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CALIBRATING A TAPER MODEL FOR ORIENTAL SPRUCE IN TURKEY

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HIGHLIGHTS

A modified form of Max and Burkhart’s (1976) taper model was used in this study.

We tested all combinations of five fitting methods and six adjustment strategies.

Mixed results were obtained when various fitting/adjustment procedures were used.

The model optimized for taper and cumulative volume and then adjusted to fit the combined estimator was the most appropriate.

ABSTRACT

In this study, Max and Burkhart (1976)’s segmented taper model was used to describe stem profile and predict stem volume of oriental spruce in Turkey. Thirty procedures were evaluated, which include five fitting methods and six adjustment strategies. The fitting methods resulted in parameters that were optimized for (1) taper, (2) cumulative volume, (3) taper and cumulative volume, (4) taper and total volume, and (5) taper, cumulative volume, and total volume. The adjustment strategies are (1) unadjusted, and adjusted to match (2) DBH, (3) predicted total volume, (4) DBH and predicted total volume, (5) a combined estimator, and (6) DBH and a combined estimator. Results showed that, without adjustment, the model with parameters optimized for taper gave good prediction for both taper and cumulative volume. Mixed results were obtained when various adjustment strategies were used on different fitting techniques. The overall best-ranked procedure for predicting both taper and volume was the model optimized for taper and cumulative volume and then adjusted to fit the combined estimator.

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INTRODUCTION

Prediction of tree volume in a stand, either total volume or merchantable volume, are essential for forest management and planning. Predicting tree merchantable volume for any utilization standard can be done by use of volume ratios (Honer 1964, Burkhart 1977, Cao and Burkhart 1980; Teshome 2005) or by integrating stem profile models. Numerous taper equations, from simple to complex, have been used to describe stem profile of various tree species (Kozak 2004; Jordan et al. 2005; Diéguez-Aranda et al. 2006; Li and Weiskittel 2010; Schröder et al. 2015; Özçelik and Crecente-Campo 2016). Flexible equations such as variable-exponent taper models (Kozak 1988; Bi 2000; Kozak 2004; Newnham 1992; Sharma and Zhang 2004) cannot be analytically integrated, and therefore need to be numerically integrated for volume computation. On the other hand, volume by integration exists in closed form for segmented taper equations (Max and Burkhart 1976; Cao et al. 1998; Clark et al. 1991; Fang and Bailey 2000), which can also be directly solved to produce an estimate of merchantable height for a given top diameter (Kozak and Smith 1993).

Demaerschalk (1972) introduced the concept of a compatible taper and volume system, in which integration of the taper model produces volume that equals the volume predicted by a volume equation. This is because taper and volume are mathematically and biologically related (Munro and Demaerschalk 1974). A compatible taper equation can be obtained either by deriving from a total or merchantable volume equation (Demaerschalk 1972, Clutter 1980), or by applying constraints to ensure that its integration produces specified stem volume (Goulding and Murray 1976; Cao et al. 1980; Van Deusen et al. 1982, 1988; Reed and Green 1984; Lenhart et al. 1987; Fang and Bailey 1999; Diéguez-Aranda et al. 2006).

A method to simultaneously fit equations in the taper and volume system substantially reduced the total estimation error (Reed 1982, Reed and Green 1984). This simultaneous estimation problem was reformulated by Van Deusen (1988) as a seemingly unrelated regressions (SUR) problem, which can be easily solved using standard statistical software packages.

Oriental spruce (Picea orientalis L.) is an important tree species in northeastern of Turkey and it occupies an area of 328,000 ha, with the standing volume about 71.4 million m³ (GDF 2018). This species is utilized for pulpwood and cellulose. However, because of ever-changing market conditions, existing equations and local volume tables that are based on fixed merchantability limits no longer suffice. In addition, environmental benefits from oriental spruce forests in northeastern Turkey include conservation of biological diversity, climate change mitigation and adaptation, and protection of soil and water resources. Therefore, forest managers need detailed information supplied by growth and yield prediction models, such as volume classified by merchantable products, for sustainable management of these forests.

Some taper models have been tested to describing stem profile and predict volume for some tree species in Turkey (Brooks et al. 2008; Sakici et al. 2008; Özçelik et al. 2014; Özçelik and Crecente-Campo 2016 and Özçelik and Cao 2017; Sakici and Ozdemir, 2018). Taper equations generally are specific to each species, meaning that a separate set of parameters is needed for each species to identify its unique bole shape (Sharma and Parton 2004).

The Max and Burkhart’s (1976) taper equation was used in this study because it is straightforward to predict height at a given stem diameter. This model included three quadratic functions, which are joined together. The result is a continuous and smooth stem profile. For a flexible model, it was relatively simple, and therefore has been frequently used to describe stem profile of many tree species (Byrne and Reed 1986; Muhairwe 1999; Jiang et al. 2005; Diéguez-Aranda et al. 2006; Schröder et al. 2014; Scolforo et al. 2018). Constraints have been applied to ensure that the taper curve go through diameter at breast height (DBH) (Cao 2009) and/or an upper-stem diameter ( Czaplewski and McClure 1988; Cao 2009; Cao and Wang 2011; Sabatia and Burkhart 2015). Working with black pine in Turkey, Özçelik and Cao (2017) found that various fitting and adjustment strategies for taper and volume predictions did not improve performance of the taper model. However, they did not consider the possibility of adjustment based on a combined estimator, which is a weighted average of predicted stem volumes from the taper and volume models.

The objective of this study was to evaluate various combinations of methods for estimating parameters and calibrating a modified form of Max and Burkhart’s (1976)’s segmented taper equation for oriental spruce.

MATERIAL AND METHODS

Data

Data used in this study consist of 5859 outside-bark diameter observations from measurements of 642
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destructively sampled oriental spruce trees. The data was collected from natural stands located throughout the area of distribution of oriental spruce in northeastern Turkey. Sample trees were selected to represent diameter and height distributions, based on information from a previous inventory. Diameter at breast height (at 1.3 m above ground level, dbh) and total bole length were measured to the nearest 0.1 cm and 0.01 m for each tree, respectively. Stump height averaged 0.30 m. The all data ranged from 5.2 to 74.3 cm for dbh and 4.1 to 37.7 m for total height. The felled trees were sectioned at 2 m intervals starting from the stump to the tree tip. Two measures of diameter outside bark (dob) perpendicular to each other were collected and averaged to obtain dob measurement up the stem. Smalian’s formula was used to calculate the volume of sections in cubic meters. The volume of the last portion of the stem profile model. The segmented taper model by Max and Burkhart (1976) is preferred because volume can be easily integrated and height prediction from diameter can be directly obtained. A modified form of this taper equation (Cao 2009) was used in this study, where: $\hat{V}_j$ predicted value of $V_j$, $d_{ij}$: outside-bark diameter in cm at height $h_i$ of location $j$ on tree $i$, $h_i$: height from the ground in m, $z_i$: relative height from the tree tip, $h_k = \begin{cases} 1 & \text{if } z_i > a_k, k = 1, 2, \\ 0 & \text{otherwise} \end{cases}$, and $a_k$ and $b_k$: regression coefficients.

Performance was improved when $d_i$ was used as a dependent variable rather than equation 3. The regression model is: $V_i = \frac{1}{2} \pi d_i^2 h_i$. A constant to convert diameter in cm to area in m$^2$.

A myriad of equations have been developed to predict stem volume of a tree. Schumacher and Hall’s (1933) model, which has been widely used for many tree species, was applied in this study to estimate total stem volume, where $V_i = \sum_{j=1}^{N} V_{ij}$, where $n_i$ is number of diameter measurements for tree $i$, $N$ is number of trees, and $d_{ij}$ is predicted bole diameter at location $j$ on tree $i$.

**Methods**

**Total volume equation**

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**Taper equation**

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**Fitting method 1 – Optimized for taper**

The least squares approach used in this method is commonly employed in fitting taper equations. The parameters were selected to minimize $\sum_{i=1}^{N} \sum_{j=1}^{n_i} (d_{ij} - \hat{d}_{ij})^2$, where $n_i$ is number of diameter measurements for tree $i$, $N$ is number of trees, and $\hat{d}_{ij}$ is predicted bole diameter at location $j$ on tree $i$.

**Fitting method 2 – Optimized for cumulative volume**

The goal was to produce good prediction for cumulative volume by integrating the taper model. This was done by minimizing $\sum_{i=1}^{N} \sum_{j=1}^{n_i} (v_{ij} - \hat{v}_{ij})^2$, where $v_{ij}$ and $\hat{v}_{ij}$ are observed and predicted cumulative volume of tree $i$ from the stump to the $j^{th}$ diameter measurement, respectively.

**Fitting method 3 – Optimized for both taper and cumulative volume**

Both $\sum_{i=1}^{N} \sum_{j=1}^{n_i} (d_{ij} - \hat{d}_{ij})^2$ and $\sum_{i=1}^{N} \sum_{j=1}^{n_i} (v_{ij} - \hat{v}_{ij})^2$ were simultaneously minimized in this approach by use of seemingly unrelated regression (SAS proc MODEL, option SUR). This method ensures that predictions for both diameter and cumulative volume are reliable.

**TABLE I** Summary statistics of data used in this study.

| Groups | Variable | Min | Mean | Max |
|--------|----------|-----|------|-----|
| Group 1 (n=321) | DBH (cm) | 5.80 | 28.29 | 72.60 |
|          | Total height (m) | 5.00 | 17.26 | 33.00 |
|          | Total volume (m$^3$) | 0.01 | 0.71 | 3.79 |
|          | Number of sections | 3.00 | 9.79 | 17.00 |
| Group 2 (n=321) | DBH (cm) | 5.20 | 27.88 | 74.30 |
|          | Total height (m) | 4.10 | 16.67 | 37.70 |
|          | Total volume (m$^3$) | 0.01 | 0.71 | 6.07 |
|          | Number of sections | 3.00 | 8.96 | 18.00 |
Fitting method 4 – Optimized for both taper and total volume

Similar to the previous approach, the objective was to simultaneously minimize \( \sum_{i=1}^{N} \left( d_{ij} - \hat{d}_{ij} \right)^2 \) and \( \sum_{i=1}^{N} (V_i - \hat{V}_i)^2 \), where \( \hat{V}_i \) are predicted total volume of tree \( i \), obtained by integrating the taper equation (3). Seemingly unrelated regression (SUR) was also used for optimizing both diameter and total volume.

Fitting method 5 – Optimized for taper, cumulative volume, and total volume

In this approach, the objective was to simultaneously minimize \( \sum_{i=1}^{N} \left( d_{ij} - \hat{d}_{ij} \right)^2 \), \( \sum_{i=1}^{N} \left( V_i - \hat{V}_i \right)^2 \), and \( \sum_{i=1}^{N} (\hat{V}_i - \tilde{V}_i)^2 \). SUR was again used for this approach.

In each of the five fitting methods, parameters were adjusted so that predictions from the resulting taper model that match various attributes.

Adjustment strategy 1 – Unadjusted

No adjustment was made; estimates of parameters remained unchanged.

Adjustment strategy 2 – Adjusted to match DBH

When the Max and Burkhart (1976) taper equation is applied to breast height \( h = 1.30 \) m, predicted diameter at this point does not necessarily equal to \( \text{DBH} \). This adjustment procedure, proposed by Cao (2009), replaced parameter \( b_1 \) with \( b_1^* \) such that predicted diameter at breast height is \( D_i \):

\[
b_1^* = b_1 + \frac{1 - \hat{V}(z_{i, \text{BIH}})}{z_{i, \text{BIH}}} \tag{8}
\]

\[
z_{i, \text{BH}} = 1 - 1.3/H_i \tag{9}
\]

\[
b_1^* = b_1 + \frac{2(\hat{V}_i - \tilde{V}_i)}{K D_i^2 H_i z_{i, \text{IS}}} \tag{10}
\]

\[
z_{i, \text{IS}} = 1 - h_{i, \text{IS}}/H_i \tag{11}
\]

Adjustment strategy 3 – Adjusted to match total volume

In this strategy, replaced \( b_1 \) such that the resulting total volume matches, which is predicted from the total volume equation (1), where \( K = 0.00007854 \), a constant to convert diameter in cm to area in m \(^2\), \( h_{i, \text{IS}} \) = stump height for tree \( i \).

Adjustment strategy 4 – Adjusted to match both DBH and total volume

In this adjustment strategy, parameters \( b_1 \) and \( b_2 \) were replaced with \( b_1^* \) and \( b_2^* \), respectively, so that

\[
b_2^* = b_2 + \frac{2(\hat{V}_i - \tilde{V}_i) - K D_i^2 H_i z_{i, \text{IS}}}{K D_i^2 H_i z_{i, \text{IS}} (2z_{i, \text{IS}}/3 - z_{i, \text{BH}})} \tag{12}
\]

\[
b_1^* = b_1 + \frac{2(\hat{V}_i - \tilde{V}_i)}{K D_i^2 H_i z_{i, \text{IS}}} - (b_1^* - b_1) z_{i, \text{BH}} \tag{13}
\]

predicted diameter at breast height matches \( D_i \) and the resulting total volume matches \( \hat{V}_i \):

Adjustment strategy 5 – Adjusted to match combined estimator for total volume

The combined estimator for total volume \( \hat{V}_i \) is the weighted average of predicted volumes from the taper equation \( (\hat{V}_i) \) and the volume model \( (\tilde{V}_i) \), where \( w \) is obtained by minimizing.

This is adjustment strategy 3, with the combined estimator \( (\hat{V}_i) \) replacing the total stem volume estimated from the total volume equation \( (\tilde{V}_i) \).

\[
h_i = w \hat{V}_i + (1 - w) \tilde{V}_i \tag{14}
\]

\[
\sum_{i=1}^{N} (V_i - \hat{V}_i)^2 \tag{15}
\]

Adjustment strategy 6 – Adjusted to match DBH and combined estimator for total volume

This strategy is identical to adjustment strategy 4, with the exception that replacing to predict total volume.

Model Evaluation

In this study, a total of 30 procedures (five fitting methods × six adjustment strategy) were evaluated. The two-fold evaluation approach was applied. For this purpose, the data were randomly divided into two groups; each containing 321 trees. The coefficients obtained by fitting data from one group were used to predict for the other group. The predicted values from both groups were then pooled to calculate evaluation statistics for both diameters and total volumes. The evaluation statistics included mean bias \( (\text{MB}) \) between measured and predicted values, mean of absolute bias \( (\text{MAB}) \) and fit index \( (\text{FI}) \). \( \text{MB} \) measures the average bias of the prediction, \( \text{MAB} \) measures the magnitude of the bias, and \( \text{FI} \) is analogous to \( R^2 \) in linear regression. These statistics are computed as follows, where \( x_i \) = either diameter \( (d_i) \) or volume \( (V_i) \), and \( \bar{x}_i \) are predicted and average values of \( x_i \), respectively.

\[
\text{MB} = \frac{\sum_{i=1}^{N} (x_i - \bar{x}_i)}{N} \tag{16}
\]

\[
\text{MAD} = \frac{\sum_{i=1}^{N} |x_i - \bar{x}_i|}{N} \tag{17}
\]

\[
\text{FI} = 1 - \frac{\sum_{i=1}^{N} (x_i - \bar{x}_i)^2}{\sum_{i=1}^{N} (x_i - \bar{x}_i)^2} \tag{18}
\]
For each procedure and for each evaluation statistic, a relative rank (Poudel and Cao 2013) was computed. The best and the worst procedures in this ranking system have relative ranks of 1 and k, respectively, where k is number of procedures being evaluated. The remaining procedures have ranks as real numbers between 1 and k. Because this scheme considers both magnitude and order of the evaluation statistic, the relative ranking system should offer more information than the traditional ordinal ranks. The sum of the relative ranks from the three evaluation statistics for each procedure was calculated and then ranked again to give an overall rank for each procedure.

RESULTS AND DISCUSSION

Total volume prediction

Table 2 shows that, based on all three statistics, total volume prediction was better from the total volume model (relative rank of 1.26) than from integrating the various taper models (relative ranks between 7.00 to 11.00). The five fitting methods attained good results in predicting total volume, explaining between 97.91% to 98.01% of the variation, but still slightly worse than 98.92% when predicted from the total volume model. Surprisingly, the inclusion of total volume in the optimization methods (fitting methods 4 and 5) did not improve total volume prediction, as compared to fitting method 1 that optimized only taper. The total volume model produced similar evaluation statistics to those obtained from the combined estimators (relative ranks between 1.00 to 1.30), which is a weighted average of total volume estimates from the total volume model and from integrating the taper equations.

Table 2 Evaluation statistics for total volume. Bold, italic numbers denote the best method for each criterion, whereas underlined numbers denote the worst method.

| Model | Optimization | Volume calculation | MD | MAD | FI | Relative rank |
|-------|--------------|--------------------|-----|-----|----|---------------|
| Total volume | Unadjusted | -0.0035 | 0.0467 | 0.9892 | 1.2622 |
| | DBH | 0.0199 | 0.0658 | 0.9796 | 9.9491 |
| | Predicted TV | 0.0226 | 0.0666 | 0.9796 | 10.5217 |
| | Combined estimator | -0.0037 | 0.0462 | 0.9899 | 1.0000 |
| Taper | Integration | 0.0060 | 0.0615 | 0.9795 | 7.0036 |
| | Combined estimator | -0.0036 | 0.0467 | 0.9893 | 1.2477 |
| Cumulative volume | Unadjusted | -0.0039 | 0.0467 | 0.9893 | 1.2974 |
| | DBH | 0.0199 | 0.0658 | 0.9796 | 9.9491 |
| | Predicted TV | 0.0226 | 0.0666 | 0.9796 | 10.5217 |
| | Combined estimator | -0.0037 | 0.0462 | 0.9899 | 1.0000 |
| Taper and cumulative volume | Unadjusted | -0.0039 | 0.0467 | 0.9893 | 1.2974 |
| | DBH | 0.0199 | 0.0658 | 0.9796 | 9.9491 |
| | Predicted TV | 0.0226 | 0.0666 | 0.9796 | 10.5217 |
| | Combined estimator | -0.0037 | 0.0462 | 0.9899 | 1.0000 |

Diameter prediction

Table 3 shows that the modified form of Max and Burkhart (1976) model was adequate in estimating tree taper for this data set, regardless of optimization method. Bold, italic numbers denote the best method for each criterion, whereas underlined numbers denote the worst method.

Table 3 Evaluation statistics for taper. Bold, italic numbers denote the best method for each criterion, whereas underlined numbers denote the worst method.

| Optimization | Adjustment | MD | MAD | FI | Relative rank |
|--------------|------------|----|-----|----|---------------|
| Taper | Unadjusted | 0.2067 | 1.2232 | 0.9845 | 3.9073 |
| | DBH | 0.2725 | 1.1693 | 0.9844 | 2.8641 |
| | Predicted TV | 0.1449 | 1.2311 | 0.9840 | 3.6802 |
| | Combined estimator | 0.1179 | 1.2161 | 0.9824 | 4.6297 |
| | DBH and Combined estimator | 0.1145 | 1.2317 | 0.9840 | 3.7004 |
| Cumulative volume | Unadjusted | 0.5902 | 1.4227 | 0.9784 | 30.0000 |
| | DBH | 0.3992 | 1.2937 | 0.9806 | 16.7619 |
| | Predicted TV | 0.0405 | 1.3717 | 0.9794 | 14.6004 |
| | Combined estimator | 0.0597 | 1.4183 | 0.9759 | 22.1091 |
| | DBH and Combined estimator | 0.0524 | 1.4219 | 0.9758 | 22.2665 |
| Taper and cumulative volume | Unadjusted | 0.4242 | 1.2789 | 0.9839 | 11.9781 |
| | DBH | 0.2251 | 1.1764 | 0.9843 | 2.3439 |
| | Predicted TV | 0.0128 | 1.2470 | 0.9839 | 4.2257 |
| | Combined estimator | 0.0136 | 1.2494 | 0.9838 | 2.0667 |
| | DBH and Combined estimator | 0.0124 | 1.2346 | 0.9815 | 6.9040 |
| Taper, cumulative volume, and total volume | Unadjusted | 0.3969 | 1.2721 | 0.9839 | 11.0792 |
| | DBH | 0.2487 | 1.1679 | 0.9845 | 2.1567 |
| | Predicted TV | 0.0643 | 1.2365 | 0.9840 | 2.2404 |
| | Combined estimator | 0.0709 | 1.2383 | 0.9840 | 2.4652 |
| | DBH and Combined estimator | 0.0705 | 1.2179 | 0.9820 | 4.2757 |
| | Unadjusted | 0.4261 | 1.2857 | 0.9836 | 12.7607 |
| | DBH | 0.1851 | 1.1688 | 0.9844 | 1.0000 |
| | Predicted TV | 0.0768 | 1.3224 | 0.9820 | 9.3875 |
| | Combined estimator | 0.0996 | 1.3270 | 0.9820 | 10.0867 |
| | DBH and Combined estimator | 0.0659 | 1.3212 | 0.9793 | 12.8685 |
techniques or adjustment strategies. The resulting taper equations produced a mean absolute difference ranging from 1.17 cm to 1.42 cm, and explained between 97.59% and 98.45% of the variation in diameter.

Without adjustment, the taper equation with parameters optimized only for cumulative volume (fitting method 2) gave the worst prediction for taper (Table 3). On the other hand, the taper optimization (fitting method 1) fared better than the other fitting methods, as expected.

The DBH adjustment constrained the taper curve to go through DBH and therefore resulted in better taper prediction than did the unadjusted taper equations (Table 3). In fact, the DBH adjustment for fitting method 5 (optimized for taper, cumulative volume, and total volume) was the best in predicting taper. The other adjustment strategies, with some exceptions, also improve taper prediction, compared to the unadjusted strategy.

Cumulative volume prediction

All of the taper equations resulting from various fitting and adjustment strategies yielded acceptable results in predicting cumulative volume, from 94.66% to 98.74% in fit index, and from 0.04 m$^3$ to 0.12 m$^3$ in mean absolute deviation (Table 4).

Table 4 also shows that, without adjustment, the taper optimization method (fitting method 1) attained the higher relative rank (1.51) than did the rest of the fitting methods (ranging from 2.48 to 3.07 in ranks).

The DBH adjustment did not help to predict cumulative volume, resulting in the worst relative ranks (23.70 to 30.00) among all methods (Table 4). Conversely, adjustment for total volume (adjustment strategies 4 and 5), by use of either prediction from a total volume model or a combine estimator, did improve the cumulative volume prediction. The improvement was enhanced when this adjustment was coupled with the DBH adjustment (adjustment strategies 4 and 6).

Diameter and cumulative volume prediction

Results from Tables 3 and 4 shows that most fitting/adjustment procedures tended to favor either cumulative volume or taper prediction, but not both. In order to evaluate each procedure based on its ability to predict both taper and volume, we summed the relative ranks for taper and cumulative volume. The sum of the relative ranks for taper and cumulative volume for each procedure was then ranked to yield an overall rank (Table 5).

Cao et al. (1980) found that a taper equation (that was optimized for taper), while being excellent in predicting predicted taper, did not predict cumulative volume as well as a volume ratio model. Results from this study tell a different story. The taper model in this study with parameters optimized for cumulative volume (fitting method 2) can be considered somewhat similar to a volume ratio model, yet ranked lower in volume prediction (3.07) than the model optimized for taper

TABLE 4 | Evaluation statistics for cumulative volume. Bold, italic numbers denote the best method for each criterion, whereas underlined numbers denote the worst method.

| Optimization | Adjustment | MD | MAD | FI | Relative rank |
|--------------|------------|----|-----|----|---------------|
| Taper        | Unadjusted | -0.0008 | 0.0445 | 0.9850 | 1.5077 |
|              | DBH        | 0.1219 | 0.1225 | 0.9466 | 29.0472 |
|              | Predicted TV | -0.0027 | 0.0447 | 0.9847 | 1.7500 |
|              | DBH and predicted TV | 0.0037 | 0.0421 | 0.9864 | 1.1377 |
|              | Combined estimator | -0.0027 | 0.0448 | 0.9846 | 1.7844 |
|              | DBH and Combined estimator | 0.0036 | 0.0421 | 0.9864 | 1.1297 |
| Cumulative volume | Unadjusted | 0.0168 | 0.0483 | 0.9857 | 3.0734 |
|              | DBH        | 0.1109 | 0.1123 | 0.9161 | 23.6973 |
|              | Predicted TV | 0.0019 | 0.0436 | 0.9865 | 1.1506 |
|              | DBH and predicted TV | 0.0063 | 0.0431 | 0.9781 | 1.3057 |
|              | Combined estimator | 0.0017 | 0.0436 | 0.9865 | 1.1347 |
|              | DBH and Combined estimator | 0.0061 | 0.0431 | 0.9781 | 1.2898 |
| Taper and cumulative volume | Unadjusted | 0.0137 | 0.0472 | 0.9860 | 2.6280 |
|              | DBH        | 0.1237 | 0.1242 | 0.9439 | 30.0000 |
|              | Predicted TV | -0.0011 | 0.0430 | 0.9865 | 1.0156 |
|              | DBH and predicted TV | 0.0054 | 0.0419 | 0.9874 | 1.0239 |
|              | Combined estimator | -0.0013 | 0.0430 | 0.9865 | 1.0315 |
|              | DBH and Combined estimator | 0.0052 | 0.0419 | 0.9874 | 1.0080 |
| Taper and total volume | Unadjusted | 0.0107 | 0.0470 | 0.9855 | 2.4791 |
|              | DBH        | 0.1219 | 0.1225 | 0.9463 | 29.1147 |
|              | Predicted TV | -0.0017 | 0.0435 | 0.9858 | 1.2803 |
|              | DBH and predicted TV | 0.0045 | 0.0419 | 0.9870 | 1.0424 |
|              | Combined estimator | -0.0019 | 0.0435 | 0.9858 | 1.2962 |
|              | DBH and Combined estimator | 0.0044 | 0.0419 | 0.9870 | 1.0345 |
| Taper, cumulative volume, and total volume | Unadjusted | 0.0138 | 0.0476 | 0.9857 | 2.7518 |
|              | DBH        | 0.1203 | 0.1212 | 0.9472 | 28.6305 |
|              | Predicted TV | -0.0005 | 0.0445 | 0.9863 | 1.1914 |
|              | DBH and predicted TV | 0.0051 | 0.0425 | 0.9872 | 1.1165 |
|              | Combined estimator | -0.0012 | 0.0443 | 0.9865 | 1.1782 |
|              | DBH and Combined estimator | 0.0045 | 0.0423 | 0.9874 | 1.0000 |
Compared to the unadjusted strategy, the DBH adjustment (adjustment strategy 2) resulted in better evaluation statistics for taper prediction, but worse statistics for prediction of cumulative volume. Adjustment for predicted total volume (from either a total volume model or a combined estimator), in many cases, bettered prediction of both taper and cumulative volume. Adding DBH to the above adjustment actually lowered the overall ranks (Table 5).

The worst overall procedure (rank of 30.00), which is the cumulative volume optimization coupled with DBH adjustment, gave poor prediction for both taper and volume. The best overall rank came from a taper equation optimized for taper and cumulative volume (fitting method 3) which was then adjusted for the combined estimator (adj ustment strategy 5). This procedure achieved relative ranks of 2.07 and 1.03 for predicting taper and cumulative volume, respectively.

Finally, the proposed taper and stem volume models for optimization alternatives were refit to the entire data set using all five fitting methods (Table 6). Figure 1 shows the observed data, overlaid with predictions from the taper model of fitting method 5.

TABLE 6

| Fitting methods | Parameters |
|------------------|------------|
|                  | $b_i$       | $b_0$       | $b_2$       | $d_i$       | $d_0$       |
| Taper            | 0.1060      | 1.5561      | -1.7349     | 53.2930     | 0.2452      | 0.9187      |
| Cumulative volume| 0.2139      | 1.4019      | -1.9383     | 1.0310      | 0.3569      | 0.4893      |
| Taper and cumulative volume| 0.0865      | 1.9549      | -1.7782     | 51.5390     | 0.2472      | 0.9136      |
| Taper, and total volume| 0.0902      | 1.9360      | -1.7187     | 56.0283     | 0.2460      | 0.9189      |
| Taper, cumulative volume, and total volume| 0.0856      | 1.9613      | -1.7736     | 54.9194     | 0.2452      | 0.9172      |

FIGURE 1 Graph of observed data and predictions from the taper model optimized for taper, cumulative volume, and total volume.
CONCLUSIONS

In this study, a simpler form of the Max and Burkhart's (1976) taper model was used to predict taper and stem volume of oriental spruce. A total of thirty procedures was evaluated, including five fitting methods (optimized for taper, cumulative volume, taper and cumulative volume, taper and total volume, and taper and both cumulative and total volumes) and six adjustment strategies (unadjusted, and adjusted to match DBH, predicted total volume predicted from either a total volume model or a combined estimator, and DBH and predicted total volume). Results of this study indicated that, without adjustment, the model with parameters optimized for taper gave good prediction for both taper and cumulative volume. Mixed results were obtained when various adjustment strategies were used on different fitting techniques. The overall best-ranked procedure for estimating both taper and stem volume was the model optimized for taper and cumulative volume and then adjusted to fit the combined estimator.

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