Multi-scale phase waveform inversion method of on-ground GPR data

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Abstract. Full waveform inversion of ground penetrating radar (GPR) based on Maxwell's equations can invert the permittivity and conductivity of the underground medium, and achieve quantitative interpretation of the target formation. Here, we present a GPR waveform inversion method based on the phase misfit function, and inverts the phase information in the GPR data. By analyzing the characteristics of phase transformation, the misfit function of PWI is given. The gradient is constructed by the convolution of the forward and reverse fields, and its gradient formula is derived. When the transmitting source and receiver of GPR are arranged on the ground, the waveform inversion can only use the scattered field to obtain the effective information of the underground medium, and the inversion will face a strong indetermination. The inversion is more dependent on the initial model than the cross-hole GPR waveform inversion, but non-destructive detection often lacks prior information. In order to solve this problem, this paper makes use of the characteristics of integral operation and formulates a multi-scale inversion strategy. In the iterative process of inversion, high integration times are preferentially used to restore long wavelength information. This can reduce the dependence of the waveform inversion on the initial model as much as possible, implement the waveform inversion method for on-ground GPR data, and verify it with two sets of synthetic data.

1. Introduction

GPR is a new geophysical exploration method, which has the advantages of simple operation, high efficiency and non-destructive detection. Unlike other geophysical methods, they are more widely used in geological surveys and resource exploration. The GPR method is mainly used in environmental and engineering exploration fields such as highways, railways, water conservancy, and construction. With the development of recent years, it has gradually become a conventional geophysical exploration technology in the above fields. Conventional GPR imaging mainly relies on conventional processing and migration of radar signals. By using radar reflected wave information, a radar profile that can reflect the position of the underground anomalous body and the stratum interface is obtained. With the development of economy and society, the application scope of GPR is getting wider, and the technical requirements of GPR are getting higher. GPR imaging is not satisfied with only obtaining the interface position, and the research is gradually developing to the inversion method which can obtain the specific electrical parameters of underground. In order to obtain the high-resolution images of dielectric constant and conductivity of underground media simultaneously, and realize the objective of intuitive and quantitative interpretation of the physical parameters of
underground media. Waveform inversion technology has been introduced into the field of GPR and has gradually become a research hotspot.

The earliest application of waveform inversion technology in geophysics is for inversion of seismic data. Tarantola [1] proposed a full waveform inversion method based on the generalized least squares inversion theory. The convolution of the forward propagation field and the residual back-propagation field was used to construct the gradient required in the iterative process, a theoretical framework of time domain waveform inversion method for seismic data was formed. Subsequently, it was further developed by Pratt [2-3] using the Gauss-Newton method and calculating Hessian, to invert seismic data in the frequency domain. Seismic waveform inversion methods have matured for more than 30 years, and have achieved certain results. They can be used to invert synthetic data from a series of classic seismic models.

Considering that there is a high degree of consistency in the propagation modes of the radar wave and the seismic wave, the practical experience of the waveform inversion method in the field of seismic data inversion is substituted into the radar inversion field, and the adjustment has been applied to the processing of GPR data. Ground penetrating radar encountered many problems in the application of waveform inversion method: 1. Radar waves as electromagnetic waves have a much stronger attenuation than seismic waves as elastic waves when they propagate in the medium. Therefore, the reflected wave intensity which can reflect the effective information of the underground target body is relatively weak; 2. Due to the shallow depth of the GPR, the time series occupied by the direct wave accounts for a higher proportion in the time series of the entire waveform, so it is often difficult to distinguish between direct and reflected waves; 3. The radar wave is affected by electromagnetic interference is relatively serious. The magnitude of the amplitude of the electrical noise is equivalent to or even higher than the magnitude of the reflected wave amplitude, which adds great difficulty to the inversion. The above problems are difficult to overcome in the short term. In order to avoid the above problems, the GPR waveform inversion method was originally used to process the cross-hole GPR data. This was done to invert the waveform of the direct wave and was successfully verified. Meles [4] proposed a vector full waveform inversion technique, and the dielectric constant and conductivity were iterated simultaneously during the inversion process. Wu [5] deduced the theoretical basis of full waveform inversion in detail, using the steepest descent method to update the model parameters, but without considering the conductivity. Meng [6] realized the simultaneous inversion of the dielectric constant and conductivity of GPR data by performing forward modeling of radar waves in the time domain and calculating gradients in the frequency domain. However, there are few studies on GPR waveform inversion methods based on ground-acquired data, and the results obtained are limited.

In this paper, a phase waveform inversion method (PWI) is proposed, and the phase-integrated multi-scale inversion strategy is used to invert the synthetic data of on-ground GPR data. During the inversion process, a ground penetrating radar vector matrix waveform inversion scheme based on Maxwell’s equations was used. The forward modeling of the model was achieved by using CPML absorption boundary of high-order finite difference in time domain. When constructing the ground penetrating radar phase waveform inversion misfit function, the amplitude spectrum and phase spectrum of the actual observation data and forward data are obtained through Fourier transform (FFT) transformation, and the amplitude spectrum of the actual observation data is used instead of the synthesized data amplitude spectrum. The amplitudes of the observation data and the forward data are normalized to eliminate the influence of the amplitude and achieve the purpose of inverting the phase. Phase waveform inversion can recover the same model information from GPR data as full wave inversion. The phase waveform inversion does not need to know the amplitude of the source wavelet, and it is difficult to estimate the amplitude when processing actual data. The phase-integrated multiscale inversion strategy allows the inversion to use a poor initial mode, which is very important for waveform inversions without prior information. Through phase multi-scale inversion method, this paper successfully inverts the ground penetrating radar synthetic data of the ground acquisition scheme, and realizes the inversion of the permittivity and conductivity respectively. In synthetic data
simulation, the phase multiscale inversion method can accurately restore the position information of anomalous bodies and formations, and the electrical parameters obtained by the inversion are very close to the actual model.

2. PWI theory

2.1. PWI misfit function

The misfit function expression of full waveform inversion is as follows:

$$S(\varepsilon, \sigma) = \frac{1}{2} \sum_{i} \sum_{j} \sum_{r} \left[ \mathbf{E}(\varepsilon, \sigma) - \mathbf{E}^{obs} \right] \cdot \left[ \mathbf{E}(\varepsilon, \sigma) - \mathbf{E}^{obs} \right]$$  \hspace{1cm} (1)

In the formula, \( \mathbf{E}(\varepsilon, \sigma) \) and \( \mathbf{E}^{obs} \) are forward data and actual data, respectively. The error value of the misfit function is the sum of all transmitting sources (represented by \( i \)), receivers (represented by \( j \)), and observation time (represented by \( \tau \)). Among them, each of the synthetic data and the actual data can obtain their amplitude spectrum \( \mathbf{A} \) and phase spectrum \( \phi \) by Fourier transform.

$$\mathbf{F} \left[ p(x, \omega; x, t) \right] = A(x, \omega, x) \exp \left[ i \phi(x, \omega, x) \right]$$  \hspace{1cm} (2)

$$\mathbf{F} \left[ p(x, \omega; x, t) \right] = A(x, \omega, x) \exp \left[ i \phi(x, \omega, x) \right]$$  \hspace{1cm} (3)

Here, the amplitude spectrum of the data obtained in the forward process is replaced with the amplitude spectrum of the actual data to ensure that the intensity of the two sets of data is equal. For two data with equal intensity, the matching is mainly based on the phase spectrum of the GPR data. The synthesized data and actual data expressions after replacing the amplitude spectrum are as follows:

$$\mathbf{F} \left[ p(x, \omega; x, t) \right] = \mathbf{F}^{-1} \left[ L(\omega) A(x, \omega, x) \exp \left[ i \phi(x, \omega, x) \right] \right]$$  \hspace{1cm} (4)

$$\mathbf{F} \left[ p(x, \omega; x, t) \right] = \mathbf{F}^{-1} \left[ L(\omega) A(x, \omega, x) \exp \left[ i \phi(x, \omega, x) \right] \right]$$  \hspace{1cm} (5)

Among them, \( A(x, \omega, x) = A(x, \omega, x) \), \( L \) is a filter for processing each set of data. Integrate the time-domain signal synthesized above and construct the misfit function of PWI:

$$S^{\text{approx}}(\varepsilon, \sigma) = \sum_{i} \sum_{j} \sum_{r} \left[ I^{*} \mathbf{E}(\varepsilon, \sigma) - I^{*} \mathbf{E}^{obs} \right] \cdot \left[ I^{*} \mathbf{E}(\varepsilon, \sigma) - I^{*} \mathbf{E}^{obs} \right]$$  \hspace{1cm} (6)

Among them, \( I^{n} \) is an integration operator, \( I = \int dt \); \( I^{n} \) indicates that the integration has been performed a total of \( n \) times, \( \mathbf{E}(\varepsilon, \sigma) \) and \( \mathbf{E}^{obs} \) are the wave fields modified according to formulas (4) and (5), respectively. If \( \mathbf{E}(\varepsilon, \sigma) \) is replaced by \( \mathbf{E}(\varepsilon, \sigma) \), the PWI misfit function is consistent with the FWI misfit function, and only low-pass filters and integration operators are added.

2.2. Misfit function gradient

The gradient of the phase waveform inversion is obtained by deriving the model parameters with formula (6), and has the following form:

$$\frac{\partial S}{\partial p} = \sum_{i} \sum_{j} \sum_{r} \frac{\partial I^{*} \mathbf{E}}{\partial p} \left( I^{*} \mathbf{E} - I^{*} \mathbf{E}^{obs} \right)$$  \hspace{1cm} (7)
In the above formula, \( I^*E \) is abbreviated as \( \mathcal{E}_I \). The convolution of the forward function’s Green’s function \( G \) and the virtual vector source \( v \), the formula (7) can be rewritten as follows:

\[
\frac{\partial S}{\partial p} = \sum_{i}^{n_r} \sum_{j}^{n_s} \left( I^*v \ast G \right) \left( \mathcal{E}_I - \mathcal{E}_I^{obs} \right)_{i,j}
\]  

(8)

In Equation (8), \( v \) is the virtual source vector. Rewrite the above formula into integral form:

\[
\frac{\partial S}{\partial p} = \sum_{i}^{n_r} \sum_{j}^{n_s} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I^*v_{i,j}(\xi - \tau) g_{i,j}(\tau) d\tau \sqrt{r_{i,j}(\xi)} d\xi
\]

(9)

In the above formula, \( ns \) and \( nr \) represent the total number of transmitting sources and receivers, respectively. Where the backward residual field source \( \sqrt{r}(\xi) = \mathcal{E}_i(\xi) - \mathcal{E}_I^{obs}(\xi) \).

Because the transmitting source represented by the virtual source vector \( v_{i,r} \) is positive in time, and the receiving represented by the Green function \( g_{i,j} \) is opposite in time, the subscripts of the two are not the same, and the opposite direction corresponds to time. Let \( \xi - \tau = t \), then \( dt = -d\tau \). Rearrange (8) formula:

\[
\frac{\partial S}{\partial p} = \sum_{i}^{n_r} \sum_{j}^{n_s} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I^*v_{i,j}(\tau) \left[ \int_{-\infty}^{\infty} g_{i,j}(\xi - \tau) r_{i,j}(\xi) d\xi \right] dt
\]

(10)

\[
\int_{-\infty}^{\infty} g(\xi - \tau) r(\xi) d\xi
\]

represents the backward propagation field with opposite time series, and the formula (10) is the zero-phase convolution of the virtual source and the backward propagation field. According to formula (10), this paper gives the gradient expressions of dielectric constant and conductivity:

\[
\begin{bmatrix}
\nabla S_x \\
\nabla S_y
\end{bmatrix} = \sum_{i}^{n_r} \sum_{j}^{n_s} \left( \frac{\partial}{\partial p} \mathcal{E}_I \right)^T \left( \hat{G} \hat{R} \right)
\]

(11)

Among them, \( \hat{G} \hat{R} \) represents the backward propagation process of the sum of the residual field \( \overline{R} \).

3. PWI Multiscale Inversion Strategy

3.1. Influence of the Integral Times

The misfit function of the PWI shown in Equation 6 includes an integration operator \( I^n \), and \( n \) represents the number of integrations of the GPR data. During the inversion process, the number of integrations in the misfit function determines the frequency component corresponding to the current inversion data. In order to verify the influence of integration times on single channel data, this paper presents a Ricker wavelet with a central frequency of 200MHz. The effect of integration is illustrated by taking different times of integration for the Ricker wavelet and observing the influence of integration times on the wavelet frequency.
Figure 1. Frequency characteristics of Ricker wavelet and its integrated wavelet (a) Ricker wavelet and (e) its spectrum; (b) wavelet and (f) its spectrum after one integration; (c) wavelet and (g) its spectrum after two integration; (d) wavelet and (h) its spectrum after three integration.

It can be seen from Fig. 1 (a-d) that as the number of integrations increases, the waveform of the wavelet in the time domain changes. Comparing the wavelets with different integration times, it can be clearly found in the frequency domain that as the integration times increase, the frequency range moves to low frequencies, and the frequency band narrows. This also shows that during the inversion process, the misfit function with high integration times is suitable for the low-frequency information of the inversion data. The phase multiscale waveform inversion can be achieved by using the misfit function with different integration times.

3.2. Multiscale inversion strategy

Based on the characteristics of integral operation, a multi-scale inversion strategy for phase waveform inversion is formulated in this paper (Figure 2). In the initial stage of the inversion, the data can be selectively filtered to purposely invert part of the frequency information of the observation data. During the phase waveform inversion stage (in the dashed box), this paper chooses to integrate the data. When the value of \( n \) is relatively large, the inversion is mainly aimed at the low frequency information in the GPR data. Usually we give a maximum value \( N_{\text{max}} \). As the iteration continues, the error value decreases. When the error value is less than our preset value \( S_i \), we believe that convergence is achieved under the current integration times. Next, we use this inversion result as the initial model and reduce the value of \( n \), perform the inversion again, and repeat the above operation. When \( n \) is equal to 0, it means that we no longer integrate the data. When the error value \( S \) is less than \( S_0 \), the inversion ends and the inversion result is output.
4. Inversion results of synthetic data

4.1. Synthetic Data: Double-layer Pipeline Anomaly Model

The model is shown in Figure 3. The size of the underground part is 3m * 6m, including 2 floors. Among them, the relative permittivity and conductivity of the first layer are 5 and 0.005 S / m, and the relative permittivity and conductivity of the second layer are 6 and 0.006 S / m, respectively. The interface between the first layer and the second layer is located at 1.9m. In the first layer of medium, a 0.3-m-diameter pipeline-shaped anomalous body is contained at a position of (3 m, 0.9 m), and the relative permittivity and conductivity are 7 and 0.007 S / m, respectively. The relative permittivity of the air layer is 1, and the electrical conductivity is 0.00001S / m. The initial model is a homogeneous medium model, and its physical properties are consistent with those of the first layer of underground medium. When the permittivity is inverted separately, the conductivity of the underground medium is set to 0.001 S / m. When the conductivity is inverted separately, the relative permittivity of the underground medium is set to 1. The on-ground GPR data collection system uses a single-transmission source and multiple-receiver acquisition. The center frequency of the antenna is 200 MHz. There are 17 transmitting sources and receiver positions, and the interval between each position is 0.3 m.

As shown in Figure 3, it can be clearly seen that the inversion result of the two-dimensional profile includes a circular anomaly. The shape, position, and electrical parameter values of the circular anomaly obtained by the inversion are very consistent with the original model. A horizontal interface can be seen below the circular anomaly, and the location of the interface is consistent with the interface of the original model. In the inversion results, the permittivity and conductivity of the formation below the interface are higher than those above the interface. The overall law is consistent with the original model, but in specific values, the substratum is not completely consistent with the original model. The reason may be that due to the attenuation of the radar signal and the existence of the absorption boundary, the receiver cannot receive the reflected wave that penetrates the interface.
and reflects back to the ground again. The conductivity inversion results of the lowermost stratum are slightly better than the dielectric constant, and are numerically closer to the original model.

![Figure 3. PWI results of Synthesized Data II. Relative permittivity (a) Original model (c) Inversion results; Conductivity (b) Original model (d) Inversion results.](image)

5. Conclusions
This paper implements phase multiscale waveform inversion for on-ground GPR data, analyzes how to eliminate the effect of amplitude, and then gives a phase misfit function. Based on the phase misfit function, its gradient formula is derived. For the iterative process, the influence of the number of integrations during the inversion is analyzed, and a multi-scale strategy for phase waveform inversion is given by using the characteristics of the integration. In order to verify the validity of the method, the inversion was performed through one set of synthetic data, and the permittivity and conductivity of on-ground GPR data were separately inverted. It is proved that the multi-scale inversion strategy can effectively reduce the nonlinear problem of inversion. The phase multi-scale waveform inversion method can realize the inversion of on-ground GPR data.

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