Observation of bulk Fermi arc and polarization half charge from paired exceptional points

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The ideas of topology have found tremendous success in closed physical systems, but even richer properties exist in the more general open or dissipative framework. We theoretically propose and experimentally demonstrate a bulk Fermi arc that develops from non-Hermitian radiative losses in an open system of photonic crystal slabs. Moreover, we discover half-integer topological charges in the polarization of far-field radiation around the bulk Fermi arc. Both phenomena are shown to be direct consequences of the non-Hermitian topological properties of exceptional points, where resonances coincide in their frequencies and linewidths. Our work connects the fields of topological photonics, non-Hermitian physics, and singular optics, providing a framework to explore more complex non-Hermitian topological systems.

We theoretically design and experimentally realize a new configuration of isolated EP pairs in momentum space, which allows us to reveal topological signatures of EPs in the band structure and far-field polarization, and to extend topological band theory into the realm of non-Hermitian systems. Specifically, we demonstrate that a Dirac point (DP) with nontrivial Berry phase can split into a pair of EPs (20–22) when radiation loss—a form of non-Hermiticity—is added to a two-dimensional (2D)–periodic photonic crystal (PhC) structure. The EP-pair generates a distinct double–Riemann sheet topology in the complex band structure, which leads to two notable consequences: bulk Fermi arcs and polarization half topological charges. First, we demonstrate that this pair of EPs is connected by an open-ended iso-frequency contour—a bulk Fermi arc—in direct contrast to the common intuition that iso-frequency contours are necessarily closed loops. The bulk Fermi arc here is a special topological signature of non-Hermitian effects in paired EPs and resides in the bulk dispersion of a 2D system. This is fundamentally different from the previously known surface Fermi arcs that arise from the 2D projection of Weyl points in 3D Hermitian systems. Moreover, we find experimentally that around the Fermi arc, the far-field polarization of the system exhibits a robust half-integer winding number (23–25), analogous to the orientation reversal on a Möbius strip. We show that this is a direct consequence of the topological band-switching properties across the Fermi arc connecting the EP pair and is direct experimental proof of the $v = \pm \frac{1}{2}$ topological index associated with an EP (8). With comprehensive comparisons between analytical models, numerical simulations, and experimental measurements, our results are a direct validation of non-Hermitian topological band theory and present its novel application to the field of singular optics.

Fig. 1. Bulk Fermi arc arising from paired exceptional points split from a single Dirac point. (A and B) Illustration of photonic crystal (PhC) structures, iso-frequency contours, and band structures. (A) Band structure of a 2D-periodic PhC consisting of a rhombic lattice of elliptical air holes, featuring a single Dirac point on the positive $k_x$ axis. (B) The real part of the eigenvalues of an open system consisting of a 2D-periodic PhC slab with finite thickness, where resonances experience radiation loss. The Dirac point splits into a pair of exceptional points (EPs). The real part of the eigenvalues is degenerate along an open-ended contour—the bulk Fermi arc (blue line)—connecting the pair of EPs. (C) Examples of the iso-frequency contours in this system, including the open bulk Fermi arc at the EP frequency (middle panel), and closed contours at higher (upper panel) or lower (lower panel) frequencies. Solid lines are from the analytical model, and circles are from numerical simulations.
Our scheme involves splitting a single DP into a pair of EPs, which directly leads to the emergence of a bulk Fermi arc. First, consider a 2D-periodic PhC with a square lattice of circular air holes introduced into a dielectric material. In this Hermitian system (no material gain, loss, or radiation loss), the crystalline symmetry ($C_{4v}$) ensures a quadratic band degeneracy at the center of the Brillouin zone (26). As this $C_{4v}$ symmetry is broken, e.g., by shearing the structure into a rhombic lattice with elliptical holes (Fig. 1A), the quadratic degeneracy point splits into a pair of DPs situated at $(\pm k_x, 0)$ along the $k_x$ axis. The same splitting behavior is shown in both analytical models and numerical simulations (21, 26, 27).

Next, we consider a non-Hermitian system consisting of a finite-thickness PhC slab (inset of Fig. 1B), where modes near the DP become resonances with finite lifetime because of radiative losses toward the top and bottom. Adopting the non-Hermitian setup, we highlight the region of interest in both panels, where the isofrequency contours measured at different wavelengths. To focus on the bulk Fermi arc, we show the shrinking and reexpanding behavior. As shown in Fig. 3, as the wavelength decreases from 794.0 nm, the corresponding isofrequency contour shrinks (top two rows), and eventually becomes an open-ended arc at 791.0 nm (middle row), consistent with our previous theoretical predictions in Fig. 1C. As the wavelength is further decreased down to 789.5 nm and 788.7 nm, the arc expands out into closed contours again (bottom two rows). The bending feature of the contours is a result of higher-order terms in the band dispersion (26). The open contour at 790.0 nm (middle row) is a clear direct observation of the bulk Fermi arc.

So far, we have shown one direct consequence of the unique double-Riemann sheet topology near paired EPs—the bulk Fermi arc. Next, we demonstrate another consequence: half-integer topological charges in the polarization configuration, which also serve as a direct experimental proof of the $v = t_{1/2}$ topological index of an EP. These topological charges describe the direction (clockwise or counterclockwise) and number of times the polarization vector winds about a point or line singularity in the optical field, and in our particular system, we observe a robust
180° winding around the Fermi arc, corresponding to a half-integer topological charge.

To fully reconstruct the far-field polarization configurations of the resonances, we perform polarimetry measurements by recording the intensity of isofrequency contours after passing through six different configurations of polarizers and/or waveplates (26). Although the incoming light is vertically polarized, the scattered light at each point along the contour is, in general, elliptically polarized, reflecting the polarization state of its underlying resonance. Taking points X and Z in Fig. 4A as examples: After passing through a vertical polarizer, the scattered light is weak (strong) at point X(Z); whereas after a horizontal polarizer, the relative intensity of the scattered light switches between points X and Z. This clearly shows that the far field of the underlying resonance at point X(Z) is mostly horizontally (vertically) polarized.

Examples of the fully reconstructed spatial polarizations (blue ellipses) at representative points along the 794-nm isofrequency contour (red solid line) are shown in the top panel of Fig. 4B, which agree well with numerical results (Fig. 4B, bottom panel). Furthermore, both experimental and numerical results show 180° winding of the polarization long axis, as illustrated by the green arrows in Fig. 4B: As the momentum point starts from point X, traverses the full contour in the counterclockwise direction, and returns to point X, the polarization long axis flips direction by rotating 180° in the clockwise direction—corresponding to a −1/2 topological charge being enclosed in the loop. These results thus indicate that the far-field emission from our PhC is a vector-vortex beam with half-integer topological charge, in stark contrast to the integer vector beams realized in photonic crystal surface-emitting lasers (24).

We now explain the fundamental connections between the half-integer topological charges observed in the far-field polarization and the half-integer topological index of an EP (8), manifested as its mode-switching property (26). Along the $k_x$ axis, the two bands forming the EP pair in our system have orthogonal linear polarizations due to the $y$–mirror symmetry: One is horizontal (e.g., mode X in Fig. 4C), whereas the other is vertical (e.g., modes Z and W). As we follow a closed path in momentum space $X \rightarrow Y \rightarrow Z \rightarrow W$ that encircles one of the EPs in the counterclockwise direction, the initial eigenstate X (horizontally polarized) on the top sheet adiabatically evolves into state Z (vertically polarized) and eventually into final state W (vertically polarized) on the bottom sheet, owing to the mode-switching topological property of the EP (10–12). The switching behavior of the eigenmodes—from X to W—directly follows from their eigenvalue swapping behavior on the complex plane (26).

Equivalently, one complex eigenvalue winds around the other one by half a circle, thus implying that the topological index of an EP is a half-integer. The orthogonal nature between the polarizations at X and Z, arising from the mode-switching property of the EP, guarantees

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Fig. 3. Experimental demonstration of a bulk Fermi arc. (A) Numerically simulated spectral density of states and (B) experimentally measured isofrequency contours at five representative wavelengths. The bulk Fermi arc appears at 791.0 nm (middle row), when the isofrequency contour becomes open-ended. The regions of interest are highlighted in all panels to emphasize the shrinking (top two rows) and reexpanding (bottom two rows) feature of isofrequency contours near the bulk Fermi arc. The numerical results are offset by 0.5 nm for better comparison.
Fig. 4. Experimental demonstration of polarization topological half charges around the bulk Fermi arc. (A) Intensity of scattered light at 794 nm after passing through a vertical (horizontal) polarizer is plotted in the left (right) panel, showing that point X(Z) is mostly horizontally (vertically) polarized. (B) Experimental reconstruction (top) and numerical simulation (bottom) of the full polarization information, showing the polarization ellipses (blue ellipses) as well as their long-axis directions (green arrows) along an isofrequency contour (red line). As shown by the green arrows in the bottom panel, the polarization long axis exhibits a $-\frac{1}{2}$ topological charge. (C) Schematic illustration of the mode switching (X to W) in the band structure, along a loop enclosing an EP (X-Y-Z-W), as a result of the double–Riemann sheet topology. This mode-switching behavior directly leads to the half-integer topological index of an EP and the half-charge polarization winding.

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SUPPLEMENTARY MATERIALS
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Supplementary Text
Figs. S1 to S4
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Exploring photonic topology

Scattering topological effects are being explored in a variety of electronic and optical materials systems owing to their robustness against defects (see the Perspective by Özdemir). Yang et al. designed and fabricated an ideal optical analog of a three-dimensional Weyl system. Angular transmission measurements revealed four Weyl points at the same energy, as well as the signature helicoidal arcs associated with such an exotic topological system. Zhou et al. theoretically proposed and experimentally demonstrated the formation of a topologically protected bulk Fermi arc. They attributed the formation of the arc to the topological nature of paired exceptional points (points at which gain and loss in the system are matched). Photonic crystals may provide a powerful platform for studying exotic properties of topological electronic systems and may also be used to develop optical devices that exploit topological properties of light-matter interactions.

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