Multi-Target Search in Euclidean Space with Ray Shooting
(Full Version)

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Abstract
The Euclidean shortest path problem (ESPP) is a well-studied problem with many practical applications. Recently a new efficient online approach to this problem, RAYSCAN, has been developed, based on ray shooting and polygon scanning. In this paper we show how we can improve RAYSCAN by carefully reasoning about polygon scans. We also look into how RAYSCAN could be applied in the single-source multi-target scenario, where logic during scanning is used to reduce the number of rays shots required. This improvement also helps in the single target case. We compare the improved RAYSCAN+ against the state-of-the-art ESPP algorithm, illustrating the situations where it is better.

Introduction
Euclidean shortest path finding (ESPP) has many variants. We examine obstacle-avoiding 2D shortest (i.e. optimal) path determination, which is a well-studied problem with many practical applications, in e.g. robotics and computer games (Algfoor, Sunar, and Kolivand 2015). Obstacles are an effective way to represent the world directly, for example, a wall can be represented as a rectangular obstacle that has to be navigated around.

ESPP algorithms can be very fast in static environments, i.e. where obstacles do not change between shortest path queries, as this allows for longer pre-processing. Search queries in a dynamic environment are considerably harder, as the obstacles change regularly, therefore any pre-processing has to be updated to account for these changes.

Most existing approaches to ESPP first convert obstacles into another representation to perform a search, either via a visibility graph (Lozano-Pérez and Wesley 1979) or a navigation mesh (Cui, Harabor, and Grastien 2017) for example. A recent approach is RAYSCAN (Hechenberger et al. 2020). It partially builds a visibility graph on the fly by using ray shooting (also called ray casting) to discover blocking obstacles, and then scanning along the edges of such obstacles to find ways around. RAYSCAN is comparable to an on-the-fly partial edge generation of a sparse visibility graph (SVG) running taut A* (Oh and Leong 2017); as such it does not require pre-processing, as it makes use of the natural polygonal representation of obstacles. Much of the runtime of RAYSCAN is taken up by the ray shooting.

In this paper we extend RAYSCAN with various improvements that are aimed at reducing the number of ray shots required. We call this advancement RAYSCAN+. We also tackle the multi-target variant, where we find the shortest path from a start point s to all target points \( t \in T \).

We compare RAYSCAN+ against the state-of-the-art ESPP algorithm Polyanya (Cui, Harabor, and Grastien 2017); including tests with static and dynamic environments, single- and multi-target search. We show RAYSCAN+ significantly improves upon RAYSCAN and that RAYSCAN+ is competitive with Polyanya (in particular in highly dynamic scenarios).

RAYSCAN
RAYSCAN is presented in this paper as a 2D ESP algorithm, made to work with a 2D environment represented as a set of non-intersecting polygonal obstacles (inner-obstacles), and a single enclosing polygon (enclosure or outer-obstacle), containing and non-intersecting all inner-obstacles. The outer-boundary is defined as the convex hull of the enclosure.

RAYSCAN does not provide the method of searching, rather it can be viewed as a fast method of producing a subset of edges of the visibility graph during the search. It uses the A* algorithm (Hart, Nilsson, and Raphael 1968) to drive the search, using start \( s \), target \( t \) and corner points on the polygons as the nodes in the search. When pushing a succes-
The approach to navigate around an obstacle is the scan routine, where we trace a scan line along the polygon in a clockwise (cw) or counterclockwise (ccw) orientation w.r.t. expanding node $u$, e.g., Figure 1 shoots ray from $u$ to $c$, finding intersection $K$; a cw-scan starts from the scan line $uK$ and has it sweep in a cw direction along polygon edge to point $d$ and then stop, finding the $d$ point which we refer to as a turning point. The ccw-scan has the scan line reach turning point $c$. Determining which point is considered a turning point can be done with two different approaches: the forward- and convex-scanning methods; that are detailed under the Scanning section. If there are no obstacles between $u$ and a turning point, it is considered a visible point and is a successor node of $u$, otherwise it is a blocked point.

The scanning process does not end at finding the turning point; it instead recurses into new scans. Referring to Figure 2 for a blocked point like $c$, the scan will split into two scans, one cw the other ccw starting at the intersection point $K$. For a visible point like $d$ (found from intersection $K$), we add $d$ as a successor and shoot the ray past the point to find the next polygon up (intersection $L$); then we continue the scan in the same orientation (cw) from that intersection ($L$). The whole recursive process starts from a single scan, where we call the whole process the full scan.

The recursive scanning process makes use of angled sectors to improve performance and as a base condition of the recursion. An angled sector is depicted as $AS(\alpha_{ccw}, \alpha_{cw})$, where the angled sector is defined as the angular region starting from the ccw pivot angle $\alpha_{ccw}$, turning ccw towards angle $\alpha_{cw}$. Referring to Figure 1 the shaded area circular to $uI$, $uM$ is an angled sector $AS(uI, uM)$. During a scan, the scan line must always remain within this area; if at any point the scan line leaves the sector, the recursion ends with no (additional) turning point discovered. For the start point we abuse notation and use a $360^\circ$ angled sector $AS(sI, sI)$.

An angled sector can also be split into two angled sectors. We use the notation $SPLIT(AS(\alpha_{ccw}, \alpha_{cw}), \vec{p}, d)$ to split the angled sector $AS(\alpha_{ccw}, \alpha_{cw})$ along $\vec{p}$, returning a d-split (ccw-split or cw-split). The ccw-split returns $AS(\alpha_{ccw}, \vec{p})$, while a cw-split returns $AS(\vec{p}, \alpha_{cw})$. This split is used by the recursive scanning, where for turning point $p$, any other ccw scan will take the ccw-split of the current angled sector along $p$, and vice versa for cw scan.

Every expanding node $u$ (excluding $s$) has a special angled sector known as the projection field. This angled sector encompasses the region that all successors of $u$ must fall within, since any shortest path via $u$ must be taut to the obstacle that $u$ is part of. In Figure 2 the projection field for $u$ is $AS(su, u\vec{a}) = AS(uI, uM)$. The full scan begins with the projection field, resulting in significant speedups by limiting the area the scan runs in.

A point on a polygon is considered to be a concave point.
if its inside angle is more than 180°, otherwise is a **convex point**. Expanded nodes, with the possible exception of s, will always be convex, as you can only bend around such points.

**RayScan Example**

Consider the example execution of RayScan illustrated in Figure 2 from [Hechenberger et al. 2020]. Initially we expand s by shooting a ray from start s to target t (Figure 2a). We find this is blocked by a polygon (the outer polygon). Then we recursively scan to move around the blocking polygon from I (Figure 2b). Scanning ccw we skip B since the ray cannot bend around the polygon here and reach node A which is on the convex boundary of the map, so we stop the scan. There is no way around the (outer) polygon in this direction. Scanning cw we skip C and reach D. We then shoot a ray towards D, which is blocked by the polygon abcd at intersection J (Figure 2c). We recursively scan this polygon. The cw scan will find a as a turning point, shooting to it reveals it is visible and adds to the queue, and shooting past it will hit the outer-boundary, thus the cw scan does not continue. The recursion in the other orientation, ccw, finds turning point d, shooting to it and adding it to the queue (Figure 2d). We shoot past d and find intersection K, scan ccw from here we leave the angled sector boundary st, ending the recursion.

Next we expand d (Figure 2e). The target is not within the projection field AS(su, dc) so we scan both extreme edges. We shoot a ray d c and find that c is visible from d, thus c is added as its successor. We scan the polygon that blocks dc in ccw and leave the angled sector without discovering any turning point and stop. The other extreme ray sd does not discover any successor. Next, we expand c in the same manner as d (Figure 2f), scanning cw to find only D. The next lowest f value is a, which we expand to find b (not shown in figure). Node b is then expanded (not shown in figure) and it finds D, although a shorter path to D is already discovered thus it is not added. Now, we expand D and find E scanning cw (Figure 2g).

Finally, we expand E and notice that t is within the projection field (Figure 2h), thus we shoot to t. We find that t is visible from E, thus we have found a path to t. This path is the shortest path because the node E had the minimum f value which guarantees that all other paths to t are no shorter than this path.

**RayScan Improvements**

The first contribution is a number of improvements to RayScan. They do not change the underlying algorithm, but improve components hence the proof of optimality of RayScan continues to apply.

**Scan Overlap**

The expansion of a node in RayScan relies on multiple full scans, two from both ends of the projection field, called the projection scans (for any expanding node not s), and an additional two if the target is within the projection field, called the target scans.

These full scans usually overlap with each other, resulting in redundant work of up to three complete scans of the region. We introduce two methods for reducing this overlap.

**Refinement by sector:** The first method, implemented in RayScan, refinement by sector, works by refining the initial angle sector given to the full scan. Normally, the angled sector for the full scan is the projection field for projection scans, and the projection field split by u and u for target scans. When we do a full scan that will overlap a previous scan for that expansion, work will become redundant when the scan line passes a ray shot by a previous full scan. To avoid this, we want to identify the closest cw or ccw to the starting scan line and split the angled sector to prevent the scan from passing it.

**Refinement by ray:** The second method, implemented in RayScan+, refinement by ray, uses the projection field for all full scans, instead ending a scan early when we try shooting a ray towards a turning point that we have already shot to this expansion.

For example, in Figure 2i starting with a projection scan cw from su, it starts with the angled sector of the projection field AS(su, su), shoots the rays from u along su, ub, u and su. The next full scan starting ccw from su will have a reduced angled sector of the projection field AS(su, su).

**Scanning**

**Forward scan:** The scan is split into two types, cw and ccw. The original RayScan method uses a scan line that rotates in the specified orientation until the line is forced to turn the opposite way when encountering a polygon vertex. This is a turning point. We call this the forward scan method.

The idea of a turning point is to find a possible point that is visible from the expanding node and also on the shortest path. The forward scan method can find a concise point, which can never be on a taut path and hence not on any shortest path. We also know that a concave turning point can never be visible due to the nature of the scan, if the scan line goes from cw to ccw and is concave, the progressing line must be blocking the point.

For example, consider Figure 11 where C, D and E are concave points, when performing a cw scan from ray sl, a forward scan will stop at E (as E to F orientates ccw) and shoot a ray. We see that E is blocked by its own polygon, though it will find F and H after the recursive scan.

**Convex scan:** We consider an alternate scanning method we call convex scan, where we only shoot at convex points, which are the only points that can appear on a shortest path. Referring back to the previous example, the scan line will instead pass over E even though it goes opposite to the scan-
Avoiding Ray Shooting

RAYS can be improved by avoiding shooting rays that are unnecessary. We list three different methods.

Blocking extension: Originally, RAYS starts expanding a node $u$ with the target scans (if $t$ is within $u$’s projection field), followed by the projection scans. It is possible to instead start with the projection scans followed by target scans. The blocking extension makes use of this switch in the order of the full scans to potentially avoid some target scans entirely. If during the projection scans, we see that the target is not visible (i.e., is blocked by an obstacle edge we scanned over), then we do not have to shoot the target.

For example, in Figure 3b, we show several possible targets $t_{1,2,3}$. Expanding node $u$ will shoot ray $s u$ from $u$ and do a CW scan to find $A$. During that scan $t_1$ is passed by the scan line behind the obstacle scanned, meaning we do not need to shoot it later. After shooting to $A$, a CW scan to find $B$, sees that $t_2$ is blocked, while a CW scan will find $C$ and pass over $t_3$, except in this case $t_3$ is in front of the obstacle so we still need to shoot towards it.

Skip extension: The skip extension attempts to skip (scan) past a turning point that cannot be on the shortest path. We can deduce a skip candidate point $v$ from $u$ if the following condition holds: $g(u) + ||u v|| > g(v)$. Formally, expanding $u$, when finding a turning point $v$ that has been reached by another point $x$, then $g(v) = g(x) + ||x v||$, and if the $g$ value $u$ will give $v$ as a successor $g(u) + ||u v||$ is larger than $v$’s current $g$ value, than this point may be skipped, as we know that candidate point $v$ is not on a shortest path via $u$. But we may need to find a shortest path to a point past $v$ so the scan may need to continue.

For example, referring to Figure 3c, the shortest path is $s - u - b - c - t$. When expanding $u$, we perform a CW projection scan that starts at intersection $I$, then finds turning point $a$. We normally shoot to $a$ and continue the CW scan from $J$, except in this case we notice $a$ has a shorter path from $s$ via $x$. Since we do not want to reach $a$ from $u$ but need to continue the scan to find the successor on the shortest path $b$, we want to avoid shooting the ray to $a$. Instead, we continue the scan without shooting, ignoring all turning points (if any). If the scan line orientation reaches what would be the ray $(u a)$ when reaching $J$ without leaving the angled sector (as is the case in this example) then we deduce that we do no need to shoot to $a$ and can continue the CW scan from $J$ to eventually find the turning point $b$.

Unlike this example, if we are unable to pass ray $u a$, either from leaving the angled sector, reaching $a$ again or encountering a certain number of points (a limit for performance reasons), then we must shoot to $a$ to continue the scan.

In any case, the scan must be continued CW from $J$ as $b$ is on the shortest path from $u$.

Bypass extension: The final extension outlined in this paper is the bypass extension, which is similar to the skip extension except it can be applied to turning points that are not skip candidates, which do not appear that often. Bypass seeks to skip shooting a ray to a node in the same way as the skip extension, except it also checks if the scan line sweeps over the target, and if it does then we can bypass the node. For example; with Figure 3d, when expanding $E$, the CW projection scan reaches $C$ and attempts to bypass it. The scan line will continue from $C$ past $B$ before reaching the ray that would be $EC$; if $t_1$ is our desired target then the scan line sweeps over it, meaning we cannot bypass $C$ (as it is on the shortest path); if however $t_2$ is our desired target; although we sweep past it, we did not sweep over $t_2$ as it is blocked by the polygon, thus we can bypass $C$, which is done the same way as skip. The scan then immediately leaves the angled sector producing no successors.

Sometimes a node on the shortest path will be bypassed when expanding another node on the shortest path; take Figure 3d as an example, when expanding node $F$: we perform a CW projection scan that finds turning point $H$ (on the shortest path), except the point can be bypassed, thus RAYS will skip over it to find turning point $J$. This is where con-
sidering the angled sector is important, as that bypass was
done with the projection field \( AS(F\hat{K}, s\hat{F}) \); however, since
we did not shoot a ray to \( H \) that angled sector remains the
same, this is important as it allows us to reach \( H \) further in
the full scan. After find turning point \( J \) (on the shortest
path), we shoot a ray and if it is blocked, thus we recur,
the \( ccw \) scan will find turning point \( K \) (also on the shortest
path), which if we can reach will get us to \( J \). We shoot
a ray to \( K \) to find it again blocked by line segment \( GH \),
which when we recurse scan \( ccw \) will again find \( H \), except
this time in the check to bypass it, we have the angled sector
\( AS(F\hat{K}, FJ) \) and thus, trying to bypass will quickly leave
the scan’s angled sector along the line segment \( HI \), there-
fore bypass has failed and we need to shoot to \( H \).

Caching Rays Shot

The ray shots are the most expensive part of \( RayScan \), as
we will show in the experimental sections; therefore, redu-
cing the cost of these shots can achieve great performance
improvements. We make use of a simple method of remem-
bering rays we shoot so that subsequent queries that shoot
the same rays can just look at the result.

For our method for caching rays, we store the results of
a ray shot from node \( u \) to node \( v \), where \( u, v \notin \{s,t\} \). The
dynamic cases changes the results of stored ray, thus any
changes in the environment invalidates the whole cache, and
we start again.

Extension to Multi-Target ESPP

\( RayScan^+ \) can be extended to support single-source multi-
target searches. A naive way to do this is, during expansion,
instead of looking for a single target within the projection
field and shooting a ray, find all targets within the projection
field and shoot rays to all. A change of the heuristic function
\( h(u) \) either to 0 or to the distance form \( u \) to the closest target
maintains the admissibility of the heuristic.

The number of additional rays shot to every target signifi-
cantly increases runtime, which can be counteracted by the
target blocking extension.

Algorithm 1 details a successor generator for \( RayScan^+ \), modi-

ified from \( RayScan \) to support multiple targets. It can find paths from a start node \( s \) to a list of target
nodes \( T \).

Function \( \text{START}_\text{SUCCESSORS}(s, T) \) (line 1) will generate
successors for source node \( s \) that will lead to all targets
\( T \). Function \( \text{SUCCESSORS}(u, T, F) \) similarly finds suc-
cessors for the expanding node \( u \) for all \( t \in T \) within its pro-
jection field \( F \).

Function \( \text{POPULATE}_\text{TARGETS}(T, F) \) (line 2 and 3) filters
the list of targets \( T \) that are within projection field \( F \) and
orders them circularly within the angled sector. These targets
are stored within the \( \text{TARGETS}_\text{REMAINING} \) global variable.

Function \( \text{SHOOTRAY}(u, a) \) handles the ray shooting and
returns the intersecting polygon \( p \), the intersection point \( I \),
and a boolean \( R \) indicating if shot in direction \( a \) from \( u \) has
already been made this expansion. Lines 23 to 25 check for
all collinear points the ray intersects with, and pushes the
closest visible one as successor (if present) (line 25). The ray
will stop when it is blocked by an edge; if it only touches the
corner of a polygon without entering (i.e. a collinear point)
it will pass through it.

When expanding the start node \( s \), we shoot to all the tar-
gets (line 3). The \( \text{TARGET}_\text{SUCCESSORS}(u, F) \) (line 12) will
handle all the target scans by shooting from the expanding
node \( u \) towards all remaining targets (when expanding node
\( u \) is the start node, all targets are the remaining targets).
The successors will continue to shoot to targets (lines 13 to 21). Specifically, it selects any of the remaining targets \( t \) (line 14), removes it from the \textsc{Targets\_Remaining} variable (line 15) and shoots towards it (line 16). If \( t \) is visible, it is pushed as a successor (line 18). Otherwise, the polygon that blocks the ray is scanned \( \text{cw} \) and \( \text{ccw} \) while potentially removing additional targets that are found to be blocked (lines 20 and 21).

Expanding any node \( u \) other than the start node will first do the projection scans by shooting along the extremes of its projection field \( F \) and scan inwards of \( F \) along the blocking polygon (lines 6 to 10) like \textsc{RayScan}. Line 9 is the refinement by ray. These scans may have blocked some targets in \textsc{Targets\_Remaining} which are removed. The function \textsc{Target\_Successors}(\( u, F \)) is called to shoot towards these remaining targets if any (line 11).

Function \textsc{Scan}(\( u, p, I, F, d \)) is used to scan the polygon \( p \) which blocked a ray shot from \( u \) at the intersection point \( I \). This function scans \( p \) starting at \( I \) in orientation \( d \) (\( \text{cw} \) or \( \text{ccw} \)) while restricted to the angled sector \( F \). We first find a turning point by sweeping a line from \( I \) in orientation \( d \) to find a suitable turning point \( n \) (line 30). \textsc{RayScan} uses convex scan to find a turning point whereas original \textsc{RayScan} used forward scan. This scan also uses the blocking, skipping and bypass extensions and removes targets that are found to be blocked during the scan (line 31).

If the scan leaves the projection field \( F \) or touches the outer-boundary, we terminate the scan (lines 32 and 33). Otherwise, we shoot a ray from \( u \) towards \( n \) (line 34). Refine by ray is enforced on line 35 to terminate the scan if a ray towards \( n \) was previously shot from \( u \). If not, we recurse the scan, which differs slightly depending on whether \( n \) is visible from \( u \) (line 36) or not. Like original \textsc{RayScan}, if the visible turning point is a forward turning point, we recurse the scan in the same orientation with a split angled sector (line 38). Otherwise, if it is a backward turning point, we recurse both ways, handling the opposite scan (lines 40 and 41) and continuing to find the next forward turning point (line 42). For a non-visible turning point, we follow the same principles as \textsc{RayScan} in that we try to scan \( \text{cw} \) around the blocking point \( p \) (line 44) and \( \text{ccw} \) (line 45).

### Experiments

The data set used for the experimentation stems from the Moving AI Lab pathfinding benchmarks (Sturtevant 2012). We make use of three representative Starcraft maps: Aftershock, ArcticStation and Aurora, which are converted from the grid representation to Euclidean polygonal obstacles.

These experiments were conducted on a machine with an Intel i7-8750H, locked at 2.2 GHz with boost disabled. Every search was run 7 times, discarding the best and worst results, then averaging the remaining 5. The source code will be made available online.

### Comparison against Original RayScan

The improvements listed in this paper can be seen in Figure 4, four different RayScan algorithms are compared against Polyanya. \textsc{RayScan} shows the original RayScan implementation (Hechenberger et al. 2020), which uses the forward-scan method and refinement by sector (although it was not detailed in the paper). \textsc{RayScan} uses an updated implementation using the convex-scan method with target blocking, skip and bypass extensions and refinement by ray. The \textsc{RayScan}+-Oracle is the same as \textsc{RayScan} except the shooting of rays cost comes for free. \textsc{RayScan}+-Cache

![Figure 4: Scatter plots of Polyanya runtime vs RAYSCAN implementations for single target search](image-url)

Table 1: Comparing separate extensions (rows) on different maps (columns); single-target average runtime per query (all times are in ms); (R) Original \textsc{RayScan}; (N) convex scan \textsc{RayScan}^+; (B) blocking extension; (S) skip extension; (P) bypass extension; and (C) ray caching.

| Algs | Aftershock | Aurora | ArcticStation |
|------|------------|--------|--------------|
| R    | 568        | 3604   | 2752         |
| N    | 400        | 2548   | 2689         |
| NB   | 388        | 2492   | 2627         |
| NS   | 375        | 2392   | 2090         |
| NP   | 262        | 1638   | 1859         |
| NC   | 150        | 736    | 683          |
| NBSP | 248        | 1560   | 1395         |
| NBSPC| 132        | 723    | 625          |

1[https://bitbucket.org/ryanhech/rayscan/](https://bitbucket.org/ryanhech/rayscan/)
caches the results of ray shots from previous queries. The implementation of the ray shooting is done by drawing Bresenham’s lines on a grid (Bresenham 1965).

Our Polyanya implementation code is modified from the implementation by Cui, Harabor, and Grastien (2017). The navigation mesh was generated by constrained Delaunay triangulation (CDT) using Fade2D. The triangle faces were greedily merged to make larger convex faces to improve Polyanya’s performance.

Figure 4 illustrates the significant advantages of RAYSCAN+ over RAYSCAN on single target problems. These results show that Polyanya is still faster for searching than RAYSCAN+ for static environments, but adding caching to RAYSCAN+ achieves very competitive results. The disadvantage of caching rays is that for large maps this can lead to higher memory usage, as the number of rays shot can be in the order of the number of edges in the SVG. In the case where memory usage gets too large, caching strategies can be employed to ensure the memory usage never exceeds a given budget.

Table 1 provides an ablation study of the extensions we propose. Caching is clearly the most important extension (since ray shots are the most expensive part of the algorithm). Each other extension has a positive effect, with bypass the most effective (since the environments we test on have many non-convex polygons) and blocking the least (for single targets). The positive effects combine so best is using all of them.

Dynamic Single Target Scenarios

We conduct experiments for dynamic environments with results shown in Figure 5. These tests show the accumulated runtime (y-axis) of the algorithms over 1000 queries (x-axis), where we insert or remove 10, 25 or 50 small convex polygons in the environment every 10 queries, while leaving the original map untouched (i.e. no polygon originally on the maps is modified).

Inserting obstacles into Polyanya’s mesh is done by splitting the faces the polygon lines intersect and constraining them. To remove them, we clear the constraint between the

faces along the obstacle edges. Polyanya also needs to find which face a point holds (for start, target and insert of new polygon). This is done by selecting a face and moving across the mesh in a direct line to the point.

Figure 5 shows that RAYSCAN+’s search performance is fairly consistent, as changes to its data structures are fast and do not degrade performance. Polyanya shows the performance is degrading as removing the obstacles is degrading the mesh; while this can be alleviated by repairing the mesh, this does incur additional costs in removal of unnecessary vertices and/or merging smaller faces together.

Polyanya’s performance degrades significantly as the environment is changed more rapidly, because the navigation mesh gets more and more complicated. Maintaining the mesh with methods by Kallmann, Bieri, and Thalmann (2004) that maintains the CDT or van Toll, Cook IV, and Geraerts (2012) that makes use of a Voronoi diagram become mandatory to maintain Polyanya’s performance. In highly dynamic maps RAYSCAN+ is faster.

Multi Target Scenarios

In Figure 6a-c, we consider multi-target scenarios where the targets are clustered fairly close together. We have two types of scenarios: Figure 6a and 6b split the Aurora map into 10x10 grid cells and randomly choose a number of different targets within one of those cells. Figure 6c evaluates the impact of a larger number of targets which are placed in a randomly chosen cell from a 6x6 grid.

Multi-target Polyanya makes use of the interval heuristic (Zhao, Taniar, and Harabor 2018), which produces a Dijkstra like expansion for Polyanya. This can impact the search performance in clustered examples since Polyanya is not actively seeking out the targets; however, Polyanya only needs to consider each target during the search when it reaches a face containing the target.

Using RAYSCAN+ for multiple targets makes heavy use of the blocking extension, as otherwise we need to shoot to all targets within the projection field of each node expanded, resulting in many additional rays. RAYSCAN+ uses the Euclidean distance to the closest target to the vertex. For a small number of targets (e.g. Figure 6a), the heuristic and selection of targets within each projection field is done by

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3https://www.geom.at/products/fade2d/
Figure 6: Multi-target tests on map Aurora; (a)-(c) are with clustered targets; (d)-(f) are with sparse targets. C-n details tests with n number of targets of Polyanya (x-axis) vs RAYSCAN+ (y-axis)

Checking all targets. For larger number of targets (e.g. Figure 6c) we use a Hilbert R-Tree (Kamel and Faloutsos 1999) to speed up these calculations.

Examining Figure 6a we see that for clustered targets Polyanya is fairly consistent as the number of targets change, compared to RAYSCAN+, which slows down as the number of targets grow. This is expected as Polyanya only needs to handle targets at the beginning (to locate each target’s face) and the end (to finalise the path to target); whereas RAYSCAN+ needs to deduce for each node expansion which targets are in its projection field.

Figure 6b illustrates that when caching the rays, RAYSCAN+ is competitive with Polyanya. It is faster than Polyanya when the targets are clustered together. The high spikes for C-5 are early searches still building up the cache.

Figure 6c compares results for a larger number of targets. This highlights a weakness of RAYSCAN+ since it has to consider each target within its projection field for every expansion. RAYSCAN+ must do this as it only produces a subset of successors, which means vital successors needed to reach a target might not be found without shooting to that target from certain nodes. Until a method of addressing this weakness is found, RAYSCAN+ will struggle with very large numbers of targets compared to Polyanya.

Figure 6d-f compares the methods for sparse target points, which are distributed at random around the map. The nearest point heuristic for RAYSCAN+ is less effective in these scenarios, resulting in RAYSCAN+ performance degrading more with each additional point, as the Polyanya interval heuristic is more useful in these cases. We see that the RAYSCAN+-Cache speeds perform slightly worse at 50 targets, but is highly competitive with fewer targets.

An ablation study shows that for multi-target blocking is an important extension: for 5-50 targets it leads to 20% improvements; for 500-2000 it can speed up by more than $2\times$.

**Conclusion**

RAYSCAN+ is an efficient method for Euclidean shortest path finding in dynamic situations, since it requires almost no pre-processing to run. In this paper we show how to substantially improve RAYSCAN+ by reversing the order of target and projection scans, and reducing the number of vertices we need to shoot rays at. We extend RAYSCAN+ to shoot at multiple targets. RAYSCAN+ is competitive with the state of the art ESPP method Polyanya when we cache ray shots, which make up the principal cost of RAYSCAN+. Future work will examine better methods to maintain ray shot caching in dynamic situations.
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