Tachyon warm inflationary universe model in the weak dissipation regimen

Ramón Herrera, Sergio del Campo and Joel Saavedra
Instituto de Física, Pontificia Universidad Católica de Valparaíso,
Casilla 4059, Valparaíso, Chile.
E-mail: joel.saavedra@ucv.cl

Abstract. Warm inflationary universe model in a tachyon field theory is studied in the weak dissipation regimen. We develop our model for an exponential potential and a dissipation parameter $\Gamma = \Gamma_0 = constant$. We describe scalar and tensor perturbations for this scenario. Also, we use recent astronomical observations for constraining the parameters appearing in our model.

1. Introduction

It is well known that many long-standing problems of the Big Bang model (horizon, flatness, monopoles, etc.) may find a natural solution in the framework of the inflationary universe model [1, 2]. One of the successes of the inflationary universe model is that it provides a causal interpretation of the origin of the observed anisotropy of the cosmic microwave background (CMB) radiation and also the distribution of large scale structures [3].

Warm inflation is an alternative mechanism for having successful inflation and avoiding the reheating period [4]. In warm inflation, dissipative effects are important during inflation, so that radiation production occurs concurrently with the inflationary expansion. The dissipating effect arises from a friction term which describes the processes of the scalar field dissipating into a thermal bath via its interaction with other fields. Also, warm inflation shows how thermal fluctuations during inflation may play a dominant role in producing the initial perturbations. In such models, the density fluctuations arise from thermal rather than quantum fluctuations [5].

On the other hand, implications of string/M-theory to Friedmann-Robertson-Walker (FRW) cosmological models have recently attracted great attention, in particular, those related to brane-antibrane configurations such as space-like branes[6]. The tachyon field associated with unstable D-branes might be responsible for cosmological inflation in the early evolution of the universe, due to tachyon condensation near the top of the effective scalar potential which could also add some new form of cosmological dark matter at late times [7].

Usually, in any model of warm inflation, the scalar field which drives inflation is the standard inflaton field. As far as we know, a model in which warm inflation is driven by a tachyonic scalar field has not yet been studied in the weak dissipation regimen.

In section 2, the dynamics of the tachyon warm inflationary model is obtained. In section 3, the cosmological perturbations are investigated. Finally, in section 4, we give some conclusions.
2. Tachyon Warm Inflationary Model
As was noted by Gibbons [8], the energy density, \( \rho_\phi \), and pressure, \( p_\phi \), associated with the tachyon field are defined by \( \rho_\phi = V(\phi)/\sqrt{1 - \dot{\phi}^2} \) and \( p_\phi = -V(\phi)/\sqrt{1 - \dot{\phi}^2} \), respectively. Here, \( \phi \) denotes the tachyon field (with unit \( 1/m_p \), where \( m_p \) represents the Planck mass) and \( V(\phi) = V \) is the effective potential associated with this tachyon field. The potential is one that satisfies \( V(\phi) \rightarrow 0 \) as \( \phi \rightarrow \infty \). It has been argued that the qualitative tachyonic potential of string theory can be described via an exponential potential of the form [6]

\[
V(\phi) = V_0 e^{-\alpha \phi},
\]

where \( \alpha \) and \( V_0 \) are free parameters. In the following we will take \( \alpha > 0 \) (with unit \( m_p \)).

The dynamics of the FRW cosmological model in the warm inflationary scenario, is described by the equations

\[
H^2 = \kappa [\rho + \rho_\gamma] = \kappa \left[ \frac{V}{\sqrt{1 - \dot{\phi}^2}} + \rho_\gamma \right],
\]

and

\[
\frac{\ddot{\phi}}{(1 - \dot{\phi}^2)} + 3H \dot{\phi} + \frac{V_\phi}{V} = -\frac{\Gamma}{V} \sqrt{1 - \dot{\phi}^2} \dot{\phi},
\]

where \( H = \dot{a}/a \) is the Hubble factor, \( a \) is a scale factor, \( \rho_\gamma \) is the energy density of the radiation field and \( \Gamma \) is the dissipation coefficient, with unit \( m_p^2 \). The dissipative coefficient is responsible for the decay of the tachyon scalar field into radiation during the inflationary regime. In general, \( \Gamma \) can be assumed as a function of \( \phi \), and thus \( \Gamma = f(\phi) > 0 \) by the second law of thermodynamics. Dots mean derivatives with respect to time, \( V_\phi = \partial V(\phi)/\partial \phi \) and \( \kappa = 8\pi/(3m_p^2) \).

During the inflationary era the energy density associated with the tachyonic field is the order of the potential, i.e. \( \rho_\phi \sim V \), and dominates over the energy density associated with the radiation field, i.e. \( \rho_\phi > \rho_\gamma \).

With \( \Gamma = \Gamma_0 = const. \) and using the exponential potential given by Eq.(1), we find that the slow roll parameter become

\[
\varepsilon = -\frac{\dot{H}}{H^2} = \frac{1}{6\kappa} \left[ \frac{V_\phi}{V} \right]^2 \frac{1}{V} = \frac{1}{6\kappa} \frac{\alpha^2}{V_0 e^{-\alpha \phi}}.
\]

Assuming the set of slow-roll conditions, \( \dot{\phi}^2 \ll 1 \), and \( \phi \ll 3H(1 + r)\dot{\phi} \sim 3H\dot{\phi} \), the Hubble parameter is given by \( H(\phi) = \sqrt{\kappa V_0 e^{-\alpha \phi}/2} \), where the rate \( r \) becomes

\[
r = \frac{\Gamma}{3HV} = \frac{m_p \Gamma_0}{\sqrt{24\pi}} \frac{1}{V_0^{3/2}} e^{3\alpha \phi/2} < 1,
\]

and parameterizes the dissipation of our model. For the weak (or high) dissipation regimen, \( r < 1 \) (or \( r \gg 1 \)).

The evolution of the \( \dot{\phi} \) during this scenario is governed by the expression \( \dot{\phi} = -V_\phi/3HV \). In the following, the subscripts \( i \) and \( f \) are used to denote the beginning and the end of inflation.

Using Eq. (2), the total number of e-folds at the end of warm inflation results as

\[
N_{\text{total}} = -3\kappa \int_{\phi_i}^{\phi_f} \frac{V^2}{V} d\phi = \frac{3\kappa}{\alpha^2} [V_i - V_f],
\]
where the initial tachyonic field satisfies $\phi_i < \phi_f$, since $V_i > V_f$. Rewriting the total number of e-folds in terms of $V_f$ and $V_i$, and using that $\varepsilon_f \approx 1$, we find $V_i = (2N_{total} + 1)V_f$. Since, the $N_{total}$ parameter could assume appropriate values (60 or so) for solving the standard cosmological puzzles. To do that, we need the following inequality to be satisfied: $V_i > 10^2 V_f$.

3. The Perturbations

In this section we will describe scalar perturbations in the longitudinal gauge, and then we will continue describing tensor perturbations.

By using the longitudinal gauge in the perturbed FRW metric, we write

$$ds^2 = (1 + 2\Phi)dt^2 - a(t)^2(1 - 2\Psi)dx^i dx^j,$$

where $\Phi = \Phi(t, x)$ and $\Psi = \Psi(t, x)$ are gauge-invariant variables introduced by Bardeen [9]. In momentum space, for the Fourier components $e^{ikx}$, with $k$ being the wave number. We obtain a set of exact equations which are derived from the perturbed Einstein field equations (we omit this equations).

Since what we need are the non-decreasing adiabatic and isocurvature modes on large scale $k \ll aH$, (which turn out to be weak time dependent quantities), without loss of generality we may consistently neglect $\dot{\Phi}$ and those terms containing two-times derivatives and, combining with the slow roll conditions, the equations for $\Phi$, $\delta \phi$, $\delta \rho_\gamma$, and $v$ becomes (we omit the subscript $k$ here)

$$\Phi \simeq \frac{4\pi}{m_p^2} \left( \frac{V \dot{\phi}}{H} \right) \left[ 1 + \frac{\Gamma}{4HV} + \frac{\Gamma, \phi \dot{\phi}}{48H^2V} \right] \delta \phi,$$

$$\left[ 3H + \frac{\Gamma}{V} \right] (\delta \phi) + \left[ (\ln(V), \phi) + \frac{\Gamma}{V} \right] \delta \phi \simeq \left[ \frac{\Gamma}{V} - 2(\ln(V)), \phi \right] \Phi,$$

$$\delta \rho_\gamma \simeq \frac{\dot{\phi}^2}{4H} \left[ \Gamma, \phi \delta \phi - 3\Gamma \Phi \right] \Rightarrow \frac{\delta \rho_\gamma}{\rho_\gamma} \simeq \frac{\Gamma, \phi \delta \phi - 3\Phi}{\Gamma, \phi},$$

and

$$v \simeq -\frac{k}{4aH} \left[ \Phi + \frac{\delta \rho_\gamma}{4\rho_\gamma} + \frac{3\Gamma \dot{\phi}}{4\rho_\gamma} \delta \phi \right],$$

respectively. where $v$ appears from the decomposition of the velocity field $\delta u_j = -\frac{ia_k}{k^2} \gamma e^{ikx}$ ($j = 1, 2, 3$) [9].

Note that in the case of the scalar perturbations the tachyon and the radiation fields are interacting. Therefore, isocurvature (or entropy) perturbations are generated, besides the adiabatic ones. This occur because warm inflation can be considered as an inflationary model with two basics fields. In this context, dissipative effects themselves can produce a variety of spectral ranging between red and blue [5], thus producing the running blue to red spectral suggested by WMAP three-year data[3].

The above equations can be solved taking $\phi$ as the independent variable instead of $t$. With the help of Eq. (2) we find

$$\left( 3H + \frac{\Gamma}{V} \right) \frac{d}{dt} = \left( 3H + \frac{\Gamma}{V} \right) \phi \frac{d}{d\phi} = -(\ln(V)), \phi \frac{d}{d\phi},$$
and introducing an auxiliary function $\varphi$ given by
\[ \varphi = \frac{\delta \phi}{\langle \ln(V) \rangle_{\phi}} \exp \left[ \int \frac{1}{(3H + \Gamma/V)} \left( \frac{\Gamma}{V} \right) d\phi \right], \]  
(13)
we obtain the following equation for $\varphi$
\[ \frac{\varphi, \phi}{\varphi} = -\frac{9}{8} \frac{(\Gamma/V + 2H)}{(\Gamma/V + 3H)^2} \times \]
\[ \left[ \Gamma + 4HV - \frac{1}{12H(3H + \Gamma/V)} \right] \frac{(\ln(V))_{,\phi}}{V}. \]
(14)
Solving Eq.(14) and using Eq.(13) we find that $\delta \phi = C \langle \ln(V) \rangle_{,\phi} \exp[\Im(\phi)]$, where $C$ is a constant.

In this way, the expression for the density perturbations for $\Gamma = \Gamma_0 = constant$, becomes
\[ \delta_H = \frac{2}{5} \frac{m_p^2}{(\ln(V))_{,\phi}} \exp[\Im(\phi)] \delta \phi. \]
(16)

We noted here that in the case $\Gamma = 0$, Eq.(16) is reduced to $\delta_H \sim V \delta \phi / (\dot{\phi}) \sim H \delta \phi / \dot{\phi}$, which coincides with that expression obtained in cool inflation.

The fluctuations of the tachyon field are generated by thermal interaction with the radiation field, instead of quantum fluctuations. We may write in the case $r < 1$, that $\langle \delta \phi \rangle^2 \simeq H T_\gamma / 2 m_p^4 \pi^2$, where $T_\gamma$ is the temperature of the thermal bath.

The generation of tensor perturbation during inflation would produce stimulated emission in the thermal background of the gravitational wave [10]. From expression (16), we may write the tensor-scalar ratio $R(k) = (A_g^2 / P_T)$ as
\[ R(k_0) = \frac{240\sqrt{3}}{25m_p^2} \left[ \frac{\varepsilon}{T_\gamma^2} e^{2\Im(\phi)} \coth \left( \frac{k}{2T_\gamma} \right) \right]_{k=k_0}, \]
(17)
where we have used that $\delta_H = 2P_{TT} / 5$, $A_g^2 = 32V \coth[k/2T_\gamma] / (3m_p^4)$ and $k_0$ is referred to as the pivot point.

From the combination of WMAP three-year data[3] with the SDSS large scale structure surveys, there is found an upper bound $R(k_0=0.002 \text{ Mpc}^{-1}) < 0.28(95\%CL)$, where $k_0=0.002 \text{ Mpc}^{-1}$ corresponds to $l = \tau_0 k_0 \simeq 30$, with the distance to the decoupling surface $\tau_0=14400 \text{ Mpc}$. SDSS measures galaxy distributions at red-shifts $a \sim 0.1$ and probes $k$ in the range $0.016 h \text{ Mpc}^{-1} < k < 0.011 h \text{ Mpc}^{-1}$. The recent WMAP three-year results give the values for the scalar curvature spectrum $P_R(k_0) \equiv 25\delta_H^2(k_0)/4 \simeq 2.3 \times 10^{-9}$ and the scalar-tensor ratio $R(k_0) = 0.095$. Using the WMAP three-year data and choosing the parameters $T \simeq T_\gamma \simeq 0.24 \times 10^{16} \text{ GeV}$ and $k_0 = 0.002 \text{ Mpc}^{-1}$. We obtained from Eqs.(16) and (17), that $V(\phi_0) \sim 10^{-12} m_p^4$ and $\alpha \sim 10^{-10} m_p$.

4. Conclusions
In this paper we have investigated the tachyonic warm inflationary scenario in the weak regimen. Our specific model is described by an exponential scalar potential where the dissipation coefficient, $\Gamma = \Gamma_0 = constant$. In relation to the corresponding perturbations, we found a
general relation for the density perturbation expressed by Eq.(16). The tensor-scalar ratio is modified by a temperature dependent factor via stimulated emission into the existing thermal background (see Eq.(17)).

By using the WMAP three-year data, we have found some constraints for the parameters appearing in our model. For example, the potential becomes of the order of $V(\phi_0) \sim 10^{-12} m_p^4$ when it leaves the horizon, at the scale of $k_0 = 0.002\text{Mpc}^{-1}$.

Dissipative effects plays a crucial role in producing the entropy mode; they can themselves produce a rich variety of spectra ranging between red and blue. The possibility of a spectrum which runs from blue to red is particularly interesting because it is not commonly seen in inflationary models, which typically predict red spectral. Models of inflation with dissipative effects and models with interacting fields have much more freedom to yield spectral. Summarizing, we have been successful in describing tachyon warm inflationary model for describing the early epoch of the universe in the weak dissipation regimen.

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