In-plane dynamic crushing of a novel bio-inspired re-entrant honeycomb with negative Poisson's ratio

Yonghui Wang\textsuperscript{1} \cdot Qiang He\textsuperscript{1} \cdot Yu Chen\textsuperscript{2} \cdot Hang Gu\textsuperscript{1} \cdot Honggen Zhou\textsuperscript{1}

Received: 29 March 2021 / Accepted: 30 August 2021 / Published online: 16 September 2021
© The Brazilian Society of Mechanical Sciences and Engineering 2021

Abstract
In order to seek higher crashworthiness and energy absorption capacity, based on biological inspiration, a novel bio-inspired re-entrant honeycomb (BRH) structure with negative Poisson’s ratio is designed by selecting lotus leaf vein as biological prototype. The numerical simulation model is established by the nonlinear dynamics software ABAQUS and further compared with the available reference results to verify the feasibility. The dynamic compression behavior and energy absorption capacity of two types of BRH (BRH-I and BRH-II) are firstly compared with conventional re-entrant honeycomb (RH). The simulation results show that BRH has better mechanical properties and energy absorption characteristics. Then, the crushing behavior of BRH-II under different impact velocities is systematically studied. Three typical deformation modes of BRH-II are observed through the analysis of deformation profile. The quasi-static plateau stress is closely related to the cellular structure. Based on one-dimensional shock theory, the empirical equations of dynamic plateau stress for BRH-II with different relative densities are given by using least square fitting. In addition, the effects of impact velocity and relative density on plateau stress and energy absorption behavior are also studied. The results show that the energy absorption capacity of BRH-II is increased nearly six times compared with RH at the same impact velocity.

Keywords  Bio-inspired re-entrant honeycomb (BRH) \cdot Numerical simulation \cdot Crushing behavior \cdot Deformation mechanism \cdot Energy absorption capacity

1 Introduction
Due to the excellent specific strength, specific stiffness and energy absorption characteristics [1, 2], honeycomb structure has been widely used in aerospace [3, 4], broadside protection structure and other fields. Therefore, a lot of research has been done on its in-plane and out-of-plane compression characteristics [5, 6]. Liang et al. [7] studied the collapse of a square honeycomb sandwich panel and revealed some mechanical properties of the square honeycomb sandwich structure. Hu et al. [8] carried out a finite element simulation on the in-plane compression of a polycarbonate circular honeycomb, and characterized the uneven deformation and its evolution in the honeycomb block. It was found that the uneven deformation of honeycomb is closely related to internal factors, such as local deformation zones and cell deformation patterns. Cricrì et al. [9] examined the in-plane compression behavior of the regular hexagonal honeycomb through experiments and numerical evaluation, and determined the buckling mode and overall strength characteristics. Yang et al. [10] proposed a reinforced bellows honeycomb structure and studied its mechanical properties under quasi-static uniform compression based on numerical simulation. The results show that under the same quality, the reinforced bellows honeycomb structure has higher energy absorption than the traditional honeycomb structure.

With the development of negative Poisson’s ratio materials, honeycomb structures with negative Poisson’s ratio performance have also been continuously researched and discovered. The honeycomb with negative Poisson’s ratio has better mechanical properties than conventional honeycomb structures due to the characteristics of horizontal shrinkage, shear modulus [11], quasi-static and dynamic
impact resistance [12–16] under lateral load. After Lakes [17] prepared polyurethane foam with Poisson's ratio of −0.7, scholars successively designed various honeycomb structures with NPR. Hou et al. [18] proposed a new isotropic negative Poisson's ratio composite structure, and used the finite element method to analyze their NPR effect and mechanical behavior. Qin et al. [19] used a topology optimization method to design a variety of honeycomb materials with arbitrary negative Poisson's ratio. Gao et al. [20] designed a three-dimensional double-V honeycomb intersected by a negative Poisson's ratio two-dimensional double-V structure, and compared the experimental, numerical and theoretical solutions of the honeycomb platform stress. The results show that the three solutions are very consistent. Based on theoretical analysis, Hu et al. [21] studied the mechanical properties of hexagonal honeycombs during compression. The results show that the negative Poisson's ratio effect can be enhanced by changing the structural parameters of the honeycomb cell wall. Xiao et al. [22] studied the dynamic response characteristics of metal honeycombs with negative Poisson's ratio under low-, medium- and high-impact velocities, and revealed a new shrinkage deformation mechanism. Tan et al. [23] proposed two hierarchical honeycomb structures and studied the mechanical properties of the two honeycomb structures under in-plane impact. The results show that these two hierarchical honeycombs have better energy absorption characteristics than traditional negative Poisson's ratio honeycombs. Yang et al. [24] studied the in-plane crushing behavior and energy absorption capacity of the petal-shaped honeycomb finite element model established under different impact velocities. Studies have shown that the energy absorption performance of this petal-shaped honeycomb is twice as high as that of the traditional honeycomb. Wang et al. [25] introduced peanut-like holes into the solid block matrix and designed a novel negative Poisson's ratio structure.

The special structure of lotus leaf vein is conducive to the reasonable transmission of load, so that it can bear strong load. Therefore, lotus leaf vein can be regarded as a potential bionic design object. So far, no relevant research has introduced this structural feature into the design of honeycomb structures. In this paper, a novel bio-inspired re-entrant honeycomb (BRH) with negative Poisson’s ratio is proposed and inspired by the branching characteristics of lotus leaf veins. Based on finite element simulation, the crashworthiness and energy absorption capacity of BRH under the same crushing load are quantitatively compared with the conventional re-entrant honeycomb (RH) with negative Poisson’s ratio. Then, the in-plane crushing behavior and energy absorption characteristics of BRH-II under different impact velocities are systematically studied, including the typical crushing modes, plateau stress, shrinkage deformation and energy absorption characteristics.

2 Geometric model of BRH

2.1 Geometry of BRH

The strong carrying capacity of lotus leaf is due to the special distribution of its vein, which realizes the reasonable transmission of the load. Therefore, the geometric characteristics of lotus leaf vein branch may open a new design idea for the design of honeycomb structure to improve their mechanical properties by introducing such structural features. Figure 1 shows the design process inspired by the branch of lotus leaf vein. It can be seen from Fig. 1a that the geometric characteristics of lotus leaf vein are mainly reflected in the main veins and branch veins. Combining this characteristic with unit cell of conventional re-entrant honeycomb (RH), two new representative unit cell can be obtained. And then two types of bio-inspired re-entrant honeycombs (BRH-I, BRH-II) are depicted in Fig. 1b.

Figure 2 depicted the geometrical characteristic of representative unit cell of BRH in detail. The height $H_0$ of the representative unit cell, the angle $\alpha$ between the hypotenuse and the horizontal side, and the branch angle $\beta$ are 8 mm, 53.15°, 73.71°, respectively. The side lengths $H$, $L$ and $L_0$ are 12 mm, 5 mm and 5 mm, respectively. Therefore, the length $L_3$ and height $L_4$ of BRH are, respectively, equal to 76 mm and 48 mm. As shown in Fig. 2a and b, the red and blue dots in the structural units of BRH-I and BRH-II are divided into midpoints and quarter points representing the corresponding edges. RH consists of 30 unit cells in six rows and five columns, which is exactly same as that in Ref. [22].
Based on the previous theoretical analysis [12], the relative densities of BRH-I and BRH-II can be calculated by:

\[ \rho_I = \frac{5(H_0 + 3L_0)}{3H_0} \frac{t_0}{L_0} \]  
\[ \rho_{II} = \frac{5(H_0 + 4L_0)}{3H_0} \frac{t_0}{L_0} \]  

where \( \rho_I, \rho_{II} \) are, respectively, the relative densities of BRH-I and BRH-II. As shown in Eqs. (1) and (2), although their relative densities can be adjusted by changing the structural parameters of BRH-I and BRH-II, changing the
thickness may be the simplest and most effective method. When studying the influence of relative density on plateau stress and energy absorption, this method is used to adjust its relative density.

2.2 Crashworthiness indicators

In order to evaluate the crashworthiness of BRH, it is very important to define the crashworthiness index. In this study, the plateau stress and specific energy absorption are two main indicators to measure the crashworthiness of all honeycomb structures. The plateau stress is the average stress between the yield strain of the structure and the densification strain. The calculation formula is as follows:

$$\bar{\sigma}_p = \frac{1}{\epsilon_d - \epsilon_y} \int \sigma(\varepsilon) d\varepsilon$$ (3)

where $\epsilon_y$ is the yield strain of the structure, $\epsilon_d$ is the densification strain. EA is the total energy absorbed by the structure during the impact, it can be given as:

$$EA(x) = \int_0^s F(x) dx$$ (4)

where $F(x)$ is the crushing force and $s$ is the total compression displacement. SEA is the energy absorption per unit mass, which can reflect energy absorption capacity of the structure. It can be calculated by:

$$SEA = \frac{EA(x)}{M}$$ (5)

where $M$ is the total mass of the structure, and these crashworthiness indexes can be obtained from the force and displacement curves. Obviously, the larger the SEA, the better energy absorption characteristics of the structure. In addition, a large plateau stress means more energy can be absorbed and the compressive stress is relatively stable. Therefore, generally the greater the plateau stress, the better the crashworthiness of the structure.

3 Finite element model

3.1 Modeling method

In order to study the crashworthiness and energy absorption capacity of BRH, the nonlinear dynamics software ABAQUS is used to simulate the in-plane dynamic compression process of BRH. Figure 3 shows the finite element model of BRH under in-plane impact. The finite element model includes three parts, namely the top plate, the bottom plate and the BRH. In the simulation process, BRH is placed between the top plate and the bottom plate. The bottom plate is set as a fixed constraint. The top plate compresses the honeycomb structure downward at a constant velocity of 1–200 m/s. The material constitutive of BRH adopts an ideal elastoplastic model. Its material parameters are density 2700 kg/m³, Young's modulus 68 GPa, Poisson's ratio 0.3, yield stress 255 MPa [22], and the top and bottom plates are set as rigid bodies. BRH, top plate and bottom plate are all modeled by four-node linear quadrilateral shell elements, and five thickness integration points are used to ensure convergence. In addition, the contact between the BRH and the top and bottom plates is set to a general contact algorithm with a friction coefficient of 0.2 and a hard contact to prevent penetration and relative sliding between the structures. Finally, the convergence analysis is carried out through simulation. The mesh size is set to 0.75 mm × 0.75 mm after a trade-off between accuracy and computational cost.

3.2 Validation of FE model

In order to verify the validity of the finite element model in this study, the compressive stress and deformation modes of conventional RH under quasi-static, low-velocity, medium-velocity and high-velocity impact are compared and analyzed with results in Ref. [22]. As shown in Fig. 4a, the nominal stress–strain curve and the energy absorption value per unit volume of the present simulation model are in good agreement with Ref. [22]. The difference lies in the peak value and fluctuation of the stress–strain curve. At the same time, the calculated plateau stress of the finite element simulation is only 1.4% lower than that in Ref. [22], which further demonstrates that the slight difference of the stress–strain curve will not greatly affect the total absorption energy and SEA. As shown in Fig. 4b, during the dynamic compression process, the RH first appeared a "Y"-shaped local shear zone, and then formed an "I"-shaped shear in the middle area, and finally RH was compacted. From the perspective of
the entire compression process, the negative Poisson’s ratio
effect of RH is due to its shrinkage deformation.

Figure 5 shows the comparison of stress–strain curves
at the impact end. Under low-, medium- and high-velocity
impact, the changing trend of stress–strain curve is con-
sistent with that in Ref. [22] and the deviations of plateau
stress and densification strain are all within the allowable
range. As the corresponding deformation modes are shown
in Fig. 6, the finite element simulation of NPRH under
medium- and high-speed impact is very consistent with
the reference results, but there are differences under low-
velocity compression. Through comparison, it is found
that the RH of Ref. [21] has a similar symmetrical defor-
mation mode to the simulation results under low-speed
 compression. This asymmetrical deformation mode in Ref.
[22] may be caused by the unfavorable sliding between the
plate and the honeycomb during low-velocity compres-
sion. Comprehensive analysis shows that the present FE
model method is accurate and effective, so it can be used
to establish BRH finite element model.

4 Comparative analysis of three honeycomb structures
In order to illustrate the excellent crashworthiness of BRH,
the dynamic response curves of BRH and conventional RH
under impact velocity of 1 m/s, 50 m/s and 100 m/s are
compared and analyzed. The material properties of BRH and RH are the same. Figure 7 shows the stress–strain curves of conventional RH, BRH-I, BRH-II under different impact velocities. The crushing process of the three honeycomb structures basically presents three stages: the initial linear elastic stage, the stable crushing stage and the densification stage. From Fig. 7a and b, it can be seen the plateau stresses of BRH-I and BRH-II at impact velocity of 1 m/s and 50 m/s are significantly greater than that of conventional RH. However, the plateau stresses of the three honeycomb structures under 100 m/s impact velocity are almost the same in Fig. 7c, indicating that the impact velocity has a great influence on the energy absorption of honeycomb structure. It is generally believed that higher plateau stress means better crashworthiness [25], so the crashworthiness of BRH-I and BRH-II is better than conventional RH at impact velocities of 1 m/s and 50 m/s. Meanwhile, the initial stress peak increases with the increase in impact velocity. This is because the greater the impact velocity, the greater the effect of inertia. As shown in Fig. 7a, the stress of BRH-I in the early stage of stable crushing is greater than that of BRH-II. With the continuous compression of the rigid plate, BRH-II shows stronger energy absorption ability. However, the difference of stress–strain curves between BRH-I and BRH-II decreases with the increase in impact velocity.

The deformation mechanism has an important influence on its energy absorption performance. Thus, the deformation modes of these three honeycombs under different impact velocities are compared. Figure 8a shows the typical deformation modes of the three honeycomb structures under low-velocity impact. The conventional RH presents a "Y" crushing mode and a "V+X" mixed crushing mode. The formation of this local shear band is due to the free edge effect on the side of honeycomb. The degree of freedom of the cells near the central area of the honeycomb structure is smaller than that of the cells near the sides, and the cross-sectional constraints may change during the crushing process. Therefore, the deformation of cells along the cross section is different from each other, resulting in the formation of local shear bands. The generation of this local deformation zone will directly affect the effective crushing force. While for the BRH-I and BRH-II, only a single "X" local shear zone appeared in the early stage, and then no obvious local shear zone is observed and the cells in the entire red box may be seriously deformed. There is no doubt that the uniform plastic deformation of all the cells within the red box will enhance its energy absorption ability.

For BRH-I and BRH-II, the crushing modes in the early and later stage of compression deformation are nearly the same. However, some differences can be seen from the dynamic response curve of the impact end. Considering that BRH-I has a larger wall thickness under the same mass, the stress level of BRH-I is higher than that of BRH-II when the "X" local shear zone appears in the early stage. With the continuous compression of the impact end, the cells of the whole honeycomb structure deform uniformly. It is easy to find that BRH-II has more cell walls than BRH-I (as shown in Fig. 2). The improvement of the strength of the whole structure is bound to improve the stress level. This is the reason that why the stress level of BRH-II may surpass BRH-I in the later stage of compression. However, all three honeycombs exhibit the same deformation mode under high-velocity impact as shown in Fig. 8b. In the compression process, the local collapse first starts from the impact end, and then an "I"-shaped shear zone is gradually formed as the stress wave propagates to the distal end. Therefore,
there is almost no difference in the plateau stress of the three honeycomb structures under high-velocity impact. As another important indicator to measure crashworthiness, Fig. 9 shows the SEA of these three honeycomb structures at different impact velocities. It can be found from Fig. 9 that under the impact velocity of 1 m/s and 50 m/s, the SEA of BRH-I and BRH-II has been greatly improved compared with the conventional RH. Specifically, the SEA of BRH-I has increased by 388.5% and 241.56%. The growth rate of BRH-II even reaches 644.5% and 346.93%. Under high-velocity impact, the inertia effect will play a leading role and the SEA gap between these three honeycomb structures may be narrowed. BRH-I and BRH-II still have stronger energy absorption capacity with growth rates reaching about 42.08% and 51.49%, respectively.

Besides, the negative Poisson’s ratio effect caused by shrinkage deformation plays an important role in enhancing crashworthiness. As shown in Fig. 8, the shrinkage deformation feature of BRH-I and BRH-II is different from the conventional RH. In order to better illustrate this difference, Fig. 10 shows the typical local deformation modes of single unit cell under quasi-static compression. It can be seen from Fig. 10 that the rotation of the plastic hinge causes the hypotenuse to bend and compact. The horizontal tensile force pulls the vertical plate, which further makes the structure shrink and deform. Compared with conventional RH, the difference is that BRH-I and BRH-II have a pair of plastic

![Stress-strain curves of three honeycomb structures under different impact velocities](image-url)
This kind of plastic hinge in the opposite direction is more conducive to induce shrinkage deformation. Therefore, the shrinkage deformability of BRH-I and BRH-II is better than conventional RH. Meanwhile, the maximum horizontal strains of the three honeycombs under different impact velocities are extracted. For the calculation of NPR of each layer, six points are taken at the same interval on both sides of the structure. The average horizontal strain of this layer is calculated according to the horizontal displacement of the left and right points of each layer during the compression process. The NPR value of each layer is the ratio of the average horizontal strain to the vertical strain. The calculation formulas and its corresponding diagram are shown in Eqs. (6–8) and Fig. 11. As shown in Fig. 12a, the maximum horizontal strains of BRH-I and BRH-II are larger than those of conventional RH under three different compression velocities. The maximum horizontal strains decrease with the increase in impact velocity. However, BRH-I and BRH-II are more sensitive to crushing.
velocity than conventional RH. It can be seen from Fig. 12b that the negative Poisson’s ratio of the honeycomb structure presents the same changing trend at low-, medium- and high-impact speeds, that is, it first decreases, then increases and finally stabilizes. It is worth noting that under the three speed shocks, BRH-II has the smallest negative Poisson’s ratio, followed by BRH-I. This means that BRH-II has the best negative Poisson’s ratio effect.

\[
\epsilon_H = \frac{x_i y_j - x_i y_{ij}}{x_i y_j}
\]  

(6)

\[
\epsilon_V = \frac{\delta_0 - \delta}{\delta_0}
\]  

(7)

\[
\nu = -\frac{\epsilon_H}{\epsilon_V}
\]  

(8)

5 Discussion

According to the above discussion and analysis, it can be concluded that BRH has better mechanical properties and energy absorption capacity compared with conventional RH. BRH-I and BRH-II have similar compression characteristics and BRH-II has stronger energy absorption ability. Therefore, this section focuses on the study of the compression behavior and energy absorption characteristics of BRH-II.

5.1 Deformation mode

5.1.1 Deformation mode under different impact velocity

It can be seen in Sect. 4 that the energy absorption capacity of BRH is higher than that of conventional RH due to the change of deformation mechanism. Considering that the deformation mode of honeycomb structure under different impact velocities may be different from each other; the crushing behavior of BRH-II under low-, medium- and high-impact velocities is studied. The results show that there are three typical deformation modes for BRH-II,

| Conventional RH |  |
|-----------------|--|
| BRH-I           |  |
| BRH-II          |  |

Fig. 10 Typical local deformation mode of single unit cell under quasi-static compression

Fig. 11 The calculation of BRH-II horizontal strain and vertical strain

(a)

(b)
namely quasi-static, transition and dynamic collapse mode. A detailed analysis is made accordingly.

Figure 13 shows the deformation mode of BRH-II under quasi-static compression. It can be seen from the map that during the compression process, plastic deformation propagates from the top or bottom to the center. The top and bottom ends have slight plastic deformation, while the center position has more obvious plastic deformation. Severe plastic deformation appears in the central area of BRH-II with further compression, as shown in Fig. 13d and e. Then, the overall plastic deformation occurs until it is finally compacted in Fig. 13f. It is worth noting that the conventional RH always has various local shear bands in the process of dynamic compression. However, there are few local shear bands in the early stage of compression for BRH-II and the whole uniform plastic deformation occurs in the later stage until compaction. This is mainly due to the enhancement of the structural strength of unit cells and the plastic deformation of more cells will make the energy absorption capacity of BRH-II significantly improved.

Figure 14 shows the deformation mode of BRH-II under 50 m/s velocity impact. The influence of inertia effect is enhanced with the increase in impact velocity. The global plastic bending deformation in the quasi-static case ($V = 1 \text{ m/s}$) disappears. It can be observed that BRH-II has a typical layer-by-layer failure process. At the beginning of compression, the upper half of the unit cells in the first row undergoes plastic deformation. After the upper half of the unit cells is fully compacted, the subsequent cells may deform in the same manner layer by layer. It is important to note that some triangular cells with smaller sides appear due to the introduction of the geometric characteristics of lotus leaf vein. There may be not enough space for the plastic buckling of cell walls in these triangular cells. The cell walls of these triangular cells do not collapse completely until densification and mix in the existed deformation band. In fact, this phenomenon also exists in the plastic deformation of BRH-II under quasi-static compression ($V = 1 \text{ m/s}$), which can be found in Fig. 13.
The deformation mode of BRH-II under high-velocity impact is shown in Fig. 15. During the compression process, BRH-II first produced an I-shaped deformation mode at the impact end, and then, this I-shaped deformation mode propagated vertically downward layer by layer. The characteristic of this compression mode is that only the single-layer unit cell undergoes dynamic compression until it is compacted, while the latter layer only undergoes slight plastic deformation, as shown in Fig. 15c and d. It is worth noting that the unit cell of the latter layer will propagate downward with the I-shaped wave and transform from small plastic deformation to large plastic deformation. This is because the strength of the upper part of the BRH-II unit is much smaller than the lower part. As the compression displacement increases, the inertial effect will be gradually weakened.

### 5.1.2 Deformation pattern diagram

After the observation and analysis of the deformation modes of BRH-II under different impact velocities, it can be found that impact velocity may have a significant influence. Like the most honeycomb structures, the deformation mechanism of BRH-II can be also summarized as quasi-static mode, transition mode and dynamic crushing mode. Figure 16
Fig. 15 Deformation mode of BRH-II under high-velocity compression ($\bar{\rho}=0.133$, $V=200$ m/s)

(a) $\varepsilon=0$
(b) $\varepsilon=0.085$
(c) $\varepsilon=0.213$
(d) $\varepsilon=0.3$
(e) $\varepsilon=0.425$
(f) $\varepsilon=0.6$

Fig. 16 The deformation mode of BRH-II with different impact velocities and relative densities

shows the deformation modes of BRH-II with different relative densities under different impact velocities. It is noticeable that the deformation mode of BRH-II depends not only on the velocity, but also on the relative density. In light of relative density $\bar{\rho}$, the corresponding empirical formula for the critical velocity can be obtained as follows:

$$V_{c1} = 10$$  \hfill (9)

$$V_{c2} = 621.06\bar{\rho} + 13.98$$  \hfill (10)

where $V_{c1}$ and $V_{c2}$ represent the critical velocity of transition from quasi-static deformation pattern to transition mode and the critical velocity of transition from transition mode to dynamic mode, respectively.

Fig. 17 The plateau stress of BRH-II with different relative densities under different impact velocities

5.2 Plateau stress at impact end

Figure 17 shows the platform stress of BRH-II with different relative densities under different impact speeds. It can be seen from the map that the increase in relative density and impact velocity will cause an increase in platform stress. Due to the curve relationship between platform stress and impact velocity, assuming that the platform stress of BRH-II under quasi-static compression can be calculated as follows:

$$\sigma(\bar{\rho}) = B\sigma_0\bar{\rho}^m$$  \hfill (11)

Considering that the impact energy can be almost completely converted into internal energy when impacted by 1 m/s, the kinetic energy can be ignored [37]. Therefore,
1 3

the condition of 1 m/s impact is considered as quasi-static condition. $B$ and $m$ in Eq. (11) are constants and can be obtained by least squares fitting. According to the data in Table 1, the fitting parameters $B$ and $m$ are 0.5 and 2, respectively. Figure 18 shows the fitting results of the relative density of BRH-II and the corresponding plateau stress under quasi-static compression. The results show that Eq. (11) can well fit the plateau stress of BRH-II under quasi-static compression.

From the previous discussion of deformation modes under different impact velocities, it can be found that the influence of inertia effect increases with the increase in impact velocity. When the impact velocity reaches a certain value, the plastic deformation of BRH-II will propagate from top to bottom in a plane plastic wave-like deformation mode. Therefore, one-dimensional shock wave theory can be used to analyze the dynamic plateau stress of BRH-II. The theoretical model created by Zou and Reid [38, 39] has been widely used in the analysis of various honeycomb structures, which can be expressed as follows:

$$\sigma = \sigma_0 + AV^2$$

where $\sigma$, $\sigma_0$, $V$ are dynamic plateau stress, static plateau stress and impact velocity, respectively. $A$ is a constant that can reflect the effect of inertia, and the expression is as follows:

$$A = \frac{\rho_s}{\varepsilon_d}$$

where $\rho_s$, $\rho_d$, $\varepsilon_d$ are the relative density, the density of the base material and the densification strain of BRH-II.

According to the data listed in Table 1, the parameters in Eq. (12) are fitted by the least square method. Figure 19 shows the plateau stress comparisons between numerical results and fitting results. The blue dot in the map is the numerical result, and the red solid line is the fitting results. It is obvious that the simulated values are in good agreement with the fitting results. Table 2 further lists the dynamic plateau stress mathematical models for BRH-II with different relative densities. It can be obtained from the table that the constant $A$ is $6.76 \times 10^{-5}$, $1.01 \times 10^{-4}$, $1.51 \times 10^{-4}$, $2.25 \times 10^{-4}$, $2.78 \times 10^{-4}$ and $3.29 \times 10^{-4}$, respectively, corresponding to different densities ($\rho = 0.044$, $\rho = 0.062$, $\rho = 0.089$, $\rho = 0.133$, $\rho = 0.177$ and $\rho = 0.222$). According to the least square method, the correlation relative density and constant $A$ of BRH-II can be further modeled as:

$$A(\rho) = C_1 \rho^2 + C_2 \rho + C_3$$

where $C_1$, $C_2$ and $C_3$ are fitting constants. Figure 20 gives the relationship between the fitting results of constant $A$ and relative density. It can be found that Eq. (14) can accurately predict the value of constant $A$ under different relative densities. The fitting constants $C_1$, $C_2$ and $C_3$ obtained by the least squares method are, respectively, $-3.13 \times 10^{-3}$, $2.3 \times 10^{-3}$ and $-2.89 \times 10^{-5}$. The $R^2$ of the fitting results of Figs. 18, 19, and 20 is shown in Table 3. Thus, the final expression of the dynamic plateau stress of BRH-II under different impact velocities can be expressed as follow:

![Table 1](image)

| Relative density | Impact velocity (m/s) | 1 | 20 | 50 | 70 | 100 | 125 | 150 | 175 |
|------------------|-----------------------|---|----|----|----|-----|-----|-----|-----|
| $\bar{\rho} = 0.044$ | 0.171 | 0.193 | 0.232 | 0.416 | 0.776 | 1.237 | 1.761 | 2.233 |
| $\bar{\rho} = 0.062$ | 0.352 | 0.372 | 0.42 | 0.632 | 1.138 | 1.928 | 2.73 | 3.496 |
| $\bar{\rho} = 0.089$ | 0.782 | 0.868 | 1.09 | 1.5 | 1.991 | 2.904 | 4.365 | 5.469 |
| $\bar{\rho} = 0.133$ | 1.749 | 1.908 | 2.216 | 2.547 | 3.591 | 4.675 | 6.989 | 9.156 |
| $\bar{\rho} = 0.177$ | 3.03 | 3.583 | 4.251 | 4.546 | 5.803 | 6.586 | 8.894 | 12.164 |
| $\bar{\rho} = 0.222$ | 4.975 | 5.426 | 5.986 | 6.475 | 7.406 | 9.802 | 12.358 | 15.502 |
| $\bar{\rho} = 0.267$ | 7.531 | 8.249 | 8.379 | 8.621 | 10.237 | 12.264 | 17.159 | 19.707 |

Fig. 18 Quasi-static plateau stress comparisons between simulation results and fitting formula Eq. (11)

![Figure](image)
5.3 Shrinkage deformation and negative Poisson’s ratio (NPR) effect

In order to systematically study the shrinkage deformation characteristics of BRH-II, the horizontal strain of each layer of BRH-II under different impact velocities will be extracted for comparative analysis. The calculation formulas and its corresponding diagram are shown in Eqs. (13–15) and Fig. 11.

Figure 21 shows the horizontal strain of each layer of BRH-II under low, medium and high impact velocities. It is noticeable that all the horizontal strains of each layer of BRH-II under different impact velocities show an upward trend with the increase in vertical strain. The first five
layers of BRH-II have uneven horizontal strains at a fixed impact velocity, which are caused by uneven horizontal deformation. Quite the opposite, the sixth layer has almost no deformation. Under low- and high-velocity impact, the maximum horizontal strain appears in the third and fourth layers, respectively. Meanwhile, the strain curves under medium- and high-velocity impact are both smoother than those under low-velocity impact. Also note that under high-velocity impact, the shrinkage deformation ability of the first layer will not be weakened like the other layers. This is because the effect of the plastic hinge with opposite rotation direction in the unit body (Fig. 10) is strengthened under high-velocity impact. Although the horizontal strain of BRH-II will be affected by the structural failure under high-velocity impact [20], because the strength of the upper half of the BRH-II unit is much smaller than the lower half, and this effect occurs before the lower half is densified, as shown in Fig. 14c and d.

As shown in Fig. 21a, the horizontal strains of the third or fourth layers of BRH-II in the early stage of low-velocity impact are all greater than other layers. This is due to the appearance of the X-shaped deformation mode, which results in a large shrinkage deformation of the middle layer. Under medium and high velocities, the horizontal strain of the (N−1)th layer of BRH-II at the initial stage of impact is greater than that of the Nth layer, which is consistent with the layer-by-layer deformation mechanism.

5.4 Energy absorption capacity

As stated above, SEA is also an important index for evaluating crashworthiness. Therefore, in this section the energy absorption capacity of BRH-II is further studied. Figure 22 shows the influence of the impact velocity and relative density on SEA of BRH-II. In order to show that the impact velocity and relative density of BRH-II have a coupling effect on the SEA, contour map of the SEA under different impact velocities and relative densities is given in Fig. 22b. As shown in Fig. 22a, the SEA of BRH-II keeps increasing as the impact velocity and relative density increases. However, as the impact velocity increases, the SEA of BRH-II under different relative densities tends to gradually converge. It demonstrates that the SEA of
BRH-II is more sensitive to impact velocity at low relative density, and the influence of relative density on the energy absorption capacity of BRH-II will gradually weaken with the increase in impact velocity.

Considering that the structural parameters have a very important influence on its crashworthiness, Fig. 23 shows the plateau stress and SEA of BRH-II with different branch angles $\beta$, and the unit cells of BRH-II under different branch angles $\beta$ correspondingly are shown. As shown in Fig. 23, the plateau stress shows an upward trend with the increase in $\beta$, while SEA first increases and then decreases. This is mainly because that when the branch angle is greater than 74°, although the total energy absorption can be increased with the further increase in $\beta$, the increments in total energy absorption is smaller than the mass increments.

Meanwhile, considering that the BRH-II unit is the up-down asymmetric structure, it is very meaningful to study its arrangement. In order to study the impact of BRH-II arrangement on its crashworthiness, three different arrangements (a) (b) (c) are designed, as shown in Fig. 24. It can be seen from Fig. 25 that different arrangements of BRH-II have different SEA and plateau stresses, and arrangement (c) has the largest SEA and plateau stress values. It shows that different arrangements have a very important influence on the SEA and plateau stress of BRH-II.
6 Conclusions

Based on biological inspiration, this paper chooses lotus leaf veins as the biological prototype, and designs a new type of bio-inspired re-entrant honeycomb (BRH) with negative Poisson’s ratio. Based on numerical simulation, the compression behavior and energy absorption capacity of BRH-I, BRH-II and RH under different impact speeds are compared. Subsequently, the mechanical properties of BRH-II were systematically studied. The following conclusions can be drawn:

1. BRH has greater plateau stress and energy absorption capacity than conventional RH, especially at impact velocities of 1 m/s and 50 m/s. The SEA of BRH-I is, respectively, 388.5% and 241.56% higher than conventional RH, and the growth rate of BRH-II, respectively, reaches 644.5% and 346.93%. Under the 100 m/s high-velocity impact, the SEA gap between the three honeycomb structures has narrowed, but BRH-I and BRH-II still have outstanding energy absorption capacity with growth rates reaching 42.08% and 51.49%, respectively. In addition, BRH shows better shrinkage deformation characteristics than conventional RH.

2. The deformation modes of BRH-II can be divided into three types, namely quasi-static mode, transition mode and dynamic crushing mode. Under low- and medium-velocity impact, the honeycomb structure shows overall plastic deformation. The obvious X-shaped local shear zone first appears in the quasi-static crushing mode. When the impact velocity is high enough, inertia effect plays a leading role and the ‘‘I’’-shaped local deformation band can be observed at the impact end. The empirical formula of critical velocity is then given based on the classification diagram of BRH-II.

3. The plateau stress and SEA of BRH-II increase with the increase in relative density and impact velocity. The SEA of BRH-II is more sensitive to impact velocity at low relative density, and the influence of relative density will gradually weaken with the increase in impact velocity. The empirical formulas are given to predict the dynamic plateau stress of BRH-II at the impact end. In addition, the branch angle $\beta$ and arrangement of BRH-II have an important influence on its plateau stress and energy absorption capacity.

4. BRH-II shows different shrinkage deformation characteristics and negative Poisson's ratio effects under different impact velocities. Under the same impact velocity, there is a big difference in the horizontal strain between the layers of BRH-II. At higher impact velocities, the shrinkage deformation of the $(N-1)$th layer is earlier than that of the $N$th layer.

Acknowledgments This work was financially supported by The National Natural Science Foundation of China (No. 51705215), The Graduate Student Scientific Research Innovation Projects in Jiangsu Province (No. KYCX21_3444). The authors would like to express their thanks.

References

1. Gibson LJ, Ashby MF (1997) Cellular solids: structure and properties. Pergamon Press, Oxford
2. Gibson LJ, Ashby MF, Schajer GS, Robertson CI (1982) The mechanics of two dimensional cellular materials. Proc R Soc A Math Phys Eng Sci 382:25–42
3. Wu Y, Fang J, He Y, Li W (2018) Crashworthiness of hierarchical circular-joint quadrangular honeycombs. Thin-Walled Struct 133:180–191
4. Smahat A, Mankour A, Slimane S, Roubache R, Bendine K, Guelailia A (2020) Numerical investigation of debris impact on spacecraft structure at hyper-high velocity. J Braz Soc Mech Sci Eng 42(3):117
5. Xie SC, Feng ZJ, Zhou H et al (2020) In-plane and out-of-plane compressive mechanical properties of Nomex honeycombs and their prediction. J Braz Soc Mech Sci Eng 42:460
6. He Q, Feng J, Zhou HG (2019) A numerical study on the in-plane dynamic crushing of self-similar hierarchical honeycombs. Mech Mater 138:103151.1-103151.15
7. Liang S, Chen HL (2006) Investigation on the square cell honeycomb structures under axial loading. Compos Struct 72:446–454
8. Hu LL, Yu TX, Gao ZY, Huang XQ (2008) The inhomogeneous deformation of polycarbonate circular honeycombs under in-plane compression. Int J Mech Sci 50(7):1224–1236
9. Crici G, Perrella M, Calì C (2013) Honeycomb failure processes under in-plane loading. Compos B 45:1079–1090
10. Yang XF, Ma JX, Sun YX, Yang JL (2018) Ripplecomb: a novel triangular tube reinforced corrugated honeycomb for energy absorption. Compos Struct 202:988–999
11. Ju J, Summers JD (2011) Compliant hexagonal periodic lattice structures having both high shear strength and high shear strain. Mater Des 32(2):512–524
12. Tan HL, He ZC, Li KX, Li E, Cheng AG, Xu B (2019) In-plane crashworthiness of re-entrant hierarchical honeycombs with negative Poisson’s ratio. Compos Struct 229:111415
13. Qi C, Remennikov A, Pei LZ, Yang S, Yu ZH, Ngo TD (2017) Impact and close-in blast response of auxetic honeycomb-cored sandwich panels: experimental tests and numerical simulations. Compos Struct 180(15):161–178
14. Wu HX, Liu Y, Zhang XC (2018) In-plane crushing behavior and energy absorption design of composite honeycombs. Acta Mech Sin PRC 34:1108–1123
15. Wang H, Lu ZX, Yang ZY, Li X (2019) A novel in-plane auxetic honeycomb with enhanced in-plane impact resistance. Compos Struct 208:758–770
16. Zied K, Osman M, Elmahdy T (2015) Enhancement of the in-plane stiffness of the hexagonal re-entrant auxetic honeycomb cores. Phys Status Solidi B 252:2685–2692
17. Lakes RS (1987) Foam structures with a negative Poisson’s ratio. Science 235:1038–1040
18. Hou X, Hu H, Silberschmidt V (2014) Numerical analysis of composite structure with in-plane isotropic negative Poisson’s ratio: effects of materials properties and geometry features of inclusions. Compos Part B 58:152–159
19. Qin H, Yang D, Ren C (2018) Modelling theory of functional element design for metamaterials with arbitrary negative Poisson’s ratio. Comput Mater Sci 150:121–133
20. Gao Q, Wang L, Zhou Z, Ma Z, Wang C, Wang Y (2018) Theoretical, numerical and experimental analysis of three-dimensional double-V honeycomb. Mater Des 139:380–391
21. Hu LL, Zhou MZ, Deng H (2018) Dynamic crushing response of auxetic honeycombs under large deformation: theoretical analysis and numerical simulation. Thin-Walled Struct 131:373–384
22. Xiao D, Kang X, Li Y, Wu W, Lu J, Zhao G, Fang D (2019) Insight into the negative Poisson’s ratio effect of metallic auxetic reentrant honeycomb under dynamic compression. Mater Sci Eng A 763:138151
23. Tan HL, He ZC, Li KX, Eric Li AG, Cheng BX (2019) In-plane crashworthiness of re-entrant hierarchical honeycombs with negative Poisson’s ratio. Compos Struct 229:111415
24. Yang XF, Xi XL, Pan QF, Liu H (2019) In-plane dynamic crushing of a novel circular-celled honeycomb nested with petal-shaped mesostructure. Compos Struct 226:111219
25. Wang H, Zhang Y, Lin W, Qin Q (2020) A novel two-dimensional mechanical metamaterial with negative Poisson’s ratio. Comput Mater Sci 171:109232
26. Yin HF, Xiao YY, Wen GL et al (2015) Crushing analysis and multi-objective optimization design for bionic thin-walled structure. Mater Des 87:825–834
27. Ha NS, Lu GX (2020) A review of recent research on bio-inspired structures and materials for energy absorption applications. Compos Part B 181:107496
28. Zou M, Xu SC, Wei CG et al (2016) A bionic method for the crashworthiness design of thin-walled structures inspired by bamboo. Thin-Walled Struct 101:222–230
29. Song JF, Xu SC, Wang HX et al (2018) Bionic design and multi-objective optimization for variable wall thickness tube inspired bamboo structures. Thin-Walled Struct 125:76–88
30. Jiang HY, Ren YR et al (2020) Crashworthiness of novel concentric auxetic reentrant honeycomb with negative Poisson’s ratio biologically inspired by coconut palm. Thin-Walled Struct 154:106911
31. He Q, Wang YH, Gu H, Feng J, Zhou HG (2021) Dynamic crushing analysis of a circular honeycomb with leaf vein branched characteristic. Mech Mater 153:103566.1-103566.14
32. Sabah SHA, Kueh ABH, Al-Fasih MY (2017) Comparative low-velocity impact behavior of bio-inspired and conventional sandwich composite beams. Compos Sci Technol 149:64–74
33. Xiang J, Du J, Li D, Scarpa F (2017) Numerical analysis of the impact resistance in aluminum alloy bi-tubular thin-walled structures designs inspired by beetle elytra. J Mater Sci 52(22):13247–13260
34. Shang JS, Ngern NHH, Tan VBC (2016) Crustacean-inspired helicoidal laminates. Compos Sci Technol 128:222–232
35. Liu Q, Ma J, He Z, Hu Z, Hui D (2017) Energy absorption of bio-inspired multi-cell CFRP and aluminum square tubes. Compos Part B 121:134–144
36. Jiang H, Ren Y, Liu Z, Zhang S, Lin Z (2019) Low-velocity impact resistance behaviors of bio-inspired helicoidal composite laminates with non-linear rotation angle based layups. Compos Struct 214:463–475
37. Zhang DH, Feic QG, Zhang PW (2017) In-plane dynamic crushing behavior and energy absorption of honeycombs with a novel type of multi-cells. Thin-Walled Struct 117:199–210
38. Zou Z, Reid SR, Tan PJ, Li S, Harrigan JJ (2009) Dynamic crushing of honeycombs and features of shock fronts. Int J Impact Eng 36:165–176
39. Reid SR, Peng C (1997) Peng C, dynamic uniaxial crushing of wood. Int J Impact Eng 19:531–570

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.