Composite Vectors at the Large Hadron Collider

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Abstract

An unspecified strong dynamics may give rise to composite vectors sufficiently light that their interactions, among themselves or with the electroweak gauge bosons, be approximately described by an effective Lagrangian invariant under $SU(2)_L \times SU(2)_R / SU(2)_{L+R}$. We study the production at the LHC of two such states by vector boson fusion or by the Drell–Yan process in this general framework and we compare it with the case of gauge vectors from a $SU(2)_L \times SU(2)_R \times SU(2)^N$ gauge model spontaneously broken to the diagonal $SU(2)$ subgroup by a generic $\sigma$-model. Special attention is payed to the asymptotic behaviour of the different amplitudes in both cases. The expected rates of multi-lepton events from the decay of the composite vectors are also given. A thorough phenomenological analysis and the evaluation of the backgrounds to such signals, aiming at assessing the visibility of composite-vector pairs at the LHC, is instead deferred to future work.
1 Introduction

The energy scale characteristic of the EW interactions, or the scale of Electro-Weak Symmetry
Breaking (EWSB), has not yet been experimentally explored in an extensive way, notwithstanding
the results of LEP and of the Tevatron. Its thorough exploration is the primary task of the LHC.
In turn this suggests a cautious attitude in judging our level of understanding of the physics in
the TeV region and beyond it.

Broadly speaking, two alternative pictures can be thought of. In the first one, the physics of
EWSB is weakly coupled, a relatively light Higgs boson exists (as part of an extended system) and,
perhaps with the embedding of the Standard Model (SM) into a proper supersymmetric extension
at the weak scale, the perturbative physics can be extrapolated to much higher energies without
significant change. In the alternative case, the SM, with or without Higgs boson(s), cannot be
perturbatively extrapolated up to energies far above the Fermi scale, because of new forces or new
degrees of freedom or even new dimensions opening up nearby. These new phenomena are in a
way or another responsible for EWSB.

If it is allowed to characterize together all the different ideas belonging to the strong-coupling
alternative, as opposed to the perturbative picture all the way up to the GUT or the Planck
scale, it is clear that they suffer by a weaker calculative power. Furthermore, explicit models
are generally harder to accommodate with existing data, like the ElectroWeak Precision Tests
(EWPT) or the flavour tests. Yet dismissing this broad alternative before seeing the LHC data
would represent a severe unreasonable limitation. In fact we find it useful to take the following
general attitude. Rather than concentrating on any specific model of strong EWSB, it looks
more useful to focus, whenever possible, on effective Lagrangian descriptions of the new particles
expected with the incorporation of the relevant symmetries, exact or approximate. Among these
particles there could be spin-0, spin-1/2 or spin-1 states. The most obvious case is the one of a
SU(2)$_{L+R}$ singlet scalar, i.e. a composite Higgs boson [2, 3, 4, 5, 6]. Here we consider new spin-1
states. These states may be the lightest non standard particles and their discovery could provide
the first clue of strong EWSB at the LHC.

Let us therefore make the assumption - pretty standard in this framework - that the new
strong dynamics supposedly breaking the EW symmetry is by itself invariant under a global
SU(2)$_{L} \times$ SU(2)$_{R}$ symmetry, spontaneously broken to the diagonal SU(2)$_{L+R}$ subgroup. We
further assume that a vector state, $V$, belonging to the adjoint representation of SU(2)$_{L+R}$, exists
as a physical degree of freedom. $V$ is sufficiently lighter than a cut-off scale $\Lambda \approx 3$ TeV, that its
main properties can be caught by a suitable SU(2)$_{L} \times$ SU(2)$_{R}$/SU(2)$_{L+R}$ invariant Lagrangian,
also locally invariant under the SM gauge group SU(2)$_{L} \times$ U(1)$_{Y}$. We shall ignore other spin-1
states that could occur below $\Lambda$, although their incorporation would be straightforward and might
be needed for a fully consistent picture. One or more vectors relatively light with respect to $\Lambda$ might be instrumental to keep the $WW$ scattering amplitude from growing too much before $\Lambda$ [1, 7, 8] and even, surprisingly enough and anyhow under suitable conditions, to provide consistency
with the EWPT [9].

If not too heavy, say below 1 TeV, the single production, either by Vector Boson Fusion (VBF)
or by the Drell–Yan (DY) process, or its production in association with a standard gauge boson are

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1^ For a pioneering work in this direction see [1] and references therein.
very likely to be the first manifestations of $V$ at the LHC \[10, 11, 12, 13, 14\]. To understand the underlying dynamics, however, further measurements and observations will certainly be required. This motivates the study of the pair production of $V$, which we are going to do in this work under the assumption that also this process at the LHC can be described by an appropriate effective Lagrangian. From a phenomenological point of view, the pretty large number of different charge channels, from VBF or from DY, is of potential interest. We shall present the cross sections for $V$-pair production and the expected rates of multi-lepton events from the decay of such heavy vectors at the LHC, deferring to a further study a detailed investigation of the SM backgrounds, wherein acceptance cuts on final-state leptons and jets, as well as detector effects, are expected to play a role.

We call these vectors composite since they should arise dynamically from the new strong interaction, which is left unspecified. As such, the interactions of the composite vectors with the standard electroweak gauge bosons or among themselves are in general less constrained than if the new spin-1 states were the gauge vectors of a spontaneously broken gauge symmetry. It is in fact interesting to study the constraints that would arise in this case, which we do by considering a gauge theory based on $G = SU(2)_L \times SU(2)_R \times SU(2)^N$ broken to the diagonal subgroup $H = SU(2)_{L+R+\ldots}$ by a generic non-linear $\sigma$-model. This gauge model includes as special cases or approximates via deconstruction many of the models in the literature \[8, 15, 16, 17, 18, 19, 20\]. As foreseeable, this proves useful in discussing the high energy behaviour of the production amplitudes of the spin-1 states. In turn this is important for a consistent description of a relatively light vector by an effective Lagrangian approach, like the one attempted here.

## 2 The basic Lagrangian

The starting point is the usual lowest order chiral Lagrangian for the $SU(2)_L \times SU(2)_R / SU(2)^N$ Goldstone fields with the addition of the invariant kinetic terms for the $W$ and $B$ bosons

$$\mathcal{L}_\chi = \frac{\nu^2}{4} \left( D_\mu U \left( D^\mu U\right)^\dagger \right) - \frac{1}{2g^2} \langle W_{\mu\nu} W^{\mu\nu} \rangle - \frac{1}{2g'^2} \langle B_{\mu\nu} B^{\mu\nu} \rangle, \quad (2.1)$$

where

$$U(x) = e^{i\tilde{\pi}(x)/\nu}, \quad \tilde{\pi}(x) = \tau^a \pi^a = \begin{pmatrix} \pi^0 & \sqrt{2} \pi^+ \\ -\sqrt{2} \pi^- & -\pi^0 \end{pmatrix}, \quad (2.2)$$

$$D_\mu U = \partial_\mu U - iB_\mu U + iU W_\mu, \quad W_\mu = \frac{g}{2} \tau^a W^a_\mu, \quad B_\mu = \frac{g'}{2} \tau^3 B^0_\mu,$$

the $\tau^a$ are the ordinary Pauli matrices and $\langle \rangle$ denotes the trace over $SU(2)_L \times SU(2)_R$. The transformation properties of the Goldstone fields under $SU(2)_L \times SU(2)_R$ are

$$u \equiv \sqrt{U} \rightarrow g_R u h^\dagger = h g^\dagger_L, \quad (2.3)$$

where $h = h(u, g_L, g_R)$ is an element of $SU(2)_{L+R}$, as defined by this very equation \[21\].

Especially in low-energy QCD studies, the heavy spin-1 states are most often described by antisymmetric tensors \[22, 23\]. Here we shall on the contrary make use of the more conventional

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\(^2\)It is $m_W = gv/2$, so that $v \approx 250$ GeV.
Lorentz vectors, belonging to the adjoint representation of $SU(2)_{L+R}$,

$$V_\mu = \frac{1}{\sqrt{2}} \tau^a V^a_\mu, \quad V^\mu \rightarrow h V^\mu h^\dagger.$$  \hfill (2.4)

The $SU(2)_L \times SU(2)_R$-invariant kinetic Lagrangian for the heavy spin-1 fields is given by

$$\mathcal{L}_\text{kin} = -\frac{1}{4} \langle \hat{V}^{\mu\nu} \hat{V}_{\mu\nu} \rangle + \frac{M^2}{2} \langle V^\mu V_\mu \rangle,$$  \hfill (2.5)

where $\hat{V}_{\mu\nu} = \nabla_\mu V_\nu - \nabla_\nu V_\mu$ in terms of the covariant derivative

$$\nabla_\mu V_\nu = \partial_\mu V_\nu + [\Gamma_\mu, V_\nu], \quad \Gamma_\mu = \frac{1}{2} \left[ u^\dagger \left( \partial_\mu - iB_\mu \right) u + u \left( \partial_\mu - iW_\mu \right) u^\dagger \right], \quad \Gamma_\mu^\dagger = -\Gamma_\mu  \hfill (2.6)$$

Note that this covariant derivative transforms homogeneously as $V_\mu$ itself does. The other quantity that transforms covariantly is $u_\mu = u_\mu^\dagger = iu^\dagger D_\mu U u$, so that indeed $u_\mu \rightarrow h u_\mu h^\dagger$.

Assuming parity invariance of the new strong interaction, the full set of interactions of the spin-1 fields relevant to our problem is

$$\mathcal{L}_\text{int}^V = \mathcal{L}_1^V + \mathcal{L}_2^V + \mathcal{L}_3^V,$$  \hfill (2.7)

where

$$\mathcal{L}_1^V = -\frac{ig_V}{2\sqrt{2}} \langle \hat{V}^{\mu [u^\mu, u^\nu]} \rangle - \frac{f_V}{2\sqrt{2}} \langle \hat{V}_{\mu\nu} (u W^{\mu\nu} u^\dagger + u^\dagger B^{\mu\nu} u) \rangle,$$  \hfill (2.8)

$$\mathcal{L}_2^V = g_1 \langle V_\mu V^{\mu} u^\alpha u_\alpha \rangle + g_2 \langle V_\mu u^\alpha V^{\mu} u_\alpha \rangle + g_3 \langle V_\mu V_\nu [u^\mu, u^\nu] \rangle + g_4 \langle V_\mu V_\nu \{ u^\mu, u^\nu \} \rangle$$

$$+ g_5 \langle V_\mu (u^{\mu} V_\nu u^\nu + u^\nu V_\nu u^\mu) \rangle + ig_6 \langle V_\mu V_\nu (u W^{\mu\nu} u^\dagger + u^\dagger B^{\mu\nu} u) \rangle,$$  \hfill (2.9)

$$\mathcal{L}_3^V = \frac{ig_K}{2\sqrt{2}} \langle \hat{V}_{\mu\nu} V^\mu V^\nu \rangle.$$  \hfill (2.10)

Every parameter in (2.7) is dimensionless. From the total Lagrangian

$$\mathcal{L}^V = \mathcal{L}_\chi + \mathcal{L}_\text{kin}^V + \mathcal{L}_\text{int}^V,$$  \hfill (2.11)

we leave out:

- Operators involving 4 $V$’s or only light fields, either $W$ or $Z$ or the Goldstone $\pi$’s, since they only contribute at sub-leading order to the amplitudes considered in this work (although relevant in $W_L W_L$ elastic scattering).

- Operators of dimension higher than 4, which we assume to be weighted by inverse powers of the cutoff $\Lambda \approx 3$ TeV, as suggested by naive dimensional analysis. As such, they would contribute to the $VV$-production amplitudes at c.o.m. energies sufficiently below $\Lambda$ by small terms relative to the ones that we are going to compute.
• Direct couplings between any fermion of the SM and the composite vectors. This is plausible if the SM fermions are elementary. The third generation doublet could be an exception here. If this were the case, with a large enough coupling, this would not change any of the $VV$-production amplitudes, but might lead to a dominant decay mode of the composite vectors into top and/or bottom quarks, rather than into $W, Z$ pairs.

The relation of $L^V$ with the Lagrangian formulated in terms of anti-symmetric tensor fields is described in Appendix A.

3 $W_L W_L \rightarrow V_\lambda V_{\lambda'}$ helicity amplitudes

In this Section we calculate the scattering amplitudes for two longitudinal $W$-bosons into a pair of heavy vectors of any helicity $\lambda, \lambda' = L, +, -$. To simplify the explicit formulae, we take full advantage of $SU(2)_{L+R}$ invariance by considering the $g' = 0$ limit, so that $Z \approx W^3$. We also work at high energy, such that

$$\sqrt{s}, \sqrt{-t}, \sqrt{-u}, M_V >> M_W,$$

which allows us to make use of the equivalence theorem, i.e.

$$A(W_L^a W_L^b \rightarrow V_{\chi_L}^c V_{\chi_L}^d) \approx -A(\pi^a \pi^b \rightarrow V_{\chi_L}^c V_{\chi_L}^d).$$

This restriction will be dropped in Sections 6 and 7, where we shall present numerical results, although the limitations of the effective Lagrangian approach will remain.

There are in fact four such independent amplitudes:

$$A(W_L^a W_L^b \rightarrow V_{\chi_L}^c V_{\chi_L}^d)$$
$$A(W_L^a W_L^b \rightarrow V_{\chi_L}^c V_{\chi_L}^d)$$
$$A(W_L^a W_L^b \rightarrow V_{\chi_L}^c V_{\chi_L}^d) = A(W_L^a W_L^b \rightarrow V_{\chi_L}^c V_{\chi_L}^d)$$

and

$$A(W_L^a W_L^b \rightarrow V_{\chi_L}^c V_{\chi_L}^d) = -A(W_L^a W_L^b \rightarrow V_{\chi_L}^c V_{\chi_L}^d).$$

By $SU(2)_{L+R}$ invariance the general form of these amplitudes is

$$A(W_L^a W_L^b \rightarrow V_{\chi_L}^c V_{\chi_L}^d) = A_{\lambda\lambda'}(s, t, u)\delta^{ab}\delta^{cd} + B_{\lambda\lambda'}(s, t, u)\delta^{ac}\delta^{bd} + C_{\lambda\lambda'}(s, t, u)\delta^{ad}\delta^{bc},$$

where, by Bose symmetry, it is simple to prove that

$$A_{\lambda\lambda'}(s, t, u) = A_{\lambda\lambda'}(s, u, t)$$
$$C_{\lambda\lambda'}(s, t, u) = B_{\lambda\lambda'}(s, u, t),$$

for $\lambda\lambda' = LL, +-, ++,$

whereas

$$A_{L+}(s, t, u) = -A_{L+}(s, u, t)$$
$$C_{L+}(s, t, u) = -B_{L+}(s, u, t).$$

These amplitudes receive contributions from:

i) contact interactions, $\pi^2 V^2$, contained in $L_{\text{kin}}^V$ and proportional to unity (with an overall $1/v^2$ factored out) or contained in $L_{2V}$ and proportional to $g_i, i = 1, \ldots, 5$;
ii) one-π exchange, proportional to \( g_\pi^2 \), contained in \( \mathcal{L}_{1V} \);
iii) one-\( V \) exchange, proportional to \( g_V^2 g_K \), with \( g_V \) contained in \( \mathcal{L}_{1V} \) and \( g_K \) in \( \mathcal{L}_{3V} \).

For ease of the reading, we keep first only the contributions with \( \mathcal{L}_{2V} \) and \( \mathcal{L}_{3V} \) set to zero, so that

- For \( \lambda\lambda' = LL \)

\[
A_{LL}^{1V} = -\frac{G_V^2 s}{v^4 (s - 4M_V^2)} \left[ \frac{(t + M_V^2)^2}{t} + \frac{(u + M_V^2)^2}{u} \right],
\]
\[
B_{LL}^{1V} = \frac{u - t}{2v^2} + \frac{G_V^2 s (u + M_V^2)^2}{v^4 u (s - 4M_V^2)}. \tag{3.10}
\]

- For \( \lambda\lambda' = +- \)

\[
A_{+-}^{1V} = \frac{2G_V^2 M_V^2 (t + u)(tu - M_V^4)}{v^4 tu (s - 4M_V^2)},
\]
\[
B_{+-}^{1V} = \frac{2G_V^2 M_V^2 (M_V^4 - tu)}{uv^4 (s - 4M_V^2)}. \tag{3.11}
\]

- For \( \lambda\lambda' = ++ \)

\[
A_{++}^{1V} = \frac{2G_V^2 M_V^2 (t + u)(M_V^4 - tu)}{v^4 tu (s - 4M_V^2)},
\]
\[
B_{++}^{1V} = \frac{(t - u)}{2v^2} - \frac{2G_V^2 M_V^2 (M_V^4 - tu)}{uv^4 (s - 4M_V^2)}. \tag{3.12}
\]

- For \( \lambda\lambda' = L+ \)

\[
A_{L+}^{1V} = \frac{\sqrt{2}G_V^2 M_V^2 (t - u) \sqrt{s(tu - M_V^4)}}{v^4 tu (s - 4M_V^2)},
\]
\[
B_{L+}^{1V} = \frac{-\sqrt{s(tu - M_V^4)} \{v^2 su + 4M_V^2 [G_V^2 (M_V^4 + u) - v^2 u]\}}{2\sqrt{2} uv^4 M_V (s - 4M_V^2)}. \tag{3.13}
\]

Here and in the following, we set

\[
G_V \equiv g_V M_V, \quad F_V \equiv f_V M_V, \tag{3.18}
\]

 adopting a notation familiar in the description of spin-1 states by anti-symmetric Lorenz tensor fields. As discussed in Appendix [A] these same amplitudes would indeed be obtained using anti-symmetric tensors instead of Lorentz vectors to describe the spin-1 states.

Switching on \( \mathcal{L}_{2V} \) and \( \mathcal{L}_{3V} \) gives an extra contribution to the various amplitudes:

\[^3\text{In all these functions the variables are in the order } (s, t, u) \text{ and are left understood.}\]
• For $\lambda\lambda' = LL$

$$\Delta A_{LL} = (g_1 - g_2) \frac{s(s - 2M^2_V)}{v^2M^2_V} + (g_4 - g_5) \frac{s[2M^2_V(3M^2_V - s) + t^2 + u^2]}{v^2M^2_V(s - 4M^2_V)},$$

$$\Delta B_{LL} = g_2 \frac{s(s - 2M^2_V)}{v^2M^2_V} + s(t - u) \left( g_3 + \frac{g_Kg_V(s + 2M^2_V)}{4} + g_5 \frac{s[2M^2_V(3M^2_V - s) + t^2 + u^2]}{v^2M^2_V(s - 4M^2_V)} \right).$$

• For $\lambda\lambda' = +-$

$$\Delta A_{+-} = 4(g_4 - g_5) \frac{(M^4_V - tu)}{v^2(s - 4M^2_V)},$$

$$\Delta B_{+-} = 4g_5 \frac{(M^4_V - tu)}{v^2(s - 4M^2_V)}.$$

• For $\lambda\lambda' = ++$

$$\Delta A_{++} = 2(g_1 - g_2) \frac{s}{v^2} + 4(g_4 - g_5) \frac{(tu - M^4_V)}{v^2(s - 4M^2_V)},$$

$$\Delta B_{++} = 2g_2 \frac{s}{v^2} + \frac{4g_5(tu - M^4_V)}{v^2(s - 4M^2_V)} - \frac{g_Kg_Vs(t - u)}{2v^2(s - M^2_V)}.$$

• For $\lambda\lambda' = L+$

$$\Delta A_{L+} = (g_4 - g_5) \frac{(t - u)\sqrt{2s(tu - M^4_V)}}{v^2M_V(s - 4M^2_V)},$$

$$\Delta B_{L+} = \frac{\sqrt{2s(tu - M^4_V)}}{v^2M_V} \left[ g_3 \frac{t - u}{s - 4M^2_V} + \left( g_3 + \frac{g_Kg_Vs}{2} \frac{s}{s - M^2_V} \right) \right].$$

### 3.1 Asymptotic behaviour of the $W_LW_L \rightarrow V_\lambda V_{\lambda'}$ amplitudes

For arbitrary values of the parameters all these amplitudes grow at least as $s/v^2$ and some as $s^2/(v^2M^2_V)$ or as $s^{3/2}/(v^2M_V)$. As readily seen from these equations, there is on the other hand a unique choice of the various parameters that makes all these amplitudes growing at most like $s/v^2$, i.e.

$$g_Vg_K = 1, \quad g_3 = -\frac{1}{4}, \quad g_1 = g_2 = g_4 = g_5 = 0,$$

whereas $f_V$ and $g_6$ are irrelevant. With this choice of parameters the various helicity amplitudes simplify to

• For $\lambda\lambda' = LL$

$$A_{LL}^{gauge} = -\frac{G^2_Vs}{v^4(s - 4M^2_V)} \left[ \frac{(t + M^2_V)^2}{t} + \frac{(u + M^2_V)^2}{u} \right],$$

$$B_{LL}^{gauge} = \frac{u - t}{2v^2} + \frac{G^2_Vs(u + M^2_V)^2}{v^4u(s - 4M^2_V)} - \frac{3s(u - t)}{4v^2(s - M^2_V)}.$$
• For $\lambda\lambda' = +-$

\[
A^{\text{gauge}}_{+-} = \frac{2G^2 V M_V^2 (t + u) (tu - M_V^4)}{v^4 tu (s - 4M_V^2)}, \tag{3.30}
\]

\[
B^{\text{gauge}}_{+-} = \frac{2G^2 V M_V^2 (M_V^4 - tu)}{uv^4 (s - 4M_V^2)}. \tag{3.31}
\]

• For $\lambda\lambda' = ++$

\[
A^{\text{gauge}}_{++} = \frac{2G^2 V M_V^2 (t + u) (M_V^4 - tu)}{v^4 tu (s - 4M_V^2)}, \tag{3.32}
\]

\[
B^{\text{gauge}}_{++} = -\frac{M_V^2 (t - u)}{2v^2 (s - M_V^2)} + \frac{2G^2 V M_V^2 (M_V^4 - tu)}{uv^4 (s - 4M_V^2)}. \tag{3.33}
\]

• For $\lambda\lambda' = L+$

\[
A^{\text{gauge}}_{L+} = \frac{\sqrt{2}G^2 V M_V^2 (t - u) \sqrt{s (tu - M_V^4)} }{v^4 tu (s - 4M_V^2)}, \tag{3.34}
\]

\[
B^{\text{gauge}}_{L+} = -\frac{\sqrt{2}G^2 V M_V (M_V^4 + u) \sqrt{s (tu - M_V^4)}}{uv^4 (s - 4M_V^2)} + \frac{M_V \sqrt{s (tu - M_V^4)}}{\sqrt{2}v^2 (s - M_V^2)}. \tag{3.35}
\]

We show in Section 3 that the relations (3.27), and so the special form of the $W_LW_L \rightarrow V_LV'_L$ helicity amplitudes, arise in a minimal gauge model for the vector $V_L$. In the generic framework considered here, some deviations from (3.27) may occur. In such a case the asymptotic behaviour of the various amplitudes will have to be improved, e.g., by the occurrence of heavier composite states, vectors and/or scalars, with appropriate couplings. Note in any event that, even sticking to the relations (3.27), the amplitudes for longitudinally-polarized vectors grow as $s/v^2$ for any value of $G^2_V$. 

### 4 Drell–Yan production amplitudes

At the parton level there are four Drell–Yan production amplitudes, related to each other by $SU(2)$- invariance (in the $g'$ limit, as usual):

\[
|A(u\bar{d} \rightarrow V^+V^0)| = |A(d\bar{u} \rightarrow V^-V^0)| = \sqrt{2}|A(u\bar{u} \rightarrow V^+V^-)| = \sqrt{2}|A(d\bar{d} \rightarrow V^+V^-)|. \tag{4.1}
\]

They receive contributions from: i) $W(Z)$-exchange diagrams, with the $W(Z)$ coupled to a pair of composite vectors either through their covariant kinetic term, $\mathcal{L}^V_{\text{kin}}$, or via $g_6$ in $\mathcal{L}_{2V}$; ii) light-heavy vector mixing diagrams proportional to $f_V g_K$ with these couplings contained in $\mathcal{L}_{1V}$ and $\mathcal{L}_{3V}$. Their modulus squared, summed over the polarizations of the final-state vectors and averaged over colour and polarization of the initial fermions, can be written as

\[
< |A(u\bar{d} \rightarrow V^+V^0)|^2 > = \frac{g^4}{1536 M_V^2 s^2 (s - M_V^2)^2} F(s, t - u, M_V^2), \tag{4.2}
\]
with $F$ organized in different powers of $s$:

$$F(s, t - u, M^2_F) = F^{(6)}(s, t - u, M^2_F) + F^{(5)}(s, t - u, M^2_F) + F^{(4)}(s, t - u, M^2_F),$$  

(4.3)

where

$$F^{(6)} = (g_K f_V - 4 g_6)^2 M^2_F s^4 [s^2 - (t - u)^2],$$

(4.4)

$$F^{(5)} = 4 M^4_F s^3 \left\{ (g_K f_V - 4 g_6)^2 \left[ 2 s^2 + (t - u)^2 \right] + (g_K f_V - 4 g_6) \left[ (2 (7 g_6 - 3) s^2 + 2 (g_6 - 1) (t - u)^2 + 2 (1 - 2 g_6) s^2 + (t - u)^2) \right] \right\},$$

(4.5)

$$F^{(4)} = 4 M^6_F \left\{ -3 s^2 f^2_v g^2_K \left[ 3 s^2 + (t - u)^2 + 4 M^2_F s \right] - 4 M^4_F \left[ (4 g_6 (g_6 + 2) - 25) s^2 + (t - u)^2 \right] + 2 f_v g_K \left( s \left\{ (26 g_6 + 9) s^2 + (4 g_6 + 7) (t - u)^2 \right\} - 6 M^2_F \left[ (4 g_6 - 3) s^2 + (t - u)^2 \right] - 24 s M^4_F \right\] + 2 M^6_F s \left[ (28 g^2_6 + 9 (8 g_6 - 3)) s^2 + (48 g^2_6 + 13) (t - u)^2 \right] - 4 s^2 \left[ 3 g_6 (g_6 + 8) s^2 + (5 g^2_6 + 4) (t - u)^2 \right] - 48 M^6_F s \right\}.$$  

(4.6)

$F^{(5)}$ is written in such a way as to make evident what controls its high-energy behaviour after the dominant $F^{(6)}$ is set to zero by taking $g_K f_V = 4 g_6$. In general, these amplitudes squared grow at high energy as $(s/M^2_F)^2$, which is turned to a constant behaviour for

$$g_K f_V = 2, \quad g_6 = \frac{1}{2}.$$  

(4.7)

In this special case the function $F$ in eq. (4.2) acquires the form

$$F^{\text{gauge}} = 4 M^6_F \left\{ s^2 \left[ s^2 - (t - u)^2 \right] + 4 M^2_F s \left[ 2 s^2 + (t - u)^2 \right] - 12 M^4_F \left[ 3 s^2 + (t - u)^2 \right] - 48 M^6_F s \right\}.$$  

(4.8)

## 5 Composite versus gauge models

Before studying the physical consequences for the LHC of the amplitudes calculated in the previous Sections, we consider the connection between a composite vector, as discussed so far, and a gauge vector of a spontaneously broken symmetry [9 23]. For concreteness we take a gauge theory based on $G = SU(2)_L \times SU(2)_R \times SU(2)^N$ broken to the diagonal subgroup $H = SU(2)_{L+R+...}$ by a generic non-linear $\sigma$-model of the form

$$\mathcal{L}_X = \sum_{I,J} \bar{v}^2_{I,J} (D_{\mu} \Sigma_{IJ} (D^\mu \Sigma_{IJ}))^\dagger, \quad \Sigma_{IJ} \rightarrow g_I \Sigma_{IJ} g_J^\dagger,$$  

(5.1)

where $g_{I,J}$ are elements of the various $SU(2)$ and $D_{\mu}$ are covariant derivatives of $G$. Both the gauge couplings of the various $SU(2)$ groups and $\mathcal{L}_X$ are assumed to conserve parity. This gauge model includes as special cases or approximates via deconstruction many of the models in the literature [15 8 17 18 19 20]. The connection between a gauge model and a composite model for the spin-1 fields is best seen at the Lagrangian level by a suitable field redefinition, as we now show.
For the clarity of exposition let us first consider the simplest \( N = 1 \) case, based on \( SU(2)_L \times SU(2)_C \times SU(2)_R \), i.e. on the Lagrangian

\[
\mathcal{L}_{\text{gauge}}^\chi = \mathcal{L}_{\chi}^\text{gauge} - \frac{1}{2g_C^2} \langle v_{\mu\nu} v_{\mu\nu} \rangle - \frac{1}{2g^2} \langle W_{\mu\nu} W_{\mu\nu} \rangle - \frac{1}{2g^2} \langle B_{\mu\nu} B_{\mu\nu} \rangle,
\]

(5.2)

where

\[
v_{\mu} = \frac{g_C}{2} v_{\mu}^a t^a
\]

is the \( SU(2)_C \)-gauge vector and the symmetry-breaking Lagrangian is described by

\[
\mathcal{L}_{\text{gauge}}^\chi = \frac{v^2}{2} \left( D_{\mu} \Sigma_{RC} (D^\mu \Sigma_{RC})^\dagger \right) + \frac{v^2}{2} \left( D_{\mu} \Sigma_{CL} (D^\mu \Sigma_{CL})^\dagger \right).
\]

(5.4)

Denoting collectively the three gauge vectors by

\[
v_{\mu}^I = (W_{\mu}, v_{\mu}, B_{\mu}), \quad I = (L, C, R),
\]

(5.5)

one has for the two bi-fundamental scalars \( \Sigma_{IJ} \)

\[
D_{\mu} \Sigma_{IJ} = \partial_{\mu} \Sigma_{IJ} - i v_{\mu}^I \Sigma_{IJ} + i \Sigma_{IJ} v_{\mu}^J.
\]

(5.6)

The \( \Sigma_{IJ} \) can be put in the form \( \Sigma_{IJ} = \sigma_I^T \sigma_J^\dagger \), where \( \sigma_I \) are the elements of \( SU(2)_I / H \), transforming under the full \( SU(2)_L \times SU(2)_C \times SU(2)_R \) as \( \sigma_I \rightarrow g_I \sigma_I h^I \).

As the result of a gauge transformation

\[
v_{\mu}^I \rightarrow \sigma_I^T v_{\mu}^I \sigma_I + i \sigma_I^T \partial_{\mu} \sigma_I \equiv \Omega_{\mu}^I, \quad \Sigma_{IJ} \rightarrow \sigma_I^T \Sigma_{IJ} \sigma_J = 1,
\]

(5.7)

the symmetry-breaking Lagrangian reduces to

\[
\mathcal{L}_{\text{gauge}}^\chi = \frac{v^2}{2} \left( (\Omega_{\mu}^R - \Omega_{\mu}^C)^2 \right) + \frac{v^2}{2} \left( (\Omega_{\mu}^L - \Omega_{\mu}^C)^2 \right),
\]

(5.8)

or, after the gauge fixing \( \sigma_R = \sigma_L^T = u \) and \( \sigma_C = 1 \), to

\[
\mathcal{L}_{\text{gauge}}^\chi = v^2 \left( (v_{\mu} - i \Gamma_{\mu})^2 \right) + \frac{v^2}{4} \left( u_{\mu}^2 \right),
\]

(5.9)

where

\[
u_{\mu} = \Omega_{\mu}^R - \Omega_{\mu}^L, \quad \Gamma_{\mu} = \frac{1}{2i} (\Omega_{\mu}^R + \Omega_{\mu}^L)
\]

(5.10)

coincide with the same vectors defined in Section 2.

We can finally make contact with the Lagrangian (2.11) by setting

\[
v_{\mu} = V_{\mu} + i \Gamma_{\mu}
\]

(5.11)

and by use of the identity

\[
v_{\mu\nu} = \hat{V}_{\mu\nu} - i \{ V_{\mu}, V_{\nu} \} + \frac{i}{4} [u_{\mu}, u_{\nu}] + \frac{1}{2} (u W_{\mu\nu} u^\dagger + u^\dagger B_{\mu\nu} u).
\]

(5.12)

With the further replacement \( V_{\mu} \rightarrow g_C / \sqrt{2} V_{\mu} \), \( \mathcal{L}_{\text{gauge}}^{\chi} \) coincides as anticipated with \( \mathcal{L}^V \) in (2.11) for

\[
g_C = \frac{1}{2g_V}
\]

(5.13)

in the special case of (3.27) and \( g_6 = 1/2, f_V = 2g_V, M_V = g_K v/2 \) (or \( G_V = v/2 \)).
5.1 More than a single gauge vector

To discuss the case of more than one vector, i.e. \( N > 1 \), one decomposes the vectors associated to \( SU(2)^N \) with respect to parity as

\[
\Omega_i^\mu = v_i^\mu + a_i^\mu, \quad \Omega_P(i) = v_i^\mu - a_i^\mu, \quad i = 1, \ldots, N, \tag{5.14}
\]

so that under \( SU(2)_L \times SU(2)_R \)

\[
v_i^\mu \to hv_i^\mu h^\dagger + ih\partial^\mu h^\dagger, \quad a_i^\mu \to ha_i^\mu h^\dagger. \tag{5.15}
\]

In terms of these fields the gauge Lagrangian becomes

\[
\mathcal{L}_{\text{gauge}} = \mathcal{L}_{\text{gauge,SM}} - \sum_i \frac{1}{2g_i^2} \left[ \langle (v_i^{\mu\nu} - i[a_i^\mu, a_i^\nu])^2 \rangle + \langle (D_i^\mu a_i^\nu - D_i^\nu a_i^\mu)^2 \rangle \right], \tag{5.16}
\]

where \( v_i^{\mu\nu} \) are the usual field strengths and

\[
D_i^\mu a_i^\nu = \partial^\mu a_i^\nu - i[a_i^\mu, a_i^\nu]. \tag{5.17}
\]

At the same time, as a generalization of eq. (5.9) in the \( N = 1 \) case, the symmetry-breaking Lagrangian will be the sum of two separated quadratic forms in the parity-even and parity-odd fields of the type

\[
\mathcal{L}_{\chi}^{\text{gauge}} = \mathcal{L}_m^V(v_i^\mu \!-\! i\Gamma^\mu) + \mathcal{L}_m^A(u^\mu, a_i^\mu). \tag{5.18}
\]

Concentrating on the parity-even fields only, by setting

\[
v_i^\mu = V_i^\mu + i\Gamma^\mu \tag{5.19}
\]

and by the replacements \( V_i^\mu \to g_i/\sqrt{2}V_i^\mu \), the Lagrangian of the \( SU(2)_L \times SU(2)_R \times SU(2)^N \) model, restricted to the parity-even vectors, becomes a diagonal sum of \( \mathcal{L}_m^V \), each with \( g_1 = g_2 = g_4 = g_5 = g_6 = 1/2 \) and \( g_{V_i} = f_{V_i}/2 = 1/g_{K_i} \), except that the \( V_i^\mu \) are not mass eigenstates. Going to the mass-eigenstate basis maintains all the couplings quadratic in the \( V_i^\mu \) unaltered as well as the relation \( f_{V_i} = 2g_{V_i} \) for the individual mass-eigenstate vectors. On the other hand, the trilinear couplings \( g_{K_i} \), get spread among the mass eigenstates (still called \( V_i^\mu \)), so that

\[
\mathcal{L}_{3V} = \frac{i\hat{g}_{imn}^{K}}{2\sqrt{2}} \left\langle \hat{V}_m^{\mu} V_m^{\nu} V_n^{\nu} \right\rangle. \tag{5.20}
\]

Picking up the lightest vector only, \( i = 1 \), this implies \( \hat{g}_K^{111} \hat{g}_{V_1} \neq 1 \), where the *hat* denotes the couplings of the physical mass eigenstates. By the orthogonality of the rotation matrix that brings to the mass basis, it is easy to prove, however, the following sum rule over the full set of vectors\(^4\)

\[
\sum_i \hat{g}_{V_i} \hat{g}_K^{imn} = \frac{1}{2} \sum_i \hat{f}_{V_i} \hat{g}_K^{imn} = 1 \tag{5.21}
\]

for any fixed \( n \). This ensures that the asymptotic behaviour of the amplitudes studied above would not be worse than in the case of a single gauge vector, but only at \( s > M_{V_1}^2 \) for any \( i \).

\(^4\)For related sum rules, see [24]
In this Section we compute the LHC production cross section at \( \sqrt{S} = 14 \text{ TeV} \) from VBF of two heavy vectors in the different charge configurations

\[
pp \to W^+W^- , ZZ, \gamma\gamma, \gamma Z + qq \to V^+V^- + qq \ (\to W^+Z W^-Z + qq), \\
pp \to W^+W^- , ZZ + qq \to V^0V^0 + qq \ (\to W^+W^-W^+W^- + qq), \\
pp \to W^\pm W^\mp + qq \to V^\pm V^\mp + qq \ (\to W^\pm Z W^\pm Z + qq), \\
pp \to W^\pm Z, W^\pm\gamma + qq \to V^\pm V^0 + qq \ (\to W^\pm Z W^+W^- + qq).
\]

(6.1)-(6.4)

In the last step of these equations we have indicated the final state due to the largely dominant decay modes of the heavy vectors into WW or WZ (See e.g. [9]). The cross sections are summed over all the polarizations of the heavy spin-1 fields. In the calculation of the cross sections we reintroduce the hypercharge coupling \( g' \neq 0 \) and we make standard acceptance cuts for the forward quark jets,

\[
p_T > 30 \text{ GeV}, \ |\eta| < 5.
\]

These cross sections depend in general on a number of parameters. Fig. 1.a shows the total cross sections for the different charge channels with all the parameters fixed as in the minimal gauge model, eq. (3.27), and \( G_V = g_V M_V = 200 \text{ GeV} \). A value of \( G_V \) between 150 and 200 GeV keeps the elastic \( W_L W_L \)-scattering amplitude from saturating the unitarity bound below \( \Lambda \), almost independently from \( M_V < 1.5 \text{ TeV} \) [1] [9]. \( M_V \) is taken to range from 400 to 800 GeV. A value of \( M_V \) above 800 GeV would lead to a threshold for the vector-boson-fusion subprocess dangerously close to the cut-off scale of the effective Lagrangian. We have checked that the typical centre-of-mass energy of \( WW \to VV \) is on average well below 2.5 TeV, even for the highest \( M_V \) that we consider.

6 Pair production cross sections by vector boson fusion

Figure 1: Total cross sections for pair production of heavy vectors by vector boson fusion in a gauge model (1.a) and a composite model (1.b) as functions of the heavy vectors masses. See text for the choice of parameters and acceptance cuts.
As discussed in Sections 3-5, the parameters of the minimal gauge model damp the high energy behaviour of the different amplitudes. Not surprisingly, therefore, any deviation from them leads to significantly larger cross sections, as it may be the case already in a gauge model with more than one vector. As an example, this is shown in Fig. 1.b, where all the parameters are kept as in Fig. 1.a, except for \( g_K = g_V = 1/\sqrt{2} \) rather than 1, having in mind a compensation of the growing amplitudes by the occurrence of (a) significantly heavier vector(s) (See eq. 5.21). Furthermore, both in the VBF case and in the DY case, to be discussed below, it must be stressed that the deviations from the minimal gauge model are quite dependent on the choice of the parameters, with cross sections that can be even higher than those in Fig. 1. In turn, these cross sections have to be considered as indicative, given the limitations of the effective Lagrangian approach.

To calculate the cross sections, we have used the matrix-element generator CalcHEP [25], which allows one to obtain the exact amplitude for a process such as \( q_1 q_2 \rightarrow V V q_3 q_4 \) via intermediate off-shell vector bosons. As a check, the results so obtained have been compared with the same cross sections in the Effective Vector Boson Approximation, using the analytic amplitudes in Sect. 3 for \( g' = 0 \) and without acceptance cuts. While being a factor of \( 1.5 \div 2 \) systematically lower, the exact results are confirmed in their \( M_V \)-dependence and in the relative size of the different charge channels.

### 7 Drell–Yan pair production cross sections

The DY process is an additional source of \( V \)-pair production at the LHC. From the elementary parton-level amplitudes \( q \bar{q} \rightarrow V^+ V^- \) and \( q_i \bar{q}_j \rightarrow V^\pm V^0 \) of Section 4, the physical cross sections for the different charge channels

\[
pp \rightarrow V^+ V^- \quad (7.1)
\]

\[
pp \rightarrow V^\pm V^0 \quad (7.2)
\]

are readily computed. In general, the cross sections depend in this case on 3 parameters other than \( M_V: f_V, g_K \) and \( g_6 \).

As for the vector boson fusion, we show in Fig. 2.a the three cross sections for the values taken by the parameters in the minimal gauge model, \( f_V g_K = 2, g_6 = 1/2 \), and for \( F_V = f_V M_V = 400 \) GeV (corresponding to \( f_V = 2 g_V \) and \( G_V = g_V M_V = 200 \) GeV as in Fig. 1.a). On the other hand, similarly to Fig. 1.b, we show in Fig. 2.b the cross sections for \( f_V g_K = \sqrt{2}, g_6 = 1/2 \) and still \( F_V = f_V M_V = 400 \) GeV.

### 8 Same-sign di-lepton and tri-lepton events

After decay of the composite vectors,

\[
V^\pm \rightarrow W^\pm Z, \quad V^0 \rightarrow W^+ W^-, \quad (8.1)
\]

each \( VV \)-production channel, either from VBF or from DY, leads to final states containing 2 \( W \)'s and 2 \( Z \)'s, from \( V^+ V^- \) and \( V^\pm V^\mp \), 3 \( W \)'s and 1 \( Z \), from \( V^+ V^0 \), or 4 \( W \)'s from \( V^0V^0 \). In fact, all\[5\]

\[5\] Or in fact multi-top events, see Section 2
Figure 2: Total cross sections for pair production of heavy vectors via Drell–Yan $q\bar{q}$ annihilation in a gauge model (2.a) and a composite model (2.b) as functions of the heavy vectors masses. See text for the choice of parameters.

Figure 2: Total cross sections for pair production of heavy vectors via Drell–Yan $q\bar{q}$ annihilation in a gauge model (2.a) and a composite model (2.b) as functions of the heavy vectors masses. See text for the choice of parameters.

final states, except for $V^+V^-$, contain at least a pair of equal sign $W$’s, i.e., after $W \rightarrow e\nu, \mu\nu$, a pair of same-sign leptons. In most cases there are at least 3 $W$’s, i.e. also 3 leptons.

Table 1: Number of events with at least two same-sign leptons or three leptons ($e$ or $\mu$ from $W$ decays) from vector boson fusion (VBF) or Drell–Yan (DY) at the LHC for $\sqrt{S} = 14$ TeV and $\int \mathcal{L}dt = 100 \text{ fb}^{-1}$ in the minimal gauge model (MGM) or in a composite model (comp) with the parameters as in Figs. 1-2 and $M_V = 500$ GeV.

|         | di-leptons | tri-leptons |
|---------|------------|-------------|
| VBF (MGM) | 16         | 3           |
| DY (MGM)  | 5          | 1           |
| VBF (comp)| 28         | 6           |
| DY (comp) | 18         | 4           |

Table 2: Cumulative branching ratios for at least two same-sign leptons or three leptons ($e$ or $\mu$) in the $W$-decays from two vectors in the given charge configuration.

|         | di-leptons(%) | tri-leptons(%) |
|---------|---------------|----------------|
| $V^0V^0$ | 8.9           | 3.2            |
| $V^\pm V^\pm$ | 4.5 | -              |
| $V^\pm V^0$ | 4.5           | 1.0            |

At the LHC with an integrated luminosity of 100 inverse femtobarns and $\sqrt{S} = 14$ TeV, putting together all the different charge configurations, one obtains from $W \rightarrow e\nu, \mu\nu$ decays the number of same-sign di-leptons and tri-lepton events given in Table 1 for $M_V = 500$ GeV. The
other parameters are fixed as in the Minimal Gauge Model (and labelled MGM) or as in Figs. 1-2 for VBF and for DY in the previous two Sections (and labelled comp). These numbers of events are based on the cross sections in Figs. 1-2 and on the branching ratios for the various charge channels listed in Table 2. The numbers of events for different values of $M_V$ are also easily obtained. As already noticed, depending on the parameters, the number of events in the composite case could also be significantly higher. No attempt is made, at this stage, to compare the signal with the background from SM sources. To see if a signal can be observed a careful analysis will be required, with a high cut on the scalar sum, $H_t$, of all the transverse momenta and of the missing energy in each event probably playing a crucial role. The use of the leptonic decays of the $Z$ might also be important.

9 Summary

To describe the phenomenology of EWSB by an unspecified strong dynamics, we have adhered to the general program based on:

- 1. Keep $SU(2) \times U(1)$ gauge invariance but leave out the Higgs boson, while insisting on $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$ as relevant (approximate) symmetry;

- 2. Introduce new composite particles of mass less than $\Lambda \approx 4\pi v$ consistently with 1 and study the related phenomenology.

More specifically, we have considered the case of a $SU(2)_{L+R}$-triplet vector and we have focussed on the pair production of such vectors at the LHC by VBF or by the DY process.

The effective Lagrangian description of the interactions of these vectors, among themselves or with the standard gauge bosons, eq. (2.11), has several free parameters and gives rise in general to scattering amplitudes with a bad asymptotic behaviour. This does not come as a surprise, given the consolidated knowledge about massive vectors in field theory. Suitable properties/relations among the various parameters must at least approximately exist to keep the asymptotic properties under control. We have found these relations and used them to partially constrain the parameter space. We have also shown how these constraints relate to the properties of a gauge vector from a $SU(2)_L \times SU(2)_R \times SU(2)^N$ gauge theory spontaneously broken to the diagonal $SU(2)$ subgroup by a generic non linear $\sigma$-model. As such, the approach followed here can be used to analyze in a unified way several different models proposed in the literature. It should also serve as a useful and unbiased mean to analyze the LHC data, if these vectors exist in nature.

In general, the extent to which the various parameters deviate from the single-vector gauge-model relations is a relevant open issue that can in principle be addressed experimentally by studying and comparing single and pair production processes. With $M_V$ below a TeV, large deviations are both unlikely and a threat to the very use of the effective Lagrangian approach described here. They are unlikely if an underlying theory (a ‘UV completion’) exists with a meaningful asymptotic behaviour of the physical amplitudes. They constitute a threat to the effective Lagrangian approach with a single $SU(2)$-triplet vector involved, since the cutoff would be reduced to an unacceptably low level. As far as we can tell, however, moderate deviations can
exist, still leading to potentially significant signatures for $M_V$ below one TeV. In the particular QCD case, which need not be copied by the putative strong dynamics of EWSB, the $\rho$ has a mass of about 2/3 of the cutoff and couplings which deviate from the gauge model at the 20 ÷ 30% level [22]. It remains to be seen to what extent these signatures can be made to emerge at the LHC from the background.

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A Vector versus tensor formulation

Especially in QCD, when discussing the low energy pion dynamics, but also in applications to the electroweak interactions, it proofs useful to describe spin-1 states by means of anti-symmetric tensors rather than by Lorentz vectors. At the level of linear spin-1 interaction terms only, $\mathcal{L}_{V1}$, it is easy to establish an exact correspondence of the vector formulation with the tensor one, as described by the Lagrangian

$$\mathcal{L}_T = \mathcal{L}_x + \mathcal{L}_{\text{kin}}^T + \mathcal{L}_{1T},$$

in terms of the tensors $T^{\mu\nu}$, belonging to the adjoint representation of $SU(2)^L_R$, $T_{\mu\nu} = \frac{1}{\sqrt{2}} \tau^a T^a_{\mu\nu}$, $T_{\mu\nu} \rightarrow h T_{\mu\nu} h^\dagger$.

The kinetic Lagrangian for the heavy spin-1 fields is given by

$$\mathcal{L}_{\text{kin}}^T = -\frac{1}{2} \langle \nabla_\mu T^{\mu\nu} \nabla_\rho T_{\nu\rho} \rangle + \frac{M_V^2}{4} \langle T^{\mu\nu} T_{\mu\nu} \rangle,$$

with the covariant derivative $\nabla_\mu T = \partial_\mu T + [\Gamma_\mu, T]$. At the same time

$$\mathcal{L}_{1T} = \frac{i G_V}{2\sqrt{2}} \langle T^{\mu\nu} [u_\mu, u_\nu] \rangle + \frac{F_V}{2\sqrt{2}} \langle T^{\mu\nu} (u \hat{W}^{\mu\nu} u^\dagger + u^\dagger \hat{B}^{\mu\nu} u) \rangle,$$

where $G_V, F_V$ are related to $g_V$ and $f_V$ by $G_V = g_V M_V$ and $F_V = f_V M_V$.

The correspondence of $\mathcal{L}_T$ with $\mathcal{L}_V$ stopped at the linear terms in $V_\mu$ would be complete with the addition of a few contact interactions only involving the Goldstone bosons or the standard electroweak gauge bosons, not relevant to the current discussion. A formal correspondence between the vector formulations can also be established at the level of the multi spin-1 interaction terms [23, 26, 27, 28, 29, 30, 31]. This would however require adding an infinite number of terms. As shown in Section 5 the vector formulation proves more useful in discussing the asymptotic behaviour of the $WW \rightarrow VV$ amplitudes and the relation with the hidden-gauge model.
References

[1] J. Bagger et al., Phys. Rev. D 49 (1994) 1246.
[2] D. B. Kaplan and H. Georgi, Phys. Lett. B 136 (1984) 183.
[3] R. S. Chivukula and V. Koulovassilopoulos, Phys. Lett. B 309, 371 (1993) [arXiv:hep-ph/9304293].
[4] R. Contino, arXiv:0908.3578 [hep-ph].
[5] G. F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, JHEP 0706 (2007) 045 [hep-ph/0703164].
[6] I. Low, R. Rattazzi and A. Vichi, arXiv:0907.5413 [hep-ph].
[7] R. S. Chivukula, D. A. Dicus and H. J. He, Phys. Lett. B 525 (2002) 175 [arXiv:hep-ph/0111016].
[8] C. Csaki, C. Grojean, H. Murayama, L. Pilo and J. Terning, Phys. Rev. D 69, 055006 (2004) [arXiv:hep-ph/0305237].
[9] R. Barbieri, G. Isidori, V. S. Rychkov and E. Trincherini, Phys. Rev. D 78 (2008) 036012, [arXiv:0806.1624 [hep-ph]].
[10] A. Birkedal, K. T. Matchev and M. Perelstein, In the Proceedings of 2005 International Linear Collider Workshop (LCWS 2005), Stanford, California, 18-22 Mar 2005, pp 0314 [arXiv:hep-ph/0508185].
[11] H. J. He et al., Phys. Rev. D 78 (2008) 031701 [arXiv:0708.2588 [hep-ph]].
[12] E. Accomando, S. De Curtis, D. Dominici and L. Fedeli, Phys. Rev. D 79 (2009) 055020 [arXiv:0807.5051 [hep-ph]]; Nuovo Cim. 123B (2008) 809 [arXiv:0807.2951 [hep-ph]].
[13] A. Belyaev, R. Foadi, M. T. Frandsen, M. Jarvinen, F. Sannino and A. Pukhov, Phys. Rev. D 79 (2009) 035006 [arXiv:0809.0793 [hep-ph]].
[14] O. Cata, G. Isidori and J. F. Kamenik, Nucl. Phys. B 822 (2009) 230 [arXiv:0905.0490 [hep-ph]].
[15] R. Casalbuoni, S. De Curtis, D. Dominici and R. Gatto, Phys. Lett. B 155 (1985) 95; Nucl. Phys. B 282 (1987) 235.
[16] R. S. Chivukula, D. A. Dicus, H. J. He and S. Nandi, Phys. Lett. B 562 (2003) 109 [arXiv:hep-ph/0302263].
[17] Y. Nomura, JHEP 0311 (2003) 050 [arXiv:hep-ph/0309189].
[18] R. Barbieri, A. Pomarol and R. Rattazzi, Phys. Lett. B 591 (2004) 141 [arXiv:hep-ph/0310285].

[19] R. Foadi, S. Gopalakrishna and C. Schmidt, JHEP 0403 (2004) 042 [arXiv:hep-ph/0312324].

[20] H. Georgi, Phys. Rev. D 71 (2005) 015016 [arXiv:hep-ph/0408067].

[21] S. R. Coleman et. al. Phys. Rev. 177, 2239, 2247 (1969); C.G. Callan, et. al. Phys. Rev. 177 (1969) 2247.

[22] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B 321 (1989) 311.

[23] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Phys. Lett. B 223 (1989) 425;

[24] R. S. Chivukula, H. J. He, M. Kurachi, E. H. Simmons and M. Tanabashi, Phys. Rev. D 78, 095003 (2008) [arXiv:0808.1682 [hep-ph]].

[25] A. Pukhov, A. Belyaev and N. Christensen, http://theory.sinp.msu.ru/ pukhov/calchep.html.

[26] E. Pallante and R. Petronzio, Nucl. Phys. B 396 (1993) 205.

[27] B. Borasoy and U. G. Meissner, Int. J. Mod. Phys. A 11 (1996) 5183 [arXiv:hep-ph/9511320].

[28] M. Harada and K. Yamawaki, Phys. Rept. 381 (2003) 1 [arXiv:hep-ph/0302103].

[29] J. Bijnens and E. Pallante, Mod. Phys. Lett. A 11 (1996) 1069 [arXiv:hep-ph/9510338].

[30] V. Cirigliano, G. Ecker, M. Eidemuller, R. Kaiser, A. Pich and J. Portoles, Nucl. Phys. B 753 (2006) 139 [arXiv:hep-ph/0603205].

[31] K. Kampf, J. Novotny and J. Trnka, Eur. Phys. J. C 50 (2007) 385 [arXiv:hep-ph/0608051].