A new numerical production well model for the low-permeability gas reservoir

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Abstract. In the development of low permeability gas reservoirs, capillary end effect has a significant impact on the flow near the production well, and physical quantities near the production well all change dramatically, which may reduce the oil and gas production. Traditional well model cannot describe this phenomenon. In this paper, based upon the analytical solution of one dimensional radial flow, an alternative numerical well model for low permeability gas reservoir considering the end effect is constructed. The numerical results show that the gas and water productions can be predicted accurately at different grid scales using the proposed well model. In contrast, the Peaceman well model for multi-phase only can get the accurate results when the grid is enough subdivided.

1. Introduction

In numerical reservoir simulation, it is necessary to consider the presence of wells. To solve this problem, a reservoir simulation system uses an analytical model to represent flow within a grid block as it enters or leaves a well. This model is called a well model [1]. It’s necessary to build a accurate numerical well model to relate the wellbore pressure and the well block pressure. In reservoir numerical simulation, the Peaceman [2] well model, based on the solution of the one-dimensional steady single-phase flow, is widely used.

When the Peaceman well model is extended to multi-phase flow, all the procedures are analogy to single-phase flow without the support of mathematical analysis. However, it has been shown that, at the interface of two different type rocks which have the different relative permeability and capillary pressure curves, multiphase flows in porous media face the problem of the discontinuities of fluid properties and transport properties [3]. This implies that the well model constructed from single-phase flow is not suitable for multiphase flow.

It has been observed that in the core experiment, when two-phase fluid flows into the non-capillary force container from the core sample containing capillary force, the phase saturations at the outlet end of the core increase significantly compared with the rest of the area [4]. This phenomenon was called as the capillary end effect. Also the capillary end effect happens near the production well. Some studies have shown that capillary may have an important effect on multiphase flow in the low-permeability reservoir. Holditch [5] found out that the gas flow would be completely blocked and water would be trapped in the reservoirs if the pressure drawdown cannot overcome the capillary end effect. Severe reduction in gas production may occur when the damage of capillary end effect is increased. Naik et al.
also found out that water blocking due to the capillary end effect near the wellbore vicinity creates significant formation damage and decreases well productivity for oil and gas [6]. These researches implies that the capillary end effect has the significant impact on the production well. As far as we know, this capillary end effect has not been considered in the existing well model for reservoir simulation. In this article, we will construct an alternative numerical well model considering capillary end effect to describe the gas recovery for the low-permeability water-wet reservoir.

2. Two flow patterns for the one-dimensional radial steady flow for immiscible two-phase fluid toward well

Define $P_{\text{inlet}}$ and $S_{\text{inlet}}$ as the gas-phase pressure and normalized water-phase saturation of the point $r=r_{\text{inlet}}$ in the vicinity of the production well. Details of these two flow patterns are provided as follows.

2.1. Flow pattern 1: both two phases flowing into wellbore

When both water and gas flow into the well, the saturation at the wellbore satisfies Eq.1 because of the exist of the capillary end effect

$$S|_{r=r_{\text{inlet}}}=1$$ (1)

2.2. Flow pattern 2: only gas phase flowing into wellbore

If $P_{\text{inlet}} \in \left( P_{wf} + P_{wf} + P_c \left( S_{\text{inlet}} \right) \right)$, then we have $P_{w}|_{r=r_{\text{inlet}}} \leq P_{wf}$ and $P_{g}|_{r=r_{\text{inlet}}} > P_{wf}$. Thus, the water phase is trapped in the reservoir and only the gas phase can flow into the wellbore. In this article, we only consider the production well case, so we have:

$$q_w = 0, \frac{dP}{dr} = 0$$ (2)

Then, we obtain:

$$P_{\text{inlet}} - P_c \left( S_{\text{inlet}} \right) = P_{wf} - P_c \left( S_{rw} \right)$$ (3)

Where $S_{rw}$ represents saturation at the entrance of wellbore.

3. Modified well model

In the following, the Ding’s [7] finite-volume method is used to construct an alternative numerical well model.

Figure 1. Block $(i, j)$ containing a well and its four neighboring blocks.

The schematic diagram is shown in Figure 1. According to Ding’s finite-volume method, the pressure $P^{(e)}$ located at $r_e$ can be determined by
\[
\begin{align*}
q_w &= \left[ T_{\text{eqw}} P_w^{(i+1,j)} + T_{\text{eqw},2} P_w^{(i,j+1)} + T_{\text{eqw},3} P_w^{(i-1,j)} + T_{\text{eqw},4} P_w^{(i,j-1)} - (T_{\text{eqw},1} + T_{\text{eqw},2} + T_{\text{eqw},3} + T_{\text{eqw},4}) P_w^{(i,j)} \right] \\
q_g &= \left[ T_{\text{eqg},j} m^{(i+1,j)} + T_{\text{eqg},2} m^{(i,j+1)} + T_{\text{eqg},3} m^{(i-1,j)} + T_{\text{eqg},4} m^{(i,j-1)} - (T_{\text{eqg},1} + T_{\text{eqg},2} + T_{\text{eqg},3} + T_{\text{eqg},4}) m^{(i,j)} \right]
\end{align*}
\]  
(4)

Where the equivalent mobility \( \lambda_{w,\text{eq}} \), equivalent relative permeability \( K_{r,g,\text{eq}} \) and equivalent transmissibilities for the water and gas phases can be expressed as:

\[
\begin{align*}
T_{\text{eqw},i} &= \frac{K h \rho c \lambda_{w,\text{eq}} \Delta y}{L_{\text{eql}}} \\
T_{\text{eqg},i} &= \frac{K h K_{r,g,\text{eq}} \Delta y}{L_{\text{eql}}}
\end{align*}
\]  
(5)

After obtaining the pressure \( P^{(e)} \) located at equivalent radius, integrating the steady two-phase flow equations on the interval \([r_w, r_c]\) in the wellblock grid, the calculation formula for two-phase flow is expressed as

\[
\begin{align*}
q_w &= \frac{[P_w^{(e)} - P_{w,f}] h}{\int_{r_w}^{r_c} 2\pi r \rho c \lambda_w dr} \\
q_g &= \frac{[m^{(e)} - m_{g,f}] h}{\int_{r_w}^{r_c} 2\pi r KK_{r,g} dr}
\end{align*}
\]  
(6)

Eq.4 and 6 constitute the two phase flow well model based on finite-volume method. And the equivalent radius can also be any value. The saturation distribution on the interval \([r_w, r_c]\) is needed to get the gas and water productions. In the following, the procedure is presented to calculate the saturation distribution for two different flow patterns.

3.1. Both of two phase flowing into wellbore (when \( P_g^{(i,j)} - P_e(S^{(i,j)}) > P_{w,f} \))

The phase pressures can be given by the following equation:

\[
\frac{d}{dr} \left( 2\pi r \rho_a h \frac{K K_{r,g}}{\mu_a} \frac{dP_a}{dr} \right) = 0
\]  
(7)

with the boundary condition:

\[
P_a \big|_{r=r_w} = P_{w,f} \quad P_a \big|_{r=r_c} = P_a^{(i,j)} \quad (\alpha = W, G)
\]  
(8)

3.2. Only gas phase flowing into wellbore (when \( P_g^{(i,j)} - P_e(S^{(i,j)}) \leq P_{w,f} \))

According the Eq.2, we obtain the following saturation equation

\[
\frac{d}{dr} \left( -2\pi r \rho_g (P_g) h \frac{K K_{r,g}(S_g)}{\mu_g} \frac{dP_g}{dS} \frac{dS}{dr} \right) = 0
\]  
(9)

with the boundary condition:

\[
S \big|_{r=r_w} = S_{rw} \quad S \big|_{r=r_c} = S^{(i,j)}
\]  
(10)
where the saturation $S_{rw}$ is determined by Eq.3 and the phase pressures are given by Eq.11

$$P_g^{(i,j)} - P_{w}^{(i,j)}(S_g^{(i,j)}) = P_{wf} - P_{w}^{(i,j)}(S_{rw})$$

(Eq. 11)

Eqs.7-11 can be solved numerically. In order to describe the dramatic changes of saturation and pressure near the wellbore, the logarithmic coordinate grids are adopted. In the following numerical tests, the solution interval $[r_w,r_e]$ is divided into enough equal parts on the logarithmic coordinate. Newton-Raphson iteration method is applied in the solution of the discretized nonlinear equations for Eqs.7-11 to obtain the saturation and pressure distributions for two-phase flow and only gas flow, respectively.

4. Numerical results

The simulation area is a 2D square reservoir with its size 500m×500m. At the center of the reservoir, there locates a well. Two-phase immiscible flow is considered here. Gas is compressible with its density varying with its pressure. In this paper, the ideal gas equation is used. The capillary pressure curve takes the form $P_c = B_0 S$, where $B=1$MPa. The outer boundary is imposed with impermeable boundary conditions. The relevant parameters of the reservoir and fluids are shown in Table 1.

| Parameters                          | Values  |
|------------------------------------|---------|
| Absolute permeability $K$(mD)      | 0.1     |
| Porosity $\phi$                    | 0.1     |
| Initial gas phase pressure $P_{init}$ (MPa) | 6      |
| Initial water phase saturation $S_{init}$ | 0.5   |
| Bottom hole pressure $P_{wf}$ (MPa) | 3.6     |
| Water phase relative permeability $K_{rw}$ | $S^4$  |
| Gas phase relative permeability $K_{rg}$ | $(1-S)^2(1-S)^2$ |
| Water phase viscosity $\mu_w$ (Pa·s) | 1e-3    |
| Gas phase viscosity $\mu_g$ (Pa·s)  | 2e-5    |
| Wellbore radius $r_w$ (m)           | 0.1     |

Since the well is far away the boundary, boundary conditions has little effect on the well flow. It means that the reference solution can be obtained by solving the steady two-phase radial flow equations for two different flow patterns, respectively. To test the accuracy of the original Peaceman and the proposed well models, the different grid sizes of 50×50, 100×100, 200×200 and 300×300 are adopted in the computations.

![Figure 2](image-url)

Figure 2. (a) Gas phase flux profiles (b) water phase flux profiles

Figure 2 shows that the results calculated by proposed well model with different grid sizes are consistent with the reference solution. However, the deviation between the calculation results adopting...
the Peaceman well model and the reference solution is even up to 40% when the grid system is 50×50. With the mesh subdivision, the results of Peaceman well model is gradually approaching the reference solution. It means that the extremely fine grid is needed to get more accurate results when adopting the Peaceman well model. In a word, the capillary end effect has the important influence for the prediction of the low-permeability reservoir recovery and the proposed well model considering capillary end effect can get better results.

5. Conclusion
In the recovery of low permeability reservoirs, the capillary pressure has an important effect, which may reduce gas production. In this paper, an alternative well model for low permeability gas reservoir is constructed, where two different flow patterns considering capillary end effect are considered. In contrast to the original Peaceman well model, the proposed well model can provide rather accurate prediction for gas and water productions. Since the capillary end effect has already been considered in the proposed well model, the production prediction based on the proposed well model is weakly dependent on the grid sizes.

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