On the local symmetries of gravity and supergravity models

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We present here a detailed analysis of the local symmetries of supergravity in an arbitrary dimension $D$, both in the component and superfield approaches, using a field–space democracy point of view. As an application, we discuss briefly how a complete description of the local gauge symmetries clarifies the properties of the supergravity–superbrane coupled system in the standard background super field approximation for supergravity.

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1. Introduction

A recent supersymmetric analysis of the supergravity–superbrane interaction \cite{1,2} in which supergravity is described by its group manifold action (not as a background), as well as other related models \cite{3–5}, makes desirable a reexamination of the role of local supersymmetry and, more generally, of the gauge symmetries in supergravity models.

We present here a detailed account of local symmetries, beginning in Sec.2 by the differential forms formulation of $D$–dimensional general relativity. We describe in Sec.3 the complete set of its spacetime gauge symmetries, including diffeomorphism invariance whose discussion, together with general coordinate transformations, becomes especially relevant when the interacting system of (super)gravity and (super–p–)brane is considered. This is so because in the super–p–brane action the spacetime variables play a dynamical rôle.

We explain how the presence of diffeomorphism invariance provides the possibility of presenting the general coordinate invariance in an equivalent form, its ‘variational version’, which does not act on the spacetime coordinates (see \cite{6} for the $D = 4 N = 1$ superfield supergravity case). In Sec. 4 we consider the second Noether theorem for general relativity. Then, in Sec. 5, we describe the general structure of the action for the standard, component formulation of supergravity, its local symmetries and their associated Noether identities.

Sec. 6 is devoted to the superspace general coordinate symmetry and other gauge symmetries for the on–shell superfield formulation of supergravity, where supergravity is described by the set of constraints on the superspace torsion, which imply dynamical equations. We apply this knowledge in Sec.7 to uncover the relation between the local supersymmetry and the $\kappa$–symmetry of the supermembrane in a $D = 11$ superfield supergravity background.

2. D–dimensional General Relativity in differential form

$D$–dimensional gravity models can be formulated in terms of the moving frame or vielbein fields $e_{\mu}^a(x)$ (tetrad in $D = 4$), which determine the vielbein one–forms on spacetime $M^D$,

$$
e^a(x) = dx^\mu e_{\mu}^a(x)
$$

A change of frame is given by a matrix of the local $SO(1, D - 1)$ group,

$$
e^a(x) \rightarrow e^a'\!(x) = e^b(x)\Lambda_b^a(x)
$$
This local Lorentz symmetry is the first gauge symmetry of gravity theory to be noted. Its infinitesimal form is
\[
\delta_L x^\mu = 0, \quad \delta_L e^a(x) = e^b(x) L^a_b(x),
\]
\[
L^{ab}(x) = -L^{ba}(x),
\]
where \( \Lambda^a(x) = \delta^a_b + L^a_b(x) + \mathcal{O}(L^2) \). It is convenient to introduce the spin connection
\[
w^{ab}(x) = d\xi^{\mu} w^{ab}_\mu(x) = -w^{ba}(x),
\]
with the transformation rule \( w^{ab}(x) \rightarrow w'^{ab}(x) = (\Lambda^{-1}(x) w(x) \Lambda(x))^{ab} \), or
\[
\delta_L w^{ab}(x) = DL^{ab} = DL^{ab}_c - 2L^{[a} w_c b^{b]}.
\]
The spin connection can be either expressed from the beginning through the vielbein field by imposing the covariant torsion constraint
\[
T^a := De^a \equiv de^a - e^b \wedge w^a_b = 0
\]
(see e.g. [12] and refs. therein).

In field theory analyses, where \( \phi \) is treated as dynamical variables, the obvious invariance can be ignored in favour of general coordinate symmetry (see below). However, when the coupled system of (super)gravity and (super)brane is considered in the framework of an action principle (see [12]) the set of dynamical variables includes, besides the fields \( e^a_\mu(x) \) etc., the local coordinate functions \( \hat{x}^\mu(\xi) \) defined by the map \( \tilde{\phi} : W^{p+1} \rightarrow M^D \), where \( W^{p+1} \) is the worldvolume with local coordinates \( \xi^1 = (\tau, \sigma^1, \ldots, \sigma^p) \). This suggests adopting a field-space democracy approach [10] where the fields \( e^a_\mu(x) \) etc. and the spacetime coordinates \( x^\mu \) are treated on equal footing.
3.2. General coordinate symmetry and its ‘variational version’

Besides being local Lorentz and diffeomorphism invariant, the EH action \( \hat{S} \) is invariant under general coordinate transformations as we discuss below.

To derive the equations of motion for a field theory from the variational principle, as e.g. eq. (14) for general relativity, one uses arbitrary variations of the fields only, e.g. \( \delta e^a(x) \), so that
\[
\delta' x^\mu = 0 , \quad \delta' e^a(x) = d x^\mu \delta' e^a(x) .
\] (16)

On the other hand, a general variation \( \delta e^a(x) \) is
\[
\delta e^a(x) = \delta' e^a(x) + \delta x e^a(x) ,
\] (17)
where \( \delta x \) denotes the variation due to the change \( x \to x' = x + \delta x \). The \( \delta x \) variation is given by the Lie derivative \( \mathcal{L}_x = d \delta x + i \delta x d \). For instance,
\[
\delta x e^a(x) := e^a(x + \delta x) - e^a(x) = \mathcal{L}_x e^a(x) = d i \delta x e^a(x) + i \delta x e^a(x) = D(i \delta x e^a(x)) + i \delta x T^a(x) + e^b i \delta x w_b^a ,
\] (18)
where
\[
i \delta x e^a = \delta x^\mu e^a_{\mu}(x) , \quad i \delta x w_{ab} = \delta x^\mu w_{ab}^\mu(x) ,
\]
\[
i \delta x T^a(x) = e^b i \delta x e^c T^a_{bc}(x) ;
\] (19)
the last term in \( \delta x \), \( e^b i \delta x w_{ab} = \delta x^\mu w_{ab}^\mu(x) \), is the local Lorentz rotation induced by \( \delta x^\mu \).

In the above notation, diffeomorphism invariance (14), (15) can be formulated as a symmetry under the transformations
\[
\delta_{\text{diff}} x^\mu = b^\mu(x) ,
\]
\[
\delta_{\text{diff}} e^a(x) = \delta_{\text{diff}}^e e^a(x) + L_b e^a(x) = 0 .
\] (21)
Thus, \( \delta_{\text{diff}} e^a(x) \) is defined by \(- L_b e^a(x) \), i.e.
\[
\delta_{\text{diff}} e^a(x) = - L_b e^a ,
\] (22)
and \( \delta_{\text{diff}} S_{D,G} = 0 \) follows as an evident consequence of (13) and (21).

In contrast, general coordinate transformations or local translations are defined as arbitrary displacements of the spacetime points,
\[
x^\mu \mapsto x'^\mu = x^\mu + \delta_{gc} x^\mu \equiv x^\mu + t^\mu(x)
\] (23)
(cf. (20)) which induce differential forms transformations as e.g.
\[
e^a(x) \mapsto e^a(x') = e^a(x + \delta_{gc} x) \equiv e^a(x + t) ,
\] (24)
i.e.,
\[
\delta_{gc} e^a(x) = \delta_{xc} e^a(x) \equiv L_t e^a(x) = \mathcal{D}(t^a(x)) + i_i T^a(x) + e^b i \delta x w_b^a
\] (25)
(as in (18)) where \( t^a(x) = i_i e^a(x) \equiv t^a(x) e^a_\mu(x) \).

Consider a \( D \)-form \( L_D \) on \( M^D \), involving the \( \wedge \) product of forms, their exterior derivatives and, possibly, the \( \ast \) Hodge operator. Then \( L_D(x) \mapsto L_D(x') = L_D(x + t) \), and
\[
\delta_{gc} L_D = L_b L_D = (i_i d + d_i) L_D = d_i L_D ,
\] (26)
since any \( D \)-form on \( M^D \) is closed. Thus, any \( S = \int_{M^D} L_D \) is general coordinate invariant. In particular, the EH action \( \hat{S} \) possesses this invariance.

Thus, one may look at \( \delta_{\text{diff}} \) and \( \delta_{gc} \), respectively, as passive and active forms of the same spacetime coordinates transformations, unaffected by any theory defined through an integral of a \( D \)-form \( L_D \) on \( M^D \). This picture changes when the action of a \( p \)-brane, which is given by an integral \( \int_{W^{p+1}} \mathcal{L}_{p+1} \) on the \( (p+1) \)-dimensional worldvolume \( W^{p+1} \), is considered together with \( \int_{M^D} L_D \).

The coupled action \( \int_{M^D} L_D + \int_{W^{p+1}} \mathcal{L}_{p+1} \) will still possess spacetime diffeomorphism invariance provided \( \mathcal{L}_{p+1} \) is formulated in terms of pull–backs of spacetime differential forms (so that eq. (14) implies \( \tilde{x}^\mu(\xi) \mapsto \tilde{x}'^\mu(\xi) = \tilde{x}^\mu(\xi) + b^\mu(\tilde{x}(\xi)) \)), but it will not be spacetime general coordinate invariant, since such an invariance means equivalence between different spacetime points, and points on the brane are not equivalent to points outside it.

Let us go back to the pure gravity case. On account of diffeomorphism invariance, one can use equivalently (rather than \( \delta_{gc}(t) \), eqs. (24)–(25)), \( \delta_{gc}(t) \) followed by a diffeomorphism (eqs. (20), (22)) with \( b^\mu(x) = - t^\mu(x) \), \( \delta_{\text{diff}}(b = - t) \). As we are dealing with a local Lorentz invariant theory, we may also add a local Lorentz transformation with parameter \( L^{ab} = - i_t w^{ab} \) and get
\[
\tilde{\delta}_{gc}(t^a) := \delta_{gc}(t^a) + \delta_{\text{diff}} (b^\mu = - t^\mu) + \delta_{gc}(L^{ab} = - i_t w^{ab}) .
\] (27)
This $\delta_{gc}(t^a)$ will be called, following \cite{6}, the 'variational version' of the general coordinate transformation $\delta_{gc}(t^a)$. Indeed, it does not act on $x^\mu$,
\begin{equation}
\delta_{gc} x^\mu := 0 ,
\end{equation}
so that, e.g. (eqs. \cite{11}, \cite{28}),
\begin{equation}
\delta_{gc} e^a(x) := Dt^a + e^c t^b T_{bc}^a (x) = \tilde{\delta}_{gc} e^a(x) .
\end{equation}
Thus $\tilde{\delta}_{gc}$ provides the complete general coordinate variation of a differential form, e.g. $\delta_{gc} e^a(x) = \delta_{gc} e^a(x)$, as a result of the field variation $\delta_{gc}$ only.

In the second order approach, where $T^a = 0$, the $\delta_{gc}$ transformations \cite{28} simplify and acquire the characteristic form of a gauge field transformation,
\begin{equation}
\tilde{\delta}_{gc} e^a(x) |_{T^a = 0} = D t^a .
\end{equation}
This provides the possibility of treating gravity as a gauge theory of local translations in their variational form $\delta_{gc}$ (see \cite{10} for early discussions of gravity as a gauge theory).

Note, finally, that since the above $D$–form $L_D$ is diffeomorphism invariant and $\delta_{gc} S = 0$ by \cite{28}, it follows directly from \cite{28} that $\delta_{gc} S = 0$ also.

4. Second Noether theorem applied to General Relativity

The invariance of the EH action \cite{10} under the variational version of the general coordinate transformations $\delta_{gc}(t^a)$,
\begin{equation}
\delta_{gc} S_{D,G} = (-1)^D \int_M D M_{(D-1)a} t^a (x) = 0 .
\end{equation}
follows from the fact that the Bianchi identity $D R^{ab} = 0$ and the torsion constraint \cite{10} imply
\begin{equation}
D M_{(D-1)a} \equiv 0
\end{equation}
(for simplicity, we use in this section the second order approach). This is the so-called Noether identity (NI) which reflects the presence of a gauge symmetry, here the symmetry under $\delta_{gc}$.

In general, the second Noether theorem states that any gauge symmetries, $\delta_{gauge} S = 0$, given by transformation rules that involve the derivatives of the local parameter up to $k$-th order, are in one–to–one correspondence with their associated NIs, i.e. with identically satisfied relations between the (l.h. sides of the) Lagrangian equations of motion and their derivatives up to $k$-th order.

To discuss also $\delta_{diff}$ and $\delta_{gc}$ in this framework, consider the second order approach to $D$–dimensional general relativity in a field–space democracy context \cite{10}, where coordinates and fields are treated on the same footing. Then, the dynamical variables are the vielbein field $e^a_{\mu}(x)$ and the spacetime coordinate $x^\mu$. Their Lagrangian equations are
\begin{equation}
N_\mu = 0 , \quad N_\mu := \frac{\delta S}{\delta x^\mu} \quad \text{and}
\end{equation}
\begin{equation}
M^\mu_a = 0 , \quad M^\mu_a := - (1)^D \frac{\delta S}{\delta e^a_{\mu}} ,
\end{equation}
where $M^\mu_a$ defines the differential $D$–form $M_{(D-1)a}$, eq. \cite{10}.

To find an explicit expression for $N_\mu$ one uses the splitting \cite{17} of the general variation of the EH action \cite{10}.
\begin{equation}
\delta S_{D,G} = - (1)^D \int_M D M_{(D-1)a} \wedge \left( \delta e^b(x) + \delta x^a \right) .
\end{equation}
The Einstein equation \cite{34} now follows from the $\delta e^a(x)$ variation, while the $\delta x^a$ variation entering
\begin{equation}
\delta x^a = D (i_{\delta x} e^a (x)) + e^b \delta x^b a \quad \text{eq.} \quad \text{eq.} \quad \text{eq.} \quad \text{eq.}
\end{equation}
with $T^a = 0$ results in eq. \cite{33} with $N_\mu$ defined by
\begin{equation}
d^D x N_\mu = (1)^D D M_{(D-1)a} e^a_{\mu} - (1)^D M_{(D-1)a} \wedge e^b w^{\mu b} a .
\end{equation}

The variational version of the general coordinate transformations $\delta_{gc}$, \cite{28}–\cite{34}, as well as the local Lorentz transformations $\delta_L$, \cite{3}, do not act on the spacetime coordinates. As a result, the NIs reflecting the invariance under $\delta_{gc}$ and $\delta_L$ only involve the l.h. side of eq. \cite{10} (eqs. \cite{34})
\begin{equation}
\delta_{gc} S_{D,G} = 0 \iff D M_{(D-1)a} = 0 ,
\end{equation}
\begin{equation}
\tilde{\delta}_L S_{D,G} = 0 \iff M_{(D-1)a} \wedge e^b \equiv 0 .
\end{equation}
In contrast, for the general coordinate symmetry in its original form $\delta_{gc}$, \cite{28}, \cite{34}, the basic transformations are arbitrary changes of the spacetime points, eq. \cite{28}. Thus,
\begin{equation}
\delta_{gc} S_{D,G} = 0 \iff N_\mu := \frac{\delta S}{\delta x^\mu} \equiv 0 .
\end{equation}
Using eq. (37) together with (38) and (39) one finds that the identity (39) holds indeed.

It might look strange that two equivalent formulations of the same general coordinate symmetry, $\delta_{gc}$ and $\delta_{ge}$, have different NIs. The reason is simple: a linear combination of these NIs reproduces the NI for diffeomorphism invariance $\delta_{diff}$, (28), (29) (or, equivalently (14), (15)). Indeed, the explicit form of $N_\mu$ (38) actually provides us with such NI

$$\delta_{diff} S = 0 \Leftrightarrow d^D x N_\mu - (-1)^D \times (\mathcal{M}(D-1)_a e^\rho_a - M(D-1)_a \wedge e^b w_{\rho b}) = 0 \,.$$ (40)

As the two terms inside the brackets are identically equal to zero due to the NIs for $\delta_{gc}$ and $\delta_{L}$, (eqs. (27) and (28)) the NI (10) implies (39) and viceversa. This translates the definition of $\delta_{gc}$ in eq. (27) to the language of the second Noether theorem.

5. $D$–dimensional supergravity

5.1. Local supersymmetry and supergravity

Supergravity (see e.g. [12] and refs. therein) is the gravity theory invariant under local supersymmetry $\delta_s$. This is a local symmetry involving a fermionic (Grassmann) spinor parameter $e^{\alpha}(x)$, $\alpha = 1, \ldots n$ is a $D$–dimensional Lorentz spinor index, $n = \text{dim}(\text{Spin}(1, D - 1))$. Hence $\delta_s e^{\alpha}(x)$ mixes the graviton field, i.e. the vielbein $e^a_{\alpha}(x)$, with a fermionic field, the gravitino or Rarita–Schwinger field $\psi^a_{\mu}(x)$. Specifically,

$$\delta_s e^{\alpha}(x) = - 2 i \psi^a_{\alpha}(x) \Gamma^a_{\mu
u\rho} e^\beta(x) .$$ (41)

Using the fermionic one–form $e^{\alpha}(x)$,

$$e^{\alpha}(x) := dx^\mu \psi^a_{\mu\alpha}(x) = e^a \psi^a_{\alpha}(x) ,$$ (42)

eq. (41) reads

$$\delta_s e^{a}(x) = - 2 i \psi^{a}(x) \Gamma^a_{\mu
u\rho} e^\beta(x) ,$$ (43)

The vector–spinor gravitino field has the proper index structure to be the gauge field for local supersymmetry. Thus it is natural to assume that

$$\delta_s \psi^{a}(x) = \mathcal{D}_\mu e^{a}(x) ,$$ (44)

or, equivalently

$$\delta_s \psi^{a}(x) = D_a \psi^{a}(x) .$$ (45)

The guess (44) or (45) is supported by the fact that the linearized form $\delta_s$ of (45)

$$\delta_s e^{a}(x) = de^{a}(x) , \quad \delta_s \psi^{a}(x) = \partial_\mu e^{a}(x) ,$$ (46)

is an evident symmetry of the free $D$–dimensional Rarita–Schwinger (RS) action on flat spacetime

$$S_{RS}^D = - \frac{D-3}{2} \int_R d^D x \wedge e^{a\beta} \wedge (dx)^{(D-3)} \Gamma^a_{\mu
u\rho} e^\beta \propto \int d^D x \psi_{\mu
u\rho} \partial_\rho \psi_{\mu
u} .$$ (47)

The first candidate for a locally supersymmetric action is the sum of the free EH action (7) and the RS action (47) ‘covariantized’ with respect to the local Lorentz transformations (4)

$$S_{D}^{2+3/2} = \int_M \mathcal{L}^2_D + \int_M \mathcal{L}^3_D ,$$ (48)

$$\mathcal{L}^2_D = R_{ab} \wedge e^a_{\beta} \wedge e^b_{\alpha} \Gamma^{abc}_{\beta} \propto \int d^D x \psi_{\mu\nu\rho} \partial_\rho \psi_{\mu\nu} .$$ (49)

For $D = 4$, $N = 1$, $S_{D}^{2+3/2}$ is indeed locally supersymmetric under (13), (14) and provides the action for the simple $D = 4$ supergravity [1] (see also [12,13]).

$$S_{4,SG} = S_{D=4}^{2+3/2} .$$ (51)

5.2. General structure of the supergravity action and equations of motion

In higher dimensions (in particular, in $D = 10,11$) the supergravity multiplet involves a set of antisymmetric tensor gauge fields $C_{\mu_1, \ldots, \mu_q}(x)$ described by differential forms

$$C_q = \frac{1}{q!} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_q} C_{\mu_1, \ldots, \mu_q}(x) ,$$ (52)

($C_3$ for $D = 11$; $C_2$, $C_4$ and $B_2$ in $D = 10$ type IIB, $C_1$, $C_3$ and $B_2$ in $D = 10$ type IIA, etc.) and, in $4 < D < 11$, scalar fields (e.g. dilaton $\phi$ in $D = 10$ type IIA and IIB and axion $C_0$ in $D = 10$ type IIB) and spinors. Thus, in general

$$S_{SG,D} = \int_M (\mathcal{L}^2_D + \mathcal{L}^3_D + \mathcal{L}^\leq_4 D) ;$$ (53)

$\mathcal{L}^\leq_4 D$ includes, in particular, the kinetic term for the $q$–form gauge fields

$$\propto \int d^D x \det(e^a_\mu) \mathcal{H}_{\mu_1, \ldots, \mu_{q+1}} \mathcal{H}^{\mu_1, \ldots, \mu_{q+1}} + \ldots ,$$ (54)
where
\[ \mathcal{H}_{q+1} := dC_q - c_1 e \wedge e \wedge \widetilde{\Gamma}^{(q-1)}_{eg} \]
(55)
\[ = \frac{1}{(q+1)!} dx^{a_{q+1}} \wedge dx^{\mu_1} \mathcal{H}_{\mu_1 \ldots \mu_{q+1}}(x), \]
(56)
\[ \widetilde{\Gamma}^{(k)}_{eg} := \frac{1}{k!} e^{a_1} \ldots e^{a_k} \Gamma_{a_1 \ldots a_k}, \]
(57)
is the generalized field strength of \( C_q \).

These kinetic terms can be written in a first order form (which is suitable for discussing the relation with superspace approach, see [8]) if one adds to every gauge \( q \)-form \( C_q \) with an auxiliary antisymmetric tensor field \( F_{a_1 \ldots a_{q+1}}(x) = F_{[a_1 \ldots a_{q+1}]}(x) \). These fields can be used to construct the \((q+1)\)-forms and the \((D-q-1)\)-forms
\[ F_{q+1} \equiv \frac{1}{(q+1)!} e^{a_{q+1}} \wedge \ldots \wedge e^{a_1} F_{a_1 \ldots a_{q+1}}(x), \]
(58)
\[ F_{D-q-1} = e_{a_1 \ldots a_{D-1}} F_{a_1 \ldots a_{D-1}} + \ldots, \]
(59)
which allow us to write the kinetic term(s) \([54]\)
as
\[ \mathcal{L}_D \leq 1 = c(\mathcal{H}_{q+1} - \frac{1}{2} F_{q+1}) \wedge \mathcal{F}_{D-q-1} + \ldots, \]
(60)
where the terms denoted by dots do not contain \( F_{a_1 \ldots a_{q+1}}(x) \).

Indeed, the variation of \( F_{a_1 \ldots a_{q+1}}(x) \) leads to the algebraic equation
\[ \mathcal{H}_{q+1} - F_{q+1} = 0 \]
(61)
which identifies the auxiliary field \( F_{a_1 \ldots a_{q+1}}(x) \) with the generalized field strength of the tensor gauge field \( C_{a_1 \ldots a_{q}}(x) \),
\[ F_{a_1 \ldots a_{q+1}}(x) = (q+1) \nabla_{a_1} C_{a_2 \ldots a_{q+1}} + \ldots =
\]
\[ = (q+1) e^{\mu_1} \ldots e^{\mu_{q+1}} \partial_{\mu_1} C_{\mu_2 \mu_3 \ldots \mu_{q+1}} + \ldots, \]
where the dots denote the terms with torsion and fermions. Substituting eq. (61) into the Lagrangian form (60) one arrives at the standard, second order representation, for the kinetic term of the gauge field \( C_{\mu_1 \ldots \mu_q}(x) \), eq. (54).

On the other hand, varying \( F_{a_1 \ldots a_{q+1}}(x) \) with respect to the gauge field(s) \( C_{\mu_1 \ldots \mu_q}(x) \) one finds
\[ G_{(D-q)} \equiv d(e^{a_1 \ldots a_{q+1}} F_{a_1 \ldots a_{q+1}}) + \ldots = 0, \]
(62)
which, after the use of eq. (51), becomes the dynamical gauge field equation.

For future reference we note that the equation \( \delta S_{D,SG}/\delta u^{ab} = 0 \) determines the ‘improved’ constraint on the spacetime torsion \( T^a \) (cf. (9)),
\[ T^a + ie^a \wedge e^b \Gamma_{bc}^a = 0, \]
(63)
while \( \delta e^a \) and \( \delta e^a \) provide the differential form expression for the RS and Einstein equations of supergravity
\[ \Psi_{(D-1)a} := \frac{4}{3} D a \wedge e^{a(D-3)} \Gamma_{abc}^a + \ldots = 0, \]
(64)
\[ M_{(D-1)a} := R^{abc} \wedge e^{a(D-3)} + \ldots = 0. \]
(65)

For simplicity, we will not consider here the cases where the supergravity multiplet involves scalar and spinor fields. Thus our basic examples are \( D = 3, 4 \) and 11 supergravity.

In the above notation a generic variation of the supergravity action reads
\[ \delta S_{D,SG} = -(-1)^D \int_M G a \wedge \delta e^a + \]
\[ +(-1)^D \int_M \Psi_{(D-1)a} \wedge \delta e^a + \]
\[ + \int_M \delta e^{a(D-3)} \wedge (\mathcal{C} + ie^a \wedge e^b \Gamma_{bc}^a) \wedge \delta u^{ab} \]
\[ + c \int_M (\mathcal{H}_{q+1} - F_{q+1}) \wedge e^{a(D-q-1)} \delta F_{a_1 \ldots a_{q+1}} + \]
\[ + c(-1)^D \int_M \mathcal{G}_{D-q} \wedge \delta C_q. \]
(66)

5.3. Local supersymmetry, general coordinate symmetry and Noether identities

The above first order form of the supergravity action (see [8]), eq. (63) with [8], [8] and [60] is written in terms of differential forms on \( M^D \) (including the covariant zero–foms \( F_{a_1 \ldots a_{q+1}}(x) \)) and thus is invariant under \( M^D \)-diffeomorphisms, defined by eqs. (24), (23) and the analogous ones for \( e^a(x) \) etc.

The action of \( D \)-dimensional supergravity [53], being a generalization of the EH general relativity action, eq. (7), possesses local Lorentz invariance [8] and general coordinate invariance under \( \delta g_{\mu \nu} \) (eqs. (23), (23)) or, equivalently, under its variational version \( \delta g_{\mu \nu} \) (eq. (23)). Moreover, it is invariant under local supersymmetry transformations \( \delta_{ts} x^\mu = 0, \)
\[ \delta_{ts} e^a(x) = -2ie^a(x) \Gamma_{a^b} e^b(x), \]
(67)
\[
\delta_{ls} \varphi(x) = D \dot{\varphi}(x) + \epsilon \varphi(x) \mathcal{M}_{12} \varphi(x) , \tag{69}
\]
\[
\delta_{ls} C_{p+1}(x) = 2 \epsilon \varphi(x) \wedge \bar{\Gamma}^{e} \alpha \varphi(x) , \tag{70}
\]
\[
\delta_{ls} \omega^{ab}(x) = W_{ab}^{(p+1)}(x) \varphi(x) , \tag{71}
\]
\[
\delta_{ls} F^{a_{1} \ldots a_{p+1}}(x) = S_{a_{1} \ldots a_{p+1}}^{1}(x) , \tag{72}
\]
where \(S_{a_{1} \ldots a_{q+1}}^{1}(x)\) and the one–forms \(\mathcal{M}_{a_{1}a_{2}}(x)\), \(W_{a_{1}a_{2}}^{(p+1)}(x)\) are constructed from the fields of the supergravity multiplet and the auxiliary fields \(F_{a_{1} \ldots a_{p+2}}(x)\) (cf. \([13] , [14] \)).

Then, the experience of Secs. 3,4 allows one to conclude (actually without any further calculations) that the general coordinate symmetry in its variational form \(\delta_{gc}\), eqs. \((28), (29)\), and the local supersymmetry \(\delta_{ls}\), eqs. \((67)–(72)\) are reflected by Noether identities relating the \(l.h.\) sides of the field equations only, namely
\[
\mathcal{D} \Psi_{(D-1)a} = 2 i M_{(D-1)a} \wedge \epsilon G_{a}^{(p+1)} + \ldots \equiv 0 , \tag{73}
\]
\[
\mathcal{D} M_{(D-1)a} = - \ldots \equiv 0 , \tag{74}
\]
where the terms denoted by dots turn out to be proportional to the \(l.h.\) sides of eqs. \((23), (24)\), but not of the Einstein equation \((65)\). For example, for \(D = 4, N = 1\) supergravity the full NIs of the theory \([14] , [15] \) read
\[
\mathcal{D} \Psi_{3a} = 2 i M_{3a} \wedge \epsilon G_{a}^{(p+1)} - \psi_{a} \mathcal{D} \Psi_{3a} - (T^{a} + i \epsilon \mathcal{A} \wedge \epsilon G_{a}^{(p+1)} = 0 , \tag{75}
\]
\[
\mathcal{D} M_{3a} = - \frac{1}{2} \epsilon_{abcd} R^{bc} \wedge (T^{d} + i \epsilon \mathcal{A} \wedge \epsilon G_{a}^{(p+1)} = 0 . \tag{76}
\]

To check that \(\delta_{ls} S_{DS, SG} = 0\) (or \(\delta_{gc} S_{DS, SG} = 0\)) implies \((73)\) (or \((74)\) and viceversa it is sufficient to insert \((67)–(72)\) (or \((28), (29)\)) in the general expression for the supergravity action variation \((60)\),
\[
\delta_{ls} S_{DS, SG} = - (1)^{D} \int_{M} \delta_{ls} (M_{(D-1)a} \wedge \epsilon \mathcal{A}^{a}) = - \Psi_{(D-1)a} \wedge \epsilon \mathcal{A}^{a} + \ldots
\]
\[
= - (1)^{D} \int_{M} (-2 i M_{(D-1)a} \wedge \epsilon G_{a} + \mathcal{D} \Psi_{(D-1)a} + \ldots) = 0. \tag{76}
\]

Then, one sees again (cf. Sec. 4) that the gauge invariance of the action and the Noether identities imply each other.

6. Supergravity in superspace

The local supersymmetry \(\delta_{ls}\), eqs. \((23)–(25)\), has a structure which resembles that of the variational copy of the general coordinate transformations, \(\delta_{gc}\) (eqs. \((23), (25)\)), but with a fermionic parameter. The similarity can be recognized also from the structure of the Noether identities, \((23), (24)\). This is one more reason for the existence of superspace \(\Sigma^{(D|N)}\) (originally introduced \([14] \) in connection with global supersymmetry) with coordinates \(Z^{M} = (x^{\mu}, \theta^{a})\), \(\alpha = 1, \ldots, n\), where, e.g. \(n = 2[D/2]\) for \(N = 1, D \neq 10\) and \(N = 2, D = 10\). The holonomic or coordinate basis for the cotangent superspace is provided by \(dZ^{M} = (dx^{\mu}, dB^{a})\), while the general unholonomic basis (with \(Spin(1, D - 1)\) indices denoted by underlined greek letters) is defined by the superviolbein forms \(\bar{e}_{\mu}^{\alpha}, \bar{e}_{\mu}^{\alpha}\). The differential geometry of spacetime can be extended to superspace \(\Sigma\). In particular, introducing the spin connection superform
\[
w_{ab} = dZ^{M} w^{ab}_{M}(Z), \quad w_{a}^{\alpha} = \frac{1}{2} w_{ab} \tilde{\Gamma}_{a b}^{\alpha} \tag{78}
\]
one can define superspace torsions and curvature
\[
T^{a} := \mathcal{D} w^{a} = d \mathcal{A}^{a} - E^{a} \wedge w_{b}^{a} \tag{79}
\]
\[
T^{\alpha} := \mathcal{D} w_{a}^{\alpha} = d \mathcal{A}^{a} - \mathcal{A}^{a} \wedge w_{b}^{a} \tag{80}
\]
\[
F_{ab} := d w_{ab} - w_{ac} \wedge w_{b}^{c} \tag{81}
\]

as well as, when the supergravity multiplet contains antisymmetric tensor gauge fields \(C_{q}(x)\), the generalized field strengths
\[
\mathcal{H}_{q+1} := d C_{q} - c_{1} E_{a} \wedge E_{a} \wedge \bar{\Gamma}^{(q+1)}_{a} \tag{82}
\]
\[
\mathcal{H}_{q+1} := \frac{1}{(q+1)!} E_{A_{1}}^{a_{1}} \wedge E_{A_{1}}^{a_{1}} \wedge \bar{\Gamma}^{(q+1)}_{a_{1}} \tag{83}
\]
of the various gauge superforms
\[
C_{q} := \frac{1}{q!} E_{A_{1}}^{a_{1}} \wedge \ldots \wedge E_{A_{q}}^{a_{q}} C_{A_{1}} \ldots C_{A_{q}} \tag{85}
\]
6.1. Local supersymmetries of supergravity in superspace

The differential forms on $\Sigma^{(D|m)}$ are invariant under arbitrary changes of coordinates, i.e., under local superspace diffeomorphisms,

\[ Z^M \mapsto Z'^M = Z^M + b^M(Z) , \]
\[ E^A(Z) \mapsto E^A'(Z') = E^A(Z) + L_b E^A = 0 , \text{ etc.}, \]

for which (cf. (21))

\[ \delta_{sdiff} Z^M = b^M(Z) , \]
\[ \delta_{sdiff} E^A = \delta_{sdiff} Z^M = t^M(Z) , \]

but, in contrast with (83),

\[ \delta_{sgc} E^A = L_t E^A = D_i E^A + i_t T^A + E^B i w_B^A , \]
\[ i_t w_B^A = t^M w_M B^A , \]
\[ i_t E^A = t^M E^A , \text{ etc.} , \]
\[ w_B^A = \text{diag}(w_a^a, w_b^b) . \]

Using the supervielbein and spin connection contain a large amount of fields (mostly unwanted). The supergravity multiplets can be extracted from the supervielbein by imposing covariant constraints on the superspace torsions, curvature and the gauge superform field strengths. The main constraints have the form

\[ T^a = i E^a \wedge E^b \bar{\Gamma}_{ab}^c ; \]
\[ H_{q+1} := dC_q - c_1 E^a \wedge E^b \wedge \Gamma_{(q-1)}^{a b} = \frac{1}{(q+1)!} E^{a_{q+1}} \wedge \ldots \wedge E^{a_1} F_{a_1 \ldots a_{q+1}}(Z) , \]

and can be derived as a straightforward extension of the component, first order form supergravity eqs. (63), (64) to superspace. This fact is not accidental. It reflects the existence of the so-called group manifold or 'rheonomic' approach to supergravity (66), which provides the bridge between the component and superfield formalism (see also Sec. 2 of 13 for a brief review).

6.2. Superspace constraints

The unrestricted supervielbein and spin connection contain a large amount of fields (mostly unwanted). The supergravity multiplets can be extracted from the supervielbein by imposing covariant constraints on the superspace torsions, curvature and the gauge superform field strengths. The main constraints have the form

\[ T^a = i E^a \wedge E^b \bar{\Gamma}_{ab}^c ; \]
\[ H_{q+1} := dC_q - c_1 E^a \wedge E^b \wedge \Gamma_{(q-1)}^{a b} = \frac{1}{(q+1)!} E^{a_{q+1}} \wedge \ldots \wedge E^{a_1} F_{a_1 \ldots a_{q+1}}(Z) , \]

and can be derived as a straightforward extension of the component, first order form supergravity eqs. (63), (64) to superspace. This fact is not accidental. It reflects the existence of the so-called group manifold or 'rheonomic' approach to supergravity (66), which provides the bridge between the component and superfield formalism (see also Sec. 2 of 13 for a brief review).

6.3. Local supersymmetry of $(D = 11)$ supergravity constraints

After the constraints (101), (102) are taken into account, the fermionic general coordinate transformations $\delta_{sgcf}$ simplify. In particular,

\[ \delta_{sgcf} Z^M = 0 , \]
\[ \delta_{sgcf} E^a(Z) = -2i E^a \bar{\Gamma}_{a d}^0 e_d , \text{ etc.} \]
Now one can easily see that \( \tilde{\delta}_{sgcf} E^a|_{\theta=0} \) becomes identical to \( \delta_i e^a \), \( \tilde{\delta}_{sgcf} E^a|_{\theta=0} = \delta_i e^a \), after the usual identification of the supergravity forms with the leading components of superforms, \( (E^a, E^a)|_{\theta=0, d\theta=0} = (e^a, e^{\hat{a}}) \), etc., is made.

To be specific, let us consider \( D = 11 \) supergravity (\( a = 0, 1, \ldots, 10 \), \( \alpha = 1, \ldots, 32 \)). Here the superspace constraints (101), (102),

\[
T^a = -iE^a \wedge E^b \tilde{\Gamma}^a_{\hat{b} \hat{b}} ,
\]

\[
\mathcal{H}_4 \equiv dC_3 - \frac{1}{2} E^a \wedge E^b \wedge \tilde{\Gamma}^{(2)}_{\hat{a} \hat{b}} = \frac{1}{4!} E^a \wedge \ldots \wedge E^c F_{e_1 \ldots e_4} ,
\]

imply

\[
T^a = \frac{1}{2} E^b \wedge E^c T_{cb} \tilde{\Gamma}^a_{\hat{b} \hat{b}} - \frac{1}{2} E^b \wedge E^d T_{bd} \tilde{\Gamma}^a_{\hat{b} \hat{b}} ,
\]

\[
T_{b\hat{a}} = \frac{i}{2} (F_{b\hat{b}b\hat{b}} (\Gamma^{b\hat{b}b\hat{b}}) \tilde{\Gamma}^a_{\hat{b} \hat{b}} + \frac{1}{4} F_{b\hat{b}b\hat{b}} (\Gamma_{b\hat{b}b\hat{b}}) \tilde{\Gamma}^a_{\hat{b} \hat{b}}) ,
\]

\[
R^{ab} = -2i E^a \wedge E^b \tilde{\Gamma}^{[a \Gamma_{b \hat{b}}} \tilde{\Gamma}^d_{\hat{b} \hat{b}]} + E^a \wedge E^b (iT^{ab} c^{[a \Gamma_{b \hat{b}}]} \tilde{\Gamma}^d_{\hat{b} \hat{b}]} + \frac{1}{4} E^a \wedge E^b R_{cde}^{ab} .
\]

Using (105)–(106), the superspace local supersymmetry (97)–(100) takes the form

\[
\tilde{\delta}_{sgcf} Z^M = 0 ,
\]

\[
\tilde{\delta}_{sgcf} E^a = -2i E^a \tilde{\Gamma}^a_{\hat{b} \hat{b}} \tilde{\epsilon}^b(Z) ,
\]

\[
\tilde{\delta}_{sgcf} C_3 = E^a \wedge \tilde{\Gamma}^{(2)}_{\hat{a} \hat{b}} \tilde{\epsilon}^a(Z) ,
\]

\[
\tilde{\delta}_{sgcf} F_{a_{1}a_{2}a_{3}a_{4}} = 3 \Gamma_{[a_{1}a_{2}a_{3}a_{4}]} \tilde{\epsilon}^{[a_{4}]}(Z) ,
\]

\[
\tilde{\delta}_{sgcf} w^{ab} = -4i E^a T_{ac} \Gamma_a \tilde{\epsilon}^{\beta}(Z) + i E^c (T^{ab} c_{c\alpha} \tilde{\epsilon}^\beta) .
\]

Setting \( \theta = 0, d\theta = 0 \) in eqs. (111)–(115), one arrives at the on–shell version of the local supersymmetry transformations characteristic of the component supergravity action, i.e., the actual local supersymmetry transformation which leaves the action invariant differs from the pull–backs of (111)–(115) to \( M^D \) by terms which vanish on the mass shell.

The discussion of the previous section suggests the following observation. The same transformation rules for superfields (superforms), (111)–(115) appear for the original form of the fermionic general coordinate transformations with

\[
\delta_{sgcf} Z^M = \tilde{\epsilon}^b(Z) E^M_b(Z) ,
\]

\[
\Leftrightarrow \begin{cases} i \delta_{sgcf} E^a = \tilde{\delta}_{sgcf} Z^M E^M_a = 0 , \\
- (\delta_{sgcf} E^a = \tilde{\delta}_{sgcf} Z^M E^M_a = \epsilon^a(Z) ,
\end{cases}
\]

\[
\delta_{sgcf} E^a = -2i E^a \tilde{\Gamma}^a_{\hat{b} \hat{b}} \tilde{\epsilon}^b(Z) ,
\]

\[
\delta_{sgcf} C_3 = E^a \wedge \tilde{\Gamma}^{(2)}_{\hat{a} \hat{b}} \tilde{\epsilon}^a(Z) ,
\]

\[
\delta_{sgcf} F_{a_{1}a_{2}a_{3}a_{4}} = 3 \Gamma_{[a_{1}a_{2}a_{3}a_{4}]} \tilde{\epsilon}^{[a_{4}]}(Z) ,
\]

\[
\delta_{sgcf} w^{ab} = -4i E^a T_{ac} \Gamma_a \tilde{\epsilon}^{\beta}(Z) + i E^c (T^{ab} c_{c\alpha} \tilde{\epsilon}^\beta) .
\]

Thus the \( D = 11 \) superspace constraints are invariant under both \( \delta_{sgcf} \) and \( \tilde{\delta}_{sgcf} \). This reflects the superdiffeomorphism invariance (86)–(88) of forms.

7. Local supersymmetry and \( \kappa \)–symmetry of a superbrane in a supergravity background

To see why a full account of the local gauge symmetries in supergravity can be relevant, let us now consider the standard description of a super–p–brane moving in a supergravity background [\textsuperscript{3}].

Consider, e.g. the supermembrane (M2–brane) in the \( D = 11 \) supergravity background defined by the constraints (103)–(104). Its action is

\[
S_{11.2} = \int_{W^{11}} \frac{1}{2} \tilde{E}_a \wedge \tilde{E}^a - \tilde{C}_3(\hat{x}, \hat{\theta}) ,
\]

\[
\text{Note that the restoration of such terms is an involved technical problem. However, the use of the second Noether theorem can simplify the proof of the local supersymmetry of the action, as it allows to work with equations of motion instead of the general variation.}
\]

\[
\text{\textsuperscript{3}} \text{Such description could be regarded as the background field approximation to a fully dynamical description of supergravity—super-p-brane system based on a coupled action including both the supergravity and super-p-brane Lagrangians.}
\]
where
\[ \hat{E}^a = d\hat{Z}^M(\xi) E_M^a(\hat{Z}) = d\xi^i \hat{\partial}_i \hat{Z}^M E_M^a(\hat{Z}) , \]  
(123)
\[ \hat{C}_3 = \frac{1}{3!} d\hat{Z}^{M_1} \wedge \ldots \wedge d\hat{Z}^{M_3} C_{M_1 M_2 M_3}(\hat{Z}) , \]  
(124)
are the pull-backs \( \hat{\psi}^*(E^a) , \hat{\phi}^*(C_3) \) of the supervielbein and gauge field superforms on the \( \Sigma^{(11|32)} \) superspace by the map
\[ \hat{\phi} : W^3 \to \Sigma^{(11|32)} , \quad \hat{\phi} : \xi^i \to \hat{Z}^M(\xi) , \]  
so that \( \hat{Z}^M = \hat{Z}(\xi) \) etc. As supergravity is treated as a background, the set of dynamical variables includes only the local supercoordinate functions \( \hat{Z}^M(\xi) = (\hat{\xi}^\mu(\xi), \hat{\theta}^a(\xi)) \), which define the worldvolume as a surface in superspace, \( Z^M \in \Sigma^{(11|32)} | Z^M = \hat{Z}^M(\xi) \). Hence the basic variations, \( \delta \hat{Z}^M(\xi) \), can be recognized as a counterpart of superspace general coordinate transformations \((10), (11)\), and can be split into the bosonic and fermionic parts
\[ i_\delta \hat{E}^a \equiv \delta \hat{Z}^M(\xi) E_M^a(\hat{Z}(\xi)) , \]
\[ i_\delta \hat{E}^3 \equiv \delta \hat{Z}^M(\xi) E_M^3(\hat{Z}(\xi)) . \]  
(126)
Taking into account the constraints, one finds that the fermionic variations of the supercoordinate functions, \( \delta \hat{\psi}^a(\xi) \), defined by (cf. \( (116) \))
\[ i_\delta \hat{E}^a = 0 , \quad i_\delta \hat{E}^3 \neq 0 , \]
\[ \Leftrightarrow \quad \delta \hat{C}_3 = i_\delta \hat{E}^3 \]  
(127)
lead to
\[ \delta \hat{E}^a = -2i \hat{E}^\alpha \hat{a}_\alpha i_\delta \hat{E}^3 , \]  
(128)
\[ \delta \hat{C}_3 = \hat{E}^3 \wedge \hat{E}^{(2)}_{\alpha\beta} i_\delta \hat{E}^3 \]  
(129)
(cf. \( (116) \), \( (117) \), \( (119) \)), where the rôle of \( \alpha(\hat{Z}) \) is played by \( i_\delta \hat{E}^3 \). Hence,
\[ \delta f S_{11,2} = \int_{W^3} \frac{1}{2} \hat{E}_a \wedge \delta f \hat{E}^a - \delta f \hat{C}_3 = \]
\[ = \int_{W^3} (-i \hat{E}_a \wedge \hat{E}^\alpha \hat{a}_\alpha - \hat{E}^a \wedge \hat{E}^{(2)}_{\alpha\beta}) i_\delta \hat{E}^3 \]
\[ = -i \int_{W^3} \hat{E}_a \wedge \hat{E}^a (\Gamma^n(I - \tilde{\gamma})) \hat{a}^a \hat{E}^3 , \]  
(130)
where
\[ \tilde{\gamma} \equiv \frac{1}{3! \sqrt{|g|}} \epsilon^{ijk} \hat{E}_i \hat{E}_j \hat{E}_k \Gamma_{abc} \]  
(131)
is the well known matrix satisfying \( \text{tr} \tilde{\gamma} = 0, \tilde{\gamma}^2 = I \), that enters in the M2-brane \( \kappa \)-symmetry projector \( \frac{1}{2}(1 + \tilde{\gamma}) \) \( [16] \). Thus, for \( i_\delta \hat{E}^a = i_\delta \hat{E}^3 \)
\[ (1 - \tilde{\gamma}) \hat{M}^\beta(\xi) \) we find \( \delta_s S_{11,2} = 0 \), which expresses the fundamental \( \kappa \)-symmetry of the supermembrane \( [16] \).

We see that when computing the fermionic variation \( \delta f \) (eqs. \( (27) \), \( (25) \), \( (29) \)) of the supermembrane action we actually perform a superspace fermionic general coordinate transformation \( \delta_{sgcf} \), eqs. \( (116) - (119) \), pulled back to \( W^3 \): \( \hat{\psi}^*(\delta_{sgcf}) = \delta f \). The variation \( \delta f \) produces the superbrane equations of motion on \( W^3 \), \( \tilde{Z}_3 := \star \hat{E}_a \wedge \hat{E}^a (\Gamma^n(I - \tilde{\gamma})) \hat{a}^a = 0 \). Thus, the whole variation \( \delta f \) is not a local symmetry of the dynamical system including the superbrane (otherwise, the brane dynamics would be trivial in the ‘fermionic’ directions). However, this fermionic equation becomes an identity when multiplied by \( (1 + \tilde{\gamma}) \), i.e. \( \tilde{Z}_3 (1 + \tilde{\gamma}) \equiv 0 \). This is the Noether identity (Sec. 4. 5.3) for \( \kappa \)-symmetry. On the other hand, as \( \hat{\psi}^*(\delta_{sgcf}) = \delta f \), this means that the breaking of \( \delta_{sgcf} \) by the supermembrane is partial and that the part of \( \delta_{sgcf} \) preserved on \( W^3 \) is given by the \( \kappa \)-symmetry transformations. Moreover, as the brane action possesses manifest local Lorentz and diffeomorphism invariance, we can use \( (27) \) to conclude that \( \delta f S_{11,2} = \delta_{sgcf} S_{11,2} \). Hence \( \delta_{sgcf} S_{11,2} = 0 \) for the superfield supersymmetry transformations \( (110) - (13) \) with the parameter restricted on \( W^3 \) to be of the form \( (\alpha(\hat{Z}) = (1 - \tilde{\gamma}) \hat{M}^\beta(\xi) \), and we can state that \( \kappa \)-symmetry is just the part of the local supersymmetry which is preserved by the brane action\(^6\).

8. Concluding remarks

The above considerations indicate that

i) The \( \kappa \)-symmetry of the superbrane in the superfield supergravity background is the part of the superfield local supersymmetry characteristic of the supergravity constraints which is not broken by the presence of the superbrane.

ii) In any complete Lagrangian description of the supergravity—superbrane coupled system which includes the standard superbrane action, the lo-

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\(^6\)See e.g. \[13\] for the relation between the local supersymmetry preserved by the bosonic brane solutions of the supergravity equations and the \( \kappa \)-symmetry of the effective superbrane actions.
Supersymmetry will be partially broken. Any coupled action describing both supergravity and the superbrane will possess not more than $1/2$ of the local supersymmetry characteristic of the ‘free’ supergravity action.

iii) As the superbrane action is written in terms of pull–backs of superspace differential forms and, possibly, the worldvolume Hodge star operator, the complete coupled action evidently possesses superdiffeomorphism symmetry $\delta_{\text{sdiff}}$.

iv) As the superfield local supersymmetry can equivalently be considered as originated either from the superspace general coordinate transformations $\delta_{\text{sgcf}}$, (116)–(121), or from their variational copy $\tilde{\delta}_{\text{sgcf}}$, (110)–(115), we conclude that the coupled system of supergravity and bosonic $p$-brane should possess $1/2$ of the local supersymmetry characteristic of the free supergravity, if the bosonic $p$-brane appears to be the $\hat{\theta}(\xi) = 0$ ‘limit’ of a superbrane $\square$.

We hope to return to these points in forthcoming publications.

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