Meissner like effect in holographic superconductors with back reaction

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Abstract

In this article we employ the matching method to analytically investigate the properties of holographic superconductors in the framework of Maxwell electrodynamics taking into account the effects of back reaction on spacetime. The relationship between the critical temperature ($T_c$) and the charge density ($\rho$) has been obtained first. The influence of back reaction on Meissner like effect in this holographic superconductor is then studied. The results for the critical temperature indicate that the condensation gets harder to form when we include the effect of back reaction. The expression for the critical magnetic field ($B_c$) above which the superconducting phase vanishes is next obtained. It is observed from our investigation that the ratio of $B_c$ and $T_c^2$ increases with the increase in the back reaction parameter. However, the critical magnetic field $B_c$ decreases with increase in the back reaction parameter.

1. Introduction

The AdS/CFT correspondence which gives a sound realization of the holographic principle, fundamentally originated from superstring theory \cite{1, 2}. The duality claims the equivalence of a strongly coupled $D$-dimensional gauge theory with a gravitational theory in $(D+1)$-dimensional anti-deSitter (AdS) spacetime. In other words this duality states the equivalence between two theories one of which is strongly coupled and the other is weakly coupled. For the past two decades many successful connections between condensed matter physics and gravitational theories have been built using this AdS/CFT duality, also known as the gauge/gravity correspondence in the literature. To describe finite temperature field theories, the gravitational part is replaced by AdS black holes. More specifically, exploiting the gauge/gravity duality one can use general relativity as a tool to describe various strongly correlated systems of condensed matter physics. One of the phenomena in condensed matter system that is explained by this correspondence is the phenomena of superconductivity \cite{3, 4}.

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Holographic superconductors have been studied extensively in the last few years and the results obtained on the field theory side from the holographic superconducting gravitational models showed considerable promise to explain some of the glaring features of high $T_c$ superconductors. The basic idea involved in holographic superconductors is the following. The local spontaneous $U(1)$ symmetry breaking of a charged black hole, minimally coupled to a complex scalar field, was first studied in [5, 6, 7]. The gravitational theory in the bulk can be mapped into a dual field theory, residing on the boundary of the AdS spacetime, using the AdS/CFT correspondence. This boundary theory suffers a global $U(1)$ symmetry breaking. In the last few years investigation on holographic superconductors have been widely explored in the context of Maxwell electrodynamics [8]-[26] as well as non-linear electrodynamics [27]-[31].

It is also well known that superconductors expel magnetic fields as the temperature is lowered below a critical temperature ($T_c$). In the presence of an external magnetic field, ordinary superconductors can be classified into two classes, namely type I and type II. Using the AdS/CFT dictionary it has been found that at low temperatures ($T < T_c$), s-wave condensate in holographic superconductors expels the magnetic field [32, 33]. Such studies were first carried out in [34, 35]. Thereafter studies of Meissner like effect in holographic superconductors have also been carried out using the matching method [36] in [37, 38].

In this paper we have investigated the influence of back reaction on holographic superconductors as well as the Meissner like effect in holographic superconductors. Our holographic superconducting theory consists of a Einstein-Hilbert gravity theory along with a complex scalar field minimally coupled to Maxwell field. The model also involves the effect of back reaction of matter fields on the bulk spacetime. Hence, we are away from the probe limit through out the entire analysis. Studies involving back reaction have been carried out earlier in [39, 40].

The paper is organized as follows. In section 2, the basic formalism for the $d$-dimensional holographic superconductor considering the effect of back reaction in the spacetime geometry is presented. In section 3, we obtain the relationship between the critical temperature and the charge density using the matching method approach. In section 4, we investigate the Meissner like effect using the same approach. We finally conclude in section 5.

2. Basic formalism

The action of the gravitational dual able to describe the phase transition in the conformal field theory living in the boundary reads

$$S \ = \ \int d^d x \ \sqrt{|g|} \ \left[ R - 2\Lambda + 2\kappa^2 L_m \right], \quad \Lambda = -\frac{(d-1)(d-2)}{2L^2}, \quad (2.1)$$

$$L_m = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \left[ (D_\mu \psi)^* D^\mu \psi + m^2 \psi^* \psi \right]$$

$$\equiv L_1 - L_2; \quad \mu, \nu = 0, 1, 2, \ldots d. \quad (2.2)$$

It is basically the Einstein-Hilbert gravity theory with a complex scalar field minimally coupled to the Maxwell field. Where the symbols have their usual meanings and $\kappa^2 = 8\pi G_d$, with $G_d$ being the $d$ dimensional Newton’s gravitational constant. $L_m$ denotes the Lagrangian density of the matter sector. The second term ($L_2$) in the above equation $(2.2)$ consists of the complex scalar field $\psi$, the covariant derivative $D_\mu$ is defined as $D_\mu \equiv \partial_\mu - iqA_\mu$ where $A_\mu$ is the gauge field. On the other hand
the first term ($\mathcal{L}_1$) gives the dynamics of the gauge field. Next we obtain the equations of motion by varying the action (2.1) with respect to the field variables. The equations of motion for the field variables $g_{\mu\nu}$, $A_\mu$ and $\psi$, respectively, are given as follows

$$
\delta_{g_{\mu\nu}} : \left[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} \right] - 2\kappa^2 \left[ \frac{1}{2} \mathcal{L}_1 g_{\mu\nu} + \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \mathcal{L}_2 g_{\mu\nu} \right.
+ \left. (D_\mu \psi)^* (D_\nu \psi) \right] = 0
$$

(2.3a)

$$
\delta_{A_\mu} : \partial_\nu \left[ \sqrt{|g|} F^{\nu\mu} \right] - 2\sqrt{|g|} q^2 A^\mu \psi^* \psi = 0
$$

(2.3b)

$$
\delta_\psi : \partial_\mu \left[ \sqrt{|g|} \partial^\mu \psi \right] - i q \sqrt{|g|} A^\mu \partial_\mu \psi - i q \partial_\mu \left[ \sqrt{|g|} A^\mu \psi \right] - \sqrt{|g|} q^2 A_\nu A^\nu \psi - \sqrt{|g|} m^2 \psi = 0
.$$  

(2.3c)

The $d$-dimensional plane-symmetric black hole metric in $AdS$ spacetime with back reaction reads

$$
ds^2 = -f(r)e^{-\chi(r)} dt^2 + \frac{1}{f(r)} dr^2 + r^2 h_{ij} dx^i dx^j
$$  

(2.4)

where $h_{ij} dx^i dx^j$ is the metric on a $(d - 2)$-dimensional hypersurface which has a flat geometry.

Now we consider a change of coordinate from $r \rightarrow z = \frac{r_+}{r}$ where $r_+$ is the horizon radius $(f(r_+) = 0)$ and make the following ansatz for the gauge field and scalar field

$$
A_\mu = (\phi(r), \ 0, \ 0, \ ...), \ \psi = \psi(r).
$$  

(2.5)

By putting $\mu = \nu = 0$ in eq.(2.3a), we obtain

$$
f'(z) - \frac{d-3}{z} f(z) + \frac{(d-1)r_+^2}{d-2} \left( z \psi'^2(z) f(z) + \frac{r_+^2}{z^3} m^2 \psi(z) + \frac{r_+^2}{z^2} \phi^2(z) \psi^2(z) e^{-\chi(z)} \frac{1}{z f(z)} + \frac{1}{2} z \phi'^2(z) \right) = 0.
$$  

(2.6)

Now setting $\mu = \nu = 1$ in eq.(2.3a) and then adding up with eq.(2.6) we obtain

$$
\chi'(z) = \frac{4\kappa^2 r_+^2}{(d-2) z^3} \left[ z^4 \psi'^2(z) + \frac{\phi^2(z) \psi^2(z) e^{\chi(z)}}{f^2(z)} \right] = 0.
$$  

(2.7)

Also putting $\mu = 0$ in eq.(2.3b), we obtain

$$
\phi''(z) + \left[ \chi'(z) - \frac{d-4}{z} \right] \phi'(z) - \frac{2 r_+^2 \phi(z) \psi^2(z)}{f(z) z^4} = 0.
$$  

(2.8)

Finally rewriting eq.(2.3c) in terms of $z$ gives

$$
\psi''(z) + \left[ \frac{f'(z)}{f(z)} - \frac{d-4}{z} - \frac{\chi'(z)}{2} \right] \psi'(z) + \frac{r_+^2}{z^4} \left[ \frac{\phi^2(z) e^{\chi(z)}}{f^2(z)} - \frac{m^2}{f(z)} \right] \psi(z) = 0.
$$  

(2.9)
3. Critical temperature \((T_c)\) in terms of charge density \((\rho)\)

The Hawking temperature of a black hole is given by

\[
T_H = \frac{f'(r_+)e^{-\chi(r_+)/2}}{4\pi}.
\] (3.1)

To solve the non-linear equations (2.6) − (2.9), we need to set the boundary conditions at the black hole horizon \(r = r_+\) and at spatial infinity \(r = \infty\). In this context we recall that \(f(r = r_+) = 0\) and \(e^{\chi(r=r_+)}\) is finite. We also set \(\lim_{r \to \infty} e^{-\chi(r)} \to 1\). The matter fields obey [5, 6]

\[
\phi(r) = \mu - \frac{\rho}{r^{d-3}} \quad (3.2)
\]

\[
\psi(r) = \frac{\psi_-}{r^{\Delta_-}} + \frac{\psi_+}{r^{\Delta_+}} \quad (3.3)
\]

where

\[
\Delta_{\pm} = \frac{(d-1) \pm \sqrt{(d-1)^2 + 4m^2L^2}}{2}. \quad (3.4)
\]

Here \(\mu\) and \(\rho\) are the duals to the chemical potential and charge density of the conformal field theory part. We also choose \(\psi_- = 0\) so that \(\psi_+\) is the expectation value of the condensation operator \(J\) at the boundary. For the matter field to be regular we require \(\phi(r_+) = 0\) and \(\psi(r_+)\) to be finite. In terms of the coordinate variable \(z\) this reads \(\phi(z = 1) = 0\) and \(\psi(z = 1)\) to be finite.

At the critical temperature \(T = T_c\), \(\psi(z) = 0\). Using this fact in eq.(2.7) we have

\[
\chi'(z) = 0. \quad (3.5)
\]

With this relation and our previous argument we get

\[
\lim_{z \to 0} e^{-\chi(z)} \to 1 \implies \chi(z) = 0. \quad (3.6)
\]

Using eq.(s)(3.5, 3.6) in eq.(2.8), we get the following behavior for the field \(\phi\) at the critical temperature

\[
\phi''(z) - \frac{d-4}{z} \phi'(z) = 0. \quad (3.7)
\]

Now using the boundary condition (3.2), we solve the above equation to get

\[
\phi(z) = \lambda r_+(1 - z^{d-3}) \quad (3.8)
\]

where

\[
\lambda = \frac{\rho}{r_+^{d-2}}. \quad (3.9)
\]

Now the field equation for \(f(z)\) (2.6) at the critical temperature \(T = T_c\) becomes

\[
f'(z) - \frac{d-3}{z} f(z) + \frac{(d-1)r_+^2}{L^2 z^3} - \frac{\kappa^2 z}{d-2} \phi^2(z) = 0. \quad (3.10)
\]

Substituting the solution of \(\phi(z)\) from eq.(3.8) in the above equation yields

\[
f'(z) - \frac{d-3}{z} f(z) + \frac{(d-1)r_+^2}{L^2 z^3} - \frac{(d-3)^2\kappa^2 \lambda^2 r_+^2}{d-2} z^{2d-7} = 0. \quad (3.11)
\]
The solution of this equation (3.11) subject to the condition \( f(z = 1) = 0 \) reads
\[
f(z) = r^2_+ \left[ \frac{1}{L^2 z^2} - \left( \frac{1}{L^2} + \frac{d - 3}{d - 2} \kappa^2 \lambda^2 \right) z^{d-3} + \frac{d - 3}{d - 2} \kappa^2 \lambda^2 z^{2(d-3)} \right]. \tag{3.12}
\]

In the rest of the analysis we shall work with \( L = 1 \). With this value of \( L \) the form of \( f(z) \) reduces to
\[
f(z) = \frac{r^2_+}{z^2} g_0(z) \tag{3.13}
\]
where
\[
g_0(z) = 1 - \left[ 1 + \frac{d - 3}{d - 2} \kappa^2 \lambda^2 \right] z^{d-1} + \frac{d - 3}{d - 2} \kappa^2 \lambda^2 z^{2(d-2)}. \tag{3.14}
\]

**Analysis by matching method**

Now we proceed to find the relationship between the critical temperature \( T_c \) and charge density \( \rho \) using the matching method [36]. For that we expand \( \phi(z) \) and \( \psi(z) \) in Taylor’s series around \( z = 1 \) and equate it with the boundary condition mentioned in eqs.(3.2, 3.3) at some point \( z = z_m \). This yields
\[
\left[ \mu - \frac{\rho}{r^d_+ z^{d-3}} \right]_{z = z_m} = \left[ \phi(1) - (1 - z) \phi'(1) + \frac{1}{2} (1 - z)^2 \phi''(1) - \mathcal{O}((1 - z)^3) \right]_{z = z_m}. \tag{3.15}
\]

Now using the values of \( \chi'(z) \) and \( \chi(z) \) at the critical temperature given in eq.(3.5) and eq.(3.6), in eq.(2.8), we end up with
\[
\phi''(z) - \frac{d - 4}{z} \phi'(z) - \frac{2r^2_+ \phi(z) \psi^2(z)}{f(z) z^4} = 0. \tag{3.16}
\]

From the above equation, we get
\[
\phi''(1) = \left[ (d - 4) + \frac{2 \psi^2(1)}{g_0(1)} \right] \phi'(1). \tag{3.17}
\]

With this result eq.(3.15) simplifies to the form
\[
\left\{ \mu - \frac{\rho}{r^d_+ z^{d-3}} z^{d-3} = -(1 - z) \phi'(1) + \frac{1}{2} (1 - z)^2 \left( (d - 4) + \frac{2 \psi^2(1)}{g_0(1)} \right) \phi'(1) \right\}_{z = z_m}. \tag{3.18}
\]

Taking derivative on both sides of the above relation and setting \( z = z_m \) gives
\[
\left\{ - (d - 3) \frac{\rho}{r^d_+ z^{d-4}} z^{d-4} = \phi'(1) - (1 - z) \left( (d - 4) + \frac{2 \psi^2(1)}{g_0(1)} \right) \phi'(1) \right\}_{z = z_m}. \tag{3.19}
\]

Setting \( \psi(1) = \alpha, \phi'(1) = -v \) and \( \tilde{v} = \frac{v}{r^d_+} \) in the above equation, we arrive at
\[
\alpha^2 = \frac{g_0(1)}{2(1 - z_m)} \left[ 1 - (1 - z_m)(d - 4) \right] \left[ 1 - \left( \frac{T_c}{T} \right)^{d-2} \right]. \tag{3.20}
\]
where
\[ T_c = \xi \rho \frac{1}{\pi^2} \]  

(3.21)

\[ \xi = \left( \frac{g'_0(1)}{4\pi} \right) \frac{1}{\bar{v} \frac{1}{\pi^2} \frac{\Delta + (1 - \zeta_m)}{\Delta + (1 - \zeta_m + 2\zeta_m)}} \frac{\Delta + (1 - \zeta_m + 2\zeta_m)}{(1 - \zeta_m)(\Delta + (1 - \zeta_m + 2\zeta_m))} \]  

(3.22)

Now we want to figure out the form of \( \bar{v} \) in terms of known parameters \( \zeta_m, d, m^2 \). Here again we use eqs (3.5, 3.6) in eq.(2.9) and end up getting

\[ \psi''(z) + \left[ \frac{f'(z)}{f(z)} - \frac{d - 4}{z} \right] \psi'(z) + \frac{r^2}{z^4} \left[ \frac{\phi^2(z)}{f^2(z)} - \frac{m^2}{f(z)} \right] \psi(z) = 0 . \]  

(3.23)

From this relation, we get

\[ \psi'(1) = \frac{m^2}{g'_0(1)} \psi(1) \]  

(3.24)

\[ \psi''(1) = \frac{1}{2} \left[ (d - 4) - \frac{g''_0(1)}{g'_0(1)} + \frac{m^2}{g'_0(1)} \right] \frac{m^2}{g'_0(1)} \psi(1) - \frac{\phi'^2(1)}{2r^2_+ g^2_0(1)} \psi(1) \]  

(3.25)

The Taylor’s series expansion of \( \psi(z) \) around \( z = 1 \) reads

\[ \psi(z) = \psi(1) - (1 - z)\psi'(1) + \frac{(1 - z)^2}{2!} \psi''(1) + .... . \]  

(3.26)

Following the matching technique as earlier, gives the expression for \( \bar{v} \) to be

\[ \bar{v}^2 = m^4 + m^2 g'_0(1) \left[ d - 4 - \frac{g''_0}{g'_0} \right] - \frac{4m^2 g'_0 \left[ \Delta + (1 - \zeta_m) + \zeta_m \right]}{(1 - \zeta_m) \left[ \Delta + (1 - \zeta_m + 2\zeta_m) \right]} + \frac{4g'^2_0 \Delta_+}{(1 - \zeta_m) \left[ \Delta_+ (1 - \zeta_m + 2\zeta_m) \right]} \]  

(3.27)

The values of \( g'_0(1), g''_0(1) \) and \( \xi \) have been calculated from eq.(3.14) and eq.(3.22) respectively. For that we have taken different values of \( \lambda \) obtained using Sturm-Liouville eigen value method corresponding to different back-reaction parameter \( \kappa \) \cite{40}. The values of \( \lambda \) and \( \xi \) are displayed in Table [1] for different back-reaction parameter \( \kappa \). In the calculations, we have chosen \( m^2 = -3 \) and \( d = 5 \).

| \( \kappa \) | \( \lambda \) | \( \xi \) |
|-------|-------|-------|
| 0     | 18.23 | 0.2016 |
| 0.05  | 18.11 | 0.1997 |
| 0.10  | 17.75 | 0.1938 |
| 0.15  | 17.16 | 0.1839 |

Table 1: Values of \( \lambda \) and \( \xi \) for different \( \kappa \) \( \{ m^2 = -3, d = 5 \} \)

In Fig[1], we have plotted the results of \( \xi \) vs. \( \kappa \). The plot shows that the critical temperature decreases with increase in the back-reaction parameter.
4. Condensate with magnetic field

In this section we shall explore the behaviour of the condensate solution in the presence of a magnetic field. To make progress we proceed with the ansatz

$$A_\mu = (\phi(r), 0, Bx, 0, 0, ...), \psi \equiv \psi(r, x).$$ (4.1)

With this choice in hand, we rewrite eq.(2.3c) as

$$r^2 f(r) \left[ \frac{\partial^2 \psi(r, x)}{\partial r^2} + \left( \frac{d-2}{r} + \frac{f'(r)}{f(r)} - \frac{\chi'(r)}{2} \right) \frac{\partial \psi(r, x)}{\partial r} + \left( \frac{q^2 e^\chi(r)}{f^2(r)} \frac{\phi^2}{f(r)} - \frac{m^2}{f(r)} \right) \psi(r, x) \right]$$

$$= - \frac{\partial^2 \psi(r, x)}{\partial x^2} + q^2 B^2 x^2 \psi(r, x).$$ (4.2)

Using the separation of variables method, we have

$$\psi(r, x) = R(r) X(x).$$ (4.3)

Eq.(4.2) now decouples into two differential equations which has the following forms

$$\frac{1}{X(x)} \frac{d^2 X(x)}{dx^2} - q^2 B^2 x^2 = -K$$ (4.4a)

$$\frac{r^2 f(r)}{R(r)} \left[ \frac{\partial^2 R(r)}{\partial r^2} + \left( \frac{d-2}{r} + \frac{f'(r)}{f(r)} - \frac{\chi'(r)}{2} \right) \frac{\partial R(r)}{\partial r} + \left( \frac{q^2 e^\chi(r)}{f^2(r)} \frac{\phi^2}{f(r)} - \frac{m^2}{f(r)} \right) R(r) \right] = K$$ (4.4b)

where $K$ is the constant of separation. Eq.(4.4a) is of the form of Schrodinger’s equation for a simple harmonic oscillator. The eigen values are $K = \lambda_n qB$ with $\lambda_n = 2n + 1$. The corresponding eigenfunctions can be given in terms of Hermite polynomials, $H_n(\sqrt{2qB}x)$. As explained in [34], for
the rest of our analysis we shall work with the lowest mode \((n = 0)\) solution, which implies \(\lambda_0 = 1\). Eq.(4.4b) can now be written as (introducing new coordinate variable \(z = \frac{r_+}{r}\))

\[
R''(z) + \left[ \frac{f'(z)}{f(z)} - \frac{d - 4}{z} - \frac{\chi'(z)}{2} \right] R'(z) + \left[ \frac{e^{\chi(z)}r_+^2\phi^2(z)}{z^4f^2(z)} - \frac{m^2r_+^2}{z^4f(z)} - \frac{B}{z^2f(z)} \right] R(z) = 0 .
\]  

(4.5)

We shall employ the matching method once again, discussed in the previous section, to find out the relation between the critical magnetic field \((B_c)\) and critical temperature \((T_c)\). For that, first we make a Taylor’s series expansion of \(R(z)\) around \(z = 1\) which reads

\[
R(z) = R(1) - R'(1)(1 - z) + \frac{1}{2}R''(1)(1 - z)^2 + O \left((1 - z)^3\right) .
\]  

(4.6)

Further the asymptotic form for \(R(z)\) reads

\[
R(z) = \frac{\langle O \rangle_+}{r_+^{\lambda_+}} z^{\lambda_+} .
\]  

(4.7)

At any interior point \(z = z_m\) we match these solutions. Hence, equating these at \(z = z_m\) yields

\[
\left[ \frac{\langle O \rangle_+}{r_+^{\lambda_+}} z^{\lambda_+} \right]_{z = z_m} = \left[ R(1) - R'(1)(1 - z) + \frac{1}{2}R''(1)(1 - z)^2 + O \left((1 - z)^3\right) \right]_{z = z_m} .
\]  

(4.8)

Differentiating eq.(s)(4.6, 4.7) with respect to \(z\) and evaluating at \(z = z_m\) yields

\[
\left[ \lambda_+ \frac{\langle O \rangle_+}{r_+^{\lambda_+}} z^{\lambda_+ - 1} \right]_{z = z_m} = \left[ R'(1) - R''(1)(1 - z) + O \left((1 - z)^3\right) \right]_{z = z_m} .
\]  

(4.9)

As was discussed in the earlier section, near the critical temperature we put \(\chi'(z) = \chi(z) = 0\) in eq.(4.5). This simplifies the form of eq.(4.5) and it reads

\[
R''(z) + \left( \frac{f'(z)}{f(z)} - \frac{d - 4}{z} \right) R'(z) + \frac{\phi^2(z)r_+^2R(z)}{z^4f^2(z)} - \frac{m^2r_+^2R(z)}{z^4f(z)} = \frac{BR(z)}{z^2f(z)} .
\]  

(4.10)

From this equation, we obtain

\[
R'(1) = \left( \frac{m^2}{g_0(1)} + \frac{B}{r_+^2g_0(1)} \right) R(1)
\]  

(4.11)

\[
R''(1) = \frac{1}{2} \left[ d - 4 + \frac{m^2}{g_0(1)} + \frac{B}{r_+^2g_0(1)} - g_0''(1) \right] \left[ \frac{m^2}{g_0(1)} + \frac{B}{r_+^2g_0(1)} \right] R(1)
\]  

\[+ \frac{BR(1)}{r_+^2g_0(1)} - \frac{\phi'^2(1)R(1)}{2r_+^2g_0^2(1)} .
\]  

(4.12)

Substituting \(R'(1)\) and \(R''(1)\) in eq.(s)(4.8, 4.9), we have
\[
\left[ \frac{\langle O \rangle_{+}^{z_{m}^{+}}}{r_{+}^{z_{m}^{+}}} \right] = R(1) - \left( \frac{m^{2}}{g_{0}(1)} + \frac{B}{r_{+}^{2}g_{0}(1)} \right) (1 - z_{m}) R(1)
+ \frac{1}{2}(1 - z_{m})^{2}\left[ \frac{1}{2} \left( d - 4 + \frac{m^{2}}{g_{0}(1)} + \frac{B}{r_{+}^{2}g_{0}(1)} - \frac{g_{0}''(1)}{g_{0}'(1)} \right) R(1)
\right. \\
\left. + \frac{BR(1)}{r_{+}^{2}g_{0}(1)} - \frac{\phi'^{2}(1)R(1)}{2r_{+}^{2}g_{0}^{2}(1)} \right].
\tag{4.13}
\]

\[
\left[ \lambda_{+} \frac{\langle O \rangle_{+}^{z_{m}^{+}}}{r_{+}^{z_{m}^{+}}} \right] = \left( \frac{m^{2}}{g_{0}(1)} + \frac{B}{r_{+}^{2}g_{0}(1)} \right) R(1)
- (1 - z_{m}) \left[ \frac{1}{2} \left( d - 4 + \frac{m^{2}}{g_{0}(1)} + \frac{B}{r_{+}^{2}g_{0}(1)} - \frac{g_{0}''(1)}{g_{0}'(1)} \right) R(1)
\right. \\
\left. + \frac{BR(1)}{r_{+}^{2}g_{0}(1)} - \frac{\phi'^{2}(1)R(1)}{2r_{+}^{2}g_{0}^{2}(1)} \right].
\tag{4.14}
\]

Eq. (s) (4.13) and (4.14) yields a quadratic equation for \( B \). This reads
\[
B^{2} + pr_{+}^{2}B + nr_{+}^{4} - \phi'^{2}(1)r_{+}^{2} = 0
\tag{4.15}
\]
where
\[
p = 2m^{2} + \left( d - 4 - \frac{g_{0}''(1)}{g_{0}'(1)} \right) g_{0}'(1) + 2g_{0}'(1) - \frac{4g_{0}'(1)(\lambda_{+}(1 - z_{m}) + z_{m})}{(1 - z_{m})(\lambda_{+}(1 - z_{m}) + 2z_{m})}
\tag{4.16}
\]
and
\[
n = m^{4} + m^{2}g_{0}'(1) \left[ \left( d - 4 - \frac{g_{0}''(1)}{g_{0}'(1)} \right) - \frac{4(z_{m} + \lambda_{+}(1 - z_{m}))}{(1 - z_{m})(2z_{m} + \lambda_{+}(1 - z_{m}))} \right]
\tag{4.17}
\]
\[
\quad + \frac{4\lambda_{+}g_{0}''(1)}{(1 - z_{m})(2z_{m} + \lambda_{+}(1 - z_{m}))}.
\]

Since at \( T = T_{c} \), the scalar field vanishes \( (\psi(z) = 0) \), this gives \( \phi'(1) \) from eq. (3.8) and reads
\[
\phi'(1) = -\lambda r_{+}(d - 3).
\tag{4.18}
\]

Substituting this in eq. (4.15) gives the critical magnetic field to be
\[
B_{c} = \left( \frac{1}{2} \right) \left( - \frac{g_{0}(1)}{4\pi} \right)^{d-4} \left( \frac{1}{\xi} \right)^{d-2} \left[ \Omega(d, m, z_{m}) - p \left( - \frac{4\pi \xi}{g_{0}(1)} \right)^{d-2} \left( \frac{T}{T_{c}} \right)^{d-2} \right]
\tag{4.19}
\]
where
\[
\Omega(d, m, z_{m}) = \left[ 4(d - 3)^{2} - (4n - p^{2}) \left( - \frac{4\pi \xi}{g_{0}(1)} \right)^{(2d-4)} \left( \frac{T}{T_{c}} \right)^{(2d-4)} \right].
\tag{4.20}
\]
In Fig.[2], we have plotted $B_c/T_c^2$ against $T/T_c$ for different back-reaction parameters $\kappa$, with $z_m = 0.5$, $m^2 = -3$, $d=5$. It is evident from the figure that there exists a certain critical temperature $T_c$ and a critical magnetic field $B_c$ above which the superconducting phase vanishes.

From Fig.[2], it is to be noted that the ratio of $B_c/T_c^2$ at $T=0$ increases with increase in the back reaction parameter. The value of $B_c$ can be estimated from the ratio $B_c/T_c^2$ and using eq.(3.21). These values have been displayed in Table [2].

| $\kappa$ | $B_c/T_c^2$ | $B_c$ |
|----------|-------------|-------|
| 0        | 77.5879     | 3.1533 $\rho_2^\frac{3}{2}$ |
| 0.05     | 78.6834     | 3.1379 $\rho_2^\frac{3}{2}$ |
| 0.1      | 82.0542     | 3.0818 $\rho_2^\frac{3}{2}$ |
| 0.15     | 89.1723     | 3.0157 $\rho_2^\frac{3}{2}$ |

From Table [2], it can be observed that the critical magnetic field $B_c$ decreases with increase in the back reaction parameter $\kappa$. This indicates that the presence of the back reaction parameter destroys the superconducting phase earlier, that is, for a smaller value of the critical magnetic field $B_c$.

## 5. Conclusion

In this paper we have studied the influence of back reaction on holographic superconductor in the framework of Maxwell electrodynamics using the matching method. The holographic superconductor model that we have considered in our analysis consists of Einstein-Hilbert gravity theory along with a complex scalar field minimally coupled to Maxwell field. Further we have investigated the effect of an external magnetic field in our holographic superconductor model. From our analysis we can conclude from the critical temperature and charge density relationship that the critical temperature depends on both charge density and the back reaction parameter. We have presented the analytical results for
the ratio of the critical temperature and charge density for $d = 5$ and $m^2 = -3$. Our results indicate that condensation gets harder to form as we include the effect of back reaction. With these findings, we then obtain the expression for the critical magnetic field above which the superconducting phase vanishes. This is obtained using matching method once again. We observe that the ratio of $B_c$ and $T_c^2$ at $T = 0$ increases with the increase in back reaction parameter $\kappa$. However, we have found that the critical magnetic field $B_c$ decreases with increase in the back reaction parameter $\kappa$. This clearly tells that the presence of the back reaction parameter destroys the superconducting phase for a smaller value of the critical magnetic field $B_c$.

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