Reconstructing late-time cosmology with kinematical models

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Abstract

The present work is based on the reconstruction of late-time cosmological dynamics using purely kinematical models. The models are constructed from different parameterizations of the deceleration parameter. The models are confronted with cosmological observations which are completely independent of any fiducial assumption about the background cosmological model. Different kinematical parameters, namely the present Hubble parameter, present value of the deceleration parameter and the redshift of transition from decelerated to accelerated phase of expansion are constrained by Markov Chain Monte Carlo (MCMC) analysis using the observation data sets. The evolution of different cosmological quantities are studied for the present models. The evolution of linear matter density contrast has been studied for the present kinematical models and the result is compared with the standard cosmological constant dark energy scenario. The thermodynamic nature of the universe has also been emphasized in the present context.

1 Introduction

The observed phenomenon of cosmic acceleration [1, 2, 3] is still an enigma for cosmologists. There are two different direction of finding a theoretical explanation of the alleged accelerated expansion of the universe. Within the framework of General Relativity (GR), the cosmic acceleration can be explained by introducing an exotic component in the energy budget of the universe. It is dubbed as dark energy. Dark energy with its characteristic negative pressure-like contribution can generate the accelerated expansion. The other way to look for a possible explanation cosmic acceleration is the modification of the theory of General Relativity. It is not yet been ascertained whether dark energy or the space-time geometry itself is responsible for the alleged accelerated expansion. But GR is highly successful to explain the local astronomical and cosmological observations than the modified gravity theories.

Though the dark energy cosmology is efficient to explain cosmological observations, there is hardly any certain knowledge about physical entity of dark energy. Various theoretical prescriptions regarding dark energy are there in the literature. The cosmological constant or vacuum energy density [5, 4], scalar field models of dark energy like quintessence [6], k-essence [7], tachyon field [8] etc, fluid model of dark energy like chaplygin gas model [9], are amongs them. Different theoretical aspects of dark energy are comprehensively reviewed byCopeland, Sami and Tsujikawa [10]. The cosmological constant (Λ) model of dark energy along with cold dark matter (CDM) is well consistent with most of the cosmological observations. Hence it is accepted as the standard cosmological paradigm, also dubbed as concordance cosmology. However, there are certain issues that urge to look for alternatives of cosmological constant model. One important theoretical issue is the humongous discrepancy between the observationally estimated value of cosmological constant and the value of vacuum energy density calculated in quantum field theory [5]. The other astonishing fact is the same order of magnitude value of cosmological constant and the matter energy density at present epoch. It is the cosmic coincidence problem. Due to these theoretical issues, time-evolving

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dark energy have gained attention in this context. Some recent cosmological observations like the Lyman-α forest BAO measurement of Hubble parameter at redshift 2.34 by Baryon Oscillation Spectroscopic Survey (BOSS) [13] and the local measurement of Hubble constant \( (H_0) \) by Hubble Space Telescope (HST) [14][15] are in disagreement with concordance cosmology. Other dark energy models are also not very successful to alleviate these disagreements.

Reconstruction of cosmological model is a reverse engineering based on cosmological observations. The idea is to figure out the evolution of certain cosmological quantities from observational data in parametric or non-parametric fashion. Aspects of reconstruction of dark energy model have been comprehensively reviewed by Sahni and Starobinsky [11]. The model can be reconstructed with some prior assumption about the dynamics of dark energy. Another way of reconstruction of cosmological model is in kinematic approach. Kinematic approach to reconstruct cosmological models are investigated in the present work. A kinematic approach of reconstruction only assumes the homogeneity and isotropy of the universe at cosmological scale. It is independent of any assumption about the dark energy model and even the theory of gravity. A kinematic approach to reconstruct cosmological evolution using a Taylor expansion of the Hubble parameter has been discussed by Mukherjee et al [16]. A model independent approach to constraint the kinematics of late-time cosmology has been discussed by Shafiello et al [17] and by Haridasu et al [18]. Parameterizations of deceleration parameter in the context of late-time cosmology has been discussed by Gong and Wang [19][20]. Campo et al [21] discussed parameterizations of deceleration parameter based on thermodynamical consequences. Kinematic reconstruction using higher order kinematic terms are discussed by Rapetti et al [22], by Zhai et al [23], and by Mukherjee and Banerjee [24][25].

In the present work, the late-time cosmological cosmological evolution is studied through some parametric forms of the deceleration parameter. Deceleration parameter is the dimensionless kinematical parameter that contains the second order time derivative of the scale factor. Deceleration parameter is a measure of cosmic acceleration in a dimensionless way. The present kinematic models are reconstructed keeping that fact in mind that the universe presently has an accelerated expansion phase and the transition form a decelerated to the present accelerated phase occurred in the recent past. The kinematical models are studied based on cosmological observations. Different kinematical parameters, namely the present value of Hubble parameter, the present value of deceleration parameter and the redshift of transition from decelerated to accelerated phase are constrained in the present context. The parameters are constrained through statistical analysis using different observational data sets. The observational measurements of Hubble parameter at different redshift, the distance modulus measurements of type Ia supernovae and the local measurement of the Hubble constant are utilized in this context. These observations are completely independent of any fiducial assumption about the background cosmological model. The evolution of different cosmological parameters are studied for the present models. The present nature of the any dark energy equation of state has also been investigated. It is always important for a cosmological model to produce a viable evolution of cosmological perturbations. The growth of matter density perturbation at linear level has been studied for the reconstructed models. Further we have studied the thermodynamics of the universe for the reconstructed kinematical models. The nature of the total entropy of the universe surrounded by a cosmological horizon, is studied.

In the following section (section 2), the phenomenological parameterizations of the deceleration parameter are discussed. The statistical analysis of the models using cosmological observations and the constrains on kinematical parameters are presented in section 3. Then the evolution of linear matter density contrast is discussed in section 4. In section (section 5), the entropic nature of the universe for the present models are discussed. Finally in section 6, it has been concluded with an overall discussion about the result.

## 2 Reconstruction of the kinematic models

The kinematic approach to reconstruct a cosmological model is purely based on the assumptions of the cosmological principle, that is the universe is spatially homogeneous and isotropic. In a kinematic approach, the parameters defined in terms of the scale factor \( (a(t)) \) and its time-derivatives are utilized to reconstruct the model. It is independent of any prior assumption about the physical nature of dark energy, the distribution of different components in the energy sector and even any assumption about the gravity theory.

The first order kinematic term is the Hubble parameter, defined as, \( H(t) = \frac{\dot{a}}{a} \), where the overhead dot
denotes differentiation with respect to time $t$. Hubble parameter gives the expansion rate at cosmological scale. It is convenient to use the redshift as the argument instead of cosmic time. Redshift is defined as $(1 + z) = \frac{a_0}{a(t)}$, where $a_0$ is the present scale factor. The Hubble parameter can also be presented as a function of redshift. The second order kinematic term, which is the measure of the cosmic acceleration in a dimensionless way, is the deceleration parameter. It is defined as $q(t) = -\frac{\ddot{a}}{a H^2}$. This can be written in terms of Hubble parameter and its derivative with respect to the redshift $z$ as,

$$q(z) = -1 + (1 + z) \frac{H'}{H},$$

(1)

where the "prime" denotes the differentiation with respect to $z$. A positive value of the deceleration parameter indicates the decelerated expansion of the universe and a negative deceleration parameter indicates an accelerated expansion. It has been confirmed by cosmological observation that the universe was going through a decelerated phase of expansion in the past and presently it is in a phase of accelerated expansion. It has also been assured that the transition from decelerated to accelerated phase of expansion happened in recent past [26]. In the present analysis, we have adopted three different parameterizations of deceleration parameter. These parameterizations are purely phenomenological and motivated from the observational facts. These parameterizations are given as, I. $q(z) = q_0 + q_1 \frac{z}{(1+z)}$, II. $q(z) = q_0 + q_1 \frac{z^2}{1+z^2}$, III. $q(z) = q_0 + q_1 \left[ 1 - \frac{1}{(1+z)^2} \right]$. Equation (1) shows that the first integral of $q(z)$ will give the expression of Hubble parameter. For these models, the expressions of Hubble parameter are obtained as,

$$I. \quad H(z) = H_0 (1+z)^{(1+q_0+q_1)} \exp \left( -q_1 \frac{z}{1+z} \right),$$

(2)

$$II. \quad H(z) = H_0 (1+z)^{(1+q_0)}(1+z^2)^{q_1/2},$$

(3)
It is interesting to note that the parameter $q_0$ represents the present value of deceleration parameter in these parameterizations of $q(z)$. But the parameter $q_1$ in the expressions of $q(z)$ are not equivalent. For a better understanding, $q_1$ can be replaced by its expression in terms of $q_0$ and the redshift of transition from decelerated to accelerated phase of expansion ($z_t$). The parameter $q_1$ is related to $q_0$ and $z_t$ for these models as, I. $q_1 = -q_0 \frac{(1+z_t)^2}{z_t}$, II. $q_1 = -q_0 \frac{1+z_t^2}{z_t(1+z_t)}$, and III. $q_1 = -q_0 \frac{(1+z_t)^2}{(1+z_t)^2-1}$. Thus the transition redshift $z_t$ has been utilized as a model parameter in the present context. Statistical analysis of the models are discussed in the following section. The parameters, constrained in the present context, are $h_0 = H_0/(100 km s^{-1} Mpc^{-1})$, the present value of deceleration parameter $q_0$ and the transition redshift $z_t$. The evolution of different cosmological quantities are also studied for the present kinematical models.

### 3 Statistical analysis and constraints on the models

An indispensable part of a reconstruction is the statistical analysis of the model based on observational data set. In the present context, statistical analysis has been carried out using different observational data sets, namely the supernova distance modulus data, observational measurements of Hubble parameter and local measurement of Hubble constant. The supernovae distance modulus measurements data from the Joint Light-curve Analysis (JLA) [27] has been utilized in the present context. The observational measurements of Hubble parameter (OHD) in the redshift range $0.07 < z < 2.36$ that include Cosmic Chronometer measurements [28], measurement of Hubble parameter from baryon acoustic oscillation galaxy distributions [29], measurement of Hubble parameter from Lyman-α forest [30] are also included in the analysis. We have also

$$III. \quad H(z) = H_0(1 + z)^{(1+q_0+q_1)} \exp \left[ \frac{q_1}{2} \left( \frac{1}{(1+z)^2} - 1 \right) \right].$$  (4)
Bayesian statistical inference is adopted here to estimate the posterior probability distribution of the parameters. Bayesian statistical inference suggests that the posterior probability distribution of a parameter is proportional to the distribution of likelihood and the prior information. Uniform prior distributions are adopted in the present analysis. The likelihood incorporates the observation data in the analysis. The parameter values are estimated by Markov chain Monte Carlo (MCMC) analysis using the PYTHON implication of MCMC sampler EMCEE, introduced by Goodman and Weare [32] and by Foreman-Mackey et al. [33]. The parameter space consist of the present Hubble parameter, scaled by 100 km s$^{-1}$ Mpc$^{-1}$ ($h_0$), the present value of the deceleration parameter ($q_0$) and the transition redshift ($z_t$). The parameters values, obtained in the present analysis are given in table 1. The statistical analysis has been carried out for two combinations of the data sets, OHD+JLA and OHD+JLA+R18. It is found that the addition of the local measurement of Hubble constant (R18) increases the value of present Hubble parameter $h_0$. The other two parameters are get slightly changed with the addition of R18 measurement. The present deceleration parameter $q_0$ value decreases and the transition redshift $z_t$ slightly increases with the addition of R18 data. The negative value of the parameter $q_0$ ensures the present accelerated expansion. The transition redshift $z_t$ is found to be $z_t < 1$ for all three kinematic models. It ensures the transition from decelerated to accelerated phase occurred in the recent past. The transition redshift predicted in model 1 is slightly higher that value predicted in other two cases, but all the values are consistent to each other within 1σ uncertainty. The two dimensional (2D) confidence contours and the posterior probability distribution of the parameters for these models are shown in figure [1],[2] and [3]. The parameter $h_0$ and $q_0$ has a sharp negative correlation. On the other hand, $z_t$ is found to be very weekly correlated with $h_0$ and $q_0$. The profile of correlations amongst the parameters are very similar in all three kinematic models, discussed in the present work.

Evolution of $H(z)/(1+z)$ are shown in figure [4]. The curves obtained for the present kinematical models and the ΛCDM cosmology are shown. There is a deviation from the ΛCDM curve at higher redshift. The

Figure 3: Marginalize posterior distribution and the 2D confidence contours of the parameters ($h_0, q_0, z_t$) for the reconstructed kinematical model III, obtained in the analysis combining OHD, JLA and R18. The 1σ, 2σ and 3σ contours on 2D parameter spaces are shown. The best fit values of the parameters and the associated 1σ uncertainties are also shown.
Table 1: Parameter values, obtained in the statistical analysis with different combinations of the data sets. The mean values of the parameters and the associated 1σ uncertainties are given.

| Model | Data Sets     | $h_0$       | $q_0$       | $z_t$       |
|-------|---------------|-------------|-------------|-------------|
| I     | OHD+JLA      | 0.700 ± 0.008 | −0.597 ± 0.064 | 0.809 ± 0.035 |
|       | OHD+JLA+R18  | 0.707 ± 0.007 | −0.640 ± 0.060 | 0.818 ± 0.047 |
| II    | OHD+JLA      | 0.699 ± 0.008 | −0.538 ± 0.058 | 0.740 ± 0.017 |
|       | OHD+JLA+R18  | 0.705 ± 0.007 | −0.576 ± 0.054 | 0.747 ± 0.017 |
| III   | OHD+JLA      | 0.702 ± 0.008 | −0.690 ± 0.075 | 0.726 ± 0.049 |
|       | OHD+JLA+R18  | 0.709 ± 0.007 | −0.736 ± 0.069 | 0.735 ± 0.047 |

Figure 4: Plots of $H(z)/(1+z)$ for the reconstructed models (black solid curves) and the ΛCDM model (dashed curves). The left panel is for model I, middle panel is for model II and right panel is for model III. The values of the parameter are kept at the values obtained in the statistical analysis combining OHD+JLA+R18. The ΛCDM curve is obtained for the parameter values obtained in Planck2018 [12].

deceleration parameter at the best fit and 1σ confidence region are shown in the upper upper panels of figure 5. As already mentioned, the reconstructed kinematical quantities are independent of any assumption of the dynamical nature of the components in the energy budget of the universe and also do not depend on the undertaken gravity theory. In the present context, the dynamical nature of different components in the energy budget of the universe are studied under the regime of GR. The effective equation of state parameter of the total fluid content of the universe is defined as,

$$w_{eff} = \frac{p_{tot}}{\rho_{tot}}.$$  (5)

The total energy density ($\rho_{tot}$) and the total effective pressure ($p_{tot}$) are connected to the expansion rate by the following relations,

$$\frac{\rho_{tot}}{\rho_c0} = \frac{H^2(z)}{H_0^2},$$  (6)

$$\frac{p_{tot}}{\rho_c0} = \frac{H(z)H'(z)}{H_0^2} + \frac{2(1+z)H(z)}{3}.$$  (7)

The $\rho_c0$ is the present critical density, defined as $\rho_c0 = 3H_0^2/8\pi G$. Thus the effective equation of state can be studied for the present kinematical models under the regime of GR. In the lower panels of figure 5 the evolution of $w_{eff}(z)$ are shown. It indicates toward an effective negative pressurelike contribution of the fluid content of the universe at the low redshift regime. At high redshift, the value of $w_{eff}$ rolls towards zero, indicating a dust matter dominated dynamics. Similarly the nature of dark energy equation of state can be studied. The additional assumption required for that is regarding the conservation of different components in the energy budget. In the present context, we assume that the dark energy and dark matter components are independently conserved. The contribution of radiation energy density can be neglected at low redshift regime. Thus in a spatially flat universe, the dark energy density can be expressed as, $\rho_{de}(z)/\rho_c0 = (H(z)/H_0)^2 - \Omega_{m0}(1+z)^3$. The present matter density parameter $\Omega_{m0}$ is defined as, $\rho_{m0}/\rho_c0$. Only the dark energy contributes to the total fluid pressure, thus $p_{de} = p_{tot}$. The dark energy equation of
state parameter \((w_{de} = p_{de}/\rho_{de})\) for the reconstructed kinematical models are shown in figure 6. It reveals a phantom nature of dark energy in the present epoch.

4 Evolution of matter density contrast at linear level

The matter density contrast is defined as, \(\delta_m = \delta\rho_m/\rho_m\), where \(\rho_m\) is the homogeneous matter density at the background and \(\delta\rho_m\) is the deviation from the homogeneous matter density. Due to the gravitational attraction, the matter over-density grows by accumulating mass from the surrounding. The evolution of \(\delta_m\) at linear level is governed by the following equation,

\[
\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G\rho_m\delta_m.
\] (8)

The evolution of \(\delta_m\) becomes non-liner near the gravitational collapse of the over-dense region. Both the linear and non-linear evolution is deeply effected by the background expansion rate. In the present analysis, the evolution of \(\delta_m\) is studied for the reconstructed kinematical models. Equation (8) is numerically studied for the present reconstructed models taking the scale factor as the argument of differentiations instead of time. The initial conditions are fixed at the scale factor \(a = 0.001\), which is close to the era of cosmic microwave background. The initial values are fixed as, \(\delta_m(a_i) = 0.001\) and \(\dot{\delta}_m(a_i) = 0\). The evolution of \(\delta_m\) for the best fit values of the parameter \(q_0\) and \(z_t\) for the present kinematical models are shown in figure 7. Though the evolution pattern is found to be similar, the values of \(\delta_m\), the values are found to be significantly
different for different models. The values of the parameters \(q_0\) and \(z_f\) can be adjusted to obtain the evolution \(\delta_m\) which is close to the \(\Lambda\)CDM cosmology. In figure 8, the \(\delta_m\) curves, which are close to the corresponding \(\Lambda\)CDM curve, are obtained by adjusting the value of \(q_0\) and \(z_f\). It is observed that only the reconstructed model I can produce \(\Lambda\)CDM like evolution of \(\delta_m\) selecting the parameter values from the 1σ confidence region of the parameter space. In case of other two models, the required values of \((q_0, z_f)\) are found to be out of 2σ confidence region on the parameter space.

5 Thermodynamics of the universe

In the present section, we have discussed about the thermodynamics of the universe for the reconstructed kinematical models. The basic idea is to consider the universe as a thermodynamic system bounded by a cosmological horizon. The idea is originated from the blackhole thermodynamics. The thermodynamical properties that hold for a blackhole, are equally valid for a system surrounded by a cosmological horizon \([34, 37, 38]\). The first law of blackhole thermodynamics can be recovered form Friedmann equations for an FLRW universe if the system (universe) is bounded by the apparent horizon. This motivates to select the apparent horizon as the cosmological horizon to study the thermodynamics of the causally connected universe. In an FLRW universe, the apparent horizon \((r_h)\) is defined as, \(r_h = (H^2 + k/a^2)^{-1/2}\) \([38]\). In a spatially flat universe \((k = 0)\), the apparent horizon coincide with the Hubble horizon, that is \(r_h = 1/H\). According to the law of thermodynamics, like any isolated macroscopic system, the entropy of the universe should not be decreasing with the expansion of the universe. The total entropy of the universe \((S)\) can be written as a summation of the entropy of the fluid \((S_f)\), contained in the volume bounded by the cosmological horizon, and the entropy of the surface of the boundary \(S_h\), so \(S = S_f + S_h\). Now thermodynamics ensure \(dS < 0\), where \(n = \ln a\). Another thermodynamic constraint on the entropy of the universe is \(dS < 0\), that ensures if there is an extrema, it is essentially a maxima. The entropy of the horizon is given as,
\[ S_h = \frac{k_B \mathcal{A}}{4l_{Pl}^2}, \]  

(9)

where the Planck length \((l_{Pl})\) is defined as \(l_{Pl} = \sqrt{\hbar G/c^3}\) and \(\mathcal{A}\) is the area of the horizon \(\mathcal{A} = 4\pi r_h^2\). Considering \(\hbar = k_B = c = 8\pi G = 1\), the horizon entropy is expressed as,

\[ S_h = 8\pi^2 r_h^2. \]  

(10)

Further more, we can relate the temperature of apparent horizon \((T_h)\) with its radius as \(T_h = 1/2\pi r_h\) \cite{37,38,39}. The entropy of the fluid content in the volume covered by the apparent horizon \(S_f\) is the summation of the entropy of the cold dark matter, ordinary baryonic matter, radiation and the dark energy. Considering only the cold dark matter and the dark energy as the component in the energy budget, we can write \(S_f = S_{cdm} + S_{de}\). From the first law of thermodynamics,

\[ T dS_{cdm} = dE_{cdm} + p_m dV = dE_{cdm}, \]  

\[ T dS_{de} = dE_{de} + p_d dV, \]  

(11) (12)

where \(T\) is the temperature of the fluid, the total volume is given as, \(V = 4\pi r_h^3/3\) and the energies can be written in terms of respective energy density \(E_{cdm} = 4\pi r_h^3 \rho_{cdm}/3\) and \(E_{de} = 4\pi r_h^3 \rho_{de}/3\). The differentiation of \(S_{cdm}, S_{de}, S_h\) with respect to time can be expressed as,

\[ \{\dot{S}_{cdm}, \dot{S}_{de}, \dot{S}_h\} = \left(\frac{\dot{E}_{cdm}}{T}, \frac{4\pi r_h^2 \dot{r}_h}{T}, 16\pi^2 r_h^2 \dot{r}_h\right). \]  

(13)

With the assumption that fluid and the horizon has the same temperature \((T_h)\), the differentiation of the total entropy can be expressed as,

\[ \dot{S} = \dot{S}_{cdm} + \dot{S}_{de} + \dot{S}_h = 4\pi H r_h^2 \rho_{cdm} + (1 + w_{de}) \rho_{de} \dot{r}_h. \]  

(14)

According to the second low of thermodynamics, \(\dot{S}\) should be positive. From equation \(\ref{14}\) the differentiation of the total entropy \(S\) with respect to \(n = \ln a\) can be expressed as,

\[ S_n = \frac{16\pi^2}{H^2} (H_n)^2. \]  

(15)

Finally, differentiating equation \(\ref{15}\) once more with respect to \(n\) yields

\[ S_{nn} = 2S_n \left(\frac{H_{nn}}{H_n} - \frac{2H_n}{H^2}\right). \]  

(16)

We denote \(\left(\frac{H_{nn}}{H_n} - \frac{2H_n}{H^2}\right) = \Psi\). As already discussed, for a thermodynamic equilibrium, \(S_{nn} < 0\) which ensures \(\Psi < 0\) for a thermodynamic equilibrium. In the present context, we have studied the evolution of \(\Psi\) for the reconstructed kinematical models. The relation of \(\dot{S}\) with other cosmological quantities (equation \(\ref{14}\)) has been emphasized in the context of interacting dark energy by Jamil \textit{et al} \cite{40} and by Pan \textit{et al} \cite{41}. The function \(\Psi\), obtained for the present kinematical models, are shown in figure \(\ref{9}\). The corresponding \(\Lambda\)CDM curve is also shown. The function \(\Psi\) is obtained to be negative in at present and it has a transition from positive to negative value in the past. The evolution of \(\dot{S}\) for the reconstructed kinematical models are found to be roughly consistent with the same for \(\Lambda\)CDM cosmology. But \(\Psi\) is observed to be rapidly decreasing near the present epoch for the reconstructed kinematic models. This behaviour is not observed in the corresponding \(\Lambda\)CDM curve.

6 Conclusion

In the present work, we have focused on a purely kinematic approach to study the late-time dynamics of the universe. The idea is to start with some phenomenological parameterizations of any kinematic quantity. We have utilized the parameterizations of the deceleration parameter. The kinematical parameters \(h_0, q_0, z_t\) are...
Figure 9: Plots of the thermodynamic function $\Psi(a)$, defined in equation \[15\], are shown by the reconstructed models (black solid curves). The left panel is for the reconstructed model I, the middle one is for model II and the right one is for model III. The plots in the middle is shown up to a value $a = 0.6$ as the function $\Psi$ diverges near the present epoch for the model II. The corresponding $\Lambda$CDM nature is shown by the dashed curves.

constrained for the reconstructed models. These parameter values, obtained in the statistical analysis of the models using cosmological data sets, are found to be consistent for different models. Further the dynamics of the universe has been investigated for the reconstructed kinematical models under the regime of GR. The evolution of the effective equation of state of the total fluid content of the universe has been studied. It is apparently clear from the nature of the effective equation of state that the present universe is dominated by the component which has a negative pressurelike contribution and at the high redshift, the universe was dominated by pressureless component. With the assumption of independent conservation of dark energy and dark matter components in a spatially flat universe, the present nature of dark energy equation of state is reconstructed through the kinematic models. The present nature of dark energy equation of state is found to be in phantom regime.

Besides the background evolution for the kinematical models, the evolution of matter perturbation at linear level has also been emphasized. It is important to study the viability of any model which is consistent at background level. The evolution of matter density contrast for the present models are studied. The reconstructed mode I is found to be consistent with the $\Lambda$CDM cosmology at linear level of matter perturbation within $1\sigma$ confidence region. For other two models, the corresponding $\Lambda$CDM evolution of $\delta_m$ remain out of $2\sigma$ confidence region. Thus it clearly makes reconstructed model I preferable over model II and model III.

The thermodynamics of the universe has also been emphasized in the present context. The prime motivation was to check whether the kinematic models fulfill the thermodynamical requirements of the expanding universe. The total entropy of the universe is the summation of the entropy of the cosmological horizon and the entropy of the fluid contained in the volume covered by the horizon. A function $\Psi$ has been defined that determines the evolution of the total entropy. The function $\Psi$ is found to have a transition from positive to negative value. Thus it indicates towards a thermodynamic equilibrium of the universe. This nature is consistent with standard $\Lambda$CDM cosmology.

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