber Bragg grating on the substrate with small grooves, through which the strain of the structure is transferred to the fiber Bragg grating. Its section is shown in figure 2. The basic hypothesis of theoretical analysis: the colloidal material is linear elastic isotropic body, and all bonding surfaces are continuous without relative slip; The substrate and fiber Bragg grating can only bear the force transferred along the direction of sticking. The fiber grating sensor has no effect on the matrix structure. The longitudinal section diagram of the substrate fiber Bragg grating sensor is shown in figure 1, and the adhesive length of the fiber is 2L.

The cross section diagram of the substrate fiber Bragg grating sensor pasted on the matrix is shown in figure 2. The strain transfer process of the sensor is from matrix - gel - substrate - gel - fiber. The strain transfer loss mainly occurs in two processes, one is substrate to substrate, the other is substrate to fiber. The losses of these two parts are calculated separately and then superimposed to obtain the total strain transfer loss.

Firstly, the strain loss from matrix to substrate is analyzed. As shown in figure 3, \( \sigma \) is the axial stress of the research object, \( \tau_{ar}, \tau_{ah} \) is the shear stress between the layers, \( d\sigma_b, d\sigma_a, d\sigma_h \) is the axial force of the substrate, the colloid and the microelement of the fiber, respectively. The width of the paste is b, the height of the substrate is H, and the thickness of the colloid layer is H.

The equilibrium equation of the microelement segment of the substrate obtained in FIG. 3 is;

Substrate layer:

\[
d\sigma_b \times b \times H + \tau_{ar} \cdot b \cdot dx = 0
\]  

(1)

The equilibrium equation of colloidal microsegment taken in FIG. 3 is:

Gel layer:

\[
d\sigma_a \cdot b \cdot h + \tau_{ar} \cdot b \cdot dx = \tau_{ah} \cdot b \cdot dx
\]  

(2)
The displacement of each layer is:

Matrix:

\[ u_h = \int_0^H \frac{\sigma_h(x)}{E_h} dx \]  

(3)

Substrate:

\[ u_b = \int_0^{H+h} \frac{\sigma_b(x)}{E_b} dx \]  

(4)

Colloid:

\[ \Delta_a = \int_H^{H+h} \frac{\tau(x,y)}{G_a} dy \]  

(5)

According to the displacement relation:

\[ u_b = u_h + \Delta_a \]  

(6)

Substitute equations (3), (4) and (5) into equations (6) to obtain

\[ \int_0^H \frac{\sigma_h(x)}{E_h} dx = \int_0^H \frac{\sigma_b(x)}{E_b} dx + \frac{\sigma_h \cdot E_u \cdot h^2}{2l \cdot G_a \cdot E_h} + \frac{h}{G_a} \cdot \tau_{ar} \]  

(7)

The derivative of equation (7) is

\[ \frac{\partial^2 \tau_{ar}}{\partial x^2} - \frac{G_a}{h \cdot E_u \cdot H} \tau_{ar} = 0 \]  

(8)

The solution of equation (8) can be obtained as follows:

\[ \epsilon_b(x) = \epsilon_b(0) \left[ 1 - \frac{\cosh(\lambda x)}{\cosh(\lambda L)} \right] \]  

(9)

The strain loss from substrate to fiber is analyzed. As shown in figure 4, the assumption in the substrate is the small groove of the triangle, the side length for \( a < \frac{3}{\sqrt{3}} H \), optical fiber radius \( r_f \), the length of the paste for 2L, \( \sigma_c \) for small groove colloid and \( \sigma_g \) for fiber axial stress, \( \tau_c \) for colloid and the shear stress between the substrate groove, \( \tau_g \) for the groove colloid and the shear stress between the optical fiber, \( r_b \) fiber optic axis to the groove edge distance.

![Figure 4. Slot schematic of the substrate](image-url)
For the balance equation of the microsegment of the fiber as shown in figure 5:

\[-\sigma_g(x) \cdot \varpi_{g}^2 + [\sigma_g(x) + d\sigma_g(x)] \cdot \varpi_{g}^2 + 2\varpi_g \cdot dx \cdot \tau_g = 0\]  

Equation (10)

The equilibrium equation of the micro-element segment of the protective layer (paste layer) taken in FIG. 5 is as follows:

\[-\sigma_r(x)\left(\frac{1}{2}ab - \varpi_{g}^2\right) + [\sigma_r(x) + d\sigma_r(x)]\left(\frac{1}{2}ab - \varpi_{g}^2\right) + \tau_r \cdot 2a \cdot dx \cdot \tau_g = \tau_g \cdot 2\varpi_g \cdot dx\]  

Equation (11)

Equation (11) is simplified as follows

\[\tau_r = -\frac{1}{2a} \left[ \frac{d\sigma_r(x)}{dx} \left(\frac{1}{2}ab - \varpi_{g}^2\right) + \frac{d\sigma_g(x)}{dx} \cdot \varpi_{g}^2 \right]\]  

Equation (12)

Formula (12) is sorted as follows:

\[\tau_r(x,r) = -\frac{E_g}{2a} \left[ \frac{E_r}{E_g} \cdot \frac{d\sigma_r(x)}{dx} \left(\frac{1}{2}ab - \varpi_{g}^2\right) + \frac{d\sigma_g(x)}{dx} \cdot \varpi_{g}^2 \right]\]  

Equation (13)

Since the elastic modulus of fiber Bragg grating and colloid differ greatly, fiber Bragg grating and colloid deform synchronously, so it can be considered \(\frac{d\sigma_r(x)}{dx} \approx \frac{d\sigma_g(x)}{dx}\) available:

\[\frac{E_r}{E_g} \cdot \frac{d\sigma_r(x)}{dx} \left(\frac{1}{2}ab - \varpi_{g}^2\right) \approx \alpha \left(\frac{d\sigma_g(x)}{dx}\right)\]

Formula (13) is arranged as follows:

\[\tau_r(x,r) = -\frac{E_r}{2a} \cdot \varpi_{g}^2 \cdot \frac{d\sigma_r(x)}{dx}\]  

Equation (14)

Considering only axial deformation, Substitute into equation (14) and integrate to get equation (15)

\[\int_{r_s}^{r_t} (G_r \cdot \frac{du}{dr}) dr = \int_{r_s}^{r_t} \left( -\frac{\varpi_{g}^2 \cdot E_g}{2a} \right) dr\]  

Equation (15)

\(u\) is the axial deformation of the colloid and \(G_r\) is the shear modulus of the colloid. \(r_s\) is the radius of the fiber grating.

Equation (15) is simplified to:

\[u_b - u_g = -\frac{\varpi_{g}^2 \cdot E_g (r_s - r_g)}{2a \cdot G_r} \cdot \frac{d\sigma_g(x)}{dx} = -\frac{\varpi_{g}^2 \cdot E_g (\sqrt{3}/6 - a - r_g)}{2a \cdot G_r} \cdot \frac{d\sigma_g(x)}{dx}\]  

Equation (16)
The values 

\[ k^2 = \frac{2\alpha G_k}{\bar{\nu}^2 E_g \left( \frac{\sqrt{3}}{6} a - r_g \right)} \]

So \( u_b - u_g = -\frac{1}{k^2} \frac{d\varepsilon}{dx} \) the derivative of both sides with respect to each other:

\[ \frac{d^2 \varepsilon_g}{dx^2} - k^2 \varepsilon_g + k^2 \varepsilon_b = 0 \]  

(17)

Equation (17) general solution: 

\[ \varepsilon_g(x) = c_1 e^{kx} + c_2 e^{-kx} + \varepsilon_b \]

The axial strain distribution of fiber is solved as follows

\[ \varepsilon_g(x) = \varepsilon_b \left( 1 - \frac{\cosh(kx)}{\cosh(kL)} \right) \]  

(18)

Based on the synthesis of (9) and (18), the strain relationship between matrix and fiber can be obtained as follows:

\[ \varepsilon_g(x) = \varepsilon_b \left[ 1 - \frac{\cosh(\lambda x)}{\cosh(\lambda L)} \right] \left[ 1 - \frac{\cosh(kx)}{\cosh(kL)} \right] \]  

(19)

The strain transfer coefficient of each point in the fiber sticking part can be expressed as:

\[ \alpha(x) = \left[ 1 - \frac{\cosh(\lambda x)}{\cosh(\lambda L)} \right] \left[ 1 - \frac{\cosh(kx)}{\cosh(kL)} \right] \]  

(20)

According to literature [8], the relationship between axial strain and wavelength change of fiber Bragg grating is as follows:

\[ \Delta \lambda_B = \alpha \varepsilon \]  

(21)

By substituting equation (21) into equation (19), the relationship between the wavelength change of fiber Bragg grating and matrix strain is obtained as follows:

\[ \varepsilon_b = \frac{\Delta \lambda_B}{\alpha \left[ 1 - \frac{\cosh(\lambda x)}{\cosh(\lambda L)} \right] \left[ 1 - \frac{\cosh(kx)}{\cosh(kL)} \right]} \]  

(22)

3. Comparison between finite element analysis and theoretical calculation

In order to verify the correctness of theoretical formula derivation, finite element analysis and theoretical calculation were used for comparison. In the finite element analysis, both matrix and substrate materials were made of steel plates with sizes of 40mm×15mm×3mm and 20mm×5mm×2mm, fiber length was 20mm, adhesive thickness was 0.2mm, slot width was 1mm, colloid elastic modulus [9] was 1GPa, 3GPa, 5GPa, 7GPa and 10GPa, and fiber elastic modulus was 72GPa. The solid model was established, and the fiber grid was divided into 8-node cells.

The strain transfer rate calculated by finite element method is compared with the strain transfer rate calculated by formula (19). The finite element calculation is close to the theoretical calculation, which verifies the correctness of the theoretical calculation.

Figure 6 shows the relationship between strain transfer rate and colloid elastic modulus. As can be seen from figure 6, the elastic modulus of the ge is the higher, the strain transfer rate is the higher. Thus, the stronger the interface bonding ability, the more fully the stress transfer. The elastic modulus
in colloids has a great influence on the strain transfer rate. It is assumed that the strain transfer rate can be maximized when the elastic modulus of the colloid reaches the elastic modulus of the matrix.

4. Experimental
In order to verify the theoretical results, the tensile test of the substrate strain sensor was carried out. The size of the steel substrate used for packaging is 40mm×15mm×1.5mm and the length of the bare fiber grating is 10mm. The fiber Bragg grating sensor encapsulated in the steel substrate is pasted on the steel plate and a high-precision resistance strain gauge is arranged in the corresponding position.

Figure 7 shows the strain curve of the fiber Bragg grating sensor encapsulated in steel substrate after theoretical calculation, the strain curve of the strain gauge and the strain curve of the bare fiber Bragg grating. As can be seen from the figure, the theoretical correction is close to the real value (the strain measured by the strain gauge), so the experiment well confirms the correctness of the theoretical derivation. Because of the multi-layer strain transfer process from the measured matrix to the bare fiber grating, there is a large error between the strain measured directly by the bare fiber grating and the real value, so it is necessary to correct the strain value of the fiber theoretically.

5. Conclusion
In this paper, the mechanical model of the substrate FBG sensor is established, the strain transfer formula of the substrate FBG sensor is derived, and the corresponding strain transfer coefficient is given. By comparing the finite element analysis with the theoretical calculation, the correctness of the theoretical calculation is verified. Through the tensile test of the fiber Bragg grating with steel substrate, the error between the uncorrected strain and the real strain is analyzed, and the correctness of the theoretical calculation is further verified. The strain transfer correction of the surface pasted fiber grating can be corrected according to formula (22) in this paper.
References

[1] Li Hongnan, Li Dongsheng, Zhao Baidong. Progress on study and application of smart health monitoring method by fiber optic sensor in civil engineering [J]. Earthquake Engineering and Engineering Vibration, 2002, 22(6):76–83.

[2] Sun Li, Li Hongnan, Ren Liang. Experiment study on shrink strain monitoring of concrete in the cure period with FBG sensors [A]. ASME, PVP, 2005. 71–91.

[3] Li Dongsheng, Li Hongnan, Ren Liang, Sun Li, Zhou Jing. Experiments on an offshore platform model by FBG sensors [A]. Sensors and Smart Structures Technologies for Civil, Mechanical and Aerospace Systems, Proceedings of SPIE – The International Society for Optical Engineering, 2004, 5391: 100–106.

[4] Wang Wei, Lin Yuchi, Research on strain transferring of surface FBG sensor. [J]. Journal of Laser & Infrared, 2008, 38(12): 1218-1220.

[5] GUO Wei, LI Xinliang, SONG Hao. Analysis of strain transfer of fiber grating sensors adhered to the structure surface. [J]. Journal of Measuring technique, 2011, 31(4); 1-4

[6] ZHOU Guang-dong, LI Hong nan. Study on influencing paramereas of strain transfer of optic fiber bragg grating sensors. [J]. Journal of Engineering mechanics, 2007, 24(6): 169-173

[7] YU Le-wen, ZHANG Da, YU Bin. A pole-rod type of fiber grating Strain sensor. [J]. Journal of Optoelectronics-Laser, 2012, 23(9): 1665-1668.

[8] Sun Li, Research of Fiber Bragg Grating Sensing Technology and Engineering APPlication [PhD Thesis]. Dalian University of Technology, 2006, 2

[9] Pak Y E. Longitudinal shear transfer in fiber optic sensors[J]. Smart Materials and Structures, 1992, 1(1): 57-62.