1 Introduction

The standard tabulation techniques for logic programming presuppose fixed order of computation: top-down and left-to-right in OLDT (Tamaki & Sato, 1986) and bottom-up and left-to-right in magic set (Beeri & Ramakrishnan, 1987). This makes it hard to deal with problems in which the distribution of input information is diverse and accordingly diverse flow of information is required. Consider natural language understanding for instance. In one case, syntactic information may be missing due to unknown words but semantic information may be abundant thanks to extralinguistic contexts, whereas in another case the situation may be the opposite. These cases should be treated by drastically different processing orders for the sake of efficiency. Integrated treatment of natural language understanding and production also calls for flexible control sensitive to various computational contexts.

Some data-driven control should be introduced in order to deal with such diverse contexts. A concern of this paper is hence tabulation in data-driven transformation of constraints. Another concern is optimization of such computation. Standard optimization methods in natural language processing and logic programming include accessibility check and last-call optimization. The former is employed in left-corner parsing and semantic-head-driven generation (Shieber et al., 1990), among others. The latter is not only used in Earley parsing and left-corner parsing, but also is a major factor in WAM (Warren, 1983; Aït-Kaci, 1991). In what follows we will address a data-driven method of constraint transformation with a sort of compilation which subsumes accessibility check and last-call optimization.

2 Dependency Reduction

A Horn-clause program is regarded as a forest of program trees. A program tree is a candidate for a proof tree. Namely, in a program tree, each node is (an instances of) a clause, the root node is a top clause (the program may have several top clauses), and each negative (body) literal is linked with a positive (head) literal carrying the same predicate. A program tree is a proof tree iff it contains no contradiction among unified terms. For instance, Figure 1 is a program tree, but not a proof tree, of the following program.

* 1st Workshop on 'Tabulation in Parsing and Deduction' (TAPD'98), pp. 26-35.
The definition clauses of $r$ are omitted for simplicity.

The thick dim curves in Figure 1 are dependency paths. A dependency path is a sequence of variables unified with each other. A program tree is a proof tree if it has no dependency path connecting two non-variable terms. The computational procedure we propose here, dependency reduction, is to transform the program by eliminating dependency paths connecting two non-variable terms, while preserving the proof trees.

A dependency path connecting two non-variable terms in a clause can be eliminated in two ways. First, if the two terms are unifiable, they are unified and eliminated; this is a valid operation if each variable appears at most twice in a clause, which we assume without loss of generality. Second, when the two terms are not unifiable, the clause is deleted without deleting any proof trees.

We must shorten dependency paths before eliminating them. A dependency path can be shortened by moving a predicate along it. For instance, some dependency paths including the one concerning $X=a$ in Figure 1 can be shortened by passing predicate $p$ across clause (A). To formulate this, let us introduce new predicate $p_1$, which is to be obtained by extracting $p$ from itself. That is, we add the following entry to the memoization table:

\[
(1) \quad \neg p_1 \equiv \exists X\{\neg p(X) \land p(X)\}
\]

Now we pass $p$ through (A) to new predicate $q_1$, replacing (A) with $(A')$:

\[
(A') \quad \leftarrow p_1 \land q_1(U,Y) \land r(Y,Z).
\]

$q_1$ has been dynamically defined here as below:

\[
(2) \quad q_1(U,Y) \equiv \exists X\{q(U,X,Y) \land p(X)\}
\]

This is entered to the memoization table, too. Note that $(A')$ is equivalent to $(A)$.

Next let us further pass $p$ across (D) and across (E), and generate new clauses $(D')$ and $(E')$, respectively:

\[
(A) \quad \leftarrow p(X) \land q(U,X,Y) \land r(Y,Z).
\]

\[
(B) \quad p(X) \leftarrow X=a.
\]

\[
(C) \quad p(X) \leftarrow X=f(Y) \land p(Y).
\]

\[
(D) \quad q(U,X,X) \leftarrow U=b.
\]

\[
(E) \quad q(U,X,Z) \leftarrow U=g(V) \land q(V,X,Y) \land Y=f(Z).
\]

The thick dim curves in Figure 1 are dependency paths.
\( (D') \) q2(U) ← U=b.

\( (E') \) q1(U,Z) ← U=g(V) \land q1(V,Y) \land Y=f(Z). \)

Folding based on (2) occurs twice when deriving (E') from (E). New predicate q2 has been defined by the below new entry of the memoization table:

\[
\neg q2(U) \equiv \exists X (\neg q1(U,X) \land p(X))
\]

Note that (D) entails (D') and (E) entails (E'), but not vice versa. So (D) and (E) remain instead of being replaced by (D') and (E').

Note also that p has been passed through (D) not only downwards but also upwards. By further passing p through (A'), an additional top clause (A'') is derived from (A'):

\( (A'') \) ← p1 \land q2(U) \land r1(Z).

New predicate r1 is defined by:

\[
r1(Z) \equiv \exists Y \{ r(Y,Z) \land p(Y) \}
\]

If only downward movement were allowed, we would have to move literal p(X) downwards into q(U,X,Y) or vice versa in (A) at the beginning, in order to shorten the dependency paths running through X. Since the resulting literal is equivalent to q1(U,Y). This time we must move it downwards into r(Y,Z) or vice versa, in order to shorten the dependency paths running through Y. In either case, however, we must create a new binary predicate.

In contrast, the above upward passing creates unary predicates q2 and r1. In general, passing a predicate across another derives a predicate whose arity is the total arity of the former two predicates minus the shared arguments (because no variable appears more than twice in a clause). So let us posit the below optimization strategy (which is effective even if a variable can occur more than twice in a clause) to suppress the arities of dynamically created predicates:

(3) Move the predicate with the smallest arity of those on a dependency path.

This calls for both downward and upward passing, as the above example shows.

Here let us turn to further optimization. We do not always have to pass predicates through adjacent clauses, but we can often pass them further at one stretch, skipping the middle of dependency paths. For instance, p in (1) can be passed into and across (D), skipping (A) and all the instances of (E). This takes us from Figure 2 (i) (the initial state of the program consisting of (A), (B), etc.) to (ii). In Figure 2, a bullet (●) connecting literals by broken edges represents the predicate they share, indicating that every upper (negative) literal (a literal calling the predicate) is unifiable with every lower (positive) literal (the head of a definition clause).

The thin dotted edge α connecting p1 and q1 in (i) is a dependency link. A dependency link connects two arguments of two predicates and represents the set of dependency paths connecting those arguments. Let us assume that α represents the set of dependency paths connecting the argument of p and the second argument of q. These dependency paths go through (A) and instances of (E). We can hence skip these dependency paths by passing p along α.

Such a skip gives rise to a gap in a program tree. Each such gap is represented by a gap link which is derived from the dependency link mediating the passing of the predicate. α' in (ii) is a gap link derived from α and represents the gap created by the skip mentioned above. Thus the gapped program tree (ii) represents all the program trees obtained by filling the gap by (A') and zero or more instances of (E').

To be more precise, we need more restricted notions of dependency path and dependency link, together with some more relevant terms. An active predicate is the predicate to move.
An **active literal** is a literal through which you can reach an active predicate in the same (non-gapped) program tree. For instance, \( p(X, X) \) in clause (D) is an active literal because you can go from (B) through \( q(U, X, X) \) to \( p \). On the other hand, \( \neg q(V, X, Y) \) (the body, negative, literal) is inactive provided that predicate \( q \) is inactive. An **active clause** is a clause containing more than two active literals. There is no active clause in Figure 2 (i) if \( p \) is the only active predicate.

We redefine a **d-path** to be a dependency path which connects two arguments of two possibly different predicates, contains no U-turn, and runs across no active predicate, no active clause, and no two clauses with different numbers of active literals. In Figure 2 (i), for instance, the path of arguments connecting the argument of \( p \) and the second argument of \( q \) without going through (D) is a d-path, but that connecting the second and the third arguments of \( q \) is not, because it contains the U-turn in (D). A **D-path** is a non-empty set of d-paths all running through the same sequence of clauses. In Figure 2, there is no D-path consisting of two or more non-null (that is, longer than zero) d-paths. A **D-link** is a link between two possibly different predicates which represents the set of all D-paths which connect the same sets of arguments of the two predicates. We say a dependency path **includes** another dependency path when the former includes the latter as a subsequence. We say a D-path **includes** another D-path when every d-path in the latter is included in some d-path in the former. A D-path is **maximal** if it is included in no other D-path.

As an invariant condition throughout the computation, we postulate:

(4) For every maximal D-path, there is a D-link containing it.
Such a D-link is called a **maximal D-link**. In the current example, \( \alpha \) is a maximal D-link. Each maximal D-link is marked as such. D-links are explicitly encoded as annotations to the program, but D-paths are not. We redefine here a gap link to be a derivative of a maximal D-link. At one step of computation, a predicate is passed along a maximal D-link, this D-link and the clause at its end derive a gap link and another clause, respectively, and the predicate is inserted into the next predicate. In the transition from Figure 2 (i) to (ii), for instance, predicate \( p \) is moved across \( \alpha \), \( \alpha \) derives a gap link \( \alpha' \), clause (D) derives \( (D'') \), and \( p \) is inserted into new predicate \( q_2 \).

(4) is to guarantee that maximal D-links exhaust the destinations of the literals to move. Some computations are necessary to maintain (4), as will be mentioned later. The exclusion of U-turns from d-paths is to simplify the representation of the program. If U-turns were contained in D-links, apparently disjoint D-links may interact, in the sense that a d-path included in one D-link and one included in another may not coexist in a program tree, which will complicate the representation of the program. It is also for the sake of simplicity of representation that we disallow a d-path to run across any active predicate, any active clause, and any two clauses with different numbers of active literals. Namely, this is to guarantee that gaps are appropriately represented by gap links. Details are omitted because they are irrelevant as far as the examples discussed in the present paper are concerned.

Back to the example, next we want to pass \( p \) from \( q_2 \) as indicated by the thick arrow in Figure 2 (ii). However, this gapped program tree does not contain any non-null D-path along which to move \( p \) this way. So we consult the part of the original program corresponding to the gap. That is, we try to find where to move \( p \) by looking at \( q \), because \( q \) derived \( q_2 \). Since \( \alpha'' \) has been derived from \( \alpha \), we must pass \( p \) along some d-path included in \( \alpha \); otherwise the resulting gap cannot be represented by gap links. We should hence pass \( p \) to the end of maximal D-links concerning the latter two arguments of \( q \). Since the only such D-link is a null D-link, we pass \( p \) upwards with no skip. So we check whether (A) and (E) are on any d-path represented by \( \alpha \). (A) meets this condition because \( X \) is shared by \( p(X) \) and \( q(U,X,Y) \) in it. So does (E) because of its \( X \), too. Thus (A’) replaces (A) and (E’’) is derived from (E), arriving at Figure 2 (iii). As mentioned before, it is because (A’) is equivalent to (A) that (A’) replaces (A). For simplicity, we have omitted the computation to transform \( p(Y) + Y = f(Z) \) to \( p(Z) \), but it is easy to formulate this as a primitive operation because this computation concerns adjacent clauses only. \( \alpha' \) is a gap link derived from \( \alpha \).

Note that we have made a new D-link \( \beta \), which represent the d-path connecting the argument of \( p \) and the second argument of \( q_1 \). This is required by (4), because \( \beta \) is a maximal D-link in this gapped program tree. We have omitted another maximal D-link, which connects the first argument of \( q_1 \) and the argument of \( q_2 \).

From Figure 2 (iii) we move \( p \) along \( \beta \), skipping (E’’). Then \( \beta' \) is derived from \( \beta \), as shown in (iv). The computation is over here because there is no more dependency paths connecting two non-variable terms. The other definition clause of \( q_2 \) can be derived from (E’’) on demand, though the current example involves no such demand.

D-links can be either statically precompiled or dynamically created. In Figure 2, \( \alpha \) has been precompiled whereas \( \beta \) is dynamically compiled. D-links tend to raise the efficiency of the computation as a whole, because one D-link usually represents several d-paths, which can be skipped by following the D-link. So let us employ this strategy:

(5) **Compile D-links and skip computation over them.**

D-links must be made to meet (4) at least. There are several alternatives about which non-maximal D-links to create. Creating a D-link for every D-path is probably inefficient. In what
follows we assume that each D-link longer than one clause is divided into two. Namely, if a D-path \( p \) running through two clauses or more is represented by a D-link, then there are two D-links representing two D-paths whose concatenation is \( p \) and which are at least one clause long. Look at Figure 3 (v) later for example.

### 3 Left-Corner Parsing

The following program addresses parsing of a sentence beginning with ‘The boy.’

\[
\begin{align*}
(a) & \quad s(X,Y) \land \text{str0}(X). \\
(b) & \quad s(X,Z) \leftarrow np(X,Y) \land \text{vp}(Y,Z). \\
(c) & \quad np(X,Z) \leftarrow \text{det}(X,Y) \land n(Y,Z). \\
(d) & \quad \text{det}(X,Y) \leftarrow X=\text{the}(Y). \\
(e) & \quad n(X,Y) \leftarrow X=\text{boy}(Y). \\
\ldots \\
(f) & \quad \text{str0}(X) \leftarrow X=\text{the}(Y) \land \text{str1}(Y). \\
(g) & \quad \text{str1}(X) \leftarrow X=\text{boy}(Y) \land \text{str2}(Y). \\
\ldots 
\end{align*}
\]

This is depicted by Figure 3 (i). The left part ((a) through (e)) of the program tree encodes the grammar and the right part ((f) and (g)) the input string. In the following discussion and particularly at each entry in Figure 3 we consider just one (gapped) program tree for expository simplicity, but of course there are lot more. The structures under \( \text{vp}(Y,Z) \) and \( \text{str2}(X) \) have been omitted for simplicity, too.

First we move \( \text{str0} \) along D-link \( \alpha \). This derives clause \( (d') \) from (d) and gap link \( \alpha' \) from \( \alpha \), reaching the gapped program tree in Figure 3 (ii). Thus the first dependency path connecting two non-variable terms, has been eliminated and the second one has been recognized, as indicated by the dim curve running through \( Y \) of \( (d') \). This second dependency link is only partially instantiated, but it may connect two non-variable terms because \( \text{det}'(Y) \) can be an active literal as \( \text{det}(X,Y) \) is. Also new maximal D-link \( \delta \) representing this d-path across \( (d') \) has been created in order to meet (4). New predicate \( \text{det}' \) has been defined by the following entry of the memoization table.

\[
\text{det}'(Y) \equiv \exists X \{ \text{det}(X,Y) \land \text{str0}(X) \}
\]

Next we go from (ii) to (iii) by moving \( \text{str1} \) along \( \delta \). New predicate \( \text{det}'' \) is defined as follows:

\[
\neg\text{det}'' \equiv \exists Y \{ \neg\text{det}'(Y) \land \text{str1}(Y) \}
\]

From (iii) we want to pass \( \text{str1} \) from \( \text{det}'' \) along \( \alpha' \), but \( \alpha' \) does not subsume any d-path along which to pass \( \text{str1} \). So the situation is similar to the one at Figure 2 (ii). We thus consult the part of the original program corresponding to the gap. That is, we try to find where to move \( \text{str1} \) by looking at \( \text{det} \), which derived \( \text{det}'' \). Since \( \alpha'' \) is a derivative of \( \alpha \), we must pass \( \text{str1} \) along some d-path included in \( \alpha \). We should hence pass \( \text{str1} \) to the end of maximal D-links concerning the two arguments of \( \text{det} \). Like in Figure 2 (ii) again, the only such D-link is a null D-link. Note that Clause (c) is on a d-path included in \( \alpha \), which is detected by noting D-link \( \beta \) connecting \( np \) and \( \text{str0} \). So we pass \( \text{str1} \) through (c) to derive \( (c') \), as shown in Figure 3 (iv). Here we have created a maximal D-link \( \pi \) by dynamic compilation.
From (iv) we can pass $\text{str1}$ downwards along any direction. Passing it through clause (e), we eliminate the second dependency path and thereby recognize the third, as in Figure 3 (v). $\theta$ and $\tau$ are D-links created now. $\tau$ is the maximal D-link composed of $\pi$ and $\theta$. $\text{str2}$ is hence moved along $\tau$, resulting in the gapped program tree in (vi). The transition from (vi) to (vii) is similar to that from (iii) to (iv). The rest of the computation goes the same way.

Note that D-links serve both accessibility check and last-call optimization. In the transition from (i) to (ii), that from (iii) to (iv), and that from (v) to (vi), D-links serve as accessibility
An accessibility link connects a nonterminal symbol with another which can appear as one of its leftmost descendant, whereas a D-link connects two arguments (of predicates) connected via one or more d-paths. In the transition from (ii) to (iii) and that from (v) to (vi), D-links allow last-call optimization, which is often employed in programming language interpreters and compilers.

So this computation as a whole is essentially the standard left-corner parsing with accessibility linking and last-call optimization, as shown in Figure 4. Each bullet is a word. Each triangle is the Horn clause encoding a binary context-free rule. The three edges of a triangle correspond to the three variables in the clause encoding points in the word string. Broken lines mean that the rule is not yet applied. Applied rules are drawn by solid lines, and the eliminated variables (variables through which predicates have been passed) are thick lines. Due to precompiled D-links, a binary rule is not instantiated before two variables in it are instantiated. Due to dynamically created D-links, the rightmost variable of each rule is not instantiated. Figure 5 shows the gapped program tree in each moment of computation.

We have thus shown that the two optimizing control strategies (3) and (5) derive the standard left-corner parsing. In particular, precompiled dependency links serve as accessibility links, and dynamically compiled dependency links support last-call optimization. Left-corner parsing is hence derived from a general procedure for constraint satisfaction, because the two strategies are just for the sake of efficient elimination of dependency paths, with no particular concern about any specific task (such as parsing and generation) or domain (such as language).

4 Other Examples

Fixed computation order may be inefficient even when the information source is homogeneous. For instance, let us consider parsing based on a TAG (tree-adjoining grammar) in a straightfor-
ward Horn-clause encoding such as below:

\[(6) \quad a(X,Z,U,W) \leftarrow b(X,Y,V,W) \land c(Y,Z,U,V).\]

\(a(X,Z,U,W)\) represents an auxiliary tree dominating two strings (differential lists) \(X-Z\) and \(U-W\). This clause decomposes it into two smaller auxiliary trees \(b(X,Y,V,W)\) and \(c(Y,Z,U,V)\), as shown in Figure 6. In top-down parsing, \(a(a,Z,U,W)\) will be called where \(a\) is some postfix of the input string. In OLDT, a problem arises here because there are too many results of executing this literal: The possible instantiations of \(Z\) are not more than the length of string \(X\), but there may be exponentially or infinitely many values for \(U-W\). To avoid such combinatorial explosion, we should pack these values incrementally, which is essentially program transformation.

Dependency reduction does such incremental packing. (6) is instantiated in two steps: first by eliminating \(X\) and \(Y\) just in the case of context-free parsing, and second by eliminating \(Z\), \(U\), and \(V\). The space complexity of this parsing is hence \(O(n^5)\), \(n\) being the sentence length, but it can be reduced to \(O(n^4)\) by structure sharing among clauses (Hasida, 1994). The time complexity is \(O(n^6)\) because there are \(O(n)\) ways of making each final instance of the clause.

The two strategies (3) and (5) also derive semantic-head-driven generation (Shieber et al., 1990). In particular, (5) accounts for the retrieval of a semantic head. The accessibility link in head-corner parsing (van Noord, 1997) is a special usage of D-link, too.

5 Concluding Remarks

We have proposed a constraint solver for Horn-clause programs, called dependency reduction, and shown that it derives standard efficient processes for parsing and generation. A formal proof of completeness and soundness of dependency reduction does not fit into the currently allocated space, but we will report on it in an earliest possible opportunity.
Dependency reduction can be straightforwardly extended to incorporate probability. The key issue is attaching probability scores to D-links, which employs a similar technique to Stolcke (1995). We would like to study a general method for controlling symbolic computation, and natural language processing in probabilistic dependency reduction will serve as an appropriate testbed for that.

Reference

Aït-Kaci, H. 1991. Warren’s Abstract Machine. MIT Press, Cambridge, MA.

Beeri, C., & Ramakrishnan, R. 1987. On the Power of Magic. *Journal of Logic Programming, 10*, 255–299.

Hasida, K. 1994. Emergent Parsing and Generation with Generalized Chart. In *Proceedings of the Fifteenth International Conference on Computational Linguistics*, pp. 468–474.

Shieber, S. M., van Noord, G., Pereira, F. C. N., & Moore, R. C. 1990. Semantic-Head-Driven Generation. *Computational Linguistics, 16*(1), 30–42.

Stolcke, A. 1995. An Efficient Probabilistic Context-Free Parsing Algorithm that Computes Prefix Probabilities. *Computational Linguistics, 21*.

Tamaki, H., & Sato, T. 1986. OLD Resolution with Tabulation. In *The 3rd International Conference on Logic Programming*, Vol. 225 of *Lecture Notes in Computer Science*, pp. 84–98. Springer Verlag.

van Noord, G. 1997. An Efficient Implementation of the Head-Corner Parser. *Computational Linguistics, 23*(4).

Warren, D. H. D. 1983. An Abstract Prolog Instruction Set. Tech. rep. 309, Artificial Intelligence Center, SRI International, Menlo Park, CA.