Gamma Rays from Heavy Neutralino Dark Matter

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We consider the gamma-ray spectrum from neutralino dark matter annihilations and show that internal bremsstrahlung of W pair final states gives a previously neglected source of photons at energies near the mass of the neutralino. For masses larger than about 1 TeV, and for present day detector resolutions, this results in a characteristic signal that may dominate not only over the continuous spectrum from W fragmentation, but also over the $\gamma \gamma$ and $\gamma Z$ line signals which are known to give large rates for heavy neutralinos. Observational prospects thus seem promising.

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INTRODUCTION

One of the most favoured dark matter candidates since 20 years or so, is the lightest supersymmetric particle [1, 2]. Most of the expected phenomenology has been worked out, and is implemented in freely available, extensive computer packages like DARKSUSY [3] and MICROMEGAS [4]. The vast majority of the analyses has been performed in various constrained versions of the minimal supersymmetric extension of the Standard Model (MSSM) (for reviews, see e.g. [5]), where either radiative breaking of supersymmetry or the GUT condition on gaugino masses (or both) have been imposed. It is clear from all these studies that a suitable neutralino candidate for dark matter is indeed available, with relic density as measured by WMAP (Ω_{CDM} h^2 = 0.113 ± 0.009, where Ω_{CDM} denotes the ratio of cold dark matter to critical density and $h$ is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}) [6], and with great prospects of being detected either at the CERN LHC, or at the various direct and indirect detection experiments of halo dark matter that are currently operational or being constructed.

However, one may also ask the unpleasant but not unrealistic question what happens if supersymmetry is not found at the LHC. If supersymmetry is still present, that would probably mean that the mass scale of the lightest supersymmetric particles is beyond the kinematical reach of the accelerator. For neutralino masses at the TeV scale, also the scattering cross section for direct detection would necessarily be small, as would many of the rates (antiprotons, positrons) for indirect detection. An exception seems to be gamma-ray detection, firstly because rates do not fall off as rapidly [7] and secondly due to eminent new gamma-ray telescopes, most clearly demonstrated by the recent spectacular performance of HESS, in particular as regards the multi-TeV signal that has been observed towards the galactic center [8]. With such new astrophysical gamma-ray instruments of unprecedented size and energy resolution either in operation [3, 14] or under construction [11, 12], it is appropriate to investigate possible levels of signals and spectral signatures of heavy dark matter particle annihilation.

Most of the previous calculations have been carried out at tree-level, with radiative corrections typically (and correctly) believed to be at the few percent level. In some cases, radiative corrections may on the contrary relieve the annihilation rate from inhibiting factors having to do with the Majorana fermion property of neutralinos and the fact that annihilation in the dark matter halo effectively takes place at rest [13]. One example of this is the radiative “correction” of $\chi \chi \rightarrow f \bar{f}$ through the emission of a photon in the final state. Here, the first-order corrected cross section can be many orders of magnitude larger than the tree-level result [13].

A second example of an unexpectedly large cross section is that of the second order, loop-induced $\gamma \gamma$ annihilation [6, 14, 15], where in the high mass, pure higgsino (or wino [16]) limit the branching ratio normalized to the lowest order rate can reach percent level, despite the naive expectation of being 2 to 3 orders of magnitude smaller. The origin of this enhancement has only recently been fully understood, and is explained by nonperturbative, binding energy effects in the special situation of having very small (i.e. galactic) velocities and very large dark matter masses as well as small mass differences between the neutralino and the lightest chargino [17].

In this Letter, we focus on gamma rays from neutralino annihilation into charged gauge boson pairs and show that there is yet another, previously neglected enhancement mechanism, appearing already at first order in $\alpha_{em}$: radiative processes containing one photon in addition to the weak bosons in the final state, give a new source of photons which peaks near the highest possible energy (the mass of the neutralino). This turns out to be a very beneficial effect for the potential detection of neutralinos with masses $m_{\chi} \gtrsim 1$ TeV, as the normally soft spectrum of continuous photons coming from the fragmentation of W or Z bosons (see, e.g., [2, 17, 18]) gets a high-energy supplement with a clearly distinguishable signature. This is reminiscent of the case of Kaluza-Klein dark matter, where internal bremsstrahlung in annihilation processes with charged lepton final states dominates the gamma-ray spectrum at the highest energies [10] (see also [21]).
GAMMA RAYS FROM NEUTRALINO ANNIHILATIONS

In most models, the lightest stable supersymmetric particle is the lightest neutralino, henceforth just “the neutralino”, which is a linear combination of the superpartners of the gauge and Higgs fields,

\[ \chi \equiv \tilde{\chi}_1^0 = N_{11} \tilde{B} + N_{12} \tilde{W}^3 + N_{13} \tilde{H}^0 + N_{14} \tilde{H}^0. \] (1)

In order not to overclose the universe, a TeV-scale neutralino must generally have a very large higgsino fraction, \( Z_h \equiv |N_{13}|^2 + |N_{14}|^2 \), if the usual GUT condition \( M_3 \sim M_2/2 \) is imposed; otherwise a heavy wino would also be acceptable. In the following, we therefore focus on higgsino-like neutralinos, with \( Z_h \approx 1 \) and \( N_{13} \approx \pm N_{14} \) \cite{20}. For the high masses we are interested in, the annihilation rate into charged gauge bosons often dominates. Internal bremsstrahlung in these final states are therefore of great interest to investigate. Moreover, we note that all our results are almost independent of the relative velocity \( v \) of the annihilating neutralino pair. Analytical expressions are therefore presented in the limit of vanishing velocity, but should be applicable both at the time of freeze-out \( (v/c \approx 1/6) \) and to annihilating neutralinos in the galactic halo today \( (v/c \approx 10^{-5}) \).

For a pure higgsino, the only contribution to the lowest order annihilation cross section into charged gauge bosons comes from a \( t \)-channel exchange of a chargino; it is given by

\[ (\sigma v)_{WW} = \frac{g^4}{32\pi} \left( \frac{m_{\chi_{1 \pm}}^2 - m_W^2}{m_{\chi_{1 \pm}}^2 + m_{\chi_{1 \pm}}^2 - 4m_W^2} \right)^2, \] (2)

where \( m_{\chi_{1 \pm}} \) and \( m_W \) are the lightest chargino and \( W \) masses, respectively.

Let us now consider radiative corrections with a photon in the final state in addition to the \( W \) pair. Just as at lowest order, the potential \( s \)-channel exchanges of \( Z \) and Higgs bosons vanish, and the only Feynman diagrams that contribute are shown in Fig. 1. To zeroth order in \( \epsilon \equiv m_W/m_{\chi_1} \), and retaining a leading logarithmic term, the resulting photon multiplicity is given by

\[ \frac{dN^W_\gamma}{dx} \equiv \frac{d(\sigma v)_{WW \gamma}}{d\epsilon} \left( \frac{\sigma v}_{WW} \right) \]

\[ \approx \frac{\alpha_{\text{em}}}{\pi} \left( \frac{4(1 - x + x^2)^2}{(1 - x)x} \ln(2/\epsilon) \right) \]

\[ - \frac{2(4 - 12x + 19x^2 - 22x^3 + 20x^4 - 10x^5 + 2x^6)}{(2 - x)^2(1 - x)x} \ln(1 - x) \]

\[ + \frac{2(8 - 24x + 42x^2 - 37x^3 + 16x^4 - 3x^5)}{(2 - x)^3(1 - x)x} \ln(1 - x) \]

\[ + \frac{\delta^2}{(2 - x)^2} \left( \frac{2x(2 - (2 - x)x)}{(2 - x)^2} + \frac{8(1 - x)\ln(1 - x)}{(2 - x)^3} \right) \]

\[ + \frac{\delta^4}{(2 - x)^2} \left( \frac{x(x - 1)(2 - 2x + x^2)}{(2 - x)^3} \ln(1 - x) \right) \],

(3)

where \( x \equiv E_\gamma/m_\chi \) and \( \delta \equiv (m_{\chi_{1 \pm}} - m_\chi)/m_W \).

Several interesting features can be identified in this expression. For large mass shifts \( \delta \) the last two terms dominate. They originate from longitudinally polarized charged gauge bosons in the final state, which are forbidden in the lowest order process because of the different CP properties of the initial and final state \cite{21}. Remember that in the limit of vanishing relative velocity, the initial state must be an \( S \)-wave with pseudoscalar quantum numbers due to the Majorana nature of the neutralino. The emission of a photon, on the other hand, will open up this channel in the \( 1S_0 \) partial wave, potentially leading to very large cross sections \cite{22}. However, in supersymmetric scenarios with a heavy higgsino-like neutralino one usually expects a mass shift \( \delta \ll \epsilon \) \cite{17, 21}, in which case the longitudinal part (last two terms) can be neglected.

For small mass shifts, the cross section is instead dominated by the production of transversely polarized gauge bosons. This results in a peak in the spectrum at high energies that becomes more and more pronounced for higher neutralino masses. The appearance of this peak can be understood by observing that for very heavy neutralino masses the transversely polarized \( W \) bosons can be treated as light and thus behave in the same way as infrared photons radiated from the neutralino/chargino line in Fig. 1. The mechanism that takes place is, in other words, an amusing reflection of QED infrared behaviour also for \( W \) bosons: The kinematical situation when the photon and one of the \( W \)s leave the annihilation point each with maximal energy, gets an enhancement, since it is automatically accompanied by a very soft \( W \). This is also reflected in the symmetric appearance of the \( x \to 0 \) and \( x \to 1 \) poles in the first terms of Eq. (3).

As an illustrative example, we have chosen a typical higgsino-like MSSM model, fulfilling all experimental constraints, as specified in Table 1 (similar models are found in, e.g., the focus point region of mSUGRA). The resulting photon spectrum from internal bremsstrahlung of \( W \) pair final states is shown in Fig. 2. The symmetry around \( x \sim 0.5 \) in the spectrum indicates the related
FIG. 2: The photon multiplicity for the radiative processes $\chi\chi \rightarrow W^+W^-\gamma$. The dots represent the MSSM model of Table I as computed with the FormCalc package [22] for a relative neutralino velocity of $10^{-3}$. The thick solid line shows the full analytical result for the pure higgsino limit of the same model but with zero relative neutralino velocity. The thin solid line is the corresponding approximation as given in Eq. (3). Also shown, as dashed and dotted lines, are two pure higgsino models with a lightest neutralino (chargino) mass of 10 TeV (10 TeV) and 1.5 TeV (2.5 TeV), respectively.

![Graph showing photon multiplicity](image1)

$E_\gamma / m_\chi$

$N$

$\gamma / d x = E_\gamma / m_\chi$

FIG. 3: The total differential photon distribution from $\chi\chi$ annihilations (solid line) for the MSSM model of Table I. Also shown separately is the contribution from radiative processes $\chi\chi \rightarrow W^+W^-\gamma$ (dashed), and the $W$ fragmentation together with the $\chi\chi \rightarrow \gamma\gamma$, $Z\gamma$ lines (dotted).

![Graph showing total differential photon distribution](image2)

$E_\gamma$ [TeV]

$E_\gamma / m_\chi$

$N$

$\gamma / d E_\gamma = [10^{-29} \text{cm}^3 \text{s}^{-1} \text{TeV}^{-1}]$

FIG. 4: The same spectra as in Fig. 3 as seen by a detector with an energy resolution of 15 percent.

![Graph showing differential photon distribution](image3)

$E_\gamma$ [TeV]

$E_\gamma / m_\chi$

$N$

$\gamma / d E_\gamma = [10^{-29} \text{cm}^3 \text{s}^{-1} \text{TeV}^{-1}]$

For comparison, again using the model of Table I, Fig. 4 shows the contributions from secondary photons and the line signals, as well as the new source of photons from the internal bremsstrahlung diagrams of Fig. 1.

The practical importance of the latter contribution can be appreciated even more, when considering a finite detector resolution of 15%, which is typical for atmospheric Cherenkov telescopes in that energy range; the result is a smeared spectrum as shown in Fig. 4. One can see that, although the strength of the $\gamma\gamma$ and $Z\gamma$ lines already are surprisingly large [7], the contribution from the internal bremsstrahlung further enhances this peak by a factor of 2. The signal is also dramatically increased at lower energies, thereby filling out the “dip” just below the peak; this latter effect will of course become even more pronounced for better detector resolutions.

| $M_2$ | $\mu$ | $m_A$ | $A_f$ | $\tan \beta$ | $m_{\chi}$ | $m_{\chi^\pm}$ | $Z_h$ | $W^\pm$ | $\Omega_ch^2$ |
|-------|-------|-------|-------|-------------|------------|-------------|------|--------|----------|
| 3.2   | 1.5   | 3.2   | 3.2   | 0.0         | 10.0       | 1.50        | 1.51 | 0.92   | 0.39     | 0.12     |

**TABLE I:** MSSM parameters for the example model shown in Fig. 2 and the resulting neutralino mass ($m_{\chi}$), higgsino mass ($m_{\chi^\pm}$), higgsino fraction ($Z_h$), branching ratio into $W$ pairs ($W^\pm$) and neutralino relic density ($\Omega_ch^2$), as calculated with DARKSUSY [3] and micrOMEGAs [4]. Masses are given in units of TeV.

The nature of the peak and the infrared divergence. For completeness, we have also included a very high mass (10 TeV) higgsino model which has received some attention recently [17, 23] (even though thermal production of such a neutralino in general gives a too large $\Omega_CDM$, unless one allows for finetuning of parameters like the pseudoscalar Higgs mass [24]). In addition, the case of a hypothetical model with a very large mass shift is shown (where the contributions from longitudinal $W$ bosons dominate at high energies).

Let us now consider those contributions to the gamma-ray spectrum from the decay of heavy neutralinos that have been studied earlier. Secondary gamma rays are produced in the fragmentation of the $W$ pairs, mainly through the decay of neutral pions. In addition to the secondary spectrum, there are line signals from the direct annihilation of a neutralino pair into $\gamma\gamma$ [14] and $Z\gamma$ [15]. Due to the high mass of the neutralino, these lines cannot be resolved but effectively add to each other at an energy equal to the neutralino mass.
CONCLUSIONS AND DISCUSSION

In this Letter, we have presented important radiative corrections to the gamma-ray spectrum from heavy neutralino annihilations. They contribute a characteristic peak shape at the highest energies, competing with the $\gamma \gamma$ and $Z\gamma$ line signals in today’s detectors.

When it comes to predicting absolute gamma-ray fluxes, there is, unfortunately, a great uncertainty which is primarily due to the unknown dark matter clustering properties of the Milky Way halo. Particle physics alone predicts, in lowest order perturbation theory, a flux which falls roughly as $m_\chi^{-2}$. On the other hand, the possible nonperturbative enhancement discussed in [17] is important for heavy neutralinos, and may give a substantial boost to the gamma-ray signal. The process discussed here should benefit from a similar boost as the line signal treated in [17], since the enhancement is due to binding effects in the initial state. Thus, even if the radiated $W$ boson is soft compared to the initial state total mass, it still causes a virtuality of one of the neutralinos which is much greater than the binding energy. Therefore, the factorization of the process into long distance and short distance kernels performed in [17] should be valid also here, and our curves for the ratios of cross sections ($dN_W^\gamma/dx$) should not be affected much.

Since the absolute gamma-ray flux - although not unlikely considerable in size - is hard to predict, it is rather the spectral shape that eventually may separate a dark matter signal from the background. The HESS observations of the galactic center, e.g., show a power-law energy spectrum with a spectral index of about $-2$ up to at least 10 TeV [8], and for Higgsinos as heavy as that, we expect a spectrum that is too hard to explain the dark matter signal from the background. The HESS observations of the galactic center, e.g., show a power-law energy spectrum with a spectral index of about $-2$ up to at least 10 TeV [8], and for Higgsinos as heavy as that, we expect a spectrum that is too hard to explain the dark matter particle’s mass. One should furthermore bear in mind that the best prospects for detection might therefore not be found near the galactic center, but rather for sources with a low or at least well understood background. Examples for this could be nearby dark matter clumps or intermediate mass black holes [22]; a thorough analysis of the detectional prospects for these candidates, however, is beyond the scope of this Letter.

Finally, we note that the radiative corrections presented in this Letter add to the total annihilation cross section, quite independently of the relative velocity of the annihilating neutralinos. This is relevant for any precise calculation of neutralino relic densities and is therefore of relevance for computer packages like DARKSUSY and MICROMEGAS.

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If the neutralino is a pure wino, one gets identical results as for the (anti-) symmetric combination of higgsinos that we consider here, apart from an overall factor of 16 that multiplies all quoted cross sections. A pure bino state, on the other hand, does not couple to $W$ at lowest order at all.

Unitarity is in general restored and therefore forbids too large cross sections, which can be understood from the equivalence theorem between longitudinal gauge bosons and would-be Goldstone modes of the Higgs sector [21].