Luminosities of High-Redshift Objects in an Accelerating Universe

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The results from the Supernova Cosmology Project indicate a relation between cosmic distance and redshift that corresponds to an accelerating Universe, and, as a consequence, the presence of an energy component with negative pressure. This necessitates a re-evaluation of such astrophysical luminosities that have been derived through conventional redshift analyses of, e.g., gamma-ray bursts and quasars. We have calculated corrected luminosity distances within two scenarios; the standard one with a non-zero cosmological constant, and the more recently proposed "quintessence", with a slowly evolving energy-density component. We find luminosity corrections from +30 to −40 per cent for redshifts with \( z = 0 \)–10. This finding implicates that the SCP data do not, by themselves, require a revision of the current, rather qualitative modeling of gamma-ray bursts and quasar properties.

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In 1938, Baade\(^1\) suggested supernovae as “standard candles" for measuring various cosmological parameters. At closer distances they should reveal the Hubble constant, while at higher redshifts they were assumed to eventually indicate a universal deceleration.\(^2\) Measurements of the Hubble constant became feasible in the 1980s, while the attempts to detect a universal deceleration failed, due to a lack of observable high-redshift supernovae. When the Supernova Cosmology Project (SCP) was initiated in 1988 its primary goal was to determine cosmological parameters through the magnitude-redshift relation of Type Ia supernovae. Goobar and Perlmutter\(^3\) showed that by studying this relation one might be able to separate the relative contributions to the density of the Universe into one part, \( \Omega_m \), due to masses (including the hypothetical “dark matter"), and another part, \( \Omega_\Lambda \), due to a non-zero value of the cosmological constant, \( \Lambda \), as given by the Einstein equations. The latter is looked upon as a density of “dark" energy hidden within the physical vacuum. As of March 1998, more than 75 supernovae of Type Ia at redshifts \( z = 0.18 – 0.86 \) had been discovered and analyzed\(^4\). The results are summarized in Fig. 1. A similar study by the “High-z Supernova Search Team"\(^5\) has produced results in agreement with those of the SCP.

A deviation from the expected magnitude-distance relation is seen. Assuming a flat universe, as predicted by the hypothesis of inflation within the standard Big Bang scenario\(^6\), it is clear that the major energy density must be of the “vacuum" type. This finding obviously implicates that a re-analysis of astrophysical data (and possibly of theoretical models) deduced from cosmologies with \( \Omega = \Omega_m \) is necessary. Examples of cosmic phenomena that need to be reconsidered are gamma-ray bursts (GRBs), where redshifts have been found in apparent host galaxies, as well as quasars and active galactic nuclei. Since one of the great mysteries of the GRBs is the enormous energy release in the form of gamma photons, it is important to estimate the corrections implied by the SCP results.

The luminosity, \( \mathcal{L} \), of high-redshift objects, such as GRBs, are determined using the luminosity distance, \( d_L \), and the flux on the detector, \( \phi \) (in erg s\(^{-1}\) cm\(^{-2}\)), through the relation

\[
\mathcal{L} = 4\pi d_L^2 \phi, \tag{1}
\]

assuming a spherically symmetric energy outflow. In this Letter, we examine the implications of the SCP results for \( d_L \) and hence also for \( \mathcal{L} \). There are two different approaches that have raised a particular interest in the current literature. The first one builds on a traditional use of the cosmological constant, \( \Lambda \), as first suggested by Einstein in a different context. The other one includes a recent proposal of an additional energy-density component, parametrized as a slowly evolving scalar field, \( \varphi \), with a positive potential energy\(^7\). This so-called quintessence (see\(^8\), and references herein) is a dynam-
ical, spatially inhomogeneous, energy, resulting in a negative pressure. Unlike the cosmological constant, this scalar field slowly changes its contribution to the energy density of the universe, not only due to the expansion, but also through its slow approach toward a lower potential energy. The equation of state, i.e., the relation between pressure, $p$, and density, $\rho$, for this energy component is parametrized as $p = w \rho$, where the constant $w \in (-1, 0)$. The case $w = -1$ corresponds to a nonzero cosmological constant. In [13], a fit was made to a wealth of cosmological data, resulting in $w \approx -0.65 \pm 0.07$. This is well in line with the limits placed on $w$ by the SCP (see Fig. 10 in [13]).

The distances to high-redshift objects have conventionally been estimated within a so-called Friedmann-Robertson-Walker (FRW) cosmology with $\Omega = \Omega_m$. In light of the recent SCP results, these assumptions have to be modified, and the analysis becomes a bit more complicated. The result stated by the SCP group [13] is $\Omega_m^{\text{fit}} = 0.28^{+0.09}_{-0.08} - 0.05$ for a flat Universe, defined by $\Omega = 1$. Hence, roughly 70 per cent of the energy density is in the “vacuum” form. This energy acts as an effective repulsive potential in the Friedmann equation, making the universe expand at an ever increasing speed, and the SCP [13] states that the data are in line only with a currently accelerating Universe. Nevertheless, this statement is limited to the redshift range of the studied supernovae, i.e., out to $z \approx 1$. We therefore assume that the FRW cosmology used by the SCP when fitting the data is valid also at higher redshifts, where we apply the two different approaches mentioned above. The basic Friedmann equation, neglecting a radiation energy density, can be written as

$$ H^2 = \left( \frac{\dot{a}^2}{a^2} \right) = \frac{8\pi G}{3} (\rho_m + \rho_\Lambda + \rho_\varphi) - \frac{k}{a^2}. \tag{2} $$

Here $a = a(t)$ is the spatial scale factor in the FRW metric, $G$ is Newton’s gravitational constant, $k$ is the Riemannian curvature parameter, and $\rho_m$ is the matter density. The vacuum-energy and quintessence densities, $\rho_\Lambda$ and $\rho_\varphi$, are defined as

$$ \rho_\Lambda = \frac{\Lambda}{8\pi G}, \tag{3} $$

$$ \rho_\varphi = \frac{1}{2} \dot{\varphi}^2 + V(\varphi), \tag{4} $$

where $\Lambda$ is the cosmological constant, while $\varphi$ and $V(\varphi)$ are the field and potential energy in the quintessence model.

The various contributions to the critical density $\Omega$ from $\rho_m$, $\rho_\Lambda$, and $\rho_\varphi$, as well as from the curvature term $k/a^2$, are given by

$$ \Omega_m = \frac{8\pi G}{3H_0^2} \rho_0, \tag{5} $$

$$ \Omega_\Lambda = \frac{\Lambda}{3H_0^2}, \tag{6} $$

$$ \Omega_\varphi = \frac{\Omega_m(1 + z)^3 + \Omega_\Lambda + \Omega_\varphi(1 + z)^2}{1 + \rho_\varphi(1 + z)^3}, \tag{7} $$

$$ \Omega_k = -\frac{k}{a_0^2 H_0^2}, \tag{8} $$

where subscript “0” stands for the current ($t = t_0$) value of each quantity, including that of the Hubble constant, $H_0$. The sum is fixed by $\Omega_m + \Omega_\Lambda + \Omega_\varphi + \Omega_k = 1$ for all cosmologies. Using the scaling relations [17] $\rho_m \propto 1/a^3$ and $\rho_\varphi \propto 1/a^{3(1+w)}$, we reformulate (3) as

$$ H^2/H_0^2 = \Omega_m(1 + z)^3 + \Omega_\Lambda + \Omega_\varphi(1 + z)^2 + \Omega_k(1 + z)^3, \tag{9} $$

where we have used the definition of redshift,

$$ 1 + z = \frac{a_0}{a}. \tag{10} $$

The luminosity distance, $d_L$, is defined by

$$ d_L = a_0 r_1 (1 + z), \tag{11} $$

where $r_1$ is the comoving distance traveled by a photon emitted from the source at time $t = t_1$. The quantities $a$, $r$ and $t$ are related by the equation of a radial, lightlike geodesic of the FRW metric,

$$ \frac{dr}{dt} = \frac{\sqrt{1 - kr^2}}{a(t)} \Rightarrow \int_{r_1}^{r_0} \frac{dr}{\sqrt{1 - kr^2}} = \int_{t_1}^{t_0} \frac{dt}{a(t)}. \tag{12} $$

The relationship

$$ H = \frac{d}{dt} \left( \log \left( \frac{a}{a_0} \right) \right) = \frac{-1}{1 + z} \frac{dz}{dt}, \tag{13} $$

can be used to transform the time integral in Eq. (12) to an integral over $z$, as

$$ \int_{t_1}^{t_0} \frac{dt}{a(t)} = \int^z \frac{dz'}{\sqrt{g(z')}} \tag{14} $$

where $g(z)$ is the expression in the rhs of Eq. (3). The integral over $r$ in Eq. (12) has the solutions

$$ \begin{cases} \frac{\arcsin(\sqrt{kr_1})}{\sqrt{k}} & (k > 0) \\ r_1 & (k = 0) \\ \frac{\arcsinh(\sqrt{-kr_1})}{\sqrt{k}} & (k < 0) \end{cases} \tag{15} $$

Combining Eqs. (12), (14) and (15) leads to an expression for $r_1$ as a function of $z$, given by

$$ r_1 = \frac{1}{\sqrt{-\Omega_k}} \left\{ \sqrt{-\Omega_k} \int^z \frac{dz'}{\sqrt{g(z')}} \right\}, \tag{16} $$
where $S\{x\}$ takes on the forms $\sin \{x\}$, $x, \sinh \{x\}$ for the three different curvatures given by $k = +1, 0, -1$, i.e., for a closed, flat and open Universe. The final expression for $d_L$ then becomes

$$d_L(z) = a_0 (1 + z) \frac{1}{\sqrt{|-\Omega_k|}} S \left\{ \sqrt{-\Omega_k} \int_0^z \frac{dz'}{\sqrt{g(z')}} \right\}.$$  \hspace{1cm} (17)

We compare the results from Eq. (17) with the standard expression for the luminosity distance in an FRW universe with $\Omega = \Omega_0$, i.e., with

$$d_L^0(z) = \frac{1}{H_0 q_0} \left[ q_0 z + (q_0 - 1) \left( \sqrt{1 + 2zq_0} - 1 \right) \right],$$  \hspace{1cm} (18)

where $q_0 = \Omega_m/2$, since $\Lambda = 0$ in this case. For simplicity we have used $\Omega_m = 0.28$ in all calculations, since this is a result in good agreement with the SCP and other observations \[13\]. The results are quantified as $\alpha$, the squared ratio between the corrected $d_L$ and the traditional $d_L^0$. According to Eq. (17) this is also equal to the ratio between the corrected energy outflows (or luminosities) and the “published” ones (assuming that $\Omega_m = 0.28$ has been used). Hence,

$$\alpha = \frac{E_{\text{corr}}}{E} = \left( \frac{d_L}{d_L^0} \right)^2.$$  \hspace{1cm} (19)

The results for both scenarios (“conventional” flat Universe, and quintessence) are shown in Fig. 2.

![FIG. 2. The luminosity correction factor, $\alpha$, as a function of redshift, $z$, for the “conventional” vacuum-energy and the quintessence scenarios. Curve a) has $(\Omega_m, \Omega_\Lambda, \Omega_\phi) = (0.28, 0.72, 0)$ and curve b) has $(\Omega_m, \Omega_\Lambda, \Omega_\phi) = (0.28, 0, 0.72)$ and $w = -0.65$.](image)

There is a clear positive correction for low $z$ values, although only in the $10 - 30$ per cent range. Such an enhanced, corrected luminosity has been intuitively expected by some groups for GRBs at those “low” redshifts \[20\]. It is therefore comforting that the correction is so small, which means that most qualitative conclusions about energy flows, drawn from the published values of GRB redshifts, remain unchanged.

For higher $z$ values, luminosities must be corrected downward. In the first scenario, this qualitative difference between low and high $z$ values has to do with the fact that the presence of $\Omega_\Lambda$ influences the development of the Universe in two different ways. First, it contributes an enhanced energy density that reduces the negative curvature of the Universe, and second, it provides a negative pressure that accelerates the expansion. At low $z$, i.e., for observations in our vicinity, the reduced, negative curvature of the open cosmology in the “denominator” of Eq. (19) is negligible, since the Universe is approximately flat in our neighborhood. The effect of the vacuum energy in the “numerator” is therefore dominating, which explains the positive correction. At high $z$, the opposite is valid, i.e., the effect due to the difference in curvature dominates, and the correction due to the repulsion is negligible. If the vacuum energy is enhanced beyond that of a flat Universe, i.e., so that $\Omega > 1$, the repulsive effect dominates the correction out to even higher $z$ values. Also, the maximal correction at $z \approx 1.5$ grows rapidly with increasing $\Omega_\Lambda$. In a hypothetical Universe with $(\Omega_m, \Omega_\Lambda) = (0, 1)$ it reaches a factor of about two.

In the quintessence scenario, the trends in Fig. 2 have the same origin as in the “conventional” case. The scalar field $\varphi$ has a repulsive effect, just as the cosmological constant $\Lambda$, and affects $a(t)$ in the same way. It should be noted that $\varphi$ is a function of time, and it is not obvious that $w$ is a constant. However, it is argued in \[16\] that the physical, observable consequences of a time-varying $w$ are negligible.

In conclusion, the luminosity correction in the redshift range of “identified” gamma-ray bursts, such as GRB990123 \[21\], is $10 - 30$ per cent, depending on the cosmological scenario. For a typical quasar at redshift $z \sim 5$, the correction is negative, giving a luminosity $80 - 90$ per cent of the one estimated for a Universe with $\Omega = \Omega_m = 0.28$. The main result of our study is that current models for luminous objects at high redshifts do not need to be qualitatively altered due to the SCP supernova results.

[1] W. Baade, Astrophys. J. 88, 285 (1938).
[2] G.A. Tammann, ESA/ESO Workshop on Astronomical Uses of the Space Telescope (eds. F. Macchetto, F. Pacini and M. Tarenghi, Geneva: ESO) 329 (1979).
[3] S. Colgate, Astrophys. J. 232, 404 (1979).
[4] A. Goobar and S. Perlmutter, Astrophys. J. 450, 14 (1995).
[5] S. Perlmutter et al., IAU Circ. No. 6270 (1995).
[6] S. Perlmutter et al., IAU Circ. No. 6596 (1997).
[7] S. Perlmutter et al., IAU Circ. No. 6540 (1997).
[8] S. Perlmutter et al., IAU Circ. No. 6646 (1997).
[9] S. Perlmutter et al., IAU Circ. No. 6804 (1997).
[10] S. Perlmutter et al., IAU Circ. No. 6881 (1998).
[11] A. Riess et al., Astron. J. 116, 1009 (1998).
[12] A.H. Guth, Phys. Rev. D 23, 347 (1981).
[13] S. Perlmutter et al., Astrophys. J., in press, also available at astro-ph/9812133.
[14] M. Hamuy et al., Astron. J. 112, 2391 (1996).
[15] R.R. Caldwell and P.J. Steinhardt, Phys. Rev. Lett. 80, 1582 (1998).
[16] L. Wang, R.R. Caldwell, J.P. Ostriker and P.J. Steinhardt, astro-ph/9901388 (1999).
[17] see, e.g., the text-book L. Bergström and A. Goobar, Cosmology and Particle Astrophysics (Wiley, Chichester, 1999).
[18] N.A. Bahcall and X. Fan, Astrophys. J., in press, also available at astro-ph/9803277.
[19] R.A. Daly, E.J. Guerra and L. Wan, preprint astro-ph/9803265 (1998).
[20] S.R. Kulkarni et al., Nature 398, 389 (1999).
[21] M. Feroci et al., IAU Circ. 7095 (1999).