THE DISCOVERY OF DIFFERENTIAL RADIAL ROTATION IN THE PULSATING SUBDWARF B STAR
KIC 3527751

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ABSTRACT

We analyze 3 yr of nearly continuous Kepler spacecraft short cadence observations of the pulsating subdwarf B (sdB) star KIC 3527751. We detect a total of 251 periodicities, most in the g-mode domain, but some where p-modes occur, confirming that KIC 3527751 is a hybrid pulsator. We apply seismic tools to the periodicities to characterize the properties of KIC 3527751. Techniques to identify modes include asymptotic period spacing relationships, frequency multiplets, and the separation of multiplet splittings. These techniques allow for 189 (75%) of the 251 periods to be associated with pulsation modes. Included in these are three sets of ℓ = 4 multiplets and possibly an ℓ = 9 multiplet. Period spacing sequences indicate ℓ = 1 and 2 overtone spacings of 266.4 ± 0.2 and 153.2 ± 0.2 s, respectively. We also calculate reduced periods, from which we find evidence of trapped pulsations. Such mode trapings can be used to constrain the core/atmosphere transition layers. Interestingly, frequency multiplets in the g-mode region, which sample deep into the star, indicate a rotation period of 42.6 ± 3.4 days while p-mode multiplets, which sample the outer envelope, indicate a rotation period of 15.3 ± 0.7 days. We interpret this as differential rotation in the radial direction with the core rotating more slowly. This is the first example of differential rotation for a sdB star.

Key words: stars: horizontal-branch – stars: individual (KIC 3527751) – stars: oscillations – stars: rotation – subdwarfs

Supporting material: figure set, machine-readable and VO table

1. INTRODUCTION

The Kepler spacecraft was launched in 2009 with a primary goal: to discover extrasolar planets by means of detecting their transits. To this end, the spacecraft has accomplished its mission; at the time of writing, Tenenbaum et al. (2014) suggest there may be 16,285 potential stars with transit or eclipse detections.

Kepler’s applications are broader than just hunting for extrasolar planets. The spacecraft has also been extremely useful for discovering binary stars, which allow us to derive the bulk characteristics of stars, such as mass, radius, distance, and luminosity (Conroy et al. 2014). Kepler has also advanced the field of asteroseismology, or the process of using a star’s vibrations to determine its physical characteristics. Kepler’s Earth-trailing orbit allows it to continuously obtain data, avoiding ground-based limitations, such as daytime gaps, atmospheric transparency variations, and annual visibility cycles. Kepler only ceased observing during monthly data transmissions and a few safing events over the course of its 4 yr program.

Despite asteroseismology being a secondary goal, Kepler has been particularly successful for studying the oscillations of subdwarf B (sdB) stars. sdB stars are extreme horizontal branch stars with temperatures in the range of 20,000–40,000 K. These stars have shed their outer layers near the tip of the red giant branch and have become the exposed cores of horizontal branch stars (for a review of the properties of sdB stars see Heber 2009). The pulsations of sdB stars are divided into two categories based on their periods. Short period pressure (p-)mode pulsators, or V361 Hya stars, have amplitudes typically less than 1% of their mean brightness and period multiplets of just a few minutes. Long period gravity (g)-mode pulsators are classified as V1093 Her stars with typical amplitudes below 0.1% and longer pulsation periods typically near an hour. Some stars exhibit both kinds of pulsation. These hybrids usually exhibit one pulsation type more strongly than the other, with one exception: KIC 9472174 which shows an abundance of both types of pulsation modes (Østensen et al. 2010a). The two classes are also separated in temperature with the V1093 Her stars being cooler than the V361 Hya stars (for a review of sdB pulsation properties, see Østensen 2010). In this paper we use the term sdBV to generically indicate pulsating sdB stars.

Kepler’s nearly continuous observations have been particularly useful for the study of the longer period g-mode pulsators, for which ground-based data have not provided seismic solutions. At the time of writing, 17 papers have been published using Kepler observations of sdBV stars. Previously published results using Kepler data include the discovery of nearly evenly spaced g-mode periods (Reed et al. 2011) and the detection of frequency multiplets (Baran et al. 2012; Østensen et al. 2012, 2014b; Telting et al. 2012; Reed et al. 2014), both of which can be used to identify pulsation modes. Asteroseismic rotation periods have been found to be on the order of tens to 100 days. This includes binary stars which have been shown to be subsynchronous rotators, even with orbital periods as short as half a day (Pablo et al. 2011, 2012; Telting et al. 2012, 2014; Østensen et al. 2014b).

The objective of this or any other asteroseismological study is to characterize the physical properties of the star in question. Model constraints include spectroscopic measurements (T_eff
and log g), as well as asteroseismic properties. Asteroseismic constraints can be as simple as a list of frequencies to compare with models (as in the papers by van Grootel et al. 2010; Charpinet et al. 2011b, using survey-phase Kepler data), or quite detailed, as in small structures in Echelle diagrams. Tools which have been applied to sdBV stars with some success include measuring stellar rotation via frequency multiplets (e.g., Telting et al. 2012); period spacings for mode identification (e.g., Reed et al. 2011); long overtone sequences observed in echelle diagrams (e.g., Reed et al. 2014), as well as small deviations (e.g., Baran & Winans 2012); regions above and below which period spacings do not behave asymptotically; reduced period diagrams for detecting trapped modes (Østensen et al. 2014b); and sliding Fourier Transforms (sFTs) to resolve amplitude variations (Telting et al. 2012; Reed et al. 2014).

The target of this paper, KIC 3527751, was examined in a preliminary study of hybrid sdB pulsators by Reed et al. (2010). They detected 41 g- and 3 p-mode periodicities from one month of Kepler survey data. 34 of the g-modes were identified as ℓ = 1 or 2 using period spacings. Subsequent to Kepler’s survey phase, KIC 3527751 was continuously observed with one minute cadence from quarter 5 (Q5) until mission end during Q17. Here we analyze all available data (1148 days), providing our best estimate of the frequency content to which we apply our asteroseismological tools, as discussed above.

2. DATA ANALYSIS

Kepler obtains data in two modes: short cadence (SC), which produces one integration every 58.85 s, and long cadence (LC), which produces an integration every 30 minutes. Short cadence observations consume more of Kepler’s limited memory, so the targets observed in this mode are far fewer. We downloaded optimally extracted lightcurves from the Mikulski Archive for Space Telescopes®, removed long-term trends (>1.5 days) with low-order spline fitting, and normalized the data by mean brightness. We sigma-clipped the data at 5σ and multiplied the modulation intensities so amplitudes would appear in the Fourier transform (FT) as parts-per-thousand (ppt).

The data span almost 1148 days and include 1.55 million data points, which is a temporal resolution of 0.010 μHz. These data have a duty cycle of 92.4%, with the largest gaps caused by spacecraft safing events during Q8, Q14, and Q16. As noted by Østensen et al. (2014a), a 4σ detection limit would contain ~50 spurious peaks, so we increased our detection limit to 5σ, to be reasonably confident that detections are real signal. We determined the mean level in the FT to be σFT = 0.032 ppt, making our 5σ limit 0.16 ppt. It should be noted that a 5σ detection limit only applies when examining the entire FT. When looking for predicted periodicities in smaller regions (of, say, a few 100 μHz), a 4σ detection limit is perfectly justified. Significant peaks range from 72 to 3704 μHz, though they are concentrated between 100 and 300 μHz. Peaks occurring below 20 μHz are attributed to residual signal from the spacecraft which was not removed by our detrending. An FT of the data is shown in Figure 1 with the detection limit indicated by a horizontal line.

As discussed in Reed et al. (2014), the previously utilized method of fitting the original lightcurve with a nonlinear least-squares program and prewhitening the resulting FT would have been not only tedious, but rather counterproductive as exceptionally few peaks could be cleanly removed. We attribute this property to the long-term instability of amplitudes and even, occasionally, of frequencies. In order to evaluate the pulsation content, we used the two tools described in Reed et al. (2014): sFTs to evaluate the pulsation content in the time domain and Lorentzian peak fitting which serves as a method for determining peak widths as an indicator of frequency errors. Frequency multiplets are readily apparent in the FT (see Figure 3) of the complete data set and were used as a guide for the sFTs. sFTs were generated using data spanning 220 days, to fully resolve frequency multiplets, and stepped by 5 days through the entire data set. Sample sFTs are shown in Figure 2 with sFTs of the entire frequency spectrum available on-line. The sFTs were used as guides for the Lorentzian fitting. If amplitudes were only detectable during a portion of the data, Lorentzian fits were obtained from only those portions (using data spanning a minimum of 200 days to ensure frequencies are resolved). In total, we fitted 251 frequencies, which were compared with known spacecraft artifacts to ensure none are in our list (except f159 as noted in Section 3.1). Table 1 provides the pulsation frequencies, Lorentzian widths, and amplitudes resulting from our FT fitting.

2.1. Combination Frequencies

A search for combination frequencies and a likelihood comparison was completed as in Telting et al. (2012). Residuals were calculated using the form Δf = f1 - f2 - f3 for all combinations of frequencies and those with Δf = 0 within the errors were reported for examination. Eight combinations were discovered having Δf < 0.0001 μHz. Of these, three only involve low-amplitude frequencies which are unlikely to produce combination modes, while five have at least one higher-amplitude frequency. We consider it likely that these five actually are combination frequencies and they are listed in Table 2 and marked in Table 1 with asterisks.

2.2. Spectroscopy

Over the 2010 and 2011 observing seasons of the Kepler field, we obtained a total of 24 spectra of KIC 3527751. Low-resolution spectra (R ~ 2000–2500) have been collected using the Kitt Peak 4 m Mayall telescope with RC-Spec/F3KB, the kpc-22 b grating and a 1.5–2.0 arcsec slit, the 2.56 m Nordic Optical Telescope with ALFOSC, grism #16 and a 0.5 arcsec slit, and the 4.2 m William Herschel Telescope with ISIS, the R600B grating and 0.8–1.0 arcsec slit. Exposure times were 600 s at KP4m and WHT, and either 500 or 300 s at the NOT. The resulting resolutions based on the width of arc lines are 1.7 Å for the KP4m and WHT setups, and 2.2 Å for the setup at the NOT. See Table 3 for an observing log.

The data were homogeneously reduced and analyzed. Standard reduction steps within IRAF include bias subtraction, removal of pixel-to-pixel sensitivity variations, optimal spectral extraction, and wavelength calibration based on arc-lamp spectra. To fully account for the blue CCD etching pattern in the NOT spectra, spectroscopic flats were constructed by interpolating between UBV imaging flats along the dispersion direction, as halogen flats suffered from stray light in the blue
part of the spectrum. The target spectra and the mid-exposure times were shifted to the barycentric frame of the solar system. The spectra were normalized to place the continuum at unity by comparing with a model spectrum for a star with similar physical parameters as we find for the target (described below).

Radial velocities (RVs) were derived with the FXCOR package in IRAF. We used the Hγ, Hδ, Hζ and Hη lines to determine the RVs, and used the spectral model fit (described below) as a template. See Table 3 for the results, with errors in the RVs as reported by FXCOR. The errors reported by FXCOR are correct relative to each other, but may need scaling depending on, among other things, the parameter settings and the validity of the template as a model of the star.

We find that the RV data have more scatter than one would assume for a non-variable star, but that the RV data are not sufficient to constrain any orbital parameters. Excluding the datapoint from the worst signal-to-noise ratio (S/N) spectrum, the average RV = −13.0 ± 3.2 km s^{-1}, with an rms scatter of 15.1 km s^{-1}, which is much larger than the errors in the data that range from 3 to 13 km s^{-1}, with median individual error of 7.8 km s^{-1}. An FT gives maximum amplitude of 22 km s^{-1} in the frequency range of 0–17.4 μHz, and lower amplitudes at higher frequencies.

As in our previous papers (e.g., Østensen et al. 2010b), we have fitted the average spectrum from each observatory to model grids, in order to determine effective temperature (\(T_{\text{eff}}\)), surface gravity (\(\log g\)), and photospheric helium abundance (\(\log y = \log N_{\text{He}}/N_{\text{H}}\)). The fitting procedure used was the same as that of Edelmann et al. (2003), using the metal-line blanketed LTE models of solar composition described in

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**Figure 1.** Fourier transform of KIC 3527751. The solid (blue) horizontal line is the 5σ detection limit and the dashed (magenta) vertical lines indicate known frequencies where spacecraft artifacts may occur.

**Figure 2.** Examples of sliding Fourier transforms (sFTs). A complete set is available in the appendix.

(The complete figure set (12 images) is available.)
Table 1: Table of Astroseismic Quantities

| ID  | Frequency ($\mu$Hz) | Period (s) | Amp. (ppt) | $\ell$ | $m$ | $n_f = 1$ | $n_f = 2$ | $\frac{\Delta \ell}{\ell}$ for $n_f = 1$ | $\frac{\Delta \ell}{\ell}$ for $n_f = 2$ | $\delta f$ ($\mu$Hz) |
|-----|---------------------|------------|------------|-------|-----|----------|----------|--------------------------------|--------------------------------|-----------------|
| f010 | 91.703 (0.008)    | 10904.73 (0.91) | 1.64       | 2     | -1 | ...       | ...       | ...                          | 0.35                          | 0.231        |
| f011 | 91.934 (0.007)    | 10877.33 (0.85) | 1.24       | 2     | 0  | ...       | ...       | 0.18                          | 0.00                          | 0.2266       |
| f012a| 92.161 (0.010)    | 10850.59 (1.21) | 1.89       | 2     | 1  | 38        | 68        | 0.07                          | 0.00                          | ...           |
| f013 | 94.590 (0.032)    | 10571.94 (3.56) | 0.38       | 1     | ... | 37        | 68        | 0.02                          | 0.00                          | ...           |
| f014 | 99.289 (0.004)    | 10071.59 (0.41) | 0.22       | 2     | 0  | ...       | 63        | 0.09                          | 0.00                          | 0.1221       |
| f015b| 99.411 (0.005)    | 10059.22 (0.47) | 0.19       | 1     | 1  | 35        | 63        | 0.09                          | 0.16                          | 0.1141       |

Notes. Column 1 provides a label for the periodicity, Columns 2 and 3 the frequency and period, and errors in parentheses, and Column 4 the observed amplitude taken from the Lorentzian fit of the entire data set. Columns 5 through 8 provide our best estimate mode identifications, Columns 9 and 10 period spacing deviations, and Column 11 the frequency splitting (from the subsequent frequency) of multiplet members.

* Indicates periodicities detected by Reed et al. (2010).

* Indicates frequencies which are listed in Table 2 as part of a combination frequency.

(This table is available in its entirety in machine-readable form.)

Table 2: Possible Combination Frequencies (in $\mu$Hz) for $f_3 - f_2 - f_1 = \delta f$ with $\delta f < 0.0001$

| $f_1$   | $f_2$   | $f_3$   | $A_1$ | $A_2$ | $A_3$ |
|---------|---------|---------|-------|-------|-------|
| 76.174  | 135.710 | 211.884 | 0.16  | 0.35  | 3.30  |
| 99.411  | 118.866 | 218.277 | 0.19  | 1.31  | 1.14  |
| 105.632 | 193.670 | 299.302 | 2.92  | 0.22  | 0.42  |
| 158.845 | 774.149 | 932.994 | 1.51  | 0.22  | 0.62  |
| 253.194 | 312.699 | 565.893 | 1.21  | 0.13  | 0.21  |

Note. Amplitudes (in ppt) are provided in Columns 4–6.

Heber et al. (1999). Mean values from these three fits were computed using the formal fitting errors as weights and systematics between observatories were then factored into the errors. The resultant measurements are indicated in Table 4 and we adopt the values of $T_{\text{eff}} = 27818 \pm 163$, $\log g = 5.35 \pm 0.03$, and $\log (N_{\text{He}}/N_{\text{H}}) = -2.99 \pm 0.04$.

3. MODE IDENTIFICATION

Prior to Kepler, observational mode identifications were extremely rare for sdB pulsators, leaving period matching between models and observations (the forward method) as virtually the only means of correlating periodicities to modes for sdB stars. However, using Kepler extended data sets, purely observational mode identifications using frequency multiplets and period spacings have become well-established techniques. We follow the examples of Baran et al. (2012) and Reed et al. (2011, 2014) in applying these methods to identify modes in KIC 3527751. To first order, stellar rotation removes azimuthal frequency degeneracy resulting in frequency multiplets appearing with $2\ell + 1$ members, with each member shifted by

$$\Delta \nu = \Delta m \Omega (1 - C_{n,\ell})$$  \hspace{1cm} (1)

from the $m = 0$ value (Ledoux 1951). $\Omega$ is the frequency of stellar rotation, and $C_{n,\ell}$ is the Ledoux constant which is nearly zero for $p$-modes and for $g$-modes depends on the mode degree as

$$C_{n,\ell} \approx \frac{1}{\ell (\ell + 1)}.$$  \hspace{1cm} (2)

Similarly, $g$-modes may show radial overtones spaced evenly in period as

$$\Delta \Pi_\ell = \frac{\Pi_\ell}{\sqrt{\ell (\ell + 1)}}$$  \hspace{1cm} (3)

where $\Delta \Pi_\ell = \Pi_{\ell,n+1} - \Pi_{\ell,n}$ and $\Pi_\ell$ is the radial fundamental period. Even period spacings imply idealized homogeneous stars using asymptotic relations ($n \gg \ell$). In practice, even period spacings have not been observed at short ($n$ is not much greater than $\ell$) or long ($\ell$) spacings where frequency multiplets become spaced similar to period spacings) periods (Baran & Winans 2012; Østensen et al. 2014; Reed et al. 2014).

3.1. Frequency Multiplets

$g$-mode multiplets. Uncovering multiplets in KIC 3527751 is extremely useful in associating periodicities with pulsation modes, $\ell$ and $m$. A cursory inspection of the most visible multiplets in the $g$-mode region reveals a common splitting near 0.23 $\mu$Hz. Many of these multiplets have four or five peaks, meaning these are most likely $\ell = 2$ modes. (A discussion of how inclination affects the number of observed multiplet members is in Section 4.4.) 22 multiplets with similar splittings were found and designated as $\ell = 2$ modes. In cases where the $m = 0$ component was ambiguous, we arbitrarily assigned it to the appropriate highest amplitude member. The average frequency splitting of the most easily distinguished $\ell = 2$ multiplets was 0.23 $\pm 0.03$ $\mu$Hz. Triplets which displayed similar splittings were likewise designated as $\ell = 2$.

Though the overwhelming majority of multiplets were designated $\ell = 2$, some had splittings smaller than 0.23 $\mu$Hz, often closer to 0.13 $\mu$Hz. This value is the splitting we expect to see from Equations (1) and (2) for $\ell = 1$ multiplets. Nine multiplets had splittings near this value and were designated as $\ell = 1$. We find the average splitting of $\ell = 1$ multiplets to be 0.12 $\pm 0.03$ $\mu$Hz. Altogether, we associated 123 frequencies with $\ell = 1$ and 2 modes using the frequency multiplet splitting method.

One multiplet present in KIC 3527751 ($f158$–$f162$) at first glance appears to be a quintuplet, but the spacing between members is about double that expected for $\ell = 2$ multiplets. Additionally, there are two multiplets ($f164$–$f169$ and $f190$–$f197$) which contain too many members (six and eight) to be...
\( \ell = 2 \) modes and the splitting is slightly wider, consistent with the smaller Ledoux constant expected for \( \ell = 4 \) g-modes. In total, 19 frequencies are associated with these three multiplets, with an average \( \Delta m = 1 \) splitting of 0.27 ± 0.04 \( \mu \)Hz. We interpret these three multiplets to be \( \ell = 4 \) modes. Sample multiplets with their Lorentzian fits are shown in Figure 3.

We note that \( f_{159} \) is at the known I/LC spacecraft artifact. Typically the I/LC is not observed, but rather 8–10/LC more commonly are, yet this gave us pause. We examined the nearest star with SC data (KIC 3323887), processing those data in the same way as for KIC 3527751, to see if the I/LC artifact appeared. There was no signal at I/LC in KIC 3323887. We also compared the sFT for \( f_{159} \) with other LC artifacts at 4531 (8/LC) and 5098 \( \mu \)Hz (9/LC), which show a harmonic frequency shift, since intrinsic frequencies become Doppler shifted due to the spacecraft’s orbit around the Sun after the heliocentric correction is applied. The sFT for \( f_{159} \) does not appear similar to those for 8/LC or 9/LC and so we suggest that \( f_{159} \) is intrinsic to KIC 3527751.

Another interesting multiplet includes \( f_{213}–f_{225} \) which is shown in Figure 4. Eight frequencies are spaced by 0.40–0.50 \( \mu \)Hz or 19 frequencies are spaced by 0.19–0.25 \( \mu \)Hz, missing members corresponding to \( m = \pm 3, 5, \) and 7 (with the \( m = -1 \) frequency listed as tentative in Table 1 since it is just below 5\( \sigma \)). If we assume the smaller spacing, then the average \( \Delta m = 1 \) frequency splitting would be 0.22 ± 0.02 \( \mu \)Hz and the mode would be \( \ell \geq 9 \). However, the splittings would be slightly smaller than the \( \ell = 2 \) ones, which is contrary to Equation (2). On the other hand, if we assume the larger frequency splitting (with the outside pair just being chance alignments), then it would be an \( \ell \geq 4 \) multiplet, but with spacings of 0.45 ± 0.03 \( \mu \)Hz which would be 67\% larger than the other \( \ell = 4 \) multiplets. We tentatively assign it to the smaller splittings as an \( \ell = 9 \) multiplet in Table 1 with the caveat that low and variable amplitude signals are difficult to extract from the FT and may be imprecise.

\( p \)-mode multiplets. Looking beyond about 6 minutes, where we expect \( p \)-mode pulsations to occur, there are 10 frequencies with amplitudes above the 5\( \sigma \) limit, not including known artifacts. These fall into sets near 2780 and 3700 \( \mu \)Hz. Additionally, the set near 2780 \( \mu \)Hz has four frequencies with amplitudes above the 4\( \sigma \) limit which perfectly fit into two sets of frequency multiplets (\( f_{239}–f_{241} \) and \( f_{244}–f_{245} \)).

These 14 frequencies bear some resemblance to Balloon 090100001 (Baran et al. 2009). The highest-amplitude frequency (\( f_{238} \)) occurs roughly where the radial fundamental is typically found in such stars, followed by a much lower amplitude triplet (\( f_{239}–f_{241} \)) and quintuplet (\( f_{242}–f_{246} \)), in order of ascending frequency. These multiplets have consistent frequency splittings of 0.7–0.8 \( \mu \)Hz. Beyond this group is another group near 3700 \( \mu \)Hz with inconsistent frequency splittings, also similar to what is observed for Balloon 090100001. The ratio of the \( f_{238}–f_{246} \) group near 2800 \( \mu \)Hz to the \( f_{247}–f_{251} \) group near 3700 \( \mu \)Hz is 0.76, which is similar to, but slightly larger than, the value of 0.7 calculated for Balloon 090100001 (Baran et al. 2009).

### Table 3

| Mid-exposure date | Barycentric JD | S/N | \( RV \) (km s\(^{-1}\)) | \( RV_{\text{err}} \) (km s\(^{-1}\)) | Telescope | Observer |
|-------------------|---------------|-----|-------------------------|-------------------------|-----------|----------|
| 2010 Jul 28 01:32:16 | 405.5669179 | 107.4 | 6.9 | 3.1 | WHT | RHO |
| 2010 Jul 28 05:02:45 | 405.7130811 | 82.7 | 6.0 | 3.1 | WHT | RHO |
| 2010 Aug 13 08:25:22 | 421.8535618 | 69.8 | 1.0 | 6.1 | KPNO | MDR/LHF |
| 2010 Aug 13 08:36:39 | 421.8613996 | 65.3 | 1.0 | 6.1 | KPNO | MDR/LHF |
| 2010 Aug 14 07:11:56 | 422.8025448 | 57.4 | 5.0 | 4.1 | KPNO | MDR/LHF |
| 2010 Aug 14 07:22:55 | 422.8101739 | 63.8 | 1.0 | 6.1 | KPNO | MDR/LHF |
| 2010 Aug 14 07:33:15 | 422.8173514 | 65.7 | 1.0 | 6.1 | KPNO | MDR/LHF |
| 2011 Jun 01 02:47:19 | 713.6181963 | 71.7 | 1.0 | 6.1 | NOT | JHT |
| 2011 Jun 07 04:11:17 | 719.6767124 | 47.4 | 5.0 | 6.3 | NOT | JHT |
| 2011 Jun 09 05:15:40 | 721.7214831 | 42.8 | 5.0 | 6.3 | NOT | JHT |
| 2011 Jun 20 00:33:42 | 732.5259579 | 51.9 | 1.0 | 6.1 | NOT | JHT |
| 2011 Jun 20 02:43:12 | 732.6158815 | 48.6 | 5.0 | 6.1 | NOT | JHT |
| 2011 Jun 20 05:22:41 | 732.7266388 | 36.2 | 5.0 | 5.7 | NOT | JHT |
| 2011 Jun 27 22:53:21 | 740.4564180 | 42.8 | 5.0 | 8.5 | NOT | JHT |
| 2011 Jul 23 00:58:51 | 765.5437440 | 43.6 | 5.0 | 9.8 | NOT | JHT |
| 2011 Jul 23 01:38:08 | 765.7710230 | 38.5 | 5.0 | 9.8 | NOT | JHT |
| 2011 Jul 23 02:18:10 | 765.7988233 | 38.3 | 5.0 | 9.8 | NOT | JHT |
| 2011 Jul 23 03:11:48 | 765.6360675 | 47.9 | 5.0 | 9.8 | NOT | JHT |
| 2011 Jul 23 04:05:02 | 765.6730338 | 43.2 | 5.0 | 9.8 | NOT | JHT |
| 2011 Jul 23 05:00:47 | 765.7117481 | 42.9 | 5.0 | 9.8 | NOT | JHT |
| 2011 Jul 24 04:51:35 | 766.7053592 | 47.8 | 5.0 | 9.7 | NOT | JHT |
| 2011 Aug 29 00:08:25 | 802.5080643 | 31.8 | 5.0 | 9.4 | NOT | JHT |
| 2011 Aug 30 00:30:11 | 803.5231446 | 42.1 | 5.0 | 9.4 | NOT | JHT |
| 2011 Aug 30 20:53:13 | 804.3724475 | 40.0 | 5.0 | 9.9 | NOT | JHT |

### Table 4

| Telescope | \( T_{\text{eff}} \) (K) | \( \log g \) (dex) | \( \log (N_{\text{He}}/N_{\text{H}}) \) (dex) |
|-----------|-------------------------|-----------------|------------------|
| KPNO      | 27945 ± 133             | 5.35 ± 0.02     | −2.97 ± 0.03     |
| NOT1      | 27715 ± 102             | 5.32 ± 0.02     | −3.01 ± 0.03     |
| NOT2      | 27709 ± 102             | 5.33 ± 0.02     | −3.02 ± 0.02     |
| WHT       | 27913 ± 82              | 5.38 ± 0.01     | −2.95 ± 0.03     |
| Adopted   | 27818 ± 163             | 5.35 ± 0.03     | −2.99 ± 0.04     |
Based on the consistent frequency splittings and pattern similarities with Balloon 090100001, we assign the first nine \( p \)-mode frequencies as an \( \ell = 0 \) singlet, an \( \ell = 1 \) triplet and an \( \ell = 2 \) quintuplet. The remaining five frequencies are most likely the next overtone sequence, but we cannot assign specific modes to frequencies as their splittings are inconsistent.

### 3.2. Period Spacings

To begin the hunt for period spacing sequences in KIC 3527751, we applied a Kolmogorov–Smirnov (KS) test, using the periods listed in Table 1. However, all 251 periods do not produce any distinguishing peaks. Since geometric cancellation increases with increasing \( \ell \) (Pesnell 1985), we did a KS test on just the 100 highest-amplitude periods. This produced a substantial peak at 265 s with a lesser one at 153 s, right where expected from Equation (3) \( 265/\sqrt{3} = 153 \) s. Both KS tests are shown in the top panels of Figure 5.

Next we produced echelle diagrams using the spacings indicated by the KS test. These are shown in Figure 6 with different symbols indicating mode identifications. Even with these tools, we imposed additional conditions on our search for period sequences. We began by examining (i) the high-amplitude periodicities under the assumption that they are \( \ell = 1 \) (as indicated by the strong peak in the KS test) and (ii) using the multiplets (both \( \ell = 1 \) and 2) as starting points for sequences. We could easily identify a sequence for both \( \ell = 1 \) and 2 modes. We detected 35 \( \ell = 1 \) periodicities and 95 \( \ell = 2 \) periodicities that follow sequences within a small margin. We assigned radial indices \( n \) to the members of these sequences, estimating that the radial fundamental mode would appear near 600 s, a rough estimate for stars with \( \log g \approx 5.2 \). This resulted in period spacings of \( 266.4 \pm 0.2 \) and \( 153.2 \pm 0.2 \) s for \( \ell = 1 \) and 2, respectively, in good agreement with the survey results of Reed et al. (2010).

Our best-estimate mode identifications, using multiplets and period spacings, are provided in Columns 5 through 8 of Table 1.

### 4. DISCUSSION

Now that mode identifications have been established, they can be examined using the tools applied in our previous papers.

#### 4.1. Unidentified Periodicities

Ideally, every periodicity detected in KIC 3527751 would be associated with a mode or a combination frequency. With some degree of certainty, we have identified three \( \ell = 4 \) multiplets, and possibly even an \( \ell = 9 \) multiplet, which invites a search for more high-degree periodicities. To do this, we prewhiten the KS test. We remove our assigned \( \ell = 1 \) modes, \( \ell = 2 \) modes, and then both. Of the 79 periodicities longer than 800 s not identified as \( \ell = 1 \) or 2, 19 are identified as \( \ell = 4 \) and 13 as part of a possible \( \ell = 9 \) multiplet, leaving just 34 of 225 g-mode periodicities \( f < 1100 \mu \)Hz as unidentified. The bottom panels of Figure 5 show the period prewhitening sequences. When the \( \ell = 2 \) sequence is removed, a second peak appears shortward of the \( \ell = 1 \) peak, which could indicate deviations or multiplet structure in that sequence. However, when both \( \ell = 1 \) and 2 are removed, some of that peak remains, indicating that we may not be catching all of the \( \ell = 1 \) sequence. However, no new peaks appear where we would expect the higher-degree sequences to be (indicated by vertical lines near 109 and 84 s for the \( \ell = 3 \) and 4 sequences, respectively). The three \( \ell = 4 \) multiplets we detect have spacings between them of 257 \( (3 \times 86) \) and 223 \( (3 \times 74) \) s which are close to 3x the expected value. Yet we have to conclude that the KS plots do not reveal obvious peaks where \( \ell = 3 \) or 4 modes should be. Since many of the unidentified periods are short (53 of these have periods below 1800 s), they could well be \( \ell = 1 \) and 2 modes which do not fit the period sequences because they are low radial \( n \) order.

#### 4.2. Trapped Modes

Structural models have indicated significant mode trapping in the He–H transition zone of sdB stars (Charpinet et al. 2002; Hu et al. 2009; Miller Bertolami et al. 2012). Period spacing sequences argued against significant mode trapping (see the discussion in Section 3 of Reed et al. 2011) until the recent discovery of trapped modes in the sdBV star KIC 10553698 A (Østensen et al. 2014b). Thus it is prudent to search for trapped modes in each sdBV star analyzed. Unfortunately, not finding trapped modes is like not finding a binary star—one can have positive detections, but it is nearly impossible to conclusively rule them out. For KIC 3527751, there are four periodicities \( (f023, f024, f084, \text{and} f085) \) with \( \ell = 1 \)-like multiplet splittings which do not fit into the period sequence. These can be seen in...
the echelle diagram (Figure 6), but are most easily seen in a reduced period plot (Figure 7), such as that in Østensen et al. (2014b). Reduced period plots are also commonly shown in modeling papers, such as those previously referenced. Periods are converted to reduced periods by multiplying by $\ell(\ell + 1)$, which also makes the period spacings degree independent. The trapped $\ell = 1$ modes deviate shortward from the main spacing near 380 s.

However, there are no corresponding trapped $\ell = 2$ modes. In a regular period, the positions where the trapped $\ell = 2$ modes would occur are near 3270 and 5430 s. There are no available periodicities near 3270 s (306 $\mu$Hz) even when...
looking below the $4\sigma$ limit. So there are no candidates for a trapped $\ell = 2$ mode corresponding to this trapped $\ell = 1$ mode. For the second trapped mode, there is a single peak at 5457 s that we assign as a trapped $\ell = 2$ mode corresponding to the trapped $\ell = 1$ mode near a reduced period of 13,000, but mark it in Figure 7 with an open symbol.

Since the sequences were not complete, we also looked for periodicities of missing overtones, to complete the sequences.

Figure 6. Echelle diagrams for $\ell = 1$ (right) and 2 (left) modes. Black circles indicate our identified $\ell = 1$ modes, blue triangles $\ell = 2$ modes, green squares $\ell = 4$ and 9 modes, and red stars indicate periods which are not associated with a mode. Periods which could be either $\ell = 1$ or 2 are indicated with an extra colored circle (black in the left panel and blue in the right panel). Point sizes are logarithmically scaled with amplitude and vertical lines indicate the extent of frequency multiplet splittings.

Figure 7. Reduced periods, $\Pi = P_{\ell} / (\ell + 1)$, indicating trapped modes. Circles (blue) indicate the $\ell = 1$ sequence and squares (red) indicate the $\ell = 2$ sequence. Open symbols indicate periodicities discovered as the result of a search for missing overtones and trapped $\ell = 2$ modes, as discussed in the text, and crosses indicate modes with the same period as that of a different mode (e.g., an $\ell = 1$ mode previously identified as $\ell = 2$). Dashed lines indicate missing overtones and dotted lines in the $\ell = 2$ sequence indicate that a trapped mode is missing.
A few more possible periods were detected with amplitudes between 4 and 5σ, and are marked with open symbols, and we also detected some which overlap the other sequence. Equation (3) predicts that sequences overlap, so mode degeneracy is expected. We have marked those periods with crosses and note that these mode identifications are not favored because of multiplet splittings.

The $\ell = 2$ sequence extends well beyond the $\ell = 1$ sequence and there are some indications of trapped modes at either end. Of course at the short end, the radial orders are small, where asymptotic behavior is not expected anyway and at the longer end, there are many missing radial orders, so a lot of ambiguity exists. We include them for completeness.

This would be the second sdBV star to show strong indications of trapped modes, contrary to what was indicated in Reed et al. (2011).

### 4.3. Radially Differential Rotation

Previous papers (e.g., Baran et al. 2012) have used multiplets to determine stellar rotation periods, and since many multiplets were detected in KIC 3527751, we do it here too. We separate the pulsations into $p$- and $g$-modes and further separate the $g$-modes by degree, $\ell$. We do this because the splitting in $g$-modes is degree-dependent whereas $p$-modes have small Ledoux constants (e.g., van Grootel et al. 2008).

For the $g$-modes, we calculated the stellar rotation period to be 46 ± 8, 41 ± 5, and 40 ± 5 days for the $\ell = 1, 2,$ and $4$ multiplets, respectively. These are all in close agreement. For the $p$-modes, using a Ledoux constant of zero, we calculated a stellar rotation period of 15.3 ± 0.7 days. We note that while several members of the two $p$-mode multiplets have low amplitudes, the frequency splittings are remarkably uniform with comparatively large splittings of 0.74 μHz. These are shown in the bottom two panels of Figure 8 with two $g$-mode multiplets in the top panels.

Here we arrive at a contradiction: the two $p$-mode multiplets indicate a spin period of 15.3 ± 0.7 days while the $g$-mode multiplets indicate a period near 43 days. We can imagine three possibilities which would generate these results: (i) the intrinsic $p$-mode multiplets are actually spaced at $1/3$ our observed value, (ii) KIC 3527751 is a pair of sdBV stars with one a $p$-mode pulsator rotating with a period near 15 days and the other a $g$-mode pulsator rotating with a period near 43 days, or (iii) KIC 3527751 is differentially rotating. We discuss all three possibilities below, but so as not to lose the reader in too much suspense, we discount options (i) and (ii) and highly favor option (iii).

For option (i) to be correct, the $f_{239}$–$f_{241}$ triplet would need to be $\ell, m = 3, -3, 3, 0$; and 3, +3 and the $f_{242}$–$f_{246}$ quintuplet would need to be $\ell, m = 6, -6; 6, -3; 6, 0; 6, +3; and 6, +6.$ This seems quite unlikely since both of these have quite high geometric cancellation factors (Pesnell 1985). In addition, there is no pulsation inclination where the $m = 1, 2, 4,$ and 5 modes have a common node. Yet we cannot summarily rule this out as we seem to detect $\ell = 4$ and possibly 9 modes in the $g$-mode regime. However, in the $g$-mode cases, there are many more higher-amplitude periodicities whereas in the $p$-mode range there would only be one. As such, we would have to question why the $\ell = 0, 1,$ and 2 modes are not excited to observable levels when their cancellation factors are so much smaller. Additionally, the frequency/amplitude pattern is similar to what has been observed in the sdBV star Balloon 090100001 (as discussed in Section 3.1), making option (i) unlikely.

Option (ii) seems unlikely from the outset as an sdB+sdB binary has never been observed, and only about 80% of cool sdB stars pulsate with $g$-modes and 10% of hotter sdB stars pulsate with $p$-modes. We could rule out this possibility immediately if members of $p$- and $g$-mode pulsations were in combination frequencies. Unfortunately, that is not the case as our possible combination frequencies are $g$-mode (or mixed character) periodicities only (Table 2). Several other sdB stars have been discovered to be binary via their Kepler lightcurves which show signs of Doppler beaming, tidal distortions, and/or eclipses (e.g., Bloemen et al. 2012; Telting et al. 2012). These effects should only occur for relatively short-period binaries, and so we reprocessed KIC 3527751’s data prior to long-term trend removal and this time detrended, month-by-month, only for periods longer than eight or ten days. This detrending leaves three, closely spaced, low-amplitude peaks in the FT at 3.933, 3.962, and 4.026 μHz (2.94, 2.92, and 2.87 days, respectively) with amplitudes of 0.051, 0.042, and 0.056 ppt, respectively. Already, with three peaks, these are highly unlikely to be signatures of binarity, but are most likely spurious noise which was not properly removed. But to complete this exercise, using the Doppler beaming procedure of Telting et al. (2012, specifically Equation (2)), we calculate a velocity amplitude of the binary’s orbital motion of 23.6 km s$^{-1}$ from the flux amplitudes. From our 23 RV measurements, we determine an RV amplitude of 16.5 ± 3.2 km s$^{-1}$ for this period. These velocities are not consistent with each other, and assuming sdB stars of canonical mass (0.48 $M_\odot$) would require inclinations below 15°. This combination of evidence makes it extremely unlikely that KIC 3527751 is a binary pair of pulsating sdB stars.

Therefore it is far more likely that KIC 3527751 is differentially rotating (option (iii)) and we consider this to be the case. $p$-mode pulsations only sample the envelope whereas $g$-mode pulsations penetrate the He core and so are sensitive to conditions deeper within the star (see example propagation diagrams in Charpinet et al. 2014). As such, the $g$-mode-derived rotation rate is sensing deep into the star and the $p$-mode rate is sensing the outer regions. There is overlap in the propagation diagram of Charpinet et al. (2014), so the core rate is likely slower than indicated by the $g$-modes, but these results conclude that the core is rotating at least three times slower than the envelope. To determine a more precise value would require appropriate models constructed using the observational constraints (mode IDs, period spacing/trapping sequences, etc) of this paper to calculate the proper propagation diagram. To date, no detailed models have been published which fully incorporate the detailed seismic constraints provided from Kepler data. Interestingly, a similar discovery has been detected in Kepler observations of a pulsating main sequence A star (Kurtz et al. 2014), although in that case, the core is only rotating about 5% slower than the envelope. Radially differential rotation has also been measured in red horizontal branch stars, though in that case the core is rotating faster than the envelope (Beck et al. 2012).

### 4.4. Pulsation Inclination Angle

As shown in Pesnell (1985), Reed et al. (2005), and Charpinet et al. (2011a), geometric cancellation is inclination- and $m$-dependent, which means we can determine the
Inclination angle of the pulsation axis to our line of sight using multiplet amplitudes. For observations which span several amplitude $e$-folding timescales, it is reasonable to presume that all amplitudes of all multiplet members have reached average amplitudes and then their relative heights indicate the inclination angle. However, as Figure 2 indicates, for KIC 3527751 it does not appear that the data span more than a single amplitude variation cycle, and so this assumption is likely not valid. Yet we can place constraints using surface nodes, where geometric cancellation is complete, to omit inclination ranges. The $\ell = 1$ multiplet amplitudes, while inconsistent between multiplets, have roughly equal amplitudes for all members, indicative of intermediate inclinations ($30^\circ$–$60^\circ$). Several of the $\ell = 2$ multiplets are quadruplets, a few are full quintuplets, and some are only triplets. Since all members are present in various multiplets, this indicates inclinations above $15^\circ$ (which would not have $m \neq 0$ components) and below $75^\circ$ (which would suppress $m = 2$ components). Additionally, there is an $m = 0$ node line at $i = 54^\circ$ which eliminates inclinations between $50^\circ$ and $65^\circ$. The $\ell = 4$ multiplets are more sensitive to the inclination angle, yet are inconsistent between multiplets. The $566 \mu$Hz multiplet has only even $m$ members while the other two have roughly equal amplitudes for all members. As such, we have to presume that no members are suppressed. Since $\ell = 4$ node lines appear at $i = 0^\circ$, $32^\circ$, $48^\circ$, $67^\circ$, $70^\circ$, and $90^\circ$, inclinations within a few degrees would be suppressed. What is left are inclinations between $35^\circ$ and $45^\circ$, based solely on multiplet members being visible.

4.5. Pulsation Density

We can also examine where the pulsations seem to be most easily driven. Near instability boundaries, it would be expected that fewer periodicities would be driven and so mapping pulsation density with overtone may provide constraints on the driving region itself (as in Figure 9 of Jeffery & Saio 2006). We use multiplet members as a proxy of driving power; the more multiplet members observed, the more driving power we presume that mode to have. There are methods for doing this involving pulsation amplitudes (e.g., Mukadam et al. 2006), however considering the variability observed over the duration of the observations, this seems a difficult proposition—Should we calculate the total power when the amplitudes are maximum or median? Should we consider multiple times per multiplet when each member is maximum, or choose only one time when most members are visible? The scheme we decided to use, number of multiplet members, is simpler in two ways: it is binary—either multiplet members are detected or they are not, and it imposes no time or amplitude constraints. The members do not have to be simultaneously detected. Once structure and pulsation models are sufficiently mature to accurately reflect the observed pulsation behavior, our scheme may prove not to be the best. However, with the information currently in hand, it
seems more robust than using amplitudes and a good beginning point for determining pulsation instability.

Figure 9 shows multiplet members by radial order with the densest region between \( n = 20 \) and 40 for \( \ell = 1 \) and 2 while the \( \ell = 4 \) multiplets have \( 15 \leq n \leq 20 \). It is interesting that the \( \ell = 4 \) modes have lower orders as they should be very sensitive to pulsation power to drive them to observable amplitudes. Yet as we are not producing models, we only indicate their relative regions as another constraint for those who do. Once all the \textit{Kepler}-observed sdBV stars are analyzed, a comparison between different pulsators should map the instability region.

5. SUMMARY AND CONCLUSIONS

We have detected 251 periodicities in \textit{KIC 3527751} using 38 months of virtually continuous \textit{Kepler} data. Most of these are in the \( g \)-mode regime of long periods, 14 should be \( p \)-modes with periodicities shorter than 6 minutes, and 38 periods are between 9 and 20 minutes and could be modes of mixed character. We recovered all but one of the periodicities detected by Reed et al. (2010), including those in the \( p \)-mode regime. We do see two peaks near the missing frequency, at 544 \( \mu \text{Hz} \) in Reed et al. (2010), but since they are both slightly below our \( 5\sigma \) limit, we did not list them.

Similar to most other \textit{Kepler}-observed sdBV stars, amplitudes and sometimes frequency/phase variations occur throughout the extended dataset, making traditional frequency-fitting and prewhitening tools unfeasible. We therefore used sFTs and Lorentzian fitting to attempt to disentangle the frequency content. Even so, there are regions which are extremely messy, with amplitude variations and frequency variations which make modes seemingly cross each other. For these regions, extractions are somewhat subjective as portions of FTs are chosen from the sFT for which the pulsations are most clearly seen. This usually means the frequencies are at their most separated which could apply a bias to the results. We encourage others to examine the accompanying figures of sFTs, or even better develop their own methods, for determining frequencies and amplitudes, and their variations.

In an attempt to understand the pulsations, we applied a wide variety of tools which have previously been used. Some of these did not reveal new relationships: prewhitening the KS test did not show any peaks indicative of \( \ell \geq 3 \) modes, and the density of mode multiplets (Figure 9) is not consistent between degrees. All three of the mode identification methods did work well, resulting in 75% of the periodicities having assigned modes.

Our multiplet analysis also revealed three \( \ell = 4 \) multiplets and possibly an \( \ell = 9 \) multiplet. Several other \textit{Kepler}-observed sdB pulsators also have high-degree (\( \ell \geq 3 \)) modes, including KIC 7668647 which has an \( \ell = 8 \) multiplet (Telting et al. 2014). High-degree modes have long been a staple of sdB asteroseismology to explain the density of periodicities of \( p \)-mode pulsations (see the discussion of Reed et al. 2007, Section 5.3 and Figure 15). The precision and duration of \textit{Kepler} data allowed us to detect periodicities with amplitudes 1/45 of the highest one. With this large dynamic range, we \textit{should} be seeing high-degree modes, yet only with confirming multiplet structure can we be certain. As such, even \textit{Kepler}-observed sdB pulsators have a large number of unidentified periodicities which could be high-degree modes, but there is no way to be certain.

Since the Ledoux constant is degree-dependent for \( g \)-modes, the rotation period (1/\( \Omega \)) determined from different degrees can be used to check if the multiplet splittings agree with predictions from Equations (1) and (2). As stated in Section 4.3, the rotation periods from the \( \ell = 1 \), 2, and 4 modes are calculated to be 46 ± 8, 41 ± 5, and 40 ± 5 days, respectively. Since these periods all agree, the mode-dependent Ledoux constants fit those predicted from Equation (2), supporting our mode identifications.

Remarkable features in \textit{KIC 3527751} include mode trapping and radically differential rotation. The case for mode trapping is less sure than that of Østensen et al. (2014b), where long complete sequences of overtones were found. In our case, there are several missing and so we have had to average across those missing overtones. While we have two nice \( \ell = 1 \) trapped regions, with mode identifications based on multiplet splittings, we do not detect one of the corresponding \( \ell = 2 \) trapped modes and the other occurs where a non-trapped \( \ell = 1 \) mode is, making its identification uncertain. So we have good examples of \( \ell = 1 \) trapped modes, but not for \( \ell = 2 \).

We detect \( g \)-mode multiplet splittings for three degrees \( \ell \) which provide the same rotation period which is substantially different from that derived from two \( p \)-mode multiplets. We examined the prospect that \textit{KIC 3527751} could actually be a binary set of pulsating stars, and while we cannot completely rule it out, it is exceedingly unlikely, as is the possibility that the \( p \)-mode multiplets are really spaced like the \( g \)-mode ones, but are just missing members. Therefore, we suggest that \textit{KIC 3527751} is differentially rotating in the radial direction with the core rotating nearly three times more slowly. This is opposite to that described by Kawaler & Hostler (2005), who predicted rapidly rotating cores via conservation of angular momentum in a contracting core. This could be a clue to the mass loss mechanism itself and a bit of a dichotomy. Either the core must preferentially spin down during mass loss, or the entire star must spin down with the envelope subsequently
being spun-up. The differential rotation of KIC 3527751 allows various mechanisms to be tested.

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APPENDIX SLIDING FOURIER TRANSFORMS FOR ALL REGIONS WITH PULSATIONS

Figure Set 10. Sliding Fourier transforms for 15 μHz regions of KIC 3527751.

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