On the quantum electrodynamics of moving bodies

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Abstract

A new synthesis of the principles of relativity and quantum mechanics is developed by replacing the Poincaré group for the de Sitter one. The new relativistic quantum mechanics is an indefinite mass theory which is reduced to the standard theory on the mass shell. The charge conjugation acquires a geometrical meaning and the Stueckelberg interpretation for antiparticles naturally arises in the formalism. So the idea of the Dirac sea in the second quantized formalism proves to be superfluous. The off-shell theory is free from ultraviolet divergences, which only appear in the process of mass shell reduction.

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The advent of quantum theory cherished the hope of reformulating electrodynamics free from anomalies. However, divergences were smoothed but not completely erased by quantization. Such a disappointment was considered as a serious trouble for the physics of that time and the progress in the area was delayed for two decades. After the great advances achieved by the end of the fifties, the new generation of physicists “have learned how to peacefully coexist with the alarming divergences of the old fashioned theory, but these infinities are still with us, even though deeply buried in the formalism” [1]. Due to this fact some workers in the field tried to start again from the beginning formulating the so called axiomatic quantum field theory. Their unsatisfaction was clearly summarized in the statement of Streater and Wightman: “...the quantum theory of fields never reached a stage where one could say with confidence that it was free from internal contradictions –nor the converse” [2]. Unfortunately as Rohrlich [3] has pointed out, this route does not fulfil all aspirations: “We now have a much deeper mathematical understanding of quantum electrodynamics, especially due to the work of axiomatic field theorists; but we have still not solved the basic problem of formulating the theory in a clean mathematical way, not even with all the complicated and highly sophisticated limiting procedures presently used to justify the results of a naive renormalization theory in simpler quantum field theories and in lower dimensionality. The hopes and aspirations indicated in the outlook of twenty years ago remain valid today.”

A renovating spirit was present in the more recent movement of string theorists who decided to change some basic principles. As a consequence of it, string models have non-local interactions which provide a way to avoid the ultraviolet divergences from the beginning. However the price payed for this desirable requirement is too high: we have lost the extraordinary power of calculus and predictability of quantum field theory. This is the reason why some theoretical physicists became conservative and, in a radical change to the optic of the problem, tried to justify “the unreasonable effectiveness of quantum field theory” [4], arguing that the phenomenologically desirable results are provided by ultraviolet divergences. As in the standard theoretical framework anomalies, as the chiral one, come from the gauge non-invariance of the infinite negative-energy sea. It is argued that “we must assign physical reality to this infinite negative-energy sea” [5]. We see such philosophical position as a new intent of rescuing the theory of the “ether.” Alternatively, Weinberg [6] has delayed the present difficulties for quantizing gravity re-
formulating the problem in this way. He holds the point of view that the
standard model and general relativity are the leading terms in effective field
theories, and so disregards the problem of renormalizability which is only
proper of a fundamental theory still unknown (perhaps a string model).

On the contrary, the creators of the quantum field theory, such as Dirac,
held a less conservative viewpoint:

“Nowadays, most of the theoretical physicists are satisfied with this sit-
tuation, but I am not. I think that theoretical physicists have taken a wrong
way with this new facts and we would not be pleased with this situation.
We must understand that we are in front of something wrong radically dis-
carding the infinities from our equations; here we need to respect the basic
laws of the logics. Thinking about this point could send us to an important
advance. QED is the branch of theoretical physics about we know more,
and presumably we have to put it in order until we can make a fundamental
progress in other field theories, although this theories continue developing
under experimental basis.”

In this work we develop the foundations of a new synthesis of the princi-
pies of relativity and quantum mechanics. Following Dirac’s advice we only
propose to reformulate QED. As our purpose is humbler than that of the
string program (conceived as the theory of everything) the change in the
basic principles is also less radical: essentially we propose to substitute once
more the standard group of external symmetries, i.e. the Poincaré group for
the de Sitter one. It is ironic that, approaching to the end of this century
after nine decades from Einstein did the same with the Galilei group, we can
motivate the new program rephrasing Einsten’s words:

It is known that Dirac’s quantum electrodynamics –as usually understood
at the present time– leads to asymmetries and inconsistencies which do not
appear to be inherent in the phenomena. Take, for example, the descrip-
tion of a pair creation in an external electromagnetic field. The observable
phenomenon here always involves finite measurable quantities and does not
make any distinction between electron and positron, whereas the customary
view draws a sharp distinction between the two particles. While the electron
is interpreted as a positive energy state of the Dirac equation, the positron
is interpreted as a hole or absence of a negative energy state in the Dirac
This sea of infinite electrons, which fills all the negative energy states of the Dirac equation, is the responsible for ultraviolet divergences in the effective action used for describing such phenomena. Moreover, from the standpoint of general relativity the zero point energy of the electromagnetic field also seems unsatisfactory since a divergent vacuum stress tensor would imply, via the Einstein field equations, an infinite curvature for the universe corresponding to an infinite cosmological constant, which cannot be removed simply by performing some sort of transfinite shift of the energy scale.

Examples of this sort, together with the unsuccessful attempts for quantizing gravity through these methods, suggest that the phenomena of electrodynamics as well as of gravity at a quantum level possess no properties corresponding to the quantum field notion of the vacuum. They rather suggest that a different route must be taken in order to accommodate the principles of relativity at the quantum level. From our point of view the main difficulty lies in the different role and interpretation of “time” in both theories. In fact, while quantum mechanics privileges an absolute parameter that labels the evolution of the system, the theory of relativity stresses the relative character of the temporal coordinate. Therefore the first concept of time should have the properties of a $c$-number, while the second should be an operator due to the mixing character of the Lorentz transformations. Thus this dual role of time poses a problem in relativistic quantum mechanics at a first quantized level. The standard solution to this dilemma is to give up this vessel and plunge into the sea of quantum field theory, relegating the role of space-time coordinates to be simple parameters of the theory. Unfortunately this mathematical artifact is achieved by means of a choice of vacuum compatible with the idea of the Dirac sea, which actually just swept the problem under the rug. This fact suggests us that such a dual role of time

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1 The asymmetry in the description is more evident from the historical point of view. In fact the holes were originally interpreted by Dirac as protons, who thought that he could explain the mass differences by means of the interaction of the electrons of the sea.

2 This is analogous to the case of chiral anomaly discussed above, and it results specially clear from the Weisskopf derivation of the Heisenberg-Euler Lagrangian. In Sec. 2 we discuss the proper time approach to this effective Lagrangian in which becomes clear that divergences appear in the transition from the off-shell theory to the mass shell.

3 As we will see we do not discard many “particle” formalisms (we find more appropriate to call them many charge formalisms) nor the notion of field. We only attack the choice of the vacua in standard quantum field theory to implement the charge conjugation symmetry.
demands the introduction of two different concepts for playing two different roles. In other words we propose that the unification of quantum principles with the theory of relativity requires the introduction of an additional label to describe the events, increasing in this way the dimension of the space-time manifold \(\text{\cite{15, 16, 17}}\). We will raise this conjecture to the status of a postulate, and also introduce another postulate, namely, laws of physics in our five-dimensional space-time obey the principles of the special theory of relativity. These two postulates suffice for the attainment of a simple and consistent theory of quantum electrodynamics, based on Dirac’s theory in a higher dimension. The introduction of a “Dirac sea” will prove to be superfluous inasmuch as the view here to be developed will not require ordinary time to be the parameter which labels the quantum evolution.

1 Kinematical Part

Nowadays, theoretical physicists seem to be more focused on internal symmetries than on external ones, in the search of a grand unified gauge theory. However in the sixties a great effort was made for unifying both symmetries, enlarging the Poincaré group. So for different motivations the simplest extensions of the Poincaré group, such as the five-dimensional Galilei group, the de Sitter group, and conformal group, began to be studied, constituting the antecedents of our program.\(^5\) However the idea of enlarging the dimension of space-time to take into account particle-antiparticle symmetries is an older fascinating idea. Perhaps the first antecedent can be found in the works of Hinton, who built a model of electricity associating positive and negative charges with right and left handed helixes in higher dimensional spaces. Curiously, this prerelativistic model developed in 1888 has an extraordinary parallelism with the theory of Klein \(\text{\cite{22}}\). In Sec. 2 we discuss these ideas through a generalization of the Schroedinger Zitterbewegung to four dimensions \(\text{\cite{23, 17}}\), which is related to the Stueckelberg \(\text{\cite{24}}\), Wheeler and Feynman \(\text{\cite{25, 26, 27, 28, 30}}\) interpretation of antiparticles. But in this

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\(^4\)Formulations of relativistic quantum mechanics with an invariant evolution parameter were discussed in the past. According to the external group of symmetry they can be classified as five-dimensional Galilean invariant formulations \(\text{\cite{11, 12, 13}}\) and de Sitter ones. See Refs. \(\text{\cite{14, 15}}\) for a critical review about them.

\(^5\)In connection with this work see Refs. \(\text{\cite{18, 19, 11, 20, 21}}\).
route, the concept of time must be revisited.

Time in physics is not an *a priori* concept in the Newton sense, but enters as a basic concept used to describe the laws of nature. The history of science shows us that physics always adapts and modifies this concept in order to simplify the laws. Then, from this point of view, there is no place to the question why the universe has five dimensions and not four. The important thing is that there is a set of phenomena which can be described in a more simple and symmetrical way if we use two times instead of one. The purpose of this work is to demonstrate that this is the case for QED.

We begin considering a five-dimensional manifold as space-time arena in which such phenomena occur. According to the first postulate, each event in our description has associated a point $P$ of the space-time determined by coordinates $x^A = (x^\mu, x^5)$ ($A = 0, 1, 2, 3, 5$), i.e. $P = P(x^A)$, which will be called a super-event. From the second postulate the space-time is endowed with a super-Minkowskian metric $g^{AB} = \text{diag}(+, -, -, -, -)$, so the square of the super-arc element $dS$ reads

$$dS^2 = g^{AB}dx_A dx_B = g^{\mu\nu}dx_\mu dx_\nu - (dx^5)^2. \quad (1)$$

Any linear transformation of coordinates $x^{A'} = L^A_{\ B}x^B + C^A$ which leaves $dS^2$ invariant will be referred to as a coordinate transformation between two super-inertial systems. The super-Poincaré group of such a transformation is the well-known inhomogeneous de Sitter group. The other implicit assumption is that all physical laws adopt the same form in all super-inertial frames, that is to say that they are de Sitter covariant.

We do not analyze here all the potentialities of such a description but our intention is to use this new framework to reformulate the physics associated to the Poincaré invariance free from inconsistencies. Keeping this in mind, let us restrict ourselves to the subset of linear transformations

$$x^{\mu'} = L^\mu_\nu x^\nu + C^\mu; \quad (2)$$

$$x^{5'} = x^5 + C^5, \quad (3)$$

which leaves the square of the standard arc element, $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, invariant, maintaining the fifth coordinate $x^5$ as a Poincaré invariant parameter. This means that we are going to describe the super-events posed in a
given super-frame, forbidding boosts and rotations between $x^5$ and any of the space-time coordinates. In this case such an evolution parameter works as a Newtonian time in each super-frame and introduces an absolute notion of simultaneity and retarded causality associated to it. The fifth coordinate $x^5$ is arbitrary in principle, however from Eq. (1) we see that for the particular case of motions on the super-light cone ($dS = 0$) the coordinate $x^5$ is reduced to $s$. We restrict our analysis of QED to this case. In Fig. 1 we show the super-light cone and its four-dimensional projection. Note that while a super-world line lies on the super-light cone its space-time projection lies inside the standard light cone.

![Figure 1: Super and standard light cones.](image)

At this point one could ask what we have gained with such a description. The immediate answer is that this description has now an invariant evolution parameter at the classical level, preparing the land for a description at the quantum level that avoids the lack of explicit covariance of the standard canonical formalism. What is not so evident is that it is a natural framework for introducing the notion of antiparticles. Moreover, as we show in Sec. 2,
the notion of retarded causality in $x^5$ for super-particles naturally leads to
the standard quantum field theoretical boundary conditions for the Green
functions on the mass-shell. That is, particles go forward and antiparticles
go backward in the coordinate time $x^0$.

Let us consider the world-line of a super-event in a given super-frame.
The Poincaré invariance suggests us to parametrize this curve with $x^5$, i.e. to
project the super-world-line in a hyper-plane $x^5 = \text{const}$ (the standard space-
time). Thus, at any point of the projected curve (a standard world-line), the
four-velocity $dx^\mu/dx^5 = \left( dx^0/dx^5, \, \frac{dx^1}{dx^5}, \, \frac{dx^2}{dx^5}, \, \frac{dx^3}{dx^5}\right)$ has a new key ingredient with respect to
the non-covariant description which takes the coordinate $x^0$ as the evolution
parameter, namely the rate $dx^0/dx^5$. This new degree of freedom allows us to
introduce the concept of antiparticle just at the classical level. Generalizing
Stueckelberg’s ideas [24, 25] we call super-particles and super-antiparticles
to those states for which $dx^0/dx^5$ is positive and negative respectively. Therefore
for causal propagation ($dx^5 > 0$), while the super-particles propagate
forward in time, the super-antiparticles propagate backward in coordinate
time. Notice that for $dx^5 = 0$ we cannot distinguish the two concepts. This
is the case of the photon in the standard framework, in which we identify
the fifth coordinate with the classical proper time. We could expect that
the evolution in $x^5$ also interchanges particle and antiparticle states at a first
glance. Nevertheless, as we will see below, for the standard electromagnetic
interactions this interchange is classically forbidden and only possible at the
quantum level as a consequence of the uncertainty principle.

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6This formalism allows us to reformulate the “localization problem” [31], by following
charges “trajectories” instead particles ones. Moreover, the recognition that this strange
notion of $x^0$—causality is the only compatible with the requirements of relativistic quantum
mechanics enables one to eliminate Hegerfeldt’s paradox [32].

7Also note that this notion is super-frame dependent, i.e. a state registered as a super-
particle from a super-inertial system can be registered as a super-antiparticle from another
super-inertial system. The same thing happens with the notion of simultaneity associated
to the coordinate $x^5$, which looses its invariant character under the full de Sitter group
transformations.
2 Electrodynamical Part

From a dynamical point of view the main difference between the Poincaré and the de Sitter groups is that for the second group the operator $p_\mu p^\mu$ is no longer a Casimir operator. The states of the new theory are off the mass shell $p_\mu p^\mu = m^2$. They are on the super-mass shell hyperboloid

$$p_A p^A = M^2,$$

where $M$ is a super-mass parameter. We are interested in the study of null-super-mass states because in the classical limit they motion is super-luminal and, as we discuss in the kinematical part, we can identify the five coordinate $x^5$ with the proper time $s$. So, let us begin considering the wave equation satisfied by the non-super-massive ($M = 0$) spin-$\frac{1}{2}$ irreducible representation of the de Sitter group $\Psi$

$$\Gamma^A i\partial_A \Psi = 0,$$

where $\Gamma^\mu = \gamma^5 \gamma^\mu$, $\Gamma^5 = \gamma^5 = \gamma^0 \gamma^1 \gamma^2 \gamma^3$, satisfy the Dirac algebra

$$\Gamma^A \Gamma^B + \Gamma^B \Gamma^A = 2 g^{AB}.$$

Multiplying on the left by $\gamma^5$, we can rewrite (5) in the Hamiltonian form

$$-i \frac{\partial \Psi}{\partial s} = \gamma^\mu i\partial_\mu \Psi,$$

where we have identified $x^5$ with $s$. Eq. (7) was originally introduced by Feynman in 1948 in his dissertation at the Pocono Conference. This is a Schroedinger equation in the invariant parameter $s$ for the evolution of states off the mass-shell. The mass-shell condition is satisfied by stationary states, $\Psi(x^\mu, s) = \psi_m(x^\mu) e^{i m s}$, solutions of the Dirac equation

$$\gamma^\mu i\partial_\mu \psi_m = m \psi_m.$$

\footnote{Feynman introduced Eq. (7) in a formal way and did not discuss its geometrical meaning. He could not solve Dirac’s doubts about the unitarity of the theory either. For a nice account of these anecdotes, see the review paper of Schweber.}

\footnote{The Dirac equation can be consistently introduced from first principles at a first quantized level interpreting antiparticles as negative energy states going backward in $x^0$-time.}
The Feynman equation minimally coupled to an external electromagnetic field is given by

\[-i \frac{\partial \Psi(x, s)}{\partial s} = \gamma^\mu (i \partial_\mu - e A_\mu) \Psi(x, s),\]  

(9)

where \(A_\mu\) is the electromagnetic potential.

The key idea of Feynman [27, 28] was that by Fourier transforming in \(s\) any solution \(\Psi(x, s)\) of Eq. (9) a solution \(\psi_m(x)\) of the corresponding Dirac equation

\[\left[\gamma^\mu (i \partial_\mu - e A_\mu) - m\right] \psi_m(x) = 0\]  

(10)

can be obtained, namely

\[\psi_m(x) = \int_{-\infty}^{+\infty} \Psi(x, s) e^{-ims} ds.\]  

(11)

Hence the Fourier transform of the retarded Green function \(G(x, x', s)\) of Eq. (9)

\[\left[\gamma^\mu (i \partial_\mu - e A_\mu) - i \frac{\partial}{\partial s}\right] G(x, x', s) = \delta(x, x') \delta(s),\]  

(12)

with \(G(x, x', s) = 0\), for \(s \leq 0\), enables one to derive the corresponding mass-shell Green function \(G_m(x, x')\), i.e.

\[\left[\gamma^\mu (i \partial_\mu - e A_\mu) - m\right] G_m(x, x') = \delta(x, x').\]  

(13)

From the path integral point of view the retarded condition for the propagator \(G(x, x', s)\) means that all the classical paths go forward in time \((ds > 0)\), so the on-shell positive (negative) kinetic energy states must go forward (backward) in coordinate time, since in the classical limit (neglecting spin effects) we have \(\frac{dx^0}{ds} = \pm \frac{1}{\sqrt{1-v^2}}\). This fact determines the well-known boundary conditions for \(G_m(x, x')\) [26].

Moreover if in the Fourier transformation

\[G_m(x, x') = \int_{0}^{+\infty} G(x, x', s) e^{-ims} ds,\]  

(14)

for the on-shell retarded Green function
\[
G(x, x', s) = -i\theta(s) \langle x | e^{i\gamma^\mu \pi_\mu s} | x' \rangle
\]  
(15)

the Schwinger formal identity
\[
i/(a + i\epsilon) = \int_0^\infty \exp[is(a + i\epsilon)]ds
\]  
(16)
is used for \( a = \gamma^\mu \pi_\mu - m \), one immediately sees that such retarded boundary condition for \( G(x, x', s) \) naturally leads to the Feynman \( i\epsilon \) prescription for avoiding the poles in the on-shell Green function
\[
G_m(x, x') = \langle x | \frac{1}{\gamma^\mu \pi_\mu - m + i\epsilon} | x' \rangle.
\]

This formal trick allowed Feynman to discuss external field problems of QED keeping up at a first quantized level.

Let us go further these formal tools in order to understand the physical grounds of them. In this formalism the state space is endowed with an indefinite Hermitian form \[14, 15\]
\[
\langle \Psi | \Phi \rangle = \int d^4x \overline{\Psi}(x)\Phi(x),
\]  
(17)
in which the covariant Hamiltonian or mass operator \( \mathcal{H} = \gamma^\mu i\partial_\mu \) is self-adjoint and the evolution operator \( e^{\mathcal{H}s} \) is unitary. It can be proved \[17\] that at a semiclassical level
\[
\text{sign } [\overline{\Psi}(x, s)\Psi(x, s)] = \text{sign } \frac{dx^0}{ds},
\]  
(18)
that is super-particles and super-antiparticles states have positive and negative norm respectively. This is the root of the indefinite character of the “inner product”. Frequently this fact is considered as an anomaly of the theory, due to it is not possible to straightforward apply the standard probabilistic interpretation. In fact this is one of the reasons why Dirac originally rejected the Klein-Gordon equation. But as was shown by Feshbach and Villars \[14\] the indefinite metric character of the Klein-Gordon theory can be

\[10\]Ironically, some years before it was Dirac himself \[33\] who introduced indefinite metric Hilbert spaces in quantum field theory with the hope of removing the true anomaly: the divergences.
reinterpreted in the framework of the theory of a charge. This is the interpretation we adopt in this work.

We have defined super-particles and super-antiparticles according to the Stueckelberg interpretation in the kinematical part. Let us now show that it is consistent with the more familiar notion based on charge conjugation. For making this let us note that the operation that conjugates the charge in Eq. (19) is

\[ C \Psi(x, s) = c \Psi(x, -s), \]

where \( c = \gamma^5 K \) is the standard charge conjugation operator. The remarkable points are that this operation coincides with the \( s \)-time reversal operation in the Wigner sense

\[ C = S, \]

and \( PcT \) looks as a “parity” operation in the five-dimensional space-time:

\[ PcT = \gamma^5 Q, \]

where

\[ Q \Psi(x) = \Psi(-x), \]

and \( \gamma^5 \) plays the role of the “intrinsic parity” operator. The identity (21) is the quantum analogous of a celebrated Feynman [25] observation at the classical level, that charge conjugation in the Lorentz force law is equivalent to a proper time reversal. In other words, charge conjugation is equivalent to an inversion of the sign of \( \frac{dx^\mu}{ds} \), according to the Stueckelberg interpretation for antiparticles.

In order to get a more intuitive insight about why this proper time formalism works, let us return to the problem of particle creation in an external electromagnetic field. In this case, the Heisenberg equations of motion are

\[ \frac{d\gamma^\mu}{ds} = 2i\gamma^\mu \mathcal{H} - 2i\pi^\mu, \]

\[ \frac{d\pi^\mu}{ds} = eF^{\mu\nu}\gamma_\nu, \]

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which form a coupled system of linear differential equations of first order in
\[ \gamma^\mu = \frac{dx^\mu}{ds} \quad \text{and} \quad \pi^\mu = p^\mu - eA^\mu, \]
where the mass operator \( H = \gamma^\mu \pi^\mu \) is a constant of motion.

Let us restrict to the case of pure electric field, and choose the coordinate system in such a way that \( \vec{E} = E \vec{e}_1 \), therefore the only non-vanishing components of the electromagnetic field tensor are \( F_{10} = \pi_0, F_{01} = -eE \), and the system of differential equations are reduced to

\[
\begin{bmatrix}
\frac{d}{ds} \gamma^0 \\
\frac{d}{ds} \gamma^1 \\
\frac{d}{ds} \pi^0 \\
\frac{d}{ds} \pi^1
\end{bmatrix} =
\begin{bmatrix}
-2i\cal{H} & 0 & -2i & 0 \\
0 & 2i\cal{H} & 0 & -2i \\
0 & -eE & 0 & 0 \\
eE & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\gamma^0 \\
\gamma^1 \\
\pi^0 \\
\pi^1
\end{bmatrix},
\tag{25}
\]

plus uncoupled equations for the components 2 and 3 identical to the free case \[23, 17\]

\[
\frac{dx^\mu}{ds} = p^\mu + \left[ \frac{dx^\mu}{ds}(0) - p^\mu \right] \cos (2ps) - \frac{1}{2p} \frac{d\gamma^\mu}{ds}(0) \sin (2ps).
\tag{26}
\]

The system of differential equations could be exactly solved diagonalizing the matrix of Eq. (25). The eigenvalues are \( z_{1,2,3,4} = i\cal{H} \pm \sqrt{-\cal{H}^2 \pm 2ieE} \). In the weak field approximation \((\cal{H}^2 \gg 2eE)\) the solution of this system adopts a specially simple form \[17\]

\[
\begin{align*}
\frac{dx^0}{ds}(s) &= \frac{dx^0}{ds}(s) \bigg|_{E=0} \cosh \left( \frac{eE}{\cal{H}} s \right) - \frac{dx^1}{ds}(s) \bigg|_{E=0} \sinh \left( \frac{eE}{\cal{H}} s \right), \\
\frac{dx^1}{ds}(s) &= \frac{dx^1}{ds}(s) \bigg|_{E=0} \cosh \left( \frac{eE}{\cal{H}} s \right) - \frac{dx^0}{ds}(s) \bigg|_{E=0} \sinh \left( \frac{eE}{\cal{H}} s \right),
\end{align*}
\tag{27, 28}
\]

where \( p = \sqrt{p^\mu p_\mu} \) is the free positive mass operator. The classical picture of Eq. (26) together with Eq. (28) is a helical motion in the space and the orbital angular momentum of this \textit{Zitterbewegung} gives rise to the normal magnetic moment of the electron \[23, 17\]. Eqs. (27) and (28) describe the classical hyperbolic motion derived from the Lorentz force law modulated by the free \textit{Zitterbewegung}. This quick oscillatory motion (of a Compton space-time wavelength order) vanishes in the classical limit. Two different s-time scales appear, one related to the inverse of the frequency of the \textit{Zitterbewegung} \( \frac{1}{2\cal{H}} \) and the other related to the inverse of the electric field strength \( \frac{4\pi}{eE} \). Then
when $\frac{H}{eE} \gg \frac{1}{2H}$, the Zitterbewegung does not feel the adiabatic changes in the mean classical motion, so it works as in the free case. The same scales also appear in the space-time trajectories. If the minimal distance $\frac{2H}{eE}$ between the two branches of the hyperbola –representing particle and antiparticle solutions at the classical level– is greater than $\frac{1}{H}$, the particle and antiparticle trajectories are distinguishable. However, when $\frac{2H}{eE} \approx \frac{1}{H}$, such trajectories overlap, increasing the probability that the particle jumps to the trajectory of the antiparticle and vice versa. These jumps are reinterpreted in the standard viewpoint –which parameterizes the dynamics with the coordinate time $x^0$– as the pair creation and annihilation processes (Dirac picture\textsuperscript{11}). Summarizing, the Schroedinger Zitterbewegung depicted above gives a very clear semiclassical interpretation of such processes, which dresses the corresponding Feynman diagrams of physical content, disregarding the concept of Dirac’s sea (see Fig. 2).

![Figure 2: Pair creation: the dark side of relativistic quantum mechanics.](image)

At this point we disagree with some recognized field theorists that regard Feynman’s graphical method as “a convenient pictorial device that enables to keep track of the various terms in the matrix elements which can rigorously derived from quantum field theory” \textsuperscript{36}. We think that their opinion is due to they do not completely take into account the genesis of Feynman’s ideas\textsuperscript{11}.

\textsuperscript{11}This picture was refined by Sauter by considering the deformation of the energy gap produced by the electric field. Pair creation is interpreted as a tunneling of a negative energy state (not a hole in a sea) to a positive energy state \textsuperscript{10}. 

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originally developed from the proper time method. Unfortunately Feynman due to the misunderstanding of his dissertation at Pocono [28] was forced to introduce his space-time visualization of quantum electrodynamical processes in the form written in his 1949 papers [26]. He relegated much of his original physical ideas and motivations to his 1950 and 1951 papers [27]. So there are a generation of field theorists that have learned the derivation of Feynman rules from Dyson’s paper [37] rather than from Feynman’s ones. In fact when Dyson’s paper appeared most of Feynman’s work was still unpublished. Unfortunately although Dyson himself remarked that “the theory of Feynman differs profoundly from that of Schwinger and Tomonaga,” the announcement of the demonstration of the equivalence (strictly speaking only at the level of the consequences) of both theories had great impact. Moreover the fine Schwinger calculations [38] using a proper time method were considered just as mathematical tools and Nambu’s claims of his deep paper of 1950 [30] “The space-time approach to quantum electrodynamics, as has been developed by Feynman, seems to offer a very attractive and useful idea to this domain of physics. His ingenious method is indeed attractive, not only because of its intuitive procedure which enables one to picture to oneself the complicated interactions of elementary particles, its ease and relativistic correctness with which one can calculate the necessary matrix elements or transition probabilities, but also because of its way of thinking which seems somewhat strange at first look and resists our minds that are accustomed to causal laws. According to the new standpoint, one looks upon the world in its four-dimensional entirety. A phenomenon that will come into play in this theatre is now laid out beforehand in full detail from immemorial past to ultimate future and one investigates the whole of it at glance. The time itself loses sense as the indicator of the development of phenomena; there are particles which flow down as well as up the stream of time; the eventual creation and annihilation of pairs that may occur now and then, is no creation nor annihilation, but only a change of directions of moving particles, from past to future, or from future to past; a virtual pair, which, according to the ordinary view, is foredoomed to exist only for a limited interval of time, may also be regarded as a single particle that is circulating round a closed orbit in the four-dimensional theatre; a real particle is then a particle whose orbit is not closed but reaches to infinity ...” received little attention.

On the other hand most of quantum field theory treatises which intent
to incorporate the Feynman space-time visualization turn out to be contradictory. For example they interpret field operators as operators that create and annihilate particles in space-time points for giving an interpretation to the Green functions. However relativistic and non-relativistic quantum fields exhibit a striking difference concerning the localizability of their respective field quanta \[39\]. In fact, while in the non-relativistic case there is in principle no limitation on the accuracy of measuring the position of a particle, the combination of relativity and quantum theory provides an intrinsic limitation on the measurability of the position due to the particle creation mechanism. The understanding of such difficulties have inclined some authors to propose the idea that Minkowsky space-time is not suitable for particle physics and its role was essentially a historical one, unlike the energy-momentum space which would be fundamental \[40\]. On the contrary, in our proposal we prefer to leave Poincaré group and retain the localizability in Minkowsky space-time.

Summarizing, those field theories which desire to keep Feynman diagrams interpretative picture, must give up the Poincaré group. There is no space-time localization of particles in this framework. There is only space time localization of charges off the mass-shell.

In order to reinforce our pictorial image of the Fig. 2 let us derive the one-loop effective action \(W^{(1)}\), which describes the pair creation in an external electromagnetic field, from an argument purely based on the proper time formalism. As \(W^{(1)}\) is \(i\) times the closed loop amplitude \(L\), let us compute \(L\) using the proper time formalism. First, let us evaluate the amplitude for a super-particle at \(x^\mu\) and polarization \(k\) at time \(s = 0\) remains in the same point and with the same polarization at time \(s\). As a consequence of the indefinite metric (17), the spectral resolution of the identity is

\[
I = \int d^4x \sum_{jk} \gamma_j^0 \langle j, x^\mu \rangle \langle k, x^\mu \rangle.
\]

Then the expression of such an amplitude per unit of proper time for all the degrees of polarization is \(\frac{1}{s} \sum_{jk} \gamma_j^0 \langle j, x^\mu \rangle e^{i(\gamma^\nu \pi^\nu)s} \langle k, x^\mu \rangle\). The above process

\[12\] Although this hypothesis could work for the Poincaré group in the case of free fields, strong difficulties arise at the time of introducing interactions. Let us bear in mind that localizability and minimal coupling are intimately linked. Moreover, this fact is not compatible with the principle of general covariance. Notice that it would be possible to extend this formulation to develop quantum field theory in curved space-time.
is represented through an open diagram in the five-dimensional space-time, but it is a closed loop in four dimensions \[41\]. Restricting the formalism to the mass-shell by means of a Fourier transformation in proper time with the causal prescription and summing the contributions of each space-time point, we finally have

\[
W^{(1)} = i \int \int_{0}^{\infty} \frac{1}{s} \sum_{jk} \gamma_{jk}^{0} \langle k, x^{\mu} \mid e^{i(\gamma^{\mu} \pi_{\mu})s} \rangle \mid j, x^{\mu} \rangle e^{-ims} dsd^{4}x. \tag{30}
\]

Schwinger, using quantum field theory, obtained Eq. (30), which became the starting point of his 1951 seminal paper \[38, 27\].

The procedure used in the calculation of \(W^{(1)}\) also shows that the ultraviolet divergences only appear after the reduction of the off-shell amplitude on the mass shell. Note that this circumstance also suggests a natural regularization method based on a small mass dispersion \[27\]. Our alternative explanation does not involve the infinite amount of energy and charge of the Dirac sea in order to consider antiparticles, and in this way it avoids the infinities introduced in the standard theory from the very beginning. This is the reason why closed loops do not appear in the off-shell theory.

Until now we have only discussed the theory of external fields. In order to concluding, let us briefly discuss the radiative process.

Using this formalism and his operator calculus, Feynman presented at Pocono a closed expression for a system of spin half charges interacting via the quantized electromagnetic field for the case in which only virtual photons are present. In the particular case of one charge it reads \[27, 28\]

\[
\Psi(x, s) = \exp \left\{ -i \int_{0}^{s} \gamma^{\mu}(s') \pi_{\mu}(s') ds' + e^{2} \int_{0}^{s} \int_{0}^{s} \gamma^{\mu}(s') \gamma^{\mu}(s'') \delta_{\pm} \{ [x_{\mu}(s') - x_{\mu}(s'')]^{2} \} ds' ds'' \right\} \Psi(x, 0), \tag{31}
\]

where \(\delta_{\pm} \{ [x_{\mu}(s') - x_{\mu}(s'')]^{2} \}\) is the Green function of the d’Alembertian with Feynman’s boundary conditions. From the second term of Eq. (31), Feynman showed that the radiative corrections of QED can be derived. The analogy between the phase of Eq. (31) and the Wheeler-Feynman action \[42, 25\] for classical electrodynamics is remarkable. In fact the only substantial difference is the boundary conditions (half-advanced and half-retarded) chosen for the d’Alembertian Green function. The right boundary conditions for QED can
be obtained from the retarded condition of the off-shell theory. This fact strongly suggests that Eq. (31) could be derived, from first principles, from a de Sitter invariant formulation of QED.

For one super-particle (antiparticle) the de Sitter invariant equations read

\[ \Gamma^A (i \partial_A - e A_A) \Psi = 0, \]  
\[ \partial_A F^{AB} = e \overline{\Psi} \Gamma^B \Psi, \]

where the super-potential \( A^A = (A^\mu, A^5) \) arises from a natural extension of the gauge principle \[43\]. The standard four-potential can be obtained from \( A^A \) integrating the first four components in the proper time

\[ A^\mu(x^\nu) = \int_{-\infty}^{+\infty} A^\mu(x^\nu, s) ds, \]

as in the case of the matter fields. (The exponential factor does not appear in this case because the photon is non-massive. Note also that the transformation \( A^\mu(x^\nu, s) \rightarrow A^\mu(x^\nu, -s), (ds \rightarrow -ds) \), leads to the standard notion of charge conjugation for the potentials.)

**Note added in proof**

After completing this work we discovered a review paper of Fanchi \[44\] and the closely related works of Herdegen \[45\] and Kubo \[46\].

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