Unravelling the role of natural weak value amplification, waveguide-plasmon strong coupling and avoided crossing on the giant Faraday effect in magneto-plasmonic crystals

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Abstract
The weak magneto-optical effects of materials can be greatly enhanced by hybridizing them with plasmonic structures which not only enables nonreciprocal nano devices but also provides another degree of freedom for control and manipulation of light in the nanometer length scale. Here, we unravel the distinctly different physical origin of the giant enhancement of Faraday rotation and ellipticity in a hybrid magneto-plasmonic system, namely, waveguided magneto-plasmonic crystal for excitation with transverse electric (TE) and transverse magnetic (TM) polarized light. For excitation with TE polarization, where the surface plasmons are not directly excited, the natural weak value amplification of Faraday effect which appears due to the spectral domain interference of Fano resonance is the dominant cause for the enhancement of Faraday rotation and ellipticity. In such scenario, optimal weak value amplification takes place for certain geometrical parameters of the plasmonic crystals for which there is a strong overlap of the resonance frequencies of TE and the TM waveguide modes. For TM polarization excitation on the other hand, waveguide-plasmon strong coupling and its universal manifestation of avoided crossing plays an intriguing role in the enhancement of the magneto-optical effects. The enhancement is observed to be the strongest in the avoided crossing regime. These results enrich the existing understanding of the observed giant enhancement of magneto-optical effects and provide potentially useful information on optimizing plasmonic crystal structure for its potential applications in nanoscale devices.
I. Introduction

Hybridization of magneto-optic (MO) materials with plasmonic nanostructures enables magnetic-tuneable multifunctional nano-devices that provides precise control over its plasmonic and magnetic properties\(^1\,^2\). Moreover, such systems are also known to exhibit giant enhancement of the MO effects (Faraday, Kerr effect etc.)\(^2\,^8\). Note that the non-reciprocal nature of the MO effects is extremely crucial for developing optical isolators\(^3\), modulators, and optical-magnetic data storage devices\(^1,^8\) etc. The very weak MO effects of the available materials are a major stumbling block towards their applications in nano scale devices. Significant efforts have therefore been delivered in the last few years to demonstrate enhancement of both the magneto-optical Kerr effect and the Faraday effect in magneto-plasmonic nanostructures\(^2\,^4\,^7\). Earlier approaches for increasing the MO activity of the systems were mostly based on the excitation of surface plasmons, where the near field enhancement associated with the surface plasmons leads to substantial enhancement of MO responses of samples\(^9\,^10\). In this regard, a recent study has attracted a lot of attention that has demonstrated giant enhancement of Faraday rotation in a waveguided magneto-plasmonic crystal (WMPC)\(^3\) system. Some other related studies have reported even larger enhancement of Faraday rotation in various magneto-plasmonic systems using similar approach\(^4\,^6\). In most of these systems, the enhancement of MO effects is attributed to either plasmon-induced near field enhancement or the coupling of waveguide-plasmon-polariton \(^2\,^4\,^6\). In either of the scenarios, the MO effect enhancement is crucially related to the excitation of the surface plasmon resonance, which in case of the investigated planar structures can only be excited using transverse magnetic (TM) polarization of light. In contrast, in our recent investigation a large enhancement of the Faraday rotation and ellipticity was shown in a Fano resonant WMPC for input TE polarization where no surface plasmons were directly excited\(^11\). This observation initially indicated towards another important mechanism of the enhancement, namely, the natural weak value amplification (WVA) of Faraday rotation and ellipticity due to spectral domain interference of Fano resonance. Proper understanding of the physical origin of the enhanced MO effects in hybrid magneto-plasmonic system and its dependence on the various physically accessible parameters of the system and excitation light is of paramount importance for both fundamental and applied interests.

The concept of weak measurement and WVA was introduced by Aharonov, Albert, and Vaidman\(^13\,^17\). This special measurement process involves three steps, quantum state preparation (pre-selection), a weak interaction, and post-selection on a final quantum state, which is nearly orthogonal to the initial state. The outcome of such measurement, the so-
called weak value may not only become exceedingly large and lie outside the eigenvalue spectrum of the observable but can also assume complex values\textsuperscript{13-15}. The quantum mechanical concept of WVA can be understood within the realm of wave optics and thus most of the experiments on weak measurements are performed in classical optical setting\textsuperscript{17-23}. The WVA concept can be interpreted as near destructive interference between the eigenstates of the measuring observable as a consequence of nearly mutually orthogonal pre and post selections of the system states\textsuperscript{13,14}. Based on this simple yet profound interferometric philosophy of WVA, we had initially shown that a similar situation arises in the spectral domain interference of Fano resonance that leads to natural WVA of small Faraday rotation and ellipticity in WMPC\textsuperscript{11}, which act as the weak interaction in this scenario. The preliminary results indicated that for TE polarization excitation, the giant enhancement of the Faraday rotation and ellipticity in Fano resonant WMPC may primarily be attributed to this natural interferometric WVA\textsuperscript{11}. In contrast, for TM polarization excitation, the underlying physical mechanisms for the MO effect enhancement are expected to be rather complex and more interesting due to simultaneous involvement and coupling of the waveguide mode and the plasmon mode in such systems. As there appears to have different possible mechanisms of the enhancement of MO effects in magneto-plasmonic crystals\textsuperscript{2-4,10} including the one of natural WVA\textsuperscript{11}, the question arises whether there is a common physical origin for the enhancement irrespective of the input polarization state or are there distinctly different mechanisms with varying strengths depending upon the excitation polarization and the geometrical parameters of the WMPC. The main purpose of this work is to address this important question and to shed light upon the competing roles of the different contributing mechanisms, namely, the natural weak value amplification and the purity of Fano resonance\textsuperscript{11}, near field enhancement effects due to the excitation of the surface plasmons\textsuperscript{2}, resonance enhanced cross coupling between the TE and TM polarization\textsuperscript{3,4}, the waveguide-plasmon strong coupling \textsuperscript{3,4} and the related phenomenon of avoided crossing and so forth on the giant enhancement of magneto-optical effects (Faraday rotation and ellipticity).

The paper is organized as follows. We first provide the details of the WMPC structures and the specifics of the numerical simulations using Finite element method (FEM). The theoretical framework of natural interferometric WVA in Fano resonance is presented next and the results of the enhancement of Faraday rotation and ellipticity with TE polarization excitation in WMPC having varying geometrical parameters and the corresponding natural weak value interpretation are presented subsequently. We then present the simulation results for the enhancement of MO effects with TM polarization excitation for
similar WMPCs with varying geometrical parameters. The intriguing roles of the electromagnetic near field enhancement due to the excitation of the surface plasmons, the waveguide-plasmon strong coupling and the avoided crossing behaviour\textsuperscript{24,25} on the enhancement of the Faraday rotation and ellipticity are analyzed and interpreted. The paper concludes with a summary of the gained quantitative understanding on the different physical mechanisms of the MO effect enhancement, which in turn provides a useful recipe for tailoring their contributions to achieve optimal enhancement through optimization of the geometrical parameters of the nanostructure.

II. Methods

\textit{Finite element Method (FEM) simulation of the WMPCs}

The WMPC system comprises of a periodic array of noble metal nanostructures on top of a dielectric waveguiding layer, which is made of MO materials that exhibit Faraday rotation and ellipticity. The coupling of the spectrally broad surface plasmon mode in the metallic nanostructures or the photon continuum (depending upon the excitation polarization of light) with the narrow quasiguided photonic modes in the waveguiding layer leads to Fano resonance. A schematic illustration of the FEM simulation\textsuperscript{26} of transmission spectra and MO responses of the WMPC is presented in \textbf{Figure 1}. The WMPC system investigated in this study consists of gold (Au) grating on top of a thin Y-BIG film and the substrate is taken to be quartz. The MO active Y-BIG film serves as the waveguiding layer and additionally exhibit Faraday effect in the presence of an external magnetic field. For input TE (y) polarization, the electric field of light is oriented parallel to the axis of the Au grating, whereas for TM (x) polarization, the electric field is perpendicular to the grating (\textbf{Fig. 1a}). All the simulations are performed for normal incidence of light and the thickness of the waveguide layer are kept constant ($t = 150 \text{ nm}$) (\textbf{Fig. 1b}). The far field optical transmittance spectra ($E = h\omega = 0.83 \text{ to } 2.067eV$, corresponding to $\lambda = 1493.78 \text{ to } 599.82 \text{ nm}$) and the corresponding spectral dependence of Faraday rotation and ellipticity of the WMPC system are studied by systematically varying the grating periodicity ($d$), width ($w$) and height ($h$). For the simulations, the permittivity tensor elements of the Y-BIG film were taken, $\epsilon_{11} = \epsilon_{33} = 6.7 + 0.053i$ and $g = 0.016 - 0.0092i$ for typical magnetic field of 140 mT\textsuperscript{3} and dielectric permittivity of Au was taken from literature\textsuperscript{27}.

The spectral variations of the Faraday rotation and ellipticity from the transmitted intensity spectra of the WMPC were generated using standard Stokes polarization method\textsuperscript{28}. 


Briefly, for a given input polarization of light (TM-x or TE-y), six different polarization resolved intensity components of the transmitted light were recorded, namely, $I_H$ – horizontal (x) linear polarization, $I_V$ – vertical (y) linear polarization, $I_{45}$ – $45^\circ$ linear polarization; $I_M$ – $135^\circ$ linear polarization; $I_R$ – right circular polarization, $I_L$ – left circular polarization. These intensities were used to yield the spectral variation of the Stokes parameters $[I, Q, U, V]^T$. The observed net Faraday rotation (which is subsequently defined as polarization vector orientation angle $\psi$) and net Faraday ellipticity (subsequently defined as $\xi$) are determined from these Stokes parameters of transmitted light using subsequent equations, Eq. 3a and Eq. 3c. Note that the angle $\psi$ is always defined with respect to the input polarization (x or y).

III. Theoretical Framework of natural weak value amplification of Faraday effects in a Fano resonant WMPC

We have recently developed a theoretical treatment for natural WVA of polarization rotation and ellipticity in the spectral domain interference of Fano resonance. This concept is based on the interpretation of the WVA phenomenon as near destructive interference between slightly shifted pointer profiles as a result of mutually nearly orthogonal pre-post selection of states. Near destructive interference refers to destructive interference of two fields (having a phase difference $\pi$) with a small amplitude offset ($\varepsilon_a \approx 1 - a^2$, $a$ is the ratio of amplitudes of the two fields) or alternatively, interference of two fields having equal amplitude but with a phase difference of $(\pi + \varepsilon_p, \varepsilon_p$ is a small phase offset). It was shown a similar situation of WVA of small Faraday rotation and ellipticity naturally arises near the Fano spectral dip corresponding to the spectral domain destructive Fano interference between an optically active (having small Faraday rotation and ellipticity) narrow quasiguided (waveguide) mode and a polarization isotropic photon continuum in WMPC. Briefly, in this model, the spectral ($\omega$) variation of the polarized electric field of Fano resonance is expressed as the interference of a $y$-polarized frequency-independent continuum mode with an optically active narrow resonance mode exhibiting small rotation ($\alpha$) and ellipticity ($\chi$). The resultant polarized field can be expressed as

$$E_x(\omega) \approx \left[ \frac{q-i}{\varepsilon(\omega)+i} \{ (\cos \alpha \cos \chi - i \sin \alpha \sin \chi) \hat{y} + (\sin \alpha \cos \chi + i \cos \alpha \sin \chi) \hat{x} \} + \hat{y} \right](1)$$

Here, $\varepsilon(\omega) = \frac{\omega - \omega_0}{(\gamma/2)}$, where $\omega_0, \gamma$ are the central frequency and the width of the narrow resonance respectively. The q parameter is related to the coupling of modes and determines the characteristic asymmetry of Fano spectral line shape. WVA of small polarization rotation
(\(\alpha\)) and ellipticity (\(\chi\)) takes place as a consequence of near destructive spectral domain interference between the two modes with slightly different polarization states (which arises due to the presence of the small optical activity of the narrow resonance mode, acting as the weak interaction parameter in this weak measurement scenario) around the frequency of the Fano spectral dip \(\omega_F = \left(\omega_0 - \frac{q\gamma}{2}\right)\) corresponding to \(\varepsilon = -q\). Note that \(\omega_F\) is the frequency of exact destructive interference where the phase difference between the two modes becomes \(\Psi(\omega_F) = \pi\) and the ratio of the amplitudes of the two modes is unity \(a(\omega_F) = 1\). At close vicinity of \(\omega_F\), near destructive spectral interference takes place between the two modes with simultaneous small amplitude offset \((\epsilon_a(\omega))\) and phase offset \((\epsilon_p(\omega))\). It is this near destructive spectral interference between the two modes that mimics the near orthogonal pre and post-selections of states (small overlap of states \(\epsilon_{a/p}\)) in conventional optical WVA. The expressions for the offset parameters are obtained in terms of the parameters of Fano resonance as

\[
\epsilon_a(\omega) \approx \frac{1-\left(\frac{q+1}{q+2}\right)}{1+\left(\frac{q+1}{q+2}\right)}; \quad \epsilon_p(\omega) = \pi - \Psi(\omega) = \pi - \tan^{-1}\left[\frac{q+\varepsilon}{1-q\varepsilon}\right]. \quad (2)
\]

Using the framework of WVA, it can be shown that the real and the imaginary WVA of the polarization rotation \(\alpha\) and ellipticity \(\chi\) are manifested as \(\epsilon_{a/p}\) dependent large changes in the polarization state of light (Stokes vector elements), which acts as the pointer here. The corresponding WVA variations are conveniently described by the intensity-based Stokes vector elements. The simultaneous real and imaginary WVAs of \(\alpha\) and \(\chi\) can be shown to manifest in the \(\epsilon_{a/p}\) dependence of the observed polarization vector orientation angle \(\psi\) (determined from the \(U\) and \(Q\) Stokes parameters) and the circular (elliptical) polarization descriptor \(4^{th}\) Stokes vector element \(\left(\frac{V}{I}\right)\), and the observed net ellipticity \(\xi\) of the transmitted light as (see Appendix A1 for details)

\[
\psi = \frac{1}{2}\tan^{-1}\left(\frac{U}{Q}\right) \approx -\frac{\alpha}{2\varepsilon_a} - \frac{1}{2} \chi \cot \epsilon_p \quad (3a)
\]

\[
\frac{V}{I} \approx \alpha \cot \epsilon_p - \frac{\chi}{\epsilon_a} \quad (3b)
\]

\[
\xi = \frac{1}{2}\sin^{-1}\left(\frac{V}{I}\right) \quad (3c)
\]

Note that we have used two sets of notations for optical activity parameters, the first set, \((\alpha)\) and \((\chi)\) correspond to intrinsic rotation and ellipticity (respectively) of the optically active narrow resonance mode. Whereas \(\psi\) and \(\xi\) represent the net optical rotation
(polarization vector orientation angle) and net ellipticity (respectively) determined from the transmitted light following natural WVA or spectral domain interference of the optically active narrow resonance mode with a polarization isotropic continuum mode. Note that this theoretical framework is a generic one, which is applicable to a broad range of Fano resonant systems irrespective of their geometrical complexities in the presence of an appropriate weak interaction parameter (like the small Faraday rotation and ellipticity of WMPCs). We have subsequently used this theoretical treatment to analyze and interpret the enhancement of the Faraday rotation and ellipticity of the Fano resonant WMPCs for TE polarization excitation. Specifically, for TE polarization excitation of the WMPCs, prominent Fano resonance occurs in the transmitted intensity profile due to the interference of a polarization isotropic photon continuum and the optically active narrow quasi-guided photonic modes. These results are presented next.

IV. Results and Discussions

a. Enhancement of MO effects in WMPCs for TE polarization excitation as a manifestation of interferometric natural weak value amplification in Fano resonance

The simulated transmittance spectra \( (E = \hbar \omega = 0.83 \text{ to } 2.067 \text{eV}) \) and Faraday rotation and ellipticity for varying geometrical parameters of the WMPC with TE polarization excitation are summarized in Figure 2. The transmission spectra (1st row), the spectral variation of the observed Faraday rotation \( \psi \) (2nd row) and the observed net ellipticity \( \xi \) (3rd row) of the WMPC are shown for varying periodicity \( (d, 1^{\text{st}} \text{ column}) \), width \( (w, 2^{\text{nd}} \text{ column}) \) and height \( (h, 3^{\text{rd}} \text{ column}) \) of the Au grating. Several observations are in place. Prominent signatures of Fano resonance with asymmetric spectral line shape are observed in all the transmission spectra with TE polarization excitation (1st row). Here, the surface plasmons are not directly excited and hence the Fano resonance arises due to the interference between the spectrally narrow quasiguided resonance mode with an ideal photon continuum. The appearance of an additional spectral dip in the high frequency region is signature of the excitation of higher order quasiguided modes. Importantly, giant enhancements of the Faraday rotation (2nd row) and ellipticity (3rd row) are observed around the Fano spectral dip \( (E = \hbar \omega_f) \) for all the geometrical parameters of the WMPC. In agreement with previous reports\(^{25}\), the Fano dip is observed to gradually move towards shorter frequency (or energy \( E = \hbar \omega \)) with increasing periodicity \( (d) \) of the grating (Fig. 2a). The magnitudes of the enhanced Faraday rotation \( \psi \) (Fig. 2b) and ellipticity \( \xi \) (Fig. 2c) exhibit an interesting trend
with varying $d$, which increase to a maximum value for a certain value of periodicity and then follows a gradual decrease. The maximum magnitudes of the enhanced Faraday rotation ($\psi \sim 6.20^\circ$) and ellipticity ($\xi \sim 5.74^\circ$) of the WMPC are obtained for a value of $d= 600$ nm (Fig. 2b), as compared to the corresponding maximum bare film rotation and ellipticity of 0.28$^\circ$ and 0.25$^\circ$, respectively. Neither the value of the width $w$ nor the height $h$ of the Au grating has any significant influence on the quasiguided modes and hence the resulting Fano spectral line shape of the WMPC is not sensitive to variations of the $w$ and $h$-parameters of the grating (Fig. 2e, 2i). Accordingly, the magnitude and spectral variation of both the Faraday rotation (Fig. 2j) and ellipticity (Fig. 2k) do not exhibit appreciable variation with the change in the height $h$ of the grating. Similarly, with varying width $w$ of the grating, the Faraday rotation $\psi$ (Fig. 2f) and ellipticity $\xi$ (Fig. 2g) exhibit only very weak variations (Faraday effects seem to be marginally stronger for $w = 120$ nm as compared to others). The spectral variation of the transmitted TM polarization intensities with TE polarization excitation are also plotted for each of the varying geometrical parameters of the WMPC (Fig. 2 4th row). The overall TM polarized intensities are considerably weak and approach their respective minima near the Fano spectral dip where maximum magnitudes of Faraday rotation and ellipticity are observed. This rules out the possibility of the role of the resonance enhanced cross-coupling of the TE and the TM quasiguided modes in the enhancement of the Faraday effect. The observed enhancement of the Faraday rotation and ellipticity around the Fano spectral dip corresponding to near destructive spectral domain interference between the quasiguided mode and the photon continuum thus points towards the vital role of the natural WVA of Fano resonance for TE polarization excitation. This aspect is therefore investigated in further details by studying the dependence of the Faraday effect enhancement on the periodicity of the grating and its natural weak value interpretation.

Figure 3 summarizes the results of natural WVA interpretation of Faraday rotation and ellipticity enhancements for TE polarized excitation for Fano resonant WMPCs with two different values of periodicity of the Au grating ($d = 600$ nm, Fig. 3a and $d= 450$ nm Fig. 3b) and having the same width and height $w= 120$ nm and $h = 60$ nm, respectively. The spectral ($E = 0.83$ to $2.067eV$) variations of both the Faraday rotation $\psi$ (Fig. 3a(i) and Fig. 3b(i)) and ellipticity $\xi$ (Fig. 3a(ii) and Fig. 3b(ii)) are plotted along with the transmitted intensity profiles of the WMPCs. In order to analyze these results using the natural WVA formalism of Fano resonance, the spectral line shape of the transmitted intensity is first fitted with Fano intensity formulae (Eq. A3 of Appendix). The relevant Fano resonance parameters, namely,
the central frequency of the interfering narrow resonance of the quasiguided mode \( \omega_0 \), width of the narrow resonance \( \gamma \), the Fano spectral asymmetry parameter \( q \) are estimated from this fitting. These are subsequently used to determine the exact values of the frequency corresponding to the Fano dip \( \omega_F = \left( \omega_0 - \frac{q\gamma}{2} \right) \) and the amplitude offset \( (\epsilon_a) \) and the phase offset\((\epsilon_p) \) parameters (using Eq. 5) in the natural WVA formalism. As discussed previously, at the frequency corresponding to the spectral domain destructive interference \( (\omega_F) \), both the offset parameters \((\epsilon_a,\epsilon_p) \) are zero and as one moves spectrally slightly away from \( \omega_F \), both the \( \epsilon_a,\epsilon_p \) weak measurement parameters vary simultaneously (shown in Fig. 3a (iv)). The complex (simultaneous real and imaginary) WVA of the weak Faraday effect is accordingly manifested as large enhancement of both the Faraday rotation and ellipticity when \((\epsilon_a,\epsilon_p \rightarrow 0) \) for the WMPC with \( d = 600 \) nm (Fig. 3a (v) and (vi)). In order to further comprehend the important role of the natural WVA of Fano resonance, we have shown the comparison of the transmitted TM and TE polarized intensities with TE polarized excitation (Fig. 3a (iii)). This clearly shows that the transmitted TM polarized intensity is much weaker than the TE polarization signal and both approach their respective minima in the vicinity of the Fano spectral dip where the rotation reaches its maximum value. This indicates that the amplification of the weak Faraday effect arises due to the near destructive interference of the TE polarized quasiguided mode with the continuum mode at close vicinity of the Fano spectral dip and it takes place at the expense of the total intensity signal, which is a universal signature of WVA. The simulated Faraday rotation \( \psi \) and ellipticity \( \xi \) for the WMPCS having \( d = 600 \) nm also follows the \((\propto \alpha \cot \frac{\epsilon_a}{\epsilon_p} \sim \frac{\alpha}{\epsilon_a/\epsilon_p}) \) behaviour (Fig. 3a (v) and (vi)) as predicted by the natural WVA formalism (Eq. 3a -3c). A quantitative comparison of the amplified Faraday rotation \( \psi \) (Fig. 3a (v)) and the ellipticity \( \xi \) (Fig. 3a (vi)) with the exact theoretical predictions of natural WVA of Faraday effect in Fano resonance (using Eq. A5 and A6 of Appendix) shows excellent agreement. This confirms that natural WVA in Fano resonance is the underlying reason for the enhancement of weak Faraday effect in the WMPC system having a value of periodicity \( d = 600 \) nm. We now move on to inspect similar results for the WMPC with periodicity \( d = 450 \) nm (Fig. 3b). An important difference between the WMPC with \( d = 450 \) nm as compared to that with \( d = 600 \) nm is that while for the latter the resonance frequencies of the TM and TE quasiguided modes perfectly overlaps \((\omega_0^{TE} = \omega_0^{TM})\), for the former these two frequencies are separated \((\omega_0^{TE} \neq \omega_0^{TM})\). Note that the ideal natural WVA of Fano resonance demands that the peak frequencies of the narrow resonance
for the $x$ and $y$ polarizations have to be similar (as per Eq. 1). Thus, according to the model of natural WVA of Faraday effect in Fano resonance, the weak value amplification of small Faraday rotation an ellipticity is expected to be more pronounced when $\omega_0^{TE} = \omega_0^{TM}$. This indeed is the case as the enhancement of both the Faraday rotation (Fig. 3b(i)) and ellipticity (Fig. 3b(ii)) are considerably weaker for the WMPCS with $d = 450$ nm as compared to that for $d = 600$ nm (Fig. 3a(i) and Fig. 3a(ii)). In such case, for obvious reasons, the exact theoretical predictions of natural WVA of Faraday effect in Fano resonance also significantly deviate from the observed Faraday rotation $\psi$ (Fig. 3b (v)) and ellipticity $\xi$ (Fig. 3b (vi)). To summarize, the results presented in Fig.2 and Fig. 3 provide conclusive evidence that natural WVA of Faraday effect in the spectral domain interference of Fano resonance is the primary mechanisms for the enhancement of Faraday rotation and ellipticity in WMPCs with TE polarization excitation. Moreover, strong spectral overlap between the TM ($x$) and TE ($y$) quasiguided resonance modes is an essential criteria for optimal enhancement of Faraday rotation and ellipticity through ideal weak value amplification of Faraday effect in Fano resonance in the WMPCs. We now turn our attention to the enhancement of Faraday effect in WMPCs with TM polarization excitation, where the surface plasmons are directly excited and hence the underlying mechanism is expected to be fundamentally different and more complex.

b. Enhancement of MO effects in WMPCs for TM polarization excitation: role of the waveguide-plasmon strong coupling and avoided crossing

With TM polarization excitation, the spectrally narrow quasiguided mode and the broad surface plamon modes are simultaneously excited, leading to hybridization and strong coupling of waveguide-plasmon modes. As a result of this, the waveguide-plasmon hybrid mode splits the resonance and two dips are observed in the transmission spectra ($E = 0.83$ to $2.067$ eV) of the WMPC (Figure 4, 1st row). Even though there is mild spectral asymmetry in the transmission spectra, the Fano resonance is not as prominent as that for TE polarization excitation. One reason for this is the fact that in case of TE polarization excitation, the photon continuum provides an ideal continuum as desirable for ideal Fano resonance, whereas the broad surface plasmon mode has finite line width and spectral profile. Despite the fact that one has hybridized waveguide-plasmon modes, the dominant contributions of the two different modes in the observed spectral dips can still be distinguished for certain geometrical parameter range of the WMPC. For example, at lower
value of the periodicity of the grating $d = 450$ nm, the transmission dip at the lower frequency end ($E = \hbar \omega \sim 1.31 \text{ eV}$) shows broader spectral feature which therefore appears to be dominated by the broad surface plasmon mode (Fig. 4a). The sharp waveguide resonance, on the other hand, dominates the behaviour of the narrower dip at the higher frequency end ($E \sim 1.77 \text{ eV}$) (discussed subsequently). With increasing value of the periodicity $d$ of the grating, the two modes (waveguide mode and the plasmon resonance mode) approach spectrally closer to each other and a clear avoided or anti-crossing behaviour is observed as a universal manifestation of the strong coupling phenomenon (Fig. 4a). It is however, important to note that large enhancement of both the Faraday rotation $\psi$ (Fig. 4b) and ellipticity $\xi$ (Fig. 4c) are always observed in the spectral dip of the lower frequency end which is primarily dominated by the surface plasmon mode for lower value of the grating periodicity (below the avoided crossing regime) and is dominated by the waveguide mode at higher grating periodicity (above the avoided crossing). Thus, both the electromagnetic field enhancement associated with the surface plasmons and the formation of the strongly coupled waveguide-plasmon polariton system play important role in this enhancement. A trend similar to that observed for the TE excitation is also observed here with varying periodicity $d$ of the grating (Fig. 4, 1st column), the magnitudes of the enhanced Faraday rotation $\psi$ (Fig. 4b) and ellipticity $\xi$ (Fig. 4c) attain a maximum value for a specific value of $d$ and then decreases. Here, the maximum enhancement of the rotation ($\psi \sim 8.60^\circ$) and ellipticity ($\xi \sim 8.84^\circ$) are obtained for the range of $d \sim 530 - 575$ nm. This is particularly interesting as this periodicity range is exactly within the window of the avoided crossing regime where the splitting is minimum and the two modes are spectrally the closest (shown subsequently). This underscores the vital role of the strong coupling and the avoided crossing of the waveguide-plasmon modes in the resulting enhancement of Faraday effect. Note that a very similar trend is also observed for varying width $w$ of the grating in the WMPC (Fig. 4, 2nd column). The avoided crossing behaviour of the two modes appears in the transmission spectra since the width parameter $w$ changes the spectral position of the plasmon resonance (Fig. 4e). The maximum enhancement of the Faraday rotation (Fig. 4f) and ellipticity (Fig. 4g) takes place for 120 nm grating width, which is once again within the window of the avoided crossing regime of minimum splitting. No significant changes in the transmission spectra (Fig. 4i) are observed for the variation of height $h$ of the grating as it neither affects the spectral response of the plasmons or the waveguide resonances (Fig. 4, 3rd column). For each of the cases of varying periodicity, width and the height of the grating in the WMPCS,
the transmitted TE polarization spectra are also shown with TM polarization excitation (Fig. 4, 4th row).

In order to further comprehend the role the avoided crossing behaviour associated with the strong coupling of the waveguide-plasmon modes, the frequencies (energy $E = h\omega$) corresponding to the two spectral dips are shown as a function of varying value of periodicity $d$ of the grating in the WMPC (Fig. 5a). A clear avoided crossing behaviour is apparent and the minimum spectral difference (minimum splitting) between the two branches appears for the grating periodicity $d \sim 530 - 575$ nm, for which the maximum enhancement of the magnitudes of the Faraday rotation $\psi$ (Fig. 5b) and ellipticity $\xi$ (Fig. 5c) are observed. To understand the role of the hybridization of the plasmon and the waveguide modes, the simulated spatial distribution of the magnetic field for input TM polarization ($H_y$) at the energies (frequencies) corresponding to the two transmission dips are shown for three different grating periods. The first one is for grating period $d = 450$ nm (Fig. 5d) below the avoided crossing regime, the second one is for $d = 550$ nm (Fig. 5e) which is exactly at the avoided crossing regime corresponding to minimum splitting, and the 3rd one is for $d = 700$ nm (Fig. 5f) above the avoided crossing. For grating period $d = 450$ nm below the avoided crossing, the field distribution corresponding to the spectral dip at the lower frequency ($E = 1.31$ eV) shows dominant character of plasmon resonance, whereas the waveguide field distribution is more visible at the longer frequency ($E = 1.77$ eV) region (Fig. 5d). Above the avoided crossing of the modes, the natures of the two modes flip which can be clearly observed from the spatial field distribution for $d = 700$ nm grating period (Fig. 5f). The lower frequency ($E = 1.08$ eV) dip now shows prominent signature of waveguide field distribution and the dip at the higher frequency ($E = 1.42$ eV) is dominated by the plasmon mode. For grating period $d = 550$ nm near the avoided crossing, a stronger hybridization of the waveguide-plasmon mode is observed, where maximum enhancement of the rotation and ellipticity are also observed. Note that the enhancement of Faraday effect is observed in the lower frequency spectral dip in the avoided crossing regime, where the electromagnetic field enhancement is maximum (Fig. 5d,e,f). It is also extremely crucial in this regard that in the window of the avoided crossing (at $\sim d = 550$ nm grating periodicity), the plasmon and the waveguide modes are spectrally nearest to each other leading to the intersection of the TE waveguide mode dispersion curve with the lower frequency dip of the TM-excited hybrid waveguide-plasmon mode (shown in Fig. 5a). This therefore provides an ideal scenario of
resonance enhanced cross-coupling between the TM-TE polarizations enabling maximum enhancement of Faraday effect in this region.

V. Conclusions

In summary, our studies have unravelled distinctly different physical origins for the enhancement of Faraday rotation and ellipticity in magneto-plasmonic crystals with TE and TM polarization excitation. Natural weak value amplification of weak Faraday effect that arises due to near destructive spectral domain interference in Fano resonance of the plasmonic crystals is identified as the primary mechanisms for enhancement with TE polarization excitation. The results also showed that the enhancement of the magneto-optical effects are maximum for those geometrical parameters of plasmonic crystals which exhibit a strong spectral overlap of the TE and TM quasiguided modes, in which case an ideal weak value amplification takes place. In contrast for TM polarization excitation, electromagnetic near field enhancement associated with the surface plasmons and the strong coupling of the waveguide-plasmon mode plays the dominant role in the Faraday effect enhancement. The enhancements are the strongest near the avoided crossing regime where the waveguide-plasmon modes are spectrally the closest and the splitting is minimum. These results are of both fundamental and applied interests. From a practical point of view of developing multifunctional non-reciprocal nanodevices, these results are important as it provides useful recipe for controllably tailoring the contributions of the different mechanisms to achieve optimal enhancement of the magneto-optical effects through optimization of the geometrical parameters of the nanostructure for either TE or TM polarizations.

APPENDIX

A1. Theoretical model of natural weak value amplification in Fano resonance

Our natural weak value amplification (WVA) formalism is founded on a simple but intuitive model of optical Fano resonance that uses coherent interference of a narrow resonance mode described by a complex Lorentzian with a frequency-independent continuum (or broad) mode. It has been observed earlier that similar simple model can capture the near field mode coupling and the Fano interference effect in various optical systems including the plasmonic crystals. The resultant Fano scattered electric field is given by

\[
E_s(\omega) \approx \left[ \frac{\psi}{\epsilon+i} + B \right] = \left[ \frac{\sqrt{q^2+1}}{\sqrt{\epsilon^2+1}} e^{i\psi(\omega)} + B \right]
\]
In this model, the spectral domain interference of the modes can be understood through the phase difference between them, which is very important for our weak value analysis. The total phase difference between two interfering modes $\Psi(\omega)$ is

$$\Psi(\omega) = \tan^{-1} \left[ \frac{q + i \epsilon}{1 - q i \epsilon} \right]$$  \hspace{1cm} (A2)

The resulting expression for the transmitted intensity of electric field described in Eq. A1 can be obtained as

$$I_s(\omega) = |E_s(\omega)|^2 = B^2 \times \left[ \frac{(q_{eff} + 1)^2}{(e^2 + 1)^2} \right] + \frac{(B - 1)^2}{(e^2 + 1)^2} \hspace{1cm} (A3)$$

The parameter $B$ is the relative amplitude of the continuum with respect to the field of the narrow resonance and determines the contrast of the spectral domain interference. For the ideal situation of $B = 1$, the perfect destructive interference occurs at the Fano frequency $\omega_F = (\omega_0 - \frac{q\gamma}{2})$ corresponding to ($\epsilon = -q$, $\Psi(\omega_F) = \pi$). Equation 1 describes the polarized electric field in the natural WVA of Faraday effect in Fano resonance, which is based on Eq. A1. As described previously, natural WVA of Faraday effect takes place due to near destructive spectral domain Fano interference between the two modes with slightly different polarization states. The WVA are subsequently interpreted through the small amplitude offset ($\epsilon_a$) and phase offset ($\epsilon_p$) parameters. In general, real WVA is observed with varying $\epsilon_a$ and imaginary WVA is manifested with varying $\epsilon_p$. However, in this case of natural WVA of Faraday effect in Fano resonance both real and imaginary WVA are observed simultaneously because both the $\epsilon_a$ and the $\epsilon_p$ parameters vary simultaneously as one moves away from $\omega_F$. The resultant Fano scattered electric field described in Eq. 1 can be written in terms of simultaneously varying $\epsilon_a$ and the $\epsilon_p$ parameters in this complex WVA scenario as

$$E_s(\omega) = \left[ (1 + \epsilon_a) e^{+i\epsilon_p} \left( \cos \alpha \cos \chi - i \sin \alpha \sin \chi \right) \hat{y} + \left( \sin \alpha \cos \chi + i \cos \alpha \sin \chi \right) \hat{x} \right] - (1 - \epsilon_a) e^{-i\epsilon_p} \hat{y}$$  \hspace{1cm} (A4)

The corresponding WVA are manifested in the Stokes vector elements $[I, Q, U, V]^T$ as

$$I = 2(1 + \epsilon_a^2) - 2(1 - \epsilon_a^2) \left( \cos \alpha \cos \chi \cos 2\epsilon_p + \sin \alpha \sin \chi \sin 2\epsilon_p \right)$$

$$Q = -(1 + \epsilon_a)^2 \cos 2\alpha \cos 2\chi - (1 - \epsilon_a)^2$$

$$+ 2(1 - \epsilon_a^2) \left( \cos \alpha \cos \chi \cos 2\epsilon_p + \sin \alpha \sin \chi \sin 2\epsilon_p \right)$$
\[ U = (1 + \epsilon_a)^2 \sin 2\alpha \cos 2\chi - 2(1 - \epsilon_a^2)\left( \sin \alpha \cos \chi \cos 2\epsilon_p - \cos \alpha \sin \chi \sin 2\epsilon_p \right) \]

\[ V = -(1 + \epsilon_a)^2 \sin 2\chi + 2(1 - \epsilon_a^2)\left( \cos \alpha \sin \chi \cos 2\epsilon_p + \sin \alpha \cos \chi \sin 2\epsilon_p \right) \]

(A5)

Using the weak interaction limit \((\alpha, \chi \to 0)\), the orientation angle of the polarization vector \((\psi)\), the circular (elliptical) polarization descriptor 4th Stokes vector element \((\frac{V}{I})\) and the ellipticity \((\xi)\) for small amplitude and phase offset can be obtained as

\[ \psi = \frac{1}{2} \tan^{-1} \left( \frac{U}{Q} \right) = \frac{1}{2} \tan^{-1} \left( -\alpha \left( 1 + \frac{1}{\epsilon_a} \right) - \chi \cot \epsilon_p \right) \approx -\frac{\alpha}{2\epsilon_a} - \frac{\chi}{2} \cot \epsilon_p \]  

(A6a)

\[ \frac{V}{I} \approx \alpha \cot \epsilon_p - \frac{\chi}{\epsilon_a} \]  

(A6b)

\[ \xi = \frac{1}{2} \sin^{-1} \left( \frac{V}{I} \right) \approx \frac{1}{2} \left( \alpha \cot \epsilon_p - \frac{\chi}{\epsilon_a} \right) \]  

(A6c)

Simultaneous real and imaginary WVAs of the tiny polarization effect are clearly manifested in the \(\epsilon_{a/p}\) dependent drastic changes of the polarization state of light. It is also important to note that simultaneous real and imaginary WVA of both Faraday rotation and Faraday ellipticity are manifested in the 1st and the 2nd terms of Eq. A6a and A6b (A6c) respectively. For small \(\epsilon_a\) and \(\epsilon_p\) both the polarization rotation \((\psi)\) and the circular (elliptical) polarization descriptor 4th Stokes vector element \((\frac{V}{I})\) increase rapidly and consequently a huge enhancement of both the Faraday rotation and ellipticity is observed around the Fano dip as a direct manifestation of the WVA.

**Acknowledgement:** The authors acknowledge Indian Institute of Science Education and Research (IISER) Kolkata for the funding and facilities. S.G. acknowledges financial support from the Council of Scientific and Industrial Research (CSIR), Government of India through a research fellowship.

J.K.N. and S.G. contributed equally to this work.
Figure Captions

Figure 1: (a) A schematic illustration of the FEM simulation of Faraday rotation $\psi$ in the transmission spectra of the WMPC system. The direction of the TM ($x$) and TE ($y$) polarized light electric fields are perpendicular and parallel, respectively to the axis of the Au gratings in the WMPC. (b) The WMPC system comprises of gold (Au) gratings on top of a thin magnetooptic Y-BIG film and the substrate is taken to be quartz. The Y-BIG film serves as the waveguiding layer and additionally exhibit Faraday effect in the presence of an external magnetic field. The thickness of the Y-BIG film ($t$) and the periodicity ($d$), width ($w$), and height ($h$) of the Au gratings are marked.

Figure 2: The dependence of the enhanced Faraday rotation and ellipticity on the geometrical parameters of the Fano resonant WMPC system with TE polarized excitation. The FEM simulated transmission spectra (1st row), the spectral variation of the Faraday rotation $\psi$ (2nd row) and ellipticity $\xi$ (3rd row) of the WMPC are shown for varying periodicity ($d$, 1st column), width ($w$, 2nd column) and height ($h$, 3rd column) of the Au gratings (shown for $E = \hbar \omega = 0.83$ to 2.067eV). Prominent signatures of Fano resonance with asymmetric spectral line shape are observed in all the transmission spectra (1st row). Large enhancements of the Faraday rotation (2nd row) and ellipticity (3rd row) are observed around the Fano spectral dip ($E = \hbar \omega F$) for all the geometrical parameters. The spectral variation of the transmitted TM polarization intensities (4th row) are also shown for each of the varying geometrical parameters of the WMPC. The $d$ parameter of the grating is varied (1st column) from 450nm to 700nm with a step size of 50nm, keeping fixed $w = 120$ nm and $h = 6$. Similarly, the $w$ parameter of the grating is varied from 100 nm to 200 nm in a step of 20nm with a fixed $d = 600$nm and $h = 65$ nm. The $h$ parameter is increased from 40nm to 120nm with a step of 20nm for a fixed $d = 600$nm and $w = 120$nm. In order to maintain clarity in the figures, the baseline magnitudes of Faraday rotation $\psi$ and ellipticity $\xi$ are shifted by 5 deg. consecutively for each sub-plots with varying periodicity, width and height of the Au gratings.

Figure 3: Interpretation of enhanced Faraday rotation and ellipticity of WMPC for TE polarized excitation through natural WVA of Faraday effect in Fano resonance. The simulation results are shown for two differrent periodicities $d$ of the grating in the WMPC: (a) $d = 600$ nm and (b) $d = 450$ nm. The spectral variations ($E = \hbar \omega = 0.83$ to 2.067eV) of Faraday rotation $\psi$ (right axis, red solid line) (a(i) and b(i)) and ellipticity $\xi$ (right axis, red solid line) (a(ii) and b(ii)) are shown along with the transmitted intensity profiles (left axis blue solid line) and the bare film Faraday rotation (left axis red dotted line). A comparison of the transmitted TM (right axis red solid line) and TE polarized intensities (left axis blue solid line) for TE polarized excitation (a(iii) and b(iii)). (a(iv) and b (iv)): Spectral variations of the weak measurement parameters, amplitude offset $\epsilon_a$ (left axis, blue solid line) and the phase offset $\epsilon_p$ (right axis, red solid line) around the Fano spectral dip ($E = \hbar \omega F$). (a(v) and b (v)): The variation of the Faraday rotation $\psi$ as a function of the $\epsilon_a$ (grey solid balls) and the corresponding theoretical predictions of natural WVA (using Eq. A5 - Eq. A6 of Appendix). (a(vi) and b (vi)): The variation of the ellipticity $\xi$ as a function of the $\epsilon_p$ (solid balls) and the corresponding theoretical predictions of natural WVA (using Eq. A5 - A6 of Appendix).

Figure 4: The dependence of the enhanced Faraday rotation and ellipticity on the geometrical parameters of the WMPC system with TM polarized excitation. The FEM simulated transmission spectra (1st row), the spectral variation of the Faraday rotation $\psi$ (2nd
and ellipticity $\xi$ (3rd row) of the WMPC are shown for varying periodicity ($d$, 1st column), width ($w$, 2nd column) and height ($h$, 3rd column) of the Au gratings (shown for $E = \hbar \omega = 0.83$ to $2.067eV$). Simultaneous excitation of waveguide and plasmon leads to the formation of hybrid plasmon-waveguide modes and consequently two dips appear in the transmission spectra (1st row). Large enhancements of the Faraday rotation (2nd row) and ellipticity (3rd row) are observed around the transmission dip at the lower frequency end. The spectral variation of the transmitted TE polarization intensities (4th row) are also shown for each of the varying geometrical parameters of the WMPC. The $d$ parameter of the grating is varied (1st column) from 450nm to 700nm with a step size of 50nm, keeping fixed $w = 120$ nm and $h = 60$ nm. Similarly, the $w$ parameter of the grating is varied from 100 nm to 200 nm in a step of 20nm with a fixed $d = 600$nm and $h = 60$ nm. The $h$ parameter is increased from 40nm to 120nm with a step of 20nm for a fixed $d = 600$nm and $w = 120$nm. In order to maintain clarity in the figures, the baselines of each of the spectra are shifted. In order to maintain clarity in the figures, the baseline magnitudes of Faraday rotation $\psi$ and ellipticity $\xi$ are shifted by 5 deg. consecutively for each sub-plots with varying periodicity, width and height of the Au gratings.

**Figure 5:** Understanding the role of the waveguide-plasmon strong coupling and avoided crossing on enhanced Faraday effect in WMPC system with TM polarized excitation. (a) The frequencies (energy $E = \hbar \omega$) corresponding to the spectral dips are shown as a function of varying value of periodicity $d$ of the grating in the WMPC width and height fixed at 120nm and 65nm respectively. The dispersion of the TE waveguide mode (black dotted line) is also shown for varying periodicities. The spectral variation ($= \hbar \omega = 0.83$ to $2.067eV$) of Faraday rotation $\psi$ (b, right axis, red solid line) and ellipticity $\xi$ (c, right axis, red solid line) for grating periodicity $d = 550$ nm within the window of the avoided crossing. The spectral variation of the transmitted TM polarized intensity are also shown (left axis, blue dashed line). The simulated spatial distribution of the magnetic field ($H$) for input TM polarization ($H_y$) at energies (frequencies) corresponding to the two transmission dips are shown for grating period $d = 450$ nm (below the avoided crossing regime) (d), for $d = 550$ nm (at the avoided crossing regime)(e), and for $d = 700$nm above the avoided crossing(f). The cross section of the WMPC is shown by black solid lines, with the rectangles representing the position of the Au gratings.
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Figures

Figure 1
Figure 2
Figure 3

a) Periodicity 600 nm

i) Transmitted intensity TE\textsubscript{0} (arb. unit)

ii) Faraday rotation \( \psi \) (deg)

iii) Transmitted intensity TM\textsubscript{0} (arb. unit)

iv) Amplitude and Phase offset

v) Faraday rotation \( \psi \) (deg)

vi) Faraday ellipticity \( \xi \) (deg)

b) Periodicity 450 nm

i) Transmitted intensity TE\textsubscript{0} (arb. unit)

ii) Faraday rotation \( \psi \) (deg)

iii) Transmitted intensity TM\textsubscript{0} (arb. unit)

iv) Amplitude and Phase offset

v) Faraday rotation \( \psi \) (deg)

vi) Faraday ellipticity \( \xi \) (deg)
| Periodicity | Width | Height |
|-------------|-------|--------|
| ![Periodicity Diagram](image1) | ![Width Diagram](image2) | ![Height Diagram](image3) |

**Figure 4**
Figure 5