Exploratory study of X(4140) in QCD sum rules

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In this work, we chose three molecular and three diquark-antidiquark currents with the quark content $c\bar{c}s\bar{s}$ and $J^{PC} = 0^{++}, 1^{++}, 2^{++}$, and estimated the masses and the meson coupling constants of the ground states coupling to these currents in the framework of QCD sum rules. In operator product expansion, we considered the terms including dimension eight, and we performed pole contribution tests carefully. According to our results, all of these currents couple to the ground states with degenerate masses which are in 10 MeV vicinity of X(4140). Therefore, with a QCD sum rules analysis, it is not possible to conclude that X(4140) has a dominant molecular or diquark-antidiquark content. However, there may be three states degenerate in mass, with positive charge conjugation and different isospins.

TABLE I: Experimental results on the mass and decay width of X(4140), given in chronological order. The average is taken from Ref. [12], and it gives the status of X(4140) measurements before the recent LHCb results.

| Year | Experiment | Mass (MeV) | Width (MeV) |
|------|------------|------------|-------------|
| 2008 | CDF [3]    | 4145.0 ± 2.9 ± 1.2 | 11.73_5.0 ± 3.7 |
| 2009 | Belle [5]  | 4143.0       | 11.7       |
| 2011 | CDF [6]    | 4143.4 ±2.9 ± 0.6 | 15.3_6.3 ± 2.5 |
| 2011 | LHCb [7]   | 4143.4       | 15.3       |
| 2013 | CMS [8]    | 4148.0 ± 2.4 ± 6.3 | 28±15 ± 19 |
| 2013 | D0 [9]     | 4159.0 ± 4.3 ± 6.6 | 19.9 ±12.6_1.0 |
| 2014 | Babar [10] | 4143.4       | 15.3       |
| 2015 | D0 [11]    | 4152.5 ± 1.7 ± 6.2 | 16.3 ± 5.6 ± 11.4 |
| 2017 | LHCb [12]  | 4146.5 ± 4.5 ± 2.8 | 83 ± 21 ± 14 |

In the literature, there are several studies exploring the structure of X(4140) [13–26]. Among these studies, the tetraquark model proposed in [13] predicted the quantum numbers and the mass of the X(4140) consistent with LHCb. However, lattice QCD calculation with diquark operators found no evidence for the existence of an axial vector X(4140) [16]. Within these theoretical efforts, studies with QCD sum rules (QCDSR) are puzzling since most of them predicts the quantum numbers of X(4140) inconsistent with the LHCb. In Refs. [17–20], the authors predicted X(4140) to be a scalar $D^*_s D^*_s$ molecule, however Refs. [21, 22] claim the opposite with similar currents. In Refs. [21, 22], using scalar and axial vector tetraquark currents, the authors calculated masses of the ground states but the results were incompatible with X(4140). However with a similar axial vector diquark-antidiquark current, a mass close to recent results of LHCb was predicted in [23]. These inconsistencies in between the predictions of QCD sum rules studies, and with the LHCb results, motivated us to perform a complete QCD sum rules investigation with the aim of contributing to this unconfirmed topic.

In the present work, we calculated the mass and the meson coupling constants of the ground states coupling to $D^*_s D^*_s$ or diquark-antidiquark currents with the content $sc\bar{c}s\bar{s}$. In order to make a comparison with the aforementioned QCD sum rules studies, we chose these currents to be scalar, axialvector and tensor. To this end, we calculated the two point correlation functions of these where the quark, gluon and mixed condensates were considered up to dimension eight. We performed a very detailed numerical analysis to achieve the convergence of the series in operator product expansion (OPE) and the dominance of the ground state resonance in the continuum. Performed investigations should allow us to find an explanation to this unconfirmed puzzle.

This work is organised as follows. In section [11], we present the sum rules calculations of masses and meson couplings of the ground states coupling to currents under investigation. In section [11], we performed a numerical analysis for the success of the evaluated sum rules and extracted the numerical values of masses and meson couplings. Here we also compared our results with other theoretical predictions. Section IV contains our final conclusions. The explicit expressions of the spectral densities

I. INTRODUCTION

In the past fifteen years, charmonium like resonances which do not seem to have a simple $cc$ structure were observed by several experiments. All of these new states, decay into final states containing charmonium, however their final decay rates into open charm pairs are unexpectedly small. Thus, they are considered as good candidates of exotic hadrons, which are assumed to have other structure than the ordinary mesons and baryons. A list of these states, and their current experimental status can be found in Refs. [1–3].

Among these newly observed resonances, X(4140) was experimentally observed by several collaborations in the invariant mass spectrum of $J/\psi \phi$ final states, and very recently its quantum numbers were announced to be $J^{PC} = 1^{++}$ by LHCb Collaboration [3,12]. However, the decay width of the state observed by LHCb is unexpectedly wider than the previous observations, and the origin of this difference still remains unsolved. In Table 1, we summarized the current experimental results which confirms the existence of X(4140). Despite these observations, its structure has not been totally understood yet, as well as other exotic mesons.

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of the two point correlators are given in Appendix.

II. QCD SUM RULES FOR MASSES AND MESON COUPLING CONSTANTS

In order to study the physical properties of X(4140), we started with six interpolating currents, interpreting X(4140) as a scalar, a vector or a tensor $D^*_sD^*_s$ molecule, or as a scalar, a vector or a tensor diquark-antidiquark. The following molecular currents

$$J^{(1)}(x) = s^\dagger(x)\gamma_\mu c^\dagger (x) \not{\tau}(x)\gamma^\mu s^\dagger(x),$$

$$J^{(1)}_\mu(x) = \frac{1}{\sqrt{2}} (s^\dagger(x)\gamma_5 c^\dagger (x) \not{\tau}(x)\gamma_\mu s^\dagger(x)$$

$$- \overline{s}(x)\gamma_\mu c^\dagger (x) \not{\tau}(x)\gamma_5 s^\dagger(x)), (2)$$

$$J^{(1)\mu\nu}(x) = \frac{1}{\sqrt{2}} (s^\dagger(x)\gamma_\mu c^\dagger (x) \not{\tau}(x)\gamma_\nu s^\dagger(x)$$

$$+ \overline{s}(x)\gamma_\nu c^\dagger (x) \not{\tau}(x)\gamma_\mu s^\dagger(x)), (3)$$

and the following diquark-antidiquark currents

$$J^{(2)}(x) = \varepsilon^{ijk}\varepsilon^{imn}(s^\dagger(x)\gamma_\mu c^\dagger (x) \not{\tau}(x)\gamma^\mu C\not{\tau^n}(x), (4)$$

$$J^{(2)\mu}(x) = \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}} (s^\dagger(x)\gamma_5 c^\dagger (x) \not{\tau}(x)\gamma_\mu C\not{\tau^n}(x)$$

$$- \overline{s}(x)\gamma_\mu c^\dagger (x) \not{\tau}(x)\gamma_5 C\not{\tau^n}(x)), (5)$$

$$J^{(2)\mu\nu}(x) = \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}} (s^\dagger(x)\gamma_\mu c^\dagger (x) \not{\tau}(x)\gamma_\nu C\not{\tau^n}(x)$$

$$+ \overline{s}(x)\gamma_\nu c^\dagger (x) \not{\tau}(x)\gamma_\mu C\not{\tau^n}(x)), (6)$$

are chosen to examine the QCDSR analyses that were done in literature, where $C$ is the charge conjugation matrix and $i, j, ...$ are color indices. In Eqs. (1) - (5), the superscripts (1) and (2) denote that the current has either molecular or diquark-antidiquark structure, and they will be denoted by superscript (a) whenever a compact formalism is required. The sum rules to obtain masses and meson coupling constants of the ground state mesons coupling to these currents are constructed from the following two point correlation functions

$$\Pi^{(a)}(q) = i \int d^4xe^{iq\cdot x} \langle 0 | \mathcal{T} (J^{(a)}(x) J^{(a)\dagger}(0)) | 0 \rangle, (7)$$

$$\Pi_{\mu}^{(a)}(q) = i \int d^4xe^{iq\cdot x} \langle 0 | \mathcal{T} (J^{(a)}_\mu(x) J^{(a)\dagger}_\mu(0)) | 0 \rangle, (8)$$

$$\Pi_{\mu\nu}^{(a)}(q) = i \int d^4xe^{iq\cdot x} \langle 0 | \mathcal{T} (J^{(a)}_{\mu\nu}(x) J^{(a)\dagger}_{\mu\nu}(0)) | 0 \rangle. (9)$$

In QCDSR approach, the correlation function's dependence on momentum $q$ enables to extract the physical properties of a hadron, by evaluating the correlator twice in different momentum regions and relating these two expressions to obtain sum rules. For $q^2 >> 0$, i.e., large distances, the interpolating current and its conjugate are interpreted as annihilation and creation operators of the mesons which has the same quark content and quantum numbers as the chosen current. In this case, the correlation functions in Eqs. (7-9) are saturated with a complete set of hadrons and integrals over $x$ are performed. These interpretations of the correlators are called “the physical (or phenomenological) side”. For $q^2 << 0$, the correlation functions can be calculated by using OPE. By using OPE, contributions from quark, gluon and mixed condensates are included in the evaluation of the correlators in Eqs. (7-9). The evaluations of the correlators with the help of OPE are oftenly called “the OPE (or QCD) side”.

The physical sides of the correlation functions in Eqs. (7-9) can be expressed as

$$\Pi^{(a)\text{phys}}(q) = \frac{\langle 0 | J^{(a)}(x) X(q) | X(q) J^{(a)\dagger}(0) | 0 \rangle}{m_X^2 - q^2} + ... , (10)$$

$$\Pi_{\mu}^{(a)\text{phys}}(q) = \frac{\langle 0 | J^{(a)}_\mu(x) X(q) | X(q) J^{(a)\dagger}_\mu(0) | 0 \rangle}{m_X^2 - q^2} + ... , (11)$$

$$\Pi_{\mu\nu}^{(a)\text{phys}}(q) = \frac{\langle 0 | J^{(a)}_{\mu\nu}(x) X(q) | X(q) J^{(a)\dagger}_{\mu\nu}(0) | 0 \rangle}{m_X^2 - q^2} + ... , (12)$$

where $m_X$ is the mass of the ground state meson coupling to the chosen current and dots denote the higher resonance contributions which are parameterized via continuum threshold parameter $s_0$. The scalar, axial vector and tensor matrix elements are defined as

$$\langle 0 | J^{(a)}(x) X(q) \rangle = \lambda^{(a)}_0,$$ (13)

$$\langle 0 | J^{(a)}_\mu(x) X(q) \rangle = \lambda^{(a)}_1 \varepsilon_\mu,$$ (14)

$$\langle 0 | J^{(a)}_{\mu\nu}(x) X(q) \rangle = \lambda^{(a)}_2 \varepsilon_{\mu\nu},$$ (15)

where subscript 0, 1, 2 denote the spin of the ground state coupling to chosen current and $\varepsilon_\mu$ and $\varepsilon_{\mu\nu}$ are vector and tensor polarizations satisfying the following relations

$$\varepsilon_\mu \varepsilon^\mu_\nu = T_{\mu\nu},$$ (16)

$$\varepsilon_{\mu\nu} \varepsilon^{\mu\nu}_{\alpha\beta} = \frac{1}{2} T_{\mu\alpha} T_{\nu\beta} + \frac{1}{2} T_{\mu\beta} T_{\nu\alpha} - \frac{1}{3} T_{\mu\nu} T_{\alpha\beta},$$ (17)

where $T_{\mu\nu} = -g_{\mu\nu} + q_\mu q_\nu/m_X^2$. 

In terms of hadronic states, the correlators of the scalar, axial vector and tensor currents are obtained as

\[ \Pi^{(a)\text{Phys}}(q) = \frac{\lambda_0^{(a)^2}}{m_X^2 - q^2}, \]  

(18)

\[ \Pi^{(a)\text{Phys}}_{\mu\nu}(q) = \frac{\lambda_1^{(a)^2}}{m_X^2 - q^2} g_{\mu\nu} + \text{other structures}, \]  

(19)

\[ \Pi^{(a)\text{Phys}}_{\mu\nu\alpha\beta}(q) = \frac{1}{2} \left( \frac{\lambda_2^{(a)^2}}{m_X^2 - q^2} \left\{ \frac{1}{2} (g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha}) \right\} \right) + \text{other structures}, \]  

(20)

On the OPE side of the QCDSR calculations, light and heavy quark fields are contracted and the correlation functions for the currents given in Eqs. (1 - 6) are obtained in terms of full s and c quark propagators as

\[ \Pi^{(1)\text{OPE}}(q) = i \int d^4x e^{iq \cdot x} \text{Tr} \left[ S_s^{i\gamma}(x) \gamma_{\mu} s^{i\gamma} c(x) \gamma_{\nu} \right] \text{Tr} \left[ S_c^{j\gamma}(x) \gamma_{\mu} c^{j\gamma} s(x) \gamma_{\nu} \right], \]  

(21)

\[ \Pi^{(1)\text{OPE}}_{\mu\nu}(q) = i \int d^4x e^{iq \cdot x} \left\{ \text{Tr} \left[ S_s^{i\gamma}(x) \gamma_{\mu} s^{i\gamma} c(x) \gamma_{\nu} \right] \text{Tr} \left[ S_c^{j\gamma}(x) \gamma_{\mu} c^{j\gamma} s(x) \gamma_{\nu} \right] - \text{Tr} \left[ S_s^{i\gamma}(x) \gamma_{\mu} s^{i\gamma} c(x) \gamma_{\nu} \right] \text{Tr} \left[ S_c^{j\gamma}(x) \gamma_{\mu} c^{j\gamma} s(x) \gamma_{\nu} \right] - \text{Tr} \left[ S_s^{i\gamma}(x) \gamma_{\mu} s^{i\gamma} c(x) \gamma_{\nu} \right] \text{Tr} \left[ S_c^{j\gamma}(x) \gamma_{\mu} c^{j\gamma} s(x) \gamma_{\nu} \right] + \text{Tr} \left[ S_s^{i\gamma}(x) \gamma_{\mu} s^{i\gamma} c(x) \gamma_{\nu} \right] \text{Tr} \left[ S_c^{j\gamma}(x) \gamma_{\mu} c^{j\gamma} s(x) \gamma_{\nu} \right] \right\}, \]  

(22)

\[ \Pi^{(1)\text{OPE}}_{\mu\nu\alpha\beta}(q) = i \int d^4x e^{iq \cdot x} \left\{ \frac{1}{2} \left[ \text{Tr} \left[ S_s^{i\gamma}(x) \gamma_{\mu} s^{i\gamma} c(x) \gamma_{\nu} \right] \text{Tr} \left[ S_c^{j\gamma}(x) \gamma_{\mu} c^{j\gamma} s(x) \gamma_{\nu} \right] \right] + \text{Tr} \left[ S_s^{i\gamma}(x) \gamma_{\mu} s^{i\gamma} c(x) \gamma_{\nu} \right] \text{Tr} \left[ S_c^{j\gamma}(x) \gamma_{\mu} c^{j\gamma} s(x) \gamma_{\nu} \right] + \text{Tr} \left[ S_s^{i\gamma}(x) \gamma_{\mu} s^{i\gamma} c(x) \gamma_{\nu} \right] \text{Tr} \left[ S_c^{j\gamma}(x) \gamma_{\mu} c^{j\gamma} s(x) \gamma_{\nu} \right] + \text{Tr} \left[ S_s^{i\gamma}(x) \gamma_{\mu} s^{i\gamma} c(x) \gamma_{\nu} \right] \text{Tr} \left[ S_c^{j\gamma}(x) \gamma_{\mu} c^{j\gamma} s(x) \gamma_{\nu} \right] \right\}, \]  

(23)

\[ \Pi^{(2)\text{OPE}}(q) = i \int d^4x e^{iq \cdot x} e^{ijk \varepsilon m n \varepsilon j' k' \varepsilon m' n'} \text{Tr} \left[ \gamma_{\nu} S^{j' s}(x) \gamma_{\mu} S^{k s}(x) \right] \text{Tr} \left[ \gamma_{\mu} S^{m' n}(x) \gamma_{\nu} S^{m m}(x) \right], \]  

(24)

\[ \Pi^{(2)\text{OPE}}_{\mu\nu}(q) = i \int d^4x e^{iq \cdot x} e^{ijk \varepsilon m n \varepsilon j' k' \varepsilon m' n'} \times \right\} \text{Tr} \left[ \gamma_{\nu} S^{j' s}(x) \gamma_{\mu} S^{k s}(x) \right] \text{Tr} \left[ \gamma_{\mu} S^{m' n}(x) \gamma_{\nu} S^{m m}(x) \right], \]  

(25)
where $S_{ij}^{\mu}(x) = CS_{ij}^{\mu T}(x)C$, and $S_{s}^{\mu}(x), S_{c}^{\mu}(x)$ are the full propagators of s and c quarks respectively. For the s quark, we chose the light quark propagator in the coordinate space which is in the form

$$S_{s}^{ij}(x) = \frac{i}{2\pi^{4}} \delta_{ij} - \frac{m_{s}}{4\pi^{2}x^{2}} \delta_{ij} - \frac{g_{s}G_{i}^{\alpha\beta}}{32\pi^{2}x^{2}} \left( \frac{\sigma^{\alpha\beta}(k + m_{s})}{4}(k^{2} - m_{s}^{2})^{2} \right) \delta_{ij} - i\frac{g_{s}^{2}m_{s}k^{2} + m_{s}k}{27648} G_{2} \delta_{ij} + \cdots \,,$$

(27)

For the c quark, we used the following heavy quark propagator

$$S_{c}^{ij}(x) = \frac{i}{2\pi^{4}} \delta_{ij} - \frac{m_{c}}{4\pi^{2}x^{2}} \delta_{ij} - \frac{\langle \bar{s}s \rangle}{12} \left( 1 - \frac{m_{s}}{4} \right) \delta_{ij} - \frac{g_{c}G_{i}^{\alpha\beta}}{32\pi^{2}x^{2}} \left( \frac{\sigma^{\alpha\beta}(k + m_{c})}{4}(k^{2} - m_{c}^{2})^{2} \right) \delta_{ij} - i\frac{g_{c}^{2}m_{c}k^{2} + m_{c}k}{27648} G_{2} \delta_{ij} + \cdots \,,$$

(28)

given in Ref. [30], where $G_{\alpha\beta} = G_{\alpha\beta}^{A} A$, $G^{2} = G_{\alpha\beta} G_{\alpha\beta}$ and $\Gamma_{A} = \lambda_{A}/2$ are the Mandel matrices with $A = 1, \ldots, 8$. Similar to physical side, the correlation functions given in Eqs. (21–26) on the OPE side are also expanded in terms of Lorentz structures as

$$\Pi^{\mu\nu\alpha\beta}_{OPE}(q) = 1^{(a)} \Gamma_{1},$$

(29)

$$\Pi^{(a)OPE}_{\mu\nu}(q) = \Gamma_{1}^{(a)} 1 + \text{other structures},$$

(30)

$$\Pi^{(a)OPE}_{\mu\nu\alpha\beta}(q) = \Gamma_{2}^{(a)} \left\{ \frac{1}{2} (g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha}) \right\} + \text{other structures} \,,$$

(31)

where $\Gamma_{j}^{(a)}$ are the coefficients of the Lorentz structures that are selected in this work. A dispersion integral of the form

$$\Gamma_{j}^{(a)}(q^{2}) = \int_{0}^{\infty} ds \frac{\rho_{j}^{(a)OPE}(s)}{s - q^{2}},$$

(32)

can be written for the selected coefficients, where $\rho_{j}^{(a)OPE} = \text{Im}[\pi_{j}^{(a)/\pi}]$ are the spectral densities, and $J$ is the total angular momentum of the state. According to quark hadron duality ansatz in QCDSR, it is assumed that the spectral density obtained from the continuum of the states given in Eq. (10) in the physical side are related to the spectral density obtained from the OPE side via relation

$$\rho_{\text{cont}}(s) = \rho_{\text{OPE}}(s - s_{0}),$$

(33)

which enables us to isolate the ground state hadron from the infinite sum[31,33]. To improve the duality of the correlators, Borel transformation with respect to $q^{2}$ is applied. After applying these steps of traditional QCDSR analysis, the sum rules for the currents given in Eqs. (1) and (6) are obtained as

$$\lambda_{j}^{(a)OPE} e^{-m_{X}^{2}/M^{2}} = \int_{(2m_{s} + 2m_{c})^{2}}^{s_{0}} ds \rho_{j}^{(a)OPE}(s) e^{-s/M^{2}} \,.$$  

(34)

In order to estimate the mass of the ground state hadron, one takes the derivative of Eq. (34) with respect to $1/M^{2}$ and divides it to Eq. (34) and obtains the mass of the ground state as

$$m_{X}^{2} = \sqrt{\int_{(2m_{s} + 2m_{c})^{2}}^{s_{0}} ds \rho_{j}^{(a)OPE}(s) e^{-s/M^{2}}}.$$  

(35)

The expressions of the spectral densities obtained in this work are given in Appendix A.

### III. NUMERICAL ANALYSIS

The sum rules obtained in previous section depend on the values of the parameters such as quark, gluon and mixed condensates, and on the masses of c and s quarks. Values of these parameters are given in Table I. The c and s quark masses are chosen in the MS scheme at the scale $\mu = m_{c}$ and $\mu = 2 \text{GeV}$ respectively, and their values are taken from the Particle Data Group [34], and the values of the condensates are taken from Ref. [17]. The expressions of the masses and meson coupling constants given in Eqs. (34) and (35) depend also on the values of the continuum threshold ($s_{0}$) and the Borel Mass ($M^{2}$), which are parameters of the theory. In general,
The masses of the ground states hadrons coupling to these masses as a function of Borel mass at fixed final results which are obtained for the masses and depict necessary to observe the dependence of the physical quantities in the final results for meson coupling constants.

Finally, we present our results for the masses and the meson coupling constants of the specified currents in Tables III and IV. The uncertainties in the obtained results are due to variations of $s_0$ and $M^2$ within the working regions specified in Table III. We also considered the errors of the input parameters given in Table IV.

IV. DISCUSSION AND CONCLUDING REMARKS

In the current work, we performed a QCD sum rules analysis for scalar, axial vector and tensor currents identifying possible $D^*_s D^*$ molecular states, and scalar, axial vector and tensor, diquark-antidiquark currents. For these currents, the corresponding spectral densities are calculated up to dimension eight, and a careful analysis is done to determine the working regions of the sum rules. The masses of the ground states coupling to these six currents are estimated within 10 MeV vicinity of the mass of $X(4140)$ measured by several experiments[12, 13, 14, 15, 16], which is acceptable with the given uncertainties. Thus we conclude that, if $X(4140)$ is an axial vector state as measured by LHCb [12], it has scalar and tensor partners with the same mass. In the literature, such scenario was introduced for $X(3872)$ and its possible partners [29]. In addition, existence of states with different spins which couple to $D^*_s D^*$ and $D^*_s D^*$ were claimed in Ref. [27]. We also calculated the meson coupling constants for these ground states, which may be $X(4140)$ and its partners. Since the chosen molecular and diquark currents estimate degenerate masses, we can not favor neither of the $D^*_s D^*$ molecular nor the diquark-antidiquark structures for these states.

The masses estimated in this work were presented in Table IV in comparison with the results of the sum rules.

| Current | $M^2_{\text{min}}$ (GeV$^2$) | $M^2_{\text{max}}$ (GeV$^2$) | $19.7 M^2 \leq s_0 \leq 21.5 M^2$ | $M^2 \geq s_0$ |
|---------|------------------------------|------------------------------|---------------------------------|----------------|
| $J^{(1)}$ | 3.69                        | 4.43                        | 3.75 $M^2 + 5.88$               |
| $J^{(1)}_{D_s^*}$ | 3.99                      | 4.74                        | 3.66 $M^2 + 5.10$               |
| $J^{(1)}_{D_s^*}$ | 3.95                      | 4.74                        | 3.53 $M^2 + 5.76$               |
| $J^{(2)}$ | 3.65                        | 4.38                        | 3.85 $M^2 + 5.65$               |
| $J^{(2)}_{D_s^*}$ | 3.89                      | 4.63                        | 3.75 $M^2 + 5.13$               |
| $J^{(3)}_{D_s^*}$ | 3.73                      | 4.57                        | 3.33 $M^2 + 7.27$               |

$s_0$ is related to the mass of the state under investigation as $(m_X + 0.3)^2 GeV^2 \leq s_0 \leq (m_X + 0.5)^2 GeV^2$. In the present case, this restricts $s_0$ as 19.7 GeV$^2 \leq s_0 \leq 21.5$ GeV$^2$. In order to have reliable sum rules, $s_0$ and $M^2$ should jointly satisfy the pole dominance and OPE convergence requirements. In addition to these criteria, one has to choose working regions for these parameters in which the dependence of the obtained results for the masses and meson coupling constants should be minimal.

In QCDSR, the contribution coming from the pole of the ground state required to be greater than the contribution of the continuum. To analyze the pole dominance of the obtained sum rules, we plot the two parameter heat graphs of the ratio $\Pi(s_0, M^2)/\Pi(\infty, M^2)$ with respect to $s_0$ and $M^2$ which are given in Fig. [1].

The OPE convergence of the obtained sum rules is analyzed as follows. In Fig. [2] we plot the ratio of the sum of the terms with dimensions up to the specified term, to the correlator. It is seen from Fig. [2] that the correlators of the currents given in Eqs. (1) - (6) satisfy the following conditions.

- The contribution of the perturbative terms are greater than 50%.
- The contribution of the terms with dimension five are smaller than 25% for $M^2 \geq 2.5$ GeV$^2$, and reduces for further values of $M^2$.
- The contribution of the terms with dimension six to eight, either converge to zero or obtained as zero.

Thus for $M^2 \geq 2.5$ GeV$^2$, a good OPE convergence is ensured. The working regions of the sum rules obtained in this work are determined by combining the analysis up to this step, and they are given in Table IV.

In traditional sum rules analysis, one last check is necessary to observe the dependence of the physical quantities to parameters $M^2$ and $s_0$. In Fig. 4 we provide the final results which are obtained for the masses and depict these masses as a function of Borel mass at fixed $s_0$ values. The masses of the ground state hadrons coupling to specified currents are stable with respect to variations of $M^2$ and $s_0$. In Fig. 4, the dependence of the meson coupling constants are plotted with respect to $M^2$ at some fixed $s_0$ values within the range of working regions of the sum rules. Even though the dependence of the meson coupling constants to these parameters are within the acceptable limits, the observed variations with respect to continuum thresholds are the main sources of uncertainties in the final results for meson coupling constants.
In summary, we presented a QCD sum rules analysis of the two point correlation function for possible $D^*D_s^*$ molecule and diquark-antidiquark currents with $J^{PC} = 0^{++}, 1^{++}$ and $2^{++}$. Our motivation is to search a possible state which can be associated with X(4140) confirmed by several experiments[4, 6, 8, 9], and measured to be an axial vector state by LHCb[12]. For both molecular and diquark-antidiquark axial vector currents, we obtained a stable mass in agreement with X(4140) observed by LHCb. Thus we confirm the existence and the mass of X(4140), but we can not predict its content. In addition, we also have a subsidiary attempt to reanalyse the scalar molecular, tensor molecular, scalar diquark and tensor diquark currents which were used in studying X(4140) within QCD sum rules. We found that, all of these four currents predict same masses as X(4140), which confirms the traditional sum rules analysis that were done with similar currents. However, interpreting these states as X(4140) contradicts with LHCb measurements. Thus, we conclude that the analyzed scalar and tensor states might be partners of X(4140) with degenerate masses which can either have a molecular or diquark-antidiquark content, and they have not been observed yet. In addition to the masses, we also predicted the meson coupling constants for the states coupling to chosen currents. Our findings can be used in further analysis for the decays of X(4140) or its possible partners. Consequently, X(4140) should be investigated more, by studying its decays, and by other approaches as well. Preliminary results of this work was also presented in [23].

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TABLE V: Meson coupling constants of the ground state hadron coupling to specified current. The results obtained in this work are given in comparison with literature.

| Current | $\lambda_X$ (GeV) | $\lambda_X$ (GeV) |
|---------|------------------|------------------|
|         | This Work        | Literature       |
| $J^{(1)}_L$ $(0^{++})$ | $3.889 \pm 0.951$ | $4.20 \pm 0.96$ [17] |
|         | $6.2 \pm 1.1$ [21] | $5.75 \pm 0.90$ [20] |
| $J^{(1)}_T$ $(1^{++})$ | $2.221 \pm 0.503$ | $4.199 \pm 0.948$ [20] |
| $J^{(1)}_T$ $(2^{++})$ | $4.510 \pm 1.099$ | $4.8 \pm 0.8$ [23] |
| $J^{(2)}_L$ $(0^{++})$ | $2.556 \pm 0.578$ | $0.94 \pm 0.16$ [20] |
| $J^{(2)}_L$ $(2^{++})$ | $4.775 \pm 1.085$ | $5.75 \pm 0.90$ [23] |
FIG. 1: Pole dominance of the sum rules obtained in this work. The variation of the ratio of the pole to pole plus continuum with respect to $s_0$ and $M^2$, for the scalar, axial vector and tensor molecular currents (left panel) and for the scalar, axial vector and tensor diquark-antidiquark currents (right panel). For each plot, on the left of the dashed line, pole dominance is achieved.
FIG. 2: The OPE convergence of the sum rules obtained in this work. The ratio of the sum of the terms up to specified dimension, to correlation function is plotted with respect to $M^2$ for the scalar, axial vector and tensor molecular currents (left panel) and for the scalar, axial vector and tensor diquark-antidiquark currents (right panel).
FIG. 3: The mass of the ground state coupling to specified current as a function of $M^2$ for different values of $s_0$, for the scalar, axial vector and tensor molecular currents (left panel) and for the scalar, axial vector and tensor diquark-antidiquark currents (right panel).
FIG. 4: Meson coupling constant ($\lambda$) of the ground state coupling to specified current as a function of $M^2$ for different values of $s_0$, for the scalar, axial vector and tensor molecular currents (left panel) and for the scalar, axial vector and tensor diquark-antidiquark currents (right panel).
Appendix: Spectral Densities

In this appendix section, analytic expressions of the spectral densities \( \rho^{OPE(a)} \), obtained by using currents given in Eqs. (1-6) are given. They are obtained from the sum rules given in Eq. (34). Spectral densities can be written as

\[
\rho_{S}^{(a)OPE} = \sum_i \rho_{S,i}^{(a)OPE},
\]

where \( \rho_{S,i}^{(a)OPE} \) are the contributions of terms with dimension denoted by \( i = 0, 2, 3, \ldots, 8 \). In Eq. (A.1), \( a = 1, 2 \) denotes the molecular or tetraquark content, and \( S = 0, 1, 2 \) denotes whether the current is scalar, axial vector or tensor, respectively. In the appendix section, \( A = (x_1 + x_2 - 1) \) and \( B = (x_2 - 1) \) are defined for simplicity.

1. Spectral densities of molecular scalar current \((J^{(1)})\)

\[
\rho_{0,0}^{(1)OPE}(s) = \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \, \frac{-3(sx_1x_2A - m_c^2(x_1^2 + x_2^2(2x_2 - 1) + 2x_1Bx_2 + Bx_2^2))^2}{1024\pi^6 A(x_1^2 + x_1B + Bx_2)^8} \\
\times \left\{ m_s^4(x_1^2 - x_1 + Bx_2)(x_1^2 + x_2^2(2x_2 - 1) + 2x_1Bx_2 + Bx_2^2)^2 \right. \\
- \left. 4m_c^3m_s(x_1 + x_2)^2(x_1^2 + x_1B + Bx_2)^3 - 2m_c^2(x_1^2 + x_1B + Bx_2)^2 \right. \\
\times \left. \left( 12m_s^2(x_1^2 + x_1B + Bx_2)^3 + sx_1x_2(2x_1^2 + x_1^2(3x_2 - 4) + x_1^2(2x_2^2 - 7x_2 + 2) \right. \\
+ \left. x_1x_2(3x_2^2 - 7x_2 + 4) + 2B^2x_2^3) \right) + 10m_c^6m_s^2x_1x_2(x_1^2 + x_1B + Bx_2)^2 \right. \\
\times \left. (x_1^2 + x_2^2(2x_2 - 1) + Bx_2) + s^2x_1^2x_2^2A^2(3x_1^2 - x_1(4x_2 + 3) + 3Bx_2) \right\}
\]

\[
\rho_{0,3}^{(1)OPE}(s) = \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \, \frac{-3(s^s)}{64\pi^6 (x_1^2 + x_1B + Bx_2)^8} \left\{ 2m_c^5(x_1^2 + x_1^2(2x_2 - 1) \right. \\
+ \left. 2x_1Bx_2 + Bx_2^2)^3 + 2m_c^4m_s(x_1^2 + x_1B + Bx_2)^2(5x_1^2 + x_1^2(13x_2 - 10) \right. \\
+ \left. x_1^2(18x_2^2 - 28x_2 + 5) + 3x_1^2x_2(6x_2^2 - 12x_2 + 5) + x_1x_2^2(13x_2^2 - 28x_2 + 15) \right. \\
+ \left. 5B^2x_2^2 - 2m_c^2A(x_1^2 + x_1^2(2x_1 - 1) + 2x_1Bx_2 + Bx_2^2)^2(m_2^2(x_1^2 + x_1B + Bx_2) \right. \\
+ \left. 3sx_1x_2) - 2m_c^2m_s^2x_1x_2(7x_1^2 + x_1^2(19x_2 - 28) + 6x_1^2(4x_2^2 - 13x_2 + 7) + 2x_1^4 \right. \\
\times \left. (11x_3^2 - 57x_2^2 + 60x_2 - 14) + x_1^2(22x_3^2 - 124x_2^3 + 177x_2^3 - 82x_2 + 7) + 3x_1^2B^2x_2 \right. \\
\times \left. (8x_3^2 - 22x_2 + 7) + x_1B^3x_2^2(19x_2 - 21) + 7B^4x_2^3) + m_c^6x_1x_2A^2(x_1^3 \right. \\
+ \left. x_1^2(2x_1 - 1) + 2x_1Bx_2 + Bx_2^2)(3m_2^2(x_1^2 + x_1B + Bx_2) + 4sx_1x_2) \right. \\
\times \left. + 4m_s^2x_1^2x_2^2A^3(x_1^2 - x_1(4x_2 + 1) + Bx_2) \right\}
\]
\( \rho_{0.4}^{(1)\text{OPE}}(s) = \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{\langle \alpha \frac{Q^2}{u} \rangle}{512 \pi^4 A (x_1^2 + x_1 B + Bx_2)^6} \times \left\{ \begin{align*} & m_0^4 \left( x_1^2 + x_1^2 (2x_2 - 1) + 2x_1 Bx_2 + Bx_2^2 \right)^2 (4x_1^2 + x_1^2 (29x_2 - 10)) + x_1^2 (38x_2^2 - 39x_2 + 6) + x_1 x_2 (29x_2^2 - 39x_2 + 12) + x_1^2 (2x_2^2 - 5x_2 + 3) \right. \\
& + 2m_0^2 m_s (x_1^2 + x_1 B + Bx_2)^2 (2x_1^2 + x_1^2 (7x_2 - 1) + x_1 x_2 (35x_2 - 26)) \\
& + 12x_1 x_2 (4x_1^2 - 5x_2 + 1) + x_1^2 x_2^2 (35x_2^2 - 60x_2 + 24) + 2x_1 x_2^3 (7x_2^2 - 13x_2 + 6) \\
& + 2Bx_2^3 + m_0^2 (x_1^2 + x_1 B + Bx_2)^2 \\
& \left. \times (x_1^4 - x_1^3 + Bx_2^3) - sx_1 x_2 (15x_1^4 + 4x_1^2 (31x_2 - 12) + x_1^4 (340x_2^3 - 301x_2 + 51)) \right. \\
& + 3x_1^4 (154x_2^3 - 214x_2^3 + 77x_2 - 6) + 2x_1^2 x_2 (170x_2^3 - 321x_2^3 + 180x_2 - 27) \\
& + x_1^2 (124x_2^3 - 301x_2^3 + 231x_2 - 54) + 3B^2 x_2 (5x_2 - 6)) \big) - 6m_0 m_s x_1 x_2 \\
& \times \left( x_1^4 + x_1^3 (7x_2 - 3) + x_1^2 (23x_2^2 - 26x_2 + 3) + x_1 (44x_2^2 - 81x_2^2 + 37x_2 - 1) \right) \right. \\
& + 3x_1^4 x_2 (18x_2^3 - 45x_2^3 + 35x_2 - 8) + x_1^3 B x_2 (44x_2^2 - 47x_2 + 6) \\
& + x_1^2 B^2 x_2^2 (23x_2 - 12) + x_1 B^2 x_2^3 (7x_2 - 12x_2 + 6) + B^3 x_2^3 \\
& \left. + 6s^2 x_1^2 x_2^2 A^2 (2x_1^2 + x_1^2 (9x_2 - 2) + x_1 x_2 (9x_2 - 4) + 2Bx_2^2) \right\} \right. \\
& (A.3) \\
\end{align*} \)

\[
\rho_{0.5}^{(1)\text{OPE}}(s) = \int_0^1 \frac{dx_1 \, m_0^2 m_s \langle ss \rangle (-8m_s^2 + m_s m_s - 3s(x - 1)x)}{64\pi^4} \\
+ \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{m_0^2 \langle \bar{s}s \rangle A}{128\pi^4 (x_1^2 + x_1 B + Bx_2)^5} \\
\times \left\{ \begin{align*} & 6m_0^4 \left( x_1^2 + x_1^2 (2x_2 - 1) + 2x_1 Bx_2 + Bx_2^2 \right)^2 - 8m_0^2 m_s x_1 x_2 \left( x_1^4 + x_1^3 (3x_2 - 2) \right) \\
& + x_1^2 (4x_2^3 - 5x_2 + 1) + x_1 x_2 \left( 3x_2^2 - 5x_2 + 2 \right) + B^2 x_2^3 \right) - 9m_s x_1 x_2 \\
& \times \left( x_1^4 + x_1^3 (3x_2 - 2) + x_1^2 (4x_2^3 - 5x_2 + 1) + x_1 x_2 \left( 3x_2^2 - 5x_2 + 2 \right) + B^2 x_2^3 \right) \\
& + 18m_s x_1^2 x_2^2 A^2 \right\} \right. \\
& (A.4) \\
\]

\[
\rho_{0.6}^{(1)\text{OPE}}(s) = \int_0^1 \frac{dx \, \langle \bar{s}s \rangle^2}{432\pi^4} \left\{ \begin{align*} & g_s^2 \left( 2m_0^2 - m_c m_s + 3s(x - 1)x \right) - 54\pi^2 \left( 2m_0^2 - m_c m_s \right) \right. \\
& - 3m_0^2 (x - 1)x + \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{g_s^2 \langle \bar{s}s \rangle x_1 x_2 A^2}{432\pi^4 (x_1^2 + x_1 B + Bx_2)^5} \\
& \times \left\{ \begin{align*} & 4m_0^2 \left( x_1^4 + x_1^3 (2x_2 - 1) + 2x_1 Bx_2 + Bx_2^2 \right) - 9sx_1 x_2 A \right\} \right. \\
& (A.5) \\
\end{align*} \right. \\
\end{align*} \)

\[
\rho_{0.7}^{(1)\text{OPE}}(s) = \int_0^1 \frac{dx \, \langle \alpha \frac{Q^2}{u} \rangle \langle \bar{s}s \rangle (m_c - 6m_s)}{192\pi^2} + \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{\langle \alpha \frac{Q^2}{u} \rangle m_c \langle \bar{s}s \rangle}{96\pi^2 B (x_1^2 + x_1 B + Bx_2)^4} \times \left\{ \begin{align*} & 2x_1^6 + 4x_2^5 B + x_1^5 (-5x_2^2 + 3x_2 + 2) - 13x_1^2 B^2 x_2 + x_1 x_2 (-13x_1^3 + 31x_2^2) \\
& - 24x_2 + 6 - x_1 B^2 x_2^2 (7x_2 - 6) + B^2 x_2^4 \right\} \right. \\
& (A.6) \\
\end{align*} \)

\[
\rho_{0.8}^{(1)\text{OPE}}(s) = 0 \right. \\
(A.7) \\
\]


2. Spectral densities of molecular axial vector current ($J_\mu^{(1)}$)

\[
\rho_{1,0}^{(1)\text{OPE}}(s) = \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{1}{4096\pi^6 A (x_1^2 + x_1 B + Bx_2^2)^5} \{sx_1 x_2 A
\]
\[\quad - m^2 (x_1^3 + x_1^2 (2x_2 - 1) + 2x_1 Bx_2 + Bx_2^2))^2 (3m_s^4 x_1 x_2 (x_1^3 + x_1^2 (2x_2 - 1)
\]
\[\quad + 2x_1 Bx_2 + Bx_2^2)^2 + 18m_s^6 m_s (x_1 + x_2)^2 (x_1^2 + x_1 B + Bx_2)^3 + 2m_s^6 (x_1^2
\]
\[\quad + x_1 B + Bx_2) (36m_s^6 (x_1^2 + x_1 B + Bx_2)^3 - 13sx_1^2 x_2^2 (x_1^2 + x_1 (2x_2 - 1)
\]
\[\quad + Bx_2)) - 54m_s^m s x_1 x_2 (x_1^2 + x_1 B + Bx_2)^2 (x_1^2 + x_1 (2x_2 - 1) + Bx_2)
\]
\[\quad + 35s^2 x_1^2 x_2 A^2 \}\]  
(A.8)

\[
\rho_{1,3}^{(1)\text{OPE}}(s) = \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{3\langle s \bar{s} \rangle}{256\pi^6 (x_1^2 + x_1 B + Bx_2)^5} \{3m_s^5 (x_1^3 + x_1^2 (2x_2 - 1)
\]
\[\quad + 2x_1 Bx_2 Bx_2^2)^3 + 2m_s^4 m_s (x_1^2 + x_1 B + Bx_2)^2 (4x_1^5 + x_1^4 (9x_2 - 8)
\]
\[\quad + x_1^3 (11x_2^2 - 21x_2 + 4) + x_1^2 x_2 (11x_2^2 - 26x_2 + 12) + 3x_1 x_2^2 (3x_2^2 - 7x_2 + 4)
\]
\[\quad + 4B^2 x_2^3) - m_s^3 A (x_1^3 + x_1^2 (2x_2 - 1) + 2x_1 Bx_2 + Bx_2^2)^2 (3m_s^2 (x_1^2 + x_1 B + Bx_2)
\]
\[\quad + 10x_1 x_2) - 8m_s^5 m_s x_1 x_2 (x_1^2 - 4x_1^6 + x_1^5 (-7x_2^2 - 3x_2 + 6)
\]
\[\quad + x_1^4 (15x_2^2 + 10x_2 + 9x_2 - 4) + x_1^3 (-15x_1^4 + 19x_1^3 + 4x_2^2 - 9x_2 + 1)
\]
\[\quad - x_1^2 B^2 x_2 (7x_2^2 + 4x_2 - 3) - 3x_1 B^3 x_2 + B^4 x_2^3) + m_s x_1 x_2 A^2 (x_1^4 + x_1^3 (2x_2 - 1)
\]
\[\quad + 2x_1 Bx_2 + Bx_2^2) (5m_s^5 (x_1^2 + x_1 B + Bx_2) + 7sx_1 x_2) - 30m_s s^2 x_1^3 x_2 A^3 \}  
(A.9)

\[
\rho_{1,4}^{(1)\text{OPE}}(s) = \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{-\langle s \bar{s} \rangle G^2}{6144\pi^4 A (x_1^2 + x_1 B + Bx_2)^5} \{6m_s^4 x_1 x_2 (x_1^2 + x_1 x_2 + x_2^2)
\]
\[\quad \times (x_1^2 + x_1^2 (2x_2 - 1) + 2x_1 Bx_2 + Bx_2^2)^2 + 9m_s^6 m_s (x_1^2 + x_1 B + Bx_2)^2
\]
\[\quad \times (2x_1^5 - 2x_1^2 (x_1^2 + 1) + x_1^6 (6 - 13x_2 x_2 - 4x_1^2 x_2) (4x_2^2 - 5x_2 + 1)
\]
\[\quad + x_1^2 x_2^2 (-13x_2^2 + 20x_2 - 8) - 2x_1 x_2 (x_2^3 - 3x_2 + 2) + 2Bx_2^2
\]
\[\quad + 8m_s^2 (x_1^2 + x_1 B + Bx_2) (3m_s^2 (x_1^2 + x_1 B + Bx_2)^2 (x_1^4 - x_1^3 + Bx_2^3)
\]
\[\quad + sx_1^2 x_2^2 (x_1^4 + 2x_1^2 (5x_2^2 - 2) + 3x_1^2 (6x_2^2 - 6x_2 + 1) + 2x_1 x_2 (5x_2^2 - 9x_2 + 3)
\]
\[\quad + x_2^2 (x_2^2 - 4x_2 + 3)) - 30m_s m_s x_1 x_2 (x_1^8 - x_1^7 (x_2 + 3) + x_1^6 (-9x_2^2 + 6x_2 + 3)
\]
\[\quad - x_1^5 (20x_2^4 - 31x_2^3 + 11x_2^2 + 1) + x_1^6 x_2 (-26x_2^2 + 57x_2^2 - 39x_2 + 8)
\]
\[\quad - x_1^3 B^2 x_2 (20x_2^6 - 17x_2^4 + 2) + x_1^4 B^2 x_2^2 (9x_2 - 4) - x_1 B^2 x_2^2 (2x_2 - 4x_2 + 2)
\]
\[\quad + B^2 x_2^2) - 30s^2 x_1 x_2 A^3 (x_1 + x_2) \}  
(A.10)

\[
\rho_{1,5}^{(1)\text{OPE}}(s) = \int_0^1 \frac{3m_s m_s x_1 x_2 \langle s \bar{s} \rangle (4m_s - m_s)}{512\pi^4} + \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{1}{512\pi^4 (x_1^2 + x_1 B + Bx_2)^5} \{-m_0^5 \langle s \bar{s} \rangle A
\]
\[\quad \times \left\{ \frac{9m_c^3 (x_1^3 + x_1^2 (2x_2 - 1) + 2x_1 Bx_2 + Bx_2^2)^2 - 8m_s^2 m_s x_1 x_2 (x_1^2 + x_1 (3x_2 - 2)
\]
\[\quad + x_1^3 (2x_2 - 1) + x_1 x_2 (3x_2^2 - 5x_2 + 2) + B^2 x_2^2) - 15m_s x_1 x_2 (x_1^4
\]
\[\quad + x_1^3 (2x_2 - 2) + x_1^2 (4x_2 - 5x_2 + 2) + x_1 x_2 (3x_2 - 5x_2 + 2) + B^2 x_2^2)
\]
\[\quad + 24m_s x_1^2 x_2 A^2 \} \}  
(A.11)
\[
\rho_{1,6}^{(1)\text{OPE}}(s) = \int_0^1 dx \frac{(\bar{s}s)^2 \left\{ g_s^2 m_c m_s + 18\pi^2 \left( 4m_c^2 - 3m_c m_s - 3m_s^2(x - 1)x \right) \right\}}{1152\pi^4} \\
+ \int_0^1 \int_0^{1-x_1} dx_1 dx_2 \frac{g_s^2(\bar{s}s)^2 x_1 x_2 A^2}{432\pi^4 (x_1^2 + x_1 B + Bx_2)^2} \left\{ m_c^2 (x_1^3 + x_2^3 (2x_2 - 1) \\
+ 2x_1 Bx_2 + Bx_2^2) - 3sx_1 x_2 A \right\}
\] (A.12)

\[
\rho_{1,7}^{(1)\text{OPE}}(s) = \int_0^1 dx - \frac{\langle \alpha_s G^2_x \rangle m_c (\bar{s}s)}{512\pi^2} + \int_0^1 \int_0^{1-x_1} dx_1 dx_2 \frac{\langle \alpha_s G^2_x \rangle m_c (\bar{s}s)}{256\pi^2 B (x_1^2 + x_1 B + Bx_2)^2} \\
\times \left\{ 2x_1^6 + 4x_1^5 B + x_1^4 (3x_2^2 - 5x_2 + 2) + 3x_1^3 B^2 x_2 + x_1^2 x_2^2 (3x_2^2 - 9x_2^2 + 8x_2 - 2) \\
+ x_1 (x_2 - 2) B^2 x_2^2 + 2B^2 x_2^2 \right\}
\] (A.13)

\[
\rho_{1,8}^{(1)\text{OPE}}(s) = 0
\] (A.14)

3. Spectral densities of molecular tensor current \((J_{\mu\nu}^{(1)})\)

\[
\rho_{2,0}^{(1)\text{OPE}}(s) = \int_0^1 \int_0^{1-x_1} dx_1 dx_2 \frac{1}{512\pi^6 A (x_1^2 + x_1 B + Bx_2)^8} \left\{ sx_1 x_2 A \\
- m_c^2 (x_1^3 + x_2^3 (2x_2 - 1) + 2x_1 Bx_2 + Bx_2^2))^2 (m_c^4 x_1 x_2 (x_1^3 + x_2^3 (2x_2 - 1) \\
+ 2x_1 Bx_2 + Bx_2^2)^2 + 6m_c^3 m_s (x_1 + x_2)^2 (x_1^2 + x_1 B + Bx_2)^3 \\
+ 2m_c^2 (x_1 + x_1 B + Bx_2) (18m_c^2 (x_2^2 + x_1 B + Bx_2)^3 - 5sx_1^2 x_2^2 \\
\times (x_1^2 + x_1 (2x_2 - 1) + Bx_2)) - 24m_c m_s x_1 x_2 (x_1^2 + x_1 B + Bx_2)^2 \\
\times (x_1^2 + x_1 (2x_2 - 1) + Bx_2) + 15s^2 x_1^2 x_2^2 A^2 \right\}
\] (A.15)

\[
\rho_{2,3}^{(1)\text{OPE}}(s) = \int_0^1 \int_0^{1-x_1} dx_1 dx_2 \frac{3sx_1 x_2 A^6}{32\pi^4 (x_1^2 + x_1 B + Bx_2)^6} \left\{ m_c^2 (x_1^3 + x_2^3 (2x_2 - 1) \\
+ 2x_1 Bx_2 + Bx_2^2)^3 + 2m_c^4 m_s (x_1^3 + x_1 B + Bx_2)^2 (2x_1^5 + x_1^4 (5x_2 - 4) \\
+ x_1^3 (7x_2^2 - 11x_2 + 2) + x_1^2 x_2 (7x_2^2 - 14x_2 + 6) + x_1 x_2^2 (5x_2^2 - 11x_2 + 6) \\
+ 2B^2 x_1^3 - m_c^4 A (x_1^3 + x_1^2 (2x_2 - 1) + 2x_1 Bx_2 + Bx_2^2)^2 \\
\times (m_s^2 (x_1^4 + x_1 B + Bx_2) + 4sx_1 x_2) - 4m_c^2 m_s x_1 x_2 (x_1^6 + x_1^6 (x_2 - 4) \\
- 3x_1^5 (x_2^2 + 2x_2 - 2) - 4x_1^4 (2x_2 - 3x_2 + 1) + x_1^3 (-8x_2^3 + 5x_2^2 + 12x_2^2 \\
- 10x_2 + 1) - 3sx_1 x_2 (x_2^2 - 4x_2^2 + 4x_2 - 1) + x_1 (x_2 - 3) B^3 x_2^2 + B^4 x_2^3) \\
+ m_s x_1 x_2 A^2 (x_1^3 + x_2^3 (2x_2 - 1) + 2x_1 Bx_2 + Bx_2^2) \\
\times (2m_c^2 (x_1^2 + x_1 B + Bx_2) + 3sx_1 x_2) - 12m_s s^2 x_1^3 x_2^3 A^3 \right\}
\] (A.16)
\( \rho_{2,4}^{(1) \text{OPE}}(s) = \int_0^1 \int_0^{1-z_1} dx_1 \, dx_2 \frac{\langle \alpha_s G^2 \rangle}{768 \pi^4 A (x_1^2 + x_1 B + B x_2)^6} \left\{ 2m_c^4 x_1 x_2 \left( x_1^2 + x_1^2 (2x_2 - 1) \right) + 2x_1 B x_2 + B x_2^2 \right\}^2 \left( x_1^2 + x_1 (2x_2 - 3) + x_2 (7x_2 - 3) \right) + 6m_s^2 m_s \left( x_1^2 + x_1 B + B x_2 \right)^2 \times \left( x_1^2 + x_1^2 (6x_2 - 2) + x_1^2 (11x_2 - 10) + 4x_1^2 x_2 \left( 4x_2^2 - 5x_2 + 1 \right) + x_1^2 x_2^2 \left( 11x_2^2 - 20x_2 + 8 \right) + 2x_1 x_2^3 \left( 3x_2^2 - 5x_2 + 2 \right) + 2B x_2^3 \right) \times \left( x_1^5 \left( 12x_2^4 - 25x_2^4 + 13x_2 - 1 \right) + x_1^4 x_2 \left( 14x_2^3 - 39x_2^3 + 33x_2 - 8 \right) + x_1^4 B x_2 \left( 12x_2^2 - 15x_2 + 2 \right) + x_1^3 x_2^3 \left( 7x_2^4 - 4 \right) + x_1^2 B x_2^2 \right) \right\} \}

(A.17)

\( \rho_{2,5}^{(1) \text{OPE}}(s) = \int_0^1 dx \frac{-m_c^5 m_s \langle \bar{s}s \rangle \left( 6m_c - m_s \right)}{64 \pi^4} + \int_0^1 \int_0^{1-z_1} dx_1 \, dx_2 \frac{m_c^5 \langle \bar{s}s \rangle A}{64 \pi^4 \left( x_1^2 + x_1 B + B x_2 \right)^5} \times \left\{ 3m_c^3 \left( x_1^2 + x_1^2 (2x_2 - 1) + 2x_1 B x_2 + B x_2^2 \right)^2 - 2m_c^2 m_s x_1 x_2 \right. \times \left( x_1^2 + x_1^2 (3x_2 - 2) + x_1^2 \left( 4x_2^2 - 5x_2 + 1 \right) + x_1^2 \left( 3x_2^2 - 5x_2 + 2 \right) + B x_2^2 \right) - 6m_s x_1 x_2 \left( x_1^2 + x_1^2 (3x_2 - 2) + x_1^2 \left( 4x_2^2 - 5x_2 + 1 \right) + x_1^2 \left( 3x_2^2 - 5x_2 + 2 \right) \right) + B x_2^2 \right\} + 8m_s x_1^2 x_2^2 A^2 \}

(A.18)

\( \rho_{2,6}^{(1) \text{OPE}}(s) = \int_0^1 dx \frac{\langle \bar{s}s \rangle^2 \left\{ 2g_s^5 m_c m_s + 27 \pi^2 \left( 8m_c^2 - 4m_c m_s - 9m_s^2 \left( x - 1 \right) \right) \right\}}{864 \pi^4} + \int_0^1 \int_0^{1-z_1} dx_1 \, dx_2 \frac{g_s^5 \langle \bar{s}s \rangle^2 x_1 x_2 A^2}{216 \pi^4 \left( x_1^2 + x_1 B + B x_2 \right)^5} \left\{ m_c^2 \left( x_1^3 + x_1^2 (2x_2 - 1) \right) + 2x_1 B x_2 + B x_2^2 \right\} - 4x_1 x_2 A \}

(A.19)

\( \rho_{2,7}^{(1) \text{OPE}}(s) = \int_0^1 dx \frac{\langle \alpha_s G^2 \rangle m_c \langle \bar{s}s \rangle}{192 \pi^2} \left( \begin{array}{l} \int_0^1 \int_0^{1-z_1} dx_1 \, dx_2 \frac{\langle \alpha_s G^2 \rangle m_c \langle \bar{s}s \rangle}{96 \pi^2 B \left( x_1^2 + x_1 B + B x_2 \right)^4} \times \left\{ m_c \left( 2x_1^2 + 4x_1^2 B + x_1^2 \left( -5x_1^2 - 3x_2 + 2 \right) - 13x_1^2 \left( x_1^2 + 3x_2 \right)^4 \left( x_1^2 + x_1 B + B x_2 \right)^3 \right) + 31x_1^2 - 24x_2 + 6 \right\} + x_1^2 B x_2 \left( 7x_2 - 6 \right) + 2B x_2^4 + m_s x_1 B x_2 \left( x_1 + x_2 \right) A^2 \right\} \}

(A.20)

\( \rho_{2,8}^{(1) \text{OPE}}(s) = 0 \)

(A.21)

4. Spectral densities of tetraquark scalar current \((J^{(2)})\)

\( \rho_{0,0}^{(2) \text{OPE}}(s) = \int_0^1 \int_0^{1-z_1} dx_1 \, dx_2 \frac{1}{256 \pi^6 A \left( x_1^2 + x_1 B + B x_2 \right)^8} \left\{ \langle s x_1 x_2 A \right\} - m_c^6 \left( x_1^2 + x_1^2 (2x_2 - 1) + 2x_1 B x_2 + B x_2^2 \right) \left\{ m_c^4 \left( x_1^2 - x_1 + B x_2 \right) \times \left( x_1^2 + x_1^2 (2x_2 - 1) + 2x_1 B x_2 + B x_2^2 \right)^2 - 4m_c^2 m_s \left( x_1 + x_2 \right)^2 \right\} \times \left( x_1^2 + x_1 B + B x_2 \right)^3 - 2m_c^2 \left( x_1^2 + x_1 B + B x_2 \right) \left\{ \left[ m_c^2 \left( x_1^2 + x_1 B + B x_2 \right)^3 \right] + s x_1 x_2 \left( 2x_1^4 + x_1^4 \left( 3x_2 - 2 \right) + x_1^2 \left( 2x_2^2 - 7x_2 + 2 \right) + x_1 x_2 \left( 3x_2^2 - 7x_2 + 4 \right) + 2B x_2^4 \right) \right\} + 10m_s m_s x_1 x_2 \left( x_1^4 + x_1 B + B x_2 \right)^2 \left( x_1^2 + x_1 \left( 2x_2 - 1 \right) + B x_2 \right) + s^2 x_1^2 x_2^2 A^2 \left( 3x_1^2 - x_1 \left( 4x_2 + 3 \right) + 3B x_2 \right) \}

(A.22)
\[ \rho_{0.3}^{(2)\text{OPE}}(s) = \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{1}{16\pi^4 (x_1^2 + x_1 B + B x_2)^6} \left(2 m_c^5 (x_1^3 + x_1^2 (2x_2 - 1)) + 2 x_1 B x_2 + B x_2^2 \right)^3 + 2 m_c^5 m_s (x_1^3 + x_1 B + B x_2)^7 (5 x_1^3 + x_1^4 (13 x_2 - 10)) + x_1^3 (18 x_2^2 - 28 x_2 + 5 + 3 x_1^4 x_2 (6 x_2^2 - 12 x_2 + 5) + x_1^1 (13 x_2^2 - 28 x_2 + 15) + 5 B^2 x_2^2) - 2 m_c^5 A (x_1^3 + x_1^2 (2x_2 - 1) + 2 x_1 B x_2 + B x_2^2)^3 (m_c^5 (x_1^3 + x_1 B + B x_2)) + 3 s x_1 x_2 (2 m_c^5 m_s x_1 x_2 (7 x_1^4 + x_1^6 (19 x_2 - 28) + 6 x_1^1 (4x_2^2 - 13 x_2 + 7) + 2 x_1^1 (11 x_2^3 - 57 x_2^2 + 60 x_2 - 14) + x_1^1 (22 x_2^4 - 124 x_2^3 + 177 x_2^2 - 82 x_2 + 7) + 3 x_1^2 B x_2^2 (8 x_2^2 - 22 x_2 + 7) + x_1^1 B^2 x_2^2 (19 x_2 - 21) + 7 B^4 x_2^2) + m_c^5 x_1 x_2^2 A^2 (x_1^3 + x_1^2 (2x_2 - 1) + 2 x_1 B x_2 + B x_2^2) (m_c^5 (x_1^3 + x_1 B + B x_2) + 4 s x_1 x_2) + 4 m_c^5 s x_1^2 x_2 A^3 (x_1^3 - x_1 (4x_2 + 1) + B x_2) \right) \] (A.23)

\[ \rho_{0.4}^{(2)\text{OPE}}(s) = \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{(\alpha_s G^2)}{1536\pi^4 B A (x_1^2 + x_1 B + B x_2)^6} \left(2 m_c^5 (x_1^3 + x_1^2 (2x_2 - 1)) + 2 x_1 B x_2 + B x_2^2 \right)^3 + 2 m_c^5 m_s (x_1^3 + x_1 B + B x_2)^7 (5 x_1^3 + x_1^4 (13 x_2 - 10)) + x_1^3 (18 x_2^2 - 28 x_2 + 5 + 3 x_1^4 x_2 (6 x_2^2 - 12 x_2 + 5) + x_1^1 (13 x_2^2 - 28 x_2 + 15) + 5 B^2 x_2^2) - 2 m_c^5 A (x_1^3 + x_1^2 (2x_2 - 1) + 2 x_1 B x_2 + B x_2^2)^3 (m_c^5 (x_1^3 + x_1 B + B x_2)) + 3 s x_1 x_2 (2 m_c^5 m_s x_1 x_2 (7 x_1^4 + x_1^6 (19 x_2 - 28) + 6 x_1^1 (4x_2^2 - 13 x_2 + 7) + 2 x_1^1 (11 x_2^3 - 57 x_2^2 + 60 x_2 - 14) + x_1^1 (22 x_2^4 - 124 x_2^3 + 177 x_2^2 - 82 x_2 + 7) + 3 x_1^2 B x_2^2 (8 x_2^2 - 22 x_2 + 7) + x_1^1 B^2 x_2^2 (19 x_2 - 21) + 7 B^4 x_2^2) + m_c^5 x_1 x_2^2 A^2 (x_1^3 + x_1^2 (2x_2 - 1) + 2 x_1 B x_2 + B x_2^2) (m_c^5 (x_1^3 + x_1 B + B x_2) + 4 s x_1 x_2) + 4 m_c^5 s x_1^2 x_2 A^3 (x_1^3 - x_1 (4x_2 + 1) + B x_2) \right) \] (A.24)

\[ \rho_{0.5}^{(2)\text{OPE}}(s) = \int_0^1 dx \frac{m_c^2 m_s \langle \bar{ss} \rangle \left(-8 m_c^6 + m_c m_s - 3s(x-1)x \right)}{48\pi^4} + \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{m_c^2 \langle \bar{ss} \rangle A}{96\pi^4 (x_1^2 + x_1 B + B x_2)^6} \left(6 m_c^3 (x_1^3 + x_1^2 (2x_2 - 1)) + 2 x_1 B x_2 + B x_2^2 \right)^3 - 8 m_c^2 m_s x_1 x_2 \left(x_1^3 + x_1^2 (3x_2 - 2) + x_1^2 (4x_2^2 - 5x_2 + 1) + x_1 x_2 (3x_2 - 5x_2 + 2) + B^2 x_2^2 \right) - 9 m_c m_s x_1 x_2 \left(x_1^3 + x_1^2 (3x_2 - 2) + x_1^2 (4x_2^2 - 5x_2 + 2) + B^2 x_2^2 \right) + 18 m_c m_s x_1^2 x_2 A^2 \right] \] (A.25)

\[ \rho_{0.6}^{(2)\text{OPE}}(s) = \int_0^1 dx \frac{\langle \bar{ss} \rangle A^2}{324\pi^4} \left(g_s^2 (2 m_c^2 - m_c m_s + 3s(x-1)x) - 54 m_c^2 (2 m_c^2 - m_c m_s) - 3 m_c^2 (x-1)x \right) + \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{g_s^2 \langle \bar{ss} \rangle x_1 x_2 A^2}{324\pi^4 (x_1^2 + x_1 B + B x_2)^6} \left(4 m_c^5 (x_1^3 + x_1^2 (2x_2 - 1) + 2 x_1 B x_2 + B x_2^2) - 9 s x_1 x_2 A \right) \] (A.26)
\[
\rho_{0,7}^{(2)\text{OPE}}(s) = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{(\alpha_s \frac{G_F^2}{\pi})(\bar{s}s)(m_c - 3m_s)}{144\pi^2} + \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{(\alpha_s \frac{G_F^2}{\pi})m_c(\bar{s}s)}{288\pi^2 B(x_1^2 + x_1 B + Bx_2)^4} \\
\times \left\{ m_c \left( 8x_1^6 + 19x_1^3 B - 2x_1^3 (x_1^2 + 6x_2 - 7) - x_1^3 (19x_2^3 - 38x_2^3 + 16x_2 + 3) \right) + x_1^2 x_2 (19x_3^3 + 43x_2^3 - 33x_2 + 9) - x_1 B^2 x_2^2 (10x_2 - 9) + B^2 x_2^3 (11x_2 - 3) \right\} \\
- 3m_s x_1 B x_2 A^2 (x_1 + x_2) \right\} \\
(A.27)
\]

\[
\rho_{0,8}^{(2)\text{OPE}}(s) = 0 \\
(A.28)
\]

5. Spectral densities of tetraquark axialvector current \( (J_\mu^{(2)}) \)
\[ \rho_{1,4}^{(2)\text{OPE}}(s) = \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{-\langle \alpha_s G^2 \rangle}{18432\pi^4 A^2 (x_1^2 + x_1 B + Bx_2)^6} \]
\[ \times \left\{ m_c^2 (x_1^2 + x_1 B + Bx_2)^2 (7287 + x_1^2 (481x_2 - 252) + x_1^3 (1277x_2^2) \right. \]
\[ - 1420x_2 + 324) + x_1^3 (393x_2^4 - 3274x_2^3 + 1533x_2^2 - 180) \]
\[ + x_1^3 (393x_2^4 - 4212x_2^3 + 2961x_2^2 - 702x_2 + 36) + x_1^5 x_2 (1277x_2^4 - 3274x_2^3 + 2961x_2^2 - 1044x_2 + 108) + x_1 x_2^3 (481x_2^4 - 1420x_2^3 + 1533x_2^2 - 702x_2 + 108) + 36B^3x_2^3(2x_2 - 1) + 9m_c^2 m_s (x_1^2 + x_1 B + Bx_2)^2 (8x_1^2 + x_1^3 (13x_2 - 16) \]
\[ - 2x_1^3 (4x_2^2 + 7x_2 - 4) + x_1^4 x_2 (25x_2^3 + 29x_2 - 2) \]
\[ + x_1 x_2^2 (25x_2^3 + 46x_2^2 - 29x_2 + 4) + x_1^2 x_2^2 (8x_2^3 + 29x_2^2 - 29x_2 + 8) \]
\[ + x_1 x_2^3 (13x_2^3 - 14x_2^2 - 3x_2 + 4) + 8B^2 x_2^2) + m_c^2 (x_1^3 + 2x_1 B + x_2 (2x_2 - 3x_2 + 1) + B^2 x_2) \]
\[ \times (16x_1^2 - x_1^3 (3x_2 + 16) + 3x_1^2 (1 - x_2) x_2 - x_1 Bx_2^2 + 16Bx_2^3) \]
\[ - sx_1 x_2 (72x_1^6 + x_1^5 (431x_2 - 252) + 4x_1^4 (201x_2^2 - 99x_2 + 81) + x_1^3 (895x_2^2 + 2014x_2^3 + 1239x_2 - 180) + 2x_1^2 (402x_2^4 - 1007x_2^3 + 894x_2^2 - 273x_2 + 18) \]
\[ + x_1 x_2 (431x_2^5 - 1196x_2^4 + 1239x_2^3 - 546x_2 + 72) + 36B^2 x_2^3 (2x_2 - 1)) \]
\[ - 15m_c m_s x_1 x_2 A^2 (8x_1^2 - x_1^6 (3x_2 + 16) + x_1^5 (-17x_2^2 + 10x_2 + 8) \]
\[ + x_1 x_2^2 (39x_2^3 + 42x_2 - 11) + x_1 x_2^2 (39x_2^3 + 68x_2 - 32x_2 + 4) \]
\[ - x_1^2 x_2 (17x_2 - 8) - x_1 Bx_2^3 (3x_2 - 4) + 8B^2 x_2^5 \]
\[ + 6s x_1^2 x_2 A^2 (5m_c^2 (x_1^2 + x_1 B + Bx_2)^2 + 11s x_1 x_2 (x_1^2 + x_1 (2x_2 - 1) + Bx_2) ) \]  
(A.31)

\[ \rho_{1,5}^{(2)\text{OPE}}(s) = \int_0^1 dx \frac{m_0^2 (ss)}{128\pi^4} \left\{ -m_c m_s (4m_c - m_s) \right\} \]
\[ + \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{m_0^2 (ss) A}{384\pi^4 (x_1^2 + x_1 B + Bx_2)^5} \left\{ 9m_c^3 (x_1^3 + x_1^2 (2x_2 - 1) \right. \]
\[ + 2x_1 Bx_2 + Bx_2^2 \right)^2 - 8m_c^2 m_s x_1 x_2 (x_1^4 + x_1^3 (3x_2 - 2) + x_1^2 (4x_2^2 - 5x_2 + 1) \]
\[ + x_1 x_2 (3x_2^2 - 5x_2 + 2) + Bx_2^2 \right) - 15m_c m_s x_1 x_2 (x_1^4 + x_1^3 (3x_2 - 2) \]
\[ + x_1^2 (4x_2^2 - 5x_2 + 1) + x_1 x_2 (3x_2^2 - 5x_2 + 2) + Bx_2^2 \]
\[ + 24m_c m_s x_1^2 x_2 A^2 \} \]  
(A.32)

\[ \rho_{1,6}^{(2)\text{OPE}}(s) = \int_0^1 dx \frac{(ss)^2}{864\pi^4} \left\{ g_2^5 m_c m_s + 18\pi^2 (4m_c^2 - 3m_c m_s - 3m_s^2 (x - x_1)) \right. \]
\[ + \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{-g_2^2 (ss)^2 x_1 x_2 A^2}{324\pi^4 (x_1^2 + x_1 B + Bx_2)^6} \left\{ m_c^2 (x_1^3 + x_1^2 (2x_2 - 1) \right. \]
\[ + 2x_1 Bx_2 + Bx_2^2 \right) - 3sx_1 x_2 A \} \]  
(A.33)

\[ \rho_{1,7}^{(2)\text{OPE}}(s) = \int_0^1 dx \frac{-\langle \alpha_s G^2 \rangle (ss) m_c}{384\pi^2} + \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{\langle \alpha_s G^2 \rangle (ss)}{1536\pi^2} \]
\[ \times \frac{1}{B (x_1^2 + x_1 B + Bx_2)^4} \left\{ m_c (-16x_1^6 - 32x_1^5 B + x_1^4 (-11x_2^2 + 27x_2 - 16) \right. \]
\[ + 2x_1 Bx_2 + x_1^2 x_2 (2x_2^2 + 9x_2 - 16x_2 + 5) + 5x_1 B^2 x_2^2 (x_2 + 1) \]
\[ - 16B^2 x_2^4 + 4m_s x_1 Bx_2 (x_1^2 + 2x_1^2 B + x_1 (2x_2^2 - 3x_2 + 1) + B^2 x_2) \} \]  
(A.34)
\[\rho_{1,8}^{(2)\text{OPE}}(s) = 0 \quad (A.35)\]

6. Spectral densities of tetraquark tensor current \((J_{\mu\nu}^{(2)})\)

\[
\rho_{2,0}^{(2)\text{OPE}}(s) = \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \left( s x_1 x_2 A - m_c^2 (x_1^3 + x_2^3 + 2(x_1 - x_2)) + 2 x_1 B x_2 + B x_2^2 \right)^2 \frac{38\pi^6 A (x_1^2 + x_1 B + B x_2)^8}{38\pi^6 A (x_1^2 + x_1 B + B x_2)^8} \times \left\{ \left( x_1^3 + x_2^3 \right) (2x_1 - 1) + 2 x_1 B x_2 + B x_2^2 \right\}^2 + 6 m_c^2 m_s (x_1 + x_2)^2 \\
\times (x_1^2 + x_1 B + B x_2)^3 + 2 m_c^2 (x_1^3 + x_1 B + B x_2) \left( 18 m_c^2 (x_1^2 + x_1 B + B x_2)^3 \right) - 5 s x_1 x_2 (x_1^2 + x_2^2 + B x_2)) - 24 m_c^2 s x_1 x_2 (x_1^2 + x_1 B + B x_2)^2 \\
\times (x_1^2 + x_1 (2x_2 - 1) + B x_2) + 15 s^2 x_1^3 x_2^2 A^2 \right\} \quad (A.36)
\]

\[
\rho_{2,3}^{(2)\text{OPE}}(s) = \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{\langle \bar{s}s \rangle}{8\pi^4 (x_1^2 + x_1 B + B x_2)^6} \left\{ m_c^2 (x_1^3 + x_2 (2x_2 - 1)) + 2 x_1 B x_2 + B x_2^2 \right\}^2 \\
\times 4 m_c^2 s x_1 x_2 A (x_1^2 + x_2 (2x_2 - 1) + 2 x_1 B x_2 + B x_2^2)^2 - 12 m_c^2 m_s s x_1 x_2 A^2 \\
\times (x_1^3 + x_2 (2x_2 - 1) + 2 x_1 B x_2 + B x_2^2)^3 - 3 m_c^2 s x_1^2 x_2^2 A^2 (x_1^3 + x_2 (2x_2 - 1)) \\
\times 2 x_1 B x_2 + B x_2^2 + 12 m_s^2 s x_1^3 x_2^2 A^3 \right\} \quad (A.37)
\]

\[
\rho_{2,4}^{(2)\text{OPE}}(s) = \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{\langle \alpha_s G_F^2 \rangle}{460\pi^4 B A (x_1^2 + x_1 B + B x_2)^6} \left\{ m_c^4 x_1 B x_2 (52 x_2^3 \right) \\
\times (66 - 7 x_2) + 52 x_2^2 + 66 x_2 - 54 (x_1^3 + x_2 (2x_2 - 1) + 2 x_1 B x_2 + B x_2^2)^2 \\
\times 12 m_c^3 m_s (x_1^2 + x_1 B + B x_2)^2 (2 x_1^3 + 2 x_1 (13 x_2 - 9) + x_2^3 (37 x_2^2 - 49 x_2 + 12) \\
\times x_1^2 (35 x_2^3 - 6 x_2^2 + 35 x_2 - 2) + x_1 x_2 (40 x_2^3 - 91 x_2^2 + 63 x_2 - 12) \\
\times x_1^2 x_2^2 (27 x_2^2 - 78 x_2^2 + 71 x_2 - 20) + 3 x_1 B x_2^3 (7 x_2 - 4) + 2 B x_2^2 (5 x_2 - 1)) \\
\times 4 m_c^2 (x_1^2 + x_1 B + B x_2)^2 (8 x_2^3 + 16 x_2^2) B \\
\times x_1^3 (7 x_2^2 - 15 x_2 + 8) - x_1 B^2 x_2 - x_1 B^2 x_2^2 + 8 B x_2^3 (7 x_2^4 \\
\times 5 x_1^3 (x_2 - 1) + x_2^3 (-4 x_2^2 + 38 x_2 - 24) + x_1 (5 x_2^2 + 38 x_2^2 - 48 x_2 + 12) \\
\times x_2 (7 x_2^2 + 5 x_2 - 24 x_2 + 12)) - 18 m_c m_s s x_1 x_2 (8 x_2^3 + 34 x_2^2) B \\
\times 7 x_1^2 (9 x_2^2 - 17 x_2 + 8) + x_1^6 (78 x_2^3 - 199 x_2^2 + 165 x_2 - 44) + x_1^5 (76 x_2^4 \\
\times - 249 x_2^3 + 278 x_2^2 - 121 x_2 + 16) + x_1^4 B^2 (62 x_2^3 - 131 x_2^2 + 49 x_2 - 2) \\
\times x_1^3 B^2 x_2 (52 x_2^3 - 73 x_2 + 12) + x_1^2 B^2 x_2^2 (38 x_2^3 - 57 x_2 + 20) \\
\times x_1 B^3 x_2^2 (23 x_2^3 - 27 x_2 + 12) + 2 B^2 x_2^2 (5 x_2 - 1)) + s^3 x_1 B x_2^3 A^2 (18 x_2^4 \\
\times 11 x_1 (x_2 + 12) + 6 (3 x_2^2 + 22 x_2 - 25)) \right\} \quad (A.38)
\]

\[
\rho_{2,5}^{(2)\text{OPE}}(s) = \int_0^1 dx \frac{m_c^2 \langle \bar{s}s \rangle m_c m_s (6 m_c - m_s)}{48\pi^4} + \int_0^1 \int_0^{1-x_1} dx_1 \, dx_2 \frac{m_c^2 \langle \bar{s}s \rangle A}{48\pi^4 (x_1^2 + x_1 B + B x_2)^5} \times \left\{ 3 m_c^3 (x_1^2 + x_2^2) (2x_2 - 1) + 2 x_1 B x_2 + B x_2^2 \right\}^2 - 2 m_c^2 m_s x_1 x_2 (x_1^2 + x_2^2) (3 x_2 - 2) \\
\times x_1^3 (4 x_2^2 - 5 x_2 + 1) + x_1 x_2 (3 x_2^2 - 5 x_2 + 2) + B^2 x_2^2 - 6 m_c x_1 x_2 (x_1^2 + x_2^2) (3 x_2 - 2) \\
\times x_1^2 (4 x_2^2 - 5 x_2 + 1) + x_1 x_2 (3 x_2^2 - 5 x_2 + 2) + B^2 x_2^2 + 8 m_s x_1^2 x_2^2 A^2 \right\} \quad (A.39)
\]
\[
\rho_{2,6}^{(2)\text{OPE}}(s) = \int_0^1 dx \frac{-\langle ss \rangle^2 (2g_s^2 m_c m_s + 27\alpha_s^2 (8m_c^2 - 4m_c m_s - 9m_s^2(x-1)x))}{64\pi^4} \\
+ \int_0^1 \int_0^{1-x_1} dx_1 dx_2 \frac{g_s^2 \langle ss \rangle x_1 x_2 A^2}{162\pi^4 (x_1^2 + x_1 B + B x_2)^3} \left\{ m_c^2 (x_1^2 + x_1^2 (2x_2 - 1)) \\
+ 2x_1 B x_2 + B x_2^2 \right\} - 4s x_1 x_2 A \}
\]

\[
\rho_{2,7}^{(2)\text{OPE}}(s) = \int_0^1 dx \frac{\langle \alpha_s G_s^2 \rangle \langle ss \rangle m_c}{144\pi^2} + \int_0^1 \int_0^{1-x_1} dx_1 dx_2 \frac{\langle \alpha_s G_s^2 \rangle \langle ss \rangle}{9216\pi^2 B (x_1^2 + x_1 B + B x_2)^4} \\
\times \left\{ 2m_c (-128x_1^6 + 240x_1^5 B + x_1^4 (-153x_2^2 + 57x_2 + 96) - x_1^3 B^2 (473x_2 - 16) \\
- 16x_1^2 x_2 (29x_2^2 - 67x_2^2 + 51x_2 - 13) - x_1 B x_2^2 (263x_2 - 199) \\
+ B^2 x_2^2 (121x_2 + 7) + m_c x_1 B x_2 (80x_1^3 + 5x_2^2 (49x_2 - 32) \\
+ 4x_1 (59x_2^2 - 79x_2 + 20) + 71B^2 x_2^2) \right\}
\]

\[
\rho_{2,8}^{(2)\text{OPE}}(s) = 0
\]

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