Fractal characteristics of 3D surface topography in laser machining

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Abstract. The measurement of the profile data from carbon steel surface machined by laser machining with the white light interferometer is discussed in this paper, the research of the surface topography characteristics of laser machining is based on fractal geometry analysis, and the Box-counting method is used for 3D fractal dimension calculation of the laser machining surface. The results indicate that a universal fractal dimension beyond scale range is unattainable, and only when the scale r is less than the maximum height of profile $R_y$, the performance of the fractal characteristics on the laser machining surface is present. When the same laser machining surface is characterized, the $R_a$ of same scale fluctuates greatly along the thickness direction, and the fractal dimension $D_b$ can be used to stably characterize the complexity of the whole surface topography. The 3D surface fractal dimension $D_b$ is sensitive to the machining surface defects, once the $D_b$ becomes smaller, deep cracks and pronounced defects are exhibited in machining surface. In this paper, the characteristic analysis of the laser machining 3D surface is obtained by characterizing the micro-structured surface complexity using the fractal dimension, which is aided by the roughness to evaluate the profile height characteristics of laser machining surface. This study indicates that the Box-counting fractal analysis is an effective method to evaluate laser machining surface comprehensively.

1. Introduction

Laser machining is a non-contact, high speed, high precision machining method which uses high density energy to carry out by gathering the energy into the tiny space [1-3]. This is a novel method to get rid of the traditional mechanical machining, heat treatment and so on, which has been widely used in industrial, medical, communications, equipment and other industries [4, 5]. Surface topography not only has a direct impact on the wear and sealing properties of the parts, but also indirectly affects the performance and service life of the equipment [6]. When the thickness of the laser machining workpiece is more than 2 mm, the roughness of the machining surface varies greatly along the thickness direction. The micro-topography can be divided into two parts: the upper machining surface is smooth and the roughness is small, the lower machining surface fringe is relatively disordered, and the surface roughness is larger [7]. The traditional evaluation methods are limited by the measurement scale and the sampling length, and it is difficult to comprehensively evaluate and characterize topography characteristics of 3D surface machined by laser [8].

In recent years, fractal theory has been widely applied in the study of rough surfaces. The surface
roughness is reflected by fractal dimension of surface profile, and to a certain extent, which overcomes the deficiency of scale correlation of traditional roughness parameters [9, 10]. At present, there are many studies on the fractal characteristics of surface topography of mechanical machining [11-17], however, the research on the fractal characteristics of laser machining surface is almost blank. In this paper, fractal theory is used to study the fractal characteristics of 3D surface topography of carbon steel machined by laser, and to explore the effective method to characterize the laser machining surface topography.

2. Three-dimensional fractal dimension calculation method

Characterization of 3D surface topography plays an important role in the field of engineering technology. Fractal dimension is an important method for analysis and characterization of 3D surface topography. With the development of 3D topography measurement instruments, the calculation method of 3D fractal dimension has made great progress [18]. At present, there are many methods to calculate the fractal dimension such as Hausdorff Method, Correlation Method, Similarity Method, Box-counting Method, Structure Function Method, et al [19]. The Box-counting method has the advantage of clear physical meaning, small data calculation, high calculation precision and wide application range, so it is used to calculate the fractal dimension of laser machining surface.

There exists a non-empty bounded subset Z in space \( \mathbb{R}^3 \), \( N_r(Z) \) is used to represent the minimum value of the 3D cube that is needed to cover the set \( Z \), and \( r \) is side length of 3D cube. If \( \exists D > 0 \), when \( r \to 0 \)

\[
N_r(Z) \propto 1/r^D
\]

Then, \( D \) is called Box-counting dimension of set \( Z \). Generally, \( 3 > D > 2 \), when and only when \( \exists \forall e > 0 \), then lead:

\[
\lim_{r \to 0} \frac{N_r(Z)}{1/r^D} = e, \lim_{r \to 0} (\log N_r(Z) + D \log r) = \log e, D = \lim_{r \to 0} \frac{\log e - \log N_r(z)}{\log r} = \lim_{r \to 0} \frac{\log N_r(z)}{\log(1/r)}
\]

(2)

Where, because when \( r \to 0 \), The denominator \( \log(1/r) \) tends to infinity ignoring \( \log e \), usually, \( D_b \) is denoted as Box-counting dimension.

In the process of Box-counting dimension calculation, rough surface is covered with different cubes with \( r_k \) length. Given a square grid on the XOY plane, the unit length of grid is \( r \), which is shown in Figure 1. In the grid \((a, b)\), four vertex height coordinates of the rough surface are \( z(a, b) \), \( z(a, b + 1) \), \( z(a + 1, b) \) and \( z(a + 1, b + 1) \). Where \( 1 \leq a \leq n - 1, 1 \leq b \leq n - 1 \), \( N \) is the number of measuring points in \( X, Y \) direction. Let \( M \) be an array with 4 rows and 1 columns, the required number of \( N_{ij} \) covering the rough surface can be written as

\[
M = [z(a, b), z(a, b + 1), z(a + 1, b), z(a + 1, b + 1)]
\]

(3)

\[
N_{ij} = \text{ceil} \left( \frac{1}{r^D} (\text{max} M - \text{min} M) \right)
\]

Where \( \text{ceil}(\cdot) \) is the integral function tendency to be \( +\infty \). So, the number of cubes covering the entire area \( N_r(Z) \) can be derived as

\[
N_r(Z) = \sum_{i=1}^{n} N_{ij}
\]

(4)

When the number of boxes at a certain scale \( r_k \) is calculated, in the coordinate system with \( -\log(r) \) as the \( X \) axis coordinate and \( -\log N_r(Z) \) as the \( Y \) axis coordinate, the points \( (-\log(r_k), \log N_{rk}(Z)) \) of all scales are described, then, the slope of fitting line is estimated by the least square linear regression method, and slope is the Box-counting dimension [20,21].

Figure 1. Fractal Box-counting method.
3 Experimental design

3.1 Experimental equipment
CO₂ laser of ELG-2500W axial flow is used as experimental machining equipment. The ordinary carbon structural steel Q235 is chosen as experimental workpiece material for its good comprehensive performance of strength, plasticity and welding and better matching. SIS2000 white light interferometer is used to measure the surface roughness, for this interferometer, its surface height measurement range is from 1 to 200000nm, vertical resolution is up to 0.1 nm, and the maximum resolution is 640*480.

3.2 Experimental process and data acquisition
The machining auxiliary gas is O₂, the maximum laser power is 3000W, the maximum wire machining speed is 30m/min, and the mechanical positioning precision is ±0.03mm, and the mechanical repeat positioning accuracy is ±0.01mm, the nozzle diameter is 1.2mm, the distance between the nozzle and the surface is 1mm, laser operation mode is normal. Laser machining process parameters such as laser power, machining speed, auxiliary gas pressure and thickness of carbon steel plate, are shown in Table 1.

Table 1. Process parameters of laser machining.

| Workpiece | Laser power (P/W) | Machining speed (v/m·min⁻¹) | Auxiliary gas pressure (p/MPa) | Thickness of carbon steel (d/mm) |
|-----------|------------------|-----------------------------|-------------------------------|---------------------------------|
| I         | 1100             | 1.6                         | 0.30                          | 3                               |
| II        | 1100             | 1.0                         | 0.42                          | 4                               |
| III       | 2400             | 1.3                         | 0.48                          | 5                               |
| IV        | 2400             | 1.6                         | 0.42                          | 7                               |

Data is collected using white light interferometer after machining the surface, which is with a measurement range of 0.55mm*0.55 mm, a total pixels of 512*512. The profile height data of the laser machining surface is introduced into the system, and the 3D profile of the laser machining surface is generated, which is shown in Figure 2.

4. Experimental results and 3D fractal characteristics analysis
By using the Box-counting method, the fractal dimension of process parameters in 4 different groups are the calculation of surface profile data generated by laser machining. The Box-counting dimension of the workpiece IV(represented with the workpiece IV, the Box-counting dimension of workpiece numberI~III is omitted) is shown in Figure 3.
Figure 3. Fractal Box-counting calculation of workpiece IV

In the Box-counting dimension calculation of workpiece IV, due to the determination coefficient $R^2$ satisfies $0 \leq R^2 \leq 1$ and $R^2 = 0.9961$, when the value $R$ is close to 1, the effect of linear fitting between $\log N_r(r)$ and $\log(1/r)$ is better. Therefore, the laser machining surface has good fractal characteristics, which can be used to characterize the surface topography of the laser machining surface by using the Box-counting dimension. The maximum profile height of workpiece IV is $R_y = 20.54\mu m$, when the measuring scale $r \geq R_y$, the ratio value between $\log N_r(r)$ and $\log(1/r)$ is 2.34, and fractal dimension $D_b$ tends to 2.34, laser machining surface does not have fractal characteristics. Only when the measurement scale $r < R_y$, laser machining surface fractal dimension satisfies $2.34 < D_b < 3$, and laser machining surface has certain fractal characteristics. Therefore, there is not the pervasive fractal dimension beyond a scale range, in theory, only when the scale $r \to 0$, the calculated fractal dimension is infinitely close to the real value. In the process of the actual calculation, the big resolution measurement instruments are chosen as far as possible to ensure that the Box-counting dimension calculation is more accurate.

Due to the surface roughness of the same incision on carbon steel surface machined by laser machining is not the same, the surface roughness $R_a$, which is along the thickness direction and near the top of the laser beam, is minimum, the farther from the bottom, the larger the roughness $R_a$.

The fractal dimension $D_b$ and surface roughness $R_a$ of three parts (upper, middle and lower) on the surface of the workpiece II are calculated respectively in Table 2.

Table 2. Fractal dimension and surface roughness of machining surface on workpiece II.

| Incision | Fractal dimension $D_b$ | Surface roughness $R_a$ |
|----------|------------------------|------------------------|
| Upper    | 2.370                  | 3.951                  |
| Middle   | 2.354                  | 5.242                  |
| Lower    | 2.376                  | 6.128                  |

The surface fractal dimension $D_b$ and the profile arithmetic-average-value $R_a$ of I ~ IV set of laser machining experiments are shown in Table 3.

Table 3. Fractal dimension and surface roughness of laser machining surface.

| Workpiece | Fractal dimension $D_b$ | Surface roughness $R_a$ |
|-----------|------------------------|------------------------|
| I         | 2.341                  | 7.656                  |
| II        | 2.376                  | 6.128                  |
| III       | 2.363                  | 6.986                  |
| IV        | 2.348                  | 6.990                  |
Table 2 shows that in the same laser machining surface and the same scale measuring, $R_a$ of the lower part is 55.1% larger than that of the upper. The $R_a$ changes more significantly along the thickness direction, the reason is that the traditional surface roughness parameter $R_a$ is affected by the factors such as sampling length, only the profile characteristics of a section on laser machining surface are analysed and it is difficult to describe accurately topography characteristics of the whole laser machining surface. While the difference ratio of fractal dimension between the lower and the upper is less than 1%, which fluctuates very little. The fractal dimension $D_b$ can be more stable in the measurement of the complex degree of the overall surface topography. Therefore, compared with the traditional evaluation method of surface topography, the fractal dimension $D_b$ is more comprehensive and stable to characterize the three-dimensional topography of laser machining surface.

Calculation results of $D_b$ and $R_a$ in Table 2 and Table 3 can be found that regardless of the same laser machining surface, or between the surface of a variety of laser machining, there is no obvious linear correspondence between Box-counting dimension $D_b$ and surface roughness $R_a$. Although fractal dimension can comprehensively reflect the micro-structures features of 3D surface topography, comprehensive embodiment of machining surface profile information is lack. Therefore, only one parameter of fractal dimension cannot be used to realize the unique characterization of surface topography.

Figure 4 shows the microscopic topography surfaces in different process parameters of laser machining, in which different material removal behaviors occurred. As shown in Figure 4(a), Figure 4(b), it is observed that the work surface exists a crack and macro-brittle fracture free topography, which shows that the workpiece material is predominantly removed in ductile mode owing to insufficient compressive stress and shear stress to induce cracks or crack propagation [22-24].
Figure 4. The topography of laser machining surfaces with different process parameters in Table 1. (a) \( R_a = 6.128 \mu m \), workpiece II; (b) \( R_a = 6.986 \mu m \), workpiece III; (c) \( R_a = 6.990 \mu m \), workpiece IV.

Qualitatively, as the \( D_b \) is larger, the micro topography of laser machining surface is more exquisite and the texture is fine and dense, which means that the better surface quality is generated. For the laser machining surface with a smaller \( D_b \), a deeper crack and more pronounced defects occur, which means that a coarse and sparse texture generated. Therefore, the surface fractal dimension \( D_b \) is sensitive to the variation of the surface defects. Furthermore, as shown in Figure 4(b) and Figure 4(c), for two surfaces with same or similar roughness \( R_a \), the surface with a larger \( D_b \) value has better surface quality. Thus, it is concluded that the \( D_b \) may serve as a good quantitative evaluation of laser machining surface, and imply on the material removal mechanism of the machining materials.

In addition, the 3D surface fractal dimension \( D_b \) is relatively stable compared with the surface roughness \( R_a \). The laser machining surface is formatted by a large number of micro peaks and troughs, resulting from laser beam acting directly in the upper part and molten metal scouring repeatedly in the lower part. Macroscopically, the generation process of laser machining surface has a dynamic interaction property. On a Microscopic level, it shows a dynamic interaction process that the fractal structures of machining surface interact. As laser machining process goes on, the fractal characteristics of laser machining surface become more obvious. The surface is occupied by more and more extremely microstructure which causes the large fractal dimension. And the fractal dimension presents a positive correlation with the space expansion of microstructure. However, once the surface defects are existed in machining surface, the fractal dimension decreases dramatically, the reason can be explained that its original microstructures are broken and a series of the big peak-valley structures (pits and cracks) are formed. The formation process of laser machining surface is irregular, but the fractal dimension would finally attain a stable value when the generation rate of microstructure keeps a balance.

5. Conclusion
Carbon steel surface of laser machining is fractal surface, which is important to characterize the surface of laser machining with fractal parameters. The laser machining surface shows the fractal characteristics only when the scale \( r \in (0, R_y) \), when the same surface are characterized, the same scale of \( R_a \) change along the thickness direction, and the fractal dimension \( D_b \) can be used to characterize the 3D profile of the laser machining surface. The fractal dimension can be used to characterize the surface microstructure complexity of the workpiece in the analysis of 3D surface topography characteristics of laser machining surface, and the height characteristics of surface profile can be aided to evaluate with surface roughness. The experiment results show that the laser machining surfaces exhibit good fractal behavior. From result analysis by using the 3D Box-counting method, the fractal dimension is sensitive to the surface defects.

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