Measuring Higgs boson self-couplings with $2 \rightarrow 3$ VBS processes

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ABSTRACT: We study the measurement of Higgs boson self-couplings through $2 \rightarrow 3$ vector boson scattering (VBS) processes in the framework of Standard Model effective field theory (SMEFT) at both proton and lepton colliders. The SMEFT contribution to the amplitude of the $2 \rightarrow 3$ VBS processes, taking $W_LW_L \rightarrow W_LW_Lh$ and $W_LW_L \rightarrow hhh$ as examples, exhibits enhancement with the energy $\frac{\Lambda_{\text{SMEFT}}}{\Lambda}$, which indicates the sensitivity of these processes to the related dimension-six operators in SMEFT. Simulation of the full processes at both hadron and lepton colliders with a variety of collision energies are performed to estimate the allowed region on $c_6$ and $c_{\Phi 1}$. Especially we find that, with the help of exclusively choosing longitudinal polarizations in the final states and suitable $p_T$ cuts, $WWh$ process is as important as the more widely studied triple Higgs production ($hhh$) in the measurement of Higgs self-couplings. Our analysis indicates that these processes can play important roles in the measurement of Higgs self-couplings at future 100 TeV pp colliders and muon colliders. However, their cross sections are generally tiny at low energy machines, which makes them much more challenging to explore.

KEYWORDS: Higgs Physics, Beyond Standard Model, Effective Field Theories

ArXiv ePrint: 2105.11500
1 Introduction

The discovery of Higgs boson at LHC [1, 2] marked a new era for particle physics. Although all experimental results so far agree with the standard model (SM), the origin of electroweak symmetry breaking (EWSB) still remains a mystery and deserves more detailed studies [3–7]. Meanwhile the absence of BSM signal also makes precise measurements of Higgs properties more important than ever. Especially, in order to determine the shape of the Higgs potential, the measurement of Higgs self-couplings becomes critical.

The most straightforward approach to measure Higgs couplings is through direct productions of Higgs boson(s). As the most notable example, the main channel to measure trilinear Higgs self-couplings at the LHC is the di-Higgs production through gluon-gluon fusion, see e.g. [8–15] and references therein. However, processes related to longitudinal vector bosons can also be used for the measurement of Higgs couplings [16, 17]. The underlying reason is as following: according to Goldstone equivalence theorem (GET) [18–20], scattering amplitudes of longitudinal vector bosons can be approximately evaluated by amplitudes of the corresponding Goldstone bosons i.e. $V_L \sim \phi$. Moreover, since Goldstone bosons and the Higgs boson form a SU(2) doublet in the SM, as well as in the Standard Model effective field theory (SMEFT) [21–23], couplings of the Goldstone bosons are related to Higgs couplings through the same parameters. Therefore, processes involving longitudinal vector bosons provide an alternative approach to measure Higgs couplings.
Recently, it was proposed in [16] that vector boson scattering (VBS) processes with multiple final states at the LHC with or even without Higgs involved (and its counterparts at lepton colliders) can be used for the measurement of trilinear Higgs coupling. It was argued that the energy increase of dim-6 operators from longitudinal vector boson enhances the sensitivity of amplitudes to related Wilson coefficients in high energy. As a result, those process can potentially be very beneficial to the precise measurement of Higgs self-couplings. In this work we follow up this proposal by studying $2 \to 3$ VBS processes extensively at different colliders. Different from [16], however, our strategy is to take GET directly and analyze the high energy behavior of $2 \to 3$ VBS amplitudes under SMEFT. The goal of our paper is two folded. First, we try to understand more clearly how higher dimension operators affect the amplitudes of $2 \to 3$ VBS. This is mainly achieved by analyzing how different Feynman diagrams (after taking GET) contribute to the amplitude. We choose $W_L W_L \to W_L W_L h$ and $W_L^+ W_L^- \to hhh$ as examples with $V_L V_L h$ and $hhh$ final states respectively. Second, guided by the results of analyzing amplitudes, we carry out simulations to study the measurement of Higgs self-couplings at the HL-LHC, as well as its future upgrades (HE-LHC), 100TeV pp colliders and lepton colliders [24–37].\footnote{Some previous studies for exploring trilinear Higgs coupling can be found in [38–50].} We set to give a qualitative picture of the sensitivity of full processes at colliders to dim-6 operators, and the potential of the measurement of Higgs self-couplings at different colliders.

Our main results are briefly summarized as following. At high energy regions, amplitudes of $2 \to 3$ VBS are indeed sensitive to dim-6 operators, with $\frac{A_{BSM}}{A_{SM}} \sim E^2$. However, there are some subtleties involved that will be discussed carefully in the paper. This sensitivity to dim-6 operators translates further to full processes at colliders, although smallness of cross sections indicates that the processes can only be useful at the future 100TeV pp colliders or high energy muon colliders. After exclusively selecting longitudinal polarizations for vector bosons in the final states and applying suitable $p_T$ cuts in the phase space of final state particles, processes with final state $WWh$ are found to be as important as triple Higgs production ($hhh$).

The rest of the paper is organized as following. In section 2, we lay down the framework of SMEFT and discuss the dim-6 operators that are relevant in this paper and then derive and discuss related scalar couplings. Then, we derive and analyze the amplitudes of $W_L W_L \to W_L W_L h$ and $W_L^+ W_L^- \to hhh$ at high energy by using GET in section 3. The dependence on Wilson coefficients are also discussed, along with other subtleties. The cross section for the full processes $pp \to j j W_L W_L h$ and $pp \to j j hhh$ at hadron colliders and $\mu^+ \mu^- \to \nu_\mu \bar{\nu}_\mu W_L^+ W_L^- h$ and $\mu^+ \mu^- \to \nu_\mu \bar{\nu}_\mu hhh$ at muon colliders are studied in section 4 through which we discuss the sensitivity of these channels on the SMEFT operators. Finally, we conclude in section 5.
2 Scalar couplings in SMEFT

2.1 Relevant Dim-6 operators in SMEFT

The null result of searching BSM signals at the LHC indicates that new physics may be hidden at the energy scale much higher than the electroweak (EW) scale. This justifies the usage of effective field theory (EFT) to constrain the possible new physics in a model independent way. Preserving the SM gauge symmetry group of SU(3)_c \times SU(2)_L \times U(1)_Y further reduces the framework to SMEFT. This framework is suitable for the scenario that the Higgs boson is an elementary particle [6]. We will focus on this case and ignore other exotic scenarios hereafter.

Generally, if we ignore the dim-5 Weinberg operator [21], the Lagrangian for SMEFT can be written as

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_i c_i \mathcal{O}_i \Lambda^2 + \mathcal{O} \left( \frac{1}{\Lambda^3} \right) \quad (2.1)$$

The first term $\mathcal{L}_{\text{SM}}$ is the Lagrangian for the SM, which includes all known physics; the second term includes all dim-6 operators that are suppressed by $\Lambda^2$, with $\Lambda$ being the energy scale of new physics and $c_i$ being Wilson coefficients of the corresponding operators $\mathcal{O}_i$.

Ignoring CP violating terms, dim-6 operators relevant to couplings of (and between) scalars and gauge bosons can be written as

$$\mathcal{L}_{\text{dim-6}} = \frac{1}{\Lambda^2} \left( c_6 (\Phi^\dagger \Phi)^3 + c_{\Phi_1} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + c_{\Phi_2} (\Phi^\dagger D^\mu \Phi)^* (\Phi^\dagger D_\mu \Phi) 
+ c_{\Phi_3} W^{a\mu} \Phi^\dagger W_a^{\mu\nu} W^{\nu\rho} 
+ c_{\Phi_4} B^{\mu\nu} \Phi^\dagger B_{\mu\nu} 
+ c_{\Phi_5} W^a \Phi^\dagger D^\mu \Phi W_{\mu\nu} W^a B^{\nu\rho} 
+ c_{\Phi_6} \epsilon^{abc} W^{a\mu} W_b^{\mu\nu} W_{\nu\rho} W^{\rho\mu} \right) \quad (2.2)$$

where $\Phi$ is the Higgs doublet and can be parameterized as

$$\Phi = \begin{pmatrix} \phi^+ \\ v + h_i / \sqrt{2} \end{pmatrix} \quad (2.3)$$

The processes considered in this paper are $2 \to 3$ VBS with initial and final vector bosons being longitudinal polarized, $V_L V_L \to V_L V_L h$ and $V_L V_L \to h h h$. In general, after EWSB, $\phi^+ / \phi^0$ are considered as unphysical, "eaten" by gauge fields $W^\pm / Z$ becoming their longitudinal components. The "real" identities of those degrees of freedom only reveal themselves at high energy through GET [18–20, 51–56]. Especially in unitary gauge, $\Phi$ becomes $\Phi = \begin{pmatrix} 0, (v + h)/\sqrt{2} \end{pmatrix}^T$ under which the would-be Goldstone bosons disappear from the physical spectrum and Feynman diagrams. The three operators in the first line of eq. (2.2), $\mathcal{O}_6$, $\mathcal{O}_{\Phi_1}$ and $\mathcal{O}_{\Phi_2}$, induce and modify Higgs self-couplings, as well as couplings between vector boson bosons and the Higgs boson. The operators in the second line and the third line in eq. (2.2) are also involved in scatterings of longitudinal vector bosons, because in this physical picture, couplings involving longitudinal vector bosons, especially vector bosons couplings to vector bosons, are mainly induced by gauge fields. However, there is a well-known problem in this physical picture of only gauge bosons being physical: power counting becomes invalid. This becomes a major obstacle to understanding the...
leading energy behavior of processes involving longitudinal vector bosons. Especially, in
the presence of SMEFT, higher dimensional operators usually have derivative couplings
that lead to unitarity violating energy increase of S-matrix which are suppressed by scale
Λ. Meanwhile, longitudinal polarization vectors also bring energy increase in the diagram
by diagram level, which nevertheless disappears in the final S-matrix as guaranteed by
GET. The real energy increase from derivative couplings and spurious energy increase
from longitudinal vector bosons mix together in unitary gauge and the related physical
picture. This obscures the underlying physics greatly.

To solve the problem discussed above, we simply take GET and identify
$V_L$ with $\phi$
directly. This is a good approximation as long as the energy scale is much larger than
EW scale. Without the spurious energy increase from longitudinal polarization vectors,
the energy behavior of Feynman diagrams becomes physical. Thus we can analyze the
amplitudes at the level of single diagrams and obtain the leading energy behavior of the
processes. Moreover, since $\phi^\pm / \phi^0$ and $h$ all belong to the same SU(2) Higgs doublet, the
couplings of (and between) Goldstone bosons and Higgs boson, are determined by the same
parameters of the Higgs potential. Thus it becomes manifest that we can measure Higgs
self-couplings through processes involving $V_L$s or $\phi$s.

Now let’s review the dim-6 operators in eq. (2.2) under GET. Higgs self-couplings
and couplings between Goldstone bosons and Higgs are induced by $O_6$, $O_{\Phi_1}$ and $O_{\Phi_2}$,together with SM Lagrangian terms. $O_6$ is the only one that contributes to 5-point and
6-point scalar vertices. $O_{\Phi_2}$ term violates custodial symmetry, the Wilson coefficient $c_{\Phi_2}$ is
strongly constrained by LEP [39]. Therefore, we will ignore it from now on. $O_{\Phi_1}$, $O_{\Phi_2 W^2}$,
$O_{\Phi_2 B^2}$ and $O_{\Phi_2 W B}$ give rise to gauge-gauge-scalar and gauge-gauge-scalar-scalar vertices.
$O_{W^3}$ doesn’t contribute to $V_L V_L \rightarrow V_L V_L h$, but can have contributions to the amplitudes if
the polarizations of vector bosons are transverse. Moreover, $O_{W^3}$’s vertices also contribute
to $2 \rightarrow 4$ process, e.g. $V_L V_L \rightarrow V_L V_L hh$. For simplicity, we are only interested in the
modification to scalar couplings in this work. Thus only $O_6$ and $O_{\Phi_1}$ are considered.

2.2 Scalar vertices

The full Feynman rules in SMEFT can be found in [57]. Here we briefly review the results
related to our processes. Before symmetry breaking, the Lagrangian in scalar sector is
\[ L_\Phi = (D_\mu \Phi)^\dagger (D^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda_h (\Phi^\dagger \Phi)^2 + \frac{c_6}{\Lambda^2} (\Phi^\dagger \Phi)^3 + \frac{c_{\Phi_1}}{\Lambda^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi), \]
(2.4)
where $\Lambda$ is the new physics scale, which we choose as 1 TeV in the paper. After symmetry
breaking, the VEV $v$ can be expressed in terms of $\mu$, $\lambda_h$ and $c_6$ by tracking the minimum
position of the Higgs potential,
\[ v = \sqrt{\frac{\mu^2}{\lambda_h}} - \frac{3}{8} \frac{\mu^3}{\lambda_h^{5/2} \Lambda^2} c_6 \]
(2.5)
The Higgs field $h$ and goldstone field $\phi^0$ need extra field renormalization, after which, we
obtain Higgs mass $m_h$ in terms of $(v, c_6, c_{\Phi_1}, \lambda_h)$:
\[ m_h^2 = \frac{1}{2} \lambda_h v^2 - (3c_6 + \lambda_h c_{\Phi_1}) \frac{v^4}{\Lambda^2} \]
(2.6)
Eliminating $(\mu, \lambda_h)$ by $(m_h, v)$ through eq. (2.5) and eq. (2.6), we can express all scalar vertices in terms of $(m_h, v, c_0, c_{\Phi_1})$, with $v^2 = \frac{v^2F}{2\xi_F} \approx (246 \text{ GeV})^2$. Then, we have all 3-point, 4-point and 5-point scalar couplings to be

\begin{equation}
\lambda_{\phi^+\phi^-} = \lambda_{\phi^0\phi^0} = -i \frac{m_h^2}{v} + ic_{\Phi_1} v \frac{p_1^2 + m_h^2}{\Lambda^2}, \tag{2.7a}
\end{equation}

\begin{equation}
\lambda_{hhh} = -i \left( \frac{3m_h^2}{v} - 6c_0 v^2 \right) + ic_{\Phi_1} v \frac{2p_1^2 + 2p_2^2 + 2p_3^2 + 3m_h^2}{\Lambda^2}, \tag{2.7b}
\end{equation}

\begin{equation}
\lambda_{\phi_0^+} = -3i \frac{m_h^2}{v^2} - 3ic_{\Phi_1} v^2 - 2ic_{\Phi_1} \frac{(p_1 + p_2)^2 + (p_1 + p_3)^2 + (p_1 + p_4)^2}{\Lambda^2}, \tag{2.7c}
\end{equation}

\begin{equation}
\lambda_{\phi_0^+\phi^-} = -i \frac{m_h^2}{v^2} - ic_{\Phi_1} v^2 - 2ic_{\Phi_1} \frac{(p_1 + p_2)^2}{\Lambda^2}, \tag{2.7d}
\end{equation}

\begin{equation}
\lambda_{\phi^+\phi^-h^2} = \lambda_{\phi_0^+h^2} = -i \left( \frac{m_h^2}{v^2} - 6c_0 v^2 \right) + ic_{\Phi_1} v^2 \frac{2p_1^2 + 2p_2^2 + 2p_3^2 + 3m_h^2}{\Lambda^2}, \tag{2.7f}
\end{equation}

\begin{equation}
\lambda_{h^4} = -12ic_{\Phi_1} v^2 \frac{2p_1^2 + 2p_2^2 + 2p_3^2 + 2p_4^2 + 6m_h^2}{\Lambda^2}, \tag{2.7g}
\end{equation}

\begin{equation}
\lambda_{\phi^+\phi^-h^2} = \lambda_{\phi_0^+h^2} = 18ic_{\Phi_1} v^2, \tag{2.7h}
\end{equation}

\begin{equation}
\lambda_{(\phi^+\phi^-)^2h} = 12ic_{\Phi_1} v^2, \tag{2.7i}
\end{equation}

\begin{equation}
\lambda_{\phi^0\phi^0h^2} = 6ic_{\Phi_1} v^2, \tag{2.7j}
\end{equation}

where $p_i$ in $\lambda_{\phi^+\phi^-}$ and $\lambda_{\phi^0\phi^0}$ is the momentum of the Higgs boson; $p_1, p_2$ in $\lambda_{\phi^+\phi^-\phi^0}^2, \lambda_{\phi_0^+h^2}$ and $\lambda_{\phi^0h^2}$ are momenta of $\phi^+ / \phi^-$ or equivalently $h$; in $\lambda_{\phi_0^+\phi^-}^2$, we can assign $p_1$ and $p_2$ to $\phi^+, \phi^-$, and $p_3$ and $p_4$ to $\phi^-$. 6-point scalar couplings are not listed as they are irrelevant to the processes we study here.

3 \textbf{Amplitudes and cross sections}

3.1 \textbf{Feynman diagrams and amplitudes with Goldstone equivalence}

In this section we analyze the amplitude of $V_L V_L \rightarrow V_L V_L h$ and $V_L V_L \rightarrow h hh$ at high energy by using GET for SM plus two dim-6 operators of SMEFT ($O_6, O_{\Phi_1}$). For simplicity, we will choose $W_L^+ W_L^- \rightarrow W_L^+ W_L^- h$ and $W_L^+ W_L^- \rightarrow h hh$ as examples to illustrate the general behavior, and work in Feynman gauge.

Following GET, $W_L^+ W_L^- \rightarrow W_L^+ W_L^- h$ and $W_L^+ W_L^- \rightarrow h hh$ can be approximated by $\phi^+ (p_1) \phi^- (p_2) \rightarrow \phi^+ (p_3) \phi^- (p_4) h (p_5)$ and $\phi^+ (p_1) \phi^- (p_2) \rightarrow h (p_3) h (p_4) h (p_5)$ at high energy. The corresponding amplitudes can be classified according to the number of propagators:
zero, one or two propagators:
\[ A = A_0 + A_1 + A_2, \]  
where \( A_i \) with \( i = 0, 1, 2 \) can be further classified as diagrams from SM vertices only and diagrams including BSM contributions: \( A_i = A_i^{\text{SM}} + A_i^{\text{BSM}} \).

Before going to the analysis in details, it’s important to understand in a general way how the high energy behavior of \( \phi^+\phi^- \to \phi^+\phi^-h \) is determined. After the unphysical energy increase from longitudinal polarization vectors getting eliminated, physical energy dependence only comes from derivative couplings in dim-6 operators as well as the \( \frac{1}{E^2} \) factor of the propagators. Understanding the overall energy behavior of the amplitude from the interplay of different factors is the main focus of our analysis. We mainly focus on \( \phi^+\phi^- \to \phi^+\phi^-h \) process, and make comments when there is difference with \( \phi^+\phi^- \to \phi^+\phi^-hh \).

Since we will focus on the high energy behavior, we also only keep the leading terms in \( \frac{1}{E^2} \).

**Diagrams with 0 propagator.** In the SM, \( \phi^+\phi^- \to \phi^+\phi^-h \) and \( \phi^+\phi^- \to hh \) have no 5-point contact diagram. There is, however, one such diagram from the \( \mathcal{O}_6 \) operator, as shown in figure 1.

\[ A_{\phi^+\phi^- \to \phi^+\phi^-h} = \lambda_{(\phi^+\phi^-)h} = 12ic_6 \frac{v}{\Lambda^2} \]  
\[ A_{\phi^+\phi^- \to hh} = \lambda_{\phi^+\phi^-h^3} = 18ic_6 \frac{v}{\Lambda^2} \]  

In both cases we obtain \( A_0 \sim \frac{v}{\Lambda^2} \).
Diagrams with 1 propagator. For any $2 \rightarrow 3$ process, the amplitude of all Feynman diagrams with one propagator can be schematically written as

$$A_1 = \sum_{\text{diagrams}} \frac{M'_1 \cdot M'_3}{q^2 - m^2} \quad (3.4)$$

The intermediate particle of the propagator can be either a scalar or a vector boson, as illustrated in figure 2 for $\phi^+ \phi^- \rightarrow \phi^+ \phi^- h$. $M'_1$ and $M'_3$ denote the amplitudes involving 4-point and 3-point vertices respectively, with Lorentz indices suppressed if the intermediate state is a vector boson. However, for the vector boson propagator, we need $\phi \phi \phi V$-like vertex, which does not exist in the SM and dim-6 operators we consider in this work ($O_6$ and $O_{8_1}$).\footnote{We could have $\phi \phi \phi V$-like vertex, once we include more operators in SMEFT, e.g. $O_{8_2}$. The analysis will be similar and they have same behavior at high energy.} Thus the propagator can only be a scalar.

For SM-only diagrams, we have

$$A_{1}^{\text{SM}} = -2i \frac{m^2_h}{v^2} \left( \frac{1}{(p_3 + p_5)^2 - m_W^2} + \frac{1}{(p_4 + p_5)^2 - m_W^2} + \frac{1}{(p_2 - p_5)^2 - m_W^2} + \frac{1}{(p_1 - p_5)^2 - m_W^2} \right)$$
$$-2i \frac{m^2_h}{v^2} \left( \frac{1}{(p_1 + p_2)^2 - m_h^2} + \frac{1}{(p_1 - p_3)^2 - m_h^2} \right). \quad (3.5)$$

Neither 4-point vertices nor 3-point vertices have any energy dependence. Since at high energy, $p_i^2 \sim E^2 \gg m_W^2, m_h^2$, the amplitude scales as $A_{1}^{\text{SM}} \sim \frac{v}{\Lambda^2}$.

For BSM contributions, keep terms up to $\frac{1}{\Lambda^2}$, we have:

$$A_{1}^{\text{BSM}} \simeq -2i c_{\Phi} \frac{m^2_h}{v\Lambda^2} \left( \frac{(p_1 + p_2)^2}{(p_4 + p_5)^2 - m_W^2} + \frac{(p_1 + p_2)^2}{(p_3 + p_5)^2 - m_W^2} + \frac{(p_1 - p_3)^2}{(p_2 - p_5)^2 - m_W^2} + \frac{(p_1 - p_3)^2}{(p_1 - p_5)^2 - m_W^2} \right)$$
$$- i c_{\Phi} \frac{m^2_h}{v\Lambda^2} \left( \frac{(p_1 + p_2)^2}{(p_3 + p_4)^2 - m_h^2} + \frac{(p_1 + p_2)^2}{(p_1 + p_2)^2 - m_h^2} + \frac{(p_1 - p_3)^2}{(p_2 - p_4)^2 - m_h^2} + \frac{(p_1 - p_3)^2}{(p_1 - p_3)^2 - m_h^2} \right)$$
$$-16i c_{\Phi} \frac{m^2_h}{v\Lambda^2} \left( \frac{(p_3 + p_4)^2}{(p_3 + p_1)^2 - m_h^2} + \frac{(p_1 + p_2)^2}{(p_1 + p_2)^2 - m_h^2} + \frac{(p_2 - p_4)^2}{(p_2 - p_4)^2 - m_h^2} + \frac{(p_1 - p_3)^2}{(p_1 - p_3)^2 - m_h^2} \right), \quad (3.6)$$

where both 3-point and 4-point scalar vertices provide momentum-dependent couplings leading to $E^2$ behavior in the numerator, which cancels the $\frac{1}{\Lambda^2}$ factor from the propagator. Therefore, we obtain $A_{1}^{\text{BSM}} \sim \frac{v^3}{\Lambda^2}$. Similarly, the behavior of amplitude for $\phi^+ \phi^- \rightarrow hhh$ also scales as $\frac{v^3}{\Lambda^2}$ in the SM and $\frac{v^3}{\Lambda^2}$ when dim-6 operators are involved.

Diagrams with 2 propagators. The amplitudes for the Feynman diagrams with two propagators can be classified according to the type of the propagators as shown in figure 3:

(a) Two scalars

(b) One scalar and one vector boson

(c) Two vector bosons
There are too many diagrams with two propagators to give a short and concise analytical expression even in high energy limit, hence we only analyze a representative diagram for each case.

For the case with two scalar propagators, a typical diagram is shown in figure 3a which has both SM and BSM contributions:

\begin{align}
A_{2,SM}^a &\simeq -i \left( \frac{m_h^2}{v} \right)^3 \frac{1}{(p_1 + p_2)^2 - m_h^2} \frac{1}{(p_4 + p_5)^2 - m_W^2}, \\
A_{2,BSM}^a &\simeq 4ic_{\Phi} \frac{m_h^4}{v^2} \frac{(p_1 + p_2)^2}{(p_1 + p_2)^2 - m_h^2} \frac{1}{(p_4 + p_5)^2 - m_W^2}.
\end{align}

(3.7)

Thus, at high energy, diagrams with two scalar propagators scale as $A_{2,SM}^a \sim \frac{v^3}{E^4}$, $A_{2,BSM}^a \sim \frac{v^3}{E^2}$. They are suppressed by $\frac{1}{E^2}$ compared with $A_{1,SM}^a$ and $A_{1,BSM}^a$ respectively.

The typical diagram for the case with one scalar and one vector boson propagators is shown in figure 3b. Summing the diagram and the diagram by exchanging $p_3 \leftrightarrow p_4$, keeping only the leading contributions, we have

\begin{align}
A_{2,SM}^b &\simeq g^2 m_h^2 \left( \frac{(p_1 + p_2 + p_3)(p_5 - p_4)}{((p_1 + p_2)^2 - m_h^2)((p_1 + p_4)^2 - m_W^2)} + \frac{(p_1 + p_2 + p_4)(p_5 - p_3)}{((p_1 + p_2)^2 - m_h^2)((p_3 + p_5)^2 - m_W^2)} \right), \tag{3.8}
\end{align}

where we only have SM contribution. At high energy, the amplitude scales as $A_{2,SM}^b \sim \frac{v}{E^2}$, which is of the same order as $A_{1,SM}^a$.

In figure 3c, we show the typical diagram for the case with two vector boson propagators. The amplitude only receives SM contributions and reads:

\begin{align}
A_{2}^c &\simeq -i \frac{g^2 m_Z c_{2W}^2}{4c_W^3} \frac{(p_1 - p_2) \cdot (p_3 - p_4)}{((p_1 + p_2)^2 - m_Z^2)((p_3 + p_4)^2 - m_Z^2)}, \tag{3.9}
\end{align}

where $c_{2W} = \cos 2\theta_W$, $c_W = \cos \theta_W$ and $\theta_W$ is the Weinberg angle. At high energy, it scales as $A_{2}^c \sim \frac{v}{E^2}$.

Combined amplitudes. Taking into account all cases discussed above, the amplitude of $W_L^+ W_L^- \rightarrow W_L^+ W_L^- h$ can be written as

\begin{align}
A(W_L^+ W_L^- \rightarrow W_L^+ W_L^- h) = A_{SM} + A_{BSM}^{SM}, \tag{3.10}
\end{align}
with $A^{\text{SM}}$ being the SM contribution which has no dependence on $c_6$ or $c_{\Phi_1}$ and $A^{\text{BSM}}$ the BSM contributions depending on $c_6$ and $c_{\Phi_1}$. We only keep the terms up to the order of $\frac{c_6}{\Lambda^2}$, higher order terms are truncated to be consistent with the EFT expansions.

The leading energy dependence of SM and BSM contributions are

$$A^{\text{SM}} \sim \frac{v}{E^2}, \quad A^{\text{BSM}} \sim \frac{v}{\Lambda^2}.$$  \hspace{1cm} (3.11)

Thus, the ratio between BSM and SM is approximately

$$\frac{A^{\text{BSM}}}{A^{\text{SM}}} \sim \frac{E^2}{\Lambda^2}.$$  \hspace{1cm} (3.12)

From eq. (3.12) we find that, the BSM contribution to the total amplitude will be enhanced relative to the SM one at high energy. Let’s stop and analyze the physical reasons behind eq. (3.12). By a naive dimensional analysis, the amplitude for $2 \to 3$ process will scale as $1/(\text{GeV})$. For the SM contributions, combining the energy dependence from the propagator and 3-point vertices, it scales as $\frac{v}{E}$ with the energy coming from the propagator. On the other hand, the BSM contribution has a different leading energy behavior — it remains constant as $\frac{v}{\Lambda^2}$ due to: (a) 5-point scalar vertices as shown in figure 1 from $O_6$, giving dependence on $c_6$; (b) The cancellation between energy suppression from propagators and energy increase from momentum dependence in 3/4-point vertices from $O_{\Phi_1}$, giving dependence on $c_{\Phi_1}$. Although this enhancement of the BSM contribution from $c_{\Phi_1}$ relative to SM one applies to many processes, it is not the case for $c_6$, which depends crucially on the 5-point scalar vertices coming solely from $O_6$. Since $O_6$ is also the only source for 6-point scalar vertices, this sensitivity of amplitude to BSM physics for $c_6$ also applies to $2 \to 4$ VBS processes. In comparison, 4-point scalar vertices can come from both SM and higher dimensional operators, therefore the amplitude of $2 \to 2$ VBS does not have the behavior of eq. (3.11) for $c_6$.

There is also a subtlety that’s related to the so-called soft-collinear singularities that originate from propagators reaching (close to) to the poles, see, for example, [58–63].

After integrating over phase space, those singularities result in logarithmic enhancement to the cross sections which will change the behavior in eq. (3.12) and can reduce the sensitivities to the Wilson coefficients. Hence those singularities require careful treatments which we will discuss in the next section.

3.2 Cross sections for subprocesses

After deriving the amplitudes of $W_L W_L \rightarrow W_L W_L h$ and $W_L W_L \rightarrow h h h$ using GET, we will now examine the behaviors of the cross sections for $W_L W_L \rightarrow W_L W_L h$ and $W_L W_L \rightarrow h h h$ which is calculated using FeynArts [64] and FormCalc [65] with a cut $p_T > 50 \text{ GeV}$ on the final states and also cross checked with MadGraph [66]. The dependence on $c_6$ and $c_{\Phi_1}$ are considered separately.

The dependence of the cross section for $W W \rightarrow W W h$ and $W W \rightarrow h h h$ on $c_6$ is shown in figure 4 for two representative energy $\sqrt{s} = 1 \text{ TeV}$ (solid lines) and $\sqrt{s} = 3 \text{ TeV}$ (dashed lines). The dependence on $c_6$ for these processes only comes from the 5-point contact terms.
as shown in figure 1 which is denoted as the yellow line in figure 4. Thus the dependence of the cross section of $\phi\phi \to \phi\phi h$ and $\phi\phi \to hhh$ (green lines), as well as $W_L W_L \to W_L W_L h$ and $W_L W_L \to hhh$ (blue lines), mainly follows the behavior of the yellow lines, except the region where $c_6$ is close to zero.

However, $WW \to WW h$ is dominated by transverse polarizations as can be seen by comparing the red and blue lines. Thus, in order for the former process to remain sensitive to $c_6$, longitudinal polarizations of $W$ boson pair need to be singled out by some specific selections. The technical details can be found in [67–71]. On the other hand $WW \to hhh$ is dominated by longitudinal polarizations, hence, the sensitivity is largely remained in this sense. In figure 5, we show the ratio $\sigma/\sigma_{SM}$ as functions of $c_6$ for $W^+W^- \to W^+W^- h$ and $W^+W^- \to hhh$. It is clear that, by just looking at these $2 \to 3$ processes, $WW \to hhh$ is more sensitive than $WW \to WW h$ on $c_6$. Note that, the sensitivity is reduced at
higher energy which is mainly due to the logarithm enhancement of the SM cross section as discussed in previous section.

Similar results for \( c_{\Phi 1} \) are shown in figure 6 and figure 7. Again, \( WW \rightarrow WWh \) is dominated by transverse polarizations. Compared with the dependence on \( c_6 \), the cross section is much more enhanced by \( c_{\Phi 1} \) due to the momentum dependence of \( \mathcal{O}_{\Phi 1} \) operator. Further, this momentum dependence of \( \mathcal{O}_{\Phi 1} \) also overcomes the logarithms in SM cross section, thus higher energy corresponds to higher sensitivities. Finally, we comment that the shapes of cross sections vs. \( c_6 (c_{\Phi 1}) \) are parabolic. This is consistent with the fact that only terms up to the order of \( \frac{1}{\Lambda^2} \) are kept in the amplitude level. As a result, the cross sections are quadratic functions of \( c_6 (c_{\Phi 1}) \).

In this section, we discussed the cross section for \( 2 \rightarrow 3 \) VBS processes and their dependence on \( c_6 \) and \( c_{\Phi 1} \) which provides the basic ideas about the sources of the sensitivities.
In the next section, we will proceed to discuss the measurement of $c_6$ and $c_{\Phi_1}$ with more realistic setup at both hadron and lepton colliders.

4 VBS processes at hadron/lepton collider

4.1 Signal processes

In section 3, we analyzed VBS processes with three bosons in the final state at high energy, taking $W^+_L W^-_L \rightarrow W^+_L W^-_L h$ and $W^+_L W^-_L \rightarrow hhh$ as two examples. We found that they are sensitive to dim-6 operators in the SMEFT as shown in figure 5 and figure 7. In this section we continue to analyze the corresponding full processes at hadron and lepton colliders using MadGraph [66] with the SMEFT model file from ref. [72].

Since the aim of this paper is to illustrate important and crucial behaviors of $2 \rightarrow 3$ VBS processes, we don’t plan to cover all related processes here. Instead, we choose the following processes as benchmarks:

$$l^+l^- \rightarrow \nu_l \bar{\nu}_l W^+_L W^-_L h$$

$$pp \rightarrow jjW^\pm_L W^\pm_L h$$

(4.1)

(4.2)

where $l$ is either $\mu$ or $e$. Processes in eq. (4.1) can be explored at a series of future lepton colliders, including CLIC [29–31] ($1\text{TeV} < \sqrt{s} < 3\text{TeV}$) and the recently proposed muon colliders [32–36] ($3\text{TeV} < \sqrt{s} < 30\text{TeV}$). Processes in eq. (4.2) can be explored at the HL-LHC, HE-LHC [14] and the future 100TeV pp colliders [37, 73]. Notice that we focus on the same-sign W-pair for hadron collider to suppress the backgrounds.

4.2 Signal cross sections

In this section, we will examine the cross section for relevant signal processes listed in last section, especially the dependence of the cross section on $c_6$ and $c_{\Phi_1}$ and the comparison with SM cross section. In the following, we will present the results for $WWh$ and $hhh$ productions respectively.

Production of $WWh$. In this category, we considered following processes for hadron and lepton colliders:

$$pp \rightarrow jjW^\pm_L W^\pm_L h,$$

$$l^+l^- \rightarrow \nu_l \bar{\nu}_l W^+_L W^-_L h,$$

(4.3)

(4.4)

where, as stated in last section, we choose same-sign W-pair for hadron collider to suppress the backgrounds.

The cross sections for $\mu^+\mu^- \rightarrow \nu_l \bar{\nu}_l W^+_L W^-_L h$ for different choices of $c_6$ and $c_{\Phi_1}$ are listed in table 1 and table 2 respectively. The cuts we imposed on the cross section calculation are listed in table 3. Note that we impose a slightly stronger cuts for the case in table 1 (as well as the case in table 4 below). As in this case, the enhancement due to $c_6$
The cuts used for processes with energetic. The cuts Table 2. Beams:

| c_6         | $-2$ | $-1$ | $0$  | $1$  | $2$  |
|-------------|------|------|------|------|------|
| 1 TeV       | $4.17 \times 10^{-9}$ | $1.57 \times 10^{-9}$ | $8.22 \times 10^{-10}$ | $1.93 \times 10^{-9}$ | $4.90 \times 10^{-9}$ |
| 3 TeV       | $1.79 \times 10^{-6}$ | $6.98 \times 10^{-7}$ | $3.71 \times 10^{-7}$ | $8.01 \times 10^{-7}$ | $2.00 \times 10^{-6}$ |
| 5 TeV       | $6.10 \times 10^{-6}$ | $2.43 \times 10^{-6}$ | $1.32 \times 10^{-6}$ | $2.74 \times 10^{-6}$ | $6.72 \times 10^{-6}$ |
| 10 TeV      | $1.94 \times 10^{-5}$ | $7.98 \times 10^{-6}$ | $4.38 \times 10^{-6}$ | $8.74 \times 10^{-6}$ | $2.09 \times 10^{-5}$ |
| 14 TeV      | $2.99 \times 10^{-5}$ | $1.25 \times 10^{-5}$ | $7.11 \times 10^{-6}$ | $1.36 \times 10^{-5}$ | $3.22 \times 10^{-5}$ |
| 30 TeV      | $6.45 \times 10^{-5}$ | $2.82 \times 10^{-5}$ | $1.58 \times 10^{-5}$ | $2.95 \times 10^{-5}$ | $6.68 \times 10^{-5}$ |

Table 1. The cross section for $\mu^+\mu^- \rightarrow \nu_\mu \bar{\nu}_\mu W^+_L W^-_L h$ with $c_{\Phi_1} = 0$ at different c.m. energies. Five benchmark points of $c_6$ are displayed in different columns. The cuts $m_{\nu\nu} > 150$ GeV, $p_T(W,h) > 150$ GeV are implemented to obtain these cross sections.

| c_{\Phi_1} | $-2$ | $-1$ | $0$  | $1$  | $2$  |
|-------------|------|------|------|------|------|
| 1 TeV       | $6.23 \times 10^{-7}$ | $6.70 \times 10^{-7}$ | $7.69 \times 10^{-7}$ | $9.27 \times 10^{-7}$ | $1.14 \times 10^{-6}$ |
| 3 TeV       | $2.65 \times 10^{-5}$ | $2.34 \times 10^{-5}$ | $2.47 \times 10^{-5}$ | $2.96 \times 10^{-5}$ | $3.95 \times 10^{-5}$ |
| 5 TeV       | $8.85 \times 10^{-5}$ | $6.71 \times 10^{-5}$ | $6.43 \times 10^{-5}$ | $8.13 \times 10^{-5}$ | $1.17 \times 10^{-4}$ |
| 10 TeV      | $4.09 \times 10^{-4}$ | $2.22 \times 10^{-4}$ | $1.71 \times 10^{-4}$ | $2.61 \times 10^{-4}$ | $4.72 \times 10^{-4}$ |
| 14 TeV      | $8.63 \times 10^{-4}$ | $3.95 \times 10^{-4}$ | $2.56 \times 10^{-4}$ | $4.38 \times 10^{-4}$ | $9.39 \times 10^{-4}$ |
| 30 TeV      | $4.97 \times 10^{-3}$ | $1.60 \times 10^{-3}$ | $5.07 \times 10^{-4}$ | $1.66 \times 10^{-3}$ | $5.04 \times 10^{-3}$ |

Table 2. The same as table 1, but for $c_6 = 0$ with five benchmark points of $c_{\Phi_1}$ at different c.m. energies. The cuts $m_{\nu\nu} > 150$ GeV is implemented to obtain these cross sections.

| Beams: | $\mu^+\mu^-$ | $pp$ | $\mu^+\mu^-$ | $pp$ |
|--------|---------------|------|---------------|------|
| Varying $c_6$: | $m_{\nu\nu} > 150$ GeV | $p_T(W,h) > 150$ GeV | $m_{\nu\nu} > 150$ GeV | $p_T(W,h) > 150$ GeV |
|         | $m_{j_1j_2} > 150$ GeV | $\eta_{j_1} \times \eta_{j_2} < 0$ | $|\Delta\eta_{j_1j_2}| > 2.5$ |
| Varying $c_{\Phi_1}$: | $m_{\nu\nu} > 150$ GeV | $\eta_{j_1} \times \eta_{j_2} < 0$ | $|\Delta\eta_{j_1j_2}| > 2.5$ |

Table 3. The cuts used for processes with $WWh$ and $hhh$ final states at $pp$ and lepton colliders respectively.
Cross sections (pb) for $pp \rightarrow jjW_L^{\pm}W_L^{\pm}h$ with $c_{\Phi_1} = 0$

| $c_6$  | $-2$     | $-1$     | 0     | 1     | 2     |
|--------|----------|----------|-------|-------|-------|
| 14 TeV | $6.35 \times 10^{-7}$ | $2.81 \times 10^{-7}$ | $1.68 \times 10^{-7}$ | $3.01 \times 10^{-7}$ | $6.79 \times 10^{-7}$ |
| 27 TeV | $3.58 \times 10^{-6}$ | $1.68 \times 10^{-6}$ | $1.09 \times 10^{-6}$ | $1.78 \times 10^{-6}$ | $3.76 \times 10^{-6}$ |
| 100 TeV| $3.28 \times 10^{-5}$ | $1.82 \times 10^{-5}$ | $1.38 \times 10^{-5}$ | $1.87 \times 10^{-5}$ | $3.34 \times 10^{-5}$ |

Table 4. The cross section for $pp \rightarrow jjW_L^{\pm}W_L^{\pm}h$ with $c_{\Phi_1} = 0$ at different c.m. energies. Five benchmark points of $c_6$ are displayed in different columns. The cuts $m_{jj} > 150$ GeV, $p_T(W,h) > 150$ GeV as well as the VBS selections ($\eta_{j_1} \times \eta_{j_2} < 0$ and $|\Delta \eta_{j_1,j_2}| > 2.5$) are implemented to obtain these cross sections.

Cross sections (pb) for $pp \rightarrow jjW_L^{\pm}W_L^{\pm}h$ with $c_6 = 0$

| $c_{\Phi_1}$ | $-2$     | $-1$     | 0     | 1     | 2     |
|---------------|----------|----------|-------|-------|-------|
| 14 TeV        | $2.52 \times 10^{-5}$ | $2.65 \times 10^{-5}$ | $2.88 \times 10^{-5}$ | $3.29 \times 10^{-5}$ | $3.90 \times 10^{-5}$ |
| 27 TeV        | $1.27 \times 10^{-4}$ | $1.20 \times 10^{-4}$ | $1.27 \times 10^{-4}$ | $1.49 \times 10^{-4}$ | $1.83 \times 10^{-4}$ |
| 100 TeV       | $1.26 \times 10^{-3}$ | $9.91 \times 10^{-4}$ | $9.70 \times 10^{-4}$ | $1.23 \times 10^{-3}$ | $1.72 \times 10^{-3}$ |

Table 5. The same as table 4, but for $c_6 = 0$ with five benchmark points of $c_{\Phi_1}$ at different c.m. energies. The cuts $m_{jj} > 150$ GeV and the VBS selections ($\eta_{j_1} \times \eta_{j_2} < 0$ and $|\Delta \eta_{j_1,j_2}| > 2.5$) are implemented to obtain these cross sections.

is not large which is overwhelmed by the Sudakov logarithms from soft/collinear behavior in the SM cross section. We thus impose additional $p_T$ cuts on final states to avoid such soft/collinear regions.

From these tables, we find that the behavior of the cross section with respect to $c_6$ and $c_{\Phi_1}$ is similar to what we observed in section 3. For $WWh$ final state, the dependence of the cross section on $c_6$ will be slightly weaker at higher energy, i.e. $\frac{\sigma(\text{c6=2})}{\sigma_{\text{SM}}}$ for $\sqrt{s} = 1(30)$ TeV. On the other hand, the cross section enhancement due to $c_{\Phi_1}$ will be stronger at higher energy, i.e. $\frac{\sigma(\text{c_{\Phi_1}=2})}{\sigma_{\text{SM}}} = 1.48(9.94)$ for $\sqrt{s} = 1(30)$ TeV. Hence, by just measuring the total events, we will have stronger constraints on $c_{\Phi_1}$ than $c_6$ with a 14 TeV or even 30 TeV muon collider machine.

In the simulation we have chosen longitudinal polarizations for $W$ bosons in the final states. The reason is that the cross sections for summing over the polarizations of the final state $W^{\pm}s$ are dominated by transverse polarizations, whereas deviation of Higgs self-couplings mainly modifies cross sections of all longitudinal vector bosons only. So in order to study the influence from high dimension operators, in practice, the longitudinal polarizations should be picked using some techniques [67–71]. Detailed comparison between cross sections summing over polarizations of $W$ bosons in the final states and the ones with longitudinal $W$ bosons in the final states can be found in section A.

The cross section for $pp \rightarrow jjW_L^{\pm}W_L^{\pm}h$ with different choices of $c_6$ and $c_{\Phi_1}$ are listed in table 4 and table 5. The relevant cuts applied on this process are also listed in table 3. The
Table 6. The cross section for $\mu^+\mu^- \to \nu_\mu \bar{\nu}_\mu hhh$ with $c_{\Phi_1} = 0$ at different c.m. energies. Five benchmark points of $c_6$ are displayed in different columns. The cut $m_{\nu\nu} > 150$ GeV is implemented to obtain these cross sections.

| $c_6$ | $c_{\Phi_1} = -2$ | $c_{\Phi_1} = -1$ | $c_{\Phi_1} = 0$ | $c_{\Phi_1} = 1$ | $c_{\Phi_1} = 2$ |
|-------|------------------|------------------|-----------------|-----------------|-----------------|
| 1 TeV | $4.42 \times 10^{-8}$ | $1.06 \times 10^{-8}$ | $3.39 \times 10^{-9}$ | $2.25 \times 10^{-8}$ | $6.76 \times 10^{-8}$ |
| 3 TeV | $1.93 \times 10^{-6}$ | $5.66 \times 10^{-7}$ | $2.78 \times 10^{-7}$ | $1.08 \times 10^{-6}$ | $2.94 \times 10^{-6}$ |
| 5 TeV | $4.91 \times 10^{-6}$ | $1.57 \times 10^{-6}$ | $9.50 \times 10^{-7}$ | $3.03 \times 10^{-6}$ | $7.74 \times 10^{-6}$ |
| 10 TeV | $1.25 \times 10^{-5}$ | $4.59 \times 10^{-6}$ | $3.46 \times 10^{-6}$ | $8.75 \times 10^{-6}$ | $1.95 \times 10^{-5}$ |
| 14 TeV | $1.80 \times 10^{-5}$ | $6.70 \times 10^{-6}$ | $5.38 \times 10^{-6}$ | $1.30 \times 10^{-5}$ | $2.88 \times 10^{-5}$ |
| 30 TeV | $3.50 \times 10^{-5}$ | $1.77 \times 10^{-5}$ | $1.41 \times 10^{-5}$ | $2.92 \times 10^{-5}$ | $5.42 \times 10^{-5}$ |

Table 7. The same as table 6, but for $c_6 = 0$ with five benchmark points of $c_{\Phi_1}$ at different c.m. energies.

| $c_{\Phi_1}$ | $c_{\Phi_1} = -2$ | $c_{\Phi_1} = -1$ | $c_{\Phi_1} = 0$ | $c_{\Phi_1} = 1$ | $c_{\Phi_1} = 2$ |
|--------------|------------------|------------------|-----------------|-----------------|-----------------|
| 1 TeV        | $2.78 \times 10^{-8}$ | $1.08 \times 10^{-8}$ | $3.39 \times 10^{-9}$ | $5.56 \times 10^{-9}$ | $1.73 \times 10^{-8}$ |
| 3 TeV        | $3.01 \times 10^{-6}$ | $1.11 \times 10^{-6}$ | $2.78 \times 10^{-7}$ | $5.43 \times 10^{-7}$ | $1.89 \times 10^{-6}$ |
| 5 TeV        | $1.33 \times 10^{-5}$ | $4.47 \times 10^{-6}$ | $9.50 \times 10^{-7}$ | $2.38 \times 10^{-6}$ | $8.76 \times 10^{-6}$ |
| 10 TeV       | $7.83 \times 10^{-5}$ | $2.38 \times 10^{-5}$ | $3.46 \times 10^{-6}$ | $1.50 \times 10^{-5}$ | $5.97 \times 10^{-5}$ |
| 14 TeV       | $1.77 \times 10^{-4}$ | $4.97 \times 10^{-5}$ | $5.38 \times 10^{-6}$ | $3.73 \times 10^{-5}$ | $1.44 \times 10^{-4}$ |
| 30 TeV       | $1.07 \times 10^{-3}$ | $2.77 \times 10^{-4}$ | $1.41 \times 10^{-5}$ | $2.44 \times 10^{-4}$ | $9.86 \times 10^{-4}$ |

overall behavior of the cross section with respect to $c_6$ and $c_{\Phi_1}$ is similar to the case at muon collider, however, the sensitivity is weaker: $\frac{\sigma(c_6=2)}{\sigma_{SM}} = 4.04(2.42)$ and $\frac{\sigma(c_{\Phi_1}=2)}{\sigma_{SM}} \approx 1.35(1.77)$ for $\sqrt{s} = 14(100)$ TeV.

**Production of $hhh$.** In this category, the processes we considered at hadron and lepton colliders are:

$$pp \rightarrow jjhhh,$$

$$l^+l^- \rightarrow \nu_\ell \bar{\nu}_\ell hhh.$$  \hfill (4.5)

$$l^+l^- \rightarrow \nu_\ell \bar{\nu}_\ell hhh.$$  \hfill (4.6)

The cross section for $\mu^+\mu^- \to \nu_\mu \bar{\nu}_\mu hhh$ for different choices of $c_6$ and $c_{\Phi_1}$ are listed in table 6 and table 7. The cuts we imposed on the process are also listed in table 3. The cross sections for $hhh$ production are slightly smaller than that of $WWh$ production, while the sensitivity on $c_{\Phi_1}$ from $hhh$ channel is much stronger than that in $WWh$ channel as we can see from table 7, $\frac{\sigma(c_{\Phi_1}=2)}{\sigma_{SM}} \approx 70$ at $\sqrt{s} = 30$ TeV. However, the enhancement due to $c_6$ is moderate: $\frac{\sigma(c_6=2)}{\sigma_{SM}} \approx 4$ at $\sqrt{s} = 30$ TeV. The cross section and its dependence on $c_6$ and
selections (c benchmark points of The cross section for we can estimate the allowed region of the O the sensitivity on the sensitivity at lepton collider will be stronger than that at hadron collider. Further, the sensitivity on O c and production mechanism is VBS. All these processes provide some sensitivities on both O sections as function of the c.m. energies for both O pp → jjhh production. At √s = 100 TeV, we have \( \frac{\sigma(c_{\Phi_1}=2)}{\sigma_{SM}} \approx 8 \) and \( \frac{\sigma(c_{\Phi_1})}{\sigma_{SM}} \approx 18 \).

### Table 8

| \( c_6 \)       | -2         | -1         | 0         | 1         | 2         |
|-----------------|------------|------------|-----------|-----------|-----------|
| 14 TeV          | 1.99 \times 10^{-6} | 5.77 \times 10^{-7} | 2.97 \times 10^{-7} | 1.16 \times 10^{-6} | 3.12 \times 10^{-6} |
| 27 TeV          | 9.46 \times 10^{-6} | 2.93 \times 10^{-6} | 1.50 \times 10^{-6} | 5.48 \times 10^{-6} | 1.45 \times 10^{-5} |
| 100 TeV         | 7.91 \times 10^{-5} | 2.65 \times 10^{-5} | 1.48 \times 10^{-5} | 4.30 \times 10^{-5} | 1.13 \times 10^{-4} |

*Table 8.* The cross section for \( pp \rightarrow jjhh \) with \( c_{\Phi_1} = 0 \) at different c.m. energies. Five benchmark points of \( c_6 \) are displayed in different columns. The cut \( m_{j_1j_2} > 150 \text{GeV} \) and VBS selections (\( \eta_{j_1} \times \eta_{j_2} < 0 \) and \( |\Delta\eta_{j_1j_2}| > 2.5 \)) are implemented to obtain these cross sections.

### Table 9

| \( c_{\Phi_1} \) | -2 | -1 | 0 | 1 | 2 |
|-----------------|----|----|---|---|---|
| 14 TeV          | 2.94 \times 10^{-6} | 1.10 \times 10^{-6} | 2.97 \times 10^{-7} | 5.33 \times 10^{-7} | 1.83 \times 10^{-6} |
| 27 TeV          | 1.97 \times 10^{-5} | 6.99 \times 10^{-6} | 1.50 \times 10^{-6} | 3.75 \times 10^{-6} | 1.35 \times 10^{-5} |
| 100 TeV         | 3.31 \times 10^{-4} | 1.02 \times 10^{-4} | 1.48 \times 10^{-5} | 7.04 \times 10^{-5} | 2.69 \times 10^{-4} |

*Table 9.* The same as table 8, but for \( c_6 = 0 \) with five benchmark points of \( c_{\Phi_1} \) at different c.m. energies.

\( c_{\Phi_1} \) for \( pp \rightarrow jjhh \) are listed in table 8 and table 9. Similar to muon collider case, the sensitivities on both \( c_6 \) and \( c_{\Phi_1} \) are stronger from \( hhh \) production than that from \( WWh \) production. At \( \sqrt{s} = 100 \text{ TeV} \), we have \( \frac{\sigma(c_{\Phi_1}=2)}{\sigma_{SM}} \approx 8 \) and \( \frac{\sigma(c_{\Phi_1})}{\sigma_{SM}} \approx 18 \).

### 4.3 Results and prospects

Based on the cross section results in section 4.2, in figure 8 and figure 9, we show the cross section as function of the c.m. energies for both \( WWh \) and \( hhh \) productions where the black curve is for the SM case and different colors (solid or dashed) are for the cases either \( c_6 \) or \( c_{\Phi_1} \) is non-zero. Notice that the cross section increases logarithmically as the main production mechanism is VBS. All these processes provide some sensitivities on both \( c_6 \) and \( c_{\Phi_1} \) as can be seen by comparing the colored curves with the SM one.

In figure 10 and figure 11, we show the cross section difference induced by \( \mathcal{O}_6 \) and \( \mathcal{O}_{\Phi_1} \) compared with the SM one for \( WWh \) and \( hhh \) production respectively. In general, the sensitivity at lepton collider will be stronger than that at hadron collider. Further, the sensitivity on \( c_6 \) (\( \mathcal{O}_6 \)) decreases as energy increases, as we have indicated in previous sections, due to the logarithmical enhancement of the SM cross section. On the other hand, the sensitivity on \( c_{\Phi_1} \) (\( \mathcal{O}_{\Phi_1} \)) increases at high energy due to the momentum dependence in the \( \mathcal{O}_{\Phi_1} \) operator.

Based on the deviation in the cross section due to \( \mathcal{O}_6 \) and \( \mathcal{O}_{\Phi_1} \) operators contributions, we can estimate the allowed region of \( c_6 \) and \( c_{\Phi_1} \) under some simplified assumptions. First,
the full signal to background analysis is beyond our scope.\footnote{However, we’d like to comment that the background at lepton collider can be well controlled, as the cut on invariant mass of all final states can help to remove most of the background due to the VBS nature of our signal process. While at hadron collider, we have chosen same-sign $W$ boson production, as the specific 2 same-sign lepton signals will also suppress the possible background at hadron collider.} In our analysis, we simply...
\[
\Delta \sigma \equiv \sigma - \sigma_{SM}
\]

Figure 10. The $\Delta \sigma / \sigma_{SM}$ for $c_6 = \pm 1, \pm 2$ with $c_{\Phi_1} = 0$ and $c_{\Phi_1} = \pm 1, \pm 2$ with $c_6 = 0$ for $\mu^+\mu^- \rightarrow \nu_\mu \bar{\nu}_\mu WWh$ from $\sqrt{s} = 1$ to 30 TeV (upper panel) and $pp \rightarrow jjWWh$ from $\sqrt{s} = 14$ to 100 TeV (lower panel) where $\Delta \sigma \equiv \sigma_{tot} - \sigma_{SM}$.

Figure 11. The $\Delta \sigma / \sigma_{SM}$ for $c_6 = \pm 1, \pm 2$ with $c_{\Phi_1} = 0$ and $c_{\Phi_1} = \pm 1, \pm 2$ with $c_6 = 0$ for $\mu^+\mu^- \rightarrow \nu_\mu \bar{\nu}_\mu hhh$ from $\sqrt{s} = 1$ to 30 TeV (left panel) and $pp \rightarrow jjhhh$ from $\sqrt{s} = 14$ to 100 TeV (right panel) where $\Delta \sigma \equiv \sigma_{tot} - \sigma_{SM}$. 
follow the strategy used in ref. [50] to count the deviation of signal events from SM ones. Further, we define the significance of non-SM events over SM ones as:

$$\text{significance} \sim \frac{|N_{\text{BSM}} - N_{\text{SM}}|}{\sqrt{N_{\text{SM}}}}. \quad (4.7)$$

where $N_{\text{SM}}$ and $N_{\text{BSM}}$ are the event number of SM processes and the case with $O_6$ or $O_{\Phi_1}$ operator respectively. For $\mu^+\mu^- \rightarrow \nu_{\mu}\bar{\nu}_{\mu}W_L^+W_L^-h$ process, $N_i \equiv \sigma_i(\mu^+\mu^- \rightarrow \nu_{\mu}\bar{\nu}_{\mu}W_L^+W_L^-h) \times [\text{BR}(h \rightarrow b\bar{b}) \times [\text{BR}(W^\pm \rightarrow \ell\nu)]^2 \times L$ where $L$ is the integrated luminosity. Here we assume that all of leptonic, semileptonic and hadronic decay modes from opposite sign $W$ boson pair can be detected at future muon collider experiments. For $pp \rightarrow jjW_L^+W_L^-h$ process, $N_i \equiv \sigma_i(pp \rightarrow jjW_L^+W_L^-h) \times [\text{BR}(h \rightarrow b\bar{b}) \times [\text{BR}(W^\pm \rightarrow \ell\nu)]^2 \times L$, where we only focus on the leptonic decay modes of same sign $W$ boson pair at hadron colliders to avoid possible huge SM backgrounds. For the $hhh$ processes, $N_i \equiv \sigma_i \times$

| $\sqrt{s}$ (TeV) | $\mathcal{L}$ (ab$^{-1}$) | $c_6$ ($c_{\Phi_1} = 0$) x-sec only | $c_{\Phi_1}$ ($c_6 = 0$) x-sec only |
|-----------------|-----------------|-----------------|-----------------|
|                 |                 | $1\sigma$      | $2\sigma$      | $1\sigma$      | $2\sigma$      |
| 1               | 1.2             | $-\text{--}$   | $-\text{--}$   | $-\text{--}$   | $-\text{--}$   |
| 3               | 4.4             | $-\text{--}$   | $-\text{--}$   | $[-2.54, 0.71]$| $[-3.07, 1.18]$|
| 5               | 12              | $[-0.65, 0.53]$| $[-0.89, 0.77]$| $[-1.02, 0.30]$| $[-1.22, 0.50]$|
| 10              | 20              | $[-0.44, 0.35]$| $[-0.60, 0.51]$| $[-0.43, 0.13]$| $[-0.50, 0.22]$|
| 14              | 33              | $[-0.37, 0.28]$| $[-0.50, 0.41]$| $[-0.23, 0.10]$| $[-0.29, 0.15]$|
| 30              | 100             | $[-0.23, 0.18]$| $[-0.31, 0.26]$| $[-0.07, 0.04]$| $[-0.09, 0.06]$|

Table 10. The range of $c_6$ with $c_{\Phi_1} = 0$ and $c_6 = 0$ for $\mu^+\mu^- \rightarrow \nu_{\mu}\bar{\nu}_{\mu}W_L^+W_L^-h$ process at lepton colliders with various benchmarks $\sqrt{s}$ and $\mathcal{L}$. The $1\sigma$ and $2\sigma$ allowed regions on $c_6$ and $c_{\Phi_1}$ are calculated from the definition in eq. (4.7) which only rely on the size of cross sections. The notation “−−” means either $N_{\text{SM}} < 1$ or $|N_{\text{BSM}} - N_{\text{SM}}| < 1$.

| $\sqrt{s}$ (TeV) | $\mathcal{L}$ (ab$^{-1}$) | $c_6$ ($c_{\Phi_1} = 0$) x-sec only | $c_{\Phi_1}$ ($c_6 = 0$) x-sec only |
|-----------------|-----------------|-----------------|-----------------|
|                 |                 | $1\sigma$      | $2\sigma$      | $1\sigma$      | $2\sigma$      |
| 1               | 1.2             | $-\text{--}$   | $-\text{--}$   | $-\text{--}$   | $-\text{--}$   |
| 3               | 4.4             | $-\text{--}$   | $-\text{--}$   | $-\text{--}$   | $-\text{--}$   |
| 5               | 12              | $[-1.00, 0.46]$| $[-1.27, 0.73]$| $[-0.34, 0.76]$| $[-0.54, 0.96]$|
| 10              | 20              | $[-0.95, 0.30]$| $[-1.14, 0.50]$| $[-0.14, 0.41]$| $[-0.23, 0.50]$|
| 14              | 33              | $[-0.93, 0.21]$| $[-1.08, 0.37]$| $[-0.10, 0.24]$| $[-0.16, 0.30]$|
| 30              | 100             | $[-0.75, 0.12]$| $[-0.84, 0.21]$| $[-0.04, 0.10]$| $[-0.06, 0.12]$|

Table 11. The same as table 10 but for $\mu^+\mu^- \rightarrow \nu_{\mu}\bar{\nu}_{\mu}hhh$ process at lepton colliders.


First, the dashed (solid) lines represent 1-\(L\) to \(c\) constraining those channels that cannot provide enough event rate as hatched region. It is clear from the color indicates the 1-\(c\) region within \(c\), \(\phi\) and \(c\) in table 10–13. Note that several entries of the allowed regions for \(N\) process at muon colliders or \(pp\) colliders. We also require both \(\sqrt{s} = 100\) ab\(^{-1}\) and \(N_{\text{SM}} > 1\) and |\(N_{\text{BSM}} - N_{\text{SM}}| > 1\) in our analysis. The allowed region for \(c_6\) and \(c_{\phi_1}\) at 1- and 2-\(\sigma\) level are summarized in table 10–13. Note that several entries of the allowed regions for \(c_{\phi_1}\) are beyond \([-2, 2]\), which should be treated with caution. As in our analysis, we only calculate the cross section within \([-2, 2]\) for both \(c_6\) and \(c_{\phi_1}\), the interpolation is only valid within this region.

The allowed regions for \(c_6\) (red) and \(c_{\phi_1}\) (blue) are also shown in figure 12 where darker color indicates the 1-\(\sigma\) region, while lighter one indicates the 2-\(\sigma\) region. We also denote those channels that cannot provide enough event rate as hatched region. It is clear from this plot that, in general, high energy muon collider is more powerful than HE-LHC in constraining \(c_6\) and \(c_{\phi_1}\). Furthermore, both \(WWh\) and \(hh\) production are more sensitive to \(c_{\phi_1}\) than \(c_6\). However, for both \(c_6\) and \(c_{\phi_1}\), we will have higher sensitivities with higher energy at both lepton and hadron colliders thanks to the increase of cross sections.

We further study the allowed parameter space on the \((c_6, c_{\phi_1})\) plane for 30 TeV muon collider with \(\mathcal{L} = 100\) ab\(^{-1}\) and 100 TeV hadron collider with \(\mathcal{L} = 30\) ab\(^{-1}\) in figure 13. First, the dashed (solid) lines represent 1-\(\sigma\) (2-\(\sigma\)) allowed regions. Then, we include four channels with \(\mu^+ \mu^- \to \nu_\mu \bar{\nu}_\mu W_L^+ W_L^- h\) (black), \(\mu^+ \mu^- \to \nu_\mu \bar{\nu}_\mu h hh\) (blue), \(pp \to jj W_L^+ W_L^- h\) (purple), and \(pp \to jj hh hh\) (red). For comparison purpose, we uniformly apply the stronger cuts in table 3 for both \(\mu^+ \mu^- \to \nu_\mu \bar{\nu}_\mu W_L^+ W_L^- h\) and \(pp \to jj W_L^+ W_L^- h\). We can find the most stringent constraint on the \((c_6, c_{\phi_1})\) plane comes from the process \(\mu^+ \mu^- \to \nu_\mu \bar{\nu}_\mu W_L^+ W_L^- h\). However, \(\mu^+ \mu^- \to \nu_\mu \bar{\nu}_\mu hh hh\) and \(pp \to jj hh hh\) processes can still help us to cover some parameter space on the \((c_6, c_{\phi_1})\) plane which the process \(\mu^+ \mu^- \to \nu_\mu \bar{\nu}_\mu W_L^+ W_L^- h\) cannot reach.

| \(\sqrt{s}\) (TeV) | \(\mathcal{L}\) (ab\(^{-1}\)) | \(c_6\) (\(c_{\phi_1} = 0\)) x-sec only | \(c_{\phi_1}\) (\(c_6 = 0\)) x-sec only |
|------------------|-----------------|-----------------|-----------------|
|                  |                 | 1\(\sigma\)     | 2\(\sigma\)     | 1\(\sigma\)     | 2\(\sigma\)     |
| 14               | 3               | –               | –               | –               | –               |
| 27               | 15              | –               | –               | –               | –               |
| 100              | 30              | \([-0.99, 0.92]\) | \([-1.38, 1.32]\) | \([-1.08, 0.23]\) | \([-1.24, 0.40]\) |

Table 12. The same as table 10 but for \(pp \to jj W_L^+ W_L^- h\) process at hadron colliders.

| \(\sqrt{s}\) (TeV) | \(\mathcal{L}\) (ab\(^{-1}\)) | \(c_6\) (\(c_{\phi_1} = 0\)) x-sec only | \(c_{\phi_1}\) (\(c_6 = 0\)) x-sec only |
|------------------|-----------------|-----------------|-----------------|
|                  |                 | 1\(\sigma\)     | 2\(\sigma\)     | 1\(\sigma\)     | 2\(\sigma\)     |
| 14               | 3               | –               | –               | –               | –               |
| 27               | 15              | \([-0.80, 0.33]\) | \([-1.00, 0.53]\) | \([-0.19, 0.79]\) | \([-0.33, 0.90]\) |
| 100              | 30              | \([-0.55, 0.14]\) | \([-0.65, 0.24]\) | \([-0.07, 0.30]\) | \([-0.13, 0.35]\) |

Table 13. The same as table 10 but for \(pp \to jj hh hh\) process at hadron colliders.
2 → 3 VBS processes are not the only channels to study the measurement of Higgs self-couplings at colliders. At the LHC, and future hadron colliders, di-Higgs production through gluon fusion remains the dominant channel [8–15]. Whereas at future lepton colliders, di-Higgs production through 2 → 2 VBS, i.e. \( VV \to hh \), become dominant [30, 31, 34–36]. In terms of Wilson coefficients, both \( c_6 \) and \( c_{\Phi_1} \) can also be measured with either \( gg \to hh \) or \( VV \to hh \). Furthermore, \( c_{\Phi_1} \) can be measured with 2 → 1 vector boson fusion, 2 → 2 VBS with gauge boson final states and etc. See also [7] and [74] for overall reviews.

A comparison between muon colliders and other future colliders was done in [35], showing clear advantages of muon colliders. Here we mainly compare our results with \( VV \to hh \) at muon colliders. In ref. [35], the projective limits on \( c_6 \) and \( c_{\Phi_1} \) at muon colliders with \( \sqrt{s} = 14 \text{ TeV} \) and the integrated luminosity of 20 ab\(^{-1} \) are obtained by combining \( VV \to hh \) and \( VV \tohhh \). The results are

\[
c_6 \in [-0.66, 0.23], \quad c_{\Phi_1} \in [-0.17, 0.30]
\]

(4.8)

at 95% for \( \Lambda = 1 \text{ TeV} \). This is close to our similar results for both \( VV \to W^+_L W^-_L h \) and \( VV \tohhh \), as can be seen in table 10 and table 11, indicating that constraints on Higgs self-couplings from 2 → 3 VBS might be comparable to \( VV \to hh \) at muon colliders. Of course, more careful and in-depth studies are obviously needed.
\[ s = 30 \text{ TeV with } 100 \text{ ab}^{-1} : \mu^+\mu^-\rightarrow \nu\mu \nu\mu \]

\[ s = 100 \text{ TeV with } 30 \text{ ab}^{-1} : \]

\[ pp \rightarrow jjW^\pm LW^\pm h \]

\[ pp \rightarrow jjh \]

\[ pp \rightarrow jjhh \]

**Figure 13.** The allowed parameter space on the \((c_6, c_{\Phi_1})\) plane for 30 TeV muon collider with \(\mathcal{L} = 100 \text{ ab}^{-1}\) and 100 TeV hadron collider with \(\mathcal{L} = 30 \text{ ab}^{-1}\). The dashed (solid) lines represent 1-\(\sigma\) (2-\(\sigma\)) allowed regions and four channels are labelled with \(\mu^+\mu^-\rightarrow \nu\mu \nu\mu W^\pm L W^\pm h\) (black), \(\mu^+\mu^-\rightarrow \nu\mu \nu\mu h hh\) (blue), \(pp \rightarrow jjW^\pm LW^\pm h\) (purple), and \(pp \rightarrow jjhh\) (red).

### 5 Conclusions

Measuring Higgs self-couplings is a crucial task for the future collider experiments. It will uncover the nature of the discovered SM-like Higgs boson, the origin of EWSB, and the shape of the Higgs potential, etc. In this work, we studied the \(2 \rightarrow 3\) VBS processes under the framework of SMEFT, using \(W_L^\pm W_L^\pm W_L^\pm h\) and \(W_L^3 W_L^3 \rightarrow hhh\) as examples.

First, the behavior of the amplitudes for those processes at high energy is analysed using GET. We found that compared with SM contributions, the BSM contribution will be enhanced at high energy as \(\frac{A_{\text{BSM}}}{A_{\text{SM}}} \sim \frac{E^2}{\Lambda^2}\). Physically, this behavior comes from the combination of the following two factors: SM amplitudes are suppressed by the energy in the propagators; BSM amplitudes can stay constant due to contact vertex from \(O_6\) or increase with energy due to the momentum dependence in \(O_{\Phi_1}\). By numerically calculating the cross section for these \(2 \rightarrow 3\) processes, we showed the sensitivities of these processes to \(c_6\) and \(c_{\Phi_1}\) in figure 4 to figure 7.

Second, the processes \(pp \rightarrow jjW^\pm LW^\pm L h\) and \(pp \rightarrow jjh hh\) at pp colliders and \(\mu^+\mu^-\rightarrow \nu\mu \nu\mu W^\pm L W^\pm L h\) and \(\mu^+\mu^-\rightarrow \nu\mu \nu\mu hh hh\) at muon colliders are simulated with various energy benchmark points. However, certain \(p_T\) cuts in the phase space of final state particles are needed for \(pp \rightarrow jjW^\pm LW^\pm L h\) and \(\mu^+\mu^-\rightarrow \nu\mu \nu\mu W^\pm L W^\pm L h\) processes, in order to reduce the SM cross sections enhanced by Sudakov logarithms from collinear divergences. We study in details on how cross sections change with energies and with the Wilson coefficients \(c_6\) and \(c_{\Phi_1}\) in figure 8 to figure 11.
Based on the simulation, we estimated the allowed regions of $c_6$ and $c_{\Phi_1}$ at different c.m. energies and types of colliders assuming that signal events are already extracted from relevant SM backgrounds for processes in eq. (4.1) and eq. (4.2). Cross sections of these processes are generally tiny at HL-LHC, HE-LHC and CLIC, which makes them very challenging to explore. On the other hand, we find at the future 100 TeV pp colliders or high energy muon colliders, these VBS processes are good processes to measure the Higgs self-couplings, parameterized by $c_6$ and $c_{\Phi_1}$ in SMEFT. The allowed 1-$\sigma$ and 2-$\sigma$ regions for $c_6$ and $c_{\Phi_1}$ are obtained based on simple event counting procedure and are summarized in figure 12. Optimistically, we expect $-0.23 < c_6 < 0.18, -0.07 < c_{\Phi_1} < 0.04$ at 1$\sigma$ can be reached in the future and $c_{\Phi_1}$ is more restrictive than $c_6$.

We find $WWh$ process is as important as the more widely studied triple Higgs production ($hhh$) [6, 44, 50, 75–84] in the measurement of Higgs self-couplings. Our analysis is only preliminary, as we aim to give an overall picture and qualitative conclusions. A more careful analysis that takes into account of decay products, relevant SM background and detector effects is obviously needed. Moreover, we only studied a partial list of all $2 \to 3$ VBS processes, with $W^\pm W^\pm h/hhh$ ($W^+W^- h/hhh$) as final states at hadron (lepton) colliders. We will devote a more complete survey for all $2 \to 3$ VBS processes in measuring Higgs self-couplings in future work.

Furthermore, the enhancement of the amplitude at high energy in the presence of BSM physics due to contact scalar vertices is not limited to the case of $2 \to 3$ VBS processes. For example, the analysis in this paper can also be applied to $2 \to 4$ VBS processes which have the same energy increase behavior. Hence, they can also be used to measure Higgs self-coupling at future high energy colliders. It would be interesting to explore further on this direction.

Acknowledgments

We would like to thank helpful discussions with Yang Ma, Tong Li. CTL is supported by KIAS Individual Grant, No.PG075301 at Korea Institute for Advanced Study. JMC is supported by Fundamental Research Funds for the Central Universities of China NO.11620330.

A Full cross sections summing over polarization vector bosons

In this appendix, we sum over polarizations for final state $W^\pm$s of $\mu^+\mu^- \to \nu_\mu\bar{\nu}_\mu W^+W^-h$ and $pp \to jjW^+W^-h$ processes in table 14 and table 15, respectively. Compared with table 2 and table 5, the relevant cross sections can be larger by one order, but the relative deviations from SM ones are not obvious, especially for the $c_6$ contributions. Therefore, a precise way to distinguish $W^\pm_L$ from $W^\pm_T$ is important to extract effects from the $O_6$ operator. In this work, we assume $W^\pm_L$ can already be successfully separated from $W^\pm_T$ experimentally. More details about the method to distinguish $W^\pm_L$ from $W^\pm_T$ can be found in [67–71].

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with different c.m. energies, varies with
Table 14.

| $c_6$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|-------|------|------|-----|-----|-----|
| 1 TeV | 2.87 x 10^{-8} | 2.51 x 10^{-8} | 2.37 x 10^{-8} | 2.44 x 10^{-8} | 2.73 x 10^{-8} |
| 3 TeV | 2.04 x 10^{-5} | 1.90 x 10^{-5} | 1.85 x 10^{-5} | 1.88 x 10^{-5} | 2.01 x 10^{-5} |
| 5 TeV | 8.18 x 10^{-5} | 7.74 x 10^{-5} | 7.60 x 10^{-5} | 7.72 x 10^{-5} | 8.07 x 10^{-5} |
| 10 TeV | 3.16 x 10^{-4} | 3.02 x 10^{-4} | 3.00 x 10^{-4} | 3.02 x 10^{-4} | 3.13 x 10^{-4} |
| 14 TeV | 5.29 x 10^{-4} | 5.12 x 10^{-4} | 5.03 x 10^{-4} | 5.06 x 10^{-4} | 5.29 x 10^{-4} |
| 30 TeV | 1.38 x 10^{-3} | 1.31 x 10^{-3} | 1.31 x 10^{-3} | 1.33 x 10^{-3} | 1.36 x 10^{-3} |

Cross sections (pb) for $\mu^+\mu^- \rightarrow \nu_\mu \bar{\nu}_\mu W^+W^- h$ with $c_{\Phi_1} = 0$

| $c_{\Phi_1}$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|-------------|------|------|-----|-----|-----|
| 1 TeV       | 5.99 x 10^{-6} | 6.85 x 10^{-6} | 7.97 x 10^{-6} | 9.40 x 10^{-6} | 1.11 x 10^{-5} |
| 3 TeV       | 2.39 x 10^{-4} | 2.51 x 10^{-4} | 2.81 x 10^{-4} | 3.29 x 10^{-4} | 3.90 x 10^{-4} |
| 5 TeV       | 7.15 x 10^{-4} | 7.00 x 10^{-4} | 7.58 x 10^{-4} | 8.96 x 10^{-4} | 1.11 x 10^{-3} |
| 10 TeV      | 2.66 x 10^{-3} | 2.12 x 10^{-3} | 2.12 x 10^{-3} | 2.66 x 10^{-3} | 3.69 x 10^{-3} |
| 14 TeV      | 5.07 x 10^{-3} | 3.48 x 10^{-3} | 3.15 x 10^{-3} | 4.22 x 10^{-3} | 6.50 x 10^{-3} |
| 30 TeV      | 2.49 x 10^{-2} | 1.10 x 10^{-2} | 6.81 x 10^{-3} | 1.25 x 10^{-2} | 2.76 x 10^{-2} |

Table 14. The $\mu^+\mu^- \rightarrow \nu_\mu \bar{\nu}_\mu W^+W^- h$ process with summing over polarizations for final $W^{\pm}$, with different c.m. energies, varies with $c_6$, $c_{\Phi_1}$ = 0 (upper table) and varies with $c_6$, $c_6$ = 0 (lower table). Same cuts listed in table 3 are applied.

Cross sections (pb) for $pp \rightarrow jjW^\pm W^\pm h$ with $c_{\Phi_1} = 0$

| $c_6$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|-------|------|------|-----|-----|-----|
| 14 TeV | 1.39 x 10^{-5} | 1.37 x 10^{-5} | 1.36 x 10^{-5} | 1.37 x 10^{-5} | 1.42 x 10^{-5} |
| 27 TeV | 9.11 x 10^{-5} | 8.93 x 10^{-5} | 8.91 x 10^{-5} | 8.98 x 10^{-5} | 9.15 x 10^{-5} |
| 100 TeV | 1.13 x 10^{-3} | 1.11 x 10^{-3} | 1.11 x 10^{-3} | 1.12 x 10^{-3} | 1.13 x 10^{-3} |

Cross sections (pb) for $pp \rightarrow jjW^\pm W^\mp h$ with $c_6 = 0$

| $c_{\Phi_1}$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|-------------|------|------|-----|-----|-----|
| 14 TeV      | 2.26 x 10^{-4} | 2.51 x 10^{-4} | 2.83 x 10^{-4} | 3.21 x 10^{-4} | 3.63 x 10^{-4} |
| 27 TeV      | 1.04 x 10^{-3} | 1.14 x 10^{-3} | 1.27 x 10^{-3} | 1.46 x 10^{-3} | 1.67 x 10^{-3} |
| 100 TeV     | 9.15 x 10^{-3} | 9.48 x 10^{-3} | 1.04 x 10^{-2} | 1.20 x 10^{-2} | 1.42 x 10^{-2} |

Table 15. The same as table 14 for the $pp \rightarrow jjW^+W^- h$ process. Same cuts listed in table 3 are applied.
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