More on Conformally Sequestered SUSY Breaking

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Abstract

We extend our models for conformal sequestering of dynamical supersymmetry breaking with decoupling vector-like matter in several different ways. These extensions enable us to simplify concrete model building, in particular, rendering large gauge group and \textit{ad hoc} global symmetry for sequestering unnecessary. Conformal sequestering appears highly natural in such circumstances.
1 Introduction

It is phenomenologically interesting to study superconformal gauge theory (see Ref. [1, 2, 3]), since if it includes a SUSY-breaking sector, conformal sequestering [4] of the SUSY breaking may take place, providing a solution to the flavor-changing neutral current (FCNC) problem in the supersymmetric standard model.\footnote{See also Ref. [5, 6]. For some other phenomenological applications of superconformal dynamics, see Ref. [7].}

In a previous paper [8], we modified vector-like gauge theories for the SUSY breaking [9] by adding massive hyperquarks to turn the full high-energy theory above the mass threshold into conformal gauge theory. More generally, our strategy to construct conformally sequestered hidden sector is as follows: We first choose a model of dynamical SUSY breaking. Next, we add vector-like supermultiplets $\Phi$ (hyperquarks) to uplift the SUSY-breaking model to a conformal field theory in the anticipation of introducing a mass term for $\Phi$.\footnote{This mass can be originated from a vacuum expectation value of another field (see section \ref{section:mass}).} Then, the theory which starts at the Planck scale flows to the infrared (IR) fixed point of the conformal theory. Finally, the mass of $\Phi$ breaks the conformal invariance and effectively leads to the above model of dynamical SUSY breaking.

However, as is detailed in the previous paper, this simple modification does not achieve the conformal sequestering due to unwanted global $U(1)$ symmetries accompanied by the introduction of hyperquarks $\Phi$. In order to eliminate the unwanted global $U(1)$ symmetries, we introduced non-abelian gauge interactions acting on the additional massive hyperquarks. These additional interactions are so determined as to be strong enough to break the $U(1)$ symmetries for sufficient sequestering effects.

Although we obtained various examples realizing the sequestering, the concrete models so constructed typically have large gauge groups and also indispensable global non-abelian symmetry of the SUSY-breaking sector. In particular, since the global non-abelian symmetries, which are thought to be enhanced at the IR fixed point, prevent a generic Kähler potential from sequestering, we were forced to assume the presence of the global symmetry from the start in the hidden sector.

In this paper, we further extend our models for conformal sequestering of dynamical SUSY breaking with decoupling vector-like matter in several different ways. These exten-
sions enable us to simplify concrete model building, in particular, rendering large gauge
group and global symmetry for sequestering unnecessary.

The rest of the paper goes as follows: In section 2, we summarize generic problems to
achieve the conformal sequestering. In section 3, we present concrete methods to eliminate
problematic $U(1)$ symmetries which disturb the conformal sequestering. In addition to the
way of breaking $U(1)$ symmetries by introducing gauge interactions discussed in Ref. [8],
we pursue another way of introducing relevant deformations in the superpotential. In
section 4, we explain the models which require no global non-abelian symmetry in the
hidden sector imposed by hand. Our models are based on a so-called non-calculable SUSY
breaking model [10] of the $SO(10)$ gauge theory which includes only one chiral superfield
in the 16-dimensional spinor representation. Finally, in section 5, we discuss a model
which is defined by adding massive vector-like matter to the SUSY breaking model in
Ref. [11]. In the construction of the final model, we need no new interactions to forbid
problematic $U(1)$ symmetries nor non-abelian symmetries imposed by hand, and hence,
the model is natural in the sense of Ref. [5].

2 Conformal sequestering

The Kähler potential interaction between hidden sector superfields $a_i$ and visible sector
ones $q_a$

$$\Delta K = \frac{C^{ab}}{M_P^2} q_a^\dagger q_b a_i^\dagger a_j$$

induces a severe FCNC problem for generic $C^{ab}_{ij}$. The conformal sequestering [4] is intended
to achieve small $C^{ab}_{ij}$ at low energy by means of manageable strong dynamics of the hidden
sector.

Let us suppose that the hidden sector flows to a strongly coupled superconformal field
theory (SCFT) through a certain high-energy scale $\Lambda_{CFT}$ to a small mass scale $m$ which
eventually sets the SUSY breaking scale. Owing to the large renormalization effects of
the SCFT, $C^{ab}_{ij}$ at low energy are expected to be suppressed (that is, the hidden sector
sequestered) as

$$C^{ab}_{ij}(m) \propto \sum_k C^{ab}_k e^{-L_{ij,k} \ln \frac{\Lambda_{CFT}}{m}}.$$
The matrix $L_{ijk}$ is to be called as a sequestering matrix, whose component values are obtained from the anomalous dimensions\(^3\) of the (possibly non-conserved) composite current superfield $[a_i a_j^\dagger]_r$. The determinant of $L_{ijk}$ is vanishing for conserved currents of the SCFT, because the conserved current is not renormalized. The Kähler term corresponding to a zero eigenvalue is not suppressed and then the sequestering is not achieved.

Our primary concern is on the $U(1)$ part of the sequestering matrix, that is, the $i = j$ part $L_{ik} = L_{iik}$. As for this part, the sequestering matrix is related to the slopes of the $\beta$ functions with respect to the coupling constants $g_k$ in the SCFT as $L_{ik} \sim \partial g_k |_{\beta_i} \mid^*$, or the slopes of the anomalous dimensions $\gamma_{a_i} = -(\partial \ln Z_{a_i} / \partial t)$ of the elementary fields as $L_{ik} \sim \partial g_k |_{\gamma a_i} \mid^*$,\(^4\) where “$^*$” indicates the values evaluated at the fixed point and $t = \ln(\mu / \Lambda_{\text{CFT}})$ with $\mu$ as the renormalization scale.\(^5\)

Therefore, to construct a conformally sequestered SUSY-breaking model, we should look for a SCFT with no (abelian) conserved currents. As is explained in the Introduction, we begin with a dynamical SUSY-breaking model and add several vector-like matters to turn it into a SCFT. Typically, the very introduction of additional vector-like matters $\Phi$ results in an enhancement of $U(1)$ symmetries.\(^6\) They stem from non-anomalous combinations of the $U(1)$ axial rotations on the $\Phi$ fields and anomalous $U(1)$ axial rotations in the SUSY-breaking part.

In the next section, we discuss a few ways to eliminate such $U(1)$ symmetries by introducing further interactions. On the other hand, in section\(^5\) we present a SUSY breaking model where the introduction of a vector-like field does not result in an enhancement of $U(1)$ symmetry, and hence, no additional interactions are required.

Finally, we consider the non-abelian part, namely $i \neq j$. As for the non-abelian part, we can forbid the corresponding Kähler couplings Eq.\(^1\) by imposing a global symmetry on the hidden sector, that is, we can take $C_{ij}^{ab} \propto C_{ij}^{ab} \delta_{ij}$ naturally in the sense of ’t Hooft.

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\(^3\)The operator mixing makes the anomalous dimensions a matrix indexed by $ij$ and $k$.

\(^4\)The relation between the $\beta$ function and the anomalous dimension of the elementary field is given by the NSVZ formula\(^12\). The slope of the $\beta$ function is related to the anomalous dimension of the composite current operator as e.g. in Ref.\(^13\).

\(^5\)See Refs.\(^4, 8\) for details. We have shown that the sequestering matrix for the $U(1)$ part is given by the Hessian of the renormalization group flow at the fixed point when there is no $U(1)$ symmetry\(^5\).

\(^6\)The small mass terms $m \ll \Lambda_{\text{CFT}}$ for these vector-like matters may be ignored in discussing the superconformal dynamics relevant for sequestering.
As such, the problem is reduced to the existence of $U(1)$ symmetries as discussed above. However, imposing such a global non-abelian symmetry seems “unnatural” as emphasized in Ref. [5].

The distinguishing feature, which solves this “naturalness” problem, of our models presented in sections 4 and 5 is that they do not require any global non-abelian symmetries imposed on the hidden sector by judiciously choosing the matter contents and interactions.

3 Eliminating unwanted $U(1)$ symmetries

In this section, we consider a few possibilities to break those unwanted $U(1)$ symmetries with conformality of the dynamics kept intact. The breaking effects should be large because the amount of sequestering follows that of the breaking. This condition implies necessity to construct another strongly coupled SCFT through relevant deformations of the above SCFT with the $U(1)$ symmetries. At the same time, we should arrange the total model so that SUSY breaking is also realized in the end. Namely, a possible recovery of SUSY through the deformations should be avoided.\textsuperscript{7}

3.1 by gauge interactions

One way to break an additional $U(1)$ symmetry (rotation of $\Phi$) is to introduce additional gauge interaction on the matter field $\Phi$. When the $U(1)$ symmetry is chiral under the additional gauge interaction, the $U(1)$ symmetry is broken via the associated chiral anomaly. This possibility was pursued in Ref. [8]. A typical example is the $SP(3) \times SP(1)^2$ gauge theory with matter contents given in table 1. The low-energy SUSY breaking is provided by the $SP(3)$ IYIT model [9], and additional quark superfields $Q'$ (as $\Phi$) are gauged under $SP(1) \times SP(1)$ to break $U(1)$ symmetry that rotates $S_{ij}$, $Q^i$, and $Q'$ simultaneously. The superpotential has a form

$$W = \lambda S_{ij}Q^i Q^j + mQ'^2,$$

where the contracted gauge indices are omitted.

\textsuperscript{7}Although SUSY-broken vacua may exist as local ones in such SUSY recovery examples, it seems hard to separate a SUSY-broken vacuum and a supersymmetric one far enough to stabilize the former, since the deformations need to be sizable for sufficient conformal sequestering.
Table 1: The matter contents in the strongly coupled $SP(N) \times SP(N')$ model with $N = 3, N' = 1$. Here, the subscripts of the fundamental representations denote the dimensions of the representations. In terms of the $SP(N)$ gauge theory, the number of the fundamental representation is given by $N_F = 2(N + 1) + 2 \times 2N' = 12$, while the number of the fundamental representations of each $SP(N')$ gauge theory is given by $N'_F = 2N = 6$.

For $m = 0$, the theory is expected to have a strongly-coupled IR fixed point for the gauge couplings $g_{SP(3)}$, $g_{SP(1) \times SP(1)}$ and Yukawa coupling $\lambda$. The slopes of the anomalous dimensions (i.e. sequestering matrix) are expected to be of order one though the explicit computation is hard to perform. We note that the structure of the gauged SCFT (anomalous dimension, central charge, etc.) is totally different from the original ungauged SCFT. As for the SUSY breaking, the mass term for $Q'$ in the superpotential exclusively causes no problem.

### 3.2 by Yukawa interactions

Another possible way to break the $U(1)$ symmetries is to add superpotential terms (Yukawa interactions) that are relevant deformations of the SCFT, which lead to a new strongly coupled CFT and break the unwanted symmetries explicitly.

Let us introduce a singlet $Y$ and try a superpotential

$$W = \lambda S_{ij} Q^i Q^j + \lambda_Y Y \Phi^2 + \lambda_n Y^n,$$

where $\lambda_Y$ and $\lambda_n$ denote the coupling constants and $3/2 \leq n \leq 3$ for unitarity at a possible fixed point. This deformation completely eliminates the axial symmetry of $\Phi$ rotation (except a $U(1)_R$ symmetry). If the deformation is relevant, that is, a choice of the coupling constants yields a nontrivial fixed point, it provides a candidate SCFT for conformal sequestering.
However, the simple addition of the mass term $m\Phi^2$ does not bring it back to the SUSY-breaking model as discussed in Appendix A. To retain broken SUSY, we further introduce another singlet $Z$ and add superpotential terms

$$\Delta W = MZ(Y - m),$$

where $M$ denotes a mass scale near $\Lambda_{CFT}$.

Under this deformation, the conformal sequestering and the subsequent dynamical SUSY breaking go as follows: The introduction of the singlet $Z$ without the superpotential would be accompanied by a new conserved $U(1)$ current due to the rotation of $Z$. However, we expect that the interaction $MZY$ will lead to a new CFT point, breaking this $U(1)$ symmetry and realizing conformal sequestering of all the hidden fields appearing in the action. The tadpole term $-mMZ$ then serves as a relevant deformation, which cannot have a nontrivial fixed point, and eventually yields a mass of order $\lambda m$ to $\Phi$. Once the $\Phi$ field becomes massive, the low-energy dynamics is described by the IYIT model and the dynamical SUSY breaking at low energy is achieved.\(^8\)

## 4 Models with no imposed flavor symmetry

Now that we have eliminated the problematic $U(1)$ symmetries of the SCFT, we turn to consider the non-abelian symmetry thereof. For example, the models that exemplified sequestering in the previous section require the $SU(4)$ symmetry for the hidden sector. The “naturalness” problem of such non-abelian symmetry for conformal sequestering is emphasized in Ref. \[^5\]. In this section, we provide a concrete example of conformally sequestered SUSY-breaking model with no global symmetry (except $U(1)_R$) in the hidden sector.

Let us construct a conformally sequestered SUSY-breaking model with no global symmetry step by step, following the procedure exposed in the preceding sections.

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\(^8\)This Yukawa-type deformation possibly has an intimate relationship with the gauge-type one. An example to indicate this is given in Appendix B.
• SUSY-breaking sector

SUSY-breaking sector is provided by a so-called non-calculable model \[10\] of the $SO(10)$ gauge theory with one chiral superfield $\psi$ in the 16-dimensional spinor representation. Non-calculable models have no classical flat direction and consequently the SUSY is expected to be broken dynamically: the 't Hooft anomaly-matching condition seems difficult to be satisfied when we have no calculable description of the low-energy dynamics at the classical level. The above model is unique in the point that the chiral content for SUSY-breaking consists of a single multiplet.\(^9\)

• Conformal extension

In order to attain the conformal sequestering, we add vector-like multiplets to make the model conformal. There are several choices of additional matter multiplets. One possibility is to add many \(10\)'s ($H_i$ for $i = 1, \ldots, N_f$). Then the theory (without superpotential terms) is expected to flow to a nontrivial conformal fixed point for $7 \leq N_f \leq 21 \[16\].\(^{10}\)

The $\beta$ function for the gauge coupling $\alpha = g^2/4\pi$ is given by

$$
\beta_\alpha = -\alpha^2 \frac{3 \times (10 - 2) - N_f(1 - \gamma_{10}) - 2(1 - \gamma_{16})}{2\pi - 8\alpha},
$$

where $\gamma_r$ is the anomalous dimension of the chiral superfield in the $r$-dimensional representation.

The model has $SU(N_f) \times U(1)$ symmetry along with the conformal $U(1)_R$ symmetry, whose charge $R$ of a chiral operator with naive dimension one is related to its anomalous dimension $\gamma$ at the fixed point by the formula $R = \frac{2}{3}(1 + \frac{2}{3}) \ [2]$. The $R$ charges can be obtained by the $a$-maximization procedure \[14\]: through maximizing the $a$-function

$$
a(R) = \sum [3(R - 1)^3 - (R - 1)]
= 16[3(R_{16} - 1)^3 - (R_{16} - 1)] + 10N_f[3(R_{10} - 1)^3 - (R_{10} - 1)]
$$

\(^9\)As far as we know, the supersymmetric $SU(2)$ gauge theory with a chiral multiplet of the five-dimensional representation \[15\] gives the other candidate with this property. Unfortunately, we do not have a conformal extension of that example.

\(^{10}\)As can be seen from table \[2\] $a$-maximization implies an enhanced $U(1)$ symmetry (see Ref. \[17\]) in the IR fixed point for $7 \leq N_f \leq 9$, which should remedy the unitarity-violating charge assignment. This class of models has been investigated recently \[18\]. Hereafter we restrict ourselves to $N_f \geq 10$. 

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under the condition that the $\beta$ function Eq.(6) vanishes, we obtain

$$R_{10} = \frac{-15 - 24N_f + 3N_f^2 + \sqrt{2885 - N_f^2}}{3(-5 + N_f^2)}.$$  \hspace{1cm} (8)

The numerical results are summarized in table 2. The above conformal extensions will reduce to the SUSY-breaking model after introducing the mass terms for the 10’s.

- **Breaking of unwanted $U(1)$**

The additional matter causes new conserved $U(1)$ currents. We should eliminate the unwanted $U(1)$ symmetries so that the conformal sequestering occurs.

  i) **gauge couplings**

As discussed in section 3, one way to do this is to gauge the flavor symmetry, which leads to the anomalous breaking of the $U(1)$ symmetry. Let us show how it works in our present setup by considering $SO(10)$ gauge theory with fourteen 10’s and sixteen 10’s in turn.

For example, we gauge the flavor symmetry by $SO(7) \times SO(7)$ gauge group with the matter contents summarized in table 3.\(^{11}\) Now we argue that the resultant theory flows to a new conformal theory in the IR. The perturbation by $SO(7) \times SO(7)$ interaction yields a relevant deformation of the original SCFT, which is expected to flow to a new fixed point in the IR. The conformal $R$ charges of this gauged theory are obtained as

$$R_{10} = R_{16} = \frac{1}{2}.$$  \hspace{1cm} (9)

\(^{11}\)As a gauged flavor symmetry, $SP$ would forbid the mass term for 10’s and $SU$ would result in unwanted vector-like $U(1)$ symmetries.

| $N_{10}$ | $N_{16}$ | $R_{10}$ | $R_{16}$ | $R_{54}$ |
|---------|---------|---------|---------|---------|
| 7       | 1       | 0.131   | 0.0415  | —       |
| 8       | 1       | 0.215   | 0.140   | —       |
| 9       | 1       | 0.285   | 0.2175  | —       |
| 10      | 1       | 0.343   | 0.285   | —       |
| 14      | 1       | 0.504   | 0.472   | —       |
| 16      | 1       | 0.558   | 0.536   | —       |
| 1       | 1       | 0.514   | 0.483   | 0.377   |

Table 2: The $R$ charge assignments which are determined from the $a$-maximization procedure. $N_r$ denotes the number of the chiral superfields in the $r$-dimensional representation.
(10)

ψ

spinor(16) 1 1

\( H_1 \)
\( \Box_{N_f/2} \) 1

\( H_2 \)
\( \Box_{N_f/2} \) 1

S
\( \Box_{54} \) 1 1

Table 3: The matter contents of the \( SO(10) \times SO(N_f/2) \times SO(N_f/2) \) model with \( \psi \) and \( H \)'s. The symmetric traceless representation \( S(54) \) is written here for later use.

corresponding to \( \gamma_{10} = \gamma_{16} = -\frac{1}{2} \).

The sequestering matrix near the fixed point is determined by first-order deferential equations for \( \Delta \ln Z_i \equiv \ln Z_i + \gamma_i^* t \) because the FCNC causing interaction in Eq. (11) is regarded as the initial value of \( \Delta \ln Z_i \supset c_{ab}^{\alpha^*} q_a^\dagger q_b \). Under the renormalization convention given in Ref. [4], they are given by

\[
\frac{d}{dt} \Delta \ln Z_i = -\sum_a \left( \frac{\partial \gamma_i}{\partial \alpha_a} \right) \Delta \alpha_a = \sum_k L_{ik} \Delta \ln Z_k,
\]

where

\[
\Delta \alpha_{SO(10)} = \frac{\alpha_{SO(10)}^*}{2\pi - 8\alpha_{SO(10)}^*} [N_f \Delta \ln Z_{10} + 2\Delta \ln Z_{16}],
\]

\[
\Delta \alpha_{SO(7)} = \frac{\alpha_{SO(7)}^*}{2\pi - 5\alpha_{SO(7)}^*} [N'_f \Delta \ln Z_{10}],
\]

with \( N_f = 14 \) and \( N'_f = 10 \) in this particular model. We expect that the sequestering matrix is of the same order in magnitude as the anomalous dimensions of order one to induce sufficient sequestering.\(^{12}\)

Another example is the \( SO(8) \times SO(8) \) gauge interaction with \( R_{10} = \frac{2}{5} \), \( R_{16} = \frac{9}{5} \). The anomalous dimensions are relatively large and considerably away from the original fixed point with a \( U(1) \) conserved current. We expect that this model yields large sequestering effects, that is, a sensible example of conformal sequestering with no imposed non-abelian flavor symmetry.\(^{13}\)

\(^{12}\)However, there is one subtlety here. Although the added gauge interaction becomes strong in the IR, the new conformal fixed point might be too close to the unganged conformal fixed point as could be inferred from the conformal \( R \) charge assignment in table 2. Thus, the conformal sequestering might be insufficient in this case. Owing to this possibility, the next example seems preferable.

\(^{13}\)The above models are governed by the strong dynamics. We also present a weakly-coupled toy model as a perturbative example of conformal sequestering in Appendix C.
After adding mass terms for $H_i$,

$$W_{\text{mass}} = mH_iH_i,$$  \hfill (12)

these models reduce to the non-calculable SUSY-breaking model discussed above. The distinguishing feature of these models is that there is no non-abelian flavor symmetry at the fixed point. Thus, we do not need to impose *ad hoc* flavor symmetry in the hidden sector from the beginning.

**ii) Yukawa couplings**

As discussed in the previous section, we can break the unwanted $U(1)$ symmetry associated with the rotation of $H_i$ by adding superpotential terms. For example, we add the following terms to the conformally extended non-calculable SUSY-breaking model:

$$W = \sum_{i=1}^{N_f} \lambda_i Y_i H_iH_i + \lambda_n Y_i^n, \hfill (13)$$

together with a deformation

$$\Delta W = \sum_{i=1}^{N_f} M_i Z_i(Y_i - m_i). \hfill (14)$$

Here $Y_i$ and $Z_i$ are additional singlets. Under the assumption of conformality, the anomalous dimensions of the fields are given by

$$\gamma_{Y_i} = -2 + \frac{6}{n}, \quad \gamma_{10} = 1 - \frac{3}{n}, \quad \gamma_{16} = -11 + \frac{3N_f}{2n}. \hfill (15)$$

A suitable choice of $N_f$ and $n$ might lead to a conformally sequestered model.

More simply, we can introduce a field in a higher representation to achieve conformality. For example, let us add one symmetric traceless representation $\Sigma(54)$ (instead of $N_f$ fundamentals) to the non-calculable SUSY-breaking model. Since its Dynkin index amounts to 12, it is equivalent to $N_f = 12$ fundamentals in the $\beta$ function of the gauge coupling

$$\beta_\alpha = -\alpha^2 \frac{3 \times (10 - 2) - 2(1 - \gamma_{16}) - 12(1 - \gamma_{54})}{2\pi - 8\alpha}. \hfill (16)$$

Under the superpotential

$$W = \lambda_Y Y\Sigma\Sigma + \lambda_n Y^n \hfill (17)$$
to break the unwanted $U(1)$ symmetry, we expect to have a SCFT with anomalous dimensions\(^\text{14}\)

\[
\gamma_Y = -2 + \frac{6}{n}, \quad \gamma_{54} = 1 - \frac{3}{n}, \quad \gamma_{16} = -11 + \frac{18}{n}. \tag{18}
\]

If the theory flows to a SCFT, we obtain a model of conformal sequestering. Then the sequestering matrix is given by

\[
\frac{d}{dt}\Delta \ln Z_i = -\sum_a \left( \frac{\partial \gamma_i}{\partial \alpha_a} \right) \Delta \alpha_a = \sum_k L_{ik} \Delta \ln Z_k, \tag{19}
\]

where $\alpha_\lambda = |\lambda|^2/4\pi$ and $\alpha_n = |\lambda_n|^2/4\pi$ with

\[
\Delta \alpha_{SO(10)} = \frac{\alpha_{SO(10)}^*}{2\pi - 8\alpha_{SO(10)}^*} [12\Delta \ln Z_{54} + 2\Delta \ln Z_{16}],
\]

\[
\Delta \alpha_\lambda = -\alpha_\lambda^* (2\Delta \ln Z_{54} + \Delta \ln Z_Y),
\]

\[
\Delta \alpha_n = -\alpha_n^* n \Delta \ln Z_Y. \tag{20}
\]

We again note that the sequestering matrix $L_{ik}$ is given by the Hessian of the renormalization group flow $\partial_{g_k \beta_i}|_*$. The deformation

\[
\Delta W = MZ(Y - m) \tag{21}
\]

makes low-energy physics effectively governed by the non-calculable SUSY-breaking model.

\section{Model with no $U(1)$ symmetry enhancement}

As promised in section 2, we now try to go beyond the models with no global symmetry constructed in the previous section. Namely, we propose to utilize a SUSY-breaking model with no anomalous $U(1)$ symmetry. Then we encounter no enhanced $U(1)$ symmetry even when we add vector-like matter to turn the model into a SCFT, which enables us to present a possibly simplest model for conformal sequestering.

Our starting point is a calculable variant to the non-calculable SUSY breaking model of the $SO(10)$ gauge theory with one $10$ representation $H$ and one $16$ representation $\psi$ \[\text{I}\]. The superpotential is given by

\[
W_{\text{tree}} = \lambda \psi \psi H + \frac{1}{2} MH^2, \tag{22}
\]

\(^{14}\)Corresponding elementary fields have positive dimensions for $3/2 \leq n \leq 2$. 

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where $M$ denotes a mass scale. The limit $M \to \infty$ corresponds to the original non-calculable model with only $\psi$. Owing to the superpotential (two couplings: $\lambda_\psi$ and $M$ for two multiplets: $\psi$ and $H$), we have no (anomalous) $U(1)$ symmetry in this SUSY-breaking model.

Now we add a 54-dimensional chiral multiplet $\Sigma$ to make the theory conformal.\footnote{We expect that the same model \textit{without} the superpotential also flows to a SCFT in the IR by calculating the conformal $R$ charge through the $a$-maximization procedure shown as the last row in table 2. The value is significantly different from the model \textit{with} the superpotential, so the $U(1)$ breaking seems large as needed.} Note that we do not require any additional gauge interactions nor superpotential terms which would break $U(1)$ rotation of $\Sigma$. All the $U(1)$ currents are already broken either by the superpotential terms in Eq.\footnote{2} or by the anomaly due to the $SO(10)$ gauge interaction. The unique possible conformal $U(1)_{R}$ assignment is given by

$$R_{10} = 1, \quad R_{16} = \frac{1}{2}, \quad R_{54} = \frac{5}{12}, \quad (23)$$

which is consistent with the unitarity of the gauge invariant operators.

If the total theory indeed flows to a nontrivial fixed point, the conformal sequestering takes place since the model contains no conserved $U(1)$ charge (except for $U(1)_{R}$) even at the fixed point. After adding a mass term for $\Sigma$, \textit{i.e.} $W_{\text{mass}} = m\Sigma \Sigma$, the low-energy physics is effectively described by the above SUSY-breaking model.

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**A SUSY recovery**

In section 3, we have discussed a way to break the unwanted $U(1)$ symmetry by means of superpotential interactions. For instance, we can add tree-level Yukawa couplings to
break the $U(1)$ symmetry in the extended IYIT model with $SP(N_c)$ gauge group

$$W_{\text{tree}} = \lambda S_{ij} Q^i Q^j - M_S^2 S_{ij} J^{ij} + \lambda_Y Y \Phi^2 - \lambda_n Y^n + m \Phi^2;$$

(24)

where $J^{ij} = i \sigma_2 \otimes 1_{N_c+1}$ is the symplectic form. The second term shifts the origin of the meson superfield $M^{ij} = Q^i Q^j$ and serves as a regularization as we will see.

For $m = M_S = 0$, the theory possibly flows to a nontrivial conformal fixed point for $g_{SP(3)}, \lambda, \lambda_Y$, and $\lambda_n$. Since all the interactions are strong for a suitable choice of $n$, the sequestering matrix is expected to be order one. Unfortunately, however, SUSY is not broken in this example. In this appendix, we confirm that the theory recovers SUSY as the existence of Higgs vacua.

To see this, we utilize the glueball superpotential technique (see Ref. [19, 20]). First of all, let us assume a supersymmetric vacuum and write down the Konishi anomaly equation [21]

$$\bar{D}^2 J = \phi_j \frac{\partial W_{\text{tree}}}{\partial \phi_i} + \frac{T(r_i)}{32 \pi^2} \text{Tr} W^2 \delta_{ij},$$

(25)

where $T(r_i)$ is the Dynkin index of $i$-th matter, evaluated at such a hypothetical vacuum:

$$2 \lambda \langle S_{ij} M^{kj} \rangle = \delta^k_i S,$$

$$\lambda \langle S_{ij} M^{kl} \rangle - M_S^2 J^{kl} \langle S_{ij} \rangle = 0,$$

$$m \langle \Phi^2 \rangle + \lambda_Y \langle Y \Phi^2 \rangle = S,$$

$$\lambda_Y \langle \Phi^2 \rangle - n \lambda_n \langle Y^{n-1} \rangle = 0,$$

(26)

where $S = -\frac{1}{32 \pi^2} \langle \text{Tr} W^2 \rangle$ is the so-called glueball superfield. When $M_S$ is 0, the supersymmetric vacuum exists if and only if $S = 0$.

For nonzero $M_S$, we can solve these equations to derive the vacuum expectation values of matter superfields in terms of $S$. The solutions have two branches, one of which corresponds to the classical unHiggsed branch:

$$\langle M^{ij} \rangle = \frac{M_S^2}{\lambda} J^{ij},$$

$$\langle S_{ij} \rangle = \frac{1}{2 M_S^2} (J^{-1})_{ij},$$

$$\langle \Phi^2 \rangle = \frac{S}{m} + \mathcal{O}(S^2),$$

$$\langle \Phi^2 \rangle = \frac{S}{m} + \mathcal{O}(S^2),$$
\[ \langle Y^{n-1} \rangle = \frac{\lambda_Y S}{n \lambda_n m} + O(S^2). \] (27)

The other one corresponds to the Higgsed branch:

\[
\begin{align*}
\langle M^{ij} \rangle &= \frac{M_S^2}{\lambda} J^{ij}, \\
\langle S_{ij} \rangle &= \frac{1}{2 M_S^2} (J^{-1})_{ij}, \\
\langle \Phi^2 \rangle &= \frac{n \lambda_n}{\lambda_Y} \left( -\frac{m}{\lambda_Y} \right)^{n-1} - \frac{n-1}{m} S + O(S^2), \\
\langle Y \rangle &= -\frac{m}{\lambda_Y} + \frac{1}{n \lambda_n} \left( -\frac{\lambda_Y}{m} \right)^{n-1} S + O(S^2).
\end{align*}
\] (28)

Now we integrate each set of equations to obtain the effective glueball superpotential up to an integration constant \( C(S) \).

In the unHiggsed branch, we obtain

\[
W_{\text{eff}}(S) = (N_c + 1) S \ln \frac{\Lambda^2 \lambda}{M_S^2} + S \ln \frac{m}{\Lambda} + O(S^{n-1}),
\] (29)

where \( \Lambda \) denotes the dynamical scale of the gauge interaction and \( C(S) \) is determined from the method explained in Ref. \([20]\). The superpotential is singular in the \( M_S \to 0 \) limit unless \( S = 0 \). However, for \( S = 0 \), the \( F \)-term condition is not satisfied to imply broken SUSY. This is consistent with the fact that the low-energy dynamics in this branch are nothing but those of the IYIT model.

On the other hand, in the Higgsed branch, the effective glueball superpotential takes a form

\[
W_{\text{eff}}(S) = (N_c + 1) S \ln \frac{\Lambda^2 \lambda}{M_S^2} - \lambda_n \left( -\frac{m}{\lambda_Y} \right)^n + S \left( \ln \frac{\lambda_Y^n S}{m^{n-1} \lambda_n \Lambda} - 1 \right) + O(S^2).
\] (30)

The extremization condition of the glueball superpotential is given by

\[
\frac{\lambda_Y^n S}{\lambda_n m^{n-1} \Lambda} = \left( \frac{M_S^2}{\Lambda^2 \lambda} \right)^{N_c+1},
\] (31)

and this gives a solution \( S \to 0 \) as \( M_S \to 0 \). Therefore, there exists a supersymmetric vacuum for this branch. This means that the model shows a recovery of SUSY due to the \( U(1) \) breaking interaction.

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We can use a different relevant deformation to obtain a SUSY-breaking model in IR, avoiding problematic Higgs vacua. Namely, instead of the mass term $m\Phi^2$, we introduce the following superpotential terms:

$$\Delta W = MZ(Y - m).$$  \hspace{1cm} (32)

Similarly the effective glueball superpotential is given by

$$W_{\text{eff}}(S) = (N_c + 1)S\ln \frac{\Lambda^2\lambda}{M^2_S} - \lambda_n m^n + S\ln \frac{\lambda_Y m}{\Lambda}$$  \hspace{1cm} (33)

to imply dynamically broken SUSY.\(^{16}\)

**B  From gauge to Yukawa**

In section 3, we have discussed two possible ways to strongly break unwanted $U(1)$ symmetry which hinders the conformal sequestering: one is to break it anomalously by gauge interaction and the other is to break it explicitly by superpotential interaction. The latter Yukawa-type deformation possibly has an intimate relationship with the former gauge-type one. In this Appendix, we give an example to indicate such an interplay.

**B.1  effective theory**

Let us consider the following example. The extended IYIT model with $SP(3)$ has an unwanted $U(1)$ symmetry which rotates $S_{ij}$, $Q^i$, and $Q'^j$ simultaneously. To attain the conformal sequestering, we have introduced $SP(1) \times SP(1)$ gauge symmetry which breaks the $U(1)$ rotation of $Q'^j$ by anomaly of $SP(1) \times SP(1)$ gauge interaction in Ref. \[8\]. The new theory with the $SP(1) \times SP(1)$ gauging also flows to a conformal field theory in the IR, ensuring the conformal sequestering of the gauged theory.

Suppose $g_{SP(3)} \ll g_{SP(1)}$ at an intermediate scale between $\Lambda_{UV}$ and $\Lambda_{CFT}$. Then we can solve the strong dynamics of the $SP(1) \times SP(1)$ theory first and consider an effective $SP(3)$ gauge theory. This effective $SP(3)$ gauge theory has the following matter contents: 8 fundamental representations $Q^i$, 28 singlets $S^{ij}$, 2 antisymmetric (14-dimensional)

\(^{16}\)In the broken SUSY case, the effective glueball superpotential here does not necessarily yield a good low-energy description of the model. An attempt to construct an effective action in a similar SUSY-breaking model can be found in Ref. \[22\].
representations $A_{ab}$ and 2 additional singlets $B$. The $A_{ab}$ and $B$ are composite meson superfields of the $Q'$ in the strong $SP(1) \times SP(1)$ dynamics:

$$Q'_a Q'_b \sim A_{ab} + J_{ab} B,$$  \hspace{1cm} (34)

where $J_{ab}$ denotes the symplectic form.

The induced effective $SP(3)$ gauge theory has the following effective superpotential:

$$W_{eff} = \lambda_1 \left( A_{ab}^{(1)} A_{cd}^{(1)} A_{ef}^{(1)} \epsilon^{abcdef} + 3B^{(1)} A_{ab}^{(1)} A_{cd}^{(1)} J_{ef} \epsilon^{abcdef} + 24(B^{(1)})^3 \right)$$

$$+ \lambda_2 \left( A_{ab}^{(2)} A_{cd}^{(2)} A_{ef}^{(2)} \epsilon^{abcdef} + 3B^{(2)} A_{ab}^{(2)} A_{cd}^{(2)} J_{ef} \epsilon^{abcdef} + 24(B^{(2)})^3 \right).$$  \hspace{1cm} (35)

This effective potential comes from the effective superpotential of the $SP(1)$ gauge theory such as $\text{Pf}(Q'_a Q'_b)$. The $SP(3)$ dynamics together with this superpotential lead to the same IR dynamics as those of the $SP(3) \times SP(1)^2$ gauge theory. The unwanted $U(1)$ symmetry which would be given by a rotation of $A$ and $B$ is broken explicitly by the superpotential terms.

Let us perform a simple consistency check: if $W_{eff}$ possesses a nontrivial IR fixed point, the conformal $R$ charge of $A$ and $B$ is given by

$$R_A = R_B = \frac{2}{3},$$  \hspace{1cm} (36)

which indeed agrees with the direct $SP(3) \times SP(1) \times SP(1)$ computation: $\gamma_{Q'} = -1$ in terms of Eq.(35). The vanishing of the $\beta$ function for the gauge coupling then shows that $\gamma_Q = -1$ also in agreement with the $SP(3) \times SP(1) \times SP(1)$ results.

### B.2 SUSY breaking

We have seen that the anomalous breaking of the $U(1)$ symmetry by gauging and the explicit breaking by the superpotential are dual descriptions of the same physics in the above example. One potential subtlety is that the phase of SUSY after adding a mass term for $Q'$, or equivalently the addition of tadpole $m^2 B$ in the superpotential. From the former viewpoint, dynamical SUSY breaking of the model is obvious due to the decoupling argument. However it could appear subtle in the dual Yukawa viewpoint. In this subsection, we explain SUSY breaking in the composite model with a particular Yukawa interaction.
We consider the $SP(3)$ gauge theory with IYIT matter sector coupled to an extra antisymmetric-tensor (14) representation $A_{ab}$ and a singlet $B$. The superpotential generically takes a form

$$W = \lambda S_{ij} Q^i Q^j - M_S^2 S_{ij} J^{ij} - m^2 B + \alpha B^3 + \beta B A_{ab} A_{cd} J_{ef} \epsilon^{abdef} + \gamma A_{ab} A_{cd} A_{ef} \epsilon^{abdef}. \quad (37)$$

The second term was introduced as a regularization as in Appendix A. In the following, we argue that when $\alpha, \beta, \gamma$ satisfy a specific relation $\alpha = \frac{\beta}{3} = \frac{\gamma}{24} = c$ inferred from Eq.(35), this model causes dynamical SUSY breaking.\(^{17}\)

Let us show this by means of the glueball superpotential technique as in Appendix A. First we assume a supersymmetric vacuum and write down the Konishi anomaly equation evaluated at such a hypothetical vacuum:

$$2\lambda \langle S_{ij} M^{kj} \rangle = \delta_i^k S$$

$$\lambda \langle S_{ij} M^{kl} \rangle - M_S^2 J^{kl} \langle S_{ij} \rangle = 0$$

$$2\langle B A_{ab} A_{cd} J_{ef} \epsilon^{abdef} \rangle + \langle A_{ab} A_{cd} A_{ef} \epsilon^{abdef} \rangle = \frac{4S}{3c}$$

$$\langle 24B^2 \rangle + 3 \langle A_{ab} A_{cd} J_{ef} \epsilon^{abdef} \rangle = \frac{m^2}{3c}. \quad (38)$$

We note that the supersymmetric vacuum exists if and only if $S = 0$ for $M_S = 0$. For $M_S \neq 0$, we can solve Eq.(38) perturbatively around $S = 0$, which results in

$$\langle M^{ij} \rangle = \frac{M_S^2}{\lambda} J^{ij}$$

$$\langle S_{ij} \rangle = \frac{S}{2M_S^2} (J^{-1})_{ij}$$

$$\langle A_{ab} A_{cd} J_{ef} \epsilon^{abdef} \rangle = \mathcal{O}(\frac{S}{\sqrt{cm^2}})$$

$$\langle B \rangle = \pm \sqrt{\frac{m^2}{12c}} + \mathcal{O}(S). \quad (39)$$

Actually, Eq. (38) have several solutions, two of which survive in the $S \to 0$ limit. The other solutions run away to infinity of $S_{ij}$ for $M_S \to 0$.\(^{18}\)

\(^{17}\)The condition of SUSY breaking is actually equivalent to the condition that this model can be derived by integrating out the strongly coupled $SP(1)$ gauge interaction as in the previous subsection.

\(^{18}\)If we adopted a general cubic potential Eq.(37) instead of the choice in Eq.(38), the other solutions would stay in the classical Higgs branch and eventually lead to the recovery of SUSY.
Now we integrate the above set of equations to obtain the effective glueball superpotential to the leading order in $S$:

$$W_{\text{eff}}(S) = \mp \frac{m^3}{9\sqrt{2}C} + c_1 S + (N_c + 1) S \ln \frac{\Lambda^2}{M_S^2} + \mathcal{O}(S^2),$$

(40)

where $c_1 = \ln(m^{12}/e^4\Lambda^{12})$. Owing to the $S \ln(\Lambda^2/M_S^2)$ term, the supersymmetric vacuum would be given by $S = 0$ in the $M_S \to 0$ limit if any. However, then, the $F$-term condition would not be satisfied in Eq. (40) to imply dynamical SUSY breaking in this model.

### C Perturbative example

In section 3, we have considered nonperturbative models for conformal sequestering. In this Appendix, we provide a perturbative example of the $SO(6) \times SO(6) \times SO(6)$ gauging of the conformally extended $SO(10)$ non-calculable models with bi-fundamentals transforming as $(10, 6_i)$ for $i = 1, 2, 3$. The conformal $R$ charges are determined to be $R_{10} = R_{16} = \frac{3}{5}$, which is close to the free field value $R = \frac{2}{3}$ and suggests a possibility of perturbative computation.

At the one-loop level, the anomalous dimension of a matter indexed by $i$ in a gauge theory is given by a formula

$$\gamma_i(\alpha) = -\frac{\alpha}{\pi} C_2(r_i) + \mathcal{O}(\alpha^2); \quad C_2(r) = \frac{|G|}{|r|} T(r),$$

(41)

where $|G|$ and $|r|$ denote the dimensions of the group and the representation, respectively, and $T(r)$ is the Dynkin index.\(^{19}\)

In this model, one-loop calculation shows

$$\gamma_{10} = -\frac{9}{2\pi} \alpha_{SO(10)} - \frac{5}{2\pi} \alpha_{SO(6)},$$

$$\gamma_{16} = -\frac{45}{8\pi} \alpha_{SO(10)}.\hspace{1cm}(42)$$

Then, the vanishing of the $\beta$ functions

$$\beta_{\alpha_{SO(10)}} = -\alpha_{SO(10)}^2 \frac{3 \times (10 - 2) - 2(1 - \gamma_{16}) - 18(1 - \gamma_{10})}{2\pi - 8\alpha_{SO(10)}},$$

\(^{19}\)For the gauge group $SO(2N)$, $T(\text{fundamental}) = 1$ and $T(\text{spinor}) = 2^{N-4}$.\)
\[ \beta_{\text{SO}(6)} = -\alpha_{\text{SO}(6)}^2 \frac{3 \times (6 - 2) - 10(1 - \gamma_{10})}{2\pi - 4\alpha_{\text{SO}(6)}} \]  

(43)

determines $\alpha_{\text{SO}(10)}^* = 8\pi/225$ and $\alpha_{\text{SO}(6)}^* = 2\pi/125$, which suggests that the one-loop approximation is not so bad in the sense of Ref. [1].

The sequestering matrix is given by the Hessian $\partial_{\alpha_i} \partial_{\alpha_k} |_{g=g^*}$. Its eigenvalues are evaluated as $(0.0004, 0.07)$, and its smallest value amounts to the efficiency of the conformal sequestering. Although we see that the perturbatively obtained conformal sequestering is too small for phenomenological applications, we have presented one example for definiteness.

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