Checkerboard test implementation into ionospheric tomography for SuGAr data

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Abstract. Available GPS data in Indonesia which coincidence with large earthquake is limited and not adequate to reconstruct a well-ionospheric tomography. Large earthquake is occur in 2004 and large GPS data is only available from Sumatra GPS Array (SuGAr). This study is intended to support ionospheric tomography for limited data by checkerboard test. Tomography result is obtained by optimizing grid size, interval, a priori models, and norm damping. The dependency to a priori model is reduce by choosing alpha value parameter (α=0.1). For case study, ionospheric tomography is implemented to preseismic analysis in Bengkulu earthquake at September 12, 2007 which expected to have long-term effects in the ionosphere. The result is validated by checkerboard test to shows the resolutions of ionospheric tomography.

1. Introduction

Indonesia is located in the geomagnetic equator, the area which known with high ionospheric activity. Ionosphere anomaly is usually driven by high solar activity. Indonesia is also situated in Pacific Ring of Fire where a large number of earthquakes occur. Large earthquake could trigger ionosphere anomaly. The effect of the earthquake on the ionosphere before it happens is still the subject of research until now. Ionospheric tomography is often used in discussing this issue.

Ionosphere anomaly after earthquake, known as coseismic, have short-term duration. Whereas, ionospheric anomalies before earthquake, known as preseismic, are have long-term duration [1]. Therefore, we could use GPS tomography to image preseismic behavior at that time.

GPS data availability that coincided with large earthquake in Indonesia is very limited. This study is intended to support ionospheric tomography when available GPS data are limited. Takeda [2] and Hirooka [3] use occultation data to increase the number of observation. Another methods, known as checkerboard test, is used to support limited data in tomography for preseismic is proposed here. Checkerboard test is used in seismic tomography.

Sumatra GPS Array (SuGAr) observation data is used to compute ionospheric tomography. While navigation data is obtained from the International GNSS Service (IGS) through Scripps Orbit and Permanent Array Center (SOPAC) website. Other GPS data is obtained from Institut Teknologi Bandung (ITB) and the National Aeronautics and Space Agency (LAPAN). GPS receiver used in this study located between -5° to 5° in latitude and 100° to 104° in longitude. This area is elongated with...
large Sumatran Fault Zone as shown in figure 1.

![Figure 1. Location of GPS station (SuGAr and IGS stations).](image)

Ionosphere is modeled using Taylor series method [4]. Absolute TEC (Total Electron Content) value is obtained by decomposing ionospheric function to reference part and an unknown correction term as described by [5]. The reference part that used in this study is given from Global Ionospheric Model (GIM) as a constraint.

Norm damping is used in the inversion step for this ionospheric tomography. Damping (explicit regularization) is used to stabilize the solution in poor sample mantle regions [6]. However, it is necessary to search for appropriate damping value so the solution to wisely lessen the dependency on the model. Three-dimensional models from NeQuick is used as a priori information [7]. Checkerboard test is used as the resolution test from our tomography as described in [8][9][10].

2. Theory

2.1. Absolute TEC

Over-determined system is acquired in tomography inversion to compute absolute TEC. However, daily SuGAr data is not always available for every station. Therefore, we pick only specific epoch which has a large number of observation.

Absolut TEC value is obtained after receiver (br) and satellite bias (bs) is determined as can be written as:

\[
P2-P1=VTEC*F(z)/\gamma+br+bs
\]

where:
\[P2\] = Pseudorange data on L1,
\[P1\] = Pseudorange data on L2,
\[VTEC\] = Vertical TEC,
\[F(z)\] = mapping function,
\[\gamma = (f_1^2 - f_2^2) / 40.3(f_2^2 - f_1^2)\]
\[f_1 = 1575.42 \text{ Hz, and}\]
\[f_2 = 1227.60 \text{ Hz.}\]

\[VTEC\] is modeled by a Taylor series of two-dimensional (in longitude and latitude) as:
\[ VTEC = \sum E_{nm} (\varphi - \varphi_0)(\lambda - \lambda_0) \] (2)

where:
- \( E_{nm} \) = unknown coefficients of Taylor series expansions,
- \( \varphi \) = latitude coordinate of IPP (Ionospheric Pierce Point),
- \( \lambda \) = longitude coordinate of IPP,
- \( \varphi_0 \) = latitude coordinate of reference, and
- \( \lambda_0 \) = longitude coordinate of reference.

An a priori model could handle limited data to overcome under-determined system to find Taylor coefficients:

\[ VTEC = VTEC_{model} - \Delta VTEC \] (3)

A priori model is required for mixed-determined problems in Weighted Damped Least Squares [11]. The search for solutions should be minimized the prediction error while adding a minimum a priori information. Such procedure leads to:

\[ m = \bar{m} + [G^T W_e G + \varepsilon^2 I]^{-1} G^T W_e [d - G \bar{m}] \] (4)

where:
- \( m \) = unknown model,
- \( \bar{m} \) = a priori model,
- \( G \) = kernel matrix,
- \( W_e \) = weighting factor as elevation function which defines contribution of each individual error to the total prediction error,
- \( \varepsilon \) = weighting factor to control prediction error and solution length,
- \( I \) = Identity matrix, and
- \( d \) = data.

Initial model is far from the actual model in the first step. The more iteration the more stable is the solution. The more data interval the more stable is the estimated bias we have. Interval data of 6, 10, 20, and 30 minutes is analyzed to the effect of the stability of the solution. Estimated receiver bias is shown in figure 2 as the result of 30 minutes interval data which more suitable for the limited SuGAr data.
2.2. Ionospheric tomography
Ionospheric tomography is a technique to reconstruct the electron density profile. Ionosphere Tomography equation is shown by:

\[ A \cdot Ne = STEC \] (5)

where:
- \( A \) = Matrix of segment length
- \( Ne \) = Unknown electron density
- \( STEC \) = Slant STEC

Matrix \( A \) is constructed from specific grid size in the limited area. Electrons in ionosphere are concentrated around a height of 80-1000 km. Matrix \( A \) is sparse matrix and contains zero value that will leads to very small or wrong value. Therefore, we use another a priori model from three-dimensional NeQuick model [12].

2.3. Norm damping
Norm damping inversion is one of the solutions to overcome limited observation data and is expressed as follows:

\[ \begin{pmatrix} A \\ \alpha I \end{pmatrix} Ne = \begin{pmatrix} STEC \\ 0 \end{pmatrix} \] (6)

While \( \alpha \) is a damping parameter as weighting factor in adding initial value to the regions that were not passed by GPS signals. We assume that ionosphere behavior in longitudinal directions is more stable than in latitudinal directions. Therefore, grid size in a longitudinal direction is enlarged to 4° to increase the resolution and lessen the dependency on the a priori models. While grid size in altitude is 100 km [13]. Overview 2D-tomography grid is shown by figure 3.
2.4. Checkerboard test
Two-dimensional ionospheric tomography results need to be tested by resolution test. We use checkerboard test because it easy to construct and easy to interpreted. Checkerboard test begins by building an early perturbation model with a value of ± 10% of the initial value of the delay (figure 4).

Figure 3. Grid of 2D-tomography grid.

Figure 4. Initial model of perturbation for checkerboard test.

Predicted delay value is constructed after passing to the perturbation models. The difference between delay value after and before passing to perturbation is obtained in percentage. The percentage value is indicating the return value to the initial model perturbation.

Example of checkerboard test can be seen in figure 5. The color indicating the returned value to the checkerboard (recovery) pattern. The good pattern (red and blue colors) shows that the area may return into the initial model perturbation. While the green color (0%) shows that the area did not return to the initial model, because the area does not passed by GPS signal. So the difference in perturbation delay and delay value before the perturbation is equal to zero.
3. Result and analysis
Norm damping can overcome the lack of GPS data in tomographic inversion. This method uses the parameter alpha (α) as a controlling influence on the results of the initial model inversion. The alpha value is obtained by trial and error. In this study some of the alpha value is then compared.

For SuGAr data, the best alpha value is 0.1 which shows the dependency from the initial model in reasonable pattern (figure 6). Layering patterns at 1° latitude with altitude of 300km have poor resolution (figure 8). Poor resolution means data used in this area is very limited. It can be concluded that for -2° latitude regions still have a good resolution.

For case study, we apply to Bengkulu earthquake case. The Epicenter of main earthquake is (101.37, -4.44) at 11:10 UT and aftershock is (100.87, -2.66) at 23:49 UT. Electron density profile in -4° can not be used as it was in the edge of the observation area that typically has a worst value in the interpolation. The more data used in tomographic inversion methods the better the results of the resolution. In this case the data is enlarged by expanding the interval of observations time (6, 10, 20, 30 minutes). Figure 7 shows the electron density profile slice at latitude -2° from 10 minutes interval data. Resolution test producing by the inversion with 10 minutes interval was in reasonable pattern (figure 8).
Figure 7. Electron density profile on the latitude -2°. Blue color tomographic result while red color is Nequick model.

Figure 8. Checkerboard test for data with 10 minutes interval shows good resolution in bottom left area.

Using wider interval data will reduce the presence of anomalies. This is indicated by the absence of anomalies in the electron density profile that originally detected when using the data in 10-minute intervals. Hirooka [3] makes an analysis of Bengkulu earthquake precursors on 12 September 2007. The results show the anomaly before the earthquake on September 9, 2007. Anomalies are defined as the difference tomography results with the model at around 7:30 - 7:45 UT. Hirooka results have vast amounts of data by adding data from Radio Occultation of LEO satellites.

4. Conclusion
Tomography requires large data to make over-determined system. Available SuGAr data that coincided with large earthquake to construct ionospheric tomography is limited. Using several methods such as increasing the size of the grid, using wide interval, a priori models in absolute TEC estimation, norm damping in tomography and a priori model in inversion shows a good results. The dependency to a priori model is reduce by choosing alpha value (α=0.1). However, this result should be validated by checkerboard test to shows the resolutions of ionospheric tomography.

For case study, tomography result is implemented to preseismic analysis in Bengkulu earthquake at September 12, 2007 which is expected to have long-term effects in the ionosphere. The result at 07: 00 - 07: 10 UT for latitude -2° contained anomalous electron density profile when compared to the
reference model. Checkerboard test could be used to support this justification by showing good resolution is such area.

Acknowledgments

The authors would like to thank to SOPAC for sharing IGS and SuGAr data, ITB for GPS data, Kemenristekdikti for the financial support of my study. We would also like to express our gratitude to Prof. Sri Widiyantoro and Dr. Andri Dian Nugraha for helping us to understand more about Tomography. Last but not least, thanks are also intent to Susumu Saito on teaching 3D-NeQuick model as well.

References

[1] M. N. Cahyadi and K. Heki, “Ionospheric disturbances of the 2007 Bengkulu and the 2005 Nias earthquakes, Sumatra, observed with a regional GPS network,” J. Geophys. Res. Sp. Phys., vol. 118, no. 4, pp. 1777–1787, 2013.
[2] T. Takeda and X. Ma, “Ionospheric Tomography by Neural Network Collocation Method,” vol. 2, pp. 1–6, 2007.
[3] S. Hirooka, K. Hattori, M. Nishihashi, and T. Takeda, “Neural network based tomographic approach to detecting the ionospheric anomalies prior to the 2007 Southern Sumatra earthquake,” 2011 30th URSI Gen. Assem. Sci. Symp. URSIGASS 2011, pp. 5–8, 2011.
[4] S. Schaer, “Mapping and Predicting the Earth’s Ionosphere Using the Global Positioning System.” Geodatisch-Geophysikalische Arbeiten in der Schweiz, Zurich, Switzerland, 1999.
[5] M. Schmidt, “Wavelet modelling in support of IRI,” Adv. Sp. Res., vol. 39, no. 5, pp. 932–940, 2007.
[6] S. Widiyantoro and R. vanderHilst, “Mantle structure beneath Indonesia inferred from high-resolution tomographic imaging,” Geophys. J. Int., vol. 130, no. 1, pp. 167–182, 1997.
[7] B. Nava, P. Coïsson, and S. M. Radicella, “A new version of the NeQuick ionosphere electron density model,” J. Atmos. Solar-Terrestrial Phys., vol. 70, no. 15, pp. 1856–1862, 2008.
[8] J.-J. Leveque, L. Rivera, and G. Wittlinger, “On the use of the checker-board test to assess the resolution of tomographic inversions,” Geophys. J. Int., vol. 115, no. 1, pp. 313–318, 1993.
[9] N. Rawlinson and M. Sambridge, “Seismic Traveltime Tomography of the Crust and Lithosphere,” Adv. Geophys., vol. 46, no. C, pp. 81–198, 2003.
[10] M. Běhounková, H. Cížkova, and C. Matyska, “Resolution tests of global geodynamic models by travel-time tomography,” Stud. Geophys. Geod., vol. 49, no. 3, pp. 343–363, 2005.
[11] W. Menke, Geophysical Data Analysis: Discrete Inverse Theory: MATLAB Edition. 2012.
[12] S. Saito, T. Yoshihara, and N. Fujii, “Study of Effets of the Plasma Bubble on GBAS by Three-Dimensional Ionospheric Delay Model,” 22nd Int. Tech. Meet. Satell. Div. Inst. Navig. 2009, ION GNSS 2009, vol. 1, pp. 267–274, 2009.
[13] G. K. Zewdie, “Tomographic Imaging of Ionospheric Electron Density Over Ethiopia Using Ground Based Gps Receivers,” no. June, p. 112, 2012.