Image segmentation application combined with DRLSE and moving mesh method

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Abstract. Aiming at the time-consuming problem of level set method in processing image segmentation, this paper presents an image segmentation algorithm combining DRLSE and moving mesh method. This method is applied to segment images into multiple blocks in different initial regions. When a large image needs to be segmented into multiple blocks, the level set method has to calculate various data at each pixel. So, there are problems such as low-efficiency and time-consuming processing. In order to solve this problem, we propose a moving mesh method to process image segmentation based on the level set method. By calculating the monitor function about the image gradient, the grid encryption is automatically realized with the change of image gradient. This makes the calculation of the image segmentation more accurate. In order to improve efficiency, our method will reduce the grid to 1/4. Then the level set method is applied to achieve the result of computational acceleration. In order to verify the performance and accuracy of the new method, our method segmented some images several times. Experiments show that our method can quickly and automatically segment some images into multiple blocks without affecting the results.

1. Introduction

Image segmentation is an indispensable method for extracting quantitative information in images. It is also a prerequisite for visualizing pre-processing. Moreover, it is of great significance in computer vision and pattern recognition. The quality of image segmentation directly affects the quality of recognition and understanding, and is a key technology for successful image analysis and description.

The level set method is mainly developed from research areas such as interface propagation. It is an effective computational tool that can handle geometric topological changes in closed motion interfaces over time. The method is stable in calculation. It can obtain a limited high-precision difference format, and it is easy to extend from low-dimensional to high-dimensional. Essentially, the level set method is based on gradient changes in the image. For contaminated images, it is difficult to give correct segmentation results based on the gradient of the image. For this reason, we can consider improving the image segmentation of the level set method by moving mesh method.

Osher and Sethian [1] first proposed the level set description of the motion interface related to time. The evolution equation of the level set function can be obtained from the evolution equation of the closed hypersurface. This embedded closed hypersurface is always its zero-level set. In the end, as long as the zero-level set is determined, the result of mobile interface evolution can be determined. Level set segmentation can handle sharp corners and has a powerful ability to change the topology. It can separate the boundaries of objects with a high complexity, which is difficult to achieve through methods such as active contours. In this way, the advantages of the level set method are more obvious.
Since the level set method was proposed, it has been widely used in the fields of image processing and computer vision. For example, Osher and Sethian [1] uses level set method to remove image noise. Bertalmio et al. [2] applied level set method to image deformation and damaged image restoration. Masouri et al. [3] applies level set method to the field of moving target tracking. Paragios and Deriche [4] used level set method for texture segmentation and moving target segmentation and tracking. Samson et al. [5] used the level set method to classify images.

In this paper, an image segmentation method combining the level set method [6-9] and the moving mesh method [10-12] is presented. First, before using the level set method, the image is filtered to effectively suppress the noise of the image. And the image edge indicator function is calculated. Then the moving mesh is calculated on the 1/4 grid. And then edge indicator function on the moving mesh is calculated. Finally, the image is segmented on computational domain by using DRLSE [6-7]. Our method uses of 1/4 grid to reduce calculation time while maintaining good results.

2. Distance regularized level set evolution (DRLSE)

In the traditional curve evolution process, the level set method needs to be re-initialized. Moreover, there are irregularities in evolution. In order to make up for the shortcomings of the traditional method, Li et al. [6-7] used a signed distance function for initialization. Distance regularized level set method is proposed to constrain the stability of curve evolution. Their algorithm is composed of a distance regularization term and an external energy term that drives the contour to the desired contour.

The distance regularization term is a penalty function for the gradient of the level set function. This function forces the gradient magnitude of each point to change to one of the minimum values (0 or 1) of the potential function in the penalty term. This potential function is called a double-well potential function. The signed distance function is used for initialization. This function constructs to force LSF to evolve into a form close to the signed distance function.

Let $\phi$ is a level set function. According to the law of motion of the curve, the energy functional is defined as:

$$
E(\phi) = \mu R_p(\phi) + E_{ext}(\phi)
$$

where $R_p(\phi)$ is the distance regularization term, $\mu$ is a constant greater than zero, and $E_{ext}(\phi)$ is the external energy term. It is the minimization target of the algorithm.

The distance regularization term of the level set is defined as:

$$
R_p(\phi) = \int p(|\nabla \phi|)\,dx
$$

where $p$ is the potential function, $p : [0, \infty) \rightarrow \mathbb{R}$. The double-well potential function used in equation (2) is $p_2$ ($p_2$ is $p$ in the previous formula):

$$
p_2(s) = \begin{cases} 
\frac{1-\cos(2\pi s)}{(2\pi)^2} & \text{if } s \leq 1 \\
\frac{(s-1)^2}{2} & \text{if } s > 1
\end{cases}
$$

where $s = |\nabla \phi|$ is the gradient magnitude. The double-well potential function (3) has two minimum values at $|\nabla \phi| = 0$ and $|\nabla \phi| = 1$. One of the features of this double-well potential function is that, when $|\nabla \phi|<0.5$, curve diffusion will make $|\nabla \phi|$ evolve to 0. When $|\nabla \phi|\geq0.5$, it will make $|\nabla \phi|$ evolve to 1.

The external energy term is defined as:

$$
E_{ext}(\phi) = \lambda L_g(\phi) + \alpha A_g(\phi)
$$

$$
L_g(\phi) = \int g\delta(\phi)\,|\nabla \phi|\,dx
$$

$$
A_g(\phi) = \int g H(-\phi)\,dx
$$

where $L_g(\phi)$ is the line integral on the contour, $A_g(\phi)$ is the energy integral inside the contour, $g$ represents the edge indicator function, $H$ and $\delta$ are the Heaviside function and the Dirac function.
respectively. The algorithm in this paper comes from [6-7], and the experimental default parameters are: $\Delta t = 1$, $\mu = 0.2$, $\lambda = 5$, $\alpha = -3$. The contour line is a collection of the points whose value of $\phi = 0$. Since the gradient magnitude is basically 0 far away from the contour, $\phi$ is basically unchanged. Therefore, each iteration of evolution will only update the value near the contour instead of updating the value of $\phi$ on the entire image, the efficiency of the algorithm can be improved. At the same time, the algorithm is very robust and the evolution is very stable. Even if the initial contour is relatively poor, good results can be segmented.

3. Moving mesh method

The level set method is based on the gradient changes in the image. The moving mesh equation can control the grid distribution on the computational grid, the grid can be encrypted where the image gradient is high. As a result, the image gradient will become more uniform on the computational grid.

In the physical domain, the encryption of the mesh can vary depending on the image gradient through a monitor function. In this paper, the moving mesh makes the image gradient uniform to achieve the purpose of segmenting the target image.

Let $(x, y)$ and $(\xi, \eta)$ denote the physical and computational coordinates, respectively. Let $u$ denotes image value. Since the grid encryption is determined by image gradients, we use the following monitor function:

$$\omega = \left[ 1 + 100(u_x^2 + u_y^2) \right]^{1/2} \quad (7)$$

In order to make the monitor function not too concentrated, we will do the following dispersions 10 times:

$$\omega_{j,k} \leftarrow \frac{1}{8} \left( 4\omega_{j,k} + \omega_{j+1,k} + \omega_{j-1,k} + \omega_{j,k+1} + \omega_{j,k-1} \right) \quad (8)$$

Ceniceros and Hou’s mesh generator is defined as in [10]:

$$\begin{align*} 
(\omega x_\xi)_\xi + (\omega y_\xi)_\eta &= 0, \\
(\omega x_\eta)_\xi + (\omega y_\eta)_\eta &= 0
\end{align*} \quad (9)$$

It follows from the above equation that

$$x_{j,k} = \frac{\omega_{j+1/2,k}x_{j+1/2,k} + \omega_{j-1/2,k}x_{j-1/2,k} + \omega_{j+1/2,k+1/2}x_{j+1/2,k+1/2} + \omega_{j-1/2,k+1/2}x_{j-1/2,k+1/2}}{\omega_{j+1/2,k + \omega_{j-1/2,k} + \omega_{j+1/2,k+1/2} + \omega_{j-1/2,k+1/2}}} \quad (10)$$

Similar approximation is used for the equation in (10) to obtain $y_{j,k}$. We will do 10 moving mesh updates by using equation (10). In order to prevent errors in the boundary region, the boundary and the 2 grids next to the boundary are not moved.

4. Application to image segmentation

The method in this paper is to introduce the moving mesh method to process image segmentation based on the DRLSE [6-7]. Considering that the image is to be segmented multiple times, it is necessary to improve efficiency without reducing accuracy. This method allows the use of 1/4 grid to significantly reduce calculation time while maintaining good results. The grid is concentrated on the high gradient region. This method allows the use of 1/4 grid to significantly reduce the computation time. In addition, the termination condition of the method in this paper is a fixed number of iterations to avoid an endless loop.

The DRLSE method is based on the gradient change in the image. So, the method of this paper takes the image data from the physical coordinates, but the image segmentation using DRLSE is calculated on the computational coordinates. When calculating image segmentation, the gradient changes will be more average.

First, the image will be subjected to a Gaussian filter. Let $Img$ denotes the image. We will use the following Matlab command to smooth the image:

$$G = \text{fspecial(’gaussian’,10,3)}$$
\[ \text{Img} = \text{conv2}(\text{Img}, G, \text{'same'}) \]

After that, the smoothed image is used to calculate the edge indicator function. We use the following edge indicator function:

\[
|\nabla G| = \left[ \left( \frac{\partial \text{Img} \text{smooth}}{\partial \xi} \right)^2 + \left( \frac{\partial \text{Img} \text{smooth}}{\partial \eta} \right)^2 \right]^{1/2}
\]

(11)

\[ g = e^{-|\nabla G|} \]

(12)

where \( g \) represents the edge indicator function. The physical coordinates \((x, y)\) are calculated from the smooth image according to the equation (10), as shown in Figure 1. Then, we use \texttt{interp2} in Matlab to calculate the two-dimensional interpolation of the edge indicator function in physical coordinates, denoted as \( g_{\text{mesh}} \), and the algorithm is \texttt{bicubic}. Finally, the DRLSE algorithm [6-7] is used to calculate the target contour. We will use \( g_{\text{mesh}} \) to replace the edge indicator function in [6-7], and the others are exactly the same. In addition, all calculations are performed on the computational coordinates.

![Figure 1. The left is the original image, and the right is the grid distribution of moving mesh.](image)

Obviously, the density of the mesh is changed in response to the image gradient.

The goal of this article is to automatically segment the image into multiple blocks, so we will draw 100 squares (the number can be changed) in the smooth image in average, as shown in Figure 2. We find out the least image difference among 100 square and set it as the initial region \( \Omega_0 \). \( \phi \) is initialized as a signed distance function:

\[
\phi = \begin{cases} -c_0 & \text{if } \mathbf{x} \in \Omega_0 \\ c_0 & \text{if } \mathbf{x} \notin \Omega_0 \end{cases}
\]

where the parameter \( c_0 = 2 \). Then we use DRLSE code once, and write down the contour after the calculation is completed. If there is a negative value of \( \phi \) in all 100 squares, or if the maximum difference in these remaining squares is greater than 10% of the maximum difference in the whole image, the code stops running. If there are any squares in which \( \phi \) is full positive, and the maximum difference of these squares is less than 10% of the maximum difference in the whole image, then the smallest of the maximum difference in these squares is set to be the new initial region. The DRLSE code is run again until the negative value of \( \phi \) exists in all 100 squares.

In order to verify the performance and accuracy of the image segmentation method in this paper, we compare the moving mesh method with the DRLSE method [6-7], and the comparison results are shown in Figure 3. Table 1 shows the results of the time taken by different algorithms.

| Image/operation time (s) | Our method (1/4 grid) | DRLSE (whole grid) |
|--------------------------|-----------------------|--------------------|
| Knee                     | 52 s                  | 404 s              |
| CT Aorta                 | 302 s                 | 2638 s             |
5. Conclusion
A new numerical method is proposed, which combines the level set method with the moving mesh method to automatically segment the image into multiple blocks. Numerical test problems validate this new method. The test problem shows that the current method is more accurate than the method with a unified grid structure. The experimental results show that no matter where the initial region is, a good image segmentation can be obtained. The traditional level set methods need to manually set the initial region. The image segmentation method proposed in this paper can set the initial region automatically. In addition, our algorithm can reduce the grid without affecting the accuracy. As a result, the efficiency of the computation can be improved significantly. This method can quickly segment images. Our future work will focus on other kinds of moving mesh method.

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