Strongly anisotropic ballistic magnetoresistance in compact three-dimensional semiconducting nanoarchitectures

Ching-Hao Chang,1,2 Jeroen van den Brink,1,3 and Carmine Ortix1

1 Institute for Theoretical Solid State Physics, IFW Dresden, Helmholtzstr. 20, 01069 Dresden, Germany
2 Department of Physics, National Tsing Hua University, Hsinchu 30043, Taiwan
3 Department of Physics, Technical University Dresden, D-1062 Dresden, Germany

We establish theoretically that in non-magnetic semiconducting thin films rolled-up into compact quasi-one-dimensional nanoarchitectures the ballistic magnetoresistance is very anisotropic: conductances depend strongly on the direction of an externally applied magnetic field. This phenomenon originates from the curved open geometry of rolled-up nanotubes which leads to a tunability of the number of one-dimensional magnetic subbands crossing the Fermi energy. The experimental significance of this phenomenon is illustrated by an anisotropy of \( \simeq 40\% \) that we predict for weak magnetic fields in experimentally relevant nanoarchitectures.

PACS numbers: 75.47.-m, 75.75.-c, 73.20.At

In 1857 Thomson discovered that the resistivity of bulk ferromagnetic metals depends on the relative angle between the electric current and the magnetization direction [1]. The prediction of this anisotropic magnetoresistance (AMR), caused by an anisotropy in the electron scattering due to spin-orbit interaction, was experimentally verified more than a century later in iron, cobalt and nickel alloys. Since then, the interest in this phenomenon has received a boost thanks to the development of AMR sensors for magnetic recording [2, 3].

It was recently proposed that this phenomenon might also occur in ferromagnetic Fe and Ni nanowires [3]. Contrary to macroscopic samples, in miniaturized objects with characteristic dimensions less than the electronic mean free path, electronic transport is ballistic rather than diffusive and electron scattering does not contribute to the conductance. The ballistic conductance of a quasi one-dimensional (1D) magnetic nanostructure with a slowly varying constriction of width of the order of the Fermi wavelength \( \lambda_F \) is indeed simply given by \( G = N e^2 / h \) where \( N \) is the number of open conducting channels [2]. Due to the strong spin-orbit coupling in these nanostructures, the number of transverse modes at the Fermi energy changes with the magnetization direction and leads ultimately to a ballistic anisotropic magnetoresistance (BAMR).

In this Letter, we show that a strong BAMR occurs in the electronic quantum transport of compact quasi-1D tubular nanostructures subject to externally applied magnetic fields. Contrary to the nanoarchitectures of magnetic materials mentioned above, the BAMR in these nanoarchitectures is entirely due to the open curved geometry of the tubes which breaks the rotational symmetry of the three-dimensional (3D) embedding space. As a result, a strong BAMR of \( \simeq 40\% \) arises for a rolled-up nanotube [6] of a conventional non-magnetic semiconducting material with a typical radius of curvature of \( \simeq 100 \text{ nm} \) subject to an external magnetic field of \( \simeq 1.5 \text{ T} \).

Our starting point is a curved two-dimensional electron gas (2DEG) [11] at the interface of a rolled-up tubular heterostructure [c.f. Figs. 1(a),(b)]. We consider the total length of this nanostructure, i.e. its characteristic dimension before rolling, to be smaller than the characteristic mean free path \( \mu \simeq 10 \text{ nm m} \). Assuming a radius of curvature \( R \simeq 100 \text{ nm} \) comparable to the large Fermi wave-length \( \lambda_F \) of the low-density electron gas (\( \lambda_F \simeq 40 \text{ nm} \) for the 2DEG of a GaAs-Al\(_{x}\)Ga\(_{1-x}\)As heterostructure) this compact 3D nanoarchitecture can also be viewed as a curved quantum point contact (QPC) [12, 13] with circular cross section and width corresponding to the diameter of the nanotube. Henceforth quantum-size effects appear that lead to ballistic conductance quantization both at zero-field and in the presence of a magnetic field [13]. As we show below, in the latter case the nature of the quantum states, and thus the ballistic conductance itself, strongly depends on whether one considers closed or open tubular nanostructures.

We first consider a cylindrical 2DEG in an external magnetic field [13, 15] for which the radius of curvature \( R \) is much larger than the Landau magnetic length \( l_B = \sqrt{\hbar / eB} \). Assuming \( R \simeq 10^2 \text{ nm} \), this regime can be reached if magnetic fields on the scale of one Tesla are externally applied. This also implies that one can neglect the Zeeman splitting removing the spin degeneracy and take only into account the orbital effect of the magnetic field. The large separation among the Landau magnetic length and the radius of curvature allows for the formation of cyclotron orbits centered at the two positions where the tangential plane of the nanostructure is orthogonal to the magnetic field direction [see Fig. 1(a)]. The appearance of cyclotron orbits is reflected, in turn, in the formation of doubly-degenerate quasi-1D Landau-like states. This is demonstrated in Fig. 2(a) where we show the energy spectrum for a cylindrical 2DEG [16] with \( R \simeq 100 \text{ nm} \) under the influence of a transversal magnetic field [17, 18] of strength 1.65 T [see Supplemental Material]. At large values of the momentum \( k_z \) along
the translationally invariant tube axis direction, the flat quasi-1D Landau-like subbands, however, start to acquire a characteristic parabolic dispersion [c.f. Fig. 2(a)]. One can identify these dispersive states as snake states centered, due to Lorentz force, at the two points where the tangential plane of the nanostructure is parallel to the magnetic field direction [c.f. Fig. 1(a)]. We emphasize that the snake shape of the orbits results from the fact that in the immediate vicinity of the orbit centers the normal component of the externally applied magnetic field switches sign [c.f. Supplemental Material].

To investigate the corresponding quantum states in an open tubular nanostructure we consider, as an example, an Archimedean spiral rolled-up nanotube (RUNT) [19] parametrized in cylindrical coordinate as \((r, \phi, z) = (l\phi, \phi, z)\) with turning length \(l\). We assume mixed periodic and open boundary conditions: \(z \in (-\infty, \infty)\) and \(\phi \in (\phi_{in}, \phi_{out})\) with \(\phi_{in} \gg 1\) in order to ensure that the inner radius of the nanotube \(R_{in} = l\phi_{in}\) is much larger than the turning length \(l\). The qualitative change in the nature of the quantum states for the open geometry is shown in Fig. 1(b) where we sketch the different quantum states appearing in a one winding RUNT that has performed a complete rotation, i.e., \(\phi_{out} = \phi_{in} + 2\pi\) with \(\phi_{in} = 16\pi\), and subject to a magnetic field whose direction \(\theta\) with respect to the edge axis [c.f. Fig. 1(c)] is set to zero. The presence of the edges does not influence the formation of the snake states. However, due to the hard walls one of the two cyclotron orbits encountered in the closed geometry fractionalizes into two skipping orbits for which a clear dispersion along \(k_z\) is expected. Fig. 1(b) shows the ensuing energy spectrum with \(l = 2 \text{ nm}\),

\[ R_{in} \equiv 16\pi l \simeq 100 \text{ nm} \]  

and a magnetic field strength, as before, of 1.65 T [see Supplemental Material]. One can identify a single non-degenerate quasi-1D Landau-like state now coexisting for \(k_z \simeq 0\) with two dispersive edge states localized at the inner and outer radius of the RUNT which distinctly differentiate the magnetic spectra of open tubular nanostructures with respect to their closed counterparts.

Having established the qualitative difference between the magnetic spectra for closed and open tubular nanostructures, we now discuss the interplay between the location of the hard-wall boundaries and the direction of the externally applied magnetic field. Since, as mentioned above, the open curved geometry of a spiral RUNT breaks the rotational symmetry of the embedding 3D space, the features of the magnetic spectra are drastically altered as the direction of the magnetic field changes.

FIG. 1. (color online). (a) Sketch of a cylindrical 2DEG subject to a transversal magnetic field with the different magnetic states and their associated orbits appearing on it. (b) Same of (a) for an Archimedean spiral rolled-up nanotube (RUNT). Its cross section is shown explicitly in (c).

FIG. 2. (color online). (a) Resulting magnetic spectrum as a function of the momentum along the tube axis for an external magnetic field of 1.65 T strength. The radius of curvature is \(R \equiv 32\pi\) nm. (b) Magnetic spectrum of a one-winding RUNT with inner radius \(R_{in} = 32\pi\) nm with an external magnetic field of 1.65 T strength oriented along the edge axis (\(\theta = 0\)). The green and orange lines explicitly show the dispersive edge states absent in a closed nanostructure. (c) and (d) show the evolution of the magnetic spectrum of panel (b) varying the magnetic field direction to \(\theta = \pi/4\) and \(\theta = \pi/2\) respectively.
With the magnetic spectra for different magnetic field directions in our hands, we have determined the ballistic conductance of our curved QPC neglecting intersubband scattering processes [22, 24]. Within this approximation the Landauer formula for the two-terminal conductance including the thermal smearing of the Fermi-Dirac distribution becomes [23, 26]

\[
G(E_F, T, \theta) = \int_0^\infty G(E, 0, \theta) \frac{\partial f}{\partial E_F} \, dE,
\]

where \( f \) indicates the Fermi-Dirac distribution, \( E_F \) is the Fermi energy while \( G(E, 0, \theta) = 2e^2/h \) is the conductance at zero temperature proportional to the number \( N(\theta) \) of occupied magnetoelastic subbands. Fig. 3(a) shows the behavior of the magnetoconductance at low temperatures as a function of the inverse of the Fermi wavelength of the 2DEG \( \lambda_F = \sqrt{2\pi/n} \). Here \( n \) is the electron density given by

\[
n(E_F, T, \theta) = \frac{2}{L} \sum_{i} \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} f [E_i(k_z, \theta) - E_F, T],
\]

where the index \( i \) runs over the occupied one-dimensional subbands, \( L \) is the total spiral arclength and the factor of \( 2 \) accounts for spin degeneracy. Independent of the direction of the externally applied magnetic field, the magnetoconductance shows a step-like increase which is a direct consequence of the effective one-dimensionality of the curved nanoarchitecture. However, the different features of the magnetic spectra reported in Fig. 3(b) lead to a substantial BAMR whose magnitude we define [4] as

\[
\text{BAMR}(\theta) = \frac{G(\theta) - G(\pi/2)}{G(\pi/2)}
\]

measured from the reference direction perpendicular to the edge axis where the ballistic conductance takes its minimum value. In Fig. 3(b) we illustrate the relevance of the BAMR effect by showing that the magnitude at \( \theta = 0 \) extends over a wide range of \( \lambda_F \) and persists up to \( \lambda_F = 40 \) nm with a 40\% value. One can indeed estimate that a sizable BAMR will be encountered down to a critical Fermi wavelength \( \lambda_F \approx l^D_0/R \approx 22 \) nm for our set of parameters [c.f. Supplemental Material]. Fig. 3 shows the angular dependence of the BAMR for \( \lambda_F \approx 100 \) nm and \( \lambda_F \approx 40 \) nm. In both cases the maximum attainable anisotropy is reached for a 90\degree tilt of the magnetic field direction while smaller tilts also yield a sizable anisotropy.

We have, in conclusion, predicted the existence of a ballistic anisotropic magnetoresistance – a change in the ballistic conductance with the direction of an externally applied magnetic field – in compact rolled-up tubular nanostructures of conventional non-magnetic semiconducting materials [27–31]. The occurrence of this phenomenon stems from the open curved geometry of

This is immediately manifested in Fig. 2 where we show the evolution of the magnetic spectrum of Fig. 2(b) by applying two consecutive 45\degree tilts of the magnetic field while keeping its strength constant. As shown in Fig. 2(c) the first 45\degree rotation of the magnetic field direction restores a quasi-1D Landau state doublet for \( k_z \sim 0 \) due to the insensitiveness of the small radii cyclotron orbits centered at \( \phi = 16\pi + \pi/4 \) to the hard-walls. The same holds true for the snake states at large values of \( k_z \approx 0 \) whose orbits are centered at \( \phi = 18\pi - \pi/4 \). On top of this, we find the two edge states with skipping orbits at the inner and outer radius of the nanotube to appear for intermediate values of momentum \( k_z > 0 \) [c.f. the green and orange lines in Fig. 2(c)]. Considering instead the magnetic field direction perpendicular to the edge axis leads to the magnetic spectrum shown in Fig. 2(d). It strongly resembles the spectrum for the closed nanostructure of Fig. 2(a) with the following caveat: due to the hard-walls the snake states at large momenta \( k_z > 0 \) are substituted by edge states. This, however, does not change qualitatively the magnetic spectrum apart from an asymmetry in the dispersion around \( k_z = 0 \) due to the different nature of the magnetic states, i.e. snake orbits for \( k_z < 0 \) and skipping orbits for \( k_z > 0 \).
these nanoarchitectures and is thus independent of their structural symmetries. We have shown that a sizable BAMR occurs if the Landau magnetic length $l_B < \min\{R, \sqrt{RA_F}\}$. This regime can be easily reached using weak magnetic fields in present day curved nanostructures manufactured with the rolled-up nanotechnology [8, 32] where radii of curvature in the hundreds of nanometer range can be achieved [33].

This work was supported by the FP7-FET-OPEN project (No: 618083 CNTQC). C.H.C. acknowledge support from the Ministry of Science and Technology of Taiwan (No: 102-2917-I-564-054).

FIG. 4. (color online). Angular dependence of the BAMR at $T = 1$ K (a) and $T = 5$ K (b). The blue line corresponds to a Fermi wavelength $\lambda_F \equiv 102$ nm whereas the dark yellow line is for $\lambda_F \equiv 46$ nm.

(a) $T = 1$ K

(b) $T = 5$ K

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