TRIPLET HIGGS BOSONS AT $e^+e^-$ COLLIDERS

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ABSTRACT

We investigate the possibility of probing the triplet Higgs boson sector via single charged Higgs production in the process $e^+e^- \rightarrow H^+\ell\nu_\ell$, at high energy electron-positron colliders, using the tree level $H^+W^-Z$ coupling which is a unique feature of such models. We find that even LEP-200 can give nontrivial information upto $M_{H^\pm} \simeq 120$ GeV if the doublet -triplet mixing is not restricted by the current value of the $\rho$ parameter which is the case in models with a custodial symmetry. Further we point out that in such models, the 4-body, tree level decay $H^+ \rightarrow W^*Z^* \rightarrow 4$ fermions dominates and hence provides a very clean signal when the four fermions are leptons. At NLC the discovery range for the charged Higgs in triplet models via this process is $\sim 400$ GeV.

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There is no evidence as yet of the existence of the scalar particle(s) which are essential for the spontaneous breakdown of the $SU(2)_L \otimes U(1)_Y$ electroweak symmetry. Although a single scalar $SU(2)$ doublet suffices for the purpose, it is by no means compulsive for elementary scalars, if they exist at all in nature, to be restricted to one doublet only. Theories with two or more doublets are often explored in this spirit \cite{1}. Also, it is a tenable hypothesis that there are not only doublets but also higher representations of $SU(2)$, such as triplets, comprising scalar particles \cite{2}. It is particularly interesting that whereas all the charged fermions must get their masses via Yukawa couplings with Higgs doublets, the vacuum expectation values (vev) of triplets can give masses to neutrinos without requiring any “sterile” right-handed neutrino species. Since the existence of nonzero neutrino masses is suggested by phenomena such as the solar neutrino puzzle \cite{3}, it might be useful to consider this possibility of a different origin of neutrino masses, which in turn might help us understand why they are so small compared to those of the other fermions.

However, one is faced with the problem that the vev of the neutral member of the triplet gives additional contributions to the parameter $\rho = m_W^2/(m_Z^2 \cos^2 \theta_W)$ (where $\theta_W$ is the Weinberg angle) at the tree-level, tending to change its value of unity which is guaranteed if the representation of the Higgs is restricted to only doublets. Since the present experimental value of $\rho$ is $(1.0004 \pm 0.0022 \pm 0.002)$ \cite{4}, any scenario with scalar triplets must be constrained accordingly.

One option to build this constraint into the model is to postulate that the vev of the neutral member of the triplet is small enough so that its contribution to $\rho$ is within the experimental limits. This smallness can then be translated into an upper limit on the mixing angle between the doublet and the triplet.

The other option is to make the ingenious assumption, first suggested by Georgi and Machacek \cite{5} and by Chanowitz and Golden \cite{6}, that there are in fact more than one triplets, arranged in such a manner that their contributions to the $\rho$-parameter cancel each other. In
order that this may happen, one needs to have one complex triplet \( \Delta \) (with hypercharge \( Y=2 \)) and one real \( \chi \) (\( Y=0 \)) triplet, in addition to the \( Y=1 \) complex doublet of the minimal standard model. Furthermore the vev’s of these two triplet fields must be equal to guarantee \( \rho = 1 \). Therefore, there is no restriction on the vev of the triplet and hence on the doublet-triplet mixing.

In this paper, we examine some testable consequences of the existence of triplets in both these situations. Several such studies have already been done in recent times in the context of \( e^+e^- \) as well as of hadronic colliders [7], [8]. However, we focus here on a specific interaction which is a distinguishing feature of triplet scalars (or those of higher representations), namely, a tree-level interaction involving the \( W \), the \( Z \) and a charged scalar and which has not been studied in detail before. This interaction vertex is absent with an arbitrary number of Higgs doublets added to the particle spectrum (for loop induced \( H^+W^-Z \) vertex see [9]). The possibility of constraining the triplet sector using the above tree level coupling in \( Z \)-factories has been discussed earlier by one of the present authors [10]. Here we investigate its observable consequences in LEP-II and more energetic \( e^+e^- \) colliders. As we shall see below, it leads to some very characteristic signals which are considerably free from standard model backgrounds.

Let us first summarize the main features of the scalar sector for the two possibilities mentioned above. With a complex triplet (\( Y = 2 \)) \( \Delta \) and a doublet \( \phi \) given by

\[
\Delta = \begin{pmatrix}
\Delta^{++} \\
\Delta^+ \\
\Delta^0
\end{pmatrix}, \quad \phi = \begin{pmatrix}
\phi^+ \\
\phi^0
\end{pmatrix}
\]

(1)

the most general Higgs potential including \( \Delta \) and \( \phi \) reads

\[
V_{\phi\Delta} = \mu_1^2(\phi^\dagger\phi) + \mu_2^2(\Delta^\dagger\Delta) + \lambda_1(\phi^\dagger\phi)^2 + \lambda_2(\Delta^\dagger\Delta)^2 + \lambda_3|\Delta^T C' \Delta|^2 + \lambda_4(\phi^\dagger\phi)(\Delta^\dagger\Delta)
\]
\[ + \lambda_5 (\Delta^\dagger T_i \Delta) (\phi^\dagger \frac{T_i}{2} \phi) - i \lambda_6 \left( \Delta_P^i (\phi^T \frac{T_i}{2} \phi) + \text{h.c.} \right) \]  \hspace{1cm} (2)

where \( \Delta_P = P^\dagger \Delta \) and \( P \) is a unitary matrix which relates the real 3-dimensional \( SU(2) \) representation with generators \( (\hat{T}_j)_{ik} = i \epsilon_{ijk} \) to the equivalent representation given by \( T_i = P \hat{T}_i P^\dagger \) with diagonal \( T_3 \). The matrices \( P \) and \( C' \) are then

\[
P = \begin{pmatrix}
-1/\sqrt{2} & i/\sqrt{2} & 0 \\
0 & 0 & 1 \\
1/\sqrt{2} & i/\sqrt{2} & 0
\end{pmatrix}, \quad C' = P^* P^\dagger = \begin{pmatrix}
0 & 0 & -1 \\
0 & 1 & 0 \\
-1 & 0 & 0
\end{pmatrix} \hspace{1cm} (3)
\]

Note here that the potential of eq. (2) breaks lepton number explicitly and the model therefore contains no majoron in the physical particle spectrum \( [11] \) (a triplet majoron is now excluded by LEP measurements). Taking the vacuum expectation values to be \( < \phi^0 > = v \) and \( < \Delta^0 > = w \) the \( \rho \) parameter in this model is \( \rho = (1 + \frac{2w^2}{v^2})/(1 + \frac{4w^2}{v^2}) \). The current experimental constraint mentioned above translates (at 99% confidence level) to

\[
\frac{w}{v} \leq 0.066 \hspace{1cm} (4)
\]

The mass eigenstates for the charged scalars can be obtained from

\[
\begin{pmatrix}
H'^+ \\
G^+
\end{pmatrix} = \begin{pmatrix}
-s_{H'} & c_{H'} \\
-c_{H'} & s_{H'}
\end{pmatrix} \begin{pmatrix}
\phi^+ \\
\Delta^+
\end{pmatrix} \hspace{1cm} (5)
\]

where \( G^+ \) is the would-be Goldstone boson. The mixing angles are \( (s_{H'} \equiv \sin \theta_{H'} \text{ etc.}) \)

\[
s_{H'} = \frac{\sqrt{2}w}{\sqrt{v^2 + 2w^2}}, \quad c_{H'} = \frac{v}{\sqrt{v^2 + 2w^2}} \hspace{1cm} (6)
\]

It is worth noting here that in this case the \( H'^+ \) has tree level couplings to fermions through the doublet component. The mass of the charged boson is given by

\[
M_{H'^+}^2 = \left( \frac{\lambda_5}{2} + \frac{\lambda_6}{\sqrt{2}w} \right) (v^2 + 2w^2) \hspace{1cm} (7)
\]
Since two scales, \( v \) and \( w \), are involved the dimensional coupling constant \( \lambda_6 \) can be either \( O(1)w \) or \( O(1)v \). In the former case we expect the charged Higgs boson mass to be \( \sim O(v^2) \) whereas in the latter case this charged particle can be much heavier \( \sim O(wv^2) \).

It is known that one has to worry about hierarchy problems because of the large splitting of the two vacuum expectation values implied by eq. (4). One way to avoid this problem would be to supersymmetrize the model (for supersymmetric model with a real \( Y = 0 \) triplet see [12]). While discussing below the phenomenological consequences of triplet models we will consider the model given by eq. (2) as a representative of models with small triplet vev (e.g. a supersymmetric version of the same). Yet another solution to the above mentioned problem suggested by the authors of refs. [3], [6] is to add a real triplet field \( \chi \) \((Y = 0)\)

\[
\chi = \begin{pmatrix} \chi^+ \\ \chi^0 \\ \chi^- \end{pmatrix}
\]

and impose on the Higgs potential a \( SU(2)_L \otimes SU(2)_R \) symmetry which at tree level forces the two vacuum expectation values, \( < \Delta^0 > \) and \( < \chi^0 > \) to be equal. As a result the \( \rho \) parameter is one at tree level. The fields carrying two group indices can be conveniently represented by

\[
\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix}, \quad \xi = \begin{pmatrix} \Delta^0 & \chi^+ & \Delta^{++} \\ \Delta^- & \chi^0 & \Delta^+ \\ \Delta^{--} & \chi^- & \Delta^{0*} \end{pmatrix}
\]

The corresponding Higgs potential is then

\[
V_{\phi\Delta\chi} = \bar{\mu}_1^2 \text{Tr}(\Phi^\dagger \Phi) + \bar{\mu}_2^2 \text{Tr}(\xi^\dagger \xi) + \bar{\lambda}_1 \text{Tr}(\Phi^\dagger \Phi)^2 + \bar{\lambda}_2 \text{Tr}(\Phi^\dagger T_i \Phi T_j) + \bar{\lambda}_3 \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(\xi^\dagger \xi)
\]

\[
+ \bar{\lambda}_4 \text{Tr}(\xi^\dagger \xi \xi^\dagger \xi) + \bar{\lambda}_5 \text{Tr}(\Phi^\dagger \Phi \frac{T_i T_j}{2}) \text{Tr}(\xi^\dagger T_i \xi T_j)
\]

\[
+ \bar{\lambda}_6 \text{Tr}(\Phi^\dagger \Phi \frac{T_i T_j}{2}) \xi^\dagger \xi + \bar{\lambda}_7 \text{Tr}(\xi^\dagger T_i T_j \xi) \xi^\dagger \xi (10)
\]
where \( \xi_P = P^\dagger \xi P \). The trilinear terms proportional to \( \tilde{\lambda}_6 \) and \( \tilde{\lambda}_7 \), are often dropped by requiring the discrete symmetry \( \Delta \to -\Delta \) and \( \chi \to -\chi \). This symmetry then has to be implemented also on the Yukawa term

\[
L_{\nu}\Delta = ih_{ab} \bar{\psi}^T_{aL} C \tau^i_i \frac{\tau_2}{2} \psi_{bL} \Delta^i_P + \text{h.c.,} \quad \psi_i = \begin{pmatrix} \nu_i \\ l_i \end{pmatrix}
\]

resulting in a neutrino spectrum with one Weyl and one massive Dirac neutrino. Hence, in principle, the model with the discrete symmetry has different physical consequences as compared to the more general model of eq. (10) as far as the neutrino mass spectrum is concerned.

Diagonalising the mass matrix of the scalar sector, in general one finds, after duly absorbing the Goldstone bosons as longitudinal components of gauge fields, a 5-plet, \( H^+_5 \), a 3-plet, \( H^+_3 \), and two singlets, \( H^0_1 \) and \( H^0_1' \), as the physical states. The different multiplets characterise their respective transformation properties under the custodial \( SU(2) \). The members of the \( H_5 \)-plet are given by (for further details on the particle spectrum and couplings see [8])

\[
\begin{align*}
H^+_5 &= \Delta^{++} \\
H^+_5 &= (\Delta^+ - \chi^+)/\sqrt{2} \\
H^0_5 &= (2\chi^0 - \sqrt{2}\Delta^0)/\sqrt{6}
\end{align*}
\]  

Clearly, none of them have any overlap with the components of the doublets \( \phi \), and as such they do not couple to fermions at the tree-level in contrast to the case of the physical charged Higgs \( H^+ \) of eq. (3) of the earlier model. The mass of the 5-plet members is given by

\[
M_5^2 = 8\tilde{\lambda}_4 w^2 - 3\tilde{\lambda}_5 v^2 - \frac{1}{2} \tilde{\lambda}_6 \frac{v^2}{w} 
\]

One constraint on the above scenario arises from the lepton-lepton couplings that the
complex Y=2 triplet possesses (see eq. (11)). The mass thus acquired by the electron neutrino will, for example with diagonal $h_{ab}$, be given by

$$M_{\nu e} = h_{ee} \frac{s_H M_W}{2 g}$$

(14)

with

$$s_H = \frac{2w}{\sqrt{v^2 + 4w^2}}$$

(15)

and $g$ is the $SU(2)$ coupling constant. The experimental constraints from neutrinoless double beta-decay imply that $M_{\nu e} < 1eV$. This means that either the doublet-triplet mixing angle or the $\Delta L = 2$ Yukawa coupling is restricted to very small values. Since in this model $\rho = 1$ at tree level a large mixing angle $s_H$ is apriori not excluded. A large vev $w$ will imply of course larger contribution to the gauge boson mass coming from the triplet sector. We will comment on this further at the end of the paper. It has also been shown that so far as scalar interactions are concerned, one can maintain the equality of vev’s at higher orders. However, no way has yet been found to prevent the custodial symmetry from being broken at the loop-level by $U(1)$ gauge couplings. This means that fine-tuning is required to make such a model work. In any case, as has been discussed in ref. [8], the degree of such fine-tuning required is not higher than that involved in connection with the naturalness problem within the standard model itself (see also [13] in this connection).

As has been mentioned before, in both the models discussed above the $H^+W^-Z$ coupling exists at the tree level. The lagrangians are

$$\mathcal{L}_{H^+W^-Z}^{(1)} = -\frac{g M_W}{\cos \theta_W} s_H c_{H'} H'^+ W^- Z^\mu + h.c.$$  

$$\mathcal{L}_{H^+W^-Z}^{(2)} = -\frac{g M_W}{\cos \theta_W} s_H H_5^+ W^- Z^\mu + h.c.$$  

(16)

We now focus on the production of $H^\pm (H'^\pm$ or $H_5^\pm$) via the above interactions. First, there is the s-channel process $e^+e^- \rightarrow Z^* \rightarrow WH^\pm$ which will dominate at lower energies.
Here we concentrate upon the final states consisting of the leptonic decay products of the W, i.e. on \( e^+e^- \rightarrow H^\pm l\bar{\nu}_l \). In such cases, the \( e\nu_e \) final state also receives contributions from a t-channel diagram with the \( H^\pm \) emitted from the propagator. As we shall see later, for high values of the centre-of-mass energy, this latter diagram is dominant.

Fig. 1 shows the cross-sections for \( H^\pm l\nu_l \)-production plotted against \( M_{H^\pm} \) for different values of \( \sqrt{s} \). The contributions due to the electronic and muonic final states are added. The curves correspond to \( s_H = 1 \) and \( c_H s_{H'} = 1 \) for the two cases of \( H_5^\pm \), \( H'^\pm \) respectively. The cross-sections for various values of the mixing angles can be read off by multiplying by the appropriate value of \( s_{H'}^2 (s_{H'}^2 c_{H'}^2 \sim s_{H'}^2) \). It is straightforward to see from the graphs that in the LEP-II case (thin solid line), assuming an integrated luminosity of \( 10^{39} cm^{-2} \) per year there will be a few hundreds of events for \( s_H = 1 \) upto at least \( M_{H_5} = 110 GeV \). If now one has only the \( Y = 2 \) triplet, the restriction from the \( \rho \)-parameter allows a maximum \( s_{H'}^2 \) of 0.009. This leaves us with about 2 events per year. In such a case, the only reasonable chance of observing this process exists in a higher-energy \( e^+e^- \) machine. As can be seen from the plot (dotted line), such a machine with a luminosity of \( 50 fb^{-1}/year \) can lead to few tens of events upto \( M_{H'} \simeq 300 GeV \) even with a value of \( s_{H'} \) well within the limits imposed by \( \rho \). For sake of completeness we also show the expected cross section for higher values of \( \sqrt{s} \) (1 TeV (dashed line) and 2 TeV (thick solid line)). On the other hand, since this restriction gets relaxed if one presupposes a complex and a real triplet, at LEP-II itself one can investigate a large area of the \( \rho - s_H \) parameter space in the latter scenario.

It is instructive to note here that while the s-channel diagram dominates at lower beam-energies (\( \sqrt{s} = 200 \) GeV) and hence the cross-sections for the electronic and muonic final states are not too different (shown in fig. 2 by the solid and dashed line, respectively), at higher energies the t-channel clearly dominates. For example, the cross-section for the electronic final state (shown by solid line) is at least a factor 5 larger than that for muonic
final state (dashed line) already at $\sqrt{s} = 500 \text{ GeV}$ for $M_{H^\pm} \leq 150 \text{ GeV}$. For higher values of $\sqrt{s}$ the cross-sections is almost completely dominated by the process $e^+e^- \rightarrow e\nu_e H^\pm$

The signals of the triplet scalars thus produced also of course depend in a rather crucial manner on their subsequent decay channels. For the (complex+real) case, as has already been mentioned, the charged scalars $H_5^\pm$ do not have tree-level interactions with fermions. Possibilities of observing them through loop-induced decays into fermion pairs have already been studied \[\text{[8]}\]. However, we would like to point out here that there also exists the tree-level decays into four-fermions mediated by a $W$ and a $Z$ coupling with $H_5$. So long as $M_{H_5} < M_W$, both the $W$ and the $Z$ in this decay are virtual. We have explicitly calculated such decay widths using methods described in ref. \[\text{[15]}\] modified appropriately. The width up to mixing angles is given by

$$\Gamma(H^+ \rightarrow W^*Z^* \rightarrow f_1\bar{f}_1f_2\bar{f}_2) = \frac{g_V^6 m_W^2}{2^9 \times 9 \times M_{H^+}^3 \times (2\pi)^5 \times \cos^4 \theta_W} (g_V^2 + g_A^2) \times \int_0^{M_{H^+}^2} dQ_1^2 \int_0^{(M_{H^+} - \sqrt{Q_1^2})^2} dQ_2^2 \lambda^{1/2}(M_{H^+}^2, Q_1^2, Q_2^2) \times [8Q_1^2Q_2^2 + (M_{H^+}^2 - Q_1^2 - Q_2^2)^2] B_Z(Q_1^2) B_W(Q_2^2)$$

(17)

where $g_V = T_{3L}^{f_1} - 2Q_{f_1} \sin^2 \theta_W$, $g_A = -T_{3L}^{f_1}$, $B_V(Q^2) = [(Q^2 - M_V^2)^2 + M_V^2 \Gamma_V^{-2}]^{-1}$ and $\lambda(x, y, z)$ is the kinematical triangle function. We show the decay widths into this channel for leptonic final states (with $s_H = 1$) as a function of $M_{H_5}$ by the dashed line in fig. 3. Here we have summed over the muons and the electrons in the final state. For purposes of comparison we also show by the dotted line one of the representative results for the loop induced partial width $\Gamma(H_5^+ \rightarrow c\bar{s})$ taken from ref. \[\text{[8]}\] (also drawn there for $s_H = 1$). The figure clearly shows that indeed the partial decay width for our four-fermion modes are of similar orders for $M_{H_5} \sim 50 \text{ GeV}$, and completely dominate for higher values of $M_{H_5}$. \[\text{[8]}\] When $M_{H_5}$ becomes

\[\text{[1]}\] It should be pointed out here that even for $M_{H_5}$ as small as 25 GeV, the four-fermion width (summed over leptons and light quarks) is somewhat larger than the loop induced $\Gamma(H_5 \rightarrow c\bar{s})$. If we further remember
greater than $M_H$, the loop-induced $W\gamma$ channel also opens up for the $H_5$. However, if we take the numbers for the partial decay width for this channel given in [16] as a very rough guideline, we find that the decay mode into the four fermions (via two gauge bosons (real or virtual)) does indeed dominate. Consequently, the branching ratio for decays into a pair of weak gauge-bosons (WZ) remains close to 100 per cent over most of the parameter space under scrutiny. The 4-lepton ($\nu l^+l^-l^-$) channel then has a healthy branching ratio of $\sim 1.4\%$ The corresponding signal is practically free from standard model backgrounds, and thus it should be considered as the principal technique in looking for triplet Higgs bosons (in this model) produced in $e^+e^-$ collider experiments. Thus using this clean lepton channel, for $s_H = 1$, LEP-200 (NLC) can have a discovery range upto $M_{H_5} = 100(350)\text{ GeV}$. The signal with four jets in the final state might seem hopeless at first glance when one thinks of multijets coming from gauge boson pair production and their decays. However, it should be remembered that the invariant mass of the four jets will be quite different from $\sqrt{s}$ and hence even the four jet final states can be used increasing the useful branching ratio to $\sim 50\%$. This would increase the discovery range of $H_5^\pm$ even further since the width into four quarks final state will be a factor $\sim 35$ higher than the four lepton channel. Hence, the discovery range at LEP-200 (NLC) gets extended to 125 (425) GeV, for $s_H = 1$. It should also be pointed out that the $e^-/e^+$ coming from the reaction $e^+e^- \rightarrow e^+H_5^-\bar{\nu}_e(c.c.)$ is likely to be lost in the beam pipe.

For the case of model 1 (i.e. $H'^+$) of course the two-fermion decay-modes can occur at tree level just like the $WZ$ decay and are thus not loop suppressed though suppressed by the mixing angle $s_{H'c_{H'}}$. As can be seen from $\Gamma(H'^+ \rightarrow c\bar{s})$ given by the solid line in fig. 3 (and dotted line in fig. 4) this decay mode will dominate over the four-lepton decay mode upto that the loop induced $\Gamma(H_5 \rightarrow \tau\nu_\tau)$ is even smaller than that for $c\bar{s}$ final state [8], this implies that the normal search strategies for a charged Higgs at LEP might have missed such a charged Higgs.
\[ M_{H'^+} = 150 \text{ GeV}. \] However the width for the 4-quark decay mode (mediated by \( WZ \) (real or virtual)), shown by dashed line in fig. 4, will be comparable to the two quark decay mode for \( M_{H'^+} \geq 120 \text{ GeV} \) as can be seen from fig. 4. For still heavier \( H'^+ \) (once the \( WZ \) channel is allowed kinematically and the \( H'^+ \rightarrow t\bar{b} \) opens up) these two decay modes (shown by the thin and thick solid line respectively in fig. 4) will take over and eventually the branching ratio into the clean 4-lepton channel will be \( \sim 0.7\% \) level. Since all the relevant decays occur only through the mixing the actual value of the mixing angle \( s_{H'} \) is immaterial for gauging the relative strengths of different channels. Of course, the statements about the tree-level decays into \( WZ, t\bar{b} \) and the four jet channels, made above are true for the case of \( H_5^+ \) as well.

At the end, we would like to make a comment regarding the production of doubly charged Higgs bosons which form an integral part of the triplet models discussed here. If one considers the process \( e^+e^- \rightarrow H^{++}l^-l^- \) (or its charge conjugate), the production rate is proportional to the square of the \( H^{++} \)-lepton-lepton coupling strength. This strength is bounded above from the non-observation of neutrinoless double beta decay. As is evident from eqs. (14,15), the maximum value of the coupling is thus inversely proportional to the quantity \( s_H \). A rather model-independent limit can be obtained from indirect contributions of \( \Delta^{++} \) to Bhabha scattering (t-channel exchange) [17]

\[
\frac{h_{ee}}{M_{H^{++}e}^2} < 1.9 \times 10^{-5} \text{ GeV}^{-2}
\] (18)

Therefore, for very small \( s_H \), the production cross-section for a single doubly-charged Higgs becomes less restricted. In the extreme cases, \( h_{ee} \) very small or \( w/v \) very small, there is an interesting dichotomy which can be used for experimental studies: either the \( HWZ \) coupling is sizeable or the coupling strength of \( \Delta^{++} \) to leptons is non-negligible. A detailed exploration of the signal of a such a singly-produced doubly charged Higgs is thus advisable. Although this production mechanism has been studied previously [18], some relevant diagrams have
been left out, which are potentially important for a high-energy $e^+e^-$ machine. A study of the full process is currently under way.

In conclusion, The tree level $HWZ$-coupling in the triplet model is found to be a rather interesting way of either uncovering or ruling out such a scenario. For a model with only complex triplets, LEP-II can have at best a marginal glimpse of the allowed region of the parameter space, and machines with higher energy and luminosity are required for a closer survey. On the other hand, with one complex and one real triplet a considerably large region of the allowed parameter space is likely to come within the purview of LEP-II, with conspicuous and testable signals.

**Acknowledgements.** We thank J.W.F.Valle, A.S.Joshipura and S.Rindani for useful discussions. We also thank the Department of Science and Technology (India) and the organisers of WHEPP-III held at Madras (India) where this work was started. R.G. and B.M. would like to thank the International Centre of Theoretical Physics for hospitality where this work was completed. The work of R.G was supported in part by the Council for Scientific and Industrial Research under a research grant. M.N. wishes to acknowledge the financial support by the HCM program under EEC contract number CHRX-CT 920026.
Figure Captions:

**Fig. 1.** $\sigma(e^+ e^- \rightarrow l\nu lH^\pm)$ (pb) as a function of $M_{H^+}$ (for $s_H(s_{H^+}c_{H^+}) = 1$) for different values of $e^+ e^-$ center of mass energies: the thin solid, dotted, dashed and thick solid line correspond to $\sqrt{s} = 200$ GeV, 500 GeV, 1 TeV and 2 TeV respectively. Cross-sections for both charges of the Higgs and muonic as well as electronic channels in the final state are added.

**Fig. 2.** $\sigma(e^+ e^- \rightarrow e\nu e H^\pm)$ (solid line) and $\sigma(e^+ e^- \rightarrow \mu\nu \mu H^\pm)$ (dashed line) for $\sqrt{s} = 200$ GeV and 500 GeV, for same values of the mixing angle as in fig. 1 and again summed over the charge of the Higgs.

**Fig. 3.** The different possible two body decay widths ($H_5^+ \rightarrow c\bar{s}$) at loop level (dotted line), tree level (solid line) and the tree level leptonic four body decay width $\Gamma(H_5^+ \rightarrow W^*Z^* \rightarrow 4$ leptons) (dashed line) for the charged Higgs. Again the mixing angles are put equal to 1 as in fig. 1. The $\Gamma(H_5^+ \rightarrow c\bar{s})$ (loop) has been taken from ref. [8]. The four body decay width is summed over electrons and $\mu's$ in the final state. For four quark final state the numbers are obtained by multiplying the dashed figure by $\sim 35$.

**Fig. 4.** The different possible tree level decay widths ($H_{R}^+ \rightarrow c\bar{s}$) (dotted line), $\Gamma(H_{R}^+ \rightarrow W^*Z^* \rightarrow 4$ quarks) (dashed line), $\Gamma(H_{R}^+ \rightarrow W^+Z)$ (thin solid line) and $\Gamma(H_{R}^+ \rightarrow t\bar{b})$ (thick solid line) for the charged Higgs. Again the mixing angles are put equal to 1 as in fig. 1. $m_b$ has been neglected.
References

[1] The standard reference for extended Higgs sectors is J. Gunion, H. E. Haber, G. Kane and S. Dawson, ‘The Higgs Hunter’s Guide’, Addison-Wesley Publishing Company 1990; for a recent study of a two Higgs doublet model see for example A. Pilaftsis and M. Nowakowski, Int. J. Mod. Phys. A9 (1994) 1097

[2] H. M. Georgi, S. L. Glashow and S. Nussinov, Nucl. Phys. B193 (1981) 297

[3] see e.g. S. M. Bilenky and S. T. Petcov, Rev. Mod.Phys. 59 (1987) 671; J. W. F. Valle, Prog. Nucl. Phys. 26 (1991) 91

[4] Review of Particle Properties, Phys. Rev. D50 (1994).

[5] H. Georgi and M. Machacek, Nucl. Phys. B262 (1985) 463

[6] S. Chanowitz and M. Golden, Phys. Lett. B165 (1985) 105

[7] R. Vega and D. Dicus, Nucl. Phys. B329 (1990) 533

[8] J. F. Gunion, R. Vega and J. Wudka, Phys. Rev. D42 (1990) 1673; D43 (1991) 2322

[9] J. A. Grifols and A. Mendez, Phys. Rev. D22 (1980) 1725; M. C. Peyranere, H. E. Haber and P. Irulegui, Phys. Rev. D44 (1991) 191

[10] B. Mukhopadhyaya, Phys. Lett. B252 (1992) 123

[11] G. B. Gelmini and M. Roncadelli, Phys. Lett. B99 (1981) 411

[12] J. R. Espinosa and M. Quiros, Nucl. Phys. B384 (1992) 113

[13] P. Bamert and Z. Kunszt, Phys. Lett. B306 (1993) 335
[14] see, e.g., M. Drees and R. M. Godbole, Z. Phys. C 59 (1993) 591; P. Chen, T. L. Barklow and M. E. Peskin, Phys. Rev. D49 (1994) 3209.

[15] H. Pois, T. J. Weiler and T. C. Yuan, Phys. Rev. D47 (1993) 3886.

[16] S. Raychaudhuri and A. Raychaudhuri, Phys. Rev. D50 (1994) 412.

[17] M. L. Swartz, Phys. Rev. D40 (1989) 1521.

[18] M. Lusignoli and S. Petrarca, Phys. Lett. B226 (1989) 397.
Fig. 1

Fig. 2
Fig. 3

\[ \Gamma(H^+ \rightarrow 4 \text{ leptons (c\bar{s})) \text{ (GeV)}} \]
\[ \Gamma(H'_+ \rightarrow c\bar{s}) \text{ (loop)} \]
\[ \Gamma(H'_+ \rightarrow c\bar{s}) \text{ (tree)} \]

Fig. 4

\[ \Gamma(H'_+ \rightarrow W^*Z^* \rightarrow \text{four q)} \]
\[ \Gamma(H'_+ \rightarrow W^Z) \]
\[ \Gamma(H'_+ \rightarrow t\bar{b}) \]
\[ \Gamma(H'_+ \rightarrow c\bar{s}) \]