Non-linear gravitational clustering in scalar field cosmologies

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Abstract. Non-linear gravitational clustering in a universe dominated by dark energy, modelled by a ‘quintessence’ scalar field, and cold dark matter with space-time varying mass is studied. Models of this type, where the variable mass is induced by dependence on the scalar field, as suggested by string theory or extra-dimensions, have been proposed as a viable solution of the coincidence problem. A general framework for the study of the non-linear phases of structure formation in scalar field cosmologies is provided, starting from a general relativistic treatment of the combined dark matter-dark energy system. As a first application, the mildly non-linear evolution of dark matter perturbations is obtained by a straightforward extension of the Zel’dovich approximation. We argue that the dark energy fluctuations may play an active role in cosmological structure formation if the scalar field effective potential develops a temporary spinodal instability during the evolution.

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1. Introduction

There is increasing agreement on the picture of the Universe on large scales: the analysis of the magnitude-redshift relation for type Ia Supernovae [1], measurements of the anisotropies of the Cosmic Microwave Background (CMB) [2], and analyses of the galaxy distribution in large redshift catalogs [3] all converge to show that the Universe is spatially flat and composed by roughly 30% of cold dark matter (DM), responsible for the structure we see via the small amount of baryons, and, for the remaining 70% by an unclustered form of dark energy (DE), responsible for its present-day accelerated expansion. In order to alleviate or avoid fine-tuning of initial conditions, an alternative solution to the usual cosmological constant has been proposed in the form of a dynamical vacuum energy component with negative equation of state, the so-called quintessence [4].

The most common candidate for quintessence is a self-interacting scalar field \( \phi \). Different choices for the effective potential have been considered in the literature. In order to avoid fine tuning on the initial conditions, particular attention has been devoted to potentials leading to scaling or tracker behavior, like those depending on the scalar field exponentially [5] or with a negative power-law [6].

In addition to self-interactions, the DE scalar field might in principle be coupled to any other particle or field present in Nature. However, given the extremely low mass typically expected for \( \phi \) today, \( m_\phi \sim H_0 \sim 10^{-33} \) eV, its couplings to common particles, such as baryons or photons, are severely bounded by experimental tests on the equivalence principle and on the time variation of coupling constants. On the other hand, the coupling to non-baryonic DM is not so tightly constrained and might play a significant role in the cosmological evolution [7]. In this paper we will take into account such a possibility by allowing the DM particles’ mass to depend on the DE scalar field. The DM particles are then Varying Mass Particles (VAMPS), a possibility that was discussed in the past in connection to the age problem of the Universe [8].

The VAMPS scenario was considered more recently in the quintessence context in [9], where it was shown that it may lead to a solution of the so-called ‘cosmic coincidence’ problem. Indeed, assuming an exponential form both for the coupling and for the effective potential, the late time cosmology may be such that the universe accelerates and the energy densities of DE and DM scale at the same rate. As expected, in this scenario the predictions for the DM abundance are drastically altered with respect to the standard freeze-out scenario [9]. A coupling of this type may arise for radii moduli in braneworld scenarios [10]. Couplings between DE and DM may also emerge in strongly coupled string theory [11].

It is generally believed that perturbations of the quintessence scalar field play a negligible dynamical role in cosmic structure formation, because of the extreme smallness of the quintessence mass (\( m_\phi \sim H \)) which implies that its spatial fluctuations only appear on very large scales and are bound to be linear. Nevertheless, a number of models have been proposed where DE perturbations might in principle grow non-linear
and play an active role down to galactic scales. These include coupled-quintessence [12], extended quintessence [13, 14], inhomogeneous Chaplygin gas [15], k-essence [16]. A key question in the context of scalar field cosmologies is whether fluctuations of the quintessence field can influence the galaxy formation process in a sensible way, affecting the dynamics of structure formation during the non-linear phases e.g. in the cores of dark matter halos. This problem has been recently addressed by Wetterich [17], but the issue of whether quintessence fluctuations can grow non-linear on small scales and actively influence the non-linear clustering of the CDM component is still completely open.

The VAMPS scenario considered here provides an useful benchmark to study situations where non-linearities in the scalar field may be induced either by the field’s self interaction or by its couplings to other cosmic components.

The plan of the paper is as follows. In Section 2 we give a full derivation of the equations which govern the non-linear evolution of DM and DE perturbations on sub-horizon scales. Then, in order to gain some intuitive insight on the content of these equations, we specialize to the VAMP model discussed in [9]. A simple solution of the equations in the mildly non-linear regime for this model is obtained in Section 3, in terms of first-order lagrangian perturbation theory. In Section 4 we discuss how the simple model considered here could be modified to give rise to non-negligible scalar field fluctuations on scales relevant for galaxy or galaxy-cluster formation.

2. General formalism

As a starting point, we will expand the metric tensor $g_{\mu\nu}$ (greek indices run from 0 to 3, while latin ones from 1 to 3) to linear order around a flat Robertson-Walker (RW) metric, while keeping the full – i.e. non-linear – energy momentum tensor on the RHS of Einstein equations. In the conformal Newtonian gauge, vector and tensor perturbations of the metric decouple from the scalar ones as long as they are treated linearly; by taking the spatial covariant divergence of the $0-i$ Einstein equations and the trace of the $i-j$ equations, we get rid of vector and tensor modes and single out scalar ones. Thus, it is not restrictive to consider the metric [18]

$$\text{ds}^2 = a^2(\eta) \left[(1 + 2\Phi)d\eta^2 - (1 - 2\Psi)\delta_{ij}dx^i dx^j \right],$$

where the scalar perturbations $\Phi$ and $\Psi$ are related to the gravitational potential, and are of order $|u|^2 \ll 1$ ($u$ being the typical velocity of DM particles) even in the presence of highly non-linear DM overdensities. Here $a(\eta)$ is the background scale-factor and $\eta$ is the conformal time.

In this approximation, Einstein’s equations read

$$\delta R^\mu_\nu = 8\pi G (S^\mu_\nu - \bar{S}^\mu_\nu),$$

where $\delta R_{\mu\nu}$ is the linear perturbation to the Ricci tensor, while $S^\mu_\nu \equiv T^\mu_\nu - \frac{1}{2}\delta^\mu_\nu T$ is the fully non-linear source and $\bar{S}^\mu_\nu$ is the background one.
The Friedmann equations for the background RW metric read

\[ H^2 = \frac{8\pi G}{3} a^2 (\bar{\rho}_m + \bar{\rho}_\phi) \]  

\[ \dot{H} = -\frac{4\pi G}{3} a^2 (\bar{\rho}_m + \bar{\rho}_\phi + 3\bar{\rho}_\phi) , \tag{4} \]

where \( H \equiv \dot{a}/a \), and the dot denotes differentiation with respect to \( \eta \).

We will take into account the contribution to the energy-momentum tensor coming from the dark matter and the scalar field. The former is made up of a discrete set of particles with field-dependent mass \( m(\phi(\eta, x)) \) and coordinates \( x_a(\eta) \) \((a = 1, 2, \ldots)\), described by the energy–momentum tensor

\[ T^\mu_\nu \simeq a^{-2} \rho_m(\phi) u^\mu u_\nu , \tag{5} \]

where

\[ \rho_m(\phi) = m(\phi) a^{-3} \sum_a \delta^{(3)}(x - x_a) , \]

\( u^\mu \equiv dx^\mu/d\eta \) \((x^0 \equiv \eta)\), and we have neglected \( O(|u|^2) \) corrections.

The scalar field energy-momentum tensor reads

\[ T^\mu_\nu = M_p^2 \phi^{i\mu}_\nu - \delta^\mu_\nu \left( \frac{M_p^2}{2} \phi^{\alpha\beta} \phi_{,\alpha \beta} - V(\phi) \right) , \tag{6} \]

where \( \phi \) is dimensionless because of the factorization of \( M_p^2 = (8\pi G)^{-1} \). After replacing the background quantities \( \bar{g}_{\mu\nu}, \bar{\phi} \), and \( \bar{x}_a^\mu \) in (5) and (6) one can read out the background energy density and pressure for our two components, \( \bar{\rho}_m, \bar{\rho}_\phi \), and \( \bar{p}_\phi \).

The Einstein equations provide a redundant set of equations for the evolution of the linearized metric perturbations; we choose to use the 0−0 equation and the spatial covariant derivative of the 0−i’s. They read, respectively,

\[ \nabla^2 \Phi + 3H(\dot{\Phi} + \dot{\Psi}) + 3\ddot{\Psi} = \frac{a^2}{2M_p^2} \left[ \rho_m(\phi) - \bar{\rho}_m + \frac{2M_p^2}{a^2} (\phi^2 - \bar{\phi}^2) - 2(V(\phi) - V(\bar{\phi})) \right] , \tag{7} \]

\[ H \nabla^2 \Phi + \nabla^2 \ddot{\Psi} = \frac{a^2}{2M_p^2} \left[ \partial_i \left( \rho_m(\phi) u^i \right) + \frac{M_p^2}{a^2} (\partial_i \phi \partial^i \phi + \phi \nabla^2 \phi) + \frac{2M_p^2}{a^2} \phi \partial_i \Psi \partial^i \phi \right] , \tag{8} \]

where primes denote differentiation with respect to \( \phi \) and, from now on, latin indices are raised with a Kronecker delta.

The evolution of the scalar field \( \phi(\eta, x) \) is given by the Klein–Gordon equation, which, for \(|\Phi|, |\Psi|, |u^i| \ll 1\), reads

\[ \dddot{\phi} + 2H \dot{\phi} - \nabla^2 \phi + \frac{a^2}{M_p^2} U'(\phi) = \nabla(\Phi - \Psi) \cdot \nabla \phi , \tag{9} \]

where the effective potential \( U(\phi) \) is the sum of the scalar field potential and the field-dependent dark matter energy density,

\[ U(\phi) \equiv V(\phi) + \rho_m(\phi) \tag{10} \]

Finally, we need the equation of motion for the dark matter particles,

\[ \frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{\sigma \beta} \frac{dx^\sigma}{ds} \frac{dx^\beta}{ds} - \frac{m'}{m} \partial^\mu \phi = 0 , \]
where the last term comes from the field dependence of the particle mass and makes the trajectories deviate from geodesics. The spatial coordinates of the $a$-th particle satisfy

$$\ddot{x}_a^i + \dot{x}_a^i \left[ \mathcal{H} + \frac{m'}{m} \dot{\phi} \right] = -\partial_i \Phi - \frac{m'}{m} \partial^i \phi,$$

(11)

where $\mathcal{H}$, $\Phi$, and $\phi$ are evaluated at $(\eta, x_a(\eta))$.

Equations (7), (8), (9) and (11) form a close system in the variables $\Phi$, $\Psi$, $\phi$ and $x_a^i$'s.

As we are interested in sub-horizon scales, there are further approximations which can be made. First of all, in Eq. (7), only the first term on the LHS will be kept, as the others are $O(\mathcal{H}^2 \Phi, \mathcal{H}^2 \Psi)$. Furthermore, the RHS of Eq. (9) is suppressed by at least an $O(\Phi, \Psi)$ factor with respect to the Laplacian on the LHS, and can then be dropped. As a result, the $\Psi$ perturbation is not needed anymore, $\Phi$ can be identified with the gravitational potential, and we can concentrate on the reduced system of Eqs. (7), (9), and (11) in their approximated forms.

To illustrate the role of the scalar field we split it into background, ‘non-relativistic’ and ‘relativistic’ contributions,

$$\phi(\eta, x) = \bar{\phi}(\eta) + \phi_{NR}(\eta, x) + \phi_R(\eta, x),$$

(12)

$\bar{\phi}(\eta)$ is the solution of the background Klein-Gordon equation,

$$\ddot{\bar{\phi}} + 2\mathcal{H} \dot{\bar{\phi}} + a^2 M_p^{-2} U'(\bar{\phi}) = 0,$$

(13)

while the non-relativistic part $\phi_{NR}(\eta, x)$ is defined as the solution of the Poisson-like equation

$$\nabla^2 \phi_{NR} = a^2 M_p^{-2} \Delta U' (\bar{\phi} + \phi_{NR}),$$

(14)

where $\Delta U(\phi) \equiv U(\phi) - V(\bar{\phi}) - \bar{\rho}_m$. Thus, $\phi_{NR}$ accounts for the response of the scalar field to the localized DM distribution, as induced by the field dependence of the DM particle mass. It behaves similarly to the gravitational potential $\Phi$ and, as we will see, in specific cases it may turn out to be just proportional to the latter.

The remaining ‘relativistic’ part solves the equation

$$\ddot{\phi}_R + 2\mathcal{H} \dot{\phi}_R - \nabla^2 \phi_R + \ddot{\phi}_{NR} + 2\mathcal{H} \dot{\phi}_{NR} + a^2 M_p^{-2} [\Delta U'(\bar{\phi} + \phi_{NR}) - \Delta U'(\bar{\phi} + \phi_{NR})] = 0.$$

(15)

The effective mass for $\phi_R$, as read out from Eq. (15), is given by $\Delta U''(\bar{\phi} + \phi_{NR})$, which, for the scales of interest for the formation of cosmic structures is much smaller than the corresponding $k^2/a^2$. Moreover, since the time-scale for $\bar{\phi}$ and $\phi_{NR}$ variation is given by $\mathcal{H}$, the $\dot{\phi}_{NR}$ and $\ddot{\phi}_{NR}$ terms in (15) provide $O(\mathcal{H}^2)$ sources. As a consequence, the $\phi_R$ component behaves essentially as the superposition of approximately massless plane waves on the scales of interest, with $O(a^2 \bar{\rho}_m / k^2 M_p^2) \ll 1$ amplitudes, and can then be neglected.

The relevant dynamics can then be described in terms of the equations
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\[ \nabla^2 \Phi = \frac{a^2}{2M_p^2} \left[ \rho_m(\bar{\phi} + \phi_{NR}) - \bar{\rho}_m - 2 \left( V(\bar{\phi} + \phi_{NR}) - V(\bar{\phi}) \right) \right], \quad (16) \]

\[ \nabla^2 \phi_{NR} = \frac{a^2}{M_p^2} \Delta U'(\bar{\phi} + \phi_{NR}), \quad (17) \]

\[ \ddot{x}_a + \left[ \mathcal{H} + \frac{m'}{m}(\dot{\bar{\phi}} + \dot{\phi}_{NR}) \right] \dot{x}_a = -\partial_i \Phi - \frac{m'}{m} \partial_i (\bar{\phi} + \phi_{NR}), \quad (18) \]

and the background ones, Eqs. (3), (4), and (13).

The equations above extend the standard Newtonian approximation from the pure DM case to the coupled system of DM and quintessence DE. All non-background DE effects are given by \( \phi_{NR} \), which solves the Poisson-like eq. (17).

In order to get a flavour of the dynamical content of the system, we specialize to a simple model which was proposed as a solution of the coincidence problem [9, 10]. The scalar potential and the DM particle mass are both exponentially dependent on \( \phi \),

\[ V(\phi) = \hat{V} \exp(\beta \phi), \quad m = \hat{m} \exp(-\lambda \phi), \quad (19) \]

with \( \beta, \lambda > 0 \). The model has a solution such that

\[ \bar{\phi} = \frac{-3}{\lambda + \beta} \log a, \quad \Omega_{\phi} = 1 - \Omega_m = \frac{3 + \lambda(\lambda + \beta)}{(\lambda + \beta)^2}, \quad (20) \]

which is an attractor in field space if \( \beta > (-\lambda + \sqrt{\lambda^2 + 12})/2 \). From the moment the attractor is reached, the energy densities in DM and in DE evolve at a constant ratio depending only on \( \lambda \) and \( \beta \), thus solving the cosmic coincidence problem.

Indeed, the solution (20) implies

\[ \bar{\rho}_m = m(\bar{\phi}) \bar{n}_m \sim \bar{\rho}_\phi \sim a^{-3(1+w)}, \quad \text{with} \quad w = -\frac{\lambda}{\beta + \lambda}, \quad (21) \]

where \( \bar{n}_m \) is the DM background number density. The negative \( w \) leads, if \( w < -1/3 \), to accelerated expansion of the universe. The \( \phi \) dependence of the DM mass modifies the usual scaling \( a^{-3} \) of non relativistic matter. Since the mass increases with the expansion, the effect is analogous to that of a fluid with negative equation of state, though DM is, as in the standard scenario, a pressureless fluid made up by non-interacting particles.

The impact on the CMB of this model is similar to the one considered in [10]. It was shown there that in order to get results in agreement with the CMB anisotropy spectrum a time-dependent \( \lambda \) should be invoked, such that the early time dependence is the standard CDM one, and the attractor behavior of Eq. (20) starts only recently. Since we use the model only for illustrative purposes, we will stick to a constant \( \lambda \) in the following.

The exponential dependences on \( \phi \) in \( \Delta U \) and Eq. (17) imply that also \( \phi_{NR} = \mathcal{O}(a^2 \bar{\rho}_m/k^2 M_p^2) \ll 1 \). Then, as dark matter fluctuations approach the non linear regime, they start dominating over those of the effective potential, i.e.

\[ \Delta U \sim m(\bar{\phi}) \left( a^{-3} \sum_a \delta^{(2)}(x - x_a) - \bar{n}_m \right). \quad (22) \]
In this approximation, the RHS of the Poisson’s equation (16) and that of Eq. (17) are proportional to each other, so that \( \phi_{NR} = -2\lambda\Phi = O(|u|^2) \).

Using the properties of the attractor solution, Eq. (20), we get the further reduced system,

\[
\begin{align*}
\nabla^2 \Phi &= \frac{\dot{m}}{2M_p^2} a^{2-3w} \left( a^{-3} \sum_a \delta(3) (x - x_a) - \bar{n}_m \right), \\
\ddot{x}_a^i + (1 - 3w)H \dot{x}_a^i &= -(1 + 2\lambda^2) \partial^i \Phi
\end{align*}
\]

The coupling between DM and DE modifies the equations from the standard ones in two ways. It changes the background evolution through the \( w \)-terms, and it gives an extra contribution to the force between two DM particles, due to the exchange of a light \( \phi \) quantum. The last effect is responsible for the \( 2\lambda^2 \) correction to the gravitational force in the LHS of Eq. (24).

3. Mildly non-linear evolution

The set of equations (23), (24) provides the required extension of the standard Newtonian approximation (e.g. [20]), describing the non-linear evolution of matter perturbations, to the case where a quintessence scalar field component is present. As a preliminary application, we can study the effect of the scalar field on the DM evolution in the mildly non-linear regime, using first-order Lagrangian perturbation theory.

Standing to standard notation [21], one can look for a time variable \( \tau = \tau(\eta) \) by which the equation of motion derived from (18) does not contain the Hubble drag term. We can then write an equation for the displacement from the initial (Lagrangian) position \( q \) to the final (Eulerian) position \( x = q + S(\tau, q) \) subjected to a force per unit mass proportional to \( \partial^i \Phi \). After taking the divergence of the resulting equation of motion, one can use the Poisson Eq. (23) to obtain an equation for the deformation tensor \( S_{ij}(\tau, q) \equiv \partial S^i/\partial q^j \) and then reconstruct the density field \( \rho(\tau, x) = \rho(q)J^{-1} \), where \( J = \det(\delta_{ij} + S_{ij}) \) is the determinant of the Jacobian \( \partial x^i/\partial q^j \).

On the attractor (20), for \( \tau = a^n \) and \( n = -\frac{1}{2}(1 + 9w) \), the DM equation of motion is simplified to

\[
\frac{d^2 S}{d\tau^2} = -\left( \frac{1 + 2\lambda^2}{n^2 \tau^2 H^2} \right) \nabla_x \Phi \equiv -\nabla_x \varphi
\]

and the Poisson equation (23) becomes

\[
\nabla_x^2 \varphi = \left( \frac{3}{2} \Omega_m \frac{(1 + 2\lambda^2)}{n^2 \tau^2} \right) \delta_m(x(\tau, q)) \equiv b(\tau)\delta,
\]

where \( \delta = \delta_m(x(\tau, q), \tau) = [\rho_m(\tau, x) - \bar{\rho}_m(\tau)]/\bar{\rho}_m(\tau) \) is the density contrast.

Expanding the resulting equation to first order in the displacement vector \( S \) and making the ansatz \( S(\tau, q) = f(\tau)\tilde{S}(q) \) we obtain

\[
\frac{d^2}{d\tau^2} f(\tau) - b(\tau)f(\tau) = 0,
\]

where \( f(\tau) \) is the solution of the gravitational equation for the scalar field. The ansatz is justified by the fact that the scalar field equation is linearized in the vicinity of the attractor solution. The ansatz is typically justified in the context of scalar field cosmology, where the scalar field potential is assumed to be slowly varying on cosmological scales.
whose solutions read
\[ f(\tau) = \tau^p, \quad p = \frac{1}{2} \pm \frac{1}{2} \sqrt{1 + \frac{24\Omega_m(1 + 2\lambda^2)}{(1 - 9w)^2}}. \] (28)

We also get \( \nabla_q \cdot \tilde{S}(q) = -\delta_0 \), where \( \delta_0 \) is the VAMPS density fluctuation linearly extrapolated to the present time.

The matter density field follows:
\[ \rho_m(z, x) = \bar{\rho}_m (1 + z)^{3(1 + w)} \prod_{i=1}^{3} [1 + \lambda_i(q)(1 + z)^{-m_+}], \] (29)
where \( \lambda_i(q) \) are the eigenvalues of \( \partial \tilde{S}_j / \partial q^i \) and
\[ m_\pm = -\frac{1}{4}(1 - 9w) \left( \mp \sqrt{1 + \frac{24\Omega_m(1 + 2\lambda^2)}{(1 - 9w)^2}} \right) \] (30)

(in agreement with [19]): the effect of the scalar field and a non-vanishing coupling constant \( \lambda \) is to anticipate (if \( m_+ > 1 \)) or delay (if \( m_+ < 1 \)) the formation of pancakes with respect to scenarios with only DM with constant mass \( (m_+ = 1) \), depending on the value of \( \lambda \) and \( \beta \). Because the DM-DE coupling in our specific model affects the motion of DM particles only through a \( 2\lambda^2 \) correction of the gravitational force, the eigenvectors of the deformation tensor \( \partial \tilde{S}_j / \partial q^i \) turn out to be exactly aligned with those of the gravitational force itself. As a consequence, no other scalar field effects appear on small scales.

4. Conclusions

In this paper we have set the stage for future studies of non-linear gravitational clustering in scalar field cosmologies, extending the Newtonian approximation to the case of a coupled DM-DE system. The non-linearity in the scalar field may emerge either as a result of its self interactions or by its coupling to DM. It obeys a Poisson-like equation, Eq. (17), quite similar to that for the gravitational potential \( \Phi \).

In order to get some analytic insight, we have specialized to the VAMP scenario discussed in [9], in which the DM particle’s mass depends exponentially on the DE field. In this particular model, we found that the scalar field perturbations remain linear inside the horizon, although the coupling plays a non-trivial dynamical role.

The mildly non-linear regime has been tested here, using first-order Lagrangian perturbation theory. The DM-DE coupling weakly affects the gravitational field, acting on the onset of pancake formation without any further consequence on the spatial distribution of the DM density field on the scales of interest.

The constant coupling \( \lambda \) considered here is, however, disfavored by comparison with the CMB anisotropy spectrum. A time- or field-dependent \( \lambda \) – smaller in the past and approximately constant in the recent epoch – would take care of this problem, as discussed in [19]. In this case, our results would not change qualitatively. Namely, the scalar field fluctuations would still remain well inside the linear regime and the onset of
structure formation would be changed with respect to ordinary quintessence, though of a lesser extent than for a constant $\lambda$.

Scalar field non-linearities are, however, not precluded in the VAMP scenario. For instance, modifying the field dependence of the DM mass in Eq. (19), the full potential $U(\phi)$ may temporarily develop a spinodal instability (negative second derivative). During this time, perturbations at scales $k/a \leq \sqrt{-U''}$ grow exponentially and easily exit the linear regime. When the spinodal epoch ends, the system can be described by our equations (16), (17), (18), assigning the developed non-linearities to the initial conditions for the non-relativistic field fluctuation $\phi_{NR}$.

The approach developed in this paper can be straightforwardly applied to the numerical study of models where the quintessence field fluctuates non-linearly, as those proposed in refs. [12, 13, 14, 15, 16, 17].

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