Nuclear fusion as a probe for octupole deformation in $^{224}$Ra

Raj Kumar, J. A. Lay and A. Vitturi

Dipartimento di Fisica e Astronomia “Galileo Galilei”,
Università di Padova, via Marzolo, 8, I-35131 Padova, Italy and
INFN, Sezione di Padova, via Marzolo, 8, I-35131 Padova, Italy

Background Nuclear fusion has been shown to be a perfect probe to study the different nuclear shapes. However, the possibility of testing octupole deformation of a nucleus with this tool has not been fully explored yet. The presence of a static octupole deformation in nuclei will enhance a possible permanent electric dipole moment, leading to a possible demonstration of parity violation.

Purpose To check whether static octupole deformation or octupole vibration in fusion give qualitatively different results so that both situations can be experimentally disentangled.

Method Fusion cross sections are computed in the Coupled-Channels formalism making use of the Ingoing-Wave Boundary Conditions (IWBC) for the systems $^{16}$O+$^{144}$Ba and $^{16}$O+$^{224}$Ra.

Results Barrier distributions of the two considered schemes show different patterns. For the $^{224}$Ra case, the octupole deformation parameter is large enough to create a sizeable difference.

Conclusions The measurement of barrier distributions can be an excellent probe to clarify the presence of octupole deformation.

PACS numbers: 25.60.Pj, 24.10.Eq

I. INTRODUCTION

The search of octupole deformations in nuclei is living a revival thanks to its impact in a possible permanent atomic electric dipole moment (EDM) $^{[1]}$ $^{[2]}$. A non-zero EDM will indicate a time-reversal (or equivalently charge-parity) violation. The magnitude or the experimental maximum limit to it can constrain the different suggested extensions to the standard model $^{[3]}$. The presence of a static octupole deformation in an odd nucleus will generate enhanced nuclear Schiff moments, which contributes to the atomic EDM so that it can be improved by several orders of magnitude $^{[4]}$.

Therefore, the experimental focus is set on the search of permanent octupole deformations in some of the regions where different theoretical approaches predict strong octupole correlations $^{[5]}$ $^{[6]}$ with the help of continuous development of the radioactive beam facilities. In the present work we will focus on $^{144}$Ba and $^{224}$Ra as representative of two different regions where possible static deformations have been predicted.

In $^{[1]}$ $^{[7]}$, Coulomb excitation has been used to measure the different electric transition probabilities. This tool provides quadrupole and octupole transition probabilities with good accuracy. Smaller dipole electric transition probabilities will carry larger error bars, even though this problem is experimentally affordable and they were able to provide some measurements. It should be kept in mind that large octupole transitions can be found also for dynamic octupole vibrations. In addition, the coupling between quadrupole deformation and octupole vibrations can lead to enhanced dipole moments $^{[8]}$.

Therefore, we would like to propose here a complimentary experimental probe for static octupole deformation. A traditional experimental tool for the study of nuclear structure is provided by subbarrier fusion. It is well known that fusion at energies around the Coulomb barrier is driven by the dynamical couplings to the internal degrees of freedom of the fusing counterparts $^{[8]}$ $^{[9]}$. The absence or presence of octupole and dipole moments in one of the fusing partners will have a certain impact in the final fusion cross section. This fact could allow us, if the minimum accuracy is reached, to distinguish between static octupole deformation and the corresponding dynamical vibration.

The work is structured as follows. In Sec. II we recall the reaction formalism for studying nuclear fusion including deformations and/or vibrations. We apply this framework in Sec. III to the reactions $^{16}$O+$^{144}$Ba and $^{16}$O+$^{224}$Ra considering quadrupole deformations for $^{144}$Ba and $^{224}$Ra looking for the differences between adding an octupole vibration or deformation to the previous quadrupole deformation. Finally, in Sec. IV the main results of this work are summarized.

II. REACTION FRAMEWORK

Fusion probabilities are calculated by solving the corresponding coupled-channel equations under ingoing-wave boundary conditions (IWBC). The coupled-channel formalism for direct reaction processes given by Austern $^{[10]}$ expands the total wave function in terms of the wavefunction for the internal state of the projectile $\phi_\beta$ and the radial wave functions $\chi_\beta$ that accounts for the relative
motion between projectile and target:

$$\Psi(+) = \sum_\beta \frac{\chi_\beta(R)}{R} \phi_\beta.$$  \hspace{1cm} (1)

This leads to a set of coupled equations for the radial wave functions:

$$\frac{d^2\chi_\beta}{dR^2} - \frac{L(L+1)}{R^2} + \frac{2\mu}{R^2} [E_\beta - V^{eff}_\beta(R)] \chi_\beta = \frac{2\mu}{R^2} \sum_{\alpha \neq \beta} V^{comp}_{\beta\alpha}(R) \chi_\alpha$$  \hspace{1cm} (2)

In these expressions $V$ is the interaction potential while, $\beta$ is a certain channel, $\mu$ is the reduced mass, and $E_\beta$ is the relative energy. Assuming only inelastic excitations, $\mu$ does not depend on the channel and $E_\beta$ is simply $E_\beta = E + Q_\beta$, being $Q_\beta$ the $Q$-value for the reaction going from the incoming channel to a certain channel $\beta$. Each channel $\beta$ correspond to a set of quantum numbers $\{I, L, J\}$, where $I$ is the angular momentum of the internal state $\phi_\beta$, $L$ is the angular momentum in the relative coordinate, and both are coupled to a total angular momentum $J$.

As a simplification we can use the same central potential for the $V^{eff}_\beta(R)$ although this potential is modified by the terms $V^{comp}_{\beta\alpha}$ known as reorientation terms. The non central terms $V^{comp}_{\beta\alpha}$ are those on charge of the coupling between two different channels $\alpha$ and $\beta$. Central, coupling and reorientation potentials have different forms depending on the physical case and physical meanings of the channels. For all the cases, we will consider the central ion-ion potential, a Woods-Saxon potential with the parametrization from Akyüz-Winther [11] [12] plus the corresponding Coulomb repulsion.

In our case we will consider one of the two ions to be a quadrupole deformed rotor. We will also look at cases where the deformed nuclei has an octupole vibration or a static octupole deformation together with the quadrupole one.

In the case of permanent deformation, we can just modify the original spherical Woods-Saxon to have a radius with a dependence on the angle. Indeed, the distance from the two surfaces (one spherical and the other one deformed) can be characterized by a function of the angle $\theta'$, defined with respect to an intrinsic body-fixed frame,

$$R(\theta') = R_0 + \sum_\lambda \delta_\lambda Y^{*}_{\lambda0}(\theta') \equiv R_0 + \hat{\Delta}(\Omega')$$  \hspace{1cm} (3)

where $R_0$ is an average radius of the core. Axial symmetry has been assumed. The remaining term (denoted $\hat{\Delta}(\Omega')$) represents the deviation of the radius for a particular point on the surface from this average radius. The function $\hat{\Delta}(\Omega')$ is sometimes referred to as shift-function. The quantities $\delta_\lambda$ are the deformation lengths. From these deformation lengths, a dimensionless deformation parameter $\beta_\lambda$ can be defined as:

$$\delta_\lambda = \beta_\lambda \cdot R_0.$$  \hspace{1cm} (4)

\textbf{TABLE I:} Theoretical deformation parameters expected for $^{144}\text{Ba}$ and $^{224}\text{Ra}$ according to [13] for the different multipoolarities.

|        | $^{144}\text{Ba}$ | $^{224}\text{Ra}$ |
|--------|-------------------|-------------------|
| $\beta_2$ | 0.149            | 0.138             |
| $\beta_3$ | 0.068            | 0.099             |

The nuclei of interest in the present work are candidates to have a permanent octupole deformation together with the quadrupole one. In Table I we show the deformation parameters used in this work following the theoretical predictions from [13].

If one assumes that the potential is still a function of the distance between the valence particle and the surface of the core, the interaction potential will follow the same functional dependence as $V(R - R_0)$, but replacing $R_0$ by $R(\theta', \phi')$ or $R(\theta')$ in case of axial symmetry:

$$V(R, \Omega') = V(R - R(\theta')).$$  \hspace{1cm} (5)

This expression is expanded in multipoles as:

$$V(R, \Omega') = \sum_\lambda V_\lambda(R) Y^*_{\lambda0}(\theta')$$  \hspace{1cm} (6)

with

$$V_\lambda(R) = 2\pi \int_{-1}^{1} V(R - R(\theta')) Y_{\lambda0}(\theta', 0) d(cos \theta')$$  \hspace{1cm} (7)

The angular variables in these expressions are referred to the axial symmetry axis, but can be converted to the laboratory frame (characterized by the variables $\theta, \phi$) by means of the transformation:

$$Y^*_{\lambda0}(\theta', 0) = \sum_\mu D^\lambda_\mu(\alpha, \beta, \gamma) Y^*_{\lambda\mu}(\theta, \phi)$$

where $D$ is the so called rotation matrix (or D-matrix) [14]. Its arguments $\alpha, \beta$ and $\gamma$ are the Euler angles describing the transformation from the body-fixed frame to the laboratory frame.

Replacing this expression in (6)

$$V(R, \Omega) = \sum_\lambda V_\lambda(R) D^\lambda_\mu(\alpha, \beta, \gamma) Y^*_{\lambda\mu}(\hat{R})$$  \hspace{1cm} (8)

The spherical harmonic will connect two states $\chi_\alpha(R)$ and $\chi_\beta(R)$ depending on the order $\lambda$. Therefore, many physical channels should be included in the full coupled channels calculation which quickly becomes too large for practical purposes.

In order to overcome this limitation, we will make use of the iso-centrifugal approximation [15] [17]. We will first neglect the effect of the change in the centrifugal barrier
due to the excitation of the intrinsic degree of freedom. Then a weighted average wavefunction is defined as:

$$\tilde{\chi}_I(R) = (-1)^I \sum_L \langle J0I0|L0 \rangle \chi_\beta(R).$$ \hspace{1cm} (9)

This average wavefunction satisfies the following coupled equations:

$$\frac{d^2 \tilde{\chi}_I}{dR^2} - \frac{L(L+1)}{R^2} + \frac{2\mu}{h^2} \left[ E_\beta - V_{\beta I}^\prime(\tilde{\chi}) \right] \tilde{\chi}_I = \frac{2\mu}{h^2} \sum_{I' \neq I} \sqrt{\frac{2\lambda + 1}{4\pi}} \langle KI||V_{\alpha \beta}^\lambda(\tilde{\chi})||KI' \rangle \tilde{\chi}_I,$$ \hspace{1cm} (10)

with same boundary conditions.

Using the definition of Bohr and Mottelson \cite{14} for reduced matrix elements, and considering the reduced matrix elements of the D-matrix between the corresponding two rotational states \cite{13} we obtain:

$$\langle KI||D_{\alpha \beta}^{\lambda \mu}(\tilde{\chi})||KI' \rangle = \hat{I} \langle IK\lambda I' K \rangle,$$ \hspace{1cm} (11)

where the extra quantum number $K$ has been added to the internal states in order to specify the bandhead to which the states $I$ and $I'$ belong. We finally arrive to \cite{14,18}:

$$\langle KI||V_{\alpha \beta}^\lambda(\tilde{\chi})||KI' \rangle = V_\lambda(R) \hat{I} \langle IK\lambda I' K \rangle.$$ \hspace{1cm} (12)

On the other hand, the coupling may also arise from a vibration of one of the nucleus. In this case this vibration is characterized as a variation on the surface of the form:

$$R(\xi) = R_0 \left[ 1 + \sum_{\lambda,\mu} \alpha_{\lambda \mu} Y_\lambda^{\mu}(\tilde{\chi}) \right] \equiv R_0 + \Delta(\tilde{\chi})$$ \hspace{1cm} (13)

with $\Delta(\tilde{\chi}) = R_0 \sum_{\lambda,\mu} \alpha_{\lambda \mu} Y_\lambda^{\mu}(\tilde{\chi})$ and where $\alpha_{\lambda \mu}$ are to be understood as dynamical variables, given in terms of phonon creation ($b_\lambda^{\mu}$) and annihilation ($b_\lambda^{\mu}$) operators as \cite{19}:

$$\alpha_{\lambda \mu} = \frac{\beta_\lambda}{\sqrt{2\lambda + 1}} \left[ b_\lambda^{\mu} + (-1)^\mu b_{\lambda - \mu} \right].$$ \hspace{1cm} (14)

The nuclear coupling between the ground state and the first one phonon state of multipolarity $\lambda$ can be obtained thanks to the properties of the creation and annihilation operators:

$$\langle 1, I\mu|V(R - R(\xi))|0, 0 \rangle = \langle 0, 0|b_{\lambda \mu}, V(R - R(\xi))|0, 0 \rangle = \frac{\beta_\lambda}{\lambda} \langle 0|\frac{\partial V}{\partial \alpha_{\lambda \mu}}|0 \rangle,$$ \hspace{1cm} (15)

where

$$\frac{\partial V}{\partial \alpha_{\lambda \mu}} = -R_0 \frac{\partial V}{\partial R} Y_{\lambda \mu}^*(\tilde{R}).$$ \hspace{1cm} (16)

However, we will consider an octupole vibration on top of a quadrupole deformed nucleus instead of a pure spherical nucleus. In this case, the derivative of the potential has a certain dependence of the orientation. We can express such dependence in the form:

$$V_{\text{coup}}(R, \xi) = -\frac{\beta_3}{3} R_0 \frac{\partial V}{\partial R} (R - R_0 [1 + \beta_2 Y_{20}]) Y_{30},$$ \hspace{1cm} (17)

which we later expand in spherical harmonics as done for the full rotor case, i.e. following Eqs. (5) and (7).

One should remind that this potential is only coupling zero octupole phonon states with one octupole phonon states. Together with these vibrational couplings, the traditional quadrupole deformed potential will only act between states with the same number of phonons.

The derivation of several non trivial cases can be found in \cite{19} and \cite{18}.

On top of all these nuclear couplings considered here, high order Coulomb couplings should be considered. However, it has been shown that Coulomb couplings has a minor role compare to the nuclear ones \cite{20} and, therefore, they will not be included in the present calculations.

Once the potentials are defined, Eq. (10) can be solved with the following boundary conditions:

$$\tilde{\chi}_I(R) \xrightarrow{R \to \infty} \delta_{10} H_1^{(-)}(k_\beta R) + r_1 H_1^{(+)}(k_1 R),$$

$$\tilde{\chi}_I(R = R_{\text{min}}) = t_1 H_1^{(-)}(k_1 R),$$ \hspace{1cm} (18)

where $H_1^{(+)}$ and $H_1^{(-)}$ are the outgoing and incoming Coulomb wave functions, respectively and $k_I = \sqrt{2\mu(E - \varepsilon_I)/\hbar^2}$ is the wave number associated with the energy $E$. The total transmission probability is then

$$T = \sum_l |T_l|^2 = \sum_l \frac{\psi_l}{\psi_0} |t_l|^2$$ \hspace{1cm} (19)

where $\psi_l$ is the velocity corresponding to channel I.

The fusion cross-section, in terms of partial waves, is given by

$$\sigma = \sum_{L=0}^{L_{\text{max}}} \sigma_L = \frac{\pi \hbar^2}{2 \mu E} \sum_{L=0}^{L_{\text{max}}} (2L + 1) T_L(E).$$ \hspace{1cm} (20)

The probability of transmission for the partial wave can also be calculated simply by a shift of energy,

$$T_L \approx T_0 \left[ E - \frac{L(L+1)\hbar^2}{2 \mu R_0^2} \right],$$ \hspace{1cm} (21)

where $R_0$ is the position of the barrier for the s-wave. Hereafter we will refer as transmission probability $T$ to the probability of transmission for the s-wave $T_0$ \cite{8,21}.
In the present work we will define as barrier distribution the energy derivative of the transmission probability. Its name is related with its sensitivity to the number and nature of the different channels included in the calculation. This quantity is approximately proportional to the second derivative of the product of the cross section and the energy ($E\sigma$), thus being a direct link between nuclear structure and this experimental observable [8, 22].

III. APPLICATIONS

A. Prolate vs. Oblate deformations

Fusion reaction is one of the experimental tests that can discriminate between prolate and oblate deformations. As seen before, a change on the sign of the deformation length will change the sign of the coupling and reorientation terms.

The sign of the off-diagonal coupling potential does not have an effect in the cross section. A simple explanation arise from thinking how to solve our Hamiltonian computing matrix elements in a certain basis and obtaining eigenvectors and eigenvalues. In our case, the coupling potential is an off-diagonal value and its sign only changes the relative sign of the components of the eigenvectors without altering the eigenvalues. Therefore, observables as the cross sections will remain unperturbed. Instead, the reorientation term is on-diagonal, so being the main responsible for the change in the cross section when one of the counter partners is either oblate or prolate.

In addition, the coupling term is equal in first order in the vibrational and the deformed case if we consider the same value for the parameter $\beta_2$. However the reorientation terms are not present in the vibrational case unless we consider higher order terms.

In Fig. 1 we show the barrier distribution for the fusion of the system $^{16}$O+$^{144}$Ba considering a positive quadrupole deformation parameter, solid line, and a negative one, dashed line. In this calculation and hereafter, we consider only the first three levels of a typical quadrupole deformed rotor, including all allowed couplings. The influence of higher energy and spin levels depends on the nucleus. For $^{144}$Ba we have studied the variation of the barrier distribution with the number of levels included, as shown in Fig. 2. From this analysis we conclude to consider only up to the $6^+$ level since including more levels does not alter dramatically the barrier distribution. Similar results are found for $^{224}$Ra.

Additional examples of fusion reactions with prolate and oblate nuclei can be found in [8, 22].

B. Coupled-Channels vs. frozen approximation

The effect of static deformation was previously studied in terms of the frozen approximation in [24] for quadrupole and in [25] for octupole deformation. Within this formalism the energies of the different states are neglected. As a consequence, the total cross section can be defined as the average cross section between those obtained in a single channel calculation for the different barriers found for each possible orientation of the deformed nucleus. This fact simplifies the calculations but it has been shown to overestimate the cross sections at energies below the barrier [8, 26].

In order to test the validity of the frozen approxima-
FIG. 3: (Color online) Cross sections (upper panel) for the fusion of $^{16}$O and $^{144}$Ba considering $^{144}$Ba as a spherical nuclei (solid line) and including a positive quadrupole deformation within the frozen approximation (dotted line) and within the coupled channels framework (dashed line). For the coupled channel calculation we have included levels up to $6^+$. The effect of including more levels is negligible and would not be distinguishible in the present figure. In the lower pannel we show the corresponding barrier distributions including a positive quadrupole deformation within the frozen approximation (dashed line) and within the coupled channels framework (solid line).

FIG. 4: (Color online) Level schemes considered for a quadrupole octupole deformed nucleus (upper panel) and for a quadrupole deformed nucleus with an octupole vibration (lower panel). Each arrow can be related to transitions with more than one possible multipolarity $\lambda$.

C. Difference between octupole deformation and vibration

A more interesting and up-to-date case is to check if barrier distributions are sensitive enough to the difference between a nucleus with both quadrupole and octupole static deformations and a quadrupole deformed nucleus with an octupole vibration. The levels and the coupling potentials between the two cases will be different. In
for the structure of \( ^{16}\text{O}^{+144}\text{Ba} \) for each multipolarity are shown in Fig. 5 again for the two different schemes considered. Differences between both schemes are small but significant. The presence of an octupole deformation affects to the quadrupole strength and viceversa. Even though Fig. 5 only includes up to \( \lambda = 6 \), larger multipolarities have been included. Hereafter we will refer to this two cases as octupole deformation case and octupole vibration case.

With the procedure discussed in section II we calculated the barrier distributions for two systems \( ^{16}\text{O}^{+144}\text{Ba} \) and \( ^{16}\text{O}^{+224}\text{Ra} \). Both \( ^{144}\text{Ba} \) and \( ^{224}\text{Ra} \) candidates to have an octupole deformation with a considerable \( \beta_3 \). States of the projectile are not considered. Results are shown in Fig. 6. Barrier distributions for both two cases are qualitatively different. The change in the \( ^{144}\text{Ba} \) case is small but for \( ^{224}\text{Ra} \) it is large enough to open the possibility of distinguishing the two situations by measuring the barrier distribution.

### IV. CONCLUSIONS

In this work we have studied the fusion reactions \( ^{16}\text{O}^{+144}\text{Ba} \) and \( ^{16}\text{O}^{+224}\text{Ra} \) under different assumptions for the structure of \( ^{144}\text{Ba} \) and \( ^{224}\text{Ra} \). We started introducing only a quadrupole deformation and studying the sensitivity of the barrier distributions to the amount of levels included and to the possibility of having a prolate or oblate shape.

Keeping just this quadrupole deformation we have compared the results of the coupled-channel calculation with the results of the frozen approximation. We show how the frozen approximation clearly overestimates the total cross section and, therefore, can give rise to misleading conclusions.

Finally, we analyze the effect of adding an octupole deformation or vibration. Even though the differences in the form factors are not large and the same levels are included, the fusion cross section is sensitive enough to change from one case to the other. Both cases considered, \( ^{16}\text{O}^{+144}\text{Ba} \) and \( ^{16}\text{O}^{+224}\text{Ra} \), show different barrier distributions. This difference is larger in the \( ^{224}\text{Ra} \) case. However, the final possibility of disentangle both distributions will depend on the available accuracy for each case in a hypothetical future experiment.

Since the search for a large EDM needs large static octupole deformations, it should be possible to ensure its presence by analyzing the subbarrier fusion cross section of the different candidates. A sizeable impact in the bar-
rier distribution is expected and, therefore, subbarrier fusion can be a valuable tool to clarify the presence of static octupole deformations.

To sum up, we would like to stress that fusion reactions have been of great help on understanding the structure of heavy ions for many years. In this particular case, it can also help the community to find the perfect candidate for large EDM’s and, consequently, to test models beyond the standard model.

Acknowledgments

This work has been supported by MIUR research fund PRIN 2009TWL3MX. The research leading to these results has received funding from the European Commission, Seventh Framework Programme (FP7/2007-2013) under Grant Agreement nº 600376. J.A.L. is a Marie Curie Piscopia fellow at the University of Padova.

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