DEVELOPMENTS IN SUPERSTRING THEORY

Ashoke Sen

Mehta Research Institute of Mathematics and Mathematical Physics,
Chhatnag Road, Jhusi, Allahabad 211019, INDIA
E-mail: sen@mri.ernet.in

In this talk I review recent developments in superstring theory. In the first half of the talk I discuss some of the earlier developments in this subject. This includes a review of the status of string theory as a unified theory of gravity and other interactions, the role of duality in string theory, and application of string theory to the problem of information loss near a black hole. In the second half of the talk I review the developments of the last two years. This includes application of duality symmetries, Matrix theory, Maldacena conjecture, and derivation of gauge theory results from string theory. In choosing the list of topics for this talk, I have focussed on those which are likely to be of some interest to non-string theorists, thereby leaving out many of the marvelous technical results in this subject.

1 Earlier Developments

1.1 String Theory and Quantum Gravity

As we all know, quantum field theory has been extremely successful in providing a description of elementary particles and their interactions. However, it does not work so well for gravity. If we naïvely try to quantize general relativity — which is a classical field theory — using the methods of quantum field theory, we run into divergences which cannot be removed by using the conventional renormalization techniques of quantum field theory. An example of such a divergent graph contributing to $e^- - e^-$ scattering has been shown in Fig.1.

![Divergent contribution to $e^- - e^-$ scattering in quantum general relativity. In this diagram the solid lines denote the electron propagator and the broken lines denote graviton propagator.](image1.png)

Figure 1: Divergent contribution to $e^- - e^-$ scattering in quantum general relativity. In this diagram the solid lines denote the electron propagator and the broken lines denote graviton propagator.

String theory is an attempt to solve this problem. The basic idea in string theory is quite simple. According to string theory, different elementary particles, instead of being point-like objects, are different vibrational modes of a string. Fig.2 shows some of the oscillation modes of closed strings and open strings. However, the typical size of a string is extremely small, being of the order of the Planck length ($\sim 10^{-33} cm.$). Thus in all present day experiments these will appear to be point-like objects, and string theory will be indistinguishable from an ordinary quantum field theory.

This simple idea has drastic consequences. One finds that

- A consistent quantum string theory does not suffer from any ultraviolet divergences.

- The spectrum of such a string theory automatically contains a massless spin two state which has all the properties of a graviton — the mediator of gravitational interaction.

Thus string theory automatically gives us a finite quantum theory of gravity! However, there are problems in the immediate application of string theory to our world. One finds that:

- String theory is consistent only in (9+1) dimensional space-time instead of the (3+1) dimensional space-time in which we live.

- Instead of a single string theory there are five consistent string theories in (9+1) dimensions. They are named as:
  - Type IIA
  - Type IIB
  - Type I
  - $E_8 \times E_8$ heterotic
  - $SO(32)$ heterotic

As we shall see, the first problem is resolved using the idea of compactification. The second problem is partially resolved using the idea of duality.
1.2 Compactification

The idea of compactification is quite simple. Although consistency of string theory tells us that it must live in (9+1) dimensions, there is no need for all the 9 space-like directions to be of infinite extent. Instead we can take six of them to be small and compact, and the other three to be of infinite extent. As long as the sizes of the compact directions are smaller than the resolution of the most powerful microscope (the accelerators), we shall not be able to see these extra directions and the world will look (3+1) dimensional. This has been explained in Fig.3 by a toy example. Here we have an intrinsically two dimensional surface, but one of the two directions is taken to be a circle of small radius. For sufficiently small radius of the circle, this two dimensional cylinder appears to be a one dimensional space.

Figure 3: Example of Compactification. The two dimensional cylinder appears to be one dimensional if the radius is very small.

It turns out that there are many different choices for this six dimensional compact manifold. Thus each of the five string theories in (9+1) dimensions gives rise to many possible string theories in (3+1) dimensions after compactification. Some of these theories come tantalizingly close to the observed universe. In particular one can construct models with:

- Gauge group containing the standard model gauge group $SU(3) \times SU(2) \times U(1)$,
- Chiral fermions representing three generations of quarks and leptons,
- $N=1$ supersymmetry,
- Gravity.

Furthermore unlike conventional quantum field theories which are ultraviolet divergent but renormalizable, and quantum general relativity which is ultraviolet divergent and not renormalizable, string theories have no ultraviolet divergence at all!

1.3 Duality Symmetries of String Theory

Existence of duality symmetries in string theory started out as a conjecture and still remains a conjecture. However so many non-trivial tests of these conjectures have been performed by now that most people in the field are convinced of the validity of these conjectures. A review of this subject and more references can be found in ref.5

A duality conjecture is a statement of equivalence between two or more apparently different string theories. Two of the most important features of duality are as follows:

- Often under the duality map, an elementary particle in one theory gets mapped to a composite particle in a dual theory and vice versa. Thus classification of particles into elementary and composite loses significance as it depends on which particular theory we use to describe the system.
- Often duality relates a weakly coupled string theory to a strongly coupled string theory and vice versa. In many simple cases the coupling constants $g$ and $\tilde{g}$ in the two theories are related via the simple relation:

$$g = \frac{\tilde{g}^{-1}}{1}.$$ (1)

Thus a perturbation expansion in $g$ contains information about non-perturbative effects in the dual theory. In particular the tree level (classical) results in one theory can contain contribution from perturbative and non-perturbative terms in the dual theory. This also clearly shows that duality is a property of the full quantum string theory, and not of its classical limit.

Thus there are two aspects of duality

- elementary ↔ composite
- classical ↔ quantum

Let me now give some examples of dual pairs of string theories.

- (9+1) dimensional SO(32) heterotic and type I string theories are conjectured to be dual to each other.\[\text{[1]}\]
- SO(32) heterotic string theory compactified on a four dimensional torus (denoted as $T^4$) is conjectured to be dual to type IIA string theory compactified on a different four dimensional manifold, denoted by $K3$.\[\text{[1]}\]
- Type IIB string theory is conjectured to be self-dual, in the sense that the type IIB string theories at two different couplings $g$ and $\tilde{g}$ related by Eq. (1) are conjectured to describe the same physical theory.\[\text{[1]}\]
Heterotic string theory compactified on a six dimensional torus, denoted by $T^6$, is conjectured to be self-dual in the same sense as above.

Figure 4: Moduli space of unified string theory. The shaded regions denote weakly coupled string theories, and can be studied using perturbation theory. Most of the region in the $U$-theory moduli space, depicted in white, does not admit such a description.

Due to the fact that a duality conjecture relates two apparently different theories, we see that it gives a unified picture of all string theories. The situation is summarized in Fig.4. According to this picture the apparently different string theories and their compactifications are just different limits of the same theory, with a large parameter space. There is no universally accepted name for this central theory, — I have chosen to call it $U$-theory for the purpose of this talk. $U$ can be taken to stand for Unknown or Unified. Some small regions of the parameter space of $U$-theory, which can be represented by some weakly coupled string theory, are reasonably well understood. This has been denoted by the shaded regions in Fig.4, and correspond to the weak coupling regime of the five different string theories and their compactifications. But for most of the parameter space $U$-theory does not have a description in terms of weakly coupled string theory. Note that in one corner of the parameter space of $U$-theory, there is a theory called $M$-theory, which has not been introduced before. At present not much is known about $M$-theory except that its low energy limit is the eleven dimensional supergravity theory, and that various string theories and their compactifications approach $M$-theory in certain limits. However, unlike string theory, $M$-theory does not have any coupling constant, and no systematic procedure for doing computations in $M$-theory beyond the low energy supergravity limit is known. (As we shall discuss later, Matrix theory is an attempt in this direction.) It is generally believed that due to the absence of a coupling constant, understanding $M$-theory will require a full understanding of $U$-theory, and for this reason $U$-theory has often been identified with $M$-theory. However, we shall keep the distinction between the two, and reserve the name $M$-theory for the unknown eleven dimensional theory which arises in a certain limit of $U$-theory.

Finally we note that since duality typically relates a strongly coupled string theory to a weakly coupled string theory, and since at present there is no independent description of string theory at strong coupling, we cannot prove duality. However one can devise various tests of duality by working out various consequences of a duality conjecture, and then verifying these predictions by explicit computation. Often such predictions take the form of highly non-trivial mathematical identities which can nevertheless be proved. (See refs. for specific examples.) As I have already stated, as a result of many such tests, most people in the field now take it for granted that duality is a genuine symmetry of string theory.

1.4 String Theory and the Information Loss Puzzle

Black holes are classical solutions of general relativity, and its extensions like string theory. They can be formed by collapse of matter under its own gravitational pull, or can be primordial, formed at the time of the big bang. Classically black holes are completely black, in the sense that they absorb everything that falls within a certain radius (known as the event horizon) and nothing can come out of a black hole once it is inside the event horizon. However this picture changes dramatically in quantum theory. It has been argued by Hawking, Bekenstein and others that in quantum theory a black hole behaves as a perfect black body at finite temperature (inversely proportional to its mass). In particular,

- Black holes emit thermal black body radiation.
- Black holes carry entropy proportional to the area of its event horizon.

This thermal description of the black hole can cause potential conflict with quantum mechanics and give rise to the information loss puzzle. For this let us consider the following thought experiment.

\[ \text{\footnote{One should keep in mind that this is only a schematic representation. Each string theory gives rise to many weakly coupled string theories after compactification. Also different compactifications of a string theory may be separated by infinite distance in the moduli space if in order to go from one to the other one needs to pass through the decompactification limit. Finally, not all components of $U$-theory may belong to the same moduli space, i.e. there may not be a continuous path connecting them such that each point on the path represents a solution of the field equations, but it is expected that they are all connected via intermediate configurations which are not necessarily solutions of field equations.}} \]
\[ \text{\footnote{In string theory parameters themselves are related to vacuum expectation values of different fields and are expected to be determined dynamically.}} \]
we start with a pure quantum state in the s-wave and let it collapse under its own gravitational pull to form a black hole.

We then wait till it evaporates via Hawking radiation. Since the Hawking radiation is thermal, the final state represents a mixed state. Thus the net result of this process is the evolution of a pure quantum state into a mixed state. In particular, most of the information about the initial state (except its mass and gauge charges) are lost during the process. This is certainly in conflict with the fundamental principles of quantum mechanics.

In order to put things in perspective, let us note that all hot objects emit thermal radiation, but they do not violate the principles of quantum mechanics. The main (apparent) difference from the case of the black hole is that for an ordinary hot object the thermal description is merely a matter of convenience, and represents our inability to explicitly study the quantum mechanics of large number of particles. Thus we choose to give a statistical description of the object by averaging over microstates. However we can, in principle, give a completely microscopic description of the radiation in terms of quantum scattering processes occurring inside the object.

The absence of such a microscopic description in case of thermal radiation from a black hole is the main cause of the information loss puzzle. Thus if one could provide a microscopic description of the Bekenstein entropy and Hawking radiation of a black hole in terms of quantum states of the hole and their scattering respectively, then the thermal radiation from the black hole will be on the same footing as the radiation from any other hot object; and the information loss problem will disappear. This is precisely what string theorists have attempted to do and have been partially successful. It was found that for a special class of black holes:

- One can count the number of quantum states $N$ of these black holes and show that:

$$S_{BH} = \ln(N),$$

where $S_{BH}$ is the Bekenstein-Hawking entropy of a black hole and $N$ is the degeneracy of states of the black hole carrying certain quantum numbers. This gives a microscopic explanation of the Bekenstein-Hawking entropy of the black hole.

- One can compute the rate of Hawking radiation from these black holes due to quantum scattering processes inside the hole and show that the rate agrees precisely with the Hawking radiation formula. Thus this provides a microscopic explanation of Hawking radiation.

Although these calculations have been done for a special class of black holes for special regions in the parameter space, at least this gives us hope that in string theory, the phenomenon of Hawking radiation and the entropy of a black hole are firmly embedded within the usual principles of quantum mechanics.

## 2 Recent Developments

Much of the recent development in the subject have focused on the study of the strong coupling regime of string theory, which might also be described as the study of $U$-theory. We shall describe some of these approaches here.

### 2.1 Application of Duality to Deriving $U$-theory Effective Action

This approach is best understood with the help of Fig.4. As we have said earlier, the shaded regions in this diagram denote various weakly coupled string theories, and hence various quantities of interest can be computed in these regions using string perturbation theory. Given these results, one can try to guess the exact form of the answer for these quantities by demanding that it correctly interpolates between the known results in various shaded regions, and is consistent with various duality symmetries and known perturbative non-renormalization theorems. This approach has yielded several concrete results in $U$-theory. Thus for example, this has been used to deduce the correct form of some specific terms in the effective action of type IIB string theory on tori of various dimensions. Here we quote one such term in the effective action of type IIB string theory:

$$\frac{1}{3 \cdot 2^{14} \cdot \pi^2 \cdot \alpha'} \int d^{10} x \sqrt{-\det g} E(\tau, \bar{\tau}) \mathcal{R}^4,$$

where $g_{\mu\nu}$ denotes the metric, $\mathcal{R}^4$ is a specific scalar constructed from the fourth power of the Riemann curvature tensor, $\tau$ is a complex scalar field, $\alpha'$ is related to the string tension $T_S$ as $T_S = (2\pi\alpha')^{-1}$, and

$$E(\tau, \bar{\tau}) = (Im\tau)^\frac{7}{2} \sum_{m,n}^\prime \frac{1}{|m + n\tau|^2},$$

$\Sigma^\prime$ denotes sum over all integers $m, n$ except for $m = n = 0$. These terms contain both perturbative and non-perturbative contributions from the viewpoint of type $U$. In both these computations the microscopic computation is done in a region of the parameter space where gravity is weak, and then continued to the regime of strong gravity with the help of a non-renormalization theorem.
IIB string theory, and whereas the perturbative terms could be explicitly derived using string perturbation theory, the non-perturbative terms are novel results which are beyond the reach of string perturbation theory.

Of all the attempts to understand U-theory, this approach has been the most successful one in deriving concrete new quantitative results in U-theory. Unfortunately this approach probably can be used to derive only a limited set of terms in the U-theory effective action — terms which enjoy some sort of perturbative non-renormalization theorem — since such guesswork becomes quite difficult as one considers more general terms in the effective action of U-theory.

2.2 Matrix Theory

Going back to Fig.2, we notice that in one corner of the moduli space of U-theory is a theory called M-theory living in eleven space-time dimensions. We have said earlier that not much is known about this theory, except that its low energy limit is the eleven dimensional supergravity theory. This theory does not contain any freely adjustable coupling constant, and hence there is no systematic perturbation expansion which allows us to compute amplitudes beyond the low energy supergravity approximation. Matrix theory is an attempt to give a nonperturbative definition of M-theory. Since most of the known regions of U-theory can be regarded as some sort of compactification of M-theory, there is also a hope that by giving a non-perturbative definition of M-theory and its various compactifications one might be able to give a description of the full U-theory.

In order to understand the basic proposal of Matrix theory, let us consider a specific scattering process in M-theory. Instead of trying to compute the amplitude in the center of momentum frame, we consider a frame which is boosted by a large (infinite) amount relative to the center of momentum frame in some given direction. This frame is sometime known as the infinite momentum frame or the light-cone frame, as all external particles participating in the scattering process travels with infinite momentum along the forward light-cone. According to the Matrix theory proposal M-theory in the infinite momentum frame is equivalent to a quantum mechanical system in the sense that computation of every scattering amplitude in M-theory can be mapped to the computation of an appropriate correlation function in this quantum mechanical system. The fundamental degrees of freedom of this quantum mechanical system are $N \times N$ matrices, the Hamiltonian is that of a particular supersymmetric quantum mechanics, and we need to take the limit $N \to \infty$ at the end of the calculation.

Since the Hamiltonian of this quantum mechanical system is given, this in principle gives an algorithm to compute any amplitude in M-theory, and thereby provides a non-perturbative definition of M-theory. The most obvious consistency check that this proposal can be subjected to is that in the low energy limit, the amplitudes computed from the matrix theory proposal should agree with those computed from eleven dimensional supergravity. This has been tested in many examples.

Unfortunately, the complexity of matrix quantum mechanics has so far prevented us from deriving any new quantitative result about M-theory using Matrix theory. In particular, the large N limit of this quantum mechanical system is not well understood. Although it has not affected the computations which have been done so far, it has been suggested that the full agreement between the supergravity and matrix theory computations may involve subtle effects of the large N limit when we consider more complicated amplitudes. For these reasons, Matrix theory is still in its infancy. It would indeed be interesting to see if Matrix theory can be used to derive some of the terms in U-theory, which have been derived using the approach outlined in the previous section.

2.3 Maldacena Conjecture

A third approach to the study of U-theory is based on the discovery of a new relationship between some region of U-theory, and IIB string theory, and ordinary quantum field theories. This series of conjectures was first put forward by Maldacena, and hence is now known as the Maldacena conjecture. In this talk I shall focus on only one of these conjectures. It states that:

Type IIB string theory on $S^5 \times \text{AdS}_5$ is equivalent to $\mathcal{N} = 4$ supersymmetric SU(N) gauge theory in (3+1) dimensions.

Since this conjecture involves several terms which might not be familiar, let me try to define each of these terms. First of all $S^5$ denotes a five dimensional sphere, which might be described by its embedding in six dimensional Euclidean space with coordinates $y^1, \ldots, y^6$ as follows:

\[
\sum_{i=1}^{6} (y^i)^2 = R^2. \tag{5}
\]

$R$ denotes the radius of the sphere. $\text{AdS}_5$, the five dimensional anti-de Sitter space — can be described by its embedding in a six dimensional flat space with signature $(-++++)$ spanned by coordinates $x^0, \ldots, x^5$ as follows:

\[
(x^0)^2 + (x^1)^2 - \sum_{i=2}^{5} (x^i)^2 = R^2. \tag{6}
\]

$R$ is a parameter labelling the size of the $\text{AdS}_5$ space. Note that we have used the same parameters $R$ for both
$S^5$ and $AdS_5$. This is intentional, since the $S^5$ and $AdS_5$ appearing in the Maldacena conjecture are related this way:

Thus $S^5 \times AdS_5$ is a ten dimensional space, and one side of the duality relation involves type IIB string theory on this ten dimensional space. Let us now turn to the other side of the duality conjecture. An $\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge theory is an ordinary $SU(N)$ gauge theory with four Majorana fermions and six scalars in the adjoint representation of the gauge group. Furthermore, all the Yukawa couplings and the scalar self-couplings are completely determined in terms of the gauge coupling constant using the requirement of supersymmetry. Thus the only free parameters in the theory are the gauge coupling constant, and the vacuum angle $\theta$ which does not affect perturbation theory.

This finishes the definition of all the terms that appeared in the Maldacena conjecture. Since the conjecture relates a ten dimensional theory to a four dimensional theory, the exact meaning of this conjecture is not transparent. The precise form of this conjecture states that there is a one to one correspondence between the physical green’s functions in type IIB string theory on $S^5 \times AdS_5$ and the correlation functions of gauge invariant operators in $\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge theory. Such a relationship is possible because the physical excitations in $AdS_5$ live on the boundary of $AdS_5$, which in turn can be identified with a four dimensional flat spacetime (after including ‘points at infinity’).

Type IIB string theory on $S^5 \times AdS_5$ has three dimensionless parameters, – the string coupling constant $g_{st}$, a constant $a$ related to the vacuum expectation value of a particular scalar field in the theory (known as the Ramond-Ramond scalar), and the ratio $R/\sqrt{\alpha'}$, where $R$ is the common radius of $S^5$ and $AdS_5$. On the other hand $\mathcal{N} = 4$ supersymmetric Yang-Mills theory is also characterized by three parameters, the gauge coupling constant $g_{YM}$, the vacuum angle $\theta$ and the number $N$ determining the gauge group $SU(N)$. The Maldacena conjecture gives a precise relation between these parameters:

$$g_{st} = g_{YM}^2, \quad a = \theta, \quad \frac{R}{\sqrt{\alpha'}} = (4\pi g_{YM}^2 N)^{1/4}.$$  

(7)

There are many other examples of such conjectures which have been found. Typically such a conjecture takes the form of an equivalence relation between a string theory / $M$-theory on a certain manifold $K$, and a quantum field theory. The precise form of this quantum field theory depends on the choice of the manifold $K$ as well as on which theory we compactify. Since quantum field theories have intrinsic non-perturbative definition (even if explicit computations may be difficult) we can use this conjecture to give non-perturbative definitions of string theories in specific backgrounds. This makes certain regions of $U$-theory accessible via the usual quantum field theory techniques. Here we shall discuss one particular application of the Maldacena conjecture to the study of $U$-theory — namely the derivation of the holographic principle.

The holographic principle, first proposed by ’t Hooft and then by Susskind, states that in a consistent quantum theory of gravity, the fundamental degrees of freedom reside at the boundary of space-time and not in the interior. Furthermore, on the boundary there is precisely one degree of freedom per Planck area. (If the original theory is $(d + 1)$-dimensional, then here Planck ‘area’ refers to a $(d - 1)$ dimensional volume of order $M_{pl}^{-d+1}$, where $M_{pl}$ is the Planck mass.) The Maldacena conjecture provides a concrete verification of this holographic principle for type IIB on $S^5 \times AdS_5$. According to this conjecture the degrees of freedom in this theory can be identified as those of an $\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge theory living on the boundary of $AdS_5$. After proper ultraviolet regularization of the gauge theory (which amounts to infrared regularization in the type IIB string theory) one can count the total number of degrees of freedom of the gauge theory. Dividing this by the volume of the boundary one finds of order one degree of freedom per Planck volume, as required by the holographic principle.

Although this illustrates the holographic principle in $AdS$ space, extension of this principle to Minkowski space remains an open and challenging problem.

3 Application of String Theory to Gauge Theories

We shall now discuss application of string theory to gauge theories. Since we have been discussing Maldacena conjecture, we shall first discuss its application, and then turn to the (historically older) subject of gauge theories from branes.

3.1 Maldacena Conjecture

Since the Maldacena conjecture relates string theory to gauge theory, we can use it to study gauge theories using known results in string theory. This may sound strange, as we understand gauge theories much better than string theory; however as we shall see, in some region in the
parameter space the string theory side is better understood, and hence can be useful in deriving gauge theory results. For this let us look at the relation (6) between the string theory and the gauge theory parameters. If we consider the 't Hooft large $N$ limit on the gauge theory side:

$$gYM \to 0, \quad N \to \infty, \quad \lambda \equiv g^2 YM N = \text{fixed}, \quad (8)$$

then it corresponds to the following limit in string theory:

$$g_{st} \to 0, \quad \frac{R}{\sqrt{\alpha'}} = (4\pi \lambda)^{1/4} = \text{fixed} . \quad (9)$$

The smallness of $g_{st}$ implies that in this limit we can use classical string theory. Let us now further take the limit when the 't Hooft coupling $\lambda$ is large. In this case $R/\sqrt{\alpha'}$ is large and hence we can approximate string theory by its supergravity limit for computation of Green’s functions with external momenta small compared to the string scale. Since these Green’s functions are related to the correlation functions of gauge invariant operators in the gauge theory, we conclude that in the 't Hooft large $N$ limit with large 't Hooft coupling $\lambda$, the gauge theory correlation functions can be computed by studying classical type IIB supergravity theory on $S^5 \times AdS_5$. Note that ordinary large $N$ perturbation theory in gauge theory is useless in this limit, as it involves a series expansion in $\lambda$. Thus this approach can give genuinely new results in gauge theory.

![Figure 5: Computation of Wilson line — or equivalently, the quark anti-quark potential — using Maldacena conjecture. Here $C$ is a closed curve in the four dimensional space-time, regarded as the boundary of $AdS_5$, and $S$ is a surface of minimal area in the interior of $AdS_5$ subject to the condition that $C$ is its boundary. The Wilson line associated with the curve $C$ in the $N = 4$ supersymmetric gauge theory is given by the area of the surface $S$ in $AdS_5$.](image)

Here I shall discuss one example — the computation of (non-dynamical) quark anti-quark potential in the $N = 4$ supersymmetric gauge theory. It turns out that using the Maldacena conjecture this problem can be mapped to the geometric problem of finding the minimal area surface bounded by a fixed curve at the boundary of $AdS_5$ as illustrated in Fig. 5. This problem is easily solved, and the final answer for the potential is:

$$V(r) = -\frac{4\pi^2(2g^2 YM N)^{1/4}}{(\Gamma(1/4))^2 r} , \quad (10)$$

where $r$ is the separation between the quark and the antiquark. Although the $r$ dependence of the potential follows from the conformal invariance of the gauge theory, the dependence on the coupling constant $gYM$ is novel since it is linear instead of quadratic as expected from perturbation theory.

One can also use the Maldacena conjecture to study $N = 4$ supersymmetric gauge theories at finite temperature $T$. In this case we need to make the time direction

- Euclidean, and
- periodic with period $2\pi T^{-1}$.

This theory turns out to be dual to type IIB string theory on $S^5 \times K$, where $K$ is a new five dimensional manifold representing the Euclidean black hole solution in $AdS_5$. The boundary of this new manifold $K$ is $R^3 \times S^1$, which is to be identified with the three space and the periodic time direction of the gauge theory. As in the case of the zero temperature theory, in the 't Hooft large $N$ limit, with large 't Hooft coupling $\lambda$ for the gauge theory, the relevant type IIB string theory can be approximated by classical type IIB supergravity theory on $S^5 \times K$. This description can be used to study various properties of the finite temperature gauge theory in this limit. Thus for example, the problem of finding the mass spectrum in the finite temperature gauge theory can be mapped to the problem of finding eigenvalues of certain differential operators on $K$. As an example we quote the relevant differential equation for determining the mass $m_n$ of the $n$th scalar ‘glueball’ (from the point of view of three dimensional gauge theory obtained in the high temperature limit of the four dimensional gauge theory)

$$\pi^2 T^2 \frac{1}{\rho} \frac{d}{d\rho} \left[ (\rho^5 - \rho) \frac{df_n}{d\rho} \right] = -m_n^2 f_n , \quad (11)$$

where $\rho$, denoting the ‘radial coordinate’ in $K$, lies in the range $1 \leq \rho < \infty$. $f_n$ satisfies the boundary conditions

$$\frac{df_n}{d\rho} = 0 \quad \text{at} \quad \rho = 1$$

$$f_n \sim \rho^{-1} \quad \text{as} \quad \rho \to \infty . \quad (12)$$

This converts a non-perturbative quantum field theory problem to a classical eigenvalue problem. This eigenvalue problem can be solved by numerical methods.
used for getting information about pure $SU(N)$ gauge theories. It was shown by Witten\textsuperscript{13} that pure $SU(N)$ gauge theory is dual to $M$-theory on $S^4 \times K_7$, where $K_7$ is a particular seven dimensional manifold related to the Euclidean black hole solution in the seven dimensional anti-de Sitter space. The temperature of this black hole, which is a parameter labelling the manifold $K_7$, corresponds to the ultraviolet cut-off in the $SU(N)$ gauge theory. In the limit

$$N \to \infty, \quad g_{YM} \to 0, \quad g_{YM}^2 N = \text{fixed but large},$$

(13)

$M$-theory can be approximated by classical eleven dimensional supergravity theory. Thus various properties of gauge theory in this limit can be studied using the classical supergravity theory on $S^4 \times K_7$. In particular one can prove confinement and existence of a mass gap in the gauge theory in this limit. However, since pure $SU(N)$ gauge theory is asymptotically free, in order to take the continuum limit, one needs to take the ultraviolet cut-off to infinity, and the ’t Hooft coupling $g_{YM}^2 N$ to zero keeping a certain combination fixed. Unfortunately in this limit the classical supergravity is no longer a good approximation to $M$-theory. Thus application of these ideas to the study of large $N$ gauge theory in the continuum limit remains an open problem.

### 3.2 Gauge Theories from Branes

Let me now discuss another approach that has been useful in deriving gauge theory results from string theory\textsuperscript{30,53}. This involves the study of branes. Historically this proceeds the Maldacena conjecture, and in fact Maldacena arrived at his conjecture by examining a special configuration of branes.

Branes are static classical solutions in string theory which are known to exist in many string theories. A $p$-brane denotes a static configuration which extends along $p$ spatial direction (the tangential directions) and is localized in all other spatial directions (the transverse directions). Thus the solution is invariant under translation along the $p$ directions tangential to the brane, as well as the time direction, and approaches the vacuum configuration as we go away from the brane in any one of the transverse direction. Thus in this language,

- 1-brane $\equiv$ string
- 2-brane $\equiv$ membrane
- 0-brane $\equiv$ particle

etc. Typically the quantum dynamics of a configuration of $p$-branes is described by a $(p+1)$ dimensional gauge field theory\textsuperscript{53} and the coupling constant of this quantum field theory is related to the coupling constant of the string theory of which the brane configuration is a solution. In this case duality symmetries relating strong and weak coupling limits of the original string theory can be used to derive duality relations involving the quantum field theories describing the dynamics of the brane. This approach has been used to derive many different results in supersymmetric gauge theories. Some example are:

- Derivation of Montonen-Olive duality\textsuperscript{59} in $\mathcal{N} = 4$ supersymmetric gauge theories\textsuperscript{53}.
- Derivation of Seiberg-Witten like results\textsuperscript{61} in $\mathcal{N} = 2$ supersymmetric gauge theories\textsuperscript{53,54,56}.
- Derivation of a special kind of symmetry, known as the mirror symmetry\textsuperscript{62} in (2+1) dimensional gauge theories\textsuperscript{3}.
- Derivation of Seiberg dualities\textsuperscript{63} involving $\mathcal{N} = 1$ supersymmetric gauge theories in (3+1) dimensions\textsuperscript{53,54,56}.

![Figure 6: Possibility of novel compactifications involving branes. Our brane hosts the standard model fields, the (9+1) dimensional 'bulk' hosts gravity and its superpartners, and the shadow brane hosts other fields which communicate to us via gravitational interaction. Supersymmetry may be broken at a high scale in the shadow brane, and yet have relatively small effect in our brane due to the weakness of gravitational interactions.](image-url)

Existence of branes in string theory has also given rise to the possibility that the standard model gauge fields arise from branes rather than in the bulk of space-time. This corresponds to novel compactifications in which gravity lives in the bulk of the ten dimensional space-time, but the other observed fields (quarks, leptons, gauge particles etc.) live on a brane of lower dimension\textsuperscript{53}. In particular we might imagine a scenario in which the standard model fields live on a three brane, the directions transverse to the three brane being compact, and the directions tangential to the three brane describing the usual three dimensional space. There may also be other branes, separated from us in the extra directions, forming 'shadow worlds'! This situation has been illustrated in Fig.\textsuperscript{6}. Since the usual gauge and matter fields live on the brane, they do not see the extra transverse directions; the only long range interaction which
sees the extra directions is gravity. This allows the extra dimensions to be much larger (\(\sim 1 \text{mm}\)) than in the conventional compactification scheme, the most stringent bound coming from tests involving the inverse square law of Newtonian gravity. This fact has been used in recent proposals for superstring model building.

4 Summary

It is now time to summarise the main results. We have seen that string theory has had a reasonable success in providing a consistent quantum theory of gravity. In particular we have achieved:

- Finiteness of perturbation theory,
- Partial resolution of the problems associated with quantum theory of black holes,
- Explicit realization of holographic principle for a special class of space-time, and
- (3+1) dimensional theories with gravity, gauge interactions, chiral fermions and \(\mathcal{N} = 1\) supersymmetry, closely resembling the standard model.

String theory has also provided us with an internally consistent and beautiful theory. In particular string duality provides us with

- Unification of many different string theories,
- Unification of elementary and composite particles, and
- Unification of classical and quantum effects.

Progress in string theory has also dramatically improved our understanding of supersymmetric quantum field theories.

Unfortunately, there are still no concrete new predictions of string theory at low energies. We shall have to wait and see if the situation improves during the next few years.

References

1. M. Green, J. Schwarz and E. Witten, Superstring Theory vol. 1 and 2, Cambridge University Press (1986);
   D. Lust and S. Theisen, Lectures on String Theory, Springer (1989);
   J. Polchinski, [hep-th/9411028].
2. A. Sen, [hep-th/9802051].
3. E. Witten, Nucl. Phys. B443 (1995) 85 [hep-th/9503124].
4. J. Polchinski and E. Witten, Nucl. Phys. B460 (1996) 525 [hep-th/9510163].
5. C. Hull and P. Townsend, Nucl. Phys. B451 (1995) 525 [hep-th/9505073].
6. A. Font, L. Ibanez, D. Lust and F. Quevedo, Phys. Lett. B249 (1990) 35;
   S. Rey, Phys. Rev. D43 (1991) 526.
7. A. Sen, Int. J. Mod. Phys. A9 (1994) 3707 [hep-th/9402003] and references therein.
8. A. Dabholkar and J. Harvey, [hep-th/9809122].
9. P. Townsend, Phys. Lett. 350B (1995) 184 [hep-th/9501008].
10. J. Schwarz, Phys. Lett. B367 (1996) 97 [hep-th/9510080].
11. P. Aspinwall, Nucl. Phys. Proc. Suppl. 46 (1996) 30 [hep-th/9508154].
12. P. Horava and E. Witten, Nucl. Phys. B460 (1996) 506 [hep-th/9510203]; Nucl. Phys. B475 (1996) 94 [hep-th/9601142].
13. A. Sen, Phys. Lett. B329 (1994) 217 [hep-th/9402032].
14. C. Vafa and E. Witten, Nucl. Phys. B431 (1994) 3 [hep-th/9408074].
15. S. Kachru and C. Vafa, Nucl.Phys. B450 (1995) 69 [hep-th/9605105];
   V. Kaplunovsky, J. Louis and S. Theisen, Phys. Lett. B357 (1995) 71 [hep-th/9506110].
16. S. Hawking, Nature 248 (1974) 30; Comm. Math. Phys. 43 (1975) 199.
17. J. Bekenstein, Lett. Nuov. Cim. 4 (1972) 737; Phys. Rev. D7 (1973) 2333; Phys. Rev. D9 (1974) 3192;
   G. Gibbons and S. Hawking, Phys. Rev. D15 (1977) 2752.
18. L. Susskind, [hep-th/9309143];
   L. Susskind and J. Uglam, Phys. Rev. D50 (1994) 2700 [hep-th/9401074];
   J. Russo and L. Susskind, Nucl. Phys. B437 (1995) 611 [hep-th/9405117].
19. A. Sen, Mod. Phys. Lett. A10 (1995) 2081 [hep-th/9504147];
   A. Peet, Nucl. Phys. B456 (1995) 732 [hep-th/9506200].
20. A. Strominger and C. Vafa, Phys. Lett. B379 (1996) 99 [hep-th/9601029].
21. C. Callan and J. Maldacena, Nucl. Phys. B472 (1996) 591 [hep-th/9602043].
22. A. Dhar, G. Mandal and S. Wadia, Phys. Lett. B388 (1996) 581 [hep-th/9605234].
23. S. Das and S. Mathur, Phys. Lett. B375 (1996) 103 [hep-th/9601152].
24. J. Maldacena and A. Strominger, Phys. Rev. D55 (1997) 861 [hep-th/9609026].
25. M. Green and M. Gutperle, [hep-th/9701093].
M. Green and P. Vanhove, Phys. Lett. B408 (1997) 122 [hep-th/9704145];
M. Green, M. Gutperle and P. Vanhove, Phys. Lett. B409 (1997) 177 [hep-th/9706173];
M. Green, M. Gutperle and H. Kwon, Phys. Lett. B421 (1998) 149 [hep-th/9710151];
M. Green and M. Gutperle, Phys. Rev. D58 (1998) 046007 [hep-th/9804123];
M. Green and S. Sethi, [hep-th/9808061].

26. N. Berkovits and C. Vafa, [hep-th/9803145].

27. E. Kiritsis and B. Pioline, Nucl. Phys. B508 (1997) 509 [hep-th/9707018]; Phys. Lett. B418 (1998) 61 [hep-th/9710078];
B. Pioline, Phys. Lett. B431 (1998) 73 [hep-th/9804023];
J. Russo and A. Tseytlin, Nucl. Phys. B508 (1997) 245 [hep-th/9707134];
J. Russo, Phys. Lett. B417 (1998) 253 [hep-th/9707241]; [hep-th/9802099];
A. Kehagias and H. Partouche, Phys. Lett. B422 (1998) 109 [hep-th/9710023].

28. T. Banks, W. Fischler, S. Shenker and L. Susskind, Phys. Rev. D55 (1997) 5112 [hep-th/9610043].

29. W. Taylor, hep-th/9801182.

30. L. Susskind, [hep-th/9704080].

31. A. Sen, Adv. Theor. Math. Phys. 2 (1998) 51 [hep-th/9709220].

32. N. Seiberg, Phys. Rev. Lett. 79 (1997) 3577 [hep-th/9710009].

33. K. Becker and M. Becker, Nucl. Phys. B506 (1997) 48 [hep-th/9705091].

34. K. Becker, M. Becker, J. Polchinski and A. Tseytlin, Phys. Rev. D56 (1997) 3174 [hep-th/9706072].

35. R. Dijkgraaf, E. Verlinde and H. Verlinde, Nucl. Phys. B500 (1997) 43 [hep-th/9703030].

36. Y. Okawa and T. Yoneya, [hep-th/9806108];
W. Taylor and M. Van Raamsdonk, [hep-th/9806066];
M. Fabbrichesi, G. Ferretti and R. Iengo, JHEP06(1998)2 [hep-th/9806122];
J. McCarthy, L. Susskind and A. Wilkins, [hep-th/9806136];
U. Danielsson, M. Kruczenski and P. Stjernberg, [hep-th/9807071];
R. Echols and J. Grey, [hep-th/9806109].

37. M. Douglas, H. Ooguri and S. Shenker, Phys. Lett. B402 (1997) 36 [hep-th/9702203];
M. Douglas and H. Ooguri, [hep-th/9701078].

38. D. Kabat and W. Taylor, Phys. Lett. B426 (1998) 297 [hep-th/9712183];
J. David, A. Dhar and G. Mandal, [hep-th/9707132].

40. M. Dine, R. Echols and J. Grey, [hep-th/9810021].

41. J. Maldacena, [hep-th/9711200].

42. L. Brink, J. Schwarz and J. Scherk, Nucl. Phys. B121 (1977) 77.

43. E. Witten, [hep-th/9802150].

44. S. Gubser, A. Polyakov and I. Klebanov, [hep-th/9802109].

45. G. ’t Hooft, gr-qc/9310026.

46. L. Susskind, J. Math. Phys. 36 (1995) 6377.

47. L. Susskind and E. Witten, [hep-th/9805114].

48. G. ’t Hooft, Nucl. Phys. B72 (1974) 461.

49. J. Maldacena, [hep-th/9803002].

50. S. Rey and J. Yee, [hep-th/9803001].

51. E. Witten, [hep-th/9803131].

52. C. Csaki, H. Ooguri, Y. Oz and J. Terning, [hep-th/9806021];
R. de Mello Koch, A. Jevicki, M. Mihailescu and J. Nunes, [hep-th/9806125];
M. Zyskin, [hep-th/9806128].

53. A. Sen, Nucl. Phys. B475 (1996) 562 [hep-th/9605150].

54. T. Banks, M. Douglas and N. Seiberg, Phys. Lett. B387 (1996) 278 [hep-th/9605199].

55. A. Hanany and E. Witten, Nucl. Phys. B492 (1997) 152 [hep-th/9611230].

56. E. Witten, Nucl. Phys. B500 (1997) 3 [hep-th/9703160].

57. J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724 [hep-th/9510017].

58. E. Witten, Nucl. Phys. B460 (1996) 335 [hep-th/9510133].

59. C. Montonen and D. Olive, Phys. Lett. B72 (1977) 117;
H. Osborn, Phys. Lett. B83 (1979) 321.

60. P. Townsend, Phys. Lett. B373 (1996) 68 [hep-th/9512062];
M. Green and M. Gutperle, Phys. Lett. B377 (1996) 28 [hep-th/9602074];
A. Tseytlin, [hep-th/9602064].

61. N. Seiberg and E. Witten, Nucl. Phys. B426 (1994) 19 [hep-th/9407087]; Nucl. Phys. B431 (1994) 484 [hep-th/9408099].

62. K. Intrilligator and N. Seiberg, Phys. Lett. B387 (1996) 513 [hep-th/9607207].

63. N. Seiberg, Nucl. Phys. B435 (1995) 129 [hep-th/9411149].

64. S. Elizur, A. Giveon and D. Kutasov, Phys. Lett. B400 (1997) 269 [hep-th/9702013];
S. Elizur, A. Giveon, D. Kutasov, E. Rabinovici and A. Schwimmer, Nucl. Phys. B505 (1997) 202 [hep-th/9704104].

65. E. Witten, Nucl. Phys. B507 (1997) 658 [hep-th/9706109].

66. E. Witten, Nucl. Phys. B471 (1996) 135 [hep-th/9802150].
67. E. Caceres, V. Kaplunovsky, I. Mandelberg, Nucl. Phys. B493 (1997) 73 [hep-th/9606036].
68. N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429 (1998) 263 [hep-ph/9803315]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, [hep-ph/9804398]; K. Dienes, E. Dudas and T. Gherghetta, [hep-ph/9803466], [hep-ph/9806292].