NEAR-INFRARED MICROLENSING OF STARS BY THE SUPERMASSIVE BLACK HOLE IN THE GALACTIC CENTER

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ABSTRACT

We investigate microlensing amplification of faint stars in the dense stellar cluster in the Galactic center by the supermassive black hole, which is thought to coincide with the radio source Sgr A*. Such amplification events would appear very close to the position of Sgr A* and could be observed, in principle, during the monitoring of stellar proper motions in the Galactic center.

We use observations of the near-infrared K-band (2.2 μm) luminosity function in the Galactic center and in Baade’s window, as well as stellar population synthesis computations, to construct empirical and theoretical K luminosity function models for the inner 300 pc of the Galaxy. These, together with the observed dynamical properties of the central cluster and inner bulge, are used to compute the rates of microlensing events that amplify stars with different intrinsic luminosities above specified detection thresholds.

We present computations of the lensing rates as functions of the event durations, which range from several weeks to a few years, for detection thresholds ranging from $K_0 = 16$ to 19 mag. We find that events with durations shorter than a few months dominate the lensing rate because of the very high stellar densities and velocities very near the black hole, where the effective lens size is small. For the current detection limit of $K_0 = 17$ mag, the total microlensing rate is $3 \times 10^{-3}$ yr$^{-1}$. The rate of events with durations $\geq 1$ yr is $1 \times 10^{-3}$ yr$^{-1}$. The median value of the peak amplification for short events is $\Delta K \sim 0.75$ mag above the detection threshold and is only weakly dependent on $K_0$. Long events are rarer and are associated with more distant stars, stars at the low-velocity tail of the velocity distribution or stars that cross closer to the line of sight to Sgr A*. Therefore, the median peak amplifications of long events are larger and attain values $\Delta K \sim 1.5$ mag above the threshold.

Recent proper-motion studies of stars in the Galactic center have revealed the possible presence of one or two variable K-band sources very close to, or coincident with, the position of Sgr A*. These sources may have attained peak brightnesses of $K \simeq 15$ mag, about 1.5–2 mag above the observational detection limits, and appear to have varied on a timescale of ~1 yr. This behavior is consistent with long-duration microlensing amplification of faint stars by the central black hole. However, we estimate that the probability that a single such event could have been detected during the course of the recent proper-motion monitoring campaigns is ~0.5%. A 10-fold improvement in the detection limit and 10 yr of monthly monitoring could increase the total detection probability to ~20%.

Subject headings: Galaxy: center — Galaxy: kinematics and dynamics — Galaxy: stellar content — gravitational lensing — infrared: stars

1. INTRODUCTION

Recent proper-motion studies of infrared-luminous stars in the Galactic center (GC; Eckart & Genzel 1996; Genzel et al. 1997; Ghez et al. 1998) have convincingly demonstrated the existence of a compact $\sim 2.6 \times 10^6 M_\odot$ dark mass in the dynamical center of the GC, which is located within 0.1 of the radio source Sgr A* (Ghez et al. 1998; 1″ = 0.039 pc at 8 kpc). Lower bounds on the compactness of this mass concentration, together with dynamical considerations, argue against the possibility of a massive cluster and point toward a supermassive black hole as the likely alternative (Genzel et al. 1997; Maoz 1998). This conclusion has important implications for basic issues such as the prevalence of massive black holes in the nuclei of normal galaxies and the nature of the accretion mechanism that makes Sgr A* so much fainter than typical active galactic nuclei (Melia 1994; Narayan, Yi, & Mahdevan 1995).

Wardle & Yusef-Zadeh (1992) considered several gravitational lensing effects that could be induced by a massive black hole in the GC. In particular, Wardle & Yusef-Zadeh pointed out that gravitational lensing would occasionally lead to the amplification and splitting of the stellar images of stars that happen to move behind the black hole and that such events would typically last from months to years, depending on the distance of such stars from the black hole. Wardle & Yusef-Zadeh also estimated that an optical depth of order unity for such events requires an observed central surface density greater than 1000 stars arcsec$^{-2}$ and that
this in turn would require an angular resolution of $\lesssim 0.01$ to individually separate the lensed images from the crowded background of faint stars. However, the photometric sensitivities and spatial resolutions required for such observations are far beyond current observational capabilities. Presently, the deepest near-infrared $K$-band (2.2 $\mu$m) images of the central arcsecond reach down to $K = 17$ mag (Ghez et al. 1998). As we will show, at this limiting magnitude the expected central surface density is $\sim 20$ stars arcsec$^{-2}$ (Davidov et al. 1997). The highest spatial resolutions obtained so far are the diffraction limited resolutions of $\sim 0.15$ arcsec (Eckart et al. 1995; Davidov et al. 1997) and $\sim 0.05$ arcsec (Ghez et al. 1998) achieved in the proper-motion surveys.

In this paper we investigate a different possibility, namely micro lensing amplification of faint stars by the central black hole. Such events occur when the amplified but unresolved images of faint stars rise above the detection threshold and then fade again as such stars move behind the black hole close to the line of sight to Sgr A*. We present a quantitative study of such microlensing events, and we compute in detail the microlensing rates as functions of the event durations for a wide range of assumed detection thresholds. In addition, we also consider the possible amplification of sources that lie above the detection thresholds, and we reexamine the limit in which the two lensed images can actually be resolved.

Our study is motivated by several recent developments. Deep infrared star counts in the inner GC (Blum, Sellgren, & DePoy 1996; Davidov et al. 1997) and infrared and optical star counts in Baade’s window (Tiede, Frogel, & Terndrup 1995; Holtzman et al. 1998) now make it possible to reliably model the infrared stellar luminosity function in the vicinity of the Galactic center. The ongoing proper-motion monitoring campaigns of the inner few arcseconds of the GC record both the positions and fluxes of individual stars in the field at a sampling rate of 1–2 observing runs per year. As a by-product, these measurements can be used to search for microlensing events, albeit at a low sampling rate. Such events would appear as time-varying sources very close to Sgr A*. It is therefore intriguing that one or two variable IR sources may have already been detected close to, or coincident with, the position of Sgr A* (Genzel et al. 1997; Ghez et al. 1998). These sources may have brightened to, or coincident with, the position of Sgr A* (Genzel et al. 1996). These sources may have brightened to, or coincident with, the position of Sgr A* (Genzel et al. 1996). These sources may have brightened to, or coincident with, the position of Sgr A* (Genzel et al. 1996).

2. LENSING BY THE SUPERMASSIVE BLACK HOLE

The effective size of a gravitational lens at the lens plane is set by the Einstein radius, $R_E$,

$$R_E = \left(\frac{4GM_\odot Dd}{c^2(D + d)}\right)^{1/2} \sim 2.2 \times 10^{15}(M_{2.6}d_4)^{1/2} \text{ cm},$$

where $G$ is the gravitational constant, $c$ is the speed of light, $M_\odot$ is the lens mass (here the black hole mass), and $D$ and $d$ are the observer-lens and lens-source distances, respectively (see, e.g., review by Bartelmann & Narayan 1998). We assume, as will be justified below, that $d \ll D$ and define $M_{2.6} = M_\odot/2.6 \times 10^6 M_\odot$ and $d_4 = d/1\text{ pc}$. The effective size of the lens at the source plane is $R_s = R_\odot (D + d)/D \sim R_E$. The angular size of the Einstein radius, $\theta_E$, is

$$\theta_E \sim 0.018D_8^{-1}(M_{2.6}d_4)^{1/2},$$

where $8D_8$ kpc is the Sun’s Galactocentric distance (Carney et al. 1995).

In our study we assume that any IR extinction associated with an accretion disk or an “atmosphere” near the event horizon is negligible.

We begin by showing that in our problem the stars can be treated as point sources and that the linear approximation (small light-bending angle approximation) holds. First, when $d \ll D$, a star can be approximated as a point source as long as its radius, $r_\star$, is much smaller than $R_E/A$, where $A$ is the required amplification to observe the lensed source above the detection threshold (see §2.1). As we show below (§3.2.1), the faintest stars that contribute significantly to the lensing are low-mass stars with $r_\star \lesssim 10^{-11}$ cm, which require amplification factors of $A \lesssim 100$. Brighter stars require progressively smaller amplification factors, down to $A \sim 1$, which for detection thresholds $K \sim 17$ mag correspond to stars with $R_s < 10^{-12}$ cm. Assuming a mean stellar mass of $\sim 1 M_\odot$, a central mass density of $\rho \sim 4 \times 10^6 M_\odot$ pc$^{-3}$ in the GC (Genzel et al. 1996) implies a mean stellar separation of $\delta \sim 0.005$ pc. Even at such a small distance from the black hole, $R_E \sim 1.5 \times 10^{14}$ cm, so that the condition $R_s < R_\odot/A$ is generally satisfied for all relevant stars in the system. We thus conclude that the point-source approximation holds over the entire range of interest.

Second, the linear approximation holds as long as the Einstein radius is much larger than the Schwarzschild radius of the black hole, $R_\bullet$:

$$R_E/R_\bullet \sim c d/(GM_\bullet)^{1/2} \sim 2.8 \times 10^3(d_4/M_{2.6})^{1/2},$$

which even for $d = \delta$ is as high as $\sim 200$.

A point lens produces two images, one within and one outside the Einstein radius. The angular separation between the two images is

$$\Delta\theta = \theta_E \sqrt{u^2 + 4},$$

and their mean angular offset from the optical axis is $\theta_{\alpha} E$, where $\theta_{\alpha} E$ is the angular separation between the unlensed source and the lens. As will be shown in §3, $u \ll 1$ for amplification above present-day detection thresholds in the GC, so that $\Delta\theta \sim 2\theta_{\alpha} E$.

Three angular scales in the problem determine the way the lensing will appear to the observer: the Einstein angle $\theta_{\alpha} E(d)$, the FWHM angular resolution of the observations, $\phi$, and the mean projected angular separation between the observed stars, $\Delta p(K_\odot)$, where $K_\odot$ is the detection threshold.
The lensed images have to be detected against the background of a very dense stellar system. This background will be low as long as the central surface density of observed stars, \(S_0 = \Delta \rho^{-2}\), is small enough so that \(\pi \phi^2 S_0 < 1\). Thus, for a given detection threshold, an angular resolution of at least
\[
\phi = \Delta \rho(K_0)/\sqrt{\pi}
\]
is required to detect the lensed star. Lower resolutions correspond to the regime of “pixel lensing” (Crotts 1992), which we do not consider here.

When \(\theta_s < \phi\), the two images will not be resolved, and the lensed source will appear as a microlensing event. For a given angular resolution, there is a maximal lens-source distance \(d_\mu\) for microlensing,
\[
d_\mu = \frac{D_\beta^2}{M_{2,6}} \left(\frac{\phi}{0.018}\right)^2 \text{ pc}.
\]
More distant stars will have \(\theta_E > \phi\), and their two images will be separately resolved.

### 2.1. The Microlensing Rate

The unresolved images of a faint star at \(d < d_\mu\) close to the line of sight will appear as a “new source” at the position of Sgr A*, to within the observational resolution. The total source amplification, \(A\), is related to \(u\) by (Paczynski 1986)
\[
u^2 = 2A/\sqrt{A^2 - 1} - 2,
\]
and for small \(u\), \(A \approx 1/u\). The maximal amplification along the star’s trajectory is reached when its projected position is closest to the lens. A star of stellar type \(s\) and an absolute \(K\) magnitude \(K_s\) is observed on a line of sight with an extinction coefficient \(A_{K}\) and distance modulus \(\Delta\) will be detected above the flux threshold \(K_0\) if it is amplified by at least \(A_t = 10^{-0.4(K_0 - K_s - \Delta - A_{K})}\). This corresponds to a maximal impact parameter of \(u_{0,s}\). As the minimal required amplification approaches 1, \(u_0\) diverges. This, and the fact that in practice the threshold is not sharply defined, leads us to introduce a cutoff at \(u_0 = 1\), which implies a minimal amplification of \(A = 1.34\).

The basic quantity that is required for predicting the lensing properties is the differential lensing rate as function of stellar type, event duration, amplification, and source distance from the black hole. We derive an explicit expression for the differential lensing rate in the Appendix. Here it is instructive to discuss the relations between these properties by considering the simpler case of the total lensing rate, regardless of the event duration. The total integrated rate for microlensing amplification of background stars above the detection threshold is
\[
\Gamma(K_0) = 2\langle u_0 \rangle \int_{r_1}^{r_2} R_E \bar{v}_2 n_s \, dr,
\]
where \(r_1\) is the inner radius of the central stellar cluster, \(r_2\) the maximal radius for producing unresolved lensed images, \(\bar{v}_2\) is the transverse two-dimensional stellar velocity averaged over the velocity distribution function, and \(n_s\) is the total number density of stars. Here and below, the angle brackets designate the average of the bracketed quantity over the stellar types with \(K_s > K_{0,s}\) weighted by their fraction in the stellar population, \(f_s\), where it is assumed that \(f_s\) does not depend on \(r\). The properties of the stellar population enter the integrated rate only through the mean impact parameter \(\langle u_0 \rangle\), which for \(u_0 \ll 1\) is simply
\[
\langle u_0 \rangle = \sum_{s} f_s u_{0,s},
\]
\[
\approx \sum_{s} f_s A_s^{-1} = \langle F_K \rangle / F_0,
\]
where \(F_K\) is the observed (dust extinguished) stellar \(K\)-band flux and \(F_0\) is the detection threshold flux corresponding to \(K_0\).

We characterize the microlensing timescale for stars of type \(s\) as the average time they spend above the detection threshold,
\[
\tau(K_0) = \frac{\pi}{2} u_0 R_E \bar{v}_2,
\]
where a \(\pi/4\) factor comes from averaging over all impact parameters with \(u \leq u_{0,s}\).

The lensing rate, amplification, and event duration are interrelated. For \(u_0 \ll 1\), the median value of the distribution of maximal amplifications is simply \(A(u_0/2) \approx 2A(u_0)\). Note, however, that the mean maximal amplification, \(\langle A_{\max} \rangle \approx \int_0^{u_0} (u_0 u)^{-1} du\), diverges logarithmically. Generally, a fraction \(x\) of the events will have a maximal amplification of \(A_{\max} / x\) or more (i.e., a peak magnitude above the threshold of \(\Delta K = 2.5 \log x \) or less). The rate of such events is smaller,
\[
\Gamma(x) = x \Gamma(K_0).
\]
The timescale of events amplified by a factor of \(1/x\) above the threshold is somewhat longer than the average timescale (eq. [10]), since they must cross closer to the line of sight,
\[
\tau(x) = \frac{2}{\pi} \left(\frac{1}{1 - x^2} + \frac{\sin^{-1} x}{x}\right) \tau(K_0),
\]
which approaches \(4\tau(K_0)/\pi\) for large amplifications. Conversely, for a given \(v_2\) and \(u_{0,s}\), even a small increase in \(\tau\) is associated with a considerable increase in the maximal amplification.

### 2.2. Resolved Lensed Images

When \(\theta_E > \phi\) at \(d > d_\mu\), the two images can be resolved. As we show in § 4, \(d_\mu\) is already quite small for present-day angular resolutions and will become smaller still as the resolution improves. This implies that there may be a non-negligible contribution to the lensing rate from regions beyond \(d_\mu\). We therefore have to consider also the case of resolved images.

For \(u \ll 1\), the two images will appear at an offset of \(~\theta_E\) from Sgr A*, on opposite sides of it. The amplifications of the individual images are related by \(A = A_1 + A_2\) and \(A_2 = A_1 + 1\), which, for the high amplifications that are required in the GC, can be approximated as \(A_1 \approx A_2 = A/2\). The formalism used for calculating the lensing of unresolved images can therefore be applied in this case simply by raising the effective detection threshold by a factor of 2 (0.75 mag). This makes resolved images harder to detect, but
on the other hand, if both images are observed, the identification of the event as lensing is much more certain.

2.3. Lensing of Observed Bright Stars

The formalism for describing unresolved and resolved lensing of stars from below the detection threshold can be also extended to cases where the microlensed source is an observed bright, \( K < K_0 \) star. At the current detection threshold, the observed central surface density is \( \sim 10 \) arcsec\(^{-2} \) (Genzel et al. 1997; Ghez et al. 1998). For such a small number of stars, whose orbits can be tracked individually, a statistical treatment is not very useful. However, our calculations indicate that a 10-fold improvement in the detection threshold will yield an observed central surface density of at least \( \sim 100 \) arcsec\(^{-2} \) (eq. [25]). For such a large sample, a statistical treatment is more meaningful. We therefore present below, for completeness, results for microlensing amplification of stars fainter than the detection threshold (‘‘faint-star lensing’’) as well as for stars brighter than the detection threshold (‘‘bright-star lensing’’). For bright-star microlensing we assume that the effective minimum magnification factor for detection is \( A = 1.34 \) (\( u_0 = 1 \)).

3. MODELING THE STELLAR POPULATION IN THE GALACTIC CENTER

Three basic quantities enter into the computation of the microlensing rate: the stellar number density distribution, the stellar velocity field, and the \( K \)-band luminosity function (KLF).

The stellar population in the central \( \sim 100 \) pc appears to consist of a mixture of old bulge stars and “central cluster” stars that may have been produced in various star formation episodes during the lifetime of the Galaxy (Genzel, Hollenbach, & Townes 1994; Serabyn & Morris 1996). Evidence for recent star formation in the GC has been provided by Krabbe et al. (1995), who found a concentration of luminous early-type stars within a few arcseconds of Sgr A*, implying a recent (\( \lesssim 10^7 \) yr) starburst in which \( \sim 10^3 \) stars were formed. Additional young stellar systems, the “Arches” and “Quintuplet” clusters, also exist close to the GC and contain large numbers of massive stars (Serabyn, Shupe, & Figer 1998). A further indication of ongoing star formation in the GC is the fact that the KLF in the central cluster and by assuming for the bulge and disk, so

In our analysis we make the simplifying assumption that the discrete young clusters in the GC can be modeled by a volume-averaged and smoothly distributed stellar population. In particular, we do not consider here lensing events that might be associated with the innermost stellar cusp, which has been identified by Eckart & Genzel (1997) and Ghez et al. (1998) in the immediate vicinity of Sgr A*. An overdensity of stars above the smoothed distribution very near the black hole may contribute to very short duration microlensing events. However, the total stellar mass and luminosity function of the stars that are associated with this “Sgr A* cusp” are poorly constrained at the present time, so that computations of the lensing rates require more extensive modeling and analysis. We will present such computations elsewhere.

Because the lensing timescales increase with distance, long-duration events, which are relevant for low sampling rates of the current proper-motion studies, tend to be associated with more distant stars. We therefore consider both stars in the central star-forming cluster and more distant old-population bulge stars in our analysis. We now proceed to discuss the stellar densities, velocities, and KLFs of these two components.

3.1. Stellar Densities and Velocities

Genzel et al. (1996, 1997) derived density and velocity models for the central cluster based on fits to the observed star counts, stellar radial velocities, and proper motions in the inner few parsecs. Their mass density distribution is parameterized by a softened isothermal distribution

\[
\rho_{\text{core}}(r) = \frac{\rho_c}{1 + 3(r/r_c)^2},
\]

where \( r_c \) is the core radius and \( \rho_c \) is the central density, with best-fit values of \( r_c = 0.38 \) pc and \( \rho_c = 4 \times 10^6 M_\odot \) pc\(^{-3} \).

The one-dimensional velocity dispersion of the late-type stars in the core, which are dynamically relaxed, is modeled by (Genzel et al. 1996)

\[
\sigma_0 = \left[ 55^2 + 10^3(r/r_{10})^{-2}\alpha \right]^{1/2} \text{ km s}^{-1},
\]

where \( \alpha = 0.6 \) and \( r_{10} \) is the projected distance corresponding to 10\( ^7 \) pc.

In the inner GC, the mass density is strongly constrained by the dynamics, whereas the \( K \)-luminosity density \( \nu (L_{K_\odot} \) pc\(^{-3} \) is not well defined because of the patchy nature of the extinction. The situation is reversed on scales larger than 1 kpc, where the observed rotation curve and velocity dispersion are harder to interpret, but \( \nu \) can be deprojected from the observed surface brightness. Kent (1992) proposed that both the rotation curve and the surface brightness along the major axis of the Galactic bulge in the midplane of the Galaxy at \( r \gtrsim 1 \) kpc can be described as a superposition of bulge and disk components with a mass-to-light ratio\(^5 \) \( \Upsilon = 17 \zeta_\odot \) and a luminosity density

\[
\nu(r) = \nu_{\text{bulge}} + \nu_{\text{disk}} = 3.53K_0 \left( \frac{r}{r_b} \right) + 3 \exp \left( -\frac{r}{\gamma} \right) L_{K_\odot} \text{ pc}^{-3},
\]

where here \( K_0 \) is a modified Bessel function (not to be confused with the detection threshold), \( r_b = 667 \) pc, and \( r_b = 3001 \) pc. Kent also suggested a \( \nu \propto \rho^{-1.85} \) extrapolation of \( \nu \) toward the center based on observations of the intensity variation in the inner 10 pc. We replace this extrapolation with an updated and nondivergent model by adopting the Genzel et al. (1996) mass density model (eq. [13]) for the core and by assuming \( \Upsilon = 17 \zeta_\odot \) for the bulge and disk, so that \( \rho_{\text{bulge}} = \nu_{\text{bulge}} \) and \( \rho_{\text{disk}} = \nu_{\text{disk}} \). We further assume that the bulge is axisymmetric and model the mass density over

\(^5\) Following Kent (1992), we define the mass-to-light ratio as \( \Upsilon = (M/M_\odot)/(L/L_\odot) \), where the solar monochromatic luminosity at 2.2 \( \mu \)m is \( L_{2.2 \mu m} = 2.154 \times 10^{25} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ s}^{-1} \). Note that Genzel et al. (1996) define this quantity as \( \Upsilon = (M/M_\odot)/(L/\odot) \), with \( \lambda = 2.2 \) \( \mu \)m. The two definitions are related by \( \Upsilon = \kappa \Upsilon_2 \).
the entire distance range by

$$\rho = \rho_{\text{core}} + \rho_{\text{bulge}} + \rho_{\text{disk}}.$$  \hfill (16)

Figure 1 shows our mass density model, and in particular the emergence of the bulge component at \( \sim 100 \) pc. The central cluster completely dominates the total mass density in the inner 10 pc, but then falls to \( \sim 85\% \) of the total at 50 pc and to \( \sim 65\% \) at 100 pc.

The velocity field on this scale includes both the bulk rotation and the random motion. We approximate Kent’s models for the Galactic rotation and the velocity dispersion in the inner 300 pc along the major axis of the bulge by log-linear fits. We assume that the rotation velocity is perpendicular to the line of sight and that its contribution to the transverse velocity is

$$v_{\text{rot}} = 80 + 20\log_{10}(r/1\text{pc}) \text{ \, km \, s}^{-1},$$  \hfill (17)

and that the one-dimensional velocity dispersion in the bulge is

$$\sigma_{\text{bulge}} = 60.9 + 18.9\log_{10}(r/1\text{pc}) \text{ \, km \, s}^{-1}.$$  \hfill (18)

We model the one-dimensional velocity dispersion over the entire distance range by

$$\sigma = \max(\sigma_{\text{core}}, \sigma_{\text{bulge}}).$$  \hfill (19)

Note that both the proper motion of Sgr A*, which is \( \lesssim 20 \text{ \, km \, s}^{-1} \) (Backer 1996), and the Sun’s Galactocentric rotation have a negligible contribution to the relative proper motion of the source and lens because of the high stellar velocities near the dynamic center and because \( d \ll D \).

3.2. K Luminosity Function

In our analysis we consider a model for the KLF that is based on the observed portions of the KLFs in the central cluster and in the bulge and on a theoretical KLF computed in a population synthesis model for the central cluster.

3.2.1. The Power-Law KLF

The KLF of the bulge has been observed through Baade’s window, which at \( l = 1^\circ, b = -3^\circ.9 \) is \( \sim 0.6 \text{ \, kpc} \) from the GC at the tangential point. The bulge KLF approximately follows a single power-law \( d \log N_K/d \log L_K = \beta \), with \( \beta = 1.695 \) (Tiede et al. 1995), from \( M_K \sim -7.5 \) to \( M_K \sim 2 \)

![Graph showing GC mass density model](image)

6 \( M_K = -2.5 \log L_K + 84.245 \text{ \, mag} \) for \( L_K \) in ergs s\(^{-1}\) \( \mu m\)^{-1}.

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Recent Hubble Space Telescope observations of the optical luminosity function in Baade’s window (Holtzman et al. 1998) probe it down to very low stellar masses (\( \sim 0.3 M_\odot \)), well below the main-sequence turn-off point. The \( V \)-band luminosity function (VLF) presented by Holtzman et al. (1998) shows that a sharp break occurs at the turn-off point \( M_V \sim 4.5 \) to \( 5 \text{ \, mag} \), corresponding to \( \sim 1 M_\odot \) stars. We extrapolated the power-law KLF of Tiede et al. (1995) down to \( M_K = 3.5 \text{ \, mag} \) (assuming \( V - K \sim 1.5 \) for \( 1 M_\odot \) stars) and compared it with the Holtzman et al. (1998) VLF at \( M_V = 5 \text{ \, mag} \). We find that the predicted \( K \) star counts somewhat underestimate the \( V \) star counts but agree with them to within a factor of 2. We thus conclude that the power-law KLF can be extended down to at least \( M_K = 3.5 \text{ \, mag} \). The observed VLF can be used to determine the behavior of the KLF at even lower luminosities by using the \( V \)-to-\( K \) conversions for low-mass stars presented by Henry & McCarthy (1993). Doing this shows that the KLF likely flattens at a break point close to \( M_K = 3.5 \text{ \, mag} \), with \( \beta \sim 1.5 \) in the range \( 3.5 \leq K \leq 6.5 \text{ \, mag} \), and turns over at \( M_K \sim 7 \text{ \, mag} \).

As we have discussed above, the stellar population in the central cluster appears to be consistent with continuous star formation throughout the Galaxy lifetime (see, e.g., Serabyn & Morris 1996). However, despite the many differences between the populations in the bulge and the central cluster, the observed KLF in the central \( 12.4 \times 11.9 \) of the GC follows a power law similar to that of the bulge KLF (Davidge et al. 1997) but extends to more luminous stars (Blum et al. 1996). The exact upper luminosity cutoff is not well-defined because of statistical fluctuations in the counts and the contribution of asymptotic giant branch (AGB) stars, which do not follow the power law.

The central cluster’s KLF is not known beyond the current detection threshold of \( K_{\odot} \sim 17 \text{ \, mag} \), which is the region of interest for the microlensing calculations. However, the similarity of the KLF power-law indices in the central cluster and bulge KLFs suggests that the two KLFs are similar, apart for having different upper luminosity cutoffs. The much lower extinction in Baade’s window, which is only \( A_K = 0.14 \text{ \, mag} \) as compared with \( 3.4 \text{ \, mag} \) in the GC (Rieke, Rieke, & Paul 1989), makes it possible to extend the KLF in the GC to lower luminosities. Indeed, Blum et al. (1996) find that the bulge KLF, after correcting for the \( A_K \) difference, can be smoothly joined to the GC luminosity function in the central \( 2 \times 2 \). Their resulting composite luminosity function has a best fit power-law index \( \beta = 1.875 \) and extends from \( M_K \sim -10 \) down to \( M_K = 2 \text{ \, mag} \). Our theoretical model for the central cluster, which we discuss below, indicates that this power-law character is further maintained down to \( M_K \sim 3.5 \text{ \, mag} \).

We therefore adopt the same \( \beta = 1.875 \text{ \, power-law index} \) for both the central cluster and bulge KLFs. For the central cluster KLF we set the upper luminosity cutoff at \( K_{\odot} = 8 \text{ \, mag} \) and for the bulge KLF at \( K_{\odot} = 10.5 \text{ \, mag} \). We set an effective low-luminosity cutoff for both at \( K_{\odot} = 21.5 \text{ \, mag} \).
which corresponds to stars with masses \( \sim 1 M_\odot \). As we show in § 3.2.3, the lensing rates are insensitive to the exact choice of the low-luminosity cutoff.

We note that the observed bulge KLF is better fitted by a somewhat flatter power law than \( \beta = 1.875 \), which implies fewer faint-star lensing candidates. On the other hand, the comparison with the optical luminosity function suggests that the KLF is a conservative estimate of the total number of stars. In addition, the flatter power law fails to account for the strong excess of horizontal branch (HB) stars above the power law (Tiede et al. 1995). These are important potential microlensing sources, as they lie in a magnitude range that is just below the threshold if the bulge population is observed through the GC extinction of \( A_K = 3.4 \) mag. We therefore consider these small discrepancies in slope and number to be within the limitations of the power-law approximation and the observational uncertainties.

### 3.2.2. The Theoretical KLF

As a check on the empirically based pure power-law KLF, we have also computed a series of theoretical KLFs using our “population synthesis” code (see, e.g., Sternberg 1998).

In our models we used the Geneva stellar evolutionary tracks (Schaerer et al. 1993) and concentrated on stellar models with twice solar metallicities, as is indicated by the enhanced abundances of the gas in the GC (Morris & Serabyn 1996). However, recent measurements of stellar spectra of cool luminous stars in the GC point to solar metallicities (Ramirez et al. 1998). We therefore also considered such stellar models and verified that our results do not depend strongly on the assumed metallicity. We computed the stellar \( K \) luminosities using the empirical bolometric corrections and \( V - K \) colors for dwarfs, giants, and supergiants compiled by Schmidt-Kaler (1982) and Tokunaga (1998). The Geneva tracks for intermediate mass stars (\( \sim 2 - 7 M_\odot \)) do not extend beyond the end of the early AGB phase. We extended these tracks to include also the more luminous thermal-pulsing AGB phases following prescriptions described by Charlot & Bruzual (1991) and Bedijn (1988). In our models we assume explicitly that stellar remnants and stars less massive than 0.8 \( M_\odot \) are negligible sources of K-band luminosity. This low-mass luminosity cutoff roughly corresponds to the low-luminosity cutoff of the empirically based power-law KLF.

We constructed theoretical models for a range of cluster parameters assuming continuous star formation lasting for 10 Gyr. We considered models with pure power-law initial mass functions (IMFs) and the Miller-Scalo IMF (Miller & Scalo 1979; Scalo 1986) for a range of lower and upper mass cutoffs. We selected the model that yields a KLF which best matches the observed KLF for the central cluster measured by Davidge et al. (1997).

We find that a model with a Miller-Scalo IMF ranging from 0.1 to 120 \( M_\odot \) provides the best overall fit to the data. In Figure 2 we compare our model KLF for the central cluster with the \( \beta = 1.875 \) power-law fit of Blum et al. (1996) to their composite KLF. The overall agreement between the theoretical KLF and the power-law KLF is remarkable for both stellar models with solar and twice solar metallicities. The model successfully reproduces both the power-law character and slope of the observed KLF of the central cluster. Furthermore, the model shows that the power-law KLF likely extends down to at least \( K = 21.5 \) mag. In this model, the most probable lensed sources are \( K \sim 21 \) mag stars with \( M_\bullet \sim 1 M_\odot \), \( R_\ast \sim 10^{-11} \) cm just off the main sequence, which require a magnification of order \( A \sim 50 \) to be detected above the threshold.

Our population synthesis model predicts a mass-to-light ratio \( \Upsilon = 0.24 \) \( \Upsilon_\odot \), in excellent agreement with \( \Upsilon \sim 0.25 \) \( \Upsilon_\odot \) measured in the central cluster by Genzel et al. (1996).

### 3.2.3. Normalizing the KLF

As shown by equation (8), the lensing rate depends on, \( n_\ast \), the total number density of stars that are effective sources of K-band luminosity. We refer to such stars as “K-emitting” stars and define

\[
n_\ast = \frac{f_\ast}{\bar{m}} \rho \,,
\]

where \( \rho \) is the total dynamical mass density, \( \bar{m} \) is the mean stellar mass of the \( K \)-emitting stars, and \( f_\ast \) is the fraction of the total dynamical mass contained in \( K \)-emitting stars, i.e., excluding low-mass stars, remnants (objects that lie below the effective low-luminosity cutoff of the KLF), and gas clouds.

The observed star counts, \( dN_\text{obs}/dL_K \), within an area \( S \), can be used to obtain a best-fit value of \( f_\ast/\bar{m} \) given the constraint

\[
\frac{dN_\text{obs}}{dL_K} = \frac{df}{dL_K} \frac{f_\ast}{\bar{m}} \int \rho dS \, dr \,,
\]

where the KLF \( df/dL_K \) is normalized to 1. The mass-to-light ratio over the integration volume used in equation (21) can then be deduced from the fit by

\[
\Upsilon = \frac{M}{L_K} = \frac{\bar{m}}{f_\ast L_K},
\]

where \( L_K \) is the average K-band luminosity of the K-emitting stars.

The power-law KLF we adopt in our computations \( df/dL_K \propto L_K^{-\beta} \) is characterized by \( 1 < \beta < 2 \), \( L_1 \ll L_\ast \), and \( L_1 \ll L_0 \ll L_\ast \), where \( L_1 \) and \( L_\ast \) are the effective lower and upper luminosity cutoffs to the KLF and \( L_0 \) is the K lumi-
nosity corresponding to the detection threshold. By approximating \( u \sim 1/A \) (where \( A \) is the required amplification), the mean stellar luminosity and mean impact parameter for such KLFs are given by the simple approximate relations

\[
L_{K} \approx \frac{(β - 1)}{(2 - β)} L_{s} \frac{L_{s}}{L_{0}}^{-β},
\]

\[
\langle u_{0} \rangle \approx \frac{(β - 1)}{(2 - β)} \left( \frac{L_{s}}{L_{0}} \right)^{-1}. \tag{23}
\]

It also follows that \( n_{s}, Y \), and the lensing rate \( Γ \) scale as

\[
n_{s} \sim f_{s}/m \sim \frac{1}{β - 1} L_{s}^{-β},
\]

\[
Y \sim (2 - β)L_{s}^{-2},
\]

\[
Γ \sim \langle u_{0} \rangle n_{s} \sim \frac{1}{2 - β} L_{0}^{-1}. \tag{24}
\]

Equation (24) shows that the total lensing rate is insensitive to the low-luminosity cutoff of the KLF. This simply reflects the fact that an increase in the number of \( K \)-emitting stars as \( L_{s} \) decreases is offset by a correspondingly smaller mean lensing impact parameter for the stellar system and vice versa. This important property allows us to robustly compute the lensing rate even if the effective lower luminosity cutoff of the power-law KLF is not well determined.

We note that since the differential contribution of stars with luminosity \( L_{K} \) to the mean impact parameter \( \langle u_{0} \rangle \) scales like \( (L_{K}/L_{s}) L_{s}^{-β} \approx L_{K}^{-1} \), the integrated contribution of stars from a 1 mag wide bin is, for \( β = 1.875 \), a very weakly decreasing function of the bin’s \( K \) magnitude. We therefore expect that the lensed stars will exhibit a wide range of intrinsic \( K \) magnitudes, with a weak trend toward lensing by stars close to the detection threshold.

It is uncertain at which radius the stellar population makes the transition from a population that is characteristic of a star-forming cluster to a more bulge-like population. However, as we argued above, the observed properties of the KLFs in the GC and the bulge suggest that they are very similar for \( K > 10.5 \) mag. Since the normalization of the KLF does not depend strongly on the upper luminosity cutoff (eq. [24]) and since the very high luminosity tail of the KLF is not relevant for the lensing calculations (the \( 8 < K < 10.5 \) mag stars have a negligible contribution to the lensing rate of observed stars), we infer the value of \( f_{s}/m \) using the star-counts observed in the core, and we adopt this normalization for the entire inner 300 pc.

In carrying out this procedure\(^7\) we used the observed number counts in the \( K = 14 \) mag bin (stars mag\(^{-1}\) arcsec\(^{-2}\)) averaged over the 12.4” \( \times \) 11.9” field observed by Davidge et al. (1997). The star counts in this luminosity range are likely complete, and this range is also well separated from the regions that are affected by AGB and HB stars, which cause deviations from the power-law behavior (Tiede et al. 1995). Using equation (21), we then find that \( f_{s}/m = 0.2 M_{⊙}^{-1} \). This value for \( f_{s}/m \) can be reproduced, for example, by a choice of \( f_{s} = 0.2 \) and \( m = 1 M_{⊙} \), which is comparable with the values \( f_{s} = 0.22 \) and \( m = 0.84 M_{⊙} \) of our population synthesis model for the central cluster. For the central cluster power-law KLF, \( L_{K} = 22 L_{K,0} \), so that equation (22) yields \( Γ \sim 0.22 \Y Γ_{0} \) again in excellent agreement with \( Γ \sim 0.25 \Y Γ_{0} \) inferred by Genzel et al. (1996) for the central cluster.

4. RESULTS

We have carried out detailed computations of the lensing event rates using the formalism described in § 2 and in the Appendix and using the stellar number and velocity distributions and the power-law \( K \)-band luminosity function as discussed in § 3. The integrations were carried out from a minimum distance \( r_{1} = 0.005 \) pc, equal to the mean central stellar separation, to a distance \( r_{2} = 300 \) pc, where the integrated lensing rates approach their asymptotic values.

The normalized central cluster KLF allows us to estimate the stellar surface density as a function of the detection threshold and from it derive the mean angular separation of the stars, \( Δp \), the required angular resolution of the observations \( φ \) and the maximal distance for observing (unresolved) microlensed stars \( d_{u} \) (as given by eqs. [5] and [6]). We find that for the central cluster KLF,

\[
\log Γ_{0} = 3.21 - 0.175K_{0},
\]

\[
\log S_{0} = -4.62 + 0.35K_{0}, \tag{25}
\]

for \( Δp \) in arcsec and \( S_{0} \) in arcsec\(^{-2}\). Thus, for a detection threshold of \( K_{0} = 17 \) mag we expect a central surface density of 21 stars arcsec\(^{-2}\) for complete counts.\(^9\)

We present our results in a way that makes it possible to flexibly estimate the actual detection probabilities for a broad range of observing strategies. We consider the generic monitoring campaign to consist of a series of \( n \) very short observing runs that are carried out during a total time period \( T \) (typically \( T \sim \) several years), with a mean interval \( ΔT \) between each observing run (typically \( ΔT \sim \) a month to a year). We now wish to distinguish between “long” and “short” events, where we define the event duration as the time the source spends above the detection threshold. Long events are those with durations \( τ > ΔT \) and would appear as time-varying sources that brighten and fade during the course of several observing runs. The microlensing origin of long events could be verified, in principle, from the shape of the light curve and its achromatic behavior. Short events are those with durations \( τ < ΔT \) and would be observed as a single “flash” provided they occur within a time \( τ \) of any one of the \( n \) observing runs. In the limit of small event rates the detection probability may be written as

\[
P = P_{short} + P_{long} = nτ_{short}Γ_{short} + TT_{long}, \tag{26}
\]

where \( Γ_{short} \) and \( τ_{long} \) are the rates of short and long events and \( τ_{short} \) is the rate averaged lensing duration for short events. In the (ideal) limit of continuous monitoring, \( Γ_{long} \) approaches the total event rate, and \( P = TT_{total} \).

\(^7\) We note that if \( β > 2 \), then the rate does depend on \( L_{s} \) with \( Γ \sim L_{s}^{-β} \). However, as we have discussed, the observations of Holtzman et al. (1998) strongly suggest that the KLF flattens, rather than steepens, below our assumed value for \( L_{s} \).

\(^8\) The volume integration in eq. (21) was carried out by approximating the rectangular field with a circular field of projected radius \( p \) having the same area. The integration was done over a cylinder of radius \( p \), centered on the black hole and extending along the line of sight 300 pc in each direction, a distance large enough to ensure that the surface density reaches its asymptotic value.

\(^9\) Genzel et al (1997) and Ghez et al (1998) reported \( S_{0} \sim 20 \) and 15 stars arcsec\(^{-2}\), respectively. However, the star counts are incomplete close to the detection thresholds, and the measured \( S_{0} \) actually indicate a significant density enhancement (the “Sgr A* cusp”) relative to the immediate surroundings.
The results of our computations are displayed in Figures 3 and 4. In Figure 3 we plot the cumulative rates, \( \Gamma_{\text{long}}(\tau > \Delta T) \), for all lensing events with durations \( \tau \) longer than the timescale \( \Delta T \), as functions of \( \Delta T \). We present results for events that produce unresolved and resolved images for stars that are either intrinsically below or above the detection thresholds. We present computations for detection thresholds ranging from 16 to 19 mag. The total lensing rates can be read off the plot from the asymptotic values of the curves as \( \Delta T \to 0 \). The curves are flat for timescales less than \( \sim 1 \) week, which is shorter than most of the event durations. As \( \Delta T \) increases, \( \Gamma_{\text{long}} \) decreases as a smaller fraction of the lensing events satisfy \( \tau > \Delta T \). As an example, Figure 3 shows that for \( K_0 = 17 \) mag the total lensing rate is equal to \( 3 \times 10^{-3} \text{ yr}^{-1} \) and that for events with durations greater than 1 yr the rate is equal to \( 1 \times 10^{-3} \text{ yr}^{-1} \).

In Figure 3 we also plot the rate-averaged lensing timescale, \( \tau_{\text{short}} \), for events with \( \tau < \Delta T \), which is needed to estimate the detection probability of short events. The values of \( \tau_{\text{short}} \) are almost independent of \( K_0 \), since the shape of the cumulative rate function is insensitive to \( K_0 \). We note also that for small timescales \( \tau_{\text{short}} \approx \Delta T/2 \), as would be expected for a rate that is independent of the timescale. The rate of short events is simply \( \Gamma_{\text{short}}(\Delta T) = \Gamma_{\text{long}}(0) - \Gamma_{\text{long}}(\Delta T) \). Thus, for \( K_0 = 17 \) mag the rate of events lasting less than 1 yr is \( 2 \times 10^{-3} \text{ yr}^{-1} \), and the average duration of such events is about 3 months.

In Figure 4 we plot \( \Delta K_{\text{long}} \), the median value of the maximal amplifications for long events (\( \tau > \Delta T \)) as a function of \( \Delta T \). Long events that amplify subthreshold stars are associated with stars at greater distances from the black hole, stars at the low-velocity tail of the velocity distributions, or stars that cross closer to the line of sight to Sgr A*. Because of the latter effect, \( \Delta K_{\text{long}} \) is greater than 0.75 mag, which is the median value for all the events. This effect is less marked for resolved lensing, which is characterized by longer timescales and is very weak for lensing of stars that lie above the detection threshold, where \( \Delta K_{\text{long}} \) approaches the limit \( \sim 0.75 \) mag.

Figure 5 shows the contributions to the lensing rate from different regions in the GC for a specific example where \( K_0 = 16.5 \) mag and \( \Delta T = 1 \) yr. Several important features are apparent in the results displayed in Figures 3 and 5. First, it is evident that the cumulative rate \( \Gamma_{\text{total}}(<r) \) approaches its asymptotic value at \( r < 10 \) pc and that short lensing events due to stars near the black hole dominate the total rate. For example, Figure 3 shows that the median timescale of unresolved amplifications of subthreshold stars for \( r < 2 \) months, and Figure 5 shows that the median distance from the black hole is \( \sim 0.3 \) pc. The run of the differential rate for long events, \( d\Gamma_{\text{long}}/dr \), with distance illustrates that the inner regions hardly contribute any long events. Second, unresolved lensing of subthreshold stars have the shortest timescales. This is because such events are due mainly to stars close to the black hole, where the velocities are high and the lensing cross sections are small. Unresolved lensing of stars that lie above the threshold, for which the cross sections are larger, have somewhat longer...
timescales, and resolved events, which are due to stars at larger distances from the black hole, are longer still. Third, the rates for resolved lensing are about an order of magnitude smaller than those for unresolved lensing events.

We now apply our results to estimate the probability that the variable $K$-band source (or sources) reported by Genzel et al. (1997; source S12) and Ghez et al. (1998; source S3) was a microlensing event. The monitoring of proper motions in the GC has been going on for about $T = 6$ yr, at a sampling interval of $\Delta T = 1$ yr, with a detection threshold of $K_0 = 16.5$ mag and an FWHM resolution of $\leq 0.15$ arcsec (Genzel et al. 1997). For this detection threshold, $\Delta \phi = 0.25$ (which corresponds to a required spatial resolution $\phi = 0.15$), and $d_s = 65$ pc. We can now use Figures 3 and 4 and equation (26) to estimate the detection probability and typical amplification of a lensing event in this experiment. The rate of unresolved and long events that amplify subthreshold stars is $7.5 \times 10^{-4}$ yr$^{-1}$, with a resulting detection probability $P_{\text{long}} \sim 0.5\%$. The median amplifications of such events is $\Delta K_{\text{long}} \sim 1.5$ mag above the detection threshold. The probability for detecting a short amplification of subthreshold stars is $P_{\text{short}} \sim 0.2\%$. The probabilities for detecting unresolved events involving above threshold stars is of the same order of magnitude. The probabilities for detecting long resolved events are an order of magnitude smaller, and the probabilities for detecting short resolved events are negligible in this experiment. Thus, we find that the behavior of the variable source (or sources) at Sgr A*, i.e., a brightening of a previously undetected source to 1.5–2 mag above the threshold with an event duration of $\sim 1$ yr, is the typical behavior that would be expected for a microlensing event. However, we also find that the a posteriori probability for detecting such an event is only $\sim 0.5\%$.

The probability for detecting a lensing event can be increased considerably by carrying out more sensitive observations at higher sampling rates. For example, 10 years of monthly observations with a detection threshold of $K_0 = 19$ mag will require a resolution of $\phi = 0.06$, which is already available (Ghez et al. 1998), and will increase the total detection probability of long lensing events to $P_{\text{long}} \sim 20\%$. These estimates are uncertain by a factor of few due to uncertainties in the stellar density distribution, the $K$ luminosity function, and the extinction.

5. DISCUSSION AND SUMMARY

In the past several years, very high resolution, very deep IR observations of the GC have regularly monitored stellar motions within few arcseconds of the radio source Sgr A*. The primary objective of these efforts is to dynamically weigh the central dark mass and set lower bounds on its compactness. As a by-product, these observations can detect and record the light curves of faint time-varying sources in the inner GC over timescales of years. These proper-motion studies have recently revealed the possible presence of one or two variable $K$-band sources very close to, or coincident with, the position of Sgr A* (Genzel et al. 1997; Ghez et al. 1998).

The first issue to resolve, when considering results from such technologically challenging observations, is whether these sources are real or merely artifacts of the complex procedures for obtaining deep diffraction limited images in the crowded GC. Assuming that such a source is indeed real and that it is not simply a variable star that happens to lie very near to the line of sight to Sgr A*, there are two interesting possibilities to consider, both directly linked to the existence of a supermassive compact object in the GC. A time-variable IR source, coincident with Sgr A*, may be the IR counterpart of the radio source, with the IR flare resulting from fluctuations in the accretion process, which has up to now eluded detection in any band other than the radio (see, e.g., Backer 1996). Another possibility is that the new source is a faint star in the dense stellar cluster in the GC, which is microlensed by the black hole. Such amplification events would appear as time-varying sources very close to the position of Sgr A*.

We note that a detection of microlensing could provide an independent probe of the compactness of the central dark mass. The innermost observed stars in the GC set an upper limit on the size of the dark mass, $r_{\text{lim}} \sim 7 \times 10^4 R_\odot$, where $R_\odot$ is the Schwarzschild radius (Ghez et al. 1998). Microlensing has the potential, given a well-sampled light curve, to improve on the dynamical limit, since for typical values of the lens-source distance, $r_{\text{lim}} \geq R_\odot$ (see eq. [3]). If the central mass is not a black hole, but rather an extended object, then the microlensing light curve will deviate from that of a point-mass lens when $r_{\text{lim}} \gtrsim R_\odot$. Because the innermost observed stars already constrain $\theta_{\text{lim}} \lesssim 0.1$, only lensed stars in the inner few parsecs have a small enough $\theta_0$ to probe possible structure in the dark mass distribution (eq. [2]). Therefore, the question whether there is a significant chance to detect such events is related to the yet-unresolved question of whether there is a strongly peaked stellar cusp in the innermost GC. We will discuss these issues elsewhere.

In this paper we investigated the possibility of microlensing amplification of faint stars in the dense stellar cluster in the GC by the supermassive black hole, which is thought to coincide with the radio source Sgr A*. We calculated in detail the rates, durations and amplifications of such events and considered separately the cases of unresolved and resolved images and of intrinsically faint and bright sources. We presented our results in a general way that can be used to estimate the detection probabilities of microlensing for a wide range of observing strategies.

The background stellar surface density increases with the detection threshold $K_0$. This determines the observational angular resolution required to detect a microlensing event and fixes the maximal distance behind the black hole for which the two lensed images of a star will appear unresolved. This maximal distance occurs before the integrated lensing rate reaches its asymptotic value, even for present-day detection thresholds. We therefore considered two manifestations of the lensing: unresolved microlensing of stars near the black hole and resolved lensing of stars farther away. We find that short lensing events of stars close to the black hole dominate the total lensing rate. This reflects the fact that the high stellar density and velocities near the black hole make it more likely for the smaller lensing cross-sections there. For this reason, and because unresolved images are on average twice as bright as the resolved images, unresolved microlensing dominates the lensing rate. We have also considered the lensing amplification of bright observed stars. The contribution of this type of microlensing to the total rate becomes progressively more important as the detection threshold decreases and at low sampling rates, which are primarily sensitive to longer events. We find that low sampling rates significantly bias the detection toward high-amplitude events.
Our predicted lensing rates are small but not so small as to be negligible. In particular, longer, deeper proper-motion monitoring done at higher rates, e.g., 10 years of monthly monitoring with $K_0 = 19$ mag, may have a significant chance of detecting such an event.

Finally, could either of the variable sources reported by Genzel et al. (1997; source S12) and by Ghez et al. (1998; source S3) be the amplified microlensed image of a faint star? The lack of evidence for related variability in the radio and X-rays, as would be expected in some accretion scenarios if the $K$-flare were due to fluctuations in the accretion process, argues against the possibility that the new source is the IR counterpart of Sgr A*.

Another possibility is that the new sources are variable stars, which are below the detection limit in their low state. Long period variable stars (LPVs), which are probably luminous Mira variables, are observed in the GC (Haller & Rieke 1989; Tamura et al. 1996). Typical amplitude variations of $\Delta K \sim 0.15$–0.5 mag are observed over 1–2 yr, although in some cases the variations are as large as $\Delta K \gtrsim 1$ mag. Haller & Rieke (1989) find 12 LPVs in a 4.5 arcmin$^2$ survey of the GC (not including the central 1' $\times$ 1') down to $K = 12$ mag ($M_K = -5.9$ mag). Of these, only one exhibited high-amplitude variations (1 mag in 4 months). At $K \sim 15$ mag, the new variable K source is much fainter than the LPVs observed in this survey. If it is an intrinsically bright LPV, it must lie on a highly extinguished line of sight. Using the observed surface density of $M_K < -5.9$ mag, $\Delta K \sim 1$ mag LPVs, it is possible to make a rough estimate of the probability for finding such a star within 0.15 of Sgr A*. Even after taking into account a factor of $\sim 40$ difference in the surface mass density between the dynamical center and the survey area at $\sim 2.5$ from the center, the probability is only $\sim 0.005%$. This of course does not rule out the possibility of an intrinsically faint but highly variable star. In any case, if the new source is a variable star, future observations should detect continued variability from this source.

We can also argue statistically against the possibility that these flares are due to microlensing of a star by another star in the GC that happens to be close to the line of sight to Sgr A*. Because $m/\mu \sim 10^{-6}$, the typical lensing timescale by a star will be of the order of 1 hr, 10$^3$ times shorter than that of lensing by the black hole (eqs. [1] and [8]). Furthermore, it is straightforward to show that the total rate of lensing of a star by a star within an angle 0'1 of the line of sight to Sgr A* is $\sim 5 \times 10^{-5}$ yr$^{-1}$ for $K_0 = 17$ mag, which is 100 times smaller than the rate of lensing by the black hole.

Our analysis has shown that the behavior of the variable $K$-band source (or sources) at Sgr A*, in particular a brightening of a previously undetected source to $\sim 1.5$–2 mag above the threshold, on a timescale of $\sim 1$ yr, is the typical behavior that would be expected for a microlensing event. However, we also estimate that the probability that such an event could have been observed during the course of the proper-motion studies that have been carried out thus far is only $\sim 0.5%$. While this probability is small, it is not so small as to rule out this possibility entirely. The probability of detecting a microlensing event at Sgr A* will increase considerably in the future as the observational sensitivities and the monitoring sampling rates improve.

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**APPENDIX**

THE MICROLENSING RATE AND AMPLIFICATION OF LONG DURATION EVENTS

Figure 3 shows that the total lensing rate is dominated by short events (shorter than a few month). Such events are shorter than the current mean time between observations and can therefore be observed only in the rare cases where they occur simultaneously with an observing run (§ 2). Of more relevance are long events, which span two or more observing runs. Longer events occur when the lensed star is either slow or when it is far away from the black hole, so that its trajectory simultaneously with an observing run ($\tau > \Delta T$). Of more relevance are long events, which span two or more observing runs.

Microlensing events with $\tau > \Delta T$ are those with transverse velocity $v_2$ and impact parameter $uR_\ast$, such that

$$\tau = \frac{2R_\ast \sqrt{u_{0,s}^2 - u^2}}{v_2} > \Delta T,$$

(A1)

where $s$ is the stellar type and $u_{0,s}$ is the maximal impact parameter for amplification above the threshold. The local rate per stellar type is

$$\frac{d^2\Gamma_2}{dr ds} = 2R_\ast n_s \int_0^{u_{0,s}} \int_0^{v_{\text{max}}(u_2)} v_2 DF_2(v_2)d^2v_2,$$

(A2)

where $v_{\text{max}} = 2(u_{0,s}^2 - u^2)^{1/2}R_\ast/\Delta T$ and $DF_2$ is the two-dimensional distribution function of velocities. The two-dimensional velocity $v_2$ is composed of an ordered component $v_{\text{rot}}$, which we assume to be perpendicular to the line of sight, and a random isotropic component, which we assume to be Gaussian with a one-dimensional dispersion $\sigma$. The two-dimensional distribution function of the projected velocity in polar coordinates is

$$DF_2(v_2)d\theta dv_2 d\theta = \frac{\tilde{v}_2}{2\pi} \exp \left[ - (\tilde{v}_2^2 + \tilde{v}_{\text{rot}}^2 - 2\tilde{v}_2 \tilde{v}_{\text{rot}} \cos \theta)/2 \right] d\tilde{v}_2 d\theta,$$

(A3)
In the limit of \( \lim_{\varepsilon_0 \to 0} \), where the weight function \( \hat{W}(\varepsilon_0) \) can be written as

\[
\hat{W}(\varepsilon_0, \varepsilon_\text{rot}) = \exp \left( -\varepsilon_\text{rot}^2 / 4 \right) \frac{1}{2} \int_{0}^{\varepsilon_\text{max}} x \left[ 1 - (x/\varepsilon_\text{max})^2 \right] \exp \left( -x^2/2 \right) I_0(x/\varepsilon_\text{rot}) dx .
\]

It is straightforward to verify that

\[
\lim_{\varepsilon_\text{max} \to \infty} W(\varepsilon_\text{max}, \varepsilon_\text{rot}) = \frac{1}{2} .
\]

The total rate-averaged impact parameter is

\[
\bar{\varepsilon} = \int_{0}^{r_2} \bar{u}(r) \frac{d\Gamma_2}{dr} dr / \Gamma_2 ,
\]

and the median amplification above the threshold can be estimated by \( A = \langle u_0 \rangle / \bar{\varepsilon} \).

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