Degenerate vacua from unification of second law of thermodynamics with other laws

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Abstract

Using our recent attempt to formulate second law of thermodynamics in a general way into a language with a probability density function, we derive degenerate vacua. Under the assumption that many coupling constants are effectively “dynamical” in the sense that they are or can be counted as initial state conditions, we argue in our model behind the second law that these coupling constants will adjust to make several vacua all having their separate effective cosmological constants or, what is the same, energy densities, being almost the same value, essentially zero. Such degeneracy of vacuum energy densities is what one of us works on a lot under the name “The multiple point principle” (MPP).

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1 Introduction

The second law of thermodynamics [1, 2, 3] concerns, contrary to the other laws, the question of initial state and further seemingly straightly violates the time-reversal symmetry of the other laws. Even if time reversal symmetry is slightly broken in the Standard Model, at last CPT is not broken, and the breaking is anyway so tiny that it does not support the violation of time reversal invariance of the order of that of the second law. This arrow of time problem [4] at first seems to violate any hope of constructing a model or theory behind the second law without violating the usual symmetries of the other (time development) laws, especially CPT or time reversal symmetry. However, we believe to have actually presented such a model, and S. Hawking and J. Hartle’s [9] no boundary initial conditions also present a model [5, 6, 7, 8] that should indeed both have the second law for practical purposes and obey the usual symmetries. Really our model ends up very close to the Hartle-Hawking’s one, but we think that ours is in principle more general. We see the connection so that by using imaginary time by Hawking et al have effectively got an imaginary part of the action come in. Our model [5, 6, 7, 8, 10] could be formulated as having a general complex action where real and imaginary parts are in principle independent functions to be chosen only respecting the symmetries and dimensionwise requirements etc.

Since we ended up with a reasonable picture for second law without too detailed assumptions about the real and imaginary parts of the action we might claim the generalization somewhat successful.

So far we worked purely classically to avoid at first the unpleasant quantum features of quantum mechanics for such a second law discussion that there does not truly exist a clean history path being true but rather a mysterious functional integral over many paths.

We did not so far go in detail with the question that such a purely classical model could definitively not be good enough at the end.

From an esthetic and simplicity point of view it would seem that a priori one should at first seek to construct models like the ones mentioned, since that is what we could consider “unification” of the second law with the rest of the laws and their symmetries. Also one could easily imagine that some law behind the second law could exist and possibly give a bit more information than just the second law itself,
so that if we could guess it or find—it is perhaps Hartle-Hawking’s no boundary—then we could use it for more. In our previous articles we in principle sought to discuss just a general formulation of such a law behind the second law by simply stating that is must—at least—be of the form of providing for every time track—i.e. equation of motion solution—a probability density $P(path)$ in phase space. We think of the paths as associated with points in a phase space by simply choosing a standard moment of time say $t = t_{st}$ and letting the phase space point associated with the path be the ordered set of generalized coordinates $q^i(t_{st})$ and the ordered set of generalized momenta $p^i(t_{st})$ for this path, called path, at that moment $t_{st}$. The density $P$ shall give the probability density for the path relative to the natural (Liouville theorem) measure on phase space. Because of Liouville theorem saying that this measure is invariant under the time development, the density $P(path)$ defined will for a given path be the same number independent of at which moment of time $t_{st}$ we choose to use the phase space (canonical) density $dqdp = \prod_i (dq^i dp^i)$. So generally formulated we have almost not assumed anything but left all assumptions to be done to the selection of the functional form of $P(path)$ as function of the path.

At first one would think \[1\] that $P(path)$ should depend in a simple way only on the very first moment $t \to 0$ or $t = t_{creation}$, the creation time of the universe. However, we are with the usual law properties used as a paradigm tempted to favor a form of the probability weight factor like

$$P(path) = \exp\left(\int P(q(t), p(t))dt\right) \quad (1.1)$$

which depends in the same way on the state along the track for all times $t$ ! But such a form immediately seems to endanger getting out a good second law, since its time translational invariance is already in danger of leading to at least some features of the path to depend is a possibly simple enough to be recognized way on even the future. Such sufficiently simple dependence on the future might be recognized as “the hand of God” or even “miraculous effects” some times. However, we believe that it is realistic with models of a reasonable nature—a reasonable nice choice of $P$—of this kind to in practice have so few miracles or “hand of God” effects that the model is phenomenologically viable. That was what we attempted to argue for in last article \[10\] and the miracles would be small under the present conditions although Higgs particles could be a special danger for them to pop up so that LHC would be a flavored target for miracles or hand of God effects. The major partly
future determined effect were there suggested to be the smallness of the cosmological constant, a phenomenologically welcome “miracle”.

It is the purpose of the present article to extend somewhat this cosmological constant prediction to not only having one cosmological constant or vacuum energy density being small, but to have several minima in the scalar field effective potential “landscape” being very close to zero, too.

This result of the present paper is what one of us (H. B. N.) and his collaborators have been announcing as the Multiple Point Principle\textsuperscript{2}. Mainly it has been claimed to give phenomenologically good results and derivations have, although being similar, been in principle quite different from the present one. In fact derivations have only been successful with some mild violation of the principle of locality. In that light the previous derivations cannot be extremely convincing, since after all, we otherwise do not find much evidence for violation of locality, except perhaps precisely in connection with the cosmological constant problem.

Indeed we shall in the present article argue for the multiple point principle, but only under a very important extra assumption: At least some coupling constants or mass parameters are “dynamical”, or one should rather say that they are to be counted as part of the “initial conditions”.

The meaning of this making the coupling constants—such as say the Higgs-quark Yukawa couplings—“dynamical” is that we consider them part of the path in the above terminology, so that $P(\text{path})$ also comes to depend on them. Thus we have to maximize the probability also allowing for the variation, and thus adjustment, of the couplings which are declared \cite{12} “dynamical”. We might either just assume then “dynamical” in this sense—really meaning counted as part of the “path”—as a brute force assumption, adding them as special generalized coordinates, or we may imagine that they in some way have come out of the ordinary dynamical variables as e.g. in baby universe theory. Really it is the way of arguing in the present article not to go into details with respect to how precisely the coupling constants became “dynamical”, rather saying:

Since we seemingly had some success—solving the cosmological constant problem—in last articles by introducing the assumption that the cosmological constant was “dynamical” in this type behind second law model, it is by analogy suggested that

\textsuperscript{2}See, e.g. \cite{2} and references therein.
also other couplings, quite analogous to the cosmological constant, are or “dynamical”.

It is our hope that allowing several possibilities for how it came that the coupling constants became “dynamical” is the sense of depending on dynamical variables or fundamentally themselves already being “dynamical”. Then the model presented has the collected probability of being true, collected from these different possible ways.

In the following section, section 2, we shall set up the formalism for the probability density, and sketch how one might ideally wish it to look very analogous with the action. In section 3 we shall make some rather general considerations about the stability and most flavored states of universe that can be relevant for surviving over exceedingly long periods of time. The main point is here to investigate, how the likelihood of a certain combination of macrostates \( \langle P e^S \rangle \) depends on the variation of the couplings, especially when a minimum in the landscape of the scalar field effective potential passes from being negative to being positive. Our point is that the minimum being close to zero is flavored. In section 4 we shortly review that the model could—as seen in last article—provide an effective Big Bang although the time before the inflation era is a crunching inflationary era with opposite second law i.e. \( \dot{S} < 0 \). It is thus “pre-Big Bang” one could say. In section 5 we review how this multiple point principle prediction has already been claimed to be phenomenologically a very good assumption leading to phenomenologically good predictions for relations between coupling constants in the Standard Model, especially the top quark mass is what is predicted. Also a detail difference between the present and the earlier “derivations” of the multiple point principal of degenerate vacua is put forward: In the present model many of the possible vacua are only realized over very small space time regions. Perhaps only one of the vacua are hugely realized. In the old competing derivations they all had to be realized over order of magnitude comparable space time 4-volumes. In section 6 we present the conclusion and further outlook.
2 Model behind second law of thermodynamics

Since the second law of thermodynamics is well-known to concern the state of the world rather than as “the other laws”, such as Hamilton equations or equivalently Newton’s second law, then a law behind this law must of course somehow assign probabilities to different states, or directly tell which one is the right one. Since the time development laws (“the other ones”) are assumed to be valid (under all circumstances) we should really think about a law behind the second law of thermodynamics as assigning probability or perhaps even validity to solutions of the equations of motion. We might to keep it very abstract think of a space of all solutions to the equations of motions. Then the law behind the second law of thermodynamics could be thought of as having the form of a probability distribution \( P \) over this space of solutions. So it (=the law behind) is required to formulate a probability measure over this space of solutions to the equations of motion. It happens that such a measure can be written down rather elegantly in as far as a solution by selection of a “standard time” \( t_{st} \) is correlated to a point in phase space namely

\[
(q_1(t_{st}), q_2(t_{st}), \ldots, q_n(t_{st}), p_1(t_{st}), \ldots, p_n(t_{st})) .
\] (2.1)

Now the phase space has the “natural” measure

\[
\prod_i dq_i \prod_i dp_i
\] (2.2)

which is the one from the Liouville theorem. It is of course suggested then to use this measure with \((q_1, \ldots, q_n, p_1, \ldots, p_n)\) taken as \((q_1(t_{st}), \ldots, q_n(t_{st}), p_1(t_{st}), \ldots, p_n(t_{st}))\) which means to use the measure

\[
\prod_i dq_i(t_{st}) \prod_i dp_i(t_{st}).
\] (2.3)

Then one could define a density \( P(path) \) using (2.3) by writing the probability density for the path

\[
\text{path} = (q_1, \ldots, q_n, p_1, \ldots, p_n) : \text{time axis} \rightarrow \text{“Phase Space”}
\] (2.4)

as

\[
\text{“probability measure”} = P(path) \prod_i dq_i(t_{st}) \cdot \prod_i dp_i(t_{st}).
\] (2.5)
One would now fear that this probability density $P(\text{path})$ defined this way would depend on the standard moment $t_{st}$ chosen. It is, however, trivial to see that this fear is without reason, since indeed $P(\text{path})$ will not depend on $t_{st}$. It is well known that the measure (2.2) or (2.3) is invariant under canonical transformations and that the time development is a canonical transformation. Thus we do not need to attach any index $t_{st}$ to $P(\text{path})$, it is only a function of the solution “path”.

Now to really produce a guess making up a law behind the second law of thermodynamics one has to make some assumptions about the defined probability density function, $P : \text{solution space} \rightarrow \mathbb{R}^+ \{0\}$. Because if one does not assume anything it is a very big class of possibilities for $P$ and there will not be much content in such a formalism. That there is not much content in just putting up such a formalism is encouraging, because it makes it (more) likely that we have not assumed anything wrong by using the formalism with such $P$.

In the present article it is our intention to a large extend to keep the model at this general level by making very general assumptions about $P$. For example we may assume that it exists for some sort of world machinery at some fundamental level, but that we do not dare to guess it—since our chance guessing it wrong by world of course be outrageously high—so that we instead should attempt to guess a statistical distribution over function of type $P$. Then the idea should be that we should be allowed to play with the formalism as if $P$ were chosen as a random one from this assumed distribution of $P$-type functions. This way of thinking of a statistical distribution for objects—here $P$—that actually are thought to make a law of nature is typical for the project which one of us called “random dynamics”. In this sense we can consider our last paper [10] a random dynamics derivation of the second law of thermodynamics.

Here we shall, however, not go on to put up a statistical distribution for $P$ as a function but just keep ourselves to a rather general discussion about $P$. In fact we may use such argumentation as: To find a big value for $\log\langle P \rangle$ where $\langle \cdots \rangle$ denotes averaging over a region in space of solutions we have less chance to find it very big when we average over a smaller region than if we average over a bigger region. There is a bigger fluctuation for a small region and thus better chance for the outrageous average value.

From this kind of statistical argument we would see that it will in all likelihood
help to produce a big probability if we can get arranged that the system would
stands around in an appropriate (not too big) region in phase space. The smaller
this region the better is the chance that we accidentally have in that region a high
average \( P \). Thus we would see that regions of phase which are metastable have high
chance to have some of the highest probabilities. As the typical region of such a
stable kind or rather metastable one we could think of the universe in a stable one
of an only slightly excited vacuum with a limited amount of field vibration on it. It
might then be metastable due to some interactions.

If we want to write down an expression for a proposal for \( P(\text{path}) \) which has
symmetry and locality properties analogous with those of the time development
laws, we would in a classical field theory model make a construction for \( \log P(\text{path}) \)
guide analogues to the action. The suggestion of such an analogy is in fact strongly
suggested by for a short moment thinking about a quantized generalization of our
model in a Feynman path integral formulation. It would be very strongly suggested
to put the \( P(\text{path}) \) in as a factor \( \sqrt{P(\text{path})} \) multiplying the path-amplitude by
suggesting the replacement

\[
e^{iS[\text{path}]} \rightarrow \sqrt{P(\text{path})} e^{iS[\text{path}]}
\]

(Here, \( S[\text{path}] \) is of course the action, and not the entropy.) for the quality occurring
in the Feynman path integral. To do this replacement one would of course need to
have a model form for \( \sqrt{P(\text{path})} \) and \( P(\text{path}) \) even for those paths which do not
obey the equations of motion. In the present article it is, however, still the intention
to use a purely classical description and we would not need such an extension. But
only the esthetic suggestion of seeing

\[
\log \sqrt{P(\text{path})} = \frac{1}{2} \log P(\text{path}) = -\text{"Im}S
\]

as really being an imaginary part of the action, so that symmetry and locality
properties of \( \log P(\text{path}) \) would be suggested to be taken to be just the same as
for the usual—i.e. the real part of—action \( S(\text{path}) \). We would therefore, say in a
general relativity setting, obtain a form

\[
\log P(\text{path}) = \int d^4x \sqrt{|g(x)|} P(\varphi, \partial_{\rho} \varphi, \psi, \partial_{\sigma} \psi, g_{\mu\nu}, \partial_{\sigma} g_{\mu\nu}, \cdots).
\]

Here we should of course have in mind that corresponding to a path one has a
development of all the field \( \varphi(x), g_{\mu\nu}(x), \psi(x), \cdots \) their derivatives \( \partial_{\sigma} \varphi(x) \), \( \cdots \) too.
Thus the expression \( (2.8) \) is a well-defined functional of the path.
We can imagine—and it would be the most esthetic an nicest—that the function $P$ of the fields and their derivative obey all the rules required from the symmetries obeyed by the usual timedevelopment laws, the ones given by the action.

For instance since gauge transformations are supposed not to cause any physical change, we should have $\int d^4x \sqrt{g}$ be gauge invariant clearly. The form as an integral the requirement of locality and thus if we can manage to get such a form work phenomenologically we could even say that the law behind the second law of thermodynamic could obey such a locality postulate.

Such a set up with a lot of symmetry requirements might at first be somewhat difficult to check and thus remain speculations, but the real immediate worry, the reader is expected to have, is that such a form of $P$(path) will have enormous difficulty in leading to the second law. Immediately one would rather think that it would lead to mysterious regularities in what will happen both in past and future and even today in order to optimize $P$. If there are too many features of the actual path predicted to be destined to organize a special future or present the model may be killed immediately.

In reality we consider it a remarkable result of our previous work [10] that we argue that this type of model is not totally out, but on the contrary looks promising even without almost assuming anything about the specific form of log $P$.

2.1 Example: Scalar fields, exercise

To provide us with an idea of how such a model will function let us imagine a theory with one or several scalar fields. If we add the further assumption that not only the Lagrangian density, but also the quite analogous density $P$ has coefficients of the dimensions required by “renormalizability”, then the “kinetic terms” in the density $P$ would be quite analogous to the ones in the Lagrangian density $L$ and no terms with higher number of derivatives would be allowed neither in $L$ in $P$. Also only an up to fourth order term in the potential $V(\varphi_1, \varphi_2, \cdots)$ and the analogous “potential” term in $P$ would be allowed.

To get an idea of what can go on we can think that if for some special value combination of the scalar fields

$$\left(\varphi_1, \varphi_2, \cdots\right) = \left(\varphi_1^{(0)}, \varphi_2^{(0)}, \cdots\right)$$

\[ (2.9) \]
where the density \( P \) has a maximum, then a configuration with the scalar fields taking that set of values will be a priori very likely. However, we shall also have in mind that most likely the fields will not stay at just that special combination for long if it has to obey the equations of motion. Unless such a maximum in the “potential” part of \( P \) (We think of the part of \( P \) independent of the derivatives of the fields thus only depending on the values of the fields) is also an extremum for the potential part of the Lagrangian density there is no reason that standing fields should be solutions. Rather the fields will roll down say—and not even any especially slow roll a priori—.

It might actually pay better to get a high probability or likelihood if the field-combination chooses to sit at a minimum in the potential \( V(\varphi_1,...) \) from the usual Lagrangian density \( L = \sum \partial_\mu \varphi_i \partial^\mu \varphi_i - V(\varphi_1,...) \) with a relatively high but not maximal \( P \)-potential-part value. At such a place we could have the fields standing virtually externally and that would count much more than a short stay at an even higher value for the “potential” part of \( P \).

The longer time of it staying there will give much more to the time integral form for \( \log P \).

But we can investigate if it could be arranged to get the gain from the very high \( P \) near some unstable combination for a relatively short time and then at another earlier and/or later time attain for long the somewhat lower but still if well-arranged reasonably high \( P \)-value from a minimum in the potential \( V \) from \( L \).

In the previous articles it were suggested that such a shorter time high \( P \) could well pay and be indeed the explanation that during some period in the middle of times there were an inflationlike Big Bang similar time with the scalar field at an unstable point. Our model is not really guaranteed to solve the problem of getting the roll slow enough—although we could say that meaning “it would like to if it could”— but even a shorter inflation period could at least provide a from outside (in time) seen Big Bang. Let us though stress two important deviations—one of which are so far experimentally accessible—between our simulated Big Bang and the conventional one:

1) Ours is in the “middle of times” so that there is a half time axis at the pre-Big Bang side actually with an inverted second law of thermodynamics \( \dot{S} < 0 \).

2) We do not have any true singularity, but rather have inflation like situation
with finite energy density all through this “middle period.”

3 The Derivation of Multiple Point Principle

3.1 Dynamical couplings and what to maximize

To derive the Multiple point principle it is very important that we take a series of coupling constants to be dynamical in the sense that they can be adjusted to take special values guaranteeing the many degenerate minima, which are by definition the point of the Multiple Point Principle. So we must take it that the $P$-probability also depends on these couplings. That means that so called different paths have as some of their degrees of freedom these couplings so that they are different for different paths.

We have already argued for that the most likely type of path i.e. development to corresponds to the scenario of an inflation era in some middle of the time axis, surrounded by asymptotic regions of an almost static big universe with thin matter and essentially zero cosmological constant operating near a minimum in the potential. Then one can get the biggest $P$ from a long asymptotic era—which though must be at least meta stable—while still getting a high $P$ concentrated contribution from a short “around Big Bang” era.

Now we should have in mind that the effective potential $V(\varphi_1, \varphi_2, \ldots)$ can and will typically have several minima. A priori, however, these minima will not be degenerate with their separate cosmological constants being zero as the Multiple Point Principle which we seek to derive.

Rather the precise height of the various minima in the effective potential will depend on the various coupling constants and mass parameters which we have just assumed that we shall —at least effectively— count as part of the “initial conditions” i.e. the solution “path”. After we assumed these couplings and mass-parameters to be “dynamical” meaning here part of the path on which $P$ depends we shall allow them to be varied too in the search for the most likely path. Now it is, however, not quite the right thing to look for just that very special path that goes with the highest $P$, because what we in practice are interested in is not really to know the special path but rather what class of paths not distinguishable by macroscopic observation. We rather look for describing the scenario in terms of macrostates meaning roughly
that sort of states that are used in thermodynamics where one characterizes systems
with huge number of degree of freedom by means of a few macro variables, energy,
numbers of various types of particles and the like, entropy e.g. Even if such a macro
state having a huge number of micro states collected under its heading does not
contain the most likely single solution to the equations of motion if it could very
well happen that the sum over all its micro states

$$P_{\text{macro}} = \sum_{\text{path-macro}} P(\text{path})$$ (3.1)

could be—even much—bigger than the single \(P(\text{path}_{\text{max}})\) for the uttermost scoring
solution \(\text{path}_{\text{max}}\). In such a case we should like in practice to consider it that the
correct scenario for us as macro-beings is the one with the macro state giving the
biggest sum (3.1). Rather than looking for the largest \(P(\text{path})\) we are therefore
looking for the largest sum over a whole or perhaps even better a whole class of
similar macro states, i.e. for the largest

$$P_{\text{macro}} = \sum_{\text{path} \in \text{macro}} P(\text{path}) = \langle P \rangle_{\text{macro}} \cdot e^S$$ (3.2)

where we introduced the average over the macro state notation

$$\langle P \rangle_{\text{macro}} = \frac{\sum_{\text{path} \in \text{macro}} P(\text{path})}{\# \text{micro states in } \text{macro}} = \frac{\sum_{\text{path} \in \text{macro}} P(\text{path})}{e^{S(\text{macro})}}$$ (3.3)

and defined the entropy of the macro state “macro” as the logarithm of the number
of micro states in it

$$S(\text{macro}) \equiv \log(\# \text{micro states in } \text{macro}).$$ (3.4)

3.2 Central derivation of many degenerate vacua.

When one characterizes the competing classes of microstates as macrostates with
some entropy \(S\), what we really shall think of as being maximized by the model, is the
quantity \(\langle P \rangle e^S \mu_2^N\) or we can say \(\log(\langle P \rangle e^S)\). Here \(\langle P \rangle\) stands for the average over
the macrostate of \(P\). This quantity \(\log(\langle P \rangle e^S)\) is expected from general smoothness
assumptions and assuming no fine tuning a priori to vary smoothly and with non-
zero slope as a function of all the parameters, especially as a function of the various
coupling constants and mass parameters. In other words these coupling constants
and mass parameters should be determined together with the class of microstates to be most likely from the maximization of $\log(\langle P \rangle e^S)$ point of view.

Now, however, we have to take into account that the appearance of a minimum in the effective potential—as function of the effective (composite or fundamental) scalar fields—in addition to that minimum that leads the exceptionally high $\log(\langle P \rangle e^S)$ which gives the highly probable asymptotic behavior can cause a destabilization. In fact the appearance of a competing different minimum means when it becomes deeper than the high $\log(\langle P \rangle e^S)$ one that the latter becomes strictly speaking unstable. It can namely in principle then happen that the high $\log(\langle P \rangle e^S)$ macrostate around this latter minimum, develops into a state around the lower energy density vacuum, a state belonging to this other minimum. One should have in mind that it is the lack of energy that keeps the “asymptotic” state of the universe to remain very close to the vacuum so as to ensure the high $\log(\langle P \rangle e^S)$. If energy can be released by the scalar fields shifted to a lower/deeper minimum then this cause of stability disappears and the universe will no longer keep at the vacuum with high $\log(\langle P \rangle e^S)$ and most likely a much lower value for $\log(\langle P \rangle e^S)$ will be reached. That means that the smooth continuous variation with the coupling constants etc. as a function gets a kink, a singularity, wherever a competing minimum passes from being above the high $\log(\langle P \rangle e^S)$ one to being deeper.

There is a very high chance that the maximum achievable $\log(\langle P \rangle e^S)$ will occur just at this type of kink. All that is needed is really that as the minimum competing with the high $\log(\langle P \rangle e^S)$ as a function of some coupling, $g$ say, is lowered—still while being above and thus no threaten to the high $\log(\langle P \rangle e^S)$ the $\log(\langle P \rangle e^S)$ is—accidentally—having appropriate sign of its rate of variation. In fact what is needed is that the $\log(\langle P \rangle e^S)$-quantity gets larger under variation of say $g$ when the competing minimum gets lower. In such a case the largest $\log(\langle P \rangle e^S)$ will be reached by bringing the competing minimum to be as low as possible before it destabilizes the high $\log(\langle P \rangle e^S)$ vacuum and thus spoils the smooth estimation. But that means that the maximum $\log(\langle P \rangle e^S)$ meaning the most likely scenario will precisely happen when the destabilization sets in. So it is very likely that seeking—as our model does—the maximal $\log(\langle P \rangle e^S)$ scenario will lead to very likely have competing minima just with the same effective potential values as the high $\log(\langle P \rangle e^S)$—vacuum. But this is precisely what we mean by the multiple point principle: There shall be
many vacua with the same energy density or we can say same cosmological constant.

In this way out of our model we have interestingly enough derived just this principle on which one of us and his collaborators have already worked a lot, seeking to show that it has very good phenomenological fitting power.

4 Review of the other good features of our model

In this section we shall review and elaborate the point that our model—although it does not at first look so—indeed is to a very good approximation a law behind the second law, even with a few extra predictions.

The most surprising is that we can get the second law of thermodynamics out of an at the outset totally time reversal invariant “law behind the second law of thermodynamics” However, that can also only be done by a slight reinterpretation:

We argued that although the bulk of the—assumed infinite—time axis is taken up by eras in which roughly the maximal contribution from these bulk eras to log(⟨P⟩e^S) is the biggest attainable for a rather limited stable region in phase space, it pays nevertheless to have a short less stable era in some smaller interval. The full development will, in this case even if not exactly, then with respect to crude features be time reversal invariant around a time-reflection point in the middle of this unstable little era. The time reversal asymmetry is now achieved by postulating that we ignore and in practical life do not take seriously one of the two half axis of the time axis. Indeed we claim that we in practice only count what happens offer the mentioned middle point of the relatively short “more unstable era”. The argument was now that by finding some small subset of microstates with very high log P-contribution from this “unstable” era a universe development with higher log(⟨P⟩e^S) could likely be found with such an unstable period than as a development of the type behaving as the asymptotically stable way at all times. Typically a very small phase space volume in the central part of the “unstable era” is expected to be statistically favorable because we expected it to be easier to find an average over P to be very big if we only average over a very small region. We almost expect a state with exceptionally high P to have to be past to make the “unstable era—excursion” from the asymptotic behavior to be the very most likely. We thus see that we expect the entropy in this “unstable era” to be very low indeed. Thinking of the especially
high $P$ being achieved by going to a highest “potential” part for $P$ and having scalar fields sliding down from there the argument for a very low entropy in the unstable era seems indeed to be justifiable in such a more concrete setting.

A priori one would now think that an analogous argumentation of most exceptionally high $\log \langle P \rangle$ occurring more likely in a small region of phase space than in a larger phase space region would also give a low entropy in the asymptotic era. Now, however, there are some phenomenological peculiarities in nature which are combined with our suggested picture of a big universe in the asymptotic era points to that for practical purposes $\log P$ gets almost constant over the relevant neighborhood or the high $\log((P) e^S)$ providing vacuum (minimum in the effective potential). This phenomenological peculiarity is that the universe even today already expanded so much and the parameters of the Standard Model are such that:

1) Interactions are relatively seldom—i.e. weak couplings,

2) All the particles around are in practice of the nature that they only acquire non-zero-masses by the Higgs field expectation value $\langle \varphi_{ws} \rangle \neq 0$,

3) Even this Higgs VEV is tiny from the presumed fundamental scale point of view.

As a caricature we may thus see the present era—which is already really the asymptotic era to first approximation—as an era with a big universe with a “gas” of massless weakly interacting particles only.

Further we should keep in mind that we phenomenologically have—locally at least—Lorentz invariance. This means by imagining the theory rewritten from the field theory description, used so far in this article, to a particle description that the contributions to $\log P$ should be integrals along the time tracks of the various particles with coefficients depending on which particle type provides the $\log P$-contribution. Now, however, for massless particles the time-track is lightlike and thus always zero. We get therefore no such contribution from the presumably almost massless particles in the Standard Model. If this is so it means that once we have got limited the set of states at which to find the true state in the asymptotic era to those with the Lorentz invariance and masslessness properties, there is no gain for $\log \langle P \rangle$ by further diminishing the class of states included. The $\log \langle P \rangle$ would anyway remain much the same even if in the asymptotic time the photons say were removed
because they due to the masslessness do not count anyway. Thus a further reduction in phase space in the future is not called for since really it is rather as earlier stressed \( \log(\langle P \rangle e^S) \) which should be maximized, and we can increase this quantity by having more particles—meaning a wider range of phase space—contributing to entropy \( S \) without changing \( \log(\langle P \rangle) \) much.

This masslessness phenomenology thus provides an argument for a much higher entropy in the future in the scenario favored by our model. Well, we should rather than future say in the numerically asymptotically big times.

In the inflation era on the other hand the typical temperatures at least after “re”heating are much higher and at least the Weinberg Salam Higgs cannot be prevented from appearing.

5 Multiple point principle already somewhat successful phenomenologically

Accidentally the derivation from our law behind the second law of thermodynamics of there being many minimal in the effective potential for the scalar fields—fundamental or bound state ones—having all very small cosmological constants\( (= \text{potential heights}) \) is just a hypothesis—called multiple point principle—on which one of us and his collaborators have worked a lot and claim a fair amount of phenomenological success.

In fact we started by fitting fine structure constants in model with a bit unusual gauge group by means of the phase transition couplings in lattice gauge theories. Now phase transition couplings would mean couplings for which more than one phase of the vacuum can coexist. So asking for vacua with the same cosmological constants is in fact equivalent to ask for some relevant coupling constant being at the phase transition point. So if we look the lattice gauge theory serious to really exist in nature, or just the lattice artifact monopoles which mainly determine the phase transition couplings, the above prediction of degenerate vacua would imply such phase transition coupling constant values. In the old times we had indeed a sort of historically probable success in the sense that we had the by that time unknown number of families of leptons and quarks as a fitting parameter relating the “family gauge group” gauge couplings taken to be just at the phase transition point, and
we fitted it to be three. Thereby we predicted by a model that had as one of its major input assumptions the equally deep minima—although formulated rather differently—just derived. The model though is just one among many possibilities first of all characterized by having the gauge group of the Standard Model $G = SU(3) \times SU(2) \times U(1) = S(U(2) \times U(3))$ repeated at a more fundamental level near the Planck energy scale once for each family of quarks and leptons. In other words, each of the $N_{gen}$ families of quarks and leptons supposed finally to be found had their own set of Standard Model gauge particles only acting on that “proto” family. Remarkably we predicted this number of families $N_{gen} \approx 3$ before the measurement at L.E.P. of the number of families of neutrinos.

In the pure Standard Model our requirement of same dept in this case of a second minimum in the Weinberg-Salam Higgs effective potential as the one of which we live $\langle \phi_{ws} \rangle \approx 24 GeV/\sqrt{2}$ leads to the Higgs particle to be the minimal one allowed by stability of vacuum. Without extra corrections pure renormalization group calculations lead to a prediction of the Higgs mass from this degeneracy principle to be $135 GeV/c^2$. This is already good in consideration of indirect Higgs mass determinations pointing to a light Higgs mass.

In works involving one of us (H.B.N.) and C.D. Froggatt and L. Laperašhivieli were developed a perhaps not so trustable story of an exceptionally strongly bound highly exotic meson of 6 top quarks and 6 antitop quarks bound together by Higgs exchange just in such a way as to produce a degenerate vacuum with this type of exotic meson forming a Bose-condensate. Remarkably enough our calculations taking that sort of bound state or exotic meson serious and imposing the degeneracy of the vacua, not only leads to an only within uncertainly too high Yukawa coupling for the top quark, but also solves the problem essentially behind the hierarchy problem! Indeed the coincidence of the top-quark-Yukawa-coupling values $g_t$ needed for

1) getting the bound state condensate just be degenerate, and for

2) getting it possible to have the second minimum in the Weinberg-Salam Higgs field effective potential degenerate with the first one;

leads to a need for the ratio of the Higgs vacuum expectation values the minima be a number given as an exponential.
That is to say that if for some reason the second minimum in the Weinberg-Salam Higgs effective potential were of the order of some grand unifying scale or the Planck scale or a fundamental scale, then the ratio of this scale to the weak scale would be explained to have to be an exponentially big ratio from the derived multiple point principle in the present article. In this sense we can claim that the multiple point principle solved the question as to why so big a scale ratio problem, a problem which is really behind the more technical hierarchy problem.

6 Conclusion and Outlook

We have worked further on the model that the second law of thermodynamics be caused by there existing a “fundamental” probability density functional $P$ assigning to each possible solution “path” of the equations of motion a probability density $P(\text{“path”})$ in phase space. Without making more than even in mild form the assumption that this “fundamental” probability assignment $P$ should obey the usual properties of laws of nature—locality (in time first of all) and translational invariance—we already got (phenomenologically) good results. In fact we roughly and practically got the second law of thermodynamics as was the initial purpose and in addition some good cosmology.

The present article obtained the further prediction of there being most likely many different states of vacuum, all having small cosmological constants. It must be admitted though that we only obtained this result with the further very important assumption that—some way or another the coupling constants and mass parameter, i.e. the coefficients in the Lagrangian density, have become or are what we call “dynamical”. This meant that it somehow were themselves or depended on ordinary dynamical variables, like fields or particle positions. Now it turned out remarkably that this prediction by one of us and his collaborators had since long been argued to be a good one phenomenologically! It must be admitted though that for all its successes a bit of helping assumptions were to be used. But even with only a mild assumption that the order of magnitude of the Higgs field in the high Higgs VEV alternative vacuum we got a very good value for the top quark mass $173 GeV \pm 6 GeV$. Taking our previous Multiple Point fitting most seriously with three degenerate vacua in the Standard Model alone we actually could claim that
Higgs-mass of 115GeV/c² seemingly found at L.E.P. is quite well matching as being our prediction. It must be admitted though that especially our correction bringing the predicted down from our older prediction 135GeV/c² to about the L.E.P. values is very doubtful and uncertain.

It is remarkable that we get such a funny and at least in future by Higgs mass, testable series of models higher scale of energy predictions about couplings constants as this multiple point principle out of modeling the second law of thermodynamics, an at first sight rather different branch in physics. Already this, provided it works (i.e. Higgs mass really be what the calculations will give etc.), would be a remarkable sort of unification of this second law with other physics information, seemingly at first quite unrelated! Taking into account that the major development of the universe into a low density, low temperature, large universe could—in the foregoing articles in this series—be considered the major “hand of God effects” predicted from of model we must say that it unifies quite far away features for the physical world!

As outlook we may list a few routes of making testing of our present unification:

1) In the light of the result of the present article testing of there being the many degenerate vacua in the various models beyond the Standard Model may if sufficiently successful be considered a confirmation of our “law behind the second law of thermodynamics”.

2) One could seek to estimate more numerically the cosmological parameters such as what size the already argued to be “small” cosmological constant should have included here could also be if some detail concerning the inflation going on predicted could be tested by say microwave background investigations.

3) A third route of testing or checking the model would be to really find rudimentary “hand of God effects”. That would of cause from the conventional theory point of view be quite shocking and thus be a strong confirmation of something in the direction of our model, if such effects were convincingly seen. It would of course be even more convincing if they were found with a predictable order of magnitude and of the right type. In previous articles we put is as an especially likely possibility that Higgs particles—special in the Standard Model by not being mass protected—were either flavored or disfavored to be produced.
That is to say there would respectively happen hand of God effects seeking to enhance or to diminish the number of Higgs particles being produced.

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