Mismatches analysis based on channel response and an amplitude correction method for time interleaved photonic analog-to-digital converters

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Abstract: A channel mismatch model for time-interleaved photonic analog-to-digital converters is built by decomposing both optical time division multiplexing channels and the wavelength division multiplexing channels into time channels with uniform and equal-amplitude sampling clock. Based on the model, the influence of power and timing mismatch of optical sampling pulse trains, optical sampling pulse temporal shape mismatches, photodetection bandwidth mismatches on the sampling results are analyzed theoretically. Depending on the found relationship between unmodulated components and modulated components, an effective online amplitude mismatch correction method applicable to wideband signals is proposed. The theoretical results are verified on a 4-channel TIPADC system. The proposed amplitude mismatch correction method can suppress the spurs caused by unmodulated components mismatch and modulated components mismatch by ~50 dB and ~30 dB for a two-tone signal, respectively.

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1. Introduction

The electronic bottlenecks have restricted the development of traditional electronic analog-to-digital converters (EADCs), while benefiting from the merits of photonics, photonic analog-to-digital converters (PADCs) technology provides a new promising way to break through the restriction [1]. After decades of development, Time interleave PADCs (TIPADCs) technology stands out from the proposed schemes, and is considered as one of the most feasible solutions to achieve high sampling rate, bandwidth, and effective number of bits (ENOB) [1–5]. However, the channel mismatches must be overcome for the TIPADCs as a multi-channel system to achieve a high performance. Similar to time interleaved EADCs (TIEADCs), the channel mismatches in TIPADCs will result in the amplitude mismatch, timing mismatch, and bandwidth mismatch in sampling results. These mismatches cause undesired additional spectral components or spurs, and degrade the ENOB. Researchers have proposed several mismatch analyses and correction algorithms for TIEADCs [6–10], which can give us helpful inspiration for the study of the channel mismatches in TIPADCs. G. Yang et al. have analyzed the mismatch of time-wavelength interleaved optical sampling train [11], and presented a mismatch compensation algorithm based on spectrum analyses of a single tone signal for TIPADCs, basically similar to that of TIEADCs [12]. However, the effects of other parts in TIPADCs such as photodetection (accounting from the photodiode to the EADC) and Mach-Zehnder modulator (MZM) on channel mismatches have not been analyzed. Meanwhile, bandwidth response mismatch in TIPADCs is also an open issue.

In this paper, we investigate the mismatches in TIPADCs in terms of the channel response by regarding TIPADC as a hybrid electronic and photonic system. A channel mismatch model is built by decomposing both the channels in optical time division multiplexing (OTDM) and wavelength division multiplexing (WDM) into time channels with uniform and equal-amplitude sampling clock. The expressions of amplitude, timing and bandwidth mismatches are presented. The correlation among different mismatches are analyzed. The results show that the timing mismatch of optical sampling pulse trains and the photodetection bandwidth mismatch will also induce the amplitude mismatch. When the bandwidth of the optical sampling pulse is much broader than that of MZM, the effect of pulse shape on the bandwidth mismatch can be ignored. The photodetection bandwidth mismatch can also be ignored when the photodetection bandwidth is broad enough to form a continuous channel frequency response. Based on the relationship between unmodulated components (UCs) and modulated components (MCs) found in the theoretical analyses, an effective online amplitude mismatch correction method applicable to wideband signals is proposed further. The theoretical analyses are verified by experiments under different conditions on a 4-channel TIPADC system. A ~50 dB UCs mismatch correction and a ~30 dB MCs mismatch correction for a two-tone signal are achieved by the proposed amplitude mismatch correction method.
2. Mismatch modeling and analysis

Figure 1 illustrates a structure of a general TIPADC [2–5]. The pulses with a repetition rate of \( f_s \) are generated from an optical pulse generator (OPG). OTDM and WDM are used to generate a repetition rate multiplied time-wavelength interleaved photonic sampling pulse train. The multiplication factors of OTDM and WDM are assumed to \( M \) and \( N \), respectively. With an MZM, the sampled radio frequency (RF) signal is modulated onto the sampling pulse train. A WDM demultiplexes the modulated optical sampling pulse train into \( N \) parallel WDM channels to reduce the pulse repetition rate of each WDM channel. Each optical pulse in a WDM channel is detected and converted into an electrical pulse by a photodiode (PD), and then sampled and quantized into digital data by a cascaded EADC. The sampled RF signal can be reconstructed by combining the digital data from WDM channels with digital signal processing (DSP).

The optical sampling pulse train in each WDM channel is equivalent to a time interleave of \( M \) sub optical sampling pulse trains with a repetition rate of \( f_s \). Accordingly, the EADC in each WDM channel performs at the sampling rate of \( Mf_s \). When there is no inter-symbol interference (ISI), one electronic pulse generated from PD will not overlap with the adjacent one [13,14], and the EADC in each WDM channel can be equivalent to a TIEADC composed of \( M \) EADCs with the sampling rate of \( f_s \). Thus, a WDM channel is decomposed to \( M \) OTDM channel. As a result, the final sampling result of TIPADCs can be regarded as a time interleave of data from \( MN \) uniformly sampling time channels.

2.1 Analysis of the amplitude and timing mismatches

In the \( m \)-th OTDM channel in the \( n \)-th WDM channel, the temporal shape of the optical sampling pulse train can be written as:

\[
p_{mn}(t) = P_{d,\text{ave}} p_{s,mn}(t) * \sum_{k=-\infty}^{\infty} \delta(t - kT_s - d_{p,mn})
\]

where \( p_{s,mn}(t) \) is the photonic sampling pulse power function normalized by the average power, \( P_{d,\text{ave}} \); \( * \) denotes the convolution operation, \( \delta(t) \) is the Dirac delta function; \( T_s \) is the sampling period of the time channel; \( d_{p,mn} \) represents the delay of the optical sampling pulse train in the channel, and can be expressed as:

\[
d_{p,mn} = d_{p,mn} + d_{\sigma,mn}.
\]
The first item, \( d_{\rho,nn} \), is the ideal delay for uniform time interleave, whose expression is

\[
d_{\rho,nn} = \frac{(m-1)M + n-1}{MN} T_s.
\]  

(3)

The second item, \( d_{\sigma,nn} \), is the nonideal time deviation from the ideal time position.

The sampling digitized result of a time channel is the summation of two parts \([13]\): UC and MC which are represented by \( v_{Q0,nn} \) and \( v_{Q1,nn}[k] \).

\[
v_{Q0,nn} = 0.5\alpha h_{E,nn}(t - d_{E,nn}) * p_{nn}(t),
\]  

(4)

\[
v_{Q1,nn}[k] = h_{t,nn}(t) * v_{t}(t),
\]  

(5)

where \( \alpha \) is the attenuation factor of MZM; \( h_{E,nn}(t) \) is the impulse response of the photodetection; \( d_{E,nn} \) is the delay induced by the photodetection; \( v_{t}(t) \) is the sampled signal; \( h_{t,nn}(t) \) is the equivalent channel impulse response and can be expressed as:

\[
h_{A,nn}(t) = -0.5\alpha h_{E,nn}(t - d_{E,nn}) p_{nn}(-t) * h_{MD}(t),
\]  

(6)

where \( h_{MD}(t) \) is the small-signal impulse response of MZM.

2.1.1 Unmodulated component

For the UC part, from Eq. (1) and (4), we have

\[
v_{Q0,nn} = 0.5\alpha P_{A,nn} h'_{E,nn}(-d_{E,nn} - d_{\sigma,nn}),
\]  

(7)

where the expression of \( h'_{E,nn}(t) \) is

\[
h'_{E,nn}(t) = p_{t,nn}(t) * h_{E,nn}(t).
\]  

(8)

From Eq. (7), one can find that the \( v_{Q0,nn} \) is determined by the average power, \( P_{A,nn} \), the shape of \( h'_{E,nn}(t) \), and the nonideal delay, \( d_{\sigma,nn} \), since the \( h_{E,nn}(-d_{E,nn} - d_{\sigma,nn}) \) is usually varied with \( d_{\sigma,nn} \). Therefore, although UC in each time channel has a constant value, the combined UC in sampling results, \( v_{Q0}[k] \), obtained by time interleaving UC in each time channel, is no longer a constant value when \( P_{A,nn} \) and \( d_{\sigma,nn} \) in time channels are the different.

The reconstructed signal spectrum of UC, \( V_{Q0}(\Omega) \), can be derived as

\[
V_{Q0}(\Omega) = \frac{1}{T_s} \sum_{n=1}^{N} \sum_{m=1}^{M} v_{Q0,nn} \sum_{l=-\infty}^{\infty} \delta(\Omega - l\Omega_s) \exp(-j\Omega d_{\rho,nn}),
\]  

(9)

where \( \Omega_s = 2\pi / T_s \).

From Eq. (9), one can figure out that the spectrum of UC not only includes the direct current signal but also the spurs at integer multiples of \( \Omega_s \) frequency.

2.1.2 Modulated component

After a derivation (detailed in Appendix), the channel equivalent impulse response of a time channel in Eq. (6) can be further expressed as
\[ h_{t,\text{in}}(t) = h_{\text{MD}}(t) \ast \sum_{k=\infty}^{\infty} c_{k,\text{in}} \delta(-t + kT_s + d_{\text{p,\text{in}}}). \] (10)

In the expression above, \( c_{k,\text{in}} \) is the channel gain coefficient with an expression of

\[ c_{k,\text{in}} = -0.5\alpha P_{A,\text{in}} h'_{E,\text{in}}(kT_s - d_{E,\text{in}} - d_{\sigma,\text{in}}) \sum_{k=\infty}^{\infty} \delta(-t + kT_s + d_{\text{p,\text{in}}}). \] (11)

In the cases without ISI, \( c_{k,\text{in}} \) is not related to \( k \) and can be denoted as \( c_{\text{in}} \). And \( h_{t,\text{in}}(t) \) and \( c_{k,\text{in}} \) can be further simplified as:

\[ h_{t,\text{in}}(t) = c_{\text{in}} h_{\text{MD}}(-t + d_{\text{p,\text{in}}}), \] (12)

\[ c_{\text{in}} = -0.5\alpha P_{A,\text{in}} h'_{E,\text{in}}(-d_{E,\text{in}} - d_{\sigma,\text{in}}). \] (13)

From Eq. (5) and Eq. (12), the MC in the sampling results of the time channel can be expressed as:

\[ v_{Q1,\text{in}}[k] = -0.5\alpha P_{A,\text{in}} h'_{E,\text{in}}(-d_{E,\text{in}} - d_{\sigma,\text{in}}) v'_i(t - d_{\text{p,\text{in}}} - d_{\sigma,\text{in}}), \] (14)

where \( v'_i(t) = h_{\text{MD}}(-t) \ast v_i(t) \).

From Eq. (14), one can find that timing mismatch in sampling results is only caused by \( d_{\sigma,\text{in}} \). The \( v_{Q1,\text{in}}[k] \), however, is related to the average power, \( P_{A,\text{in}} \), the shape of \( h'_{E,\text{in}}(t) \) and the nonideal delay, \( d_{\sigma,\text{in}} \), since the \( h'_{E,\text{in}}(-d_{E,\text{in}} - d_{\sigma,\text{in}}) \) is usually varied with \( d_{\sigma,\text{in}} \). It means that the timing mismatch of channels, \( d_{\sigma,\text{in}} \), will also induce amplitude mismatch in the combined sampling result. This comes from the fact that the timing mismatch will lead to the deviation of EADC sampling points on the electric pulses for different channels, and the values on different sampling points are different for a non-flat electric pulse.

When the digitizer sampling points on the electric pulses are located at the peak, both the UC value and the MC amplitude have maximum values. When the timing mismatch induce the deviation of the digitizer sampling points away from the peak, both the UC value and the MC amplitude decease.

The spectrum of MC in reconstructed signal is the sum of the spectrum of MC in each time channel, and can be derived as

\[ V_Q(\Omega) = \frac{1}{T_s} \sum_{s=1}^{N} \sum_{m=1}^{M} H_{s,m}(\Omega) \ast V'_i(\Omega), \] (15)

where \( H_{s,m}(\Omega) \) is the Fourier transformation of \( h_{t,\text{in}}(t) \) , \( V'_i(\Omega) \) is the Fourier transformation of \( v'_i(t) \).

There exists the aliasing in the spectrum of sampling result in each channel since each time channel performs under-sampling. In the absence of mismatches, aliasing parts are canceled out after summation and no spurs are generated. However, when there are mismatches, parts of the aliasing remain in \( V_Q(\Omega) \) due to the imperfect cancellation [7].

Taking a TIPADC with two time channels for example, the effect of mismatch on the combined spectrum is illustrated in Fig. 2, where \( V_{Q1,1}(\Omega) \) and \( V_{Q1,2}(\Omega) \) are the spectrum of MC in time channel 1 and time channel 2, respectively.
2.2 Analysis of the bandwidth mismatches

From Eq. (6), the equivalent channel frequency response of the m-th OTDM channel in the n-th WDM channel can be written as [15,16]:

\[
H_{l,m,n}(\Omega) = -0.5\alpha P_{l,m,n}H_{MD}(\Omega)P_{s,m,n}(\Omega)\exp(j\Omega d_{p,m,n}),
\]

where \(H_{MD}(\Omega)\) is the Fourier transform of \(h_{MD}(t)\), \(P_{s,m,n}(\Omega)\) is the Fourier transform of \(p_{s,m}(t)\). \(R_{mn}(\Omega)\) denotes the ISI in the channel and can be expressed as

\[
R_{mn}(\Omega) = \frac{1}{T_f} \sum_{k=-\infty}^{\infty} H_{E,m,n}(\Omega + k\Omega_s)\exp(-j\Omega d_{E,m,n})\exp[-j(\Omega + k\Omega_s)d_{p,m,n}],
\]

where \(H_{E,m,n}(\Omega)\) is the Fourier transform of \(h_{E,m}(t)\).

According to Nyquist ISI criterion, when there is no ISI, the equivalent channel frequency response can form a continuous passband, and \(R_{mn}(\Omega)\) is a constant value. Equation (16) can be simplified as

\[
H_{l,m,n}(\Omega) = -0.5\alpha P_{l,m,n}H_{MD}(\Omega)P_{s,m,n}(\Omega)\exp(j\Omega d_{p,m,n}).
\]

From Eq. (18), the equivalent channel frequency response is independent of the photodetection frequency response in the case without ISI, which means the bandwidth mismatches of the EADCs won’t lead to the bandwidth mismatches of the time channels. Moreover, because the modulator is shared in the system, \(H_{MD}(\Omega)\) is the same for all time channels, and hence will not result in a bandwidth mismatch. On the other hand, the bandwidth mismatches will be caused by the mismatch of optical sampling pulse temporal shapes for different channel characteristics such as WDM channel uniformity, nonlinearity of fiber, and so on. In the case, however, where the bandwidth of the modulator is far less than that of the optical sampling pulses, \(H_{l,m,n}(\Omega)\) will mainly depend on the \(H_{MD}(\Omega)\), and bandwidth mismatches for the frequency response differences among WDM channels can be ignored.

3. Amplitude correction method based on the relationship of UC and MC

From Eq. (9), in the combined signal spectrum, the mismatched UC results in the spur at zero frequency and integer multiples of \(\Omega_s\) frequency in the combined spectrum. The spur at zero frequency is easy to be removed by subtracting the mean from the original data, because the sampled signal is hardly at zero frequency. However, the spurs at integer multiples of \(\Omega_s\)
frequency perhaps overlap with the sampled signal, so they cannot be removed with filters directly.

The UC for each time channel is only at the zero frequency due to the uniform sampling. In addition, the spurs at integer multiples of \( \Omega \), frequency in the combined spectrum result from the time interleaving of UCs from different time channels. Considering the UC is irrelevant to the sampled signal, it can be removed from the sampling result in each time channel before combination. Thus, the time interleaved UCs have the equal value (all are zero), and all spurs induced by UC can be suppressed.

Contrasting Eq. (7) and Eq. (14), one can find that the relationship between UC and MC:

\[
q_{\Omega, m, n} = -v_{\Omega, m, n} \cdot v_{i, k} T + d_{p, m, n}.
\]  

Equation (19) indicates that the amplitude of MC is a product of the original amplitude of the sampled signal and a gain coefficient (the negative UC). In other words, all the amplitude mismatch information of the MC can be obtained from the corresponding UC which is unaffected by sampled signal. The amplitude mismatch of MC in sampling results can be corrected by excluding the effect of gain coefficient, and we can obtain the amplitude corrected signal of the \( m \)-th OTDM channel in \( n \)-th WDM channel as

\[
y_{\Omega, m, n}[k] = -\frac{v_{\Omega, m, n}[k]}{v_{\Omega, m, n}}.
\]  

The flow chart for the proposed amplitude correction method is drawn in Fig. 3. For any channel, for example the \( m \)-th OTDM channel in \( n \)-th WDM channel, the sampling results, \( x_{mn}[k] \), is taken the mean value to extract the UC of the time channel by the corresponding UC extractor. The UC is then subtracted from \( x_{mn}[k] \) to obtain the MC. The amplitude correction of MC is achieved through dividing MC by UC. Finally, the amplitude corrected MCs from all channels are time interleaved to obtain the combined digital signal \( y[k] \).

![Fig. 3. The flow chart for the proposed amplitude correction method.](image)

4. Experiment results

In order to verify the theoretical analyses and correction method above, we set up a TIPADC experimental system with two OTDM channels and two WDM channels, shown in Fig. 4. The passive mode-locked fiber laser (MLL) produces pulses at a repetition rate of 36.456 MHz with an average power of about 40 mW. After an optical coupler (OC), the pulses are split apart and combined by the other OC to multiply the repetition rate of pulses to 72.912 MHz. The repetition rate multiplied pulse train is fed in to an arrayed waveguide grating (AWG) to split it into two trains with different center wavelength, and combined together by another AWG to construct an optical sampling pulse train with a repetition rate of 145.824
MHz. Optical delay lines (ODLs) and variable delay lines (VDLs) are used to tune the delays of channel coarsely and precisely, respectively. The delay adjustment range of VDL is 0 to 560 ps with the precision of 0.01 ps. The power differences between channels are adjusted with a VOA.

A single tone signal is generated from a microwave signal generator (SG) and then fed into a 10 GHz MZM with a half wave voltage of ~4.5 V. To ensure the validity of the small signal approximation [17], the MZM is biased at quadrature and the power of the input microwave is set to 0 dBm. After modulated, the optical sampling pulse train is demultiplexed by an AWG according to wavelength and photodetected by PDs with a 5 GHz bandwidth and a 20 dBm maximum acceptable peak power. The input average power to photodiode is controlled at ~6 dBm by an attenuator to avoid nonlinearity. Signals after the photodiode are sampled by a digitizer (Keysight, M9703A). The synchronizing signal from MLL is frequency multiplicated with a phase locked loop (PLL) for providing the sampling clock of the digitizer. VDLs before the PDs are used to adjust the digitizer sampling points on the electric pulses. In the experiments, the digitizer sampling points are chosen at the peak of the electric pulses. CH\textsubscript{mn} denotes the time channel of the m-th OTDM channel in n-th WDM channel.

Figure 5(a) shows the normalized amplitudes of UC and MC variation vs. the relative power mismatch ratio in CH11 and CH12. The relative power mismatch ratio is defined as the ratio of the average power differences between two channels to the average powers of CH11. In the experiments, in order to only observe the effect of the power mismatch of optical sampling pulse trains in different channels, the delays of channels are tuned to ideal value as possible by the VDLs. The attenuations of CH12 is tuned by the VOA in the corresponding WDM channel while the attenuations of other channels are unchanged. The frequency of the applied microwave signal on MZM is 55 MHz. The amplitude of MC in each channel is obtained from the Fast Fourier Transformation (FFT), and the components are normalized by the average of UC in CH11. From the figure, one can see that both the UC value & the MC amplitude in CH12 increase with the increase of relative power mismatch ratio. This is because the UC value & the MC amplitude is proportional to the optical pulse power, as Eq. (7) and Eq. (14) indicate, which increases with the increase of the relative power mismatch ratio. The ratios of MC to UC of CH11 and CH12 are shown in Fig. 5(b). One can see that, as the Eq. (20) indicates, the ratios of MC to UC in each channel are almost unchanged with the relative power mismatch ratio, and also the same for different channels.
Figure 5. (a) The amplitude variation vs relative power mismatch ratio and (b) The ratio of MC to UC of CH11 and CH12.

Figure 6(a) shows the normalized amplitude of UC and MC variation vs. relative time deviation of channels, where the components are normalized by the average of CH11 UC. The relative time deviation is defined as ratio of the nonideal delay, \( d_{\text{rel,non}} \), to the sampling period, \( T_s \). In the experiment, in order to only observe the effect of the timing mismatch of optical sampling pulse trains in different channels, the average powers of channels are tuned to equal as possible by the VOAs. The delay of CH12 is tuned by the VDL in the corresponding WDM channel while the delays of other time channels are unchanged. The frequency of the applied microwave signal on MZM is 55 MHz. From the figure, one can see that both the UC value and the MC amplitude in CH12 have the maximum value for the relative time deviation of zero, and decrease with the increase of the relative time deviation. This is consistent with Eq. (7) and Eq. (14) and reasonable since the digitizer samples the electric pulses at its peak and obtains the maximum amplitude values of both the UC and the MC for the relative time deviation of zero. As the increase of the relative time deviation, the deviation of the digitizer sampling points away from the peak also increase, and the UC value and the MC amplitude decrease for single peak pulses. The ratios of MC to UC in CH11 and CH12 are shown in Fig. 6(b), respectively. Similar to Fig. 5(b), one can see that, as Eq. (20) indicates, the ratios of MC to UC in each channel are almost unchanged with the relative time deviation, and also the same for different channels.

A 300 MHz low pass filter (LPF) is cascaded after the PD in CH11 and CH12 to limit the photodetection bandwidth and other channels are unlimited. Figure 7(a) shows the measured photodetection bandwidths with/without the LPF, where the magnitude of channel response is normalized by respective maximum. The measured equivalent channel frequency responses in the two cases are shown in Fig. 7(b), where the magnitude of channel response is normalized
by the maximum of the equivalent channel frequency responses without LPF. From the figure, one can see the measured equivalent channel frequency responses under two photodetection bandwidths are very similar to each other except a magnitude offset, although the difference of the two photodetection bandwidths is more than ~1.5 GHz. This is because both the two photodetection bandwidths meet the condition of no ISI [13].

The effects of the optical sampling pulse temporal shapes on equivalent channel frequency responses are measured by cascading an optical tunable filter (Alnair Labs, CVF-220CL) after the MLL. The optical pulses width is changed by adjusting the linewidth of the pulses through the optical tunable filter. Figure 8 shows the equivalent channel frequency responses in different filter linewidths. All the curves are normalized by their own peak value. One can see that the equivalent channel frequency responses changes obviously with the increase of pulse linewidth at the first, and then tend to the same after the pulse linewidth is larger enough. The results confirm the theoretical analyses of Eq. (18). A narrower pulse linewidth corresponds to a longer temporal pulse with a narrower bandwidth. When the bandwidth of pulse is comparable with that of the adopted MZM, the equivalent channel frequency responses will be affected by the bandwidth of pulse. After the pulse bandwidth is far larger than the bandwidth of the adopted MZM, the effect the pulse bandwidth can be ignored and the equivalent channel frequency response will trend to the same for different pulse bandwidth.

In the experiment system setup, the 3 dB linewidths of optical pulses after the two WDM channels are ~1.35 nm and ~1.17 nm, respectively, shown in Fig. 9(a). The channel frequency responses of four time channels are measured and shown in Fig. 9(b). We can see that the four channel frequency responses are almost the same, and the effect of pulse bandwidth can
be ignored since the pulse bandwidths under the two linewidths are broad enough for the bandwidth of the adopted MZM.

![Image](image1.png)

**Fig. 9.** (a) The normalized power responses of the adopted WDM channels and (b) the measured channel frequency responses in different pulse shapes.

The power spectrum of the sampling data in each time channel is shown in Fig. 10, where the power is normalized by the UC in CH11 and the frequencies are normalized by a half of the single-channel sampling rate. The channel delays are set to ideal time position as possible for only investigating the amplitude mismatch. A two-tone signal with frequencies of 30 MHz and 60 MHz is fed into MZM. In the figure, the components at the normalized frequencies of 0.334, 0.708 are under-sampled signals in each time channel because the signal frequencies are beyond a half of the sampling rate. The components at the zero frequency is the UC. One cannot observe other obvious spurs in each time channel, which indicates that each time channel can be considered as a uniform sampling channel. The ratio among UCs calculated by averaging sampling data in each time channels is ~1:0.95:0.90:0.84.

![Image](image2.png)

**Fig. 10.** The power spectrum of the sampling result in each time channel.

The sampling data in four channels are combined with and without the proposed amplitude correction. The power spectrum of the corresponding combined sampling data is shown in Fig. 11. In the figure, the spectrum power is normalized by the digital power of the 30 MHz signal, and the frequency is normalized by a half of the system sampling rate. In the power spectrum of the directly combined sampling data shown in Fig. 11(a), the spectrum components at the normalized frequencies of 0.411 and 0.823 correspond to the sampled signals. The spurs at the normalized frequencies of 0.5 and 1 are induced by the amplitude mismatch of UC, as Eq. (9) indicates. The spurs at the normalized frequencies of 0.089, 0.177, 0.323, 0.589, 0.677 and 0.912 are induced by the amplitude mismatch of MCs, as Eq. (15) indicates. Figure 11(b) shows the power spectrum of the combined sampling data after the proposed correction. From the spectrum, the powers of spurs induced by UCs can be
suppressed by ~50 dB and the powers of spurs induced by MCs can be suppressed by ~30 dB at the same time.

Moreover, the nonlinearity of the MZM biased at quadrature causes an odd harmonic distortion of the sampled signals. The frequencies of the third harmonic distortional signals with the largest power are 90 MHz and 180 MHz, respectively, for the applied 30 MHz and 60 MHz signals, the frequencies of the third harmonic distortional signals are 90 MHz and 180 MHz. Since the frequencies of the two third harmonic distortional signals have been beyond the first Nyquist zone (0 ~ 72.912 MHz), the two distortional signals of 90 MHz and 180 MHz are down-sampled and aliased to 55.824 MHz and 34.176 MHz, respectively, relative to system sampling rate (145.824 MHz) [18]. After normalized by 72.912 MHz, 55.824 MHz and 34.176 MHz cause the spurs at 0.766 and 0.469 in Fig. 11.

Fig. 11. (a) The power spectrum of the sampling data without the proposed amplitude correction; (b) The power spectrum of the sampling data with the proposed amplitude correction.

5. Conclusion

Considering TIPADCs as a hybrid electronic and photonic system, we build a channel mismatch model for TIPADC. The factors related to amplitude, timing and bandwidth mismatches are analyzed and the theoretical expressions are presented. The theoretical analyses show that the timing mismatch of optical sampling pulse trains and the photodetection bandwidth mismatch will also result in the amplitude mismatch. The channel bandwidth mismatch may be induced by the pulse shape mismatch and the photodetection bandwidth mismatch, but can be ignored when the bandwidth of the optical sampling pulse is much broader than that of electro-optic modulator and the photodetection bandwidth is broad enough to form a continuous channel frequency response. Based on the relationship between unmodulated components and modulated components, an effective online wideband amplitude mismatch correction method is proposed. The theoretical analyses are verified on a 4-channel TIPADC experimental system. An unmodulated components correction and a modulated components mismatch correction are achieved by using the proposed correction method for a two-tone signal. The UCs mismatch and MCs mismatch for a two-tone signal are corrected ~50 dB and ~30 dB, respectively, by the proposed correction method.

Appendix

This appendix presents the derivation of Eq. (10), which gives the value of MC.

Equation (6) can be rewritten as

\[ h_{\text{un}}(t) = -0.5\alpha p_{\text{un}}^2 h_{MD}(t) * T_{\text{un}}(t), \]  

(21)
where

\[
T_{k,m}(t) = h_{E,m}(t-d_{E,m}) \left[ p_{s,m}(-t) * \sum_{k=-\infty}^{\infty} \delta(-t + kT_s + d_{p,m}) \right].
\]  

(22)

The optical sampling pulse temporal power, \( p_{s,m}(t) \), can be considered as typically sufficient narrow compared to the impulse response of the photodetection, \( h_{E,m}(t) \), and symmetrical, because the fiber nonlinearity in TIPADCs can be ignored [13]. The approximation as follows can be taken:

\[
T_{k,m}(t) = \left[ p_{s,m}(t) * h_{E,m}(t-d_{E,m}) \right] \sum_{k=-\infty}^{\infty} \delta(-t + kT_s + d_{p,m}).
\]  

(23)

Applying the sampling property of the \( \delta(t) \), Eq. (10) is obtained.

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