Online Resource Allocation with Buyback: Optimal Algorithms via Primal-Dual

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Motivated by applications in cloud computing spot markets and selling banner ads on popular websites, we study the online resource allocation problem with costly buyback. To model this problem, we consider the classic edge-weighted fractional online matching problem with a tweak, where the decision maker can recall (i.e., buyback) any fraction of an offline resource that is pre-allocated to an earlier online vertex; however, by doing so not only the decision maker loses the previously allocated reward (which equates the edge-weight), it also has to pay a non-negative constant factor $f$ of this edge-weight as an extra penalty. Parameterizing the problem by the buyback factor $f$, our main result is obtaining optimal competitive algorithms for all possible values of $f$ through a novel primal-dual family of algorithms. We establish the optimality of our results by obtaining separate lower-bounds for each of small and large buyback factor regimes, and showing how our primal-dual algorithm exactly matches this lower-bound by appropriately tuning a parameter as a function of $f$. The optimal competitive ratio $\Gamma_{\text{gen}}(f)$ and the optimal competitive ratio $\Gamma_{\text{det-int}}(f)$ of deterministic integral algorithms are as follows,

\[
\Gamma_{\text{gen}}(f) = \begin{cases} 
\frac{e}{e-(1+f)} & \text{if } f \leq \frac{e^2}{2} \\
-W_1\left(-\frac{1}{e(1+f)}\right) & \text{if } f \geq \frac{e^2}{2},
\end{cases} \quad \Gamma_{\text{det-int}}(f) = \begin{cases} 
\frac{2}{1-f} & \text{if } f \leq \frac{1}{3} \\
1 + 2f + 2\sqrt{f(1+f)} & \text{if } f \geq \frac{1}{3}
\end{cases}
\]

where $W_1 : [-1/e, 0) \rightarrow (-\infty, -1]$ is the non-principal branch of the Lambert W function.

In a nutshell, our main technical (and conceptual) contribution is developing a new connection between the primal-dual framework for the online bipartite allocation problems and the costly buyback problem. We then use this new understanding to obtain the optimal competitive algorithms in all the regimes of the buyback parameter $f$. Our development relies on several novel technical ideas, such as considering instances with a continuum of weights, and proposing careful construction of dual variables corresponding to the offline resources (based on these instances) for the general case. We further study lower and upper bounds on the competitive ratio in variants of this model, e.g., single-resource with different demand sizes, or matching with deterministic integral allocations. We show how algorithms in the our family of primal-dual algorithms can obtain the exact optimal competitive ratio in all of these variants — which in turn demonstrates the power of our algorithmic framework for online resource allocations with costly buyback.

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CCS Concepts: • Theory of computation → Online algorithms; Approximation algorithms analysis.

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