Intelligent reflecting surface-assisted single-input single-output Golden codeword-based modulation schemes

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Abstract
In this paper, two intelligent reflecting surface (IRS)-assisted single-input single-output Golden codeword-based modulation schemes for next-generation networks are investigated. In Scheme 1, one pair of Golden codeword super-symbols are transmitted over two consecutive time slots with the assistance of an IRS to maximise the received signal-to-noise ratio (SNR) in each slot. Scheme 2 applies component-interleaving to the pair of Golden codeword super-symbols and transmits the modified super-symbols over four consecutive time slots with the assistance of an IRS to maximise the received SNR in each slot. A significant improvement in error performance is demonstrated over their counterparts, for both conventional/non-access point (AP) or AP-based IRSs. Scheme 2 exhibits superior error performance over Scheme 1, especially for small and medium IRSs. The theoretical average bit error probabilities for the proposed schemes are formulated and validated by simulation results. Low-complexity detectors for the schemes under study are provided and the computational complexity imposed by these detectors are analysed and found to be extremely low.

1 | INTRODUCTION

Next-generation wireless communication systems require drastic improvements in reliability and capacity. Multiple-input multiple-output (MIMO) systems theoretically address these requirements. However, several factors limit the extent of their practical realisation. On this note, the research community is constantly investigating and proposing innovative techniques that may be employed to meet these requirements.

Meta-surfaces that modify the propagation environment to enhance wireless communication system objectives was first proposed by Kaina, et al. [1]. These meta-surfaces are commonly referred to as intelligent reflecting surfaces (IRSs), reconfigurable intelligent surfaces or large intelligent surfaces and can be used to improve reliability, capacity, security, energy and spectrum efficiency [2]. In this paper, we will refer to these surfaces as IRSs. An IRS is typically composed of a large number of reflecting elements, each associated with an adjustable parameter, for example, amplitude, phase, frequency or polarisation. The elements are supposed to be low-cost and energy-efficient. The key idea in the use of these surfaces is that the adjustable parameter may be reconfigured, such as to effect a change on the impinging electromagnetic (EM) wave, thereby improving upon a particular communication system objective [2]. For example, the phase parameter of an IRS element could be adjusted to maximise the received signal-to-noise ratio (SNR), consequently optimising error performance.

In the current literature on IRSs, researchers have considered placement of these surfaces on walls, buildings, ceilings, people and vehicles. This is highly practicable because these EM materials can be used to coat objects like building and vehicle surfaces, furniture and clothes [3]. Furthermore, the majority of IRSs have been envisioned as passive (green) and low-cost due to the absence of the need for analog-to-digital or digital-to-analog converters and power amplifiers. Some significant challenges also exist. For example, Yuan, et al. [3] highlighted some of the key challenges of IRSs of which channel state information acquisition was flagged as a significant issue. In a typical IRS-based communication system, channel state information corresponding to the IRS element-transmit antenna link and the receive antenna-IRS element link is required. This is difficult to acquire, since the IRS is supposed to be passive...
and low-cost, thus rendering it impossible to employ pilot symbol estimation. Several possible solutions to this challenge were highlighted in [3]. Nevertheless, estimation of the channel state information for an IRS equipped with a large number of elements imposes significant complexity. Furthermore, [3] proposed IRSs as a solution for edge intelligence, wireless power transfer, physical-layer security, unmanned-aerial-vehicle communication and device-to-device communication.

In [4], hardware impairments (HWIs) were identified as a serious challenge for large IRSs. Specifically, the study investigated capacity and utility degradations stemming from HWIs due to large IRSs. To address this challenge, the authors proposed a distributed implementation of a typical IRS by segmentation into smaller IRSs. A notable drawback is the additional complexity introduced in this implementation. On the same note, [5] presented a study into the real-time implementation of an IRS as an alternative to massive MIMO, where a distributed architecture with multiple smaller units is employed. Further, the merits and demerits of several implementation strategies were discussed.

Zhao, et al. [6] also presented a survey of numerous contributions in the open literature. Contributions have been classified into several categories addressing capacity analyses, power/spectral efficiency optimisation, channel estimation, deep-learning for optimisation and channel estimation, secure communication, terminal positioning and implementation.

Some other useful mentions are: In [7], phase adjustable IRS elements were employed to redirect incident EM waves in a desired spatial direction between a base station and mobile station in a cellular network. The optimal phase adjustment was determined to maximise ergodic capacity by employing statistical channel information. Jiang, et al. [8] investigated IRSs for over-the-air computation for fast data aggregation in a wireless setting and a unique algorithm was developed to solve the resulting intractable non-convex bi-quadratic programming problem. The IRS-based solution was shown to outperform the state-of-the-art solutions. In [9], the authors identify practical challenges for realising the phase shift required for full signal reflection in IRSs. Further, a method for beamforming optimisation is devised based on a more practical phase shift model for an IRS-based multiple antenna AP to single antenna user downlink.

Basar [10] formulated a framework for error performance analysis of an IRS with and without (blind) knowledge of the channel phases. An access point (AP)-based IRS was further proposed in [10], were both information transmission and phase adjustment can be accomplished by an IRS when coupled with a radio-frequency source. This IRS required only channel state information corresponding to the single receive antenna-IRS element link. The AP-based scheme was shown to exhibit a significant SNR gain over the conventional or non-AP-based IRS. In [11], Basar applied the concept of index modulation to an AP-based IRS setup with multiple receive antennas. Index modulation was performed by mapping bits to a target receive antenna. Phases at the IRS elements are then adjusted in a manner to maximise the SNR at this target receive antenna. Both the space-shift keying and spatial modulation classes of index modulation were considered.

The Golden code is a full-rate, full-diversity linear dispersion space-time block code [12] applied to a MIMO setting that employs two transmit antennas and two or more receive antennas. The Golden codeword which is transmitted over two consecutive time slots, conveys four amplitude/phase modulation (APM) symbols. In the encoding process, the four APM symbols are used to create four super-symbols. Then, an arrangement of two super-symbols that carry all four APM symbols are transmitted in each time slot by the two independent antennas.

Recently, Xu, et al. [13] proposed that a pair of super-symbols, which carry the same two APM symbols may be transmitted over two consecutive time slots with a single transmit antenna, such that the distance properties of Golden encoding may be exploited even in a SIMO setting. Given $N_R$ receive antennas, numerical results for a $1 \times N_R$ setting demonstrated that the scheme yields identical diversity order as signal-space diversity (SSD) [14], while matching its error performance. Evidently, this pair of super-symbols from the Golden codeword is rotated by an angle as determined in the signal rotation in SSD. However, SSD uses both signal rotation and component-interleaving. Hence, this motivated the further application of component-interleaving between pairs of super-symbols, thus resulting in additional diversity gain and consequently an improved error performance. Furthermore, in [13], low-complexity detectors were proposed for the schemes based on the use of a symbol detection subset. The proposed detectors exhibit identical error performance compared to their maximum-likelihood (ML) detector counterparts when the appropriate size of symbol detection subset is chosen.

### 1.1 Motivation

Based on the above background, IRSs with phase adjustable elements have gained much interest. Furthermore, the majority of works are focusing on large IRSs. However, small and medium IRSs may also be used to access the potential of intelligent surfaces, while minimising system complexity involved in channel estimation and deployment. These smaller IRSs can be easily deployed on vehicles, people and smaller objects making them more ubiquitous and accessible. The attractiveness of the SIMO Golden codeword modulation is undoubtable. Its application to IRS-aided communication systems is valid, since it may be used to enhance the error performance for small and medium IRSs, when such error performance is not saturated. Further, we are motivated to limit the number of antennas at the receiver to primarily decrease the complexity of channel estimation. Hence, in this paper, we investigate two single-input single-output (SISO) IRS-aided Golden codeword-based modulation schemes for next-generation networks.

An additional motivation for SISO is as follows: Although MIMO systems have numerous advantages, as stated earlier, some features are difficult to realise due to several factors, for example, power consumption, computational complexity and system complexity. Further, in battery constrained devices, power consumption is a crucial factor. The motivation of small IRSs for placement on small objects and people and with high
occurrence also requires low power consumption and system complexity. In this regard, small IRSs in a SISO setting hold an advantage. It is also possible that a system may support several modes for example, operating in MIMO and when transitioning to power saving modes etc., a SISO system could be employed, allowing the extension of battery life. For example, specific systems that employ several types of links like direct, backhaul or access, incorporate SISO settings for direct links to terrestrial users [15]. Further, the performance improvement of increasing the number of IRS elements compared to increasing the number of antennas in an IRS system can still be considered.

1.2 Contributions

The contributions of this paper are as follows: (A) Two IRS-assisted SISO Golden codeword-based modulation schemes are investigated as candidates for improving error performance. (B) The theoretical average bit error probabilities (ABEPs) for the schemes are formulated. (C) The schemes are accompanied by their associated low-complexity detectors, and (D) An analysis of computational complexity is presented.

1.3 Organisation

The structure of the remainder of the paper is as follows: In Section 2, the system models for the proposed schemes are presented including the formulation of their theoretical ABEPs. Furthermore, their low-complexity detectors and associated computational complexity analyses are presented. The numerical results are presented in Section 3. Finally, concluding remarks are drawn in Section 4.

Notation: Upper-case bold symbols represent sets or matrices, while lower-case bold symbols represent vectors. \( | \cdot | \) represents the Euclidean norm and \((\cdot)^* \) is the complex conjugate. \( E\{\cdot\} \) is the expectation operator. \( D(\cdot) \) is the constellation demodulation function. \( Q(\cdot) \) represents the Gaussian Q-function. \( \arg\min \{\cdot\} \) represents the argument of the minimum with respect to \( w \). \( \Re\{\cdot\} \) represents the real or in-phase part of a number in the Argand plane and \( \Im\{\cdot\} \) represents its imaginary or quadrature part. \( j \) is the complex number \( \sqrt{-1} \).

2 IRS-ASSISTED SISO SCHEMES

2.1 System model

Consider a single antenna at the transmitter and receiver and the presence of an IRS in its conventional/non-AP form (refer Figure 1(a)) or as an AP-based surface (refer Figure 1(b)), where the IRS is coupled with the transmit antenna. The presence of small, medium or large IRSs is assumed. We assume IRS elements have knowledge of the channel state information and are capable of intelligently adjusting their phase. The conventional Golden code encoder produces two pairs of super-symbols, which carry the same two APM symbols [13]. One of these pairs of super-symbols is considered in the proposed Schemes 1 and 2.

2.1.1 Scheme 1

Given two \( M \)-ary quadrature amplitude modulation (\( M \)-QAM) or \( M \)-ary phase shift keying (\( M \)-PSK) symbols \( x^i_1 \) and \( x^i_2 \), \( p \in [1 : M] \) with \( E[|x^i_1|^2] = 1 \), \( i \in [1 : 2] \), consider two sets \( \Omega^i_1 \) and \( \Omega^i_2 \) each of cardinality \( M^2 \). The entries of \( \Omega^i_1 \) are given as \( x^i_q = \frac{1}{\sqrt{5}} (j \alpha + x_2^\prime \theta), q \in [1 : M^2] \), while the entries of \( \Omega^i_2 \) are given as \( x^i_q = \frac{1}{\sqrt{5}} (j \alpha + x_2^\prime \theta), q \in [1 : M^2] \), where \( \theta = \frac{1 + \sqrt{5}}{2}, \)  
\[ \theta = 1 - \theta, \alpha = 1 + j \theta \text{ and } \bar{\theta} = 1 + j \bar{\theta}. \]  
Then an \( m \)-tuple vector \( b = [b_1 b_2 \ldots b_m] \) of \( m = 2 \log_2 M \) bits is mapped to entries in \( \Omega^i_1 \) and \( \Omega^i_2 \), respectively, to produce the symbols \( x^i_3 \) and \( x^i_4 \), where \( x^i_q = 1 + \sum_{k=1}^{m} 2^{q-k} b_k \). In Scheme 1, the transmission of the symbols \( x^i_3 \) and \( x^i_4 \) are performed over two consecutive time slots. Considering the non-AP IRS (Figure 1(a)), in each time slot, the transmitted symbol is reflected of an IRS element in the conventional/non-AP form (refer Figure 1(a)).
represents additive white Gaussian noise (AWGN) with distribution $CN(0, 1)$ at the receive antenna in the $\ell$th time slot.

Let $A_{\ell} = \sum_{i=1}^{N} h_i^\ell e^{j\phi_i^\ell}$, then in the case of the AP IRS (Figure 1(b)), $A_{\ell} = \sum_{i=1}^{N} e^{j\phi_i^\ell} \delta_i^\ell$.

The phase $\phi_i^\ell$ is adjusted, such as to maximise the received SNR [10]. Hence, given $h_i^\ell = \alpha_i^\ell e^{j\theta_i^\ell}$ and $\delta_i^\ell = \beta_i^\ell e^{j\phi_i^\ell}$, we set $\phi_i^\ell = -\theta_i^\ell - \phi_i^\ell$ for the non-AP IRS and $\phi_i^\ell = -\phi_i^\ell$ for the AP IRS.

The received signal for the non-AP or AP settings can therefore be expressed as:

$$ j_{\ell} = \sqrt{\gamma A_{\ell}} x_{\ell} + \eta_{\ell}, \ell \in [1 : 4], \quad (1b) $$

where $A_{\ell} = \sum_{i=1}^{N} \alpha_i^\ell \beta_i^\ell$ for non-AP and $A_{\ell} = \sum_{i=1}^{N} \beta_i^\ell$ for AP.

Assuming complete channel state information at the receiver, the corresponding ML detector is given as:

$$ \left( x_{1,\ell}^*, x_{2,\ell}^* \right) = \arg \min_{q \in [1:M^2]} \sum_{\ell=1}^{2} \left| j_{\ell} - \sqrt{\gamma A_{\ell}} x_{\ell}^q \right|^2, \quad (2a) $$

$$ = \arg \min_{q \in [1:M^2]} \sum_{\ell=1}^{2} \left\{ |j_{\ell} - \sqrt{\gamma A_{\ell}} \Re(x_{\ell}^q)|^2 - 2 \sqrt{\gamma A_{\ell}} \Re(y_{\ell}) \{ y_{\ell}^* \} \right\}. \quad (2b) $$

### 2.2.2 | Scheme 2

Given four M-QAM or M-PSK symbols $x_1^i, x_2^i, x_3^i$ and $x_4^i$, $i \in [1 : M]$ with $E[|x_i|^4] = 1$, $i \in [1 : 4]$, consider four sets $\Omega_1^M, \Omega_2^M, \Omega_3^M$ and $\Omega_4^M$ each of cardinality $M^2$. The entries of $\Omega_{2i-1}^M, i \in [1 : 2]$ are given as $x_{1i}^M = \frac{1}{\sqrt{2}} \alpha(x_{2i-1}^M + x_{2i}^M \theta), \gamma \in [1 : M^2]$, while the entries of $\Omega_{2i}^M, i \in [1 : 2]$ are given as $x_{2i}^M = \frac{1}{\sqrt{2}} \alpha(x_{2i-1}^M + x_{2i}^M \theta), \gamma \in [1 : M^2]$. Then $m$-tupple vectors $b_i = [b_1^i b_2^i \cdots b_{Mn}^i], i \in [1 : 2]$ of $m = 2 \log_2 M$ bits are mapped to entries in $\Omega_{2i-1}^M$ and $\Omega_{2i}^M$, to produce the symbols $s_{1i}^M, s_{2i}^M$ and $s_{3i}^M, s_{4i}^M$, where $\tilde{q} = 1 + \sum_{k=1}^{m} 2^{m-k} b_{ki}, i \in [1 : 2]$. Component-interleaving is then applied to the symbols to yield the modified symbols:

$$ s_1 = \Re(x_{1i}^M) + j\Im(x_{2i}^M), i \in [1 : 2], $$

$$ s_{4+2} = \Re(x_{3i}^M) + j\Im(x_{4i}^M), i \in [1 : 2]. $$

In Scheme 2, the transmission of the symbols $s_1, i \in [1 : 4]$ are performed over four consecutive time slots. Considering the non-AP (Figure 1(a)) and AP (Figure 1(b)) IRSs, in each time slot, the transmitted symbol is reflected of an $N$-element IRS. As earlier, the channels $h_i^\ell = \alpha_i^\ell e^{j\theta_i^\ell}$ and $\delta_i^\ell = \beta_i^\ell e^{j\phi_i^\ell}$ are Rayleigh frequency-flat fading with distribution $CN(0, 1)$ and change from one time slot to the next. Setting the phase at the $i$-th IRS element in the $\ell$-th time slot $\phi_i^\ell = -\theta_i^\ell - \phi_i^\ell$ for the non-AP IRS and $\phi_i^\ell = -\phi_i^\ell$ for the AP IRS, such that the received SNR is maximised [10], the received signal may be defined as:

$$ j_{\ell} = \sqrt{\gamma A_{\ell}} e_{\ell} + \eta_{\ell}, \ell \in [1 : 4], \quad (4) $$

where $\gamma$ is the average SNR at the receive antenna, $A_{\ell} = \sum_{i=1}^{N} \alpha_i^\ell \beta_i^\ell$ for non-AP and $A_{\ell} = \sum_{i=1}^{N} \beta_i^\ell$ for AP. AWGN at the receive antenna is represented by $\eta_{\ell}, \ell \in [1 : 4]$ and is distributed as $CN(0, 1)$.

Assuming complete channel state information at the receiver, the corresponding ML detector is given as:

$$ \left( x_1^\ell, \ldots, x_4^\ell \right) = \arg \min_{(q, \tilde{q}) \in [1 : 4 \times M^2]} \sum_{\ell=1}^{4} \left| j_{\ell} - \sqrt{\gamma A_{\ell}} x_{\ell}^q \right|^2, \quad (5a) $$

$$ = \arg \min_{(q, \tilde{q}) \in [1 : 4 \times M^2]} \sum_{\ell=1}^{4} \left\{ |j_{\ell} - \sqrt{\gamma A_{\ell}} \Re(x_{\ell}^q)|^2 - 2 \sqrt{\gamma A_{\ell}} \Re(y_{\ell}) \{ y_{\ell}^* \} \right\}. \quad (5b) $$

### 2.2.2 | Theoretical ABEP

In this section, we present the theoretical formulation of the ABEP for the proposed schemes.

#### 2.2.2.1 | Scheme 1

Using a union bound [16], the ABEP for Scheme 1 may be upper-bounded as:

$$ P_{\text{unbound}} \leq \frac{1}{2^m} \sum_{\tilde{q}=1}^{4M^2} \sum_{q=1}^{M} \frac{N(q, \tilde{q})}{M} \mathbb{P}(x_q \rightarrow y_{\tilde{q}}), \quad (6) $$

where $m = 2 \log_2 M$, $P(x_q \rightarrow y_{\tilde{q}})$ is the pairwise error probability (PEP) assuming $x_q = [x_q^1 x_q^2]$ is transmitted and $y_{\tilde{q}} = [y_{\tilde{q}}^1 y_{\tilde{q}}^2]$ is received and $N(q, \tilde{q})$ is the number of bit errors for the associated pairwise error event.

The conditional PEP may be formulated as:

$$ P(x_q \rightarrow y_{\tilde{q}} | A_1, A_2) = P\left( \frac{1}{2} \sum_{\ell=1}^{4} \left| j_{\ell} - \sqrt{\gamma A_{\ell}} x_{\ell}^q \right|^2 > \frac{1}{2} \sum_{\ell=1}^{4} \left| j_{\ell} - \sqrt{\gamma A_{\ell}} x_{\ell}^{\tilde{q}} \right|^2 \right). \quad (7a) $$

Simplifying, we have:

$$ P(x_q \rightarrow y_{\tilde{q}} | A_1, A_2) \leq P\left( \frac{1}{2} \sum_{\ell=1}^{4} \Re\left( \sqrt{\gamma A_{\ell}} (x_{\ell}^{\tilde{q}} - x_{\ell}^q)^* \eta_{\ell} \right) > \frac{1}{2} \sum_{\ell=1}^{4} \Re\left( \sqrt{\gamma A_{\ell}} (x_{\ell}^{\tilde{q}} - x_{\ell}^q)^* \eta_{\ell} \right) \right). \quad (7b) $$
Now, \( \mathbf{R}\{\sqrt{2} A_r(x_{1j}^2 - x_{2j}^2)\eta_j\} \) and \( \mathbf{R}\{\sqrt{2} A_r(x_{2j}^2 - x_{2j}^2)\eta_j\} \) are Gaussian random variables (RVs) with distribution \( \mathcal{N}(0,2A_r^2 |x_{1j}^2-x_{2j}^2|)^2 \) and \( \mathcal{N}(0,2A_r^2 |x_{2j}^2-x_{2j}^2|)^2 \), respectively. Hence, the conditional PEP can be expressed using the Gaussian upper-tail probability as:

\[
P(x_j \rightarrow x_{\hat{j}} | A_1, A_2) = Q\left( \frac{1}{2} \sum_{\ell = 1}^{2} A_r^2 |x_{1j}^2-x_{2j}^2| \right).
\] (7c)

Exploiting the central limit theorem, \( A_\ell \) can be approximated as a Gaussian RV with mean \( \mu \) and variance \( \sigma^2 \) when \( N \gg 1 \) [10]. Let \( a_\ell = \frac{1}{2} |x_{1j}^2-x_{2j}^2|, \ell \in [1:2] \) and invoke the RV \( x_\ell \) with distribution \( \mathcal{N}(0,1) \) and probability density function (PDF)

\[
f_{x_\ell}(x_\ell) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x_\ell^2}{2}\right).
\]

Since we may define \( A_\ell^2 = (c_{x_\ell} + \mu)^2 \) as a non-central chi-square RV with mean \( \mu = \frac{N\pi}{4} \) and standard deviation \( \epsilon = (N(1 - \frac{n^2}{16}))^{\frac{1}{2}} \) in the case of the non-AP IRS and \( \mu = \frac{N\sqrt{\pi}}{2} \) and \( \epsilon = (\frac{N(4-\pi)}{4})^{\frac{1}{2}} \) for the AP IRS, we then have:

\[
P(x_j \rightarrow x_{\hat{j}}) = \int_0^\infty \int_0^\infty Q\left( \frac{1}{2} \sum_{\ell = 1}^{2} a_\ell (c_{x_\ell} + \mu)^2 \right)
\times f_{x_1}(x_1) f_{x_2}(x_2) \, dx_1 \, dx_2.
\] (7d)

Employing the well-known Q-function approximation \( Q(\sqrt{x}) \leq \frac{1}{12} \exp\left(-x^2\right) + \frac{1}{2} \exp\left(-\frac{2x^2}{3}\right) \) [16] and solving using the moment generating function approach [16], the PEP is approximated as:

\[
P(x_j \rightarrow x_{\hat{j}}) \approx \frac{1}{12} \prod_{\ell = 1}^{2} \exp\left(-\frac{1}{2} a_\ell \mu^2 \right) \left(1 + a_\ell \epsilon^2\right)^{-\frac{1}{2}}
+ \frac{1}{4} \prod_{\ell = 1}^{2} \exp\left(-\frac{2}{3} a_\ell \mu^2 \right) \left(1 + \frac{4}{3} a_\ell \epsilon^2\right)^{-\frac{1}{2}}.
\] (7e)

### 2.2.2 Scheme 2

In order to simplify the derivation of the ABEP for Scheme 2, we assume that only \( x_{1j}^1 \) and \( x_{2j}^2 \) are detected erroneously at the receiver [13]. Hence, \( s_1 = \mathbf{R}\{x_{1j}^1\} \), \( s_2 = \mathbf{R}\{x_{2j}^2\} \), \( s_3 = j\mathbf{F}\{x_{1j}^1\} \) and \( s_4 = j\mathbf{F}\{x_{2j}^2\} \) with the corresponding received signals given by (4).

Given \( x_{1j} = [x_{1j}^1, x_{1j}^2] \) and \( x_{2j} = [x_{2j}^1, x_{2j}^2] \), as earlier, the upper-bound on ABEP may be expressed as:

\[
P_{\text{scheme2}} \leq \frac{1}{2} \sum_{q_{1j} = 1}^{M} \sum_{q_{2j} = 1}^{M} \frac{N_j(q_{1j}, q_{2j})}{m} P(x_{1j} \rightarrow x_{1j}),
\] (8)

where as earlier, \( m = 2 \log_2 M \), \( P(x_{1j} \rightarrow x_{1j}) \) is the PEP assuming \( x_{2j} \) is transmitted and \( x_{2j} \) is received and \( N_j(q_{1j}, q_{2j}) \) is the number of bit errors for the associated pairwise error event.

Similar to the earlier approach, the conditional PEP may be formulated as:

\[
P(x_{1j} \rightarrow x_{1j} | A_\ell , \ell \in [1:4]) = Q\left( \frac{1}{2} \sum_{\ell = 1}^{4} A_\ell^2 \mathbf{R}\{x_{1j}^\ell\}
- \mathbf{R}\{x_{1j}^\ell\}^2 + \frac{1}{2} \sum_{\ell = 3}^{4} A_\ell^2 (3|x_{1j}^\ell - x_{2j}^\ell|^2) \right)^{\frac{1}{2}}
\] (9a)

Let the variables \( a_\ell = \frac{1}{2} (\mathbf{R}\{x_{1j}^\ell\} - \mathbf{R}\{x_{2j}^\ell\})^2 \), \( \ell \in [1:2] \) and \( a_\ell = \frac{1}{2} (\mathbf{F}\{x_{1j}^\ell - x_{2j}^\ell\}^2) \), \( \ell \in [3:4] \). Invoke the RV \( x_\ell \), \( \ell \in [1:4] \) with distribution \( \mathcal{N}(0,1) \) and PDF \( f_{x_\ell}(x_\ell) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x_\ell^2}{2}\right), \ell \in [1:4] \). Since we may define \( A_\ell^2 = (c_{x_\ell} + \mu)^2 \), \( \ell \in [1:4] \) as non-central chi-square RVs with mean \( \mu = \frac{N\pi}{4} \), standard deviation \( \epsilon = (N(1 - \frac{n^2}{16}))^{\frac{1}{2}} \) for non-AP IRS and mean \( \mu = \frac{N\sqrt{\pi}}{2} \), standard deviation \( \epsilon = ((N(4-\pi))/4)^{\frac{1}{2}} \) for AP IRS, as earlier, we may approximate the PEP as:

\[
P(x_{1j} \rightarrow x_{1j}) \approx \frac{1}{12} \prod_{\ell = 1}^{4} \exp\left(-\frac{1}{2} a_\ell \mu^2 \right) \left(1 + a_\ell \epsilon^2\right)^{-\frac{1}{2}}
+ \frac{1}{4} \prod_{\ell = 1}^{4} \exp\left(-\frac{2}{3} a_\ell \mu^2 \right) \left(1 + \frac{4}{3} a_\ell \epsilon^2\right)^{-\frac{1}{2}}.
\] (9b)

### 2.2.3 Note on the ABEP

Consider the first term of (7.5) and ignoring the constant coefficients, we have:

\[
\rho = 2 \prod_{\ell = 1}^{2} \exp\left(-\frac{1}{2} |x_{1j}^\ell - x_{2j}^\ell|^2 \right) \left(1 + \frac{1}{2} |x_{1j}^\ell - x_{2j}^\ell|^2 \right)^{-\frac{1}{2}}.
\] (10a)
Now assume $\gamma \gg 1$, then we may simplify $\rho$ as:

$$
\rho \approx \sum_{\ell=1}^{2} \exp \left( -\frac{N\pi^2}{2(16-\pi^2)} \left( \frac{\gamma N(16-\pi^2)}{32} |x_{q_{\ell}} - x_{p_{\ell}}|^2 \right)^{\frac{1}{2}} \right),
$$

(10b)

where the non-AP IRS is assumed.

It is clear from (10b) that $\rho$ is mainly dependent on the product Euclidean distance $P_{EED}$ for a given value of $N$. This may be extended to the overall PEP approximation given by (7c).

In a similar manner, it may be shown that the PEP for Scheme 2 is mainly dependent on the product Euclidean distance: $P_{EED} = \prod_{\ell=1}^{2} |\mathfrak{R}(x_{q_{\ell}}) - \mathfrak{R}(x_{p_{\ell}})|^2$. This is consistent with [13], and since both $P_{EED}$ and $P_{EED}$ consider a pair of symbols, Definition 1 in [13], is employed using the product Euclidean distance threshold $\delta > 0$ to form the symbol detection subsets that are used in the low-complexity detectors.

Based on the above, the symbol detection subsets $\Omega_{p}^{\delta}(y_{1})$ and $\Omega_{q}^{\delta}(y_{1}, y_{2})$ and their corresponding index subsets $Q_{p}^{\delta}(y_{1})$ and $Q_{q}^{\delta}(y_{1}, y_{2})$ for Scheme 1 and Scheme 2, respectively, we may modify the ABEPs in (6) and (8) as:

$$
P_{\text{Scheme 1}} = \frac{1}{2^m} \sum_{y_{1}=1}^{M} \sum_{q_{1} \in Q_{p}^{\delta}(y_{1})} N(y_{1}, q_{1})^2 P(x_{q_{1}} \rightarrow x_{p_{1}}),
$$

(11)

$$
P_{\text{Scheme 2}} = \frac{1}{2^m} \sum_{y_{1}=1}^{M} \sum_{q_{2} \in Q_{q}^{\delta}(y_{1}, y_{2})} N(y_{1}, q_{2})^2 P(x_{q_{1}} \rightarrow x_{p_{2}}).
$$

(12)

Lastly, (10b) clearly shows that significant exponential decay in the error performance is expected, especially for large $N$.

## 2.3 Low-complexity detection

In this subsection, the low-complexity detectors for Schemes 1 and 2 are presented. These detectors are based on the formulations presented in [13].

### 2.3.1 Scheme 1

Given complete channel state information at the receiver and the received signals $y_{1}$ and $y_{2}$, we may process these as follows [13]:

**Step 1:** Estimate all transmitted symbols:

$$
\hat{z}_{q_{\ell}} = \frac{\sqrt{\gamma}}{\sqrt{A_{e}}}, \ell \in [1 : 2].
$$

(13a)

**Step 2:** Compute the un-quantised estimates of the $M$-QAM or $M$-PSK symbols $x_{1}$ and $x_{2}$:

$$
\hat{u}_{1} = \sqrt{5}e^{-1} \frac{\hat{z}_{1}}{\hat{z}_{1}} \left( \hat{\theta} \right) - \sqrt{5}e^{-1} \frac{\hat{z}_{1}}{\hat{z}_{1}} \left( \hat{\theta} \right),
$$

(13b)

$$
\hat{u}_{2} = \sqrt{5}e^{-1} \frac{\hat{z}_{2}}{\hat{z}_{2}} \left( \hat{\theta} \right) - \sqrt{5}e^{-1} \frac{\hat{z}_{2}}{\hat{z}_{2}} \left( \hat{\theta} \right).
$$

(13c)

**Step 3:** Using $x_{1} = D(\hat{u}_{1})$ and $x_{2} = D(\hat{u}_{2})$, employ the symbol detection subset $\Omega_{p}^{\delta}(y_{1})$ and $\Omega_{q}^{\delta}(y_{1}, y_{2})$, then compute metrics for the joint estimation:

$$
m_{\gamma_{1}, \gamma_{2}} = |\gamma_{1} A_{1} \mathfrak{R}(\hat{z}_{1}) - \mathfrak{R}(x_{1})| + j\gamma_{1} A_{1} \mathfrak{I}(\hat{z}_{1}) - \mathfrak{I}(x_{1})| + j\gamma_{1} A_{1} \mathfrak{I}(\hat{z}_{1}) - \mathfrak{I}(x_{1})|,
$$

(14f)
where $x^{1}_{q_1}, x^{2}_{q_1}$ are from the set $\Omega^x_{q_1}(x^{1}_{q_2})$ of cardinality $l_1$ and $x^{2}_{q_2}$, $x^{2}_{q_2}$ are from the set $\Omega^x_{q_2}(x^{2}_{q_2})$ of cardinality $l_2$.

Step 4: Select the $N_1, N_2 < l_1, l_2$, smallest elements from $m_{q_1, l_1}$ and $m_{q_2, l_2}$ corresponding to the most likely estimations and perform ML detection:

$$\left(x^{1}_{l_1}, ..., x^{2}_{l_2}\right) = \arg\min_{\tilde{q}_1(N_1), \tilde{q}_2(N_1)} \sum_{l=1}^{4} \left| yA^{2}_{x^{l}} \right|^2 - 2\sqrt{yA^{2}_{x^{l}}} \Re\{y^{2}_{x^{l}}\},$$

(14h)

where $\tilde{q}_1(N_1)$ represents the symbol pairs corresponding to the $N_1$ smallest elements in $m_{q_1, l_1}$ and $\tilde{q}_2(N_1)$ represents the symbol pairs corresponding to the $N_1$ smallest elements in $m_{q_2, l_2}$.

### 2.4 Detection computational complexity

It is evident that the ML detector complexity for Scheme 1 is $O(M^2)$, while for Scheme 2 is $O(M^4)$. In the case of the low-complexity detectors, the respective complexities are $O(7)$ and $O(N^2 + 7)$. In this sub-section, the computational complexity of the proposed schemes is formulated in terms of the number of real operations (multiplications and additions) required during detection. We consider that a complex multiplication requires only three real multiplications and five real additions [17].

#### 2.4.1 Scheme 1—ML detector

The complexity is imposed by the computation of (2b). We assume the computation of $|x^{l}_q|^2$, $\ell \in [1 : 2]$, $q \in [1 : M^2]$ can be previously performed and stored. The computation of $A^{2}_{x^{l}_q}$ requires $2M^2 + 2$ real multiplications. The computation of $\Re\{y^{2}_{x^{l}_q}\}$ only requires the real part, then $6M^2$ real operations are needed. The remaining computations require 5 real additions. Hence, the overall complexity is given as $C_{\text{scheme 1}} = 8M^2 + 7$.

#### 2.4.2 Scheme 1—Low-complexity detector

The computation of Step 1 requires two real operations. Step 2 requires six real operations. In Step 3, the complexity is dependent on the average cardinality of $\Omega^x_{q_1}(x^{1}_{q_1})$, which we denote as $\bar{l}$. Based on the complexity imposed by the ML detector, the complexity of Step 4 imposes $8\bar{l} + 7$ real operations. Hence, the overall complexity is given by $C_{\text{scheme 1}} = 8\bar{l} + 15$.

It is immediately evident that since $\bar{l} \ll M^2$ the complexity imposed by the low-complexity detector is drastically reduced compared to the ML detector.

#### 2.4.3 Scheme 2—ML detector

The complexity is imposed by the computation of (5b). As earlier, $|x^{l}_q|^2$, $\ell \in [1 : 4]$ can be computed previously and stored. The computation of $A^{2}_{x^{l}_q}$ requires $4M^4 + 4$ real multiplications. The computation of $\Re\{y^{2}_{x^{l}_q}\}$ imposes 12 real operations, while the remaining computations require 11 real operations. Hence, the overall complexity is given as $C_{\text{scheme 2}} = 4M^4 + 27$.

#### 2.4.4 Scheme 2—Low-complexity detector

The computation of Step 1 requires 4 real operations. Step 2 requires 16 real operations. In Step 3, each term in (14f) and (14g) requires seven real operations. Denoting the average cardinalities of $\Omega^x_{q_1}(x^{1}_{q_1})$ and $\Omega^x_{q_2}(x^{1}_{q_2})$ as $\bar{l}$, the complexity imposed by Step 3 is $28\bar{l}$. In Step 4, similar to the ML detector, we may verify that the complexity imposed is $4N^2 + 27$. Hence, the overall number of real operations is given by $C_{\text{scheme 2}} = 4N^2 + 28\bar{l} + 47$.

Given that $N^2 + 7 \ll M^4$, this leads to a substantial reduction in computational complexity compared to the ML detector.

In the next section, we numerically compare the complexities for the proposed schemes.

### 3 NUMERICAL RESULTS

In this section, we present the numerical results for the proposed schemes. For error performance comparisons, the chosen figure-of-merit is the average bit error rate (BER) versus average normalised SNR. Comparisons are drawn at a BER of $10^{-5}$. We consider a Rayleigh frequency-flat fading channel, which changes from one time slot to the next and the presence of AWGN at the receiver. Results are demonstrated for both the non-AP IRS and the AP IRS. We consider $N = 2, 4, 8, 16, 32$ and 64 in the majority of investigations. The $M$-QAM modulation scheme is employed with $M = 16$ and 64. The corresponding spectral efficiencies given by $\frac{M}{2}$ are 4 b/s/Hz and 6 b/s/Hz. The simulation of Schemes 1 and 2 always assume the low-complexity detectors are employed at the receiver. In the case of detection for Scheme 1 and Scheme 2, we employ the threshold $\delta = 28.8$. This yields the optimised error performance result at all values of $N$. For Scheme 2, we set $N_1 = 4$ (refer Step 4 of Section 2.3.2).

#### 3.1 Validation of the theoretical ABEP

In the first investigation, we aim to validate the theoretical ABEP; hence, we draw comparison between the error
performances from simulation and the numerical evaluation of the ABEP given by (11) and (12).

Figure 2 presents the results for Scheme 1 in non-AP and AP settings with $M = 16$, while Figure 3 demonstrates the results for Scheme 2 for the same settings. For the non-AP Schemes 1 and 2, it is evident that the simulation matches the theory very well at moderate-to-high SNRs for $N = 16$, $N = 32$ and $N = 64$. For the smaller values of $N$, it is evident that the theoretical
expression fails. This is expected, since the analysis assumes a large value of $N$, such that the central limit theorem may be employed. For the AP Scheme 1 and Scheme 2, it is evident that the simulation results match the theory very closely at moderate-to-high SNRs for $N = 8$, $N = 16$, $N = 32$ and $N = 64$. For $N = 2$ and $N = 4$, the theory fails. Once again this is expected.

In Figures 4 and 5, the corresponding results are demonstrated for $M = 64$. Similar behaviour is demonstrated. For the
non-AP Schemes 1 and 2, the theory is validated by simulation at moderate-to-high SNRs for \( N = 16, N = 32 \) and 64, while for AP Schemes 1 and 2, the theory is validated at moderate-to-high SNRs for \( N = 8, N = 16, N = 32 \) and 64.

### 3.2 Benchmark comparison of error performance

In Figures 6–9, we draw comparison between the error performance of the proposed schemes and the schemes of [13]. However, we consider \( N_R = 1 \) for the schemes in [13], hence we refer to the schemes as GC-SISO and CI-GC-SISO. In addition, we draw comparison with the corresponding non-AP [10] or AP [10] IRS scheme for several values of \( N \). We also include the curves for SSD [14], SISO media-based modulation (SISO-MBM) [18], space-shift keying media-based modulation (SSK-MBM) [18] and the AWGN channel to show the goodness of the results from an “ideal” wireless channel perspective. For the MBM schemes, \( M_{rf} \) is the number of radio frequency mirrors. We consider \( M = 16 \) and \( M = 64 \). Note that the error performance using the low-complexity detectors for GC-SISO and CI-GC-SISO includes an error floor at higher SNRs for \( N_R = 1 \); hence, for \( M = 16 \), we have employed the corresponding ML detectors and in the case of \( M = 64 \), due to very long simulation time, we have employed the numerical evaluation of the theoretical ABEPs presented in [13].

Figures 6 and 7 present the results for non-AP and AP IRSs with \( M = 16 \). Substantial SNR gains are evident for the proposed schemes. For example, considering Figure 6, with \( N = 2, 4, 8, 16, 32 \) and 64, SNR gains of 9, 21, 30, 45 and 51 dB are evident when comparing Scheme 1 and GC-SISO. When comparing Scheme 2 and CI-GC-SISO, for the same values of \( N \), SNR gains of 10, 20, 35, 42 and 48 dB are evident. In comparison to the IRS scheme, with \( N = 2, 4, 8, 16, 32 \) and 64, SNR gains of 10, 4.5, 2.6, 1.4, 0.7 and 0.5 dB are yielded for Scheme 1 and SNR gains of 14, 6.6, 3.6, 2.2, 1.3 and 0.7 dB are yielded for Scheme 2. It is evident that at larger values of \( N \), the SNR gains yielded by Schemes 1 and 2 over the IRS schemes becomes marginal.

This is due to the error performance being dominated by the IRS size for large \( N \) (refer (10b)). Hence, the most benefit from Schemes 1 and 2 when compared to the IRS schemes, is attained for small to medium values of \( N \). As expected, the curve for SSD matches that of GC-SISO and the MBM schemes, which exploit channel modulation are significantly inferior to all other systems. The AWGN error performance is shown to be superior to the GC-SISO, CI-GC-SISO and IRS schemes for \( N = 2 \) but becomes inferior with an increase in \( N \). In Figure 7, similar behaviour is evident; however, for \( N = 2 \), Scheme 2 which was inferior to the AWGN curve earlier, now exhibits a 1.5 dB SNR gain.

Figures 8 and 9 present the error performance comparisons for \( M = 64 \) non-AP and AP. The behaviour is similar to that discussed for Figures 6 and 7. For example, considering Figure 8, with \( N = 2, 4, 8, 16, 32 \) and 64, SNR gains of 10, 23, 32, 39, 46 and 52 dB are evident when comparing Scheme 1 and GC-SISO. When comparing Scheme 2 and CI-GC-SISO, for the same values of \( N \), SNR gains of 9.3, 22, 30, 37, 43 and 49 dB are evident. In comparison to the IRS scheme, with \( N = 2, \)
4, 8, 16, 32 and 64, SNR gains of 7.7, 4, 2.5, 1.2, 0.7 and 0.3 dB are yielded for Scheme 1 and SNR gains of 10, 5.93, 3.6, 2, 1.1 and 0.54 dB are yielded for Scheme 2. The SSD, MBM and AWGN curves all demonstrate similar behaviours as in the case of $M = 16$. In Figure 9, similar behaviour as shown in Figure 8 is evident.

In terms of implementation complexity, compared to GC-SISO and CI-GC-SISO, the proposed schemes require the
deployment of $N$-element non-AP or AP IRSs. Note, even with $N = 2$, substantial SNR gains were evident. Furthermore, these IRSs are essentially passive and do not require additional hardware, for example, converters and amplifiers [2, 3, 6]. The implementation complexity of the proposed schemes is similar to that of the IRS schemes. However, with a smaller number of IRS elements in the proposed schemes, equivalent error performance could be achieved, thus reducing implementation complexity.

3.3 Comparison of non-AP and AP IRSs

In this subsection, comparison is drawn between the error performance of the proposed schemes for the non-AP and AP IRSs. Results are demonstrated for $M = 16$ and $M = 64$ in Figures 10 and 11, respectively. In all instances, as expected, the AP IRS outperforms the corresponding non-AP IRS. For example (consider $M = 16$), in the case of Scheme 1, for increasing values of $N$, SNR gains of 6.1, 4, 2.2, 1.7 and 1.3 dB are yielded by the AP IRS, while for Scheme 2, SNR gains of 5.6, 2.9, 2.3, 1.9, 1.5 and 1.2 dB are evident. Considering $M = 64$, in the case of Scheme 1, for increasing values of $N$, SNR gains of 6.1, 3.8, 2.7, 2, 1.6 and 1.24 dB are yielded by the AP IRS, while for Scheme 2, SNR gains of 6.2, 3.4, 2, 1.7, 1.6 and 1.2 dB are evident.

3.4 Comparison of Schemes 1 and 2

Finally, we compare Schemes 1 and 2 (refer Figures 10 and 11). As expected, we see that Scheme 2 outperforms Scheme 1 for small and medium values of $N$. For example, with $M = 16$ and increasing values of $N$, SNR gains of 4.6, 2.3, 1.2, 0.8, 0.4 and 0.2 dB are evident for the non-AP IRS, while SNR gains of 3.75, 1, 0.6, 0.4, 0.2 and 0.1 dB are evident for the AP IRS. For $M = 64$ and increasing values of $N$, SNR gains of 2.1, 2, 1.3, 1, 0.6 and 0.2 dB are evident for non-AP, while SNR gains of 2.3, 1.7, 0.9, 0.4, 0.2 and $\approx 0$ dB are evident for the AP IRS. It is clear that Scheme 2 achieves a larger gain over Scheme 1 for the non-AP IRS.

3.5 Impact of channel estimation error

In this subsection, we consider the impact of channel estimation error on the error performance of the proposed schemes. Since AP IRSs have superior error performance over non-AP IRSs, we demonstrate the results only for the former. Further, we focus only on the small and medium IRS sizes, that is, $N = 2, 4, 8$ and 16. For the IRS schemes, channel information is required at the IRS for phase adjustment and at the receiver for detection. This is a greater requirement when compared to conventional schemes. The estimated channel may be expressed as: $\hat{g}_i = g_i + e$, where $e$ represents the estimation error, and is a RV distributed according to $\mathcal{CN}(0, \sigma_e^2)$, with $\sigma_e^2$ the corresponding variance. We assume $\sigma_e^2$ and $e$ are mutually independent, such that $\hat{g}_i$ is distributed according to $\mathcal{CN}(0, 1 + \sigma_e^2)$. We assume a 10% error at both the IRS and receiver, hence $\sigma_e^2 = 0.01$. The results are also demonstrated for GC-SISO, CI-GC-SISO and the corresponding AP IRS scheme.

Figure 12 presents the results for $M = 16$, while the results for $M = 64$ are demonstrated in Figure 13. For both $M = 16$ and $M = 64$, Schemes 1 and 2 are more robust compared to the
AP IRS and SISO schemes. In the case of $M = 16$, error floors are easily visible for $N = 2$ and $N = 4$ (IRS) and with $N = 4$ Scheme 1 incurs an SNR penalty of approximately 6 dB, while Scheme 2 incurs approximately 3 dB. For $M = 64$, the error floors for $N = 2$, $N = 4$ and $N = 8$ are easily visible. It is evident that the impact of channel estimation error increases as $M$ increases. At $M = 64$, CI-GC-SISO is slightly worse than GC-SISO and Scheme 2 shows a marginal performance improvement over Scheme 1 for $N = 2$. For both settings of $M$, it is evident that as the IRS size increases the robustness improves.
and in Figure 12, less than 1 dB penalty is evident for Schemes 1 and 2 with \( N = 16 \), while in Figure 13, less than 3 dB penalty is evident. The investigation demonstrates the importance of effective channel estimation techniques for IRS-based schemes and even more so for small and medium IRSs.

### 3.6 Comparison of detection computational complexity

In Table 1, the computational complexity in terms of the number of real operations required by the detectors is numerically
TABLE 1 Comparison of computational complexity (in terms of the required number of real operations) for ML and low-complexity detectors for Schemes 1 and 2 with $M = 16, M = 64$

| Detector                  | Scheme 1 $M = 16$ | Scheme 1 $M = 64$ | Scheme 2 $M = 16$ | Scheme 2 $M = 64$ |
|---------------------------|-------------------|-------------------|-------------------|-------------------|
| ML                        | 2055              | 32,775            | 262,171           | 67,108,891        |
| Low-complexity detection  | 375               | 703               | 1371              | 2519              |
| % Reduction               | 81.8              | 97.9              | 99.5              | $\approx$100     |

represented for $M = 16$ and $M = 64$. For Schemes 1 and 2, employing $\delta = 28.8$, with $M = 16$, $7 = 45$ and with $M = 64$, $7 = 86$.

The percentage reduction in computational complexity offered by the low-complexity detectors compared to their ML counterparts is shown. It is evident that a substantial reduction is achieved. For example, for Scheme 1, reductions of 81.8% and 97.9% are evident for $M = 16$ and $M = 64$, respectively. In the case of Scheme 2, reductions of 99.5% and $\approx$100% are evident for $M = 16$ and $M = 64$, respectively.

Furthermore, it is evident that Scheme 1 exhibits a much lower complexity compared to Scheme 2. For ML detection, 99–100% reduction is evident, while for low-complexity detection, a 72–73% reduction is yielded. Hence, a trade-off between the SNR gain achieved by Scheme 2 over Scheme 1 and the computational complexity exists.

4 Conclusion

In this paper, we have investigated two IRS-assisted SISO Golden codeword-based modulation schemes. Compared to their counterparts, the proposed schemes were shown to exhibit superior error performances. The second scheme was superior to the first scheme, especially for smaller IRSs. The theoretical ABEPs for the schemes were derived and validated by simulation results. The associated low-complexity detectors for the schemes were presented. The complexity analysis showed that the schemes imposed an extremely low-complexity compared to their corresponding ML detectors and that a trade-off between SNR gain and complexity exists. Possible future work includes: (A) Derivation of an exact ABEP expression for the non-AP and AP IRS-based systems, which holds even for small surface sizes, (B) formulation and analysis of the corresponding lower bounds for the IRS-based systems based on the BER contribution corresponding to the dominating minimum distance, and (C) investigating the proposed schemes with multiple antennas in conjunction with methods for low-overhead channel estimation.

REFERENCES

1. Kain, N., et al.: Shaping complex microwave fields in reverberating media with binary tunable metasurfaces. Sci. Rep. 4(6693), 1–7 (2014)
2. Liang, Y.-C., et al.: Large intelligent surface/antennas (LISA): Making reflective radios smart (2019). J. Commun. Inf. Netw. 4(2), 40–50 (2019)
3. Yuan, X., et al.: Reconfigurable-intelligent-surface empowered 6G wireless communications: Challenges and opportunities. https://arxiv.org/pdf/2001.00364
4. Hu, S., et al.: Capacity degradation with modeling hardware impairment in large intelligent surface. In: Proceedings of IEEE Global Communications Conference (GLOBECOM), pp. 1–6. IEEE, Piscataway (2018)
5. Tatarkia, H., et al.: Real-time implementation aspects of large intelligent surfaces. https://arxiv.org/pdf/2003.01672
6. Zhao, J., Liu, Y.: A survey of intelligent reflecting surfaces (IRSs): Towards 6G wireless communication networks. https://arxiv.org/abs/1907.04789
7. Han, Y., et al.: Large intelligent surface-assisted wireless communication exploiting statistical CSI. IEEE Trans. Veh. Technol. 68(8), 8238–8242 (2019)
8. Jiang, T., Shi, Y.: Over-the-air computation via intelligent reflecting surfaces. In: 2019 IEEE Global Communications Conference (GLOBECOM), pp. 1–6. IEEE, Piscataway (2019)
9. Abeywickrama, S., et al.: Intelligent reflecting surface: Practical phase shift model and beamforming optimization. IEEE Trans. Commun. 68(9), 5849–5863 (2020)
10. Basar, E.: Transmission through large intelligent surfaces: A new frontier in wireless communications. In: 2019 European Conference on Netwaroks and Communications. IEEE, Piscataway (2019)
11. Basar, E.: Reconfigurable intelligent surface-based index modulation: A new beyond MIMO paradigm for 6G. IEEE Trans. Commun. 68(5), 3187–3196 (2019)
12. Belfiore, J.-C., et al.: The Golden code: A $2 \times 2$ full-rate space-time code with non-vanishing determinants. IEEE Trans. Inf. Theory 51(4), 1432–1436 (2005)
13. Xu, H., Pillay, N.: Golden codeword-based modulation schemes for single-input multiple-output systems. Int. J. Commun. Syst. 32(10), e3963 (2019)
14. Boutros, J., Viterbo, E.: Signal space diversity: A power and bandwidth efficient diversity technique for the Rayleigh fading channel. IEEE Trans. Inf. Theory 44(4), 1453–1467 (1998)
15. Fouad, A., et al.: Interference management in UAV-assisted integrated access and backhaul cellular networks. IEEE Access 7, 104553–104566 (2019)
16. Proakis, J.G., Salehi, M.: Digital Communications, 5th ed. McGraw-Hill, New York (2008)
17. Winograd, S.: On the number of multiplications necessary to compute certain functions. Commun. Pure Appl. Math. 23(2), 165–179 (1970)
18. Basar, E.: Media-based modulation for future wireless systems: A tutorial. IEEE Wirel. Commun. 26(5), 160–166 (2019)

How to cite this article: Pillay N, Xu H. Intelligent reflecting surface-assisted single-input single-output Golden codeword-based modulation schemes. *IET Commun.* 2021;15:78–92. https://doi.org/10.1049/cmu2.12060