Presenting the mathematical model to optimize the reliability of the satellite attitude determination and control system

Akbar Mansouri\textsuperscript{a}, Akbar Alem-Tabriz\textsuperscript{b}\textsuperscript{*}

\textsuperscript{a} Department of Industrial Engineering, Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran

\textsuperscript{b} Department of Industrial Management, Management and Accounting Faculty, Shahid Beheshti University, Tehran, Iran

Abstract

The key issue in this study has been the integration of redundancy allocation and optimization of failure rates. The fact that we only need to work with parallel allocation and increase the number of components in a subsystem in parallel to improve the reliability or availability of parallel-series systems is necessary, but not enough. It is worth mentioning that in this research, the improvement of failure rates of different components in the system has been studied. In the meantime, it is important to note that with careful study of the effects of each of these approaches and the costs imposed on the system, the design problem data will be formed. Considering that more effort to improve the reliability of components leads to less redundancy allocation and vice versa, the optimization problem is performed to determine the exact number of redundancies along with determining the exact amount of improvements in complete failure rates. In this research, the satellite attitude determination and control system, the structure of the studied system and its components is introduced, then the reliability in this system is modeled and optimized with a mathematical approach based on the combination of reliability allocation and redundancy allocation.

Keyword: Reliability, Satellite, Attitude determination and control system, redundancy allocation problem, Genetic algorithm.

\* Corresponding author Email: A-tabriz@sbu.ac.ir
Tel: +98 912 157 8305
1- Introduction

Satellites are human-made devices that are deliberately sent into space to go around the earth or other planets. The importance of satellites for telecommunications and the study of terrestrial resources and research and military and espionage applications is growing. Part of the scientific and specialized research that is being done in space-based laboratories has never been able to take on a practical dimension on Earth. To control the safety of sensitive systems, such as those in nuclear power plants, chemical processes, and the astronaut system, it is necessary to have ready-made safety systems to complete on-line regulatory systems. Ready-to-use systems can automatically operate to keep the system safe and to prevent catastrophic outcomes under various conditions, however, ready-to-operate safety systems may not be able to perform the expected performance due to hidden failures. Therefore, it is very important to investigate these problems during testing and maintenance, and diagnose and resolve maintenance testing in accordance with the manufacturer's recommendations, which are generally cautious recommendations. In sensitive safety applications, the periodic maintenance period is sometimes determined by the supervisory team, for example, strict rules apply to intermittent maintenance to keep the level of uncertainty low. In general, intermittent and frequent testing can increase the likelihood of damage being detected. However, it may cause the system to crash faster due to the increased unnecessary costs imposed on resources. Therefore, logical and efficient testing of maintenance strategy is of great importance in ready-made safety systems.

In this research, by identifying the structure of the satellite attitude determination and control system, as well as the components used in it, the reliability of this system is analyzed and optimized. For this purpose, by investigating the information related to the system under study, the assumptions of modeling and optimizing the system reliability is determined. In this regard, practical and systemic constraints is considered. Finally, report the obtained results.

In this paper, a joint reliability–redundancy optimization approach for satellite attitude determination and control system is presented. This model is developed on a multiple-mode system that each mode has a specific reliability block diagram. The various subsystems of the system under study have active, cold-ready or k-out-of-n strategy, which are identified in the reliability block diagram of each mode. In this study, the percentage of improvements made to the failure rates of the components are integrated, and the failure rate of the components used in the closed continuous interval can be changed.
2- Literature Review

In this part, the studies conducted in the field of optimizing the reliability of satellites are mentioned and each of them is examined. Satellite is a separate system with limited communication that is difficult to access after launch. That's why it's very difficult to repair if the system breaks down. Therefore, despite the risk of failure, the satellite system must perform well during its lifetime. Many factors include satellite threats, including the environment, network problems, software errors, and so on [1]. Environmental threats include the effects of solar proton and electron damage caused by cosmic rays, leading to incorrect commands and incorrect data. [2] Also, network threats can lead to command error caused by viruses and Aplinks [3] software errors in satellites in particular are among the most important issues for long-lived systems and are a priority. Increasing the lifespan of software causes the disclosure of confidential resources and leads to a gradual reduction in system performance [4].

The study, conducted by Castet et al., [5] noted the limited access to information and failure rates associated with various satellite subsystems, and used the Kaplan-Meier estimator method to calculate system reliability. Not only did they use non-parametric methods for modeling, but they also used the maximum accuracy estimation method (MLE) to use the Weibel distribution parameters to distribute life under different systems, and finally compared the results with each other. The 11 subsystems on the satellite focused on one of the most important achievements of the study. Category 1: One year later, Castet et al. [6] considered the subsystems of a satellite to be multi-layered components. In that case, each case is named briefly. Nagiya and Ram published a study in which it used the Markov model to evaluate and optimize a satellite with specific information. The assumptions of this research include the following [7].

✓ Initially, all components of the system are intact.
✓ The satellite under study generally includes 8 states
✓ All breakdown and repair rates are fixed over time.
✓ At one point, only one transfer from one state to another is allowed.
✓ The necessary equipment is available for repair.
✓ Each component after repair is like a new component.
✓ All satellite operating states are repairable.

In this study, the level of redundancy and failure rates in the satellite attitude determination and control system are optimized. In general, the issue of over-allocation has been studied by many
researchers. Fyffe et al., [8] were the first to present the mathematical model of the general problem of redundancy allocation. The goal of their proposed model was to maximize system reliability by considering weight and cost constraints. They solved this problem with the help of dynamic planning. Nakagawa and Miyazaki [9] presented a non-linear planning problem with a solution to optimize reliability. In fact, by changing the example of Fyffe et al., They solved the problem by using the exact method of substituting constraints, and showed that in the case of multiple constraints, this approach is better than the dynamic planning method. However, in order to increase the reliability, one or a set of the following measures can be performed. These measures can be implemented based on the assumptions and requirements of each system.

- Optimization of redundancy level.
- Optimizing the selection of the type of components used in the system.
- Optimization of existing technical activities in order to regulate failure and repair rates.

In the following, some conducted research on the optimization of the redundancy allocation problem will be showed in Table 1.

Please insert Table 1 about here

3- Satellite attitude determination and control system

In this research, the satellite attitude determination and control system will be investigated. High efficiency, reliability and health are the most important criteria in the engineering of space systems. Reliability refers to the possibility of the system working properly over a period of time. The first and most important step in preventing a system from malfunctioning is to detect failure in that system. Because in this case, there will be a good time to prevent problems in the system. Satellites are examples of self-contained, important, and costly space systems that are sent on relatively long-term missions. One of the most important parts of satellite that is directly related to health is the subsystem of satellite attitude determination and control system. The task of this subsystem is to determine and control the situation in space and neutralize the disturbing environmental disturbances and torques on the satellite. Therefore, this subsystem is always known as a subject of study to provide breakdown identification because improving the subsystem reliability of satellite attitude determination and control system directly affects the reliability of the satellite in space. In order to design the subsystem of satellite attitude determination and control system, in most cases, three reaction wheels are used in line with the three main axes of the satellite. Each of
these actuator has an electric motor and a heavy disk. By applying current to the electric motor, torque is produced, which changes the speed of the angles of the motor axis. The opposite direction is produced. Changing the speed of the reaction wheel by applying the necessary control algorithm to the motors causes the motor to reach the required speed from zero speed. After producing the necessary torque, the motor shuts off again. This change in speed causes the required tower radiation to be generated to achieve the desired state of the satellite. In order to increase reliability, a reaction or spare reaction cycle is used under the systems based on the reaction wheel. According to Figure 1, this actuator is located next to the main reaction cycle and as soon as one of the wheels fails or irreparable damage occurs in them, the spare reaction wheel will replace the defective cycle in the system. In general, the modules of system studied in this research are shown in the figure 2.

Please insert Figure 1 about here
Please insert Figure 2 about here

4- Problem Statement
In this section, the satellite attitude determination and control system, the structure of the system under study and its components are introduced, then model and optimize the reliability in this system according to the mathematical approach based on the combination of reliability allocation and redundancy allocation. In the following, we will introduce the components, structure and functions of the satellite attitude determination and control system.

4-1- Functions of attitude determination and control system
The functions of the attitude determination and control system include the following:

- Adjust the satellite in the desired direction despite external disturbance torques
- Determine the attitude of satellites using sensors
- The situation switching by actuators
- Satellite orientation for the mission

The sub-system for determining and controlling the situation to perform each of its tasks in the form of control modes and obtaining the required attitude in each mode, consists of two parts: determination and control. The attitude determination section includes sensors and attitude determination algorithms, and the attitude control section includes actuators and attitude control algorithms.

4-2- Components of attitude determination and control system
The attitude determination and control system consists of several parts, which include the following:

- Electronic Control Unit (ECU)
- Interface (INT)
- Sun sensors (SS)
- Magnetic Sensor (MM)
- Gyro sensor (Gyro)
- Star Sensor (ST)
- Magnetic torquer (MT)
- Reaction wheel (RW)

4-3- **Modes of attitude determination and control system**

In general, the satellite attitude determination and control system that is analyzed in this research has five different functional modes. Including DE tumbling mode, Coarse pointing mode, Fine pointing mode, Sun pointing mode, and safe mode. Reliability block diagram of modes in Figures 3 to 7 are provided.

In these Reliability block diagrams, Subsystems marked with gate s (Before the subsystem) are cold-standby. Values of K and N for k-out-of-n subsystems is specified. For example reaction wheel subsystem in the Coarse pointing mode is 2-out-of-4. Subsystems with active redundancy are unmarked. Other subsystems are series.

Please insert Figure 3 about here
Please insert Figure 4 about here
Please insert Figure 5 about here
Please insert Figure 6 about here
Please insert Figure 7 about here

4-4- **The parameters and variables of the problem**

Table 2 introduces the parameters and Table 3 introduces the variables of optimizing the reliability of satellite attitude determination and control problem.

Please insert Table 2 about here
Please insert Table 3 about here
4-5- **Assumptions**

- The function of the components used in the system is independent of each other
- The components failure used in the system is independent of each other
- The components of the system under study are binary
- The components cannot be repaired and returned to the system after failure
- The parameters related to the cost and weight of the components in the system are deterministic and definite
- The system has the maximum cost and maximum weight allowed for the components used in it
- The system has different functional modes
- The various subsystems of the system under study are active, cold-ready or k-out-of-n
- The failure rate of the components used in the system is constant
- The lifetime distribution of the components used in the system is exponential
- The percentage of improvements made to component failure rates is an integer number
- Under systems where the components are cold-standby, there is a possibility that the switch will fail
- The failure rate of the components in a closed continuous interval can be changed.

4-6- **Mathematical modeling**

Based on the structure of the attitude determination and control system under each of the functional modes and also based on the arrangement of components according to the reliability block diagrams presented in the section 3-4, the reliability of the system under each functional modes is calculated. As can be seen from the reliability block diagram provided, the structure of the attitude determination and control system in the various functional modes of the hybrid structure includes active, cold-standby and k-out-of-n. In this section, we first model the reliability level of the attitude determination and control system in each of the functional modes under the conditions that the components used in the system follow any desired life distribution. Then, by placing the probability density function and the reliability of the exponential distribution in equations 1 to 5, the reliability of the system is presented in the functional modes of de tumbling, Coarse pointing, fine pointing, sun pointing and safe.
\[
R_D (t) = \left[ R_{ECU} (t) + \sum_{j=1}^{N_{ECU}} \rho_{ECU} (t) \int_{0}^{t} f_{ECU} (u) (j) R_{ECU} (t - u) du \right] \times \left[ 1 - (1 - R_{Int} (t))^{N_{aw}} \right] \\
\times \left[ R_{MM} (t) + \sum_{j=1}^{N_{MM}} \rho_{MM} (t) \int_{0}^{t} f_{MM} (u) (j) R_{MM} (t - u) du \right] \\
\times \left[ R_{Gyro} (t) + \sum_{j=1}^{N_{Gyro}} \rho_{Gyro} (t) \int_{0}^{t} f_{Gyro} (u) (j) R_{Gyro} (t - u) du \right] \times \left[ R_{MT} (t)^{N_{aw}} \right] 
\]

\[
R_C (t) = \left[ R_{ECU} (t) + \sum_{j=1}^{N_{ECU}} \rho_{ECU} (t) \int_{0}^{t} f_{ECU} (u) (j) R_{ECU} (t - u) du \right] \times \left[ 1 - (1 - R_{Int} (t))^{N_{aw}} \right] \\
\times \left[ R_{MM} (t) + \sum_{j=1}^{N_{MM}} \rho_{MM} (t) \int_{0}^{t} f_{MM} (u) (j) R_{MM} (t - u) du \right] \\
\times \left[ R_{Gyro} (t) + \sum_{j=1}^{N_{Gyro}} \rho_{Gyro} (t) \int_{0}^{t} f_{Gyro} (u) (j) R_{Gyro} (t - u) du \right] \times \left[ R_{SS} (t)^{N_{SS}} \right] \times \left[ R_{MT} (t)^{N_{aw}} \right] \\
\times \left[ \sum_{k=2}^{N_{RW}} \left( \frac{N_{RW}}{k} \right) R_{RW} (t)^{k} (1 - R_{RW} (t))^{N_{aw} - k} \right]
\]

\[
R_F (t) = \left[ R_{ECU} (t) + \sum_{j=1}^{N_{ECU}} \rho_{ECU} (t) \int_{0}^{t} f_{ECU} (u) (j) R_{ECU} (t - u) du \right] \times \left[ 1 - (1 - R_{Int} (t))^{N_{aw}} \right] \\
\times \left[ R_{MM} (t) + \sum_{j=1}^{N_{MM}} \rho_{MM} (t) \int_{0}^{t} f_{MM} (u) (j) R_{MM} (t - u) du \right] \\
\times \left[ R_{Gyro} (t) + \sum_{j=1}^{N_{Gyro}} \rho_{Gyro} (t) \int_{0}^{t} f_{Gyro} (u) (j) R_{Gyro} (t - u) du \right] \times \left[ R_{SS} (t)^{N_{SS}} \right] \\
\times \left[ R_{ST} (t) + \sum_{j=1}^{N_{ST}} \rho_{ST} (t) \int_{0}^{t} f_{ST} (u) (j) R_{ST} (t - u) du \right] \times \left[ R_{MT} (t)^{N_{aw}} \right] \\
\times \left[ \sum_{k=3}^{N_{RW}} \left( \frac{N_{RW}}{k} \right) R_{RW} (t)^{k} (1 - R_{RW} (t))^{N_{aw} - k} \right]
\]
\[
R_{SU}(t) = \left[ R_{ECU}(t) + \sum_{j=1}^{N_{ECU}-1} \rho_{ECU}(t) \int_0^t f_{ECU}(u) \, R_{ECU}(t-u) \, du \right] \times \left[ 1 - (1 - R_{int}(t))^{N_{int}} \right] \\
\times \left[ R_{MM}(t) + \sum_{j=1}^{N_{MM}-1} \rho_{MM}(t) \int_0^t f_{MM}(u) \, R_{MM}(t-u) \, du \right] \\
\times \left[ R_{Gyro}(t) + \sum_{j=1}^{N_{Gyro}-1} \rho_{Gyro}(t) \int_0^t f_{Gyro}(u) \, R_{Gyro}(t-u) \, du \right] \times \left[ R_{SS}(t)^{N_{SS}} \right] \times \left[ R_{MT}(t)^{N_{MT}} \right] \\
\times \sum_{k=2}^{N_{RW}} \left( \begin{array}{c} N_{RW} \\ k \end{array} \right) R_{RW}(t) \left( 1 - R_{RW}(t) \right)^{N_{RW} - k} 
\]

\[
R_{SE}(t) = \left[ R_{ECU}(t) + \sum_{j=1}^{N_{ECU}-1} \rho_{ECU}(t) \int_0^t f_{ECU}(u) \, R_{ECU}(t-u) \, du \right] \times \left[ 1 - (1 - R_{int}(t))^{N_{int}} \right] \\
\times \left[ R_{MM}(t) + \sum_{j=1}^{N_{MM}-1} \rho_{MM}(t) \int_0^t f_{MM}(u) \, R_{MM}(t-u) \, du \right] \times \left[ R_{SS}(t)^{N_{SS}} \right] \times \left[ R_{MT}(t)^{N_{MT}} \right] \\
\times \sum_{k=2}^{N_{RW}} \left( \begin{array}{c} N_{RW} \\ k \end{array} \right) R_{RW}(t) \left( 1 - R_{RW}(t) \right)^{N_{RW} - k} 
\]

Now, under the assumptions provided in Section 4-5, according to the information provided about the system for determining and controlling the attitude of the satellite base, as well as calculations for the reliability of the system under different functional modes, the mathematical optimization model allocates reliability and redundancy allocation for the system under study is presented below. In this model, the reliability of each functional mode based on the arrangement of components and system structure in the block of other names provided for each of these modes is calculated. Also, the failure rate of the components used in the fixed system is assumed. It is used in the display system, so in the set of equations presented to calculate the reliability of the system under different functional modes, the probability density function and the reliability function of the display distribution are placed.

\[
\max \left\{ \min \left\{ R_D, R_C, R_F, R_{SU}, R_{SE} \right\} \right\} 
\]

S.T
\[ R_D(t) = \left[ \exp(-\lambda_{ECU} t) + \rho_{ECU}(t) \exp(-\lambda_{ECU} t) \right] \sum_{j=1}^{N_{ECU}-1} \left( \frac{\lambda_{ECU} t}{j!} \right)^j \times \left[ 1 - (1 - \exp(-\lambda_{Int} t))^{N_{Int}} \right] \]

\[ \times \left[ \exp(-\lambda_{MM} t) + \rho_{MM}(t) \exp(-\lambda_{MM} t) \right] \sum_{j=1}^{N_{MM}-1} \left( \frac{\lambda_{MM} t}{j!} \right)^j \times \exp(-\lambda_{MT} t)^{N_{MT}} \] (7)

\[ R_C(t) = \left[ \exp(-\lambda_{ECU} t) + \rho_{ECU}(t) \exp(-\lambda_{ECU} t) \right] \sum_{j=1}^{N_{ECU}-1} \left( \frac{\lambda_{ECU} t}{j!} \right)^j \times \left[ 1 - (1 - \exp(-\lambda_{Int} t))^{N_{Int}} \right] \times \left[ \exp(-\lambda_{MM} t) + \rho_{MM}(t) \exp(-\lambda_{MM} t) \right] \sum_{j=1}^{N_{MM}-1} \left( \frac{\lambda_{MM} t}{j!} \right)^j \times \exp(-\lambda_{MT} t)^{N_{MT}} \times \exp(-\lambda_{SS} t)^{N_{SS}} \] (8)

\[ \times \left[ \exp(-\lambda_{MT} t)^{N_{MT}} \right] \times \left[ \sum_{k=2}^{N_{RW}} \binom{N_{RW}}{k} \exp(-\lambda_{RW} t)^k (1 - \exp(-\lambda_{RW} t))^{N_{RW} - k} \right] \]
\[ R_F (t) = \left[ \exp(-\lambda_{ECU} \ t) + \rho_{ECU} (t) \cdot \exp(-\lambda_{ECU} \ t) \cdot \sum_{j=1}^{N_{ECU}} \frac{(\lambda_{ECU} \ t)^j}{j!} \right] \times \left[ 1 - (1 - \exp(-\lambda_{Int} \ t))^N_{Int} \right] \times \left[ \exp(-\lambda_{MM} \ t) + \rho_{MM} (t) \cdot \exp(-\lambda_{MM} \ t) \cdot \sum_{j=1}^{N_{MM}} \frac{(\lambda_{MM} \ t)^j}{j!} \right] \times \left[ \exp(-\lambda_{Gyro} \ t) + \rho_{Gyro} (t) \cdot \exp(-\lambda_{Gyro} \ t) \cdot \sum_{j=1}^{N_{Gyro}} \frac{(\lambda_{Gyro} \ t)^j}{j!} \times \left[ \exp(-\lambda_{SS} \ t)^{N_{SS}} \right] \times \left[ \exp(-\lambda_{ST} \ t) + \rho_{ST} (t) \cdot \exp(-\lambda_{ST} \ t) \cdot \sum_{j=1}^{N_{ST}} \frac{(\lambda_{ST} \ t)^j}{j!} \times \left[ \exp(-\lambda_{MT} \ t)^{N_{MT}} \right] \times \left[ \sum_{k=3}^{N_{RW}} \binom{N_{RW}}{k} \exp(-\lambda_{RW} \ t)^k (1 - \exp(-\lambda_{RW} \ t))^{N_{RW} - k} \right] \] 

\[ R_{SU} (t) = \left[ \exp(-\lambda_{ECU} \ t) + \rho_{ECU} (t) \cdot \exp(-\lambda_{ECU} \ t) \cdot \sum_{j=1}^{N_{ECU}} \frac{(\lambda_{ECU} \ t)^j}{j!} \right] \times \left[ 1 - (1 - \exp(-\lambda_{Int} \ t))^N_{Int} \right] \times \left[ \exp(-\lambda_{MM} \ t) + \rho_{MM} (t) \cdot \exp(-\lambda_{MM} \ t) \cdot \sum_{j=1}^{N_{MM}} \frac{(\lambda_{MM} \ t)^j}{j!} \right] \times \left[ \exp(-\lambda_{Gyro} \ t) + \rho_{Gyro} (t) \cdot \exp(-\lambda_{Gyro} \ t) \cdot \sum_{j=1}^{N_{Gyro}} \frac{(\lambda_{Gyro} \ t)^j}{j!} \times \left[ \exp(-\lambda_{SS} \ t)^{N_{SS}} \right] \times \left[ \exp(-\lambda_{ST} \ t) + \rho_{ST} (t) \cdot \exp(-\lambda_{ST} \ t) \cdot \sum_{j=1}^{N_{ST}} \frac{(\lambda_{ST} \ t)^j}{j!} \times \left[ \sum_{k=2}^{N_{RW}} \binom{N_{RW}}{k} \exp(-\lambda_{RW} \ t)^k (1 - \exp(-\lambda_{RW} \ t))^{N_{RW} - k} \right] \] 

\[ R_{SE} (t) = \left[ \exp(-\lambda_{ECU} \ t) + \rho_{ECU} (t) \cdot \exp(-\lambda_{ECU} \ t) \cdot \sum_{j=1}^{N_{ECU}} \frac{(\lambda_{ECU} \ t)^j}{j!} \right] \times \left[ 1 - (1 - \exp(-\lambda_{Int} \ t))^N_{Int} \right] \times \left[ \exp(-\lambda_{MM} \ t) + \rho_{MM} (t) \cdot \exp(-\lambda_{MM} \ t) \cdot \sum_{j=1}^{N_{MM}} \frac{(\lambda_{MM} \ t)^j}{j!} \right] \times \left[ \exp(-\lambda_{SS} \ t)^{N_{SS}} \right] \times \left[ \exp(-\lambda_{MT} \ t)^{N_{MT}} \right] \]
\[ C_{ECU} \cdot N_{ECU} + C_{Int} \cdot N_{Int} + C_{MM} \cdot N_{MM} + C_{Gyro} \cdot N_{Gyro} \]
\[ + C_{ST} \cdot N_{ST} + C_{RW} \cdot N_{RW} \]
\[ D_{ECU} \cdot x_{ECU} + D_{Int} \cdot x_{Int} + D_{MM} \cdot x_{MM} + D_{Gyro} \cdot x_{Gyro} \]
\[ + D_{SS} \cdot x_{SS} + D_{ST} \cdot x_{ST} + D_{MT} \cdot x_{MT} + D_{RW} \cdot x_{RW} \leq C_{\text{max}} \]
\[ W_{ECU} \cdot N_{ECU} + W_{Int} \cdot N_{Int} + W_{MM} \cdot N_{MM} + W_{Gyro} \cdot N_{Gyro} \]
\[ + W_{ST} \cdot N_{ST} + W_{RW} \cdot N_{RW} \leq W_{\text{max}} \]
\[ L_{ECU} \leq N_{ECU} \leq U_{ECU} \]
\[ L_{Int} \leq N_{Int} \leq U_{Int} \]
\[ L_{MM} \leq N_{MM} \leq U_{MM} \]
\[ L_{\text{Gyro}} \leq N_{\text{Gyro}} \leq U_{\text{Gyro}} \]
\[ L_{ST} \leq N_{ST} \leq U_{ST} \]
\[ L_{RW} \leq N_{RW} \leq U_{RW} \]
\[ \lambda_{ECU} = \lambda_{ECU}^{\text{max}} \left( 1 - \frac{x_{ECU}}{100} \right) \]
\[ \lambda_{Int} = \lambda_{Int}^{\text{max}} \left( 1 - \frac{x_{Int}}{100} \right) \]
\[ \lambda_{MM} = \lambda_{MM}^{\text{max}} \left( 1 - \frac{x_{MM}}{100} \right) \]
\[ \lambda_{\text{Gyro}} = \lambda_{\text{Gyro}}^{\text{max}} \left( 1 - \frac{x_{\text{Gyro}}}{100} \right) \]
\[ \lambda_{SS} = \lambda_{SS}^{\text{max}} \left( 1 - \frac{x_{SS}}{100} \right) \]
\[ \lambda_{ST} = \lambda_{ST}^{\text{max}} \left( 1 - \frac{x_{ST}}{100} \right) \]
\[ \lambda_{MT} = \lambda_{MT}^{\text{max}} \left( 1 - \frac{x_{MT}}{100} \right) \]
\[ \lambda_{RW} = \lambda_{RW}^\text{max} \left(1 - \frac{x_{RW}}{100}\right) \]  \hspace{1cm} (27)

\[ \lambda_{ECU}^\text{min} \leq \lambda_{ECU} \leq \lambda_{ECU}^\text{max} \]  \hspace{1cm} (28)

\[ \lambda_{Int}^\text{min} \leq \lambda_{Int} \leq \lambda_{Int}^\text{max} \]  \hspace{1cm} (29)

\[ \lambda_{MM}^\text{min} \leq \lambda_{MM} \leq \lambda_{MM}^\text{max} \]  \hspace{1cm} (30)

\[ \lambda_{Gyro}^\text{min} \leq \lambda_{Gyro} \leq \lambda_{Gyro}^\text{max} \]  \hspace{1cm} (31)

\[ \lambda_{SS}^\text{min} \leq \lambda_{SS} \leq \lambda_{SS}^\text{max} \]  \hspace{1cm} (32)

\[ \lambda_{ST}^\text{min} \leq \lambda_{ST} \leq \lambda_{ST}^\text{max} \]  \hspace{1cm} (33)

\[ \lambda_{MT}^\text{min} \leq \lambda_{MT} \leq \lambda_{MT}^\text{max} \]  \hspace{1cm} (34)

\[ \lambda_{RW}^\text{min} \leq \lambda_{RW} \leq \lambda_{RW}^\text{max} \]  \hspace{1cm} (35)

\[ x_{ECU}, x_{Int}, x_{MM}, x_{Gyro}, x_{SS}, x_{ST}, x_{MT}, x_{RW} \in \text{Integer} \]  \hspace{1cm} (36)

\[ N_{ECU}, N_{Int}, N_{MM}, N_{Gyro}, N_{ST}, N_{RW} \in \text{Integer} \]  \hspace{1cm} (36)

\[ \lambda_{ECU}, \lambda_{Int}, \lambda_{MM}, \lambda_{Gyro}, \lambda_{SS}, \lambda_{ST}, \lambda_{MT}, \lambda_{RW} \geq 0 \]

The objective function of the above mathematical model (6) is to maximize the minimum reliability of the attitude determination and control system under different functional modes.

Constraints (7 to 11) calculates system reliability in the functional modes of de tumbling, coarse pointing, fine pointing, sun pointing and safe respectively. Constraint (12) guarantees that the total costs incurred in the system includes the costs of redundancy allocation and failure rate improvement does not exceed the maximum allowable cost. Constraint (13) ensures that the weight of the components allocated to the system does not exceed the maximum allowable weight specified for it. Constraints (14) to (19) ensure that redundant components to the system do not exceed the minimum and maximum allowable values specified for each. Constraints (20) to (27) are a set of calculative constraints that calculate the failure rate of each component based on the maximum possible failure rate and the percentage of improvement created for them. Constraints (28) to (35) ensure that the failure rate of each component does not exceed the minimum and maximum value specified for each of them.

5- Solution procedure
The mathematical optimization model presented in this study falls into the category of optimization problems associated with mixed integer nonlinear programming (MINLP). In most cases, on the other hand, the issue of over-allocation in terms of computational time falls into the category of NP-HARD problems. Therefore, metaheuristic algorithms should be used to solve the proposed mathematical model. In this section, a genetic algorithm is used to solve the proposed model.

The general form of the solutions related to the mathematical model presented consists of a matrix. The main focus of this research to solve the problem is on the genetic algorithm, which we will explain and interpret in the following search operators. The structure of the solution presented in this study, according to Figure 8, consists of a two-row matrix, in the first row the number of redundant components in the system is specified, and in the second row the percentage of improvement in the failure rate of each component relative to the maximum failure rate is determined.

Please insert Figure 8 about here

After generating the chromosome, the evaluation of each chromosome is calculated. For this purpose, the values of Z, Cost, and Weight are calculated based on the presented mathematical model, then the evaluation of each solution is determined as Equation 37.

\[
f = \frac{z}{1 + \max(\text{Cost} - C_{\text{max}}, 0) + \max(\text{Weight} - W_{\text{max}}, 0)}
\]  

(37)

Where Z, Cost, and Weight functions are calculated as relations 38, 39, and 40. Thus, solutions that do not meet the maximum budget and maximum weight constraints will be penalized.

\[
Z = \max \left\{ \min \left\{ R_D, R_C, R_F, R_{SU}, R_{SE} \right\} \right\}
\]  

(38)

\[
\text{Cost} = C_{\text{ECU}} \cdot N_{\text{ECU}} + C_{\text{Int}} \cdot N_{\text{Int}} + C_{\text{MM}} \cdot N_{\text{MM}} + C_{\text{Gyro}} \cdot N_{\text{Gyro}} + C_{\text{ST}} \cdot N_{\text{ST}} + C_{\text{RW}} \cdot N_{\text{RW}} + D_{\text{ECU}} \cdot x_{\text{ECU}} + D_{\text{Int}} \cdot x_{\text{Int}} + D_{\text{MM}} \cdot x_{\text{MM}} + D_{\text{Gyro}} \cdot x_{\text{Gyro}} + D_{\text{SS}} \cdot x_{\text{SS}} + D_{\text{ST}} \cdot x_{\text{ST}} + D_{\text{MT}} \cdot x_{\text{MT}} + D_{\text{RW}} \cdot x_{\text{RW}}
\]  

(39)
\[
W_{\text{eight}} = W_{\text{ECU}} \cdot N_{\text{ECU}} + W_{\text{Int}} \cdot N_{\text{Int}} + W_{\text{MM}} \cdot N_{\text{MM}} + W_{\text{Gyro}} \cdot N_{\text{Gyro}} + W_{\text{SS}} \cdot N_{\text{SS}} + W_{\text{ST}} \cdot N_{\text{ST}} + W_{\text{MT}} \cdot N_{\text{MT}} + W_{\text{RW}} \cdot N_{\text{RW}}
\] (40)

In this research, the roulette wheel mechanism has been used for the selection strategy. Choosing a roulette wheel was first suggested by the Holland [29].

Parents are first selected to perform the crossover operator, then the children are generated using a uniform crossover operator. The operation of this operator is described in references [30] and [31]. In this operator, for each gene in the selected parent chromosome, a number between zero and one is randomly generated, then the child's chromosomes are quantified in linear composition from the parent chromosomes.

Mutations are also performed on each array of the chromosome matrix. In this operator, after selecting the desired parent, a random number between zero and one is generated for each gene in the parent chromosome, and the values of the parent chromosome genes are mutated at a certain rate of mutation. Now, if the random number generated is less than the desired mutation rate, the corresponding gene on the parent chromosome will be randomly mutated, but if the random number generated is larger than the mutation rate, the gene will not be mutated on the parent chromosome [31]. For example, Figure 9 shows how the mutation operator executes on a chromosome.

Please insert Figure 9 about here

Using a random search method (RS) to solve the presented model can be a lower bound for minimization problems and a lower bound for maximization problems compared to other solution methods. In fact, proving the intelligent performance of meta-heuristic algorithms can be demonstrated by comparing them to an RS. So that, these algorithms must always be more powerful than an RS. Therefore an RS is provided to validate the proposed algorithm. The proposed RS Pseudocode is shown in Figure 10 [32].

Please insert Figure 10 about here

6- Results and discussions

In this section, the proposed model is solved. For this purpose, first a numerical example is presented on the case study. The table 4 shows the parameters and the information needed to solve the problem for a numerical sample example.
To solve the above example, the parameters of the genetic algorithm must first be tuned. The purpose of tuning the input parameters of algorithms is to achieve appropriate criteria for the objective function of the algorithm. The result of meta-heuristic algorithms depends on the values of their input parameters. Therefore, we explain in detail how to set the values of these parameters. Input parameters of the genetic algorithm are population size ($n_{pop}$), crossover rate ($P_c$), and mutation rate ($P_m$). Each of these parameters is of particular importance and affects the performance of this algorithm. In order to recognize the appropriate values of the parameters in such a way that the criterion of the objective function leads to the appropriate solutions, the response surface methodology (RSM) technique has been used [33]. The main parameters of this algorithm are considered in Table 5 to tune on the appropriate levels. Due to the choice of two-level experimental factor design, each of the experiments is considered at two levels, high and low. The method of advancing the response surface methodology is such that in addition to the upper and lower limits, the axial points using the middle limits as well as a number of central points (in this research 5 central points are added to the design) is also considered. For this algorithm, according to the three available parameters, factor $2^3$ is considered. For this purpose, we performed the experiment in MINITAB 16 software for the algorithm and tune the best level for the test result.

According to the explanations provided, the nonlinear regression equation for the proposed genetic algorithm, which shows the relationship between the parameters of the algorithm and the value of the objective function, is obtained. Now it is enough to solve the model (41) to get the optimal parameters of the genetic algorithm.

\[
\begin{align*}
\text{Max} \quad & 0.875248 - 0.000396038 \times n_{\text{pop}} - 0.0368956 \times P_c \\
& - 0.0567818 \times P_m + 3.03127e^{-06} \times n_{\text{pop}} \times n_{\text{pop}} + 0.0348687 \times P_c \times P_c \\
& - 0.189545 \times P_m \times P_m + 0.000768 \times n_{\text{pop}} \times P_m - 0.000241333 \times n_{\text{pop}} \times P_c
\end{align*}
\]

\begin{align*}
& + 0.128 \times P_m \times P_c
\end{align*}

S.T.
$50 \leq n_{pop} \leq 100$
$0.4 \leq P_c \leq 0.7$
$0.1 \leq P_c \leq 0.3$
$n_{pop} \in Z$

Solving the above model in Lingo software determined the values of the parameters of the genetic algorithm, which can be seen in Table 6.

Please insert Table 6 about here

After obtaining the tuned parameters of the genetic algorithm, the example presented in Table 4 is solved using the genetic algorithm developed in this study and its results are reported. As shown in Figure 11, the convergence diagram of the genetic algorithm is shown in consecutive iterations, and in Tables 7 and 8, the information about the variables of the mathematical optimization model provided under the available maximum budget is clearly presented.

Please insert Figure 11 about here
Please insert Table 7 about here
Please insert Table 8 about here

7- Sensitivity analysis and validation

In this section, in order to validate the genetic algorithm developed to solve the mathematical model, which was discussed in Section 4-6, we compare the results of this algorithm with the solutions obtained from the random search method obtained under different amounts of the maximum budget. Given that we know that the solutions obtained from the random search method are always solutions far from global optimization, so these solutions are a good criterion for evaluating the performance of the genetic algorithm. As can be seen in Table 9, genetics in all cases has found better solutions than random search, which indicates the efficient operation of this algorithm to solve the current problem. Figure 12 also shows the effect of increasing the maximum budget available on the reliability of the system under study. It is worth mentioning that in this paper, the 2018 version of MATLAB software has been used to implement the random search method and genetic algorithm.

Please insert Table 9 about here
Please insert Figure 12 about here
8- Conclusions

In this research, the satellite attitude determination and control system, the structure of the studied system and its components is introduced, then the reliability in this system was modeled and optimized according to the mathematical approach based on the combination of reliability allocation and redundancy allocation. By studying this system, it was determined that in general, the satellite attitude determination and control system has five different functional modes. These modes include de tumbling, coarse pointing, fine pointing, sun pointing and safe. In the following, each of these functional modes is introduced and the reliability block diagram of the system is presented under each of these modes. In this system, the function of the components used in the system and the components failure are independent of each other. Failed components do not damage the system as a whole. Also component are binary. In the attitude determination and control system, components cannot be repaired and returned to the system after failure. In the mathematical optimization model presented in this study, the parameters related to the cost and weight of the components in the system are deterministic and definite, and the system has the maximum cost and maximum allowable weight for the components used in it. The various subsystems of the system under study have active, cold-ready or k-out-of-n strategy, which are identified in the reliability block diagram of each mode. Also, systems where the components are cold-standby, there is a possibility that the switch will fail. In this study, the failure rate of the components used in the system is constant, so the life distribution of the components used in the system is exponential. In this study, the percentage of improvements made to the failure rates of the components is integer, and the failure rate of the components used in the closed continuous interval can be changed. The mathematical optimization model presented in this study falls into the category of optimization problems related to mixed integer nonlinear programming (MINLP). Solving these problems always involves a lot of mathematical complexity. Therefore, the development of exact solution methods for these problems is very difficult and in most cases impossible. On the other hand, the redundancy allocation problem in terms of computational time falls into the class of NP-HARD problems. Therefore, metaheuristic algorithms should be used to solve the proposed mathematical model. In this study, a genetic algorithm was used to solve the proposed model. Also, to valid the results obtained from the developed genetic algorithm, a random search algorithm was used, which according to the obtained outputs, the genetic algorithm
had significantly better performance in all cases than random search. In this study, in order to tune the parameters of the genetic algorithm, the response surface methodology was used, based on which the parameters of the number of population in each iteration, crossover rate and mutation rate were tuned.

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Biographies

Akbar Alam Tabriz received his PhD degree in Management from Turkey in 1989. He is currently an Associated Professor in Shahid Beheshti University. His research interests are performance evaluation in production industries, implementation of Total Quality Management (TQM), and efficiency and productivity measurement in Industry. He has published several papers in national and international journals.

Akbar Mansouri is a PhD candidate at the Department of Industrial Engineering at Islamic Azad University, Qazvin Branch (QIAU) in Iran. He received his BSc and MSc degrees in Industrial Engineering from Amirkabir University of technology (Tehran polytechnic) in Iran. His research interests are in reliability engineering, combinatorial optimization, multi-objective optimization, computational intelligence, and data mining.
Table 1. Assumption of mathematical optimization models in the redundancy allocation problem area

| Authors                    | State   | Element type | Algorithm             | Fuzzy availability | Fault elements | Penalty function | Objective | Parameter setting | Cost discount strategy |
|----------------------------|---------|--------------|-----------------------|--------------------|----------------|------------------|-----------|-------------------|------------------------|
| Garg et al. [10]           | Binary  | Heterogeneous| Bee colony           | No                 | Non-repairable | Yes              | Single    | No                | No                     |
| Khalili-Damghani et al. [11]| Binary  | Heterogeneous| e-constraint         | No                 | Non-repairable | No               | Multiple  | No                | No                     |
| Chambari et al. [12]       | Binary  | Heterogeneous| SA                   | No                 | Non-repairable | Yes              | Single    | No                | No                     |
| Gago et al. [13]           | Binary  | Heterogeneous| Greedy, Walk back Fuzzy inference system (FIS) Imperfect repair model | No | Non-repairable | No | Single | No | No |
| Ebrahimipour et al. [14]   | Binary  | Heterogeneous|                       |                    |                |                  |           | No                | No                     |
| Liu et al. [15]            | Multi-state| Heterogeneous| GA                   | No                 | Non-repairable | Yes              | Repairable | No  | Single | No | No |
| Ding and Lisnianski [16]   | Multi-state| Heterogeneous| GA                   | No                 | Non-repairable | No | Single | No | No |
| Ouzineb et al. [17]        | Multi-state| Heterogeneous| GA                   | No                 | Non-repairable | No | Single | No | No |
| Sharma and Agarwal [18]   | Multi-state| Heterogeneous| ACO                  | No                 | Non-repairable | No | Single | No | No |
| Ouzineb et al. [19]        | Multi-state| Homogeneous  | TS                   | No                 | Non-repairable | No | Single | No | AUD |
| Levitin et al. [20]        | Multi-state| Heterogeneous| GA                   | No                 | Non-repairable | No | Single | No | No |
| Lins and Drogue [21]       | Binary  | Heterogeneous| GA                   | No                 | Repairable     | No | Multiple | No | No |
| Lins and Drogue [22]       | Binary  | Heterogeneous| ACO                  | No                 | Repairable     | No | Multiple | No | No |
| Maatouk et al. [23]        | Multi-state| Heterogeneous| GA                   | No                 | Repairable     | No | Single | No | No |
| Garg and Sharma [24]       | Binary  | Heterogeneous| GA                   | No                 | Non-repairable | No | Multiple | No | No |
| Ebrahimipour and Sheikhalishahi [25] | Binary  | Heterogeneous| PSO                  | Yes               | Non-repairable | No | Multiple | No | AUD |
| Mirha et al. [26]          | Binary  | Heterogeneous| NSGA-II MOEA/D       | No | Non-repairable | Yes | Multiple | Taguchi | No |
| Mousavi et al. [27]        | Multi-state| Homogeneous  | CE-NRGA             | Yes               | Non-repairable | Yes | Multiple | Taguchi | AUD and IQD |
| Zaretalab et al. [28]      | Multi-state| Homogeneous  | MOSA                 | No | Non-repairable | Yes | Multiple | No | No |

Table 2: Reliability optimization problem parameters of attitude determination and control system

| Parameters | Description |
|------------|-------------|
| $t$        | System mission time |
| $\rho_{ECU}$ | Safe operation possibility of the switch for the electric control unit |
| $\rho_{MM}$ | Safe operation possibility of the switch for magnetic sensor |
| $\rho_{Gyro}$ | Safe operation possibility of the switch for the gyro sensor |
| $\rho_{ST}$ | Safe operation possibility of the switch for the star sensor |
| $\lambda_{max}^{ECU}$ | Maximum electronic control unit failure rate |
| $\lambda_{min}^{ECU}$ | Minimum electronic control unit failure rate |
| $\lambda_{max}^{int}$ | Maximum interface failure rate |
| Symbol   | Description                                      |
|----------|--------------------------------------------------|
| $\lambda_{\text{Int}}^{\text{min}}$ | Minimum interface failure rate                     |
| $\lambda_{\text{MM}}^{\text{max}}$ | Maximum magnetic sensor failure rate               |
| $\lambda_{\text{MM}}^{\text{min}}$ | Minimum magnetic sensor failure rate               |
| $\lambda_{\text{Gyro}}^{\text{max}}$ | Maximum gyro sensor failure rate                   |
| $\lambda_{\text{Gyro}}^{\text{min}}$ | Minimum gyro sensor failure rate                   |
| $\lambda_{\text{SS}}^{\text{max}}$ | Maximum sun sensors failure rate                   |
| $\lambda_{\text{SS}}^{\text{min}}$ | Minimum sun sensors failure rate                   |
| $\lambda_{\text{ST}}^{\text{max}}$ | Maximum star sensor failure rate                   |
| $\lambda_{\text{ST}}^{\text{min}}$ | Minimum star sensor failure rate                   |
| $\lambda_{\text{MT}}^{\text{max}}$ | Maximum magnetic torquer failure rate              |
| $\lambda_{\text{MT}}^{\text{min}}$ | Minimum magnetic torquer failure rate              |
| $\lambda_{\text{RW}}^{\text{max}}$ | Maximum reaction wheel failure rate                |
| $\lambda_{\text{RW}}^{\text{min}}$ | Minimum reaction wheel failure rate                |
| $U_{\text{ECU}}$ | Maximum number of redundant electronic control units in the system |
| $L_{\text{ECU}}$ | Minimum number of redundant electronic control units in the system |
| $U_{\text{Int}}$ | Maximum number of redundant interfaces in the system |
| $L_{\text{Int}}$ | Minimum number of redundant interfaces in the system |
| $U_{\text{MM}}$ | Maximum number of redundant magnetic sensors in the system |
| $L_{\text{MM}}$ | Minimum number of redundant magnetic sensors in the system |
| $U_{\text{Gyro}}$ | Maximum number of redundant gyro sensors in the system |
| $L_{\text{Gyro}}$ | Minimum number of redundant gyro sensors in the system |
| $U_{\text{ST}}$ | Maximum number of redundant star sensors in the system |
| $L_{\text{ST}}$ | Minimum number of redundant star sensors in the system |
| $U_{\text{RW}}$ | Maximum number of redundant reactive wheels in the system |
| $L_{\text{RW}}$ | Minimum number of redundant reactive wheels in the system |
| $C_{\text{ECU}}$ | The cost of each electric control unit |
| $W_{\text{ECU}}$ | The weight of each electric control unit |
| $D_{\text{ECU}}$ | The cost of one percent improvement in the failure rate of the electric control unit compared to the maximum possible failure rate |
| $C_{\text{Int}}$ | The cost of each interface |
| $W_{\text{Int}}$ | The weight of each interface |
| $D_{\text{Int}}$ | The cost of one percent improvement in interface failure rate compared to the maximum possible failure rate |
| $C_{\text{MM}}$ | The cost of each magnetic sensor |
| $W_{\text{MM}}$ | The weight of each magnetic sensor |
| $D_{\text{MM}}$ | The cost of one percent improvement of the magnetic sensor failure rate compared to the maximum possible failure rate |
$C_{\text{Gyro}}$ The cost of each gyro sensor

$W_{\text{Gyro}}$ The weight of each gyro sensor

$D_{\text{Gyro}}$ The cost of one percent improvement of the Gyro sensor failure rate compared to the maximum possible failure rate

$D_{SS}$ The cost of one percent improvement of the solar sensor failure rate compared to the maximum possible failure rate

$C_{ST}$ The cost of each star sensor

$W_{ST}$ The weight of each star sensor

$D_{ST}$ The cost of one percent improvement of the star sensor failure rate compared to the maximum possible failure rate

$D_{MT}$ The cost of one percent improvement in the magnetic torquer failure rate compared to the maximum possible failure rate

$C_{RW}$ The cost of each wheel reacts

$W_{RW}$ The weight of each wheel reacts

$D_{RW}$ The one percent cost improvement of the reaction wheel failure rate compared to the maximum possible failure rate

$C_{\text{max}}$ Maximum allowable cost

$W_{\text{max}}$ Maximum allowable weight

$N_{SS}$ The number of sun sensors in the system

$N_{MT}$ Number of magnetic torquer in the system

| Variables | Descriptions |
|-----------|--------------|
| $R_D(t)$  | Reliability of the attitude determination and control system under the De tumbling functional mode |
| $R_C(t)$  | Reliability of the attitude determination and control system under the Coarse pointing functional mode |
| $R_F(t)$  | Reliability of the attitude determination and control system under the fine pointing functional mode |
| $R_{SU}(t)$ | Reliability of the attitude determination and control system under the Sun pointing functional mode |
| $R_{SE}(t)$ | Reliability of the attitude determination and control system under the Safe functional mode |
| $N_{ECU}$ | Number of electronic control units in the system |
| $\lambda_{ECU}$ | The failure rate of each electronic control unit in the system |
| $x_{ECU}$ | Percentage of improvement in the electric control unit failure rate compared to the maximum possible failure rate |
| $N_{Int}$ | The number of interfaces in the system |
| $\lambda_{Int}$ | The failure rate of each interface in the system |
| $x_{Int}$ | Percentage of improvement in the interface failure rate compared to the maximum possible failure rate |
| $N_{MM}$ | The number of magnetic sensors in the system |
| $\lambda_{MM}$ | The failure rate of any magnetic sensor in the system |
| $x_{MM}$ | The percentage improvement in the magnetic sensor failure rate compared to the maximum possible failure rate |
| $N_{Gyro}$ | The number of gyro sensors in the system |
| $\lambda_{Gyro}$ | Failure rate of any gyro sensor located in the system |
| $x_{Gyro}$ | Percentage of improvements made to the Gyro sensor failure rate compared to the maximum possible failure rate |
| $\lambda_{SS}$ | The failure rate of each solar sensor in the system |
The number of star sensors in the system

The failure rate of each star sensor in the system

Percentage of improvement in star sensor failure rate compared to the maximum possible failure rate

The failure rate of each magnetic torquer located in the system

Percentage of improvement in magnetic torquer failure rate compared to the maximum possible failure rate

The number of reaction wheels in the system

The failure rate of each reaction wheel is located in the system

Percentage of improvement in wheel failure rate Reaction to the maximum possible failure rate

Table 4: Parameters in the numerical example

| Parameter  | Interval | Lower bound | Upper Bound |
|------------|----------|-------------|-------------|
| $x_{SS}$   |          |             |             |
| $N_{ST}$   |          |             |             |
| $\lambda_{ST}$ |        |             |             |
| $x_{ST}$   |          |             |             |
| $\lambda_{MT}$ |      |             |             |
| $x_{MT}$   |          |             |             |
| $N_{RW}$   |          |             |             |
| $\lambda_{RW}$ |     |             |             |
| $x_{RW}$   |          |             |             |

Table 5: Parameters levels of genetic algorithm

| Parameter | Interval | Lower bound | Upper Bound |
|-----------|----------|-------------|-------------|
| npop      | [50-100] | 50          | 100         |
| $p_c$     | [0.4-0.7]| 0.4         | 0.7         |
| Parameter | Optimal value |
|-----------|---------------|
| npop      | 100           |
| $p_c$     | 0.7           |
| $p_m$     | 0.2891614     |

Table 6: Optimal value of genetic algorithm parameters

| Parameter | Value |
|-----------|-------|
| $R_D(t)$  | 0.9696 |
| $\lambda_{\text{Int}}$ | 0.0019 |
| $x_{SS}$  | 97    |
| $R_C(t)$  | 0.8548 |
| $x_{\text{Int}}$ | 37    |
| $N_{ST}$  | 4     |
| $R_F(t)$  | 0.8547 |
| $N_{MM}$  | 6     |
| $\lambda_{ST}$ | 0.0001 |
| $R_{SU}(t)$ | 0.8548 |
| $\lambda_{MM}$ | 0.00052 |
| $x_{ST}$  | 90    |
| $R_{SE}(t)$ | 0.8550 |
| $x_{MM}$  | 87    |
| $\lambda_{MT}$ | 0.0001 |
| $N_{\text{ECU}}$ | 5     |
| $N_{\text{Gyro}}$ | 5     |
| $x_{MT}$  | 95    |
| $\lambda_{\text{ECU}}$ | 0.0001 |
| $\lambda_{\text{Gyro}}$ | 0.0003 |
| $N_{\text{RW}}$ | 9     |
| $x_{\text{ECU}}$ | 98    |
| $x_{\text{Gyro}}$ | 95    |
| $\lambda_{\text{RW}}$ | 0.0006 |
| $N_{\text{Int}}$ | 8     |
| $\lambda_{SS}$ | 0.00021 |
| $x_{\text{RW}}$ | 88    |

Table 7: the variables of the mathematical optimization model provided by GA under $C_{max}=35000$

| Parameter | Value |
|-----------|-------|
| $R_D(t)$  | 0.5410 |
| $\lambda_{\text{Int}}$ | 0.003  |
| $x_{SS}$  | 90    |
| $R_C(t)$  | 0.3553 |
| $x_{\text{Int}}$ | 0      |
| $N_{ST}$  | 6     |
| $R_F(t)$  | 0.3553 |
| $N_{MM}$  | 4     |
| $\lambda_{ST}$ | 0.004 |
| $R_{SU}(t)$ | 0.3553 |
| $\lambda_{MM}$ | 0.004 |
| $x_{ST}$  | 0     |
| $R_{SE}(t)$ | 0.3572 |
| $x_{MM}$  | 0     |
| $\lambda_{MT}$ | 0.002 |
| $N_{\text{ECU}}$ | 5     |
| $N_{\text{Gyro}}$ | 5     |
| $x_{MT}$  | 0     |
| $\lambda_{\text{ECU}}$ | 0.005 |
| $\lambda_{\text{Gyro}}$ | 0.006 |
| $N_{\text{RW}}$ | 11    |
| $x_{\text{ECU}}$ | 0     |
| $x_{\text{Gyro}}$ | 0     |
| $\lambda_{\text{RW}}$ | 0.005 |
| $N_{\text{Int}}$ | 5     |
| $\lambda_{SS}$ | 0.0007 |
| $x_{\text{RW}}$ | 0     |

Table 8: the variables of the mathematical optimization model provided by GA under $C_{max}=5000$

| Parameter | Value |
|-----------|-------|
| $R_D(t)$  | 0.3533 |
| $\lambda_{\text{Int}}$ | 0.0019 |
| $x_{SS}$  | 97    |
| $R_C(t)$  | 0.8248 |
| $x_{\text{Int}}$ | 0      |
| $N_{ST}$  | 4     |
| $R_F(t)$  | 0.8443 |
| $N_{MM}$  | 4     |
| $\lambda_{ST}$ | 0.001  |
| $R_{SU}(t)$ | 0.8461 |
| $\lambda_{MM}$ | 0.00052 |
| $x_{ST}$  | 90    |
| $R_{SE}(t)$ | 0.8501 |
| $x_{MM}$  | 87    |
| $\lambda_{MT}$ | 0.0001 |
| $N_{\text{ECU}}$ | 5     |
| $N_{\text{Gyro}}$ | 5     |
| $x_{MT}$  | 95    |
| $\lambda_{\text{ECU}}$ | 0.005 |
| $\lambda_{\text{Gyro}}$ | 0.006 |
| $N_{\text{RW}}$ | 11    |
| $x_{\text{ECU}}$ | 0     |
| $x_{\text{Gyro}}$ | 0     |
| $\lambda_{\text{RW}}$ | 0.005 |
| $N_{\text{Int}}$ | 8     |
| $\lambda_{SS}$ | 0.00021 |
| $x_{\text{RW}}$ | 88    |

Table 9: Comparison of the results obtained from the genetic algorithm and random search to solve the problem presented under different values of the maximum budget available

| $C_{max}$ | Genetic Algorithm | Random search |
|-----------|-------------------|---------------|
| 5000      | 0.3533            | -             |
| 7500      | 0.6075            | 0.0468        |
| 10000     | 0.8248            | 0.1588        |
| 12500     | 0.8443            | 0.4527        |
| 15000     | 0.8461            | 0.5792        |
| 17500     | 0.8484            | 0.7001        |
| 20000     | 0.8501            | 0.7923        |
| 22500     | 0.8516            | 0.7793        |
| Value  | Column1 | Column2 |
|--------|---------|---------|
| 25000  | 0.8524  | 0.7996  |
| 27500  | 0.8540  | 0.8438  |
| 30000  | 0.8543  | 0.8177  |
| 32500  | 0.8546  | 0.8249  |
| 35000  | 0.8547  | 0.8331  |
Figure 1. Reaction wheel

Satellite attitude determination and control system

- Processor
- Operators
- Sensors

Hardware and Processor
Software
Magnetic Torque
Reaction wheel
Magnetic sensor
Sun sensor
Gyro sensor
Star sensor

Attitude determination and estimation algorithms
Attitude control algorithms

Figure 2. The modules of the satellite attitude determination and control system
Figure 3. The reliability block diagram of the satellite attitude determination and control system under DE tumbling mode.

Figure 4. The reliability block diagram of the satellite attitude determination and control system under Coarse pointing mode.
Figure 5. The reliability block diagram of the satellite attitude determination and control system under Fine pointing mode

Figure 6. The reliability block diagram of the satellite attitude determination and control system under Sun pointing mode
Figure 7. The reliability block diagram of the satellite attitude determination and control system under Safe mode.

Figure 8: chromosome structure

Figure 9: Uniform mutation
Initialize $x$ with a random position in the search space. Until a termination criterion (number of iterations) is met, repeat the following:

Sample a new position $y$ from the hypersphere of a given radius surrounding the current position $x$.

If $f(y) < f(x)$ then move to the new position by setting $x = y$.

Now $x$ holds the best-found position.

Figure 10: Pseudocode of random search method

Figure 11: Convergence diagram of genetic algorithm
Figure 12: Sensitivity analysis of the results obtained from the genetic algorithm to solve the problem presented under different values of the maximum budget available.