Analysis and experiment of HMT stationary shift control considering the effect of oil bulk modulus

Xueliang Li, Lu Zhang, Shujun Yang and Nan Liu

Abstract
In order to improve the shift quality of hydro-mechanical continuously variable transmission, the effect of tangent bulk modulus and different control methods on the shift quality were analyzed. Theoretical analysis and experimental study on the tangent bulk modulus of oil were carried out to obtain the effect law of air content on the tangent bulk modulus of oil. A four-cavity model of a closed hydraulic circuit was established based on a two-stage arithmetic type hydro-mechanical transmission. By means of simulation analysis and experimental study, the effect of the tangent bulk modulus of oil on the shift quality is studied. The lean control method of reasonably controlling displacement ratio and prolonging the reverse time of load torque is put forward. The results show that this method can reduce the fluctuations of the speed of the fixed displacement motor and the oil pressure of the original low-pressure side. This method can also improve the shift quality and provide reference for the study of the shift process of hydro-mechanical continuously variable transmission.

Keywords
Hydro-mechanical continuously variable transmission, shift process, bulk modulus, shift quality, control method

Date received: 21 May 2020; accepted: 2 October 2020

Handling Editor: James Baldwin

Highlights
- Theoretical analysis and experimental study on the tangent bulk modulus of oil were carried out to obtain the effect law of air content on the tangent bulk modulus of oil.
- By means of simulation analysis and experimental study, the effect of the tangent bulk modulus of oil on the shift quality is studied.
- The control method of reasonably controlling displacement ratio and prolonging the reverse time of load torque is put forward.

Introduction
Hydro-mechanical continuously variable transmission (HMT) is a dual-power flow transmission composed of hydraulic power transmitting unit and mechanical power transmitting unit in parallel. With the combination of hydraulic and mechanical units, continuous stepless changes in transmission ratio can be realized to make the engine work in the efficient zone. In addition, the oil can buffer and reduce the transient load on the transmission system to effectively improve the service life of vehicles, which is especially important for engineering vehicles working in harsh environments for a
long time. HMT, a high-power, high-efficiency, and adaptable stepless transmission, is one of the ideal transmission forms for vehicles that not only meet the need of basic driving, but also the need of high-power operation.2

In order to improve the power and economy of vehicles equipped with HMT, scholars have done a lot of research on the influencing factors, control strategies, and layout structure.3–14 However, the poor dynamic quality of the HMT during the shifting between different ranges affects its wide application in heavy vehicles. Thus, Ni et al.,3 Zhu et al.,4 and Hu et al.5 analyzed the order of the effects of different physical parameters of the transmission on the shift quality by experiments. Yuan and Hu6 and Wei et al.7 found that the change of the volumetric efficiency of the hydraulic system led to the shifting impact. Wang and Wang8 and other researchers found that the change in the function of hydraulic components during the shift affects the stability of the fixed displacement motor speed as well as the shift quality. To solve the above problems, Zhang9 Yang et al.,10 and Yin11 respectively proposed control strategies for the shifting timing and the shifting time of multiple sets of brakes, and the rationality of the strategies was verified by experiments. Irfan et al.,12 Sung et al.,13 and Zhang et al.,14 respectively, used GT-Suite model, network analysis, and genetic algorithm to optimize the structure of HMT, improving the transmitting efficiency of the system and reducing the fuel consumption. However, all of the above studies are based on the mechanical structure, without considering the effect of oil characteristics on the shift quality.

The compressibility of the hydraulic oil in the closed pressure cavity has a great effect on the working performance of the system. The bulk modulus of the oil (compression resistance) is an important basic parameter of the pressure cavity. Its value and its variation law directly affect the establishment of the system pressure, thus affecting the operating characteristics of the system. Casoli et al.15 and Ruan16 studied the effects of different oil bulk modulus models on the performance of the pump, so as to optimize the pump structure and reduce the liquid flow vibrations in the pipeline. Yang et al.,17 Wei et al.,18 and Zhang19 proposed that the bulk modulus of oil is an important factor affecting the fluctuation of oil pressure in the hydraulic circuit and the speed of the fixed displacement motor. The research results of Hass20 and Kim and Murrenhoft21 show that the effective bulk modulus, as an important input parameter of hydraulic components, whose accurate value must be considered to improve the simulation accuracy. However, the above researches only analyze the effect of bulk modulus of oil on the response characteristics of the system, and does not put forward the control method to improve the dynamic response characteristics of the system. Controlling the shift process and improving the effect of bulk modulus of oil on the shift quality of HMT are still in the early stage of development.

In order to reduce the effect of the bulk modulus of oil on the dynamic characteristics of HMT, the tangent bulk modulus model of oil was established. The closed hydraulic circuit was divided into four end-to-end cavities, and a four-cavity model of HMT closed hydraulic circuit was built. By means of simulation analysis and experimental study, the effect laws of oil bulk modulus on the shift quality is studied. According to the above effect laws, an lean control method of displacement ratio and the flipping time of two brakes is put forward; a mathematical model of displacement ratio of variable displacement pump and a mathematical model of flipping time of two brakes are established; through simulation analysis and experiment verification, the results show that in the shift process of HMT, considering the effect of oil bulk modulus, the lean control of displacement ratio and the shifting time can effectively reduce the fluctuation of oil pressure in closed hydraulic circuit and the fluctuation of speed of fixed displacement motor, eliminate the speed difference of the motor before and after the shift, effectively improve the shift quality.

**Modeling and experiment of tangent bulk modulus of oil**

Oil bulk modulus is an important physical parameter of hydraulic oil, which directly affects the working characteristics of hydraulic system. In the actual design and analysis of hydraulic system, the bulk modulus of hydraulic oil is usually regarded as a constant.22 However, for the HMT closed hydraulic system, which requires high precision, dynamic characteristics, and stability, there will be large errors, which will affect the design of control strategy and the determination of parameter values of the hydraulic system. Therefore, in the design and analysis of hydraulic systems, especially HMT, it is particularly important to consider the change of bulk modulus of oil, see Figure 1 below for stress–strain curve.

For point A, its secant bulk modulus is calculated by the stress and strain at point A, namely the slope of string OA,

\[ E_s = -\frac{\Delta p}{\Delta V_s} V_s \]  

(1)

Where, \( p \) is the current pressure, MPa; \( V_s \) is the total volume of oil with secant bulk modulus, m³.

The tangent bulk modulus of point A is calculated by the stress derivative and strain derivative of point A, namely the slope of the tangent line AB.23

\[ E_t = -\frac{dp}{dV_t} V_t \]  

(2)
Where, $V_1$ is the total volume of oil with tangent bulk modulus, m$^3$.

The solution of the above differential equation is:

$$V_1 = V_{10}e^{-(p-p_{0})/E_i}$$

Where, $V_{10}$ is the initial total volume of oil with tangent bulk modulus, m$^3$; $p_{0}$ is current oil pressure, MPa.

The bulk modulus of oil can be divided into isentropic modulus and isothermal modulus according to the different thermodynamic properties in the process of oil compression.

Studies show that the isentropic modulus is usually greater than the isothermal modulus, and the tangent bulk modulus is usually greater than the secant bulk modulus. In the design analysis and calculation of hydraulic system, the tangent bulk modulus is usually adopted as the value of bulk modulus.\(^{24}\)

**Mathematical model of the tangent bulk modulus of oil**

When HMT shifts, the oil pressure changes at a faster rate, which can be regarded as an adiabatic process. The isentropic tangent bulk modulus is adopted in consideration of the dynamic pressure change process in the cavity. Generally, oil exists as a mixture of pure oil and air dissolved and not dissolved in the oil. Since the air dissolved in the oil has no effect on the bulk modulus of the oil, it is ignored, and only the effect of air suspended in the oil on the bulk modulus of the oil is considered.\(^{17}\)

Defined $C_u$ as the initial volume ratio of air suspended in the oil, that is, the air content.

$$C_u = \frac{V_{a0}}{V_{10}}$$

Where, $V_{a}$ is the volume of air suspended in the oil, m$^3$; $V_{a0}$ is the initial volume of air suspended in the oil, m$^3$.

The bulk modulus of pure oil is:

$$E_i = - \frac{dp}{dV_1}V_1$$

Where, $V_1$ is the volume of pure oil, m$^3$.

The solution of the differential equation above is:

$$V_1 = V_{10}e^{-(p-p_{0})/E_i}$$

Where, $V_{10}$ is the initial volume of pure oil, m$^3$.

Bubbles in oil have complex dissolution and compression processes. The equation of air state:

$$\frac{p_0V_{a0}^k}{T_0} = \frac{pV_{a}^k}{T}$$

Where, $V_{a}$ is the volume of air suspended in the oil, m$^3$; $k$ is air polytropic index; $T_0,T$ are initial temperature and current temperature of the oil respectively, K.

The total volume of the fluid is:

$$V_t = V_a + V_1 = V_{a0}\left(\frac{pT}{p_{0}T_0}\right)^{\frac{1}{k}} + V_{10}e^{-(p-p_{0})/E_i}$$

According to equations (2) to (7), the tangent bulk modulus of the fluid is:

$$E_i = \frac{e^{-(p-p_{0})/E_i} + C_u\left[\left(\frac{pT}{p_{0}T_0}\right)^{\frac{1}{k}} - e^{-(p-p_{0})/E_i}\right]}{e^{-(p-p_{0})/E_i} + C_u\left[\left(\frac{pT}{p_{0}T_0}\right)^{\frac{1}{k}} - e^{-(p-p_{0})/E_i}\right]}E_i$$

The performance parameters of the oil in the standard state can be queried, $p_{0}$ and $T_{0}$ in the above equation can be replaced by oil pressure $p_{STP}$ and temperature $T_{STP}$ of the standard state. In the closed hydraulic circuit, $(p - p_{0}) \ll E_i$, thus:

$$- e^{-(p-p_{0})/E_i} = 1$$

Equation (9) can be simplified as:

$$E_i = \frac{1 + C_u\left[\left(\frac{p_{STP}}{p_{STP}}\right)^{\frac{1}{k}} - 1\right]}{1 + C_u\left[\left(\frac{p_{STP}}{p_{STP}}\right)^{\frac{1}{k}} - 1\right]}E_i$$

where $p_{STP}$ and $T_{STP}$ are oil pressure and temperature in standard state.

Similarly, the density of air containing oil is

**Figure 1.** Stress–strain curve.
\[
\rho = \frac{1 + \frac{\rho_{\text{STP}}}{\rho_{\text{STP}}} \frac{C_a}{1-C_a}}{\frac{1}{\rho_{\text{STP}}} + \frac{1}{\rho_{\text{STP}}} \left( \frac{\rho_{\text{STP}} T}{\rho_{\text{STP}}} \right) \frac{C_a}{1-C_a}}
\]

(12)

Where, \(\rho_{\text{STP}}\) is air density in standard state, kg/m\(^3\); \(\rho_{\text{ISTP}}\) is pure oil density in standard state, kg/m\(^3\).

When HMT works, the fluid temperature in the closed hydraulic circuit will reach a dynamic equilibrium. When the temperature is 90°C, the relationship curve between the oil tangent bulk modulus and the air content and oil pressure can be calculated from the equation (11), as shown in Figure 2. Compared with oil, the elastic modulus of air is very small, so the volume elastic modulus of oil decreases as the air content increases; with the increase of working pressure, the air solubility of oil increases, and the free air content in oil decreases, so the elasticity bulk modulus of oil increases with the increase of working pressure; due to the effect of temperature, the viscosity and molecular state of oil change, so the bulk modulus of oil decreases slightly with the increase of oil temperature.

This paper discusses the shift process of H-range to HM-range. The oil pressure of the low-pressure side of the HMT closed hydraulic circuit is about 2.55 MPa, and the load torque of the fixed displacement motor of the H-range is 147 N⋅m; because of the change of the power transmitting route, the load torque of the fixed displacement motor of the HM-range is 99 N⋅m, and the oil pressure of the high-pressure side (the original low-pressure side) is about 8.1 MPa. In the shift process, the direction of load torque of fixed displacement motor changes, and the pressure of high- and low-pressure side of hydraulic circuit changes rapidly. The pressure at the original low-pressure side increased from 2.55 MPa to the high-pressure side pressure value of HM-range, 8.1 MPa, and the pressure at the original high-pressure side decreased to 2.55 MPa. It can be seen from Figure 2 that the tangent bulk modulus increases rapidly in the pressure range of 0 to 5 MPa, and is basically stable after 5 MPa. When system shifts, the bulk modulus of oil is in the fast-changing area, and the smaller pressure change will produce the larger bulk modulus change, which will cause the output speed of fixed displacement motor to fluctuate. The hydraulic transmitting unit and mechanical transmitting unit of HMT output the speed by the power merging planetary gear set. The speed fluctuation of fixed displacement motor will make the output speed of HMT fluctuate. Therefore, it is necessary to analyze the response characteristics of the closed hydraulic circuit under the changing bulk modulus of the oil, so as to provide the basis for theoretical analysis and engineering practice.

**Experimental study on tangent bulk modulus of oil**

According to Henry’s law, a measuring device for the air content of oil is designed, including a flat bottom flask, a vacuum pump and a constant temperature bath, as shown in Figure 3. According to the definition of tangent bulk modulus of oil, the following measuring devices of bulk modulus of oil are designed, mainly including: pressure sensor, linear displacement sensor, single piston rod hydraulic cylinder, loading jack, bracket and data acquisition system, as shown in Figure 4. The air content and bulk modulus of the hydraulic oil stirred at different times are measured with the above device.

Figure 5 is the theoretical curve and experiment curve of tangent bulk modulus changing with pressure when the temperature is 90°C and the air content is 0.01 and 0.017 respectively.

It can be seen from Figure 5 that the theoretical results of tangent bulk modulus of oil are basically consistent with the experiment results. When the air content is 0.01, the maximum deviation is 7.17%, and when the air content is 0.017, the maximum deviation is 8.06%. Therefore, the tangent bulk modulus model...
can accurately reflect the oil bulk modulus during the shift process of HMT closed hydraulic circuit.

**Modeling and simulation of closed hydraulic circuit**

The principle of two-stage arithmetic type hydro-mechanical transmission is shown in Figure 6, and the following analysis is carried out in this structural form. In the Figure 6, variable displacement hydraulic pump \( P \) and fixed displacement hydraulic motor \( M \) constitute the hydraulic circuit; planetary gear sets \( K_1, K_2, K_3 \) and brakes \( C_H, C_{HM} \) constitute the mechanical unit and the power merging set; \( n_i, n_o \) represent the input and output speed respectively, see Table 1 for the range shift logic.

During the shift process of HMT, the functions of the hydraulic components are interchanged, and the high- and low-pressure sides of the closed hydraulic circuit are interchanged. That is, in the H-range, the variable displacement hydraulic pump drives the fixed displacement hydraulic motor, and the outlet of the variable displacement hydraulic pump and inlet of the fixed displacement hydraulic motor are connected with the oil circuit at the high pressure side; after changing to the HM-range, the torque of the fixed displacement hydraulic motor is reversed, the fixed displacement hydraulic motor drives the variable displacement hydraulic pump, and the outlet of the fixed displacement hydraulic motor and inlet of the variable displacement hydraulic pump are connected with the oil circuit at the high pressure side.

In order to study the effect of bulk modulus of oil on the speed of fixed displacement motor during the shifting, the four-cavity model of closed hydraulic circuit is built, and its composition is shown in Figure 7. When modeling, the pressure loss of oil flow along the pipeline is ignored, and the external leakage is considered. The closed hydraulic circuit is simplified into four closed cavities. In the Figure 7, the four-cavity model of the hydraulic circuit consists of the cavity mass flow model and the cavity pressure model.

\( V_1, V_2, V_3, \text{ and } V_4 \) are respectively the volumes of closed cavities 1, 2, 3, and 4, m\(^3\); \( p_1, p_2, p_3, \text{ and } p_4 \) are the oil pressure of corresponding cavities, MPa; \( q_{m1o} \text{ and } q_{m2i} \) are respectively the output mass flow and input mass flow of variable displacement pump, kg/s; \( q_{m4o} \text{ and } q_{m3i} \) are respectively the output mass flow and input mass flow of fixed displacement motor, kg/s; \( n_p \) and \( n_m \) are respectively the speed of variable and fixed displacement hydraulic elements, rpm; \( T_p \text{ and } T_m \) are...
respectively torque of variable and fixed displacement hydraulic elements, N \cdot m.

**Speed and torque model of hydraulic components**

When shifting from H-range to HM-range, the speed of the variable hydraulic pump is
\[ n_p = n_i / i_1 \]  
(13)

Where, \( n_i \) is the input speed of HMT, rpm; \( i_1 \) is the transmission ratio from input shaft to hydraulic transmitting unit.

The torque balance equation of fixed displacement motor is:
\[ T_m = J \dot{\omega}_m + D \omega_m + V_m (p_3 - p_4) \]  
(14)

Where, \( V_m \) is the displacement of the fixed displacement motor, m³/rad; \( J \) is the equivalent moment of inertia of the fixed displacement hydraulic motor and its connected shaft, kg \cdot m²; \( \omega_m \) is the angular velocity of the fixed displacement motor, rad/s; \( D \) is the damping coefficient of the fixed displacement motor, N-s/m.

**Mass flow model of cavity**

Variable displacement hydraulic pump and fixed displacement hydraulic motor are simplified as external leakage mode. When considering the effect of oil pressure, temperature and air content on oil, the equivalent leakage of hydraulic component is
\[ \Delta q = \frac{C_s V_g \Delta p}{2 \pi \mu} \left( 1 + \frac{\Delta p}{E_i} \right) \]  
(15)

Where,
\[ \mu = \mu_c (1 + 0.015 C_u) e^{\alpha (p_3 - p_4)} \]  
(16)

Where, \( C_s \) is leakage coefficient; \( V_g \) is displacement of hydraulic element, m³/r; \( \Delta p \) is pressure difference between inlet and outlet of hydraulic element, MPa; \( \gamma \) is thermal coefficient of expansion, \( \gamma = 1 + \gamma_c (T - T_0) \); \( \gamma_c \) is thermal coefficient of expansion of pure oil; \( \mu_c \) is dynamic viscosity of pure oil, N-s/m²; \( \alpha \) is pressure-viscosity coefficient of oil, m²/N; \( \beta \) is viscosity-temperature coefficient of oil, K⁻¹.

The two hydraulic components work in different modes before and after the HMT shift process, so the mass flow models of the cavity are also different.

1. **Mass flow model of H-range cavity**

In H-range, the input mass flow of cavity-2 is
\[ q_{m2i} = \frac{\rho_2 n_2 V_p}{60} \]  
(17)

Where, \( \rho_2 \) is the oil density of volume cavity-2, kg/m³; \( V_p \) is the displacement of variable displacement pump, m³/rad

The output mass flow of cavity-1 is
\[ q_{m1o} = \frac{\rho_1 n_1 V_p}{60} - \rho_1 \Delta q_p \]  
(18)
Where, \( \rho_1 \) is oil density of volume cavity-1, kg/m\(^3\); \( \varepsilon \) is displacement ratio.

The input mass flow of cavity-3 is

\[
q_{m3i} = \frac{\rho_1 n_m V_m}{60} + \rho_3 \Delta q_m
\]  
(19)

Where, \( \rho_3 \) is oil density of volume cavity-3, kg/m\(^3\).

The output mass flow of cavity-4 is

\[
q_{m4o} = \frac{\rho_4 n_m V_m}{60}
\]  
(20)

Where, \( \rho_4 \) is the oil density of volume cavity-4, kg/m\(^3\).

(2) Mass flow model of HM-range cavity

In HM-range, the mass flow relationship between variable and fixed displacement hydraulic components is

\[
\frac{\eta_p V_p}{\eta_p} = n_m V_m \eta_{vm}
\]  
(21)

Where, \( \eta_p \) is volumetric efficiency of variable displacement pump; \( \eta_{vm} \) is volumetric efficiency of fixed displacement motor.

The input mass flow of cavity-3 is

\[
q'_{m3i} = \frac{\rho_1 n_m V_p}{60 \eta_p \eta_{vm}}
\]  
(22)

The output mass flow of cavity-4 is

\[
q'_{m4o} = \frac{\rho_4 n_m V_m}{60} - \rho_4 \Delta q_m
\]  
(23)

The input mass flow of cavity-2 is

\[
q'_{m2i} = \frac{\rho_2 n_m V_p}{60} + \rho_2 \Delta q_p
\]  
(24)

The output mass flow of cavity-1 is

\[
q'_{m1o} = \frac{\rho_1 n_p V_p}{60}
\]  
(25)

**Model of cavity pressure**

The oil temperature has almost no change in a short period of the shift, considering the change of bulk modulus, density and air content of the oil in the cavity, the pressure model of the cavity is

\[
\frac{dp}{dt} = \frac{E_1}{\rho V} (q_{mi} - q_{mo} - \rho V)
\]  
(26)

Where, \( q_{mi}, q_{mo} \) are mass flow of oil into and out of the cavity, kg/s; \( \rho \) is density of oil, kg/m\(^3\); \( V \) is volume of the cavity, m\(^3\).

(1) Model of H-range cavity pressure

In the H-range, the cavities 1 and 3 are high-pressure cavities, and the pressure changes of cavities 1 and 3 are:

\[
\frac{dp_1}{dt} = \frac{E_1}{\rho_1 V_{10}} (-q_{m1o} - \rho_1 \Delta q_p + \rho_1 n_p V_p / 60)
\]  
(27)

\[
\frac{dp_3}{dt} = \frac{E_3}{\rho_3 V_{30}} (q_{m3i} - \rho_3 \Delta q_m - \rho_3 n_m V_m / 60)
\]  
(28)

Ignoring the pressure loss in the high- and low-pressure circuits, the oil pressures in the cavities 1 and 3 are the same, which can be regarded as a whole cavity-5, and its oil pressure model is:

\[
\frac{dp_5}{dt} = \frac{E_5}{\rho_5 (V_{10} + V_{40})} \left( -\rho_1 \Delta q_p - \rho_3 \Delta q_m + \frac{\rho_1 n_p V_p}{60} - \frac{\rho_3 n_m V_m}{60} \right)
\]  
(29)

Where, \( \rho_5 \) is the density of oil in cavity-5, kg/m\(^3\); \( E_5 \) is the bulk modulus of oil in cavity-5, MPa.

(2) Model of HM-range cavity pressure

Similarly, in HM-range, the oil pressures of high-pressure cavities 2 and 4 are the same, which can be regarded as a whole cavity-6, and its oil pressure model is:

\[
\frac{dp_6}{dt} = \frac{E_6}{\rho_6 (V_{20} + V_{40})} \left( -\rho_2 \Delta q_p - \rho_4 \Delta q_m - \rho_2 \eta_p V_p / 60 + \rho_4 n_m V_m / 60 \right)
\]  
(30)

Where, \( \rho_6 \) is the density of oil in cavity-6, kg/m\(^3\); \( E_6 \) is the bulk modulus of oil in cavity-6, MPa. \( V_{20} \) is initial volume of cavity-2, m\(^3\); \( V_{40} \) is initial volume of cavity-4, m\(^3\).

The four-cavity model of hydraulic circuit built in Simulink is shown in Figure 8.

**Effect of bulk modulus of oil on shift quality**

Based on the four-cavity model of closed hydraulic circuit, different air content is substituted into the bulk modulus model of oil, and the effects of different bulk modulus of oil on the shift quality are simulated and analyzed. When HMT shifts from the H-range to the HM-range, the speed of the variable displacement pump is 1000 rpm, the oil temperature is 90 \(^\circ\)C, and the pressure at the low-pressure side of the closed hydraulic circuit is 2.55 MPa. The displacement ratio is constant at 0.98, and the load torque from -147 N \( \cdot \) m step changes to 99 N \( \cdot \) m (Figure 9). The bulk modulus of
oil with air content of 0.01, 0.02, 0.03, 0.04, and 0.05 are substituted into the model, and the simulation results are shown in Figure 10. As the effect of air content on the original high-pressure side oil pressure is not obvious, it is not stated here.

Figure 10(a) shows the oil pressure curves at the original low-pressure side with different air content. The higher the air content of the oil is, the greater the fluctuation of the oil pressure is, and the higher the oil pressure of the original low-pressure side of HM-range is.

Figure 10(b) shows the output speed curves of fixed displacement motor with different air content. The larger the air content of the oil is, the greater the fluctuation of the speed is; the greater the difference of the speed before and after the shift is.

It can be seen from the above that the bulk modulus of oil is an important factor affecting the shift quality.

In this paper, considering the effect of the bulk modulus of the oil, by precisely controlling the displacement ratio and the slipping time of the two brakes shifting, the fluctuation of the oil pressure at the original low-pressure side is reduced, so that the pressure value of the HM-range is kept at the target value, and the fluctuation of the speed of the fixed displacement motor is reduced, so as to eliminate the speed difference of the motor before and after the shift process.

**Lean control model of closed hydraulic circuit shift process**

### Mathematical model of displacement ratio of variable displacement pump

In H-range, when the pressure in the high-pressure cavity \( p_5 \) is stable, that is, \( \frac{dp_5}{dt} = 0 \), according to the equations (15) and (29), the displacement ratio of the variable displacement motor in H-range is:

\[
\varepsilon_H = n_m / n_p - \left( 1 + \frac{\Delta p_H}{E_x} \right) \frac{60 \Delta p_H C_s y}{n_p \pi \mu} \tag{31}
\]

Where, \( \Delta p_H \) is the oil pressure difference between high- and low-pressure sides of H-range, MPa.

In HM-range, when \( \frac{dp_6}{dt} = 0 \), according to the equations (15) and (30), the displacement ratio of variable displacement pump is:

\[
\varepsilon_{HM} = n_m / n_p - \left( 1 + \frac{\Delta p_{HM}}{E_x} \right) \frac{60 \Delta p_{HM} C_s y}{n_p \pi \mu} \tag{32}
\]

Where, \( \Delta p_{HM} \) is the oil pressure difference between high- and low-pressure sides of HM-range, MPa.

The change amount of displacement ratio before and after shifting is:
\[ \Delta e = \varepsilon_{HM} - \varepsilon_{H} = \frac{60C_{g} \gamma}{n_{1} \pi \mu} \left( \Delta p_{HM} + \Delta p_{H} + \frac{\Delta p_{HM}^{2}}{E_{6}} + \frac{\Delta p_{H}^{2}}{E_{5}} \right) \] (33)

From the above equation, it can be seen that the smaller the bulk modulus of the oil is, the greater the \( \Delta e \) is; from the relationship between the bulk modulus of the oil and the air content mentioned above, the greater the air content in the oil is, the smaller the bulk modulus is; therefore, the greater the air content in the oil is, the greater the \( \Delta e \) is.

In this paper, considering the effect of bulk modulus of oil, the displacement ratio of variable displacement pump is adjusted to improve the shift quality.

**Mathematical model of flipping time of two brakes**

The shifting time of HMT is the time required from the completely disengagement of the brake \( C_{H} \) to the completely engagement of the brake \( C_{HM} \).

The torque relationship of the sun gear, ring gear, and planet carrier in each planetary row is:

\[ T_{si} : T_{ri} : T_{ci} = 1 : k_{i} : -(1 + k_{i}) \] (34)

where, \( T_{si}, T_{ri}, \) and \( T_{ci} \) are the torques of the sun gear, ring gear, and planet carrier of the planet row \( K_{i} \), where \( i = 1,2,3; \) \( k_{i} \) is characteristic parameter of planetary row \( K_{i} \).

According to the structure of HMT in Figure 6, when the two brakes are engaged and overlapped, the expressions of the torque of each planetary row and the torque of brakes and fixed displacement motor are:

\[ \begin{align*}
T_{1} &= T_{CH} \\
T_{2} &= -T_{CHM} \\
T_{3} &= -T_{c3} \\
T_{1} + T_{2} + T_{m} &= 0
\end{align*} \] (35)

Where, \( T_{CH}, T_{CHM} \) are the torque produced by the brakes \( C_{H}, C_{HM}, \) N-m.

According to the equations (34) and (35), the relationship between the load torque of fixed displacement motor and the torque provided on brakes \( C_{H} \) and \( C_{HM} \) can be deduced as follows:

\[ \frac{T_{CH}}{k_{1}} - \frac{(1 + k_{1})T_{CHM}}{k_{3}(1 + k_{2})} + T_{m} = 0 \] (36)

Combining equations (14) and (26), the relationship between the shifting time required for the two brakes and the bulk modulus of the oil is as follows:

\[ \frac{\Delta}{\Delta t} \left[ \frac{(1 + k_{1})T_{CHM}}{k_{1}(1 + k_{2})} - \frac{T_{CH}}{k_{3}} - J\dot{\omega}_{m} - D\omega_{m} \right] = E_{1} \frac{\Delta q}{\rho} \] (37)

It can be seen from the above equation that the larger the bulk modulus of the oil is, the shorter the flipping time of the double brakes is required for the stable output of the speed of the fixed displacement motor; from the relationship between the bulk modulus of the oil and the air content, it can be seen that the larger the air content is, the longer the flipping time of the double brakes is, that is, the longer the shifting time is.

In this paper, considering the effect of bulk modulus of oil, the lean control method of two brakes flipping time was proposed to improve the shift quality.

**Figure 10.** Simulation results of shift when air content is different: (a) the oil pressure of original low-pressure side and (b) the speed of fixed displacement motor.
Effect of the lean control method on the shift quality

When HMT shifts from the H-range to the HM-range, according to the above displacement ratio mathematical model and the two brakes flipping time mathematical model, the displacement ratio and the lean control value of the shifting time are shown in Table 2 below by substituting the oil bulk modulus with the air content of 0.01, 0.02, 0.03, 0.04, and 0.05.

The data in Table 2 is substituted into the simulation model of HMT hydraulic circuit mentioned in section “Modeling and simulation of closed hydraulic circuit,” and the simulation results are shown in Figure 11.

Figure 11(a) shows the oil pressure curves of the original low-pressure side. Compared with Figure 10(a), the fluctuations are reduced by 85.29% to 94.9%; through the lean control, the oil pressures of HM-range with different air content are basically consistent with the target value.

Figure 11(b) shows the output speed curves of fixed displacement motor. Compared with Figure 10(b), the speed fluctuations are reduced by 72.95% to 89.58%; through the lean control for different air content, the speed differences before and after the shifting are less than 1 rpm.

When shifting, considering the effect of the bulk modulus of the oil, the reasonable controls of the displacement ratio and the shifting time can effectively reduce the pressure fluctuation in the hydraulic circuit and the speed fluctuation of the fixed displacement motor; keep oil pressure of the original low-pressure side consistent with the target value of the HM-range; keep the motor speed basically unchanged before and after the shifting.

Experiment and analysis of the HMT shift quality

In order to verify the simulation results, an experiment bench for the HMT shifting is built, as shown in Figure 12. The relevant parameters of the main equipment of the bench are listed in Table 3. The control oil source is provided by the pump station. By adjusting the current of displacement control proportional solenoid valve, the displacement ratio of variable displacement hydraulic pump can be adjusted. By controlling the voltage of the solenoid valve of transmission brakes $C_H$ and $C_{HM}$, the shifting time can be controlled.
Due to the limitation of the experiment condition, the validity of the simulation model and the proposed theory is only verified when the air content is 0.02. In order to facilitate the comparison, the simulation and experimental results are translated on the time axis, so that the starting time of the shifting process is aligned at about 0.25 s.

During the experiment, the speed of variable displacement hydraulic pump is 1000 rpm, the system temperature is constant at 90°C, and the load torque of HMT is 357 N·m. The experiment is started after the oil is stirred for 30 min, and the air content of the oil is 0.02, which can be considered as a constant during the experiment.

When shift from the H-range to the HM-range, the pressure control signals of the two brakes occur step change at the same time, and the oil charging and discharging process of the two brakes does not overlap; control the displacement ratio of the variable displacement hydraulic motor is constant 0.98 (Figure 13(a), and the simulation and experiment results are shown in Figure 13(b)).

In Figure 13(b), the fluctuation of oil pressure in the original low-pressure oil circuit is 8.786 MPa; the fluctuation of speed of fixed displacement motor is 43.9 rpm, and the speed difference before and after the shifting is 68.2 rpm. The state value and fluctuation value of the simulation curve before and after the shifting are basically consistent with the experiment curve, so the simulation model can describe the shifting characteristics of HMT.

When shifting, the displacement ratio and the flipping time of the two brakes are controlled by the value calculated by the mathematical models in sections “Mathematical model of flipping time of two brakes” and “Effect of the lean control method on the shift quality.” The oil charging and discharging process of the two brakes overlaps; the displacement ratio decreases linearly from 0.98 to 0.9125 (Figure 14(a)), and the simulation and experimental results are shown in Figure 14(b).

In Figure 14(b), the fluctuation of oil pressure in the original low-pressure oil circuit is 0.92 MPa; the fluctuation of speed of the fixed displacement motor is 8.8 rpm, and the speed difference before and after the shifting is less than 1 rpm. The simulation curve is basically consistent with the experiment curve. Compared with Figure 13(b), the fluctuation of oil pressure wave is reduced by 79.95%; the fluctuation of speed is reduced by 89.53%, and the speed difference before and after the shifting is reduced by 98.54%. Therefore, the lean control of displacement ratio and flipping time of two brakes proposed in this paper is effective when considering the bulk modulus of oil.

Table 3. Parameters of the main equipment of the bench.

| Name                        | Parameter               | Value         |
|------------------------------|-------------------------|---------------|
| Electric motor               | Rated power             | 250 kW        |
| Torque meter                 | Rated torque            | 1000 N·m      |
| Dynamometer                  | Rated power             | 200 kW        |
| Variable displacement pump   | Displacement            | 125 ml/r      |
| Fixed displacement motor     | Displacement            | 125 ml/r      |
| Three-phase asynchronous     | Rated power             | 3 kW          |
| Speed sensor                 | Response frequency      | 0–25 KHz      |
| Torque sensor                | Displacement            | 0 to ±1000 N·m |
| Temperature sensor           | Displacement            | −50°C–150°C   |
| Pressure sensor              | Displacement            | 0–60 MPa      |
| Data acquisition card        | Resolution              | 12 bit        |

Conclusion

In this paper, the closed hydraulic circuit of HMT is equivalent to four cavities, and the mathematical model of closed hydraulic circuit shift process considering the air content of oil is established. The effects of air content of oil on the shift quality are simulated and analyzed. The mathematical model of displacement ratio and the mathematical model of the shift time are established. The effect of displacement ratio and the flipping time of two brakes on the characteristics of the HMT
shift is analyzed and experimented. The conclusions are as follows:

1. Through the verification of the experiment, the tangent bulk modulus model can correctly reflect the bulk modulus of the oil in the shift process of HMT closed hydraulic circuit; when shifting, the bulk modulus of the oil changes greatly, so the effect of the bulk modulus of the oil should be fully considered.

2. The effect of the bulk modulus of the oil on the shift quality is analyzed. The results show that during shifting, the smaller the bulk modulus of the oil is, the greater the pressure fluctuation of the original low-pressure side is, and the greater the difference between the original low-pressure side and the target value is; the greater the air content is, the greater the speed fluctuation of the fixed displacement motor is, and the greater the speed difference before and after the shifting is. The smaller the bulk modulus of the oil is, the worse the quality of the shift is. The bulk modulus of oil, as an important factor, must be considered when studying the characteristics of HMT shifting process.

3. The mathematical models of displacement ratio and the shifting time are established, and the lean control method is put forward. The effectiveness of HMT four-cavity model and the lean control method are verified by experiments. It is concluded that the pressure fluctuation of hydraulic circuit and the speed fluctuation of fixed displacement motor can be effectively reduced, the speed difference before and after the shifting can be eliminated, and the shift quality can be improved when considering the oil bulk modulus during the shift process.
Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by the National Natural Science Foundation of China [Grant number 51675462].

ORCID iD
Xue-lian Li https://orcid.org/0000-0002-0628-779X

References
1. Yu J, Cao Z, Cheng M, et al. Hydro-mechanical power split transmissions: progress evolution and future trends. Proc Inst Mech Eng D J Automob Eng 2019; 233: 727–739.
2. Liu XJ. Vehicle transmission system analysis. Beijing: National Defense Industry Press, 1998, p.29.
3. Ni XD, Zhu SH, Zhang HJ, et al. Test of factors affecting the quality of hydromechanical continuously variable transmission. J Agric Mach 2013; 44: 29–34.
4. Zhu Z, Chen L, Cao Ll, et al. Analysis of gear shift quality factors of hydromechanical continuously variable transmission. Mech Des 2018; 35: 39–45.
5. Hu JB, Wei C and Yuan SH. Characteristics on hydromechanical transmission in power shift process. Chin J Mech Eng 2009; 22: 50–56.
6. Yuan SH and Hu JB. The efficiency of multirange hydro-mechanical continuously variable transmission. J Beijing Inst Technol 1998; 7: 129–134.
7. Wei C, Ma ZY, Yin XF, et al. Research on impact factors of hydraulic mechanical continuously variable transmission. J Beijing Inst Technol 2015; 35: 1122–1127.
8. Wang J and Wang JL. Research on the effect of hydraulic components on the dynamic characteristics of hydromechanical transmission. Mach Tools Hydraul 2010; 38: 69–71.
9. Zhang XS. Research on steady speed change and control strategy of hydro-mechanically continuously variable transmission. PhD Thesis, Jinlin University, China, 2011.
10. Yang SJ, Bao Y, Tang XZ, et al. Integrated control of hydro-mechanical variable transmissions. Math Probl Eng 2015; (7): 1-11.
11. Yin YX. Research on stationary smoothness of synchronous change of hydro-mechanical continuously variable transmission. PhD Thesis, Henan University of Science and Technology, China, 2017.
12. Irfan M, Berbyuk V and Johansson H. Minimizing synchronization time of a gear shifting mechanism by optimizing its structural design parameters. Proc IMechE, Part D: J Automobile Engineering 2020; 234: 488–504.
13. Sung D, Hwang S and Kim H. Design of hydromechanical transmission using network analysis. Proc Inst Mech Eng D J Automob Eng 2005; 219: 53–63.
14. Zhang Q, Sun DY and Qin DT. Optimal parameters design method for power reflux hydro-mechanical transmission system. Proc Inst Mech Eng D J Automob Eng 2019; 233: 585–594.
15. Cosoli P, Vacea A, Franzoni G, et al. Modelling of fluid properties in hydraulic positive displacement machines. Simul Model Pract Theory 2006; 14: 1059–1072.
16. Ruan J. Bulk modulus of air content oil in a hydraulic cylinder. In: Proceedings of the 2006 ASME international mechanical engineering congress and exposition, Chicago, IL, 5–10 November 2006, paper no. IMECE2006-15854, pp.259–269. ASME.
17. Yang SJ, Jiao XJ, Bao Y, et al. Effect of oil gas content on hydraulic machinery segment change performance. J Mech Eng 2015; 51: 123–130.
18. Wei C, Zhou JJ and Yuan SH. Comparison between steady model and dynamic model of hydraulic oil bulk modulus. J Mil Eng 2015; 30: 1153–1159.
19. Zhang XY. Research on the effect of oil characteristics on the performance of hydraulic machinery stepless transmission. PhD Thesis, Yanshan University, China, 2012.
20. Hass R. Compressibility measurements of hydraulic fluids in the low-pressure range. In: Proceedings of the 6th FPNI PhD symposium. West Lafayette, IN, June 2010, pp. 681–690, Purdue University.
21. Kim S and Murrenhoff H. Measurement of effective bulk modulus for hydraulic oil at low-pressure. J Fluids Eng 2012; 134: 1–10.
22. Toshio T. Hydraulic fluid mechanics. Beijing: Science Press, 1980.
23. Feng B. Research on effective bulk modulus of hydraulic oil and its measuring device. PhD Thesis, Zhejiang University, China, 2011.
24. Fan SL. Experimental study on bulk modulus of elasticity of oil. PhD Thesis, Zhejiang University, China, 1989.