Effect of radial magnetic field on peristaltic transport of Jeffrey fluid in curved channel with heat/mass transfer

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Abstract: In this paper, we deal with the impact of radialiy magnetic field on the peristaltic transport of Jeffrey fluid through a curved channel with two dimensional. The effect of slip condition on velocity, the non-slip condition on temperature and conversation is performed. The heat and mass transfer are considered under the influence of various parameters. The flow is investigated under the assumption of long wave length and low Reynolds number approximations. The distribution of temperature and concentration are discussed for various parameters governing the flow with the simultaneous effects of Brinkman number, Soret number and Schmidt number.

1- Introduction

Peristaltic transport of fluid is quite popular topic of research amongst the mathematicians, physiologists and engineers. Such popularity of this topic is due to occurrence of peristalsis in the physiological and engineering processes. The peristaltic pumping is a mechanism for fluid transport induced by progressive wave of contraction and relaxation along the distensible tube. Fluid transport in view of peristalsis is an important biological mechanism responsible for various physiological functions of the organs in the human body. Particularly such mechanism is in urine passage from kidney to bladder through ureter, chyme movement in the gastrointestinal tract, ovum movement in the female fallopian tube, transport of spermatozoa in ducts efferent of male reproductive tract, transport of lymph in lymphatic vessels such as arterioles, capillaries, venules and in esophagus during food swallowing process. Practically the peristaltic pumps are designed by engineers for pumping corrosive fluids without contact with the walls of the pumping machinery. In nuclear industry the peristaltic pumping has been found in corrosive fluid or sensitive fluids, sanitary fluids, transport of slurries and noxious fluids. Latham [1], Jaffrin and Shapiro [2], Shapiro et al. [3] and Fung [4] were the first who made a detailed analysis on peristaltic pumping. It is also noted that initial attempts for
Peristalsis have been made for viscous liquids. This is not adequate since most of the materials in the physiological and engineering processes are non-Newtonian. There are three types of non-Newtonian fluids (i.e.), 1. Differential type. 2. Rate type. 3. Integral type. The non-Newtonian fluids which exhibit the characteristic of relaxation or retardation times are belong to rate type fluids. Maxwell fluid is one of the subclass of rate type fluids which contains only relaxation time behavior. The only drawback of this fluid model is that it does not explain the retardation time behavior. Therefore to fill this gap, Jeffrey fluid model is considered this model shows the behavior of linearly viscoelastic polymer industries. Moreover the Jeffrey fluid model is comparatively simple linear model using time derivatives instead of convective derivatives for example the oldroyd-B fluid model does, it represents a different rheological behavior from that of the Newtonian fluid. In view of diverse characteristics of non-Newtonian materials, various constitutive equations have been suggested. Among such constitutive equations there is one for Jeffrey fluid which has been already utilized for peristaltic transport in both symmetric and asymmetric channel (see [5, 6, 7, 8, 9].

Influence of applied magnetic field on peristaltic activity is important in connection with certain problems of the movement of the conductive physiological fluids, e.g., blood and the blood pump machines, magnetic drug targeting and relevant process of human digestive system. Such consideration is also useful in treating gastro paresis, chronic constipation and morbid obesity, for more details can one see [10, 11]. The convective heat transfer is of excessive significance in procedures in which high temperatures are involved for instance, gas turbines, nuclear plants, storage of thermal energy etc. Referring to numerous industrial and engineering processes. The convective boundary conditions are also more practical in material drying, transpiration cooling process etc. Also impact of heat transfer in peristaltic transport of fluid is quite significant in food processing, oxygenation, hemodialysis, tissues conduction, heat convection for blood flow from the pores of tissues and radiation between environment and its surface. Mass transfer is useful in the afore mentioned processes. Especially mass transfer cannot be under estimated when nutrients diffuse out from the blood to neighboring tissues. Further mass transfer involvement is quite prevalent in distillation, chemical impurities diffusion, membrane separation and combustion process. It should be noted that relationships between fluxes and driving potentials occur when both heat and mass transfer act simultaneously. Here temperature gradient generates energy flux. However mass flux and composition gradients are due to temperature gradient (which is called soret effect). It is noted that all the afore mentioned studies on peristaltic transport have been conducted for peristalsis in straight channels which is not realistic always since most of the pipes, arteries and glandular ducts are curved. Thus some advancements have been made for peristalsis using curvilinear coordinates. Abbasi et al. [12] initiated such analysis for peristaltic transport of viscous fluids. Sato et al. [13] extend the work of sato et al. in wave frame of reference. Later some attempts [14, 15, 16, 17] have been presented to address. The curvature effects on peristalsis of fluids in a channel. In these attempts mostly the constant magnetic field are considered. Recently, Hayat et al. [18] is given in their work to explore the characteristics of radial magnetic field on peristaltic transport of Jeffrey fluid in a curved channel. Heat transfer is characterized there by utilizing convective condition. Hayat et al. [19] investigated the effect of radial magnetic field on the peristaltic flow of Jeffrey liquid in curved channel with compliant walls.

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tissues and radiation between environment and its surface. Mass transfer is useful in the aforementioned processes. Especially mass transfer cannot be under estimated when nutrients diffuse out from the blood to neighboring tissues. Further mass transfer involvement is quite prevalent in distillation, chemical impurities diffusion, membrane separation and combustion process. It should be noted that relationships between fluxes and driving potentials occur when both heat and mass transfer act simultaneously. Here temperature gradient generates energy flux. However mass flux and composition gradients are due to temperature gradient (which is called soret effect). It is noted that all the aforementioned studies on peristaltic transport have been conducted for peristalsis in straight channels which is not realistic always since most of the pipes, arteries and glandular ducts are curved. Thus some advancements have been made for peristalsis using curvilinear coordinates. Abbasi et al.\cite{12} initiated such analysis for peristaltic transport of viscous fluids. Sato et al.\cite{13} extend the work of Sato et al. in wave frame of reference. Later some attempts\cite{14,15,16,17} have been presented to address. The curvature effects on peristalsis of fluids in a channel. In these attempts mostly the constant magnetic field are considered. Recently, Hayat et al.\cite{18} is given in their work to explore the characteristics of radial magnetic field on peristaltic transport of Jeffrey fluid in a curved channel. Heat transfer is characterized there by utilizing convective condition. Hayat et al.\cite{19} investigated the effect of radial magnetic field on the peristaltic flow of Jeffrey liquid in curved channel with compliant walls.

Now, in our work, we investigated the effect of radial magnetic field on the peristaltic flow of Jeffrey fluid in curved channel by using the effect of heat and mass transfer. The effects of viscous dissipation and thermophoresis are considered in the transport equations such as the effect of Brinkman number (Br), Schmidt number (Sc) and soret number (Sr). the slip boundary conditions on velocity, and non-slip boundary conditions on temperature and conservation are considered. The equations are simplified by using long wave length and low Reynolds number approximations. The graphical results are obtained to explain the effects of parameters entering in the problem.

2- Mathematical formulation

Consider two-dimensional motion of an viscous incompressible Jeffrey fluid in a curved channel of width $(2a)$, centre at $0^\circ$ and radius at $R$ as shown in figure (1). The flow is generated due to the transverse deflections of sinusoidal waves of small amplitudes $(b)$ that are imposed on the flexible walls of the channel. The inertial effects are assumed to be small. The lower and upper walls of the channel are maintained at the same temperature $T_0$ and concentration $C_0$. The equations of the walls of channel are described as follows:

$$\tilde{r} = \mp H (\tilde{X}, \tilde{t}) = \mp a \mp b \cos(\frac{2\pi}{\lambda} (\tilde{X} - C_0 \tilde{t}))$$

.....(1)

Where $\tilde{X}$ is the axial distance, $\tilde{r}$ is the radial distance, $a$ is the radius of the stationary curved channel, $b$ is the wave amplitude, $\lambda$ is the wave length, $\tilde{t}$ is the time and the wave length is large compared with the channel width $(a)$ that is $(\frac{a}{\lambda} << 1)$.
3- Constitutive equations

The constitutive equations for a Jeffrey fluid given by: [20]

\[ \ddot{\mathbf{I}} = -P \mathbf{I} + \dddot{\mathbf{S}}, \quad \dddot{\mathbf{S}} = \frac{\mu}{1 + \lambda_1} \left( \dot{\gamma} + \lambda_2 \dddot{\gamma} \right) \]

Where \( \dddot{\mathbf{I}} \) and \( \dddot{\mathbf{S}} \) are Cauchy stress tensor and extra stress tensor, respectively, \( P \) is the pressure, \( \mathbf{I} \) is the identity tensor, \( \mu \) is dynamic viscosity, \( \lambda_1 \) is the ratio of relaxation to retardation times, \( \lambda_2 \) is the retardation time, \( \dot{\gamma} \) is the shear rate and dots over the quantities indicate differentiation with respect to time.

Let \( \mathbf{V} = \mathbf{V}(r, \mathbf{X}, t), \mathbf{V}(r, \mathbf{X}, t), 0 \) be the velocity vector in the curvilinear coordinates \((r, \mathbf{X})\).

\[
(\mathbf{grad}\mathbf{V}) = (\nabla\mathbf{V}) = \begin{pmatrix}
\frac{\partial \mathbf{V}}{\partial r} & \frac{\partial \mathbf{V}}{\partial \mathbf{X}} & \frac{\partial \mathbf{V}}{\partial t}
\end{pmatrix}
\]

The strain \( \mathbf{E} \) is defined by:

\[
e = \frac{1}{2}[((\nabla\mathbf{V}) + (\nabla\mathbf{V})^T)]
\]

The shear strain or shear rate \( \dot{\gamma} \) is defined by:
\[
\gamma = 2e = \left( \frac{2 \frac{\partial \nabla}{\partial r}}{\frac{\partial U}{\partial r}} + \frac{R}{r} \frac{\partial \nabla}{\partial \bar{X}} - \frac{\bar{U}}{r + R} \right) - \left( \frac{2 \frac{R \frac{\partial \nabla}{\partial \bar{X}}}{r + R}}{r + R} \right) \quad \text{....(6)}
\]

So, we have:

\[
\gamma'_{rr} = 2 \frac{\partial \nabla}{\partial r}, \quad \gamma'_{rr} = 2 \frac{\partial \nabla}{\partial r} + \frac{R}{r} \frac{\partial \nabla}{\partial \bar{X}} - \frac{\bar{U}}{r + R}, \quad \gamma'_{XXX} = 2 \left( \frac{R \frac{\partial \nabla}{\partial \bar{X}}}{r + R} + \frac{\nabla}{r + R} \right) \quad \text{....(7)}
\]

Now, define \( \gamma \) as follows:

\[
\gamma = \frac{D}{Dt} \gamma = (\frac{\partial}{\partial t} + \bar{V} \cdot \nabla) \gamma = \frac{\partial}{\partial t} \gamma + (\bar{V} \cdot \nabla) \gamma, \quad \text{......(8)}
\]

in which

\[
\bar{V} \cdot \nabla = \frac{R}{r + R} \frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial r}, \quad \text{......(9)}
\]

Thus we have:

\[
\gamma'_{rr} = \frac{\partial}{\partial t} \gamma'_{rr} + \left( \frac{R}{r + R} \frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial r} \right) \gamma'_{rr}
\]

\[
= 2 \frac{\partial}{\partial t} \left( \frac{\partial}{\partial r} + \left( \frac{R}{r + R} \frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial r} \right) \frac{\partial \nabla}{\partial r} \right), \quad \text{......(10)}
\]

\[
\gamma'_{rr} = \frac{\partial}{\partial t} \gamma'_{rr} + \left( \frac{R}{r + R} \frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial r} \right) \gamma'_{rr}
\]

\[
= \left[ 1 + \left( \frac{R}{r + R} \frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial r} \right) \left( \frac{\partial \nabla}{\partial r} \right) \right] \frac{\partial \nabla}{\partial r}, \quad \text{......(11)}
\]

\[
\gamma'_{xx} = \frac{\partial}{\partial t} \gamma'_{xx} + \left( \frac{R}{r + R} \frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial r} \right) \gamma'_{xx}
\]

\[
= 2 \frac{\partial}{\partial t} \left( \frac{\partial}{\partial r} + \left( \frac{R}{r + R} \frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V} \frac{\partial}{\partial r} \right) \frac{\partial \nabla}{\partial r} \right), \quad \text{......(12)}
\]

The components of shear tensor (S) are:
\[
S = \begin{pmatrix}
S_{rr} & S_{rX} \\
S_{Xr} & S_{XX}
\end{pmatrix}
\]

\[
S_{rr} = \frac{\mu}{1 + \lambda_1} \left( \gamma_{rr} + \lambda_2 \gamma_{rr} \right)
= \frac{\mu}{1 + \lambda_1} \left( 2 \frac{\partial \bar{V}}{\partial r} + 2 \lambda_1 \frac{\partial}{\partial t} + \left( \frac{R}{r + R} \frac{\partial}{\partial X} + \bar{V} \frac{\partial}{\partial r} \right) \frac{\partial \bar{V}}{\partial r} \right)
= \frac{2\mu}{1 + \lambda_1} \left( 1 + \lambda_2 \frac{\partial}{\partial t} + \left( \frac{R}{r + R} \frac{\partial}{\partial X} + \bar{V} \frac{\partial}{\partial r} \right) \frac{\partial \bar{V}}{\partial r} \right)
\]

\[
S_{XX} = \frac{\mu}{1 + \lambda_1} \left( \gamma_{XX} + \lambda_2 \gamma_{XX} \right)
= \frac{\mu}{1 + \lambda_1} \left( \frac{\partial \bar{U}}{\partial r} + \frac{R}{r + R} \frac{\partial \bar{V}}{\partial X} - \frac{\bar{U}}{r + R} \right) + \lambda_2 \left( \frac{\partial}{\partial t} + \left( \frac{R}{r + R} \frac{\partial}{\partial X} + \bar{V} \frac{\partial}{\partial r} \right) \frac{\partial \bar{U}}{\partial r} \right)
= \frac{\mu}{1 + \lambda_1} \left( 1 + \lambda_2 \frac{\partial}{\partial t} + \left( \frac{R}{r + R} \frac{\partial}{\partial X} + \bar{V} \frac{\partial}{\partial r} \right) \frac{\partial \bar{U}}{\partial r} \right) + \left( \frac{R}{r + R} \frac{\partial}{\partial X} - \frac{\bar{U}}{r + R} \right)
\]

\[
\frac{\partial}{\partial X} = \left( 1 + \lambda_2 \right) \frac{\partial}{\partial t} + \left( \frac{R}{r + R} \frac{\partial}{\partial X} + \bar{V} \frac{\partial}{\partial r} \right) \frac{\partial \bar{U}}{\partial r} + \left( \frac{R}{r + R} \frac{\partial}{\partial X} + \bar{V} \frac{\partial}{\partial r} \right) \frac{\partial \bar{V}}{\partial r}
\]

\[
\frac{\partial}{\partial X} = \left( 1 + \lambda_2 \right) \frac{\partial}{\partial t} + \left( \frac{R}{r + R} \frac{\partial}{\partial X} + \bar{V} \frac{\partial}{\partial r} \right) \frac{\partial \bar{U}}{\partial r} + \left( \frac{R}{r + R} \frac{\partial}{\partial X} + \bar{V} \frac{\partial}{\partial r} \right) \frac{\partial \bar{V}}{\partial r}
\]

\[
\text{4- Calculation of Lorentz force}
\]

Fluid in this problem is flowing under the influence of radially varying magnetic field of the form [19]:

\[
\vec{B} = \frac{RB_0}{R + r} \hat{e}_r
\]

The type of magnetic field given through eq.(1) satisfies the Maxwell equations. Velocity field for present flow configuration is taken of the form:

\[
\bar{V} = \left[ \bar{U}(r, \bar{X}, t), \bar{V}(r, \bar{X}, t), 0 \right] \quad \text{where} \quad \bar{U} \quad \text{and} \quad \bar{V} \quad \text{are the axial and radial components of the velocity respectively.}
\]

The Lorentz force \( \vec{F} \) in view of the magnetic and velocity fields mentioned above takes the following form:


\[
\vec{J} = \nabla \times \vec{B} = \begin{vmatrix} e_x & e_r & e_z \\ \vec{U} & \vec{V} & 0 \\ 0 & \frac{R}{r + B_0} & 0 \end{vmatrix} = \frac{R}{R + r} \vec{U} B_0 e_x \\
\]

\[ \sigma \times \vec{J} = \frac{R}{r + B_0} \sigma \vec{U} B_0 e_x \]  

Utilization of ohms law gives the following expression :

\[
\vec{F} = \nabla \times \vec{B} = \begin{vmatrix} e_x & e_r & e_z \\ 0 & 0 & \frac{R}{r + B_0} \sigma \vec{U} B_0 \\ 0 & \frac{R}{r + B_0} & 0 \end{vmatrix} = -\left(\frac{R}{r + B_0}\right)^2 \sigma \vec{U} B_0 e_x \\
\]

That is

\[
\vec{F} = [-\sigma \left(\frac{R}{r + B_0}\right)^2 \vec{U} B_0^2, 0, 0] \\
\]

where \( B_0 \) is the strength of applied magnetic field, \( e_r \) is the unit vector in the radial direction, \( \vec{J} \) is the current density and \( \sigma \) is the electric conductivity of fluid, \( \vec{B} \) is the magnetic field. It is observed that the effect of magnetic field appear in the flow of axial direction.

5- Basic equations

The basic equations governing the non-Newtonian in compressible viscous Jeffrey fluid are given by:

The continuity equation is given by:

\[
\rho \frac{\partial \vec{U}}{\partial t} + \rho \frac{\partial \vec{U}}{\partial r} + \frac{\rho \vec{U}}{r + R} \frac{\partial \vec{U}}{\partial r} \frac{c_x}{c_x} + \frac{\vec{U} \vec{V}}{r + R} = -\frac{R}{r + R} \frac{\partial \vec{P}}{\partial r} + \frac{R}{r + R} \frac{\partial}{\partial \vec{P}} S_{xx} + \frac{1}{(r + R)^2} \]

\[
\frac{\partial}{\partial r} \left( (r + R)^2 S_{xx} \right) - \sigma (r + R)^2 B_0^2 \vec{U} \\
\]

The momentum equations are given by:

\[
\rho \frac{\partial \vec{V}}{\partial t} + \rho \frac{\partial \vec{V}}{\partial r} + \frac{\rho \vec{V}}{r + R} \frac{\partial \vec{V}}{\partial r} \frac{c_x}{c_x} + \frac{\vec{U} \vec{V}}{r + R} = -\frac{\partial \vec{P}}{\partial r} + \frac{1}{r + R} \frac{\partial}{\partial \vec{P}} \left( (r + R) S_{xx} \right) + \frac{R}{r + R} \frac{\partial}{\partial \vec{P}} S_{xx} \\
\]

The temperature equation is given by :
\[ \rho C_p \left( \frac{\partial}{\partial t} + \nabla \cdot \mathbf{U} \right) + \frac{R \mathbf{U}}{r + R} \frac{\partial \mathbf{U}}{\partial r} + \frac{1}{r + R} \frac{\partial}{\partial r} \left( \frac{R}{r + R} \frac{\partial \mathbf{U}}{\partial r} \right) \mathbf{U'} = k \left[ \frac{\partial^2 \mathbf{U}}{\partial r^2} + \frac{1}{r + R} \frac{\partial \mathbf{U}}{\partial r} + \left( \frac{R}{r + R} \right)^2 \frac{\partial^2 \mathbf{U}}{\partial X^2} \right] + \frac{1}{r + R} \frac{\partial}{\partial r} \left( \frac{R}{r + R} \mathbf{U} \right) \]

\[ (S_{\text{in}} - S_{\text{out}}) \frac{\partial \mathbf{U}}{\partial r} + S_{\text{in}} \mathbf{U} = \frac{R}{r + R} \frac{\partial \mathbf{U}}{\partial X} \]

The concentration equation is given by:

\[ \frac{\partial}{\partial t} + \nabla \cdot \mathbf{U} = Dk \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r + R} \frac{\partial C}{\partial r} + \left( \frac{R}{r + R} \right)^2 \frac{\partial^2 C}{\partial X^2} \right) \]

\[ \frac{DK_T}{T_m} \left[ \frac{\partial^2 C}{\partial r^2} + \frac{1}{r + R} \frac{\partial C}{\partial r} + \left( \frac{R}{r + R} \right)^2 \frac{\partial^2 C}{\partial X^2} \right] \]

Where \( P \) is the pressure, \( \rho \) is the density, \( C_p \) is the specific heat, \( K \) is the thermal conductivity, \( D \) is the diffusion coefficient of the diffusing species, \( T_m \) the mean fluid temperature, \( K_T \) is the thermal diffusion ratio, \( T \) and \( C \) denote the fluid temperature and concentration respectively.

### 6- Method of solution:

In order to simplify the governing equations of motion, temperature and concentration we may introduce the following dimensionless transformations as follows:

\[ x = \frac{x}{\lambda}, \quad r = \frac{r}{\lambda}, \quad u = \frac{U}{c}, \quad v = \frac{V}{c}, \quad t = \frac{t}{\lambda}, \quad h = \frac{H}{a}, \quad \tau = \frac{b}{a}, \quad \delta = \frac{a}{\lambda}, \quad K = \frac{R}{a}, \quad p = \frac{a^3 P}{\mu}, \quad \text{Re} = \frac{\rho \lambda a}{\mu}, \quad \text{Ma} = \frac{\sigma \beta \lambda a^2}{\mu}, \quad S = \frac{a S}{a C}, \quad F = \frac{Q}{a C}, \quad \text{Pr} = \frac{T - T_0}{T_0}, \quad \text{Sc} = \frac{C - C_0}{C_0}, \quad \text{Sc} = \frac{C}{C_0}, \quad \text{Sr} = \frac{C}{C_0}, \quad \alpha_s = \frac{C}{C_0}, \quad \alpha_i = \frac{C}{C_0}, \quad \text{Ec} = \frac{C^2}{C_0 T_0}, \quad \text{Pr} = \frac{C}{C_0}, \quad \text{Sr} = \frac{C}{C_0}, \quad \text{Pr} = \frac{C}{C_0}, \quad \text{Sr} = \frac{C}{C_0}. \]

Now substituting (21) into equations (13)-(15) and into equations (16)-(20) we have:

\[ \frac{k}{(r + k) \lambda} \frac{\partial u}{\partial x} + \frac{C}{\lambda} \frac{\partial v}{\partial r} + \frac{C}{(r + k) \lambda} \frac{\nu}{u} = 0 \]

Multiplying both sides of (22) by \( \frac{\lambda}{C} \) we get:

\[ \frac{k}{(r + k) \lambda} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{\nu}{u} = 0 \]

From eq.(17) we have:
\[
\rho \frac{C^2}{\lambda} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{k}{(r + k)} u \frac{\partial u}{\partial x} + \frac{u v}{(r + k)} \right) = -\frac{k}{r} C \mu \frac{\partial P}{\partial x} + \frac{k}{(r + k)}
\]

\[
\frac{1}{\lambda} \frac{\mu C}{a^2} \frac{\partial}{\partial x} \left\{ \frac{(r + k)^2 S_{xx}}{\lambda} \right\} - \sigma \left( \frac{k}{(r + k)} \right)^2 B_0^2 C u.
\]

..... (24)

Now, multiplying sides of (24) by \( \left( \frac{a^2}{C \mu} \right) \) we get:

Now, in the laboratory frame \((r, X)\) the flow is unsteady, however if treated it as steady flow in the wave frame \((x, r)\), thus we have:

\[
\text{Re} \delta \left( \frac{\partial u}{\partial r} + \frac{k}{(r + k)} u \frac{\partial u}{\partial x} + \frac{u v}{(r + k)} \right) = -\frac{k}{r} \frac{\partial P}{\partial x} + \frac{k}{(r + k)} \delta \frac{\partial}{\partial x} \left\{ \frac{(r + k)^2 S_{xx}}{\lambda} \right\} + \frac{1}{(r + k)^2}.
\]

\[
\frac{\partial}{\partial r} \left\{ \frac{(r + k)^2 S_{xx}}{\lambda} \right\} - \left( \frac{k}{(r + k)} \right)^2 M^2 u.
\]

..... (25)

From eq. (18) we have:

\[
\rho \frac{C^2}{a} \left( \delta^2 \frac{\partial v}{\partial t} + \delta^2 \frac{\partial v}{\partial r} + \frac{k}{(r + k)} \delta^2 u \frac{\partial v}{\partial x} - \frac{u^2}{(r + k)} \frac{\partial v}{\partial x} \right) = -\frac{C \lambda \mu}{a^2} \frac{\partial P}{\partial r} + \frac{\mu C}{a^2 (r + k)} \frac{\partial}{\partial x} \left\{ \frac{(r + k)^2 S_{xx}}{\lambda} \right\} + \frac{k}{(r + k)} \delta^2
\]

\[
\frac{\partial}{\partial r} \left\{ \frac{(r + k)^2 S_{xx}}{\lambda} \right\} + \frac{k}{(r + k)} \frac{\mu C}{a \lambda} \frac{\partial}{\partial x} S_{xx} - \frac{\mu C}{a^2 (r + k)} S_{xx}
\]

..... (26)

Now, multiplying both sides of (26) by \( \left( \frac{a^3}{C \lambda \mu} \right) \) we get:

\[
\text{Re} \delta \left( \delta^2 \frac{\partial v}{\partial r} + \frac{k}{(r + k)} \delta^2 u \frac{\partial v}{\partial x} - \frac{u^2}{(r + k)} \frac{\partial v}{\partial x} \right) = -\frac{\partial P}{\partial r} + \delta \frac{\partial}{\partial r} \left\{ \frac{(r + k)^2 S_{xx}}{\lambda} \right\} + \frac{k}{(r + k)} \delta^2
\]

\[
\frac{\partial}{\partial x} S_{xx} - \delta \frac{\partial}{\partial r} S_{xx}
\]

..... (27)

From eq. (19) we have:

\[
\rho C_p \frac{C}{\lambda} \left[ \frac{\partial T}{\partial t} + \frac{\partial T}{\partial r} + \frac{k}{(r + k)} \frac{\partial T}{\partial x} \right] = \frac{k}{a^2} \frac{\partial^2 T}{\partial r^2} + \frac{1}{(r + k)} \frac{\partial T}{\partial r} + \frac{k}{(r + k)^2} \delta^2 \frac{\partial^2 T}{\partial r^2} + \frac{\mu C}{a \lambda} \frac{C}{\lambda}
\]

\[
(S_n - S_{xx}) \frac{\partial v}{\partial r} + \frac{\mu C}{a} \frac{C}{\lambda} \frac{\partial S_n}{\partial r} + \delta^2 \frac{\partial}{\partial x} \left\{ \frac{k}{(r + k)} \frac{\partial v}{\partial x} - \frac{u}{(r + k)} \right\}
\]

..... (28)

Now, multiplying both sides of (28) by \( \left( \frac{a^2}{k_1} \right) \) we get:
\[ \rho c_p \frac{a^2}{\lambda k_1 \mu} \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial r} + \frac{k}{(r + k)} \frac{\partial T}{\partial x} \right) = \frac{\partial^2 T}{\partial r^2} + \frac{1}{(r + k)} \frac{\partial T}{\partial r} + \left( \frac{k}{(r + k)} \right)^2 \frac{\partial^2 T}{\partial x^2} ]+ \]

\[ \frac{\mu C_p}{\lambda k_1} \left( \frac{\partial v}{\partial r} - S_{\alpha} \frac{\partial v}{\partial x} \right) + \frac{\mu C_{\alpha}^2}{k_1} \left( \frac{\partial u}{\partial r} + \frac{k}{(r + k)} \frac{\partial v}{\partial x} - \frac{u}{(r + k)} \right) \]

\[ \text{Now, since } \theta = \frac{T - T_0}{T_0} \Rightarrow T - T_0 = \theta T_0 + T_0 \Rightarrow \partial T = T_0 \partial \theta \]

\[ \rho c_p \frac{a^2}{\lambda k_1 \mu} \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial r} + \frac{k}{(r + k)} \frac{\partial T}{\partial x} \right) = \frac{\partial^2 T}{\partial r^2} + \frac{1}{(r + k)} \frac{\partial T}{\partial r} + \left( \frac{k}{(r + k)} \right)^2 \frac{\partial^2 T}{\partial x^2} ]+ \]

\[ \frac{\mu C_p}{\lambda k_1} \left( \frac{\partial v}{\partial r} - S_{\alpha} \frac{\partial v}{\partial x} \right) + \frac{\mu C_{\alpha}^2}{k_1} \left( \frac{\partial u}{\partial r} + \frac{k}{(r + k)} \frac{\partial v}{\partial x} - \frac{u}{(r + k)} \right) \]

Multiplying both sides of (31) by \( \frac{1}{T_0} \) we have:

\[ \text{Re Pr } \frac{\partial \theta}{\partial r} = \frac{-k}{(r + k)} \frac{\partial \theta}{\partial x} = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{(r + k)} \frac{\partial \theta}{\partial r} + \left( \frac{k}{(r + k)} \right)^2 \frac{\partial^2 \theta}{\partial x^2} ]+ \]

\[ \frac{C^2}{C_p T_0 k_1} \delta(S_{\alpha} - S_{\alpha}) \frac{\partial v}{\partial r} + \frac{C^2}{C_p T_0 k_1} S_{\alpha} \left( \frac{\partial u}{\partial r} + \frac{k}{(r + k)} \frac{\partial v}{\partial x} - \frac{u}{(r + k)} \right) \]

Eq.(32) can be written as the following form:

\[ \text{Re Pr } \frac{\partial \theta}{\partial r} = \frac{-k}{(r + k)} \frac{\partial \theta}{\partial x} + \text{Ec Pr } \delta(S_{\alpha} - S_{\alpha}) \]

\[ \frac{\partial v}{\partial r} = \text{Ec Pr } S_{\alpha} \left( \frac{\partial u}{\partial r} + \frac{k}{(r + k)} \frac{\partial v}{\partial x} - \frac{u}{(r + k)} \right) \]

From eq.(20) we have:

\[ \frac{\overline{C}}{\lambda} \left[ \frac{\partial^2 \overline{C}}{\partial t^2} + \overline{C} \frac{\partial^2 \overline{C}}{\partial r} + \frac{k}{(r + k)} \frac{\partial \overline{C}}{\partial x} \right] = \frac{D \overline{C}^2}{\lambda} \left[ \frac{\partial^2 \overline{C}}{\partial r^2} + \frac{1}{(r + k)} \frac{\partial \overline{C}}{\partial r} + \left( \frac{k}{(r + k)} \right)^2 \frac{\partial^2 \overline{C}}{\partial x^2} \right] + \frac{DK_T}{T_m} \frac{1}{a^2} \]

\[ \frac{\partial \overline{T}}{\partial t} + \frac{1}{(r + k)} \frac{\partial \overline{T}}{\partial r} + \left( \frac{k}{(r + k)} \right)^2 \frac{\partial^2 \overline{T}}{\partial x^2} ]+ \]

\[ \frac{a^2}{D} \left[ \frac{\partial^2 \overline{T}}{\partial r^2} + \frac{1}{(r + k)} \frac{\partial \overline{T}}{\partial r} + \left( \frac{k}{(r + k)} \right)^2 \frac{\partial^2 \overline{T}}{\partial x^2} \right] \]
But we have \( \theta = \frac{T - T_0}{T_0} \) and \( \phi = \frac{C - C_0}{C_0} \) thus \( \partial T = T_0 \partial \theta \) and \( \partial C = C_0 \partial \phi \)

So, we can write (35) by the following form:

\[
\frac{\bar{C}}{C_0} a^2 \rho \left( \frac{1}{\rho} \frac{\partial^2 \phi}{\partial t^2} + v \frac{\partial \phi}{\partial r} + \frac{k}{(r + k)} u \frac{\partial \phi}{\partial x} \right) = \frac{C_o}{C_0} \left[ \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{(r + k)} \frac{\partial \phi}{\partial r} + \left( \frac{k}{(r + k)} \right)^2 \frac{\partial^2 \phi}{\partial x^2} \right]
\]

\[
+ \frac{DK_f}{T_m} T_0 \left( \frac{1}{\rho} \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{(r + k)} \frac{\partial \theta}{\partial r} + \left( \frac{k}{(r + k)} \right)^2 \frac{\partial^2 \theta}{\partial x^2} \right).
\]

\[\text{......}(36)\]

Multiplying both sides of (36) by \( \frac{1}{C_0} \) we get:

\[
\text{Re} \delta S[v \frac{\partial \phi}{\partial r} + \frac{k}{(r + k)} u \frac{\partial \phi}{\partial x}] = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{(r + k)} \frac{\partial \phi}{\partial r} + \left( \frac{k}{(r + k)} \right)^2 \frac{\partial^2 \phi}{\partial x^2} + S \text{Re} \delta \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{(r + k)} \frac{\partial \theta}{\partial r} + \left( \frac{k}{(r + k)} \right)^2 \frac{\partial^2 \theta}{\partial x^2}.
\]

\[\text{......}(37)\]

From eq.(13) we have:

\[
\frac{\mu C}{a} S_\alpha = \frac{2 \mu}{1 + \lambda_2} C \frac{1}{\lambda} \left( \frac{1}{\lambda} \frac{\partial}{\partial t} + \frac{k}{(r + k)} \frac{\partial}{\partial x} + v \frac{\partial}{\partial r} \right) \frac{\partial \nu}{\partial r}.
\]

\[\text{......}(38)\]

Multiplying both sides of (38) by \( \frac{a}{C \mu} \) we get:

\[
S_\alpha = \frac{2 \mu}{1 + \lambda_2} C \frac{a}{\lambda \mu} \left( \frac{1}{\lambda} \frac{\partial}{\partial t} + \frac{k}{(r + k)} \frac{\partial}{\partial x} + v \frac{\partial}{\partial r} \right) \frac{\partial \nu}{\partial r}.
\]

\[\text{......}(39)\]

From eq.(14) we have

\[
\frac{\mu C}{a} S_\alpha = \frac{\mu}{1 + \lambda_2} \frac{C}{a} \left( \frac{1}{\lambda} \frac{\partial}{\partial t} + \frac{k}{(r + k)} \frac{\partial}{\partial x} + v \frac{\partial}{\partial r} \right) \frac{\partial u}{\partial r} + \frac{k}{(r + k)} \left( \frac{\partial^2 \nu}{\partial x^2} - \frac{u}{(r + k)} \right).
\]

\[\text{......}(40)\]

Multiplying both sides of (40) by \( \frac{a}{\mu C} \) we have:

\[
S_\alpha = \frac{1}{1 + \lambda_2} \frac{C}{a} \left( \frac{1}{\lambda} \frac{\partial}{\partial t} + \frac{k}{(r + k)} \frac{\partial}{\partial x} + v \frac{\partial}{\partial r} \right) \frac{\partial u}{\partial r} + \frac{k}{(r + k)} \left( \frac{\partial^2 \nu}{\partial x^2} - \frac{u}{(r + k)} \right).
\]

\[\text{......}(41)\]
From eq.(15) we have:

\[
\frac{\mu C}{a} S_{xx} = \frac{2\mu}{1 + \lambda_1} \left[ 1 + \frac{k}{\lambda} \left( \frac{\partial}{\partial t} + \frac{v}{(r+k)} \frac{\partial}{\partial r} \right) \left( \frac{k}{\lambda} \frac{\partial u}{\partial x} + \frac{v}{(r+k)} \right) \right],
\]

.....(42)

Multiplying both sides of (42) by \( \frac{a}{\mu C} \) we get:

\[
S_{xx} = \frac{2\delta}{1 + \lambda_1} \left[ \frac{k}{a \lambda} \left( \frac{\partial}{\partial t} + \frac{v}{(r+k)} \frac{\partial}{\partial r} \right) \left( \frac{k}{\lambda} \frac{\partial u}{\partial x} + \frac{v}{(r+k)} \right) \right],
\]

.....(43)

The general solution of the governing equations (25)-(43) in the general case seems to be impossible, therefore we shall can fine the analysis under the assumption of small wave length \((\delta << 1)\) and low Reynolds number approximation, thus we can write the above equations in the form of stream function:

\[
\frac{\partial \psi}{\partial r} = 0
\]

.....(44)

\[
\frac{\partial \psi}{\partial x} = \frac{1}{k(r+k)} \frac{\partial}{\partial r} \left\{ (r+k)^2 S_{xx} \right\} - \frac{k}{r+k} M \frac{\partial \psi}{\partial r}
\]

.....(45)

\[
S_\alpha = 0, \ S_{xx} = 0, S_n = \frac{1}{1 + \lambda_1} \left( \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{(r+k)} \frac{\partial \psi}{\partial r} \right)
\]

.....(46)

\[
0 = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{(r+k)} \frac{\partial \theta}{\partial r} + BrS_\alpha \left( \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{(r+k)} \frac{\partial \psi}{\partial r} \right)
\]

.....(47)

\[
0 = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{(r+k)} \frac{\partial \phi}{\partial r} + SrSc \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{(r+k)} \frac{\partial \theta}{\partial r} \right)
\]

.....(48)

The corresponding dimensionless boundary conditions are given by:

\[
\psi = \frac{F}{2}, \ \frac{\partial^2 \psi}{\partial r^2} + \frac{\partial \psi}{\partial r} = 0, \ \theta = 0, \ \phi = 0 \ \text{at} \ r = \mp h = \mp(1 + \varphi \cos 2\pi(x - t))
\]

.....(49)

The relation between volume flow rate and time average flow rate is []:

\[
F(x,t) = Q + 2(h(x,t) - 1)
\]

.....(50)

The non-dimensional expression for the average rise in pressure \(\Delta P\) is given as follows:

\[
\Delta P = \left( \frac{1}{0} \frac{\partial P}{\partial x} \right)_{r=0.1=0.5}
\]

.....(51)

**Solution of the problem**

Equation (40) shows that \(P\) depends on \(x\) only. Thus if we diff. equation (41) with respect to \(r\), we have the closed form solution as follows:
\[ \psi = a_4 + a_3 K_r + a_2 \frac{r^2}{2} - a_1 \frac{(K + r)^{1+n_1}}{1+n_1} + a_1 \frac{(K + r)^{1+n_1}}{1+n_1}; \]

where \( n_1 = \sqrt{1+k^2M^2(1+\lambda_i)} \); \hspace{1cm} \ldots(52)

\[ \theta = -a_2^2(1+n_1)^2(K + r)^{2n_1} \alpha_1 - a_1^2(1+n_1)^2(K + r)^{2n_1} \alpha_1 + c_2 + c_1 \text{Log}(k + r) + \]
\[ a_1 a_2 (1+n_1)^2 \alpha_1 \text{Log}(k + r)^2 \]
\[ \frac{4n_1^2(1+\lambda_i)}{1+\lambda_i} \] \hspace{1cm} \ldots....(53)

\[ \phi = a_2^2(1+n_1)^2(K + r)^{2n_1} \alpha_1 \alpha_2 \alpha_3 + a_1^2(1+n_1)^2(K + r)^{2n_1} \alpha_1 \alpha_2 \alpha_3 + c_4 + c_3 \text{Log}(k + r) - \]
\[ a_1 a_2 (1+n_1)^2 \alpha_1 \alpha_2 \alpha_3 \text{Log}(k + r)^2 \]
\[ \frac{4n_1^2(1+\lambda_i)}{1+\lambda_i} \] \hspace{1cm} \ldots....(54)

In which \( c_i, i = 1, 2, 3, 4 \) in eq.(6 -53) and (6 -54) can be found by using boundary conditions(49).

7- Results and Discussion

In this section, the numerical and computation results are discussed for the problem of an incompressible viscous non-Newtonian Jeffrey fluid in the curved channel with heat and mass transfer through the graphical illustrations of some important results. (MATHEMATICA) program is used to find out numerical and illustrations.

7-1 Pumping characteristics

We plot the expression for \( \Delta P \) in eq.(51) against \( Q \) for various values of parameters of interest in figures (2)-(6). The effect of these parameters on \( \Delta P \) have been evaluated numerically using (MATHEMATICA) program and the results are presented graphically.

Impact of Hartmann number (M). The amplitudes ratio (\( \varphi \)), Jeffrey parameter (\( \lambda_i \)), curvature parameter (k) and slip parameter (\( \alpha \)) have been pointed out. Pumping regions can be divided into three regions which are (retrograde pumping which is described by \( \Delta P > 0, Q < 0 \)), Co-pumping which is described by \( \Delta P < 0, Q > 0 \) and free pumping which is described by \( \Delta P = 0 \). In figure (2) the effect of parameter M on \( \Delta P \) against \( Q \) are seen, observed that an increase in M causes increase on the regions of \( \Delta P > 0, Q \in (-1,0) \), \( \Delta P < 0, Q \in (0,0.3) \), \( \Delta P = 0 \) and decrease in the region of \( \Delta P < 0, Q \in (0.5,1) \), also we observed that the curves of pumping are intersected in the point (0.288, -1.504) in the fourth quadrant. Influence of parameter (\( \varphi \)) on \( \Delta P \) is plotted in figure (3) which is showed that an increase in (\( \varphi \)) causes rise in all regions of pumping that is in the regions of \( \Delta P > 0, Q \in (-1,0) \), \( \Delta P < 0, Q \in (0,0.5) \), \( \Delta P = 0 \) and the curves of pumping are intersected in the point (0.7674,-2.863) in the fourth quadrant. The effects of parameters \( \lambda_i \) and \( \alpha \) are plotted in 'figure 4', 'figure 5' respectively, which is noted that an increase in these parameters lead to
increase in all pumping of regions. Opposite behavior is shown for the effect of \( k \) on pressure rise against \( Q \), which is illustrated in 'figure.6'.

### 7-2 pressure gradient distribution

Effects of various parameters on the pressure gradient versus \( x \) have been illustrated in figures (7)-(12). These figures are scratched at the fixed values of \( (r=0.2, t=0.5) \). From figure (7) displays the effect of parameter \( M \) on pressure gradient, it is noticed that an increase in \( M \) leads to reduce in pressure gradient. Figure (8) illustrates the effect of the parameter \( \varphi \) on pressure gradient, it is observed that pressure gradient decrease on \( (-0.2, 0.2) \) and increase on the regions \( (-0.4, -0.2) \) and \( (0.2, 0.4) \). The impact of parameters \( \lambda_1 \) and \( \alpha \) are plotted in figures (9) and (10) respectively which is observed that an increase in these parameters lead to rise in pressure gradient. Figure (11) displays the effect of \( k \) on pressure gradient, it is noticed that there is slightly increase in pressure gradient with an increase of \( k \). Opposite behavior is observed for the effect of \( Q \) on pressure gradient and it is displayed in the figure (12).

### 7-3 velocity distribution

Influence of different parameters on the velocity distribution have been illustrated in figures (13)-(18). These figures are scratched at the fixed value of \( (x=0.2, t=0.05) \). From figure (13) displays the effect of Hartmann number parameter \( M \) on velocity \( u \), it is noticed that the velocity increase at upper wall on region of \( r \in [0.5, 1] \) and decrease at lower wall on region of \( r \in [0.5, 1] \) and decrease at lower wall on the region of \( r \in [-1, 0] \). Figure (14), illustrates the effect of the parameter \( \varphi \) on velocity, we see that velocity \( u \) increase on upper and lower walls of channel with an increase of \( \varphi \). From figure (15), it observed that there is similar behavior of Jeffrey parameter \( \lambda_1 \) of parameter \( M \) on velocity \( u \). It is noticed from figures (13), (14) and (15) of effects of parameters \( M, \varphi \) and \( \lambda_1 \) that the velocity \( u \) is not symmetric in curved channel (for small values of culvature parameter \( k \)) and it is symmetric in straight channel (for large values of \( k \)). Figure (16) show that velocity distribution increase at upper part of channel and decrease at lower part of channel with an increase of culvature parameter \( k \), and it is noticed that for large values of \( k \) (straight channel) the velocity profiles are symmetric figure (17) show that velocity distribution \( u \) decrease at the central line and increase at the walls of channel with an increase values of slip parameter \( (\alpha) \). The effect of parameter \( Q \) on velocity is display in figure (18), it is observed that an increase in \( Q \) leads to increase in velocity at the central line and walls. In all graphs of figures of velocity distribution are parabolic graph.

### 7-4 trapping phenomenon

The effects of various parameter like \( M, \varphi, \lambda_1, k, \alpha \) and \( Q \) on trapping can be seen through figures (19)-(24). 'Figure 19' show that the size of trapped bolus increase with an increase of
value of $M$ in the upper and lower part of channel. 'Figure 20' is plotted for the effect of $\varphi$ on trapping. It can be seen that there is a change in shape of trapping bolus and these bolus taken protraction in shape and increasing in the both sides of channel with an increase of $\varphi$. 'Figure 21' show that the size of bolus decrease with an increase of $\lambda_i$ on trapping. It means that $\lambda_i$ plays the resistive role for stream lines flow. It is further observed that for viscous fluid ($\lambda_i = 0$) this size of trapping bolus is larger than Jeffrey fluid ($\lambda_i \neq 0$). The effect of culverture parameter $k$ on trapping is displayed in 'figure 22' which is noted that there is same behavior of effect $\lambda_i$ on trapping. The influence of slip parameter $\alpha$ on trapping is analyzed in figure (23), it is noticed that there is clear decreasing in size of bolus with an increase of $\alpha$. The effect of parameter $Q$ on trapping is shown in figure (24), it is noticed that there is clear increasing in size of bolus with an increase of $Q$.

7-5 Temperature characteristics

The expressions for temperature are given by eq.(53). The effects of various parameters on temperature for fixed values of $(x=0.2, t=0.05)$ are shown, the results are presented in fig (25)-(31). From 'figure 25' it can found that temperature profile decrease at the central line with an increase of parameter $M$. The effect of parameter $\varphi$ on temperature profile is plotted in figure26), it is noticed that temperature increase at the central line and the walls of channel with an increase of $\varphi$. 'Figure 27' showed the influence of parameter $\lambda_i$, it is observed that an increase in $\lambda_i$ leads to decrease in temperature profile at the central line of channel, that is temperature profile $\theta$ is smaller for non-Newtonian fluid ($\lambda_i \neq 0$) when compared with viscous fluid ($\lambda_i = 0$). The effect of parameter $k$ is noticed in 'figure 28', it is observed that an increase in $k$ leads to decrease in temperature profile at the central line of channel with an increase of k. figure (29), illustrate the influence of parameter $\alpha$ which is behaved similar to effect of $M$ on temperature. The effect of Brinkman number (Br) on temperature is displayed in figure (30), which is showed that an increase in (Br) lead to rising up on temperature, it is only due to the fact that (Br) incorporates viscous dissipation effects which expands the fluid temperature. 'Figure 31' illustrated the influence of parameter (Q) on temperature, which is noticed that the temperature increasing with an increase of Q at the central line of channel. In all graphs of temperature distribution of effects of all parameters mentioned above that the profiles of velocity are not symmetric in curved channel and it is symmetric in straight channel.

7-6 Mass transfer distribution

The expression for concentration are given by eq.(54). The effects of various parameters on concentration for fixed values of $(x=0.2, t=0.05)$ are shown, the results are presented in fig (32)-(40). The profile of concentration is reverse of profile of temperature and the parameters behaved opposite manner on concentration than a temperature distribution. The effects of parameters $M, \lambda_i, k, \varphi, \alpha, Br, Sr, Sc$ and $Q$ on concentration are plotted in figures (32)-(40). The effect of
$M$, $\lambda_1$, $k$ and $\alpha$ are plotted in figures (32), (33), (34) and (35) respectively. It is noticed that an increase in these last parameters lead to an increase on concentration opposite behavior is obtained for the parameters $\varphi, Br, Sr, Sc$ and $Q$ which is illustrated into figures (36), (37), (38), (39), (40). In fact the reason behind the reducing of concentration when we increase the values of (Sc) is due that the mass diffusion decrease which show decrease in concentration. We observed that all graphs of concentration distribution are not symmetric in curved channel and it has symmetry characteristic in straight channel.

8- concluding Remarks

The present study deals with the combined effects of radial magnetic and heat/mass transfer on the peristaltic transport of viscous incompressible Jeffrey fluid in curved channel. We obtained the exact solution of the problem under long wave length and low Reynolds number assumptions. The results are analyzed for different values of parameters namely Hartmann number ($M$), amplitude ratio $\varphi$, Jeffrey parameter $\lambda_1$, culvature parameter ($k$), slip parameter $\alpha$, time flow rate ($Q$), Brinkman number($Br$). Thus through our work we observe the following notations:

1. At the upper part of curved channel, the axial velocity increase with an increase of $M$, $\varphi$, $\lambda_1$, $k$ and $\alpha$.
2. At the lower part of channel, the axial velocity increase with an increase of $\varphi$ and $\alpha$ and decreasing with an increase of $M$, $\lambda_1$, $k$.
3. At the central line, the axial velocity increase with an increase of $Q$ and decrease with an increase of $\alpha$.
4. The profiles of velocity are parabolic and they are symmetric for large values of culvature parameter ($k$) (straight channel) and non-symmetric for curved channel.
5. The size of trapping bolus increase at both parts of channel with an increase of $M$, $\varphi$, $Q$ and decrease with an increase of $\lambda_1$, $k$ and $\alpha$.
6. The pressure gradient of fluid increase with an increase of $\lambda_1$, $k$, $\alpha$ and decrease with an increase of $M$, $Q$.
7. The pressure gradient increase into regions of $0.2 < x < 0.4$, $-0.4 < x < -0.2$ with an increase of $\varphi$ and decrease into region $-0.2 < x < 0.2$.
8. The temperature profile increase with an increase of $\varphi$, $Br$ and $Q$ and decrease with an increase of $M$, $\lambda_1$, $k$ and $\alpha$.
9. The concentration distribution decrease with an increase of $Sr$ and $Sc$. Opposite behavior for concentration profile is noted when compared with temperature.
10. The profiles of temperature and concentration are symmetric for large values of culvature parameter $k$ (straight channel) and non-symmetric for curved channel for small values of $k$.
11. Pressure rise of fluid increase with an increase of $\varphi$, $\lambda_1$, $\alpha$ and decrease with an increase of $k$.
12. The impact of Hartmann number on pressure rise against mean flow rate $Q$ is wobbling.
13. The curves of pressure rise are intersected at different regions.
Figure 2: Effect of $M$ on $\Delta p$
$$\varphi = 0.2, t = 0.5, \lambda_1 = 1, k = 2, \alpha = 0.1$$

Figure 3: Effect of $\varphi$ on $\Delta p$
$$M = 1.5, t = 0.5, \lambda_1 = 1, k = 2, \alpha = 0.1$$

Figure 4: Effect of $(\lambda_1)$ on $\Delta p$
$$M = 1.5, t = 0.5, \varphi = 0.2, k = 2, \alpha = 0.1$$

Figure 5: Effect of $(\alpha)$ on $\Delta p$
$$M = 1.5, t = 0.5, \varphi = 0.2, k = 2, \lambda_1 = 1$$

Figure 6: Effect of $(k)$ on $\Delta p$
$$M = 1.5, t = 0.5, \varphi = 0.2, \lambda_1 = 1, \alpha = 0.1$$

Figure 7: Effect of $M$ on gradient
$$\varphi = 0.2, t = 0.5, \lambda_1 = 1, k = 2, \alpha = 0.1,\quad Q = 2, r = 0.2$$

Figure 8: Effect of $\varphi$ on gradient
$$\varphi = 0.15, \varphi = 0.2, \varphi = 0.3$$

Figure 9: Effect of $(\lambda_1)$ on gradient
$$\lambda_1 = 1.2, \lambda_1 = 1.5, \lambda_1 = 2$$
\[ M = 1, t = 0.5, \lambda_1 = 1, k = 2, \alpha = 0.1, \]
\[ Q = 2, r = 0.2 \]

\[ \varphi = 0.2, M = 1, t = 0.5, k = 2, \alpha = 0.1, \]
\[ Q = 2, r = 0.2 \]

Figure 10. Effect of \((\alpha)\) on gradient

\[ \varphi = 0.2, M = 1, t = 0.5, k = 2, \lambda_1 = 1, \alpha = 0.1 \]
\[ Q = 2, r = 0.2 \]

Figure 11. Effect of \((k)\) on gradient

\[ \varphi = 0.2, M = 1, t = 0.5, \lambda_1 = 1, \alpha = 0.1 \]
\[ Q = 2, r = 0.2 \]

Figure 12. Effect of \((Q)\) on gradient

\[ \varphi = 0.2, M = 1, t = 0.5, k = 2, \lambda_1 = 1, \alpha = 0.1 \]
\[ k = 2, r = 0.2 \]

Figure 13-a. Effect of \((M)\) on velocity \(u\)

\[ \varphi = 0.2, t = 0.05, \lambda_1 = 1, \alpha = 0.1, Q = 1.5 \]
\[ x = 0.2 \text{ when } (k = 2), \text{ (curved channel)} \]

Figure 13-b. Effect of \((M)\) on velocity \(u\)

\[ \varphi = 0.2, t = 0.05, \lambda_1 = 1, \alpha = 0.1, Q = 1.5 \]
\[ x = 0.2 \text{ when } (k = 10), \text{ (straight channel)} \]

Figure 14-a. Effect of \((\varphi)\) on velocity \(u\)

\[ \varphi = 0.1, \varphi = 0.2, \varphi = 0.3 \]

Figure 14-b. Effect of \((\varphi)\) on velocity \(u\)
\[ M = 1.5, t = 0.05, \dot{\lambda} = 1, \alpha = 0.1, Q = 1.5 \]
\[ x = 0.2 \text{ when } (k = 2), \text{ (curved channel)} \]
Figure 19. Stream lines for $t = 0.05, \varphi = 0.2, \lambda_1 = 1, k = 2,$

$\alpha = 0.1, Q = 1.5$

(a) $M = 0.9$, (b) $M = 1.5$, (c) $M = 1.6$

Figure 20. Stream lines for $M = 1.5, t = 0.05, \lambda_1 = 1, k = 2,$

$\alpha = 0.1, Q = 1.5$

(a) $\varphi = 0.2$, (b) $\varphi = 0.4$, (c) $\varphi = 0.6$
Figure 21. Stream lines for 
\[ M = 1.5, t = 0.05, \varphi = 0.2, k = 2, \]
\[ \alpha = 0.1, Q = 1.5 \]
(a) \( \lambda_4 = 1 \), (b) \( \lambda_4 = 1.5 \), (c) \( \lambda_4 = 2 \)

Figure 22. Stream lines for 
\[ M = 1.5, t = 0.05, \varphi = 0.2, \lambda_4 = 1, \]
\[ \alpha = 0.1, Q = 1.5 \]
(a) \( \hat{k} = 2 \), (b) \( \hat{k} = 3 \), (c) \( \hat{k} = 4 \)
Figure 23. Stream lines for 
\( M = 1.5, t = 0.05, \varphi = 0.2, \lambda_1 = 1, \)  
\( k = 2, Q = 1.5 \)  
(a) \( \alpha = 0.1 \), (b) \( \alpha = 0.3 \), (c) \( \alpha = 0.5 \)  

Figure 24. Stream lines for 
\( M = 1.5, t = 0.05, \varphi = 0.2, \lambda_1 = 1, \)  
\( \alpha = 0.1, k = 2 \)  
(a) \( Q = 1.5 \), (b) \( Q = 1.6 \), (c) \( Q = 1.62 \)
Figure 25-a. Effect of $M$ on Temperature
$\phi = 0.2, t = 0.05, \lambda_i = 1, \alpha = 0.1, \alpha_i = 0.01,$
$Q = 1.5x = 0.2$ when $(k = 2), (\text{curved channel})$

Figure 25-b. Effect of $M$ on Temperature
$\phi = 0.2, t = 0.05, \lambda_i = 1, \alpha = 0.1, \alpha_i = 0.01,$
$Q = 1.5x = 0.2$ when $(k = 10), (\text{straight channel})$

Figure 26-a. Effect of $\phi$ on Temperature
$t = 0.05, M = 1.5, \lambda_i = 1, \alpha = 0.1, \alpha_i = 0.01,$
$Q = 1.5, x = 0.2$ when $(k = 2), (\text{curved channel})$

Figure 26-b. Effect of $\phi$ on Temperature
$t = 0.05, M = 1.5, \lambda_i = 1, \alpha = 0.1, \alpha_i = 0.01,$
$Q = 1.5, x = 0.2$ when $(k = 10), (\text{straight channel})$

Figure 27-a. Effect of $\lambda_i$ on Temperature
$\phi = 0.2, t = 0.05, M = 1.5, \alpha = 0.1, \alpha_i = 0.01,$
$Q = 1.5, x = 0.2$ when $(k = 2), (\text{curved channel})$

Figure 27-b. Effect of $\lambda_i$ on Temperature
$\phi = 0.2, t = 0.05, M = 1.5, \alpha = 0.1, \alpha_i = 0.01,$
$Q = 1.5, x = 0.2$ when $(k = 10), (\text{straight channel})$

Figure 28. Effect of $k$ on Temperature
$\phi = 0.2, t = 0.05, M = 1.5, \lambda_i = 1, \alpha = 0.1,$
$\alpha_i = 0.01, Q = 1.5, x = 0.2$
Figure 29-a. Effect of $\alpha$ on Temperature
$\varphi = 0.2, \tau = 0.05, M = 1.5, \lambda_1 = 1, \alpha_1 = 0.01, Q = 1.5, x = 0.2$ when ($k = 2$), (curved channel)

Figure 29-b. Effect of $\alpha$ on Temperature
$\varphi = 0.2, \tau = 0.05, M = 1.5, \lambda_1 = 1, \alpha_1 = 0.01, Q = 1.5, x = 0.2$ when ($k = 10$), (straight channel)

Figure 30-a. Effect of $\alpha_1$ on Temperature
$\varphi = 0.2, \tau = 0.05, M = 1.5, \lambda_1 = 1, \alpha = 0.1, \alpha_1 = 0.01, x = 0.2$ when ($k = 2$), (curved channel)

Figure 30-b. Effect of $\alpha_1$ on Temperature
$\varphi = 0.2, \tau = 0.05, M = 1.5, \lambda_1 = 1, \alpha = 0.1, \alpha_1 = 0.01, x = 0.2$ when ($k = 10$), (straight channel)

Figure 31-a. Effect of $Q$ on Temperature
$\varphi = 0.2, \tau = 0.05, M = 1.5, \lambda_1 = 1, \alpha = 0.1, \alpha_1 = 0.01, x = 0.2$ when ($k = 2$), (curved channel)

Figure 31-b. Effect of $Q$ on Temperature
$\varphi = 0.2, \tau = 0.05, M = 1.5, \lambda_1 = 1, \alpha = 0.1, \alpha_1 = 0.01, x = 0.2$ when ($k = 10$), (straight channel)

Figure 32-a. Effect of $M$ on mass transfer
$M = 0.5, M = 1, M = 1.5$

Figure 32-b. Effect of $M$ on mass transfer
$M = 0.5, M = 1, M = 1.5$
\( \varphi = 0.2, t = 0.05, \lambda_1 = 1, \alpha = 0.1, \alpha_i = 0.01, \)
\( \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2 \) when \( k = 2 \),
(curved channel)

\( \varphi = 0.2, t = 0.05, \lambda_1 = 1, \alpha = 0.1, \alpha_i = 0.01, \)
\( \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2 \) when \( k = 10 \),
(straight channel)

**Figure 33-a. Effect of \( \varphi \) on mass transfer**

\( M = 1.5, t = 0.05, \lambda_1 = 1, \alpha = 0.1, \alpha_i = 0.01, \)
\( \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2 \) when \( k = 2 \),
(curved channel)

\( \varphi = 0.2, M = 1.5, t = 0.05, \alpha = 0.1, \alpha_i = 0.01, \)
\( \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2 \) when \( k = 10 \),
(straight channel)

\( \varphi = 0.2, t = 0.05, \lambda_1 = 1, \alpha = 0.1, \alpha_i = 0.01, \)
\( \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2 \) when \( k = 2 \),
(curved channel)

**Figure 34-a. Effect of \( \lambda_1 \) on mass transfer**

\( \varphi = 0.2, M = 1.5, t = 0.05, \alpha = 0.1, \alpha_i = 0.01, \)
\( \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2 \) when \( k = 2 \),
(curved channel)

\( \varphi = 0.2, M = 1.5, t = 0.05, \alpha = 0.1, \alpha_i = 0.01, \)
\( \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2 \) when \( k = 10 \),
(straight channel)

\( \varphi = 0.2, t = 0.05, \lambda_1 = 1, \alpha = 0.1, \alpha_i = 0.01, \)
\( \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2 \) when \( k = 2 \),
(curved channel)

**Figure 35-a. Effect of \( \alpha \) on mass transfer**

\( \varphi = 0.2, t = 0.05, M = 1.5, \lambda_1 = 1, \alpha_i = 0.01, \)
\( \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2 \) when \( k = 2 \),
(curved channel)

\( \varphi = 0.2, t = 0.05, M = 1.5, \lambda_1 = 1, \alpha_i = 0.01, \)
\( \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2 \) when \( k = 10 \),
(straight channel)
Figure 36. Effect of $k$ on mass transfer

$\varphi = 0.2, t = 0.05, M = 1.5, \lambda = 1, \alpha_1 = 0.01,$

$\alpha = 0.1, \alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2$

Figure 37-a. Effect of $\alpha_j$ on mass transfer

$\varphi = 0.2, t = 0.05, M = 1.5, \lambda = 1, \alpha = 0.1,$

$\alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2$ when ($k = 2$),

(curved channel)

Figure 37-b. Effect of $\alpha_j$ on mass transfer

$\varphi = 0.2, t = 0.05, M = 1.5, \lambda = 1, \alpha = 0.1,$

$\alpha_2 = 1, \alpha_3 = 1, Q = 1.5, x = 0.2$ when ($k = 10$),

(straight channel)

Figure 38-a. Effect of $\alpha_2$ on mass transfer

$\varphi = 0.2, t = 0.05, M = 1.5, \lambda = 1, \alpha = 0.1,$

$\alpha_1 = 0.01, \alpha_3 = 1, Q = 1.5, x = 0.2$ when ($k = 2$),

(curved channel)

Figure 38-b. Effect of $\alpha_2$ on mass transfer

$\varphi = 0.2, t = 0.05, M = 1.5, \lambda = 1, \alpha = 0.1,$

$\alpha_1 = 0.01, \alpha_3 = 1, Q = 1.5, x = 0.2$ when ($k = 10$),

(straight channel)

Figure 39-a. Effect of $\alpha_3$ on mass transfer

$\varphi = 0.2, t = 0.05, M = 1.5, \lambda = 1, \alpha = 0.1,$

$\alpha_1 = 0.01, \alpha_2 = 1, Q = 1.5, x = 0.2$ when ($k = 2$),

(curved channel)

Figure 39-b. Effect of $\alpha_3$ on mass transfer

$\varphi = 0.2, t = 0.05, M = 1.5, \lambda = 1, \alpha = 0.1,$

$\alpha_1 = 0.01, \alpha_2 = 1, Q = 1.5, x = 0.2$ when ($k = 10$),

(straight channel)
REFERENCES

[1] Latham, T.W., "Fluid motion in peristaltic pump," "M. S Thesis, Massachusetts Institute of Technology, Cambridge Massachusetts, U.S.A (1966).
[2] Shapiro, A. H., Jaffrin M. Y. and Weinberg S. L., "Peristaltic pumping with long wavelength at low Reynolds number," *Journal of Fluid Mechanics*, 37, pp. 799-825 (1969).
[3] Shapiro, A. H., “Pumping and retrograde diffusion in peristaltic waves,” *Proceedings of the Workshop Ureteral Reflux Children*, National Academy of Science, Washington, D.C., U.S. (1967).
[4] Fung, Y. C., “Peristaltic pumping: a bio engineering model,” *Proceedings of the Workshop Ureteral Reflux Children*, National Academy of Science, Washington, D.C., U.S. (1971).
[5] Hayat, T., Ali, N. and Asghar, S., "An analysis of peristaltic transport for flow of a Jeffrey fluid," *Acta Mechanica*, 193, pp. 101-112 (2007).
[6] Hayat, T., Yasmin, H., Alhuthali, M. S. and Kutbi, M.A., "Peristaltic flow of a non-Newtonian fluid in an asymmetric channel with convective boundary conditions," *Journal of Mechanics*, 29, pp. 599-607
[7] . Ebaid, A., "Remarks on the homotopy perturbation method for the peristaltic flow of Jeffrey fluid with nano-particles in an asymmetric channel," *Computers& Mathematics with Applications*, 68, pp. 77-85(2014).
[8] . Nadeem, S. and Akram, S., "Peristaltic flow of a Jeffrey fluid in a rectangular duct," *Nonlinear Analysis: Real World Applications*, 11, pp. 4238-4247(2010).
[9] Kothandapani, M. and Srinivas, S., "Peristaltic transport of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel," *International Journal of Non-Linear Mechanics*, 43, pp. 915-924(2008).
[10] Tripathi, D. and Beg, O. A., "A study of unsteady physiological magneto-fluid flow and heat transfer through a finite length channel by peristaltic pumping," *Proceedings of the institution of mechanical Engineers, Part H: Journal of Engineering in Medicine*, DOI: 10.1177/0954411912449946 (2012).
[11] Jalilian, E., Onen, D., Neshve, E. and Mintchev, M.P., "Implantable neural electrical stimulator for external control of gastrointestinal motility," *Medical Engineering & Physics*, 29, pp. 238-252 (2007).
[12] Abbasi, F. M., Hayat, T. and Ahmad, B., "Peristaltic flow in an asymmetric channel with convective boundary conditions and Joule heating," *Journal of Central South University*, 21, pp. 1411-1416 (2014).
[13] Sato, H., Kawai, T., Fujita, T. and Okabe, M., "Two dimensional peristaltic flow in curved channels," *The Japan Society of Mechanical Engineers B*, 66, pp. 679-685 (2000).
[14] Ali, N., Sajid, M. and Hayat, T., "Long wave length flow analysis in a curved channel," *Zeitschrift für Naturforschung A*, 65, pp. 191-196 (2010).
[15] Ali, N., Sajid, M., Javed, T. and Abbas, Z., "Non-Newtonian fluid flow induced by peristaltic waves in a curved channel," European Journal of Mechanics - B/Fluids, 29, pp. 387-394 (2010).

[16] Hayat, T., Quratulain., Rafeq, M., Alsaadi, F. and Ayub, M., "Soret and Dufour effects on peristaltic transport in curved channel with radial magnetic field and convective conditions," Journal of Magnetism and Magnetic Materials, 405, pp. 358-369 (2016).

[17] Hayat, T., Abbasi, F. M., Ahmed, B. and Alsaedi, A., "Peristaltic transport of Carreau-Yasuda fluid in a curved channel with slip effects," PLOS ONE, 9, e95070 (2014).

[18] T. Hayat, S. Farooq, A. Alsaedi, "MHD Peristaltic flow in a curved channel with convective condition" Nonlinear Analysis and Applied Mathematics Research Group Department of Mathematics King Abdulaziz University Jeddah, Saudi Arabia

[19] Tasawar Hayat1, Hina Zahir2, Anum Tanveer2, Ahmad Alsaedi2, "Soret and Dufour effects on MHD peristaltic transport of Jeffrey fluid in a curved channel with convective boundary conditions" Department of Mathematics, Quaid-I-Azam University 45320, Islamabad 44000, Pakistan, 2 Department of Electrical and Computer Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia.

[20] V. K. Narla, K. M. Prasad, J. V. Ramanamurthy, "Peristaltic transport of Jeffrey Nano fluid in curved channels" Gitam university, Department of mathematics, Hayderabad, India, National institute of technology, Department of mathematics, waragal-506004, India.