Defragmentation of Student’s Thinking Structures in Solving Mathematical Problems based on CRA Framework

Kadek Adi Wibawa\textsuperscript{1,2}, Toto Nusantara\textsuperscript{2}, Subanji\textsuperscript{2}, I Nengah Parta\textsuperscript{2}
\textsuperscript{1}Department of Mathematics Education, Universitas Mahasaraswati Denpasar, Denpasar, 80233, Indonesia
\textsuperscript{2}Department of Mathematics, Universitas Negeri Malang, Malang, 65145, Indonesia
adi_math@yahoo.co.id

Abstract: Fragmentation of thinking structures is a phenomenon of constructing information in the brain that is inefficient, thus inhibiting the process of solving mathematical problems. Fragmentation of thought structures can be traced through erroneous constructs in solving mathematical problems. To overcome fragmentation, students need to defragment the thinking structures. Defragmentation is the thinking structure rearrangement through the process of checking, repairing, and ascertaining (CRA) in solving mathematical problems. Students defragment the thinking structure under the researcher assistance through the provision of "limited intervention". Limited interventions are conducted by facilitating the occurrence of disequilibrium, cognitive conflicts, and scaffolding. In this study, five types of defragmentation of thinking structures are described: (1) defragmentation scheme knitting to arrange the occurrence fragmentation of meaningless connections, (2) defragmentation of scheme activation to arrange the occurrence fragmentation of false-pseudo construction, (3) defragmentation of connections emertion to arrange the occurrence fragmentation of nothing connection, (4) defragmentation of scheme translation adjustment to arrange the occurrence fragmentation of translation of thinking structure, and (5) defragmentation of schemaemertion to arrange the occurrence of fragmentation of construction holes. In this study, students were asked to complete the work sheet and spoke out what was being thought (thinkaloud). In addition, students were also interviewed to explore the data that had been collected. The research data used was the result of think aloud (in the form of written data, verbal, and expression) and interview result.

1. Introduction
Problem solving is at the core of mathematics learning [17, 12,15, 4]. Problem-solving abilities can be used to solve real life problems. The better a person's problem-solving ability, the greater his chances of being able to cope with life's ever-changing challenges [16]. The importance of mastery of problem solving is of interest to many [6, 11, 3, 9]. Even NCTM [10] states that problem solving is not only the goal of mathematical learning but also the main meaning of mathematical work, so through learning problem solving on mathematics, students acquire ways of thinking, persistent habits and sharpening curiosity and confidence in the situation that is not familiar. In fact, the problem solving ability of students is still low. Low problem-solving skills as a result of "less" meaningful learning [15].

In solving mathematical problems, students often experience difficulties and produce incorrect answers [8, 23, 5, 12]. Complex issues require variations of ideas, strategies and mathematical formulations used. It causes students to think hard in order to arrive at the correct answer and in
accordance with the problems encountered. Difficulties are often the first thing the students experienced and felt, because the solution of the problem is not immediately known by using the usual procedures. Situations like this have been studied by some researchers such as [8], [23], [24], [5], [21], [16], [19], and [12]. Those researcher had been done only to identify errors that occur based on the work of students. The researches had not yet reached the discovery of the source of error through the disclosure of the students’ thought processes in solving mathematical problems. Specifically, [12] said that further research is needed to investigate students’ thinking processes in solving mathematical problems.

[20] states that in the learning process, when students receive information in the form of concepts, procedures, and others, the student actually managed to construct what is taught. Some are well constructed (concepts are fully understood) and some are poorly constructed (concepts are not fully understood). The condition of information that is not fully understood, according to [20] resulted in the information being not connected well or his position “messy” (unorganized). [20] called it a fragmentation in the student's storage system.

The term fragmentation is very popular in the area of computer. It defined as the condition of a file placed on a storage system that does not occupy the sector (storage space) efficiently. The same was also stated[17] that fragmentation in computers is defined as a phenomenon in storage space used inefficiently, reducing storage capacity.

In the computer system, fragmentation that occurs can be fixed through the defragmentation process. Stages that occur during defragmentation are checking, repairing, and ascertaining [2]. At the checking stage occurs the process of scanning and checking some errors, which in the scanning process is done through monitoring of the condition of the files in the storage system and checking some errors process is also done by conducting data collection process for errors that occur. The next step is repairing, which is defined as the process of errors repair that occur or in the context of defragmentation, repairing is defined as the process of arranging the files contained in storage systems that have fragmentation. The last stage is ascertaining, which happens to notice that the condition of the files on the storage space after the checking and repairing process has been reorganized.

In the construction process, checking can be viewed as a process of re-examination of the schemes that have been constructed. The examination is a response that occurs when researchers question the predetermined answer. The results of this re-examination of the realization that there has been a mistake in the construction process. Repairing is seen as a process of adaptation or reintegration by generating schemes, generating scheme connections, knitting unconstructed schemes, adjusting schemes, and activating existing schemes. Ascertaining is seen as self-assurance that the new construction process or after repairing is appropriate or correct.

In the process of learning, students can do the arrangement of the structure of thought, through the application of limited intervention, so that fragmentations can be resolved. Defragmentation of student’s thinking structures can be done through facilitating the occurrence of disequilibration and scaffolding [17, 7]. Defragmentation through facilitating the occurrence of disequilibration can be done by asking questions that can arouse suspicion or thinking gaps in students so that students will reflect on the answers given, such as: "Are you sure of the answer?" "Why is the formula like that?”, and so forth. The suspicion or thinking gap that occurs is his lack of confidence in the answers given. After facilitating the occurrence of disequilibration, researchers are easier to provide scaffolding. Scaffolding given can be the creation of cognitive conflict as well as a number of assistance or support to students in the process of restructuring the structure of thinking. Assistance provided by researchers is slowly reduced and stopped when students are able to continue problem solving faced [22, 1]. Defragmentation through cognitive conflict, in the form of questions that will cause conflict in the minds of students, so that students will eventually rearrange the structure of thinking and examine more deeply the problems faced [22].

Based on the above explanation, this study examines the defragmentation of student’s thinking structures in solving mathematical problems. Defragmentation of thinking structures under study is a planned defragmentation (Subanji, 2016). In this case, defragmentation of thinking structures occurs
due to limited interventions conducted by researcher by facilitating the occurrence of disequilibration and providing scaffolding.

2. Research Methods
This research is descriptive explorative research because it describes the result of defragmentation exploration of student's thinking structures [14]. The type of data collected were verbal data, expression, and writing. This research was carried out in Department of Mathematics Education FMIPA Universitas Negeri Malang on student of even semester in academic year of 2014 and 2015 (Semester 4 and 6). Research subjects were chosen using by purposive sampling. In purposive sampling, researchers deliberately select individuals and places for information-rich research. In this study, researcher conducted in-depth exploration on three subjects, namely subject 1 (S1), subject 2 (S2), and subject 3 (S3). S1 characteristic represents category type 1, S2 represents category type 2, and S3 represents category type 3.

In this study, students were asked to speak aloud what they were thinking (think alouds) when solving the problem. After the students get the answer, the researcher classified the students' answer based on the type of error that the researcher has determined before. Those types of error refers to the preliminary study findings that were conducted regarding the fragmentation that occurred. The instrument of this research is researcher as main instrument assisted with work sheet as supporting instrument.

3. Results and Discussion
Based on the given problem and the result of the interview, it is found that defragmentation of thinking structures that occurs in each subject is defragmentation of scheme knitting, defragmentation of scheme activation, defragmentation of connection emertion, defragmentation of scheme translation adjustment, and defragmentation of schema emertion. S1 performs a very essential error in solving the given problem, being unable to understand the problem well and unable to connect with the appropriate procedures that can be used to solve mathematical problems. In solving the problem of integral applications, of course, the volume of the rotary body S1 has fragmented the thinking structures, among others, the fragmentation of meaningless connections with two errors, the fragmentation of false-pseudo construction with one error, the fragmentation of nothing connection with one error, and fragmentation of the translation of thinking structure with two errors. S2 experiences fragmentation of thinking structures, among others: fragmentation of nothing connections with 1 error, fragmentation of false-pseudo construction with two errors, fragmentation of the translation of thinking structure with two errors and fragmentation of construction hole with 1 error. And S3 experiences fragmentation of thinking structures, e.g: fragmentation of false-pseudo construction with 1 error, fragmentation of translation thinking structure with three errors and fragmentation of construction hole with 1 error.

Each subject defragmented the thinking structure through the limited intervention provided by the researcher. Defragmentation of thinking structures that occurs, adjusted to the type of fragmentation of thinking structures experienced by students.

Defragmentation of schemes knitting undertaken by students (S1 and S2) begins with the existence of fragmentation of connections without meaning. Researchers provide limited interventions that aim to enable students to be aware of the mistakes and correct their own mistakes. When undertaking a scheme defragmentation, students initially check by looking back at ideas that are not connected significantly. Through the attention given, the student raises his awareness that the error in constructing the problems encountered. Researchers provide limited intervention for students to do repairs or re-arrange the problem solving done. Students re-assemble the schemes they possess with clear meanings, meaning that they can be used to solve problems correctly. Students do ascertaining by ensuring that previously raised schemes are constructed meaningfully.

Defragmentation of scheme activation by students (S1, S2, and S3) begins with fragmentation of false-pseudo construction. Researchers provide limited interventions that aim to enable students to be
aware of the mistakes and correct their own mistakes. Students check by rethinking ideas that have been constructed. The student raises his awareness that it takes an activated idea in order to correct the mistakes. Students do repairing with a quick response "oh .. it can use this way/strategy ...". Students re-structured by adding new activated schemes. The student then performs ascertaining by ensuring that the activated scheme is appropriate to the problem at hand.

For example: S2 has difficulty in determining half the height of the tube or S2 calling it "the length from here (center) to \(x\)." S2 actually already has a scheme to solve the problems encountered, but the scheme has not been activated by S2. Problems faced by S2 can be solved by Pythagorean theorem. In terms of the structure of the thinking structure, S2 experiences fragmentation of false-pseudo construction, in which S2 can actually solve the problems encountered but have difficulties because there are schemes that have not been activated. S2 arranges the structure of thinking through defragmentation of scheme activation. Defragmentation of scheme activation occurs after the researcher provides limited intervention through scaffolding of scheme activation.

Here is an explanation that illustrates the relationship between misconception of S2, limited intervention provided by the researcher, and correct understanding of S2.

Table 1. Understanding S2 Before and during Defragmentation Scheme Activation

| Wrong Understanding | Limited Intervention | Right Understanding |
|---------------------|----------------------|---------------------|
| "The purpose of the matter is not to calculate the area here (pointed picture). To calculate the volume is rotated against the X axis. Well here confused here, how many x it. You can intervene zero to x it." | Gives Scaffolding • If from here to here how much? • What are the features of a circle? | "This distance to her fingers is the same. His fingers are ten. So here five. Five root three " "A hundred squared minus two five." |

Table 1 shows a change in understanding of S2 after limited intervention. In the interview footage there is an indication that the S2 was initially wrong in understanding the problem and then the S2 made improvements, resulting in a correct understanding. The process that happens to arrive at the correct understanding is the defragmentation process of activating the scheme, ranging from checking process, repairing, to ascertaining.

Defragmentation of connection emertion made by the student begins with the fragmentation of nothing connection. Researchers then provide limited interventions that aim to enable students to be aware of the error and correct their own mistakes. Students pay attention to all the ideas that have been raised and realize that there are schemes that are not connected. Students perform repairing by generating connections between these schemes. The connection that emerges is the connection between the definition of a space wake with an image that is part of the wake of the space. Students perform ascertaining by ensuring themselves that the connections that appear have been in accordance with the problems encountered.

Defragmentation of scheme translation adjustment (S1, S2, and S3) at the time students experience fragmentation of translation of thinking structures from verbal representations to graphs begins with
the unpacking the source process, which students understand the (conceptual) ideas present in the source representation. Micro-concepts constructed by students form a scheme. Students understand the gap between schemes called scheme gaps during unpacking the source. Through the scheme gaps that occur, students have the urge to create a new representation. Students conduct preliminary coordination process by planning to create a graph, in order to determine the function of the problems encountered. The student then performs the process of constructing the target by drawing the cartesian field. Images created as new representations are not compatible with the micro-concepts that have been constructed on the source representation or in this case an error has occurred. The error occurs because the student does not consider all the micro-concepts that exist in the source representation. Students do not do determining equivalence. This is apparent when students are unaware of any mistakes made. Researchers provide limited intervention so that students are aware of the mistakes and correct their mistakes in their own way. Students check with the target representation that has been constructed, and realize there are still schema gaps. Students are aware that there is a mismatch between source representation with target representation. The student performs repairing by taking into account the source representation to consider in constructing the target representation again. Students conduct preliminary coordination process to construct the target. Finally, the student can correct his mistake by making the target representation corresponding to the source representation. Students do ascertaining by ensuring that in constructing the target representation they have considered all the facts in the source representation.

Defragmentation of translation of scheme adjustments when students experience fragmentation of the translation thinking structure from graph to algebraic form has the same process as previous defragmentation. Only, at the time of the unpacking the source process there is actually confounding schemas or schemes that are ambiguous but students are not aware of it. Students conduct preliminary coordination process by planning to create equations from graphs that have been made. Students construct the target on the wrong basis. In this case, the non-conformity is not because the student does not consider the source representation but the student is wrong in making assumptions when constructing the source representation. Students perform the determining equivalence process after the researcher provides limited intervention. Students checking with regard to the representation of sources that have been constructed and aware of any mistakes made assumptions. The student then repairs by improving the source representation and performing the unpacking the source process again. Unpacking the source does not cause schema gaps. Through preliminary coordination the students construct the target representation correctly and in accordance with the source representation. Students do ascertaining by ensuring that previously incorrect assumptions have been realized and corrected by generating new assumptions. Through these new assumptions, the student realizes that he has done the construction process on the target representation correctly.

Defragmentation of translation of scheme adjustments when students experience fragmentation of the translation thinking structure from graphs and algebraic forms to mathematical models has higher complexity. Students construct two source representations that are used to serve as the basis for constructing a target representation. The process happens almost the same as the previous defragmentation. Only, when conducting the preliminary coordination process the students plan to create a mathematical model or formulate the graphs and equations that have been constructed. Fragmentation of the structure of translation thinking appears when the student is wrong in determining the mathematical model or making a new representation of the previous representation. This is because the scheme owned by students to construct mathematical model is not enough. Therefore, students perform defragmentation of schemes and defragmentation schemes to activate the construction of target representation. Students consider all the micro-concepts that exist in the source representation to construct the target representation correctly. Students do ascertaining by making sure that the mathematical model or the integral shape of the constructed course is in accordance with the problem at hand. It is seen when students are able to give explanation on each element that is in the integral form.
Defragmentation of scheme emertion by students (S2 and S3) begins with fragmentation of construction holes. Researchers provide limited intervention so that students realize that the arguments are not strong. Students checking by paying attention to the mathematical model created and trying to give an argument in accordance with their understanding. Students are able to realize that the arguments given are incorrect which resulted in a mistake in the subsequent problem construction. Students then perform repairing after the researchers provide limited intervention. Repairing the students is to come up with the schemes that form the basis of the argumentation made. Through the appearance of these schemes students can reorganize their thinking structures into the correct thinking structures (in accordance with the problem structure). Students perform ascertaining by re-justifying the mathematical model that has been constructed. Justification given already has a solid foundation.

4. Conclusion

Based on the results of research can be concluded that, in solving mathematical problems students often make mistakes. Student construction when making mistakes can be observed intact through the disclosure of the fragmentation of thinking structures. Through findings on the fragmentation of thinking structures, it allows researchers to provide limited intervention. Limited intervention is done to accelerate the process of arrangement the thinking structures or defragmentation of thinking structures. Defragmentation of thinking structures occurs through the stages of checking, repairing, and ascertaining. Knowledge of defragmentation of thinking structures to make educators wise and able to provide appropriate response or intervention when students make mistakes. There are five types of defragmentation of thinking structure, (1) defragmentation scheme knitting, (2) defragmentation of scheme activation, (3) defragmentation of connections emertion, (4) defragmentation of scheme translation adjustment, and (5) defragmentation of scheme emertion.

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