Subleading Isgur-Wise Function of $\Lambda_b \to \Lambda_{c1}$ using QCD sum rules

Jong-Phil Lee∗ and Gye T. Park†

Department of Physics and IPAP, Yonsei University, Seoul, 120-749, Korea

Abstract

Subleading Isgur-Wise form factor $\tau(v \cdot v')$ at $O(1/m_Q)$ for $\Lambda_b \to \Lambda_{c1}^{1/2,3/2}$ weak transition is calculated by using the QCD sum rules in the framework of the heavy quark effective theory (HQET), where $\Lambda_{c1}^{1/2}$ and $\Lambda_{c1}^{3/2}$ are the orbitally excited charmed baryon doublet with $J^P = (1^-/2, 3^-/2)$. We consider the subleading contributions from the weak current matching in the HQET. The interpolating currents with transverse covariant derivative are adopted for $\Lambda_{c1}^{1/2}$ and $\Lambda_{c1}^{3/2}$ in the analysis. The slope parameter $\rho^2$ in linear approximation of $\tau$ is obtained to be $\rho^2 = 2.76$ and the interception to be $\tau(1) = -1.27$ GeV.
I. INTRODUCTION

The ground state bottom baryon Λ_b weak decays [1] provide a testing ground for the standard model (SM). They reveal some important features of the physics of bottom quark. The experimental data on these decays have been accumulated to wait for reliable theoretical calculations. With the discovery of the orbitally excited charmed baryons Λ_c(2593) and Λ_c(2625) [2], it would be of great interest for one to investigate the Λ_b semileptonic decays into these baryons.

From the phenomenological point of view, these semileptonic transitions are interesting since in principle they may account for a sizeable fraction of the inclusive semileptonic rate of Λ_b decay. In addition, the properties of excited baryons have attracted attention in recent years. Investigation on them will extend our ability in the application of QCD. It can also help us foresee any other excited heavy baryons that have not been discovered yet.

The heavy quark symmetry [3] is a useful tool to classify the hadronic spectroscopy containing a heavy quark Q. In the infinite mass limit, the spin and parity of the heavy quark and that of the light degrees of freedom are separately conserved. Coupling the spin of light degrees of freedom \( j_\ell \) with the spin of heavy quark \( s_Q = 1/2 \) yields a doublet with total spin \( J = j_\ell \pm 1/2 \) (or a singlet if \( j_\ell = 0 \)). This classification can be applied to the Λ_Q-type baryons. For the charmed baryons the ground state Λ_c contains light degrees of freedom with spin-parity \( j_\ell^P = 0^+ \), being a singlet. The excited states with \( j_\ell^P = 1^- \) are spin symmetry doublet with \( J^P(1^-/2, 3^-/2) \). The lowest states of such excited charmed states, \( \Lambda_{c1}^{1/2} \) and \( \Lambda_{c1}^{3/2} \), have been observed to be identified with Λ_c(2593) and Λ_c(2625) respectively [2].

However, the difficulties in the SM calculations are mainly due to the poor understanding of the nonperturbative aspects of the strong interaction (QCD). The heavy quark effective theory (HQET) based on the heavy quark symmetry provides a model-independent method for analyzing heavy hadrons containing a single heavy quark [3]. It allows us to expand the physical quantity in powers of \( 1/m_Q \) systematically, where \( m_Q \) is the heavy quark mass.
Within this framework, the classification of the $\Lambda_b$ exclusive weak decay form factors has been greatly simplified. The decays such as $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$ [4], $\Lambda_b \rightarrow \Sigma_c^{(*)} l \bar{\nu}$ [5], $\Lambda_b \rightarrow \Sigma_c^{(*)} \pi l \bar{\nu}$ [6], $\Lambda_b \rightarrow p(\Lambda)$ [7] have been studied.

To obtain detailed predictions for the hadrons, at this point, some nonperturbative QCD methods are also required. We have adopted QCD sum rules [8] in this work. QCD sum rule is a powerful nonperturbative method based on QCD. It takes into account the nontrivial QCD vacuum which is parametrized by various vacuum condensates in order to describe the nonperturbative nature. In QCD sum rule, hadronic observables can be calculated by evaluating two- or three-point correlation functions. The hadronic currents for constructing the correlation functions are expressed by the interpolating fields. In describing the excited heavy baryons, transverse covariant derivative is included in the interpolating field. The static properties of $\Lambda_b$ and $\Lambda_{c1}$ ($\Lambda_{c1}$ denotes the generic $j^P_l = 1^-$ charmed state) have been studied with QCD sum rules in the HQET in Ref. [9] and Ref. [10,11], respectively. Recently, the leading order Isgur-Wise (IW) function is also calculated in the HQET QCD sum rule in Ref. [12].

In $\Lambda_b \rightarrow \Lambda_{c1}$ decay, $1/m_Q$ corrections are very important. At the heavy quark limit of $m_Q \rightarrow \infty$, the transition matrix elements should vanish at zero recoil since the light degrees of freedom change their configurations. Nonvanishing contribution to, say, $\mathcal{B}(\Lambda_b \rightarrow \Lambda_{c1} \ell \bar{\nu})$ at zero recoil appears at $1/m_Q$ order. Since both $\Lambda_b$ and $\Lambda_{c1}$ are heavy enough, the behavior of the matrix elements near the zero recoil is very important. That explains why people pay attention to the next-to-leading order (NLO) contributions. The same situation occurs in heavy mesons. As for $B \rightarrow D_1(D_2^*) \ell \bar{\nu}$ decay, leading and subleading Isgur-Wise (IW) functions have been computed using QCD sum rule in Ref. [13–17]. They showed that the branching ratio is enhanced considerably when the subleading contributions are included.

In HQET, $1/m_Q$ corrections appear in a two-fold way. At the Lagrangian level, subleading terms are summarized in $\lambda_1$ and $\lambda_2$. $\lambda_1$ parametrizes the kinetic term of higher derivative, while $\lambda_2$ represents the chromomagnetic interaction which explicitly breaks the heavy quark spin symmetry. At the current level, $1/m_Q$ corrections come from the small
portion of the heavy quark fields which correspond to the virtual motion of the heavy quark. In this work, the subleading IW function from the latter case, i.e., at the current level, is analyzed in the HQET QCD sum rules.

In Sec. II, the weak transition matrix elements are parametrized by the leading and subleading IW functions. By evaluating the three-point correlation function, we give the subleading IW function in Sec. III. We present, in Sec. IV, the numerical analysis and discussions. The summary is given in Sec. V.

II. WEAK TRANSITION MATRIX ELEMENTS AND THE SUBLEADING ISGUR-WISE FUNCTIONS

The weak transition matrix elements for $\Lambda_b \rightarrow \Lambda_c$ are parametrized by the 14-form factors as

$$\frac{\langle \Lambda_c^3(v',s')|V_\mu|\Lambda_b(v,s)\rangle}{\sqrt{4M_{\Lambda_c(1/2)}M_{\Lambda_b}}} = \bar{u}_{\Lambda_c}(v',s')\left[F_1\gamma_\mu + F_2v_\mu + F_3v'_\mu\right]\gamma_5u_{\Lambda_b}(v,s),$$  \hspace{1cm} (1a)

$$\frac{\langle \Lambda_c^3(v',s')|A_\mu|\Lambda_b(v,s)\rangle}{\sqrt{4M_{\Lambda_c(1/2)}M_{\Lambda_b}}} = \bar{u}_{\Lambda_c}(v',s')\left[G_1\gamma_\mu + G_2v_\mu + G_3v'_\mu\right]u_{\Lambda_b}(v,s),$$  \hspace{1cm} (1b)

$$\frac{\langle \Lambda_c^3(v',s')|V_\mu|\Lambda_b(v,s)\rangle}{\sqrt{4M_{\Lambda_c(3/2)}M_{\Lambda_b}}} = \bar{u}_{\Lambda_c}^\alpha(v',s')\left[v_\alpha(K_1\gamma_\mu + K_2v_\mu + K_3v'_\mu) + K_4g_\alpha\mu\right]u_{\Lambda_b}(v,s),$$  \hspace{1cm} (1c)

$$\frac{\langle \Lambda_c^3(v',s')|A_\mu|\Lambda_b(v,s)\rangle}{\sqrt{4M_{\Lambda_c(3/2)}M_{\Lambda_b}}} = \bar{u}_{\Lambda_c}^\alpha(v',s')\left[v_\alpha(N_1\gamma_\mu + N_2v_\mu + N_3v'_\mu) + N_4g_\alpha\mu\right]\gamma_5u_{\Lambda_b}(v,s),$$  \hspace{1cm} (1d)

where $v(v')$ and $s(s')$ are the four-velocity and spin of $\Lambda_b(\Lambda_c)$, respectively. And the form factors $F_i$, $G_i$, $K_i$ and $N_i$ are functions of $y \equiv v \cdot v'$. In the limit of $m_Q \rightarrow \infty$, all the form factors are related to one independent universal form factor $\xi(y)$ called Isgur-Wise (IW) function. A convenient way to evaluate hadronic matrix elements is by introducing interpolating fields in HQET developed in Ref. [18] to parametrize the matrix elements in Eqs. (1). With the aid of this method the matrix element can be written as [19]

$$\bar{c}\Gamma b = \bar{h}_c^{(c)}(\nu)\Gamma h_b^{(b)} = \xi(y)v_\alpha\bar{\psi}_c^\alpha\Gamma\psi_b$$  \hspace{1cm} (2)
at leading order in $1/m_Q$ and $\alpha_s$, where $\Gamma$ is any collection of $\gamma$-matrices. The ground state field, $\psi_v$, destroys the $\Lambda_b$ baryon with four-velocity $v$; the spinor field $\psi_v^\alpha$ is given by

$$\psi_v^\alpha = \psi_v^{3/2\alpha} + \frac{1}{\sqrt{3}}(\gamma^\alpha + v^\alpha)\gamma_5 \psi_v^{1/2},$$

where $\psi_v^{1/2}$ is the ordinary Dirac spinor and $\psi_v^{3/2\alpha}$ is the spin $3/2$ Rarita-Schwinger spinor, they destroy $\Lambda_{b}^{1/2}$ and $\Lambda_{c1}^{3/2}$ baryons with four-velocity $v$, respectively. To be explicit,

$$F_1 = \frac{1}{\sqrt{3}}(y - 1) \xi(y), \quad G_1 = \frac{1}{\sqrt{3}}(y + 1) \xi(y),$$

$$F_2 = G_2 = -\frac{2}{\sqrt{3}} \xi(y), \quad K_1 = N_1 = \xi(y),$$

(others) $= 0$.

In general, the IW form factor is a decreasing function of the four velocity transfer $y$. Since the kinematically allowed region of $y$ for heavy to heavy transition is very narrow around unity,

$$1 \leq y \leq \frac{M_{\Lambda_{b}}^2 + M_{\Lambda_{c1}}^2}{2M_{\Lambda_{b}}M_{\Lambda_{c1}}} \simeq 1.3,$$

and hence it is convenient to approximate the IW function linearly as

$$\xi(y) = \xi(1)(1 - \rho_\xi^2(y - 1)),$$

where $\rho_\xi^2$ is the slope parameter which characterizes the shape of the leading IW function.

The $\Lambda_{QCD}/m_Q$ corrections come in two ways. One is from the subleading Lagrangian of the HQET while the other comes from the small portion of the heavy quark field to modify the effective currents. We only consider the latter case here.

Including $\Lambda_{QCD}/m_b$ and $\Lambda_{QCD}/m_c$, the weak current is given by

$$\bar{c}\Gamma b = \bar{h}_v^{(c)}(\Gamma - \frac{i}{2m_c}\Gamma_\mu + \frac{i}{2m_b}\Gamma_{\mu\nu})h_v^{(b)}.$$  

Keeping the Lorentz structure, the subleading terms are expanded in general as

$$\bar{h}_v^{(c)}i\Gamma_\mu h_v^{(b)} = \bar{\psi}_v^\alpha (\tau_1^{(c)} v_\alpha \psi + \tau_2^{(c)} v_\alpha \psi' + \tau_3^{(c)} \gamma_\alpha) \Gamma \Lambda_v,$$

$$\bar{h}_v^{(c)}\Gamma i\phi h_v^{(b)} = \bar{\psi}_v^\alpha \Gamma (\tau_1^{(b)} v_\alpha \psi + \tau_2^{(b)} v_\alpha \psi' + \tau_3^{(b)} \gamma_\alpha) \Lambda_v.$$
where \( \tau_i^{(Q)} \) are the subleading IW functions to be evaluated.

The matrix elements of these currents modify Eq. (4) as

\[
\sqrt{3} F_1 = (y - 1) \xi - \epsilon_c \left[ (y - 1)(-\tau_1^{(c)} + \tau_2^{(c)}) + 3\tau_3^{(c)} \right] + \epsilon_b \left[ (y - 1)(\tau_1^{(b)} - \tau_2^{(b)}) - \tau_3^{(b)} \right],
\]

\[
\sqrt{3} F_2 = -2 \xi + \epsilon_c \left[ 2y\tau_1^{(c)} + 2\tau_2^{(c)} \right] + \epsilon_b \left[ -2\tau_1^{(b)} + 2\tau_2^{(b)} \right],
\]

\[
\sqrt{3} F_3 = -2\epsilon_b \left[ (1 + y)\tau_2^{(b)} + \tau_3^{(b)} \right],
\]

\[
\sqrt{3} G_1 = (y + 1) \xi - \epsilon_c \left[ (y + 1)(\tau_1^{(c)} + \tau_2^{(c)}) + 3\tau_3^{(c)} \right] + \epsilon_b \left[ (y + 1)(\tau_1^{(b)} + \tau_2^{(b)}) + \tau_3^{(b)} \right],
\]

\[
\sqrt{3} G_2 = -2 \xi + \epsilon_c \left[ 2y\tau_1^{(c)} + 2\tau_2^{(c)} \right] - 2\epsilon_b \left[ \tau_1^{(b)} + \tau_2^{(b)} \right],
\]

\[
\sqrt{3} G_3 = 2\epsilon_b \left[ (y - 1)\tau_2^{(b)} + \tau_3^{(b)} \right],
\]

\[
K_1 = \xi + \epsilon_c \left[ \tau_1^{(c)} - \tau_2^{(c)} \right] + \epsilon_b \left[ \tau_1^{(b)} - \tau_2^{(b)} \right],
\]

\[
N_1 = \xi - \epsilon_c \left[ \tau_1^{(c)} + \tau_2^{(c)} \right] + \epsilon_b \left[ \tau_1^{(b)} + \tau_2^{(b)} \right],
\]

\[
K_2 = N_2 = -2\epsilon_c\tau_1^{(c)},
\]

\[
K_3 = -N_3 = 2\epsilon_b\tau_2^{(b)},
\]

\[
K_4 = -N_4 = 2\epsilon_b\tau_3^{(b)},
\]

\[
(9)
\]

where \( \epsilon_Q \equiv 1/2m_Q \). It is quite convenient to define

\[
\Omega^{(c\Gamma)}_{\alpha\beta} \equiv (\gamma_\alpha + v_\alpha')\gamma_5 \left( \frac{1 + \gamma'}{2} \right) \gamma_\beta\Gamma \left( \frac{1 + \gamma'}{2} \right), \tag{10a}
\]

\[
\Omega^{(b\Gamma)}_{\alpha\beta} \equiv (\gamma_\alpha + v_\alpha')\gamma_5 \left( \frac{1 + \gamma'}{2} \right) \Gamma \gamma_\beta \left( \frac{1 + \gamma'}{2} \right). \tag{10b}
\]

Possible contractions of \( \Omega_{\alpha\beta} \) are listed in the Appendix. From the Eqs. (3) and (8), Eq. (1) can be reexpressed in terms of \( \tau_i^{(Q)} \) and \( \Omega_{\alpha\beta} \):

\[
\frac{\langle \Lambda_{a1}^b(v', s')|\Gamma|\Lambda_a(v, s) \rangle}{\sqrt{4M_{a1(1/2)}M_b}} = \frac{1}{\sqrt{3}} \bar{u}_{\Lambda_{a1}}(v', s') \left[ \xi v^\alpha v'^\alpha \Omega^{(c\Gamma)}_{\alpha\beta} - \epsilon_c \left( \tau_1^{(c)} v^\alpha v'^\beta + \tau_2^{(c)} v^\alpha v'^\beta + \tau_3^{(c)} g^\alpha{}_{\alpha\beta} \right) \Omega^{(c\Gamma)}_{\alpha\beta} \right.
\]

\[
\left. + \epsilon_b \left( \tau_1^{(b)} v^\alpha v'^\beta + \tau_2^{(b)} v^\alpha v'^\beta + \tau_3^{(b)} g^\alpha{}_{\alpha\beta} \right) \Omega^{(b\Gamma)}_{\alpha\beta} \right] u_{\Lambda_b}(v, s), \tag{11}
\]

\[
(12)
\]

A similar expression can be obtained for the spin-3/2 final states.
\[
\frac{\langle \Lambda^3_{c1}(v', s') | \Gamma \Lambda_b(v, s) \rangle}{\sqrt{4M_{\Lambda_{c1}(3/2)}M_{\Lambda_b}}} = u^a_{\Lambda_{c1}}(v', s') \left[ \xi v_\alpha \Gamma - \epsilon_c \left( \tau_1^{(c)} v_\alpha v_\beta + \tau_2^{(c)} v_\alpha v'_\beta + \tau_3^{(c)} g_{\alpha\beta} \right) \gamma^\beta \Gamma 
+ \epsilon_b \left( \tau_1^{(b)} v_\alpha v_\beta + \tau_2^{(b)} v_\alpha v'_\beta + \tau_3^{(b)} g_{\alpha\beta} \right) \Gamma \gamma^\beta \right] u_{\Lambda_b}(v, s). \tag{13}
\]

### III. QCD SUM RULE EVALUATION

As a starting point of QCD sum rule calculation, let us consider the interpolating field of heavy baryons. The heavy baryon current is generally expressed as

\[
\tilde{j}_{I,P}^\mu(x) = \epsilon_{ijk} [q^{jT}(x) C \Gamma_{I,P} \tau q^i(x)] \Gamma'_{I,P} h^k_v(x), \tag{14}
\]

where \(i, j, k\) are the color indices, \(C\) is the charge conjugation matrix, and \(\tau\) is the isospin matrix while \(q(x)\) is a light quark field. \(\Gamma_{I,P}\) and \(\Gamma'_{I,P}\) are some gamma matrices which describe the structure of the baryon with spin-parity \(J^P\). Usually \(\Gamma\) and \(\Gamma'\) with least number of derivatives are used in the QCD sum rule method. The sum rules then have better convergence in the high energy region and often have better stability. For the ground state heavy baryon, we use \(\Gamma_{1/2,+} = \gamma_5\), \(\Gamma'_{1/2,+} = 1\). In the previous work \[10\], two kinds of interpolating fields are introduced to represent the excited heavy baryon. In this work, we find that only the interpolating field of transverse derivative is adequate for the analysis. Nonderivative interpolating field results in a vanishing perturbative contribution. The choice of \(\Gamma\) and \(\Gamma'\) with derivatives for the \(\Lambda_{c1}^{1/2}\) and \(\Lambda_{c1}^{3/2}\) is then

\[
\begin{align*}
\Gamma_{1/2,-} &= (a + b\gamma_5) \gamma_5, \quad \Gamma'_{1/2,-} = \frac{i\gamma_\mu}{M} \gamma_5, \\
\Gamma_{3/2,-} &= (a + b\gamma_5) \gamma_5, \quad \Gamma'_{3/2,-} = \frac{1}{3M} (i\gamma_\mu \gamma_5 + i\gamma_\mu \gamma_5) 
\end{align*}
\tag{15}
\]

where a transverse vector \(A^\mu_v\) is defined to be \(A^\mu_v \equiv A^\mu - \nu^\mu v \cdot A\), and \(M\) in Eq. (15) is some hadronic mass scale. \(a, b\) are arbitrary numbers between 0 and 1.

The baryonic decay constants in the HQET are defined as follows,

\[
\langle 0 | \tilde{j}_{1/2,+}^\mu | \Lambda_b \rangle = f_{\Lambda_b} \psi_v, \tag{16a}
\]
\[
\langle 0 | j_{1/2}^{\nu} | \Lambda_{c1}^{1/2} \rangle = f_{1/2} \psi_{\nu}^{1/2} , \quad (16b)
\]
\[
\langle 0 | j_{3/2}^{\nu \mu} | \Lambda_{c1}^{3/2} \rangle = \frac{1}{\sqrt{3}} f_{3/2} \psi_{\nu}^{3/2} , \quad (16c)
\]
where \( f_{1/2} \) and \( f_{3/2} \) are equivalent since \( \Lambda_{c1}^{1/2} \) and \( \Lambda_{c1}^{3/2} \) belong to the same doublet with \( j_{\ell}^P = 1^- \). The QCD sum rule calculations give [9]
\[
f_{\Lambda}^2 e^{-\Lambda/T} = \frac{1}{20 \pi^4} \int_0^{\omega_c} d\omega \omega^5 e^{-\omega/T} + \frac{1}{6} \langle \bar{q}q \rangle^2 e^{-m_c^2/8T^2} + \frac{\langle \alpha_s G G \rangle}{32 \pi^3} T^2 , \quad (17)
\]
and [10]
\[
M^2 f_{1/2}^2 e^{-\Lambda'/T'} = \int_0^{\omega_c} d\omega \frac{3N_c!}{4 \pi^4} \omega^7 (24a^2 + 40b^2) e^{-\omega/T'} + \frac{\langle \alpha_s G G \rangle}{32 \pi^3} T'^4 (-a^2 + b^2)
\]
\[
+ \frac{N_c!}{2 \pi^2} \left[ \langle \bar{q}q \rangle T'^5 (16ab) - \langle \bar{q}g \sigma \cdot Gq \rangle T'^3 ab \right] - \frac{\langle \bar{q}g \sigma \cdot Gq \rangle}{4 \pi^2} T'^3 (3ab) . \quad (18)
\]
In the above equations, \( T^{(i)} \) are the Borel parameters and \( \omega_c^{(i)} \) are the continuum thresholds, and \( N_c = 3 \) is the color number. In the heavy quark limit, the mass parameters \( \Lambda \) and \( \Lambda' \) are defined as
\[
\Lambda' = M_{\Lambda Q} - m_Q , \quad \Lambda = M_{\Lambda Q} - m_Q . \quad (19)
\]
The main point in QCD sum rules for the IW function is to study the analytic properties of the 3-point correlators,
\[
\Xi_1^{\mu}(\omega, \omega', y) = i^2 \int d^4x d^4z e^{i(k' - k \cdot x - k \cdot z)} \langle 0 | T_{j_{1/2}^{\nu}}(x) \bar{h}_c^{(c)}(0) \Gamma h_v^{(b)}(0) \bar{j}_{1/2}^{\nu}(z) | 0 \rangle
\]
\[
= \Xi_{\text{hadron}}(\omega, \omega', y) \left[ \xi v^\alpha v^\alpha \Omega^{(cT)}_{\alpha \beta} - \epsilon_c \left( \tau_1^{(c)} v^\alpha v^\beta + \tau_2^{(c)} v^\alpha v^\beta + \tau_3^{(c)} g_{\alpha \beta} \right) \Gamma \right]
\]
\[
+ \epsilon_b \left( \tau_1^{(b)} v^\alpha v^\beta + \tau_2^{(b)} v^\alpha v^\beta + \tau_3^{(b)} g_{\alpha \beta} \right) \Gamma \right] , \quad (20a)
\]
\[
\Xi_2^{\alpha}(\omega, \omega', y) = i^2 \int d^4x d^4z e^{i(k' - k \cdot x - k \cdot z)} \langle 0 | T^{\nu}_{j_{3/2}^{\nu}}(x) \bar{h}_c^{(c)}(0) \Gamma h_v^{(b)}(0) \bar{j}_{3/2}^{\nu}(z) | 0 \rangle
\]
\[
= \Xi_{\text{hadron}}(\omega, \omega', y) A_+^{\mu \nu} \left[ \xi v^\alpha \Gamma - \epsilon_c \left( \tau_1^{(c)} v^\alpha v^\beta + \tau_2^{(c)} v^\alpha v^\beta + \tau_3^{(c)} g_{\alpha \beta} \right) \Gamma \right]
\]
\[
+ \epsilon_b \left( \tau_1^{(b)} v^\alpha v^\beta + \tau_2^{(b)} v^\alpha v^\beta + \tau_3^{(b)} g_{\alpha \beta} \right) \Gamma \right] \left( \frac{1 + y^2}{2} \right) . \quad (20b)
\]
The variables \( k, k' \) denote residual “off-shell” momenta which are related to the momenta \( P \) of the heavy quark in the initial state and \( P' \) in the final state by \( k = P - m_Q v \), \( k' = P - m_Q v' \), respectively.
The coefficient \( \Xi(\omega, \omega', y)_{\text{hadron}} \) in Eq. (20) is an analytic function in the “off-shell energies” \( \omega = v \cdot k \) and \( \omega' = v' \cdot k' \) with discontinuities for positive values of these variables. It furthermore depends on the velocity transfer \( y = v \cdot v' \), which is fixed at its physical region for the process under consideration. By saturating with physical intermediate states in HQET, one finds the hadronic representation of the correlators as following

\[
\Xi_{\text{hadron}}(\omega, \omega', y) = \frac{f_{1/2} f_{\Lambda}^{*}}{(\Lambda' - \omega')(\Lambda - \omega)} + \text{higher resonances} \, .
\]  

(21)

In obtaining the above expression the Dirac and Rartia-Schwinger spinor sums

\[
\Lambda_+ = \sum_{s=1}^{2} u(v, s) \bar{u}(v, s) = \frac{1 + \gamma'}{2},
\]

\[
\Lambda^\mu_+ = \sum_{s=1}^{4} u^\mu(v, s) \bar{u}'(v, s) = \left( - g_\mu^\nu + \frac{1}{3} \gamma_\nu \gamma_\mu \right) \frac{1 + \gamma'}{2},
\]  

(22)

have been used, where \( g_\mu^\nu = g_\mu^\nu - v_\mu v_\nu \).

In the quark-gluon language, \( \Xi(\omega, \omega', y)_{\frac{1}{2}, \frac{3}{2}} \) in Eq. (20) is written as

\[
\Xi(\omega, \omega', y)_{\frac{1}{2}, \frac{3}{2}} = \int_{0}^{\infty} dv dv' \rho^{\text{pert}}(v, v', y) \left( \frac{\rho^{\text{pert}}(v, v', y)}{(v - \omega)(v' - \omega')} \right) + \text{(subtraction)} + \Xi^{\text{cond}}(\omega, \omega', y),
\]  

(23)

where the perturbative spectral density function \( \rho^{\text{pert}}(v, v', y) \) and the condensate contribution \( \Xi^{\text{cond}}(\omega, \omega', y) \) are related to the calculation of the Feynman diagrams depicted in Fig. 1. In Eq. (23), the \( \gamma \)-structures of spin-1/2 and 3/2 are the same as those in Eq. (20), respectively. Subleading IW functions, \( \tau_i^{(Q)} \) obtained from spin-1/2 and 3/2 are therefore identical.

The six \( \tau_i^{(Q)} \) \( (Q = c, b, \ i = 1, 2, 3) \) are not independent. From the fact that

\[
i \partial_\alpha(\bar{h}_{\nu'}^{(c)} \Gamma h_{\nu'}^{(b)}) = \bar{h}_{\nu'}^{(c)}(i \gamma_\alpha \Gamma + \Gamma i \gamma_\alpha)h_{\nu'}^{(b)} = (\bar{\Lambda} v_\alpha - \bar{\Lambda}' v_\alpha') \bar{h}_{\nu'}^{(c)} \Gamma h_{\nu'}^{(b)},
\]  

(24)

Eq. (8) implies

\[
(\tau_1^{(c)} + \tau_1^{(b)}) v_\alpha v_\beta + (\tau_2^{(c)} + \tau_2^{(b)}) v_\alpha v_\beta' + (\tau_3^{(c)} + \tau_3^{(b)}) g_{\alpha \beta} = (\bar{\Lambda} v_\beta - \bar{\Lambda}' v_\beta') v_\alpha \xi(y).
\]  

(25)

The above expression relates \( \tau_i^{(c)} \) with \( \tau_i^{(b)} \) as

\[
\tau_1^{(c)} + \tau_1^{(b)} = \bar{\Lambda} \xi, \quad \tau_2^{(c)} + \tau_2^{(b)} = -\bar{\Lambda}' \xi, \quad \tau_3^{(c)} + \tau_3^{(b)} = 0.
\]  

(26a)

(26b)

(26c)
Other relations are obtained from the equation of motion of the heavy quark, \( v \cdot D h_{\nu}^{(Q)} = 0 \):

\[
\bar{h}_\nu^{(c)} i v \cdot \vec{D}_\nu h_{\nu}^{(b)} = \bar{\psi}_\nu^{(c)} \left( y \tau_1^{(c)} + \tau_2^{(c)} \right) \Lambda_v = 0 ,
\]

\[
\bar{h}_\nu^{(c)} i v \cdot \vec{D}_\nu h_{\nu}^{(b)} = \bar{\psi}_\nu^{(b)} \left( \tau_1^{(b)} + y \tau_2^{(b)} + \tau_3^{(b)} \right) \Lambda_v = 0 ,
\]

(27a)

(27b)

From the above 5 equations in Eq. (26), (27), all the six subleading IW functions are reduced to only one independent form factor. We just pick up \( \tau_1^{(b)}(y) \equiv \tau(y) \), then others are

\[
\tau_1^{(c)} = \bar{\Lambda} \xi - \tau ,
\]

(28a)

\[
\tau_2^{(c)} = -y \bar{\Lambda} \xi + y \tau ,
\]

(28b)

\[
\tau_3^{(c)} = y(y \bar{\Lambda} - \bar{\Lambda}') \xi - (y^2 - 1) \tau ,
\]

(28c)

\[
\tau_2^{(b)} = (y \bar{\Lambda} - \bar{\Lambda}') \xi - y \tau ,
\]

(28d)

\[
\tau_3^{(b)} = -y(y \bar{\Lambda} - \bar{\Lambda}') \xi + (y^2 - 1) \tau ,
\]

(28e)

Now that all the subleading IW functions are related to \( \tau(y) \), we have only to extract the coefficient of \( v^\alpha v^\beta \Omega_{\alpha\beta}^{(b)} \) (or \( \Lambda_+^{\mu\alpha} v_\alpha v_\beta \Gamma^\beta \) for spin 3/2) in Eqs. (20) and (23).

The QCD sum rule is obtained by equating the phenomenological and theoretical expressions for \( \Xi \). In doing this the quark-hadron duality needs to be assumed to model the contributions of higher resonance part of Eq. (21). Generally speaking, the duality is to simulate the resonance contribution by the perturbative part above some thresholds \( \omega_c \) and \( \omega'_c \), that is

\[
\text{res.} = \int_{\omega_c}^{\infty} \int_{\omega'_c}^{\infty} d\nu d\nu' \frac{\rho_{\text{pert}}(\nu, \nu', y)}{(\nu - \omega)(\nu' - \omega')} .
\]

(29)

In the QCD sum rule analysis for \( B \) semileptonic decays into ground state \( D \) mesons, it was argued by Neubert in [20], and Blok and Shifman in [21] that the perturbative and the hadronic spectral densities can not be locally dual to each other, and therefore the necessary way to restore duality is to integrate the spectral densities over the “off-diagonal” variable \( \nu_\pm = \sqrt{\frac{\nu + 1}{y - 1}} (\nu - \nu')/2 \), keeping the “diagonal” variable \( \nu_+ = (\nu + \nu')/2 \) fixed. It is in \( \nu_+ \)
that the quark-hadron duality is assumed for the integrated spectral densities. The same prescription shall be adopted in the following analysis. On the other hand, in order to suppress the contributions of higher resonance states a double Borel transformation in $\omega$ and $\omega'$ is performed to both sides of the sum rule, which introduces two Borel parameters $T_1$ and $T_2$.

Combining Eqs. (21), (23), our duality assumption and making the double Borel transformation, one obtains the sum rule for $\xi(y)$ as follows;

\[
M f_{1/2} f_{\Lambda_0}^* e^{-N/2T'} e^{-\Lambda/2T} \left( \frac{1 + y^0}{2} \right) C_T \left( \frac{1 + y'}{2} \right) = 2 \left( \frac{y - 1}{y + 1} \right)^{1/2} \int_0^{\omega(y)} d\nu \int_{-\nu+}^{\nu+} d\nu_+ \exp \left( -\frac{\nu+ - \sqrt{\frac{y-1}{y+1}} \nu_-}{2T'} - \frac{\nu+ + \sqrt{\frac{y-1}{y+1}} \nu_-}{2T'} \right) \rho(\nu+, \nu_-; y) + \hat{B}_{2T}^s \hat{B}_{2T}^s \Xi^{\text{cond}},
\]

where $\nu = \nu+ + \sqrt{\frac{y-1}{y+1}} \nu_-$, $\nu' = \nu- - \sqrt{\frac{y-1}{y+1}} \nu_-$, and

\[
C_T = \begin{cases} 
\frac{1}{\sqrt{3}} \left[ \xi \tau^a \tau^a \Omega((\Gamma^{(c)})_{\alpha_\beta} - \epsilon_c \left( \tau_1^{(c)} \tau^a \tau^a + \tau_2^{(c)} \tau^a \tau^a + \tau_3^{(c)} g^{c\alpha_\beta} \right) \Omega^{(c)}_{\alpha_\beta} ight] 
+ \epsilon_b \left( \tau_1^{(b)} \tau^a \tau^a + \tau_2^{(b)} \tau^a \tau^a + \tau_3^{(b)} g^{c\alpha_\beta} \right) \Omega^{(b)}_{\alpha_\beta} 
\end{cases}
\]

Now the remaining thing is to evaluate the relevant diagrams in Fig. 1. The leading contributions are given in [12]. For the subleading corrections to the perturbative spectral density function $\rho(\omega, \omega'; y)$, we have

\[
\rho(\omega, \omega'; y) = \hat{B}_{1/\omega} \hat{B}_{1/\omega} \hat{B}_{1/\omega} \hat{B}_{1/\omega} \Xi^{\text{pert}} = \left( \frac{6N_c \alpha_i}{\pi^4} \right) \Omega_{\alpha_\beta}^{(s)} \left( \frac{1}{2 \sinh^2 \theta} \Theta(\omega) \Theta(\omega') \Theta(2y\omega' - \omega^2 - \omega'^2) \right) 
\]

\[
\left[ \frac{2v^a v'^a}{\sinh^2 \theta} \left( \frac{2 \cosh \theta A^3 B^3}{3!3!} - \frac{e^{-\theta} A^2 B^4}{2!4!} - \frac{e^{\theta} A^2 B^4}{4!2!} \right) \right] 
\]

\[
+ \frac{2v^a v'^a}{\sinh^2 \theta} \left( \frac{e^{2\theta} A^2 B^4}{4!2!} + \frac{e^{-2\theta} A^2 B^4}{2!4!} - \frac{2 A^3 B^3}{3!3!} \right) - g^{\alpha_\beta} A^3 B^3 \right],
\]

from the perturbative diagram Fig. 1 (a), where
\[ \Omega_{\alpha\beta} \equiv -i\epsilon_c \Omega_{\alpha\beta}^{(cT)} + i\epsilon_b \Omega_{\alpha\beta}^{(bT)} , \]  
(33a)

\[ A \equiv \omega' - \omega e^{-\theta} , \quad B \equiv \omega e^{\theta} - \omega' , \]  
(33b)

\[ e^{\theta} \equiv y + \sqrt{y^2 - 1} . \]  
(33c)

For the condensate contributions we just give results when \( T' = T \) for simplicity;

\[ \hat{B}_{2T} \hat{B}_{2T} \Xi^{(\bar{q}q)} = -\frac{i b c_{\alpha\beta} \Omega_{\alpha\beta}}{2\pi^2(1+y)^2} \left[ 64\langle \bar{q}q \rangle T^5 - \frac{1}{3}\langle \bar{q}g\sigma \cdot Gq \rangle T^3(4y + 5/2) \right] 
\]  
\( - \frac{i b v c_{\alpha\beta} \Omega_{\alpha\beta}}{4\pi^2(1+y)^3} \left[ - 128\langle \bar{q}q \rangle T^5(3v + 2v')^\beta \right.\]  
\[ + \frac{4}{3}\langle \bar{q}g\sigma \cdot Gq \rangle \left\{ (6y + 7/2)v^\beta + (y - 3/2)v'^\beta \right\} \right] , \]  
(34a)

\[ \hat{B}_{2T} \hat{B}_{2T} \Xi^{(\bar{q}g\sigma \cdot Gq)} = -\frac{i b c_{\alpha\beta} \Omega_{\alpha\beta}}{12(1+y)^3} \left[ - 2g_{\alpha\beta}(2y^2 + 3y + 1) + (10y + 6)v^\alpha v^\beta + 4yv^\alpha v'^\beta \right] , \]  
(34b)

\[ \hat{B}_{2T} \hat{B}_{2T} \Xi^{(\alpha GG)} = \frac{i a (\alpha GG) T^4}{192\pi^3(1+y)^4} \Omega_{\alpha\beta} \left[ 8(y + 1)^2(y - 2)\left\{ - g_{\alpha\beta} + 5v^\alpha (v + v')^\beta \right\} 
\]  
\[ + 24(y - 1)v^\alpha v'^\beta - 16(y + 1)(y + 4)v^\alpha v^\beta \right] 
\]  
\[ - \frac{i a (\alpha GG) T^4}{512\pi^3(1+y)^4} \Omega_{\alpha\beta} \left[ - 2(1+y)g_{\alpha\beta} + 6v^\alpha (v + v')^\beta \right] . \]  
(34c)

Note that these results are from \( \Lambda_{\bar{c}1}^{3/2} \). If \( \Lambda_{\bar{c}1}^{3/2} \) were the final state, \( \Omega_{\alpha\beta} \) would be replace by a proper \( \gamma \)-structure, leaving all the other things unchanged.

**IV. RESULTS AND DISCUSSIONS**

For the numerical analysis, the standard values of the condensates are used;

\[ \langle \bar{q}q \rangle = -(0.23 \text{ GeV})^3 , \]  
\[ \langle \alpha GG \rangle = 0.04 \text{ GeV}^4 , \]  
\[ \langle \bar{q}g\sigma \cdot Gq \rangle \equiv m_0^2\langle \bar{q}q \rangle , \quad m_0^2 = 0.8 \text{ GeV}^2 . \]  
(35)

There are many parameters engaged in the QCD sum rule calculations. The key point in the numerical analysis is to find a reasonable parameter space where the QCD sum rule results are stable. First, the continuum threshold \( \omega_c' \) in \( f_{3/2}^{3/2}(\bar{\Lambda}) \) can differ from that in \( f_{\Lambda_b}(\bar{\Lambda}) \).
However, it is expected that the values of $\omega_c$ and $\omega'_c$ would not be different significantly. This is because the mass difference $\bar{\Lambda}' - \bar{\Lambda}$ is fairly small [10], $\bar{\Lambda}' - \bar{\Lambda} \simeq 0.2$ GeV. Indeed the central values of them were close to each other in the sum rules analysis for $f_{\Xi(1/2)} (\bar{\Lambda}')$ and $f_{\Lambda_b} (\bar{\Lambda})$. One more thing to be noticed here is that the continuum threshold $\omega_c$ in Eq. (30) can be a function of $y$ in general. But for simplicity, we take it to be a constant $\omega_c(y) = \omega_c = \omega'_c = \omega_0$ in the numerical analysis. In this sense, we use only one constant continuum threshold throughout the analysis. An alternative choice of $\omega_c(y) = (1 + y)\omega_0/2y$ is suggested in Ref. [20]. We find that this choice yields almost no numerical differences. This is because the kinematically allowed region is very narrow around the zero recoil.

Second, there are input parameters of $a$ and $b$ in the interpolating fields in Eq. (15). They are the parameters that generalize pseudoscalar or axial-vector nature of the light degrees of freedom ($\Gamma_{1/2,3/2}$ in Eq. 15). In Ref. [10], a particular choice of $(a,b) = (1,0)$ gives the best stability for the mass parameter $\bar{\Lambda}'$. We adopt the same choice of $(a,b) = (1,0)$ in the present analysis.

Third, there are two Borel parameters $T_1$ and $T_2$ distinct in general, corresponding to $\omega$ and $\omega'$ in $\Xi(\omega, \omega', y)$, respectively. We have taken $T_1 = T_2$ in the analysis. In Ref. [16] for $B$ into excited charmed meson transition, the authors found a 10% increase in the leading IW function at zero recoil when $T_2/T_1 = 1.5$ as compared to the case when $T_1 = T_2$. It seems quite reasonable for one to expect that in the case of heavy baryon, the numerical results should be similar for the small variations around $T_2/T_1 = 1$.

In short, we adopt the same parameters used in [10,12] where the mass parameter and the leading IW function are calculated. It makes sense because the observables involved are directly related to the subleading IW function $\tau(y)$ through Eq. (30).

In Fig. 2, $\tau$ is plotted as a function of $(y, T)$. Figure 3 shows the stability of $\tau(y = 1)$ for the Borel parameter. The sum rule window is

$$0.1 \leq T \leq 1.0 \text{ (GeV)} .$$

(36)

The upper and lower bounds are fixed such that the pole contribution amounts to 50% while
the condensate one to 12%. One notes that the window given in Eq. (36) overlaps those obtained in the Refs. [9,10,12]. Of course, this reflects the self-consistency of the sum rule analysis. In Fig. 4, we present the shape of $\tau(y)$ for a fixed Borel parameter. We found that

$$\tau(y) = \tau(1)[1 - \rho^2(y - 1)],$$

$$\tau(1) = -1.27^{+0.17}_{-0.18} \text{ GeV, for } \omega_0 = 1.4 \pm 0.1 \text{ GeV },$$

$$\rho^2 = 2.76^{+0.004}_{-0.008}, \text{ for } \omega_0 = 1.4 \pm 0.1 \text{ GeV } .$$

(37)

V. SUMMARY

Subleading contributions of $O(1/m_Q)$ to the $\Lambda_b \to \Lambda_c \ell \bar{\nu}$ weak form factors are important because some of the form factors do not survive at the heavy quark limit, and other remaining form factors vanish at zero recoil. Using the QCD sum rules, we calculate the subleading IW function $\tau(y)$ which appears in the current matching in the HQET at $O(1/m_Q)$. We obtain $\tau(y)$ given by

$$\tau(y) = -1.27[1 - 2.76(y - 1)] \text{ GeV} .$$

(38)

The best stability is attained when the continuum threshold $\omega_0 = 1.4$ GeV. The parameter space for the analysis is the same as previous one for the leading IW function. The fact that by using the same set of parameters the present sum rule window for the mass parameter, leading and NLO IW function overlaps the previous ones ensures the self-consistency of the QCD sum rules. Our results can be applied directly to the decay mode $\Lambda_b \to \Lambda_c \ell \bar{\nu}$, along with the use of the previous LO IW function, but a complete analysis at $O(1/m_Q)$ requires the information on another NLO contributions from the HQET Lagrangian.

Acknowledgements

This work was supported by the BK21 Program of the Korean Ministry of Education. The work of GTP was supported in part by Yonsei University Research Fund of 2000.
APPENDIX A: CONTRACTIONS OF $\Omega_{\alpha\beta}$

After a simple algebra, possible contractions for $\Omega_{\alpha\beta}$ are given by

\begin{align*}
v^{\mu}_{\alpha}\Omega_{\alpha\beta} &= 0 , \\
v^{\alpha}_{\mu}v^{\beta}_{\nu}\Omega^{(cV)}_{\alpha\beta} &= \left(\frac{1 + \gamma^\mu}{2}\right) \left[ -2yv^\mu\gamma_5 \right] \left(\frac{1 + \gamma^\mu}{2}\right) , \\
v^{\alpha}_{\mu}v^{\beta}_{\nu}\Omega^{(cV)}_{\alpha\beta} &= \left(\frac{1 + \gamma^\mu}{2}\right) \left[ (y - 1)\gamma^\mu\gamma_5 - 2v^\mu\gamma_5 \right] \left(\frac{1 + \gamma^\mu}{2}\right) , \\
g^{\alpha\beta}\Omega^{(cV)}_{\alpha\beta} &= \left(\frac{1 + \gamma^\mu}{2}\right) \left[ 3\gamma^\mu\gamma_5 \right] \left(\frac{1 + \gamma^\mu}{2}\right) , \\
v^{\alpha}_{\mu}v^{\beta}_{\nu}\Omega^{(cA)}_{\alpha\beta} &= \left(\frac{1 + \gamma^\mu}{2}\right) \left[ (y + 1)\gamma^\mu - 2v^\mu \right] \left(\frac{1 + \gamma^\mu}{2}\right) , \\
g^{\alpha\beta}\Omega^{(cA)}_{\alpha\beta} &= \left(\frac{1 + \gamma^\mu}{2}\right) \left[ 3\gamma^\mu \right] \left(\frac{1 + \gamma^\mu}{2}\right) , \\
v^{\alpha}_{\mu}v^{\beta}_{\nu}\Omega^{(bV)}_{\alpha\beta} &= \left(\frac{1 + \gamma^\mu}{2}\right) \left[ (y - 1)\gamma^\mu\gamma_5 - 2v^\mu\gamma_5 \right] \left(\frac{1 + \gamma^\mu}{2}\right) , \\
v^{\alpha}_{\mu}v^{\beta}_{\nu}\Omega^{(bV)}_{\alpha\beta} &= \left(\frac{1 + \gamma^\mu}{2}\right) \left[ (1 - y)\gamma^\mu\gamma_5 + 2v^\mu\gamma_5 - 2(y + 1)v^\mu\gamma_5 \right] \left(\frac{1 + \gamma^\mu}{2}\right) , \\
g^{\alpha\beta}\Omega^{(bV)}_{\alpha\beta} &= \left(\frac{1 + \gamma^\mu}{2}\right) \left[ -\gamma^\mu\gamma_5 - 2v^\mu\gamma_5 \right] \left(\frac{1 + \gamma^\mu}{2}\right) , \\
v^{\alpha}_{\mu}v^{\beta}_{\nu}\Omega^{(bA)}_{\alpha\beta} &= \left(\frac{1 + \gamma^\mu}{2}\right) \left[ (y + 1)\gamma^\mu - 2v^\mu \right] \left(\frac{1 + \gamma^\mu}{2}\right) , \\
v^{\alpha}_{\mu}v^{\beta}_{\nu}\Omega^{(bA)}_{\alpha\beta} &= \left(\frac{1 + \gamma^\mu}{2}\right) \left[ (y + 1)\gamma^\mu - 2v^\mu + 2(y - 1)v^\mu \right] \left(\frac{1 + \gamma^\mu}{2}\right) , \\
g^{\alpha\beta}\Omega^{(bA)}_{\alpha\beta} &= \left(\frac{1 + \gamma^\mu}{2}\right) \left[ \gamma^\mu + 2v^\mu \right] \left(\frac{1 + \gamma^\mu}{2}\right) ,
\end{align*}

(A1)

where $V(A) \equiv \gamma^\mu(\gamma^\mu\gamma_5)$.
REFERENCES

[1] D. E. Groom et al. (Particle Data Group), Eur. Phys. J C 15 (2000) 1.

[2] H. Albrecht et al. (ARGUS Collaboration), Phys. Lett. B 317 (1993) 227; P. L. Frabetti et al. (E687 Collaboration), Phys. Rev. Lett. 72 (1994) 961; K. W. Edwards et al. (CLEO Collaboration), Phys. Rev. Lett. 74 (1995) 3331; J.P. Alexander et al. (CLEO Collaboration), Phys. Rev. Lett. 83 (1999) 3390.

[3] N. Isgur and M. B. Wise, Phys. Lett. B 232 (1989) 113; ibid. 237 (1990) 527;
E. V. Shuryak, ibid. 93B (1980) 134;
H. Georgi, ibid. B 240 (1990) 447;
E. Eichten and B. Hill, ibid. B 234 (1990) 511;
M. B. Voloshin and M. A. Shifman, Yad. Fiz. 45 (1987) 463; ibid. 47 (1988) 801;
S. Nussinov and W. Wetzel, Phys. Rev. D 36 (1987) 130;
A. F. Falk, H. Georgi, B. Grinstein and M. B. Wise, Nucl. Phys. B343 (1990) 1.

[4] N. Isgur and M. B. Wise, Nucl. Phys. B348 (1991) 276; H. Georgi, ibid. (1991) 293;
J.-P. Lee, C. Liu, and H. S. Song, Phys. Rev. D 58 (1998) 014013.

[5] T. Mannel, W. Roberts, and Z. Ryzak, Phys. Lett. B 271 (1991) 421; Y.-B. Dai, X.-H. Guo, and C.-S. Huang, Nucl. Phys. B421 (1994) 277.

[6] P. Cho, Phys. Lett. B 285 (1992) 145; J.-P. Lee, C. Liu, and H.S. Song, Phys. Rev. D 61 (1999) 014006.

[7] C.-S. Huang, C.-F. Qiao, and H.-G. Yan, Phys. Lett. B 437 (1998) 403; C.-S. Huang and H.-G. Yan, Phys. Rev. D 59 (1999) 114022.

[8] M. Shifman, A. Vainshtein and V. Zakharov, Nucl. Phys. B147 (1979) 385; ibid. (1979) 448.

[9] A. G. Grozin and O. I. Zakovlev, Phys. Lett. B 285 (1992) 254; E. V. Shuryak, Nucl. Phys. B198 (1982) 83; B. Bagan, M. Chabab, H. G. Dosch, and S. Narison, Phys. Lett.
[10] J.-P. Lee, C. Liu, and H. S. Song, Phys. Lett. B 476 (2000) 303.

[11] S.-L. Zhu, Phys. Rev. D 61 (2000) 114019;
    C.-S. Huang, A. Zhang and S.-L. Zhu, Phys. Lett. B 492 (2000) 288.

[12] M.-Q. Huang, J.-P. Lee, C. Liu, H.S. Song, Phys. Lett. B 502 (2000) 133.

[13] A.K. Leibovich, Z. Ligeti, I.W. Stewart, M.B. Wise, Phys. Rev. Lett. 78 (1997) 3995;
    Phys. Rev. D 57 (1997) 308.

[14] P. Colangelo, F. De Fazio, N Paver, Phys. Rev. D 58 (1998) 116005.

[15] M.-Q. Huang, Y.-B. Dai, Phys. Rev. D 59 (1999) 034018.

[16] M.-Q. Huang, C.-Z. Li, Y.-B. Dai, Phys. Rev. D 61 (2000) 054010; M.-Q. Huang, Y.-B.
    Dai, Phys. Rev. D 64 (2001) 014034.

[17] W.Y. Wang, Y.L. Wu, Int. J. Mod. Phys. A 16 (2001) 2505.

[18] A. F. Falk, Nucl. Phys. B378 (1992) 79.

[19] A. K. Leibovich and I.W. Stewart, Phys. Rev. D 57 (1998) 5620.

[20] M. Neubert, Phys. Rev. D 45 (1992) 2451.

[21] B. Blok and M. Shifman, Phys. Rev. D 47 (1993) 2949.
FIGURE CAPTIONS

Fig. 1
Feynman diagrams for the three-point function with derivative interpolating fields. Double
line denotes the heavy quark.

Fig. 2
Three dimensional plot of $\tau$ as a function of $y$ and $T$ in units of GeV. The continuum
threshold is chosen to be $\omega_c(y) = 1.4$ GeV.

Fig. 3
$\tau(1)$ as a function of the Borel parameter $T$. Each graph corresponds to $\omega_0 = 1.2, 1.3, 1.4, 1.5, 1.6$ GeV, respectively, from the top.

Fig. 4
$\tau(y)$ at a fixed Borel parameter $T = 0.34$. Each graph corresponds to $\omega_0 = 1.2, 1.3, 1.4, 1.5, 1.6$ GeV, respectively, from the top.
FIGURES

FIG. 1.
FIG. 2.

FIG. 3.
\( \tau(y,T=0.55 \text{ GeV}) \)

FIG. 4.