Slot Order Matters for Compositional Scene Understanding

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Abstract

Empowering agents with a compositional understanding of their environment is a promising next step toward solving long-horizon planning problems. On the one hand, we have seen encouraging progress on variational inference algorithms for obtaining sets of object-centric latent representations (“slots”) from unstructured scene observations. On the other hand, generating scenes from slots has received less attention, in part because it is complicated by the lack of a canonical object order. A canonical object order is useful for learning the object correlations necessary to generate physically plausible scenes similar to how raster scan order facilitates learning pixel correlations for pixel-level autoregressive image generation. In this work, we address this lack by learning a fixed object order for a hierarchical variational autoencoder with a single level of autoregressive slots and a global scene prior. We cast autoregressive slot inference as a set-to-sequence modeling problem. We introduce an auxiliary loss to train the slot prior to generate objects in a fixed order. During inference, we align a set of inferred slots to the object order obtained from a slot prior rollout. To ensure the rolled out objects are meaningful for the given scene, we condition the prior on an inferred global summary of the input. Experiments on compositional environments and ablations demonstrate that our model with global prior, inference with aligned slot order, and auxiliary loss achieves state-of-the-art sample quality.

1 Introduction

Humans can look at a scene and parse what objects are present and how they relate to each other seemingly effortlessly. We can also imagine highly complex scenes based on the objects we perceive. To achieve this, our visual system segregates raw perceptions into meaningful entities, binds these entities to object-centric representations (e.g., object files \cite{36}), and infers object relationships \cite{4, 64}. A major goal of artificial intelligence is to develop agents capable of such human-like compositional scene understanding \cite{5, 40, 82}. For example, this would enable robots to reason about the effects of their actions on different objects which is essential for long-term planning and decision-making.

Variational autoencoders (VAEs) \cite{38, 59} are a natural model family for compositional scene understanding since they offer a principled unsupervised learning framework based on scene inference and generation \cite{78}. However, inferring composable and modular representations from noisy observations of multi-object scenes and generating scenes by composing these representations remains challenging. Inference requires segregating the input into object-centric entities and binding them to a set of latent variables (i.e., the \textit{binding problem} \cite{22}). This appears to be fundamentally difficult for VAEs that encode observations into monolithic unstructured representations (e.g., DALL-E \cite{56}) or into fixed-size grids of symbolic variables \cite{17, 10, 46}. Other VAEs instead represent observations as...
mixture distributions whose components are in correspondence with a set of object-centric latent variables, or “slots”. These slot VAEs have demonstrated encouraging segmentation and disentanglement performance on static [6; 21; 15; 16; 14], multi-view [41; 65; 77; 76; 42], and dynamic visual scenes [74; 70; 45; 79; 11; 35; 83]. However, slot VAEs generate low-quality scenes [15; 16; 32].

Compositional scene generation is difficult because it requires learning how objects are correlated in the data. For example, consider an image dataset of cars driving on a road. A failure to learn the spatial correlation between the cars, sky, and road leads to generating physically implausible scenes with flying cars or two cars occupying the same physical space. Learning multi-object correlations is uniquely challenging because objects have no canonical ordering. In general, it is easier to learn correlations between variables when the variable order is fixed, e.g., to a canonical order. Consider pixel-level autoregressive models based on raster scan order [60; 67; 53; 9; 8]. These models essentially learn an autoregressive distribution for each pixel conditioned on all previous pixels in the order. This enables the n-th distribution to learn correlations between the n-th pixel and the previous n–1 pixels across training images, resulting in a highly expressive image density. VAEs that autoregressively model slots have evidently ignored the issue of order by their use of random and arbitrary orders for inference [15; 16]. Others simply assume slots are independent and cannot generate coherent scenes [21; 14]. We attribute the poor sample quality of slot VAEs to their limited ability to learn object correlations.

In this work, we propose to address the lack of a canonical object order by learning a fixed object order, which we use to estimate complex correlations and achieve high-quality scene generation. We cast slot inference as a set-to-sequence modeling problem; specifically, we impose a learned object order on a randomly ordered segregation slot posterior to obtain an autoregressive slot posterior. To learn an object order suitable for high-quality scene generation and to avoid the complexity of introducing discrete latent variables for order inference, we use an implicit approach that is generative in nature. In detail, we introduce a two-level hierarchical slot VAE with a level of autoregressive slot variables conditioned on a global scene-level variable. We use the global prior to aid with learning the object order and also to increase the expressiveness of the autoregressive distributions for learning higher-order correlations. We introduce an auxiliary loss that trains the slot prior to use a fixed object generation order. During set-to-sequence slot inference, the autoregressive slot order is aligned to the object order extracted from a slot prior imagination rollout. We condition the prior on an inferred global summary of the input to ensure the rolled out object order is meaningful for inference on the given observation. Our experiments show that our model, SRI (Segregate, Relate, Imagine), achieves state-of-the-art generative performance among slot VAEs across multiple compositional environments. Detailed ablation studies demonstrate that the global prior, inference with aligned slot order, and auxiliary loss all work in concert to achieve the best performance, suggesting that slot order matters for compositional scene understanding.

2 Related Work

Slot VAEs: Slot VAEs are essentially VAEs with a set of K latent variables and a mixture model observation likelihood. IODINE [21] and EfficientMORL (EMORL) [14] use iterative inference to refine a randomly ordered posterior distribution over slots; EMORL also initializes iterative refinement with probabilistic slot attention [47]. Neither are capable of coherent scene generation due to using independent slot priors. GENESIS-v2 (GENv2) [16] also infers a randomly ordered slot posterior (via differentiable clustering) yet has an autoregressive slot prior able to capture object correlations. However, GENv2 assumes inferred slots are randomly ordered (i.e., uncorrelated) which hurts the sample quality of generated scenes. GENESIS (GEN) [15] initially proposed the autoregressive prior and uses sequential attention to segregate observations following an arbitrary object order; however, arbitrarily ordered attention works poorly on complex environments [16]. By contrast, SRI aligns its autoregressive slots to a learned, fixed object order and has a global prior for expressive modeling of higher-order correlations. Other related VAEs for scene generation [2; 73] make simplifying independence assumptions that prevent them from scaling to non-trivial environments.

Spatial-symbolic VAEs: Generative Neurosymbolic Machine (GNM) [32] is a VAE that combines a global scene-level prior with symbolic latent variables for compositional scene understanding. SRI is the first to explore using a global prior in the context of slot VAEs and for learning slot order. GNM’s symbolic variables differ from slots in that they have explicit semantic meaning (e.g., as bounding box coordinates) and are arranged as a fixed-size grid [17; 10; 46]. We demonstrate in this work that
GNM fails to discover objects in environments where objects vary widely in size and/or occlude each other heavily. The related model GSGN [12] attempts to learn part-whole object hierarchies but has the same limited symbolic variables as GNM.

**Compositional GANs:** Certain GANs can compose noise variables to generate scenes with object-level controllability [68; 51; 44; 13; 52; 30; 3]; however, they are fundamentally limited without a mechanism for addressing inference (i.e., the binding problem). Interestingly, RELATE [13] and GANformer2 [3] also learn a good generation order, which lends support to our claim that aligning the slot order to a fixed object order improves correlation estimation (i.e., that slot order matters).

**Set-to-sequence modeling:** A variety of methods have been proposed for sequential modeling where the input data has no canonical order (e.g., it is an arbitrarily ordered set) but the output is ordered, starting from the seminal work of Vinyals et al. [72]. A popular approach is to use Pointer Networks [71] or to make use of continuous relaxations of permutations [1; 48; 81; 43] or differentiable sorting [24]. See Jurewicz & Derczynski [33] for a survey. Unlike these methods, we implicitly learn a slot order using the slot prior to bias learning towards orders useful for high-quality scene generation and to avoid the complexity of discrete order inference.

## 3 Segregate, Relate, Imagine (SRI)

The statistical problem we are concerned with is fitting the underlying data-generating process $p(D)$ for an unlabeled dataset $D$ of i.i.d. scene observations $x \in \mathbb{R}^{N \times C}$ containing multiple objects. In this work we restrict our focus to collections of RGB images so that $N = H \times W$ and $C = 3$. Like previous slot VAEs, we assume that the marginal likelihood for an image $p(x)$ is augmented with $K$ slots $z_k \in \mathbb{R}^s$ [21; 15; 16; 14]. Typically, $K$ is chosen to be larger than the number of objects in any given scene in $D$. We introduce a latent variable $s \in \mathbb{R}^s$ to serve as a hierarchical prior on the slots and whose posterior distribution summarizes global statistics such as higher-order correlations between objects. This gives the following joint distribution: $p(x, z_{\pi_{1:K}}, s)$, where $\pi_{1:K} := \pi_1, \pi_2, \ldots, \pi_K$ is some permutation of the integers $\{1, \ldots, K\}$. We use this notation to make slot order explicit. In what follows, $\theta$ are neural net parameters for variational distributions and $\psi, \phi$ are neural net parameters for variational posterior distributions. Following common practice all latents are Gaussian with diagonal covariance.

### 3.1 Generative Model

SRI factorizes the joint distribution into a two-level hierarchy where we assume the observation $x$, when conditioned on the slots, is independent of the scene variable (Figure 1):

$$p_\theta(x, z_{\pi_{1:K}}, s) = p(s)p_\theta(z_{\pi_{1:K}} | s)p_\theta(x | z_{\pi_{1:K}}).$$  \hspace{1cm} (1)

For $p(s)$ we use a standard Gaussian. The slot prior is autoregressive with order $\pi_{1:K}$:

$$p_\theta(z_{\pi_{1:K}} | s) = p_\theta(z_{\pi_1} | s) \prod_{k=2}^{K} p_\theta(z_{\pi_k} | z_{\pi_{k-1}}, s).$$  \hspace{1cm} (2)

Unlike the autoregressive slot prior used by GEN and GENv2, our prior is conditioned on $s$ which increases its expressiveness for modeling higher-order correlations. The conditional image likelihood can be a pixel-wise Gaussian (left) or Mixture-of-Gaussians (right):

$$p_\theta(x | z_{\pi_{1:K}}) := \prod_{i=1}^{N} \mathcal{N}\left( \sum_{k=1}^{K} m_{i,k} x_{i,k}, \sigma^2 \right) \text{ or } \prod_{i=1}^{N} \sum_{k=1}^{K} m_{i,k} \mathcal{N}(x_{i,k}, \sigma^2),$$  \hspace{1cm} (3)

where $x_{i,k} \in \mathbb{R}^C$ is an RGB pixel, $m_{i,k} \in [0, 1]$ is a mask, and $\sigma^2$ is a shared variance. While a Gaussian likelihood is easier to optimize, the Mixture-of-Gaussians can achieve better segmentation and reconstruction quality [6; 21; 15; 16]. We consider both in our experiments. To facilitate comparing SRI with previous slot VAEs, we use the same spatial broadcasting decoder (SBD) [75; 14; 16] to map each slot $z_{\pi_i}$ to mixture components $(x_k, m_k)$, leaving the exploration of recently proposed advanced compositional decoders for future work [63]. The autoregressive prior $p_\theta(z_{\pi_{1:K}} | s)$
Segregate, Relate, Imagine (SRI). SRI learns a fixed object generation order which we extract for autoregressive inference. This helps the prior and posterior autoregressive distributions for the $k$th slot learn correlations with respect to the previous $k - 1$ slots in the learned order. In the encoding pass, SRI first segregates the observation $x$ (e.g., using GENv2 [16]) and obtains randomly ordered slots $z_{o1:K}$. An order-invariant relational embedding of this set is encoded into a global posterior $q(\phi)(s | x)$. An imagination rollout conditioned on a sample $s$ gives us slots with fixed order $\pi_{1:K}$. We extract order $\hat{\pi}_{1:K}$ by matching the randomly ordered and imagined slots, then transform the segregation posterior into an autoregressive posterior with order $\hat{\pi}_{1:K}$ by predicting correlated variances $\bar{\sigma}_{\hat{\pi}_{1:K}}$. The autoregressive slots $z_{\hat{\pi}_{1:K}}$ are decoded into RGB images and masks.

$s$ is implemented with an LSTM [28]. We project the scene variable via $W^T s, W_s \in \mathbb{R}^s x z$ followed by a nonlinear activation function to pass as input to $p(\theta)(z_{\pi_1} | s)$. Two shared linear layers are used to map the $K$ LSTM outputs to Gaussian means and variances.

To generate an image, we first sample from the scene prior and then sample sequentially from the autoregressive slot prior. The $K$ sampled slots are passed to the SBD to predict masks and RGB images. The $K$ masks are normalized with a softmax before the sum aggregation in Eq. 3.

3.2 Set-to-Sequence Inference

During inference, we infer variational posteriors for the autoregressive slots and global variable. The lack of a canonical object order to use for autoregressive inference makes learning correlations between objects fundamentally difficult. We address this by casting slot inference as a set-to-sequence modeling problem with learned object order (Figure 2 and see Algorithm 2 in the appendix). In the first stage of inference, which we describe in this section, we estimate a randomly ordered slot posterior over segregations of the given observation, compute order-invariant relations between the slots, and finally summarize relation-aware slots in a slot-order-invariant global posterior. In the second stage of inference we transform the segregation posterior into an autoregressive posterior by imposing a learned object order (Section 3.2.1) and then updating the slot posterior parameters to introduce correlations. Aligning the autoregressive slots to a learned, fixed object order helps the $k$th autoregressive distribution learn correlations between the $k$th slot and the previous $k - 1$ slots.

**Segregate:** The segregation posterior is $q(\psi)(z_{o1:K} | x) = \prod_{k=1}^{K} q(\psi)(z_{o_k} | x)$ with random order $o_{1:K}$. This posterior treats slots as independent. We can use any existing method to infer this posterior (e.g., GENv2 or EMORL—non-exhaustive list, details can be found in the respective papers).

**Relate:** We use methods from set representation learning [69; 80] to compute relations between slots in the set $z_{o1:K}$ and to encode this set in the scene posterior $q(\phi)(s | x)$. This posterior summarizes the
We do not update the permuted segregation posterior means \( \mu_{s_{1:K}} \) and only correlate the variances, which we find sufficient. The scene-conditional autoregressive slot posterior is

\[
q_\phi(z_{\tilde{s}_{1:K}} \mid s, x) = q_\phi(z_{\tilde{s}_1} \mid s, x) \prod_{k=2}^{K} q_\phi(z_{\tilde{s}_k} \mid z_{\tilde{s}_{1:k-1}}, s, x) := \prod_{k=1}^{K} \mathcal{N}(\mu_{\tilde{s}_k}, \sigma_{\tilde{s}_k}^2).
\]
To ensure that the segregation VAE is not biased by ordered gradients, we prevent gradients from flowing back through $\mu_{o_1:k}$ and $z_{o_1:k}$.

### 3.3 Training

We present SRI’s negative ELBO objective as the sum of four terms here (see the appendix for its derivation). First, we have a negative log-likelihood (reconstruction error) term:

$$L_{\text{NLL}} = -E_{q_\phi(s|x)} \left[ E_{q_\pi(s_{1:k} | s, x)} \log p_\theta(x | z_{\pi_{1:k}}) \right].$$

(10)

Next is a scene-level reverse Kullback-Leibler (KL) divergence term:

$$L_{\text{sceneKL}} = D_{KL}(q_\phi(s) \parallel p(s)).$$

(11)

The slot-level reverse KL term is:

$$L_{\text{slotKL}} = \mathbb{E}_{q_\phi(s|x)} \left[ D_{KL}(q_\phi(z_{\pi_1} | s, x) \parallel p(z_{\pi_1} | s)) \right]$$

$$+ \mathbb{E}_{q_\phi(s|x)} \left[ \sum_{k=2}^{K} \mathbb{E}_{q_\phi(z_{\pi_1:k-1} | s, x)} \left[ D_{KL}(q_\phi(z_{\pi_k} | z_{\pi_{1:k-1}}, s, x) \parallel p(z_{\pi_k} | z_{\pi_{1:k-1}}, s)) \right] \right].$$

(12)

Auxiliary loss: We introduce a cross-order KL term between the imagination rollout with order $\pi_{1:K}$ and autoregressive posterior with order $\hat{\pi}_{1:K}$:

$$L_{\text{rolloutKL}} = \mathbb{E}_{q_\phi(s|x)} \left[ D_{KL}(q_\phi(z_{\hat{\pi}_1} | s, x) \parallel p_\theta(z_{\pi_1} | s)) \right]$$

$$+ \mathbb{E}_{q_\phi(s|x)} \left[ \sum_{k=2}^{K} \mathbb{E}_{q_\phi(z_{\pi_1:k-1} | s, x)} \left[ D_{KL}(q_\phi(z_{\hat{\pi}_k} | z_{\pi_{1:k-1}}, s, x) \parallel p_\theta(z_{\pi_k} | z_{\pi_{1:k-1}}, s)) \right] \right],$$

(13)

where $q_\phi(z_{\hat{\pi}_k} | \star) := q_\phi(z_{\hat{\pi}_k} | z_{\pi_{1:k-1}}, s, x)$ and $p_\theta(z_{\pi_k} | \star) := p_\theta(z_{\pi_k} | z_{\pi_{1:k-1}}, s)$. By pushing the rollout distribution towards the posterior (which is obtained from a transformation of the segregation posterior), the rollout distribution learns to adopt a fixed object order and to generate high-quality scenes (Figure 3). This loss also trains $\hat{f}_\phi$ to extract relations from $z_{o_1:k}$; intuitively, if the imagined observation resembles the input $x$, then the global posterior—which we use to condition the imagination rollout—must accurately summarize correlations between $z_{o_1:k}$.

Summary: We minimize $L$, the sum of the objectives for the segregation VAE ($L_{\text{seg}}$) and SRI:

$$L_{\text{SRI}} = L_{\text{NLL}} + L_{\text{sceneKL}} + L_{\text{slotKL}} + L_{\text{rolloutKL}}$$

$$L = L_{\text{seg}} + L_{\text{SRI}}.$$  

(14)

(15)

We directly use the objective $L_{\text{seg}}$ as defined by the chosen segregation VAE. Similar to previous models [14; 16] we dynamically balance the reconstruction and KL terms with GECO [58]. For simplicity, we use the same SBD decoder for $L_{\text{seg}}$ and $L_{\text{SRI}}$.

### 4 Experiments

In this section we analyze SRI in terms of generation quality and ability to learn correlations. Section 4.1 qualitatively evaluates random sample quality and the slot order, and Section 4.2 quantitatively compares against key baselines. We conduct ablation studies in Section 4.3. In the supplementary material we also provide videos of random walks in the learned global latent space.\(^1\)

Datasets: We use three synthetic multi-object datasets to evaluate SRI: Objects Room and CLEVR6 from the standard Multi-Object Dataset [34] and ShapeStacks [23]. All three datasets consist of rendered images of 3D scenes containing 2-6 objects and have variable illumination, shadow, and camera perspectives. Generating scenes from these datasets requires reasoning about non-trivial correlations between objects (e.g., occlusion in CLEVR6, objects resting on the floor in Objects Room, block towers in ShapeStacks).

Setup: On all datasets we train SRI with GENv2 as the segregation VAE using both Gaussian (SRI-G) and Mixture-of-Gaussians (SRI-MoG) likelihoods. We fix $|z_k| = 64$, $|s| = 128$, and $L = 3$ across

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\(^1\) Code and videos are available at [https://github.com/pemami4911/segregate-relate-imagine](https://github.com/pemami4911/segregate-relate-imagine)
environments. Following standard practice, on CLEVR6 and Objects Room we use $K = 7$ and on ShapeStacks we use $K = 9$.

**Baselines:** We compare against the state-of-the-art slot VAEs for scene generation GEN and GENv2, as well as EMORL and GNM (not slot-based). Results for GEN and GENv2 (GENv2-MoG) are taken from Engelcke et al. [16] (both use the Mixture-of-Gaussians by default). GEN and GENv2-MoG results were missing for CLEVR6 so we trained these ourselves with the official code releases. We also train GENv2 with the Gaussian likelihood (GENv2-G) on all three datasets to provide a direct comparison for SRI-G. GNM and EMORL are trained on all datasets using official code releases. Remaining architecture, hyperparameters, and compute details can be found in the appendix.

### 4.1 Qualitative Evaluation

First, we visualize segmentation masks for randomly generated ShapeStacks scenes which show that SRI learns a fixed generation order (Figure 4). We also show that GENv2 fails to learn a fixed order which hurts its ability to learn object correlations (Figure 4d). Then, we compare random scenes sampled from each generative model in Figure 5 with addition samples from SRI in the appendix (Figures 7, 8). Across environments, SRI generates the highest quality scenes. GENv2’s samples contain artifacts and structural inaccuracies (e.g., extra walls and missing floors in Objects Room for GENv2-G and floating blocks in ShapeStacks). Specifically, on Objects Room GENv2-MoG tends to segregate the walls, floor, and sky into a single slot while GENv2-G segments the background across multiple slots. This increases the difficulty of learning multi-object correlations for GENv2-G, which SRI-G addresses. GEN’s sequential attention occasionally settles on a semi-consistent order (e.g., placing the background in the first slot for CLEVR6 and Objects Room) but in general this order is arbitrary and its attention mechanism fails on challenging scenes. As expected due to its independent slot prior, EMORL generates incoherent scenes. GNM generates images of high quality yet does so at the expense of poor segmentation quality. It is particularly challenged by Objects Room and ShapeStacks due to object occlusion and variable object sizes. See Appendix A.5 for extra random samples, test time generation with different numbers of slots $K$, imagination rollouts, and more.

### 4.2 Quantitative Evaluation

We use the Fréchet Inception Distance (**FID**) [27] to quantify the visual similarity of generated scenes to training scenes. Inspired by the evaluation in GNM [32], we measure the ability to learn higher-order correlations by manually labeling 100 ShapeStacks scenes generated by each model and computing the percent of scenes that contain a physically plausible stack of blocks, i.e., all blocks in the stack are touching each other (structural accuracy, or **S-Acc**). We use officially released model weights to compute S-Acc for GEN and GENv2-MoG. We follow standard practice for evaluating segregation quality, which is to use the adjusted rand index [57; 29] for foreground objects only with ground truth masks (**ARI-FG**).
Results are in Table 1. Out of the slot VAEs, SRI achieves the lowest FID scores on all environments and the best S-Acc score. SRI-G improves GENv2-G’s FID by an average of 24% and SRI-MoG improves GENv2-MoG’s FID by an average of 19%. On CLEVR6, we observe that SRI achieves the smallest improvement in FID over GENv2, which we believe is caused by i) CLEVR6 only having simple correlations to learn (e.g., occlusion) and ii) small artifacts introduced by GENv2’s encoder which seems to negatively impact the FID score on this dataset. To check this, and to demonstrate the flexibility of SRI, we also train SRI with EMORL as the segregation VAE on CLEVR6. We see a significant improvement in FID from EMORL to SRI-EMORL (244 ± 19 to 48.9 ± 7, also see Figure 12). GENv2-G segregates the walls, floor, and sky into different slots on Objects Room which makes correlation learning difficult, leading to a poor FID score and which SRI-G improves by 36%. In general, we find that the segregation quality of SRI closely matches that of the slot VAE used for scene segregation, and that better ARI-FG correlates with better sample quality for SRI. GNM’s difficulty with segregating Objects Room and ShapeStacks scenes is reflected by its comparatively low ARI-FG scores on two out of three environments.

4.3 Ablation Study

On ShapeStacks’s validation set we carefully ablate the core aspects of SRI (Table 2). a) Removing the global variable and instead using a slot prior with $p(z_1) := \mathcal{N}(0, 1)$ significantly hurts sample
While there are not immediate negative societal implications of this work, future work that builds on our ideas could be applied to realistic data which could plausibly cause harm. Generative models capable of synthesizing photo-realistic scenes could be used, for example, as part of a disinformation campaign. Research on mitigation strategies for such scenarios is ongoing.

### Table 1: Main quantitative results. Mean ± std. dev. over 3 training runs. Results* are from Engelcke et al. [16]. Best generation results for slot VAEs are in bold.

| Model      | Slot VAE | CLEVR6 | Objects Room | ShapeStacks |
|------------|----------|--------|--------------|-------------|
|            |          | FID₁   | ARI-FG₁      | FID₁        | ARI-FG₁      | FID₁        | ARI-FG₁      | S-Acc (%) |
| GNM        |          | 27.5 ±1 | 0.97 ±0.01  | 51.6 ±5    | 0.63 ±0.00  | 49.3 ±2    | 0.37 ±0.07  | 100.0 ±0.0 |
| GEN        | ✓        | 116.9 ±4 | 0.82 ±0.07  | 62.8 ±5    | 0.63 ±0.01  | 186.8 ±8   | 0.70 ±0.05  | 0 ±0.0    |
| EMORL      |          | 244.0 ±1 | 0.96 ±0.02  | 178.3 ±7   | 0.47 ±0.22  | 258.4 ±5   | 0.60 ±0.04  | 0 ±0.0    |
| GENv2-G    | ✓        | 61.0 ±3  | 0.98 ±0.00  | 87.6 ±4    | 0.75 ±0.02  | 115.3 ±6   | 0.68 ±0.02  | 50.7 ±3   |
| GENv2-MoG  | ✓        | 61.0 ±3  | 0.98 ±0.00  | 52.6 ±3    | 0.84 ±0.01  | 112.7 ±3   | 0.81 ±0.00  | 59 ±0.0   |
| SRI-G      | ✓        | 60.6 ±2  | 0.96 ±0.02  | 55.7 ±4    | 0.74 ±0.01  | 74.7 ±5    | 0.70 ±0.01  | 72.0 ±3   |
| SRI-MoG    | ✓        | 54.4 ±2  | 0.97 ±0.01  | 48.4 ±4    | 0.83 ±0.02  | 70.4 ±3    | 0.78 ±0.02  | 80.7 ±5   |

### Table 2: Ablations with SRI-G on ShapeStacks. Mean ± std. dev. over 3 runs. The negative ELBO in bits-per-dim (bpd) is a single sample Monte Carlo estimate averaged over 5k validation scenes.

| Learned order | Global variable | $L_{\text{rolloutKL}}$ | Attention | FID₁ | $L_{\text{sri(bpd)}}$ | S-Acc (%) |
|---------------|----------------|------------------------|-----------|------|------------------------|-----------|
| a)            | ✓              | ✓                      | -         | 132.2 ±2  | -0.59 ±0.01             | 31.7 ±4   |
| b)            | ✓              | ✓                      | -         | 123.2 ±2  | -0.48 ±0.01             | 43.7 ±10  |
| c)            | ✓              | ✓                      | ✓         | 106.3 ±3  | -0.31 ±0.03             | 61.3 ±5   |
| d)            | ✓              | ✓                      | ✓         | 77.4 ±2   | -0.24 ±0.01             | 72.0 ±3   |
| e)            | ✓              | ✓                      | ✓         | 75.0 ±6   | +0.0 ±0.0               | 72.0 ±3   |

quality. Without the global variable, higher-order correlations are difficult to learn and the imagination rollout is unconditional meaning it is unrelated to the input x. As a result SRI fails to learn a fixed generation order. b) Here, we do not try to align the slot order for inference and instead train the autoregressive slot posterior using the random order $o_{1:K}$ (therefore also removing $L_{\text{rolloutKL}}$ from the loss). SRI struggles to learn object correlations and sample quality is also poor. c) If we remove $L_{\text{rolloutKL}}$ from the loss, SRI's prior fails to learn a fixed order and correlation estimation suffers. This results in a significant drop in sample quality (see Figure 4c). d) We remove the relational attention by dropping $z_{o_{1:k}}$ from Eq. 6. This incurs only a small increase in FID, likely because the DeepSets MLP is reasonably expressive for learning correlations and because other aspects such as segregation quality seem to have a larger impact on FID. In general, we see that the global variable, learned order, and cross-order KL loss all play a critical role in SRI.

### 5 Conclusion

In this work we demonstrated that slot order matters for compositional scene understanding because it facilitates learning complex multi-object correlations. We contributed SRI, a hierarchical VAE with autoregressive slots and a global scene-level prior. Inference is cast as a set-to-sequence problem which allows us to learn a good object order for autoregressive inference. Our empirical results demonstrate that SRI significantly improves the sample quality of compositionally generated scenes for slot VAES. Moreover, with SRI, future advances that improve our ability to segregate scenes should now directly translate to better sample quality.

One limitation of SRI is the lack of 3D reasoning. Although unsupervised 3D reasoning is challenging due to the ambiguities introduced by perspective geometry [7; 65; 76; 19], a 3D understanding of objects should further improve generation quality. Another limitation is that SRI does not learn causal dependencies between objects but rather only correlations, which only weakly generalize outside the training distribution [62]. Extending SRI to dynamic settings would enable exploring causal inference of dependencies. Finally, we suspect that the standard Gaussian global prior we used for simplicity is sub-optimal for learning higher-order correlations. Exploring more flexible global priors could further improve results.

While there are not immediate negative societal implications of this work, future work that builds on our ideas could be applied to realistic data which could plausibly cause harm. Generative models capable of synthesizing photo-realistic scenes could be used, for example, as part of a disinformation campaign. Research on mitigation strategies for such scenarios is ongoing.
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A Appendix

A.1 Matching Pseudocode

Algorithm 1 Greedy approximate matching. Uses $O(K)$ time and $O(K^2)$ space.

1: **Input:** Randomly ordered segregation posterior means $\mu_{\alpha_1:K}$, imagination rollout means $\mu_{\pi_1:K}$
2: $C[i, j] = \| \mu_{\pi_i} - \mu_{\alpha_j} \|_2$, $\forall i,j = 1, \ldots, K$
3: $\sigma := \text{zeros}(K, K)$
4: for index $i = 0 \ldots K - 1$ do
5: \quad $j^* = \arg \min_i C[i, :]$
6: \quad $C[:, j^*] = + \infty$
7: \quad $\sigma[j^*, i] = 1$
8: end for
9: **return** Permutation $\sigma$

Alternative matching algorithms such as the Hungarian algorithm [50] can be used instead at a higher computational cost; however, we leave an empirical comparison of these algorithms for future work.

A.2 SRI Inference Pseudocode

Algorithm 2 SRI Inference. All sampling uses the Gaussian reparameterization trick [38].

1: **Input:** Scene observation $x$, segregation VAE encoder $e_{\psi}(x)$, scene-level encoder $f_{\theta}(z)$, autoregressive prior $\text{LSTM}_{\theta_0}$, autoregressive posterior $\text{LSTM}_{\theta}$
2: $z_{\alpha_1:K} \sim q_{\theta}(z_{\alpha_1:K} \mid x)$ \hspace{1em} /* Segregate */
3: $s \sim q_{\theta}(s \mid x) \leftarrow f_{\theta}(z_{\alpha_1:K})$ \hspace{1em} /* Relate */
4: $\pi_k \sim \text{LSTM}_{\theta_0}(\{s\})$ \hspace{1em} /* Imagine */
5: $z_{\pi_k} \sim p_{\theta}(z_{\pi_k} \mid z_{\pi_{k-1}}, s) \leftarrow \text{LSTM}_{\theta_0}(\{s, z_{\pi_{k-1}}\})$ for all $k = 2, \ldots, K - 1$
6: $\sigma \leftarrow \text{Matching}(z_{\alpha_1:K}, z_{\pi_1:K})$ \hspace{1em} /* we pass the posterior means to Algorithm 1 */
7: $\mu_{\alpha_1:K} \rightarrow \mu_{\pi_1:K}$ \hspace{1em} /* apply the permutation to the $K$ means of $q_{\theta}(z_{\alpha_1:K} \mid x)$ */
8: $\sigma_{\pi_1:K} \leftarrow \text{LSTM}_{\theta_0}(\{s\})$ \hspace{1em} /* predict new correlated variances */
9: $\sigma_{\alpha_1:K} \leftarrow \text{LSTM}_{\theta_0}(\{s, \mu_{\pi_{k-1}}\})$ for all $k = 2, \ldots, K - 1$
10: $q_{\theta}(z_{\pi_1:K} \mid s, x) \coloneqq \prod_{k=1}^{K} \mathcal{N}(\mu_{\pi_k}, \sigma_{\pi_k}^2)$ \hspace{1em} /* autoregressive slot posterior */
11: **return** The sufficient statistics (e.g., means and variances) for the segregation posterior $q_{\theta}(z_{\alpha_1:K} \mid x)$, scene posterior $q_{\theta}(s \mid x)$, autoregressive slot posterior $q_{\theta}(z_{\pi_1:K} \mid s, x)$

Our proposed set-to-sequence slot inference involves aligning the object order of the segregation posterior to the learned object generation order (Lines 6-7) and then transforming this posterior into an autoregressive distribution by predicting a sequence of correlated variances (Lines 8-10).

A.3 ELBO Derivation

We derive a lower bound on the log marginal scene likelihood as follows:

\[
\log p(x) = \log \int p_{\theta}(x, s, z_{\pi_1:K}) ds, dz_{\pi_1:K} \\
= \log \int q_{\theta}(s, z_{\pi_1:K} \mid x)p_{\theta}(x, s, z_{\pi_1:K}) ds, dz_{\pi_1:K} \\
= \log \mathbb{E}_{q_{\theta}(s, z_{\pi_1:K} \mid x)} \left[ \frac{p_{\theta}(x, s, z_{\pi_1:K})}{q_{\theta}(s, z_{\pi_1:K} \mid x)} \right].
\]

Applying Jensen’s inequality and then factorizing gives:

\[
\log p(x) \geq \mathbb{E}_{q_{\theta}(s, z_{\pi_1:K} \mid x)} \left[ \log \frac{p_{\theta}(x, s, z_{\pi_1:K})}{q_{\theta}(s, z_{\pi_1:K} \mid x)} \right] \\
= \mathbb{E}_{q_{\theta}(s \mid x)} \left[ \mathbb{E}_{q_{\theta}(z_{\pi_1:K} \mid s, x)} \left[ \log \frac{p(s)p_{\theta}(z_{\pi_1:K} \mid s)p_{\theta}(x \mid z_{\pi_1:K})}{q_{\theta}(s \mid x)q_{\theta}(z_{\pi_1:K} \mid s, x)} \right] \right].
\]
We use the fact that \( \log(\frac{A}{B}) = \log(A) + \log(\frac{1}{B}) \) to factor Eq. 20 into a sum of three losses. The autoregressive slot posterior order obtained for inference \( \hat{\pi}_{1:K} \) is shown in blue.

The first term is a negative log-likelihood loss:

\[
\mathcal{L}_{\text{NLL}} = -\mathbb{E}_{q_\phi(s|x)} \left[ \mathbb{E}_{q_\phi(z_{1:K} | s, x)} \left[ \log p_\theta(x | z_{\hat{\pi}_{1:K}}) \right] \right].
\]

(21)

The second is a slot-level reverse KL loss with the approximate order:

\[
\mathcal{L}_{\text{slotKL}} = -\mathbb{E}_{q_\phi(s|x)} \left[ \mathbb{E}_{q_\phi(z_{1:K} | s, x)} \left[ \log \frac{p_\theta(z_{\hat{\pi}_{1:K}} | s)}{q_\phi(z_{\hat{\pi}_{1:K}} | s, x)} \right] \right]
\]

(22)

\[
= \mathbb{E}_{q_\phi(s|x)} \left[ \mathbb{E}_{q_\phi(z_{1:K} | s, x)} \left[ \log \frac{q_\phi(z_{\hat{\pi}_{1:K}} | z_{\hat{\pi}_{1:k-1}, s, x})}{p_\theta(z_{\hat{\pi}_{1:K}} | s)} \right] \right]
\]

(23)

\[
= \mathbb{E}_{q_\phi(s|x)} \left[ \sum_{k=2}^{K} \mathbb{E}_{q_\phi(z_{1:k-1} | s, x)} \left[ D_{KL}(q_\phi(z_{\hat{\pi}_{1:k-1}} | z_{\hat{\pi}_{1:k-1}, s, x}) \| p_\theta(z_{\hat{\pi}_{1:k-1}} | s)) \right] \right].
\]

(24)

The third term is a scene-level reverse KL loss:

\[
\mathcal{L}_{\text{sceneKL}} = -\mathbb{E}_{q_\phi(s|x)} \left[ \mathbb{E}_{q_\phi(z_{1:K} | s, x)} \left[ \log \frac{p(s)}{q_\phi(s | x)} \right] \right]
\]

(25)

\[
= -\mathbb{E}_{q_\phi(s|x)} \left[ \mathbb{E}_{q_\phi(s | x)} \left[ \log \frac{p(s)}{q_\phi(s | x)} \right] \right]
\]

(26)

\[
= \mathbb{E}_{q_\phi(s|x)} \left[ \mathbb{E}_{q_\phi(s | x)} \left[ \log \frac{q_\phi(s | x)}{p(s)} \right] \right]
\]

(27)

\[
= D_{KL}(q_\phi(s | x) \| p(s)).
\]

(28)

We also define an auxiliary slot-level reverse KL loss similar to Eq. 26 except that the inner expectation is now taken with respect to the slot posterior and the \( K \)-step rollout with order \( \hat{\pi}_{1:K} \):

\[
\mathcal{L}_{\text{rolloutKL}} = \mathbb{E}_{q_\phi(s|x)} \left[ D_{KL}(q_\phi(z_{\hat{\pi}_{1}} | s, x) \| p_\theta(z_{\pi_{1}} | s)) \right]
\]

(29)

\[
+ \mathbb{E}_{q_\phi(s|x)} \left[ \sum_{k=2}^{K} \mathbb{E}_{q_\phi(z_{1:k-1} | s, x)} \left[ D_{KL}(q_\phi(z_{\hat{\pi}_{1:k-1}} | s) \| p_\theta(z_{\pi_{1:k-1}} | s)) \right] \right],
\]

(30)

where \( q_\phi(z_{\hat{\pi}_{1:k-1}} | s) := q_\phi(z_{\hat{\pi}_{1:k-1}} | z_{\hat{\pi}_{1:k-1}, s, x}) \) and \( p_\theta(z_{\pi_{1:k-1}} | s) := p_\theta(z_{\pi_{1:k-1}} | z_{\pi_{1:k-1}, s}) \). The auxiliary KL term is always non-negative by definition, so adding it to the negative ELBO only increases the upper bound on \( -\log p(x) \) (equivalently, loosens the lower bound on \( \log p(x) \)). Summed together, the four terms form the SRI loss:

\[
\mathcal{L}_{\text{SRI}} = \mathcal{L}_{\text{NLL}} + \mathcal{L}_{\text{slotKL}} + \mathcal{L}_{\text{rolloutKL}} + \mathcal{L}_{\text{sceneKL}}.
\]

In practice, we approximate the expectations with a single sample and average the loss over a minibatch of size \( B = 32 \) of dataset samples. We leave a formal characterization of how the accuracy of the matching between \( \hat{\pi} \) and \( \pi \) affects the lower bound on the marginal log likelihood for future work. Intuitively, as long as the two slot orders are "close", we can expect these lower bounds to be similar as well.

A.4 Experiment details

A.4.1 Hyperparameters

**Architecture:** We use \( |z_s| = 64 \) for all environments following GENv2 and EMORL. We set \( |s| = 128 \) to be twice the slot dimension. The number of attention layers for scene posterior estimation is fixed at \( L = 3 \); we found that varying \( L \) incurred little change in performance. We use the same hyperparameters for the image encoder and decoder architectures as used by GENv2 and EMORL for SRI-X and SRI-EMORL, respectively. Both of the GENv2 and EMORL implementations
are based on open source released by the respective authors.\textsuperscript{23} For SRI-EMORL we fix the number of iterative refinement steps to 2. Following GEN and GENv2, SRI’s autoregressive prior and posterior LSTM uses 256 hidden units. When training GENv2 jointly with SRI we replace its autoregressive prior with a slot-wise independent prior. SRI uses GELU non-linear activations \textsuperscript{26} after each linear projection of the scene variable and in the attention layers. All layers use Xavier weight initialization \textsuperscript{18}.

\textbf{Optimization:} We mostly adopt the same hyperparameters here as used by GENv2 and EMORL. SRI based on GENv2 uses the Adam optimizer \textsuperscript{37} with default hyperparameters and a learning rate of 1e-4 but without any learning rate schedule. SRI-EMORL uses an initial learning rate of 4e-4, grad norm clipping to 5, and learning rate schedule consisting of a linear warmup for 10K steps then multiplicative decay with by a rate of 0.5 every 100K steps. A batch size of 32 is used for all models. Both GENv2 and EMORL use GECO to balance reconstruction and KL during optimization; we discuss the GECO hyperparameters in Section A.4.2.

\textbf{Datasets:} Both CLEVR6 and Objects Room can be accessed freely online under the Apache License 2.0.\textsuperscript{4} The ShapeStacks dataset is also freely available for download online under the GNU License 3.0.\textsuperscript{5} We use the same preprocessing protocol for CLEVR6 as Emami et al. \textsuperscript{14}, which is to center crop the images to 192x192 and the resize them to size 96x96. Objects Room and ShapeStacks have 64x64 RGB images.

A.4.2 Balancing reconstruction and KL with GECO

Both GENv2 and EMORL use GECO to balance reconstruction and KL losses \textsuperscript{58}. Each code base has its own GECO implementation and GECO hyperparameter schedule which we use to train SRI-X and SRI-EMORL.

SRI-G (unnormalized Gaussian image likelihood and global $\sigma = 0.7$) uses a per-pixel and per-channel GECO target of -0.353 on Objects Room and ShapeStacks and -0.356 on CLEVR6. We decreased the GECO learning rate from 1e-5 to 1e-6. Once the GECO target is reached the learning rate is multiplied by a speedup factor of 10 to accelerate the decay of the GECO Lagrange parameter.

SRI-MoG uses the same GECO hyperparameters as GENv2-MoG on ShapeStacks and Objects Room (target is 0.5655—note that the MoG likelihood here is normalized) except we also use the decreased GECO learning rate of 1e-6. However, on Objects Room we notice that this increased training instability, likely because this warms up the KL less aggressively than the faster GECO learning rate does. We use restarting from model checkpoints to mitigate this here.

For SRI-EMORL on CLEVR6 we used a target of -2.265 for unnormalized Gaussian image likelihood with global $\sigma = 0.1$. The rule of thumb used to tune the GECO target is that the target should be reached after about 20% of the training steps. This gives ample time for the GECO Lagrange parameter to automatically decay back to 1 so that a valid ELBO is eventually maximized. We use a GECO learning rate of 1e-6 and speedup of 10 as well.

A.4.3 Baselines and compute

\textbf{GENESIS and GENESIS-v2:} The authors of GENv2 have released pre-trained weights for GEN trained on ShapeStacks and GENv2-MoG trained on Objects Room and ShapeStacks. We use these weights for model visualizations and to compute the structure accuracy metric. To train GENv2-MoG on CLEVR6 we had to lower the standard deviation to 0.1. We adjusted the GENv2-MoG GECO target accordingly by tuning it to -2.265. To compute the FID score, we follow the same protocol as GENv2 \textsuperscript{16} and use 10K real and generated samples.

\textbf{GNM:} We use the official GNM PyTorch implementation provided by the authors.\textsuperscript{6} We train GNM on the 128x128 resolution version of CLEVR6 so that we could use the default GNM hyperparameters as the authors used for the 128x128 resolution CLEVR-based environment from their paper. This worked well for CLEVR6. By default, GNM organizes its symbolic variables into a 4x4 spatial
grid, which we maintained. We treat each spatial grid cell as one slot for a total of 16 slots. To train GNM on the 64x64 Objects Room and ShapeStacks scenes, we removed one downsampling and one upsampling layer from the encoder and decoder to account for the resolution being one-half that of CLEVR6. For all environments, we used the default global latent dim of 32 and z_dim of 64.

**Compute:** In general, the majority of the compute is taken up by the segregation VAE (GENv2 and EMORL in this work). SRI adds a few lightweight neural networks and therefore only marginally increases training time.

- On Objects Room and ShapeStacks, SRI-X takes about 20 hours to reach 500K steps using 2 NVIDIA A100 GPUs. On CLEVR6, it about 24 hours to reach 425K steps, at which point we stopped training since the model showed signs of convergence and so as to keep training times at about one day or faster on equivalent hardware to ease reproducibility.
- The GENv2 and GEN baselines are negligibly faster to train. We train these baselines using the same setup as SRI.
- SRI-EMORL uses 8 NVIDIA A100 GPUs to reach 225K CLEVR6 which takes about 27 hours, at which point we cut off training to keep a similar compute budget to SRI with GENv2. The memory footprint of the two steps of iterative refinement used to estimate the segregation posterior is large, hence the need for 8 GPUs.
- GNM takes about 9 hours to reach 500K steps with a batch size of 32 on one A100 GPU for Objects Room and ShapeStacks. It takes about 12 hours to reach 500K steps on CLEVR6 on one A100 GPU.

### A.4.4 Open source software

This project was conducted using the following open source Python packages: PyTorch [54], numpy [25], jupyter [39], matplotlib [31], scikit-learn [55], and sacred [20].

### A.5 Additional Qualitative Results

![Figure 6: Mask color to slot number legend](image)

**Additional random samples:** We visualize extra randomly sampled scenes from SRI-MoG (Figure 7), SRI-G (Figure 8), and GENv2-G (Figure 9).

**Generalizing to different numbers of slots:** We demonstrate that the number of slots can be changed at test time to generate scenes with fewer or more objects than seen during training in Figure 10.

**Effect of temperature scaling:** We investigate whether temperature scaling provides any further qualitative improvement in sample quality (Figure 11).

**Random samples from SRI-EMORL:** Random samples from SRI-EMORL on CLEVR6 (visualized with temperature scaling) (Figure 12).

**Imagination rollouts:** We provide more examples of pairs of reconstructed scenes with the corresponding imagination rollout used by SRI for slot order estimation (Figure 13).

**Reconstruction and segmentation examples:** Examples of reconstructed and segmented scenes from each dataset for SRI-MoG (Figure 14), SRI-G (Figure 15), GENv2-G (Figure 16), and GNM (Figure 17).
Figure 7: Additional random samples generated by SRI-MoG.
Figure 8: Additional random samples generated by SRI-G.
Figure 9: Additional random samples generated by GENESISv2-G.
Figure 10: Generalization to different numbers of slots $K$. Random ShapeStacks samples. SRI-MoG was trained with $K = 9$. 

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Figure 11: Randomly sampled ShapeStacks images from SRI-MoG with different temperature scaling parameters $\tau$ in the scene-level and slot prior. Scaling down the variances in a hierarchical Gaussian prior is known to help samples stay in regions of high probability [66]. We confirm here that values of $\tau$ near 1.0 slightly improve sample quality on challenging datasets with minimal impact on sample diversity. Note that we use $\tau = 1.0$ in this work for all evaluations unless stated otherwise.

Figure 12: Random samples generated by SRI-EMORL. These images are generated with temperature $\tau = 0.8$. 
Figure 13: Imagination examples. Side-by-side comparisons of reconstructed RGB images and masks from SRI-MoG’s aligned autoregressive posterior with imagined RGB images and masks.
Figure 14: SRI-MoG reconstruction and segmentation.
Figure 15: SRI-G reconstruction and segmentation.
Figure 16: GENv2-G reconstruction and segmentation.
Figure 17: GNM reconstruction and segmentation. The symbolic variables, which are bound to a spatial grid of fixed resolution, are unable to properly bind to the foreground objects in the Objects Room and ShapeStacks datasets. In Objects Room, the shapes are segmented into strips and grouped with the ground. The wall is erroneously considered one of the foreground objects. In ShapeStacks, distinct blocks are segmented together when they fall within the same grid cell.