Measure liquid viscosity by tracking falling ball
Automatically depending on image processing algorithm

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Abstract. A fast method presented within this paper to find transparent liquid viscosity. An introduced tracking algorithm in image processing used to find a terminal velocity automatically of a falling ball in a tube that filled with a tested liquid. Terminal velocity considered an important parameter to find a liquid viscosity. Therefore, four iron balls, with different diameters and a smartphone camera are used to measure the falling ball terminal velocity. The estimated velocity from the introduced algorithm is compared with the calculated velocity and shows a good matching. The second algorithm used to estimate the viscosity of the tested liquid. The limitation of the ball diameter around 5mm to measure the terminal velocity. The result shows error matching between the fitting data and the estimated data ranging (0.00-0.1%). This method is a faster and easier experiment to use to find any transparent liquid viscosity comparing with a viscometer.

Keywords: viscosity measurement, terminal velocity, falling ball.

1. Introduction

Viscosity is an important property of a fluid produced by the resistance between its own particles layer while flowing. It would not only suggest a vital effect on the motion characteristics of the fluid but also affects its application potentials \cite{1}. Therefore, it is a fluid property that provides an indication of liquid resistance and considers one of the liquid specifications, and an important concept that is taken into consideration in different fields. This property used as an index in quality to control applications using oils, paints and other fluids where liquid flows consider a critical property. A falling ball viscometer is commercially available to measure the viscosity of fluids and has been used for testing petroleum products, pharmaceutical drinks, silicate glass and food products \cite{2}. In addition, study of a falling ball is important in several engineering domains which involve multiphase flows i.e. sedimentation, improvement of combustion, minimization of erosion by droplets in large turbines, hydrodynamic chromatography, membrane transport, hydraulic and pneumatic transport of coarse particles in pipes, effects that utilize electric fields to enhance transport phenomena and separations in multiphase systems \cite{3,4}.

Many studies were introduced to measure liquid viscosity. A system measure absolute value considers a complex method which combined nano-technology with a classical falling ball using capillary
viscometers [5]. Terminal velocity was studied with seven measurements along a tube, but it was not accurate enough to measure the viscosity [6]. A falling ball used to measure viscosity depending on numerical code is introduced but with iteration system [7]. The ratio of drag force which sensed by the falling ball in an infinite medium to that in the Stokes limit as a function of Reynolds number alone is used to calculate the viscosity [8]. Characterized Newtonian fluids such as mixtures of glycerol and water are used with a ball drop device experiment to measure fluid viscosity. The slope angles of falling ball determine the viscosity coefficient of the test liquid [9].

In this paper, the idea was suggested to find viscosity for any transparent liquid by using glass tube with height 150cm and diameter 1.592cm because this volume is optimal to reached terminal velocity with the suggested different ball size. The metal balls were selected because they gave a better description comparing with the rest of the suggested models in previous studies. Terminal velocity is an important standard parameter and calculates viscosity for any liquid. Therefore, this study concreted to design a special system to evaluate the viscosity and measure the terminal velocity. This system depends on a suggested code within in MATLAB software to be fit to measure terminal velocity. This algorithm describes later and shows efficiency in performance. The proposed system designed with high accuracy in terms of height and diameter of the tube, size of balls and the distance of the camera from the tube. An empty space left in the upper end of the tube about 40 cm to take into consideration the amount of liquid displacement due to the mass of the ball.

Ball motion must be vertical so that it does not touch the wall of the tube to reaches its terminal velocity. This contact with the wall of the tube will generate a reverse force to the terminal velocity. The algorithm designed to find terminal velocity depending on distance and time. The ball has been located depending using built-in MATLAB software (region probe). The ball velocity is determined after located the next position that takes Δt period time. Terminals velocity is calculated from the velocity with time diagram. Finally, the viscosity calculated using Stokes’ law and depending on the terminal velocity.

2. Mathematical Background

Falling ball in a liquid is one of the methods to determine dynamic viscosity depend on Stokes’ law [1]. By measuring the velocity of constant motion of a small ball in a liquid under study, it is easy to estimate the viscosity of that liquid.

The principle to measure viscosity experiment is shown in figure (1). It is shown that there are three forces acting on the iron ball, these are gravity force $F_g$, upward buoyant $F_b$, and the force of viscous (internal) friction $F_d$. In our cases, a cylinder glass filled with studied liquid and four iron ball with different diameters are used.
Figure (1): Sketch showing affecting forces on a falling ball

The viscous drag of a falling ball results in the creation of a restraining force ($F$) described by Stokes’ law [10]

$$F_d = 6\pi \mu r v$$

where $F_d$ is the drag force, $\mu$ is the viscous friction, $r$ is the radius of the falling ball, and $v$ terminal ball velocity in liquid.

At velocity equilibrium state, the net force is zero, so the ball moves in a constant velocity called terminal velocity where the force equilibrium equation for the falling ball given by [11]:

$$F_b + F_d + F_g = 0$$

where $F_b$ equal to $(mg = \frac{4}{3}\pi r^3 \rho_{\text{liquid}} g)$ [10].

The first two forces in equation 2 arise from the buoyancy effect of displacing the liquid ($F_b$) and from the viscous drag of the liquid on the ball ($F_d$), respectively. Both forces ($F_b$) and ($F_d$) act upwards tendency to float the ball. The only force acting downwards is the body force resulting from gravitational attraction ($F_g = mg$) [12]. By summing forces in the vertical direction, we can write the following equation [12]

$$F_b + F_b = F_g$$

Combining all of the previous relationships that describe the forces acting on the ball in a fluid we can write the following expression [10,12],

$$\frac{4}{3}\pi r^3 \rho_{\text{liquid}} g + 6\pi \mu r v = mg$$
Rearranging equation 4 to get the equation of viscosity as [13]:

\[ \mu = \frac{D^2 (\rho_b - \rho_L) g}{18 \nu} \]  

(5)

where \( D \) is ball diameter, \( \rho_b \) is ball density, \( \rho_L \) is the liquid density of the used and \( g \) is the earth gravity (9.8 \( N/m^2 \)).

3. Experimental Tools

The velocity of the falling ball in viscous liquid very important in several applications. In this study, the image processing technique used to compute ball velocity in liquids. There is limitation condition must be taken into consideration and several important required tools, software and theoretical equations used to compute falling ball velocity in liquids accurately. In this study, the following items are used to perform the measurements and analysis:

1. iPhone7 camera to record the falling ball video clips.
2. Four different iron balls with density 7800 kg/m\(^3\) and diameter 15.60, 13.16, 12.00 and 9.53 mm.
3. A cylinder glass tube of length (\( l=150 \) cm) and diameter (\( d=1.592 \) cm).
4. Liquids test are water, vegetable oil, olive oil and car oil.
5. MATLAB 2017 software to build the processing algorithm running on window 7 corei5, RAM 2GB.

The scale factor (scf) code is used to convert the 2D image in pixels from a real-world image dimension in meter (m) using [14]

\[ scf = \frac{l_{cm}}{l_{pixel}} = \frac{l_m}{l_p} \]  

(6)

where \( l_m \) is the length between two points in the real world in meter, while \( l_p \) is the length between the same two points in image world (image plane) in pixels, this scf can be used to estimate the real lengths of any object in the image plane.

4. Recommended System Procedure

The suggested system is shown in fig. 2 which consist of a tube filled with a tested liquid and the iPhone camera fixed at a distance (1 m) from the glass tube. Newtonian fluid is considered within this experiment and has to be transparent. It is recommended to use a high-quality digital camera and high visibility of the ball during its fall within the vertical tube. The camera is placed in front of the tube with good lightning and the good object plane. Lightening conditions are vital for the visibility and the quality of images in order to avoid artefacts which can degrade the extracted video frames and the ball.
The procedure steps to determine viscosity using the suggested system as follow:

1. Fill the tube with one of the studied liquids.
2. Choose one ball with a known diameter.
3. Drop the ball freely in the liquid without touching the wall, otherwise, repeated again.
4. Start recording a video clip for the motion of the free ball falling using the iPhone7 camera, then finish the recording when the ball reaches the bottom of the tube.
5. Analyzing the video using the algorithm (1) to compute terminal ball velocity then compute the liquid viscosity.

5. Determine the terminal velocity

The objective of this part is calculating the velocity of the falling ball. However, it is necessary to set all parameters equally in all cases like video frame rate which is set to 120 fps. The first step is recording the movement of the falling ball to extract the ball position automatically using the algorithm (1). Algorithm (1) automatically shaped and locate the position of the free-falling ball after selecting manually tube border from the first frame, then used the same location part in the other video frames to determine automatically the centre of the falling ball in each frame (x, y). The second step determine center location of the falling ball (x, y) as a function of time depending on sequence frames in video clip (frame rate) which equal to 1, 2, 5, 10, 15, 20, 30 and 60 fps as shown in figure (3) and (4). The frame rate of the recorded video is 120 fps.

Figure (3) shows a red circular shape of the falling ball which different from the time the y-axis is important to locate the difference in position (Δy) to determine the velocity of the falling ball depend on the frame rate. The x-axis is not important parameter, therefore it is shown there is a difference in the scale which caused by a change in extracting frame manually.

Figure (4) shows tube border and ball position and describe the motion of the falling ball with the frame number. In this case the relation of velocity calculated by subtracting $y_2$ from $y_1$ and divided by interval time. When interval time is changed, the Δy and number of frames is changed respectively.
Figure 3: The center location of the free ball represented by \((x, y)\) as a function of \(y\) and \(x\) axis with different frame rate (a)=1, (b)=2, (c)=5, (d)=10, (e)=15, (f)=20, (g)=30, (h)=60.
The last step is calculating the terminal velocity ($V_e$) within the algorithm (1). The relation between estimated velocity ($V_e$) of the falling ball as a function of a time interval ($\Delta t=5$) for tested liquids and all ball diameter shown in figure (5). The figure can be divided into three section. The first section (I), the falling ball starts with velocity zero and begun to accelerate slowly and it can be described as a linear relationship. The second section (II), The drag force ($F_d$) started to act on the falling ball as well as buoyant force ($F_b$) which these forces are smaller than gravitation force ($F_g$) in this section. Therefore, the falling ball velocity increase gradually with all cases, where the three forces is varying and depending on liquid details ($\rho, \mu$). Third section (III), the affecting forces on the falling ball equal to ($F_g$). At this point, the ball stops to accelerate and continues falling at a constant speed called the terminal velocity. To be note, in figure (5), the y-axis is plotted to show the important section only and it is not the velocity value. i.e. vegetable oil, olive oil and car oil are plotted with waterfall option in Origin software.
Figure 5: The estimated velocities ($V_e$) for the four liquids with frame rate ($\Delta t=5$).

The inset shows the velocity for all tested liquid with ball diameter 15.6mm become it is the optimal size to find $V_t$, figure plotted with a frame rate of 5 fps i.e. every 5 frames is considered. This inset is comparable with figure (7), where it shows that ball velocity in the water is faster than others.

The mode velocity ($V_{md}$) and the average velocity ($V_m$) are calculated at velocity reaches the terminal level for all tested liquids. Figure (6) shows the relation between ($V_{md}$) and ($V_m$) as a function of a frame rate, this figure is the same as in fig. (5) for the (III) section. The terminal velocity value became steady and it is accurate with the big diameter of the tested ball. The III section in figure (5) is extended in fig (6), where, the frame rate represented in the X-axis to show the terminal velocity in terms of ($V_e$).

$V_{md}$ and $V_m$ are calculated using Excel built-in function and plotted in fig (6) for each frame rate. This figure shows that the effects of changing time (*1/120fps) shows the same behaviour.
Figure 6: The mode and the average velocities of the falling ball in liquid as a function of frame rate, where the different interval time multiply (1/120) between the two successive frames, in tested liquid (a) water (b) vegetable oil (c) Olive oil (d) car oil

Figure 7 explains the behaviour of the average and mode velocity with ball diameter for all tested liquids. This figure is important to improve the inset in figure 5 and the velocity of the ball inside the liquids. Moreover, the relationship is linear, and the best ball diameter is the biggest.
The relation between the computed terminal velocities ($V_e$) from the algorithm (1) and real terminal velocities ($V_t$) calculated from Eq. 3 shown in fig (7), the relation between ($V_e$) and ($V_t$) were the four tested liquid. This relation is very important to correct the real terminal velocity value.

So, the suggested fitting function between $V_e$ and $V_t$ is introduced to be used in fig. 8 as:

$$V_t = a * V_e + b$$  \hspace{1cm} (7)

where $a$ and $b$ are fitting constant.
This linear velocity equation used in MATLAB software, namely Curve Fitting (cf) tool, to fit the terminal velocity data \( V_t \) with the computed \( V_e \).

The fitting parameters values \( (a \ and \ b) \) from equation 5 for different cases are extracted. The error percentage fitting process using \( cf \) tools gives different types like R-square, Adjusted R-square and Root mean squared error (RMSE). These values are listed in the table (1) for water, vegetable oil, olive oil and car oil cases, respectively.

| liquid       | a    | b     | R_square | AdjR_square | RMSE  |
|--------------|------|-------|----------|-------------|-------|
| Water        | 2.221| -1.866| 0.9635   | 0.9452      | 0.4921|
| Vegetable oil| 3.12 | -1.492| 0.9393   | 0.909       | 0.01311|
| Olive oil    | 0.8984| -0.1918| 0.9953   | 0.9929      | 0.01284|
| Car oil      | 0.9874| -0.2634| 0.998    | 0.9969      | 0.00793|

Figure (9) shows the real velocities (black curve) and the fitting velocities (red curve) that obtained from Eq. (5). The behaviour in figure 8 shows a linear relationship which consists of equation 5.

The fitting line is generated using the built-in fitting function in Origin software, this line shows that if the ball diameter reaches to almost 5 mm, the terminal velocity cannot be measured. Therefore, it is recommended to avoid small ball size if someone wants to repeat this method in the same tube volume.

6. Viscosity Calculations

The viscosity \( \mu_d \) of the tested liquids is measured in the ministry of science and technology, Environment and water department using viscosity device. These data written in the table (2) under the liquid name. The resulted viscosity from the algorithm (2) depending on equation (3) listed in the table (2) with the ball diameter. The error value \( ARE \) calculated using equation (6) to be compared with.
\( \mu_d \). ARE data shows a good match in all cases, and as the average range for all tested liquid, it is acceptable.

\[
ARE = \left| \frac{\mu_e - \mu_d}{\mu_d} \right|
\]  

(8)

where \( \mu_d \) is the professional device viscosity and \( \mu_e \) is the estimated viscosity using the suggested method.

| Table (2): Viscosity with the absolute relative error and ball diameter. |
|-----------------|-----------------|-----------------|-----------------|
| Liquid                        | Ball diameter (mm) | \( \mu_e \) (N², s) | ARE          |
| Water                        | 15.60             | 0.001            | 0.000          |
| \( \mu_d = 0.001 \)          | 13.16             | 0.001            | 0.000          |
|                              | 12.00             | 0.001            | 0.000          |
|                              | 9.53              | 0.0009           | 0.100          |
| vegetable oil                | 15.60             | 0.52             | 0.060          |
| \( \mu_d = 0.490 \)          | 13.16             | 0.45             | 0.080          |
|                              | 12.00             | 0.50             | 0.020          |
|                              | 9.53              | 0.49             | 0.000          |
| olive oil                     | 15.60             | 0.140            | 0.000          |
| \( \mu_d = 0.140 \)          | 13.16             | 0.143            | 0.020          |
|                              | 12.00             | 0.135            | 0.030          |
|                              | 9.53              | 0.142            | 0.010          |
| car oil                       | 15.60             | 0.149            | 0.006          |
| \( \mu_d = 0.150 \)          | 13.16             | 0.153            | 0.020          |
|                              | 12.00             | 0.147            | 0.020          |
|                              | 9.53              | 0.149            | 0.006          |

7. Discussion

This work has some bullet points in terms of preparing the setup and calculating the viscosity of the tested liquids. A new algorithm is designed to fit with the suggested setup. The running time to find viscosity (\( \mu \)) using professional device takes 15 minutes while in this study it takes 5 minutes only. The ball diameter plays a critical parameter to find \( \mu \) as found in this study, and the effect of this parameter is shown earlier. The bigger ball diameter shows a good matching for the \( \mu_e \) comparing with \( \mu_d \). The relationship between velocity and ball diameter is plotted to focus on this behavior. This behavior has a linear relationship which interpret by the three forces effect. Also, it is recommended to avoid a small diameter size to measure the \( \nu_L \) because the effect of the three forces will change. It is necessary to provide a good lighting environment during recording the video clip to avoid shadow effect. This effect causing a problem in recognize the ball in image processing. It is also recommended to fix the temperature during the measurement because the viscosities value has a proportional relation with the temperature has three main sections affected by the three forces. These sections can be extended and modeled using sufficient equations which is out of the scope of this study.
8. Conclusions

The suggested setup to measure $\mu$ chosen carefully and designed to be fit with this work. The same liquids were tested in the ministry of science and technology, environment and water department using the facility of a professional device. This device use pendulum technique and shows the result in a digital window, taking 15 minutes. In this study, the terminal velocity of falling ball calculated using the designed algorithm. Terminal velocity ($v_t$) is an important parameter to measure the $\mu$. Therefore, an intensive study focused to find ($v_t$) using a different ball diameter. The $v_t$ values different with ball size, but all of them have the same behavior. The best ball size diameter is the biggest (15.6mm). The frame rate of the recording video (120fps) doesn’t have any effect to measure the $\mu$. The frame rate should be enough to compute the ball location and determine the $v_t$. The relationship between velocity and ball diameter is plotted to focus on this behavior. This behavior has a linear relationship which interpret of the three forces effect.

9. References

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Algorithm (1) Calculate the Terminal ball velocity

Input:
- Video clip of ball liquid (vid).
- Length of known object appear video image frames completely: in meter (Lm) to measure image scale factor.
- Number of frames interval different Δt between an successive two frames.

Output:
- Image scale factor (sfc).
- the Determine the terminal velocity (v_e).

- Start algorithm-
1. Determine video frames rate using MATLAB STATEMENT(M.S.)
   \[ t_1 = \text{Vid. Frame rate} \]

2. Determine total no. of frames in video clip using M.S.
   \[ lv = \text{vid number of frames} \]

3. Compute number of processed frames per second.
   \[ tf = \frac{2}{t_1} \]

4. Extract video frames of set lv using (M.S.):
   \[ \text{frames} = \text{read}(\text{vid}, \text{lv}) \]

5. Using imcrop M.S. to extract only the important region in video frames manually only for final video frame.
   \[ [x, y, I, \text{rect}] = \text{imcrop}(\text{frame}) \]

6. From the extracted important image region(I) manually determined from known object length in pixel by select two ended point in the object.
   \[ [a \ b] = \text{ginput}(2) \]
   Then compute the length of the object
   \[ L_p = \sqrt{(a(1) - a(2) + b(1) - b(2))^2} \]

7. Compute scale factor scf
   \[ \text{scf} = \frac{L_m}{L_p} \]

8. Set initial for the parameters
   \[ ii = 0, lv1 = 1, \text{new indecies for the processed frame only.} \]
   \[ lv1 = \text{round}(lv/3) \] index of started frame processed.

9. Start loop for \( i = 1 \) to \( lv \)
10. Read frames of index \( i \) using (MS):
    \[ \text{frames} = \text{read}(\text{vid}, i) \]

11. Used \( x, y \) and rect] values from step 5 to automatically extract important image region (I)
    using M.S.
    \[ I = \text{imcrop } (\text{frames},[\text{x}, \text{y}], \text{rect}). \]

12. Convert (I) image in to binary image(I).
13. Using region prop statement properties to determine the number of targets in binary image frame using M.S.
    \[ st = \text{region props}(-I) \]

14. Set \( ii = ii + 1 \)
15. Determine the center \((x_{ii}, y_{ii})\) of the largest target in \(st\).
16. end for.
17. Compute the vertical \(y\)-axis in meter using
   \[yy_{1} = yy_{1} \times scf\]
18. Compute ball velocity vector
   \[v_{t} = \frac{\text{diff}(yy_{1})}{t_{f}}\]
19. Compute terminal velocity using mode value or average of the final (10) velocity vector value to find terminal speed \((v_{e})\).
20. end algorithm.

**Algorithm 2: Algorithm of correction terminal velocity model**

**input:**
1. Terminal velocity vector ball diameters \((v_{e})\)
2. Balls diameter vector \((D_{i})\)
3. Density of the ball \((\rho_{b})\)
4. density of the liquid \((\rho_{L})\).
5. gravitated acceleration \((g)\).

**output:**
- viscosity of liquid.

- **Start algorithm** -
1. Compute the correction (real) terminal velocity vector using
   \[(\rho_{b} - \rho_{L}) \times g \times D_{i}^{2} / (18 \times v_{t})\]
2. Compute the correction fitting model between \(v_{e}\) and \(v_{t}\) using MATLAB cf tool \(v_{tt} = (v_{p}, v_{t})\)
3. save the fitting parameters \((a, and b)\)
4. end algorithm.