Formation of phase clusters and chimera states in hierarchical networks of Josephson junctions

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Abstract. We demonstrate regimes such as cluster synchronization and chimera states in a hierarchical network based on Josephson Junctions. We find that the fractal spectrum of possible system frequencies of the system forms a devils staircase. Hierarchical networks have interesting fractal features and could be used for scaling of some properties of fractal networks. In our structure, Josephson Junctions are connected through electric currents in the form of a Cayley tree.

1. Introduction
The study of large networks consisted of mutually coupled nonlinear oscillators are attractive and important in various fields of science. Among the various types of oscillators, such as van der Pole, Rössler, Chua, etc., special attention is paid to rotators made through Josephson Junctions (JJ) [1]. Ensembles based on JJ have been extensively investigated since they are widely used in various fields of science, from stable frequency self–oscillators [2] to with the neuromorphic processors [3]. An essential feature of these systems is that of synchronization. Synchronization is a fundamental phenomenon of nature observed when a large number of different rhythmic objects are interacting with each other. Examples involve circadian rhythms, actions of neurons as well as swarm behavior in birds and fish. In engineering, Josephson Voltage Standard, power combining, and phase antenna arrays, realized with van der Pole’s oscillators, depend also significantly on synchronization processes. Various effects have been studied for oscillator ensembles, for example a load influence to synchrony stability [4], different topologies of oscillator arrays [5]–[8], and hysteresis phenomenon [9].

Chimera states have been discovered relatively recent, but they attracted great attention immediately. Since the first reports [10] on the phenomenon of simultaneous coexistence of synchrony and non-synchrony regimes, it have been seen and described in physical, biological, and chemical systems [11, 12]. They have been found, in particular, in SQUID metamaterials [13] and neural networks [14]. Investigations of chimera states could be important for improvement of some technical characteristics of physical systems such as semiconductor lasers [15] and for diagnostics of brain pathologies because of strong connection of chimera states with Parkinson’s disease, Alzheimer’s disease, schizophrenia, brain tumors, and epileptic seizures [16].

In the present work we introduce dynamics of hierarchical networks based on JJ and connected by phase locking and demonstrate different regimes and global states of the systems. We present
some features of hierarchical networks, viz. the possibility of phase cluster synchronization and chimera states.

2. Construction & Model
The model we investigate is presented in the Fig. 1. The structure is an ensemble of coupled JJ in the form of the Cayley tree. The external source consists of direct current and alternating current in the form of $I_{\text{source}} = I_{ac} + I_{ac} \cos(\Omega t)$. When the electric current $I_0$ arrives to the first node, it divides into two currents $I_1$ and $I_2$, which in turn divide further into two currents, and so on. We thus form a fractal–like structure where the expression for the $k$–th electrical current is given by

$$I_k = I_{2k} + I_{2k+1}. \quad (1)$$

The current flowing through a $k$–element is given by the following expression

$$I_j = i_{cj} \sin \varphi_j + i_{cj} \frac{d\varphi_j}{dt} + i_{cj} \beta_c \frac{d^2\varphi_j}{dt^2}, \quad (2)$$

where $\varphi_j$ is a phase of a $j$–th JJ, $i_{cj}$ is a critical current of a $j$–th JJ, and $\beta_c$ is a McCumber parameter.

![Figure 1. The investigated structure.](image)

![Figure 2. A spectrum of normal modes of the investigated system in a linear regime.](image)

3. Results
Initially we examined the system in the linear regime in order to determine all possible states of the system. We found that the spectrum of normal modes of the network is a Cantor set. The form of the spectrum is presented in the Fig. 2. Subsequently we investigated the system in the nonlinear nonlinear regime and we observed the formation of the phase clusters, which are shown in the Fig. 3. When the distribution of initial phases is uniform we found oscillator synchronization, provided a threshold in nonlinearity is exceeded; in this case the oscillators located on the equal levels are locked. Chimera states shown in Fig. 4 are also exist in hierarchical networks in other parameter regimes.

4. Acknowledgement
The work was carried out with financial support from the Ministry of Science and Higher Education of the Russian Federation in the framework of Increase Competitiveness Program of NUST "MISiS" (grant N 2–2019–010), implemented by a governmental decree dated 16th of March 2013, N 211.
Figure 3. A cluster synchronization.

Figure 4. A chimera state.

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