Current induced local spin polarization due to the spin-orbit coupling in a two dimensional narrow strip

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(Dated: today)

The current induced local spin polarization due to weak Rashba spin-orbit coupling in narrow strip is studied. In the presence of longitudinal charge current, local spin polarizations appear in the sample. The spin polarization perpendicular to the plane has opposite sign near the two edges. The in-plane spin polarization in the direction perpendicular to the sample edges also appears, but does not change sign across the sample. From our scaling analysis based on increasing the strip width, the out-of-plane spin polarization is important mainly in a system of mesoscopic size, and thus appears not to be associated with the spin-Hall effect in bulk samples.

In a spin-orbit coupled electron system, an external electric field can induce a transverse spin current, giving rise to the so-called spin Hall effect (SHE). The SHE may offer a new way to control electron spins in semiconductors, and so have potential applications in spintronic devices. Depending on its origin, the SHE is generally divided into two categories: the extrinsic SHE, which originates from spin-dependent electron anomalous scattering by impurities, and the intrinsic SHE, which occurs even in the absence of impurities. The extrinsic SHE, was first proposed by D'yakonov and V. I. Perel in 1971 and reexamined recently by Hirsch and Zhang. The intrinsic SHE was predicted by Murakami, Nagaosa, and Zhang for p-type semiconductors and by Sinova et al. for n-type semiconductors in two-dimensional heterostructures. The intrinsic SHE has attracted much theoretical interest.\(^1\)\(^2\)\(^3\)\(^4\)\(^5\)\(^6\)\(^7\)\(^8\)\(^9\)\(^10\)\(^11\)\(^12\)\(^13\)\(^14\)\(^15\)\(^16\)\(^17\)\(^18\)\(^19\)\(^20\) Very recently, two independent groups have reported experimental evidence\(^19\)\(^20\) that an electric field can cause out-of-plane spin accumulations of opposite sign on opposite edges of semiconductor films, which is considered to be a signature of the SHE. Several analytical and numerical works have been published on the subject of spin accumulation in a semiconductor with spin-orbit coupling. Governale and Zülicke\(^22\) were the first to investigate spin accumulation. They studied the spin structure of electron states in a quantum wire with parabolic confining potential and strong Rashba spin-orbit coupling. Usaj and Balseiro\(^23\) showed that in a semi-infinite system with spin-orbit coupling, a current flowing parallel to the edge induces a net magnetization close to the edge. Using the Landauer-Büttik formula for a tight-binding model, Nikolić et al.\(^23\) showed numerically that in a two-dimensional bar with a width of 30 lattice constant, the Rashba spin-orbit coupling can induce opposite spin accumulation near the two edges, which is qualitatively similar to that observed in the experiment. In order to clarify whether such spin polarizations are related to the SHE, it is important to investigate their scaling behavior with increasing sample size, and to reveal the parameters that control the relative magnitude of the spin accumulation or polarization.

In this paper, the electron wave function in a continuous model is obtained for an infinite long conducting strip with finite width \(L\). Using the Kubo formula, we show that a longitudinal electrical current induces both out-of-plane spin polarization \((S_z)\) and in-plane spin polarization \((S_y)\). Near the two edges, the spin polarization \(S_z\) has opposite sign, whereas \(S_y\) has the same sign. When sample width \(L\) increases, its scaling behavior indicates that \(S_z\) near the edges decreases and \(S_y\) becomes dominant for given fixed electrical current density. Therefore, the out-of-plane spin polarization is an effect due to boundary reflections from the two opposite edges, and appears not to be related to the SHE in a bulk sample. Let us consider a system of a two-dimensional (2-D) infinite long conducting strip with finite width \(L\). The Hamiltonian for the system with Rashba spin-orbit coupling can be written as by

\[
H = \frac{k^2}{2m} + \lambda(\sigma_x k_y - \sigma_y k_x),
\]

where \(\lambda\) is the coupling constant of spin-orbit interaction, \(\sigma_x\) and \(\sigma_y\) are the Pauli matrices, \(m\) is the electron effective mass, and we take units with \(\hbar = 1\).

The eigenstates of plane waves are

\[
|E_{\pm}, \vec{k} \rangle = |E_{\mp}, k_x, k_y \rangle = \frac{1}{\sqrt{2}} e^{i\vec{k} \cdot \vec{r}} \left( \begin{pmatrix} e^{-i\phi} \\ 1 \end{pmatrix} \right),
\]

where \(\phi = \arctan(k_y/k_x)\), \((+ (-)\) labels lower (higher) energy eigenstate with eigenvalue \(E_{\pm} = k^2/2m \mp \lambda k\) for a given \(\vec{k}\).

Assuming hard-wall boundary conditions, the wave function at the two edges \((y = 0\) and \(y = L)\) is zero. Since the system is uniform along the \(x\) direction, \(k_x\) commutes with the Hamiltonian and is a good quantum number. We can write a eigenstate, with eigen-energy \(E\), of the system as a superposition of four plane waves, with same \(E\) and \(k_x\). Suppose the system is in universal region as defined in Ref. [5], the wave function near the Fermi level is given by

\[
\Psi(E, k_x, y) = |E, k_x > = \alpha_{k_x} |E, k_x, k_y^- > + \beta_{k_x} |E, k_x, -k_y^- > + \gamma_{k_x} |E, k_x, k_y^+ > + \delta_{k_x} |E, k_x, -k_y^+ >,
\]
where \( k^\pm = \sqrt{k^2 + k_F^2} \) and \( k^\pm = \pm \lambda n + \sqrt{\lambda^2 m^2 + 2mE} \) with boundary conditions \( \Psi(k_x, 0) = \Psi(k_x, L) = 0 \). One can solve the boundary conditions and find the eigenvalues of \( k_x \), which are a discrete set of values in the interval of \((-k_F^+, k_F^+)\), here \( k_F^+ = k^+ \) with \( E = E_F \) (the Fermi energy). In Usaj and Balseiro’s work, there is only one edge, the eigenfunctions are propagating waves written as a superposition of one incident and two reflected waves. \( k_x \) can take any value between \((-k_F^+, k_F^+)\).

In our current study, the interference due to the two edges of the strip limits number of eigenvalues for \( k_x \) at Fermi level, which could inject rather different physics for the problem.

While the four plane waves have different spin polarizations within the two-dimensional plane, their interference leads to nonzero out-of-plane local spin density. The local spin polarization depends on the sign of the conserved longitudinal wave vector \( k_z \). For any given energy \( E \) and a positive eigenvalue \( k^+_{\text{nx}} \) for \( k_x \), \(-k^+_{\text{nx}}\) is also an eigenvalue for \( k_x \). In the ground state where both positive and negative \( k_z \) states are occupied, the total local spin density is zero since the contribution of each spin band is zero. However, if there are a longitudinal current flowing in the strip, which causes a small shift of the Fermi circles. The numbers of occupied states with positive \( k_x \) and negative \( k_x \) are no longer equal, which can induce net spin polarizations in the strip.

The net local spin polarization can be calculated using Kubo formula\(^{24,25}\),

\[
\frac{\vec{S}(y)}{\ell} = \frac{i e}{\cal V} \sum_{\ell_x, E, E'} \langle < E, k_x, \frac{1}{2} \vec{\sigma}(y)|E, k_x \rangle < E, k_x |v_x|E', k_x \rangle |\delta(E'-E)\delta(E-E_F)
\]

which is a finite quantity.

All the coefficients in Eq. \(4\) can be determined numerically. However, we found that the eigenfunctions (standing waves) can not be expressed as a superposition of the two eigenfunctions\(^{22}\) of same \( k_x \) obtained in the case of only one edge. We also found that, for \(|k_x| < k_F^+\), \(|\alpha| = |\beta| \) and \(|\gamma| = |\delta|\); for \(|k_F^-| > |k_x| > k_F^+, |\gamma| = |\delta|\).

Plots in Fig. 1 show local spin polarizations \( s_z = \frac{h}{2} < E_F, k_x |\sigma_z|E_F, k_x > \) and \( s_y = \frac{h}{2} < E_F, k_x |\sigma_y|E_F, k_x > \) as functions of position \( y \) in all the eigenstates that have positive \( k_x \) with eigen-energy at the Fermi level for three different values of Rashba coupling \( \lambda \), which is in the units of \( k_F \). The width of the strip is set to be \( 8/k_F \), where \( k_F \) is the Fermi wave vector when there is no spin-orbit coupling. \( k_F \) is related to electron density in the sample by \( k_F^2 = 2\pi n \). Using typical value of two-dimensional electron density \( 10^{12} \text{ cm}^{-2} \) [see Ref. 26], we estimate \( k_F \approx 10^5 / \text{m} \) and \( L \approx 80 \text{ nm} \). The spin polarizations \( s_x \) and \( s_y \) vanish at the two edges as required by the boundary conditions. For each eigenstate, \( s_z(y) = -s_z(L-y) \), whereas \( s_y(y) = s_y(L-y) \). We have also obtained \( s_z \) and \( s_y \) and it is zero across the sample.

Without spin-orbit coupling, \( k_y \) is quantized to values \( k_{yn} = n\pi / L \), where \( n = 1, 2, 3, \ldots \). For each \( k_{yn} \), the eigenstates for two spin directions are degenerate. When spin-orbit coupling is in presence, the two spin bands are no longer degenerate. However, the spin polarization increases as the Rashba coupling increases. When we further increase \( \lambda \), some values of \( k_x \) are larger than Fermi wave vector of the higher spin band, as shown in Figs. 1(e) and (f). Under this case, decaying waves show up in the wave functions along the \( y \)-direction for the higher spin band.

The net spin polarizations are calculated by using the Kubo formula in Eq. \(4\). Figures 2(a), (c) and (e) show the net \( S_z(y)L/I \) when we sum the contribution from all the positive \( k_x \) modes at the Fermi level. The longitudinal charge current also induces a local in-plane polarization \( S_{xy} \), as shown in Figs. 2(b), (d) and (f), whereas \( S_x \equiv 0 \). Unlike \( S_z \), \( S_y \) has the same sign across the sample. At weak Rashba couplings \( \lambda = 0.01 \) or 0.05, we see from Figs. 2(a)-(d) that \( S_z \) is one or two order greater than \( S_y \) in magnitude. With increasing the Rashba coupling, \( S_y \) increases much faster than \( S_z \). As a consequence, \( S_y \) becomes comparable to \( S_z \) at relatively large Rashba coupling \( \lambda = 0.1 \), as seen from Figs. 2(e) and (f).

We have also obtained the local spin polarization for the case of \( L = 16/k_F \). Figure 3 shows the results for \( \lambda = 0.01, 0.05, \) and 0.1. There are 10 eigenvalues for \( k_x \) for each \( \lambda \). Similarly to Fig. 2 the magnitude of spin polarization \( S_z \) near the edges increase as \( \lambda \) increases when \( \ell \) is small. However, when \( \ell \) is large (\( \lambda = 0.1 \)),
FIG. 1: $s_x$ [figure (a), (c), and (e)] and $s_y$ [figure (b), (d), (f)] as function of position $y$ for eigenvalues of positive $k_z$ at the Fermi level. $s_x$ and $s_y$ are in units of $\frac{\hbar}{2m}$. $L = 8/k_F$, where $k_F$ is the Fermi wave vector when there is no spin-orbit coupling.

For each of the three values of $\lambda$, there are 4 eigenvalues of $h$.

Magnitude of the polarization $S_z$ near the edges becomes smaller and large oscillations appears deep inside of the sample. $S_y$ also increases as $\lambda$ increases. When $\lambda = 0.1$, in-plane spin polarization $S_y$ dominates out-of-plane polarization $S_z$. Comparing with the results of $L = 8/k_F$, $S_z / I$ is larger near the edges for $\lambda = 0.01$ and 0.05. But when $\lambda = 0.1$, it is smaller. $S_y / I$ is larger for all $\lambda$’s we chose. We conclude that for fixed finite sample size, the out-of-plane spin polarization $S_z$ dominates at relatively weak Rashba coupling and in-plane spin polarization overwhelsms for relatively strong Rashba coupling. It is interesting to examine how the spin polarization changes as the sample width $L$ increases. In Fig. 2(a), we show $S_z / I$ for various $L$ with $\lambda = 0.05$. The peak magnitude of $S_z / I$ near the $y=0$ edge increases as $L$ increases at small $L$ and decreases as $L$ increases at large $L$. The plots for $\lambda = 0.1$ in Fig. 2(b) show a similar pattern, but the width, which has the biggest $S_z / I$ near the edge, is shorter than that of $\lambda = 0.05$.

FIG. 2: Total local $S_z / I$ and $S_y / I$ as a function of $y$ for $L = 8/k_F$. In this and following figures they are in units of $\hbar/(2\pi k_F^2)$. In (a) and (b) $\lambda = 0.01$. In (c) and (d), $\lambda = 0.05$. In (e) and (f), $\lambda = 0.1$.
of the electrons at the Fermi level. The charge current along the strip induces both out of plane and in plane local spin polarizations. Near the two edges, the spin polarization $S_z$ has opposite sign, whereas $S_y$ has the same sign. When the sample width $L$ increases, the peak magnitude of $S_z L/I$ near the edges increases at small $L$ and decreases at large $L$ for weak $\lambda$. And at large $L$, our numerical results indicate that $S_y L/I$ becomes dominant. From our scaling analysis based on varying $L$, the out-of-plane spin polarization is important mainly in systems of mesoscopic sizes, and thus appears not to be associated with the SHE in bulk samples.

Acknowledgement—We wish to thank J. Sinova for helpful discussion. This work is supported by a grant from the Robert A. Welch Foundation and the Texas Center for Superconductivity at the University of Houston.

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FIG. 4: Plots of $S_z(y) L/I$ for various $L$ with $\lambda = 0.05$ [panel (a)] and 0.1 [panel (b)]. The value of $L$ of each curve can be identified from the right end of the curve. In panel (a), $L=4, 8, 12, 16, 24$; in panel (b), $L=4, 8, 12, 16, 20$. 