QUANTUM COSMOLOGY AND THE EMERGENCE OF A CLASSICAL WORLD

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La pendule s’est arrêtée
Personne ne se bouge ...
Comme sur les images
Il n’y aura plus de nuit
Pierre Reverdy, from La Réalité Immobile

1 Why quantum cosmology?

Quantum cosmology is the application of quantum theory to the Universe as a whole. At first glance such an attempt seems surprising since one is used to apply quantum theory to microscopic systems. Why, then, does one wish to extrapolate it to the whole Universe? This extrapolation is based on the assumption that quantum theory is universally valid, in particular that there is no a priori classical world. The main motivation for this assumption comes from the kinematical non-locality (or non-separability) of quantum theory, i.e. from the fact that one cannot in general assign a wave function to a given system since it is not isolated but coupled to its natural environment, which is again coupled to another environment, and so forth. The extrapolation of this quantum entanglement thus leads inevitably to the concept of a wave function for the Universe. Many experiments, notably those which contradict Bell’s inequalities, have impressively demonstrated this fundamental non-separability of quantum theory. As we will see in section 3, it is this entanglement between many degrees of freedom which is
also responsible for the emergence of a classical world. The importance of this effect seems to have been overlooked in the traditional discussion of quantum theory, and led to the belief in an independently existing classical world.

Since the dominating interaction in the cosmological realm is gravity, this extrapolation immediately has to address the problem of quantizing the gravitational field. In fact, since all other known interactions are successfully described by quantum field theories it seems unavoidable to quantize gravity, too, since the gravitational field is coupled to all other fields and it would appear strange, and probably even inconsistent, to have a drastically different framework for this one field. Many technical and conceptual difficulties have, however, as yet prevented the construction of a consistent and predictive theory of quantum gravity, some of which we will briefly describe in the course of this article.

Quantum cosmology is only meaningful as a physical theory if one can agree on how to interpret a universal quantum theory. The traditional “Copenhagen interpretation,” for example, strictly denies the idea of a fundamental quantum world and assumes from the outset the existence of a classical world whose concepts have to be used to interpret the results of quantum measurements. Such an attitude appears to be ad hoc even in ordinary quantum mechanics since measurement apparatus are built from atoms which are known to obey the laws of quantum theory. Although not being consistent, such a hybrid description of Nature has been sanctioned by using purely verbal constructs like “complementarity.” The conceptual inconsistency of the “Copenhagen interpretation” becomes even more evident in the framework of quantum cosmology, where the whole Universe including all observers and apparatus has to be described in quantum terms from the very beginning. To quote Gell-Mann and Hartle (1990): “Quantum mechanics is best and most fundamentally understood in the framework of quantum cosmology.”

But since it is impossible to prepare an ensemble of universes, can there be any fundamental meaning of a wave function of the Universe? The situation can be compared to ordinary quantum field theory where the concept of a vacuum state vector is used to derive properties of elementary particles but not to perform experiments with an ensemble of vacua. One might expect that in an analogous sense a wave function of the Universe, \( \Psi \), determines structures of the Universe which can be checked by observations. One example is outlined in the last section of this article and is concerned with the direction of time. Another example is the claim (supported by heuristic
considerations only) that \( \Psi \) is peaked around a vanishing cosmological constant (Baum, 1983) leading to the prediction that the cosmological constant is zero. Since such a wave function of the Universe is independent of any observation, one must attribute to it the status of reality.

As mentioned above, any attempt to understand quantum cosmology must focus on the construction of a quantum theory of gravity. One can distinguish several levels of the relationship between quantum theory and gravity which we briefly wish to describe. The most fundamental level should be described by a quantum theory of all interactions including gravity. A promising candidate in recent years has been the theory of superstrings which is constructed by using the assumption that the fundamental entities are one-dimensional objects instead of local quantum fields. Since many mathematical and conceptual problems in this framework have not yet been solved, no final agreement on the physical status of this “theory of everything” has been reached so far. A less ambitious level consists in the application of formal quantization rules to the general theory of relativity. Here one does not attempt to provide a unified theory of all interactions but tries to focus on the specific quantum aspects of the gravitational field. The scale where such aspects are expected to become relevant is found by setting the Compton wavelength associated with a mass equal to its Schwarzschild radius, and is called the Planck length:

\[
l_{Pl} = \sqrt{\frac{\hbar G}{c^3}} \approx 10^{-33}\text{cm}.
\]

One would expect that the level of quantum general relativity can be recovered from superstring theory in an appropriate low-energy limit. It might, however, also be the case that quantum general relativity is the fundamental theory, and impressive formal developments have taken place recently which support this possibility. We will comment on this below. In either case it is justified to study the implications of quantum general relativity which is the very conservative framework of tying together quantum theory and general relativity. Both theories have up to now passed all tests in their respective realms.

On the next level we find the framework of quantum field theory for non-gravitational fields on a fixed classical background spacetime. How this level can be recovered from the more fundamental level of quantum general relativity and in particular how the Schrödinger equation for non-gravitational fields can be derived from quantum gravity, is the subject of
the third section. On this level one still has no experimental test but at least
a definite prediction – the Hawking effect: Black holes are not really black
if quantum effects are taken into account, but radiate with a temperature

\[ T = \frac{\hbar c^3}{8\pi G k_B M} = \frac{\hbar k}{2\pi c k_B} \approx 10^{-6} \text{Kelvin} \times \frac{M_{\odot}}{M}, \tag{2} \]

where \( M \) is the mass of the black hole, \( k_B \) Boltzmann’s constant, and \( \kappa \) the
surface gravity of the hole. It is remarkable that all fundamental constants
of nature appear in this formula. The significance of the relation (2) for
a quantum theory of gravity might well turn out to be analogous to the
significance of de Broglie’s relation \( p = \hbar k \) for the development of quantum
mechanics (Zeh, 1992).

The experimental level of the relation between quantum theory and grav-
ity is only reached at the very modest level of the Schrödinger equation with
a Newtonian potential where impressive experiments using neutron interfer-
ometry have been performed.

What are the main obstacles in constructing a quantum theory of grav-
ity? An important formal problem is the non-renormalizability of a pertur-
bative expansion of quantum general relativity around a given background
spacetime, i.e. the fact that an infinite number of parameters would have
to be determined by experiment to absorb all the infinities of the theory. It
is therefore not surprising that recent developments mainly focus on non-
perturbative approaches. A serious obstacle for experiments in this field is
the extreme smallness of the Planck length (1). One would have to extrap-
olate current particle accelerators to the dimensions of the Milky Way to be
able to probe such a scale – a hopeless enterprise. It is thus hoped that a
consistent theory of quantum gravity will reveal what the observable effects
are. Such effects might well be found in a direction not imagined hitherto.

One recent investigation, for example, claims that effects coming from the
wave function of the Universe have already been observed by the COBE
satellite (Salopek, 1993). Independent of the lack of experiments, there re-
main many conceptual problems which have to be addressed if gravity has to
be quantized. Most of these problems are connected in one way or the other
with the concept of time. We will discuss this issue in the next section but
will first briefly outline the formal framework in which most of the recent
investigations are made – the framework of canonically quantizing general
relativity.

What does it mean to “quantize” a classical theory? The prominent role
in quantum mechanics is played by the position and momentum operators
(the “\(p\)’s and the \(q\)’s”) of a given system. The central property is their non-commutativity which gives rise to the famous uncertainty relations. In the Schrödinger formulation of quantum mechanics one uses wave functions which are defined on configuration space, i.e. the space of all position coordinates (or, alternatively, on momentum space). “Canonically quantizing” a classical theory now means to identify the “\(p\)’s and the \(q\)’s” of the theory under consideration and imposing on them non-trivial commutation relations. In the case of the general theory of relativity this involves some preparatory steps since the \(p\)’s can only be defined after an appropriate time parameter can be distinguished. Since the theory treats all space- and time-coordinates on an equal footing one has to formally rewrite the theory in such a manner that time-coordinates appear. This does, of course, not alter the physical content of the theory which is invariant under general coordinate transformations.

Basically, the steps in canonically quantizing general relativity are the following: The first step consists in foliating spacetime into a family of spacelike three-dimensional hypersurfaces – one decomposes spacetime into space and time. The metric on these hypersurfaces, \(h_{ab}(x)\), will play the role of the canonical variable (the \(q\)). All quantities are then decomposed into variables which live on such a hypersurface and variables which point in the fourth, timelike, dimension. It then turns out that the canonical momentum, \(\pi^{ab}(x)\), is essentially given by the extrinsic curvature of a three-dimensional hypersurface, i.e. the quantity which describes the embedding of space into spacetime. Loosely speaking, one can say that intrinsic and extrinsic geometry are canonically conjugate to each other.

A major feature of general relativity is its invariance under arbitrary coordinate transformations. As a consequence one finds that there exist – at each space point – four constraints. Three of them generate coordinate transformations on the three-dimensional space and are analogous to Gauss’ law in electrodynamics. The fourth, so-called *Hamiltonian constraint*, plays a double role: Although being a constraint, it generates the dynamics. Its explicit form is

\[
\mathcal{H} = \frac{16\pi G}{c^2} G_{abcd} \pi^{ab} \pi^{cd} - \frac{c^4}{16\pi G} \sqrt{\mathcal{h}} R + \mathcal{H}_m = 0,
\]

where \(\sqrt{\mathcal{h}}\) is the square root of the determinant of the three-metric, \(R\) is the curvature scalar on three-space, and \(\mathcal{H}_m\) is the Hamiltonian density for non-gravitational fields. The coefficients \(G_{abcd}\) depend explicitly on the metric and play itself the role of a metric in the space of all metrics. The existence
of the constraint (3) is directly connected to the invariance of the theory under reparametrizations of time. It is therefore not surprising that it is quadratic in the momenta (the same happens, for example, in the case of the relativistic particle – invariance under reparametrizations of the world line parameter leads to the constraint \( p^2 + m^2 = 0 \)). The presence of the invariance under coordinate transformations and its associated constraints lies at the heart of the quantization problem. To quote Pauli (1955):

\[ \text{Es scheint mir . . . , daß nicht so sehr die Linearität oder Nicht-linearität Kern der Sache ist, sondern eben der Umstand, daß hier eine allgemeinere Gruppe als die Lorentzgruppe vorhanden ist ...[3]} \]

Quantization now proceeds, at least formally, by elevating the metric and its momentum to the status of operators and imposing the commutation relations

\[ [h_{ab}(x), \pi^{cd}(y)] = i\hbar \delta^c_{\left(a \delta^d_{\left(b \delta h_{ab}\right)}\right)} \delta(x, y) \] (4)

in full analogy to the commutation relation \([q, p] = i\hbar\) in quantum mechanics. One specific realization of (4) is provided by the substitution (in analogy to substituting \( p \to \hbar \frac{d}{dq} \))

\[ \pi^{ab} \to \hbar \frac{\delta}{i \delta h_{ab}}. \] (5)

The classical constraint (3) is then formally implemented in the quantum theory by inserting (4) and (5) into (3) and applying it on wave functionals \( \Psi \) which depend – apart from non-gravitational fields denoted by \( \phi \) – on the three-metric, i.e.

\[ \mathcal{H}\Psi[h_{ab}(x), \phi(x)] = \left(-\frac{16\pi\hbar^2 G}{c^2} G_{abcd} \frac{\delta^2}{\delta h_{ab} \delta h_{cd}} - \frac{c^4}{16\pi G} \sqrt{\hbar R + \mathcal{H}_m} \right) \Psi = 0. \] (6)

This equation is called the Wheeler-DeWitt equation in honour of the pioneering work of DeWitt (1967) and Wheeler (1968). It has the form of a zero-energy Schrödinger equation. There are of course many technical problems such as factor ordering or regularization which we will not be able to address in this article (see, e.g., Kuchař, 1992).
The quantization of the remaining three constraints leads to the condition that this wave functional actually does not change under a coordinate transformation of the three-metric but is a function of the geometry only. The configuration space is thus the space of all three-geometries and is called superspace. An important physical consequence of the commutation rules (4) is the “uncertainty” between the three-dimensional space and its embedding in the fourth dimension: The concept of spacetime is a classical concept only with no fundamental meaning in quantum theory. This is fully analogous to the concept of a particle trajectory which has no fundamental meaning in quantum mechanics due to the uncertainty between position and momentum. When applied to cosmology, the wave functional $\Psi$ in (6) is the desired “wave function of the Universe” which has been mentioned at the beginning of this section.

An important development has been the discovery of appropriate variables which enables one to find exact formal solutions of Eq. (6) in the absence of matter (Ashtekar, 1991). This became possible since the complicated potential term of (6) disappears when the equation is written in terms of the new variables, which have a strong similarity to Yang-Mills variables. The solutions can be classified in terms of loops and knots and exhibit an interesting structure of space, of which may be the most important is the existence of a minimal length of the order of the Planck length (1). Consequently, smaller scales do not have any operational meaning.

We have not yet addressed the issue of boundary conditions to be imposed on the Wheeler-DeWitt equation. In contrast to systems in the laboratory, they are not at our disposal. In fact, the question of boundary conditions has been of great interest in recent years, basically because of the no boundary proposal by Hartle and Hawking (1983). These two authors express the wave functional as a formal path integral where the sum is over euclidean (instead of lorentzian) geometries. The no boundary proposal then consists in the condition that one performs a sum over compact manifolds with one boundary only – the boundary which is given by the considered universe. The lack of a second boundary (for example at a small size of the universe) saves one from the need to find appropriate boundary conditions there. Unfortunately, these path integrals can only be evaluated in a semiclassical approximation and, moreover, do not lead to a unique wave function. We will return to the question of boundary conditions in the last section in connection with a discussion of the arrow of time.

Since the full equation (6) is in general difficult to handle, most applications have been performed in a restricted framework where only finitely
many degrees of freedom are quantized. A typical example in the cosmological context is the quantization of the scale factor $a$ of a Friedmann Universe. If in addition a (conformally coupled) scalar field $\phi$ is taken into account to simulate the matter content of the Universe we find instead of (6) the much simpler equation

$$H\psi(a,\phi) = \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \phi^2} - a^2 + \phi^2\right)\psi(a,\phi) = 0. \quad (7)$$

This equation has the form of an indefinite harmonic oscillator and can serve as a useful tool in studying conceptual questions in quantum cosmology (see the following sections).

The canonical quantization scheme outlined in this section still contains the structure of a differentiable three-dimensional manifold. There exist more ambitious approaches to quantum gravity which give up even this structure and start with a quantization of topology, see e.g. Isham, Kubyshin and Renteln (1990) who construct wave functions which live on a family of topologies. We will not, however, follow these approaches any further.

2 Time in quantum gravity

In the preface to Max Jammer’s book on the Problem of Space, Einstein writes

"Es hat schweren Ringens bedurft, um zu dem für die theoretische Entwicklung unentbehrlichen Begriff des selbständigen und absoluten Raumes zu gelangen. Und es hat nicht geringerer Anstrengung bedurfte, um diesen Begriff nachträglich wieder zu überwinden – ein Prozeß, der wahrscheinlich noch keineswegs beendet ist."

Although Einstein refers in this quotation to space only, the same can be said about time. Absolute time, as well as absolute space, was an indispenable ingredient in Newton’s theory of motion and more powerful in the development of dynamics than, for example, the notion of relative time which was put forward by Leibniz. Time kept its absolute status even in

\footnote{It was a hard struggle to gain the concept of independent and absolute space which is indispensable for the theoretical development. And it has not been a smaller effort to overcome this concept later on, a process which probably has not yet come to an end.}
non-relativistic quantum mechanics – the $t$ in Schrödinger’s equation is still an external parameter and not turned into a quantum operator.

Although the causal structure of spacetime has drastically changed with the advent of the special theory of relativity by dropping the notion of absolute simultaneity, spacetime is there still understood as a non-dynamical entity which provides an arena for the laws of physics but does not take part in the play. There is hence still no quantum operator for time in relativistic quantum field theory, although one now has the possibility to choose any spacelike hypersurface as a time parameter. These spacelike hypersurfaces can be deformed independently at each space point, which gives rise to a local or “many-fingered” time $\tau(x)$ instead of one single $t$. This local time appears also in the field theoretic Schrödinger equation, which is an equation for wave functionals depending on fields $\phi(x)$ (which play the role of the “q’s” in quantum mechanics). This equation reads

$$i\hbar \frac{\delta \psi[\phi(x)]}{\delta \tau(x)} = \mathcal{H}_m \psi[\phi(x)],$$

where $\mathcal{H}_m$ is the Hamiltonian density connected with $\phi(x)$.

Spacetime becomes a dynamical object, in analogy to a particle trajectory, only in the general theory of relativity where the spacetime metric describes the gravitational field, which is subject to Einstein’s field equations. The quantization of gravity therefore unavoidably has to address the quantization of time. Why would one expect any problems to occur in this step? One has to recall that the presence of an external time is an essential ingredient of quantum mechanics – matrix elements are calculated at a given time, and measurements are performed at a given time. Moreover, probabilities are preserved in time. The absence of any time parameter is a fundamental property of the Wheeler-DeWitt equation (6). This is not surprising since, as we have argued above, the concept of spacetime has no fundamental meaning in quantum gravity. The question therefore arises: Can one introduce a viable concept of time on the fundamental level of quantum gravity itself or only in a semiclassical approximation? A critical investigation into the concept of time may lead to fruitful insights into the structure of the desired theory. To quote Einstein again:

\[ \ldots \text{und doch ist es im Interesse der Wissenschaft nötig, daß immer wieder an diesen fundamentalen Begriffen Kritik geübt wird,} \]
The emergence of a semiclassical time from quantum gravity will be discussed in the next section. Here we focus on the level of Equation (6) itself. An important property of the Wheeler-DeWitt equation is its hyperbolic nature, i.e. its behaviour as a wave equation. This can be recognized from the coefficients $G_{abcd}$ by treating them – at each space point separately – as the elements of a symmetric $6 \times 6$ matrix which after diagonalization has the signature $(-, +, +, +, +, +)$. It is important to note that there remains one global minus sign after gauge degrees of freedom have been eliminated. This minus sign is basically given by the size of the Universe, which may thus be considered as an intrinsic time variable. $\Psi$ has now to be interpreted as a probability amplitude for time, but not in time. All other degrees of freedom, including physical clocks, are correlated with intrinsic time. The hyperbolic nature of (6) allows the formulation of a Cauchy problem with respect to this time. In spite of its static appearance, the Wheeler-DeWitt equation describes an intrinsic dynamics! This has drastic consequences for the behaviour of wave packets in the case of a recollapsing universe (Zeh, 1988). In the classical theory, the recollapsing leg of the history of the universe can be considered as the deterministic successor of the expanding leg. This is no longer true in quantum cosmology! The wave equation (6) describes dynamics with respect to intrinsic time, which in simple models like (7) is the radius of the universe. The expanding and recollapsing legs of a wave packet concentrated near the classical trajectory in configuration space can thus not be distinguished intrinsically. With respect to the Cauchy problem the returning wave packet must be present initially.

This can already be seen in the simple model (7). To construct a wave tube which follows the collapsing trajectory, one must use solutions to (7) which fall off for large values of $a$ and $\phi$. This forces one to use the normalisable harmonic oscillator eigenfunctions in a wave packet solution which thus reads

$$
\psi(a, \phi) = \sum_n A_n \frac{H_n(a)H_n(\phi)}{2^n n!} \exp \left( -\frac{1}{2} a^2 - \frac{1}{2} \phi^2 \right),
$$

where $A_n$ are coefficients which are peaked around some $n = \bar{n}$, and $H_n$ are the Hermite polynomials. The solution (9) automatically describes two packets at $a = 0$ (see figure 1). While in the simple oscillator model (7) there is no dispersion of the wave packet, this is no longer true in more...
realistic examples like the case of a massive scalar field in a Friedmann universe (Kiefer, 1988; Zeh, 1992). The demand for the wave packet to go to zero at large radii (otherwise it cannot correspond to a recollapsing universe) unavoidably leads to a smearing of the packet in regions close to the classical turning point. This demonstrates that a WKB approximation cannot hold in the whole region of an expanding and recollapsing trajectory in configuration space. This has important consequences for the discussion of the arrow of time (see the last section).

The above considerations are at present only of a heuristic nature. It is still unclear what the Hilbert space structure of quantum theory is, if it is necessary at all. Imposing the Schrödinger inner product onto the solutions of (6) in general leads to a diverging result if all variables are integrated over. Recalling the hyperbolic structure of this equation, it would seem to be more appropriate to choose a Klein-Gordon inner product like in relativistic quantum mechanics and to integrate over “spacelike” hypersurfaces $a = \text{constant}$. Unlike relativistic quantum mechanics it is, however, here not possible to make a decomposition into positive and negative frequencies (see Kuchař, 1992). This has prompted some authors to invoke a third quantization of the theory, i.e. to elevate the wave functional $\Psi$ to an operator in some “new” Hilbert space. We will, however, not follow these approaches any further. We also mention that there are attempts which try to first solve the classical constraint (3) and only then make the transition to quantum theory. This creates its own problems, and we refer to the excellent review articles by Isham (1992) and Kuchař (1992) for details.

3 Decoherence and the recovery of the Schrödinger equation

We now address the issue of how the level of quantum field theory in a classical spacetime background can be recovered from quantum gravity where there is no spacetime. This will basically involve two steps. The first step is the derivation of the Schrödinger equation (8) from the Wheeler-DeWitt equation (6). The second step is to understand the unobservability of non-classical states for the gravitational field. A detailed review of these steps can be found, e.g., in Kiefer (1993b).

The basic observation which goes into the development of the first step is the fact that the length scale contained in (6), the Planck length (1),
is much smaller than any relevant scale of non-gravitational physics. This enables one to make a formal expansion of the wave functional in (6) in powers of the Planck length (or, equivalently, the gravitational constant). If there were no non-gravitational fields in (6), an expansion with respect to $G$ would be fully equivalent to an expansion in powers of $\hbar$ which is the usual semiclassical (“WKB”) expansion, since both constants would appear in the combination $G\hbar$ only. As far as gravity is concerned, the present expansion scheme thus is a WKB expansion. This is no longer true for other fields in which case the situation is analogous to a Born-Oppenheimer approximation in molecular physics: The gravitational part in (6) corresponds to the heavy nuclei whose kinetic terms are neglected in a first approximation while the remaining part corresponds to the light electrons. To highest order, the wave functional depends only on the gravitational field,

$$\Psi_0 = C[h_{ab}] \exp \left( \frac{ic^2}{32\pi G\hbar} S_0[h_{ab}] \right),$$  

(10)

where $S_0$ obeys the Hamilton-Jacobi equation for gravity. This equation is equivalent to all of Einstein’s field equations and describes a classical gravitational background in the sense that one can assign classical “trajectories” to it. Each trajectory, which represents a whole spacetime, runs orthogonally to hyperspaces $S_0 = \text{constant}$ in configuration space. Formally, this is the same as the recovery of geometrical optics from Maxwell’s equations.

In the next order of approximation the wave functional assumes the form

$$\Psi_1 = \Psi_0 \chi[h_{ab}, \phi],$$

(11)

where the wave functional $\chi$ also depends on non-gravitational fields and obeys the equation (Banks, 1985)

$$i\hbar G_{abcd} \frac{\delta S_0}{\delta h_{ab}} \frac{\delta \chi}{\delta h_{cd}} \equiv i\hbar \frac{\delta \chi}{\delta \tau(x)} = \mathcal{H}_m \chi.$$  

(12)

This is nothing else but the functional Schrödinger equation (8) for non-gravitational fields propagating on one of the classical spacetimes described by $S_0$. This spacetime is parametrized by the “many-fingered time” $\tau(x)$ appearing in (12).

As Julian Barbour (1992) emphasized, this WKB-time corresponds exactly to the notion of ephemeris time used by astronomers. Ephemeris time is the extraction of time, in retrospect, from actual observations of celestial
bodies, see for example Clemence (1957). The semiclassical time in (12) is thus defined by the actual motion of bodies in the real world. It is amazing that this exactly corresponds to the concept of time used by the ancient Greeks who defined time by the motion of the celestial bodies (already Plato used the term ephemeris time). It is clear that there can be no emergence of a semiclassical time for flat Minkowski space which demonstrates the absence of any concept of absolute time. It is the three-dimensional geometry which carries information about time, see Baierlein, Sharp and Wheeler (1962). To determine ephemeris time, eventually all available information about motion in the Universe has to be taken into account. An impressive example is the case of the binary pulsar PSR 1913+16.

Using general relativity, ephemeris time can only be consistently extracted from the orbital motion, if the gravitational pull of the whole Galaxy on the pulsar is taken into account, as was demonstrated by Damour and Taylor (1991).

If one proceeds with the above approximation scheme to the next order (Kiefer and Singh, 1991), one can derive correction terms to the Schrödinger equation (12) which are proportional to the gravitational constant. In addition one finds a back reaction of the non-gravitational fields onto the gravitational background which modifies the definition of semiclassical time. One can calculate concrete results from these correction terms, such as the quantum gravitational correction to the trace anomaly in De Sitter space (Kiefer, 1993b).

Is the recovery of the Schrödinger equation in a classical spacetime sufficient for the understanding of the classical behaviour of the spacetime geometry in our world? The answer must be no since one still has to focus on the issue of superpositions of different “classical” states of the gravitational field. This problem is even of direct relevance in the above derivation: If one takes a superposition of two semiclassical states, for example the state (10) and its complex conjugate, one cannot recover the Schrödinger equation (Barbour, 1993). This equation follows only if a special, complex, state like (10) is taken as the starting point. As one recognizes immediately, this issue is directly connected with the emergence of the $i$ and the use of complex wave functions in ordinary quantum theory – the $i$ in the Schrödinger equation is taken directly from the state (10) of the gravitational field. How, then, can one justify the use of such a special state? A possible answer is provided by the mechanism of decoherence (Kiefer, 1993a). The key ingredient is the quantum entanglement of the wave function of the Universe, i.e. the
existence of correlations between a large number of degrees of freedom, which we discussed in the first section. Since only very few degrees of freedom in this wave function are accessible to observations, the relevant object is not the full wave function but the reduced density matrix which is obtained by tracing out all irrelevant (unobservable) degrees of freedom. If the only relevant degree of freedom were the scale factor, this density matrix would read

$$\rho(a, a') = \text{Tr}_\phi \Psi^* [a', \phi] \Psi[a, \phi],$$

(13)

where $\phi$ stands for the irrelevant degrees of freedom. These can be gravitational degrees of freedom like gravitational waves as well as, for example, matter density perturbations. They “measure” the gravitational background and force it to become classical (Zeh, 1986; Kiefer, 1987). In this way “quasiclassical domains” emerge from the fundamental quantum world (Gell-Mann and Hartle, 1990). When applied to simple models, the tracing out of such irrelevant degrees of freedom suppresses interference terms between different WKB components of the form (10). A conformally coupled scalar field in a Friedmann universe, for example, leads to a suppression factor of the interference between (10) and its complex conjugate given by

$$\exp \left( -\frac{\pi m H_0^2 a^3}{128} \right),$$

(14)

where $m$, $H_0$, and $a$ are, respectively, the mass of the scalar field, the Hubble parameter, and the scale factor (Kiefer, 1992). This is very tiny, except for small radii of the Universe and near the region of the classical turning point where the Hubble parameter vanishes. Quantum gravity itself thus contains the seeds for the emergence of a classical geometry, but also describes its limit.

4 The direction of time

It is an obvious fact that most phenomena in Nature distinguish a direction of time (see, for example, Zeh, 1992): Electromagnetic waves are observed in their retarded form only, where the fields causally follow from their sources. The increase of entropy, as it is expressed in the second law of thermodynamics, also defines a time direction. This is directly connected with the psychological arrow of time – we remember the past but not the future. In quantum mechanics it is the irreversible measurement process and in
cosmology the expansion of the Universe, as well as the local growing of inhomogeneities, which determine a direction of time.

And yet, the fundamental laws of physics are invariant under time reversal (the only exception being the small CP violation in weak interactions). How, then, can one understand that most phenomena distinguish a direction of time? The answer lies in the possibility of very special boundary conditions such as an initial condition of low entropy.

One assumes now that the occurrence of such boundary conditions can be understood within the dynamical laws of physics and that it makes sense to search for a common root of the various arrows of time – the master arrow of time. Such an assumption transcends the Newtonian separation into laws and boundary conditions by also seeking physical explanations for the latter.

Where lies the key to the understanding of the irreversibility of time? As in particular Penrose (1979) has convincingly emphasized, it is primarily the high unoccupied entropy capacity of the gravitational field which allows for the emergence of structure far from thermodynamical equilibrium. Whereas the non-gravitational part of the entropy reaches its maximum for a homogeneous state, the opposite is true for the gravitational part which tries to develop a highly clumpy state. It is therefore a cosmological problem to justify the presence of an initial state of very low gravitational entropy, i.e. a very homogeneous state. This has provoked Penrose to formulate his Weyl tensor hypothesis that the Weyl tensor vanishes at singularities in the past but not at those in the future. The Weyl tensor is that part of the Riemann tensor which is not fixed by the field equations (in which only the Ricci tensor enters) but by the boundary conditions only. It describes the degrees of freedom of the gravitational field. Since it vanishes exactly for a homogeneous and isotropic Friedmann Universe, it can be taken as a heuristic measure for inhomogeneity and, therefore, for gravitational entropy.

The arrow of time can of course only be explained by the Weyl tensor hypothesis if it can be derived from some fundamental theory. As we have argued in the previous sections, the fundamental framework to address such questions is quantum cosmology. In the configuration space for the wave function of the Universe there is no intrinsic distinction between Big Bang and Big Crunch since both just correspond to regions of low scale factor \( a \). The desired boundary condition is then a boundary condition for the wave function at small scales. It has therefore been suggested (Zeh, 1993) to impose the boundary condition that for small scales the wave function

\[ \psi(\mathbf{x}) \big|_{a_{\text{small}}} = \psi_{\text{low entropy}}(\mathbf{x}) \]
depends only on $a$ but not on any other degrees of freedom. These degrees of freedom emerge only with increasing $a$ when they are in their ground state, as can be seen from a discussion of the Wheeler-DeWitt equation (Conradi, 1992). A similar behaviour was also derived from the no boundary condition (Hawking, 1985). At least heuristically, this impl
ements a low gravitational entropy at small scales. Complexity, and therefore entropy, increases with increasing scale factor. Since more and more degrees of freedom come into play, decoherence also increases and the Universe becomes more and more classical. The thermodynamical arrow of time is thus inextricably tied to the cosmological arrow of time.

Such a boundary condition has important physical consequences. Since the thermodynamical arrow of time is correlated with the scale factor but not with any classical trajectory (which is absent in quantum cosmology), this would mean that in the case of a closed universe time would formally reverse its direction near the turning point. We call such a reverse formal since no information gathering system could survive this reversal. All observers in any branch of the wave function would consider their universe as expanding. This boundary condition would also have drastic consequences for the behaviour of black holes (Zeh, 1993) since they would formally become white holes during the recontraction phase, and the formation of a horizon would be prevented. The whole quantum universe would be singularity free in this case.

These considerations are of course still speculative. They are, however, only based on the established formal framework of quantum theory and cosmology. They demonstrate that quantum cosmology, if taken seriously, can yield a picture of the Universe which drastically modifies the classical Big Bang model.

Acknowledgement

I am grateful to H.-Dieter Zeh for many stimulating discussions and a critical reading of this manuscript. I also thank Claudia Pertzborn and Tejinder Singh for their comments on this manuscript.

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Figure Caption

Wave packet solution of the Wheeler-DeWitt equation (7).
This figure "fig1-1.png" is available in "png" format from:

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