TRANSVERSE SHOCKS IN THE TURBULENT GLUON PLASMA PRODUCED IN ULTRA-RELATIVISTIC A+A

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Mini-jet production in ultra-relativistic nuclear collisions leads to initial conditions characterized by large fluctuations of the local energy density (hot spots) and of the collective flow field (turbulence). Assuming that local equilibrium is reached on a small time scale, $\sim 0.5 \text{ fm/c}$, the transverse evolution of those initial conditions is computed using hydrodynamics. We find that a new class of collective flow phenomena (hadronic volcanoes) could arise under such conditions. This could be observable via enhanced azimuthal fluctuations of the transverse energy flow, $d^2 E_\perp / d\phi dy$.

1 Introduction

At energies $\sqrt{s} > 100$ AGeV, mini-jet production in central nuclear collisions is expected to be the primary dynamical source of a plasma of gluons with an initial energy density an order of magnitude above the deconfinement and chiral symmetry scale, $\epsilon_c \sim 1 \text{ GeV/fm}^3$. Many observable consequences of the formation of this new phase of matter have been predicted based on a variety of assumptions, and experiments are currently under construction to search for evidence for that so called quark-gluon plasma (QGP) at the Relativistic Heavy–Ion Collider (RHIC) at Brookhaven. Evidently, signatures depend sensitively on the assumed ensemble of initial conditions generated in such collisions. Most often it is assumed for simplicity that a homogeneous, cylindrically symmetric, and longitudinally boost-invariant quark-gluon plasma is created and thus that signatures can be computed ignoring fluctuations of the initial conditions themselves.

In this talk we point out, however, that the mini-jet formation mechanism leads to a rather inhomogeneous and turbulent ensemble of initial conditions that is characterized by a wide fluctuation spectrum of the local energy density (hot spots) and of the collective flow field (turbulence). In this case, some of the proposed signatures will be washed out while new ones will certainly arise. We show below that in fact a new type of collective “shock” phenomena may occur under these conditions that could be readily observed as unusual transverse energy fluctuations.

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This topic is well suited to this symposium honoring the 60th birthday of Professor Greiner because he was the first to propose in 1974 the “shocking” idea that nuclear shock waves may be produced in nuclear collisions and serve as an ideal probe of the equation of state of dense matter. Since the experimental discovery of nuclear shocks in the 1 AGeV range in 1984 and in the AGS energy range last year much theoretical and experimental effort has been devoted to this topic over a large energy range: \( E_{\text{lab}} = 100 \text{ AMeV to } 100 \text{ ATeV} \). The present work focuses on a new class of shock phenomena that could occur in quark-gluon plasmas near the top end of that energy scale.

To illustrate the novel type of collective flow patterns we have in mind, the hydrodynamic evolution of initial conditions consisting of three static cylindrical “hot spots” are shown in Fig. 1.

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\[ \epsilon(x, y, \tau_0) \quad \epsilon(x, y, \tau_t) \quad \frac{dE_T}{d\Phi} \]

Figure 1: Hadronic volcanoes erupt due to shock formation in regions where expanding shells of matter evolved from inhomogeneous “hot spots” initial conditions (left panels) intersect. The signature of such volcanoes is the strong anisotropic azimuthal angle dependence of the transverse energy flow \( dE_T/\Phi \) (right panels). Contours of constant energy density, \( \epsilon(x, y, \tau) \), at the breakup time \( \tau_f \) are shown in the middle panels. The top row shows the evolution from a symmetric initial configuration of hot spots. The bottom rows shows the evolution from a more realistic asymmetric configuration.

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\[ b \text{see talks by J. Symons, J. Stachel, and P. Braun-Munzinger in these proceedings.} \]
An ideal gas equation of state \( p = \epsilon/3 \) is assumed. Shock waves are seen to be created where the three rapidly expanding shells of matter intersect at six points. The collective acceleration of matter in those shock zones can be clearly seen as strong peaks in the transverse energy flow. For the symmetric configuration (Fig. 1a), a “mercedes” pattern emerges (Fig. 1b), leading to three primary and three secondary shocks in Fig. 1c. For the asymmetric case in Figs. 1d,e the azimuthal angle dependence of the shock peaks is spread out and modified. The search for analogous enhanced anisotropic transverse energy fluctuations (or hadronic volcanoes) as a function of rapidity and azimuth is the subject of the following discussion.

2 HIJING Initial Conditions

Mini-jet production is thought to be the dominant mechanism controlling the initial conditions in \( A + A \) because the pQCD inclusive cross section for jets with moderate \( p_\perp > p_0 \sim 1 - 2 \) GeV rises to the value \( \sigma_{jet}(p_0) > 10 \) mb at RHIC energies. Compelling evidence for pQCD mini-jet dynamics has been observed in \( pp \) and \( p\bar{p} \) reactions at collider energies. Due to the nuclear geometry, the total number of mini-jet gluons produced in central \( A + A \) collisions is expected to be \( \sim A^{4/3} \sim 10^3 \) times larger than in \( pp \) collisions. This simple geometric enhancement causes the rapidity density of mini-jet gluons to reach \( dN_g/dy \sim 300 - 600 \) in \( Au + Au \) depending on the mini-jet transverse momentum cut-off scale \( p_0 = 1 - 2 \) GeV/c. The hadronization of this mini-jet gluon plasma via the string fragmentation mechanism in HIJING approximately doubles the final hadron transverse energy distribution due to the pedestal or “string” effect. In the pre-hadronization phase of the evolution we therefore include a “soft” component in terms of a background gas of soft gluons normalized such that together with the mini-jet component the final transverse energy distribution predicted by HIJING is reproduced.

The average initial energy density of the mini-jet gluon plasma can be estimated using the Bjorken formula: \( \langle \epsilon(\tau) \rangle \approx \langle p_0/\tau_0 \pi R^2 \rangle dN_g/dy \langle \tau_0/\tau \rangle \sim 40 (\tau_0/\tau) \) GeV/fm\(^3\). This applies until the thermalization time is reached and work due to expansion must be considered.

To see that the initial conditions are in fact highly inhomogeneous and are dominated by a few “hot spots”, we must calculate the local energy density \( \epsilon(\tau, x_\perp) \) instead of averaging over the transverse coordinate as in the Bjorken formula. The local energy density and transverse momentum density at \( z = 0 \) are given by

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(the $y_{cm} = 0$ frame) can be computed from the HIJING list of produced gluons as

$$
\left( \epsilon(\tau, x_\perp) \right) = \sum_\alpha \frac{1}{v_{\perp \alpha}} \frac{\tau p_{\perp \alpha}^3}{1 + (\tau p_{\perp \alpha})^2} \delta^2(x_\perp - x_{\perp \alpha}(\tau)) \delta(y_\alpha) . \quad (1)
$$

The sum is over the produced gluons with transverse and longitudinal momentum ($p_{\perp \alpha}, p_{z \alpha} = p_{\perp \alpha} \sinh y_\alpha$). The longitudinal and transverse coordinates of the production, ($z_\alpha = 0,x_{\perp \alpha}$), are determined from the coordinates of the binary nucleon-nucleon collision from which the gluons originate. The above formula takes into account the free streaming of gluons not only along the $z$ direction via the volume element $\tau \Delta y$ but also in the transverse direction via $x_{\perp \alpha}(t) = x_{\perp \alpha} + v_{\perp \alpha} \tau$, where $v_{\perp \alpha} = p_{\perp \alpha}/p_\alpha$. The factor $1/(1 + (\hbar/\tau p_{\perp \alpha})^2)$ is the formation probability of the gluon in the comoving frame. High-$p_T$ gluons are produced first and gluons with lower $p_T$ later according to the uncertainty principle leading to the so-called inside-outside picture of the dynamics. Before $\tau \sim \hbar/p_{\perp \alpha}$ that component of the radiation field is still part of the coherent Weizsäcker-Williams field of the passing nuclei.

The expression (1), when averaged over transverse coordinates, recovers the Bjorken expression $\langle \epsilon(\tau) \rangle \approx \epsilon_0(\tau_0/\tau)$ with vanishing average transverse momentum density, $\langle \tilde{m}(\tau) \rangle = 0$.

To study the transverse coordinate dependence of the initial conditions, we must specify a transverse, $\Delta r_\perp$, and longitudinal, $\Delta y$, resolution scale. The densities coarse-grained on that resolution scale are obtained from (1) substituting

$$
\delta^2(x_\perp - x_{\perp \alpha}(\tau)) \delta(y_\alpha) \rightarrow \frac{\Theta(\Delta y/2 - |y_\alpha|)}{\Delta r_\perp^2 \Delta y} \Theta(\Delta r_\perp/2 - |x_{\perp \alpha}(\tau) - x_\perp|) . \quad (2)
$$

To determine the relevant resolution scale, we note that at time $\tau$ the local horizon for any gluon in the comoving ($y_g = 0$) frame has a radius $c\tau$. Thus at the thermalization time, $\tau_{th}$, each gluon can be influenced by only a small neighborhood of radius $c\tau_{th} \approx 0.5$ fm of the mini-jet plasma. We take the transverse resolution scale to be the maximal causally connected diameter, $\Delta r_\perp = 2c\tau_{th} \approx 1$ fm and the rapidity width to be $\Delta y = 1$. Gluons moving with larger relative rapidity $y$ are produced later, $\tau(y, p_\perp) \sim \cosh(y)/p_\perp$ due to time dilation at the boundary of the local horizon. The above choice of the resolution scale is the most optimistic from the point of view of minimizing fluctuations of the thermodynamic variables between the causally disconnected domains in the transverse plane. We note that the number, $N_d(\tau) = (R/\tau)^2$, of such causally disconnected domains in a nuclear area, $\pi R^2$, is initially very
large. Even at later times, $\tau \sim 3\,\text{fm}/c$, when the mean energy density falls below $\epsilon_c$, several disconnected domains remain.

The soft component is modeled by $\approx 1700$ low-$p_{\perp}$ gluons per unit rapidity with a Gaussian transverse momentum distribution with rms $p_{\perp} = 0.3$ GeV/c. This leads to about 500 GeV of transverse energy per unit rapidity as needed to reproduce the HIJING final $dE_T/dy$. However, these soft gluons only add a relatively small, approximately homogeneous contribution to the energy density, $\epsilon_s \approx 4\,\text{GeV}/\text{fm}^3$, at $\tau_{th} = 0.5\,\text{fm}/c$.

3 Hot Spots and Turbulence

Figure 2: (a) Energy density distribution of hot spots in a typical HIJING event. Note the large fluctuations of the initial energy density across the transverse plane. (b) Momentum density fluctuations indicate considerable initial state turbulence. (c) Distribution of energy density on a $\Delta r_{\perp} = 1\,\text{fm}$ transverse coordinate resolution scale averaged over 200 HIJING events. Note that this resolution scale corresponds to the maximal allowed one by causality at proper time, $t = 0.5\,\text{fm}/c$. (d) The thermal profile distribution on the same resolution scale has a rather large width. The mean and shape of course depend also on the equation of state (assumed here to be an ideal gas).
The inhomogeneous nature of the mini-jet plasma initial conditions in central \(Au + Au\) collisions at RHIC is illustrated in Fig. 2. In Fig. 2a the energy density profile at an assumed thermalization time \(\tau = 0.5 \text{ fm/c}\) of a typical HIJING \((b = 0)\) event at \(\sqrt{s} = 200 \text{ AGeV}\) is shown. The plasma in the central rapidity slice \((\Delta y_{cen} = 1)\) exhibits large fluctuations (hot spots) because the average number of hard mini-jets produced per nucleon is only \(\sim 1\) per unit rapidity at RHIC energies. The hot spots are also associated with a chaotic velocity field (turbulence) as seen in Fig. 2b. When averaged over many events the distribution of the proper energy density and temperature on a transverse resolution scale \(\Delta r_\perp = 1 \text{ fm}\) are shown in Figs. 2c,d. Note that at that formation time the plasma cannot be characterized by a unique temperature. In fact the widths of those distributions which are controlled by the Glauber geometry of finite nuclei and the size of the mini-jet cross section obviously cannot be neglected. Signatures of plasma formation will differ considerably in this turbulent gluon scenario than in the conventional homogeneous hot-glue scenario. For example high-\(p_T\) direct photon production and heavy-quark production will be greatly enhanced in the hot spots.

In the next section we find that hadronic volcanoes similar to those in Fig. 1 also arise.

### 4 Hadronic Volcanoes and Transverse Energy Fluctuations

In Figure 3 the azimuthal angle dependence of the transverse energy flow, \(dE_\perp/dy d\phi\), is shown for a typical HIJING initial condition. The left panels (Figs. 3a,c) correspond to an ideal gas equation of state, while the right panels (Figs. 3b,d) correspond to a first order Bag model equation of state. The top two panels evolve the initial distribution in Fig. 2a with the momentum field in Fig. 2b artificially set to zero \((\vec{m}(x_\perp, \tau_0) = 0)\). This static inhomogeneous initial condition is most similar to the one in Fig. 1. The initial isotropic nature of the thermally folded transverse energy distribution is seen by the dashed lines in Figs. 3a,b. The lower panels (Figs. 3c,d) correspond to the evolution of the turbulent initial condition in Figs. 2a,b. The turbulence of the initial condition is revealed by the anisotropy of the dashed curves.

In all cases the magnitude of the transverse energy decreases with time due to the work done by the plasma undergoing longitudinal (Bjorken) expansion. More work is done by an ideal gas case than a plasma with first order transition because the latter equation of state is softer. In the static hot spot case the production of hadronic volcanoes is indeed similar to Fig. 1. In the turbulent case, the effect is obscured somewhat by the large initial anisotropy. Nevertheless, in the turbulent case also several sharper peaks appear. Evidence
for the collective origin of the hadronic volcanoes in the turbulent case must be looked for in higher order $E_T$ correlation analysis, e.g., $\langle E_T(\phi)E_T(0) \rangle$. For a more detailed discussion of these results we refer to Ref. 8.

Figure 3: The evolution of transverse energy $dE_T/d\phi$ at $y = 0$ is shown for different inhomogeneous HIJING initial conditions and equations of state. In parts (a,b) the fluid velocity field shown in Fig. 2b is set to zero while in parts (c,d) the turbulent velocity field is included. The evolution in parts (a,c) assumes an ideal gas law $p = \epsilon/3$, while in parts (b,d) a first order Bag model transition is included. The dashed curve is the initial transverse energy distribution at the onset of hydrodynamic evolution. In the static hot spot cases the initial transverse energy is uniformly distributed in azimuth, while in the turbulent case large initial fluctuations are generated by the mini-jets. The solid curves show the final transverse energy distribution.

Acknowledgments

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