New twists in compact binary waveform modelling: a fast time domain model for precession

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Compact Binary GW Signals

Compact binary coalescing signals:

- High dimensional parameter space: individual masses, spins, orientation parameters, sky location, matter parameters, eccentricity ...
- Knowledge about signals from different approaches during different stages of the evolution:
  - Early inspiral: Post-Newtonian theory, Self-force
  - Numerical relativity: late inspiral, plunge, merger and ringdown
  - Ringdown: BH perturbation theory

Waveform models are crucial for extracting the best info from the detectors data:

- Parameter estimation of source properties
- Tests of general relativity
- Searches/event rates, ...

Accurate, general and efficient waveform models needed for the challenges of next observing runs and future observatories (LISA, ET)
**Phenomenological waveform modelling program**

**Phenom(enological) waveform modelling**: accurate and fast representations of GW signals.

- Extreme compression of available information (PN theory, BH perturbation theory, Numerical Relativity) in terms of fast closed-form expressions for the waveforms.

In Fourier domain (best suited for most data analysis applications).

Continuous development towards modelling generic CBC signals:
- Non-spinning (PhenomA/B)
- Spin aligned (dominant mode): PhenomC/D/X
- Precessing: PhenomP/Pv2/Pv3/XP
- Higher harmonics: PhenomHM/XHM/Pv3HM/XPHM
- Eccentricity: PhenomXE
- Matter: PhenomNRTidal/NSBH

**Motivation for a time domain Phenom family:**
- Dispense with the Stationary Phase Approximation (SPA) for modelling the precession transfer functions.
- Closer relation to system dynamics (aims to help in the modelling of generic systems).
- Easier to approximate precessing ringdown.
- Cleaner inspiral-merger-ringdown separation for testing GR.
- While maintaining Phenom philosophy:
  - Efficient and accurate representation of the waveforms.
Phenom modelling in the time domain: non-precessing

GW polarisations decomposed in (spin-weighted) spherical harmonic basis:

\[ h_+(t) - i h_\times(t) = \sum_{l} \sum_{-l \leq m \leq l} h_{lm}(t) \times Y_{lm}(\iota, \phi) \]

Model separately each mode (2,±2), (2,±1), (3,±3), (4,±4), (5,±5):

- Piecewise C$^1$ expressions for amplitude and phase (derivative) of each mode.

\[ H_{lm} = |h_{lm}|, \phi_{lm} = \arg(h_{lm}), \omega_{lm} = \dot{\phi}_{lm} \]

- Inspiral: PN analytical expressions (3.5PN spinning TaylorT3 for orbital frequency, 3PN amplitudes from Blanchet+, 2PN corrections from Buonanno+, 1.5PN corrections from Arun+ + 3.5PN for (2,2) amplitude from Faye+)

- Intermediate/plunge: phenomenological expressions based on hyperbolic functions.

- Ringdown: adaptation of analytical expressions based on QNM decomposition from Damour+

- Calibrated with 531 BBH non-precessing NR simulations from SXS Collaboration Catalog (2019) Boyle+, 15 BAM simulations and 61 numerical waveforms from TeukCode.

\[ C_1 \]

[Image of graphs and tables related to the text]
Phenom modelling in the time domain: precessing

Precessing extension based on “twisting-up” technique:

\[ h_x(t) - i h_y(t) = \sum_{l} \sum_{-l \leq m \leq l} h_{lm}^l(t) 2Y_{lm}(\theta, \phi) \]

- Inertial frames modes obtained from rotation of non-inertial (co-precessing) modes with simpler morphology:
  \[ h_{lm}^l(t) = \mathcal{D}_{nm}^l \left[ \alpha(t), \beta(t), \gamma(t) \right] h_{lm'}^{coprec}(t) \]

- Euler angles encode precessing motion of the orbital plane:
  \[ \alpha = \arctan(\hat{\ell}_y, \hat{\ell}_x) \]
  \[ \cos \beta = \hat{J} \cdot \hat{\ell} = \hat{\ell}_z \]
  \[ \gamma = -\dot{\alpha} \cos \beta \]

- Co-precessing modes approximated from non-precessing model (with modified precessing final state):
  \[ h_{lm}^{coprec}(t; m_1/m_2, \chi_1, \chi_2) \approx h_{lm}^{nonprec}(t; m_1/m_2, \chi_1, \chi_2) \]

\( \hat{\ell} \) : Unit vector perpendicular to the orbital plane (Newtonian orbital angular momentum).

\( J \) : Total angular momentum of the system.
Main analytical approaches to precessing Euler angles:

- Next-to-next-to-leading order (NNLO) (Bohe+) effective single spin.
- Multiscale analysis (MSA) (Chatziioannou+) double spin.

Aimed for more direct comparisons with other Phenom models.

Evaluated with non-precessing analytical orbital frequency:

\[ v(t) = \Omega_{orb}^{1/3}, \quad \Omega_{orb} = \frac{1}{2} \omega_{22} T \]

Improvements over previous implementations:

- Numerical evaluation of minimal rotation condition (recovering of nonprecessing limit in MSA).
- Smooth plunge behaviour with linear continuation.
Euler angles: numerical evolution

Numerical evolution approach:

- Solve evolution equations for \( \ell \) (implies evolving individual spin vectors):
  \[
  \frac{d\ell}{dt} = \Omega(v(t), q, S_1, S_2) \times \ell \quad \ell = -\dot{S}_1 - \dot{S}_2
  \]
  \[
  \frac{dS_{1,2}}{dt} = \Omega(v(t), q, S_1, S_2) \times S_{1,2}
  \]

- Orbit averaged \( N^4 LO \) PN expressions included.

- Tracking all degrees of freedom: improvement over previous analytical expressions.

- Efficient evaluation in terms of analytical non-precessing orbital frequency: fast implementation.

- Simple analytical approximation attached at ringdown:
  \[
  \alpha^{RD}(t) \approx (\omega^{RD}_{122} - \omega^{RD}_{121})t + \alpha_0^{RD}
  \]
Comparison with other state-of-the-art waveform models

Unfaithfulness comparison with other state-of-the-art precessing multimode models: IMRPhenomXPHM, SEOBNRv4PHM and NRSur7dq4.

Great agreement with TD models (median $\sim 0.2\%$): more consistent treatment of merger-RD.

Better agreement of numerical approach with SEOB: more accurate inspiral than analytical approaches.

Disagreement for large mass asymmetry and high spins norm: possibly caveat of non-precessing orbital frequency.
Comparison with Numerical Relativity

Unfaithfulness comparison with Numerical Relativity precessing simulations:

- Bulk of cases below 1% mismatch.
- 1 outlier (SXS:0165) with challenging parameters. Need to include further physics (mode asymmetry).

Parameter estimation of NR injected signals:

- Correct recovery at $M_T = 100M_\odot$.
- Individual masses in 90% contour levels for higher masses.
- Need of more detailed systematic studies towards identifying recovery bias.
Parameter estimation: GW190412 re-analysis

GW190412: first reported GW event with confident mass asymmetry: interesting to compare different multimode models.

Non-precessing IMRPhenomTHM employed in published re-analysis (Colleoni+): great consistency with other NR-calibrated models.

Precessing re-analysis:

- Recovered medians and CI consistent with previous results.
- Slightly better agreement with SEOB results (in terms of medians and CIs). (SEOB results obtained with RIFT PE code)
- Higher SNR, likelihood and BF that for Fourier domain models.
Re-analysis of GWTC-1 with a new generation of phenomenological waveform models (Maite Mateu-Lucena, presented today at 12:40):

- Re-analysis with nonprecessing model IMRPhenomTHM for all events and precessing IMRPhenomTPHM for some of them.
- Consistent results with IMRPhenomXPHM, better inference for GW170729.

A detailed analysis of GW190521 with phenomenological waveform models (Marta Colleoni, Friday at 15:10):

- Discussion of recovery of high mass ratio support, higher mode content effects, probability of PISN mass gap, association with AGN flare ZTF19abanrhr ...
**Conclusions & future work**

| New precessing multimode model in the time domain for BBH signals: |
|---------------------------------------------------------------|
| • Phenom philosophy: fast and accurate representation of the waveforms. |
| • Improved inspiral Euler angle description: numerical evolution of spin evolution equations. |
| • Simple analytical approximation for the ringdown. |
| • Fast implementation. |
| • Candidate model for BBH coalescing signals. |
| • Reviewed by the LVC, publicly available with LALSuite. |

| Caveats and future improvements: |
|----------------------------------|
| • Improve efficiency: |
| • Inefficient evaluation time for low mass signals. |
| • Bottleneck in ringdown evaluation for highly redshifted massive systems. |
| • Improve physics: |
| • Consistent evolution of orbital frequency in terms of the evolving spin magnitudes. |
| • Include mode asymmetry effects. |
| • Better understanding of precessing ringdown. |