CAVITATION EROSION MECHANISM: NUMERICAL STUDY OF THE INTERACTION BETWEEN PRESSURE WAVES AND VAPOR BUBBLES

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Abstract. Investigations on vapor bubble collapses dynamic were carried out by three-dimension simulations with the software Prototype Homogène Code_Saturne. The code solves, by applying a compressible homogeneous approach, the Euler’s equations coupled with transport equations (for the volume, mass and energy fractions) that model the phase changes and the thermodynamic effects. To close the system, the Stiffened Gas EOS (equation of state) was applied to link the pressure and the temperature to the internal energy and the density. Different mesh types and computational domains were tested to study four configurations in water: a) the vapor bubble collapse in free-field case; b) the vapor bubble collapse near a solid wall; c) the collapse of a vapor bubble in free-field case and impacted by an external pressure wave; d) the collapse of a vapor bubble placed near a solid wall and impacted by an external pressure wave. Pressure waves generated during these bubble implosions were studied and characteristic parameters (such as wave passage time and amplitude of the pressure peak applied at the solid wall) were calculated under different hydrodynamic conditions. Simulations led to some expected phenomena such as: the bubble asymmetrical shape evolution when a rigid wall is present; observation of toroidal vapor cavities; generation of high amplitude pressure waves during bubble collapses and rebounds. Original results obtained concern mainly the study of the interaction between external pressure waves and collapsing vapor bubbles. Analyses on the influence of the incoming external pressure wave amplitude on the bubble collapse time and on the pressure peak reached on the wall are also presented in the paper. The numerical study showed an amplification of the collapse pressure by the incoming pressure wave. This interaction mechanism between pressure waves and vapor structures leading to pressure amplitude amplification could be responsible for the material damages due to cavitation.

1. Introduction

The analysis and prediction of cavitation erosion mechanisms remain a major challenge for researchers and industrial partners to improve the design of hydraulic machineries and to develop efficient maintenance processes.

The University of Grenoble (France) and EDF (Electricité de France) have been working together for several years to characterize cavitation erosion phenomena, as well as to develop and validate prediction models [1-4]. In previous works [1], it was found that high pressure waves emitted during the collapses of vapor structures were the main factor contributing to cavitation damage. The emission of
the pressure waves could be generated either by bubble or vortex collapses (as observed by [5-10]), as well as by microjet formation [9-12]. According to [1], to damage the nearby solid materials, the impacting pressure wave should reach high amplitudes (~GPa) and very short duration (ns to μs). Those numerical results have been supported by experimental works proposed by [13-14].

In spite of the complex physical phenomena observed experimentally during cavitation structure collapses, theoretical and numerical studies are frequently based on spherically collapsing bubbles [15-17]. Nevertheless, bubbles may deform under the effect of nearby surfaces, gravity, or passing pressure waves.

Various numerical studies have analyzed the bubble dynamics and emitted pressure waves [18-19]. Some articles present simulations of bubble collapses under passing pressure waves [20-22], subjected to inertial forces [23] or free-surface [24]. The major part of those numerical studies considers 2D approaches, non-condensable gas inside the bubbles (generally air), neglects phase changing and does not take into account cavitation phenomena.

Then, from bibliographic study, it appears that more numerical developments are needed to obtain reliable 3D simulations of the pressure waves emitted by collapsing vapor bubbles and to evaluate the effect of external passing pressure waves on these collapses. In this context, the present work proposes an original physical and numerical modeling to try to deepen investigations and to contribute to improve prediction models for cavitation intensity [25-26].

The article is structured as follows: the section 2 describes the numerical tool and physical modeling applied in the study. The considered configurations, numerical domains and meshing are presented in section 3. Section 4 describes the numerical parameters used. Results are illustrated and discussed in sections 5 (for the vapor bubble collapse in free-field case), 6 (for the vapor bubble collapse near a solid wall) and 7 (for the vapor bubble collapse near a solid wall and impacted by an external pressure wave). Finally, the investigation conclusions are presented in section 8.

Results concerning the collapse of a vapor bubble in free-field case and impacted by an external pressure wave are not addressed in this article for sake of brevity. Detailed study of all configurations is described in [27].

2. Description of the numerical code

The simulations have been carried out using the software Prototype Homogène Code_Saturne, which applies an inviscid compressible homogeneous approach [28-29] to solve Euler’s equations coupled with transport equations (for the volume, mass and energy fractions) for the modelization of the phase changes between liquid water and its vapor, and taking into account the thermodynamic effects. The numerical code has been verified and some validation results are discussed in [29].

The simulations presented in this article did not take into account surface tension either non-condensable gasses.

2.1. System of equations

In the following, a subscript \(v\) will denote a vapor quantity, and a subscript \(l\) a liquid one. The quantities describing the mixture of the two phases are: the specific mass \(\rho\), the velocity \(u\) and the specific internal energy \(e\). Three fractions define the way the two phases are mixed in terms of: the volume through the vapor volumic fraction \(\alpha_v=\alpha\), the mass through the vapor mass fraction \(\gamma_v\), and the energy through the vapor energy-fraction \(z_v\) (equations 1). In fact these fractions allow to express the phasic quantities in terms of the mixture quantities. The multiphase fluid is considered as a homogeneous mixture characterized by a specific mass \(\rho\) varying between vapor one \(\rho_v\) and liquid one \(\rho_l\), as a function of the vapor volume fraction \(\alpha\):

\[
\rho = \alpha \rho_v + (1 - \alpha) \rho_l \quad \gamma_v = \alpha \rho_v / \rho \quad z_v = \gamma_v e_v / e
\]  

(1)

No equilibrium assumption is made except for the kinematic equilibrium \((u=\bar{u}=u_v)\). The specific total energy is defined by:

\[
E = e + \frac{u^2}{2}
\]  

(2)
The solved system of equations associating Euler’s equations and transport equations for the vapor volume, mass and energy fractions is given by:

\[ \frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0, \]  
\[ \frac{\partial \rho u}{\partial t} + \frac{\partial (\rho u^2 + p)}{\partial x} = 0, \]  
\[ \frac{\partial \rho E}{\partial t} + \frac{\partial (u(\rho E + p) + \rho u^2)}{\partial x} = 0, \]  
\[ \frac{\partial Y}{\partial t} + u \frac{\partial Y}{\partial x} = \Gamma_Y \]  

where \( p = p(Y, \rho, e) \) is the mixture pressure, and \( Y = (\alpha, y_v, z_v)^T \), with \( 0 \leq Y' \leq 1 \) for \( i = 1, 2 \) or 3. From the Gibbs’ relations, the mixture pressure can be expressed by [28]:

\[ p = \frac{\alpha}{\tau_p} p_v + \frac{1-\alpha}{\tau_l} p_l \]  

The three source terms in equation 6 (one for each fraction) rule the thermodynamic exchange between the phases and allow the system to return to the thermodynamic equilibrium. Hence they must be chosen to comply with the second principle of thermodynamics.

The source terms are given by:

\[ \Gamma_Y^i = \frac{Y_{eq}^i (1/\rho, e) - Y^i}{\lambda} \]  

where \( \lambda \) is a characteristic time-scale, taken similar for the three components. \( Y_{eq}^i \) defines the equilibrium fractions. This equilibrium state \( Y_{eq} \) maximizes the specific entropy of the mixture \( s \) for a given specific internal energy \( e \) and a given specific volume \( \tau = 1/\rho \):

\[ s(Y_{eq}(\tau, e), \tau, e) = \max_{0 \leq Y \leq 1} (s(Y, \tau, e)) \]  

### 2.2. Stiffened Gas EOS

To close the system, an equation of state is necessary to link the pressure and the temperature to the internal energy and the density. In the present study, we have applied the Stiffened Gas EOS for each pure phase, that is:

\[ s_k(\tau_k, e_k) = C_{v,k} \ln \left( \frac{e_k - Q_k - \Pi_k \tau_k}{\Pi_k} \right)^{\gamma_k-1} + s_k^0 \]  

where \( C_{v,k} \) is the heat capacity, \( Q_k \) is a reference value of the enthalpy \( h_k = e_k + P_k \tau_k \), the parameter \( -\Pi_k \) corresponds to the minimal pressure, \( \gamma_k \) is the adiabatic coefficient and \( s_k^0 \) is the reference entropy. This phasic entropy allows to define the pressure and temperature laws through the phasic Gibbs’ relations:

\[ P_k(\tau_k, e_k) = \frac{e_k-Q_k}{\tau_k} (\gamma_k - 1) - \Pi_k \gamma_k \]  
\[ T_k(\tau_k, e_k) = \frac{e_k-Q_k-\Pi_k \tau_k}{C_{v,k}} \]  

In the present study, we considered the following parameters, corresponding to a reference pressure of 2340 Pa and reference temperature of 20°C:

\[ C_{v,v} = 8091.4 \ J K^{-1}, \ \gamma_v = 1.07, \ \Pi_v = 0 \ Pa \text{ and } C_{v,l} = 9.41 \ J K^{-1}, \ \eta_l = 28.8, \ \Pi_l = 7.65 \ 10^7 \ Pa. \]

The validity zone of this equation of state is between 0 Pa and 0.5 MPa.

### 2.3. Numerical scheme

The system of equations has a hyperbolic nature and is resolved using a finite volume (FV) scheme. The overall numerical scheme is a fractional step method, as proposed in [28]: the convection part of the
system is first solved using a Godunov type explicit scheme, and the source terms are then integrated. A VFRoe-ncv scheme is used to compute the numerical fluxes at the interface between two cells of the mesh. Moreover, a partial WFRoe scheme is performed to improve the prediction of the speed of the contact wave in the linearized problem at the interface between two cells. More details are given in [29].

3. Studied configurations, numerical domains and meshing

3.1. Configurations

Figure 1 illustrates the four configurations considered in the present study: a) the vapor bubble collapse in free-field case; b) the vapor bubble collapse near a solid wall; c) the collapse of a vapor bubble in free-field case and impacted by an external pressure wave; d) the collapse of a vapor bubble placed near a solid wall and impacted by an external pressure wave.

Figure 1. Studied configurations. Detailed results are presented in [27].

3.2. Meshes

Depending on the considered configuration, different mesh strategies have been applied (figure 2): a) for free-field case calculations, a spherical approach has been initially applied (figure 2a). Tests of the mesh influence carried out with this approach have been used as reference for the 3D calculations done with the “complete” domain (figure 2b), “1/8” domain (figure 2c) and “1/4” domain (figure 2d). b) based on the results obtained for free-field case (presented in section 5), the other configurations have been calculated with the “1/4” domain (figure 2d).

Figure 2. Mesh types: (a) spherical domain (cone angle of 2°), (b) 3D “complete”, (c) “1/8” and (d) “1/4” domains.

Hexahedral meshes are done. Due to storage data constraint, the number of cells is limited to 50M elements. The meshes are characterized by the parameters represented in figure 3.

3.3. Boundary conditions

The numerical treatment of the boundary conditions is based on the use of the characteristic relations of the Euler’s equations. Two kinds of boundary conditions have been applied:
a) Non-reflecting boundary conditions, to avoid the reflections of the waves at the fluid domain outlet
b) Conditions of symmetry (“wall”): the normal velocity is equal to zero.
More information is given in [27].

Figure 3. Mesh characteristic parameters: BUL (number of cells inside of the bubble initial radius); LIM (number of cells corresponding to the refined mesh zone); TOT (total number of cells in a direction).

4. Calculation parameters
This section presents physical and numerical parameters applied in the study.

4.1. Initial conditions
The bubble is initially filled of pure vapor and has a radius of \( R_0 = 750 \, \mu\text{m} \).
The center of the bubble is located at \((0,0,0)\) coordinates.
Initial pressure inside of the bubble is \( p_b = p_{\text{sat}}(20\, ^\circ\text{C}) = 2340 \, \text{Pa} \).
Water temperature is \( T_l = 20\, ^\circ\text{C} \).
Initial pressure in the liquid outside of the bubble is \( p_l = 10^5 \, \text{Pa} = 1 \, \text{bar} \).
Inside of the bubble: \( \alpha = y_v = z_v = 1 \)
Outside of the bubble: \( \alpha = y_v = z_v = 0 \)
For calculations taking into account a solid wall, the initial distance between the bubble center and the wall has been fixed as \( d = 1.4 R_0 = 1050 \, \mu\text{m} \), i.e. \( \gamma = d/R_0 = 1.4 \).
These initial conditions are similar to ones applied by [30].

4.2. Numerical parameters
The applied numerical scheme is explicit and the maximum value for the Courant-Friedrichs-Levy condition is \( \text{CFL}_{\text{max}} = 0.45 \).
The physical simulation duration corresponds to \( \tau^* = 200\, \mu\text{s} \).
The maximum length extension of the domain is \( L_D = 0.3\, \text{m} \) (equivalent to 400 \( R_0 \)). By supposing the wave celerity in liquid water \( C_L \) about 1500m/s, one obtains \( C_L \tau^* = L_D \). It means that the mesh extent is chosen so as a pressure wave, generated at the mesh center at the initial time, reaches the mesh extremity at the end of the simulation time, avoiding wave reflections at the domain outer boundaries.
In the literature, [31] applied \( L_D = 40 \, R_0 \) in [32] \( L_D = 100 \, R_0 \).
It is important to note that sensitivity analyses have been carried out concerning the mesh and the numerical parameters. This paper presents the most relevant results obtained after carrying out the numerical tests. The detailed study is described in [27].

5. Numerical results for the free-field case
The first tests performed have applied the spherical approach (figure 3a) with four different meshes: BUL = 250, 500, 750, 1000 cells. These tests had as objective to find the better compromise between the accuracy of the results and the calculation time. The convergence was reached for BUL = 750 cells.
Numerical data (\( \alpha, \rho, p, T \) and \( e \)) have been recorded at each 100 iterations, in 11 numerical sensors located as indicated in figure 4c. The first sensor is located at the bubble center. The other ones are placed at a distance of \( R_0/5 \) from each other.
Figures 4 illustrate also some results obtained with BUL = 750 cells. In figure 4a, comparisons are done with the theoretical Rayleigh’s model [33] (which considers, unlike the present numerical study, incompressible fluid, instantaneous condensation and tension surface). One can observe that the bubble collapse time is slightly overestimated by the simulations (Table 1). It could be caused by the different
hypotheses considered in theoretical and numerical approaches. Moreover, the amplitudes of the pressure wave reached during simulations are very high and go beyond the validity limit of the EOS considered (figures 4d and 4e). Another approach based on the Thermodynamic Table for Water and Steam IAPWS-97 to replace the Stiffened Gas EOS is being tested currently and results will be presented in a future article.

**Figure 4.** Results for vapor bubble collapse in free-field case. a) time evolution of the bubble radius: comparison with Rayleigh’s model; b) pressure field calculated at a given time: the bubble radius (indicated in the figure by the black arrow) corresponds to $\alpha = 0.5$; c) location of the numerical sensors; d) time evolution of the pressure for each sensor: visualization of the pressure wave emitted during the vapor bubble collapse; e) zoom of the figure 4d; f) evaluation of the celerity of the pressure wave; g) evolution of the maximum amplitude of the pressure wave as a function of the distance “r” from the bubble center.

From figures 4d and 4e, one can analyze the characteristic of the pressure wave emitted during the bubble collapse. The maximum amplitude (observed near the bubble center) is about 300 MPa. This value is probably underestimated because only 1/100 iterations have been recorded. The celerity of the
pressure wave is 1557 m/s (figure 4f) and the maximum wave amplitude decreases as $p_{\text{max}} \sim 4250/r$ (figure 4g). These numerical results agree well with experimental studies proposed by [34-35].

Results for the bubble collapse time $\tau_{\text{bubble}}$ obtained by applying the four calculation domains illustrated in figure 2 are summarized in table 1. Calculations with “1/8” mesh have presented some defaults of symmetry. The “complete” mesh was not enough refined to capture all physical phenomena. The best results have been obtained with the “1/4” mesh. This mesh has given results similar to spherical approach and has been applied for the other configurations presented here below.

| Mesh       | BUL [-] | LIM [-] | TOT [-] | Collapse time [$\mu$s] |
|------------|---------|---------|---------|------------------------|
| Spherical  | 1 000   | 2 000   | 4000    | 83                     |
| 1/8        | 180     | 230     | 372     | 80                     |
| 1/4        | 120     | 170     | 295     | 83                     |
| “complete” | 60      | 75      | 190     | 93                     |

Table 1. Bubble collapse time computed for the different meshing approaches considered in figure 2. For comparisons, Rayleigh’s collapse time is 69 $\mu$s.

6. Numerical results for the vapor bubble collapse near a solid wall

Some results obtained for this configuration are illustrated in figure 5.

Figure 5. Temporal evolution of the pressure, of the $\alpha = 0.5$ iso-surface (bubble shape) and of the velocity vectors (scale 30) before the bubble collapse (figures 5a to 5f) and after the rebound (figures 5g to 5i). Cut planes correspond to $z=0$ and $y = -1 050 \mu$m (solid wall).

The bubble collapse time is $\tau_{\text{bubble}} \sim 93.5 \mu$s, higher than one observed in the free-field case. The bubble center moves towards the solid wall during the collapse and becomes non-symmetrical. The first
pressure wave emitted propagates in the fluid, reaches the wall, where the maximum pressure peak $P_{\text{max, wall}}$ is $\sim 4.7$ MPa. Other pressure waves are generated during the bubble rebounds, as observed experimentally by [9]. The micro-jet is also detected by simulations, but not the counter-jet (observed by [36] for $1.1 < \gamma < 2.6$).

7. Numerical results for the vapor bubble collapse near a solid wall and impacted by an external pressure wave

Pressure waves emitted by bubbles collapsing under the effect of a passing pressure wave have been shown to be sensitive to the latter’s timing and strength [37]. Moreover, in real cavitating flows, bubble clouds can be frequently observed and seem to be closely related to the severe cavitation damage [38-40]. In a bubble cloud, the interaction between bubbles leads to a consecutive sequence of collapses: pressure wave emitted during the collapse of a bubble interacts with neighboring bubbles, amplifies their collapses, and then, increases the erosive power of the flow. Some studies [17,41-43] pointed out amplification phenomena of the emitted pressure waves during the bubble collapses in cascade, but more investigation are needed to better understand these phenomena. In this context, we carried out some numerical simulations to study the collapse of vapor bubbles exposed to a passing pressure wave (figure 6). The passing pressure wave (with an initial amplitude $P_{\text{wave}}$) is modelled by the generation of an instantaneous pressure peak (during a numerical time step) along a mesh length, located at a distance $L_{\text{wave}}$ of the bubble center. The pressure wave is generated at a time $t_{\text{wave}}$ after the beginning of the simulation. Table 2 summarizes the tests performed. Further investigations are needed to study the influence of other characteristic parameters of the incoming pressure wave, as for example the wave passage time $\delta_{\text{wave}}$ [17].

![Figure 6](image_url)  
**Figure 6.** Applied configuration to study the interaction between passing pressure wave and the bubble collapses. $R_0=750\,\text{mm}$, $\gamma=1.4$, $L_{\text{wave}}=20\,\text{mm}$.

| $t_{\text{wave}}$ [\(\mu\text{s}\)] | $P_{\text{wave}}$ [MPa] | $\tau_{\text{bubble}}$ [$\mu\text{s}$] | $P_{\text{max, wall}}$ [MPa] |
|----------------------------------|-----------------|-----------------|-----------------|
| No wave                          | -               | 93              | 4.7             |
| 20                               | 5               | 63              | 12.1            |
| 40                               | 5               | 74              | 13.2            |
| 60                               | 5               | 84              | 13.1            |

| $t_{\text{wave}}$ [\(\mu\text{s}\)] | $P_{\text{wave}}$ [MPa] | $\tau_{\text{bubble}}$ [$\mu\text{s}$] | $P_{\text{max, wall}}$ [MPa] |
|----------------------------------|-----------------|-----------------|-----------------|
| No wave                          | -               | 93              | 4.7             |
| 20                               | 5               | 63              | 12.1            |
| 20                               | 10              | 52              | 24.7            |
| 20                               | 20              | 44              | 50.7            |
| 20                               | 50              | 38              | 82.6            |

**Table 2.** Configurations considered in the study. Results of the collapse time and $P_{\text{max, wall}}$.

Figure 7 illustrates time evolution of the bubble shape during the collapses. One can observe the generation of the microjet and of a toroidal vapor cavity, even if the geometry lost some angular symmetry due to still insufficient mesh refinement. According to the performed numerical simulations, the collapse time decreases when the $P_{\text{wave}}$ increases and for wave generation times $t_{\text{wave}}$ smaller than the collapse time without incoming pressure wave (93\(\mu\text{s}\)). For all the considered tests, one can observe the amplification of the pressure peak applied on the solid wall due to the interaction between the incoming wave and the vapor bubble. In certain cases, the pressure peak at the solid wall was amplified by \(\sim 10\), or even by \(\sim 17\). Nevertheless, the pressure amplitudes evaluated at the wall (less than 100 MPa) are not enough to damage metallic materials and quantitative analyzes have to be enhanced by applying more fine meshes (which leads to very expensive calculation costs) or by improving physical modelling, especially by using more realistic equations of state.
8. Conclusions
In order to deepen the analyses of the physical mechanisms responsible for cavitation aggressiveness, an original numerical approach (taking into account phase change between liquid and vapor) has been developed to study the collapse of vapor bubbles under different configurations. In particular, the interaction between passing pressure waves and vapor bubbles located near a solid wall has been analyzed. For all considered patterns, the simulations have pointed out the amplification of the pressure peak applied on the solid wall due to the interaction between incoming pressure wave and vapor bubble. Qualitative numerical results are in good agreement with experimental investigations found in the literature. To enhance quantitative analyses, several improvements can be considered, as for example, the employment of more physical EOS or the development of bi-fluid modelizations allowing to take into account tension surface among others.

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