Optical frequency combs have become indispensable tools for precision measurements including atomic/molecular spectroscopy, low-phase-noise microwave generation, and ranging. In optical clocks, linking different atomic clocks or distant clocks via optical fibers with a fractional uncertainty of $10^{-15}$ is of significant concern with a future redefinition of the second in the International System of Units (SI). Such endeavors, in turn, offer intriguing opportunities for testing the constancy of the fundamental constants and for relativistic geodesy. These applications necessitate ultralow-noise optical frequency combs that allow long-term and robust operation. Titanium–sapphire-based frequency combs have demonstrated outstanding stability. However, their bulky optical setups require regular maintenance, thus hampering long-term and robust operation, which limits their potential applications. In contrast, erbium (Er) fiber combs enable all-fiber architecture for robust operation. Although a typical Er: fiber comb uses nonlinear polarization rotation (NPR) to acquire mode-locking with excellent noise performance, the operational condition for such NPR-based Er: fiber oscillators can be sensitive to environmental conditions. For practical applications, such as the long-term operation of clocks to generate the optical second and field and space applications, the all-polarization-maintaining (PM) architecture is preferred. However, such architecture has shown relatively large phase noise. Low intrinsic phase noise with PM architecture is demonstrated by applying a nonlinear amplifying loop mirror (NALM).

In linking multiple optical frequencies, Er: fiber combs with a multibranch configuration, where each port consists of an Er-doped fiber amplifier (EDFA) and a highly nonlinear fiber (HNLF), have been employed as it allows sufficient output power per comb tooth optimized for the single frequency. In such a multibranch comb, the phase noise in different branches introduces the instability of $\sim 10^{-16}$ at 1 s. A record high instability of $4 \times 10^{-16}$ is recently demonstrated for synchrononous clock comparison between strontium (Sr)- and ytterbium (Yb)-based optical lattice clocks, in which the instability is mainly limited by the fiber effect due to the frequency noise of the multibranch comb.

The single-port architecture is advantageous for suppressing such interbranch relative phase noise that is caused by the optical path length fluctuation. Moreover, in order to access multiple optical clocks with different frequencies, an octave-spanning super-continuum (SC) output with a sufficient signal-to-noise ratio (SNR) is favored. In this work, we develop a low-noise and single-port Er: fiber comb by utilizing an NALM-based all-PM architecture. A uniformly broadened high-SNR comb over 135–285 THz allows linking Sr-, Yb-, and mercury (Hg)-based optical lattice clocks, which operate at 429–1129 THz, with a modified Allan deviation below $10^{-17}$ at $\tau = 1$ s. By applying synchronous operation, we show that an optical lattice clock comparison with $2 \times 10^{-17}$ is possible, which is one order of magnitude smaller than the best instability of the frequency ratio of optical lattice clocks.
than 30 dB measured with the resolution bandwidth (RBW) of 100 kHz or the phase-noise floors smaller than $-80 \text{dBc/Hz}$. This sets the noise floor of the frequency ratio measurement with the power spectrum density (PSD) of $1 \times 10^{-18} f/\text{Hz}$ $1/\sqrt{\text{Hz}^2}$ where $f$ is the Fourier frequency of the noise component. The SNR degradation from the Er:fiber oscillator output to the SC output was observed to be less than 20 dB, which is in agreement with the spectral broadening of about 15 ($=150 \text{THz}/10 \text{THz}$). The carrier-envelop offset frequency $f_{\text{CEO}}$ is obtained using a self-referencing interferometer with the phase-noise floor of $-100 \text{dBc/Hz}$, as shown in the lower panel in Fig. 1(b).

The spectral shape and bandwidth of the SC output is optimized by the current supplied to pumping laser diodes (LDs) for the EDFA. Once optimized, the beat signals, as shown in Fig. 1(b), maintain their SNRs without daily adjustment, which demonstrates the long-term stable operation of the system. In about 85% of the spectral region in the range of 135–285 THz, we observed a tooth power of $>10 \text{nW}$ at $215 \text{THz}$, which is locked to a 40-cm-long reference cavity.21) Lasers at 259 and 282 THz are stabilized to respective teeth of Er comb (1) in Fig. 1(a). In the following, we analyze the beat signals of these lasers and Er comb (2) with the measurement bandwidth of 2 MHz by using in- and quadrature-phase demodulators based on analog frequency mixers and the RF reference. Most of the optical paths connecting the two combs with PM fibers and free-space optics are stabilized using interferometer-based Doppler noise cancellers (DNCs),22) where an approximately 10-cm-long optical path remains uncompensated. Since the frequency noise of the spectral transfer via the comb typically shows white phase noise characteristics, we use the modified Allan deviation to indicate the instability. Figure 3 shows the modified Allan deviation of the fractional frequency noise observed in the spectral transfer from 215 to 259 THz (blue) and to 282 THz (red). This shows instabilities of $(5–7) \times 10^{-18}$ at $\tau = 1 \text{s}$ and $1 \times 10^{-19}$ at $\tau = 100 \text{s}$, which is similar to those described in Refs. 12 and 13. Compared with the instability using a multibranch NPR-based Er:fiber comb10,21) as shown by a black line, the spectral transfer instability at $\tau = 1 \text{s}$ is improved by more than 30 times. At $\tau > 100 \text{s}$, the

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**Fig. 1.** (a) Optical configuration of a single-port, all-PM Er:fiber comb and signal detections for frequency ratio measurement of the optical lattice clocks consisting of Sr, Yb, and Hg. Periodically poled lithium niobate (PPLN) is used for the self-referencing $f/2f$ interferometer. PR, partial reflectors; SHG, second-harmonic generator; BS, beam splitter; PD, photodetector; DBM, double-balanced mixer; OSC, oscillator. (b) RF spectra of beat signals at 215, 259, and 282 THz, and $f_{\text{CEO}}$ signal measured with RBW = 100 kHz in free-running operation.

**Fig. 2.** Optical spectra of the oscillator (blue) and the super-continuum (SC) output (red). The vertical axis stands for the optical power per comb tooth. The vertical lines show optical frequencies used to obtain the SC output (red). The vertical axis stands for the optical power per comb tooth. The phase-noise power spectrum density (PSD) of $1 \times 10^{-18} f/\text{Hz}$ is the Fourier frequency of the noise component. The phase-noise PSD in units of rad$^2$/Hz. This small RMS phase noise allows the tight locking of $f_{\text{CEO}}$ and $f_{\text{REP}}$ to the references without the need for extra devices such as transfer oscillators to reduce the noise bandwidth or frequency dividers to expand the frequency capture range. In our experiment, we stabilize $f_{\text{CEO}}$ by controlling the current of the pump LD with a unity-gain frequency of 0.2 MHz, and $f_{\text{REP}}$ by using a PZT for a slow signal (DC $-1 \text{kHz}$) and an EOM with a unity-gain frequency of 1.3 MHz. The residual contributed phase noise (integrated from 3 Hz to 5 MHz) of the in-loop beat signals for both $f_{\text{CEO}}$ and the beat signal at 215 THz is $\leq 0.2 \text{rad}$, which is comparable to the values reported for non-PM NPR-based Er:fiber combs.10) This residual phase noise is sufficiently small to remain locked for longer than a few days.17)
instability reaches the floor of around $10^{-19}$, which is most likely caused by the fluctuation of the residual uncompensated optical path length. Since this measurement utilizes an RF reference at 10MHz with a fractional frequency uncertainty of $10^{-12}$, this corresponds to a sub-$10^{-19}$ uncertainty. Within this statistical uncertainty, there is no obvious frequency offset.

Figure 4(a) shows the fractional frequency noise PSD of the frequency spectral transfer. For $f' \geq 100$ Hz, the spectral transfer noise is close to the limit estimated by the SNR of the beat signals except for a bump at around 1 kHz, which is caused by the insufficient control gain of DNCs. On the other hand, excess noise above the SNR limit is observed for $f < 100$ Hz. This is possibly due to the fluctuation of the uncompensated optical path length. Consequently, we achieved the fractional frequency PSD of $(1 - 2) \times 10^{-17} 1/Hz^{1/2}$ at 1 Hz for the spectral transfer between 215, 259, and 282 THz.

Finally, we discuss the fractional instability of the frequency ratio $R = \nu_1/\nu_2$ in clock comparison, which offers a benchmark for testing the short-term stability of the frequency comb used in atomic clocks. The fractional instability is limited by the quantum projection noise (QPN)\(^{23}\), $\sigma_{QPN}$, the Dick effect $\sigma_{Dick}$\(^{20}\), and the spectral transfer noise of the comb $\sigma_{comb}$ to bridge the two clock frequencies $\nu_1$ and $\nu_2$.

The overall instability is described as

$$\sigma = \sqrt{\sigma_{QPN}^2 + \sigma_{Dick}^2 + \sigma_{comb}^2}.\quad (1)$$

The QPN-limited instability for respective clocks $j = 1, 2$ is given by $\sigma_{QPN} \sim \nu_i / T_i (N_i / T_i)^{1/2}$, where $\nu_i$ is the transition frequency, $T_i$ the interrogation time of the clock transition, and $N_i$ the number of atoms interrogated in cycle time $T_i = T_i + 1$ s including the atom preparation time of 1 s, resulting in the total instability of $(\sigma_{QPN1}^2 + \sigma_{QPN2}^2)^{1/2}$ for two clocks. In optical lattice clocks interrogating $N_j > 10^3$ atoms, a QPN-limited instability better than $\sim 10^{-16}(r/s)^{-1/2}$ is achievable, which is comparable to or smaller than the Dick-effect-limited instabilities.\(^{24}\) Moreover, $\sigma_{QPN}$ can be further reduced by increasing $N_j$ and/or $T_i / T_C$.

The Dick-effect-limited instability $\sigma_{Dick}$ is caused by the down-conversion of the frequency noise of the “clock laser” that periodically interrogates the clock transition with a dead time,

$$\sigma_{Dick} = \left[ \frac{1}{T_s} \sum_{n=1}^{\infty} \left( \frac{g_n^c}{g_0} \right)^2 + \left( \frac{g_n^s}{g_0} \right)^2 \right]^{1/2} S_c(n/T_C)^{1/2},\quad (2)$$

where $S_c(f)$ is the fractional frequency noise PSD in units of $1/Hz$, $g_0$ is the 1-cycle average of a sensitivity function $g(t)$, and $g_n^c$ and $g_n^s$ are the cosine and sine components of the $n$-th Fourier series expansion of $g(t)$, respectively.\(^{20}\) As the sensitivity $[(g_n^c/g_0)^2 + (g_n^s/g_0)^2]^{1/2}$ rapidly decreases for $f \gg 1/T_i$, frequency noise with low Fourier components of a few Hz solely affects the clock instability $\sigma_{Dick}$. Note that the frequency noise of the clock laser with the state-of-the-art instability of $\sigma_c \sim 1 \times 10^{-16}$ at $\tau = 1$ s\(^{25-27}\) [green line in Fig. 4(a)] is an order of magnitude larger than the frequency transfer noise of the comb for $f < 100$ Hz, where the Dick effect plays a decisive role. Assuming this laser noise, the Dick effect limit of comparison of Yb (Hg) and Sr optical lattice clocks is calculated to be $\sigma_{Dick} \sim 10^{-16}/(r/s)^{-1/2}$ for each clock $j = 1$ and 2, as shown by the black circles in Fig. 4(b). This indicates that the thermal noise of an optical cavity\(^{28}\) used to stabilize the clock laser severely degrades the short-term instability of optical clocks, and the frequency transfer instability of the comb is not fully utilized.

The ratio measurement beyond the Dick effect limit of the “clock laser” is possible by applying synchronous interrogation to reject the laser frequency noise.\(^{1,24}\) When the
two clocks share a single cavity by transferring its spectral characteristics via the comb, the Dick effect term $\sigma_{\text{Dick}}$ in Eq. (1) is given by

$$
\sigma_{\text{Sync}}^2 = (\sigma_{\text{Dick}(\text{Cavity})}^2 + \sigma_{\text{Dick(Comb)}}^2)^{1/2},
$$

where the Dick effect due to the cavity-induced laser noise is partially rejected and reduced to $\sigma_{\text{Dick(Cavity)}}^2$ and the spectral transfer via the comb adds an extra Dick effect $\sigma_{\text{Dick(Comb)}}^2$. Employing a comb with lower frequency noise than the cavity thermal noise will allow $\sigma_{\text{Sync}}^2 < \sigma_{\text{Async}}^2$, as demonstrated in Ref. 3, where the synchronous frequency ratio measurement of optical lattice clocks.

The green circles in Fig. 4(b) indicate the instability for the multibranch Er:fiber comb. This work is partially supported by the Photon Frontier Network Program of the Ministry of Education, Culture, Sports, Science and Technology, Japan. We thank Dr. Takamoto and Dr. Nemitz for providing lasers of Sr and Yb clocks.

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