Quenching factor and electronic LET in a gas at low energy

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Abstract. Nuclear quenching factors for recoil ions of 1-200 keV in He, Ne, Ar and Xe are discussed. The Bragg-like curve for Time Projection Chamber (TPC) in relation with dark matter search is obtained using the electronic linear energy transfer (LET). The quenching factors for C, N, O, Ar, and Pb ions in Ar gas are also reviewed.

1. Introduction
The interaction of low energy heavy ions (\( v < v_0 = e^2/h \approx c/137 \) ) with matter is a significant issue in the search for dark matter such as Weakly Interacting Massive Particles (WIMPs) [1]. Most of energy loss is due to nuclear collisions, i.e., elastic scattering in the screened electric field of atom [2]. The secondaries, recoil atoms and electrons, may again go to the collision process and transfer the energy to new particles. After this cascade process is complete, the energy of incident particle \( E \) is given to atomic motion \( v \) and electronic excitation \( \eta \). Only the part of energy \( \eta \) can be used as ionization and/or scintillation signals. The nuclear quenching factor (the Lindhard factor, \( q_{nc} = \eta / E \) ), the ratio of energy given to the electronic excitation to the total energy, is important for the determination of the sensitivity of WIMP detectors.

Lindhard theory [2] gives \( q_{nc} \) in the case of that the incoming particle belongs to the target medium, \( Z_1 = Z_2 \), for all the elements. On the other hand, for \( Z_1 \neq Z_2 \), the recoil ions in \( \alpha \) decay [3] for example, only few calculations or approximate equation are available. Some results reported for C, N, O, Ar and Pb ions in Ar will be reviewed and compared with the power law approximation proposed by Lindhard et al. Also we present an empirical form for \( Z_1 < Z_2 \) using experimental results by Phipps et al [4].

The linear energy transfer (LET) is a key factor for understanding the radiation effects such as quenching in liquid and solid scintillators. The LET is simply \(-dE/dx\) for fast ions since the total energy loss is due almost exclusively to the electronic stopping; \( S_T \approx S_e \). However, it is necessary to introduce the electronic LET (\( \text{LET}_{el} = -d\eta/dx \) ) [5] because the nuclear stopping \( S_n \) and the electronic stopping \( S_e \) become the same order of magnitude at low energy; \( S_T = S_n + S_e \). The Bragg-like curves have been obtained using \( \text{LET}_{el} \). The curves may be used for detecting the direction of low energy recoil ions in gas TPC.

2. Stopping power and nuclear quenching
Lindhard et al. treated the nuclear and electronic collisions separately. The nuclear process follows the usual procedure of a screened Rutherford scattering. The electronic process is based on the Thomas-Fermi treatment. \( S_i \) is expressed as \((de/d\rho)_e = k_1 \varepsilon^{1/2} \) where \( \varepsilon \) and \( \rho \) are dimensionless measures of energy and range. When the projectile and the target are the same element, \( Z_1 = Z_2 \), \( k_1 \) is given by the
atomic number, Z, and mass A, as \( k = 0.133Z^{2/3}A^{-1/2} \). Charge transfer processes; capture and loss are included in the stopping calculations.

The nuclear \( S_n \) and electronic \( S_e \) stopping powers are illustrated in Fig. 1. The energy \( E \) in keV is converted to \( \varepsilon \) by \( \varepsilon = C_k E \). The values of constant \( C_k \) are 2.28, 0.0534, 0.0135, 0.00104 for He, Ne, Ar and Xe ions, respectively. The energy ranges in keV for recoil ions in rare gases are indicated with horizontal lines. \( \varepsilon \) is smaller for heavier particles at the same energy. The electronic stopping is shown for \( k=0.15 \). Most elements have \( k \) values close to 0.15. Light elements, such as He, have a small \( k \) and less steep slope. The figure shows the stopping end of the Bragg curve. The Bragg peak is far right, out side of the figure. The nuclear stopping is dominant for Xe ions of < 200 keV in Xe whereas the nuclear stopping is small for He ions in He with > 10 keV.

Lindard et al. gave numerical results for the nuclear quenching factors \( q_{nc} = \eta/E \) for \( Z_1 = Z_2 \) for \( k = 0.1, 0.15 \) and 0.2. They also gave an asymptotic form for \( k = 0.1-0.2 \). The asymptotic form reproduces the numerical \( \nu \) within an accuracy of several %. The quenching factor \( q_{nc} \) are shown for recoil ions in He, Ne, Ar and Xe of 1-50 keV in Fig. 2. It should be noted that Xe ions in Xe less than 10 keV are out of the energy range \( \varepsilon < 0.01 \) valid for Lindhard model (shown with a broken curve in Fig. 2). The Thomas-Fermi treatment becomes a crude approximation at the extreme low energy. Light ions, He and Ne satisfy this criterion; however, the number of ions produced becomes very small. The W-values, the average energy required to form ab ion pair, for He and Ne are an order of magnitude larger than those for semiconductors. The distribution of number of ions produced may not be expressed with a Gaussian distribution. The W-values can be different from those at higher energy. The W-value for \( \gamma \)-rays usually has a considerable energy dependence and even has some structures around K- and L-edges of the target atom.
The energy balance [6] gives
\[ W = \bar{E}_i + (N_{ex} / N_i) \bar{E}_{ex} + \bar{E}_{s}, \]
where \( \bar{E}_i \) and \( \bar{E}_{ex} \) are average energies for ionization and excitation, \( N_{ex} / N_i \) is the ratio of numbers of excited states and ions, and \( \bar{E}_s \) is the average kinetic energy of subexcitation electrons. Those values can be different for slow ions since the slow ions can excite the metastable state, etc.

3. Quenching factor for \( Z_1 \neq Z_2 \)

The evaluation of \( q_{nc} \) for \( Z_1 \neq Z_2 \) is quite hard. The recoil ions in –decay in rare gases give one of the simplest cases. The \( \alpha \)-decay produces a very heavy ion with 100-150 keV. The ionization measurements in gas basically give the Lindhard factor \( q_{nc} \) both in gas and condensed phases. Madsen [3] has measured the ionization by recoil ions in \( \alpha \)-decay from Po, ThC, and ThC’.

Ling and Knipp [7] have presented an approximate equation for a heavy particle (Z=82, A= 208) in Ar. For \( v / v_0 < 1 \),

\[ q_{nc} = \eta / E \approx \left[ \frac{2}{3} a + \frac{16}{21} \right] \gamma^3 \]

where \( \gamma' = 2A_1 / (A_1 + A_2) \). They found measurement by Madsen is well represented by Eq. (1) with the proportional factor \( 2/3a + 16\gamma'/21 = W_a/(15.4 \text{ eV})v_0 \), where \( W_a \) is the W-value for \( \alpha \)-particle.

Lindhard et al. [2] gave a power law approximation for very low energy. Two characteristic energies, \( E_{1c} \) and \( E_{2c} \), associated with the target atom and the projectile atom, set the upper boundary. We have,

\[ \eta = CE^{3/2}, \quad \text{for } E < E_{1c}, E_{2c} \]

where \( C = \frac{5}{2} \left( E_{1c}^{-1/2} + \frac{1}{2} \gamma^2 E_{2c}^{-1/2} \right), \quad \gamma = 4A_1 A_2 / (A_1 + A_2)^2 \) and \( E_c = \gamma/E_{2c} \). The power low approximation gives for Pb ions produced in \( \alpha \)-decay in Ar,

\[ \eta = 0.019E^{3/2} \quad (E: \text{keV}), \quad \text{for } E \ll 660 \text{ keV}. \]

Lindhard et al. also gave a numerical calculation for this particular case in Ar. The results are shown in Fig. 3 for comparison. The experimental points come in between the numerical result and the power low approximation.

Phipps et al. [4] measured the ionization due to H to Ar ions in gas Ar. The results are also shown in Fig. 3. The result for Ar ions in Ar agrees well with the Lindhard numerical values with \( k = 0.15 \) only about 10 % smaller than the calculation. However, Those for C, N and O ions show considerably small values predicted by Eq. (2). The energy restriction is rather severe for those light ions. Phipps et al. plotted the data in \( W - v_0/v \). The light ions, C, N and O, lie quite well on a straight line as shown in Fig. 4. We have an empirical representation for light ions in Ar for \( v_0/v > 2.5 \),

![Figure 3. The nuclear quenching factor \( q_{nc} \) for various ions in Ar [4]. The ionization measurements [3] in Ar gas are also shown. Solid curves are Lindhard numerical calculation [2] and dashed curves are the power low approximation Eq. (2). Dot-dashed curve is by Ling and Knipp [7], Eq. (1).](image-url)
Figure 4. The $W - \nu_0/\nu$ plot for various ions measured in gas Ar [4]. The solid line shows, Eq. (4), fit for C, N and O ions (close symbols).

\[ q_{nc} = W_\alpha / W_{RN} = 26.4 / [15.3(\nu_0 / \nu) + 6.1] \]  

Where, $W_\alpha$ and $W_{RN}$ are $W$-values for $\alpha$-particles and ions, respectively.

4. The electronic LET and the Bragg-like curve

The linear energy transfer LET ($= -dE/dx$) generally gives the excitation density along the particle track and is a good measure for evaluating radiation effects on the medium such as scintillation efficiency in condensed phase. However, the energy loss due to electronic excitation and nuclear collisions becomes the same order of magnitude for slow ions. The energy $\eta$ used for ionization and scintillation is only a part of particle energy $E$ as discussed above. We introduce here the electronic LET which represents the electronic energy given to the target material per unit length along the particle track.

A particle of energy $E_0$ can deposit an electronic energy $\eta_0$ in a range $R_0$. The particle energy becomes $E_1$ after an energy loss of $\Delta E$, then it can deposit $\eta_1$ in $R_1$ as shown in fig. 5a). We have simply $\text{LET}_e = -d(q(e)c)/d\rho$. However, a little complication comes since $dE/dx$ is usually given as a function of energy $E$ not $x$. Then, we have,

\[ \text{LET}_e = -d\eta/dR \approx -(\eta_1 - \eta_0)/(R_1 - R_0) = -\Delta \eta/\Delta R \]  

Then we can obtain $\text{LET}_e$ as a function of the energy. The idea here becomes useful when we calculate the Bragg-like curve for TPC.

There are also two choices of $R$, i.e., the true range $R_T$ and the projected range $R_{PRJ}$ according to the purpose as discussed below. The ion track have a cylindrical structure and is almost straight for fast heavy ions, then $R_T \approx R_{PRJ}$. However, the track for slow particles becomes crooked because of the elastic scattering with target atoms and may have some branches as shown in Fig. 5b). Then $R_T$ is no longer equal to $R_{PRJ}$.

4.1 The electronic LET for quenching calculations. The true range $R_T$ is obtained by the total stopping power as usual manner,

\[ R_T = \int (dE/dx)^{-1} dE \]  

Then, we have $\text{LET}_e$ with Eq. (5). $\text{LET}_e(R_T) = -d\eta/dR_T$ gives the excitation density (ionization included) along the track. The result obtained for He ions in He is shown in Fig. 6a) together with the stopping powers. $\text{LET}_e(R_T)$ comes in between $S_i$ and $S_{TR}$. 

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\[ q_{nc} = W_\alpha / W_{RN} = 26.4 / [15.3(\nu_0 / \nu) + 6.1] \]
4.2 The Bragg-like curve for TPC. For the directional detection of recoil ions in gas TPC, it is useful to evaluate the ionization density as a function of depth – the projected range $R_{PRJ}$. The Bragg-like curve is given by $-d\eta/dR_{PRJ}$. It may be convenient to have the number of ions produced $N_i$ as a function of the range. The number of ions is calculated using $W$-value. The Bragg-like curve obtained in this way for He ions in He is shown in Fig. 6b). The projectile enters from the right hand side. Points are plotted at every 1 keV. The vertical axis is plotted in $N_i$, the number of ions produced per unit length. The area below the curve shows the number of ions produced.

5. Discussion

The Lindhard factor $q_{nc}$ basically give the recoil to $\gamma$ ratio, $RN/\gamma$, for ionisation measured in gas. It also sets an upper limit for $RN/\gamma$ in liquids. The $RN/\gamma$ is expressed as $(q_{nc}+q_{el})/L(\gamma)$ in liquid, where $q_{el}$ is the electronic quenching factor and $L(\gamma)$ is the scintillation efficiency for $\gamma$-rays [5].

The relation between stopping powers and electronic LET is shown in Fig. 6. Two LET$_{el}$ are presented; the microscopic LET$_{el}$ ($R_T$) (Fig. 6a) and the macroscopic LET$_{el}$ ($R_{PRJ}$) (Fig. 6b). LET$_{el}$ ($R_T$) is larger then $S_0$ because a part of energy given to atomic motion may again be converted to the electronic excitation as the secondary ion may again excite atoms. The electronic LET for Xe has been used for the calculation of electronic quenching in liquid Xe [5] as well as CsI(Tl) [8] and showed considerable success.

Figure 5. Schematic presentation of a) the idea of obtaining LET$_{el}$ and b) the relation between two ranges, $R_T$ and $R_{PRJ}$.

Figure 6. a) The stopping powers [9] and electronic LET in He ions in He; b) The Bragg-like curve. The points are plotted at every 1 keV. The area below the curve shows $N_i$, the number of ions produced.
The LET$_e$ ($R_{PRJ}$) is a practical representation used for TPC. In principle, the Monte-Carlo simulation gives the three dimensional presentation of an ion track. However, the accurate information needed for the calculation, such as the cross sections for fundamental processes are not fully available in the energy ranges concerned here. The Bragg-like curve proposed here give no less information than the Monte-Carlo if one dimensional presentation is good enough such as the head and tail detection of the recoil ions. Furthermore, it is obtained much easier if $q_{nc}$ can be estimated.

The total stopping power $S_T$ for He ions decreases as the energy of He ions decreases, reaches a minimum at about 3 keV then turns to increase as the contribution from the nuclear stopping power increases. The Bragg-like curve on the other hand, decreases monotonically towards the stopping end of the track. It should be noted that the LET$_e$ ($R_{PRJ}$) can be larger than the total stopping power. In fact, the value of LET$_e$ ($R_{PRJ}$) for He in He is quite close to $S_T$ at $> 7$ keV and becomes larger than $S_T$ around 20 keV.

6. Summary

The relation between the stopping power and the electronic LET were discussed. The nuclear quenching factor for recoil ions in He, Ne, Ar and Xe were presented. The treatment of the nuclear quenching factor, $q_{nc}$, for $Z_1 \neq Z_2$ was discussed for various ions in Ar. For low energy, or for heavy ions, the power low approximation by Lindhard, and by Ling and Knipp may be applied. An empirical representation has been introduced for light ions.

The Bragg-like curve, $-\frac{d\eta}{dR_{PRJ}}$, which can be used for recoil ions in gas TPC for the directional measurements of dark matter search, has been presented.

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