Quantum Global Strings
and Their Correlation Functions

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Abstract

A full quantum description of global vortex strings is presented in the framework of a pure Higgs system with a broken global U(1) symmetry in 3+1D. An explicit expression for the string creation operator is obtained, both in terms of the Higgs field and in the dual formulation where a Kalb-Ramond antisymmetric tensor gauge field is employed as the basic field. The quantum string correlation function is evaluated and from this, the string energy density is obtained. Potential application in cosmology (cosmic strings) and condensed matter (vortices in superfluids) are discussed.

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1) Introduction

Quantized vortex lines or string-like excitations appear in a wide variety of systems, ranging from condensed matter (vortices in Helium II) to cosmology (cosmic strings). In the case of superfluid He, there is still no microscopic theory which can explain several relevant issues such as the dynamics of quantum vortices, the dragging processes, the vortex nucleation or even the structure of quantized vortex lines \[1\]. On the other hand, among the topological defects that were left behind by cosmological phase transitions as the universe expanded and cooled, cosmic strings show very interesting properties. For instance, they may have played an important role in the structure formation by acting as seeds of galaxies and other structures which can be observed today in the universe \[4\].

In all the standard approaches only classical or semiclassical strings have been considered. In a recent paper \[3\] a full quantum theory of local (magnetic) strings in the Abelian-Higgs model was introduced. In this work we present a fully quantized formulation for strings or vortices in a theory with a spontaneously broken global $U(1)$ symmetry. Applying the same strategy of refs.\[3\]-\[7\] we construct the corresponding quantum creation operator whose correlation functions are local (i.e. they only depend on the positions of the string excitations).

The paper is organized as follows. In sec. 2 we consider the simplest model which supports global string-like excitations i.e. a complex scalar field with a symmetry breaking potential and present a dual formulation in terms of the Kalb-Ramond (K-R) antisymmetric tensor potential. In sec. 3 we introduce the operator, $\sigma(C, t)$, that creates quantum topological strings along the curve $C$, which in the K-R language are “charge” strings. In sec. 4 we compute the correlation functions of this operator both in the Kalb-Ramond and Higgs formulations. In particular, we obtain the string tension for a long straight string. Conclusions and final remarks are presented in Section 5.
Global Strings in the Higgs and Kalb-Ramond Representations

Global string excitations appear in theories with a spontaneously broken continuous global symmetry. We shall consider the simplest theory exhibiting string solutions, namely, that of a complex scalar field $\phi(x)$ described by the lagrangian density

$$L[\phi] = |\partial_\mu \phi|^2 - V(|\phi|^2),$$

which has a global U(1) symmetry. The potential is the standard symmetry breaking one:

$$V(|\phi|^2) = -m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4.$$

Using the polar representation for the complex scalar field $\phi = \frac{\phi}{\sqrt{2}} e^{i\theta}$, the lagrangian density (2.1) can be written as

$$L[\rho, \theta] = \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}\rho^2 (\partial_\mu \theta)^2 - V(\rho^2).$$

The global U(1) invariance of (2.2) implies the conservation of the current

$$j_\mu = \rho^2 \partial_\mu \theta = -i\phi^* \partial_\mu \phi.$$  

Field configurations containing strings correspond to a multivalued Goldstone $\theta$-field i.e. are such that the value of $\theta$ is defined up to $2\pi$ times an integer. It is convenient to split the $\theta$-field into two parts

$$\theta(t, x) = \bar{\theta}(t, x) + \alpha(t, x),$$

where $\bar{\theta}$ describes a given configuration of vortices and $\alpha$ corresponds to the single valued part. For a configuration of $N$ vortices, the antisymmetric tensor vortex current is given by \footnote{Topological currents are denoted by capital letters while Noether currents are denoted by lower case letters.}

$$J^{\mu \nu}(x) \equiv \epsilon^{\mu \nu \rho \sigma} \partial_\rho \partial_\sigma \theta(x) = \epsilon^{\mu \nu \rho \sigma} \partial_\rho \partial_\sigma \bar{\theta}(x) =$$

\footnote{Topological currents are denoted by capital letters while Noether currents are denoted by lower case letters.}
\[
\sum_{a=1}^{N} \gamma_a \int d\tau d\sigma (\dot{X}_a^\mu X_a'^\nu - \dot{X}_a^\nu X_a'^\mu) \delta^{(4)}(x - X_a(\tau, \sigma)),
\]  

(2.5)

where the subindex \( a \) labels the vortices, \( \gamma_a \) is the quantized circulation or vorticity, \( X_a \) denotes the vortex position and the dot and prime indicate, respectively, differentiation with respect to \( \tau \) and \( \sigma \). The integrals are taken over the universe surfaces of the strings. In the case of a superfluid \( \gamma_a = n_a \frac{h}{M} \) with \( n_a = \pm 1, \pm 2, \ldots \), where \( M \) is the mass of the superfluid atoms. Observe that acting on the multivalued \( \theta \)-field, different components of the derivative operator no longer commute. The tensor topological current \( J^{\mu \nu} \) is identically conserved:

\[
\partial_\mu J^{\mu \nu} \equiv 0.
\]

(2.6)

This is because \( \partial_\sigma \bar{\theta}(x) \) is no longer multivalued. It is possible to work in the equivalent dual representation expressed in terms of the two-index antisymmetric Kalb-Ramond tensor field \( B_{\mu \nu}(x) \) instead of the Goldstone boson \( \theta(x) \). The connection with this dual formulation is provided by

\[
\frac{1}{2} \epsilon_{\mu \nu \alpha \beta} \partial^\nu B^{\alpha \beta} = \rho^2 \partial_\mu \theta,
\]

(2.7)

which can also be written as

\[
^*H_\mu = \rho^2 \partial_\mu \theta,
\]

or

\[
H^{\mu \alpha} = \epsilon^{\mu \nu \alpha \beta} (\rho^2 \partial_\beta \theta)
\]

(2.8)

where \( ^*H_\mu = \frac{1}{6} \epsilon_{\mu \nu \alpha \beta} H^{\nu \alpha \beta} \) is the dual of the Kalb-Ramond field strength, \( H^{\alpha \beta \gamma} = \partial^\alpha B^{\beta \gamma} + \partial^\beta B^{\gamma \alpha} + \partial^\gamma B^{\alpha \beta} \). The \( \theta \) term in (2.2) can be expressed in terms of a Kalb-Ramond field, by means of the following Gaussian identity

\[
\exp \left\{ i \int d^4x \left[ \frac{1}{2} \rho^2 (\partial_\mu \theta)^2 \right] \right\} = \int [\rho^{-1} d^*H_\mu] \exp \left\{ i \int d^4x \left[ -\frac{1}{2\rho^2} (^*H_\mu)^2 + ^*H_\mu \partial^\mu \bar{\theta} + ^*H_\mu \partial^\mu \alpha \right] \right\}
\]

Note that we have split the \( \theta \)-field in two parts, according to (2.4). Integrating over the single valued \( \alpha \) we get a \( \delta(\partial^\mu H_\mu) \) which is identically satisfied, according to the definition of \( ^*H_\mu \).
Substituting (2.9) in the vacuum functional corresponding to the lagrangian (2.2), integrating by parts the $^*H_\mu$ derivatives in the $\bar{\theta}$-term and using (2.5), we obtain the following dual lagrangian density

$$L[\rho, B_{\mu\nu}] = \frac{1}{2}(\partial_\mu \rho)^2 - V(|\rho|^2) + \frac{1}{12\rho^2}H^2_{\mu\nu\alpha} + \frac{1}{2}B_{\mu\nu}J^{\mu\nu}. \quad (2.10)$$

The quantum theory corresponding to the above lagrangian density is well defined because $<\rho> \neq 0$ (spontaneously broken symmetry). On the contrary, for a non-spontaneously broken theory, if $<\rho> = 0$ the division by zero in the kinetic $H^2$ term would make the theory undefined. In the approximation where the field $\rho$ has a constant value $\rho_0$, i.e. the large $\lambda$ limit, (2.10) reduces to

$$L[B_{\mu\nu}] = \frac{1}{12\rho_0^2}H^2_{\mu\nu\alpha} + \frac{1}{2}B_{\mu\nu}J^{\mu\nu}. \quad (2.11)$$

The corresponding operator field equation is:

$$\partial_\alpha H^{\alpha\mu\nu} = \rho_0^2 J^{\mu\nu}. \quad (2.12)$$

The theory possesses an identically conserved topological current:

$$J^\mu = \frac{1}{2}e^{\mu\nu\alpha\beta}\partial_\nu B_{\alpha\beta}. \quad (2.13)$$

which from (2.7) is identical to the Noether current (2.3): $J^\mu \equiv j^\mu$, i.e. the topological current of the antisymmetric field coincides with the electric current (2.3) of the scalar field representation. On the other hand, the reciprocal result also does hold: the topological current of the Goldstone field representation (2.5) appears as the “electric” current in the dual representation according to (2.12). We see that the electric and topological currents in the original and dual representations are interchanged, as usual.

3) The Vortex String Creation Operator

Let us introduce now the creation operator for a fully quantized string state. We have seen in the previous section that the topological charge associated to the string excitations become “electric” charges in the dual K-R formulation. Hence in
this formulation we need a charge creation operator. This kind of operator has been introduced in \([4, 8]\) and, for a closed string \(C\) it is given by

\[
\sigma(C, t) = \exp \left\{ \frac{1}{2} \int d^4 x H_{\alpha \mu \nu} \tilde{C}^{\alpha \mu \nu} \right\},
\]

where the 3-tensor external field \(\tilde{C}^{\alpha \mu \nu}\) is of the form

\[
\tilde{C}^{\alpha \mu \nu} = \partial^\alpha \tilde{C}^{\mu \nu}
\]

with

\[
\tilde{C}^{\mu \nu} = ia \int_{S(C)} d^2 x \frac{1}{-\Box} (z - \xi).
\]

In the above expressions, \(a\) is an arbitrary real number and \(S(C)\) is a space-like surface bounded by the closed string at \(C\). \(d^2 \xi_{ij}\) is the surface element of \(S(C)\), the directions \(i, j\) being along the surface.

Substituting \((3.2)\) in \((3.1)\) we get for the \(\sigma(C)\) operator the expression of ref.\([8]\):

\[
\sigma(C, t) = \exp \left\{ \frac{ia}{2} \int_{S(C)} d^2 \xi_{ij} \frac{\partial^\alpha H^{\alpha \mu \nu}}{-\Box} \right\}
\]

or

\[
\sigma(C, t) = \exp \left\{ \frac{-ia}{2} \int_{S(C)} d^2 \xi_{ij} B^{ij} + \text{gauge terms} \right\}.
\]

The gauge terms in \((3.4)\) guarantee the gauge invariance of \(\sigma\) which is explicit in \((3.3)\). Later on, it will become clear that both the correlation functions and commutation rules of \(\sigma\) are independent of the surface \(S\): they just depend on \(C\). The generalization for an open string is straightforward. The operator \(\sigma(C)\) creates a string along the curve \(C\). In order to prove this, let us consider the topological charge operator along a surface \(R\) (i.e. an analogous operator to the magnetic flux operator of ref.\([8]\), namely,

\[
\Phi_R = \int_R d^2 \tilde{x}^i J^{i0}(\tilde{x}, t)
\]

Let us evaluate the commutator \([\Phi_R, \sigma]\). To do this, let us observe that according to \((2.12)\)

\[
J^{i0} = \partial_j \Pi^{ij}
\]
where $\Pi^{ij}$ is the momentum canonically conjugate to the Kalb-Ramond field $B^{ij}$, satisfying the equal-time commutator

$$[B^{ij}(\vec{x}, t), \Pi^{kl}(\vec{y}, t)] = i(\delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk})\delta^3(\vec{x} - \vec{y})$$

Using the above relation, the Baker-Hausdorff formula and Stokes’ theorem, we immediately get

$$\left[ \int_R d^2x^i J^{i0}(\vec{x}, t), \sigma(C, t) \right] = a\sigma(C, t) \int_R d^2x^i \oint_C d\xi^i \delta^3(\vec{x} - \vec{y}) \quad (3.6)$$

The above integrals give $\pm 1$ whenever the curve $C$ pierces the surface $R$ in the positive or negative sense, respectively. Otherwise they vanish. Hence, we if we choose the “magnetic” flux surface and the string in such a way that it pierces the surface positively, we get

$$[\Phi_R, \sigma] = a\sigma \quad (3.7)$$

This shows that the $\sigma$ operator carries $a$ units of “magnetic” flux along the curve $C$ and indeed creates a topological string along this curve.

### 4) Quantum Strings Correlation Functions

#### 4.1) Kalb-Ramond Representation

In this section we compute the Euclidean correlation functions of the operator $\sigma$ introduced above in the Kalb-Ramond representation. Using the expression (3.1) for the $\sigma$ operator and the lagrangian (2.11), we can express the correlation function in Euclidean space (in a completely analogous way as in ref. [4]) as

$$<\sigma(C_x)\sigma^\dagger(C_y)> = Z^{-1} \int DB_{\mu\nu} \exp \left\{ -\int d^4z \left[ \frac{1}{12\rho^2_0} H^{\mu\nu\alpha} H_{\mu\nu\alpha} + \right. \right.$$  

$$\left. + \frac{1}{6} \tilde{C}_{\mu\nu\alpha}(z; x, y) H^{\mu\nu\alpha} + \mathcal{L}_R \right]\right\}, \quad (4.1)$$

where $\mathcal{L}_R$ is a surface renormalization factor, to be determined below, which ensures the locality of this correlation function i.e. the fact that it depends only on the string position, namely, on the border of the surface, $C$, and not on the surface $S(C)$ itself.
We see that \(< \sigma \sigma^\dagger > = e^{W(\tilde{C}_{\mu\nu})}\) is the vacuum functional in the presence of the external field \(\tilde{C}_{\mu\nu}\). This property of the correlation functions of \(\sigma\) is common to all of the topological charge bearing related operators [3]-[7] and follows from the general fact that topological charge carrying operators are closely related to the disorder variables of Statistical Mechanics [9]. Indeed, treating these operators as disorder variables one can demonstrate in general [10] that the \(\sigma\) operator correlation functions can be expressed in terms of the coupling of the lagrangian field to an external field like \(\tilde{C}_{\mu\nu\alpha}\) as in (3.1). It is not difficult to see, using (3.4) and (2.5) that the second term in the exponent in (4.1) can be written, up to gauge terms, as

\[
\frac{1}{2} \int d^4 z J_{\mu\nu} B^{\mu\nu},
\]

(4.2)

if we choose \(a = \gamma_\alpha\) and \(N = 1\) in (2.5). Going back to (2.10) or (2.11), and comparing with the above expression, we immediately conclude that if we retain the \(\bar{\theta}\) part in (2.9) and just integrate over the single valued part \(\alpha\), the functional thereby obtained is precisely the above string correlation function \(< \sigma \sigma^\dagger >\). This would provide an alternative way for obtaining the string operator.

One can show in general [10] that the appropriate renormalization factor consists of the corresponding self-coupling of the external field. Also here, we will see explicitly that the renormalization counterterm

\[
\mathcal{L}_R = \frac{\rho^2_0}{6} \tilde{C}_{\mu\nu\alpha} \tilde{C}_{\mu\nu\alpha}.
\]

(4.3)

will absorb all the hypersurface dependence of the correlation function, thereby making it completely local. Indeed, performing the change of functional integration variable

\[
B_{\mu\nu} \to B_{\mu\nu} + \rho^2_0 \Omega_{\mu\nu}
\]

(4.4)

with

\[
\Omega_{\mu\nu} = \tilde{C}_{\mu\nu}(S') - \tilde{C}_{\mu\nu}(S)
\]

(4.5)

– where \(S'\) is an arbitrary surface also bounded by the curve \(C\) – in the integral (4.1), and choosing \(\mathcal{L}_R\) as given by (4.3), we conclude, after a straightforward calculation,
that \( < \sigma \sigma^\dagger > (S) = < \sigma \sigma^\dagger > (S') \), thereby establishing its surface invariance in general.

Let us now explicitly compute the correlation function (4.1), with the choice (4.3) made for \( L_R \). Before performing the functional integration in (4.1), note that we can rewrite the linear term as

\[
\frac{1}{6} \int d^4 z \tilde{C}_{\mu\nu\alpha} H^{\mu\nu\alpha} = \frac{1}{2} \int d^4 z K_{\mu\nu} B^{\mu\nu},
\]

where

\[
K_{\mu\nu} = \frac{1}{2} \tilde{C}_{\alpha\beta\gamma\delta} F^{\alpha\beta\gamma\delta}_{\mu\nu},
\]

and

\[
F^{\alpha\beta\gamma}_{\mu\nu} = \partial^\alpha \Delta^{\beta\gamma}_{\mu\nu} + \partial^\beta \Delta^{\gamma\alpha}_{\mu\nu} + \partial^\gamma \Delta^{\alpha\beta}_{\mu\nu}.
\]

with

\[
\Delta^{\mu\nu\alpha\beta} = \delta^{\mu\alpha} \delta^{\nu\beta} - \delta^{\mu\beta} \delta^{\nu\alpha}.
\]

Inserting (4.3) and a gauge fixing term

\[
\mathcal{L}_{GFB} = -\frac{\xi}{8 \rho_0^2} B_{\mu\nu} K^{\mu\nu\alpha\beta} (-\Box)^{-1} B_{\alpha\beta}
\]

where \( K^{\mu\nu\alpha\beta} = \partial^\mu \partial^\alpha \delta^{\nu\beta} + \partial^\nu \partial^\beta \delta^{\mu\alpha} - (\alpha \leftrightarrow \beta) \) and \( \xi \) is gauge fixing parameter – in (4.1), we can perform the quadratic integration over \( B_{\mu\nu} \) with the help of the euclidean propagator of this field, namely

\[
D^{\mu\nu\alpha\beta}(x) = \frac{\rho_0^2}{4} \left[ (\Box)^{-1} K^{\mu\nu\alpha\beta} (\Box)^{-1} \right] \left( \frac{1}{\Box} \right)^2
\]

The result is

\[
< \sigma(C_x) \sigma^\dagger(C_y) > = \exp \left\{ \frac{1}{2} \int d^4 z d^4 z' K^{\mu\nu}(z) K^{\alpha\beta}(z') D_{\mu\nu\alpha\beta}(z - z') - S_R \right\}.
\]

where \( S_R \) is the action corresponding to the renormalization counterterm \( \mathcal{L}_R \). We immediately see that only the first term of (4.11) contributes to (4.12). In particular all the gauge dependence disappears. This happens because of the gauge invariant way in which the external field is coupled in (4.1) which results in the form of \( K_{\mu\nu} \) given by (4.7). Using the identity

\[
F^{\mu\nu\alpha}_{\sigma\tau\lambda\chi} = -4 \epsilon^{\mu\nu\alpha\sigma} \epsilon^{\gamma\rho\beta\lambda} \left[ -\delta^{\sigma\lambda} + \partial^\sigma \partial^\lambda \right]
\]

and performing the \( z \) and \( z' \) integrals in (4.12), we get
\[
< \sigma(C_x) \sigma^\dagger(C_y) > = \exp \left\{ \frac{a^2 \rho_0^2}{2} \sum_{i,j=1}^{2} \lambda_i \lambda_j \int_{S_i(C)} d^2 \xi_{\alpha\beta} \int_{S_j(C)} d^2 \eta_{\mu\nu} \right.
\]
\[
\partial_i \partial_{\lambda} \left[ \frac{1}{4 \Box} \right]^2 \epsilon^{\alpha\beta\gamma\rho} \epsilon_{\mu\nu\lambda\rho} \left[ \Box \delta^{\alpha\rho} + \partial^n \partial^\rho \right] \left[ \frac{1}{\Box} \right] - S_R \right\}. \tag{4.14}
\]

Since we are evaluating the two-point function, actually in (4.1), \( \tilde{C}_{\mu\nu\alpha}(z; x, y) = \tilde{C}_{\mu\nu\alpha}(z; x) - \tilde{C}_{\mu\nu\alpha}(z; y) \). Hence, in the above expression, \( i, j = 1, 2 \) correspond to \( x, y \), respectively, and \( \lambda_1 \equiv +1 \) and \( \lambda_2 \equiv -1 \).

Only the \( \delta^{\alpha\rho} \) term of the above expression gives a nonzero contribution. Inserting the identity
\[
\epsilon^{\alpha\beta\gamma\rho} \epsilon_{\mu\nu\lambda\rho} = \delta^{\lambda\gamma} \Delta^{\alpha\beta\mu\nu} - \delta^{\lambda\beta} \Delta^{\alpha\gamma\mu\nu} + \delta^{\lambda\alpha} \Delta^{\beta\gamma\mu\nu} \tag{4.15}
\]
in (4.14) we immediately see that the first term above produces an expression which is exactly canceled by the surface renormalization counterterm \( S_R \). Applying Stokes’ theorem to the expression yielded by the two remaining terms we obtain
\[
< \sigma(C_x) \sigma^\dagger(C_y) > = \lim_{m, \epsilon \to 0} \exp \left\{ \frac{-a^2 \rho_0^2}{2} \sum_{i,j=1}^{2} \lambda_i \lambda_j \oint_{C_i} d\xi^\alpha \oint_{C_j} d\eta^\alpha \right.
\]
\[
\left[ -\frac{1}{8\pi^2} \ln \mu [||\xi - \eta|| + |\epsilon|] \right]. \tag{4.16}
\]

The expression between brackets is \( \frac{1}{(-\Box)^2} = \mathcal{F}^{-1} \left[ \frac{1}{K^2} \right] \) and \( \mu \) and \( \epsilon \) are respectively an infrared and an ultraviolet regulator. We see that for the above topological charge conserving correlation function, all the \( \mu \)-dependence is cancelled because \( \sum_i \lambda_i = 0 \). The \( \epsilon \) dependence associated with the self-interacting \( \tilde{i} - \tilde{i} \) terms can be eliminated by a multiplicative renormalization of the string field operator \( \sigma \). Expression (4.16) is precisely the one found for the large distance behavior of a magnetic vortex in the case of a local U(1) Higgs theory \( [3] \). This result, which at first sight might seem surprising, is actually to be expected, according to the following argument. In ref. \( [4] \) it was proved that Maxwell lagrangian
\[
\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \tag{4.17}
\]
can be formulated as a particular sector of a Kalb-Ramond theory with lagrangian
\[ \mathcal{L}' = -\frac{1}{12} H_{\mu\nu\alpha} (\Box)^{-1} H^{\mu\nu\alpha}. \]

In a similar way, it is not difficult to show [8] that the lagrangian
\[ \mathcal{L} = -\frac{1}{12} H_{\mu\nu\alpha} H^{\mu\nu\alpha}, \quad (4.18) \]
appearing in (2.11), corresponds to
\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} (\Box)^{-1} F^{\mu\nu}. \]

It turns out that this lagrangian has precisely the form of a gauge invariant mass term which appears in the local U(1) Higgs theory of ref. [3] and which determines the large distance behavior of the vortex correlation function in the local case. This explains the coincidence of results for the quantum vortices in both models.

Considering the case of a straight string along the z-direction and piercing the \( z = 0 \) plane at the point \( (\vec{x}, 0) \), the global string creation operator \( \sigma(C, t) \) can be written as \( \sigma(\vec{x}, t) \). Starting from (4.16) and following exactly the same steps as in [3], we obtain, for a long straight string of length \( L \),
\[ < \sigma(\vec{x}) \sigma^\dagger(\vec{y}) > = \exp \left\{ -\frac{La^2 \rho_0^2}{8\pi} |\vec{x} - \vec{y}| \right\}, \quad (4.19) \]

From this expression, one can infer that the string energy is given by \( E(L) = \frac{La^2 \rho_0^2}{8\pi} \) which means that the string energy density \( \epsilon = E(L)/L \) is given by the following expression:
\[ \epsilon = \frac{a^2 \rho_0^2}{8\pi}. \quad (4.20) \]

The large distance behavior of the correlation function of a genuine quantum vortex seems to be “universally” governed - whether they are local or global - by an exponential decay. This is not the case, of course, in the symmetric phase of the U(1) Higgs model, where we have \( \rho_0 = 0 \) and consequently \( E = 0 \). Hence it follows that \( < \sigma \sigma^\dagger > \to \text{const} \neq 0 \) when \( |\vec{x} - \vec{y}| \to \infty \) and the conclusion is that there are no true physical vortex excitations in this phase.
4.2) Higgs Representation

Let us evaluate here, for the sake of completeness, the string correlation function in the Higgs language. As a subproduct, we will obtain the global string creation operator in terms of the scalar Higgs field $\phi$. Let us start from the correlation function (4.1), with the $\mathcal{L}_R$ given by (4.3). Using the fact that $\partial_\mu H^\mu \equiv 0$, we can write (4.1) as

$$< \sigma(C_x)\sigma^\dagger(C_y) > = \int D^*H^\mu D\theta \exp \left\{ - \int d^4z \left[ \frac{1}{2\rho_0^2} (H^\mu)^2 + \partial_\mu \phi \partial_\mu \phi + \frac{\rho_0^2}{6} \hat{C}_{\mu\alpha\beta} \hat{C}^\mu_{\mu\alpha\beta} \right] \right\}$$

(4.21)

where

$$\hat{A}_\mu = \epsilon^{\mu\alpha\beta} \hat{C}_{\nu\alpha\beta} \equiv \epsilon^{\mu\alpha\beta} \partial_\nu \hat{C}_{\alpha\beta},$$

(4.22)

with $\hat{C}_{\alpha\beta}$ given by (3.2). Integrating over $^*H^\mu$, we obtain

$$< \sigma(C_x)\sigma^\dagger(C_y) > = \int D\theta \exp \left\{ - \int d^4z \left[ \frac{1}{2} \rho_0^2 \partial_\mu \theta \partial_\mu \theta + \frac{\rho_0^2}{6} \hat{C}_{\mu\alpha\beta} \hat{C}^\mu_{\mu\alpha\beta} \right] \right\}$$

(4.23)

We immediately recognize in the above expression the minimal coupling to the external field $\hat{A}_\mu$. Generalizing for the case of an arbitrary value of $\rho$, we can write

$$< \sigma(C_x)\sigma^\dagger(C_y) > = \int D\phi D\phi^* \exp \left\{ - \int d^4z \left[ |D_\mu \phi|^2 + V(\phi) + \mathcal{L}_R \right] \right\}$$

(4.24)

where $D_\mu = \partial_\mu + i \hat{A}_\mu$ with $\hat{A}_\mu$ given by (4.22). Here, of course $\hat{A}_\mu \equiv \hat{A}_\mu(x) - \hat{A}_\mu(y)$ in order to describe the two-point function.

>From this expression we can infer the explicit form of the string creation operator in the Higgs representation:

$$\sigma(C,t) = \exp \left\{ \frac{1}{2} \int d^4x (\partial^\mu \phi) \hat{D}_\mu \phi \right\}$$

(4.25)

Let us now evaluate (4.24) or (4.23) in the polar representation of $\phi$, using the approximation of constant $\rho = \rho_0$. Performing the quadratic $\theta$ integration in (4.23), we get

$$< \sigma(C_x)\sigma^\dagger(C_y) > = \exp \left\{ \frac{\rho_0^2}{2} \int d^4z \hat{A}_\mu \left[ \frac{(-\Box + \partial_\mu \partial^\nu) \hat{A}_\nu}{-\Box} \right] \hat{A}_\mu - S_R \right\}$$

(4.26)
where the first term above is the third term in (4.23) which did not participate in the $\theta$-integration. Inserting the explicit form of $\tilde{A}_\mu$, (4.22) in (4.26), we immediately reobtain (4.14). From here on the calculation is identical as before and we again arrive at expression (4.19) for the string correlation function. This establishes the equivalence of the Higgs and KR formulations for the quantum string correlation functions.

5) Conclusions

The present method allows a description of the quantum dynamics of vortices or strings in a theory with a spontaneously broken global $U(1)$ symmetry. Temperature can be easily introduced in the usual way, as the string correlation functions are simply given by functionals of given peculiar external field configurations [10]. This would allow one to treat in a unified way both thermal and quantum fluctuations. The potential applications of the present formalism are many. A finite-temperature version of this treatment, for instance, might be useful to study the creation of topological defects in the course of cosmological phase transitions [11] in the Kibble/Zurek scenario. Another interesting field of application can be found in condensed matter. For example, by introducing an external Lorentz-noninvariant background field [12] this approach can be applied to a system like a superfluid helium film where point-like quantum vortices are induced at zero temperature as quantum fluctuations. In particular, it seems specially suited for the problem of vortex nucleation. In general, this approach works in the dilute gas approximation i.e. as long as the core radius is negligible compared with the vortex separation.

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