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Study of Thermodynamic Quantities in Horava-Lifshitz and \( f(R) \) gravity Theories

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Abstract. The present paper reports studies on thermodynamic quantities like temperature of the universe, heat capacity and squared speed of sound in \( f(R) \) and Horava-Lifshitz gravity theories. Considering the universe to be filled with dark matter and dark energy we have shown that in all cases the equation of state behaves like quintessence. The thermodynamic quantities have been studied graphically by plotting them against redshift \( z \).

1. Introduction

Accelerated expansion of the universe is well documented in literature [1]. There are two classes of models to explain this accelerated expansion. The classes are: (i) Dark energy and (ii) Modified gravity. In the first class, a new matter component characterized by negative pressure is invoked and is dubbed as “dark energy”. This is described by an equation of state \( w = p/\rho \), namely the ratio of the homogeneous dark energy pressure over the energy density. For a review of dark energy see [1]. In the second case, one modifies the laws of gravity whereby a late-time acceleration is produced [2]. Different models of modified gravity have been suggested till date. A review on modified gravity theory is available in [3]. In the present work, two types of modified gravity are considered. They are \( f(R) \) gravity [4, 5] and Horava-Lifshitz gravity [6, 7]. The \( f(R) \) gravity has the interesting feature that choosing an observationally reliable \( f(R) \), it is possible to describe the early inflation as well as the late time acceleration of the universe in a fascinating manner [8]. The basic idea of Horava-Lifshitz (HL) gravity is to modify the UV behavior of gravity so that the theory is perturbatively renormalizable [9]. The action of the HL gravity is given by [10]

\[
I = dt \int d^3x \left( L_0 + L_1 + L_m \right) ; \quad L_0 = \sqrt{g} N \left[ \frac{2}{\kappa^2} (K_{ij}K^{ij} - \lambda K^2) + \frac{\kappa^2 \rho^2 (\Lambda R - 3 \Lambda^2)}{8(1 - 3 \lambda)} \right] \\
L_1 = \sqrt{g} N \left[ \frac{\kappa^2 \rho^2 (1 - 4 \lambda)}{32(1 - 3 \lambda)} R^2 - \frac{\kappa^2}{2 \lambda^3} (C_{ij} - \frac{\mu \omega}{2} R_{ij}) (C^{ij} - \frac{\mu \omega^2}{2} R^{ij}) \right] 
\]  

(1)
where, $\kappa^2$, $\lambda$, $\mu$, $\omega$ and $\Lambda$ are constant parameters, and $C_{ij}$ is Cotton tensor (conserved and traceless, vanishing for conformally flat metrics). $\mathcal{L}_m$ stands for the Lagrangian of other matter field. Modified field equations for this case are thoroughly explained in the reference [10].

The action of $f(R)$ gravity is given by [11]

$$S = \int d^4x \sqrt{-g} \left[ \frac{f(R)}{2\kappa^2} + \mathcal{L}_{\text{matter}} \right]$$

where $g$ is the determinant of the metric tensor $g_{\mu\nu}$, $\mathcal{L}_{\text{matter}}$ is the matter Lagrangian and $\kappa^2 = 8\pi G$. The $f(R)$ is a non-linear function of the Ricci curvature $R$ that incorporates corrections to the Einstein-Hilbert action which is instead described by a linear function $f(R)$. The gravitational field equations are elaborated in the reference [12].

2. Thermodynamic quantities

We consider the FRW universe treated as a thermodynamical system. Then from Gibb’s equation of thermodynamics, we have [13] $T dS = d(\rho V) + pdV = d((\rho + p)V) - Vdp$, where $S$ is the entropy, $T$ is the temperature and $V$ is the volume of the universe. The integrability condition of thermodynamic system is given by [13] $\frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T}$. Subsequently, we get $S = \frac{(\rho+p)V}{T}$. However, for adiabatic process entropy is constant and consequently $d((\rho + p)) = Vdp$. The expressions for temperature $T$, squared speed of sound $v_s^2$ and heat capacity $C_V$ can be obtained as $T = \frac{(\rho+p)V}{S}$; $v_s^2 = \frac{\partial p}{\partial \rho}$; $C_V = V \frac{\partial p}{\partial T}$ (see ref. [14] for details).

Figure 1. This figure shows the evolution of $w_{\text{total}}$ with evolution of the universe for HL gravity.

Figure 2. This figure shows the behaviour of $T$ with evolution of the universe for HL gravity.

Figure 3. This figure shows the behavior of $C_V$ with evolution of the universe for HL gravity.

Figure 4. This figure shows $v_s^2$ with evolution of the universe for HL gravity.
2.1. Thermodynamic quantities under Horava-Lifshitz gravity

Under HL gravity, the total energy density and total pressure are expressed in terms of redshift \( z = \frac{1}{a} - 1 \) as

\[
\rho(z) = \rho_m + \rho_D = \rho_{m0}(1 + z)^{3(1+w_m)} + \frac{1}{16\pi G_c} \left( \frac{3k^2(1+z)^4}{\Lambda} + 3\Lambda \right)
\]

and

\[
p(z) = p_m + p_D = \rho_{m0}w_m(1 + z)^{3(1+w_m)} + \frac{1}{16\pi G_c} \left( \frac{k^2(1+z)^4}{\Lambda} - 3\Lambda \right)
\]

Where, \( G_c = \frac{\kappa^2c^4}{16\pi(3\Lambda-1)} \) and \( c = \frac{\kappa^2\mu}{4} \sqrt{\frac{\Lambda}{1-3\Lambda}} \). Subsequently, the thermodynamic quantities are expressed in terms of redshift \( z \) as

\[
v_s^2(z) = \frac{k^2(1+z)\kappa^2\mu^2 + 6(1+z)^3w_m(-1 + 3\Lambda)w_m\rho_{m0}(1+w_m)}{3(1+z)\kappa^2\mu^2 + 2(1+z)^3w_m(-1 + 3\Lambda)(1+w_m)\rho_{m0}}
\]

and

\[
C_v(z) = \frac{3S_0(k^2(1+z)\kappa^2\mu^2 + 2(1+z)^3w_m(-1 + 3\Lambda)(1+w_m)\rho_{m0})}{k^2(1+z)\kappa^2\mu^2 + 6(1+z)^3w_m(-1 + 3\Lambda)w_m\rho_{m0}(1+w_m)}
\]

Using the expressions of thermodynamic quantities expressed in the earlier section we can get \( T \) and \( w_{\text{total}} \) in terms of \( z \). The thermodynamic quantities obtained above are plotted against \( z \) in figures 1 to 4. In the figures, we make the plots for \( k = -1 \) (red), 1 (green) as well as 0 (blue).

2.2. Thermodynamic quantities under \( f(R) \) gravity

In the present section, while considering the \( f(R) \) gravity, we have illustrated with a solution \( f(R) = \beta R + \alpha R^n, \quad R = \frac{A}{m}, \quad m > 1, \quad \alpha > 0, \quad \beta > 0 \). Using the above form of \( f(R) \) in we have computed \( \rho_1, p_1, \rho_c \) and \( p_c \) in terms of \( z \) as

\[
\rho_1 = \frac{(1 + z)^{3(1+w_m)}\rho_{m0}}{m(A(1 + z)^n)^{-1+m}\alpha + \beta}; \quad p_1 = \frac{w_m(1 + z)^{3(1+w_m)}\rho_{m0}}{m(A(1 + z)^n)^{-1+m}\alpha + \beta}
\]

\[
\rho_c = \frac{\alpha(-1 + m)A^n(1 + z)^{mn}}{16\pi(mA^{-1+m}(1 + z)^{n(-1+m)}\alpha + \beta)} \left( 1 + \frac{mn}{A^2} \left( 3C_1 - \frac{3k}{(1+z)^2} + \frac{A(1+z)^{4+n}}{n-4} \right) \right)
\]

\[
p_c = \frac{A(1+z)^{2n}}{8\pi m} \left( -3A + 6(-2 + m)mn(1 + n)^2(1 + z)^{4-n} + 6mn(1 + z)^{2-n}(-5 + n + z + nz) \sqrt{C_1 - \frac{k}{(1+z)^2} + \frac{A(1+z)^{-4+n}}{3(-4+n)}} \right)
\]

Using \( \rho_1, p_1, \rho_c \) and \( p_c \) as above we have derived the equation of state parameter and the thermodynamic quantities and plotted them in figures 5 to 8.

3. Concluding remarks

In this study, we have considered Horava-Lifshitz and \( f(R) \) gravity theories. In each case we have considered that the universe is filled with dark matter and dark energy which are not interacting. Prior to discussing the thermodynamic quantities we have studied the behaviors of the equation of state parameters. In both the cases the equation of state parameter is found to stay above \(-1\) that indicates quintessence like behavior. In both of the cases, the \( C_v \) remains at the positive level throughout the evolution of the universe. Temperature \( T \) is always found to fall with evolution of the universe. It is known [15] that deviation from \( v_s^2 = 1 \) is a direct sign that the dark energy is a complex, dynamical fluid rather than an inert cosmological constant. In the case of \( f(R) \), it stays near 1 in the early stage of universe. However, at late stages, it stays much above 1. A negative heat capacity is impossible for any system in stable equilibrium [16]. The positive heat capacity in all of the gravities indicate equilibrium.
Figure 5. This figure shows the evolution of $w_{total}$ with evolution of the universe for $f(R)$ gravity.

Figure 6. This figure shows the behaviour of $T$ with evolution of the universe for $f(R)$ gravity.

Figure 7. This figure shows the behavior of $C_v$ with evolution of the universe for $f(R)$ gravity.

Figure 8. This figure shows $v_s^2$ with evolution of the universe for $f(R)$ gravity.

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