Dispersion effect on the single-photon wave-packet

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After a laser pulse propagates through the dispersive media, its temporal-spectral function will be inevitably modified. Although such dispersion effect can be well described in classical optics, its effect on a single-photon wave-packet, i.e. the matter wave of a single-photon, has not yet been revealed. In this paper, we investigate the dispersion effect on the single-photon wave-packet through the Hong-Ou-Mandel (HOM) interference. By dispersively manipulating two indistinguishable single-photon wave-packets before the interference with each other, we observe that the difference of the second-order dispersion between two arms of the HOM interferometer can be mapped to a HOM curve, suggesting that (1) with the same dispersion effect on both wave-packets, the HOM curve must be only determined by the intrinsic width of the wave-packets; (2) extra dispersion effect from an unknown optical medium will broaden the HOM curve thus providing a way to measure the second-order dispersion coefficient of the medium. Our results give a deeper understanding of the single-photon wave-packet and pave the way to explore further applications of the HOM interference.

I. INTRODUCTION

The matter wave theory describes the wave property of physical objects [1]. Especially, it gives a comprehensive understanding of the wave-particle duality of quantum objects, for instance single-photons [2]. Similar with procedures for analyzing the classical counterpart of the matter wave, i.e. the electromagnetic wave, properties of the matter wave of single-photons have been investigated through the schemes of interference. Lots of exciting demonstrations, which certify the genuine quantum nature of the single-photons behavior, have been realized in recent years [3–5]. However, the dispersion effect on the single-photon wave-packet, which should take place after a single-photon propagating through the dispersive environment, has not yet been revealed so far. Generally, the dispersion effect will influence the indistinguishability of a single-photon wave-packet in its spectral-temporal joint distribution, thus suggesting that such an effect can be evaluated through Hong-Ou-Mandel (HOM) interference [6, 7]. In fact, HOM interference has been widely applied to investigate the indistinguishability of a single-photon wave-packet in the temporal mode [8, 9], polarization mode [10], spectral mode [11–13], and orbital angular momentum mode [14]. Moreover, the HOM interference has been adopted in quantum metrology, for instance the measurement of the coherent time of light field [15], pulse duration [16] and time delays [3, 18, 19], and also lies in quantum communications with dispersive quantum channels, such as quantum teleportation [20, 21] and measurement device independent quantum key distribution through fiber channels [22–24].

In this paper, we theoretically analyze the dispersion effect on a single-photon wave-packet via HOM interference. Our analyses show that the difference of the second-order dispersion effects between two arms of a HOM interferometer can be accurately mapped to HOM curves. Such phenomena provide ways to obtain the original width of single-photon wave-packets after they propagate through the dispersive channels, and to measure the second-order dispersion coefficient of an unknown dispersive optical medium. Proof-of-principle demonstrations are carried out with single-photon wave-packets which are prepared by attenuating mode-locked laser pulses. On one hand, the mode-locked laser pulses propagate along a 50-kilometers long fiber spool which serves as a dispersion module to manipulate the spectral-temporal joint distribution of the wave-packets. Then, they are separated into two parts and attenuated into single-photon level, and then sent into a 50:50 beam splitter, in which the HOM interference happens. The theoretical prediction is experimentally verified that the widths of HOM curves are not influenced by the dispersion effect on the wave-packets, thus restoring the original temporal information of the wave-packet. On the other hand, the mode-locked laser pulses are divided into two paths. After attenuated into the single-photon level, both of beams are sent into two fiber pieces, then interfering with each other in the HOM interferometer. The measured width of a HOM curve is wider than that of the previous one, with that the widened amount is exactly corresponding to the difference of the second-order dispersion effect in two optical paths, thus providing a novel method for measuring the second-order dispersion coefficient of an unknown optical medium.
FIG. 1. Conceptual illustration of HOM interference with the manipulated spectral-temporal joint distribution of single-photon wave-packets. (a) HOM interferometer with dispersion manipulation modules. Two identical single-photon wave-packets are manipulated with dispersion modules along two optical paths, i.e. path A and path B, and then are sent into a HOM interferometer; (b) HOM interference curves without the second-order dispersion along two paths; (c) with the same second-order dispersion along two paths; (d) with different second-order dispersions along two paths. To guide eyes, envelopes of three sub-wave-packets are depicted with solid and dash lines in red, green, and blue respectively. To obtain the HOM interference curve, the propagation time of the wave packet in path A is fixed, and that in path B is varied.

II. THEORETICAL MODEL AND ANALYSIS

Figure 1(a) gives the conceptual illustration of our proposal. In the theoretical model, we consider two genuine Gaussian-shaped single-photon wave-packets with the pulse width of $T_0$, which propagate along path A and path B respectively. Two dispersion modules are utilized to manipulate the spectral-temporal joint distributions of two wave-packets. Then the wave-packets are input to a HOM interferometer, consisting of a 50: 50 beam splitter (BS), two single photon detectors, and a coincidence circuit. To observe the HOM interference curve, an optical delay is introduced in path B. The dispersion effect on the single-photon wave-packet is measured by comparing the HOM interference curves with different dispersion manipulations.

Firstly, we calculate the coincidence count per trial $P(\tau)$ without any dispersion, as expressed by\[25, 26],

$$P(\tau) = \frac{1}{2} - \frac{1}{2} e^{-\frac{\tau^2}{2T_0^2}}, \quad (1)$$

where $\tau$ is the relative delay between two optical paths. The detailed calculation of Eq. (1) is given in Appendix A. According to Eq. (1), a conceptual HOM interference curve without dispersion manipulation is shown in Fig. 1(b). A HOM interference curve with a full width at half maximum (FWHM) which is $\sqrt{2}$ times of the FWHM of a laser pulse can be obtained by scanning $\tau$ from $-t_1$ to $t_1$ in the axis of the relative delay as illustrated in Fig. 1(b). Figure 1(b) also shows the cartoon process of how to obtain the HOM interference, in which the fixed single-photon wave-packet is indicated by solid lines, while the moving one is indicated by dash lines.

After the dispersive manipulation of a single photon wave-packet, the temporal width of wave-packet will be broadened because of the spectral-temporal joint redistribution which is attributed to different group velocities for different frequencies. One might expect a HOM dip to be changed due to the group-velocity time delay and the width of HOM interference curve to be broadened due to the second-order dispersion effect, respectively. Ignoring the third and above order dispersion effects, we can obtain the coincidence count per trial $P'(\tau)$, as given by (the details are introduced in Appendix A),

$$P'(\tau) = \frac{1}{2} - \frac{T_0^2}{\sqrt{4T_0^4 + \alpha^2}} \exp \left[ -\frac{2(\tau - \delta\tau)^2}{4T_0^2 + (\alpha/T_0)^2} \right], \quad (2)$$

where $\delta\tau = \beta_1 A L_A - \beta_1 B L_B$, $\alpha = \beta_2 A L_A - \beta_2 B L_B$ represent the difference of total group-velocity time delay and the difference of the total group-velocity dispersion propagating along two optical paths respectively, and $\beta_1 A$ and $\beta_1 B$ are group-velocity time delays, $\beta_2 A$ and $\beta_2 B$ are the group-velocity dispersions, $L_A$ and $L_B$ are the lengths
of two dispersive optical paths.

As shown in Eq. (3), with the dispersive manipulation, the HOM curve would be changed, such as the minimum point of the curve would be shifted, the width of the curve would be broadened. It can be concluded that all the changes in the HOM curve are determined by the difference of dispersion manipulation between two wave-packets. In Figs. 1(c)-(d), we give an interpretation with the redistributed spectral-temporal joint distribution, in which three walk-off frequency components are indicated by red, green, and blue lines, respectively. When two single-photon wave-packets experience the same dispersive manipulation in Fig. 1(c), different individual colors of the wave-packet in path A begin and end to interfere with its partner who owns the same ‘color’ in path B at the same time. The HOM interference curve will keep the same as that without dispersive manipulation, thus providing an effective method to measure the original width of optical pulses even though the pulses have propagated through dispersive environment. Note that although we only involve the second-order dispersion effect in the model, the result is still valid when extending to all-order dispersion effects on this occasion. For the case with different amount of dispersion manipulation, the width of a HOM curve will be broadened. The interference extends from the encounter of the fastest component to the separation of the slowest one and the position with the maximum interference appears to shift. The broadened value of the HOM curve can accurately describe the difference of the second-order dispersion effect between path A and B in Eq. (3),

$$\alpha = T_0 \sqrt{(d^2_{FWHM}/2 \ln 2 - 4T_0^2)},$$  

where \(d_{FWHM}\) is the width of a HOM interference curve. If the second-order dispersion property in one path is assumed to be known, the unknown dispersion property in the other path can be measured by HOM interference.

III. PROOF-OF-PRINCIPLE DEMONSTRATIONS

We construct a proof-of-principle experimental setup as shown in Fig. 2 (see more details in Appendix 4) and experimentally demonstrate the dispersion effect on single-photon wave-packets. We then illustrate the feasibility of measuring the original width of laser pulses and the second-order dispersion coefficient of an unknown dispersive optical medium. In the proof-of-principle demonstrations, genuine single-photon wave-packets in our theoretical model are substituted with attenuated mode-locked laser pulses with a mean photon number of 0.03 per pulse [23]. The pulse width is 0.798 ± 0.010 ps, measured by a second-order autocorrelator (FEMTOCHROME, FR-103XL), and the period \(T\) is 200 ns. The dispersion manipulation is realized by using pieces of fiber as dispersion modules.

In our experiment, we firstly measure the HOM interference curve at the output port of a mode-locked laser as shown in Fig. 2 which is used as a reference for our demonstrations. The HOM curve is obtained as shown in Fig. 3(a). The blue dots denote the normalized experimental results and the solid lines denote the Gaussian fitting curves with the Monte Carlo method [23]. From the fitted HOM interference curve, a FWHM of 1.035 ± 0.008 ps can be obtained. From Eq. (1), we also obtain the laser pulse width 0.732 ± 0.006 ps.

To show that the intrinsic width of a laser pulse can be measured with the HOM interference even with the dispersion broadening, we connect a 50 km-long single-mode fiber spool (Yangtze Optical Fibre and Cable Co. LTD, G.652.D ULL) at the output port of our mode-locked laser. After propagating through the dispersive environment, the laser pulses would be broadened to about 3.9 ns with a second-order dispersion coefficient of 17.1 ps/(km-nm). In the experiment, we employ a superconducting nanowire single photon detector (SNSPD, Photon Technology Co., P-CS-6) [23] and a time to digital convertor (TDC, ID900, ID Quantique) to measure the width of the broadened laser pulses after the broadened laser pulses are attenuated to the single-photon level. The waveform of the single-photon wave-packet is shown in Appendix 4. It shows that after the dispersion manipulation the measured pulse width is 4.1 ns including a total jitter of about 270 ps in the experiment. By considering the dispersive manipulation only changes the width of laser pulses, the width of HOM curve would be 5.57 ns according to Eq. (1). However, the width of the measured HOM curve is 1.032 ± 0.009 ps, corresponding to a FWHM width of 0.730 ± 0.006 ps for the photon wave-packets, as shown in Fig. 3(b). It can be seen that the obtained widths are the same for with or without the dispersive manipulation, which is in good agreement with our theoretical analyses. Thus, the HOM interference must recovery the original wave-packet of the light.
FIG. 3. HOM interference curves (a) without a dispersion module, and (b) with 50 km long fiber as the dispersion module at the output of mode-locked laser, respectively. The blue dots denote experimental results. The solid purple lines are 1000-time Gaussian fitting curves with the Monte Carlo method.

FIG. 4. HOM interference curves with a 80 m-long single-mode fiber inserted in one path, which correspond to two periods of mode-locked laser pulses. The width of HOM interference curves have been broadened to $4.123 \pm 0.124 \text{ ps}$ with the degraded visibility of $0.062 \pm 0.01$.

even though the dispersive manipulation has changed the spectral-temporal joint distribution inevitably, offering us a dispersion immune method to measure the original pulse width especially for femtosecond laser pulses.

Next, we show that the HOM interference can be applied to measure the second-order dispersion coefficient of an unknown optical material. We use a piece of the single-mode fiber with the length of 80 m-long to serve as the unknown optical material, which is inserted in one arm of the HOM interferometer as shown in Fig. 2. One may think that an extra optical path should be added in the other arm to balance transmission times in two arms. Fortunately, the extra optical path can be removed according to the result from Y. S. Kim et al. [30], i.e. the HOM interference can occur between periodically delayed mode-locked laser pulses. In our experiment, the HOM interference curves are measured between laser pulses with a two-period delay. The measured results are shown in Fig. 4. The measured FWHM of HOM interference curves is $4.123 \pm 0.124 \text{ ps}$ with the visibility of $0.062 \pm 0.01$. According to our theoretical analyses, the second-order dispersion coefficient of the inserted fiber is $15.04 \pm 0.48 \text{ ps/(km-nm)}$, which is consistent with the dispersion parameter from the manufacturer.

IV. DISCUSSION AND CONCLUSIONS

In this paper, we theoretically investigate the dispersion effect on single-photon wave-packets. Based on our investigation, we found that two important applications for HOM interference for the first time, i.e. measuring the original pulse width of an ultrashort laser pulse and measuring the second-order dispersion coefficient of an unknown optical material. In principle, precision metrology of ultrashort laser pulses can be realized with the second-order autocorrelation method [31], while in order to trigger the nonlinear optical effect employed in the second-order autocorrelation process, the peak power of the ultrashort laser should be amplified by the sophisticated chirped pulse amplification [32]. In 1993, Y. Miyamoto et al. have introduced the HOM interference scheme to the measurement of the pulse width of ultrashort laser pulses [16]. However, the dispersion effect on the measured laser pulses has not yet been discussed, which was comprehensively addressed in his work. In our experiment, the pulse width from HOM interference is $0.732 \pm 0.006 \text{ ps}$, smaller than $0.798 \pm 0.010 \text{ ps}$ from the second-order autocorrelation measurement. This would be caused by the dispersive broadening before the second-order autocorrelation measurement. It is also important to measure the second-order dispersion coefficient of an unknown optical material. In our proof-of-principle demonstration, the second-order dispersion coefficient of a piece of 80 m-long fiber is obtained through HOM interference. There have been several methods to measure the dispersion of optical materials, for example, the pulse-delay measurement, phase-shift measurement, interferometric measurement, etc [33–38]. Comparing with the previous approaches, the minimum dis-
We have established a theoretical model to reveal the dispersion effect on single-photon wave-packets via HOM interference. Our proof-of-principle demonstrations have been implemented that such a scheme can be used to measure the original pulse width of ultrashort laser pulses even after propagating through the dispersive environment and to measure the second-order dispersion coefficient of an unknown optical material simultaneously. Our results show that the measurement accuracy can respectively reach 6 fs and 0.458 ps/(km-nm) for the measurement of the pulse width and the second-order dispersion coefficient, suggesting that our schemes can be considered as the standard calibration methods. In summary, our work provides a procedure for understanding the spectral-temporal joint property of single-photon wave-packets, i.e. property of a matter wave of the single-photon, and opens up new applications for HOM interference.

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Appendix A: Theoretical scheme

Considering two single-photon wave-packets propagating along paths A and B, we can write their quantum state as

\[ |\varphi\rangle_{A,B} = \int d\omega \varphi_{A,B}(\omega) \hat{a}_{A,B}^\dagger(\omega)|0\rangle, \tag{A1} \]

where \( \varphi_{A,B} \) is the spectral amplitude function and \( \hat{a}_{A,B}(\omega) \) the creation operator of the optical field propagating along the corresponding path. The state is normalized such that \( \int d\omega |\varphi(\omega)|^2 = 1 \).

After the dispersive manipulation by the dielectric mediums in paths A and B, the quantum state in Eq. (A1) turns into

\[ |\psi\rangle_{A,B} = \int d\omega \varphi_{A,B}(\omega) \hat{a}_{A,B}^\dagger(\omega)e^{i\beta_{A,B}(\omega)L_{A,B}}|0\rangle_{A,B}, \tag{A2} \]

where \( \beta_{A,B}(\omega) \) and \( L_{A,B} \) are the frequency-dependent propagation constant and propagation distance in dispersive optical path.

To implement HOM interference, a time delay \( \tau \) is introduced in path B, as shown in Fig. 1(a). After the time delay, the creation operator of optical field in this path evolves into

\[ \hat{a}_{B}(\omega) \rightarrow \hat{a}_{B}(\omega)e^{-i\omega\tau}. \tag{A3} \]

When the wave-packets arrive at the BS, the input state of the BS can be treated as a two photon state with expression of

\[ |\psi_{in}\rangle_{AB} = \int d\omega_1 \varphi_A(\omega_1) \hat{a}_A^\dagger(\omega_1)e^{i\beta_A(\omega_1)L_A} \int d\omega_2 \varphi_B(\omega_2) \hat{a}_B^\dagger(\omega_2)e^{i\beta_B(\omega_2)L_B}e^{-i\omega_2\tau}|0\rangle_{AB}. \tag{A4} \]
The evolution of the creation operators on the BS can be modelled with a unitary transformation $\hat{U}_{BS}$ leading to
\begin{equation}
\hat{a}_A^\dagger(\omega) \frac{\hat{U}_{BS}}{\sqrt{1 - \eta\hat{a}_A^\dagger(\omega)}} + \sqrt{\eta}\hat{a}_B^\dagger(\omega), \tag{A5}
\end{equation}
\begin{equation}
\hat{a}_B^\dagger(\omega) \frac{\hat{U}_{BS}}{\sqrt{\eta\hat{a}_A^\dagger(\omega)}} - \sqrt{1 - \eta}\hat{a}_B^\dagger(\omega), \tag{A6}
\end{equation}
where $\eta$ is the reflectivity of the beam splitter. With this unitary transformation, the output two photon state of the BS can be expressed as $|\psi^{\text{out}}\rangle_{AB} = \hat{U}_{BS} |\psi^{\text{in}}\rangle_{AB}$. After substituting Eq. (A4), Eq. (A5), and Eq. (A6) into this expression and letting $\eta = 1/2$, we can obtain
\begin{equation}
|\psi^{\text{out}}\rangle_{AB} = -\frac{1}{2} \int d\omega_1 \varphi_A(\omega_1) e^{-i\beta_A(\omega_1)\omega_A} \int d\omega_2 \varphi_B(\omega_2) e^{-i\beta_B(\omega_2)\omega_B} e^{-i\omega_2}\tau \times \left[ \hat{a}_A^\dagger(\omega_1) \hat{a}_A^\dagger(\omega_2) + \hat{a}_B^\dagger(\omega_1) \hat{a}_B^\dagger(\omega_2) - \hat{a}_B^\dagger(\omega_1) \hat{a}_B^\dagger(\omega_2) - \hat{a}_B^\dagger(\omega_1) \hat{a}_B^\dagger(\omega_2) \right] |0\rangle_{AB}. \tag{A7}
\end{equation}

The projection operators describing the detection of one photon in paths A and B have expressions of
\begin{equation}
\hat{P}_A = \int d\omega \hat{a}_A^\dagger(\omega) |0\rangle_A \langle 0|_A \hat{a}_A(\omega), \tag{A8}
\end{equation}
\begin{equation}
\hat{P}_B = \int d\omega \hat{a}_B^\dagger(\omega) |0\rangle_B \langle 0|_B \hat{a}_B(\omega). \tag{A9}
\end{equation}
Moreover, the coincidence count per trail of detecting one photon in the two paths simultaneously is
\begin{equation}
P(\tau) = \langle \psi^{\text{out}} | \hat{P}_A \otimes \hat{P}_B | \psi^{\text{out}} \rangle_{AB}. \tag{A10}
\end{equation}
Substituting Eq. (A8) and Eq. (A9) into Eq. (A10), we can get
\begin{equation}
P(\tau) = \frac{1}{4} \int d\omega_a \int d\omega_b \int d\omega_1 \int d\omega_2 \int d\omega_1' \int d\omega_2' \varphi_A^* (\omega_1) \varphi_A (\omega_1') \varphi_B^* (\omega_2) \varphi_B (\omega_2') \times e^{-i\beta_A(\omega_1)\omega_A} e^{-i\beta_B(\omega_2)\omega_B} e^{i\omega_2}\tau \times \delta(\omega_1 - \omega_2) \delta(\omega_1' - \omega_2') \times \left[ \hat{a}_A(\omega_1) \hat{a}_A(\omega_2) + \hat{a}_B(\omega_1) \hat{a}_B(\omega_2) - \hat{a}_B(\omega_1) \hat{a}_B(\omega_2) - \hat{a}_B(\omega_1) \hat{a}_B(\omega_2) \right] \langle 0 |_A \langle \hat{a}_A^\dagger(\omega_a) \hat{a}_B^\dagger(\omega_b) |0\rangle_{AB}. \tag{A11}
\end{equation}

In Eq. (A11), terms with an odd number of creation operators in one path goes to zero, while those with an even number equals to delta functions. Thus, we obtain
\begin{equation}
P(\tau) = \frac{1}{4} \int d\omega_a \int d\omega_b \int d\omega_1 \int d\omega_2 \int d\omega_1' \int d\omega_2' \varphi_A^* (\omega_1) \varphi_A (\omega_1') \varphi_B^* (\omega_2) \varphi_B (\omega_2') \times e^{-i\beta_A(\omega_1)\omega_A} e^{-i\beta_B(\omega_2)\omega_B} e^{i\omega_2}\tau \times \delta(\omega_a - \omega_b) \delta(\omega_1 - \omega_2) \delta(\omega_1' - \omega_2') \times \delta(\omega_a - \omega_b) \delta(\omega_1' - \omega_2') \delta(\omega_1 - \omega_2). \tag{A12}
\end{equation}

After $\omega_a, \omega_b, \omega_1'$, and $\omega_2'$ in Eq. (A12) are eliminated by utilizing the property of delta functions, and the normalization condition of $\varphi_{A,B}$ is applied, Eq. (A12) is simplified to
\begin{equation}
P(\tau) = \frac{1}{2} - \frac{1}{2} \int d\omega \varphi_A^2 (\omega) \varphi_B^2 (\omega) e^{-i(\omega\tau + \beta_A(\omega)\omega_A - \beta_B(\omega)\omega_B)} \tag{A13}
\end{equation}
When the two single photon wave-packets are indistinguishable, their spectral amplitude functions are identical, and Eq. (A13) turns into
\begin{equation}
P(\tau) = \frac{1}{2} - \frac{1}{2} \int d\omega |\varphi(\omega)|^2 e^{-i(\omega\tau + \beta_A(\omega)\omega_A - \beta_B(\omega)\omega_B)} \tag{A14}
\end{equation}
Usually, the effect of higher order dispersions is negligible. After truncating the Taylor expansion to the second order, we can approximate $\beta_{A,B}$ as

$$
\beta_{A,B}(\omega) = \beta_{0A,B} + \beta_{1A,B}(\omega - \omega_0) + \frac{\beta_{2A,B}(\omega - \omega_0)^2}{2},
$$

where $\omega_0$ is the central frequency of the wavepacket; $\beta_{0A,B}$ and $\beta_{1A,B}$ is the propagation constant and the group-velocity time delay at $\omega_0$, respectively. We can substitute Eq. (A15) into Eq. (A14), and obtain

$$
P(\tau) = \frac{1}{2} - \frac{1}{2} \int d\omega |\varphi(\omega)|^2 e^{-i\omega T} \left[ \beta_0 + \beta_1(\omega - \omega_0) + \frac{\beta_2}{2}(\omega - \omega_0)^2 \right] L_A - \left[ \beta_0 + \beta_1(\omega - \omega_0) + \frac{\beta_2}{2}(\omega - \omega_0)^2 \right] L_B \right] e^{-i\omega T},
$$

where $\Omega = \omega - \omega_0$.

In our theoretical model, we consider two indistinguishable Gaussian wave-packets with width of $T_0$ and their amplitude can be represented in time domain as

$$
\varphi(t) = e^{-\frac{t^2}{2T_0^2}} e^{-i\omega_0 t}.
$$

Then, we get the corresponding frequency domain representation of wavepacket amplitude as

$$
\varphi(\omega) = \sqrt{\frac{1}{\sqrt{\pi}T_0}} e^{-\frac{(\omega - \omega_0)^2}{2T_0^2}}.
$$

The substitution of Eq. (A18) into Eq. (A16) can give us

$$
P(\tau) = \frac{1}{2} - \frac{T_0^2}{\sqrt{4T_0^2 + \alpha^2}} \left[ \frac{2(\tau - \delta \tau)^2}{4T_0^2 + (\alpha/T_0)^2} \right],
$$

where $\delta \tau = \beta_1 A L_A - \beta_1 B L_B$ and $\alpha = \beta_2 A L_A - \beta_2 B L_B$ represent the difference of total group-velocity time delay and the difference of the total group-velocity dispersion between the paths A and B, respectively.

It is obvious that Eq. (A19) implies a dip of photon coincidence count for two encountering single photon wavepackets to go toward different pathways, and this is the typical characteristic of HOM interference. According to this equation, the full width at half maximum (FWHM) of the HOM interference curve equals to

$$
d_{FWHM} = 2\sqrt{\ln 4 \sqrt{T_0^2 + (\alpha/2T_0)^2}}.
$$

When the dispersive manipulation in two optical paths are the same, the Eq. (A20) is simplified into

$$
d_{FWHM} = 2\sqrt{\ln 4T_0} = \sqrt{2}T_{FWHM},
$$

where $T_{FWHM}$ is the full width at half maximum of the wave-packets. In this case, the FWHM of interference curve is the same with the absent dispersion case. Thus, we can obtain the original pulse width is $1/\sqrt{2}$ times of the HOM interference curve even though the wave-packets have been broadened by the dispersive manipulation. When the dispersive manipulation in two paths are different, the difference of the second-order dispersion effect between two paths can accurately mapped to the HOM interference curve by

$$
\alpha = T_0 \sqrt{\frac{d_{FWHM}^2}{2\ln 2} - 4T_0^2},
$$

which can be utilized to measure the second-order dispersion effect of an unknown optical material.

**Appendix B: Methods**

The pulsed light source is selected as a passively mode-locked fiber laser with the central wavelength of 1565 nm, the pulse duration of 0.798 ± 0.010 ps by second-order autocorrelation method, the repetition rate of 5 MHz and the 3-dB
spectral bandwidth of 4.3 nm as shown in Fig. 5(a). The laser pulses are attenuated to the single photon level with the mean photon number of 0.03 per wave-packet through a variable optical attenuator (VOA1). To simulate the two single-photon wave-packets, a 50:50 single-mode fiber coupler (SMC) is used to separate the attenuated laser pulses into two paths, resulting in the average photon number of 0.015 per wave-packet in each path. Two wave-packets are injected into a HOM interferometer, which consists of two variable optical attenuators (VOA2 and VOA3), two polarization beam splitters (PBS1 and PBS2), a 50:50 polarization-maintaining fiber coupler (PMC), and a fiber pigtailed variable optical delay line (Delay). VOA2 and VOA3 in two arms are used to adjust the mean photon number in the two optical paths, which ensure that the mean photon numbers of two paths are the same. PBS1 and PBS2 are used to ensure the indistinguishability in polarization. The relative time delay is introduced by an optical delay-line (MDL-002, General Photonics) with the accuracy of 10 fs and a maximum range of 560 ps. Two output ports of HOM interferometer are connected with two superconducting nanowire single photon detectors (SNSPDs) with a detection efficiency of the 68%. The electronic signals generated by photon detection events are input into a time to digital converter (TDC, ID 900, ID Quantique) to obtain the coincidence counts. The HOM dip can be observed in the HOM interference curve, i.e. coincidence counts versus the relative time delay between two paths.

In the experiment, to demonstrate that the intrinsic width of a mode-locked laser can be measured after the dispersive manipulation, we connect the output port of a mode-locked laser with a 50 km-long fiber spool, and attenuate the laser pulses into the single-photon level with VOA1, VOA2 and VOA3, then send the attenuated laser pulses into HOM interferometer. To demonstrate that the second-order dispersion coefficient of an unknown optical medium can be measured through HOM interference, we first attenuate the mode-locked laser pulses and then separate them into two paths. In one of the two paths we connect a piece of 80 meter-long fiber as the dispersive optical medium for the test. These connections are switched in the experiment. In our demonstration, the widths of laser pulses after the dispersive manipulation are also measured through the single-photon detection and a TDC. The measured results are shown in Fig. 5(b). It shows that after propagating along a 50 km-long fiber, the width of pulse laser has been broadened to 4.1 ns including the jitters in the single-photon detection and TDC.

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