Squeezed back-to-back correlation between boson and antiboson with different in-medium masses in high-energy heavy-ion collisions

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We derive the formulas for calculating squeezed back-to-back correlation (SBBC) between boson and antiboson with different in-medium masses in high-energy heavy-ion collisions. The influence of the in-medium mass difference between boson and antiboson on the SBBC is investigated. We calculate the SBBC functions of D-meson pairs for the hydrodynamic sources described by VISH2+1 code for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Our results indicate that the SBBC strengths of $D^+D^-$ and $D^0\bar{D}^0$ are different if there are the charge-dependent in-medium interactions.

PACS numbers: 25.75.Gz, 25.75.Ld, 21.65.jk

\section{Introduction}

In high-energy heavy-ion collisions, the in-medium mass shifts of bosons may cause a squeezed back-to-back correlation (SBBC) between detected boson and antiboson \cite{1-4}. This SBBC is related to the in-medium energies of the bosons, through a Bogoliubov transformation between the creation (annihilation) operators of the quasiparticles in medium and the corresponding free particles \cite{1,3}. The study of the SBBC can provide information about the bosons formations and in-medium interactions in high-energy heavy-ion collisions.

In previous studies of the SBBC, the mass shifts of boson and antiboson are taken to be the same \cite{1-4}. More generally, the interactions of boson and antiboson in medium are different, especially in the medium with a finite baryon chemical potential \cite{10,12}. The in-medium energy difference between boson and antiboson leads to a mass difference between the quasiparticles in medium. It is necessary to check the validity of the previous formulas of SBBC calculation in this case.

In this work, we derive the formulas for calculating the SBBC function of boson-antiboson with different in-medium masses. The influence of the in-medium mass difference on the SBBC functions of D-meson pairs are investigated. Because containing a charm quark, which is believed to experience the whole evolution of the quark-gluon plasma (QGP) created in relativistic heavy-ion collisions, D-meson measurements are recently of great interest \cite{16-21}. We calculate the SBBC functions of D-meson pairs for the hydrodynamic sources described by the VISH2+1 code \cite{22} and find that the SBBC strengths of $D^+D^-$ and $D^0\bar{D}^0$ are different in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV if there are the charge-dependent in-medium interactions.

In next section, we shall present the formula derivations of SBBC function for the boson and antiboson with different in-medium masses. Then, we shall show the SBBC results of D-meson pairs in section III. Finally, we shall present the summary in section IV.

\section{Formulism}

In the framework of complex scalar field, the Hamiltonian density of system can be written as \cite{1}

\begin{equation}
\mathcal{H} = \dot{\phi}\phi^\dagger + \nabla\phi^\dagger \cdot \nabla\phi + m^2 \phi^\dagger \phi,
\end{equation}

where

\begin{equation}
\phi(x) = \sum_P (2V\omega_P)^{-\frac{1}{2}} (e^{-p_x a_P} + e^{p_x b_P^\dagger}),
\end{equation}

\begin{equation}
\phi^\dagger(x) = \sum_P (2V\omega_P)^{\frac{1}{2}} (e^{p_x a_P^\dagger} + e^{-p_x b_P}),
\end{equation}

where $a_P$ and $b_P^\dagger$ ($b_P$ and $b_P^\dagger$) are creation and annihilation operators of the free boson (antiboson), $p = (\omega_P, \mathbf{p})$, and $\omega_P = \sqrt{\mathbf{p}^2 + m^2}$.

For the boson and antiboson in a medium with the same mass $m' = \sqrt{m^2 \pm m_1^2}$, where "+" or "--" represents the case that $m'$ is larger or smaller than $m$, the Hamiltonian density of system can be written as \cite{1}

\begin{equation}
\mathcal{H}_M = \dot{\phi}\phi^\dagger + \nabla\phi^\dagger \cdot \nabla\phi + (m^2 \pm m_1^2)\phi^\dagger \phi,
\end{equation}

and the Hamiltonian of system can be diagonalized through a Bogoliubov transformation \cite{1,2}.

Generally speaking, the interactions of boson and antiboson with medium are somewhat different. Assuming the energy split between the boson and antiboson in the medium is $2\delta'$, we consider the transformation,

\begin{equation}
\phi \rightarrow e^{i\delta' t} \phi, \quad \phi^\dagger \rightarrow e^{-i\delta' t} \phi^\dagger,
\end{equation}

and have

\begin{equation}
\mathcal{H}_M = \dot{\phi}\phi^\dagger + \nabla\phi^\dagger \cdot \nabla\phi + m^2 \phi^\dagger \phi \pm m_1^2 \phi^\dagger \phi + \delta'^2 \phi^\dagger \phi - i\delta'(\phi\phi^\dagger - \phi\phi^\dagger).
\end{equation}
It will be seen that $\delta'^2$ provides an additional term of mass square in average energy of boson and antiboson, which is associated with the different in-medium interactions for the boson and antiboson, while $m'^2$ reflects the in-medium interaction that is the same for the boson and antiboson.

Using the Bogoliubov transformation between the operators $(a_p^\dagger, a_p, b_p, b_p^\dagger)$ for the free particles and $(a_p'^\dagger, a_p', b_p', b_p'^\dagger)$ for the quasi-particles,

$$a_p = c_p a_p' + s_p^* b_p'^\dagger, \quad b_p = \bar{c}_p b_p' + \bar{s}_p^* a_p'^\dagger,$$

where

$$c_{\pm p} = c_{\pm p}' = \bar{c}_{\pm p} = \cosh r_p, \quad s_{\pm p} = s_{\pm p}' = \bar{s}_{\pm p} = \sinh r_p,$$

$$r_p = \frac{1}{2} \ln(\omega_p/\Omega_p), \quad \Omega_p = \sqrt{\mathbf{p}^2 + m^2 + m'^2 + \delta'^2},$$

we can diagonalize the Hamiltonian of system for the boson and antiboson with energy split $2\delta'$ in the medium as

$$H_M = \sum_p \left[ (\Omega_p + \delta') a_p'^\dagger a_p' + (\Omega_p - \delta') b_p'^\dagger b_p' \right].$$

The in-medium masses of the boson and antiboson are

$$m'_p = (\Omega_p + \delta')|_{p=0} = \sqrt{m^2 + m'^2 + \delta'^2},$$

and the in-medium mass difference between boson and antiboson with the same momentum is also the split $2\delta'$.

One can see that the Bogoliubov transformation involves with only the average in-medium energy of boson and antiboson. And, the average energy $\Omega_p$ is not only related to $m_1$ associated with the in-medium interaction as the same for the boson and antiboson but also related to $\beta'$ associated with the in-medium interactions different for the boson and antiboson.

The SBBC function of boson-antiboson with momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ is defined as

$$C(\mathbf{p}_1, \mathbf{p}_2) = 1 + \frac{|G_s(\mathbf{p}_1, \mathbf{p}_2)|^2}{G_c(\mathbf{p}_1, \mathbf{p}_2)G_c(\mathbf{p}_2, \mathbf{p}_1)}.$$

where $G_c(\mathbf{p}_1, \mathbf{p}_2)$ and $G_s(\mathbf{p}_1, \mathbf{p}_2)$ are the so-called chaotic and squeezed amplitudes. They are given by

$$G_c(\mathbf{p}_1, \mathbf{p}_2) = \int \frac{d^4\sigma_\mu(r)}{(2\pi)^3} K_{1,2}^\mu \left\{ |c_{p_1, p_2}'|^2 n_{p_1, p_2}' + |s_{p_1, p_2}'|^2 n_{p_1, p_2}' + 1 \right\},$$

$$G_s(\mathbf{p}_1, \mathbf{p}_2) = \int \frac{d^4\sigma_\mu(r)}{(2\pi)^3} K_{1,2}^\mu e^{2iK_{1,2}^\mu} \left\{ |s_{p_1, p_2}'|^2 c_{p_1, p_2}' + |c_{p_1, p_2}'|^2 s_{p_1, p_2}' + 1 \right\},$$

for an evolving source. Here, $d^4\sigma_\mu(r)$ is the four-dimension element of freeze-out hypersurface, $q_{1,2}^\mu = p_{1,2}^\mu - p_{1,2}'^\mu$, $K_{1,2}^\mu = (p_{1,2}^\mu + p_{1,2}'^\mu)/2$, and $p_i'$ is the local-frame momentum corresponding to $\mathbf{p}_i$ ($i = 1, 2$). In Eqs. (15) and (16), the quantities $c_{p_i, p_i}'$ and $s_{p_i, p_i}'$ are the coefficients of Bogoliubov transformation between the creation (annihilation) operators of the quasiparticles and the free particles, and $n_{p_i, p_i}'^*$ is the boson distribution of the quasiparticle pair.

### III. RESULTS

We consider first a simple case, a rest particle-emitting source with a fixed freeze-out temperature $T_f$, a Gaussian spatial distribution $[e^{-r^2/2R^2}/(\sqrt{2\pi}R^3)]$, and a temporal distribution of exponential decay $[\theta(t-t_0)e^{-(t-t_0)/\Delta t}/\Delta t]$. In this case, the SBBC function of boson-antiboson emitted from the source with momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, and under the condition $|\mathbf{p}_1| = |\mathbf{p}_2| = |\mathbf{p}|$, can be given analytically by

$$C(\mathbf{p}_1, \mathbf{p}_2) = 1 + e^{-2p^2R^2[1+\cos(\alpha)]B(p)} = 1 + f(\alpha)B(p),$$

where $\alpha$ ($0 < \alpha < \pi$) is the angle between momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, and

$$B(p) = \frac{|c_{p} s_{p} n_{p} + c_{p} s_{p} (n_{p} + 1)|^2}{(1 + 4\omega_p^2\Delta t^2)n_1(\mathbf{p})n_1(\mathbf{p})},$$

where $n_{p}$ is boson distribution of the quasiparticle pair with average energy $\Omega_p$, and $n_1(\mathbf{p}) = |c_{p}|^2 n_{p} + |s_{p}|^2 (n_{p} + 1)$. Here, it should be mentioned that we have used an approximation, which replacing the boson or antiboson momentum distribution with the pair momentum distribution $n_{p}$, in the denominator of Eq. (15). The SBBC function $C(\mathbf{p}_1, \mathbf{p}_2)$ approaches its maximum $[1 + B(p)]$ when the boson and antiboson approach antiparallel, and decreases with increasing cos $\alpha$ exponentially. For the case of incompletely antiparallel $\mathbf{p}_1$ and $\mathbf{p}_2$, there is still the mass-shift caused SBBC except for very large source.

The SBBC is expected to be strong for the mesons with large masses under the same source size and freeze-out temperature. We plot $B(p)$ in Fig. 1 as functions of mass shift $\Delta m_1 = (m' - m_0)$ for $D^+D^-$ pairs with different momenta and in-medium mass differences. Here the solid and dashed lines are for $\delta' = 0$ and 60 MeV, respectively. In the calculations the source freeze-out temperature is taken to be 150 MeV and we take $\Delta t = 2$ fm. It can be seen that $\delta'$ leads to a shift of $B(p)$ towards decreasing $\Delta m_1$. The function width decreases with increasing momentum.

In Fig. 2 we plot $B(p)$ as functions of $\delta'$ for $D^+D^-$ pairs with momenta 0.8 and 1.2 GeV/$c$. Here, the source parameters are the same as in Fig. 1. Based on the results calculated in the FMK framework, the mass of $D$ meson in hadronic medium in relativistic heavy-ion
collisions is about $3 \sim 5$ MeV/$c^2$ smaller than its value at free state. So, we compare the $B(p)$ functions at $\Delta m_1 = -3$ and $-5$ MeV/$c^2$. For the lower momentum $|p| = 0.8$ GeV/$c$, $B(p)$ decreases with increasing $\delta'$. However, for the higher momentum $p = 1.2$ GeV/$c$, the result of $B(p)$ for $\Delta m_1 = -5$ MeV/$c^2$ increases with increasing $\delta'$, while the result for $\Delta m_1 = -3$ decreases more rapidly with increasing $\delta'$ when $\delta' > 40$ MeV. The results of $B(p)$ are sensitive to the mass shift $\Delta m_1$, mass difference $\delta'$, and particle momentum $|p|$.

We show in Fig. 3 the SBBC functions of D-meson pairs for the source as in Figs. 1 and 2 and with a Gaussian radius $R = 3$ fm. One can see that the influences of $\delta'$ on the SBBC functions at the higher momentum are different when $\Delta m_1 = -3$ and $-5$ MeV/$c^2$. For $\Delta m_1 = -3$ MeV/$c^2$, $\delta'$ makes the SBBC function at high momentum decrease. However, $\delta'$ makes the SBBC function at high momentum increase for $\Delta m_1 = -5$ MeV/$c^2$.

As discussed above, $\delta'$ is associated with the in-medium interactions which are different for boson and antiboson. If assuming these in-medium interactions are particle-charge dependent, the split $\delta'$ will be zero for $D^0\bar{D}^0$ pair. For $D^+D^-$ pair, the split may reach a few tens of MeV [11, 12]. In this case, one can see that the SBBC of $D^+D^-$ at high momentum is weaker or stronger than that of $D^0\bar{D}^0$ when $\Delta m_1 = -3$ MeV/$c^2$ or $\Delta m_1 = -5$ MeV/$c^2$ in the simple source model, where $\Delta m_1$ is the mass-shift related to the in-medium interactions which are the same for the particles and antiparticles.

We investigate next the SBBC functions for the evolving sources described by the viscous hydrodynamic model VISH2+1 [22] under the MC-Glb initial conditions fluctuating event-by-event [28]. Figures 4 and 5 show the SBBC functions $C(\Delta \phi)$ of D-meson pairs for the hydrodynamic sources for $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions with centralities 0–80% and 40–80%, respectively. Here,
\[ \delta' = 0 \]

FIG. 4: (Color online) SBBC functions of D-meson pairs for the viscous hydrodynamic sources for \( \sqrt{s_{NN}} = 200 \) GeV Au+Au collisions with 0–80% centrality.

FIG. 5: (Color online) SBBC functions of D-meson pairs for the viscous hydrodynamic sources for \( \sqrt{s_{NN}} = 200 \) GeV Au+Au collisions with 40–80% centrality.

\( \Delta \phi \) is the angle between the transverse momenta of the two D mesons, the ratio of the shear viscosity to entropy density of QGP is taken to be 0.08 \[24, 30\], and we take the freeze-out temperature to be 150 MeV based on comparisons of the transverse-momentum spectra of D meson \[24\] with the RHIC experimental data \[16\].

One can see that the results of SBBC function for nonzero \( \delta' \) are smaller than those for zero \( \delta' \) when \( \Delta m_1 = -3 \) MeV. And, the results of SBBC function for nonzero \( \delta' \) are slight larger than those for zero \( \delta' \) when \( \Delta m_1 = -5 \) MeV. On the assumption that there is the split \( \delta' \) associated with the charge-depend in-medium interactions, we conclude that the SBBC of \( D^+D^- \) and \( D^0\bar{D}^0 \) pairs are different for the different \( \Delta m_1 \). This may provide a probe to study the in-medium interactions in detail.

The dependence of SBBC on particle momentum is complicated for the hydrodynamic sources with fluctuating initial conditions. The more serious oscillations of single-event SBBC functions at higher momentum \[4\] may lead to a lower SBBC function after being averaged over events \[7, 24\], although the intercept of the SBBC function \( C(p, -p) \) increases with increasing particle momentum \[1, 2\]. One can see that the widths of the SBBC functions \( C(\Delta \phi) \) for the higher momentum are narrower than those for the lower momentum, which is similar to that for the simple source in Fig. 3. By comparing the SBBC functions in Figs. 4 and 5, we find that the SBBC functions for the peripheral collisions are higher than those for the central collisions. It is because the averaged source lifetime is smaller for the peripheral collisions \[7\].

IV. SUMMARY

We have derived the formulas of SBBC between boson and antiboson with different in-medium masses. The SBBC is related to the average in-medium energy of the boson and antiboson, the same as in the case that the quasiparticles having the same mass. However, the SBBC now is associated with both the in-medium interactions which are the same and different for boson and antiboson. Because of the high strength, the SBBC of heavy-meson pairs provide a possible probe to study the in-medium interactions of the heavy mesons in detail in relativistic heavy-ion collisions. Our results calculated with the VISH2+1 code for Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV indicate that the SBBC strengths of \( D^+D^- \) and \( D^0\bar{D}^0 \) pairs are different if there are charge-dependent in-medium interactions.

Acknowledgments

This research was supported by the National Natural Science Foundation of China under Grant Nos. 11675034, 11647166.
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