σ and κ in Scattering Processes and New π⁰π⁰ Phase Shift Data

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The evidences for σ(600) and κ(900) observed in our analyses on the ππ and Kπ scattering phase shift data are described, briefly. The analysis have been performed by the interfering amplitude method, which satisfies the unitarity requirement, using physically meaningful parameters. The introduction of the negative phase shifts (repulsive force) are essential in the analysis. New data for the π⁰π⁰ scattering amplitudes and the I=0 S wave phase shifts are presented. The data have been obtained in the p-p charge exchange reaction, π⁻ p → π⁰π⁰ n at 9 GeV by the E135 experiment at the KEK PS. The amplitude analysis are performed. The behavior of the I=0 S wave phase shifts below KK threshold are consistent with those of the π⁺π⁻, so called, standard data and those of the down-flat solution of the CERN-Cracow-Münich polarization data. The analysis of the π⁺π⁻ phase shift data observes σ(600) with the B-W parameters, \(M_\sigma = 588 \pm 12\) MeV and \(\Gamma_\sigma = 281 \pm 25\) MeV, which are in good agreement with those in our analysis on the π⁺π⁻ data.

§1. Introduction

The σ particle, the chiral partner of pion as Nambu-Goldstone boson, has long been expected for confirmation of its existence in the Nambu-Jona-Lasinio type model\(^1\). Though its mass is expected to be twice of the quark mass, no \(I = 0\) scalar meson below 1 GeV had not been observed in past two decades, leaving confusions in the studies\(^2\). The confusions were due solely to the fact that σ was not observed in the conventional analysis\(^3\) of the \(I = 0\) S wave ππ scattering phase shifts. The phase shift rises slowly up to 90 degrees from the threshold to 1 GeV and did not allow to accommodate a resonance simply in the region. In the review of scalar states, Morgan\(^4\) attributes, excluding σ, a very broad resonance around 1 GeV to the slowly varying ππ phase shift data. Correspondingly, it is also stated clearly that the huge events of the ππ system below 1GeV observed in the pp central collision process should not be recognized as a resonance.

When we presented the result of the S wave resonant state below 1 GeV in the π⁰π⁰ system produced in the pp central collision process observed by the GAMS spectrometer at CERN\(^5\), Pennington cast a comment\(^6\) against it on the basis of

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the "universality argument" through the scattering and the production processes. However, the argument loses its base, when the right $S$ matrix bases are taken into account$^7$.

Meanwhile, we tried to re-analyze$^8,9$ the $\pi \pi$ phase shifts data to study $S$ wave resonance below 1 GeV in the scattering process, having got the evidence for the sigma particle. Efforts to study the $S$ wave resonance in re-analyses of the $\pi \pi$ scattering process have been reported by several authors$^{10}$, concluding also evidences for the existence of $\sigma$.

We present in the next chapter a brief summary of the results of our reanalysis on the $I = 0$ $S$ wave $\pi \pi$ phase shift data to show the evidence for the existence of $\sigma(600)$ in the scattering process. We would also show an evidence for $\kappa(900)$ in the results of our analysis$^{11}$ on $K \pi$ scattering phase shift data obtained by Aston et al.$^{12}$ with the LASS spectrometer at SLAC.

The new data on the $\pi^0\pi^0$ system have been obtained in the $\pi^{-}P$ charge exchange process by the E135 experiment using the Benkei spectrometer$^{13}$ at the KEK 12 GeV proton synchrotron. After an amplitude analysis of the $\pi^0\pi^0$ system is performed, the $\pi^0\pi^0$ scattering phase shifts and their analysis are described in the chapter 3. The study of the $\pi^0\pi^0$ final state has an advantage that the $\pi^0\pi^0$ scattering amplitude has no odd wave and has no odd isospin state. The analyses are not suffered from the huge contribution from $\rho^0$ and free from the ambiguity in the phase shift solutions. Though the $\pi^0\pi^0$ data have long been desired, the high statistic data by Cason et al.$^{14}$ are only those used for the analysis in these two decades.

§2. $\sigma$ and $\kappa$ in the $\pi\pi$ and $K\pi$ scattering phase shifts.

The $I = 0$ $S$ wave phase shift data of CERN b)$^{15}$, of Srinivassan et al.$^{16}$, of Rosselet et al.$^{17}$ and of Bel'kov et al.$^{18}$, so called the standard phase shift data, are used in our analysis. The new method, interfering amplitude (IA) method$^{8,9}$ is applied for the analysis. It uses a few physically meaningful parameters. $S$ matrix is written as $S = e^{2i\delta(s)} = 1 + 2i\alpha(s)$, where $\delta(s)$ is the sum of phase shifts coming from resonant states, $\delta_R$'s and the background phase shift due to a repulsive force below $K\bar{K}$ threshold, $\delta_{BG}$, i.e. $\delta(s) = \delta_R + \delta_{BG} = \delta_{f_0} + \delta_{\sigma} + \delta_{BG}$. $f_0(980)$ and $\sigma$ are considered for the relevant resonant states. The relativistic Breit-Wigner form is taken for $\alpha_R(s)$, i.e. $\alpha_R(s) = s\Gamma_R(s)/(m_R^2 - s - is\Gamma_R(s))$. Then, the total $S$ matrix is expressed by the product of $S$ matrices of resonances, $S_R = S_{f_0}S_{\sigma}$ and that of the background, $S_{BG}$, i.e. $S = S_RS_{BG} = S_{f_0}S_{\sigma}S_{BG}$. It is clear that unitarity for the total $S$ matrix is automatically satisfied by each $S$ matrix. A hard core type is adopted for the negative phase shift, $\delta_{BG}$, as $\delta_{BG} = -r_c|p_1|$, where $r_c$ is a hard core radius and $p_1$ is the CM momentum of pion. The introduction of the negative phases, the repulsive force is not a matter of arbitrariness$^{19}$ but has a physical base. There exists the experimental fact that the negative phase shifts$^{20}$ are observed in $I = 2$ $S$ wave $\pi\pi$ scattering amplitudes where no resonant state is expected. The $I = 2$ $S$ wave $\pi\pi$ scattering phase shift, $\delta^{(2)}_0$ decreases linearly from threshold to 1 GeV.
It can be expressed by hard core parameters. The same type of the background phase shifts may appear in the $I = 0$ state. In theoretical consideration, the repulsive force are derived from the compensating $\lambda \phi^4$ contact interaction based on current algebra and PCAC.

The fitting by the IA method with an introduction of the negative background phase shifts reprodces the standard data for $I = 0$ S wave scattering phase shifts, excellently, as shown by a solid line in Fig. 1(a). The B-W parameters for $\sigma$ are obtained to be $M_\sigma = 585 \pm 20$ MeV, $\Gamma_\sigma^p = 385 \pm 70$ MeV and $r_c = 3.03 \pm 0.35$ GeV$^{-1}$ (0.60±0.07 fm). The reduced $\chi^2$ value is 23.6/(34-4). The existence of the low mass $S$ wave resonance with broad width, $\sigma(600)$ has been confirmed. A comment has been given by Klempt on our results. The essential role and the physical origin of the negative background phase shifts is ignored and our results are looked down upon a trivial physics of a $\chi^2$ fitting in the comment.

We analyzed the data of CERN c) and of the $\pi^0\pi^0$ scattering phase shifts obtained by Cason et al. in the same way that for the standard data to get an idea for the upper and the lower bounds of the parameters in the experimental uncertainties. We obtained 540 MeV and 675 MeV for mass values of $\sigma$ and 440 MeV and 345 MeV for widths of it for upper and lower bounds, respectively. The curves fitted are also shown by dotted lines in Fig.1 c).

We have performed a reanalysis of the $K\pi$ scattering phase shift data obtained by Aston et al. in the reaction $K^-P \rightarrow K^-\pi^+n$ with the LASS spectrometer at SLAC. The $K^-\pi^+$ scattering amplitudes are the sum of the $I=1/2$ and $I=3/2$ components. The $I=1/2$ S wave amplitude $\delta_0^{(1/2)}$ was determined by subtraction of $I=3/2$ component obtained by Esterbrook et al. The phase shift, $\delta_0^{(1/2)}$ and the amplitude, $\alpha_0^{(1/2)}$ are shown in Fig. 2 a) and b), respectively. The dot-dash lines in the figure a) show the contribution of $\kappa$ meson and $K^0\pi(1430)$ in the analysis.
The negative values of $\delta_{BG}^{(1/2)}$ are shown by dotted line in the figure. The parameters obtained in the fitting are as follows, $M_\kappa = 905^{+65}_{-30}$ MeV, $I_\kappa = 545^{+235}_{-110}$ MeV and $r_{oc}^{(1/2)} = 3.57^{+0.40}_{-0.45}$ GeV$^{-1}$ ($0.7^{+0.08}_{-0.09}$ fm). The existence of $\kappa(900)$ has been confirmed.

It is interesting in the SU(3) flavor symmetry point of view to note that value of the core radius is almost the same that for $I=0$ $S$ wave $\pi\pi$ phase shifts.

§3. New data for the $\pi^0\pi^0$ system

The phase shift data for the $\pi^0\pi^0$ system has long been desired. However, the experimental difficulties and probably the strong argument on no existence of $\sigma$ in past two decades suppressed efforts to get data of the $\pi^0\pi^0$ scattering phase shifts with enough quality for the analysis. The high statistic data of Cason et al.\textsuperscript{14} have only been used for analysis, so far. The data, however, show different behavior, apparently, from the $I=0$ $S$ wave $\pi^+\pi^-$ phase shift data as seen in Fig. 1 c). We have analyzed the data of the $\pi^0\pi^0$ final state produced in the $\pi^-P$ charge exchange process, $\pi^-P \rightarrow \pi^0\pi^0n$ at 9 GeV studied by E135 at the KEK 12 GeV proton synchrotron. We have obtained the $\pi^0\pi^0$ scattering amplitudes up to 1.5 GeV and the $\pi^0\pi^0$ scattering phase shifts below $K\bar{K}$ threshold. Data was taken by Benkei Spectrometer late in 1980’s. The spectrometer system is described elsewhere\textsuperscript{13}. We used the data of the all neutral final state (four $\gamma$’s) for analysis. The acceptance corrected mass distribution of the $\pi^0\pi^0$ (reconstructed from four $\gamma$’s) system is shown in Fig. 3.

The off mass-shell scattering amplitude, $T_{\pi\pi}(m_{\pi\pi}^2, \cos \theta, t)$ are extrapolated to
the on mass-shell scattering amplitude at the pion pole, \( T_{\pi\pi}(m^2_{\pi\pi}, \cos \theta, m^2_\pi) \), as the process can be considered to proceed through one pion exchange. A linear extrapolation is adopted. The on mass-shell scattering amplitude can be described by the \( S \) and \( D \) waves taken in consideration, as follows, 

\[
T_{\pi\pi}(m^2_{\pi\pi}, \cos \theta, m^2_\pi) = A_S + A_D 5(3\cos^2 \theta - 1)/2,
\]

where \( A_S \) and \( A_D \) are \( S \) and \( D \) wave scattering amplitudes, respectively. The partial waves for the \( S \) and \( D \) waves, obtained are shown in Fig. 4 a) and b) respectively.

The \( S \) wave \( \pi^0\pi^0 \) scattering amplitudes can be written in terms of the of \( I = 0 \) \( S \) wave scattering phase shift, \( \delta^{(0)}_0 \) and of the \( I = 2 \), \( \delta^{(2)}_0 \), \( |A_S|^2 \sim \sin^2(\delta^{(0)}_0 - \delta^{(2)}_0) \) below \( K\bar{K} \) threshold. We used a hard core type for \( \delta^{(2)}_0 \), as \( \delta^{(2)}_0 = -r^{(2)}_c|q_1| \), where \( q_1 \) is the CM momentum of pion and \( r_c \) is the core radius. The parameter, \( r^{(2)}_c \) has been obtained\(^{25} \) from \( \pi^+\pi^- \) data to be 0.87 GeV\(^{-1} \) (0.17 fm). The \( S \) wave \( \pi^0\pi^0 \) phase shifts obtained below \( K\bar{K} \) threshold are shown in Fig. 5 with open squares. The results are consistent with so called the standard data of the \( \pi^+\pi^- \) phase shifts and also with those of the down-flat solution\(^{26} \) obtained in the analysis performed recently by Kamiński et al. on the CERN-Cracow-München polarization data\(^{27} \).

The phase shift difference \( \delta^{(0)}_0 - \delta^{(2)}_0 \) at the neutral kaon mass in our analysis gives 42.5±5 degrees, which is consistent with the value obtained from the threshold pion production\(^{28} \).

The \( \pi^0\pi^0 \) phase shift data are analyzed by the IA method. Negative background phase shifts are introduced in the analysis. A hard core is used for them. The fit is shown in Fig. 5 by the solid line. The Breit-Wigner parameters obtained for \( \sigma \) are as follows; \( M_\sigma = 588 \pm 12 \) MeV, \( \Gamma_\sigma = 281 \pm 25 \) MeV and \( r_c = 2.76 \pm 0.15 \) GeV\(^{-1} \). The reduced \( \chi^2 \) value is 20.4/12. These values are in good agreement with those which we have obtained in our analysis\(^{8},9 \) on the standard \( \pi^+\pi^- \) phase shift data. The dotted line in the figure is the result obtained with no negative background (no hard core, \( r_c =0 \)). The B-W parameters obtained are \( M_{\pi^+\pi^-} = 890 \pm 16 \) MeV, and
\( \Gamma_{\sigma^0} = 618 \pm 51 \text{ MeV} \) which deviate appreciably from those with the hard core. The reduced \( \chi^2 \) becomes worse to be 85.0/13.

\section*{§4. Conclusions}

Evidences for \( \sigma(600) \) and \( \kappa(900) \) in our analyses on the scattering processes are summarized. The existence of them suggests strongly that they form a chiral nonet. The new data for the \( \pi^0\pi^0 \) scattering amplitudes are presented for the \( \pi^-P \) charge exchange reaction at 9 GeV/c. The \( \pi^0\pi^0 \) \( S \) wave scattering phase shifts below 1 GeV show the consistent behavior with the \( \pi^+\pi^- \) standard phase shift data and with those of the CERN Cracow Münich polarization data. \( \sigma(600) \) is observed in the analysis of the \( \pi^0\pi^0 \) scattering phase shifts. Its Breit-Wigner parameters are 588 MeV and 276 MeV for mass and width, respectively, which are in excellent agreement with those for the \( \pi^+\pi^- \) phase shift data.

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