Phase control of SBS PCM seeding by optical interference pattern clarified: Direct applicability for IFE laser driver

Ondrej Slezak and Milan Kalal
Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague, Brehova 7, 115 19 Prague 1, Czech Republic
E-mail: slezaond@fjfi.cvut.cz

Hong Jin Kong
Department of Physics, Korea Advanced Institute of Science and Technology, Yuseong-gu, Daejeon, Korea

Abstract. Theoretical background of stimulated Brillouin scattering (SBS) seeded by a spatially localized optical interference pattern was investigated. Performed analysis revealed that the SBS may have its origin in the acoustic standing wave produced as a transient state between the initial optical interference field and the resulting stationary density modulation. Acquired knowledge can be found valuable in optimization of laser beam combination scheme considered e.g. for IFE applications.

1. Introduction
Construction of high energy high repetition laser drivers belongs to important IFE challenges. One particular approach to this goal considers a combination of less energetic laser beams into more powerful ones using stimulated Brillouin scattering (SBS) phase conjugating mirrors (PCM). The main obstacle of this approach is a randomness of the phase of the reflected Stokes field exhibited by individual laser beams before their combination [1, 2]. Hence it became necessary to find some mechanisms which could control the phase of the SBS PCM reflected waves. Over the last twenty years many techniques dealing with this problem were suggested [3]–[9]. Probably the most suitable among them was the technique adopted by Kong et al [10]. In this approach the feedback concave mirrors placed behind the SBS cells are used for a suitable phase locking. This technique was found both efficient as well as relatively insensitive to an optical alignment. It makes possible, in principle, to lock the phase of an arbitrary number of simultaneously working SBS PCMs.

2. Phase locking using feedback mirror
To get the SBS reflected Stokes field phase under control it is necessary to drive properly the process of an arising acoustic Brillouin wave. In particular, the initial time and position of such an acoustic wave have to be set the right way. It was proven experimentally that the phase difference between two or more SBS PCM reflected Stokes fields may be controlled with
a high degree of accuracy by the back-seeding concave mirror [10]. Theoretical description of mechanisms playing role in this phase locking technique was presented recently [11]. In that paper the formula was found describing how the Brillouin acoustic wave initial time depends on the gradual laser energy increase (starting the integration time from the pump pulse leading edge). This way the initial time was firmly established. However, the second condition, the exact spatial location of this wave, was still awaiting for its correct mathematical description as well as proper understanding.

When the concave mirror is placed behind the SBS cell (through which the parallel laser beam is traveling) the corresponding interference pattern is produced. In fact, this pattern is an optical standing wave, oscillating with the optical frequency. These oscillations are far too fast for any immediate response by SBS medium molecules. Therefore, the electrostriction source term coming from the time-averaged optical intensity modulation is employed to drive the acoustic wave [12]. However, such a stationary optical intensity field cannot become the source term for the moving density grating.

Let us analyze the SBS PCMs system shown in the Fig. 1.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** N simultaneously working SBS PCMs with back-seeding concave mirrors (BCM). The incident pump beam $E_{1j}$ for the $j$-th SBS PCM is going through the SBS cell unfocused. It is reflected by the $j$-th concave mirror and focused into the SBS cell as $E_{2j}$. The SBS reflected Stokes wave is denoted as $E_{3j}$. The $z$-position of the $j$-th BCM is given by $\delta_j$.

In order to comprehend the basic mechanism of the phenomenon as such, let us analyze the simplest model possible. All the waves ($E_{ij}$, $i = 1 \ldots 3$, $j = 1 \ldots N$) are considered to be the corresponding components of linearly polarized monochromatic plane waves which are switched on at the time $t = 0$ with constant amplitudes. The fact, that the beam is focused by the concave mirror and so $E_{2j}$ becomes the Gaussian beam, allows for further simplification considering just the Rayleigh region close to the focal plane of the mirror. In this area the electromagnetic wave reflected by the mirror may be approximated by the plane wave provided the pump beam cross section radius $r_p$ is significantly smaller then the concave mirror curvature radius $R_m$ ($r_p \ll R_m$). The reflected wave amplitude is then multiplied by the factor $\mu$ which represents the amplitude increase caused by focusing.

Under these conditions any optical wave in the system under consideration may be expressed as

$$E_{ij} = \frac{1}{2} A_{ij} \exp \left[ i \left( k_{ij} z - \omega_{ij} t + \phi_{ij} \right) \right] + \text{c.c.},$$

where $A_{ij}$, $k_{ij}$, $\omega_{ij}$, and $\phi_{ij}$ denote the amplitude, wave number, frequency, and phase of the $ij$-th wave, respectively. The waves reflected by the mirror are counter-propagating and so $k_{3j} = -k_{1j} \equiv k_j$. The mirror reflection does not change the waves frequency $\omega_{3j} = \omega_{1j} \equiv \omega_j$. The phase after the reflection is given by $\phi_{2j} = \phi_{1j} + \pi + (k_{1j} - k_{2j}) \delta_j$. Substitution for $k_{2j}$ provides $\phi_{2j} = \phi_{1j} + \pi - 2k_j \delta_j$. The amplitude after the reflection is given as $A_{2j} = \mu_j A_{1j} \equiv \mu_j A_j$. 


By using these formulae it is possible to derive the intensity interference pattern in the SBS PCM proximity in the form

$$
\langle E_j^2 (z, t) \rangle = (E_{1j} + E_{2j}) (E_{1j} + E_{2j})^* = 
\frac{1}{4} |A_j|^2 \left[ \frac{1}{2} (1 + \mu_j^2) - \mu_j \exp [i2k_j (z - \delta_j)] \right] + c.c.,
$$

(2)

It should be noted that the intensity interference pattern nodal point position does not depend on the pump beam phase $\phi_{1j}$. It is fully determined by the actual mirror position $\delta_j$.

The acoustic wave equation right-hand side driven by the electrostriction is given as [12]

$$
g_j (z, t) = - \frac{\gamma}{8\pi} \frac{\partial^2}{\partial z^2} \langle E_j^2 (z, t) \rangle ,
$$

(3)

where the electrostriction coupling constant $\gamma$ was introduced. Substitution of (2) into (3) provides

$$
g_j (z, t) = - \frac{\gamma \mu_j}{4\pi} |A_j|^2 k_j^2 \exp [i2k_j (z - \delta_j)] + c.c.
$$

(4)

The acoustic waves $g_j (z, t)$ in SBS medium are solutions of the acoustic wave equation (for the sake of simplicity no damping was considered)

$$
\frac{\partial^2 g_j}{\partial t^2} - v^2 \frac{\partial^2 g_j}{\partial z^2} = - \frac{\gamma \mu_j}{4\pi} |A_j|^2 k_j^2 \exp [i2k_j (z - \delta_j)],
$$

(5)

where $v$ stands for the sound velocity in SBS medium.

The initial conditions may be determined as the acoustic noise waves with random phases. Considering only the SBS phase matched waves and denoting $q_j = 2k_j$ and $\Omega_j = q_j v$ these initial conditions can be expressed the following way

$$
\varrho_j (z, 0) = \frac{1}{2} S_j \left\{ \exp \left[ i \left( q_j z + \varphi_j^- \right) \right] + \exp \left[ i \left( q_j z + \varphi_j^+ \right) \right] \right\} + c.c.,
$$

$$
\frac{\partial \varrho_j}{\partial t} (z, 0) = \frac{1}{2} iS_j \Omega_j \left\{ - \exp \left[ i \left( q_j z + \varphi_j^- \right) \right] + \exp \left[ i \left( q_j z + \varphi_j^+ \right) \right] \right\} + c.c.,
$$

(6)

where $S_j$ is the acoustic noise amplitude dependent on geometry, temperature, material parameters, and frequency $[1]$ and $\varphi_j^\pm$ denotes the thermal noise random phases.

The general solution of the problem (5)+(6) can be found in the form

$$
\varrho (z, t) = \varrho_0 - \left\{ \frac{\gamma \mu_j}{32\pi v^2} |A_j|^2 \exp [i(q_j (z - \delta_j)] + 
\frac{1}{2} \left( \frac{\gamma \mu_j}{32\pi v^2} |A_j|^2 \left[ \exp [-iq_j \delta_j] + S_j \exp [i\varphi_j^+] \right] \right) \exp [i(q_j z - \Omega_j t)] + 
+ \frac{1}{2} \left( \frac{\gamma \mu_j}{32\pi v^2} |A_j|^2 \left[ \exp [-iq_j \delta_j] + S_j \exp [i\varphi_j^+] \right] \right) \exp [i(q_j z + \Omega_j t)] \right\} + c.c.,
$$

(7)

This solution consists of three distinct components. The first one ($\varrho_0$) represents the mean density value of the medium. The second component (remaining part of the first line) stands for the stationary density modulation. The terms on the second and the third line express the standing acoustic wave written in the form of a superposition of two counter-propagating acoustic waves. One of these waves exactly matches the SBS wave.

Moreover, the acoustic waves from thermal noise background with their phase close to $\varphi_j^\pm = -q_j \delta_j$ undergo the constructive interference with the interference field drive wave and become dominant in the acoustic noise background. Such waves have the highest probability to become the SBS seed. However, in the case of $\frac{\gamma \mu_j}{32\pi v^2} |A_j|^2 \gg S_j$ it is possible to neglect the thermal noise background compared to the standing wave. It should be apparent from (7) that the relative phase difference between any SBS cells may be tuned by the change of $\delta_j$ parameter.
3. Conclusions
A simple model of the phase locking controlled by SBS PCM scheme using a back-seeding concave mirror was analyzed. It was found that the SBS process can be efficiently seeded by a standing acoustic wave produced by an optical interference pattern in SBS PCM proximity. Compared to the thermal noise random phase wave seeding considered so far this is a new important result. Formulae were found to determine under which conditions one of these seeding processes would prevail. The corresponding acoustic grating induced by the interference field is spatially localized uniquely with respect to the concave mirror position. Using this approach it is possible to control the relative phase of the backscattered Stokes field for an arbitrary number of SBS cells simultaneously employed thus offering many interesting applications. This standing acoustic wave is in fact a transient phenomenon as for the equation with viscosity included this wave would become gradually extinct and only the stationary density modulation would remain. The solution found is an important contribution to proper understanding of this already observed and in practice employed mechanism of the phase locking.

4. Acknowledgments
This research was supported by the Ministry of Schools, Youth and Sports of the Czech Republic (project No. LC528 and grant KONTAKT ME933), by the IAEA Research Contract No. 13781, and No. 13758/R0, by the Nuclear Research & Development Program of the Korea Science and Engineering Foundation (KOSEF) grant funded by the Korean government (MEST) (grant code: M20090078160), directly by CTU in Prague, Czech Republic, and also by KAIST, Daejeon, Republic of Korea.

References
[1] Boyd R W, Rzazewski K, and Narum P 1990 Noise initiation of stimulated Brillouin scattering Phys. Review A 42 5514
[2] Basov N G, Zubarev I G, Mironov A B, Mikhailov S I, and Okulov A Yu 1980 Phase fluctuations of the Stokes wave produced as a result of stimulated scattering of light Zh. Eksp. Teor. Fiz. 31 645
[3] Rockwell D A and Giuliano C R 1986 Coherent coupling of laser gain media using phase conjugation Opt. Lett. 11 147
[4] Loree T R, Watkins D E, Johnson T M, Kurnit N A, and Fisher R A 1987 Phase locking two beams by means of seeded Brillouin scattering Opt. Lett. 12 178
[5] Moyer R H, Valley M, and Cimolino M C 1988 Beam combination through stimulated Brillouin scattering J. Opt. Soc. Am. B 5 2473
[6] Andreev N F, Khazanov E A, Kuznetsov S V, Pasmanik G A, Shklovsky E I, and Sidorin V S 1991 Locked phase conjugation for two-beam coupling of pulse repetition rate solid-state lasers IEEE J. Quant. Elect. 27 135
[7] Ridley K D and Scott A M 1996 Phase-locked phase conjugation using a Brillouin loop scheme to eliminate phase fluctuations J. Opt. Soc. Am. B 13 900
[8] Kong H J, Lee H J, Shin Y S, Byun J O, Park H S, and Kim H 1997 Beam recombination characteristics in array laser amplification using stimulated Brillouin scattering phase conjugation Opt. Review 4 277
[9] Bowers M W and Boyd R W 1998 Phase locking via Brillouin-enhanced four-wave-mixing phase conjugation IEEE J. Quant. Elect. 34 634
[10] Kong H J, Lee S K, Yoon J W, and Beak D H 2006 Beam combination using stimulated Brillouin scattering for the ultimate high power-energy laser system operating at high repetition rate over 10Hz for laser fusion driver Opt. Review 13 119
[11] Ostermeyer M, Kong H J, et al. 2008 Trends in stimulated Brillouin scattering and optical phase conjugation Las. Part. Beams 26 297
[12] Boyd R W 1992 Nonlinear Optics San Diego: Academic press Inc.