ОЦЕНКА ТОЧНОСТИ ИЗМЕРЕНИЯ ВЕКТОРНОГО ИНФОРМАЦИОННОГО ПАРАМЕТРА СИГНАЛА ПРИ ВОЗДЕЙСТВИИ МУЛЬТИПЛИКАТИВНЫХ НЕГАУССОВСКИХ ПОМЕХ

В статье рассматривается оценка погрешности измерения параметров движения протяженных объектов при воздействии на обрабатываемый сигнал мультипликативных негауссовских помех.

Показано, что на полезный сигнал наряду с аддитивными помехами воздействуют также и мультипликативные помехи. При этом и аддитивные, и мультипликативные помехи могут иметь как гауссовскую, так и негауссовскую плотность распределения вероятности (ПРВ).

Для оценки информационных параметров сигнала используются метод максимума апостериорной ПРВ, а также нижние границы неравенства Рао-Крамера. Показано, что воздействие мультипликативной помехи на полезный сигнал в общем случае приводит к смещению оценки измеряемых параметров. Наиболее просто определить смещение, когда оно не зависит от оцениваемого параметра, что позволяет перейти к несмещенной оценке.

Рассмотрена оценка точности измерения векторного информационного параметра обрабатываемого сигнала. При этом в качестве векторного информационного параметра рассмотрено изменение частоты, производной частоты и фазы полезного сигнала, которые несут информацию о параметрах движения локируемого объекта. Оценка точности измерения информационного параметра сигнала осуществлена при воздействии мультипликативных помех, имеющих как независимый, так и коррелированный характер.

Получены расчетные соотношения для оценки частоты, производной частоты и фазы полезного сигнала для различных условий осуществления оценки, в частности, при движе-
ESTIMATION OF MEASUREMENT ACCURACY OF THE VECTOR INFORMATION SIGNAL PARAMETER UNDER INFLUENCE MULTIPLICATIVE NON-GAUSSIAN NOISE

The article considers estimating the measurement error of motion parameters of extended objects when the processed signal is exposed to multiplicative non-Gaussian noise.

It is shown that, in practice, as a rule, the useful signal is influenced by both additive and multiplicative noise. In this case additive and multiplicative noise can have Gaussian as well as non-Gaussian density of probability distribution (PDF).

To estimate information parameters of the signal the method of maximum of posterior PDF and the lower bound of Cramer-Rao inequality are used. It is shown that the influence of multiplicative noise on the useful signal results in a bias of measured parameters estimator. It is easiest to determine a bias of an information parameter when it is not dependent on the estimated parameter. When distribution parameters of multiplicative noise are precisely known it is possible to determine the value of a bias made by this multiplicative noise and go to an unbiased estimate.

Estimating the accuracy of measurement of vector information parameter of the processed signal is considered. In this case, as a vector information parameter, the change in the frequency, derivative frequency and phase of the useful signal, which carry information about the parameters of the movement of the object being located, is considered. The estimation of the accuracy of an information parameter of the signal is carried out under the influence of multiplicative noise that has both independent and correlated nature.

The calculated ratios are obtained to estimate the frequency, its derivative and the phase of the processed signal under various conditions, for example, when a detected object moves with a constant velocity, when the initial phase of the signal is known, and when an extended object is motionless.

It is shown that in the General case the multiplicative noise leads to the displacement of the estimated information parameters of the useful signal; moreover, the greater the value of the displacement, the measurement accuracy is worse. It is proved that the accuracy of measurement of motion parameters increases with the increase in the difference between the PDF of the estimated parameter and the acting multiplicative noise from the Gaussian. It is shown that the measurement error of the information parameters of the useful signal can be significantly reduced by taking into account the Fisher dispersion information (information) about the modulating noise.

**Key words:** probability density function, non-Gaussian noise, multiplicative noise, noise with independent values, correlated noise.
Introduction

The issues of measurement (estimation) of signal parameters are very traditional and considered in detail in [1–3, etc.]. In the majority of research works devoted to the issues of estimation of signal parameters, it was thought that the useful signal is affected only by the additive noise, described, usually, by a Gaussian probability density function (PDF). However, as it is shown by the conducted research [4–6, etc.], a signal received by a measuring instrument is affected not only by additive but also by multiplicative noise [7–11, etc.]. This noise has a pronounced non-Gaussian character. For measuring devices it is rather important to estimate the impact of multiplicative noise on informational parameters of signal.

1. Estimation of measurement accuracy of the vector information parameter in multiplicative noise with independent values

Now we analyze the estimation of the vector information parameter \( \hat{\lambda} = \{ \lambda_1, \ldots, \lambda_m \} \) of the useful signal in multiplicative noise with independent values, using the method described in [12, 13].

As an example, we consider the case of measurement of \( \omega, \varphi \) and \( \theta \) signal

\[
s(\tilde{\lambda},t_h) = U_{m,h} \sin[(\omega + 0.5 \hat{\varphi}_h)t_h + \varphi], \quad (1)
\]

where \( \lambda_1 = \varphi, \lambda_2 = \omega, \lambda_3 = 0.5 \hat{\varphi} \).

Note that in our case, when we estimate parameters of posterior PDF, there should be three equations:

\[
\frac{dW_y(\tilde{\lambda})}{d\lambda_1} \bigg|_{\lambda_1 = \tilde{\lambda}_1} = 0; \quad \frac{dW_y(\tilde{\lambda})}{d\lambda_2} \bigg|_{\lambda_2 = \tilde{\lambda}_2} = 0; \\
\frac{dW_y(\tilde{\lambda})}{d\lambda_3} \bigg|_{\lambda_3 = \tilde{\lambda}_3} = 0.
\]

The expression of lower bound of the Cramer-Rao inequality to estimate combined parameters of the useful signal by the maximum posterior PDF will have the form [14]:

\[
\sigma_{\tilde{\lambda}ij}^2 \geq \frac{|J_{ij}|}{|J|} \quad i, j = 1, 2, 3, \quad (2)
\]

where \( |J_{ij}| \) is the cofactor of the element \( J_{ij} \) of the Fisher information matrix \(|J|\); \(|J|\) is the determinant of the matrix \(|J|\).

However, the elements of Fisher information matrix in general, taking into account the bias of estimated parameters

\[
b(\hat{\lambda}_i) = \lambda_i + \Delta(\lambda_i), \quad i = 1, \ldots, m
\]

will be determined as:

\[
J_{ij} = \left\{ \begin{array}{l}
db(\hat{\lambda}_i) \times d\hat{\lambda}_j \end{array} \right\} \left\{ \sigma_{\tilde{\lambda}ij}^2 - I_{ij}^* + I_{F,ij}^* \right\}^{-1}, \quad (3)
\]

where

\[
\sigma_{\tilde{\lambda}ij}^2 = \lim_{\|H\| \to \infty} H^{-1} \sum_{h=1}^{H} \left[ s_{\tilde{\lambda}ij}(\tilde{\lambda},t_h)s_{\tilde{\lambda}ij}(\tilde{\lambda},t_h) \right] s^2(\tilde{\lambda},t_h),
\]

\[ I_{F,ij} \]

is the component of Fisher information matrix relative to the estimated (measured) information parameters in the PDF of the measured parameters

\[ W_\lambda(\tilde{\lambda}), \quad I_{ij}^* = -m_1 \left\{ \eta^2 \frac{\partial^2}{\partial \eta^2} W_\eta(\eta) \right\}, \]

is Fisher variance information [12], concluded relative to multiplicative noise in one-dimensional PDF \( W_\eta(\eta) \).

Note that if the estimated parameters are not biased \( \Delta(\hat{\lambda}_i) = 0 \), then (3) will be:

\[
J_{ij} = \left\{ \sigma_{\tilde{\lambda}ij}^2 - I_{ij}^* + I_{F,ij}^* \right\}^{-1}. \quad (4)
\]

If the information parameters \( \tilde{\lambda} \) are estimated by the maximum likelihood method, then (4) can be written:

\[
J_{ij} = \left\{ \sigma_{\tilde{\lambda}ij}^2 - I_{ij}^* \right\}^{-1}. \quad (5)
\]

Note that if the bias from multiplicative noise is known and does not depend on the estimated parameters, then, like when we estimate just one parameter, we can pass to unbiased estimates [1, 13]

\[
b(\hat{\lambda}_i) - \Delta = \hat{\lambda}_i.
\]

We assume that the useful signal \( s(\tilde{\lambda},t_h) \) is affected by multiplicative noise \( \eta(t_h) \) described by the Nakagami PDF [7]:

\[
W(U) = \frac{2}{\Gamma(m)}(m/\Omega)^m \exp\left\{ -mU^2/\Omega \right\}, \quad U \geq 0,
\]

where

\[
m = \Omega^2 / \left( \langle U^2 - \Omega^2 \rangle \right) \geq 0.5, \quad \Omega = \left\langle U^2 \right\rangle
\]

are distribution parameters, \( \Gamma(.) \) is a gamma function.
Estimation of parameters is carried out on the observation interval \([0, T]\), the beginning and the end of which is accurately known and coincide with the moments of time corresponding to the beginning and end of the desired signal.

We assume that within the measurement interval, the estimated parameters remain unknown, and their value is equal to the values taken at the end of the measurement interval. The number of samples in the observation interval is large \(H = T / \Delta_h \gg 1\) (where \(\Delta_h = h - (h - 1)\) is the interval of taking the samples), in this case \(\omega T \gg 1\).

Using the results of [15], after the necessary mathematical transformations we obtain an expression to define the elements of the Fisher information matrix:

\[
J_{ij} = \left\{ \begin{array}{c} \sigma_{k,ij}^2 4m(-1)^{i+j-2} T^{i+j-1} \\
(i+j-1)^{-1} + F_{ij}^\lambda \end{array} \right. \]  
\tag{5}

The Fisher information matrix would be:

\[
\|J\| = \begin{bmatrix}
\sigma_{k,ij}^2 4m & -0.5T^2 & \frac{1}{3}T^3 & -0.25T^4 \\
-0.5T^2 & \frac{1}{3}T^3 & -0.25T^4 & 0,25T^5 \\
\frac{1}{3}T^3 & -0.25T^4 & 0,25T^5 & + I_{ij}^\lambda \\
I_{F,11} & I_{F,12} & I_{F,13} & I_{F,14} \\
I_{F,21} & I_{F,22} & I_{F,23} & I_{F,24} \\
I_{F,31} & I_{F,32} & I_{F,33} & I_{F,34}
\end{bmatrix}
\]  
\tag{6}

If the estimation of the measured parameters is performed by the maximum likelihood method [12, 16], the correlation matrix of errors is simplified

\[
\|J\| = \begin{bmatrix}
T & -0.5T^2 & \frac{1}{3}T^3 \\
-0.5T^2 & \frac{1}{3}T^3 & -0.25T^4 \\
\frac{1}{3}T^3 & -0.25T^4 & 0,25T^5
\end{bmatrix}
\]  
\tag{6}

The determinant of this matrix is:

\[
|J| = \left( \sigma_{k,ij}^2 4m \right)^3 T^9 / 2160 . \]  
\tag{7}

Substituting (5) and (6) into (2) taking into account the rules of operation with matrices, we obtain an expression for finding the lower bounds of Cramer-Rao inequality of the information parameters of the signal (1) in multiplicative noise, described by a one-dimensional Nakagami PDF.

Thus [13], for the estimate of the frequency

\[
\sigma_{\omega,\eta}^2 \geq \left\{ \sigma_{k,22}^2 4mT^3 / 192 \right\}^{-1} ; \]  
\tag{8a}

– for the derivative of the frequency

\[
\sigma_{\omega,\eta}^2 \geq \left\{ \sigma_{k,33}^2 4mT^5 / 720 \right\}^{-1} ; \]  
\tag{8b}

– for the phase

\[
\sigma_{\phi,\eta}^2 \geq \left\{ \sigma_{k,11}^2 4mT / 9 \right\}^{-1} . \]  
\tag{8c}

In the case, when the values \(\omega, \dot{\omega}\) and \(\phi\) are equal in the middle of the measurement interval, the expression (5) takes the form:

\[
J_{ij} = \left\{ \begin{array}{c} \sigma_{k,ij}^2 4m(-1)^{i+j-2} \\
(i+j-1)^{-1} + F_{ij}^\lambda \end{array} \right. \]  
\tag{5}

When estimating the parameters by the maximum likelihood method, the Fisher information matrix \(\|J\|\) will be written

\[
\|J\| = \sigma_{k,ij}^2 4m \begin{bmatrix}
T & 0 & T^3/12 \\
0 & T^3/12 & 0 \\
T^3/12 & 0 & T^5/80
\end{bmatrix}
\]  
\tag{6}

Then \(|J|\) is determined from (7) and in this case [10]:

\[
\sigma_{\omega,\eta}^2 \geq \left\{ \sigma_{k,22}^2 4mT^3 / 12 \right\}^{-1} ; \]  
\tag{9a}

\[
\sigma_{\omega,\eta}^2 \geq \left\{ \sigma_{k,33}^2 4mT^5 / 720 \right\}^{-1} ; \]  
\tag{9b}

\[
\sigma_{\phi,\eta}^2 \geq \left\{ \sigma_{k,11}^2 4m/9 \right\}^{-1} . \]  
\tag{9c}

Comparing (8) and (9), we see that, like when estimating information parameters of the signal in additive non-Gaussian noise with independent values [13], due to the fact that, with the exception of \(J_{ij}\) and \(J_{ji}\) non-diagonal elements equal to zero, the variance of the estimates \(\hat{\omega}\)
and \( \varphi \) is much smaller, if the measurement reference is carried out in the middle of the observation interval. It completely coincides with the results obtained before [14, 17].

Fig. 1 shows the dependencies
\[
\delta^2_{\lambda, \eta} = f(T, m) \quad \text{where:} \quad a - \hat{\lambda} = \hat{\omega};
\]
\[
b - \hat{\lambda} = \hat{\omega}; \quad c - \hat{\lambda} = \hat{\varphi}
\]

of the given errors
\[
\delta^2_{\lambda, \eta} = \frac{\sigma^2_{\lambda, \eta}}{\sigma^2_{\lambda, m-1}}
\]
of measurement
\[\hat{\lambda} = \omega, \; \hat{\omega}, \; \hat{\varphi} \]

of the processed signal under the influence of multiplicative noise, described by a one-dimensional Nakagami PDF for different values of distribution parameter \( m \) and the measurement interval \( T \).

From the expressions (8) and presented graphs it is seen that with increasing \( m \) and \( T \) the reduced error decreases. Therefore, accuracy of estimation of the information parameters \( \omega, \; \hat{\omega}, \; \hat{\varphi} \) characterizing the parameters of movement of detected objects, increases.

\[a)\]

\[b)\]
Fig. 1. Dependence of the reduced variance estimation of the signal information parameters $\delta^2_{\lambda, \eta}$ on the values $m$ and $T$

(\text{where}: a - \hat{\lambda} = \hat{\omega}; b - \hat{\lambda} = \hat{\phi}; c - \hat{\lambda} = \hat{\varphi})

2. Estimation of measurement accuracy of the information parameters on the background of correlated multiplicative noise

Let us analyze estimation of the information parameter in correlated multiplicative noise. To simplify the calculations, but still preserving the obtained results we assume that estimation is unbiased. We assume that at the input of measuring instrument we have a multiplicative mixture of the type $y_h = \eta_h s(\lambda, t_h)$ of the useful signal $s(\hat{\lambda}, t_h)$ carrying the information about one of the parameters of useful signal and multiplicative non-Gaussian noise $\eta_h$.

We also assume that noise is described by the transition PDF $W_{\eta}(\eta | \eta_{h-1})$. Measuring (estimating) the information parameter in the interval $[0, T]$ is carried out in discrete time, and $\lambda = \lambda_h = \lambda_{h-1}$. The logarithm of the likelihood function exists and is determined by:

$$B_{\eta}(\eta) = \ln W_{\eta}(y(t_h)/s(\lambda, t_h), y(t_{h-1})/s(\lambda, t_{h-1}));$$

$$s(\lambda, t_{h-1}) = s^{-1}(\lambda, t_h),$$

where $y(t_{h-i}) = \eta_h$; $i = 0, 1, W_{\eta}$. Is the PDF of non-Gaussian multiplicative noise with independent values.

We believe that the likelihood function meets the conditions of regularity:

$$\left\langle \frac{d}{d\lambda} B_{\eta}(\eta) \right\rangle = 0;$$

$$\left\langle \frac{d}{d\lambda} B_{\eta}(\eta) \right\rangle = \left[ \left\langle \frac{d}{d\lambda} B_{\eta}(\eta) \right\rangle \right]^2.$$

Using the method described by derivating a posteriori error of measurement of information parameters in additive non-Gaussian correlated noise [18], omitting the cumbersome mathematical calculations, we write:

$$\tilde{B}_{\lambda} = \text{tr}[IP] = \sum_{\alpha=1}^{2} \sum_{\beta=1}^{2} I_{\alpha \beta} \rho_{\alpha \beta};$$

$$\alpha, \beta = 1, 2;$$

(10)

where $I_{\alpha \beta}$ are the components of the information matrix and its elements are equal:

when $\alpha = \beta$: $I_{11} = I_{22} = [I^*_{\nu, \alpha \beta} - 1];$

when $\alpha \neq \beta$: $I_{12} = I_{21} = I^*_{\nu, \alpha \beta};$ $\alpha, \beta = 1, 2.$

Here $I^*_{\nu, \alpha \beta}$ are the components of the Fisher information matrix relative to the multiplicative noise, in transition PDF $W_{\eta}(\eta | \eta_{h-1})$; $P_{\nu}$ are the elements of the matrix $||P||$:

when $\alpha = \beta$: $P_{\alpha \beta} = \sigma_{\alpha i}^2 = K_{\alpha i}^2 = H^{-1}.$

$$\sum_{i=1}^{H} s_{\lambda}(\lambda_{h-i}) s^{-1}(\lambda_{h-i})^2;$$

when $\alpha \neq \beta.$
When estimation is carried out by the maximum likelihood:

\[
\sigma^2_{\lambda,\eta} \geq \left[ \text{tr}[\Pi]\right]^{-1}.
\]

Since \[7\]
\[
I_{F,nc} \left(W_i(\Pi_h)\right) \leq I_{F,c} \left(W_i(\Pi_h|\Pi_{h-1})\right),
\]

where \(I_{F,nc}\) and \(I_{F,c}\) is, respectively, the amount of Fisher information in a one-dimensional PDF \(W_i(\Pi_h)\) and transition \(W_i(\Pi_h|\Pi_{h-1})\) PDF of noise, then it can be shown, that

\[
|\frac{\text{\tilde{B}n}}{\text{\tilde{B}nc}}| \geq |\frac{\text{\tilde{B}n}}{\text{\tilde{B}nc}}|.
\]

Here \(\frac{\text{\tilde{B}n}}{\text{\tilde{B}nc}}\) and \(\frac{\text{\tilde{B}n}}{\text{\tilde{B}nc}}\) are, respectively the second derivatives of the LLF in the information parameter under the influence of correlated multiplicative noise and noise with independent values.

Then, taking into account (10) it can be written:

\[
\sigma^2_{\lambda,\eta} < \sigma^2_{\lambda,\eta}.
\]

As can be seen from (11), taking into account the correlations of multiplicative noise a posteriori error of the measurement of the information parameters is reduced, which leads to an increase in accuracy of their estimation, which coincides with the results of the works [8].

**Conclusion**

Thus, estimating the accuracy of measurement of vector information parameters of the processed signal is considered. A change of frequency, its derivative and the phase of the useful signal are considered as a vector parameter. The estimation of the accuracy of an information parameter of the signal is carried out under the influence of multiplicative noise that has both independent and correlated nature.

The calculated ratios are obtained to estimate the frequency, its derivative and the phase of the processed signal under various conditions.

It is shown that in general, multiplicative noise leads to a bias of estimated information parameters of the useful signal; and the bigger is the bias, the worse is the measurement accuracy. It is proved that the measurement accuracy of useful parameters can be significantly improved if the PDF of both information parameters and multiplicative noise affecting them is taken into account. Moreover, the more the PDF of an estimated parameter and multiplicative noise is different from Gaussian, the measurement accuracy will be higher.

**Список литературы**

1. Сосулин Ю.Г. Теоретические основы радиолокации и радионавигации: учеб. пособие для вузов. – М.: Радио и связь, 1992. – 304 с.

2. Финкельштейн М.И. Основы радиолокации. – М.: Радио и связь, 1983. – 536 с.

3. Radar handbook / Ed. by M. I. Scolnik. 2nd ed. – New York: McGraw-Hill, 1990. – 1199 p.

4. Kassam S.A. Signal Detection in Non-Gaussian Noise. – New York: Springer Verlag, 1989. – 242 p.

5. Lu N.H. Eisenstein Bruce A. Detection of weak signals in non-Gaussian noise // IEEE Trans. Microwave Theory Tech. – Nov. 1981. – Vol. 27. – No. 6. – P. 755–771.

6. Теория обнаружения сигналов / П.С. Акимов, П.А. Бакут, В.А. Богданович и др.; Под ред. П.А. Бакута. – М.: Радио и связь, 1984. – 440 с.

7. Тихонов В.И. Кульман Н.К. Нелинейная фильтрация и квазикогерентный приём сигналов. – М.: Сов. радио, 1975. – 704 с.

8. Кремер И.Я., Владимиров В.И., Карпухин В.И. Модулирующие (мультипликативные) помехи и прием радиосигналов / Под ред. И.Я. Кремера. – М.: Сов. радио, 1972. – 480 с.
9. Васильев К.К. Прием сигналов при мультипликативных помехах. — Саратов: Изд-во Саратовского ун-та, 1983. — 128 с.
10. Артюшенко В.М., Воловач В.И. Квазиоптимальная обработка сигналов на фоне аддитивной и мультипликативной негауссовских помех // Радиотехника. — 2016. — № 1. — С. 124–130.
11. Артюшенко В.М., Воловач В.И. Квазиоптимальная дискретная демодуляция сигналов на фоне коррелированных негауссовских флюктуационных мультипликативных помех // Радиотехника. — 2016. — № 6. — С. 106–112.
12. Новоселов О.Н., Фомин А.Ф. Основы теории и расчета информационно-измерительных систем. — 2-е изд., перераб. и доп. — М.: Машиностроение, 1991. — 336 с.
13. Артюшенко В.М., Воловач В.И. Анализ влияния некоррелированных аддитивных негауссовских помех на точность измерения скоростных параметров // Автометрия. — 2017. — Т. 53. — № 3. — С. 36–43. — DOI: 10.15372/AUT20170305
14. Валеев В.Г. Оптимальная оценка параметров сигнала при наличии аддитивных негауссовских помех // Изв. АН СССР. Техническая кибернетика. — 1974. — № 2. — С. 135–146.
15. Tuzlukov V.P. Signal Processing Noise. — Boca Raton, London, New York, Washington D.C.: CRC Press, Taylor & Francis Group, 2002. — 635 p.
16. Артюшенко В.М., Воловач В.И. Измерение информационных параметров сигнала в условиях воздействия аддитивных негауссовских коррелированных помех // Автометрия. — 2016. — Т. 52. — № 6. — С. 22–28. — DOI: 10.15372/AUT20160603

References
1. Sosulin Ju.G. Teoreticheskoe osnovy radiolokacii i radionavigacii: ucheb. posobiе dlja vzov. — M.: Radio i svjaz', 1992. — 304 s.
2. Finkel'shtein M.I. Osnovy radiolokacii. — M.: Radio i svjaz', 1992. — 304 s.
3. Radar handbook / Ed. by M. I. Scolnik. 2nd ed. — New York: McGraw-Hill, 1990. — 1199 p.
4. Kassam S.A. Signal Detection in Non-Gaussian Noise. — New York: Springer Verlag, 1989. — 242 p.
5. Lu N.H. Eisenstein Bruce A. Detection of weak signals in non-Gaussian noise // IEEE Trans. Microwave Theory Tech. — Nov. 1981. — Vol. 27. — No. 6. — P. 755–771.
6. Teorija obnaruzhenija signalov / P.S. Akimov, P.A. Bakut, V.A. Bogdanovich and dr.; Pod red. P.A. Bakuta. — M.: Radio i svyaz’, 1984. — 440 s.
7. Tihonov V.I., Kulin N.K. Nelinejnye fil'stry i kvazikogerentnyj priem signalov. — M.: Sov. radio, 1975. — 704 s.
8. Kremer I.Ya., Vladimirov V.I., Karpuhin V.I. Moduliruushchij pomeh // Radiotexnika. — 2016. — № 6. — С. 124–130.
9. Vasil’jev K.K. Priem signalov pri mulp'liifikativnyh pomehah. — Saratov: Izd-vo Saratovskogo un-ta, 1983. — 128 s.
10. Artyushenko V.M., Volovach V.I. Kvazioptimal’naja obrabotka signalov na fone additivnoj i mulp'tliifikativnoj nегауссовских pomeh // Radiotexnika. — 2016. — № 1. — С. 124–130.
11. Artyushenko V.M., Volovach V.I. Kvazioptimal’naja diskretnaja demoduljacija signalov na fonе korrelirovannych nегауссовских pomeh // Radiotexnika. — 2016. — № 6. — С. 106–112.
12. Novosjolov O.N., Fomin A.F. Osnovy teorii i raschjota informacionno-izmeritel’nyh sistem / 2-e izd., pererab. i dop. — M.: Mashinostrojenije, 1991. — 336 s.
13. Artjushenko V.M., Volovach V.I. Analysis of influence of uncorrelated additive non-Gaussian noise on accuracy of motion parameters measurement in short-range radio systems // Proceedings of IEEE International Siberian conference on control and communications (SIBCON-2015). — P. 7147279. — May 2015. — DOI: 10.1109/SIBCON.2015.7147279
14. Tihonov V.I. Нелинейные преобразования случайных процессов. М.: Radio i svjaz’, 1986. – 296 с.

15. Artjushenko V.M., Volovach V.I. Идентификация параметров распределения аддитивных и мультипликативных негауссовских помех // Автометрия. – 2017. – Т. 53. – № 3. – С. 36–43. – DOI: 10.15372/AUT20170305

16. Valejev V.G. Оптимальная оценка параметров сигнала при наличии негауссовских помех // Изв. AN SSSR. Техническая кибернетика. – 1974. – № 2. – С. 135–146.

17. Tuzlukov V.P. Signal Processing Noise. – Boca Raton, London, New York, Washington D.C.: CRC Press, Taylor & Francis Group, 2002. – 635 p.

18. Artjushenko V.M., Volovach V.I. Измерение информационных параметров сигнала в условиях воздействия аддитивных негауссовских коррелированных помех // Автометрия. – 2016. – Т. 52. – № 6. – С. 22–28. – DOI: 10.15372/AUT20160603