PROGRESS ON BEAM-PLASMA EFFECT SIMULATIONS IN MUON IONIZATION COOLING LATTICES

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Abstract

New computational tools are essential for accurate modeling and simulation of the next generation of muon-based accelerators. One of the crucial physics processes specific to muon accelerators that has not yet been simulated in detail is beam-induced plasma effect in liquid, solid, and gaseous absorbers. We report here on the progress of developing the required simulation tools and applying them to study the properties of plasma and its effects on the beam in muon ionization cooling channels.

INTRODUCTION

Though muon accelerator simulation codes have been steadily improving over the years, there is still much room for improvement. Many single-particle processes and collective effects in vacuum and matter, such as space charge, beam-beam effects, plasma effects from ionized electrons and ions have not been implemented in any single current code. These effects have to be either deemed negligible or taken into account to ensure the proper accuracy of simulations.

Ionization cooling (principle illustrated in Fig. 1) is a method by which the emittance of a muon beam can be reduced. A beam is sent through a material, losing momentum through multiple scattering and ionization processes, and reducing its emittance. By re-accelerating the beam through RF cavities, the longitudinal momentum is restored, and any lost energy is regained so that the process can be repeated.

The evolution of the normalized transverse emittance can be described by the following equation:

$$\frac{d\epsilon_n}{dz} \approx -\frac{1}{\beta^2} \frac{dE_\mu}{dz} \frac{\epsilon_n}{E_\mu} + \frac{1}{\beta^3} \frac{\beta_\perp E_s^2}{2E_\mu mc^2X_0},$$

where $\epsilon_n$ is the normalized emittance, $z$ is the path length, $E_\mu$ is the muon beam energy, $\beta = v/c$, $X_0$ is the radiation length of the absorber material, $\beta_\perp$ is the betatron function, and $E_s$ is the characteristic scattering energy [1]. Here, two competing effects can be seen: the first term is the cooling (reduction of phase space beam size) component from ionization energy loss and the second term is the heating (increase of phase space beam size) term from multiple scattering. For minimizing heating, a small betatron function from a strong magnetic field and a large radiation length are needed. To maximize cooling, a large stopping power is needed, $(\frac{dE_n}{dz})$. Hydrogen gives the best balance between a large radiation length and a large stopping power.

Muons will ionize material as they travel through absorbers. This will generate a plasma, and it is the interaction of the muon beam with the generated plasma that is studied here. Beam-plasma interaction is not taken into account currently in a majority of muon accelerator simulation codes. This interaction is especially important when simulating ionization cooling in the hybrid cooling channels with medium-to-high pressure gas-filled RF cavities (Fig. 2).

The plasma effects have been studied by plasma physicists, but have not been studied extensively from a beam physics point of view. The plasma has been shown not to disrupt the beam or make it blow up dramatically [2], however, for ionization cooling purposes beam-plasma effects may have a large impact on the cooling rates for both charges of muons. Essentially, the head of a bunch sees a material with different properties than the tail of the bunch and whole bunches may see materials with different properties than the previous bunches. Ionization rates vary from material to material so they must be described by individual formulas for each material.

Figure 1: Principle of ionization cooling: 1) The overall momentum is reduced through ionization where $(\frac{dE}{dz})$ is the mean energy loss of the muons. 2) Transverse momentum increases through multiple scattering. 3) Through re-acceleration, longitudinal momentum is regained.

Figure 2: Example of a matter-dominated hybrid cooling channel with gas-filled RF cavities, nearly all the cell length has material: either medium-pressure gaseous hydrogen or LiH absorber. (Image courtesy of D. Stratakis).

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the effects may be more prominent in some materials than others.

**SIMULATIONS**

After several simulation packages were considered, the one found to best suit our needs was WARP [3]. WARP is an actively developed particle-in-cell (PIC) simulation code designed to simulate particle beams with high space-charge intensity.

Several ionization models were considered to generate the plasma including multiple ways to introduce the plasma manually, but ultimately the ionization module contained within WARP was used. Starting with an ionization cross section,

$$\sigma = \left( \frac{dE}{dx} \right) \frac{1}{W_i \rho n}$$

WARP will generate the plasma on its own. Here, \( \left( \frac{dE}{dx} \right) \) is the mean rate of energy loss by the muons, \( W_i \) is the average energy to produce an ion pair, and \( \rho \) and \( \rho n \) are the mass and atomic densities of the medium, respectively.

In the proof-of-concept simulations with a dense muon beam, it was seen that beam-plasma effects can significantly alter the results. The bunch shape varied drastically when comparing the simulation results with and without plasma effects [4]. WARP proved to calculate the desired effects fairly efficiently, with a factor of six slowdown when plasma effects were included.

**PROGRESS ON SIMULATIONS**

Beam-plasma effects have been shown to potentially have a significant impact on the shape of a muon bunch. This impact needs to be quantified, and the effect on cooling rates needs to be studied. To do this, a section of a realistic cooling channel has to be simulated.

In the previous simulations, scattering and straggling have not been taken into account due to the lack of these features in WARP. Subsequently, a WARP-ICOOL wrapper has been used [5], incorporating into WARP the scattering and straggling processes from ICOOL [6]. At the end of each step inside the material, WARP calls the relevant ICOOL processes and applies them to the particles in the simulation. A sample beam of \( p = 220 \text{ MeV/c} \) muons was simulated in WARP and G4beamline [7] to test the validity of the muon behavior in matter. The simulation consisted of 2 m of drift, followed by 2 m liquid Hydrogen (LH2), and another 2 m of drift, all contained in a 3 T solenoidal field. The results are summarized in Fig. 3, confirming the observed muon behavior is consistent between codes. This is still without ionization, only verifying the stochastic effects, hence the slight discrepancy. Due to the way the virtual detectors read out particle information, data points near the beginning and end of simulations may not be accurate and are cut from plots. Similar simulations confirmed that the behavior of electrons also matched across simulation packages.

A complete cooling cell based on the first stage of the current version of the rectilinear cooling channel [8] has been modeled with the layout similar to two of the cells in Fig. 2, one after another, with magnetic field polarity reversed in the second half. This cell consists of four solenoidal coils producing a maximum magnetic field of 2.36 T, six 325 MHz RF cavities with a maximum gradient of 22 MV/m and accelerating phase of 14°, and 10.5 cm of LH2 absorber between solenoids. Due to current limitations, flat absorbers are used in place of wedge absorbers. With the use of flat absorbers instead of wedge absorbers, the magnetic coils are not tilted. This channel was simulated in WARP, both with and without ionization. A Gaussian beam similar to the previous one was used, but with \( n = 2 \times 10^{12} \) muons. Results were compared using Ecalc9 [9] and are summarized in Fig. 3.
in Fig. 4 and Table 1. A Z-X cross section of the beam and ionized electrons can be seen in Fig. 5.

![6D emittance](image1)

![Longitudinal emittance](image2)

![Transverse emittance](image3)

Figure 4: Comparison of the six-dimensional (a), longitudinal (b), and transverse (c) emittances in WARP for the case with (blue) and without (green) ionization effects through 10 cells of the rectilinear cooling channel with flat absorbers and non-tilted coils. Stars indicate the end of the 9th cell where the data was taken for Table 1.

Table 1: Emittance values taken after 9 cells with and without ionization and the difference between them.

|                      | w/ Ionization | w/o Ionization | Increase |
|----------------------|---------------|----------------|----------|
| 6D emittance (m^3)   | 5.61×10^{-7}  | 2.88×10^{-7}  | 94.5%    |
| longitudinal emittance (m) | 1.38×10^{-2} | 1.16×10^{-2} | 18.4%    |
| transverse emittance (m) | 6.38×10^{-3} | 4.97×10^{-3} | 28.1%    |

![Z-X Cross Section](image4)

Figure 5: Z-X cross section of the beam muons and ionized electrons. A majority of the plasma stays contained within the LH2 absorbers (at 1 m and 2 m). The magnetic field changes polarity at the absorbers and is weakest at those points, allowing some plasma to escape.

beam and electromagnetic fields will be much stronger in the later stages. It is expected the beam-induced plasma effects will be more substantial.

Several effects are still not taken into account here, including beam-plasma effects on subsequent bunches and plasma recombination. These are currently being investigated, along with expanding the limits of the simulation to include wedged absorbers.

A few real world experiments have been identified to potentially model with WARP. This could provide us with a verification of our models.

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