ASYMmetry and non-random ORIENTATION OF THE INFlight EFFECTive BEAM PATTERn IN THE WMAP DATA

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ABSTRACT

Tentative evidence for statistical anisotropy in the Wilkinson Microwave Anisotropy Probe data was alleged to be due to “insufficient handling of beam asymmetries.” In this paper, we investigate this issue and develop a method to estimate the shape of the inflight effective beam, particularly the asymmetry and azimuthal orientation. We divide the whole map into square patches and exploit the information in the Fourier space. For patches containing bright extragalactic point sources, we can directly estimate their shapes, from which the inflight effective beam can be estimated. For those without, we estimate the pattern from iso-power contours in two-dimensional Fourier space. We show that the inflight effective beam convolving the signal is indeed non-symmetric for most of the sky, and it is not randomly oriented. Around the ecliptic poles, however, the asymmetry is smaller due to the averaging effect from different orientations of the beam from the scan strategy. The orientations of the effective beam with significant asymmetry are parallel to the lines of ecliptic longitude. In the foreground-cleaned Internal Linear Combination map, however, the systematics caused by beam effect is significantly lessened.

Key words: cosmic background radiation — cosmology: observations — methods: data analysis

Online-only material: color figures

1. INTRODUCTION

The measurement of the cosmic microwave background (CMB) by the NASA Wilkinson Microwave Anisotropy Probe (WMAP; Bennett et al. 2003a, 2013; Spergel et al. 2003, 2007; Hinshaw et al. 2007, 2009, 2013; Komatsu et al. 2009, 2011; Jarosik et al. 2011) and ESA Planck Surveyor (Planck Collaboration et al. 2011a, 2011b, 2011c, 2013a, 2013b) has enabled us to probe cosmology with high precision. The CMB signal is observed through the convolution of an antenna beam, an effect, and possible systematic error, which must be carefully created in the data analysis. The main beam of the WMAP is not shown to be azimuthally symmetric about the line of sight, but rather elliptical (see Table 1) due to the fact that they cannot all be placed on the center of the focal plane. However, it was assumed that the inflight effective beam convolving the signal should be averaged to become symmetric and circular after each pixel on the map having more than at least 400 hits during the one year observation (Page et al. 2003). Prior to the WMAP data release, the issue of beam asymmetry in a full-sky CMB experiment was already discussed in Burigana et al. (1998), Souradeep & Ratra (2001), Fosalba et al. (2002). The issue of the asymmetry of the WMAP beam was later tackled in Bennett et al. (2013) with a new map-making procedure, which as a result deconvolves the beam sidelobes to produce maps with the true sky signal convolved by symmetrized beams. While the issue of beam asymmetry is considered in the estimation of the CMB angular power spectrum (Mitra et al. 2004, 2009) and is demonstrated to have less of an effect (Hinshaw et al. 2007; Wehus et al. 2009), it poses a serious issue for statistical isotropy, with the foremost example being the quadrupolar power modulation (Hanson et al. 2010; Bennett et al. 2013; Joshi et al. 2012). It is because systematic alignment in the orientation of even a mildly asymmetric beam can result in large-scale anisotropy.

To investigate this issue, we would like to see what effective beam pattern is convolving the signal. Note that for each pixel, the asymmetric inflight beam with different azimuthal orientation (around the line-of-sight direction) convolves the underlying signal thousands of times with some pointing uncertainty, so we are particularly interested in the asymmetry and azimuthal orientation of the effective elliptical main beam, which is derived from the collective effect of the inflight beam. Theoretically, one can gather all the inflight beam information through the time-ordered data and formulate the effective beam pattern for each point on the sky; nevertheless, it is difficult to get the effective beam pattern in the final product such as the Internal Linear Combination (ILC) map due to the complex noise properties. Throughout this paper, we define $r \equiv r_{\text{maj}}/r_{\text{min}}$, where $r_{\text{maj}}$ and $r_{\text{min}}$ are the major and minor axes of an elliptical shape, respectively, and the asymmetry of the beam as $K \equiv r - 1$.

In this paper, we develop a method that can reveal the asymmetry and azimuthal orientation of the effective main beam in different small patches of the sky, based on the flat sky approximation. There are two ways the effective beam convolving the signal manifests itself in the signal. One is through the shape of the bright extragalactic point sources and the other is in the Fourier amplitude, the latter of which is particularly useful for the processed map such as the ILC map.

This paper is arranged as follows. In Section 2, we introduce the Fourier method, and in Section 3 we test the accuracy and precision of the method. We then employ the methods on the WMAP data in Section 4, and the conclusion and discussion are given in Section 5.

2. MANIFESTATION OF THE ORIENTATION AND ASYMmetry of the inflight EFFECTive BEAM in the FOURIER SPACE

In a CMB experiment, the temperature measured in the sky can be written as $S = (C + F) \ast B + N$, where $C$ and $F$ are the CMB and foreground signal, respectively, $B$ is the effective beam convolving the signal, $N$ is the noise, and the star sign $\ast$ denotes convolution. The convolving beam was often assumed to be azimuthally symmetric and thus...
Figure 1. Reciprocity of the beam in real and Fourier space. The top row shows that the elliptical beam profile with major and minor FWHM of 36 and 30 arcmin (left and middle panel) and 21 and 18 arcmin (right), respectively, is used to convolve a 24 $\times$ 24 deg$^2$ simulated CMB patch (without adding noise). The convolution is performed with a fixed orientation of 0° (left), 60° (middle), and 120° (right) of the major axis against the x-axis. The bottom row shows the Fourier amplitude contour of the corresponding beam-convolved patches. We only show the amplitude contour down to $-30$ dB in order to display the shape and orientation. Note that the reciprocal property between real and Fourier space has caused the orientation manifested in the Fourier space to shift $\pi/2$; also, the larger FWHM of the beam, the smaller it manifests itself in the Fourier domain.

(A color version of this figure is available in the online journal.)

| WMAP Beam Asymmetry | R | K | Kα | Q1 | V1 | W1 |
|----------------------|---|---|----|----|----|----|
| A                    | 0.4258 | 0.2950 | 0.3217 | 0.1409 | 0.0970 |
| B                    | 0.4192 | 0.2913 | 0.3305 | 0.1366 | 0.0918 |

Notes. The WMAP beam maps are from Jupiter observation and we list the A and B sides of the focal planes for the K, Kα, Q1, V1, and W1 bands. The asymmetry is high because the feeds are away from the primary focus. The asymmetry is significantly lessened from multiple observations with different orientations in each pixel.

To estimate the inflight effective beam, we assume the main beam is of an elliptical Gaussian shape, which can be expressed in Cartesian coordinate

$$B(x, y) = \frac{1}{2\pi \sigma_x \sigma_y} \exp\left[-\frac{(x \cos \gamma + y \sin \gamma)^2}{2\sigma_x^2} - \frac{-(x \sin \gamma + y \cos \gamma)^2}{2\sigma_y^2}\right],$$

where $\gamma$ is the orientation against the x-axis.

We have measured the WMAP beam asymmetry and list it in Table 1. The WMAP beam maps are from observation of Jupiter and we list the A and B sides for the K, Kα, Q1, V1, and W1 differencing assembly (DA). The high asymmetry is due to the feeds being away from the primary focus. The asymmetry is significantly lessened from multiple observations with different orientations in each pixel.

In Chiang et al. (2002), it is demonstrated that the beam profile manifests itself in the two-dimensional Fourier space. In Figure 1, we show three different orientations of the beam profile.
convolving on a simulated CMB signal in a $24 \times 24$ deg$^2$ patch and in the bottom row we show the corresponding Fourier amplitudes of the beam-convolved patch. One of the important characteristics shown in the Fourier space is that the beam orientation turns to $\pi/2$ due to the reciprocity between real and Fourier space. It is thus possible to extract the inflight effective beam information contained in the patches.

Below, we present some methods that can reveal the inflight effective beam convolving the patch of the sky.

2.1. Direct Effective Beam Estimate via Bright Extragalactic Point Sources

For patches with bright radio point sources, following the assumption of slow rotation of the beam, we can further assume that the point source is the representative of the effective beam for the whole patch. The bright point sources are manifestations of the beam similar to the standard measurement of Jupiter for the beam profile, but in point sources we can get to at best $-10$ dB for most cases. Nevertheless, it is still useful for providing the asymmetry and orientation of the inflight effective beam. In Figure 2, we show the bright point source GB6 J2253+1608 in $Q1$ DA as an example. The estimated asymmetry is $R = 0.179$ and the angle is 98:98 for the major axis against the (positive) ecliptic equator.

2.2. Fourier Method for Estimation of the Effective Beam

As shown in Figure 1, the effective beam manifests itself in the Fourier domain, so we can extract the beam asymmetry and orientation directly from the Fourier amplitude. Below, we present two methods using different methods of defining the iso-power contours used for fitting. The first method (hereafter Method I) uses the Fourier amplitudes directly. Assuming the foreground power spectrum $\propto k^{-2}$ (Bennett et al. 2003b), we take the grid points $k \equiv (k_x, k_y)$ in the Fourier domain, where $|k|^{-2} \exp(-|k|^2 \sigma^2) > r$, $\sigma \equiv$ FWHM$/2\sqrt{2\ln 2}$, and FWHM is the size of the nominal beam of the frequency band. Even for the same $r$, the number of grid points varies for different frequency bands due to different beam sizes. Also, note that the level of $r$ cannot be lower than the pixel noise, and thus simulation is required in order to get the optimal $r$.

The other method is by phase perturbation (hereafter Method II), developed in Chiang et al. (2002). For the measured signal $S$ in Equation (2), we can add controlled white noise $W$: $M = S + W$ to perturb the phases of the patch and calculate the phase shift $\Psi_k^M - \Psi_k^S$, where $\Psi_k^M$ and $\Psi_k^S$ are the Fourier phase at mode $k$ for patches $M$ and $S$, respectively. Note that the controlled noise level cannot be lower than that of the pixel noise (the $N$ in Equation (2)). We can then calculate the mean from an ensemble, say $n = 200$, of such a perturbation:

$$\Delta^2(k) = \langle (\Psi_k^M - \Psi_k^S)^2 \rangle_n.$$  (4)

For the Fourier modes where the Fourier amplitudes of the convolved signal are much higher than that of the controlled noise, the phases are not perturbed much, so the $\Delta^2(k) \simeq 0$. 

![Figure 2](image_url)

Figure 2. For patches with bright point sources, we can estimate the asymmetry of the effective beam directly. The point source here is GB6 J2253+1608, which appears in $Q1$ DA. Direct estimate of its asymmetry and orientation results in $R = 0.179$ and the orientation angle is 98:98 for the major axis against the ecliptic equator. We can also apply the Fourier method on the patches with point sources for consistency, which renders $R = 0.110$ and an angle of 96:14. (A color version of this figure is available in the online journal.)

![Figure 3](image_url)

Figure 3. Demonstration of the estimate of the effective beam from the Fourier method. The left panel shows the contour of the Fourier power ($mK^2$) up to $10^{-6}$, which is the level equivalent to around $-5$ dB of any beam on the best-fit for the input beam $R = 0.1$ and major axis angle $50^\circ$ convolving the simulated CMB signal. Note that one can clearly see that the asymmetry and major axis in the Fourier space are shift by $90^\circ$ due to the reciprocal property between the real and Fourier space. Middle and right: estimation from the points of Method I collected down to $-5$ dB and from Method II, respectively. The dashed line is the input shape and the solid line is the estimated shape. We make the input and output ellipses different sizes in order to show how small the difference is.
Figure 4. We test the accuracy and precision of the two Fourier methods for estimating the orientation angle and ratio of the axes using simulation (see the text). We show the scatter plot for the estimated angle against the input angle (top row) and the ratio of the axes (middle row) for three different ratios. In the top two rows, Method I is denoted with a large sign and Method II with a small one, and the straight black line denotes the target estimation. Note that the plots in the top row have a periodic property. In the bottom row, we show the error from the estimation compared to the simulation input. In all three panels, the plus, triangle, and diamond symbols denote $r_0 = 1.10, 1.15, \text{ and } 1.20$, respectively. The left panel shows the estimate from Method I: error in asymmetry $\langle \Delta r \rangle / r_0 = 0.055 \pm 0.038$, error in angle $\langle \Delta \theta \rangle = 3.15 \pm 21.08 \text{ deg}$. The middle panel is that from Method II: $\langle \Delta r \rangle / r_0 = 0.056 \pm 0.049, \langle \Delta \theta \rangle = 0.05 \pm 27.74 \text{ deg}$. For the mean from both estimates, the error is shown in the right panel: $\langle \Delta r \rangle / r_0 = 0.056 \pm 0.036, \langle \Delta \theta \rangle = 1.60 \pm 18.05 \text{ deg}$. (A color version of this figure is available in the online journal.)

When the controlled noise level is close to that of the signal, the phases are perturbed so much that they can be approximated as

$$\Delta^2(k) \simeq \frac{|\langle |W_k|^2| \rangle}{2|S_k|^2},$$

(5)

where the controlled noise level is set as the same as that of the pixel noise (Chiang et al. 2002). In the following demonstration, we take the grid points in the Fourier domain where $\Delta < 9^\circ$.

3. DEMONSTRATION AND ACCURACY AND PRECISION OF THE FOURIER METHOD

To demonstrate the Fourier method for estimating the effective beam, an elliptical beam whose major and minor axis FWHM is 33 and 30 arcmin, respectively (thus asymmetry $R = 0.1$), is used to convolve a $24 \times 24 \text{ deg}^2$ CMB map simulated with the best-fit $\Lambda$CDM model before adding noise at a level of $\sigma = 0.152 \text{ mK}$. The orientation of the major axis is $50^\circ$.
Figure 5. Estimation of the effective beam of the WMAP Q1 DA in the ecliptic coordinate. The bar denotes the asymmetry $R$, and the orientation of the major axis is denoted by that of the bar. The bottom two panels show the histogram of the asymmetry $R$ and the orientation angle. The angle is defined with that between the bar and the ecliptic equator. The reciprocal property causes the orientation of the beam to turn $\pi/2$ in the Fourier space, now $140^\circ$. One can see that the errors from the angle are $-3.01$ and $-16.8^\circ$, respectively, whereas the errors of the asymmetry are $0.019$ and $0.034$.

One source of error in such an estimation comes from the small number of points on an equal-spaced grid in the Fourier domain. The less points that are available in the Fourier space, the higher the error in estimation. The number of points available from the Fourier grid is limited by the beam size and the noise level. For estimating a small beam size such as in the WMAP W band, there are more points in the Fourier grids (due to reciprocity), but the noise level is higher than in the low-frequency bands. For a larger beam in the K band there are fewer points. Thus, to gain more available grid points, we resort to large patches so that the white noise level can be lowered.

We conduct simulations on the WMAP Q band with a nominal beam size of FWHM 30.6 arcmin for three different axis ratios $r_0 = 1.10$, 1.15, and 1.20, each with 18 different orientations with a 10° increment. We apply the two Fourier methods to test their accuracy and precision. In Figure 4, we show the scatter plot for the estimated angle (top row) and the ratio of the axes (middle row) from Method I (large sign) and Method II (small sign). The error from each method is shown in the bottom row.

For the top row of Figure 4, the estimated angle is plotted against the input from the simulation. One should note the periodicity of the plots, e.g., points close to $180^\circ$ are equivalently close to $0^\circ$. The middle row is the estimated ratio, which is plotted against the input angle. If the angle or the ratio is estimated correctly, then the point should fall on the black line in both rows. One can see qualitatively that most of the estimations for the angle are scattered around the line, and some of the estimates for the angle deviate from the line due to the fact that its ratio is also notably underestimated close to unity, e.g., $r_0 = 1.1$, 70°, and 160° from Method I, and $r_0 = 1.15$, 130° from Method II. This is because for grid points forming a shape with an axis ratio that is close to unity (whether or not the Fourier method picking up the targeted shape), the angle can easily be severely misestimated due to the discretization of the grids.

In the bottom row of Figure 4, we plot the error in asymmetry against the error in orientation angle for Method I (left panel) and Method II (middle) in the simulations. Method I skews the...
estimate in orientation angle for $r_0 = 1.10$ (plus sign) but has better accuracy and precision for $r_0 = 1.15$ (triangle sign) and $1.20$ (diamond sign), whereas Method II has greater accuracy but less precision. Since each use different information from the Fourier space (though not independently), we take the mean from both estimates and plot it in the right panel. The estimated error in the angle is $\langle \Delta \theta \rangle = 1.60 \pm 18.05$ deg and the error in the axis ratio is $\langle \Delta r \rangle / r_0 = 0.056 \pm 0.036$, indicating that our estimate for the orientation angle is reasonably good, while the asymmetry is underestimated by 5.6%.

We can also use the Fourier method mentioned above for the patch containing the bright point source GB6 J2253+1608 (Figure 2). The asymmetry from the Fourier method is $R = 0.110$ and angle is $96^\circ 14$. The difference in the estimated asymmetry $-0.069$ is as predicted: our Fourier method consistently underestimates the asymmetry, whereas the difference in the estimated angle $-2^\circ 84$ is negligible.

There are a few factors causing the systematic underestimation of the axis ratio. One factor is related to the non-periodic condition in the cut patch. In an ideal (periodic) condition, we can choose the right band of points (say $6^\circ < \Delta < 9^\circ$ for Method II) forming a band of ellipse for the estimation. The non-periodic condition, however, induces significant artificial power depending on the morphology around the Fourier grid points ($\pm k_x$, 0) and (0, $\pm k_y$). For stability, we thus resort to using all of the inner grid points ($\Delta < 9^\circ$) to reduce the weighting of those points with artificial power. Those points along the $k$ axes nevertheless cause an underestimate. Another culprit is the equal-spaced grid from which we are to estimate an elliptical shape. The points around the two ends of the major axis are fewer than around those of the minor axis. Everything being equal, random fluctuation will produce some more points statistically causing the underestimate.

4. ASYMMETRY AND NON-RANDOM ORIENTATION OF THE INFLIGHT EFFECTIVE BEAM OF THE WMAP DATA

We can now employ the methods that we demonstrated in the previous section to estimate the inflight effective beam on the WMAP data. We divide the whole map into $24^\circ \times 24^\circ$ patches and apply the Fourier method described above to the patches. The reason why we estimate the beam on such a large patch is that the noise level in the WMAP data is quite high, which inevitably sets the limit of the available Fourier grid points. We show our estimate of the inflight effective beam of the WMAP 9 year Q1 DA.

Due to the scan strategy that the WMAP observes from a Lissajous orbit about the L2 Sun–Earth Lagrange point, and that the telescope line of sight is around $70^\circ$ off the WMAP spinning axis, the path swept out on the sky by a given line of sight resembles a spirograph pattern that reaches from the north to the south ecliptic poles, and hence the inflight beam pattern is closely related to the ecliptic coordinate. In Figure 5, we plot the estimated inflight effective beam pattern. The lengths of the bars indicate the asymmetry in $R$ and the inclination denotes that of the major axis. The bottom two panels show the histogram of the asymmetry $R$ and the orientation angle. The angle is defined
with that between the bar and the ecliptic equator. We plot the same for the WMAP ILC map in Figure 6.

Our results confirm that the inflight effective beams that convolve the underlying signal are asymmetric for most parts of the sky and are not randomly oriented. The alignment is most severe around the ecliptic equator. Near the ecliptic poles, however, the asymmetry is small due to the averaging effect from different orientations from the scan strategy. The histogram of the orientation angle shows a high concentration around $\pi/2$.

On the other hand, the ILC map results from the combination of different frequency band maps, which have different beam orientations and beam sizes, and hence the asymmetry of the orientation alignment of the effective beam on the ILC map is ameliorated by the internal combination.

5. CONCLUSION AND DISCUSSION

In this paper, we have developed a method to reveal the asymmetry and orientation of the inflight effective beam in the WMAP data. We divide the whole sky map into patches and exploit the information residing in the Fourier domain. We test the accuracy and consistency of our Fourier method using simulations and the bright point source in the patch, where the shape of the latter is representative of the inflight effective beam. We then apply the method to the WMAP $Q_1$ DA and ILC map, and it is confirmed that the effective beam is rather asymmetric with strong alignment in their orientation around the ecliptic equator in the $Q_1$ DA map. The fact that the asymmetry of the effective beam in the ILC map is lessened does not guarantee that the ILC map is not without the systematic error from the beam effect. A highly aligned beam with significant asymmetry combing through the measured signal, which includes strong emission of the galactic foreground, can have an elongated effect on the signal, which will cause some systematic error in the foreground cleaning process for the CMB if it is not carefully treated. The method developed in this paper can be used on the Planck data.

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