We find a mixed chirality $d$-wave superconducting state in the coexistence region between antiferromagnetism and interaction-driven superconductivity in lightly doped honeycomb materials. This state has a topological chiral $d + id$-wave symmetry in one Dirac valley but $d - id$-wave symmetry in the other valley and hosts two counter-propagating edge states, protected in the absence of intervalley scattering. A first-order topological phase transition, with no bulk gap closing, separates the chiral $d$-wave state at small magnetic moments from the mixed chirality $d$-wave phase.

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Topological Superconductivity in Two Dimensions with Mixed Chirality

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Superconducting (SC) pairing driven by electron repulsion in two dimensions (2D) has spin-singlet $d$-wave symmetry and appears close to, or even coexists with, an antiferromagnetic (AF) phase in materials ranging from cuprates to heavy fermion compounds and organic superconductors [1–6]. In materials with three- and sixfold rotational lattice symmetries, the two $d$-wave states, $d_1 = d(x^2 - y^2)$ and $d_2 = d(xy)$, are dictated to be degenerate at the transition temperature $T_c$ and at lower temperatures the chiral $d_1 \pm id_2$ combinations are favored [7–12]. These chiral $d$-wave states are fully gapped, topologically non-trivial with finite Chern (or winding) numbers, and have two chiral edge states [13–15]. Thus, $d$-wave SC pairing on honeycomb and triangular lattices naturally blend strong electron interactions and topology, resulting in highly unconventional states of matter.

The honeycomb lattice near half-filling adds further versatility by having two disjoint Fermi surfaces, or valleys, centered at the inequivalent Brillouin zone corners $\pm K$. Recently, several materials with honeycomb lattice have been proposed to be chiral $d$-wave superconductors, including In$_3$C$_2$V$_2$O$_9$ [16–19], SrPtAs [20–22], MoS$_2$ [23–25], graphene and silicene [11, 12, 26–28], and (111) bilayer SrIrO$_3$ [29, 30]. Two Fermi surfaces allow for the fascinating idea of having $d_1 + id_2$ symmetry in one valley but $d_1 - id_2$ symmetry in the other; a novel state with mixed chirality, even in a translationally invariant system [19, 31]. However, as noted in Ref. [32], the sign change of the $d_2$ component between the valleys requires it to be spin-triplet in order to fulfill Fermi-Dirac statistics. Since the $d$-wave SC state usually arises from distinctly spin-singlet mechanisms, such as in the hole-doped Mott insulator, this appears to pose an unsurmountable obstacle for actually realizing a mixed chirality $d$-wave state.

In this Letter we show that mixed chirality $d$-wave superconductivity is allowed, and even likely, in the coexistence region between AF and $d$-wave SC order in lightly doped honeycomb materials. A finite AF moment $M$ in a spin-singlet superconductor has been shown to spontaneously generate a so-called $\pi$-triplet state [33–36]. We find that this spin-triplet state facilitates a phase transition at a critical $M_c$ from the chiral $d$-wave state to the mixed chirality $d$-wave state. $M_c$ is well within the AF-SC coexistence region found in the hole-doped honeycomb Mott insulator [37, 38]. In terms of properties, the mixed chirality state is topological in each Dirac valley, although the topological invariant cancels when summed over the full Brillouin zone. We find that the phase transition between the chiral and mixed chirality $d$-wave phases occurs without the bulk energy gap closing, otherwise a necessity for any topological phase transition [39, 40]. At the phase transition the two co-propagating edge states of the chiral $d$-wave state are discontinuously converted to two counter-propagating states. By introducing disorder, we show that the mixed chirality edge states are only protected in the absence of intervalley scattering. These results establish both the existence and properties of a highly unconventional mixed chirality SC state. In the same way that the chiral $d$-wave SC state has many properties common with quantum Hall states [41], the mixed chirality $d$-wave state is similar to a quantum valley Hall state [42, 43].

We start by creating a spin-singlet chiral $d$-wave SC state on the honeycomb lattice using [15]:

$$\mathcal{H}_{SC} = -t \sum_{\langle i,j \rangle, \sigma} (a_{i\sigma}^\dagger b_{j\sigma} + \text{H.c.}) + \mu \sum_i (a_{i\uparrow}^\dagger a_{i\uparrow} + b_{i\uparrow}^\dagger b_{i\uparrow}) + \sum_{i,\alpha} \Delta_\alpha (a_{i\uparrow}^\dagger b_{i+R_{\alpha}\downarrow} - a_{i\downarrow}^\dagger b_{i+R_{\alpha}\uparrow}),$$

where $a$ ($b$) is the annihilation operator on sublattice A (B). As is well-known from graphene [44], the nearest neighbor hopping $t$ gives for small doping levels ($|\mu| < t$) a normal-state band structure with two Dirac valleys centered at the Brillouin zone corners $K = 4\pi/(3a)$ and $K' = -K$, where $a = 1$ is the length of the unit cell vectors. The mean-field SC order parameters (OPs) $\Delta_\alpha$ ($\alpha = 1, 2, 3$) on the three nearest-neighbor bonds $R_\alpha$ can be expressed compactly as $\Delta = (\Delta_1, \Delta_2, \Delta_3)$. Each OP is independently determined by the self-consistency equations $\Delta_\alpha = -J (a_{i\uparrow} b_{i+R_{\alpha}\downarrow} - a_{i\downarrow} b_{i+R_{\alpha}\uparrow})$, where $J$ is the pair potential. Eq. (1) is the BCS Hamiltonian for
the $t$-$J$ model within renormalized mean-field theory [19, 45–47], where statistical weighting factors renormalize $t$ and $J$ to implement the Gutzwiller projection to single-occupancy states [45, 48]. It can also be viewed quite generally as a minimal model for the spin-singlet chiral $d$-wave SC state [15]. We add a finite AF moment $M$ using $H_{\text{AF}} = M \sum_i (a_i^\dagger a_i - a_i a_i^\dagger)$. 

To proceed, we first Fourier transform and then rewrite $H$ self-consistently for $\Delta$, see e.g. Refs. [11, 14]. To proceed, we first Fourier transform and then rewrite $H$ in the basis where the kinetic and magnetic terms are fully diagonal:

$$H = \sum_{k, \sigma} (\mu - E_k) c_k^\dagger c_{k,\sigma} + (\mu + E_k) d_k^\dagger d_{k,\sigma}$$

$$+ \sum_{k, \alpha} (\Delta_{k+} + \Delta_{k+}^M) c_k^\dagger c_{-k,\alpha} + (-\Delta_{k-} + \Delta_{k-}^M) d_k^\dagger d_{-k,\alpha}$$

$$+ \Delta_{k+} c_{k+}^\dagger c_{-k,\alpha} - \Delta_{k-} c_{k-}^\dagger d_{-k,\alpha}.$$  (2)

Here $c (d)$ is the annihilation operator in the lower (upper) band with band dispersion $E_k = \sqrt{(t\varepsilon_k)^2 + M^2}$, where $\varepsilon_k = \sum_{\alpha} \varepsilon_k R_{\alpha}$ and $\varepsilon_k = \arg(\sum_{\alpha} \varepsilon_k R_{\alpha})$. Diagonalizing the normal state results in intraband pairing

$$\Delta_{k}^i = \sum_{\alpha} \Delta_{\alpha}(\mathbf{k} \cdot \mathbf{R}_{\alpha} - \varphi_{\alpha}),$$

$$\Delta_{k}^M = -i \frac{M}{E_k} \sum_{\alpha} \Delta_{\alpha}(\mathbf{k} \cdot \mathbf{R}_{\alpha} - \varphi_{\alpha})$$  (3)

and also a less important interband pairing term $\Delta_{k}' = (it\varepsilon_k)/E_k \sum_{\alpha} \Delta_{\alpha}(\mathbf{k} \cdot \mathbf{R}_{\alpha} - \varphi_{\alpha}).$

**OPs in SC phase.**—We start by analyzing the pure SC phase at $M = 0$. The generally favored $\Delta_{\alpha}$ belongs to the 2D $E_{2g}$ irreducible representation of the $D_{6h}$ lattice point group [11]. The state can be written as a combination of $\Delta \sim (2, -1, -1)$, which gives $d_1$-wave intraband (4-wave) pairing, and $\Delta \sim (0, 1, -1)$, giving $d_2$-wave intraband pairing. The $d_1,2$-wave solutions are degenerate at $T_c$, but below $T_c$, the time-reversal symmetry breaking chiral combinations $d_{1,2} = d_1 \pm id_2$ have the lowest energy [11, 12, 19, 32]. This follows from a simple energy argument since the $d_{1,2}$ states have nodal quasiparticles, whereas the $d_{p,m}$ states are fully gapped, see Fig. 1. The chiral $d$-wave states have $N = \pm 2$ Chern numbers, which can be viewed as the winding numbers for the intraband OP around the Brillouin zone center [13, 15]. We can, alternatively, consider the symmetry of the intraband OP around $K, K'$. The $d_0$ state has $-p_x + ip_y$-wave symmetry around $K$, but $p_y - ip_x$-wave symmetry around $K'$ [49], such that each valley contribute $-1$ to $N$. The sign change between the valleys is dictated by the spin-singlet nature, which enforces even parity with respect to the zone center. The interband (4-wave) pairing, on the other hand, has odd spatial parity, but is a spin-singlet, as it is odd under band index exchange. The interband pairing is not important for the energy spectrum [11, 32] or the behavior at finite $M$ and we thus here only focus on the intraband pairing.

**OPs in AF-SC phase.**—At finite $M$, the spin-triplet odd spatial parity OP $\Delta^{IM}$ is added to the intraband pairing. A spin-triplet state, often referred to as a $\pi$-triplet, is known to be spontaneously generated when a finite AF moment coexists with a spin-singlet SC state [33–36]. Here we find the spin-triplet state directly proportional to $M$ and $\pi/2$ phase-shifted relative to $\Delta^i$. This has dramatic consequences on the honeycomb lattice. Adding $\Delta^{IM}$ to the $d_1$ state makes the intraband OP develop an imaginary part, such that it has $-p_y + ip_x$-wave symmetry around $K$, but the opposite $p_y + ip_x$-wave chirality around $K'$, see Fig. 1. Thus at finite $M$, the $d_1$
state becomes a time-reversal breaking mixed chirality SC state with opposite OP winding in the two valleys, but \( N = 0 \) if summed over the whole Brillouin zone. The mixed chirality state requires the combination of a spin-singlet \((p_0)\) and a spin-triplet \((p_2)\) component, since the spin-singlet pairing changes sign between \( K \) and \( K' \), but the spin-triplet does not. This is why a magnetic moment \( M \), with its accompanying spin-triplet \( \Delta^{IM} \), is necessary for generating mixed chirality \( d\)-wave superconductivity. Instead starting from the \( d_2 \)-wave state just interchanges \( x \) and \( y \). With the \( \Delta^{IM} \) contribution, the \( d_2 \) state become fully gapped, as shown in Fig. 1. Quite the opposite happens for the \( d_p \) state. At finite \( M \) it still has a \( N = -2 \) winding. However, the \( \Delta' \) and \( \Delta^{IM} \) intraband terms start to cancel around \( K' \), as seen in Fig. 1. This results in low-lying quasiparticle excitations around \( K' \) (\( K \) for \( d_m \)) as \( M \) increases.

**Critical magnetic moment.**—From the quasiparticle energy spectrum it is clear that a phase transition likely occurs at a critical \( M_c \) between the chiral \( d \)-wave state, favorable at \( M = 0 \), and the mixed chirality \( d\)-wave state. By minimizing the free energy of \( \mathcal{H} \) with respect to the three bond OPs \( \Delta_\alpha \) for fixed pair potential \( J \) and doping \( \mu \), we find both the magnitude and symmetry of the favored SC state, and can thus locate any \( M_c \). We have investigated a wide range of \( J \) and \( 0 < |\mu| < t \), and find that the phase transition into the mixed chirality state always takes place for small enough \( M \) such that superconductivity is not suppressed by the magnetic order. To quantify the behavior of \( M_c \), we plot \( M_c \) as a function of the filling fraction \( \delta \) in Fig. 2(a). As seen, \( M_c \) decreases when approaching half-filling. This is highly advantageous for achieving the mixed chirality state, since the AF moment is largest at half-filling. \( M_c \) does not depend strongly on \( J \) for weak superconductivity, but for a stronger SC state we find that \( M_c \) slightly decreases. Since strong superconductivity can suppress the AF moment, also this trend is beneficial for realizing the mixed chirality state. In fact, a large AF-SC coexistence region has very recently been found in the \( t-J \) model on the honeycomb lattice for \( \delta \lesssim 0.1 \), with \( M \gtrsim 0.2 \) for \( \delta < 0.05 \) [37, 38]. According to Fig. 2(a) results in the mixed chirality state for all investigated parameters.

**Phase transition.**—Having established the existence of \( M_c \) for realizing the mixed chirality state, we now analyze the phase transition itself. In Fig. 2(b) we plot the bond OPs as function of \( M \). The perfect \( d_p \) solution at \( M = 0 \), with equal parts of \( d_1 \) and \( d_2 \)-wave character (inset), becomes a slightly imperfect \( d_p \) state for \( 0 < M < M_c \), because the imaginary part of the bond OPs is somewhat suppressed at finite \( M \). However, as long as the imaginary part of the bond OP is finite, the chiral topology is preserved. At \( M_c \) there is a sudden jump in the bond OPs, such that the imaginary part disappears and the real part increases. This results the \( d_1 \)-wave character of the bond OP jumping from 0.56 to 1, with only a 0.6% change in the magnetization at \( M_c \). This large discontinuous change of the OP at \( M_c \) is accompanied with a similarly discontinuous derivative in the free energy and strongly supports a first order phase transition between the chiral \( d \)-wave state and the mixed chirality \( d \)-wave state. The phase transition to the mixed chirality state is driven by minimizing the number of low-lying quasiparticle excitations. The mixed chirality state is always fully gapped, with the gap increasing with \( M \), but the chiral state develops low-energy quasiparticle excitations at finite \( M \), as seen in Fig. 1. However, the phase transition takes place well before the chiral \( d \)-wave state develops any zero-energy states. Thus, the system remains fully gapped in the whole the AF-SC coexistence phase [55].

The fully gapped bulk energy spectrum is very interesting because of the topological nature of the phase transition. At \( M = 0 \), the chiral \( d \)-wave state belongs to symmetry class \( C \), as it has full spin-rotation symmetry [50, 51]. The \( C \) classification allows for \( Z \) different topological states in 2D. However, as soon as we add the AF moment, only rotations around \( S_z \) are left invariant. This results in symmetry class \( A \) for finite \( M \), which also has \( Z \) topological classification [50, 51]. The chiral \( d \)-wave state is therefore always classified by a \( |\mathcal{N}| = 2 \) winding number. On the other hand, the mixed chirality state at \( M > M_c \) has opposite OP windings in the two valleys. Thus, while in each valley is topologically non-trivial, the topological invariant cancels when summed over the full Brillouin zone. Thus, the phase transition at \( M_c \) is a topological phase transition between two topologically distinct phases of matter. Topological phase transitions

![FIG. 2: (Color online). (a) Critical magnetic moment \( M_c \) as a function of filling fraction \( \delta \) for different pair potentials \( J \). Highest \( \delta \) reported corresponds to \( \mu = 0.8t \). (b) Bond OPs \( \Delta_\alpha \) as a function of magnetic moment \( M \) with a phase transition at \( M_c = 0.168t \), where imaginary part (purple) of the bond OPs disappears while real part (black) increases for \( J = 1.3t \), \( \mu = 0.5t \). Inset shows \( d_1 \)-wave (red) and \( d_2 \)-wave (blue) characters of \( \Delta \).](image)
have widely been assumed to require the bulk energy gap to close, as that is the only generic way to change topological order [39, 40, 52]. The topological transition between the chiral and the mixed chirality $d$-wave states provides an explicit counterexample. This is possible because here the topological transition is the result of a first-order transition within the Ginzburg-Landau paradigm.

**Edge states.**—A defining property of topological states is the existence of edge modes, with the bulk-boundary correspondence equaling the number of edge states with the change of topological number at the edge [39]. To investigate the edge band structure we solve self-consistently for the SC OPs at every site in a thick ribbon. For $M < M_c$ the Dirac valleys at $K, K'$ each give rise to one state per edge, which are co-propagating, see Fig. 3(a), in agreement with the change of topological number at the edge. This result is for zigzag edges, but armchair edges has been shown to behave very similarly [14]. At $M = M_c$ the direction of the edge states at $K'$ changes discontinuously due to the first-order transition, and the edge states instead become counter-propagating reflecting the mixed chirality, see Fig. 3(b).

![FIG. 3: (Color online). Self-consistent band structures for zigzag ribbons. Thick ribbon with $M = 0$ (a) and $M = 0.3t > M_c$ (b), with left (red) and right (green) edge states. Thin (16a thick) ribbon (c) with $M = 0$ (black), $M = 0.14t < M_c$ (orange), and $M = 0.3t > M_c$ (blue). Extended supercell (20a wide) with $M = 0.42t > M_c$ (d) with on-site Anderson disorder $W = 0.1t$ (purple) compared to clean system (black). Here $J = 1.3t$, $\mu = 0.8t$ ($|\Delta| = 0.27t$ at $M = 0$).](image)

Reducing the width of the ribbon allows the study of finite size effects on superconductivity. We find $M_c$ to be significantly reduced in all but extremely thick ribbons. For example, in a $J = 1.3t$, $\mu = 0.8t$ ribbon, perfect chiral $d$-wave symmetry is reached already within 15 unit cells from the edge. Still, $M_c = 0.25t$ even in a ribbon 100 unit cells thick, compared to $M_c = 0.29t$ in the bulk. Thus, ribbons provide a promising route for inducing the mixed chirality state below the bulk $M_c$. For very thin ribbons we find that the phase transition is not sharply defined, instead there is gradual suppression of the imaginary part of the bond OPs as $M$ increases. Reducing the ribbon thickness also gives hybridization between the left and right edge states. This is a scattering process within each Dirac valley, which thus gaps the edge state spectra in both phases. We find no notable difference between the thin ribbon energy gaps deep within the chiral and the mixed chirality $d$-wave phases, see Fig. 3(c). However, for $M \lesssim M_c$ low-lying bulk excitations are present at $K'$ (see Fig. 1), resulting in a larger gap at $K'$ just prior to the phase transition.

Finally, we also study the effect of disorder. We find no influence of weak to moderate disorder on $M_c$. The two chiral $d$-wave edge states are topologically protected and also not sensitive to disorder [14]. However, the mixed chirality edge states should only be protected in the absence of intervalley processes: the edge states, one from each valley, are counter-propagating and thus intervalley scattering opens a back-scattering channel. We here demonstrate the sensitivity to intervalley scattering for the mixed chirality edge states by using Anderson disorder, i.e. site fluctuating chemical potential $\mu + \delta \mu$, with $\delta \mu$ distributed randomly within the interval $[-W, W]$. Such disorder captures charge inhomogeneity, known to be important for transport properties in graphene [53]. We here create disordered samples by using a $T = 20a$ wide (perpendicular to edges) ribbon supercell, which reduces the 1D Brillouin zone to $k \in [-\pi/T, \pi/T]$. With the supercell momentum always preserved, intervalley scattering is only present for edge states centered at $\Gamma$ or $\pi/T$. Although this sounds as if it might require fine-tuning, we find that finite disorder often locks the edge state to $\pi/T$, as exemplified for a particular disorder configuration in Fig. 3(d). While the clean system has its zero-energy crossing located away from $\pi/T$, finite disorder gives a gapped edge state centered at $\pi/T$.

In summary, we have shown that a mixed chirality topological $d$-wave SC state exists in the AF-SC coexistence region in lightly doped honeycomb materials. At a finite AF moment there is a first-order topological phase transition from the chiral to the mixed chirality $d$-wave SC state. The mixed chirality state hosts two counter-propagating edge modes, protected in the absence of intervalley scattering. These results should also be applicable to bilayer honeycomb materials, which hosts a similar chiral $d$-wave state [26–28]. Our work show that interaction-driven superconductivity in honeycomb materials harbors multiple fascinating SC states. Recently developed novel numerical approaches for time-reversal broken superconductivity in hole-doped Mott insulators [37, 54] could offer the possibility of more precise treatments.

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[1] D. J. Van Harlingen, Rev. Mod. Phys. 67, 515 (1995).
[2] C. C. Tsuei and J. R. Kirtley, Rev. Mod. Phys. 72, 969 (2000).
[3] J. D. Thompson and Z. Fisk, J. Phys. Soc. Jpn. 81, 011002 (2012).
[4] T. Arai, K. Ichimura, K. Nomura, S. Takasaki, J. Yamada, S. Nakatsuji, and H. Anzai, Phys. Rev. B 63, 104518 (2001).
[5] K. Ichimura, M. Takaki, and K. Nomura, J. Phys. Soc. Jpn 77, 114707 (2008).
[6] T. O. Wehling, A. M. Black-Schaffer, and A. V. Balatsky, Adv. Phys. 63, 1 (2014).
[7] M. Sigrist and K. Ueda, Rev. Mod. Phys. 63, 239 (1991).
[8] G. Baskaran, Phys. Rev. Lett. 91, 097003 (2003).
[9] B. Kumar and B. S. Shastry, Phys. Rev. B 68, 104508 (2003).
[10] C. Honerkamp, Phys. Rev. B 68, 104510 (2003).
[11] A. M. Black-Schaffer and S. Doniach, Phys. Rev. B 75, 134512 (2007).
[12] R. Nandkishore, L. S. Levitov, and A. V. Chubukov, Nature Phys. 8, 158 (2012).
[13] G. E. Volovik, JETP Lett. 66, 522 (1997).
[14] A. M. Black-Schaffer, Phys. Rev. Lett. 109, 197001 (2012).
[15] A. M. Black-Schaffer and C. Honerkamp, J. Phys.: Condens. Matter 26, 432301 (2014).
[16] A. Möller, U. Lüö, T. Taetz, M. Kriener, G. André, F. Damay, O. Heyer, M. Braden, and J. A. Mydosh, Phys. Rev. B 78, 024420 (2008).
[17] Y. J. Yan, Z. Y. Li, T. Zhang, X. G. Luo, G. J. Ye, Z. J. Xiang, P. Cheng, L. J. Zou, and X. H. Chen, Phys. Rev. B 85, 085102 (2012).
[18] D.-Y. Liu, Y. Guo, X.-L. Zhang, J.-L. Wang, Z. Zeng, H.-Q. Lin, and L.-J. Zou, EPL 103, 47010 (2013).
[19] W. Wu, M. M. Scherer, C. Honerkamp, and K. Le Hur, Phys. Rev. B 87, 094521 (2013).
[20] Y. Nishikubo, K. Kudo, and M. Nohara, J. Phys. Soc. Jpn 80, 055002 (2011).
[21] P. K. Biswas, H. Luetkens, T. Neupert, T. Stürrer, C. Baines, G. Pascua, A. P. Schnyder, M. H. Fischer, J. Goryo, M. R. Lees, et al., Phys. Rev. B 87, 180503 (2013).
[22] M. H. Fischer, T. Neupert, C. Platt, A. P. Schnyder, W. Hanke, J. Goryo, R. Thomale, and M. Sigrist, Phys. Rev. B 89, 020509 (2014).
[23] J. T. Ye, Y. J. Zhang, R. Akashi, M. S. Bharamy, R. Arita, and Y. Iwasa, Science 338, 1193 (2012).
[24] K. Taniguchi, A. Matsumoto, H. Shiomotani, and H. Takagi, Appl. Phys. Lett. 101, 042603 (2012).
[25] N. F. Q. Yuan, K. F. Mak, and K. T. Law, Phys. Rev. Lett. 113, 097001 (2014).
[26] J. Vučičević, M. O. Goerbig, and M. V. Milovanović, Phys. Rev. B 86, 214505 (2012).
[27] F. Liu, C.-C. Liu, K. Wu, F. Yang, and Y. Yao, Phys. Rev. Lett. 111, 066404 (2013).
[28] O. Vafek, J. M. Murray, and V. Cvetkovic, Phys. Rev. Lett. 112, 147002 (2014).
[29] S. Okamoto, Phys. Rev. Lett. 110, 066403 (2013).
[30] S. Okamoto, Phys. Rev. B 87, 064508 (2013).
[31] M.-T. Tran and K.-S. Kim, Phys. Rev. B 83, 125416 (2011).
[32] A. M. Black-Schaffer, W. Wu, and K. Le Hur, Phys. Rev. B 90, 054521 (2014).
[33] G. C. Psaltakis and E. W. Fenton, J. Phys. C 16, 3913 (1983).
[34] M. Murakami and H. Fukuyama, J. Phys. Soc. Jpn 67, 2784 (1998).
[35] B. Kyyung, Phys. Rev. B 62, 9083 (2000).
[36] E. Demler, W. Hanke, and S.-C. Zhang, Rev. Mod. Phys. 76, 909 (2004).
[37] Z.-C. Gu, H.-C. Jiang, D. N. Sheng, H. Yao, L. Balents, and X.-G. Wen, Phys. Rev. B 88, 155112 (2013).
[38] Y. Zhong, L. Zhang, H.-T. Lu, and H.-G. Luo, Physica B 462, 1 (2015).
[39] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. 82, 3045 (2010).
[40] X.-L. Qi and S.-C. Zhang, Rev. Mod. Phys. 83, 1057 (2011).
[41] R. B. Laughlin, Phys. Rev. Lett. 80, 5188 (1998).
[42] A. Rycerz, J. Tworzydło, and C. W. J. Beenakker, Nature Phys. 3, 172 (2007).
[43] A. F. Young, C. R. Dean, L. Wang, H. Ren, P. Cadden-Zimansky, K. Watanabe, T. Taniguchi, J. Hone, K. L. Shepard, and P. Kim, Nature Phys. 8, 550 (2012).
[44] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, Rev. Mod. Phys. 81, 109 (2009).
[45] F. C. Zhang, C. Gros, T. M. Rice, and H. Shiba, Supercond. Sci. Techn. 1, 36 (1988).
[46] P. W. Anderson, P. A. Lee, M. Randeria, T. M. Rice, N. Trivedi, and F. C. Zhang, J. Phys.: Condens. Matter 16, R755 (2004).
[47] B. Edegger, V. N. Muthukumar, and C. Gros, Adv. Phys. 56, 927 (2007).
[48] D. Vollhardt, Rev. Mod. Phys. 56, 99 (1984).
[49] J. Linder, A. M. Black-Schaffer, T. Yokoyama, S. Doniach, and A. Sudbo, Phys. Rev. B 80, 094522 (2009).
[50] A. P. Schnyder, S. Ryu, A. Furusaki, and A. W. W. Ludwig, Phys. Rev. B 78, 195125 (2008).
[51] A. Kitaev, AIP Conf. Proc. 1134, 22 (2009).
[52] M. Ezawa, Y. Tanaka, and N. Nagaosa, Sci. Rep. 3, 2790 (2013).
[53] S. Das Sarma, S. Adam, E. H. Hwang, and E. Rossi, Rev. Mod. Phys. 83, 407 (2011).
[54] D. Poilblanc, P. Corboz, N. Schuch, and J. I. Cirac, Phys. Rev. B 89, 241106 (2014).
[55] The spectrum is fully gapped until superconductivity is ultimately killed at $|M| = |\mu| \gg M_c$, as then the normal-state Fermi surface only consists of the $K, K'$ points.