An integrated method for hybrid distribution with estimation of demand matching degree

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Abstract
Timely and effective distribution of relief materials is one of the most important aspects when fighting with a natural or a man-made disaster. Due to the sudden and urgent nature of most disasters, it is hard to make the exact prediction on the demand information. Meanwhile, timely delivery is also a problem. In this paper, taking the COVID-19 epidemic as an example, we propose an integrated method to fulfill both the demand estimation and the relief material distribution. We assume the relief supply is directed by government, so it is possible to arrange experts to evaluate the situation from aspects and coordinate supplies of different sources. The first part of the integrated method is a fuzzy decision-making process. The demand degrees on relief materials are estimated by extending COPRAS under interval 2-tuple linguistic environment. The second part includes the demand degrees as one of the inputs, conducts a hybrid distribution model to decide the allocation and routing. The key point of hybrid distribution is that each demand point could be visited by different vehicles and each vehicle could visit different demand points. Our method can also be extended to include both relief materials and medical staffs. A real-life case study of Wuhan, China is provided to illustrate the presented method.

Keywords COPRAS · Interval 2-tuple linguistic · Hybrid distribution · Relief materials · Multi-criteria decision-making · Vehicle routing
1 Introduction

From the end of 2019, the emergence of 2019 novel coronavirus (COVID-19) has caused a large global outbreak and a major public health issue. China is the first country to discover and report the existing of COVID-19. Till March 9, 2020, 80,890 people have been confirmed being infected by COVID-19 in China. Wuhan, the region of epidemic in China, has 49,448 people infected, accounting for 61% of total infected people in China and 73% of total infected people in Hubei province. Because of its sudden outbreak and severe infectivity, the shortage of relief materials and medical staffs are very serious. Although there are lots of donations and support from all sectors of the society, many difficulties of rescue management are exposed as shown in the following.

As the main rescue force, hospitals need a variety of materials to support their rescue work. However, as the number of confirmed cases increased, the demand for materials is also increasing rapidly. The consumption of various materials in each hospital is very large and many hospitals are under the threat of shortage of materials in the early days of the epidemic. Because of the severe situation and the panic caused by the epidemic, material shortage and over-protection exist in the same time. So it is hard for government to estimate accurately on the relief material demand or rely on the report handed in by the hospitals.

The efficiency of distribution is low and always mismatched. In Wuhan, there is a government department—the Red Cross, being responsible for the distribution of donated relief materials. But some big hospitals could arrange their own vehicles to take part in the distribution. The government decision-makers could not make reasonable overall planning for the routes and packings of vehicles, which often lead the waste of vehicles and distribution time. Some hospitals have to wait for a long time for vehicles to serve them, while some hospitals get certain type of supplies more than they need.

Generally, the relief materials can be classified into three types: daily necessities, drug medical materials and protection materials. As for the medical apparatus and instruments, we do not include them to our study because the amount involved is relatively small. For this multiple materials type, multiple demands without accurate estimation, the problems to be solved are a proper decision of item allocation and distribution routing.

In order to solve the above problems, this paper puts forward an integrated method from two aspects. In the first stage, we first extend a multi-criteria decision-making method named COPRAS (COmplex PRoportional ASsessment) to evaluate the demand degree of each type of materials for each hospital respectively. The interval 2-tuple linguistic variables are used as the evaluation language. Then a hybrid distribution model is explored in the second stage to decide the distribution of materials for designated hospitals. The demand degree output in the first stage is employed as one of the inputs of the distribution model. The final decision of overall allocation of relief materials and the distribution routing is output in the second stage.
The rest of paper is organized as follows. Section 2 briefly reviews the literatures related to multi-criteria decision-making method and hybrid distribution problem. Section 3 presents the terminologies and definitions about the interval 2-tuple linguistic, which is the fuzzy language applied in our paper. In Sect. 4, the extended COPRAS under interval 2-tuple linguistic variables environment is proposed to evaluate the demand degree. Section 5 models the hybrid distribution model of relief materials. Section 6 considers the distribution model with both relief materials and medical staffs being considered. Section 7 gives a real-life case study of Wuhan anti-epidemic in China. Section 8 is the conclusion of this paper.

# 2 Literature review

We now review the literatures related to our problem from two aspects: (1) The multi-criteria decision-making methods and the fuzzy approaches; (2) The problem related to hybrid distribution of materials.

Multi-Criteria Decision-Making (MCDM) method provides a methodology that uses both decision criteria (including benefit and cost information) and the decision maker’s perspective to give rankings on a set of alternatives, or select the best ones from the set of alternatives. Lots of MCDM methods have been proposed in literatures, such as AHP, TOPSIS, VIKOR, etc. As one member of MCDM, COPRAS was first proposed by Zavadskas et al. (Zavadskas et al. 1994) in 1994. They used this method to evaluate the building life cycles in order to select an optimal alternative. It has the following advantages: (1) It can handle both positive and negative criteria in the decision system; (2) Compared with other MCDM methods such as AHP or TOPSIS, the process of COPRAS is simpler and more efficient. Lots of literatures have applied it to the multi-criteria decision-making problems. For instance, Mulliner et al. (2013) used it to make an assessment of sustainable housing affordability. Kaklauskas et al. (2005) used this method to choose the best option to design an efficient building refurbishment. Mulliner et al. (2016) used the COPRAS method for the evaluation of sustainable housing affordability and compared it with other multi-criteria decision-making problem approaches. Pitchipoop et al. (2014) implemented the COPRAS decision-making model to find the optimal blind spot in heavy vehicles. These literatures all develop the method based on the exact numbers. But most times a fuzzy linguistic is more useful to express the evaluation ideas. Peng et al. (2019) extended the COPRAS method into Pythagorean fuzzy environment. Zheng et al. (2018) made a severity assessment of chronic obstructive pulmonary disease based on hesitant fuzzy linguistic COPRAS method.

As for the distribution problem, the classical model is called Transportation Problem (TP), which was first proposed by Frank and Hitchcock (1941). This problem tries to minimize the total transportation cost by allocating the transportation amount between supply and demand places. It does not consider the distribution routing issue. After that, many researchers expended it based on this model. Hirsch and Dantzig (1968) proposed a Fixed-Charge Transportation Problem (FCTP) with the consideration of opening cost. Over the years, many extensions have been proposed,
like multi-item FCTP (Vinay et al. 2019), solid FCTP (Pravash et al. 2015), multi-objective FCTP (Amiya et al. 2019).

Different from Transportation Problem, Travelling Salesman Problem (TSP) and Vehicle Routing Problem (VRP) take the routing problem into consideration. TSP was first proposed by Menger (1932). The process of TSP is as follows: A salesman wants to go to several cities to sell products. He starts from one city and needs to go back to the starting place after passing through all the cities. The goal is to choose the route to make the total journey shortest. VRP was first proposed by Dantzig et al. (1959). The basic description of classical VRP is as follows: The warehouse needs to distribute commodities to several customer nodes. The location and demand of the customers are public information. Vehicles start from the warehouse, visit the designated customer nodes (each customer is served by one vehicle) in a certain order and return to the warehouse. The objective is to minimize the transportation cost by designing a proper vehicle route. For the extension for VRP, Tillman (1969) proposed multi-depot vehicle routing problem (MDVRP) where the number of warehouse is more than one. Chen et al. (2009) proposed the vehicle routing with time window (VRPTW) for perishable food and put a penalty into the objective function. Liu et al. (2020) and Alkaabneh et al. (2020) considered inventory routing problem (IRP) which determine additionally the quantity of commodity to each service node. Biswas et al. (2019) considered the case that there are different transportation modes between the origin and destination. It is noteworthy that these VRP are all with an assumption that one demand point is served by exactly one vehicle. Dror et al. (1989) relaxed this assumption and proposed VRP with split delivery (SDVRP), they assumed that each demand point can be served by multiple vehicles. For the case of uncertain demand information, Jorge et al. (2017) made a review for the stochastic vehicle routing problem.

In summary, in the above transportation problem or vehicle routing problem, the demand is a known value or a random variable subject to a known distribution, and the objective is generally to minimize the total cost. But for the relief material distribution problem, it is hard to predict the demand like in the commercial case. Meanwhile, besides the distribution cost, whether or not the relief materials are matching to demand is more important. So in this paper, we extend the multi-criteria method to estimate the demand, decide the distribute routing, minimize the distribution cost on the premise of maximum matching degrees.

3 Terminologies and definitions

In order to estimate the demand degree of each demand point for each kind of relief material, multiple criteria need to be considered. Experts from several areas are invited to give the evaluation and estimation. Their opinions are collected and processed to generate a unified conclusion. Comparing to the exact real number, fuzzy numbers and fuzzy language can embrace more uncertainty and speculation, which is especially crucial for the complex decision process. Another issue has to be taken into account is that usually experts from different departments tend to use different linguistic term sets to express their judgments on criteria. And they have different
scoring habits even when they use the same linguistic term sets. These problems could not be resolved by general fuzzy number like triangular fuzzy number, but it can be well handled by interval 2-tuple linguistic variables (Herrera and Martínez 2000; Chen and Tai 2005). Besides of these, the computational processes of dealing with interval 2-tuple linguistic variables can avoid loses of information (You et al. 2015). Therefore, interval 2-tuple linguistic variable is a suitable fuzzy language for our evaluation problem. Related definitions are presented in the following.

The 2-tuple linguistic variable was first proposed by Herrera and Martinez (2000) based on the concept of symbolic translation. It is used to represent the linguistic information by means of a pair of values \((s_i, \alpha)\), where \(s_i\) is a linguistic term from the linguistic term set \(S\) defined in advance. For example, a linguistic term set \(S\) composed of five linguistic terms can be written as: \(S = \{s_0, s_1, s_2, s_3, s_4\}\). \(\alpha\) is a numerical value representing the deviation between the value representing the result of a symbolic aggregation operation (called \(\beta\)) and its closest index label \(s_i\).

**Definition 1** Let \(S = \{s_0, s_1, \ldots, s_g\}\) be a linguistic term set and \(\beta \in [0, 1]\) a value representing the result of a symbolic aggregation on operation. Then the translation function \(\Delta\) used to obtain the 2-tuple linguistic variable equivalent to \(\beta\) can be defined as follows:

\[
\Delta : [0, 1] \rightarrow S \times \left[\frac{-1}{2g}, \frac{1}{2g}\right]
\]

\[
\Delta(\beta) = (s_i, \alpha), \quad \left\{ \begin{array}{l}
    s_i, i = \text{round}(\beta \cdot g), \\
    \alpha = \beta - \frac{i}{g}, \alpha \in \left[\frac{-1}{2g}, \frac{1}{2g}\right],
\end{array} \right.
\]

where round (\(\ast\)) is the usual round operation, \(s_i\) has the closest index label to \(\beta\) and \(\alpha\) is the value of symbolic translation in the interval of \([-1/2g, 1/2g]\) which is determined by the number of linguistic terms in \(S\).

**Definition 2** There exists a reverse function \(\Delta^{-1}\) which is able to transform a 2-tuple linguistic variable \((s_i, \alpha)\) into an equivalent value \(\beta \in [0, 1]\). The function \(\Delta^{-1}\) is defined as follows:

\[
\Delta^{-1} : S \times \left[\frac{-1}{2g}, \frac{1}{2g}\right] \rightarrow [0, 1]
\]

\[
\Delta^{-1}(s_i, \alpha) = \frac{i}{g} + \alpha
\]

It should be noted that the conversion of a linguistic term into a linguistic 2-tuple consists of adding a value 0 as symbolic translation:

\[s_i \in S \Rightarrow (s_i, 0)\]
(1) If \( k < 1 \) then \( (s_k, \alpha_1) \) is smaller than \( (s_l, \alpha_2) \);
(2) If \( k = 1 \) then.
(a) if \( \alpha_1 = \alpha_2 \) then \( (s_k, \alpha_1) \) is equal to \( (s_l, \alpha_2) \);
(b) if \( \alpha_1 < \alpha_2 \) then \( (s_k, \alpha_1) \) is smaller than \( (s_l, \alpha_2) \);
(c) if \( \alpha_1 > \alpha_2 \) then \( (s_k, \alpha_1) \) is bigger than \( (s_l, \alpha_2) \).

On the basis of the definition by Chen and Tai (2005), Zhang (2012) put forward the interval 2-tuple linguistic representation model and extended above definitions in the condition of interval 2-tuple linguistic variables as follows:

Definition 4 Let \( S = \{s_0, s_1, \ldots, s_g\} \) be a linguistic term set. An interval 2-tuple linguistic variable is composed of two 2-tuples, denoted by \([(s_k, \alpha_1), (s_l, \alpha_2)]\), where \((s_k, \alpha_1) \leq (s_l, \alpha_2), s_k(s_l)\) and \(\alpha_1(\alpha_2)\) represent the linguistic label of the predefined linguistic term set \(S\) and symbolic translation, respectively. The interval 2-tuple that expresses the equivalent information to an interval value \([\beta_1, \beta_2] = [\beta_1, \beta_2] \in [0, 1], \beta_1 < \beta_2\) is derived by the following function:

\[
\Delta[\beta_1, \beta_2] = [(s_k, \alpha_1), (s_l, \alpha_2)] \quad \text{where} \quad \begin{align*}
s_k, k &= \text{round} (\beta_1 \cdot g) \\
s_l, l &= \text{round} (\beta_2 \cdot g) \\
\alpha_1 &= \beta_1 - \frac{k}{g}, \alpha_1 \in \left[ -\frac{1}{2g}, \frac{1}{2g} \right] \\
\alpha_2 &= \beta_2 - \frac{l}{g}, \alpha_2 \in \left[ -\frac{1}{2g}, \frac{1}{2g} \right]
\end{align*}
\]

(6)

Symmetrically, there also exists a reverse function \(\Delta^{-1}\) which is able to transform an interval 2-tuple into an equivalent value \([\beta_1, \beta_2]. (\beta_1, \beta_2 \in [0, 1], \beta_1 < \beta_2)\) as follows:

\[
\Delta^{-1}[(s_k, \alpha_1), (s_l, \alpha_2)] = [(\alpha_1 + \frac{k}{g}, \alpha_2 + \frac{l}{g}]
\]

(7)

4 Extended COPRAS under interval 2-tuple linguistic variables environment for the evaluation of demand degree

The COPRAS is a method of multi-criteria decision-making problems. Comparing with other multi-criteria decision making methods like TOPSIS, TODIM, it is simple and efficiency. It can process the information when both positive criteria and negative criteria exist (Zheng et al. 2018). And in our evaluation, criterion like the number of patients has a positive relationship with the demand degree; criterion like the materials inventory level has a negative relationship with the demand degree. Therefore, COPRAS is an appropriate way to applied for the evaluation. We extend
the COPRAS method under the interval 2-tuple linguistic variable environment to evaluate each hospital’s demand degree for different kinds of materials respectively.

Suppose that there are $H$ decision makers $DM_h (h = 1, 2, ..., H)$, $P$ hospitals $A_p (p = 1, 2, ..., P)$, $Q$ criteria $C_q (q = 1, 2, ..., Q)$ for the evaluation of demand degree for a kind of material. Each decision maker $DM_h$ is given a weight $\lambda_h > 0$ satisfying $\sum_{h=1}^{H} \lambda_h = 1$ to reflect the importance of each decision maker. Let $S = \{s_0, s_1, \ldots, s_g\}$ be the linguistic term set, $D_h = (d_{pq}^h)_{P \times Q}$ be the linguistic decision matrix of decision maker $h$, where $d_{pq}^h$ is the linguistic information provided by $DM_h$ on the assessment of criteria $q$ for hospital $p$. Let $W = (w_{hq})_{H \times Q}$ be the linguistic weight decision matrix given by decision makers, where $w_{hq}$ is the weight of criteria $q$ of $DM_h$. It is noteworthy that different decision makers can employ different linguistic term set. Based upon assumptions and notations, the procedure of interval 2-tuple linguistic COPRAS method for the evaluation of material’s demand degree for each hospital can be defined as follows:

**Step1:** Convert the linguistic decision matrix $D_h$ into interval 2-tuple linguistic decision matrix $R_h = \left( \left[ \left( r_{pq}^h, 0 \right), \left( r_{pq}^h, 0 \right) \right] \right)_{P \times Q}$, where $r_{pq}^h, r_{pq}^h \in S$, and $r_{pq}^h \leq r_{pq}^h$.

**Step2:** Convert the linguistic weight vector $w_{hq}$ into 2-tuple linguistic weight vector $W = \left( \left( k_{hq}, 0 \right), \left( n_{hq}, 0 \right) \right)_{H \times Q}$, where $k_{hq}, n_{hq} \in S$, $S = \{s_1, s_2, \ldots, s_g\}$ and $k_{hq} \leq n_{hq}$.

**Step3:** Convert each element in the above two interval 2-tuple linguistic decision matrix to its equivalent numerical value with the reverse function $\Delta^{-1}$ and the new matrix are separately written as $R'_h$ and $W'_h$.

**Step4:** Aggregate the all decision makers’ ratings on each criterion to construct a collective interval 2-tuple linguistic decision matrix $R' = (r')_{P \times Q}$, where

$$
\begin{align*}
    r' &= \left[ \sum_{h=1}^{H} \lambda_h \Delta^{-1} \left( r_{pq}^h, 0 \right) \right]_{p=1, 2, \ldots, P, \quad q=1, 2, \ldots, Q,}
\end{align*}
$$

(8)

**Step5:** Aggregate the all decision makers’ ratings on each criterion weights to construct a collective interval 2-tuple linguistic decision matrix $w' = [(w_1, 0), (w_2, 0), \ldots, (w_Q, 0)]^T$, where

$$
\begin{align*}
    (w_q, 0) &= \left( \sum_{h=1}^{H} \lambda_h \Delta^{-1} \left( w_{hq}^h, 0 \right) \right), \quad q = 1, 2, \ldots, Q,
\end{align*}
$$

(9)

**Step6:** Defuzzy the interval by the following equation:

$$
\begin{align*}
    \Delta^{-1} \left[ \left( r_{pq}^h, 0 \right), \left( r_{pq}^h, 0 \right) \right] = \left[ \beta_1, \beta_2 \right] = \frac{\beta_1 + \beta_2}{2}
\end{align*}
$$

(10)

The final collective decision matrix is written in the form of $R'' = [r''_{pq}]_{P \times Q}$.

The final collective weight vector is written as $w'' = [w_1'', w_2'', \ldots, w_Q'']^T$. 
**Step 7:** Let $E = \{e_{pq}\}_{P \times Q}$ be the Normalization matrix of the decision-making, where

$$
e_{pq} = \frac{w_q^r}{\sum_{p=1}^{P} r_{pq}^r} r_{pq}^r \tag{11}$$

**Step 8:** Calculate the sums of weighted normalized criteria for every hospital. The criteria are always composed of positive criteria and negative criteria. The higher the positive criteria’s values are, the more demand degree of hospital is. Reversely, the lower the negative criteria’s values are, the more demand degree of hospital is. The sums of positive and negative weighted normalized criteria are calculated by the following equation:

$$S_p^+ = \sum_{z_q=+} e_{pq} \tag{12}$$

$$S_p^- = \sum_{z_q=-} e_{pq} \tag{13}$$

where $z_q = \{+, if \ criteria \ q \ is \ positive \} \ - , if \ criteria \ q \ is \ negative$.

**Step 9:** Calculate the relative significance $Q_p$ of each hospital by the following equation:

$$Q_p = S_p^+ + \frac{S_{min}}{S_p^-} \sum_{p=1}^{P} S_p^- \tag{14}$$

**Step 10:** Calculate the normalization number $Q_p^\sim$ by the following equation

$$Q_p^\sim = \frac{Q_p}{\sum_{p=1}^{P} Q_p} \tag{15}$$

In summary, this section proposes a COPRAS method based on interval 2-tuple linguistic variable for the evaluation of each hospital’s demand degree for each kind of material. On the basis of it, we introduce a model for mixed distribution of the relief materials in next section.

### 5 Hybrid distribution of relief materials

In the following we will introduce our model for the hybrid distribution of relief materials. We assume that a number of homogenous vehicles distribute multiple kinds of relief materials starting from a warehouse. The first goal of our model is to
maximize the demand degree times the number of materials that hospital receives, which can be denoted as the matching degree. The second goal is to minimize the total distribution time of vehicles. The assumptions are as follows:

1. The definition of unit number of materials refers to each non-separable package. The number of each material means the number of non-separable package of each kind of material;
2. The volume of each package is same;
3. Different materials can be packed on a same vehicle;
4. The amount of relief materials in the warehouse is limited;
5. Each hospital can be visited by multiple vehicles;
6. Vehicles must come back to the warehouse;
7. The number of vehicles is limited;
8. The distribution time of each vehicle refers to the total time required for each vehicle to start from the warehouse, visit some hospitals in a certain order, and return to the warehouse.

5.1 Notations

In order to avoid confusion with the notations in previous sections, we claim that the notations used in Sect. 4 and Sect. 5 are defined formally in the following.

5.1.1 Sets

\[ I = \{1, 2, \ldots, I\} \] The set of hospitals.
\[ V \] The set of warehouse and hospitals.
\[ R = \{1, 2, \ldots, r, \ldots, R\} \] The set of \( R \) kinds of materials.
\[ K = \{1, 2, \ldots, k, \ldots, K\} \] The set of homogeneous vehicles.
\[ E = \{(i,j) | i, j \in V, i \neq j\} \] The set of edges between node \( i \) and node \( j \).

5.1.2 Parameters

\[ C \] The capacity of vehicles.
\[ A \] The number of hospitals.
\[ D_{ir} \] The demand of material \( r \) reported by hospital \( i \).
\[ \omega_{ir} \] The demand degree of hospital \( i \) for material \( r \) output in Sect. 4.
\[ t_{ij} \] Travel time between node \( i \) and \( j \).
\[ B_r \] Total amount of material \( r \) in the warehouse.
5.1.3 Decision variables

$x_{ir}^k$ The number of material $r$ to hospital $i$ delivered by vehicle $k$.

$l_{ij}^k$ 1 if node $i$ is visited before node $j$ by vehicle $k$; 0 otherwise.

$y_{ik}$ 1 if node $i$ is visited by vehicle $k$; 0 otherwise.

5.2 Hybrid distribution model

$max f_1 = \sum_{i \in I} \sum_{r \in R} \sum_{k \in K} o_{ir} x_{ir}^k y_{ik}$ (16)

$min f_2 = \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} t_{ij} l_{ij}^k$ (17)

$\sum_{i \in I} \sum_{k \in K} x_{ir}^k \leq B_r, \forall r \in R$ (18)

$\sum_{i \in I} y_{ik} \leq A, \forall k \in K$ (19)

$\sum_{i \in I} \sum_{r \in R} x_{ir}^k \geq (1 - \rho)C, \forall k \in K$ (20)

$\sum_{i \in I} \sum_{r \in R} x_{ir}^k \leq C, \forall k \in K$ (21)

$\sum_{k \in K} x_{ir}^k \geq a D_{ir}, \forall i \in I, r \in R$ (22)

$\sum_{j \in I} l_{ij}^k - l_{ji}^k = 0, \forall k \in K, i = 0$ (23)

$y_{ik} y_{jk} \geq l_{ij}^k, \forall k \in K, i, j \in V$ (24)

$y_{ik} \in \{0, 1\}, \forall k \in K, i \in I$ (25)

$l_{ij}^k \in \{0, 1\}, \forall k \in K, i, j \in V$ (26)

$x_{ir}^k \in Z^+ \cup \{0\}, \forall k \in K, i \in I$ (27)
Equation (16) and (17) are the objective functions of our model. Objective function (16) means a hospital with higher demand degree will receive more materials of that kind. Objective function (17) is to minimize the total distribution time. Equations (18)–(27) are the constraints of our model. Constraint (18) shows the resources limitation of materials. Constraint (19) means the number of hospitals that is served by one vehicle does not exceed the total number of hospitals. Constraint (20) and (21) show the minimum and maximum limitation of the total number of materials that would be loaded in each vehicle, where \( \rho \) is the least no-load rate of each vehicle. Constraint (22) means the minimum number of materials to be allocated to each hospital, \( \alpha \) is a real number whose value range is 0 to 1. Constraint (23) means each vehicle must come back to the warehouse. Constraint (24) shows that if \( l_{ij}^k = 1 \), then node i and j are both visited by vehicle k. Constraints (25)–(27) are the types and ranges of variables.

6 Hybrid distribution containing both relief materials and medical staffs

In Sect. 5, we introduce a distribution model with the consideration of demand matching degree, to ensure the relief materials are with targeted and fair distribution. We set the objective such that a hospital with higher demand degree is also allocated more relief materials. Although this is for sure an effective method for fair allocation, we found materials waste in real world application. This happens because the processing ability of the hospital does not keep up. And that is true in the early days of COVID-19 epidemic, a large number of medical staffs from several provinces went to Wuhan to support the hospitals there. Therefore, in this section, we additionally take the distribution of medical staffs into consideration.

We still assume that there are several kinds of relief materials should be distributed to some hospitals. The amount of each kind material that the hospital could be allocated is a decision variable. Different from the model in Sect. 5, the number of medical staffs should be transported to the hospital is also a decision variable. The objective of our new model is consistent with that in Sect. 5. Additional assumptions are as follows:

1. The relief materials and the medical staffs should not take the same vehicle;
2. There are two types of vehicles. Type \( a \) is for the distribution of relief materials and type \( b \) is for the transportation of medical staffs;
3. Assume the medical staffs also set out from the warehouse.

Following are the notations of terminologies and variables.
6.1 Sets

\[ I = \{1, 2, \ldots, I\} \]

The set of hospitals.

\[ V \]

The set of warehouse and hospitals.

\[ W = \{1, 2, \ldots, w, \ldots, W\} \]

The set of relief materials.

\[ H = \{0\} \]

The set of medical staffs.

\[ R = W \cup H \]

The set of relief materials and medical staffs, named as support.

\[ K_a = \{1, 2, \ldots, n, \ldots, N\} \]

The set of vehicles for type \( a \).

\[ K_b = \{N + 1, N + 2, \ldots, m, \ldots, M\} \]

The set of vehicles for type \( b \).

\[ K = K_a \cup K_b, k \in K \]

The set of all vehicles.

\[ E = \{(i,j)|i,j \in V, i \neq j\} \]

The set of edges between node \( i \) and node \( j \).

6.2 Parameters

\( C_k \)

The capacity of vehicle \( k \).

\( A \)

The number of hospitals.

\( D_{ir} \)

The demand of support \( r \) reported by hospital \( i \).

\( \omega_{ir} \)

The demand degree of hospital \( i \) for support \( r \) calculated in Sect. 4

\( t_{ij} \)

Travel time between node \( i \) and \( j \).

\( B_r \)

Total number of relief materials and medical staffs in warehouse.

6.3 Decision variables

\( x_{ir}^k \)

The number of support \( r \) to hospital \( i \) delivered by vehicle \( k \).

\( l_{ij}^k \)

1 if node \( i \) is visited before node \( j \) by vehicle \( k \); 0 otherwise.

\( y_{ik} \)

1 if vehicle \( k \) visits node \( i \); 0 otherwise.

Our hybrid distribution model containing both relief materials and medical staffs can be formulated as follows:

\[
\max f_1 = \sum_{i \in I} \sum_{r \in R} \sum_{k \in K} \omega_{ir} x_{ir}^k y_{ik} 
\]

\[
\min f_2 = \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} t_{ij} l_{ij}^k 
\]

\[
\sum_{i \in I} \sum_{k \in K} x_{ir}^k \leq B_r, \forall r \in R 
\]

\[
\sum_{i \in I} y_{ik} \leq A, \forall k \in K 
\]

\[
x_{iw}^n = 0, \forall w \in W, n \in K_b, i \in I 
\]
Equations (28) and (29) are the objective functions of our model. Objective function (28) means a hospital with higher demand degree will receive more supports. Objective function (29) is to minimize the distribution time.

Equations (30)-(41) are the constraints of our model. Constraint (30) shows the resource limitation of supports. Constraint (31) means the number of hospitals that are served by one vehicle does not exceed the total number of hospitals. Constraints (32) and (33) show that the relief materials and medical staffs are not in a same vehicle. Constraint (34) and (35) show the minimum and maximum limit the total number of materials that could be loaded in vehicle $k$, where $\rho_k$ is the least no-load rate of vehicle $k$. Constraint (36) means the minimum number of materials to be allocated to each hospital, $\alpha$ is a real number whose value range is 0 to 1. Constraint (37) means each vehicle must come back to the warehouse. Constraint (38) shows the relationship between $y_{ik}$ and $l_{ij}^k$. Constraints (39)-(41) are the types and ranges of variables.

7 Real-life case study of Wuhan anti-epidemic

In this section we conduct a real-life case study in Wuhan, China for the anti-epidemic of COVID-19. Three kinds of relief materials and medical staffs are to be distributed to designated hospitals. In order to clear the results, we select 4
representative hospitals as our object. Hospital 1 is *Wuhan Third Hospital-Tongren Hospital of Wuhan University*. It is a 3A hospital with 2000 beds; Hospital 2 is *Tianyou Hospital affiliated to Wuhan University of Science & Technology*. It is a 3A hospital with 630 beds; Hospital 3 is *Wuhan Jinyintan Hospital*. It is a 3A specialized hospital for infectious diseases with 2000 hospital beds; Hospital 4 is a newly built mobile cabin hospital.

As for the allocation of relief materials, based on our method, hospitals propose the requirement, experts evaluate the overall situation and rate the hospitals demand degree on each kind of material, the bi-objective mathematical programming output the final amount and distribution routing. Experts are invited from departments of government and hospitals. The rating language is interval 2-tuple variables.

The whole process of the demand degree evaluation is placed in Appendix I. The Parameters for the real case study experiment are placed in Appendix II. Following are some analysis and conclusion on the results output by our method.

From Table 1, we can see that a hospital’s demand degree is varied on different kinds of materials. In general the hospitals with higher demand degree get comparatively more materials. Take the allocation for hospital 4 as an example. Hospital 4 is a mobile cabin hospital. There are two main characteristics of this hospital: the number of patients is larger than most of the general designated hospitals and the clinical symptom of these COVID-19 patients are milder than those of designated hospitals. So it is reasonable that the demand of hospital 4 for daily necessities and drug medical materials are the highest. As for the demand of hospital 4 for protection materials is also the highest, this phenomenon may relate to the reason that the inventory level of this newly-built hospital is much lower than other hospitals.

### Table 1
The number of material $r$ to hospital $i$

| Hospital, $i$ | Material degree of hospital $i$ for material $r$, $w_{ir}$ | The number of material $r$ to hospital $i$ |
|--------------|--------------------------------------------------------|------------------------------------------|
|              | 1 2 3                                                   | 1 2 3                                    |
| 1            | 0.280 0.260 0.267                                       | 30 25 25                                 |
| 2            | 0.189 0.142 0.161                                       | 20 20 15                                 |
| 3            | 0.229 0.186 0.203                                       | 25 25 20                                 |
| 4            | 0.302 0.413 0.369                                       | 45 45 40                                 |

### Table 2
The number of each kind of material to each hospital for vehicle 1

| Materials, $r$ | Hospitals, $i$ |
|----------------|----------------|
|                | 1 2 3 4        |
| 1              | 3 0 25 45      |
| 2              | 0 0 0 0        |
| 3              | 0 0 0 40       |
From Tables 2, 3 and 4, we can see that all kinds of materials are mixed and distributed together by vehicles. Take the distribution for hospital 4 as an example, the demand of each kind of material for hospital 4 is satisfied by two vehicles. The combination of materials in each vehicle is different. In another word, the distribution of materials for hospital 4 is finished by the coordination of these vehicles with a variety number of materials. Table 5 shows the route of each vehicle.

Due to the materials shortages at the beginning of epidemic, Wuhan government could not satisfy the total demand reported by hospitals. The methodology they adopted is to discount the reported demand by some ratio, while this results in higher reporting by hospitals. Comparing with the method adopted by Wuhan government in the initial stage, we have a clear advantage. The results of allocation in our model and the method adopted by government are shown in Table 6. We can see that the matching degree in our model is higher than it in the method adopted by government.

The model proposed in our paper can help government to make an overall planning on the vehicles used for distribution. As we mentioned before, the government actually distributed the materials with the method of TP model. We can see that in this case study, if the government method is used, the entire materials distribution requires five vehicles (hospital 4 needs two vehicles because of its high demand). On the contrary, by planning the packing and the

| Materials, \( r \) | Hospitals, \( i \) |
|---------------------|------------------|
|                     | 1    | 2    | 3    | 4    |
| 1                   | 25   | 0    | 0    | 0    |
| 2                   | 25   | 0    | 25   | 0    |
| 3                   | 25   | 0    | 20   | 0    |

| Materials, \( r \) | Hospitals, \( i \) |
|---------------------|------------------|
|                     | 1    | 2    | 3    | 4    |
| 1                   | 2    | 20   | 0    | 0    |
| 2                   | 0    | 20   | 0    | 45   |
| 3                   | 0    | 15   | 0    | 0    |

| Vehicle, \( k \) | Route |
|------------------|-------|
| 1                | 0 → 1 → 4 → 3 → 0 |
| 2                | 0 → 1 → 3 → 0 |
| 3                | 0 → 4 → 1 → 2 → 0 |
routing of vehicles, we can accomplish the distribution under the constraint of three available vehicles.
It can also be noticed that when the number of vehicles is limited, that is the total number of materials to be distributed is approximately the total capacity of vehicles, the number of used vehicles in our model is always less than it used in TP. Let $\theta$ be the ratio of the total number of materials to the sum of vehicle’s capacity, the corresponding number of vehicles used in our model and TP are shown in Fig. 1.

We also solve the case with considering the distribution of medical staffs. The results are shown in Tables 7, 8, 9, 10, 11, 12, 13, 14 and 15 which are in Appendix III. From Table 7 we can see that hospital 1 and hospital 3 have higher demand degree in medical staffs. This is reasonable because the patients of COVID-19 in these two hospitals are much more serious than other hospitals. They need more medical treatment. Tables 8, 9, 10, 11, 12, 13 and 14 are the situation of materials in each vehicle. We can see the relief materials are still mixed distributed by type $a$.

### Table 6 The result of allocation by using two methods

| Hospital, $i$ | The demand degree of hospital $i$ for material $r$, $w_{ir}$ | The number of material $r$ to hospital $i$ in our model | The number of material $r$ to hospital $i$ in the method adopted by government |
|--------------|-------------------------------------------------------------|----------------------------------------------------------|--------------------------------------------------------------------------------|
| 1            | 0.280 0.260 0.267                                         | 30 25 25                                                | 40 30 30                                                                        |
| 2            | 0.189 0.142 0.161                                         | 20 20 15                                                | 20 20 10                                                                        |
| 3            | 0.229 0.186 0.203                                         | 25 25 20                                                | 30 30 20                                                                        |
| 4            | 0.302 0.413 0.369                                         | 45 45 40                                                | 20 20 25                                                                        |
| Objective $f_i$ | 91.98                                                  |                                                          | 75.28                                                                           |

![Fig. 1](https://example.com/figure1.png)  
**Fig. 1** Number of used vehicles in our model and TP ($\theta = 0.958$)
**Table 7**  The number of material \( r \) to hospital \( i \)

| Hospital, \( i \) | The demand degree of hospital \( i \) for material \( r \), \( w_{ir} \) | The number of material \( r \) to hospital \( i \) |
|------------------|----------------------------------|-----------------|
|                  | 0                               | 1               | 2   | 3   | 0  | 1  | 2   | 3   |
| 1                | 0.307                           | 0.280           | 0.260| 0.267| 95 | 30 | 25  | 25  |
| 2                | 0.219                           | 0.189           | 0.142| 0.161| 25 | 20 | 20  | 15  |
| 3                | 0.284                           | 0.229           | 0.186| 0.203| 60 | 25 | 25  | 20  |
| 4                | 0.191                           | 0.302           | 0.413| 0.369| 20 | 45 | 45  | 40  |

**Table 8**  The number of each kind of material to each hospital for vehicle 1

| Materials, \( r \) | Hospitals, \( i \) |
|---------------------|-------------------|
|                     | 1     | 2     | 3     | 4     |
| 0                   | 0     | 0     | 0     | 0     |
| 1                   | 3     | 0     | 25    | 45    |
| 2                   | 0     | 0     | 25    | 0     |
| 3                   | 0     | 0     | 0     | 40    |

**Table 9**  The number of each kind of material to each hospital for vehicle 2

| Materials, \( r \) | Hospitals, \( i \) |
|---------------------|-------------------|
|                     | 1     | 2     | 3     | 4     |
| 0                   | 0     | 0     | 0     | 0     |
| 1                   | 25    | 0     | 0     | 0     |
| 2                   | 25    | 0     | 25    | 0     |
| 3                   | 25    | 0     | 20    | 0     |

**Table 10**  The number of each kind of material to each hospital for vehicle 3

| Materials, \( r \) | Hospitals, \( i \) |
|---------------------|-------------------|
|                     | 1     | 2     | 3     | 4     |
| 0                   | 0     | 0     | 0     | 0     |
| 1                   | 2     | 20    | 0     | 0     |
| 2                   | 0     | 20    | 0     | 45    |
| 3                   | 0     | 15    | 0     | 0     |
| Materials, $r$ | Hospitals, $i$ | 1 | 2 | 3 | 4 |
|---------------|----------------|----|----|----|----|
| 0             | 55             | 0  | 0  | 0  | 0  |
| 1             | 0              | 0  | 0  | 0  | 0  |
| 2             | 0              | 0  | 0  | 0  | 0  |
| 3             | 0              | 0  | 0  | 0  | 0  |

| Materials, $r$ | Hospitals, $i$ | 1 | 2 | 3 | 4 |
|---------------|----------------|----|----|----|----|
| 0             | 0              | 10 | 14 | 20 |    |
| 1             | 0              | 0  | 0  | 0  | 0  |
| 2             | 0              | 0  | 0  | 0  | 0  |
| 3             | 0              | 0  | 0  | 0  | 0  |

| Materials, $r$ | Hospitals, $i$ | 1 | 2 | 3 | 4 |
|---------------|----------------|----|----|----|----|
| 0             | 40             | 15 | 0  | 0  | 0  |
| 1             | 0              | 0  | 0  | 0  | 0  |
| 2             | 0              | 0  | 0  | 0  | 0  |
| 3             | 0              | 0  | 0  | 0  | 0  |

| Materials, $r$ | Hospitals, $i$ | 1 | 2 | 3 | 4 |
|---------------|----------------|----|----|----|----|
| 0             | 0              | 0  | 46 | 0  | 0  |
| 1             | 0              | 0  | 0  | 0  | 0  |
| 2             | 0              | 0  | 0  | 0  | 0  |
| 3             | 0              | 0  | 0  | 0  | 0  |
vehicles, and the medical staffs are distributed by type $b$ vehicles. Table 15 is the route of each vehicle.

### 8 Conclusions

The distribution of relief materials after a natural or man-made disaster is one of the most important things in emergency rescue. This paper proposes an integrated method for the distribution of relief materials when facing an emergency situation. Taking the COVID-19 rescue work as an introductory example, this study first extends COPRAS with interval 2-tuple linguistic variables to evaluate the demand degree of each hospital for each kind of relief material. Then, based on the degrees, we distribute all kinds of materials together. Different from classical transportation problem and vehicle problem, we not only try to minimize the travel cost, but also consider the demand of each hospital as a decision variable which aims to maximize the matching degree between the demand degree of each hospital and the number of materials it receives. As of now, we have solved the two problems mentioned in our background. Compared to the method adopted by the government in Wuhan, we do not only fairly distribute various kinds of relief materials, but also save the number of used vehicles. Finally, to match the ability of processing materials, we also additionally take the medical staffs as a kind of material and consider the distribution with both relief material and medical staffs.

### Appendix I

**The process of evaluation for the demand degree**

We shortly name the four designated hospital as $A_1, A_2, A_3, A_4$. An expert committee composed of four decision makers, $DM_1, DM_2, DM_3, DM_4$, has been formed to evaluate each hospital’s demand degree for each kind of material. $DM_1, DM_2$ are from hospitals. $DM_3, DM_4$ are from government sectors. The evaluation is made on the basis of the following criteria:

- $C_1$: The number of hospitalizations who get COVID-19 infected every day;

| Vehicle, $k$ | Route |
|-------------|-------|
| 1           | $0 \rightarrow 1 \rightarrow 4 \rightarrow 3 \rightarrow 0$ |
| 2           | $0 \rightarrow 1 \rightarrow 3 \rightarrow 0$ |
| 3           | $0 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 0$ |
| 4           | $0 \rightarrow 1 \rightarrow 0$ |
| 5           | $0 \rightarrow 2 \rightarrow 4 \rightarrow 3 \rightarrow 0$ |
| 6           | $0 \rightarrow 1 \rightarrow 2 \rightarrow 0$ |
| 7           | $0 \rightarrow 3 \rightarrow 0$ |
$C_2$: The proportion of critical patients who get COVID-19 every day;
$C_3$: The overall protection level of hospital every day;
$C_4$: The inventory level of relief materials for COVID-19 every day;
$C_5$: The number of hospitalizations every day;
$C_6$: The ratio of the number of existing medical staff to the daily number of critical COVID-19 patients in the hospital.

It is noteworthy that for the evaluation of the demand degree for different type of materials, the criteria or its weight that need to be considered may be different. The above criteria are all the criteria that may be used. For example, criterion like ‘The inventory level of relief materials for COVID-19 every day’ is not suitable for the evaluation of demand degree for medical staffs.

Firstly, each expert provides their linguistic term sets before the evaluation starts. Linguistic term sets employed by $DM_1$, $DM_2$, $DM_3$, $DM_4$ are denoted in the following.

$O_1, O_2, O_3$ are the linguistic term sets used by $DM_1$:

$O_1 = \{ a_{01} = \text{Very few} (VF), a_{11} = \text{Few} (F), a_{21} = \text{Medium} (M), a_{31} = \text{Large} (L),$

$a_{41} = \text{Very large} (VL) \},$

$O_2 = \{ a_{02} = \text{Very low} (VL), a_{12} = \text{Low} (L), a_{22} = \text{Medium} (M), a_{32} = \text{High} (H),$

$a_{42} = \text{Very high} (VH) \},$

$O_3 = \{ a_{03} = \text{Very unimportant} (VU), a_{13} = \text{Unimportant} (U), a_{23} = \text{Medium} (M)$

$a_{33} = \text{Important} (I), a_{43} = \text{Very important} (VI) \},$

The linguistic term sets employed by $DM_2$, $DM_3$, $DM_4$ are denoted as $B_1$, $B_2$, $B_3$, $T_1$, $T_2$, $T_3$, $D_1$, $D_2$, $D_3$:

$B_1 = \{ b_{01} = \text{Very few} (VF), b_{11} = \text{Few} (F), b_{21} = \text{Medium few} (MF), b_{31} = \text{Medium} (M), b_{41} = \text{Medium large} (ML), b_{51} = \text{Large} (L), b_{61} = \text{Very large} (VL) \},$

$B_2 = \{ b_{02} = \text{Very low} (VL), b_{12} = \text{Low} (L), b_{22} = \text{Medium low} (ML), b_{32} = \text{Medium} (M), b_{42} = \text{Medium high} (MH), b_{52} = \text{High} (H), b_{6} = \text{Very high} (VH) \}$

$B_3 = \{ b_{03} = \text{Very unimportant} (VU), b_{13} = \text{Unimportant} (U), b_{22} = \text{Medium} \}$
Important (I), \( b_{63} = \text{Veryimportant (VI)} \),
\[ T_1 = \{ c_{01} = \text{Extremefew (EF)}, c_{11} = \text{Veryfew (VF)}, c_{21} = \text{Few (F)}, c_{31} = \text{Mediumfew (MF)}, c_{41} = \text{Medium (M)}, c_{51} = \text{Mediumlarge (ML)} \} \]
\[ c_{61} = \text{Large (L)}, c_{7} = \text{Verylarge (VL)}, c_{8} = \text{Extremelarge (EL)} \],
\[ T_2 = \{ c_{02} = \text{Extremelow (EL)}, c_{12} = \text{Verylow (VL)}, c_{22} = \text{Few (F)}, c_{32} = \text{Mediumfew (MF)}, c_{42} = \text{Medium (M)}, c_{52} = \text{Mediumlarge (ML)} \} \]
\[ c_{62} = \text{Large (L)}, c_{72} = \text{Verylarge (VL)}, c_{82} = \text{Extremelarge (EL)} \],
\[ T_3 = \{ c_{03} = \text{Extremeunimportant (EU)}, c_{13} = \text{Veryunimportant (VU)}, c_{23} = \text{Unimportant (U)}, c_{33} = \text{Mediumunimportant (MU)}, c_{43} = \text{Medium (M)}, c_{53} = \text{Mediumimportant (MI)}, c_{63} = \text{Important (I)} \} \]
\[ c_{73} = \text{Veryimportant (VI)}, c_{83} = \text{Extremearbitrary (EI)} \].
\[ D_1 = \{ d_{01} = \text{Veryfew (VF)}, d_{11} = \text{Few (F)}, d_{21} = \text{Medium (M)}, d_{31} = \text{Large (L)} \}, \]
\[ d_{41} = \text{Verylarge (VL)} \],
\[ D_2 = \{ d_{02} = \text{Verylow (VL)}, d_{12} = \text{Low (L)}, d_{22} = \text{Medium (M)}, d_{32} = \text{High (H)} \}, \]
\[ d_{42} = \text{Veryhigh (VH)} \],
\[ D_3 = \{ d_{03} = \text{Veryunimportant (VU)}, d_{13} = \text{Unimportant (U)}, d_{23} = \text{Medium (M)}, d_{33} = \text{Important (I)}, d_{43} = \text{Veryimportant (VI)} \} \].

Take the evaluation on the demand degree of four hospitals to protection material as an example, \( C_1, C_2, C_3, C_4 \) are the relative criteria that would be used for the assessment. The rates of the four hospitals on each criterion and the weight of each criterion provided by the four decision makers are presented in Table and Table.
Considering the difference of each decision maker’s domain knowledge and expertise, the four decision makers are assigned the following relative weights: 0.20, 0.35, 0.30, 0.15 in the decision process. Then we use the interval 2-tuple linguistic-COPRAS method to evaluate each hospital’s demand degree for medical protection material, which includes the following steps:

**Step1**: Convert the linguistic decision matrix $D_h$ into interval 2-tuple linguistic decision matrix $R_h = \left( \left[ \left( \begin{array}{c} r_{pq}^h \\ 0 \end{array} \right), \left( \begin{array}{c} r_{pq}^h \\ 0 \end{array} \right) \right] \right)_{4 \times 4}$. Taking $DM_1$ as an example, we can get the interval 2-tuple linguistic decision matrix $R_h$ as shown in Table.

**Step2**: Convert the linguistic weight vector $\omega_h$ into 2-tuple linguistic weight vector $w_h = \left( \left[ \left( k_{hp}, 0 \right), \left( n_{hp}, 0 \right) \right] \right)_{4 \times 4}$, which is presented in Table.

**Step3**: Convert each element in the above two interval 2-tuple linguistic decision matrix to its equivalent numerical value with the reverse function $\Delta^{-1}$ and the new matrix are separately written as $R'_h$ and $w'_h$, taking $DM_1$ as an example, we can get $R'_h$ as shown in Table and $w'_h$ can be written as:

$$w'_1 = [(0.500, 0.500), (0.750, 1.000), (0.750, 1.000), (0.000, 0.250)]$$

**Step4**: Aggregate the all decision makers’ ratings on each criterion to construct a collective interval 2-tuple linguistic decision matrix $R' = \left( r' \right)_{4 \times 4}$ which is presented in Table.

**Step5**: Aggregate all decision makers ratings on each criteria's weight to construct a collective interval 2-tuple linguistic weight matrix.

$w'$, which is presented as follows:

| Table 16 Linguistic assessment of hospitals by four decision makers | Decision makers | Criteria | Hospitals | A₁ | A₂ | A₃ | A₄ |
|---|---|---|---|---|---|---|---|
| DM₁ | C₁ | M-L | M-M | L-L | VL-VL |
| | C₂ | H-VH | M-H | M-M | L-M |
| | C₃ | H-VH | M-M | M-H | L-M |
| | C₄ | VL-VL | L-VL | VL-L | VL-VL |
| DM₂ | C₁ | L-L | M-ML | ML-L | VL-VL |
| | C₂ | H-VH | MH-H | H-VH | ML-M |
| | C₃ | H-VH | MH-H | H-VH | M-M |
| | C₄ | VL-L | L-L | VL-L | VL-VL |
| DM₃ | C₁ | ML-ML | M-M | L-VL | VL-EL |
| | C₂ | H-VH | MH-H | H-H | ML-M |
| | C₃ | H-VH | VH-H | V-VH | M-M |
| | C₄ | EL-VL | VL-EL | VL-L | EL-EL |
| DM₄ | C₁ | M-L | M-M | L-VL | VL-EL |
| | C₂ | H-VH | H-H | H-H | ML-M |
| | C₃ | H-VH | H-H | H-VH | M-M |
| | C₄ | VL-L | VL-L | VL-VL | EL-EL |
Table 17  Linguistic assessments of criteria for medical protection material

| Decision makers | Criteria          | $C_1$   | $C_2$   | $C_3$   | $C_4$   |
|-----------------|-------------------|---------|---------|---------|---------|
| $DM_1$          | M-M               | I-VI    | I-VI    | VI-VI   | MU-VU   |
| $DM_2$          | MU-MU             | I-VI    | VI-VI   | MI-M    | MU-M    |
| $DM_3$          | MU-MU             | I-VI    | VI-EI   | MI-M    | MI-M    |
| $DM_4$          | M-M               | I-VI    | VI-VI   | U-M     | U-M     |

Table 18  Interval 2-tuple linguistic decision matrix for $DM_1$

| Criteria | Hospitals |
|----------|-----------|
|          | $A_1$     | $A_2$     | $A_3$     | $A_4$     |
| $C_1$    | $[(a_2, 0), (a_3, 0)]$ | $[(a_2, 0), (a_3, 0)]$ | $[(a_3, 0), (a_3, 0)]$ | $[(a_4, 0), (a_4, 0)]$ |
| $C_2$    | $[(a_3, 0), (a_4, 0)]$ | $[(a_2, 0), (a_3, 0)]$ | $[(a_2, 0), (a_2, 0)]$ | $[(a_1, 0), (a_2, 0)]$ |
| $C_3$    | $[(a_3, 0), (a_4, 0)]$ | $[(a_2, 0), (a_2, 0)]$ | $[(a_2, 0), (a_3, 0)]$ | $[(a_1, 0), (a_2, 0)]$ |
| $C_4$    | $[(a_0, 0), (a_0, 0)]$ | $[(a_1, 0), (a_1, 0)]$ | $[(a_0, 0), (a_1, 0)]$ | $[(a_0, 0), (a_0, 0)]$ |

Table 19  2-tuple linguistic criteria weights

| Decision makers | Criteria | $C_1$       | $C_2$       | $C_3$       | $C_4$       |
|-----------------|----------|-------------|-------------|-------------|-------------|
| $DM_1$          |          | $[(a_2, 0), (a_2, 0)]$ | $[(a_3, 0), (a_4, 0)]$ | $[(a_3, 0), (a_4, 0)]$ | $[(a_0, 0), (a_1, 0)]$ |
| $DM_2$          |          | $[(b_2, 0), (b_2, 0)]$ | $[(b_5, 0), (b_5, 0)]$ | $[(b_6, 0), (b_6, 0)]$ | $[(b_2, 0), (b_3, 0)]$ |
| $DM_3$          |          | $[(c_3, 0), (c_3, 0)]$ | $[(c_6, 0), (c_7, 0)]$ | $[(c_7, 0), (c_8, 0)]$ | $[(c_2, 0), (c_3, 0)]$ |
| $DM_4$          |          | $[(d_2, 0), (d_2, 0)]$ | $[(d_3, 0), (d_4, 0)]$ | $[(d_4, 0), (d_4, 0)]$ | $[(d_1, 0), (d_2, 0)]$ |

Table 20  Convert interval 2-tuple linguistic decision matrix to equivalent numerical value for $DM_1$

| Criteria | Hospitals |
|----------|-----------|
|          | $A_1$     | $A_2$     | $A_3$     | $A_4$     |
| $C_1$    | $[0.500, 1.000]$ | $[0.500,0.500]$ | $[0.750,0.750]$ | $[1.000,1.000]$ |
| $C_2$    | $[0.750,1.000]$ | $[0.500,0.750]$ | $[0.500,0.500]$ | $[0.250,0.500]$ |
| $C_3$    | $[0.750,0.750]$ | $[0.500,0.500]$ | $[0.500,0.750]$ | $[0.250,0.500]$ |
| $C_4$    | $[0.000,1.000]$ | $[0.250,0.250]$ | $[0.000,0.250]$ | $[0.000,0.000]$ |

$w' = [(0.404, 0.404), (0.779, 0.904), (0.913, 1.000), (0.229, 0.413)]$

Step6: Defuzzy the interval. The final collective decision matrix $R''$ and the final collective weight vector $w''$ are presented in Table and $w''$ can be written as: $w'' = [0.404, 0.842, 0.956, 0.321]$. 
### Table 21  Collective interval 2-tuple linguistic decision matrix $R'$

| Criteria | Hospitals         | $A_1$          | $A_2$          | $A_3$          | $A_4$          |
|----------|-------------------|----------------|----------------|----------------|----------------|
| $C_1$    |                   | [0.654,0.742]  | [0.500,0.596]  | [0.721,0.817]  | [0.963,1.000]  |
| $C_2$    |                   | [0.779,0.963]  | [0.633,0.779]  | [0.729,0.827]  | [0.354,0.500]  |
| $C_3$    |                   | [0.779,0.963]  | [0.671,0.729]  | [0.729,0.913]  | [0.450,0.538]  |
| $C_4$    |                   | [0.000,0.133]  | [0.108,0.183]  | [0.038,0.183]  | [0.000,0.075]  |

### Table 22  Defuzzy collective interval 2-tuple linguistic decision matrix $R''$

| Criteria | Hospitals         | $A_1$          | $A_2$          | $A_3$          | $A_4$          |
|----------|-------------------|----------------|----------------|----------------|----------------|
| $C_1$    |                   | 0.698          | 0.548          | 0.769          | 0.981          |
| $C_2$    |                   | 0.871          | 0.706          | 0.777          | 0.427          |
| $C_3$    |                   | 0.871          | 0.700          | 0.821          | 0.494          |
| $C_4$    |                   | 0.067          | 0.146          | 0.110          | 0.038          |

### Table 23  The normalization matrix of the decision-making $E$

| Criteria, $q$ | Weight, $w^r$ | Hospitals, $p$ |
|---------------|---------------|----------------|
| $C_1$         | $+$           | 0.404          | 0.094          | 0.074          | 0.104          | 0.132          |
| $C_2$         | $+$           | 0.842          | 0.264          | 0.214          | 0.235          | 0.129          |
| $C_3$         | $+$           | 0.956          | 0.289          | 0.232          | 0.272          | 0.164          |
| $C_4$         | $-$           | 0.321          | 0.059          | 0.130          | 0.098          | 0.033          |

### Table 24  Results of COPRAS evaluation

| Hospitals | $A_1$          | $A_2$          | $A_3$          | $A_4$          |
|-----------|----------------|----------------|----------------|----------------|
| $S^*_p$   | 0.646          | 0.520          | 0.611          | 0.425          |
| $S^-_p$   | 0.059          | 0.130          | 0.098          | 0.033          |
| $Q_p$     | 1.036          | 0.698          | 0.846          | 1.118          |
| $Q^-_p$   | 0.280          | 0.189          | 0.229          | 0.302          |

### Table 25  The demand degree of each kind of material for each hospital

| Hospital, $i$ | The demand degree of hospital $i$ for material $r,w_{ir}$ |
|---------------|--------------------------------------------------------|
| 0             | 1           | 2           | 3           |
| 1             | 0.307       | 0.280       | 0.259       | 0.267       |
| 2             | 0.219       | 0.189       | 0.142       | 0.161       |
| 3             | 0.284       | 0.229       | 0.186       | 0.203       |
| 4             | 0.191       | 0.302       | 0.413       | 0.369       |
Step 7: Calculate the normalization matrix of the decision-making $E = [e_{pq}]_{4 \times 4}$, which is presented in Table.

Step 8, 9 and 10: Calculate the sum of weighted normalized criteria $S^+_p, S^-_p$, the relative significance $Q_p$, and the normalization number $Q^\sim_p$, the results are shown in Table. $Q^\sim_p$ is the result of our evaluation for each hospital’s demand degree. The demand degree for all three kinds of materials of four hospitals is shown in Table (Tables 16, 17, 18, 19, 20, 21, 22, 23, 24 and 25).

Appendix II

Parameters for the real case study experiment

Type of materials and the number of each kind of material in the warehouse are shown in Table.

The number of available vehicles of type $a$ is 3; The number of available vehicles of type $b$ is 4; The number of hospitals is 4. Capacity of vehicles is shown in Table. Travel time $t_{ij}$ between node $i$ and $j$ is presented in Table (Tables 26, 27 and 28).

| Table 26 | Total number of material $r$ in warehouse |
|----------|------------------------------------------|
| Materials, $r$ | 0 | 1 | 2 | 3 |
| Type | Medical staffs | Protection materials | Daily necessities | Drug medical materials |
| $B_r$ | 200 | 120 | 115 | 100 |

| Table 27 | Capacity of each vehicle |
|----------|--------------------------|
| $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ | $C_7$ |
| 120 | 120 | 120 | 55 | 55 | 55 | 55 |

| Table 28 | The value of $t_{ij}$ (min) |
|----------|-----------------------------|
| $i$ | $j$ | 0 | 1 | 2 | 3 | 4 |
| 0 | / | 20 | 20 | 30 | 18 |
| 1 | 20 | / | 10 | 30 | 13 |
| 2 | 20 | 10 | / | 34 | 20 |
| 3 | 30 | 30 | 34 | / | 21 |
| 4 | 18 | 13 | 20 | 21 | / |
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References

Alkaabneh F, Diabat A, Gao HO (2020) Benders decomposition for the inventory vehicle routing problem with perishable products and environmental costs. Comput Oper Res 113:104751. https://doi.org/10.1016/j.cor.2019.07.009

Amiya B, Ali AS, Seyed TAN (2019) Multi-objective non-linear fixed charge transportation problem with multiple modes of transportation in crisp and interval environments. Appl Soft Comput 80:628–649

Biswa A, Shaikh AA, Niaki ST (2019) A, Multi-objective non-linear fixed charge transportation problem with multiple modes of transportation in crisp and interval environments. Appl Soft Comput 80:628–649

Chen CT, Tai WS (2005) Measuring the intellectual capital performance based on 2-tuple fuzzy linguistic information. The 10th annual meeting of APDSI, Asia Pacific region of decision sciences institute. Vol. 20

Chen HK, Hsueh CF, Chang MS (2009) Production scheduling and vehicle routing with time windows for perishable food products. Comput Oper Res 36:2311–2319

Dantzig G, Ramser J (1959) The truck dispatching problem. Manage Sci 6:80–91

Dror M, Trudeau P (1989) Savings by split delivery routing. Transp Sci 23:141–145

Frank L (1941) Hitchcock, The distribution of a product from several sources to numerous localities. J Math Phys 20:224–230

Herrera F, Martínez L (2000) A 2-tuple fuzzy linguistic representation model for computing with terms. IEEE Trans Fuzzy Syst 8(6):746–752

Hirsch WM, Dantzig GB (1968) The fixed charge problem. Naval Res Logistics Quarterly 15(3):413–424

Jorge O, Halvard A, David LW (2017) The stochastic vehicle routing problem, a literature review, Part II: solution methods. EURO J Transp Logist 6(4):349–388

Kaklauskas A, Zavadskas EK, Raslanas S (2005) Multivariate design and multiple criteria analysis of building refurbishments. Energy Buildings 37(4):361–372

Liu W, Ke GY, Chen J, Zhang L (2020) Scheduling the distribution of blood products: a vendor-managed inventory routing approach. Transp Res Part E: Logist Transp Rev 140:101964

Menger K (1932) Das Botenproblem. Ergebnisse Eines Mathematischen Kolloquiums 2:11–12

Mulliner E, Smallbone K, Maliene V (2013) An assessment of sustainable housing affordability using a multiple criteria. Omega 41:270–279

Mulliner E, Malys N, Maliene V (2016) Comparative analysis of MCDM methods for the assessment of sustainable housing affordability. Omega 59:146–156

Panicker VV, Sarin IV (2019) Multi-product multi-period fixed charge, transportation problem: an ant colony optimization approach. IFAC-PapersOnline 52:1937–1942

Peng X, Selvachandran G (2019) Pythagorean fuzzy set: state of the art and future directions. Artif Intell Rev 52:1873–1927

Pitchipoo P, Vincent DS, Rajini N, Rajakarunakaran S (2014) COPRAS decision model to optimize blind spot in heavy vehicles: a comparative perspective. Procedia Engineering 97:1049–1059

Pravash KG, Manas KM, Manoranjan M (2015) Fully fuzzy fixed charge multi-item solid transportation problem. Appl Soft Comput 27:77–91

Tillman F (1969) The multiple terminal delivery problem with probabilistic demands. Transp Sci 3:192–204

You XY, You JX, Liu HC, Zhen L (2015) Group multi-criteria supplier selection using an extended VIKOR method with interval 2-tuple linguistic information. Expert Syst Appl 42(4):1906–1916

Zavadskas EK, Kaklauskas A, Sarka V (1994) The new method of multicriteria complex proportional assessment of projects. Technol Econ Dev Econ 1(3):131–139

Zhang H (2012) The multi-attribute group decision making method based on aggregation operators with interval-valued 2-tuple linguistic information. Math Comput Model 56:27–35

Zheng Y, Xu Z, He Y, Liao H (2018) Severity assessment of chronic obstructive pulmonary disease based on hesitant fuzzy linguistic COPRAS method. Appl Soft Comput 69:60–71

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