Probing the dynamical phase transition with a superconducting quantum simulator

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Non-equilibrium quantum many-body systems, which are difficult to study via classical computation, have attracted wide interest. Quantum simulation can provide insights into these problems. Here, using a programmable quantum simulator with 16 all-to-all connected superconducting qubits, we investigate the dynamical phase transition in the Lipkin-Meshkov-Glick model with a quenched transverse field. Clear signatures of the dynamical phase transition, merging different concepts of dynamical criticality, are observed by measuring the non-equilibrium order parameter, nonlocal correlations, and the Loschmidt echo. Moreover, near the dynamical critical point, we obtain the optimal spin squeezing of $-7.0 \pm 0.8$ decibels, showing multipartite entanglement useful for measurements with precision five-fold beyond the standard quantum limit. Based on the capability of entangling qubits simultaneously and the accurate single-shot readout of multi-qubit states, this superconducting quantum simulator can be used to study other problems in non-equilibrium quantum many-body systems.

I. INTRODUCTION

Quantum simulation uses a controllable quantum system to mimic complex systems or solve intractable problems. Emergent phenomena in out-of-equilibrium quantum many-body systems, e.g., thermalization versus localization, and time crystals, have attracted considerable attentions using quantum simulation. Recently, the dynamical phase transition (DPT) and the non-equilibrium phase transition in transient time scales have been theoretically studied in the transverse-field Ising model with all-to-all interactions. These two transitions can be characterized by a non-equilibrium order parameter and the Loschmidt echo associated with the Lee-Yang-Fisher zeros in statistical mechanics, respectively. Moreover, recent experimental progress has allowed for the controllable simulation of these exotic phenomena with cold atoms and trapped ions. Yet, experimental explorations for the dynamics of entanglement, which is valuable resource in quantum information processing, remain limited in the presence of a DPT.

In our experiments, applying a sudden change of the transverse field with a controllable strength, we drive the system, initially in its ground state, out of equilibrium. Accurate single-shot readout techniques enable us to synchronously record the dynamics of all qubits and observe essential signatures of DPTs and spin squeezing from the dynamical criticality in the Lipkin-Meshkov-Glick (LMG) model.

This work presents a systematic quantum simulation of DPTs with two different concepts, providing an evidence of the relation between the non-equilibrium order parameter and the Loschmidt echo. More importantly, we verify entanglement in spin-squeezed states generated from the dynamical criticality, directly observing squeezing of $-7.0 \pm 0.8$ decibels for 16 qubits.

II. RESULTS

Our quantum simulator is a superconducting circuit with 20 fully-controllable transmon qubits capacitively coupled to a resonator bus $R$ (Fig. 1A). Sixteen qubits ($Q_1$–$Q_{16}$), with XY-control lines, are selected to perform experiments (see Materials and Methods). The resonant frequency of $R$ is fixed at about 5.51 GHz, while the qubit frequencies are individually tunable via their Z-control lines, enabling us to engineer the qubit-qubit interactions induced by $R$. We detune all 16 qubits from $R$ by, e.g., $\Delta/2\pi \approx -450$ MHz, to switch on the resonator-mediated interactions between two arbitrary qubits. Simultaneously, identical resonant microwave drives, with a magnitude of $h^x$, are imposed on all qubits to generate the local transverse fields for the control of a DPT (Fig. 1C). The crosstalk effects of microwave pulses have been precisely corrected to ensure the uniformity of the local fields (see Supplementary Materials). The effective Hamiltonian of the quenched system is

$$H_{1}/\hbar = \sum_{i<j}^{N} \lambda_{ij}(\sigma_{i}^{x}\sigma_{j}^{x} + \sigma_{i}^{z}\sigma_{j}^{z}) + \hbar^x \sum_{j=1}^{N} \sigma_{j}^{x},$$

where $N = 16$, $\lambda_{ij} \equiv g_{ij}/\Delta + \lambda_{ij}^0$ is the qubit-qubit coupling strength, $g_{ij}$ represents the coupling strength be-
When necessary, single-qubit rotation pulses \( R \) the 16-qubit joint readout is executed, yielding the probabilities \( \{ P_{00...0}, P_{00...1}, ..., P_{11...1} \} \), from which \( \{ \sigma_j^+ \} \) can be calculated. When necessary, single-qubit rotation pulses \( R_j(\theta_j, \phi_j) = \exp[-i\hbar\theta_j(\cos \phi_j \sigma_j^+ + \sin \phi_j \sigma_j^x)]/2 \) (in black dotted box) are applied in advance to bring the axis defined by \( (\theta_j, \phi_j + \pi/2) \) in the Bloch sphere of \( Q_j \) to the \( \sigma^z \) direction before the readout.

The experimental pulse sequences for simulating the DPT. The sequences are divided into three steps. First, the qubits are initialised at the eigenstate \( |00...0\rangle \) state at their corresponding idle frequencies. Then, rectangular pulses are applied to quickly bias each qubit to the same detune from the common resonator bus, to turn on the qubit-qubit interactions. Almost simultaneously, the system is quenched by driving the qubits with resonant microwave pulses for a time \( t \). Finally, the qubits are biased back to their idle frequencies before the 16-qubit joint readout is executed, yielding the probabilities \( \{ P_{00...0}, P_{00...1}, ..., P_{11...1} \} \), from which \( \{ \sigma_j^+ \} \) can be calculated.

Parameters are shown in Supplementary Materials. Since the values of \( \lambda_{ij} \) are nearly the same for most pairs of qubits and do not decay over a distance \( |i-j| \) (Fig. 1B), the quenched system can be reasonably approximated by the LMG model, whose Hamiltonian is \( H_{\text{LMG}} = (J/N)(S^z)^2 + \mu S^x \), with \( S^x = \sum_j \sigma_j^x \). Recent studies have shown that \( H_{\text{LMG}} \) has a dynamical critical point \( \mu J = 1/2 \), separating the dynamical paramagnetic phase (DPP) and the dynamical ferromagnetic phase (DFP) with and without a global \( Z_2 \) symmetry, respectively.

First, we show that our programmable superconducting qubits can simulate and verify the DPT by measuring the magnetisation and the spin correlation. The system is initialised at the eigenstate \( |00...0\rangle \) of \( H_1 \) with \( h^x = 0 \), where \( |0\rangle \) denotes the ground state of a qubit. Then, we quench the system by suddenly adding a transverse field and monitor its dynamics from the time \( t = 0 \) to 600 ns. With the precise full control and the high-fidelity single-shot readout of each qubit, we are able to omnidirectionally track the evolutions of the average magnetisation

\[
\langle \sigma^\alpha(t) \rangle = \frac{1}{N} \sum_{j=1}^{N} \langle \sigma_j^\alpha(t) \rangle,
\]

along the \( x,y,z \) axes for different strengths of the quenched transverse fields, with \( \alpha \in \{x,y,z\} \). By depicting the trajectory of the Bloch vector \( \langle \vec{\sigma} \rangle = [\langle \sigma^x \rangle, \langle \sigma^y \rangle, \langle \sigma^z \rangle] \), the dynamics of our quantum simulator with two distinct transverse fields is visualised in Fig. 2A. For a small transverse field, e.g., \( h^x/2\pi \approx 2 \text{ MHz} \), \( \langle \sigma^z(t) \rangle \) exhibits a slow relaxation (Fig. 2B). However, given a strong transverse field, e.g., \( h^x/2\pi \approx \)
8 MHz, $\langle \sigma^z(t) \rangle$ exhibits a large oscillation at an early time and approaches zero in the long-time limit (Fig. 2B). In Fig. 2C, we show the behaviour of the time-averaged magnetisation, $\langle \sigma^z \rangle \equiv \langle 1/t_f \rangle \int_0^{t_f} dt \langle \sigma^z(t) \rangle$, that is defined as the non-equilibrium order parameter. It demonstrates that $\langle \sigma^x \rangle \neq 0$ and $\langle \sigma^x \rangle = 0$ in the DFP and the DPP, respectively. The experimental data of $\langle \sigma^z \rangle$ for qubits with different detunings $\Delta$ are presented in Supplementary Materials. In addition, the Bloch vector length $|\langle \sigma \rangle|$ also depends on the strength of the transverse field $h^x$. For large $h^x$, $|\langle \sigma \rangle|$ decays rapidly to a small value, indicating strong quantum fluctuations in the DPP\(^1\) (Fig. 2D). Figure 2E shows the averaged spin correlation function, $C_{zz} \equiv (1/t_f) \int_0^{t_f} dt \sum_{ij} \langle \sigma_i^z(t) \sigma_j^z(t) \rangle/N^2$, versus $h^x$ with a final time $t_f = 600$ ns, where the DPT is characterised by the local minimum of two-spin correlations\(^6\). We can observe the critical behaviours of $\langle \sigma^z \rangle$ and $C_{zz}$ as the signatures of the DPT, when the transverse field strength is set near the theoretical pre-

**FIG. 2. Magnetisation and spin correlation.** A, Experimental and numerical data of the time evolution of the average spin magnetisation shown in the Bloch sphere for different strengths of the transverse fields. B, The time evolution of the magnetisation $\langle \sigma^z \rangle(t)$. C, The nonequilibrium order parameter, $\langle \sigma^z \rangle$, as a function of $h^x/2\pi$. D, The dynamics of the Bloch vector length $|\langle \sigma \rangle|$. E, The averaged spin correlation $C_{zz}$ versus $h^x/2\pi$. The regions with light red and light blue in C and E show the dynamical ferromagnetic phase and dynamical paramagnetic phase, respectively, separated by a theoretically predicted critical point $h_c^x/2\pi = 5.7$ MHz. The solid curves in B–E are the numerical results using Eq. (1) without considering decoherence.

Another perspective on dynamical criticality is based on the Loschmidt echo, defined as $\mathcal{L}(t) = ||(00 \ldots 0) e^{-iH_1 t/H} |00 \ldots 0\rangle||^2$, where the time $t$, satisfying $\mathcal{L}(t) = 0$, is a Lee-Yang-Fisher zero. The zero will cause the nonanalytical behaviour of the rate function, $r(t) = -N^{-1} \log[\mathcal{L}(t)]$, regarded as the complex-plane generalisation of the free energy density\(^11\). Recent numerical studies\(^7,9\) have revealed that the existence of Lee-Yang-Fisher zeros closely relates to the DPT between the DFP and the DPP in long-range interacting systems. In Fig. 3A, we show distinct behaviours of the Loschmidt echo in different dynamical phases. Here, we consider the first minimum of the Loschmidt echo $\mathcal{L}^{(1)}_{\min}$ (Fig. 3A), which is sufficient for judging whether the zeros exist or not. It approaches zero in the DPP, and remains relatively large in the DFP\(^9\). In Fig. 3B, we plot $\mathcal{L}^{(1)}_{\min}$ versus $h^x$ to show the relationship between the DPT and the Loschmidt echo. Similar as the numerical results of the LMG model with finite-size effects (see Methods), we experimentally observe that $\mathcal{L}^{(1)}_{\min} \leq 0.01$ in the DPP, and it becomes relatively large $\mathcal{L}^{(1)}_{\min} > 0.01$ in the DFP. Our work is the first experiment to combine the non-
equilibrium order parameter and the Loschmidt echo for verifying a DPT.

In addition to demonstrating a DPT, the LMG model is also useful for generating the spin-squeezed state with twist-and-turn dynamics. Near the equilibrium critical point, spin squeezing can be achieved, originating from quantum fluctuations, according to the Heisenberg uncertainty principle. Similarly, we show that the spin-squeezed state can also be generated from the dynamical criticality. During the dynamics of the quenched Hamiltonian, we can visualise the spin-squeezed state by measuring the quasidistribution $Q$-function $Q(\theta, \phi) \propto \langle \theta, \phi | \rho(t) | \theta, \phi \rangle$, where $| \theta, \phi \rangle \equiv \bigotimes_{j=1}^{N} (\cos \frac{\theta}{2} | 0 \rangle + \sin \frac{\theta}{2} e^{i\phi} | 1 \rangle)$ is the spin coherent state. The measurement is realised by applying a single-qubit rotation to bring the axis defined by $(\theta, \phi)$ in the Bloch sphere to the z-axis for each qubit before the joint readout. The experimental and numerical data of $Q(\theta, \phi)$ are compared in Fig. 4A, which clearly show spin squeezing with a large strength of the external field, due to stronger quantum fluctuations in the DPP (see also Fig. 2C).

We also measured the time-evolved spin-squeezing parameter (see Supplementary Materials)

$$\xi^2 = 4 \min_{\hat{n}_i} \left[ \text{Var}(S^z) \right] / N,$$

(2)

where $\hat{n}_i$ denotes an axis perpendicular to the mean spin direction, and $\text{Var}(S^z) = \langle (S^z) \rangle^2 - \langle S^z \rangle^2$. In Fig. 4B, we show that $\xi^2 < 1$ (a sufficient condition for particle entanglement) occurs in the time interval $t \lesssim 46$ ns when $\hbar/2\pi \approx 3$ MHz, and for $t \lesssim 38$ ns when $\hbar/2\pi \approx 6$ MHz. The minimum spin squeezing parameter over time, $\xi_{\text{min}}^2$, as a function of $\hbar^2$ is shown in Fig. 4C, where the minimum value $\xi_{\text{min}}^2 \approx 0.2 (-7.0 \text{ dB})$ is attained very close to the critical point of the DPT. Compared with the theoretical limit, about $N^{-2/3}$, of the squeezing parameter for an $N$-body one-axis twisting Hamiltonian, our 16-qubit system achieves a spin squeezing parameter satisfying $\xi_{\text{min}}^2 \approx N^{-\alpha}$, with $\alpha \approx 0.58$. It indicates the high-efficiency generation of the spin-squeezed state from dynamical criticality, and reveals the potential application of the DPT in quantum metrology.

III. DISCUSSION

We have presented clear signatures and entanglement behaviors of the DPT in the LMG model with a superconducting quantum simulator featuring all-to-all connectivity, including the non-equilibrium order parameter, the Loschmidt echo, and spin squeezing. Based on its high degree of controllability, precise measurement, and long decoherence time, our platform with all-to-all connectivity is powerful for generating multipartite entanglement and investigating non-trivial properties of out-of-equilibrium quantum many-body systems, such as many-body localization, quantum chaos in Floquet systems, and quantum annealing.

IV. MATERIALS AND METHODS

A. Device information and system Hamiltonian

The device used here consists of 20 frequency-tuneable superconducting qubits capacitively coupled to a central resonator bus. It is the same circuit presented...
in Ref.20, where more details about the device, the qubit manipulation, and the readout can be found. In Table I (see Supplementary Materials), we present the characteristics for the quantum simulator involving 16 out of the 20 qubits, with XY-control lines, which have been relabelled in the experiments.

The unused four qubits in this device, without XY-control lines, are detuned far off resonance from the other 16 qubits to avoid interacting with them during the experiments. Thus, they will not be included in the following descriptions. The system Hamiltonian, without applying external transverse fields, can be written as

\[ H_{s1}/\hbar = \omega_R a_1^\dagger a + \sum_{j=1}^{16} \left[ \omega_j(t)\{1_j\} + g_j(\sigma_j^+ a + \sigma_j^- a^\dagger) \right] \]

\[ + \sum_{i<j} \lambda_{ij}(\sigma_i^+ \sigma_j^- + \sigma_j^+ \sigma_i^-), \]

where \( \omega_R \) and \( \omega_j \) represent the fixed resonant frequency and the tuneable frequency of \( Q_j \), respectively, while \( g_j \) is the coupling strength between the \( Q_j \) and resonator bus. The magnitude of the crosstalk coupling between \( Q_i \) and \( Q_j \) beyond the resonator-induced virtual coupling is denoted as \( \lambda_{ij} \). When equally detuning all the 16 qubits from the resonator bus by about \( \Delta/2\pi = -450 \text{ MHz} \), and simultaneously applying resonant microwaves to each qubit, the system Hamiltonian can be transformed to

\[ H_{s2}/\hbar = \sum_{i<j} \left( \lambda_{ij}^c + g_i g_j/\Delta \right)(\sigma_i^+ \sigma_j^- + \sigma_j^+ \sigma_i^-) \]

\[ + \sum_{j=1}^{16} \left( h_j^x(\sigma_j^+ e^{i\phi_j} + \sigma_j^- e^{-i\phi_j}) \right) \]

with \( g_i g_j/\Delta \) being the magnitude of the resonator-mediated coupling between \( Q_i \) and \( Q_j \). It acts as a dominant part of the qubit-qubit interaction terms, because the crosstalk coupling \( \lambda_{ij}^c \) is much smaller. In Fig. 1b, we plot the connectivity graph of the total coupling strength \( \lambda_{ij} \) for all the combinations of pairs of qubits. The individually-controllable amplitude and the phase of the microwave drive on each \( Q_j \) are represented by \( h_j^x \) and \( \phi_j \), respectively. In our experiments, we set the uniform amplitude and phase for all qubits, leading to the Hamiltonian in Eq. (1) in the main text. To ensure this uniformity, the calibration process for the microwave drives is described in the Supplementary Materials.

### B. Relation between the quantum simulator and the LMG model

The experimental technologies ensure that the device can be described via the Hamiltonian in Eq. (1) (\( H_1 \)) in the main text. With uniform couplings \( \lambda \equiv \lambda_{ij} \), the first term of Eq. (1) can be written as

\[ \lambda \sum_{i<j} (\sigma_i^+ \sigma_j^- + \text{H.c.}) = (J/N)[S^z(S^z)^\dagger], \]

where \( J \equiv \lambda N \). The second term can be directly rewritten as \( h^2 \sum_{i=1}^{16} \sigma_i^x = g S^x \), with \( g = 2h^2 \). According to \([S^z, S^\alpha] = 0 (\alpha \in \{x, y, z\})\), and the fact that the initial state \([00\ldots0] \) is an eigenstate of \( S^z \), we have

\[ \exp[-i(H_1/\hbar)t][00\ldots0] \propto \exp(-iH_{\text{LMG}}t)[00\ldots0], \]

indicating that the dynamical properties of the device \( H_1 \) can be approximately expressed as the one of the LMG model

\[ H_{\text{LMG}} = -(J/N)(S^z)^2 + \mu S^x. \]

The location of the DPT critical point of the LMG model is \( \mu_c = |J|/2 \), leading to \( h^2 = N\lambda/4 \). Note that we only roughly estimate the location of the dynamical critical point of the LMG model. The numerical simulations in the main text are based on the Hamiltonian of the quantum simulator described by Eq. (1) in the main text.

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Correction of XY crosstalk. The characterisation of the Z-crosstalk effect and its correction have been described in Ref. 3, which are also the same as in this experiment. In addition, the XY-drive crosstalks between qubits must be corrected, as all qubits are driven by identical microwave drives to quench the system. Different from the Z-crosstalk effects, the characterisation of the XY-crosstalk effects includes the phase calibration of the microwave drives. Fig. 5 shows the calibration process taking the measurement of the XY crosstalk effect of $Q_2$ to $Q_1$ as an example.

In Fig. 5a, to characterise the crosstalk amplitude, $Q_1$ is biased to the interacting frequency $\omega_l/2\pi$, while $Q_2$’s frequency is tuned to a nearby one, e.g., $\omega_l/2\pi - 0.08$ GHz (other qubits are decoupled from $Q_1$ and $Q_2$ by tuning their resonant frequencies far away). We apply a strong flattop-envelop microwave pulse with frequency $\omega_l$ to $Q_2$’s XY-control line, generating a crosstalk excitation on $Q_1$. We then monitor the evolution of $Q_1$’s excitations for different resonant frequencies $(\omega_l - \delta_1)$ of $Q_1$ ($\delta_1$ is a small deviation). By fitting the Rabi oscillation of the measured excited probabilities of $Q_1$, we can obtain the crosstalk amplitude of $Q_2$ to $Q_1$.

In Fig. 5b, we characterise the crosstalk phase of $Q_2$ to $Q_1$, with the same frequency arrangement as that in Fig. 5a. However, to cancel the crosstalk effect of $Q_2$ to $Q_1$, a microwave pulse on $Q_1$’s XY-control line, with amplitude equal to the crosstalk amplitude of $Q_2$ to $Q_1$, should also be added. In this process, we monitor the evolution of $Q_1$’s excitations for different phases of the microwave pulses on $Q_2$’s XY-control line, while fixing the microwave phase of $Q_1$ to zero. The excitations of $Q_1$ are almost completely inhibited during the whole evolution process at a specific phase, as can be seen from Fig. 5b, showing that the phase difference of the microwave drives on the XY-control lines of $Q_1$ and $Q_2$ is $\pi$.

Calibrations of other pairs of qubits are performed with a similar method. After quantifying these crosstalk effects, we correct these issues in experiments by considering the case which only involves two qubits $Q_1$ and $Q_2$. We bias these two qubits to $\omega_l/2\pi$ and simultaneously apply resonant microwave pulses on their XY-control lines with amplitudes $Ae^{i\phi_A}$ and $Be^{i\phi_B}$ for $Q_1$ and $Q_2$, respectively (other qubits are decoupled from $Q_1$ and $Q_2$ by tuning their resonant frequencies far away). Here $\phi_A$ and $\phi_B$ represent the microwave phases of $Q_1$ and $Q_2$, respectively. If no correction is made, the microwave amplitude and phase of each qubit can be represented as

$$
\begin{bmatrix}
Ae^{i\phi_A} + a_{12}Be^{i(\phi_{12}+\phi_B)} \\
a_{21}Be^{i(\phi_{21}+\phi_B)} + Be^{i\phi_B}
\end{bmatrix} = \hat{M}_{xy}^{Q_1,Q_2} \begin{bmatrix}
Ae^{i\phi_A} \\
Be^{i\phi_B}
\end{bmatrix},
$$

where

$$
\hat{M}_{xy}^{Q_1,Q_2} = \begin{bmatrix}
1 & a_{12}e^{i\phi_{12}} \\
a_{21}e^{i\phi_{21}} & 1
\end{bmatrix}
$$

is the XY-crosstalk matrix measured with the technique described above. On the contrary, if we set the microwave amplitudes and phases of the qubits to $A'e^{i\phi_A'}$ and $B'e^{i\phi_B'}$ in advance, the microwaves we apply to the XY-control lines should be corrected as

$$
(\hat{M}_{xy}^{Q_1,Q_2})^{-1} \begin{bmatrix}
A' e^{i\phi_A'} \\
B' e^{i\phi_B'}
\end{bmatrix}.
$$

We have performed experiments to verify the validity of this XY-crosstalk correction, where we bias $Q_1$ and $Q_2$ to $\omega_l$ to open the interaction and simultaneously apply identical microwaves to these two qubits. The results are displayed in Fig. 6, demonstrating the validity of this correction. The same treatment can be easily extended to the multi-qubit case.

Note that in the experiments we apply two kinds of microwave drives on the qubits. One establishes the transverse field with the same driving frequency $\omega_l$ for all qubits, and the other is the rotation pulse applied at each qubit’s idle frequency. The XY-crosstalk correction is only applied when we impose microwave drives on all the qubits, to generate the identical transverse fields. As for the rotation pulses, the XY-crosstalk effects are negligible due to the large detuning between the idle points of qubits.

Calibration of the transverse field. The term of the local transverse field $\hbar^2 \sigma_j^x$ for each $Q_j$ in Eq. (1) in the main text is enabled by the resonant microwave drives with calibrated magnitude and phase. To ensure the uniformity of the driving magnitude, we perform Rabi oscillation measurements on each stand-alone qubit $Q_j$ at the interacting frequency $\omega_l$. The qubit exposed to the resonant microwave drive will experience an oscillation of its excited-state probability, where the oscillation frequency $\hbar^2 / \pi$ can be adjusted by the driving amplitude. For the phase calibration of the transverse field, when applying microwave drives with a flattop envelop to each $Q_j$, we actually obtain the form $\hbar^2 \sigma_j^x(e^{-i\phi_j} \sigma_j^z + e^{i\phi_j} \sigma_j^z)$, where $\phi_j$ is the microwave phase sensed by each $Q_j$ and may be different from each other, because of the length disparities between each $Q_j$’s XY-control wires. The experiments require the uniformity of $\phi_j$, which can be
XY-control line with an adjustable phase. After tuning and calibrating the sequence and results for measuring the XY-crosstalk amplitude. In our experiments, we add a microwave drive on one of the qubits with the slowest Rabi oscillation characterises the crosstalk. The selection of \( \phi \) can induce an enhancement or neutralisation effect (red dotted vertical line) on \( Q_1 \)'s Rabi oscillation, which can help us identify the XY-crosstalk phase.

| \( Q_1 \) | \( T_1, k \) (\( \mu s \)) | \( T_2, k \) (\( \mu s \)) | \( g_1/2\pi \) (MHz) | \( \omega_1/2\pi \) (GHz) | \( \omega_1''/2\pi \) (GHz) | \( F_{0,j} \) | \( F_{1,j} \) |
|---|---|---|---|---|---|---|---|
| \( Q_1 \) | 4.835 | 33 | 1.7 | 27.6 | 6.768 | 4.815 | 0.979 | 0.928 |
| \( Q_2 \) | 5.290 | 21 | 1.8 | 27.4 | 6.741 | 5.292 | 0.970 | 0.913 |
| \( Q_3 \) | 5.330 | 37 | 1.8 | 29.1 | 6.707 | 5.350 | 0.978 | 0.920 |
| \( Q_4 \) | 4.460 | 36 | 2.0 | 26.5 | 6.469 | 4.420 | 0.953 | 0.907 |
| \( Q_5 \) | 4.791 | 32 | 2.8 | 29.2 | 6.611 | 4.792 | 0.980 | 0.893 |
| \( Q_6 \) | 4.870 | 30 | 2.1 | 30.1 | 6.513 | 5.245 | 0.976 | 0.925 |
| \( Q_7 \) | 4.569 | 25 | 2.2 | 26.5 | 6.551 | 4.583 | 0.972 | 0.946 |
| \( Q_8 \) | 5.250 | 31 | 2.0 | 27.7 | 6.513 | 5.245 | 0.978 | 0.925 |
| \( Q_9 \) | 4.680 | 23 | 2.4 | 27.3 | 6.524 | 4.765 | 0.967 | 0.926 |
| \( Q_{10} \) | 4.960 | 23 | 1.5 | 26.9 | 6.550 | 4.735 | 0.972 | 0.946 |
| \( Q_{11} \) | 4.899 | 32 | 1.6 | 29.1 | 6.568 | 4.880 | 0.985 | 0.924 |
| \( Q_{12} \) | 5.176 | 22 | 2.0 | 26.3 | 6.640 | 4.310 | 0.993 | 0.941 |
| \( Q_{13} \) | 5.220 | 34 | 2.0 | 26.5 | 6.659 | 5.205 | 0.987 | 0.942 |
| \( Q_{14} \) | 4.490 | 43 | 2.0 | 27.4 | 6.741 | 5.292 | 0.970 | 0.913 |
| \( Q_{15} \) | 4.415 \( >30 \) | 1.9 | 24.6 | 6.788 | 4.375 | 0.967 | 0.944 |
| \( Q_{16} \) | 4.766 | 37 | 1.5 | 27.5 | 6.758 | 4.906 | 0.970 | 0.945 |

TABLE I. Quantum simulator characteristics. Here, \( \omega_j \) is the idle frequency of \( Q_j \), where single-qubit rotation pulses are applied. \( T_{1,j} \) and \( T_{2,j} \) are the energy relaxation time and Ramsey dephasing time (Gaussian decay) of \( Q_j \), respectively, which are measured at the interacting frequency \( \omega_j = \omega_R + \Delta \); \( g_j \) denotes the coupling strength between \( Q_j \) and the resonator bus \( R \); \( \omega_j^\prime \) is the resonant frequency of \( Q_j \)'s readout resonator; \( \omega_j'' \) is the resonant frequency of \( Q_j \) at the beginning of the measurement process, when its readout resonator is pumped with microwave pulse. Finally, \( F_{0,j} \) (\( F_{1,j} \)) is the probability of detecting \( Q_j \) in the \( |0\rangle \) (\( |1\rangle \)) state, when it is prepared in the \( |0\rangle \) (\( |1\rangle \)) state.

![Quantification of XY-crosstalk effects](image_url)

**FIG. 5.** Quantification of XY-crosstalk effects. a. Experimental sequence and results for measuring the XY-crosstalk amplitude. After tuning \( Q_1 \) to the interacting point \( \omega_j \), we apply a strong microwave drive (\( \hbar \omega_j/2\pi \approx 15 \) MHz) on \( Q_2 \)'s microwave drive line with resonant frequency \( \omega_j \). The strong drive will generate a crosstalk Rabi oscillation on \( Q_1 \). We measure the Rabi oscillations for different values of \( \delta_1 \), among which the one with the slowest Rabi oscillation characterises the crosstalk amplitude, as shown by the red dotted line. b. Experimental sequence and results for the measurement of the XY-crosstalk phase. In our experiments, we add a microwave drive on \( Q_1 \)'s XY-control line with an adjustable phase \( \phi \). The selection of \( \phi \) can induce an enhancement or neutralisation effect (red dotted vertical line) on \( Q_1 \)'s Rabi oscillation, which can help us identify the XY-crosstalk phase.

Phase calibration of the rotation pulse. As can be seen from Fig. 1c, after the evolution under the quenched Hamiltonian, we apply the rotation pulse on each qubit before the joint readout to measure the physical quantities, including the average spin magnetisations \( \langle \sigma^x(t) \rangle \).
FIG. 6. Experimental test of the XY-crosstalk correction. a, The experimental sequence. We tune two qubits on resonance at \( \omega_I \), while other qubits are arranged in the vicinity, and apply resonant microwave drives \( \hbar x/2\pi \approx 5 \text{ MHz} \) for each qubit) on these two qubits’s XY-control lines with a controllable phase difference of \( \phi \). b, The measured probabilities \( P_{11} \) of the \( |11\rangle \)-state, versus \( t \) and \( \phi \), in cases with and without applying the XY-crosstalk correction, compared with the numerical results. When no XY-crosstalk correction is made, the measured oscillation periods of \( P_{11} \) for different \( \phi \) values show an obvious inconsistency, indicating an enhancement (black dotted vertical line) or neutralisation (red dotted vertical line) effects induced by the XY crosstalk. After applying the XY-crosstalk correction, the experimental results are in good agreement with the numerical results.

and \( \langle \sigma^y(t) \rangle \). The rotation operation on each qubit is realised by a Gaussian-envelope microwave pulse with a full width at half maxima of 20 ns, which has been characterised by randomised benchmarking with a fidelity above 0.99 for both \( X_{\pi/2} \) and \( Y_{\pi/2} \) rotation gates.

To mainly compensate for the dynamic phase caused by frequency tuning through the sequence, the phase of each rotation pulse needs to be corrected. The calibration process is presented in Fig. 8a, taking \( Q_1 \) as an example. The calibrated qubit is biased to the interacting frequency \( \omega_I \) with a rectangular pulse, while the frequencies of other qubits are arranged in the vicinity to minimise the Z-crosstalk effect. Almost simultaneously, \( Q_1 \) is driven by a flattop-envelope microwave pulse with frequency \( \omega_I \). Then, we bias \( Q_1 \) back to its idle frequency and apply a \( \pi/2 \)-rotation pulse before the readout. We record the probabilities of \( Q_1 \)’s excited state during this dynamics for different phases \( \phi \) of the rotation pulse. The results are displayed in Fig. 8b, where the phase offset used for the correction is highlighted by the red dotted vertical line.

FIG. 7. Phase alignment of the transverse field. a, Experimental sequence. Two qubits (\( Q_1 \) and \( Q_2 \)) are detuned from the resonator bus \( R \) by the same amount \( \Delta/2\pi \approx -450 \text{ MHz} \), while other qubits are arranged in the vicinity of this point to minimise Z-crosstalk effects. We then apply resonant microwave drives on these two qubits with the same magnitude \( \hbar x/2\pi \approx 2 \text{ MHz} \) but a phase difference of \( \phi \) and monitor the dynamics from 40 ns to 120 ns by recording the probabilities of \( Q_1 \) and \( Q_2 \), denoted as \( P_{11}^{Q_1} \) and \( P_{11}^{Q_2} \). b, 2D graph of \( \delta P = P_{11}^{Q_2} - P_{11}^{Q_1} \) as a function of \( t \) and \( \phi \) (top) and the time-averaged \( \delta P \) (bottom). We fit this curve with a sine function to extract the phase offset (red dotted vertical line), which will be added to the microwave drive of \( Q_2 \) to ensure the phase alignment between these two qubits.

Measurement of the spin-squeezing parameter. The calculation of the spin-squeezing parameter \( \xi^z \) consists of the following steps. The first step is to calculate the mean-spin direction \( \bar{n}_0 = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \), where

\[
\theta = \arccos \left( \frac{S_z}{|S|} \right),
\]
optimisation of \( \theta \) Eq. (2) of the main text is actually equivalent to the optimisation procedure 

\[
\overline{\xi}^2 = \frac{2}{N} \left[ \langle (\mathcal{S}^{n_1})^2 \rangle + \langle (\mathcal{S}^{n_2})^2 \rangle \right] \\
+ \sqrt{\langle (\mathcal{S}^{n_1})^2 \rangle - \langle (\mathcal{S}^{n_2})^2 \rangle^2 + \langle S^{n_1}, S^{n_2} \rangle^2},
\]

(3)

with \( \{\mathcal{S}^{n_1}, \mathcal{S}^{n_2}\} = \mathcal{S}^{n_1} \mathcal{S}^{n_2} + \mathcal{S}^{n_2} \mathcal{S}^{n_1} \).

We measure \( \langle (\mathcal{S}^{n_1})^2 \rangle \) and \( \langle (\mathcal{S}^{n_2})^2 \rangle \) by applying single-qubit rotations to move the \( n_1 \) (\( n_2 \)) axis in the Bloch sphere to the z-axis before readout. For \( \langle S^{n_1}, S^{n_2} \rangle \), it boils down to the measurement of two-spin correlators

\[
\langle \{S^{n_1}, S^{n_2}\} \rangle = \frac{1}{4} \left( \sum_{i,j} \sigma_i^{n_1} \sigma_j^{n_2} + \sum_{i,j} \sigma_i^{n_2} \sigma_j^{n_1} \right).
\]

To characterise the two spin correlators for all combinations \((16 \times 15 \times 2)\), we adopt the following methods: First, we divide the 16 qubits into 2 groups randomly, e.g., \( G_1 = \{Q_1-Q_8\} \) and \( G_2 = \{Q_9-Q_{16}\} \). Next, we apply rotation pulses on the qubits in \( G_1 \) to bring the \( n_1 \)-axis to the z-axis, and simultaneously apply other rotation pulses on qubits in \( G_2 \) to bring the \( n_2 \)-axis to the z-axis, after which the 16-qubit joint readout is executed, yielding the probabilities

\[
P_{n_1n_2}(G_1^i, G_2^j) = \sum_{i\in G_1^i,j \in G_2^j} \langle \sigma_i^{n_1} \sigma_j^{n_2} \rangle.
\]

Moreover, by exchanging the rotation pulses applied to qubits in these two groups, we can obtain

\[
P_{n_2n_1}(G_1^i, G_2^j) = \sum_{i\in G_1^i,j \in G_2^j} \langle \sigma_i^{n_2} \sigma_j^{n_1} \rangle.
\]

After repeating this process 5 times, where 16 qubits are divided into two different groups of equal size, we can approach \( \langle \{S^{n_1}, S^{n_2}\} \rangle \) by averaging the overall results

\[
\frac{16 \times 15}{64 \times 5} \sum_{i=1}^{5} \left[ P_{n_1n_2}(G_1^i, G_2^i) + P_{n_2n_1}(G_1^i, G_2^i) \right].
\]

This method has been verified by numerical simulations that possess a very high precision, as illustrated in Fig. 9.

**Finite-size effect of the Loschmidt echo in the LMG model.** The results in the main text are in good agreement with the theoretical predictions based on the LMG...
model. It has been shown that the Loschmidt echo cannot be strictly equal to 0 in a finite-size LMG model\textsuperscript{28}. In Fig. 10a, we present the first minimum value of the Loschmidt echo $\mathcal{L}^{(1)}_{\text{min}}$ as a function of the LMG model’s size $N$ with $J = 1$ and different $g$, showing a perimeter law $\mathcal{L}^{(1)}_{\text{min}} \sim \exp(-\alpha N)$, with $\alpha > 0$. Although $\mathcal{L}^{(1)}_{\text{min}} \to 0$ as $N \to \infty$ for arbitrary $g > 0$, we can still observe a drastic difference of $\mathcal{L}^{(1)}_{\text{min}}$ in the two phases (Fig. 10b). Based on the above discussions, we believe that for the 16-qubit system, the value of $\mathcal{L}^{(1)}_{\text{min}}$ smaller than $\sim 10^{-2}$ can be a characteristic of the dynamical paramagnetic phase.

Possible signature of the anomalous dynamical phase. In addition to $\mathcal{L}^{(1)}_{\text{min}}$, including short-time properties of the Loschmidt echo $\mathcal{L}(t)$, the long-time evolution of $\mathcal{L}(t)$ provides more information, such as the signature of the anomalous dynamical phase, i.e., the $\mathcal{L}(t)$ may approach zero for long times, suggesting the existence of the nonanalytical point of the rate function $r(t) = -N^{-1}\log[\mathcal{L}(t)]$ in the dynamical ferromagnetic phase. The anomalous dynamical phase only exists for models with long-range interactions, and is absent for models with short-range interactions. In Fig. 11, we present the experimental data of $\mathcal{L}^{(1)}_{\text{min}}$ compared with the global minimum value of $\mathcal{L}(t)$, i.e., $\mathcal{L}^{(\text{glob})}_{\text{min}}$, during its dynamics with a final time $t_f \approx 600$ ns, showing a possible signature of the anomalous dynamical phase, which can enlighten further works for investigating the anomalous dynamical phase.

Additional experimental data. In Fig. 12, we plot the experimentally measured non-equilibrium order parameter as a function of the transverse field magnitude for different values of the detuning $\Delta$. In Fig. 13, we display the evolution of the experimental quasidistribution $Q$-function for two transverse field magnitudes.
FIG. 12. Experimental data for non-equilibrium order parameter with different values of the detuning $\Delta$. a, The order parameter $\langle \sigma^z \rangle$ as a function of the field strength $h^x$. The theoretically predicted critical points for $\Delta/2\pi \approx -450$ MHz, $-500$ MHz and $-550$ MHz are $h^x_c/2\pi \sim 5.7$ MHz, 5.0 MHz, and 4.4 MHz, respectively, as highlighted by the dashed vertical lines. b, The same data in a but with $h^x$ normalized by its critical value $h^x_c$. 
FIG. 13. Dynamics of the quasidistribution $Q$-function. The quasidistributions $Q_{\text{exp}}(\theta, \phi)$ at different time intervals, for $\hbar \omega / 2\pi = 2$ MHz (up) and 6 MHz (down), respectively.