Gravitomagnetic effects in Kerr-de Sitter space-time

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ABSTRACT

We explicitly worked out the orbital effects induced on the trajectory of a test particle by the weak-field approximation of the Kerr-de Sitter metric. It results that the node, the pericentre and the mean anomaly undergo secular precessions proportional to $k$, which is a measure of the non-linearity of the theory. We used such theoretical predictions and the latest observational determinations of the non-standard precessions of the perihelia of the inner planets of the Solar System to put a bound on $k$ getting $k \leq 10^{-29}$ m$^{-2}$. The node rate of the LAGEOS Earth’s satellite yields $k \leq 10^{-26}$ m$^{-2}$. The periastron precession of the double pulsar PSR J0737-3039A/B allows to obtain $k \leq 3 \times 10^{-21}$ m$^{-2}$. Interpreting $k$ as a cosmological constant $\Lambda$, it turns out that such constraints are weaker than those obtained from the Schwarzschild-de Sitter metric.

Subject headings: Classical general relativity–Approximation methods; equations of motion–Experimental tests of gravitational theories–Orbit determination and improvement
1. Introduction

The General Theory of Relativity (GTR) has passed with excellent results many observational tests, as Solar System and binary pulsars observations show (Ni 2005; Will 2006; Turyshhev 2008). As a matter of fact, the current values of the PPN parameters are in agreement with GTR predictions.

However, some observations seem to question the general relativistic model of gravitational interaction on larger scales. On the one hand, the data coming from the galactic rotation curves of spiral galaxies (Binney and Tremaine 1987) cannot be explained on the basis of Newtonian gravity or GTR: the existence of dark matter is postulated to reconcile the theoretical model with observations; furthermore, dark matter can explain the mass discrepancy in galactic clusters (Clowe et al. 2006). On the other hand, a lot of observations, such as the light curves of the type Ia supernovae and the cosmic microwave background (CMB) experiments (Riess et al. 1998; Perlmutter et al. 1999; Bennet et al. 2003), firmly state that our Universe is now undergoing a phase of accelerated expansion. Actually, the present acceleration of the Universe cannot be explained, within GTR, unless the existence of a cosmic fluid having exotic properties is postulate, i.e. the so called dark energy.

The cosmological constant is one of the candidates to explain (in the GTR framework) what the dark energy is (see e.g. Peebles and Ratra 2003 and references therein). On the other hand, modified gravity models that go beyond GTR have been proposed to try to explain current observations and, among these models, $f(R)$ theories of gravity (Sotiriou and Faraoni 2008) received much attention in recent years. In these theories the gravitational lagrangian depends on an arbitrary function $f$ of the scalar curvature $R$; they are also referred to as “extended theories of gravity”, since they naturally generalize, on a geometric ground, GTR: namely, when $f(R) = R$ the action reduces to the usual
Einstein-Hilbert action, and Einstein’s theory is obtained. It is interesting to point out that the vacuum solutions of GTR with a cosmological constant are also solutions of $f(R)$ gravity vacuum field equations: this is always true in the Palatini formalism, while in metric $f(R)$ gravity this holds for the solutions with constant scalar curvature $R$ (Ferraris et al. 1993; Allemandi et al. 2005; Magnano 1995).

The relevance of the cosmological constant in modern gravitational physics is manifest, and it is interesting to focus on the solutions of Einstein’s field equations with cosmological constant, to investigate its role on different scales. For instance, the Schwarzschild-de Sitter metric, which describes a point-like mass in a space-time with a cosmological constant, has been recently studied by Kagramanova et al. (2006); Sereno and Jetzer (2006); Jetzer and Sereno (2006); Iorio (2006b). In particular, the Schwarzschild-de Sitter metric has been studied to investigate the influence of the cosmological constant on gravitational lensing in Rindler and Ishak (2007); Sereno (2008a); Ruggiero (2007); Sereno (2008b).

In this paper we are concerned with the Kerr-de Sitter metric, which describes a rotating black-hole in a space-time with a cosmological constant (Demianski 1973; Carter 1973; Kerr et al. 2003; Kraniotis 2004, 2005, 2007). In particular, we want to study the gravito-magnetic (GM) effects in Kerr-de Sitter metric. GM effects are due to the rotation of the sources of the gravitational field: this gives raise to the presence of off-diagonal terms in the metric tensor, which are responsible for a variety of effects concerning orbiting test particles, precessing gyroscopes, moving clocks and atoms and propagating electromagnetic waves (Mashhoon et al. 2001; Ruggiero and Tartaglia 2002; Schäfer 2004; Mashhoon 2007). They are expected in GTR, but are generally very small and, hence, very difficult to detect (Iorio 2007a). In recent years, there have been some attempts to measure the Lense-Thirring effect (Lense and Thirring 1918) with the LAGEOS and LAGEOS II laser-ranged satellites in the gravitational field of the Earth (Ciufolini and Pavlis 2004); the evaluation of the
realistic accuracy reached in such a test and other topics related to it are still matter of
debate (Iorio 2006a, 2007b). For other attempts to measure the Lense-Thirring effect in
other Solar System scenarios with natural and artificial satellites, see (Iorio 2007a). In
April 2004 the Gravity Probe B spacecraft (Everitt et al. 2001) was launched to accurately
measure the gravito-magnetic (and geodetic) precession of an orbiting gyroscope (Pugh
1959; Schiff 1960) in the terrestrial space environment: the final results are going to be
published. We focus on the GM effects in Kerr-de Sitter metric (the GM precession of an
orbiting gyroscope was investigated by Ruggiero (2008)): in particular we work out the
GM effects in the weak-field and slow-motion approximation on the orbit of a test particle,
working out explicitly the perturbations of the Keplerian orbital elements; furthermore,
by using the EPM2004 (Pitjeva 2005a) and EPM2006 (Pitjeva 2008) ephemerides, we put
constraints on the parameter $k$, which is the cosmological constant in GTR and a measure
of the non linearity of the theory in $f(R)$ gravity.

2. Gravito-magnetic field in Kerr-de Sitter metric

The Kerr-de Sitter metric in the standard Boyer-Lindquist coordinates $x^\mu = (t, \rho, \theta, \phi)$
has the form\(^1\)

$$ds^2 = \left\{ 1 - \frac{2GM\rho}{c^2\Sigma} - \frac{k}{3}\left[ \rho^2 + \left( \frac{J}{Mc} \right)^2 \sin^2\theta \right] \right\} c^2 dt^2 +$$

$$+ 2 \left( \frac{J}{Mc} \right) \left\{ \frac{2GM\rho}{c^2\Sigma} + \frac{k}{3}\left[ \rho^2 + \left( \frac{J}{Mc} \right)^2 \sin^2\theta \right] \right\} \sin^2\theta c\dd t \dd \phi + \frac{\Sigma}{\Delta} d\rho^2 + \frac{\Sigma}{\chi} d\theta^2 +$$

\(^1\)The space-time metric has signature (1, −1, −1, −1), greek indices run from 0 to 3, and
latin ones run from 1 to 3, boldface letters like $\mathbf{r}$ refers to three-vectors.
\[ + \left\{ \frac{2GM\rho}{c^2\Sigma} \left( \frac{J}{Mc} \right)^2 \sin^2 \theta + \left[ 1 + \frac{k}{3} \left( \frac{J}{Mc} \right)^2 \right] \left[ \rho^2 + \left( \frac{J}{Mc} \right)^2 \right] \right\} \sin^2 \theta d\phi^2, \]

where
\[ \Sigma = \rho^2 + \left( \frac{J}{Mc} \right)^2 \cos^2 \theta, \quad \chi = 1 + \frac{k}{3} \left( \frac{J}{Mc} \right)^2 \cos^2 \theta, \]

\[ \Delta = \rho^2 - 2 \frac{GM\rho}{c^2} + \left( \frac{J}{Mc} \right)^2 - \frac{k}{3} \rho^2 \left[ \rho^2 + \left( \frac{J}{Mc} \right)^2 \right]. \]

The mass of the source is \( M \), while \( J \) is its angular momentum (which is perpendicular to the \( \theta = \pi/2 \) plane); \( k \) is the cosmological constant in GTR framework, while in \( f(R) \) theories (Ruggiero 2008) it is a parameter related to the non linearity of the gravity lagrangian (namely, when \( f(R) = R \) then \( k = 0 \)). When \( k = 0 \) the Kerr-de Sitter metric given by eq. (1) reduces to the Kerr metric. Other limiting cases can be checked: for instance, when \( J = 0 \), we obtain the Schwarzschild-de Sitter solution, and when \( M = J = 0 \) we have the de Sitter space-time.

In order to study GM effects a weak field approximation eq. (1) is sufficient; furthermore, it is useful to introduce the isotropic radial coordinate \( r \) denotes the defined as
\[ r = \rho \left( 1 - \frac{GM}{c^2 \rho} - \frac{k \rho^2}{12} \right), \]

where \( \rho \) is the standard Boyer-Lindquist radial coordinate. Then, up to linear terms in \( \frac{GM}{c^2r} \), \( kr^2 \), \( \frac{kJr}{cM} \), the metric is

\[ ds^2 = \left( 1 - \frac{2GM}{c^2r} - \frac{k}{3} r^2 \right) c^2 dt^2 - \left( 1 + \frac{2GM}{c^2r} - \frac{k}{6} r^2 \right) \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) + \\
+ 2 \frac{J}{Mc} \left[ \frac{2GM}{c^2r} + \frac{kr}{3} \left( r + \frac{5GM}{2c^2} \right) \right] \sin^2 \theta d\phi dt. \]

By differentiating
\[ \frac{y}{x} = \tan \phi \]
and using
\[ \cos \phi = \frac{x}{r \sin \theta}, \]  
(7)

it turns out that
\[ \sin^2 \theta d\phi = -\frac{y}{r^2} \, dx + x \, dy; \]  
(8)

thus, the off-diagonal, i.e. gravito-magnetic, components \( g_{0i}, \, i = 1, 2, 3 \) of the metric tensor of the weak-field approximation of the Kerr-de Sitter space-time are, in cartesian coordinates,

\[
\begin{align*}
g_{01} &= -\frac{J}{Mc} \left[ \frac{2GM}{c^2 r^3} + \frac{k}{3} \left( 1 + \frac{5GM}{2c^2 r} \right) \right] y, \\
g_{02} &= \frac{J}{Mc} \left[ \frac{2GM}{c^2 r^3} + \frac{k}{3} \left( 1 + \frac{5GM}{2c^2 r} \right) \right] x, \\
g_{03} &= 0;
\end{align*}
\]  
(9-11)

In the weak-field and slow-motion linear approximation the spatial components of the geodesic equations of motions yielding gravito-magnetic accelerations are (\text{Brumberg 1991})

\[
d^2 x^i \bigg/ dt^2 = c (\partial_j h_{0i} - \partial_i h_{0j}) \, \frac{dx^j}{dt}, \quad i = 1, 2, 3
\]  
(12)

where \( h_{0l} = g_{0l} - \eta_{0l} = g_{0l}, \, l = 1, 2, 3 \). It can be straightforwardly showed that the terms not containing \( k \) yield the usual Lense-Thirring acceleration in cartesian coordinates (\text{Soffel 1989}). The components of the acceleration containing \( k \) in cartesian coordinates are

\[
\begin{align*}
A_x &= 2GJk \left\{ - \left[ \frac{c^2}{GM} + \frac{5}{4} \left( \frac{x^2 + y^2 + 2z^2}{r^3} \right) \right] \dot{y} + \left( \frac{5yz}{4r^3} \right) \dot{z} \right\}, \\
A_y &= 2GJk \left\{ \left[ \frac{c^2}{GM} + \frac{5}{4} \left( \frac{x^2 + y^2 + 2z^2}{r^3} \right) \right] \dot{x} - \left( \frac{5xz}{4r^3} \right) \dot{z} \right\}, \\
A_z &= 5GJk \left[ z(\dot{x}y - \dot{y}x) \right] / r^3.
\end{align*}
\]  
(13-15)

which can be cast into the vectorial form (\text{Mashhoon 2007})

\[
\mathbf{A} = -2\frac{v}{c} \times \mathbf{B},
\]  
(16)
where the gravito-magnetic field $\mathbf{B}$ is

$$\mathbf{B} = \frac{Jkc}{3M} \hat{J} + \frac{5GJk}{12c} \left[ \hat{J} + \left( \hat{J} \cdot \hat{r} \right) \hat{r} \right],$$

(17)

with $\hat{J} = \hat{z}$; the unit vector in the radial direction is defined as

$$\hat{r} = \frac{r}{r}.$$

(18)

We notice that the gravito-magnetic field consists of two contributions, the first one that is everywhere constant and parallel to $\mathbf{J}$, the second one whose position and directions are position-dependent.

Furthermore, by defining the gravito-magnetic potential $\mathbf{A}$ as

$$\mathbf{A} = kr^2 \left[ \frac{c^2 r}{6GM} + \frac{5}{12} \right] \frac{G \mathbf{J} \times r}{c r^3},$$

(19)

and using

$$\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}),$$

(20)

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C}) \mathbf{B} - (\mathbf{A} \cdot \mathbf{B}) \mathbf{C},$$

(21)

$$\nabla \times (\psi \mathbf{H}) = \nabla \psi \times \mathbf{H} + \psi \nabla \times \mathbf{H},$$

(22)

$$\nabla \cdot r = 3,$$

(23)

$$\nabla \frac{1}{r} = -\hat{r}$$

(24)

with $\psi = 1/r$, it is possible to express the gravito-magnetic field $\mathbf{B}$ in terms of $\mathbf{A}$ as

$$\mathbf{B} = \nabla \times \mathbf{A}.$$  

(25)

In order to calculate the impact of eq. (16) on the orbit of a test particle, let us project it onto the radial ($\hat{r}$), transverse ($\hat{t}$) and normal ($\hat{n}$) directions of the co-moving frame
picked out by the three unit vectors\(^2\) (Montenbruck and Gill 2000)

\[
\begin{align*}
\hat{r} &= \cos u \hat{x} + \cos i \sin u \hat{y} + \sin i \sin u \hat{z}, \\
\hat{t} &= -\sin u \hat{x} + \cos i \cos u \hat{y} + \sin i \cos u \hat{z} \\
\hat{n} &= -\sin i \hat{y} + \cos i \hat{z},
\end{align*}
\] (26)

where \(i\) is the inclination of the orbital plane to the equator of the central mass and \(u = \omega + f\) is the argument of latitude defined as the sum of the argument of the pericentre \(\omega\), which fixes the position of the pericentre with respect to the line of the nodes, and the true anomaly \(f\) which reckons the position of the test particle from the pericentre. Thus,

\[
\begin{align*}
A_r &= A \cdot \hat{r} = -\frac{GJk}{6e^2} \left(5 + 4\frac{e^2r}{GM}\right) \cos i \dot{u}, \\
A_t &= A \cdot \hat{t} = 0, \\
A_n &= A \cdot \hat{n} = \frac{GJk}{3e^2} \left(5 + 2\frac{e^2r}{GM}\right) \sin i \sin u \dot{u};
\end{align*}
\] (27)

they must be inserted into the right-hand-side of the Gauss equations (Bertotti et al. 2003) of the variations of the Keplerian orbital elements

\[
\begin{align*}
\frac{da}{dt} &= \frac{2}{n\sqrt{1-e^2}} \left[ eA_r \sin f + A_t \left(\frac{p}{r}\right) \right], \\
\frac{de}{dt} &= \frac{n}{na\sqrt{1-e^2}} \left\{ A_r \sin f + A_t \left[ \cos f + \frac{1}{e} \left(1 - \frac{r}{a}\right)\right] \right\}, \\
\frac{di}{dt} &= \frac{1}{na\sqrt{1-e^2}} A_n \left(\frac{r}{a}\right) \cos u, \\
\frac{d\Omega}{dt} &= \frac{1}{na \sin i \sqrt{1-e^2}} A_n \left(\frac{r}{a}\right) \sin u, \\
\frac{d\omega}{dt} &= \frac{n}{nae} \left[ -A_r \cos f + A_t \left(1 + \frac{r}{p}\right) \sin f \right] - \cos i \frac{d\Omega}{dt}, \\
\frac{d\mathcal{M}}{dt} &= n - \frac{2}{na} A_r \left(\frac{r}{a}\right) - \sqrt{1-e^2} \left(\frac{d\omega}{dt} + \cos i \frac{d\Omega}{dt}\right),
\end{align*}
\] (32)

\^2Here we have chosen the \(x\) axis coincident with the line of the nodes, i.e. \(\Omega = 0\).
where \(a, e, \Omega\) and \(\mathcal{M}\) are the semi-major axis, the eccentricity, the longitude of the ascending node and the mean anomaly of the orbit of the test particle, respectively, \(p = a(1 - e^2)\) is the semi-latus rectum and \(n = \sqrt{GM/a^3}\) is the un-perturbed Keplerian mean motion. By evaluating them onto the un-perturbed Keplerian ellipse

\[
r = \frac{a(1 - e^2)}{1 + e \cos f}
\]

and averaging\(^3\) them over one orbital period \(P_b\) of the test particle by means of

\[
\frac{dt}{P_b} = \frac{(1 - e^2)^{3/2}}{2\pi(1 + e \cos f)^2}df,
\]

it is possible to obtain the secular effects induced by eq. (16)

\[
\langle \dot{a} \rangle = 0, \quad (40)
\]
\[
\langle \dot{e} \rangle = 0, \quad (41)
\]
\[
\langle \dot{i} \rangle = 0, \quad (42)
\]
\[
\langle \dot{\Omega} \rangle = \frac{Jk}{3M} \left(1 + \frac{5GM}{2c^2a}\right), \quad (43)
\]
\[
\langle \dot{\omega} \rangle = -\frac{Jk \cos i}{3M} \left(2 + \frac{5GM}{2c^2a}\right), \quad (44)
\]
\[
\langle \dot{\mathcal{M}} \rangle = n + \frac{5Jk \cos i}{3M} \left(1 + \frac{GM}{c^2a}\right). \quad (45)
\]

In the calculation we have neglected terms of order \(\mathcal{O}(e^2)\).

The correction \(\Delta P_b\) to the orbital period \(P_b\) due to eq. (16) can be calculated using the mean longitude

\[
\lambda = \mathcal{M} + \omega + \cos i \, \Omega. \quad (46)
\]

For small eccentricities eq. (35)-eq. (37) yield

\[
\frac{d\lambda}{dt} \approx n - \frac{2}{na} A_r \left(\frac{r}{a}\right). \quad (47)
\]

\(^3\)We used \(\dot{u} = \dot{f}\) because over one orbital revolution the pericentre \(\omega\) can be assumed constant.
By using eq. (29) it is possible to obtain, for $e \to 0$,

$$P_b \approx \frac{2\pi}{n} \left[ 1 - \frac{GJk}{3c^2na} \left( \frac{4c^2a}{GM} + 5 \right) \cos i \right], \quad (48)$$

so that

$$\Delta P_b = -\frac{2\pi Jka^2}{3c^2M} \left( \frac{4c^2a}{GM} + 5 \right) \cos i. \quad (49)$$

We will, now, put constraints on $k$ from the corrections to the standard Newtonian/Einsteinian precessions of the longitudes of the perihelia $\varpi = \omega + \cos i \Omega$ of the inner planets of the Solar System, quoted in Table I estimated by E.V. Pitjeva by fitting more than 400000 observations of various kinds with the EPM2004 (Pitjeva 2005a,b) and EPM2006 (Pitjeva 2008) ephemerides. No gravito-magnetic terms of any kind were included in the dynamical force models used, so that, in principle, they account for the effects investigated by us. For the spin angular momentum of the Sun we will use the value $J_\odot = (190.0 \pm 1.5) \times 10^{39}$ kg m$^2$ s$^{-1}$ determined from helioseismology (Piopers 1998, 2003), i.e. independently of the planetary dynamics which we want to test. From

$$\langle \dot{\varpi} \rangle = -\frac{Jk \cos i}{3M}, \quad (50)$$

and Table I it is possible to obtain

$$k \leq 1 \times 10^{-29} \text{ m}^{-2}. \quad (51)$$

In the case of the laser-ranged LAGEOS satellite (Smith and Dunn 1980), orbiting at about 6000 km above the Earth’s surface, by assuming $J_\oplus = 5.85 \times 10^{33}$ kg m$^2$ s$^{-1}$ (McCarthy and Petit 2004) and an uncertainty of the order of 1 cm or less (Lucchesi 2007) in reconstructing its orbit, which translates into an uncertainty in the nodal rate of $\delta \dot{\Omega} \sim 0.1$ milliarcseconds per year, the bound which can be obtained from eq. (43) is $k \leq 4 \times 10^{-26}$ m$^{-2}$. 
Concerning the double pulsar system PSR J0737-3039A/B (Bur Gay et al. 2003), by assuming for the moment of inertia of A the value $I \approx 10^{38}$ kg m$^2$ (Lorimer and Kramer 2005), since its rotational period is 22 ms (Kramer et al. 2006) its angular momentum can be evaluated as $J_A = 2.8 \times 10^{40}$ kg m$^2$ s$^{-1}$. The overall uncertainty (including also the mismodelling in the usual 1PN term) in the periastron precession amounts to 0.03 deg yr$^{-1}$ (Iorio 2009), so that eq. (44) and the system’s parameters (Kramer et al. 2006) yield $k \leq 3 \times 10^{-21}$ m$^{-2}$.

In order to evaluate such bounds it is useful remember that studying the Schwarzschild-de Sitter non-gravitomagnetic precession various authors (Kagramanova et al. 2006; Iorio 2006b; Sereno and Jetzer 2006; Jetzer and Sereno 2006; Ruggiero and Iorio 2007) have obtained bounds on the cosmological constant that range from $10^{-40}$ m$^{-2}$ to about $10^{-42}$ m$^{-2}$; furthermore, the data in the general relativistic ΛCDM (cosmological constant plus cold dark matter) model suggest that a value of the cosmological constant of $10^{-52}$ m$^{-2}$ is required in order to explain the accelerated expansion of the Universe (Kochanek 1996). As a consequence, the bounds that we have just obtained are not competitive. The same conclusions apply to Palatini $f(R)$ gravity: in fact when the formation of large-scale structure is studied in this context (i.e. when the non linear part of $f(R)$ is supposed to drive cosmic acceleration) the results are practically indistinguishable from the ΛCDM model; see e.g. Koivisto (2006) who used large scale structure cosmological data from Sloan Digital Sky Survey (SDSS). However, these tight constraints on $f(R)$ gravity are loosened when dark matter with inherent stresses (generalized dark matter, GDM) is allowed (Koivisto 2007).
3. Discussion and Conclusions

We explicitly worked out the effects induced on the orbit of a test particle by the weak-field approximation of the Kerr-de Sitter metric, which is a solution of the vacuum field equations both in GTR and in $f(R)$ gravity. It turns out that the semi-major axis, the eccentricity and the inclination do not experience secular, i.e. averaged over one orbital period, changes; instead, the longitude of the ascending node, the argument of pericentre and the mean anomaly undergo secular precessions. Interestingly, all such effects consist of two kinds of contributions. The first type is given by terms proportional to $GMc^{-2}$, which vanish in the limits $c \to \infty$, $G \to 0$, $M \to 0$. Instead, the second kind consists of terms proportional to $Jk/M$, which are independent of the speed of light $c$, the constant of gravitation $G$ and the source’s mass $M$, so that they do not vanish in the limits for $c \to \infty$, $G \to 0$ and $M \to 0$. Concerning the dependence on the orbital geometry, both kinds of effects depend on the inclination and vanish for polar orbits; while the $O(c^{-2})$ terms depend also on the size of the orbit through the semi-major axis, it is not so for the $O(Jk/M)$ ones which are, indeed, independent of it. Then, we compared our predictions to the latest observational determinations of the corrections to the standard Newtonian/Einsteinian precessions of the perihelia of the inner planets of the Solar System obtaining the constrain $k \leq 10^{-29}$ m$^{-2}$. The node of the terrestrial LAGEOS satellite yields $k \leq 10^{-26}$ m$^{-2}$, while the bound from the periastron of the double pulsar system PSR J0737-3039A/B is $k \leq 3 \times 10^{-21}$ m$^{-2}$. Such bounds are not competitive with the ones which can be obtained from the Schwarzschild-de Sitter non-gravitomagnetic precessions ($k \leq 9 \times 10^{-43}$ m$^{-2}$ from Solar System data) and from those deriving from cosmological observations.
Table 1: Inner planets. First row: estimated perihelion extra-precessions in $10^{-4}''$ cy$^{-1}$ ("cy$^{-1}$→ arcseconds per century), from Table 3 of [Pitjeva 2005b] (apart from Venus). The quoted errors, in $10^{-4}''$ cy$^{-1}$, are not the formal ones but are realistic. The formal errors are quoted in square brackets (E.V. Pitjeva, personal communication to L.I., November 2005). The units are $10^{-4}''$ cy$^{-1}$. Second row: semi-major axes, in Astronomical Units (AU). Their formal errors are in Table IV of [Pitjeva 2005a], in m. Third row: eccentricities. Fourth row: orbital periods in years. The result for Venus have been recently obtained by including the Magellan radiometric data (E.V. Pitjeva, personal communication to L.I., June 2008).

|         | Mercury | Venus | Earth | Mars |
|---------|---------|-------|-------|------|
| $\langle \Delta \dot{\omega} \rangle$ ($10^{-4}''$ cy$^{-1}$) | $-36 \pm 50[42]$ | $-4 \pm 5[1]$ | $-2 \pm 4[1]$ | $1 \pm 5[1]$ |
| $a$ (AU) | 0.387   | 0.723 | 1.000 | 1.523 |
| $e$      | 0.2056  | 0.0067 | 0.0167 | 0.0934 |
| $P$ (yr) | 0.24    | 0.61  | 1.00  | 1.88 |
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