Efficient Privacy-Preserving Nonconvex Optimization

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Abstract

While many solutions for privacy-preserving convex empirical risk minimization (ERM) have been developed, privacy-preserving nonconvex ERM remains under challenging. In this paper, we study nonconvex ERM, which takes the form of minimizing a finite-sum of nonconvex loss functions over a training set. To achieve both efficiency and strong privacy guarantees with efficiency, we propose a differentially-private stochastic gradient descent algorithm for nonconvex ERM, and provide a tight analysis of its privacy and utility guarantees, as well as its gradient complexity. We show that our proposed algorithm can substantially reduce gradient complexity while matching the best-known utility guarantee obtained by Wang et al. (2017). We extend our algorithm to the distributed setting using secure multi-party computation, and show that it is possible for a distributed algorithm to match the privacy and utility guarantees of a centralized algorithm in this setting. Our experiments on benchmark nonconvex ERM problems and real datasets demonstrate superior performance in terms of both training time and utility gains compared with previous differentially-private methods using the same privacy budgets.

1 Introduction

For many important domains such as health care and medical research, the datasets used for training machine learning models contain sensitive personal information. There is a risk that models trained on this data can reveal private information about individual records in that training data (Fredrikson et al., 2014; Shokri et al., 2017; Carlini et al., 2019). This motivates the research on privacy-preserving machine learning.

Much of this work has focused on achieving differential privacy (Dwork et al., 2006), a rigorous definition of privacy that provide statistical data privacy for individual records. In the past decade, many differentially private machine learning algorithms for solving the empirical risk minimization (ERM) problem (3.1) have been proposed (Chaudhuri and Monteleoni, 2009; Chaudhuri et al., 2011; Kifer et al., 2012; Bassily et al., 2014; Zhang et al., 2017; Wang et al., 2017; Jayaraman et al., 2018; Wang and Gu, 2019a,b). Almost all of them are for ERM with convex loss functions, but many important machine learning approaches such as deep learning are formulated as ERM problems with nonconvex loss functions. In addition, these learning problems are often of large-scale (the number of training examples, \( n \), is large), requiring the use of stochastic optimization algorithms such as stochastic gradient descent (SGD). Due to the nonconvex objective function and the corresponding optimization problem at scale, defferentially-private algorithms for solving large-scale nonconvex ERM are in high demand.
Recently, several studies have advanced the application of differential privacy in deep learning (Abadi et al., 2016; Papernot et al., 2016; Xie et al., 2018). They focus on satisfying differential privacy and evaluating utility experimentally, but lack theoretical bounds on utility. Only a few differentially-private algorithms for solving nonconvex optimization problems have proven utility bounds (Zhang et al., 2017; Wang et al., 2017). For example, Wang et al. (2017) proposed a differential private gradient descent (DP-GD) algorithm with both privacy and utility guarantees. However, in each iteration of DP-GD, it needs to compute the full gradient, which is too expensive for use on large training sets. Zhang et al. (2017) proposed a random round private stochastic gradient descent (RRPSGD), which has better runtime complexity than DP-GD to achieve the same privacy guarantee. However, its utility bound is slightly worse than that of DP-GD.

Our work develops a method for differentially-private nonconvex ERM that provides both strong utility guarantees and low runtime complexity. Inspired by the recent progress on stochastic variance reduced nonconvex optimization algorithms (Johnson and Zhang, 2013; Reddi et al., 2016a; Allen-Zhu and Hazan, 2016; Lei et al., 2017; Nguyen et al., 2017; Fang et al., 2018; Zhou et al., 2018), we propose a Differentially-Private Stochastic Recursive variance reduced Gradient Decent (DP-SRGD) algorithm for nonconvex ERM. At the core of our algorithm is the stochastic path-integrated differential estimator proposed by Fang et al. (2018), which tracks the full gradient with significantly reduced variance compared with the standard stochastic gradient, and an analysis of the privacy guarantees using Rényi Differential Privacy (RDP) (Mironov, 2017).

Contributions. We develop a differentially-private stochastic variance reduced gradient algorithm for nonconvex ERM and provide a sharp analysis of the privacy guarantee (Section 4). Our algorithm matches the best-known utility guarantee (Wang et al., 2017) for nonconvex optimization with lower computational complexity compared with prior methods (Zhang et al., 2017; Wang et al., 2017). To achieve the same utility guarantee, the gradient complexity (i.e., the number of stochastic gradients calculated in total) of our algorithm is $O(n^{3/2})$, which outperforms the state-of-the-art results (Zhang et al., 2017; Wang et al., 2017) by a factor of $\Theta(n^{1/2})$.

We further extend our algorithm to the distributed setting (sometimes known as federated learning or distributed learning) using a multi-party computation protocol (Section 5). This results in the first differentially-private distributed optimization algorithm with theoretical guarantees that match that of the centralized differentially private nonconvex optimization algorithm.

We implement our proposed methods on two nonconvex ERM problems: nonconvex logistic regression and convolutional neural networks. We report on experiments on several real datasets (Section 6), finding that our methods can produce the models that are closest to the non-private models in terms of model accuracy.

Notation. We use curly symbol such as $\mathcal{B}$ to denote the index set. For a set $\mathcal{B}$, we use $|\mathcal{B}|$ to denote its cardinality. For a finite sum function $F = \sum_{i=1}^n f_i/n$, we denote $F_{\mathcal{B}}$ by $\sum_{i\in \mathcal{B}} f_i/|\mathcal{B}|$. For a $d$-dimensional vector $x \in \mathbb{R}^d$, we use $\|x\|_2$ to denote its $\ell_2$-norm. Given two sequences $\{a_n\}$ and $\{b_n\}$, if there exists a constant $0 < C' < \infty$ such that $a_n \leq C b_n$, we write $a_n = O(b_n)$. In addition, if there exist constants $0 < C_1, C_2 < \infty$ such that $C_1 b_n \leq a_n \leq C_2 b_n$, we write $a_n = \Omega(b_n)$.

2 Related Work

Plenty of differentially-private machine learning algorithms for convex ERM have been proposed in the past decade. Specifically, there are three main approaches to achieve differential privacy in such settings, including output perturbation (Wu et al., 2017; Zhang et al., 2017), objective perturbation (Chaudhuri et al., 2011; Kifer et al., 2012; Iyengar et al., 2019), and gradient perturbation (Bassily et al., 2014; Wang et al., 2017;
Table 1: Comparison of different \((\epsilon, \delta)\)-DP algorithms for nonconvex optimization. We report the utility bound in terms of \(E\|\nabla F(\theta^p)\|_2\), where \(\theta^p\) is the output of differentially private algorithm, \(E\) is taken over the randomness of the algorithm. For the Distributed DP-SRGD, \(m\) is the number of party and \(\tilde{n}\) is the dataset size of each party.

| Algorithm                  | Utility                                      | Gradient Complexity |
|----------------------------|----------------------------------------------|---------------------|
| DP-GD (Wang et al., 2017)  | \(O\left(\frac{(d \log (1/\delta))^{1/4}}{(n\epsilon)^{1/2}}\right)\) | \(O\left(n^2\epsilon\right)\) |
| RRPSGD (Zhang et al., 2017)| \(O\left(\frac{(d \log (n/\delta) \log (1/\delta))^{1/4}}{(n\epsilon)^{1/2}}\right)\) | \(O\left(n^2\right)\) |
| DP-SRGD (This paper)       | \(O\left(\frac{(d \log (1/\delta))^{1/4}}{(n\epsilon)^{1/2}}\right)\) | \(O\left(n^{3/2}/\epsilon\right)\) |
| Distributed DP-SRGD (This paper) | \(O\left(\frac{(d \log (1/\delta))^{1/4}}{(m\tilde{n}\epsilon)^{1/2}}\right)\) | \(O\left((m\tilde{n})^{3/2}/\epsilon\right)\) |

Jayaraman et al., 2018). However, it is very hard to generalize these methods to nonconvex ERM except for the gradient perturbation approach. Therefore, most of the differentially private algorithms for nonconvex ERM are based on the gradient perturbation approach, including our work in this paper. The problem with gradient perturbation approaches is their iterative nature quickly consumes any reasonable privacy budget. Hence, the main challenge for nonconvex ERM with differential privacy is to develop algorithms that can provide sufficient utility while maintaining privacy with high computational efficiency.

Several recent works (Abadi et al., 2016; Papernot et al., 2016; Xie et al., 2018) studied deep learning with differential privacy. Abadi et al. (2016) proposed a method called moments accountant to keep track of the privacy cost of stochastic gradient descent algorithm during the training process, which provides a strong privacy guarantee. Papernot et al. (2016) established a Private Aggregation of Teacher Ensembles (PATE) framework to improve the privacy guarantee of deep learning for classification tasks. Xie et al. (2018) and Yoon et al. (2019) investigated the differentially private Generative Adversarial Nets (GAN) with different distance metrics. However, none of these works provide utility guarantees for their algorithms.

Table 2 summarizes provable differentially private nonconvex optimization algorithms for nonconvex ERM. The only studies to date giving utility guarantees of their differentially private algorithms for nonconvex optimization problems are Zhang et al. (2017) and Wang et al. (2017). The random round private stochastic gradient descent (RRPSGD) developed by Zhang et al. (2017) is the first differentially private nonconvex optimization algorithm with the utility guarantee. Inspired by the random round SGD algorithm (Ghadimi and Lan, 2013), Zhang et al. (2017) proposed to perform the perturbed SGD, i.e., adding Gaussian noise to stochastic gradient in each iteration, for a random number of iterations, which is drawn from some distribution. They showed that RRPSGD is able to find a stationary point in expectation with a diminishing error \(O\left((d \log (n/\delta) \log (1/\delta))^{1/4}/(n\epsilon)^{1/2}\right)\), where \(\epsilon\) is the privacy budget, \(n\) is the number of observations, \(d\) is the dimension, and \(\delta \in (0, 1)\) is confidence parameter for \((\epsilon, \delta)\)-DP. In addition, the gradient complexity of RRPSGD is \(O\left(n^2\right)\). Their analysis of the privacy guarantee is mainly based on the previous framework (Bassily et al., 2014), which makes use of the standard privacy amplification via sampling and strong
composition theorem. Although such an analysis can be easily adopted to the nonconvex setting with stochastic optimization algorithms, it will result in a large bound on the variance of the adding noise compared with relaxed definitions such as the moments accountant (Abadi et al., 2016).

Wang et al. (2017) proposed a differentially private gradient descent (DP-GD) algorithm for nonconvex optimization. They showed that DP-GD has a comparable gradient complexity $O(n^2\epsilon)$, and an improved utility guarantee $O((\log(n/\delta))^{1/4}/(n\epsilon)^{1/2})$ compared with that of RRPSGD. The reason that DP-GD can achieve such improvement, i.e., by a factor of $O((\log(n/\delta))^{1/4})$, is that DP-GD uses full gradient rather than stochastic gradient. Nevertheless, this makes DP-GD computationally very expensive or even intractable for large-scale machine learning problems ($n$ is big). Recently, Wang et al. (2019a) also proposed a differentially private method for solving the nonconvex optimization problem based on stochastic gradient descent. Their goal is to find the local minima, while our aim is to find the stationary point. In addition, their utility guarantee is asymptotic, i.e., they need to run an infinite number of iterations to get the proposed utility guarantee. In sharp contrast, our utility guarantee holds for a finite number of iterations.

Although distributed machine learning enables collaborative learning across different parties, the privacy requirement becomes more acute. In such setting, each party not only needs to provide protection against inference attacks on the learned model but also wishes to share nothing about its own records with other parties throughout the learning process. There is a series of work (Pathak et al., 2010; Bindschaedler et al., 2017; Heikkilä et al., 2017; Shi et al., 2017; Chase et al., 2017; Jayaraman et al., 2018) studying differential privacy in distributed setting. The most relevant one to ours is the multi-party computation (MPC) based framework studied in Jayaraman et al. (2018). They proposed to aggregate different parties’ local gradients with a secure computation, and perturbed it within the MPC. Although their method can achieve the state-of-the-art theoretical guarantees in the setting of convex optimization, it does not have privacy or utility guarantee in the nonconvex optimization settings.

3 Preliminaries

We consider the empirical risk minimization (ERM) problem: given a training set $S = \{(x_1, y_1), \ldots, (x_n, y_n)\}$ drawn from some unknown but fixed data distribution with $x_i \in \mathbb{R}^D$, $y_i \in \mathcal{Y} \subseteq \mathbb{R}$, we aim to find a solution $\hat{\theta} \in \mathbb{R}^d$ that minimizes the empirical risk,

$$
\min_{\theta \in \mathbb{R}^d} F(\theta) := \frac{1}{n} \sum_{i=1}^{n} f_i(\theta),
$$

where $F(\theta)$ is the empirical risk function (i.e., training loss), $f_i(\theta) = \ell(\theta; x_i, y_i)$ is the loss function defined on the $i$-th training example $(x_i, y_i)$, and $\theta \in \mathbb{R}^d$ is the model parameter we want to learn.

Here, we provide some definitions and lemmas that will be used in our theoretical analysis.

**Definition 3.1.** $\theta \in \mathbb{R}^d$ is an $\zeta$-approximate stationary point if $\|\nabla f(\theta)\|_2 \leq \zeta$.

**Definition 3.2.** A function $f: \mathbb{R}^d \to \mathbb{R}$ is $G$-Lipschitz, if for all $\theta_1, \theta_2 \in \mathbb{R}^d$, we have

$$
|f(\theta_1) - f(\theta_2)| \leq G\|\theta_1 - \theta_2\|_2.
$$
**Definition 3.3.** A function \( f : \mathbb{R}^d \to \mathbb{R} \) has \( L \)-Lipschitz gradient, if for all \( \theta_1, \theta_2 \in \mathbb{R}^d \), we have

\[
\| \nabla f(\theta_1) - \nabla f(\theta_2) \|_2 \leq L \| \theta_1 - \theta_2 \|_2.
\]

Differential privacy provides a formal notion of privacy, introduced by Dwork et al. (2006):

**Definition 3.4 (\((\epsilon, \delta)\)-DP (Dwork et al., 2006)).** A randomized mechanism \( \mathcal{M} : S^n \to \mathcal{R} \) satisfies \((\epsilon, \delta)\)-differential privacy if for any two adjacent data sets \( S, S' \in S^n \) differing by one element, and any output subset \( O \subseteq \mathcal{R} \), it holds that

\[
P[\mathcal{M}(S) \in O] \leq e^\epsilon \cdot P[\mathcal{M}(S') \in O] + \delta.
\]

To achieve \((\epsilon, \delta)\)-DP for a given function \( q : S^n \to \mathcal{R} \), we can use Gaussian mechanism (Dwork and Roth, 2014) \( \mathcal{M} = q(S) + u \), where \( u \) is sampled from Gaussian distribution with variance that is proportional to the \( \ell_2 \)-sensitivity of the function \( q \), \( \Delta(q) \), which is defined as follows.

**Definition 3.5 (\(\ell_2\)-sensitivity (Dwork and Roth, 2014)).** For two adjacent datasets \( S, S' \in S^n \) differing by one element, the \( \ell_2 \)-sensitivity \( \Delta(q) \) of a function \( q : S^n \to \mathcal{R} \) is defined as

\[
\Delta(q) = \sup_{S, S'} \| q(S) - q(S') \|_2.
\]

Rényi differential privacy. Although the notion of \((\epsilon, \delta)\)-DP is widely used in the output and objective perturbation methods, it suffers from the loose composition and subsample amplification results, which make it unsuitable for the stochastic iterative learning algorithms. In this work, we will make use of the notion of Rényi Differential Privacy (RDP), which is proposed by Mironov (2017), and is particularly useful when the dataset is accessed by a sequence of randomized mechanisms (Wang et al., 2019b).

**Definition 3.6 (RDP (Mironov, 2017)).** For \( \alpha > 1, \rho > 0 \), a randomized mechanism \( \mathcal{M} : S^n \to \mathcal{R} \) satisfies \((\alpha, \rho)\)-Rényi differential privacy, i.e., \((\alpha, \rho)\)-RDP, if for all adjacent datasets \( S, S' \in S^n \) differing by one element, we have \( D_\alpha(\mathcal{M}(S) \| \mathcal{M}(S')) := \log \mathbb{E}(\mathcal{M}(S)/\mathcal{M}(S'))^{\alpha/2} \leq \rho \), where the expectation is taken over \( \mathcal{M}(S') \).

To achieve RDP, we can use Gaussian mechanism and its corresponding subsample amplification results.

**Lemma 3.7.** Given a function \( q : S^n \to \mathcal{R} \), the Gaussian Mechanism \( \mathcal{M} = q(S) + u \), where \( u \sim N(0, \sigma^2 I) \), satisfies \((\alpha, \Delta^2(q)/(2\sigma^2))\)-RDP. In addition, if we apply the mechanism \( \mathcal{M} \) to a subset of samples using uniform sampling without replacement, \( \mathcal{M} \) satisfies \((\alpha, 5\tau^2 \Delta^2(q)\alpha/\sigma^2)\)-RDP given \( \tau^2 = \sigma^2/\Delta^2(q) \geq 1.5 \), \( \alpha \leq \log(1/\tau(1 + \sigma^2)) \), where \( \tau \) is the subsample rate.

**Remark 3.8.** According to Lemma 3.7, let \( \Delta^2(q) = 1 \), to achieve \((\alpha, 5\tau^2 \alpha/\sigma^2)\)-RDP of the subsampled Gaussian mechanism, we need the conditions that \( \sigma^2 \geq 1.5, \alpha \leq \log(1/\tau(1 + \sigma^2)) \). For the moment accountant based method (Abadi et al., 2016), it can achieve the following asymptotic privacy guarantee of \((\alpha, \tau^2 \alpha/(1 - \tau)\sigma^2 + O(\tau^3 \alpha^3/\sigma^3))\)-RDP when \( \tau \) goes to zero and \( \sigma^2 \geq 1, \alpha \leq \sigma^2 \log(1/\tau\sigma) \). In contrast to moment accountant, our method has a closed-form bound on the privacy guarantee.

We also have the following composition rule for RDP.
Algorithm 1 Differentially-Private Stochastic Recursive variance reduced Gradient Descent (DP-SRGD)

**input** \( \theta^0, T, G, L, \delta, \beta \), privacy budget \( \epsilon \), accuracy for first-order stationary point \( \zeta \)

1: \textbf{for} \( t = 0, 1, 2, \ldots, T - 1 \) \textbf{do}  
2: \quad \textbf{if} \ mod (t, l) = 0 \textbf{then}  
3: \quad \quad \quad \quad \quad \quad \text{v}^t = \nabla F(\theta^t), \text{draw } u^t \sim N(0, \sigma^2 I_d) \text{ with } \sigma^2 = 2TG^2\alpha/(\beta n^2 \epsilon), \alpha = \log(1/\delta)/((1 - \beta) \epsilon) + 1  
4: \quad \quad \quad \quad \quad \quad \text{Release the differentially private gradient } v_p^t = v^t + u^t  
5: \quad \textbf{else}  
6: \quad \quad \quad \text{Uniformly sample } b \text{ examples without replacement indexed by } B_t  
7: \quad \quad \quad \quad \quad \quad \text{v}^t = \nabla F_{B_t}(\theta^t) - \nabla F_{B_t}(\theta^{t-1}) + v_p^{t-1}, \text{ where } \nabla F_{B_t}(\theta^t) = \sum_{i \in B_t} \nabla f_i(\theta^t)/b  
8: \quad \quad \quad \quad \quad \quad \text{Draw } u^t \sim N(0, \sigma^2 I_d) \text{ with } \sigma^2 = 2T\zeta^2\alpha/(\beta n^2 \epsilon), \alpha = \log(1/\delta)/((1 - \beta) \epsilon) + 1  
9: \quad \quad \quad \quad \quad \quad \text{Release the differentially private gradient } v_p^t = v^t + u^t  
10: \quad \textbf{end if}  
11: \quad \theta^{t+1} = \theta^t - \eta_t v_p^t, \text{ where } \eta_t = \min\{\zeta/(L\|v_p^t\|_2), 1/(2L)\}  
12: \textbf{end for}

**output** \( \theta \) chosen uniformly at random from \( \{\theta^t\}_{t=0}^{T-1} \).

**Lemma 3.9** (Mironov (2017)). If \( k \) randomized mechanisms \( M_i : S^n \to R \) for \( i \in [k] \), satisfy \((\alpha, \rho_i)\)-RDP, then their composition \( (M_1(S), \ldots, M_k(S)) \) satisfies \((\alpha, \sum_{i=1}^k \rho_i)\)-RDP. Moreover, the input of the \( i \)-th mechanism can base on the outputs of previous \((i - 1)\) mechanisms.

Based on Lemmas 3.7 and 3.9, we will use RDP-based analysis to establish the privacy guarantee of our algorithms, which can give us a strong utility guarantee. Finally, we illustrate the relationship between RDP and \((\epsilon, \delta)\)-DP in the following lemma.

**Lemma 3.10** (Mironov (2017)). If a randomized mechanism \( M : S^n \to R \) satisfies \((\alpha, \rho)\)-RDP, then \( M \) satisfies \((\rho + \log(1/\delta)/(\alpha - 1), \delta)\)-DP for all \( \delta \in (0, 1) \).

### 4 Algorithm

Our proposed algorithm for nonconvex ERM can be seen as an extension of the stochastic recursive variance-reduced gradient descent algorithm (Nguyen et al., 2017; Fang et al., 2018; Zhou et al., 2018) for finite-sum nonconvex optimization. The detailed algorithm is illustrated in Algorithm 1. The main idea is that we will construct the gradient estimator \( v^t \) iteratively based on the information obtained from the previous update. More specifically, we initialize the \( v^t \) to the current full gradient every \( l \) iterations, and inject Gaussian noise \( u^t \) with covariance matrix \( \sigma^2 I_d \) into \( v^t \), i.e., line 3, to make it differentially private. In the subsequent \((l - 1)\) iterations, we recursively update \( v^t \), i.e., line 7, as \( v^t = \nabla F_{B_t}(\theta^t) - \nabla F_{B_t}(\theta^{t-1}) + v_p^{t-1}, \) where \( \nabla F_{B_t}(\theta^t) \) and \( \nabla F_{B_t}(\theta^{t-1}) \) are mini-batch stochastic gradients, and \( v_p^{t-1} \) is the private gradient estimator released at the last iteration. After updating \( v^t \), we again inject Gaussian noise \( u^t \) with covariance matrix \( \sigma^2 I_d \), i.e., line 9, to make it differentially private. The variance \( \sigma^2 \) of the Gaussian random vectors are determined by our later RDP-based analysis.
4.1 Main Theoretical Results

Here, we present formal results on the privacy and utility guarantees of Algorithm 1. Our proof involves new techniques for the privacy and utility guarantees that are of general use for variance reduction-based algorithms. In particular, we propose an RDP-based analysis (the full proof is in Appendix B.1) for characterizing the privacy loss of our method. It considers two different cases to characterize the sensitivity of our method: (1) at the beginning of each epoch, i.e., \( \mod(t, l) = 0 \); and (2) the subsequent \((l - 1)\) iterations in each epoch. To provide the utility guarantee (the full proof is in Appendix B.2), we provide a new bound for the difference between the full gradient and our differentially private gradient estimator \( v^t \). Equipped with this bound, we show that the utility guarantee of our method depends on the accuracy of the first-order stationary point \( \zeta \) as well as the error introduced by our privacy mechanism. By solving for the smallest \( \zeta \), we obtain the utility guarantee for our method.

**Theorem 4.1.** Suppose that each component function \( f_i \) is \( G \)-Lipschitz and has \( L \)-Lipschitz gradient. Given the total number of iterations \( T \) and the accuracy for the first-order stationary point \( \zeta \), for any \( \delta > 0 \) and the privacy budget \( \epsilon \), Algorithm 1 satisfies \((\epsilon, \delta)\)-differential privacy with \( \sigma_1^2 = 2TG^2\alpha/(\beta n^2\epsilon) \) and \( \sigma_2^2 = 20T\zeta^2\alpha/(\beta n^2\epsilon) \), where \( \alpha = \log((1/\delta)/(\log(1/\delta)) + 1, \) if there exists \( \beta \in (0, 1) \) such that \( \alpha \leq \log \left( \frac{\beta n^2\epsilon}{(5b^2T\alpha + \beta n^2\epsilon)} \right) \) and \( 5b^2T\alpha/(\beta n^2\epsilon) \geq 1.5 \).

**Remark 4.2.** According to Theorem 4.1, there exist a constraint on the parameter \( \alpha \), which is due to the subsample amplification result in Lemma 3.7, and is similar to the constraint given by moments accountant (Abadi et al., 2016). This constraint can be removed if we use the analytic framework proposed by Bassily et al. (2014). However, such an analysis would introduce an extra \( \log(T/\delta) \) factor in the variance \( \sigma^2 \) of the noise, which will lead to a worse utility guarantee. Since \( \beta \) is a constant, we will omit the dependence of \( \beta \) in our results in the rest of the paper.

The following theorem shows the utility guarantee and the gradient complexity, which is the total number of the stochastic gradients we need to estimate during the training process of Algorithm 1.

**Theorem 4.3.** Under the same conditions of Theorem 4.1 on \( f_i, \sigma^2, \alpha \), if we choose \( l = b = \sqrt{n} \), the number of iterations \( T = C_1n\epsilon\sqrt{LD_F}/(G\sqrt{d}\log(1/\delta)) \), where \( D_F = F(\theta^0) - F(\theta^*) \) and \( F(\theta^*) \) is a global minimum of \( F \), the accuracy for first-order stationary point \( \zeta = C_2G^{1/2}(LD_F\log(1/\delta))^{1/4}/(n\epsilon)^{1/2} \), the output \( \hat{\theta} \) of Algorithm 1 satisfies the following

\[
\mathbb{E}\|\nabla F(\hat{\theta})\|_2 \leq C_3G^{1/2}(LD_F\log(1/\delta))^{1/4}/(n\epsilon)^{1/2},
\]

where \( C_1, C_2, C_3 \) are absolute constants, and the expectation is taken over all the randomness of the algorithm, i.e., the random Gaussian noise and the subsample gradient. Since we have \( T = O(n\epsilon) \) and \( l = b = \sqrt{n} \), the total gradient complexity of Algorithm 1 is \((T/l \cdot n + Tb)\), which is at the order of \( O(n^{3/2}\epsilon) \).

**Remark 4.4.** Our method can achieve a utility guarantee in \( O(G^{1/2}(dLD_F\log(1/\delta))^{1/4}/(n\epsilon)^{1/2}) \), which matches the best known result for differentially private nonconvex optimization method (Wang et al., 2017). However, their method is based on gradient descent, which is computationally very expensive in large-scale machine learning problems. Compared with the stochastic gradient method proposed by Zhang et al. (2017), our utility guarantee is better by a factor of \( O(\log(n/\delta)) \). Furthermore, the gradient complexity of our method, i.e., \( O(n^{3/2}\epsilon) \), is smaller than \( O(n^2\epsilon) \) gradient complexity provided by Zhang et al. (2017) and Wang et al. (2017).
5 Extension to Distributed Private Learning

For many important problems, the training data is owned by different parties but too sensitive to expose to a single party to perform the learning. The primary goal of distributed machine learning is to enable a group of independent data owners to develop a model from their combined data without exposing that data to others. Any differentially private ERM method in distributed setting must also satisfy this primary goal.

5.1 Problem Setup and Algorithm Description

More formally, suppose that we have \( m \) parties, and without loss of generality, each party has \( \tilde{n} \) examples. In specific, party \( j \) has its own data set \( S_j = \{(x_{j1}^1, y_{j1}^1), \ldots, (x_{j\tilde{n}}^j, y_{j\tilde{n}}^j)\} \), which may contain sensitive information. We use \( S = \{S_1, S_2, \ldots, S_m\} \) to denote the combined dataset. Our goal is to solve the following nonconvex ERM problem such that each party does not release their own data to others and the learned model satisfies differential privacy.

\[
\min_{\theta \in \mathbb{R}^d} \left\{ F(\theta) : = \frac{1}{m} \sum_{j=1}^{m} F_j(\theta) = \frac{1}{m} \sum_{j=1}^{m} \sum_{i=1}^{\tilde{n}} f_{ji}(\theta) \right\},
\]

where \( F_j(\theta) \) denotes the empirical loss function for party \( j \), where each component function \( f_{ji}(\theta) = \ell(\theta; x_{ji}^j, y_{ji}^j) \) for party \( j \) is \( G \)-Lipschitz and has \( L \)-Lipschitz continuous gradient.

A straightforward way to extend Algorithm 1 to the distributed setting would be to add noise to each party’s local gradient before the aggregation. Similar ideas have been previously exploited in Pathak et al. (2010). However, this requires that the variance \( \sigma^2 \) of the adding noise scales inversely proportional to \( \tilde{n} \). To further reduce the noise magnitude, we incorporate secure multi-party computation (MPC) into our algorithmic framework, as was done by Jayaraman et al. (2018) for convex ERM.

By using cryptographic techniques, MPC protocols enable participants to compute a functionality in a joint way over their private inputs. Several recent works (Nikolaenko et al., 2013; Gascón et al., 2017; Ma et al., 2018; Wang et al., 2018), have shown that MPC protocols can be efficiently employed in distributed machine learning, including deep learning. However, MPC protocols by themselves only protect the training data throughout the learning process; they provide no protection on the resulting model against inference attacks. Thus, we propose to combine differential privacy with MPC protocols in the distributed setting.

The key modification of our Distributed Differentially-Private Stochastic Recursive variance reduced Gradient Descent (DDP-SRGD) is that each party first computes local gradients and then the local gradients are aggregated within the MPC. Differential privacy is ensured by perturbing the aggregated gradients with Gaussian noise inside the MPC framework (line 3 and line 7 of Algorithm 2).

5.2 Main Results for Distributed Setting

In this subsection, we present the privacy and the utility guarantees for our distributed method.

**Theorem 5.1.** Suppose that each component function \( f_{ji} \) is \( G \)-Lipschitz and has \( L \) Lipschitz gradient. Given the number of iterations \( T \) and the accuracy for the first-order stationary point \( \zeta \), for any \( \delta > 0 \) and the privacy budget \( \epsilon \), Algorithm 1 satisfies \((\epsilon, \delta)\)-differential privacy with \( \sigma_1^2 = 2TG^2\alpha/(\beta m\tilde{n}^2\epsilon) \).
Algorithm 2 Distributed Differentially-Private Stochastic Recursive variance reduced Gradient Descent (DDP-SRGD)

\begin{algorithm}
\textbf{input} $\theta^0, T, G, L, \delta, \beta$, privacy budget $\epsilon$, accuracy for first-order stationary point $\zeta$
\begin{algorithmic}[1]
\State \textbf{for} $t = 0, 1, 2, \ldots, T - 1$ \textbf{do}
\If{mod $(t, l) = 0$}
\State Inside MPC: $v^t = \nabla F^j(\theta^t)$, $u^t \sim N(0, \sigma^2 I_d)$, $\sigma^2 = 2TG^2\alpha/(m^2\bar{n}^2\epsilon^2)$, $\alpha = \log(1/\delta)/(1 - \beta)e + 1$
\State Release the differentially private gradient $v_p^t = v^t + u^t$
\Else
\State Generate index set $B^j_t$ with $|B^j_t| = b_j$ for each party $j$ by sampling without replacement
\State Inside MPC: $v^t = \sum_{j=1}^m \sum_{i \in B^j_t} (\nabla f^j_i(\theta^t) - \nabla f^j_i(\theta^{t-1}))/(m\bar{b}) + v^t_{p-1}$, where $\bar{b} = \sum_{j=1}^m b_j/m$
\State $u^t \sim N(0, \sigma^2 I_d)$, $\sigma^2 = 20T\zeta^2\alpha/(m^2\bar{n}^2\epsilon^2)$, $\alpha = \log(1/\delta)/(1 - \beta)e + 1$
\State Release the differentially private gradient $v_p^t = v^t + u^t$
\EndIf
\EndFor
\textbf{output} $\theta$ chosen uniformly at random from $\{\theta^t\}_{t=0}^{T-1}$.
\end{algorithmic}
\end{algorithm}

and $\sigma^2 = 20T\zeta^2\alpha/(\beta\bar{m}\bar{n}^2\epsilon)$, where $\alpha = \log(1/\delta)/(1 - \beta)e + 1$, if there exists $\beta \in (0, 1)$ such that $\alpha \leq \log((\beta\bar{n}^3\epsilon)/(5\bar{b}^3T\alpha + \beta\bar{m}^2\epsilon)))$ and $5\bar{b}^3T\alpha/(\beta\bar{n}^2\epsilon) \geq 1.5$.

Remark 5.2. In distributed settings, we can ensure differential privacy if we add the noise that scales inversely proportional to $m\bar{n}$ (instead of $\bar{n}$, which would be the case with the straightforward approach if noise were added to each party’s subsampled gradients before the aggregation).

Next, we give the utility guarantee and gradient complexity of our proposed distributed method.

Theorem 5.3. Under the same conditions of Theorem 5.1 on $f^j, \sigma^2, \alpha$, if we choose $l = \sqrt{m\bar{n}}, \bar{b} = \sqrt{n/m},$ the total number of iterations $T = C_1m\bar{n}\epsilon\sqrt{LDF}/(G\sqrt{d\log(1/\delta)})$, and the accuracy for the first-order stationary point $\zeta = C_2G^{1/2}(LDF \log(1/\delta))^{1/4}/(m\bar{n}e)^{1/2}$, where $D_F = F(\theta^0) - F(\theta^*)$ and $F(\theta^*)$ is a global minimum of $F$, the output $\theta$ of Algorithm 2 satisfies

$$\mathbb{E}\|\nabla F(\bar{\theta})\|_2 \leq C_3G^{1/2}(LDF \log(1/\delta))^{1/4}/(m\bar{n}e)^{1/2},$$

where $C_1, C_2, C_3$ are absolute constants, and the expectation is taken over all the randomness of the algorithm. In addition, the gradient complexity of distributed method is $O((m\bar{n})^{3/2}\epsilon)$.

Theorem 5.3 establishes that the utility guarantee of our distributed method is $\tilde{O}(G^{1/2}(LDF)^{1/4}/(m\bar{n}e)^{1/2}),$ which matches the utility guarantee of Algorithm 1 if $n = m\bar{n}$.

6 Experiments

In this section, we present numerical experiments on different nonconvex ERM problems and on different datasets to evaluate the performance of our method. All experiments are implemented in Pytorch platform version 0.4.0 within Python 3.6.4. on Amazon AWS p3.2xlarge servers which come with Intel Xeon E5 CPU
Table 2: Comparison of different algorithms on $a9a$ dataset under different privacy budgets $\epsilon \in \{0.2, 0.5\}$ and $\delta = 10^{-5}$. Note that the non-private baseline denotes the test error of the non-private SPIDER algorithm (Fang et al., 2018). For DDP-SRGD, we choose the number of parties $m = 10$.

and NVIDIA Tesla V100 GPU (16G GPU RAM). For the distributed learning setting, we randomly split the training dataset into $m$ subsets, where $m$ is the number of parties, with the same number of training examples. Details on our MPC implementation can be found in Appendix A.

6.1 Nonconvex Logistic Regression

The first nonconvex ERM problem we consider is the binary logistic regression problem with a nonconvex regularizer (Reddi et al., 2016b)

$$\min_{\theta \in \mathbb{R}^d} \frac{1}{n} \sum_{i=1}^{n} y_i \log \phi(x_i^\top \theta) + (1 - y_i) \log [1 - \phi(x_i^\top \theta)] + \lambda \sum_{i=1}^{d} \frac{\theta_j^2}{(1 + \theta_j^2)},$$

where $\phi(x) = 1/(1 + \exp(-x))$ is the sigmoid function, $\theta_j$ is the $j$-th coordinate of $\theta$, and $\lambda > 0$ is the regularization parameter. We set the regularization parameter as $\lambda = 0.001$ in this experiment.

Datasets. We consider two commonly-used binary classification benchmarks: $a9a$ dataset and $ijcnn1$ dataset. More specifically, $a9a$ dataset has 32561 training examples, 16281 test examples, 123 features, and $ijcnn1$ dataset has 9990 training examples, 91701 test examples, 22 features.

Baseline methods. We compare our method (DP-SRGD) with random round private stochastic gradient descent (RRPSGD) proposed by Zhang et al. (2017), differentially private gradient descent (DP-GD) proposed by Wang et al. (2017), and differentially private adaptive gradient descent (DP-AGD) proposed by Lee and Kifer (2018).

Parameters. For all the algorithms, the step size is tuned around the theoretical values to give the fastest convergence using grid search. For our method, we tune the batch size $b$ by searching the grid $\{50, 100, 200\}$. We set $C = 2$ for the $a9a$ experiments and $C = 1$ for $ijcnn1$ experiments. We choose two different privacy
Table 3: Comparison of different algorithms on *ijcnn1* dataset under different privacy budgets $\epsilon \in \{0.2, 0.5\}$ and $\delta = 10^{-5}$. Note that the non-private baseline denotes the test error of the non-private SPIDER algorithm (Fang et al., 2018). For DDP-SRGD, we choose the number of party $m = 10$.

| Privacy Budget | Non-private Baseline | Method | Test Error | Data Passes | CPU time | Gradient Norm |
|----------------|----------------------|--------|------------|-------------|----------|---------------|
| $\epsilon = 0.2$ |                      | DP-GD  | 0.2954 (0.0049) | 20 | 0.4760 | 0.0120 (0.0018) |
| (0.002)         |                      | DP-AGD | 0.2650 (0.0041) | 346 | 24.81  | 0.0103 (0.0016) |
|                 | DP-SRGD              | 0.2475 (0.0036) | 5 | 0.3750 | 0.0120 (0.0018) |
|                 | DDP-SRGD             | 0.2520 (0.0033) | 5 | 10.68  | 0.0080 (0.0009) |
| $\epsilon = 0.5$ |                      | DP-GD  | 0.2581 (0.0034) | 20 | 0.4750 | 0.0009 (0.0011) |
| (0.002)         |                      | DP-AGD | 0.2376 (0.0029) | 365 | 95.42  | 0.0385 (0.0025) |
|                 | DP-SRGD              | 0.2349 (0.0027) | 5 | 32.89  | 0.0120 (0.0018) |
|                 | DDP-SRGD             | 0.2396 (0.0028) | 5 | 12.19  | 0.0052 (0.0007) |

Gradient clipping and privacy tracking. We use the gradient clipping technique of Abadi et al. (2016) to ensure that $\|\nabla f_i\|_2$ is upper bounded by some predefined value $C$. This will ensure that the Lipschitz constant $G$ is upper bounded by $C$, and give us the desired privacy protection. At each iteration, we add the Gaussian noise with variance $\sigma^2$, and keep track of the RDP according to Lemmas 3.7 and 3.9. We then transfer it to the $(\epsilon, \delta)$-DP according to Lemma 3.10.

Results. Due to the randomized nature of all the algorithms, the experimental results are obtained by averaging the results over 10 runs. Figure 1 and Figure 2 show the objective function value and the gradient norm of different algorithms for privacy budgets $\epsilon \in \{0.2, 0.5\}$ on *a9a* and *ijcnn1* datasets respectively. Our DP-SRGD algorithm outperforms other three baseline algorithms in terms of objective loss, gradient norm, and convergence rate by a large margin, which aligns with our theoretical results. Table 6.1 and Table 6.1 demonstrate the test error of different algorithms as well as the CPU time (in seconds) of the training process on *a9a* and *ijcnn1* dataset respectively. Our experimental results show that our algorithm convergences faster...
and can achieve a better test error on the test set than the comparison baselines.

![Graphs showing experiment results for nonconvex logistic regression on ijcnn1 dataset.](image)

**Figure 2**: Experiment results for nonconvex logistic regression on ijcnn1 dataset. (a), (b) Objective loss versus number of epoch. (c), (d) Gradient norm versus number of epoch.

### 6.2 Convolutional Neural Networks

We compare our algorithm with the differentially-private stochastic gradient descent (DP-SGD) algorithm proposed by Abadi et al. (2016) on training a convolutional neural network for image classification on the MNIST dataset (Schölkopf et al., 2002). We use a network architecture that has two convolutional layers followed by one fully-connected layer with the output size 10. More specifically, the convolutional layers use 16 and 32 filters of size 5 respectively, followed by a ReLU and $2 \times 2$ max pooling.

#### Parameters.
We set the the gradient clipping threshold $C = 2$, choose two different privacy budgets $\epsilon \in \{2, 4\}$, and set $\delta = 10^{-5}$. For DP-SGD, we tune the batch size by searching the grid $\{256, 512, 1024\}$ and the step size by $\{0.01, 0.05, 0.1, 0.5, 1\}$. For DP-SRGD, we tune the batch size $b$ by searching the grid $\{256, 512, 1024\}$, step size by $\{0.01, 0.05, 0.1, 0.5, 1\}$, and use $\{2b, 4b, 8b\}$ to approximate the full gradient.

#### Results.
Figures 3(a) and 3(b) illustrate the average test error of different methods versus the number of data epochs under different privacy budgets over 10 trials. The CNN trained by the non-private SGD can achieve 2% test error after 20 epochs. The result shows that our proposed method can achieve 4.88% and 3.47% test error under privacy budget $\epsilon = 2$ and $\epsilon = 4$ correspondingly, which are better than DP-SGD. In addition, it can be seen that our method converges faster than DP-SGD. Figures 3(c) and 3(d) demonstrate the test error versus CPU time of distributed DP-SRGD with different number of parties $m \in \{1, 2, 4, 8, 16\}$ under different privacy budgets. We do not consider the MPC cost for generating noise here and defer it to Section A. We can see from the result that the proposed algorithm can achieve similar performance as the centralized DP-SRGD algorithm.

### 7 Conclusions

In this paper, we propose an efficient differentially private algorithm for nonconvex ERM. We prove both privacy and utility guarantees for our proposed algorithm, and extend it to distributed learning setting. Both theoretical analyses and experiments demonstrate the advantage of our algorithms compared with the state-of-the-art.
Figure 3: Experimental results for CNN on MNIST dataset. (a), (b) depict the test error versus data epochs under different privacy budgets. (c), (d) illustrate the test error versus CPU time with different number of parties under different privacy budgets for distributed learning.

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A Implementation of Secure Aggregation

We implemented the secure aggregation and noise generation inside the MPC framework using Obliv-C (Zahur and Evans, 2015). Obliv-C converts any computation into garbled circuit which can be securely evaluated. This is essentially a two-party protocol where one party (generator) encodes the secure computation into garbled circuit and sends it to the other party (evaluator) along with his share of input keys. The evaluator then securely requests his share of input keys from the generator via oblivious transfer protocol. Evaluator then evaluates the garbled circuit using the input shares of both parties to obtain the final output of the computation. Similar to the approach of Jayaraman et al. (2018), we first aggregate the gradients of all the parties and then sample the noise which is added to the aggregated gradients, all within the MPC framework. The main difference is that we sample Gaussian noise instead of the Laplace noise as used by Jayaraman et al. We use the ratio of uniforms method (Kinderman and Monahan, 1977) to sample Gaussian noise, where the parties collaboratively generate two uniform random numbers which are then used to generate Gaussian random number. Given two uniform random numbers $u_1$ and $u_2$ in the range $(0, 1)$, then the corresponding Gaussian value in the same range is given by their ratio $x = (2u_2 - 1) \sqrt{2e^{-1}/u_1}$ provided the inequality $x^2 \leq -4 \ln u_1$ is satisfied.

We consider 5 parties collaboratively running Algorithm 2 where they invoke secure aggregation to aggregate their local gradients with differential privacy. Table A gives the cost of single invocation of secure aggregation for nonconvex logistic regression on $a9a$ and $ijcnn1$ datasets. The secure computation takes 18 seconds for $a9a$ dataset and 3.5 seconds for $ijcnn1$ dataset. The MPC time scales linearly with the number of parties and the size of the gradients to be aggregated. While the actual aggregation takes only small fraction of time, the major computation overhead is due to the noise generation process which requires costly operations like multiplication, division and logarithm using big integer library (Doerner, 2017). The garbled circuit for $a9a$ dataset consists of around 53.5 million garbled gates which corresponds to 1021.60 MB of
data transfer. Additional 22.88 MB bandwidth is used for oblivious transfer of key shares. For a9a dataset, garbled circuit has around 9.5 million gates amounting to 182.71 MB. Oblivious transfer takes 3.89 MB. While Yao’s garbled circuit protocol provides security against semi-honest adversaries, we also implemented the dual execution protocol (Huang et al., 2012) which provides security against malicious adversaries but leaks one bit of information. In this protocol, the garbled circuit protocol is executed twice such that the parties exchange their roles in the second execution. Dual execution protocol takes 21.5 seconds for a9a dataset and 4.4 seconds for ijcnn1 dataset.

Table 5 gives the cost of secure computation for convolutional neural networks over MNIST dataset. Our CNN model consists of 10 layers of different sizes. We report the bandwidth cost and execution time of aggregating the gradients for each individual layer. The shape of each layer defines the size of the corresponding gradients, which in turn dictates the MPC cost. The total bandwidth of garbled circuit of all the layers is 235.50 GB and the oblivious transfer bandwidth is 8.23 GB.

B Proofs of Main Results

In this section, we lay out the main proofs of our results.
B.1 Proofs of Theorem 4.1

Proof. In this subsection, we will provide the privacy guarantee of Algorithm 1. We first show that our proposed algorithm satisfies RDP using Lemma 3.7 and Lemma 3.9. Then we will transform it into \((\epsilon, \delta)\)-DP based on Lemma 3.10. In the following discussion, we use \(t_0\) to denote the iteration satisfying \(\text{mod}(t_0, l) = 0\). For the given dataset \(S\), we use \(S'\) to denote its neighboring dataset with one different example indexed by \(i'\).

We use \(\mathcal{M}_t\) to denote the mechanism of Algorithm 1 after \(t\)-th iteration. Therefore, our goal is to show the privacy guarantees of \(\mathcal{M}_t\) for \(t = 1, \ldots, T\). Note that Algorithm 1 will reset the semi-stochastic gradient \(v^t\) to the current full gradient when \(t = t_0\), and then iteratively update it in the subsequent \((l - 1)\) iterations.

**Case 1:** If \(t = t_0\), we have \(v^{t_0} = \nabla F(\theta^{t_0})\), which implies the following Gaussian mechanism at \(t_0\)-th iteration

\[
G_{t_0} = \nabla F(\theta^{t_0}) + u^{t_0},
\]

where \(u^{t_0} \sim N(0, \sigma^2_{t_0} I_d)\). We first show that this mechanism satisfies RDP given appropriate \(u^{t_0}\).

**Sensitivity.** Consider the following query on the dataset \(S\) as follows \(q_{t_0}(S) = \nabla F(\theta^{t_0})\), where \(q_{t_0}(S)\) denotes that \(q_{t_0}\) is queried based on the dataset \(S\). Thus, we have

\[
q_{t_0}(S) - q_{t_0}(S') = \frac{1}{n}(\nabla f_i(\theta^{t_0}) - \nabla f_i'(\theta^{t_0})),
\]

which implies the following \(\ell_2\)-sensitivity of the \(t\)-th query

\[
\Delta_t = \frac{1}{n} \| \nabla f_i(\theta^{t_0}) - \nabla f_i'(\theta^{t_0}) \|_2 \leq \frac{2G}{n},
\]

(B.1)

where the last inequality is due to the \(G\)-Lipschitz of each component function \(f_i\).

**Privacy Guarantee of \(G_{t_0}\).** By Lemma 3.7, if we add noise with

\[
\sigma^2_{t_0} = \frac{2TG^2}{\beta n^2 \epsilon},
\]

(B.2)

the mechanism \(G_{t_0}\) satisfies \((\alpha, \rho_0)\)-RDP, \(\rho_0 = \beta \epsilon / T\).

**Case 2:** If \(t \neq t_0\), i.e., \(t_0 < t < t_0 + l\), we have the following mechanism

\[
\mathcal{M}'_{t_0} = \nabla F_{B_t}(\theta^t) - \nabla F_{B_t}(\theta^{t-1}) + u^t + v^{t-1},
\]

where \(u^t \sim N(0, \sigma^2 I_d)\). First, we consider the following Gaussian mechanism

\[
G_t = \nabla F_{B_t}(\theta^t) - \nabla F_{B_t}(\theta^{t-1}) + u^t.
\]

Now, we are going to show that \(G_t\) satisfies RDP given appropriate \(u^t\). Note that the mechanism \(G_t\) is based on the subsample approach, thus we will use the subsample amplification result, i.e., Lemma 3.7, to show
that $G_t$ satisfies RDP. To this end, we consider the following Gaussian mechanism without subsampling
\[
\tilde{G}_t = \frac{1}{b} \sum_{i=1}^{n} \nabla f_i(\theta^t) - \frac{1}{b} \sum_{i=1}^{n} \nabla f_i(\theta^{t-1}) + u^t.
\]

**Sensitivity.** We consider the following query without subsampling
\[
\tilde{q}_t(S) = \frac{1}{b} \sum_{i=1}^{n} \nabla f_i(\theta^t) - \frac{1}{b} \sum_{i=1}^{n} \nabla f_i(\theta^{t-1})
\]
which implies that
\[
\tilde{q}_t(S) - \tilde{q}_t(S') = \frac{1}{b} (\nabla f_i(\theta^t) - \nabla f_i(\theta^{t-1}) - \nabla f_i'(\theta^t) + \nabla f_i'(\theta^{t-1})).
\]
As a result, we can obtain the $\ell_2$-sensitivity of the query $\tilde{q}_t$ as follows
\[
\tilde{\Delta}_t = \frac{1}{b} \|\nabla f_i(\theta^t) - \nabla f_i(\theta^{t-1}) - \nabla f_i'(\theta^t) + \nabla f_i'(\theta^{t-1})\|_2
\leq \frac{2L}{b} \|\theta^t - \theta^{t-1}\|_2,
\]
where the inequality is due to the $L$-Lipschitz continuous gradient of each component function. Furthermore, according to the update rule of Algorithm 1 and the definition of $\eta_t$, we have
\[
\|\theta^t - \theta^{t-1}\|_2 \leq \eta_t \|\theta^{t-1}\|_2 \leq \min \left\{ \frac{\zeta}{L\|\theta^{t-1}\|_2}, \frac{1}{2L} \right\} \|\theta^{t-1}\|_2 \leq \frac{\zeta}{L},
\]
which implies that
\[
\tilde{\Delta}_t \leq \frac{2L}{b} \|\theta^t - \theta^{t-1}\|_2 \leq \frac{2\zeta}{b}.
\] (B.3)

**Privacy Guarantee of $G_t$.** By Lemma 3.7, if we add noise with
\[
\sigma^2_t = \frac{20T\alpha\zeta^2}{\beta n^2 \epsilon},
\] (B.4)
the mechanism $\tilde{G}_t$ satisfies $(\alpha, \beta \epsilon n^2/(10b^2T))$-RDP. Therefore, according to Lemma 3.7, we have that the mechanism with subsampling $G_t$ will satisfy $(\alpha, \rho_1)$-RDP if $\sigma^2_t/\tilde{\Delta}^2_t \geq 1.5$, where $\rho_1 = \beta \epsilon / T$. Furthermore, the variance $\sigma^2_t$ should satisfy the following condition
\[
\frac{\sigma^2_t}{\tilde{\Delta}^2_t} = \frac{5b^2T\alpha}{\beta n^2 \epsilon} \geq 1.5.
\]
As a result, we show that $G_t$ satisfies $(\alpha, \rho_1)$-RDP.
Privacy Guarantee of $\mathcal{M}_{t_0}$. Recall that, we have

$$\mathcal{M}_{t_0} = \nabla F_{B_t}(\theta^t) - \nabla F_{B_t}(\theta^{t-1}) + u_t + v_{\rho}^{t-1}. \tag{B.5}$$

Start from $t_0$, we have that the mechanism $\mathcal{M}_{t_0}^{t_0+1}$ is a composition of $G_{t_0}$ and $G_{t_0+1}$, i.e., $\mathcal{M}_{t_0}^{t_0+1} = (G_{t_0}, G_{t_0+1})$. Therefore, according to the definition of $\mathcal{M}_{t_0}$, we can get $\mathcal{M}_{t_0}^{t_0} = (G_{t_0}, G_{t_0+1}, \ldots, G_t)$. According to the composition property of RDP, i.e., Lemma 3.9, we have $\mathcal{M}_{t_0}^{t_0}$ satisfies $(\alpha, (t - t_0)\rho_1 + \rho_0)$-RDP. Privacy Guarantee of Algorithm 1. According to Algorithm 1, when $t = t_0$, we will reset the semi-stochastic gradient to the current full gradient, and iteratively update it in the subsequent $(t - 1)$ iterations. In addition, we have shown that from $t_0$-th iteration to $t$-th iteration, where $t \in [t_0, t_0 + l)$, the mechanism satisfies $(\alpha, (t - t_0)\rho_1 + \rho_0)$-RDP. Therefore, according to Lemma 3.9, after $T$ iterations, we have $\mathcal{M}_{T'}$ satisfies $(\alpha, \rho')$-RDP, where we have

$$\rho' \leq \frac{T'}{l} \rho_0 + \frac{T'}{l} (l - 1) \rho_1,$$

which implies that $\mathcal{M}_{T'}$ satisfies $(\alpha, T'\beta\epsilon/T)$-RDP, where $\alpha = \log(1/\delta)/(1 - \beta)\epsilon + 1$. According to Lemma 3.10, we have $\mathcal{M}_{T'}$ satisfies $(T'\beta\epsilon/T + (1 - \beta)\epsilon, \delta)$-DP. Finally, according to the definition of $\tilde{\theta}$, we have $\tilde{\theta}$ satisfies $(\epsilon, \delta)$-DP. Recall that we have the following constraint on $\alpha$ according to Lemma 3.7

$$\frac{1}{\tau(1 + \sigma^2)} = \frac{\beta n^3 \epsilon}{5b^3 T \alpha + \beta b n^2 \epsilon},$$

which implies that $\alpha = \log(1/\delta)/(1 - \beta)\epsilon + 1 \leq \log (\beta n^3 \epsilon/(5b^3 T \alpha + \beta b n^2 \epsilon)). \tag*{\Box}$

### B.2 Proofs of Theorem 4.3

**Proof.** Note that we choose $l = \sqrt{n}$ and $b = \sqrt{n}$. According to the assumption that each component function has $L$-Lipschitz continuous gradient, we can obtain that

$$\|\nabla F(x) - \nabla F(y)\|_2 = \frac{1}{n} \sum_{i=1}^{n} \|\nabla f_i(x) - \nabla f_i(y)\|_2 \leq L\|x - y\|_2,$$

which implies that $F(x)$ has $L$-Lipschitz continuous gradient. Thus we have

$$F(\theta^{t+1}) \leq F(\theta^t) + \langle \nabla F(\theta^t), \theta^{t+1} - \theta^t \rangle + \frac{L}{2}\|\theta^{t+1} - \theta^t\|_2^2$$

$$= F(\theta^t) - \eta_t \langle \nabla F(\theta^t), v_p^t \rangle + \frac{\eta_t^2 L}{2}\|v_p^t\|_2^2$$

$$= F(\theta^t) - \eta_t \langle \nabla F(\theta^t) - v_p^t, v_p^t \rangle - \eta_t \left(1 - \frac{\eta_t L}{2}\right)\|v_p^t\|_2^2.$$
By Cauchy-Schwartz inequality, we can further obtain that

$$F(\theta^{t+1}) \leq F(\theta^t) + \frac{\eta t}{2}\|\nabla F(\theta_t) - v^t_p\|^2 - \eta t \left(\frac{1}{2} - \frac{\eta t L}{2}\right)\|v^t_p\|^2,$$

$$\leq F(\theta^t) + \frac{\eta t}{2}\|\nabla F(\theta_t) - v^t_p\|^2 - \frac{\eta t}{4}\|v^t_p\|^2,$$

where the last inequality comes from the fact that $\eta t \leq 1/(2L)$. In addition, we have

$$\frac{\eta t}{4}\|v^t_p\|^2 = \frac{\zeta^2}{8L}\min\{2\|v^t_p/\zeta\|_2, \|v^t_p/\zeta\|_2^2\} \geq \frac{\zeta\|v^t_p\|_2^2 - 2\zeta^2}{4L}.$$  

Thus we have

$$F(\theta^{t+1}) \leq F(\theta^t) + \frac{1}{4L}\|\nabla F(\theta_t) - v^t_p\|^2 - \frac{\zeta\|v^t_p\|_2^2}{4L} + \frac{\zeta^2}{2L}.$$  

Let $t_0 = \lceil t/\sqrt{n} \rceil \cdot \sqrt{n}$, taking expectation given the previous observations after iterations 0, . . . , $(t_0 - 1)$, we have

$$\mathbb{E}_{t_0} F(\theta^{t+1}) - \mathbb{E}_{t_0} F(\theta^t) \leq \frac{1}{4L}\mathbb{E}_{t_0}\|\nabla F(\theta_t) - v^t_p\|^2 - \frac{\zeta\|v^t_p\|_2^2}{4L} + \frac{\zeta^2}{2L}, \quad (B.6)$$

where $\mathbb{E}_{t_0}$ is taken over the randomness of $t_0, \ldots, t$ given the observations after iterations 0, . . . , $(t_0 - 1)$. Next, we are going to bound the term $I_1$. Note that we have

$$\mathbb{E}_t\|\nabla F(\theta^t) - v^t_p\|^2$$

$$= \mathbb{E}_t\|\nabla F(\theta^t) - \nabla F(\theta^{t-1}) + \nabla F(\theta^{t-1}) - \nabla F_{B_t}(\theta^t) + \nabla F_{B_t}(\theta^{t-1}) - v^{t-1}_p - u^t\|^2$$

$$= \mathbb{E}_t\|\nabla F(\theta^t) - \nabla F(\theta^{t-1}) - \nabla F_{B_t}(\theta^t) + \nabla F_{B_t}(\theta^{t-1})\|^2 + \mathbb{E}_t\|\nabla F(\theta^{t-1}) - v^{t-1}_p - u^t\|^2$$

$$= \mathbb{E}_t\|\nabla F(\theta^t) - \nabla F(\theta^{t-1}) - \nabla F_{B_t}(\theta^t) + \nabla F_{B_t}(\theta^{t-1})\|^2 + \|\nabla F(\theta^{t-1}) - v^{t-1}_p - u^t\|^2 + \mathbb{E}_t\|u^t\|^2,$$

where $\mathbb{E}_t$ is taken over the randomness at the $t$-th iteration given the observations after $(t - 1)$-th iteration, the first equality is due to the definition of $v^t$, the second and last equality comes from the independence of the random variables. In addition, we have

$$\mathbb{E}_t\|\nabla F(\theta^t) - \nabla F(\theta^{t-1}) - \nabla F_{B_t}(\theta^t) - \nabla F_{B_t}(\theta^{t-1})\|^2$$

$$\leq \frac{1}{\sqrt{n}} \cdot \frac{1}{n} \sum_{i=1}^n \|\nabla F(\theta^t) - \nabla F(\theta^{t-1}) - \nabla f_i(\theta^t) + \nabla f_i(\theta^{t-1})\|^2$$

$$\leq \frac{1}{\sqrt{n}} \cdot \frac{1}{n} \sum_{i=1}^n \|\nabla f_i(\theta^t) - \nabla f_i(\theta^{t-1})\|^2$$

$$\leq \frac{L^2}{\sqrt{n}}\|\theta^t - \theta^{t-1}\|_2^2,$$
where the first inequality is due to Lemma D.1, the second one comes from the fact that $\mathbb{E}\|X - \mathbb{E}X\|^2 \leq \mathbb{E}\|X\|^2$ for any random variable $X$, and the last one is due to the gradient Lipschitz property of each component function. According to the updating rule, we have

$$\|\theta^t - \theta^{t-1}\|_2 \leq \eta_t \|v_p^{t-1}\|_2 \leq \min \left\{ \frac{\zeta}{L\|v_p^{t-1}\|_2}, \frac{1}{2L} \right\} \cdot \|v_p^{t-1}\|_2 \leq \frac{\zeta}{L},$$

which implies

$$\mathbb{E}_t \|\nabla F(\theta^t) - \nabla F(\theta^{t-1}) - \nabla F_B_t(\theta^t) + \nabla F_B_t(\theta^{t-1})\|_2^2 \leq \frac{\zeta^2}{\sqrt{n}}.$$  

As a result, we can obtain

$$\mathbb{E}_t \|\nabla F(\theta^t) - v_p^t\|_2^2 \leq \frac{\zeta^2}{\sqrt{n}} + \|\nabla F(\theta^{t-1}) - v_p^{t-1}\|_2^2 + \mathbb{E}_t \|u^t\|_2^2 \leq \frac{\zeta^2}{\sqrt{n}} + \|\nabla F(\theta^{t-1}) - v_p^{t-1}\|_2^2 + d\sigma_t^2. \tag{B.7}$$

Taking expectation over $t_0, \ldots, (t - 1)$, we can obtain that

$$\mathbb{E}_{t_0} \|\nabla F(\theta^t) - v_p^t\|_2^2 \leq \frac{(t-t_0)\zeta^2}{\sqrt{n}} + \mathbb{E}_{t_0} \|\nabla F(\theta^{t_0}) - v^{t_0}\|_2^2 + d \sum_{i=t_0}^t \sigma_i^2 \leq \zeta^2 + \mathbb{E}_{t_0} \|\nabla F(\theta^{t_0}) - v^{t_0}\|_2^2 + d \sum_{i=t_0}^t \sigma_i^2 \leq \zeta^2 + d \sum_{i=t_0}^t \sigma_i^2, \tag{B.8}$$

where the second inequality is due to the fact that $t - t_0 \leq \sqrt{n}$ and the last one is due to that fact that $v^{t_0} = \nabla F(\theta^{t_0})$. As a result, we can get

$$I_1 = \mathbb{E}_{t_0} \|\nabla F(\theta^t) - v_p^t\|_2^2 \leq \zeta^2 + d \sum_{i=t_0}^t \sigma_i^2. \tag{B.9}$$

Plugging this result into (B.6), we can obtain

$$\mathbb{E}_{t_0} F(\theta^{t+1}) - \mathbb{E}_{t_0} F(\theta^t) \leq \frac{3\zeta^2}{4L} - \frac{\zeta}{4L} \mathbb{E}_{t_0} \|v_p^t\|_2^2 + \frac{d}{4L} \sum_{i=t_0}^t \sigma_i^2 \leq \frac{3\zeta^2}{4L} - \frac{\zeta}{4L} \mathbb{E}_{t_0} \|v_p^t\|_2^2 + \frac{d}{4L} \sum_{i=1}^{\sqrt{n}-1} \sigma_i^2.$$
where the last inequality is due to the definition of $T$

$$T = \frac{3T \zeta^2}{4L} + \sum_{t=0}^{T-1} \frac{d\sigma_0^2}{4L} + \frac{d}{4L} \sum_{t=0}^{T-1} \sum_{i=1}^{\sqrt{n}-1} \sigma_i^2,$$

$$\leq F(\theta^0) - F(\theta^*) + \frac{3T \zeta^2}{4L} + \sum_{t=0}^{T-1} \frac{d\sigma_0^2}{4L} + \frac{d}{4L} \sum_{t=0}^{T-1} \sum_{i=1}^{\sqrt{n}-1} \sigma_i^2,$$

(B.10)

which implies

$$\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} \| \nabla F(\theta^t) \|_2 \leq \frac{4L}{T \zeta} (F(\theta^0) - F(\theta^*)) + 3\zeta + \sum_{t=0}^{T-1} \frac{d\sigma_0^2}{T \zeta} + \sum_{t=0}^{T-1} \frac{d\sqrt{n} \sigma_1^2}{T \zeta},$$

(B.11)

where the last inequality is due to the fact that $T = [4L(F(\theta^0) - F(\theta^*)) / \zeta^2] + 1$.

According to the definition of $\tilde{\theta}$, we have

$$\mathbb{E} \| \nabla F(\tilde{\theta}) \|_2 = \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} \| \nabla F(\theta^t) \|_2$$

$$\leq \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} \| \nabla F(\theta^t) \|_2 + \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} \| \nabla F(\theta^t) - \nabla F(\theta^t) \|_2$$

$$\leq 5\zeta + \sum_{t=0}^{T-1} \frac{d\sigma_0^2}{T \zeta} + \sum_{t=0}^{T-1} \frac{d\sqrt{n} \sigma_1^2}{T \zeta} + \sum_{t=0}^{T-1} \frac{\sqrt{d} \sigma_0}{T} + \sum_{t=0}^{T-1} \frac{\sqrt{d} \sqrt{n} \sigma_1}{T},$$

(B.12)

where the last inequality is due to (B.9), (B.11), and Jensen’s inequality.

According to (B.2) and (B.4), we have

$$\sigma_0^2 \leq \frac{4T G^2 \log(1/\delta)}{(1 - \beta)\beta n^2 \epsilon^2} \quad \text{and} \quad \sigma_1^2 = \frac{40T \zeta^2 \log(1/\delta)}{\beta (1 - \beta) n^2 \epsilon^2}.$$

Plugging these quantities into (B.12), we can get

$$\mathbb{E} \| \nabla F(\tilde{\theta}) \|_2 \leq 5\zeta + \frac{4T d G^2 \log(1/\delta)}{(1 - \beta)\beta n^2 \epsilon^2 \zeta} + \frac{40T d \sqrt{n} \zeta \log(1/\delta)}{(1 - \beta)\beta n^2 \epsilon^2} + \frac{2G \sqrt{d} \log(1/\delta)}{\sqrt{(1 - \beta)\beta n \epsilon}}$$

$$+ \frac{7\zeta \sqrt{d} \log(1/\delta) n^{1/4}}{\sqrt{(1 - \beta)\beta n \epsilon}}$$

$$\leq 5\zeta + \frac{32LD F d G^2 \log(1/\delta)}{\beta (1 - \beta) n^2 \epsilon^2 \zeta^3} + \frac{320LD F d \sqrt{n} \log(1/\delta)}{\beta (1 - \beta) n^2 \epsilon^2 \zeta},$$

where the last inequality is due to the definition of $T$ and the fact that the second and third terms are the dominate terms. If we choose $\zeta^2 = G \sqrt{d} LD F \log(1/\delta) / (n \epsilon)$, we have $T = 4n \epsilon \sqrt{LD F} / (G \sqrt{d} \log(1/\delta))$,
and we can obtain that
\[ \mathbb{E}\|\nabla F(\tilde{\theta})\|_2 \leq C \frac{G^{1/2} dLD_F \log(1/\delta)^{1/4}}{(ne)^{1/2}}, \]
where \( C \) is an absolute constant.

**Gradient Complexity.** Every \( \sqrt{n} \) iterations, we need to compute one full gradient, and \( \sqrt{n} \) times stochastic gradient with batch size \( \sqrt{n} \). Therefore, the total gradient complexity is
\[
\left\lceil \frac{T}{\sqrt{n}} \right\rceil \cdot n + T \sqrt{n} \leq \left\lceil \frac{T}{\sqrt{n}} \right\rceil \cdot n + T \sqrt{n} \leq \frac{4L \sqrt{n} (F(\theta^0) - F(\theta^*))}{\zeta^2},
\]
where the last inequality is due to the fact that \( T \leq 4L (F(\theta^0) - F(\theta^*))/\zeta^2 \). As a result, the total gradient complexity is less than \( O(n^{3/2}/\epsilon) \).

\[ \square \]

**B.3 Proofs of Theorem 5.1**

Proof. In order to make use of Lemma 3.7 in the distributed setting, our subsample procedure should be uniform sampling without replacement. This can be achieved by the following procedure. According to the setup in section 5.1, we can define the following index set \( I = \{j_1, j_2, \ldots, j_{\tilde{n}}\}_{j=1}^{m} \) with \( |I| = m \tilde{n} \) and \( j_i \) corresponding to the \( i \)-th example from party \( j \). Therefore, at \( t \)-th iteration, we can uniformly sample \( mb \) index without replacement from \( I \) such that each party has a corresponding subsampled index set \( B^j_t \) with \( |B^j_t| = b_j \) and \( \tilde{b} = \sum_{j=1}^{m} b_j / m \) as illustrated in line 6 of Algorithm 2.

Now, we are ready to prove the privacy guarantee of Algorithm 2. According to the proof in the single party setting, i.e., equations (B.1) and (B.3), we have
\[
\frac{1}{mn} \|\nabla f_i(\theta^0)\|_2 \leq \frac{G}{mn} \quad \text{and} \quad \frac{1}{mb} \|\nabla f_i(\theta^t) - \nabla f_i(\theta^t-1)\|_2 \leq \frac{\zeta}{mb}.
\]
Therefore, following the same proof as in the centralized setting, we only need to replace the parameter \( n \) with \( m \tilde{n} \) is the distributed setting to get our privacy guarantee. \[ \square \]

**B.4 Proofs of Theorem 5.3**

Proof. Note that we choose \( l = \sqrt{m \tilde{n}} \) and \( \tilde{b} = \sqrt{n/m} \). The utility analysis of multi-party setting is similar to the analysis of single party setting. According to (B.2) and (B.4), we have
\[
\sigma_0^2 \leq \frac{2TG^2 \alpha}{\beta m^2 \tilde{n}^2 \epsilon} \quad \text{and} \quad \sigma_1^2 = \frac{40T \zeta^2 \alpha}{\beta m^2 \tilde{n}^2 \epsilon}.
\]
If we choose $\zeta^2 = G \sqrt{dLD_F \log(1/\delta) / (m\tilde{\epsilon})}$, we have $T = 4m\tilde{\epsilon} \sqrt{LD_F / (G \sqrt{d \log(1/\delta)})}$, and we can obtain that

$$\mathbb{E}\|\nabla F(\tilde{\theta})\|_2 \leq C \frac{G^{1/2} (dLD_F \log(1/\delta))^{1/4}}{(m\tilde{\epsilon})^{1/2}},$$

where $C$ is an absolute constant.

\[\Box\]

## C Proof of Lemma 3.7

**Proof.** To prove the result of subsample amplification, we want to show that if a Gaussian mechanism $M$ satisfying $(\alpha, \rho(\alpha))$-RDP, by applying it to a subset of samples with subsample rate $\tau$, it satisfies $(\alpha, \rho'(\alpha))$-RDP, and we will provide an explicit upper bound for $\rho'(\alpha)$. Without loss of generality, we assume $\Delta(q) = 1$. According to Theorem 9 in Wang et al. (2019b), we have

$$\rho'(\alpha) \leq \frac{1}{\alpha - 1} \log \left(1 + \tau^2 \left(\frac{\alpha}{2}\right) \min \left\{4(e^{\rho(2)} - 1), 2e^{\rho(2)}\right\} + \sum_{j=3}^{\alpha} \tau^j \left(\frac{\alpha}{j}\right) 2e^{(j-1)\rho(j)}\right), \quad (C.1)$$

where $\tau$ is the subsample rate, $\rho(j) = j / (2\sigma^2)$. Next, we will show that the summation term on the right hand side of the above inequality is dominated by the second term under certain conditions. First of all, when $\sigma^2$ is large, i.e., $\sigma^2 \geq 1.5$, we have

$$\min \left\{4(e^{\rho(2)} - 1), 2e^{\rho(2)}\right\} = 4(e^{\rho(2)} - 1) \leq 8/\sigma^2,$$

which implies that

$$\tau^2 \left(\frac{\alpha}{2}\right) \min \left\{4(e^{\rho(2)} - 1), 2e^{\rho(2)}\right\} \leq \tau^2 \left(\frac{\alpha}{2}\right) 8/\sigma^2.$$

Next, we consider the summation term in (C.1), and we have

$$\sum_{j=3}^{\alpha} \tau^j \left(\frac{\alpha}{j}\right) 2e^{(j-1)\rho(j)} \leq 2\tau^2 \left(\frac{\alpha}{2}\right) \left(\sum_{j=3}^{\alpha} \tau^j \alpha^{j-2} e^{\frac{(\alpha-1)j}{2\sigma^2}}\right)$$

$$\leq 2\tau^2 \left(\frac{\alpha}{2}\right) \tau \alpha e^{\frac{3(\alpha-1)}{2\sigma^2} \frac{\alpha-1}{1 - \tau e^{\frac{\alpha-1}{2\sigma^2}}}},$$

where the first inequality is due to the fact that

$$e^{(j-1)\rho(j)} = e^{\frac{(j-1)j}{2\sigma^2}} \leq e^{\frac{(\alpha-1)j}{2\sigma^2}} \quad \text{and} \quad \binom{\alpha}{j} = \frac{\alpha!}{j!(\alpha - j)!} \leq \frac{\alpha!}{2!(\alpha - 2)!}.$$
In addition, the last inequality comes from the condition that \( \tau \alpha \exp \left( (\alpha - 1)/(2\sigma^2) \right) < 1 \) and the sum of the geometric sequence. Therefore, as long as

\[
\alpha - 1 \leq \sigma^2 \log \frac{1}{\tau \alpha (1 + \sigma^2)},
\]

we have

\[
\sum_{j=3}^{\alpha} \tau^j \binom{\alpha}{j} 2e^{(j-1)\rho(j)} \leq \tau^2 \left( \frac{\alpha}{2} \right)^2 \sigma^2.
\]

A sufficient conditions to ensure the (C.2) holds is that

\[
\alpha \leq \log \frac{1}{\tau (1 + \sigma^2)}.
\]

In addition, we require that \( \tau \alpha \exp \left( (\alpha - 1)/(2\sigma^2) \right) < 1 \). By plugging the condition of \( \alpha \) into the above requirement, we can obtain that this condition can hold if \( \tau < 1 \).

As a result, under the conditions that \( \sigma^2 \geq 1.5, \alpha \leq \log(1/\tau (1 + \sigma^2)) \), we can obtain that

\[
\rho'(\alpha) \leq \frac{1}{\alpha - 1} \log \left( 1 + \tau^2 \left( \frac{\alpha}{2} \right)^{10/\sigma^2} \right) \leq \frac{1}{\alpha - 1} \tau^2 \left( \frac{\alpha}{2} \right)^{10/\sigma^2} \leq 5\alpha \tau^2/\sigma^2.
\]

\[\square\]

D Auxiliary Lemmas

Lemma D.1. (Lei et al., 2017) Consider vectors \( a_i \) satisfying \( \sum_{i=1}^{n} a_i = 0 \). Let \( B \) be a uniform random subset of \( \{1, 2, \ldots, n\} \) with size \( m \), we have

\[
\mathbb{E} \left\| \frac{1}{m} \sum_{i \in B} a_i \right\|_2^2 \leq \frac{1}{mn} \sum_{i=1}^{n} \|a_i\|_2^2.
\]

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