TREATING THE LIFETIMES OF CHARM AND BEAUTY HADRONS WITH QCD PLUS A BIT MORE! a

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The heavy quark expansion implemented through an operator product expansion provides us with a treatment of inclusive decays of beauty and charm hadrons that is genuinely derived from QCD, though it requires one additional assumption, namely that of 'local' quark-hadron duality. Subtleties in the application of factorization to hadronic expectation values are pointed out. The observed pattern in the charm lifetime ratios is reproduced in a semi-quantitative manner. The ratio \( \tau(\Lambda_b)/\tau(B_d) \) cannot be pushed significantly below 0.9 – unless one invokes a new hitherto unknown paradigm for evaluating baryonic matrix elements. One confidently predicts \( \tau(B^-) \) to exceed \( \tau(B_d) \) by several percent only. Failure of those predictions would force us to pay a hefty theoretical price, namely ultimately to abandon local duality as a practical concept.

The notion that the weak lifetimes of beauty and charm hadrons have to be measured accurately will hardly be challenged. For on the one hand the decay widths constitute a defining property of hadrons; on the other hand one has to know their size to translate the semileptonic branching ratio into a semileptonic width from which one can extract the KM parameters etc.; lastly a precise recording of the lifetime evolution is essential in probing \( B^0 - \bar{B}^0 \) oscillations as described by \( \Delta m \) and \( \Delta \Gamma \). The answer to "How well do we need to understand the lifetimes theoretically?" is however less obvious. For one can measure total widths accurately without theoretical input. Furthermore no apparent qualitative disaster has occurred since the expected pattern has indeed been observed, namely that the relative lifetime differences among beauty hadrons are smaller than among charm hadrons. Finally one can recite several reasons why a theoretical treatment of nonleptonic widths could – or even should – fail on a quantitative level. I, however, view the task of describing lifetimes of heavy flavour hadrons as a no-lose situation. For the general

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concept of quark-hadron duality implies that total decay widths represent the ‘safest’ quantity theoretically after semileptonic widths. Once one has developed a description that is genuinely based on QCD, then even a failure of it will teach us a valuable – albeit disappointing – lesson on QCD, namely on quantitative limitations for the concept of duality. In Sect.1 I sketch the relevant methodology of the heavy quark expansion; in Sect.2 and 3 I compare the predictions on the widths of charm and beauty hadrons, respectively, with the available data before presenting a summary in Sect.4.

1 Methodology of the Heavy Quark Expansion

The experimental findings in 1979 that the semileptonic branching ratio of \( D^+ \) mesons is much higher than that of \( D^0 \) mesons and that therefore the \( D^+ \) is much longer lived than the \( D^0 \) caused quite a stir in the community since it ran counter to some strongly held convictions. These data enforced revisions in our descriptions that first took shape in the form of phenomenological models: the concepts of Pauli Interference (PI)\(^1\), Weak Annihilation (WA)\(^2\,3\), in meson and W Scattering (WS)\(^4\,5\,6\) in baryon decays, respectively, were born. The foundations for a truly theoretical description of heavy flavour decays were laid already in 1983 by Shifman and Voloshin\(^7\): they argued that an expansion in powers of \( 1/m_Q \) with \( m_Q \) being the heavy flavour quark mass can be performed for inclusive decay rates. The analysis involves a sequence of steps. In analogy to the treatment of \( e^+ e^- \rightarrow \text{hadrons} \) one first describes the transition rate into an inclusive final state \( f \) through the imaginary part of a forward scattering operator evaluated to second order in the weak interactions\(^7\,8\,9\):

\[
\hat{T}(Q \rightarrow f \rightarrow Q) = i \text{Im} \int d^4x \{ \mathcal{L}_W(x) \mathcal{L}^\dagger_W(0) \} \tag{1}
\]

\( \{,\}_T \) denotes the time ordered product and \( \mathcal{L}_W \) the relevant effective weak Lagrangian expressed on the parton level. If the energy released in the decay is sufficiently large one can express the non-local operator product in eq.(1) as an infinite sum of local operators \( O_i \) of increasing dimension with coefficients \( \tilde{c}_i \) containing higher and higher powers of \( 1/m_Q \). The width for \( H_Q \rightarrow f \) is then obtained by taking the expectation value of \( \hat{T} \) between the state \( H_Q \):

\[
\langle H_Q | \hat{T}(Q \rightarrow f \rightarrow Q) | H_Q \rangle \propto \Gamma(H_Q \rightarrow f) = G_F^2 | K M |^2 \sum_i \tilde{c}_i^{(f)}(\mu) \langle H_Q | O_i | H_Q \rangle(\mu) \tag{2}
\]

\(^b\)It should be kept in mind, though, that it is primarily the energy release rather than \( m_Q \) that controls the expansion.
The c number coefficients $c_i^{(f)}(\mu)$ are determined by short-distance dynamics whereas long-distance dynamics controls the expectation values of the local operators $O_i$. Such a separation necessitates the introduction of an auxiliary scale with long distance $> \mu^{-1} >$ short distance. While this is a conceptually and often also practically important point I will not refer to it explicitly anymore in this article. The coefficients $c_i^{(f)}(\mu)$ depend on the KM parameters and the quark masses; in particular, they contain powers of $1/m_Q$ that increase with the dimension of the local operators $O_i$.

After a stumbling block concerning the $1/m_Q$ scaling in the presence of gluon radiation had been removed, this expansion was more fully performed in $8, 9$ with the following result:

$$\Gamma(H_Q \rightarrow f) = \frac{G_F^2 m_Q^3}{192\pi^3} |KM|^2 \left[ c_4^f \langle H_Q|\bar{Q}Q|H_Q \rangle + c_5^f \frac{\langle H_Q|\bar{q}\sigma \cdot GQ|H_Q \rangle}{m_Q^2} + \sum_i c_6^{f,i} \frac{\langle H_Q|\bar{q}\Gamma_i q|H_Q \rangle}{m_Q^3} + \mathcal{O}(1/m_Q^4) \right] \quad (3)$$

with $KM$ denoting the product of the KM parameters. As already stated, the quantities $c_i^f$ can be calculated within short-distance dynamics; furthermore the operators appearing on the right hand side of eq.(3) are known and their dimensions control the scaling in $1/m_Q$. Using the equations of motion one finds for the leading operator $\bar{Q}Q$:

$$\bar{Q}Q = \bar{Q}_{\gamma_0} Q - \frac{Q (i\bar{D})^2 - (i/2)\sigma \cdot GQ}{2m_Q^2} + g_s \bar{Q}_{\gamma_0} t^i q \sum_q \bar{q}_{\gamma_0} t^i q + \mathcal{O}(1/m_Q^4) \quad (4)$$

with the sum in the last term running over the light quarks $q$; the $t^i$ denote the colour $SU(3)$ generators. Total derivatives are ignored in this expansion since they do not contribute to the expectation values. Since $\bar{Q}_{\gamma_0} Q$ constitutes the Noether current for the heavy-flavour quantum number one has

$$\langle H_Q|\bar{Q}_{\gamma_0} Q|H_Q \rangle_{\text{norm}} = 1$$

leading to

$$\langle H_Q|\bar{Q}Q|H_Q \rangle_{\text{norm}} = 1 + \mathcal{O}(1/m_Q^2) \quad (5)$$

From eqs.(3) and (5) we read off two important general results:

Observables do not depend on $\mu$. Yet we have to choose $\Lambda_{QCD} \ll \mu \ll m_Q$ if we want to calculate perturbative as well as nonperturbative corrections in a self-consistent fashion.

Contributions of order $1/m_Q^3$ arise also from expanding $\langle H_Q|Q_{\sigma \cdot GQ}|H_Q \rangle/m_Q^2$; those are practically insensitive to the light quark flavours.

The relativistic normalization is used: $\langle H_Q|O_i|H_Q \rangle_{\text{norm}} \equiv \langle H_Q|O_i|H_Q \rangle/2M_{H_Q}$. 

3
• The naive spectator contribution $\Gamma_{\text{spect}}(H_Q) \propto G_F^2 m_Q^5$ emerges from $\langle H_Q|\bar{Q}Q|H_Q \rangle$ as the leading term for $m_Q \to \infty$.

• There is no pre-asymptotic correction of order $1/m_Q$! For the only locally gauge invariant operator of dimension-four – $\bar{Q}i\gamma_\mu D_\mu Q$, $D_\mu = \partial_\mu - ig A_\mu^a t^a$ – can be reduced to $m_Q \bar{Q}Q$ due to the equation of motion. This yields the general result that the leading non-perturbative corrections to beauty decays are of order $(\mu_{\text{had}}/m_b)^2 \sim O (1 \text{ GeV}/m_b^2) \sim \text{few }\%$, i.e., quite small. Among other things this implies that $B_c$ decays exhibit a short lifetime below 1 psec and that their decays are dominated by charm decays.

• Two dimension-five operators emerge, namely $\bar{Q}(i\vec{D})^2 Q$ and $\bar{Q}\sigma \cdot GQ$, which had been overlooked in the phenomenological approaches. The first one represents the square of the spatial momentum of the heavy quark $Q$ moving in the soft gluon background and thus describes its kinetic energy $f$. The second one constitutes the chromomagnetic operator.

• PI, WA and WS that had been anticipated in the phenomenological descriptions enter through $\langle H_Q|(\bar{Q}\Gamma_i q)(\bar{q}\Gamma_i Q)|H_Q \rangle$ in order $1/m_Q^3$. The formally leading contributions to WA are helicity suppressed.

These points can be summarized as follows:

$$\Gamma(H_Q) = \Gamma_{\text{decay}}(H_Q) + \Gamma_{PI,WA,WS}(H_Q) + O(1/m_Q^4)$$

$$\Gamma_{\text{decay}}(H_Q) = \Gamma_{\text{spect}}(H_Q) + O(1/m_Q^3) \quad (6)$$

$\Gamma_{\text{spect}}$ is universal for all hadrons of a given flavour, but $\Gamma_{\text{decay}}$ is not: $\Gamma_{\text{decay}}(P_Q) \neq \Gamma_{\text{decay}}(\Lambda_Q) \neq \Gamma_{\text{decay}}(\Omega_Q)$.

The mesonic matrix elements of the chromomagnetic operator can be extracted from the hyperfine splitting:

$$\langle \mu_G^2 \rangle_{\text{norm}} \equiv \langle P_Q|\bar{Q}\frac{i}{2}\sigma \cdot GQ|P_Q \rangle_{\text{norm}} \simeq \frac{3}{2} m_Q (M_{VQ} - M_{PQ}) \simeq \frac{3}{4} (M_{VQ}^2 - M_{PQ}^2)$$

(7a)

where $V_Q = B^*$, $D^*$ and $P_Q = B$, $D$ $\ddagger$. Thus

$$\langle \mu_G^2 \rangle_D \simeq 0.41 \text{ (GeV)}^2, \quad \langle \mu_G^2 \rangle_B \simeq 0.37 \text{ (GeV)}^2, \quad \frac{\langle \mu_G^2 \rangle_B}{m_c^2} \simeq 0.21, \quad \frac{\langle \mu_G^2 \rangle_D}{m_b^2} \simeq 0.016$$

(7b)

$\ddagger$Since it is not a Lorentz scalar, it cannot appear in eq.(3).

$\ddagger$We have also assumed here that the mass of the antiquark in the meson is light and can be neglected to this order: $m_Q \simeq (M_{VQ} + M_{PQ})/2$. For $B_c$ mesons one obviously has to go beyond this approximation.
A measure for the numerical reliability of the expansion is then provided by \( \sqrt{\langle \mu^2_G \rangle_D / m_c^2} \approx 0.46 \) and \( \sqrt{\langle \mu^2_G \rangle_B / m_b^2} \approx 0.13 \), respectively. This parameter is certainly small compared to unity for beauty decays; on the other hand a \( 1/m_c \) expansion is of uncertain numerical value.

The light di-quark system in \( \Lambda_Q \) and \( \Xi_Q \) baryons carries no spin; therefore

\[
\langle \Lambda_Q | \bar{Q} \sigma \cdot GQ | \Lambda_Q \rangle \approx 0 \equiv \langle \Xi_Q | \bar{Q} \sigma \cdot GQ | \Xi_Q \rangle (8)
\]

This operator thus generates width differences between mesons and baryons in order \( 1/m_Q^2 \), see eqs.(7b) vs. (8).

The value of \( \langle H_Q \rangle_{\Lambda Q} \) is not known accurately. An analysis based on QCD sum rules yields \( \langle H_Q \rangle_{\Lambda Q} \sim 0.5 \pm 0.1 \) (GeV) \(^2\) in agreement with a rigorous lower bound \(15\), \(16\), \(17\)

\[
\langle (\vec{p}_c)^2 \rangle_H \sim (0.015 \pm 0.030) \) (GeV) \(^2\)
\]

The differences in the mesonic and baryonic expectation values can be related to the ‘spin averaged’ meson and baryon masses:

\[
\langle (\vec{p}_c)^2 \rangle_{\Lambda Q} - \langle (\vec{p}_c)^2 \rangle_{\Lambda c} \approx \frac{2 m_b m_c}{m_b - m_c} \cdot \left( \langle M_D \rangle - m_c \right) - \left( \langle M_B \rangle - m_c \right) \]

Present data yield:

\[
\langle (\vec{p}_b)^2 \rangle_{\Lambda Q} - \langle (\vec{p}_b)^2 \rangle_{\Lambda c} = -(0.015 \pm 0.030) \) (GeV) \(^2\)
\]

i.e., no significant difference. In deriving eq.(10) it was assumed that the \( c \) quark can be treated as heavy; in that case \( \langle (\vec{p}_c)^2 \rangle_{\Lambda c} \approx \langle (\vec{p}_c)^2 \rangle_H \) holds.

The expectation values of the four-fermion operators are not reliably known. To estimate their size for mesons one usually invokes factorization:

\[
\langle H_Q(p) | (\bar{Q}L \gamma_\mu q_L) | (\bar{q}L \gamma_\nu Q_L) | H_Q(p) \rangle_{\text{norm}} \approx
\]

\[
\langle H_Q(p) | (\bar{Q}L \gamma_\mu q_L) | 0 \rangle_{\text{norm}} | (\bar{q}L \gamma_\nu Q_L) | H_Q(p) \rangle_{\text{norm}} = \frac{1}{8 M_{H_Q}} f_{H_Q}^2 p_{\mu} p_{\nu} \) (11a)
\]

\[
\langle H_Q(p) | (\bar{Q}L \gamma_\mu \lambda_q q_L) | (\bar{q}L \gamma_\nu \lambda_q Q_L) | H_Q(p) \rangle_{\text{norm}} \approx
\]

\[
\langle H_Q(p) | (\bar{Q}L \gamma_\mu \lambda q_L) | 0 \rangle_{\text{norm}} | (\bar{q}L \gamma_\nu \lambda Q_L) | H_Q(p) \rangle_{\text{norm}} = 0 \) (11b)
\]

However such an ansatz cannot be an identity. It can hold as an approximation, but only for certain scales. Invoking it at \( \sim m_Q \) does not make sense at all.

For as far as QCD is concerned, \( m_Q \) is a completely foreign quantity, only moderately less so than the mass of an elephant. A priori it has a chance to
hold at ordinary hadronic scales $\mu_{\text{had}} \sim 0.5 \div 1$ GeV: various theoretical analyses based on QCD sum rules, QCD lattice simulations, $1/N_C$ expansions etc. have indeed found it to apply in that regime. It would be inadequate conceptually as well as numerically to renormalize merely the decay constant: $f_Q(m_Q) \to f_Q(\mu_{\text{had}})$. Instead the full set of operators has to be evaluated at $\mu_{\text{had}}$. One proceeds in three steps (for details see [25]):

(A) Ultraviolet renormalization translates the weak Lagrangian defined at $M_W$, $\mathcal{L}_W(M_W)$, into one effective at $m_Q$, $\mathcal{L}_W(m_Q)$.

(B) All operators $O_i$ in eq.(2) undergo hybrid renormalization down to $\mu_{\text{had}}$.

(C) At scale $\mu_{\text{had}}$ one invokes factorization.

Some comments are in order to elucidate the situation that is not properly reflected in [20]:

- It has been known for more than 16 years now that the factorizable contributions to PI almost cancel – apparently for accidental reasons – at scales around $m_Q$ making the ratio of non-factorizable to factorizable contributions large and numerically unstable there.

- No such cancellation occurs around scales $\mu_{\text{had}}$ making factorizable contributions numerically stable and dominant over non-factorizable ones.

- Contributions that are factorizable (in colour space) at $\mu_{\text{had}}$ are mainly non-factorizable at $m_Q$.

- The role of non-factorizable terms has been addressed in the literature over the years, most explicitly and in a most detailed way in [21, 22].

The situation becomes much more complex for baryon decays. To order $1/m_Q^3$ there are several different ways in which the valence quarks of the baryon can be contracted with the quark fields in the four-quark operators; furthermore WS is not helicity suppressed and thus can make a sizeable contribution to lifetime differences; also the PI effects can now be constructive as well as destructive. Finally one cannot take recourse to factorisation as a limiting case. Thus there emerge three types of numerically significant mechanisms at this order in baryon decays – in contrast to meson decays where there is a single dominant source for lifetime differences – and their strength cannot be expressed in terms of a single observable like $f_{HQ}$. At present we do not know how to determine the relevant matrix elements in a model-independent way. The best available guidance and inspiration is to be derived from quark model calculations with their inherent uncertainties. This analysis had already been undertaken in the framework of phenomenological models [4, 5]. One thing
should be obvious already at this point: with terms of different signs and somewhat uncertain size contributing to differences among baryon lifetimes one has to take even semi-quantitative predictions with a grain of salt!

2 Lifetimes of Charm Hadrons – ‘A Painting in Broad Brush Strokes’

In discussing heavy quark expansions one should start with three caveats:

(i) Since the charm quark mass is not much larger than hadronic scales, the expansion parameter is uncomfortably large, though smaller than unity: \( \mu_{\text{had}}/m_c \sim 0.5 \).

(ii) By the same token the evaluation of hybrid renormalization that turns out to be quantitatively important is of uncertain numerical reliability.

(iii) Equating the observed semileptonic width of D mesons with the theoretical expression through order \( 1/m_c^2 \) yields \( m_c \simeq 1.6 \) GeV. The theoretically more reasonable value \( m_c \simeq 1.4 \) GeV reproduces only half of \( \Gamma_{\text{SL}}(D)|_{\text{exp}} \).

Terms of order \( 1/m_c^3 \) in \( \Gamma_{\text{SL}}(D)|_{\text{theor.}} \) do not seem to bridge the gap. This discrepancy can be interpreted as signaling that quark-hadron duality does not generally hold even in semileptonic charm decays. I will adopt the working hypothesis that it still applies – with reasonable accuracy – to the ratios of lifetimes and semileptonic branching ratios. With these caveats one can dare to make predictions on the charm lifetime ratios.

The dominant source for the \( D^+ - D^0 \) lifetime difference is destructive PI in nonleptonic \( D^+ \) decays with WA enhancing the \( D^0 \) width as a secondary effect. More specifically one finds:

\[
\Gamma(D^+) \simeq \Gamma_{\text{decay}}(D) + \Gamma_{\text{PI}}(D^+) \quad (12a)
\]

\[
\Gamma_{\text{PI}}(D^+) \simeq \Gamma_0 \cdot 24 \pi \frac{f_D^2}{m_c^4} \kappa^{-4} \left[ (c_+^2 - c_-^2) \kappa_2^2 + \frac{c_+^2 + c_-^2}{3} - \frac{1}{9} (\kappa_2^2 - 1)(c_+^2 - c_-^2) \right], \quad (12b)
\]

where \( \kappa \equiv \left[ \alpha_S(\mu_{\text{had}})/\alpha_S(m_c) \right]^{1/b} \), \( b = 11 - 2n_F/3 \) represents hybrid renormalization. Large cancellations no longer occur among the factorizable terms in Eqs.(12); their overall contribution is destructive and large. We then arrive at

\[
\frac{\tau(D^+)}{\tau(D^0)} \simeq 1 + \left( \frac{f_D}{200 \text{ MeV}} \right)^2 \sim 2 \quad (13)
\]

A priori \( \tau(D_s) \) and \( \tau(D^0) \) could differ substantially from each other, in particular due to a different weight of WA in the two transitions. Yet using the heavy quark expansion and assuming factorization one predicts \( \tau(D_s) \) and \( \tau(D^0) \) to agree within several percent due to a compensation among various competing smallish effects.
Table 1: QCD Predictions for Charm Lifetime Ratios

| Observable          | QCD Expectations (1/m_c expansion)                                                                 | Ref. | Data from |
|---------------------|---------------------------------------------------------------------------------------------------|------|-----------|
| \(\tau(D^+)/\tau(D^0)\) | \(\sim 2\) [for \(f_D \simeq 200\) MeV] (mainly due to destructive interference)                  |      | 2.547 ± 0.043 |
| \(\tau(D_s)/\tau(D^0)\) | 1 ± few \(\times 0.01\)                                                                           | 23   | 1.12 ± 0.04 |
| \(\tau(\Lambda_c)/\tau(D^0)\) | \(\sim 0.5^*\)                                                                                  | 23   | 0.51 ± 0.05 |
| \(\tau(\Xi^+ c)/\tau(\Lambda_c)\) | \(\sim 1.3^*\)                                                                                  | 23   | 1.75 ± 0.36 |
| \(\tau(\Xi^0 c)/\tau(\Xi^+ c)\) | \(\sim 2.8^*\)                                                                                  | 23   | 3.57 ± 0.91 |
| \(\tau(\Xi^0 c)/\tau(\Omega_c)\) | \(\sim 4^*\)                                                                                    | 23   | 3.9 ± 1.7  |

The main differences in the lifetimes of baryons on one hand and of mesons on the other and also among the various baryons arise in order 1/m_c^3 due to WS and destructive as well as constructive PI:

\[
\Gamma(\Lambda^+_c) = \Gamma_{\text{decay}}(\Lambda^+_c) + \Gamma_{WS}(\Lambda^+_c) - |\Gamma_{PL,-}(\Lambda_c)| \tag{14a}
\]
\[
\Gamma(\Xi^0_c) = \Gamma_{\text{decay}}(\Xi^0_c) + \Gamma_{WS}(\Xi^0_c) + |\Gamma_{PL,+}(\Xi^0_c)| - |\Gamma_{PL,-}(\Xi^0_c)| \tag{14b}
\]
\[
\Gamma(\Xi^+_c) = \Gamma_{\text{decay}}(\Xi^+_c) + |\Gamma_{PL,+}(\Xi^+_c)| - |\Gamma_{PL,-}(\Xi^+_c)| \tag{14c}
\]
\[
\Gamma(\Omega_c) = \Gamma_{\text{decay}}(\Omega_c) + |\Gamma_{PL,+(\Omega_c)}| \tag{14d}
\]

with both quantities on the right-hand-side of eq.(14d) differing from the corresponding ones for \(\Lambda_c\) or \(\Xi_c\) decays. On rather general grounds one concludes:

\[
\tau(\Xi^0_c) < \tau(\Xi^+_c), \quad \tau(\Xi^0_c) < \tau(\Lambda^+_c) \tag{15}
\]

To go beyond this qualitative prediction one has to evaluate the expectation values of the various four-fermion operators. No model-independent manner is known for doing that for baryons; we do not even have a concept like factorization allowing us to lump our ignorance into a single quantity. Instead we have to rely on quark model computations and thus have to be prepared for additional very sizeable theoretical uncertainties. In Table 1 I juxtapose the data with the theoretical expectations obtained from the heavy quark expansion described above. The numbers for baryon lifetimes are based on quark model evaluations of the four-fermion expectation values; this is indicated by an asterisk. Details can be found in [25].

The agreement between the expectations and the data, within the uncertainties, is respectable or even remarkable considering the large theoretical
expansion parameter and the fact that the lifetimes for the apparently shortest-lived hadron – Ω_c – and for the longest-lived one – D^+ – differ by an order of magnitude! Of course the experimental uncertainties in τ(Ξ_c) and τ(Ω_c) are still large; the present agreement could fade away – or even evaporate – with the advent of more accurate data. Yet at present we conclude:

- The observed difference in τ(D^0) vs. τ(D^+) is understood as due mainly, though not exclusively, to a destructive interference in Γ_{NL}(D^+) arising in order 1/m_c^3. This is not contradicted by the data showing BR_{SL}(D^+) ≃ 17%. For the corrections of order 1/m_c^2 reduce the number obtained in the naive spectator model – BR_{SL}(D) ≃ BR_{SL}(c) – from around 16% down to around 9%.

- The observed near-equality of τ(D^0) and τ(D_s) provides us with circumstantial evidence for the reduced weight of WA. It puts a severe bound on the size of the non-factorizable parts in the expectation values of the four-fermion operators, as given in [21].

- The lifetimes of the charm baryons reflect the interplay of destructive as well as constructive PI and WS.

- The Ω_c naturally emerges as the shortest-lived charm hadron due to spin-spin interactions between the decaying c quark and the spin-one ss di-quark system.

Finally one should note that the ratios Γ_{SL}(Ξ_c)/Γ_{SL}(D^0) and Γ_{SL}(Ω_c)/Γ_{SL}(D^0) will not reflect their lifetime ratios; for Γ_{SL}(Ξ_c) and Γ_{SL}(Ω_c) get significantly enhanced relative to Γ_{SL}(D^0) in order 1/m_c^3 due to constructive PI in Γ_{SL}(Ξ_c, Ω_c) among the s quarks. Thus Ω_c – despite its short lifetime – could well exhibit a larger semileptonic branching ratio than D^+!

3 Lifetimes of Beauty Hadrons – ‘Hic Rhodus, Hic Salta!’

Most of the caveats stated for charm decays cannot be used as excuses for failures in beauty decays. Due to m_b ≫ μ_{had} the heavy quark expansion would be expected to yield fairly reliable predictions on lifetime ratios among beauty hadrons. The actual computations proceed in close analogy to the charm case and can be found in [25]. The B_d – B^- lifetime difference is again driven mainly by destructive PI, namely in the b → cūd channel; similarly, τ(Λ_b) is reduced relative to τ(B_d) by WS winning out over destructive PI in b → cūd:

\[ Γ(B_d) ≃ Γ_{decay}(B_d), \quad Γ(Λ_b) ≃ Γ_{decay}(Λ_b) + Γ_{WS}(Λ_b) - |Γ_{PI,-}(Λ_b)| \]
Table 2: QCD Predictions for Beauty Lifetimes

| Observable | QCD Expectations (1/m_b expansion) | Ref. | Data from |
|------------|-----------------------------------|------|-----------|
| $\tau(B^-)/\tau(B_d)$ | $1 + 0.05 (f_B/200 \text{ MeV})^2 [1 \pm \mathcal{O}(30\%)] > 1$ (mainly due to destructive interference) | 11 | 1.03 ± 0.06 |
| $\bar{\tau}(B_s)/\tau(B_d)$ | $1 \pm \mathcal{O}(0.01)$ | 29 | 0.97 ± 0.08 |
| $\tau(\Lambda_b)/\tau(B_d)$ | $\simeq 0.9^*$ | 29 | 0.73 ± 0.06 |

In Table 2 I list presently available data together with quantitative predictions. Several comments are in order here:

- These are predictions in the old-fashioned sense, i.e. they were made before data (or data of comparable sensitivity) became available.

- As far as the meson lifetimes are concerned, data and predictions are completely and non-trivially consistent.

- A careful evaluation of the radiative corrections and analysis of non-factorizable contributions allows to predict that $\tau(B^-)$ exceeds $\tau(B_d)$ by several percent, as stated in Table 2. Contrary to the claims of future experimental findings that $\tau(B^-) < \tau(B_d)$ or $\tau(B^-) \simeq 1.2 \cdot \tau(B_d)$ could not naturally be accommodated within the heavy quark expansion. One more cross check can be performed to make this case conclusive by closing a possible loophole in the argument: contrary to presently available theoretical evidence factorization might be a poor ansatz. One can extract the factorizable as well as non-factorizable contributions from a difference observed in the endpoint energy spectra for semileptonic decays of $B_d$ and $B^-$ mesons. Comparing the inclusive lepton spectra in semileptonic $D^0$, $D^+$ and $D_s$ decays would provide us with similar information.

- The average $B_s$ lifetime, i.e. $\bar{\tau}(B_s) = [\tau(B_{s,\text{long}}) + \tau(B_{s,\text{short}})]/2$, as measured in $B_s \rightarrow l\nu D_{s}^{(*)}$, is practically identical to $\tau(B_d)$.

- The largest lifetime difference among beauty mesons is expected to occur due to $B_s - \bar{B}_s$ oscillations. One predicts

$$ \frac{\Delta \Gamma(B_s)}{\Gamma(B_s)} = \frac{\Gamma(B_{s,\text{short}}) - \Gamma(B_{s,\text{long}})}{\Gamma(B_s)} \simeq 0.18 \cdot \frac{(f_{B_s})^2}{(200 \text{ MeV})^2} \quad (16) $$

- The prediction on $\tau(\Lambda_b)/\tau(B_d)$ seems to be in conflict with the data.
For proper evaluation of the last point one has to keep the following in mind:

(i) The experimental situation has not been settled yet. In Table 2 I have listed the world average of already published data on $\tau(\Lambda_b)/\tau(B_d)$. It should be pointed out that a recent preliminary CDF study finds $\tau(\Lambda_b)/\tau(B_d) = 0.85 \pm 0.12$. While this value is quite consistent with the stated world average, it would also satisfy the theoretical prediction.

(ii) The difference between $\tau(\Lambda_b)/\tau(B_d)|_{exp} \simeq 0.73$ and $\tau(\Lambda_b)/\tau(B_d)|_{theor} \simeq 0.9$ represents a large discrepancy. For once one has established – as we have – that $\tau(\Lambda_b)$ and $\tau(B_d)$ have to coincide for $m_b \to \infty$, then the predictions really concern the deviation from unity; finding a $\sim 27\%$ deviation when one around $10\%$ was predicted amounts to an error of order 300%!

(iii) A failure of that proportion cannot be rectified unless one adopts a new paradigm in evaluating baryonic expectation values. Two recent papers [27, 20] have re-analyzed the relevant quark model calculations and found:

$$
\tau(\Lambda_b)/\tau(B_d) \equiv 1 - \text{DEV}, \quad \text{DEV} \sim 0.03 \div 0.12
$$

(17)

i.e., indeed there are large theoretical uncertainties in DEV, yet one cannot boost its size much beyond the $10\%$ level. To achieve the latter one had to go beyond a description of baryons in terms of three valence quarks only.

4 Summary

During the last few years considerable conceptual progress has been achieved in our theoretical description of the decays of heavy flavour hadrons in general and their lifetimes in particular. Questions that could hardly be raised before can be tackled now. A failure to describe the weak lifetimes will of course never rule out QCD – yet it can and will teach us significant lessons on the inner workings of QCD.

Such failures can actually occur at different levels thus leading to different layers of lessons which I am going to list now in ascending order of depth:

(i) It seems quite unlikely that future data could contradict the predicted qualitative pattern, namely $\tau(D^+) > \tau(D^0) \simeq \tau(D_s) > \tau(\Lambda_c)$ and $\tau(\Xi_c^0) < \tau(\Xi^{+}_c)$, $\tau(\Xi_c^0) < \tau(\Lambda_c)$.

(ii) An inability to quantitatively reproduce the observed lifetime ratios for charm baryons can be rationalized most easily. For their widths receive contributions with different signs from several mechanisms whose intervention reflects the rather complex internal structure of baryons; furthermore contributions that are formally of higher order in $1/m_c$ are numerically reduced only in a moderate fashion; it would therefore seem unrealistic to expect any success beyond purely qualitative considerations.
(iii) If however one succeeds in describing charm baryon lifetime ratios in a semi-quantitative fashion at least, then one can use this information to probe the internal structure of these baryons in a novel way, namely concerning the behaviour of the diquark system, as briefly referred to above for $\Omega_c$.

(iv) If a future determination of $f_D$ revealed a significant discrepancy in the prediction for $\tau(D^+)/\tau(D^0)$, one could blame that on $m_c$ being too low. More specifically it would – like $\Gamma_{SL}(D)$ signal the limitation of quark-hadron duality at the relatively low scale $m_c$.

(v) For inclusive beauty decays no plausible deniability exists and one had to face up to harder lessons.

(vi) As discussed before one has to allow for considerable numerical uncertainties in the predictions on $\tau(\Xi^+_b) \text{ vs. } \tau(\Xi^-_b) \text{ vs. } \tau(\Lambda_b) \text{ vs. } \tau(B_d)$. Yet their differences should not exceed the 10 % level. To reproduce larger lifetime differences – as suggested by the present world average on $\tau(\Lambda_b)/\tau(B_d)$ – would require a new paradigm in evaluating at least baryonic matrix elements that goes beyond the usual valence quark description. Instead one might go one step further and argue that quark-hadron duality does not operate here with sufficient accuracy.

(vii) Discrepancies concerning $B$ meson lifetime ratios would lead to unequivocal lessons. Namely a failure in $\tau(B^-)/\tau(B_d)$ would first cast serious doubts on the applicability of factorization even at the natural low scale. However if an extraction of the expectation values of the four-fermion operators from semileptonic decays without imposing factorization had closed this loophole, one would be forced to conclude that local duality is not realized in nonleptonic beauty decays; local duality means that the rates for inclusive processes involving hadrons can be calculated from the corresponding quark reactions without the ‘smearing’ or averaging in energy advocated in $\text{[11]}$. The same negative conclusion would follow if $\widetilde{\tau}(B_s)$ and $\tau(B_d)$ were found to differ by more than a few percent. This would certainly be a disappointing lesson – in particular since we have not spotted any previous sign for trouble – but it would be an important one nevertheless! A more detailed discussion of these points can be found in $\text{[10,25]}$.

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