Models of the in-hour equation for fast reactors surrounded by a physically thick neutron reflector for upgrading the reactor safety

G G Kulikov, A N Shmelev, V A Apse and E G Kulikov
Institute of Nuclear Physics and Engineering, National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe highway, 31, Moscow, Russia

E-mail: GGKulikov@mephi.ru

Abstract. Main purpose of the study is to analyse some models of the in-hour equation for fast reactors surrounded by a physically thick neutron reflector that would be capable to slow down chain fission reaction and, thus, upgrade the reactor safety. The paper proposes the mathematical model that takes into consideration the neutrons coming back from the neutron reflector to the reactor core in the form of delayed neutrons and compares it with one-point model of neutron kinetics and with model that includes some additional groups of delayed neutrons. It was revealed that, under rather slow power excursions when asymptotic periods were significantly longer than lifetime of the neutrons coming back from the neutron reflector, all three models mentioned above were equivalent ones. Under relatively quicker power excursions, the developed model gives a conservative evaluation for the reactivity, i.e. with some safety margin acceptable for scientific and advanced studies.

1. Introduction
In papers [1–4] new approach was proposed to upgrade nuclear safety of fast reactors. The approach is based on the increased fraction of delayed neutrons caused by temporary delay of prompt fission neutrons during their transport to and from the neutron reflector. It was shown that fraction of delayed neutrons may be increased by using heavy material with weak neutron absorption (lead isotope $^{208}$Pb, for example) as the neutron reflector. Such a neutron reflector should be a physically thick one. This approach requires creating a new correct model of the in-hour equation that plays important role in kinetics of nuclear reactors [5].

Within the frames of the one-point kinetics model with six groups of delayed neutrons the in-hour equation links asymptotic period of power excursion $T$ with reactivity $\rho$ inserted into the reactor core in the following form [5]:

$$\rho = \frac{\Lambda_{\text{pert}}}{T} + \sum_{i=1}^{6} \frac{\Lambda_i \beta_i}{T + \Lambda_i}$$

(1)

where: $\Lambda_{\text{pert}}$ – mean lifetime of prompt neutrons in the reactors with traditional neutron reflectors; $\Lambda_i$ – mean lifetime of delayed neutrons emitters for the $i$-th time group; $\beta_i$ – effective delayed neutrons fraction in the $i$-th time group.

2. Developed model of reflector neutrons in form of delayed neutrons
The neutrons coming back from the neutron reflector to the reactor core may be characterized as prompt fission neutrons in their physical nature and as delayed neutrons for chain fission reaction because they contribute to the chain fission reaction progress with some time delay needed for their transport, slowing down and diffusion from the reactor core to the neutron reflector and back. By analogy with traditional delayed neutrons, these neutrons can be divided onto several time groups each of them corresponds to the neutrons coming back from the \( j \)-th layer of the neutron reflector. The in-hour equation may be re-written in the following form with accounting for the neutrons coming back from the neutron reflector:

\[
\rho = \frac{\Lambda_C}{T} + \sum_{i=1}^{J-1} \frac{\Lambda_j \cdot \beta_i}{T + \Lambda_i} + \sum_{i=1}^{J} \frac{\Lambda_j \cdot \rho_{R_i}^j}{T + \Lambda_R^j}
\]

(2)

where: \( J \) – the number of spatial layers in the neutron reflector, \( \Lambda_R^j \) – lifetime of the neutrons coming back from the \( j \)-th layer of the neutron reflector, i.e. sum of neutron lifetime in the reactor core before leakage, time for neutron diffusion from the reactor core to the \( j \)-th layer of the neutron reflector, time for neutron staying in the \( j \)-th layer of the neutron reflector, time for neutron diffusion from the \( j \)-th layer of the neutron reflector to the reactor core and neutron lifetime in the reactor core after return from the neutron reflector; \( \rho_{R_i}^j \) – the reactivity increment caused by surrounding the reactor core with the \( j \)-th layer of the neutron reflector. It should be noted here that the first summand of the in-hour equation (1) contains the term \( \Lambda_{pr} \) as neutron lifetime in the reactor as a whole including the neutron reflector while the first summand of the in-hour equation (2) contains the term \( \Lambda_C \) as neutron lifetime in the reactor core without the neutron reflector (the unreflected core).

As is known, delayed neutrons and prompt neutrons, which did not stay in the neutron reflector, begin contributing to the chain fission reaction just after fission, though their contributions are stretched in time. On the contrary, the neutrons coming back from the neutron reflector are always characterized by a certain “dead-time”, when they are not able, in principle, to contribute to the chain fission reaction development. The “dead-time” is a sum of time for neutron diffusion from the reactor core to the neutron reflector and time for neutron diffusion from the neutron reflector to the reactor core. This peculiarity of the neutrons coming back from the neutron reflector is a favorable factor for the reactor safety. Unfortunately, this peculiarity is not described by the developed in-hour equation (2) where the neutrons coming back from the neutron reflector are considered as analogues of delayed neutrons.

3. One-point model of the neutron reflector

The monograph [6] analyzes the reactivity effect of neutron reflection from external objects (walls, equipment units and so on) into the core of experimental pulsed reactor and describes the mathematical method that takes neutron “dead-time” into consideration. Two models of neutron kinetics in the reflected reactor were considered in the monograph, namely two-point model of neutron kinetics (one additional spatial point), and introduction of additional time groups for delayed neutrons emitters.

The first model is based on the coupled system of two neutron kinetics equations defined separately for the reactor core and for the neutron reflector [7]. Under assumption that all the neutrons coming into the neutron reflector can come back to the reactor core (this condition is mainly satisfied in the case of thick \(^{208}\)Pb-reflector), the following in-hour equation was derived:

\[
\rho = \frac{\Lambda_C}{T} + \sum_{i=1}^{J-1} \frac{\Lambda_j \cdot \beta_i}{T + \Lambda_i} + \frac{\Lambda_k \cdot \rho_{R_i}^j}{T + \Lambda_R^j}
\]

(3)

Similar model may be formed either for several neutron reflectors or for several layers of one neutron reflector. Then, equation (3) can be reduced to equation (2) for the proposed model of a reactor with the thick reflector where contributions to the reactor reactivity from individual layers of the neutron reflector are defined by additional time groups of delayed neutrons emitters.
4. Model of additional time groups for delayed neutrons emitters

The second model describes the reactivity effects produced by the neutron reflector through introduction of several additional time groups of delayed neutrons emitters [8-9]. Applications of the model are limited by only those neutron reflectors which are able to produce the reactivity increments below several dollars (effective delayed neutrons fractions). The model is also based on the following assumptions. Firstly, one-point neutron kinetics equations are applicable for analysis of the unreflected reactor cores. Secondly, disposition of the neutron reflector in the vicinity of the reactor core can induce only insignificant deformations in space-energy distribution of neutrons in the reactor core. Thirdly, one-point neutron kinetics equations may be generalized on analysis of the reflected reactor cores by introduction of a time-dependent neutron source which accounts for the neutrons coming back from the neutron reflector to the reactor core and participating in the chain fission reaction development. Under these assumptions, the following form of the in-hour equation was derived, where energy distribution of the reflected neutrons is divided on J additional groups of delayed neutrons:

\[
\rho = \frac{\Lambda_{pr}}{T} + \sum_{i=1}^{e} \frac{\Lambda_{i} \cdot \beta_{i}}{T + \Lambda_{i}} + \sum_{j=1}^{i} \rho_{j} \int t_{j}^{k} + T \cdot \left(1 - \exp \left[-\frac{t_{j}^{k}}{T}\right]\right)
\]

where: \( t_{j}^{k} \) – time of neutron staying in the reflector, \( \tau_{j}^{k} \) – total time for neutron diffusion from the reactor core to the neutron reflector and back, i.e. the “dead-time” during which neutrons can’t contribute to the chain fission reaction development.

This model describes energy distribution of the reflected neutrons while the proposed model describes the following parameters of the neutrons coming back from the neutron reflector: effective fraction and lifetime. Anyway, the reactivity effects produced by the neutron reflector are described by two models in a similar manner, namely through introduction of additional groups of delayed neutrons emitters. Nevertheless, the in-hour equations (2) and (4) of these models differ each other in the form of the first and the last summands.

The first summand of the in-hour equation (4) contains mean neutron lifetime in the reflected reactor core while the first summand in the in-hour equation (2) contains mean neutron lifetime in the unreflected (bare) reactor core. These lifetimes may be quite different in their values (on several orders of magnitude). This difference is explained by the mentioned assumptions of the second model. Namely, the model considers only those neutron reflectors which are able to produce the reactivity increments below several effective delayed neutrons fractions, and those neutron reflectors which are able to induce only insignificant changes in mean neutron lifetime (i.e. it is assumed that \( \Lambda_{pr} \approx \Lambda_{c} \)). However, if the reactor core is surrounded by thick (four meters) layer of weak neutron absorber \((^{208}\text{Pb})\), then these assumptions are considerably violated. Indeed, so thick \(^{208}\text{Pb}\)-reflector produces the very large reactivity increment (~35 · β) and increases mean neutron lifetime by about three orders of magnitude [1, 2]. That is why the first summand of the in-hour equation (4) is not applicable for analysis of the physically thick neutron reflectors. It should be noted here that two-point model of neutron kinetics in the reflected reactor core does not limit the reactivity effects induced by the neutron reflectors. That is why the first summands of the in-hour equations (3) and (2) are the same ones.

Now the differences between the third summands of the in-hour equations (2) and (4) should be considered. The first of all, index \( j \) in these summands designates different parameters, namely index \( j \) in the in-hour equation (2) designates the number of spatial layer while index \( j \) in the in-hour equation (4) designates the number of energy group. However, it seems evident that one means another, i.e. the neutrons coming back from the \( j \)-th layer of the neutron reflector may be characterized by the certain group energy. Thus, total time of neutron diffusion from the reactor core to the neutron reflector and back \( t_{j}^{k} \) and time of neutron staying in the neutron reflector \( \tau_{j}^{k} \) are related with total lifetime of the reflected neutrons \( \Lambda_{j}^{k} \) by the following simple formula: \( \Lambda_{j}^{k} = \tau_{j}^{k} + t_{j}^{k} \). Neutron lifetimes in the reactor core before leakage and after return from the neutron reflector are so short that they may be neglected [6]. If the neutron reflector is divided on sufficiently large number of thin layers, then...
neutrons will spend main time fraction on diffusion from the reactor core to a certain layer and back (“dead-time”) and very small time fraction on the intra-layer diffusion, i.e. $\tau^R_k \ll t^R_k$. Under these conditions, i.e. $\Lambda_{det} = \Lambda_C$, $\tau^C_j = \Lambda^C_j$, $t^R_k = 0$, the in-hour equation (4) may be re-written in the following form:

$$\rho = \frac{\Lambda_C}{T} + \sum_{j=1}^{6} \frac{\Lambda_j \cdot \beta_j}{T + \Lambda_j} + \sum_{j=1}^{J} \rho^j \cdot 1 \exp \left( -\Lambda^C_j / T \right) \right]$$ (5)

Numerical calculations were carried out for rather thick (4 m) $^{208}$Pb-reflector. The results showed that the use of the reflected neutron lifetime $\Lambda^R_j$ as the “dead-time” in equation (5) as compared with the in-hour equation (2) can increase the third summand on 17-20% and the necessary reactivity $\rho$ on 8-15% for relatively short asymptotic periods of power excursions (0.1-0.01 s). These values of the asymptotic periods are comparable or even shorter than the “dead-time” for the most long-lived neutrons coming back from the neutron reflector. This means that the in-hour equation (2) gives a conservative evaluation of reactivity, i.e. with a certain margin for safe reactor operation.

Under sufficiently slow power excursions, when asymptotic period is significantly longer than lifetime of the reflected neutrons ($T \ll \Lambda^R_j$, $j = 1,...,J$), the third summands of the in-hour equations (2), (3) and (5) coincide each other. The in-hour equations may be re-written in the following common form:

$$\rho = \frac{\Lambda_C}{T} + \sum_{j=1}^{6} \frac{\Lambda_j \cdot \beta_j}{T + \Lambda_j} + \sum_{j=1}^{J} \rho^j \cdot \Lambda^R_j / T$$

So, all three models of neutron kinetics in the reflected reactors may be transformed one to another, i.e. they are equivalent ones under certain conditions.

5. Conclusion

The proposed model of neutron kinetics in the reflected reactors (see the in-hour equation (2)) is a symbiosis of the two-point model of neutron kinetics and the model of additional time groups of delayed neutrons emitters. In the proposed model the neutron reflector is divided on several spatial layers (points), and each layer (point) is characterized by appropriate delayed neutrons parameters. The proposed model is a simple and non-contradictory one because the neutron reflector is taken into consideration in the form of additional time groups of delayed neutrons emitters, and these groups are characterized by the same parameters as traditional delayed neutrons emitters, namely fraction of delayed neutrons group (the reactivity increment induced by the $j$-th layer of the neutron reflector) and mean lifetime of delayed neutrons emitters (mean lifetime of neutrons coming back from the $j$-th layer of the neutron reflector). The model of additional time groups of delayed neutrons emitters in the monograph [6] includes, in addition to the delayed neutrons parameters mentioned above, the parameter of neutron “dead-time”, unusual parameter for traditional delayed neutrons. So, there is a terminological contradiction between these models.

Finally, by comparing different models of neutron kinetics, it was shown that the proposed model is not only obvious and simple in usage but also the model is able to give a reliable conservative evaluation for the reactivity, i.e. the model provides a certain safety margin that is an acceptable option for advanced scientific studies.

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