DESIGN OF PROBABILISTIC $l_2 - l_\infty$ FILTER FOR UNCERTAIN MARKOV JUMP SYSTEMS WITH PARTIAL INFORMATION OF THE TRANSITION PROBABILITIES

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Abstract. In this paper, the problem of $l_2 - l_\infty$ probabilistic filtering for uncertain Markov jump systems with partial information of the transition probabilities is studied, where the uncertainties are caused by randomly changing interior parameters. Combining the original system and the filtering system, an augmented error system is proposed. Some concepts of probability theory are introduced to handle the uncertainties. Due to the complicated structure of real practical systems, only partial information on the transition probabilities are available. In this paper, by using Lyapunov functional method and probability theory, linear matrix inequalities (LMIs) type of sufficient conditions are derived. Based on these sufficient conditions, a probability filter is constructed such that the augmented error system with partial information of the transition probabilities is stochastically stable with a given confidence level and satisfying an $l_2 - l_\infty$ performance index. Furthermore, the gain matrices of the filter are obtained through the introduction of slack matrices. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

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1. **Introduction.** Markov jump systems, which compose of modes and state-space equations, are typical multi-model systems. Each mode corresponds to a state-space equation, and the switching between the state-space equations is governed by a transition probability matrix. Furthermore, the switching order among state-space equations is random. Thus, a Markov jump system is also a stochastic system. In practice, many dynamic systems, such as economic systems [24], network control systems [2] and oil catalytic cracking process [18], are subject to abrupt variation in their structure or parameters causing undesirable effects to these systems. Clearly, Markov jump systems appear to be suitable to characterize these dynamic systems, where their structures or parameters can change suddenly due to the components failures, data loss or time delay. Due to the practical significance of Markov jump systems, they have received an increasing attention over the past several decades. Important results obtained for Markov jump systems include stochastic stability and stabilization problems [27, 16], control problems [9, 14], filtering problems [10, 25] and fault detection problems [11, 26]. The transition probability plays a critical role in a Markov jump system, because it determines the probability that the system will be operated in certain mode at the next moment. However, for all the papers mentioned above, they assume that the transition probabilities are fully available. However, in many practical situations, due to the complex structure of the systems, many parameters of the systems cannot be measured, causing the transition probabilities not being fully known. For these reasons, many results related to unknown transition probabilities are available in the literature. See, for example, [26, 12, 21, 13].

In a complicated industrial process, its system parameters can change due to severe environment. These changes are generally referred to uncertainties of the system. Uncertainties can also occur in a dynamic systems caused by inaccurate modelling. In addition, a large deviation of the measurement data from the actual data can induce uncertainties. Due to the existence of uncertainties, the theory and methods used for deterministic systems are not applicable to systems with uncertainties. Thus, it is necessary to design a controller such that the uncertain system has a satisfactory performance. A common approach to the analysis of uncertain systems is to add a $\Delta$ to the parameters of the mathematical model of the system [28]. Many important results have been obtained for these uncertain systems [28, 17, 15]. However, for these studies, the uncertainties are assumed to be norm bounded. The designed controller or filter is effectively robust to all possible uncertainties within the norm bounded constraints. In other words, the controller or filter obtained are for the worst-case scenario, and hence can be over conservative. In addition, the norm bounded method presented in [28, 17] requires that the bounds exist and these bounds are fully known to the controller designers. In reality, for a high-dimensional uncertain system, it is difficult to have access to the size of the bound. Generally, these uncertainties usually depend on some vital parameters, which have significant effects on their bound. These parameters are bounded within a given range. Each parameter changes continuously within its own range. Although some uncertainties are within the assumed bounds, the probability for these situations is very small, close to zero. If the robust controller or filter to be designed takes these uncertainties into consideration, it will lead to the wastage of resources and the corresponding cost function value will increase. Clearly, it is more natural to make use of probability theory to deal with the uncertainty. More specially, a fixed number of uncertain instances are selected based on probabilistic
randomization method. Then, the probabilistic controller or filter is designed such that the system is stable with a high confidence level and that a certain performance index is achieved. This method is regarded as a soft processing mechanism for uncertainties [23]. The controller or filter being designed based on this method will be robust with respect to a large majority of uncertainties. A computational method has been proposed to solve the resulting optimization problems based on randomization approach [23, 5].

Filtering and state estimation are popular topics in control community and have been extensively applied to practical systems, such as signal processing and target tracking. Many useful results concerning filtering and estimation have been proposed [1, 22, 3, 8]. One of the most famous filtering method is Kalman filtering [19], which is used extensively in control theory. However, it was developed based on an exact mathematical model subject to the disturbance of Gaussian noise. Besides Kalman filtering, $H_\infty$ filtering [20], $H_2$ filtering [6], $l_2-l_\infty$ filtering [7] have also been used to deal with noises in control theory, which can effectively estimate the states of uncertain systems. Compared with Karman filtering, these filtering methods are more suitable to estimate the states of uncertain systems, because they do not rely on an exact mathematical model and Gaussian noise. However, the design of $l_2-l_\infty$ robust filter for uncertain Markov jump systems with partial information of the transition probabilities by using probabilistic randomization approach has not been studied previously.

In this paper, we design an $l_2-l_\infty$ probabilistic filter for uncertain Markov jump systems with partial information of the transition probabilities based on probabilistic randomization approach. Sufficient conditions are obtained such that under these sufficient conditions the augmented error system is probabilistically stochastically stable with a high confidence level and that a prescribed performance index is satisfied. The filter gains are obtained through introducing slack matrices. The main contributions of this paper are summarized as follows: (1) a probabilistic robust filter is designed for uncertain Markov jump systems with partial information of the transition probabilities; (2) the designed probabilistic filter will ensure that the augmented system is probabilistically stochastically stable with a high confidence level and that a prescribed performance index is satisfied; (3) gain matrices of the probabilistic filter are obtained through introducing slack matrices.

**Notation.** Throughout this paper, $P > 0$ means that $P$ is positive definite. * represents the symmetric block of a symmetric matrix. 0 and $I$ are, respectively, the zero matrix and identity matrix with appropriate dimensions. $M^T$ and $M^{-1}$ represent, respectively, the transpose and the inverse of the matrix $M$. $E\{\cdot\}$ denotes the expectation operator, $l_2[0, \infty)$ denotes the space of square summable infinite vector sequences over $[0, \infty)$, $\|\cdot\|$ refers to the Euclidean norm. $R^n$ denotes the set of real $n$-dimensional vectors.

2. Problem statement and preliminaries. In this paper, we consider a class of uncertain Markov jump systems (S) with partial information of the transition probabilities in a given probability space $(\Theta, F, P)$, described by

$$
\begin{align*}
\begin{cases}
x(k+1) &= A(v(k))x(k) + B(v(k))w(k) + p(x(k), w(k), v(k), \alpha), \\
y(k) &= C(v(k))x(k) + D(v(k))w(k), \\
z(k) &= E(v(k))x(k) + F(v(k))w(k),
\end{cases}
\end{align*}
$$

(1)
where \( x(k) \in \mathbb{R}^m \) is the system state vector; \( y(k) \in \mathbb{R}^p \) is the measurement; \( z(k) \in \mathbb{R}^q \) is the signal to be estimated; \( w(k) \in \mathbb{R}^m \) is the external disturbance belonging to \( l_2[0, \infty] \); \( p(x(k), w(k), v(k), \alpha) \) is the uncertainty, which is described by \( p(x(k), w(k), v(k), \alpha) = g_1(v(k), \alpha)x(k) + g_2(v(k), \alpha)w(k) \), where \( g_1(v(k), \alpha) \) and \( g_2(v(k), \alpha) \) are known matrices involving random parameter vector \( \alpha = [\alpha_1, \alpha_2, \cdots, \alpha_r]^T \in \mathbb{R}^r \). The Markov chain \( \{v(k)\} (k \in \mathbb{Z}^+) \) takes values in a finite state set \( \varphi = \{1, 2, \ldots, N\} \) with transition probability matrix \( \Pi \triangleq \{\pi_{ij}\} \), where the transition probability is defined by \( \pi_{ij} \triangleq \Pr\{v(k+1) = j|v(k) = i\} \geq 0, \forall i, j \in \varphi \), which represents the transition probability from mode \( i \) at time \( k \) to mode \( j \) at time \( k+1 \), satisfying \( 0 \leq \pi_{ij} \leq 1 \) and \( \sum_{j=1}^{N} \pi_{ij} = 1, i \in \varphi \); \( A(\varphi(k)) \in \mathbb{R}^{n \times n}, B(\varphi(k)) \in \mathbb{R}^{n \times m}, C(\varphi(k)) \in \mathbb{R}^{p \times n}, D(\varphi(k)) \in \mathbb{R}^{p \times m}, E(\varphi(k)) \in \mathbb{R}^{m \times n}, \) and \( F(\varphi(k)) \in \mathbb{R}^{m \times m} \) are known real constant system matrices.

For convenience, when \( p(k) = i \), the system matrices are shown by \( A_i, B_i, C_i, D_i, E_i \) and \( F_i \). Let \( g_1(p(k), \alpha)x(k) = g_{1i}(\alpha)x(k), \) \( g_2(p(k), \alpha)x(k) = g_{2i}(\alpha)w(k) \). Therefore, system (1) can be written as

\[
\begin{cases}
x(k+1) = (A_i + g_{1i}(\alpha))x(k) + (B_i + g_{2i}(\alpha))w(k), \\
y(k) = C_i x(k) + D_i w(k), \\
z(k) = E_i x(k) + F_i w(k).
\end{cases}
\]

(2)

In this paper, there are only partial information of the transition probability matrix \( \Pi \). For convenience, define

\[
K^i = \{j : \pi_{ij} \text{ is known}\}, \quad UK^i = \{j : \pi_{ij} \text{ is unknown}\}.
\]

Then, we construct the following mode-dependent full order filter (F), described by

\[
\begin{cases}
\dot{x}(k+1) = \hat{A}_i \dot{x}(k) + \hat{B}_i y(k), \\
\dot{z}(k) = \hat{C}_i \dot{x}(k) + \hat{D}_i y(k),
\end{cases}
\]

(3)

where \( \dot{x}(k) \in \mathbb{R}^m \) is the filter system state; \( \dot{z}(k) \in \mathbb{R}^q \) is the filter system output; \( \hat{A}_i, \hat{B}_i, \hat{C}_i \) and \( \hat{D}_i \) are filter system matrices to be determined with appropriate dimensions.

Combining system (2) with filter system (3), we obtain an augmented error system (S-F) with following form:

\[
\begin{cases}
\dot{\hat{x}}(k+1) = \hat{A}_i(\alpha)\hat{x}(k) + \hat{B}_i(\alpha)w(k), \\
\dot{\hat{z}}(k) = \hat{C}_i \hat{x}(k) + \hat{D}_i w(k),
\end{cases}
\]

(4)

where \( \hat{x}(k) = [x^T(k), \dot{x}^T(k)]^T \), \( \hat{z}(k) = z(k) - \hat{z}(k), \)

\[
\hat{A}_i(\alpha) = \begin{bmatrix} A_i + g_{1i}(\alpha) & 0 \\ B_i C_i & \hat{A}_i \end{bmatrix}, \quad \hat{B}_i(\alpha) = \begin{bmatrix} B_i + g_{2i}(\alpha) \\ \hat{B}_i D_i \end{bmatrix}, \quad \hat{C}_i = \begin{bmatrix} E_i - \hat{D}_i C_i & -\hat{C}_i \end{bmatrix}, \quad \hat{D}_i = F_i - \hat{D}_i D_i.
\]

Let \( G_f = \{G_{f1}, G_{f2}, \cdots, G_{fN}\} \), where \( G_{fi} = \{\hat{A}_i, \hat{B}_i\}, \) \( i \in \varphi \). Define a binary function \( q(G_f, \alpha) \),

\[
q(G_f, \alpha) = \begin{cases} 0, & \text{if (4) is stochastically stable under } G_f, \\
1, & \text{otherwise.}
\end{cases}
\]

(5)
Our purpose is to find a feasible solution $G_f$ such that the augmented error system (4) is probabilistically stochastically stable. Let $Pr$ denote a given probability measure over the uncertainty set $U$, $\epsilon \in (0, 1)$ be a given small number representing the probabilistic level. Then, some necessary concepts are given below.

For a given $G_f$, let the probability of violation be defined as

$$\Theta(G_f, \alpha) = Pr(q(G_f, \alpha) = 1).$$

(6)

We wish to find a feasible $G_f$ such that following inequality holds:

$$Pr(q(G_f, \alpha) = 1) \leq \epsilon$$

(7)

However, it is difficult to solve the problem subject to the probabilistic constraint given by (7). Thus, we shall make use of the theory of confidence level to handle this probabilistic constraint. Let $\rho \in (0, 1)$ be a confidence parameter, $\epsilon \in (0, 1)$ be an accuracy parameter. Our objective is to find a feasible solution $G_f$ such that the probability of $Pr(q(G_f, \alpha) = 1) \leq \epsilon$ is not smaller than $1 - \rho$. In order to specify an appropriate $\rho$, we need a sufficient number of samples so as to guarantee that the designed filter will ensure the error system (4) to be stochastically stable with a given confidence level. The following lemma gives indication on the required minimum number of samples $M(\epsilon, \rho)$ such that the probability of $Pr(q(G_f, \alpha) = 1) \leq \epsilon$ is not smaller than $1 - \rho$.

**Lemma 2.1.** [4] Let an accuracy parameter $\epsilon \in (0, 1)$ and a confidence parameter $\rho \in (0, 1)$ be given. Suppose that the number of samples is such that the following inequality

$$M \geq \tilde{M}(\epsilon, \rho) = \left\lceil \inf_{v \in (0, 1)} \frac{1}{1 - v} \left( \frac{1}{\epsilon} \ln \frac{1}{\rho} \right) + q + \frac{q}{\epsilon} \ln \frac{1}{\epsilon v} + \frac{1}{\epsilon} \ln \frac{2}{4q} \right\rceil,$$

(8)

is satisfied, where $\lceil \cdot \rceil$ denotes the smallest integer greater than or equal to the argument, $q$ is the number of free variables for constraints, $v$ can be freely selected in $(0, 1)$. Then, by selecting $M$ independently identically distributed samples \{\alpha^1, \alpha^2, \cdots, \alpha^M\}, a feasible solution $G_f$ is obtained such that $Pr(q(G_f, \alpha) = 1) \leq \epsilon$ holds with probability not smaller than $1 - \rho$.

**Definition 2.2.** Let $\tilde{M}$ possible extractions \{\alpha^1, \alpha^2, \cdots, \alpha^{\tilde{M}}\} be selected randomly according to Lemma 2.1. Suppose that $\epsilon \in (0, 1)$ and $\rho \in (0, 1)$. Then, for a given initial state $\bar{x}(0)$, system (4) is said to be $\epsilon$-level stochastically stable, if for $w(k) \equiv 0$, the following inequality is satisfied:

$$E\left\{\sum_{k=0}^{\infty} \|\bar{x}(k)\|^2 |\bar{x}(0), p(0), \alpha^p\right\} < \infty, \ p = 1, 2, \cdots, \tilde{M}.$$  

(9)

**Definition 2.3.** Let Lemma 2.1 be used to randomly select $\tilde{M}$ possible extractions \{\alpha^1, \alpha^2, \cdots, \alpha^{\tilde{M}}\}. Suppose that $\epsilon \in (0, 1)$, $\rho \in (0, 1)$ and $\gamma > 0$. Then, the error system (4) is said to be $\epsilon$-level stochastically stable with an $l_2 - l_\infty$ performance index $\gamma$, if inequality (9) holds and furthermore, for any nonzero $w(k) \in l_2[0, \infty)$, there exists a scalar $\gamma$, such that

$$\sup_k E\{\bar{z}(k)\} \leq \gamma^2 E\left\{\sum_{t=0}^{\infty} w^T(t)w(t)\right\}.$$  

(10)
3. Main results. In this section, our purpose is to design a filter such that the resulting system (4) with partial information of the transition probabilities is $\epsilon$-level stochastically stable with an $l_2-l_\infty$ performance index $\gamma$. First, the sufficient conditions for ensuring $\epsilon$-level stochastic stability is presented for the augment system (4).

Theorem 3.1. Let Lemma 2.1 be used to randomly select $\tilde{M}$ possible extractions $\{\alpha^1, \alpha^2, \cdots, \alpha^{\tilde{M}}\}$. Let $\epsilon \in (0, 1)$ and $\rho \in (0, 1)$. Then, the error system (4) with partial information of the transition probabilities is $\epsilon$-level stochastically stable, if $w(k) \equiv 0$, and there exist positive definite symmetric matrices $P_i > 0$ for each $i \in \varphi$, such that

$$\Gamma_{1i}(\alpha^p) = \tilde{A}_i(\alpha^p)P_{i}^K\tilde{A}_i(\alpha^p) - P_i^K < 0, \quad (11)$$

$$\Gamma_{2i}(\alpha^p) = \tilde{A}_i(\alpha^p)P_j\tilde{A}_i(\alpha^p) - P_j < 0, \quad j \in UK^i, \quad (12)$$

where $P_j^K = \sum_{j \in K^i} \pi_{ij}P_j$, $P_i^K = \sum_{j \in K^i} \pi_{ij}P_i$, $p = 1, 2, \cdots, \tilde{M}$.

Proof. Denote

$$P_{j}^{UK} = \sum_{j \in U K^i} \pi_{ij}P_j, \quad P_i^{UK} = \sum_{j \in U K^i} \pi_{ij}P_i.$$  

Let $\tilde{M}$ possible extractions $\{\alpha^1, \alpha^2, \cdots, \alpha^{\tilde{M}}\}$ be selected randomly according to Lemma 2.1. Consider the following Lyapunov functional candidate

$$\rho \Delta V(k, i, k, \alpha^p) = \tilde{x}^T(k)P_i\tilde{x}(k), \quad p = 1, 2, \cdots, \tilde{M}. \quad (13)$$

Define

$$E\{\Delta V(k, \alpha^p)\} = E\{V(\tilde{x}(k + 1), j, k + 1, \alpha^p|x(k), i, k, \alpha^p)\} - V(\tilde{x}(k), i, k, \alpha^p).$$

Consider $w(k) \equiv 0$ and $\sum_{j=1}^{N} \pi_{ij} = 1, i \in \varphi$. Then, we have

$$E\{\Delta V(k, \alpha^p)\} = E\{V(\tilde{x}(k + 1), j, k + 1, \alpha^p|x(k), i, k, \alpha^p)\} - V(\tilde{x}(k), i, k, \alpha^p)$$

$$= \tilde{x}^T(k)\tilde{A}_i^T(\alpha^p)\left(P_{j}^{UK} + P_{j}^{K}\right)\tilde{A}_i(\alpha^p)\tilde{x}(k)$$

$$- \tilde{x}^T(k)\left(P_{j}^{K} + P_{j}^{UK}\right)\tilde{x}(k)$$

$$= \tilde{x}^T(k)\left(\tilde{A}_i^T(\alpha^p)P_{j}^{K}\tilde{A}_i(\alpha^p) - P_i^K\right)\tilde{x}(k)$$

$$+ \sum_{j \in U K^i} \pi_{ij}\tilde{x}^T(k)\left(\tilde{A}_i^T(\alpha^p)P_{j}\tilde{A}_i(\alpha^p) - P_j\right)\tilde{x}(k). \quad (14)$$

From Theorem 3.1, it follows that $E\{\Delta V(k, \alpha^p)\} < 0$. Denote

$$\sigma_{1}(\alpha^p) = \min_{i}\{\lambda_{\min}(-\Gamma_{1i}(\alpha^p))\},$$

$$\sigma_{2}(\alpha^p) = \min_{i}\{\lambda_{\min}(-\Gamma_{2i}(\alpha^p))\}, \quad \forall i \in \varphi, \quad p = 1, 2, \cdots, \tilde{M},$$

where $\lambda_{\min}(-\Gamma_{1i}(\alpha^p))$ is the minimal eigenvalue of $-\Gamma_{1i}(\alpha^p)$, $\lambda_{\min}(-\Gamma_{2i}(\alpha^p))$ is the minimal eigenvalue of $-\Gamma_{2i}(\alpha^p)$.

Then, we have

$$E\{\Delta V(k, \alpha^p)\} \leq \left(-\sigma_{1}(\alpha^p) - \left(1 - \sum_{j \in K^i} \pi_{ij}\right)\sigma_{2}(\alpha^p)\right)\tilde{x}_k^T\tilde{x}_k.$$
Denote
\[ \tau(\alpha^p) = \left( \sigma_1(\alpha^p) + \left( 1 - \sum_{j \in K^i} \pi_{ij} \right) \sigma_2(\alpha^p) \right). \]

Then,
\[ E\{\Delta V(k, (\alpha^p))\} \leq -\tau(\alpha^p)\bar{x}_k^T\bar{x}_k. \]

Hence, we have
\[ E\left\{ \sum_{k=0}^{T} \Delta V(k, \alpha^p) \right\} = V(\bar{x}(T + 1), \alpha^p) - V(\bar{x}_0, \alpha^p) \leq -\tau(\alpha^p)E\left\{ \sum_{k=0}^{T} ||\bar{x}(k)||^2 \right\}. \]

It follows that
\[ E\left\{ \sum_{k=0}^{T} ||\bar{x}(k)||^2 \right\} \leq \frac{1}{\tau(\alpha^p)}(V(\bar{x}_0, \alpha^p) - V(\bar{x}(T + 1), \alpha^p)) \leq \frac{1}{\tau(\alpha^p)}V(\bar{x}_0, \alpha^p), \]

which implies \( \lim_{T \to \infty} E\left\{ \sum_{k=0}^{T} ||\bar{x}(k)||^2 \right\} < \infty \) for \( p = 1, 2, \cdots, \hat{M} \). From Definition 2.2, the augmented error system (4) with partial information of the transition probabilities is \( \epsilon \)-level stochastically stable with \( w(k) \equiv 0 \). This completes the proof. \( \square \)

Then, an \( l_2 - l_\infty \) performance index is considered for the error system (4) with \( w(k) \not\equiv 0 \).

**Theorem 3.2.** Let Lemma 2.1 be used to select randomly \( \hat{M} \) possible extractions \( \{\alpha^1, \alpha^2, \cdots, \alpha^{\hat{M}}\} \). Let \( \epsilon \in (0, 1) \), \( \rho \in (0, 1) \) and \( \gamma > 0 \). Then, the error system (4) with partial information of the transition probabilities is \( \epsilon \)-level stochastically stable and satisfying an \( l_2 - l_\infty \) performance index \( \gamma \), if there exist positive definite symmetric matrices \( P_i > 0 \) for each \( i \in \varphi \), such that

\[ \Sigma_{1i}(\alpha^p) = \begin{bmatrix} -P_j^K & P_j^K \hat{A}_i(\alpha^p) & P_j^K \hat{B}_i(\alpha^p) \\ * & -P_i^K & 0 \\ * & * & -\sum_{j \in K^i} \pi_{ij} \end{bmatrix} < 0, \quad (15) \]

\[ \Sigma_{2i}(\alpha^p) = \begin{bmatrix} -P_j & P_j \hat{A}_i(\alpha^p) & 0 \\ * & -P_i & 0 \\ * & * & -I \end{bmatrix} < 0, \quad j \in U K^i, \quad (16) \]

\[ \Sigma_{3i} = \begin{bmatrix} -I & \hat{C}_i & \hat{D}_i \\ * & -\gamma^2 P_i & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0, \quad (17) \]

where \( P_j^K = \sum_{j \in K^i} \pi_{ij} P_j, P_i^K = \sum_{j \in K^i} \pi_{ij} P_i, p = 1, 2, \cdots, \hat{M}. \)

**Proof.** Let \( \hat{M} \) possible extractions \( \{\alpha^1, \alpha^2, \cdots, \alpha^{\hat{M}}\} \) be selected randomly according to Lemma 2.1. By Schur complement, \( \Gamma_{1i}(\alpha^p) < 0 \) and \( \Gamma_{2i}(\alpha^p) < 0 \) can be derived from Theorem 3.2. Therefore, the augmented system (4) with partial information of the transition probabilities is \( \epsilon \)-level stochastically stable with \( w(k) \equiv 0 \) according to Theorem 3.2.
Next, we consider the following performance index for the augmented error system (4).

\[ J = V(\tilde{x}(k), i, k) - E \left\{ \sum_{t=0}^{k-1} w^T(t)w(t) \right\}. \] (18)

It can be written as:

\[ J = E \sum_{t=0}^{k-1} (\Delta V(\tilde{x}(t), i, t) - w^T(t)w(t)). \] (19)

Then,

\[ J = \sum_{t=0}^{k-1} \left\{ \tilde{x}^T(t) \left( \tilde{A}_i^T(\alpha^p) \sum_{j=1}^{N} \pi_{ij} P_j \tilde{A}_i(\alpha^p) - P_i \right) \tilde{x}(t) + 2\tilde{x}^T(t) \left( \tilde{A}_i^T(\alpha^p) \sum_{j=1}^{N} \pi_{ij} P_j \tilde{B}_i(\alpha^p) \right) w(t) + w^T(t) \left( \tilde{B}_i^T(\alpha^p) \sum_{j=1}^{N} \pi_{ij} P_j \tilde{B}_i(\alpha^p) - I \right) w(t) \right\}. \] (20)

Let \( J \) be more concise as given below:

\[ J = \sum_{t=0}^{k-1} \xi^T(t)\Omega_i(\alpha^p)\xi(t), \] (21)

where

\[ \xi^T(t) = [\tilde{x}^T(t) \ w^T(t)], \]

\[ \Omega_i(\alpha^p) = \begin{bmatrix} \tilde{A}_i^T(\alpha^p) \sum_{j=1}^{N} \pi_{ij} P_j \tilde{A}_i(\alpha^p) - P_i & \tilde{A}_i^T(\alpha^p) \sum_{j=1}^{N} \pi_{ij} P_j \tilde{B}_i(\alpha^p) \\ * & \tilde{B}_i^T(\alpha^p) \sum_{j=1}^{N} \pi_{ij} P_j \tilde{B}_i(\alpha^p) - I \end{bmatrix}. \]

It follows that

\[ \Omega_i(\alpha^p) = \begin{bmatrix} \tilde{A}_i^T(\alpha^p) P_j^K \tilde{A}_i(\alpha^p) - P_i^K & \tilde{A}_i^T(\alpha^p) P_j^K \tilde{B}_i(\alpha^p) \\ * & \tilde{B}_i^T(\alpha^p) P_j^K \tilde{B}_i(\alpha^p) - \sum_{j \in K_i} \pi_{ij} \end{bmatrix} + \sum_{j \in \tilde{K}_i} \pi_{ij} \begin{bmatrix} \tilde{A}_i^T(\alpha^p) P_j \tilde{A}_i(\alpha^p) - P_i & \tilde{A}_i^T(\alpha^p) P_j \tilde{B}_i(\alpha^p) \\ * & \tilde{B}_i^T(\alpha^p) P_j \tilde{B}_i(\alpha^p) - I \end{bmatrix}. \] (22)

By Schur complement, \( \Omega_i(\alpha^p) < 0 \) can be derived from Theorem 3.2, which implies that \( J < 0 \).

Thus, we have

\[ V(\tilde{x}(k), i, k) < E \left\{ \sum_{t=0}^{k-1} w^T(t)w(t) \right\}. \] (23)

Obviously, there exists a positive scalar \( \gamma \) such that the following inequality holds:

\[ \gamma^2 V(\tilde{x}(k), i, k) + \gamma^2 w^T(k)w(k) < \gamma^2 E \left\{ \sum_{t=0}^{k} w^T(t)w(t) \right\}. \]

In addition,

\[ E \left( \tilde{z}^T(k) \tilde{z}(k) \right) = \left( \tilde{C}_i \tilde{x}(k) + \tilde{D}_i w(k) \right)^T \left( \tilde{C}_i \tilde{x}(k) + \tilde{D}_i w(k) \right) = \tilde{z}^T(k)\tilde{C}_i^T \tilde{C}_i \tilde{x}(k) + 2\tilde{z}^T(k)\tilde{C}_i^T \tilde{D}_i w(k) + w^T(k)\tilde{D}_i^T \tilde{D}_i w(k). \] (24)
From Theorem 3.2, we conclude that
\[
E(\hat{z}^T(k)\hat{z}(k)) < \gamma^2 V(\hat{z}(k), i, k) + \gamma^2 w^T(k)w(k) < \gamma^2 E\left\{\sum_{t=0}^{k} w^T(t)w(t)\right\}.
\]

Thus, the resulting system (4) with partial information of the transition probabilities is \(\epsilon\)-level stochastically stable and satisfying a prescribed \(l_2 - l_\infty\) performance index. This completes the proof.

**Theorem 3.3.** Let \(\tilde{M}\) possible extractions \(\{\alpha^1, \alpha^2, \ldots, \alpha^{\tilde{M}}\}\) be selected randomly according to Lemma 2.1. Let \(\epsilon \in (0, 1), \rho \in (0, 1)\) and \(\gamma > 0\). Then, the error system (4) with partial information of the transition probabilities is \(\epsilon\)-level stochastically stable and satisfying an \(l_2 - l_\infty\) performance index \(\gamma\), if there exist positive definite symmetric matrices \(P_i\) and matrices \(G_i\) for each \(i \in \varphi\), such that

\[
\Upsilon_{1i}(\alpha^p) = \begin{bmatrix}
P_i^K - G_i - G_i^T & G_i\hat{A}_i(\alpha^p) & G_i\hat{B}_i(\alpha^p) \\
* & -P_i^K & 0 \\
* & * & -\sum_{j \in K^i} \pi_{ij}
\end{bmatrix} < 0, \quad (25)
\]

\[
\Upsilon_{2i}(\alpha^p) = \begin{bmatrix}
P_j - G_i^T & -G_i\hat{A}_i(\alpha^p) & G_i\hat{B}_i(\alpha^p) \\
* & -P_i & 0 \\
* & * & -I
\end{bmatrix} < 0, \quad j \in UK^i, \quad (26)
\]

\[
\Upsilon_{3i} = \begin{bmatrix}
-I & \hat{C}_i & \hat{D}_i \\
* & -\gamma^2 P_i & 0 \\
* & * & -\gamma^2 I
\end{bmatrix} < 0, \quad (27)
\]

where \(P_j^K = \sum_{j \in K^i} \pi_{ij} P_j\), \(P_i^K = \sum_{j \in K^i} \pi_{ij} P_i\), \(p = 1, 2, \ldots, \tilde{M}\).

**Proof.** Left-and-right multiplying inequality (15) by diagonal matrix \(\text{diag}\{G_i (P_j^K)^{-1} I I\}\) and its transpose, respectively, we obtain

\[
\Phi_{1i}(\alpha^p) = \begin{bmatrix}
-G_i(P_j^K)^{-1}G_i^T & G_i\hat{A}_i(\alpha^p) & G_i\hat{B}_i(\alpha^p) \\
* & -P_i^K & 0 \\
* & * & -\sum_{j \in K^i} \pi_{ij}
\end{bmatrix}, \quad (28)
\]

Recalling that \(-G_i(P_j^K)^{-1}G_i^T < P_j^K - G_i - G_i^T\), we have \(\Phi_{1i}(\alpha^p) < 0\) from condition (25).

Left-and-right multiplying inequality (16) by the diagonal matrix \(\text{diag}\{G_i P_j^{-1} I I\}\) and its transpose, respectively, gives

\[
\Phi_{2i}(\alpha^p) = \begin{bmatrix}
-G_iP_j^{-1}G_i^T & G_i\hat{A}_i(\alpha^p) & G_i\hat{B}_i(\alpha^p) \\
* & -P_i & 0 \\
* & * & -I
\end{bmatrix}, \quad j \in UK^i, \quad (29)
\]

Recalling that \(-G_iP_j^{-1}G_i^T < P_j - G_i - G_i^T\), we have \(\Phi_{2i}(\alpha^p) < 0\) from condition (26).

Thus, the resulting system (4) with partial information of the transition probabilities is \(\epsilon\)-level stochastically stable and satisfying an \(l_2 - l_\infty\) performance index \(\gamma\) from Theorem 3.3. This completes the proof. \(\square\)
It is difficult to deal with the inequalities in Theorem 3.3, because they are non-convex. In Theorem 3.4, slack matrices will be introduced to these inequalities so that the conditions obtained become solvable.

**Theorem 3.4.** Let $\bar{M}$ possible extractions $\{\alpha^1, \alpha^2, \cdots, \alpha^{\bar{M}}\}$ be selected randomly according to Lemma 2.1. Suppose that $\epsilon \in (0,1), \rho \in (0,1)$ and $\gamma > 0$. Then, the error system (4) with partial information of the transition probabilities is $\epsilon$-level stochastically stable and satisfying an $l_2 - l_\infty$ performance index $\gamma$, if there exist positive definite symmetric matrices $P_{i1} > 0$ and $P_{3i} > 0$, and matrices $P_{2i}, U_i, V_i, X_i, A_{fi}, B_{fi}, C_{fi}, D_{fi}$ for each $i \in \varphi$ such that

$$
\Psi_{i1}(\alpha^p) = 
\begin{bmatrix}
P_{1j} - U_i - U_i^T & P_{i1}^K - X_i - X_i^T & H_{13i} & A_{fi} & H_{15i} \\
* & * & -p_{i1}^K & 0 & 0 \\
* & * & * & -p_{3i}^K & 0 \\
* & * & * & * & -\sum_{j \in K^i} \pi_{ij}
\end{bmatrix} < 0,
$$

and

$$
\Psi_{i2}(\alpha^p) = 
\begin{bmatrix}
P_{ij} - U_i - U_i^T & P_{2j} - X_i - X_i^T & H_{13i} & A_{fi} & H_{15i} \\
* & * & -p_{i1}^2 & -p_{3i} & 0 \\
* & * & * & -p_{3i}^2 & 0 \\
* & * & * & * & *
\end{bmatrix} < 0, \quad i, j \in U K^i.
$$

where $P_{ij}^K = \sum_{j \in K^i} \pi_{ij} P_{ij}, P_{i1}^K = \sum_{j \in K^i} \pi_{ij} P_{1i}, l = 1, 2, 3, H_{13i} = U_i A_{1i}(\alpha^p) + B_{1i} C_i, H_{15i} = U_i B_i(\alpha^p) + B_{fi} D_{fi}, H_{23i} = V_i A_{1i}(\alpha^p) + B_{fi} C_i, H_{25i} = V_i B_i(\alpha^p) + B_{fi} D_{fi}, A_{1i}(\alpha^p) = A_i + g_{1i}(\alpha^p), B_{i}(\alpha^p) = B_i + g_{2i}(\alpha^p), p = 1, 2, \cdots, \bar{M}$. Furthermore, the gain matrices of the filter are $\hat{A}_i = X_i^{-1} A_{fi}, \hat{B}_i = X_i^{-1} B_{fi}, \hat{C}_i = C_{fi}, \hat{D}_i = D_{fi}.$

**Proof.** Let

$$
P_i = \begin{bmatrix} P_{i1} & P_{i2} \\ * & P_{3i}\end{bmatrix}, P^K_i = \begin{bmatrix} P_{i1}^K & P_{i2}^K \\ * & P_{3i}^K \end{bmatrix}, P^K_j = \begin{bmatrix} P_{j1}^K & P_{j2}^K \\ * & P_{3j}^K \end{bmatrix}, G_i = \begin{bmatrix} U_i & X_i \\ V_i & X_i \end{bmatrix}.
$$

Denote $A_{fi} = X_i A_{fi}, B_{fi} = X_i B_{fi}, C_{fi} = X_i C_{fi}, D_{fi} = X_i D_{fi}$. Then, $\Psi_{i1}(\alpha^p) < 0$ implies $\Upsilon_{1i}(\alpha^p) < 0$. Similarly, $\Psi_{i2}(\alpha^p) < 0$ and $\Psi_{3i} < 0$ imply that $\Upsilon_{2i}(\alpha^p) < 0$ and $\Upsilon_{3i} < 0$, respectively. Therefore, if condition (30)-(32) hold, the resulting system (4) with partial information of the transition probabilities is $\epsilon$-level stochastically stable and satisfying an $l_2 - l_\infty$ performance index $\gamma$. This completes the proof. $\square$

**4. Numerical example.** In this section, a numerical example is given to illustrate the effectiveness of the proposed method. Consider a four modes Markov jump system with the following parameters:

$$
A_1 = \begin{bmatrix} 0.37 & 0.34 \\ -0.27 & 0.52 \end{bmatrix}, A_2 = \begin{bmatrix} -0.21 & 0.13 \\ 0.34 & 0.24 \end{bmatrix}, A_3 = \begin{bmatrix} -0.25 & -0.51 \\ 0.48 & -0.43 \end{bmatrix},
$$

$$
A_4 = \begin{bmatrix} 0.43 & 0.38 \\ -0.48 & 0.35 \end{bmatrix}, B_1 = \begin{bmatrix} 0.15 \\ 0.21 \end{bmatrix}, B_2 = \begin{bmatrix} 0.29 \\ -0.25 \end{bmatrix}, B_3 = \begin{bmatrix} 0.21 \\ 0.18 \end{bmatrix}.
$$
where $\zeta \in \mathbb{R}$.

The external disturbance is given by

$$E_1 = E_2 = E_3 = E_4 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad F_1 = F_2 = F_3 = F_4 = 1.$$ 

The transition probability matrix containing unknown elements is given by

$$\Pi = \begin{bmatrix}
0.5 & 0.3 & ? & ? \\
0.1 & 0.4 & 0.3 & 0.2 \\
? & 0.2 & ? & 0.1 \\
0.1 & 0.1 & 0.1 & 0.7
\end{bmatrix}.$$ 

The uncertainties in system (2) are given below:

$$g_{11}(\alpha) = \begin{bmatrix}
\alpha_1 \alpha_2 & 0.3 \alpha_1 \alpha_4 \zeta \\
-5 \alpha_1 \alpha_2 \alpha_4 & -\alpha_1 (1 + \alpha_4) \zeta
\end{bmatrix}, \quad g_{21}(\alpha) = \begin{bmatrix}
\alpha_1 \zeta \\
\alpha_3 \alpha_5
\end{bmatrix},$$

$$g_{12}(\alpha) = \begin{bmatrix}
\alpha_3 \alpha_4 \alpha_5 & \alpha_1 \alpha_2 (1 + \zeta) \\
0.5 \alpha_3 \alpha_5 & -0.9 \alpha_1 (1 + \alpha_5) \zeta
\end{bmatrix}, \quad g_{22}(\alpha) = \begin{bmatrix}
(\alpha_1 + \alpha_2) \alpha_3 \alpha_4 \zeta \\
\alpha_4 \zeta
\end{bmatrix},$$

$$g_{13}(\alpha) = \begin{bmatrix}
\alpha_2 \alpha_3 & \alpha_3 \alpha_4 (\alpha_5 + \zeta) \\
0.1 \alpha_1 \zeta & \alpha_3 \alpha_4
\end{bmatrix}, \quad g_{23}(\alpha) = \begin{bmatrix}
0 \\
\alpha_1 \alpha_4
\end{bmatrix},$$

$$g_{14}(\alpha) = \begin{bmatrix}
\alpha_1 (1 + \alpha_4) \zeta & 0.5 (1 + \alpha_1) \zeta \\
0.3 (1 + \alpha_1) \zeta & \alpha_4 \alpha_5
\end{bmatrix}, \quad g_{24}(\alpha) = \begin{bmatrix}
\alpha_1 \zeta \\
(1 + \alpha_3) \zeta
\end{bmatrix},$$

where $\zeta = \cos(\alpha_1 + 0.77) - \sin(\alpha_2 + 0.23)$, $\alpha_i \in [-0.1, 0.1]$, $i = 1, 2, \ldots, 5$. For given probabilistic levels $\epsilon = 0.1$, $\rho = 0.01$ and scalar $\nu = 0.5$, we have $\hat{M} \geq 3898$ from Lemma 2.1.

Then, an $l_2 - l_\infty$ full filter is designed such that the augmented error system (4) is $\epsilon$-level stochastically stable with an $l_2 - l_\infty$ performance index $\gamma$.

From Theorem 3.4, the filter gains are obtained with $\gamma = 1$ as given below:

$$\hat{A}_1 = \begin{bmatrix}
0.0098 & 0.0412 \\
-0.0212 & 0.0704
\end{bmatrix}, \quad \hat{A}_2 = \begin{bmatrix}
-0.041 \alpha_1 \alpha_4 & 0.0163 \\
0.0621 & 0.0572
\end{bmatrix}, \quad \hat{B}_1 = \begin{bmatrix}
-1.0915 \\
-0.1982
\end{bmatrix},$$

$$\hat{A}_3 = \begin{bmatrix}
-0.048 & -0.1073 \\
0.0131 & -0.0866
\end{bmatrix}, \quad \hat{A}_4 = \begin{bmatrix}
0.1753 & 0.2184 \\
-0.3697 & 0.18
\end{bmatrix}, \quad \hat{B}_2 = \begin{bmatrix}
-0.0549 \\
-0.2773
\end{bmatrix},$$

$$\hat{B}_3 = \begin{bmatrix}
-0.2096 \\
-1.8493
\end{bmatrix}, \quad \hat{B}_4 = \begin{bmatrix}
-0.0121 \\
-3.8901
\end{bmatrix}, \quad \hat{C}_1 = \begin{bmatrix}
-0.0198 \\
-0.2168
\end{bmatrix},$$

$$\hat{C}_2 = \begin{bmatrix}
0.0055 & -0.4876 \\
-0.0247 & 0.2777
\end{bmatrix}, \quad \hat{C}_3 = \begin{bmatrix}
0.0121 & -0.3455
\end{bmatrix},$$

$$\hat{D}_1 = 10.3173, \quad \hat{D}_2 = 9.8253, \quad \hat{D}_3 = 10.3597, \quad \hat{D}_4 = 9.598.$$ 

To test the system performance, we run simulations for 5000 times of the system with random uncertain parameters. The initial states of the original system and the filter system are set as $x(0) = [0.2 \ 0.2]^T$ and $\hat{x}(0) = [-0.2 \ -0.2]^T$, respectively. The external disturbance is given by $w(k) = 5e^{-0.2k} \sin(0.5k)$.

Figure 1-Figure 4 show the system mode trajectory, the system states curve, the filtering system states curve and the error response curve, respectively.

From the results obtained, it is clearly seen that the designed filter can guarantee that all tested systems are stochastically stable, which means that the proposed method is effective.
Figure 1. The system mode trajectory.

Figure 2. The system states curve.

Figure 3. The filtering system state.
5. Conclusion. In this paper, we discussed the problem of $l_2 - l_\infty$ probabilistic filtering for uncertain Markov jump systems with partial information of the transition probabilities, where the uncertainties were caused by randomly changing interior parameters. Combining the original system and the filtering system, an augmented error system was constructed. Some concepts of probability theory were introduced to deal with the uncertainties. Due to the complicated structure of real practical systems, only partial information on the transition probabilities were available. In this paper, by using Lyapunov functional method and probability theory, linear matrix inequalities (LMIs) type of sufficient conditions were derived. Based on these sufficient conditions, a probability filter was constructed such that the augmented error system with partial information of the transition probabilities was stochastically stable with a given confidence level and satisfying an $l_2 - l_\infty$ performance index. Furthermore, the gain matrices of the filter were obtained through the introduction of slack matrices. Finally, a numerical example was given to illustrate the effectiveness of the proposed method.

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