The nature of the recently detected high-energy and very high-energy $\gamma$-ray emission of Tycho’s supernova remnant (SNR) is studied. A nonlinear kinetic theory of cosmic-ray (CR) acceleration in SNRs is employed to investigate the properties of Tycho’s SNR and their correspondence with the existing experimental data, taking into account that the ambient interstellar medium (ISM) is expected to be clumpy. It is demonstrated that the overall steep $\gamma$-ray spectrum observed can be interpreted as the superposition of two spectra produced by the CR proton component in two different ISM phases: the first $\gamma$-ray component, extending up to about $10^{14}$ eV, originates in the diluted warm ISM, whereas the second component, extending up to 100 GeV, comes from numerous dense, small-scale clouds embedded in this warm ISM. Given the consistency between acceleration theory and the observed properties of the nonthermal emission of Tycho’s SNR, very efficient production of nuclear CRs in Tycho’s SNR is established. The excess of the GeV $\gamma$-ray emission due to the clouds’ contribution above the level expected in the case of a purely homogeneous ISM is inevitably expected in the case of Type Ia SNe.

Key words: acceleration of particles – cosmic rays – gamma rays: general – radiation mechanisms: non-thermal, shock waves – supernovae: individual (Tycho’s SNR)

1. INTRODUCTION

The detection of high-energy (HE; 100 MeV $\leq E \leq$ 100 GeV) and very high-energy (VHE; $E \geq$ 100 GeV) $\gamma$-ray emission from supernova remnants (SNRs) is extremely important because it provides direct evidence for the acceleration of charged particles (atomic nuclei and/or electrons) inside SNRs to energies that are comparable to those of the $\gamma$-rays. Based on such detections one can hope to eventually confirm the idea that SNRs are indeed the main source of nuclear cosmic rays (CRs) up to energies of about $10^{17}$ eV in the Galaxy, as widely expected (Bell & Lucek 2001; Ptuskin & Zirakashvili 2003). Such a relation is extremely important because it provides direct evidence for the acceleration of charged particles (atomic nuclei and/or electrons) inside SNRs to energies that are comparable to those of the $\gamma$-rays.

Nonlinear kinetic theory allows one to simulate the effect of the external magnetic field configuration, expected for Type Ia SNRs in general and for Tycho’s SNR in particular (Völk et al. 2008). However, as shown earlier (Völk et al. 2003; Berezhko et al. 2009), this means that only approximately 20% of the total shock surface will contribute to nuclear CR production from a Type Ia SNR and nonlinear shock-modification effects like magnetic field amplification in a Type Ia SN, whereas about 80% of the shock surface corresponds to a quasi-perpendicular shock without significant injection of nuclei and to suppressed acceleration. A spherically symmetric acceleration model therefore has to be “renormalized” in the sense that the integrated particle spectrum obtained in spherical symmetry needs to be diminished by a factor $\approx 0.2$. Making this correction, the sum of the multi-wavelength observations and their theoretical interpretation suggests that nuclear CRs are produced in SN 1006 with high efficiency, as required for the Galactic CR sources (Berezhko et al. 2012).

With significant detections at GeV as well as at TeV energies of otherwise simple objects like remnants of Type Ia SNe in the Galactic gas disk, the question remains whether the circumstellar environment is uniform in gas density or not, and what role this nonuniformity plays in the overall observed $\gamma$-ray spectrum and morphology. This question is discussed here for Tycho’s SNR. The remnant is spatially unresolved in $\gamma$-rays but otherwise very well studied. The spatially integrated TeV spectrum, detected by the VERITAS array (Acciari et al. 2011), is quite compatible with a theoretical model for a Type Ia—explosion in a strictly uniform interstellar medium (ISM), studied in detail in a previous paper (Völk et al. 2008, hereafter VBK08). However, the recent Fermi Large Area Telescope (Fermi LAT) detection of HE $\gamma$-rays above 400 MeV (Giordano et al. 2012) disagrees with this simple nonlinear model, showing a significant GeV excess. Morlino & Caprioli (2012, hereafter MC12) have attempted to understand this result assuming a spectrum of CR protons $N \propto \epsilon^{-\gamma}$ with a spectral index $\gamma \approx 2.2$. This spectrum is considerably steeper than the spectrum predicted by VBK08, which implies $\gamma \approx 2$. To obtain this result, MC12 assumed Bohm diffusion for all the accelerated particles. However, as will be shown…

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5 This argument should hold also for the probably significantly more complex and smaller-scale inhomogeneous but topologically equivalent external magnetic field configuration, expected for Type Ia SNRs in general and for Tycho’s SNR in particular (Völk et al. 2003).
below, such an interpretation contains an internal contradiction: Bohm diffusion even at the highest particle energies involved is inconsistent with such a steep proton spectrum.

In this paper, a new interpretation of the detected $\gamma$-ray spectrum of Tycho’s SNR will be given. It is based on the expectation that the actual ISM is clumpy instead of being purely homogeneous (Field 1965; Wolfire et al. 2003).

### 2. HOMOGENEOUS ISM

The present form of the solutions of the nonlinear acceleration equations considered here (see Berezhko 2008 for a review) assumes spherical symmetry. In this approximation it is possible to predict the temporal and radial evolution of gas density, pressure, and mass velocity, together with that of the energy spectrum, as well as the spatial distribution of CR nuclei and electrons, including the properties of their nonthermal radiation.

This theoretical model has been used in detail to investigate Tycho’s SNR as the remnant of a Type Ia SN (Krause et al. 2008) in a homogeneous ISM in order to compare the results with the existing data (VBK08). It was argued that consistency of the standard value of stellar ejecta mass $M_{\text{ej}} = 1.4 M_\odot$ and a total hydrodynamical explosion energy $E_{\text{kin}} = 1.2 \times 10^{51}$ erg (Badenes et al. 2006) with the gas dynamics, acceleration theory, and the existing $\gamma$-ray measurements required the source distance $d$ to exceed 3.3 kpc in order to be consistent with the existing HEGRA upper limit for TeV $\gamma$-ray emission. The corresponding ambient gas number density $N_g = \rho/m$ (where $\rho$ is the gas density and $m$ is the proton mass) had then to be lower than 0.4 cm$^{-3}$. On the other hand, the rather low distance estimates from independent measurements together with internal consistency arguments of the theoretical model made it even more likely that the actual $\gamma$-ray flux from Tycho is “only slightly” below the HEGRA upper limit. The strong magnetic field amplification produced by accelerated CRs implied a mean field strength of $\approx 400 \mu$G (e.g., Völk et al. 2005) and as such implied in addition that the $\gamma$-ray flux is hadronically dominated. The shock was modified with an overall compression ratio $\sigma \approx 5.2$ and a subshock compression ratio $\sigma_s \approx 3.6$; the latter is consistent with the observed radio index (Reynolds & Ellison 1992). The differential proton spectral index was $\gamma \approx 2$.

The TeV $\gamma$-ray emission from Tycho detected by VERITAS (Acciari et al. 2011) corresponds very well to the above expectation. As can be seen from Figure 1, a new $\gamma$-ray spectrum calculated within the kinetic nonlinear theory (shown by the dashed line) is well consistent with the VERITAS measurements. This new calculation was performed following the usual procedure as described in VBK08. For the proton injection rate $\eta = 3 \times 10^{-4}$ is still compatible with the above-mentioned shock modification and softening of the observed radio synchrotron emission spectrum. The new distance $d = 3.8$ kpc and the corresponding new ambient ISM number density $N_g = 0.25$ cm$^{-3}$ were taken in order to fit the observed TeV $\gamma$-ray emission (see also Acciari et al. 2011).

However, as mentioned in the Introduction, the $\gamma$-ray spectrum measured by Fermi LAT at energies 400 MeV to 100 GeV (Giordano et al. 2012) is considerably (by a factor 2–5) above the value predicted by the kinetic theory (see Figure 1). This excess of GeV $\gamma$-ray emission, when compared with the theoretical predictions, requires a more detailed consideration of this object and its environment, taking into account new physical factors which had been hitherto neglected.

### 3. A STEEP CR SPECTRUM?

The first possibility to explain the complete $\gamma$-ray data might be the search for a solution with an essentially steeper CR spectrum for all energies above $\approx 1$ GeV, compared with the hard-spectrum solution discussed in the last section. Such a soft-spectrum solution was proposed by MC12. These authors also used a nonlinear approach, assuming a homogeneous circumstellar environment of the SNR. However, their model differs essentially from the present one in terms of internal consistency.

For some aspects of the nonlinear problem, which require a consistent determination of the CR spectrum together with the time-dependent dynamics of the expanding spherical shock, MC12 substitute a simplified semi-analytical approach. This is not a principle issue. In addition, there are several basic differences between the present approach and theirs. First of all, MC12 do not permit that some part of SN shock should be locally quasi-perpendicular, where the injection of suprathermal, ionized nuclear gas particles from downstream is strongly suppressed. Their magnetic field model is instead topologically equivalent to assuming a monopolar magnetic field. As a consequence they suggest equally efficient injection/acceleration of CRs all over the shock surface. Second, these authors ignore any nonadiabatic gas heating within the shock precursor. Therefore the magnetic field pressure in the resulting model plays a more important role than the gas pressure for the determination of the shock parameters. On the other hand, numerical simulations (Bell 2004; Zirakashvili et al. 2008) of the nonresonant instability, caused by CRs within the precursor, clearly demonstrate significant gas heating that is roughly consistent with the model considered here (e.g., Berezhko 2008).

Under the above assumptions of MC12 it is indeed formally possible to obtain a solution with a steep CR spectrum, consistent with the existing measurements. However, apart from the differences between the two approaches, the scheme of MC12 in addition contains the assumption of Bohm diffusion for all particles, including those of the highest energies. For the steep
spectrum obtained, this amounts to an internal contradiction which renders such a solution inconsistent.

To make this clear requires a brief analysis of the general properties of CR acceleration in spherical blast waves. The expanding spherical SN shock produces a power-law momentum spectrum of accelerated CRs which extends up to a cutoff momentum determined by the expression (Berezhko 1996)

\[ \kappa(p_{\text{max}}) = R_s v_s / A, \]  

(1)

where \( \kappa \) denotes the CR diffusion coefficient, \( R_s \) and \( V_s \) are the radius and the speed of the shock, respectively, and \( A \) is a numerical factor whose value depends on the shock expansion law. For a rough estimate one can use the value \( A = 10 \). Note that the above expression is also roughly valid for a nonspherical shock, for example for the shock transmitted into a dense clump. In this case half of the mean size of the clump \( l \), i.e., \( l/2 \), plays the role of the shock radius \( R_s \).

When CRs are scattered mainly by Alfvén waves, as assumed by MC12, the diffusion coefficient can be written in the form

\[ \kappa(p) = \kappa_{\text{Bohm}}(p) E_B / E_w(k = 1/\rho_B), \]  

(2)

where

\[ \kappa_{\text{Bohm}} = \rho_B v / 3 \]  

is the theoretically minimal diffusion coefficient, the so-called Bohm diffusion coefficient, \( v \) and \( \rho_B \) are the particle velocity and gyroradius, respectively, \( E_B = B^2 / (8 \pi) \) is the energy density of the large-scale magnetic field, and \( E_w(k) \) is the Alfvén wave magnetic energy density per unit logarithm of wavenumber \( k \).

Bell (2004) found that a nonresonant scattering instability will occur within the precursor of a strong, accelerating shock. The diffusive streaming of accelerated CRs is expected to be so strong in this region that a purely growing MHD mode appears with a growth rate that is, at least in the outer part of the precursor, larger than the growth rate of the well-known resonant Alfvénic mode. In addition, it was demonstrated (Reville & Bell 2012) that CRs form filamentary structures in the precursor due to their self-generated magnetic field. This filamentation results in the growth of a long-wavelength instability. It is expected that due to these instabilities the external magnetic field \( B_{\text{ISM}} \) is amplified within the entire precursor structure up to \( B \gg B_{\text{ISM}} \), since the main effect is produced by the most energetic CRs with momenta \( p \sim p_{\text{max}} \), which populate the whole precursor diffusively during their acceleration.

The Alfvén wave excitation in the shock precursor (Bell 1978) corresponds to an additional unstable mode. It leads, in particular, to a high level of resonantly scattering waves. Therefore, the overall field amplification will be the result of all instabilities operating in the precursor.

These mechanisms should lead to the Bohm limit \( \kappa(p) = \kappa_{\text{Bohm}}(p) \) for CR diffusion which is achieved when the condition \( E_w(k = 1/\rho_B) \) is reached.

CRs with the momentum spectrum \( N \propto p^{-\gamma} \) produce an Alfvén wave spectrum \( E_w \propto k^{\gamma-2} \propto p^{2-\gamma} \), where \( k \) and \( p \) are approximately connected by the relation \( k = 1/\rho_B \). It can be represented in the form

\[ E_w(p) = E_m \times (p/p_{\text{max}})^{2-\gamma}, \]  

where \( E_m = E_w(p_{\text{max}}) \).

The amplified magnetic field \( B \) in the case of a steep CR spectrum is produced by CRs with energy \( p \sim m_p c \) because it is these CRs that provide the main contribution to the overall CR energy content; here \( m_p \) denotes the proton rest mass. Therefore the Bohm limit condition \( E_w(p) = E_B \) can be fulfilled for \( p \sim m_p c \), whereas for CRs with \( p \approx 10^6 m_p c \) one would have \( E_w \approx 10^{2(\gamma-2)} E_B \approx 0.1 E_B \) for \( \gamma = 2.2 \). This means that the CR diffusion coefficient of such HE particles would exceed the Bohm limit by roughly an order of magnitude. As a consequence, this amplified field would play almost no role for CRs with high momenta \( p \gg m_p c \), and therefore the maximal CR momentum that can be achieved would be considerably lower than \( 10^6 m_p c \). This is in contradiction to the assumption of MC12 that \( \kappa(p) = \kappa_{\text{Bohm}}(p) \) for the entire range of CR momenta \( p \leq 10^6 m_p c \) considered.

This argument demonstrates that a considerable increase of the maximum CR momentum due to magnetic field amplification is expected only in the case of a hard CR spectrum \( N \propto p^{-\gamma} \) with \( \gamma \lesssim 2 \), where the CRs with the highest momenta \( p \sim p_{\text{max}} \) provide the main contribution to the overall CR energy content. Fortunately, such a spectrum is expected to be produced by SN shocks for injection rates that considerably exceed those used by MC12.

4. CR AND \( \gamma \)-RAY PRODUCTION IN A CLUMPY ISM

The physics aspect which is not included in the present kinetic and (“renormalized”) spherically symmetric model is an essential inhomogeneity of the ambient ISM on spatial scales that are smaller than the SNR radius. This inhomogeneity is not the result of the progenitor star’s evolution toward the final SN explosion, for example in the form of a wind and a corresponding modification of the circumstellar environment. It is rather an inherent nonuniformity of the average ISM on account of (1) the interplay between its radiative heating by the diffuse Galactic UV field and the radiative cooling of the gas (Field 1965) and (2) the stochastic agitation of the ISM by the mechanical energy input and gas heating from SN explosions (Cox & Smith 1974; McKee & Ostriker 1977). The first effect is a thermal instability and thus a mechanism for small-scale cloud formation in the ISM driven by runaway radiative cooling (Field et al. 1969). Specifically, the balance between line-emission cooling and gas heating due to the UV background radiation leads to two thermally stable equilibrium ISM phases (Wolfire et al. 2003). One of them is the so-called warm ISM with a typical gas number density \( N_g \approx 0.1 \text{ cm}^{-3} \) and temperature \( T \approx 8000 \text{ K} \), the other one a cold neutral medium with \( N_g \approx 10 \text{ cm}^{-3} \) and \( T \approx 100 \text{ K} \). According to simulations, the scale of dense clouds is typically \( l_c \approx 0.1 \text{ pc} \) (e.g.,Audit & Hennebelle 2008, 2010; Inoue et al. 2012). The second effect is a general compressible turbulence of the ISM, at least on scales in excess of \( \sim 1 \text{ pc} \), driven by the Galactic SN explosions. According to MHD simulations, it involves high- and low-temperature gas components out of ionization equilibrium on all larger scales (de Avillez & Breitschwerdt 2005); for a review see Breitschwerdt et al. (2012). While the overall picture of the ISM is clearly not simple, the evolution of a young SNR like that of Tycho’s SN will encounter a single realization of the stochastic ensemble of density fluctuations. For the energy spectrum of energetic particles, accelerated at the blast wave, small-scale high-density fluctuations of the ISM play the most conspicuous role because they produce mainly particles with energies far below the cutoff energies for a uniform circumstellar medium. To estimate

\[ 4 \text{ Gravitationally bound molecular clouds are not considered here because they are large-scale massive elements of the ISM. If present, they would require an individual treatment.} \]
the spectral changes due to upstream density variations in an analytical model, the typical ISM is therefore treated here as a generalized two-phase medium composed of a pervasive warm/hot ISM (phase I)—called here for brevity “warm” ISM—and small-scale dense clouds (phase II) embedded in this warm ISM.

In order to determine the specific properties of the CRs and their nonthermal emission in the case of such a generalized two-phase ISM, the latter is approximated here in a simple form as a uniform warm phase with gas number density \( N_{g1} \) plus an ensemble of small-scale dense clouds with gas number density \( N_{g2} \). The warm diluted ISM phase is assumed to have a volume filling factor \( F_1 \approx 1 \), whereas the clouds occupy a small fraction of space with filling factor \( F_2 \ll 1 \). It is, in addition, assumed that most of the gas mass is contained in the warm phase, which means that \( F_1 N_{g1} \gg F_2 N_{g2} \). Then the SN shock propagates in the two-phase ISM without essential changes compared with the case of a purely homogeneous ISM with number density \( N_{g1} \) (Inoue et al. 2012). Therefore it produces inside the phase I of the ISM roughly the same amount of CRs and nonthermal emission as in the case of a homogeneous ISM. Then one has to estimate the additional contribution of the clouds in order to determine the overall spectrum of CRs.

The large-scale SN blast wave, interacting with each single cloud, produces a pair of secondary transmitted and reflected shocks. The reflected shock propagates in the warm isobaric ISM already heated by the blast wave. Due to this fact its Mach number is quite low and therefore its contribution to the overall CR production can be neglected.

The size \( R_{s2} = L_c/2 \) and the speed \( V_{s2} \approx (N_{g1}/N_{g2})^{1/2}V_s \sim 10^{-1}V_s \) of the transmitted shock are both considerably smaller than the corresponding values of the SN blast wave. Therefore, according to Equation (1), and using \( A = 10 \) for both shocks, the maximum momentum of CRs produced inside the cloud,

\[
p_{\text{max2}} = R_{s2} V_{s2}/(R_s V_s) p_{\text{max1}},
\]

is much smaller than the maximal momentum \( p_{\text{max1}} \) of the CRs produced by the SN shock in the warm ISM: \( p_{\text{max2}} \ll p_{\text{max1}} \). Since the ram pressure \( \rho V_s^2 \) is expected to be the same in both cases, and since the amplified magnetic field pressure reaches roughly the same fraction of the ram pressure, the magnetic field values are roughly the same in these two cases: \( B_2 \sim B_1 \).

An estimate of the CR spectrum produced by the transmitted shock starts from the expression for the CR distribution function

\[
f = \frac{q \eta N_{g2}}{4\pi p_{\text{max}}^2} \left( \frac{p}{p_{\text{max}}} \right)^{-q},
\]

which is valid at all momenta \( p \geq p_{\text{max}} \) up to the cutoff momentum \( p_{\text{max}} \) in the case of an unmodified shock, and is roughly valid within the momentum range \( p_{\text{max}} < p < 10 m_p c \) of the subshock in the case of a modified shock. Using this expression for the case of the SN blast wave and for the transmitted shock, the ratio of the two corresponding distribution functions can be found as

\[
\frac{f_2}{f_1} = \frac{q_2}{q_1} \left( \frac{N_{g2}}{N_{g1}} \right)^{1-(q_2-3)/2} \left( \frac{p}{p_{\text{max}}} \right)^{-q_2},
\]

for \( p_{\text{max}} < p \leq 10 m_p c \), where the expression for the injection momentum (Berezhko et al. 1996)

\[
p_{\text{inj}} \approx m_p V_s
\]

was also used.

According to the calculation of Section 2, the power-law spectrum with index \( q_1 \approx 4.3 \), determined by the subshock compression ratio, extends up to CR momenta \( p \sim 10 m_p c \).

Since the cutoff momentum \( p_{\text{max2}} \ll p_{\text{max1}} \) of the CR spectrum, produced by the transmitted shock, is much lower than the corresponding value for the blast wave, \( p_{\text{max1}} \approx 10^5 m_p c \), one can neglect the modification of the transmitted shock. This leads to \( q_2 \approx 4 \).

Taking into account that the \( \gamma \)-ray production is proportional to the gas density, for \( \epsilon_\gamma \geq 1 \text{ GeV} \) one can write the relation between the fluxes of \( \gamma \)-rays produced due to the two shocks considered:

\[
F_{\gamma 2}(\epsilon_\gamma) = a F_{\gamma 1}(\epsilon_\gamma) \exp(-\epsilon_\gamma/\epsilon_{\gamma \text{max2}}).
\]

Here the factor \( a \) is determined by the expression

\[
a = \frac{q_2}{q_1} \left( \frac{N_{g2}}{N_{g1}} \right)^{1.5} \left( \frac{10 c}{V_s} \right)^{0.3}
\]

and the \( \gamma \)-ray cutoff energy is \( \epsilon_{\gamma \text{max2}} \approx 0.1 p_{\text{max2}} c \) since on average the energy of the \( \gamma \)-rays resulting from inelastic proton–proton collisions is about one-tenth of the proton energy: \( \epsilon_\gamma \sim 0.1 p_c \).

Substituting the values of the SN shock speed \( V_s \approx 5000 \text{ km s}^{-1} \) (Katsuda et al. 2010; Völk et al. 2008), the number density for the warm phase of the ISM \( N_{g1} = 0.25 \text{ cm}^{-3} \), as well as suitable fit parameter values for the cold phase of the ISM in the form of \( N_{g2} \approx 23 N_{g1} = 6 \text{ cm}^{-3} \) and \( F_2 = 0.005 \), results in \( \epsilon_{\gamma \text{max2}} \approx 10^{-3} \epsilon_{\gamma \text{max1}} = 10^3 m_p c \) and \( a \approx 800 \). It is noted here that such parameter values for the cold ISM phase correspond rather well to the results of numerical modeling of the two-phase ISM (Inoue et al. 2012).

Then the flux of \( \gamma \)-rays produced inside the clouds can be written in the form

\[
F_{\gamma 2}(\epsilon_\gamma) = 4 F_{\gamma 1}(\epsilon_\gamma) \exp(-\epsilon_\gamma/100 \text{ GeV}).
\]

The total \( \gamma \)-ray flux \( F_\gamma = F_{\gamma 1} + F_{\gamma 2} \), expected from Tycho’s SNR for a two-phase ISM, is shown in Figure 1. One can see that it fits the existing data in a satisfactory way. Note that the considerable increase (by a factor of five) of the \( \gamma \)-ray emission at energies \( \epsilon_\gamma < 100 \text{ GeV} \) over and above the case of a purely homogeneous ISM is due to the contribution of clumps which contain only 10% of the ISM mass.

5. DISCUSSION

Small-scale dense clumps of sizes \( l_c \ll 0.1 R_s \) can also be produced by the accelerating CRs themselves within the precursor as the result of the so-called acoustic instability (Dorfi & Drury 1985; Drury 1984; Berezhko 1986).

Small-scale bright structures of angular size \( 10'' \) were recently detected in nonthermal X-rays (Eriksen et al. 2011). For a source distance \( d = 3.8 \text{ kpc} \) the corresponding spatial size is \( l \approx 0.2 \text{ pc} \), which would be roughly consistent with the sizes of the expected clumps. However, the acceleration in such clouds is not expected to reach electron energies in the TeV range which could lead to synchrotron X-ray emission. Therefore these small-scale X-ray structures cannot be considered as an indication that dense gas clumps of size \( l_c \sim 0.1 \text{ pc} \) indeed exist inside Tycho’s SNR. Their existence rather derives from the general properties of the ISM, as discussed above.

The contribution of dense gas clumps in different kinds of emission can be roughly estimated as follows. First, consider the
The expected luminosity of individual clumps is considerably higher (by a factor of about 300) compared with the surrounding diluted gas of the same volume. However, each instrument sees the remnant in projection. Therefore, the expected ratio of X-ray fluxes from the projection volume $V$ to the two gas phases is not an essential factor for the soft X-rays with energies below 2 keV (e.g., Hamilton et al. 1983), we conclude that the soft thermal X-ray emission should be dominated by the contribution of dense gas clumps. In the hard X-ray range above 2 keV, on the other hand, the luminosity $L$ of thermal X-ray emission is small compared to the $f/c$ of the remnant where $L \approx 10$ pc to about $L \approx 25$ at the limb region with $L \approx 1.4$ pc. Therefore we conclude that these clumps could be detected in soft X-rays from the limb regions. In their study with Chandra, Cassam-Chenaï et al. (2007) observed a small contribution of thermal X-rays from the regions occupied by the swept-up ambient gas. They concluded that it is not clear whether the faint lines or other residual emission come from the ejecta or whether they arise from the shocked ambient medium. In line with the idea of the present paper, it is suggested here that this emission comes from small-scale clouds in the surrounding ISM. The difficulty is of course that the X-ray emission of Tycho’s SNR is dominated by the nonthermal X-ray emission. In the context of X-ray emission also the observed irregularities in the blast wave position around the remnant (e.g., Warren et al. 2005) could at least be partly due to the interaction with ambient clouds.

Second, the contribution of dense gas clumps to the synchrotron emission of SNRs is estimated here. From their measurements of the variations in the expansion parameter in the radio range and comparison with X-ray features, Reynoso et al. (1997) suggested the presence of ambient clouds, shocked by the blast wave. Since the synchrotron emission is produced by electrons with energies less than 1 GeV, Equation (7) with $p = m_e c$ is used in order to estimate the ratio of the total synchrotron fluxes originating within the two gas phases: $F_{\text{2g}}/F_{\text{1g}} \propto (F_{\text{2}}/F_{\text{1}})(f_{\text{2}}/f_{\text{1}}) \approx 0.1$. This shows that the contribution of dense gas clumps to the radio emission is expected to be small. The luminosity of individual clumps compared with the neighboring region of the same volume is about 15. Therefore one should be able to detect the clumps with an instrument of corresponding angular resolution from the limb region where the contrast is expected to be $r \approx 2.9$. The synchrotron X-ray energy flux scales as $F_{\nu} \propto K_{\nu} \rho N_{\gamma} V_{s}^{2} \propto F$. Therefore the expected X-ray flux from all the clumps within the SNR, $F_{\text{X2}} \approx (F_{\text{2}}/F_{\text{1}})F_{\text{X1}} \approx 5 \times 10^{-3}F_{\text{X1}}$, is small compared with the total flux $F_{\text{X1}}$.

The dense clumps of size $l_\text{c} \approx 0.1$ pc have an angular size of about $6^\circ$. Structures of such sizes (or even smaller) near the outer shock of Tycho’s SNR have been studied in optical Hα emission (e.g., Ghavamian et al. 2000; Lee et al. 2010). It is difficult to conclude whether the dense clumps can be detected in optical emission, even though the lack of smoothness of the optical filaments points in this direction.

It should also be noted that there are some observations which show that Tycho’s SNR is expanding into an ISM with large-scale density gradients and is possibly interacting with a dense ambient medium toward the northern direction (Lee et al. 2004). If confirmed, this makes the overall picture of this SNR even more complicated. It is nevertheless suggested here that the spherically symmetric approach assuming a uniform ISM on a large scale is roughly valid.

According to the above considerations, the excess of $\gamma$-ray GeV emission above the level expected in the case of a purely homogeneous ISM due to the contribution of small-scale interstellar clouds is inevitably expected in the case of Type Ia SNe situated in a relatively dense ISM inside the Galactic disk. It is not clear whether such an effect is expected in the case of an SNR situated in a much more rarefied ISM, like SN 1006, in the uppermost part of the Galactic gas disk. On a speculative basis, similar effects may also take place in Cassiopeia A, where the observed $\gamma$-ray spectrum (Abdo et al. 2010) looks very similar to that of Tycho’s SNR. The difference would be that in the case of Cassiopeia A numerous observed knots from the SN ejecta might take over the role of pre-existing interstellar clumps.

6. SUMMARY

The $\gamma$-ray spectrum of Tycho’s SNR, consistent with the measurements by Fermi and VERITAS, is proposed to be the superposition of two spectra: the first part, extending to about 100 TeV, is produced by the SN blast wave within the dilute “warm” phase of the ambient ISM, whereas the second part, with a cutoff at about 100 GeV, originates in dense clouds embedded in this warm ISM. The remarkable connection between CR production and the physical nature of the Galactic ISM becomes evident through the characteristics of the spatially integrated $\gamma$-ray emission of the SNR sources.

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