Investigation of Single-Phase Inverter and Single-Phase Series Active Power Filter with Sliding Mode Control

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1. Introduction

The effective operation of the power converters of electrical energy is generally determined from the chosen operational algorithm of their control system. With the expansion of the application of the Digital Signal Processors in these control systems, gradually entering of novel operational principles such as space vector modulation, fuzzy logic, genetic algorithms, etc, is noticed. The method of sliding mode control is applicable in different power electronic converters – DC/DC converters, resonant converters (Sabanovic et al., 1986). The method’s application is expanded in the quickly developing power electronic converters such as active power filters and compensators of the power factor (Cardenas et al., 1999; Hernandez et al., 1998; Lin et al., 2001; Mendalek et al., 2001).

In this chart, results of the study of a single-phase inverter and single-phase active power filter both with sliding mode control are discussed. A special feature is the use of control on only one output variable.

2. Single-phase inverter with sliding mode control

2.1 Schematic and operational principle

Different methods to generate sinusoidal voltage, which supplies different types of consumers, are known. Usually, a version of a square waveform voltage is generated in the inverter output and then using a filter the voltage first order harmonic is separated. Unipolar or bi-polar pulse-width modulation, selective elimination of harmonics, several level modulation – multilevel inverters are applied to improve the harmonic spectrum of the voltage (Antchev, 2009; Mohan, 1994). Inverters with sinusoidal output voltage are applicable in the systems of reserve or uninterruptible electrical supply of critical consumers, as well as in the systems for distributed energy generation.

In this sub-chart an implementation of sliding mode control of a single-phase inverter using only one variable – the inverter output voltage passed to the load, is studied. As it is known, two single-phase inverter circuits – half-bridge and bridge, are mainly used in practice (see Fig.1). The inverters are supplied by a DC voltage source and a LC-filter is connected in their outputs. The output transformer is required at use of low DC voltage, and under
certain conditions it may be missing in the circuit. The voltage passed to the load is monitored through a reverse bias using a voltage transducer. The use of a special measuring converter is necessitated by the need of correct and quick tracing of the changes in the waveform of the load voltage at the method used here. In the power electronic converters studied in the chart, measuring converter CV 3-1000 produced by LEM is applied.

Fig. 1. a) half-bridge and b) bridge circuits of an inverter

Fig. 2 displays a block diagram of the control system of the proposed inverter. The control system consists of a generator of reference sinusoid $U_m \sin \Theta$ (it is not shown in the figure), a comparing device, a comparator with hysteresis and drivers. The control system compares
the transitory values of the output voltage of the inverter to these values of the reference sinusoid and depending on the result (negative or positive difference) control signal is generated. The control signal is passed to the gate of the transistor VT1 or VT2 (for half-bridge circuit) or to the gates of the transistor pairs – VT1-VT4 or VT2-VT3 (for bridge circuit).

Fig. 2. Block diagram of the control system with hysteresis control

The process of sliding mode control is illustrated in Fig. 3. Seven time moments are discussed – from $t_0$ to $t_6$. In the moment $t_0$ the transistor VT1 of the half-bridge schematic, transistors VT1 and VT4 of the bridge schematic, respectively, turns on. The voltage of the inverter output capacitor C increases fed by the DC voltage source. At the reach of the upper hysteresis borderline $U_m \sin \Theta + H$, where in $H$ is the hysteresis size, at the moment $t_1$, VT1 turns off (or the pair VT1-VT4 turns off) and VT2 turns on (or the pair VT2-VT3).

The voltage of the capacitor C starts to decrease till the moment $t_2$, when it is equal to the lower hysteresis borderline $U_m \sin \Theta - H$. At this moment the control system switches over the transistors, etc. Therefore the moments $t_0, t_2, t_4 \ldots$ and moments $t_1, t_3, t_5 \ldots$ are identical.

Fig. 3. Explanation of the sliding-mode control
2.2 Mathematical description
Fig. 4 displays the circuit used to make the analysis of sliding mode control of the inverter. The power devices are assumed to be ideal and when they are switched over the supply voltage $U_d$ with altering polarity is passed to the $LC$-filter.

![Circuit diagram](image)

Fig. 4. Circuit used to make the analysis of sliding mode control of the inverter

The load current and the current of the output transformer, if it is connected in the schematic, is marked as $i_L$. From the operational principle, it is obvious that one output variable is monitored – the voltage of the capacitor $u_C$. Its transient value is changed through applying the voltage $U_d$ with an altering sign. The task (the model) is:

$$u_{REF} = U_M \sin \omega t$$  \hspace{1cm} (1)

As a control variable, the production $uU_d$ may be examined, where in:

$$u = \text{sgn}[(u_C - u_{REF}) - H]$$

$$u = +1 \text{ when } (u_C - u_{REF}) < H$$

$$u = -1 \text{ when } (u_C - u_{REF}) > H$$  \hspace{1cm} (2)

The following relationships are valid for the circuit shown in Fig.4:

$$U_d = u_C + L \frac{di_d}{dt}$$

$$i_d = i_C + i_L$$

$$i_C = C \frac{du_C}{dt}$$  \hspace{1cm} (3)

Using (3) and after several transformations, it is found:

$$\dot{u}_C = \frac{du_C}{dt} = \frac{1}{LC} \int U_d dt - \frac{1}{LC} \int u_C dt - \frac{i_L}{C}$$  \hspace{1cm} (4)

In conformity with the theory of sliding mode control, the following equations are written (Edwards & Spurgeon, 1998):

$$\begin{align*}
    x_d &= u_{REF} \\
    x &= u_C \\
    \dot{x} &= \dot{u}_C 
\end{align*}$$  \hspace{1cm} (5)
The control variable $u_{eq}$ corresponding to the so-called “equivalent control” may be found using the following equation (Utkin, 1977):

$$\dot{s} = \ddot{x} - \dot{x} = 0$$

(6)

Using (1) and (4) and taking in consideration (5) and (6), it is found:

$$u_{eq} = u_{d} = u_{C} + L \frac{d i_{L}}{d t} - L C \omega^{2} U_{M} \sin \omega t$$

(7)

The value found may be considered as an average value when the switching is between the maximum $U_{\text{MAX}}$ and minimum $U_{\text{MIN}}$ values of the control variable (Utkin, 1977; Utkin, 1992). If they could change between $+\infty$ and $-\infty$, in theory, there is always the probability to achieve a mode of the sliding mode control in a certain range of a change of the output variable. In order to be such a mode, the following inequalities have to be fulfilled:

$$U_{\text{MIN}} < u_{eq} < U_{\text{MAX}}$$

(8)

for physically possible maximum and minimum values. In this case they are:

$$U_{\text{MIN}} = -U_{d}$$

$$U_{\text{MAX}} = +U_{d}$$

(9)

Resolving (7) in respect to the variable, which is being monitored $u_{C}$, and substituting in (9), the boundary values of the existence of the sliding mode control could be found:

$$u_{C} = \pm U_{d} - L \left( \frac{d i_{L}}{d t} \right)_{t = \frac{\pi}{2} \omega} + L C \omega^{2} U_{M} \sin \omega t$$

(10)

The equation (10) may be interpreted as follows: a special feature of the sliding mode control with one output variable – the capacitor voltage, is the influence of the load current changes upon the sliding mode, namely, at a sharp current change it is possible to break the sliding mode control within a certain interval. From this point of view, it is more suitable to operate with a small inductance value. As the load voltage has to alter regarding a sinusoid law, let (10) to be analyzed around the maximum values of the sinusoid waveform. It is found:

$$u_{C} = \pm U_{d} - L \left( \frac{d i_{L}}{d t} \right)_{t = \frac{\pi}{2} \omega} + L C \omega^{2} U_{M} (\pm 1)$$

(11)

Where in (11) the positive sign is for the positive half period and the negative one – for the negative half period. After taking in consideration the practically used values of $L$ and $C$ (scores microhenrys and microfarads), the frequency of the supply source voltage ($f = 50$ or $60Hz$) and its maximum value $U_{M} (=325$ or $156V$), it is obvious that the influence of the last term could be neglected. Thus the maximum values of the sinusoidal voltage of the load are mainly limited from the value of the supply voltage $U_{d}$ and the speed of a change of the load current. So, from the point of view of the sliding mode control,
it is good the value of $U_d$ to be chosen bigger. Of course, the value is limited and has to be considered with the properties of the power switches implemented in the circuit.

2.3 Study through computer simulation

Study of the inverter operation is made using an appropriate model for a computer simulation. Software OrCad 10.5 is used to fulfill the computer simulation.

Fig.5 displays the schema of the computer simulation. The inverter operation is simulated with the following loads – active, active-inductive (with two values of the inductance – smaller and bigger ones) and with a considerably non-linear load (single-phase bridge uncontrollable rectifier with active-capacitive load). Only the load is changed during the simulations. The supply voltage of the inverter $U_d$ is 250V, $C= 120\ \mu F$ and $L = 10\ \mu H$.

![Fig. 5. Schematic for the computer simulation study of sliding mode control of the inverter](image)

The simulation results are given in Fig.6, Fig.7, Fig.8 and Fig.9. The figures display the waveform of the voltage feeding the load, and the load current, which is displayed multiplied by 100 for the first three cases and by 40 for the last one.
Fig. 6. Computer simulation results of the inverter operation with an active load equal to 500Ω using sliding mode control. Curve 1 – the voltage feeding the load, curve 2 – the load current

Fig. 7. Computer simulation results of the inverter operation with an active-inductive load equal to 400Ω/840μΗ using sliding mode control. Curve 1 – the voltage feeding the load, curve 2 – the load current

Fig. 8. Computer simulation results of the inverter operation with an active-inductive load equal to 400Ω/2Η using sliding mode control. Curve 1 – the voltage feeding the load, curve 2 – the load current
The results support the probability using sliding mode control on one variable – the output voltage, in the inverter, to obtain a waveform close to sinusoidal one of the inverter output voltage feeding different types of load.

### 2.4 Experimental study

Based on the above-made study, single-phase inverter with output power of 600VA is materialized. The bridge schematic of the inverter is realized using IRFP450 transistors and transformless connection to the load. The value of the supply voltage of the inverter is 360V. Fig.10, Fig.11, Fig.12 and Fig.13 display the load voltage and load current waveforms for the load cases studied through the computer simulation.

Fig. 10. The load voltage and load current in the case of active load
Fig. 11. The load voltage and load current in the case of active-inductive load with the smaller inductance.

Fig. 12. The load voltage and load current in the case of active-inductive load with the bigger inductance.
All results show non-sinusoidal part of the output voltage less than 1.5% as well as high accuracy of the voltage value – (230V ± 2%).

3. Single-phase series active power filter with sliding mode control

3.1 Schematic and operational principle

Active power filters are effective means to improve the energy efficiency with respect to an AC energy source as well as to improve energy quality (Akagi, 2006). Series active power filters are used to eliminate disturbances in the waveform of a network source voltage in such a way that they compliment the voltage waveform to sinusoidal voltage regarding the load. Usually pulse-width modulation is used to control the filters, but also researches of sliding mode control of the filters on several variables are known (Cardenas et al, 1999; Hernandez et al, 1998). In this sub-chart sliding mode control of a single-phase series active power filter on one variable – the supply voltage of the load is studied (Antchev et al, 2007; Antchev et al, 2008).

Fig. 14 shows the power schematic of the active power filter with the block diagram of its control system.

Synchronized to the source network and filtering its voltage, the first order harmonic of the source voltage is extracted. This harmonic is used as a reference signal $U_{\text{ref}}$. This signal is compared with a certain hysteresis to the transient value of the load voltage $U_{\text{real}}$. Depending on the sign of the comparison, the appropriate pair of diagonally connected transistors (VT1-VT4 or VT2-VT3) of the inverter is switched on.
3.2 Mathematical description

Fig. 15 displays the circuit used to make the analysis of sliding mode control of the series active power filter. The power switches are assumed to be ideal and in their switching the source voltage $U_d$ with an alternating polarity is passed to the $LC$-filter.

Fig. 15. Circuit used to make the analysis of sliding mode control of the series active power filter
The analysis is similar to those made for the single-phase inverter. The load current is marked as \( i_L \). From the operational principle, it is clear that one output variable – the load voltage \( u_L \) is monitored. Its transient value is changed through applying the voltage \( U_d \) with an altering sign. The task (the model) is:

\[
U_{REF} = U_M \cdot \sin \omega t
\]  

(12)

As a control variable, the production \( u \cdot U_d \) may be examined, where in:

\[
u = \text{sgn}\left( (u_L - u_{REF}) - H \right)
\]

\[u = +1 \quad \text{when} \quad (u_L - u_{REF}) < H\]

\[u = -1 \quad \text{when} \quad (u_L - u_{REF}) > H\]

(13)

The following relationships are valid for the schematic shown in Fig.15:

\[
U_d = u_C + L \frac{di_d}{dt}
\]

\[i_C = i_L + i_d\]

\[i_C = C \frac{du_C}{dt}\]

\[u_s + u_C = u_L\]

(14)

Using (14), it is found:

\[
\dot{u}_L = \frac{du_L}{dt} = \dot{u}_S + \frac{1}{L C} \int U_d dt - \frac{1}{L C} \int u_C dt + \frac{i_L}{C}
\]

(15)

In conformity with the theory of sliding mode control, the following equation is written (Edwards & Spurgeon, 1998):

\[
x_d = u_{REF}
\]

\[x = u_L\]

\[\dot{x} = \dot{u}_L\]

(16)

The control variable \( u_{eq} \) corresponding to the so-called “equivalent control” may be found using the following equation (Utkin, 1977, Utkin, 1992):

\[
\dot{s} = \ddot{x} - \dot{x}_d = 0
\]

(17)

Using (12) and (15) and taking in consideration (16) and (17), it is found:

\[
u_{eq} = u \cdot U_d = u_L - u_s - L C u_s - L \frac{di_L}{dt} - L C \omega^2 U_M \cdot \sin \omega t
\]

(18)

The value found may be considered as an average value when the switching is between the maximum \( U_{MAX} \) and minimum \( U_{MIN} \) values of the control variable (Utkin, 1977; Utkin 1978). If they could change between \(+\infty\) and \(-\infty\), in theory, there is always the probability...
to achieve a mode of the sliding mode control in a certain range of a change of the output variable. In order to be such a mode, the following inequalities have to be fulfilled:

\[ U_{\text{MIN}} < u_{eq} < U_{\text{MAX}} \]  

(19)

for physically possible maximum and minimum values. In this case they are:

\[ U_{\text{MIN}} = -U_d \]
\[ U_{\text{MAX}} = +U_d \]  

(20)

Resolving (18) with respect to the variable, which is being monitored \( u_L \), and substituting in (20), the boundary values of the existence of the sliding mode control could be found:

\[ u_L = \pm U_d + u_S + L \frac{di}{dt} + L.C.i_S + L.C.\omega^2 D M \sin \omega t \]  

(21)

The equation (21) found could be interpreted in the following way: a special feature of the sliding mode control with one output variable – the load voltage, is the influence of the load current changes upon the sliding mode, namely, at a sharp current change it is possible to break the sliding mode control within a certain interval leading to distortion in the transient value of the voltage feeding the load. It is worthy to be mentioned that, for example, rectifiers with active-inductive load consume current with sharp changes in its transient value from the source. From this point of view, to reduce this influence it is more suitable to operate with a small inductance value. As the load voltage has to change regarding a sinusoid law, let (21) to be analyzed around the maximum values of the sinusoidal waveform. It is found:

\[ u_L = \pm U_d + \left( u_S \right)_{t=\pi/2\omega} + L \left( \frac{di}{dt} \right)_{t=\pi/2\omega} + L.C.(i_S)_{t=\pi/2\omega} + L.C.\omega^2 D M (\pm 1) \]  

(22)

Where in (22) the positive sign is for the positive half period and the negative one – for the negative half period. After taking in consideration the practically used values of \( L \) and \( C \) (scores microhenrys and microfarads), the frequency of the supply source voltage \( f = 50 \text{ or } 60 \text{Hz} \) and its maximum value \( U_M \) \( (\approx 220 \text{ or } 156 \text{V}) \), it is obvious that the influence of the last two terms could be neglected. Thus the maximum values of the sinusoidal voltage of the load is mainly limited from the value of the supply voltage \( U_d \), the transient value of the load voltage and the speed of a change of the load current. So, from the point of view of the sliding mode control, it is good the value of \( U_d \) to be chosen bigger.

Concerning the conclusion of the influence of the load current change made based on the equations (21) and (22), the following may be commented: let us assume the worst case – short circuit of the load terminals (for example during break down regime or commutation processes in the load). Then the speed of the current change will be of maximum value and it will be limited only by the impedance of the AC supply source in a case of transformless active power filter. When an output transformer is present, its inductance will be summed to this of the source and it will additionally decrease the speed. Taking in consideration the maximum value of the voltage of single-phase network at a low voltage, as well as the range
of the inductance possible values, the speeds tentatively of $1 \frac{A}{\mu S}$ order may be expected.

The value of the filter inductance is within the range of 1 to 3mH. Therefore, the influence of the third term in equations (21) and (22) will be approximately 10 times lower than the influence of the supply voltage $U_d$.

3.3 Study through computer simulation

In this part, software PSIM is used to study single-phase active power filter. The operation of the single-phase active power filter is studied at a trapezoidal waveform of the voltage of the supply source. The computer simulation schematic is shown in Fig.16. The results from the simulation are shown in Fig.17. Total harmonic distortion of the source voltage is assumed to be 20%. The altitude of the trapezium is given equal to 300V. The values of the elements in the output of the single-phase uncontrolled rectifier are $1200 \mu F$ and $50 \Omega$.

At so chosen waveform of the AC source, the results put show good reaction of APF and also show its effective operation. So chosen trapezium form of the voltage is very close to the real cases of distortion of the source voltage. As it is seen from the results included, in this case of the source voltage waveform the system voltage supplying the load is obtained to be very closed to the ideal sinusoidal waveform without distortions around the maximum value of the sine wave and without presence of over voltages.

Fig. 16. Simulation schematic of operation of the single-phase APF with single-phase uncontrolled rectifier with active-capacitive load
Fig. 17. Results of the simulation of the schematic shown in Fig. 16. The upper waveform is the source voltage, the middle one – APF voltage, and the lower waveform – the voltage passed to the load

3.4 Experimental study

A precise stabilizer-filter for single-phase AC voltage for loads with power upto 3kVA is materialized. The device is realized using the block diagram shown in Fig. 18. The source voltage $U_{dc}$ for the active power filter is provided from a bi-directional converter connected to the network. Fig. 19 shows the general appearance of the precise stabilizer-filter. Its basic blocks are marked.

Fig. 18. Block diagram of a precise stabilizer-filter of AC voltage
Fig. 19. Single-phase precise stabilizer-filter of AC voltage

Fig. 20. Parameters of the load voltage when stabilizer – APF is switched off
Fig. 20 displays the parameters of the load voltage – its value, harmonic spectrum, total harmonic distortion, when the stabilizer – APF is switched off. Fig. 21 displays the same results when the stabilizer – APF is switched on. Fig. 20 shows decreased effective value of the voltage with 13%, increased fifth harmonic and total harmonic distortion 4.5%. Fig. 21 shows the stabilization of the effective value of the load voltage to (230V - 1.2%), decrease of the values of all harmonics, as well as a decrease in the total harmonic distortion to 1.6%.

Fig. 22 displays results when the stabilizer – APF is switched off, the effective value of the source voltage is increased with approximately 10% and the total harmonic distortion of 3.2%. Fig. 23 displays results when the stabilizer – APF is switched on. It is seen a stabilization of the voltage feeding the load to (230V +1.8%), decrease of the values of all harmonics, as well as a decrease in the total harmonic distortion to 1.8%.

Fig. 21. Parameters of the load voltage when stabilizer – APF is switched on

Fig. 24 shows transient processes at a sharp change of the source voltage. The reason that the sinusoidal waveform of the voltage is not seen is that the scale of the X-axis is 1s/div. The aim of this presentation is to be more clear that the value of the voltage feeding the load do not change significantly at a sharp change of the source voltage (both when its value decreases or increases) when the precise stabilizer-filter is switched on.
Fig. 22. Parameters of the load voltage when stabilizer – APF is switched off
Fig. 23. Parameters of the load voltage when stabilizer – APF is switched on

| Harmonics Table | Volt | L1 | THD% | H3% | H5% | H7% | H9% | H11% | H13% | H15% |
|-----------------|------|----|------|-----|-----|-----|-----|------|------|------|
|                 | 1.8  | 98.4| 0.6  | 2.1 | 1.2 | 2.5 | 0.8 | 2.4  | 0.1  | 2.1  |

Fig. 24. Experimental results at a sharp change of the value of the source voltage. a) without APF, b) with APF. The upper oscillograms present the source voltage, the lower ones – the load voltage.
4. Conclusion

The included results in the chart prove the effective operation of the single-phase inverter and single-phase active power filter studied with sliding mode control on one output variable – the voltage feeding the load.

The results found concerning the sliding mode control of inverters and series active power filters based on only one variable may be expanded and put into practice for three-phase inverters and three-phase series active power filters.

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The main objective of this monograph is to present a broad range of well worked out, recent application studies as well as theoretical contributions in the field of sliding mode control system analysis and design. The contributions presented here include new theoretical developments as well as successful applications of variable structure controllers primarily in the field of power electronics, electric drives and motion steering systems. They enrich the current state of the art, and motivate and encourage new ideas and solutions in the sliding mode control area.

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