Scalable quantum computation architecture using always-on Ising interactions via quantum feedforward

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Here, we propose a way to control the interaction between qubits with always-on Ising interaction. Unlike the standard method to change the interaction strength with unitary operations, we fully make use of non-unitary properties of projective measurements so that we can effectively turn on/off the interaction via feedforward. We then show how to generate a two or three-dimensional cluster state that are universal resource for fault tolerant quantum computation with this scheme. Our scheme provides an alternative way to realize a scalable quantum processor.

I. INTRODUCTION

Quantum computation is a new paradigm of information processing. Known algorithms give superior performances for tasks such as factoring [1,2], searching an unsorted database [3,4], quantum simulation [5,6], other algorithms [7,12] and more. All these algorithms require a large scale quantum computer. A quantum computer is composed of a sequence of implementation of single-qubit gates and two-qubit gates [13,16]. The single-qubit gate denotes a rotation of the qubit around arbitrary axis and degree. A control-phase gate is one of the typical examples of two-qubit gates. This gate flips the phase of the target-qubit if and only if the state of the control-qubit is |1⟩. The role of control and target qubit are reversible for control-phase gate. Individual qubits should be efficiently addressed and the interaction between two-qubits should be controlled by some external apparatus.

The challenge is how to design and build a quantum computer with a realistic technology. This requires quantum architecture. There have been a number of these for relevant physical systems, such as nitrogen-vacancy centre [17,18], ion traps [19], superconducting systems [20]. Many of these have assumed isolating system and excellent controllability. However, in realistic circumstances, turning on/off the interaction in a reliable way is one of the hardest parts in such architectures. For example, two-qubit gates require in-situ turn on/off the interaction between qubits by the external control apparatus. Since imperfection of the interaction control tends to induce correlated errors between qubits, sophisticated technology is required to suppress such error rate below the threshold of fault tolerant quantum computation [21,22]. However, varying the interaction between qubits in-situ is not possible for all physical systems.

One of the ways to reduce the required level of technology is to use a system with always-on interaction. There are a couple of theoretical proposals for this direction. Zhou et al suggested a system with always-on Heisenberg interaction [24]. They use interaction free subspace to protect the target encoded qubit from the residual interaction, and they show that only local manipulations on the system actually provide universal quantum computation. Simon et al also suggested to use always-on Heisenberg interaction system for scalable quantum computation by collectively tuning the qubits [21,24]. These approach look attractive due to its simplicity that could reduce potential decoherence from the interaction.

Here, we propose a novel way to perform universal quantum computation with a system having an always-on Ising interaction. In quantum mechanics, there are two type of operations, unitary operations such as applying microwave pulses and non-unitary operations such as readout of the qubit. While most of the authors in previous papers use unitary operation to control the interaction [24,26], we exploit the non-unitary properties that the projective measurement have. We will assume an always-on Ising interaction between nearest neighbor qubits, and will insert an ancillary qubit between the qubits that process quantum information. We show that it is possible to effectively turn on/off the interaction via quantum measurement and feedforward on the ancillary qubits. Furthermore, we explain how scalability is achieved in this scheme, and suggest a way to construct a large two or three-dimensional cluster state which enables us to perform fault tolerant quantum computation with high error threshold [23]. Since quantum feedforward technology is becoming matured technology [27,30], our proposal provides a feasible and reliable way to control the interaction, which is a crucial step for the realization of quantum information processing.

The remainder of this paper is organized as follows. In Sec. III we review the preliminaries of this paper. Section IV presents the detail of our scheme to show how
always-interaction is effectively turned on/off via projective measurement to ancillary qubits and quantum feedforward. In Sec. IV and Sec. V we propose the way to generate two and three-dimensional cluster states using qubits arranged on a plane. Section VI concludes our discussion.

II. GRAPH STATES AS A RESOURCE FOR QUANTUM COMPUTATION

Let us review the concept of a graph state introduced in Ref. 40. A graph $G(V, E)$ is composed of vertices $V$ and edges $E$ where an edge connects two vertices. By regarding the vertex as a qubit and edge as an entanglement between the qubits, we can associate the graph with a graph state $|\Phi\rangle_{G(V, E)}$ defined as the following equation

$$|\Phi\rangle_{G(V, E)} = \prod_{(a, b) \in E} \hat{U}_{CZ}^{(a,b)} |+\rangle^\otimes N$$

where $(a, b) \in E$ denotes an edge between the vertices $a$ and $b$. Also, $\hat{U}_{CZ}^{(a,b)}$ denotes a controlled-phase ($\pi$) operation between them. It is worth mentioning that, a specific type of a graph state such as a two or three-dimensional cluster state can be a universal resource for measurement-based quantum computation (MBQC) [41–44] and topological quantum computation [21–23]. The controlled-phase gate can be realized by Ising type interaction [41, 42]. When a graph $G(V, E)$ is given, the necessary Hamiltonian to create a cluster state corresponding $G$ is as follows

$$\hat{H}_{G(V, E)} = \sum_{(l, l') \in E} g_{(l, l')} \left( \frac{1 + \hat{Z}_l}{2} + \frac{1 + \hat{Z}_{l'}}{2} \right)$$

where $g_{(l, l')}$ denotes the interaction strength between qubit $l$ and $l'$. By letting a separable state $\otimes_{l \in V} |+\rangle_l$ evolve for $g_{(l, l')} t = \pi$ according to this Hamiltonian, the following unitary operator will be applied to the initial state

$$S_{G(V, E)} = \exp \left( -i \pi \sum_{(l, l') \in E} \frac{1 + \hat{Z}_l + \hat{Z}_{l'}}{2} \right)$$

$$= \prod_{(l, l') \in E} \hat{U}_{CZ}^{(l, l')}.$$ (3)

and hence we can create the target graph state.

Although there are many proposal to realize Ising type interaction such as ultracold atoms in an optical lattice [45, 52], ion traps [53–58], superconducting charge qubit [59], superconducting spin qubit [60], superconducting flux qubit [61], resonator waveguide [62], nitrogen-vacancy center [17, 63–68], quantum dot [69–72] and electronic spins coupled to the motion of magnetized mechanical resonators [73], the major challenge for experimental realization is to switch on/off the interaction with a high fidelity. Only a few experiments have demonstrated a high fidelity controllable two-qubit gate with a fidelity above the threshold of fault tolerant quantum computation [74–76]. One of the possible ways to overcome the experimental difficulties for demonstrating the high-fidelity two-qubit gates is to use an always-on interaction scheme [24–26, 77–79]. Since there are no need for the additional controlling operations to switch the interaction, these scheme may scale well for a large number of qubits. Here, we propose a new approach to implement fault tolerant quantum computation with always-on interaction by using the non-unitary properties of projective operations and quantum feedforward.

III. EFFECTIVE INTERACTION CONTROL VIA PROJECTIVE MEASUREMENTS AND QUANTUM FEEDFORWARD

A. Interaction switching with quantum feedforward

We introduce the Hamiltonian to realize our scheme to turn on/off the interaction effectively via projective measurements and quantum feedforward. The physical device that we consider is a general solid-state system where every qubit can be individually controlled by a microwave pulse and there are always-on interaction between nearest neighbor qubits. Throughout of this paper, we assume the following Hamiltonian corresponding to $G(V, E)$

$$\hat{H} = \hat{H}_{G(V, E)} + \hat{H}_{interaction}$$

$$\hat{H}_{G(V, E)} = \sum_{l \in V} g_{l} \hat{Z}_l$$

$$\hat{H}_{interaction} = \sum_{(l, l') \in E} \frac{g_{(l, l')}}{4} \hat{Z}_l \hat{Z}_{l'}$$

where $\omega$, $\lambda(t)$, $\omega'$, and $\theta$ denote the qubit energy, Rabi frequency, microwave frequency, and a phase of the microwave. In most of the solid-state systems, it is possible to control the value of $\lambda(t)$ by changing the power of microwave with much higher accuracy than the case of two-qubit gates. We move to a rotating frame defined by

$$\hat{U} = \exp \left( -i \sum_{l} \frac{\omega_l}{2} \hat{Z}_l t \right)$$

where $\omega_l$ denotes its angular frequency of the rotating frame at the site $l$, and use a rotating wave approximation so that we could obtain the following Hamiltonian

$$\hat{H} \simeq \sum_{l \in V} \left( \frac{\omega_l - \omega'}{2} \hat{Z}_l + \frac{\lambda(t)}{2} \hat{A}_l \right) + \sum_{(l, l') \in E} \frac{g_{(l, l')}}{4} \hat{Z}_l \hat{Z}_{l'}$$

where

$$\hat{A}^\theta = \begin{pmatrix} 0 & e^{i\theta} \\ e^{-i\theta} & 0 \end{pmatrix}.$$ (9)
Unless when required to perform single qubit gates, we turn off the microwave and set $\lambda = 0$, and therefore the Hamiltonian introduced here is effectively the same as an Ising model with always-on interaction. On the other hand, for the implementation of accurate single-qubit rotations, we assume a large Rabi frequency as $\lambda \gg g$ so that the coupling strength from the nearest neighbor qubit can be negligible. We will later discuss an error accumulation due to imperfect single qubit rotation in detail.

The Hamiltonian described above has an interesting property that an interaction from other qubits can be turned off by preparing the state of a qubit in a ground state. To explain this, we consider a specific qubit $A$ and other qubits interacting with the qubit $A$, and the Hamiltonian of those is described as follows.

$$
\hat{H}_A' = \sum_{(A,j) \in E} \frac{\omega_j - \omega'_j}{2} \hat{Z}_j + \frac{g(A,j)}{4} \hat{Z}_A \hat{Z}_j + \frac{\omega_A - \omega'_A}{2} \hat{Z}_A \tag{10}
$$

$$
= \sum_{(A,j) \in E} \left( g(A,j) + \frac{1}{2} \hat{Z}_A + \frac{1}{2} \hat{Z}_j + \frac{\omega_j - \omega'_j - g(A,j)}{2} \hat{Z}_j \right) \tag{11}
$$

with

$$
\omega_A - \omega'_A = \sum_{(A,j) \in E} \frac{1}{2} g(A,j). \tag{12}
$$

Interestingly, if the qubit $A$ is prepared in a ground state, the interaction from the qubit $A$ cancels out because of

$$
g(A,j) + \frac{1}{2} \hat{Z}_A + \frac{1}{2} \hat{Z}_j |0\rangle_A = 0. \tag{13}
$$

This means that preparing a specific qubit in a ground state effectively turn off the interaction between this qubit and nearest-neighbor qubit. Therefore, if all nearest-neighbor qubits are ground state, the qubit is not affected by any interactions, which is the striking feature of our scheme.

Although we will later discuss the case of two or three-dimensional cluster state that is a universal resource for quantum computation [23], we start by explaining how to generate a one-dimensional three-qubit cluster state (Fig. 1) to illustrate our concept about how to control quantum computation [23]. We start by explaining how to perform a projective measurement onto the middle qubit, and rotate the middle qubit back into a ground state so that a C-Phase can be implemented between the qubit $A$ and $C$. Due to the engineered Hamiltonian form that we make, this guarantees that the qubit $A$ and $C$ does not evolve anymore even under the effect of the always-on Ising type Hamiltonian.

For a time $t = \frac{\pi}{g}$, and perform $\hat{Y}$ measurement on the middle qubit $B$. The state after the measurement is written as

$$
|\rho'\rangle = \hat{P}_B^1 e^{-i\hat{H}t} \rho e^{i\hat{H}t} \hat{P}_B^\perp \tag{17}
$$

where $\pm$ denotes the measurement result. Here,

$$
\hat{P}_B^{\pm} = \frac{1}{2} \left( 1 \pm \hat{Y} \right) \tag{18}
$$

denotes a projection operator on the qubit $B$. Finally, we perform a quantum feedforward operation, that is an implementation of different local operations depending on the measurement results, onto the qubit $B$ so that the qubit $B$ can be prepared in a ground state. We define a feedforward operator as

$$
\hat{F}_{ABC}^\pm = \hat{S}_A^\pm \hat{U}_B^{\pm} \hat{S}_C' \tag{19}
$$

where $\hat{S}_\pm$ denotes a shift gate defined as

$$
\begin{pmatrix}
1 & 0 \\
0 & \pm i
\end{pmatrix} \tag{20}
$$

and $\hat{U}^{\theta, \hat{X}}$ denotes a single-qubit rotating around $x$-axis rotation with an angle of $\theta$. The state after the quantum feedforward is described as

$$
\rho_{\text{final}} = \hat{P}_{ABC}^+ \rho \hat{P}_{ABC}^+ \hat{F}_{ABC}^+ + \hat{F}_{ABC}^- \hat{P}_{AC}^- \rho \hat{F}_{ABC}^- \tag{21}
$$

$$
= \hat{U}^{(A,C)}_C |\phi_A \rangle |\phi_C \rangle \mathcal{O}_{AC} \otimes |0\rangle_B. \tag{22}
$$

Therefore, after these operations, controlled-phase operations are performed between the qubit $A$ and $C$, and the state does not evolve anymore because the qubit $B$ is prepared in a ground state.
FIG. 2. Controlled-phase operations using a spin echo technique with asymmetric coupling strength. $\hat{U} = \hat{U}_{CZ}^{(A,B)}\hat{U}_{CZ}^{(B,C)}$ is performed on the initial state at a specific timing due to the implementation of a $\pi$ pulse where $\hat{U}_{CZ}^{(j,k)}$ denotes a controlled-phase operation between qubit $j$ and $k$. Meanwhile, if we set the qubit B in an excited state by applying a spin echo technique [80–82] to balance the interaction. In the spin echo technique, we assume $\hat{U}_{CZ}^{(A,B)}\hat{U}_{CZ}^{(B,C)}$ is performed on the initial state at a specific timing due to the implementation of a $\pi$ pulse where $\hat{U}_{CZ}^{(j,k)}$ denotes a controlled-phase operation between qubit $j$ and $k$. We let the state evolve for a time $t_1$, perform $\pi$ pulse to qubit $A$, and let the state evolve for a time $t_2$. The total unitary evolution can be described by

$$\hat{U} = \exp\left(-ig_1(t_1 - t_2)\frac{1 + \hat{Z}_A}{2}\frac{1 + \hat{Z}_B}{2}\right).$$

and so,

$$\hat{U}|\phi\rangle_{ABC} = \hat{U}_{CZ}^{(B,C)}\hat{U}_{CZ}^{(A,B)}|\phi\rangle_{ABC}$$

so that we can perform controlled-phase gates even if the coupling strength is asymmetric. After this evolution, we use projective measurements and quantum feedback to effectively turn off the interaction as long as qubit $B$ is prepared in a ground state. Therefore, we succeed in performing controlled-phase operation between qubit $A$ and $C$.

C. One dimensional cluster state

We only explain about three-qubit case here. However, it is straightforward to generalize this idea to an arbitrary size of one-dimensional cluster state, because we can ignore the coupling from the other qubits as long as we insert an ancillary qubit prepared in a ground state as we discussed before. It is worth mentioning that the necessary number of the $\pi$ pulses increases linearly against the number of the qubits, due to the use of such ancillary qubits to stop the interaction from the other qubits. Additionally, our scheme can be applied to two or three-dimensional cluster state. In these cases, we repeatedly implement the similar procedure as we use in the case of a one-dimensional cluster state so that we can balance the interactions just by adding a few operations. We discuss the details of those interaction-balancing schemes in Appendix A. Hence, throughout of this paper, we assume that all interactions are equal.

D. Unavoidable error of feedforward operation

It is worth mentioning that we could not avoid a detuning error to perform a single qubit rotation in our always-on interaction system. In a solid-state system, it is typically possible to perform a high-fidelity single qubit rotation by applying a on-resonant microwave pulse whose frequency is the same as the qubit energy. However, in our case, the target qubit has an unknown energy shift due to the interaction when a state of nearest neighbor qubits contains a superposition. As an example, we again consider a case of three-qubit one-dimensional chain, and estimate the fidelity to perform a $\frac{\pi}{2}$ pulse on the middle qubit prepared in a ground state, and the Hamiltonian

$$\hat{H} = \frac{1}{2g}g_1\hat{Z}_A + \frac{1}{2g}g_2\hat{Z}_B + \frac{1}{2g}g_3\hat{Z}_C$$

to satisfy

$$g_1(t_1 - t_2) = g_2(t_1 + t_2) = \pi.$$
TABLE I. The effective Hamiltonian of qubit B depends on the states of qubit A and C in Fig. 1. When the states of the qubit A and C contains superposition, the resonant frequency of the qubit B is not uniquely determined. Since the microwave frequency is fixed, $Z_B$ component induce the detuning error $\epsilon_{\pi/2}$ when we rotate qubit B. As the table shows, the detuning error $\epsilon_{\pi/2}$ becomes maximum for the case of $|\uparrow\uparrow\rangle_{AC}$.

| Qubit A and C (state) | Qubit B (effective Hamiltonian) |
|-----------------------|----------------------------------|
| $|\uparrow\uparrow\rangle_{AC}$ | $H_B = gZ_B + \frac{\lambda_B}{2}Y_B$ |
| $|\uparrow\downarrow\rangle_{AC}$, $|\downarrow\uparrow\rangle_{AC}$ | $H_B = g'Z_B + \frac{\lambda_B}{2}Y_B$ |
| $|\downarrow\downarrow\rangle_{AC}$ | $H_B = \frac{\lambda_B}{2}Y_B$ |

of this system is described as follows.

$$\hat{H}_{ABC} \simeq \sum_{l \in A,B,C} \frac{\omega_l - \omega'_l}{2} \hat{Z}_l + \frac{\lambda_B(t)}{2} \hat{Y}_B$$

$$+ \sum_{(l,l') \in (A,B),(B,C)} \frac{g_{l,l'}}{4} \hat{Z}_l \hat{Z}_{l'}.$$  \hspace{1cm} (29)

Here, we set mixing angle

$$\theta = \frac{\pi}{2}$$  \hspace{1cm} (30)

and microwave frequency

$$\omega'_B = \omega_B - g$$  \hspace{1cm} (31)

to obtain the following Hamiltonian.

$$\hat{H}'_{ABC} = \sum_{l \in A,C} \frac{1}{2} \left( \omega_l - \omega'_l - \frac{g}{2} \right) \hat{Z}_l + \frac{\lambda_B}{2} \hat{Y}_B$$

$$+ \sum_{(l,l') \in (A,B),(B,C)} \frac{1 + \hat{Z}_l + 1 + \hat{Z}_{l'}}{2} \hat{Y}_l.$$  \hspace{1cm} (32)

We show the effective Hamiltonian of the qubit B of this case in Table I. Since the resonant frequency of the qubit B depends on the state of the qubit A and C, it becomes impossible to apply on-resonant pulse on the qubit B if one of these states have a superposition. To implement our scheme to control the interaction, we have already chosen the frequency of $(\omega_B - g)$ for the microwave $\frac{\pi}{2}$ pulse and the worst fidelity (when the actual effective Hamiltonian of the qubit B is

$$H_B = g'Z_B + \frac{\lambda_B}{2}Y_B$$  \hspace{1cm} (33)

can be calculated as follows.

$$\epsilon_{\pi/2} = 1 - \left| \langle \downarrow | e^{-iH_B t} | \uparrow \rangle \right|^2$$

$$= 1 - \left| \cos \left( \frac{1}{2} t \sqrt{4g^2 + \lambda_B^2} \right) + \sin \left( \frac{1}{2} t \sqrt{4g^2 + \lambda_B^2} \right) \frac{2g + \lambda_B}{\sqrt{4g^2 + \lambda_B^2}} \right|^2$$

$$= 1 - \frac{1}{4}$$

where

$$t = \frac{\pi}{2\lambda}$$  \hspace{1cm} (34)

denotes the duration of the microwave $\frac{\pi}{2}$ pulse. This means that, by increasing the Rabi frequency $\lambda_B$, we can suppress this detuning error. We plot this error $\epsilon_{\pi/2}$ against the coupling strength $g$ and the Rabi frequency $\lambda_B$ in Fig. 3.

FIG. 3. The worst rotating error ($\epsilon_{\pi/2}$) and the interaction strength $(gT_2)$ between each pair of nearest neighbor qubits in switching scheme (Fig. 1) against various Rabi frequency ($\lambda_B T_2$). Here, $T_2$ denotes the coherence time of the qubit.

Throughout of this paper, when we calculate a fidelity, we always consider the worst case for detuning error as discussed above. The effective Hamiltonian of the target qubit to be rotated by the microwave is described by

$$H_{\text{target}} = \frac{\lambda}{2} \hat{X}_{\text{target}}$$  \hspace{1cm} (35)

when all nearest neighbor qubits are in a ground state while the worst case of the Hamiltonian is

$$H_{\text{target}} = \frac{g}{2} n \hat{Z}_{\text{target}} + \frac{\lambda}{2} \hat{X}_{\text{target}}$$  \hspace{1cm} (36)

when all nearest neighbor qubits is in an excited state, and we fix the frequency of

$$\omega_{\text{target}} = \frac{g}{2} n$$  \hspace{1cm} (37)

for the microwave $\frac{\pi}{2}$ pulse. Here, $\omega_{\text{target}}$ is the original resonant frequency of the target qubit and $n$ denotes the number of qubits interacting with the target qubit. This will enable us to evaluate the performance of our scheme for the fault tolerant quantum computation.

E. Optimal interaction strength

Here, we discuss the optimal interaction strength between the qubits to perform a high fidelity controlled-phase gate. Since the coherence of the quantum states degrades due to decoherence, we need to perform a controlled-phase operation much faster time scale than...
the coherence time of the qubit. For this purpose, we need to increase the coupling strength to realize a fast controlled-phase gate. However, the strong coupling strength makes it difficult to perform an accurate quantum feedback operations because the always-on coupling between qubits induces unknown energy detuning of the qubit frequency as described before. So there should exist an optimal interaction strength to minimize the controlled-phase gate error that comes from the decoherence of the qubits and imperfect quantum feedback operations. Decoherence error that we consider is general Markovian noise. We assume that the error rate increases exponentially against time as

$$\epsilon_d = \frac{1}{2}(1 - e^{-\frac{T_{CZ}}{T_2}}).$$

(38)

Here, $T_2$ denotes the coherence time of the qubit and

$$T_{CZ} = \frac{\pi}{g}$$

(39)

denotes the gate operation time. Since we consider a parameter regime for $T_{CZ} \ll T_2$, we can simplify the decoherence error as

$$\epsilon_d = \frac{T_{CZ}}{T_2}. \quad (40)$$

We assume that the single-qubit operations can be implemented much faster than the coherence time, and hence the decoherence effect during the single qubit operations is negligible compared with other effect such as decoherence during the controlled-phase gate. The setup we consider for the estimation of the optimal coupling strength is as follows. As described in Fig. 1 to perform a controlled-phase gate, we use two main qubits A and C and one ancillary qubit B that is inserted between the main qubits. Initially, an ancillary is prepared in a ground state and main qubit are prepared in arbitrary states. Also, for simplicity, we assume that all nearest-neighbor coupling strength between qubits are equal between these three qubits. We evaluate the achievable fidelity during the implementation of a controlled-phase gate in our scheme (Fig. 1). Firstly, by performing $\frac{\pi}{2}$ pulse, we rotate the ancillary qubit B from ground state into $|+\rangle$ state. At this time, the qubit B have unknown energy shift due to the coupling from qubit A and C during the rotation so that a detuning error occurs. Secondly, let evolve the system according to the Hamiltonian. During this time evolution, every qubit is affected by environmental noise, and so decoherence error accumulates. Finally, we measure $\hat{Y}$ and perform quantum feedback on the qubit B. Again, due to the coupling from nearest neighbor qubits, qubit A and C suffers the detuning error for the feedback rotations while the qubit B can be accurately rotated by a resonant microwave pulse. Therefore, the achievable fidelity is calculated as $F = 1 - (2\epsilon_d + 3\epsilon_d)$, where we assume that the error makes the state orthogonal to the ideal one to consider the worst case. We plot the achievable fidelity $F$ and interaction strength $g$ corresponding to the range of the Rabi frequency $\lambda$ in Fig. 4(a). Also, we plot the relationship between an achievable fidelity, the optimal interaction strength and Rabi frequency in Fig. 4(b). This shows that an achievable fidelity ($F$) monotonically increase with the increasing Rabi frequency ($\lambda T_2$) and interaction strength $g$ has the optimal point against $\lambda$.

In this paper, we do not discuss about the details of the errors in projective measurements and quantum feedback operations. But we can treat these errors as a type of additional dephasing error. For example, in our switching scheme of III A, we assume that we fail to perform measurement or feedforward operation with a probability of $\epsilon_m$. The ancillary qubit $B$ become

$$\rho_B = (1 - \epsilon_m)|\uparrow\rangle\langle\downarrow|_B + \epsilon_m|\uparrow\rangle\langle\uparrow|_B.$$  

(41)
At this time, the state of the total system is written as
\[ \rho_{ABC} = (1 - \epsilon_m) U_{CZ}^{(A,C)} |\phi\rangle \langle \phi| U_{CZ}^{(A,C)} \otimes |\downarrow\rangle \langle \downarrow| B \]
\[ + \epsilon_m e^{-i\hat{H}t} U_{CZ}^{(A,C)} |\phi\rangle \langle \phi| U_{CZ}^{(A,C)} \otimes |\uparrow\rangle \langle \uparrow| B e^{i\hat{H}t} \]
where
\[ \hat{H} = \sum_{(l,l') \in (A,B),(B,C)} \frac{1 + \hat{Z}_l + \hat{Z}_{l'}}{2} \]
denotes Ising type interactions between qubit A, B and B, C. By trace out of the ancillary qubit B, we can treat the effects of the interactions \( \hat{H} \) as dephasing errors at most \( \epsilon_m \sigma_Z \) on the qubit A and C. Such error can be corrected by the quantum error correction as long as the error is less than the threshold.

**IV. GENERATION OF A TWO DIMENSIONAL CLUSTER STATE UNDER THE EFFECT OF ALWAYS-ON INTERACTION FOR SURFACE CODING SCHEME**

In this section, we show how to apply our scheme to generate a two-dimensional cluster state, which is a universal resource \([21, 22]\) for quantum computation. We now give the overview of our setting in Fig. 5(a) and the physical implementation in Fig. 5(b). There are three types of main qubits. One of them is called a logical qubit that contains quantum information. The other qubits are called syndrome qubits. Half of syndrome qubits are to detect the dephasing errors on the logical qubits while the other ancillary qubits are to detect bit-flip noises. Main qubits are set on a grid point of square lattice and ancillary qubits for effectively turning on/off interactions are set on the midpoint of these main qubits. There are Ising type interactions between each pair of nearest neighbor main qubit and ancillary qubit. To generate a whole two-dimensional cluster state, we need to perform controlled-phase operations between every pair of the nearest logical and syndrome qubits as described in the following procedure. Firstly, we prepare all ancillary qubits between main and syndrome qubit to \( |+\rangle \) state. Secondly, we let the states evolve for a time \( t = \frac{\pi}{g} \). Finally, we perform projective measurements and quantum feedforward operations to all ancillary qubits for generating two-dimensional cluster state. In these operations, each ancillary qubit has no effect on the state of other ancillary qubits, so that we can handle the effect of each operation as individual three-qubits system and we can proceed all controlled-phase operations simultaneously. Furthermore, similar to the case of a one-dimensional cluster state, the energies of ancillary qubits have unknown energy shifts as described in Table I so that the upper bound fidelities and optimal interaction strengths of each controlled-phase operations coincide with three-qubit case shown in Fig. 4(a). Since scalable surface coding scheme require the error rate around below 1 \%, this result shows that the Rabi frequency should be tens of thousands times larger than the decay rate \( \sqrt{\frac{1}{T_1}} \) and the coupling strength should be thousands times larger than that.

**V. GENERATION OF A THREE DIMENSIONAL CLUSTER STATE UNDER THE EFFECT OF ALWAYS-ON INTERACTION FOR TOPOLOGICAL QUANTUM COMPUTATION**

Although we discussed how to generate a two-dimensional cluster state above, we can apply our scheme to generate a three-dimensional cluster state, which is a universal resource for the topological quantum computation \([23]\). Topological quantum computation is known to have a high threshold for quantum error correction especially when there is a finite probability to lose a qubit \([33, 34]\). The overview of this scheme is shown in Fig. 6 where three-dimensional cluster state is used as a resource for the computation. In the 3D cluster state, qubits connected in a \( z \)-axis direction are used for the logical qubit that contains the information for the computation (Fig. 2). The other qubits located between the logical qubits are used for detecting error syndrome. In order to process the computation with error corrections, we measure qubits by layer. Syndrome qubits are measured in \( X \)-basis, and the outcomes are used for detecting the location and type of the error so that we can correct
the error after analyzing the syndromes by classical computation.

We can use either a cubic cluster state or bilayer cluster state for the topological quantum computation, and we choose the latter one to avoid an unnecessary decoherence effect, as previous authors did [18]. In this case, it is necessary to generate the bilayer cluster state again and again, after the implementation of the measurements on one of the layers. We discuss how to generate a bilayer 3D cluster state under the effect of always-on Ising interaction by using projective measurements and quantum feedforward. We again assume that the Ising type Hamiltonian described in Eq. 8 dominates this system, and there is an interaction between every nearest neighbor qubit pairs. In our approach, unit cell to generate a bilayer three-dimensional cluster state is composed of 28 qubits (6 main qubits and 22 ancillary qubits), and we repeatedly put these cells on the same plane as shown in the Fig. 6. Interestingly, although these qubits are located in a two-dimensional plane, it becomes possible to implement a 3D topological quantum computation.

Let us consider qubits located on the cross-shape structure, which is a part of the unit cell (See the illustration (c) in the Fig. 7). The 5 ancillary qubits are used to implement a controlled-phase gate between an arbitrary pair of two main qubits in this cross shape structure without changing the states of the other main qubits. Interestingly, by preparing two of the ancillary qubits in ground states and preparing the other ancillary qubits in the $|+\rangle$ state as described in Fig. 7, only two main qubits will be involved in the implementation of the controlled-phase gate while the other main qubits do not affect the operation due to the existence of the ancillary qubits prepared in a ground state, which has the same analogy with the case of a one-dimensional cluster state.

Next, we estimate the optimal interaction strength of the above system for generating a three-dimensional cluster state. We consider the same noise as in [18]. For this estimation, we introduce the following setup. As described in Fig. 7, we use three ancillary qubits inserted between the target main qubits for performing controlled-phase gate. We name these qubits as qubit A, B, C, D, and E where A and E denote the main qubits and B, C, and D denote ancillary qubits (See Fig. 8).

Initially, all ancillary qubits are prepared in a ground state and main qubit are prepared in arbitrary states. We evaluate the error accumulation during the implementation of a controlled-phase gate in our scheme using three ancillary qubits (Fig. 8). Firstly, by performing $\frac{\pi}{2}$ pulse, we rotate the ancillary qubit C into $|\rangle$ state, and subsequently rotate the other ancillary qubit B and D into $|+\rangle$ state. In this case, since all nearest neighbor qubits for the qubit C are prepared in a ground state, the qubit C is not affected by the coupling strength from any other qubits and can be accurately rotated by a microwave resonant pulse. However, the qubit B (D) have unknown energy shift due to the coupling from qubit A (E) and C during the rotation so that a detuning error occurs. Secondly, we wait the appropriate time evolution of system with the Hamiltonian. During this time evolution, decoherence error of every qubit accumulates. Finally, we measure $\hat{Y}$ and perform quantum feedforward on the
FIG. 8. Error accumulation during the implementation of a controlled-phase gate between the main qubits. There are two main qubits A and E initially prepared in arbitrary state. Between these qubits, we insert three ancillary qubits B, C, and D initially prepared in a ground state. As long as the state of the nearest neighbor qubit contains a superposition, we cannot determine a resonant frequency of the qubit due to the always-on interaction, which induces a detuning error $\epsilon_{\pi}^{2}$ to rotate the qubit. Also, we assume a decoherence error $\epsilon_d$ that occurs during the time evolution to entangle nearest neighbor qubits by the interaction.

FIG. 9. The optimal coupling strength ($gT_2$) and an achievable fidelity of a controlled-phase operation in our scheme using three ancillary qubits against the Rabi frequency ($\lambda T_2$). The solid line denotes $F$ and the dashed line denotes $gT_2$. as with Fig. 4(a).

In this appendix, we discuss how to generate a cluster state with asymmetric coupling strength. Suppose that only three qubits are involved in, and the other qubits are set not to interact with these three qubits by controlling the state of the ancillary qubits. As we described before, by using unitary evolution, implementation of a spin echo, quantum measurements and quantum feedforward, we can perform a controlled-phase gate between two main qubits where a single ancillary qubit is inserted between two main qubits for this case. This is a two-step elementary operation to implement controlled-phase gate between main qubits under the effect of always-on interaction. We will use this operation recursively to make a large cluster state.

1. Generating one-dimensional cluster state

First, we aim for generating a one-dimensional cluster state using qubits that are arranged in a raw. In order to avoid an exponentially large number of implementations of $\pi$ pulses, we use the two-step procedure for generating should be thousands times larger than that.

VI. CONCLUSION

Here we show a scalable way to generate two and three-dimensional cluster state with always-on Ising interaction. Here, we use projective measurements and quantum feedforward to effectively turn on/off the interaction in this system. Our schemes provide a novel way to construct a surface code quantum computation and topological quantum computation.

Appendix A: Generating cluster state with asymmetric interaction strength

In this appendix, we discuss how to generate a cluster state with asymmetric coupling strength. Suppose that only three qubits are involved in, and the other qubits are set not to interact with these three qubits by controlling the state of the ancillary qubits. As we described before, by using unitary evolution, implementation of a spin echo, quantum measurements and quantum feedforward, we can perform a controlled-phase gate between two main qubits where a single ancillary qubit is inserted between two main qubits for this case. This is a two-step elementary operation to implement controlled-phase gate between main qubits under the effect of always-on interaction. We will use this operation recursively to make a large cluster state.
one-dimensional cluster state with $m$ main qubits and $m - 1$ ancillary qubits as shown in Fig. 11. In this setting, interaction strengths differ from each other. Firstly,

we perform controlled-phase operation between $2n - 1^{th}$ main qubit and $2n^{th}$ main qubit so that $\frac{m}{2}$ bell pairs are created. Since the $2k^{th}$ ancillary qubits are prepared in a ground state, we only need to consider the dynamics of three-qubit during the time evolution, and so we can apply the scheme to turn on/off effective interaction for three-qubit case as described in Fig. 1. Secondly, we perform controlled-phase operations between $2n^{th}$ main qubit and $2n + 1^{th}$ main qubit so that a one-dimensional cluster state can be created. Here, the $2k - 1^{th}$ ancillary qubits are in a ground state, and $2k^{th}$ ancillary qubits interact with two main qubits. Again, only three qubits interact with each other, and so we can apply the scheme described in Fig. 11.

2. Generating a two-dimensional cluster state

Next, we aim for generating a two-dimensional cluster state using $m^2$ main qubits and $2m(m - 1)$ ancillary qubits that are arranged on two-dimensional lattice with asymmetric coupling strength. For this purpose, we suggest a four-step procedure as shown in Fig. 12. Firstly, we generate $\frac{m^2}{2}$ Bell-pairs between each nearest pairs of logical and syndrome qubits at each column. At every step, ancillary qubits, which are not used for controlled-phase operations, prepared in ground state. Secondly, we perform controlled-phase operations between each Bell-pairs in the same column to generate one-dimensional cluster states. Thirdly, we perform controlled-phase operations in horizontal direction. At each controlled-phase operation, the syndrome qubit is arranged at the left side and the logical qubit is arranged at the right side. Finally, we perform controlled-phase operation between all the remaining nearest-neighbor pair of logical and syndrome qubits so that we can obtain a two-dimensional cluster state.

3. Generating a three-dimensional cluster state

In this subsection, we suggest a procedure to make a three-dimensional cluster state as shown in Fig. 13. For this procedure, we use $\frac{m^2}{4}$ main qubits and $\frac{3m^2}{4}$ ancillary qubits that are arranged on a two-dimensional plane with asymmetric coupling strength. Here, we show how to perform controlled-phase operation between two-main qubits $A$ and $E$ via three-ancillary qubits $B$, $C$, and $D$ with asymmetric interactions as in Fig. 14. Firstly, we perform controlled-phase operation between qubit $A$ and $C$ using our technique in Fig. 2. Secondly, we perform controlled-phase operation between qubit $C$ and $E$ using the same technique. Finally, we perform $Y$ basis measurement quantum feedforward operations on qubit $C$ so that controlled-phase operation can be implemented between qubit $A$ and $E$. We use this three-step controlled-phase operation recursively to make a large three-dimensional cluster state in following procedure.

The details of procedure for generating a three-dimensional cluster state are described as follows. Firstly, we generate $m$-qubit one-dimensional cluster states composed of $\frac{m}{2}$ logical qubits and $\frac{m}{2}$ syndrome qubits in vertical direction. For these operations, we use the same
process as described in Appendix A. Secondly, we generate two separable two-dimensional graph states as shown in Fig. 14. One of the two-dimensional cluster state is composed of logical qubits and bit-flip-detection syndrome qubits. The other one is composed of logical qubits and dephasing-detection syndrome qubits. Interactions between these two-dimensional cluster states are effectively turned off by ancillary qubits prepared in a ground state. Finally, we perform controlled-phase operations on a slant direction as shown in Fig. 7(c) between pairs of logical qubits to connect these two-dimensional cluster states so that we can generate a three-dimensional cluster state.

![Diagram](attachment:image.png)

FIG. 14. Controlled-phase operations via three-ancillary qubits using our technique with asymmetric coupling strength. Qubit \(A\) and \(E\) are main qubits, and qubit \(C\), \(D\), and \(E\) are ancillary qubits. Firstly, we prepare ancillary qubits \(D\) to ground state. Secondly, we perform controlled-phase operation between qubit \(A\) and \(C\) by applying our technique described in Fig. 2. Thirdly, we perform controlled-phase operation between qubit \(C\) and \(E\) in the same way. Finally, we perform \(Y\) basis measurements to qubit \(C\) and feedforward operations.

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