Continuous variable B92 quantum key distribution protocol using single photon added and subtracted coherent states

S. Srikara · Kishore Thapliyal · Anirban Pathak

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Abstract
In this paper, a continuous variable B92 quantum key distribution protocol is proposed using single photon added and subtracted coherent states, which are prepared by adding and subsequently subtracting a single photon on a coherent state. It is established that in contrast to the traditional discrete variable B92 protocol, this protocol for quantum key distribution is intrinsically robust against the unambiguous state discrimination attack, which circumvents the requirement for any uninformative states or entanglement used in corresponding discrete variable case as a remedy for this attack. Further, it is shown that the proposed protocol is intrinsically robust against the eavesdropping strategies exploiting classical communication during basis reconciliation, such as beam splitter attack. Security against some individual attacks, key rate, and bit-error rate estimation for the proposed scheme are also provided. Specifically, the proposed scheme ensures very small bit-error rate due to properties of the states used. Thus, the proposed scheme is shown to be preferable over the corresponding discrete variable B92 protocol as well as some similar continuous variable quantum key distribution schemes.

Keywords Continuous variable QKD · B92 type QKD · Application of engineered quantum states · Single photon · Added and subtracted coherent states

Anirban Pathak
anirban.pathak@jiit.ac.in

S. Srikara
srikara.s@students.iiserpune.ac.in

Kishore Thapliyal
kishore.thapliyal@upol.cz

1 Indian Institute of Science Education and Research, Pune, India
2 RCPTM, Joint Laboratory of Optics of Palacky University and Institute of Physics of Academy of Science of the Czech Republic, Faculty of Science, Palacky University, 17. listopadu 12, 771 46 Olomouc, Czech Republic
3 Jaypee Institute of Information Technology, A-10, Sector-62, Noida 201307, India
1 Introduction

Quantum key distribution (QKD) is a method by which quantum states and features of quantum mechanics are used to distribute an unconditionally secure key (see [1–3] for review). A scheme for QKD was first proposed by Bennett and Brassard in 1984, which is now known as BB84 protocol for QKD [4]. This was followed by an entangled state-based protocol for QKD [5] introduced by Ekert in 1991, which later formed the basis of device independence. This protocol involved measurement in three bases and thus uses six states in contrast to four states used in BB84 protocol. Almost immediately after the introduction of Ekert’s protocol, in 1992, Bennett established that neither four nor six states are essential to accomplish the QKD task in a secure manner, and two non-orthogonal states are sufficient to perform QKD [6]. The protocol drew considerable attention of the community as it successfully provided some fundamental insights into the origin of the security in the schemes of QKD. This scheme is now known as B92 protocol. In the original form, this protocol was designed for discrete variable (DV) QKD; here we aim to extend it to continuous variable (CV) QKD for certain advantages that CV-QKD provides over its DV counterpart. Before we elaborate on the advantages and limitations of CV-QKD, it would be of use to briefly recall the original B92 protocol.

In the B92 protocol [6], Alice prepares and sends a string of qubits individually prepared randomly in state $|0\rangle$ (representing the bit value 0) or $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ (representing the bit value 1) to Bob. Bob measures each incoming qubit randomly either in the computational $\{|0\rangle, |1\rangle\}$ or diagonal $\{|+\rangle, |-\rangle\}$ basis, and designates the measured bit as 0 (1) if his measurement outcome is $|-\rangle$ ($|1\rangle$). For the measurement outcomes $|0\rangle$ or $|+\rangle$, he declares the result as inconclusive, and discards these results. A part of the remaining conclusive bits are used to check eavesdropping and the rest are retained by both Alice and Bob, thus sharing a symmetric secret key. This protocol is unconditionally secure under an ideal lossless channel. However, it can be attacked by Eve in a lossy channel using the unambiguous state discrimination (USD) attack [7,8], which makes this protocol insecure in extremely lossy channels, thus making it difficult to be practically implemented. A modification of the B92 protocol, which was robust against the USD attack, was introduced in [9], using a non-maximally entangled Bell pair and two additional decoy states, called the uninformative states. A device independent version of the B92 protocol [9] was also proposed in the recent past [10] by allowing the entanglement generation in the hands of an untrusted party present between Alice and Bob. Recently, B92 protocol was found relevant in designing a quantum private query scheme [11], which was later shown insecure in lossy channel [12]. Further, an advanced USD attack is also designed for cryptanalysis of B92 scheme [13].

In contrast to the above-mentioned DV quantum cryptography schemes, where photon counters are used, a set of CV quantum communication schemes is also proposed in which information is encoded on quadratures and decoded by homodyne or heterodyne detection (see [14–16] for review). The first set of CV cryptography schemes was proposed using squeezed state [17], Einstein–Podolsky–Rosen correlated states [18], and coherent state [19–22]. The initial set of proposals was focused on encoding discrete quantum key in CV quantum states; such hybrid schemes were later extended
to all CV schemes distributing CV quantum key [23]. The CV quantum cryptography has several advantages over its DV counterpart as it involves multi-photon pump beams and does not require single photon detectors, which omit the limitations of single photon source and detector. On top of that, implementation with easily available light source(s) and detector(s), compatibility with the existing optical communication technology, and advantage of CV schemes at short distances make them preferred candidates for metropolitan secure networks. Historical development and current status of the CV cryptography can be found in a set of review articles [14,16,24,25]. Because of these advantages of CV-QKD, a CV counterpart of the BB84 scheme [4] has recently been proposed using single photon added and subtracted coherent states (PASCS) [26]. This scheme uses four states analogous to the corresponding DV BB84 scheme and also involves classical communication at the end to discard the cases where their choices of quadratures are different (see Ref. [26] for more detail). The security of CV QKD schemes is analyzed in detail (see [24,27] for review). For instance, different studies have reported security against general attack [28], composable security [29], machine learning for parameter estimation [30], using whole raw key for parameter estimation without compromising security [31]. On the other hand, initial CV QKD schemes have also led to the schemes for measurement device independent [32], device independent [33], entanglement-distribution-based [34], atmospheric [35], satellite-based [36] CV-QKD as well as hybrid CV- and DV-QKD [37]. More recently, proposal to completely eliminate information leakage in CV communication performed over lossy channels has been reported [38], which has no DV analogue. The proposals for CV cryptography are not restricted to QKD as CV schemes for quantum signature [39], direct secure quantum communication [40,41], quantum secret sharing [42], and position-based quantum cryptography [43] are also proposed.

Motivated by the above facts, in this paper, we propose a CV version of the B92 protocol [6], which can be viewed as a B92 type modification of a QKD protocol [26]. The protocol proposed in this paper is intrinsically robust against the USD attack, i.e., it does not require any uninformative state or entanglement. Also, unlike the protocol discussed in Ref. [26], the proposed protocol is intrinsically (i.e., without any additional conditions on any parameters) robust against the beam splitter attack, too. Such discrete modulation CV-QKD schemes are shown to perform better by using efficient error correction codes [44].

The rest of the paper is structured as follows. In Sect. 2, we briefly introduce PASCS and the mathematical tools that are used in this work. Section 3 describes the protocol, and Sect. 5 is dedicated to the security analysis of the proposed protocol. Finally, the paper is concluded in Sect. 6.

2 Photon added and subtracted coherent states

In this section, we aim to introduce the mathematical tools and the quantum states that will be required to explain the protocol. To begin with we may note that coherent state, which is usually obtained as the state of the radiation field at the output of a laser source, is essentially an eigen-ket of the annihilation operator. It can also be described as a displaced vacuum state. Subsequent photon addition and subtraction on a coherent
state leads to PASCS. This state seems physically realizable as the photon addition and subtraction operations are experimentally feasible (see [45–47] and references therein). A single photon added then subtracted coherent state $|\psi (\gamma)\rangle$ is defined as

$$|\psi (\gamma)\rangle = N_{\gamma}^{-1/2} a a^\dagger |\gamma\rangle,$$

(1)

where $|\gamma\rangle$ is the initial coherent state having an average photon number $|\gamma|^2$ with $a$ and $a^\dagger$ corresponding to annihilation and creation operators, respectively. Further, $N_{\gamma}$ is the normalization constant which can be expressed as

$$N_{\gamma} = |\gamma|^4 + 3|\gamma|^2 + 1.$$

In what follows, we propose a protocol of QKD using two PASCSs described by Eq. (1) and characterized by $\gamma = \alpha$ (i.e., $|\psi_0 (\alpha)\rangle$) and $\gamma = i\alpha$ (i.e., $|\psi_1 (\alpha)\rangle$) for $\alpha \in \mathbb{R}^+$. A phase space description of quantum mechanics was introduced by Wigner in 1932 [48]. As this distribution function can have negative values (normalized to unity), it is not a true probability distribution function, and is often referred to as quasidistribution function. Much later, Glauber and Sudarshan introduced the notion of nonclassicality in terms of negative values of $P$ function [49,50], and it was realized that the states having negative values of Wigner function must be nonclassical. The Wigner function for an arbitrary state, with density matrix $\rho$, is given by [51]

$$W(\xi) = \frac{2}{\pi} \sum_{n=0}^{\infty} (-1)^n \langle n | \hat{D}^{-1}(\xi) \hat{\rho} \hat{D}(\xi) | n \rangle,$$

(2)

where $\hat{D}(\xi) = e^{(\zeta a^\dagger - \xi^* \hat{a})}$ is the displacement operator defined in terms of complex number $\zeta = \zeta_r + i\zeta_i$. Here $\zeta_r$ and $\zeta_i$ are the phase space coordinates corresponding to the position and momentum quadratures, respectively, and $|n\rangle$ is the Fock state. Now, using Eq. (2), we can obtain Wigner function for the PASCS of our interest. Specifically, Wigner function for $|\psi_j (\alpha)\rangle$ is obtained as [26,52]

$$W_j (\xi, \alpha) = \frac{2e^{\frac{-2\left[(\zeta_a - \alpha)^2 + \xi_b^2\right]}{\pi \left(\alpha^4 + 3\alpha^2 + 1\right)}} \left\{ \left(\alpha^2 - 1 - 2\zeta_a \alpha\right)^2 + (2\zeta_b \alpha)^2 - \alpha^2 \right\},$$

(3)

where $\{a = r, b = i\}$ for $j = 0$, while $\{a = i, b = r\}$ for $j = 1$.

It is already mentioned in the previous section that a CV quantum communication scheme is different from a DV scheme designed for the same task as it involves homodyne measurement(s). This can be performed by mixing the quantum signal with a classical beam at a beam splitter and measuring the difference of the currents at two output ports. By controlling the phase of the input classical beam, we can address one of the quadratures $\zeta_r$ (i.e., position) or $\zeta_i$ (i.e., momentum).

For the sake of completeness, we have also shown the surface plots of the Wigner functions of the PASCS $|\psi_0 (\alpha)\rangle$ and coherent state $|\alpha\rangle$ with $\alpha = 0.8$ in Fig. 1a and b, respectively. Clearly, the Wigner function of the PASCS has a negative region (shown...
by gray color in Fig. 1a), indicating that the state is nonclassical. Due to applications of two non-Gaussianity inducing operations, namely photon addition and subtraction, in generation of PASCS, the obtained Wigner function is non-Gaussian and also shows a shift toward the positive real side as compared to the corresponding coherent state. In a homodyne detection, one may choose either to measure quadrature $\zeta_r$ or quadrature $\zeta_i$. In analogy of Fig. 1a obtained for $|\psi_0(\alpha)\rangle$, one can also obtain the Wigner function for PASCS $|\psi_1(\alpha)\rangle$. In what follows, we will propose our CV B92 QKD scheme using PASCS described in this section.

### 3 Continuous variable B92 protocol

Our discrete modulation non-Gaussian state-based CV-QKD protocol described below is a CV counterpart of B92 protocol. The protocol can be described as follows.

1. Alice prepares an $n$ bit random string ($K_A$) and prepares signal pulses in PASCS either as $|\psi_0(\alpha)\rangle$ corresponding to the bit value 0 or $|\psi_1(\alpha)\rangle$ for bit value 1 in $K_A$. Subsequently, she sends the sequence $S_A$ of the prepared states to Bob.

2. Bob also prepares a random string $K_B$ of $n$ bit. Corresponding to the bit value 0 (1) in $K_B$, he selects to measure the position (momentum) quadrature of the signal pulses in $S_A$ in the homodyne measurement resulting in the real (imaginary) part of $\zeta$.

3. If Bob performs homodyne measurement on the position (momentum) quadrature and obtains $\zeta_r < -\zeta_c$ ($\zeta_i < -\zeta_c$), he designates it as conclusive, where $\zeta_c$ is the post-selection threshold chosen by him. Otherwise, he discards the result and terms the corresponding outcome as inconclusive.
4. Bob then declares the coordinates of the retained (conclusive) results, using which both Alice and Bob obtain bit strings $K^R_A \in K_A$ and $K^R_B \in K_B$, respectively, after discarding the bit values corresponding to the inconclusive results.

At the end of the quantum communication, Alice and Bob are expected to share an unconditionally secure quantum key, but in ideal conditions $K_B = K_A$. Therefore, it is predecided that at the end of Step 4, Bob will flip his key to ensure $K_B = K_A = K$.

Bit error rate (when $K_B = K_A$) is expected to be low due to negligibly small nonzero value of marginal distribution in the region $\zeta_r < -\zeta_c$ for $|\psi_0(\alpha)\rangle$. However, this can be circumvented by using error correction and privacy amplification [24,44], we will discuss this in the next section.

4 Information gain per transmitted state

In this section, we are going to calculate the average amount of information $G_{ab}$ (in bits) gained transferred Alice and Bob every time Alice sends a PASCS through a lossy channel [53], which can be modeled as passing through a beam splitter with transitivity $T$ and reflectivity $R$. Without any loss of generality, all the losses can be attributed to the eavesdropping attempts by Eve.

Suppose Alice transmits $|\psi_j(\alpha)\rangle$ to Bob. The Wigner function of the attenuated signal in this model can be described by two mode Wigner function of the state after $|\psi_j(\alpha)\rangle$ passes through the beam splitter as

$$\tilde{W}_j(\zeta, \epsilon) = W_j(T\zeta - R\epsilon, \alpha) \times W_j(R\zeta + T\epsilon, 0).$$

(4)

Here, we assume the other input of the beam splitter as a single mode vacuum state. Also, $W_j(\zeta, \alpha)$ is the Wigner function for the PASCS states sent by Alice corresponding to $j = 0$ and $1$ defined in Eq. (3). Using this the joint probability distribution can be computed as [26,53]

$$P_j(\zeta_x) = \int \tilde{W}_j(\zeta, \epsilon)d\zeta_x d^2\epsilon,$$

(5)

where $x, y \in \{r, i\} : x \neq y$. It can further be used to calculate the probability $P_j$ that Bob correctly infers the state $|\psi_j(\alpha)\rangle$ sent by Alice (i.e., gets bit $j$)

$$P_j = \int_{-\infty}^{-\zeta_c} P_j(\zeta_b)d\zeta_b,$$

(6)

while the probability $\bar{P}_j$ that Bob wrongly infers the state (i.e., gets bit $\bar{j}$) is

$$\bar{P}_j = \int_{-\infty}^{-\zeta_c} P_j(\zeta_a)d\zeta_a.$$

(7)
where notation is same as in Eq. (3). Hence, the fraction of accepted bits $r_{\text{acc}}$ is given by

$$r_{\text{acc}} = \frac{P_j + P_j}{2}$$

(8)

with factor $1/2$ corresponding to the probability that $i$th term in $K_A$ and the same in $K_B$ are the same. Further, the bit-error rate $\delta$ per conclusive result can be given as

$$\delta = \frac{P_j}{P_j + P_j}.$$  

(9)

Therefore, Shannon information between Alice and Bob per conclusive result can be calculated by averaging over Bob’s measurement outcomes

$$I_{ab} = \int_{-\infty}^{-\zeta_c} d^2 \xi \frac{P_j(\xi_b) + P_j(\xi_a)}{P_j + P_j} \left\{1 + \Phi(\xi) \log_2 \Phi(\xi) + (1 - \Phi(\xi)) \log_2 (1 - \Phi(\xi))\right\},$$

(10)

where error function $\Phi(\xi) = \frac{P_j(\xi_a)}{P_j(\xi_b) + P_j(\xi_a)}$. Thus, the average information $G_{ab}$ gained by Bob per transmitted state by Alice is

$$G_{ab} = I_{ab} r_{\text{acc}}.$$  

(11)

Privacy amplification of the shared key provides a lower bound to the secret information transmitted in one pulse as [54]

$$S_{ab} = r_{\text{acc}} (I_{ab} - \tau)$$

(12)

at the cost of reduction of the key size by $\tau = 1 + \log_2 P_{\text{coll}}$, where collision probability is

$$P_{\text{coll}} = \int d^2 \epsilon \frac{P_j(\epsilon \mid \zeta_c < |\zeta_b|)^2 + P_j(\epsilon \mid \zeta_c < |\zeta_a|)^2}{P_j(\epsilon \mid \zeta_c < |\zeta_b|) + P_j(\epsilon \mid \zeta_c < |\zeta_a|)}$$

(13)

with $P_j(\epsilon \mid \zeta_c < |\zeta_x|) = \int_{-\zeta_c}^{\zeta_c} \frac{P_j(\zeta_x, \epsilon)}{P_j + P_j} d\zeta_x$. Variation of fraction of accepted bits $r_{\text{acc}}$, mutual information $I_{ab}$, average information $G_{ab}$, and bit-error rate $\delta$ with threshold value $\zeta_c$ are shown in Fig. 2a and b which shows similar variation as reported in Ref. [53]. Here, the noteworthy point is extremely low bit-error rate provided by the present scheme (cf. Fig. 2b). Further, smaller value of $r_{\text{acc}}$, and thus $G_{ab}$, can be attributed to the fact that due to very small errors, reflected through small bit-error rate, contribution of wrong accepted bits is low as well as only non-unity value of transmittivity $T$ is considered here. However,
the larger value of information transmitted per accepted bit is due to the high value of \( \alpha \) (to visualize this compare Fig. 2a and b). The same set of parameters is also shown as functions of the transmission distance over an optical fiber with absorption rate 0.02 and Bob’s homodyne detection efficiency 0.9 in Fig. 2c, where one can clearly see the increase in the bit-error rate and decrease in both information transmitted per accepted bit and average information shared with losses in the channel. Finally, security of the extracted key can be enhanced by reducing the size of key in privacy amplification.

The present scheme is foundationally and operationally different from the CV-BB84 schemes based on squeezed states [53] and PASCS [26]. Specifically, a bit value 1 (0) is encoded as positive (negative) values of displacement in position and momentum quadratures in CV-BB84 schemes, while in the present scheme, the choice of position and momentum quadratures itself corresponds to the bit value. Specifically, the squeezed state-BB84 (SS-BB84) scheme [53] transmits information using quantum state $|\alpha| \exp \{i \theta \} \exp \left( s \exp \{-2i \theta \} \hat{a}^2 - s \exp \{2i \theta \} \hat{a}^\dagger \hat{a} \right) |0\rangle$ with $\theta = 0, \pi/2$ and $\theta = \pi, 3\pi/2$ for sending 0 and 1, respectively. Thus, Alice and Bob have to keep quadrature information secure in CV-B92 scheme to extract a key. A comparative study of the different parameters discussed for the CV-B92 scheme also establishes distinct features of the present scheme (cf. Fig. 3). Specifically, the fraction of accepted bits is relatively smaller for the present scheme when compared with CV-BB84 and SS-BB84 (shown in Fig. 3a). However, mutual information between Alice and Bob is higher in our scheme than for CV-BB84 schemes (cf. Fig. 3b), which can be attributed to the higher bit-error rate and nonzero probability of inconclusive measurement results by Bob. Specifically, the bit-error rate for smaller values of $\xi_c$ is less than that of SS-BB84, while for larger $\xi_c$ the CV-B92 scheme has maximum error rate as shown in Fig. 3c. This leads to smaller average information transmitted using CV-B92 schemes compared to CV-BB84 and SS-BB84 schemes (as shown in Fig. 3d). This is a limitation of the present scheme. However, it does not undermine the fact that our work shows feasibility of a foundationally unique scheme resistant to certain attacks possible on CV-BB84 and DV-B92 schemes further discussed in the next section.
5 Security analysis of the protocol

The security of the proposed QKD protocol against an adversary will be discussed under three specific attacks. We begin with the beam splitter attack in which Eve exploits the transmission losses by using a beam splitter. In another attack, Eve may intercept the transmitted signal pulses to measure both the quadratures. Finally, we will consider a particular attack referred to as USD attack which was successfully used to attack the original B92 protocol (DV version of our scheme).

5.1 Beam splitter attack

In the context of CV-QKD, a commonly discussed attack strategy is the beam splitter attack or superior channel attack [26,53,55]. In this attack, Eve uses a beam splitter to split the incoming signal pulses from Alice to Bob. She sends one of the outputs of the beam splitter to Bob, while keeps the other output in a quantum memory. Subsequently, during quadrature reconciliation at the end of quantum transmission, Bob announces
his choice of quadrature measurement and keeps only the outcomes when both Alice and Bob had chosen the same quadratures. During this step, Eve also comes to know the choice of quadrature by Alice and Bob. Subsequently, Eve performs measurement of the same quadratures in the corresponding pulses to infer the bit values shared by Alice and Bob.

It is easy to show that the protocol described in Sect. 3 is robust against the above mentioned attack. This is so because, to succeed in the beam splitter attack, Eve needs access to the classical information that Alice and Bob share during quadrature reconciliation. In the proposed protocol, there is no need for this classical communication at the end of the measurement, so Eve cannot obtain the information accessible to her in the CV-BB84 scheme [26] and looses the advantage of the beam splitter attack. This observation also establishes supremacy of the proposed CV B92 protocol over the earlier proposed CV-BB84 protocol [26].

5.2 Intercept and resend attack

Though beam splitter attack is not useful for Eve in the present case, she can adopt different strategies. For example, she may attempt an intercept and resend attack or simultaneous quadrature measurement attack [26,53]. In this attack, Eve uses a symmetric beam splitter (with $R = T$) to perform homodyne measurements of two different quadratures (say $\beta_r$ and $\epsilon_i$) in the two outputs of the beam splitter. Using the measurement outcomes, she prepares and sends a state to Bob, for which the joint probability distribution $P_j(\beta_r, \epsilon_i)$ is maximum. Both $|\psi_0(\alpha)\rangle$ and $|\psi_1(\alpha)\rangle$ have independent regions in phase space with $\beta_r \geq |\epsilon_i|$ and $\epsilon_i > |\beta_r|$ of area $A_0$, respectively. The probability that Eve will infer the state successfully, given the incoming state, $P_{\text{corr}}$ can be computed using Eq. (5) as

$$P_{\text{corr}} = \int_{A_0} P_j(\beta_r, \epsilon_i) d\beta_r d\epsilon_i. \quad (14)$$

Initially, before starting the protocol, Bob chooses a post-selection threshold $\zeta_c$ and Alice chooses the optimum $\alpha$ for a given fixed bit-error rate $\delta$. Eve sends the correct PASCS only with a probability $P_{\text{corr}}$ to Bob. Hence, lesser the value of $P_{\text{corr}}$, higher the probability of Eve getting detected by a significant change in the bit-error rate $\delta$ or the rate $r_{\text{acc}}$ of accepted bits or by a change in the probability distribution of Bob’s quadrature measurements.

5.3 Unambiguous state discrimination (USD) attack

This is a version of the intercept and resend attack which is known to be a major drawback for the DV B92 protocol [8]. In the DV B92 protocol, Eve intercepts all the transmitted qubits and measures the state sent by Alice randomly either in the computational ($Z$) basis or diagonal ($X$) basis. If she obtains a conclusive result, she re-prepares and sends the freshly prepared state to Bob by an ideal channel. On the other hand, if she obtains an inconclusive result, she does not resend anything.
and thus mimics a lossy channel. This attack cannot be performed in the proposed CV scenario because in the CV case, Bob performs homodyne measurement instead of the polarization measurement or the photon number measurement. Hence, he will always expect some signal coming towards him instead of vacuum. If Eve performs the USD attack on the protocol described in Sect. 3 and finds an inconclusive result, she would send a vacuum state and will easily get caught as Bob will not receive any signal. Therefore, this protocol is robust against the USD attack without any requirement of using an alternate basis measurements, uninformative states or entanglement [9,10].

However, Eve may modify the USD attack by replacing the lossy channel by an ideal channel. If she measures \( n' \) out of the \( n \) incoming states via homodyne detection and obtains a conclusive result, she re-prepares and sends the freshly prepared state to Bob. Otherwise, she prepares the state \(|\psi(\alpha e^{i\frac{\pi}{4}})\rangle\) and sends it to Bob.

For instance, in the presence of a lossy channel and in the absence of Eve, the probability of Bob getting an inconclusive results \( g/n \) is

\[
\frac{g}{n} = (1 - r_{acc}). \tag{15}
\]

However, if Eve performs this attack and the channel now is ideal, the probability \( Q_j \) that Eve guesses the correct state is

\[
Q_j(\alpha) = \frac{1}{2} \int_{-\infty}^{\xi_c} \int_{-\infty}^{\infty} W_j(\xi, \alpha) d\xi_a d\xi_b, \tag{16}
\]

where \( W_j(\xi, \alpha) \) is defined in Eq. (3), and 1/2 is the probability of choosing the right quadrature for measurement.

Similarly, the probability \( Q_{\bar{j}} \) that Eve guesses the wrong state is

\[
Q_{\bar{j}}(\alpha) = \frac{1}{2} \int_{-\infty}^{\xi_c} \int_{-\infty}^{\infty} (\int_{-\infty}^{\infty} W_j(\xi, \alpha) d\xi_b) d\xi_a. \tag{17}
\]

Thus, fraction \( k/n' \) of inconclusive results from Eve’s measured \( n' \) states will be

\[
\frac{k}{n'} = (1 - (Q_j(\alpha) + Q_{\bar{j}}(\alpha)))). \tag{18}
\]

Eve prepares state \(|\psi(\alpha e^{i\frac{\pi}{4}})\rangle\) corresponding to these cases and sends them to Bob. Out of these cases, fraction \( m/k \) for which Bob will get an inconclusive result is

\[
\frac{m}{k} = (1 - (Q_j(\alpha e^{i\frac{\pi}{4}}) + Q_{\bar{j}}(\alpha e^{i\frac{\pi}{4}}))). \tag{19}
\]

Notice that there is negligibly small but nonzero probability for Bob getting a conclusive result from the state \(|\psi(\alpha e^{i\frac{\pi}{4}})\rangle\) as well. Subsequently, for the remaining \( n' - k \)
Fig. 4 The probability of Bob’s inconclusive measurement results in lossy channel in absence of eavesdropping $g/n$ as function of the beamsplitter transmittivity $T$ and due to USD attack $N/n$ varying with Eve’s attack fraction $n'/n$ for $\alpha = 0.7$ and $\zeta_c = 0.6$.

$$\frac{N}{n} = \frac{m}{n} + \frac{k}{n'} \left(1 - \frac{k}{n'}\right).$$  \hspace{1cm} (22)
to exploit Eve’s inability to perform simultaneous measurement in more than one mutually unbiased basis. The inherent foundational aspect of B92 protocol, motivated many other works from the perspective of foundational issues. For example, in 2000, Phoenix et al. [60] tried to answer whether addition of a third state to the B92 protocol can improve its performance. He obtained a positive answer and that led to a set of cryptographic protocol which may be referred to as trine spherical code QKD protocol [61].

The present work addressed a couple of foundationally relevant questions in context of CV-QKD motivated by the conclusions drawn for DV-QKD. Firstly, the fact that B92 scheme established that two non-orthogonal states suffice to provide quantum security led to a question: Does such a situation persist or is it lost when going from DV to CV? The present work establishes that yes, the same move is possible. Secondly, the fact that secure key rate of B92 is less than BB84 protocol, led to another foundational question: Can the situation change in going from DV to CV? The present work addresses this question, and shows that this is not the case. Finally, the answers of the last two questions led to another relevant question: Is there any benefit of moving from DV to CV as far as the B92 protocol is concerned? The answer is obtained in affirmative, and it is established that CV-B92 gives a superior performance than DV-B92—with respect to lossy channels. This makes the present work interesting. It would be worth stressing here that additional uninformative states are used in DV-B92 protocol as a countermeasure for USD attack, which is an additional limitation in comparison to BB84. In contrast, there is no such additional limitation in corresponding CV counterpart. Finally, it would be apt to note that despite the fact that BB84 performs better than B92, it (B92) may still have some yet to be understood advantages as the fact that only coordinate information but not basis information is revealed places a severe constraint on Eve, and the implications of this in more general attacks (collective/coherent) merits further scrutiny.

6 Conclusion

A CV counterpart of B92 protocol is proposed here using PASCs. The proposed protocol is shown to be free from the limitations of the DV B92 protocol, which is prone to USD attack. Due to this fact that the proposed B92 scheme omits the requirement for any uninformative states or entanglement. On top of that, the proposed protocol is also resistant to all the eavesdropping strategies which exploit classical communication during basis reconciliation; for instance, it is intrinsically robust against the beam splitter attack. Therefore, the proposed scheme provides security against some of the individual attacks and omits requirement of a classical communication channel without compromising with the key rate and bit-error rate estimated for other similar CV-QKD schemes [26,53]. We have also established the security of our scheme against other individual attacks such as intercept and resend attack. However, practical implementation of the proposed scheme is expected to work over metropolitan distances only, while corresponding DV counterpart can be performed over longer distances [56]. Due to nonzero probability of inconclusive results, the average information gain per pulse becomes less than CV-BB84 schemes.
Security of our scheme over collective attacks remains an open problem and will be attempted in the future. The proposed scheme helps in sharing a discrete key between two parties by encoding it on a continuous quantum carrier and can also be extended to design an all continuous B92 scheme where continuous key can be shared with the help of a continuous quantum carrier [23]. With the well-known advantages of CV-QKD schemes over DV-QKD schemes, this protocols adds some additional benefits (robustness against certain attacks), and thus provides a potential scheme for practical implementation.

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