Final State Interactions and the Effects of Potentials on Particle Reactions

G. Castellani, S. Reucroft, Y.N. Srivastava†,
J. Swain and A. Widom
Physics Department, Northeastern University
Boston Massachusetts, USA
† Physics Department & INFN, University of Perugia
Perugia Italy

Abstract

In nuclear physics, it is well known that the electromagnetic (Coulomb) interaction between final state products can drastically affect particle reaction rates. Near thresholds, for example, nuclear alpha decay is suppressed while nuclear beta decay is enhanced by final state Coulomb interactions. Here we discuss high energy physics enhancement and/or suppression of reactions wherein the potentials must include weak and strong as well as electromagnetic interactions. Potentials due to the exchange of gluons and the exchange of a hypothetical Higgs particle are explicitly considered.

1 Introduction

The Coulomb interaction

\[ U_{\text{Coul}} = \left( \frac{e^2}{4\pi\epsilon_0} \right) \left( \frac{Z_1 Z_2}{r} \right) = Z_1 Z_2 \left( \frac{\hbar c \alpha}{r} \right) \]  \hspace{1cm} (1)

between the final products of nuclear reactions can have a large effect on particle reaction rates and cross sections. If the final state Coulomb potential is repulsive, then the reaction is suppressed. Such is the case for (say) nuclear alpha decay or inverse nuclear beta decay. If the Coulomb final state interaction is attractive, then the reaction is enhanced. Such is the case for nuclear beta decay. The effects of the final state Coulomb potential is (i) particularly large near threshold and (ii) requires methods far beyond standard low order perturbation theory for a proper calculation.

Although the application of final state interaction theory to problems of nuclear physics is by now fairly routine, the theory is not yet quite standard practice in high energy physics wherein perturbation theory perhaps too often reigns supreme. Yet the potentials of the weak and strong interactions, if not
the gravitational potential

\[ U_{\text{Newton}} = -G \left( \frac{M_1 M_2}{r} \right), \]  

(2)
surely play a final state interaction role similar to the Coulomb interaction in nuclear physics. In particular, we wish to discuss these final state interactions which are derived from both weak and strong forces. There has been considerable earlier work\[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\] with applications to the \( W^+W^- \) and heavy flavor \( q\bar{q} \) production.

The strong force potential, presumed due to gluon exchange, has the form

\[ U_{\text{Glue}} = \left( \frac{g^2}{4\pi\varepsilon_0} \right) \left( \frac{T_1 \cdot T_2}{r} \right) = \left( \frac{\hbar \alpha_s}{r} \right) T_1 \cdot T_2, \]  

(3)
in which the matrices \( \{T\} \) are the color SU(3) group generators. The gluon exchange potential Eq. (3) is written down in close analogy to the photon exchange potential Eq. (1). It reads

\[ U_{\bar{q}q} = V_{q\bar{q}} = -\frac{4}{3} \left( \frac{\hbar \alpha_s}{r} \right) (\text{quark anti-quark}), \]

\[ U_{q\bar{q}} = V_{qq} = -\frac{2}{3} \left( \frac{\hbar \alpha_s}{r} \right) (\text{quark quark}). \]  

(4)
However, Eqs. (3) and (4) hold true only in the \( r \to 0 \) limit. For large \( r \), the presumed confinement (linear) portion of the potential is presently only partially understood. The details of the full quark potentials are summarized in \( \text{A.} \)

The weak Higgs exchange potential has the form

\[ U_{\text{Higgs}} = - \left( \frac{\sqrt{2}G_F}{4\pi} \right) \left( \frac{M_1 M_2}{r} \right) e^{-(M_H c/\hbar)r} \]  

(5)
in close analogy to the graviton exchange potential Eq. (2). Here, the Fermi interaction strength \( G_F \) plays a role analogous to the Newtonian gravitational coupling \( G \) while the mass \( M_H \) of the Higgs particle plays the role of an inverse screening length. That the graviton exchange potential should bear a strong resemblance to the Higgs exchange potential (apart from screening) is due to the fact that gravitational mass is the source and sink of the gravitational field while inertial mass is the source and sink of the Higgs field. The principle of equivalence between gravitational and inertial mass dictates that the Higgs particle (if it exists) is intimately connected with gravity.

To compute the final state interaction effects of the effective exchange potentials which may enhance or may suppress the reaction, it is convenient to employ the quasi-classical relativistic Hamilton-Jacobi equation. If the potential is repulsive and the reaction suppressed, then the effect lies mainly in the classically disallowed region (quantum tunneling). If the potential is attractive and the reaction is enhanced, then the effect arises due to the strong overlap of
the attracted particle wave functions. This point is illustrated in Sec.2 wherein
the amplification of beta decay and the suppression of inverse beta decay will be
reviewed. In Sec.3 the attractive gluon exchange potential will be discussed with
regard to enhancement factors for the production of quark anti-quark pairs, i.e.
quark jets. Final state interactions induced by the Higgs field are discussed in
Sec.4 for $Z\bar{Z}$ and $W^+W^-$ production. The Higgs effects become more important
as the mass increases. In principle these effects may be of use in experimental
probes which seek to verify that the Higgs field exists. This point is briefly
discussed in the concluding Sec.5.

2 The Coulomb Potential

Consider the inverse beta decay of a nucleus written as the reaction

\[ \bar{\nu}_e + (Z + 1, A) \rightarrow (Z, A) + e^+. \]  
(6)

The finally produced positron interacts with the final nucleus via the repulsive
Coulomb potential

\[ U_+(r) = \frac{\hbar c\alpha Z}{r}. \]  
(7)

Since the nucleus is much more massive than is the positron, it is normally
sufficient to treat the Coulomb interaction potential as if it were external. The
positron energy equation then reads

\[ (E - U_+(r))^2 = m^2 c^4 + c^2 |p|^2. \]  
(8)

Relativistic Hamilton-Jacobi dynamics assert that the momentum is the gradi-
ent of the positron action

\[ p = \text{grad} W(r, E). \]  
(9)

The radial solution of Eqs.8 and 9 reads

\[ c^2 p(r, E)^2 = \left[ \frac{\partial W(r, E)}{\partial r} \right]^2 = (E - U_+(r))^2 - m^2 c^4, \]
(10)

\[ c^2 p(r, E)^2 = \left[ E - \frac{\hbar c\alpha Z}{r} + mc^2 \right] \left[ E - \frac{\hbar c\alpha Z}{r} - mc^2 \right]. \]

The classically allowed ($p^2 > 0$) and classically disallowed ($p^2 < 0$) regions in
the radial coordinate $r$ are defined by

\[ 0 < r < a \text{ or } r > b \implies \text{(allowed)}, \]
\[ a < r < b \implies \text{(disallowed)}, \]  
(11)

wherein

\[ a = \frac{\hbar c\alpha Z}{E + mc^2} \text{ and } b = \frac{\hbar c\alpha Z}{E - mc^2}. \]  
(12)
The reaction suppression is described by the barrier factor $B$ for the regime in which classical motion is forbidden. In detail

$$B = \frac{2}{\hbar} 3m |W(b, E) - W(a, E)| = \frac{2}{\hbar} \int_a^b \left| \Im m[p(r, E)] \right| dr,$$

$$B = \frac{2}{\hbar c} \int_a^b \sqrt{E - \frac{\hbar c Z \alpha r}{r} + mc^2} \left| E - \frac{\hbar c Z \alpha r}{r} - mc^2 \right| dr,$$

$$B(E, Z\alpha) = 2\pi Z\alpha \left[ \frac{E}{\sqrt{E^2 - m^2c^4}} - 1 \right] = 2\pi Z\alpha \left[ \left( \frac{E}{c}\right) - 1 \right], \quad (13)$$

where $v$ is the positron velocity. In the non-relativistic limit $v \ll c$, the Coulomb barrier factor $B \approx (2\pi Z\alpha c/v)$ is well known. Eq. (13) represents the relativistic theory in which the barrier factor vanishes in the high energy limit ($v \to c$).

The physical picture in the relativistic theory is worthy of note. The “tunnelling” through the barrier is in reality electronic “pair creation” under the barrier for ($a < r < b$). When the pair is created the positron half of the pair rushes off to infinity ($b < r < \infty$). The electron half of the pair falls into the center ($0 < r < a$) converting one of the nuclear protons into a neutron and emitting an electron neutrino. The total inverse beta decay reaction may then be represented as

$$(\text{vacuum}) \to e^- + e^+,$$

$${\bar}{\nu}_e + e^- + (Z + 1, A) \to (Z, A), \quad (14)$$

for which Eq. (14) is the total reaction. The full suppression factor cross section ratio induced by the Coulomb repulsion between the positron and the final state nucleus is given by

$$S(E, Z) = \frac{\sigma \left[ {\bar}{\nu}_e + (Z + 1, A) \to (Z, A) + e^+ \right]}{\sigma^{(0)} \left[ {\bar}{\nu}_e + (Z + 1, A) \to (Z, A) + e^+ \right]},$$

$$S(E, Z) = \frac{B(E, Z\alpha)}{e^{B(E, Z\alpha)} - 1}. \quad (15)$$

Eq. (15) concludes our discussion for the case of inverse beta decay.

For the case of beta decay

$$(Z - 1, A) \to (Z, A) + e^- + {\bar}{\nu}_e, \quad (16)$$

the Coulomb potential between the outgoing electron and the nucleus is attractive

$$U_-(r) = -\frac{\hbar c Z \alpha}{r}. \quad (17)$$

The Hamilton-Jacobi equation for the attractive Coulomb energy reads

$$(E - U_-(r))^2 = m^2c^4 + e^2|p|^2 \text{ wherein } p = \text{grad} W(E, r). \quad (18)$$
Since there is a particle anti-particle “duality” corresponding to positive and negative energy solutions in any relativistic theory, if an electron sees an attractive potential then the positron will see a repulsive potential. Relativistic dynamics with Poincaré symmetry automatically includes both particle and antiparticle dynamics. Employing this duality of solutions one finds that the beta decay for the electron is again described by Eq. (13) but this time with an amplification factor. The full ratio of decay rates corresponds to

$$A(E, Z) = \frac{\Gamma[(Z-1, A) \rightarrow (Z, A) + e^- + \bar{\nu}_e]}{\Gamma^{(0)}[(Z-1, A) \rightarrow (Z, A) + e^- + \bar{\nu}_e]},$$

$$A(E, Z) = \frac{B(E, Z\alpha)}{1 - \exp(-B(E, Z\alpha))}.$$  \hspace{1cm} (19)

The suppression factor for an outgoing positron and the amplification factor for an outgoing electron are plotted in Figure 1. For the inverse beta decay of Eq. (15), the positron emerges with velocity

$$v = \frac{c\sqrt{E^2 - m^2c^4}}{E}$$ \hspace{1cm} (20)

and the cross section is suppressed by the coulomb interaction factor $S$. For the beta decay case in Eq. (15), the electron can still emerge with the velocity in Eq. (20) but the decay rate is enhanced with an amplification factor $A$.

## 3 The Gluon Exchange Potential

Consider the production of a quark and an anti-quark with momenta $p$ and $\bar{p}$. The pair interacts with an attractive gluon exchange potential $U_{\bar{q}q}(r)$. On a short distance scale one expects a Coulomb-like potential with a strong interaction charge which dominates the actual Coulomb potential; i.e.

$$U_{\bar{q}q}(r) = -\frac{4}{3} \left( \frac{g^2}{4\pi\alpha_s r} \right) = -\frac{4}{3} \left( \frac{\hbar c\alpha_s}{r} \right) \text{ as } r \rightarrow 0.$$ \hspace{1cm} (21)

On a larger distance scale, the potential is discussed in A.

The total mass $\sqrt{s}$ of the final state pair is determined by

$$c^2 s = -P^2 = -(p + \bar{p})^2 = 2(c^2 m^2 - \bar{p} \cdot p).$$ \hspace{1cm} (22)

In the center of mass reference frame of the pair ($P = p + \bar{p}$ = 0), kinematics dictates

$$-c^2 \bar{p} \cdot p = \bar{E}E - c^2 \bar{p} \cdot p = c^4 m^2 + 2c^2 |p|^2;$$ \hspace{1cm} (23)

In detail, the relative momentum of the quark anti-quark pair is given by

$$|p| = c\sqrt{(s/4) - m^2}.$$ \hspace{1cm} (24)
Figure 1: For an outgoing beta decay electron or inverse beta decay positron with energy $E = \frac{mc^2}{\sqrt{1 - (v/c)^2}}$ there will be, respectively, an attraction or repulsion from the central nuclear final state charge $Ze$. Shown are the curves for the electron rate amplification $A(Z, E)$ and the positron rate suppression $S(Z, E)$ implicit in the conventional Coulomb final state corrections.

Figure 2: The gluon exchange potential amplification of quark anti-quark jet production is plotted as a function of the invariant mass squared. The amplification begins at threshold. A reasonable but approximate value for the strong coupling strength $\alpha_s$ has been employed.
The enhancement factor for the quark anti-quark jet production then follows a form closely analogous to the Coulomb case in Eqs. (17) and (19). The production amplification is

\[ A_{\bar{q}q}(s) = \frac{\Gamma_{\bar{q}q}(s)}{\Gamma_{\bar{q}q}^{(0)}(s)}, \]

\[ B_{\bar{q}q}(s) = \frac{4\pi\alpha_s}{3} \left[ \sqrt{\frac{s}{s-4m^2}} - 1 \right], \]

\[ A_{\bar{q}q}(s) = \frac{B_{\bar{q}q}(s)}{1 - \exp\left(-B_{\bar{q}q}(s)\right)}, \]  

which has been plotted in Figure 2. The amplification is particularly strong near the threshold value \( s_0 = 4m^2 \).

4 The Higgs Exchange Potential

The calculation of Higgs exchange amplification factor from the potential in Eq. (5) is a bit more delicate due to the screening effect of the Higgs mass \( M_H \). As shown in what follows, it turns out that the Higgs mass drops out of the result since the amplification factor is determined by the wave function of the two produced particle at zero distance for a fixed time. In effect, this represents a “zero space time interval” for the exchange and it is well known that the nature of the light cone singularity in the mass propagator is mass independent. The Higgs boson exchange Feynman diagram producing the exchange potential is shown in Figure 3.

The action associated with this exchange is given by

\[ S_{Higgs} = \frac{\sqrt{2}G_F}{2c^5} \int \int T(x)D(x-y)T(y)d^4xd^4y, \]  

wherein \( T(x) \) is the trace of the stress tensor and \( D(x-y) \) is the Higgs boson propagator

\[ D(x-y) = \hbar^2 \int \frac{e^{ip.(x-y)/\hbar}}{p^2 + (M_Hc)^2 - i0^+} \frac{d^4p}{(2\pi\hbar)^4}. \]  

A more physical space-time representation of the Higgs boson propagation follows from the Schwinger proper time representation

\[ D(x-y) = \frac{M_H}{8\pi^2\hbar} \int_0^\infty e^{[iM_H/2\hbar][(-c^2\tau^2 + (x-y)^2)/\tau]} \left( \frac{d\tau}{\tau^2} \right). \]  

For two particles moving at uniform velocities the trace of the stress tensor quasi-classical sources reads \( T_{a,b}(x) = -M_{a,b}c^3 \int \delta(x - v_{a,b}\tau)d\tau \). Eq. (26) now yields the action

\[ S_{ab} = \left( \frac{\sqrt{2}G_F M_a M_b}{c} \right) c^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D(v_a \tau_a - v_b \tau_b)d\tau_ad\tau_b. \]  

7
If Eq. (28) is substituted into Eq. (29), then the resulting Gaussian integrals over $d\tau_a$ and $d\tau_b$ can be performed yielding

$$S_{ab} = \left(\frac{\sqrt{2} G_F m_a m_b}{c}\right) \int_0^\infty \tilde{F}(v_a, v_b, \tau) \left(\frac{d\tau}{\tau}\right),$$

(30)

wherein

$$\tilde{F}(v_a, v_b, \tau) = \left(\frac{c^2}{4\pi \sqrt{(v_a \cdot v_b)^2 - c^4}}\right) e^{-i M_H c^2 \tau / 2\hbar}.$$  

(31)

The Higgs mass $M_H$ drops out of the final expression for the imaginary part of the action,

$$\Im m \, S_{ab} = -\left(\frac{\sqrt{2} G_F M_a M_b}{8c}\right) \left(\frac{M_a M_b c^2}{\sqrt{(p_a \cdot p_b)^2 - (M_a M_b c^2)^2}}\right),$$

(32)

wherein the momenta $p_a = M_a v_a$ and $p_b = M_b v_b$ have been introduced.

Suppose the production of a particle anti-particle pair each of mass $M$. Associated with such a mass is a weak coupling strength

$$\alpha_F(M) = \left(\frac{\sqrt{2} G_F M^2}{4\pi \hbar c}\right)$$

(33)

such that

$$B_{\text{pair}}(s) = -\frac{2}{\hbar} \Im m S_{\text{pair}} = 2\pi \alpha_F(M) \left(\frac{M^2}{\sqrt{s(s - 4M^2)}}\right).$$

(34)

The resulting Higgs induced amplification factor is determined by

$$A_{\text{pair}}(s) = \frac{B_{\text{pair}}(s)}{1 - \exp(-B_{\text{pair}}(s))}.$$  

(35)

In this regard one may consider the reactions

$$e^+ + e^- \rightarrow W^+ + W^-,$$

$$e^+ + e^- \rightarrow Z + \bar{Z}.$$  

(36)

The amplification coupling strengths for the above reactions are, respectively,

$$2\pi \alpha_F(M_W) \approx 0.0532,$$

$$2\pi \alpha_F(M_Z) \approx 0.0687.$$  

(37)

For these massive particles the Higgs boson exchange induced amplification is somewhat larger than the photon exchange amplification which contributes in the $W^+ W^-$ production case.
Figure 3: The exchange of a Higgs boson between two particles gives rise to the attractive potential $U_{ab} = -\left(\frac{\sqrt{2}}{4\pi}\right)(G_F M_a M_b/r) \exp(-M_H r/\bar{h} c)$. The action $S_{ab}$ of the exchange is examined in detail.

Figure 4: Shown is the amplification factor $A_{(Z\text{-pair})}(s)$ of the $Z$ pair production reaction $e^+ + e^- \rightarrow Z + \bar{Z}$ due to a Higgs boson exchange.
Figure 5: Shown is the amplification factor $A_{W^+W^-}(s)$ of the $W^+W^-$ pair production reaction $e^+ + e^- \rightarrow W^+ + W^-$ due to both Higgs boson exchange and photon exchange. Both Higgs exchange and photon exchange contribute to the amplification factor yielding a somewhat larger effect than for the case of $e^+ + e^- \rightarrow Z + \bar{Z}$.

In Figure 4, we exhibit the amplification factor for $Z\bar{Z}$ production due to the exchange potential of the Higgs boson; It is

$$A_{(Z-\text{pair})}(s) = \frac{\Gamma(e^+ + e^- \rightarrow Z + \bar{Z})}{\Gamma(0)(e^+ + e^- \rightarrow Z + Z)} ,$$

$$A_{(Z-\text{pair})}(s) = \frac{B_{(Z-\text{pair})}(s)}{1 - \exp[-B_{(Z-\text{pair})}(s)]} ,$$

$$B_{(Z-\text{pair})}(s) = 2\pi\alpha_F(M_Z) \left( \frac{M_Z^2}{\sqrt{s(s-4M_Z^2)}} \right) , \tag{38}$$

The case of $W^+W^-$ production, the enhancement is due to both photon exchange (which surely exists) and Higgs boson exchange (which may exist). The complete answer for $W^+W^-$ amplified production reads

$$A_{(W-\text{pair})}(s) = \frac{\Gamma(e^+ + e^- \rightarrow W^+ + W^-)}{\Gamma(0)(e^+ + e^- \rightarrow W^+ + W^-)} ,$$

$$A_{(W-\text{pair})}(s) = \frac{B_{(W-\text{pair})}(s)}{1 - \exp[-B_{(W-\text{pair})}(s)]} ,$$

$$B_{(W-\text{pair})}(s) = 2\pi\alpha_F(M_W) \left( \frac{M_W^2}{\sqrt{s(s-4M_W^2)}} \right) + \pi\alpha \left( \frac{\sqrt{s} - \sqrt{s-4M_W^2}}{\sqrt{(s-4M_W^2)}} \right) , \tag{39}$$

which is plotted in Figure 5. The amplification factor for $W^+W^-$ production is
more pronounced than the amplification factor for $Z\bar{Z}$ production since photon exchange contributes to the former process but not contribute to the later.

5 Conclusions

The threshold amplification and/or suppression factors familiar from the theory of final state interactions has been applied in this work in a higher energy regime. In particular we have considered final state interactions involving the Higgs boson under the supposition that it exists. Even below the threshold for the physically real Higgs particle production, the Higgs field can act as a messenger field entering into enhanced production rates for pairs of heavy particles such as $Z\bar{Z}$, $W^+W^-$ or $t\bar{t}$ pairs. The sharp peaks shown in the plots of enhancement factors will be considerably “rounded” due to (i) particle lifetime effects, (ii) radiative corrections and (iii) energy resolution factors from the energy distributions in incoming beams. Nevertheless, even if a sharp peak no longer appears, the physically “smoothed” threshold regime will be shifted. Since the production amplification is above the threshold mass squared, i.e. $s > s_0 \equiv 4M^2$, it follows that the threshold transition region will occur at a mass slightly higher that the threshold to be expected if the amplification were ignored. For example, experimental reaction threshold mass shifts of order

$$e^+ + e^- \rightarrow Z + \bar{Z} \quad \Rightarrow \quad \Delta M_Z \approx M_Z\alpha_F(M_Z),$$

$$e^+ + e^- \rightarrow W^+ + W^- \quad \Rightarrow \quad \Delta M_W \approx M_W[\alpha_F(M_W) + 0.5\alpha], \quad (40)$$

would not be unreasonable and might constitute an unexpected probe of the Higgs field existence.

A Quark Potentials

The one gluon exchange potential between a quark and anti-quark has been approximated as

$$V_{Glu} = \left(\frac{\hbar c\alpha_s}{r}\right)(\mathbf{T}_1 \cdot \mathbf{T}_2) = \int \left(\frac{4\pi\hbar c\alpha_s}{|k|^2}\right) e^{ik \cdot r} \frac{d^3k}{(2\pi)^3}(\mathbf{T}_1 \cdot \mathbf{T}_2). \quad (41)$$

In reality, the strong interaction coupling strength itself depends on $|k|^2$ so that the Coulomb-like potential is modified to read

$$\tilde{V}_{Glu}(r) = 4\pi\hbar c\int \left(\frac{\alpha_s(|k|^2)}{|k|^2}\right) e^{ik \cdot r} \frac{d^3k}{(2\pi)^3}(\mathbf{T}_1 \cdot \mathbf{T}_2). \quad (42)$$

More simply,

$$\tilde{V}_{Glu}(r) = \left[\frac{\hbar c(\mathbf{T}_1 \cdot \mathbf{T}_2)}{r}\right] \chi(r),$$

$$\chi(r) = \frac{2}{\pi} \int_0^\infty \frac{\alpha_s(k^2) \sin(kr) dk}{k}. \quad (43)$$
If $\alpha_s(k^2)$ were a constant, then Eqs. (43) would reduce to Eq. (3). However the Coulomb-like law from gluon exchange breaks down at large distances.

To see what happens as $r \to \infty$, one may presume a finite limit in the form

$$\lim_{k^2 \to 0^+} \{hck^2\alpha_s(k^2)\} = 2\sigma,$$

and differentiate Eq. (43) twice with respect to $r$; i.e.

$$\chi''(r) = -\frac{2}{\pi} \int_0^{\infty} k\alpha_s(k^2)\sin(kr)dk,$$

$$\lim_{r \to \infty} \chi''(r) = -\frac{2\sigma}{\hbar c}.$$ (45)

What is called a “QCD motivated potential” results from the assertion that $\chi''(r) = -(2\sigma/\hbar c)$ for all of the important distance scales. If this is indeed the case, then

$$\tilde{V}_{\text{Glu}}(r) = T_1 \cdot T_2 \left\{ \frac{\hbar \alpha_s}{r} - \frac{\sigma}{r} \right\},$$ (46)

wherein the long range linear part of the potential describes the intrinsic tension $\sigma$ in a QCD string. In detail, for the quark anti-quark potential

$$U_{\bar{q}q}(r) = -\frac{4}{3} \left( \frac{\hbar \alpha_s}{r} \right) + \tau_{\bar{q}q}r \quad \text{where} \quad \tau_{\bar{q}q} = \frac{4\sigma}{3},$$ (47)

and for the quark-quark potential

$$U_{qq}(r) = -\frac{2}{3} \left( \frac{\hbar \alpha_s}{r} \right) + \tau_{qq}r \quad \text{where} \quad \tau_{qq} = \frac{2\sigma}{3}.$$ (48)

References

[1] M. Strassler and M. Peskin, Phys. Rev. D43, 1500(1991).
[2] K. Melnikov and O. Yakovlev, Phys. Lett. B324, 217(1994).
[3] V. Fadin, V. Khoze and A. Martin, Phys. Rev. D49, 2247(1994).
[4] Y. Sumino, Acta Phys. Polonica B25, 1837(1994).
[5] V. Khoze and W. Stirling, Phys. Lett. B356, 373(1995).
[6] K. Melnikov and O. Yakovlev, Nuc. Phys. B471, 90(1996).
[7] V. Khoze and W. Sjöstrand, Z. Phys. C70, 625(1996).
[8] W. Beenakker, A. Chapovsky and F. Berends, arXiv:hep-ph/9706339 and arXiv:hep-ph/9707326
[9] “Top quark physics: Future Measurements”, by R. Frey et al. arXiv:ph/9704243.

[10] R. Harlander, M. Jezábek, J. Kühn and M. Peter, Z. Phys. C73, 477(1997).

[11] M. Peter and Y. Sumino. arXiv:hep-ph/9708223

[12] N. Fabiano and G. Pancheri, Euro Phys. J. C25, 421(2002).

[13] R. N. Lee, A. I. Milstein and V. M. Strakhovenko. arXiv:hep-ph/0307388