NUCLEAR STRUCTURE FUNCTIONS AT SMALL $x$
IN MULTIPLE SCATTERING APPROACHES

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A simple model for nuclear structure functions in the region of small $x$ and small and moderate $Q^2$, is presented. It is a parameter-free extension, in the Glauber-Gribov approach to nuclear collisions, of a saturation model for the nucleon. A reasonable agreement with experimental data on ratios of nuclear structure functions is obtained. The unintegrated gluon distribution and the behavior of the saturation scale which result from this model are discussed.

1 Motivation

Nuclear structure functions are key tools to study the behavior of partons inside nuclei. It is well known that the ratio $R(A/B) = BF_{2A}/(AF_{2B})$ of structure functions per nucleon in different nuclei shows a complex behavior depending on the region of $x$. We will be interested in the small $x$ region, $x \leq 0.01$ corresponding to high $\gamma^*\text{-nucleon}$ energies, where $R(A/B) < 1$ (shadowing region). In this region and in the unpolarized case, the structure function $F_2$ can be related with the transversely or longitudinally polarized virtual photon-target cross section,

$$F_{2A}(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}(\sigma^A_T + \sigma^A_L).$$

As the nucleon structure function increases very strongly with decreasing $x$, $F_2 \propto x^{-\Delta}$, $\Delta \sim 0.2$ ÷ 0.3, a large number of partons is expected and non-linear effects may start to play a role.

Different approaches to this problem have been essayed (see [2] for a brief review and [3] for a description of the different approaches). On the one hand, parton densities have been parameterized at some small $Q_0^2 \gg \Lambda_{QCD}^2$ and then evolved to larger $Q^2$ using evolution equations. On the other hand, there are models which try to describe the form of the initial condition itself (see e.g. [4]). For this description of the small $Q^2$ region, two alternative but equivalent pictures
exist: In the rest frame of the nucleus the virtual photon develops a hadronic $q\bar{q}$ fluctuation with a coherence length which at small $x$ is larger than the nuclear size, so multiple scattering becomes important. In a frame in which the hadron is moving rapidly, the $q\bar{q}$ fluctuation sees simultaneously the overlapping parton clouds of different nucleons in the nucleus, and non-linear effects are expected to appear. In all these models saturation, meaning either an upper bound in parton densities and fields or the blackness of the scattering matrix, appears for $Q^2 < Q^2_s$, with $Q^2_s$ the saturation scale. All these studies have great influence on the description of multiparticle production in nuclear collisions, particularly in view of the recent RHIC data, see 3, 5, 6.

In this contribution we will describe a very simple model 4 for nuclear structure functions, based on the multiple scattering picture in the dipole model 7. It is a parameter-free extension of a model for the nucleon to the nuclear case using the Glauber-Gribov approach. The model will be presented in the next Section, and the saturation scale it implies will be analyzed in Section 3. Finally, some conclusions will be drawn in Section 4.

2 Description of the model

We are going to work in the dipole model 7 valid for small $x$, in which a $q\bar{q}$ fluctuation of the virtual photon develops with a coherence length $l_c \propto (xm_N)^{-1} > R_A$ and interacts hadronically with the target. In this model (see Fig. 1)

$$
\sigma_{T,L}^A(x) = \int d^2r \int_0^1 dz |\psi_{T,L}(r,z,Q^2)|^2 \sigma_{dA}(x,r),
$$

(2)

with $\psi_{T,L}(r,z,Q^2)$ the $q\bar{q}$-component of the $\gamma^*$ wave function 7. For the dipole-proton cross section $\sigma_{dp}(x,r)$ we will use the model of 8,

$$
\sigma_{dp}(x,r) = \sigma_0 \left[ 1 - \exp \left( -\frac{Q^2_s(x)r^2}{4} \right) \right],
$$

(3)

with $Q^2_s(x) = Q^2_0 \left( \frac{m}{\bar{x}} \right)^{\lambda}$, $\bar{x} = x \left( 1 + \frac{4m^2_f}{Q^2} \right)$, $Q^2_0 = 1$ GeV$^2$, $m_{u,d,s} = 0.14$ GeV, $m_c = 1.5$ GeV, $\sigma_0 = 23.03$ (29.12) mb, $\lambda = 0.288$ (0.277) and $x_0 = 3.04 \cdot 10^{-4}$ (0.41 $\cdot 10^{-4}$) for the 3(4)-flavor version of the model.

![Figure 1: Interaction of the dipole fluctuation of the virtual photon with a nuclear target.](image)
The extension to the nuclear case will be done in a parameter-free way using the Glauber-Gribov approach:

\[
\sigma_{dA}(x,r) = \int d^2 b \left[ 1 - \exp \left( -\frac{1}{2} A T_A(b) \sigma_{dp}(x,r) \right) \right].
\] (4)

The region of validity of the model, Eqs. (1)-(4), is related with that of the model for the proton and with the absence of non-linear effects: \(0.02 > x > 10^{-5} \div 10^{-6}, Q^2 < 20 \text{ GeV}^2\). In results of this model are presented. The comparison with experimental data for different ratios \(R(A/B)\) is reasonable, although at \(x \sim 0.01\) the model tends to overestimate shadowing. The nuclear effects on the longitudinal-to-transverse cross section ratios are moderate, in any case smaller than 15% in the region of applicability of the model.

3 Unintegrated gluon and \(Q_s^2\)

The dipole-target cross section can be related, in leading-order \(k_T\)-factorization, with the unintegrated gluon distribution \(\phi_A(x,k,b)\):

\[
\phi_A(x,k,b) = -\frac{N_c}{4\pi^2\alpha_s} k^2 \int \frac{d^2 r}{2\pi} \exp (ik \cdot r) \sigma_{dA}(x,r,b),
\] (5)

whose integral over \(d^2 k\) is related to the usual gluon density. In our model, the unintegrated gluon depends on the transverse momentum \(k\) just through the combination \(k^2/Q_s^2\), it goes to 0 both for \(k \to 0\) and \(k \to \infty\), and presents a maximum whose position in \(k\) can be identified with the saturation scale \(Q_s\). Thus the size of \(Q_s^2\) with respect to \(\Lambda_{\text{QCD}}^2\) indicates the applicability of perturbative methods in the saturation picture. In our model the saturation scale (for quarks, for gluons a color factor \(9/4\) is naively expected) in nuclei \(Q_s^2\) can be related to the saturation scale in proton \(Q_s^2\). Estimations give

\[
Q_s^2 A \simeq \left[ 4 \ln \left( \frac{2 A T_A(b) \sigma_0}{2 A T_A(b) \sigma_0 - 1} \right) \right]^{-1} Q_s^2 \propto A^{1/3} \text{ for } A \to \infty.
\] (6)

Numerical results are shown in Fig. 2 compared with the results for the saturation scale in non-linear equations.

Some comments are in order. First, the results of the non-linear equations give a similar asymptotic \(A\)-dependence but a much steeper \(x\)-dependence (characterized by an exponent \(\lambda \simeq 0.78\) to be compared with \(\simeq 0.28\) in the model of and ours). Second, in our model the saturation scale \(Q_s^2\) for central \((b = 0)\) Pb is 2 times the saturation scale \(Q_s^2\) for proton, notably smaller than the factor \(208^{1/3} \simeq 6\) naively expected; this result is in agreement with the conclusions in. Finally, the structure function \(F_2\) in our model shows in nuclei explicit geometrical scaling in the variable \(\tau = Q^2/Q_s^2\), at the same level as the model in for the proton. Indeed, such scaling has been seen in proton experimental data and searched for in nuclear data; it has been interpreted as a possible hint on the existence of a saturating, high-density regime in small \(x\), small \(Q^2\) data on scattering of leptons on nucleons or nuclei.

4 Conclusions

A simple model for nuclear structure functions, in the form of a parameter-free extension of a saturating model for the proton to the nuclear case in the Glauber-Gribov approach, has been presented. The model shows a reasonable agreement with experimental data. It has been
Figure 2: Upper plot: saturation momentum in the model for proton (solid line), and for Pb in three cases: central ($b = 0$, dashed-dotted line), peripheral ($b = 7$ fm, dashed line), and integrated over $b$ (dotted line). Lower plot: saturation momentum in the numerical solution of the non-linear equation, for $A = 1$ (solid line) and $A = 208$ (dotted line). Notice the difference in vertical scales between the plots.

used to estimate the saturation scale in nuclei, relevant for the application of saturation ideas to multiparticle production in ultrarelativistic heavy ion collisions\footnote{3}. From the results of the model the saturation scale $Q^2_{sA}$ results $< 2$ GeV$^2$ for $x$-values $\sim 0.01$ relevant for RHIC.

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References

1. M. Arneodo, Phys. Rept. \textbf{240}, 301 (1994); D.F. Geesaman, K. Saito and A.W. Thomas, \textit{Ann. Rev. Nucl. Part. Sci.} \textbf{45}, 337 (1995).
2. N. Armesto and C.A. Salgado, [hep-ph/0301200].
3. \textit{QCD Perspectives on Hot and Dense Matter}, Eds. J.-P. Blaizot and E. Iancu, NATO Science Series, Kluwer Academic Publishers 2002.
4. N. Armesto, \textit{Eur. Phys. J. C} \textbf{26}, 35 (2002).
5. N. Armesto \textit{et al.}, [hep-ph/0304119]. J. López-Albacete, in these Proceedings.
6. Proceedings of the \textit{XVIth International Conference On Ultrarelativistic Nucleus-Nucleus Collisions: Quark Matter 02} (Nantes, France, 18-24 July 2002), \textit{Nucl. Phys.} A to appear.
7. N.N. Nikolaev and B.G. Zakharov, \textit{Z. Phys. C} \textbf{49}, 607 (1991); A.H. Mueller, \textit{Nucl. Phys. B} \textbf{415}, 373 (1994); A.H. Mueller and B. Patel, \textit{Nucl. Phys. B} \textbf{425}, 471 (1994).
8. K. Golec-Biernat and M. Wüsthoff, \textit{Phys. Rev. D} \textbf{59}, 014017 (1999).
9. B. Andersson \textit{et al.}, \textit{Eur. Phys. J. C} \textbf{25}, 77 (2002); J.C. Collins, [hep-ph/0304122].
10. N. Armesto and M.A. Braun, \textit{Eur. Phys. J. C} \textbf{20}, 517 (2001); C \textbf{22}, 351 (2001).
11. A. Freund \textit{et al.}, [hep-ph/0210139]. H. Kowalski and D. Teaney, [hep-ph/0304189].
12. A.M. Staśto, K. Golec-Biernat and J. Kwieciński, \textit{Phys. Rev. Lett.} \textbf{86}, 596 (2001).