Irreversible thermodynamics of dark energy on the entropy-corrected apparent horizon

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Abstract

We study the irreversible (non-equilibrium) thermodynamics of the Friedmann–Robertson–Walker (FRW) universe containing only dark energy. Using the modified entropy–area relation that is motivated by loop quantum gravity, we calculate the entropy-corrected form of the apparent horizon of the FRW universe.

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1. Introduction

Observations of type Ia supernovae suggest that the universe is dominated by two dark components: dark matter (DM) and dark energy (DE) [1]. DM, a matter without pressure, is mainly used to explain galactic rotation curves and the formation of large-scale structure, while DE, an exotic energy with negative pressure, is used to describe the present cosmic-accelerated expansion. However, the nature and origin of DE are still unknown, and people have proposed several candidates to describe it (for reviews, see [2] and references therein).

The holographic DE (HDE) is one of the interesting DE candidates that were proposed based on the holographic principle [3]. According to the holographic principle, the number of degrees of freedom in a bounded system should be finite and has a relationship with the area of the boundary of the system [4]. By applying the holographic principle to cosmology, one can obtain the upper bound on the entropy contained in the universe [5]. Following this line, Li [6] argued that in quantum field theory, the ultraviolet cut-off $\Lambda$ could be related to the infrared cut-off $L$ due to the limit set by forming a black hole, i.e. the quantum zero-point energy of a system with size $L$ should not exceed the mass of a black hole with the same size, i.e. $L^3\Lambda^3 \leq (M_p L)^{3/2}$. This last expression can be rewritten as $L^3\rho_\Lambda \leq L M_p^2$, where $\rho_\Lambda \sim \Lambda^4$ is the energy density corresponding to the zero-point energy and cut-off $\Lambda$. Now the last inequality takes the form $\rho_\Lambda \leq M_p^2 L^{-2}$ or $\rho_\Lambda = 3c^2 M_p^2 L^{-2}$. Here, $M_p^2 = (8\pi G)^{-1}$ is the modified Planck mass and $3c^2$ is constant and attached for convenience. The HDE models have been studied widely in the literature [7–9]. Obviously, in the derivation of HDE, the black hole entropy $S_{BH}$ plays an important role. As is well known, $S_{BH} = A/(4G)$, where $A \sim L^2$ is the area of the horizon. However, as per the literature, this entropy–area relation can be modified to the equation [10]

$$S_{BH} = \frac{A}{4G} + \tilde{\alpha} \ln \frac{A}{4G} + \tilde{\beta},$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ are dimensionless constants of the order of unity. These corrections can appear in the black hole entropy in loop quantum gravity (LQG) [11]. They can also be due to thermal equilibrium fluctuation quantum fluctuation or mass and charge fluctuations (for a good review, see [11] and references therein).

An interesting aspect in the cosmological context is the study of non-equilibrium thermodynamics of DE. Das et al [12] computed leading-order corrections to the entropy of any thermodynamic system due to small statistical fluctuations around equilibrium. They obtained a general logarithmic correction to black hole entropy. Wang et al [13] studied a thermodynamical description of the interaction between HDE and DM. Resorting to the logarithmic
correction to the equilibrium entropy, they arrived at an expression for the interaction term that was consistent with observational tests. Pavón and Wang [14] considered a system composed of two subsystems, (DM and DE) at different temperatures. By virtue of the extensive property, the entropy of the whole system is the sum of the entropies of the individual subsystems, which (being equilibrium entropies) are just functions of the energies of DE and DM. Zhou et al [15] have further employed the second law of thermodynamics to study the coupling between the DE and DM in the universe by resorting to the non-equilibrium entropy of extended irreversible thermodynamics. Karami and Ghaffari [16] extended the work of Zhou et al [15] and investigated the validity of the generalized second law in irreversible thermodynamics for the interaction of DE with DM in a non-flat FRW universe enclosed by the dynamical apparent horizon. It was shown that for the present time, the generalized second law in non-equilibrium thermodynamics is satisfied for the special range of energy transfer constants. More recently, Gang and Wen-Biao [17] studied the non-equilibrium thermodynamics of DE on the cosmic apparent horizon. They clarified that if the irreversible process is considered, the proper position for building thermodynamics will not be the apparent horizon anymore. The new position is related to the DE state equation and the irreversible process parameters.

In this paper, our aim is to extend the work of Gang and Wen-Biao [17] using the quantum-corrected entropy–area relation [11]. We investigate how the original apparent horizon can be modified to keep the non-equilibrium thermodynamic laws in effect. This paper is organized as follows. In section 2, we study the non-equilibrium thermodynamics of DE on the entropy-corrected apparent horizon. Section 3 is devoted to the conclusions.

2. Non-equilibrium thermodynamics of dark energy (DE) on the entropy-corrected apparent horizon

We consider the Friedmann–Robertson–Walker (FRW) metric for the non-flat universe as

\[ ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right). \] (2)

where \( k = 0, 1, -1 \) represents a spatially flat, closed or open FRW universe, respectively. Define \( \tilde{r}(t) = a(t)r \); the metric (2) can be rewritten as \( ds^2 = h_{ij} dx^i dx^j + \tilde{r}^2 d\Omega^2 \), where \( x^i = (t, r), h_{ij} = \text{diag}(-1, a^2/(1 - kr^2), 1, j = 0, 1, 2). \) By definition,

\[ f := h^{ij} \tilde{\partial}_i \tilde{\partial}_j \tilde{r} = 1 - \left( \frac{H^2 + \frac{k}{a^2}}{\tilde{r}} \right) \tilde{r}^2 \] (3)

when \( f = 0 \), the location of the horizon in the FRW universe is found to be \( \tilde{r} = R_A = (H^2 + k/a^2)^{-1/2} \) [18]. Observational evidence suggests that our universe is spatially flat with \( k = 0 \) [19], so the apparent horizon is the same as the Hubble horizon, i.e. \( R_A = 1/H \).

With the help of equation (3), the surface gravity \( \kappa \) and the Hawking temperature \( T \) are defined as [20]

\[ \kappa = -\frac{\partial f}{\partial t} = \frac{\tilde{r}}{R_A^2}, \] (4)

\[ T = \frac{\kappa}{2\pi} = \frac{\tilde{r}}{2\pi R_A^2}. \] (5)

So, the Hawking temperature at the apparent horizon is obtained as

\[ T_A = T \bigg|_{\tilde{r}=R_A} = \frac{1}{2\pi R_A}. \] (6)

Recently, Cai et al [21] proved that the apparent horizon of the FRW universe has an associated Hawking temperature \( T_A = 1/2\pi R_A \). They also showed that this temperature can be measured by an observer with the Kodoma vector inside the apparent horizon.

The first Friedmann equation for the flat FRW universe takes the form

\[ H^2 = \frac{8\pi}{3} \rho, \] (7)

where we take \( G = 1 \) and \( \rho \) is the energy density of DE. The continuity equation for DE is

\[ \dot{\rho} + 3H(1 + \omega)\rho = 0, \] (8)

where \( \omega \) is the parameter of the equation of state (EoS) of DE, and when \( \omega < -1/3 \) it describes an accelerating universe. For a constant \( \omega \), from equations (7) and (8) we obtain

\[ a(t) = t^{1/\epsilon}, \quad \epsilon := \frac{3}{2}(1 + \omega) \] (9)

and the apparent (Hubble) horizon is obtained as \( R_A = 1/H = \epsilon t \). Note that to ensure that \( R_A \) is positive, it is also needed to assume \( \epsilon > 0 \) or \( \omega > -1 \). Therefore, the DE with \( -1 < \omega < -1/3 \) behaves like a quintessence scalar field model [22]. The flux of energy across the apparent horizon within the time interval \( dt \) is

\[ -dE_A = 4\pi R_A^2 \rho (1 + \omega)dt = \epsilon dt. \] (10)

The process of the energy flux crossing the apparent horizon is irreversible. Therefore, in the presence of interaction between DE and its surroundings enveloped by the horizon, the time derivative of the non-equilibrium entropy is given by

\[ \dot{S} = \dot{S}_i + \dot{S}_e, \] (11)

where \( \dot{S}_i \) is the rate of change in internal entropy production of the universe and \( \dot{S}_e \) arises from the heat flow between the universe and the horizon. Following [17], we have

\[ \dot{S}_i = \int_{\Sigma} \sigma dV, \] (12)

\[ \dot{S}_e = -\int_{\Sigma} \tilde{J}_S \cdot d\Sigma, \] (13)

where \( \sigma \) is an internal entropy source production density and \( \tilde{J}_S \) is an entropy flow density. If we consider only the heat conduction between the universe and the horizon, then \( \sigma \) and \( \tilde{J}_S \) are given as [17]

\[ \sigma = \tilde{J}_q \cdot \frac{1}{T} \bigg|_{\tilde{r}=R_A}, \] (14)

\[ \tilde{J}_S = \frac{\tilde{J}_q}{T_A}. \] (15)
Here $\tilde{J}_\beta$ is the heat current. Substitution of equation (15) into (13) yields
\[
\dot{S}_c = -\frac{1}{T_\Lambda} \oint_{S_c} \tilde{J}_\beta \cdot d\Sigma = \frac{1}{T_\Lambda} J_q 4\pi R_\Lambda^2,
\] (16)
where we assume that $\tilde{J}_\beta$ takes the same value at every point of the apparent horizon surface $A = 4\pi R_\Lambda^2$.

From the modified entropy–area relation (1), one obtains
\[
\frac{d}{dt} \left( \frac{A}{4} + \tilde{\alpha} \ln \frac{A}{4} + \tilde{\beta} \right) = 2\pi R_\Lambda \tilde{R}_\Lambda + \frac{2\tilde{\alpha}}{R_\Lambda} \dot{R}_\Lambda
= 2\pi R_\Lambda \epsilon \left( 1 + \frac{\tilde{\alpha}}{\pi R_\Lambda^2} \right),
\] (17)
where $\tilde{R}_\Lambda = \epsilon$. Like [17], considering the relation
\[
\dot{S}_c = \frac{d}{dt} \left( \frac{A}{4} + \tilde{\alpha} \ln \frac{A}{4} + \tilde{\beta} \right),
\] (18)
from equations (16) and (17), one obtains
\[
J_q = \frac{\epsilon}{4\pi R_\Lambda^2} \left( 1 + \frac{\tilde{\alpha}}{\pi R_\Lambda^2} \right).
\] (19)

Following Fourier’s law,
\[
\tilde{J}_\beta = -\lambda \nabla T \bigg|_{i=R_\Lambda},
\] (20)
where $\lambda$ is the thermal conductivity [17]. This shows that there will be spontaneous heat flow between the horizon and the DE and the thermal equilibrium will no longer hold [16].

Substituting equations (19) and (20) in (14) gives
\[
\sigma = \frac{\epsilon^2}{4\pi R_\Lambda^4} \left( 1 + \frac{\tilde{\alpha}}{\pi R_\Lambda^2} \right)^2.
\] (21)
while from equation (12) we obtain
\[
\tilde{S}_c = \frac{\epsilon}{3} \pi R_\Lambda^3 \left( 1 + \frac{\tilde{\alpha}}{\pi R_\Lambda^2} \right)^2.
\] (22)

Finally, from equations (18) and (22), the time derivative of the irreversible entropy can be obtained as
\[
\dot{S} = \dot{S}_e + \dot{S}_c = \frac{\epsilon^2 \pi R_\Lambda}{3\lambda} \left( 1 + \frac{\tilde{\alpha}}{\pi R_\Lambda^2} \right)^2 + 2\pi R_\Lambda \epsilon \left( 1 + \frac{\tilde{\alpha}}{\pi R_\Lambda^2} \right).
\] (23)

Therefore, the first law in irreversible thermodynamics holds if we define
\[
\frac{dS}{d\epsilon} = -\frac{dE_A}{T_\Lambda} = 2\pi R_\Lambda \epsilon d\tau,
\] (24)
where $\tilde{T}_\Lambda := 1/2\pi R_\Lambda$ and
\[
\tilde{R}_\Lambda = R_\Lambda \left( 1 + \frac{\tilde{\alpha}}{\pi R_\Lambda^2} \right) \left[ 1 + \left( \frac{\epsilon}{6\lambda} \right) \left( 1 + \frac{\tilde{\alpha}}{\pi R_\Lambda^2} \right) \right],
\] (25)
which is the so-called entropy-corrected apparent horizon. If we set $\tilde{\alpha} = 0$, then
\[
\tilde{R}_\Lambda = R_\Lambda \left( 1 + \frac{\epsilon}{6\lambda} \right),
\] (26)
which is the same as the result obtained by [17].

\section{Conclusions}

In this paper, we investigated the FRW universe as a non-equilibrium (or irreversible) thermodynamical system by considering the logarithmic correction term to the horizon entropy. We assumed that the universe contains only DE in dominant form compared to other cosmic components; moreover, the universe is bounded by the apparent horizon. In a non-equilibrium scenario, there is a flux of energy across the horizon. As energy goes outside the horizon (and cannot come in due to irreversibility), an internal entropy production term is taken into account. Note that this term is zero in the equilibrium setting. Recently, Gang and Wen-Biao [17] studied this problem and showed that when the irreversible process is considered, the original apparent horizon is no longer perfect for building non-equilibrium thermodynamic laws exactly and should be modified. They obtained the modified apparent horizon expression that depended on the state parameter of DE $\epsilon$ and a non-equilibrium factor $\lambda$.

We have extended their study by considering the ‘corrected’ entropy motivated by the LQG. The addition of correction terms to the horizon entropy is a fundamental prediction of LQG and hence must be taken into account while studying the thermodynamics of horizons for the background geometry. Using this modified entropy–area relation, we obtained the entropy-corrected form of the apparent horizon of the FRW universe.

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