Modal identification of a building structure by frequency domain decomposition

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Abstract. Frequency domain decomposition (FDD) and Enhanced frequency domain decomposition (EFDD) methods were adopted to estimate modal parameters of an 18-story reinforced concrete building. Ambient vibration test was conducted on the 18-story reinforced concrete building and the two modal identification methods above were applied. The results show that FDD and EFDD methods can estimate modal parameters accurately and effectively from ambient vibration data.

1. Introduction
Modal identification techniques can identify modal parameters from dynamic measurements and provide important information reflecting the dynamic characteristics of structures. Traditionally, input-output modal identification methods are used to estimate modal parameters. However, input-output modal identification methods require artificial excitations to produce input signals. As structures are becoming higher and larger, it is difficult or even impossible to excite a large structure artificially nowadays.

Output-only modal identification techniques become popular as the difficulties obtaining input signals are avoided. Based on the white noise assumption, various output-only modal identification methods have been developed. In the frequency domain, Peak-picking (PP) method has been applied for decades. Frequency domain decomposition (FDD) method was later developed by Brincker et al. [1,2] and it overcame disadvantages of the PP method. Enhanced frequency domain decomposition (EFDD) method was subsequently developed to estimate modal damping [3].

In this paper, FDD and EFDD methods were adopted to estimate modal parameters of an 18-story reinforced concrete building. Ambient vibration test was conducted on the building, and the two modal identification methods were employed. The application procedure was presented. The results demonstrate that FDD and EFDD methods can effectively and accurately estimate modal parameters from ambient vibration data.

2. Frequency domain decomposition method
Frequency domain decomposition method removes limitations of the classical approaches. By taking singular valuedecomposition (SVD) of the spectral matrix, the spectral matrix is decomposed into a set of auto spectral densityfunctions, and each function corresponds to a single degree of freedom (SDOF) system. Modal parameters can then be obtained.
Considering an $n$ degree of freedom structure subjected to an input $x(t)$. The relationship between the measured output response $y(t)$ and the unknown input can be expressed as

$$ G_y(j\omega) = H(j\omega)G_x(j\omega)H(j\omega)^T $$  \hspace{1cm} (1)

where $G_x(j\omega)$ is the power spectral density matrix of the input, $G_y(j\omega)$ is the power spectral density matrix of the output, $H(j\omega)$ is the frequency response function (FRF), the overbar and the superscript $T$ denote the complex conjugate and transpose, respectively.

The frequency response function can be written as

$$ H(j\omega) = \sum_{k=1}^{n} \frac{R_k}{j\omega - \lambda_k} + \overline{R_k} \frac{1}{j\omega - \overline{\lambda}_k} $$  \hspace{1cm} (2)

with

$$ R_k = \phi_k \gamma_k^T $$  \hspace{1cm} (3)

where $n$ is the DOF number and the mode number, $R_k$ is the residue, $\lambda_k$ is the pole, $\phi_k$ is the mode shape vector, $\gamma_k$ is the modal participation vector.

If the input $x(t)$ is ambient excitation, with the white noise assumption, $G_x(j\omega)$ is reduced to a constant matrix $C$. Based on the Heaviside partial fraction theorem, Eq. (1) is reduced to a form as follows

$$ G_y(j\omega) = \sum_{k=1}^{n} A_k \frac{R_k}{j\omega - \lambda_k} + \overline{A_k} \frac{R_k}{j\omega - \overline{\lambda}_k} + \frac{R_k}{j\omega - \lambda_k} + \frac{\overline{R_k}}{j\omega - \overline{\lambda}_k} $$  \hspace{1cm} (4)

where $A_k$ is the $k$th residue matrix of $G_y(j\omega)$, and can be written as

$$ A_k = R_k C \left( \sum_{i=1}^{n} \frac{R^T_i}{\lambda_i - \lambda_k} + \frac{R^T_i}{\overline{\lambda}_i - \lambda_k} \right) $$  \hspace{1cm} (5)

In case that the structural damping is small and there are no close modes, Eq. (5) can be simplified as follows

$$ A_k = d_k \phi_k \phi_k^T $$  \hspace{1cm} (6)

where $d_k$ is scalar constant.

Taking Eq. (6) into Eq. (4), $G_y(j\omega)$ can be simplified as follows

$$ G_y(j\omega) = \sum_{k=1}^{n} \left( d_k \phi_k \phi_k^T + d_k \phi_k \phi_k^T \right) = \Phi \cdot \text{diag} \left( 2\text{Re} \left( \frac{d_k}{j\omega - \lambda_k} \right) \right) \cdot \Phi^T $$  \hspace{1cm} (7)

The power spectral density matrix of the $k$th order can be represented as follows

$$ G_{y_k}(j\omega) = \phi_k \cdot \text{diag} \left( \frac{2d_k \zeta_k \omega_k}{(\zeta_k \omega_k)^2 + (\omega - \omega_k)^2} \right) \cdot \phi_k^T $$  \hspace{1cm} (8)

From Eq. (8), the singular value decomposition form can be noticed, and it is noticed that only one mode can contribute to the maximum of power spectral energy. Therefore, with the measured response data induced by ambient excitation, the modal parameters can be estimated by the FDD method.

In order to estimate structural damping as well, Enhanced frequency domain decomposition method was developed. In EFDD method, the piece of the SDOF density function obtained around the peak of the PSD is taken back to the time domain by the Inverse discrete Fourier transform.
(IDFT) method, and the damping ratio can simply be obtained through inspection of the decay of autocorrelation functions.

3. FDD and EFDD methods applied to a building
FDD and EFDD methods were applied to ambient vibration data of an 18-story building. The test building extends from 5.3m underground to 53.1m above the basement level. The plan of a standard story is 22.2m long by 11.4m wide, and the story height is 2.95m. The lateral and gravity load-resisting system consists of reinforced concrete shear walls. Layout of the structural system is shown in Figure 1. Ambient vibration test was conducted on the building. Two accelerometers were set horizontally, one along the length direction and the other along the width direction, and the remaining accelerometer was set vertically. The response acceleration data along the width direction and the associated PSD function are shown in Figure 2. Due to the ambient excitation, the response amplitude is relatively small, and it can be observed that the modes are scatteredly distributed.

Singular values of the PSD matrix of the output are obtained, as shown in Figure 3. The close modes can be clearly indicated in the plot. The 1st mode and the 2nd mode are coupled, and the 4th mode and 5th mode are coupled. The first six modes are indicated in the figure and the natural frequencies are listed in Table 1.

The singular values corresponding to the first mode are obtained, as shown in Figure 4. A reasonable part of the PSD functions is identified, and the IDFT technique is applied on the identified part to obtain the normalized autocorrelation function. Finally, the damping ratio is obtained by the logarithmic decrement technique. The estimated damping ratios of the first six modes are listed in Table 1.

Figure 1. Layout of the structural system

Figure 2. Acceleration record along width direction

Figure 3. Power spectral density

- Acceleration along width direction
- Power spectral density

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Figure 3. Singular values of the output response PSD matrix

Figure 4. Identification of the first SDOF system

Table 1. Natural frequencies and damping ratios of the building.

| Mode | Natural frequency (Hz) | Damping ratio (%) |
|------|------------------------|-------------------|
| 1st  | 1.07                   | 1.11              |
| 2nd  | 1.27                   | 0.81              |
| 3rd  | 2.44                   | 1.36              |
| 4th  | 4.98                   | 1.35              |
| 5th  | 5.47                   | 1.31              |
| 6th  | 8.30                   | 2.29              |

4. Conclusions
In this paper, FDD and EFDD methods are introduced and are applied to ambient vibration data of a building. The results clearly indicate that FDD technique is able to extract modal information from ambient vibration data, and can estimate close modes with high accuracy. The EFDD method extends the application of FDD method and leads to an estimation of structural damping. By combined utilization of FDD method, EFDD method, and the ambient excitation technique, modal parameters can be identified, and the testing cost is significantly reduced.

References
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