The Non-Abelian Coulomb Phase of the
Gauged Vector Model at Large N

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Abstract: The renormalization group flows of the coupling constants for the gauged
$U(N)$ vector model, with $N_f$ massless fermions in the defining representation, are stud-
ied in the large $N$ limit, to all orders in the scalar coupling $\lambda$, leading order in $1/N$, and
lowest two orders in the gauge coupling $g^2$. It is shown that the restrictions of
asymptotic freedom, and the reality of the coupling constants throughout the flows,
places important restrictions on $N_f/N$. For the case with massless mesons, these condi-
tions are sufficiently restrictive to imply the existence of an infrared fixed-point $(g_*, \lambda_*)$
in both couplings. Thus, the consistent massless theory is scale invariant, and in a
non-abelian Coulomb phase. The case of massive mesons, and of spontaneously broken
symmetry is also discussed, with similar, but not identical, conclusions. Speculations
related to the possibility that there is a non-perturbative (in $g^2$) breakdown of chiral

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I. Introduction

In spite of the enormous progress that has been made in developing non-perturbative methods for supersymmetric gauge theories [1], there remains considerable interest in approaches suitable for non-supersymmetric theories. A frequently used technique for that purpose is the $1/N$ expansion for a theory with internal symmetry, such as SU($N$) or O($N$), say. Examples include ’t Hooft’s analysis of gauge theories [2], string behavior in two-dimensional QCD [3], and O($N$)-invariant $\lambda\phi^4$ theory [4, 5].

The $1/N$ expansion for $\lambda\phi^4$ theory (in 3+1 dimensions) with O($N$) or U($N$) symmetry (the so-called vector model) has been extensively studied as a renormalized field theory. However, the renormalized vector model encounters a number of problems [5], reviewed in ref. [6].

In [6], the U($N$) vector model was extended by gauging the theory, and adding $N_f$ massless fermions in the defining representation. The model was considered to all orders in $\lambda$, and leading order in $1/N$ and the gauge coupling $g^2$. It was shown that this theory has two phases, one of which is asymptotically free and the other not, with the asymptotically-free phase consistent in that it avoids the difficulties found by Abbott, et al. [5]. To the order considered in [6], the asymptotically free sector requires $0 < \lambda/g^2 < 4/3 \left( N_f/N - 1 \right)$ and $N_f/N < 11/2$ in the large $N$ limit. If these conditions are not satisfied, one returns to all the problems of the non-gauged model.

In this paper we examine the renormalization group (RG) flows for $\lambda$ and $g^2$ for the same model, but now extended to all orders in $\lambda$, leading order in $1/N$, and lowest two orders in $g^2$. Once again one finds a range of parameters for which the theory remains asymptotically free, and thus we concentrate on this phase, as this appears to be the only consistent phase. The contribution of the next order in $g^2$ narrows the “window” for asymptotic freedom somewhat. We find that the
scalar coupling is bounded above by

\[ \frac{\lambda}{g^2} \leq \frac{2}{3} \left( \frac{N_f}{N} - 1 \right) + \left[ \frac{4}{9} \left( \frac{N_f}{N} - 1 \right)^2 - 3 \right]^{1/2} \tag{1} \]

in the ultraviolet (UV) limit if the theory is to be asymptotically free. Since \( \frac{\lambda}{g^2} \) in Eq. (1) must be real, this implies that \((\frac{3\sqrt{3}}{2} + 1) \leq (N_f/N)\). It is also possible to have an infrared (IR) fixed-point \[ \text{in } g^2 \text{ if} \]

\[ \frac{34}{13} < \frac{N_f}{N} < \frac{11}{2} \tag{2} \]

in which case the gauge-coupling at the IR fixed point is

\[ \left( \frac{g_*}{4\pi} \right)^2 = \frac{\left( \frac{11}{2} - \frac{N_f}{N} \right)}{13 \left( \frac{N_f}{N} - \frac{34}{13} \right)} \tag{3} \]

[Note that lower-bound of eq. (2) is less than that imposed on \( N_f/N \) by the reality of (1).]

Perturbation theory in \( g^2 \), can always be justified if \( N_f/N \) is sufficiently close to \( 11/2 \). Further, for massless scalars, we explore a possible infrared fixed-point for the scalar coupling as well, which would mean that the theory is scale invariant at the infrared fixed-point \( (g_*, \lambda_*) \).

Some consideration of cases with non-vanishing masses are also presented. This allows us to comment on the RG flows to the IR when various masses appear. The results of this paper lead us to argue that the massless theory is in a non-Abelian Coulomb phase, if parameters are chosen such that the theory is asymptotically free. An analogous discussion is also given for certain massive cases, as well as speculations related to a possible non-perturbative (in \( g^2 \)) breakdown of chiral symmetry.

II. The Model

Consider the theory of gauged complex scalar fields in the defining representation of U(N), and \( N_f \) massless fermions in the defining representation as well, with Lagrangian
\[ N^{-1} \mathcal{L} = |\partial_\mu \phi + igA_\mu \phi|^2 + \frac{1}{2\lambda} \chi^2 - \frac{\mu^2}{\lambda} \chi - \chi |\phi|^2 - \frac{1}{4} Tr(F_{\mu
u} F^{\mu\nu}) + i \sum_{i=1}^{N_f} (\bar{\psi}_i \gamma \cdot D \psi_i) \] (4)

In (4) \( \phi \) and \( \psi_i \) transform in the defining representation of U(N), the gauge field \( A_\mu \) in the adjoint, \( \chi \) is a singlet, and \( D \) is the covariant derivative. [We do not write gauge fixing terms explicitly.] The field \( \chi \) serves as a Lagrange multiplier, which if eliminated reproduces the usual \( \lambda \phi^4 \) interaction. The coupling constants and fields have been rescaled so that \( N \) is an overall factor of the Lagrangian, and hence \( 1/N \) is a suitable expansion parameter. Note that there is no Yukawa coupling between \( \phi \) and \( \psi \), since both are in the defining representation.

It is convenient to consider the model in Landau gauge, so that the gauge parameter will not be renormalized. The renormalization of the theory may be carried out in modified minimal subtraction. This introduces an arbitrary mass-scale \( M \) as a result of the renormalization process. [For more details pertaining to the renormalization of this model, see ref. [6].] Particularly relevant for us are the renormalized gauge and scalar coupling constants \( g(M) \) and \( \lambda(M) \) respectively. These coupling constants satisfy renormalization group equations

\[ \beta_g = M \frac{dg}{dM} \] (5a)

and

\[ \beta_\lambda = M \frac{d\lambda}{dM} \] (5b)

It is possible to obtain these beta-functions to all orders in \( \lambda \), leading order in \( 1/N \), and a perturbation expansion in \( g^2 \), directly from the work of Machacek–Vaughn (MV) [7]. We evaluated this in the large \( N \) limit, and find
\[
16\pi^2 \beta_g = -g^3 \left( \frac{22}{3} - \frac{4}{3} \frac{N_f}{N} \right) - \frac{4}{3} \frac{g^5}{(4\pi)^2} \left( 34 - 13 \frac{N_f}{N} \right) + \ldots ,
\]
(6)

with \(N_f/N\) fixed, and

\[
\beta_\lambda = a_0 \lambda^2 - a_1 g^2 \lambda + a_2 g^4 ,
\]
(7)

where the \(g^2\) dependent coefficients are

\[
\begin{align*}
(4\pi)^2 a_0 &= 2 + 16 \left( \frac{g}{4\pi} \right)^2 + \ldots \\
(4\pi)^2 a_1 &= 12 + \frac{1}{3} \left[ 256 - 40 \left( \frac{N_f}{N} \right) \right] \left( \frac{g}{4\pi} \right)^2 + \ldots \\
(4\pi)^2 a_2 &= 6 + \frac{1}{3} \left[ 304 - 64 \left( \frac{N_f}{N} \right) \right] \left( \frac{g}{4\pi} \right)^2 + \ldots
\end{align*}
\]

Equations (6)-(8) are obtained by applying the MV results to our model. Note that in (6) the scalar mesons make no contribution to \(\beta_g\) in the large \(N\) limit, as expected from 't Hooft’s analysis of gauge theories at large \(N\). Recall that conventional coupling constants have been rescaled as \(g^2 N \to g^2\) and \(\lambda N \to N\), (and fields rescaled as well) so as to give the overall factor of \(N^{-1}\) in (4).

The remainder of the paper is devoted to exploring the consequences of the coupled set of coupled equations (5)–(8).

III. The Renormalization Group Flows

Since \(g(M)\) does not depend on \(\lambda\), we may solve for it first. Define

\[
t = \ln M
\]
(9a)

and

\[
x(t) = g^2(M) ,
\]
(9b)

then

\[
\frac{dx}{dt} = -b_0 x^2 + b_1 x^3 + \ldots
\]
(10)

where
\[(4\pi)^2 b_0 = \frac{4}{3} \left( 11 - \frac{2N_f}{N} \right) \]
\[(4\pi)^2 b_1 = \frac{8}{3} \left( 13 \frac{N_f}{N} - 34 \right) \]  

(11)

If Eq. (8) is satisfied, then \(g^2(M)\) is asymptotically free, and there is an IR fixed-point for \(g^2\) given by Eq. (3).

In order to solve the equation \(M \frac{d\lambda}{dM} = \beta \lambda\), we first need to find an explicit solution for \(g^2(M)\). Make the change of variables

\[ds = x(t)dt\]

(12)

suggested by Calloway [8]. It is then straightforward to show that

\[x(s) = \left[ A e^{sb_0} + \frac{b_1}{b_0} \right]^{-1}, \]

(13)

where

\[A = \left( \frac{1}{x_0} - \frac{b_1}{b_0} \right) \]

(14)

with

\[x(s = 0) \equiv x_0 = g^2(s = 0). \]

(15)

The explicit relation between \(t\) and \(s\) is also easily found to be

\[t = \frac{A}{b_0} (e^{sb_0} - 1) + \frac{b_1}{b_0} s, \]

(16)

so that \(t\) versus \(s\) is single-valued, and \(s \to \pm \infty\) when \(t \to \pm \infty\). The integration constants were chosen so that \(t = 0\) implies that \(s = 0\). Then \(g_0^2 = g^2(M_0)\), where \(M_0\) is the mass-scale at which coupling constants are defined.

Define

\[y(s) = \lambda(s)/g^2(s) \]

(17)
Then the renormalization group equation for $\lambda$ becomes, in terms of eq. (13),

$$\frac{dy}{ds} = [a_0 y^2 - a_1 y + a_2] + [b_0 - b_1 x(s)]y \tag{18}$$

In the ultraviolet (UV) limit $s \to \infty$, $(g^2/4\pi) \to 0$, so that for large $s$,

$$\frac{dy}{ds} \simeq a_0 y^2 - (a_1 - b_0)y + a_2 \tag{19}$$

where $b_0$ is given in (11), and $a_0, a_1, \text{ and } a_2$ are now just the leading terms of (8). It is convenient to write (19) as

$$\frac{dy}{ds} = a_0[y - y_+(\infty)][y - y_-(\infty)] \tag{20}$$

with

$$y_\pm(\infty) = \frac{2}{3} \left( \frac{N_f}{N} - 1 \right) \pm \left[ \frac{4}{9} \left( \frac{N_f}{N} - 1 \right)^2 - 3 \right]^{1/2}. \tag{21}$$

Reality of the coupling constants in (19) requires

$$(a_1 - b_0)^2 \geq 4a_2a_0 \tag{22}$$

which is the requirement that (21) be real. This implies, when combined with (2), that

$$3.6 \simeq \left( \frac{3\sqrt{3}}{2} + 1 \right) \leq N_f/N \leq \frac{11}{2} \tag{23}$$

which therefore forces $b_1 > 0$.

Equation (21) implies the following renormalization group flows to the UV, where $y_0$ is an initial value with sufficiently large $s_0$.

a) If $y_0 = y_+ (+\infty)$, then $dy/ds = 0$ for large $s$, and $y(s)$ remains at $y_+(\infty)$ for increasing $s$.

b) If $y_0 < y_+ (+\infty)$, then $y(s) \xrightarrow{s \to \infty} y_-(\infty)$.

c) If $y_0 > y_+ (+\infty)$, then $y(s) \xrightarrow{s \to \infty} \infty$, and asymptotic freedom is lost. Therefore we see that $y_+(\infty)$

\footnote{More precisely in case (a) $y(s_0)$ lies on the boundary $y_+(s)$ defined in eq. (33).}
defines the phase-boundary which separates an asymptotic free theory (in both couplings) from the inconsistent non-asymptotically free theory. From (21), we see that this phase boundary is given by

\[ y_+ (\infty) = \frac{2}{3} \left( \frac{N_f}{N} - 1 \right) + \left[ \frac{4}{9} \left( \frac{N_f}{N} - 1 \right)^2 - 3 \right]^{1/2}, \tag{24} \]

which then gives the upper-bound for \( (\lambda/g^2) \) as stated in eq. (1). [The inclusion of higher order corrections in \( g^2 \) has reduced somewhat the upper-bound \( \lambda/g^2 < 4/3(N_f/N - 1) \) given in ref. [6].]

We now turn to the consideration of the IR region. Let us set the renormalized value of the mass-parameter \( \mu^2/\lambda = 0 \) in (4), so that the scalar fields remain massless. For this part of the discussion, we assume that the \( N_f \) massless fermions do not acquire a mass by means of a non-perturbative process. [Later in the paper we return to the possibility that the fermions get a mass non-perturbatively.] With these assumptions, we can use the beta functions, as given by (6) and (7), all the way to the IR limit, \( s \to -\infty \).

Since (23) requires \( b_1 > 0 \), we consider the consequences of a fixed-point \( g_* \). The IR limit of eq. (18) gives approximately

\[ \frac{dy}{ds} \simeq a_0 y^2 - a_1 y + a_2 \tag{25} \]

where the coefficients \( a_0, a_1, \) and \( a_2 \) in (8) are to be evaluated for \( s \to -\infty \). [To obtain (25) note that \( x(-\infty) = b_0/b_1 \).] The reality of \( (\lambda/g^2) \) for \( s \to -\infty \) requires

\[ a_1^2 - 4a_2a_0 \geq 0. \tag{26} \]

For convenience define \( z = (g_*/4\pi)^2 \). Then using (8), (26) can be cast into the inequality

\[ 1 + \left[ 8.9 - 1.6 \left( \frac{N_f}{N} \right) \right] z + \left[ 8.3 - 9.5 \left( \frac{N_f}{N} \right) + 1.9 \left( \frac{N_f}{N} \right)^2 \right] z^2 \geq 0 . \tag{27} \]
Since we are considering the consequences of a fixed point \( g^* \), \( N_f/N \) is not independent of \( z \). One must use (3) to solve the constraints of (27). Then (27) is satisfied for all values allowed by (23), so that

\[
0 \leq z = \left( \frac{g^*}{4\pi} \right)^2 \leq 0.15 ,
\]

which justifies perturbation theory in \( g^2 \). We have seen that for the massless theory, \( g^2(s) \to +\infty \) as \( s \to -\infty \) is not allowed for our system of equations. One must have an IR fixed-point for \( g^2 \), to the order we are working. Higher correction in \( g^2 \) might alter specific numerical values, but a change in qualitative conclusions would be surprising.

We also want to know whether \( \lambda(-\infty) \) has a fixed-point when one reaches the IR fixed-point \( g^* \). Let us write (25) as

\[
\frac{dy}{ds} = a_0[y - y_+(-\infty)][y - y_-(\infty)] .
\]

Let \( y(s_0) \) be an initial value for the solution of (29), taken at sufficiently large negative \( s_0 \)

\[\text{a)} \quad \text{If } y(s_0) > y_+(-\infty), \text{ then } dy/ds > 0, \text{ and } y(s) \text{ decreases towards } y_+(-\infty). \quad \text{[Of course that means } y(s) \text{ will increase as } s \text{ increases, which is not asymptotically free, and thus outside the consistent phase.]}\]

\[\text{b)} \quad \text{If } y(s_0) = y_+(-\infty), \text{ then } dy/ds = 0, \text{ and } y \text{ remains at } y_+(-\infty).\]

\[\text{c)} \quad \text{If } y_+(-\infty) \geq y(s_0) \geq y_-(\infty), \text{ dy/ds < 0, and } y(s) \text{ flows to } y_+(-\infty) \text{ in the IR limit.}\]

\[\text{d)} \quad \text{If } y(s_0) = y_-(\infty), \text{ then } y \text{ remains at } y_-(\infty).\]

\[\text{e)} \quad \text{If } y(s_0) < y_-(\infty), \text{ then } dy/ds > 0, \text{ and } y(s) \to -\infty, \text{ which is not allowed.}\]

Therefore, the values of \( y_{\pm}(-\infty) \) are of interest. From (29)

\[
y_{\pm}(-\infty) = \frac{a_1}{2a_0} \pm \frac{1}{2a_0} [a_1^2 - 4a_0 a_2]^{1/2}
\]

\[^5\text{More precisely for cases (b) and (d) } y(s_0) \text{ lies on the boundaries } y_{\pm}(s) \text{ respectively, as defined in eq. (33).}\]
where the coefficients are evaluated in the IR limit. Since \( y_\pm(-\infty) > 0 \), we see that we reach a fixed point \( \lambda_* = \lambda(-\infty) \) if for very large negative \( s_0 \), \( y_-(\infty) \leq y(s_0) \leq y_+(\infty) \). [Cases (b), (c) and (d).] The actual values of \( (\lambda_*/g_*) \) are obtained from (30), together with (8) evaluated at \( g_* \) where the allowed range of \( g_* \) is obtained from (28). Thus, consistency in the IR of the massless theory, with no non-perturbative generation of masses for the fermions, leads to fixed points \( (g_*, \lambda_*) \) for the (gauge, scalar) couplings, and hence presumably to a scale-invariant theory. At the IR fixed point \( (g_*, \lambda_*) \) we have massless interacting non-Abelian gauge bosons, so that this is a non-Abelian Coulomb phase. We shall argue below that this certainly occurs for \( N_f/N \) sufficiently close to \( 11/2 \).

Suppose now that the renormalized mass-parameter in (4), \( (\mu^2/\lambda) > 0 \). How does the analysis of the IR region change? In that case, all the elementary scalar mesons acquire a mass. Of course, the criteria for the consistency of the theory in the UV does not change, but for momenta \( |p| << \mu \), the mesons decouple, and the effective low-energy theory is that of massless gluons and fermions, again assuming that there is no non-perturbation generation of masses for the fermions. The scalar-coupling “freezes” at the renormalization-scale \( M = \mu \). It is reasonable to take the arbitrary mass-scale \( M_0 = \mu \) as well. With this convention, we have

\[
\lambda(s) = \lambda(s = 0) \quad \text{for} \quad s \leq 0 ,
\]

while the evolution of \( g^2(s) \) towards the IR is still governed by (10)–(16). In this case \( g^2(s) \) will have an IR fixed-point. Since consistency in the UV restricts one to the case where \( N_f/N \) satisfies (28), \( b_1 > 0 \), this is a non-Abelian Coulomb phase as well.

Next suppose \( (\mu^2/\lambda) < 0 \) for the renormalized mass-parameter. Then symmetry is spontaneously broken to \( SU(N-1) \), with a Higgs boson which gives mass to some of the mesons and gauge bosons. The massless mesons and gauge bosons transform as the defining and adjoint representations of \( SU(N) \). Since there is no Yukawa coupling, the \( N_f \) fermions remain massless in
perturbation theory, and transform as the $(N-1) \oplus 1$ representation of SU$(N-1)$. The RG flow to the UV is unchanged from our previous analysis for momenta large compared to the masses. In the IR, the RG flows are now appropriate to SU$(N-1)$. These RG equations only differ from those of the massless SU$(N)$ theory by terms of order $1/N$. Thus, we expect an IR fixed-point $(g^*, \lambda^*)$ for the massless SU$(N-1)$ sector, as the massive mesons and gauge bosons uncouple in the low-energy region.

We have discussed the phase-boundaries in the far UV and IR, which gave useful constraints on the parameters of the model. The phase-boundaries can be described for arbitrary $s$, not just the asymptotic values $\pm \infty$. To do so, consider (18) which can be written as

$$\frac{dy}{ds} = a_0[y - y_+(s)][y - y_-(s)]$$

(32)

where the $s$-dependent coefficients $a_0$, $a_1$ and $a_2$ are given by (8), and $b_0$ and $b_1$ by (11). Therefore

$$y_\pm(s) = \left[\frac{a_1 - b_0 + b_1 x(s)}{2a_0}\right] \pm \left\{\left[\frac{a_1 - b_0 + b_1 x(s)}{2a_0}\right]^2 - \frac{a_2}{a_0}\right\}^{1/2} > 0$$

(33)

where reality of the couplings is required for all values of $s$. The curve $y_+(s)$ separates the asymptotically free from the non-asymptotic free phase. Flows for $y(s) > y_+(s)$ grow in the UV to $y(s)_{s \to \infty} \to + \infty$, which is not a consistent phase of the model. Flows for $y(s) < y_-(s)$ evolve in the IR to $y(s)_{s \to -\infty} \to - \infty$, which is also not permitted, as negative couplings are not allowed. Therefore, $y_-(s) \leq y(s) \leq y_+(s)$ is required for consistency of the theory, which as we have argued above describes a non-Abelian Coulomb phase of the theory [see figure].

It should be mentioned, based on earlier work [4]–[6], that there is a singlet scalar meson bound-state in meson-meson scattering. This bound-state is massive in the asymptotic free-phase, and becomes massless on the phase-boundary $y_+$, in accord with the fact that $\lambda$ increases relative
to $g^2$ as one approaches the phase-boundary $y_+$. If $\lambda$ increases further (with $g^2$ fixed), one leaves the asymptotic free phase, and the theory is inconsistent with all the problems of the ungauged model. In particular, the bound-state becomes “over-bound”, i.e., a tachyon, if one is in the non-asymptotically free sector. Since there is no Yukawa coupling, this scalar meson bound-state does not couple to the fermions.

Finally we address the possibility that chiral symmetry is broken non-perturbatively in $g^2$, and that the fermions acquire a mass. [It is not obvious whether the scalars become massive by this mechanism, as the scalars do not couple to the fermions. We shall discuss the consequences of the scalar becoming or not becoming massive.] Since the scalars do not contribute to gluon dynamics in large $N$, we speculate that the fate of the fermions is the same as that of the theory without scalars. Banks and Zaks expressed the view that chiral symmetry is broken spontaneously for all values of $N_f/N < 11/2$ consistent with asymptotic freedom. However, more recently it has been argued that there is a critical value $(N_f/N)_{cr}$ above which the chiral symmetry is restored. Since we speculate that the value of $(N_f/N)_{cr}$ is not altered in large $N$ by the scalars of the model, as one would expect from the 't Hooft analysis one obtains from eq. (94) of the estimate in the large $N$ limit

$$(N_f/N)_{cr} = 4 .$$

As the UV structure of the theory is not altered by these considerations, we would have

$$3.6 \simeq \left( \frac{3\sqrt{3}}{2} + 1 \right) \leq N_f/N < 4$$

as the range of values for which the fermions become massive, and

$$4 \leq N_f/N < 11/2$$

for which the fermions remain massless. Let us concentrate on the case $\mu^2/\lambda = 0$. Then the scalars
remain massless for $N_f/N$ in the range (36), so that this is a massless theory, with RG flow to the IR fixed-point $(g_*, \lambda_*)$, as described by (29)–(31), a non-Abelian Coulomb phase.

Let us now consider the theory with $\mu^2/\lambda = 0$, but with $N_f/N$ in the range (35):

(i) Assume that the scalars get a mass by the same non-perturbative mechanism in $g^2$ by which the fermions become massive. Then, both the fermions and scalars uncouple in the IR region, and the RG flow towards the IR is just that of the pure gluon theory. This is presumably a confining phase.

(ii) The alternate possibility is that the scalars remain massless in the phase described by (35). If this is the case, only the fermions uncouple in the infrared region. For this possibility, the RG flow for $s \to -\infty$ is described by (10) and (18), but with $(N_f/N) = 0$ in (11) and (8). Equation (18) evaluated in the IR region now gives the reality condition,

$$[a_1 + b_1 x(s)]^2 \geq 4a_2 a_0,$$

(37)

since $a_0$, $a_1$, $a_2$ increase as $x(s)$ as $s \to -\infty$. One may verify that (37) is not satisfied in the IR! Thus, it appears that non-perturbative generation of masses for the fermions, without a concurrent generation of masses for the scalars, is not a logical possibility.

In conclusion, we have studied the RG flows of the gauged vector model, with $N_f$ massless fermions, in the large $N$ limit, with particular emphasis on the asymptotic free phase of the model, which is the only consistent phase of the theory. For the case of massless mesons, we showed that an IR fixed point $(g_*, \lambda_*)$ is extremely likely if certain required restrictions on $N_f/N$ are met. For the case of massive mesons, the scalar meson sector decouples at momenta small compared to the meson mass $\mu$. In this case, the low-energy effective theory is that of gauge bosons coupled to massless fermions. Nevertheless, the restrictions on $N_f/N$ inherited from the model in the UV region again make it likely that the theory has a IR fixed-point $g_*$ for the gauge coupling. The
case with spontaneously broken symmetry to SU(N-1) in the large $N$ limit is closely related to the massless SU(N) theory. The predictions of IR fixed points is predicated on the assumption that higher order corrections in $g^2$ will not qualitatively change our analysis. This is certainly true for $(N_f/N)$ sufficiently close to $11/2$, where $g^2/4\pi$ is quite small. We also speculated on the consequences of non-perturbative (in $g^2$) generation of masses for the fermions.

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Graph of the renormalization group flow for $N_f/N = 5$, where ultraviolet to infrared flow progressing from left to right. The upper and lower dashed lines, $y_+$ and $y_-$ respectively, brackets flows consistent with asymptotic freedom and stability of the theory. The vertical dashed line at the right marks the value of the infrared fixed-point $g_\ast$. 
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