Dynamics of acoustic waves in a liquid with solid particles and gas bubbles

D A Gubaidullin, D D Gubaidullina, Yu V Fedorov

Institute of Mechanics and Engineering - Subdivision of the Federal State Budgetary Institution of Science "Kazan Scientific Center of the Russian Academy of Sciences", 2/31 Lobachevsky str., Kazan 420111, Russia
Kazan (Volga Region) Federal University, 18 Kremlyovskaya str., Kazan 420008, Russia
E-mail: gubaidullin@imm.knc.ru

Abstract. A mathematical model that determines the propagation of acoustic waves in a mixture of a liquid with gas bubbles and solid particles is presented. The system of differential equations is written, the dispersion equation is obtained. Low-frequency and high-frequency asymptotics of the sound speed in the specified mixture are found and illustrated. The influence of solid particles and gas bubbles on the dispersion and dissipation of acoustic waves is indicated. For a mixture of liquid and solid particles, a comparison of the speed of sound with known experimental data is presented.

1. Introduction

It is known that the presence of impurities of a gas bubble or of solid or liquid particles in a liquid has a significant effect on its acoustic properties. By this time there is a significant amount of work on the acoustics of a bubbly liquid. Most of them are reflected in the reviews [1, 2]. The works [3] – [9] can be attributed to the first works on the experimental and theoretical study of the propagation of sound in a liquid with solid or liquid particles. Analytical expressions are obtained for the attenuation coefficient based on the theory of scattering of waves from the surface of the inclusions [6], [7], [9], and the speed of sound in the mixture was determined from known values of the density and compressibility of the medium [5], [8].

Theories of propagation of acoustic waves in suspensions and emulsions based on continuum mechanics are presented in [10] – [14].

The reflection of low-amplitude pressure waves at the interface between a liquid and a mixture of a liquid with solid particles and gas bubbles is experimentally investigated [15]. An experimental study of the reflection of shock waves of moderate intensity pressure from a solid wall in a three-phase mixture was carried out in [16]. The calculated dependencies for the reflection coefficient are obtained without taking into account dispersion and dissipation in a three-phase medium. A comparison with experimental data is presented.

Problems of acoustics of bubble liquids in a particular setting are considered in [17] - [33].

In this paper, based on the mathematical model [34], some results on the dispersion and dissipation of acoustic waves in a mixture of a liquid with gas bubbles and solid particles are presented.
2. Some results.

Fig. 1 shows the dependencies of the phase velocity and the attenuation coefficient on the frequency of disturbances for a mixture of water with glass particles and air bubbles at various volumetric gas contents. The presence of gas bubbles in the mixture leads to the appearance of a local minimum for the curves of the phase velocity and a local maximum for the curves of the attenuation coefficient in the vicinity of the resonant frequency associated with the natural oscillations of the bubbles.

![Figure 1](image_url)

**Figure 1.** Dependencies of the phase velocity and attenuation coefficient on the frequency of disturbances for a mixture of water with air bubbles and glass particles for different values of the volume content of air bubbles: 1 – 4 – \( \alpha_2 = 0, 10^{-6}, 2 \cdot 10^{-6}, 5 \cdot 10^{-6} \), \( \alpha_s = 0.5 \), \( r_s = 10^{-4} \) m, \( r_g = 10^{-3} \) m.

The resonant frequency is determined by the formula

\[ f_0 = \frac{1}{2\pi r_g} \sqrt{\frac{3\gamma p_0}{\rho_1}} \]

For the considered mixture we find \( f_0 \approx 3261 \) Hz. An increase in the volumetric gas content leads to a decrease in the equilibrium sound speed and an increase in the attenuation coefficient. Note that the addition of gas bubbles to a liquid has a noticeable effect not only on the dynamics of acoustic waves, but also on the nature of the flow [18]. Summarizing the results, we arrive at the fact that one of the methods for determining the presence of solid particles in a bubble mixture is the following circumstance. If the frozen sound speed is greater than the sound speed in the carrier phase, then with high probability we can expect the presence of solid particles in the bubble mixture.

Fig. 2 presents a comparison of the theoretical dependencies of the phase velocity on the volume content of solid particles with experimental data for a mixture of water with kaolin particles (data 3, 4) and glass particles (data 5). The mixture pressure is \( p_0 = 0.1 \) MPa, the density of kaolin is \( \rho_s = 2600 \) kg/m\(^3\), the speed of sound is \( C_s = 4100 \) m/s. Since the data 3, 4 were obtained at frequencies corresponding to the equilibrium speed of sound, therefore, theoretical curve 1 is non-monotonic, which is well confirmed by experimental data. Data 5 was obtained at a frequency where the frozen speed of sound is realized, therefore, a monotonously increasing curve 2 is observed. Thus, the proposed continual model describes the experimental data quite well not only at low but also high volume contents of solid particles in a liquid.

Fig. 3 shows the dependencies of phase velocity on disturbance frequency for a mixture of water with glass particles at different particle radii. The mixture pressure is \( p_0 = 0.1 \) MPa,
temperature $T_0 = 293$ K, $\rho_1 = 1000$ kg/m$^3$, $\rho_{2s} = 2800$ kg/m$^3$, $\rho_2 = 0.9$ kg/m$^3$, $C_1 = 1500$ m/s, $C_{2s} = 5200$ m/s. Note that these curves are qualitatively consistent with the phase velocity curves for a gas mixture with solid particles: 1) at low and high disturbance frequencies, the phase velocity does not change and takes certain values $C_{el}$ and $C_{eh}$, and which are analogs of the equilibrium and frozen sound speed; 2) with an increase in the frequency of disturbances, an increase in the phase velocity from the value $C_{el}$ to the value $C_{eh}$ is observed (velocity inflection); 3) with a decrease in the particle size, the bend of the phase velocity is shifted towards an increase in the frequency of disturbances. The difference is that for a mixture of gas with solid particles the equilibrium value $C_{el}$ is always less than the speed of sound in the carrier phase $C_1$, since $C_{eh} < C_1$. For a mixture of liquid with solid particles, the inequality $C_{eh} > C_1$ is satisfied, while the equilibrium speed of sound can be both smaller and larger than $C_1$, depending on the volume content of the particles.

Conclusions

It is shown that the presence of gas bubbles in a mixture of liquid with solid particles leads to appearance of a minimum in the dependence of the phase velocity and a maximum of the dependencies of the attenuation coefficient on the perturbation frequency in the neighborhood of the bubble resonance frequency. It is found that if the velocity of high-frequency perturbations is higher than the speed of sound in the pure liquid, this testifies on the presence of an admixture of solid particles not only in the pure liquid but also in the bubbly liquid.

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