QCD corrections to lifetime differences of $B_s$ mesons

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Abstract

The calculation of QCD corrections to the width difference $\Delta \Gamma$ in the $B_s$-meson system is presented. The next-to-leading order corrections reduce the dependence on the renormalization scale significantly and allow for a meaningful use of hadronic matrix elements from lattice gauge theory. At present the uncertainty of the lattice calculations limits the prediction of $\Delta \Gamma$. The presented work has been performed in collaboration with Martin Beneke, Gerhard Buchalla, Christoph Greub and Alexander Lenz.

1 Introduction

Precision analyses of flavor-changing transitions are of experimental top priority in the forthcoming years. Decays of $B$ mesons provide an especially fertile testing ground for various reasons: they allow for a high precision determination of three of the four parameters characterizing the Cabibbo-Kobayashi-Maskawa (CKM) matrix [[1]], including the CP-violating phase $\gamma$. Since flavor-changing transitions of $B$ mesons are always suppressed by small CKM elements and heavy electroweak gauge boson masses, it is well possible that $B$ physics experiments will reveal new physics. The large mass $m_b$ of the $b$-quark further allows us to control hadronic uncertainties. Fermilab’s CDF, D0 [2] and the planned BTeV [3] experiment prepare a dedicated $B$ physics program. Other studies are in

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Figure 1: $B_s - \overline{B}_s$ mixing in the Standard Model. The zigzag lines represent $W$-bosons or charged pseudo-goldstone bosons.

progress at CLEO, LEP and at HERA-B [4] or planned for the future LHCb [5] experiment. While $B$-factories [6] only produce $B_d, \overline{B}_d$ and $B^\pm$ mesons, LEP and the hadron colliders also provide $B_s$ mesons. Like their $K$, $D$ and $B_d$ counterparts $B_s$ mesons mix with their antiparticles. Therefore the two mass eigenstates $B_H$ and $B_L$ (for “heavy” and “light”) are linear combinations of $B_s$ and $\overline{B}_s$ and differ in their mass and width. In the Standard Model $B_s - \overline{B}_s$ mixing is described in the lowest order by the box diagram depicted in Fig. 1. The dispersive part of the $B_s - \overline{B}_s$ mixing amplitude is called $M_{12}$. In the Standard Model it is dominated by box diagrams with internal top quarks. The absorptive part is denoted by $\Gamma_{12}$ and mainly stems from box diagrams with light charm quarks. $\Gamma_{12}$ is generated by decays into final states which are common to $B_s$ and $\overline{B}_s$. While $M_{12}$ can receive sizable corrections from new physics, $\Gamma_{12}$ is induced by the CKM-favored tree-level decay $b \rightarrow c\bar{s}s$ and insensitive to new physics. Experimentally $B_s - \overline{B}_s$ mixing manifests itself in damped oscillations between the $B_s$ and $\overline{B}_s$ states which are governed by $M_{12} - i\Gamma_{12}/2$. We denote the mass and width differences between $B_H$ and $B_L$ by

$$\Delta m = M_H - M_L, \quad \Delta \Gamma = \Gamma_L - \Gamma_H.$$  

By solving the eigenvalue problem of $M_{12} - i\Gamma_{12}/2$ one can relate $\Delta m$ and $\Delta \Gamma$ to $M_{12}$ and $\Gamma_{12}$:

$$\Delta m = 2|M_{12}|, \quad \Delta \Gamma = 2|\Gamma_{12}| \cos \phi,$$

(1)

where $\phi$ is defined as

$$\frac{M_{12}}{\Gamma_{12}} = -\left|\frac{M_{12}}{\Gamma_{12}}\right| e^{i\phi}.$$  

(2)

$\Delta m$ equals the $B_s - \overline{B}_s$ oscillation frequency and has not been measured yet, but we know the lower bound $\Delta m \geq 14.9$ ps$^{-1}$ from LEP data [7]. It can be shown that this bound implies $|\Gamma_{12}|/|M_{12}| \ll$
0.01. In deriving (1) terms of order \( |\Gamma_{12}/M_{12}|^2 \) have been neglected. \( \phi \) in (2) is a CP-violating phase, which is tiny in the Standard Model, so that \( \Delta \Gamma_{SM} = 2|\Gamma_{12}| \). In the presence of new physics \( \arg M_{12} \) and thereby \( \phi \) can assume any value. \( \phi \) can be measured from CP-asymmetries, which requires the resolution of the rapid \( B_s^{-}\bar{B}_s \) oscillations and tagging, i.e. the discrimination between \( B_s \) and \( \bar{B}_s \) mesons at the time \( t = 0 \) of their production. From (1) one verifies that a non-vanishing \( \phi \) also affects \( \Delta \Gamma \), which can be measured from untagged data samples and therefore involves better efficiencies than tagged studies. Unlike in the case of \( B_d \) mesons, the Standard Model predicts a sizable width difference \( \Delta \Gamma \) in the \( B_s \) system, roughly between 5 and 30\% of the average total width \( \Gamma = (\Gamma_L + \Gamma_H)/2 \). Now the decay of an untagged \( B_s \) meson into the final state \( f \) is in general governed by two exponentials:

\[
\Gamma[f,t] \propto e^{-\Gamma_L t} |\langle f | B_L \rangle|^2 + e^{-\Gamma_H t} |\langle f | B_H \rangle|^2.
\]

(3)

If \( f \) is a flavor-specific final state like \( D_s^{-}\pi^+ \) or \( X\ell^+\nu \), the coefficients of the two exponentials in (3) are equal. A fit of the corresponding decay distribution to a single exponential then determines the average width \( \Gamma \) up to corrections of order \((\Delta \Gamma)^2/\Gamma\). In the Standard Model CP violation in \( B_s^{-}\bar{B}_s \) mixing is negligible, so that we can simultaneously choose \( B_L \) and \( B_H \) to be CP eigenstates and the \( b \to c\bar{s}s \) decay to conserve CP. Then \( B_H \) is CP-odd and cannot decay into a CP-even double-charm final state \( f_{CP+} \) like \( (J/\psi\phi)_{L=0,2} \), where \( L \) denotes the quantum number of the orbital angular momentum. Thus a measurement of the \( B_s \) width in \( B_s \to f_{CP+} \) determines \( \Gamma_L \). By comparing the two measurements one finds \( \Delta \Gamma/2 \). In the presence of a non-zero CP-violating phase \( \phi \) this procedure measures \( \Delta \Gamma \cos \phi \):

\[
\Delta \Gamma \cos \phi = \Delta \Gamma_{SM} \cos^2 \phi.
\]

(4)

The extra factor of \( \cos \phi \) stems from the fact that in the presence of CP violation both \( B_L \) and \( B_H \) can decay into \( f_{CP+} \). CDF will perform this measurement with \( B_s \to D_s^{-}\pi^+ \) and \( B_s \to J/\psi\phi \) in Run-II of the Tevatron [9].

### 2 QCD effects

The \( B_s^{-}\bar{B}_s \) mixing amplitude of Fig.1 and the \( B_s \) decay amplitude are affected by strong interaction effects. \( \Delta \Gamma_{SM} = 2|\Gamma_{12}| \) involves various different mass scales and the the QCD corrections associated with these scales require different treatments. In the first step an operator product expansion at the scale \( M_W \) is performed to integrate out the \( W \)-boson. The Standard Model \( b \to c\bar{s}s \) amplitude is matched to its counterpart in an effective field theory in which \( \Delta B = 1 \) transitions \( (B \) is the bottom number) are described by four-quark operators. The corresponding effective hamiltonian reads

\[
\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} \left( \sum_{r=1}^{6} C_r Q_r + C_8 Q_8 \right),
\]

(5)
with the operators

\[ Q_1 = (\bar{b}_i c_j)_{V-A}(\bar{c}_j s_i)_{V-A}, \quad Q_2 = (\bar{b}_i c_i)_{V-A}(\bar{c}_j s_j)_{V-A}, \]

\[ Q_3 = (\bar{b}_i s_i)_{V-A}(\bar{q}_j q_j)_{V-A}, \quad Q_4 = (\bar{b}_i s_j)_{V-A}(\bar{q}_j q_i)_{V-A}, \]

\[ Q_5 = (\bar{b}_i s_i)_{V-A}(\bar{q}_j q_j)_{V+A}, \quad Q_6 = (\bar{b}_i s_j)_{V-A}(\bar{q}_j q_i)_{V+A}, \]

\[ Q_8 = \frac{g}{8\pi^2} m_b \bar{b}_i \sigma^{\mu\nu} (1 - \gamma_5) T_{ij} s_j G_{\mu\nu}^a. \]  

Here the \( i, j \) are colour indices and a summation over \( q = u, d, s, c, b \) is implied. \( V \pm A \) refers to \( \gamma^a (1 \pm \gamma_5) \) and \( S - P \) (which we need below) to \( (1 - \gamma_5) \). The current-current operators \( Q_1 \) and \( Q_2 \) stem from \( W \)-boson exchange between the \( \bar{b}c \) and \( \bar{t}s \) lines. \( Q_{3-6} \) are four-quark penguin operators and \( Q_8 \) is the chromomagnetic penguin operator. The Wilson coefficients \( C_i \) contain the short-distance physics and are functions of the heavy \( W \) and top quark masses. Since they do not depend on long-distance QCD effect, they can be calculated in perturbation theory. \( C_{3-6} \) are very small. The matching calculation determines the \( C_i \)'s at a high renormalization scale \( \mu = \mathcal{O}(M_W) \). The renormalization group (RG) evolution of the coefficients down to \( \mu = \mathcal{O}(m_b) \) sums the large logarithm \( \alpha_s \ln(M_W/m_b) \) to all orders in perturbation theory. The operator product expansion leading to (6) and the RG improvement amount to a simultaneous expansion in \( m_b^2/M_W^2 \), \( \alpha_s(M_W) \) and \( \alpha_s(m_b) \) of the \( b \to c\bar{t}s \) amplitude. \( \mathcal{H}_{\text{eff}} \) in (5) reproduces the leading term in the power expansion in \( m_b^2/M_W^2 \).

The second step to predict \( \Delta \Gamma_{SM} \) involves an operator product expansion at the scale \( m_b \). The corresponding formalism has been formulated long ago by the hosts of this conference [10]. The starting point for the calculation of the widths \( \Gamma_H \) of some \( b \)-flavored hadron \( H \) is the optical theorem, which relates \( \Gamma_H \) to the absorptive part of the forward scattering amplitude of \( H \). Neglecting CP violation in the decay amplitude the optical theorem implies

\[ \Gamma_H \propto \text{Im} \langle H | i \int d^4x T \mathcal{H}_{\text{eff}}(x) \mathcal{H}_{\text{eff}}(0) | H \rangle. \]  

Now the Heavy Quark Expansion (HQE) [10] is an operator product expansion of the forward scattering amplitude in (10). Schematically

\[ \Gamma_H \propto G_F^2 \sum_j m_b^{8-d_j} F_j (\mu/m_b) \langle H | \mathcal{O}_j (\mu) | H \rangle \]

\[ \mathcal{O} \left( \Lambda_{\text{QCD}}^{d_j-3} \right) \]

Here new Wilson coefficients \( F_j \) have appeared. They contain the physics associated with scales above the matching scale \( \mu = \mathcal{O}(m_b) \), at which the HQE is performed. The \( \mathcal{O}_j \)'s are local operators.
with dimension $d_j \geq 3$. The HQE is a simultaneous expansion of $\Gamma_H$ in $\Lambda_{QCD}/m_b$ and $\alpha_s(m_b)$. Increasing powers of $\Lambda_{QCD}/m_b$ correspond to increasing dimensions $d_j$ of the local operators $O_j$.

To calculate $\Delta \Gamma$ from the HQE one must extend the above formalism to the two state system $(B_s, \bar{B}_s)$:

$$
\Delta \Gamma_{SM} = 2|\Gamma_{12}| = -\frac{1}{M_{B_s}} \text{Im} \langle \bar{B}_s | i \int d^4x T H_{\text{eff}}(x) H_{\text{eff}}(0) | B_s \rangle
$$

The corresponding leading-order diagrams are shown in Fig. 2. (12) is matched to local operators in analogy to (11):

$$
\text{Im} \langle \bar{B}_s | i \int d^4x T H_{\text{eff}}(x) H_{\text{eff}}(0) | B_s \rangle = -\frac{G_F m_b^2}{12\pi} |V_{cb} V_{cs}|^2 \cdot
$$

$$
\left[ F \left( \frac{m_c^2}{m_b^2} \right) \langle \bar{B}_s | Q | B_s \rangle + F_S \left( \frac{m_c^2}{m_b^2} \right) \langle \bar{B}_s | Q_S | B_s \rangle \right] \left[ 1 + \mathcal{O} \left( \frac{\Lambda_{QCD}}{m_b} \right) \right]. \quad (13)
$$

The HQE for the $\Delta B = 2$ transition in (12) requires four-quark operators involving both the $b$-quark and the $s$-quark field, i.e. operators with dimension six or higher. The two dimension-6 operators appearing in (13) are

$$
Q = (\bar{b}_i s_i)_{V-A} (\bar{b}_j s_j)_{V-A}, \quad Q_S = (\bar{b}_i s_i)_{S-P} (\bar{b}_j s_j)_{S-P}. \quad (14)
$$

In the leading order of QCD the RHS of (13) is pictorially obtained by simply shrinking the $(c, c)$ loop in Fig. 2 to a point. The Wilson coefficients $F$ and $F_S$ also depend on the charm quark mass $m_c$, which is formally treated as a hard scale of order $m_b$, since $m_c \gg \Lambda_{QCD}$. Strictly speaking, the HQE in (13) is an expansion in $\Lambda_{QCD}/\sqrt{m_b^2 - 4m_c^2}$. For the calculation of $F$ and $F_S$ it is crucial that these

![Figure 2: Leading-order diagrams for $\Gamma_{12}$](image)
coefficients do not depend on the infrared structure of the process. In particular they are independent of the QCD binding forces in the external \( B_s \) and \( \bar{B}_s \) states in (13), so that they can be calculated in perturbation theory at the parton level. The non-perturbative long-distance QCD effects completely reside in the hadronic matrix elements of \( Q \) and \( Q_S \).

The third and final step in the prediction of \( \Delta \Gamma_{SM} \) is the calculation of the hadronic matrix elements with non-perturbative methods such as lattice gauge theory. It is customary to parametrize these matrix elements as

\[
\langle B_s | Q | B_s \rangle = \frac{8}{3} f_{B_s}^2 M_{B_s}^2 B \\
\langle B_s | Q_S | B_s \rangle = -\frac{5}{3} f_{B_s}^2 M_{B_s}^2 \frac{M_{B_s}^2}{(\bar{m}_b + \bar{m}_s)^2} B_S. \tag{15}
\]

In the so called vacuum insertion approximation \( B \) and \( B_S \) are equal to 1.

### 3 Next-to-leading order QCD corrections to \( \Delta \Gamma \)

The discussion of \( \Delta \Gamma \) in sect. 2 has been restricted to the leading order (LO) of QCD \([11]\). The only QCD effects included in this order are the leading logarithms \( \alpha_s^n \ln^n(M_W/m_b), n = 0, 1, 2, \ldots \), contained in the \( C_j \)'s of the effective \( \Delta B = 1 \) hamiltonian in (5). To predict \( \Delta \Gamma \) with next-to-leading order (NLO) accuracy one must first include the corrections of order \( \alpha_s^{n+1} \ln^n(M_W/m_b), n = 0, 1, 2, \ldots \), to these coefficients \([12]\). Second corrections of order \( \alpha_s(m_b) \) must be included in \( F \) and \( F_S \) \([13]\). This step requires the inclusion of hard gluon exchange on both sides of (13). The corresponding diagrams are depicted in Fig. 3. The motivations for this cumbersome calculation are

1. to verify the infrared safety of \( F \) and \( F_S \),
2. to allow for an experimental test of the HQE,
3. a meaningful use of lattice results for hadronic matrix elements,
4. to reduce the sizable \( \mu \)-dependence of the LO,
5. a consistent use of \( \Lambda_{\overline{MS}} \),
6. the large size of QCD corrections, typically of order 30%.
We will now comment on these points: A necessary condition for the validity of any operator product expansion is the disappearance of all infrared regulators from the Wilson coefficients. From our explicit calculation we have verified that this is indeed the case at order $\alpha_s$. We found IR-singularities to cancel via two mechanisms:

- Bloch-Nordsiek cancellations among different cuts of the same diagram,
- factorization of IR-singularities, which end up in $\langle \bar{B}_s|O|B_s \rangle$, $\langle \bar{B}_s|O_s|B_s \rangle$.

Early critics of the HQE had found power-like infrared divergences in individual cuts of diagrams. In response the cancellation of these divergences has been shown [14], long ago before we have performed the full NLO calculation. The second type of IR-cancellations occurs between the diagrams in the first and second row of Fig. 3. Thus when the external meson states in (13) are replaced by quark states, both sides of the equation are infrared divergent. Yet the IR-divergences factorize rendering $F$ and $F_S$ infrared safe. Point 2 above addresses the conceptual basis of the HQE, which
is sometimes termed *quark-hadron duality*. It is not clear, whether the HQE reproduces all QCD effects completely. Exponential terms like \(\exp(-\kappa m_b/\Lambda_{\text{QCD}})\), for example, cannot be reproduced by a power series. The relevance of such corrections to the HQE can at present only be addresses experimentally, by confronting HQE-based predictions with data. The only QCD informations contained in the LO prediction for \(\Delta \Gamma\) are the coefficients of \(\alpha_s^n \ln^n M_W\), associated with hard gluon exchange along the \(W\)-mediated \(b \to c\) amplitude. The question of quark-hadron duality, however, has nothing to do with these logarithmic terms. A meaningful test of this aspect of the HQE therefore requires a NLO calculation, which includes non-logarithmic terms of order \(\alpha_s\). In view of the success of the HQE in accurately measured \(B\) physics observables it is conceivable that the uncertainty due to violations of quark-hadron duality is well below the uncertainty from the non-perturbative calculation of the hadronic \(B\)-parameters. At present lattice calculation of \(B\) and \(B_s\) are only possible in the quenched approximation, neglecting the effect of dynamical fermions. Unquenched calculations of \(f_{B_s}\) are now available, but still a new subject in the field. The third point in our list above refers to the fact that QCD predictions obtained on the lattice must be matched to the continuum. This involves the calculation of the diagrams in the first row on Fig. 3 in lattice perturbation theory. A meaningful prediction for \(\Delta \Gamma\) with a proper cancellation of the renormalization scale and scheme dependences between \(F,F_s\) and \(B,B_s\) then requires a full NLO calculation. The renormalization scale \(\mu\) is an unphysical parameter and observables do not depend on \(\mu\). The truncation of the perturbation series, however, introduces a \(\mu\)-dependence, which diminishes order-by-order in \(\alpha_s\). As mentioned in point 4, the LO result for \(\Delta \Gamma\) suffers from a huge scale dependence, which is substantially reduced in the NLO prediction. Further a LO calculation cannot use the fundamental QCD scale parameter \(\Lambda_{\overline{\text{MS}}}\), which is an intrinsic NLO quantity. Finally, as mentioned in point 6, the calculated QCD corrections are sizable, of the order of 30%, and therefore necessary to keep up with the precision of the forthcoming experiments.

Including corrections of order \(\Lambda_{\text{QCD}}/m_b\) to \(\Delta \Gamma\) we predict

\[
\left(\frac{\Delta \Gamma_{\text{SM}}}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{245 \, \text{MeV}}\right)^2 [0.008 B + 0.204 B_S - 0.086]
\]

with \(B\) and \(B_S\) defined in the \(\overline{\text{MS}}\)-scheme at \(\mu = m_b\). With \(\overline{B} (\mu = m_b) = 0.80 \pm 0.15, \overline{B}_S (\mu = m_b) = 1.19 \pm 0.20\)

one finds

\[
\left(\frac{\Delta \Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{245 \, \text{MeV}}\right)^2 (0.162 \pm 0.041 \pm ???)
\]

The questions marks address the unknown error from the quenching approximation.
If $\Delta \Gamma$ is found below the Standard Model prediction, it will be interesting to find out, whether this is due to a breakdown of the HQE prediction in (16) or a new CP-violating phase $\phi$ in (4). To this end we note that one can determine $\phi$ without using the theory prediction for $\Delta \Gamma_{SM}$, even from untagged data alone [8, 20]. Further the HQE prediction for other width differences, e.g. between the $B^+$ and $B_d$ or between the $B_s$ and $B_d$ mesons, involve a similar structure than the prediction for $\Delta \Gamma$. The corresponding diagrams are similar to those in Fig. 2 and Fig. 3, but involve $\Delta B = 0$ transitions [10, 19]. The width difference between $B^+$ and $B_d$ is insensitive to new physics and therefore directly tests the HQE and the lattice calculations of hadronic matrix elements. The small width difference between $B_s$ and $B_d$ is mildly sensitive to new physics from penguin contributions [21].

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