Coherent Control of Superfluidity in a Fermi Gas of Atoms

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We theoretically examine photoassociation of a two-component Fermi degenerate gas, focusing on light-induced atom-atom interactions as a means to raise the critical temperature of the BCS transition to a superfluid state. As it stands, photoassociation-induced superfluidity is limited by spontaneous decay to experimentally inconvenient light intensities [Mackie et al., Opt. Express 8, 118 (2000)]. We therefore propose to use coherent control in photoassociation to a pair of molecular levels to cancel spontaneous emission, whereby the BCS transition should occur within reach of current experimental techniques.

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Studies of degeneracy in Fermi gases[1-4] presently face bottlenecks in reaching temperatures cold enough to form Cooper pairs. Below the Fermi temperature, evaporative cooling of a dual Fermi gas begins to stall as the near-unit-occupied low energy states exhibit Pauli blocking[3], while in a mixture of Fermi and Bose gases the sympathetic cooling provided by ultracold bosons loses efficiency when the bosonic heat capacity falls below its fermionic counterpart[4]. The lowest achieved temperatures are thus about a third of the Fermi temperature[3,4], whereas Cooper pair formation requires a temperature of at least an order of magnitude lower[5,6]. However, by adjusting atom-atom interactions, it could theoretically examine photoassociation of a two-component Fermi degenerate gas, focusing on light-induced atom-atom interactions as a means to raise the critical temperature of the BCS transition to a superfluid state. As it stands, photoassociation-induced superfluidity is limited by spontaneous decay to experimentally inconvenient light intensities [Mackie et al., Opt. Express 8, 118 (2000)]. We therefore propose to use coherent control in photoassociation to a pair of molecular levels to cancel spontaneous emission, whereby the BCS transition should occur within reach of current experimental techniques.

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The Hamiltonian density for the atom-molecule system described above is

\[ \hat{H}^{-1} = \sum_i \left[ \hat{\phi}_i^\dagger \left( -\frac{\hbar \nabla^2}{2m} + \frac{\hbar}{4m} + \hat{b}_i \right) \hat{\psi}_i \right] - \sum_i \kappa_i(\mathbf{r}) \left( \hat{\psi}_i^\dagger \hat{\phi}_1 + \hat{\psi}_i \right) \]

\[ -\frac{1}{2} \gamma \sum_{i,i'} \hat{\psi}_i^\dagger \hat{\psi}_{i'} + \lambda \hat{\phi}_2^\dagger \hat{\phi}_1 \hat{\phi}_2, \]  

(1)

The detuning of the photoassociating laser from the respective vibrational levels is \( \delta_l = \omega_{l - \omega_L - \Delta_l} \), where the binding energy of the \( l \)th molecular state is \( \hbar \Delta_l \), the energy of the photon is \( \hbar \omega_L \), and \( \hbar \omega_{l - \omega_L - \Delta_l} \) is the asymptotic energy difference between the two electronic manifolds (\( l = 1, 2 \)). A low-momentum approximation is implicit, whereby relevant atom-atom collisions are described by a contact interaction of strength \( \lambda = 4\pi \hbar a/m \), with a
being the s-wave scattering length. Similarly, correcting the bosonic result with a statistical factor of $\sqrt{2}$, the (real) free-bound contact interaction strength $\kappa_l$ is given as

$$\kappa_l(r) = \frac{\epsilon_R \lambda^{3/2}}{\sqrt{2}} \left[ I(r) / I_0 l_1 \right]^{1/2}.$$  \hfill (2)

Here $\epsilon_R = \hbar / 2m \lambda^2$ is the usual photon recoil frequency, $2\pi \lambda$ is the wavelength of the photoassociating light, and $I(r)$ is the prevailing light intensity at the position $r$. Finally, if the photoassociating rate coefficient $a_l$ is known (in cm$^{-5}$) at a temperature $T$ and detuning $\delta$, the characteristic intensity $I_0 l_1$ is given in W/cm$^2$ as

$$I_0 l_1 = \frac{\sqrt{\pi} \sqrt{\delta \epsilon} \hbar^4}{2 \lambda \omega^2 m^2 (k_B T)^{3/2} \lambda^2} e^{-\hbar \delta/k_BT}.$$  \hfill (3)

Assuming that $|\delta|_1$ is the largest frequency scale in the Heisenberg equations of motion, we solve adiabatically for the molecular fields $\psi_l$. Substituting the result into Eq. (2) and keeping also the leading order of the imaginary part in the energy, we obtain an effective Hamiltonian density involving only fermions

$$h^{-1} H_{\text{eff}} = \sum_l \phi_l^\dagger \left( -\frac{\hbar \nabla^2}{2m} \right) \phi_l + \lambda_{\text{eff}} \phi_l \phi^\dagger_l \phi_1 \phi_2.$$  \hfill (4)

The influence of photoassociating light on atom-atom interactions is now evident in the effective collisional interaction strength

$$\lambda_{\text{eff}} = \lambda - \left( \frac{\kappa_1^2 + \kappa_2^2}{\delta_1 + \delta_2} \right) - \frac{1}{2} i \gamma \left( \frac{\kappa_1}{\delta_1} + \frac{\kappa_2}{\delta_2} \right)^2.$$  \hfill (5)

From the imaginary term in Eq. (6), it is clear that spontaneous decay of (virtual) excited molecules will limit the lifetime of the superfluid state through inelastic atom-atom scattering events. Combining Eqs. (3) and (5) gives the radiative Cooper pair lifetime as

$$\epsilon_R \tau = \frac{2}{\rho \lambda^2 \gamma R R_\text{l} (1 + R)^2},$$  \hfill (6)

where the $R = (\delta_1/\delta_2) \sqrt{I_0 l_2 / I_0 l_1}$ and $(\rho/2)^2$ was used for the dual-atom density term $\phi_l \phi^\dagger_l \phi_1 \phi_2$. We are evidently free to adjust the lifetime according to the ratio $R$ and, in particular, $\tau = \infty$ is achieved by choosing $R = -1$. For $\delta_2 = \delta_1 + \omega_2$, where $\hbar \omega_2 = \hbar (\Delta_1 - \Delta_2) > 0$ is the separation in energy between the molecular levels, and a characteristic intensity that scales with binding energy as $I_0 l_2 / I_0 l_1 = \sqrt{\Delta_2 / \Delta_1}$, we find $\delta_1 = -\omega_2 / (1 + \sqrt{\Delta_1 / \Delta_2}) \approx -\omega_2 / (1 + \omega_2) - 1 / \omega_2$, i.e., the photoassociating laser should be tuned roughly halfway between the molecular levels. By exerting coherent control over photoassociation, not only is spontaneous decay hereafter a non-issue, but the detuning and light intensity will assume reasonable values.

Turning to the sought-after increase in the BCS transition temperature, we ignore the native scattering length $a$ on the assumption that the associated collisional interaction alone is too weak for experimental utility. The atom-atom interactions are now due solely to the light-induced scattering length

$$\frac{a_L}{\lambda} = \frac{1}{8\pi} \frac{\epsilon_R}{\delta} \frac{I}{I_0 l_1} \left( 1 + \sqrt{\frac{\Delta_1 - \Delta_2}{\Delta_2}} R \right).$$  \hfill (7)

Canceling spontaneous decay as above yields $(1 + \sqrt{\Delta_1 / \Delta_2} R) / \delta_1 > 0$, and therefore the attractive interaction necessary for Cooper pairing. Having assumed the detuning is large enough to allow for adiabatic elimination of the molecular field, the rigorous Fermi-Bose thermodynamics will reduce to the usual BCS theory; hence, the critical temperature for the superfluid transition is approximately $T_c = T_F \exp[-i/\sqrt{\lambda/k_B \epsilon_L}]$, where $T_F = \hbar^2 k_B^2 / 2 m k_B$ and $k_B = (3\pi^2 \rho)^{1/3}$ are the Fermi temperature and wave number, respectively. Substituting Eq. (6) along with $R = -1$ gives

$$\frac{T_c}{T_F} = \exp \left[ -\frac{18.1329 |\delta|_1 I_0 l_1}{(\rho \lambda^2)^{1/3} \epsilon_R} \frac{I_0 l_1}{I} \left( 1 - \sqrt{\frac{\Delta_1 - \Delta_2}{\Delta_2}} \right)^{-1} \right].$$  \hfill (8)

Degeneracy has been observed for $^{40}$K [4] and $^{6}$Li [12], but the spectroscopic data are more plentiful for $^{6}$Li and we discuss explicit numbers for this species only. In our example, we use the vibrational states $\nu = 79$ and $\nu = 80$, with respective dissociation energies $\Delta_1 = 31.58$ and $\Delta_2 = 22.61 \times 2 \pi$ GHz, which leads to the detuning $|\delta|_1 = 4.3 \times 2 \pi$ GHz. The fixed parameters have the following values: the wavelength is $2 \pi \lambda = 671$ nm; the photon recoil frequency is $\epsilon_R = 63.3 \times 2 \pi$ MHz; the density is assumed such that $\rho \lambda^2 = 1$; and, finally, the characteristic intensity for the $\nu = 79$ state is $9.8$ mW/cm$^2$. Hence, the estimated intensity required to make $T_c/T_F = 0.1$ is $I = 30$ kW/cm$^2$, which is marginal at best.

To make further progress, we must appreciate the physics behind the canceling of spontaneous emission. Let us regard the transitions from an atom pair to molecular levels 1 and 2 as charged harmonic oscillators excited by light. The amplitudes of the oscillators may be adjusted by tuning the laser frequency. When the light is tuned between the two transition frequencies, it also drives one of the oscillators below resonance and the other one above resonance. The oscillations, and the radiation from the two oscillators, therefore have opposite phases. By a suitable choice of the amplitudes (i.e., light frequency) the spontaneous emission cancels. Analogous cancellations are of course well known in quantum-optical few level atomic systems [21,22]. Although a molecular system has many more degrees of freedom, the proliferation of rovibrational states— and even dissociation continua— does not in principle invalidate our scheme. Despite a large number of oscillators, there still exists a
driving frequency that results in cancellations of the radiation [22]. As the oscillators closest to resonance have the largest amplitudes, such a frequency should be close to the value deduced for the oscillators nearest to resonance.

Meanwhile, the change in the scattering length reflects light shifts of the molecular levels. In the model with two molecular levels of Eq. (5), the two light shift terms proportional to $1/\delta l$ work against each other. At the point of vanishing spontaneous emission there is a nonzero change in the scattering length, and of the right sign for Cooper pairing, which is the usual case in a molecule. Among the further off-resonance states, we expect the ones above resonance to dominate. The resulting added change in the scattering length then has the right sign for Cooper pairing, and may be substantial since the light shift from a level decreases only inversely proportional to the detuning.

With these considerations in mind, the multitude of states in a molecule actually helps our cause. For instance, if we were to take into account in our argument four additional states below the state $\nu = 79$ and four states above the state $\nu = 80$ (see Ref. [23] for the dissociation energies), and assume the $\sqrt{\Delta}$ scaling of the characteristic intensities, we find that the detuning needed to cancel spontaneous emission becomes $|\Delta_1| = 2.8 \times 2\pi$ GHz, and the intensity to make $T_l/T_F = 0.1$ is now $I = 4$ kW/cm$^2$. This value is indeed an experimentally feasible.

A truly quantitative prediction of the intensity required for the BCS transition amounts to a calculation of an off-resonance light shift, a notoriously difficult assignment. In this task experimental trial and error may be an easier way to proceed. Should the dipole matrix elements turn out to be uncooperatively small, one may also envisage a two-color scheme where the primarily photoassociated molecules are coupled with second laser to another molecular state. Cancellation of spontaneous decay occurs (to lowest nontrivial order) as in the one-color case, and the added shifts from off-resonant two-photon transitions will give a handle to control atom loss and atom-atom interactions more independently than is possible with just one laser.

All told, by allowing for the cancellation of spontaneous decay for reasonable values of detuning and laser intensity, coherently controlled photoassociation should provide a means for creating a superfluid state in a Fermi gas of atoms that is well within the reach of current experimental techniques.

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[1] B. DeMarco and D. S. Jin, Science 285, 1703 (1999).
[2] F. Schreck, G. Ferrari, K. L. Corwin, J. Cubizolles, L. Khaykovich, M.-O. Mewes, and C. Salomon, Phys. Rev. A 64 011402 (R) (2001).
[3] M. J. Holland, B. DeMarco, and D. S. Jin, Phys. Rev. A 61, 053610 (2000).
[4] A. G. Truscott, K. E. Strecker, W. I. McAlexander, G. B. Partridge, and R. G. Hulet, Science 291, 2570 (2001).
[5] H. T. C. Stoof, M. Houbiers, C. A. Sackett, and R. G. Hulet, Phys. Rev. Lett. 76, 10 (1996).
[6] M. Houbiers, H. T. C. Stoof, R. Ferwerda, W. I. McAlexander, C. A. Sackett, and R. G. Hulet, Phys. Rev. A 56, 4864 (1997).
[7] E. Tiesinga, A. J. Moerdijk, B. J. Verhaar, and H. T. C. Stoof, Phys. Rev. A 46, R1167 (1993).
[8] J. M. Vogels, C. C. Tsai, R. S. Freeland, S. J. J. M. F. Kokkelmans, B. J. Verhaar, and D. J. Heinzen, Phys. Rev. A 56, R1067 (1997).
[9] S. L. Cornish, N. R. Claussen, J. L. Roberts, E. A. Cornell, and C. E. Wieman, Phys. Rev. Lett. 85, 1795 (2000).
[10] A. J. Moerdijk, B. J. Verhaar, and T. M. Nagtegaal, Phys. Rev. A 53, 4343 (1996).
[11] M. Marinescu and L. You, Phys. Rev. Lett. 81, 4596 (1998).
[12] P. O. Fedichev, Yu. Kagen, G. V. Shlyapnikov, and J. T. M. Walraven, Phys. Rev. Lett. 77, 2913 (1996).
[13] J. L. Bohn and P. S. Julienne, Phys. Rev. A 56, 1486 (1997).
[14] M. Kostrun, M. Mackie, R. Cote, and J. Javanainen, Phys. Rev. A 62, 063616 (2000).
[15] E. Timmermans, K. Furuya, P. W. Milonni and A. K. Kerman, LANL e-print cond-mat/0103327.
[16] M. Mackie, E. Timmermans, R. Cote, and J. Javanainen, Opt. Express 8, 118 (2000).
[17] J. L. Bohn, Phys. Rev. A 61, 053409 (2000).
[18] M. Holland, S. J. J. M. F. Kokkelmans, M. Chiofalo, and R. Walser, LANL e-print cond-mat/0103473.
[19] R. Friedberg and T. D. Lee, Phys. Rev. B 40, 6745 (1989).
[20] S.-Y. Zhu and M. O. Scully, Phys. Rev. Lett. 76, 388 (1996).
[21] E. Paspalakis and P. L. Knight, Phys. Rev. Lett. 81, 293 (1998).
[22] Normally, this kind of many-level cancellations of spontaneous-emission fail on angular momenta: Radiation from two oscillators that have different polarizations (say, one going in the x direction and the other in the y direction) cannot cancel. However, to zeroth order, vibrational level structure makes replicas of the same electronic transitions separated by vibrational level spacings, and the cancellation should indeed work.
[23] E. R. I. Abraham, N. W. M. Ritchie, W. I. McAlexander, and R. G. Hulet, J. Chem. Phys. 103, 7773 (1995).