Demonstration of essentiality of entanglement in a Deutsch-like quantum algorithm

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Quantum algorithms can be used to efficiently solve certain classically intractable problems by exploiting quantum parallelism. However, the effectiveness of quantum entanglement in quantum computing remains a question of debate. This study presents a new quantum algorithm that shows entanglement could provide advantages over both classical algorithms and quantum algorithms without entanglement. Experiments are implemented to demonstrate the proposed algorithm using superconducting qubits. Results show the viability of the algorithm and suggest that entanglement is essential in obtaining quantum speedup for certain problems in quantum computing. The study provides reliable and clear guidance for developing useful quantum algorithms.

quantum computing, quantum entanglement, quantum algorithm, Deutsch’s problem

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1 Introduction

Quantum information has undergone a revolutionary change in recent years. In 1982, the legendary physicist, R. P. Feynman noted that simulating \( n \) qubits on a classical computer needs exponential resources, as it requires storing and processing of \( 2^n \) complex amplitudes [1]. However, a quantum computer based on the laws of quantum physics can naturally simulate \( n \) qubits, and this attractive advantage has since driven the field of quantum computing. To date, considerable effort has been made to realize the dream of practical quantum computers [2-18].

Quantum computers harness the intrinsic nature of quantum mechanics, the quantum superposition principle, and compared to their classical counterparts they promise to enable exponential speedup for certain tasks [19, 20]. Quantum algorithms run on a realistic model of quantum computing and are the core of the speedup. For many years, researchers have been posed with the intellectual challenge of designing well-performing quantum algorithms, and notable achievements include Shor’s algorithm [21], Grover-Long algorithm [22,23], Simon’s algorithm [24], quantum simulation [1,25], solving linear systems [26], and quantum machine learning [27, 28]. However, to enable the design of further quantum algorithms, it is extremely important to investigate the quantum-mechanical effects within the quantum algorithm; in particular, the role of different types of quantum resources therein.
Entanglement is a specific and magical type of quantum superposition, and it is the quantum property of multiparticle systems that cannot be written as a tensor product of individual quantum states. However, although entanglement has been used as a useful quantum resource in several quantum cryptographic and communication tasks [29-31], its role in enabling quantum speedup has not yet been established.

It has been determined that several quantum algorithms, such as Bernstein-Vazirani [32] and Grover search [22], do not require entanglement for their implementation [33, 34], and Biham et al. [35] showed that certain advantages of quantum algorithms remain, even in the absence of entanglement. In addition, Biham et al. [36] demonstrated that how well a state performs as an input to Grover’s search algorithm depends critically upon the entanglement present in that state, and they found that the more the entanglement in the input, the less well the algorithm performs. However, although some works have been proposed to study the mechanism of quantum speedup [37], and many [38-40] have argued that entanglement is necessary for quantum algorithms, no specific examples have yet been shown to provide clear evidence of the role of entanglement.

In this paper, a quantum algorithm is presented to solve the Deutsch-like problem [41, 42] for two black boxes of two functions, which is completely dependent on entanglement for quantum speedup. To present the role of entanglement, classical algorithms and quantum algorithms without entanglement are proved to require at least three queries to functions, whereas only two queries are required in the proposed quantum algorithm with entanglement. Furthermore, a proof-of-principle demonstration is reported to show the viability of the proposed algorithm. For the first time, our work clearly demonstrates the necessity of entanglement in the quantum speedup for solving certain problems.

2 Theory

Prior to introducing the proposed algorithm, Deutsch’s algorithm is briefly described [41]. Given a black box executing certain unknown function \( f : \{0, 1\} \rightarrow \{0, 1\} \), assume that one wishes to know whether function \( f \) is constant \((f(0) \oplus f(1) = 0)\) or balanced \((f(0) \oplus f(1) = 1)\). Classically, two queries to the function \( f \) are required to solve this problem, but Deutsch’s algorithm solves it using only a single query, as follows (see Figure 1):

1. Initializing two qubits to \( |0\rangle_{a_1} \otimes |1\rangle_{a_2} \) and applying a Hadamard gate to each qubit yields

\[
\frac{|0\rangle_{a_1} + |1\rangle_{a_1}}{\sqrt{2}} \otimes \frac{|0\rangle_{a_2} - |1\rangle_{a_2}}{\sqrt{2}}.
\]

2. By applying the function \( f \) to the current state, the following state is obtained,

\[
|0\rangle_{a_1} + (-1)^{f(0) \otimes f(1)} |1\rangle_{a_1} \otimes \frac{|0\rangle_{a_2} - |1\rangle_{a_2}}{\sqrt{2}}.
\]

3. Apply Hadamard gate, and subsequent measure the qubit \( a_1 \) on a computational basis.

It is evident that \( f(0) \oplus f(1) = 0 \), when zero is measured; and \( f(0) \oplus f(1) = 1 \), when one is measured. Therefore, it is possible to decide with certainty whether the function is constant or balanced. Deutsch’s algorithm solves this problem in only one query, and it is therefore faster than the classical algorithm.

However, Deutsch’s algorithm cannot be used to prove the advantage of entanglement, as no entanglement is generated in the algorithm. A modified problem is therefore firstly proposed here. We then show that to solve this specific problem, entanglement provides advantages over the classical algorithm and even over the quantum algorithm without entanglement.

If we consider a similar problem and assume that Alice has black boxes of the two unknown functions \( f : \{0, 1\} \rightarrow \{0, 1\} \) and that \( g : \{0, 1\} \rightarrow \{0, 1\} \). She has been assured that both functions \( f \) and \( g \) are either constant or balanced, that is, \( f(0) \oplus f(1) = g(0) \oplus g(1) \). Alice wants to compute the following two quantities with minimum possible queries to the functions \( f \) and \( g \).

\[
(1) f(0) \oplus f(1) \text{ or } g(0) \oplus g(1), \text{ functions } f \text{ and } g \text{ are constant or balanced; (2) } f(0) \oplus g(0) \text{ or } f(1) \oplus g(1), \text{ functions } f \text{ and } g \text{ are same or different.}
\]

It is clear that, classically, two queries to the function \( f \) (or \( g \)) are required to compute \( f(0) \) and \( f(1) \) (or \( g(0) \) and \( g(1) \)), and one query to the function \( g \) (or \( f \)) is required to compute \( g(0) \) (or \( f(0) \)).

A quantum algorithm is proposed here to exploit quantum entanglement, and it requires only one query to each function \( f \) and \( g \). The proposed algorithm therefore spares one query compared to the classical one. We then subsequently show that this quantum advantage is not possible without entanglement. In the following, the proposed algorithm is presented step by step (see Figure 2).
(1) We begin with one qubit |0⟩_A and two ancilla qubits |0⟩_{a_1} and |0⟩_{a_2}, and initialize these qubits to

\[ |0⟩_A + \frac{1}{\sqrt{2}} |1⟩_A \otimes |0⟩_{a_1} |0⟩_{a_2} - |1⟩_{a_1} |1⟩_{a_2}. \]

In the initialization step, two ancilla qubits are entangled.

(2) Quantum state of the composite system after applying functions \( f \) and \( g \) (ignoring normalization coefficients) is given by

\[
|0⟩_A (|0⟩ \oplus f(0)⟩_{a_1}) |0⟩ \oplus g(0)⟩_{a_2} - |1⟩ \oplus f(0)⟩_{a_1} |1⟩ \oplus g(0)⟩_{a_2})
+ |1⟩_A (|0⟩ \oplus f(1)⟩_{a_1}) |0⟩ \oplus g(1)⟩_{a_2} - |1⟩ \oplus f(1)⟩_{a_1} |1⟩ \oplus g(1)⟩_{a_2}).
\]

**Case -1:** If \( f(0) \oplus g(0) = f(1) \oplus g(1) = 0 \), that is, \( f(0) = g(0) \) and \( f(1) = g(1) \), the above equation reads

\[
|0⟩_A (-1)^{f(0)}(|0⟩_{a_1} |0⟩_{a_2} - |1⟩_{a_1} |1⟩_{a_2})
+ |1⟩_A (-1)^{f(1)}(|0⟩_{a_1} |0⟩_{a_2} - |1⟩_{a_1} |1⟩_{a_2}),
\]

which is equivalent to

\[
|0⟩_A + (-1)^{f(0)}(1)|1⟩_A (|0⟩_{a_1} |0⟩_{a_2} - |1⟩_{a_1} |1⟩_{a_2}).
\]

**Case -2:** For \( f(0) \oplus g(0) = f(1) \oplus g(1) = 1 \), that is, \( f(0) \neq g(0) \) and \( f(1) \neq g(1) \), it reads

\[
|0⟩_A (-1)^{f(0)}(|0⟩_{a_1} |1⟩_{a_2} - |1⟩_{a_1} |0⟩_{a_2})
+ |1⟩_A (-1)^{f(1)}(|0⟩_{a_1} |1⟩_{a_2} - |1⟩_{a_1} |0⟩_{a_2}),
\]

which is equivalent to

\[
|0⟩_A + (-1)^{f(0)g(1)}(1)|1⟩_A (|0⟩_{a_1} |1⟩_{a_2} - |1⟩_{a_1} |0⟩_{a_2}).
\]

(3) Apply Hadamard gate on qubit A, and then measure the three qubits in computational basis.

It is obvious that the measurement outcome of qubit A determines whether the functions \( f \) and \( g \) are constant or balanced, while the measurement outcomes of the two ancilla qubits determines whether they are same or different, as depicted in Table 1.

**Table 1** Measurement results of proposed algorithm that determine properties of functions \( f \) and \( g \)

| Measurement outcome (first qubit A) | \( f(0) \oplus f(1) \) |
|-------------------------------------|-----------------|
| \( |0⟩ \) | 0 |
| \( |1⟩ \) | 1 |

| Measurement outcome (ancilla qubits) | \( f(0) \oplus g(0) \) |
|--------------------------------------|-----------------|
| \( |0⟩_{a_1} |0⟩_{a_2} \) or \( |1⟩_{a_1} |1⟩_{a_2} \) | 0 |
| \( |0⟩_{a_1} |1⟩_{a_2} \) or \( |1⟩_{a_1} |0⟩_{a_2} \) | 1 |

Entanglement is generated during computing in the proposed algorithm above, and only two queries are required. In computational complexity theory, it is customary to analyze algorithms with respect to the number of queries involved. This method of analyzing algorithms is known as the query model. If an algorithm in the query model makes minimal queries of the oracle, it is said to be more efficient [43]. Next, we will prove that at least three queries are required when entanglement is not generated during computing.

**Theorem** It is impossible to compute \( f(0) \oplus f(1) \) and \( f(0) \oplus g(0) \) together using a total of two queries if quantum entanglement is not generated in the algorithm.

**Proof** Here, we assume that it is possible to compute \( f(0) \oplus f(1) \) and \( f(0) \oplus g(0) \) together in one query for each function \( f \) and \( g \), without generating entanglement in any intermediate stage of the algorithm, and to derive a contradiction. In this case, the structure of the algorithm can be concluded as follows.

(1) Initial state is \( |ϕ₀⟩ |ϕ₁⟩ |ϕ₂⟩ \). Please begin a new paragraph (2) After applying the function \( f \), the state of the composite system 0, 1 and 2 changes into \( U_{f₀}(|ϕ₀⟩ |ϕ₁⟩ |ϕ₂⟩) = |ϕ₀₁⟩ |ϕ₂⟩ \).

(3) Assume that some local quantum operations could be performed to change the state \( |ϕ₁₀⟩ \) into \( |ξ₁₀⟩ \) after applying \( f \). The state of the composite system 0, 1 and 2 after applying function \( g \) is given by \( U_{g₀}(|ξ₁₀⟩ |ϕ₂⟩) \).

Let us now consider the second step of the algorithm: as it has been assumed that algorithm does not utilize entanglement, \( |ϕ₁₀⟩ \) must be a product state, \( U_{f₀}(|ϕ₀⟩ |ϕ₁⟩) = |ϕ₀₁⟩ = |ϕ₀⟩ ⊗ |ϕ₁⟩ \).

If function \( f \) is constant, it is easy to see that \( |ϕ₀₁⟩ \) will always be a product state. Thus, the above relation will be true for any initial state \( |ϕ₀⟩ |ϕ₁⟩ \). However, a problem arises when function \( f \) is balanced. Without loss of generality, let us take \( f(0) = 0 \) and \( f(1) = 1 \). In this case unitary \( U_{f₁} \) is controlled-NOT (CNOT). If \( |ϕ₀⟩ = α |0⟩ + β |1⟩ \), \( |ϕ₁⟩ = γ |0⟩ + δ |1⟩ \) then,

\[
U_{f₀}(|ϕ₀⟩ |ϕ₁⟩) = αγ |00⟩ + αδ |01⟩ + βδ |10⟩ + βγ |11⟩.
\]
For it to be a product state, $a\beta(y^2 - \delta^2) = 0 \Rightarrow \alpha = 0$ or $\beta = 0$ or $\gamma = \pm \delta$. Thus, the possible states that do not generate entanglement in this case read as:

$$|0\rangle (\gamma |0\rangle + \delta |1\rangle),$$

$$|1\rangle (\gamma |0\rangle + \delta |1\rangle),$$

$$(\alpha |0\rangle + \beta |1\rangle) \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right),$$

$$(\alpha |0\rangle + \beta |1\rangle) \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right).$$

We now determine the information that can be computed using these states.

1. If the state is of type $|0\rangle (\gamma |0\rangle + \delta |1\rangle)$ or $|1\rangle (\gamma |0\rangle + \delta |1\rangle)$, then, is the corresponding output after applying $U_{f_0}$ is $|0\rangle (\gamma |0 + f(0)\rangle + \delta |1 + f(0)\rangle)$ or $|0\rangle (\gamma |0 + f(1)\rangle + \delta |1 + f(1)\rangle)$. At best, we can learn the value of $f(0)$ or $f(1)$, when either $\gamma = 0$ or $\delta = 0$ is true.

2. If the state is of type $|0\rangle (\alpha |0\rangle + \beta |1\rangle) \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)$, after applying $U_{f_0}$ output will be

$$\alpha |0\rangle \left(\frac{|0 + f(0)\rangle + |1 + f(0)\rangle}{\sqrt{2}}\right) + \beta |1\rangle \left(\frac{|0 + f(1)\rangle + |1 + f(1)\rangle}{\sqrt{2}}\right).$$

This would always be $|0\rangle (\alpha |0\rangle + \beta |1\rangle) \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)$ independent of mapping $f$. Since the input and output are the same, no information about the function $f$ is obtained after the computation.

3. If the state is of type $|0\rangle (\alpha |0\rangle + \beta |1\rangle) \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)$, after applying $U_{f_0}$ the output will be

$$(\alpha |0\rangle + (-1)^{f(0)+f(1)}\beta |1\rangle) \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right).$$

At best, it is possible to obtain $f(0) + f(1)$, when $\alpha = \beta = \frac{1}{\sqrt{2}}$.

Therefore, it is only possible to make one computation out of $f(0)$, $f(1)$, and $f(0) \oplus f(1)$ in one query without generating entanglement in the second step of algorithm. After execution of function $f$, the 0-th and 1-st qubits are in a certain product state $|\psi_0\rangle |\psi_1\rangle$. Assume that some local quantum operations could be applied on state $|\psi_0\rangle \otimes |\psi_1\rangle$ to change it into $|\psi_0\rangle \otimes |\psi_1\rangle$ before applying the function $g$. However, after applying $U_{g_0}$ ($|\psi_0\rangle |\psi_1\rangle \rightarrow |\psi_2\rangle$) and again demanding that the output should be a product state, it is only possible to conduct one computation out of $g(0)$, $g(1)$, and $g(0) \oplus g(1)$ in one query, which provides a similar result to that obtained in the second step.

Hence, in two queries, it is only possible to compute one quantity from each set $A = \{f(0), f(1), f(0) \oplus f(1)\}$ and $B = \{g(0), g(1), g(0) \oplus g(1)\}$, without generating entanglement in any stage of the algorithm. In addition, as there were no combinations of $(x, y)$, where $x \in A$, $y \in B$ gave $f(0) \oplus f(1)$ and $f(0) \oplus g(0)$ together, a contradiction was derived.

Therefore, when restricted to only one query to each of the functions $f$ and $g$, it is not possible to calculate logical quantities of $f(0) \oplus f(1)$ and $f(0) \oplus g(0)$ together (given that both functions are either constant or balanced) using either a classical computer or a quantum computer without entanglement. Figure 3 provides an example of a quantum algorithm without entanglement that requires three queries. By implementing the algorithm in Figure 3, we can determine that functions $f$ and $g$ are constant (balanced) if the measurement result of the first qubit is $|0\rangle (|1\rangle)$, and functions $f$ and $g$ are the same (different) when the measurement results of the second and third qubits are the same (different). In fact, the measurement results of the algorithm in Figure 3 are the same as the results presented in Table 1. It is of note that both the proposed algorithms in Figures 2 and 3 are deterministic. However, the algorithm in Figure 2 only needs two queries, whereas the algorithm in Figure 3 needs three queries. We have therefore shown that entanglement is essential for quantum speedup of certain problems. With the exception of the number of queries for the function, the number of quantum gates in the quantum algorithm in Figure 2 is also less than that in the quantum algorithm in Figure 3.

### 3 Experimental realization

We also implemented a proof-of-principle experiment to demonstrate the proposed algorithm using the superconducting system (http://www.research.ibm.com/quantum/). In our implementation, we chose two types of balanced functions, as shown in Figure 4(a) and (b), and two types of constant functions, as shown in Figure 4(c) and (d). Without loss of generality, the following four cases were considered:

1. $f = B_1$ and $g = B_1$,
2. $f = B_1$ and $g = B_2$,
3. $f = C_1$ and $g = C_1$.

![Figure 3](https://example.com/figure3.png) (Color online) Circuit for proposed algorithm without entanglement. Information determining whether functions are constant or balanced is stored in the first qubit, while information determining whether they are same or different is stored in the second and third qubits. $U_f$ is applied on the 1-st and 2-nd qubits, and $U_g$ is applied on the 1-st and 3-rd qubits.
(4) $f = C_1$ and $g = C_2$, where both functions $f$ and $g$ are balanced in case-(1) and case-(2). However, $f = g$ in case-(1) and $f \neq g$ in case-(2). Similarly, both functions $f$ and $g$ are constant in case-(3) and case-(4), but $f = g$ in case-(3) and $f \neq g$ in case-(4). By substituting the functions in the cases into the circuits in Figures 2 and 3, it is possible to realize one algorithm with entanglement and one without entanglement, respectively.

Figure 5(a)-(d) shows both the ideal (red bar) and experimentally obtained (blue bar) probabilities for each outcome when implementing the version of the algorithm with entanglement for cases-(1)-(4). In this respect, we use Figure 5(a) as an example to explain the results. Ideally, according to Table 1, the output is in $|100\rangle$ with a probability of 50%, and another 50% probability yields $|111\rangle$. With a measurement result of either $|100\rangle$ or $|111\rangle$, it is possible to determine that functions $f$ and $g$ are balanced, and $f = g$ according to Table 1. To quantify the experimental performance, we use the statistical fidelity $F = \sum_{k=0}^{3} \sqrt{p_{k}^{\exp} p_{k}^{\text{th}}}$ [44] to characterize the overlap between experimental and theoretical values, where $p_{k}^{\exp}$ and $p_{k}^{\text{th}}$ are the experimental and theoretical output probabilities of the state $|k\rangle$, respectively. From the data in Figure 5, the fidelities are calculated as $F_1 = 0.891(7)$, $F_2 = 0.873(7)$, $F_3 = 0.952(7)$, and $F_4 = 0.953(7)$. Thus, the algorithm is proven to be successful: the version of the algorithm with entanglement is able to solve the task by asking only two queries.

We also implemented the version of the algorithm without entanglement for cases-(1)-(4) experimentally: Figure 6(a)-(d) shows the measurement results for cases-(1)-(4). The data in Figure 6 show that the experimental results agree with the theoretical prediction. The fidelities of the results for cases-(1)-(4) are $F_1 = 0.837(9)$, $F_2 = 0.863(10)$, $F_3 = 0.916(10)$, and $F_4 = 0.929(10)$. Our experiments show that it is possible to solve the task using the version of the algorithm without entanglement; however, at least three queries are required. These experiments show that the use of entanglement can provide advantages over the quantum algorithm without entanglement.

The errors of our experiments are mainly related to errors
in the quantum gates and readout, and Table 2 shows an error analysis conducted in relation to experiments.

4 Conclusions

In summary, a new quantum algorithm is proposed and experimentally demonstrated to show that quantum entanglement is essential for obtaining quantum speedup in solving certain problems. It is also shown that when restricted to a total of two queries for the functions \( f \) and \( g \), a classical computer or a quantum computer without entanglement is unable to compute logical functions, \( f(0) \oplus f(1) \) or \( f(0) \oplus g(0) \) together. However, a quantum computer that has entanglement as a resource is able to compute these quantities deterministically with one query for each of the functions \( f \) and \( g \). The algorithm demonstrated here succinctly illustrates how entanglement can be used in a simple way with quantum algorithms, and it could thus be used as a prototype in the future to develop useful quantum algorithms. Furthermore, the proposed algorithm could be used directly to learn the property of two Boolean functions (testing whether two Boolean functions are the same), and could be easily generalized to test more Boolean functions [45].

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