Neutrino masses and baryogenesis in SO(10) unified theories

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Abstract

We report on some phenomenological implications of a class of unified models based on $SO(10)$ gauge group, with intermediate symmetry group containing $SU(2)_R$. Interesting predictions for neutrino masses are discussed, which are relevant both for solar neutrino and dark matter problems, as well as a model for the formation of the baryon asymmetry of the universe required by primordial nucleosynthesis.

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1 Introduction

It is now over twenty years, when the $SU(5)$ model was proposed by Georgi and Glashow, that unification programme has been carried over, looking for a Gauge Unified Theory (GUT) based on a larger symmetry simple group $G$ embedding the standard one, $G \supset G_{321} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, to which it breaks down.

The most peculiar signature of these theories is the non conservation of baryonic number which, in particular, allows for proton instability. For long time the comparison of the predicted proton lifetime with the experimental lower bound has been the only crucial test that GUT’s should satisfy. In this respect the minimal version of the $SU(5)$ model, in which Higgs scalars are classified in \[24 \oplus 5 \oplus \bar{5},\] is at variance with the very precise measurements of the three coupling constants $\alpha_s$, $\alpha_L$ and $\alpha_Y$ of, respectively, $SU(3)_c$, $S(2)_L$ and $U(1)_Y$ at the $Z^0$ mass scale. Actually in 1990-1991 \[1, 2\] it became clear that the three couplings do not match at a single point, and this rules out the minimal $SU(5)$ model.

The big interplay between particle physics and cosmology, as well as the increase in precision of cosmological measurements, open the perspective of using the history of universe as a natural laboratory in which to check the predictions at high energies of classes of unified theories. Items as the production of heavy monopoles, the origin of baryonic matter-antimatter asymmetry or the nature of dark matter in the galactic halos, provide a set of additional constraints which are, sometimes, quite stringent.

In this paper we would like to account for the main features of a class of models based on the $SO(10)$ gauge group. Interestingly, they are able to predict a value for $\tau_{p\rightarrow e^+\pi^0}$ in agreement with the experimental lower limit, as well as neutrino masses of the order of magnitude required to explain solar neutrino problem using MSW \[3\], and to give the status to $\tau$ neutrinos of interesting candidates for the hot component of the dark matter. Moreover, we will discuss how, within $SO(10)$ models, the three conditions necessary to have production of a baryon asymmetry \[4\] appears to be naturally satisfied and a value for the baryon to photon density in agreement with data on primordial nucleosynthesis emerges.
2 SO(10) Gauge Theories

SO(10) has been proposed as a unifying gauge group with three main motivations:

a) models based on this gauge group are naturally anomaly free. What appeared as a chance in SU(5) model, because of their exact compensation due to the use of 10 and 5 to classify fermions, is a general feature of all orthogonal groups, with the only exception of SO(6);

b) the 10 and 5 representations of SU(5) are contained in the 16 (spinorial) representation of SO(10) together with a SU(5) singlet with the quantum numbers of a $\nu_L$;

c) the 16 of SO(10) decomposes under $SU(4) \otimes SU(2) \otimes SU(2)$, the gauge group first introduced by Pati and Salam, into the $(4, 2, 1) + (\bar{4}, 1, 2)$ representation, which displays the quark-lepton universality of weak interactions.

Quite recently other interesting features of the models have been also investigated:

d) the possibility, through the see-saw mechanism, to give cosmologically and astrophysically interesting Majorana mass to neutrinos;

e) the explanation of baryon asymmetry of the universe through out-of-equilibrium decays of heavy Higgs bosons and of low energy sphaleron-like processes.

A study of the low dimensional Irreducible Representations (IR’s) of SO(10) shows that their singlets (with the only exception of the vector-spinor representation 144) have symmetry larger than $G_{321}$. Therefore, at least two of them are necessary to drive the symmetry breaking process of SO(10) down to $G_{321}$, via an intermediate symmetry stage with an unbroken subgroup $G'$, whose energy scale we will denote with $M_R$. Beyond $SU(5)$ (or $SU(5) \otimes U(1)$), there are four intriguing cases which correspond to several $G'$. They are listed in Table I together with the direction of the minimum of Higgs potential.
Table I

|    | \(G'\) | Higgs direction | IR |
|----|--------|-----------------|----|
| A  | \(SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R \times D\) | \(\omega_L = \frac{2}{\sqrt{60}} (\omega_{11} + \ldots + \omega_{66}) - \frac{3}{\sqrt{60}} (\omega_{77} + \ldots \omega_{00})\) | 54 |
| B  | \(SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \times D\) | \(\Phi_L = \frac{\Phi_{1234} + \Phi_{1256} - \Phi_{3456}}{\sqrt{3}}\) | 210 |
| C  | \(SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R\) | \(\Phi_T = \Phi_{7890}\) | 210 |
| D  | \(SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}\) | \(\Phi(\theta) = \cos \theta \Phi_L + \sin \theta \Phi_T\) | 210 |

\(\omega_{ab}\) is a second-rank traceless symmetric tensor; \(\Phi_{abcd}\) is a fourth-rank antisymmetric tensor, and the indices 1...6 correspond to \(SO(6) \sim SU(4)_{PS}\), whereas 7...0 correspond to \(SO(4) \sim SU(2)_L \otimes SU(2)_R\).

For the models in Table I it is possible to show that, at one loop approximation, and within the Extended Survival Hypothesis (ESH), i.e. by only considering scalars in the renormalization group equations (RGE) which are required to drive symmetry breaking at \(M_R\) and at the electroweak scale, the first breaking scale \(M_X\) corresponding to \(SO(10) \rightarrow G'\) is not higher than the meeting point of \(\alpha_3\) and \(\alpha_2\) in the SU(5) minimal model \[7\]. In particular, with two doublets of scalars in the 10 (to avoid the prediction \(m_t = m_b\)), one can get the inequality

\[
M_X \leq M_Z \exp \frac{\pi}{2\alpha(M_Z)} \left( \sin^2 \theta_W(M_Z) - \frac{\alpha}{\alpha_s}(M_Z) \right)
\]

Since the lepto-quarks responsible for proton decay take mass at \(M_X\), this inequality translates into an upper limit for exclusive processes lifetime, since \(\tau_p \propto M_X^4\). A more restrictive bound, by a factor of about \(\frac{4}{3}\), can be obtained using a two-loop approximation \[8\].

It is also worth reminding that the intermediate scale \(M_R\) at which \(G'\) breaks down to the standard group \(G_{321}\) is connected, via the see-saw mechanism \[6\], to Majorana mass for \(\tilde{\nu}_L\). This allows for quite interesting prediction for \(\nu_\tau\) and \(\nu_\mu\) masses. We will come back to this point later on. For the value of \(M_R\), at one loop and still using ESH, it was possible
to find an upper limit for $M_R$

$$M_R \leq M_Z \exp \frac{\pi}{6\alpha(M_Z)} \left[ \frac{3}{2} - 3\sin^2\theta_W(M_Z) - \frac{\alpha}{\alpha_S}(M_Z) \right],$$  \hspace{1cm} (2)

where the equality holds only for the model with $G' \supset SU(4)_{PS} \times D$, whose prediction for $M_X$, however, is too small when compared with experimental lower limit.

Because of the richness of the mass spectrum of $SO(10)$ models, it may seem that ESH is a too restrictive assumption. Actually this observation led Dixit and Sher to claim that huge uncertainties are introduced in the $SO(10)$ predictions if ESH is removed \[^9\]. However, as it has been explicitly shown in a recent paper \[^8\], by carefully studying the mass spectrum, it is possible to deduce rather restrictive conditions on the contributions of the scalars to RGE. In this way, by requiring for $M_X$ a value sufficiently high to be in agreement with the lower limit on $\tau_{p\rightarrow e^+\pi^0}$, it is possible to find upper limits on $M_R$, which correspond to lower limits on the masses of the (almost) left-handed neutrinos.

3 Prediction on symmetry breaking scales

To obtain the values of $M_X$ and $M_R$ it is necessary to study the mass spectrum of scalars that, according to the decoupling theorem of Appelquist-Carrazzone, would contribute to RGE. In the following we quote the results of a detailed study performed on the mass spectrum characterizing the various models of Table I. (for a complete analysis see \[^8\]). The mass of the multiplet $(d_1, ..., d_n)$, transforming as a $d_i$ dimensional IR under the factor $G_i \subset G'$, will be denoted with $m(d_1, ..., d_n)$

3.1 $SU(4) \otimes SU(2) \otimes SU(2) \times D$

In the following we will use the values for $\sin^2\theta_W$, $\alpha_s$ and $\alpha$ at the $M_Z$ mass energy scale \[^10\],

$$\sin^2\theta_W(M_Z) = 0.2315 \pm 0.0002 \pm 0.0003$$ \hspace{1cm} (3)

$$\alpha_s(M_Z) = 0.123 \pm 0.004 \pm 0.002$$ \hspace{1cm} (4)

$$\alpha(M_Z) = 1/(127.9 \pm 0.2)$$ \hspace{1cm} (5)

where the second error on the first two results is due to the uncertainty on the Higgs mass.
For this model we find, at one loop,
\[
M_R = M_Z \exp B = M_{SU(5)} = 5.3 \cdot 10^{13} \text{ GeV}
\]
with \( B \equiv \frac{\pi}{\alpha(M_Z)} \left( \frac{3}{2} - 3 \sin^2 \theta_W(M_Z) - \frac{\alpha_S}{\alpha}(M_Z) \right) \).

Thus, in this case, it is not possible to increase the value of \( M_R \). If one tries to get \( M_X \geq 3.2 \cdot 10^{15} \text{ GeV} \) it is necessary to take into account the request of having minimum in the \( SU(4) \otimes SU(2) \otimes SU(2) \times D \) direction, which gives, still at one loop,
\[
M_X < M_Z e^{\frac{7}{2} (A + 9B + 3 \ln 1.35)} = (2.46 \cdot 10^{15}) \text{ GeV}
\]
with \( A \equiv \frac{6\pi}{\alpha(M_Z)} (\sin^2 \theta_W(M_Z) - \frac{\alpha_S}{\alpha}(M_Z)) \).

The same study can be performed at two loops in RGE. In this case we find (with all the multiplets at the scale \( M_R \) but the \((1,3,3)\) one, for which we have \( m(1,3,3) \leq 1.35 M_R \)):
\[
M_R^{(2)} = 4.9 \cdot 10^{13} \text{ GeV}, \quad M_X^{(2)} = 0.74 \cdot 10^{15} \text{ GeV}
\]
\( M_X \) is too small (about two standard deviations) to comply with the lower limit on proton lifetime.

3.2 \( G' \equiv SU(4)_c \otimes SU(2)_L \otimes SU(2)_R \)

In this case, one obtains for \( M_R \):
\[
\ln \frac{M_R}{M_Z} = B - \frac{5}{44} \left[ 3 \ln \frac{m(15,3,1)}{m(15,1,3)} + 4 \ln \frac{m(10,3,1)}{m(10,1,3)} \right]
\]

The ESH would imply \( m(1,2,2)_{10} \sim M_Z, m(10,1,3) \sim M_R \) and all the other scalars with mass \( \sim M_X \). One would obtain for \( M_R \) (this time \( M_X \) is consistent with the experimental limit)
\[
M_R = 2.3 \cdot 10^{11} \text{ GeV}, \quad M_X = 6.2 \cdot 10^{15} \text{ GeV}
\]

To establish how strongly this result depend on ESH, we look for the highest value for \( M_R \) consistent with a sufficiently high value for \( M_X \) (\( M_X^{(2)} \geq 3.2 \cdot 10^{15} \text{ GeV} \)).

From eqs. (3), by taking \( m(10,3,1) \geq M_R \), in order to have the highest \( M_R \) and the absolute minimum in the \( G_{422} \)-invariant direction one has:
\[
m(10,3,1)_{210} = m(10,1,3)_{210}, \quad m^2(15,3,1)_{210} = \frac{2 - \sqrt{3}}{2 + \sqrt{3}} m^2(15,1,3)_{210},
\]
\[
\frac{m^2(15,3,1)_{210} m^2(15,1,3)_{210}}{m^4(15,1,1)_{210}} \leq 9.
\]
From eqs. (11), (12) and (13), one gets the following inequality:

$$M_R \leq 3\pi\frac{1}{3}M_Z \left( \frac{M_Z}{M_X} \right)^{\frac{10}{3}} e^{\frac{\pi}{\alpha(M_Z)} \left( \frac{3}{\alpha(M_Z)} - \frac{8}{\alpha(M_Z)} \right)} \leq 4.87 \cdot 10^{13} \text{ GeV},$$  \hspace{1cm} (14)

the last one coming from $M_X \geq 3.2 \cdot 10^{15} \text{ GeV}$.

The highest value for $M_R$ is found by taking $(1,2,2)_{10}$ at the scale $M_Z$, the multiplets $(10,2,2)_{210}$, $(6,1,1)_{10}$ and all the states of the 126 at the scale $M_R$ and all the other states of the 210 at the scale $M_X$: in such conditions one finds:

$$M_R^{(2)} = 1.6 \cdot 10^{13} \text{ GeV}, \quad M_X^{(2)} = 3.2 \cdot 10^{15} \text{ GeV}$$  \hspace{1cm} (15)

### 3.3 $G' \equiv SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \times D$

In this case, by keeping into account the $SU(2)_L \leftrightarrow SU(2)_R$ symmetry above $M_R$, ESH would imply $m(1,2,2,0)_{10} \sim M_Z$, $m(1,3,1,-2)_{126} \sim M_R$ and all the other multiplets at the scale $M_X$. One would get at two loops

$$M_R^{(2)} = 5.3 \cdot 10^{10} \text{ GeV}, \quad M_X^{(2)} = 1.98 \cdot 10^{15} \text{ GeV}$$  \hspace{1cm} (16)

a too small value for $\tau_{p \to e^+\nu}^{(2)}$. In considering the contribution of the other scalars, the constraints on the mass spectrum, which follow from the requirement that the absolute minimum falls in the desired direction, implies the following inequalities for the masses

$$\frac{m(8,3,1,0)_{210}}{m(3,3,1,4/3)_{210}} > \sqrt{\frac{37}{14}}, \quad \frac{m(6,2,2,2/3)_{210}}{m(1,2,2,2)_{210}} > \frac{1}{\sqrt{7}},$$  \hspace{1cm} (17)

$$1 < \frac{m(8,3,1,0)_{210}}{m(6,2,2,2/3)_{210}} < \frac{2}{\sqrt{3}}, \quad 1 \leq \frac{m(3,3,1,-2/3)_{126}}{m(3,2,2,4/3)_{126}} \leq 2.$$  \hspace{1cm} (18)

In this way it is found:

$$M_R \leq \left( \frac{2 \cdot 7^{\frac{3}{2}}}{37} \right) \frac{\pi}{3} M_{SU(5)}^{\frac{44}{M_X}}$$  \hspace{1cm} (19)

$$M_R \leq M^\frac{3}{5} Z M^\frac{9}{2} X e^{\frac{\pi}{\alpha(M_Z)} \left( \frac{3}{\alpha(M_Z)} - \frac{8}{\alpha(M_Z)} \right)}$$  \hspace{1cm} (20)

where the equality holds only in the extreme case with $m(8,1,1,0)_{210}$, $m(6,3,1,2/3)_{126}$ and $m(8,2,2,0)_{126}$ at the scale $M_R$ and $m(3,2,2,4/3)_{126}$ at the scale $M_X$ for eq. 19, and $m(3,2,2,-2/3)_{210}$, $m(3,3,1,-2/3)_{126}$ at the scale $M_R$ and $m(1,2,2,2)_{210}$, $m(8,1,1,0)_{210}$ and $m(3,1,1,-2/3)_{126}$ at the scale $M_X$ for eq. 20. The two requirements may not be
satisfied at the same time, since they imply a different scale for $m(8, 1, 1, 0)_{210}$ and disagree with eq. (18), which implies that the concerned masses are at the same scale.

By eliminating $M_X$ in eqs. (19) and (20) one finds the inequality

$$M_R < 3.8 \cdot 10^{11} \text{ GeV}. \quad (21)$$

It would be possible, of course, to get a lower bound for $M_R$ since the one just written has been obtained by multiplying inequalities which cannot be both equalities. So it is not surprising that, by looking for the highest value for $M_R$ consistent with $M_X^{(2)} \geq 3.2 \cdot 10^{15} \text{ GeV}$ and with the constraints on the spectrum following, we find

$$M_R^{(2)} = 2.7 \cdot 10^{10} \text{ GeV}. \quad (22)$$

### 3.4 $G' \equiv SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

In the ESH limit

$$M_R^{(2)} = 5 \cdot 10^9 \text{ GeV} \quad M_X^{(2)} = 1.3 \cdot 10^{16} \text{ GeV}. \quad (23)$$

Due to the complexity of the conditions, we have not been able to deduce, as in the previous cases, intriguing inequalities for the one-loop equations, and a lengthy numerical analysis has been needed to get the highest value for $M_R$ [8]. As a result, we found with the scalars of the $126 \oplus \overline{126}$ at the scale $M_R$ and with the scalars of the 210 at the scale $M_X$

$$M_R^{(2)} \leq 0.48 \cdot 10^{11} \text{ GeV} \quad (24)$$

### 4 Baryogenesis

As shown by Sakharov [4], three conditions should be satisfied to produce the baryon asymmetry, which is required in order to explain the result on baryon to photon density $\eta_B$ in the universe, coming from primordial nucleosynthesis data, $\eta_B \sim (3 \div 4) \cdot 10^{-10}$

1) baryon number $(B)$ violating interactions;

2) $C$ and $CP$ violation;

3) non equilibrium conditions.
Because of 1), it was soon realised that GUT theories may be the natural framework for baryon asymmetry generation. In the standard scenario this asymmetry is produced by out of equilibrium decays of heavy Higgs scalars or gauge bosons into light fermions \[1\]. However it was pointed out by several authors \[12\] that anomalous $B + L$ violating processes ($L$ is lepton number) mediated by sphaleronic $SU(2)_L \otimes U(1)_Y$ gauge and Higgs configurations at low energy scale can almost completely wash out any asymmetry in $B$ or $L$ produced at GUT scales. Nevertheless, it is worth observing that these kind of effects cannot affect an asymmetry produced for $B - L$, since the corresponding current is anomaly free. A possible scenario for the production of $\eta_B$ is therefore based on the idea that an asymmetry in this quantum number is produced at GUT scales and then it is eventually transformed into $B$ and $L$ asymmetries at low scales via the *shuffling* effect of sphaleronic configurations.

This possibility has been studied in the framework of $SO(10)$ models in \[13\], to which we refer for details. Notice that for minimal $SU(5)$ model, $B - L$ represents an accidental global symmetry and, therefore, no corresponding asymmetry can be produced in this case.

An interesting feature of $SO(10)$ models is the possibility to define a charge conjugation operator $C$ whose corresponding symmetry remains unbroken even after $M_X$ if $G'$ has rank 5 \[14\]. Notice that all models considered in the previous sections are just of this kind. It follows that any asymmetry in $C$-odd quantum number, as $B - L$, can only be produced after $C$ is broken, i.e. when $G'$ is spontaneously broken down to the standard model at $M_R$. Interestingly enough, this scale is also the one at which $B - L$ is no more a symmetry of the vacuum, what allows, if $CP$ violating effects are also present, for the production of microscopic asymmetries $\delta_{B-L}$ in decay or scattering processes. In particular we have considered decays of heavy Higgs bosons $\Phi$ in channels containing massive $\bar{\nu}_L$. Tipically one expects for $\delta_{B-L}$ values of the order

$$\delta_{B-L} \sim h^2 \frac{M^2_\nu}{M^2_\Phi} \epsilon_{CP}$$

where $h$ denotes the order of magnitude of the Yukawa couplings which are involved in the process, $M_\nu$ are the Majorana neutrino masses, $M_\Phi$ is the mass of the decaying particle and finally $\epsilon_{CP}$ parametrizes the magnitude of $CP$ violation. However, this microscopic asymmetry traduces into a macroscopic one only if decay processes occur out of equilib-
rium. If this is not the case, in fact, inverse decay processes and scattering mediated by virtual propagating $\Phi$, would produce an opposite value for $\delta_{B-L}$. Actually, from a thermodynamical point of view, no asymmetry corresponding to non exactly conserved quantum numbers can be generated in equilibrium conditions, since maximal entropy is obtained for vanishing corresponding chemical potentials.

We remind that a quite accurate indications of whether a decay process occurs in equilibrium or not is to compare the corresponding width $\Gamma$ to the current value of the Hubble parameter $H$ when the decaying massive particle becomes non-relativistic ($T \sim M_\Phi$). In particular out of equilibrium conditions corresponds to $\Gamma \lesssim H$. In radiation dominated epoch, it holds $H \sim \sqrt{g} T^2/M_{Pl}$ with $T$ the temperature, $m_{Pl}$ the Planck mass and $g$ the number of effective degrees of freedom ($g \sim 100$ at GUT scales). A more sophisticated analysis, using Boltzmann equations gives, in general, similar results \cite{11}. The most natural candidates to produce a $B-L$ asymmetry are the multiplets in the 126 IR which takes mass at $M_R$ and can decay in $\bar{\nu}_L$ and a fermion $f$ pairs. However, because of the quite low value of $M_R$ compared with Planck mass, these decays occurs in equilibrium since

$$\Gamma(126 \rightarrow \bar{\nu}_L f) = \frac{h^2}{32\pi} M_R > H(T = M_R) \sim 10 \frac{M_R}{M_{Pl}}$$

unless $h$ is chosen unnaturally small.

For this reason, it seems that more suitable candidates for the $\Phi$ we are looking for, should have larger mass than $M_R$, taken at the first symmetry breaking stage at $M_X$. In addition they also have to be weakly coupled in order to survive down to $M_R$, at which $B-L$ violating processes become possible. A natural candidate satisfying the above conditions are the Higgs bosons in the 210 IR. They cannot be coupled to pair of fermions at tree level because 210 IR is not contained in the product $16 \times 16$. Moreover effective terms in the lagrangian at higher order in perturbation theory are also absent if the intermediate group $G'$ has rank 5. Since the fast decay channel in fermions pairs is forbidden, it has been shown in \cite{13}, for the case $G' = SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$, that the most efficient channel is $210 \rightarrow 126 f f$, whose rate is sufficiently small to have an out of equilibrium overabundant population of these bosons at $M_R$. When these decays eventually

\footnote{To evaluate the residual density as a function of temperature it is also necessary to take into account annihilation processes. A study of these effects has been considered in details in \cite{14}, showing that they are frozen out and unable to strongly reduce the 210 population at $M_R$ if their mass is larger than $10^{12}$ GeV, as it can be expected since they take mass at $M_X \geq 3.2 \cdot 10^{15}$ GeV.}
occur, for temperature below this scale, the particular channels \(210 \rightarrow 126 \bar{\nu}_L f\) produce a microscopic \(B - L\) asymmetry \(\delta_{B-L}\) of the order of magnitude reported in \(\text{(23)}\). The macroscopic value for \(B - L\) asymmetry, \(\Delta(B - L)\), in particular the ratio \(\Delta(B - L)/s\), where \(s\) is the specific entropy, can be evaluated by considering the dilution effect due to entropy release of decaying heavy bosons to radiation degrees of freedom \(\text{(11)}\).

\[
\frac{\Delta(B - L)}{s} \sim h_{10}^2 \frac{M_\nu^2}{M_\Phi^2} \frac{T_{RH}}{M_\Phi} \quad \text{(27)}
\]

where \(h_{10}\) and \(h_{126}\) are the Yukawa coupling of 10 and 126 IR to fermions, respectively and \(T_{RH}\) is the reheating temperature. This value, due to sphaleronic effects, traduces into a corresponding value for baryon asymmetry \(\Delta B \sim \Delta(B - L)/3\), which for \(M_\Phi\) in the range \(10^{12} \div 10^{14} \text{ GeV}\) gives a result in agreement with the experimental value coming from the results on primordial nucleosynthesis of light nuclei, provided the \(CP\) parameter \(\epsilon_{CP}\) is larger than \(10^{-2}\), which sounds quite reasonable.

### 5 Conclusions

In TABLE II and III we have reported the result on \(M_X\) and \(M_R\) for the models discussed in the previous sections, while in Figure 1 are shown the curves of level, in the plane \(\sin^2 \theta_W - \alpha_s\) corresponding to several values of \(M_R\).

The results of the analysis can be summarized as follows:

- In case \(A\) of Table I the resulting value of \(M_X\), both with and without the ESH hypothesis, is too small in order to comply with the experimental limit on proton lifetime, so this possibility is ruled out from the present determinations of \(\sin^2 \theta_W\) and \(\alpha_s\);
- case \(B\) gives a value for \(M_X\) which is in agreement with \(\tau_{\mu \rightarrow e + \pi^0}\) only if ESH is released. Even in this case, the upper limit on \(M_R\) gives, via the see-saw mechanism, a lower limit on \(\nu_\tau\) mass, namely \(m_{\nu_\tau} \geq 80 \div 140 \text{ eV}\), depending on the values chosen for \(\sin^2 \theta_W\) and \(\alpha_s\), which is too high if compared with cosmological constraints on the age of the universe;
- in case \(C\), in the ESH limit, one has \(m_{\nu_\tau} \sim 15 \text{ eV}\), which is in agreement with the idea that \(\nu_\tau\) gives a relevant contribution to the hot component of dark matter.
By releasing ESH one gets instead the lower limit \( m_{\nu_e} \geq 0.2 \div 0.3 \ eV \) and \( m_{\nu_\mu} \geq (1.8 \div 2.2) \cdot 10^{-5} \ eV \);

- for case \( D \), the ESH analysis gives too high values for neutrino masses, while the lower limit one gets on \( m_{\nu_\mu} \) in the general case, namely \( m_{\nu_\mu} \geq (2 \div 6) \cdot 10^{-3} \ eV \), is just of the right order of magnitude to allow for a solution of solar neutrino problems in the framework of MSW model. The results on \( \tau \) neutrino, \( m_{\nu_\tau} \geq 25 \div 75 \ eV \), are quite large, though the smallest one is still marginally compatible with analysis on dark matter composition and perturbation spectra. Moreover, in this case, we have shown how baryon asymmetry can be efficiently produced by combining \( B-L \) violating processes at high scales and sphaleronic effects at low scales, with a result for \( \eta_B \) which is in agreement with data from primordial nucleosynthesis.

It is worth stressing, to conclude, the strong uncertainties on these predictions coming from the corresponding ones on the gauge couplings at \( M_Z \), as can be seen from Figure 1.
| Table II | First-Loop (ESH) | Second-Loop (ESH) | upper limit for $M_R$ with $M_X \geq 3.2 \cdot 10^{15} \text{GeV}$ and without ESH |
|----------------|------------------|------------------|---------------------------------------------------------------|
| $SU(4) \otimes SU(2) \otimes SU(2) \times D$ | | | |
| $M_X = M_Z \exp \left[ \frac{\pi (0.78 + 1.08 \sin^2 \theta_W - 3.16 \alpha_s)}{11 \alpha} \right]$ | $1.3 \cdot 10^{15}$ | $\frac{M_X}{3.2 \cdot 10^{16} \text{GeV}} = 0.21$ | $\frac{M_X}{3.2 \cdot 10^{16} \text{GeV}} \leq 0.23$ |
| $M_R = M_Z \exp \left[ \frac{\pi (1.5 - 3 \sin^2 \theta_W - \frac{1}{11 \alpha})}{10 \alpha} \right]$ | $5.4 \cdot 10^{13}$ | $\frac{M_R}{10^{11} \text{GeV}} = 490$ | |
| $SU(3) \otimes SU(2) \otimes SU(2) \otimes U(1) \times D$ | | | |
| $M_X = M_Z \exp \left[ \frac{\pi (3 \pm 18 \sin^2 \theta_W - 26 \alpha_s)}{16 \alpha} \right]$ | $5.1 \cdot 10^{15}$ | $\frac{M_X}{3.2 \cdot 10^{16} \text{GeV}} = 0.62$ | |
| $M_R = M_Z \exp \left[ \frac{\pi (3 - 12 \sin^2 \theta_W + 4 \alpha_s)}{10 \alpha} \right]$ | $1.8 \cdot 10^{10}$ | $\frac{M_R}{10^{11} \text{GeV}} = 0.53$ | $\frac{M_R}{10^{11} \text{GeV}} \leq 0.27$ |
| $SU(4) \otimes SU(2) \otimes SU(2) \otimes U(1)$ | | | |
| $M_X = M_Z \exp \left[ \frac{\pi (1.5 + 15 \sin^2 \theta_W - 19 \alpha_s)}{4 \alpha} \right]$ | $8.7 \cdot 10^{15}$ | $\frac{M_X}{3.2 \cdot 10^{16} \text{GeV}} = 1.93$ | |
| $M_R = M_Z \exp \left[ \frac{\pi (10.5 - 36 \sin^2 \theta_W + 8 \alpha_s)}{47 \alpha} \right]$ | $7.7 \cdot 10^{11}$ | $\frac{M_R}{10^{11} \text{GeV}} = 2.3$ | $\frac{M_R}{10^{11} \text{GeV}} \leq 163$ |
| $SU(3) \otimes SU(2) \otimes SU(2) \otimes U(1)$ | | | |
| $M_X = M_Z \exp \left[ \frac{\pi (\sin^2 \theta_W - \frac{1}{2 \alpha})}{2 \alpha} \right]$ | $4.1 \cdot 10^{16}$ | $\frac{M_X}{3.2 \cdot 10^{16} \text{GeV}} = 4.06$ | |
| $M_R = M_Z \exp \left[ \frac{\pi (12 - 51 \sin^2 \theta_W + 19 \alpha_s)}{34 \alpha} \right]$ | $1.4 \cdot 10^9$ | $\frac{M_R}{10^{11} \text{GeV}} = 0.05$ | $\frac{M_R}{10^{11} \text{GeV}} \leq 0.48$ |

The values of $M_X$ and $M_R$ in the case $\sin^2 \theta_W = 0.2315 \pm 0.0002$ and $\alpha_s = 0.123 \pm 0.004$. When $M_X$ is less than the lower limit we omit to write the upper limit on $M_R$. 


The same as Table II but with \( \sin^2 \theta_W = 0.2302 \pm 0.0005 \) and \( \alpha_s = 0.117 \pm 0.008 \)
The figure shows the curves of level at $M_R = 10^9, 10^{10}, 10^{11}, 10^{12}, 5.4 \cdot 10^{13}$ GeV for the models of Table I. The star indicates the value $M_R^{sup} = 5.4 \cdot 10^{13}$.

Note that in case A only the upper value appears in the figure because the others are too high.

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