Gauge Mediation in Metastable Vacua

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Abstract

Until recently, dynamical supersymmetry breaking seemed an exceptional phenomenon, involving chiral gauge theories with a special structure. Recently it has become clear that requiring only metastable states with broken supersymmetry leads to a far broader class of theories. In this paper, we extend these constructions still further, finding new classes which, unlike earlier theories, do not have unbroken, approximate $\mathcal{R}$ symmetries. This allows construction of new models with low energy gauge mediation.
1 Introduction: Retrofitting O’Raifeartaigh Models

Until recently, dynamical supersymmetry breaking seemed an exceptional phenomenon [1]. Analysis of the Witten index indicated that such breaking can only occur in chiral gauge theories, and even then only under rather special circumstances. Recently, however, Intriligator, Shih and Seiberg[2] exhibited a class of vector-like gauge theories which possess metastable, supersymmetry-breaking minima. Feng, Silverstein, and one of the present authors showed that this phenomenon is quite common[3]. One can take a generic Fayet-Iliopoulos model, and simply replace the scales appearing there with dynamical scales associated with some underlying, supersymmetry-conserving, dynamics.

Consider, for example, a theory with chiral fields, $A$, $Y$ and $Z$, and superpotential:

$$W = \lambda Z (A^2 - \mu^2) + mYA. \quad (1)$$

This is a theory which breaks supersymmetry. The scale, $\mu$, can be generated dynamically by introducing a dynamical gauge theory and replacing $\mu^2$ by the expectation value of some suitable composite operator. One simple possibility is to take the extra sector to be a pure gauge theory, say $SU(N)$, and introduce a coupling

$$\int d^2\theta \frac{Z}{4M} W^2. \quad (2)$$

This structure can be enforced by discrete symmetries. The gauge theory has a $Z_N$ discrete symmetry, so if $A$ and $Y$ transform like $W_\alpha$, while $Z$ is neutral, the only couplings of dimension three or less which are invariant are those above. Integrating out the gauge fields, leaves a superpotential:

$$W = \lambda Z A^2 + \frac{A^3 e^{-8\pi Z/b_0}}{M} + mYA \quad (3)$$

The resulting potential has a minimum at $Z \to \infty$, i.e. it exhibits runaway behavior. But the Coleman-Weinberg corrections give rise to a local minimum at $Z = 0$.

This simple theory can be used in an interesting way as a hidden sector in a supergravity theory. Previously, most known models of dynamical supersymmetry breaking contained no gauge singlets. As a result, one could not write dimension five operators giving rise to gaugino masses, and the leading contributions arose from anomaly mediation.\(^1\) But in these “retrofitted”

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\(^1\)An exception was provided by the Intriligator-Thomas models\([4, 5]\). In general, naturalness arguments would require an anomalous discrete R symmetry in these theories, but this will be required in our models below, as well. Models with local minima with broken supersymmetry have been considered in the past as well, e.g. \([6, 7]\)
models, there is no obstruction to the existence of a coupling of $Z$ to the various gauge fields, so there is no difficulty generating gaugino masses. One still faces the problem of large potential flavor violation.

Various strategies were discussed in [3] to break supersymmetry at lower scales. However, it was difficulty within the examples presented there, to build realistic models. (Alternative strategies based on the ISS models were put forward in [8].) The difficulty is illustrated by our simple model. At low energies, the model has a continuous $U(1)$ $R$ symmetry. Under this symmetry, the fields $Y$ and $Z$ carry charge two. This symmetry forbids gaugino masses. Within simple models with chiral fields only, it is difficult to find examples where the analogs of the $Y$ and $Z$ fields acquire expectation values, breaking the symmetry.

On the other hand, at least since Witten’s work on the “inverted hierarchy” long ago[9], gauge interactions have been known to destabilize the origin of moduli potentials in O’Raifeartaigh models. In this note, we exhibit examples with gauge symmetries where the potential has a local minimum away from, but not far away from, the origin. The symmetry is broken and a rich phenomenology is possible.

In the next section, we generalize further the constructions of [3]. It is troubling that the simplest “retrofitted” models introduce additional mass parameters in the lagrangian, and we explain how these can also be understood dynamically, in terms of a single set of gauge interactions. In section three, we introduce the gauged model, compute the potential, and verify that $R$ symmetry is broken for a range of parameters. We then turn to the construct of low energy (direct) models of gauge mediation. We implement a solution of the $\mu$ problem which following on ideas of Giudice, Rattazzi and Slavich[10, 11]. In the conclusions we discuss issues of stability, fine tuning, and directions for future work.

2 Naturalness in the Retrofitted Models

The model of eqn. 1 has, in addition to $\mu^2$, the dimensionful parameter $m$. Some strategies to obtain both mass terms dynamically were discussed in [3], but these often run afoul of naturalness criteria. A simple variant of the ideas above works here as well, however. Suppose, again, that one has a pure gauge theory with a large scale, $\Lambda$. Then the scales $\mu^2$ and $M$ can be replaced by couplings:

$$W_\Lambda = Z A^2 + \frac{1}{M_p^4}Z W_\alpha^4 + \frac{1}{M_p^2}W_\alpha^2 A Y.$$  

(4)
We have taken the scale here to be the Planck scale, but one could well imagine that some other large scale (the GUT scale, for example) would determine the size of these operators. Now

$$\mu^2 = \frac{\Lambda^6}{M_p^4}; \ M = \frac{\Lambda^3}{M_p^2}$$

and $M$ and $\mu$ are naturally of the same order. This structure can readily be compatible with a discrete $Z_N$ R-symmetry. For example, if $\alpha$ is an $N$’th root of unity,

$$W_\alpha^2 \rightarrow \alpha W_\alpha^2; Z \rightarrow \alpha^{-1} Z; A \rightarrow \alpha A; B \rightarrow \alpha^{-1} B.$$  

It is also necessary to impose a $Z_2$, under which $A$ and $B$ are odd, to prohibit the coupling $A^2 B$ (additional restrictions may be necessary for particular values of $N$). In the model we consider in the next section, the extra $Z_2$ is not necessary; the gauge symmetries forbid the unwanted coupling.

In [3], still another mechanism to obtain dimensional parameters naturally was described: it was shown that one can naturally obtain Fayet-Iliopoulos terms. We will not exploit this in our model building in this paper, but this may also be a useful tool.

3 Including Gauge Interactions: Coleman-Weinberg Calculation

The basic model is a $U(1)$ gauge theory, with charged fields $Z^\pm$ and $\phi^\pm$, and a neutral field, $Z^0$. The superpotential of the model is

$$W = M_+ Z^+ \phi^- + M_- Z^- \phi^+ + \lambda Z^0 (\phi^+ \phi^- - \mu^2).$$

The model breaks supersymmetry. For simplicity, we take $M_+ = M_- = M$. If $|M^2| < |\lambda^2 \mu^2|$, at the minimum of the potential:

$$\phi^+ = \phi^- = v \quad v^2 = \frac{\lambda^2 \mu^2 - M^2}{\lambda^2}$$

(up to phases) while

$$F_{Z^+} = F_{Z^-} = Mv; \ F_{Z^0} = \frac{M^2}{\lambda}.$$  

There is a flat direction with

$$Z^\pm = -\frac{\lambda Z^0 \phi^\pm}{M}.$$
As in the previous section, both the parameters $M$ and $\mu$ can arise from dynamics at some much larger scale; the structure can be enforced by discrete symmetries.

It is easy to compute the potential at large $Z$. In this limit, the theory is approximately supersymmetric, and the gauge fields, as well as certain linear combinations of the $Z$’s, are massive. It is then possible to integrate out the massive fields, writing a supersymmetric effective action for the light fields. It is also helpful to work in a limit of large $M$, $M \gg \lambda \mu$, so that the $F$ components of $Z^\pm$ are larger than that of $Z^0$. Any supersymmetry breaking should show up from the $F$ components of the various fields, i.e. as terms of the form

$$\int d^4 \theta (Z^{+\dagger} Z^+ f + Z^{-\dagger} Z^- g + Z^{0\dagger} Z^0 h),$$

where $f, g, h$ are functions of the $Z$’s.

So we need to compute Feynman diagrams (supergraphs) with external $Z$ fields. These are greatly simplified by isolating the pieces of the form $F^{\dagger} F$, noting that the external lines all have two $\theta$’s. We need the propagators in a generalized ’t Hooft-Feynman gauge:

$$\langle Z^+ Z^{+\dagger} \rangle = \frac{i}{p^2 - M_V^2} \times (1 + \theta - \text{dependent}) \quad \langle V(\theta_1) V(\theta_2) \rangle = -\frac{1}{2} \frac{i \delta(\theta_1 - \theta_2)}{p^2 - M_V^2}. \quad (12)$$

There are essentially two graphs. In the diagram with two external $Z^0$’s an internal $\phi^+$ and $\phi^-$ (with mass $\lambda Z^0$), the theta’s at the interaction vertices are soaked up by the $F$’s, and the graph is simply

$$\lambda^2 |F_{Z^0}|^2 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 - |\lambda^2 Z^0|^2)^2} = \frac{\lambda^2}{16\pi^2} |F_{Z^0}|^2 \ln(|\lambda Z^0|^2). \quad (13)$$

For the gauge interactions, the diagrams are equally simple. The leading interaction is $Z^{±\dagger} Z^{±}(2gV)$. Now one has $\int d^4 \theta$ at each vertex, but the delta function in the gauge boson propagator soaks up the remaining $\theta$’s. So we have

$$-2g^2 (|F_{Z^+}|^2 + |F_{Z^-}|^2) \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 - M_V^2} \quad (14)$$

$$= -\frac{4g^2 |F_{Z^+}|^2}{16\pi^2} \ln(g^2 |Z^+|^2).$$

So overall, the asymptotic behavior of the potential is given by:

$$V = \frac{1}{16\pi^2} \left( |F_{Z^0}|^2 \lambda^2 - 4g^2 |F_{Z^+}|^2 \right) \ln(Z^{02}/\Lambda^2) \quad (15)$$

for a cutoff, $\Lambda$. 5
For a range of $g$ and $\lambda$, then, the potential grows at large $Z^0$. We wish to determine whether, within this range, there is a range for which the potential has negative curvature at small $Z$. The answer is yes. At small $Z$, it is simplest to do the Coleman-Weinberg calculation directly. In fig. 1, we have plotted the potential for several values of $g$ and $\lambda$, and, indeed, for a range of parameters, there is a minimum at non-zero $Z^0$.

![Figure 1: $Z$ potential with $g = 0.4, \lambda = 1, M = 1, \mu = 1.5$.](image)

In general, for small $Z^0$,

$$V(Z) = \text{const} + m_Z^2 |Z^0|^2.$$  

The constant is obtained by diagonalizing the full mass matrix. The condition that $m_Z^2 < 0$ is a condition on the ratio of gauge to Yukawa couplings, as is the condition that the potential should rise at $\infty$. The bands of allowed $g$ and $\lambda$, for different values of $h$, are indicated in fig. 2.

Note that $Z^0, Z^\pm$ at the minimum are of order $\mu$. So by dialing the dynamical scale, one can obtain supersymmetry breaking at any scale. In addition, the accidental $R$ symmetry of the low energy theory is spontaneously broken. The would-be Goldstone boson gains a substantial mass once couplings to supergravity are included, as explained in [13]. This, then, is just the sort of structure we would like in gauge mediated models[12]. We can now couple $Z^0$, for example, to messenger fields. We will initiate a study of such models in the next section.

4 Low Energy, Direct Mediation

In the model we have described above, the scale of supersymmetry breaking is a free parameter, and the scale of $R$-symmetry breaking is of the same order. This is clearly only one of a large class of possible models. By making different choices of charges, for example, one can avoid
Figure 2: Graphs of the regions of parameter space where there is a local R-breaking minimum. Below the blue line is the region where the potential grows positive at large $Z_0$ and above the black line is the region where the potential curves down at the origin. The values of $h$ are $h = .5$, $h = .1$, $h = .66$ and $h = 2$ in (a),(b),(c), and (d), respectively.
introducing the dimensionful parameter, $M$. (One can, and in general should, introduce two independent mass parameters in the original model). One can now make a model of direct gauge mediation by introducing a set of messenger fields, $M$ and $\bar{M}$, with the quantum numbers of a 5 and $\bar{5}$ of $SU(5)$, and with couplings to $Z^0$:

$$\lambda' Z^0 \bar{M} M.$$  

(17)

Then the standard gauge mediation computation yields a positive mass-squared for squarks and sleptons. Note that in this case, if $Z^0$ is neutral under the discrete $R$ symmetry, no symmetry forbids a large mass term for $\bar{M}M$. This problem arises because of our choice of coupling in equation 2. In order that symmetries forbid a $\bar{M}M$ mass term, one needs that $Z$ transform non-trivially under the discrete symmetry, as discussed in section 2.

One might worry that in this model, in addition to the far away supersymmetric minimum, there is a close by one with

$$\lambda' \bar{M}M - \lambda \mu^2 = 0.$$  

(18)

For suitable $\lambda$ and $\lambda'$, our candidate minimum remains a local minimum of the potential, however; it is also sufficiently metastable. To see this, note that in the metastable minimum, the quadratic terms in the $M, \bar{M}$ potential are:

$$\lambda' F_{Z^0}^* \bar{M} M + \mathrm{c.c.} + |\lambda' Z^0|^2 (|M|^2 + |\bar{M}|^2).$$  

(19)

We can work in a regime where $\lambda \mu \gg M$. In this regime, at the minimum

$$Z^\pm \sim \mu \quad Z^0 \sim M/\lambda \quad F_{Z^0} \sim \frac{M^2}{\lambda}$$  

(20)

so the curvature of the $M, \bar{M}$ potential is positive provided $\lambda' \gg \lambda$. (The first relation in eqn. 20 follows from the fact that the potential for $Z^\pm$ will exhibit structure on the scale of the $\phi^\pm$ expectation values; the second from the vanishing of $F_{\phi^\pm}$.) Numerically, we find that the situation is better than this; $F_{Z^0}$ is typically significantly smaller than $|Z_0|^2$, even when $\lambda \mu \sim M$.

Note that the energy difference between the metastable and the supersymmetric vacuum is of order:

$$\Delta E = M^2 v^2.$$  

(21)

The barrier height, on the other hand, is of order the shift in $\phi^\pm$ times the $\phi^\pm$ masses in the metastable minimum, or

$$V_0 \sim \lambda^2 \mu^4.$$  

(22)
So a thin walled treatment is appropriate\[14\], and the bounce action is at least as large as

\[ S \sim \frac{\pi^2 S^4}{2\Delta E^3} \]  

(23)

where

\[ S_1 \sim \lambda \mu^3. \]  

(24)

This gives an estimate for the bounce action:

\[ S \sim \pi^2 \lambda^{-2} \left( \frac{\lambda^6 \mu^6}{M^6} \right). \]  

(25)

\( \lambda \) and \( M^2/\mu^2 \lambda^2 \) were, by assumption, our small parameters, and the decay amplitude can easily be extremely small, even if the small parameters are not.

The problems of generating suitable \( \mu \) and \( B_\mu \) terms in gauge-mediated models are well-known. If we simply couple \( Z^0 \) to \( \bar{H}H \), with a small coupling, this can generate a small \( B_\mu \) but this will lead to a very small \( \mu \) term. In this framework, we can also generate a \( \mu \) and \( B_\mu \) term of a reasonable order of magnitude, following the ideas of \[10\] and \[11\]. This approach involves introducing a singlets, \( S \) with couplings similar to those of the NMSSM\[10\]. It is also necessary to double the messenger sector, i.e. to have fields \( M_i, \bar{M}_i, i = 1, 2 \). We can take the additional terms in the superpotential to be (we avoid giving names to all of the various couplings at this stage):

\[ W = Z^0(y_1 M_1 \bar{M}_1 + y_2 M_2 \bar{M}_2) + hSM_1 \bar{M}_2 + S^3. \]  

(26)

The renormalizable couplings can be restricted to this form by a discrete \( R \) symmetry. For example, taking

\[ \alpha = e^{\frac{2\pi i}{N}} \]  

(27)

and supposing \( Z^0 \) transforms with phase \( \alpha^{-1} \), we can take:

\[ M_1 \to \alpha^2 M_1 \quad \bar{M}_1 \to \bar{M}_1 \quad M_2 \to M_2 \quad \bar{M}_2 \to \alpha^2 \bar{M}_2 \quad S \to \alpha^{-3} S \quad H_U H_D \to \alpha^{-1} H_U H_D. \]  

(28)

This still allows some dangerous couplings; in particular, \( Z^0 S' \). This can be forbidden by an additional \( Z_2 \), for example, under which \( S' \) is odd, and one of \( H_U \) or \( H_D \) is odd.

In \[10, 11\], it was shown that the one loop corrections to the \( S \) mass vanish (to order \( F^2 \)) in a model such as this, and the two loop contributions can be negative. As a result, the \( S \) vev is one loop order, the \( \mu \) term one loop order, and the \( B_\mu \) term two loop order.
5 Conclusions

This paper can perhaps be viewed as the culmination of the program initiated by ISS[2]. ISS exhibited vector-like models with non-vanishing Witten index, in which there are metastable states in which supersymmetry is dynamically broken. Ref. [3] enlarged the set of possible models, by simply taking O’Raifeartaigh theories and replacing all mass parameters by dynamically generated scales. As noted in [3], because these models often contain singlets neutral under gauge symmetries and discrete $R$ symmetries, they open up new possibilities for building supergravity models with supersymmetry broken dynamically in a hidden sector. This may be particularly interesting in light of recent studies of the landscape of string vacua[15]. In many string constructions, large chiral theories of the type previously thought needed for dynamical supersymmetry breaking seem rare. Non-chiral theories with singlets seem far more common. $R$-symmetries may be rare, however[16].

One can ask about the cosmology of the susy-breaking vacuum states. First, there is the issue of metastability. We would expect the susy breaking states in the retrofitted models to be highly metastable. Even before coupling to gravity, the supersymmetric minimum lies far away in field space, and the amplitudes are suppressed by huge exponentials. Before worrying about gravity, the decay amplitudes are typically extremely small,

$$\Gamma \sim e^{-cM^4/\mu^4},$$

where $c$ is a number of order 1. Once coupled to gravity, the standard Coleman-DeLuccia analysis will give vanishing amplitude in most cases for decays to big crunch spacetimes. A number of papers have appeared recently discussing the question: can the system find its way into the metastable vacuum. In the ISS case, if one assumes that the system was in thermal equilibrium after inflation, one finds that the broken susy minimum, is thermodynamically favored[17]. In the O’Raifeartaigh models, the same can be true; the analysis is in fact simpler. For example, in our models, we have large numbers of messenger fields, which are light in the metastable vacuum. More generally, the low energy theory has accidental, approximate symmetries near the origin of field space, and even non-thermal effects (e.g. in cosmologies with low reheating temperatures) may favor these points.

Our principle interest in this paper was to construct models of direct mediation. For this, in both the models of [2] and [3] there was an obstacle: the low energy theories possessed an accidental continuous $R$ symmetry. In this paper, we have shown how, by adding gauge interactions, one can break the $R$ symmetry spontaneously. It is straightforward to add messengers to
implement dynamical supersymmetry breaking. In this way, one could construct models with susy breaking scale as low as 10’s of TeV. One still must confront the standard difficulties in gauge mediation, especially the $\mu$ problem and the question of fine tuning. We have considered one mechanism for solving the $\mu$ problem in these theories, and argued that it is technically natural. The models, at low energies, look like the conventional NMSSM. Ameliorating the tuning problems will require a hidden sector with more fields, in which the squarks are not parameterically heavy compared to the doublet sleptons. This question will be explored elsewhere[18], as will further studies of the $\mu$ term.

Acknowledgements: We thank Tom Banks, Jonathan Feng and Eva Silverstein for conversations. We are grateful to R. Rattazzi, Nathan Seiberg, David Shih and Yuri Shirman for criticisms of an early version of this paper. This work supported in part by the U.S. Department of Energy. M. D. thanks the Kavli Institute for Theoretical Physics for hospitality during the course of this project.

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