Study of coupling loss on bi-columnar BSCCO/Ag tapes by a.c. susceptibility measurements

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Coupling losses were studied in composite tapes containing superconducting material in the form of two separate stacks of densely packed filaments embedded in a metallic matrix of Ag or Ag alloy. This kind of sample geometry is quite favorable for studying the coupling currents and in particular the role of superconducting bridges between filaments. By using a.c. susceptibility technique, the electromagnetic losses as function of a.c. magnetic field amplitude and frequency were measured at the temperature $T = 77 \text{ K}$ for two tapes with different matrix composition. The length of samples was varied by subsequent cutting in order to investigate its influence on the dynamics of magnetic flux penetration. The geometrical factor $x_0$ which takes into account the demagnetizing effects was established from a.c. susceptibility data at low amplitudes. Losses vs frequency dependencies have been found to agree nicely with the theoretical model developed for round multifilamentary wires. Applying this model, the effective resistivity of the matrix was determined for each tape, by using only measured quantities. For the tape with pure silver matrix its value was found to be larger than what predicted by the theory for given metal resistivity and filamentary architecture. On the contrary, in the sample with a Ag/Mg alloy matrix, an effective resistivity much lower than expected was determined. We explain these discrepancies by taking into account the properties of the electrical contact of the interface between the superconducting filaments and the normal matrix. In the case of soft matrix of pure Ag, this is of poor quality, while the properties of alloy matrix seem to provoke an extensive creation of intergrowths which can be actually observed in this kind of samples.

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I. INTRODUCTION

BSCCO/Ag tapes usually contain many superconducting filaments embedded in a silver or silver alloy matrix. For achieving thermal stability and quench protection, the metallic matrix with high thermal conductivity should have a good interface with the superconducting material. When the tape is immersed in alternating (a.c.) magnetic field, an energy loss is observed that is caused by various mechanisms. The irreversible magnetisation is an intrinsic property of type II superconductors, and the energy loss related to it is usually called "hysteretic loss". The a.c. magnetic field induces also an electrical field which drives currents in the loops formed by pairs of filaments and closed by crossing the metallic matrix. The ohmic loss generated in the metallic part of the loop is accordingly called "coupling loss". Finally, the currents, which flow in metallic matrix only, give rise to the so called "eddy currents loss". The latter contribution can be often omitted at the power grid frequency. In general, it is desirable to take measures that reduce the different kinds of losses. To reduce the hysteretic losses, the dimensions of superconducting filaments have to be diminished. To cut down the coupling losses, it is essential to reduce the area of induced flux e.g. by twisting the filaments. Moreover, the matrix resistivity should rise, for example by manufacturing an artificial resistive barrier around the filaments. The effective resistivity of the paths that the coupling currents follow generally differs from the bare matrix resistivity, because it depends on the tape geometry and on the particular arrangement of the superconducting filaments. It is often used to characterize the coupling currents by the time constant $\tau$ of magnetic flux penetration. As shown by Kwasnitza, the coupling loss can be expressed as a function of $\omega \tau$ ($\omega = 2\pi \nu$) at constant a.c. magnetic field amplitude ($B_0$). In other words, the coupling loss is well characterized by this time constant. For example, in an application working at frequency $\nu$ the tape should be used that exhibits the time constant such that $2\pi \nu \tau \ll 1$ to avoid the saturation of the filaments just by the induced coupling currents. Existing theories provide formulas for the time constant in a tape with known geometry and matrix resistivity. Unfortunately, in HTS tapes the experimental data on the time constant can be very different from the calculated value because of the irregularities in the interface between the filaments and the matrix. Therefore it is necessary to understand how the coupling loss works in real BSCCO/Ag tapes. Moreover it is necessary to understand what is the metallic matrix
which has the best effective resistivity.

Thorough investigations have been carried out on coupling loss in HTS tapes. However, it is still not evident whether the coupling mechanism in BSCCO/Ag can be analyzed in the same manner as for low Tc, wires. In this paper, this kind of losses has been studied in tapes with a geometry resembling just two filaments separated by a metallic matrix. In reality, each “filament” consists in dense stack of extremely flat filaments. In this way a simple geometry has been achieved, allowing us to study the coupling currents without other spurious effects. We have measured the time constant and determined the effective resistivity for two such bi-columnar tapes that differ significantly in the matrix composition and thus in its resistivity. The external magnetic field \( B \) (related to demagnetizing factor) has been measured. In order to study the effect of the sample length on the losses, the measurements were repeated for the same tape, cut several times in always shorter pieces. In this way we could discuss the various aspects of coupling losses in this system and make a comparison with the most used model in this field. The importance of the intergrows between the superconducting filaments on the coupling losses is also pointed out.

II. THEORETICAL BACKGROUND

The coupling loss per unit volume and per cycle \( Q_c \) for a sinusoidal external field:

\[
Q_c = \frac{B_0^2}{2\mu_0} \left[ 2\pi \chi_0 \left( \frac{\omega \tau}{1 + \omega^2 \tau^2} \right) \right] 
\]  

where \( \chi_0 \) is the demagnetization factor in the actual sample vs. field geometry. The expression in square brackets is independent of magnetic field amplitude. Moreover we can suppose that the frequency dependence predicted by eq. \( Q \) is still valid also in presence of few filaments. In this case we can expect that the eq. \( Q \) has to be corrected by a numerical factor:

\[
Q' = \gamma Q_c 
\]  

On the other hand the a.c. losses are related to the imaginary part of the first harmonic of the a.c. susceptibility, and can be expressed as:

\[
Q = \pi \chi'' \chi_0 B_0^2 / \mu_0 
\]  

where \( \chi'' \) is the imaginary part of the a.c. susceptibility. By supposing that the losses are dominated by coupling mechanism it holds:

\[
\chi'' = \gamma \frac{\omega \tau}{1 + \omega^2 \tau^2} 
\]  

By measuring the loss as a function of frequency \( \nu \) at fixed \( B_0 \), the time constant can be determined by finding the frequency \( \nu_m \) where the maximum occurs, corresponding to \( \omega \tau = 1 \). On the other hand, in the low frequency limit \( Q_c \) depends linearly on \( \tau \) and therefore, \( \tau \) can be evaluated by measuring magnetisation loops for different sweep rates of a small external field. Nevertheless the linear dependence has to be separated from the logarithmic dependence due to the flux creep in the filaments, which has not been considered in the Campbell model. The equation \( Q \) is no more valid when the filaments become saturated by currents, i.e. the current density is equal to the critical current density in the whole section of the filament. In fact for \( B_0 \) values larger than the full penetration field of a single superconducting filament \( B_{pf} \), or as \( B_0 \) becomes quite large, the currents saturate the outer filaments. Then the filaments behave as fully coupled and the coupling loss turns to be hysteretic. Since the coupling currents propagate inside the specimen, the field inside the sample is no more uniform, which was the key assumption for the validity of the equation \( Q \). On the other hand, for increasing \( B_0 \), the magnetic moment of the sample rises. Nevertheless, it cannot exceed the magnetic moment of an equivalent superconducting tape with a single core offering the identical cross section and the same \( J_c \) of the multifilamentary tape. Therefore at high field, the losses of a multifilamentary tape can be found from the area of the hysteresis loop of the equivalent monocoire tape. This upper limit of loss can be exactly evaluated by taking into account the sample geometry. Nevertheless, for \( B_0 \gg B_{ps} \) (where \( B_{ps} \) is
the magnetic field where the saturation in the magnetisation occurs) one can neglect the actual shape of $M(B_a)$ loop and the extremities of $M(B_a)$ loops can be approximate by upright straight lines. Thus, the saturation loss is written as:

$$Q_c = 4\mu_0 M_s B_0$$  \hspace{1cm} (6)$$

where $M_s$ is the saturation magnetisation.

As far as the frequency dependence is concerned, for $B_0 \gg B_{ps}$, as the frequency increases, the losses saturate at a constant value\textsuperscript{13} given by eq. (6), which is, in a first approximation, frequency independent. Also in this case, going more insight, $Q_c$ should be considered as frequency dependent due to the frequency dependence of $M_s$ which is not always negligible in high $T_c$ superconductors\textsuperscript{15}.

**A. Expressions for time constant**

Theoretical expressions for $\tau$ can be found by calculating the electrical field $E$ inside the tape and the power loss density $p = E^2/\rho_m$, where $\rho_m$ is the effective resistivity of the metallic matrix. At low frequencies this quantity can be compared to the expression given by the Campbell model\textsuperscript{13}:

$$p = \chi_0 \tau \frac{\dot{B}^2}{2\mu_0}$$  \hspace{1cm} (7)$$
to extract $\tau$. In literature, expressions for $\tau$ are reported\textsuperscript{13},\textsuperscript{14} for twisted and untwisted flat cable, for external magnetic field either perpendicular or parallel to the broad face of the tape. In our case, the samples are untwisted BSCCO tapes and the expression for $\tau$ is:

$$\tau = \frac{\ell^2 \mu_0}{\pi^2 \chi_0 \rho_m}$$  \hspace{1cm} (8)$$

showing that the time constant depends on the square of the length, on the resistivity and on the tape geometry (represented by $\chi_0$ factor).

If $\chi_0$ is 1 (the value for a slab geometry) the previous expression is identical to the expression for $\tau$ found solving the flux diffusion equation in a slab.

**B. Effective resistivity**

In the expression (8), the effective resistivity depends also on the arrangement of the filaments in the tape. Carr\textsuperscript{15} has shown that, in the continuum anisotropic model, the effective resistivity is defined in terms of the matrix resistivity $\rho_m$ and the superconducting volume fraction $\eta_{eff}$ related to the filamentary zone alone. In BSCCO tape $\eta_{eff}$ is larger than the superconducting fraction $\eta$ due to a presence of an external metallic sheath that embeds the filaments. In the particular case of a round wire, and supposing that the electrical contact between the metallic matrix and the superconducting filaments is good, the effective resistivity is lower than $\rho_m$ and it is given by\textsuperscript{18}:

$$\rho_{eff} = \rho_m \frac{1 - \eta_{eff}}{1 + \eta_{eff}}$$  \hspace{1cm} (9)$$

If the electrical contact is bad, giving a high resistive barrier around each filament, $\rho_{eff}$ increases in comparison with the resistivity of the matrix, according to the expression\textsuperscript{18}:

$$\rho_{eff} = \rho_m \frac{1 + \eta_{eff}}{1 - \eta_{eff}}$$  \hspace{1cm} (10)$$

In our sample, there was no artificially involved resistive barrier. We also assume that heat treatments commonly increases the quality of the electrical contact between filaments and matrix. Modification of the expressions (9) and (10), taking into account geometries more similar to our samples, have been calculated\textsuperscript{19}, yielding for the geometry of our samples the ratio $\rho_{eff}/\rho_m$ as:

$$\frac{\rho_{eff}}{\rho_m} = \frac{t_u w_u - t_u w_f}{t_u w_u - t_u w_f + t_f w_f}$$  \hspace{1cm} (11)$$

where the meaning of $t_u$, $t_f$, $w_u$, and $w_c$ is shown in Fig. 1. Introducing the effective superconducting fraction $\eta_{eff} = (w_f t_f) / (w_u t_u)$, the last equation can be written as:

$$\frac{\rho_{eff}}{\rho_m} = \frac{1 - \frac{w_f}{w_u}}{1 - \frac{w_f}{w_u} + \eta_{eff}}$$  \hspace{1cm} (12)$$

**C. Theoretical values of $\chi_0$**

For a superconductor in Meissner state, the $\chi_0$ expressions for infinite strip and finite x-z array have been reported by Fabbricatore et al.\textsuperscript{14}
If our samples are considered like a single strip with rectangular cross section, \( \chi_0 \) is given by:

\[
\chi_0 = \frac{\pi w_z}{4d_z} \quad (13)
\]

where \( w_z \) and \( d_z \) are respectively the width and the thickness of the filamentary zone. For a more realistic geometry like a finite x-z array with \( 2 \times 8 \) filaments, the expression for \( \chi_0 \) is very complicate and is given by:

\[
\chi_0 = \chi_0(1)\frac{f_x}{f_z} \quad (14)
\]

where \( \chi_0(1) = \pi w_f/4t_f \), is the geometrical factor calculated according to Eq. (13), and

\[
f_x = \left[ 1 + 2\pi (\beta - 1) \text{arcsec} \left( \frac{n_x + 5\beta - 1}{5\beta} \right) \right] ; \quad (15)
\]

\[
f_z = \chi_0(1) \left( \chi_0(z - \infty) + \frac{\chi_0(1) - \chi_0(z - \infty)}{n_z^{0.8}} \right)^{-1} \quad (16)
\]

where \( n_x \) is the number of filaments in the \( x \) direction and \( n_z \) is the number of filaments in \( z \) direction. Moreover

\[
\beta = \frac{\chi_0(x - \infty)}{\chi_0(1)} \frac{D_c d_f + D_c - w_f}{w_f D_c - w_f} \quad (17)
\]

where

\[
\chi_0(x - \infty) = -\frac{2D_c^2}{\pi w_f t_f} \ln \left[ \cos \left( \frac{\pi w_f}{2D_c} \right) \right] ; \quad (18)
\]

\[
\chi_0(z - \infty) = \frac{2t_u^2}{\pi w_f t_f} \ln \left[ \cosh \left( \frac{\pi w_f}{2t_u} \right) \right] ; \quad (19)
\]

and \( D_c \) is the distance between the centers of each column, whereas \( t_u \) is the distance between the centers of two neighbor filaments.

### III. EXPERIMENTAL DETAILS

#### A. The samples

The behaviour of two different kinds of bi-columnar BSCCO(2223) tapes has been investigated by means of the a.c. susceptibility technique. The first BSCCO/Ag tape (named sample A) was prepared with 16 filaments in pure silver matrix with a stack of 8 filaments for each column separated by about 0.3 mm of pure silver. The external sheath is also made with the same material. The geometry of the second sample (sample B) is very similar to that of the sample A, but the number of filaments is 15 and therefore there are 8 filaments in one column and 7 in the other. The metallic sheet between filaments is a Ag/Mg(0.4%) alloy and the matrix which embeds the whole filamentary zone is a Ag/Mg(0.4%)/Ni(0.22%) alloy. For both samples, strong bridging could be present between filaments in the same column. The initial length of the sample A was 61.7 mm, whereas the sample B was 61.6 mm long. Preliminary transport measurements have shown that sample A has a higher critical current than sample B. This indicates that the grains alignment or the grains connectivity in sample B is worse than in sample A. The main features of the samples are summarized in Tab. I and in Fig. 2, the cross section of the sample A is shown. In Tab. II the different sizes are reported.

| Sample | A | B |
|--------|---|---|
| matrix | Ag | Ag/Mg(0.4%) |
| External sheath | Ag | Ag/Mg(0.4%)/Ni(0.22%) |
| fill factor (\( \eta \)) | 32 % | 32 % |
| width (\( w \)) | 3.3 mm | 3.1 mm |
| thickness (\( t \)) | 0.25 mm | 0.24 mm |
| initial length (\( \ell \)) | 61.7 mm | 61.6 mm |
| critical current (\( I_c \)) | 23.5 A | 17.5 A |
| c. c. density (\( J_c \)) | 9653 A/cm² | 7173 A/cm² |
| exponent (\( n \)) | 15 | 15 |

TABLE I: Main features of the samples A and B

![FIG: 2: Cross section of BSCCO/Ag sample A](image)

In the samples A and B, the resistivity of the different matrices was experimentally measured by a D.C. transport measurement on samples without superconducting material. For pure Ag matrix \( \rho_m = 0.27 \mu\Omega \text{ cm} \), while the resistivity of the Ag/Mg alloy is \( \rho_m = 1.08 \mu\Omega \text{ cm} \). By using the expression (8) and the dimensions reported in Tab. II the expected ratio \( \rho_{eff}/\rho_m \) is 0.25. Therefore the expected effective resistivity for the Sample A is 0.068 \( \mu\Omega \text{ cm} \), while it is 0.27 \( \mu\Omega \text{ cm} \) for the sample B. Finally, the estimated value for \( \eta_{eff} \) is 71%.

| \( w_z \) (mm) | \( t_z \) (mm) | \( D_c \) (mm) | \( w_x \) (mm) | \( t_x \) (mm) |
|---|---|---|---|---|
| 1.0 | 0.21 | 1.3 | 2.3 | 0.21 |
| \( w_u \) (mm) | \( t_u \) (mm) | \( w_f \) (mm) | \( t_f \) (mm) | \( t_u - t_f \) (mm) |
| 1.3 | 0.027 | 1.0 | ~0.025 | ~0.002 |

TABLE II: Geometrical dimensions of the tapes as defined in Fig. II
B. A.C. loss measurements

The a.c. losses were measured by using the standard a.c. susceptibility technique. An a.c. susceptometer with a system of coils suitable for measurements on sample with length up to 12 cm is sketched in Fig. 3. The susceptometer comprises an a.c. magnetic field which produces an a.c. magnetic field with $B_0$ up to 50 mT, with a field homogeneity within 1% on a 8 cm length and 2% on 12 cm. The power to the a.c. magnet is delivered by an amplifier controlled by the signal from the internal oscillator of the lock-in amplifier. The a.c. field induces a voltage in two coils: the pick-up coil, which is very close to the sample surface, while the second, the null coil, which is 1 cm apart. The a.c. magnet and both the coils are placed in a reinforced plastic cryostat, so no eddy currents are induced in the cryostat walls. The system is cooled by liquid nitrogen and all the measurements have been performed at 77 K. Since the pick-up coil and the null coil are not perfectly identical, a variable compensation system is also used. The input signal for the lock-in amplifier is the result of the difference between the pick-up voltage and the sum of the null and the compensation voltages. The current which flows in the a.c. magnet is inductively measured by the current coil.

The two original full length samples were subsequently cut several times to vary the sample lengths. In each of these operations, the tape boundaries were polished and protected with grease. Measurements were performed in the frequency range from 1 Hz to 1000 Hz in the field amplitude ranging from 0.05 mT to 45 mT.

C. Experimental measurement of $\chi_0$

To experimentally evaluate the $\chi_0$ value of the samples, the technique described in detail by Fabbricatore et al. was used. By performing the measurement at very low magnetic field amplitude (such as to assure the Meissner state), the constant $\chi_0$ can be determined by using the relation:

$$\chi_0 = G_c \frac{V_{coil}}{U_{meas}} \frac{U_{sample}}{V_{coil}}$$

where $U_{meas}$ is the in input signal of the lock-in amplifier when the sample is placed in the pick-up coil, $U_{coil}$ is the voltage of null coil only, $V_{sample}$ is the volume of superconducting fraction, $V_{coil}$ is the volume of the pick-up coil and $G_c$ is a constant depending on the geometry of the coils.

IV. EXPERIMENTAL RESULTS

A. Determination of $\chi_0$

The value of $\chi_0$ was measured on a samples of 6.4 mm length at the frequencies of 7 Hz and 21 Hz.

In order to check the frequency dependence of $\chi_0$, measurements in the frequency range between 7 Hz and 710 Hz, reported in Fig. 4, were performed on the sample A. All these measurements were done at the magnetic field amplitude of 1.3 G, enough to achieve a complete shielding of the sample. At low frequency this shielding corresponds to the Meissner state of the separated filaments in the tape. As the frequency increases, the shielding is achieved first in the whole filamentary zone and then, at higher frequency, in the whole tape.

From the measurements at 7 Hz, the value of $\chi_0$ for the sample A is 8.9, while it is 8.8 for the sample B. This is not surprising if we consider the very similar geometry of the two tapes.

The comparison between the theoretical and measured values of $\chi_0$ reported in Tab. III shows that the experimental values are close to those theoretically calculated for an x-z array. Looking at the Fig. 4 the $\chi_0$ value is 9.0 at low frequency, it increases up to 9.9 at 510 Hz, and becomes 10.37 at 710 Hz. By comparing these values with the theoretical values reported in Tab. III, we can observe that the value measured at 510 Hz is closer to the one calculated for a infinite strip with rectangular cross section, by taking the width and thickness of the superconducting zone. Furthermore, the value measured at 710 Hz is closer to that calculated for a strip with the

| $\chi_0$ | strip $w_z \times t_z$ Eq. 13 | 9.8 |
| $\chi_0$ | strip $w \times t$ Eq. 15 | 10.37 |
| $\chi_0$ | x-z array Eq. 14 | 9.0 |
| Measured value sample A | | 8.9 |
| Measured value sample B | | 8.8 |

TABLE III: Theoretical and measured $\chi_0$. 
whole tape dimensions. Therefore the behaviour of the experimental $\chi_0$, suggest that the whole tape dimensions can be considered only at higher frequencies.

The interpretation is straightforward: as the frequency increases, a larger volume of the tape is shielded by the current flowing in the metallic matrix, both in the superconducting core and in the central metallic core. Therefore, as the frequency increases, the effective geometry of the sample changes from an x-z array geometry to a monofilamentary strip and therefore, the $\chi_0$ values measured changes accordingly.

B. A.C. susceptibility measurements

In Fig. 5 and in Fig. 6, the real ($\chi'$) and the imaginary ($\chi''$) part of the a.c. susceptibility as function of $B_0$ are shown as measured on the 61.7 mm long sample A at different frequencies. In Fig. 5 we can observe that a larger part of the tape is screened as the frequency of the magnetic field increases. Such frequency dependence has not been observed when $\chi_0$ was determined on much shorter samples. Now, the coupling currents contribute to the overall shielding and thus to $\chi'$. Nevertheless, the correctness of experimental technique is tested by checking that the value of $\chi'$ approaches -1 at low field and rises up to 0 at high field.

In Fig. 6 a set of $\chi''(B_0)$ curves, measured in the frequency range from 1 Hz up to 70 Hz, is shown. For frequencies ranging from 1 Hz to 6 Hz our power supply does not allow to reach magnetic fields higher than 1.5 mT. We can observe that $\chi''$ strongly depends on the frequency. In the low field region ($B_0 \leq 1$ mT) $\chi''$ depends slightly on $B_0$ and if we analyze the its frequency behaviour, at fixed $B_0$, we find that $\chi''$ reaches its maximum value around 4.8 Hz. However this maximum value is equal to about 0.15 whereas the Campbell model predicts a value of 0.5 (see Eq. (3)). For these reason ,the $\gamma$ factor introduced in Eq. (3) is equal to 3.3. In the low field region, the light dependence on $B_0$ and the large frequency dependence are in agreement with the expected behaviour in presence of coupling losses dominant in comparison with the filaments hysteretic losses.

In the high field region, the full coupling among filaments is expected, leading to a behaviour similar to a critical state with the typical peak of $\chi''$ in the amplitude dependence. In this region, as the frequency rises, the maximum value ($\chi''_{max}$) increases and shifts at higher field amplitudes. Such increasing is due to a larger contribution of resistive effects with respect to the critical state. In order to compare the experimental curves with the analytical results for the a.c. susceptibility, the data reported in Fig. 6 have been plotted in Fig. 7 in terms of normalized units $\chi''/\chi''_{max}$ and $B_0/B_{0,max}$ (where $B_{0,max}$ is the magnetic field amplitude where $\chi''$ has its maximum value). In the same figure, the dotted, dashed and continuous lines are the a.c. suscepti-
bility as calculated in the framework of the critical state model, respectively for a slab,\textit{} a thin strip\textit{}, and a thin disk.\textit{} In the high field range, in particular above \(B_{0,\text{max}}\), the experimental measurements fall on a single curve. The used analytical models do not fit very well the experimental data. However the scaling of the data shows that the sample behaviour, at least in the high field range, can be treated in good approximation within the critical state model framework. Numerical calculations of a.c. susceptibility of a superconducting thick strip\textit{} could give a better agreement, also taking into account the field dependence of the critical current density (\(J(B)\))\textit{} and thermal activated creep phenomena.\textit{}

V. DISCUSSION

In this section the total losses are determined by using the equation (\ref{eq:total_loss}) and discussed in the subsection \texttt{V A}. In the subsection \texttt{V B} the losses are analyzed as function of the frequency, at fixed magnetic field amplitudes much lower than the full penetration field of the composite specimen, in order to be in the limit for the validity of the Campbell model. Finally in the last section the effective resistivities are determined and discussed.

A. Total losses

According to equation (\ref{eq:total_loss}), in Fig.\texttt{3} the total losses densities (\(Q\)) measured in the sample \texttt{A}, 61.7 mm long are reported as function of \(B_0\), for different frequencies. Very similar dependencies were found for the sample \texttt{B}. In Fig.\texttt{3} we also report the theoretical slopes expected for the coupling losses in the low field (dotted line) and in the high field (dashed line) regimes. In fact, according to Eq. (\ref{eq:coupling_loss}) for \(B_0\) much lower than \(B_{0,\text{max}}\) the coupling losses should have a quadratic dependence on \(B_0\). At higher fields, in a critical state description, a cubic dependence can be expected for \(B_0 < B_{0,\text{max}}\), whereas in the high field region \(B_0 > B_{0,\text{max}}\) the dependence is linear. Nevertheless for \(B_0 \ll B_{0,\text{max}}\), a contribution due to the hysteretic losses in the single filaments (\(Q_h \propto B_0^2\)) should be added to the coupling losses. Therefore in this field region \(Q \propto k_1 \times B_0^3 + k_2 \times B_0^2\) where \(k_1\) and \(k_2\) are two proportionality constants and in a log-log plot the slope of the loss density can range from 2 to 3. Our experimental data for fields lower than 1 mT, show a slope very close to the expected value for a pure coupling regime. Deviations from the square slope are observed as the length of the samples is reduced. This is in agreement with the reduction of the coupling loss density as the sample length is reduced, whereas the hysteretic loss density does not change.

B. Frequency dependence of the losses at low magnetic fields

Losses have been investigated as function of the frequency at field much lower than \(B_{0,\text{max}}\), by measuring several pieces with different lengths, cut from our original samples \texttt{A} and \texttt{B}. As reported in Fig.\texttt{3} the losses exhibit a maximum that shifts towards higher frequencies as the samples length decreases. The experimental data are compared with the results derived from the Campbell model.\texttt{} In particular, the data have been fitted by adding to the coupling loss a frequency independent contribution (\(\beta\)) due to the filaments hysteretic loss:

\[
Q_{\text{fit}}(\omega) = \alpha \frac{\omega \tau}{1 + \omega^2 \tau^2} + \beta \tag{21}
\]
For each fit we have used three different parameters, which were not free. In fact, for $\beta$ we have employed the same value in the three different fits, performed for the samples with different lengths. These values are respectively equal to $\beta = 0.29$ J/m$^3$ cycle for the samples A and $\beta = 0.215$ J/m$^3$ cycle for the samples B. Since the hysteretic losses are proportional to the $J_c$, we compared the ratio $(J_{c,B}/J_{c,A})$, with the ratio of the $\beta$ values found by the fit. $J_{c,B}/J_{c,A} = 0.743$, that is in good agreement with the ratio $\beta_B/\beta_A = 0.741$.

The parameter $\alpha$ is determined by the maximum value of $Q$ which is equal to $\alpha/2 + \beta$. The $\alpha$ values are in the range $(1.52 \pm 0.58)$ J/m$^3$ cycle for the sample A and in the range $(1.13 \div 1.23)$ J/m$^3$ for the sample B.

The $\tau$ values, obtained from the fits, are reported in Tab. IV whereas in the inset of Fig. 10 its linear dependence on the square of the sample length is shown. The linear fit is very satisfactory for both samples. As shown in Fig. 9 the fits for the samples B are better than those for the samples A. The largest deviations are observable for the sample A of 15.4 mm length. In fact, for this sample, in Fig. 9 we show the measured losses for a field amplitude of 0.1 mT, because at 0.5 mT the observed discrepancy between the fit and the experimental data is even larger. This can be ascribed to some other frequency effect which becomes important in the frequency range (100 - 1000 Hz) for pure Ag matrix sample. Indeed, as pointed out by Takács $^{29,30}$, the introduction of two different time constants, $\tau_0$ and $\tau_1$, in the expression for the coupling losses (see formula (2)) is necessary in the case of flat composite cables, leading to:

$$Q_c = \frac{\gamma B_0^2}{2\mu_0} \left[ \frac{2\pi \chi_0}{1 + \omega^2 T_1^2} \left( \frac{\omega \tau_0}{1 + \omega^2 T_1^2} \right) \right]$$

In Eq. (22) $\tau_0$ is related to the resistivity of the coupling currents loops and it is determined by the zero frequency limit of $Q(\nu)$, while $\tau_1$ is related to the mean inductance of these loops, and it is determined by the position of the maximum in $Q(\nu)$. Unfortunately, for most of the samples, at low frequencies there are not enough data to be linearly fitted in order to determine the value of $\tau_0$. Nevertheless, for the 15.5 mm long sample A this fitting procedure has been possible, and the value for $\tau_0$ has been estimated to be 1.5 ms, which is not so different from 1.7 ms found for $\tau_1$ by using the same data. Increasing the length of the sample, the frequency where the maximum in $Q(\nu)$ occurs decreases and therefore a smaller difference between $\tau_0$ and $\tau_1$ is expected.

**C. Effective Resistivity**

By using the experimental values of $\tau$ and $\chi_0$, the effective resistivity of the metallic matrix has been determined.

For samples with the same length and the same geometry the relation $\rho_A/\rho_B = \rho_B/\rho_A$ is valid. If $\rho_A < \rho_B$ we should have $\tau_A > \tau_B$, which means that the frequency ($\nu_{\text{max}}$) where the maximum in the $Q_c(\nu)$ occurs

![Graph](image-url)
crease the coupling losses.

due to the use of a matrix with higher resistivity in order to de-
growths reduces or cancels the advantage coming from
flowing of the coupling currents. The presence of inter-
columns can generate paths with lower resistivity for the
in the metallic matrix are visible. Intergrowths between
A

Fig. 11 the micrographs of a section of both the samples
B

to determined if the structure of the sample is considered. In
A
as determined from the experimental data for all the considered
samples.

is lower for the sample with lower resistivity. The di-
rectly measured matrix resistivity of the sample A is 4
times smaller than that for the sample B (see section
III.A). Since the structure of the samples is very similar
(as confirmed by the experimentally found values of χ0),
we expect (vmax,A < vmax,B). It is striking to see that
the experimental data in Fig. 11 demonstrate the oppo-
site behaviour. The obtained results are summarized in
Tab. IV. For the sample A, the value of the effective
resistivity is 3 times higher than the value expected from
Eq. (11). Moreover the effective resistivity found for the
sample B is lower than both the ρeff of the sample A, and
its ρeff determined by Eq. (11). This can be un-
derstood if the structure of the sample is considered. In
Fig. 11 the micrographs of a section of both the samples
A and B are shown. In the sample B many intergrowths
in the metallic matrix are visible. Intergrowths between
columns can generate paths with lower resistivity for the
flowing of the coupling currents. The presence of inter-
growths reduces or cancels the advantage coming from
the use of a matrix with higher resistivity in order to de-
crease the coupling losses.

| ℓ (mm) | τ (ms) | χ0 | ρ_{eff} (μΩ cm) |
|--------|--------|----|----------------|
| sample A |
| 61.7   | 31     | 8.9 | 0.176          |
| 30.9   | 7.8    | 8.9 | 0.175          |
| 15.5   | 1.75   |     | 0.192          |
| 6.5    | 0.32   | 9.8 | 0.172          |
| sample B |
| 61.8   | 42     | 8.8 | 0.132          |
| 30.9   | 8.9    | 8.8 | 0.155          |
| 15.6   | 1.9    | 9.0 | 0.182          |

TABLE IV: Values of the quantities τ, χ0 and ρ_{eff} as de-
determined from the experimentally found values of χ0)

On the contrary, the sample A shows a value of ρ_{eff}
which is higher than the expected one. This high value
can be due to a contact resistance between the supercon-
ducting filaments and the metallic matrix.

VI. CONCLUSIONS

In this work we have studied the a.c. coupling losses
on two different sets of bicolumnar BSCCO tapes.

For the geometrical factor χ0, we used the experi-
mental value instead of the demagnetizing factor (t_z + w_z/t_z)
which should give the found value 13.5 instead of the value 8.9, for both samples. Our
experimental value is in agreement with the theoretical
value computed for an x-z finite array. In this way the
main physical quantity involved in the coupling losses
have been experimentally estimated.

For increasing frequencies the measured χ0 approaches
the value calculated for a superconducting strip with the
width and the thickness equal to the dimensions of the
filamentary zone of the tape. By further frequency in-
creasing, χ0 approaches the value calculated for a super-
conducting strip with dimensions equal to those of the
tape.

A.C. susceptibility and losses, measured as function of
the magnetic field amplitude and frequency, confirm that
the coupling losses dominate in these samples over the
hysteretic losses of the single filaments. The frequency
dependence of the experimentally measured losses at low
field has been analyzed by using the Campbell model
and a good agreement between the model and the experi-
mental data has been obtained. The time constants τ of
the tape has been determined for different lengths of the
sample, finding, as expected, a linear dependence of τ on
the square of the sample length. The final task has been
the experimental evaluation of the effective resistivity. In
the sample A with a pure silver matrix, the high value
measured for ρ_{eff} is interpreted as bad electrical con-
tacts between the filaments and the metallic matrix. In
the sample B, the experimentally found value ρ_{eff} lower
than the expected one in an ideal sample, is explained
with the presence of intergrowths. This particular result
suggests that in this kind of samples, the enhancement
of the effective matrix resistivity does not reduce intrin-
sically the coupling losses.

Finally it is worth to point out that for low magnetic
field amplitudes, the Campbell model can be successfully
used also for BSCCO tapes containing few flat filaments.
This can be used as a starting point to analyze all the
other factors which influence the losses in more complex
geometries.

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