Optimization of Time and Cost for a Research Project by Project Crashing Method

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Abstract. The aim of current study is to access trade off between time and cost for the duration of a research project. A linear programming model (LPP) was developed and used for the same purpose. Normal and Crashing Costs for the project activities were obtained from the principal investigator of the project and were used for conducting the analysis in the excel solver. Maximum and minimum duration for the project was obtained through critical path method and crashing method respectively. The result of the study indicated significant reduction in project duration with normal increase in project cost. A small increase in the project cost with reduction of nearly 45 days in the project interval was observed. Nearly 5% increase in cost of the project was noticed with reduction of 15% in total project duration.

1. Introduction
Project completion within minimum time incurring minimum cost is desirable in every project. Heuristic and mathematical programmings are used to achieve the same purpose. Genetic Algorithm, Fuzzy multi objective genetic algorithm (FMOGA) were observed to be Heuristic Approach whereas Linear Programming, Integer Programming and Dynamic programming were considered as Mathematical techniques [1]. The most reliable approach to analyze time-cost tradeoff is LP Solver technique was suggested by Ragsdale [2]. The use of excel solver to facilitate identification of activities to be crashed was also demonstrated by Mantel et al. [3].

Based on the past study and previous information about crashing project activities using Linear Programming model with excel solver, we had applied this technique to reduce project duration for a research project- Development of heat transfer Correlation in micro channels (a TEQIP Phase III sponsored project). In the current study, time cost tradeoff for the research project was conducted using Linear Programming method. The Critical Path and activities of the research Project was first identified. LP equations were prepared and solved using excel solver to obtain the optimal cost for reducing the project duration. The increase in cost due to crashing was also presented [4].

The technique used here is easy to understand and simple for implementation as well as simple to solve if large numbers of decision variables are present which is quite difficult to solve, if we solve them manually.
2. Activity Time-Cost Tradeoff

![Figure 1: Time - cost trade-off for an activity.](image)

From Fig.1 Slope of the activity can be defined as the slope of the line which is joining from Normal Time and Crash Time [5]. Mathematically, Slope of the activity or Cost Slope can be written as

\[
\text{Cost Slope} = \frac{\text{Crashing Cost} - \text{Normal Cost}}{\text{Normal Time} - \text{Crashing Time}}
\] (1)

3. Problem Formulation for considered Prototype Example

Consider the following project data given in Table 1, obtained from our considered Project by discussion with the Project Manager. Here, the main objective is to Crash the Project duration from its earlier expected completion time. The following table gives data on Normal Time, Normal Cost, Crash Time and Crash Cost of our considered Project.

Firstly The Gantt Chart of Project activities is drawn as shown in Figure 2 for formulation of LPP model as stated by Perera S. [6]. In Gantt Chart along x-axis Dates (in dd/mm format) are represented and along y-axis activities are represented.

Table 1: Data for Normal and Crash Times and their associated costs.

| Activity Name                  | Activity | Predecessor | Normal Time (Day) | Crash Time (Day) | Normal Cost (Rs.) | Crash Cost (Rs.) |
|--------------------------------|----------|-------------|-------------------|------------------|-------------------|-----------------|
| Literature Reading             | A        | -           | 180               | 150              | 1500              | 4000            |
| Formulation of Problem         | B        | -           | 15                | 10               | 300               | 400             |
| Initialization of Procurement of Instrument | C        | B           | 60                | 55               | 150000            | 180000          |
| Further Procurement of Instruments during Setup | D        | C           | 90                | 50               | 250000            | 275000          |
| Setup of Experimental Apparatus | E        | C           | 105               | 60               | 60000             | 72000           |
| Validation of Experimental Setup | F        | D,E         | 45                | 38               | 6000              | 7200            |
| Data Generation                | G        | A,F         | 60                | 45               | 2500              | 3200            |
| Conclusion & Results           | H        | G           | 15                | 15               | 0                 | 0               |
By drawing the Project Network Diagram and Obtaining Earliest Start Time, Earliest Finish Time, Latest Start Time, Latest Finish Time, and Critical Path Length or Project Duration we had formulated LPP model. This data is represented in Table 2. The Project Network diagram is shown in Figure 3.
Table 2: Calculation of EST, EFT, LST, LFT, and Slack.

| Activity | Duration | Earliest Start Time | Earliest Finish Time | Latest Start Time | Latest Finish Time | Slack | Critical Path |
|----------|----------|---------------------|---------------------|-------------------|-------------------|-------|---------------|
| A        | 180      | 0                   | 180                 | 45                | 225               | 45    |               |
| B        | 15       | 0                   | 15                  | 0                 | 15                | 0     | CP            |
| C        | 60       | 15                  | 75                  | 15                | 75                | 0     | CP            |
| D        | 90       | 75                  | 165                 | 90                | 180               | 15    | CP            |
| E        | 105      | 75                  | 180                 | 75                | 180               | 0     | CP            |
| F        | 45       | 180                 | 225                 | 180               | 225               | 0     | CP            |
| G        | 60       | 225                 | 285                 | 225               | 285               | 0     | CP            |
| H        | 15       | 285                 | 300                 | 285               | 300               | 0     | CP            |

From the data obtained in Table 2 and Fig. 3, we get Critical Path. The Critical Path of the Project is B-C-E-F-G-H. The observed duration of Project or Length of Critical Path is 300 days.

4. Project Crashing by using Linear Programming Technique

In this technique we first construct a mathematical model of our project crashing problem by using Linear Programming for minimizing the total cost of crashing activities. If we estimate the project duration by drawing network diagrams like CPM or in Management software with exact activity duration we get it 300 days (i.e. before crashing).

Now, we are defining the decision variables, Let $X$ is the time when an event occurs, it is measured from starting (or beginning) of the project. Therefore,

$X_1 =$ time at which event 1 occurs
$X_2 =$ time at which event 2 occurs
$X_3 =$ time at which event 3 occurs
$X_4 =$ time at which event 4 occurs
$X_5 =$ time at which event 5 occurs
$X_6 =$ time at which event 6 occurs
$X_7 =$ time at which event 7 occurs
$X_8 =$ time at which event 8 occurs

$Y_A =$ No. of days activity A can be crashed.

Similarly, $Y_B$, $Y_C$, $Y_D$, $Y_E$, $Y_F$, $Y_G$, $Y_H$ No. of days activity B,C,D,E,F,G,H can be crashed respectively.
Table 3: Maximum Crash Time and Cost-Time Slope of the activities.

| Activity | Normal Time | Crash Time | Normal Cost | Crash Cost | Max. Crash Time | Crash Cost per day |
|----------|-------------|------------|-------------|------------|-----------------|-------------------|
| A        | 180         | 150        | 1500        | 4000       | 30              | 83.33             |
| B        | 15          | 10         | 300         | 400        | 5               | 20                |
| C        | 60          | 55         | 150000      | 180000     | 5               | 6000              |
| D        | 90          | 50         | 250000      | 275000     | 40              | 625               |
| E        | 105         | 60         | 6000        | 72000      | 45              | 266.67            |
| F        | 45          | 38         | 6000        | 7200       | 7               | 171.43            |
| G        | 60          | 45         | 2500        | 3200       | 15              | 46.67             |
| H        | 15          | 15         | 0           | 0          | 0               | 0                 |

Objective Function:
The starting time of each activity is dependent on the starting time and duration of its immediate predecessors as we seen in Fig. 3. We also get crash cost per day (or cost slope) as shown in Table 3 by using equation (1). Using this cost slope and decision variable Y (i.e. number of days an activity should be crashed) we have developed the following objective function:

Minimize Crash Cost = 83.33Y_A + 20Y_B + 6000Y_C + 625Y_D + 266.67Y_E + 171.43Y_F + 46.67Y_G + 0Y_H

Constraint: Based on data in Table 3 we have several Constraints.

Crash Time Constraint:
- Y_A ≤ 30
- Y_B ≤ 5
- Y_C ≤ 5
- Y_D ≤ 40
- Y_E ≤ 45
- Y_F ≤ 7
- Y_G ≤ 15
- Y_H ≤ 0

Start Time Constraint:
- For 1st event, X_1 = 0
- For 2nd event, X_2 ≥ 15 - Y_B
- For 3rd event, X_3 ≥ 60 - Y_C + X_2
- For 4th event, X_4 ≥ 90 - Y_D + X_3
- For 5th event, X_5 ≥ 105 - Y_E + X_3
- X_5 ≥ X_4
- For 6th event, X_6 ≥ 180 - Y_A
- X_6 ≥ 45 - Y_F + X_5
- For 7th event, X_7 ≥ 60 - Y_G + X_6
- For 8th event, X_8 ≥ 15 - Y_H + X_7

Project Completion Constraint:
- X_8 ≤ 255 (Specified Deadline)

Non-Negativity Constraint: X_1 ≥ 0, X_2 ≥ 0, X_3 ≥ 0, X_4 ≥ 0, X_5 ≥ 0, X_6 ≥ 0, X_7 ≥ 0, X_8 ≥ 0

Similarly, Y_A, Y_B, Y_C, Y_D, Y_E, Y_F, Y_G, Y_H ≥ 0.
5. Solution of the LPP using Excel Solver

We have defined the objective function and all the constraints for the problem stated in Kunpeng li et al. (2012) [7]. To determine the minimum crash cost we have to minimize the objective function. We can get the solution for this LP model by the help of Excel Solver. For the solution of LP model using Excel Solver, enter the objective function and constraints as shown in the Figure 4. This LP model can also be solved by MATLAB, STORM, LINDO or by other Linear Programming computer programs which are available in the market.

![Figure 4: LPP Model using Excel Solver (Screenshot of LP solution Procedure in Excel)](image-url)
6. Results Obtained by Solving LPP

The Linear Programming Problem is solved by Excel Solver to determine the Minimum Crashing cost for reducing project duration. Figure 5 is the graph between activities on abscissa and duration of these activities on ordinates. This graph is representing the duration of activity before crashing and after crashing. The main aim of our calculations is to reduce the project duration with minimum crashing cost. The Solution for model is shown in table 4:

| Objective | Final Value |
|-----------|-------------|
| Z         | 23622.065   |

| Variable | Final Value |
|----------|-------------|
| Y_A      | 0           |
| Y_B      | 4.76        |
| Y_C      | 0           |
| Y_D      | 19.13       |
| Y_E      | 39.33       |
| Y_F      | 6.30        |
| Y_G      | 0           |
| Y_H      | 0           |
| X_1      | 0           |
| X_2      | 10.43       |
| X_3      | 70.43       |
| X_4      | 141.30      |
| X_5      | 141.30      |
| X_6      | 180         |
| X_7      | 240         |
| X_8      | 255         |

Table 4
Solution of the LP Problem using Excel Solver.

Our findings suggest that if project is completed without crashing i.e. in 300 days then the total cost of the project is Rs 470300 while if it is completed in 255 days by using crashing method then total cost of the project increases to Rs. 493922. The following Table 5 is showing these findings.
### 7. Conclusion

From the above data, we can conclude that *Project Duration is reduced by 45 days* with the expenditure of Rs. 23622 in order to crash the activities of Project. The Project Duration can be further reduced but we have to spent more money to achieve that task.

Here, The Percentage Reduction in Project Duration is 15\% which is a significant value obtained on the expenditure of just Rs.23622. This increased crash cost is not affecting too much because here crashing cost is increasing but subsequently indirect cost (or running cost) associated with the project activities reduces for the 45 days. Therefore, the net cost required for reduction in project duration by crashing is optimum.

Further, The Project activities duration can be varied due to several external factors. So in future studies, we can convert this CPM Problem into PERT by taking activities duration (or Normal Time) as most likely occurring time, Optimistic times as 70\% of normal duration and Pessimistic times as 125\% of normal duration [8]. The PERT Technique follows Normal Distribution Curve. And then Analyze the above Problem in PERT. In PERT first we determine the new Critical Path and standard deviation in duration of each activity, and then measures Standard deviation (SD) of the project. By using this SD and Critical Path Length we can measure the Probability of the Completion of the project on the scheduled date.

### 8. References

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