Swing wave-wave interaction: Coupling between fast magnetosonic and Alfvén waves

T.V. Zaqarashvili and B. Roberts

School of Mathematics and Statistics, University of St Andrews, St Andrews, Fife KY16 9SS, Scotland

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We suggest a mechanism of energy transformation from fast magnetosonic waves propagating across a magnetic field to Alfvén waves propagating along the field. The mechanism is based on swing wave-wave interaction [T.V. Zaqarashvili, Astrophys. J. Lett. 552, 107 (2001)]. The standing fast magnetosonic waves cause a periodical variation in the Alfvén speed, with the amplitude of an Alfvén wave being governed by Mathieu’s equation. Consequently, sub-harmonics of Alfvén waves with a frequency half that of magnetosonic waves grow exponentially in time. It is suggested that the energy of nonelectromagnetic forces, which are able to support the magnetosonic oscillations, may be transmitted into the energy of purely magnetic oscillations. Possible astrophysical applications of the mechanism are briefly discussed.

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I. INTRODUCTION

Many observed phenomena can be associated with wave-like motions, increasing interest in the study of wave dynamics. Linear perturbation theory considers an arbitrary disturbance as a superposition of independently evolving eigenmodes, thus simplifying the description of the process. However, interactions between different harmonics as well as between different kind of waves leads to the appearance of substantially new phenomena.

In the case of large-amplitude acoustic waves, nonlinearity leads to the generation of higher harmonics which cause steepening of the wave front and consequently the formation of shock waves. Also developments in plasma theory raise interest in the study of interactions between different waves. It is shown that nonlinear interaction leads to the generation of resonant triplets (or multiplets) in the plasma [8, 9, 10, 11, 12]. Or a background flow [13, 14, 15] has also been developed.

Recently, a new kind of interaction between sound and Alfvén waves has been discussed by Zaqarashvili [16]. The physical basis of this interaction is the parametric influence; sound waves cause a periodical variation in the medium’s parameters, which affects the velocity of transversal Alfvén waves and leads to a resonant energy transformation into certain harmonics. In a high β plasma, it is shown that periodical variations of the medium’s density, caused by the propagation of sound waves along an applied magnetic field, results in Alfvén waves being governed by Mathieu’s equation (here \( \beta = 8 \pi p / B^2 \gg 1 \), where \( p \) is the plasma pressure and \( B \) is the magnetic field). Consequently, harmonics with half the frequency of sound waves grow exponentially in time. The same phenomenon was developed in the case of standing sound waves [17]. The process of energy exchange between these different kinds of wave motion is called swing wave-wave interaction. This terminology arises from an analogy with a swinging pendulum, as described below.

In this paper we further develop the theory for interactions between fast magnetosonic waves and Alfvén waves. For clarity of presentation we first recall the pendulum analogy and show that under certain conditions the energy of spring oscillations along the pendulum axis is transformed into the energy of transversal oscillations of the pendulum, and vice versa. Following a discussion of the general physics of swing interaction we go on to consider the example of coupling between fast magnetosonic waves propagating across an applied magnetic field and Alfvén waves propagating along the field. Finally, we briefly describe the applications of the theory to various astrophysical situations.

II. SWING PENDULUM

It is useful to begin with a mechanical analogy of the wave dynamics in a medium (see [3] in the case of three-wave interaction). Consider a mathematical pendulum with mass \( m \) and equilibrium length \( L \) (see Fig. 1). Part of the pendulum length consists of a spring with stiffness constant \( \sigma \). There are two kinds of oscillation in this system: transversal oscillations due to gravity and spring oscillations along the pendulum axis due to the elasticity of the spring. This is a swing pendulum.

In equilibrium, gravity is balanced by the stiffness force \( T_0 \) of the spring so that

\[
T_0 = \sigma h = mg,
\]

where \( g \) is the gravitational acceleration and \( h \) is the equilibrium length of the spring (the natural length of the spring is supposed negligible). For displacement \( x \)
along the pendulum axis, the stiffness force becomes
\[ T = \sigma(h + x) = mg + \sigma x. \]

Newton’s second law applied along the pendulum axis, when the pendulum makes an angle \( \Theta \) with the vertical (see Fig.1), gives the equation of motion (the centrifugal force due to the transversal oscillation is neglected)
\[ \ddot{x} + \frac{\sigma}{m}x = g(\cos\Theta - 1). \]

Due to the oscillation of the spring along the axis, the pendulum length is a function of time and the equation of transversal motions of the pendulum under gravity is
\[ \ddot{\Theta} + \frac{g}{L + x}\sin\Theta = 0. \]

For clarity of presentation a term \( 2i\dot{\Theta}/(L + x) \) is here neglected; it does not affect the physical nature of the phenomenon (for general consideration, see \[15\,19\]). So we have two different oscillations of the pendulum, which are coupled, and each oscillation influences the other. Considering small amplitude oscillations, we find two coupled equations governing the dynamics of the pendulum:
\[ \ddot{x} + \omega_1^2x = -\frac{1}{2}g\Theta^2, \quad (1) \]
\[ \ddot{\Theta} + \omega_2^2(1 - \frac{x}{L})\Theta = 0, \quad (2) \]
where \( \omega_1 = \sqrt{g/h} \) and \( \omega_2 = \sqrt{g/L} \) are the fundamental frequencies of the system.

From equations (1) and (2) we can see that longitudinal oscillations of the pendulum causes a periodical variation of the pendulum length. In certain conditions this can lead to the well known parametric amplification of transversal oscillations. When \( x \) is a periodical function of time, then equation (2) becomes Mathieu’s equation and it has a resonant solution when
\[ \omega_2 = \frac{1}{2}\omega_1, \quad (3) \]

corresponding to \( L = 4h \).

Under these conditions, initial spring oscillations, \( x \), along the pendulum axis can amplify small transversal perturbations, \( \Theta \) (see equation (2)). On the other hand, transversal oscillations may be considered as an external periodic force (see equation (1)) which causes the damping and consequent amplification of longitudinal oscillations. So, in the absence of dissipation, there is a subsequent energy exchange between different oscillations in the system. But if some kind of external force supports the spring oscillations then they can amplify the transversal oscillations until nonlinear effects became significant.

### III. SWING WAVE-WAVE INTERACTION

The generalisation of the above analogy to waves in a medium leads to interesting phenomena. Spring oscillations do work against gravity and cause periodical variations of the parameter (pendulum length \( L \)) of transversal oscillations. As a result of this work, the energy of spring oscillations transforms into the energy of transversal oscillations. So we may expect a similar process in a medium when one kind of waves cause a periodical variation of another wave parameters.

There are three main forces in the equation of motion for an ideal conductive fluid: the pressure gradient \(-\nabla p\), gravity \( \rho\nabla \phi \), and the Lorentz force \( j \times B \). Here \( p \) and \( \rho \) denote the plasma pressure and density, \( \phi \) is the gravitational potential and \( j \) is the current in a magnetic field \( B \). Each of these forces represents the restoring force against the fluid inertia and thus leads to the generation of different kinds of wave motion. Of these forces, only the Lorentz force does not include the density (in the pressure gradient the density arises from the equation of state). This fact leads to the appearance of the density in the expression for the magnetic speed, the Alfvén speed \( V_A = B/\sqrt{\pi\rho} \), which describes the propagation of magnetic waves and depends on the medium density \( \rho \). For a similar reason, the frequency of pendulum oscillations does not depend on the pendulum mass (because the gravitational force depends on it), while the frequency of spring oscillation does depend on the mass (because the stiffness force does not depend on it). On the other hand, compressible waves cause density variations in the medium and therefore they may affect the propagation properties of magnetic waves. This suggests a coupling between longitudinal, compressible waves (leading to density perturbations) and transversal magnetic waves propagating with a velocity which depends on the density. The later can be associated with Alfvén waves which are transversal and represent the purely electromagnetic properties of the medium. The compressible waves cause a periodical variation of the density and so of the Alfvén speed, and may lead to the effective energy transmission into certain harmonics of Alfvén waves. The swing coupling between sound and Alfvén waves propagating along an applied magnetic field [16-17] is a good
example of this phenomenon. On the other hand, magnetosonic waves propagating at an angle to the magnetic field also cause periodical variations of the Alfvén speed and may lead to similar phenomena. It is worth noticing that, contrary to the Alfvén waves, the magnetosonic waves can be easily excited in a medium by any force (even of non electromagnetic origin). Therefore, the coupling between magnetosonic and Alfvén waves allows the transmission of energy into purely transversal magnetic oscillations through compressible magnetosonic oscillations.

To show the mathematical formalism of swing wave interaction we consider the case of magnetosonic wave propagation across the applied magnetic field. In this case we have fast magnetosonic waves. For simplicity we consider a rectangular geometry, which then can be generalised to cylindrical and spherical symmetries.

### A. Coupling between fast magnetosonic and Alfvén waves

Consider motions of a homogeneous medium, with zero viscosity and infinite conductivity, as described by the ideal MHD equations:

\[
\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{B} = (\mathbf{B} \cdot \nabla) \mathbf{u} - \nabla p - \rho \frac{\mathbf{B}^2}{8\pi} \nabla \cdot \mathbf{B} = 0, \tag{4}
\]

\[
\frac{\rho}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla \left( p + \frac{\mathbf{B}^2}{8\pi} \right) + \frac{(\mathbf{B} \cdot \nabla) \mathbf{B}}{4\pi}, \tag{5}
\]

\[
\frac{\partial \rho}{\partial t} + (\mathbf{u} \cdot \nabla) \rho + \rho \nabla \cdot \mathbf{u} = 0, \tag{6}
\]

where \(\mathbf{u}\) is the fluid velocity. We consider adiabatic processes, so the pressure \(p\) and density \(\rho\) are connected by the relation

\[
p = p_0 \left( \frac{\rho}{\rho_0} \right)^\gamma, \tag{7}
\]

where \(p_0\) and \(\rho_0\) are the unperturbed uniform pressure and density and \(\gamma\) is the ratio of specific heats. We neglect gravity, though it may be of importance under some astrophysical conditions.

Linear analysis of equations (4)-(7) show the existence of three kinds of MHD waves: Alfvén and magnetosonic (fast and slow) waves. The difference between these waves is that the restoring force of Alfvén waves is the tension of magnetic field lines, \((\mathbf{B} \cdot \nabla)\mathbf{B}/4\pi\), acting alone, while the restoring force of magnetosonic waves is mainly the gradient of ordinary and magnetic pressures, \(-\nabla \left[ p + \mathbf{B}^2/8\pi \right]\). The various waves can be distinguished by their different speeds and polarizations. The linear evolution of the waves in a homogeneous medium is governed by the usual linear wave equations.

Consider a uniform, unperturbed, magnetic field \(\mathbf{B}_0 = (0, 0, B_0)\) directed along the \(z\)-axis, and the case of magnetosonic wave propagation across the field in the \(x\) direction (see Fig. 2). Then there are only fast magnetosonic waves (the slow wave is absent) which in the linear approximation is described by the equations:

\[
\frac{\partial b_z}{\partial t} = -B_0 \frac{\partial u_x}{\partial x}, \tag{8}
\]

\[
\rho_0 \frac{\partial u_x}{\partial t} = -\frac{\partial}{\partial x} \left[ c_s^2 \rho + \frac{B_0 b_z}{4\pi} \right], \tag{9}
\]

\[
\frac{\partial \rho}{\partial t} = -\rho_0 \frac{\partial u_x}{\partial x}, \tag{10}
\]

where \(b_z\) and \(u_x\) are the perturbations of magnetic field and velocity, respectively, and \(c_s = \sqrt{\gamma p_0 / \rho_0}\) is the sound speed. Here and afterwards \(\rho\) denotes the perturbation of density (in equations (4)-(5) \(\rho\) was the total density).

The wave equation for linear fast magnetosonic waves then follows,

\[
\frac{\partial^2 u_x}{\partial t^2} - V_f^2 \frac{\partial^2 u_x}{\partial x^2} = 0, \tag{11}
\]

where \(V_f = \sqrt{c_s^2 + V_A^2}\) is the phase velocity of fast waves and \(V_A = \sqrt{B_0^2/4\pi \rho_0}\) is the Alfvén speed.

The solution of the wave equation can be either propagating or standing patterns. The boundedness of the medium leads to the formation of a discrete spectrum of harmonics which represent the normal modes (eigenmodes) of the system. We consider the standing fast magnetosonic waves, which have a straightforward extension to cylindrical (pulsating magnetic tube) and spherical (pulsating sphere with dipole-like magnetic field) geometries. The solutions for standing (plane) fast magne-
tosonic waves are:

\[
\begin{align*}
    u_x &= \alpha V_f \sin(\omega_n t) \sin(k_n x), \\
    \rho &= \alpha \rho_0 \cos(\omega_n t) \cos(k_n x), \\
    b_z &= \alpha B_0 \cos(\omega_n t) \cos(k_n x),
\end{align*}
\]

(12)

where \( k_n = \frac{n \pi}{l} \) \((n = 1, 2, ...)\) is the eigenvalue for a system of size \( l \) in the \( x \) direction, \( \omega_n \) is the corresponding eigenfrequency, and \( \alpha \) is the relative amplitude of the waves. Eigenvalues and eigenfrequencies are related by the dispersion relation \( \omega_n/k_n = V_f \).

It is seen from the expressions (12) that standing fast magnetosonic waves cause a local periodical variation in both the density and the magnetic field. This variation is maximal near the nodes of the velocity and approaches to zero near the antinodes. The amplitude of the variation is considered to be small \((\alpha \ll 1)\), and so does not affect the fast magnetosonic wave itself.

Consider now the influence of the density and the magnetic field variations (12) on Alfvén waves, considered to be polarised in the \( yz \) plane. Then the velocity fields of fast magnetosonic and Alfvén waves are decoupled. The linear equations for Alfvén waves are:

\[
\frac{\partial b_y}{\partial t} = B_0 \frac{\partial u_y}{\partial z},
\]

(13)

\[
\rho_0 \frac{\partial u_y}{\partial t} = B_0 \frac{\partial b_y}{\partial z},
\]

(14)

where \( b_y \) and \( u_y \) are small perturbations of the magnetic field and the velocity. These equations lead to the wave equation

\[
\frac{\partial^2 b_y}{\partial t^2} - V_A^2 \frac{\partial^2 b_y}{\partial z^2} = 0.
\]

(15)

The influence of the fast magnetosonic waves can be expressed by modifying equations (13) and (14), which now became

\[
\frac{\partial b_y}{\partial t} = (B_0 + b_z) \frac{\partial u_y}{\partial z} - \frac{\partial u_z}{\partial x} b_y,
\]

(16)

\[
(\rho_0 + \rho) \frac{\partial u_y}{\partial t} = \frac{B_0 + b_z}{4\pi} \frac{\partial b_y}{\partial z}.
\]

(17)

Here we have neglected the advective terms \( u_z \partial b_y/\partial x \) and \((\rho_0 + \rho)u_x \partial b_y/\partial x\) for several reasons. At the initial stage, the perturbations \( b_y \) and \( u_y \) of Alfvén waves propagating along the \( z \) axis do not depend on the \( x \) coordinate; each magnetic surface across \( x \) evolves independently. The \( x \) dependence arises due to the action of the fast magnetosonic waves, and so the neglected terms are second order in \( \alpha^2 \). Moreover we can consider the Alfvén waves at the velocity node of standing fast magnetosonic waves, where these terms are zero. In principle, the coordinate \( x \) stands as a parameter in equations (16) and (17) of Alfvén waves.

Equations (16) and (17) lead to the Hill type second order differential equation

\[
\frac{\partial^2 b_y}{\partial t^2} - \frac{(2B_0 + b_z) \partial b_y}{B_0 (B_0 + b_z)} \frac{\partial b_y}{\partial t} - \frac{(B_0 + b_z) \partial b_y}{B_0 (B_0 + b_z)} b_y - \frac{(B_0 + b_z)^2}{4\pi (\rho_0 + \rho)} \frac{\partial^2 b_y}{\partial z^2} = 0,
\]

(18)

where \( \dot{b}_z \) denotes the time derivative of the perturbing field. Introducing

\[
b_y = h_y(z, t) \exp \int \frac{(2B_0 + b_z) \dot{b}_z}{2B_0 (B_0 + b_z)} dt
\]

(19)

and neglecting terms of order \( \alpha^2 \) leads to the equation

\[
\frac{\partial^2 h_y}{\partial t^2} - \frac{V_A^2}{\omega_n^2} \left[ 1 + \alpha \cos(k_n x) \cos(\omega_n t) \right] \frac{\partial^2 h_y}{\partial z^2} = 0.
\]

(20)

Comparing equations (20) and (15) we can see that the influence of standing fast magnetosonic waves is expressed through a periodical variation of the Alfvén speed.

Performing a Fourier transform of \( h_y \) with \( h_y = \int h_y(k_z, t) e^{ik_z z} dk_z \), equation (20) leads to Mathieu’s equation

\[
\frac{\partial^2 \hat{h}_y}{\partial t^2} + \left[ V_A^2 k_z^2 + \delta \cos(\omega_n t) \right] \hat{h}_y = 0,
\]

(21)

where

\[
\delta = \alpha V_A^2 k_z^2 \cos(k_n x),
\]

(22)

with \( x \) playing the role of a parameter. Equation (21) has main resonant solution if

\[
\omega_A = \frac{B_0 k_z}{\sqrt{4\pi \rho_0}} = \frac{\omega_n}{2}
\]

(23)

and it can be expressed as

\[
\hat{h}_y = h_0 e^{i k_z z} \left[ \cos \left( \frac{\omega_n t}{2} \right) - \sin \left( \frac{\omega_n t}{2} \right) \right],
\]

(24)

where \( h_0 = h(0) \). The solution has a resonant character within the frequency interval

\[
\left| \omega_A - \frac{\omega_n}{2} \right| < \frac{\delta}{\omega_n}.
\]

(25)

Equation (24) shows that the harmonics of Alfvén waves with half the frequency of fast magnetosonic waves grow exponentially in time. The growth rate of Alfvén waves is maximal at the velocity nodes of fast magnetosonic waves and tends to zero at the antinodes (see equations (12) and (22)). The amplitude of the magnetic field component in Alfvén waves depends on the
x coordinate, i.e. there is the periodical magnetic pressure gradient along this direction. Energy conservation implies that this gradient leads to the damping of initial fast magnetosonic waves, i.e. the energy transformed into Alfvén waves is extracted from fast magnetosonic waves. To show this, we consider the backreaction of amplified Alfvén waves on the initial fast magnetosonic waves.

The dependence of \( b_y \) on the \( x \) coordinate leads to an additional term in the equation of motion (9) for fast waves,

\[
\frac{\rho_0}{\partial t} = -\frac{\partial}{\partial x} \left[ \frac{c_s^2 \rho + B_0 b_z}{4\pi} \right] - \frac{\partial}{\partial x} \left[ \frac{b_y^2}{8\pi} \right].
\]  

Therefore the wave equation (11) now becomes

\[
\frac{\partial^2 u_x}{\partial t^2} - V_f^2 \frac{\partial^2 u_x}{\partial x^2} = -\frac{\partial^2}{\partial t \partial x} \left[ \frac{b_y^2}{8\pi} \right].
\]

The additional term has the frequency of the initial fast magnetosonic waves \( \omega_0 \) (within the order of \( \alpha^2 \)) and can be considered as the external, periodic force. At the initial stage it can be neglected as of second order of smallness. However, it becomes significant because of the exponential growth of amplitudes (see equation (24)). It oscillates out of phase with respect to the initial fast waves (12), thus leading to their damping (as expected from physical considerations).

Note that equations (1) and (2) describing the pendulum oscillations are similar to equations (20) and (27) describing Alfvén waves and fast magnetosonic waves; the longitudinal oscillations of the pendulum correspond to the fast magnetosonic waves and the transversal oscillations correspond to the Alfvén waves.

Swing coupling between fast magnetosonic and Alfvén waves may be generalised from rectangular geometry to other symmetries, though a detailed description is beyond the scope of this paper.

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nuclear reactions, tidal forces in binary stars, convection, etc. (e.g., [22]). Then the transformation of pulsational energy into torsional oscillations may lead to new sources for stellar magnetic activity.

V. CONCLUDING REMARKS

The swing wave-wave interaction [16] is developed here in the case of fast magnetosonic waves propagating across a magnetic field and Alfvén waves propagating along the field. In the case of oblique propagation, slow magnetosonic waves also exist and they may transmit their energy into Alfvén waves. In some cases the coupling between slow magnetosonic and Alfvén waves may be of importance. Also, the coupling in the case of different geometries (cylindrical, spherical) may be important in astrophysical situations. The most important result of swing wave interaction is that it reveals a new energy channel for Alfvén waves, permitting the transformation of energy of nonelectromagnetic origin into the energy of electromagnetic oscillations.

Acknowledgments

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