RESEARCH ARTICLE

Mathematical model for the novel coronavirus (2019-nCOV) with clinical data using fractional operator

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Abstract
Coronavirus infection (COVID-19) is a considerably dangerous disease with a high demise rate around the world. There is no known vaccination or medicine until our time because the unknown aspects of the virus are more significant than our theoretical and experimental knowledge. One of the most effective strategies for comprehending and controlling the spread of this epidemic is to model it using a powerful mathematical model. However, mathematical modeling with a fractional operator can provide explanations for the disease’s possibility and severity. Accordingly, basic information will be provided to identify the kind of measure and intrusion that will be required to control the disease’s progress. In this study, we propose using a fractional-order SEIARPQ model with the Caputo sense to model the coronavirus (COVID-19) pandemic, which has never been done before in the literature. The stability analysis, existence, uniqueness theorems, and numerical solutions of such a model are displayed. All results were numerically simulated using MATLAB programming. The current study supports the applicability and influence of fractional operators on real-world problems.

KEYWORDS
Adams–Bashforth–Moulton method, COVID-19 disease, existence theorems, numerical simulations, stability analysis, uniqueness theorems
1 | INTRODUCTION

Classical mathematical models for infectious diseases are a tool that can support state officials in undertaking the necessary measures to combat this disease [1–4]. These models help us understand how this epidemic spread. Recently, the entire world is suffering from a new virus called coronavirus infection (COVID-19), which is suspected to have appeared first in Wuhan, China, and then spread to different countries [5]. This type of disease poses a significant danger to individuals and a country’s economy. This pandemic is distinguished by its quick spread and the absence of symptoms. Specifically, the incubation time is from 2 to 14 days, according to WHO [6]. In addition, there is no antiviral therapy or inoculation that has been authoritatively confirmed for the administration or prevention of this pandemic. To combat the pandemic, general health decision-makers and overall policymakers should adopt and follow a strategic and medical patronage management approach. Adequate awareness of the severity of this disease and methods to combat it are key elements in minimizing the spread of this infection. Since the outbreak of COVID-19, various statistical simulations have provided health officials with valuable insights into how to best prevent the disease’s spread. They recommended a continual increase in the rate of diagnosis, a doubling of the quarantine rate due to increased touch tracing, and strict safety measures in hospitals.

Fredj and Farouk investigated the COVID-19 pandemic model using real data from Tunisia and reported that the entire population is divided into seven stages of illness [7]. Fractional calculus has emerged as a powerful mathematical tool for classifying the memory and inherited features of complex systems in recent decades [8–17]. Moreover, the fractional calculus results are of a more generic type [18–26]. Therefore, several fractional models have been used to investigate the spread of the novel COVID-19 (2019-nCOV) [27–33]. They concluded that fractional models propagate faster than classical models and have useful scaling qualities that make them desirable for applications. The fractional model has a memory impact if its outlook status is based on the current status and the historical backdrop of the states. Further, the fractional operator has this memory impact advantage, making it ideal for showing the COVID-19 diffusion model [34–45]. Some fractional calculus definitions include Riemann–Liouville, Caputo, Caputo–Fabrizio, Atangana Baleanu, and many more [8–17].

Currently, the dynamics of the novel COVID-19 with fractional order take the following form:

\[
\begin{align*}
D_\sigma^\alpha t S(t) &= -\frac{\lambda_1}{N} SE - \frac{\lambda_2}{N} SI - \lambda_3 S, \\
D_\sigma^\alpha t E(t) &= \frac{\lambda_1}{N} SE + \frac{\lambda_2}{N} SI - (\alpha_1 + \alpha_2) E, \\
D_\sigma^\alpha t I(t) &= \alpha_1 E - (\beta_1 + \beta_2) I, \\
D_\sigma^\alpha t A(t) &= \alpha_2 E, \\
D_\sigma^\alpha t R(t) &= \beta_1 I, \\
D_\sigma^\alpha t P(t) &= \beta_2 I, \\
D_\sigma^\alpha t Q(t) &= \lambda_3 S.
\end{align*}
\]

The initial conditions are \(S_0(0) > 0, E_0(0) \geq 0, A_0(0) \geq 0, I_0(0) \geq 0, R_0(0) \geq 0, P_0(0) \geq 0\) and \(Q_0(0) \geq 0\), where \(S(t)\) represents the susceptible population, \(E(t)\) indicates the exposed population, \(I(t)\) represents the infected population, \(A(t)\) denotes the recovered–exposed population who are not counted by the government, \(R(t)\) indicates the recovered population, \(P(t)\) represents the deceased population, \(Q(t)\) represents the quarantined population, and \(N = S(t) + E(t) + I(t) + A(t) + R(t) + P(t) + Q(t)\). Each model parameters are positive values and have been evaluated in [7] using genuine data. The COVID-19 (SEIARLPQ model) parameters have the following genuine concept:
• $\sigma$ denotes the fractional-order operator.
• $\lambda_1$ denotes the rate of infection resulting from the contact between exposed and susceptible cases.
• $\lambda_2$ denotes the rate of infection resulting from the contact between infected and susceptible cases.
• $\lambda_3$ denotes the home quarantine rate of a susceptible case.
• $\alpha_1$ signifies the incubation rate of the epidemic.
• $\alpha_2$ signifies the recovered rate of an exposed case.
• $\beta_1$ signifies the recovered rate of an infected case.
• $\beta_2$ signifies the death rate.

This paper is organized as follows: Section 2 provided some basic definitions and concepts. Section 3 investigates the stability and sensibility analyses of reproduction number ($\mathcal{R}_0$), without model control. Section 4 presents the indicator corrector PECE technique for Adams–Bashforth–Moulton. Section 5 provides the numerical simulation and discussion of the 2019-nCOV fractional model. Finally, some conclusions are provided.

2 | BASIC DEFINITIONS AND CONCEPTS

Several concepts of fractional calculus have been recently proposed, including various definitions [8–11].

Definition 1 The fractional integral operator of order $\sigma > 0$, of a function $S(t) \in C$, $t \geq -1$ is denoted by $I_0^\sigma$ and defined as follows:

$$I_0^\sigma S(t) = \frac{1}{\Gamma(\sigma+1)} \int_0^t (t-\xi)^{\sigma-1} S(\xi) d\xi, \quad t, \sigma > 0$$

Definition 2 We can write the definition of Caputo fractional derivative as follows [8]:

$$D_0^\sigma S(t) = \frac{\partial^\sigma S(t)}{\partial t^\sigma} = \frac{1}{\Gamma(m-\sigma)} \int_0^t (t-\xi)^{m-\sigma-1} \frac{\partial^m S(t)}{\partial \xi^m} d\xi, \quad (m-1 < \sigma \leq m, \quad m \in \mathbb{N}).$$

3 | ANALYSIS OF THE SEIAR Model of Fractional Order

3.1 | Stability and sensibility analysis

To determine the disease-free equilibrium state of the suggested model, equate the right-hand side of the model (1) to zero.

$$\begin{cases} -\frac{\lambda_1}{N} SE - \frac{\lambda_2}{N} SI - \lambda_3 S = 0, \\ \frac{\lambda_1}{N} SE + \frac{\lambda_2}{N} SI - (\alpha_1 + \alpha_2) E = 0, \\ \alpha_1 E - (\beta_1 + \beta_2) I = 0, \alpha_2 E = 0, \\ \beta_1 I = 0, \beta_2 I = 0, \lambda_3 S = 0. \end{cases}$$

Finding equilibrium points and studying stability analysis are similar between classical and fractional systems. Therefore, we discover that the stability analysis of this system has been studied in a previous research [7]. The disease-free equilibrium state is given as follows:

$$\mathcal{F}^0 \left( S^0, E^0, I^0, A^0, R^0, P^0, Q^0 \right) = (N, 0, 0, 0, 0, 0).$$
The dimension of the suggested model is substantially higher; thus, it is difficult to examine the stability of the suggested model in connection with equilibrium states. Subsequently, we will investigate the stability analysis dependent on the essential propagation number in a later subsection. We can write the basic reproduction number as follows [7]:

$$\mathcal{R}_0 = \frac{\alpha_1 \lambda_2 + (\beta_1 + \beta_2) \lambda_1}{(\alpha_1 + \alpha_2)(\beta_1 + \beta_2)}.$$  \hspace{1cm} (4)

Because epidemiological system parameters are either predetermined or fitted, there is some skepticism about the values used to deduce the fundamental pestilence. Therefore, it is essential to evaluate the individual effects of each parameter on the dynamics of the pestilence and, thus, identify the parameters with the greatest impact on limiting or spreading the epidemic. In this subsection, we use a sensitivity indicator to perform a sensitivity analysis of the epidemiical parameters included in the proposed SEIARPQ model. The sensitivity indicator strategy will help measure the most sensitive parameters for the basal reproductive number, \(\mathcal{R}_0\). The standardized sensitivity indicator of \(\mathcal{R}_0\) with respect to the parameters \((\lambda_1, \lambda_2, \beta_1, \beta_2, \alpha_1, \alpha_2)\) used in the SEIARPQ model is given by the following equations, where, for example, \(\Omega_{\lambda_1}^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \lambda_1} \times \frac{1}{\mathcal{R}_0}\), where \(\Omega\) represents the assessment of the singular effects of each parameter on the dynamics of the epidemic

$$\Omega_{\lambda_1}^{\mathcal{R}_0} = \frac{1}{\alpha_1 + \alpha_2} = 0.960918 > 0,$$

$$\Omega_{\lambda_2}^{\mathcal{R}_0} = \frac{\alpha_1}{(\alpha_1 + \alpha_2)(\beta_1 + \beta_2)} = 0.0390821 > 0,$$

$$\Omega_{\beta_1}^{\mathcal{R}_0} = -\frac{\alpha_1 \lambda_2}{(\alpha_1 + \alpha_2)(\beta_1 + \beta_2)^2} = -0.0174995 < 0,$$

$$\Omega_{\beta_2}^{\mathcal{R}_0} = -\frac{\alpha_1 \lambda_2}{(\alpha_1 + \alpha_2)(\beta_1 + \beta_2)^2} = -0.0215827 < 0,$$

$$\Omega_{\alpha_1}^{\mathcal{R}_0} = \frac{\alpha_2 \lambda_2 - (\beta_1 + \beta_2) \lambda_1}{(\alpha_1 + \alpha_2)^2 (\beta_1 + \beta_2)} = -0.0592046 < 0,$$

$$\Omega_{\alpha_2}^{\mathcal{R}_0} = -\frac{\alpha_1 \lambda_2 + (\beta_1 + \beta_2) \lambda_1}{(\alpha_1 + \alpha_2)^2 (\beta_1 + \beta_2)} = -0.901713 < 0.$$  \hspace{1cm} (5)

Equation (5) shows that a portion of the sensitivity indicator is positive, such as \(\lambda_1\) and \(\lambda_2\) while others are negative, such as \(\beta_1, \beta_2, \alpha_1, \) and \(\alpha_2\). Moreover, the greatest aspects of these indicators are the functions of the parameters of the SEIARPQ model. This implies that even a minor change in a parameter will alter the dynamics of the pestilence. The value \(\Omega_{\alpha_2}^{\mathcal{R}_0} = -0.901713\) shows that increasing (decreasing) \(\alpha_2\), for example, by 10%, decreases (increases) the basic reproductive number \(\mathcal{R}_0\) by about 10%. A slight alteration in a parameter may result in relatively enormous quantitative alterations, thus requiring knowledge of these sensitive parameters. From Equation (5) shows that the minimum and maximum sensitivity epidemiical parameters, \(\mathcal{R}_0\) are the parameters \(\alpha_2\) (the recovered rate of an exposed case) and \(\lambda_1\) (the rate of infection resulting from contact between an exposed case and a susceptible case), respectively.

### 3.2 | Existence and uniqueness of solutions

Because summarizing the entire model (1) yields zero, the model is divided: as can be seen

$$D_t^\sigma S(t) + D_t^\sigma E(t) + D_t^\sigma I(t) + D_t^\sigma A(t) + D_t^\sigma R(t) + D_t^\sigma P(t) + D_t^\sigma Q(t) = 0,$$  \hspace{1cm} (6)
this signifies that the overall population (the sum of all whole case variables) is constant. Let

\[ Z = [S(t), E(t), I(t), A(t), R(t), P(t), Q(t)]^T. \]  

be the state variables vector. Since the case variables connote the populace fractions, we can assume that \( \sum_{\nu} Z(i) = 1 \), where 1 implies the overall population.

We assume the following:

\[
\begin{align*}
D_t^2 S &= -\dfrac{\lambda_1}{N} SE - \dfrac{\lambda_2}{N} SI - \lambda_3 S = M_1(S, E, I, A, R, P, Q), \\
D_t^2 E &= \dfrac{\lambda_1}{N} SE + \dfrac{\lambda_2}{N} SI - (\alpha_1 + \alpha_2) E = M_2(S, E, I, A, R, P, Q), \\
D_t^2 I &= \alpha_1 E - (\beta_1 + \beta_2) I = M_3(S, E, I, A, R, P, Q), \\
D_t^2 A &= \alpha_2 E = M_4(S, E, I, A, R, P, Q), \\
D_t^2 R &= \beta_1 I = M_5(S, E, I, A, R, P, Q), \\
D_t^2 P &= \beta_2 I = M_6(S, E, I, A, R, P, Q), \\
D_t^2 Q &= \lambda_3 S = M_7(S, E, I, A, R, P, Q).
\end{align*}
\]  

Suppose that the following equation holds for a constant \( \mathcal{N} \)

\[
\Theta = \{ (S(t), E(t), I(t), A(t), R(t), P(t), Q(t)) \in \mathbb{R}^7 : |Z(i)| \leq \mathcal{N}, 0 \leq t \leq T \}.
\]

At that point, we get the following:

\[
\begin{align*}
\frac{\partial M_1}{\partial S} &= -\frac{(N - S)}{N^2} (\lambda_1 E + \lambda_2 I) - \lambda_3 \Rightarrow \left| \frac{\partial M_1}{\partial S} \right| = \frac{N - S}{N^2} (\lambda_1 E + \lambda_2 I) + \lambda_3 \leq g_{11}; \\
\frac{\partial M_1}{\partial E} &= -\lambda_1 S(N - E) \frac{N^2}{N^2} + \lambda_2 SI \frac{N^2}{N^2} \Rightarrow \left| \frac{\partial M_1}{\partial E} \right| = \lambda_1 S(N - E) \frac{N^2}{N^2} + \lambda_2 SI \frac{N^2}{N^2} \leq g_{12}; \\
\frac{\partial M_1}{\partial I} &= \lambda_1 SE \frac{N^2}{N^2} - \lambda_2 S \left( \frac{N - I}{N^2} \right) \Rightarrow \left| \frac{\partial M_1}{\partial I} \right| = \lambda_1 SE \frac{N^2}{N^2} + \lambda_2 S \left( \frac{N - I}{N^2} \right) \leq g_{13}; \\
\frac{\partial M_1}{\partial A} &= \frac{\partial M_1}{\partial R} = \frac{\partial M_1}{\partial P} = \frac{\partial M_1}{\partial Q} \\
&= S \left( \lambda_1 E + \lambda_2 I \right) \leq g_{14}; \\
\frac{\partial M_2}{\partial S} &= (N - S) \frac{N^2}{N^2} (\lambda_1 E + \lambda_2 I) \Rightarrow \left| \frac{\partial M_2}{\partial S} \right| = \frac{N - S}{N^2} (\lambda_1 E + \lambda_2 I) \leq g_{21}; \\
\frac{\partial M_2}{\partial E} &= -\lambda_1 S(N - E) \frac{N^2}{N^2} + \lambda_2 SI \frac{N^2}{N^2} - (\alpha_1 + \alpha_2) \Rightarrow \left| \frac{\partial M_2}{\partial E} \right| = \lambda_1 S(N - E) \frac{N^2}{N^2} + \lambda_2 SI \frac{N^2}{N^2} + (\alpha_1 + \alpha_2) \leq g_{22}; \\
\frac{\partial M_2}{\partial I} &= \lambda_1 SE \frac{N^2}{N^2} - \lambda_2 S \left( \frac{N - I}{N^2} \right) \Rightarrow \left| \frac{\partial M_2}{\partial I} \right| = \lambda_1 SE \frac{N^2}{N^2} + \lambda_2 S \left( \frac{N - I}{N^2} \right) \leq g_{23}; \\
\frac{\partial M_2}{\partial A} &= \frac{\partial M_2}{\partial R} = \frac{\partial M_2}{\partial P} = \frac{\partial M_2}{\partial Q} \\
&= S \left( \lambda_1 E + \lambda_2 I \right) \leq g_{24};
\end{align*}
\]
\[
\frac{\partial M_3}{\partial S} = 0 \Rightarrow M_3(S) = g_{31}; \quad \frac{\partial M_3}{\partial E} = \alpha_1 \Rightarrow \left| \frac{\partial M_3}{\partial E} \right| = \alpha_1 \leq g_{32}; \quad \frac{\partial M_3}{\partial I} = - (\beta_1 + \beta_2) \Rightarrow \left| \frac{\partial M_3}{\partial I} \right| = (\beta_1 + \beta_2) \leq g_{33};
\]
\[
\frac{\partial M_3}{\partial A} = 0 \Rightarrow M_3(A) = g_{34}; \quad \frac{\partial M_3}{\partial R} = 0 \Rightarrow M_3(R) = g_{35}; \quad \frac{\partial M_3}{\partial P} = 0 \Rightarrow M_3(P) = g_{36}; \quad \frac{\partial M_3}{\partial Q} = 0 \Rightarrow M_3(Q) = g_{37};
\]
\begin{equation}
\frac{\partial M_4}{\partial S} = 0 \Rightarrow M_4(S) = g_{41}; \quad \frac{\partial M_4}{\partial E} = \alpha_2 \Rightarrow \left| \frac{\partial M_4}{\partial E} \right| = \alpha_2 \leq g_{42}; \quad \frac{\partial M_4}{\partial P} = 0 \Rightarrow M_4(I) = g_{43};
\end{equation}
\[
\frac{\partial M_4}{\partial A} = 0 \Rightarrow M_4(A) = g_{44}; \quad \frac{\partial M_4}{\partial R} = 0 \Rightarrow M_4(R) = g_{45}; \quad \frac{\partial M_4}{\partial P} = 0 \Rightarrow M_4(P) = g_{46}; \quad \frac{\partial M_4}{\partial Q} = 0 \Rightarrow M_4(Q) = g_{47};
\]
\begin{equation}
\frac{\partial M_5}{\partial S} = 0 \Rightarrow M_5(S) = g_{51}; \quad \frac{\partial M_5}{\partial E} = \beta_1 \Rightarrow \left| \frac{\partial M_5}{\partial E} \right| = \beta_1 \leq g_{53};
\end{equation}
\[
\frac{\partial M_5}{\partial A} = 0 \Rightarrow M_5(A) = g_{54}; \quad \frac{\partial M_5}{\partial R} = 0 \Rightarrow M_5(R) = g_{55}; \quad \frac{\partial M_5}{\partial P} = 0 \Rightarrow M_5(P) = g_{56}; \quad \frac{\partial M_5}{\partial Q} = 0 \Rightarrow M_5(Q) = g_{57};
\]
\begin{equation}
\frac{\partial M_6}{\partial S} = 0 \Rightarrow M_6(S) = g_{61}; \quad \frac{\partial M_6}{\partial E} = \beta_2 \Rightarrow \left| \frac{\partial M_6}{\partial E} \right| = \beta_2 \leq g_{63};
\end{equation}
\[
\frac{\partial M_6}{\partial A} = 0 \Rightarrow M_6(A) = g_{64}; \quad \frac{\partial M_6}{\partial R} = 0 \Rightarrow M_6(R) = g_{65}; \quad \frac{\partial M_6}{\partial P} = 0 \Rightarrow M_6(P) = g_{66}; \quad \frac{\partial M_6}{\partial Q} = 0 \Rightarrow M_6(Q) = g_{67};
\]
\begin{equation}
\frac{\partial M_7}{\partial S} = \lambda_3 \Rightarrow \left| \frac{\partial M_7}{\partial S} \right| = \lambda_3 \leq g_{71}; \quad \frac{\partial M_7}{\partial E} = 0 \Rightarrow M_7(E) = g_{72}; \quad \frac{\partial M_7}{\partial I} = 0 \Rightarrow M_7(I) = g_{73};
\end{equation}
\[
\frac{\partial M_7}{\partial A} = 0 \Rightarrow M_7(A) = g_{74}; \quad \frac{\partial M_7}{\partial R} = 0 \Rightarrow M_7(R) = g_{75}; \quad \frac{\partial M_7}{\partial P} = 0 \Rightarrow M_7(P) = g_{76}; \quad \frac{\partial M_7}{\partial Q} = 0 \Rightarrow M_7(Q) = g_{77};
\]
where \(g_{xy} > 0\) and \(1 \leq x, y \leq 7\). We deduce from the above that the condition of Lipschitz is true in every of the functions from (9) to (15).

We will show the existence of solutions of the model (1) in the next segment. Let \(\Theta = \Lambda(\eta) \times (\eta)\) and \(\Lambda(\eta)\) be a Banach space of real valued continuous function \(\mathbb{R} \rightarrow \mathbb{R}\) on \(\eta\) with the norm \(\|S, E, I, A, R, P, Q\| = \|S\| + \|E\| + \|I\| + \|A\| + \|R\| + \|P\| + \|Q\|\), by using a Volterra type integral equation, we can transform Equation (1) as
\[
S(t) - S(0) = \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \left\{ - \frac{\lambda_1}{N} S(\theta) E(\theta) - \frac{\lambda_2}{N} S(\theta) I(\theta) - \lambda_3 S(\theta) \right\} d\theta,
\]
The model (1) can be written as:

\[
E(t) - E(0) = \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \left\{ \frac{\lambda_1}{N} S(\theta) E(\theta) + \frac{\lambda_2}{N} S(\theta) I(\theta) - (\alpha_1 + \alpha_2) E(\theta) \right\} d\theta, \quad (17)
\]

\[
I(t) - I(0) = \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \left\{ \alpha_1 E(\theta) - (\beta_1 + \beta_2) I(\theta) \right\} d\theta, \quad (18)
\]

\[
A(t) - A(0) = \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \left\{ \alpha_2 E(\theta) \right\} d\theta, \quad (19)
\]

\[
R(t) - R(0) = \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \left\{ \beta_1 I(\theta) \right\} d\theta, \quad (20)
\]

\[
P(t) - P(0) = \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \left\{ \beta_2 I(\theta) \right\} d\theta, \quad (21)
\]

\[
Q(t) - Q(0) = \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \left\{ \lambda_3 S(\theta) \right\} d\theta. \quad (22)
\]

Presume for simplicity

\[
\begin{aligned}
\Lambda_1(t, S) &= -\frac{\lambda_1}{N} S E - \frac{\lambda_2}{N} S I - \lambda_3 S, \\
\Lambda_2(t, E) &= \frac{\lambda_1}{N} S E + \frac{\lambda_2}{N} S I - (\alpha_1 + \alpha_2) E, \\
\Lambda_3(t, I) &= \alpha_1 E - (\beta_1 + \beta_2) I, \\
\Lambda_4(t, A) &= \alpha_2 E, \\
\Lambda_5(t, R) &= \beta_1 I, \\
\Lambda_6(t, P) &= \beta_2 I, \\
\Lambda_7(t, Q) &= \lambda_3 S.
\end{aligned}
\quad (23)
\]

**Theorem 1** Solutions to model (1) are exists and unique for \( t \in [0, \infty) \)

**Proof.** The model (1) can be written as:

\[
D_r^\sigma Y(t) = F(Y(t)), \quad (24)
\]

with a given initial data \( Y(0) \) where \( Y(t) = (S(t), E(t), I(t), A(t), R(t), P(t), Q(t)) \) and \( F(Y(t)) \) indicates the right hand expressions. The conclusion is simple: using the existence and uniqueness theorem in [46], it is simple to prove that there is a unique solution for model (1) in the bounded region \( \mathbb{R}^7_+ \). This may be verified by ensuring that in the bounded region \( \mathbb{R}^7_+ \), each of the partial derivatives on the right side of the model (1) with regard to \( S, E, I, A, R, P, \) or \( Q \) are bounded in \( \mathbb{R}^7_+ \). \( \blacksquare \)

According to the following theorem, the solutions of the model (1) are bounded from above and nonnegative with given initial conditions.

**Theorem 2** With a nonnegative initial value, all model (1) solutions stay nonnegative for all \( t \geq 0 \) and

\[
\lim_{t \to \infty} \sup N(t) = C \quad (25)
\]

**Proof.** We demonstrate that, in accordance with [47, 48], every solution to the model (1) is nonnegative. We demonstrate that the model’s state variable \( S \) is positive for all
\( t \geq 0 \) using the proof by contradiction. We assume that at time \( t_1 \), a trajectory crosses the positive cone as follows:

\[
t_1 : S(t_1) = 0, \frac{dS}{dt}(t_1) < 0, E(t) > 0, I(t) > 0, A(t) > 0, R(t) > 0, P(t) > 0, Q(t) > 0
\]

for \( t \in (0, t_1) \).

Employing the first equation of (1), the initial presumption leads to

\[
\frac{dS}{dt}(t_1) = \frac{\lambda_1}{N} E + \frac{\lambda_2}{N} I > 0
\]

which goes against the first presumption that \( \frac{dS}{dt}(t_1) < 0 \). Thus, \( S(t) \) remains positive for all \( t \geq 0 \). We select \( t_1 \) in this case so that \( E(t_1) \) and \( I(t_1) \) are positive and our point is on the positive axis of \( S(t) \).

According to the second equation of model (1),

\[
\frac{dE}{dt} = \frac{\lambda_1}{N} SE + \frac{\lambda_2}{N} SI - (\alpha_1 + \alpha_2) E \geq -(\alpha_1 + \alpha_2) E
\]

because \( S(t) \) cannot be negative for all \( t \geq 0 \). By resolving this equation, we get

\[
E(t) \geq E(0) \exp(-(\alpha_1 + \alpha_2) t) \geq 0
\]

In accordance with this, we derive from the third equation of model (1)

\[
\frac{dI}{dt} = \alpha_1 E - (\beta_1 + \beta_2) I \geq - (\beta_1 + \beta_2) I
\]

By solving this equation, we get

\[
I(t) \geq I(0) \exp(-(\beta_1 + \beta_2) t) \geq 0
\]

The final four equations of model (1), in a similar manner, lead to

\[
\frac{dA}{dt} = \alpha_2 E, \quad \frac{dR}{dt} = \beta_1 I, \quad \frac{dP}{dt} = \beta_2 I, \quad \frac{dQ}{dt} = \lambda_3 S,
\]

because \( S(t), E(t), \) and \( I(t) \) are nonnegative for all \( t \geq 0 \). By solving the above equations, we get

\[
\begin{align*}
A(t) & \geq A(0) + \alpha_2 E(0) \exp(-(\alpha_1 + \alpha_2) t) \geq 0, \\
R(t) & \geq R(0) + \beta_1 I(0) \exp(-(\beta_1 + \beta_2) t) \geq 0, \\
P(t) & \geq P(0) + \beta_2 I(0) \exp(-(\beta_1 + \beta_2) t) \geq 0, \\
Q(t) & \geq Q(0) + \lambda_3 S(t) \geq 0,
\end{align*}
\]

respectively. Thus, any solution of model (1) is nonnegative with nonnegative initial data for all \( t \geq 0 \). Furthermore, adding the right-hand sides of model (1) together, we obtain \( \frac{dN}{dt} = 0 \). It follows that \( N(t) = C \), where \( C \) is any constant. Considering \( t \rightarrow \infty \), we have

\[
\lim_{t \rightarrow \infty} \sup N(t) = C.
\]

Thus, the model (1) is bounded.

**Theorem 3** The kernels Equation (23) \( \Lambda_i, i = 1, 2, \ldots, 7 \) satisfy Leipchitz’s condition if the next inequality holds:

**Proof.** We will begin the analysis with \( \Lambda_1(t, S) \). Let \( S \) and \( S^* \) be two functions, so that

\[
\begin{align*}
\| \Lambda_1(t, S) - \Lambda_1(t, S^*) \| &= \left\| \left( -\frac{\lambda_1}{N} E - \frac{\lambda_2}{N} I - \lambda_3 \right) (S - S^*) \right\| \\
&\leq \left( \frac{\lambda_1}{N} \| E \| + \frac{\lambda_2}{N} \| I \| + \lambda_3 \right) \| S - S^* \| \\
&\leq \varphi \| S - S^* \| ,
\end{align*}
\]
Likewise, for the another kernels, we get

\[
\begin{align*}
\|\Lambda_2(t, E) - \Lambda_2(t, E^*)\| &\leq \varphi_2 \|E - E^*\|, \\
\|\Lambda_3(t, I) - \Lambda_3(t, I^*)\| &\leq \varphi_3 \|I - I^*\|, \\
\|\Lambda_4(t, A) - \Lambda_4(t, A^*)\| &\leq \varphi_4 \|A - A^*\|, \\
\|\Lambda_5(t, R) - \Lambda_5(t, R^*)\| &\leq \varphi_5 \|R - R^*\|, \\
\|\Lambda_6(t, P) - \Lambda_6(t, P^*)\| &\leq \varphi_6 \|P - P^*\|, \\
\|\Lambda_7(t, Q) - \Lambda_7(t, Q^*)\| &\leq \varphi_7 \|Q - Q^*\|.
\end{align*}
\]

(35)

where \(s = \max_{t\in S(t)} \|S(t)\|, e = \max_{t\in E(t)} \|E(t)\|, i = \max_{t\in I(t)} \|I(t)\|\) and

\[
\begin{align*}
\varphi_1 &= \frac{\lambda_1}{\alpha_1} e + \frac{\lambda_2}{\alpha_2} i + \lambda_3, \\
\varphi_2 &= \frac{\lambda_1}{\alpha_1} s - (\alpha_1 + a_2), \\
\varphi_3 &= - (\beta_1 + \beta_2), \\
\varphi_4 &= \alpha_2, \\
\varphi_5 &= \beta_1, \\
\varphi_6 &= \beta_2, \\
\varphi_7 &= \lambda_3.
\end{align*}
\]

(36)

We can write the kernels of the model (1) as

\[
\begin{align*}
S(t) &= S(0) + \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \Lambda_1(\theta, S) d\theta, \\
E(t) &= E(0) + \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \Lambda_2(\theta, E) d\theta, \\
I(t) &= I(0) + \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \Lambda_3(\theta, I) d\theta, \\
A(t) &= A(0) + \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \Lambda_4(\theta, A) d\theta, \\
R(t) &= R(0) + \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \Lambda_5(\theta, R) d\theta, \\
P(t) &= P(0) + \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \Lambda_6(\theta, P) d\theta, \\
Q(t) &= Q(0) + \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \Lambda_7(\theta, Q) d\theta.
\end{align*}
\]

(37)

Consider the next initial conditions and Iterative formula as

\[
\begin{align*}
S_{m+1}(t) &= S(0) + \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \Lambda_1(\theta, S_m) d\theta, \\
E_{m+1}(t) &= E(0) + \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \Lambda_2(\theta, E_m) d\theta, \\
I_{m+1}(t) &= I(0) + \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \Lambda_3(\theta, I_m) d\theta, \\
A_{m+1}(t) &= A(0) + \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \Lambda_4(\theta, A_m) d\theta, \\
R_{m+1}(t) &= R(0) + \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \Lambda_5(\theta, R_m) d\theta, \\
P_{m+1}(t) &= P(0) + \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \Lambda_6(\theta, P_m) d\theta, \\
Q_{m+1}(t) &= Q(0) + \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \Lambda_7(\theta, Q_m) d\theta.
\end{align*}
\]

(38)

and \(S(0) > 0, E(0) \geq 0, I(0) \geq 0, A(0) \geq 0, R(0) \geq 0, P(0) \geq 0, Q(0) \geq 0\). The difference between the consecutive components takes the form below
\[ S(t) = S_{m+1}(t) - S_m(t) = \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} [\lambda_1(\theta S_m) - \lambda_1(\theta, S_{m-1})] d\theta, \quad (39) \]

\[ \mathcal{E}(t) = E_{m+1}(t) - E_m(t) = \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} [\lambda_2(\theta E_m) - \lambda_2(\theta, E_{m-1})] d\theta, \quad (40) \]

\[ \mathcal{F}(t) = I_{m+1}(t) - I_m(t) = \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} [\lambda_3(\theta I_m) - \lambda_3(\theta, I_{m-1})] d\theta, \quad (41) \]

\[ \mathcal{A}(t) = A_{m+1}(t) - A_m(t) = \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} [\lambda_4(\theta A_m) - \lambda_4(\theta, A_{m-1})] d\theta, \quad (42) \]

\[ \mathcal{R}(t) = R_{m+1}(t) - R_m(t) = \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} [\lambda_5(\theta R_m) - \lambda_5(\theta, R_{m-1})] d\theta, \quad (43) \]

\[ \mathcal{P}(t) = P_{m+1}(t) - P_m(t) = \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} [\lambda_6(\theta P_m) - \lambda_6(\theta, P_{m-1})] d\theta, \quad (44) \]

\[ \mathcal{Q}(t) = Q_{m+1}(t) - Q_m(t) = \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} [\lambda_7(\theta Q_m) - \lambda_7(\theta, Q_{m-1})] d\theta. \quad (45) \]

One can see from the previous equations and calculations that

\[
\begin{align*}
S_{m+1}(t) & = \sum_{i=0}^{m+1} S_i(t), E_{m+1}(t) = \sum_{i=0}^{m+1} E_i(t), I_{m+1}(t) = \sum_{i=0}^{m+1} I_i(t), \\
A_{m+1}(t) & = \sum_{i=0}^{m+1} A_i(t), R_{m+1}(t) = \sum_{i=0}^{m+1} R_i(t), P_{m+1}(t) = \sum_{i=0}^{m+1} P_i(t), \\
Q_{m+1}(t) & = \sum_{i=0}^{m+1} Q_i(t).
\end{align*}
\]

Utilizing the norm property on Equation (39), we find that

\[
\| S(t) \| = \| S_{m+1}(t) - S_m(t) \| \leq \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \| \lambda_1(\theta S_m) - \lambda_1(\theta, S_{m-1}) \| d\theta, \quad (47)
\]

Because in the kernels, the Lipschitz condition is fulfilled, we find that

\[
\| S_m(t) - S_m(\theta) \| \leq \frac{\varphi_1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \| S_m(\theta) - S_m(\theta) \| d\theta, \quad (48)
\]

then we have

\[
\| S_m(t) - S_{m-1}(t) \| \leq \frac{\varphi_1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \| S_m(\theta) - S_{m-1}(\theta) \| d\theta, \quad (49)
\]

Likewise for Equations (40)–(45), we get the next

\[
\| E_{m+1}(t) \| \leq \frac{\varphi_2}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \| E_m(\theta) - E_{m-1}(\theta) \| d\theta, \quad (50)
\]

\[
\| F_{m+1}(t) \| \leq \frac{\varphi_3}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \| I_m(\theta) - I_{m-1}(\theta) \| d\theta, \quad (51)
\]

\[
\| A_{m+1}(t) \| \leq \frac{\varphi_4}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma - 1} \| A_m(\theta) - A_{m-1}(\theta) \| d\theta, \quad (52)
\]
\[ \| \mathcal{R}_{m+1}(t) \| \leq \frac{\phi_5}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} \| R_m(\theta) - R_{m-1}(\theta) \| \, d\theta, \]  

(53)

\[ \| \mathcal{P}_{m+1}(t) \| \leq \frac{\phi_6}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} \| P_m(\theta) - P_{m-1}(\theta) \| \, d\theta, \]  

(54)

\[ \| \mathcal{Q}_{m+1}(t) \| \leq \frac{\phi_n}{\Gamma(\sigma)} \int_0^t (t - \theta)^{n-1} \| Q_m(\theta) - Q_{m-1}(\theta) \| \, d\theta, \]  

(55)

**Theorem 4**  
The model (1) as unique solutions under the conditions that we can obtain \( t_{\text{max}} \) such that \( \frac{t_{\text{max}}}{\Gamma(\sigma)} \phi_i < 1, i = 1, 2, \ldots, 6 \)

**Proof.** We have proven the facts that the functions \( S(t), E(t), I(t), A(t), R(t), P(t), \) and \( Q(t) \) are bounded, and their kernels fulfilled the condition of the Lipshtz. In view of Equations (49)–(55) and by using the recursive technique, we extract the following

\[ \| S_{m+1}(t) \| \leq \| S_{m+1}(0) \| \left[ \frac{t_{\text{max}}}{\Gamma(\sigma)} \phi_1 \right]^{m+1}, \]  

\[ \| I_{m+1}(t) \| \leq \| I_{m+1}(0) \| \left[ \frac{t_{\text{max}}}{\Gamma(\sigma)} \phi_3 \right]^{m+1}, \]  

\[ \| R_{m+1}(t) \| \leq \| R_{m+1}(0) \| \left[ \frac{t_{\text{max}}}{\Gamma(\sigma)} \phi_5 \right]^{m+1}, \]  

\[ \| Q_{m+1}(t) \| \leq \| Q_{m+1}(0) \| \left[ \frac{t_{\text{max}}}{\Gamma(\sigma)} \phi_6 \right]^{m+1}. \]  

(56)

Consequently, Equation (46) are smooth functions and exist. Following, we offer that they are particular solutions of Equation (1). Assume

\[
\begin{aligned}
& S(t) - S(0) = S_{m+1}(t) - s_{m+1}(t), \\
& I(t) - I(0) = I_{m+1}(t) - i_{m+1}(t), \\
& R(t) - R(0) = R_{m+1}(t) - r_{m+1}(t), \\
& Q(t) - Q(0) = Q_{m+1}(t) - q_{m+1}(t). \\
\end{aligned}
\]

(57)

Our purpose is to illustrate that at infinity, \( \| s_{\infty} \| \to 0, \| e_{\infty} \| \to 0, \| i_{\infty} \| \to 0, \| a_{\infty} \| \to 0, \) \( \| r_{\infty} \| \to 0, \| p_{\infty} \| \to 0, \) and \( \| q_{\infty} \| \to 0. \) So, we have to start with the first state, then

\[ \| s_{m+1}(t) \| \leq \left\| \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} (\Lambda_1(\theta s_{m+1}) - \Lambda_1(\theta s_m)) \, d\theta \right\| \leq \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} \| \Lambda_1(\theta s_{m+1}) - \Lambda_1(\theta s_m) \| \, d\theta, \]  

(58)

where \( \frac{t_{\text{max}}}{\Gamma(\sigma)} \phi_1 \| S_{m+1} - S_m \| < 1. \) When we recursively apply this process, we get

\[ \| s_{m+1}(t) \| \leq \| s_{m+1}(0) \| \left[ \frac{t^{\sigma}}{\Gamma(\sigma)} \phi_1 \right]^{m+1} \phi_1^{m+2} N, \]  

(59)
at $t_{\text{max}}$, we get
\[
\|s_{m+1}(t)\| \leq \|S_{m+1}(0)\| \left[ \frac{r_{\text{max}}}{\Gamma(\sigma)} \varphi_1 \right]^{m+2} \varphi_1^{m+2} N.
\] (60)

By taking the limit at $m + 1 \to \infty$, we find that $\|s_\infty\| \to 0$. The proof is therefore confirmed. In the same manner, one enable prove that $\|e_\infty\| \to 0$, $\|a_\infty\| \to 0$, $\|i_\infty\| \to 0$, $\|r_\infty\| \to 0$, $\|p_\infty\| \to 0$, and $\|q_\infty\| \to 0$.

**Theorem 5**  
The solution of system presented by model (1) is unique.

**Proof.**  
We will obtain another solutions for the model (1), like $S^*(t)$, $E^*(t)$, $I^*(t)$, $A^*(t)$, $R^*(t)$, $P^*(t)$ and $Q^*(t)$, then
\[
\begin{align*}
S(t) - S^*(t) &= \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} [\Lambda_1(\theta S) - \Lambda_1(\theta, S^*)] \, d\theta, \\
E(t) - E^*(t) &= \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} [\Lambda_2(\theta E) - \Lambda_2(\theta, E^*)] \, d\theta, \\
I(t) - I^*(t) &= \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} [\Lambda_3(\theta I) - \Lambda_3(\theta, I^*)] \, d\theta, \\
A(t) - A^*(t) &= \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} [\Lambda_4(\theta A) - \Lambda_4(\theta, A^*)] \, d\theta, \\
R(t) - R^*(t) &= \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} [\Lambda_5(\theta R) - \Lambda_5(\theta, R^*)] \, d\theta, \\
P(t) - P^*(t) &= \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} [\Lambda_6(\theta P) - \Lambda_6(\theta, P^*)] \, d\theta, \\
Q(t) - Q^*(t) &= \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} [\Lambda_7(\theta Q) - \Lambda_7(\theta, Q^*)] \, d\theta.
\end{align*}
\]
(61)

Application of the norm on all sides of Equation (61), we find that
\[
\begin{align*}
\|S(t) - S^*(t)\| &\leq \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} \|\Lambda_1(\theta S) - \Lambda_1(\theta, S^*)\| \, d\theta, \\
\|E(t) - E^*(t)\| &\leq \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} \|\Lambda_2(\theta E) - \Lambda_2(\theta, E^*)\| \, d\theta, \\
\|I(t) - I^*(t)\| &\leq \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} \|\Lambda_3(\theta I) - \Lambda_3(\theta, I^*)\| \, d\theta, \\
\|A(t) - A^*(t)\| &\leq \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} \|\Lambda_4(\theta A) - \Lambda_4(\theta, A^*)\| \, d\theta, \\
\|R(t) - R^*(t)\| &\leq \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} \|\Lambda_5(\theta R) - \Lambda_5(\theta, R^*)\| \, d\theta, \\
\|P(t) - P^*(t)\| &\leq \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} \|\Lambda_6(\theta P) - \Lambda_6(\theta, P^*)\| \, d\theta, \\
\|Q(t) - Q^*(t)\| &\leq \frac{1}{\Gamma(\sigma)} \int_0^t (t - \theta)^{\sigma-1} \|\Lambda_7(\theta Q) - \Lambda_7(\theta, Q^*)\| \, d\theta.
\end{align*}
\]
(62)

Provided the conclusions of Theorems 1 and 2, we have
\[
\begin{align*}
\|S(t) - S^*(t)\| &\leq \frac{q_{\text{max}}}{\Gamma(\sigma)} \|S(t) - S^*(t)\|, &\|E(t) - E^*(t)\| &\leq \frac{q_{\text{max}}}{\Gamma(\sigma)} \|E(t) - E^*(t)\|, \\
\|I(t) - I^*(t)\| &\leq \frac{q_{\text{max}}}{\Gamma(\sigma)} \|I(t) - I^*(t)\|, &\|A(t) - A^*(t)\| &\leq \frac{q_{\text{max}}}{\Gamma(\sigma)} \|A(t) - A^*(t)\|, \\
\|R(t) - R^*(t)\| &\leq \frac{q_{\text{max}}}{\Gamma(\sigma)} \|R(t) - R^*(t)\|, &\|P(t) - P^*(t)\| &\leq \frac{q_{\text{max}}}{\Gamma(\sigma)} \|P(t) - P^*(t)\|, \\
\|Q(t) - Q^*(t)\| &\leq \frac{q_{\text{max}}}{\Gamma(\sigma)} \|Q(t) - Q^*(t)\|.
\end{align*}
\]
(63)

This gives
\[
\begin{align*}
\|S(t) - S^*(t)\| \left(1 - \frac{q_{\text{max}}}{\Gamma(\sigma)}\right) &\leq 0, &\|E(t) - E^*(t)\| \left(1 - \frac{q_{\text{max}}}{\Gamma(\sigma)}\right) &\leq 0, \\
\|I(t) - I^*(t)\| \left(1 - \frac{q_{\text{max}}}{\Gamma(\sigma)}\right) &\leq 0, &\|A(t) - A^*(t)\| \left(1 - \frac{q_{\text{max}}}{\Gamma(\sigma)}\right) &\leq 0, \\
\|R(t) - R^*(t)\| \left(1 - \frac{q_{\text{max}}}{\Gamma(\sigma)}\right) &\leq 0, &\|P(t) - P^*(t)\| \left(1 - \frac{q_{\text{max}}}{\Gamma(\sigma)}\right) &\leq 0, \\
\|Q(t) - Q^*(t)\| \left(1 - \frac{q_{\text{max}}}{\Gamma(\sigma)}\right) &\leq 0.
\end{align*}
\]
(64)
\[
\begin{align*}
\|S(t) - S^*(t)\| &= 0 \rightarrow S(t) = S^*(t), \\
\|E(t) - E^*(t)\| &= 0 \rightarrow E(t) = E^*(t), \\
\|I(t) - I^*(t)\| &= 0 \rightarrow I(t) = I^*(t), \\
\|A(t) - A^*(t)\| &= 0 \rightarrow A(t) = A^*(t), \\
\|R(t) - R^*(t)\| &= 0 \rightarrow R(t) = R^*(t), \\
\|P(t) - P^*(t)\| &= 0 \rightarrow P(t) = P^*(t),
\end{align*}
\]
(65)

The proof is therefore confirmed.

4 | NUMERICAL ALGORITHM WITH CAPUTO FRACTIONAL DERIVATIVE

Numerous physical and realistic applications developed using fractional PDEs cannot be solved precisely; nonetheless, in engineering and science, a numerical approximation to the solution is frequently sufficient to address the problem. The technique calculated here can be used to register a solution using the indicator corrector PECE technique for the Adams–Bashforth–Moulton method. In this section, we examine the 2019-ncov using data from Tunisia known as the SEIARPQ model with the Caputo fractional derivative to demonstrate the potentiality, commonness, and supremacy of our algorithm. MATLAB software package was used to complete all analytical and numerical computations during the calculation time.

The following general fractional differential equation with blurring memory is considered using the Caputo fractional derivative:

\[
\begin{align*}
\frac{CD_T^\sigma}{\Gamma(\sigma)} \varphi(t) &= Y(t, \varphi(t)), \\
\varphi(0) &= \varphi_0.
\end{align*}
\]
(66)

By stratifying the essential theorem of calculus, we transform Equation (66) to the following:

\[
\varphi(t) - \varphi(0) = \frac{1}{\Gamma(\sigma)} \int_0^t Y(\chi, \varphi(\chi))(t - \chi)^{\sigma - 1} d\chi,
\]
(67)

so that, at \( t = t_{n+1}, n = 1, 2, \ldots, \) we get

\[
\varphi(t_{n+1}) - \varphi(0) = \frac{1}{\Gamma(\sigma)} \int_0^{t_{n+1}} Y(t, \varphi(t))(t_{n+1} - t)^{\sigma - 1} dt,
\]
(68)

and

\[
\varphi(t_n) - \varphi(0) = \frac{1}{\Gamma(\sigma)} \int_0^{t_n} Y(t, \varphi(t))(t_n - t)^{\sigma - 1} dt.
\]
(69)

By subtracting (69) from (68), we obtain the following:

\[
\varphi(t_{n+1}) - \varphi(t_n) = \frac{1}{\Gamma(\sigma)} \int_0^{t_{n+1}} Y(t, \varphi(t))(t_{n+1} - t)^{\sigma - 1} dt + \frac{1}{\Gamma(\sigma)} \int_0^{t_n} Y(t, \varphi(t))(t_n - t)^{\sigma - 1} dt
\]
(70)

This suggests the following:

\[
\varphi(t_{n+1}) - \varphi(t_n) = \mathcal{A}_\sigma + \mathcal{B}_\sigma,
\]
(71)

where

\[
\mathcal{A}_\sigma = \frac{1}{\Gamma(\sigma)} \int_0^{t_{n+1}} Y(t, \varphi(t))(t_{n+1} - t)^{\sigma - 1} dt,
\]

and

\[
\mathcal{B}_\sigma = \frac{1}{\Gamma(\sigma)} \int_0^{t_n} Y(t, \varphi(t))(t_n - t)^{\sigma - 1} dt.
\]
By applying the Lagrange interpolation on the function \( Y(t, \varphi(t)) \), we get

\[
\mathcal{P}(t) \equiv \frac{t - t_{n-1}}{t_n - t_{n-1}} Y(t_n, \varphi_n) + \frac{t - t_n}{t_{n-1} - t_n} Y(t_{n-1}, \varphi_{n-1}) = Y(t_n, \varphi_n) \frac{(t - t_{n-1})}{h} + Y(t_{n-1}, \varphi_{n-1}) \frac{(t - t_n)}{h}.
\]  

(72)

Then, we have

\[
\mathcal{A}_\sigma = \frac{Y(t_n, \varphi_n)}{h \Gamma(\sigma)} \int_0^{t_{n+1}} (t - t_{n-1})(t_{n+1} - t)^{\sigma-1} dt + \frac{Y(t_{n-1}, \varphi_{n-1})}{h \Gamma(\sigma)} \int_0^{t_{n+1}} (t - t_n)(t_{n+1} - t)^{\sigma-1} dt
\]

\[
= \frac{Y(t_n, \varphi_n)}{h \Gamma(\sigma)} \int_0^{t_{n+1}} (t_{n+1} - \varphi - t_{n-1}) \varphi^{\sigma-1} d\varphi + \frac{Y(t_{n-1}, \varphi_{n-1})}{h \Gamma(\sigma)} \int_0^{t_{n+1}} (t_{n+1} - \varphi - t_n) \varphi^{\sigma-1} dt
\]  

(73)

Subsequently,

\[
\mathcal{A}_\sigma = \frac{Y(t_n, \varphi_n)}{h \Gamma(\sigma)} \left\{ \frac{2ht^\alpha}{\sigma} - \frac{t^\alpha_{n+1}}{\sigma + 1} \right\} - \frac{Y(t_{n-1}, \varphi_{n-1})}{h \Gamma(\sigma)} \left\{ \frac{ht^\alpha}{\sigma} - \frac{t^\alpha_n}{\sigma + 1} \right\}
\]  

(74)

Similarly likewise, we obtain

\[
\mathcal{B}_\sigma = \frac{Y(t_n, \varphi_n)}{h \Gamma(\sigma)} \int_0^{t_n} (t - t_{n-1})(t_n - t)^{\sigma-1} dt + \frac{Y(t_{n-1}, \varphi_{n-1})}{h \Gamma(\sigma)} \int_0^{t_n} (t - t_n)(t_n - t)^{\sigma-1} dt
\]

\[
= \frac{Y(t_n, \varphi_n)}{h \Gamma(\sigma)} \int_0^{t_n} (t - \varphi - t_{n-1}) \varphi^{\sigma-1} d\varphi + \frac{Y(t_{n-1}, \varphi_{n-1})}{h \Gamma(\sigma)} \left( \frac{t^\alpha}{\sigma} \right)
\]  

(75)

Subsequently,

\[
\mathcal{B}_\sigma = \frac{Y(t_n, \varphi_n)}{h \Gamma(\sigma)} \left\{ \frac{2ht^\alpha}{\sigma} - \frac{t^\alpha_{n+1}}{\sigma + 1} \right\} - \frac{Y(t_{n-1}, \varphi_{n-1})}{h \Gamma(\sigma)} \left( \frac{t^\alpha_n}{\sigma} \right)
\]  

(76)

Thus the approximate solution is given as

\[
\varphi(t_{n+1}) = \varphi(t_n) + \frac{Y(t_n, \varphi_n)}{h \Gamma(\sigma)} \left\{ \frac{2ht^\alpha}{\sigma} - \frac{t^\alpha_{n+1}}{\sigma + 1} \right\} - \frac{Y(t_{n-1}, \varphi_{n-1})}{h \Gamma(\sigma)} \left( \frac{t^\alpha_n}{\sigma} \right)
\]  

(77)

Consequently, the solution of the model (1) is

\[
S_{n+1} = S_n + \frac{1}{h \Gamma(\sigma)} \left\{ \frac{2ht^\alpha}{\sigma} - \frac{t^\alpha_{n+1}}{\sigma + 1} + \frac{2ht^\alpha}{\sigma} - \frac{t^\alpha_{n+1}}{\sigma + 1} \right\}
\]

\[
\left\{ - \frac{\lambda_1}{N} S_n(t_n) E_n(t_n) - \frac{\lambda_2}{N} S_n(t_n) I_n(t_n) - \lambda_3 S_n(t_n) \right\} - \left\{ \frac{ht^\alpha}{\sigma} - \frac{t^\alpha_{n+1}}{\sigma + 1} + \frac{t^\alpha_n}{\sigma} \right\}
\]

\[
\left\{ \frac{-\lambda_1}{N} S_{n-1} (t_{n-1}) E_{n-1} (t_{n-1}) - \frac{\lambda_2}{N} S_{n-1} (t_{n-1}) I_{n-1} (t_{n-1}) - \lambda_3 S_{n-1} (t_{n-1}) \right\}
\}
\]

(78)

\[
E_{n+1} = E_n + \frac{1}{h \Gamma(\sigma)} \left\{ \frac{2ht^\alpha}{\sigma} - \frac{t^\alpha_{n+1}}{\sigma + 1} + \frac{2ht^\alpha}{\sigma} - \frac{t^\alpha_{n+1}}{\sigma + 1} \right\}
\]

\[
\left\{ \frac{\lambda_1}{N} S_n (t_n) E_n (t_n) + \frac{\lambda_2}{N} S_n (t_n) I_n (t_n) - (\alpha_1 + \alpha_2) E_n (t_n) \right\} - \left\{ \frac{ht^\alpha}{\sigma} - \frac{t^\alpha_{n+1}}{\sigma + 1} + \frac{t^\alpha_n}{\sigma} \right\}
\]

\[
\left\{ \frac{\lambda_1}{N} S_{n-1} (t_{n-1}) E_{n-1} (t_{n-1}) + \frac{\lambda_2}{N} S_{n-1} (t_{n-1}) I_{n-1} (t_{n-1}) - (\alpha_1 + \alpha_2) E_{n-1} (t_{n-1}) \right\}
\}
\]

(79)
\[ I_{n+1} = I_n + \frac{1}{h \Gamma(\sigma)} \left\{ \frac{2h r_{n+1}^\sigma}{\sigma} - \frac{r_{n+1}^{\sigma+1}}{\sigma + 1} + \frac{2h t_n^\sigma}{\sigma} - \frac{t_n^{\sigma+1}}{\sigma + 1} \right\} \{ \alpha_1 E_n(t_n) - (\beta_1 + \beta_2) I_n(t_n) \} \\
- \left\{ \frac{h t_{n+1}^\sigma}{\sigma} - \frac{t_{n+1}^{\sigma+1}}{\sigma + 1} + \frac{t_n^{\sigma+1}}{\sigma + 1} \right\} \{ \alpha_1 E_{n-1}(t_{n-1}) - (\beta_1 + \beta_2) I_{n-1}(t_{n-1}) \} \right\} \]

\[ A_{n+1} = A_n + \frac{1}{h \Gamma(\sigma)} \left\{ \frac{2h r_{n+1}^\sigma}{\sigma} - \frac{r_{n+1}^{\sigma+1}}{\sigma + 1} + \frac{2h t_n^\sigma}{\sigma} - \frac{t_n^{\sigma+1}}{\sigma + 1} \right\} \{ \alpha_2 E_n(t_n) \} \\
- \left\{ \frac{h t_{n+1}^\sigma}{\sigma} - \frac{t_{n+1}^{\sigma+1}}{\sigma + 1} + \frac{t_n^{\sigma+1}}{\sigma + 1} \right\} \{ \alpha_2 E_{n-1}(t_{n-1}) \} \right\} \]

\[ R_{n+1} = R_n + \frac{1}{h \Gamma(\sigma)} \left\{ \frac{2h r_{n+1}^\sigma}{\sigma} - \frac{r_{n+1}^{\sigma+1}}{\sigma + 1} + \frac{2h t_n^\sigma}{\sigma} - \frac{t_n^{\sigma+1}}{\sigma + 1} \right\} \{ \beta_1 I_n(t_n) \} \\
- \left\{ \frac{h t_{n+1}^\sigma}{\sigma} - \frac{t_{n+1}^{\sigma+1}}{\sigma + 1} + \frac{t_n^{\sigma+1}}{\sigma + 1} \right\} \{ \beta_1 I_{n-1}(t_{n-1}) \} \right\} \]

\[ P_{n+1} = P_n + \frac{1}{h \Gamma(\sigma)} \left\{ \frac{2h r_{n+1}^\sigma}{\sigma} - \frac{r_{n+1}^{\sigma+1}}{\sigma + 1} + \frac{2h t_n^\sigma}{\sigma} - \frac{t_n^{\sigma+1}}{\sigma + 1} \right\} \{ \beta_2 I_n(t_n) \} \\
- \left\{ \frac{h t_{n+1}^\sigma}{\sigma} - \frac{t_{n+1}^{\sigma+1}}{\sigma + 1} + \frac{t_n^{\sigma+1}}{\sigma + 1} \right\} \{ \beta_2 I_{n-1}(t_{n-1}) \} \right\} \]

\[ Q_{n+1} = Q_n + \frac{1}{h \Gamma(\sigma)} \left\{ \frac{2h r_{n+1}^\sigma}{\sigma} - \frac{r_{n+1}^{\sigma+1}}{\sigma + 1} + \frac{2h t_n^\sigma}{\sigma} - \frac{t_n^{\sigma+1}}{\sigma + 1} \right\} \{ \lambda_3 S_n(t_n) \} \\
- \left\{ \frac{h t_{n+1}^\sigma}{\sigma} - \frac{t_{n+1}^{\sigma+1}}{\sigma + 1} + \frac{t_n^{\sigma+1}}{\sigma + 1} \right\} \{ \lambda_3 S_{n-1}(t_{n-1}) \} \right\} \]

5 | NUMERICAL SIMULATION AND DISCUSSION

In this section, we resolve the fractional operator SEIARQPQ model mathematically using the indicator corrector PECE technique for the Adams–Bashforth–Moulton method [49, 50]. The approach can be used for both linear and nonlinear problems, and multiterm equations (involving more than one differential operator) can also be generalized.

The properties of the parameters used for the mathematical simulation are evaluated from the Tunisian genuine life statistics [7] as follows:

\[ \lambda_1 = 0.8 \text{ (day}^{-1} \text{)} , \lambda_2 = 0.02 \text{ (day}^{-1} \text{)} , \lambda_3 = 0.166 \text{ (day}^{-1} \text{)} , \beta_1 = 0.003 \text{ (day}^{-1} \text{)} , \beta_2 = 0.0037 \text{ (day}^{-1} \text{)} , \alpha_1 = 0.0109 \text{ (day}^{-1} \text{)} , \text{and } \alpha_2 = 0.1 \text{ (day}^{-1} \text{)} . \]

The total population is assumed to be 11 million people, and the initial estimates of the various population stages after normalization are \( N = 11E6 \), \( S(0) = N - E(0) - I(0) \), \( E(0) = 200 \), \( I(0) = 18 \), \( A(0) = R(0) = P(0) = Q(0) = 0 \). The results of the numerical simulation of the SEIARQPQ model (without control) are shown next.

Figure 1a shows the comparison between our fractional solutions, classical solution [7], and clinical data of the infected population, \( I(t) \) with different values of fractional order \( \sigma \) and constant, \( \lambda_3 \). The results obtained when \( \sigma = 1 \) using the proposed algorithm are consistent with the classical results [7].
The authors in [7] deduce from their classical numerical results that the modification of the parameter $\lambda_3$ generates a consequent variation in the rate of the infected population. They demonstrated that the simulation curve model of the infected population, $I(t)$, is consistent with only the clinical data at $\lambda_3 = 0.166$. Our fractional results obtained for the confirmed infected population, $I(t)$, are more consistent with the clinical data at $\lambda_3 = 0.166$ and $\sigma = 0.85$ than with the classical results [7]. More accurate solutions can be obtained by changing the value of the parameter, $\lambda_3$, and the fractional order, $\sigma$ as shown in Figure 1b–d. Figure 2a shows the comparison between our fractional solutions, classical solution [7], and clinical data of the dead population, $P(t)$, with different values of fractional order, $a$, and constant, $\lambda_3$. The results obtained when $\sigma = 1$ using the proposed algorithm are consistent with the classical results [7]. The authors in [7] conclude from their classical numerical results that the dead population, $P(t)$, is consistent with only the clinical data at $\lambda_3 = 0.166$. Our fractional results show that the numerical solutions do not depend only on the value of $\lambda_3$ but also on the fractional order, $\sigma$ as shown in Figure 2b–d.

Figures 3–6 show the solution behavior with time history for $S(t)$, $E(t)$, $I(t)$, $A(t)$, $R(t)$, $P(t)$, and $Q(t)$ obtained for different values of $\sigma$. The numerical solutions depend on the value of the fractional order, $\sigma$. Figure 7 shows the behavior of the solution when the average of parameters $\lambda_1$ and $\lambda_2$ is changed in all population stages at the fractional derivative order $\sigma = 0.85$. Figure 8 shows the behavior of the
FIGURE 2  Comparison of solution behavior between dead data and dead population $P(t)$ with different fractional derivative order $\sigma$ and constant $\lambda_3$.

FIGURE 3  Solution behavior with time history of susceptible population $S(t)$ and exposed population $E(t)$. The fractional derivative order increases.
FIGURE 4  Solution behavior with time history of infected population $I(t)$ and recovered–exposed population $A(t)$

FIGURE 5  Solution behavior with time history of recovered population $R(t)$ and deceased population $P(t)$

FIGURE 6  Solution behavior with time history of quarantine population $Q(t)$
FIGURE 7  The impact of changing the average of parameter $\lambda_1$ and parameter $\lambda_2$ on all populace stages after some time where the fractional derivative order $\sigma = 0.85$.

FIGURE 8  The impact of changing the average of parameter $\beta_1$ and parameter $\beta_2$ on all populace stages after some time where the fractional derivative order $\sigma = 0.85$.

FIGURE 9  The impact of changing the average of parameter $\alpha_1$ and parameter $\alpha_2$ on all populace stages after some time where the fractional derivative order $\sigma = 0.85$. 
solution when the average of parameters \( \beta_1 \) and \( \beta_2 \) is changed in all population stages at the fractional derivative order \( \sigma = 0.85 \). Figure 9 shows the behavior of the solution when the average of parameters \( \alpha_1 \) and \( \alpha_2 \) is changed in all population stages at the fractional derivative order \( \sigma = 0.85 \). Figure 10 shows the behavior of the solution when the average of parameter \( \lambda_3 \) is changed in all population stages at the fractional derivative order \( \sigma = 0.85 \).

6 | CONCLUSION

The SEIARPQ model with the Caputo fractional derivative was used in this study to show the COVID-19 pandemic. The proposed fractional model is numerically solved using the indicator corrector PECE methodology for the Adams–Bashforth–Moulton method. The numerical results obtained do not only depend on the value of the SEIARPQ model parameters but also on that of the fractional order \( \sigma \). This shows that the fractional-order model is more consistent with the clinical data than with the integer-order model. Consequently, the fractional-order model offers solutions that rapidly converge to real problems.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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