Gluonic colour singlets in QCD

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1 Introduction

Identifying the internal Abelian directions has been of interest to studies of the QCD vacuum since Savvidy’s landmark paper [1] demonstrating the energetic favourability of a magnetic condensate. This led to a long-running controversy surrounding the condensate’s stability [2, 3, 4, 5, 6, 7], with recent papers [7, 8, 9, 10, 11, 12] concluding in the positive.

What concerns this work is the manner in which the necessarily Abelian internal direction(s) of the condensate were identified. Two-colour studies typically assigned the Abelian direction to $c_3 [13, 14, 15, 16]$, in a blatant violation of gauge invariance that always left doubts that the calculated effects might be gauge artefacts. A further defect was that these papers were unable to prove that the magnetic background is due to monopoles.

These problems are avoided by the Cho-Faddeev-Niemi-Shabanov decomposition [14, 15, 16], which identifies the Abelian degrees of freedom in a gauge-invariant means of specifying the Abelian dynamics of a gauge-invariant condensate. This led to a long-running controversy surrounding the condensate’s stability, with recent papers [7, 8, 9, 10, 11, 12] concluding in the positive.

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These problems are avoided by the Cho-Faddeev-Niemi-Shabanov decomposition [14, 15, 16], which identifies the Abelian directions without choosing a special gauge. It does this by introducing the Cho connection, a topologically generated contribution to the gluon field which represents a monopole potential. Thus the problems of gauge invariance and demonstrating the magnetic condensate to be of monopole origin are solved simultaneously.

Identifying the Abelian directions of freedom in a gauge invariant manner allows us to consider them as physical entities, and not as gauge artefacts. Furthermore, these particular physical entities are colour-neutral, and the primary claim of this paper is that they are not confined. We therefore refer to them as Free Abelian Gluons (FAGs).

Unconfined gluonic colour-singlets have been discussed in the QCD literature as glueballs for some time [18, 19]. However, the FAGs discussed in here are different because they consist of just one gluon, rather than the two or three confined within gluonium glueballs. Of course, when observed from the outside they are only distinguished by their masses and, as we shall see in section 4, their decay modes.

Section 2 presents the CFNS decomposition for general $SU(N)$ gauge groups. Section 3 justifies the claim that the Abelian generators are colourless and unconfined, while section 4 uses the condensate coupling and dual-superconductor analogy to put an upper limit on the FAG’s mass, and then goes on to discuss other properties such as stability and decay modes. Some experimental signatures are proposed. The paper concludes with a discussion in section 5.

2 Specifying Abelian Directions

The CFNS decomposition was first presented by Cho [17], and later by Faddeev and Niemi [15] and by Shabanov [16], as a gauge-invariant means of specifying the Abelian dynamics of two-colour QCD. These authors [14, 15] also applied it to three-colour QCD. In this section we adapt it to general $SU(N)$, although we are not the first to do so [20, 21], and establish our notation.

The Lie group $SU(N)$ for $N$-colour QCD has $N^2 - 1$ generators $\lambda^{(i)}$, of which $N - 1$ are Abelian generators $A^{(i)}$. For simplicity, we specify the gauge transformed Abelian directions with $\hat{n}_i = U \lambda^{(i)} U$. Fluctuations in the $\hat{n}_i$ directions are described by $c_{i\mu}^{(i)}$. The gauge field of the covariant derivative which leaves the $\hat{n}_i$ invariant is implicitly defined by

$$ g V_\mu \times \hat{n}_i = -\partial_\mu \hat{n}_i, \tag{1} $$

for which the general form is

$$ V_\mu = \epsilon^{(i)}_{\mu} \hat{n}_i + B_\mu, \quad B_\mu = g^{-1} \partial_\mu \hat{n}_i \times \hat{n}_i, \tag{2} $$

where $\epsilon^{(i)}_{\mu}$ is the totally antisymmetric tensor and $B_\mu$ is the magnetic field.
where the last term vanishes. It is easily shown that the monopole field strength
\[ H_{\mu\nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu} + gB_{\mu} \times B_{\nu}, \]
has only \( \hat{n}_i \) components, i.e.
\[ H_{\mu\nu}^{(i)} \hat{n}_i = H_{\mu\nu}, \]
where \( H_{\mu\nu}^{(i)} \) has the eigenvalue \( H^{(i)} \). Since we are only concerned with magnetic backgrounds, \( H^{(i)} \) is considered the magnitude of a background magnetic field \( H \).

\[ X_{\mu} \] contains the dynamical degrees of freedom (DOF) perpendicular to \( \hat{n}_i \), so if \( A_\mu \) is the gluon field then
\[ A_\mu = V_\mu + X_\mu = c_\mu^{(i)} \hat{n}_i + B_\mu + X_\mu, \]
where
\[ X_{\mu} \perp \hat{n}_i. \]  

This appears to leave the gluon field with additional DOF due to \( \hat{n}_i, B_\mu \), but detailed analyses can be found in \cite{8, 11, 22, 23} demonstrating that these fields are not fundamental, but a compound of dynamic fields. Hence \( \hat{n}_i, B_\mu \) are dynamic but do not constitute extra DOFs.

Substituting the CFN decomposition into the QCD field strength tensor gives
\[ F^2 = \delta c_{\mu}^{(i)} - \delta c_{\mu}^{(i)} \right)^2 + (\partial_\mu B_\nu - \partial_\nu B_\mu + gB_\mu \times B_\nu)^2 \]
\[ + 2(\partial_\mu c_\mu^{(i)} - \delta c_{\mu}^{(i)}) \hat{n}_i \cdot (\partial_\mu B_\nu - \partial_\nu B_\mu + gB_\mu \times B_\nu) \]
\[ + (\partial_\mu X_\nu - \partial_\nu X_\mu + gB_\mu \times B_\nu)^2 \]
\[ + 2g(\partial_\mu c_\mu^{(i)} - \delta c_{\mu}^{(i)}) \hat{n}_i + \delta c_{\mu}^{(i)} B_\nu - \partial_\nu B_\mu + gB_\mu \times B_\nu \]
\[ + g^2 (X_\mu \times X_\nu)^2 + 2g(\partial_\mu X_\nu - \partial_\nu X_\mu) \cdot (X_\mu \times X_\nu). \]

This expression holds for all \( N \)-colour QCD except \( N = 2 \) where the last term vanishes.

The kinetic terms for \( c_\mu^{(i)} \) are unmistakably those of Abelian fields. Eq. (5) has its analogue in studies \cite{12, 27, 24} utilising the maximal Abelian gauge. However, dependence on a particular gauge casts a shadow on any analysis and makes it impossible to consider the corresponding DOFs as physically significant. However the CFNS decomposition allows the Abelian dynamics to be specified in a gauge-invariant, well-defined manner that makes it physically meaningful to say that the fields \( c_\mu^{(i)} \) describe the Abelian component of the gluon field.

The CFN decomposition also introduces the additional gauge symmetry \( SU(N) \} / \{ U(1)^{(N-1)} \} \) corresponding to the gauge transformations of the \( \hat{n}_i \), in addition to the original \( SU(3) \). These additional degrees of freedom can be removed by imposing the condition eq. (7) with the gauge condition \cite{22}
\[ \hat{D}_\mu X_\mu = 0. \]

Alternatively, one may impose a stronger condition \cite{23} if Gribov copies are a concern, such as in calculations beyond one loop or in lattice studies. Either way, the decomposition is left invariant under the “active” \cite{22} gauge transformation
\[ \delta c_\mu^{(i)} = \hat{n}_i \cdot (\partial_\mu \alpha), \]
\[ \delta B_\mu = (\partial_\mu \alpha) \times \hat{n}_i + gB_\mu \times \alpha, \]
\[ \delta X_\mu = gX_\mu \times \alpha. \]

and we are back to the original \( SU(3) \) gauge symmetry as conventional QCD, i.e. without the CFNS decomposition, before gauge fixing. One may now impose a conventional gauge condition, such as Landau gauge \cite{23}, by fixing the gauge for \( c_\mu^{(i)} \) and \( B_\mu \).

A crucial observation of eq. (10) is that \( X_\mu \) transforms like a source, such as a quark, while the transformation of \( c_\mu^{(i)} \) is photon-like, consistent with both colour-neutrality \cite{8} and the form of the corresponding kinetic terms in eq. (8). This interpretation in \( SU(3) \) QCD leaves six off-diagonal, or valence, gluons \( X_\mu \) corresponding to the six non-white colour-anticolour combinations, and two colour-neutral Abelian gluons. Consistent with this picture, the quark states may be chosen to be eigenvectors of all the Abelian generators simultaneously, since they commute with each other, providing a gauge-invariant way to define colour charge \cite{25, 26}.

3 Abelian Gluons are Unconfined

The previous section demonstrated two important things. The first is that the Abelian components of the gluon field can be meaningfully separated from the off-diagonal components. The second is that these two different gluon types have different gauge transformation properties (see eq. (10)), which indicate that the Abelian components are photon-like while the off-diagonal gluons should be regarded as coloured sources.

We let consider this point in the context of the dual-Meissner effect model of confinement. The coloured sources, quarks and valence gluons, are connected to each other by flux tubes which bind them into either glueball or hadron states. It follows from both the infrared Abelian dominance \cite{25, 26, 27, 28, 29, 30} and the dual-Meissner analogy that this tube will be filled predominantly with the Abelian gluons, at confinement scales. However, as we shall now argue, the Abelian gluons themselves are not confined by this mechanism.

Crucially, the Abelian gluons do not carry colour charge, as indicated by equation (10), and nor do they couple to each other, as required for consistency. This follows from the definition of Abelian generator and is easily seen from the structure constants.

This is not to say that Abelian gluons can travel without restriction. The lagrangian \cite{8} contains the term
\[ 2(\partial_\mu c_\mu^{(i)} - \delta c_{\mu}^{(i)}) \hat{n}_i \cdot (\partial_\mu B_\nu - \partial_\nu B_\mu + gB_\mu \times B_\nu), \]

which clearly indicates that the monopole background acts like a sink/source for Abelian fields whose gauge bosons must therefore have an effective mass. It follows that an Abelian gluon
attempting to leave its flux tube must possess enough energy to overcome its mass-gap, as expected by a model based on analogy with conventional magnetic fields restricted to flux tubes in a type II superconductor, but is then free to propagate throughout space.

Significantly, there is no corresponding sink/source term for the valence gluons, although it has been argued [11,31] that a mass-gap term is generated for them from the interaction term

\[(\partial_\mu B_\nu - \partial_\nu B_\mu + g B_\mu \times B_\nu) \cdot (X_\mu \times X_\nu).\] (12)

4 The Properties of FAGs

As discussed above, FAGs are Abelian fields, but massive ones. The mass, in principal, is calculable from the properties of QCD. Specifically, the FAG mass is analogous to the photon mass in a superconductor, which varies inversely as the London penetration depth.

As is well-known (e.g. [32]), in type II superconductors the London penetration depth is greater than \(\frac{\lambda}{\sqrt{2}}\) times the correlation length, or

\[\lambda > \frac{\xi}{\sqrt{2}}.\]

This makes the corresponding photon/FAG mass less than \(\sqrt{2}\) times the mass gap, given by

\[\Delta = k_B T_c.\]

According to lattice and other numerical studies [33], this lies in the energy range 151-193 MeV, giving an upper limit to the FAG mass of

\[m_{FAG} < 193 \times \sqrt{2} = 274\text{MeV}.\] (13)

This is comparable to the \(\pi^0, \pi^\pm\) masses of 135,140 MeV respectively.

Apart from the specific mass value, the FAG has properties very similar to the \(Z_0\), except that it couples only to quarks. Hence it can couple to a quark-antiquark pair, and from there to a photon and an \(e^+ - e^-\) pair. Another decay mode is into a \(\pi^0\), with a photon to conserve angular momentum. While such decay products are by no means unique, the existence of FAGs would provide a resonance to these products, providing a signal that would be unmistakably stronger at the LHC than at any \(e^+ - e^-\) collider at the same energy, due to the quark-only coupling at the bare level.

Another interesting possibility is the interception of a FAG by a virtual pion emitted by a hadron. The virtual pion could absorb the FAG and use its energy to become real while emitting a photon. Note that both neutral and charged pions can participate in this reaction, so a proton could stimulate a FAG to become a \(\pi^0\) and emit a photon, or to become a \(\pi^\pm\) (and emit a photon) while turning itself into a neutron.

This is quite different from the double-meson decay mode of conventional gluonium.

We end this section with a brief discussion of the internucleon potential. In principle, FAGs ought to contribute a spin-dependent, short-range, van der Waals interaction between hadrons.

Unfortunately, any FAG signal in polarized hadron-hadron scattering will certainly be swamped by meson interactions, since mesons have approximately half of the mass expected for FAGs, and obscured by the uncertainties in the inter-hadron potential as well as by the hadrons’ complicated spin structure [34,35].

5 Discussion

We have made a case for non-gluonium gluonic colour singlets. It is based on the observation that two of the eight gluon generators in three-colour QCD are without colour charge, and that it is colour which is confined. The argument requires the CFNS decomposition to identify the Abelian directions in a gauge invariant way. Without this it is impossible to claim that the Abelian degrees of freedom have real physical meaning. It has been noted that the analysis is not sensitive to the number of colours, with one Abelian degree of freedom in the two-colour case and \(N - 1\) of them for \(N\)-colour QCD.

An attractive feature of the CFNS decomposition, which makes it useful in dual-Meissner effect studies, is that it unambiguously identifies the gluon’s monopole degrees of freedom [15][16][17]. It is easily shown furthermore, at least to one-loop order, that the corresponding monopole condensate is non-zero [1][24]. Especially important for this work, a term describing the condensate acting as a sink/source appears. This not only allows the condensate to restrict the chromoelectric flux to flux tubes as required by the dual Meissner effect, but also provides a mass gap for unconfined gluon fluctuations. This mass gap has, according to the dual superconductor analogy, an upper bound, given by eq. (13), of 274 MeV.

As mentioned in the introduction, the gluonic colour-singlets discussed here are conceptually distinct from conventional glueballs made up of multiple, bound gluons (gluonium), despite being experimentally distinguishable only by the decays. The ideal FAG consists of one single gluon whose decay modes, discussed in section 3, do not include gluonium’s decay to meson pairs (see [19] and references therein).

Finally, we have identified some experimental signatures of FAGs. Firstly, as a resonance to \(e^+ - e^-\) pairs or to photon-\(\pi^0\) production which would be more apparent at the LHC than at an \(e^+ - e^-\) detector. Secondly, the catalysis of pion production by hadrons.

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References

1. G.K. Savvidy. Infrared instability of the vacuum state of gauge theories and asymptotic freedom. *Phys. Lett.*, B71:133, 1977.
2. N.K. Nielsen and P. Olesen. An unstable yang-mills field mode. *Nucl. Phys.*, B144:376, 1978.
3. K.-I. Kondo and Y. Taira. Non-abelian stokes theorem and quark confinement in su(n) yang-mills gauge theory. *Prog. Theor. Phys.*, 104:1189–1265, 2000.
4. J. Honerkamp. The question of invariant renormalizability of the massless Yang-Mills theory in a manifest covariant approach. *Nucl. Phys.*, B48:269–287, 1972.

5. V. Schanbacher. Gluon propagator and effective Lagrangian in QCD. *Phys. Rev.*, D26:489, 1982.

6. W. Dittrich and M. Reuter. Effective QCD Lagrangian with zeta function regularization. *Phys. Lett.*, B128:321.

7. C.A. Flory. Covariant constant chromomagnetic fields and elimination of the one loop instabilities. *SLAC-PUB-3244*, 1983.

8. Y.M. Cho and D.G. Pak. Monopole condensation in SU(2) QCD. *Phys. Rev.*, D65:074027, 2002.

9. Y.M. Cho, M.L. Walker, and D.G. Pak. Monopole condensation and confinement of color in SU(2) QCD. *JHEP*, 05:073, 2004.

10. Y.M. Cho and M.L. Walker. Stability of monopole condensation in SU(2) QCD. *Mod. Phys. Lett.*, A19:2707–2716, 2004.

11. K.-I. Kondo. Magnetic condensation, abelian dominance and instability of Savvidy vacuum. *Phys. Lett.*, B600:287–296, 2004.

12. D. Kay, A. Kumar, and R. Parthasarathy. Savvidy vacuum in SU(2) Yang-Mills theory. *Mod. Phys. Lett.*, A20:1655–1662, 2005.

13. G. ’t Hooft. Topology of the gauge condition and new confinement phases in nonabelian gauge theories. *Nucl. Phys.*, B190:455, 1981.

14. Y.M. Cho. Colored monopoles. *Phys. Rev. Lett.*, 44:1115, 1980.

15. L.D. Faddeev and A.J. Niemi. Decomposing the Yang-Mills field. *Phys. Lett.*, B464:90–93, 1999.

16. S.V. Shabanov. An effective action for monopoles and knot solitons in Yang-Mills theory. *Phys. Lett.*, B458:322–330, 1999.

17. Y.M. Cho. A restricted gauge theory. *Phys. Rev.*, D21:1080, 1980.

18. H. Fritzsch and P. Minkowski. PSI resonances, gluons and the Zweig Rule. *Nuovo Cim.*, A30:393, 1975.

19. V. Mathieu, N. Kochelev, and V. Vento. The Physics of Glueballs. *Int. J. Mod. Phys.*, E18:1–49, 2009.

20. L.D. Faddeev and A.J. Niemi. Partial duality in SU(n) Yang-Mills theory. *Phys. Lett.*, B449:214–218, 1999.

21. S. Li, Y. Zhang, and Z.-Y. Zhu. Decomposition of SU(n) connection and effective theory of SU(n) QCD. *Phys. Lett.*, B487:201–208, 2000.

22. W.S. Bae, Y.M. Cho, and S.W. Kim. QCD versus Skyrme-Faddeev theory. *Phys. Rev.*, D65:025005, 2002.

23. K.-I. Kondo, T. Murakami, and T. Shinohara. BRST symmetry of SU(2) Yang-Mills theory in Cho-Faddeev-Niemi decomposition. *Eur. Phys. J.*, C42:475–481, 2005.

24. H. Flyvbjerg. Improved QCD vacuum for gauge groups SU(3) and SU(4). *Nucl. Phys.*, B176:379, 1980.

25. K.-I. Kondo. Abelian magnetic monopole dominance in quark confinement. *Phys. Rev.*, D58:105016, 1998.

26. Y.M. Cho. Abelian dominance in Wilson loops. *Phys. Rev.*, D62:074009, 2000.

27. A. di Giacomo. Monopole condensation and colour confinement. *Prog. Theor. Phys. Suppl.*, 131:161–188, 1998.

28. M.I. Polikarpov. Recent results on the abelian projection of lattice gluodynamics. *Nucl. Phys. Proc. Suppl.*, 53:134–140, 1997.

29. R.C. Brower, K.N. Orginos, and C.-I. Tan. Magnetic monopole loop for the Yang-Mills instanton. *Phys. Rev.*, D55:6313–6326, 1997.

30. Z.F. Ezawa and A. Iwazaki. Abelian dominance and quark confinement in Yang-Mills theories. *Phys. Rev.*, D25:2681, 1982.

31. M.L. Walker. Stability of the magnetic monopole condensate in three- and four-colour QCD. *JHEP*, 01:056, 2007.

32. A. A. Abrikosov. Nobel Lecture: Type-II superconductors and the vortex lattice. *Rev. Mod. Phys.*, 76:975–979, 2004.

33. Y. Aoki et al. The QCD transition temperature: results with physical masses in the continuum limit II. *JHEP*, 06:088, 2009.