Floating of critical states and the QH to insulator transition

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Abstract

The transition from the quantum Hall state to the insulator is considered for non-interacting electrons in a two-dimensional disordered lattice model with perpendicular magnetic field. Using correlated random disorder potentials the floating up of the critical states can be observed in a similar way as in the continuum model. Thus, the peculiar behaviour of the lattice models reported previously originates in the special choice of uncorrelated random disorder potentials.

Keywords: Quantum Hall to insulator transition, floating of critical states, correlated random potentials

1. Introduction

Instead of floating up in energy, as has been proposed [1, 2] and shown to exist in continuum models [3, 4], a peculiar annihilation of the current carrying states takes place in lattice models when the quantum Hall to insulator transition is approached. According to recent investigations [5, 6, 7, 8, 9], increasing the disorder causes states associated with a negative Chern number to start moving outwards from the band centre and to destroy states with corresponding positive Chern numbers at the band edges. This behaviour is a special feature of lattice models which implicitly contain a periodic potential, but is not possible in the continuum model without a periodic potential usually considered to explain the levitation of the current carrying states.

The transition from the quantum Hall phase to the insulator is strongly affected by the way the critical states disappear. While in the continuum model the crossover takes place when the last current carrying state floats across the Fermi level, a direct transition to the insulator is possible in the lattice model also from higher Hall plateaus. The latter possibility is not contained in the global phase diagram for the quantum Hall effect [10] since it was based on the levitation scenario [1, 2].

Up to now the results of experimental investigations are not conclusive [11, 12, 13, 14]. Both, direct transitions from higher Hall plateaus to the insulator have been reported as well as experiments that are in accord with the global phase diagram. Recently, it has been pointed out that the low field quantum Hall to insulator transition might be unaccessible for the present time [15], because the available temperature range and sample sizes refuse to enter the scaling regime. However, a QH to insulator transition as a function of disorder strength at higher magnetic fields is more likely within experimental reach.
In the present paper we consider the possible origin of both the diverging experimental and theoretical results reported so far. In particular, the fate of the current carrying states is studied when the disorder is increased. We also investigate, how the QH to insulator transition depends on the correlation length of the disorder potentials.

2. Model with correlated disorder potentials

A two-dimensional tight binding model describing non-interacting electrons moving in a perpendicular magnetic field and random disorder potentials associated with the lattice sites $k$ is given by the Hamiltonian

$$H = \sum_k w_k |k\rangle \langle k| + \sum_{<k\neq l>} V_{kl} |k\rangle \langle l|. \quad (1)$$

The transfer terms $V_{kl}$ connect only neighbouring sites and contain the magnetic field $B = \alpha h/ea^2$ which is taken to be commensurate with the lattice. It is expressed as the number $\alpha = p/q$ of flux quanta $h/e$ per plaquette of area $a^2$ and enters via the complex phases, $V_{kl} \sim \exp(i 2\pi\alpha k_x/a)$. We take $p/q = 1/8$ so that the tight binding band splits into 8 sub-bands. The correlated disorder potentials $w_k$ are generated iteratively site by site using a generalisation of the Gibbs representation of Markov random chains \[15\].

The Gibbs distribution for the configuration of random variables $\mu$ is

$$P(\mu) = Z^{-1} \exp(-C\Theta(\mu)), \quad (2)$$

where $\Theta(\mu) = -\sum_{<kl>} \sigma_k(\mu) \sigma_l(\mu)$ is the Gibbs potential and $Z$ the partition function. Here, $\sigma_k(\mu) = \mu_k \in [-1/2, 1/2]$ is the value of the random variable on site $k$. For a given configuration $\mu^0$ the conditional probability that the new variable on the new site $n$ takes a value between $-1/2$ and $\Delta \leq 1/2$ is

$$P(\mu_n < \Delta | \mu^0) = \frac{e^{\Delta C \Sigma_n} - e^{-1/2 C \Sigma_n}}{2 \sinh(1/2 C \Sigma_n)}, \quad (3)$$

where $\Sigma_n = \sum_{i \in N_n} \sigma_i$, and $N_n$ is the set of neighbours of $n$ already determined. The correlation parameter $C$ tunes the strength of the potential correlations. $C = 0$ gives the uncorrelated case.

The correlated random numbers are then multiplied by the disorder strength $W$ which results in the set of disorder potentials $w_k$ used in Eqn. (1). An example of correlated potentials generated by this method with $C = 1.3$ is shown in Fig. 1 where a gray-scale plot of a $128 \times 128$ array is displayed. White regions correspond to $w_k \approx +W$ and black areas to $w_k \approx -W$ with the gray spots for values in between.

To further characterise the disorder potentials, we have calculated the correlation function $K(\rho) = \langle w_k w_l \rangle_{\rho = |r_k - r_l|}$ averaged over all pairs of sites which are a given distance $|r_k - r_l|$ apart,

$$K(\rho) = \frac{\sum_{i,j}^N w_i w_j \delta(|r_i - r_j| - \rho)}{\left( \sum_{i}^N w_i^2 \right)^{1/2} \left( \sum_{j}^N w_j^2 \right)^{1/2}}. \quad (4)$$

For $C = 0.5, 0.7, 1.0, 1.3, 1.7, \text{ and } 2.0$, we find an exponential form, $K(\rho) \sim \exp(-|r_k - r_l|/\eta)$.  

![Gray-scale plot for a 128×128 array of correlated disorder potentials with correlation parameter $C = 1.3$.](image-url)
The spatial decay of the correlations is governed by the correlation length $\eta$ which in the range $0.7 \lesssim C \lesssim 2.5$ can approximately be fitted by $\eta(C) \approx \exp(2C)$. The results for the correlation function $K(p)$ and the correlation length $\eta$ describing the decay of the potential correlations are shown in Fig. 2.

3. Hall conductivity and localisation length

The zero temperature Hall conductivity

$$\sigma_{xy}(E, W) = - \lim_{\varepsilon \to 0^+} \lim_{L \to \infty} \frac{e^2}{h} \frac{2}{\Omega} \text{Tr} \left\{ \sum_{i} i \varepsilon (G_{ii}^+ - G_{ii}^-) x_i \hat{y} - 2 \sum_{i,j} \varepsilon^2 G_{ij}^+ \hat{y} G_{ji}^- x_i \right\}$$

and the localisation length

$$\lambda_M^{-1}(E, W) = - \lim_{L \to \infty} \frac{1}{2L} \ln \text{Tr}|G_{1,L}|^2$$

are calculated numerically for large disordered two-dimensional systems of width $M$ and length $L$ using a recursive Green function method developed previously [17, 18]. Depending on the correlation length $\eta$ the width is varied between $M/a = 32$ and $M/a = 160$. The length of the system necessary for convergence is typically between $10^3$ and $10^7$ in units of the lattice constant.

a. In calculating the Hall conductivity one has strictly to observe the correct order of limits.

4. Results and discussion

The Hall conductivity was calculated near filling factor $\nu = 2$ as a function of increasing disorder $W$. For correlation parameters in the range $0.0 < C < 1.0$ a continuous decay from $\sigma_{xy} = 2e^2/h \to \sigma_{xy} = 0$ is observed. Although a direct transition is seen in the Hall conductivity for these correlation parameters, a closer look at the corresponding localisation length indicates a totally different behaviour of the current carrying states. For $C = 0.2$, which corresponds to a correlation length smaller than the magnetic length, no floating up of the critical state of the lowest Landau band can be observed. In fact, as in the uncorrelated case it slightly moves downward in energy with increasing disorder strength until it gets annihilated by the corresponding state with negative Chern number moving faster downward from the band centre. This is shown for a system of width $M/a = 48$ in Fig. 3 where the normalised localisation length $\lambda_M(E, W)/M$ is plotted versus energy in the filling factor range $0 \lesssim \nu \lesssim 1.5$ and several disorder strength $W$. The transition to the insulator takes place at $W_c/V \approx 2.5$ where the Chern and anti-Chern state touch each other.
A completely opposite behaviour is obtained for a larger correlation parameter, $C = 1.0$. This corresponds to a correlation decay length of 1.9 $a$ which is almost twice as large as the magnetic length $l_B/a = (2\pi\alpha)^{-1}$. While for $C = 0.2$ the last current carrying state disappeared at a disorder strength of about $W/V \approx 2.5$ two critical states are still observed at $W/V \approx 3.5$ and $C = 1.0$. This is shown in Fig. 4 where the disorder dependence of the localization length for energies corresponding to filling factors $1.5 \lesssim \nu \lesssim 2.5$ is displayed. Fig. 4 clearly reveals that there still exist two distinct critical points which move to higher disorder values when the filling factor is increased. In other words, the current carrying states of the two lowest Landau bands float up in energy and cross the Fermi energy one by one. Within this energy (filling factor) range the shift in disorder is weaker for the critical state of the lowest Landau band while it is stronger for the next critical state so that the two states get closer with increasing filling factor.

Since the two critical points are close in disorder the calculated Hall conductivity even for a system of width $M/a = 160$ does not reflect the situation of two separate transitions. The reason for this behaviour is that the divergence of the localization length of the lowest and that of the next current carrying state overlap because for a given Fermi energy the two critical disorders are close (see abscissa in Fig 4) so that the Hall steps cannot be resolved. This overlap will eventually disappear in the thermodynamic limit and similarly will the Hall conductivity exhibit the missing Hall plateau at $\sigma_{xy} = e^2/h$.

In conclusion, a lattice model with sufficient correlation in the disorder potentials shows similar behaviour as the continuum model. Hence, a lattice model can perfectly be in agreement with the proposed levitation scenario and the predictions of the global phase diagram and the references.

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