Creation of coherent atomic superpositions by fractional STIRAP

N. V. Vitanov\textsuperscript{1,a} K.-A. Suominen\textsuperscript{1,\textsuperscript{b}}, and B. W. Shore\textsuperscript{2}

\textsuperscript{1}Helsinki Institute of Physics, PL 9, FIN-00014 Helsingin yliopisto, Finland
\textsuperscript{2}Lawrence Livermore National Laboratory, Livermore, CA 94550, USA

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We discuss a simple scheme for preparing atoms and molecules in an arbitrary preselected coherent superposition of quantum states. The technique, which we call fractional stimulated Raman adiabatic passage (f-STIRAP), is based upon (incomplete) adiabatic population transfer between an initial state $\psi_1$ and state $\psi_3$ through an intermediate state $\psi_2$. As in STIRAP, the Stokes pulse arrives before the pump pulse, but unlike STIRAP, the two pulses terminate simultaneously while maintaining a constant ratio of amplitudes. The independence of f-STIRAP from details of pulse shape and pulse area makes it the analog of conventional STIRAP in the creation of coherent superpositions of states. We suggest a smooth realization of f-STIRAP which requires only two laser pulses (which can be derived from a single laser) and at the same time ensures the automatic fulfillment of the asymptotic conditions at early and late times. Furthermore, we provide simple analytic estimates of the robustness of f-STIRAP against variations in the pulse intensity, the pulse delay, and the intermediate-state detuning, and discuss its possible extension to multistate systems.

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I. INTRODUCTION

Atoms and molecules prepared in a well defined preselected initial state have applications in many branches of atomic, molecular and optical physics. There are various techniques available for preparing a single initial state \[1\]. Optical pumping is the standard method when the desired state is the $m = -J$ or $m = +J$ sublevel of a ground state with a total angular momentum of $J$. $\pi$-pulse and chirped-pulse techniques are used to prepare atoms and molecules in a particular excited state, accessible via an electric-dipole single-photon transition. A state, accessible via a two-photon transition (which can be an excited, metastable, or another ground state) can be populated by using generalized $\pi$-pulses, chirped pulses, or the robust and efficient technique of stimulated Raman adiabatic passage (STIRAP) \[2\,3\]. A state accessible via a multiphoton transition can be populated by means of generalizations of the above techniques. Some of these techniques are robust against moderate variations in the interaction parameters, as the chirped-pulse method and STIRAP, while others, such as the $\pi$-pulse method, are not.

It is considerably more difficult to prepare atoms and molecules in a preselected coherent superposition of states. Creating an initial atomic coherence is essential for such effects as dark resonances \[3\,4\], subrecoil laser cooling \[10\,13\], electromagnetically induced transparency \[14\,16\], light amplification without inversion \[17\,19\], refraction index enhancement without absorption \[20\], harmonic generation \[21\,22\], and quantum information \[23\]. Modifications of the above techniques can still be used but they are generally very sensitive to small variations in the interaction parameters, such as pulse areas and detunings. In this paper, we discuss a technique, based on ideas similar to those of STIRAP, which guarantees the creation of any desired coherent superposition of two states. The technique, which we call fractional STIRAP (f-STIRAP), is based upon adiabatic population transfer between the initially populated state $\psi_1$ and state $\psi_3$ through an intermediate state $\psi_2$. It requires a two-photon resonance between states $\psi_1$ and $\psi_3$ and uses two laser pulses, a pump pulse $\Omega_P$, linking states $\psi_1$ and $\psi_2$, and a Stokes pulse $\Omega_S$, linking states $\psi_2$ and $\psi_3$. As in STIRAP, the Stokes pulse arrives before the pump pulse, but unlike STIRAP, where the Stokes pulse vanishes first, here the two pulses vanish simultaneously. This “incompleted STIRAP” evolution provides the possibility of ending with the population residing in both states $\psi_1$ and $\psi_3$, rather than being transferred entirely to state $\psi_3$, as in STIRAP. Moreover, since the population transfer is carried out through an adiabatic state which is a linear superposition of states $\psi_1$ and $\psi_3$ only (often referred to as the trapped or dark state), state $\psi_2$ remains unpopulated, even transiently, and hence its properties, including spontaneous decay, do not affect the process. This guarantees the coherence of the created final superposition of states. The f-STIRAP has all the advantages that STIRAP has in population transfer to a single state, regarding robustness, efficiency and simplicity, and can be considered as its analog in creating coherent superpositions of states.

The idea of interrupted STIRAP was proposed for the first time by Marte, Zoller and Hall \[24\] as a way to create an atomic beam splitter, which is an essential part of an atomic interferometer \[24\,25\]. This idea was later discussed by Lawall and Prentiss \[26\] and analyzed in more detail by Weitz, Young and Chu \[27\], who have demonstrated it experimentally \[28\] achieving the interruption of STIRAP evolution by simultaneously and abruptly turning to zero the intensities of both the pump and the Stokes fields. In the present paper, we discuss
the potential of f-STIRAP for preparing *coherent superpositions* of states. This aspect is slightly different because it puts a particular emphasize on the *robustness* of the process. This is so because variations in the parameters of the created superposition (the populations and the relative phase) are very undesirable when the goal is to create a well defined coherent superposition for subsequent use as an initial state in a certain process, whereas such variations are less important in an atom interferometer. Following this argument, we propose a *smooth*, rather than abrupt, realization, which is advantageous in achieving adiabaticity (and hence, robustness) more easily. It makes use of only two laser pulses (which can be derived from a single laser) and at the same time ensures the automatic fulfillment of the asymptotic conditions for f-STIRAP. Furthermore, we derive simple analytic estimates of the robustness of f-STIRAP against variations in the pulse intensity, the pulse delay, and the intermediate-state detuning, and discuss a possible extension of this scheme to multistate systems.

We note that another extension of STIRAP aimed at creating a coherent superposition of states has been proposed very recently [32]. It is based on adiabatic transfer in a four-state system with a tripod linkage by means of three laser pulses.

This paper is organized as follows. The idea of f-STIRAP is presented in Sec. II. The robustness of the process is examined in Sec. III. The extension of STIRAP to multistate systems is discussed in Sec. IV. The conclusions are summarized in Sec. V.

II. FRACTIONAL STIRAP

A. The idea

The probability amplitudes $c_k(t)$ of the three states $\psi_k$ ($k = 1, 2, 3$) satisfy the Schrödinger equation,

$$i\hbar \frac{dc}{dt} = H(t)c(t),$$

where $c(t)$ is a column vector comprising $c_k(t)$. In the rotating-wave approximation, the Hamiltonian of our three-state system is

$$H(t) = \hbar \begin{bmatrix} 0 & \frac{1}{2} \Omega_P(t)e^{-i\phi_P} & 0 \\ \frac{1}{2} \Omega_P(t)e^{i\phi_P} & \frac{1}{2} \Omega_S(t)e^{-i\phi_S} & \Delta \\ 0 & \frac{1}{2} \Omega_S(t)e^{i\phi_S} & 0 \end{bmatrix},$$

where $\Delta$ is the single-photon detuning of the intermediate state, while $\Omega_P(t)$ and $\Omega_S(t)$ are the Rabi frequencies of the pump and Stokes pulses, respectively, and $\phi_P$ and $\phi_S$ are the phases of the two fields. An important condition for the success of the scheme discussed below is the two-photon resonance between states $\psi_1$ and $\psi_3$, already assumed in $H(t)$. The system is supposed to be initially in state $\psi_1$,

$$\Psi(-\infty) = \psi_1,$$

and we wish to transform it at the end of the interaction into the coherent superposition

$$\Psi(+\infty) = \psi_1 \cos \alpha - \psi_3 e^{i\phi} \sin \alpha,$$

where $\alpha$ is a constant mixing angle ($0 \leq \alpha \leq \frac{1}{4}\pi$), and $\phi$ is a constant phase. The minus sign, which does not limit the generality of the superposition, is taken for the sake of later convenience.

Following STIRAP, we are going to use the fact that one of the eigenvalues of $H(t)$ is equal to zero and the corresponding eigenstate (the trapped state) is

$$\Phi_T(t) = \frac{1}{\Omega(t)} [\Omega_S(t)e^{-i\phi_S}\psi_1 - \Omega_P(t)e^{i\phi_P}\psi_3],$$

where

$$\Omega(t) = \sqrt{\Omega_P^2(t) + \Omega_S^2(t)}.$$

In STIRAP, the Stokes pulse precedes the pump pulse, that is $\Omega_P(t)/\Omega_S(t) \to 0$ as $t \to -\infty$ and $\Omega_S(t)/\Omega_P(t) \to 0$ as $t \to +\infty$. Consequently, we have $\Phi_T(-\infty) = e^{-i\phi_S}\psi_1$ and $\Phi_T(+\infty) = -e^{i\phi_P}\psi_3$, which ensures complete population transfer from state $\psi_1$ to state $\psi_3$ in the adiabatic limit. Moreover, insofar as the trapped state $\Phi_T(t)$ does not involve state $\psi_2$, the latter is not populated during the transfer. This implies that its properties, including its detuning $\Delta$ and decay rate, do not affect the transfer efficiency in the adiabatic limit. Furthermore, since STIRAP is an adiabatic process, it is robust against moderate variations of laser parameters (intensities, detunings, pulse shapes, pulse widths, pulse delay).

Note that as far as STIRAP is concerned the phases of the pump and Stokes fields do not affect the transfer efficiency in the adiabatic limit. Fur-
phase difference between these two states is determined is initially in the sublevels in STIRAP is the three-state chain which is formed by the pump pulse $\Omega_P$ should be $\sigma^+$ polarized and the Stokes pulse $\Omega_S$ should be $\sigma^-$ polarized. The f-STIRAP creates a coherent superposition of states $m = -1$ and $m = +1$.

The case when $\Omega_P/\Omega_S \to 1$ at $+\infty$ and $\phi = 0$ is particularly interesting for quantum information and atom optics; then $\alpha = \frac{1}{4}\pi$ and $\Psi(+\infty) = \frac{1}{\sqrt{2}}(\psi_1 - \psi_3)$. The creation of this superposition corresponds to the Hadamard transform of a quantum bit in quantum information. In atom optics, if the pump and Stokes pulses propagate in opposite directions, the creation of this coherent superposition is accompanied with a momentum transfer of $2\hbar k$ for a half of the atoms and f-STIRAP works in this case as a coherent beam splitter \[24,31\].

### B. The system

A particularly suitable system for realization of f-STIRAP is the three-state chain which is formed by the sublevels in $J = 1 \leftrightarrow J = 0$ transition by using a couple of circularly polarized laser pulses. If, for example, the system is initially in the $m = -1$ sublevel of the lower level, then the pump pulse $\Omega_P$ should be $\sigma^+$ polarized and the Stokes pulse $\Omega_S$ should be $\sigma^-$ polarized. The f-STIRAP creates a coherent superposition of states $m = -1$ and $m = +1$. If, for example, the system is initially in the $m = -1$ sublevel, then the pump pulse should be $\sigma^+$ polarized and the Stokes pulse $\sigma^-$ polarized. The convenience of this system derives from the fact that the two-photon resonance condition, which is essential for f-STIRAP as well as for the standard STIRAP, is automatically fulfilled, provided there are no magnetic fields. Moreover, the two pulses can be derived from the same laser pulse by beam splitting. This fact, along with the robustness of the scheme against single-photon detuning and laser power, makes f-STIRAP insensitive to phase and energy fluctuations. Moreover, the equal energies of states $m = -1$ and $m = +1$ mean that the phase difference between these two states is determined entirely by f-STIRAP and remains fixed after its completion. Subsequently, if necessary, the relative phase between states $\psi_1$ and $\psi_3$ can be altered by applying a pulsed magnetic field or off-resonant laser pulses.

### C. The pulse sequence

One of the possible realizations of the f-STIRAP asymptotic conditions \[6\] is to take a Stokes pulse of longer duration than the pump pulse, with the maxima of both pulses occurring at the same time ($t = 0$), and truncate both of them there, e. g. $\Omega_P(t) = \begin{cases} \Omega_0 \sin \alpha e^{-t/T}^2, & t \leq 0 \\ 0, & t > 0 \end{cases}$, $\Omega_S(t) = \begin{cases} \Omega_0 \cos \alpha e^{-(t^2/T)}^2, & t \leq 0 \\ 0, & t > 0 \end{cases}$, where $\alpha < 1$. This case, a variation of which has been implemented in \[28\], is a straightforward example of interrupted evolution.

A more elegant, smooth realization of condition \[\frac{6}{2}\] can be achieved by using three pulses, a pump pulse and two Stokes pulses – one with the same time dependence as the pump pulse and another coming earlier, e.g. $\Omega_P(t) = \Omega_0 \sin \alpha e^{-(t^2-T^2)/T^2}$, $\Omega_S(t) = \Omega_0 e^{-(t^2-T^2)/T^2} + \Omega_0 \cos \alpha e^{-(t^2-T^2)/T^2}$. In fact, this pulse timing can be achieved by using only two pulses – one with $\sigma^-$ polarization and Rabi frequency $\Omega_0 e^{-(t^2-T^2)/T^2}$, and another with time dependence $\Omega_0 e^{-(t^2-T^2)/T^2}$ and elliptic polarization in the $x$-plane, whose electric field (in the complex representation $E = E_x + iE_y$) is given by \[\frac{6}{33}\] $E = A_P e^{i\omega t + i\phi_P} + A_S e^{i\omega t - i\phi_S}$, where $A_P/A_S = \tan \alpha$. The former term represents the $\sigma^+$ polarized component and the latter term represents the $\sigma^-$ one. Here $\phi = \frac{1}{2}(\phi_P + \phi_S)$ is the angle of rotation of the polarization ellipse and $|1 - \tan \alpha/(1 + \tan \alpha)$ is its axial ratio \[\frac{31}{33}\]. Thus, the desired final superposition of states \[2\] is controlled entirely by the polarization of the delayed pulse. Using a linear polarization and $\phi = 0$ would lead to $\alpha = \frac{1}{2}\pi$ and a superposition $\Psi(t \to \infty) = \frac{1}{\sqrt{2}}(\psi_1 - \psi_3)$.

Note that the phase of the first $\sigma^-$ pulse is irrelevant in the present context [because it does not affect conditions \[\frac{6}{2}\]] and it is assumed to be the same as that of the $\sigma^-$ component of the second pulse.

A typical example of time evolution in fractional STIRAP is shown in Fig. 3 (lower figure). The pulse shapes (upper figure) are defined by Eqs. \[3\] with $\alpha = \frac{1}{2}\pi$, $\tau = 0.7T$, $\Omega_0 T = 20$. The population evolves smoothly from state $\psi_1$ initially to the coherent superposition $\frac{1}{\sqrt{2}}(\psi_1 - \psi_3)$ finally, very similarly to the manner in which the population is transferred completely from state $\psi_1$ to state $\psi_3$ in the standard STIRAP.
where $\vartheta$ is the adiabatic condition, which for $\Delta = 0$ reads \[2,3\]

The pulse shapes (upper figure) are defined by Eqs. (8) with $\alpha = \frac{\pi}{4}$, $\tau = 0.7T$, $\Omega_0 T = 20$.

III. ROBUSTNESS

A. Pulse delay and laser intensity

1. Adiabaticity condition

The starting point in our analysis of the robustness of f-STIRAP against variations in the interaction parameters is the adiabatic condition, which for $\Delta = 0$ reads \[4\]

where $\dot{\vartheta}(t) = \arctan[\Omega_P(t)/\Omega_S(t)]$ and $\Omega(t)$ is defined by Eq. (4). For the shapes \[5\] with $\phi = 0$, we find

$$\dot{\vartheta}(t) = \frac{4\tau}{T^2} \frac{\zeta(t) \sin \alpha}{\sin^2 \alpha + [\cos \alpha + \zeta(t)]^2},$$

$$\Omega(t) = \Omega_0 e^{-(t-\tau)^2/T^2} \sqrt{\sin^2 \alpha + [\cos \alpha + \zeta(t)]^2},$$

where $\zeta(t) = e^{-4\tau t/T^2}$. For appreciable non-adiabatic transitions to occur, two conditions have to be satisfied,

$$\left| \dot{\vartheta}(t) \right| \lesssim \frac{1}{2} \Omega(t), \quad \left| \dot{\vartheta}(t) \right| \lesssim \frac{1}{T}.$$  \[10a\]  \[10b\]

The former of these means that the adiabatic condition \[4\] has to be violated while the latter requires appreciable non-adiabatic coupling. Non-adiabatic transitions are most likely to occur in the region around the maximum of $\dot{\vartheta}(t)$ which is situated at $\zeta(t_0) = 1$, i.e. at

$$t_0 = 0.$$  \[12\]

At $t = t_0$, $\dot{\vartheta}(t)$ and $\Omega(t)$ are equal to

$$\dot{\vartheta}_{\text{max}} = \dot{\vartheta}(t_0) = \frac{2\tau}{T^2} \tan \frac{\alpha}{2},$$

$$\Omega(t_0) = 2\Omega_0 e^{-\tau^2/T^2} \cos \frac{\alpha}{4}.$$  \[13a\]  \[13b\]

Note that this is not necessarily the maximum of $\Omega(t)$. Since both $\Omega(t)$ and $\dot{\vartheta}(t)$ are pulse-shaped, it is useful to find their widths (full widths at half maximum),

$$T_\vartheta \approx \frac{T^2}{\tau} \ln \left( \frac{\sqrt{1 + \cos^2 \frac{\alpha}{2} \cos \frac{\alpha}{4}}} \right),$$

$$T_{\Omega} \approx 2\tau + 2T \sqrt{\ln 2}.$$  \[14a\]  \[14b\]

2. Lower limit on $\tau$

When $\tau \to 0$, the non-adiabatic coupling $\dot{\vartheta}(t)$ becomes a very broad function, broader than $\Omega(t)$ [see Eqs. \[13\]], and there are early times as well as late times, when conditions \[11\] are satisfied. The interference between these two non-adiabatic zones leads to oscillations. This problem will be avoided if the width of $\dot{\vartheta}(t)$ is smaller than the width of $\Omega(t)$, $T_\vartheta \lesssim T_{\Omega}$. From here we find a lower bound for $\tau$, which for $\alpha = \frac{\pi}{4}$ reads as

$$\tau \gtrsim 0.35T.$$  \[15\]

The same arguments applied to STIRAP lead to the estimate $\tau \gtrsim 0.30T$. Hence, fractional STIRAP requires slightly larger pulse delays than STIRAP.

3. Upper limit on $\tau$

Since the maximum \[13a\] of the non-adiabatic coupling $\dot{\vartheta}(t)$ increases with $\tau$ whereas its width \[14a\] decreases, $\dot{\vartheta}(t)$ approaches a $\delta$-function behavior for large $\tau$, which increases the probability for non-adiabatic transitions near $t_0$. The situation is aggravated by a "hole" in $\Omega(t)$, which appears around $t_0$ for large $\tau (\gtrsim 0.8T)$. Hence, the pulse delay should not be very large. To suppress the non-adiabatic transitions, we must have $\frac{1}{2} \Omega(t_0) \gtrsim n \dot{\vartheta}(t_0)$, where $n$ is a "sufficiently large" number. By assuming that both $\dot{\vartheta}(t)$ and $\Omega(t)$ are nearly constant near $t_0$, we find that the probability for nonadiabatic transitions in this region is $\lesssim 1/(n^2 + 1)$. Hence, the choice of $n$ depends on how much nonadiabaticity we
can allow, that is how much deviation from the desired superposition is acceptable. Thus, we find an (implicitly defined) upper bound on \( \tau \),

\[
\Omega_0 T \gtrsim \frac{2n \sin \frac{\tau}{2} \alpha \tau \cos^2 \frac{\alpha}{2} T^2}{\tau^2},
\]

which is similar to the one for standard STIRAP.\(^{33}\)

This inequality can also be seen as a lower bound for the peak Rabi frequency \( \Omega_0 \). Obviously, the Rabi frequency needed to ensure sufficient adiabaticity increases exponentially with \( \tau \).

The conclusion is that although fractional STIRAP should work for any pulse delay \( \tau > 0 \) for sufficiently strong laser pulses, there is an optimal range of \( \tau \), in which adiabaticity is most easily achieved. For example, for \( n = 5 \) this range is \( 0.35T \lesssim \tau \lesssim 0.93T \) for \( \Omega_0 T = 20 \), \( 0.35T \lesssim \tau \lesssim 1.2T \) for \( \Omega_0 T = 40 \), and \( 0.35T \lesssim \tau \lesssim 1.28T \) for \( \Omega_0 T = 60 \).

In Fig. 3, the final population \( P_3 \) of state \( \psi_3 \) is plotted against the time delay \( \tau \) and the peak Rabi frequency \( \Omega_0 \). Comparison is made between standard STIRAP (upper figure) and fractional STIRAP (lower figure). The plateau for STIRAP corresponds to \( P_3 = 1 \) and that for f-STIRAP to \( P_3 = 0 \).

The results in \(^{35}\) for the detuning dependence in STIRAP suggest that in the near-adiabatic regime (when \( \Omega_0 T \gg 1 \)), the range of detunings which do not affect significantly the transfer efficiency is large compared with the peak Rabi frequency, \( \Delta \gg \Omega_0 \). Then \( \varphi(t) \approx \Omega(t)/2\Delta \), and the adiabatic condition (17) reduces to

\[
\Delta \lesssim \frac{\Omega^2(t)}{4n |\dot{\varphi}(t)|}.
\]

It is most important to satisfy this condition in the region around \( t = 0 \), where \( \dot{\varphi}(t) \) is maximal. There we have

\[
\Omega_0 T \gg \frac{1}{\Delta}.
\]
Hence, the acceptable range of intermediate detunings \( \Delta \) is proportional to the squared peak Rabi frequency \( \Omega_0^2 \).

In Fig. 4, the final population \( P_3 \) of state \( \psi_3 \) is plotted against the single-photon detuning \( \Delta \) and the squared peak Rabi frequency \( \Omega_0^2 \). Comparison is made between STIRAP (upper figure) and f-STIRAP (lower figure). The plateau for STIRAP corresponds to \( P_3 = 1 \), while that for f-STIRAP corresponds to \( P_3 = 0.5 \). The plateaus are described well by our simple formula (18).

To conclude this section, we point out that it is possible to derive simple estimates also for the robustness of f-STIRAP against the intermediate state loss rate in the manner in which this has been done for STIRAP [67].
\[
\Psi(1 \leftrightarrow 0) = \sqrt{\frac{1}{4}} \psi_1 - \sqrt{\frac{1}{2}} \psi_2,
\]
(22)
\[
\Psi(1 \leftrightarrow 1) = \sqrt{\frac{1}{4}} \psi_1 + \sqrt{\frac{1}{2}} \psi_2,
\]
(23)
\[
\Psi(2 \leftrightarrow 1) = -\sqrt{\frac{3}{4}} \psi_2 - \sqrt{\frac{1}{4}} \psi_0 + \sqrt{\frac{1}{2}} \psi_3,
\]
(24)
\[
\Psi(2 \leftrightarrow 2) = -\sqrt{\frac{3}{4}} \psi_2 + \sqrt{\frac{1}{4}} \psi_0 + \sqrt{\frac{3}{2}} \psi_2,
\]
(25)
\[
\Psi(3 \leftrightarrow 2) = -\sqrt{\frac{5}{4}} \psi_3 - \sqrt{\frac{3}{16}} \psi_1 + \sqrt{\frac{1}{16}} \psi_1 - \sqrt{\frac{3}{8}} \psi_3,
\]
(26)
\[
\Psi(3 \leftrightarrow 3) = \sqrt{\frac{5}{4}} \psi_3 + \sqrt{\frac{1}{16}} \psi_1 + \sqrt{\frac{3}{4}} \psi_1 + \sqrt{\frac{3}{8}} \psi_3,
\]
(27)
\[
\Psi(4 \leftrightarrow 3) = -\sqrt{\frac{5}{128}} \psi_4 - \sqrt{\frac{7}{128}} \psi_2 + \sqrt{\frac{1}{64}} \psi_0 - \sqrt{\frac{7}{64}} \psi_1 + \sqrt{\frac{1}{128}} \psi_1,
\]
(28)
\[
\Psi(4 \leftrightarrow 4) = \sqrt{\frac{35}{128}} \psi_4 + \sqrt{\frac{35}{128}} \psi_2 + \sqrt{\frac{1}{64}} \psi_0 + \sqrt{\frac{7}{64}} \psi_1 + \sqrt{\frac{1}{128}} \psi_1.
\]
(29)

Such multistate systems provide the possibility of robust creation of well-defined radiatively stable coherent superpositions of more than two states. From the viewpoint of beam splitting, f-STIRAP applied to such systems achieves coherent splitting of the initial atomic beam into \( J + 1 \) components. Moreover, we can alter the components in each superposition by changing the mixing angle \( \alpha \), and we can alter their phases by changing the phase \( \phi \), i.e., we have two degrees of freedom. Finally, if necessary, the populations of the intermediate states can be removed (stored into metastable states or ionized) by subsequent laser pulses, thus leaving the atom in a coherent superposition of two states (\( \psi_{-J} \) and \( \psi_J \)) with a large momentum difference. It is seen from the superpositions listed above that, due to the nature of the Clebsch-Gordan coefficients, this is relevant for the transitions with \( J' = J \) only, because for \( J' = J - 1 \) the coefficients of \( \psi_{-J} \) and \( \psi_J \) are too small.

V. CONCLUSIONS

We have discussed the properties of a technique which can create any preselected coherent superposition of two states. The technique, which we have called fractional STIRAP (f-STIRAP), is based upon (incomplete) adiabatic population transfer between the initial state \( \psi_1 \) and state \( \psi_3 \) through an intermediate state \( \psi_2 \). As in STIRAP, the Stokes pulse arrives before the pump pulse, but unlike STIRAP, the two pulses vanish simultaneously. This “incompleted STIRAP” evolution provides the possibility of ending with the population residing in both states \( \psi_1 \) and \( \psi_3 \), rather than being transferred entirely to state \( \psi_3 \), as in STIRAP. We have suggested a smooth realization of f-STIRAP which requires only two laser pulses (which can be derived from a single laser) and in the same time ensures the automatic fulfillment of the asymptotic conditions for f-STIRAP. Fractional STIRAP has all the properties that STIRAP has in population transfer to a single state, regarding robustness, efficiency, and simplicity, and can thus be considered as its analog in creating coherent superpositions of states. The robustness against variations in the laser parameters (such as detunings and pulse areas) is a particularly important feature, which makes f-STIRAP insensitive to phase and energy fluctuations of the laser. In the frequently implemented setup, where an atomic beam crosses two slightly displaced laser beams at right angles, the atoms with different velocities have different interaction times and then the robustness of the technique against the pulse area ensures that virtually all atoms undergo the same excitation. We have derived simple analytic estimates of the robustness of f-STIRAP against variations in the pulse intensity, the pulse delay, and the intermediate-state detuning, and have discussed a possible extension of f-STIRAP to multistate systems. In principle, the method can be implemented without significant complications to any system where STIRAP has been done.

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a Electronic address: vitanov@rock.helsinki.fi
b Permanent address: Department of Physics, University of Helsinki, PL 9, FIN-00014 Helsingin yliopisto, Finland
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