Demonstration of a programmable source of two-photon multipartite entangled states

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(Dated: March 26, 2010)

We suggest and demonstrate a novel source of two-photon multipartite entangled states which exploits the transverse spatial structure of spontaneous parametric downconversion together with a programmable spatial light modulator (SLM). The 1D SLM is used to perform polarization entanglement purification and to realize arbitrary phase-gates between polarization and momentum degrees of freedom of photons. We experimentally demonstrate our scheme by generating two-photon three-qubit linear cluster states with high fidelity using a diode laser pump with a limited coherence time and power on the crystal as low as \( \sim 2.5\text{mW} \).

PACS numbers: 03.65.Bg,42.50.Ex,42.50.Dv

I. INTRODUCTION

Multiqubit entangled states, e.g. cluster states, are key resources to realize several protocols of quantum information processing, including measurement based quantum computation [1,3], quantum communication [4] and quantum error correction [5]. Besides, they found applications in advanced fundamental tests of quantum non-locality [6–9]. Basically, there are two ways to generate multiqubit entangled states, e.g. cluster states. On one hand, one may increase the number of entangled photons [10–13]. On the other hand one may use different degrees of freedom of the same pair of photons [3,14–16] achieving so-called hyperentanglement. The second method offers a larger robustness against decoherence and nonunit detector efficiency. Four and six multiphoton cluster states have been experimentally created [10–12] as well as two-photon four- [3,14,15,17,25] and six-qubit cluster states [26].

In this paper we suggest and demonstrate a novel scheme to generate two-photon multipartite entangled states which exploits the transversal spatial structure of spontaneous parametric downconversion and a programmable spatial light modulator (1D SLM) based on a liquid crystal display. This kind of devices have been already used as pulse shaper for Bell state generation [27], as amplitude modulators for momentum imaging and qudit generation [28] as well as diffractive elements to operate on orbital angular momentum [29]. Here we employ SLM in an innovative way to realize two-photon multiqubit/qudit entangled states and demonstrate its use in the generation of two-photon three-qudit linear cluster states with high fidelity.

The novelty of our setup is twofold. On the one hand, we use the SLM for purification, and this allows us to dramatically decrease the spectral and angular filtering of downconverted photons, which is the method generally used to prevent the degradation of the purity. Moreover the SLM may be externally controlled, via software, and this makes our method more easily adjustable for the different implementations, compared to purification schemes that involve the use of suitably prepared crystals along the path of the downconverted photons [30]. On the other hand, we fully exploit the properties of the SLM to realize arbitrary phase-gates between polarization and momentum degrees of freedom. In this way, we obtain an effective, low cost, source of two-photon multiqubit entanglement using a pump with low power and a limited coherence time.

The paper is structured as follows: In the next Section we describe our PDC system in some details and illustrate the purification method based on the use of SLM. In Section III we address the generation of two-photon multiqubit/qudit entangled states and describe our experimental setup, used to demonstrate the generation of two-photon three-qubit linear cluster states with high fidelity. Section IV closes the paper with some concluding remarks.

II. POLARIZATION ENTANGLEMENT AND PURIFICATION

The first step in our scheme is the generation of polarization entangled states by spontaneous parametric down conversion (SPDC) in two adjacent BBO crystals oriented with their optical axes aligned in perpendicular planes [31,32]. The state outgoing the two crystals may be written as

\[
|\Phi\rangle = \frac{1}{\sqrt{2}} \int d\omega_s d\omega_f f(\omega_p, \omega_s, \theta) A(\omega_p) \\
\left[ e^{i k_p(\omega_p)} L e^{i (\phi(\theta)+s'f(\theta'))} |H, \theta, \omega_s\rangle |H, \theta', \omega_p - \omega_s \rangle + \right.
\left. e^{i k_p(\omega_p, \omega_s, \theta)} L |V, \theta, \omega_s\rangle |V, \theta', \omega_p - \omega_s \rangle \right],
\]

(1)

where \( L \) is the crystals length. The complex amplitude spectrum of the pump laser is \( A(\omega_p) \) whereas the downconverted photons are generated with the two photons spectral and angular amplitude \( f(\omega_p, \omega_s, \theta) \), as defined in [33]. We call \( \omega_p = \omega_p^0 + \omega_p \) and \( \omega_s = \omega_p^0/2 + \omega_s \) the frequencies of the pump laser and of the signal, where \( \omega_p \) and \( \omega_s \) are the shift from the central frequencies \( \omega_p^0 \) and \( \omega_s \).
and \( \omega_0^2 / 2 \). Likewise signal and idler generation angles are respectively \( \theta = \theta_0 + \theta' \) and \( \theta' = -\theta + \gamma \omega_s + \gamma' \omega_p \). Within the spectral width of our pump the dependence on \( \omega_p \) is negligible and thus we have \( \theta' \simeq -\theta + \gamma \omega_s \), with \( \gamma = \beta \theta'/\omega_0 \).

The phase term \( k_p^0 (\omega_p) L \) is due to the pump traversing the first crystal before it generates photons in the second one, whereas the term \( k_2^0 (\omega_p, \omega_s, \theta) L \) appears because the photons generated in the first crystal must traverse the second one. The perpendicular part \( k_2^\perp \) disappears for conservation of the transverse momentum, as it is guaranteed by the large pump spot on the crystals \((\sim 1.5 \text{mm})\). The other phase terms are common and are grouped out. The phase-shifts \( \phi(\theta) \) and \( \phi'(\theta') \) are introduced by a spatial light modulator (SLM) respectively on the signal and on the idler and depend on the generation angles \( \theta \) and \( \theta' \). These will be discussed in details in the following. It can be shown numerically that \( f(\omega_p, \omega_s, \theta) \approx f(\theta, \omega_s, \theta) = f(\omega_s, \theta) \) for crystals length \( L \lesssim 1 \text{mm} \). Upon expanding all the contributions to the optical paths to first order and after some algebra we arrive at

\[
|\Phi\rangle = \frac{1}{\sqrt{2}} \int \frac{d\theta}{\Delta \theta} d\omega_s \int d\omega_p f(\omega_s, \theta) A(\omega_p) \cdot \\
[|H, \theta, \omega_s\rangle |H, \theta', \omega_p - \omega_s\rangle + \\
e^{i\epsilon(\omega_p, \omega_s, \theta)} |V, \theta, \omega_s\rangle |V, \theta', \omega_p - \omega_s\rangle]
\]

where \( \Delta \theta \) is the angular acceptance and \( \omega_{s,1,2} \approx 1/\gamma (\theta \pm \Delta \theta/2) \) are the integration limits for \( \omega_s \), as determined by \( \Delta \theta \) and \( \gamma \). The phase function between the \( H \) and the \( V \) component is given by

\[
\varphi(\omega_p, \omega_s, \theta) = \phi_0 + \alpha L \omega_p + \beta L \omega_s - \delta L \theta - \phi(\theta) - \phi'(\theta'),
\]

where \( \phi_0 \) includes all the zero-order terms of the expansion. The phase term \( \alpha L \omega_p \) accounts for the delay time between horizontal and vertical downconverted photons. The two subsequent terms rise for conservation of the transverse momentum. The term \( \beta L \omega_s \) may be understood by considering the signal at the fixed angle \( \theta \): for different \( \omega_s \), the idler sweeps different \( \theta' \) and this means different optical path. Likewise fixing \( \omega_s \), a positive variation of \( \theta \) correspond to a negative variation of \( \theta' \) and this introduces an optical path dependent on \( \theta \), i.e. the phase shift \( \delta L \theta \). The delay time between the photons may be compensated upon the introduction of a proper combination of birefringent crystals on the pump path, as already demonstrated in Ref. 32 (see Fig. 2). Let us now focus attention on the action of the SLM, i.e. on the phase function \( \phi(\theta) \) and \( \phi'(\theta') \). At first, since \( \theta' \simeq -\theta + \gamma \omega_s \) we note that the choice

\[
\phi'(\theta') = \beta L \theta'/\gamma + \epsilon \quad \phi(\theta) = -\beta \gamma L \theta + \epsilon,
\]

with \( \epsilon \approx \phi_0 \) allows one to compensate all the remaining phase terms in \( \varphi(\omega_p, \omega_s, \theta) \) and to achieve purification of the state. In this way, we may generate polarization entangled states as in Eq. (2) with \( \phi(\omega_p, \omega_s, \theta) = 0 \). Experimentally we obtain a visibility of about \( \sim 0.9 \) starting from \( \sim 0.42 \).

### III. TWO-PHOTON MULTIPARTITE ENTANGLEMENT

In order to generate multipartite entangled states, and in particular cluster states, purification is just the first step. Here we suggest a new technique based on the use of the SLM. We consider the signal and the idler beams divided in \( N \) and \( M \) subdivisions (see Fig. 1(d)), which individuate different momentum qudits, and write the signal and idler momentum state as

\[
|s\rangle = \sum_n a_n |n\rangle, \quad |i\rangle = \sum_m a_m |m\rangle_i
\]

with \( n = 0, 1, \ldots, N - 1 \) and \( m = 0, 1, \ldots, M - 1 \). The total momentum state is \( |\Psi\rangle = |s\rangle \otimes |i\rangle \). This is not an entangled state in the momentum since for a certain signal angle \( \theta \), the idler sweeps a wide interval of \( \theta' \), actually covering all the angular acceptance \( \Delta \theta \) due to the broad down conversion spectrum. The global state is thus given by \(|\Phi\rangle \otimes |\Psi\rangle\), where polarization provides two qubits, and the rest of information is encoded onto the momentum degrees of freedom. 

![FIG. 1: Generation of multipartite entangled/cluster states by the use of SLM.](image)

The action of the SLM corresponds to impose a phase shift only on the horizontal component of polar-
ization, leaving the vertical part undisturbed. We exploit this property, leaving a different constant phase, besides the purification ones $\phi(\theta)$ and $\phi'(\theta')$, for each portion of signal and idler. This corresponds to the action of a set of controlled phase-gates $C_{\Phi_i}, \phi = \{\phi_0, ..., \phi_{M-1}, \phi_0, ..., \phi_{N-1}\}$ to the state $|\Phi\rangle \otimes |\Psi\rangle$. Using a suitable number of sectors (power of two) one may generate multiqubit entangled states of the form $|\Xi\rangle = C_{\Phi_i} |\Phi\rangle \otimes |\Psi\rangle$.

The simplest example, which we implemented experimentally, is obtained using $M = 1$ and $N = 2$, i.e. by dividing the signal beams in two parts exploiting the SLM to apply a phase $\phi$ to only one of them (see Fig. 1(a) and (b)). This leads to the generation of a two-photons three-qubit entangled state of the form $\frac{1}{\sqrt{2}}(|\Phi^+\rangle|0\rangle - |\Phi^-\rangle|1\rangle)$, where $|\Phi^\pm\rangle$ are standard Bell states. In order to highlight the power of our method let us consider another example, with four qubits: for $M = N = 2$ (see Fig. 1(c)) and applying $\phi_0 = -\phi_1$, $\phi_1 = \pi - \phi_{1s}$ we achieve the four-qubit entangled state

$$|\Xi_4\rangle = \frac{1}{2}(|\Phi^+\rangle|00\rangle - |\Phi^-\rangle|11\rangle + |\Delta^+(\phi_1)|01\rangle - |\Delta^-(\phi_1)|10\rangle),$$

where $\phi_1 = \phi_{0i} + \phi_{1s}$ and $|\Delta^\pm(\phi_1)\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm e^{i\phi_1}|11\rangle)$. We foresee that using a narrower spectral filter for the downconverted photons it is possible to select different regions of the angular distribution in a way that allow us to engineer entanglement also for the momentum degrees of freedom. Using a 10 nm bandpass filter and coupling $\Delta \theta \simeq 1.6 mrad$ on the momentum channels $|n\rangle_s$, $|m\rangle_i$, for $N, M = 2$, we would have $|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle + |10\rangle + |01\rangle)$. In such a way the total state would be the two-photon four-qubit cluster state reported in [14, 15].

A. Experiments

The experimental setup is shown in Fig. 2. The pump derives from a 405 nm CW laser diode (Newport LQC-405 – 40 P). After a half wave plate (HWP) that rotates its polarization in order to balance the generated state, the pump passes through two BBO crystals that compensate the delay time $\alpha L$ between $|H\rangle$ and $|V\rangle$ generated photons. Two BBO crystals, each cut for type-I down conversion, stacked back-to-back and orientated with their optical axes aligned in perpendicular planes, are used to generate the polarization entangled state. As shown in Fig. 2(a) a portion of the output cones passes through the spatial light modulator (SLM), which is a crystal liquid phase mask (64 x 10 mm) divided in 640 horizontal pixels, each wide $d = 100 \mu m$ and with the liquid crystal 10 $\mu m$ deep. The SLM is set at a distance $D = 500 nm$ from the two generating crystals. Driven by a voltage the liquid crystal orientation in correspondence of a certain pixel changes. The photons with an horizontal polarization feel an extraordinary index of refraction depending on the orientation, and this introduce a phase-shift between the two polarizations. Since each pixel is driven independently we can introduce a phase function dependent on the position on the SLM, i.e. on the generation angle $\theta$ and $\theta'$.

![FIG. 2: Experimental setup.](image)

It is worth noting that the SLM also replaces the birefringent plate used for the optimal generation of photon pairs [31]. After SLM, on the signal and the idler paths, there are a slit, an iris, two longpass filters (cut-on wavelength = 715 nm), a coupler with an $1/e^2$ output beam diameter of 7.14 mm and a multimode optical fiber that directs the photons to the detector. The detectors are home-made single-photon counting modules (D1, D2), based on an avalanche photodiode operated in Geiger mode with passive quenching. For tomographic reconstruction we insert a quarter-wave plate, a half-wave plate and a polarizer and for the optimization of the phase functions only the polarizers.

Our experimental setup allows us to collect the downconverted photons within a wide spectrum and angular distribution. In order to underline this fact the pump power on the crystals has been intentionally left very low (2.5 mW) by using an amplitude modulator. To collect
as many photons as possible we make the imaging of the pump spot on the crystals ($\simeq 1.5\, \text{mm}$) into the optical fibers core (diameter of $62.5\, \text{mm}$) using the coupler lenses. Setting the slits at $4\, \text{mm}$ ($\Delta \theta \simeq 6.5\, \text{mrad}$) and the iris with a diameter of $9\, \text{mm}$ we collect up to 100 coincidence counts per second. It is worthwhile to note that such an angular acceptance $\Delta \theta$ acts as a $100\, \text{nm}$ band-pass spectral filter for the down converted photons. In order to purify the state we insert the phase functions
\[
\phi(x) = a_2(x - x_{c2}) + b_2, \quad \phi'(x) = a_1(x - x_{c1}) + b_1,
\]

where $x$ is the pixel number, $x - x_{c2} = \frac{D}{a} \theta$ and $x - x_{c1} = \frac{D}{b} \theta'$, $x_{c1}$ and $x_{c2}$ are the central pixels on idler and signal, i.e. the pixels corresponding to the central angles $\theta = -\theta^0$ and $\theta' = \theta^0$. The values of the parameters $a_1, b_1, a_2$ and $b_2$ has been optimized upon inserting two polarizers set at $\alpha_1 = 45^\circ$ and $\alpha_2 = -45^\circ$ in front of the couplers and then searching for the minima in the coincidence counts, corresponding to the values of $b_{1,2}$ compensating the constant phase difference $\phi_0$ and $a_{1,2}$ removing the angular dependence on $\theta$ and on $\theta'$, and in turn on $\omega_z$. For our configuration we have $a_1 = -a_2 = \beta L d/\gamma D \simeq -0.05$, $b_1 + b_2 = \phi_0$.

![Coincidence counts on a time window of 30s with the polarizers in front of the couplers set at 45° and -45°. (a) Coincidence counts as a function of $b_{1,2}$ (blue/red) with $b_{2,1} = 0$ and optimal $a_{1,2}$; (b) Coincidences as a function of $a_1 = -a_2$ with optimal $b_1$ and $b_2 = 0$.](image)

FIG. 3: Coincidence counts on a time window of 30s with the polarizers in front of the couplers set at 45° and -45°. (a) Coincidence counts as a function of $b_{1,2}$ (blue/red) with $b_{2,1} = 0$ and optimal $a_{1,2}$; (b) Coincidences as a function of $a_1 = -a_2$ with optimal $b_1$ and $b_2 = 0$.

In Fig. 3(a) we report the coincidence counts on a time window equal to 30s as a function of $b_1$ ($b_2$) (blue/red) with $b_2 = 0$ ($b_1 = 0$) and with $a_{1,2}$ set to their optimal values. In Fig. 3(b) we report the coincidence counts on a time window of 30s as a function of $a_1 = -a_2$ with $b_2 = 0$ and $b_1 = \phi_0$. The agreement with the theoretical model is excellent. In turn, the purification of the state works as follows: starting from a visibility equal to $0.423 \pm 0.016$ we achieve $0.616 \pm 0.012$ after the delay compensation with the crystals and $0.886 \pm 0.012$ after the spatial compensation with the SLM. Finally, by closing the iris at the same width of the slits we obtain $0.899 \pm 0.008$. Actually, we verified experimentally that variations of the phase in the azimuthal direction have only a minor effect. The residual lack of visibility is in turn due to the low spatial coherence of the pump, which is spatially multimode.

B. State reconstruction

Upon properly programming the SLM, i.e. by setting $M = 1$, $N = 2$, and $\phi = \pi$ as in Fig. 3(b), our scheme may be set to generate, in ideal conditions, the cluster state $|C_3\rangle$. In order to characterize the output state, denoted by $R_3$, and to check the effects of the decoherence processes, we have performed state reconstruction by (polarization) quantum tomography [37, 38]. The experimental procedure goes as follows: upon measuring a set of independent two-qubit projectors $P_\mu = |\psi_\mu\rangle\langle\psi_\mu|$ ($\mu = 1, ..., 16$) corresponding to different combinations of polarizers and phase-shifters, the density matrix may be reconstructed as $\rho = \sum_\mu p_\mu \Gamma_\mu$, where $p_\mu = \text{Tr}[\rho P_\mu]$ are the probabilities of getting a count when measuring $P_\mu$ and $\Gamma_\mu$ the corresponding dual basis, i.e. the set of operators satisfying $\text{Tr}[P_\mu \Gamma_\nu] = \delta_{\mu\nu}$ [39]. Of course in the experimental reconstruction the probabilities $p_\mu$ are substituted by their experimental samples i.e. the frequencies of counts obtained when measuring $P_\mu$. In order to minimize the effects of fluctuations and avoid non physical results we use maximum-likelihood reconstruction of two-qubit states [37, 38].

At first we reconstruct the purified state prior the action of the SLM phase-gate, i.e. without addressing the momentum qubit. Then, we reconstruct the two reduced states $g_j = \frac{1}{\text{Tr}[R_3]} \text{Tr}_3[|j\rangle\langle j| R_3]$ obtained by measuring the momentum qubit after the phase-gate. This is obtained by moving the slit on the signal to select the corresponding portion of the beam. Results are summarized in Fig. 4.

As it is apparent from the plots our scheme provide a reliable generation of the target states. Fidelity of the purified polarization state is about $F \simeq 0.90 \pm 0.01$, whereas fidelities of the conditional states $F_0 = \langle \Phi^+ | \rho_0 | \Phi^+ \rangle$ and $F_1 = \langle \Phi^- | \rho_1 | \Phi^- \rangle$ are given by $F_0 = 0.92 \pm 0.01$ and $F_1 = 0.90 \pm 0.01$ respectively. In order to achieve this precision we have employed a long acquisition time ($\sim 60$s) thus also demonstrating the overall stability of our scheme. We also report the visibility of the state prior the action of the SLM phase-gate, which confirms the entanglement purification process [40].

IV. CONCLUSIONS

We have suggested and implemented a novel scheme for the generation of two-photon multipartite entangled
FIG. 4: Characterization of the output state. In (a) we report the tomographic reconstruction (real part) of the global purified polarization entangled state prior the action of the phase-gate, whereas in (b) we show the corresponding visibility curve and the fit with the curve \( \cos^2(\alpha - 45^\circ) \) (solid line). In (c) and (d) we report the tomographic reconstructions (real part) of the reduced states \( \rho_0 \) and \( \rho_1 \).

In our device a programmable spatial light modulator acts on different spatial sections of the overall downconversion output and provides polarization entanglement purification as well as arbitrary phase-gates between polarization and momentum qubits. It should be mentioned that also measurements on the momentum qubits benefits from our configuration. In fact, addressing momentum is equivalent to select portions of the signal (idler) beam and then make them interacting, say by a beam splitter and other linear optical elements, to perform arbitrary momentum measurements. In our scheme this may be implemented in a compact form since the portions of the beam are quite close each other, and we may work with beam splitter at non normal incidence. Overall, our scheme represents an effective, low cost, source of two-photon multiqubit/qudit entanglement. We foresee applications in one-way quantum computation and quantum error correction.

Acknowledgments

The authors thank I. Boscolo for encouragement and support.

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