relations for c-type RR Lyrae Variables based upon Fourier Coefficients

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ABSTRACT

[Fe/H] - $\phi_{31}$ - $P$ relations are found for c-type RR Lyrae stars in globular clusters. The relations are analogous to that found by Jurcsik & Kovács (1996) for field ab-type RR Lyrae stars, where a longer period correlates with lower metallicity values for similar values of the Fourier coefficient $\phi_{31}$. The relations obtained here are used to determine the metallicity of field c-type RR Lyrae stars, those within $\omega$ Cen, the LMC and toward the galactic bulge. The results are found to compare favorably to metallicity values obtained elsewhere.

Key words: stars: abundances – stars: variables: other – globular clusters: general

1 INTRODUCTION

For nearly one hundred years, Fourier functions have been used to determine the pulsation period, $P$, of regular variables. Simon & Lee (1981) were able to show that other useful information apart from the period could be derived from Fourier functions. Their format of the relation is commonly used:

$$V(t) = A_0 + \sum_{i=1}^{n} A_i \cos(\omega t + \phi_i)$$

where the terms $A_i$ and $\phi_i$ are the Fourier coefficients of the fit of degree $n$, and $\omega = 2\pi/P$. Generally the values of $A_i$ and $\phi_i$ were ignored when this function was used to determine the value of $P$. Simon & Lee (1981) showed that these terms when combined in the following manner $-R_{ij} = A_i/A_j$ and $\phi_{ij} = j\phi_i - i\phi_j$, could provide more information about pulsating variables than at first thought. Currently, the coefficients derived from the light curves of Cepheids and RR Lyrae have been used to provide information about pulsation modes and resonance effects, as well as physical characteristics of the stars such as mass, luminosity, and metallicity.

Jurcsik & Kovács (1996, hereafter JK96) surveyed field ab-type RR Lyrae (RRab) and derived a relationship between $P$, $\phi_{31}$ and [Fe/H] for these variables which has been used in a variety of applications, in particular to determine the metallicity of stars found in large scale surveys (Morgan, Simet & Bargerquast 1998, hereafter MSB98), in globular clusters (Cacciari, Corwin & Carney 2005) and in other galaxies (Di Fabrizio et al. 2005). Sandage (2004) describes this relationship as an aspect of the Oosterhoff–Arp–Preston period-metallicity effect, in which the apparent shift of the value of $P$ or $\phi_{31}$ is dependent upon the influence metallicity has on horizontal branch morphology. Kovács & Walker (2001) expanded upon the work of JK96 by investigating possible relationships between the physical parameters of RRab stars and other Fourier coefficients. They found several trends in the value of photometric colours and magnitudes with the coefficients.

The utilization of the Fourier coefficients of the c-type RR Lyrae variables (RRc) has not been as extensive as that of the RRab stars. Simon (1989) used hydrodynamic models of RRc stars to show relationships exist between the values of helium abundance ($Y$), luminosity, mass, $P$ and $\phi_{31}$. In particular, he found that $\phi_{31}$ is directly related to the luminosity-mass ratio, $L/M^{1.81}$, for these stars. Simon (1990) showed that the distribution of $\phi_{31}$ with period would vary according to Oosterhoff group, with Oosterhoff I cluster stars having larger values of $\phi_{31}$ for a given value of $P$ than stars from Oosterhoff II clusters. Clement, Jankulak & Simon (1992) found strong evidence for the trend of $\phi_{31}$ increasing with period in their observations of several globular clusters, and also noted the trend for lower metallicity clusters to have their $\phi_{31}$ values shifted to longer periods than the higher metallicity clusters on a $\phi_{31}$-$P$ diagram. This result was also observed in the RRc variables in NGC 4590 (M68) by Clement, Ferance & Simon (1993). The general nature of this aspect of the variation of $\phi_{31}$ with metallicity has previously been used to estimate the metallicity of RRc stars in the OGLE surveys of the galactic bulge (MSB98) and toward 47 Tuc (Morgan & Dickerson 2000), however these results were at best only general approximations of [Fe/H].

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The lack of a metallicity relationship for the RRc stars analogous to that derived by JK96 is likely due to the scarcity of accurate metallicity and light curve data for field RRc variables. Generally metallicities for field RRc stars are estimated using the ΔS method of [Preston (1959)], with only a few metallicities derived from high resolution spectroscopy ([Butler et al. 1982] and [Lambert et al. 1996]; [Fernley & Barnes 1997]; [Solano et al. 1997]). In several cases, ΔS values obtained for a single RRc star can vary significantly, making the accuracy of the metallicity for such stars suspect.

Fortunately, there is an abundance of data for globular cluster RRc variables, including values for cluster metallicity and a significant number of Fourier coefficients. We will use these data to derive relationships for RRc stars relating the Fourier coefficient φ31, pulsation period and [Fe/H]. The resulting relations will also be applied to several test cases, including field RRc stars, as well as variables in ω Cen, the Large Magellanic Cloud and toward the galactic bulge.

2 METHOD

The Fourier coefficients for globular cluster RRc variables were obtained from the Fourier coefficient website [Morgan 2003] as well as several recent publications. The clusters used here are listed in Table 1. In order to avoid possible errors with conversion from one photometric system to another, only Fourier coefficients derived from V magnitudes were considered. Where ever possible the light curve data for the individual stars were examined and Fourier fits that were based upon sparsely sampled light curves or with poorly defined maxima/minima were excluded. Fourier coefficients with large uncertainties were also excluded. The sources for the Fourier coefficients, and the number of stars from each cluster ultimately used in this study are included in Table 1 as well. There are several sources for metallicity that could be used for globular clusters. Two metallicity scales that are frequently cited in the literature are those of Zinn & West (1984, hereafter ZW84) and Carretta & Gratton (1997, hereafter CG97). Both of these metallicity scales will be used here. For some clusters, the updated values of Zinn (1983) are used in place of the ZW84 values where available. When values were not available for clusters based on the system of CG97, the values found by ZW84 were converted to the CG97 scale using the relation of [Carretta et al. 2001]. The metallicity values and their corresponding uncertainties are also given in Table 1.

Overtone pulsators such as RRc stars generally have a lower order fit (small value of n in equation 1) used for the light curve, which limits the number of Fourier coefficients available. As was outlined in the introduction and shown in [Clement et al. 1992] and MSB98, the coefficient that appears to depend strongest upon [Fe/H] is the φ31 term. This trend is also observed in RRab stars as was noted by JK96 and [Clement & Shelton 1999]. Our study concentrated exclusively on this coefficient. The φ31 values for the 106 stars in our sample are plotted relative to P in Figure 1 with the coding in the diagram based upon the [Fe/H] values from Table 1.

The relation between [Fe/H] - P - φ31 found by JK96 for RRab stars is linear as is a similar relation found by [Sandage 2004]. It should be noted that Sandage (2004) used log P rather than P with virtually the same degree of accuracy as that obtained with the JK96 relation. We found the use of a P term to be slightly better than the use of a log P term when comparing the residuals of the derived relations to the observed values. The form of the relationship between [Fe/H], P and φ31 appears to depend upon the metallicity scale that is used, ZW84 or CG97. This is seen when the P values for the RRc stars in each cluster are increased by the same amount to place the φ31 values along a common distribution, as is illustrated in Figure 2. The dependence of

2 METHOD

| Cluster | Symbol | Source | N | [Fe/H]ZW | [Fe/H]CG |
|---------|--------|--------|---|----------|----------|
| NGC 6171 | × | 1 | 7 | −0.99 ± 0.06 | −0.97 ± 0.04* |
| NGC 6362 | × | 2 | 13 | −1.08 ± 0.09 | −0.96 ± 0.01 |
| NGC 1851 | □ | 3 | 4 | −1.33 ± 0.09 | −1.18 ± 0.05* |
| NGC 5904 | □ | 4 | 14 | −1.40 ± 0.06 | −1.11 ± 0.11 |
| NGC 6934 | △ | 5 | 6 | −1.54 ± 0.09 | −1.32 ± 0.07* |
| NGC 7089 | △ | 6 | 3 | −1.62 ± 0.07 | −1.38 ± 0.06* |
| NGC 5272 | △ | 7 | 17 | −1.66 ± 0.06 | −1.34 ± 0.06 |
| NGC 6333 | + | 8 | 5 | −1.78 ± 0.15 | −1.52 ± 0.16* |
| NGC 4147 | + | 9 | 8 | −1.80 ± 0.26 | −1.55 ± 0.28* |
| NGC 6099 | + | 10 | 5 | −1.82 ± 0.15 | −1.57 ± 0.17* |
| NGC 4590 | ▽ | 11 | 14 | −2.09 ± 0.11 | −1.99 ± 0.10 |
| NGC 7078 | ▽ | 12,13 | 10 | −2.15 ± 0.08 | −2.12 ± 0.01 |

Sources: 1 - Clement & Shelton (1997), 2 - Olech et al. (2003), 3 - Walker (1998), 4 - Kaluzny et al. (2000), 5 - Kaluzny, Olech & Stanek (2001), 6 - Clement & Shelton (1998), 7 - Cacciari et al. (2005), 8 - Clement & Shelton (1999), 9 - Stetson, Catelan & Smith (2003), 10 - Olech et al. (1999), 11 - Walker (1994), 12 - Clement & Shelton (1999), 13 - Arellano Ferro, Garcia Lugo, & Rosenweig (2006)

* - calculated using Carretta et al. (2001)
the $\phi_{31}$ values on $P$ is apparent, while the amount that was added to the period of each cluster’s RRc stars indicates the dependence of $P$ on the cluster’s metallicity. The amounts added to the periods of the RRc in each cluster in order to produce the distribution in Figure 2 are shown in Figure 3. For the CG97 scale, a linear relation appears to exist between the shift in $P$ and $[\text{Fe/H}]$, while for the ZW84 scale a non-linear relation appears to be appropriate. In the case of the ZW84 data, the standard deviation of the points from a linear fit is $\sigma = 0.145$, while a quadratic fit has a standard deviation of $\sigma = 0.092$. Applying a quadratic fit to the CG97 data results in a function that is nearly identical to the linear one, indicating that no improvement to the fit is made with a higher order function. Even though the CG97 scale does not appear to require a high-order non-linear relationship between $[\text{Fe/H}]$ and $P$, those relations were nonetheless examined.

In order to determine the best relationship between the three variables, all possible permutations were made between them with functions of the form

$$[\text{Fe/H}] = a P^2 + b P + c \phi_{31}^2 + d \phi_{31} + e P \phi_{31} + f,$$

(2)

tested for quality of fit to the data. All possible combinations of zero and non-zero values for the coefficients $a - f$ were examined using both the ZW84 and CG97 scales. Only relations which included at least one $P$ and $\phi_{31}$ term were considered. A summary of the resulting fits to the various formulae are shown in Table 2. This table shows the range of values for the sample standard deviation of the best fitting formula to the 106 data points. The best solution for the ZW84 scale is one with all six terms used in equation (2), while the quality of the best solutions for the CG97 scale does not vary significantly when fewer terms are used in equation (2). This result is expected given the linear relation of the $[\text{Fe/H}]$ values on $P$ as is shown in Figure 3 for the CG97 system.

Table 2. Sample Standard Deviations ($\sigma$) for solutions to equation (2)

| Metallicity | Terms | Range of $\sigma$ |
|-------------|-------|------------------|
| ZW84        | 6     | 0.145            |
|             | 5     | 0.152 – 0.161    |
|             | 4     | 0.160 – 0.170    |
|             | 3     | 0.162 – 0.174    |
| CG97        | 6     | 0.142            |
|             | 5     | 0.142 – 0.144    |
|             | 4     | 0.142 – 0.154    |
|             | 3     | 0.143 – 0.169    |

Two criteria were used for determining which formulae would be the most useful for calculating RRc metallicities. First was the quality of the fit to the data, which is summarized in Table 2. The second criterion was the simplicity of the formula. This criterion was only relevant for the CG97 solutions, where the quality of the formula varied insignificantly when the number of terms used in the solution were changed. The best fit formula for equation (2) to the clusters in Table 1 using the ZW84 scale is

$$[\text{Fe/H}]_{ZW} = 52.466P^2 - 30.075P + 0.131\phi_{31}^2 + 0.982\phi_{31} - 4.198\phi_{31} P + 2.424$$

(3)

which has a sample standard deviation of 0.145 dex. The best formula based upon the CG97 scale is

$$[\text{Fe/H}]_{CG} = 0.0348\phi_{31}^2 + 0.196\phi_{31} - 8.507P + 0.367$$

(4)

which has a sample standard deviation of 0.142 dex. 84% and 87% of the $[\text{Fe/H}]$ values based upon the above for-
Figure 4. Lines of constant [Fe/H], based upon the fits to the ZW84 (equation 3) and the CG97 (equation 4) metallicity scales are shown, along with the RRc data from Figure 1. [Fe/H] values for each line are given in the lower left corner of each graph.

Figure 5. Average cluster metallicities based upon ZW84 and equation 3 (top), and CG97 and equation 4 (bottom) are shown, along with a line of value unity.

3 TEST OF THE [FE/H] RELATIONS

Equations 3 and 4 were applied to several test cases to examine the quality of the relationships in determining [Fe/H] values in other environments. Field RRc stars were examined first. The number of field RRc stars with [Fe/H] values and good quality V magnitude light curves that could be used is very small, comprised of only 15 stars. [Fe/H] values for the stars were taken either from Fernley & Barnes (1997) or were calculated using published ∆S values and the ∆S - [Fe/H] relation of Fernley & Barnes (1997). Some stars have only one measured value of ∆S, while others have widely divergent values. The comparison of the [Fe/H] values from the literature and equations 3 and 4 is shown in Figure 6. The estimated uncertainty of the individual values of [Fe/H] from the literature varies with each source, but typical uncertainties are 0.2 dex or less. This uncertainty is similar to that found in the derivation of equations 3 and 4 (approximately 0.15 and 0.14 dex respectively). These uncertainties are displayed in Figure 6. The sample standard deviation of the average metallicity values from the literature and those derived using our formulae is 0.41 dex for equation 3 and 0.42 dex for equation 4. There are several stars that are well removed from the unity relation in Figure 6. These are TV Boo, ST CVn and V487 Sco. TV Boo ([Fe/H] = −2.44) has a metallicity value from the literature outside of the range used to derive equations 3 and 4 which may explain its divergent value. ST CVn (P = 0.329025 days) has a relatively large value of φ31 (5.11) for its value of P, which accounts for its abnormally high calculated values of [Fe/H]. V487 Sco’s metallicity ([Fe/H] = −1.89) is based upon a single value for ∆S. When these stars are excluded, the sample standard deviations reduce to 0.26 dex (equation 3) and 0.15 dex (equation 4).

The RRc stars in ω Cen were examined next. Data from the variables in this cluster were collected from the Fourier coefficient website (Morgan 2003) and an initial data set was selected based upon the method of observation, where recent, CCD based coefficients were favored over pho-
Figure 6. Values of [Fe/H] from the literature for field RRc stars compared to values based upon equations 3 (top) and 4 (bottom). TV Boo, V487 Sco, and ST CVn are indicated. The line indicates a ratio of unity.

Figure 7. The range of metallicity values for RRc stars in ω Cen, based upon equations 3 (top) and 4 (bottom).

Figure 8. Individual RRc stars in ω Cen with metallicities from the literature, compared to those based upon equations 3 (top) and 4 (bottom). The line indicates a ratio of unity. The typical errors associated with the data points are shown.

tographic observations. These included Fourier coefficients from Morgan & Dickerson (2000), and Olech et al. (2003). As with the globular cluster variables used to derive equations 3 and 4, the RRc stars in ω Cen were examined and poor quality fits or sparse light curves were excluded. This reduced the number of variables to 67, and equations 3 and 4 were applied to these stars. The resulting distribution of metallicity is shown in Figure 7. The relatively wide spread in metallicity is not surprising given the broad range of metallicity observed in ω Cen (Sollima et al. 2005). Values that we calculated for [Fe/H] range from −2.04 to −0.97, with a mean of −1.65 ± 0.24 using equation 3 and a range of −2.22 to −0.83, with a mean of −1.44 ± 0.25 for equation 4. The largest concentration of values is near −1.77 and −1.50 for equations 3 and 4 respectively. These peaks are near the value Sollima et al. (2005) found for the dominant metallicity population of ω Cen, [Fe/H] ∼ −1.7 dex.

There have been several studies that have found the metallicities of individual stars within ω Cen using a variety of methods, such as Cαy photometry (Rey et al. 2000), high-resolution spectroscopy (Sollima et al. 2006) and line indices (Gratton et al. 2004). These methods cover a total of 54 stars that also have good quality light curves and derived Fourier coefficients. The individual stars are compared to the metallicities based on equations 3 and 4 in Figure 8. The values based upon the ZW84 relation (equation 3) have a sample standard deviation from the published metallicity values of 0.25 dex, while the CG97 values (equation 4) have a deviation of 0.23 dex. Errors for individual stars are not shown in Figure 8 due to the crowded graph, however most values for errors from the literature are approximated at 0.2 dex. Errors from equations 3 and 4 are again taken to be 0.15 and 0.14 dex respectively.

Alcock et al. (2004) provided V-band Fourier coefficients for 682 RRc in the Large Magellanic Cloud. Equations 3 and 4 were applied to all of these stars and respective metallicity averages of [Fe/H] = −1.61 ± 0.40 and [Fe/H] = −1.42 ± 0.37 were found. There were significant deviations from these averages, with some positive metallicity values calculated. When stars with large errors (> 0.5) in their values of ϕ31 were removed from the sample, the resulting average metallicity changes slightly, to −1.66 ± 0.29
Gratton et al. (2004), who calculated metallicities using line indices. The statistical comparison to the 14 stars’ metallicities results in sample standard deviations of approximately 0.28 for both equation 3 and 4, with the star-by-star comparison illustrated in Figure 9. It should be noted that the metallicities found in the LMC and ω Cen from Gratton et al. (2004) study were calibrated using the [Fe/H] scale of Harris (1996), which is typically closer in value to the ZW84 system.

The Fourier coefficients for 60 RRc stars observed in the direction of the galactic bulge were derived from MSB98. These values were obtained from I-band photometry, and were transformed to the V-band using the relations from MSB98. The average metallicities obtained using equations 3 and 4 were −1.03 ± 0.50 and −0.98 ± 0.37 respectively. There are several notable outliers in the sample, some of which have been previously noted by MSB98 as likely having unusual metallicity values. These include BW9 V38, and BW11 V55, both of which have abnormally high values of [Fe/H], and BW1 V11, BW2 V8, BW2 V10, BW4 V46, BW7 V30, BW10 V45 and BW11 V34, all with at least one value of [Fe/H] > −1.75. When these 9 stars are excluded, the metallicity averages become −0.97 ± 0.36 (equation 3) and −0.91 ± 0.22 (equation 4). These metallicities compare favorably to the value obtained by Smolec (2003) for RRab stars in the galactic bulge ([Fe/H] = −1.04 ± 0.03).

4 CONCLUSIONS

Two formulae were derived that show the relationship between the Fourier coefficient φ31, pulsation period and [Fe/H] for RRc stars in 12 globular clusters. The formulae are based on the widely used metallicity scales of Zinn & West (1984) and Carretta & Gratton (1997). These relations (equations 3 and 4) were able to provide reliable estimates for the value of [Fe/H] for RRc stars in other environments, including ω Cen, the LMC, the galactic bulge and field RRc stars. Even though these results are encouraging, we do realize that the metallicity relations found here will no doubt evolve as more high quality light curves for globular cluster RRc stars are made available. In particular, values of φ31 for RRc stars in clusters with very high or very low metallicities would be useful to refine and improve the formulae derived here, which were based upon cluster [Fe/H] values between approximately −1 and −2. At the present time it is hoped that the formulae presented here will be of use to others investigating the characteristics of RR Lyrae variables.

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Figure 9. Individual RRc stars in the LMC with metallicities taken from Gratton et al. (2004), compared to those based upon equations (top) and (bottom). The line indicates a ratio of unity.

(42x-559)–(42x-429)
[Fe/H] relations for c-type RR Lyrae Variables

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