Determination of $|V_{ub}|$ from exclusive baryonic $B$ decays

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Abstract

We use the exclusive baryonic $B$ decays to determine the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{ub}$. From the relation $|V_{ub}|^2/|V_{cb}|^2 = (B_\pi/B_D)R_{ff}$ based on $B^- \to p\bar{p}\pi^-$ and $\bar{B}^0 \to p\bar{p}D^0$ decays, where $|V_{cb}|$ and $B_\pi/B_D \equiv B(B^- \to p\bar{p}\pi^-)/B(\bar{B}^0 \to p\bar{p}D^0)$ are the data input parameters, while $R_{ff}$ is the one fixed by the $B \to p\bar{p}$ transition matrix elements, we find $|V_{ub}| = (3.48^{+0.87}_{-0.63} \pm 0.40 \pm 0.07) \times 10^{-3}$ with the errors corresponding to the uncertainties from $R_{ff}$, $B_\pi/B_D$ and $|V_{cb}|$, respectively. Being independent of the previous results, our determination of $|V_{ub}|$ has the central value close to those from the exclusive $\bar{B} \to \pi\ell\bar{\nu}_\ell$ and $\Lambda_b \to p\mu^-\bar{\nu}_\mu$ decays, but overlaps the one from the inclusive $\bar{B} \to X_u\ell\nu_\ell$ with the current uncertainties. The extraction of $|V_{ub}|$ in the baryonic $B$ decays is clearly very useful for the complete determination of the CKM matrix elements as well as the exploration of new physics.
I. INTRODUCTION

In the standard model (SM), the unique physical phase in the $3 \times 3$ unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix [1, 2] provides the only source for CP violation. However, it is known that this CP phase is not sufficient to solve the mystery of the matter-antimatter asymmetry in the universe. To test the SM and look for other CP violation mechanisms, many CP violating processes, proceeding through the CKM matrix element $V_{ub} = |V_{ub}|e^{-i\gamma}$ with $\gamma$ the CP phase, have been extensively explored by both experimental and theoretical studies. Nonetheless, with $\gamma$ more precisely analyzed from the present data [3], the $|V_{ub}|$ determination is not conclusive. In particular, the experiments in the inclusive $\bar{B} \to X_u \ell \bar{\nu}_\ell$ and exclusive $\bar{B} \to \pi \ell \bar{\nu}_\ell$ decays give [3]

$$|V_{ub}| = (4.41 \pm 0.15^{+0.15}_{-0.17}) \times 10^{-3},$$  \hspace{1cm} (1)

$$|V_{ub}| = (3.28 \pm 0.29) \times 10^{-3},$$  \hspace{1cm} (2)

respectively, where the first result has $3\sigma$ deviation from the second one. This is the well-known long-standing tension between $V_{ub}$ measured by inclusive and exclusive decays at the $B$-factories, which triggers the theoretical studies in the SM [4, 5].

To resolve the problem, it has been proposed that there exists some new physics, such as the right-handed quark current with the form of $\bar{u}\gamma_\mu(1 + \gamma_5)b$ in $\bar{B} \to X_u \ell \bar{\nu}_\ell$ [6, 7], but not supported by the test of $B \to \rho \ell \bar{\nu}_\ell$ [8]. It is also not sustained by the recent measurement of $|V_{ub}| = (3.27 \pm 0.15 \pm 0.17 \pm 0.06) \times 10^{-3}$ [9] in the exclusive baryonic decay of $\Lambda_b \to p\mu^- \bar{\nu}_\mu$, which contains both contributions from the vector and axial-vector quark currents as $B \to X_u \ell \bar{\nu}_\ell$. Clearly, the resolution of the dual nature of $|V_{ub}|$ in Eqs. (1) and (2) is one of the most important tasks in particle physics and it would lead to physics beyond the SM.

The exclusive baryonic $B$ decays is worthwhile to have its own version for the extraction of $|V_{ub}|$, which can be independent of the previous ones from $\bar{B} \to \pi \ell \bar{\nu}_\ell$ and $\Lambda_b \to p\mu^- \bar{\nu}_\mu$. For $\Lambda_b \to p\mu^- \bar{\nu}_\mu$, the extraction of $|V_{ub}|$ relies on the relation of $|V_{ub}|^2/|V_{cb}|^2 = [B(\Lambda_b \to p\mu^- \bar{\nu}_\mu)/B(\Lambda_b \to \Lambda_c^+ \mu^- \bar{\nu}_\mu)] R_{FF}$ with $R_{FF}$ as the ratio of the $\Lambda_b \to p$ and $\Lambda_b \to \Lambda_c$ transition form factors calculated in the lattice QCD [10]. Likewise, by connecting $B^- \to p\bar{p}\pi^-$ and $\bar{B}^0 \to ppD^0$ decays that proceed through $b \to u\bar{u}d$ and $b \to c\bar{u}d$ at the quark level, respectively, we obtain $|V_{ub}|^2/|V_{cb}|^2 = (B_\pi/B_D) R_{ff}$ with $B_\pi/B_D \equiv B(B^- \to p\bar{p}\pi^-)/B(\bar{B}^0 \to pp\pi^-)$. 


$p\bar{p}D^0$) and $R_{ff}$ the parameter related to the hadronic effects including those from the $B \to p\bar{p}$ transition matrix elements. Note that the momentum dependences of these transition elements have been well studied in the literature \cite{11-14} to explain the threshold effect in the baryonic $B$ decays with fully accounted theoretical uncertainties. On the other hand, the decay of $\bar{B} \to \pi\ell\bar{\nu}_\ell$ can not be isolated from the uncertainty caused by the momentum dependences of the form factors in the $\bar{B} \to \pi$ transition, calculated in different QCD models \cite{17,18}. Besides, since $|V_{ub}|^2/|V_{cb}|^2 = (B_\pi/B_D)R_{ff}$ receives the contributions from both vector and axial vector currents, it can also be used to test new physics in the form of the axial vector current. It is clear that once $R_{ff}$ is obtained in the baryonic $B$ decays, one can determine $|V_{ub}|$ from $|V_{ub}|^2/|V_{cb}|^2 = (B_\pi/B_D)R_{ff}$, which is independent of the previous cases.

II. DATA ANALYSIS

A. Amplitudes

In terms of the quark level effective Hamiltonian for the $b \to c\bar{u}d$ and $b \to u\bar{u}d$ transitions, the amplitudes of the $B^- \to p\bar{p}\pi^-$ and $\bar{B}^0 \to p\bar{p}D^0$ decays can be derived as \cite{13-16}

$$A(B^- \to p\bar{p}\pi^-) \simeq i\frac{G_F}{\sqrt{2}}V_{ub}V_{ud}a_1f_\pi\langle p\bar{p}|\bar{u}(1-\gamma_5)b|B^-\rangle,$$

$$A(\bar{B}^0 \to p\bar{p}D^0) = i\frac{G_F}{\sqrt{2}}V_{cb}V_{ud}a_2f_D\langle p\bar{p}|\bar{d}(1-\gamma_5)b|\bar{B}^0\rangle,$$  \hspace{1cm} (3)

with $G_F$ the Fermi constant, $V_{ij}$ the CKM matrix elements, and $\not{p} = p^\mu\gamma_\mu$, where the decay constants $f_\pi$ and $f_D$ along with the momentum transfer $p^\mu$ come from the matrix elements of $\langle \pi|\bar{u}\gamma^\mu(1-\gamma_5)d|0\rangle = if_\pi p^\mu$ and $\langle D|\bar{c}\gamma^\mu(1-\gamma_5)u|0\rangle = if_Db^\mu$, respectively, and $a_i \equiv c_i^{eff} + c_i^{eff}/N_c$ for $i=$odd (even) are composed of the effective Wilson coefficients $c_i^{eff}$ defined in Refs. \cite{13-16}. The matrix elements for $B \to B\bar{B}'$ transition in Eq. (3) can be parameterized as \cite{13}.

$$\langle B\bar{B}'|\bar{q}\gamma_\mu b|B\rangle = i\bar{u}[g_1\gamma_\mu + g_2i\sigma_{\mu\nu}p^\nu + g_3p_\mu + g_4q_\mu + g_5(p_{B'} - p_B)_\mu]\gamma_5v,$$

$$\langle B\bar{B}'|\bar{q}\gamma_\mu\gamma_5 b|B\rangle = i\bar{u}[f_1\gamma_\mu + f_2i\sigma_{\mu\nu}p^\nu + f_3p_\mu + f_4q_\mu + f_5(p_{B'} - p_B)_\mu]v.$$  \hspace{1cm} (4)
where $g_i(f_i)$ ($i = 1, 2, 3, 4, 5$) are the $B \to \bar{B}B'$ transition form factors, of which the momentum dependences can be written as \[11–14\]

$$f_i(t) = \frac{D_{f_i}}{t^3}, \quad g_i(t) = \frac{D_{g_i}}{t^3}.$$  \(5\)

In terms of the $SU(3)$ flavor and $SU(2)$ spin symmetries, $D_{f_i}$ and $D_{g_i}$ for different $B \to \bar{B}B'$ transitions can be related, given by \[15\]

$$D_{g_i(f_i)} = \frac{5}{3} D_{||} \mp \frac{1}{3} D_{\perp}, \quad D_{g_j(f_j)} = \pm \frac{4}{3} D_{||}^j,$$

$$D_{g_i(f_i)} = \frac{1}{3} D_{||} \mp \frac{2}{3} D_{\perp}, \quad D_{g_j(f_j)} = \pm \frac{1}{3} D_{||}^j,$$  \(6\)

for $\langle p\bar{p}|\bar{u}\gamma_5(\gamma_5)b|B^-\rangle$ and $\langle p\bar{p}|\bar{d}\gamma_\mu(\gamma_5)b|B^0\rangle$, respectively, with the constants $D_{||}^j$ and $D_{||}^j$ ($j = 2, 3, 4, 5$) to be determined by the global fit with all available data of $B^- \to p\bar{p}e^-\bar{\nu}_e$ and $B \to p\bar{p}M_e$ ($M = \pi, K$ and $K^*$ and $M_c = D^{(*)}$).

Subsequently, in terms of the amplitudes in Eq. \[3\], we derive $|V_{ub}|^2/|V_{cb}|^2$ as

$$|V_{ub}|^2/|V_{cb}|^2 = (\mathcal{B}_\pi/\mathcal{B}_D)\mathcal{R}_{ff},$$  \(7\)

where $\mathcal{B}_\pi/\mathcal{B}_D \equiv \mathcal{B}(B^- \to p\bar{p}\pi^-)/\mathcal{B}(\bar{B}^0 \to p\bar{p}D^0)$, and $\mathcal{R}_{ff}$ is given by

$$\mathcal{R}_{ff} = \int \int \frac{|a_2 f_D\bar{d}\bar{\gamma}_5(\gamma_5)b|\bar{\mathcal{B}}^0|^2}{\int \int |a_1 f_\pi\bar{p}\bar{u}\gamma_5(\gamma_5)b|\mathcal{B}^-|^2} dm^2_{pp}dm^2_{\pi\pi},$$  \(8\)

with $m^2_{ij} = (p_i + p_j)^2$, in which the allowed ranges over the phase space can be referred in the PDG \[3\].

**B. The $|V_{ub}|$ extraction**

Since the relation in Eq. \[7\] can be used to extract $|V_{ub}|$, we adopt the data from PDG as the experimental inputs, given by \[3\]

$$(f_D, f_\pi) = (204.6 \pm 5.0, 130.4 \pm 0.2) \text{ MeV},$$

$$\mathcal{B}(B^- \to p\bar{p}\pi^-) = (1.60 \pm 0.18) \times 10^{-6},$$

$$\mathcal{B}(\bar{B}^0 \to p\bar{p}D^0) = (1.04 \pm 0.07) \times 10^{-4},$$  \(9\)

which result in $\mathcal{B}_\pi/\mathcal{B}_D = (1.54 \pm 0.17) \times 10^{-2}$. 
For the theoretical inputs, we use the new extraction for the $B \rightarrow p \bar{p}$ transition form factors, which includes the new observation of $\mathcal{B}(B^{-} \rightarrow p \bar{p} e^{-} \bar{\nu}_{e})$ [19], such that the overestimation in Ref. [20] can be fixed. The results from the global fitting with all available data of $B^{-} \rightarrow p e^{-} \bar{\nu}_{e}$ and $B \rightarrow p \bar{p} M(c)$ ($M = \pi$, $K$ and $K^{*}$ and $M_{c} = D^{(*)}$) are given by $^{[15, 20, 21]}

(D_{\|}, D_{\|}) = (37.1 \pm 68.9, -356.5 \pm 22.2) \text{ GeV}^{5}, \n
(D_{\|}, D_{\|}, D_{\|}, D_{\|}) = (16.6 \pm 30.7, -274.7 \pm 171.9, 4.0 \pm 29.5, 137.8 \pm 37.4) \text{ GeV}^{4}. \quad (10)

The parameter $a_{1}$ for the charmless $B^{-} \rightarrow p \bar{p} \pi^{-}$ decay is given by $a_{1} = c_{1}^{eff} + c_{2}^{eff} / N_{c}$ with $N_{c}$ the color number, where the effective Wilson coefficients $c_{1,2}^{eff}$ have been adopted to be $(c_{1}^{eff}, c_{2}^{eff}) = (1.168, -0.365) \quad [13, 16]$. In the generalized version of the factorization, one is able to float $N_{c}$ from 2 to $\infty$ to estimate the non-factorizable effects, resulting in $a_{1} = 1.05 \pm 0.12$. Since the parameter $a_{2}$ for $\bar{B}^{0} \rightarrow p \bar{p} D^{0}$ is sensitive to the non-factorizable effects, the fitting with the all available data gives $a_{2} = 0.42 \pm 0.04 \quad [15, 21]$. We then estimate $\mathcal{R}_{ff}$ in Eq. (8) to be

$$\mathcal{R}_{ff} = 0.50^{+0.13}_{-0.09},$$

which leads to $|V_{ub}|/|V_{cb}| = 0.088^{+0.022}_{-0.016} \pm 0.010$ with the errors from $\mathcal{R}_{ff}$ and $B_{s}/B_{D}$, respectively. Since $|V_{cb}|$ has been well measured, with $|V_{cb}| = (39.5 \pm 0.8) \times 10^{-3} \quad [3]$, we obtain

$$|V_{ub}| = (3.48^{+0.87}_{-0.63} \pm 0.40 \pm 0.07) \times 10^{-3}, \quad (12)$$

with the third error for $|V_{cb}|$.

III. DISCUSSIONS AND CONCLUSIONS

Our result in Eq. (12) is close to the exclusive $\bar{B} \rightarrow \pi \ell \bar{\nu}_{\ell}$ and $\Lambda_{b} \rightarrow p \mu^{-} \bar{\nu}_{\mu}$ cases; particularly, nearly the same as $|V_{ub}| \approx |A\lambda^{2}(\rho - i\eta)| \approx 3.56 \times 10^{-3}$ in the Wolfenstein parameterization [3]. Nonetheless, the complete estimation of the theoretical uncertainties from the $B \rightarrow p \bar{p}$ transitions gives the biggest error of $0.87 \times 10^{-3}$, such that our result also overlaps the inclusive value in Eq. (11). While the tension between the exclusive and inclusive extractions in Eqs. (1) and (2) is suspected to be due to the underestimated theoretical uncertainties [8], in our case the range of $|V_{ub}| = (2.73 - 4.43) \times 10^{-3}$ seems to reconcile the difference.
In sum, by relating $B^- \to p\bar{p}\pi^-$ and $B^0 \to p\bar{p}D^0$ decays, we have obtained $|V_{ub}|^2/|V_{cb}|^2 = (\mathcal{B}_\pi/\mathcal{B}_D)\mathcal{R}_{ff}$ for the extraction of $|V_{ub}|$, where $|V_{cb}|$ and $\mathcal{B}_\pi/\mathcal{B}_D \equiv \mathcal{B}(B^- \to p\bar{p}\pi^-)/\mathcal{B}(B^0 \to p\bar{p}D^0)$ are given by data, while $\mathcal{R}_{ff}$ is the parameter related to the $B \to p\bar{p}$ transition matrix elements. With $\mathcal{R}_{ff} = 0.50^{+0.13}_{-0.09}$ estimated from the global fitting of all available data of $B^- \to p\bar{p}e^-\bar{\nu}_e$ and $B \to p\bar{p}M_c$ ($M = \pi$, $K$ and $K^*$ and $M_c = D^{(*)}$), we have found that $|V_{ub}| = (3.48^{+0.87}_{-0.63} \pm 0.40 \pm 0.07) \times 10^{-3}$, where the errors correspond to the uncertainties from $\mathcal{R}_{ff}$, $\mathcal{B}_\pi/\mathcal{B}_D$ and $|V_{cb}|$, respectively. Being independent of the previous results, the extraction of $|V_{ub}|$ in the baryonic $B$ decays is clearly very useful for the complete determination of the CKM matrix elements as well as the exploration of new physics.

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