The Inclusive $\bar{B} \to \tau \bar{\nu} X$ Decay in Two Higgs Doublet Models

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Abstract

We calculate in the framework of two Higgs doublet models the differential decay rate for inclusive $\bar{B} \to \tau \bar{\nu} X$ transitions to order $1/m_b^2$ in the heavy quark expansion, for both polarized and unpolarized tau leptons. In contrast to the situation in the standard model, we find sizeable $1/m_b^2$ corrections. A systematic heavy quark expansion significantly reduces the theoretical uncertainties in the calculation compared to the existing free quark decay model results. From the experimental measurement of the branching ratio, we derive the bound $\tan \beta < 0.51 m_{H^\pm} \; [\text{GeV}]$. We point out that the tau polarization is potentially a more sensitive probe of multi scalar models than the branching ratio itself.
I. INTRODUCTION

Semileptonic $B$ meson decays into a tau lepton are sensitive to extensions of the standard model (SM) with several Higgs fields. In two Higgs doublet models (2HDM) [1] the experimental measurement of the decay rate $\bar{B} \to \tau \bar{\nu} X$ provides the strongest upper bound on $\tan \beta / m_{H^\pm}$ [2–3], for large values of $\tan \beta$. Theoretically, charged scalar contributions could increase $BR(\bar{B} \to \tau \bar{\nu} X)$ by as much as an order of magnitude compared to the SM [2,3,6]. This could have provided an exciting solution to the semileptonic $B$ meson branching ratio problem [7,8]. While this scenario is ruled out by the recent ALEPH measurement [9], the ratio of decay rates into the tau and light lepton channels still provides a sensitive probe to multi scalar models. Moreover, the possibility to study the tau polarization offers a greater variety in both the experimental and the theoretical analyses [3].

Recently, it has been observed that inclusive semileptonic decays of hadrons containing a single heavy quark allow for a systematic, QCD-based expansion in powers of $1/m_Q$ [10]. The heavy quark limit ($m_Q \to \infty$) coincides with the free quark decay model and there are no corrections to this result at order $1/m_Q$ [10,11]. The leading nonperturbative corrections are of order $1/m_Q^2$ and depend on only two hadronic parameters, which parameterize forward matrix elements of local dimension-five operators in the heavy quark effective theory (HQET) [12]. These corrections have been computed for a number of processes [11,13–18].

In the framework of the SM it was shown [18] that the heavy quark expansion provides more precise predictions than the spectator quark model for inclusive $\bar{B} \to \tau \bar{\nu} X$ decays. The effect of including the nonperturbative $1/m_Q^2$ corrections is numerically small. More important is that HQET gives an unambiguous definition of the quark masses [19] that determine the inclusive decay rates. Using the relation between the masses of the heavy quarks in terms of the HQET parameters leads to a significant reduction in the theoretical uncertainties [18].

In this paper, we use a combination of the operator product expansion and HQET to study the decay $B \to \tau \bar{\nu} X$ in the framework of 2HDM. In section 2 we define the model.
In section 3, we present the analytic results to order $1/m_b^2$ for the branching ratio and for the tau polarization. In section 4, we study the numerical predictions for the total decay rate and for the tau polarization. Section 5 contains our conclusions.

II. THE MODEL

Extensions of the standard model scalar sector are constrained by the experimental value of $\rho \equiv m_W^2/(m_Z^2 \cos^2 \theta_W) \simeq 1$ [20] and by the strong limits on flavor changing neutral currents (FCNC). The first of these constraints is naturally fulfilled if the Higgs sector contains only doublets (and singlets), providing the tree level prediction $\rho = 1$. FCNC are eliminated at tree level if we assume natural flavor conservation (NFC) [21], a construction that requires fermions of a given electric charge to couple to a single Higgs doublet. The phenomenology of such models has been widely studied [1]. In these models weak decays are mediated besides the $W^{\pm}$ bosons by charged scalar particles as well.

For simplicity we consider 2HDM with NFC and comment on the more general case below. After spontaneous symmetry breaking only two physical charged scalars remain, $H^{\pm}$. Such a model has still several variants. If the Higgs doublet that couples to up-type quarks also couples to down-type quarks or to charged leptons then the experimental results on the decays $B \to X_s \gamma$ [22] and $Z \to b \bar{b}$ [23] yield very strong constraints and the scalar contribution to $\bar{B} \to \tau \bar{\nu} X$ is negligible. Therefore, we only consider models in which one Higgs doublet couples to down-type quarks and charged leptons and the other to up-type quarks (model $\Pi$ in the language of [1]). The minimal supersymmetric standard model belongs to this class, so our results also apply to this model (up to radiative corrections).

The Yukawa interaction of the physical charged scalars with fermions is determined by $\tan \beta$ (the ratio of the vacuum expectation values of the two Higgs doublets), by the fermion masses and by the CKM matrix. The terms in the effective lagrangian relevant for $\bar{B} \to \tau \bar{\nu} X$ decays are

$$
\mathcal{L} = -V_{cb} \frac{4G_F}{\sqrt{2}} \left[ (\bar{c} \gamma^\mu P_L b) (\bar{\tau} \gamma_\mu P_L \nu_\tau) - R (\bar{c} P_R b) (\bar{\tau} P_L \nu_\tau) \right],
$$

(2.1)
where
\[ R = r^2 m_r m_b^Y, \quad r = \frac{\tan \beta}{m_H^\pm}, \quad (2.2) \]
and \( P_{R,L} = \frac{1}{2}(1 \pm \gamma_5) \). We denoted by an upper index \( Y \) the running quark masses (this distinction is unimportant for the tau, see discussion in section 4). The first term gives the SM contribution, while the second one gives that of the charged scalars. We neglect a term proportional to \( m_Y^c \): first, it is suppressed by the mass ratio \( m_Y^c/m_b^Y \); second, it cannot be enhanced by the possibly large factor \( \tan^2 \beta \).

Our analysis holds with minor modifications for general multi Higgs doublet models with NFC (for a recent comprehensive analysis, see [5]). In such models, instead of the single parameter \( \tan \beta \), three complex coupling constants determine the Yukawa interactions of the charged scalars. The parameters \( X, Y \) and \( Z \) describe the couplings to up-type quarks, down-type quarks and charged leptons, respectively. Then in the effective lagrangian \( (2.1) \)
\[ r^2 = XZ^*/m_H^2, \quad \text{where} \quad m_H \text{ is the lightest charged scalar mass (assuming that the heavier charged scalars effectively decouple from the fermions). Thus, although } r^2 \text{ is real in 2HDM, we treat it as complex so that our results apply to general multi Higgs doublet models.} \]

**III. HEAVY QUARK EXPANSION**

In this section we present the analytic results necessary for our numerical analysis. The techniques by which they were obtained are described in detail elsewhere [13–16], so we give only a brief summary, followed by the results of the computation.

The inclusive differential decay distribution is determined by the imaginary part of the time-ordered product of two flavor-changing currents,
\[ T = -i \int d^4x e^{-iq\cdot x} \langle B | T \{ \mathcal{O}^1(x), \mathcal{O}(0) \} | B \rangle, \quad \mathcal{O} = \bar{c} \left( \gamma^\mu P_L - m_b^Y r_P \right) b. \quad (3.1) \]
Since over most of the Dalitz plot the energy release is large (of order \( m_b \)), the time-ordered product can be written as an operator product expansion, in which higher-dimension operators are suppressed by powers of \( \Lambda/m_b \), where \( \Lambda \) is a typical low energy scale of the strong
interactions. To this end it is necessary to separate the large part of the \(b\) quark momentum by writing \(p_b = m_b v + k\), where \(v\) is the velocity of the decaying \(B\) meson, and its residual momentum \(k\) is of order \(\Lambda\). This separation is most conveniently performed by using the formalism of the heavy quark effective theory \([12]\). One can then evaluate the matrix elements of the resulting tower of nonrenormalizable operators with the help of the heavy quark symmetries.

The leading term in the expansion reproduces the result of the free quark decay model \([10]\), while giving an unambiguous definition of the heavy quark mass \([19]\). The leading nonperturbative corrections are of relative order \(1/m_b^2\) and can be written in terms of two parameters, \(\lambda_1\) and \(\lambda_2\), which are related to the kinetic energy \(K_b\) of the \(b\) quark inside the \(B\) meson, and to the mass splitting between \(B\) and \(B^*\) mesons:

\[
K_b = -\frac{\lambda_1}{2m_b}, \quad m_{B^*}^2 - m_B^2 = 4\lambda_2.
\]

The operator product expansion for the SM contribution has been presented in refs. \([17,18]\). Including the charged Higgs contribution is, in principle, a straightforward generalization. We shall present only the final results. The tau lepton can have spin up (\(s = +\)) or spin down (\(s = -\)) relative to the direction of its momentum, and it is convenient to decompose the corresponding decay rates as

\[
\Gamma(B \to \tau(s = \pm) \bar{\nu} X) = \frac{1}{2} \Gamma \pm \tilde{\Gamma}.
\]

The total rate, summed over the tau polarizations, is given by \(\Gamma\), while the tau polarization is \(A_{pol} = 2\tilde{\Gamma}/\Gamma\). Let us further decompose

\[
\Gamma = \frac{|V_{cb}|^2 \, G_F^2 \, m_b^5}{192\pi^3} \left[ \Gamma_W + \frac{|R|^2}{4} \, \Gamma_H - 2 \text{Re}(R) \frac{m_\tau}{m_b} \, \Gamma_I \right],
\]

and similarly for \(\tilde{\Gamma}\). The subindices \(W, H,\) and \(I\) denote the \(W\) mediated (standard model), Higgs mediated and interference contributions, respectively. This notation is particularly convenient, because in the spectator model \(\Gamma_W = \Gamma_H, \tilde{\Gamma}_H = -\tilde{\Gamma}_W\) and \(\tilde{\Gamma}_I = 0\). In the \(m_\tau, m_c \to 0\) limit of the spectator model \(\Gamma_W = \Gamma_H = \Gamma_I = 2\tilde{\Gamma}_H = -2\tilde{\Gamma}_W = 1\). After
integration over the invariant mass of the lepton pair and the neutrino energy, the differential decay rate depends only on a single kinematic variable $E_\tau$, which denotes the tau energy in the rest frame of the decaying $B$ meson. We will use the dimensionless variables

$$y = \frac{2E_\tau}{m_b}, \quad \rho = \frac{m_\tau^2}{m_b^2}, \quad \rho_\tau = \frac{m_\tau^2}{m_b^2}.$$  \hfill (3.5)

The standard model contribution to the lepton spectrum ($d\Gamma_W/dy$ and $\tilde{d}\Gamma_W/dy$) has been calculated in ref. [18], and we do not present it here. In terms of these quantities, however, the Higgs mediated contribution is quite simple. The reason is that the kinetic operator does not violate the heavy quark spin symmetry, thus the corrections proportional to $\lambda_1$ are the same for the $W$ and Higgs mediated terms. We obtain

$$\frac{d\Gamma_H}{dy} = \frac{d\Gamma_W}{dy} + \frac{\lambda_2}{m_b^2} 24x_0(1 - x_0)[y(3 - y) - \rho_\tau(8 - 3y)]\sqrt{y^2 - 4\rho_\tau},$$  \hfill (3.6)

$$\frac{d\tilde{\Gamma}_H}{dy} = - \frac{d\tilde{\Gamma}_W}{dy} + \frac{\lambda_2}{m_b^2} 12x_0(1 - x_0)(y^2 - 4\rho_\tau)(3 - y - \rho_\tau),$$  \hfill (3.7)

where

$$x_0 = 1 - \frac{\rho}{1 + \rho_\tau - y}.$$  \hfill (3.8)

For the interference term we get:

$$\frac{d\Gamma_I}{dy} = \sqrt{y^2 - 4\rho_\tau} \left\{ 6x_0^2(1 + \rho_\tau - y)\right. - \frac{\lambda_1}{m_b^2(1 + \rho_\tau - y)} \left[ 2(y^2 - 4\rho_\tau) + 2x_0(y^2 - 3y(1 + \rho_\tau) + 8\rho_\tau) \right. \left. - x_0^2(3 + 14\rho_\tau + 3\rho_\tau^2 - 9y(1 + \rho_\tau) + 4y^2) \right] 
- \frac{3\lambda_2}{m_b^2(1 + \rho_\tau - y)} \left[ 2x_0(12 + 8\rho_\tau - 3y(5 + \rho_\tau) + 4y^2) \right. \left. - x_0^2(15 + 10\rho_\tau + 3\rho_\tau^2 - 3y(7 + 3\rho_\tau) + 8y^2) \right]\right\},$$  \hfill (3.9)

and

$$\frac{d\tilde{\Gamma}_I}{dy} = - \frac{3\lambda_2}{m_b^2} 3x_0(1 - x_0)(y^2 - 4\rho_\tau)(2 - y).$$  \hfill (3.10)
The explicit form of the total decay rates are obtained by integrating the above expressions over the kinematic range $2\sqrt{\rho} \leq y \leq 1 + \rho_r - \rho$. We do not reproduce here the explicit expressions for the standard model contributions $\Gamma_W$ and $\tilde{\Gamma}_W$ that can be found in ref. [18].

For the Higgs mediated term we obtain:

$$
\Gamma_H = \Gamma_W + \frac{12\lambda_2}{m_b^2} \left\{ \sqrt{\lambda} \left[ 2(1 - \rho_r)^2 + \rho(5 - \rho + 5\rho_r) \right] - 6\rho(1 - \rho_r^2)A - 6\rho(1 + \rho_r^2)B \right\},
$$

(3.11)

$$
\tilde{\Gamma}_H = -\tilde{\Gamma}_W + \frac{6\lambda_2}{m_b^2} \left\{ \left[ (1 - \hat{m}_r)^2 - \rho \right] \left[ 2(1 - \rho_r)(1 + \hat{m}_r)^2 - \rho^2 \right.ight.
\left.\left. + \rho(5 - 3\hat{m}_r - \rho_r + 15\rho_r\hat{m}_r)/(1 - \hat{m}_r) \right] - 2\rho(3 + 2\rho_r - 4\rho_r - 5\rho_r^2) \ln \frac{(1 - \hat{m}_r)^2}{\rho} \right\},
$$

(3.12)

where we defined

$$
A = \ln \frac{2\hat{m}_r \rho}{(1 - \rho_r)^2 - \rho(1 + \rho_r) - (1 - \rho_r)\sqrt{\lambda}}, \quad B = \ln \frac{2\hat{m}_r}{1 + \rho_r - \rho + \sqrt{\lambda}},
$$

(3.13)

$$
\lambda = 1 - 2(\rho_r + \rho) + (\rho_r - \rho)^2, \quad \text{and} \quad \hat{m}_r = m_r/m_b = \sqrt{\rho_r}. \quad \text{For the contribution of the interference term we obtain}
$$

$$
\Gamma_I = \sqrt{\lambda} \left\{ \left( 1 + \frac{\lambda_1}{2m_b^2} \right) (1 - 5\rho - 2\rho^2 + 10\rho_r - 5\rho_r \rho + \rho_r^2) \right.
\left. - \frac{3\lambda_2}{2m_b^2} (11 - \rho + 2\rho_r + 8\rho^2 + 35\rho_r \rho - \rho_r^2) \right\}
+ 6 \left( 1 + \frac{\lambda_1}{2m_b^2} \right) \left[ \rho^2(1 - \rho_r)A + \left( (\rho^2 + 2\rho_r)(1 + \rho_r) - 4\rho_r \rho \right) B \right]
+ \frac{9\lambda_2}{m_b^2} \left[ \rho(2 + \rho - 5\rho_r \rho - 2\rho_r^2)A + \left( 2\rho(1 - \rho_r)^2 - 6\rho_r + 2\rho_r^2 + \rho^2 + 5\rho_r \rho^2 \right) B \right],
$$

(3.14)

and

$$
\tilde{\Gamma}_I = -\frac{3\lambda_2}{2m_b^2} \left\{ \left[ (1 - \hat{m}_r)^2 - \rho \right] \left[ (1 - \rho_r)(1 + \hat{m}_r)^2 - \rho^2 - 2\hat{m}_r \rho(1 - 5\rho_r)/(1 - \hat{m}_r) \right]
- 2\rho(1 - \rho - \rho_r)(1 + 3\rho_r) \ln \frac{(1 - \hat{m}_r)^2}{\rho} \right\}.
$$

(3.15)

In the limit $\lambda_1, \lambda_2 \rightarrow 0$, corresponding to the free quark decay model, our results agree with [3]. However, we disagree with [4] on the size of the interference term. There it was presumably multiplied with an extra factor of $m_r/m_b$. As expected, the parameter $\lambda_1$ only
enters the total decay rate proportional to the spectator model result, namely the total decay rate depends on $\lambda_1$ only through the combination $(1 + \lambda_1/2m_b^2)$. By virtue of the coupling of the charged scalar to the lepton pair (2.1), the Higgs mediated term produces tau leptons with positive polarization. This indicates that, in addition to the decay rate, the tau polarization is also sensitive to new physics.

IV. NUMERICAL RESULTS

1. Input parameters

When we neglect the tiny contribution from $b \to u$ transitions, the parameters entering our calculations are the mass of the tau lepton $m_\tau = 1.777$ GeV [24], the heavy quark masses $m_b$ and $m_c$, the hadronic parameters $\lambda_1$ and $\lambda_2$, the quark mixing parameter $|V_{cb}|$, and the combination of the unknown Higgs sector parameters $R$ (2.2). To utilize the full power of the heavy quark expansion it is important that not all of these parameters are independent [18,25]. In fact, the same HQET parameters $\lambda_1$ and $\lambda_2$ appear in the expansion of the heavy meson masses in terms of the charm and bottom quark masses (up to the running of $\lambda_2$) [26]:

$$m_B = m_b + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b} + \ldots,$$

$$m_D = m_c + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_c} + \ldots. \tag{4.1}$$

The parameter $\bar{\Lambda}$ can be associated with the effective mass of the light degrees of freedom inside the heavy meson [23,19]. For each set of values $\{\bar{\Lambda}, \lambda_1, \lambda_2\}$, eq. (4.1) determine $m_c$ and $m_b$. The consistency of the heavy quark expansion requires that these values of the

*It has been argued recently [28] that the definition of $\bar{\Lambda}$ is ambiguous. If this ambiguity turns out to be numerically significant that could probably be accounted for by taking a larger range for $\bar{\Lambda}$ in our numerical calculations and would slightly increase the uncertainties.
quark masses are used in the theoretical expressions for the decay rates. Using only three independent parameters \( \{\bar{\Lambda}, \lambda_1, \lambda_2\} \) instead of the four quantities \( \{m_b, m_c, \lambda_1, \lambda_2\} \) reduces the theoretical uncertainties significantly.

These HQET parameters are genuinely nonperturbative. While the value of \( \lambda_2 \) is related to the \( B^* - B \) mass splitting \( \lambda_2 = (m_{B^*}^2 - m_B^2)/4 \simeq 0.12 \text{GeV}^2 \), there is no similarly simple way to determine \( \bar{\Lambda} \) and \( \lambda_1 \). We expect this value of \( \lambda_2 \) to be accurate to within 10%, as a result of the finite \( b \) quark mass and the experimental uncertainties. The parameters \( \bar{\Lambda} \) and \( \lambda_1 \) can only be estimated in models of QCD at present, or constrained from phenomenology \cite{25}. QCD sum rules have been used to compute \( \lambda_1 \) \cite{29,30}, but these calculations suffer from large uncertainties. There is increasing theoretical evidence that, in accordance to its definition \cite{3.2}, \( \lambda_1 \) is negative \cite{31}, and its magnitude cannot be too large \cite{32}. Here we use \( 0 < -\lambda_1 < 0.3 \text{GeV}^2 \), which is also supported by a recent QCD sum rule calculation \cite{33}. Assuming \( \lambda_1 < 0 \), the phenomenological analysis of ref. \cite{25} prefers values of \( \bar{\Lambda} \) lower than QCD sum rules \cite{29,34}. Here we take \( 0.4 < \bar{\Lambda} < 0.6 \text{GeV} \).

Finally, a subtle point is that of the perturbative QCD corrections. While we know exactly the \( \mathcal{O}(\alpha_s) \) corrections to the \( W \) mediated term in the spectator model, analogous calculations for the \( H \) and \( I \) terms have not been carried out. For the \( W \) term \( \eta_r \simeq 0.90 \) \cite{35}; we shall also use the \( \mathcal{O}(\alpha_s) \) correction to \( \bar{B} \to e \bar{\nu} X : \eta_e \simeq 0.88 \). The leading corrections to the Higgs coupling are incorporated by running the Yukawa coupling. Therefore, \( R \) depends on the running mass of the \( b \) quark \( m_b(\mu) \). We take a conservative range \( 0.9 m_b \leq m_b^Y \leq 1.05 m_b \) corresponding to \( m_c \leq \mu \leq m_b \). For the interference term we include an extra \((-5 \pm 10)\%\) correction to represent the uncertainty in the QCD correction.

To conclude the above discussion we summarize the ranges for the various input parameters that we use in our analysis:

\[
0.4 < \bar{\Lambda} < 0.6 \text{GeV}, \quad 0 < -\lambda_1 < 0.3 \text{GeV}^2, \\
0.9 m_b < m_b^Y < 1.05 m_b, \quad 0.11 < \lambda_2(m_b) < 0.13 \text{GeV}^2.
\]
2. Branching ratio

Normalizing the branching ratio in the tau channel to that into light leptons separates the theoretical and experimental uncertainties in the numerical analysis, eliminates the otherwise significant uncertainties from the values of $|V_{cb}|^2$ and $m_b^5$, and also reduces the sensitivity to the unknown QCD corrections. We plot the theoretical prediction for the branching ratio as a function of $r$ in Fig. 1. Our result is given by the shaded region between the solid lines.

Using the recent measurement of the branching fraction into final states with a tau lepton [9] and that with an electron [20],

\[
\begin{align*}
BR(\bar{B} \to \tau \bar{\nu} X) &= 2.76 \pm 0.63 \%, \\
BR(\bar{B} \to e \bar{\nu} X) &= 10.7 \pm 0.5 \%,
\end{align*}
\]

we obtain the 1\(\sigma\) upper bound

\[r < 0.51 \text{ GeV}^{-1}.\] (4.4)

The 95\% CL upper bound is $r < 0.55 \text{ GeV}^{-1}$.

There are a number of points to be made regarding this result:

a. The $1/m_b^2$ corrections to the free quark decay model turn out to be more significant than in the SM: while the $W$ mediated contribution is suppressed by $-4\%$ and $-8\%$ in the electron and tau channels respectively, the Higgs mediated term is enhanced by approximately $14\%$, and the interference term is suppressed by as much as $25\%$. As a result, the difference between the 2HDM and the SM becomes more significant than expected from the free quark decay model.

b. The uncertainty in our result (corresponding to the width of the shaded region in Fig. 1) takes into account the theoretical uncertainties in (4.2), as well as an estimate of the $1/m_Q^3$ and more importantly the perturbative QCD corrections. If the latter terms were known, the width of the shaded region would be reduced by about a factor of two for not very small values of $r$. 

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The improvement of our calculation over the spectator model originates from large $1/m_b^2$ corrections, and from using $m_b$ and $m_c$ as determined by eq. (4.1) rather than treating them as independent input parameters. To illustrate this, we plotted with dashed lines in Fig. 1 the prediction of the free quark decay model corresponding to $1.4 < m_c < 1.5 \text{GeV}$ and $4.6 < m_b < 5 \text{GeV}$. Our predictions are not very sensitive to the values of $\bar{\Lambda}$ and $\lambda_1$. Adopting the range $0.3 < \bar{\Lambda} < 0.7 \text{GeV}$ and $-0.5 < \lambda_1 < 0.5 \text{GeV}^2$, which covers a range for $m_c$ and $m_b$ larger than the ones previously mentioned, makes the allowed bands slightly wider than in Fig. 1.

As the interference term gives rise to a dip in the branching ratio for small values of $r$, this measurement cannot probe values of $r$ smaller than $0.43 \text{GeV}^{-1}$. Within the spectator model, this limit is $r < 0.6 \text{GeV}^{-1}$.

In addition to our improvement of the theoretical calculation we would like to point out that some of the earlier analyses quoted too strong bounds on $r$ by (a) neglecting the uncertainties in the quark masses; (b) underestimating the size of the interference term by about a factor of three based on [4]; and (c) overestimating the SM prediction for the branching ratio.

**3. tau polarization**

The polarization of the tau lepton, $A_{\text{pol}} = 2\tilde{\Gamma}/\Gamma$, being a ratio of decay rates, is subject to much smaller uncertainties than the rates themselves. We find that the numerical value for $A_{\text{pol}}$ is rather insensitive to variations in $\bar{\Lambda}$ and $\lambda_1$. Allowing these parameters to vary within the ranges (4.2), we find the allowed values of the tau polarization as a function of $r$. Our result is given by the shaded region between the solid lines in Fig. 2.

A few points are in order regarding this result:

a. The $1/m_b^2$ corrections to the free quark decay model increase $A_{\text{pol}}$ for small values of $r$. More importantly, these corrections make the tau polarization almost a monotonically increasing function of $r$. While the interference term contribution to $\tilde{\Gamma}$ is small (it is exactly
zero in the spectator model), it is important that the contribution of the interference term to $\Gamma$ is significantly suppressed while that of the Higgs mediated term is enhanced due to the $1/m_b^2$ corrections.

$b.$ The uncertainty in our result (corresponding to the width of the shaded region in Fig. 2) takes into account the same theoretical uncertainties as in Fig. 1. The most important of these are the perturbative QCD corrections: if they were known, that would make the prediction for $A_{\text{pol}}$ accurate at the 1% level for $r \leq 0.3 \text{GeV}^{-1}$.

c. Besides the suppression of the interference term and the enhancement of the Higgs contribution to the total rate, the improvement of our calculation over the spectator model is again due to using $m_b$ and $m_c$ as determined by eq. (4.1) rather than treating them as independent input parameters. For comparison, we plot (with dashed lines) the prediction of the free quark decay model corresponding to the same ranges of the quark masses as in Fig. 1.

d. Most important is that a tau polarization measurement of an accuracy of about 10% would be sensitive to values of $r$ as small as $0.3 \text{GeV}^{-1}$. As pointed out in the previous section, such a small value of $r$ cannot be probed with the total branching ratio. The theoretical prediction for $A_{\text{pol}}$ is accurate at the 1% level in the standard model $-0.70 \leq A_{\text{pol}}^{\text{SM}} \leq -0.71$ \cite{18}. A precise measurement of this quantity would be a very stringent test of the SM, or alternatively provide a determination of the parameter $r$.

Finally we would like to compare the bounds that can be achieved from the inclusive decay $\bar{B} \to \tau \bar{\nu} X$ to that from the exclusive decays $\bar{B} \to \ell \bar{\nu}$. In ref. \cite{36} it was shown that the purely leptonic decays of $B$ mesons are enhanced compared to the SM for $r > 0.27 \text{GeV}^{-1}$, and a bound $r \leq 0.52 \text{GeV}^{-1}$ has also been claimed. It is important to note that deriving a bound on $r$ from this decay requires an independent estimate of $|V_{ub}|$ and the $B$ meson decay constant $f_B$. Uncertainties related to the values of these parameters have been neglected in \cite{36}. These uncertainties are, however, very significant; taking them into account ($f_B > 140 \text{MeV}$ and $|V_{ub}| > 0.0024$) allows one to obtain only $r < 0.80 \text{GeV}^{-1}$. Concerning future bounds from this process we estimate that an enhancement of $BR(\bar{B} \to \ell \bar{\nu})$ will only be

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observable if $r \geq 0.35 \text{GeV}^{-1}$. We conclude that a stronger bound on $r$ will be provided by a measurement of the tau polarization in the inclusive $\bar{B} \to \tau \bar{\nu} X$ decay, while at present the best bound comes from the measurement of the branching ratio in the same decay.

V. SUMMARY

We investigated in detail the effects of a charged scalar on the inclusive decay $\bar{B} \to \tau \bar{\nu} X$. We used the heavy quark expansion to incorporate the nonperturbative $1/m_b^2$ corrections, and the relation between the heavy quark masses implied by the HQET. This resulted in a reduction of the theoretical uncertainties by more than a factor of two compared to the calculation in the free quark decay model. While the $1/m_b^2$ corrections are usually less than 5–10% in standard model calculations, in the present case they turned out to be more significant. To derive a bound on $r$, special care has to be taken of the uncertainties in the theoretical calculation which have been mostly neglected so far. From the presently available experimental data the strongest bound that can be obtained is $r = \tan \beta/m_H < 0.51 \text{GeV}^{-1}$. We pointed out that the tau polarization is more sensitive to small values of $r$ than the branching ratio. While $BR(\bar{B} \to \tau \bar{\nu} X)$ is insensitive to $r \leq 0.4 \text{GeV}^{-1}$, a measurement of $A_{\text{pol}}$ could probe $r \sim 0.3 \text{GeV}^{-1}$ or even lower. This is particularly interesting since the recent ALEPH measurement of the branching ratio excludes very large deviations from the standard model. We hope that this will encourage experimentalists to measure the tau polarization in $B$ decays with the best achievable accuracy.

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FIGURES

FIG. 1. $\Gamma (\bar{B} \to \tau \bar{\nu} X)/\Gamma (\bar{B} \to e \bar{\nu} X)$ as a function of $r = \tan \beta/m_{H^\pm}$. The shaded area between the solid lines is our result. The area between the dashed lines gives the free quark decay model result. The dash-dotted lines give the experimental 1σ bounds.

FIG. 2. The tau polarization, $A_{pol} = 2\tilde{\Gamma}/\Gamma$, as a function of $r = \tan \beta/m_{H^\pm}$. The shaded area between the solid lines is our result. The area between the dashed lines gives the free quark decay model result.
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