Data Transformation for Super-totalstation Positioning System Integrated by GPS and Totalstation

GUO Jiming  ZHANG Zhenglu  LUO Nianxue  HUANG Quanyi

1 Introduction

In densely built urban area, the signals from GPS satellites are severely interrupted by buildings around the stations. Hence, GPS is limited in such environment, especially for detail surveying. Thus totalstation has its advantage of catching detail points in this case, but we have to set up a series of control points used for stations and back sight reference points to locate other unknown detail points. If there is no control point around the surveying area, the surveyor has to develop a control network from some known points so far away, that an extra control surveying task is appended. The integrated GPS and totalstation system (SPS) is a solution to the above problem. First, a GPS receiver and a radio are put on a reference station with known coordinates and good sky view to receive GPS satellite signals. Then a totalstation with built-in GPS receiver is set up at any point under open sky to receive satellite signals and oriented to another known point which has been previously determined by SPS or GPS only. Last, the built-in GPS receiver will record the phase, pseudo range and navigation data from GPS satellites, at the same time, the surveyor may operate the totalstation to get horizontal direction, slope distance and zenith angle to any detail point. The GPS raw data of reference station will be transferred to the rover by radio or GSM system in real time. All the data will be processed with the software in the notebook computer in the field. The totalstation will get the coordinates from GPS on line and then determine the coordinates of other detail points. In this way, the dense control network is exempted.
2 A transformation model of Cartesian coordinates from WGS84 to local ellipsoid

The data-carrier phase, code and peseudo range received by the GPS receiver is with reference to WGS84 (G873). The calculated three-dimensional Cartesian coordinates (X, Y, Z) and baseline vector (dX, dY, dZ) from these measurements both are in the same coordinate system. But in China, we use different reference ellipsoid, BJ54 developed from Krassovsky ellipsoid and GDZ80 which has the same parameters as the International Reference Ellipsoid published in 1975. So the GPS results need to be transferred to the local system. The transformation parameters can be solved through static GPS measurement on common control points according to three-dimensional or two-dimensional models.

2.1 Three-dimensional transformation parameters

According to Bursa-Wolf transformation model[11], we can obtain

\[
\begin{bmatrix}
X_T \\
Y_T \\
Z_T
\end{bmatrix} =
\begin{bmatrix}
\Delta X_0 \\
\Delta Y_0 \\
\Delta Z_0
\end{bmatrix} +
(1 + m) \begin{bmatrix}
1 & \varepsilon_Z & -\varepsilon_Y \\
-\varepsilon_Y & 1 & \varepsilon_X \\
\varepsilon_X & -\varepsilon_Y & 1
\end{bmatrix}
\begin{bmatrix}
X_S \\
Y_S \\
Z_S
\end{bmatrix}
\]

(1)

where (\Delta X_0, \Delta Y_0, \Delta Z_0) are the shift parameters; m is the scale factor; \((\varepsilon_X, \varepsilon_Y, \varepsilon_Z)\) are the rotation parameters; \((X_S, Y_S, Z_S)\) are the three-dimensional Cartesian coordinates in WGS84; \((X_T, Y_T, Z_T)\) are the corresponding three-dimensional Cartesian coordinates for the same control point in local reference ellipsoid system from \((X_S, Y_S, Z_S)\).

Let \(m + 1 = a_{11}, \varepsilon_X = a_{12}/a_{11}, \varepsilon_Y = a_{13}/a_{11}, \varepsilon_Z = a_{14}/a_{11}\), then we can obtain

\[
\begin{bmatrix}
X_T \\
Y_T \\
Z_T
\end{bmatrix} =
\begin{bmatrix}
\Delta X_0 \\
\Delta Y_0 \\
\Delta Z_0
\end{bmatrix} +
\begin{bmatrix}
a_{11} & a_{14} & -a_{13} \\
-a_{14} & a_{11} & a_{12} \\
a_{13} & -a_{12} & a_{11}
\end{bmatrix}
\begin{bmatrix}
X_S \\
Y_S \\
Z_S
\end{bmatrix}
\]

(2)

Let \((X'_T, Y'_T, Z'_T)\) be the known three-dimensional Cartesian coordinates in local reference ellipsoid system, then we can obtain

\[
\begin{bmatrix}
X'_T \\
Y'_T \\
Z'_T
\end{bmatrix} =
\begin{bmatrix}
X_T \\
Y_T \\
Z_T
\end{bmatrix} +
\begin{bmatrix}
V_{X'_T} \\
V_{Y'_T} \\
V_{Z'_T}
\end{bmatrix} =
\begin{bmatrix}
\Delta X_0 \\
\Delta Y_0 \\
\Delta Z_0
\end{bmatrix}
\]

(3)

\[
\begin{bmatrix}
a_{11} & a_{14} & -a_{13} \\
-a_{14} & a_{11} & a_{12} \\
a_{13} & -a_{12} & a_{11}
\end{bmatrix}
\begin{bmatrix}
X_S \\
Y_S \\
Z_S
\end{bmatrix} +
\begin{bmatrix}
V_{X'_T} \\
V_{Y'_T} \\
V_{Z'_T}
\end{bmatrix}
\]

If there are more than three common known points, the most probable values of the 7 transformation parameters can be solved according to least squares principle. And the error equation of Eq. (3) will be

\[
\begin{bmatrix}
V_{X'_T} \\
V_{Y'_T} \\
V_{Z'_T}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & X_S & 0 & -Z_S & Y_S \\
0 & 1 & 0 & Y_S & Z_S & 0 & -X_S \\
0 & 0 & 1 & Z_S & -Y_S & X_S & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta X_0 \\
\Delta Y_0 \\
\Delta Z_0 \\
a_{11} & a_{12} & a_{13} & a_{14}
\end{bmatrix}
\begin{bmatrix}
X'_T \\
Y'_T \\
Z'_T
\end{bmatrix}
\]

(4)

The matrix expression of Eq. (4) is:

\[\mathbf{V} = \mathbf{B} \cdot \delta \mathbf{X} + \mathbf{L}, \text{ weight matrix } \mathbf{P}\]

where

\[\mathbf{V} = (V_{X'_T}, V_{Y'_T}, V_{Z'_T})^T;\]

\[\delta \mathbf{X} = (\Delta X_0, \Delta Y_0, \Delta Z_0, a_{11}, a_{12}, a_{13}, a_{14})^T;\]

\[\mathbf{L} = (X'_T, Y'_T, Z'_T)^T;\]

\[\mathbf{B} \text{ is the coefficient matrix.}\]

The normal equation is

\[\mathbf{B}^T \mathbf{P} \mathbf{B} \cdot \delta \mathbf{X} + \mathbf{B}^T \mathbf{PL} = 0\]

The solution is

\[\delta \mathbf{X} = - (\mathbf{B}^T \mathbf{P} \mathbf{B})^{-1} \mathbf{B}^T \mathbf{PL}\]

The above method is based on the two sets of coordinates of common points. Another method is taking the transformation parameters as additional unknown parameters at the stage of three-dimensional combined adjustment, then they can be solved by the least squares adjustment. The latter is similar to the calculation of two-dimensional transformation parameters in Section 2.2.

2.2 Two-dimensional transformation parameters

The three-dimensional Cartesian coordinates of control points in local reference ellipsoid system are transferred from the corresponding geodetic latitude, longitude and ellipsoid height. The ellip-
sold height is obtained from the orthometric height and height abnormal. But the accuracy of height abnormal is very low due to the lack of precise geoid height for most areas of China, which will affect the accuracy of transformation parameters of three-dimensional transformation model. In addition, the shift parameters \((\Delta X_0, \Delta Y_0, \Delta Z_0)\) are governed by the GPS84 original coordinates. In some cases, the point positioning WGS84 coordinates are used as the reference coordinates. In this way, the shift parameters \((\Delta X_0, \Delta Y_0, \Delta Z_0)\) will change significantly for different control networks. In order to solve this problem, a practical way is to process plane coordinates and height separately. The plane transformation is achieved through a two-dimensional transformation model and the height transformation by mathematic matching model.

To get the two-dimensional transformation parameters, one way is to transfer the adjusted WGS84 Cartesian coordinates to plane coordinates with WGS84 ellipsoid as the reference, together with the known plane coordinates in local coordinate system to calculate the parameters (similar to Section 2.1). Another way is to take the parameters as additional unknown parameters for combined adjustment and resolved by the least squares method. The latter is discussed in detail here. At least two common points (or one point, one distance and one azimuth) are needed in this process. For the field work, we do static GPS measurement, then calculate GPS baseline vectors and transfer these vectors to the reference plane to get plane vectors and ellipsoid height differences, and the covariance matrices are transferred as well.

After the transformation\(^{[2,3]}\) of three-dimensional Cartesian baselines and covariance matrices to vectors of plane coordinates differences and the corresponding covariance matrices, the scale factor \((m)\) and the rotation parameter \((a)\) are introduced as additional unknown parameters for the combination adjustment of GPS data and terrestrial data on the reference plane. Then transformation parameters \((m\) and \(a)\) can be solved together with other unknown coordinates according to the least squares principle.

The measurement equation for the vector of plane coordinates differences is
\[
\begin{pmatrix}
    dx_{ij} + v_{dx_{ij}} \\
    dy_{ij} + v_{dy_{ij}}
\end{pmatrix} = \begin{pmatrix}
    \cos a & \sin a \\
    -\sin a & \cos a
\end{pmatrix} \begin{pmatrix}
    dx'_{ij} \\
    dy'_{ij}
\end{pmatrix} = \begin{pmatrix}
    a & b \\
    -b & a
\end{pmatrix} \begin{pmatrix}
    dx^0 + \delta x_i - \delta x_j \\
    dy^0 + \delta y_j - \delta y_i
\end{pmatrix}
\]
where \((dx_{ij}, dy_{ij})\) are the plane coordinates differences from point \(i\) to point \(j\) under the WGS84 reference system; \((v_{dx_{ij}}, v_{dy_{ij}})\) are the corresponding corrections; \((dx'_{ij}, dy'_{ij})\) are the differences of the plane coordinates from point \(i\) to point \(j\) under the local reference system; \((dx^0, dy^0)\) are the approximate values of \((dx_{ij}, dy_{ij})\); \(\delta x_i, \delta y_i, \delta x_j, \delta y_j\) are the unknown coordinates parameters.

Let
\[
a = (m + 1) \cos a \\
b = (m + 1) \sin a
\]
then
\[
m + 1 = \sqrt{a^2 + b^2}
\]
\[
a = \arctan \frac{b}{a}
\]
The error equation of Eq. (5) can be written as
\[
\begin{pmatrix}
    v_{dx_{ij}} \\
    v_{dy_{ij}}
\end{pmatrix} = \begin{pmatrix}
    -a & -b & a & b & dx^0 & dy^0 \\
    b & a & -b & a & dy^0 & -dx^0
\end{pmatrix} \begin{pmatrix}
    \delta x_i \\
    \delta y_i \\
    \delta x_j \\
    \delta y_j \\
    a \\
    b
\end{pmatrix} - \begin{pmatrix}
    dx_{ij} \\
    dy_{ij}
\end{pmatrix}
\]
The weight matrix \(P = \sigma_0^2 \cdot \Sigma_{\text{dxy}}^{-1}\)

Let \(B = \begin{pmatrix}
    -a & -b & a & b & dx^0 & dy^0 \\
    b & a & -b & a & dy^0 & -dx^0
\end{pmatrix}, \quad L = -\begin{pmatrix}
    dx_{ij} \\
    dy_{ij}
\end{pmatrix}, \quad \Delta X = \begin{pmatrix}
    \delta x_i \\
    \delta y_i \\
    \delta x_j \\
    \delta y_j \\
    a \\
    b
\end{pmatrix}^T\)
supposing that the initial values of \(a\) and \(b\) are 1 and 0, respectively, then we can obtain the normal equation: \(B^T PB \cdot \Delta X + B^T PL = 0\). By iteratively calculating until \(|m_i - m_{i-1}| < 0.0001 \times 10^{-6}\) and \(|a_i - a_{i-1}| < 0.001\", \Delta X\) is solved.

2.3 The model for height interpolation
The height from GPS is the WGS84 ellipsoid height. Its accuracy is about 1 cm to 2 cm. If GPS receivers with higher precision are used and a longer period of measuring time is adopted, the precision may reach mm-order. In spite of the precision of GPS height is so high, it is normally not the
final result used by real engineering projects. In fact, we have to transfer the GPS height to orthometric height. The relationship of orthometric height \( H_r \) and ellipsoid height \( H_e \) is: \( H_r = H_e - \xi \). Here, \( \xi \) is the height abnormal. The geoid and the surface of the reference ellipsoid are not parallel due to the gravity abnormal, so the height abnormal is varying from point to point. But the variation is normally smooth, and we can get a mathematic model from the GPS height and the corresponding known orthometric height to match it. The mathematic model may be a plane or a curve model. If the number of common points is less than 6, a plane model may be adopted (Eq. (7)); if there are 6 or more common height points, a conicoid model may be adopted (Eq. (8)).

\[
\begin{align*}
\xi_i &= a_0 + a_1x_i + a_2y_i + e_i \quad (7) \\
\xi_i &= a_0 + a_1x_i + a_2y_i + a_3x_i^2 + a_4y_i^2 + \\
&+ a_5x_ix_j + e_i \quad (8)
\end{align*}
\]

where \( a_0, a_1, a_2, a_3, a_4, a_5 \) are the coefficients, which are solved according to the least squares principle. For moving the conicoid model, a weight function\(^4\) denoting the influence of the known height abnormal to the interpolated point is determined according to the distance. The better result will be obtained if more common height points exist and are distributed in the whole area. For the area with big undulation, an additional topographic correction will improve the matching precision\(^5\).

3 Solving the station coordinates in real time

The station coordinates of SPS can be solved in real time from the GPS measurements and the transformation parameters (see Section 2).

3.1 The systematic structure of SPS

A set of SPS consists of one or more reference stations (GPS receiver and radio), rover (GPS, TPS, radio, computer), detail points (prism).

3.2 Acquiring the plane coordinates in local reference system

1) Setting up the reference station of SPS on known point A of the local coordinate system, 2) Setting up the TPS station (rover) of SPS on unknown point B, the GPS units for both A and B receive satellite signals synchronously.

3) The horizontal direction, slope distance and zenith angle from rover to detail point are measured by the TPS.

4) About 15-20 minutes later, the GPS data of reference station and rover are transmitted to the computer through radio. Then the baseline \((dX, dY, dZ)_{WGS84}\) of AB is calculated from the synchronous GPS measurements.

5) If there are more than one reference station, several baselines to rover will be available, then the most probable coordinates \((X, Y, Z)_{WGS84}\) of B are calculated by three-dimensional adjustment. The most probable value of a baseline \((dX, dY, dZ)_{WGS84}\) from a reference station to the rover is the difference of the rover and the reference station.

6) Transforming the three-dimensional baseline to plane coordinates difference and ellipsoid height difference; i.e., \((dX, dY, dZ)_{WGS84} \rightarrow (dx, dy, dh)_{WGS84}\).

7) Performing a similar transformation with the transformation parameters \((m, a)\) acquired in Section 2 to get the local plane coordinate difference like: \((dx, dy)_{WGS84} \rightarrow (dx, dy)_{local}\).

8) Calculating the rover plane coordinates in local reference system according to Eq. (9).

\[
(x, y)_{\text{rover \ local}} = (x, y)_{\text{reference \ local}} + (dx, dy)_{\text{local}} \quad (9)
\]

3.3 Acquiring the orthometric height of rover

1) Calculating the WGS84 ellipsoid height \((H_r)\) of rover.

2) Calculating the height abnormal \((\xi)\) by the matching model (see Section 2.3).

3) Calculating the orthometric height of rover: \(H_o = H_r - \xi\).

4 Acquiring coordinates of detail points

One or more reference stations are set up on known points and a rover is set up in the surveying area. First, locating a point sighting through to the rover by GPS method as the back sight. Second,
performing the orientation with the theodolite and other preparation. Third, pointing at the prism on a detail point to measure and record the horizontal direction, slope distance and zenith angle. Forth, about 15-20 minutes later, all of the GPS data are transmitted to the computer through radio and the baseline is computed. Last, the position of the rover in local coordinates system is calculated by the method described in Section 3 and all the coordinates of detail points will be solved just a few seconds later. In this way, both the station and detail points are acquired nearly at the same time, so we do not need to set up a new control network for the detail surveying.

As for setting out, we have to wait for about 15-20 minutes to solve the rover station firstly. Then the setting out data will be calculated according to the designed coordinates and the position of the rover by the computer in the field. Last, the setting out are performed.

5 Conclusions

SPS integrated with GPS and TPS has the advantage of solving station position and detail points at the same time. It will make the concept of “surveying without setting up control network” become true for the real surveying activity. This new surveying model will make surveying work easier and with a higher efficiency.

References
1 Kong X Y, Mei S Y (1996) Control surveying(2). Wuhan: Press of Wuhan Technical University of Surveying and Mapping, 117-121 (in Chinese)
2 Liu J Y, Li Z H, Wang Y H, et al. (1993) The principle and application of GPS. Beijing: Publishing House of Surveying and Mapping, 220-226
3 Zhou Z M, Yi J J, Zhou Q (1992) The principle and application of GPS satellite surveying. Beijing: Publishing House of Surveying and Mapping, 192 (in Chinese)
4 Xu S Q, Li Z H, Wu Y S (1999) Research of GPS abnormal height interpolation system. Journal of Wuhan Technical University of Surveying and Mapping, 24 (4): 356-339 (in Chinese)
5 Zhao J H, Liu J N, Zhang H M (1999) Height determination with GPS leveling data considering non-grid data and effect of terrain. Journal of Wuhan Technical University of Surveying and Mapping, 24 (4): 346-349 (in Chinese)
6 Inge R H (1999) A model for the transformation of satellite vectors to the plane of the map. Survey Review, 35: 274
7 Twigg D R (2000) OSGB36/WGS84 coordinate transformations. Survey Review, 35: 275