Max Plus Algebra and Petri Net Application on Scheduling of Ship Engine Component’s Spare Part Ordering

Farah Azizah and Subiono

Abstract—Shipping company is a company that runs its business by operating the ships or other businesses that are closely related to the ship. A ship has a main engine and some auxiliary engines to support the ship’s performance. It needs to do maintenance of engines so that the ship can operate properly. This engine maintenance is replacement of the old engine components with the new ones if the running hours of the components are over. Therefore, in the ship, the spare parts must always be available at least one for each engine component. During this time, the company has experienced a difficulty in determining the time of spare part ordering. When the running hours of engine components are over, the spare parts were not yet available. Then, Petri Net and Max Plus Algebra model will be built to schedule the ordering of ship engine component’s spare part based on the ordering flow and the running hours of engine components. The Petri Net based on the Max Plus Algebra obtains maximum time to order the spare part so that it produces the ship engine component’s spare part ordering schedule in running hour form and date. Therefore, spare part of each ship engine component is always available so that the installation can be timely and never be late.

Index Terms—Max-plus algebra, Petri nets, Scheduling of spare part ordering.

I. INTRODUCTION

MAX plus algebra is the useful approach to represent the discrete event systems. This approach makes us possible to determine and analyze various kinds of system properties. Therefore, the model of these ones will be linear over max plus algebra. But in conventional algebra, it is not linear. We can analyze the systems in max plus algebra easier and simpler than the conventional systems because of this linearity [1]. One of applications of max plus algebra is a scheduling of crystal sugar production system [2].

A Petri net is a mathematical modeling tool which can be applied to represent the state evolution of the discrete event systems. Petri net is called autonomous if every transition in this Petri net has at least an input state. This means that there is no transition which is enabled without any condition. In other words, autonomous Petri net does not have a transition which is always enabled [3]. Timed Petri net is an extension of Petri net. In this paper, we use the autonomous timed Petri net and max plus algebra model for scheduling of ship engine components’ spare part ordering. Furthermore, we will build a model of Max Plus Algebra using Supply Chain model to obtain the date of spare part ordering. For more detailed discussion of supply chain using max plus algebra, the interested reader is referred to [4].

Shipping company is a company that runs its business by operating the ships or other businesses that are closely related to the ship. The ship becomes a very important part in this company. Therefore, the company must maintain the performance of the ship so that operations run optimally. The most important thing in keeping the performance of the ship is to make sure all of the ship’s engine run properly so as not to cause delays in shipping time. Inside the ship there are two large groups of machines, the main engine and auxiliary engine. In order for ship engines to function normally, the system required periodic maintenance. Thus, the ship’s engine does not get a breakdown. Periodic maintenance is generally in the form of checking up the replacement of components in the engine according to running hours of the components. Therefore, in the ship must always be available at least one of each component parts for the engine, so that when it is needed in the periodic maintenance, the spare parts can be used directly without disturbing the shipping schedule.

The spare part of each ship engine component is ordered from various suppliers. When the running hours of ship engine components will end, the ship crew will start to require the spare parts. Furthermore, the request will be processed by purchasing division until the spare parts are ready to be sent to the warehouse. Purchasing division will inform the spare parts requirement to suppliers who already have a working relationship with the company. The suppliers will offer the spare parts requested by different price, time availability, and quality. From some offers, purchasing division makes a summary of the offers which will be submitted to the ship manager to determine which supplier will be chosen. Following an agreement, the chosen supplier will provide the spare parts and send to the company’s warehouse in a given time period. Thus, the spare part is available and ready to be sent to the ship.

During this time the company experienced difficulties in determining the time of spare parts ordering. When running hours of engine components are over, the spare parts are not available. As a result, the spare parts must be ordered from within the country or abroad and delivered by plane. Thus, the purchase cost is increasing. Meanwhile, the company expects the spare parts are always available before running hours of ship engine components end so that the ship can operate...
optimally. In addition, the purchase cost of spare parts become cheaper if ordering in the right time and not in a hurry because it can be delivered by land or sea transportation. Therefore, Petri Net and Max Plus Algebra model will be built to schedule the ordering of ship engine component’s spare part based on the ordering flow and the running hour of engine components. It is expected that the spare part of each ship engine component is always available so that the installation can be timely.

II. PRELIMINARIES

A. Max Plus Algebra

Given $\mathbb{R}_e \overset{\text{def}}{=} \mathbb{R} \cup \{\varepsilon\}$ where $\mathbb{R}$ is a set of real numbers and $\varepsilon \overset{\text{def}}{=} -\infty$. In $\mathbb{R}_e$, two operations are defined by:

$$x \oplus y \overset{\text{def}}{=} \max\{x, y\} \quad \text{and} \quad x \otimes y \overset{\text{def}}{=} x + y, \quad \forall x, y \in \mathbb{R}_e.$$

Furthermore, $(\mathbb{R}_e, \oplus, \otimes)$ is a semiring with neutral element $\varepsilon$ and unit element $\varepsilon \overset{\text{def}}{=} 0$. For $x \in \mathbb{R}_e$ and $n \in \mathbb{N}$ with $n \neq 0$, where $\mathbb{N}$ is the set of all positive integers, we define

$$x \otimes^n = \underbrace{x \otimes \cdots \otimes x}_{n},$$

whereas for $n = 0$, it is defined by $x \otimes^0 = e (= 0)$. Therefore, for each $n \in \mathbb{N}$, $x \otimes^n$ is defined by

$$x \otimes^n = x \otimes \cdots \otimes x = n \otimes x.$$

Furthermore,

$$x \otimes^\alpha = \alpha \otimes x, \quad \text{for} \ \alpha \in \mathbb{R}.$$

Addition and multiplication of two matrices with appropriate size over max-plus algebra are defined by

$$[A \oplus B]_{ij} = [A]_{ij} \oplus [B]_{ij},$$

$$[A \otimes B]_{ij} = \bigoplus_{k} ([A]_{ik} \otimes [B]_{kj})$$

where $A$ and $B$ are matrices of appropriate size. As an example, the multiplication of two matrices of size $2 \times 2$ is given by

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \otimes \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a \otimes e \oplus b \otimes g & a \otimes f \oplus b \otimes h \\ c \otimes e \oplus d \otimes g & c \otimes f \oplus d \otimes h \end{bmatrix}$$

and the identity matrix of size $2 \times 2$ over max plus algebra is

$$\begin{bmatrix} 0 & \varepsilon \\ \varepsilon & 0 \end{bmatrix}.$$

B. Petri Nets

Definition 1 ([11]): Petri net is 4-tuple $(P, T, A, w)$ with

- $P$ : a finite set of places, $P = \{p_1, p_2, \ldots, p_n\}$;
- $T$ : a finite set of transitions, $T = \{t_1, t_2, \ldots, t_n\}$;
- $A$ : a set of arcs, $A \subseteq (P \times T) \cup (T \times P)$;
- $w$ : a weight function, $w \{1, 2, 3, \ldots\}$.

Definition 2 ([11]): Marking $x$ in a Petri net is a function $x : P \rightarrow \{0, 1, 2, \ldots\}$.

Definition 3: Transition $t_j \in T$ in a Petri net is enabled if $x(p_i) \geq w(p_i, t_j)$, for all $p_i \in I(t_j)$.

C. Prioritized Petri Nets

Each transition in a Petri net has a priority value and denoted by $\pi$. Prioritized Petri Net is a Petri Net which prioritizes one or several transition(s) so that in certain condition, the prioritized transition is selected from a set of enabled transitions. This situation is shown in Fig. 1 and 2.

![Fig. 1. Petri Net which has two transitions with the same priority.](image1)

![Fig. 2. Petri Net which has two transitions with different priority.](image2)

D. System of Ship Engine Component’s Spare Part Ordering

The flow of ship engine component’s spare part ordering is described in Fig. 3. Furthermore, the process of ship engine component’s spare part ordering will be represented by a Petri Net and use it to construct a Max Plus Algebra model for obtaining the maximal time of spare part ordering.

![Fig. 3. Flow Chart of Ship Engine Component’s Spare Part Ordering.](image3)
III. RESULTS AND DISCUSSIONS

A. Model of Ship Engine Component’ Spare Part Ordering in Max Plus Algebra

Based on Fig. 3, we obtain Petri Net on Fig. 4 that represents the process of ship engine component’s spare part ordering, where the interpretation of transitions is as follows:

- $T_1$: spare part ordering by the ship’s crew to purchasing division ($\pi = 1$).
- $T_2$: dissemination of ordering information by purchasing division to suppliers ($\pi = 2$).
- $T_3$: offering from suppliers ($\pi = 2$).
- $T_4$: manager checks the offering summary from suppliers ($\pi = 2$).
- $T_5$: the offering of type A ($\pi = 8$).
- $T_6$: the offering of type A is not available or ignored ($\pi = 8$).
- $T_7$: the offering of type C ($\pi = 7$).
- $T_8$: the offering of type B is not available or ignored ($\pi = 7$).
- $T_9$: the offering of type C is not available or ignored ($\pi = 6$).
- $T_{10}$: the offering of type C is not available or ignored ($\pi = 6$).
- $T_{11}$: the offering of type B ($\pi = 5$).
- $T_{12}$: the offering of type D is not available or ignored ($\pi = 4$).
- $T_{13}$: the offering of type A is picked ($\pi = 3$).
- $T_{14}$: the offer other than type A is picked ($\pi = 2$).
- $T_{15}$: the offer rejected ($\pi = 2$).
- $T_{16}$: manager instruct the re-dissemination of information offering by purchasing division to other suppliers ($\pi = 2$).
- $T_{17}$: spare parts start to order ($\pi = 2$).
- $T_{18}$: spare parts arrived at the warehouse ($\pi = 2$).

The meaning of each place is given by

- $P_1$: purchasing division receives spare parts ordering
- $P_2$: suppliers receive ordering information from purchasing division
- $P_3$: the offering summary is submitted to the manager by purchasing division
- $P_4$: the offering of type A
- $P_5$: the offering of type B
- $P_6$: the offering of type C
- $P_7$: the offering of type D
- $P_8$: a decision on the acceptance or rejection of the offering
- $P_9$: saving temporary offering summary that is rejected
- $P_{10}$: the offering of type A is prioritized
- $P_{11}$: selected one supplier
- $P_{12}$: selected suppliers provide spare parts ordered

Furthermore, this Petri Net is used to make the following Max Plus Algebra model which produces maximum time of ship engine component’s spare part ordering.

$$T_1 (k) = \psi_{T_1, k} \otimes T_1 (k - 1)$$  \hspace{1cm} (1)
$$T_2 (k) = \psi_{T_2, k} \otimes T_1 (k) \oplus \psi_{T_2, k} \otimes T_{16} (k - 1)$$  \hspace{1cm} (2)
$$T_3 (k) = \psi_{T_3, k} \otimes T_2 (k)$$  \hspace{1cm} (3)
$$T_4 (k) = \psi_{T_4, k} \otimes T_3 (k)$$  \hspace{1cm} (4)
$$T_5 (k) = \psi_{T_5, k} \otimes T_4 (k)$$  \hspace{1cm} (5)
$$T_6 (k) = \psi_{T_6, k} \otimes T_4 (k)$$  \hspace{1cm} (6)
$$T_7 (k) = \psi_{T_7, k} \otimes T_4 (k)$$  \hspace{1cm} (7)
$$T_8 (k) = \psi_{T_8, k} \otimes T_4 (k)$$  \hspace{1cm} (8)
$$T_9 (k) = \psi_{T_9, k} \otimes T_4 (k)$$  \hspace{1cm} (9)
$$T_{10} (k) = \psi_{T_{10}, k} \otimes T_4 (k)$$  \hspace{1cm} (10)
$$T_{11} (k) = \psi_{T_{11}, k} \otimes T_4 (k)$$  \hspace{1cm} (11)
$$T_{12} (k) = \psi_{T_{12}, k} \otimes T_4 (k)$$  \hspace{1cm} (12)
$$T_{13} (k) = \psi_{T_{13}, k} \otimes [T_3 (k) \oplus T_7 (k) \oplus T_9 (k) \oplus T_{11} (k)]$$  \hspace{1cm} (13)
$$T_{14} (k) = \psi_{T_{14}, k} \otimes [T_5 (k) \oplus T_7 (k) \oplus T_9 (k) \oplus T_{11} (k)]$$  \hspace{1cm} (14)
$$T_{15} (k) = \psi_{T_{15}, k} \otimes [T_5 (k) \oplus T_7 (k) \oplus T_9 (k) \oplus T_{11} (k)]$$  \hspace{1cm} (15)
$$T_{16} (k) = \psi_{T_{16}, k} \otimes T_{15} (k)$$  \hspace{1cm} (16)
$$T_{17} (k) = \psi_{T_{17}, k} \otimes [T_{13} (k) \oplus T_{14} (k)]$$  \hspace{1cm} (17)
$$T_{18} (k) = \psi_{T_{18}, k} \otimes T_{17} (k)$$  \hspace{1cm} (18)

Based on equation (1), (16), and (18), we obtain

$$\begin{bmatrix}
T_1 (k) \\
T_{16} (k) \\
T_{18} (k)
\end{bmatrix} =
\begin{bmatrix}
\psi_{T_1, k} & c_n & d_n \\
a & b & e \\
\end{bmatrix}
\begin{bmatrix}
T_1 (k - 1) \\
T_{16} (k - 1) \\
T_{18} (k - 1)
\end{bmatrix} \quad (19)$$

where

- $\psi_{T_1, k}$: duration of spare part offering by crew that is accepted by purchasing division
- $\psi_{T_2, k}$: duration of dissemination offering process from purchasing division to suppliers
- $\psi_{T_3, k}$: duration of offering from the first supplier since receiving the request information
- $\psi_{T_4, k}$: duration of offering from the second supplier since receiving the request information
- $\psi_{T_5, k}$: duration of offering from the third supplier since receiving the request information
- $\psi_{T_6, k}$: duration of offering from the fourth supplier since receiving the request information
- $\psi_{T_7, k}$: duration of submission of offering summary to the manager
- $\psi_{T_8, k}$: duration of the manager determines all offerings being rejected
- $\psi_{T_9, k}$: duration of offering from other suppliers since receiving the information
- $\psi_{T_{10}, k}$: duration of submitting a new offering summary to manager
- $\psi_{T_{11}, k}$: duration of the manager determines one of the offerings received
- $\psi_{T_{12}, k}$: duration of spare parts began to be ordered since supplier determination
- $\psi_{T_{13}, k}$: duration of spare parts began to be ordered since the determination of selected suppliers
- $\psi_{T_{14}, k}$: duration of spare parts arrived in the warehouse since it was ordered.
and spreading to the suppliers. Therefore, for $n = 2$, we obtained

$$c_2 = 39 \otimes v_{T_{16},k}$$

$$d_2 = 38 \otimes v_{T_{18},k}$$

The maximal time of spare part ordering from equation (19) is in day unit so it has to be converted to hour unit by multiplying it with 10 hours which is the average of ship’s running hours every day, then we get the ordering duration. To compute the time when the spare part starts to be ordered, we have to subtract the interval for overhaul by the ordering duration and the result is showed in Table I.

Furthermore, we will construct a model of Max Plus Algebra using Supply Chain model to obtain the date of spare part ordering. Figure 5 represents the process of spare part ordering using a Supply Chain model.

Based on Fig. 5, we get the following Max Plus Algebra model:

$$t_2 (k) = w_6 \otimes t_3 (k - n) \oplus t_1 (k)$$
$$t_3 (k) = w_9 \otimes t_2 (k)$$
$$y (k) = t_3 (k)$$

where $n = 1$ is the number of spare parts.

The model can be changed into matrix equations as follow.

$$X (k) = A_0 \otimes X (k) \oplus A_1 \otimes X (k - n) \oplus B_0 \otimes U (k)$$
$$Y (k) = C \otimes X (k)$$

where

$$U (k) = \begin{bmatrix} t_1 (k) \\ t_3 (k) \end{bmatrix}$$
$$X (k) = \begin{bmatrix} t_2 (k) \\ t_3 (k) \end{bmatrix}$$
Fig. 5. Petri Net of Spare Part Ordering Using Supply Chain Model.

\[ Y(k) = [t_4(k)] \]

\[ A_0 = \begin{bmatrix} \varepsilon & 0 \\ w_b & \varepsilon \end{bmatrix} \]

\[ A_1 = \begin{bmatrix} \varepsilon & w_a \\ \varepsilon & \varepsilon \end{bmatrix} \]

\[ B_0 = \begin{bmatrix} 0 \\ \varepsilon \end{bmatrix} \]

\[ C = \begin{bmatrix} \varepsilon & 0 \end{bmatrix} \]

Since \( A = A_0^* \otimes A_1 \) and \( B = A_0^* \otimes B_0 \) [3], then

\[ A = A_0^* \otimes A_1 \]

\[ = \begin{bmatrix} 0 & \varepsilon \\ w_b & 0 \end{bmatrix} \otimes \begin{bmatrix} \varepsilon & w_a \\ \varepsilon & \varepsilon \end{bmatrix} \]

\[ = \begin{bmatrix} 0 \otimes \varepsilon \otimes \varepsilon & 0 \otimes w_a \otimes \varepsilon \otimes \varepsilon \\ w_b \otimes \varepsilon \otimes 0 \otimes \varepsilon & w_b \otimes w_a \otimes 0 \otimes \varepsilon \end{bmatrix} \]

\[ = \begin{bmatrix} \varepsilon & w_a \\ w_b \otimes w_a \end{bmatrix} \]

\[ = \begin{bmatrix} \varepsilon & w_a \otimes w_b \\ \varepsilon \end{bmatrix} \]

and

\[ B = A_0^* \otimes B_0 \]

\[ = \begin{bmatrix} 0 & \varepsilon \\ w_b & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ \varepsilon \end{bmatrix} \]

\[ = \begin{bmatrix} 0 \otimes \varepsilon \otimes \varepsilon & 0 \otimes 0 \otimes \varepsilon \\ w_b \otimes 0 \otimes \varepsilon & w_b \otimes 0 \otimes \varepsilon \end{bmatrix} \]

\[ = \begin{bmatrix} 0 \otimes \varepsilon \\ w_b \otimes \varepsilon \end{bmatrix} \]

\[ = \begin{bmatrix} 0 \\ w_b \end{bmatrix} \]

so that

\[ CB = \begin{bmatrix} \varepsilon & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ w_b \end{bmatrix} \]

\[ = \begin{bmatrix} \varepsilon \otimes 0 \otimes 0 \otimes w_b \end{bmatrix} \]

and

\[ CAB = \begin{bmatrix} \varepsilon & 0 \end{bmatrix} \otimes \begin{bmatrix} \varepsilon & w_a \\ \varepsilon & \varepsilon \end{bmatrix} \otimes \begin{bmatrix} 0 \\ w_b \end{bmatrix} \]

\[ = \begin{bmatrix} \varepsilon \otimes \varepsilon \otimes 0 \otimes \varepsilon & \varepsilon \otimes w_a \otimes 0 \otimes w_a \otimes w_b \end{bmatrix} \otimes \begin{bmatrix} 0 \\ w_b \end{bmatrix} \]

\[ = \begin{bmatrix} \varepsilon \otimes w_a \otimes w_b \\ \varepsilon \otimes w_a \otimes w_b \end{bmatrix} \]

\[ = \begin{bmatrix} \varepsilon \otimes w_a \otimes w_b \otimes 2 \end{bmatrix} \]

According to [1]

\[ y(k) = \sum_{i=0}^{\alpha} C \otimes A^{\otimes i} \otimes B \otimes u(k - i \cdot n) \]

so that for \( n = 1 \) and \( \alpha = \lfloor k/n \rfloor = \lfloor k \rfloor = k \), we obtain

\[ y(1) = C \otimes A^{\otimes 0} \otimes B \otimes u(1 - 0 \cdot 1) = C \otimes A^{\otimes 1} \otimes B \otimes u(1 - 1 \cdot 1) = C \otimes B \otimes u(1) \]

\[ = C \otimes A^{\otimes 2} \otimes B \otimes u(2 - 2 \cdot 1) = C \otimes B \otimes u(2) \]

\[ y(2) = C \otimes A^{\otimes 0} \otimes B \otimes u(2 - 0 \cdot 1) = C \otimes A^{\otimes 1} \otimes B \otimes u(2 - 1 \cdot 1) \]

Moreover,

\[ y = H \otimes U \]
where
\[
Y = \begin{bmatrix}
y(1) \\
y(2)
\end{bmatrix},
\]
\[
H = \begin{bmatrix}
CB & \varepsilon \\
CAB & CB
\end{bmatrix},
\]
\[
U = \begin{bmatrix}
u(1) \\
u(2)
\end{bmatrix}.
\]

Notice that \( Y \) is time when the running hours of engine component over and \( U \) is the time when the spare part ordering starts.

Therefore, the solution is \[1\]
\[
U = -H^T \otimes' Y
\]
(20)

where
\[
u(1) = \min \{ y_1 - h_{1,1}, y_2 - h_{2,1} \},
\]
\[
u(2) = \min \{ y_1 - h_{1,2}, y_2 - h_{2,2} \}.
\]

**B. Schedule of Engine Component’s Spare Part Ordering**

Based on the ordering duration and overhaul interval of each spare part which are gotten before, we can determine the running hours when the ordering is started that is shown in Table I.

| Spare part’s name          | Ordering duration (hours) | Overhaul for interval (hours) | Running hours when the ordering is started (hours) |
|---------------------------|---------------------------|-------------------------------|--------------------------------------------------|
| CRANKPIN BEARING SHELL    | 520                       | 16000                         | 15480                                            |
| CROSSHEAD BEARING SHELL   | 650                       |                               | 15350                                            |
| O-RING - EN17M6220        | 940                       | 8000                          | 7060                                            |
| SCRAPER RING (LOWER)      | 940                       |                               | 7060                                            |
| SCRAPER RING (UPPER)      | 940                       |                               | 7060                                            |
| TIGHTENING RING           | 940                       |                               | 7060                                            |
| PISTON RING - 3169804     | 730                       | 8000                          | 7270                                            |
| PISTON RING - 3169805     | 1400                      |                               | 6600                                            |
| GUIDE RING                | 880                       | 4000                          | 3120                                            |
| O-RING – 4511913          | 490                       |                               | 3510                                            |
| O-RING – 4511912          | 490                       |                               | 3510                                            |
| O-RING – EN17M340         | 730                       |                               | 3270                                            |
| O-RING – 4183312         | 940                       |                               | 3060                                            |

**TABLE I. Running Hours of Engine Component When the Spare Part Starts to Be Ordered.**

Furthermore, the running hours when the ordering is started is converted into day unit by dividing it by 10 which is the average of ship’s running hour every day. By using equation (20), we obtain the date when each spare part starts to be ordered that is presented on Table II until Table V below.

| Spare part’s name          | Date when spare part starts to be ordered |
|---------------------------|-------------------------------------------|
| CRANKPIN BEARING SHELL    | 11 January 2020                           |
| CROSSHEAD BEARING SHELL   | 29 December 2019                          |
| O-RING                    | 02 December 2016                          |
| SCRAPER RING (LOWER)      | 02 December 2016                          |
| SCRAPER RING (UPPER)      | 02 December 2016                          |
| TIGHTENING RING           | 02 December 2016                          |
| PISTON RING – 3169804     | 13 October 2017                           |
| PISTON RING – 3169805     | 1400                                      |
| GUIDE RING                | 880                                       |
| O-RING – 4511913          | 490                                       |
| O-RING – 4511912          | 490                                       |
| O-RING – EN17M340         | 730                                       |
| O-RING – 4183312         | 940                                       |
| Spare part's name | Date when spare part starts to be ordered | Date when spare part starts to be ordered |
|------------------|----------------------------------------|----------------------------------------|
| PISTON RING - 3169805 | 07 August 2017 | 16 October 2019 |
| GUIDE RING | 23 August 2016 | 27 September 2017 |
| O-RING - 4511913 | 01 October 2016 | 05 November 2017 |
| O-RING - 4511912 | 01 October 2016 | 05 November 2017 |
| O-RING - EN17M340 | 07 September 2016 | 12 October 2017 |
| O-RING - 4183312 | 17 August 2016 | 21 September 2017 |
| O-RING - EN17M365 | 07 September 2016 | 12 October 2017 |
| SEAL RING - 4184389 | 07 September 2016 | 12 October 2017 |
| SEAL RING - 4184390 | 07 September 2016 | 12 October 2017 |
| SPACER RING | 07 September 2016 | 12 October 2017 |
| O-RING - 4181145 | 16 January 2016 | 26 March 2018 |
| O-RING - 4181146 | 26 December 2015 | 05 March 2018 |
| O-RING - 4183002 | 26 December 2015 | 05 March 2018 |
| SLIDE VALVE ASS | 16 January 2016 | 26 March 2018 |
| SPINDLE GUIDE ASS | 16 January 2016 | 26 March 2018 |
| SPACER RING | 13 February 2016 | 01 July 2020 |
| SCRAPER RING | 20 January 2016 | 07 June 2020 |
| O-RING - 4181145 | 30 December 2015 | 17 May 2020 |
| O-RING - 4181452 | 20 January 2016 | 07 June 2020 |
| O-RING - 4181455 | 20 January 2016 | 07 June 2020 |
| SLIDE VALVE | 12 February 2016 | 30 June 2020 |
| SPRING | 12 February 2016 | 30 June 2020 |
| THRUST PIECE | 12 February 2016 | 30 June 2020 |
| CYLINDER COMPLETE | 08 February 2016 | 26 June 2020 |

| Spare part’s name | Date when spare part starts to be ordered | Date when spare part starts to be ordered |
|------------------|----------------------------------------|----------------------------------------|
| CRANKPIN BEARING SHELL | 23 August 2018 | 09 January 2023 |
| CROSSHEAD BEARING SHELL | 10 August 2018 | 27 December 2022 |
| O-RING | 05 May 2016 | 14 July 2018 |
| SCRAPER RING (LOWER) | 05 May 2016 | 14 July 2018 |
| SCRAPER RING (UPPER) | 05 May 2016 | 14 July 2018 |
| TIGHTENING RING | 05 May 2016 | 14 July 2018 |
| PISTON RING - 3169804 | 04 August 2018 | 12 October 2020 |
| PISTON RING - 3169805 | 29 May 2018 | 06 August 2020 |
| GUIDE RING | 06 July 2016 | 18 September 2017 |
| O-RING - 4511913 | 14 August 2016 | 18 September 2017 |
| O-RING - 4511912 | 14 August 2016 | 18 September 2017 |
| O-RING | 21 July 2016 | 25 August 2017 |
| O-RING | 30 June 2016 | 04 August 2017 |
| O-RING - EN17M340 | 21 July 2016 | 25 August 2017 |
| O-RING - 4183312 | 21 July 2016 | 25 August 2017 |
| SEAL RING - 4184389 | 21 July 2016 | 25 August 2017 |
| SEAL RING - 4184390 | 21 July 2016 | 25 August 2017 |
| PISTON RING | 21 July 2016 | 25 August 2017 |
| SEAL RING - 4184389 | 21 July 2016 | 25 August 2017 |
| SEAL RING - 4184390 | 21 July 2016 | 25 August 2017 |
| PISTON RING | 21 July 2016 | 25 August 2017 |
| SEAL RING - 4184389 | 21 July 2016 | 25 August 2017 |
| SEAL RING - 4184390 | 21 July 2016 | 25 August 2017 |
| SCRAPER RING | 21 July 2016 | 25 August 2017 |
| O-RING - 4181145 | 07 January 2016 | 17 March 2018 |
| O-RING - 4181146 | 17 December 2015 | 24 February 2018 |
| O-RING - 4183002 | 17 December 2015 | 24 February 2018 |
| SLIDE VALVE ASS | 07 January 2016 | 17 March 2018 |
| SPINDLE GUIDE ASS | 07 January 2016 | 17 March 2018 |
| SPACER RING | 26 January 2016 | 12 January 2023 |
| SCRAPER RING | 02 August 2018 | 19 December 2022 |
| O-RING - 4181145 | 12 July 2018 | 28 November 2022 |

**TABLE II.** The date of spare part’s ordering for cylinder 1.
| Spare part’s name      | Date when spare part starts to be ordered |
|------------------------|------------------------------------------|
| O-RING - 4181452       | 02 August 2018 19 December 2022          |
| O-RING - 4181455       | 02 August 2018 19 December 2022          |
| SLIDE VALVE            | 25 August 2018 11 January 2023          |
| SPRING                 | 25 August 2018 11 January 2023          |
| THRUST PIECE           | 25 August 2018 11 January 2023          |
| CYLINDER COMPLETE      | 21 August 2018 07 January 2023          |

**TABLE III.** The date of spare part’s ordering for cylinder 2.

| Spare part’s name      | Date when spare part starts to be ordered |
|------------------------|------------------------------------------|
| CRANKPIN BEARING SHELL | 18 January 2020 05 June 2024            |
| CROSSHEAD BEARING SHELL | 05 January 2020 23 May 2024           |
| O-RING                 | 25 June 2016 03 September 2018         |
| SCRAPER RING (LOWER)   | 25 June 2016 03 September 2018         |
| SCRAPER RING (UPPER)   | 25 June 2016 03 September 2018         |
| TIGHTENING RING        | 25 June 2016 03 September 2018         |
| PISTON RING - 3169804  | 16 July 2016 24 September 2018         |
| PISTON RING - 3169805  | 10 May 2016 19 July 2018               |
| GUIDE RING             | 09 October 2015 12 November 2016        |
| O-RING - 4511913       | 17 November 2015 21 December 2016       |
| O-RING - 4511912       | 17 November 2015 21 December 2016       |
| O-RING - EN17M340      | 24 October 2015 27 November 2016        |
| O-RING - 4183312       | 03 October 2015 06 November 2016        |
| O-RING - EN17M365      | 24 October 2015 27 November 2016        |
| PISTON RING            | 24 October 2015 27 November 2016        |
| SEAL RING - 4184389    | 24 October 2015 27 November 2016        |
| SEAL RING - 4184390    | 24 October 2015 27 November 2016        |
| SPACER RING            | 24 October 2015 27 November 2016        |

**TABLE IV.** The date of spare part’s ordering for cylinder 3.

| Spare part’s name      | Date when spare part starts to be ordered |
|------------------------|------------------------------------------|
| CRANKPIN BEARING SHELL | 23 August 2018 09 January 2023          |
| CROSSHEAD BEARING SHELL | 10 August 2018 27 December 2022        |
| O-RING                 | 03 May 2016 12 July 2018                |
| SCRAPER RING (LOWER)   | 03 May 2016 12 July 2018                |
| SCRAPER RING (UPPER)   | 03 May 2016 12 July 2018                |
| TIGHTENING RING        | 03 May 2016 12 July 2018                |
| PISTON RING - 3169804  | 24 May 2016 02 August 2018              |
| PISTON RING - 3169805  | 18 March 2016 27 May 2018               |
| GUIDE RING             | 12 June 2016 17 July 2017               |
| O-RING - 4511913       | 21 July 2016 25 August 2017             |
| O-RING - 4511912       | 21 July 2016 25 August 2017             |
| Spare part’s name | Date when spare part starts to be ordered |
|------------------|------------------------------------------|
| O-RING EN17M340  | 27 June 2016 01 August 2017              |
| O-RING - 4183312 | 06 June 2016 11 July 2017               |
| O-RING EN17M365  | 27 June 2016 01 August 2017              |
| PISTON RING      | 27 June 2016 01 August 2017              |
| SEAL RING 4184389 | 27 June 2016 01 August 2017            |
| SEAL RING 4184390 | 27 June 2016 01 August 2017            |
| SPACER RING      | 27 June 2016 01 August 2017              |
| O-RING - 4181145 | 16 January 2016 26 March 2018           |
| O-RING - 4181146 | 26 December 2015 05 March 2018          |
| O-RING - 4183002 | 26 December 2015 05 March 2018          |
| SLIDE VALVE ASS  | 16 January 2016 26 March 2018            |
| SPINDLE GUIDE ASS| 16 January 2016 26 March 2018            |
| SPACER RING      | 26 August 2018 12 January 2023          |
| SCRAPER RING     | 02 August 2018 19 December 2022         |
| O-RING - 4181145 | 12 July 2018 28 November 2022           |
| O-RING - 4181452 | 02 August 2018 19 December 2022         |
| O-RING - 4181455 | 02 August 2018 19 December 2022         |
| SLIDE VALVE      | 25 August 2018 11 January 2023          |
| SPRING           | 25 August 2018 11 January 2023          |
| THRUST PIECE     | 25 August 2018 11 January 2023          |
| CYLINDER COMPLETE| 21 August 2018 07 January 2023          |

TABLE V. The date of spare part’s ordering for cylinder 4.

IV. Conclusions

Petri net can be used to represent a process of ship engine component’s spare part ordering. Based on the Petri net model, we can build a max-plus-algebra model to find the maximal time of spare part ordering. Therefore, we obtain the date when the spare part should be ordered.

REFERENCES

[1] Subiono, Aljabar Min-Max-Plus dan Terapannya. Surabaya: Institut Teknologi Sepuluh Nopember, 2015.
[2] D. Indriyani and Subiono, “Scheduling of the crystal sugar production system in sugar factory using max-plus algebra,” International Journal of Computing Science and Applied Mathematics, vol. 2, no. 3, pp. 33-37, 2016.
[3] F. Baccelli, G. Cohen, G. Olsder, and J.-P. Quadrat, Synchronization and linearity: an algebra for discrete event systems. John Wiley & Sons Ltd, 1992.
[4] Subiono and K. Fahim, “On computing supply chain scheduling using max-plus algebra,” Applied Mathematical Sciences, vol. 10, no. 10, pp. 477-486, 2016.