Rapid Nonlinear Analysis for Electrothermal Microgripper Using Reduced Order Model based Krylov Subspace

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Abstract. The conventional numerical analysis methods could not perform the rapid system-level simulation to MEMS especially when containing sensing and testing integrated circuit. Using reduced-order model can simulate the behavior characteristic of multiphysical energy domain models including nonlinear analysis. This paper set up the reduced-order model of electrothermal microgripper using Krylov subspace projection method. The system functions were assembled through finite element analysis using Ansys. We took the structure-electro-thermal analysis to microgripper finite element model and reduced model order through second-order Krylov subspace projection method based on Arnoldi process. The simulation result from electrothermal reduced order model of microgripper is accurate compared with finite element analysis and even more has a few computing consuming.

1. Introduction

Most of Micro-Electro-Mechanical Systems have complicated structure and cross multiple physical coupled energy domains. The development of microtechnologies in the field of microsystems has generated wide interest in exploiting fast and accurate design methods to simulate entire system static and dynamic behavior. The typical MEMS can exchange or transform energy between different energy domains. The coupled effect between mechanical, electrical and thermal field used in MEMS transducer is shown in table 1. A diagonal entry represents a single physical field effect; an off-diagonal entry describes a coupling between different energy domains. Finite element and boundary element tools have been successfully applied for the design of micro electromechanical devices on the multiphysical coupled modeling. These numerical methods are highly accurate but are cumbersome and time consuming, i.e. cannot perform the system-level simulation to overall system. Furthermore, if the systems contain the microelectronic element for switching the non-electric signals to electric signals or some else signal processing units, finite element or boundary element models are inappropriate. Therefore, the model order reduction technique has been extensively studied in recent years.
Table 1. Coupled effect between mechanical, electrical and thermal field.

| Causes       | Effects               |
|--------------|-----------------------|
| Mechanical   | Thermal               | Electrical           |
| Elasticity   | Piezocaloric          | Piezoelectricity     |
| Plasticity   | Heat dissipation      | Piezoresistivity     |
| Thermal      | Specific heat         | Pyroelectricity      |
| expansion    | Thermal conductivity  |                     |
| Electrostatic forces | Thermoelectric effect |                     |
| inverse piezoelectricity | (Peltier,Thomson,Joule,Bridgman) effects |                     |

The reduced order model (ROM), also called macromodel, is a low-order behavioral representation of a device that has some basic attributes [1], it can be described with hardware language and directly application in EDA environment. There are two strategies to create ROM: analytic solution [2-3] and numerical solution [4-6]. One principal numerical method is matrix subspace projection [7] in which the system matrices calculated using finite element tools and represented by state-space model. The technique based on Krylov subspace projection [8] has become an important aspect of model order reduction.

2. Theory background
The dynamics of MEMS are represented by partial-differential equations (PDEs) and associated boundary conditions. Discretization using finite element methods (FEM), boundary element methods (BEM) or finite difference methods (FDM) can transfer PDEs to ordinary-differential equations (ODEs) which can be written in the state space form (1)

$$\mathbf{\Omega}: \begin{cases} \frac{dx}{dt} = A \cdot x + B \cdot u \\ y = C \cdot x \end{cases}$$  \hspace{1cm} (1)

where $A \in \mathbb{R}^{n \times m}$ is system matrix, $B \in \mathbb{R}^{n \times m}$ is input matrix, $C \in \mathbb{R}^{p \times n}$ is output matrix, $u \in \mathbb{R}^m$ is input variable, $y \in \mathbb{R}^p$ is output variable and $x \in \mathbb{R}^n$ is state variable. Matrix subspace projection method for reduce the original system order indicate to find a transition matrix $V$, $V \in \mathbb{R}^{n \times r}$, so as to $x = V \cdot z$, $z \in \mathbb{R}^r$. The $n$-dimensional column vectors of $V$, which the number is $r$, can compose a basis subspace, called projection subspace. If $r \ll n$, than the $r$-dimensional vector $z$ could be considered as the reduced order state variable, the reduced order state function is (2)

$$\begin{cases} \frac{dz}{dt} = \hat{A} \cdot z + \hat{B} \cdot u \\ \hat{y} = \hat{C} \cdot z \end{cases}$$  \hspace{1cm} (2)

where $\hat{A} = V^{-1} AV$, $\hat{B} = V^{-1} BV$, $\hat{C} = CV$.
Perform a Laplace transform for equation (1) can obtain its frequency domain transfer function $G(s)$ as...
\[ G(s) = C \cdot (sI - A)^{-1} \cdot B = -C \cdot A^{-1} (I - sA^{-1})B \]  

(3)

If (2) want to be used as the reduced order model of (1), it must satisfy \( \min \| y - \hat{y} \| \) and \( \min \| G(s) - \hat{G}(s) \| \) in time and frequency domain separately.

A r-dimension Krylov subspace \( \Psi_r \) is defined as below

\[ \Psi_r(A, b) = \text{span} \{ b, A_1 b, \ldots, A^{r-1} b \} \]  

(4)

Where \( A_1 \in R^{n \times n} \), \( b_1 \in R^n \) is called starting vector.

The Arnoldi process [9] can create the orthonormal vectors to construct the unit basic Krylov subspace. One can expand \( G(s) \) in Taylor series about some point, e.g. \( s=0 \), so get \( G(s) \) in polynomial format

\[ G(s) = -CA^{-1}(I + sA^{-1} + s^2A^{-2} + \ldots)B = -\sum_{k=0}^{\infty} m_k s^k \]  

(5)

the kth coefficient \( m_k = CA^{-1}(A^{-1}B) \) is called the kth moment of the transfer function.

The previous research [10] have indicated that if transition matrix \( V \) is a unit basic Krylov subspace spanned by Arnoldi process, then the first rth moment between \( G(s) \) and \( \hat{G}(s) \) are matching then minimum of \( \| G(s) - \hat{G}(s) \| \) is achieved, so we can project the original system function matrix to Krylov subspace and implement the model order reduction. Furthermore in many cases the system dynamical behavior are described with second-order differential functions given in the equation (6)

\[
\begin{bmatrix}
M \ddot{x} + D \dot{x} + K x = b u \\
y = c^T x
\end{bmatrix}
\]  

(6)

where \( M, D, K \in R^{n \times n} \), \( b, c \in R^n \), \( u \in R^m \), \( y \in R^p \) and \( x \in R^n \). One could convert the second-order model into the state-space model but it destroyed the physical structure and increased the functions number. In order to reduce second-order model with certain numbers of matching moment there is a new second-order Arnoldi algorithm based on second-order Krylov subspace [11], which can project (6) directly and the reduced order model is still second-order type.

Second-order Krylov subspace is defined as follows:

\[ \Psi_r(A_1, A_2, b_1) = \text{span} \{ k_0, k_1, \ldots, k_{r-1} \} \]  

(7)

where

\[
\begin{bmatrix}
k_0 = b_1 \\
k_1 = A_1 b_1 \\
k_2 = A_1 k_{1} + A_2 k_{1}
\end{bmatrix}
\]  

and \( A_1, A_2 \in R^{n \times n} \), \( b_1 \in R^n \). \( b_1 \) is called starting vector and \( k_i \) is called basic vector.

For the second order system (6), the input and output second-order Krylov subspace defined in (8) and (9)

\[ \Psi_r(-K^{-1}D, -K^{-1}M, -K^{-1}b) \]  

(8)

\[ \Psi_r(-K^{-T}D^T, -K^{-T}M^T, -K^{-T}c) \]  

(9)
Using modified Arnoldi process can also calculate the basic second-order Krylov subspace for projection, consider the projection \( x = V \chi \), \( V \in \mathbb{R}^{n \times r} \), \( r \ll n \), the matrix \( V \) is a unit basic input second Krylov subspace as (8). Multiply projected system by the transpose of \( W \), \( W \in \mathbb{R}^{n \times r} \), and if \( W = V \), the first \( r \) moments of the original and reduced order modal match then get the reduced order system functions as follows

\[
\begin{align*}
W^T MV \ddot{x} + W^T DV \dot{x} + W^T KV x = W^T bu \\
y = e^T V x
\end{align*}
\]

(10)

3. Microgripper finite element model

Microgripper is one of the key elements in microrobot and microassembly technologies. Chu [12] designed and fabricated the polysilicon microgripper based on electrothermal principle using bulk silicon process in figure 1. Micrograph of microgrippers before ICP etching is shown in figure 2. V-shaped beam array microactuator is adopted to generate displacements and gripper is designed by topology optimization, both of them anchor in electrode. The finite element microgripper model is coupled between mechanical, electrical and thermal field, so it is a strong nonlinear problem. Because of the thickness is far less than length and width we consider it as a plane problem and mesh it with 2D element.

The finite element analysis contain three steps: Firstly, the electric field equilibrium equation (11) is solved to get current density.

\[
\nabla \vec{J} + i = 0, \vec{J} = \frac{1}{\rho} \vec{E}, \vec{E} = -\nabla V
\]

(11)

where \( i \) is current supply in unit volume, \( \vec{J} \) is unknown current density vector, \( \rho \) is resistivity, \( \vec{E} \) is density of electric field and \( V \) is electric potential. Secondly Joule heat can be calculated in follow (12) after known the current distribution.

\[
Q_J = \vec{J} \cdot \vec{E} = \rho \| \vec{J} \|^2
\]

(12)

The Joule heat \( Q_J \) then used as thermal load in temperature field simulation.

\[
k \nabla^2 T + Q_J = 0
\]

(13)
k is heat conductivity, T is temperature, so thermal stress can be known through temperature distribution. At last the elastic function (14) could be solved and obtain the solution of microgripper deflection.

\[ \nabla \tilde{\sigma} = 0, \tilde{\sigma} = D(\tilde{\varepsilon} - \varepsilon) \]  

(14)

where \( \tilde{\sigma} \) is stress, \( \varepsilon \) is strain, \( D \) is elasticity matrix.

We establish the finite element model of microgripper using Ansys. The mesh model is shown in figure 3 and the number of elements is 1584 and nodes number is 2083. Ansys perform the Electro-Thermal-Structural analysis using sequential coupling [13], where the coupling in the matrix equation is shown in the most general form as (15).

\[
\begin{bmatrix}
M & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\dot{u} \\
\dot{T} \\
\dot{V} \\
\end{bmatrix} +
\begin{bmatrix}
C & 0 & 0 \\
0 & C & 0 \\
0 & 0 & C \\
\end{bmatrix}
\begin{bmatrix}
\dot{u} \\
\dot{T} \\
\dot{V} \\
\end{bmatrix} +
\begin{bmatrix}
K & K^{UT} & 0 \\
0 & K^T & 0 \\
0 & K^V & K^V \\
\end{bmatrix}
\begin{bmatrix}
u \\
T \\
V \\
\end{bmatrix} = \begin{bmatrix}
F \\
Q \\
0 \\
\end{bmatrix}
\]

(15)

where \( u, T, V \) are displacement, temperature and voltage degrees of freedom and the assembly of system matrix \( M, C, K, C^T, K^T, C^V, K^V \) follow standard finite element methodology. \( C^{TU}, K^{UT}, K^V \) represent the coupled effect attributes. Ansys solve these nonlinear functions using Newton-Raphson method.

Figure 3. Finite element models of electrothermal microgripper.

4. Model order reduction for microgripper

In Ansys command prompt, one can input the command line as follows:

/solu
allsel
antype,static
eqslv,sparse
nsubst,1
wrfull,1
ematwrite,yes
solve
fini

It makes ANSYS write FULL and EMAT file only and not perform a real solve. A FULL file contains the load vector, the stiffness matrix, the Dirichlet and equation constraints. A EMAT file contains the element matrices in order to assemble the global system matrices. Mor4ansys [14] is a command-line tool. It can read FULL and EMAT files and then write systems, input and output matrices directly as shown in (6).

We took steady-state analysis to microgripper using Ansys, the polysilicon characteristic parameter shown in table 2. We also use mor4ansys to get the microgripper finite element model system matrix M, K and input matrix B, C is set only output the largest Y direction node.
displacement, the damp effect is ignored so \( D = 0 \). The input \( u \) is a single input voltage and output \( y \) only the largest \( Y \) direction node displacement which node number in Ansys is 1386. Second-order Arnoldi algorithm is executed in Matlab language as shown in figure 4.

| Polysilicon characteristic parameter | Value |
|--------------------------------------|-------|
| Young’s modulus                     | \( 1.6 \times 10^6 \) (MPa) |
| Poisson’s ratio                      | 0.22  |
| density                              | \( 2.23 \times 10^{-15} \) (kg/\( \mu \)m\(^3\)) |
| coefficient of thermal expansion    | \( 4.7 \times 10^{-6} \) |
| thermal conductivity                | \( 1.48 \times 10^9 \) (pw/\( \mu \)m·K) |
| Convective heat-transfer coefficient | 25 (pw/\( \mu \)m\(^2\)·K) |
| Black body coefficient              | 0.6   |
| Boltzmann constant                  | \( 5.67 \times 10^{-8} \) (w/(m\(^2\)·K\(^4\))) |
| resistivity                          | \( \rho = \rho_0 \left[ 1 + \beta (T - T_0) \right] \) |

Figure 4. Second-order Arnoldi process matlab programm flow diagram.

We reduced original model to 30 and 15 order using Matlab program and also calculate the reduced order model output value. The calculating time is fast than Ansys. The microgripper output displacement result from Ansys, 30 and 15 order reduced model is shown in figure 5.
It shows the calculation error from 30 order model is less 5% than Ansys result and the result error is increment along with the order number reduction. It means that the order of reduced system should be chosen taking the integrated consider in aspects of precision and speed need. In generally speaking, 30 order reduced model is suitable.

If this microgripper reduced-order model put into EDA tools with description of hardware language, then one can simulate the whole microoperation system including voltage drive circuit and displacement sensor. This work is carrying through in our research group.

5. Conclusions
From the computing solution it is concluded that using reduced-order model based Krylov subspace has become a favorite topic for MEMS system-level simulation. It also can solve the coupled field effect and nonlinear problem combined with FEA, but this algorithm itself only used in time-invariant dynamic system and could not reduce nonlinear system immediately. The reduced order methods based Krylov subspace will be proved in later research.

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References
[1] Senturia S D 1998 Proc. of the IEEE vol 86 (New Jersey: IEEE)pp 1611-26
[2] Zhou N, Clark J V and Pister K S J 1998 Int. Conf. on Modeling and Simulation of Microsystems Semiconductors, Sensors and Actuators (Santa Clara: NSTI)pp 308-13
[3] Vandemeer J E 1998 Tech. Report (Pittsburgh: University of Carnegie Mellon)
[4] Gabby L D 1998 Ph.D. dissertation (Cambridge, MA: MIT)
[5] Lin W Z, Lee K H, Lim S P and Lu P 2001 Int. J. Nonlinear Sci 289-100
[6] Antoulas A C, Sorensen D C 2001 Int. J. Appl. Math. Comput. Sci 11 1093-21
[7] Kung S Y 1980 Proc. Joint Auto. Contr. Conf. (San Francisco: American Automatic Control Council) F48D
[8] Freund R W 2000 J. Comput. Appl.Math 123 395-421
[9] Arnoldi W E 1951 *Quart. J. Appl Math* 9 17-29
[10] Grimme E J 1997 Ph.D. dissertation (Urbana: University of Illinois at Urbana Champaign)
[11] Bai Z J, Su Y F 2004 *J. of Shanghai Univ.* 8 378-390
[12] Chu J K, Hao X C and Wang L D 2007 *Chinese J. of Mech. Engi* 43 116-21
[13] Kohnke P (ed) 2004 *ANSYS Theory Reference* (Canonsburg, PA: ANSYS, Inc.)
[14] Rudnyi E B, Lienemann J Greiner A and Korvink J G 2004 *Tech.Proc.of Nanotech. Conf. and Trade Show* vol 2(Boston: NSTI) pp 279-82