Quantum searching a classical database
(or how we learned to stop worrying and love the bomb)

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We show how to perform a quantum search for a classical object, specifically for a classical object which performs no coherent evolution on the quantum computer being used for the search. We do so by using interaction free measurement as a subroutine in a quantum search algorithm. In addition to providing a simple example of how non-unitary processes which approximate unitary ones can be useful in a quantum algorithm, our procedure requires only one photon regardless of the size of the database, thereby establishing an upper bound on the amount of energy required to search an arbitrarily large database. Alternatively, our result can be interpreted as showing how to perform an interaction free measurement with a single photon on an arbitrarily large number of possible bomb positions simultaneously. We also provide a simple example demonstrating that in terms of the number of database queries, the procedure outlined here can outperform the best classical one.

The nefarious Dr. Strangelove has created a doomsday bomb so sensitive that if it absorbs a single photon it will explode. More distressing than this, even more distressing in fact than Strangelove’s strange accent, is that you are his prisoner. In his typical dastardly manner, he toys with you by informing you that the bomb is in one of a large number \( N \) of boxes, and he will release you and disarm the bomb only if you can determine which one. However, in the clichéd manner of all evil overlords, Strangelove decides to grant you (and the rest of the world for that matter) some chance at life, by allowing you standard optical elements and photodetectors to aid in your challenge.

When you fail to exhibit the abject desperation upon which Strangelove thrives, in fact you even look a little smug, Strangelove becomes suspicious. He calls in his evil assistant Petriskudo, who is, as all evil assistants should be, a rather good physicist. Petriskudo informs Strangelove that in fact there is a way, using ‘interaction-free’ measurement [1] for you to locate the box holding the bomb. However, she suggests that if Strangelove changes the rules, as is the clear prerogative of an evil overlord, in such a way as to let you use only one photon, then you will only be able to search a single box. Unfortunately, Petriskudo is not as smart as she thinks she is; she should have abandoned reading refereed journals and started reading only the quant-ph archive – on which she would have found a long precedent for quantum information theorists trying to rescue the imprisoned [2], and in particular she’d have found the paper [3] which explains how to determine the box holding the bomb using only one photon...

The quantum search algorithm [3] was originally phrased in terms of searching an unsorted database for a marked item. This was unfortunate; it allowed particularly polemic people such as Charlie Bennett, to argue that such a database would have to be a specially constructed “quantum” database, and could not be a regular classical database. The question as to exactly what a classical database is does not seem to have been addressed. Accordingly, as with so many things in quantum information theory, we choose here to make up our own definition – a definition which of course conveniently coincides with a problem we can solve. We should point out that the original paper on quantum searching in fact contained a completely cryptic comment about interaction free measurement (IFM) and its potential utilization as part of a quantum search. We therefore pretend here that this was not recalled with hindsight, but rather formed part of the motivation for this work.

We will imagine that some classical object is used to mark a particular one of \( N \) items, the other items have no such object\(^1\). Obviously, if the classical object is a specially chosen phaseshifter, then a quantum search for the phaseshifter is simple. However, this is generally considered “not cricket”, as an (East) Indian might say. We therefore assume the object provides no potential coherent evolution for a photon; it is some form of incoherent scatterer (or doomsday bomb). By using interaction free measurement as a subroutine in a quantum search algorithm we will show how the marked item can be located using only a single photon, thereby setting a fairly small upper bound on the total amount of energy required to

\(^1\) Since empty space is a particularly good quantum channel for photons (although not for two-level atoms), one might argue that this is not a completely classical database. We have no reply, except to say that all information (quantum or classical if your personal philosophy insists on a distinction), is encoded in/carried by physical systems and is thus describable within quantum theory. However the abstract notion of ‘a classical object’ is not axiomatized within quantum theory, and thus defining a classical database in a manner unobjectionable to all will be difficult. Moreover, since this problem is ultimately responsible for the employment of truckloads of physicists and philosophers working in quantum foundations, we suspect that concerns over job security has prevented most of them making any sort of sensible effort on this problem.

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search an arbitrarily large database. Our algorithm also provides a simple demonstration of how a series of non-unitary operations, which asymptotically approach a unitary one, can be used as part of a coherent quantum computation. Finally, we will give a simple example which demonstrates that in terms of the number of database "queries" the algorithm presented here can outperform the best classical one.

![Fig.1](image1)

Fig.1. 'Interaction free' detection of a single bomb.

We begin by briefly reviewing the standard scheme for IFM\(^2\), as shown in Fig.1. A horizontally polarized photon (in state \(|H⟩\)) enters an interferometer through a switchable mirror, and is rotated an angle \(\theta\) by a polarization rotator (described by the matrix \(\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}\) in the \(|H⟩, |V⟩\) basis). It then passes to a polarizing beam splitter (PBS), which transmits horizontally polarized light and reflects vertically polarized light. If there is no bomb in the vertical arm of the polarization interferometer then the photon emerges from the second PBS in the state \(|\theta⟩ = \cos \theta |H⟩ + \sin \theta |V⟩\). If there is a bomb, then either the bomb explodes (with probability \(\sin^2 \theta\)), or the photon is collapsed back to the state \(|H⟩\). The photon is fed back through the whole system \(M\) times, after which it exits the circuit via the switchable mirror. If there is no bomb then it exits in the state \(|Mθ⟩\), if there is a bomb, then with probability \(\cos^2 Mθ\) it exits in the state \(|H⟩\) and with probability \(1 - \cos^2 Mθ\) the bomb explodes. If we choose \(θ = π/2M\), then the probability of not exploding the bomb approaches 1 for large \(M\); moreover the photon emerges in the state \(|H⟩\) or \(|V⟩\) according to whether or not a bomb is present, and these two states are orthogonal and thus distinguishable.

We note that it is irrelevant where in the vertical arm of the interferometer the bomb is located, in effect the photon is "sniffing out" a large number of spacetime points. Thus we intuitively expect that there should be some way of modifying the above procedure to provide information on an arbitrarily large number of potential bomb positions. To do so we first change the above procedure to choose \(θ = π/M\). Thus, after \(M\) cycles the state will be \(±|H⟩\) according to whether or not a bomb is present. That is, the presence of the bomb introduces a \(π\) phase shift to the probing photon.

![Fig.2](image2)

Fig.2. A series of \(N\) boxes, one of which contains the bomb.

We assume the bomb is known to be located in one of an arbitrarily large number \(N\) of boxes, as depicted in Fig.2. An IFM device similar to the one in Fig.1 is built around each box, with \(θ\) chosen as discussed above. A horizontally polarized photon is placed in a superposition of states, such that it has equal amplitude for being in each of the spatial modes 1 through \(N\). This state can be written \(|ψ^{(0)}⟩ = \frac{1}{\sqrt{N}} ∑_{i=1}^{N} |H_i⟩\). The photon then goes through the \(N\) different IFM setups simultaneously, the \(M\) cycles of which we will refer to as "small cycles". Assuming the bomb does not explode, which occurs with probability \(P_{x}^{(1)} = 1 - \frac{1 - \cos^2 Mθ}{N}\), the photon emerges after the \(M\) small cycles in the state |ψ(0)⟩ = \[\left(∑_{i≠t} |H_i⟩ - \cos Mθ |H_t⟩\right) / √(P_{x}^{(1)})\], where we label the ‘target’ mode holding the bomb by \(t\) and drop an overall phase of \(π\). We arrange for the photon to pass through an array of beamsplitters designed \(\tilde{γ}\) to perform the “inversion about average” operation \(\tilde{γ}\). This completes the first “large cycle”, the state at the end of which is given by \(|ψ^{(1)}⟩ = ∑_{i≠t} α^{(1)}(i)|H_i⟩ + τ^{(1)}|H_t⟩\). If we go through \(k\) large cycles (without exploding the bomb), then it can be shown that the state of the photon is |ψ(k)⟩ = \[\sum_{i≠t} α^{(k)}(i)|H_i⟩ + τ^{(k)}|H_t⟩\], where \(α^{(k)} = \frac{α^{(k-1)}}{sin(k+1)θ}\), \(τ^{(k)} = \frac{sin(k+1)θ}{√(N-1)sin(kθ)sin(k+1)θ}\), and

\[
\tilde{γ}^{(k)} = -cos Mθ sin kθ + √cos Mθ sin(k+1)θ
\]
\[
\tilde{τ}^{(k)} = sin kθ + √cos Mθ sin(k+1)θ.
\]  

Here \(φ\) is defined by

\[
\cos φ = \frac{(1 - \frac{2}{N}) (1 + \cos Mθ)}{2√cos Mθ},
\]

and we recall that \(α^{(0)} = τ^{(0)} = 1/√N\) and \(θ = π/M\). The cumulative probability that we get to large cycle \(k\)
without the bomb exploding is given by

\[
P(k) = \prod_{i=1}^{k} P[i] = \prod_{i=1}^{k} \left(1 - \tau^{(i-1)^2} (1 - \cos 2M \theta)\right)
\]

\[\geq \prod_{i=1}^{k} \cos 2M \theta = \cos 2kM \theta.
\] (2)

In the case of large \( N \), it is not difficult to show from (1) and (2) that by choosing \( k = O(\sqrt{N}) \) and \( M \) to be asymptotically greater than \( k \), the probability of the photon ending up in mode \( t \) without exploding the bomb goes to 1. However, the evolution described by the equations (1), (2) is quite complicated in general.

In Fig. 3, we show how \( \tau(k) \) (dashed line) and \( P(k) \) (dotted line) vary with \( k \), along with the overall probability of successful detection \( P(k)|\tau(k)|^2 \) (solid line). For large values of \( M \) the evolution approaches that of the standard quantum search. However for small \( M \) the evolution can be such that the amplitude for the target state saturates and does not oscillate – this corresponds to the point where the parameter \( \phi \) changes from being imaginary to real. In Fig. 3, this transition between the saturation and oscillation occurs for \( M \) between 9 and 12.

We conclude by making some remarks about interpreting this algorithm as a search of a classical database. We presume we need not search the database in fear of our lives, i.e. that the classical item marking a certain box is not explosive. After \( k \) large cycle iterations, the total number of database queries will be \( kM \). We have not, so far, been able to establish the extent to which this algorithm may be used to out perform a classical one. That it can somewhat do so is implied by the following examples.

Let us take \( N = 4 \) and examine the case when we make only 1 query to the database. Classically, our probability of identifying the marked item in our 1/4 or 25%. For a regular quantum search it would be 100%. If instead we run an algorithm such as the one presented here, then the probability of successfully identifying the marked item is 56.25%. If instead we take \( N = 15 \) and consider the case when we make only 3 queries to the database, then classically our probability of obtaining the marked item in our three queries is 3/15 or 20%. A regular quantum search would find the marked item with 93.5% probability of success. However, if we run the algorithm presented here with \( M = 3 \) and \( k = 1 \), we find that the probability of successfully identifying the marked item is now about 26%, still marginally better than the classical case.

Acknowledgments

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[1] A. Elitzur and L. Vaidman, Found. Phys. 23, 987 (1993); P. Kwiat et. al., Phys. Rev. Lett. 74, 4763 (1995).

[2] Quantum triangulation and violation of conservation of trouble. Terry Rudolph. quant-ph/9902010 Citation of this article is performed as a mild protest against ridiculous self-citations that pervade certain authors’ papers.

[3] Quantum searching a classical database (or how we learned to stop worrying and love the bomb). Terry Rudolph and Dr. (Strange)Lov Grover, quant-ph/XXXXXX Gotcha.

[4] L. Grover, Phys. Rev. Lett. xx,xxxx (199x).

[5] M. Reck et. al., Phys. Rev. Lett. 73, 58 (1994).