Analysis of Virtual Impedance Control Characteristics of Microgrid Power Supply

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Abstract. In order to solve the power decoupling problem of micro source, virtual impedance control is widely used in micro source. In this paper, the virtual impedance control is analysed in detail, and the equivalent circuit model of micro-source system using virtual impedance control is established. The effect of virtual impedance control on micro-source system is analysed from three aspects. The influence of virtual impedance on the output voltage of micro-source is analysed by calculating the output voltage of micro-source in synchronous rotating coordinate system. A small signal model of the system is established according to the droop control characteristics of the micro source, and the influence of the change of virtual impedance value on the stability of the system is analysed. Finally, the simulation model of micro-source system is established, and the analysis results of this paper are verified based on the simulation model.

1. Introduction

With the increasing demand for renewable energy, that makes the renewable energy penetration method become a research focus [1]. Compared with other technologies, distributed generation (DG) technology has the characteristics of small investment, reliable power supply and flexible power generation methods, making it the most ideal way to develop and utilize renewable energy so far. As the DGs can provide effective support for power grids, which make the DGs became more and more important to the grid [2]. Due to all the advantages of DG mentioned above, more and more of DGs have been penetrated into the grid. However, because DGs are mostly located near to the load of the power system, the power flow direction of the traditional power grid will be changed by the DG penetration. The operating characteristics of traditional power generation systems will also be changed. The DGs connected to the power grid increasing, which brought some impacts and challenges to the power grid’s acceptance and dispatching capabilities, as well as the power grid’s voltage control, power supply reliability, power quality, and relay protection. Therefore, the concept of microgrid was proposed.

In order to make the full use of DGs and at the same time weaken the adverse effects brought about by the integration of DGs, the Consortium for Electric Reliability Technology Solutions (CERTS) proposed the microgrid method in 2002 [3]. The microgrid incorporates multiple DGs and loads into the same network, which can either be connected to the grid as a controllable unit or operate independently from the grid. The microgrid achieves the goal of reducing the adverse effects of DGs by making the grid into a controllable small system [4]. Current research shows that integrating DGs into the grid through a microgrid can not only improve the utilization rate of distributed power, but also effectively guarantee the reliability of power supply for important loads. Microgrid has shown great potential and advantages in the efficient application of distributed energy and intelligent control, making it one of the
important strategies for many developed countries to develop the power industry and solve energy problems.

The microgrid located in the low-voltage distribution network, and the distribution network line impedance usually exhibits resistance characteristics. Based on this kind of characteristics, if the original P-F and Q-V droop control methods are still used, it will lead to incomplete decoupling of power control, thereby reducing micro-sources (microgrid converter) dynamic performance and stability [5]. In order to realize the power decoupling control of micro-sources, some papers use hardware filter circuit transformation to complete the decoupling, but this method increases the hardware investment. Therefore, the method of using software control strategy to achieve decoupling control has gained more applications. Nowadays, the methods that can realize micro-source power decoupling control are mainly divided into virtual power droop control methods, active-voltage, reactive-frequency (P-V, Q-f) droop control methods, virtual frequency droop control methods and virtual impedance control methods. [6-7]. Among them, the virtual frequency droop control method cannot cover the frequency and voltage operating range of the actual droop characteristic, so that the micro-source operating area is reduced; and the P-V and Q-f droop control methods cannot realize the parallel connection with the rotating motor.

This article starts with the influence of virtual impedance control on micro-source control, establishes a micro-source system model based on virtual impedance control method, calculates the equivalent circuit of the system, and calculates the small signal model of the micro-source system based on the equivalent circuit. Based on the system model, the effect of the virtual impedance on the output voltage and stability of the micro-source is analysed, and a guiding conclusion is obtained. Finally, a simulation model is established. The simulation method verified the analysis results in this paper.

2. Virtual impedance control of micro-source

The micro-source mostly controlled by droop control method. According to the droop control, the droop characteristics of the micro-source are expressed as follows:

\[
\begin{align*}
\omega_{\text{ref}} &= \omega_\text{ref}^* - k_{\text{ref},p} P_{\text{dms}} \\
V_{\text{ref}} &= V_\text{ref}^* - k_{\text{ref},q} Q_{\text{dms}}
\end{align*}
\]

Where, \(k_{\text{ref},p}\) represents the frequency droop gain of the micro-source command voltage, \(k_{\text{ref},q}\) represents the voltage droop gain of the micro-source command voltage, \(\omega_\text{ref}^*\) represents the reference angular frequency of the micro-source command voltage, and \(V_\text{ref}^*\) represents the reference voltage of the micro-source command voltage, \(\omega_\text{ref}\) represents the angular frequency of the micro-source command voltage, and \(V_\text{ref}\) represents the voltage amplitude of the micro-source command voltage. In practical applications, due to the high control performance of the three-loop control method, it has been widely used. The principle of micro-source control based on three-loop control is as follows:

Figure 1. Three-loop droop control diagram

The three control loops of the micro source are the droop control loop, the voltage control loop and the current control loop. The droop control loop first calculates the output power of the micro source, and obtains the command voltage amplitude and phase angle of the micro source according to the droop characteristics. The input of the voltage control loop is the difference between the command voltage and the feedback voltage. The output of voltage loop controller goes to the current controller. The input of
the current control loop is the difference between the command current and the feedback inductor current. The output of current controller goes to the PWM controller.

The idea of virtual impedance control comes from the definition of equivalent impedance in circuit theory. Based on this control method, the equivalent output impedance of the micro-source to exhibit the desired characteristics of the system, and the method of controlling the micro-source to generate this equivalent impedance is called virtual impedance [8]. The virtual impedance adopted in this paper is to calculate the expected impedance voltage drop from the product of expected impedance and output current, subtract the expected impedance voltage drop from the micro-source command voltage, and use it as an inner loop control command. When the system achieves a good command voltage during tracking, a voltage output that is the same as adding an actual expected impedance on the basis of the original micro-source can be obtained, and then the simulation of the expected impedance characteristic can be realized. The expected impedance is the virtual impedance. The topology circuit using virtual impedance control is as follows:

\[
\begin{align*}
    \frac{d}{dt} v &= i_L - i \\
    L \frac{d}{dt} i_L &= u \cdot v_{in} - v
\end{align*}
\]

Based on equation (2), the open-loop dynamic average output voltage of the micro source can be expressed as follows:

\[
LC \frac{d}{dt} \langle v \rangle + \langle v \rangle + L \frac{d}{dt} \langle i \rangle = \langle u \cdot v_{in} \rangle
\]

Where, <> represents a switching cycle. Based on Figure 1 and Figure 2, the transfer function block diagram of the three-loop micro-source system based on virtual impedance control can be obtained as follows:

Based on Figure 3, the output voltage of the micro source can be obtained as follows:

\[
v = G_o(s)v_{ref} - Z_{ei}(s)i - Z_{eo}(s)i
\]

where, \(G_o(s)\) represents the transfer function of the command voltage coefficient of the micro-source, \(Z_{ei}(s)\) represents the internal impedance transfer function and \(Z_{eo}(s)\) represents the external impedance transfer function, its expression is as follows:
\[ G_u(s) = \frac{G_i(s)G_o(s)}{LCs^2 + (r_L + G_i(s))Cs + G_i(s)G_o(s) + 1} \]  \hspace{1cm} (5) \\
\[ Z_{ei}(s) = \frac{r_L + Ls + G_i(s)}{LCs^2 + (r_L + G_i(s))Cs + G_i(s)G_o(s) + 1} \]  \hspace{1cm} (6) \\
\[ Z_{ei}(s) = G_u(s)Z_{vir}(s) \]  \hspace{1cm} (7)

In practical applications, the gain of the micro-source command voltage coefficient transfer function is mostly designed to be 1\(^5\). According to equation (3), the equivalent circuit of the micro-source based on virtual impedance control is shown in Figure 4.

It can be inferred from Figure 4 that the internal impedance of the micro-source is in series with the virtual impedance. Under the premise of a certain internal impedance, when the virtual impedance is set to 0, the output impedance of the micro-source is determined by the internal impedance, and when the virtual impedance is not 0, the value and characteristics of the output impedance can be changed by changing the value and characteristics of the virtual impedance. In practical applications, micro-sources mostly use virtual impedances to achieve power decoupling control to ensure their stable operation in the distribution network.

3. Influence of virtual impedance on the output voltage

According to the previous description, the micro-source three-loop control principle with virtual impedance can be obtained as follows:

\[ v_{refv} = v_{ref} - Z_{vir}(s)i \]  \hspace{1cm} (8)

Where, \( Z_{vir}(s) \) represents the virtual impedance transfer function of the micro source. The virtual impedance is selected as the inductance characteristic, that is \( L_{vir}s \), and the voltage vector of \( v_{ref} \) is expressed as \( V_{ref} \), the voltage vector of \( v_{refv} \) is expressed as \( V_{refv} \), the voltage vector of \( V_{PCC} \) is expressed as \( V_{PCC} \), and the voltage vector of \( i \) is expressed as \( I \). According to the complex vector analysis and coordinate transformation method, the equation (8) in the fundamental wave positive sequence synchronous rotating coordinate system can be obtained as follows:
Where, \( v_{\text{ref}_d} \) represents the d-axis component of vector \( V_{\text{ref}} \), \( v_{\text{ref}_q} \) represents the q-axis component of vector \( V_{\text{ref}} \), \( i_d \) represents the d-axis component of vector \( I \), and \( i_q \) represents the q-axis component of vector \( I \) and \( \omega \) represent the angular frequency of the micro source. In practical application, \( V_{\text{ref}} \) is mostly oriented on the d-axis, the q-axis component of \( V_{\text{ref}} \) is 0, and the fluctuation of the current can be ignored under steady-state conditions, so the micro-source vector relationship can be obtained as follows:

\[
\begin{align*}
 v_{\text{ref}_d} &= v_{\text{ref}_d} - L_{\text{vir}} i_d + \omega L_{\text{vir}} i_q \\
 v_{\text{ref}_q} &= v_{\text{ref}_q} - L_{\text{vir}} i_q - \omega L_{\text{vir}} i_d
\end{align*}
\]

(9)

Figure 6. The vector diagram in the synchronous rotating coordinate system

Where, \( \delta \) represents the angle between \( V_{\text{ref}} \) and \( V_{\text{PCC}} \), and \( \delta_v \) represents the angle between \( V_{\text{ref}_v} \) and \( V_{\text{PCC}} \).

The relationship shown in Figure 7 and equation (9) can be obtained as follows:

\[
\begin{align*}
 v_{\text{ref}_d} &= v_{\text{ref}} + i_q \omega L_{\text{vir}} \\
 v_{\text{ref}_q} &= -i_d \omega L_{\text{vir}} \\
 v_{\text{ref}} &= \sqrt{(v_{\text{ref}_d})^2 + (v_{\text{ref}_q})^2}
\end{align*}
\]

(10)

Since the angle between \( V_{\text{ref}} \) and \( V_{\text{PCC}} \) is very small [91], we can conclude that the angle between \( V_{\text{ref}} \) and \( V_{\text{ref}_v} \) is also very small. We can approximate that \( V_{\text{ref}_v} \) has the same amplitude as \( V_{\text{ref}} \), and the amplitude difference between \( V_{\text{ref}_v} \) and \( V_{\text{ref}} \) can be obtained as follows:

\[
\Delta V = V_{\text{ref}_v} - V_{\text{ref}} = v_{\text{ref}_v} - v_{\text{ref}} = i_q \omega L_{\text{vir}}
\]

(11)

Assuming that the output of the micro-source can track the command voltage well, considering the small q-axis component of \( V_{\text{ref}_v} \), the output power of the micro-source can be approximated as follows: amplitude difference between \( V_{\text{ref}_v} \) and \( V_{\text{ref}} \) can be obtained as follows:

\[
\begin{align*}
 P_{\text{dms}} &= \frac{3}{2} v_{\text{ref}_v}^2 + \frac{3}{2} v_{\text{ref}_v} i_q \approx \frac{3}{2} v_{\text{ref}_v} i_d \\
 Q_{\text{dms}} &= -\frac{3}{2} v_{\text{ref}_v} i_d + \frac{3}{2} v_{\text{ref}_v} i_q \approx -\frac{3}{2} v_{\text{ref}_v} i_d
\end{align*}
\]

(12)

From equations (11) and (12), the amplitude difference between \( V_{\text{ref}_v} \) and \( V_{\text{ref}} \) can be calculated as follows:

\[
\Delta V = i_q \omega L_{\text{vir}} \approx -\frac{2}{3} \frac{Q_{\text{dms}}}{v_{\text{ref}_v}} \omega L_{\text{vir}} \approx -\frac{2}{3} \frac{Q_{\text{dms}}}{V_{\text{ref}}} \omega L_{\text{vir}}
\]

(13)

The equations (14) shows that when the output reactive power of the micro-source is positive, the value of \( \Delta V \) is positive, in the opposite case, the value of \( \Delta V \) is negative. Considering that the output reactive power of the micro-source under normal working conditions is positive, the available value of \( \Delta V \) is
mostly negative, that is, after adopting virtual impedance control, the actual output voltage of the micro-source is lower than the command voltage.

4. Influence of virtual impedance on the stability of micro-source
In practical applications, in order to achieve the goal of reducing the output frequency and voltage fluctuation, a low-pass filter is usually added to the power measurement link, and the bandwidth is mostly selected as 2-10Hz [9]. Then we can get the power expression of droop control characteristics is as follows:

\[
\begin{align*}
P_{\text{dms}} &= \frac{1}{\tau s + 1} P_{\text{dms}} \\
Q_{\text{dms}} &= \frac{1}{\tau s + 1} Q_{\text{dms}}
\end{align*}
\]

(14)

Where, \( \tau \) represents the time constant of the power low-pass filter, \( p \) represents the instantaneous active power, \( q \) represents the instantaneous reactive power. Select the state variables and input variables of the micro-source as follows:

\[
\begin{align*}
\mathbf{X} &= [\omega \ V]^T \\
\mathbf{U} &= [P_{\text{dms}} \ Q_{\text{dms}}]^T
\end{align*}
\]

(15)

Based on equation (14) and equation (15), the state equation of the micro source can be obtained as follows:

\[
\begin{align*}
\dot{\omega} &= -\frac{1}{\tau} \omega - \frac{1}{\tau} k_p P_{\text{dms}} \\
\dot{V} &= -\frac{1}{\tau} V - \frac{1}{\tau} k_q Q_{\text{dms}}
\end{align*}
\]

(16)

From equation (15) and Figure 8, it can be seen that when calculating the small signal model. We should first calculate the output power of the micro-source. The output power is calculated based on Figure 5.

![Figure 7 Equivalent circuit of micro-source](image)

The state equation of the micro-source is linearized by the Taylor series expansion method, the quadratic term is omitted, and only the first-order and constant terms are retained. The resulting small-signal model of the micro-source is as follows:

\[
\begin{align*}
\Delta \omega &= -\frac{1}{\tau} \Delta \omega - \frac{1}{\tau} k_p \Delta P_{\text{dms}} \\
\Delta V &= -\frac{1}{\tau} \Delta V - \frac{1}{\tau} k_q \Delta Q_{\text{dms}}
\end{align*}
\]

(17)

Suppose the input variables and state variables of the micro-source are expressed as follows:

\[
\begin{align*}
\mathbf{X}_{\text{dms}} &= [\delta \ \omega_{\text{ref}} \ V_{\text{ref}}]^T \\
\mathbf{U}_{\text{dms}} &= [P_{\text{dms}} \ Q_{\text{dms}}]^T
\end{align*}
\]

(18)

Where, \( X_{\text{dms}} \) represents the input variable of the micro-source system, \( U_{\text{dms}} \) represents the state variable of the micro-source system, and \( \delta \) represents the phase angle of the micro-source command voltage vector.
The calculation method of the small signal model, and considering the differential relationship between the command voltage and the angular frequency, the micro-source small signal model can be obtained as follows:

\[
\begin{align*}
\Delta \delta &= \Delta \omega_{ref} \\
\dot{\Delta \omega}_{ref} &= -\frac{1}{\tau} \Delta \omega_{ref} - \frac{1}{\tau} k_{ref,p} \Delta P_{dsm} \\
\dot{\Delta V}_{ref} &= -\frac{1}{\tau} \Delta V_{ref} - \frac{1}{\tau} k_{ref,q} \Delta Q_{dsm}
\end{align*}
\]  

(19)

Where, the symbol \( \Delta \) represents small deviation. According to the small signal model shown in formula (21), the influence of the virtual impedance change on the stability of the micro-source system is analysed. With the virtual impedance is gradually increased from 0mH, and the characteristic value changes of the system can be obtained as below:

![Figure 8. The influence of virtual impedance changes on the characteristic of the micro-source system](image)

Figure 8 shows that when the virtual impedance value is 0mH, the dominant pole of the eigenvalue of the system is a pair of conjugate complex numbers, and the corresponding damping coefficient is 0.32. When the virtual impedance value gradually increases, the system's distance between the dominant pole of the eigenvalue and the real axis gradually decreases, and the damping coefficient of the system gradually increases. When the value of the virtual impedance increases to \( L_{virc} \), the characteristic of the dominant pole of the eigenvalue changes from a pair of conjugate complex numbers to a pair of real numbers. If continue increase the value of the virtual impedance, one of the dominant poles will move towards the virtual axis. From the above analysis, it can be inferred that when the virtual impedance value in the system increases from 0 to \( L_{virc} \), the damping coefficient of the system gradually increases, and the overshoot and adjustment time of the system are also reduced, and the stability of the system is improved. When the virtual impedance continues to increase on the basis of \( L_{virc} \), the distance between the dominant pole of the system and the virtual axis will decrease, and the stability margin of the system will also decrease.

5. Simulation results and analysis
In order to test the influence of virtual impedance control on the output voltage and stability of the micro-source, a simulation study is carried out in this section.
5.1. Simulation example 1
At the beginning of the simulation, the active and reactive power loads of the system are 40% and 12% of the rated capacity. At 0.5s, the reactive power load of the system suddenly changes to 24% of the rated capacity. When the impedance is selected, the reactive power and voltage amplitude response of the micro-source are shown in Figure 10.

From Figures 10, we can conclude that when the virtual impedance of the micro-source become larger, the voltage drop amplitude of the micro-source will larger. From this, it can be inferred that when the reactive load of the system is positive, the increase of the reactive load will cause the output voltage of the micro-source reduced, and the increase of the virtual impedance of the micro-source will also cause the output voltage of the micro-source to decrease.

5.2. Simulation example 2
Due to the increasing of the frequency droop gain of the micro source will reduce the stability of the micro source. Therefore, in this section, the frequency droop gain of the micro-source will be used as the disturbance source, and the MATLAB simulation is used to verify that the virtual impedance control can improve the system stability. The simulation uses the micro-grid topology composed of micro-sources and concentrated loads shown in Figure 9. The results and analysis are as follows:

Figure 9. Topology for simulation
Comparing Figure 11(a) to Figure 11(c), we can see that the smaller the virtual impedance of the micro-source is, the easier it is for the micro-source to enter an unstable operating state after being disturbed. As the value of the virtual impedance increases, the smaller the overshoot of the micro-source after being disturbed, the shorter the adjustment time. It can be inferred that as the value of the virtual impedance of the micro-source increases, the damping of the micro-source increases, and the stability of the system also increases.

6. Conclusion
This paper describes the principle of virtual impedance control and establishes a micro-source model using virtual impedance control. Based on the control model, the equivalent circuit of the virtual impedance is obtained, and a small signal model of the micro-source system using virtual impedance control is constructed. Based on the above model, the influence of virtual impedance control on the micro-source is analysed, and the virtual impedance control can reduce the output voltage of the micro-source and improve the stability of the system. The larger the value of the virtual impedance, the more obvious the impact on the system output voltage and stability. In order to verify the correctness of the theoretical analysis in this paper, a simulation model is established, and the response of the system under different virtual impedance values is verified and analysed. The simulation results verify the correctness of the theoretical analysis.

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