Residual Networks as Flows of Velocity Fields for Diffeomorphic Time Series Alignment

Hao Huang
New York University Abu Dhabi
hh1811@nyu.edu

Boulbaba Ben Amor
Inception Institute of Artificial Intelligence
boulbaba.amor@inceptioniai.org

Xichan Lin
New York University
xl3417@nyu.edu

Fan Zhu
Inception Institute of Artificial Intelligence
fan.zhu@inceptioniai.org

Yi Fang*
New York University Abu Dhabi
yfang@nyu.edu

Abstract

Non-linear (large) time warping is a challenging source of nuisance in time-series analysis. In this paper, we propose a novel diffeomorphic temporal transformer network for both pairwise and joint time-series alignment. Our ResNet-TW (Deep Residual Network for Time Warping) tackles the alignment problem by composing a flow of incremental diffeomorphic mappings. Governed by the flow equation, our Residual Network (ResNet) builds smooth, fluid and regular flows of velocity fields and consequently generates smooth and invertible transformations (i.e. diffeomorphic warping functions). Inspired by the elegant Large Deformation Diffeomorphic Metric Mapping (LDDMM) framework, the final transformation is built by the flow of time-dependent vector fields which are none other than the building blocks of our Residual Network. The latter is naturally viewed as an Eulerian discretization schema of the flow equation (an ODE). Once trained, our ResNet-TW aligns unseen data by a single inexpensive forward pass. As we show in experiments on both univariate (84 datasets from UCR archive) and multivariate time-series (MSR Action-3D, Florence-3D and MSR Daily Activity), ResNet-TW achieves competitive performance in joint alignment and classification.

1 Introduction

Aligning temporal observations remains a challenging problem in time-series analysis [35], spacetime scene understanding in computer vision [1], behavioral analysis in medical imaging, and so on and so forth. Indeed, time-series data often presents a significant amount of misalignment, also known as non-linear time warping, which is usually caused by differences in execution or sampling rates. For instance, actions such as walking performed by different actors have different execution rates and different starting points in a periodic process owing to physiological and bio-mechanical factors. Veeraraghavan et al. [40] showed that ignoring such temporal variability can greatly decrease recognition performance. Temporal alignment seeks to find a plausible temporal transformation between a query sequence and a target sequence, such that their temporal variability is minimized (synchronizing observations in the previous walking example). In this work, we propose ResNet-TW, a novel diffeomorphic temporal transformer network for both pairwise and joint time-series alignment.

*Corresponding author
Inspired by the geometric LDDMM (Large Deformation Diffeomorphic Metric Mapping) framework [3], we introduce the flow equation as an additional constraint to govern the construction of the transformation on the basis of fluid flow of vector fields. To this end, we accommodate Deep Residual Networks to predict and integrate such flows of non-stationary vector fields. Then, similarly to [43], we formulate the joint alignment problem as to simultaneously compute the centroid and align all sequential data within a class, under a semi-supervised schema.

**Contributions and paper’s organization.** 1) We restate training Residual Networks as integrating non-stationary velocity fields in temporal LDDMM to compute/learn warping functions for time-series alignment. A similar interpretation of ResNets as incremental flows of diffeomorphisms was recently presented in [29] in the context of supervised learning with application to image classification. 2) We propose ResNet-TW, a diffeomorphic temporal transformer network for both pairwise and joint alignment of time-series. Compared to existing transformers (e.g. DTAN [43] and TTN [23]), ResNet-TW guarantees diffeomorphic warping under large misalignment, 3) We conduct extensive experiments on several publicly available datasets to validate the generalization ability of our models to unseen data for time-series joint alignment and classification. The rest of our paper is organized as follows: In Section 2, we provide a mathematical formulation of both pairwise and joint alignment problems in time-series analysis and review existing works. Section 3 describes the proposed approach. Implementation details and experimental evaluations are reported in Section 4. Section 5 provides some concluding remarks and opens some perspectives.

2 Problem formulation and related work

**Pairwise alignment.** Given two time-series $f$ and $g : \Omega \rightarrow \mathbb{R}^d$, observations of the same temporal process where $\Omega \subseteq \mathbb{R}$ is the time domain, i.e., a time interval $[\tau_1, \tau_2]$, on which the sequential data are defined, the key problem is to find an optimal (plausible) warping function $\gamma^* : \Omega \rightarrow \Omega$, such that the data fit between $f$ and $g \circ \gamma^*$ is high (i.e., their discrepancy according to a distance measure $D$ is minimum). This problem is typically solved as an optimization problem [35], as in Eq. (1),

$$\gamma^* = \arg \min_{\gamma \in \Gamma} D(f, g \circ \gamma) + R(\gamma) \ . \ (1)$$

While the first data term evaluates the similarity between $f$ and $g \circ \gamma$, $\gamma \in \Gamma$, and the second regularization imposes constraints on the warping function $\gamma$, as smoothness, monotonicity preserving and boundary conditions. The set $\Gamma$ is then the set of all monotonically-increasing functions from $\Omega$ to itself with $\gamma(0) = 0$ and $\gamma(1) = 1$. In time-series analysis, it is desirable to have a warping function $\gamma$ that is invertible, and both $\gamma$ and $\gamma^{-1}$ (its inverse) sufficiently smooth, i.e., $\gamma$ is a diffeomorphism.

**Joint alignment.** The goal of joint time-series alignment is to find an average temporal sequence $\bar{g}$ for a given set of $N$ time-series observations $G = \{g_1, g_2, \cdots, g_N\}$ in a space $E$ induced by a distance measure $D$, such that $\bar{g}$ minimizes the sum of distances to all elements in the set $G$ (Eq. 2),

$$\bar{g} = \arg \min_{g \in E} \sum_{i=1}^{N} D(g, g_i) \ . \ (2)$$

In analogy to Eq. (1), the joint alignment can be solved by finding a set of optimal warping functions $\{\gamma^*_i\}_{i=1}^N$ verifying Eq. (3),

$$\{\gamma^*_i\}_{i=1}^N = \arg \min_{\{\gamma_i \in \Gamma\}_{i=1}^N} \sum_{i=1}^{N} \left[D(\bar{g}, g_i \circ \gamma_i) + R(\gamma_i)\right] \ , \ (3)$$

where each optimal warping function $\gamma^*_i : \Omega \rightarrow \Omega$ minimizes the discrepancy between $g_i$ and $\bar{g}$. Solve Eq. (3) jointly computes a consensual time-series and align all times-series to it.

**Dynamic Time Warping (DTW) and variants.** A popular approach for pairwise alignment of time-series is DTW [31, 32] which, by solving the Bellman’s recursion via dynamic programming [4], finds an optimal monotonic alignment between two time-series. The time cost for DTW to align two time-series of respective length $n$ and $m$ is $O(nm)$. In [10] Cuturi et al. have defined Soft-DTW, a differentiable loss function, that can be integrated into neural networks. It is also used for averaging time series using DTW discrepancy [10]. However, it is only capable of aligning currently-available data, and compute from scratch for new data, which is especially computation-consuming when the
new data is much larger than the original one. Srivastava et al. have derived in [34] an interesting
representation, termed SRVF (for Square Root Velocity Function) and an elastic metric that is
invariant to the reparameterization group. Optimal warping functions are computing using Dynamic
Programming. In [1], Ben Amor et al. accommodated this representation to skeletal trajectories on
the Kendall’s shape space for action recognition. Advanced variants of DTW such as Canonical Time
Warping (CTW) [47] finds the optimal reduced dimensionality subspace such that the sequences are
maximally linearly correlated. Generalized Time Warping (GTW) [46] uses a combination of CTW
and a Gauss-Newton temporal warping method that parameterizes the warping path as a combination of
monotonic functions. Deep learning versions of CTW have also been recently proposed [38].

Temporal Transformers Networks. Following the spatial transformer methodology proposed
first in [20], several temporal transformers nets have been proposed recently. Among them, the
Diffeomorphic Temporal Alignment Nets (or DTAN) and its recurrent variant (R-DTAN) are recently
proposed in [43] for pairwise and joint time-series alignment. DTAN learns and applies an input-
dependent transformation to the input signal to minimize a loss function, including a regularizer.
That is, in single-class problems, DTAN turns to an unsupervised method for registration (i.e. solve
Eq. (1)), while in multi-class case, it yields into a semi-supervised approach where class labels are
considered (i.e. solve Eq. (3)). To deal with large misalignment, a recurrent variant of DTAN is also
proposed. R-DTAN guarantees diffeomorphic transformations by incrementally composing smaller
diffeomorphic transformations, however, this aspect was omitted in [43]. To reduce the number of its
parameters, R-DTAN integrates stationary velocity fields by sharing the learned parameters by all the
temporal transformer layers. Taking a different direction, in [23] a separate temporal transformer
was integrated at the front end of a classification network. This temporal transformer network (TTN)
lead to rate-robust representations that reduce intra-class variability. In [26] a deep architecture for
learning warping functions is proposed for elastic shape analysis (using SRVF representation as in
[34] and [1]). A temporal transformer network (TTN) is trained on shape representations derived
from the original shapes then used to predict optimal warping functions separating unseen shapes.
Except for R-DTAN, approaches cited above belong to the family of Elastic Models in which warping
functions $\gamma : \Omega \rightarrow \Omega$ are generated by perturbations from the identity, i.e. $\gamma(t) = Id(t) + v^\theta(t)$,
$t \in \Omega$ and $\theta$ are parameters of the network. Consequently, they can not guarantee diffeomorphisms
for large misalignment (see [3] for further argumentation).

Consensus sequence problem (Joint alignment). In the context of statistical inference and time-
series classification, many algorithms require a method to represent information from a set of objects
in a sample average. DTW is intractable for large-scale joint alignment problems due to a quadratic
time cost for pairwise alignment. Averaging under the DTW distance is nontrivial, as it involves
solving the joint-alignment problem. A congealing algorithm solves iteratively for the joint alignment
by gradually aligning one signal towards the rest [21]. Typical alignment criteria used in congealing
are entropy minimization [24, 19] or least squares [8, 9]. Dalca et al. [12] proposed a learning-based
method for building deformable conditional templates based on diffeomorphisms. While several
authors proposed smart solutions for the averaging problem [37, 28, 11, 27, 43], none of them except
for [43] does not require solving a new optimization problem each time for aligning new sequential
data. Recently, [23] proposed a temporal transformer network based on 1D diffeomorphisms for
time series classification, but does not scale well with the signal’s length. [13] showed it is possible
to explicitly incorporate flexible and efficient diffeomorphisms [14, 15] within deep learning models
via a Spatial Transformer Network (STN) [20]; particularly, they focused on supervised learning for
image recognition and classification.

3 Proposed approach

Registration problems, including temporal alignment, have been widely stated as the estimation of
diffeomorphic transformations between input and output data. By parameterizing the transformation
via integration of sufficiently regular stationary or time-dependent velocity fields [3, 41, 18, 7] and
imposing sufficient regularization, diffeomorphic transformations can be assured. This was the
central idea of Large Deformation Diffeomorphic Metric Mapping (LDDMM) [3] which tackles the
registration issue through a composition of a series of incremental diffeomorphic mappings, each
individual mapping being close to an identity mapping. More formally, we refer to $g$ as a query and $f$
as a target time-series. It is desirable to have the warping function $\gamma$ between $f$ and $g$ to be invertible,
and both $\gamma$ and $\gamma^{-1}$ to be sufficiently smooth, i.e., $\gamma$ is a diffeomorphism. This set of diffeomorphisms

forms a group with the identity mapping as the neutral element. As derived in [3], the transformation map is time-dependent and is generated as the result of the integration over time of smooth velocity field \( v : [0, 1] \times \Omega \rightarrow \Omega \), governed (or constrained) by the the Flow Equation (Eq. (4)),

\[
\dot{\gamma}(t, \tau) := \frac{\partial \gamma(t, \tau)}{\partial t} = v(t, \gamma(t, \tau)), \quad \gamma(0, \tau) = \tau ,
\]

for all \( \tau \in \Omega, \ t \in [0, 1] \), and \( v(t, \gamma(t, \tau)) \) is a time-dependent velocity field. Accordingly, a time-dependent transformation \( \gamma : [0, 1] \times \Omega \rightarrow \Omega \) is modeled where \( \gamma(t, \tau) \in \Omega \) describes the position of a particle at time \( t \) that was at time 0 in \( \tau \in \Omega \). The warping \( \gamma(t, \tau) \) is obtained by integration of the flow equation as follows (Eq. (5)),

\[
\gamma(t, \tau) = \gamma(0, \tau) + \int_0^t v(s, \gamma(s, \tau)) ds .
\]

This gives a path \( \gamma_t = \gamma(t, .) : \Omega \rightarrow \Omega, t \in [0, 1] \) in the space of warping functions starting with \( \gamma_0 = \gamma(0, .) \) as an identity mapping and terminating at the end-point \( t = 1 \) of a particular warping function \( \gamma(1, .) = \gamma_1 = \gamma_0 + \int_0^1 v_t(\gamma_t) dt \) transforming \( g \) to \( f \). Accordingly, Eq. (1) can be phrased as follows: find a flow of velocity fields \( v^* : [0, 1] \rightarrow \Omega \) as:

\[
v^* = \arg \min_{v: \gamma_t = v \circ \gamma_t} \left( \mathcal{D}(f, g \circ \gamma_1) + \alpha \int_0^1 \| v_t \|^2 dt \right) ,
\]

where \( \alpha \) balances the weight between data and regularization terms. In LDDMM formulation, \( V \) is an Reproducing Kernel Hilbert Space (or RKHS) defined by the Gaussian kernel to guarantee smoothness and regularity of computed velocity fields and \( \| . \|_V \) is the induced RKHS norm. In addition, the quantity \( \int_0^1 \| v_t \|^2 dt \) plays the role of a Kinetic energy of the velocity field \( v \) and allows to measure the length of the path separating \( \gamma_0 \) and \( \gamma_1 \). While the LDDMM framework have been widely used in shape and image registration, to our knowledge its extension to time-series (except for time series of images e.g. [17]) is not explored yet. Furthermore, this work is among very few recent works (e.g. [29]) to revise the LDDMM framework using Deep Residual Networks.

**Eulerian discretization using deep residual networks.** Deep learning models have been applied for a wide range of problems. Among the proposed architectures, *Residual Networks* (also called ResNets) have achieved state-of-the-art performance in supervised learning. In this paper, inspired by insights into ResNets from an aspect of ordinary/partial differential equation (ODE/PDE) [16, 30, 44], we regard ResNets as numerical schemes of differential equations and relate the incremental mapping defined by ResNets to diffeomorphic registration models, especially to Large Deformation Diffeomorphic Metric Mapping (LDDMM) [29]. Concretely, in our ResNet-TW, the \( l \)-th residual block computes an update in the form of,

\[
\gamma_l = \gamma_{l-1} + F(\gamma_{l-1}; W_l).
\]

where \( \gamma_{l-1} \) is the input to the \( l \)-th residual block \( F(\cdot; W_l) \) (\( \gamma_0 \) is the identity mapping), and \( W_l \) is a set of weights and biases associated with the \( l \)-th residual block. Specifically, the \( l \)-th residual block predicts the velocity field \( v_l = F(\gamma_{l-1}; W_l) \) that is added to the warping function \( \gamma_{l-1} \). By imposing regularity on the norm of \( v \), which lies to the space of neural functions (assimilated to a RKHS [5]), the entire ResNet implements the composition of a series of incremental diffeomorphic mappings, which is a discretized version of Eq. (5), i.e., by replacing integral with summation. An instantiation of ResNet-TW is shown in Figure 1 which builds on three main steps (only two building blocks are illustrated),

- An embedding step consisting of a single convolutional layer which embeds the input time series data from an initial low-dimensional space to a higher dimensional space driven by the number of filters used.
- A series of identical residual blocks $F(\cdot; W_l)$ which computes time-dependent (non-stationary) velocity fields ($W_l$ are different). In the core of each block, a point-wise ReLu activation function is applied to introduce non-linearity.
- A series of projection operations (i.e. dimensionality reduction) ends each of residual blocks and allow to cast estimated a velocity fields such that $v_l : \Omega \rightarrow \Omega$. Consequently, the outputs $\gamma_l$ are also $\Omega \rightarrow \Omega$ by summation of $v_l$ over the residual blocks $l \in [1, l]$ and the initial warping function $\gamma_0$.

By constraining our temporal transformer to control the amount of kinetic energy introduced by elementary velocity $v_l$ (i.e. the network activity), we guarantee (1) diffeomorphic intermediate $\gamma_l$ and final warping functions $\gamma_1$, and (2) optimal warping functions, in terms of length of the path $\gamma_t \in \Gamma$ connecting $\gamma_1$ and $\gamma_0$. To our knowledge, ResNet-TW is the first temporal (transformer) alignment method that propose this solution inspired by the LDDMM framework [3]. We notice that unlike ResNet-TW, previous approaches (e.g. DTAN [43] and TTN [23]) compute perturbations from the identity and thus do not guarantee diffeomorphic warping functions. As far as R-DTAN (the recurrent version of DTAN [43]) is concerned, it predicts stationary velocity fields (i.e. a soothing approach) which make the approach more efficient, by computing an initial velocity $v_0$, but less flexible than our ResNet-TW (i.e. a relaxation approach). We will further illustrate these advantages in Section 4.2. In single-class cases, this yields an unsupervised method for joint-alignment learning. In multi-class cases, this forms a semi-supervised method in which only class labels are used during training to align data from multiple categories.

**Loss function for multiple alignment.** Following the formulation in [43], the data term for single-class joint alignment is defined as Eq. (8),

$$
\frac{1}{N} D(\hat{g}, g_i \circ \gamma_i) = \frac{1}{N} \sum_{i=1}^{N} 1 - \frac{1}{N} \sum_{j=1}^{N} \| \hat{g}_j \|_2^2 ,
$$

where $\hat{g}_i = g_i \circ \gamma_i$. Note this setting is unsupervised and $\bar{g} = \frac{1}{N} \sum_{j=1}^{N} \hat{g}_j$ is the average sequence of the warped data. For multi-class joint alignment, the data term is the sum of the within-class variances (Eq. (9)),

$$
\frac{1}{N} D(\bar{g}, g_i \circ \gamma_i) = \frac{1}{K} \sum_{k=1}^{K} \frac{1}{N_k} \sum_{i : z_i = k} 1 - \frac{1}{N_k} \sum_{j : z_j = k} \| \hat{g}_j \|_2^2 ,
$$

where $K$ is the number of classes, $z_i$ takes values in $\{1, \ldots, K\}$ and is the class label associated with $g_i$, i.e., $z_i = k$ if and only if $g_i$ belongs to class $k$, and $N_k = |\{i : z_i = k\}|$ is the number of observations in class $k$. This is a semi-supervised setting as proposed in [43]. That is, the labels $\{z_i\}_{i=1}^{N}$ are available during the training, but not during the testing. Importantly, note that the same single network is responsible for aligning each of the classes, i.e., $\bar{W}$ does not vary with $k$. Following [43] and [15], the regularization term in Eq. (3) in respectively joint alignment is defined as,

$$
\mathcal{R}(\gamma) = \sum_{l=1}^{L} a_l^T \Sigma^{-1} a_l ,
$$

where $a_l = [a_l^1, \ldots, a_l^{N_T}]$ and $\Sigma$ is a zero-mean Gaussian with a $N_T \times N_T$ covariance matrix whose correlations decay with inter-cell distances. Notice that due to $v_t^1 = a_l^1 \tau + b_l$, if we choose $\Sigma$ as an identity matrix, Eq. (10) is the discretized version of the regularization term in Eq. (6). Similar to [15], $\Sigma$ has two parameters: $\lambda_{var}$, which controls the overall variance, and $\lambda_{smooth}$, which controls the smoothness of velocity fields. A small $\lambda_{var}$ favors small warps (i.e., close to the identity) and vice versa; similarly, a large $\lambda_{smooth}$ favors velocity fields that are almost purely affine and vice versa.

In Figure 2, we illustrate within-class joint alignment results of sequences taken from the UCR archive Plane and Car datasets. The illustrated results are of previously unseen test samples. Both misaligned and aligned data are shown. The colors of short bars at the right-bottom on the right part correspond to signals on the left part. (More examples are provided in our supplementary materials.)

**Monotonic constraint in Time Warping (TW).** We focus on a specific set $\Gamma$ of warping functions. Given a 1-differentiable function $\gamma_t$ obtained at any $t \in [0, 1]$ and defined on the domain $\Omega = [0, 1]$,
These properties imply that any $\gamma_t \in \Gamma$ is a monotonically increasing function. This property is also known as order-preserving which is important to time series alignment. It is easy to show that:

- $\forall \eta_1, \eta_2 \in \Gamma \land \eta_1 \circ \eta_2 \in \Gamma$,
- $\gamma_0 = \gamma_{id} \in \Gamma$,
- $\forall \gamma_t \in \Gamma \exists \gamma_t^{-1} \in \Gamma$ s.t. $\gamma_t \circ \gamma_t^{-1} = \gamma_{id}$, where $\gamma_{id}(\tau) = \tau$ is an identity warping function.

These properties imply that $\Gamma$ is a group under the operation of warping function composition. While in Dynamic Time Warping both constraints are satisfied as a monotonic path is computed from the initial to the endpoint, we have imposed these constraints to our ResNet-TW as will describe next. Inspired by [15] and on top of our Residual architecture, we define a finite tessellation, denoted by $\mathcal{T} = \{T_i\}_{i=1}^{N_T}$ where $N_T \in \mathbb{Z}_+$, to be a set of $N_T$ closed subsets of $\Omega$, also named cells. The union of $\mathcal{T}$ is $\Omega$ and the intersection of any adjacent cells is their shared border. For time-series, each cell is a 1-dimensional interval and has two vertices as borders. A velocity field $v_t$ for a given $t \in [0, 1]$ is a map viewed as the mapping of points $\tau \in \Omega$ to $\tau' \in \Omega$. An affine velocity field w.r.t. $\mathcal{T}$ in cell $T_i$ is defined as $v_t^f(\tau) = A_t^f \tilde{\tau}$ where,

$$\tilde{\tau} \triangleq \begin{bmatrix} \tau \\ 1 \end{bmatrix} \in \mathbb{R}^{n+1}, \quad A_t^f \in \mathbb{R}^{n \times (n+1)}.$$

For time series, $n = 1$ and thus $A_t^f = [a_t^f, b_t^f]$. The velocity field $v_t$ is continuous if the continuity is ensured for points on borders, i.e., $A_{t+1/2}^f \tilde{\tau}_i = A_{t+1/2}^f \tilde{\tau}_{i+1/2}$.

\[ a_t^f \tau_i + b_t^f \equiv a_{t+1/2}^f \tau_{i+1/2} + b_{t+1/2}^f \]

where $\tau_i$ is on the border of two pre-defined adjacent cells $T_i$ and $T_{i+1}$. All $\{b_t^f\}_{i=1}^{N_T}$ can be computed through recursion by knowing $b_t^f$ and $\{a_t^f\}_{i=1}^{N_T}$. Based on the above definitions, we design each projection step as MLP which outputs $b_t^f$ and $\{a_t^f\}_{i=1}^{N_T}$ (We discretize $t \in [0, 1]$ to correspond to $L$ blocks). The monotonic increasing (order-preserving) property of $\gamma_t$ can be assured by forcing each $a_t^f > 0$, which is achieved by using a ReLU or exponential operation before the output of each projection block. The boundary condition is realized by scaling $\gamma_t$ into the interval of $[0, 1]$.

4 Experiments

In this section, we provide some experimental illustrations, validations, and relative evaluations of the proposed method on real datasets. We use a subset of 84 datasets from the UCR Archive [6] for univariate time-series classification archive and three 3D Action/Activity recognition datasets – and Florence 3D [33], MSR Action3D [22] and MSR Daily Activity [42] for multivariate time-series classification.

\[The second [0, 1] has different meaning with the first [0, 1].\]
4.1 Univariate time-series (UCR archive datasets)

The UCR time series classification archive contains 85 real-world datasets and we use a subset containing 84 datasets, as in [43]. These datasets differ from each other in the number of examples, time-series length, application domain, (e.g., ECG, medical imaging, motion sensors), and number of classes ranging from 2 to 60. We experiment with the train and test split provided with the archive. Here we report a summary of our results which are fully detailed in our supplementary materials. For each of the UCR datasets, we train our ResNet-TW for joint alignment, where $\lambda_{var} \in \{10^{-3}, 10^{-2}\}$ and $\lambda_{smooth} \in \{0.5, 1\}$. Our ResNet-TW is composed of 4 to 8 building blocks for different datasets and each block consists of 3 convolutional layers with kernel size set to 51 and channel number set to 128. We set $\alpha = 0.1$. The network is initialized by Xavier initialization using a normal distribution. We optimize our network with learning rate set to $10^{-4}$ and without weight decay. During the training, our ResNet-TW jointly aligns training samples for each class and computes a sample mean. In testing, we adopt the Nearest Centroid Classification (NCC) in which NCC is conducted by aligning first each test sample through the trained ResNet-TW (i.e. our learned metric) and thus returns a distance to each of the training set centroids.

In Figure 3, we provide a t-SNE visualization of the original and aligned data [39], illustrates how our ResNet-TW decreases intra-class variance while increasing inter-class one, thus improving the performance of classification. We compare our ResNet-TW to: (1) the sample mean of the misaligned sets (Euclidean); (2) DBA [28]; (3) SoftDTW [10] and (4) DTAN [43]. DBA and SoftDTW were measured by DTW distance, and DTAN is measured by Euclidean distance as ours. Figure 11 shows the NCC experimental results. Each point above the diagonal stands for an entire dataset of which our ResNet-TW correct classification rate is better than (or equal to) the competing method. This was the case for 89% of the datasets when compared to Euclidean, 72% for DBA, and 66% for SoftDTW and 61% for DTAN. These results (1) illustrate the importance of aligning the misaligned data for classification and (2) indicate that the average sequences of unwarped sequences synchronized by our ResNet-TW is usually more representative than other compared methods.

Figure 4: Correct classification rates using NCC. Each point above the diagonal indicates an entire UCR archive dataset where our ResNet-TW achieves better (or no worse) results than the comparing method. From left to right, our test accuracy compared with: Euclidean (ResNet-TW was better or no worse in 89% of the datasets), DBA (72%), SoftDTW (66%) and DTAN (61%).

4.2 Multivariate time-series (3D Action Recognition Datasets)

Florence 3D-Actions dataset [33] is captured using a Kinect camera. It includes 9 activities: wave, drink from a bottle, answer phone, clap, tight lace, sit down, stand up, read watch, bow. During
acquisition, 10 subjects were asked to perform the above actions for two or three times. **MSR Action-3D dataset** [22] consists of a total of 20 types of segmented actions. It consists of a total of 20 types of segmented actions: high arm wave, horizontal arm wave, hammer, hand catch, forward punch, high throw, draw x, draw tick, draw circle, hand clap, two hand wave, side boxing, bend, forward kick, side kick, jogging, tennis swing, tennis serve, golf swing, pick up & throw. Each action starts and ends with a neutral pose and is performed two or three times by each of the 10 actors. **MSR Daily Activity dataset** [42] is captured by a Kinect device and it consists of 16 activity types: drink, eat, read book, call cellphone, write on a paper, use laptop, use vacuum cleaner, cheer up, sit still, toss paper, play game, lay down on sofa, walk, play guitar, stand up, sit down. The challenging part here is that each subject performs an activity in two different poses: sitting and standing. Following the cross-subject experimental setting, where the first five actors are taken as training and the last five for testing, we provide our quantitative results. Before, that we present some qualitative results.

**Ablative study using synthetic warping.** For selected action sequences in Florence 3D, we randomly generate a synthetic warping function using `utility_functions.rgam` function in fdasrsf³, a python package for functional data analysis. We set the variance of warping functions to 10, such that we can synthesize warping functions with large temporal deformations. Next, we warp original action sequences \( g \) using these synthetic warping functions as target sequences \( f \). Finally, we adopt our ResNet-TW (for pairwise alignment) to estimate the synthetic warping functions \( \gamma \) by minimizing the Euclidean distance between the warped original sequences \( g \circ \gamma \) and target sequences \( f \).

![Figure 5: Estimation of synthetic warping functions for examples from Florence 3D dataset. Top: Sampled frames from original action sequences. Middle: Estimated warping functions by each block of our ResNet-TW. Bottom: Corresponding sampled frames from synthetically reparameterized sequences.](image)

![Figure 6: Effect of the # of building blocks in ResNet-TW on action sequences alignment.](image)

Four examples of estimated warping functions along with the ground-truth are shown in Figure 5. In particular, intermediate transformations \( \gamma_l \), derived at each building block are shown. The overlaps of solid blue lines with red dashed lines show that our ResNet-TW is able to estimate (large) temporal deformations in an incremental fashion, i.e., the final warping function is sum of a serial of small velocity fields. We also notice that the smoothness of all dashed lines indicate each warping function is diffeomorphic, i.e., both the function and its inverse are differentiable.

We also evaluate the effect of the number of composed diffeomorphic mappings by varying the number of blocks in ResNet-TW from 1 to 8. We adopt the Euclidean distance between the target sequence \( f \) and warped original sequence

³https://fdasrsf-python.readthedocs.io
We proposed a diffeomorphic temporal transformer for both pairwise (unsupervised) and joint (semi-supervised) sequences alignment. Our ResNet-TW estimates time-warping functions through an integration of smooth and regular velocity fields, building blocks of the residual architecture, and thus incrementally computes the final warping. Geometrically, our ResNet-TW is an Eulerian discretization of the ODE (i.e. the flow equation) which governs the final reparameterization (temporal warping) transformation. Regularized neural functions guarantees smooth and regular velocity fields. Intermediate warping functions are also diffeomorphic. Experiments on several datasets validates our ResNet-TW. It does not only align pairwise sequential data but also is capable of learning representative sample average sequences for multi-class joint alignment. Our ResNet-TW builds a step further in bridging between Deep Residual Networks and geometric diffeomorphic frameworks (i.e., LDDMM [3]). We leave the comparison of the static (Gaussian) kernel used in LDDMM and the
dynamic kernels inferred by the building blocks of our ResNet and induced RKHS [5] to guarantee smooth and regular velocity fields [25] for a future investigation.

References

[1] Boulbaba Ben Amor, Jingyong Su, and Anuj Srivastava. Action recognition using rate-invariant analysis of skeletal shape trajectories. IEEE Transactions on Pattern Analysis and Machine Intelligence, 38(1):1–13, 2016.

[2] Vincent Arsigny, Olivier Commowick, Xavier Pennec, and Nicholas Ayache. A log-euclidean framework for statistics on diffeomorphisms. In International Conference on Medical Image Computing and Computer-Assisted Intervention, pages 924–931. Springer, 2006.

[3] M Faisal Beg, Michael I Miller, Alain Trouvé, and Laurent Younes. Computing large deformation metric mappings via geodesic flows of diffeomorphisms. International Journal of Computer Vision, 61(2):139–157, 2005.

[4] Richard Bellman and Robert Kalaba. On adaptive control processes. IRE Transactions on Automatic Control, 4(2):1–9, 1959.

[5] Alberto Bietti and Julien Mairal. Invariance and stability of deep convolutional representations. In NIPS 2017-31st Conference on Advances in Neural Information Processing Systems, pages 1622–1632, 2017.

[6] Yanping Chen, Eamonn Keogh, Bing Hu, Nurjahan Begum, Anthony Bagnall, Abdullah Mueen, and Gustavo Batista. The ucr time series classification archive. 2015. http://www.cs.ucr.edu/~eamonn/time_series_data/.

[7] Zhuoyuan Chen, Hailin Jin, Zhe Lin, Scott Cohen, and Ying Wu. Large displacement optical flow from nearest neighbor fields. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, pages 2443–2450, 2013.

[8] Mark Cox, Sridha Sridharan, Simon Lucey, and Jeffrey Cohn. Least squares congealing for unsupervised alignment of images. In Conference on Computer Vision and Pattern Recognition, pages 1–8. IEEE, 2008.

[9] Mark Cox, Sridha Sridharan, Simon Lucey, and Jeffrey Cohn. Least-squares congealing for large numbers of images. In International Conference on Computer Vision, pages 1949–1956. IEEE, 2009.

[10] Marco Cuturi and Mathieu Blondel. Soft-dtw: a differentiable loss function for time-series. In International Conference on Machine Learning, pages 894–903. PMLR, 2017.

[11] Marco Cuturi and Arnaud Doucet. Fast computation of wasserstein barycenters. In International Conference on Machine Learning, pages 685–693. PMLR, 2014.

[12] Adrian V Dalca, Marianne Rakic, John Guttag, and Mert R Sabuncu. Learning conditional deformable templates with convolutional networks. In Advances in neural information processing systems, 2019.

[13] Nicki Skafte Detlefsen, Oren Freifeld, and Soren Hauberg. Deep diffeomorphic transformer networks. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, pages 4403–4412, 2018.

[14] Oren Freifeld, Soren Hauberg, Kayhan Batmanghelich, and John W Fisher. Highly-expressive spaces of well-behaved transformations: Keeping it simple. In Proceedings of the IEEE International Conference on Computer Vision, pages 2911–2919, 2015.

[15] Oren Freifeld, Soren Hauberg, Kayhan Batmanghelich, and John W Fisher. Transformations based on continuous piecewise-affine velocity fields. IEEE Transactions on Pattern Analysis and Machine Intelligence, 39(12):2496–2509, 2017.

[16] Eldad Haber and Lars Ruthotto. Stable architectures for deep neural networks. Inverse Problems, 34(1):014004, 2017.

[17] Mehdi Hadj-Hamou, Marco Lorenzi, Nicholas Ayache, and Xavier Pennec. Longitudinal analysis of image time series with diffeomorphic deformations: a computational framework based on stationary velocity fields. Frontiers in neuroscience, 10:236, 2016.
[18] Gabriel L Hart, Christopher Zach, and Marc Niethammer. An optimal control approach for deformable registration. In *IEEE Computer Society Conference on Computer Vision and Pattern Recognition Workshops*, pages 9–16. IEEE, 2009.

[19] Gary B Huang, Vidit Jain, and Erik Learned-Miller. Unsupervised joint alignment of complex images. In *2007 IEEE 11th International Conference on Computer Vision*, pages 1–8. IEEE, 2007.

[20] Max Jaderberg, Karen Simonyan, Andrew Zisserman, and Koray Kavukcuoglu. Spatial transformer networks. In *Advances in Neural Information Processing Systems*, pages 2017–2025, 2015.

[21] Erik G Learned-Miller. Data driven image models through continuous joint alignment. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 28(2):236–250, 2005.

[22] Wanqing Li, Zhengyou Zhang, and Zicheng Liu. Action recognition based on a bag of 3d points. In *IEEE Computer Society Conference on Computer Vision and Pattern Recognition Workshops*, pages 9–14. IEEE, 2010.

[23] Suhas Lohit, Qiao Wang, and Pavan Turaga. Temporal transformer networks: Joint learning of invariant and discriminative time warping. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 12426–12435, 2019.

[24] Erik G Miller, Nicholas E Matsakis, and Paul A Viola. Learning from one example through shared densities on transforms. In *Proceedings IEEE Conference on Computer Vision and Pattern Recognition*, volume 1, pages 464–471. IEEE, 2000.

[25] Michael I Miller, Alain Trouvé, and Laurent Younes. Geodesic shooting for computational anatomy. *Journal of mathematical imaging and vision*, 24(2):209–228, 2006.

[26] Elvis Nunez and Shantanu H Joshi. Deep learning of warping functions for shape analysis. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition Workshops*, pages 866–867, 2020.

[27] François Petitjean, Germain Forestier, Geoffrey I Webb, Ann E Nicholson, Yanping Chen, and Eamonn Keogh. Dynamic time warping averaging of time series allows faster and more accurate classification. In *2014 IEEE International Conference on Data Mining*, pages 470–479. IEEE, 2014.

[28] François Petitjean, Alain Ketterlin, and Pierre Gançarski. A global averaging method for dynamic time warping, with applications to clustering. *Pattern Recognition*, 44(3):678–693, 2011.

[29] François Rousseau, Lucas Drumetz, and Ronan Fablet. Residual networks as flows of diffeomorphisms. *Journal of Mathematical Imaging and Vision*, pages 1–11, 2019.

[30] Lars Ruthotto and Eldad Haber. Deep neural networks motivated by partial differential equations. *Journal of Mathematical Imaging and Vision*, pages 1–13, 2019.

[31] Hiroaki Sakoe. Dynamic-programming approach to continuous speech recognition. In *1971 Proc. the International Congress of Acoustics, Budapest*, 1971.

[32] Hiroaki Sakoe and Seibi Chiba. Dynamic programming algorithm optimization for spoken word recognition. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 26(1):43–49, 1978.

[33] Lorenzo Seidenari, Vincenzo Varano, Stefano Berrett, Alberto Bimbo, and Pietro Pala. Recognizing actions from depth cameras as weakly aligned multi-part bag-of-poses. In *IEEE Computer Society Conference on Computer Vision and Pattern Recognition Workshops*, pages 479–485, 2013.

[34] Anuj Srivastava, Eric Klassen, Shantanu H Joshi, and Ian H Jermyn. Shape analysis of elastic curves in euclidean spaces. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 33(7):1415–1428, 2010.

[35] Anuj Srivastava and Eric P Klassen. *Functional and shape data analysis*, volume 1. Springer, 2016.

[36] Bing Su and Ying Wu. Learning meta-distance for sequences by learning a ground metric via virtual sequence regression. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2020.
[37] Guo-Zheng Sun, Hsing-Hen Chen, and Yee-Chun Lee. Time warping invariant neural networks. In Advances in neural information processing systems, pages 180–187, 1993.

[38] George Trigeorgis, Mihalis A Nicolaou, Stefanos Zafeiriou, and Bjorn W Schuller. Deep canonical time warping. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, pages 5110–5118, 2016.

[39] Laurens Van der Maaten and Geoffrey Hinton. Visualizing data using t-sne. Journal of Machine Learning Research, 9(11), 2008.

[40] Ashok Veeraraghavan, Anuj Srivastava, Amit K Roy-Chowdhury, and Rama Chellappa. Rate-invariant recognition of humans and their activities. IEEE Transactions on Image Processing, 18(6):1326–1339, 2009.

[41] Tom Vercauteren, Xavier Pennec, Aymeric Perchant, and Nicholas Ayache. Diffeomorphic demons: Efficient non-parametric image registration. NeuroImage, 45(1):S61–S72, 2009.

[42] Jiang Wang, Zicheng Liu, Ying Wu, and Junsong Yuan. Mining actionlet ensemble for action recognition with depth cameras. In IEEE Conference on Computer Vision and Pattern Recognition, pages 1290–1297. IEEE, 2012.

[43] Ron Shapira Weber, Matan Eyal, Nicki Skafte Detlefsen, Oren Shriki, and Oren Freifeld. Diffeomorphic temporal alignment nets. In Advances in Neural Information Processing Systems, pages 6570–6581, 2019.

[44] E Weinan. A proposal on machine learning via dynamical systems. Communications in Mathematics and Statistics, 5(1):1–11, 2017.

[45] Laurent Younes. Shapes and diffeomorphisms, volume 171. Springer, 2010.

[46] Feng Zhou and Fernando De la Torre. Generalized canonical time warping. IEEE Transactions on Pattern Analysis and Machine Intelligence, 38(2):279–294, 2015.

[47] Feng Zhou and Fernando Torre. Canonical time warping for alignment of human behavior. Advances in neural information processing systems, 22:2286–2294, 2009.
A Additional Qualitative Results on MSR Action-3D

In this section, we provide more qualitative results on MSR Action-3D dataset for both pairwise and multi-class joint alignment.

A.1 Pairwise Alignment

For each action category, we randomly pick two sequences performed by different actors, thus, there exists temporal misalignment between these two sequences. We treat one sequence as a query $g$ and another as a target $f$. We adopt our ResNet-TW to estimate the warping function by minimizing the Euclidean distance between the warped query sequence $g \circ \gamma$ and target sequence $f$. Figure 7 illustrates three examples of pairwise alignment.

A.2 Joint Alignment

In this subsection, we investigate whether our ResNet-TW can learn the “average” action sequences within categories. We use the five first subjects for training and the last five for testing. All sequences are up-sampled to 50 frames using the pipeline proposed in [1]. Six action sequences (green) from three different categories are sampled and visualized in Figure 8. We also visualize the average action sequences (red). Note that the average sequences are averaged over all testing samples in the corresponding categories, not the sampled two sequences. From Figure 8, we notice that the average sequences capture key poses of each action, showing the ability of our ResNet-TW to capture essential patterns of each action. Although the movement amplitudes of the average sequences decrease, we attribute such phenomena to the counteraction of spatial variability of actions performed by different actors.
Figure 7: Results of pairwise alignment on MSR Action-3D dataset. In each case, $f$ and $g$ (first column), $f$ and $g \circ \gamma^*$ (second column). We note that the temporal misalignment of the red and the green skeleton sequences is clear in the left column. In contrast, they are well aligned in the second column after temporal registration. The plot in the third column shows the optimal warping functions $\gamma^*$ obtained using our ResNet-TW.

(a) Hammer; top: a03_s01_e01; middle: a03_s06_e01.

(b) High arm wave; top: a01_s05_e02; middle: a01_s07_e01.

(c) Side tick; top: a15_s07_e03; middle: a15_s09_e01.

Figure 8: Joint multi-class alignment of MSR Action dataset. Green: original action sequences. Red: averaged sequences.
B Additional Alignment Results of Test Data

In this section, we provide more qualitative results of joint alignment of test data in different datasets from the UCR archive [6].

B.1 Within-class Variance Reduction

Figure 9: Within-class joint alignment of four datasets from the UCR archive [6]. All results are randomly selected from previously unseen test data. **First column**: misaligned signals (top), the average signal and ± standard deviation shown in red shaded area (bottom). **Second column**: aligned signals (top), the average signal and ± standard deviation shown in blue shaded area (bottom). **Third and fourth columns**: warping functions $\gamma$ produced by each layer of our ResNet-TW. The colors of short bars at the right-bottom on the third and fourth columns correspond to signals on the first and second columns.
Figure 9: Within-class joint alignment of four datasets from the UCR archive [6]. All results are randomly selected from previously unseen test data. **First column:** misaligned signals (top), the average signal and ± standard deviation shown in red shaded area (bottom). **Second column:** aligned signals (top), the average signal and ± standard deviation shown in blue shaded area (bottom). **Third and fourth columns:** warping functions $\gamma$ produced by each layer of our ResNet-TW. The colors of short bars at the right-bottom on the third and fourth columns correspond to signals on the first and second columns.
Figure 10: Examples of ECG200 dataset from the UCR archive [6]. First row: 10 randomly selected test data from each class of the dataset. Second until the last rows: aligned signals using the warping functions \( \gamma \) produce by each block of our ResNet-TW. The blue curve represents the sample mean of the signals.

C Nearest Centroid Classification (NCC) Results

In this section, we show detailed quantitative results of the NCC experiment on UCR archive [6] in Figure 11 and Table 2\(^4\). For the baseline experiment we used the Euclidean mean of the misaligned set. We compare our ResNet-TW with DTW Barycenter Averaging (DBA) [28], SoftDTW [10], DTAN [43].

\(^4\)We update our results in this document.
Figure 11: Correct classification rates using NCC on UCR archive [6]. Each point above the diagonal indicates an entire UCR archive dataset where our ResNet-TW achieves better (or no worse) results than the comparing method. From left to right, our test accuracy compared with: Euclidean (ResNet-TW was better or no worse in 96% of the datasets), DBA (79%), SoftDTW (69%) and DTAN (68%).

| Dataset Baseline | Softdtw | DBA | DTAN | Ours |
|-----------------|---------|-----|------|------|
| 50words         | 0.516484 | 0.615385 | **0.652747** | 0.516484 |
| Adiac           | 0.549872 | 0.501279 | **0.615385** | 0.549872 |
| ArrowHead       | 0.611429 | 0.520000 | **0.748571** | 0.754286 |
| Beef            | 0.533333 | 0.566667 | **0.633333** | 0.633333 |
| BeetleFly       | 0.850000 | 0.850000 | **0.900000** | 0.954000 |
| BirdChicken     | 0.550000 | 0.700000 | 0.600000 | **0.800000** |
| CBF             | 0.763333 | **0.971111** | 0.965556 | 0.850000 |
| Car             | 0.616667 | 0.683333 | 0.633333 | **0.816667** |
| ChlorineConcentration | 0.333073 | 0.348177 | 0.323698 | 0.333073 |
| CinC_ECG_torso  | 0.385507 | 0.398551 | 0.445652 | 0.615942 |
| Coffee          | 0.964286 | 0.964286 | **0.970588** | 0.964286 |
| Computers       | 0.416000 | 0.640000 | 0.616000 | **0.816667** |
| Cricket_X       | 0.238462 | **0.602564** | 0.574359 | 0.423077 |
| Cricket_Y       | 0.348718 | 0.571795 | 0.541026 | 0.415385 |
| Cricket_Z       | 0.305128 | **0.615385** | 0.605128 | 0.420513 |
| DiatomSizeReduction | 0.957516 | 0.950980 | 0.950980 | **0.973856** |
| DistalPhalanxOutlineAgeGroup | 0.817500 | 0.850000 | 0.840000 | **0.875000** |
| DistalPhalanxOutlineCorrect | 0.471667 | 0.490000 | 0.483333 | **0.505000** |
| DistalPhalanxTW  | 0.747500 | 0.760000 | 0.755000 | **0.797500** |
| ECG200          | 0.750000 | 0.730000 | 0.720000 | 0.790000 |
| ECG5000         | 0.860444 | 0.853778 | 0.834667 | **0.891333** |
| ECGFiveDays     | 0.689895 | 0.670151 | 0.658537 | **0.977933** |
| Earthquakes     | 0.754658 | 0.822981 | 0.757435 | 0.773292 |
| ElectricDevices | 0.482687 | 0.539748 | 0.539748 | **0.534820** |
| FaceAll         | 0.491716 | 0.827811 | 0.796450 | **0.840909** |
| FaceFour        | 0.840909 | 0.852273 | 0.852273 | **0.851522** |
| FacesUCR        | 0.539512 | 0.812683 | 0.774634 | **0.857143** |
| FISH            | 0.560000 | 0.697143 | 0.651429 | 0.902857 |
| FordA           | 0.495973 | 0.552902 | 0.549570 | **0.604832** |
| FordB           | 0.499725 | **0.591309** | 0.568482 | 0.579758 |
| Gun_Point       | 0.753333 | 0.733333 | 0.700000 | **0.880000** |
| Ham             | 0.761905 | 0.733333 | 0.723810 | **0.790476** |
| HandOutlines    | 0.818000 | 0.812000 | 0.804000 | **0.850000** |
| Haptics         | 0.392857 | 0.373377 | 0.350649 | **0.457792** |
| Herring         | 0.546875 | 0.609375 | 0.546875 | **0.703125** |
| InlineSkate     | 0.192727 | 0.252727 | 0.232727 | **0.260000** |
| InsectWingbeatSound | **0.601010** | 0.328283 | 0.289394 | 0.538734 |
| ItalyPowerDemand| 0.918367 | 0.750243 | 0.730807 | **0.962099** |
| LargeKitchenAppliances | 0.440000 | **0.733333** | 0.728000 | 0.842667 |
| Lighting2       | 0.688525 | 0.622951 | 0.639344 | **0.721311** |
| Lighting7       | 0.589041 | **0.726027** | 0.698630 | 0.712329 |
| MALLAT          | 0.966738 | 0.953945 | 0.952665 | **0.968870** |

18
| Dataset                                | Baseline | Softdtw | DBA    | DTAN  | Ours  |
|----------------------------------------|----------|---------|--------|-------|-------|
| Meat                                   | 0.93333  | 0.93333 | 0.91667| 0.93333| 0.93333|
| MedicalImages                          | 0.385526 | 0.461842| 0.436842| 0.468421| 0.473684|
| MiddlePhalanxOutlineAgeGroup           | 0.732500 | **0.795000** | 0.712500 | 0.737500 | 0.752500 |
| MiddlePhalanxOutlineCorrect            |          |         |        |       | 0.551667|
| MiddlePhalanxTW                        | 0.591479 | 0.581454 | 0.556391 | 0.596491 | 0.531667 |
| MoteStrain                             | 0.861022 | 0.843450 | 0.826767 | 0.904153 | 0.912939 |
| NonInvasiveFetalECGThorax1             | 0.769466 | 0.710941 | 0.712977 | 0.853435 | 0.838677 |
| NonInvasiveFetalECGThorax2             | 0.802036 | 0.773028 | 0.763868 | 0.905344 | 0.838680 |
| OliveOil                               | 0.866667 | 0.800000 | 0.766667 | 0.866667 | 0.866667 |
| OSULeaf                                | 0.359504 | 0.475207 | 0.438017 | 0.462810 | 0.458678 |
| PhalangesOutlinesCorrect               | 0.625874 | 0.637529 | 0.632867 | 0.642191 | 0.663170 |
| Phoneme                                | 0.078586 |          |        |       | 0.078586 |
| Plane                                  | 0.961905 | 0.990476 | 1.000000 | 1.000000 | 1.000000 |
| ProximalPhalanxOutlineAgeGroup         | 0.819512 | 0.853659 | 0.843902 | 0.853659 | 0.873171 |
| ProximalPhalanxOutlineCorrect          | 0.646048 | 0.725086 | 0.649485 | 0.642612 | 0.687285 |
| ProximalPhalanxTW                      | 0.707500 | 0.747500 | 0.735000 | 0.817500 | 0.822500 |
| RefrigerationDevices                   | 0.354667 | 0.586667 | 0.584000 | 0.466667 | 0.482667 |
| ScreenType                             | 0.442667 | 0.389333 | 0.378667 | 0.445333 | 0.469333 |
| ShapeletSim                            | 0.500000 | **0.588889** | 0.522222 | 0.538889 | 0.588889 |
| ShapesAll                              | 0.513333 | 0.628333 | 0.603333 | 0.628333 | 0.681667 |
| SmallKitchenAppliances                 | 0.418667 | 0.658667 | **0.661333** | 0.621333 | 0.560000 |
| SonyAIBORobotSurface                   | 0.811980 | **0.893511** | 0.835275 | **0.893511** | 0.860233 |
| SonyAIBORobotSurfaceII                 | 0.793284 | 0.772298 | 0.766002 | 0.811123 | 0.830100 |
| Strawberry                             | 0.668842 | 0.649266 | 0.616667 | 0.843393 | 0.786297 |
| SwedishLeaf                            | 0.702400 | 0.723200 | 0.681600 | 0.806400 | 0.836800 |
| Symbols                                | 0.864322 | **0.954774** | **0.954774** | 0.857286 | 0.906533 |
| Synthetic_control                      | 0.916667 | 0.980000 | 0.980000 | 0.950000 | 0.950000 |
| ToeSegmentation1                       | 0.574561 | **0.671053** | 0.614035 | 0.640351 | 0.653509 |
| ToeSegmentation2                       | 0.546154 | **0.853846** | 0.838462 | 0.753846 | 0.746154 |
| Trace                                  | 0.580000 | **0.970000** | **0.970000** | 0.780000 | 0.800000 |
| Two_Patterns                           | 0.464750 | 0.989750 | **0.975000** | 0.555750 | 0.700500 |
| TwoLeadECG                             | 0.554873 | 0.801580 | 0.81238 | **0.956102** | 0.955224 |
| uWaveGestureLibrary_X                  | 0.631212 | 0.706868 | 0.676438 | 0.681184 | **0.721943** |
| uWaveGestureLibrary_Y                  | 0.548297 | 0.564768 | 0.525405 | 0.611669 | **0.617253** |
| uWaveGestureLibrary_Z                  | 0.537409 | 0.604132 | 0.592406 | 0.642099 | **0.646287** |
| UWaveGestureLibraryAll                 | 0.849725 | 0.836313 | 0.83937 | **0.920715** | 0.911502 |
| wafer                                  | 0.654445 | 0.649416 | 0.511032 | **0.988968** | 0.982803 |
| Wine                                   | 0.555556 | 0.574074 | 0.518519 | 0.574074 | **0.592593** |
| WordsSynonyms                          | 0.271160 | 0.412226 | 0.344828 | 0.474922 | **0.501567** |
| Worms                                  | 0.215470 | 0.408840 | **0.41365** | 0.259669 | 0.342541 |
| WormsTwoClass                          | 0.541436 | 0.651934 | 0.591160 | 0.618785 | **0.618785** |
| yoga                                   | 0.497000 | 0.574000 | 0.557000 | 0.631667 | **0.696667** |

Table 2: Quantitative results of nearest centroid classification on UCR archive [6].