Superior dark-state cooling via nonreciprocal couplings in trapped atoms

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Abstract

Cooling the trapped atoms toward their motional ground states is key to applications of quantum simulation and quantum computation. By utilizing nonreciprocal couplings between two atoms, we present an intriguing dark-state cooling scheme in Λ-type three-level structure, which is shown superior than the conventional electromagnetically-induced-transparency cooling in a single atom. The effective nonreciprocal couplings can be facilitated either by an atom–waveguide interface or a free-space photonic quantum link. By tailoring system parameters allowed in dark-state cooling, we identify the parameter regions of better cooling performance with an enhanced cooling rate. We further demonstrate a mapping to the dark-state sideband cooling under asymmetric laser driving fields, which shows a distinct heat transfer and promises an outperforming dark-state sideband cooling assisted by collective spin–exchange interactions.

1. Introduction

Cooling the trapped atoms toward their motional ground states engages a series of improving laser cooling techniques from Doppler to Sisyphus and subrecoil cooling schemes \cite{1}. Upon approaching the motional ground state of atoms, the linewidth or the spontaneous decay rate of the transition puts a limit on the temperature that these cooling schemes can achieve. Their ultimate performance depends on the balance between cooling and heating mechanisms, where the fluctuations of spontaneously emitted and rescattered photons give rise to the constraint that forbids further cooling. In one of the subrecoil cooling platforms, the resolved sideband cooling \cite{2–6} in the Lamb–Dicke (LD) regime suppresses the carrier transition and specifically excites the transition with one phononic quanta less. Effectively, it moves the atoms toward the zero-phonon state via spontaneous emissions, and essentially, the steady-state phonon occupation is determined by a squared ratio of the spontaneous emission rate over the trapping frequency, which can be much smaller than one.

Alternatively, the dark-state sideband cooling \cite{7–22} utilizes Λ-type three-level structure of atoms with two laser fields operating on two different hyperfine ground states and one common excited state. This leads to an effective dark-state picture owing to quantum interference between two ground states and forms an asymmetric absorption profile under the two-photon resonance condition with a large detuning. This profile allows a narrow transition in the resolved sideband to cool down the atoms by removing one phonon, similar to the sideband cooling scheme with a two-level structure. In contrast to the two-level structure, the effective laser drivings and decay rates in dark-state sideband cooling can be tunable, which makes it a flexible and widely-used cooling scheme in ultracold atoms.

Recently, a novel scheme of using nonreciprocal couplings in optomechanical systems \cite{23, 24} manifests motional refrigeration, which gives insights in heat transfer by utilizing unequal decay channels. This coupling can be facilitated as well in a nanophotonics platform of atom–waveguide interface \cite{25–27} which
Figure 1. A schematic plot of nonreciprocal couplings between trapped atoms. The Λ-type three-level atoms are trapped in harmonic potentials under the dark-state cooling scheme with two laser fields Ωg,l and Ωr,l operating on the transitions |g⟩j→|e⟩j(l) and |r⟩j→|e⟩j(l) respectively at two-photon resonance Δg=Δr. Respective total decay rates for individual atoms to the states |g⟩ and |r⟩ are γg,r along with nonreciprocal decay channels γL=g,L+g,R and γR=g,R+g,L. These left (L)- and right (R)-propagating decay rates represent the nonreciprocal couplings γg,L≠γg,R and γr,L≠γr,R, which effectively facilitates spin–exchange hopping between the jth and lth sites of trapped atoms via an atom–waveguide interface.

allows quantum state engineering via mediating collective nonreciprocal couplings between atoms [27–47]. In this quantum interface, a strong coupling regime in light–matter interaction can be reached by coupling atoms with evanescent fields at the waveguide surface, where the directionality of light exchange processes can be further manipulated and tailored by external magnetic fields [34]. This gives rise to exotic phenomena of collective radiation behaviors [38–40] and a new topological waveguide-QED platform that can host photonic bound states [48].

In this work, we theoretically investigate the dark-state sideband cooling scheme in trapped atoms with nonreciprocal couplings, as shown in figure 1. The nonreciprocal couplings can be facilitated in an atom–waveguide interface or via a photonic quantum link in free space [49], which leads to collective spin–phonon correlations [18, 50] and distinct heat exchanges. This cooperative mechanism of cooling has been considered as well in two ions in free space [51], cavity-mediated atoms [52, 53], and one-dimensional chain of optically bound cold atoms [54]. Here we present a superior dark-state cooling compared to the conventional single atom results by utilizing the nonreciprocal couplings between two atoms. We denote the first atom as the target atom and the second as residual or spectator one, and place the first atom left to the second atom without loss of generality. We explore various tunable parameter regimes which are allowed in dark-state cooling and identify the parameter region that gives better cooling performance without compromising its cooling rate. We further obtain an analytical form of the phonon occupation for the target atom under the asymmetric driving conditions. Our results demonstrate a distinct heat transfer within the atoms, which leads to a superior dark-state sideband cooling assisted by collective spin–exchange interactions. This opens new avenues in surpassing the cooling obstacle [18, 55, 56], which is crucial in scalable quantum computation [55, 57–59], quantum simulations [60, 61], and preparations of large ensemble of ultracold atoms [21, 62] or molecules [63]. This paper is organized as follows. We present the theoretical model of dark-state cooling with nonreciprocal couplings in section 2. We then identify the parameter regimes that demonstrate superior cooling behaviors and their cooling dynamics in section 3. In section 4, we further show the analytical prediction of the steady-state phonon occupation of the target atom and confirm its validity under dark-state mapping. Finally we discuss possible experimental realizations and conclude in section 5.

2. Theoretical model

We start with a conventional EIT cooling scheme [7] with nonreciprocal couplings between constituent trapped atoms, as shown in figure 1. Each atom involves Λ-type three-level atomic structure, where two
hyperfine ground states $|g\rangle$ and $|r\rangle$ for $j$th atom couple to their common excited state $|e\rangle_j$ with laser fields of Rabi frequencies $\Omega_{e1}$ and $\Omega_{e2}$, respectively. The intrinsic decay rates from the excited state are $\gamma_e$ and $\gamma_r$. With the nonreciprocal couplings introduced as $\gamma_L$ and $\gamma_R$, the dynamics of a density matrix $\rho$ for $N$ atoms with mass $m$ can be expressed as $(\hbar = 1)$

$$\frac{d\rho}{dt} = -i[H_{LD} + H_L + H_R, \rho] + L_{\rho} [\rho] + L_{\rho}[\rho],$$

where $H_{LD}$ for the EIT cooling in the LD regime (in the first order of LD parameter $\eta_{gr}$) reads

$$H_{LD} = -\sum_{j=1}^{N} \Delta_j |e\rangle_j\langle e| + \nu \sum_{j=1}^{N} a_j^\dagger a_j + \sum_{j=1}^{N} \left( \frac{\Omega_{e1}}{2} |e\rangle_j\langle g| + \frac{\Omega_{e2}}{2} |e\rangle_j\langle r| + \text{H.c.} \right)$$

$$+ i\eta L \cos(\psi) \sum_{j=1}^{N} \left[ \frac{\Omega_{e1}}{2} |e\rangle_j\langle g| (a_j + a_j^\dagger) - \text{H.c.} \right] + i\eta R \cos(\psi) \sum_{j=1}^{N} \left[ \frac{\Omega_{e2}}{2} |e\rangle_j\langle r| (a_j + a_j^\dagger) - \text{H.c.} \right],$$

with the common laser detuning $\Delta_j \equiv \omega_{ej} - \omega_{er} = \omega_{gj} - \omega_{gr}$ as required in EIT cooling scheme, i.e. the same difference between the $j$th-site laser central frequencies $(\omega_{gj}, \omega_{er})$ and the atomic transition frequencies $(\omega_{gj}, \omega_{gr})$. Projection angles of the laser fields to the motional direction are denoted as $\psi$. The harmonic trap frequency is $\nu$ with a creation (annihilation) operator $a_j^\dagger$ ($a_j$) in the quantized phononic states $|n\rangle$, and LD parameters are $\eta_{gr} = k_{gr}/\sqrt{2m\nu}$ with $k_{gr} \equiv \omega_{gr}/\nu$. The coherent and dissipative nonreciprocal couplings in the zeroth order of $\eta_{gr}$ are [35]

$$H_{LR} = -i \sum_{m,g,r} \frac{\gamma_{LR}}{2} \sum_{j>l} \left( e^{i\phi_{x}} |e\rangle_j\langle g| (|m\rangle \otimes |l\rangle - |l\rangle \otimes |m\rangle) - \text{H.c.} \right),$$

and

$$L_{\rho} [\rho] = - \sum_{m,g,r} \frac{\gamma_{LR}}{2} \sum_{j>l} e^{-i\phi_{x}} \left( \langle e|_{j} (|m\rangle \otimes |l\rangle - |l\rangle \otimes |m\rangle) - \text{H.c.} \right),$$

respectively, where $k_{gr}$ denotes the wave vector in the guided mode that mediates nonreciprocal couplings $\gamma_{gj} \equiv \gamma_{Lg} + \gamma_{Rg}$ and $\xi \equiv k_{x}[j_{x+1} - j_{x}]$ quantifies the light-induced dipole–dipole interactions between the relative positions of trap centers $j_{x}$ and $j_{x}$. Throughout the paper, we define spin–exchange interactions specifically denoted to two lower states ($|g\rangle$ and $|r\rangle$) and excitation–exchange in general for the effect of nonreciprocal couplings. We also note that in actual atom–waveguide interfaces, a small but finite nonguided coupling $\gamma_{ng}$ can not be avoided entirely, which can be quantified and included in equation (4), for example of the probe field transition, as $-\gamma_{ng}/2 |e\rangle_j\langle g| \rho + |g\rangle_j\langle e| - 2|g\rangle_j\langle e| \rho |g\rangle_j\langle g|$ for all individual atoms. We will elaborate more on its role in experimental realizations in section 5.

In the terms of the eigenstates with only atomic spin degrees of freedom in equation (2), we can further diagonalize the EIT Hamiltonian [64] in a single-particle limit, where three eigenstates are

$$|\pm\rangle = \sin \phi |e\rangle - \cos \phi |g\rangle, \quad |\mp\rangle = \cos \phi |e\rangle + \sin \phi |g\rangle, \quad |d\rangle = \cos \theta |g\rangle - \sin \theta |r\rangle,$$

with $|b\rangle \equiv \sin \theta |g\rangle + \cos \theta |r\rangle$, and the corresponding eigenenergies are

$$\omega_{\pm} = \left( -\Delta + \sqrt{\Omega_e^2 + \Omega_r^2 + \Delta^2} \right)/2, \quad \omega_{d} = \left( -\Delta - \sqrt{\Omega_e^2 + \Omega_r^2 + \Delta^2} \right)/2, \quad \omega_{d} = 0.$$

The angles $\theta$ and $2\phi$ are defined as $\tan^{-1}(\Omega_r/\Omega_e)$ and $\tan^{-1}(-\sqrt{\Omega_e^2 + \Omega_r^2}/\Delta)$, respectively. Under the EIT cooling condition of $\omega_{\pm} = \nu [6, 7]$, a red sideband transition between $|d, n\rangle \leftrightarrow |+, n - 1\rangle$ becomes resonant, and we can safely ignore the influences of off-resonant state $|\mp\rangle$ and blue sideband transition between $|d, n\rangle \leftrightarrow |+, n + 1\rangle$. 

\[3\]
The effective Hamiltonian within the subspace $|d⟩$ and $|+⟩$ can then be obtained from equation (2), which reads in an interaction picture [6, 22],

$$H_{\text{eff}}^i = i\hbar \sum_{j=1}^{N} \frac{\Omega_{\text{eff}}}{2} \left[ |d⟩⟨+| (a_j + a_j^{\dagger}) - \text{H.c.} \right],$$

(7)

with $\eta_{\text{eff}} \equiv \eta_\epsilon \cos(\psi_j) - \eta_\gamma \cos(\psi_j)$, $\Omega_{\text{eff}} \equiv \Omega_{g,j} \Omega_{\epsilon,j} \sin(\phi_j)/\sqrt{\Omega_{g,j}^2 + \Omega_{\epsilon,j}^2}$, $\tan(2\phi_j) \equiv -\sqrt{\Omega_{g,j}^2 + \Omega_{\epsilon,j}^2}/\Delta_j$, and $\Delta_j = -\nu + (\Omega_{g,j}^2 + \Omega_{\epsilon,j}^2)/(4\nu)$. The associated nonreciprocal coupling terms as in equations (3) and (4) reduce to

$$H_{\text{eff}}^{L(R)} = -i \sum_{j<n} \frac{\gamma_{\text{eff},L(R)}(j,l)}{2} \left( e^{i\phi_j - \phi_j^{\dagger}} |+⟩_j ⟨d| \otimes |d⟩_j |+⟩ - \text{H.c.} \right),$$

(8)

$$\mathcal{L}_{\text{eff}}^{L(R)}[\rho] = - \sum_{j=1}^{N} \frac{\gamma_{\text{eff},L(R)}(j,l)}{2} \left( e^{i\phi_j - \phi_j^{\dagger}} (|+⟩_j ⟨d| \otimes |d⟩_j) |+⟩_j |+⟩_j + |+⟩_j |d⟩_j + |d⟩_j |+⟩ - 2 |d⟩_j |+⟩_j |d⟩_j \right),$$

(9)

where

$$\gamma_{\text{eff},L(R)}(j,l) \equiv \sin \phi_j \sin \phi_l (\gamma_g \cos \theta_j \cos \theta_l + \gamma_{l,L(R)} \sin \theta_j \sin \theta_l),$$

(10)

$$\gamma_{\text{eff},J} \equiv \sin^2 \phi_l (\gamma_\epsilon \cos^2 \theta_j + \gamma_\gamma \sin^2 \theta_j),$$

(11)

with $\gamma_g = \gamma_{g,L} + \gamma_{g,R}, \gamma_{l} = \gamma_{l,L} + \gamma_{l,R}$, and $\tan \theta_j = \Omega_{g,j}/\Omega_{l,j}(\gamma_g)$. We note that $\gamma_{\text{eff},L,R}(j,l) \neq [\gamma_{\text{eff},L,R}(j,l)]^2$ in general, unless $\theta_j = \theta_l$, which would otherwise be assured in quantum systems with collective and pairwise dipole–dipole interactions [35]. This inequality arises owing to the mapping to the reduced Hilbert space, where different $\phi_j$ determined by external laser fields and $\theta_j$ by laser Rabi frequencies can further allow extra and tunable degrees of freedom for cooling mechanism. We note that to have different $\theta_j$ requires a spatially inhomogeneous field condition.

The above dark-state mapping sustains when the state $|−⟩$ can be detuned significantly from the subspace $|+, n − 1⟩$ and $|d, n⟩$, which leads to the requirement of $\Delta_j \gg \Omega_{l,j} \gg \nu$ for the $j$th atom based on the condition of $|ω_−| \gg \nu$, the EIT cooling condition $ω_+ = \nu$, and the usual assumption of a weak probe field $\Omega_{g,j} \ll \Omega_{l,j}$. The resultant effective dark-state sideband cooling should also be legitimate by fulfilling the sideband cooling condition of $\nu \gg \eta_{\text{eff}} \Omega_{\text{eff},l,j}$ and $\gamma_{\text{eff},l,j}$. This can be guaranteed when $\phi_j$ and $\theta_j$ are made small enough, which can easily be achieved respectively by choosing large enough laser detuning $\Delta_j \gg \Omega_{l,j}$ and control fields $\Omega_{l,j} \gg \Omega_{g,j}$. In the setting of EIT cooling with nonreciprocal couplings, it is these tunable excitation parameters and the controllable directionality of excitation–exchange interactions that give rise to distinct cooling behaviors we explore in the following section.

3. Superior dark-state cooling

Here we consider a setting of two atoms with $A$-type three-level configurations with nonreciprocal couplings, which represents the building block for a large-scale atomic array. In this basic unit under EIT cooling with collective excitation–exchange interactions, we numerically obtain the steady-state phonon occupations $\langle n_j(t \to \infty) |a_j^{\dagger} a_j \rangle = \langle n_j \rangle_{ss}$ from equation (1), under the Hilbert space $|α, n⟩$ with $α \in \{g, r, e\}$ and $n \in \{0, 1, 2\}$. This truncation of phonon numbers is valid when the composite two-atom system is close to their motional ground state, that is when $\langle n_j \rangle_{ss} \ll 1$.

As a comparison to the single atom result ($\langle n_j \rangle_{ss}^1$) without nonreciprocal spin exchange interactions, we calculate the ratio $\langle n_j \rangle_{ss}^2/\langle n_j \rangle_{ss}^1$ to account for the superior or inferior cooling regime when $\langle n_j \rangle_{ss}^2$ is less or more than 1. In figure 2, we explore various parameter regimes of $\Omega_{g,j}^2, \Omega_{l,j}^2, \gamma_g, \gamma_{l,j}^2, R_j$ and look for superior cooling regions in the allowable parameters under EIT cooling conditions. An essential parameter $\gamma_R$ is defined as $\gamma_R = \gamma_{g,R} + \gamma_{l,R}$ with $\gamma_L = \gamma_{g,L} + \gamma_{l,L}$, a total decay rate $\gamma_R + \gamma_L = \gamma_{g} + \gamma_{\gamma}$, and

$$\gamma_{g} = \gamma_{g,L} + \gamma_{g,R}, \gamma_{\gamma} = \gamma_{l,L} + \gamma_{l,R}, \text{ and}$$

with $\gamma_{g} = \gamma_{g,L} + \gamma_{g,R}, \gamma_{l} = \gamma_{l,L} + \gamma_{l,R}$, and $\tan \theta_j = \Omega_{g,j}/\Omega_{l,j}(\gamma_g)$.
where \( \gamma \) is an excitation rate. Cooling at \( \sim \) transitions. As \( \Omega_{\beta}/\gamma \) increases along with a finite nonreciprocal coupling \( \gamma_{\beta} \), a superior cooling region emerges and allows an impressive performance of almost twofold improvement (\( \bar{n}_1 \approx 0.5 \)) compared to the case without excitation–exchange interactions. Finally in figure 2(d), we find the optimal operations of EIT cooling. For \( \tilde{n}_1 \) on the finite motional states to guarantee the convergence in numerical simulations. As shown in figure 3, we select three contrasted regimes for heating, cooling, and neutral time dynamics. The characteristic times to reach the steady-states are shown shorter in cooling dynamics in figure 3(b). This presents an enhancement in cooling rate compared to the EIT cooling scheme in a single atom, in addition to the outperforming cooling behaviors in the steady-state phonon occupations. By contrast, the time dynamics in the heating regime shows a longer time behavior than the single atom case, while the neutral regime has a

\[
\begin{align*}
\gamma_{\beta} &= \gamma_{x\beta} + \gamma_{y\beta} \\
\gamma_{x} &= \gamma_{x\beta} + \gamma_{xR} \\
\end{align*}
\]
similar characteristic timescale. We attribute the enhanced cooling rate or prolonged time dynamics in heating to the collective spin–phonon couplings between the atoms.

4. Dark-state sideband cooling under asymmetric driving

In equations (7)–(9), we have demonstrated the mapping from EIT cooling with three-level structures to the effective dark-state sideband cooling scheme. Within $n \in \{0, 1\}$, the mapping has reduced the Hilbert space dimensions in two atoms from 36 to 16 with $16^2$ coupled linear equations. We follow the wisdom in sideband cooling with chiral couplings by considering asymmetric driving conditions when $\eta_{EIT} \ll \eta_{EIT,1}$ [66]. In this way, we are able to perform a partial trace on the motional degree of freedom (m) and focus on the target atom behavior $\langle n_1 \rangle_{SS}$, which further diminishes the dimension of the Hilbert space to 8. The dynamics of this two-atom system can then be determined by the reduced density matrix $\rho_{m(m)}(\rho)$. This neglect of $O_{eff,2}$ is legitimate since its effect becomes a higher order correction compared to the motions of the target atom owing to a relatively small driving field in the second atom.

The steady-state solutions for the density matrix elements of $\rho_{m(m)}(\rho) = \rho_{\alpha_1 \alpha_2 \alpha_1 \alpha_2, m_1 m_2} = \{\alpha_1, \alpha_2 | \rho_{\alpha_1 \alpha_2, m_1 m_2}, \beta_1, \beta_2 \}$ can further be simplified by taking advantage of the fact that $\rho_{dddd} = O(1)$ when $\gamma_{ij}^2 / \nu^2$ and $\eta_{EIT,1}^2 \Omega_{EIT,1}^2 / \nu^2$ are much smaller than one. Here we use $\alpha_{1(2)}$ and $\beta_{1(2)}$ to denote the first (second) atomic internal levels $|d\rangle$ and $|+\rangle$ in the dark-state mapping, and $n_1$ and $m_1$ for phonon occupation numbers of the first atom. The essential focus of this section is that (a) with the assistance of dark-state mapping, we are able to reduce three-level EIT cooling scheme to an effective two-level sideband cooling scheme in two atoms, (b) we can simplify the coupled equations of all density matrix elements by tracing over the second atom’s motional degrees of freedom and keeping the leading order terms, and obtain some analytical forms, and (c) we later prove that a dark-state mapping is legitimate in figure 4, where we directly compare numerical simulations of three-level EIT cooling scheme in two atoms with equation (15).

To proceed, we can neglect those $\rho_{\alpha_1 \alpha_2 \alpha_1 \alpha_2, m_1 m_2}$ whose leading terms are higher than the second order. Setting $\epsilon_{kin} = 1$ and keeping the terms of $O(1), O(\gamma_{eff,1}), O(\gamma_{eff,2}), O(\eta_{EIT} \Omega_{EIT}), O(\gamma_{EIT,1}, \gamma_{EIT,2}, L, R, \eta_{EIT} \Omega_{EIT})$, we obtain

\[
\begin{align*}
0 &= \eta_{EIT} \Omega_{EIT,1}(\rho_{dd++} - \rho_{dd++}) + 2\gamma_{eff,1}(1, 2)(\rho_{dd++} + \rho_{dd++}) + 2\gamma_{eff,2}(1, 2)(\rho_{dd++} + \rho_{dd++}) + 2\gamma_{EIT,1}(1, 2)(\rho_{dd++} + \rho_{dd++}) + 2\gamma_{EIT,2}(1, 2)(\rho_{dd++} + \rho_{dd++}), \\
0 &= -\eta_{EIT} \Omega_{EIT,1}(\rho_{dd++} - \rho_{dd++}) - 2\gamma_{EIT,1}(1, 2)(\rho_{dd++} + \rho_{dd++}) - 2\gamma_{EIT,2}(1, 2)(\rho_{dd++} + \rho_{dd++}) - i(\gamma_{EIT,1} + \gamma_{EIT,2})\rho_{dd++} + (\rho_{dd++}), \\
0 &= \eta_{EIT} \Omega_{EIT,1}(\rho_{dd++} - \rho_{dd++}) - 2\gamma_{EIT,1}(1, 2)(\rho_{dd++} + \rho_{dd++}) + \gamma_{EIT,1} \rho_{dd++} - i(\gamma_{EIT,1} + \gamma_{EIT,2})\rho_{dd++} + (\rho_{dd++}), \\
0 &= -2\gamma_{EIT,2}(1, 2)(\rho_{dd++} + \rho_{dd++}) - 2\gamma_{EIT,2}(1, 2)(\rho_{dd++} + \rho_{dd++}) - i\gamma_{EIT,2} \rho_{dd++} + (\rho_{dd++}), \\
0 &= \eta_{EIT} \Omega_{EIT,1}(\rho_{dd++} - \rho_{dd++}) + 2\gamma_{EIT,1}(1, 2)(\rho_{dd++} + \rho_{dd++}) + \gamma_{EIT,1} \rho_{dd++} + i(\gamma_{EIT,1} + \gamma_{EIT,2})\rho_{dd++} + (\rho_{dd++}), \\
0 &= \eta_{EIT} \Omega_{EIT,1}(\rho_{dd++} - \rho_{dd++}) + i\gamma_{EIT,1} \rho_{dd++} + (\rho_{dd++}),
\end{align*}
\]

along with three other relationships.

Figure 3. Time dynamics of average phonon occupations for the target atom. We numerically simulate the time dynamics of $\langle n_1 \rangle$ (blue-solid line) and $\langle n_2 \rangle$ (red-solid line) with specific parameters denoted by star polygons in the insets plotted extracted from figure 2(c), for (a) $\gamma_{EIT,1}/\gamma_{EIT,2} = (18, 0.3)$, (b) $\gamma_{EIT,1}/\gamma_{EIT,2} = (0.7, 18)$, and (c) $\gamma_{EIT,1}/\gamma_{EIT,2} = (2, 0.7)$, in heating, cooling, and neutral regimes, respectively. The respective dotted lines represent the corresponding results of a single atom under the EIT cooling scheme without nonreciprocal couplings. All $\langle n_2 \rangle$ almost overlap with respective single atom results, which signifies no significant cooling or heating effect as evidenced in the steady-state solutions in figure 2(c). The lower inset plot in (a) shows long-time behaviors of the heating region approaching the steady states. The numerical simulation uses $\eta_{EIT} = 0.7$ and truncates the phonon states to $n = 2$. 

\[n \in \{0, 1\}\]
The above shows the minimum of steady-state phonon occupation for a single atom under the sideband cooling [6]. The remaining terms are single atom result, we obtain corrections of $\gamma$ rate as in the single atom result of $S$. Solving the above equations leads to the solutions of density matrix elements expressed in terms of where the approximate form assumes $\eta eff,1 \rightarrow \gamma_1$ and (b) when $\gamma_1 \gamma_2 \eta_2$. Figure 4. Steady-state occupations of two-atom system under dark-state sideband cooling. Under the asymmetric driving condition in the dark-state sideband cooling scheme, the motional degrees of freedom of the residual atom $a_2$ can be traced out and equation (15) can be obtained analytically. For the target atom, we numerically calculate $\langle n_1 \rangle_a$ from the EIT cooling scheme with nonreciprocal couplings with three-level atomic structures (solid line) and compare it with equation (15) from the effective dark-state sideband cooling scheme under the dark-state mapping (dashed line). The driving fields in the two-atom EIT cooling with a light blue to a darker blue line are chosen as $\Omega_{L(1,2)}$, $\Omega_{R(1,2)}$ and (3, 15, 0.03, 30), respectively, with corresponding larger two-photon detunings $\Delta_{L(1,2)}$ in the latter case. We choose $\eta_{1(2)} \approx 0, \gamma_1 = 16 \nu, \gamma_2 = 2 \nu$, and the other relevant parameters are the same as the common parameters we apply in figure 2.

\[ \rho_{dd} = \frac{\eta_2^2 \eta_1^2}{16 \nu^2} \rho_{dd}, \]
\[ \rho_{dd} = -\frac{\gamma_2 \Omega_{L(1,2)}(1, 2) \Omega_{R(1,2)}}{8 \nu^2} \rho_{dd}, \]
\[ \rho_{dd} = -\frac{4 \nu - \gamma_2 \Omega_{L(1,2)}}{16 \nu^2} \eta_1 \Omega_{eff,1} \rho_{dd}. \]

Solving the above equations leads to the solutions of density matrix elements expressed in terms of $\rho_{dd}$. Finally we obtain the steady-state phonon occupation for the target atom as ($\rho_{dd} \approx 1$)

\[ \langle n_1 \rangle_a = \rho_{1d+1d} + \rho_{1d+1d} + \rho_{1d+1d} + \rho_{1d+1d}, \]
\[ \approx \frac{\gamma_1^2}{16 \nu^2} + \frac{\eta_1^2 \Omega_{eff,1}^2}{8 \nu^2} \frac{\gamma_2 \Omega_{L(1,2)}(1, 2) \Omega_{R(1,2)}}{4 \nu^2} \]
\[ + \frac{\gamma_1 \gamma_2 \Omega_{L(1,2)}(1, 2) \Omega_{R(1,2)}(1, 2)}{\gamma_2 \Omega_{L(1,2)}(1, 2) \Omega_{R(1,2)}(1, 2) + \eta_1^2 \Omega_{eff,1}^2 + \eta_2^2 \Omega_{eff,2}^2} \eta_1 \Omega_{eff,1} \rho_{dd}. \]

where the approximate form assumes $\nu \gg \gamma_{eff,L(1,2)}(1, 2)$ and the first two terms are exactly the steady-state phonon occupation for a single atom under the sideband cooling [6]. The remaining terms are the modifications originating from the nonreciprocal couplings. At unidirectional coupling $\gamma_{eff,L(1,2)}(1, 2) = 0$, equation (15) again reduces to the single atom result, which is also shown in figures 2(a) and (b) when $\gamma_1 / \gamma_2 = 1$ or 0.

In the LD regime as $\eta eff \rightarrow 0$, we can estimate equation (15) as

\[ \langle n_1 \rangle_{a | \eta eff \rightarrow 0} \approx \frac{\gamma_1^2}{16 \nu^2} - \frac{\gamma_2 \Omega_{L(1,2)}(1, 2) \Omega_{R(1,2)}(1, 2)}{4 \nu^2} \frac{\gamma_2 \Omega_{L(1,2)}(1, 2) \Omega_{R(1,2)}(1, 2)}{4 \nu^2}, \]

where the phonon occupation of the target atom is no longer limited by the effective spontaneous emission rate as in the single atom result of $\gamma_{eff,1}^2/(16 \nu^2)$. If we assume $\theta_1 = \theta_2 \approx 0$ and $\gamma \gg \gamma_r$, by comparing to the single atom result, we obtain

\[ \langle n_1 \rangle_{a | \eta eff \rightarrow 0} \approx 1 - \frac{4 \gamma_{g,R} \gamma_{g,L} \gamma_r}{\gamma_r^2}. \]

The above shows the minimum of $\langle n_1 \rangle_{a | \eta eff \rightarrow 0} = 0$ when $\gamma_{g,L(1,2)} = 1/2$. This coincides with the predictions of the target ion under chiral-coupling-assisted sideband cooling [66] when an extreme LD regime as $\eta \rightarrow 0$ is applied. Furthermore in this extreme regime as in an infinite mass limit, a reciprocal coupling emerges to host the heat removal in the target atom from the mechanism of collective spin–exchange interaction. The values of $\gamma_{g,R}$ at the minimum of $\langle n_1 \rangle$ would shift to nonreciprocal coupling regimes of $\gamma_{g,L} \lesssim 0.5$ when finite corrections of $\eta eff$ in equation (15) are retrieved, where two minimums arise from a quartic dependence of $\gamma_{g,R}$ in equation (15). These optimal conditions can be obtained [66] with a perturbation of $\eta_{eff}$. 

\[ \text{Figure 4.} \]
where the optimal condition $\gamma_{g,R} = 1/2$ migrates toward nonreciprocal coupling regime as a square root function of $\eta_{eff}\Omega_{eff}$.}

In figure 4, we show the superior dark-state sideband cooling under nonreciprocal couplings, where two lowest minimums can be identified as superior cooling compared to the single atom results at $\gamma_R = 1$. We compare the exact results from EIT cooling scheme with three-level atomic structures and the predictions from the effective dark-state sideband cooling, which are well described by an analytical form in equation (15) under asymmetric driving conditions. This confirms the validity of dark-state mapping when large enough two-photon detunings $\Delta$ are applied. The symmetry of the curves in figure 4 results from the asymmetric driving condition, where the residual atom behaves as an entity to allow spin−exchange interactions only. In this scenario, the target atom exchanges spin excitations with the residual one at finite directional coupling strengths. An interference from nonreciprocal couplings can build up owing to the collective nature of spin−exchange interactions, which leads to a multiplication factor of $\gamma_{eff}\gamma_{eff}$ in equation (16) or $\gamma_{g,R}\gamma_{g,R}$ in equation (17), and therefrom the symmetric profiles in figure 4.

5. Discussion and conclusion

A superior cooling performance in the constituent atoms emerges owing to the presence of nonreciprocal couplings. With this extra degree of freedom, the composite system is allowed to access new parameter regimes with potentially better cooling behaviors. These unequal coupling channels provide a directional heat transfer from the target atom to the residual one under an asymmetric driving condition. This distinct heat removal can be attributed to an effective heat sink of the residual atom, where an asymmetric driving strengthens an imbalance of the heat distribution in the composite systems. Due to the collective nature of excitation−exchange interactions, a finite directional decay channel allows a superior cooling region that surpasses the single atom results. For experimental realizations, we propose to apply a side excitation scheme in an atom−waveguide system [34], where a quantum-state−controlled directional coupling strength can be realized and tunable in a strong coupling regime of $\beta \equiv (\gamma_L + \gamma_R)/(\gamma_L + \gamma_R + \gamma_{ef})$ [33, 38] greater than 90% with a nonguided mode quantified by $\gamma_{ef}$. In this system, $\gamma_{L(R)}/\gamma$ can be tailored between 0 and 1 [34], which we have theoretically explored here. The nonguided mode coupling would further provide an extra dissipation channel that can turn the heating mechanism at the reciprocal coupling regime ($\gamma_R = \gamma_L$) to cooling instead, indicating a reduction in the spin−spin correlation that is otherwise more significant in the heating regime [66].

For experimental consideration of EIT cooling schemes, we have several options in the level transitions of rubidium atoms [21], where various hyperfine states or field polarizations can be utilized. In addition to the tunable strengths of the probe and control fields with a large enough detuning, the essential requirement for superior cooling relies on the ratio of decay rates as shown in figure 2(c). We take $^{85}$Rb atoms as an example: the probe to control field transitions with polarizations $\sigma^+ \to \sigma^-$ can be chosen as $|F = 2, m_F = 1 \rangle \to |F = 3, m_F = 2 \rangle \to |F = 3, m_F = 3 \rangle$ or $|F = 2, m_F = 0 \rangle \to |F = 3, m_F = 1 \rangle \to |F = 2, m_F = 2 \rangle$ for $D_2$ transition and $|F = 2, m_F = 1 \rangle \to |F = 3, m_F = 2 \rangle \to |F = 3, m_F = 3 \rangle$ or $|F = 3, m_F = 0 \rangle \to |F = 2, m_F = 3 \rangle \to |F = 3, m_F = 1 \rangle$ for $D_1$ transition, where the branching ratios $\gamma_f/\gamma_r$ can be 32 : 15, 6 : 1, 10 : 3, and 15 : 1, respectively. This ratio can be tuned by using different polarizations, for example $\sigma^+ \to \sigma^+$ transition, or different atomic species of $^{133}$Cs [62]. For $^{85}$Rb atoms, we have $\gamma \approx 2 \pi \times 6$ MHz and $\nu \approx 2 \pi \times 0.3$ MHz, which can be satisfied in typical EIT settings and optical traps with a weak confinement.

In summary, we have shown theoretically that a superior EIT cooling behavior can be feasible when nonreciprocal couplings between constituent atoms are introduced. This provides a way to get around the cooling bottleneck [18, 55, 56] that an atom−based quantum computer or other applications of quantum technology may encounter. The superior EIT cooling in a single atom is limited by the steady-state phonon occupation $\sim (\gamma/\Delta)^2$ [22], while the corresponding dark-state sideband cooling is by $\sim (\gamma_{eff}/\nu)^2$. By utilizing nonreciprocal couplings between constituent atoms, an intriguing dark-state cooling scheme in $\Lambda$-type three-level structure manifests a distinct heat transfer to further cool the target atom surpassing its single atom limit. We explore the allowable system parameters especially under asymmetric driving conditions and identify the regions for superior cooling performance with an enhanced cooling rate. Our results unravel the mechanism of collective spin−exchange interactions between trapped atoms, which enable new possibilities in driving the atoms toward their motional ground states. Future work will consider a multatom platform, where more complex spin−phonon correlations may arise to further enhance the performance of the EIT cooling scheme.
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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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