On Mechanism of Charmed $c$–Quarks Fragmentation in Hadronic Collisions

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We propose a modification of the mechanism of charmed quarks fragmentation into $D$ mesons in hadronic collisions. It is shown that the distinction in valence quark distributions in the initial $\pi^\pm$ and $K^\pm$ mesons leads to different inclusive spectra of $D$ and $D_s$ mesons produced in $\pi^\pm$ and $K^\pm$ beams.

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1 INTRODUCTION

Hadroproduction of the particles with open charm, among other reactions ($e^+e^- \rightarrow c\bar{c}$ etc) is of interesting possibility to study a hadronization process of the heavy quarks, such as $c \rightarrow D, \Lambda_c, ...$.

It is well known that the production of charmed particles in $e^+e^-$-annihilation,

$$e^+e^- \rightarrow D X,$$

may be separated into a process of $c\bar{c}$-pair production and a process of independent fragmentation of each $c$ ($\bar{c}$) quark into $D$-meson:

$$c(\bar{c}) \rightarrow D(\bar{D}) X.$$

In so doing the fragmentation process is described by means of fragmentation function $D(z)$, where $z = p_D/p_c$ ($p_D$ and $p_c$ are the momenta of $D$-meson and $c$-quark, respectively).

It worth to note that a scale–invariant description becomes available at sufficiently high energies, i.e. in the limit of $\sqrt{s} \gg m_D$. For the low energy case ($\sqrt{s} \geq m_D$) the role of non–scaling (power) corrections is very essential. As a result at that energy region one can not represent the cross section in a simple factorized form:

$$\frac{d\sigma}{dz} \approx \sigma_{c\bar{c}} \otimes D(z).$$

The question arises of what extent a mechanism of fragmentation of heavy quarks, produced in $e^+e^-$ annihilation, can be applied for the case of hadronic interactions ?

Note, that in hadronic interactions the presence of light partons from the initial hadrons will affect the hadronization scenario. One should distinguish two kinematical regions, namely, the region of $p_\perp < p_0$ and the region of $p_\perp > p_0$. Like in $e^+e^-$ annihilation, the fragmentation process above $p_0$ has a fragmentation character and the cross section has the form as follows:

$$\frac{d\sigma_D}{dp_\perp} = \int \frac{d\sigma_c}{dk_\perp} \otimes D(z)dz.$$  

At the region of $p_\perp < p_0$ the presence of light partons, produced simultaneously with $c\bar{c}$ pair, may radically altered the factorization in the form of (3).
In our previous publication [2] we made an effort to take into account this fact by consideration of a some contribution into hadronization as a process of recombination with valence quarks. The rest part is described by a fragmentation mechanism. In the framework of such a consideration one can properly describe the asymmetry in the production of leading and non-leading charmed hadrons [2]. Note, that our description of total spectrum is not well adequate. It explains because the excess of our calculated distribution at low $x$. This fact is caused by the application of the fragmentation model for the whole kinematical region.

In the present article we make an effort to improve the description of $c$–quark hadronization at low $p_T$ region. We try do not use the fragmentation model as a basic mechanism. We apply this model only at boundaries of the phase space, where $c$–quark momenta achieve their maximum values and where the hadronic accompaniment of $c$–quarks is very small.

We note also, that in the present article we do not consider the question related to an absolute magnitude of the cross section of charmed particle production. The matter is that such a problem is determined by taking into account the higher order corrections of the perturbative QCD as well as the choice of the strong coupling constant of $\alpha_s(\mu^2)$. Indeed, the analysis of $O(\alpha_s^3)$ corrections to $c\bar{c}$ pair production cross section (see, for example, [3]) exhibits that their inclusion practically do not alter the form of the inclusive distributions of $c$–quarks. Therefore, in what follows we shall restrict our consideration to study the form of differential spectra of the charmed particles.

The article is organized as follows. We consider the modification of the fragmentation mechanism in the Section 2. We compare in Section 3 the results of our calculations with the experimental data on $D$–meson production in $\pi$–beam. Section 4 presents the model predictions for the $K$ beam. The main results are summarized in Conclusion.
\section{c–QUARK HADRONIZATION IN HADRONIC COLLISIONS}

In the parton model, the cross section for the production of heavy quarks $Q$ in hadron–hadron collisions has the form

$$\sigma(h_1 h_2 \to Q \bar{Q} X) = \sum_{i,j} \hat{\sigma}(ij \to Q \bar{Q}) f_i^{h_1}(x_1) dx_1 f_j^{h_2}(x_2) dx_2,$$

where summation is performed over all partons participating in $Q$ quark production $ij \to Q \bar{Q}$, $\hat{\sigma}$ is the cross section for the corresponding hard subprocess, $f_i^{h}(x)$ is the distribution of $i$–type partons in the hadron $h$, $x_1(2)$ is the fraction of the initial-hadron momentum carried away by the corresponding parton.

As it mentioned in the Introduction, the presence of light partons from the initial hadrons leads to radically different scenarios of the hadronization of charmed $c$–quarks, produced in hadron–hadron collisions and in $e^+ e^−$ annihilation. In particular, the interaction in final state with valence quarks from initial hadrons (recombination) makes possible to explain the leading particle effect in the charmed hadrons production \cite{2, 3} (i.e. the difference in $x$–distributions of $D$ and $\bar{D}$ mesons, as well as $\Lambda_c$ and $\bar{\Lambda}_c$ baryons).

Such an inclusion of interaction of charmed quarks with valence quarks from the initial hadrons is carried out by the introduction of the quark recombination function of $R(x_V, z; x)$ \cite{3}. Below we present the basic concepts of the recombination mechanism. The detail description of this mechanism is given elsewhere \cite{2, 3}. The recombination of $q_V$ and $\bar{Q}$ quarks into $M_{\bar{Q}}$ meson is described by the function of $R(x_V, z; x)$:

$$R(x_V, z; x) = \rho(\xi_V, \xi_Q) \delta(1 - \xi_V - \xi_Q),$$

\hspace{1cm}

$$\rho(\xi_V, \xi_Q) = \frac{\Gamma(2 - \alpha_V - \alpha_Q)}{\Gamma(1 - \alpha_V) \Gamma(1 - \alpha_Q)} \xi_V^{1-\alpha_V} \xi_Q^{1-\alpha_Q},$$

where $\xi_V = x_V/x$ and $\xi_Q = z/x$, while $x_V$, $z$, and $x$ are the fractions of the initial-hadron c.m. momentum that are carried away by the valence quark, quark $\bar{Q}$, and the meson $M_{\bar{Q}}$, respectively. $\alpha_V$ and $\alpha_Q$ are the intercepts of the leading Regge-trajectories for the $q_V$ and $\bar{Q}$ quarks, respectively. In our calculations we use $\alpha_u = \alpha_d = 1/2$, $\alpha_s \approx 0$, $\alpha_c \approx -2.2$. \hfill (8)
With the aid of the function $R(x_V, z; x)$ describing the recombination of the quarks $q_V$ and $\bar{Q}$ into a meson, we represent the corresponding contribution to the inclusive spectrum of $M_Q$ mesons as follows:

$$x^* \frac{d\sigma^{rec}}{dx} = R_0 \int x_V z^* \frac{d^2\sigma}{dx_V dz} R(x_V, z; x) \frac{dx_V dz}{x_V z},$$  \hspace{1cm} (9)

where $x^* = 2E/\sqrt{s}$ and $x = 2p_l/\sqrt{s}$ (here, $E$ and $p_l$ are the energy and longitudinal momentum of the $M_Q$ meson in the c.m.s. of the initial hadrons); $x_V$ and $z$ are the momentum fractions carried away by the valence quark and heavy antiquark, respectively, and $x_V z^* \frac{d^2\sigma}{dx_V dz}$ is the double-differential cross section for the simultaneous production of the quarks $q_V$ and $\bar{Q}$ in a hadronic collision.

The parameter, $R_0$, is the constant term of the model, that determines the relative contribution of recombination. In the present article the best description of the experimental data could be achieved at

$$R_0 \approx 0.8.$$  \hspace{1cm} (10)

Note, that using of the recombination with the valence quarks is indispensable to an explanation of the leading particle effect. At the same time its contribution in the total inclusive cross section production of the charmed particles is sufficiently small ($\sim 10\%$). This mechanism dominates in the high $x$ region.

In the conventional fragmentation mechanism the inclusive cross section for the charmed hadrons ($D$ mesons) production has the form as follows:

$$E_H \frac{d^3\sigma^F}{d^3p_H} = \int E_c \frac{d^3\sigma(h_1 h_2 \rightarrow cX)}{d^3p_c} D(z) \delta(p_H - zp_c) \frac{d^3p_c}{d^3p_c}.$$  \hspace{1cm} (11)

In the low $x$ region, which determines the basic contribution into the total cross section for charmed hadron production, the hadronization process of $c$ quark has more complicated nature.

Indeed, when evaluating the spectra of charmed particles produced in hadronic collisions one assumes that the fragmentation function $D(z)$ is well known from other experiments (in particular, from $e^+e^-$ annihilation). One of the widely used parameterization is as follows $[4]$: \hspace{1cm} (12)

$$D(z) \sim [z(1 - \frac{1}{z} - \frac{\varepsilon}{1-z})^{-2],}$$
where the parameter \( \varepsilon \approx m_q^2/m^2_Q \) is determined by the type of a hadron (for instance, one has \( \varepsilon_{D^0} = 0.135 \pm 0.010 \) and \( \varepsilon_{D^*} = 0.078 \pm 0.008 \)).

Another parameterization, proposed by us early \[3\], takes into account the Regge-asymptotic at \( z \to 0 \):

\[
D(z) \sim z^{-\alpha_Q}(1 - z)^\gamma, \tag{13}
\]

where \( \gamma \approx 1 \), while \( \alpha_Q \) is the intercept of the leading Regge-trajectory for the \( Q \) quark (\( \alpha_c \approx -2.2 \), see \[8\]).

Both of these parameterizations provide a reasonably fair description of the experimental data. The comparison of description of the reaction \( e^+e^- \to DX \) by means of the fragmentation functions in both form of \((12)\) and \((13)\) is presented in \[8, 9\].

As it mentioned in Introduction, the use of the fragmentation function is justified at asymptotically large values of the invariant mass of \( c\bar{c} \) pair, namely at \( M_{c\bar{c}} \gg 2m_c \). However, this condition is not obeyed in the hadronic production of the charmed particles. For such a case the principal contribution into inclusive charm production cross section results from \( c \) quarks with low values of the invariant mass of \( c\bar{c} \) pair \( (M_{c\bar{c}} \geq 2m_c) \). These quarks dominate in the central region on Feynman variable of \( x \). The pairs of \( c \) quarks with a large invariant mass, where one may apply the fragmentation formalism, give a noticeable contribution at high \( x \) and high \( p_T \), as well.

These arguments are illustrated in Fig. 1. This picture presents the inclusive \( x \)-distribution for \( c \) quark for all \( M_{c\bar{c}} \) (the upper histogram) and for the \( c \) quarks with the invariant mass of \( M_{c\bar{c}} \geq M_0 = 10 \text{ GeV} \) (the lower histogram). As is seen from the figure just the charmed quarks with small invariant masses of \( c\bar{c} \) pair give the dominant contribution in the charm production cross section in the central region, while the region of \( x \to 1 \) corresponds to the contribution due to a large values of mass of \( M_{c\bar{c}} \).

In this figure we also present the spectrum of charmed particles summed over all types \( D \) and \( D \) mesons (for the reaction of \( \pi^-N \) collisions at \( E_\pi = 250 \text{ GeV} \)). This experimental spectrum should be compared with the distribution of \( c \) quarks. One may deduces from the Fig. 1 that the correspondence, like a duality relation, takes place. Namely, the spectrum of charmed hadrons, summed over all types of charmed mesons, is well described by the inclusive spectrum of \( c \)-quarks.

Such a satisfactory description of summed spectra of \( D \) mesons by pure \( c \)-quark spectra was also pointed out early \[4, 14\]. However, it is evident that
in the framework of pure fragmentational mechanism one should expect the identical spectra of both \( D \) and \( \bar{D} \) mesons (as well as charmed baryons and antibaryons). As a result, one can not reproduce the leading particle effect. As it mentioned above, this effect can be described with the help of the recombination mechanism. Therefore, one could assume that the inclusive \( D \) meson production cross section is described by the sum of two mechanisms as follows:

\[
\frac{d\sigma_D}{dx} = \frac{d\sigma_D^{HF}(\vec{p}_D = \vec{p}_c)}{dx} + \frac{d\sigma_D^{rec}}{dx},
\]

where the first term corresponds to the "hard" (HF) fragmentation (in that mechanism a charmed quark does not lose its momentum in hadronization process), while the second term corresponds to recombination contribution (that mechanism takes into account the charmed \( c \)-quark interaction with valence quarks from initial hadrons).

However, such a simple addition of the recombination contribution to the \( c \) quark spectrum (i.e. \( D \) meson) does not provide to reproduce the behavior of \( x \)-dependence of the corresponding asymmetry \( A \):

\[
A = \frac{\frac{d\sigma}{dx}(\text{leading})}{\frac{d\sigma}{dx}(\text{leading}) + \frac{d\sigma}{dx}(\text{non-leading})}.
\]

Indeed, the Fig. 2 presents the description of the asymmetry (15) by means of the equation (14). The different histograms in this figure correspond to the different values of the parameter \( R_0 \) (see the equation (9) for the recombination mechanism). As is seen from this figure it is impossible to obtain the simultaneously proper description of the asymmetry \( A \) both at small \( x \) (0 \( \leq \) \( x \) \( \leq \) 0.4) and at high \( x \) (0.5 \( \leq \) \( x \) \( \leq \) 0.8) as well as with the description of the \( D \) mesons inclusive \( x \)-distributions.

This is because the simple equating of \( D \) meson spectrum to the \( c \) quark spectrum (i.e. \( D(z) \sim \delta(1 - z) \)) leads to exclusively "hard" spectra of \( D \) mesons at high \( x \). Therefore, we get the conclusion that one needs to use a more softer (in compare with \( \delta(1 - z) \)) fragmentation function for the description of charmed quark hadronization at high values of the Feynman variable of \( x \).

Bearing in mind the preceding, we consider the modification of the conventional fragmentation scenario of charmed quarks hadronization. It seem
likely that in the region of small invariant mass of $c\bar{c}$ pair the description of the hadronization in terms of fragmentation function (evaluating from the $e^+e^-$ annihilation) is not justified. Indeed, one has large amount of partons from initial hadrons in the central region of $x$. Therefore, the $c$ quark in combination with one of such a parton can easily produce a charmed hadron. Such a process occurs practically without any loss of $c$ quark momentum (i.e. $\vec{p}_D \approx \vec{p}_c$). Therefore, in the small $x$ region one should expect the coincidence of the spectra of $D$ mesons and $c$ quarks. Whereas at high $x$ region one may use the conventional fragmentation mechanism (and a recombination mechanism, as well).

Hence, we consider two regimes of the charmed quark fragmentation.

a) Close to the threshold of $c$ quarks production, i.e. at $M_{c\bar{c}} \geq 2m_c$, the momentum of the produced $D$ meson should practically coincides with the momentum of the charmed parent-quark.

b) For $c\bar{c}$ pair with the invariant mass $M_{c\bar{c}}$ greater that certain scale of $M_0$ (where $M_0 \gg 2m_c$) the $c$ quark hadronization process may be described with the help of fragmentation function (for instance, in the form of (12) or (13)).

In terms of the fragmentation mechanism these two regimes may be represented in a uniform way by introduction of the dependence of fragmentation function from an invariant mass of $c\bar{c}$ pair as follows:

$$D^{MF}(z, M_{c\bar{c}}) = \begin{cases} \sim \delta(1-z) & \text{at } M_{c\bar{c}} \approx 2m_c \\ D(z) \text{ from (12) or (13)} & \text{at } M_{c\bar{c}} \geq M_0 \end{cases} \quad (16)$$

It worth to note, that assumed $M_{c\bar{c}}$-dependence of the $c$-quark fragmentation function has nothing to do with logarithmic violation of scaling in the fragmentation function.

To reproduce in a uniform way the both two fragmentation regimes (16) we use the simplest expression for $D(z, M_{c\bar{c}})$ in the from of (13) as follows:

$$D^{MF}(z, M_{c\bar{c}}) \sim z^{-\alpha(M_{c\bar{c}})}(1-z), \quad (17)$$

with two additional conditions on $\alpha(M_{c\bar{c}})$:

$$\begin{align*}
\alpha(M_{c\bar{c}}) &\to -\infty \quad \text{at } M_{c\bar{c}} \to 2m_c, \\
\alpha(M_{c\bar{c}}) &\to \alpha_c \quad \text{at } M_{c\bar{c}} \approx M_0.
\end{align*} \quad (18)$$
Our parameterization for $\alpha(M_{c\bar{c}})$ is presented in the Appendix 1. A fit to experimental data exhibits that the magnitude of the parameter $M_0$,

$$M_0 \approx 10 \text{ GeV},$$

provides a satisfactory description of experimental results. Such a magnitude does not contradict to the experiments in $e^+e^-$ annihilation, where already at the energy of $\sqrt{s} \approx 10 \text{ GeV}$ one can describe the process in terms of fragmentation mechanism by using the dependence in the form of $^3$

Thus, the summed differential cross section production of charmed hadron ($D$-meson) can be represented as follows:

$$\frac{d\sigma_D}{dx} = \frac{d\sigma_{D}^{MF}}{dx} + \frac{d\sigma_{D}^{rec}}{dx}, \quad (19)$$

where $\frac{d\sigma_{D}^{MF}}{dx}$ is the differential cross section production for $D$-meson due to $c$-quark fragmentation (that is described by the equation ([4]) with the modified fragmentation function of $D^{MF}(z,M_{c\bar{c}})$), while $\frac{d\sigma_{D}^{rec}}{dx}$ is the differential cross section production for $D$-meson resulted from $c$-quark recombination with valence quarks (see ([5])).

Like in the previous article([2]), we assume that a mesonic state ($c\bar{q}$) goes over into a vector $M_V$ or a pseudoscalar $M_{PS}$ meson with the probability proportional to the spin factor:

$$M_{PS} : M_V = 1 : 3. \quad (20)$$

## 3 COMPARISON OF THE MODEL PREDICTIONS WITH THE EXPERIMENTAL RESULTS IN $\pi^-$ BEAMS

As it mentioned in the Introduction our article deals with the description of two types of the inclusive $x$--distributions, namely, the differential cross section for $D$ meson production (i.e. $\frac{d\sigma}{dx}$) and the asymmetry $A(x)$.

Fig. 3 presents the description of the differential distribution $\frac{d\sigma}{dx}$ for the reaction

$$\pi^- N \rightarrow (D + \bar{D}) X.$$
The experimental data is summed with respect to all types of $D$ meson. The beam energy equals $E_\pi = 250$ GeV.

As is seen from the figure our model (the modified fragmentation + recombination) reproduces satisfactorily the experimental data. Note, also, that although the recombination contribution into the total cross section is rather small ($\leq 10\%$), it plays a substantial role at high $x$ (see Fig. 3). In addition, one may deduce from this figure that pure modified fragmentation can not provide the proper description of the inclusive spectrum in the whole kinematic region.

Fig. 4 presents the description of the corresponding asymmetry (the leading particle effect). In this case, too, the considered model provides the description of the experimental data [10, 11].

It should be particularly emphasized once more that the "hard" fragmentation (i.e. $\vec{p}_D = \vec{p}_c$) makes it possible the description of the differential spectrum in the whole kinematic region but can not reproduce the $x$–dependence of the asymmetry.

In the Table 1 we compare our model predictions with the experimental data on the total yields of the charmed mesons as well as with the predictions of the Lund model [12] (the experimental results and the Lund-model predictions are taken from [13]).

One may deduce from the Table 1 that our evaluations for the ratios of the cross sections production of $D$ mesons are in agreement with the experimental results within the experimental errors. Remind, that we obtain these results by consideration of the two types of the hadronization of charmed $c$ quarks. The behavior of these processes is determined by two parameters $R_0$ and $M_0$, as well as the distribution functions of the partons in the initial hadrons.

The decisive test of the considered model should be the comparison of the theoretical predictions with the experimental results in $K^\pm$ and $\Sigma^-$ beams. The valence quarks distributions in these hadrons differ substantially from those in $\tau^\pm$ and $p$ beams (see below). Consequently, one should expect the different contributions due to recombination mechanism in the inclusive spectra of charmed hadrons.
4 CHARM PRODUCTION IN THE BEAMS OF CHARGED $K^{\pm}$ MESONS

From the view of parton model the difference of $K^{\pm}$ mesons from $\pi^{\pm}$ mesons not only resides in the replacement of the valence $d$–quark by the strange valence $s$–quark. The distribution function of valence quarks in $K^{\pm}$ mesons should change substantially, as well.

The simplest (without scaling violation) parameterization of the distribution functions of the valence $q_1$ quark in the meson of $M(q_1\bar{q}_2)$ with the valence quarks of $q_1$ and $\bar{q}_2$ has the form as follows [3]:

$$V_{q_1}^{M(q_1\bar{q}_2)}(x) = \frac{\Gamma(2 + \gamma_0 - \alpha_1 - \alpha_2)}{\Gamma(1 - \alpha_1)\Gamma(1 + \gamma_0 - \alpha_2)} x^{-\alpha_1} (1 - x)^{\gamma_0 - \alpha_2},$$

(21)

where $\alpha_i$ is the intercept of the leading Regge-trajectory for the $q_i$ quark, while $\gamma_0$ is certain parameter.

The coefficient in the above equation is determined by the normalization condition:

$$\int_0^1 V_q^M(x) dx = 1.$$

The choice of the parameter of $\gamma_0$ in (21) is governed by asymptotic behavior of the structure functions at $x \to 1$. The quark counting rules predicts the value as follows:

$$\gamma_0 - \alpha_2 = 1.$$

From the well-known asymptotic for $\pi$–meson,

$$V^\pi(x)|_{x \to 1} \sim \frac{1}{\sqrt{x}} (1 - x)^{1},$$

if follows that $\gamma_0 = \frac{3}{2}$. Taking into account that $\alpha_u = \alpha_d = 1/2$, while $\alpha_s \approx 0$ (see [3]), we obtain the following distributions of the valence $u$ and $s$ quarks in $K^{\pm}$ meson:

$$V_u^K \sim \frac{1}{\sqrt{x}} (1 - x)^{3/2},$$

(22)

$$V_s^K \sim (1 - x)^{1}.$$
From the form of the distributions of (22) and (23) it is evident that the valence s–quark in the K meson is much "harder" than u quark:

\[ < x^K_{s_v} > = 0.33, \]
\[ < x^K_{u_v} > = 0.166. \]

From the given parameterization of the K meson structure functions it follows that the total momentum fraction carried away by the valence quarks is equal to:

\[ < x^K_{v} > = < x^K_{s_v} > + < x^K_{u_v} > = 0.5. \]
\[ (24) \]

Such a magnitude should be compared to the similar value for the \( \pi \) meson:

\[ < x^{\pi}_{v} > = < x^{\pi}_{d_v} > + < x^{\pi}_{u_v} > = 0.4. \]
\[ (25) \]

We assume further that the distribution of the gluons in \( \pi^\pm \) and \( K^\pm \) mesons are identical in form. The previous analysis [14] of the evolution of structure functions of \( \pi^\pm \) and \( K^\pm \) mesons, where the evolution starts from the different distributions of valence quarks, provides an argument in favor of such an assumption. Other arguments in support of such an assumption are the identical form of the spectra of charmed mesons produced in \( \pi^\pm \) and \( K^\pm \) beams [10].

As mentioned above, the form of the distributions of the rest sea partons in \( K^\pm \)–meson coincides with the form of corresponding parton distributions in \( \pi^\pm \) mesons:

\[ f^K_{sea}(x) = \epsilon f^{\pi}_{sea}(x). \]

Here, the parameter \( \epsilon \) takes into account the variation of the momentum fraction carried away by the valence quarks in K meson with respect to to analogous value in \( \pi^\pm \) meson:

\[ \epsilon = \frac{1 - < x^K_{v} >}{1 - < x^{\pi}_{v} >} \approx 0.8. \]

Due to the different distributions of (22) and (23) we should expect the different \( x \)–distributions of \( D(c\bar{u}) \) and \( D_s(c\bar{s}) \) mesons, produced in the \( K^\pm \) beams (that resulted from the recombination with valence quarks). Two–particle distributions of the partons in K meson, needed for such a calculation, can be easily evaluated with the help of equation (21). Their explicit form is presented in the Appendix 2.
Fig. 5 presents our model predictions for the leading particle effect in $K^-$ meson beam at the energy of $E_K = 250$ GeV. As expected, our model predicts the different $x$-dependence of the asymmetry for $D$ and $D_s$ mesons. For the strange–charmed $D_s$–mesons the leading particle effect is more pronounced as compared with usual charmed mesons. The observation of such a difference would be evidence in favor of the considered model.

Unfortunately, only the integral asymmetry magnitude is measured in the experiment [15]:

$$A_{K}^{\text{exp}}(D_s) = 0.25 \pm 0.11,$$

that should be compared with our prediction:

$$A_{K}^{\text{theor}}(D_s) = 0.29.$$  \hfill (27)

As is seen our theoretical estimate is in agreement with the experimental value.

5 CONCLUSION

This article presents the ”improved” model of charmed quark hadronization. Our model provides the consistent description of the inclusive differential spectra of $D$ mesons, produced in $\pi^-p$ collisions. Further step forward in the understanding of a hadronization mechanism of heavy quarks would be related with a consideration of the processes of charmed particle production in $K$ and $\Sigma$ interactions. It caused, in particular, the different distributions of valence quarks in $K$ and $\Sigma$ hadrons as compared with $\pi$ and $p$ beams. The next step of such an investigation should be a consideration of the process of charmed baryons production, where the diquarks from the initial hadrons play a substantial role.

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APPENDIX 1.

In order to deduce the parameterization of $\alpha(M_{cc})$, which takes into account the conditions (18), i.e. $\alpha(2m_c) = \infty$ and $\alpha(M_0) = \alpha_c$, we consider the expression for the first moment of $\mu$ from the fragmentation function of $D^{MF}(z, M_{cc})$ from (17):

$$
\mu(M_{cc}) \equiv \int_0^1 z D(z, M_{cc}) dz = \frac{1 - \alpha(M_{cc})}{3 - \alpha(M_{cc})}.
$$

The desired expression for $\alpha(M_{cc})$ equals:

$$
\alpha(M_{cc}) = \frac{1 - 3\mu(M_{cc})}{1 - \mu(M_{cc})}.
$$

We assume the following (QCD–motivated) dependence for $\mu(M_{cc})$

$$
\mu(M_{cc}) = \left(\frac{\ln\left(\frac{M_{cc}}{2m_c q_0}\right)}{\ln q_0}\right)^d,
$$

where $d \approx 0.464$ is the parameter, similar to anomalous dimension.

The new parameter $q_0$ is expressed through $M_0$ as follows:

$$
q_0 = \left(\frac{M_0}{2m_c}\right)^{\frac{\nu}{1-\nu}},
$$

where

$$
\nu = \mu(M_0)^{-\frac{1}{4}} \quad \text{and} \quad \mu(M_0) = \frac{1 - \alpha_c}{3 - \alpha_c}.
$$
APPENDIX 2.

For completeness sake we present here the explicit form for two–particle distribution functions $f_{V_i}^h(x_V, x_1)$ in $\pi^\pm$ and $K^\pm$ mesons.

Note, that two-particle parton distributions (like the single–particle functions) can not be theoretically evaluated. Therefore, we use the simplest phenomenological expression that takes into account the momentum conservation, the $(1 - x_1)^n$ dependence of sea partons, as well as the normalization conditions as follows (see [3] for the details):

$$\int_0^{(1-x_1)} f_{V_i}^h(x_V, x_1) dx_V = f_i^h(x_1),$$

where $f_i^h(x_1)$ is the single-particle distribution of an $i$–type parton in the hadron $h$.

For two valence quarks such a distribution has the form [3]:

$$f_{VV}(x_1, x_2) = \frac{\Gamma(2 + \gamma_0 - \alpha_1 - \alpha_2)}{\Gamma(1 - \alpha_1)\Gamma(1 - \alpha_2)\Gamma(\gamma_0)} x_1^{-\alpha_1} x_2^{-\alpha_2} (1 - x_1 - x_2)^{\gamma_0 - 1}.$$

For the case of one valence and one sea parton the corresponding distribution equals:

$$f_{Vj}(x_v, x_j) = N_j \frac{\Gamma(2 + n_v - \alpha_v)}{\Gamma(1 - \alpha_v)\Gamma(1 + n_v)} x_v^{-\alpha_v} x_j^{-1} (1 - x_v - x_j)^{n_v}(1 - x_j)^k,$$

where $N_j$ is the corresponding normalization factor of a sea parton, $n_v = \gamma_0 - \alpha_1 - \alpha_2 + \alpha_v$, and $k = n_j - 1 - \gamma_0 + \alpha_1 + \alpha_2$.

The parameters of the function $f_{Vj}(x_1, x_2)$ for $\pi$ and $K$ mesons are presented in the Table 2.
Table 1. The ratios of the total yields (at $x > 0$) of the charmed mesons in the considered model, the experimental results [13] and the Lund–model predictions (taken also from [13]).

|        | $\frac{D^+ + D^-}{D^0 + D^{*0}}$ | $\frac{D^+ + D^-}{D^0 + D^{*0} + D^0}$ | $\frac{D^+}{D^0}$ | $\frac{D^0}{D^0}$ |
|--------|----------------------------------|----------------------------------|-----------------|-----------------|
| our model | 0.332                           | 0.102                            | 1.16            | 1.0             |
| experiment | 0.416 ± 0.016                     | 0.129 ± 0.012                     | 1.35 ± 0.05     | 0.93 ± 0.03     |
| Lund    | 0.472                            | 0.077                            | 2.25            | 1.09            |

Table 2. Values of the parameters appearing in the functions $f_{Vj}(x_1, x_2) = Ax_1^{-\alpha_1}x_2^{-\alpha_2}(1 - x_1 - x_2)^n(1 - x_2)^k$.

|        | $A$ | $\alpha_1$ | $\alpha_2$ | $n_v$ | $k$ |
|--------|-----|------------|------------|-------|-----|
| $\pi^\pm$ meson |      |            |            |       |     |
| partons |      |            |            |       |     |
| $u_vd_v$ | 0.477 | 0.5 | 0.5 | 0.5 | 0   |
| $u_vg$    | 1.50  | 0.5 | 1.0 | 1.0 | 1.5 |
| $u_v(u, d)_{sea}$ | 0.090 | 0.5 | 1.0 | 1.0 | 3.5 |
| $u_vs_{sea}$ | 0.045 | 0.5 | 1.0 | 1.0 | 3.5 |
| $K^\pm$ meson |      |            |            |       |     |
| partons |      |            |            |       |     |
| $u_vs_v$  | 1.27  | 0.5 | 0.0 | 0.5 | 0   |
| $u_vg$    | 1.34  | 0.5 | 1.0 | 1.0 | 1.5 |
| $s_vg$    | 3.22  | 0.0 | 1.0 | 1.5 | 1.5 |
| $u_v(u, d)_{sea}$ | 0.08 | 0.5 | 1.0 | 1.0 | 3.5 |
| $u_vs_{sea}$ | 0.04 | 0.5 | 1.0 | 1.0 | 3.5 |
| $s_v(u, d)_{sea}$ | 0.192 | 0.0 | 1.0 | 1.5 | 3.5 |
| $s_vs_{sea}$ | 0.096 | 0.0 | 1.0 | 1.5 | 3.5 |
Figure 1: Differential distributions $\frac{d\sigma}{dx}$ for charmed $c$ quarks for all values of $M_{c\bar{c}}$ (the upper histogram) and for $M_{c\bar{c}} \geq M_0 = 10$ GeV (the lower histogram). The experimental points correspond to charmed particles yield, summed over all types of $D$ and $\bar{D}$ mesons (the reaction of $\pi^- N$ collisions at $E_\pi = 250$ GeV [10].)
Figure 2: The description of the asymmetry $A(x)$ (the leading particle effect) in $\pi^- p$ collisions [10, 11] in the mechanism of the "hard" fragmentation (see (14)). The different histograms correspond to different values of the parameter $R_0$ for the recombination mechanism (see (9)). The value of $R_0 = 3.0$ corresponds to the upper histogram and so on.
Figure 3: Differential distributions $\frac{d\sigma}{dx}$ for the energy of $E_\pi = 250$ GeV. The experimental data are taken from [10]. The dotted (dashed) histogram corresponds to the recombination (fragmentation) contribution. The solid histogram represents their sum. The cross sections are presented in $\mu$b.
Figure 4: The description of the asymmetry $A(x)$ in $\pi^- p$ collisions [10, 11] in the mechanism of the modified fragmentation (see (19)) with the help of fragmentation function from (17).
Figure 5: The predictions of the modified fragmentation mechanism for the asymmetry $A(x)$ in $K^- p$ collisions at $E_K = 250$ GeV. The histograms correspond to the ratios of: 1) $D_s(\bar{c}s)$ to $D_s(c\bar{s})$ mesons, 2) all charmed $D + D_s$ mesons, 3) $D_s(\bar{c}s)$ to all charmed $D + D_s$ mesons.