An efficient Lagrangian-based heuristic to solve a multi-objective sustainable supply chain problem

Camila P.S. Tautenhain\textsuperscript{a}, Ana Paula Barbosa-Povoa\textsuperscript{b}, Bruna Mota\textsuperscript{b}, Mariá C.V. Nascimento\textsuperscript{a,*}

\textsuperscript{a}Instituto de Ciência e Tecnologia, Universidade Federal de São Paulo, São José dos Campos, Brasil
\textsuperscript{b}Centro de Estudos de Gestão, Instituto Superior Técnico, Universidade Técnica de Lisboa, Lisboa 1049-101, Portugal

Abstract

Sustainable Supply Chain (SSC) management aims at integrating economic, environmental and social goals to assist in the long-term planning of a company and its supply chains. There is no consensus in the literature as to whether social and environmental responsibilities are profit-compatible. However, the conflicting nature of these goals is explicit when considering specific assessment measures and, in this scenario, multi-objective optimization is a way to represent problems that simultaneously optimize the goals. This paper proposes a Lagrangian matheuristic method, called \textit{AugMathLagr}, to solve a hard and relevant multi-objective problem found in the literature. \textit{AugMathLagr} was extensively tested using artificial instances defined by a generator presented in this paper. The results show a competitive performance of \textit{AugMathLagr} when compared with an exact multi-objective method limited by time and a matheuristic recently proposed in the literature and adapted here to address the studied problem. In addition, computational results on a case study are presented and analyzed, and demonstrate the outstanding performance of \textit{AugMathLagr}.

Keywords:
Sustainable Supply Chain Management, Multi-Objective Optimization, Matheuristic, Lagrangian Heuristic

1. Introduction

The change in mentality of organizations to go beyond profit maximization and consider the long-term economic success by taking into account social and environmental responsibilities has raised the interest of researchers in the subject of Sustainable Supply Chain (SSC) management [38, 15, 2]. In particular, multi-objective SSC management models have relied on the integration of social, environmental and economic issues – the so-called triple bottom line – to explicitly define the long-term planning of sustainable companies [8]. Multi-objective optimization plays an important role in approaching SSC management since a compromise among conflicting goals related to profit, environment and social issues can be reached.

A measure commonly used as an economic criterion in SSC management optimization models is the Net Present Value (NPV) [10]. There is a plethora of measures [11] to quantitatively assess the environmental impacts related to, for example, greenhouse gas emissions, pollution and resource usage. Additionally, there exist some methods specifically designed to quantify these impacts such as Eco-Indicator 99 [20] and ReCiPe 2008 [19], which are based on the Life Cycle Analysis (LCA) methodology.

*Corresponding author.

Email addresses: santos.camila@unifesp.br (Camila P.S. Tautenhain), apovoa@tecnico.ulisboa.pt (Ana Paula Barbosa-Povoa), bruna.mota@tecnico.ulisboa.pt (Bruna Mota), mcv.nascimento@unifesp.br (Mariá C.V. Nascimento)
In spite of the relevance of the social impacts in the SSC management, only a few models in the literature optimize this indicator along with environmental and economic indicators, e.g. in [34, 4, 7, 35, 10]. Eskandarpour et al. [15] discuss various indicators to assess social responsibilities, most of which related to human rights and social justice laws. In particular, the authors suggest metrics that encourage the creation of job vacancies [34, 4, 35], employment stability [4] and working conditions. Recently, Mota et al. [34] introduced the social benefit indicator whose calculation relies on the total number of jobs created in each location across the supply chain. This indicator weighs preferably the creation of entities in less developed countries. Mota et al. [35] have optimized this social indicator in the multi-objective SSC management problem under investigation. In addition to optimizing the social benefit indicator, the SSC management problem introduced in [35] aims at maximizing the NPV and minimizing the environmental impacts quantified by ReCiPe 2008. The problem in question models a generic SSC management problem that manufactures multiple products in a planning horizon composed of multiple periods. The appeal of the model lies on its applicability to a wide variety of industries, since it integrates a number of strategic and tactical decisions related to, for example, the use of technologies to manufacture and refurbish products and the shipment of items through different transport modes, among others. The authors solved mono-objective problems to minimize the economic, i.e., the NPV, and environmental objective functions and to maximize the social objective function. Moreover they have also considered two scenarios to maximize the social objective function with additional constraints that require the NPV value to be at least 85% and 95% of the value found when optimizing the mono-objective problem that maximizes the economic objective function. The computational difficulty to solve the resulting problem was the main reason why the authors did not consider the three objective functions simultaneously when optimizing the problems in the case studies.

In this context, the primary contributions of this paper are:

- The development of an efficient Lagrangian matheuristic for SSC management problems, here called AugMathLagr, to tackle the SSC problem proposed in [35].
- The adaptation of the AugMathFix matheuristic introduced in [40] to the SSC management problem put forward in [35].
- A test bed of artificial instances and an instance generator loosely based on the study of real data.

The present paper also reports the computational experience with AugMathLagr on a case study and a set of medium-sized randomly generated instances. Results from of AugMathLagr were compared with results obtained with the Augmented $\epsilon$-Constraint method (AUGMECON2) [33], which is an enhancement of AUGMECON [32], and with the adapted AugMathFix. The comparison involved evaluating the solution quality and time-to-solution. The proposed matheuristic achieved very good results in significantly lesser computational times than AUGMECON2 and AugMathFix.

The remainder of this paper is organized as follows. Section 2 reviews the related methods for solving multi-objective SSC management problems; Section 3 presents the SSC model to which we propose a Lagrangian heuristic; Section 4 describes the proposed multi-objective matheuristic, AugMathLagr and briefly discusses the adapted AugMathFix; Section 5 thoroughly explains the random instance generator; Section 6 presents the computational experiments and Section 7 sums up the paper drawing some conclusions and suggesting future works.
2. Related Works

A vast body of literature on SSC management multi-objective solution methods is based on scenario analysis, the $\epsilon$-Constraint or weighting aggregation methods and heuristics. To approach a multi-objective minimization problem, the $\epsilon$-Constraint method \cite{31, 23} optimizes one of its objective functions and adds constraints to assign upper bounds for the other objective functions. Among the strategies based on the $\epsilon$-Constraint method, we can cite the multi-parametric method proposed by Hugo and Pistikopoulos \cite{27}, the two-phase method by Neto et al. \cite{36}, the decomposition methods by Guillén-Gosálbez et al. \cite{22} and Gao and You \cite{18}. In particular, Mavrotas \cite{32} introduced the Augmented $\epsilon$-Constraint method (AUGMECON) that solves a sequence of mono-objective problems. Later, Mavrotas and Florios \cite{33} proposed AUGMECON2 as an improved version of AUGMECON. Examples of works that employ AUGMECON to approach multi-objective SSC management problems are found in \cite{34, 7}.

Regarding heuristic methods, it is worth noting that a significant amount of studies focuses on evolutionary population-based algorithms, among which we can cite the adaptation of the classical Non-dominated Sorting Genetic Algorithm (NSGAII) \cite{11} proposed by Dehghanian and Mansour \cite{12}, the genetic algorithm by Soleimani et al. \cite{39}, the memetic multi-objective algorithm by Jamshidi et al. \cite{29} and the evolutionary algorithms hybridized with the Variable Neighborhood Search (VNS) metaheuristic proposed by Devika et al. \cite{13} and Zhalechian et al. \cite{42}.

In the literature, one can find a number of multi-objective optimization algorithms for SSC management problems that are not population-based as, for example, those based on hybridizations with the VNS \cite{16, 21}, Simulated Annealing \cite{9} and AUGMECON2 with a local search method \cite{40}. Tautenhain et al. \cite{40}, in particular, introduced a multi-objective matheuristic that combines AUGMECON2 with a model-based heuristic that iteratively solves SSC management problems with the strategic constraints relaxed. In this case, for the method to find a feasible solution to the original problem in a reduced computational time, the variables related to the tactical decisions are fixed at the values of the solution of the relaxed problem. Moreover, at each iteration a local search strategy is applied to the feasible solutions to enhance the quality of the solutions found.

Heidari-Fathian and Pasandideh \cite{25} and Yousefi-Babadi et al. \cite{41}, on the other hand, studied the Lagrangian decomposition of multi-objective supply chain problems, which is more related to the multi-objective matheuristic introduced here. Heidari-Fathian and Pasandideh \cite{25} employed a $\epsilon$-Constraint method to transform the multi-objective SSC management problem into a mono-objective problem. In order to define the constraints to be relaxed in the mono-objective problem, Heidari-Fathian and Pasandideh \cite{25} chose the pair that resulted in the lowest computational running times when removed from the original problem. In the introduced mathematical formulation, Yousefi-Babadi et al. \cite{41} did not consider environmental issues in any of the objective functions, however, they included recycling centers. The authors employed a weighting strategy to aggregate the objective functions into a mono-objective function and then relaxed the most complex constraints of the problem to then solve it using an optimization solver.

Lagrangian-based heuristics are extensively studied to approach mono-objective supply chain problems \cite{17, 30, 11, 43, 37}. Eskigun et al. \cite{17} reduced the original capacitated network design problem into simpler and independent subproblems to approach the design of vehicle distribution centers of several instances constructed from industrial data by a Lagrangian heuristic. In the Lagrangian heuristic proposed by Lidestam and Rönqvist \cite{39}, the authors decomposed the supply chain planning problem into two different subproblems
associated with different stages of cellulose production. Elhedhli and Merrick [14] studied a supply chain network design problem that aims to minimize environmental costs due to carbon dioxide emissions. The authors employed a Langragian relaxation by decomposing the original problem according to entity types and warehouse site. Zhang et al. [43] studied a supply chain problem in which the flow between suppliers, factories and customers required a distribution center. The authors relaxed the constraints which ensured that each customer and supplier was assigned to only one distribution center in the Lagrangian heuristic they suggested. Rafie-Majd et al. [37] studied a supply chain of perishable products that takes into consideration fuel consumption and product wastage. The Lagrangian heuristic presented by the authors considers the relaxation of the vehicle capacity constraints, a nonlinear constraint and a constraint related to the allocation of distribution centers to customers.

As an extensive review is out of the scope of this paper, we refer to Eskandarpour et al. [15] for further details on solution methods to SSC management problems. The next section describes the SSC management problem, the focus of this paper.

3. Problem description

The studied (SSC) is composed of suppliers, factories, warehouses, customers, airports and seaports, and is tailored to support multi-period planning. Moreover, there are three types of items in the SSC: raw materials, final products and recovered products.

Figure 1 illustrates a flow network whose nodes are the entities of the SSC, and the arcs represent the flow of goods between the sites. In this figure, the rectangles represent airports and seaports whereas the ellipses represent the remaining entities. The double-sided arrows indicate that the corresponding arcs represent the flow of both final products and recovered products between the sites. The rightwards arrow represents the arc indicating the flow of raw materials from suppliers to factories.

![Figure 1: An illustration with a general representation of the studied SSC.](image)

Factories employ production technologies to manufacture final products from a bill of raw materials and may also use remanufacturing technologies to reuse products. The problem allows the storage of final
products sent from factories to warehouses. Factories and warehouses can ship final products to meet customer demands, indicated by the arc linking these entities. Customers return recovered products to factories and warehouses after the end of their lifetime. Warehouses can then return the recovered products to the factories.

Land transportation is responsible for transporting raw materials from suppliers to factories. The possible transportation modes between factories, warehouses and customers are by land, air or sea. Trucks carry items from and to entities through land transportation, whereas air and sea transportation are only allowed from/to airports or seaport hubs.

The goals of the studied SSC management are related to the triple bottom line: (i) the maximization of NPV as the economic function; (ii) the minimization of the environmental impact evaluated by ReCiPe 2008 as the environmental function; and (iii) the maximization of the social benefit indicator as the social function. The Gross Domestic Product (GDP) [5] was used to assess the country’s industrial base development since it is a widely accepted index to measure the economic activity of a nation.

In the studied SSC, the set of periods is given by \( T = \{1, 2, \ldots, |T|\} \) and the set of entities by \( I = \) \( I_{\text{sup}} \cup I_f \cup I_w \cup I_c \cup I_{\text{air}} \cup I_{\text{port}}, \) where \( I_{\text{sup}}, I_f, I_w, I_c, I_{\text{air}} \) and \( I_{\text{port}} \) are, respectively, the sets of suppliers, factories, warehouses, customers, airports and seaports. The set of items is given by \( M = M_{\text{rm}} \cup M_{\text{fp}} \cup M_{\text{rp}}, \) where \( M_{\text{rm}}, M_{\text{fp}} \) and \( M_{\text{rp}} \) are the sets of raw materials, final products and recovered products, respectively. The set of transportation modes is given by \( A = A_{\text{truck}} \cup A_{\text{plane}} \cup A_{\text{boat}}, \) where \( A_{\text{truck}}, A_{\text{plane}} \) and \( A_{\text{boat}} \) are the set of trucks, airplanes and ships, respectively. The technologies set is given by \( G = G_{\text{prod}} \cup G_{\text{rem}}, \) where \( G_{\text{prod}} \) and \( G_{\text{rem}} \) are, respectively, the production and the remanufacturing technologies sets. Moreover, consider \( H_{\text{prod}} = \{(m, g) \text{ such that } m \in M_{\text{fp}} \text{ can be manufactured using technology } g \in G_{\text{prod}}\} \) and \( H_{\text{rem}} = \{(m, g) \text{ such that } m \in M_{\text{fp}} \text{ can be remanufactured using technology } g \in G_{\text{rem}}\}. \)

The decisions associated with the management of the studied SSC relate to the amount of raw materials acquired from each supplier; the opening of factories and warehouses and their capacities; production and remanufacturing technologies assigned to factories and recycling centers, respectively; production and remanufacturing levels in factories; storage levels in warehouses; and shipment of items between entities using different transportation modes. Table 1 presents a complete list of decision variables.

The following section presents the multi-objective formulation of the SSC management problem studied in [35].

### 3.1. Multi-objective formulation

Let \( n', n'', b' \) and \( b'' \) be natural numbers that define the dimension of the following decision variables: \( u \in \mathbb{R}_{\geq 0}^{n'}, v \in \mathbb{Z}_{\geq 0}^{n''}, w' \in \{0, 1\}^{b'} \) and \( w'' \in \{0, 1\}^{b''}. \) Moreover, consider \( f'_{\text{eco}}, f'_{\text{env}}, f'_{\text{soc}} : \mathbb{R}^{n'} \times \mathbb{Z}_{\geq 0}^{n''} \times \{0, 1\}^{b'} \times \{0, 1\}^{b''} \rightarrow \mathbb{R} \) as the economic, environmental and social functions of the problem, respectively. The economic function \( (f'_{\text{eco}}) \) and social function \( (f'_{\text{soc}}) \) must be maximized to achieve the best values of NPV and of the social benefit indicator, respectively. Thereby, without loss of generality, we define \( f_{\text{eco}} = -f'_{\text{eco}}, f_{\text{env}} = f'_{\text{env}} \) and \( f_{\text{soc}} = -f'_{\text{soc}} \) to describe the multi-objective minimization problem [1]-[4].

\[
\begin{align*}
\min \quad & f_{\text{eco}}(u, v, w', w''), f_{\text{env}}(u, v, w', w''), f_{\text{soc}}(u, v, w', w'') \\
\text{s.t.} \quad & A' u + A'' v \leq \beta' w' + \beta'' w'' \\
& E' u + E'' v \leq d \\
& u \in \mathbb{R}_{\geq 0}, v \in \mathbb{Z}_{\geq 0}^{n''}, w' \in \{0, 1\}^{b'}, w'' \in \{0, 1\}^{b''}
\end{align*}
\]
Table 1: Decision variables of the SSC formulation introduced in [35].

| Variable | Description |
|----------|-------------|
| $S_{mit}$ | Amount of product $m$ stocked in entity $i$ in time period $t \in T$. |
| $X_{majt}$ | Amount of item $m \in M$ transported from entity $i$ to entity $j$ by transportation mode $a$ in time period $t \in T$. |
| $P_{mgit}$ | Amount of product $m \in M$ produced by technology $g \in G_{prod}$ in factory $i$ in time period $t \in T$. |
| $R_{mgit}$ | Amount of product $m \in M$ remanufactured by technology $g \in G_{rem}$ in factory $i$ in time period $t \in T$. |
| $YCT_{it}$ | Capacity of entity $i$. |
| $K_{ait}$ | Effective use of capacity in entity $i$ in time period $t \in T$. |
| $K_{ait}$ | Upper bound to the number of transportation modes $a \in A$ from entity $i$ to another in time period $t \in T$. |

Continuous decision variables

| $Q_{aijt}$ | Number of trips from entity $i$ to $j$ by transportation mode $a$ in time period $t \in T$. |

Integer decision variables

| $Y_{i}$ | value 1 indicates that entity $i$ is installed and 0, otherwise. |
| $Z_{gmi}$ | value 1 indicates that the technology $g$ is selected to produce product $m$ in factory $i$ and 0, otherwise. |

where $p$ and $q$ are natural numbers, $\mathcal{A} \in \mathbb{R}^{p \times n'}$ and $E' \in \mathbb{R}^{q \times n'}$ are parameters associated with the real variables; $\mathcal{A}'' \in \mathbb{R}^{p \times n''}$ and $E'' \in \mathbb{R}^{q \times n''}$ are parameters associated with the integer variables; $\beta' \in \mathbb{R}^{p \times b'}$ and $\beta'' \in \mathbb{R}^{p \times b''}$ are parameters associated with the binary variables $Y$ and $Z$ of the problem, respectively; $\mathcal{A}'u + \mathcal{A}''v \leq \beta'w' + \beta''w''$ are the constraints associated with the binary variables; and $E'u + E''v \leq d, d \in \mathbb{R}^q$, are the remaining constraints.

To fully understand the proposed method, we show the constraints $\mathcal{A}'u + \mathcal{A}''v \leq \beta'w' + \beta''w''$ in (5)-(23).

Let $s_{c_{mi}}^{\text{max}}$ and $s_{c_{mi}}^{\text{min}}$ be, respectively, the maximum and minimum amounts of raw material $m$ provided by supplier $i$. Constraints (5) and (6) ensure that the amount of raw materials acquired by the factories from the selected suppliers are within the interval $[s_{c_{mi}}^{\text{min}}, s_{c_{mi}}^{\text{max}}]$.

\[
\sum_{a \in A, j \in I_f} X_{majt} \leq s_{c_{mi}}^{\text{max}}Y_{i}, \quad i \in I_{\text{sup}}, m \in M_{\text{rm}}, t \in T \tag{5}
\]

\[
\sum_{a \in A, j \in I_f} X_{majt} \geq s_{c_{mi}}^{\text{min}}Y_{i}, \quad i \in I_{\text{sup}}, m \in M_{\text{rm}}, t \in T \tag{6}
\]

Constraints (7) and (8) define, respectively, $ec_{i}^{\text{max}}$ as the maximum in- and out-flow of products between a pair of installed entities $i, j \in I$.

\[
\sum_{m \in M, a \in A, j \in I} X_{majt} \leq ec_{i}^{\text{max}}Y_{i}, \quad i \in I, t \in T \tag{7}
\]

\[
\sum_{m \in M, a \in A, j \in I} X_{majit} \leq ec_{i}^{\text{max}}Y_{i}, \quad i \in I, t \in T \tag{8}
\]

Let $ie_{mi}^{\text{max}}$ and $ie_{mi}^{\text{min}}$ be, respectively, the maximum and the minimum amounts of product $m$ in storage in factory or warehouse $i$. Constraints (9) and (10) ensure that the amount of final products $m$ stored at
installed factory or warehouse \( i \) is within the interval \([ic_{mi}^{\min}, ic_{mi}^{\max}]\).

\[
S_{mit} \leq ic_{mi}^{\max}Y_i, \quad m \in M_{fp}, i \in I_f \cup I_w, t \in T \tag{9}
\]

\[
S_{mit} \geq ic_{mi}^{\min}Y_i, \quad m \in M_{fp}, i \in I_f \cup I_w, t \in T \tag{10}
\]

Constraints (11) and (12) ensure that the installation area of each factory or warehouse \( i \) is within the interval \([ea_{mi}^{\min}, ea_{mi}^{\max}]\), where \( ea_{mi}^{\min} \) and \( ea_{mi}^{\max} \) are non-negative scalars.

\[
YC_i \leq ea_{mi}^{\max}Y_i, \quad i \in I_f \cup I_w \tag{11}
\]

\[
YC_i \geq ea_{mi}^{\min}Y_i, \quad i \in I_f \cup I_w \tag{12}
\]

Constraints (13) and (14) guarantee that only entities selected to be installed can receive or send items.

\[
\sum_{m \in M, a \in A, i \in I, t \in T} X_{maijt} \geq Y_j, \quad j \in I \tag{13}
\]

\[
\sum_{m \in M, a \in A, j \in I, t \in T} X_{maijt} \geq Y_i, \quad i \in I \tag{14}
\]

Constraints (15) and (16) guarantee that if the number of trips to transport items from/to an entity is higher than 0, the entity must be installed.

\[
Q_{aijt} \leq BigMY_i, \quad a \in A, i, j \in I, t \in T \tag{15}
\]

\[
Q_{aijt} \leq BigMY_j, \quad a \in A, i, j \in I, t \in T \tag{16}
\]

Constraints (17) restrict the purchase of trucks \( a \in A_{\text{truck}} \) only at installed entities \( i \in I \).

\[
K_{ai} \leq BigMY_i, \quad a \in A_{\text{truck}}, i \in I \tag{17}
\]

Let \( pc_{g}^{\max} \) and \( pc_{g}^{\min} \) be, respectively, the maximum and minimum amounts of products that technology \( g \in G \) can produce. For each factory \( i \) and time period \( t \), constraints (18) and (19) ensure that the production levels of the final product \( m \) using technology \( g \), \( \forall (m, g) \in H_{\text{prod}} \), are within the interval \([pc_{g}^{\min}, pc_{g}^{\max}]\). Analogously, constraints (20) and (21) ensure that the remanufacturing levels of technology \( g \) are within the interval \([pc_{g}^{\min}, pc_{g}^{\max}]\).

\[
P_{mgit} \geq pc_{g}^{\min}Z_{gmi}, \quad i \in I_f, (m, g) \in H_{\text{prod}}, t \in T \tag{18}
\]

\[
P_{mgit} \leq pc_{g}^{\max}Z_{gmi}, \quad i \in I_f, (m, g) \in H_{\text{prod}}, t \in T \tag{19}
\]

\[
R_{mgit} \geq pc_{g}^{\min}Z_{gmi}, \quad i \in I_f, (m, g) \in H_{\text{rem}}, t \in T \tag{20}
\]

\[
R_{mgit} \leq pc_{g}^{\max}Z_{gmi}, \quad i \in I_f, (m, g) \in H_{\text{rem}}, t \in T \tag{21}
\]

Constraints (22) and (23) define that production or remanufacturing technologies can only be selected in
installed factories.

\[ \sum_{g:(m,g) \in H_{prod}} Z_{gmi} \leq Y_i, \quad m \in M_{fp}, i \in I_f \]  \hspace{1cm} (22)

\[ \sum_{g:(m,g) \in H_{rem}} Z_{gmi} \leq Y_i, \quad m \in M_{fp}, i \in I_f \]  \hspace{1cm} (23)

We refer to [35] for details about the functions \( f'_{eco}, f'_{env} \) and \( f'_{soc} \) and remaining constraints \( E'u + E''v \leq d \), which are related to the tactical planning.

The following section discusses the solution methods proposed in this paper.

4. Proposed method

This section describes the proposed Lagrangian matheuristic – called AugMathLagr — to approach the multi-objective SSCM problem discussed in the previous section.

AugMathLagr heuristically solves the multi-objective SSCM problem by following the same core strategy as the Augmented \( \epsilon \)-Constraint Method (AUGMECON2) [33]. AugMathLagr introduces the Lagrangian-based heuristic for solving mono-objective problems as an innovation in relation to AUGMECON2.

Mavrotas and Florios [33] proposed AUGMECON2 as an improvement of the \( \epsilon \)-Constraint method. Its aim is to identify the Pareto set of a multi-objective problem by systematically solving a sequence of mono-objective problems (\( \epsilon \)-Constrained problems). In line with this, the multi-objective problem (1)-(4) can be approached by AUGMECON2 through the solution of the mono-objective problem (24)-(30), referred here to as MOP.

\[ \text{(MOP): min } f_{eco}(u, v, w', w'') - \epsilon \text{ps}\left( \frac{l_{env}}{r_{env}} + 0.1 \frac{l_{soc}}{r_{soc}} \right) \]  \hspace{1cm} (24)

\[ s.t. \quad \mathcal{A}'u + \mathcal{A}''v \leq \beta'w' + \beta''w'' \]  \hspace{1cm} (25)

\[ E'u + E''v \leq d \]  \hspace{1cm} (26)

\[ f_{env}(u, v, w', w'') + l_{env} = \epsilon_{env} \]  \hspace{1cm} (27)

\[ f_{soc}(u, v, w', w'') + l_{soc} = \epsilon_{soc} \]  \hspace{1cm} (28)

\[ u \in \mathbb{R}_{\geq 0}, v \in \mathbb{Z}_{\geq 0}^{n'}, w' \in \{0,1\}^{l'}, w'' \in \{0,1\}^{l''} \]  \hspace{1cm} (29)

\[ l_{env}, l_{soc} \in \mathbb{R}_{\geq 0} \]  \hspace{1cm} (30)

where \( l_{env} \) and \( l_{soc} \) are the slack variables of the \( \epsilon \)-Constraints (27) and (28); \( \epsilon_{env} \) and \( \epsilon_{soc} \) are scalar values defined as thresholds of \( f_{env} \) and \( f_{soc} \), respectively; \( r_{env} \) and \( r_{soc} \) are positive scalars which are the absolute value of the difference between the best and worst possible values of the functions \( f_{env} \) and \( f_{soc} \), respectively; and \( \epsilon \text{ps} \in \mathbb{R}_+ \) is a small value to promote alternative optimal solutions for \( f_{eco} \) with the best possible values of \( f_{env} \) and \( f_{soc} \). The coefficients 1 and 0.1 on the objective function mean that \( f_{env} \) must be prioritized over \( f_{soc} \). Their values were defined according to Mavrotas and Florios [33].

To define the values of \( \epsilon_{env} \) and \( \epsilon_{soc} \), AUGMECON2 creates a grid of evenly distributed points in the Cartesian plane limited by the best and worst possible values for \( f_{env} \) and \( f_{soc} \). The Pareto frontier approximation is composed of the solutions of the MOP considering pre-defined values of \( \epsilon_{env} \) and \( \epsilon_{soc} \).
Let us consider AUGMECON2 to solve the MOP. As AUGMECON2 is an iterative strategy, an upper index is used on the functions and variables of the MOP to indicate the solutions of a given iteration. Moreover, to assign values to $\epsilon_{soc}$ considering an imposed number of points of the grid, the step value of $\epsilon_{soc}$ is defined and referred to as $step_{soc}$. In the first iteration, iteration 0, an initial value for $\epsilon_{soc}$ must be considered, here denoted by $\epsilon_{soc}^{(0)}$, which can be, for example, the nadir point of $f_{soc}$. According to constraints (24), $f_{soc}(u, v, w', w'') + l_{soc}^{(1)} = \epsilon_{soc}^{(0)}$, hence, $l_{soc}^{(0)} = \epsilon_{soc}^{(0)} - f_{soc}(u, v, w', w'')$. Then, to update $\epsilon_{soc}$, i.e., to define $\epsilon_{soc}^{(1)}$, $\epsilon_{soc}^{(0)}$ is decremented by an scalar $step_{soc}$ and in the next iteration, the values are updated according to: $f_{soc}(u, v, w', w'') + l_{soc}^{(1)} = \epsilon_{soc}^{(0)} - step_{soc}$. First let $l_{soc}^{(0)} - step_{soc} \geq 0$ or, equivalently, $l_{soc}^{(0)} \geq step_{soc}$. Thereby, when $l_{soc}^{(0)} = l_{soc}^{(0)} - step_{soc}$, constraint (28) ensures $f_{soc}(u, v, w', w'') + l_{soc}^{(0)} - step_{soc} = \epsilon_{soc}^{(0)} - step_{soc}$, i.e., $f_{soc}(u, v, w', w'') + l_{soc} = \epsilon_{soc}$, that is the same problem solved in the current iteration. Therefore, to avoid solving redundant problems, it is necessary to choose a step size whose $l_{soc} < step_{soc}$ holds. AUGMECON2 does that by selecting the step size where $l_{soc} = step_{soc}$.

Mavrotas [32] observed that if a problem is infeasible for a given value of $\epsilon_{soc}$, for smaller values of $\epsilon_{soc}$, it will also be infeasible. Therefore, in this case, AUGMECON2 halts at decrementing $\epsilon_{soc}$ to avoid solving unnecessary problems. This mechanism enables AUGMECON2 to investigate a lower number of problems than an enumerative $\epsilon$-Constraint method.

In the next section, we thoroughly explain the MathLagr matheuristic, which is the Lagrangian heuristic to solve mono-objective problems in AugMathLagr.

4.1. MathLagr

MathLagr finds heuristic solutions through the Lagrangian relaxation of each MOP. The constraints associated with binary variables $A'u + A''v \leq \beta' w' + \beta'' w''$ can be decomposed into equivalent constraints $A'u + A''v \leq \beta' + \beta'' w''$ and $A'u + A''v \leq Mw + \beta'' w''$, $M \in \mathbb{R}^{p \times b'}$ being a matrix whose elements are BigM values. The method keeps constraints $A'u + A''v \leq \beta' + \beta'' w''$ in the problem and relaxes constraints $A'u + A''v \leq Mw' + \beta'' w''$.

As the elements of $\beta'$ are a trivial estimation of the BigM values of $M$, we shall refer to the relaxed constraints as $A'u + A''v \leq \beta' w' + \beta'' w''$.

Let $\lambda \in \mathbb{R}^p, \lambda \geq 0$, be the $p$-dimensional vector of Lagrange multipliers associated with the relaxed constraints. Each Lagrange multiplier, $\lambda_i$, penalizes the corresponding violation of constraints $a'_i u + a''_i v \leq \beta'_i w' + \beta''_i w''$ in the objective function.

MathLagr solves the Lagrangian mono-objective problems $P_{RL}$ defined by Equations (31)–(37).

\[
(PRL) : \min L(u, v, w', w'') = f_{eco}(u, v, w', w'') - eps \left( \frac{1}{r_{env}} l_{soc} \right) + \lambda^T \left( A'u + A''v - \beta' w' - \beta'' w'' \right)
\]

\[1\] $a'_i, a''_i, \beta'_i$ and $\beta''_i$ indicate the $i$-th row of, respectively, $A', A''$, $\beta'$ and $\beta''$. 


\[ \begin{align*}
\text{s.t. } & \quad \mathcal{A}'u + \mathcal{A}''v \leq \beta' + \beta''w'' \\
& \quad E'u + E''v \leq d \\
& \quad f_{\text{env}}(u, v, w', w'') + l_{\text{env}} = \epsilon_{\text{env}} \quad (33) \\
& \quad f_{\text{soc}}(u, v, w', w'') + l_{\text{soc}} = \epsilon_{\text{soc}} \quad (34) \\
& \quad u \in \mathbb{R}_{\geq 0}, v \in \mathbb{Z}''_{\geq 0}, w' \in \{0, 1\}^b', w'' \in \{0, 1\}^b'' \quad (36) \\
& \quad l_{\text{env}}, l_{\text{soc}} \in \mathbb{R}_{\geq 0} \quad (37)
\end{align*} \]

The resulting problem is nonlinear. In order to approach it numerically using mixed integer program methods, \textit{MathLagr} employs the subgradient method \cite{26}. This iterative method assigns, in its first iteration, initial values to the Lagrange multipliers, which are usually null values. Then, at each iteration it relies on the resulting best lower bounds found along the iterations and on estimated upper bounds, possibly from heuristics incorporated in the method, to update the Lagrange multipliers in an attempt to refine them. In this paper, \( \lambda^{(k)} \) refers to the values of the Lagrange multipliers at iteration \( k \). Moreover, for ease of notation we shall refer to the solutions, that is, the values of the decision variables \( u, v, w', w'' \), as \( x \).

A solution to the Lagrangian problem is not necessarily feasible for the original problem. Therefore we apply a feasibility heuristic to the solution of the Lagrangian problem obtained at each iteration of the subgradient method. Since the feasibility heuristic guarantees that the variables are integer, we solve Lagrangian problems with the integer variables relaxed to speed CPLEX up. Algorithm 1 presents a pseudocode of the proposed Lagrangian heuristic named \textit{Mathlagr} to solve PRL.

\begin{algorithm}
\caption{Mathlagr.}
\begin{algorithmic}[1]
\Require Maximum number of iterations, \( k_{\text{Max}} \), step size \( st \), an initial value for the Lagrange multipliers, \( \lambda^{(0)} \)
\Ensure Best feasible solution \( x^{\text{best}} \)
\For{\( k = 0 \) to \( k = k_{\text{Max}} \) and \( UB \neq LB \)}
\State \( P^{(k)}_{\text{RL}} := \) Relaxed problem (31)-(37) considering \( \lambda^{(k)} \)
\State \( x^{(k)}_{\text{RL}} := \) Solve the linear relaxation of \( P^{(k)}_{\text{RL}} \)
\State \( P^{(k)}_{F} := \) Assign 0 to real variables \( X_{\text{maijt}}, m \in M, a \in A, i, j \in I, t \in T \), with value 0 at \( x^{(k)}_{\text{RL}} \) in MOP
\State \( x^{(k)}_{f} := \) Solve \( P^{(k)}_{F} \)
\State \( UB := \min\{f_{\text{eco}}(x^{(k)}_{f}), UB\} \)
\If{\( f_{\text{eco}}(x^{(k)}_{f}) < UB \)} \( x^{\text{best}} := x^{(k)}_{f}; \)
\State \( LB := \max\{L(x^{(k)}_{\text{RL}}), \lambda^{(k)}, LB\} \)
\State \( \lambda^{k+1} := st \frac{UB - L(x^{(k)}_{\text{RL}}, \lambda^{(k)})}{||g(x^{(k)}_{f})||} \)
\EndFor
\Ensure \( x^{\text{best}} \)
\end{algorithmic}
\end{algorithm}

Algorithm 1 uses the following as input: (i) the maximum number of iterations of the method, \( k_{\text{Max}} \); (ii) the step size to adjust the Lagrange multipliers at each iteration, \( st \); and (iii) an initial value for the Lagrange multipliers, \( \lambda^{(0)} \). Then, CPLEX \cite{28} is employed to solve the Lagrangian problem \( P_{\text{RL}} \) with the
integer variables relaxed at iteration 0, \( P^{(0)}_{RL} \). In line 4, problem \( P_F^{(k)} \) is created. It is the MOP with the real variables \( X_{maijt}, m \in M, a \in A, i, j \in I, t \in T, \) which are null in the relaxed solution of the iteration \( k \), i.e. \( x^{(k)}_{RL} \), fixed at zero. In line 5, CPLEX returns a solution for \( P_F^{(k)} \) and a feasible solution to the MOP, i.e., \( x^k_f \).

In line 6, the algorithm updates the upper bound \( (UB) \) with the best economic objective function value of a solution encountered up to that iteration. In line 7, \( x^{best} \) is updated if the value of its economic objective function is better than \( UB \). In the first iteration, \( UB \) is exactly the value of \( f_{eco}(x^0_f) \), since it is the first upper bound obtained by the method. The lower bound \( (LB) \), on the other hand, is the Lagrangian function value of the relaxed solution obtained in that iteration, that is, \( L(x^{(0)}_{RL}, \lambda^{(0)}) \), defined in line 8. In the next iterations, \( LB \) can be worse than those found in former iterations. Therefore, \( LB \) is the largest lower bound up to that iteration. Let \( g(x^{(k)}_f) \) be defined as \( \mathcal{A} u + \mathcal{A}'' v - \beta' w' - \beta'' w'' \), where \( u, v, w' \) and \( w'' \) are the values of the continuous, integer and binary variables related to \( Y \) and \( Z \) in solution \( x^{(k)}_f \). Then, the Lagrange multipliers are updated as indicated in line 9. The process is repeated until either the upper bound is equal to the lower bound or the maximum number of iterations has been reached. Algorithm 1 returns the best feasible solution found over the iterations referred to as \( x^{best} \) in line 11.

4.2. Multi-objective matheuristic

Algorithm 2 presents a pseudocode of the AugMathLagr solution method that creates a sequence of mono-objective problems using the multi-objective method AUGMECON2. This algorithm has as input the maximum number of iterations of the Lagrangian heuristic, \( kMax \), the step size of the subgradient method, \( st \), the initial value of the Lagrange multipliers, \( \lambda^{(0)} \), and the number of points of the grid, \( dg \).

Mavrotas [32] suggests the use of lexicographic optimization for estimating the lower and upper bounds of each objective function. However, this approach is computationally expensive. Therefore, due to the complexity of the studied model, to define these bounds in line 1 of Algorithm 2, each objective function is individually optimized by applying the Lagrangian heuristic described in Algorithm 1 to the problems

\[
\min f_i(x) + \lambda^T(g(x)), \ s.t. \ Constraints (32), (33), and (36), \ \forall i \in \{eco, env, soc\}.
\]

The lower bounds \( fL_i \) and the upper bounds \( fU_i \) are the lowest and highest values of each objective function \( f_i \), \( i \in \{eco, env, soc\} \), and are respectively applied to a relaxed solution and an incumbent solution found along the iterations.

In line 2, the algorithm estimates the ranges of the objective functions by calculating the difference between the lower and upper bounds estimated in line 1. The number of grid points required for each objective function is pre-defined by \( dg \). The initial values of \( \epsilon_{env} \) and \( \epsilon_{soc} \) are set as the estimated upper bound values for the respective objective functions. In line 3, the algorithm calculates the step values for \( \epsilon_{env} \) and \( \epsilon_{soc} \) according to the number of grid points and the ranges of the corresponding objective functions. In line 4, the algorithm initializes the Pareto frontier approximation \( P \) as an empty set and in line 5 a control variable \( gr_{env} \) indicating the point in the grid is initialized as zero.

In line 10, AugMathLagr employs Algorithm 1 to heuristically solve the Lagrangian mono-objective problem MOP, constructed in line 9. If solution \( x_f \) is feasible, in line 12 the approximation of the Pareto frontier, \( P \), is updated with \( x_f \). In lines 13 and 14, the algorithm avoids solving redundant problems by checking the ratio between the slack variable of the social constraint and the step size. Otherwise, if \( x_f \) is infeasible, it means that further decreasing \( \epsilon_{soc} \) will only result in infeasible problems. Therefore, the algorithm sets \( gr_{soc} = dg \) in line 15, preventing the method from solving these problems and making it proceed to the next value of \( \epsilon_{env} \).
Algorithm 2: AugMathLagr.

**Input**: maximum number of iterations, $kMax$, the step size $st$, an initial value for the Lagrange multipliers, $\lambda(0)$, the number of grid points $dg$

**Output**: Pareto frontier approximation $\mathcal{P}$

1. Estimate the lower bounds $fL_i$ and the upper bounds $fU_i$, $i \in \{eco, env, soc\}$ for the objective functions
2. Identify ranges $r_j = fU_j - fL_j$, $j \in \{env, soc\}$ of the environmental and social objective functions, respectively
3. $step_j = \frac{r_j}{dg}$, $j \in \{env, soc\}$
4. $\mathcal{P} := \emptyset$
5. $gr_{env} = 0$
6. for $\epsilon_{env} = fU_{env}$ until $gr_{env} < dg$ do
7.     $gr_{soc} = 0$
8.     for $\epsilon_{soc} = fU_{soc}$ until $gr_{soc} < dg$ do
9.         $MOP := \text{Problem (24)-(30)}$
10.        $x_f := \text{solve MOP by the Algorithm 1 considering } \epsilon_{env}$ and $\epsilon_{soc}$
11.         if $x_f$ is feasible then
12.             $\mathcal{P} := \mathcal{P} \cup x_f$
13.             $gr_{soc} = gr_{soc} + 1 + \lfloor \frac{\epsilon_{soc}}{step_{soc}} \rfloor$
14.             $\epsilon_{soc} = \epsilon_{soc} - step_{soc} \left( 1 + \frac{\epsilon_{soc}}{step_{soc}} \right)$
15.         else
16.             $gr_{soc} = dg$
17.         end
18.     end
19.     $gr_{env} = gr_{env} + 1$
20.     $\epsilon_{env} = \epsilon_{env} - step_{env}$
21. end
22. $\mathcal{P} := \text{Remove dominated solutions from } \mathcal{P}$
23. return $\mathcal{P}$

In line 20, the dominated solutions are deleted from $\mathcal{P}$ and the updated Pareto frontier approximation $\mathcal{P}$ is returned.

In addition to introducing AugMathLagr, we have also adapted the multi-objective matheuristic AugMathFix \cite{40} to find solutions to the target SSC management problem. Next section briefly explains this adaptation.

4.3. Adaptation of AugMathFix

To better evaluate the proposed matheuristic, AugMathLagr, we compare its performance with an AUGMECON2-based matheuristic recently proposed to approach an SSC problem, the AugMathFix. As the SSC problem is different from the one studied in this paper, we had to adapt such matheuristic, the AugMathLagr, to the target SSC management problem. The adapted AugMathFix selects the same constraints to be relaxed as AugMathLagr. To find feasible solutions to relaxed problems, in addition to fixing the binary variables at value 1, as proposed by Tautenhain et al. \cite{40}, we introduce a procedure to estimate feasible lower bounds for the integer variables related to the transportation through entities using trucks.

Details about the implementation of AugMathFix to the supply chain studied in this paper are presented in Supplementary Material. The mono-objective heuristic in AugMathFix is called MathFix, and is also adapted in this paper. It is described in the Supplementary Material.

Next section describes the instance generator introduced in this paper.
5. Instance Generator

This section describes at length the introduced instance generator. The optimization models found in the literature for sustainable supply chains are usually problem-specific [2]. As a consequence, defining a generic instance generator is not an easy task since it should incorporate several traits from this diverse range of characteristics. In this context, this paper introduces a methodology to generate random instances to target the SSC formulation proposed in [35].

In order to define the ranges of the parameters, the data distribution, and so forth, we relied on real data, in particular, from two case studies [34, 35]. One of the case studies [35] is discussed in Section 6.2. The generator estimates the maximum capacities according to the demand of the SSC customers. The minimum capacities are always estimated as a fraction of the maximum capacities.

The following sections present the guidelines for estimating the data parameters of the uniform random variables.

5.1. Demands and specification of items

Table 2 reports the parameters related to demand, bills and items. The demand of the first period of the planning horizon is defined as a random variable drawn from a uniform distribution in the interval \([lbdc, ubdc]\), where \(lbdc\) and \(ubdc\) are scalars such that \(lbdc < ubdc\). In each subsequent period, the demand of a customer \(i\) for a given final product \(m\) increases by 100\(\text{vart}\)%, where \(\text{vart} \in [0, 1]\). The supplementary material presents suggestions to define all the input values required by the instance generator.

Let \(lbBOM^{prod}, ubBOM^{prod}, lbBOM^{rem}\) and \(ubBOM^{rem}\) be scalars such that \(lbBOM^{prod} < ubBOM^{prod}\) and \(lbBOM^{rem} < ubBOM^{rem}\). The bill of raw materials \(m \in M_{rm}\) required to manufacture product \(n \in M_{fp}\) using technology \(g \in G_{prod}\) is chosen with constant probability in the interval \([lbBOM^{prod}, ubBOM^{prod}]\). The bill of recovery products \(m \in M_{rp}\) to remanufacture a final product \(n \in M_{fp}\) follows a uniform distribution which falls within the interval \([lbBOM^{rem}, ubBOM^{rem}]\). The weight \((pw_m)\) and necessary area per unit \((apu_m)\) of raw material \(m\) are defined by random variables \(Xpw_{rm}\) and \(Xapur_{rm}\) drawn from uniform distributions in the intervals \([lbpw, ubpw]\) and \([lbapu, ubapu]\), respectively. The values of \(pw_m\) and \(apu_m\) of a final product \(m\) are the average values of, respectively, \(pw_n\) and \(apu_n\) of raw materials \(n\) required to manufacture \(m\). The remanufacturing bill must require several recovery products to refurbish a new final product. Therefore, the values of \(pw_m\) and \(apu_m\) for a recovered product \(m\) are the sums of, respectively, \(pw_n\) and \(apu_n\) of final products \(n\) in the bill to remanufacture \(m\).

The return rate depends on the type of industry and is a value randomly chosen between 0 and an input upper bound \(ubret\). In the case study of the electrical industry, for example, the return rate is approximately 15%. The number of workers required to manage a technology \(g\) is defined as the operating cost of \(g\) \((opc_g)\) multiplied by a scalar \(fracwg\).

5.2. Maximum and minimum capacities of entities

In an attempt to avoid an empty solution space when generating artificial instances and when estimating the installation area of entities, customer demands are taken into account. In line with this, Table 3 presents the parameters related to the capacity of the entities.

The higher the demands, the larger the inventory of products, production, acquisition of raw materials, and return of used products in the reverse flow must be. The storage capacity for a product \(m\) in a given
entity \( i \) (\( ic_{mi}^{\text{max}} \)) can be estimated as a fraction \( icfrac_{g}^{\text{max}} \) of the maximum total demand for the product \( (\sum_{j \in I_c} dmd_{mj}) \) for each period \( t \in T \) in the planning horizon. A random variable with uniform distribution which lies between 0 and \( \sum_{j \in I_c} dmd_{mj} \) is summed to the storage capacity estimation of each period. The minimum stock level of a product \( m \) (\( ic_{mi}^{\text{min}} \)) is defined as a fraction \( icfrac_{g}^{\text{min}} \) of the maximum inventory capacity.

The maximum technology capacities are estimated by the amount of final products that must be produced in the factories to meet the customer demands. Let us suppose the production is evenly distributed among the factories. In this case, the capacity of a technology \( g \) to produce final products \( m \) (\( pc_{g}^{\text{max}} \)) can be estimated as \( [\sum_{j \in I_c} dmd_{mj}] / |I_p| \). Since the production shall not necessarily be evenly distributed among the factories, we subtract 10\% from this estimation for each factory and period in the planning horizon and multiply the resulting value by a random variable with a uniform distribution between 1 and 2. The capacity of a technology \( g \) is the maximum estimation over all final products in the planning horizon. The minimum use required by a technology \( g \) (\( pc_{g}^{\text{min}} \)) is a fraction \( lbpc_{g}^{\text{min}} \) of the maximum technology capacity. The minimum and maximum installation area of factories and warehouses are selected randomly from uniform distributions in pre-defined intervals.

Each supplier is allowed to provide \( \frac{2}{|I_p|} \) of the total amount of raw material necessary to produce the maximum quantity of final products due to the technology capacities. The minimum order of a raw material \( m \in M_{rm} \) from a supplier \( i \in I_p \) (\( sc_{mi}^{\text{min}} \)) is drawn from a uniform distribution whose range depends on scalars...
The distances between the entities have uniform distribution that falls in the interval $[lbdist, ubdist]$, where $lbdist$ and $ubdist$ are scalars such that $lbdist < ubdist$.

Table 3: Instance generator description: maximum and minimum capacities of entities and technologies.

| Parameter | Value | Variables |
|-----------|-------|-----------|
| Storage capacity of a product $m \in M_{fp}$ in entity $i \in I_f \cup I_w$ | $ic_{mi}^{max} = \max_{t \in T} \left( icfrac_{mi}^{max} \left[ \sum_{j \in I_c} dmd_{mjt} \right] + \frac{1}{\lambda_{icmax}} \left[ \sum_{j \in I_c} dmd_{mjt} \right] \right)$ | $icfrac_{mi}^{max} \in \mathbb{R}_{\geq 0}$ |
| Minimum stock level of a product $m \in M_{fp}$ in entity $i \in I_f \cup I_w$ | $ic_{mi}^{min} = icfrac_{mi}^{min} ic_{mi}^{max}$ | $icfrac_{mi}^{min} \in \mathbb{R}_{\geq 0}$ |
| Maximum capacity of a technology $g \in G$ | $pc_{g}^{max} = \max_{m \in M_{fp}, t \in T} \left[ \left( \sim U[1, 2] \right) \left[ \left( \sum_{j \in I_c} dmd_{mjt} \right) - 0.1 \left( \sum_{j \in I_c} dmd_{mjt} \right) \right] \right]$ | $icfrac_{mi}^{max} \in \mathbb{R}_{\geq 0}$ |
| Minimum use required by a technology $g \in G$ | $pc_{g}^{min} = lbpc_{g}^{min} pc_{g}^{max}$ | $lbpc_{g}^{min} \in \mathbb{R}_{\geq 0}$ |
| Maximum installation area of entity $i \in I_f \cup I_w$ (m$^2$) | $ea_{i}^{max} \sim \left\{ \begin{array}{ll} U[lbaf_{i}^{max}, ubaf_{i}^{max}], & \text{if } i \in I_f \\ U[lbeaw_{i}^{max}, ubeaw_{i}^{max}], & \text{if } i \in I_w \end{array} \right.$ | $lbaf_{i}^{max}, ubaf_{i}^{max}, lbeaw_{i}^{max}, ubeaw_{i}^{max} \in \mathbb{R}_{\geq 0}$ |
| Minimum installation area of entity $i \in I_f \cup I_w$ (m$^2$) | $ea_{i}^{min} \sim \left\{ \begin{array}{ll} U[lbaf_{i}^{min}, ubaf_{i}^{min}], & \text{if } i \in I_f \\ U[lbeaw_{i}^{min}, ubeaw_{i}^{min}], & \text{if } i \in I_w \end{array} \right.$ | $lbaf_{i}^{min}, ubaf_{i}^{min}, lbeaw_{i}^{min}, ubeaw_{i}^{min} \in \mathbb{R}_{\geq 0}$ |
| Maximum amount of a raw material $m \in M_{rm}$ that can be supplied by the supplier $i \in I_p$ | $sc_{mi}^{max} = 2 \left\{ \sum_{(s,a) \in H_{pcr}} \frac{pc_{g}^{min} BOM_{m_{rm}}^{min}}{|I_p|} \right\}$ | |
| Minimum order of a raw material $m \in M_{rm}$ from a supplier $i \in I_p$ | $sc_{mi}^{min} \sim U[lbsc_{mi}^{max}, ubsc_{mi}^{max}]$ | $lbsc, ubsc \in \mathbb{R}_{\geq 0}$ |
| Distance between two entities $i$ and $j$ (km) | $dist_{ij} \sim U[lbdist, ubdist]$ | $lbdist, ubdist \in \mathbb{R}_{\geq 0}$ |

5.3. Parameters related to the objective functions.

In Table 4, one may observe the possible values of the parameters related to the costs associated with the production, operation and inventory of items. The cost of installing a technology $g \ (tec_{g})$ is randomly defined with constant probability in an interval that depends on scalar multipliers $lbtech$ and $ubtech$ and on the maximum technology capacity, $pc_{g}^{max}$. The inventory price of a final product $m$ is defined by the weight of $m$ plus a given scalar $scfrac$. The cost to recover a product $m$ considers the bill ($BOM_{m_{fm}}^{rem}$), the price of the resulting final products ($psu_{m}$) and a random variable drawn from a uniform distribution ranging between two scalars $lbpc$ and $ubpc$. Since the cost to recover a product must be substantially smaller than its selling cost, we suggest $ubpc$ to be 0.5 at most. The average fuel consumption is randomly selected between 14 and 18 liters per 100 km. This interval was chosen in accordance with the case studied by Mota et al. [35].

Table 5 shows the parameters related to social and labor aspects of the supply chain. The construction cost of an entity $i$, $sqmc_{i}$, is randomly drawn from a uniform distribution whose ranges depend on the average labour cost of the area where the entity is located ($lc_{i}$) and the maximum installation area of the entity.
Table 4: Instance generator description: production, operation and inventory costs.

| Parameter                                      | Value                                                                 |
|------------------------------------------------|-----------------------------------------------------------------------|
| Cost of installing a technology $g \in G$ (€) | $t_{ecg} \sim U[\text{lbtec} \cdot p_{g}^{\text{max}}, \text{ubtech} \cdot p_{g}^{\text{max}}]$ $\text{lbtec, ubtech} \in \mathbb{R}_{\geq 0}$ |
| The operating cost of a technology $g \in G$ (€) | $o_{pcg} \sim U[\text{lbopc}, \text{ubopc}]$ $\text{lbopc, ubopc} \in \mathbb{R}_{\geq 0}$ |
| Selling price of a final product $m \in M_{fp}$ (€) | $ps_{um} \sim U[\text{lbpsu}, \text{ubpsu}]$ $\text{lbpsu, ubpsu} \in \mathbb{R}_{\geq 0}$ |
| Inventory price of a final product $m \in M_{fp}$ (€) | $sc_{m} = \text{scfrac} + p_{wm}$ $\text{scfrac} \in \mathbb{R}_{\geq 0}$ |
| Cost of recovery product $m \in M_{rp}$ (€) | $r_{pc_{m}} = BOM_{rm} \cdot ps_{u} \cdot X_{rp} \sim [\text{lbpc} \cdot \text{ubpc}]$ $\text{lbpc, ubpc} \in \mathbb{R}_{\geq 0}$ |
| Cost of raw material $m \in M_{rm}$ (€) | $r_{mc_{m}} \sim U[\text{lbmc}, \text{ubmc}]$ $\text{lbmc, ubmc} \in \mathbb{R}_{\geq 0}$ |
| Average vehicle fuel consumption of $a \in A_{\text{truck}}$ (l per 100km) | $avc_{a} \sim U[14, 18]$ |

$(ec_{i}^{\text{max}})$. The lower bound for the distribution is half the value of $lc_{i}ec_{i}^{\text{max}}$ whereas the upper bound also depend on scalars $ubsqmc$ and $sqmcfac$. The average labor cost, GDP and unemployment indexes strongly depend on the area where the entity is located. In the case study used to base these parameters on, the authors set them according to the development of the country wherein the entity is installed.

The environmental impacts due to the installation of entities, production and remanufacturing of products, and transportation depend on the analysis of the life cycle assessment of the product. Table 6 presents guidelines for generating these parameters. The lower and upper bound values of the uniform distribution intervals from which we base the values on were proposed by Mota et al. [35].

In the next section, we illustrate a small case study created by the instance generator introduced in this paper.

5.4. Illustrative example

This section presents a small example to illustrate how the instance generator works. In this example, the SSC has one supplier, one factory, two warehouses, two customers, two airports and two seaports. Moreover, it produces only one final product using two types of raw materials. The example also considers three production and remanufacturing technologies. In our example, we refer to the production technologies as $g_0, g_1$ and $g_2$ and to the remanufacturing technologies as $g_3, g_4$ and $g_5$. The transportation modes are trucks, airplanes and boats; and there are two types of trucks to perform land transportation, each of them specified by $k_0$ and $k_1$.

Table 7 shows the results (time to solutions and upper bounds) achieved by the Lagrangian matheuristic whose pseudocode is in Algorithm 1. MathLagr when optimizing each objective function of the MOP individually. CPLEX [28] was the tool that solved the Lagrangian relaxed problems with a stopping criterion of 1% of optimality gap. The first column of Table 7 identifies the optimized function. The third, fourth and fifth columns report, respectively, the values of the economic, environmental and social functions of the
Table 5: Instance generator description: social and labor aspects.

| Parameter                                           | Value                                                                 |
|-----------------------------------------------------|------------------------------------------------------------------------|
| Construction costs of each entity \( i \in I_f \cup I_w \) (€) | \( sqmc_i \sim U(0.5c_i, cc_i^{max}), (ubsqmc + sqmcfac \cdot lc_i, ec_i^{max}) \) | \( u_bsqmc, sqmcfac \in \mathbb{R}_{\geq 0} \) |
| Labor cost of an entity \( i \in I \) (€)                   | \( lc_i \sim U[3.5, 30.4] \)                                           |
| GDP index of an entity \( i \in I \)                      | \( u_{gdp} \sim U[0.355, 1.24] \)                                      |
| Unemployment index of an entity \( i \in I \)             | \( pUnempInd \sim U[4.8, 24.5] \)                                      |
| Minimum number of workers in entity \( i \in I_f \cup I_w \) | \( w_i \) = \{ \begin{align*} & Xwf \sim [lbwf, ubwf], & \text{if } i \in I_f \nonumber \quad \text{lbwf, ubwf}, \\ & Xww \sim [lbww, ubww], & \text{if } i \in I_w \nonumber \quad \text{lbww, ubww} \in \mathbb{R}_{\geq 0} \end{align*} \quad \text{if } i \in I_w \nonumber \quad \text{lbww, ubww} \in \mathbb{R}_{\geq 0} \nonumber \quad \text{lbww, ubww} \in \mathbb{R}_{\geq 0} \) |
| Minimum number of workers per square meter in entity \( i \in I_f \cup I_w \) | \( wpsqi = \{ \begin{align*} & Xwpsqf \sim [lbwpsqf, ubwpsqf], & \text{if } i \in I_f \nonumber \quad \text{lbwpsqf, ubwpsqf}, \\ & Xwpsqw \sim [lbwpsqw, ubwpsqw], & \text{if } i \in I_w \nonumber \quad \text{lbwpsqw, ubwpsqw} \in \mathbb{R}_{\geq 0} \end{align*} \quad \text{lbwpsqw, ubwpsqw} \in \mathbb{R}_{\geq 0} \) |

Table 6: Instance generator description: environmental impacts.

| Parameter                                           | Value                                                                 |
|-----------------------------------------------------|------------------------------------------------------------------------|
| Entity installation environmental impact according to category \( c \in C \) (per \( m^2 \)) | \( et_c \sim U[0, 0.83200] \);                                         |
| Production environmental impact of manufacturing \( m \in M_{fp} \) using technology \( g \in G \) according to category \( c \in C \) (per product) | \( et_{mgc} \sim U[0.0000049, 457000] \);                              |
| Impact of the transportation mode \( a \in A \) according to the category \( c \in C \) (per \( kg \)) | \( et_{ac} \sim U[0, 0.00314] \)                                      |

heuristic solution to the mono-objective problem whose objective function is that indicated in the first column. Observe that \( f'_{eco} \) and \( f'_{soc} \) are the original economic and social functions to be maximized and \( f'_{env} \) is the environmental function to be minimized.

Table 7: Results for optimizing each objective function individually.

| Function to be optimized | Time(s) | Values of the objective functions |
|--------------------------|---------|-----------------------------------|
|                          |         | \( f'_{eco} \)                   | \( f'_{env} \) | \( f'_{soc} \) |
| max \( f'_{eco} \)       | 0.065   | 3048630.231                      | 65121273406.186 | 40.851 |
| min \( f'_{env} \)       | 0.182   | -18094916.800                    | 65120597510.939 | 533.959 |
| max \( f'_{soc} \)       | 0.194   | -96170518.625                    | 113078751769.982 | 621.963 |

Figures 2 to 4 display the decision variable values regarding the last time-period obtained by MathLagr when optimizing the economic, environmental and social functions, respectively. These figures illustrate the flow of items indicated by the values of the variables of production, remanufacturing and storage on the arcs of the network. Labels \( S, F, W_1, W_2, C_1, C_2, Air_1, Air_2, Sea_1 \) and \( Sea_2 \) represent, respectively, the supplier, the factory, the first warehouse, the second warehouse, the first customer, the second customer, the first airport, the second airport, the first seaport and the second seaport.
Moreover, to identify these entities in the indices of the variables, we assign integer numbers to them. Entity \( S \) corresponds to index 0, \( F \) to 1, and so on until \( \text{Sea}_2 \), which is identified by index 9 in the variables. For the same reason, technologies \( g_0, g_1, g_2, g_3, g_4 \) and \( g_5 \) are assigned to sequential indices 0 to 5 and trucks \( k_0 \) and \( k_1 \) to indices 0 and 1. The labels on the arcs correspond to the decision variables, according to Table 1.

In this example, warehouse \( W_2 \), customer \( C_2 \), airport \( \text{Air}_2 \) and \( \text{Port}_2 \) are located in a continent different from the remaining entities. Thereby to transport items between entities, in this case, air or sea transportation is mandatory.

![Figure 2: Illustrative example of the solution obtained by the optimization of the economic criterion of an SSC generated by the proposed instance generator.](image)

![Figure 3: Illustrative example of the solution obtained by the optimization of the environmental criterion of an SSC generated by the proposed instance generator.](image)

No warehouse is opened in the solution that optimizes the economic function, i.e., maximizes the NPV. In the solution that minimizes the environmental function, environmental impacts are lower in comparison to the economic optimization solution, since the installation of the warehouses is also avoided. Since the decisions to open warehouses contribute positively to the social objective function, the solution to the maximization of this indicator indicates the opening of the two warehouses at maximum capacity.

Truck \( k_1 \) is selected for land transportation in all the solutions. In particular, overall, truck \( k_1 \) has a lower price and environmental impact than truck \( k_0 \). In the environmental optimization solution, apart from land transportation, only airports are selected due to the lower environmental impact they cause. In the economic optimization solution, seaports are responsible for delivering products to customer \( C_2 \) whereas airports are employed to return used products to the factories. In the solution that optimizes the social function, in contrast, the stock in warehouse \( W_2 \) at the end of the second time period is used to meet the demand by customer \( C_2 \) on the third and last time period.
The three solutions employ the manufacturing and remanufacturing technologies $g_0$ and $g_4$, respectively. The remanufacturing levels at factories are equal in both the economic and environmental solutions, suggesting that the reverse logistics can improve both functions.

Next section presents the computational experiments carried out with a real case study and artificially generated instances.

6. Computational Experiments

In this section we shall discuss two experiments carried out using a case study and artificial instances drawn from the instance generator presented in this paper. The results obtained by the proposed matheuristic, AugMathLagr, are contrasted with two multi-objective methods found in the literature: AUGMECON2 and an adaptation of AugMathFix. Note that we refer to the heuristics employed in AugMathLagr and in AugMathFix to solve mono-objective problems by, respectively, MathLagr and MathFix. A maximum time limit of 7200s was imposed for CPLEX to solve each mono-objective problem, MOP. The first experiment concerns the case study presented by Mota et al. [35]. The second presents an analysis of the computational results achieved by the methods for a set of benchmark instances.

The number of grid points were set at 10 for all the methods. The AugMathLagr parameters were: $k_{Max} = 10$; $\lambda^{(0)} = 0$; and $st = 1 \times 10^{-6}$, $st = 1 \times 10^{-10}$ and $st = 1 \times 10^{-2}$ when optimizing the economic, environmental and social functions, respectively. All experiments were performed on a computer with an Intel Xeon E5-2680v2 2.8 GHz processor and 128 GB of main memory. CPLEX was limited to use only 15GB of memory in all the experiments. Moreover, CPLEX is limited to use 8 threads in both experiments.

Before detailing the results of the experiments, we shall describe the measures used to evaluate the multi-objective optimization methods.

6.1. Measures of Assessment

GAP, a measure of proximity, was used to evaluate the quality of solutions with respect to their optimal value. GAP quantifies how close a solution $S'$ is from another $S^*$, considering an objective function $f_i, i \in \{eco, env, soc\}$, as indicated in Equation (38).

$$GAP(S', S^*) = \frac{|f_i(S') - f_i(S^*)|}{\max(f_i(S^*), f_i(S'))}$$  (38)
The stopping criterion for CPLEX was when it reached a solution whose GAP between the upper and lower bounds was less than or equal to 1%. The remaining parameters were set as the default of the solver.

To better assess the results achieved by MathLagr, similar to Tautenhain et al. [40], we used a set of multi-objective metrics to evaluate the quality of the Pareto frontiers obtained by the methods. They were: the R2 Indicator and two variations of the Mean Ideal Distance (MID) and Spread of Non-Dominated Solutions (SNS).

The R2 Indicator [24] checks the quality of a Pareto frontier approximation, $P^A$, by comparing it with a representative set of Pareto frontier, $P^Z$, as shown by Equation (39). In the experiments reported in this paper, $P^Z$ was estimated by AUGMECON2.

$$I_{R2}(P^A, U) = \frac{1}{|U|} \sum_{\mu \in U} \max_{S \in P^Z} \{\mu(S)\} - \sum_{\mu \in U} \max_{a \in P^A} \{\mu(a)\}$$

(39)

where $U$ is the set of utility functions. A utility function $\mu \in U$, $\mu : \mathbb{R}^3 \to \mathbb{R}$, maps a solution of the multi-objective problem to a scalar value. Lower values of $I_{R2}(P^A, U)$ indicate better approximations. Negative values of the R2 Indicator express that solutions from $P^A$ are closer to the Ideal point than solutions from $P^Z$. Let the Ideal point be defined by $f_I = \{f^*_\text{eco}, f^*_\text{env}, f^*_\text{soc}\}$, such that $f^*_i$ is the optimal value of the problem that optimizes objective function $f_i, i \in \{\text{eco, env, soc}\}$.

Hansen and Jaszkiewicz [24] suggest using the Weighted Sum and weighted Tchebycheff utility functions. The Weighted Sum function only takes into account points inside the convex hull of a feasible region and therefore is unsuitable for cases where the solution space is not convex. Thereby, Brockhoff et al. [6] suggest using the weighted Tchebycheff function, formulated as presented in Equation (40).

$$u_\gamma(S) = \max_{i \in \{\text{eco, env, soc}\}} \gamma_i |f^I - f_i(S)|$$

(40)

where $\gamma \in \Gamma, \Gamma = (\gamma_{\text{eco}}, \gamma_{\text{env}}, \gamma_{\text{soc}})$, is a weight vector.

For ease of notation, we refer to $I_{R2}(P^A)$ as $I_{R2}(P^A, U)$, with $U$ representing the Tchebycheff utility function.

The Mean Ideal Distance (MID) and Spread of Non-Dominated Solutions (SNS) [3] calculate, respectively, the mean and the standard deviation of the distance between the solutions of an approximation of the Pareto frontier, $P^A$, and the Ideal point. Due to the different scales of the objective function values, Tautenhain et al. [40] employed the adaptation of these measures to calculate the multidimensional GAP instead of the distance between the solutions. The multidimensional GAP is presented in Equation (41).

$$GAPM(f(S'), f^I) = \sqrt{\sum_{i \in \{\text{eco, env, soc}\}} (GAP(f_i(S'), f^*_i))^2}$$

(41)

Equations (42) and (43) present the adapted $aMID$ and $aSNS$ metrics for $P^A$.

$$aMID(P^A) = \sum_{S' \in P^A} \frac{\|GAPM(f(S'), f^I)\|}{|P^A|}$$

(42)
\[ aSNS(P^A) = \sqrt{\sum_{S' \in P^A} \frac{(aMID(P^A) - \|GAPM(f(S'), f_I)\|)^2}{|P^A| - 1}} \]  

(43)

Lower values of \(aMID(P^A)\) and \(aSNS(P^A)\) indicate better solutions in \(P^A\).

6.2. Experiment I: case study

In the first experiment, a case study of an electronics company is investigated (see the supplementary material or [35] for additional details on this case study). Table 8 only shows details regarding entities in this case study – location, \(GDP\) index, cost related to construction labor – in order to contrast such information with the results obtained by the methods.

Table 8: Details about the entities of the supply chain.

| Entity   | Label | Location         | Region    | \(\frac{1}{GDP}\) | Construction cost | Labor cost |
|----------|-------|------------------|-----------|---------------------|-------------------|------------|
| Supplier | \(S_1\) | Verona, Italy    | Europe    | 0.98                | -                 | 28.1       |
|          | \(S_2\) | Hannover, Germany| Europe    | 1.24                | -                 | 30.4       |
|          | \(S_3\) | Leeds, United Kingdom | Europe | 1.06                | -                 | 15.3       |
| Factories| \(F_1\) | Verona, Italy    | Europe    | 0.98                | 595               | 28.1       |
|          | \(F_2\) | Hannover, Germany| Europe    | 1.24                | 661               | 30.4       |
|          | \(F_3\) | Leeds, United Kingdom | Europe | 1.06                | 601               | 15.3       |
| Warehouses| \(W_1\) | Verona, Italy    | Europe    | 0.98                | 595               | 28.1       |
|          | \(W_2\) | Hannover, Germany| Europe    | 1.25                | 661               | 30.4       |
|          | \(W_3\) | Leeds, United Kingdom | Europe | 1.06                | 601               | 15.3       |
|          | \(W_4\) | Zaragoza, Spain  | Europe    | 0.95                | 373               | 21         |
|          | \(W_5\) | Lisbon, Portugal  | Europe    | 0.75                | 318               | 12.2       |
|          | \(W_6\) | São Paulo, Brazil | Brazil    | 0.355               | 538               | 8.98       |
|          | \(W_7\) | Recife, Brazil    | Brazil    | 0.355               | 538               | 8.98       |
|          | \(W_8\) | Budapest, Hungary | Europe    | 0.67                | 282               | 7.5        |
|          | \(W_9\) | Sofia, Bulgaria   | Europe    | 0.47                | 270               | 3.7        |
| Customers | \(C_1\) | Italy            | Europe    | 0.98                | -                 | 28.1       |
|          | \(C_2\) | Germany          | Europe    | 1.24                | -                 | 30.4       |
|          | \(C_3\) | United Kingdom   | Europe    | 1.06                | -                 | 15.3       |
|          | \(C_4\) | Spain            | Europe    | 0.95                | -                 | 21         |
|          | \(C_5\) | Portugal         | Europe    | 0.75                | -                 | 12.2       |
|          | \(C_6\) | São Paulo, Brazil| Brazil    | 0.355               | -                 | 8.98       |
|          | \(C_7\) | Recife, Brazil    | Brazil    | 0.355               | -                 | 8.98       |
| Airports | \(Air_1\) | Zaragoza         | Europe    | 1.19                | -                 | 21         |
|          | \(Air_2\) | Paris-Charles de Gaulle | Europe | 1.08                | -                 | 32.4       |
|          | \(Air_3\) | Kortrijk-Wevelgem | Europe    | 0.95                | -                 | 37.2       |
|          | \(Air_4\) | São Paulo         | Brazil    | 0.355               | -                 | 8.98       |
| Seaports | \(Sea_1\) | Hamburg, Germany | Europe    | 1.24                | -                 | 30.4       |
|          | \(Sea_2\) | Santos, Brazil    | Brazil    | 0.355               | -                 | 8.98       |

The first, second, third and fourth columns respectively indicate the entities, their labels, geographical locations and regions. The fifth, sixth and seventh columns respectively report the values of \(\frac{1}{GDP}\), where \(GDP\) stands for GDP index, and construction and labor costs of entities.

The supply chain entities are located in several countries in Europe and Brazil. The company has its factory \(F_1\) and warehouse \(W_1\) installed in Verona. Since the suppliers and factories are located in Europe, the company must use either air or sea transportation to carry the final products to Brazil.
The company produces only two types of final products from four different raw materials. The returned final products are modeled as recovered products. There are also four technologies available for factories to manufacture products from raw materials and two technologies for recovering used products into final products.

Table 9 depicts the results of the exact method employed in AUGMECON2, of MathLagr and of MathFix in the optimization of each objective function of the problem.

The third, fourth and fifth columns indicate the results obtained by optimizing the economic, environmental and social functions. For each method, the results reported are the computational running time to find the solution and the values of economic, \( f'_{eco} \), environmental, \( f'_{env} \), and social, \( f'_{soc} \), functions. Recall that we minimize \( f_{eco} = -f'_{eco} \) and \( f_{soc} = f'_{soc} \), which are equivalent to maximizing \( f'_eco \) and \( f'_soc \). To calculate the GAPs, we assume the solutions achieved by the exact method to be the optimal ones. Moreover, the rows identified by “Speed up” report the ratio of the exact method execution time and the matheuristics execution time. All the methods estimate the ideal and nadir points using, respectively, the best and worst values of each objective solution.

| Function to be optimized | Exact method | MathLagr | MathFix |
|--------------------------|--------------|----------|---------|
|                          | Time (s)     | Values of the objective functions | Time (s)     | Values of the objective functions | Time (s)     | Values of the objective functions |
|                          |              | \( f_{eco} \) | 106.235  | 1575453941.043 | 1562117002.338 |
|                          |              | \( f'_{eco} \) | 1575453941.043 | 1575453941.043 | 1562117002.338 |
|                          |              | \( f_{env} \) | 99395333.696 | 925092666.284 | 927404494.163 |
|                          |              | \( f'_{env} \) | 99395333.696 | 925092666.284 | 927404494.163 |
|                          |              | \( f_{soc} \) | 943.427   | 716.414      | 891.994     |
|                          |              | \( f'_{soc} \) | 943.427   | 716.414      | 891.994     |
|                          |              | GAP (%)     | 0.001732  | 0            | 0.001732    |
|                          |              | Speed up    | 2.746     | 2.746        | 2.746       |

MathLagr is approximately 2.746, 4.055 and 12.180 times faster than the exact method for optimizing the economic, environmental and social objective functions, respectively. Despite being slower than MathLagr, MathFix was still 1.676, 0.486 and 1.187 times faster than the exact method for optimizing the economic, environmental and social objective functions, respectively. We highlight that the adaptation of MathFix to set lower bound constraints related to transportation allowed this method to perform well to solve the target SSC management problem which, different from the model introduced by Tautenhain et al. [40], has a large number of integer variables in addition to the binary variables.

MathLagr and MathFix obtained solutions whose GAPs to the solutions obtained by the exact method were lower than 0.003384% and 0.004312%, respectively. Even though both MathLagr and MathFix output solutions are very close to those found by the exact method, MathLagr, in particular, was faster and obtained
a smaller GAP concerning the economic function optimization solution.

Table 10 presents the numbers of non-dominated solutions found by the three multi-objective methods AUGMECON2, AugMathLagr and AugMathFix as well as the aMID, aSNS, and R2 Indicator values. It also reports the total computational running times in seconds that the methods took to approximate the Pareto frontier for the case study.

Table 10: Results of the multi-objective metrics for the Pareto frontier approximations obtained by AUGMECON2, AugMathLagr and AugMathFix.

| Method         | Number | aMID   | aSNS   | R2   | Time (s)  |
|----------------|--------|--------|--------|------|-----------|
| AUGMECON2      | 46     | 7.973  | 1.994  | 0.000| 78921.849 |
| AugMathLagr    | 50     | 8.541  | 1.961  | 0.002058| 4442.895  |
| AugMathFix     | 81     | 6.468  | 1.135  | 0.003590| 16394.983 |

Table 10 shows that the solutions obtained by AugMathLagr and AugMathFix presented lower values of aMID and aSNS than AUGMECON2. In addition, the R2 indicator of the solutions found by AugMathLagr and AugMathFix were low. These results attest the good quality of the Pareto frontier approximations obtained by the methods when compared to AUGMECON2.

On the one hand, AugMathLagr is the fastest algorithm, taking 17.764 and 3.690 times less than the running time of AUGMECON2 and AugMathFix, respectively, to estimate the Pareto frontier. On the other, AugMathFix finds more non-dominated solutions than the other methods.

Figures 5(a), 5(b) and 5(c) exhibit the Pareto frontier approximations achieved by AUGMECON2, AugMathLagr and AugMathFix, respectively, in a three dimensional space.

![Figure 5: Pareto frontier approximation obtained by the methods for the case study.](image)

Figure 5 graphically demonstrates the good quality of the Pareto frontier approximations found by the methods, since it is possible to visualize that the non-dominated solutions are well distributed in the hyperplane. In particular, Figures 5(b) and 5(c) show that AugMathLagr did not find some solutions found by AugMathFix.

Table 11 shows which factories and warehouses are opened in the solutions found by MathLagr, MathFix and the exact method in AUGMECON2 to optimize the economic, environmental and social criteria. Additionally, in this table we point out the entities installed at maximum and minimum capacities. The marks A, L and M indicate, respectively, the exact method, MathLagr, and MathFix.
Table 11: Case study: location and installation capacities of factories and warehouses in the solutions obtained by the exact method, \textit{MathLagr} and \textit{MathFix}.

| Function to be optimized | max $f_{eco}$ | min $f_{env}$ | max $f_{soc}$ |
|--------------------------|---------------|---------------|---------------|
| Location | Maximum capacity | Minimum capacity | Location | Maximum capacity | Minimum capacity | Location | Maximum capacity | Minimum capacity |
| Factories |            |               |             |               |               |             |               |               |
| $F_1$ | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - |
| $F_2$ | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - |
| $F_3$ | $A; L; M$ | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - |
| Warehouses |            |               |             |               |               |             |               |               |
| $W_1$ | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - |
| $W_2$ | $M$ | - | $M$ | - | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - |
| $W_3$ | $M$ | - | $M$ | - | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - |
| $W_4$ | $M$ | - | $M$ | - | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - |
| $W_5$ | $M$ | - | $M$ | - | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - |
| $W_6$ | $M$ | - | $M$ | - | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - |
| $W_7$ | $M$ | - | $M$ | - | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - |
| $W_8$ | $M$ | - | $M$ | - | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - |
| $W_9$ | $M$ | - | $M$ | - | $A; L; M$ | - | $A; L; M$ | - | $A; L; M$ | - |

It is possible to note in Table 11 that there is no difference in the selection of entities to be opened at maximum or minimum capacity by \textit{MathLagr} and the exact method. In the solution obtained by \textit{MathFix}, on the other hand, warehouses $W_2$ to $W_9$ are opened at minimum capacity.

Except for factory $F_1$ and warehouse $W_1$, required by the case study to be opened at a fixed capacity in every solution, all the remaining entities are opened at maximum capacity in the solutions that optimize the environmental and social functions.

In the solutions obtained by \textit{MathLagr} and \textit{MathFix} to optimize the objective functions individually and sea transportation are employed by using the four airports and the two seaports. In the solution obtained by the exact method when maximizing the economic function, apart from the two seaports, only airports $Air_2$ and $Air_3$ operate.

6.3. Experiment II: artificial instances

In this experiment, we created artificial instances through the introduced instance generator. For this purpose, we considered instances with a predefined number of entities, items and technologies. The instances had 3 suppliers, 3 factories, 3 warehouses, 4 customers, 2 airports and 2 seaports. In particular, 1 warehouse, 1 customer, 1 airport and 1 seaport were located in a continent different from the remaining entities and, therefore, required air or sea transportation so that the flow of items among entities was possible. There were 2 raw materials available to produce a single final product. The number of production and remanufacturing technologies was 3, totalling 6 technologies. The supplementary material presents the values chosen for the parameters discussed in Section 5 to generate such instances.

The generated set consisted of 15 artificial instances, whose primary characteristics are summarized in Table 12. The first and second columns of this table express, respectively, the names and the number of periods in the planning horizon of the instances. For ease of identification, we included in the name of the instances information regarding the number of periods in the planning horizon and specific patterns for the parameters, as described in the following paragraph.

In the instances prefixed by "TECHC", the technology acquisition costs were the same for all technologies.
Table 12: Number of periods of each set of instances.

| Instances                  | Number of periods |
|----------------------------|-------------------|
| STD_T3, TECHC_T3, RAWC_T3, SUP_T3, CAP_T3 | 3                 |
| STD_T5, TECHC_T5, RAWC_T5, SUP_T5, CAP_T5 | 5                 |
| STD_T10, TECHC_T10, RAWC_T10, SUP_T10, CAP_T10 | 10                |

The manufacturing costs were the same for all the suppliers in the instances prefixed by "RAWC". In the instances prefixed by "SUP", no minimum order was imposed for the suppliers. There was no minimum use of technologies in the instances prefixed by "CAP". The corresponding remaining parameters of the instances prefixed by "STD", "TECHC", "RAWC", "SUP" and "CAP" were generated as described in Section 5.

Let $P_{AUG2}$, $P_{MathLagr}$ and $P_{MathFix}$ be, respectively, the Pareto frontier approximations obtained by AUGMECON2, AugMathLagr and AugMathFix. Figures 6, 7 and 8 present, respectively, the $aMID$ measure, the $aSNS$ measure and the R2 Indicator for $P_{AUG2}$, $P_{MathLagr}$ and $P_{MathFix}$. Figure 9 presents the total computational running times for obtaining $P_{AUG2}$, $P_{MathLagr}$ and $P_{MathFix}$.

As can be noted in Figure 6, the $aMID$ measures for the solutions of AugMathLagr and AugMathFix are lower than or approximately the same as the solutions achieved by AUGMECON2 for 11 and 15, respectively, out of the 15 instances. Although the $aMID$ measures of AugMathLagr are unexpectedly high for 1 instance, their overall results indicate that the solutions provided by AugMathLagr present low GAPs in comparison to the Ideal point for most of the instances.

The standard deviations of $aMID$, measured by the $aSNS$ measure, for solutions obtained by AugMathFix were consistently close or even lower than those that considered the solutions found by AUGMECON2. AugMathLagr, on the other hand, obtained considerably larger values for the $aSNS$ measure than those...
achieved by AUGMECON2 which was significantly high for two instances. AUGMECON2, in particular, presented unexpectedly large aSNS values for instance CAP_T10.

The R2 indicator value for the solutions obtained by AUGMECON2 is null, since we considered $P_{AUG2}$ as the set of representative solutions for the measure. The maximum distance from a solution obtained by AugMathLagr and AugMathFix to the Ideal point is close to the maximum distance from a solution found by AUGMECON2 to the Ideal point.

The total computational running times required by AUGMECON2 were at least 27.313 and 6.683 times higher than those required by AugMathLagr and AugMathFix. Concerning the running times, the advantages of AugMathLagr and AugMathFix over AUGMECON2 are even more evident when considering the largest instances with 10 time periods in the planning horizon. AugMathLagr, in particular, is from 1.556 to 69.209 times faster than AugMathFix on instances with 10 time periods in the planning horizon.

The results of this experiment attest the good quality of the Pareto frontier approximations obtained by AugMathLagr. Moreover, the adaptation of AugMathFix presented significantly inferior running times in comparison to AUGMECON2. AugMathLagr, in particular, is even faster than AugMathFix. This enables us to infer that the proposed Lagrangian matheuristic is suitable for solving large instances.

7. Final remarks

SSC management has recently drawn the attention of industrial and academic sectors to new research developments. Most studies approach SSC management by multi-objective optimization, focusing on the construction of optimization models based on case studies. This approach, however, is commonly very time-consuming and, even though it belongs to the strategical and tactical levels of planning, to refine, validate and thoroughly study the supply chain, it is necessary to employ efficient solution methods [15]. Perhaps
due to the difficulty in solving these problems and because the studies are usually problem-oriented, little effort has been devoted to keeping a benchmark data repository. This paper introduces an artificial instance generator and an efficient Lagrangian matheuristic, here called AugMathLagr, for SSC management multi-objective problems. In particular, AugMathLagr is implemented to the optimization model introduced by Mota et al. [35]. In addition to introducing AugMathLagr, this paper also efficiently adapts the matheuristic AugMathFix proposed by Tautenhain et al. [40] to the target problem by including a procedure to estimate lower bounds to integer variables.

Experiments conducted with a case study found in the literature and with a test bed of artificial instances indicate that in comparison with a classical exact method for multi-objective problems, known as AUGMECON2, AugMathLagr and AugMathFix are significantly faster. In particular, AugMathLagr is faster than AugMathFix and performs even better when approximating the Pareto frontier in cases where larger instances are considered. We assessed the Pareto frontier approximations using three multi-objective measures to evaluate how close the solutions from the Pareto frontier approximation were from the ideal point. These results indicated that the solutions of the Pareto frontier approximation achieved by AugMathLagr and AugMathFix are closer to the ideal point, in comparison with the solutions from AUGMECON2. In particular, the values of the R2 indicator expressed the high quality of the solutions found by AugMathLagr and AugMathFix.

This paper also contributes with a framework for generating artificial instances. Artificial instances are particularly useful for assessing the performance of optimization models since they allow statistical inferences on a wide variety of numerical scenarios. Due to the specificity of each optimization model, the implementation of the instance generator to other SSC management optimization models is a suggestion for future work.

Another direction for future work regards the proposed matheuristic. The primary goal of such study would be to further improve the quality of the solutions in order to obtain even closer values to the optimum ones achieved by AUGMECON2 using local search procedures.

Acknowledgements

Authors Camila P. S. Tautenhain and Mariá C. V. Nascimento would like to acknowledge the funding granted by São Paulo Research Foundation (FAPESP), grant numbers: 14/27334-9, 15/21660-4 and 16/02203-4; Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), grant numbers: 306036/2018-5; and Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brazil (CAPES) – Finance Code 001. The authors Ana Paula Barbosa-Póvoa and Bruna Mota would like to acknowledge the financial support from FCT and Portugal 2020 FCT under the project PTDC/EGEOGE/28071/2017, Lisboa -01.0145-Feder-28071. The author Mariá C.V. Nascimento is also grateful to Leonardo V. Rosset for giving her a hand. Research carried out using the computational resources of the Center for Mathematical Sciences Applied to Industry (CeMEAI) funded by FAPESP (grant 2013/07375-0).

References

[1] Ahi, P., Searcy, C., 2015. An analysis of metrics used to measure performance in green and sustainable supply chains. Journal of Cleaner Production 86, 360 – 377.

[2] Barbosa-Póvoa, A. P., da Silva, C., Carvalho, A., 2018. Opportunities and challenges in sustainable
supply chain: An operations research perspective. European Journal of Operational Research 268 (2), 399 – 431.

[3] Behnamian, J., Ghomi, S. F., Zandieh, M., 2009. A multi-phase covering pareto-optimal front method to multi-objective scheduling in a realistic hybrid flowshop using a hybrid metaheuristic. Expert Systems with Applications 36 (8), 11057–11069.

[4] Boukherroub, T., Ruiz, A., Guinet, A., Fondrèvelle, J., 2015. An integrated approach for sustainable supply chain planning. Computers & Operations Research 54, 180–194.

[5] Brezina, C., 2011. Understanding the gross domestic product and the gross national product. The Rosen Publishing Group.

[6] Brockhoff, D., Wagner, T., Trautmann, H., 2012. On the properties of the r2 indicator. In: Proceedings of the 14th Annual Conference on Genetic and Evolutionary Computation. ACM, pp. 465–472.

[7] Cambero, C., Sowlati, T., 2016. Incorporating social benefits in multi-objective optimization of forest-based bioenergy and biofuel supply chains. Applied Energy 178, 721–735.

[8] Carter, C. R., Rogers, D. S., 2008. A framework of sustainable supply chain management: moving toward new theory. International Journal of Physical Distribution & Logistics Management 38 (5), 360–387.

[9] Chibeles-Martins, N., Pinto-Varela, T., Barbosa-Póvoa, A. P., Novais, A. Q., 2016. A multi-objective meta-heuristic approach for the design and planning of green supply chains- MBSA. Expert Systems with Applications 47, 71–84.

[10] Copeland, T. E., Weston, J. F., Shastri, K., et al., 1983. Financial theory and corporate policy. Vol. 3. Addison-Wesley Massachusetts.

[11] Deb, K., Pratap, A., Agarwal, S., Meyarivan, T., 2002. A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Transactions on Evolutionary Computation 6 (2), 182–197.

[12] Dehghanian, F., Mansour, S., 2009. Designing sustainable recovery network of end-of-life products using genetic algorithm. Resources, Conservation and Recycling 53 (10), 559–570.

[13] Devika, K., Jafarian, A., Nourbakhsh, V., 2014. Designing a sustainable closed-loop supply chain network based on triple bottom line approach: A comparison of metaheuristics hybridization techniques. European Journal of Operational Research 235 (3), 594–615.

[14] Elhedhli, S., Merrick, R., 2012. Green supply chain network design to reduce carbon emissions. Transportation Research Part D: Transport and Environment 17 (5), 370–379.

[15] Eskandarpour, M., Dejaz, P., Miemczyk, J., Péton, O., 2015. Sustainable supply chain network design: An optimization-oriented review. Omega 54, 11–32.

[16] Eskandarpour, M., Zegordi, S. H., Nikbakhsh, E., 2013. A parallel variable neighborhood search for the multi-objective sustainable post-sales network design problem. International Journal of Production Economics 145 (1), 117–131.
[17] Eskigun, E., Uzsoy, R., Preckel, P. V., Beaujon, G., Krishnan, S., Tew, J. D., 2005. Outbound supply chain network design with mode selection, lead times and capacitated vehicle distribution centers. European Journal of Operational Research 165 (1), 182–206.

[18] Gao, J., You, F., 2017. Modeling framework and computational algorithm for hedging against uncertainty in sustainable supply chain design using functional-unit-based life cycle optimization. Computers & Chemical Engineering 107, 221–236.

[19] Goedkoop, M., Heijungs, R., Huijbregts, M., De Schryver, A., Struijs, J., van Zelm, R., 2009. Recipe 2008 - a life cycle impact assessment method which comprises harmonised category indicators at the midpoint and the endpoint level. A life cycle impact assessment method which comprises harmonised category indicators at the midpoint and the endpoint level 1.

[20] Goodkoop, M., 1999. The eco-indicator 99 a damage oriented method for life cycle impact assessment. Methodology Report.

[21] Govindan, K., Jafarian, A., Nourbakhsh, V., 2015. Bi-objective integrating sustainable order allocation and sustainable supply chain network strategic design with stochastic demand using a novel robust hybrid multi-objective metaheuristic. Computers & Operations Research 62, 112–130.

[22] Guillén-Gosálbez, G., Mele, F. D., Grossmann, I. E., 2010. A bi-criterion optimization approach for the design and planning of hydrogen supply chains for vehicle use. AIChE Journal 56 (3), 650–667.

[23] Haimes, Y. Y., 1971. On a bicriterion formulation of the problems of integrated system identification and system optimization. IEEE Transactions on Systems, Man, and Cybernetics 1 (3), 296–297.

[24] Hansen, M. P., Jaszkiewicz, A., 1998. Evaluating the quality of approximations to the non-dominated set. IMM, Department of Mathematical Modelling, Technical University of Denmark.

[25] Heidari-Fathian, H., Pasandideh, S. H. R., 2018. Green-blood supply chain network design: Robust optimization, bounded objective function & lagrangian relaxation. Computers & Industrial Engineering 122, 95 – 105.

[26] Held, M., Wolfe, P., Crowder, H. P., 1974. Validation of subgradient optimization. Mathematical Programming 6 (1), 62–88.

[27] Hugo, A., Pistikopoulos, E., 2005. Environmentally conscious long-range planning and design of supply chain networks. Journal of Cleaner Production 13 (15), 1471–1491.

[28] IBM, 2014. ILOG CPLEX Optimization Studio 12.6.1 - User’s Manual. IBM Corp.

[29] Jamshidi, R., Ghomi, S. F., Karimi, B., 2012. Multi-objective green supply chain optimization with a new hybrid memetic algorithm using the taguchi method. Scientia Iranica 19 (6), 1876–1886.

[30] Lidestam, H., Rönnqvist, M., 2011. Use of Lagrangian decomposition in supply chain planning. Mathematical and Computer Modelling 54 (9), 2428–2442.

[31] Marglin, S. A., 1967. Public investment criteria. Allen & Unwin London.
[32] Mavrotas, G., 2009. Effective implementation of the $\varepsilon$-constraint method in multi-objective mathematical programming problems. Applied Mathematics and Computation 213 (2), 455–465.

[33] Mavrotas, G., Florios, K., 2013. An improved version of the augmented $\varepsilon$-constraint method (AUGMECON2) for finding the exact pareto set in multi-objective integer programming problems. Applied Mathematics and Computation 219 (18), 9652–9669.

[34] Mota, B., Gomes, M. I., Carvalho, A., Barbosa-Povoa, A. P., 2015. Towards supply chain sustainability: economic, environmental and social design and planning. Journal of Cleaner Production 105, 14–27.

[35] Mota, B., Gomes, M. I., Carvalho, A., Barbosa-Povoa, A. P., 2018. Sustainable supply chains: An integrated modeling approach under uncertainty. Omega 77, 32 – 57.

[36] Neto, J. Q. F., Walther, G., Bloemhof, J., Van Nunen, J., Spengler, T., 2009. A methodology for assessing eco-efficiency in logistics networks. European Journal of Operational Research 193 (3), 670–682.

[37] Rafie-Majd, Z., Pasandideh, S. H. R., Naderi, B., 2018. Modelling and solving the integrated inventory-location-routing problem in a multi-period and multi-perishable product supply chain with uncertainty: Lagrangian relaxation algorithm. Computers & Chemical Engineering 109, 9–22.

[38] Seuring, S., 2013. A review of modeling approaches for sustainable supply chain management. Decision Support Systems 54 (4), 1513–1520.

[39] Soleimani, H., Govindan, K., Saghafi, H., Jafari, H., 2017. Fuzzy multi-objective sustainable and green closed-loop supply chain network design. Computers & Industrial Engineering 109, 191–203.

[40] Tautenhain, C. P., Barbosa-Póvoa, A. P., Nascimento, M. C. V., 2018. A multi-objective matheuristic for designing and planning sustainable supply chains. Computers & Industrial Engineering.

[41] Yousefi-Babadi, A., Tavakkoli-Moghaddam, R., Bozorgi-Amiri, A., Seifi, S., 2017. Designing a reliable multi-objective queuing model of a petrochemical supply chain network under uncertainty: a case study. Computers & Chemical Engineering 100, 177–197.

[42] Zhalechian, M., Tavakkoli-Moghaddam, R., Zahiri, B., Mohammadi, M., 2016. Sustainable design of a closed-loop location-routing-inventory supply chain network under mixed uncertainty. Transportation Research Part E: Logistics and Transportation Review 89, 182–214.

[43] Zhang, Z.-H., Li, B.-F., Qian, X., Cai, L.-N., 2014. An integrated supply chain network design problem for bidirectional flows. Expert Systems with Applications 41 (9), 4298–4308.
Supplementary material to the paper "An efficient Lagrangian-based heuristic to solve a multi-objective sustainable supply chain problem"

1 Adapted AugMathFix

This appendix shows the matheuristic AugMathFix originally introduced in [Tautenhain et al., 2018], and which we adapted in this paper to solve the model introduced in [Mota et al., 2018]. The pseudocode of the adapted method is presented in Algorithm 1.

First, let us present the problem with \( \epsilon \)-Constraints to be solved by the adapted AugMathFix – referred to as MOP in the paper.

\[
\begin{align*}
\text{(MOP)}: \min & \quad f_{\text{eco}}(u, v, w', w'') - \epsilon \text{ps}(\frac{l_{\text{env}}}{r_{\text{env}}} + 0.1 \frac{l_{\text{soc}}}{r_{\text{soc}}}) \\
\text{s.t.} & \quad A' u + A'' v \leq \beta' + \beta'' w'' \\
& \quad E' u + E'' v \leq d \\
& \quad f_{\text{env}}(u, v, w', w'') + l_{\text{env}} = \epsilon_{\text{env}} \\
& \quad f_{\text{soc}}(u, v, w', w'') + l_{\text{soc}} = \epsilon_{\text{soc}} \\
& \quad u \in \mathbb{R}_{\geq 0}, v \in \mathbb{Z}_{\geq 0}^{n'}, w' \in \{0, 1\}^{b'}, w'' \in \{0, 1\}^{b''} \\
& \quad l_{\text{env}}, l_{\text{soc}} \in \mathbb{R}_{\geq 0}
\end{align*}
\]

The adapted AugMathFix is an iterative AUGMECON2-based matheuristic that solves a sequence of mono-objective problems MOP by a heuristic method known as MathFix. These problems are generated by varying the values of \( \epsilon \) as in AugMathLagr. Therefore, the pseudocode of AugMathFix – Algorithm 1 – is the same as AugMathLagr except for line 10, which corresponds to the solution method to solve the MOP.

MathFix is a heuristic to solve mono-objective problems based on the fixing and re-solving strategy. Algorithm 2 shows a pseudocode of this heuristic.

The inputs of Algorithm 2 are the original MOP, referred to as \( P(0) \), the maximum number of iterations of the algorithm, \( IT \), and positive real scalars \( \alpha, \beta, \theta, \text{deca}, \text{dec}\beta \) and \( \text{dec}\theta \) such that \( \alpha + \beta + \theta = 1 \) and \( \text{deca} + \text{dec}\beta + \text{dec}\theta = 0 \). More details about these scalars can be found in the paper that first proposed the method. The values employed are the ones suggested in [Tautenhain et al., 2018]. Therefore, the following parameter values were employed in the experiments described in this paper: \( IT = 5, \alpha = 0.5, \beta = 0.5, \theta = 0, \text{deca} = -0.25, \text{dec}\beta = 0 \) and \( \text{dec}\theta = -0.25 \).

In the first line of Algorithm 2, a relax-and-fix heuristic in MathFix, called Heuristic, is used to solve the initial problem \( P(0) \). The pseudocode of the Heuristic is presented in Algorithm 3 and shall be discussed later. Alike the original MathFix, a control variable \( ch \) dictates the choice for defining the closing of entities. In this adaptation, we consider two alternatives: if the choice is 0, i.e., if value 0 is assigned to \( ch \) the option is to close a warehouse; if \( ch \) is 1, the option is to close a factory. Moreover,
Algorithm 1: Adapted AugMathFix.

Input: maximum number of iterations, kMax, step size st, number of grid points dg
Output: Pareto frontier approximation P

1. Estimate the lower bounds fL_i and the upper bounds fU_i, i ∈ {eco, env, soc} for the objective functions
2. Identify ranges r_j = fU_j − fL_j, j ∈ {env, soc} of the environmental and social objective functions, respectively
3. step_j = r_j / π, j ∈ {env, soc}
4. P := ∅
5. gr_{env} = 0
6. for ε_{env} = fU_{env} until gr_{env} < dg do
7.  gr_{soc} = 0
8.  for ε_{soc} = fU_{soc} until gr_{soc} < dg do
9.     MOP := Problem (1)-(7)
10.    x_f := solve MOP using Algorithm 2 – MathFix – considering ε_{env} and ε_{soc}
11.   if x_f is feasible then
12.      P := P ∪ x_f
13.      gr_{soc} = gr_{soc} + 1 + ⌊ε_{soc} / step_{soc}⌋
14.      ε_{soc} = ε_{soc} − step_{soc} (1 + ⌊ε_{soc} / step_{soc}⌋)
15.  else
16.      gr_{soc} = dg ;
17. end
18. gr_{env} = gr_{env} + 1
19. ε_{env} = ε_{env} − step_{env}
20. P := Remove dominated solutions from P
21. return P

In line 2, auxiliary variables of the algorithm related to this choice are initialized: num_o = 0 (number of times that choice o was applied in the heuristic) and max_o = IT (maximum number of times that choice o can be applied in the heuristic before restarting the problem), for each option o ∈ {0, 1}.

At each iteration of MathFix, veco_i, venv_i and vuse_i are defined in lines 6, 7 and 8 as, respectively, the economic cost, the environmental impact and the storage or production related to warehouse or factory i.

In line 7, wwh, wpt and wpsqi are, respectively, the number of weekly working hours, the number of weeks per time period and the minimum number of workers per square meter in entity i ∈ I_f ∪ I_w. In line 9, if ch = 1, I is defined as the warehouse i whose sum a_{veco} + β_{venv} + θ_{vuse} is the minimum amongst all i ∈ I_w. If ch = 1, in line 10, I is defined as the factory i whose value of a_{veco} + β_{venv} + θ_{vuse} is the minimum amongst all i ∈ I_f. In line 11, P^{(it)} is obtained by fixing Y_i = 0, YC_i = 0, KT_{ait} = 0, K_{ai} = 0 and S_{mit} = 0 and removing the constraints S_{mit} ≥ i_{min} Y_i, m ∈ M_f, a ∈ A, t ∈ T and YC_i ≥ e_{a}^{min} Y_i at P^{(it−1)}. Moreover, if ch = 1, i.e., i is a factory, P_{mgit} = 0, (m, g) ∈ H_{prod}, and R_{mgit} = 0, (m, i) ∈ H_{rem} and the constraints P_{mgit} ≥ p_{g}^{min} Z_{gm}, (m, g) ∈ H_{prod} and R_{mgit} ≥ p_{g}^{min} Z_{gm}, (m, g) ∈ H_{rem}, t ∈ T are also removed. In line 12, Heuristic is applied to P^{(it)} and its solution is called x^{FS}_{FS}. If solution x^{FS}_{FS} is feasible, num_{ch} is incremented in line 14. The best solution achieved up to the current iteration x^{best} is checked for update in line 15. In lines 16 and 17, ch is updated to either 1 or 0, depending on its current value. In case num_{ch} ≥ max_{ch}, the problem and feasible solution of the current iteration are reset to the respective problem and feasible solution of the first iteration. Moreover, max_{ch} is limited by num_{ch}.

In case x^{FS}_{FS} is infeasible, α, β and θ are updated in line 23. At the end, Algorithm 2 returns the best solution found over the iterations, x^{best}.

Algorithm 3 presents the heuristic to solve an input problem P^{(it)}.

In line 1 of Algorithm 3, the linear relaxation of problem P^{(it)} is assigned to PR^{(it)}. In line 2, the
Algorithm 2: MathFix.

Input: Problem $P(0)$, maximum number of iterations, $IT$ and positive scalars $\alpha$, $\beta$, $\theta$, $\text{deca}$, $\text{dec} \beta$ and $\text{dec} \theta$ such that $\alpha + \beta + \theta = 1$ and $\text{deca} + \text{dec} \beta + \text{dec} \theta = 0$

Output: The best feasible solution $x_{\text{best}}$

1. $x_{FS}^{(0)} := \text{Heuristic}(P(0))$
2. numch = 0, $\maxch = IT, o \in \{0, 1\}$
3. $x_{best} := x_{FS}^{(0)}$
4. it = 1, $\text{ch} = 0$
5. while $\text{it} < IT$ do
6.  $\text{vuse}_i = \sum_{t \in T} \sum_{m \in M_{fw} \cup M_{tp}} S_{mit}, \forall i \in I_f \cup I_w$
7.  $\text{veco}_i = \sum_{t \in T} \sum_{m \in M_{fw} \cup M_{tp}} w_{pt} w_{tps}, \forall i \in I_f \cup I_w$
8.  $\text{venv}_i = \sum_{t \in T} \sum_{m \in M_{fw} \cup M_{tp}} \text{wuh}, \forall i \in I_f \cup I_w$
9.  if $\text{ch} = 0$ then $\mathcal{I} := \arg\min_{\text{ch} \in I_w} (\alpha \text{vuse}_i + \beta \text{veco}_i + \theta \text{venv}_i)$
10. else if $\text{ch} = 1$ then $\mathcal{I} := \arg\min_{\text{ch} \in I_f} (\alpha \text{vuse}_i + \beta \text{veco}_i + \theta \text{venv}_i)$
11. $P^{(it)} := \text{Fix non-binary variables index by } \mathcal{I} \text{ at problem } P^{(it-1)}$
12. $x_{FS}^{(it)} := \text{Heuristic}(P^{(it)})$
13. if $x_{FS}^{(it)}$ is feasible then
14.  $\text{numch} = \text{numch} + 1$
15.  if $f_{\text{eco}}(x_{FS}^{(it)}) > f_{\text{eco}}(x_{\text{best}})$ then $x_{\text{best}} := x_{FS}^{(it)}$
16.  if $\text{ch} = 0$ then $\text{ch} = 1$
17.  else if $\text{ch} = 1$ then $\text{ch} = 0$
18.  if $\text{numch} \geq \maxch$ then
19.    $P^{(it)} := P^{(0)}$, $x_{FS}^{(it)} := x_{FS}^{(0)}$
20.    $\maxch = \text{numch}$, $\text{numch} = 0, o \in \{0, 1\}$
21.  end
22. else
23.    $\alpha = \alpha + \text{deca}, \beta = \beta + \text{dec} \beta, \theta = \theta + \text{dec} \theta$
24. end
25. it = it + 1
26. end
27. return $x_{\text{best}}$

Consider the transportation constraints given by Equations (8) - (10). Constraints (8) calculate the transportation constraints given by Equations (8) - (10). Constraints (8) calculate the maximum investment of $\text{invt}$ ($\text{€}$) in trucks.

$$KT_{ait} = \sum_{a \in A_{\text{truck}}} \sum_{i \in I} 2d_{ij} Q_{aijt}, \quad a \in A_{\text{truck}}, i \in I, t \in T$$

(8)

$$K_{ai} \geq KT_{ait}, \quad a \in A_{\text{truck}}, i \in I, t \in T$$

(9)

$$\sum_{a \in A_{\text{truck}}, i \in I} ftc_i K_{ai} \leq \text{invt}$$

(10)

From lines 12 to 23, the values of the variables $Q_{aijt}, a \in A_{\text{truck}}, i, j \in I, t \in T$ are inspected to set lower bounds to the related variables $K_{aijt}$ and $K_{ai}$ that satisfy Constraint (10). In line 17, $Q_{aijt}^{(it)}$ is the value of $Q_{aijt}$ in $x_{RL}^{(it)}$. Problem is solved using the CPLEX solver. Then, the problem given as input is assigned to $P_{FS}^{(it)}$ and in the next lines of the algorithm some variables of $P_{FS}^{(it)}$ are fixed according to the values of the variables of the solution of the linear relaxation. Accordingly, the variables $Y_i$, for all the values of $i$ that meet the conditions in lines 4 to 7, are fixed at 1 in problem $PR_{RS}^{(it)}$. Variables $Y_i$ and $Z_{gmi}$ that are not fixed at 0 are fixed at 1 in lines 8 and 9.
Algorithm 3: Heuristic.

Input : Problem $P^{(it)}$

Output: Feasible solution $x^{(it)}_{FS}$

1. $P^{(it)}_{RL} :=$ Linear relaxation of problem $P^{(it)}$
2. $x^{(it)}_{RL} :=$ Solve $P^{(it)}_{RL}$
3. $P^{(it)}_{FS} := P^{(it)}$
4. forall $i,j$ such that $\sum_{t \in T} \sum_{m \in M_{m}, o \in A} X_{maijt} > 0$ in $x^{(it)}_{RL}$ do Fix $Y_{i} = 1$ and $Y_{j} = 1$ at problem $P^{(it)}_{FS}$;
5. forall $i \in I_{w}$ such that $S_{m_{w}} > 0, m \in M_{fp}, t \in T$, in $x^{(it)}_{RL}$ do Fix $Y_{i} = 1$ at problem $P^{(it)}_{FS}$;
6. forall $i \in I_{f}, (m, g) \in H_{prod}, t \in T$ such that $P_{mg_{it}} > 0$, in $x^{(it)}_{RL}$ do Fix $Y_{i} = 1$ and $Z_{g_{mi}} = 1$ at problem $P^{(it)}_{FS}$;
7. forall $i \in I_{f}, (m, g) \in H_{rem}, t \in T$ such that $R_{mg_{it}} > 0$ in $x^{(it)}_{RL}$ do Fix $Y_{i} = 1$ and $Z_{g_{mi}} = 1$ at problem $P^{(it)}_{FS}$;
8. if $Y_{i}$ is not fixed at 1 then Fix $Y_{i} = 0$ at problem $P^{(it)}_{FS}$;
9. if $Z_{g_{mi}}$ is not fixed at 1 then Fix $Z_{g_{mi}} = 0$ at problem $P^{(it)}_{FS}$;
10. acc = 0
11. for $a \in A_{truck}, i \in I$
12. for $t \in T$
13. for $j \in I$
14. sum = 0
15. sum = sum + $\frac{2d_{j}Q_{aij}}{\text{avg}_{i,j}}$
16. if $(\text{acc} + \lfloor \text{sum} \rfloor) \leq \text{invt}$ then
17. Add the following constraint to problem $P^{(it)}_{FS}$: $Q_{aij} \geq Q_{aij}^{(it)}$
18. end
19. Add the following constraint to problem $P^{(it)}_{FS}$: $K_{ai} \geq \text{sum}$
20. end
21. max = maximum lower bound of $K_{ai}$ over $t \in T$
22. acc = acc + max
23. Add the following constraint to problem $P^{(it)}_{FS}$: $K_{ai} \geq \text{max}$
24. end
25. $x^{(it)}_{FS} :=$ Solve $P^{(it)}_{FS}$
26. return $x^{(it)}_{FS}$

A feasible solution $x^{(it)}_{FS}$ is obtained by solving problem $P^{(it)}_{FS}$ using CPLEX in line 25 and is returned in line 26.

2 Generator parameters values

This section presents the values of the generator used in the experiments reported here.

- $\text{lbdc} = 8336.8$, $\text{ubdc} = 34481.28$, and $\text{vart} = 0.1$;
- $\text{lbBOM}_{\text{prod}} = 0.015$, $\text{ubBOM}_{\text{prod}} = 0.45$, $\text{lbBOM}_{\text{rem}} = 4$ and $\text{ubBOM}_{\text{rem}} = 5$;
- $\text{lbpw} = 0.1$ and $\text{ubpw} = 0.9$;
- $\text{lbapu} = 0.001$ and $\text{ubapu} = 0.004$;
- $\text{wbret} = 0.15$ and $\text{fracwg} = 10$;
- $\text{icfrac}_{\text{max}} = 0.5$ and $\text{icfrac}_{\text{min}} = 0.01$;
- $\text{lbpc}_{\text{min}} = 0.0005$;
- \( lbeaf^{max} = 10000, ubeaf^{max} = 12500, lbeaw^{max} = 4000 \) and \( ubeaw^{max} = 4750; \)
- \( lbeaf^{min} = 100, ubeaf^{min} = 1000, lbeaw^{min} = 25 \) and \( ubeaw^{min} = 250; \)
- \( lbsc = 0.00001 \) and \( ubsc = 0.0001; \)
- \( lbtec = 0.025 \) and \( ubtech = 0.037; \)
- \( lbopc = 0.150 \) and \( ubopc = 0.4. \)
- \( lbpsu = 23 \) and \( ubpsu = 37; \)
- \( scfrac = 0.01; \)
- \( lbrpc = 0.005 \) and \( ubrpc = 0.5; \)
- \( lbrmc = 0.01 \) and \( ubrmc = 0.27; \)
- \( lbsqmc = 0.5, ubsqmc = 50 \) and \( sqmcfac = 1.25; \)
- \( lbwf = 5, ubwf = 6, lbww = 4 \) and \( ubww = 5; \)
- \( lbpsqf = 0.01, ubpsqf = 0.01, lbwpsqw = 0.03 \) and \( ubwpsqw = 0.03. \)

Moreover, the remaining parameters of the model studied by Mota et al. [2018] and their fixed values are given by:

- fuel price \( fp = 1.7 \) (€ per l);
- vehicle maintenance costs \( vmc = 0.3 \) (€);
- average vehicle speed \( avs = 60 \) (km/h);
- maximum driving hours per week \( mhw = 45; \)
- number of weeks per time period \( wpt = 17.33; \)
- number of weekly working hours \( wwh = 40; \)
- average vehicle consumption \( avc_a \in [14, 18], a \in A_{\text{truck}} \) (l per 100km);
- number of workers in the transportation \( w_a = 1, a \in A_{\text{truck}}; \)
- Maximum investment in trucks \( invt = 2000000 \) (€);
- interest rate \( ir = 0.1; \)
- savage value \( sv = 0.2; \)
- tax rate \( tx = 0.3. \)
3 Illustrative example data

Tables 1 to 7 show the data regarding the illustrative example presented in Section 5.4 of the paper.

Table 1 presents the parameters of each item. We refer to raw materials as $rm_0$ and $rm_1$, to the final product as $fp_0$ and to the recovered product as $rp_0$. The lifetime of the final product is one period.

Table 1: Illustrative example: data related to items.

| Data                     | $rm_0$ | $rm_1$ | $fp_0$ | $rp_0$ |
|--------------------------|--------|--------|--------|--------|
| Return rate              | -      | -      | -      | 0.0549277 |
| Recovery cost (€)        | -      | -      | -      | 9.55536   |
| Stock cost (€)           | -      | -      | 0.0149574 | 0      |
| Selling price (€)        | -      | -      | 34.995 | -      |
| Product weight (kg)      | 0.127  | 0.185  | 0.050  | 0.249  |
| Required area ($m^2$)    | 0.001  | 0.002  | 0.001  | 0.003  |

Table 2 reports the production and remanufacturing technologies. We refer to technologies as $G = G_{prod} \cup G_{rem}$, where $G_{prod} = \{g_0, g_1, g_2\}$ and $G_{rem} = \{g_3, g_4, g_5\}$.

Table 2: Illustrative example: technology specifications.

| Data                     | $g_0$ | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ |
|--------------------------|-------|-------|-------|-------|-------|-------|
| Maximum capacity         | 44676 | 45679 | 39978 | 38236 | 44339 | 46553 |
| Minimum capacity         | 0     | 0     | 0     | 0     | 0     | 0     |
| Installation Cost (€)    | 1247  | 1329  | 1439  | 1028  | 1146  | 1322  |
| Operation costs per unit (€) | 0.354 | 0.303 | 0.376 | 0.187 | 0.400 | 0.396 |

Table 3 defines the bill of raw materials required by a production technology to produce final products.

Table 3: Illustrative example: bill of raw materials for production.

| $fp_0$          |
|-----------------|
| $rm_0$ | $g_0$ | 0.123 |
|       | $g_1$ | 0.381 |
|       | $g_2$ | 0.105 |
| $rm_1$ |       | 0.391 |
|       | $g_3$ | 0.113 |
|       |       | 0.165 |

The amount of recovered products $rp_0$ required by any remanufacturing technology to obtain the final product $fp_0$ is 5.

Table 4 presents the maximum capacity, the minimum order and the unit cost of the raw materials for each supplier.

Table 4: Illustrative example: supplier related data.

| Data                     | $rm_0$ | $rm_1$ |
|--------------------------|--------|--------|
| Maximum supplier capacity| 108928 | 96840  |
| Minimum order            | 5      | 4      |
| Raw material cost (€)    | 0.156347 | 0.0258353 |

Table 5 presents the parameters related to the installation and operation of the factories and warehouses.
Table 5: Illustrative example: installation and operation of the factory and warehouses.

| Data                              | Factory | Warehouse 1 | Warehouse 2 |
|-----------------------------------|---------|-------------|-------------|
| Maximum installation area (m²)    | 2272    | 892         | 836         |
| Minimum installation area (m²)    | 54      | 30          | 43          |
| Construction cost (€)             | 6228    | 5836        | 5841        |
| Fixed number of workers           | 1       | 1           | 0           |
| Fixed number of workers per square meter | 0.002  | 0.001       | 0.002       |
| Maximum storage                   | -       | 39146       | 36893       |
| Minimum storage level             | -       | 391         | 368         |

Table 6 details the labour cost and GDP index of each entity.

Table 6: Illustrative example: labour cost and GDP index of each entity.

| Entity     | Labour cost | GDP index |
|------------|-------------|-----------|
| Supplier   | 30.245      | 0.572     |
| Factory    | 10.958      | 0.74      |
| Warehouse 1| 21.299      | 0.861     |
| Warehouse 2| 27.92       | 0.421     |
| Customer 1 | 27.277      | 1.188     |
| Customer 2 | 5.933       | 0.773     |
| Airport 1  | 13.639      | 0.915     |
| Airport 2  | 28.207      | 0.956     |
| Seaport 1  | 7.663       | 1.226     |
| Seaport 2  | 18.628      | 0.803     |

Table 7 describes the transportation specifications for each mode, which we refer to as $K = \{ k_0, k_1, k_2, k_3 \}$.

Table 7: Illustrative example: data related to transportation.

| Data                              | $k_0$   | $k_1$   | $k_2$   | $k_3$   |
|-----------------------------------|---------|---------|---------|---------|
| Maximum transportation capacity   | 52377   | 54504   | 400000  | 500000  |
| Minimum transportation            | 0       | 0       | 0       | 0       |
| Maximum contracted units          | -       | -       | 3000000 | 3000000 |
| Fixed transportation cost (€)     | 96627   | 87103   | 4000000 | 4000000 |
| Fixed number of workers           | 1       | 1       | 0       | 0       |
| Vehicle consumption (l per 100km) | 18      | 15      | -       | -       |
| Maximum investment on trucks (€)  | 13716710| 13716710| -       | -       |

The investments, as well as their savage values and depreciation rates; and the fixed parameter values are defined according to the case study presented in [Mota et al., 2018]. They are shown in Section 4 of this supplementary material.

Table 8 details the demand of the customers for final products in each period of the planning horizon.

Table 8: Illustrative example: demand for final products.

| Final Product | Customer    | Planning period $t \in T$ |
|---------------|-------------|----------------------------|
|               | 1           | 2         | 3           |
| $f_{p_0}$     | Customer 1  | 14895     | 16384       | 18023       |
|               | Customer 2  | 12528     | 13781       | 15159       |
Tables 9, 10 and 11 show the environmental impacts of installing entities by using, respectively, production and remanufacturing technologies and transportation modes.

Table 9: Illustrative example: environmental impact on entity installation, per $m^2$ of entity area.

| Impact category | Environmental impact per $m^2$ of entity area. |
|-----------------|-----------------------------------------------|
| CC              | 82612.041                                     |
| OD              | 21939.939                                     |
| TA              | 24797.047                                     |
| FE              | 81280.902                                     |
| ME              | 46016.221                                     |
| HT              | 76412.059                                     |
| POF             | 63497.697                                     |
| PMF             | 49721.387                                     |
| TET             | 37345.676                                     |
| FET             | 64471.149                                     |
| MET             | 65295.073                                     |
| IR              | 52890.036                                     |
| ALO             | 66265.249                                     |
| ULO             | 39267.008                                     |
| NLT             | 69005.33                                      |
| MRD             | 8356.2                                        |
| FRD             | 5347.654                                      |

Table 10: Illustrative example: environmental impact resulting from the use of technology per kilogram of product produced or remanufactured.

| Impact category | $g_0$  | $g_1$  | $g_2$  | $g_3$  | $g_4$  | $g_5$  |
|-----------------|--------|--------|--------|--------|--------|--------|
| CC              | 21020.507 | 230767.701 | 74750.833 | 27157.828 | 93404.670 | 379750.088 |
| OD              | 301007.67 | 345759.524 | 200319.033 | 350671.788 | 401081.367 | 117987.282 |
| TA              | 247888.673 | 430041.032 | 448535.444 | 127199.718 | 218901.199 | 330934.938 |
| FE              | 440598.94  | 138700.425 | 109190.333 | 360359.71  | 70537.035  | 68253.735  |
| ME              | 19388.193  | 288075.111 | 225731.139 | 181922.07  | 414621.059 | 442205.277 |
| HT              | 1075.716   | 243544.61  | 33124.89   | 394432.162 | 220422.137 | 397460.129 |
| POF             | 64068.708  | 434766.741 | 23947.812  | 99142.95   | 427939.753 | 374738.739 |
| PMF             | 67938.75   | 297073.265 | 115906.014 | 21040.483  | 157449.8   | 198709.26  |
| TET             | 91691.729  | 251985.538 | 277920.455 | 44441.083  | 193805.298 | 450702.193 |
| FET             | 83609.791  | 289347.153 | 357246.65  | 375536.064 | 67408.427  | 152868.785 |
| MET             | 48082.5    | 91851.341  | 386452.911 | 107381.762 | 53418.982  | 274171.031 |
| IR              | 159791.308 | 148911.536 | 284027.889 | 355751.987 | 99959.859  | 8312.942   |
| ALO             | 408108.968 | 409499.795 | 11510.356  | 174853.967 | 86683.948  | 2608.231   |
| ULO             | 74783.198  | 118635.371 | 195645.937 | 392006.597 | 196628.775 | 154000.17  |
| NLT             | 300366.11  | 452523.988 | 429418.414 | 350896.322 | 12951.12   | 106293.507 |
| MRD             | 100833.497 | 19961.464  | 170858.402 | 447148.05  | 348070.812 | 59490.558  |
| FRD             | 302240.943 | 17385.897  | 5063.264   | 7082.493   | 177477.589 | 322209.56  |
Table 11: Illustrative example: environmental impact resulting from the transportation per kg of item and per km.

| Impact category | Environmental impact per kg and km transported |
|-----------------|-----------------------------------------------|
|                 | $k_0$ | $k_1$ | $k_2$ | $k_3$ |
| CC              | 0.002 | 0.003 | 0.001 | 0.002 |
| OD              | 0.001 | 0.001 | 0.002 | 0.002 |
| TA              | 0     | 0.002 | 0.002 | 0.003 |
| FE              | 0     | 0.002 | 0.003 | 0.002 |
| ME              | 0     | 0.001 | 0.002 | 0.001 |
| HT              | 0.003 | 0.001 | 0 | 0.002 |
| POF             | 0.001 | 0.001 | 0.001 | 0.001 |
| PMF             | 0.001 | 0.002 | 0 | 0.001 |
| TET             | 0.003 | 0.002 | 0.003 | 0.001 |
| FET             | 0     | 0.003 | 0.001 | 0.002 |
| MET             | 0.002 | 0.001 | 0.002 | 0.001 |
| IR              | 0     | 0.002 | 0.003 | 0.001 |
| ALO             | 0.002 | 0.003 | 0.001 | 0.002 |
| ULO             | 0.001 | 0.002 | 0.002 | 0.003 |
| NLT             | 0.001 | 0 | 0.002 | 0.001 |
| MRD             | 0.002 | 0.003 | 0.002 | 0.002 |
| FRD             | 0.001 | 0.001 | 0.001 | 0 |

Table 24 presents the normalization factors for each impact category.

4 Case study from [Mota et al., 2018]

This section presents the data employed the case study from [Mota et al., 2018].

Table 12 presents the parameters of each item. We refer to raw materials as $rm_0, rm_1, rm_2$ and $rm_3$, to final products as $fp_0$ and $fp_1$, and to recovered products as $rp_0$ and $rp_1$.

| Raw material | Final product | Recovered product |
|--------------|---------------|-------------------|
| $rm_0$ | $rm_1$ | $rm_2$ | $rm_3$ | $fp_0$ | $fp_1$ | $rp_0$ | $rp_1$ |
| Product recovery cost (€) | - | - | - | - | - | 0.15 | 0.15 |
| Inventory cost per unit (€) | - | - | - | 0.01 | 0.01 | - | - |
| Price per unit sold (€) | - | - | - | 23 | 37 | - | - |
| Product weight (kg) | 0.118 | 0.184 | 0.365 | 0.913 | 0.4 | 0.5 | 0.4 | 0.5 |
| Necessary area per unit of product (m²) | 0.002 | 0.001 | 0.004 | 0.003 | 0.007 | 0.009 | 0.007 | 0.009 |

Table 13 specifies the production and remanufacturing technologies. The production technologies are $gp_1, gp_1alt, gp_2$ and $gp_2alt$, whereas the remanufacturing technologies are $gr_1$ and $gr_2$. 
Table 13: Case study: technology specifications.

| Technology | Production | Remanufacturing |
|------------|-----------|-----------------|
|            | gp1       | gp1alt          | gp2     | gp2alt | gr1 | gr2 |
| Maximum capacity | 5800000   | 6000000         | 4600000 | 5200000 | 2900000 | 2300000 |
| Minimum capacity   | 30000     | 30000           | 30000   | 30000   | 0    | 0   |
| Installation Cost (€) | 150000   | 175000          | 167000  | 186000  | 50000 | 45000 |
| Operation costs per unit (€) | 0.212   | 0.196           | 0.324   | 0.267   | 0.116 | 0.134 |
| Number of workers | 2         | 1               | 4       | 3       | 1    | 1   |

Table 14 defines the bill of raw materials required by a production technology to produce final products. Technologies gp1 and gp1alt can produce fp0 and gp2 and gp2alt can produce fp1.

Table 14: Case study: bill of raw materials for production.

|         | fp0     | fp1     |
|---------|---------|---------|
| gp1     | 0.25    | 0.45    |
| gp1alt  | 0.25    | 0.42    |
| gp2     | 0.4     | 0.2     |
| gp2alt  | 0.025   | 0.015   |
| rm0     | 0.3     | 0.3     |
| rm1     | 0.3     | 0.4     |
| rm2     | 0.15    | 0.42    |
| rm3     | 0.2     | 0.3     |

Table 15 defines the amount of recovered products required by a remanufacturing technology to obtain a final product.

Table 15: Case study: remanufacturing bill.

|         | fp0     | fp1     |
|---------|---------|---------|
| gr1     | 4       | 0       |
| gr2     | 0       | 5       |

Table 16 presents the maximum capacity, the minimum order and the unit cost of the raw materials for each supplier.

Table 16: Case study: raw material parameters for each supplier.

| Supplier     | Raw material |         |         |         |         |
|--------------|--------------|---------|---------|---------|---------|
| Maximum capacity | rm0  | rm1  | rm2  | rm3  |         |
| Verona       | 3600000     | 3600000 | 1000000 | 4000000 |         |
| Hannover     | 3800000     | 3800000 | 1200000 | 5000000 |         |
| Leeds        | 3800000     | 3800000 | 1200000 | 5000000 |         |
| Minimum order | Verona | 1000 | 1000 | 200 | 1000 |
| Hannover     | 0          | 0     | 0     | 0     |         |
| Leeds        | 0          | 0     | 0     | 0     |         |
| Unit cost (€) | Verona | 0.01 | 0.025 | 0.03 | 0.09 |
| Hannover     | 0.035      | 0.0875 | 0.105 | 0.315  |         |
| Leeds        | 0.03       | 0.075  | 0.09  | 0.27   |         |

The factory and the warehouse in Verona are already installed with capacities of 20000 m² and 5000 m², respectively. Table 17 presents the parameters related to the installation and operation of the remaining factories and warehouses.
Table 17: Case study: installation and operation of the factory and warehouses.

| Data                                      | Factories | Warehouses |
|-------------------------------------------|-----------|------------|
| Maximum installation area (m$^2$)         | 25000     | 8000       |
| Minimum installation area (m$^2$)         | 2000      | 500        |
| Fixed number of workers                   | 11        | 9          |
| Fixed number of workers per square meter  | 0.01      | 0.01       |
| Maximum storage                           | -         | 1200000    |
| Minimum storage level                      | -         | 391        |

Only warehouses stock products in this case study. The maximum storage capacity for final products $fp_0$ and $fp_1$ are, respectively, 1200000 and 1000000. The minimum inventory levels for $fp_0$ and $fp_1$ are, respectively, 12000 and 10000 units.

Table 18 describes the transportation specifications for each mode, that is, land ($A_{truck}$), air ($A_{plane}$) and sea ($A_{boat}$). The land transportation mode uses trucks $Truck1$ and $Truck2$; the air transportation mode uses airplanes $Air1$, $Air2$, $Air3$ and $Air4$; and the sea transportation uses boats $Boat1$ and $Boat2$. In this table, (*) indicates that the transportation cost per kilogram in truck $a \in A_{truck}$ is calculated according to Equation (11).

Table 18: Case study: transportation specifications.

|                  | Land      | Air      | Sea       |
|------------------|-----------|----------|-----------|
|                  | $Truck1$  | $Truck2$ | $Air1$    | $Air2$    | $Air3$    | $Air4$    | $Boat1$  | $Boat2$  |
| Maximum capacity | 35000     | 55000    | 400000    | 500000    |
| Minimum units transported | 200 | 200 | 1000 | 1000 |
| Maximum contracted capacity per time period | - | - | 3000000 | 3000000 |
| Fixed costs (€)  | 30000     | 50000    | 108000    | 126000    | 126000    | 72000     | 144000    | 72000    |
| Handling costs at hub terminals per unit (€) | - | - | 0.100 | 0.125 | 0.125 | 0.075 | 0.150 | 0.075 |
| Transportation cost per kg (€)               | * | * | 0.04 | 0.01 |
| Necessary workers per unit                    | 1 | 1 | 0 | 0 |
| Average vehicle consumption (L/100km)        | 14| 18| - | - | - | - | - | - |

$t_{ca} = \frac{avc_a}{100} fp + vmc$ \hspace{1cm} (11)

There are three types of investments: investments in entities, technologies and transportation. Their savage values are, respectively, 0.5, 0 and 0. The depreciation rates of the investments at each time period are presented in Table 19.

Table 19: Case study: depreciation rates of entities.

| Investment  | Planning period $t \in T$ |
|-------------|---------------------------|
|             | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| Entities    | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Technologies| 0.125 | 0.125 | 0.125 | 0.125 | 0.125 | 0.125 | 0.125 | 0.125 | 0 | 0 |
| Transportation| 0.2 | 0.2 | 0.2 | 0.2 | 0 | 0 | 0 | 0 | 0 | 0 |
Table 20 details the demand of the customers for final products in each period of the planning horizon.

Table 20: Case study: demand for final products.

| Final Product | Customer    | Planning period \( t \in T \) |
|---------------|-------------|-----------------------------|
|               | Verona      | 2873448                     |
|               | Hannover    | 1789152                     |
|               | Leeds       | 1138848                     |
|               | Zaragoza    | 1221432                     |
|               | Lisbon      | 1128840                     |
|               | São Paulo   | 919464                      |
|               | Recife      | 1443216                     |
|               | Verona      | 1518120                     |
|               | Hannover    | 1570152                     |
|               | Leeds       | 1033632                     |
|               | Zaragoza    | 825308                      |
|               | Lisbon      | 1065288                     |
|               | São Paulo   | 1390704                     |
|               | Recife      | 833688                      |

The analysis of the environmental impact assessed by ReCiPe2008 takes into account all the impact categories, i.e. Climate Change (CC), Ozone Depletion (OD), Terrestrial Acidification (TA), Freshwater Eutrophication (FE), Marine Eutrophication (ME), Human Toxicity (HT), Photochemical Oxidant Formation (POF), Particulate Matter Formation (PMF), Terrestrial Ecotoxicity (TE), Freshwater Ecotoxicity (FE), Marine Ecotoxicity (MET), Ionising Radiation (IR), Agricultural Land Occupation (ALO), Urban Land Occupation (ULO), Natural Land Transformation (NLT), Metal Depletion (MRD) and Fossil Depletion (FD).

Tables 21, 22 and 23 show the environmental impacts of installing entities that use production and remanufacturing technologies and transportation modes, respectively. Table 24 defines the normalization factors for each impact category.

Table 21: Case study: environmental impact on entity installation, per \( m^2 \) of entity area.

| Impact category | Environmental impact per \( m^2 \) of entity area |
|-----------------|-----------------------------------------------|
| CC              | 3.78E+02                                      |
| OD              | 2.23E-05                                      |
| TA              | 4.02E+00                                      |
| FE              | 1.24E-01                                      |
| ME              | 1.66E-01                                      |
| HT              | 8.32E+02                                      |
| POF             | 2.18E+00                                      |
| PMF             | 1.38E+00                                      |
| TET             | 4.66E-01                                      |
| FET             | 1.21E+00                                      |
| MET             | -1.84E+01                                     |
| IR              | 7.67E+00                                      |
| ALO             | 2.22E+02                                      |
| ULO             | 3.38E+00                                      |
| NLT             | 3.20E-02                                      |
| MRD             | 1.31E+02                                      |
| FRD             | 1.02E+02                                      |
Table 22: Case study: environmental impact on the technology use, per kg of unit produced or remanufactured.

| Impact category | Environmental impact per kg produced or remanufactured. | gp1 | gp1al | gr1 | gp2 | gp2al | gr2 |
|-----------------|--------------------------------------------------------|-----|-------|-----|-----|-------|-----|
| CC              |                                                        | 5.36E+02 | 3.75E+02 | 1.34E+02 | 5.14E+02 | 4.11E+02 | 1.03E+02 |
| OD              |                                                        | 1.96E+05 | 1.37E+05 | 4.90E+06 | 1.66E+05 | 1.33E+05 | 2.66E+06 |
| TA              |                                                        | 4.93E+00 | 3.45E+00 | 1.23E+00 | 3.96E+00 | 3.17E+00 | 6.33E+01 |
| FE              |                                                        | 2.51E+00 | 1.76E+00 | 6.28E-01 | 7.15E-01 | 5.72E-01 | 1.14E-01 |
| ME              |                                                        | 3.43E-01 | 2.40E-01 | 1.23E+00 | 3.96E+00 | 3.17E+00 | 6.33E+01 |
| HT              |                                                        | 1.10E+03 | 7.70E+02 | 2.75E+02 | 8.71E+02 | 6.97E+02 | 1.39E+02 |
| POF             |                                                        | 3.23E+00 | 2.26E+00 | 8.07E-01 | 1.96E+00 | 1.57E+00 | 3.14E-01 |
| PMF             |                                                        | 1.85E+00 | 1.30E+00 | 4.63E-01 | 1.57E+00 | 1.25E+00 | 2.51E-01 |
| TET             |                                                        | 5.02E-01 | 3.51E-01 | 1.26E-01 | 3.99E-01 | 3.19E-01 | 6.39E-02 |
| FET             |                                                        | 8.51E+00 | 5.96E+00 | 2.13E+00 | 3.99E-01 | 3.19E-01 | 6.39E-02 |
| MET             |                                                        | 4.57E+00 | 3.20E+00 | 1.14E+00 | 3.99E+00 | 3.19E+00 | 6.39E-02 |
| IR              |                                                        | 1.14E+02 | 7.95E+01 | 2.84E+01 | 1.36E+00 | 8.48E+00 | 1.70E+00 |
| ALO             |                                                        | 1.49E+02 | 1.04E+02 | 3.72E+01 | 1.45E+02 | 1.16E+02 | 2.32E+01 |
| ULO             |                                                        | 1.65E+01 | 1.16E+01 | 4.13E+00 | 1.06E+01 | 8.48E+00 | 1.70E+00 |
| NLT             |                                                        | 1.15E-01 | 8.05E-02 | 2.88E-02 | 7.57E-02 | 6.06E-02 | 1.21E-02 |
| MRD             |                                                        | 7.17E+02 | 5.02E+02 | 1.79E+02 | 1.47E+02 | 1.18E+02 | 2.35E+01 |
| FRD             |                                                        | 1.38E+02 | 9.64E+01 | 3.44E+01 | 1.24E+02 | 9.94E+01 | 1.99E+01 |

Table 23: Case study: environmental impact on transportation per kg and km transported.

| Impact category | Environmental impact per kg and km transported | truck1 | truck2 | plane | boat |
|-----------------|-----------------------------------------------|-------|-------|-------|------|
| CC              |                                               | 1.80E-03 | 4.34E-04 | 1.03E-03 | 1.09E-05 |
| OD              |                                               | 1.28E-10 | 3.31E-11 | 7.70E-11 | 7.31E-13 |
| TA              |                                               | 8.29E-06 | 1.26E-06 | 3.35E-06 | 2.34E-07 |
| FE              |                                               | 5.04E-07 | 4.00E-08 | -1.07E-08 | 1.45E-09 |
| ME              |                                               | 4.43E-07 | 7.07E-08 | 1.89E-07 | 6.13E-09 |
| HT              |                                               | 3.14E-03 | 4.19E-04 | 6.54E-04 | 1.04E-05 |
| POF             |                                               | 1.25E-05 | 2.18E-06 | 5.93E-06 | 1.73E-07 |
| PMF             |                                               | 3.87E-06 | 7.02E-07 | 1.08E-06 | 6.89E-08 |
| TET             |                                               | 3.18E-06 | 9.77E-07 | 3.01E-07 | 4.00E-09 |
| FET             |                                               | 5.80E-07 | 1.81E-07 | 2.02E-07 | 2.13E-09 |
| MET             |                                               | 2.22E-03 | 3.98E-04 | 3.56E-04 | 1.21E-05 |
| IR              |                                               | 1.61E-04 | 2.53E-05 | 6.39E-05 | 1.25E-06 |
| ALO             |                                               | 3.68E-04 | 1.09E-04 | 2.00E-04 | 1.27E-06 |
| ULO             |                                               | 7.43E-05 | 2.85E-05 | 8.67E-06 | 1.03E-07 |
| NLT             |                                               | 5.12E-07 | 1.39E-07 | 3.02E-07 | 2.87E-09 |
| MRD             |                                               | 3.71E-04 | 5.25E-05 | 5.85E-06 | 4.89E-07 |
| FRD             |                                               | 6.64E-04 | 1.68E-04 | 3.79E-04 | 3.74E-06 |
Table 24: Case study: normalization factors for environmental impact categories.

| Impact category | Normalization factor |
|-----------------|----------------------|
| CC              | 1.81E-04             |
| OD              | 2.66E+01             |
| TA              | 2.37E-02             |
| FE              | 3.45E+00             |
| ME              | 1.36E-01             |
| HT              | 6.89E-04             |
| POF             | 1.76E-02             |
| PMF             | 7.11E-02             |
| TET             | 1.23E-01             |
| FET             | 2.20E-01             |
| MET             | 1.48E-03             |
| IR              | 7.59E-04             |
| ALO             | 1.84E-04             |
| ULO             | 1.29E-03             |
| NLT             | 8.31E-02             |
| MRD             | 2.25E-03             |
| FRD             | 7.75E-04             |

References

B. Mota, M. I. Gomes, A. Carvalho, and A. P. Barbosa-Povoa. Sustainable supply chains: An integrated modeling approach under uncertainty. *Omega*, 77:32 – 57, 2018. ISSN 0305-0483.

C. P. Tautenhain, A. P. Barbosa-Póvoa, and M. C. V. Nascimento. A multi-objective matheuristic for designing and planning sustainable supply chains. *Computers & Industrial Engineering*, 2018.