Topcolor Dynamics and
The Effective Gluon-Gluon-Higgs Operator

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We discuss the production of the composite Higgs boson in topcolor models via the gluon fusion process. We consider the contribution of color-octet massive gauge bosons (colorons) strongly interacting with the top quark, in addition to nonstandard contributions of the top-Yukawa coupling and heavy colored fermions other than the top quark. In order to estimate the contribution of colorons, we derive the low-energy effective theory by eliminating colorons by using the equation of motion for colorons. We replace the composite operator \((\bar{q}L_L R)L_R q_L\) in the effective theory by the composite Higgs operator. We then obtain the effective gluon-gluon-Higgs \((ggH)\) operator induced by colorons and find that its coefficient \((\mathcal{A}_{\text{col}})\) is proportional to \(m_{\text{dyn}}^2/M^2\), where \(M\) and \(m_{\text{dyn}}\) denote the coloron mass and the mass dynamically generated by colorons, respectively. The contribution of colorons \(\mathcal{A}_{\text{col}}\) becomes comparable to the top-loop effect \(\mathcal{A}_{\text{top}}\) for \(M \sim \mathcal{O}(1\text{TeV})\) and \(m_{\text{dyn}} \sim \mathcal{O}(0.6\text{TeV})\). Such a large dynamical mass can be realized in top-seesaw (TSS) models consistently with the experimental value of the top quark mass \(m_t\), while the dynamical mass itself is adjusted to \(m_t\) in topcolor assisted technicolor models (TC2). We find that the coloron contribution \(\mathcal{A}_{\text{col}}\) can be sizable in a certain class of TSS models: the contribution of colorons (the top-loop) is dominant in the real (imaginary) part of the \(H \to gg\) amplitude for the Higgs boson mass \(m_H\) of the order of 1 TeV. On the other hand, enhancement of the top-Yukawa coupling becomes important in TC2. We can observe signatures of the Higgs boson in TC2 with \(m_H \sim 200\) GeV even at the Tevatron Run II as well as at the LHC. We estimate \(S/\sqrt{B} = 3 - 6\) for an integrated luminosity of 2 \(\text{fb}^{-1}\) and \(m_H = 190\) GeV at the Tevatron Run II.

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I. INTRODUCTION

The gauge interaction properties of the Standard Model (SM) have been measured quite precisely in the last decade. However, the Higgs particle has not yet been discovered in spite of much effort. The physics behind the electroweak symmetry breaking (EWSB) and the origin of masses of quarks and leptons are left as unresolved problems. The idea of the top quark condensate \(\Phi\), in which the 4-top-quark interaction is introduced to trigger the EWSB, explains naturally the large mass of the top quark at the EWSB scale. (See also the earlier attempt \(\Phi\).) This model is often called the “top mode standard model” (TMSM), because the scalar bound state of \(\Phi\) plays the role of the Higgs boson in the SM.

It is known that the original version of the TMSM has some difficulties: the top quark mass \(m_t\) is predicted about 10\%–30\% larger than the experimental value \(m_t = 174\) GeV, even if we take the ultraviolet cutoff (or the compositeness scale) to the Planck or the GUT scale \(\Lambda\). In addition, such a huge cutoff causes a serious fine-tuning problem. In topcolor models (TCMs) \(\Phi\), the 4-top-quark interaction, whose origin is not specified in the original version of the TMSM, is provided by exchange of colorons which are color-octet massive gauge bosons strongly interacting with the top quark. \(\Phi\) If we assume that the coloron mass \(M\) is \(\mathcal{O}(1\text{TeV})\), we need not tune the coupling of the 4-top-quark finely to its critical value. Although the mass generated by the dynamics of colorons (the dynamical mass \(m_{\text{dyn}}\)) becomes quite large, typically \(m_{\text{dyn}} = 0.6 - 0.7\) TeV, in TCMs where only the top-condensate is responsible for the EWSB, the problem \(m_{\text{dyn}} \gg m_t\) can be resolved in some classes of models such as topcolor assisted technicolor models (TC2) \(\Phi\) and top-seesaw (TSS) models \(\Phi\). In TC2, we introduce techni-fermions in addition to colorons. We assume that the topcolor interaction is responsible for the top-quark mass, while the EWSB occurs mainly due to technicolor interactions. Namely, the dynamical mass itself is adjusted to \(m_t\) with a small vacuum expectation value (VEV) of the top quark condensate \(v_{\text{top}} \approx v/(3 - 4)\), where \(v\) denotes the EWSB scale. The top-Yukawa coupling in TC2 is about 3 – 4 times larger than the value in the SM. This enhancement of the top-Yukawa is quite important to the Higgs boson production via the gluon fusion process. In TSS models, we introduce a vector-like heavy colored fermion \(\chi\), whose mass term has nothing to do with the EWSB, in order to have a seesaw-type mass matrix. The dynamical mass \(m_{\text{dyn}}\) in TSS models does not correspond to the mass eigenvalue of the top-quark. The experimental value \(m_t\) is obtained after diagonalization of the mass matrix for the top-quark via the seesaw mechanism. Namely, the large dynamical mass of the order of 0.6 TeV can be realized in TSS models consistently.
with \( m_t^{\text{exp}} \). Here, we note that the mixing between \( t_L \) and \( \chi_L \) is severely constrained from the custodial symmetry violation, i.e., the \( T \)-parameter, which usually requires \( \chi \) to have a mass of the order of several-TeV. Although the TMSM with extra dimensions proposed by Arkani-Hamed et al. [11] is also a candidate to resolve the fine-tuning problem etc., it is a pure bulk gauge theory without 4-fermion interactions in the bulk rather than a TCM [12]. The phenomenology of the TMSM with extra dimensions will be studied elsewhere [12].

In this paper, we study the production of the Higgs boson in TCMs. We regard the Higgs boson as the tightly bound state of \( \bar{t}t \) and the dynamical mass \( m_\chi \) is very large in TSS models, while it cannot be sizable in TCMs with the coloron mass \( m_\chi \sim \mathcal{O}(10\text{TeV}) \) [11]. We then find that the contribution of colorons can be sizable in TSS models and TC2. We find that the contribution of colorons can be sizable in TSS models with the coloron mass \( M \sim \mathcal{O}(1\text{TeV}) \) and the mass of \( \chi, m_\chi \sim \mathcal{O}(10\text{TeV}) \), and that the effect of the top-loop is almost same as in the SM. We estimate numerically the effects of colorons and the top-loop as \( A_{\col} = -(1-3) + (0.1-0.2)i \times 10^{-2} \text{TeV}^{-1} \) and \( A_{\top} = [(0.4 - 0.1) + (1.4 - 1.1)i] \times 10^{-2} \text{TeV}^{-1} \) for \( m_H = 0.8 - 1 \text{ TeV} \), respectively. Namely, the contribution of colorons (the top-loop) is dominant in the real (imaginary) part of the \( H \to gg \) amplitude. In TC2, the contribution of colorons becomes small, while the effect of the top-loop is strongly enhanced. We find that signatures of the Higgs boson in TC2 can be observed even at the Tevatron Run II as well as at the LHC. In particular, we evaluate \( S/\sqrt{B} = 3 - 6 \) for an integrated luminosity of 2 fb\(^{-1} \) and \( m_H = 190 \text{ GeV} \) at the Tevatron Run II.

This paper is organized as follows: In Sec. II we derive the low-energy effective theory by eliminating colorons by using the EOM in a TCM with only the top- and bottom-quarks, for illustration. We find that the contribution of colorons is proportional to \( m_{\text{dyn}}^2/M^2 \). In Sec. III, we study expected signals of the Higgs boson in TC2 can be observed even at the Tevatron Run II and at the LHC. Sec. IV is devoted to summary and discussion.

### II. THE EFFECTIVE \( ggH \)-OPERATOR INDUCED BY COLORONS

We define the amplitude of \( H \to gg \) as

\[
A(H \to g\mu(p)G^\alpha_v(k)) \equiv -4\delta^{\alpha\nu} \epsilon^\mu_\nu(p)\epsilon^\nu_\nu(k)((p \cdot k)g^{\mu\nu} - k^\mu p^\nu)A,
\]

where \( G^\mu_\nu(p) \) denotes a gluon with momentum \( p \) and the suffix \( a \) is the index of colors for the adjoint representation. In the SM, we can obtain \( A = A_{\SM} \) at the \( 1 \)-loop level,

\[
A_{\SM}(\tau) = \frac{\alpha_s}{8\pi v} \tau(1 - (\tau - 1)f(\tau)) \quad (2)
\]

with \( v = 246 \text{ GeV} \), where we have defined \( \tau \equiv 4m_t^2/m_H^2 \) and

\[
f(\tau) \equiv \left\{ \begin{array}{ll}
\arcsin^2 \frac{1}{\sqrt{1 - \tau}}, & (\tau \geq 1) \\
\frac{1}{2} \left[ \ln \frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} + i\pi \right]^2, & (\tau < 1)
\end{array} \right.),
\]

Since the factor \( A_{\SM} \) in the \( \tau \to \infty \) limit becomes independent on \( \tau \),

\[
A_{\SM}(\infty) = \frac{\alpha_s}{12\pi v}, \quad (4)
\]

the \( H \to gg \) amplitude can be written as the local operator

\[
O_{ggH} = A_{\SM}(\infty)HG^\mu_\nu G^{\alpha\nu}.
\]

In this sense, the factor \( A \) corresponds to the coefficient of the local operator \( O_{ggH} \). We also note that the \( H \to gg \) amplitude has the imaginary part above the threshold of the top-pair production \( m_H > 2m_t \).
The gluon fusion process is obviously sensitive to the top-Yukawa coupling and the number of heavy quarks. In this paper, we also consider the effect of colorons strongly interacting with the top quark. We hence classify non-standard contributions to the $H \rightarrow gg$ amplitude into three categories:

(a) Loop effects of the top quark;

In the SM, the lowest order amplitude for the gluon fusion arises from the triangle diagram of the top quark. On the other hand, the top-Yukawa coupling can be larger than the SM one in TCMs with multi composite Higgs doublets. The enhancement of the top-Yukawa coupling is quite important for the gluon fusion process.

(b) Loop effects of other heavy quarks;

Generally, heavy quarks other than the top are also introduced in TCMs. They contribute to the $H \rightarrow gg$ amplitude at the 1-loop level.

(c) Contributions of strongly interacting colorons;

Since colorons are strongly coupled to the top quark, effects of colorons are potentially sizable.

Corresponding to the three contributions (a), (b) and (c), we split the factor $A$ in Eq. (1) into three parts, $A_{\text{top}}$, $A_{\text{heavy}}$ and $A_{\text{col}}$,

$$A = A_{\text{top}} + A_{\text{heavy}} + A_{\text{col}}.$$  

In this section, we roughly estimate the size of (c). For this purpose, we derive the low-energy effective theory by eliminating colorons by using the equation of motion (EOM) for colorons. For illustration, we take a toy model of TCMs with only the top- and bottom-quarks. This model is not realistic, but it is sufficient to understand the origin of the coloron contribution $A_{\text{col}}$. In the next section, we evaluate quantitatively the sizes of $A_{\text{top}}$, $A_{\text{heavy}}$, and $A_{\text{col}}$ in typical TCMs such as TC2 and TSS models.

After the topcolor symmetry is spontaneously broken down, we write down the model,

$$L = L_{\text{top}} + L_{\text{int}} + L_{\text{col}},$$  

with

$$L_{\text{top}} = \bar{q}_L igq_L + \bar{t}_R ig t_R - \frac{1}{2} \text{tr}(G_{\mu\nu} G^{\mu\nu}),$$

$$L_{\text{int}} = g' A^a_L (J^a_{\mu\nu} + J^a_{\rho\sigma}),$$

$$L_{\text{col}} = -\frac{1}{2} \text{tr}(A_{\mu\nu} A^{\mu\nu}) + M^2 \text{tr}(A_{\mu} A^{\mu})$$

$$+ i g_{GAA} \text{tr}(G_{\mu\nu}[A^{\mu}, A^{\nu}]) + i g_{A^a} \text{tr}(A_{\mu} A^{\mu}[A^{\mu}, A^{\nu}]) + g_{A^a} \text{tr}([A^{\mu}, A^{\nu}]^2)$$

up to dimension four operators, where $g'$ is the coupling constant between the top quark and colorons, and $A_{\mu}$ and $G_{\mu\nu}$ denote the coloron field and the gluon, respectively. The current $j^\mu_L(R)$ for the weak doublet $q_L$ (the weak singlet $t_R, b_R$) is defined as usual. We note that $G_{\mu\nu}$ and $A_{\mu}$ are matrices defined as $G_{\mu} \equiv G^a T^a, A_{\mu} \equiv A^a T^a$ with generators $T^a$ of $SU(N_c)$, where $N_c$ is the number of colors. In Eqs. (8)–(10) with the gauge symmetry of QCD, the covariant derivative $D_{\mu}$, and the field strength $G_{\mu\nu}$ and $A_{\mu\nu}$ for the gluon and the coloron are written as

$$D_{\mu} \equiv \partial_{\mu} - ig_s G_{\mu}, \quad G_{\mu\nu} \equiv \frac{i}{g_s} [D_{\mu}, D_{\nu}], \quad A_{\mu\nu} \equiv \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} - ig_s [G_{\mu}, A_{\nu}] + ig_s [G_{\nu}, A_{\mu}].$$

We do not incorporate higher dimensional operators in $L_{\text{col}}$. If we would allow them, we could freely choose the coefficient $A_{\text{col}}$ of the operator $O_{4\mu\nu}$, as we will see later on. We note that the triple coupling $g_{GAA}$ plays an important role in the following analysis and that it is equal to the QCD coupling,

$$g_{GAA} = g_s.$$  

1 In order to avoid the bottom quark condensation, we have to introduce a new strong $U(1)$-gauge interaction, for example. Instead of specifying the manner for suppression of the bottom condensation, we adjust the 4-fermion interactions so that only the top condensation takes place.
in usual TCMs.

Now, we derive the effective theory at the coloron scale in the classical approximation. We eliminate colorons from Eq. (17) by using the EOM for $A_{\mu}$,

$$M^2 A_{\mu} = -g' J_{\mu} + [D^\nu, A_{\mu\nu}] - ig_{GAA}[A^\nu, G_{\mu\nu}]$$

$$-ig_{A^3}[D^\nu, [A_{\mu}, A_{\nu}]] - ig_{A^5}[A^\nu, [A_{\mu}, A_{\nu}]] - g_{A^7}[A^\nu, [A_{\mu}, A_{\nu}]]$$

(13)

with $J^\mu \equiv J^\mu_0 + J^\mu_R$, where we can expand $A_{\mu}$ in powers of $1/M^2$ with recursive use of Eq. (13) and can rewrite $A_{\mu}$ in terms of $D_{\mu}, J_{\mu}$ and $G_{\mu\nu}$:

$$A_{\mu} = -\frac{g'}{M^2} J_{\mu} + \frac{g'}{(M^2)^2}[g_{GAA}[J^\nu, G_{\mu\nu}]]$$

$$+ \frac{g'}{(M^2)^2}[g_{GAA}[J^\nu, G_{\mu\nu}]] - \frac{g'}{(M^2)^2}[D_{\nu}, [D_{\mu}, J^\nu]]$$

$$+ \frac{g'}{(M^2)^2}[g_{GAA}[D_{\nu}, [D_{\mu}, J^\nu]], G_{\mu\nu}]] - \frac{g'}{(M^2)^2}[D_{\nu}, [D_{\mu}, [D_{\lambda}, J^\nu]]]$$

$$- \frac{g'}{(M^2)^2}[D_{\nu}, [D_{\lambda}, [D_{\mu}, J^\nu]]]$$

$$- \frac{(g')^2}{(M^2)^3}[g_{A^3}[J^\nu, [D_{\mu}, J^\nu]]]$$

$$+ \mathcal{O}\left(\frac{1}{M^2}\right)$$

(14)

Noting that $J_{\mu}$ is the conserved current, i.e. $[D_{\mu}, J_{\nu}] = \partial_{\mu} J^\nu - ig_{A}[G_{\mu\nu}, J^\nu] = 0$, we find

$$A_{\mu} = -\frac{g'}{M^2} J_{\mu} + \frac{g'}{(M^2)^2}[(g_{GAA} + g_5)J_{\nu}, G_{\mu\nu}]] + \frac{g'}{(M^2)^3}(g_{GAA} + g_5)^2G_{\mu\nu}, [G^\nu, [G^\lambda, J^\lambda]] + \cdots ,$$

(15)

where we neglected irrelevant terms to the effective $ggH$-coupling. Substituting Eq. (13) for Eqs. (1) and (10), we obtain the effective Lagrangian written in terms of local composite operators of $J_{\mu}$ and $G_{\mu\nu}$:

$$\mathcal{L}_{\text{int}} + \mathcal{L}_{\text{col}} \rightarrow \mathcal{L}_{\text{comp}}$$

(16)

with

$$\mathcal{L}_{\text{comp}} = -\frac{g'}{2M^2} J_{\mu} J_{\nu} - \frac{(g_{GAA} + g_5)g'g^2}{2(M^2)^2} f_{abc} G_{\mu b} f_{b c \alpha} J_{\nu} - \frac{(g_{GAA} + g_5)^2g'g}{2(M^2)^3} f_{abc} f_{c d e} G_{\mu b} G_{\nu c} G_{\mu d} J_{\nu} + \cdots .$$

(17)

Since only scalar operators with large anomalous dimension are relevant in the low-energy effective theory, we neglect vectorial and tensorial terms of 4-fermion operators. The part including the 4-top-quark operator is given by

$$\mathcal{L}_{A-\text{top}} = \frac{g_0}{M^2}(\bar{q}L_{tR})(\bar{t}LR_{L}) + \frac{N_c}{4} \frac{(g_{GAA} + g_5)^2g'g^2}{2(M^2)^3} G_{\mu b} G_{\nu c} G_{\mu d} J_{\nu} + \cdots .$$

(18)

at the leading order of the $1/N_c$-expansion, where we have used the Fierz transformation. The first term in Eq. (18) is the driving force of the top condensation, while the second one is the source of the $ggH$-operator. (See also Fig. 1.) Here, we note that the coefficient of the 4-top-quark (4-bottom-quark) operator is generally required to be larger (smaller) than $(g')^2$, which is nearly equal to the critical coupling $(g_{\text{crit}})^2$, in order to produce the desirable pattern of condensations. We thus replaced the coefficient by $g_0^2$, $(g_0 \gtrsim g')$ in Eq. (18), assuming a certain mechanism for suppression of the bottom condensation.

In order to obtain the low-energy effective theory with the Higgs doublet $\phi$, we rewrite the effective Lagrangian Eq. (18) at the coloron scale as follows:

$$\mathcal{L}_{\phi_0} = -M^2 \phi^0_0 \phi_0 - [g_0(\bar{q}L_{tR})\phi_0 + \text{h.c.}]$$

$$- \frac{N_c}{4} \frac{g'g^2}{g_0^2} \frac{(g_{GAA} + g_5)^2}{M^2} G_{\mu b} G_{\nu c} \phi_0^0 \phi_0 - \frac{N_c}{4} \frac{g'g^2}{g_0^2} \frac{(g_{GAA} + g_5)^2}{(M^2)^3} G_{\mu b} G_{\nu c} (g_0(\bar{q}L_{tR})\phi_0 + \text{h.c.})$$

(19)
with the bare Higgs doublet $\phi_0$. We can confirm the equivalence of Eq. (19) and Eq. (18) through the EOM for $\phi_0^*$ and $\phi_0$. The kinetic term of the Higgs doublet develops below the coloron scale. We then obtain the low-energy effective theory at the weak scale:

$$L_{\text{eff}} = L_{\text{top}} + L_{\phi}$$

with

$$L_{\phi} \equiv \partial_\mu \phi^\dagger \partial^\mu \phi - m_\phi^2 \phi^\dagger \phi - \lambda_\phi (\phi^\dagger \phi)^2 - g_t ([\bar{t}_L t_R] \phi + \text{h.c.}] - \frac{N_c - 1}{4Z_\phi} g_t^2 (g_{GAA} + g_s)^2 \frac{M^2}{g_0^2} G_\mu^a G_\nu^a \phi^\dagger \phi - \frac{N_c - 1}{4Z_\phi} g_t^2 (g_{GAA} + g_s)^2 \frac{M^2}{g_0^2} G_\mu^a G_\nu^a [\bar{t}_L t_R] \phi + \text{h.c.}],$$

$$\phi \equiv Z_\phi^{1/2} \phi_0, \quad \text{and} \quad g_t \equiv g_0 / Z_\phi^{1/2}.$$  

(21)

Since the neutral component of $\phi$ can be written as $(v + H(x))/\sqrt{2}$, the coefficient $A_{\text{col}}$ of the effective $ggH$-operator is given by

$$A_{\text{col}} = -\frac{N_c - 1}{4Z_\phi} g_t^2 (g_{GAA} + g_s)^2 \frac{M^2}{g_0^2} \frac{v}{Z_\phi}.$$  

(22)

Here, we comment on the reason why we did not allow higher dimensional operators in the Lagrangian for colorons. If the dimension six operator $\text{tr}([G_{\mu\nu}, A^\mu][G_{\mu\lambda}, A^\lambda])$ was not forbidden in $L_{\text{col}}$, it would lead to the same composite operator as the third term of $L_{\text{comp}}$ in Eq. (17). This means that the coefficient $A_{\text{col}}$ of the operator $O_{ggH}$ at the weak scale is not constrained. Namely, tolerance of higher dimensional operators in $L_{\text{col}}$ is equivalent to adding $O_{ggH}$ with a free coefficient by hand.

The VEV of $\bar{t}t$ has a nonzero value only when the coefficient $(g_0)^2$ of the 4-top quark interaction is larger than its critical coupling [16, 17],

$$(g_0)^2 \geq (g_{\text{crit}})^2, \quad (g_{\text{crit}})^2 = \frac{8\pi^2}{N_c}.$$  

(24)

The top quark then acquires the dynamical mass,

$$m_{\text{dyn}} \equiv \frac{g_t}{\sqrt{2}} v = \frac{g_0}{\sqrt{2} Z_\phi^{1/2}} v.$$  

(25)
Within the usual large $N_c$-bubble approximation, we find the wave function renormalization of the Higgs doublet,

$$ Z_\phi(\mu^2) \simeq \frac{N_c g_\text{top}^2}{4\pi^2} \ln \frac{M^2}{\mu^2} $$

(26)

with the renormalization point $\mu$. Substituting Eq. (26) for Eq. (23), we obtain the relation

$$ v^2 = \frac{N_c}{8\pi^2} m^2_{\text{dyn}} \ln \frac{M^2}{m^2_{\text{dyn}}} $$

(27)

which is called the Pagels-Stokar (PS) formula [18], and can evaluate the dynamical mass as $m_{\text{dyn}} \sim \mathcal{O}(0.6 \text{ TeV})$ for $M \sim \mathcal{O}(1 \text{ TeV})$. Although the top quark mass is predicted too large ($m_t = m_{\text{dyn}}$) in this toy model, we consider realistic TCMs such as TSS models and TC2 in the next section.

Now, we estimate roughly the size of $\mathcal{A}_{\text{col}}$ and compare it with the size of $\mathcal{A}_{\text{top}}$. Using $g_\text{top} \approx g_\text{crit}$, Eq. (26) reads $Z_\phi \sim \mathcal{O}(1)$ up to a logarithmic term. Namely, the expression of $\mathcal{A}_{\text{eq}}$ in Eq. (23) has no loop-suppression factor $1/(4\pi)^2$. It is, in other words, a consequence of the large dynamical mass obtained from the PS formula, $m_{\text{dyn}} \sim 4\pi v$ up to a logarithmic term. Actually, we can explicitly rewrite Eq. (23) as

$$ \mathcal{A}_{\text{col}} = -\frac{(g_{\text{GAA}} + g_s)^2 g^2 v^2 m^2_{\text{dyn}}}{2 N_c g_\text{top}^2 v^2 g_\text{top}^2 M^2} $$

(28)

in terms of $m_{\text{dyn}}$, where $g_\text{top} \approx g' \approx g_\text{crit}$ and the suppression factor $2 N_c g_\text{top}^2 \sim (4\pi)^2$ is canceled by $m_{\text{dyn}} \sim 4\pi v$. We find roughly that the contribution of colorons is $\mathcal{A}_{\text{col}} \sim -g_\text{top}^2 v^2/M^2$ with $g_{\text{GAA}} = g_s$, while the top-loop effect is $\mathcal{A}_{\text{top}} \sim g_\text{top}^2/(4\pi)^2 v^2$ from Eq. (28). In the toy model, we can estimate the contribution of the top quark loop as $\mathcal{A}_{\text{top}} = \mathcal{A}_{\text{SM}(1)} = \alpha_s/(8\pi v)$ by using the relation $m^2_H = 4m^2_{\text{dyn}}$ in the bubble approximation. In any case, we obtain the ratio of $\mathcal{A}_{\text{col}}$ and $\mathcal{A}_{\text{top}}$,

$$ \left| \frac{\mathcal{A}_{\text{col}}}{\mathcal{A}_{\text{top}}} \right| \propto \frac{m^2_{\text{dyn}}}{M^2}. $$

(29)

where we used $g_\text{top} \approx g' \approx g_\text{crit}$ and $g_{\text{GAA}} = g_s$. This relation suggests that $\mathcal{A}_{\text{col}}$ becomes comparable to $\mathcal{A}_{\text{top}}$ in the situation of $M \sim \mathcal{O}(1 \text{ TeV})$ and $m_{\text{dyn}} \sim \mathcal{O}(0.6 \text{ TeV})$.

The coloron mass $M \sim \mathcal{O}(1 \text{ TeV})$ is favorable in order to avoid the fine-tuning problem, while the large dynamical mass $m_{\text{dyn}} \sim \mathcal{O}(0.6 \text{ TeV})$ is obtained from a single VEV triggering the EWSB. The large dynamical mass can be realized within TSS models. However, the dynamical mass is not necessarily large in TCMs with non-minimal Higgs sectors such as TC2. In the next section, we estimate quantitatively the effective $ggH$-coupling in TSS models and TC2.

### III. SIZE OF THE EFFECTIVE $ggH$-COUPLING IN TSS MODELS AND TC2

#### A. Analysis for TSS models

In TSS models, it is possible that the EWSB occurs via the “top-condensate” only. The large dynamical mass can be tuned consistently with the experimental value $m_t^{(\text{exp})} (= 174 \text{ GeV})$ through the seesaw mechanism, which requires an additional vector-like heavy quark $\chi$. We hence consider the original version of TSS models (the $U(1)$ tilting model [1]) where the SM is embedded into a topcolor scheme, $SU(3)_c \times SU(3)_L \times SU(2)_W \times U(1)_{1} \times U(1)_{2} \times U(1)_{B-L}$ gauge group. The gauge group $SU(3)_c \times U(1)_{1}$ and $SU(3)_c \times U(1)_{2}$ acts only on the third (first and second) generation(s) of quarks and leptons. The $U(1)_{B-L}$ changes are $x (-1/3 < x < 0)$ for $t_R$ and $\chi_L, 1/3$ for the other quarks, and $-1$ for leptons.

The charge assignment is also shown in Table 1. The gauge symmetry breaking $SU(3)_c \times SU(3)_L \rightarrow SU(3)_{\text{QCD}}, U(1)_{1} \times U(1)_{2} \rightarrow U(1)_{Y}$, and $U(1)_{B-L}$ leave colorons, and two massive gauge bosons $Z_1$ and $Z_{B-L}$, respectively. We note that the mass term $\chi_L \chi_R$ is related to the $U(1)_{B-L}$ breaking VEV in the $U(1)$ tilting model. We may take common masses $\Lambda$ for $Z_1$ and $Z_{B-L}$ of the order of a multi-TeV and the coloron mass $M$ of the order of 1 TeV. In this situation, we can expect that the contribution of colorons becomes sizable and that the constraint on the $T$-parameter can be satisfied. Here, we comment on the Higgs boson mass $m_H$ in the $U(1)$ tilting model: it is possible that the Higgs boson has a mass of order 100 GeV with tuning of some parameters [2]. However, the Higgs boson mass is likely of the order of $m_{\text{dyn}}$, i.e. $m_H \sim \mathcal{O}(1 \text{ TeV})$. We should note that the $WW$-fusion process becomes comparable to the gluon fusion in the mass range $m_H \sim \mathcal{O}(1 \text{ TeV})$ in the SM.

Under certain conditions [2], only the VEV $\langle \tilde{L}_R \chi_R \rangle$ acquires dynamically a non-zero value. We thus refer to

| SU(3)$_c$ | SU(3)$_L$ | SU(2)$_W$ | U(1)$_1$ | U(1)$_2$ | U(1)$_{B-L}$ |
|---|---|---|---|---|---|
| $g_L$ | 3 | 1 | 2 | 1/6 | 0 |
| $t_R$ | 3 | 1 | 1 | -2/3 | 0 |
| $b_R$ | 3 | 1 | 1 | 1/3 | 0 |
| $\tau_R$ | 1 | 1 | 1 | 1 | 1 |
| $\nu_{\nu R}$ | 1 | 1 | 1 | 0 | 1 |

**TABLE I:** The representation of the third-generation and $\chi$ fermion in the $U(1)$ tilting model.

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2 In the case of $m_H = m_{\text{dyn}}$, we obtain $\mathcal{A}_{\text{top}} = (4 - \pi^2/3) \cdot \alpha_s/(8\pi v) \simeq 0.71 \cdot \alpha_s/(8\pi v)$.

3 We also incorporate right-handed neutrinos.
mass term of $t_{LX_R}$ ($m_{tX}$) as the dynamical mass. The mass matrix for $t$ and $\chi$ is diagonalized by rotating the left- and right-handed fields as follows:

\[
\begin{pmatrix}
  t_L \\
  \chi_L
\end{pmatrix} =
\begin{pmatrix}
  c_L & s_L \\
  -s_L & c_L
\end{pmatrix}
\begin{pmatrix}
  t'_L \\
  \chi'_L
\end{pmatrix},
\]

\[
\begin{pmatrix}
  t_R \\
  \chi_R
\end{pmatrix} =
\begin{pmatrix}
  -c_R & s_R \\
  s_R & c_R
\end{pmatrix}
\begin{pmatrix}
  t'_R \\
  \chi'_R
\end{pmatrix},
\]

where $s_{L(R)} = \sin \theta_{L(R)}, c_{L(R)} = \cos \theta_{L(R)}$ and the prime $X' (X = t, \chi)$ denotes the mass eigenstate. The relations between the mass eigenvalues of $m_t$ and $m_\chi$, and the dynamical mass $m_{\text{dyn}}$ are obtained as

\[
\frac{m_t}{m_{\text{dyn}}} = \frac{s_R}{c_L}, \quad \frac{m_\chi}{m_{\text{dyn}}} = \frac{s_L}{c_R}.
\]

We can determine the mixing angles $s_L, s_R$ and the dynamical mass $m_{\text{dyn}}$ through Eq. (31) and the PS formula for the VEV $\langle t_{LX_R} \rangle$. Since we obtain Yukawa vertices of $t$ and $\chi$ in mass eigenstates from

\[
\frac{g_{tX}}{\sqrt{2}} (c_L t'_L + s_L \chi'_L) (s_R t'_R + c_R \chi'_R) H
\]

with

\[
m_{\text{dyn}} = \frac{g_{tX}}{\sqrt{2}} v, \quad g_{tX} \equiv g_{tX0} Z_H^{-1/2},
\]

where $g_{tX0}$ is nearly equal to the critical coupling $g_{\text{crit}}$, we find the wave function renormalization $Z_H(p^2)$ of the Higgs boson with the momentum $p$ in 1-loop approximation.

\[
Z_H(p^2) = - \frac{N_c g_{tX0}^2}{8\pi^2} \left( \frac{1}{2} + 3 (s_L^2 s_R^2 + c_L^2 c_R^2) \int_0^1 dx x (1-x) \log \frac{D_2(m^2, m^2; p^2)}{3^2 + D_2(m^2, m^2; p^2)} \right)
\]

\[
+ 3 s_L^2 c_L^2 \int_0^1 dx x (1-x) \log \frac{D_2(m^2, m^2; p^2)}{3^2 + D_2(m^2, m^2; p^2)} ,
\]

\[
+ 3 c_L^2 s_R^2 \int_0^1 dx x (1-x) \log \frac{D_2(m^2, m^2; p^2)}{3^2 + D_2(m^2, m^2; p^2)}
\]

where we defined

\[
D_2(M_1^2, M_2^2; p^2) = x M_1^2 + (1-x) M_2^2 - x (1-x) p^2,
\]

and we used the regularization with the naive ultraviolet-cutoff $\Lambda$. Noting that $\Lambda^2, m_\chi^2 \gg m_t^2, p^2$, we can obtain a more convenient expression for $Z_H$:

\[
Z_H(p^2) \simeq \frac{N_c g_{tX0}^2}{(4\pi)^2} \left( (s_L^2 + c_L^2 c_R^2) \log (1 + \Lambda^2/m_\chi^2) \right.
\]

\[
+ c_L^2 s_R^2 \left\{ \log \frac{\Lambda^2}{m_t^2} + \frac{5}{3} + \frac{4 m_t^2}{p^2} \right\} \left( 1 + \frac{2 m_t^2}{p^2} \right) + k \right)
\]

with

\[
F(m^2, p^2) \equiv \left\{ \begin{array}{cc}
\sqrt{1 - \frac{4 m_t^2}{p^2}} \log \frac{1 + \sqrt{1 - \frac{4 m_t^2}{p^2}}}{1 - \sqrt{1 - \frac{4 m_t^2}{p^2}}} & (p^2 > 4 m_t^2), \\
2 \sqrt{\frac{4 m_t^2}{p^2} - 1} \arctan \frac{1}{\sqrt{\frac{4 m_t^2}{p^2}}} & (p^2 \leq 4 m_t^2),
\end{array} \right.
\]

where we introduced a constant term $k(\approx 1)$ arising from ambiguities of various regularizations. Substituting Eq. (36) with $p^2 = 0$ for Eq. (33), we obtain the PS formula,

\[
\frac{t_{LX_R}^2}{m_t^2 c_L^2} = \frac{N_c}{8\pi^2} \left[ (s_L^2 + c_L^2 c_R^2) \log (1 + \Lambda^2/m_\chi^2) + c_L^2 s_R^2 \log \frac{\Lambda^2}{m_t^2} + k \right].
\]

Since we find that Eq. (38) is a closed equation for $s_R$ due to the relations (31), $(s_L s_R)/(c_L c_R) = m_t/m_\chi$, we can solve Eq. (38) numerically,

\[
m_{\text{dyn}} = 0.5 - 0.9 \text{ TeV}, \quad s_R = 0.2 - 0.3, \quad s_L \simeq 0.1
\]
with \( v = 246 \text{ GeV} \) and \( m_t = 174 \text{ GeV} \), where we used \( k = 0, 1, 2, \Lambda = 20 - 30 \text{ TeV} \) and \( m_\chi = 5 - 10 \text{ TeV} \), for example. These parameters are fairly safe from the present constraint on the \( T \)-parameter with \( m_H = \pm 1 \text{ TeV} \) [3].

Now, we estimate the \( H \to gg \) amplitude \( A \). Using Eq. (32), the contribution of the top-quark loop is given by

\[
A_{\text{top}}(\text{TSS}) = \frac{c_L^2 A_{\text{SM}}(\tau)}{v}, \tag{40}
\]

which is almost equivalent to the SM contribution. Since we consider the Higgs boson mass to be larger than the threshold of the top-pair production, \( A_{\text{top}}(\text{TSS}) \) has an imaginary part. (See also Eqs. (3) and (4).) Eqs. (31), (22) and (33) lead to the contribution of the \( \chi \) field

\[
A_{\text{heavy}}(\text{TSS}) = s_{L,R} \frac{m_{\text{dyn}}}{m_\chi} \frac{\alpha_s}{12\pi v} \left( \frac{c_L}{2} \right) \tag{41}
\]

at the leading order of \( m_{\text{dyn}}^2/m_\chi^2 \). Although we do not explicitly integrate out the heavy fermion \( \chi \), this effect is translated in the estimate of \( A_{\text{heavy}}(\text{TSS}) \) in Eq. (11).

We evaluate the coloredon contribution \( A_{\text{col}} \) in TSS models from Eq. (23).

\[
A_{\text{col}}(\text{TSS}) = - 4 \frac{g_{AA}^2 + g_v^2}{4Z_H(m_H^2)} g_{\chi_0}^2 \left( \frac{v}{M} \right)^2 \tag{42}
\]

with \( p^2 = m_H^2 \), \( g_{AA} = g_v \), \( g_{\chi_0} \approx g' \approx g_{\text{crit}} \) and \( N_{g_{\chi_0}} \approx 8\pi^2 \). We note that \( A_{\text{col}}(\text{TSS}) \) has an imaginary part arising from \( Z_H(m_H^2) \) due to \( m_H^2 > 4m_t^2 \). Numerically, we find \( A_{\text{top}}, A_{\text{col}} \) and \( A_{\text{heavy}} \),

\[
A_{\text{top}}(\text{TSS}) = \left[ (1.7 - 0.1) + (1.5 - 1.1)i \right] \times 10^{-2} \tag{43}
\]

\[
A_{\text{col}}(\text{TSS}) = \left[ -(1 - 3) + i(0 - 0.2) \right] \times 10^{-2} \tag{44}
\]

and

\[
A_{\text{heavy}}(\text{TSS}) \sim \frac{10^{-4}}{(1 \text{ TeV})}, \tag{45}
\]

for \( m_H = 0.5 - 1 \text{ TeV} \) with \( \alpha_s = 0.12, \Lambda = 20 - 30 \text{ TeV} \) and \( m_\chi = 5 - 10 \text{ TeV} \), where we used the reference value of the coloron mass \( M = 2.1 \text{ TeV} \), corresponding to the expected bound for the direct production at TeV 33 [20]. (See also Table I.) The \( H \to gg \) amplitude \( A \) is given by

\[
A(\text{TSS}) = \frac{([+1 - 1] + (1 - 2)i) \times 10^{-2}}{(1 \text{ TeV})}, \tag{46}
\]

for \( m_H = 0.5 - 1 \text{ TeV} \). Although the real part of \( A_{\text{top}} \) is comparable to the size of its imaginary part up to \( m_H = 0.6 \text{ TeV} \), the imaginary part becomes dominant around \( m_H = 1 \text{ TeV} \). The real part of \( A_{\text{col}} \) is sizable for \( m_H = 0.5 - 1 \text{ TeV} \), while the imaginary part of \( A_{\text{col}} \) is negligible. The contribution \( A_{\text{heavy}} \) of the \( \chi \) field is also suppressed due to the constraint on the \( T \)-parameter (\( s_L < 1 \)). We thus find that the real (imaginary) part of the \( H \to gg \) amplitude \( A \) is dominated by \( A_{\text{col}}(\text{TSS}) \) for \( m_H = 0.8 - 1 \text{ TeV} \).

We have shown that the contribution of colorons can be sizable in the \( U(1) \) tilting model in the situation of \( M \sim O(\text{1 TeV}) \) and \( \Lambda \sim m_\chi \sim O(10 \text{ TeV}) \). However, we have not included higher order loop corrections. Since colorons are strongly coupled to the top quark, diagrams including top-quark loops are possibly sizable. In TSS models, the amplitude of \( H \to V_L V_L (V_L = W_L, Z_L) \) is also likely to receive non-perturbative effects due to the heavy Higgs boson mass near the perturbative unitarity bound, \( m_H \sim O(1 \text{ TeV}) \). Thus, the Higgs boson in TSS models may be studied in non-perturbative approaches such as Bethe-Salpeter equations. This, however, is out of the scope of this paper. An analysis of the Higgs boson production of TSS models at the LHC will be performed elsewhere [22].

### B. Analysis for TC2

In TC2, the condensate of the techni-fermion \( T \) mainly triggers the EWSB,

\[
v^2 = v_{\text{TC}}^2 + v_{\text{top}}^2, \quad v_{\text{TC}} \sim v, \tag{47}
\]
While the dynamical mass is small in TC2, \( m_{\text{dyn}} \) due to enhancement of the top-Yukawa coupling, \( m_{\text{top}} \), leads to the Higgs boson mass in TC2 as not suffering from the contribution of colorons. We estimate \( m_{\text{top}} \) in Ref. [13]. There is no study of backgrounds for Tevatron. In the table, we estimate signals of the Higgs boson in TC2 by using Eq. (52) and the SM value shown in Table 29 in Ref. [13]. We assume that the main decay mode of the Higgs boson in TC2 is pair production of weak bosons.

| \( m_H [\text{GeV}] \) | 180 | 190 |
|----------------------|-----|-----|
| signals of \( H \) in TC2 [fb] | 8 – 14 | 7 – 12 |
| signals in the SM [fb] | 0.85 | 0.73 |
| backgrounds [fb] | 3.8 | 7.5 |
| \( S/\sqrt{B} \) for 2 fb\(^{-1} \) in TC2 | 6 – 10 | 3 – 6 |
| \( S/\sqrt{B} \) for 2 fb\(^{-1} \) in the SM | 0.62 | 0.38 |

TABLE III: Expected signals of \( gg \rightarrow H \rightarrow W^+W^- \rightarrow \ell\ell\nu\bar{\nu} \) in TC2 after the kinematical cuts and the likelihood cut at the Tevatron. In the table, we estimate signals of the Higgs boson in TC2 by using Eq. (52) and the SM value shown in Table 29 in Ref. [13]. There is no study of backgrounds for mH ≥ 200 GeV in Ref. [13]. We assume that the main decay mode of the Higgs boson in TC2 is pair production of weak bosons.

| \( m_H [\text{GeV}] \) | 200 | 240 | 280 | 320 |
|----------------------|-----|-----|-----|-----|
| signals of \( H \) in TC2 for 30 fb\(^{-1} \) | 486 – 864 | 792 – 1408 | 810 – 1440 | 810 – 1440 |
| signals in the SM for 30 fb\(^{-1} \) | 54 | 88 | 90 | 90 |
| backgrounds for 30 fb\(^{-1} \) | 7 | 15 | 17 | 16 |
| \( S/\sqrt{B} \) for 30 fb\(^{-1} \) in TC2 | 184 – 327 | 204 – 364 | 196 – 349 | 203 – 360 |
| \( S/\sqrt{B} \) for 30 fb\(^{-1} \) in the SM | 20.4 | 22.7 | 21.8 | 22.5 |

TABLE IV: Expected signals of \( H \rightarrow ZZ \rightarrow 4\ell \) in TC2 after \( p_T \) cut at the LHC. In the table, we estimate signals of the Higgs boson in TC2 by using Eq. (52) and the SM value shown in Table 19-21 in Ref. [14]. We assume that the main decay mode of the Higgs boson in TC2 is pair production of weak bosons.

where condensates of \( \bar{T}T \) and \( \bar{t}t \) provide \( v_{TC} \) and \( v_{\text{top}} \), respectively. Adjusting the value of \( v_{\text{top}} \), we can obtain the experimental value of the top quark mass \( m_t^{\text{exp}} \); the PS formula,

\[
v_{\text{top}}^2 = \frac{N_c}{8\pi^2} (m_t^{\text{exp}})^2 \ln \frac{M^2}{(m_t^{\text{exp}})^2},
\]

which is same as Eq. (27), leads to \( v_{\text{top}}/v = 1/(3 - 4) \). While the dynamical mass is small in TC2, \( m_{\text{dyn}} = m_t^{\text{exp}} = 174 \text{ GeV} \), the top-loop effect becomes very large due to enhancement of the top-Yukawa coupling \( (g_t \propto m_t^{\text{exp}}/v_{\text{top}}) \). As a result, the gluon fusion process does not suffer from the contribution of colorons. We estimate the Higgs boson mass in TC2 as \( m_t^{\text{exp}} < m_H < 2m_t^{\text{exp}} \).

We easily obtain the contribution of colorons \( A_{\text{col}} \) in TC2 from Eq. (28):

\[
A_{\text{col}}(TC2) \sim -\frac{\alpha}{\pi v_{\text{top}}} \left(\frac{m_t^{\text{exp}}}{M}\right)^2 \sim -\frac{0.3 \times 10^{-2}}{(1 \text{ TeV})},
\]

with \( g_{t0} \approx g' \sim g_{\text{crit}}' \) and \( M = 2.1 \text{ TeV} \). On the other hand, the effect of enhancement of the top-Yukawa coupling is given by

\[
A_{\text{top}}(TC2) = \frac{v}{v_{\text{top}}} A_{\text{SM}}(\tau), \quad \frac{v}{v_{\text{top}}} \simeq 3 - 4,
\]

where the value of the SM is numerically found,

\[
A_{\text{SM}}(\tau) = \frac{(1.4 - 1.7) \times 10^{-2}}{(1 \text{ TeV})},
\]

for \( m_H = 180 - 320 \text{ GeV} \). We emphasize that the value of \( v/v_{\text{top}} \) is not changed, even if the coloron mass \( M \) is taken to the order of a multi-TeV. Unless we specify the matter content of TC2 in detail, we cannot estimate contributions of techni-fermions having QCD charges, i.e. \( A_{\text{heavy}} \) in our notation. For example, techni-fermions in models of Ref. [8] do not have QCD charges, so that they do not contribute to the gluon fusion process. We then estimate the enhancement factor to the SM value as

\[
A(\text{TC2}) = A_{\text{top}} + A_{\text{col}} \sim (3 - 4) \times A_{\text{SM}}
\]

with \( A_{\text{heavy}} = 0 \) and find that the coloron effect \( A_{\text{col}} \) is negligible compared to the contribution of the top-loop \( A_{\text{top}} \).
Now, we study the signature of Higgs boson production at hadron colliders. Since masses of top-pions $\pi_t$ much below $\sim 165$ GeV are phenomenologically forbidden \cite{2} due to the absence of the decay mode $t \rightarrow \pi_t^+ + b$, the main decay mode of the Higgs boson is expected to be pair production of weak bosons. In this situation, we can apply the SM analysis at the Tevatron \cite{3} and at the LHC \cite{4} directly to TC2. Although $HWW$- and $HZZ$-couplings in TC2 are suppressed,

$$g_{HV}(TC2)/g_{HV}(SM) = v_{top}/v, \quad V = W, Z$$ \hspace{1cm} (53)$$

the effects are irrelevant under the narrow width approximation, because the branching ratio of $H \rightarrow VV$ is not changed from the SM in the mass range $m_{\text{top}}(\text{exp}) < m_H < 2m_{\text{top}}(\text{exp})$. The considerable enhancement of the top-Yukawa gives a chance to observe the Higgs boson in TC2 even at the Tevatron Run II. (See Table II) While we cannot expect to find any evidence of the SM Higgs boson with $m_H \sim 200$ GeV for an integrated luminosity of 2 fb$^{-1}$ at the Tevatron Run II, we can estimate $S/\sqrt{B} = 3 - 6$ for the Higgs boson in TC2 with 2 fb$^{-1}$ and $m_H = 190$ GeV. We show expected signals of the Higgs boson in TC2 up to $m_H = 190$ GeV at the Tevatron, since there is no background estimate for $m_H \geq 200$ GeV in Ref. \cite{13}. At the LHC, signals of the Higgs boson in TC2 are considerably enhanced. (See Table IV) The Higgs boson in TC2 with the mass range $m_{\text{top}}(\text{exp}) < m_H < 2m_{\text{top}}(\text{exp})$ can be discovered at the LHC much more easily than the SM Higgs boson.

\section*{IV. SUMMARY AND DISCUSSION}

In this paper, we have studied the effective $ggH$-coupling in TCMs. We have considered the effect of colorons, in addition to the loop contributions of the top quark and other heavy quarks. In order to estimate the coloron effect, we have derived the low-energy effective theory by eliminating colorons by using the EOM for colorons. We have found that the contribution of colorons $\mathcal{A}_{\text{col}}$ is proportional to $m_{\text{dyn}}^2/M^2$. Thus, $\mathcal{A}_{\text{col}}$ becomes sizable for the coloron mass $M \sim O(1 \text{TeV})$ and the dynamical mass $m_{\text{dyn}} \sim O(0.6 \text{TeV})$. An important point is that the large dynamical mass $m_{\text{dyn}} \sim O(0.6 \text{TeV})$ can be realized consistently with the experimental value of the top-quark mass $m_{\text{top}}(\text{exp})$ in TSS models, while the dynamical mass itself is adjusted to $m_{\text{top}}(\text{exp})$ in TC2. We have shown that the contribution of colorons is actually sizable in TSS models with $M \sim O(\text{TeV})$ and $m_\chi \sim O(10 \text{TeV})$: we have evaluated the contributions of colorons and the top-loop as $\mathcal{A}_{\text{col}} = [-1(3) + (0.1 - 0.2)i] \times 10^{-3} \text{(TeV)}^{-1}$ and $\mathcal{A}_{\text{top}} = [(0.4 - 0.1) + (1.4 - 1.1)i] \times 10^{-2} \text{(TeV)}^{-1}$, which is almost same as the SM value, for $m_H = 0.8 - 1 \text{ TeV}$ with some parameters being safe from the constraint on the $T$-parameter. Namely, $\mathcal{A}_{\text{col}}$ is numerically dominant in the real (imaginary) part of the $H \rightarrow gg$ amplitude for $m_H \sim O(1 \text{TeV})$. In TC2, we have found that the effect of colorons is negligible compared to the contribution of the top-loop, because the dynamical mass $m_{\text{dyn}}$ itself is small and the top-Yukawa coupling is enhanced by a factor of $3 - 4$. Since the Higgs boson production is considerably enhanced in TC2, we can obtain signatures of the Higgs boson in TC2 with $m_H \sim 200$ GeV even at the Tevatron Run II as well as at the LHC. In particular, we have evaluated $S/\sqrt{B} = 3 - 6$ for an integrated luminosity of 2 fb$^{-1}$ and $m_H = 190$ GeV at the Tevatron Run II.

We have not studied the expected signals of the Higgs boson in TSS models at hadron colliders. Since the $WW$-fusion process becomes comparable to the gluon fusion in the mass range $m_H \sim O(1 \text{TeV})$, we would also needed to study the $WW$-fusion process for TSS models. In addition, we may investigate the Higgs boson production in non-perturbative approaches, because $m_H \sim O(1 \text{TeV})$ is near the perturbative unitarity bound. A detailed analysis of the expected signals of the Higgs boson in TSS models will be performed elsewhere \cite{12}.

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