Thunder-cell as Source of Energetic Protons

Aleš Berkopec*

September 29, 2020

Abstract

In this article we present the following hypothesis: thunder-cell ejects highly energetic protons, each of which creates a tree-structure of weakly ionized trajectories that can develop into a lightning channel. The tree-structure and the channel have the same geometry so the mean free path of a proton corresponds to the average length of the channel between two successive nodes (branching points). We show this length is around 660 m in lower Earth atmosphere. Effects of Coulomb interaction and various outcomes of proton-nucleus reaction are taken into account. A prediction of CG/CC ratio that follows agrees well with the available data, but only measurements of lightning geometry can reveal whether the hypothesis is any closer to the correct explanation of the phenomenon.

Keywords: lightning initial phase, CG and CC lightning

1 Introduction

The reported values of potential gradient before and during a lightning strike are at least one order of magnitude smaller compared to the ones required to induce sparks under controlled conditions in laboratories [1 2 3]. We tried to explain this difference with theoretical prediction [4] involving fast charged particles that precede stepped leader. This idea was later encouraged by a report about weak correlation between lightning frequency and solar wind intensity [5]. Since every CG (cloud-to-ground) and CC (intra-cloud, inter-cloud or cloud-to-air) lightning channel starts in a thunder-cell the intense freezing of super-cooled water inside thunder-cell was proposed as a process that might lead to ejection of charged projectiles [6].

In view of the hypothesis, trajectory of the primary projectile and its collision products, all electrically charged, determine the geometry of the stepped leader and subsequent lightning channel. Interaction of the projectile with electrons in air slows the projectile down and ionizes the trajectory, while collisions with the nuclei may produce higher-order projectiles and result in branching of the channel. At the end of the process, weakly ionized tree structure forms a conducting path between cloud and ground (CG lightning, Fig. 1), or cloud and a point in the air (CC lightning).

Figure 1: Thunder-cell ejects a projectile. Why protons are best candidates for projectiles is explained in Section 2, for the estimation of length between two successive collisions $l$ see Sections 3 and 5, and for a role of Coulomb interaction see Section 4.

The projectile and the ionized tree-structure it leaves behind provide explanation for at least three aspects of CG and CC lightnings: a) lower values of potential gradient sufficient to initiate lightning, b) tree-structure of a lightning channel with loosely defined direction, and c) dependence of CG/CC ratio on height of the cloud-base and consequently on latitude.

2 Projectile: type of particle

We reason the projectile is expected to have the following characteristics:

a) electrically charged

Only a charged particle leaves behind an ionized trajectory. Coulomb interaction has only a minor effect on the geometry of the channel, as we show later.

b) induces secondary projectiles of the same type

Lightning channel is often split, however, the branches that grow from the split point are indistinguishable.

*contact: ales.berkopec@gmx.com
ATOM 14 16 40 were used in calculation of $\lambda_{p^+}$. For the volume density of the nuclei we assumed $n = 5.33 \cdot 10^{25} / \text{m}^3$, and for the rates $\eta_N = 78.39\%$, $\eta_0 = 21.11\%$, and $\eta_A = 0.502\%$.

### 4 Coulomb interaction

Coulomb interaction is responsible for the loss of projectile’s kinetic energy and ionization of its trajectory. The rate of change in kinetic energy is expressed by Bethe equation [7]. The range dependence for a proton in lower Earth atmosphere is derived in [6] and shown on Fig. 2 for $\lambda_{p^+} = 660$ m. Since the height of a thunder-cell is around 1 km or higher, graph on Fig. 2 suggests the minimum initial energy for a proton that reaches the ground is about 1 GeV.
5 Proton-nucleus reactions

Collision of p+ projectile with a nucleus X discussed so far was assumed to be of the pick-up type \( p^+ + X \rightarrow X^- + p^+ + p^+ \), or \( (p^+, 2p^+) \), plus arbitrary number of neutral particles. According to the hypothesis, such reactions correspond to the observed binary-tree geometry of lightning channels. Two types of reactions, swap and capture, produce the results of collisions not accounted for: swap can not be distinguished from a part of a branch that has no split, and capture of a proton by a nucleus looks like the end of a branch.

Now we extend the survival probability for a projectile and one target \( i \) to reactions that are not necessarily of pick-up type. Let us presume that the average distance between two successive nodes as a function of kinetic energy at the first node – taking into account stopping power due to Coulomb interaction – was derived in [6] and is shown on Fig. 3. Its impact on the mean free path diminishes with increasing initial kinetic energy \( W_0 \). Since protons with \( W_0 < 1 \text{ GeV} \) do not reach the ground, unless significant part of the projectiles has initial energies \( W_0 \) in \([1 \text{ GeV}, 2 \text{ GeV}]\) range, we may assume Coulomb interaction can be neglected in the first approximation.

\[
\eta = \frac{R - h}{R + h}
\]

Taking the ratios \( \eta_i \) and \( \eta_j \) at two different latitudes, where the heights of the thunder-cells are \( h_i \) and \( h_j \), respectively, one finds the estimation for the ratio of thunder-cell heights reads

\[
\frac{h_i}{h_j} = \frac{1 + \eta_i}{1 + \eta_j} \approx \frac{1 + \eta_i}{1 - \eta_j}
\]

Since a thunder-cell is always located close to the base of its thundercloud we take that in the first approximation the ratios of the heights \( h_i/h_j \) and \( H_i/H_j \) are close enough \( H_i/H_j \approx h_i/h_j \). At Equator at \( \Lambda = 0^\circ \) experimental data gives \( \eta_0 \approx 0.1 \) and \( H_0 \approx 1100 \text{ m} \). Together with \( \eta \) this leads to prediction for CG/CC ratio from height of cloud-base:

\[
\eta(H) = \frac{a - H/H_0}{a + H/H_0}
\]

where \( a = (1 + \eta_0)/(1 - \eta_0) \approx 11/9 \).

Based on empirical data Prentice and Mackerras [9] for an estimate for CG/CC ratio suggest

\[
\eta^{(a)}(\Lambda) = \frac{1}{4.16 + 2.16 \cos(3\Lambda)}
\]

while Pierce [10] proposes

\[
\eta^{(b)}(\Lambda) = \left[ \frac{1}{0.1 + 0.25 \sin \Lambda} - 1 \right]^{-1}
\]

Values of CG/CC ratios for three latitudes from [7] and [8] along with our prediction [6] are given in Table 4.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Latitude} & \text{Experimental} & \text{Prediction} & \text{Prentice} & \text{Pierce} \\
\hline
\text{Equator} & 0.1 & 0.1 & 0.1 & 0.1 \\
\text{20°} & 0.05 & 0.05 & 0.05 & 0.05 \\
\text{40°} & 0.02 & 0.02 & 0.02 & 0.02 \\
\hline
\end{array}
\]

For projectiles ejected in a random direction the CG/CC ratio can be estimated from the ratio of the areas defined by the intersection of the sphere, whose center is a thunder-cell at height \( h \), and the plane, representing the ground (see Fig. 4). The area of the sphere below the plane correlates with incidence of CG lightnings (corresponding spherical angle is shaded), the area above the plane to CC lightnings. The ratio of the areas and corresponding spherical angles equals

\[
\eta = \frac{R - h}{R + h}
\]

The incidence of CG lightnings (corresponding spherical angle shaded), the area above the plane to CC lightnings. The ratio of the areas and corresponding spherical angles equals

\[
\eta = \frac{R - h}{R + h}
\]

The area of the sphere below the plane correlates with incidence of CG lightnings (corresponding spherical angle shaded), the area above the plane to CC lightnings. The ratio of the areas and corresponding spherical angles equals

\[
\eta = \frac{R - h}{R + h}
\]

The area of the sphere below the plane correlates with incidence of CG lightnings (corresponding spherical angle shaded), the area above the plane to CC lightnings. The ratio of the areas and corresponding spherical angles equals

\[
\eta = \frac{R - h}{R + h}
\]
7 Expected value of $\langle l \rangle$

Let us make a rough Fermi-type estimate about average length $\langle l \rangle$ from lightning photos and experience as observers. We aim at higher confidence level and are less concerned with error margin.

It is fairly safe to assume that less than 20% of channels have no nodes, and that no channel has more than 20 nodes. Consequently, the remaining channels with number of nodes between 1 and 19 occur with probability between 80% and 100%. If $M$ is the number of the nodes, the minimum and the maximum expected values for $M$ are

$$E(M)_{\text{min}} = 0 \cdot 0.2 + 1 \cdot 0.8 + 0 \cdot 20 = 0.8$$

$$E(M)_{\text{max}} = 0 \cdot 0.0 + 19 \cdot 1.0 + 0 \cdot 20 = 19$$

Cloud-base heights range from $H_{\text{min}} = 500$ m to $H_{\text{max}} = 1200$ m [8]. It is impossible to find the height of the cloud for a given lightning from its photograph, so we take $L_{\text{min}} \approx H$ because the length of the main channel can not be shorter than the minimum distance between the cloud base and the ground, while the maximum length is taken to be five times that, or $L_{\text{max}} \approx 5 \cdot H$ (in such case the average direction of the channel is $80^\circ$ from vertical, and the lightning strikes the ground around 4.8 km from the cloud).

The lower and upper expected values for the average length between successive nodes are then

$$E(\langle l \rangle)_{\text{min}} = \frac{L_{\text{min}}}{E(M)_{\text{max}+1}} = \frac{500}{19+1} \approx 25 \text{ m}$$

$$E(\langle l \rangle)_{\text{max}} = \frac{L_{\text{max}}}{E(M)_{\text{min}+1}} = \frac{1200}{5+1} \approx 3.3 \text{ km}$$

The range spans over three orders of magnitude and includes our prediction of 660 m for protons and $(p^+, 2p^+)$ reactions. For longest expected value $\langle l \rangle \approx 3333$ m the probability $p_{\text{nu}}$ according to [4] equals $p_{\text{nu}} \approx 660/3333 \approx 20\%$.

8 Discussion

Estimation of $\langle l \rangle$ above has a wide error margin but the particles that may be involved in the process do not come in arbitrary sizes. Classical prediction from [3] for projectile of zero size gives $\lambda \approx 1311$ m, and charged particles larger than proton are either composite or short-lived.

Secondary projectiles from proton-nucleus reactions often include pions and kaons. Their classical radii give $\lambda_p \approx 700$ m for pions and $\lambda_K \approx 810$ m for kaons. Short-lived pions decay in muons, and muons decay in positrons or electrons, so these can only contribute a non-branched part to a channel. One specific decay of kaons $K^\pm \rightarrow 3\pi^\pm$ may lead to channel branching but it is less likely to occur as kaons themselves are the rarest secondary products among the p$^+$, $\pi^\pm$, and $K^\pm$, and since this type of decay for kaons has only 6% rate [11]

Experimental verification of the hypothesis should involve measurements of the average length between successive nodes by reconstruction of 3D channel geometry. Correlation between intensity of freezing/precipitation and frequency of lightning is expected, as well as the ejection of elementary particles from super-cooled water during freezing.

References

[1] R. Gunn, Electric field intensity inside of natural clouds. *Journal of Applied Physics*, 19(5):481–484, 1948.

[2] T.C. Marshall and W.D. Rust, Electric field soundings through thunderstorms. *Journal of Geophysical Research: Atmospheres*, 96(D12):22297–22306, 1991.

[3] W.P. Winn et al., Measurements of electric fields in thunderclouds, *Journal of Geophysical Research*, 79(12):1761–1767, 1974.

[4] A. Berkopec, Fast particles as initiators of stepped leaders in CG and IC lightnings, *Journal of Electrostatics*, 70(5):462–467, 2012.

[5] C.J. Scott et al., Evidence for solar wind modulation of lightning, *Environmental Research Letters*, 9(5), 2014.

[6] A. Berkopec, About Geometry and Initial Phase of Cloud-to-Ground Lightning, arXiv:1602.02496, physics.ao-ph, 2016.

[7] H. Bethe and W. Heitler, On the stopping of fast particles and on the creation of positive electrons. *Proceedings of the Royal Society of London. Series A*, 146(856):83–112, 1934.

[8] V.A. Rakov and M.A. Uman, Lightning: physics and effects, 2003, Cambridge University Press.

[9] S.A. Prentice, D. Mackerras, The ratio of cloud to cloud-ground lightning flashes in thunderstorms, *J. Appl. Meteor.*, 545–550, 1977.

[10] E.T. Pierce, Latitudinal variation of lightning parameters, *J. Appl. Meteor.*, 194–195, 1970.

[11] J. Beringer et al. (Particle Data Group), *Phys. Rev. D*86, 010001, 2012.

| $\Lambda$ | $H$ [m] | $\eta^{(a)}(\Lambda)$ | $\eta^{(b)}(\Lambda)$ | $\eta(H)$ |
|---|---|---|---|---|
| $0^\circ$ | 1100 | 0.16 | 0.11 | 0.10 |
| $45^\circ$ | 700 | 0.38 | 0.38 | 0.32 |
| $60^\circ$ | 500 | 0.50 | 0.46 | 0.46 |

Table 1: Height $H$ of a thunder-cloud base and CG/CC ratios for three latitudes $\Lambda$ (see [8]). Values of $\eta^{(a)}$ and $\eta^{(b)}$ follow from [7] and [8]. Our prediction [6] is in column $\eta(H)$. 

\[\Lambda \quad H \quad \eta^{(a)}(\Lambda) \quad \eta^{(b)}(\Lambda) \quad \eta(H)\]

\[\begin{array}{cccc}
0^\circ & 1100 & 0.16 & 0.11 & 0.10 \\
45^\circ & 700 & 0.38 & 0.38 & 0.32 \\
60^\circ & 500 & 0.50 & 0.46 & 0.46 \\
\end{array}\]