Issues related to an attempt to recreate the geometry of a non-standard spur gear

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Abstract. Modern machine manufacturers are making the design and technology of their products more and more complicated. This is to protect against a frequently used practice at customers, i.e. making extra parts on your own. This is because entrepreneurs often cannot afford to order expensive original spare parts and - using reverse engineering - prepare working drawings and commission the components to be made in their own machine park or externally from local suppliers. However, the matter is more complex in the case of gears, which so far have been designed on the basis of the selection of standard geometric parameters. A small modification of one or more of these parameters is enough and it becomes very difficult to recreate the geometry of such a gear. This paper presents the issues related to the reverse engineering of a spur involute gear with very non-standard parameters \( m = 4.98 \), \( \alpha = 26.325 \, ^\circ \), \( x = 0.0695 \), \( y = 0.795 \), \( c^* = 0.383 \). Further metrological steps were proposed that should be taken to correctly identify at least the fact that the test object is not a part produced by standard modular tools (Fellows cutter, Maag rack cutter, worm cutter, etc.). The work also includes short graphical analyzes of the recreated geometry.

Keywords: spur gear, gear design, gear geometry, gear reverse engineering, gear modification, non-standard gear, involute profile, gear measurement

1. Introduction

It has been common practice to make parts on your own for a long time - it is especially known in the automotive industry, where you can find a multitude of replacements for original car parts. The process of such making parts is usually done according to a simple scheme:

- metrological measurements of the new or used / defective part,
- creation of a 3D model based on measurements and preparation of an technical drawing,
- ordering the parts based on the developed documentation,
- quality control of the manufactured part and its installation in a machine set,
- diagnosis to verify the correctness of the designed spare part.

However, machine manufacturers try to make such a process as difficult as possible in order to protect themselves against the practice of making spare parts on your own. This is usually manifested
in the design of dedicated parts instead of the selection of standardized ones (e.g., a non-standard prismatic groove or a spline coupling with non-standard geometric parameters). The matter is even more complicated in the case of gears, which in themselves, with the recommended parameters, are difficult to recreate the geometry. If even one gear parameter is deviated from the standard values, it becomes very difficult to reverse engineer that part. There is also a case when it is necessary to regain the geometry of the inch gear, where converting to the metric system is associated with less structural dimensions.

This paper presents the problem of recreating the geometry of a spur gear with very non-standard parameters. It is only an attempt, because - as the research results will show - the recreated gear only seemingly agrees with the real physical one. The article was prepared mainly for average enterprises that cannot afford expensive measuring devices. Subsequent works from this series will show how to deal with recreating such a complex gear using standard measuring instruments and techniques.

Most of the steps taken in the study were based on the knowledge and methodology presented by [1], [2] and [3]. The standard [4] was used to select the nominal modules. No similar works on reverse engineering of geometrically complex spur gears have been found in the available scientific databases.

2. Presentation of the tested gear

For the purpose of validating the results of the initial identification of the tested object, the authors have technical documentation of the gear wheel. Most of the geometrical parameters are included in the detailed drawing. The rest, also important from the point of view of construction and metrology, have been calculated. All key figures are presented in Table 1.

| Parameter                              | Symbol | Value         |
|----------------------------------------|--------|---------------|
| Module                                 | m      | 4.98          |
| Number of teeth                        | z      | 18            |
| Tip diameter                           | \(d_a\) | 98.250\(\pm0.06\) |
| Pitch diameter                         | \(d\)  | 89.640        |
| Base diameter                          | \(d_b\) | 80.344        |
| Root diameter                          | \(d_f\) | 78.600\(\pm0.8\) |
| Operating pressure angle               | \(\alpha\) | 26.325\(^\circ\) |
| Shifting factor                        | \(x\)  | +0.0695       |
| Total depth                            | \(h\)  | 10.955        |
| Addendum                               | \(h_{ak}\) | 5.326        |
| Dedendum                               | \(h_{fk}\) | 5.629        |
| Radial clearance factor                | \(c^*\) | 0.383         |
| Addendum modification                  | \(y\)  | 0.795         |
| Circular tooth thickness at pitch diameter | \(\overline{s}_{ck}\) | 8.165\(\pm0.025\) |
| Span measurement across 3 teeth        | \(M_{n1}\) | 38.201\(\pm0.022\) |
| Chordal addendum                       | \(\overline{h}_{ck}\) | 4.491        |
| Chordal tooth thickness                | \(\overline{s}_{ck}\) | 8.154        |

3. Gear measurement

The following experimental part is divided into successive assumptions, in which certain geometrical relationships defining a specific type of tooth are made in advance and the correctness of
this assumption is checked. If it is completely incorrect, it goes on to the next assumption. First of all, the assumption of the so-called “Zero” is that we know nothing about the gear except the number of teeth that can be directly counted. Measurements were carried out using basic measuring tools, because, as mentioned earlier in the introduction – this article is to serve average companies that cannot afford expensive metrological devices.

**Assumption I:** Standard gear (addendum modification $y = 1$, shifting factor $x = 0$)

Having a gear wheel physically, you cannot immediately guess that it is non-standard. The only thing that is certain is the number of teeth $z = 18$. Therefore, the first step in the measurement will be a common practice recommended, among others by [1], i.e. using a caliper to measure the tip diameter $d_t$ and root diameter $d_r$ (Figure 2) and the basic conversion of the total depth $h$ according to the formula 1. The results of the measurements are presented in Table 2.

**Figure 1.** Initial measurement of the tested gear.

**Table 2.** Results from the first measurement.
| Parameter       | Symbol | Cross-section measurement [mm] | Average [mm] |
|-----------------|--------|-------------------------------|--------------|
| Tip diameter    | $d_a$  | 98.23                         | $\approx 98.22$ |
| Root diameter   | $d_f$  | 77.82                         | $\approx 77.83$ |

\[
h = \frac{d_a - d_f}{2} = \frac{98.23 - 77.82}{2} = 10.195 \text{ mm}
\] (1)

Assuming that the clearance factor can assume values $c^* = 0.1-0.3$ [1], you can check the value of the nominal module $m$ from formula 2.

\[
m = h \frac{2.1 + 2.3}{2.1 + 2.3} = \frac{10.195}{2.1 + 2.3} = 4.85 \div 4.43 \text{ mm}
\] (2)

According to the standard [2], the closest values from the series of normalized nominal modules are:

- value $m = 4.5$ (series II)
- value $m = 5.0$ (privileged module)

The value of 4.5 is in the calculated range, while the modulus of 5.0 is close to the right side of the range. Further assumptions should be considered, however, it should be remembered that the root diameter $d_f$ is often tolerated with a relatively large tolerance field $T_{d_f}$. Therefore, the nominal value of this diameter may significantly differ from the average value of the measurement, which directly complicates the matter of reconstructing the geometry.

**Assumption II:** Standard gear (addendum modification $y = 1$, shifting factor $x = 0$), module $m = 4.5$, clearance factor $c^* = 0.2$

Theoretical values of individual geometric parameters were calculated from the formulas 3-5 (including the pitch diameter $d$).

\[
d = zm = 18 \cdot 4.5 = 81 \text{ mm}
\] (3)

\[
d_a = d + 2h_a = zm + 2m = (x + 2)m = (18 + 2)4.5 = 90 \text{ mm}
\] (4)

\[
d_f = d - 2h_f = zm - 2(m + c) = zm - 2(m + c^*m) = zm - 2(m + 0.2m)
\] (5)

\[
d_f = zm - 2 \cdot 1.2m = zm - 2.2m = (x - 2.2)m = (18 - 2.2)4.5 = 71.1 \text{ mm}
\]

The calculated value of 90 mm of the tip diameter $d_a$ is much smaller than the actual measured 98.22 mm. Therefore, there is no chance that this is standard gear with the nominal module $m = 4.5$.

**Assumption III:** Addendum modification $y > 1$, shifting factor $x = 0$, module $m = 4.5$, clearance factor $c^* = 0.2
Subsequent calculations using formulas 6-9.

\[ h_a = \frac{d_a - d}{2} = \frac{98.22 - 81}{2} = 8.61 \text{ mm} \] (6)

\[ h_a = y \cdot m \Rightarrow y = \frac{h_a}{m} = \frac{8.61}{4.5} = 1.91 \] (7)

\[ h_f = y \cdot m + c = 1.91 \cdot 4.5 + 0.2 \cdot 4.5 = 9.495 \] (8)

\[ d_f = d - 2h_f = 81 - 2 \cdot 9.495 = 62.01 \text{ mm} \] (9)

First of all, such large addendum modification are not used in practice \((y = 1.91)\). In addition, the root diameter \(d_f = 62.01 \text{ mm}\) strongly differs from the measured value of 77.83 mm.

**Assumption IV:** Addendum modification \(y = 1\), shifting factor \(x \neq 0\), module \(m = 4.5\), clearance factor \(c^* = 0.2\)

Subsequent calculations using formulas 10-11.

\[ h_{a0} = m = 4.5 \text{ mm} \] (10)

\[ h_{ak} = 8.61 \text{ mm} \] (11)

The above values already suggest that the shifting factor \(x\) would have to be horrendously high in order to modify the size of the addendum so much, so further calculations were abandoned. This is preceded by visual examination of the teeth which do not look significantly modified.

**Assumption V:** Addendum modification \(y = 1\), shifting factor \(x \neq 0\), module \(m = 5.0\), clearance factor \(c^* = 0.2\), standard operating pressure angle \(\alpha = 20^\circ\)

Subsequent calculations using formulas 12-17.

\[ d = zm = 18 \cdot 5 = 90 \text{ mm} \] (12)

\[ h_{a0} = 5.0 \text{ mm} \] (13)

\[ h_{ak} = \frac{d_a - d}{2} = \frac{98.22 - 90}{2} = 4.11 \text{ mm} \] (14)

\[ h_{ak} = (y + x)m = ym + xm \Rightarrow x = \frac{h_{ak} - ym}{m} = \frac{h_{ak} - y}{m} \] (15)

\[ x = \frac{4.11}{5.0} - 1 = -0.178 \]

This is a satisfactory value at that time. Therefore, the root diameter was checked \(d_f\) for different coefficients of the clearance factor \(c^*\).

\[ h_f = (y - x)m + c \text{ (for } c^* = 0.1) \] (16)

\[ h_f = (y - x)m + c = [1 - (-0.178)]5 + 0.1 \cdot 5 = 6.39 \text{ mm} \]
\[ d_f = d - 2h_f = 90 - 2 \cdot 6.39 = 77.22 \text{ mm} \]

\[ h_f = (y - x)m + c \text{ (for } c^* = 0.3) \]  

(17)

\[ h_f = (y - x)m + c = [1 - (-0.178)]5 + 0.3 \cdot 5 = 7.39 \text{ mm} \]

\[ d_f = d - 2h_f = 90 - 2 \cdot 7.39 = 75.22 \text{ mm} \]

So

\[ d_f = 75.22 \div 77.22 \text{ mm} \]

The measured value of \( d_f = 77.83 \text{ mm} \) is outside the given range but is close to the right side of the range. There can be many conclusions. For example, either the clearance factor \( c^* \) was used, exceeding the recommended range of 0.1 - 0.3, or the gear was made on the root diameter \( d_f \) within an unknown range of the tolerance field \( T_{d_f} \).

To verify the correctness of the parameters generated so far, the tooth thickness was measured with a vernier caliper, previously determining the appropriate parameters from formulas 18-22, including tooth thickness \( \overline{s}_{ck} \) measured along the pitch circle arc, center angle \( \varphi \) corresponding to half the tooth thickness, chordal thickness \( \overline{s}_{ck} \) and chordal addendum \( \overline{h}_{ck} \).

\[ \overline{s}_{ck} = \left( \frac{\pi}{2} + 2 \cdot x \cdot \tan \alpha \right) \cdot m \]  

(18)

\[ \overline{s}_{ck} = \left[ \frac{\pi}{2} + 2 \cdot (-0.178) \cdot \tan 20^\circ \right] \cdot 5 = 7.206 \text{ mm} \]

\[ \varphi = \frac{180 \cdot \overline{s}_{ck}}{\pi \cdot d} = \frac{180 \cdot 7.206}{\pi \cdot 90} = 4.587^\circ \]  

(19)

\[ \overline{s}_{ck} = d \cdot \sin \varphi = 90 \cdot \sin 4.587^\circ = 7.197 \text{ mm} \]  

(20)

\[ \overline{h}_{ck} = h_{ak} = 4.11 \text{ mm} \]  

(21)

\[ \overline{h}_{ck} = \overline{h}_{ck} + r \cdot (1 - \cos \varphi) = 4.11 + \frac{90}{2} \cdot (1 - \cos 4.587^\circ) = 4.254 \text{ mm} \]  

(22)

The measurement showed:

\[ \overline{s}_{ck} = 7.96 \text{ mm} \]

Thus, the measured result of 7.96 mm does not coincide with the theoretical value of 7.197 mm generated on the basis of preliminary assumptions regarding the basic geometric parameters. However, assuming no. V, the most steps succeeded. Therefore, it is further assumed that the nominal modulus is \( m = 5 \text{ mm} \).

**Assumption VI:** Addendum modification \( y = 1 \), shifting factor \( x = -0.178 \), module \( m = 5.0 \), clearance factor \( c^* = 0.2 \).
non-standard operating pressure angle $\alpha \neq 20^\circ$

The span measurement across “n” and “n-1” teeth with a disc micrometer was taken in order to
determine the base pitch $p_b$. The number of teeth measured by formula 23 was roughly determined,
temporarily using standard values as parameters.

$$n = \frac{z}{9} + 0.5 = \frac{18}{9} + 0.5 = 2.5 \rightarrow 3$$ (23)

Measurement results across 3 teeth $M_{n1}$ and across 2 teeth $M_{n2}$ are:

$$M_{n1} = 38.18 \text{ mm}$$
$$M_{n2} = 24.08 \text{ mm}$$

The value of the base pitch was calculated from the formula

$$p_b = \bar{t}_{p} = M_{n1} - M_{n2} = 38.18 - 24.08 = 14.10 \text{ mm}$$ (24)

The nominal pressure angle has now been calculated from the formula

$$p_b = \pi \cdot m \cdot \cos \alpha \rightarrow \alpha = \cos^{-1} \left( \frac{p_b}{\pi \cdot m} \right) = \cos^{-1} \left( \frac{14.10}{\pi \cdot 5} \right) = 26.151^\circ$$ (25)

All the necessary parameters for measuring with a vernier caliper from the formulas
26-30 were recalculated.

$$s_{\text{ck}} = \left( \frac{\pi}{2} + 2 \cdot x \cdot \tan \alpha \right) \cdot m$$ (26)

$$s_{\text{ck}} = \left[ \frac{\pi}{2} + 2 \cdot \left( -0.178 \right) \cdot \tan 26.151^\circ \right] \cdot 5 = 6.98 \text{ mm}$$

$$\varphi = \frac{180 \cdot s_{\text{ck}}}{\pi \cdot d} = \frac{180 \cdot 6.98}{\pi \cdot 90} = 4.44^\circ$$ (27)

$$s_{\text{ck}} = d \cdot \sin \varphi = 90 \cdot \sin 4.44^\circ = 6.967 \text{ mm}$$ (28)

$$h_{\text{ck}} = h_{\text{ak}} = 4.11 \text{ mm}$$ (29)

$$h_{\text{ck}} = h_{\text{ck}} + r \cdot (1 - \cos \varphi) = 4.11 + \frac{90}{2} \cdot (1 - \cos 4.44^\circ) = 4.245 \text{ mm}$$ (30)

The tooth thickness on the pitch diameter after measurement is $s_{\text{ck}} = 8.00 \text{ mm}$. This value again
does not equal the calculated one, this time 6.967 mm. It is concluded that any of the parameters $x$, $y$,
$m$ or $\alpha$ still has a value different from that assumed initially. In the next assumption, the nominal
modulus $m = 5$ is still assumed, therefore the same pressure angle is prescribed $\alpha = 26.151^\circ$, but this
time the addendum modification $y < 1$ are assumed. Therefore, the shifting factor $x$ is re-assumed as the
unknown.

Assumption VII: Addendum modification $y < 1$,
shifting factor $x \neq 0$, module $m = 5.0$, clearance factor undefined ($c^* \neq 0.2$),
non-standard operating pressure angle ($\alpha = 26.151^\circ$)

Formula 31 calculates the theoretical value of the measurement $M$ across “$n$” teeth.

$$M = m \cdot \cos \alpha \cdot [(n - 0.5) \cdot \pi + z \cdot \text{inv } \alpha] + 2 \cdot x \cdot m \cdot \sin \alpha$$  \hspace{1em} (31)

The involute unfold angle was calculated from formulas 32-33.

$$\hat{\alpha} = \frac{\alpha^\circ \cdot \pi}{180^\circ} = \frac{26.151^\circ \cdot \pi}{180^\circ} = 0.456421 \text{ rad}$$  \hspace{1em} (32)

$$\text{inv } \alpha = \tan \alpha - \hat{\alpha} = \tan 26.151^\circ - 0.456421 = 0.034578 \text{ rad}$$  \hspace{1em} (33)

At the initial assumption of the module $m = 5$, the shifting factor $x$ was calculated. The previously measured value of $M$ was inserted in the formula below for the measurement across 3 teeth.

$$M = m \cdot \cos \alpha \cdot [(n - 0.5) \cdot \pi + z \cdot \text{inv } \alpha]$$

$$2 \cdot x \cdot m \cdot \sin \alpha = M - m \cdot \cos \alpha \cdot [(n - 0.5) \cdot \pi + z \cdot \text{inv } \alpha]$$

$$x = \frac{M - m \cdot \cos \alpha \cdot [(n - 0.5) \cdot \pi + z \cdot \text{inv } \alpha]}{2 \cdot m \cdot \sin \alpha}$$

$$x = \frac{38.18 - 5 \cdot \cos 26.151^\circ \cdot [(3 - 0.5) \cdot \pi + 18 \cdot 0.034578]}{2 \cdot 5 \cdot \sin 26.151^\circ} = +0.0309$$

Again, simple calculations of the basic parameters from formulas 34-44 were made. It was assumed that the dedendum must reach the same height as the measured tooth, and thus the clearance factor $c^*$ was determined. Then, it was assumed that the root diameter would be made relatively close to tolerance, so that a similar geometric effect could be obtained in the end as in the tested gear.

$$h_{ak} = d_a - d = \frac{98.22 - 90}{2} = 4.11 \text{ mm}$$  \hspace{1em} (34)

$$h_{ak} = (y + x)m = ym + xm \rightarrow x = \frac{h_{ak} - ym}{m} = \frac{h_{ak} - y}{m}$$  \hspace{1em} (35)

$$x = \frac{h_{ak}}{m} - y \rightarrow y = \frac{h_{ak}}{m} - x = \frac{4.11}{5} - 0.0309 = 0.7911$$

$$d_f = d - 2h_{fk} \rightarrow h_{fk} = \frac{d - d_f}{2} = \frac{90 - 77.83}{2} = 6.085$$

$$h_{fk} = (y - x)m + c \rightarrow c = h_{fk} - (y - x)m = 6.085 - (0.7911 - 0.0309)5 = 2.28 \text{ mm}$$  \hspace{1em} (36)

$$c^* = \frac{c}{m} = \frac{2.28}{5} = 0.456$$  \hspace{1em} (37)

$$s_{ck} = \left(\frac{\pi}{2} + 2 \cdot x \cdot \tan \alpha\right) \cdot m$$  \hspace{1em} (38)

$$s_{ck} = \left[\frac{\pi}{2} + 2 \cdot 0.0309 \cdot \tan 26.151^\circ\right] \cdot 5 = 8.0057 \text{ mm}$$

$$\phi = \frac{180 \cdot s_{ck}}{\pi \cdot d} = \frac{180 \cdot 8.0057}{\pi \cdot 90} = 5.0965^\circ$$  \hspace{1em} (39)

$$\phi = \frac{180 \cdot s_{ck}}{\pi \cdot d} = \frac{180 \cdot 8.0057}{\pi \cdot 90} = 5.0965^\circ$$  \hspace{1em} (40)
\[ \bar{s}_{ck} = d \cdot \sin \varphi = 90 \cdot \sin 5.0965^\circ = 7.995 \text{ mm} \]  
(41)

\[ \bar{h}_{ck} = h_{ak} = 4.11 \text{ mm} \]  
(42)

\[ \bar{h}_{ck} = \bar{h}_{ck} + r \cdot (1 - \cos \varphi) = 4.11 + \frac{90}{2} \cdot (1 - \cos 5.0965^\circ) = 4.2879 \text{ mm} \]  
(43)

The measurement came out \( \bar{s}_{ck} = 8.04 \text{ mm} \). The difference was calculated from formula 44.

\[ \Delta \bar{s}_{ck} = |7.995 - 8.04| = 0.045 \text{ mm} \]  
(44)

This is a relatively small difference, so you could (but only seemingly) say that the geometry was correctly recovered. It should be remembered that during the calculations, some parameters were burdened with manufacturing errors, some measurement errors, so there is no guarantee that everything turned out correctly.

4. Summary

In the tested gear it was only fortunate that the nominal module 4.98 is very close to the standard module 5.00 and relatively the recreation of the gear geometry can be considered successful. It is envisaged that the application of the preliminary identification presented would not be applicable if this deviation from the normalized value would be even further different, e.g. if the nominal modulus would be 4.88.

There is a need to develop a special methodology for recreating such complex gears with non-standard geometric parameters. It is already planned to create a dedicated algorithm, on which the authors are working and which can be the basis for a special computer program. This, in turn, will generate the correct geometric parameters only on the basis of a photographic image of the gear.

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