Assignments of $\Lambda_Q$ and $\Xi_Q$ baryons in the heavy quark-light diquark picture

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We derive analytically a mass formula for the excited heavy-light hadrons in the relativistic flux tube model. Then the formula is applied to the mass spectrum of $\Lambda_Q$ and $\Xi_Q$ ($Q = c$ or $b$ quark) baryons, where the heavy quark-light diquark picture is considered. We find that the masses of the available $\Lambda_Q$ and $\Xi_Q$ baryons can be understood well. All assignments to these baryons do not appear to contradict their strong decay properties. $\Lambda_c(2760)^+$ and $\Xi_c(2980)$ are assigned to the first radial excitations with $J^P = 1/2^+$. $\Lambda_c(2940)^+$ and $\Xi_c(3123)$ might be the $2P$ states. The $\Lambda_c(2880)^+$ and $\Xi_c(3080)$ are the $1D$ candidates with $J^P = 5/2^+$. $\Xi_c(3055)$ is also likely to be a $1D$ state with $J^P = 3/2^+$. The newly reported resonance, $B_c(3212)$, could be classified as a $2D$ or $1F$ charmed baryons temporarily. $\Lambda_b(5912)^0$ and $\Lambda_b(5920)^0$ favor the $1P$ assignments with $J^P = 1/2^-$ and $3/2^-$, respectively. We propose a search for the $\Lambda_c(5/2^-)$ state which can help to distinguish the diquark and three-body schemes.

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I. INTRODUCTION

The so-called missing resonances problem has never been understood for the baryons physics. In the constituent quark model, a baryon contains three confined quarks. If this is true, the observed baryon resonances are much less than predictions. For the light baryons, Galatà and Santopinto have pointed out that the established and tentative states listed by the Particle Data Group (PDG) are much less than the theoretical predictions. For the heavy baryons, we take the $qqq$ ($q = u$ or $d$ quark) as an example. More than 50 states are allowed for the $\Lambda_c$ and $\Sigma_c$ baryons up to $N = 2$ shell in the three-body picture. But only 9 $\Lambda_c$ and $\Sigma_c$ candidates have been listed by PDG at present (see Table I).

A heuristic and possible solution to this problem is to introduce “diquarks” [2, 3]. Since the degree of freedom of the two quarks in the diquark is frozen, the excited states will be greatly reduced. For the nonstrange light baryons, Santopinto et al. have provided good descriptions of the masses up to 2 GeV by two different quark-diquark models [4, 5]. Based on a QCD motivated quark potential model [6], the mass spectra of heavy baryons have been calculated in the heavy quark-light diquark picture. There the light diquarks were completely relativistic and the heavy quarks were expanded in $v/c$ up to the second order. An improved method without any expansions was adopted later [7]. Masses for the higher excited heavy baryon states were presented. Based on the predictions, the Regge trajectories for orbital and radial excitations were constructed. The linearity, parallelism, and equidistance were verified. Most importantly, the authors concluded that “all available experimental data on heavy baryons fit nicely to the constructed Regge trajectories” [7].

To our knowledge, only Ref. [6, 7] have focused on the mass spectra of the high excited heavy baryons systematically in the quark-diquark picture. In the two works, a relativistic quark potential model was used. So it is required to test the quark-diquark picture for the heavy baryons in other models. In addition, the strong decay behavior has not been discussed in Ref. [6, 7].

The relativistic flux tube (RFT) model is not a potential approach because the interaction is mediated by a dynamical tube [8, 9]. Selem and Wilczek employed the relativistic flux tube model to investigate whether there are diquark in the baryons [10]. For the heavy baryons, they presented a simple mass formula by the computer simulations as

$$E = M + \sqrt{\frac{\sigma L}{2} + 2^{1/4} \kappa L^{-1/4} \mu^{3/2}}. \tag{1}$$

where $E$, $M$, and $\mu$ refer to the masses of baryon system, heavy quark, and light-diquark respectively. $L$ is the orbital angular momentum. The string tension is denoted by $\sigma/(2\pi)$. The parameter $\kappa$ is dependent on $\sigma$: $\kappa \equiv \frac{2\sigma^{1/2}}{\sqrt{\pi}}$. Obviously, the formula above does not include the ground state because of the singularity. In addition, the hyperfine interactions have not been incorporated. This formula has been used to study the mass spectrum of $D$, $D^*$, $\Omega^-$ [11] and $\Lambda^+$ [12], where the spin-orbit interactions have been taken into account.

In this work, we will derive the mass formula for the heavy-light hadrons will be derived analytically within the relativistic flux tube model. Then we will apply the new formula to the $\Lambda_Q$ and $\Xi_Q$ baryons, where the two light quarks are treated as a scalar diquark. The hyperfine term will be borrowed from the QCD-motivated constituent quark models. We will also discuss their strong decays for completeness.

The paper is organized as follows. In Section III we derive the mass formula of the heavy-light hadrons in the relativistic flux tube model. In Section IV the spectrum and decays of $\Lambda_Q$ and $\Xi_Q$ baryons are discussed. In Section V we further

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explore how to test the diquark and three-body schemes for
the heavy baryons. The last section contains the summary and
outlook.

II. THE MASS FORMULA OF HEAVY-LIGHT HADRONS
IN THE RFT MODEL

The quark and antiquark of a meson are treated as spinless
particles in the RFT model. They are connected by a relativis-
tic color flux tube which carries both energy and momentum.
The prototype of this model is the Nambu-Goto QCD string
model [14][16]. The RFT model has been studied carefully
by Olsson et al. [8][9][17][21]. An interesting research topic
of this model is to reproduce the Regge trajectories behav-
ior of different hadrons [10][17][18]. Besides the heavy-light
hadrons, the RFT model has also been applied to the charmo-
rion [22], pentaquark [23], and glueball [24].

The energy $\varepsilon$ and the angular momentum $L$ for a hadron
system in the RFT model are given by [8]

$$
\begin{align*}
\varepsilon &= \sum_{i=1}^{2} \left( \frac{m_i}{\sqrt{1 - (\omega r_i)^2}} + \frac{T}{\omega} \arcsin v_i \right) \\
L &= \sum_{i=1}^{2} \left( \frac{m_i r_i^2}{\sqrt{1 - v_i^2}} + \frac{T}{2\omega^2} (\arcsin v_i - v_i \sqrt{1 - v_i^2}) \right).
\end{align*}
$$

In Eqs. (2), we have omitted the velocity of light, $c$, for
simplicity. When we define the $v_i = \omega r_i$, the equations (2)
become

$$
\begin{align*}
\varepsilon &= \sum_{i=1}^{2} \left( \frac{m_i}{\sqrt{1 - v_i^2}} + \frac{T}{\omega} \arcsin v_i \right) \\
L &= \sum_{i=1}^{2} \left( \frac{m_i}{\omega} \frac{v_i^2}{\sqrt{1 - v_i^2}} + \frac{T}{2\omega^2} (\arcsin v_i - v_i \sqrt{1 - v_i^2}) \right).
\end{align*}
$$

For heavy-light hadron systems, we assume $m_1 \ll m_2$, and
define $m_1 = m_1/\sqrt{1 - v_1^2}$, $m_2 = m_2/\sqrt{1 - v_2^2}$. Since $m_1$ and
$m_2$ have included the relativistic effect, they are similar to the
constituent quark mass.

The velocity of heavy quark is small for its large mass. But
the light quark (or diquark) is relativistic. We take the rest
of $v_1$ as 1 for approximation, and expand Eqs. (3) up to the
second order in the parameter $v_2$. Then we obtain

$$
\begin{align*}
\varepsilon &= m_Q + m_l + m_Q v_2^2 + \frac{\pi T l}{2\omega}, \\
L &= \frac{1}{\omega} (m_l + m_Q v_2^2) + \frac{\pi T l}{4\omega}.
\end{align*}
$$

Here we have used the following relationship

$$
\frac{T}{\omega} = \frac{m_2 v_2}{1 - v_2^2} \approx m_Q v_2.
$$

Based on the equations (4), the spin-averaged mass formula
for the orbital excited states is given directly

$$
(\varepsilon - m_Q)^2 = \frac{1}{2} \sigma L + (m_l + \zeta Q)^2,
$$

where $\sigma = 2\pi T$ and $\zeta_Q = m_Q v_2^2$. The equation (6) is the
Chew-Frautschi formula for heavy-light systems. A formula without
intercept was found in Ref. [8]. Another formula with a dif-
ferent intercept was shown as: $(\varepsilon - m_2)^2 = \sigma L/2 + \sigma/6$ [25],
where the effect of string fluctuations was considered. Both
formulae in Refs. [8][25] were realized in the physical limits
that $m_1 \to 0$ and $m_2 \to \infty$. So it seems unreasonable to apply
them to the ordinary heavy-light hadrons for the finite masses
of quarks. The intercept of equation (6) is also different from
the equation (1). Since the singularity no longer appears in
the equation (6), we expect that the equation also includes the
case of $L = 0$. Of course, the angular momentum $L$ can not be
understood in the classical picture when we use the Eq. (6) to
describe the heavy-light hadrons.

The hyperfine terms can not be given by the semi-classical
way above. For $\Lambda_Q$ and $\Xi_Q$, the hyperfine interactions
are much simpler because the primary couplings exist between
the spin of heavy quark and the orbital angular momentum.
With the axial-vector diquark (see Section III), $\Sigma_Q$ and $\Xi_Q$
have more complicated hyperfine structure. In this work, we
will only study the $\Lambda_Q$ and $\Xi_Q$ baryons for simplicity. The
$\vec{s}_Q \cdot \vec{L}$ couplings can be borrowed from the QCD-motivated
quark potential models. Similar to heavy-light mesons [26],
the $\vec{s}_Q \cdot \vec{L}$ couplings for $\Lambda_Q$ and $\Xi_Q$ have the form

$$
H_{sL} = \frac{4 \alpha_s}{3 r^3} \frac{1}{m_Q m_\Xi} \vec{s}_Q \cdot \vec{L}.
$$

Here the the second and higher orders of $1/m_Q$ are ignored. $m_d$
refers to the mass of light diquark. $\alpha_s$ is the coupling constant.
By the second one of Eqs. (4) above, the angular velocity $\omega$
could be expressed as

$$
\omega = \frac{\varepsilon_1 - \varepsilon_0}{2L} \approx \frac{\sigma}{8\omega L}.
$$

When the $r \omega = v_1 + v_2 \approx 1$ is assumed, we obtain

$$
\frac{1}{r} \approx \omega = \sqrt{\frac{\sigma}{8L}}.
$$

The relations (9) have also been reached in a ultrarelativistic
limit [9]. By substituting the expressions (9) into equation (7), we
find

$$
H_{sL} = \frac{\alpha_s}{3 \times 25/2} \left( \frac{\sigma}{L} \right)^{3/2} \frac{1}{m_Q m_\Xi} \vec{s}_Q \cdot \vec{L}.
$$

Based on the numerical analysis [17][20], and the semi-
classical quantization scheme [27], the RFT model implied
that the linearity, parallelism, and equidistance may exist in
the Regge trajectories of the heavy-light hadrons. Inspired
by these results, we would like to extend the equation (6) to the
radial excited heavy baryons

$$
(\varepsilon - m_Q)^2 = \frac{1}{2} \sigma (\kappa l + L) + (m_l + \zeta Q)^2.
$$
The coefficient, κ, will be determined directly by the experimental data. Accordingly, the hyperfine term are revised as

$$H_{nl}^{so} = \frac{\alpha_s}{3 \times 27^{1/2}} \left( \frac{\sigma}{k_n + L} \right)^{3/2} \frac{1}{m_Q m_{\bar{Q}}} \hat{s}_Q \cdot \hat{L}. \quad (12)$$

In the next Section, we will test the phenomenological Eqs. (11) and (12) by the $\Lambda_Q$ and $\Xi_Q$ baryons. There the heavy quark-light diquark picture will be considered.

### III. APPLICATION

In the $\Lambda_Q$ and $\Xi_Q$ baryons, the total wave functions of the light diquark should be antisymmetric when the flavor SU(3) symmetry is considered for $u$, $d$, and $s$ quarks. Since the color and spatial parts of diquark are always antisymmetric and symmetric, respectively, the functions of $|\text{flavor}\rangle \times |\text{spin}\rangle$ should be symmetric. For this constraint, the scalar diquark ($S = 0$) is always flavor antisymmetric, and the axial-vector ($S = 1$) flavor symmetric. Following [6, 7], we denote the scalar diquark as $[q_1, q_2]$, and the axial-vector diquark as $[q_1, q_2]$. All $\Lambda_Q$ and $\Xi_Q$ candidates listed by PDG [28] are collected in Table I. Most of their quantum numbers are not yet been determined experimentally. Thus, they are usually prescribed following the quark model predictions. The different aspects of heavy baryons have been reviewed in detail in Ref. [2, 3].

One notice that the mass gaps of the corresponding $\Xi_c$ and $\Lambda_c^+$ are about 180–200 MeV. The mass gap of $\Xi_b(5790)$ and $\Lambda_b(5620)$ is about 170 MeV. For illustrating such law clearly, we present the $\Lambda_Q$ and $\Xi_Q$ baryons alongside in Table I. The mass gaps will be explained further by Eqs. (11) and (12) in Section IV. The phenomena of mass gaps reflects the similar dynamics between the $\Lambda_Q$ and $\Xi_Q$ baryons. In the following, we will discuss $\Lambda_Q$ and $\Xi_Q$ parallel.

In our fitting procedures, $\Lambda_c(2880)$ and $\Xi_c(3080)$ are identified as the 1$D$ - wave $\Lambda_c$ and $\Xi_c$ states with the $J^P = 5/2^+$. The $\Xi_b(3055)$ is regarded as the doublet partner of $\Xi_b(3080)$ with $J^P = 3/2^+$. These three baryons have been reported by the different collaborations [29, 34]. Their decay properties will constrain the assignments strongly. Especially, the ratio of $\Lambda_c(2880)^+$ partial widths [31]

$$\frac{\Gamma(\Lambda_c(2880) \to \Sigma_c^+(2520)\pi)}{\Gamma(\Lambda_c(2880) \to \Sigma_c^+(2455)\pi)} = (24.1 \pm 6.4_{-4.5}^{+1.1})\%, \quad (13)$$

does never been understood well. So it is necessary to discuss their strong decays for completeness.

**TABLE I: The experimental information of the $\Lambda_Q$ and $\Xi_Q$ baryons.** The average values of mass and decay width (in units of MeV) are taken from PDG [28]. Here the predicted $\Lambda_c(2860)^+$, $\Xi_b(6080)^+$, and $\Xi_b(6090)^+$ are listed for comparison and completeness. The mass differences between $\Lambda_Q$ and $\Xi_Q$ baryons are listed in the last column.

| Names          | Status | Mass     | Width | Names          | Status | Mass     | Width | ΔM (MeV) |
|----------------|--------|----------|-------|----------------|--------|----------|-------|----------|
| $\Lambda_c(2286)^+$ | ⋆      | 2286.46 ± 0.14 | –     | $\Xi_c(2468)^0$ | ⋆      | 2470.88 ± 0.34 | –     | 184.42 ± 0.37 |
| $\Lambda_c(2595)^+$ | ⋆      | 2592.25 ± 0.28 | 2.6 ± 0.6 | $\Xi_c(2790)^0$ | ⋆      | 2791.8 ± 3.3 | < 12 | 199.6 ± 3.3 |
| $\Lambda_c(2625)^+$ | ⋆      | 2628.11 ± 0.19 | < 0.97 | $\Xi_c(2815)^0$ | ⋆      | 2819.6 ± 1.2 | < 6.5 | 191.5 ± 1.2 |
| $\Lambda_c(2765)^+$ | ⋆      | 2766.6 ± 2.4 | 50    | $\Xi_c(2980)^0$ | ⋆      | 2968.0 ± 2.6 | < 20 | 201.4 ± 3.5 |
| $\Lambda_c(2860)^+$ | ⋆      | 2881.53 ± 0.35 | 5.8 ± 1.1 | $\Xi_c(3080)^0$ | ⋆      | 3079.9 ± 1.4 | < 5.6 | 198.4 ± 1.4 |
| $\Lambda_c(2940)^+$ | ⋆      | 2939.31 ± 1.5 | 17.8 ± 8 | $\Xi_c(3123)^+$ | ⋆      | 3122.9 ± 1.3 | < 4 | 183.6 ± 1.9 |
| $\Lambda_b(5619)^0$ | ⋆      | 5619.4 ± 0.6 | –     | $\Xi_b(5790)^-$ | ⋆      | 5791.1 ± 2.2 | –     | 171.7 ± 2.3 |
| $\Lambda_b(5912)^0$ | ⋆      | 5912.0 ± 0.6 | < 0.66 | $\Xi_b(6080)^-$ | ⋆      | ⋆        | ⋆     | ⋆        |
| $\Lambda_b(5920)^0$ | ⋆      | 5919.8 ± 0.8 | < 0.63 | $\Xi_b(6090)^-$ | ⋆      | ⋆        | ⋆     | ⋆        |

For studying the two-body strong decay widths of these heavy baryons, we employ a simple formula based on the heavy quark limit. In the $m_Q \to \infty$ limit, the light degrees of freedom will decouple from the heavy quark spin. Then the transitions from one doublet with total spins $J = j_1/2$ to another doublet with $J' = j_1/2$ are governed by a single amplitude [30]. Here, $j_1$ and $j'_1$ are the total angular momentum of the light degrees of freedom for the initial and final baryons. Thus, like the heavy-light mesons [37], the decay width for heavy baryons is written as

$$\Gamma^{B \to B' M} = \xi(0) \left| C_{j_1; j'_1; j; J}^{j_1; j'_1; j; J} \right|^2 \frac{m_B}{8\pi^2} \exp(-\frac{s^2}{2m_B^2}). \quad (14)$$

The recoupling coefficient, $C_{j_1; j'_1; j; J}^{j_1; j'_1; j; J}$, is

$$C_{j_1; j'_1; j; J}^{j_1; j'_1; j; J} = \sqrt{(2j_1 + 1)(2j + 1)} \left\{ \begin{array}{ccc} s_Q & J'_1 & J \\ J & j & j_1 \end{array} \right\}.$$
and \( \hat{j}_M = \hat{s}_M + \hat{l} \). Here we denote the \( J_a \) and \( J'_a \) as \( J \) and \( J' \) respectively. The flavor coefficient \( \xi \) is determined by

\[
\xi = \frac{(2I_b + 1)(2I_C + 1)}{2} \left\{ \begin{array}{ccc}
t_a & I_b & I_a \\ I_C & I_b & 1/2 \end{array} \right\}^2.
\]

In the following calculations, \( \hat{\beta} \) is taken as 0.38 GeV, which is consistent with the harmonic oscillator parameter in the pseudoscalar-meson emission model \[38\], the chiral quark model \[39, 40\], and the \( \Lambda P_0 \) model \[41, 42\]. We emphasize that the results in this work are insensitive to the magnitude of \( \hat{\beta} \).

A. \( \Lambda_c^+ \) and \( \Xi^{0+,+}_c \) baryons

In Eqs. (11) and (12), there are six parameters which should be fixed, i.e., \( m_Q \), \( m_a \), \( \sigma \), \( \kappa \), \( \alpha_s \), and \( \epsilon_Q \). Firstly, we fix the \( m_Q = 1.470 \text{ GeV} \), \( \sigma_{\Lambda_c} = 1.295 \text{ GeV}^2 \) and \( \sigma_{\Xi_c} = 1.558 \text{ GeV}^2 \) with the spin-averaged mass of \( 1S, 1P, \) and \( 1D \) \( \Lambda_c \) and \( \Xi_c \) states. We also fix the \( m_{[u,d]} + \epsilon_Q = 0.815 \text{ GeV} \) and \( m_{[u,s]} + \epsilon_Q = 0.997 \text{ GeV} \). Secondly, we set \( \Delta M([u,s] - [u,d]) = 182 \text{ MeV} \) which is reasonable for the experimental data. Then we obtain \( m_{[u,d]} = 451 \text{ MeV} \), \( m_{[u,s]} = 633 \text{ MeV} \), and \( \epsilon_Q = 364 \text{ MeV} \). The masses of scalar diquarks in Ref. \([7]\) were taken as \( m_{[u,d]} = 710 \text{ MeV} \), \( m_{[u,s]} = 948 \text{ MeV} \), which are much large. We fix the coupling coefficient, \( \alpha \), as 0.20 according with the hyperfine splitting. Since the predicted mass of \( 1/2^+(2S) \) state are around \( 2770 \text{ MeV} \) (see Table II), the \( \Lambda_c(2760)^+ \) seems to be a possible candidate as the first radial excited state of \( \Lambda_c(2286)^+ \). Lastly, the coefficient \( \kappa \) is extracted as 1.57 when \( \Lambda_c(2760)^+ \) is taken as the \( 1/2^+(2S) \) state.

With these parameters in hand, the masses of the \( \Lambda_c^+ \) and \( \Xi^{0+,+}_c \) baryons are shown in the Table II and IV. The results from other groups \[7, 43, 44\] are also listed for comparison. In Ref. \[43\], a QCD-motivated quark potential model was employed and the diquark picture has been considered. In Ref. \[44\], the mass spectra were explored in the three-body picture by the nonrelativistic quark model. Masses were also studied in the three-body picture by a relativized version of the quark potential model \[44\]. The kinetic term of quarks and the spin-orbit piece are different in Refs. \[43, 44\]. In the Tables II and IV, the predicted values in the square brackets belong to the corresponding states with the \( J^P \) listed in the first column. But the \( nL \) listed in the parentheses are not the quantum numbers of these states. We list them for showing the difference between the two- and three-body pictures of baryons (for details see Section IV).

Our results totally coincide with those presented by Ref. \[7\] (see Tables II and IV). For these low-lying excited states, both the flux tube model and the quark potential models can reproduce the masses. The \( \Lambda_c(2595)^+, \Lambda_c(2625)^+, \Xi_c(2790)^+ \), and \( \Xi_c(2815)^+ \) are the natural candidates for the \( 1P \) wave \( \Lambda_c \) and \( \Xi_c \) baryons. These assignments were also supported by the strong decay analysis \[39, 41, 45\].

As shown in Table II the \( \Lambda_c(2880)^+ \) is a good candidate of \( 1D \) state. This is supported by the results of Belle \[31\], that the \( \Lambda_c(2880)^+ \) favors \( J = 5/2 \) over \( J = 1/2 \) and \( J = 3/2 \). However, if the \( \Lambda_c(2880)^+ \) is a \( 1D \) state with the \( J^P = 5/2^+ \), it seems difficult to explain the ratio of (14). When only the contribution of the \( F \)-wave partial width was considered, the ratio can be understood well \[43\]. But the decay channel of \( \Lambda_c(2880)^+ \rightarrow \Sigma_c^*(2520)\pi \) can proceed through \( P \)-wave with the larger phase space. So it is necessary to deal with this question because we have taken the \( \Lambda_c(2880)^+ \) as an input. The \( 1D \) \( \Xi_c \) partners of \( \Lambda_c(2880)^+ \) are \( \Xi_c(3055)^- \) and \( \Xi_c(3088)^+ \) (see Table IV). In heavy quark limit, the processes of \( \Lambda_c(2880)^+ \rightarrow \Sigma_c(2520)/\Sigma_c(2455) + \pi \) and \( \Xi_c(3055)/\Xi_c(3088) \rightarrow \Sigma_c(2520)/\Sigma_c(2455) + K \) are governed by two independent transition strengths, \( F_{1/2}^{2,1}(0) \) and \( F_{3/2}^{2,1}(0) \). For these allowed decay channels, the c.m. momenta of the final states and the square of the coefficient \( C_{[u,s],J}^{F,P} \) are listed in the Table III. The equation (15), we obtain two chains of ratios for the \( 1D \) \( \Lambda_c^+ \) and \( \Xi_c \) baryons.

For \( P \)-wave decays,

\[
\Gamma_p(\Lambda_c(2600) \rightarrow \Sigma_c(2455)\pi) : \Gamma_p(\Lambda_c(2600) \rightarrow \Sigma_c(2520)\pi) : \Gamma_p(\Lambda_c(2880) \rightarrow \Sigma_c(2520)\pi) : \Gamma_p(\Xi_c(3055) \rightarrow \Sigma_c(2520)\pi) : \Gamma_p(\Xi_c(3080) \rightarrow \Sigma_c(2520)\pi) = 1.72 : 0.23 : 1.65 : 1.49 : 0.08 : 1.
\]

For \( F \)-wave decays,

\[
\Gamma_F(\Lambda_c(2600) \rightarrow \Sigma_c(2520)\pi) : \Gamma_F(\Lambda_c(2880) \rightarrow \Sigma_c(2455)\pi) : \Gamma_F(\Lambda_c(2880) \rightarrow \Sigma_c(2520)\pi) : \Gamma_F(\Xi_c(3055) \rightarrow \Sigma_c(2520)\pi) : \Gamma_F(\Xi_c(3080) \rightarrow \Sigma_c(2455)\pi) : \Gamma_F(\Xi_c(3080) \rightarrow \Sigma_c(2520)\pi) = 7.29 : 18.37 : 5.29 : 0.39 : 12.83 : 1.
\]
The constant $\mathcal{G}$ is defined as $\mathcal{G} = \gamma^3 \frac{\tilde{M}_A \tilde{M}_B}{\tilde{M}_C}$, which absorbs the dimensionless parameter $\gamma$ of the $^3P_0$ model. The constant $\mathcal{G}$ and the parameter $\tilde{\beta}$ in the Eqs. (15) and (16) could be fixed by other models. $\tilde{M}_A$, $\tilde{M}_B$, and $\tilde{M}_C$ are effective masses for the initial and final hadrons.

If the $P$-wave decay amplitude is accidentally near a node, the partial amplitude of $F$-wave decays is

$$F_{2,3}^\pm(0) = \frac{2^{4/3} \times 3}{\sqrt{13^2}} \mathcal{G} \frac{1}{\beta^3}.$$  \hspace{1cm} (16)

The broad $\Lambda_c(2765)^+$ ($\Gamma \approx 50$ MeV) was first reported by the CLEO Collaboration [29]. The possible signal was also seen by Belle Collaboration [31]. Recently, Joo et al. reanalyzed the full data collected by Belle. They found that the $\Lambda_c(2765)^+$ was visible in the $\Sigma_c(2455)\pi$ channel [48]. In the previous fitting procedure, we have assumed the $\Lambda_c(2765)^+$ to be the $1/2^-(2S)$ state. For the $\Xi_c$ partner of $\Lambda_c(2765)^+$ the predicted mass is 2959 MeV (see Table V), which supports the $\Xi_c(2980)^0$ as the first radical excited state of $\Xi_c(2468)^0$. $\Lambda_c(2940)^+$ and $\Xi_c(3123)^+$ might be the $2P$ charmed and charm-strange baryons. The predicted masses in our work

| $J^P$ | Candidates | Decay channels | $p$ (MeV) | $l = 1$ | $l = 3$ |
|-------|------------|----------------|---------|---------|---------|
| $^+4^+$ | $\Lambda_c(2860)^+$ | $\Sigma_c(2455) + \pi$ | 353 | × | $\frac{4}{5}$ |
| | | $\Sigma_c(2520) + \pi$ | 292 | $\frac{4}{5}$ | 1 |
| | | $\Sigma_c(2455) + K$ | 301 | $\frac{4}{5}$ | × |
| | | $\Sigma_c(2520) + K$ | 181 | $\frac{4}{5}$ | 1 |
| $^+3^+$ | $\Xi_c(3055)^+$ | $\Sigma_c(2455) + \pi$ | 375 | × | $\frac{3}{5}$ |
| | | $\Sigma_c(2520) + \pi$ | 316 | 1 | $\frac{3}{5}$ |
| | $\Sigma_c(2455) + K$ | 342 | × | $\frac{3}{5}$ |
| | | $\Sigma_c(2520) + K$ | 236 | 1 | $\frac{3}{5}$ |

TABLE III: The strong decays of 1D $\Lambda_c^+$ and $\Xi_c$. The momenta of final states, $p$, are shown in the column 4. The values of $(C_{\psi \Omega}^{\psi \Omega})^2$ corresponding to $P$- and $F$-wave decays are listed in the column 5 and 6, respectively. The mass of the predicted state, $\Lambda_c(2860)^+$, is taken as 2857 MeV. The forbidden decay modes are marked by "x" 

The $\Gamma(\Xi_c(3055))$ is 1.60 $\pm$ 0.30 MeV. This answers the question why we haven’t captured the signal of $\Lambda_c(2860)^+$ so far. The ratio of branching fraction, $\Gamma(\Xi_c(3080)) \rightarrow \Sigma_c(2520)K)/\Gamma(\Xi_c(3080) \rightarrow \Sigma_cK)$, has not been spined down by experiments [33]. Finally, the theoretical ratio is predicted as

$$\frac{\Gamma(\Xi_c(3080) \rightarrow \Sigma_c(2520)K)}{\Gamma(\Xi_c(3080) \rightarrow \Sigma_c(2455)K)} \approx 36.4\%,$$

which can be test in future.

| $J^P(nL)$ | Exp. [28] | This work | Ref. [7] | Ref. [43] |
|---------|---------|----------|---------|---------|
| $^+1(1S)$ | 2470.88 | 2467 | 2476 | 2466 |
| $^+1(2S)$ | 2968.0 | 2959 | 2959 | 2924 |
| $^+1(3S)$ | 3325 | 3323 | 3183 |
| $^+1(4S)$ | 3629 | 3632 | |
| $^+1(1P)$ | 2791.8 | 2779 | 2792 | 2773 |
| $^+1(1P)$ | 2819.6 | 2814 | 2819 | 2783 |
| $^+1(2P)$ | 3122.9 | 3195 | 3179 | |
| $^+1(2P)$ | 3204 | 3201 | |
| $^+1(3P)$ | 3521 | 3500 | |
| $^+1(3P)$ | 3525 | 3519 | |
| $^+1(1D)$ | 3054.2 | 3055 | 3059 | 3012 |
| $^+1(1D)$ | 3079.9 | 3076 | 3076 | 3004 |
| $^+1(2D)$ | 3407 | 3388 | |
| $^+1(2D)$ | 3416 | 3407 | |
| $^+1(1F)$ | 3286 | 3278 | |
| $^+1(1F)$ | 3302 | 3292 | |
| $^+1(1G)$ | 3490 | 3469 | |
| $^+1(1G)$ | 3503 | 3483 | |

TABLE V: The predicted masses of the $\Xi_c$ baryons (in MeV). We also collect the experimental values [28] and other theoretical results [7, 43] for comparison.
and in Ref. [7] are about 50–70 MeV larger than the experimental values (see Table III and IV). This problem is expected to be solved by the coupled-channel effects. If \(\Lambda_c(2940)^+\) and \(\Xi_c(3123)^+\) are 2P states, they can decay through \(D^*(D^0)p/\Xi_cK\) and \(D^*(D^+)\Lambda_c\), respectively, in \(S\)-wave. Because the \(\Xi_cK\) and \(D^0, p/\Lambda_c\) thresholds locate nearly below the predicted values, the coupled-channel effects are expected to be significant. The coupled-channel effects have been considered as the responsibility for the anomalously low masses of \(D^+_c(2317)\) and \(D_{s1}(2460)\) [49–51].

Recently, Cheng et al. examined the invariant-mass spectrum of \(D^0p\) in \(B \to D^0\bar{p}p\) decays measured by BaBar [52]. They found a new charmed baryon resonance with

\[
m = 3212 \pm 20\text{MeV}; \quad \Gamma = 167 \pm 34\text{MeV},
\]

and denoted it as \(B_c(3212)^+\) [53]. If the \(B_c(3212)^+\) is a \(\Lambda_c^+\) baryon, it may be a possible 2\(D\) or 1\(F\) state according to the predicted masses.

B. \(\Lambda_c^0\) and \(\Xi_c^{0-}\) baryons

The masses of \([u, d]/[q, s]\) diquarks and the value of \(\zeta_Q\), which have been previously extracted, are taken as inputs for the \(\Lambda_b\) and \(\Xi_b\) baryons. The mass gap between the spin-averaged masses of 1\(P\) \(\Lambda_b\) and \(\Xi_b\) is also assumed as 182 MeV. Then we obtained \(m_b = 4.804\text{GeV}\), \(\sigma_{\Lambda_c} = 1.147\text{GeV}^2\) and \(\sigma_{\Lambda_b} = 1.386\text{GeV}^2\). With these values, the masses of orbital excited \(\Lambda_b^0\) and \(\Xi_b^{0-}\) are predicted, separately, in Table VI and VII.

| \(J^P(nL)\) | \(\text{Exp. [28]}\) | This work | Ref. [2] | Ref. [43] | Ref. [44] |
|---|---|---|---|---|---|
| \(\frac{1}{2}^+\) (1S) | 5619.4 | 5619 | 5620 | 5612 | 5585 |
| \(\frac{1}{2}^+\) (1P) | 5912.0 | 5911 | 5930 | 5939 | 5912 |
| \(\frac{1}{2}^-\) (1P) | 5919.8 | 5920 | 5942 | 5941 | 5920 |
| \(\frac{1}{2}^-\) (1D) | 6147 | 6149 | 6180 | 6181 | 6145 |
| \(\frac{3}{2}^+\) (1D) | 6153 | 6196 | 6181 | 6165 |
| \(\frac{3}{2}^-\) (1F) | 6346 | 6408 | 6206 | 6205 |
| \(\frac{5}{2}^-\) (1F) | 6351 | 6411 | - | 6360 |
| \(\frac{1}{2}^-\) (1G) | 6525 | 6598 | 6433 | 6445 |
| \(\frac{3}{2}^-\) (1G) | 6526 | 6599 | - | 6580 |

TABLE VI: The predicted masses of the \(\Lambda_b^0\) baryons (in MeV). We also collect the experimental values [28] and other theoretical results [2, 43, 44] for comparison.

Two narrow states, named \(\Lambda_b(5912)^0\) and \(\Lambda_b(5920)^0\), were observed in the \(\Lambda_c^0\pi^-\pi^+\) spectrum by LHCb [54]. The \(\Lambda_b(5920)^0\) was later confirmed by CDF Collaboration [55]. The hyperfine of \(\Lambda_b(5912)^0\) and \(\Lambda_b(5920)^0\) given by equation (12) is consistent with the experimental data. The \(\Lambda_b(5912)^0\) and \(\Lambda_b(5920)^0\) are the good candidates for 1\(P\) \(\Lambda_b\) baryons with \(J^P = 1/2^-\) and 3/2-, respectively.

The 1\(P\) and higher excitations of \(\Xi_b\) have not been observed so far. The predictions presented in Table VII will be helpful to the future experimental searches. One notice that the predictions of two 1\(P\) \(\Xi_b\) baryons here and in Ref. [2, 43] are

| \(J^P(nL)\) | \(\text{Exp. [28]}\) | This work | Ref. [2] | Ref. [43] |
|---|---|---|---|---|
| \(\frac{1}{2}^+\) (1S) | 5795.8 | 5801 | 5803 | 5806 |
| \(\frac{1}{2}^\mp\) (1P) | 6097 | 6120 | 6090 |
| \(\frac{1}{2}^-\) (1P) | 6106 | 6130 | 6093 |
| \(\frac{3}{2}^-\) (1D) | 6344 | 6366 | 6311 |
| \(\frac{3}{2}^-\) (1P) | 6349 | 6373 | 6300 |
| \(\frac{1}{2}^-\) (1F) | 6555 | 6777 | - |
| \(\frac{3}{2}^-\) (1F) | 6559 | 6581 | - |
| \(\frac{1}{2}^-\) (1G) | 6743 | 6760 | - |
| \(\frac{3}{2}^-\) (1G) | 6747 | 6762 | - |

TABLE VII: The predicted masses of the \(\Lambda_b^0\) baryons (in MeV). We also collect the experimental values [28] and other theoretical results [2, 43] for comparison.

above the \(\Xi_c(5945)\pi\) threshold. Thus, the 1\(P\) \(\Xi_b\) states can be searched in the strong decay channel of \(\Xi_c(5945)\pi\).

IV. FURTHER DISCUSSIONS

A. Distinctions between the heavy quark-light diquark and three-body pictures for heavy baryons

In the Section III we have studied the mass spectrum and decay properties of heavy baryons in the heavy quark-light diquark picture. Our results do not appear to contradict with this picture. However, we still cannot exclude the three-body picture. In fact, there exist other possible mechanisms for the “missing resonances problem”. As an example, the authors of Ref. [38, 42] pointed out that the missing \(N^*\) and \(\Delta\) resonances were due to the weak couplings to the \(N\pi\) channel which was used predominantly for production of excited \(N^*\) and \(\Delta\) baryons. So what can be used as criteria to distinguish these two pictures for heavy baryons? In this section, we will take the \(\Lambda_c\) baryons to illustrate this issue.

Firstly, the masses of \(\Lambda_c\) are predicted to be strikingly different in the diquark and three-quark models. In the second row of Table VII we list the \(S, P, D, F\), and \(G\)-wave \(\Lambda_c\) states in the diquark model. Besides these states, there are other possible excitations in three-quark model, which are listed in the rows 3 to 9. Details of these denotations can be found in Ref. [41]. In the diquark models, the lowest excitation with \(J^P = 5/2^-\) is a \(F\)-wave state, which is denoted as \(\Lambda_c(5/2^-)\). Differently, the three-quark models allow a \(P\)-wave state to be \(5/2^-\). Here, we denote it as \(\Lambda_c(5/2^-)\). The predicted masses of \(\Lambda_c(5/2^-)\) is in the range of 2870–2900 MeV [43, 44], which is much lower than the predictions of \(\Lambda_c(5/2^-)\) in diquark models (see Table I). In fact, the states with negative parity are not allowed to locate in the energy range from 2650 to 2930 MeV in the diquark models [3]. If any of these states were found to have negative parity in this mass range, the hypothesis of the quark-diquark picture would be excluded.

Secondly, we point out that the decay properties of the
\[ \Lambda_{c} \left( \frac{5}{2}^{-} \right) \text{ and } \Lambda_{c} \left( \frac{3}{2}^{-} \right) \text{ states are also different because their total angular momentum of the light degrees of freedom, } j_{f}, \text{ are not the same. In the heavy quark limit, two states of one doublet will be degenerate. Then they have the same mass and decay properties. In this limit, two model-independent ratios are shown below for the } \Lambda_{c} \left( \frac{5}{2}^{-} \right) \text{ and } \Lambda_{c} \left( \frac{3}{2}^{-} \right) \text{ states by Eq. (14).} \]

For three-body picture,

\[ R = \frac{\Gamma(\Lambda_{c} \left( \frac{5}{2}^{-} \right) \to \Sigma_{c}(2455)\pi)}{\Gamma(\Lambda_{c} \left( \frac{3}{2}^{-} \right) \to \Sigma_{c}(2520)\pi)} = \frac{2}{7} < 1. \]  

(17)

For diquark picture,

\[ R = \frac{\Gamma(\Lambda_{c} \left( \frac{5}{2}^{-} \right) \to \Sigma_{c}(2455)\pi)}{\Gamma(\Lambda_{c} \left( \frac{3}{2}^{-} \right) \to \Sigma_{c}(2520)\pi)} = \frac{2}{7} > 1. \]  

(18)

Here, we ignore the decay channel of \( \Sigma_{c}(2520)\pi \) in G-wave for the small phase space. In practice, the heavy quark symmetry is broken for the finite mass of \( m_{Q} \). When the different momentums of the final states are considered, the ratio of \( R \) is predicted about 0.54--0.59. However, the ratio of \( R \) is about 4.56 which is much large. Here the predicted masses shown in Table III have been used.

Finally, we stress that \( \Lambda_{c} \left( \frac{5}{2}^{-} \right) \) is a nice criterion to test the diquark picture for charmed baryons. (1) The mass of \( \Lambda_{c} \left( \frac{5}{2}^{-} \right) \) is not very high for the further experiments. (2) \( \Lambda_{c} \left( \frac{5}{2}^{-} \right) \) is the only state with \( J^{P} = 5/2^{-} \) in the range of 2760 to 2900 MeV. So the mixing effects is insignificant. (3) The primary decay channels of \( \Lambda_{c} \left( \frac{5}{2}^{-} \right) \) and \( \Lambda_{c} \left( \frac{3}{2}^{-} \right) \) are \( \Sigma_{c}(2455)\pi \) and \( \Sigma_{c}(2520)\pi \) which are used predominantly to search for the high excited \( \Lambda_{c}^{*} \) states.

B. Mass gaps between \( \Lambda_{Q} \) and \( \Xi_{Q} \) baryons

In this subsection, the mass gaps between \( \Lambda_{Q} \) and \( \Xi_{Q} \) will be explained by the Eqs. (11) and (12). We take the \( \Lambda_{c}/\Xi_{c} \) as example, and show that the main mass gap between the corresponding states originates from the different masses of diquark. To this end, we combine these two equations in the form

\[ e = M_{Q} + m_{d} + \bar{\Lambda}_{nl}(kn + L) + V_{nl}^{m_{Q}} \cdot \hat{L}, \]  

(19)

where

\[ M_{Q} = m_{Q} + \zeta_{Q}; \quad \bar{\Lambda}_{nl} = \frac{\sigma}{2(\epsilon - m_{Q} + m_{d} + \zeta_{Q})}; \]

and,

\[ V_{nl}^{m_{Q}} = \frac{\alpha_{s}}{3 \times 2^{3/2}} \left( \frac{\sigma}{kn + L} \right)^{3/2} \frac{1}{m_{Q} m_{Q}}. \]

We treated \( \zeta_{c} \) and \( m_{c} \) as constants in our calculations, then \( M_{Q} \) are equal for \( \Lambda_{c} \) and \( \Xi_{c} \). At present, only the possible 1P, 2S, 1D, and 2P \( \Lambda_{c}/\Xi_{c} \) candidates have been reported. We just calculate the values of \( \bar{\Lambda}_{nl} \) and \( V_{nl}^{m_{Q}} \) for these states. The results are shown in Table IX. Obviously, the differences of \( \bar{\Lambda}_{nl} \) and \( V_{nl}^{m_{Q}} \) are small for \( \Lambda_{c} \) and \( \Xi_{c} \). The Eq. (19) reflects the similar dynamics between \( \Lambda_{c} \) and \( \Xi_{c} \).

Table IX: The values of \( \bar{\Lambda}_{nl} \) and \( V_{nl}^{m_{Q}} \) for the 1P, 2S, 1D, and 2P \( \Lambda_{c}/\Xi_{c} \) states.

| states (nL) | \( \bar{\Lambda}_{nl} \) (GeV) | \( V_{nl}^{m_{Q}} \) (GeV) |
|-------------|-----------------|-----------------|
| 1 P         | 0.330           | 0.334           |
| 2 S         | 0.307           | 0.313           |
| 1 D         | 0.292           | 0.300           |
| 2 P         | 0.277           | 0.286           |

It is interesting to compare the Eq. (19) with the formula given by the heavy-quark effective theory (HQET). In HQET, the mass formula for heavy baryons is written as [56]

\[ e = m_{Q} + \bar{a}/m_{Q} + m_{d} + H_{d} + H_{hyp}. \]  

(20)

Here, \( m_{Q} \) and \( m_{d} \) are the masses of heavy quark and light-diquark, respectively. The second term, \( \bar{a}/m_{Q} \), arises from the kinetic energy of the heavy quark inside the heavy baryons. \( H_{d} \) denotes the energy density of diquark in baryon systems. \( H_{hyp} \) represents the hyperfine interactions. The bound energy, \( H_{d} \), can not be given in the HQET framework. Comparing the Eqs. (19) and (20), we find that \( H_{d} \) is equal to \( \bar{\Lambda}_{nl}(kn + L) \) in the RFT model.

V. SUMMARY

In this work, we have analytically derived a formula for the mass spectrum of the excited heavy-light hadrons within the relativistic flux tube model. Then the formula is applied to the mass spectrum of \( \Lambda_{Q} \) and \( \Xi_{Q} (Q = c \text{ or } b \text{ quark}) \) baryons, where the heavy quark-light diquark picture is considered. The hyperfine term was borrowed directly from the QCD-motivated potential models. Our results totally coincide with the predictions of Ref. [7]. We found that the available \( \Lambda_{Q} \) and \( \Xi_{Q} \) baryons can be understood well in the heavy quark-light diquark picture. The main conclusions are given as follows.

\[ \text{Table VIII: Allowed } \Lambda_{c} \text{ states in the quark-diquark and three-body picture.} \]

| \( S \) | \( P \) | \( D \) | \( F \) | \( G \) |
|-------|-------|-------|-------|-------|
| None  | \( \bar{\Lambda}_{c} \left( \frac{5}{2}^{-} \right) \) | \( \Lambda_{c} \left( \frac{5}{2}^{-} \right) \) | \( \Lambda_{c} \left( \frac{3}{2}^{-} \right) \) | \( \Lambda_{c} \left( \frac{1}{2}^{-} \right) \) |

\[ \text{None } \bar{\Lambda}_{c} \left( \frac{5}{2}^{-} \right) \ldots \ldots \]
\(\Lambda_c(2760)^+\) and \(\Xi_c(2980)\) can be assigned to the 2s states with \(J^P = 1/2^+\). \(\Lambda_c(2940)^+\) and \(\Xi_c(3123)\) might be the 2\(P\) excitations of \(\Lambda_c\) and \(\Xi_c\). The question that the experimental values are about 50 ~ 80 MeV lower than the predictions can be explained by the coupled-channel effects. The assignments for \(\Lambda_c(2880)^+\) and \(\Xi_c(3080)\) are the 1D \(\Lambda_c\) and \(\Xi_c\) states with \(J^P = 5/2^+\). The \(\Xi_c(3055)\) is the doublet partner of \(\Xi_c(3080)\) with \(J^P = 3/2^+\). We assign the new resonance, \(B_c(3212)\), as a 2\(D\) or 1\(F\) \(\Lambda_c\) states temporally. The \(\Lambda_b(5912)^0\) and \(\Lambda_b(5920)^0\) are the 1\(P\) bottom baryons with \(J^P = 1/2^+\) and 3\(/2^+\), respectively.

It is important to emphasize that these assignments do not appear to contradict the present strong decay properties. The narrow structures of \(\Lambda_c(2880)^+\), \(\Xi_c(3055)\), and \(\Xi_c(3080)\) have been understood. The node effects may be significant for the decays of 1\(D\) \(\Lambda_c\) and \(\Xi_c\). We partially interpreted the ratio of \(\Lambda_c(2880)^+\) partial widths which was measured by Belle. The ratio of \(\Gamma(\Xi_c(3080) \to \Sigma_c(2520)K)/\Gamma(\Xi_c(3080) \to \Sigma_c(2455)K)\) is predicted about 36.4\%, which can be test in future.

At present, we still cannot exclude the three-body picture for the heavy baryons. The distinctions between the heavy quark-light diquark and three-body pictures were discussed. We propose a search for the \(\Lambda_c^+\) state which can help to distinguish the diquark and three-body schemes. In a word, the investigations in this work are expected to be helpful for the heavy baryon physics.

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