Right-handed Neutrinos in Low-Energy Neutrino-Electron Scattering

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In this paper a scenario admitting the participation of the exotic scalar coupling of the right-handed neutrinos in addition to the standard vector and axial couplings of the left-handed neutrinos in the weak interactions is considered. The research is based on the low-energy \( (\nu_e e^-) \) and \( (\nu_e e^-) \) scattering processes. The main goal is to show how the presence of the right-handed neutrinos in the above processes changes the laboratory differential cross section in relation to the Standard Model prediction. Both processes are studied at the level of the four-fermion point interaction. Neutrinos are assumed to be polarized Dirac fermions and to be massive. In the laboratory differential cross section, the new interference term between the standard vector coupling of the left-handed neutrinos and exotic scalar coupling of the right-handed neutrinos appears which does not vanish in the limit of vanishing neutrino mass. This additional contribution, including information on the transverse components of neutrino polarization, generates the azimuthal asymmetry in the angular distribution of the recoil electrons. This regularity would be a signature of the presence of the right-handed neutrinos in the neutrino-electron scattering.

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The \( (V - A) \) structure of weak interactions describes only what has been measured so far. We mean here the measurement of the electron helicity \( \bar{e} \), the indirect measurement of the neutrino helicity \( \bar{v} \), the asymmetry in the distribution of the electrons from \( \beta \)-decay \( \bar{e} \) and the experiment with muon decay \( \bar{e} \) which confirmed parity violation \( \bar{v} \). Feynman, Gell-Mann and independently Sudarshan, Marshak established that only left-handed vector \( V, A \) couplings can take part in weak interactions because this yields the maximum symmetry breaking under space inversion, under charge conjugation; the two-component neutrino theory of negative helicity; the conservation of the combined symmetry \( CP \) and of the lepton number. In consequence it led to the conclusion that produced neutrinos in \( V - A \) interaction can only be left-handed. However Wu \( \bar{e} \) indicated that both standard left-handed \( (V, A)_L \) couplings and exotic right-handed \( (S, T, P)_R \) couplings may be responsible for the negative electron helicity observed in \( \beta \)-decay. It would mean that generated neutrinos in \( (S, T, P) \) interactions may also be \textit{right-handed} (antineutrinos \textit{left-handed}, respectively). Recent tests do not provide a unique answer as to the presence of the exotic weak interactions.

So Shimizu et al. \( \bar{e} \) determined the ratio of the strengths of scalar and tensor couplings to the standard vector coupling in \( K^+ \to \pi^0 + e^+ + \nu_e \) decay at rest assuming the only left-handed neutrinos for all the interactions. Their results indicated the compatibility with the Standard Model (SM) \( \bar{e} \) \( \bar{e} \) \( \bar{e} \) \( \bar{e} \) prediction. Bodek et al. at the PSI \( \bar{e} \) looked for the evidence of the violation of time reversal invariance measuring \( T \)-odd transverse components of the positron polarization in \( \mu^- \) decay. They also admitted the presence of the only left-handed neutrinos produced in the standard \( V - A \) and scalar interactions. The recent results presented by the DELPHI Collaboration \( \bar{e} \) concerning the measurement of the Michel parameters and the neutrino helicity in \( \tau \) lepton decays indicated the consistency with the standard \( V - A \) structure of the charged current weak interaction. However on the other hand, the achieved precision of measurements still admits the deviation from the pure \( V - A \) interaction, i.e. the possible participation of the exotic couplings of the right-handed neutrinos beyond the SM. There exist the models of the spontaneous symmetry breaking under time reversal in which the non-standard scalar weak interaction can appear \( \bar{e} \). Recently Berezhiani et al. \( \bar{e} \) analysed the scenario with the participation of the non-standard interactions of neutrinos with electrons in the case of solar neutrinos.

It is necessary to carry out the new high-precision tests of the Lorentz structure and of the handedness structure of the weak interactions at low energies in which the \textit{transverse components of the neutrino polarization} would be measured, because in the conventional observables the interference terms between the standard \( (V, A)_L \) and the exotic \( (S, T, P)_R \) couplings vanish in the limit of vanishing neutrino mass \( \bar{e} \) \( \bar{e} \) \( \bar{e} \) \( \bar{e} \). Frauenfelder et al. \( \bar{e} \) pointed out that one has to measure either the neutrino polarization (spin) or the neutrino-electron correlations to determine the full Lorentz structure of the weak interactions. Because the direct measurement of the transverse neutrino polarization is difficult now, the low-energy neutrino-electron scattering as the detection process of the exotic couplings of the right-handed neutrinos can be used. In practice, the low-energy strong and polarized neutrino source (e. g. \( ^{51}Cr \) or other artificial neutrino sources) could be used to search for the exotic effects. The process of the neutrino-electron scattering can also be used to search for the neutrino magnetic moments and to probe the flavor composition of...
a (anti)neutrino beam \cite{20}. Barbieri and Fiorentini \cite{21} analysed the conversions $\nu_{eL} \to \nu_{eR}$ in the Sun as a result of spin-flip by a toroidal magnetic field in the convective zone. They calculated the differential cross section for the $(\nu_e e^-)$ scattering with the solar $^8B$ neutrinos, assuming that the initial neutrino flux is a mixture of both the left-handed neutrinos and the right-handed ones. In this case an interference term between the weak interaction and electromagnetic amplitudes, proportional to $\mu_\nu$, appears which generates the azimuthal asymmetry in the recoil electron event rates. Pastor et al. \cite{22} calculated the azimuthal asymmetry for the low-energy pp-neutrinos and discussed the sensitivity of planned solar experiments to the expected azimuthal asymmetries in event number. Beacom nad Vogel \cite{23} derived a new limit on the neutrino magnetic moment using the 825-days SuperKamiokande solar neutrino data: $|\mu_\nu| \leq 1.5 \cdot 10^{-10} \mu_B$ at 90% CL compatible with the existing reactor limit of $|\mu_\nu| \leq 1.8 \cdot 10^{-10} \mu_B$.

The first concept of the use of the artificial neutrino source comes from Alvarez who proposed a $^{65}Zn$ \cite{24}. The $^{51}Cr$ and $^{37}Ar$ neutrino sources were proposed by Raghavan \cite{25} in 1978 and Haxton \cite{26} in 1988, respectively. The idea of the use artificial neutrino source to search for the neutrino magnetic moments was first proposed by Vogel and Engel \cite{27}. The strong $^{51}Cr$ source was used for the calibration of the GALLEX neutrino experiment \cite{28}. Miranda et al. \cite{17} proposed the use of the $^{51}Cr$ source to probe the gauge structure of the electroweak interaction. Currently at Gran Sasso, the Borexino neutrino experiment \cite{29} with the unpolarized $^{51}Cr$ source is designed to search for the neutrino magnetic moment. This experiment will use the $(\nu_e e^-)$ scattering as the detection process. There are also proposed the other experiments to test the non-standard properties of neutrinos with the use of both the solar neutrinos and neutrinos coming from the beta-radioactive neutrino sources: the Hellaz \cite{30}, the Heron \cite{31} and the new project with the use of the tritium neutrino emitter \cite{32} \cite{33}.

The main goal is to show how the presence of the right-handed neutrinos in the neutrino-electron scattering changes the laboratory differential cross section in relation to the Standard Model prediction.

In our considerations the system of natural units with $\hbar = c = 1$, Dirac-Pauli representation of the $\gamma$-matrices and the $(+, -, -, -)$ metric are used \cite{34}.

The research is based on the low-energy $(\nu_\mu e^-)$ and $(\nu_e e^-)$ scattering processes. Now, we will analyse the $(\nu_\mu e^-)$ scattering. This process is studied at the level of the four-fermion point (contact) interaction. Muon-neutrinos are assumed to be massive Dirac fermions and to be polarized. In these considerations, the incoming neutrinos come from the muon-capture, where the production plane is spanned by the direction of the initial muon polarization $\hat{P}_\mu$ and of the outgoing neutrino momentum $\hat{q}$ (this is a production process). $\hat{P}_\mu$ and $\hat{q}$ are assumed to be perpendicular to each other, Fig. 1. It is important from the experimental point of view because one has the unique situation as to the possible participation of the right-handed neutrinos. The initial neutrino flux is the mixture of the left-handed neutrinos produced in the standard $V - A$ charged weak interaction and the right-handed ones produced in the exotic scalar $S$ charged weak interaction. Govaerts and Lucio-Martinez \cite{35} considered the nuclear muon capture on the proton and $^3He$ both within and beyond SM admitting the most general Lorentz invariant four-fermion contact interaction. They calculated the different observables assuming the Dirac massless neutrino. In my paper, the minimal version of the extension of the standard $V - A$ structure is analysed to indicate the possibility of new-type high-precision tests of the Lorentz structure of the charged and neutral weak interactions. The amplitude for the $\mu^-$-capture and the formula on the magnitude of the transverse neutrino polarization vector $|\eta^T_\nu|$, in the limit of vanishing neutrino mass, are as follows:

\[
\mathcal{H}_\mu^- = C_V^R (P_\nu \gamma_\lambda (1 - \gamma_5) \Psi_\mu (\nabla_n \gamma^6 \Psi_\nu) + C_A^R (P_\nu i \gamma_\lambda (1 - \gamma_5) \Psi_\mu (\nabla_n i \gamma^6 \Psi_\nu) + C_S^R (P_\nu (1 - \gamma_5) \Psi_\mu (\nabla_n \Psi_\nu),
\]

\[
|\eta^T_\nu| = \frac{\sqrt{<S_\nu \cdot (\hat{P}_\mu \times \hat{q})^2>^2_f + <S_\nu \cdot \hat{P}_\mu>^2_f}}{s < 1 >^2_f} = \frac{|P_\mu| C^R}{C_V} (1 + \frac{q}{2M})
\]
where $C_{LV}^L, C_{LA}^L, C_{SA}^R$ - the complex fundamental coupling constants for the standard vector $V$, axial $A$ and exotic scalar $S$ weak interactions denoted respectively to the outgoing neutrino handedness; $s$ - the neutrino spin (s=1/2); $<S_\nu \cdot (\hat{P}_\mu \times \hat{q})>_f \equiv Tr(S_\nu \cdot (\hat{P}_\mu \times \hat{q})\rho_f) = -\frac{|\phi_\mu(0)|^2}{4\pi}|P_\mu|(1 + \frac{q}{2M})Im(C_{LV}^L C_{SA}^R)$, (3)

$<S_\nu \cdot \hat{P}_\mu>_f \equiv Tr(S_\nu \cdot \hat{P}_\mu \rho_f) = \frac{|\phi_\mu(0)|^2}{4\pi}|P_\mu|(1 + \frac{q}{2M})Re(C_{LV}^L C_{SA}^R)$, (4)

where $C_{LV}^L, C_{LA}^L, C_{SA}^R$ can be expressed by Fetscher’s couplings $g_{\mu}^V$, $g_{\mu}^A$, $g_{\mu}^S$ normal and inverse muon decay $3\beta$, assuming the universality of weak interactions. Here, $\gamma = S, V, T$ indicates a scalar, vector, tensor interaction; $\epsilon, \mu = L, R$ indicate the chirality of the electron or muon and the neutrino chiralities are uniquely determined for given $\gamma, \epsilon, \mu$. We get the following relations:

$$C_{LV}^L = A(g_{\mu}^V + g_{\mu}^R),$$

$$C_{LA}^L = A(g_{\mu}^L - g_{\mu}^R),$$

$$C_{SA}^R = A(g_{\mu}^S + g_{\mu}^R),$$

where $A = (4G_F/\sqrt{2})\cos\theta_c$, $G_F = 1.16639(1) \times 10^{-5} GeV^{-2}$ is the Fermi coupling constant $A$, $\theta_c$ is the Cabibbo angle. In this way, the lower limits on the $C_{LV}^L$ and upper limit on the $C_{SA}^R$ can be calculated, using the current data $3\beta$: $|C_{LV}^L| > 0.850 A$, $|C_{LA}^L| > 1.070 A$, $|C_{SA}^R| < 0.974 A$. In consequence, one gives the upper bound on the magnitude of the transverse neutrino polarization vector proportional to the value of the muon polarization: $|\eta_\nu^T| \leq 0.318|P_\mu|$ for $C_{LV}^L = \pi$. The obtained limit has to be divided by the $|P_\mu|$ to have the upper bound on the physical value of the transverse neutrino polarization vector generated by the exotic scalar interaction: $|\eta_\nu^T/|P_\mu| \leq 0.318$.

From the above, we see that the analysis for the $\mu^{-}$-capture $1S$ led to the conclusion that the production of the right-handed neutrinos in the exotic scalar interaction manifests the non-vanishing transverse components of the neutrino polarization in the limit of vanishing neutrino mass, so one expects the similar regularity in the $(\nu_e e^-)$ scattering. We assume that the incoming left-handed neutrinos are detected in the $V - A$ weak interaction, while the initial right-handed neutrinos are detected in the exotic scalar $S$ weak interaction. In the final state all the neutrinos are left-handed. One assumes that the initial neutrino beam has the assigned direction of the transverse neutrino polarization with the respect to the production plane. The couplings constants are denoted as $g_{\mu}^V, g_{\mu}^A, g_{\mu}^S$ respectively to the incoming neutrino handedness:

$$M = \frac{G_F}{\sqrt{2}} \{(\bar{\nu}_e\gamma^\alpha)(g_{\nu}^V - g_{\nu}^L(1 - \gamma_5)u_{\nu_e})$$

$$+ \frac{1}{2} g_{\mu}^S(\bar{\nu}_e u_e)\gamma^\alpha(1 + \gamma_5)u_{\nu_e}\}$$

(6)
where $u_e$ and $u_{e'}$ ($u_{\nu_e}$ and $u_{\nu_e'}$) are the Dirac bispinors of the initial and final electron (neutrino) respectively.

The analysis of the general Lorentz invariant four-fermion point interaction for the $2 \rightarrow 2$ processes involving two neutrinos and two charged fermions is presented by [37].

To describe $(\nu_e e^-)$ scattering the following observables are used: $\eta_\nu$ - the full 3-vector of the initial neutrino polarization in the rest frame, $q$ - the incoming neutrino momentum, $p_{e'}$ - the outgoing electron momentum.

The laboratory differential cross section for the $\nu_e e^-$ scattering, in the limit of vanishing neutrino mass, is of the form:

$$\frac{d^2\sigma}{dy d\phi} = (\frac{d^2\sigma}{dy d\phi})_{(V,A)} + (\frac{d^2\sigma}{dy d\phi})_{(S)} + (\frac{d^2\sigma}{dy d\phi})_{(V S)},$$

$$\frac{d^2\sigma}{dy d\phi}_{(V,A)} = B((1 - \eta_\nu \cdot \hat{q}))[g^L_\nu + g^A_\nu]^2 + (g^T_\nu - g^L_\nu)(1 - y)^2 - \frac{m_y}{E_\nu}((g^L_\nu)^2 - (g^A_\nu)^2)],$$

$$\frac{d^2\sigma}{dy d\phi}_{(S)} = B(\frac{1}{8} y(y + 2 \frac{m_e}{E_\nu})[|g^R_\nu|^2(1 + \eta_\nu \cdot \hat{q})],$$

$$\frac{d^2\sigma}{dy d\phi}_{(V S)} = B\{\sqrt{y(y + 2 \frac{m_e}{E_\nu})}[-\eta_\nu \cdot (\hat{p}_{e'} \times \hat{q})]Im(g^L_\nu g^R_\nu) + (\eta_\nu \cdot \hat{p}_{e'})Re(g^L_\nu g^R_\nu)\}$$

$$- y(1 + \frac{m_e}{E_\nu})(\eta_\nu \cdot \hat{q})Re(g^L_\nu g^R_\nu),$$

where $y$, the ratio of the kinetic energy of the recoil electron $E_{e'}^R$ to the incoming neutrino energy $E_\nu$, $\phi_{e'}$ - the angle between the direction of the outgoing electron momentum $\hat{p}_{e'}$ and the direction of the incoming neutrino momentum $\hat{q}$ (recoil electron scattering angle), $m_e$ - the electron mass, $\eta_\nu \cdot \hat{q}$ - the longitudinal polarization of the incoming neutrino, $\phi_{e'}$ - the angle between the production plane and the reaction plane spanned by the $\hat{p}_{e'}$ and $\hat{q}$ (see Fig. 1). All the calculations are made with the Michel-Wightman density matrix $\Sigma$ for the polarized incoming neutrinos in the limit of vanishing neutrino mass (see Appendix E).

It can be noticed that the main non-standard contributions to the laboratory differential cross section come from the interference between the standard left-handed vector $g^L_\nu$ coupling and exotic right-handed scalar $g^R_\nu$ coupling, whose occurrence does not depend explicitly on the neutrino mass. This interference may be understood as the interference between the neutrino waves of the same neutrino mass. This interference may be understood as the interference between the neutrino waves of the same neutrino mass. This interference may be understood as the interference between the neutrino waves of the same neutrino mass.

The other correlation $\eta_\nu \cdot (\hat{p}_{e'} \times \hat{q})$ proportional to $Im(g^L_\nu g^R_\nu)$ lies along the direction perpendicular to the reaction plane and it includes only $T$-odd transverse component of the initial neutrino polarization:

$$\eta_\nu \cdot (\hat{p}_{e'} \times \hat{q}) = \eta_\nu^T \cdot (\hat{p}_{e'} \times \hat{q}).$$

It can be shown that in the full interference term, Eq. (10), the contributions from the longitudinal components of the neutrino polarization annihilate, and in consequence one gives the interference including only the transverse components of the initial neutrino polarization, both $T$-even and $T$-odd:

$$\frac{d^2\sigma}{dy d\phi}_{(V S)} = B\{\sqrt{y(y + 2 \frac{m_e}{E_\nu})}[|g^L_\nu||g^R_\nu||\eta_\nu^T|\cos(\phi - \alpha)]\}$$

where $\alpha \equiv \alpha^L_\nu - \alpha^R_\nu$ - the relative phase between the $g^L_\nu$ and $g^R_\nu$ couplings, $\phi$ - the angle between the reaction plane and the transverse neutrino polarization vector and it is connected with the $\phi_{e'}$ in the following way; $\phi = \phi_0 - \phi_{e'}$, where $\phi_0$ - the angle between the production plane and the transverse neutrino polarization vector (see Fig. 1). It can be noticed that the contribution from the interference between the $g^L_\nu$ and $g^R_\nu$ couplings, involving the transverse neutrino polarization components, will be substantial at lower neutrino energies $E_\nu \leq m_e$ but negligibly small at large energies and vanishes for $\theta_{e'} = 0$ or $\theta_{e'} = \pi/2$. The occurrence of the interference term in the cross section depends on the relative phase between the angle $\phi$ and phase $\alpha$ and does not vanish for
FIG. 2: Plot of the $\frac{d^2\sigma}{dyd\phi}$ as a function of $y$ for the $(\nu_e e^-)$ scattering; a) SM with the left-handed neutrino (SM(y) - solid line), b) the case of the exotic scalar coupling of the right-handed neutrino for $\phi - \alpha = 0$ (ESI1(y) - dashed line), for $\phi - \alpha = \pi$ (ESI2(y) - dashed line) and for $\phi - \alpha = \pi/2$ (ESI3(y) - dotted line), respectively.

FIG. 3: Plot of the $\frac{d\sigma}{dy}$ as a function of $y$ for the $(\nu_e e^-)$ scattering; a) SM with the left-handed neutrino (SMd(y) - solid line), b) the case of the exotic scalar coupling of the right-handed neutrino after integration over the $\phi,\phi'$ (SRd(y) - dashed line).

FIG. 4: Plot of the $\frac{d^2\sigma}{dyd\phi}$ as a function of $y$ for the $(\nu_e e^-)$ scattering; a) SM with the left-handed neutrino (SMn(y) - solid line), b) the case of the exotic scalar coupling of the right-handed neutrino for $\phi - \alpha = 0$ (ESIn1(y) - dashed line), for $\phi - \alpha = \pi$ (ESIn2(y) - dashed line) and for $\phi - \alpha = \pi/2$ (ESIn3(y) - dotted line), respectively.

$\phi - \alpha \neq \pi/2$. The azimuthal asymmetry in the angular distribution of the recoil electrons is illustrated in the Fig. 2 when $\phi - \alpha = 0$ (ESI1(y) - dashed line), $\phi - \alpha = \pi$ (ESI2(y) - dashed line) and $\phi - \alpha = \pi/2$ (ESI3(y) - dotted line), respectively. All the plots (Fig. 2 - Fig. 5) both for the $(\nu_e e^-)$ and $(\nu_e e^-)$ scattering processes are made for $m_e = 511$ keV, $E_\nu = 746$ keV and $y \in [0, 0.745]$. Because the right-handed neutrinos are produced and detected in the exotic scalar S interaction, one uses the same upper limit on the $gS^2$ as for the $C^S_{33}$, i.e. $\left| gS^2 \right| \leq 0.974$, assuming the universality of weak interaction. We take the upper bound on the $\left| \eta^T_{\nu} \right| \leq 0.318$ for the neutrinos coming from the muon-capture (however the phase $\alpha$ is still unknown). The value $\left| \eta^T_{\nu} \right| = 0.318$ is used to get the upper limit on the expected effect from the right-handed neutrinos in the cross section for the $(\nu_e e^-)$ scattering. It means that the value of the longitudinal neutrino polarization is equal to $\eta^T_{\nu} \equiv \eta_{\nu} \cdot \hat{q} = -0.948$. The plot for the SM is made with the use of the present experimental values for $gS^V = -0.040 \pm 0.015$, $gS^L = -0.507 \pm 0.014$ [33], when $\eta_{\nu} \cdot \hat{q} = -1$, Fig. 2 and Fig. 3 (SM(y), SMd(y) - solid lines). If one integrates over the $\phi,\phi'$, the interference term vanishes and the cross section consists of only two terms:

$$\frac{d\sigma}{dy} = \left( \frac{d\sigma}{dy} \right)_{(V, A)} + \left( \frac{d\sigma}{dy} \right)_{(S)},$$

$$\left( \frac{d\sigma}{dy} \right)_{(V, A)} = B'\{ (1 - \eta_{\nu} \cdot \hat{q}) [(gS^V + gS^L)^2 + (gS^V - gS^L)^2 (1 - y)^2 - \frac{m_\nu y}{E_\nu} (gS^V)^2 - (gS^L)^2] \},$$

$$\left( \frac{d\sigma}{dy} \right)_{(S)} = B'\{ \frac{1}{8} y (y + 2\frac{m_\nu}{E_\nu}) \times \left| \left[ gS^S \right|^2 (1 + \eta_{\nu} \cdot \hat{q}) \right| \},$$

where $B' = 2\pi B$. The situation is illustrated in the Fig. 4 (SRd(y) - dashed line). If the only left-handed neutrinos are produced in the standard $V - A$ interaction and non-standard scalar S one, they should be detected in the same interactions. In this case there is no interference between the $(V, A)$ and $S_L$ couplings in the differential cross section, when $m_\nu \rightarrow 0$, and the angular distribution of the recoil electrons has the azimuthal symmetry. Because the left-handed scalar $S_L$ coupling is absent in the production process, so this scenario is not considered for the $(\nu_e e^-)$ scattering.

Taking into account the possibilities of the future low-energy neutrino experiments, we consider the $(\nu_e e^-)$ process. If the azimuthal asymmetry in the cross section for the $(\nu_e e^-)$ scattering appears, the similar regularity for the $(\nu_e e^-)$ process should occur, when the electron-neutrinos come from the polarized artificial neutrino source (31Cr). The 31Cr-decay proceeds by the
electron capture and is similar to the $\mu^-\text{capture}$. It is well-known that the $^{51}$Cr decays with a $Q$-value of 751 keV to the ground state of $^{51}$V (90.14% branching ratio) and to its first excited state (9.86 ± 0.05%), which deexcites to the ground state with the emission of a 320 keV $\gamma$-ray. The neutrino spectrum consists of four monoenergetic lines: 746 keV (81%), 751 keV (9%), 426 keV (9%), 431 keV (1%) [38]. If the chromium neutrino source would be polarized, one would have the fixed direction of the transverse neutrino polarization with respect to the production plane spanned by the initial polarization $J_\mu$ of the $^{51}$Cr and the outgoing electron-neutrino momentum $q$. The reaction plane is the same as for the $(\nu_\mu,e^-)$ scattering. It would allow to measure the azimuthal asymmetry in the angular distribution of the recoil electrons. In the amplitude for the $(\nu_\mu,e^-)$ scattering, the couplings constants are denoted as $c^L_\nu$, $c^R_\nu$ and $c^S_\nu$ respectively to the incoming neutrino handedness, where $c^L_\nu = g^L_\nu + 1$, $c^R_\nu = g^R_\nu + 1$ (the charged current weak interaction is included). The plot for the SM is made with the same values of the standard coupling constants as for the $(\nu_\mu,e^-)$ process, i.e. $c^L_\nu = -0.040 + 1$, $c^R_\nu = -0.507 + 1$, when $\eta_\nu \cdot \hat q = -1$, Fig. 4 and Fig. 5 (SMn(y), SMnd(y) - solid lines). The upper limit on the $c^S_\nu$ is the same as for the $g^S_\nu$, i.e. $|c^S_\nu| < 0.974$, assuming the universality of weak interaction. We also use both $|\eta_\nu^T\nu| = 0.318$ and $\eta_\nu^R \equiv \eta_\nu \cdot \hat q = -0.948$ to obtain the upper bound on the possible effect from the R-handed neutrinos for the $(\nu_\mu,e^-)$ scattering. The azimuthal asymmetry in the angular distribution of the recoil electrons is illustrated in the Fig. 4 (ESIn1(y), ESIn2(y) - dashed lines, ESIn3(y) - dotted line). If one integrates over the $\phi_\nu$, the interference term vanishes and the cross section has the azimuthal symmetry, Fig. 5 (SRnd(y) - dashed line).

In summary, it is known that in the SM the angular distribution of the recoil electrons does not depend on the $\phi_\nu$. It is necessary to observe the direction of the recoil electrons and to analyse all the possible reaction planes corresponding to the given recoil electron scattering angle to verify if the azimuthal asymmetry in the cross section appears. The regularity of this type would be a signature indicating the possible participation of the right-handed neutrinos in the neutrino-electron scattering. The future low-energy high-precision neutrino-electron scattering experiments using the intense and polarized neutrino source (e.g. $^{51}$Cr or other artificial neutrino sources) would allow to search for new effects coming from the right-handed neutrinos (to be published).

**APPENDIX A: MUON NEUTRINO OBSERVABLES**

The formulas for the transverse components of neutrino polarization (T-odd and T-even, respectively) in case of non-vanishing neutrino mass ($m_\nu \neq 0$), when the induced couplings are enclosed and $\hat P_\nu$, $\hat q$ are perpendicular to each other, are as follows:

$$(A1) \quad < S_\nu \cdot (\hat P_\nu \times \hat q) >_f = Tr\{S_\nu \cdot (\hat P_\nu \times \hat q) \rho_f\} = \frac{|\phi_\nu(0)|^2}{4\pi}|P_\nu|(\frac{q}{E_\nu} + \frac{q}{2M})Im((C^L_\nu + 2Mg^L_\nu)C^R_\nu)
$$

$$(A2) \quad < S_\nu \cdot \hat P_\nu >_f = Tr\{S_\nu \cdot \hat P_\nu \rho_f\} = \frac{|\phi_\nu(0)|^2}{4\pi}|P_\nu|((1 + \frac{q}{E_\nu} + \frac{q}{2M})Re((C^L_\nu + 2Mg^L_\nu)C^R_\nu)
$$

$$+ \frac{1}{2}\frac{m_\nu}{E_\nu}(|C^L_\nu + 2Mg^L_\nu| - 2|C^L_\nu + 2Mg^L_\nu|C^R_\nu + m_\nu^2 + |C^S_\nu| ^2),
$$

where $g^L_\nu, g^R_\nu$ - the induced couplings of the left-handed neutrinos, i.e. the weak magnetism and induced pseudoscalar, respectively; $m_\mu, q, E_\nu, m_\nu, M$ - the muon mass, the value of the neutrino momentum, its energy, its mass and the nucleon mass. If $m_\nu \to 0$, $q/E_\nu \to 1$ and the mass terms vanish in all the observables.
**APPENDIX B: GENERAL FORM OF MUON NEUTRINO OBSERVABLES**

General results for the transverse components of the neutrino polarization, in the limit of vanishing neutrino mass \((m_\nu \rightarrow 0)\), when the induced couplings are enclosed and \(\hat{P}_\mu, \hat{q}\) are not perpendicular to each other, are as follows:

\[
< S_\nu \cdot (\hat{P}_\mu \times \hat{q}) >_f = \left| \frac{\phi_\mu(0)}{4\pi} \right|^2 |P_\mu| |(\hat{P}_\mu \cdot \hat{q})^2 - 1| \left| \frac{q}{2M} \right| Re((C_{L_\mu}^T + 2Mg_{L_\mu}^T)C_{S_\mu}^{R*}) \tag{B1}
\]

\[
< S_\nu \cdot \hat{P}_\mu >_f = \left| \frac{\phi_\mu(0)}{4\pi} \right|^2 |P_\mu| |(\hat{P}_\mu \cdot \hat{q})^2 - 1| \left| \frac{q}{2M} \right| Re((C_{L_\mu}^T + 2Mg_{L_\mu}^T)C_{S_\mu}^{R*}) + \left| \frac{q}{2M} \right| Re((C_{L_\mu}^T + 2Mg_{L_\mu}^T)(C_{L_\mu}^{R*} + m_\mu \frac{q}{2M}g_{L_\mu}^{R*})) + \frac{1}{2}(1 + \left| \frac{q}{M} \right| |C_{L_\mu}^T + 2Mg_{L_\mu}^T|^2 - \frac{1}{2} |\frac{q}{M} - 1| |C_{L_\mu}^T + m_\mu \frac{q}{2M}g_{L_\mu}^{R*}]^2
+ (\hat{P}_\mu \cdot \hat{q}) \left| \frac{q}{2M} \right| |C_{L_\mu}^T + 2Mg_{L_\mu}^T|^2 - \frac{1}{2} (3 + \left| \frac{q}{M} \right| |C_{L_\mu}^T + m_\mu \frac{q}{2M}g_{L_\mu}^{R*}]^2\right) \}
\]

It can be seen that if \(\hat{P}_\mu || \hat{q}, m_\nu \rightarrow 0\), one gives \(< S_\nu \cdot (\hat{P}_\mu \times \hat{q}) >_f = 0\) (Eq. B1), and \(< S_\nu \cdot \hat{P}_\mu >_f \) (Eq. B2) \( \Rightarrow < S_\nu \cdot \hat{q} >_f \) (Eq. D3).

**APPENDIX C: NUCLEAR OBSERVABLES AND LONGITUDINAL NEUTRINO POLARIZATION**

The formulas for the longitudinal and transverse components of neutron polarization (T-even and T-odd components respectively), and for the longitudinal neutrino polarization (T-even quantity), in the case of non-vanishing neutrino mass \((m_\nu \neq 0)\), when the induced couplings are enclosed and \(\hat{P}_\mu, \hat{q}\) are perpendicular to each other, are as follows:

\[
< J_n \cdot \hat{q} >_f = \left| \frac{\phi_\mu(0)}{4\pi} \right|^2 \left( \frac{q}{E_\nu} |C_{L_\mu}^T + m_\mu \frac{q}{2M}g_{L_\mu}^T| - Re[(\left| \frac{q}{2M} \right| + \frac{q}{E_\nu})|C_{L_\mu}^T + 2Mg_{L_\mu}^T)](C_{L_\mu}^{R*} + m_\mu \frac{q}{2M}g_{L_\mu}^{R*}) \right) \tag{C1}
\]

\[
< J_n \cdot (\hat{P}_\mu \times \hat{q}) >_f = \left| \frac{\phi_\mu(0)}{4\pi} \right|^2 |P_\mu| Re[\left| \frac{q}{E_\nu} \right| (C_{L_\mu}^T + 2Mg_{L_\mu}^T)C_{S_\mu}^{R*}] \}
\]

\[
< S_\nu \cdot \hat{q} >_f = \left| \frac{\phi_\mu(0)}{4\pi} \right|^2 \left[ \frac{3}{2} \left( \frac{q}{E_\nu} \right) + \frac{q}{2M} |C_{L_\mu}^T + m_\mu \frac{q}{2M}g_{L_\mu}^T|^2 - \left| \frac{q}{E_\nu} + \frac{q}{2M} \right| |C_{L_\mu}^T + 2Mg_{L_\mu}^T|^2 + \frac{1}{2} \left| \frac{q}{M} \right| \frac{q}{E_\nu} |C_{L_\mu}^T + 2Mg_{L_\mu}^T)|C_{S_\mu}^{R*}] \}
\]

It can be seen that in these observables the occurrence of the interference term between the standard \(C_{L_\mu}^T\) couplings and exotic \(C_{S_\mu}^{R*}\) coupling depends explicitly on the neutrino mass. This dependence causes the "conspiracy" of the interference term and makes the measurement of the relative phase between the standard \(C_{L_\mu}^T\) and exotic \(C_{S_\mu}^{R*}\) impossible because \((m_\nu/E_\nu)(q/2M)\) is very small.

**APPENDIX D: GENERAL FORM OF NUCLEAR OBSERVABLES AND OF LONGITUDINAL NEUTRINO POLARIZATION**

General results for the longitudinal and transverse components of neutron polarization and for the longitudinal neutrino polarization, in the limit of vanishing neutrino mass \((m_\nu \rightarrow 0)\), when the induced couplings are enclosed and \(\hat{P}_\mu, \hat{q}\) are not perpendicular to each other, are as follows:

\[
< J_n \cdot \hat{q} >_f = \left| \frac{\phi_\mu(0)}{4\pi} \right|^2 \left[ \left| -\frac{q}{M} \right| Re[(C_{L_\mu}^T + 2Mg_{L_\mu}^T)](C_{L_\mu}^{R*} + m_\mu \frac{q}{2M}g_{L_\mu}^{R*}) + |C_{L_\mu}^T + m_\mu \frac{q}{2M}g_{L_\mu}^T|^2 \right]
\]
\[ < J_n \cdot (\vec{p}_\mu \times \vec{q}) >_f = \frac{|\phi_\mu(0)|^2}{4\pi} |P_\mu| (|\vec{p}_\mu \cdot \vec{q}|) [Re((C_\nu^f + 2Mg^f)(C_\nu^f + m_\nu)) + |C_\nu^f + m_\nu|^2], \quad (D1) \]

\[ < S_\nu \cdot \vec{q} >_f = \frac{|\phi_\mu(0)|^2}{4\pi} |P_\mu| (|\vec{p}_\mu \cdot \vec{q}|)^2 - (1 + \frac{q}{M}) Im((C_\nu^f + 2Mg^f)(C_\nu^f + m_\nu)) + \frac{1}{2} |C_\nu^f|^2 \quad (D2) \]

\[ \frac{1}{2}(1 + \frac{q}{M}) |C_\nu^f + 2Mg^f|^2 - \frac{1}{2}(3 + \frac{q}{M}) |C_\nu^f + m_\nu|^2 \]

\[ + |P_\mu| (|\vec{p}_\mu \cdot \vec{q}|) [\frac{q}{M} Re((C_\nu^f + 2Mg^f)(C_\nu^f + m_\nu)) \quad (D3) \]

\[ + \frac{1}{2} |C_\nu^f|^2 + (1 + \frac{q}{M}) |C_\nu^f + 2Mg^f|^2 + (\frac{q}{M} - 1) |C_\nu^f + m_\nu|^2 \]

APPENDIX E: FOUR-VECTOR NEUTRINO POLARIZATION AND MICHEL-WIGHTMAN DENSITY MATRIX

The formulas for the 4-vector initial neutrino polarization in its rest frame and for the initial neutrino moving with the momentum q, respectively, are as follows:

\[ S = (0, \eta_\nu) , \]

\[ S' = \frac{\eta_\nu \cdot q}{E_\nu} \cdot \frac{1}{m_\nu} \left( \frac{E_\nu}{q} \right) + \left( \begin{array}{c} 0 \\ \eta_\nu \cdot \eta_\nu \\ \eta_\nu \cdot q \\ E_\nu(E_\nu + m_\nu) \end{array} \right) \left( \begin{array}{c} 0 \\ \eta_\nu \cdot q \\ \eta_\nu \cdot \eta_\nu \\ \eta_\nu \cdot \eta_\nu \cdot q \end{array} \right) , \quad (E1) \]

\[ \text{where } \eta_\nu \text{ - the full 3-vector of the initial neutrino polarization in its rest frame. The formulas for the Michel-Wightman density matrix in the case of the polarized neutrinos with the non-zero neutrino mass and in the limit of vanishing neutrino mass, respectively, are as follows:} \]

\[ \Lambda^A_{\nu}(s) = \sum_\nu a_\nu \bar{\pi}_\nu \sim [1 + \gamma_5 S^\prime \cdot \gamma_\mu](q^\nu \gamma_\mu) + m_\nu = [(q^\nu \gamma_\mu) + m_\nu + \gamma_5(S^\prime \cdot \gamma_\mu)(q^\nu \gamma_\mu) + \gamma_5(S^\prime \cdot \gamma_\mu)m_\nu] , \quad (E3) \]

\[ (S^\prime \cdot \gamma_\mu) = \frac{\eta_\nu \cdot q}{E_\nu m_\nu}(q^\nu \gamma_\mu) - (\eta_\nu \cdot q) \eta_\nu \cdot \eta_\nu \cdot q \cdot \gamma , \quad (E4) \]

\[ [1 + \eta_5 (S^\prime \cdot \gamma_\mu)](q^\nu \gamma_\mu) + m_\nu = [1 + \gamma_5(\eta_\nu \cdot q) \eta_\nu \cdot \eta_\nu \cdot q \cdot \gamma] (q^\nu \gamma_\mu) , \quad (E7) \]

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