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QUANTUM MECHANICAL INTERACTION-FREE MEASUREMENTS

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A novel manifestation of nonlocality of quantum mechanics is presented. It is shown that it is possible to ascertain the existence of an object in a given region of space without interacting with it. The method might have practical applications for delicate quantum experiments.

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1. INTRODUCTION

Nonlocality is an intriguing aspect of quantum mechanics. Bell’s inequality\(^1\) showed that nonlocality must exist, and Aspect\(^2\) provided an experimental proof. We shall present here yet another manifestation of the nonlocality of quantum mechanics. We shall describe a measurement which, when successful, is capable of ascertaining the existence of an object in a given region of space, though no particle and no light “touched” this object. This is a new type of an interaction-free quantum measurement which has no classical analog.

Let us begin with a brief review of nonlocal measurements which yield information about the existence of an object in a given region of space.

If an object is charged or has an electric (magnetic) moments, then its existence in a given region can be inferred without any particle passing through that region, but rather by the measurement of the electric (magnetic) field the object creates outside the region. Quantum mechanics allows inferring the existence of an object in a nonlocal way via Aharonov-Bohm effect\(^3\) even when the object creates no electromagnetic field outside a certain space region, but only an electromagnetic potential.

Even if the object creates no detectable change at a distance, i.e., it interacts with the external world only locally, its location can often be found in a simple nonlocal interaction-free measurement (i.e., without interacting with the object). For example, assume it is known that an object is located in one out of two boxes. Looking and not finding it in one box tells us that the object is located inside the other box. A more sophisticated example of obtaining information in a nonlocal way is the measurement performed on a system prepared in the Einstein-Podolsky-Rosen state. If two objects are prepared in an eigenstate of relative position, the measurement of the position of one object yields the position
of the other.

In the above cases, what allowed us to infer that an object is located in a given place by performing an interaction-free measurement was the information about the object prior to the measurement. In the first example we knew that the object is located inside one of the two boxes, and in the second example we knew about the correlation between the position of one object and that of another. The question we address in this Letter is this: Is it possible to obtain knowledge about the existence of an object in a certain place using interaction-free measurements without any prior information about the object? The answer is, indeed, in the affirmative as we proceed to show.

Our method is based on a particle interferometer which is analogous to the Mach-Zehnder interferometer of classical optics. In principle, it can work with any type of particles. A particle reaches the first beam splitter which has the transmission coefficient \( \frac{1}{2} \). The transmitted and reflected parts of the particle’s wave are then reflected by the mirrors in such a way that they are reunited at another, similar beam splitter (Fig. 1). Two detectors collect the particles after they pass through the second beam splitter. We can arrange the positions of the beam splitters and the mirrors so that, due to the destructive interference, no particles are detected by one of the detectors, say \( D_2 \) (but all are detected by \( D_1 \)). If, without changing the positions of the mirrors and the beam splitters, we block one of the two arms of the interferometer, the particles which succeeded to pass through the interferometer are detected with equal probability by both detectors \( D_1 \) and \( D_2 \). Thus, detector \( D_2 \) detects particles only if something stands on the way of particles in one of the routes of the interferometer.

A practical realization of such an interferometer with electrons and protons is hampered by strong electromagnetic interaction with the environment, but neutron interferometers operate in many laboratories\(^4\). However, our method
requires a *single particle interferometer*, i.e. an interferometer with one particle passing through it at a time, and there is no appropriate neutron source which produces a single particle states. Recently experiments were performed with a source of single photon states. Thus we propose to use the Mach-Zehnder interferometer with such a source of single photons.

2. **HOW TO FIND AN OBJECT WITHOUT INTERACTING WITH IT?**

Our procedure for finding out about the existence of an object in a given place, without passing even one photon through it, is as follows: We arrange a photon interferometer as described above, i.e. no photons are detected by $D_2$ when both routes of the interferometer are open, and position it in such a way that one of the routes of the photon passes through the region of space where we want to detect the existence of an object (Fig. 1). We send a single photon through the system. There are three possible outcomes of this measurement:

i) no detector clicks,

ii) detector $D_1$ clicks,

iii) detector $D_2$ clicks. In the first case, the photon has been absorbed (or scattered) by the object and never reached the detectors. The probability for this outcome is $\frac{1}{2}$. In the second case (the probability for which is $\frac{1}{4}$), the measurement has not succeeded either. The photon could have reached $D_1$ in both cases: when the object is, and when the object is not located in one of the arms of the interferometer. In this case there has been no interaction with the object so we can try again. Finally, in the third case, when the detector $D_2$ clicks (the probability for which is $\frac{1}{4}$), we have achieved our goal: we know that there is an object inside the interferometer without having “touched” the object. Indeed, we saw that the necessary condition for $D_2$ to detect a photon is that
one of the routes of the interferometer is obstructed; therefore the object must be there. This is an interaction-free measurement because we had only one photon and has it interacted with the object, it could never reach detector $D_2$.  

The quantum mechanical formalism describing the operation of our device is simple. Let us designate the state of the photon moving to the right by $|1\rangle$, and the state of the photon moving up by $|2\rangle$. In a model which illustrates the essential aspects of the procedure every time a photon is reflected the phase of its wave function changes by $\frac{\pi}{2}$. Thus, the operation of the half-silvered plate on the state of the photon is

$$
|1\rangle \rightarrow \frac{1}{\sqrt{2}}[|1\rangle + i|2\rangle], \\
|2\rangle \rightarrow \frac{1}{\sqrt{2}}[|2\rangle + i|1\rangle].
$$

(1)

The operations of the two fully-silvered mirrors are described by

$$
|1\rangle \rightarrow i|2\rangle,
$$

(2a)

and

$$
|2\rangle \rightarrow i|1\rangle.
$$

(2b)

If the object is absent, i.e. we have a standard (undisturbed) photon interferometer, the evolution of the photon’s state is described by:

$$
|1\rangle \rightarrow \frac{1}{\sqrt{2}}[|1\rangle + i|2\rangle] \rightarrow \frac{1}{\sqrt{2}}[i|2\rangle - |1\rangle] \rightarrow \frac{1}{2}[i|2\rangle - |1\rangle] - \frac{1}{2}[|1\rangle + i|2\rangle] = -|1\rangle.
$$

(3)

The photon, therefore, leaves the interferometer moving to the right towards detector $D_1$, which then clicks. If, however, the object is present, the evolution
is described by:

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}[|1\rangle + i|2\rangle] \rightarrow \frac{1}{\sqrt{2}}[i|2\rangle + i|\text{scattered}\rangle] \rightarrow \frac{1}{2}[i|2\rangle - |1\rangle] + \frac{i}{\sqrt{2}}|\text{scattered}\rangle,$$

where $|\text{scattered}\rangle$ is the state of the photon scattered by the object. According to the standard approach to quantum measurement, the detectors cause the collapse of the quantum state (4):

$$\frac{1}{2}[i|2\rangle - |1\rangle] + \frac{i}{\sqrt{2}}|\text{scattered}\rangle \rightarrow \begin{cases} |2\rangle, & D_2 \text{ clicks, } \text{probability } \frac{1}{4}, \\ |1\rangle, & D_1 \text{ clicks, } \text{probability } \frac{1}{4}, \\ |\text{scattered}\rangle, & \text{no clicks, } \text{probability } \frac{1}{2}. \end{cases}$$  

(5)

We see that the photon can be detected by detector $D_2$ only if the object is present. Thus, the click of the detector $D_2$ yields the desired information, namely, that the object is located somewhere along the arms the interferometer. If we wish to specify by the interaction-free procedure an exact position of the object inside the interferometer, we can test (locally) that all but that region inside the interferometer is empty.

The information about the existence of the object was obtained without “touching” it. Indeed, we had a single photon. Had it been scattered or absorbed (i.e. “touched”) by the object it would not be detected by $D_2$. Our procedure is, therefore, an interaction-free measurement of the existence of the object.

The argument which claims that this is an interaction-free measurement sounds very persuasive, but is, in fact, an artifact of a certain interpretation of quantum mechanics. (The interpretation which is usually adopted in discussions of Wheeler’s delayed-choice experiment.) The paradox of obtaining information without interaction appears due to the assumption that only one “branch” of a quantum state exists. This paradox can be avoided in the framework of the
Many-Worlds Interpretation\textsuperscript{8} (MWI) which, however, has paradoxical features of its own. In the MWI there is no collapse and all “branches” of the photon’s state (5) are real. These three branches correspond to three different “worlds”. In one world the photon is scattered by the object and in two others it does not. Since all worlds take place in the physical universe we cannot say that nothing has “touched” the object. We get information about the object without touching it in one world but we “pay” the price of interacting with the object in the other world.

3. INTERACTION-FREE COLLAPSE OF A QUANTUM STATE

We use here the term “interaction-free measurement” following Dicke\textsuperscript{9}. Simplifying an example presented in his paper, we may consider replacement the object of the previous discussion by a particle being in a superposition of two states

$$\Psi = \alpha |A\rangle + \beta |B\rangle,$$

where $|A\rangle$ is a state in which the particle is located in a small region of space A, and $|B\rangle$ is a state in which the particle is located in a disjoint small region B. Looking (via photons) and \textit{not finding} a particle in the region A is an interaction-free measurement of the existence of the particle in B. The photons passing through the region A are neither scattered nor absorbed by the particle, therefore there is no interaction with the particle in this measurement.

One might think in a loose way that the photons passing through the region A “push” the wave function of the particle out of region A causing its collapse into region B. Consider, however, a modification of this experiment using our procedure for \textit{finding} the particle inside region A without any photon interacting with it. We place the interferometer in such a way that one of the photon’s routes
passes through region A. We send a photon through the interferometer. Then, the quantum state of the photon and the particle becomes:

$$|1\rangle |\Psi\rangle \rightarrow \alpha \left( \frac{1}{\sqrt{2}} (|2\rangle - |1\rangle) + \frac{i}{\sqrt{2}} |\text{scattered}\rangle \right) |A\rangle + \beta |1\rangle |B\rangle. \quad (7)$$

Assuming that detectors cause the collapse of the quantum state, the evolution of the state (the right hand side of Eq.(7)) continues:

R.H.S. of (7) → \[
\begin{cases}
|1\rangle \left[ \frac{2\beta}{\sqrt{\alpha^2 + 4\beta^2}} |B\rangle - \frac{\alpha}{\sqrt{\alpha^2 + 4\beta^2}} |A\rangle \right], & D_1 \text{ clicks, probability } \frac{\alpha^2}{4} + \beta^2, \\
|2\rangle |A\rangle, & D_2 \text{ clicks, probability } \frac{\alpha^2}{4}, \\
|\text{scattered}\rangle |A\rangle & \text{no clicks, probability } \frac{\alpha^2}{2}.
\end{cases}
\]

We see that in the case that detector $D_2$ clicks, the quantum state of the particle collapses into the state $|A\rangle$ and the photon does not “touch” the particle. What we have here is an interaction-free collapse of the quantum state of a particle in the box, not only when the particle is not there, but even when it \textit{is} there.$^{10}$

4. HOW TO TEST A BOMB WITHOUT EXPLODING IT?

The idea of our Letter is most dramatically illustrated in a way which is free from any specific interpretation of quantum theory and any specific meaning of the words “interaction-free”, “without touching”, etc. Consider a stock of bombs with a sensor of a new type: if a single photon hits the sensor, the bomb explodes. Suppose further that some of the bombs in the stock are out of order: a small part of their sensor is missing so that photons pass through the sensor’s hole without being affected in any way, and the bomb does not explode. Is it possible to find out bombs which are still in order?

Of course, we can direct some light at each bomb. If it does not explode it is not good. If it does, it \textit{was} good. But we are interested in finding a good
bomb without destroying it. The trouble is that the bomb is designed in such a way that any interaction with light, even a very soft photon bouncing on bomb’s sensor, causes an explosion. The task therefore seems to be impossible, and in classical physics it surely is. However, our interaction-free quantum measurement yields a solution.

We place a bomb in such a way that its sensor is located in one of the possible routes of the photon inside the interferometer. We send photons one by one through the interferometer until either the bomb explodes or detector \( D_2 \) detects the photon. If neither of the above happens, we stop the experiment after a large number of photons have passed the interferometer. In the latter case we can conclude that this given bomb is not good, and we shall try another one. If the bomb is good and exploded, we shall also start all over again with the next bomb. If, however, \( D_2 \) clicks, then we achieved what we promised: we know that this bomb is good and we did not explode it. Let us see what is the probability for such an outcome.

If the bomb is good, then for the first photon there is the probability of \( \frac{1}{2} \) to explode the bomb and \( \frac{1}{2} \) to reach the second half-silvered mirror through the second route. The photon which reaches the second half-silvered mirror has equal probability to be detected by \( D_1 \) and by \( D_2 \). In case the photon is detected by \( D_2 \) we know that the bomb is good and has not being destroyed. If \( D_1 \) detects the photon we get no information about the bomb, it might be good or bad. (In fact the detector \( D_1 \) can be removed.) Thus, if the bomb is good, we have the following probabilities: \( \frac{1}{4} \) to learn that it is good without destroying it, \( \frac{1}{2} \) to explode it, and \( \frac{1}{4} \) to leave it without getting decisive information. In the latter case we should continue and send the next photon. The probabilities for the next photon are the same. Thus, the good bomb will be found by the second photon with the probability of \( \frac{1}{4} \cdot \frac{1}{4} \), by the third photon with the probability
of $\frac{1}{4}$, $\frac{1}{4} \cdot \frac{1}{4}$, etc. The total probability that a good bomb will be found by this method without being destroyed is, then, $\sum_{n=1}^{\infty} \frac{1}{4^n} = \frac{1}{3}$.

The probability of one third follows from the fact that for each photon

$$\frac{\text{probability of hitting detector } D_2}{\text{probability of hitting the bomb}} = \frac{1}{2}. $$

This ratio, however, can be improved through small modification of the procedure. Let us change the half-silvered plates in the interferometer such that the first mirror is almost transparent and the second one is almost not transparent. The action of the first beam splitter is described, then, by:

$$|1\rangle \rightarrow a|1\rangle + ib|2\rangle,$$

$$|2\rangle \rightarrow a|2\rangle + ib|1\rangle;$$

and the action of the second beam splitter is:

$$|1\rangle \rightarrow b|1\rangle + ia|2\rangle,$$

$$|2\rangle \rightarrow b|2\rangle + ia|1\rangle. $$

where $a$ and $b$ are real and positive, $a^2 + b^2 = 1, a >> b$. Now, it is easy to see, similarly to Eq.(3), that if the bomb is not good (in which case it is transparent) then, the photon starting in the state $|1\rangle$ ends up in the state $-|1\rangle$ and, therefore, detector $D_2$ never clicks. If, however, the bomb is good, then the evolution of the state of the photon passing through the interferometer is:

$$|1\rangle \rightarrow a|1\rangle + ib|2\rangle \rightarrow ia|2\rangle + ib|absorbed\rangle \rightarrow ia[b|2\rangle + ia|1\rangle] + ib|absorbed\rangle.$$

And, due to the collapse, it continues:

R.H.S. of (11) $\rightarrow$ \begin{align*}
|1\rangle, & \quad D_1 \text{ clicks, no explosion} \quad \text{probability } a^4, \\
|2\rangle, & \quad D_2 \text{ clicks, no explosion} \quad \text{probability } a^2b^2, \\
|absorbed\rangle, & \quad \text{no clicks, explosion} \quad \text{probability } b^2.
\end{align*}

(12)
Thus,
\[
\frac{\text{probability of hitting detector } D_2}{\text{probability of hitting the bomb}} = a^2.
\]

Since \(a\) is close to 1, we can test without destroying about a half out of the good bombs. Note that in this case the probability to test the bomb by one photon is small, so we will need many photons, or we can use the same photon over and over again.

In one respect the experiment which tests a bomb without exploding it is easier than the experiment of testing the existence of an object in a given place without touching it. For the latter, in order to ensure that we indeed do not touch the object, we need a single-particle interferometer. We could deduce that no photon scattered by the object because there was only one photon and had it been scattered by the object it would not be detected by \(D_2\). For the experiment with the bomb, however, the source of single particle states is not necessary. We know that no photon had touched it simply by the fact that it did not explode. A weak enough source, which is stopped once detector \(D_2\) clicks, serves our purpose. Even the probability of finding a good bomb remains the same: in the optimal regime about one half of the good bombs are tested without being destroyed.

5. CONCLUSIONS

Our method allows to detect the existence of any unstable system without disturbing its internal quantum state. It might, therefore, have practical applications. For example, one might select atoms in a specific excited metastable state. Let us assume that the atom has very high crossection for absorbing photons of certain energy while it is in one out of several metastable states into which it can be “pumped” by a laser, and that the atom is practically transparent for these photons when it is not in this state. Then, our procedure selects atoms in the
specific state without changing their state in any way.

It is common to think that unlike classical mechanics, quantum mechanics poses severe restrictions on the minimal disturbance of the system due to the measurement procedure. We, however, have presented here an ultimately delicate quantum measurement which is impossible to perform classically. We found that it is possible to obtain certain information about a region in space without any interaction in that region neither in the past nor at present.

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NOTE ADDED IN PROOF

A few preprints of this paper, written at 1991 (TAUP 1865-91) were circulated among our colleagues and gave rise to a number of publications.12–15
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6. There is also a possibility that the object is transparent and the photon's wave function have
passed through the object while changing its phase. The “click” in $D_2$ tells us that there is an
object in the specified region, but only if later we have found that the object is not transparent,
we can claim that this information was obtained without interaction. In Sec. 4 we shall consider
a situation in which we ensure that the measurement is interaction-free by another means.

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10. This experiment gives a clear demonstration of the violation of conservation laws by the
collapse of a quantum state. For example, if the potentials in regions A and B are different,
then the expectation value of the particle’s energy changes though no change happens to the
photon (see also Ref.9). Conservation laws are restored when all branches of the quantum state
are considered together.

11. The MWI presents also a natural explanation why we cannot do better. Consider the world
in which the photon hits the bomb. The world which replaces it in case that the bomb is trans-
parent interfere destructively with the world in which the detector $D_2$ clicks. Since the latter is
completely eliminated it could not have probability larger than that of the former.

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**Fig. 1** Mach-Zehnder type particle interferometer. Detector $D_2$ clicks only if one of the arms of the interferometer is blocked by an object.