Effects of Nuclear Medium on the Sum Rules in Electron and Neutrino Scattering

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In this work, we study the influence of nuclear medium effects on various parton model sum rules in nuclei and compare the results with the free nucleon case. We have used relativistic nucleon spectral function to take into account Fermi motion, binding and nucleon correlations. The pion and rho meson cloud contributions have been incorporated in a microscopic model. The effect of shadowing has also been considered.

KEYWORDS: parton-model sum rules, deep inelastic scattering, nuclear medium effect

1. Introduction

In the deep inelastic scattering, the target nucleon is considered to be a collection of quarks and gluons (described as partons). The deep inelastic cross sections for charged lepton-nucleon scattering or (anti)neutrino-nucleon scattering are described in terms of nucleon structure functions which depend upon the momentum distribution of these quarks and gluons. Using the appropriate relations between these structure functions one can obtain certain relations. These relations are better known as parton-model sum rules. Some of these sum rules are Gross-Llewellyn Smith sum rule(GLS) [1], Adler sum rule(ASR) [2], Gottfried sum rule(GSR) [3].

GLS [1] is defined for an isoscalar nucleon target ‘N’ and a symmetric sea as

\[
S_{GLS} = \int_{0}^{1} F_{3N}^{\nu l}(x) dx = 3.
\]  

ASR [2] predicts the difference between the quark densities of the neutron and the proton, and is given by

\[
S_{ASR} = \int_{0}^{1} \frac{dx}{x} [F_{2n}^{\nu l}(x) - F_{2p}^{\nu l}(x)] = 2.
\]  

GSR [3] also known as valence isospin sum rule is given by

\[
S_{GSR} = \int_{0}^{1} \frac{dx}{x} [F_{2p}^{eN}(x) - F_{2n}^{eN}(x)] = \frac{1}{3} + \frac{2}{3} \int_{0}^{1} dx (\bar{u} - \bar{d})
\]

where \(F_{2}^{eN}(x)\) is the electromagnetic nucleon structure function and \(F_{i}^{\nu(l)N}(x); l = e, \mu, i=2,3\) is the weak nucleon structure functions.

With the development of high precision neutrino/antineutrino experiments as well as high luminosity electron beam experiments it is possible to verify these sum rules. These experiments are being done with moderate and heavier nuclear targets. For a nucleus, these sum rules are expressed in
terms of nuclear structure functions like $F_{1A}^{EM}(x, Q^2)$ and $F_{2A}^{EM}(x, Q^2)$ for electromagnetic processes and $F_{1A}^{Weak}(x, Q^2)$, $F_{2A}^{Weak}(x, Q^2)$ and $F_{3A}^{Weak}(x, Q^2)$ for weak interaction induced processes, which get modified because of the nucleons bound inside the nucleus. In the present work, we have taken into account nuclear medium effects like Fermi motion, binding energy, nucleon correlations, etc., using a relativistic nucleon spectral function in an interacting Fermi sea and local density approximation is then applied to obtain the results for finite nuclei. Furthermore, mesonic contributions and shadowing effects have also been taken into account. The results are compared with the free nucleon case as well as with some of the available experimental data.

The details of the present formalism are given in Ref. [4]. We are presenting the formalism in brief.

2. Formalism

For the charged lepton induced deep inelastic scattering process \( (l(k) + N(p) \rightarrow l(k') + X(p') \); \( l = e^-, \mu^- \)), the differential scattering cross section is given by

\[
\frac{d^2\sigma^N}{d\Omega dE_l} = \frac{\alpha^2}{q^4} \left| \mathbf{k}' \right| L_{\mu\nu} W_{N}^{\mu\nu},
\]  

(4)

where the hadronic tensor \( W_{N}^{\mu\nu} \) is defined in terms of nucleon structure functions \( W_i^N(i=1,2) \) as

\[
W_{N}^{\mu\nu} = \left( \frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) W_i^N + \left( p_N^\mu - \frac{P_N q}{q^2} q^\mu \right) \left( p_N^\nu - \frac{P_N q}{q^2} q^\nu \right) \frac{W_i^N}{2M^2}.
\]

(5)

with \( M \) as the mass of nucleon.

For the lepton scattering taking place with a nucleon moving inside the nucleus, the expression of the cross section is modified as

\[
\frac{d^2\sigma^A}{d\Omega dE_l} = \frac{\alpha^2}{q^4} \left| \mathbf{k}' \right| L_{\mu\nu} W_{A}^{\mu\nu},
\]

(6)

where \( W_{A}^{\mu\nu} \) is the nuclear hadronic tensor defined in terms of nuclear hadronic structure functions \( W_i^A(i=1,2) \) as

\[
W_{A}^{\mu\nu} = \left( \frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) W_i^A + \left( p_A^\mu - \frac{P_A q}{q^2} q^\mu \right) \left( p_A^\nu - \frac{P_A q}{q^2} q^\nu \right) \frac{W_i^A}{2M_A^2}.
\]

(7)

with \( M_A \) as the mass of nucleus.

To get \( d\sigma \) for \( (l, l') \) scattering on the nucleus, we are required to evaluate imaginary part of lepton self energy \( \Sigma(k) \) which is written using Feynman rules as [4]

\[
\Sigma(k) = \alpha e^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2} \frac{1}{2m} L_{\mu\nu} \frac{1}{k^2 - m^2 + ie} \Pi^{\mu\nu}(q),
\]

(8)

where \( \Pi^{\mu\nu}(q) \) the photon self energy and \( L_{\mu\nu} \) is the leptonic tensor \( L_{\mu\nu} = 2(k_\mu k_\nu' + k_\nu k_\mu' - k \cdot k' g_{\mu\nu}) \). Now we shall use the imaginary part of the lepton self energy i.e. \( Im\Sigma(k) \), to obtain the results for the cross section and for this we apply Cutkosky rules

\[
\begin{align*}
\Sigma(k) & \rightarrow 2i \, Im\Sigma(k), \\
\Pi^{\mu\nu}(q) & \rightarrow 2i\theta(q^0) \, Im\Pi^{\mu\nu}(q), \\
D(k') & \rightarrow 2i\theta(k'^0) \, ImD(k'), \\
G(p) & \rightarrow 2i\theta(p^0) \, ImG(p)
\end{align*}
\]

(9)
which leads to
\[ Im \Sigma(k) = e^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{2E_i} \delta(q^0) \ Im(\Pi^{\mu\nu}) \frac{1}{q^0} \frac{1}{2m} \ L_{\mu\nu} \] (10)

Notice from Eq. 10, \( \Sigma(k) \) contains photon self energy \( \Pi^{\mu\nu} \), which is written in terms of nucleon propagator \( G_l \) and meson propagator \( D_j \) and using Feynman rules this is given by

\[ \Pi^{\mu\nu}(q) = e^2 \int \frac{d^4p}{(2\pi)^4} G(p) \sum_X \sum_{s_p,s_i} \prod_{i=1}^N \int \frac{d^4p_i'}{(2\pi)^4} \ \prod_i G_i(p_i') \ \prod_j D_j(p_j') \]

\[ \langle X|J^\mu|H \rangle \langle X|J^\nu|H \rangle^* \ (2\pi)^4 \ \delta^4(q + \sum_{i=1}^N p_i') , \] (11)

where \( s_p \) is the spin of the nucleon, \( s_i \) is the spin of the fermions in \( X \), \( \langle X|J^\mu|H \rangle \) is the hadronic current for the initial state nucleon to the final state hadrons, index \( l \) runs for fermions and index \( j \) runs for bosons in the final hadron state \( X \).

The relativistic nucleon propagator \( G(p) \) in a nuclear medium is obtained as [6,7]:

\[ G(p) = \frac{M}{E(p)} \sum_r u_r(p) \bar{u}_r(p) \left[ \int_\infty^\mu d\omega \frac{S_h(\omega, p)}{p_0 - \omega - i\eta} + \int_\mu^\infty d\omega \frac{S_p(\omega, p)}{p_0 - \omega + i\eta} \right] , \] (12)

where \( S_h(\omega, p) \) and \( S_p(\omega, p) \) being the hole and particle spectral functions respectively, which are given in Ref. [7].

The cross section is then obtained as:

\[ \frac{d\sigma^A}{d\Omega_d dE'_I} = -\frac{\alpha}{q^0} \frac{|k'|}{|k|} \frac{1}{(2\pi)^2} L_{\mu\nu} \int Im \Pi^{\mu\nu} d^3r \] (13)

After performing some algebra, the expression of the nuclear hadronic tensor for an isospin symmetric nucleus in terms of nucleonic hadronic tensor and spectral function, is obtained as [4]

\[ W_A^{\alpha\beta} = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \int_\infty^\mu dp_0 S_h(p_0, p, \rho(r)) \left( \sum_{i=1}^2 \frac{W_A^{\nu/l}}{M} \right) \] (14)

Accordingly the dimensionless nuclear structure functions \( F_{i=1,2}^A(x, Q^2) \), are defined in terms of \( W_{i=1,2}^A(\nu, Q^2) \) as

\[ F_1^A(x, Q^2) = M A W_1^A(\nu, Q^2) \]

\[ F_2^A(x, Q^2) = \nu A W_2^A(\nu, Q^2) \] where \( \nu_A = \frac{P_A \cdot \bar{q}}{M_A} = \frac{P_0, q_0}{M_A} \) is the mass of a nucleus. \( M_A \) is the mass of a nucleus. \( \bar{\nu} \) and \( M_A \)

For weak interaction, we follow the same procedure, formalism for which is given in accompanying paper by Haider et al. [5] in this proceeding. For a nonisoscalar nuclear target the expression for the dimensionless structure functions \( F_{1,A}^A(x, Q^2) \) and \( F_{2,A}^A(x, Q^2) \) are obtained as

\[ F_{1,A}^{EM/Weak}(x_A, Q^2) = 2 \sum_{\tau = p,n} A M \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \int_\infty^\mu dp_0 S_h^T(p_0, p, \rho(r)) \left[ F_{1,A}^{EM/Weak,\tau}(x_N, Q^2) \right] + \frac{1}{M_A^2} P_A^2 \frac{F_{2,A}^{EM/Weak,\tau}(x_N, Q^2)}{\nu} \] (15)
\[ F_{EM/Weak}^{2A}(x_A, Q^2) = 2 \sum_{\tau=p,n} \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \int_{-\infty}^{\mu} dp_0 S_h^p(p_0, p, \rho^\tau(r)) \times \left[ \frac{Q^2}{q^2} \left( \frac{|p^2 - p_0^2|}{2M^2} \right) \right. \\
\left. + \frac{(p_0 - p_z \gamma)^2}{M^2} \left( \frac{p_z Q^2}{(p_0 - p_z \gamma)q_0 q_\gamma} + 1 \right) \right] \frac{M}{p_0 - p_z \gamma} F_{EM/Weak,\tau}^{2}(x, Q^2), \quad (17) \]

where \( \gamma = \frac{q_\gamma}{q_0} \).

\[ F_{EM/Weak}^{3A}(x_A, Q^2) = 2 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \left[ \int_{-\infty}^{\mu} dp_0 S_h^p(p_0, p, k_{p,\rho}) F_3^p(x_N, Q^2) \right. \\
\left. + \int_{-\infty}^{\mu} dp_0 S_h^n(p_0, p, k_{p,\rho}) F_3^n(x_N, Q^2) \right] \times \left( \frac{p_0^\gamma - p_z}{(p_0^\gamma - p_z \gamma)p_0} \right). \quad (18) \]

The nucleon structure functions \( F_i^N(x, Q^2) \) (i=1-3), are expressed in terms of parton distribution functions (PDFs), for which we have taken the parameterization of CTEQ6.6 [8]. The evaluations are performed at the Leading-Order (LO) as well as Next-to-Leading-Order (NLO). In the evaluation of sum rules (Eqs.1-3), the mesonic contribution cancels out if we follow the parameterization of Gluck et al. [9] for pion PDFs.

### 3. Results

In Fig.1, we have presented the results for the GLS sum rule in \(^{56}Fe\) at both LO and NLO and the results are also compared with CCFR experimental data [10, 11].

![Fig. 1. Results for GLS sum rule in \(^{56}Fe\) at both LO and NLO and the results are also compared with CCFR experimental data [10, 11].](image)

In Fig.1, we have presented the results for the GLS sum rule in the free nucleon evaluated at LO as well as in \(^{56}Fe\) nucleus, using spectral function of the nucleon. We find that when calculations are done using spectral function, the value of GLS integral decreases from the free nucleon case which is around 7 – 8% at all values of \(Q^2\). When shadowing effects are included following Ref. [12] (note that there is no mesonic contribution to \(F_3^p(x, Q^2)\)), \(S_{GLS}\) further reduces by \(\sim 10\%\) at low \(Q^2\) and about 3 – 4% at \(Q^2 = 12 – 15 \text{ GeV}^2\). This is the result of our full calculation at LO. When we evaluate the results at NLO, \(S_{GLS}\) further decreases by \(\sim 10\%\) at low \(Q^2\) and 6% at \(Q^2 = 10 – 15 \text{ GeV}^2\).

Similarly in Fig. 2, the results are presented for Adler and Gottfried sum rules. We find that \(S_{ASR}\) decreases from free nucleon case when spectral function is used, and the decrease is \(\sim 14 – 18\%\)
Fig. 2. Left panel: Results for Adler sum rule in $^{56}Fe$ at both LO and NLO and the results are also compared with the results of free nucleon. Right panel: Results for Gottfried sum rule in $^{56}Fe$ at both LO and NLO.

in $2 \, GeV^2 < Q^2 < 15 \, GeV^2$. When shadowing and mesonic effects are included there is further reduction of $\sim 10\%$ at low $Q^2$ and $\sim 3 - 4\%$ at high $Q^2$. The evaluation at NLO results in a very small change in the Adler sum rule.

In the right panel of this figure, we show the results for GSR evaluated with four quark flavors (u,d,s,c). We find that the inclusion of spectral function results in an increase of $S_{GSR}$ which is $\sim 14 - 15\%$ at $Q^2 = 2 - 3 \, GeV^2$ which becomes $\sim 30 - 35\%$ at $Q^2 = 10 - 15 \, GeV^2$ from free nucleon case. However, when shadowing and mesonic effects are taken into account, the net increase in $S_{GSR}$ is 7-8% at low $Q^2$ (for $Q^2 = 2 - 3 \, GeV^2$) which becomes 30-32% at higher $Q^2$ (for $Q^2 = 10 - 15 \, GeV^2$). There is 2-3% reduction at low $Q^2$ when the calculations are performed at NLO which becomes negligible at high $Q^2$.

We conclude that there is significant dependence of nuclear medium effects in the sum rules studied in this work. Moreover, we find that nuclear medium effects lead to $Q^2$ dependence in these sum rules. This study may be useful in the future analysis of experiments looking for the validity of sum rules.

References

[1] D. J. Gross and C. H. Llewellyn Smith: Nucl. Phys. B 14 (1969) 337.
[2] S. L. Adler: Phys. Rev. 143 (1966) 1144.
[3] K. Gottfried: Phys. Rev. Lett. 18 (1967) 1174.
[4] H. Haider, F. Zaidi, M. Sajjad Athar, S. K. Singh and I. Ruiz Simo: Nucl. Phys. A 943 (2015) 58.
H. Haider, I. Ruiz Simo, M. Sajjad Athar and M. J. Vicente Vacas: Phys. Rev. C 84 (2011) 054610,
M. Sajjad Athar, I. Ruiz Simo and M. J. Vicente Vacas: Nucl. Phys. A 857 (2011) 29.
[5] H. Haider, F. Zaidi, M. Sajjad Athar, S. K. Singh and I. Ruiz Simo, in this Proceeding.
[6] E. Marco, E. Oset and P. Fernandez de Cordoba: Nucl. Phys. A 611 (1996) 484.
[7] P. Fernandez de Cordoba and E. Oset, Phys. Rev. C 46 (1992) 1697.
[8] Pavel M. Nadolsky et al.: Phys. Rev. D 78 (2008) 013004; http://hep.pa.msu.edu/cteq/public.
[9] M. Gluck, E. Reya and A. Vogt, Z. Phys. C 53 (1992) 651.
[10] W. C. Leung et al., Phys. Lett. B 317 (1993) 655.
[11] J. H. Kim et al., Phys. Rev. Lett. 81 (1998) 3595.
[12] S. A. Kulagin and R. Petti, Phys. Rev. D 76 (2007) 094023.