Particle acceleration at relativistic shock waves... 
... and cosmic rays...

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Cosmic rays from relativistic shocks

→ energy budget: do relativistic sources contribute to the cosmic ray flux?

→ microphysics: how does relativistic shock acceleration operate (→ maximum energy)?
Cosmic rays from relativistic shocks

→ to match the flux at energies >10^{15} eV and <10^{18} eV:

\[ \dot{N}_\text{MW} \propto E^{18} \tau \text{ injected} \text{ in } \text{cr at } E \text{ flux matching } E \]

(assumes \( V_c = 10^{17} \text{ cm}^3 \))

→ for the long gamma-ray burst population:

\[ \dot{N}_{\text{GRB}} \sim 10^{-4} \text{ yr}^{-1}, \ E_{\text{GRB}} \sim 10^{51} \text{ erg}, \ \tau_{\text{conf}} \gtrsim 10^2 \text{ yr} \]

... however, rare sources: \( \dot{N}_{\text{GRB}} \tau_{\text{conf}} \ll 1 \) at \( \sim 10^{18} \text{ eV} \)?

→ for low luminosity gamma-ray bursts/trans-relativistic SNe:

\[ \dot{N}_{\text{LLGRB}} \sim 10^{-3.5\pm0.5} \text{ yr}^{-1}, \ E_{\text{LLGRB}} \sim 10^{50} \text{ erg} \]

⇒ relativistic supernovae and GRBs may potentially contribute to the CR flux at high energies (e.g. Milgrom + Usov 96, Dermer 02, Budnik et al. 08, Wang et al. 08, Calvez et al. 09, Chakraborti et al. 11, see also Eichler & Pohl 10, 11)

... main uncertainties: source rate, confinement time, injected E distribution

→ for pulsar winds: potentially interesting, but what is \( E_{\text{cr}} \)?

Relativistic Fermi acceleration - ultra vs non-relativistic

| Non-relativistic shock waves | Ultra-relativistic shock waves |
|-----------------------------|--------------------------------|
| \( \beta_{\text{sh}} \ll 1, \gamma_{\text{sh}} \approx 1 \) | \( \beta_{\text{sh}} \approx 1, \gamma_{\text{sh}} \gg 1 \) |
| *diffusive* shock acceleration | *non diffusive* (upstream) because \( \beta_{\text{sh}} \sim \beta \sim 1 \) |
| *parallel, oblique or perpendicular configuration* | *nearly always perpendicular* (=superluminal) |
| *scattering in large scale turbulence* | *scattering in small scale turbulence* \( (l \ll r) \) |
| *self-amplification of turbulence* | *self-generation of e.m. microturbulence* |
| *possibly Bohm scattering* in self-amplified field | *no Bohm scattering* in self-generated field: \( \text{precursor size } \sim r_{\text{max}}/\gamma_{\text{sh}}^3 \) |
Relativistic Fermi acceleration - energy gain, kinematics

**Energy gain:**
\[
\frac{\Delta E_{up}}{E_{up}} = \gamma_{\text{bl} \text{ext}}^2 \left(1 + \beta_{\text{bl} \text{ext}} \cos \theta_{\text{down-up}} \right) \left(1 - \beta_{\text{bl} \text{ext}} \cos \theta_{\text{up-down}} \right) - 1
\]

with relative up-down Lorentz factor: \(\gamma_{\text{bl} \text{ext}} \equiv \gamma_{sh}/\sqrt{2}\)

**Crucial difference w.r.t. non-relativistic Fermi:**
- shock wave velocity \(\sim c \sim \) particle velocity

\(\Rightarrow\) distance between particle and shock wave after time \(t\):
\[(1-\beta_{sh})ct \sim c \frac{t}{(2\gamma_{sh})^2}\]

\(\iff \) deflection by \(1/\gamma_{sh}\)

\(\Rightarrow\) energy gain \(\sim \gamma_{sh}^2\) at first interaction, then \(\sim 2\)

\(\Rightarrow\) precursor length scale \(\sim r_L/(2\gamma_{sh})^3\)

Note: for parallel shock waves, precursor length scale \(\sim \) infinite as some particles may escape along field lines

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Non-relativistic shock waves

- \(\beta_{sh} \ll 1, \gamma_{sh} \approx 1\)
- **diffusive** shock acceleration
- parallel, oblique or perpendicular configuration
- scattering in **large scale** turbulence
- self-amplification of turbulence
- **possibly Bohm scattering** in self-amplified field

Ultra-relativistic shock waves

- \(\beta_{sh} \sim 1, \gamma_{sh} \gg 1\)
- **non diffusive** (upstream) because \(\beta_{sh} \sim \beta \sim 1\)
- **nearly always perpendicular** (=superluminal)
- scattering in **small scale** turbulence \((l \ll r_L)\)
- self-generation of e.m. microturbulence
- **no Bohm scattering** in self-generated field:
  precursor size \(\sim r_{L,\text{max}}/\gamma_{sh}^3\)
Ultra-relativistic superluminal shock waves

\[ B_{\perp|\text{sh}} = \gamma_{\text{sh}} B_{\perp|u} \]
\[ B_{|||\text{sh}} = B_{|||u} \]

⇒ ultra-relativistic shock waves are mostly perpendicular (superluminal)

Note: behind the shock: \( \langle E_\perp \rangle \) close to \( \langle E_\parallel \rangle \)
\( \gamma_e \) close to \( \gamma_p \) or \( m_e/m_p \) (depending on \( \sigma, \gamma_{\text{sh}} \))

(\( \Leftrightarrow \) efficient heating, indicated by PIC simulations)
Sironi & Spitkovsky 11

Relativistic Fermi acceleration - ultra vs non-relativistic

| Non-relativistic shock waves | Ultra-relativistic shock waves |
|-----------------------------|--------------------------------|
| \( \beta_{\text{sh}} \ll 1, \gamma_{\text{sh}} \approx 1 \) | \( \beta_{\text{sh}} \approx 1, \gamma_{\text{sh}} \gg 1 \) |
| • **diffusive** shock acceleration | • **non diffusive** (upstream) because \( \beta_{\text{sh}} \approx \beta \ll 1 \) |
| • parallel, oblique or perpendicular configuration | • nearly always perpendicular (=superluminal) |
| • scattering in **large scale** turbulence | • scattering in **small scale** turbulence (\( l \ll r_L \)) |
| • self-amplification of turbulence | • self-generation of e.m. microturbulence |
| • **possibly Bohm scattering** in self-amplified field | • **no Bohm scattering** in self-generated field: \( \text{precursor size} \sim r_{\text{max}}/\gamma_{\text{sh}}^3 \) |
Relativistic Fermi acceleration - oblique shock waves

Ultra-relativistic shock waves are generically superluminal:
- the intersection between a magnetic field line and the shock front moves faster than c
- if a particle is tied to a field line, this particle cannot return to the shock front...

Scattering from turbulence:
- large scale turbulence does not help:
  - the particle cannot execute more than 1 1/2 cycle...
  - up → down → up → down then advection to ~∞ (ML et al. 06, Niemiec et al. 06)

large scale: w.r.t. typical Larmor radius (downstream frame) of accelerated particles

conditions for successful relativistic Fermi acceleration: $\delta B \gg B_0$, $\ell_c \ll r_L$

for reference: $r_L \sim 10^{12}$ cm $\left( \frac{B_0}{1 \mu G} \right)^{-1} \left( \frac{\gamma_{sh}}{300} \right)$ $\ell_c \sim 10^{20}$ cm in ISM

Relativistic Fermi acceleration - ultra vs non-relativistic

Non-relativistic shock waves $\frac{\beta_{sh}}{1} < 1, \gamma_{sh} \approx 1$

- diffusive shock acceleration
- parallel, oblique or perpendicular configuration
- scattering in large scale turbulence
- self-amplification of turbulence
- possibly Bohm scattering in self-amplified field

Ultra-relativistic shock waves $\beta_{sh} \approx 1, \gamma_{sh} \gg 1$

- non diffusive (upstream) because $\beta_{sh} \sim \beta \sim 1$
- nearly always perpendicular (=superluminal)
- scattering in small scale turbulence ($l \ll r_L$)
- self-generation of e.m. microturbulence
- no Bohm scattering in self-generated field:
  precursor size $\sim r_{L,max}/\gamma_{sh}^{3}$
Micro-instabilities at a relativistic shock front

→ shock reflected and shock accelerated particles move in upstream background field with Lorentz factor $\gamma^2_{sh}$ along shock normal, forming an unmagnetized beam of Lorentz factor $\gamma^2_{sh}$ and opening angle $1/\gamma_{sh}$

→ neutral beam instabilities: (e.g. Bret 09)
  - Weibel/filamentation (Gruzinov & Waxman 99, Medvedev & Loeb 99, Lyubarsky & Eichler 06, Wiersma & Achterberg 04, 07, 08; ML & Pelletier 10, 11; Rabinak et al. 10, Shaitsultanov et al. 11)
  - Cerenkov resonance with plasma eigenmodes: ML & Pelletier 10,11
  - oblique two stream with electrostatic modes $\omega_p = k_x v_{beam}$
  - Whistler waves $\omega_{WH} = k_x v_{beam}$

→ charged current instabilities:
  - Buneman mode (ML & Pelletier 11) ... efficient source of electron heating
  - Bell instability... for parallel shocks (Bell 04, Reville et al. 06, ML & Pelletier 10)

→ main limitation: very short precursor, length $\sim r_D/\gamma_{sh}^3 \sim \gamma_{sh}^{-1} c/\omega_{ci}$

Phase diagram for relativistic shock acceleration

at high magnetisation, e.m. precursor → wakefield heating /acceleration e.g.Hoshino et al. 92, Gallant et al. 92, Lyubarsky 06, Hoshino 08

too short precursor... no micro-instabilities... no Fermi acceleration...

GRB int. shocks blazar shocks?

micro-instabilities grow ⇒ Fermi acceleration...

GRB in ISM

PWNe

ML & Pelletier 10,13
Relativistic shock acceleration to UHE?

Relativistic Fermi acceleration - ultra vs non-relativistic

Non-relativistic shock waves
\[ \beta_{sh} \ll 1, \gamma_{sh} \approx 1 \]
- **small** energy gain per cycle \( \sim \beta_{sh} \ll 1 \)
- **small** escape probability \( \sim \beta_{sh} \ll 1 \)
- powerlaw spectrum, index \( s \approx 2 \)
- magnetized turbulence seeded by accelerated particles...
  - ... on **large** scales \( \sim r_\Lambda \gg c/\omega_p \)
- Maximal energy:
  \[ E_{\text{max}} \sim \beta_{sh} Z e B_{\text{amp}} R \]

Ultra-relativistic shock waves
\[ \beta_{sh} \approx 1, \gamma_{sh} \gg 1 \]
- **large** energy gain per cycle \( \sim 2 \) if magnetization is low enough
- **large** escape probability \( \sim 0.3 - 0.4 \)
- powerlaw spectrum, index \( s \approx 2.3? \)
- magnetized turbulence seeded by accelerated particles...
  - ... on **microscopic** scales \( \sim r_\Lambda / \gamma_{sh}^3 \gg c/\omega_p \)
- Maximal energy:
  \[ E_{\text{max}} \sim \gamma_{sh} Z e B_0 R \]
  \( \sim 10^{17} \text{ eV} Z B_{-5} R_{17} \gamma_{sh, 2.5} \)
Summary

→ at low magnetization (e.g., ISM), relativistic shock acceleration develops; accelerated particles build the microturbulence in which they scatter and get accelerated...

→ some crucial differences wrt non-relativistic diffusive shock acceleration:

  → acceleration is not diffusive upstream, but shock drift
  → ultra-relativistic shock waves are very nearly perpendicular
  → the precursor is strongly limited in extent: \( r_{\text{L,max}} / \gamma_{\text{sh}}^3 \)
  → Bohm scattering in the self-amplified cannot take place
  → \( E_{\text{max}} \sim \gamma_{\text{sh}} Z e B_0 R \)

→ acceleration at mildly relativistic shock waves is yet another story:

  → superluminal nature not generic
  → precursor size may come close to \( r_{\text{L,max}} \)
  → near Bohm scattering in the self-amplified cannot be excluded
  → large \( E_{\text{max}} \)?