Note on color neutral solutions of the $K^0$ condensed
color-flavor locked phase

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In the presence of nonzero strange quark mass $m_s$, we investigate color neutrality in the $K^0$ condensed phase of color-flavor locked quark matter. By treating the $m_s$ effects on both kaon condensate and Fermi-surface phenomenon self-consistently, we develop a new treatment to evaluate color neutral solutions within the model-independent framework. It is pointed out that, in the general sense, the expectation values of gluon fields obtained from dynamics of Goldstone bosons solely are not identified with the factual color chemical potentials.

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At very high densities, the lowest energy state of three-flavor, three-color quark matter is widely expected to be the color-flavor locked (CFL) phase, where the original color and flavor $SU(3)_{color} \times SU(3)_L \times SU(3)_R$ symmetries of QCD are broken down to the diagonal subgroup $SU(3)_{color+L+R}$ [1, 2, 3]. In the ideal situation where flavor asymmetry of quarks is ignored, the CFL matter should not carry any color charges so that color neutrality is realized automatically. In realistic situations, flavor asymmetry leads to some differences among the Fermi momenta for nine (flavor 3 $\times$ color 3) paired quarks. In the CFL matter, nonzero color and electric charges may appear and color/electric neutral problem needs to examined seriously. In the presence of strange quark mass $m_s$, Alford and Ragagopal have investigated color/electric neutrality by introducing the chemical potentials associated with color charges [4]. As far as color is concerned, they found that the nontrivial values of color chemical potential (the so-called color neutral solutions) cancels the $m_s$-induced color charges and guarantees the enforced neutrality of the bulk CFL matter. In the presence of $m_s$, on the other hand, a less-symmetric CFL phase has been predicted where the Goldstone bosons related to the color-flavor-locked symmetry pattern become condensed [5, 6]. For the CFL phase with maximal $K^0$ condensate, Kryjeski has discussed its color neutrality by using a different method: the color neutral solutions in the form of the expectation values of gluon fields were solved from dynamics of Goldstone bosons [7].

In the present note, we would like to reexamine color neutral problem in the CFL matter with arbitrary $K^0$ condensate (hereafter CFL$K^0$ for short). The question needs to be answered, whether the method used by Alford and Rajagopal in the conventional CFL phase...
play its role in the CFL $K^0$ case. In other words, it is whether the Kryjevski method used in the maximal $K^0$ condensed phase be sufficient for color neutral solutions. We will point out that, for a general CFL $K^0$ phase, the Kryjevski-type solutions do not correspond to the Alford-Rajagopal-defined chemical potentials strictly. Then, the factual color neutral solutions in $K^0$ condensed environment are obtained by treating the $m_s$ effects self-consistently. These conclusions clarify the links between the two kinds of color neutral solutions and correct the popular suppositions on color neutral problem.

Before going to specifics, let’s briefly review the basic lines of Refs. [4] and [7]. Within the model-independent framework, Alford and Rajagopal introduced the chemical potentials coupled with diagonal generators of $SU(3)_{\text{color}}$ to examine the CFL color neutrality in the presence of $m_s$ [4].¹ Assuming that the strange quark is far heavier than the light quarks ($m_s >> m_{u,d}$) and the quark chemical potential is far larger than the quark masses ($\mu >> m_s$), the mismatch between the Fermi momenta for strange- and light-flavor quarks is equal to $m_s^2/2\mu$ at the leading order. For four kinds of quark pairs, the common Fermi momenta were found to be [4]

\begin{align}
 p_{F, (gs, bd)}^{\text{com}} &= \mu - \frac{1}{4}\mu_3 - \frac{1}{6}\mu_8 - \frac{m_s^2}{4\mu}, \\
 p_{F, (rs, bu)}^{\text{com}} &= \mu + \frac{1}{4}\mu_3 - \frac{1}{6}\mu_8 - \frac{m_s^2}{4\mu}, \\
 p_{F, (vd, gu)}^{\text{com}} &= \mu + \frac{1}{3}\mu_8, \\
 p_{F, (ru, gd, bs)}^{\text{com}} &= \mu - \frac{m_s^2}{6\mu},
\end{align}

where $\mu_3$ and $\mu_8$ are defined to be coupled with the generators $T_3 = \lambda_3/2$ and $T_8 = \lambda_8/\sqrt{3}$ respectively ( $\lambda_\alpha$ denotes the Gell-Mann matrices ). Based on the Fermi surface phenomenon described by Eq.(1), they obtained the color neutral solutions

\begin{equation}
 \mu_3 = 0, \quad \mu_8 = -\frac{m_s^2}{2\mu},
\end{equation}

from vanishing derivatives of the CFL free energy with respect to $\mu_3$ and $\mu_8$. By inserting Eq.(2) into (1), it is found that the Fermi momenta for all pairs are reduced to $p_F^{\text{com}} = \mu - \frac{m_s^2}{6\mu}$ which corresponds to the Fermi surface phenomenon in the color-neutral CFL phase.

Different from the Alford-Rajagopal method, the Kryjeski introduced color static potentials are the expectation values of gluon fields, namely the gluon condensates. In general, our

¹As stressed in Ref.[8], no electrons are required for the CFL matter despite the unequal quark masses. Even in the presence of an electric chemical potential, electric neutrality is easier realized with respect to color neutral problem. For simplicity, we ignore the electron chemical potential and only concern color neutrality all through this work.
discussion does not limited in the case of the maximal $K^0$ condensate studied in Ref. [7]. For this purpose, it is practical to consider the effective chemical potential associated with strangeness $\mu_S$ that equal to the $K^0$-mode chemical potential due to the chemical equilibrium. At the leading order, $\mu_S$ might be given by $m^2_{s}/2\mu$ (if without the external strangeness chemical potential [9]). Once $\mu_S$ exceeds the kaon mass $m_{K^0}$, $K^0$ condensation occurs and its condensate strength is characterized by $\cos \theta = m^2_{K^0}/\mu_S^2$. Correspondingly, the chiral field $\Sigma = \exp[i/\rho_{\pi}\lambda_{\alpha}\pi_{\alpha}]$ ($\pi_{\alpha}$ denotes the Goldstone octet and $f_{\pi}$ is the in-CFL-medium decay constant) takes the form of [6]

$$\Sigma_{K^0} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & i \sin \theta \\ 0 & i \sin \theta & \cos \theta \end{pmatrix}$$

(3)

in chiral effective Lagrangian accounting for the Goldstone bosons. By extrapolating the original Kryjeski treatment, we find that the temporal components of color-diagonal gluon fields have the nonzero expectation values

$$-gA^0_3 = -\frac{m^2_{s}}{4\mu} + \frac{m^2_{s}}{4\mu} \cos \theta,$$

$$-\frac{\sqrt{3}}{2}gA^0_8 = -\frac{m^2_{s}}{8\mu} - \frac{3m^2_{s}}{8\mu} \cos \theta,$$

(4)

where $g$ is the QCD coupling coefficient. The above result reflects the anisotropic ”vacuum” for Goldstone excitations, since it is obtained from the free energy of the Goldstone bosons solely.

As above mentioned, our concerned question is actually whether Eq.(4) be adequate for the CFL$K^0$ color neutrality. Note that a gluon field should attach to the quark loop, which accounts for the color charge density at mean-field level. Therefore, it is reasonable that the expectations values $gA^0_3$ and $gA^0_8$ behave like the color chemical potentials $\mu_3$ and $\mu_8$ respectively. Superficially, one might identify the two kinds of color static potentials with each other. In the literature, the assumption like

$$\mu_3 = -gA^0_3, \quad \mu_8 = -\frac{\sqrt{3}}{2}gA^0_8,$$

(5)

has been widely adopted, where the factor $\frac{\sqrt{3}}{2}$ arises to adapt the non-standard form of the color generator $T_8$ in Ref.[4]. In the maximal $K^0$ condensed case ( $\theta = \pi/2$ ), for instance, $-gA^0_3 = -m^2_{s}/4\mu$ and $-\frac{\sqrt{3}}{2}gA^0_8/2 = -m^2_{s}/8\mu$ were supposed to be equal to the color chemical potentials $\mu_3$ and $\mu_8$ respectively [10, 11]. With the help of Eq.(5), the above question seems to have been well answered and the two kinds of methods seems equivalent towards the CFL$K^0$ neutral solutions. However, it is not the whole story yet. Eq.(5) itself does not mean
the one-to-one correspondence between the two kinds of color neutral solutions (although it might be available in somewhat conditions). In fact, Alford and Rajagopal evaluated color neutrality from the Fermion free energy while Kryjevski did from the free energy of Goldstone bosons. Since quarks and Goldstone bosons belong to the distinct degrees of freedom, there is no a direct reason to identify the two kinds of solutions with each other. Secondly and perhaps more importantly, the Goldstone-mode condensation makes sense in the vicinity of the Fermi surface of CFL quark matter. This implies that the Goldstone-excitation vacuum Eq. (4) becomes possible for a specific Fermi surface phenomenon, i.e. a kind of Fermion accumulation made up of the paired quarks. According to the Alford-Rajagopal method, nevertheless, Fermi surface phenomena are relevant for color neutrality also. Indeed, it is mismatches in the Fermi momentum space to induce color chemical potentials essentially. In this case, the factual neutral solutions should be determined by not only Eq. (4) obtained from chiral Lagrangian solely but also the Alford-Rajagopal-type solutions. Therefore, the correspondence which is realized by Eq. (5) is problematic, unless the Fermion accumulation in $K^0$ condensed environment is proved irrelevant for color neutrality.

Now we do not presume Eq. (5) but keep it in mind that the Kryjevski’s (expectation values of) gluon fields play the similar roles as the color chemical potentials. As a starting point, we suppose that $-gA^0_3$ and $-\sqrt{3}gA^0_8$ are only the parts of $\mu_3$ and $\mu_8$ respectively while the remaining parts are given by the color static potentials $\mu'_3$ and $\mu'_8$ respectively. Explicitly, we define the latter as

$$
\mu'_3 = \mu_3 + gA^0_3, \quad \mu'_8 = \mu_8 + \frac{\sqrt{3}}{2}gA^0_8,
$$

(6)

which actually generalizes Eq. (5). Note that the total free energy for the CFL$K^0$ phase consists of the contributions from the Fermion accumulation, the diquark condensates as well as the Goldstone bosons. In Ref. [4], the color superconducting gaps were treated as parameters so that the second contribution (the free energy from diquark condensates) is not important. As for the third contribution (the free energy of Goldstone bosons), it has been used in yielding the expectation values of gluon fields. To evaluate the factual color neutral solutions expressed by $\mu_{3,8}$ eventually, we will solve the fictional chemical potentials $\mu'_{3,8}$ from the first contribution (the Fermion free energy) along the line of Ref. [4] while taking the Kryjevski-type solutions Eq. (4) into account simultaneously. As shall be seen below, $\mu'_{3,8}$ have usually the nonzero values and their introduction is relevant for studies of the CFL$K^0$ color neutrality.

As functions of $\mu'_{3,8}$ and $A^0_{3,8}$, the chemical potentials for the color-flavor locked quarks
\[
\mu_{bd} = \left[ \mu - \frac{2}{3} \left( -\frac{\sqrt{3}}{2} g A_3^0 \right) \right] - \frac{2}{3} \mu'_s,
\]
\[
\mu_{gs} = \left[ \mu - \frac{1}{2} \left( -g A_3^0 \right) + \frac{1}{3} \left( -\frac{\sqrt{3}}{2} g A_8^0 \right) \right] - \frac{1}{2} \mu'_3 + \frac{1}{3} \mu'_8; \quad (7)
\]
\[
\mu_{rs} = \left[ \mu + \frac{1}{2} \left( -g A_3^0 \right) + \frac{1}{3} \left( -\frac{\sqrt{3}}{2} g A_8^0 \right) \right] + \frac{1}{2} \mu'_3 + \frac{1}{3} \mu'_8,
\]
\[
\mu_{bu} = \left[ \mu - \frac{2}{3} \left( -\frac{\sqrt{3}}{2} g A_8^0 \right) \right] - \frac{2}{3} \mu'_8; \quad (8)
\]
\[
\mu_{gu} = \left[ \mu - \frac{1}{2} \left( -g A_3^0 \right) + \frac{1}{3} \left( -\frac{\sqrt{3}}{2} g A_8^0 \right) \right] - \frac{1}{2} \mu'_3 + \frac{1}{3} \mu'_8,
\]
\[
\mu_{rd} = \left[ \mu + \frac{1}{2} \left( -g A_3^0 \right) + \frac{1}{3} \left( -\frac{\sqrt{3}}{2} g A_8^0 \right) \right] + \frac{1}{2} \mu'_3 + \frac{1}{3} \mu'_8; \quad (9)
\]
\[
\mu_{ru} = \left[ \mu + \frac{1}{2} \left( -g A_3^0 \right) + \frac{1}{3} \left( -\frac{\sqrt{3}}{2} g A_8^0 \right) \right] + \frac{1}{2} \mu'_3 + \frac{1}{3} \mu'_8,
\]
\[
\mu_{gs} = \left[ \mu - \frac{2}{3} \left( -\frac{\sqrt{3}}{2} g A_8^0 \right) \right] - \frac{2}{3} \mu'_8; \quad (10)
\]

which are essentially equivalent to the conventional expressions of \( \mu_i \). In the CFL phase, \( m_s \) takes effect on the Fermion accumulation in the way of a mismatch between the strange- and light-flavor Fermion momenta (see Eq.(1)). In the \( K^0 \) condensed environment, nevertheless, \( m_s^2/2\mu \) has been regarded as the strangeness chemical potential to trigger the condensation. Thus, the \( m_s \) effect on the CFL\( K^0 \) Fermion accumulation is expected to become modified in somewhat way.

To treat the \( m_s \) effects self-consistently, we examine the influence of \( K^0 \) condensate on the quark properties firstly. It is well known that Goldstone boson condensation(s) may be realized via axial flavor transformation of quark field
\[
q \to q' = \exp[i(\theta_\alpha \lambda_\alpha)\gamma_5]q.
\]
For our purpose, the indices \( \alpha = 6, 7 \) are considered in Eq.(11) while \( \sqrt{\theta_6^2 + \theta_7^2} \) is just the condensate strength angle \( \theta \) [12]. Under this transformation, the quark-antiquark condensates for three flavors become
\[
\langle \bar{u}'u' \rangle = \langle \bar{u}u \rangle, \quad \langle \bar{d}'d' \rangle = \langle \bar{d}d \rangle \cos \theta, \quad \langle \bar{s}'s' \rangle = \langle \bar{s}s \rangle \cos \theta. \quad (12)
\]
Within the model-independent framework, Eq.(12) may be attributed to the modifications in quark masses

\[ m'_u = m_u, \quad m'_d = m_d \cos \theta, \quad m'_s = m_s \cos \theta, \]  

(13)
at the mean-field level.\(^2\) Although themselves are not the quark masses, \(m'_{u,d,s}\) do reflect the actual contributions of quark masses on the Fermi surface phenomenon in the \(K^0\) condensed environment. Ignoring the light masses, it is not \(m_s^2/2\mu\) but \(m'_s^2/2\mu\) to cause the CFL\(^0\) Fermi-momentum mismatches at the leading order. In analogy with Ref.[4], the common Fermi momenta are rewritten as

\[
\begin{align*}
 p_{\text{com}}^{F,(gs,bd)} &= \left[ \mu - \frac{1}{4}(-gA^0_3) - \frac{1}{6}\left(-\frac{\sqrt{3}}{2}gA^0_8\right) \right] + \frac{1}{4}\mu_3 - \frac{1}{6}\mu'_s - \frac{m'_s^2}{4\mu}, \\
 p_{\text{com}}^{F,(rs,bs)} &= \left[ \mu + \frac{1}{4}(-gA^0_3) - \frac{1}{6}\left(-\frac{\sqrt{3}}{2}gA^0_8\right) \right] + \frac{1}{4}\mu'_3 - \frac{1}{6}\mu'_s - \frac{m'_s^2}{4\mu}, \\
 p_{\text{com}}^{F,(rd,gu)} &= \left[ \mu + \frac{1}{3}\left(-\frac{\sqrt{3}}{2}gA^0_8\right) \right] + \frac{1}{3}\mu'_s, \\
 p_{\text{com}}^{F,(ru,gd,bs)} &= \mu - \frac{m'_s^2}{6\mu};
\end{align*}
\]  

(14)

where Eqs.(7-10) have been considered.

With the help of \(\mu_i\) and \(p_{\text{com}}^{F,i}\), the free energy relevant for the CFL\(^0\) Fermion accumulation reads

\[
\Omega'_{\text{CFL}}(\mu'_3, \mu'_s, m'_s) = \frac{1}{\pi^2} \sum \int_{0}^{p_{\text{com}}^{F,i}} \left( \sqrt{p^2 + m'_s^2 - \mu_i} \right) p^2 dp + \frac{1}{\pi^2} \sum \int_{0}^{p_{\text{com}}^{F,i}} \left( p - \mu_i \right) p^2 dp,
\]  

(15)

where the first term of RHS involves the sum of \(i = gs, rs, bs\) while the second term does the sum of \(i = bd, bu, rd, ru, gu, gd\). Although Eq.(15) has the similar form as the CFL free energy, \(m'_s, \mu'_3\) and \(\mu'_s\) have replaced \(m_s, \mu_3\) and \(\mu_s\) to become the variables respectively while \(A^0_3\) and \(A^0_8\) are actually the invariables being independent of \(\mu'_3, \mu'_s\). As in Ref.[4], we concern the \(\mu'_3, \mu'_s\)-related terms up to order \(m'_s^4\) to the purpose of yielding the leading-order solutions. Explicitly, the strange-quark-involved part in Eq.(15) reads

\[
-\frac{1}{12\pi^2} \sum [p_{F,i}^{\mu_3^3} - \frac{5}{2}p_{F,i}^{\mu_3 m'_s^2}] + \frac{3}{8\pi^2} m'_s^4 \log\left(\frac{m'_s^2}{2\mu}\right) + \ldots,
\]  

(16)

while the light-quark-involved part is

\[
-\frac{1}{12\pi^2} \sum p_{F,i}^{\mu_3^3}.
\]  

(17)

\(^2\)When the quark-antiquark condensates (as well as the diquark condensates) are taken into account explicitly, however, Eq.(13) no longer holds valid and thus the method developed in this work is not available yet. Actually, the descriptions of color superconducting phases (including the CFL\(^0\) phase) have been studied in NJL-type models (see, e.g. Refs.[12, 13, 14]). There, the color neutrality is imposed by hand, e.g. by the numerical tuned values of color chemical potentials, which is very different from the treatments within the model-independent framework.
Further expanding the above equations, it is found that there are the components like $A_{3,8}^0(\mu_{3,8}')^2\mu$, $(A_{3,8}^0)^2\mu_{3,8}'\mu$ and $A_{3,8}^3\mu_{3,8}'m_s'^2$ in our concerned terms. Noticing that $\mu_{3,8}'$ and $A_{3,8}^3$ have order $m_s'$, the order of these components is beyond $m_s^4$ and they are actually irrelevant for evaluating $\mu_{3,8}'$. To the order at which we are working, it is practical to ignore the $A_{3,8}^0$-related terms in the expansion of $\Omega_{CFL}$. Equivalently, we can simplify the square-bracket parts in the expressions of $\mu_i$ and $p_{F,i}^{com}$ (i.e. Eqs.(7-10) and (14)) as $\mu$. Under the above approximation, it becomes easy to yield the Alford-Ragagopal-type solutions $\mu_{3,8}'$. If replacing $\mu_3$, $\mu_8$ and $m_s$ by $\mu_3'$, $\mu_8'$ and $m_s'$ respectively, it is obvious that the conventional CFL expressions of $\mu_i$ and $p_{F,i}^{com}$ hold (approximately) unchanged in the CFL$^0$ case. Correspondingly, $\mu_3'$ and $\mu_8'$ should have the formally same result as Eq.(2) as long as the replacements $\mu_3 \rightarrow \mu_3'$, $\mu_8 \rightarrow \mu_8'$ and $m_s \rightarrow m_s'$ are considered. Instead of evaluating $\partial\Omega_{CFL}/\partial\mu_{3,8}' = 0$, we find that the fictional chemical potentials are

$$\mu_3' = 0, \quad \mu_8' = -\frac{m_s^2}{2\mu},$$

at the leading order, by comparing with the well-known result Eq.(2).

As an example, we examine the case of the maximal kaon condensate (i.e. $\theta = \pi/2$). In such a CFLK$^0$ phase, the value of $m_s'$ becomes zero so that its effect on the common Fermi momenta vanishes. Thus, the introduction of $\mu_{3,8}'$ is no longer necessary and we have $\mu_3' = \mu_8' = 0$, which is consistent with Eq.(18). Correspondingly, the factual color chemical potentials become $\mu_3 = -gA_0^s = -m_s^2/4\mu$ and $\mu_8 = -\sqrt{3}/2gA_0^s = -m_s^2/8\mu$, i.e. Eq.(5) is reasonable. Even if so, it is interesting to investigate the Fermion surface phenomenon in the resulting color-neutral phase. From Eq.(14), the common Fermi momenta are found to become $p_{F,(gs,bd)}^{com} = \mu + m_s^2/12\mu$, $p_{F,(rs,bu)}^{com} = \mu - m_s^2/24\mu$, $p_{F,(ru,gu)}^{com} = \mu - m_s^2/24\mu$, and $p_{F,(ru,gs,bs)}^{com} = \mu$. Therefore, the average Fermi momentum for all the paired quarks is equal to $\mu$ in the maximal $K^0$ condensed phase (if simply adopting our approximation, indeed, it is obvious that the common Fermi momenta are reduced to $\mu$). This result is different from the conventional case where the average Fermi momentum is $\mu - m_s^2/6\mu$, but comparable with the ideal CFL case in the absence of $m_s$. In this sense, we conclude that the Fermion accumulation in the maximal $K^0$ condensed environment behaves like that in the ideal CFL matter. Physically, the reason lies in the fact that the strange quark mass does not involve the Fermi surface phenomenon directly since its effect causes the maximal kaon condensation completely. This conclusion was not drawn in the previous literatures yet. Actually, one would have $p_{F,(gs,bd)}^{com} = \mu - m_s^2/6\mu$, $p_{F,(rs,bu)}^{com} = \mu - 7m_s^2/24\mu$, $p_{F,(ru,gu)}^{com} = \mu - m_s^2/24\mu$ and $p_{F,(ru,gs,bs)}^{com} = \mu - m_s^2/6\mu$ by inserting the color neutral solutions into Eq.(1). Thus, the $m_s$ effect is improper to be counted twice and the resulting Fermion accumulation is not correct.
Thus, the treatment developed in the present work is necessary to avoid the double counting on the $m_s$ effect and illuminate the $K^0$ condensed phase self-consistently.

In principle, there are possibilities except for the maximal $K^0$ condensate. In realistic situations, e.g. for not-very-large $\mu$, the instanton contribution on the kaon mass needs to be taken into account [15]. In this case, the $K^0$ condensation is suppressed (and even no longer occurs) in CFL quark matter. The $K^0$ condensation (if it exists) has an arbitrary strength $\theta$ even if in the situation of $m_s >> m_{u,d}$. In the CFL$K^0$ phase, the nonzero $m'_s$ leads to the nonvanishing $\mu'_s$ (see Eq.(18)). Therefore, Eq.(5) no longer holds valid and the factual color chemical potentials become

$$\mu_3 = -\frac{m^2_s}{4\mu} + \frac{m^2_s}{4\mu} \cos \theta,$$

$$\mu_8 = -\frac{m^2_s}{8\mu} - \frac{3m^2_s}{8\mu} \cos \theta - \frac{m^2_s}{2\mu} (\cos \theta)^2,$$

(19)

at the leading order. In the resulting color-neutral phase the average Fermi momentum becomes $\mu - m'_s^2/6\mu$, which manifests that the Fermi surface behavior is influenced by $m_s$ partly. Although it was seldom discussed in the literature, such a CFL$K^0$ phase is likely to exist in realistic situations. More importantly, it is definitely pointed out that the Kryjevski’s (expectation values of) gluon fields do not correspond to the factual color chemical potentials. In fact, the Fermi surface phenomenon influenced by kaon condensation leads to the extra color static potentials, which are independent from $A^0_{3,8}$ and connected with the $K^0$ condensate strength. These conclusions clarify some presumptions on the $K^0$ CFL condensed phase in the literature and might be important for fully understanding the unconventional CFL phases. The present method could be extrapolated if more physics involving Goldstone mode condensations is considered. When taking the electron chemical potential and the light quark masses into account, the electric-charged boson condensations and electric neutrality of the CFL matter need to be examined. As stressed in this work, both the dynamics of the condensed mode and the Fermi-surface behavior of color-flavor locked quarks should take effect on the color/electric neutrality.

In summary, we investigate color neutral problem in the CFL quark matter with arbitrary $K^0$ condensate. In order to treat the $m_s$ effect self-consistently, we introduce the fictional variables $\mu'_3, \mu'_8$ and $m'_s$ and reexamine the Fermi-surface behavior in $K^0$ condensed environment. In this way, the calculations of $A^0_{3,8}$ (from the Goldstone-mode Effective Lagrangian) and $\mu'_3, \mu'_8$ (from the Fermion free energy $\Omega_{CFL}$) become independent from each other. The factual color chemical potentials are obtained, which implies the breaking of the previous ansatz Eq.(5). Correspondingly, the Fermion accumulation in $K^0$ condensed environment is
investigated and it is found to differ from the conventional case. Since the results of $A_{3,8}^0$ are adopted directly, our treatment is actually based on the Alford-Ragagopal method and is valid only at the mean-field level. It is still unclear how to understand the present solutions (in particular the extra chemical potentials $\mu'_{3,8}$) in the framework of the "full" effective field theory including both Goldstone bosons and the paired quarks [16]. Furthermore, the intrinsic link between our discussed CFL$K^0$ phase and the $p$-wave $K^0$ condensed phase [17] needs to be examined seriously. Another important issue that is not discussed in this work involves the gapless formation. In the presence of $m_s$, the gapless CFL phase was predicted in Ref.[18] and the influence of maximal $K^0$ condensate on it was investigated in Ref.[11, 16]. With the color neutral solutions Eq.(19), the gapless formation might warrant further investigation in the environment with arbitrary $K^0$ condensate, which is beyond the scope of the present work.

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