A study about the F-estimator for the neutrino mass hierarchy in the JUNO experiment

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At present, it is still unknown whether the correct mass ordering of the neutrino mass eigenstates is either $m_1, m_2, m_3$ (Normal Hierarchy, NH), or $m_3, m_1, m_2$ (Inverted Hierarchy, IH). The new analysis method proposed by Stanco et al. [1] should fix some issues of the currently most used estimator, $\Delta \chi^2$, and make it possible to reach $5\sigma$ measurements in less than six years of data taking with JUNO (Jiangmen Underground Neutrino Observatory, [2], [3]) if a degeneracy in the atmospheric mass, $\Delta m^2_{\text{atm}}$, is accepted. In this note, the analysis introduced in the paper above was extended to more detailed studies on the dependence of the new F estimator to $\Delta m^2_{\text{atm}}$. A fit to the values of the new estimator as a function of $\Delta m^2_{\text{atm}}$, calculated for both the true hierarchy and the wrong hierarchy, was performed.

The study of the fitting function showed that the average minimum counts, corresponding to the best value for $\Delta m^2_{\text{atm}}$, either for the true hierarchy or for the wrong hierarchy, are well separated, and allow to distinguish easily between NH and IH.

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1 Introduction

One of the most compelling issues in Neutrino Physics is the determination of the neutrino mass hierarchy. So far, for the three neutrino mass eigenstates, $\nu_1$, $\nu_2$ and $\nu_3$, only the following quantities have been measured: the "solar" mass term $\Delta m_{sol}^2 = \Delta m_{12}^2 = m_2^2 - m_1^2$, and the absolute value of $|\Delta m_{31}^2| \simeq |\Delta m_{32}^2|$. As a consequence, two possible mass ordering are allowed: the normal hierarchy (NH), with $m_1^2 < m_2^2 < m_3^2$, and the inverted hierarchy (IH), with $m_3^2 < m_1^2 < m_2^2$. In this paper the notation "$\Delta m_{atm}^2$" (which indicates the atmospheric mass term) will be used, either to indicate $\Delta m_{31}^2 = m_3^2 - m_1^2$ in the case of normal hierarchy, or to indicate $\Delta m_{23}^2 = m_2^2 - m_3^2$ in the case of inverted hierarchy.

For the studies regarding the mass hierarchy determination, only one estimator has been extensively used so far, the $\Delta \chi^2$ test:

$$\Delta \chi^2 = \chi^2_{\text{min}}(IH) - \chi^2_{\text{min}}(NH)$$

(1)

where $\chi^2_{\text{min}}(IH)$ and $\chi^2_{\text{min}}(NH)$ come from the best-fit values for IH and NH respectively. The fit is performed over the whole set of the uncertainty parameters, namely the neutrino oscillation ones and the systematic errors. Nevertheless, this estimator has caused some concerns [4].

2 The F-estimator for reactor neutrinos

Reactor anti-neutrinos with an energy $E_\nu$ can be detected in a scintillator at a distance $L$ from the reactor where they produced from the inverse $\beta$ decay $\nu_e + p \rightarrow e^+ + n$. The rate of detected events is given by

$$\frac{dN}{dE_\nu} = T \times \Phi(E_\nu) \times \sigma_{\nu_e p} \times P_{\nu_\tau \rightarrow \nu_e},$$

(2)

where $T$ is the thermal power of the reactor, $\Phi$ and $\sigma_{\nu_e p}$ are respectively the flux and the cross section of the anti-neutrinos, and $P_{\nu_\tau \rightarrow \nu_e}$ is their survival probability. For every fixed value of the atmospheric mass, either equal or distinct for NH or IH, the survival probability is different for NH and IH. Its difference brings to the quantity $\Delta N(E_\nu) = \left(\frac{dN}{dE_\nu}\right)_{NH} - \left(\frac{dN}{dE_\nu}\right)_{IH}$, which is modulated at first order by $\sin(\frac{L}{2E} \Delta m_{atm}^2)$. By using this fact, Stanco et al. [1] introduced a new estimator, called F-estimator, defined as

$$F_{MO} = \int_{1.8}^{8.0} |\Delta N(E_\nu)| dE_\nu$$

(3)
where the limits of the integral are chosen to include the energy interval in which the modulation induced by $\Delta m_{atm}^2$ is observable.

In order to obtain the value of $F$, two kinds of intervals have to be defined:

1) $I^+ = \{ E_\nu : N_{NH,exp}(E_\nu) > N_{IH,exp}(E_\nu) \}$, i.e. the values of energy where the number of expected events for NH is greater than the number of expected events for IH;

2) $I^- = \{ E_\nu : N_{NH,exp}(E_\nu) < N_{IH,exp}(E_\nu) \}$, i.e. the values of energy where the number of expected events for NH is less than the number of expected events for IH;

$F$ is then computed both for the normal and the inverted hierarchy in the following way:

$$F_{IH} = \int_{1.8}^{8.0} (N_{obs} - N_{IH,exp}) dE_\nu$$ in $I^+$ if $N_{obs} > N_{IH,exp}$ ;

$$F_{IH} = \int_{1.8}^{8.0} (N_{IH,exp} - N_{obs}) dE_\nu$$ in $I^-$ if $N_{obs} < N_{IH,exp}$ ;

$$F_{NH} = \int_{1.8}^{8.0} (N_{obs} - N_{NH,exp}) dE_\nu$$ in $I^-$ if $N_{obs} > N_{NH,exp}$ ;

$$F_{NH} = \int_{1.8}^{8.0} (N_{NH,exp} - N_{obs}) dE_\nu$$ in $I^+$ if $N_{obs} < N_{NH,exp}$ .

In the ideal case, if NH is true, $F_{NH}$ is equal to zero, and $F_{IH} \sim 6500$ in 6 years of JUNO data taking [1], [2]; vice versa if IH is true. In the real case, both $F_{NH}$ and $F_{IH}$ are different from zero, yet they are different enough to have a high sensitivity, if a long exposure is performed and a good energy resolution is obtained.

3 The analysis

The analysis presented in this paper makes use of 2000 toys, generated by Monte Carlo simulations of JUNO-like events (1000 toys assuming NH and 1000 toys assuming IH). $\Delta m_{atm}^2$ was set equal to $2.56 \cdot 10^{-3}$ eV$^2$, for both NH and IH, at the generation level. Further, for each toy $F_{NH}$ and $F_{IH}$ were computed assuming 101 different values of $\Delta m_{atm}^2$. In this way it could be checked whether the values of $\Delta m_{atm}^2$ for which $F_{NH}$ and $F_{IH}$ present a minimum correspond to the true minimum and are distinguishable with a good level of confidence. In Figure 3 the plots of $F$ vs $\Delta m_{atm}^2$ are shown, both for the universe with NH and the one with IH. For every toy a fit to $F_{NH}$ and $F_{IH}$ with the function $F_{fit}(\Delta m_{atm}) = A \cos(\omega_1 \Delta m_{atm} + \Phi_1) \cos(\omega_2 \Delta m_{atm} + \Phi_2) + h$ was performed. $A$, $\omega_1$, $\omega_2$, $\Phi_1$, $\Phi_2$ and $h$ are the free parameters for the fit. The positions of minimum $\min_{true/false}(F_{NH})$ and maximum $\max_{true/false}(F_{IH})$ for the fitting function
were evaluated for every toy. Before any further probe, the values obtained were bias corrected. In fact, because of the presence of a bias due to the energy resolution that depends on the energy itself, F was shifted by some quantity \[1\]. Hence, it was necessary to restore it to the right position, such that the minimum \(\min_{true}\) (maximum \(\max_{false}\)) corresponded to \(\Delta m^2_{atm} = 2.56 \times 10^{-3} \text{ eV}^2\). The position of the minima for the wrong hierarchy was then determined, and its distance from the true value, \(2.56 \times 10^{-3} \text{ eV}^2\), was evaluated for every toy: \(\Delta \min = |\min_{false} - \min_{true}|\).

The values obtained for both NH true and IH true are (see also Figure 2):

- For NH: 
  \[<\Delta \min> = (12.06 \pm 0.02) \times 10^{-5}, \sigma_{\Delta \min} = (0.57 \pm 0.01) \times 10^{-5}\]

- For IH: 
  \[<\Delta \min> = (11.43 \pm 0.02) \times 10^{-5}, \sigma_{\Delta \min} = (0.54 \pm 0.01) \times 10^{-5}\]

These results are perfectly consistent with the suggested value of \(12 \times 10^{-5} \text{ eV}^2\) \[1\]. This allows to discriminate between the two hierarchies if the value of \(\Delta m^2_{atm}\) is known with a precision of \(\sim 4\%\). By comparison, the \(\Delta \chi^2\) test requires a precision of \(\sim 2\%\) on the \(\Delta m^2_{atm}\) value.
4 Conclusions

From the analysis shown in this work, the new estimator introduced in [1] seems to provide promising results. In particular, the precision required on $\Delta m^2$ results to be more relaxed than the precision needed for the $\Delta \chi^2$ test. Even if not mentioned here, an even more relevant feature of $F$ is the increase of its significance with regards to the collected luminosity (see [1]). In contrast, the $\chi^2$ tends asymptotically to a limited significance. We have further studied the characteristics of the $F$ estimator, as described in this note. By focusing on the precision on $\Delta m^2_{\text{atm}}$ with which $F$ estimates the best value, either for NH or IH, we were able to conclude that an excellent precision is achievable, confirming the rough estimation done in [1].

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