Spin-orbit-locke lockdown hyperbolic polariton vortices carrying reconfigurable topological charges

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Abstract
The topological features of optical vortices have been opening opportunities for free-space and on-chip photonic technologies, e.g., for multiplexed optical communications and robust information transport. In a parallel but disjoint effort, polar anisotropic van der Waals nanomaterials supporting hyperbolic phonon polaritons (HP\textsuperscript{2}s) have been leveraged to drastically boost light-matter interactions. So far HP\textsuperscript{2} studies have been mainly focusing on the control of their amplitude and scale features. Here we report the generation and observation of mid-infrared hyperbolic polariton vortices (HP\textsuperscript{2}Vs) associated with reconfigurable topological charges. Spiral-shaped gold disks coated with a flake of hexagonal boron nitride are exploited to tailor spin–orbit interactions and realise deeply subwavelength HP\textsuperscript{2}Vs. The complex interplay between excitation spin, spiral geometry and HP\textsuperscript{2} dispersion enables robust reconfigurability of the associated topological charges. Our results reveal unique opportunities to extend the application of HP\textsuperscript{2}s into topological photonics, quantum information processing by integrating these phenomena with single-photon emitters, robust on-chip optical applications, sensing and nanoparticle manipulation.

Keywords: Hyperbolic polariton vortex, Light-matter interactions, Spin-orbit interaction, Topological charge, Reconfigurability

1 Introduction
Low-dimensional materials form an emerging platform for exotic light-matter interactions, ideally suited for various photonic technologies [1–5]. In these materials, light hybridizes with matter to form quasiparticles known as polaritons [6, 7], which feature deeply subwavelength field confinement and broadband responses spanning from terahertz to infrared and visible frequencies. The hyperbolicity of polaritons supported by various natural materials, especially in anisotropic van der Waals (vdW) crystals, is particularly promising for next-generation nanophotonics. The hyperbolicity originates from extreme anisotropy of the atomic interaction between in-plane covalent and out-of-plane vdW bondings [8], enabling unusually large wave momenta associated with giant field confinement and slow group velocities over broad bandwidths. While hyperbolic phenomena have been originally explored in engineered metamaterials, such as metal–insulator-metal multilayers, this approach suffers from inevitable fabrication imperfections and finite granularity, resulting in enhanced loss and nonlocality hindering their widespread applicability [9, 10]. Natural hyperbolic polaritons in polar vdW crystals [11] can overcome these challenges: for instance, hexagonal boron nitride (hBN) conical nanoresonators enable remarkably large quality-factors (~283) and light confinement (~\(\lambda/86\)) [12]. In addition, the dispersion...
of hyperbolic phonon polaritons (HP2) can be engineered with large flexibility, as shown in hBN [12–17] and α-MoO3 [18–23], facilitating super-resolution imaging [14, 15], light canalization [17, 20], molecular sensing [24, 25], reconﬁgurable propagation [26, 27], for a variety of polaritonic applications [11, 28].

So far, the manipulation of hyperbolic phonon polaritons (HP2) has been mainly limited to amplitude control, while adding new degrees of freedom may dramatically expand the impact of polaritons for information processing and transport. For instance, surface plasmon polaritons (SPPs) have recently been explored to control phase, spin and orbital angular momentum (OAM) as independent degrees of freedom [29–35]. Based on these principles, a plasmonic vortex has been explored to enhance the channel capacity of on-chip optical communication networks [36]. Plasmonic OAMs can selectively couple to the spin of light excitation, inducing spin–orbit interactions [37, 38] and spin-controlled plasmonic phenomena, such as unidirectional routing [39], plasmonic vortex generation [29, 30, 40, 41] and information detection, opening new opportunities for low-energy information processing and computing. Extending these explorations to the polaritonic regime in the mid-IR context of phase and angular momentum control may be intrinsically demonstrated. Our results reveal new degrees of freedom and enhanced robustness in HP2 control based on optical spin–orbit interactions and topological charges, ideally suited for super-resolution sensing and imaging, enhanced light-matter interactions, communications and multiplexing, and particle manipulation [44] at mid-IR frequencies.

As schematically illustrated in Fig. 1a, our sample is composed of a thin hBN ﬂake transferred onto a shaped Au disk. The edge of the nanodisk acts as an HP2 launcher, tailored to convert incident free-space light into strongly conﬁned near ﬁelds at the disk boundary, resulting in efﬁcient launching of HP2s [14, 15]. hBN ﬂakes are known to support discrete waveguide modes based on bulk HP2s [13], denoted by the mode index s associated with their in-plane momentum of k⊥, t refers to the axis parallel to the hBN ﬂake. Unlike free-space beams or SPPs, HP2 propagation is characterized by a large wavevector k conﬁned to the angle θ = arctan(√−εt/εz) with respect to the optical z axis, i.e., the axis perpendicular to the hBN ﬂake, as illustrated in Additional ﬁle: Fig. S1. εz and εt are the hBN permittivity components along the z axis and in the orthogonal plane, respectively. Polariton fringes are expected to form with periodicity δz = 2d tanθ on the top hBN surface, where d is the ﬂake thickness.

In order to generate a polariton vortex in hBN, we control the optical path of the launched hyperbolic polaritons by tailoring the disk shape to follow an Archimedean spiral, rAS(φ) = rmin + m mod (φ/2π) * g, where g indicates the rate of increase of the spiral radius with the azimuthal angle φ, and the integer parameter m denotes the number of spiral arms. As an example, a Au disk with outer shape mapping a four-arm spiral was fabricated, and its atomic force microscope (AFM) image is shown in Fig. 1b, where rmax and rmin are the maximum and minimum radius and g ≡ rmax − rmin. Based on this design, the polaritons induced on the top surface are expected to support a phase distribution as a function of position (r,φ) in the z = z0 plane following

\[ \varphi(r, \phi, z_0) = k_{\text{ref}} \left( r_{\text{AS}}(\phi) - r_{\text{min}} + \frac{\delta z}{2} \right) + s'2\pi \]  

(1)

where s' ≡ s − 1 is the modiﬁed HP2 order on the ﬂake with a Au substrate (see Sect. 4). As experimentally veriﬁed in Fig. 1, the induced HP2s indeed follow spiral orbitals in space to form a vortex with topological features. In
particular, if the Archimedean spiral shape satisfies the condition
\[ k_{ls} \left( g + \frac{\delta r}{2} \right) = l2\pi, \]  
(2)
the phase accumulation from \( \phi = 0 \) to \( \phi = 2\pi \) is \( m(l + s') \times 2\pi \) for an Archimedean spiral with \( m \) arms. The integer \( l \) is the orbital topological charge supported by the induced HP\(^2\) spatial distribution.

Under a circularly polarized plane wave excitation, due to spin–orbit interactions in the near-field, the total phase accumulation is \( (l \times m + \sigma)2\pi \), associated with a topological charge of total angular momentum
\[ L = m(l + s') + \sigma, \]  
(3)
where \( \sigma = +1 \) or \(-1\) or left-handed (LCP) or right-handed circularly polarized (RCP) incident light, as we assumed a left-handed spiral. Equation 3 outlines the several degrees of freedom available to control our HP\(^2\)Vs, as a result of the complex interplay between incident spin and geometry-induced control of the polariton orbitals. Nontrivial control parameters consist of the spiral geometry (e.g., \( g \)), the working frequency (i.e., \( k_{ls} \) and \( l \) when \( g \) is fixed), the number of spiral arms \( m \), the excited polaritonic mode \( s' \), and the excitation spin \( \sigma \). In our demonstration, we focus on the dominant mode, \( s' = 0 \).

HP\(^2\)Vs carrying a discrete topological charge \( L \) are expected to support a spiral phase accumulation of \( L \times 2\pi \) and a phase singularity at their center [45], controlled by the spin of impinging light [29, 46–48]. As described by the previous analysis, this spin-dependent phase originates from spin-to-orbital angular momentum conversion enabled by the coupling between the spin angular momentum (SAM) of the excitation and the OAM [37].
To observe HP\(^2\)Vs and the associated spin–orbit interactions in real space, we used s-SNOM measurements to map HP\(^2\)s on the top surface of a hBN flake in Fig. 1a. The setup of our experiment is schematically shown in Fig. 2: we fabricated a Au four-arm Archimedean spiral disk over a SiO\(_2\) top layer on a silicon substrate, shown in Fig. 1b, with \(g=650\) nm, \(r_{\text{min}}=2.275\) \(\mu\)m and height \(h=65\) nm (details in Methods). Consistent with the previous analysis, the parameter \(h\) and \(r_{\text{min}}\) do not affect the total topological charge induced in the HP\(^2\)V. An exfoliated hBN flake with a thickness of \(\sim 295\) nm was transferred on top of the Au disk. By acquiring multiple near-field images with excitation frequency spanning from 1510 to 1570 cm\(^{-1}\) (see examples in Additional file 1: Fig. S2), we retrieved (Fig. 1c) the in-plane momentum \((k)\) of the fundamental HP\(^2\) mode at different frequencies, which agrees well with the analytical dispersion calculated based on the parameters in ref. [13].

Figure 1d, e show the measured near-field distribution of polariton fringes at \(\sim 1546\) cm\(^{-1}\), yielding a clear spiral shape profile emerging from the interference of HP\(^2\) fields launched by the disk boundary with the incident field at the tip. At this frequency, \(\lambda_{l} \approx 761\) nm = \(\lambda_{0}/8.5\), where \(\lambda_{l}\) and \(\lambda_{0}\) are the in-plane and free-space wavelength, respectively, confirming a deeply subwavelength confinement of the induced HP\(^2\)V. The simulated electric field \(\langle |E_{l}| \rangle\) profile shown in Fig. 1f, g matches well with experimental results (see more details in Methods). Consistent with Eq. 3, when \(l=1\) a left-handed spiral with \(m=4\) yields polariton vortices with topological charge \(L=3\) and \(L=5\) for LCP and RCP excitation, respectively. By comparing Fig. 1d, e (or Fig. 1f, g), we notice a distinct difference in induced field profile at the vortex center, associated with the different topological charge. Both experimental and simulation results reveal that HP\(^2\)V with \(L=3\) has a smaller ring-shaped profile at its center than \(L=5\), when \(l\) is fixed. This is expected, since its size increases with the spiral phase, consistent with other optical and plasmonic vortex studies [29, 45, 49].

Our setup imprints this dynamic into nanoscale phonon polaritons. It should be mentioned that the regions of the dotted circles in Fig. 1f–i and below figures are determined by the spiral phase at the center in near-field phase images and their positions are chosen to have a minimum deviation from their center to that of the Au patterns. The calculated phase distributions at \(\sim 1546\) cm\(^{-1}\) under RCP and LCP excitation, shown in Fig. 1h (\(L=3\)) and Fig. 1i (\(L=5\)), reveal clockwise 3\(^\ast\)2\(\pi\) and 5\(^\ast\)2\(\pi\) spiral phase variations at the origin, further confirming the nature of spin–orbit interactions for \(\sigma=+1\) and \(-1\). The discrete topological charge associated with these polariton vortices introduces inherent robustness to geometric perturbations associated with topological features [50], and new degrees of freedom for polaritonic science and technology.

Next, we experimentally demonstrate precise reconfigurability of HP\(^2\)Vs, besides the incident spin demonstrated in Fig. 1. According to Eq. 3, both orbit topological charge \((l)\) and the Archimedean spiral geometry \((m)\) control the total induced angular momentum of the polariton vortex. Here, \(l\) is controlled by the frequency of impinging light when \(g\) and \(d\) are fixed, following the polariton dispersion. In Fig. 3, using geometric arguments we predict the phase accumulation \(\phi_{pp}\) along the azimuthal angle \(\phi\) for \(m=1, 2, 4\) when \(l=1, 2\), revealing a broad range of opportunities in terms of polariton profiles and induced topological charges that can be generated using this platform. In these plots, we do not explicitly take into account the additional degree of freedom stemming from the excitation spin, which enables an additional knob for manipulation. Because a large \(L\) leads to more dramatic amplitude and phase variations, and higher-order HP\(^2\)Vs correspond to sharper field profiles, which are more difficult to image [30], in the following we demonstrate this broad reconfigurability for \(m=1, 2\) and \(l=1, 2\).

We first explore the near-field profile of the induced HP\(^2\)Vs and their dependence on the number of Archimedean spiral arms \(m\). Figure 3b, c show AFM images of Au disks with one- and two-arm Archimedean spirals, i.e., \(m=1, 2\), respectively. The parameters \(g\), \(r_{\text{min}}\), and \(h\) are the same as in the previous examples, and a thinner exfoliated hBN flake with \(d \sim 170\) nm was transferred on top of the Au disks. Our experimentally measured polariton dispersion (see examples of amplitude images in Additional file 1: Fig. S3) agrees well with the analytical one also for this flake (Fig. 3d), showing larger transverse wavevectors, i.e., more confined polariton fields, compared to the results in Fig. 1c, due to the thinner nature of this flake. As seen in Eq. 2, larger wavevectors enable
access to larger $l$ for fixed $g$, hence supporting vortices with $l=1$ and $l=2$ around 1542 cm$^{-1}$ and 1561 cm$^{-1}$.

Based on Eq. 3, for $l=1$ we expect $L=0$ (1) when $m=1$ (2) under RCP light excitation, and $L=2$ (3) when $m=1$ (2) for LCP light.

The corresponding near-field images, showcasing the wide reconfigurability of polariton vortices enabled by these structures, are shown in Fig. 4. For $L=0$, the phase distribution around the HP$^2$V center is uniform, supporting a subwavelength polariton focal spot, rather than a ring profile as expected for nonzero topological charges (Fig. 4a). Figure 4e shows the corresponding numerical simulations, showing a good agreement. By varying the number of spiral arms or the excitation SAM, we can reconfigure the topological charge $L$. For example, increasing $m$ from 1 to 2 makes $L$ increase from 0 to 1, resulting in a different size of the central spot, which becomes ring-shaped and grows larger, as seen comparing Fig. 4a, g (Fig. 4c, i for simulations). Further growth of the central ring profile is obtained under LCP illumination (Fig. 4b and h), as this SAM yields larger values of $L$, again confirmed by our numerical simulations (Fig. 4d, j).

It should be noticed that as revealed by the associated spiral phase in Fig. 4f the upper bright lobe close to the central amplitude profile in Fig. 4d is not a part of the central pattern while at an outer layer of the spiral pattern, so $L$ is 2 not 3 in Fig. 4d. As $L$ increases, we also verify that the polariton vortex phase accumulation grows in discrete $2\pi$ steps, as illustrated in Fig. 4e, f, k, l. We find 0 to $3\times2\pi$ spiral phase accumulation around the center, confirming a discrete growth of topological charge $L$ from 0 to 3. These images also show a $2\times2\pi$ phase difference between LCP and RCP excitation, demonstrating strong spin–orbit polariton interactions.

Extreme HP$^2$V reconfigurability can also be achieved by controlling the orbital topological charge $l$, which can be tuned through the excitation frequency. Figure 5 shows results with a similar trend as in Fig. 4 retrieved from the same samples at $\sim 1561$ cm$^{-1}$, for which the increased $k_t$ of the supported polaritons yields an orbital topological charge $l=2$. As a result, the total angular momentum of HP$^2$V correspondingly increases compared to the $l=1$ scenario. Since HP$^2$'s support a broad range of frequency-dependent $k_t$ thanks to their hyperbolic dispersion [11,
13], a wide range of $l$ can be attained, enabling broad reconfigurability of HP$^2$Vs without varying the sample geometry. The demonstrated nano-vortices have deeply subwavelength spatial profile, again supported by the extreme HP$^2$ field confinement. A comparison between experimental and simulated images in Figs. 4 and 5 shows that even denser fringes are obtained when $l = 2$, consistent with the larger $k_t$ and shorter polariton wavelength (Eq. 1) combined with increased topological charge. At $\sim 1550$ cm$^{-1}$ (Fig. 4a, b, g, h), $\lambda_t \approx \lambda_0/14.8$ ($l = 1$), while at $\sim 1566$ cm$^{-1}$ (Fig. 5a, b, g, h) $\lambda_t \approx \lambda_0/19.5$ ($l = 2$). The near-field profiles at the vortex center are supported through the superposition of HP$^2$s excited from the Au disk boundary, so we can expect that the size of the HP$^2$ central profile narrows as $l$ increases. To further explore how the phase profile evolves with $l$, we compare Fig. 4h, j, and l ($l = 1$) with Fig. 5g, i, and k ($l = 2$). A sharper phase variation and a smaller central profile can indeed be observed at the higher frequency, despite the fact that in both scenarios the induced topological charge is the same ($L = 3$). For the $L = 5$ scenario when $l = 2$ in Fig. 5l the spiral phase is less well defined compared to the one in Fig. 1i, associated with the fact that the spiral phase is less stable at larger $L$ when $l = 2$ than $l = 1$. Overall, Fig. 5 features HP$^2$s with very large transverse momentum, realizing polariton vortices with highly confined features, offering unprecedented opportunities for super-resolution mid-IR imaging, high-precision particle manipulation, sensing, and high-density information storage and transmission. When $l$ is larger than 2 the associated frequencies become too close to the 1610 cm$^{-1}$, i.e., the high-frequency edge of the upper Reststrahlen band of [13]. Therefore, it is not possible to obtain a clean central profile in this scenario, because of increased loss and reduced group velocity of the associated polaritons (more discussions can be found in the Additional file 1).

During the review of this study, Xiong et al. reported topological spin textures of phonon polaritons by

Fig. 4 Experimental and simulation results for polariton vortices when $l = 1$. a, b, g, h Near-field amplitude images at $\sim 1550$ cm$^{-1}$ for right-handed (a and g) and left-handed (b and h) circularly polarized light. c, d, i, j Numerical $E_z$ and e, f, k, l phase distributions at $\sim 1542$ cm$^{-1}$ for right-handed (c, i, e, and k) and left-handed (d, j, f, and l) circularly polarized light. The zoom-in images of the central profile are in the lower column. The cyan/red dotted circles with numbers in the zoom-in amplitude/simulated $E_z$ images label the central interference profiles. The black dotted circles in the phase images with numbers indicate the spiral phases. The scale bars are 2 $\mu$m in the full images and 0.67 $\mu$m in the zoom-in ones.
illuminating a ring-shaped antenna located on top of an hBN flake with circularly polarized light [51]. In their study, meron-like textures with a half-integer topological charge, determined by the handedness of the incident circularly polarized light, were observed, linking transverse SAM to skyrmion/meron-like textures in polaritons [51–53]. Here, we explore the spin–orbit interaction between OAM and out-of-plane SAM and the control of total topological charges via multiple degrees of freedom.

3 Conclusions

Overall, in this work we have conclusively demonstrated spin–orbit locked HP2Vs emerging in natural polar vdW crystal hBN flakes coupled to spiral structures with tailored in-plane chirality. The resulting devices generate broadly reconfigurable topological charges, showcasing exotic polaritonic features, including spin–orbit interactions and nanofocusing. The demonstrated polariton vortices are highly tunable using various degrees of freedom, including the excitation spin, the geometry of the polariton launcher, and the hyperbolic features of the underlying material controlled by the wavelength of excitation. Our demonstration of HP2V opens unique opportunities to robustly process multiplexed information at mid-IR wavelengths and has great potential for super-resolution imaging systems with multiplexing capabilities, ultracompact mid-IR sensors, and miniaturized polaritonic devices with robust features associated with their topological nature, as well as enhanced nonlinearities and sensing. Overall, our findings broadly enrich the nanophotonic and polaritonic platforms enabled by vdW nanomaterials with phase, spin, and orbital angular momentum engineering capabilities, of great interest for a variety of classical and quantum applications.
4 Methods

4.1 Sample preparations

Gold (Au) disks with Archimedean spiral shapes were prepared through standard electron beam lithography (EBL). EBL resist (PMMA) was first spin-coated on a silicon dioxide (SiO₂) layer on a silicon substrate, and the desired patterns were created by exciting the resist with the electron beam. After resist developing in an MIBK-IPA solution, a 65-nm-thick Au layer was deposited on top of the sample via electron beam evaporation, and the resist was lifted off. Atomic force microscope (AFM) images of the realized Au disks are shown in Figs. 1b and 3b, c. Hexagonal boron nitride (hBN) flakes were obtained through mechanical exfoliation of high-quality hBN crystals. hBN flakes were transferred on top of the Au disks via a dry transfer method, as shown in Additional file 1: Fig. S4.

4.2 Near-field measurements

A commercial scattering-type scanning near-field optical microscope (s-SNOM) (neaspec GmbH) was used to record the near-field images of the hyperbolic polaritonic vortex (HP²V) in hBN. A quantum cascade laser (QCL) with tunable wavelength (MIRcat-QT by DRS Daylight Solutions) was employed as the mid-infrared light source. Near-field measurements were demodulated at the 2nd harmonic of the oscillation frequency, which is about 285 kHz. A schematic presentation of the setup is shown in Fig. 2.

As shown in Fig. 2, a circularly polarized mid-IR beam was produced by passing a linearly polarized beam through a mid-IR quarter-wave plate, and then focused on the sample with a parabolic mirror. In order to verify whether the beam focused by a parabolic mirror still preserves circular polarization, we compared the output intensities of two setups: with and without a parabolic mirror, as schematically illustrated in Additional file 1: Fig. S5. The results obtained at 1510 cm⁻¹ are listed in Additional file 1: Tables S1 and 2, which prove the preservation of circular polarization after the beam is focused by the parabolic mirror. An Au disk without spiral arms covered by an hBN flake was then measured to confirm that HP² with σ = +1 and -1 can be excited under LCP and RCP incident light, respectively. The results are shown in Additional file 1: Fig. S7 and are in good agreement with the simulations. In addition, Additional file 1: Fig. S8 unveils that there are important differences between the amplitude images retrieved under linearly and circularly polarized light. Please see more discussions in Additional file 1.

4.3 Theoretical analysis and numerical simulations

For a hBN thin layer on top of a perfect electric conductor (PEC) substrate, the phonon polariton (PhP) mode in hBN can be obtained by finding the poles of the dyadic Green's function, which is equivalent to the transverse resonance condition:

\[ 2k_{z,s}d + \angle R_1 + \angle R_2 = 2\pi s, \quad s = 0, 1, 2, \ldots \]  

(4)

where \( d \) is the thickness of hBN, \( R_1 = \frac{\epsilon_1k_0 - i\epsilon_{k1}}{\epsilon_1k_0 + i\epsilon_{k1}} \) and \( R_2 = \frac{\epsilon_2k_0 - i\epsilon_{k2}}{\epsilon_2k_0 + i\epsilon_{k2}} \) are the reflection coefficient at the top surface and bottom surface; The top and bottom material are assumed to have permittivity \( \epsilon_1 \) and \( \epsilon_2 \), respectively.

Here, \( k_{z1} = \sqrt{\epsilon_1k_0^2 - k_t^2} \); \( k_{z2} = \sqrt{\epsilon_2k_0^2 - k_t^2} \); and \( k_{z,s} = \sqrt{\epsilon_1k_0^2 - \epsilon_{z,s}k_t^2} \), where \( k_t \) is the in-plane wavevector and the index \( s \) denotes the \( s \)-th waveguide mode. For large wavevector, we have:

\[ k_{z,s} \approx k_{t,s}\sqrt{-\frac{\epsilon_t}{\epsilon_z}} = k_{t,s}\tan\theta \]  

(5)

i.e., \( k_{t,s} = \frac{k_{z,s\max}}{\tan\theta} \), where \( \theta \) is the angle with respect to the optical axis (z axis) for a Type II hyperbolic band. A fringe at the top surface can be formed with evolving periodicity \( \delta_r \equiv 2d\tan\theta \). Alternatively, Eq. (4) can be written as

\[ k_{t,s} = \frac{\Psi}{d} \left[ \arctan\left( \frac{\epsilon_1}{\epsilon_t}\Psi \right) + \arctan\left( \frac{\epsilon_2}{\epsilon_t}\Psi \right) + s\pi \right] \]

where \( \Psi \equiv \sqrt{\frac{\epsilon_2}{\epsilon_1}} \). If the substrate is gold at mid-infrared frequency, \( \Re[\epsilon_2] < 0 \) and \( |\epsilon_2| \gg 0 \), we thus have \( \arctan\left( \frac{\epsilon_2}{\epsilon_t}\Psi \right) \rightarrow -\frac{\pi}{2} \) and \( |R_2| \rightarrow 1 \) while \( \angle R_2 \rightarrow -2\pi \). Therefore, the lowest-order mode is \( s = 1 \). This differs from the case of dielectric substrates with positive permittivity. Nevertheless, one can rewrite this mode as

\[ k_{t,s} = \frac{\Psi}{d} \left[ \arctan\left( \frac{\epsilon_1}{\epsilon_t}\Psi \right) + (s - 1)\pi \right] \]

\[ = -\frac{\Psi}{d} \left[ \arctan\left( \frac{\epsilon_1}{\epsilon_t}\Psi \right) + s'\pi \right] \]

where we defined \( s' \equiv s - 1 \) for brevity. In order to form a vortex, we design the Au disk to have an Archimedean spiral shape, which can be mathematically written as \( r_{AS}(\phi) = r_0 + m\mod(\phi,2\pi)g \), where \( r_0 \) is the minimum value of the radius and \( g \) is a geometric parameter indicating the rate of increase of the radius with the
azimuthal angle $\phi$, defined as $g = r_{AS}(\phi = 2\pi) - r_0$. The integer parameter $m$ denotes the number of consecutive Archimedean arms in the pattern. We can calculate the phase of the polariton distribution at the top surface, given by the evolution of the polaritons launched from the edge:

$$\varphi(r, \phi) = k_{sl}(r_{AS} - r) + k_{s}d + \text{mod}(r_{AS} - r - d\text{cot}\theta, 2d\text{cot}\theta) \cdot (\angle R_1 + \angle R_2),$$

(6)

where the first and second term refer to the propagation phase of polaritons. The third term refers to the phase accumulation due to the reflection from the top and bottom surface, which should also consider a reflection phase shift at the substrate, since what is measured is the polariton distribution at the top. Considering the quantized model of the polariton waveguide in Eq. 4 and the dispersion relation in Eq. 5, we could further simplify this expression as

$$\varphi(r, \phi) = k_{sl}(r_{AS} - r + \frac{s}{2}) + s'2\pi.$$  

(7)

If we design the Archimedean spiral such that $k_{sl}(r_{AS} - r + \frac{s}{2}) = 2\pi l$, the phase accumulation of one Archimedean spiral is $(l + s')2\pi$. Therefore, the propagation phase accumulation after a rotation from $\phi = 0$ to $\phi = 2\pi$ is

$$\varphi_{pp} = (l + s') \cdot m2\pi$$  

(8)

Here, we focus on the $s' = 0$ mode, which is also more easily excited because it has smaller momentum mismatch with free-space plane waves. Under a circularly polarized plane wave excitation, due to the spin–orbit interactions in the near-field, we have the total phase accumulation

$$\varphi_{pp} = (l + s') \cdot m2\pi + \sigma 2\pi$$  

(9)

which yields the total topological charge

$$L = (l + s') \cdot m + \sigma$$  

(10)

where $s' = 0$ is the dominant mode. For left-handed structures, $\sigma = +1$ or $-1$ for left-handed circularly polarized (LCP) or right-handed circularly polarized (RCP) light, respectively.

We performed full-wave numerical simulations using the finite-difference time-domain commercially-available software Lumerical FDTD (https://www.lumerical.com/products/fdtd/). Two orthogonal linearly polarized waves with phase difference $\pm \pi/2$ were used to generate the circularly polarized excitation. An incident angle of $10^\circ$, which is chosen for achieving the best agreement between experimental results and simulations, is applied because an off-axis parabolic mirror is used to focus the beam on the samples in the experiments. A monitor was placed at 20 nm above the top surface to calculate the near-field images.

**Supplementary Information**

The online version contains supplementary material available at https://doi.org/10.1186/s43593-022-00018-y.

**Additional file 1: Figure S1.** Schematic illustration of the isofrequency contour for type II hyperbolic materials and cross-section view of the sample. Figure S2. Experimental results for 295 nm hBN flake covering Au disks with $m = 4$. Figure S3. Experimental results for 170 nm hBN flake covering Au disks with $m = 2$. Figure S4. Optical microscope images of hBN flakes. Figure S5. Setup to verify the preservation of circular polarization after a parabolic mirror. Table S1. Output power vs degree of linear polarizer for the setup in Figure S5a. Table S2. Output power vs degree of linear polarizer for the setup in Figure S5b. Figure S6. Near-field amplitude images at 1590.5 cm$^{-1}$. Figure S7. Simulation and experimental results for an hBN flake excited through an Au disk without spiral arms. Figure S8. Simulation and experimental results for an hBN flake excited through an Au spiral disks with $m = 1, 2,$ and $4$ under linearly polarized light.

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**Author contributions**

MW performed the experiments with the assistance from SC, MC, YA, GH. MW, GH, AA analyzed the data and wrote the manuscript with input from all authors. All authors discussed the results and commented on the manuscript. All authors read and approved the final manuscript.

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**Availability of data and materials**

All data are available upon request to the corresponding author.

**Declarations**

**Ethics approval and consent to participate**

Not applicable.

**Consent for publication**

Not applicable.

**Competing interests**

Cheng-Wei Qiu is an Editor for the journal, no other author has reported any competing interest.
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