Measurements in quantum physics: towards a physical picture of relevant processes

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We propose a new structure of ensembles in quantum theory, based on the recently introduced intrinsic properties of electrons and photons. On this statistical basis the spreading of a wave-packet, collapse of the wave function, the quantum eraser, and interaction-free measurements are re-analyzed and the usual conceptual problems removed.

I. MEASUREMENTS AND ENSEMBLES

It took Albert Einstein, in 1935 [1], to formulate the basic conclusion to Heisenberg’s uncertainty relations [2]: the systems, we are dealing with in quantum theory (QT), are not completely defined by our mathematical representations (In case this is new to you, please read Sheldon Goldstein’s recent articles in Physics Today [3]). However, given the proof by John von Neumann [4], that the linearity of the main equations in QT entails that no conventional ensemble can lie underneath the statistical description of measurements, we are left with a somewhat puzzling consequence: although there must be an ensemble, we do not know its structure. Most lucidly, this dilemma has been expressed by David Bohm [5]:

Yet it is not immediately clear, how the ensembles, to which ... probabilities refer, are formed and what their individual elements are. For the very terminology of quantum mechanics contains an unusual and significant feature, in that what is called the physical state of a quantum mechanical system is assumed to manifest itself only in an ensemble of systems.

By now the way Bohm proposed to amend this deficiency in the logical structure of QT is well researched: if the wavefunction at a given location and moment is a complex number defined by

\[ \psi = R \cdot e^{iS} \]  

then the Schrödinger equation gives rise to a "quantum potential"

\[ Q = \frac{\nabla^2 R}{R} \]  

which, in the same way a physical potential \( V \) determines the solutions of Schrödinger’s equation, determines the mathematical form of the wavefunction \( \psi \). There is an important difference, though: since \( Q \) is proportional to the second derivative of \( R \), the value of \( \psi \) at one point of the system influences \( Q \) throughout the whole system and vice versa: the very ansatz of Bohm thus gives rise to strongly non-local effects [6].

This non-locality of Bohm’s theory was, initially, taken as an argument against its theoretical soundness (see Goldstein’s article), until John Bell and Alain Aspect proved that the same applies to QT itself [7,8]. Since then the community is divided: while the more orthodox faction believes in physical non-locality regardless of the contradiction with Special Relativity (see e.g. the recent article in Nature on "quantum teleportation" [9]), the more cautious view is that we are dealing with a conceptual non-locality due to our representation of micro physical systems. It seems that only a clear perception of the ensemble structure in QT will allow a decision in this issue. As previous work revealed, the features of the ensembles in quantum theory could arise from intrinsic fields due to particle propagation [10]. This approach goes beyond the proposed modifications of Everett [11], Ghirardi [12], Omnès [13], Hartle [14], Griffiths [15], Zurek [16] and others, who, although pioneering new ways to solve the measurement problem in QT, confined themselves more or less to the question of wavefunction collapse.

The strategy employed in developing this new framework was roughly the following: since the mathematical description in QT yields correct results, it should be feasible to complete these - mathematical - expressions by a physical basis. It is evident that the conventional framework of QT allows for no such basis, since the wavefunction has no physical meaning. In the new framework, on the contrary, the wavefunction gains a double meaning: a physical as well as a statistical one, corresponding to the description of single particles (single elements in Bohm’s words) and ensembles of particles. The argument developed in the present paper will be that this double meaning is responsible for some of the most puzzling results in quantum theory. To see, where this double meaning comes from, let me briefly sketch the development of the theoretical framework, for a more thorough view please refer to the original paper [10].
II. THEORETICAL FOUNDATIONS

That the energy of a particle depends on its intrinsic frequency $\omega$ is the basis of Planck’s quantum hypothesis, thus:

$$E = h \cdot \omega$$

(3)

For photons, the frequency is equal to the frequency of its electromagnetic $\vec{E}$ and $\vec{B}$ fields. For massive particles like electrons there is no such connection, one either has to refer to de Broglie’s harmony of phases [17], or seek a solution in terms of intrinsic and rotational components of motion (e.g. in [18]). However, if the wave equation describes the intrinsic density of mass distribution $\rho = \rho(\vec{r}, t)$, oscillating with the characteristic frequency $\omega$, then the total energy of the electron is split into two distinct components [10]:

$$E = h \cdot \omega =: m_eu^2 \quad E_{QT} = E_K = \frac{m_e}{2}u^2 \quad E_F = \frac{m_e}{2}u^2$$

(4)

where $u$ is the velocity, $m_e$ the mass, and $E_{QT} = E_K$ the kinetic energy of the electron, equaling the expression in quantum theory. $E_F$, the field energy results from the intrinsic wave features of electron motion. While the latter is, initially, purely hypothetical, it can be justified, a posteriori, by calculating the solutions to Schrödinger’s equation under the condition that $E_F$ in QT is generally undefined. From the time-independent equation:

$$\left( -\frac{\hbar^2}{2m_e} \nabla^2 + V \right) \psi = E \cdot \psi$$

(5)

and in a plane wave basis set the undefined field component of electron energy leads to an uncertainty in the range of the wavevector $\vec{k}$, and the consequence of this uncertainty can be expressed in Heisenberg’s uncertainty relations [10,2]:

$$\Delta P \Delta X \geq \frac{\hbar}{2}$$

(6)

If the field energy $E_F$ is described by an intrinsic scalar field $\phi_F$ and vector fields $\vec{E}$, $\vec{B}$ so that:

$$\phi_F = \frac{1}{2} \left( \frac{1}{u^2} \vec{E}^2 + \vec{B}^2 \right) \quad E_F = \int_V dV \phi_F \quad \vec{E} \cdot \vec{u} = \vec{B} \cdot \vec{u} = \vec{E} \cdot \vec{B} = 0$$

(7)

then it can be established that these $\vec{E}$ and $\vec{B}$ fields comply with the Maxwell equations [11,19]. Please note that the shape of the particles or their volume do not enter the picture, they remain completely undefined. In addition, all the conventional Planck and de Broglie relations are valid, although the latter refer to phase velocity rather than group velocity. Based on these intrinsic features the physical meaning of the wavefunction is that the square of its real part $\Re(\psi)$, for a single particle, is proportional to its density of mass $\rho$. The statistical meaning is different for electrons and photons, since only the electron has an undefined intrinsic energy component.

In the following we shall simplify the model by assuming that the density of mass of any single particle is constant for a specific type (electron, photon), and described by $\rho = |\psi|^2$. The undefined field component of energy $E_F$ shall be accounted for by a range of particle energies rather than its development with time. The model treated in the next sections is therefore a static account of an essentially dynamical problem. However, as will be seen presently, this simplified account is sufficient to shed new light on some of the most interesting problems in measurement theory. The statistics in measurement processes result from the following unknown variables:

- **Photons**: unknown phase, unknown amplitude of wavefunction, unknown number of particles.
- **Electrons**: unknown phase, unknown amplitude of wavefunction, unknown intrinsic energy, unknown number of particles.

III. QUANTUM ENSEMBLE OF FREE ELECTRONS

Due to periodic wave functions, the intrinsic potential at a moment $t$ can take any value, and the wave vector of the problem therefore is not exactly determined, but covers the whole range from $k^2 = 0$ to $k^2 = \frac{m_e}{\hbar^2} E_T$, where $E_T = \hbar \omega$. 

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Consider a point $\vec{r}$, where the external potential vanishes $V(\vec{r}) = 0$. Due to the disregard for intrinsic potentials the Schrödinger equation at this location applies for all wavelets described by:

$$\vec{k}^2(\vec{r}) + k_t^2(t) = \frac{m}{\hbar^2} E_T \quad 0 \leq k_t^2(t) \leq \frac{m}{\hbar^2} E_T \quad (8)$$

Here $k_t^2(t)$ shall denote the intrinsic potential, not accounted for in QM. $E_T$ is the total energy of the particle. The two variables are given by:

$$E_T = mu^2 \quad \frac{\hbar^2}{m} k^2 = \phi_t(t) V_p \quad (9)$$

where $u$ again is the velocity and $V_p$ the volume of a particle. The quantum ensemble shall be the integral over allowed wave states. Then the wave function $\psi(\vec{r})$ can be written as:

$$\psi(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} d^3 r \chi_0(\vec{k}) e^{i\vec{k}\vec{r}} \quad k_0 = \sqrt{\frac{m}{\hbar^2} E_T} \quad (10)$$

Using a Fourier transformation the amplitudes of the ensemble are:

$$\chi_0(\vec{k}) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} d^3 r \psi(\vec{r}) e^{-i\vec{k}\vec{r}} \quad (11)$$

In this case an ensemble of electrons is described by identical solutions of the Schrödinger equation, although in QT the mathematical representation seems to refer to one particle and one particle only. Bohm’s puzzlement with the individual system, which is in fact an ensemble of systems then is quite justified.

**IV. QUANTUM ENSEMBLE IN EXTERNAL POTENTIALS**

An even more interesting consequence of the same feature is observed in external potentials. The external potential at a point $\vec{r}$ has two effects: the range of allowed wavelets and therefore the quantum ensemble will be changed, and the internal properties of single wavelets will be altered. If the potential at $\vec{r}$ equals $V(\vec{r})$, the allowed $k$–values will comply with:

$$k^2(\vec{r}) + k_t^2(t) = \frac{m}{\hbar^2} (E_T - V(\vec{r})) \quad 0 \leq k_t^2(t) \leq \frac{m}{\hbar^2} \quad (12)$$

$$(E_T - V(\vec{r})) k_t^2 = \frac{m}{\hbar^2} (E_T - V(\vec{r})) \quad E_T = mu^2 \quad (13)$$

For reasons of consistency the potential $V(\vec{r})$ is double the potential if only kinetic properties are considered. The range of allowed $k$–values in this case depends on the energy $E_T$ of a single particle as well as the potential applied. There are two distinct cases: $E_T - V(\vec{r})$ is either a positive or a negative value, corresponding to wavelets in a potential or to exponential decay of single waves.

For $E_T - V(\vec{r}) > 0$ the potential can either be a positive or a negative value, leading to an enhancement or a reduction of the quantum ensemble of valid solutions. The general solution for both cases is then:

$$\psi(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \int_0^{k_1} d^3 k \chi_0(\vec{k}) e^{i\vec{k}\vec{r}} k_1 = \sqrt{\frac{m}{\hbar^2}} \sqrt{(E_T \pm |V(\vec{r})|)} \quad E_T = mu^2 \quad (14)$$

The range of individual wavelets defines, as in the case of a vanishing external potential, an ensemble of particles, which comply with the differential Schrödinger equation. Any integration of the equation therefore also contains a – hidden – manifold of individual wavelets. A positive potential essentially limits the number of individual waves contained in the ensemble, because it diminishes the range of $k$. A negative potential has the opposite effect: the number of waves in the ensemble is increased, and their statistical weight in the whole system is equally higher. Fig. displays the quantum ensembles for different external potentials.

For energies $E_T < V(\vec{r})$ the mathematical formalism of Schrödinger’s equation allows for solutions with a negative square of $k$, equivalent to an exponential decay of single wavelets:
\[ k^2(\vec{r}) + k_0^2(t) = \frac{m}{\hbar^2} (E_T - V(\vec{r})) \leq 0 \quad 0 \geq k_0^2(t) \geq \frac{m}{\hbar^2} (E_T - V(\vec{r})) \]  

(15)

\[ \psi(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \int_0^{k_1} d^3 k \chi_0(\vec{k}) e^{-ikr} \quad k_1 = \sqrt{\frac{m}{\hbar^2} (V(\vec{r}) - E_T)} \quad E_T = mu^2 \]  

(16)

The question in this case concerns not so much the mathematical formalism but the physical validity. Considering, that electrodynamics is a description of intrinsic properties of single particles, applicable to photons as well as electrons, the results of electrodynamics in different media should also have relevance for the wave properties of single particles. And considering, furthermore, that an exponential decay into a medium at a boundary is one type of solution, the same must generally hold for single wavelets. It is a basically classical solution to boundary value problems and, so far, no specific feature of a quantum system.

V. WAVE FUNCTION NORMALIZATION

We have not yet defined the amplitude \( \chi_0(\vec{k}) \) of single components in the quantum ensemble. This can be done by requiring single wavelets to comply with the mass relations of particles. Since:

\[ \chi^*(\vec{k})\chi(\vec{k}') = \frac{1}{(2\pi)^3} \chi_0(\vec{k})\chi_0(\vec{k}') e^{i\vec{r}(\vec{k}' - \vec{k})} \]  

(17)

an integration over infinite space yields the result:

\[ \int_{-\infty}^{+\infty} d^3 r \chi^*(\vec{k})\chi(\vec{k}') = \delta^3(\vec{k} - \vec{k}')\chi_0(\vec{k})\chi_0(\vec{k}') \quad m = \int_{-\infty}^{+\infty} d^3 r |\chi(\vec{k})|^2 = \chi_0^2(\vec{k}) \]  

(18)

Using this amplitude the square of the wave function \( \psi(\vec{r}) \) in different external potentials can be calculated.

\[ \int_{-\infty}^{+\infty} d^3 r |\psi(\vec{r})|^2 = \frac{m}{(2\pi)^3} \int_{0}^{k} d^3 k d^3 k' e^{i\vec{r}(\vec{k}' - \vec{k})} \]

\[ = m \int_{0}^{k} d^3 k d^3 k' \delta^3(\vec{k} - \vec{k}') = \frac{4\pi m}{3} k^3 \]  

(19)

If we consider a system, where the potential \( V = V(\vec{r}) \), the wavefunction will depend, via the normalization condition, on the potentials in all parts of this system. And if we formalize two space-like separated events within a single system in QT (there is no way to avoid this in any experiment aimed at proving or refuting non-locality), these events are no longer logically (or physically) separate: non-locality is therefore an inherent conceptual feature of quantum theory.

VI. BOUNDARY CONDITIONS

One of the easiest examples in quantum theory, which suffices to demonstrate the effect of boundary conditions, is the square potential well. We take a one dimensional potential well, the external potentials described by:

\[ V = 0 \quad \forall |x| \leq x_0 \quad V = V_0 \quad \forall |x| \geq x_0 \]  

(20)

To solve the problem we have two consider the behavior of single members of the quantum ensemble as well as the behavior of the whole ensemble. The limiting \( k \) values can again be inferred from the solution of the one-dimensional Schrödinger equation, they will be for \( E_T < V_0 \):

\[ k_0^2 \leq \frac{m}{\hbar^2} E_T \quad |x| \leq x_0 \]

\[ k_0^2 \leq \frac{m}{\hbar^2} (V_0 - E_T) \quad |x| \geq x_0 \]  

(21)

For incident, reflected and penetrating waves, the three components of the wave are a wave of positive and a wave of negative propagation in the region \( |x| < x_0 \), and a decaying component in the region \( |x| > x_0 \). Considering individual
members of the ensemble, the lowest \( k \) value should correspond to maximum decay in the potential, while the member with maximum total energy should display maximum penetration. The relation between an arbitrary wave vector \( k_1 \) and its corresponding member \( k_2 \) must therefore be:

\[
k_1^2 + k_2^2 = \frac{m}{\hbar^2} V_0
\]

The wave functions in the three separate regions shall be described by standard solutions. Accounting for the boundary conditions for steady transition of the particle wave the coefficients can be determined and the solution for an individual wave is therefore, equivalent to the solution in quantum theory [20]:

\[
\chi_0 \cdot e^{k_2 x} \quad x \leq -x_0
\]

\[
\chi(x) = \chi_0 \cdot e^{-k_2 x_0} \frac{\cos k_1 x}{\cos k_1 x_0} \quad |x| \leq x_0
\]

\[
\chi_0 \cdot e^{-k_2 x} \quad x \geq x_0
\]

The normalization condition an ensemble member yields the amplitude of the particle wave:

\[
\chi_0(k_1, k_2) = \sqrt{\frac{mk_2}{1 + k_2 x_0}} e^{k_2 x_0} \cos k_1 x_0
\]

And the total ensemble can equally be calculated by integrating over the full range of allowed \( k \) values.

\[
\int_{-\infty}^{\infty} dx |\psi(x)|^2 = 2 \int_{0}^{\infty} dx \left[ \theta(x_0 - x) \int_{0}^{k_0} dk_1 \chi_0^2(k_1) e^{-2k_1 x_0} \frac{\cos^2(k_1 x)}{\cos^2(k_1 x_0)} + \right.

+ \left. \theta(x - x_0) \int_{0}^{k_0} dk_2 \chi_0^2(k_2) e^{-2k_2 x} \right]
\]

The amplitude \( \chi_0(k) \) must finally be renormalized, and the square of the wave function then describes the probability distribution of the whole ensemble. The procedure described is similar to the standard procedure in quantum theory, although in this model \( k \) space is not unlimited, the cutoff is determined by the energy \( E_T \). The structure of the ensembles in different environments provides a means to analyze the interplay between physical aspects (the intrinsic properties of single electrons) and statistical ones (the way quantum theory contains a hidden ensemble of single electrons).

**VII. SPREADING OF A WAVE–PACKET**

The effect is a common source of irritation, and concepts have been put forth to eliminate it in a modified version of quantum theory (see, for example, Mackinnon [21,22], or the GRW model [12]). In a one dimensional model the development of an amplitude \( \hat{\psi}(k) \) is described by the integral:

\[
\psi(x, t) = \int dk e^{i(kx - \omega t)} \hat{\psi}(k)
\]

Two initial distributions are calculated: a Gaussian distribution, with a wave function centered around a value \( x_0 = 0 \) at \( t = 0 \), and an ensemble with exactly defined energies (such an ensemble can be obtained by energy measurements, as shown further down):

\[
\psi_1(x, t = 0) = e^{-\frac{x^2}{2} + ik_0 x} \quad \psi_2(x, t = 0) = e^{ik_0 x}
\]

The evaluation of the integral yields:
\[ \hat{\psi}_1(k) = \int dx \psi_1(x, 0) e^{-ikx} = e^{-(k-k_0)^2b^2/2} \]

\[ \hat{\psi}_2(k) = \int dx \psi_2(x, 0) e^{-ikx} = \delta(k-k_0) \] (30)

The square of the two wave functions at \( t > 0 \) is consequently:

\[ |\psi_1(x, t > 0)|^2 = \left( 1 + \frac{\hbar^2 t^2}{m^2 b^4} \right)^{-1} \exp \left[ -b^2 \left( 1 + \frac{\hbar^2 t^2}{m^2 b^4} \right)^{-2} \left( x - \frac{\hbar k_0}{m} t \right)^2 \right] \] (31)

\[ |\psi_2(x, t > 0)|^2 = 1 \] (32)

The development of the two ensembles is shown in Fig. 2. As it is possible to decompose an arbitrary distribution of initial \( k \) values in Gaussian distributions, the result holds quite generally. There are two possibilities to account for this feature: (i) Either the restructuring is referred to some potential not covered by field theories, in this case we would have to recur to Bohm’s quantum potential \(^2\). (ii) The initial conditions contain an assumption which affects the physical properties of particles. In the single particle case (see section II), where \([\mathcal{R} \psi(x)]^2 \propto \rho(x) \propto \phi(x)\), an inhomogeneous distribution like \( \psi_1(x, 0) \) gives rise to an intrinsic potential \( \phi(x) \), described by:

\[ \psi_1(x, 0) = \psi_0 e^{ik_0x} \quad \psi_0 = e^{-x^2/2b^2} \phi(x, 0) = \psi_0^2 = e^{-x^2/b^2} \] (33)

And the system is therefore not, as implied by the mathematical formulations, free of forces, but will experience a force along the direction \( x \):

\[ F_x = -\frac{\partial \phi}{\partial x} = \frac{2x}{b^2} e^{-x^2/b^2} \] (34)

Along this line of reasoning we may reconsider the question of quantum ensembles from the viewpoint of intrinsic potentials and forces. The most general form of a wave function is given by the integral:

\[ \psi(\vec{r}) = \int d^3k \psi_{0,k}(\vec{r}, \vec{k}) e^{i\vec{k} \cdot \vec{r}} \] (35)

The potentials due to the qualities of the amplitude are then responsible for intrinsic energy components in addition to the purely periodic fields of a plane wave. They are:

\[ \phi_{i}(\vec{r}, \vec{k}) = u^2 \left| \psi_{0,k}(\vec{r}, \vec{k}) \right|^2 = \frac{\hbar^2 k^2}{m^2} \left| \psi_{0,k}(\vec{r}, \vec{k}) \right|^2 \] (36)

The forces within the propagating wave are either periodic – the total energy density of the plane wave is constant –, or they are forces due to the properties of the amplitude. These forces will be:

\[ F = \frac{\hbar^2 k^2}{m^2} \left[ \psi_{0,k}^* \nabla \psi_{0,k} + \psi_{0,k} \nabla \psi_{0,k}^* \right] \] (37)

A stable state of the system can only be expected, if these forces vanish. The equilibrium condition for a system of particles described as plane waves is therefore:

\[ \psi_{0,k}^* \nabla \psi_{0,k} + \psi_{0,k} \nabla \psi_{0,k}^* = 0 \] (38)

The two ensembles defined, the ensemble with exact energies as well as the quantum ensemble (equal amplitude of all partial waves), comply with this condition since in both cases the amplitudes \( \psi_{0,k} \) do not depend on \( \vec{r} \). But the distribution used for the calculation of the spreading wave packet is incompatible with this condition.
VIII. COLLAPSE OF THE WAVE FUNCTION

In his rather fundamental and comprehensive analysis of measurement processes in quantum theory Ballentine [24-25] proceeded from two mutually exclusive statements on the quality of the state concept, i.e. that (i) a pure state provides a complete and exhaustive description of an individual system, and (ii) a pure (or mixed) state describes the statistical properties of an ensemble of similarly prepared systems. The subsequent analysis of measurement processes proved that any interpretation of the type (i) ... is untenable.

If the state vector of a system, or the wave function in QT were an exhaustive information about the system, the logical problems seem indeed severe if not unsurmountable. The situation changes, though, if one conceives that the quantum mechanical description does not provide a full account of physical variables. To a greater or lesser extent all the proposed modifications of QT to account for the measurement problems, cited in the introduction, use this feature. As a simple example of the reduction of the wave function in a measurement process we consider a retarding field analyzer frequently employed in LEED (low energy electron diffraction) measurements. A retarding field analyzer is a positive potential, assumed rectangular for simplicity, which selects only electrons above a certain energy threshold. We equally assume, that the electrons initially are free, their energy shall be given by a value $E_k$ (see Fig. [3]). From a strictly causal point of view, the electrons below an exactly defined threshold value $E_{rfa} < E_k$ cannot pass the filter and the number of electrons after the filter is therefore reduced to single particles with an energy above the threshold value. As the calculation in quantum theory integrates over all possible particle states at a given location $\vec{r}$, and since the range of allowed $k$–values depends on the level of kinetic energy, the total density $\rho(\vec{r})$ at any location before the analyzer will be:

$$\rho(\vec{r}) = \psi^*(\vec{r})\psi(\vec{r}) = \int_0^{k_0} d^3k \chi^*(\vec{k},\vec{r})\chi(\vec{k},\vec{r})k_0^2 = \frac{2m}{\hbar^2}E_k \tag{39}$$

while after the analyzer the wave function will be limited to states with energy values higher than the threshold:

$$\rho'(\vec{r}) = \psi'^*(\vec{r})\psi'(\vec{r}) = \int_{k_1}^{k_0} d^3k \chi^*(\vec{k},\vec{r})\chi(\vec{k},\vec{r})k_1^2 = \frac{2m}{\hbar^2}(E_k - E_{rfa}) \tag{40}$$

Clearly the quantum ensemble has been reduced. The measurement of particle energies by retarding fields therefore leads to a reduction of the statistical ensemble, but in a causal and deterministic manner. The ensemble wave function is reduced due to the removal, in an equally causal and deterministic way, of partial waves. It should be noted that the collapse of the wave function in real space cannot, presently, be treated within the same model.

IX. THE QUANTUM ERASER

The polarization of the intrinsic fields is decisive for interference measurements as can be demonstrated by an analysis of quantum eraser phenomena. In this case the which–path information of the photon is said to preclude interference. Conventionally, the measurement is formalized as follows [26]. The amplitude of an incident photon of horizontal polarization is split coherently in two separate beams, described by the quantum state vector (1 denotes the first, 2 the second of the two paths)

$$\psi_{12}^0 = \frac{1}{\sqrt{2}}(\psi_{1,H} + \psi_{2,H}) \tag{41}$$

The square of $\psi$, or the probability density in this case contains an interference term $\psi_{1,H}^*\psi_{2,H}$:

$$|\psi_{12}^0|^2 = \frac{1}{2}( |\psi_{1,H}|^2 + |\psi_{2,H}|^2 + \psi_{1,H}^*\psi_{2,H} + \psi_{1,H}\psi_{2,H}^*)$$

If a polarization rotator changing the polarization of the beam to vertical (V) orientation is placed in path 1, the interference pattern is no longer observable, and the measurement yields random results for the local probability density on the measurement screen. In quantum theory the result is referred to the orthogonality of the two states $H$, and $V$, and the state vector of the photon described by:

$$\psi_{12}^1 = \frac{1}{\sqrt{2}}(\psi_{1,V} + \psi_{2,H}) |\psi_{12}^1|^2 = \frac{1}{2}( |\psi_1|^2 + |\psi_2|^2) \tag{42}$$
The result can be changed by inserting a diagonal polarizer into the path of the recombined beams, in this case the wave function and probability density will again show interference effects: the which–path information, connected to the polarization of the two separate beams is said to have been "erased".

\[ \psi_{12}^2 = \frac{1}{2}\sqrt{2} (\psi_1 + \psi_2) (\psi_V + \psi_H) |\psi_{12}|^2 = \frac{1}{4} (|\psi_1|^2 + |\psi_2|^2 + 2\Re[\psi_1^*\psi_2]) \] (43)

In the context of intrinsic properties and polarizations of intrinsic fields, since the intensity of electromagnetic radiation is described by the electromagnetic potential \( \phi_{em} \) (for a general description the field vectors are assumed complex):

\[ \phi_{em} = \frac{1}{2} \left( \frac{1}{\epsilon^2} |\vec{E}|^2 + |\vec{B}|^2 \right) \] (44)

If the beam of horizontal polarization (direction \( x \)) is split into two separate beams, the electric and magnetic fields after recombination will be:

\[ \vec{E}_{12}^0 = \frac{1}{\sqrt{2}} (E_1\vec{e}_x + E_2\vec{e}_x) \quad \vec{B}_{12}^0 = \frac{1}{\sqrt{2}} (B_1\vec{e}_y + B_2\vec{e}_y) \] (45)

where the fields \( E_2, B_2 \) contain the phase information \( e^{i\phi} \). The intensity measured after recombination will consequently contain interference terms:

\[ \phi_{em}^0 = \frac{1}{2} \left( \frac{1}{\epsilon^2} |E_1|^2 + |B_1|^2 \right) (1 + \cos\phi) \] (46)

A polarization rotator in path 1 changes the polarization of the electric and magnetic fields to \( \vec{e}_y \) and \( -\vec{e}_x \) respectively, and the intensity after recombination is then not affected by the phase \( \phi \):

\[ \vec{E}_{12}^1 = \frac{1}{\sqrt{2}} (E_1\vec{e}_y + E_2\vec{e}_x) \quad \vec{B}_{12}^1 = \frac{1}{\sqrt{2}} (-B_1\vec{e}_x + B_2\vec{e}_y) \] (47)

\[ \phi_{em}^1 = \left( \frac{1}{\epsilon^2} |E_1|^2 + |B_1|^2 \right) \] (48)

If the recombined beam is passing through a diagonal polarizer (plane of polarization in \( xy \)-direction), the electromagnetic fields after polarization are:

\[ \vec{E}_{12}^2 = \frac{1}{2} (E_1\vec{e}_{xy} + E_2\vec{e}_{xy}) \quad \vec{B}_{12}^2 = \frac{1}{2} (-B_1\vec{e}_{yx} + B_2\vec{e}_{yx}) \] (49)

And the intensity of the beam shows again the interference pattern of the phase \( \phi \):

\[ \phi_{em}^2 = \frac{1}{4} \left( \frac{1}{\epsilon^2} |E_1|^2 + |B_1|^2 \right) (1 + \cos\phi) \] (50)

Mathematically, the description by way of intrinsic potentials and polarizations yields the same result as the conventional calculation in quantum theory. However, the interesting aspect of the effect is its interpretation. While in the conventional framework the which–path information (and its relation to the conception of complementarity) is seen as the ultimate reason for the experimental results, it is, in the new theory, the intrinsic information due to the electromagnetic fields and their vector features, which are held responsible.

X. INTERACTION–FREE MEASUREMENTS

An interaction–free measurement, which is based on a thought experiment by Renninger [31], provides information about the existence of an object in a closed system without necessarily interacting with this object. The essentials of such a measurement, recently undertaken by Kwiat et al. [29,30], can be seen in Fig. 4. Interaction free measurements, usually performed with down–converted photons, are interesting due to two features: the wave function of the system
and consequently system energy is changed, even if no interaction occurs. And the results are seemingly incompatible
with classical field theories, because the trajectories of single particles through the measurement apparatus can be
identified.

The first feature was modeled by Dicke [32] using a modified Heisenberg microscope and calculating the state vectors
of photons and a non–interacting atom in first order perturbation theory. The result of Dicke’s calculation seemed
to prove that even interaction free measurements correlate with an exchange of virtual photons or, in Dicke’s words:
*The apparent lack of interaction between the atom and the electromagnetic field is only illusionary.* On the statistical
basis developed in this paper, the result must be modified. The local modification of ensemble ranges means, in
this context, that an interaction free measurement corresponds to a different ensemble, i.e. an ensemble which has
zero probability in the range, where an interacting particle is appreciable. It is therefore the limitation imposed, the
change of boundary conditions, which is the ultimate reason for the change of the wave function. And if this local
range affects system energy like in Dicke’s model of an harmonic oscillator in a magnetic field [32], then the energy of
the system changes.

The second feature of interaction free measurements, the assertion by Kwiat et al. [30] that ”complementary is
essential” to the experimental results achieved, requires a critical analysis. What the argument indicates, is the
impossibility for a single photon in the interferometer to trigger the bomb and detector D2 (see Fig. 4). But as
detection efficiency is only two percent, about 98 % of the incident energy (triggering a detector by one of the down–
converted photons) is not accounted for. And in this case the argument of complementarity as well as the whole
argumentation of interaction–free measurements seems questionable.

XI. CONCLUSION

We have shown in this paper that the intrinsic energy component plays a substantial role in the resolution of
some of the most important paradoxes in quantum theory. In particular we found that: (i) the spreading of a wave
packet is due to a peculiar choice of initial conditions which, in the present model, are physically problematic. (ii)
The collapse of the wavefunction in k-space is due to a removal of partial waves from the full ensemble during the
measuring process. (iii) The quantum eraser can be seen as an example, where the polarizations of intrinsic fields
become decisive. (iv) And interaction-free measurements are not paradoxical, if the local extension of an ensemble is
considered.

ACKNOWLEDGEMENTS

Thanks are due to the Österreichische Forschungsgemeinschaft for their generous support to attend the Wigner
Symposium.

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FIG. 1. Quantum ensemble of wavelets for three different potentials. A negative potential increases the number of allowed $k$-values, a positive potential has the opposite effect.
FIG. 2. Development of ensembles. An ensemble of arbitrary energy and Gaussian distribution will develop into a quantum ensemble of equal probability for all energy values (top). The development of a particle with exactly defined energy due to the Schrödinger equation or the wave equation leads to a local ensemble over the whole local range of the system (bottom).
FIG. 3. Reduction of the wave function due to retarding field analyzer. The positive potential $V_{rfa}$ is lower than the total energy of the ensemble limit $E_k$ (top), the selection of members of the ensemble leads to a reduction of the statistical ensemble after the measurement (bottom)
FIG. 4. Mach–Zehnder interferometer with or without a sensitive bomb-trigger in one path