Effect of Al₂O₃ nanoparticle additives on the performance characteristics of rough journal bearings

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Abstract

This paper investigates the influence of Al₂O₃ nanoparticle additives on static and dynamic characteristics of water-lubricated rough journal bearings under thermo-hydrodynamic lubrication (THL). The time-dependent modified Reynolds equation and the approximated adiabatic energy equation have been formulated and solved numerically with boundary conditions and initial conditions. The results are presented for dimensionless maximum film pressure, dimensionless maximum film temperature, dimensionless minimum film thickness, eccentricity ratio, attitude angle, dimensionless friction force, spring constants, damping coefficients and mass parameters for both journal bearings with transverse roughness patterns and with longitudinal roughness patterns under a dimensionless load of 7. The results show that journal bearings with Al₂O₃ nanoparticle additives tend to significantly increase load carrying capacity and minimum film thickness by decreasing both the eccentricity ratio and temperature rise for longitudinal roughness patterns. For journal bearings with longitudinal roughness patterns and lubricated with Al₂O₃ nanoparticle additives, the stability region tends to increase.

Keywords: Journal bearing, Al₂O₃ nanoparticle additives, Longitudinal rough surface, Transverse rough surface

1. Introduction

In modern machines with minimized energy losses, ceramics are used as bearing materials for high-speed water-lubricated bearings. In ceramic bearings, the surface may be covered with the desired surface roughness patterns. Thermal effects in lubrication are important to the performance characteristics of bearings, see examples; Pinkus and Bupara [1], Smith and Tichy [2]. The performance characteristics of journal bearings have been studied by numerous researchers. Hashimoto and Mongkolwongrojn [3] studied the performance characteristics of turbulent partial journal bearings including surface roughness effects. They found that surface roughness significantly affected the bearing...
characteristics, especially for operating under turbulent film conditions. In 2004, Bouyer and Fillon [4] presented the incompressible thermoelastohydrodynamic film pressure has a significant effect on the performance characteristics of a plain journal bearing. The water film thickness can be very thin due to the thermohydrodynamic lubrication; therefore, the effect of surface roughness is very important. Majumdar and Ghosh [5], Tur Turaga, Sekhar [6] investigated the stability of a rough journal bearing using the perturbation technique. They found that stability can be improved significantly for rough surface journal bearings. Gururajan [7] studied the influence of surface roughness on bearing characteristics.

In recent years, many researches on nanoparticles show the enhancement of the heat transfer rate by increasing thermal conductivity and increasing thermal dispersion properties for cooling liquids (Nguyen, Desgranges [8], Xuan and Roetzel [9], Yu, Xie [10]). Recent research at the microscale is necessary to learn about the microstructure of nanofluids in order to understand the flow characteristics and heat transfer process of the nanofluid. Nanoparticle additives have attracted interest because of their enhanced thermal properties. Peng, Kang [11] showed that the use of diamond and SiO₂ nanoparticles as additives in liquid paraffin oil have better anti-wear and anti-friction properties than paraffin oil. The thermal conductivity and viscosity of nanofluids increase significantly with nanoparticle concentration (Mursheed, Leong [12], Chandrasekar, Sures [13]). Shenoy, Binu [14] presented the effect of nanoparticle additives on hydrodynamic characteristics of an externally adjustable fluid film bearing. The bearing operated with API-SF engine oil blended with TiO₂ nanoparticles. The results showed that bearings with nanoparticle additives have better load-carrying capacity, reduced end leakage and increased friction when compared to API-SF engine oil and base oil without nanoparticle additives. Mongkolwongrojn and Rattapasakorn [15] studied a rough cylinder on a flat surface under TEHL with non-Newtonian lubricant blended with Al₂O₃ nanoparticle additives. The addition of Al₂O₃ nanoparticles significantly increased the film thickness both under normal loads and under sudden overloads due to the squeezing effect. The results have shown that nanoparticles significantly affected thermal elastohydrodynamic performance, especially under transient conditions. Mongkolwongrojn and Aiumpronsin [16] showed the effects of surface roughness in journal bearings under EHL; for longitudinal pattern, the increase in temperature was higher than both transverse and smooth patterns. As a result, the stability regions of the longitudinal pattern were smaller than a transverse pattern. Gunnuang, Aiumprornsin [17, 18] showed that Al₂O₃ nanoparticle additives can reduce the temperature.

The objective of this study is to investigate the performance characteristics of a journal bearing with minimize energy losses. In this analysis, the rough journal bearings with water blended with Al₂O₃ nanoparticle additives under THL have been formulated and solved using finite difference techniques.

2. Theory

2.1 Physical properties of nanofluids

The thermo-physical properties of nanofluids in this research assume that the nanoparticles are well dispersed in water and can be written as Equation (1) and equation (2) (Pak and Cho [19], Chandrasekar, Suresh [13])

\[ \rho_{nf} = \phi \rho_{par} + (1-\phi) \rho_{water} \]  
\[ C_{P,nf} = \phi C_{P,par} + (1-\phi) C_{P,water} \]

Brinkman [20] suggested an equation for calculating the viscosity of the suspension, which is defined as follows:

\[ \mu_{nf} = \frac{1}{(1-\phi)^{2.5}} \mu_{water} \]  

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The lubricant density and viscosity depend on the temperature and can be written as follows:

\[ \rho = \rho_{nf} \left(1 - \beta(T - T_0)\right) \]  

(4)

\[ \mu = \mu_{nf} \exp\left[-\gamma(T - T_0)\right] \]  

(5)

2.2 Linearization of bearing reaction

According to Lund [21], for the x-z coordinates in Figure 1, using the perturbation technique, five non-dimensional equations of bearing reaction can be obtained as follows:

\[
\begin{bmatrix}
\frac{\partial}{\partial \theta} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \theta} \\
\frac{\partial}{\partial \varphi} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial \varphi} \\
\frac{\partial}{\partial \psi} & \frac{\partial}{\partial \psi} & \frac{\partial}{\partial \psi}
\end{bmatrix}
\begin{bmatrix}
\tilde{p}_x \\
\tilde{p}_y \\
\tilde{p}_z
\end{bmatrix}
= 
\begin{bmatrix}
\frac{1}{2} \frac{\partial}{\partial \theta} (\rho \cos \vartheta) & -\frac{1}{2} \frac{\partial}{\partial \theta} (\rho \sin \vartheta) & 0 \\
\frac{1}{2} \frac{\partial}{\partial \varphi} (\rho \cos \vartheta) & \frac{1}{2} \frac{\partial}{\partial \varphi} (\rho \sin \vartheta) & 0 \\
\frac{1}{2} \frac{\partial}{\partial \psi} (\rho \cos \vartheta) & 0 & \frac{1}{2} \frac{\partial}{\partial \psi} (\rho \sin \vartheta)
\end{bmatrix}
\begin{bmatrix}
\rho h_x \\
\rho h_y \\
\rho h_z
\end{bmatrix}
\]  

(6)

where \( p = (p)_s \), \( \tilde{p} = \left(\frac{\partial p}{\partial X}\right)_s \), \( \tilde{p} \) = \( \left(\frac{\partial \tilde{p}}{\partial Z}\right)_s \), \( \tilde{p} \) = \( \left(\frac{\partial \tilde{p}}{\partial Z}\right)_s \), \( \rho = \frac{\rho}{\rho_0} \), \( \mu = \frac{\mu}{\mu_0} \)

The film thickness can be expressed as

\[ h = (1 + \varepsilon \cos(\theta - \Phi)) - \delta h \]  

(7)

where \( \delta h \) is the roughness height of the journal bearing and can be written as follows:

\[ \delta h = \frac{A_h}{c} \sin(2\pi \theta / \lambda_h) \]  

(for transverse roughness)  

(8)

\[ \delta h = \frac{A_h}{c} \sin(2\pi y / \lambda_h) \]  

(for longitudinal roughness)  

(9)
The boundary conditions for the film pressure at the start of cavitation and the bearing sides are

\[ p(\theta_e) = \frac{\partial p}{\partial \theta} \bigg|_{\theta_e} = 0, \quad p(\theta=0) = 0 \quad \text{and} \quad p(\gamma=0) = 0 \] (10)

2.3 Energy equation
According to Majumdar [22], the dimensionless adiabatic energy equation can be written as shown in equation (11)

\[ \left( 6h - \frac{\tilde{k}^3 \tilde{c}_p}{\tilde{h}} \right) \frac{\partial \tilde{T}_m}{\partial \theta} - \left( \frac{1}{\tilde{\lambda}^2} \frac{\tilde{k}^3 \tilde{c}_p}{\tilde{h}} \right) \frac{\partial \tilde{T}_m}{\partial \tilde{\gamma}} = \frac{12\tilde{\mu}}{\tilde{h}} + \frac{\tilde{h}^3}{\tilde{h}^2} \left( \frac{\partial \tilde{p}}{\partial \theta} \right)^2 + \frac{1}{\tilde{\lambda}^2} \left( \frac{\partial \tilde{p}}{\partial \tilde{\gamma}} \right)^2 \] (11)

The boundary conditions for the film temperature at the film inlet and the gradient of the film temperature at the half of length are as follows:

\[ T(\theta=0, \gamma=0) = 0, \quad \frac{\partial T}{\partial \gamma} \bigg|_{\gamma=\frac{\pi}{2}} = 0 \] (12)

2.4 Load-carrying capacity and Friction force
The forces due to the hydrodynamic pressure on the journal in the x-z coordinate system are calculated as

\[ (F_x)_s = -\int_0^{\pi} \tilde{p}_s \cos \theta d\theta d\tilde{\gamma}, \quad (F_z)_s = -\int_0^{\pi} \tilde{p}_s \sin \theta d\theta d\tilde{\gamma} \] (13)

The friction force in the fluid film is obtained by integrating the shear stress on the journal surface. The non-dimensional friction force \( F_{fric} \) is given as follows:

\[ F_{fric} = \frac{12\pi}{0} \tilde{p}_s \cos \theta d\theta d\tilde{\gamma} + \frac{12\pi}{0} \frac{\tilde{h}^3}{\tilde{h}^2} \frac{\partial \tilde{p}}{\partial \theta} d\theta d\tilde{\gamma} \] (14)

2.5 Journal bearing stability
The linearized equation of journal motion can be written as

\[ \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Z \end{bmatrix} + \begin{bmatrix} B_{xx} & B_{xz} \\ B_{zx} & B_{zz} \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Z \end{bmatrix} + \begin{bmatrix} K_{xx} & K_{xz} \\ K_{zx} & K_{zz} \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Z \end{bmatrix} = 0 \] (15)

where the non-dimensional spring and damping coefficients can be obtained as follows:

\[ \begin{bmatrix} K_{xx} & K_{xz} \\ K_{zx} & K_{zz} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \tilde{p}_s \cos \theta d\theta d\tilde{\gamma} & \frac{1}{2} \tilde{p}_s \sin \theta d\theta d\tilde{\gamma} \\ \frac{1}{2} \tilde{p}_s \sin \theta d\theta d\tilde{\gamma} & \frac{1}{2} \tilde{p}_s \cos \theta d\theta d\tilde{\gamma} \end{bmatrix} \] (16)

\[ \begin{bmatrix} B_{xx} & B_{xz} \\ B_{zx} & B_{zz} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \tilde{p}_s \cos \theta d\theta d\tilde{\gamma} & \frac{1}{2} \tilde{p}_s \sin \theta d\theta d\tilde{\gamma} \\ \frac{1}{2} \tilde{p}_s \sin \theta d\theta d\tilde{\gamma} & \frac{1}{2} \tilde{p}_s \cos \theta d\theta d\tilde{\gamma} \end{bmatrix} \] (17)
where \( \bar{p}_x = \left( \frac{\partial p}{\partial x} \right), \bar{p}_y = \left( \frac{\partial p}{\partial y} \right), \bar{p}_z = \left( \frac{\partial p}{\partial z} \right), \bar{q}_z = \left( \frac{\partial q}{\partial z} \right) \).

The solutions for dimensionless critical mass parameter \( M_{CR}^2 \Omega^2 \)

\[
M_{CR}^2 \Omega^2 = \frac{K_{xx}B_{zz} + K_{zz}B_{xx} - K_{zz}B_{xx} - K_{xx}B_{zz}}{B_{xx} + B_{zz}}
\]

(18)

3. Numerical Solution

In the present study, the characteristics of full journal bearing with roughness surface were examined with the bearing parameters and lubricant properties as shown in Table 1. The diameter to width ratio of the bearing is 1, and the clearance to radius ratio is 0.002. The static pressure; in Equation (6) and temperature in Equation (11) are solved simultaneously. The second order central difference schemed described by Majumdar [22], were employed for the discretization of pressure equation and energy equation. The applied load is assumed to be constant in magnitude and implemented in the vertical direction. The pressure, temperature, eccentricity ratio, film thickness, viscosity, density and hydrodynamic force are also simultaneously calculated within the thermohydrodynamic regime. The relative accuracy in pressure, temperature and force components were less than or equal to \(1 \times 10^{-6}\).

Based on the analysis, a computer program was developed to solve the thermohydrodynamic problems of finite journal bearing using 270 by 90 nodes.

| Parameter                     | Detail          |
|-------------------------------|-----------------|
| Journal radius, \( r \)       | 0.015 m         |
| Radius clearance, \( c \)     | 30 µm           |
| Reference viscosity, \( \mu_0 \) | 0.693 m Pa s |
| Viscosity-Temperature coefficient, \( \gamma \) | 0.0153 K\(^{-1}\) |
| Bearing length, \( L \)       | 0.03 m          |
| Inlet lubricant temperature, \( T_0 \) | 313 K          |
| coefficient of thermal expansivity, \( \beta \) | 0.000385 K\(^{-1}\) |
| Wavelength of roughness, \( \lambda_1 \) | 5 mm           |
| Density of Al\(_2\)O\(_3\), \( \rho_{par} \) | 3890 kg m\(^{-3}\) |
| Specific heat of Al\(_2\)O\(_3\), \( C_{p,par} \) | 729 J kg\(^{-1}\)K\(^{-1}\) |

4. Results

Figures 2-9 show the effects of the Al\(_2\)O\(_3\) nanoparticle concentration on the steady-state characteristics of water-lubricated journal bearings under the dimensionless load \( W=7 \) for smooth bearings, transverse roughness pattern bearings and longitudinal roughness pattern bearings.

Figure 2 shows the effects of the variations in Al\(_2\)O\(_3\) nanoparticle concentrations on the dimensionless maximum film pressure under dimensionless load \( W \) equal to 7. For the transverse roughness pattern, the dimensionless maximum film pressure of journal bearings lubricated with clean water and lubricated with water mixed with 2% Al\(_2\)O\(_3\) nanoparticles are 13.56 and 13.16 respectively. The decrease in the maximum film pressure is 2.95%. For journal bearings with 2% Al\(_2\)O\(_3\) nanoparticles, the dimensionless maximum film pressures of the longitudinal roughness pattern and smooth surface decrease 1.78% and 1.38% respectively, when compared with journal bearings lubricated without Al\(_2\)O\(_3\) nanoparticles. The dimensionless maximum film pressure decreases slightly with an increase in Al\(_2\)O\(_3\) nanoparticles added.
Figure 3 shows the effect of variations in the Al$_2$O$_3$ nanoparticle concentration on dimensionless maximum film temperature. The results show that addition of the Al$_2$O$_3$ nanoparticle additive decreases the dimensionless maximum film temperature, especially for the longitudinal roughness pattern. For smooth surface journal bearings, the dimensionless maximum film temperature of journal bearings lubricated with clean water and lubricated with water mixed with 2% Al$_2$O$_3$ nanoparticles are 66.92 and 63.08, which is a decrease of 5.74%. For transverse patterns, the addition of 0% to 2% Al$_2$O$_3$ nanoparticle additives can reduce dimensionless maximum film temperature from 82.33 to 76.18, a decrease of 7.47%. In the case of longitudinal patterns, the dimensionless maximum film temperature of journal bearings lubricated with clean water and lubricated with water mixed with 2% Al$_2$O$_3$ nanoparticles are 240.20 and 205.78, respectively, a decrease of 14.33%. The Al$_2$O$_3$ nanoparticle additive has a great effect on dimensionless maximum film temperature for journal bearings with longitudinal roughness patterns.

![Figure 2](image2.png)  
**Figure 2.** Variation of dimensionless maximum film pressure with Al$_2$O$_3$ nanoparticles concentration when journal bearing operated under dimensionless load $W$ equal to 7

![Figure 3](image3.png)  
**Figure 3.** Variation of dimensionless maximum temperature with Al$_2$O$_3$ nanoparticles concentration when journal bearing operated under dimensionless load $W$ equal to 7

Figure 4 shows the variation of dimensionless maximum film temperature with surface roughness for longitudinal and transverse roughness patterns when the journal bearing operated under dimensionless load $W$ equal to 7. The dimensionless maximum film temperature is nonlinearly increased with an increase of roughness amplitude. For journal bearings with 2% Al$_2$O$_3$ nanoparticle additive, the decrease of the dimensionless maximum film temperature for longitudinal roughness patterns with roughness amplitudes 5µm, 4µm, and 3µm are 23.3%, 14.33%, and 10.13%, respectively, when compared to journal bearings lubricated without nanoparticles added. The decreases in the dimensionless maximum film temperature for the transverse roughness pattern with roughness amplitudes 5 µm, 4 µm, and 3 µm are 7.66%, 7.47%, and 6.69%, respectively, when compared to journal bearings lubricated with clean water. The roughness surface has a significant effect on the increase of the film temperature, especially for the longitudinal roughness pattern. From the results in Figures 2-4, it is clear that for roughness amplitudes higher than 3 µm or larger than 10% of the clearance; $A_h/c > 0.1$, the addition of Al$_2$O$_3$ nanoparticles has a significant effect on the reduction of the temperature rise for longitudinal roughness patterns; the reduction of the temperature rise for transverse roughness...
patterns is small when compared with that for longitudinal roughness patterns. These results are useful in understanding the influence of Al₂O₃ nanoparticles on the longitudinal roughness patterns of journal bearings.

The effects of the concentration of Al₂O₃ nanoparticles on eccentricity ratio and attitude angle of the journal bearings are shown in Figure 5 and Figure 6. With the Al₂O₃ nanoparticle additive concentration ranging from 0% to 2%, the eccentricity ratio of journal bearings decreases from 0.793 to 0.786, a decrease of 0.88% for a smooth surface; the eccentricity ratio decreases from 0.747 to 0.739, a decrease of 1.07% for the transverse roughness pattern; and the eccentricity ratio decreases from 0.792 to 0.783, a decrease of 1.14% for the longitudinal roughness pattern, as shown in Figure 5. The attitude angle of the journal bearing increases from 36.69 to 37.36, an increase of 1.83% for a smooth surface; the attitude angle increases from 32.47 to 33.02, an increase of 1.69% for the transverse roughness pattern; and the attitude angle increases from 37.14 to 37.8, an increase of 1.78% for the longitudinal roughness pattern as shown in Figure 6.

Figure 4. Variation of dimensionless maximum film temperature with roughness amplitude when the journal bearing operated under dimensionless load W equal to 7

Figure 5. Variation of eccentricity ratio with Al₂O₃ nanoparticles concentration when journal bearing operated under dimensionless load W equal to 7

Figure 7 shows the variation of dimensionless minimum film thickness with roughness amplitude for longitudinal and transverse roughness patterns when the journal bearing is operated under dimensionless load W equal to 7. For a longitudinal roughness pattern at high roughness amplitude, the decrease of the dimensionless minimum film thickness is higher than for the transverse pattern; however, the addition of Al₂O₃ nanoparticles significantly affects the increase of the dimensionless minimum film thickness. For journal bearings with 2% Al₂O₃ nanoparticles, the increase of the dimensionless minimum film thickness for transverse roughness patterns with roughness amplitudes 5 µm, 4 µm and 3 µm are 6.67%, 6.35% and 6.12%, respectively, when compared to journal bearings lubricated without nanoparticles added. For journal bearings with 2% Al₂O₃ nanoparticles added, the increases of the dimensionless minimum film thickness are 21.4%, 10.53% and 8.33% for longitudinal roughness patterns with roughness amplitudes 5 µm, 4 µm and 3µm, respectively, when compared with bearings lubricated without nanoparticles added. The addition of Al₂O₃ nanoparticles has a significant effect on the increase of dimensionless minimum film thickness, as shown in Figure 8. The addition of Al₂O₃ nanoparticles increases the dimensionless friction force due to the increase of viscosity. Figure 9 shows that the
addition of Al₂O₃ nanoparticles from 0% to 2%, the dimensionless friction force of a longitudinal bearing with 5 µm in amplitude increases from 13.82 to 13.99, respectively, this is an increase of 1.23%. For transverse roughness patterns 5 µm in amplitude, the dimensionless friction force increases from 13.25 to 13.61, an increase of 2.72%.

**Figure 6.** Variation of attitude angle with Al₂O₃ nanoparticles concentration when journal bearing operated under dimensionless load W equal to 7

**Figure 7.** Variation of dimensionless minimum film thickness with roughness amplitude when journal bearing operated under dimensionless load W equal to 7

**Figure 8.** Variation of dimensionless minimum film thickness with Al₂O₃ nanoparticles concentration when journal bearing operated under dimensionless load W equal to 7

**Figure 9.** Variation of dimensionless friction force with roughness amplitude when journal operated under dimensionless load W equal to 7
The addition of Al₂O₃ nanoparticles decreases film temperature and increases minimum film thickness by reducing the eccentricity ratio or increasing the attitude angle. The Al₂O₃ nanoparticle additives have a significant effect on temperature when the roughness amplitude to clearance ratio, A/h, is higher than 0.1. The effects of Al₂O₃ nanoparticle additives on the dynamic characteristics of journal bearings under the dimensionless load W equal to 7 are shown in Figures 10-12.

Figure 10 shows the variation of dimensionless spring coefficients with the concentration of Al₂O₃ nanoparticles for different roughness patterns. The spring constants Kₓₓ and K zza for longitudinally rough surfaces increase significantly with Al₂O₃ concentration, but all the other dimensionless spring coefficients, Kₓₓ, K zzx, K zzz and K zzz, for both transversely rough surfaces and smooth surfaces decrease with the concentration of Al₂O₃ nanoparticles. To compare bearings lubricated with water blended with 2% Al₂O₃ nanoparticles and lubricated with clean water for the longitudinally rough surface, the spring constant, Kₓₓ, increases from 0.65 to 0.93, an increase of 43%, and K zzx increases from -2.94 to -2.88, an increase of 2.04%. For longitudinally rough surfaces, spring constant K zzx decreases slightly from 4.33 to 4.31, a decrease of 0.46%. For both transversely rough surfaces and smooth surfaces, the spring constant K zzx decreases significantly by 1.45% and 2.09%, respectively, when the use of 2% Al₂O₃ nanoparticles is compared with the use of clean water. This means that the Al₂O₃ nanoparticle additive has great effect on increasing the spring constants Kₓₓ and K zzx for journal bearings with longitudinal roughness patterns.

Figure 11 shows the variation of dimensionless damping coefficients with Al₂O₃ nanoparticle concentrations for different roughness patterns. The results clearly show that dimensionless damping coefficients Bₓₓ, B zzx and B zzz decrease with the increase in the Al₂O₃ nanoparticle concentration, but B zzx increases with the increase in Al₂O₃ nanoparticle concentration for both transverse and longitudinal roughness patterns. For the transverse roughness patterns, damping coefficients are small when compared with the damping coefficients for smooth surfaces, but the damping coefficients for longitudinally rough surfaces are close to the damping coefficients for smooth surfaces. At 2% Al₂O₃ concentration, the damping coefficients Bₓₓ, B zzx, B zzx and B zzz for a transversely rough surface are 14%, 29.3%, 29.3%, and 26% smaller, respectively, than those for smooth surfaces.

Figure 12 shows the variation of the critical mass parameter with Al₂O₃ nanoparticle concentration and roughness amplitude when journal bearings are operated under dimensionless load W equal to 7; the critical mass parameter indicates the stability region of the journal bearing. The stability region of the bearing with transverse roughness pattern increases with an increase in roughness amplitude, but the stability region of the longitudinal roughness bearing decreases with an increase in roughness amplitude, especially when the value of roughness amplitude to clearance ratio; A/h, is more than 0.1.

For the longitudinally rough surface, the mass parameter of water-blended Al₂O₃ is larger than that for clean water by 1.31%, 4.20% and 13.79% for roughness amplitudes of 3 μm, 4 μm and 5 μm, respectively. For the transversely rough surface, the mass parameter of water-mixed Al₂O₃ is smaller than that for clean water. The values of mass parameters decrease 0.62%, 0.69% and 0.78% for roughness amplitudes of 3 μm, 4 μm and 5 μm, respectively. The addition of Al₂O₃ nanoparticles tends to slightly decrease the stability region for transverse roughness patterns, but it is significant in increasing the stability region for longitudinal roughness patterns, especially when the roughness amplitude is higher than 0.3 μm. Comparing the mass parameter of water-blended 2% Al₂O₃ with the mass parameter of clean water, the transverse roughness pattern decreases 0.69% (from 2.464 to 2.447), the smooth surface decreases 0.52% (from 2.319 to 2.307) and the longitudinal roughness pattern significantly increases 4.20% (from 1.737 to 1.81). The results show that the concentration of Al₂O₃ nanoparticles tends to slightly decrease the stability region of both smooth surfaces and transverse roughness patterns, but for bearings with longitudinal roughness patterns, the stability region increases significantly with an increase in Al₂O₃ particle concentration.
Figure 10. Variation of dimensionless spring coefficients with Al$_2$O$_3$ nanoparticles concentration when journal bearing operated under dimensionless load $W$ equal to 7 for (a) $K_{xx}$, $K_{xz}$ and (b) $K_{zz}$, $K_{zx}$.

Figure 11. Variation of dimensionless damping coefficients with Al$_2$O$_3$ nanoparticles concentration when journal bearing operated under dimensionless load $W$ equal to 7 for (a) $B_{xx}$, $B_{xz}$ and (b) $B_{zz}$, $B_{zx}$.
5. Conclusions
The static and dynamic characteristics of rough journal bearings with Al₂O₃ nanoparticle additives under thermohydrodynamic lubrication were theoretically examined. The time-dependent modified Reynolds equation and the approximated adiabatic energy equation were formulated and solved numerically. This research can be summarized as follows.

1) For journal bearings with smooth and rough surfaces with both transverse roughness patterns and longitudinal roughness patterns under static characteristics, the increase of Al₂O₃ nanoparticles significantly affects on the reduction of both the film temperature and the eccentricity ratio. Therefore, the load-carrying capacity and minimum film thickness are increased for journal bearings with Al₂O₃ nanoparticles.

2) For dynamics characteristics, journal bearings with longitudinally roughness and lubricated with Al₂O₃ nanoparticles tend significantly to increase the stability regions.

Nomenclature

- \( A_h \): Roughness amplitude, m
- \( b \): Damping coefficient, N s/m
- \( B_{xx} \ldots B_{zz} \): Dimensionless damping coefficient, \( B = \frac{c\omega b}{\nu} \)
- \( c \): Radius clearance, m
- \( C_{P,\text{nf}} \): Specific heat capacity of nanofluids, J/kg K
- \( C_{P,\text{par}} \): Specific heat capacity of solid particles, J/kg K
- \( C_{P,\text{water}} \): Specific heat capacity of water, J/kg K
- \( F_{\text{fric}} \): Friction Force, N
- \( \bar{F}_{\text{fric}} \): Dimensionless friction force, \( \bar{F}_{\text{fric}} = \left( \frac{c^2}{\mu_{\text{nf}} \omega r^2} \right) F_{\text{fric}} \)
- \( h \): Film thickness, m
Dimensionless film thickness, \( \bar{h} = \frac{h}{c} \)

Stiffness coefficient, \( k \) N/m

Dimensionless stiffness coefficient, \( K = \frac{k}{c/w} \)

Bearing length, m

Dimensionless mass, \( M = \left( \frac{c^2}{w} \right) m \)

Fluid film pressure, Pa

Dimensionless film pressure, \( \bar{p} = \left( \frac{c^2}{\mu_\omega r^2} \right) \left( p - p_{atm} \right) \)

Journal radius, m

Mean temperature of lubricant across the film, K

Dimensionless mass, \( M = \left( \frac{c^2}{w} \right) m \)

Load, N

Dimensionless load, \( W = \left( \frac{c^2}{\mu_\omega r^2 L} \right)_w \)

Coordinate in a vertical direction

Coordinate in an axial direction

Coordinate in a horizontal direction

Angular velocity of whirl, rad/s

Whirl ratio, \( \Omega = \Omega/c \)

Angular velocity, rad/s

Eccentricity ratio of journal bearing, \( \varepsilon = e/c \)

The circumferential angle measured from the vertical axis, rad

Attitude angle, rad

Particle volume concentration

The wave length of roughness, m

Coefficient of thermal expansivity, 1/K

Viscosity-Temperature coefficient, 1/K

The apparent density of lubricant at the reference temperature, kg/m^3

The density of nanofluids, kg/m^3

The density of solid particles, kg/m^3

The density of water, kg/m^3

The apparent viscosity of the lubricant, Pa.s

The viscosity of nanofluids, Pa.s

The viscosity of water, kg/m^3

Dimensionless displacement in the x-direction, \( \Delta X = \Delta x/c \)

Dimensionless displacement in the z-direction, \( \Delta Z = \Delta z/c \)

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