Deuteron-Like Correlation for the $T = 0$ Channel in $^{18}$F Studied within the Continuum Contributions

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Abstract. We study the deuteron-like correlation of the valence proton and neutron for the $T = 0$ channel in $^{18}$F using the cluster-orbital shell model approach. In this study, the deuteron-like correlation is examined in terms of two key ingredients, cross-shell configuration and continuum contribution. We discuss the cross-shell configuration contributes to the restoration of the symmetry to form the $LS$-coupling in $1^+$, and the continuum contributions plays an important role of the spatial localization of the proton and neutron.

1. Introduction
The proton and neutron correlation in a nucleus has been investigated in the viewpoint of competition of the $T = 1$ and $T = 0$ channels [1, 2, 3]. For the $T = 0 (S = 1)$ channel, the correlation can be related to the deuteron, since it has the same spin and isospin with the deuteron. In the nucleus, the coupling of the spin-orbit partners $|j > \otimes |j < |$ which we call the “cross-shell” configuration, is hindered by the the spin-orbit interaction from the core nucleus different from the nucleons in the deuteron.

We study the effect of the coupling of the spin-orbit partners $|j > \otimes |j < |$ to the deuteron-like correlation in $^{18}$F and also investigate the effect of the continuum contribution to the spatial localization of the proton and neutron. For this purpose, we employ the cluster-orbital shell model (COSM) approach [4] and discuss the deuteron-like correlation in the $^{16}$O+$p$+$n$ three-body model.

2. Model and formalism
The Hamiltonian of the $^{16}$O+$p$+$n$ three-body model in the COSM formalism is formulated as follows:

$$\hat{H} = \sum_{i=1}^{2} (\hat{t}_i + \hat{v}_{i}^{CN}) + \left( \hat{T}_{12} + \hat{V}_{12} \right),$$

where $\hat{T}_{12} = \hat{p}_1 \cdot \hat{p}_2 / M_C$ is the recoil term coming from the subtraction of the center of mass motion due to the finite mass $M_C$ of the core nucleus. For $\hat{v}_{i}^{CN}$, we use the same potential applied in Ref. [5]. For the two-body interaction part $\hat{V}_{12}$, we use an effective nucleon-nucleon interaction, i.e. the Minnesota potential [7]. The potential parameters are adjusted to reproduce the $5/2^+$, $1/2^+$ and $3/2^+$ states of $^{17}$O and $^{17}$F, and $1^+$ and $0^+$ state of $^{18}$F.
The basis set of COSM is constructed in the $jj$-coupling scheme using basis functions $\phi_{i}$ defined in the coordinates of the core+$N$ subsystems as

$$\Phi_{m} = A \left\{ [\phi_{i} \otimes \phi_{o}]^{JM}_{TM} \right\}_{m} = A \left\{ u_{1}(r_{1}) u_{2}(r_{2}) \cdot [j_{1} \otimes j_{2}]^{JM} \cdot [\chi^{1}_{L} \otimes \chi^{2}_{L}]^{TM} \right\}_{m},$$

where $\phi_{i}$ denotes a set of the angular momentum and the isospin of the $i$th particle. For the radial part of the basis function, we apply the Gaussian expansion method [6]. We take the maximum angular momentum of the basis function as $\ell_{\text{max}} = 6$ in order to include the continuum contribution sufficiently.

3. Deuteron-like correlation in $^{18}\text{F}$

3.1. Examine the wave function

First, to examine the wave function of $^{18}\text{F}$ for the $1^+$ and $0^+$ states. We calculate several physical quantities of $^{18}\text{F}$ and obtain the $M1$-transition strength as $B(M1; 0^+ \rightarrow 1^+) = 18.30 \mu_{N}^{2}$, the magnetic moment as $\mu = 0.811 \mu_{N}$ and the r.m.s. radius as $R_{\text{rms}} = 2.64$ fm.

For the $B(M1)$-value, other theoretical calculations are obtained as $15.18 \mu_{N}^{2}$ [9] and $18.15 \mu_{N}^{2}$ [3], where the latter is consistent to our result, and experimentally observed value is $19.71 \mu_{N}^{2}$ [8]. Here, the essential difference of the theoretical calculations for Ref. [9] and [3] is the inclusion of the cross-shell configuration to the model space, and the former one does not include the cross-shell. The magnetic moment of other theoretical calculations are also consistent to our result, $0.834 \mu_{N}$ [3] and $0.82 \mu_{N}$ [10]. Since the magnetic moment is sensitive to the spin-coupling scheme, we decompose it into the orbital angular momentum and spin parts; $\mu^{L}$ and $\mu^{S}$ as $\mu = \langle \sum_{i} g_{i}^{(i)} l_{i} \rangle + \langle \sum_{i} g_{s}^{(i)} s_{i} \rangle \equiv \mu^{L} + \mu^{S}$. The angular momentum and spin parts are obtained as $\mu^{L} = 0.090 \mu_{N}$ and $\mu^{S} = 0.721 \mu_{N}$. Therefore, we can confirm that the $1^+$ state of $^{18}\text{F}$ has the $S = 1$ dominance in the wave function.

3.2. Configuration dependence of the deuteron-like correlation

Next, we discuss the importance of two ingredients; the cross-term and continuum contributions, to the deuteron-like correlation in $^{18}\text{F}$. To this end, we classify the set of basis functions into four groups labeled as “CN#”. CN1 and CN2 is a set of $(\ell_{j})^{2}$ type basis sets. CN1 consists only the sd-orbits, $(d_{5/2})^{2}$, $(s_{1/2})^{2}$, and CN2 includes higher-orbital $(\ell_{j})^{2}$ configurations up to $\ell_{\text{max}} = 6$. Since CN1 and CN2 can be applied to the $1^+$ and $0^+$ states, we can compare the correlation of the valence nucleons within the same model space. CN3 includes the “cross-shell” configurations, $(d_{5/2})(d_{3/2})$ and $(s_{1/2})(d_{3/2})$ in addition to CN2, and CN4 is the full configurations up to $\ell_{\text{max}} = 6$, which corresponds to the inclusion of the continuum contributions.

We calculate the correlation energy and magnetic moment by changing the model space from CN1 to CN4. Here, we define the correlation energy as $E_{\text{Corr}}^{\text{RPA}}(J^{\pi}) \equiv \langle E_{3b}^{\pi} + E_{c}^{\pi} \rangle - E(J^{\pi})$, where $E(J^{\pi})$ is the three-body energy. The obtained wave functions are decomposed into the partial waves. Results for the correlation energy, magnetic moment and the decomposition of the wave function are shown in Figs. 1(a), (b) and (c), respectively.

First, we discuss the importance of the “cross-shell” configurations. We compare the correlation energy of CN1 and CN2 for the $1^+$ and $0^+$ states. Without the cross-shell configurations, the correlation energy of the $0^+$ state is systematically larger than that of $1^+$ as shown in Fig. 1(a). Once the cross-shell configuration is added to the model space of the $1^+$ state (CN3), the correlation energy is increased by the same amount of the continuum contribution from CN1 to CN2, but the energy is still smaller than that of $0^+$. This shows the $T = 1$ channel dominance where the $(\ell_{j})^{2}$-pair is favored in the system.

From Fig. 1(b), the decomposition of the magnetic momentum to the angular momentum and spin part, we find the $(\ell_{j})^{2}$-type model space, CN1 and CN2, give the small magnetic
moment and the spin part. Even though the CN2 includes the continuum contribution in the model space. A drastic change of the coupling scheme of the wave function can be achieved by adding the cross-shell configuration to the model space, CN3 and CN4. Since the value of the magnetic momentum for CN4, $\mu = 0.811\mu_N$, is close to the exact value of the sum of the $g$-factors of the proton and neutron, 0.880$\mu_N$, we can consider the component of the $LS$-coupling with $L = 0, S = 1$ channel is dominant in the coupling scheme of the CN4 wave function.

The other important ingredient is the continuum contribution. For the correlation energy, further enhancement can be obtained by adding the continuum to the model space (CN4). The CN4 result of $1^+$ gives the correlation energy as $E_{Corr}^{0d_5/2}(1^+) = 4.67$ MeV, which is much larger than the $0^+$ (CN2) result, 3.61 MeV. The energy changes 1.56 MeV from CN3 to CN4, that is more than 30% of the correlation energy in CN4.

The decomposition of the wave functions are shown in Fig. 1(c). From CN3 to CN4, the cross-shell configuration increases, and on the other hand, the other configurations ($d_5/2$)$^2$, ($s_1/2$)$^2$, ($d_3/2$)$^2$ decrease. This indicates that the cross-shell and continuum configurations are coherently contribute to the wave function of the $1^+$ state.

4. Summary
We study the deuteron-like correlation of the valence neutron and proton in $^{18}\text{F}$ using the COSM approach. From the analysis, we showed the essential ingredients of the deuteron-like correlation are the cross-shell configuration, which is the coupling of the spin-orbit partners $j_>$ and $j_<$, and the continuum contributions and its coherence to the correlation energy.

References
[1] Poves A and Martines-Pinedo G 1998 Phys. Lett. B 430 203
[2] Sagawa H, Tanimura Y and Hagino K 2013 Phys. Rev. C 87 034310
[3] Tanimura Y, Sagawa H and Hagino K 2014 Prog. Theor. Exp. Phys. 2014 053D02
[4] Suzuki Y and Ikeda K 1988 Phys. Rev. C 38 410
[5] Masui H, Katō K and Ikeda K 2006 Phys. Rev. C 73 034318
[6] Hiyama E, Kino Y and Kamimura M 2003 Prog. Part. Nucl. Phys. 51 223
[7] Thompson D R, LeMere M and Tang Y C 1977 Nucl. Phys. A 286 53
[8] National Nuclear Data Center, Chart of Nuclides (National Nuclear Data Center, New York, n.d
[9] Lisetskiy A F, Jolos R V, Pietral N and von Bretano P 1999 Phys. Rev. C 60 064310
[10] Kanada-En’yo Y and Kobayashi F 2014 Phys. Rev. C 90 054332