A shoving model for collectivity in hadronic collisions

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Abstract: An extension of the rope hadronization model, which has previously provided good descriptions of hadrochemistry in high multiplicity pp collisions, is presented. The extension includes a dynamically generated transverse pressure, produced by the excess energy from overlapping strings. We find that this model can qualitatively reproduce soft features of Quark Gluon Plasma in small systems, such as higher $\langle p_{\perp} \rangle$ for heavier particles and long range azimuthal correlations forming a ridge. The effects are similar to those obtained from a hydrodynamic expansion, but without assuming a thermalized medium.

Recent precise measurements of pp and pA collisions at the LHC show flow-like effects \cite{1,2,3} similar to those found in high energy nucleus collisions. Examples are ridge-like structures, quantified in different flow coefficients, and measurements of strangeness enhancement with increasing event activity \cite{4}. These are regarded as two important characteristics of the soft features of the Quark Gluon Plasma, and are often described in a hydrodynamical framework assuming thermal equilibrium.

Dynamical models based on string \cite{5,6} or cluster \cite{7} hadronization models, \textit{e.g.} PYTHIA8 \cite{8,9} and HERWIG7 \cite{10}, are able to describe the general soft features of pp collisions in a very satisfactory way. The need for imposing new dynamics at a macroscopic level, only present at soft scales, is complicated, as it breaks the principle of jet universality by introducing a scale below which new dynamics should be ”switched on”. A quite successful model based on this principle, is the core–corona model \cite{11}, implemented in the EPOS generator \cite{12}, where events are subdivided into ”core” and ”corona” events.

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based on event activity. Recently, attempts have been made to incorporate a “thermal” exponential $m_\perp$-spectrum for the string break-ups in the Lund hadronization model [13] with promising results, but it is still unclear whether such a "microscopization" can capture the essential features of hydrodynamics, and if so, if this picture can correctly describe both hadrochemistry and flow.

To provide a description of the hadrochemistry in the underlying event of pp collisions, we recently suggested a "rope hadronization" model [14], based on work by Biro, Knoll and Nielsen [15]. This model provides corrections to the string hadronization model, by allowing strings overlapping in transverse space to act coherently as a "rope". The model is implemented in the DIPSY event generator [16], which provides a dynamical picture of the event structure in impact parameter space, allowing for a calculation of the colour field strength in each small rope segment. This formalism also includes all fluctuations. The colour field is characterized by two quantum numbers $\{p, q\}$, which together signifies its SU(3) multiplet structure. Lattice calculations have shown [18], that the string tension – energy per unit length – scales with the quadratic Casimir operator of the multiplet, such that the ratio of the enhanced rope tension ($\tilde{\kappa}$) to the triplet string tension in vacuum ($\kappa$) is:

$$\frac{\tilde{\kappa}}{\kappa} = \frac{1}{4} \left( p^2 + q^2 + pq + 3(p + q) \right).$$  

The enhancement of string tension was shown [19] to greatly influence the ratio of strange to non–strange hadrons, and to give the correct dependence on event activity as measured by ALICE [4].

In this letter we show how this enhanced string tension can also be employed to generate a flow–like effect. Since the energy density in the overlap region is higher than outside, a pressure will be dynamically generated, pushing strings outwards. In figure 1 this principle is illustrated; we sketch several overlapping strings in impact parameter space at some initial time $t_1$. The density is larger towards center, giving a pressure gradient. We start a spatio-temporal evolution and let the strings pick up transverse momentum from the excess energy in the overlap regions. As the strings move further apart, the excess energy will decrease, and so will the transverse pressure. From time $t_1$ to $t_2$ the strings pick up some transverse momentum, as indicated with arrows, and move a little bit. From $t_2$ to $t_3$ the strings move, but picks up less transverse momentum, as the overlap is now smaller. From $t_3$ to $t_4$ the strings only move, and picks up no transverse momentum, as there is no overlap. The strings should of course hadronize at some point along the way, and we interrupt the evolution at some given time, where strings are no longer allowed to pick up $p_\perp$ or propagate.

The partonic state obtained from the DIPSY MC is formulated in rapidity ($y$) and

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3A Lund string is in its simplest form, a straight piece stretched between a quark and an anti-quark, or a colour triplet and anti-triplet. As gluons are added to the string, they act as point-like "kinks" on the string, carrying energy and momentum [17]. We will denote all straight pieces between gluons or (anti)quarks string segments. A $q - q - \bar{q}$ string thus has two segments.
transverse coordinate space ($b_\perp$). Colour-connected partons separated by a distance $\Delta b_\perp$ are also given opposite transverse momenta $p_\perp \approx \Delta b_\perp/(\Delta b_\perp)^2$. The initial state is two Lorentz contracted pancakes colliding at $z = 0$, and the string segments are then stretched out mainly along the $z$ direction. The distribution of gluons is approximately boost invariant, and to visualize the effect of the transverse repulsion, it is most easy to study a string segment stretched between two gluons in a system where they have rapidities $\pm \Delta y/2$. The endpoints of this string segment will then move out with longitudinal velocities $v_L = \pm \tanh(\Delta y/2)$, and the length of the segment in coordinate space, at time $t$, is consequently $t \tanh(\Delta y)$. The repulsive transverse force between two strings is proportional to the length of the overlapping region, and is therefore proportional to $f \cdot t \cdot \Delta y$, where $f$ is the force per unit string length.

The cartoon in figure 1 represents in a schematic way a "slice" in rapidity. The result of the repulsion will be a transverse velocity for the string, which might be represented by very many very soft gluons. The breakup of such a string state cannot be handled by current implementations of string hadronization, as in e.g. PYTHIA8. As the DIPSY generator interfaces to the PYTHIA8 hadronization implementation, this must be remedied. A transverse gluon will give momentum to hadrons within one unit of rapidity on either side of the gluon. It is therefore possible to simulate the effect of the continuous distribution of infinitely soft gluons by finite gluons separated by at most one rapidity unit. In our calculations we cut the event into many rapidity slices, and in each slice we let the strings "shove" each other apart. The mechanism for shoving is to add a small excitation (i.e. a gluon) to each string in each slice. In each time–step $\delta t$ a string within a slice $\delta y$ (and thus length $\delta l = t \delta y$) will get a kick in the transverse direction $\delta p_\perp = f t \delta y \delta t$. As the mass of the string piece is $\approx \kappa \delta l = \kappa t \delta y$ also is proportional to the time $t$, we note that the factors $t$ drop out in the result for the transverse velocity boost. When the strings no longer overlap, the many small kicks are added to a set of gluons, which can be handled.

4In reality the strings are, of course, not distributed symmetrically, instead there are large fluctuations in the transverse positions of the strings.
by Pythia8. The $p_{\perp}$ of these gluons are chosen sufficiently small, so that they have lost their energy before the string hadronizes. This implies that their transverse momenta do not produce a jet, but just some extra $p_{\perp}$ within a rapidity range $\pm 1$ unit. The result is then not sensitive to the exact number of gluons within such an interval, as long as their transverse $p_{\perp}$ add up to the same value.

In our current implementation, the shoving is implemented as the sum of many small kicks between all pairs of string segments in different rapidity intervals spread out evenly in the available phase space. This is done in several time steps, and in each such kick, the momentum is conserved as the inserted gluons will get equal and opposite transverse kicks, while the longitudinal recoils are absorbed by the original partons in the end of the string segments.

If the string is similar to a flux tube in a type I superconductor, the field is approximately constant within a cylindrical tube. Such a picture was used in analytic studies by Abramovsky and coworkers as early as 1988 [20]. At that time there was no experimental evidence for long range azimuthal correlations, but the model was recently revived in ref. [21], and implemented as a Monte Carlo toy model. In this model the increased energy per unit string length, $\frac{\delta E}{\delta l}$, scales with the overlap area in transverse space. This gives (in our notation):

$$\frac{\delta E}{\delta l} = \Theta(R - d) \sqrt{\left(\frac{\kappa}{\pi R^2} + \frac{\lambda}{\pi R^2} A(R, d_{\perp})\right)^2 - \kappa^2}, \quad (2)$$

where $R$ is a characteristic transverse radius of the cylinder, and $A(R, d_{\perp})$ is the overlap area between two circles of radius $R$, sitting at a distance $d_{\perp}$ apart in the transverse plane. Due to the repulsion, this energy is transferred to kinetic energy, giving a transverse velocity boost to the strings.

For a type II superconductor the field strength falls off more smoothly away from a central core. Lattice calculations favour a QCD string with properties on the border between type I and II [22]. In our implementation we have assumed a smooth Gaussian form, similar to the lattice result, and furthermore added a temporal part to the evolution, as described above:

$$\frac{dp_{\perp}}{dt \cdot dl} = \frac{g\kappa}{R\sqrt{2\pi}} \exp\left(-\frac{d_{\perp}^2(t)}{2R^2}\right). \quad (3)$$

Here $g$ is a free parameter controlling the strength of the shoving, which should be of order unity. The transverse distance between the strings has acquired explicit time dependence. As discussed above, such a repulsion gives a transverse boost to the string segments, and thus extra $p_{\perp}$ to heavier hadrons. The $\langle p_{\perp}\rangle$ dependence on the hadronic mass, is an observable which is often connected to hydrodynamics, as a thermodynamic pressure would provide the same physical effect.

In figure 2 we show the $\langle p_{\perp}\rangle$ for several hadron species, divided by $\langle p_{\perp}\rangle$ for pions, in pp collisions at $\sqrt{s} = 7$ TeV. We show results for DIPSY without ropes, with ropes but no shoving, and with both ropes and shoving. By choosing a ratio, rather than the raw $\langle p_{\perp}\rangle$,
we minimize effects from small differences in the tuning of the three models. Even DIPSY without ropes shows a rise. This is expected, as lighter hadrons are more likely to be decay products – consequently with lower $\langle p_{\perp} \rangle$ – than heavy particles originating directly from the string breaking. When ropes are switched on, the $\langle p_{\perp} \rangle$ rises slightly. This is an effect of the enhanced string tension. When the string breaks, the emerging hadron obtains a $p_{\perp}$ taken from a Gaussian distribution. The width of this distribution rises with the effective string tension as [14]:

$$\tilde{\sigma}_\perp = \sigma_\perp \sqrt{\frac{\kappa}{\kappa}}. \quad (4)$$

The rise in $\langle p_{\perp} \rangle$ from ropes is, however, not directly mass dependant, while the expected mass dependence in the effect from shoving is clearly seen in figure [2].

Di-hadron correlations, which in data show a ridge effect, are of particular interest. We know that the DIPSY generator has problems reproducing the high-$p_{\perp}$ end of charged particle spectra; it generates too many hard partons. This introduces a potential problem for our string shoving model, which assumes parallel strings. To study the correlation effects on pairs of soft hadrons, we have therefore biased the generated events (pp at $\sqrt{s} = 7$ TeV) by only considering strings that span a rapidity range larger than $\Delta y = 8$, 

Figure 2: Average $p_{\perp}$ as a function of hadronic mass, for several species. Results are presented for DIPSY without ropes, DIPSY with ropes and DIPSY with ropes and shoving.
and with no partons above $p_\perp = 3$ GeV. Thus we get events with long strings, almost parallel in rapidity.

To calculate the correlations, we employ an analysis similar to the one chosen by experiments, where a signal distribution $S(\Delta \phi, \Delta \eta)$ is divided by a random background distribution, $B(\Delta \phi, \Delta \eta)$, constructed by combining particles from two different events in the same centrality class. In figure 3 we show results for $2 < \Delta \eta < 4$, for particles with transverse momentum between 0.5 and 3 GeV, using the rope model in DIPSY without (left) and with (right) shoving effects. We see the emergence of a clear "ridge" around $\Delta \eta = 0$. Although the emerging ridge is roughly of the same relative size as the one recently measured by ATLAS in high multiplicity pp events at $\sqrt{s} = 13$ TeV, we stress that the results presented here can only be taken as a qualitative proof-of-concept, due to the event bias we have introduced.

In this letter we have demonstrated that by introducing shoving in the rope hadronization mechanism implemented in DIPSY, we are able to qualitatively describe collective phenomena in pp collisions, in addition to the quantitatively correct description of hadrochemistry already provided by the ropes. We remark that the mechanism does not require any medium or thermalization, but is composed solely of microscopic interactions.

To get a better quantitative description of collective phenomena, further studies are needed. High $p_\perp$ jets are expected to rapidly leave the dense system of strings, and more work is needed for a realistic description of the interplay between the jet and the rope. Also, as it stands, the model introduces a number of parameters, e.g., the strength of the shoving, the number of rapidity intervals, and the number and size of the time steps. Although all of them have physically motivated values, their influence on the results must be studied in

\footnote{Note that the results on the mass dependence of $\langle p_\perp \rangle$ are fairly insensitive to this bias, and in figure 2 the bias was not applied.}
more detail and, in the end, they need to be tuned to data. Finally we note that, if successfully tuned to pp data, the model can be directly applied to collisions involving heavy ions in DIPSY, and in that way provide a complementary picture to the conventional hydrodynamical description of pA and AA collisions. In addition we plan to implement the model to our developing heavy ion event generator based on PYTHIA8 presented in ref. [23].

References

[1] M. Aaboud et al., “Measurements of long-range azimuthal anisotropies and associated Fourier coefficients for pp collisions at $\sqrt{s} = 5.02$ and 13 TeV and $p+$Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV with the ATLAS detector,” arXiv:1609.06213, 2016.

[2] V. Khachatryan et al., “Evidence for collectivity in pp collisions at the LHC,” arXiv:1606.06198, 2016.

[3] B. Abelev et al., “Long-range angular correlations on the near and away side in p-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV,” Phys. Lett., vol. B719, pp. 29–41, 2013.

[4] J. Adam et al., “Multiplicity-dependent enhancement of strange and multi-strange hadron production in proton-proton collisions at $\sqrt{s} = 7$ TeV,” arXiv:1606.07424, 2016.

[5] B. Andersson, G. Gustafson, and B. Söderberg, “A General Model for Jet Fragmentation,” Z. Phys., vol. C20, p. 317, 1983.

[6] B. Andersson, G. Gustafson, G. Ingelman, and T. Sjöstrand, “Parton Fragmentation and String Dynamics,” Phys. Rept., vol. 97, pp. 31–145, 1983.

[7] G. Marchesini and B. R. Webber, “Simulation of QCD Jets Including Soft Gluon Interference,” Nucl. Phys., vol. B238, pp. 1–29, 1984.

[8] T. Sjöstrand, S. Mrenna, and P. Z. Skands, “PYTHIA 6.4 Physics and Manual,” JHEP, vol. 05, p. 026, 2006.

[9] T. Sjöstrand, S. Ask, J. R. Christiansen, R. Corke, N. Desai, P. Ilten, S. Mrenna, S. Prestel, C. O. Rasmussen, and P. Z. Skands, “An Introduction to PYTHIA 8.2,” Comput. Phys. Commun., vol. 191, pp. 159–177, 2015.

[10] J. Bellm et al., “Herwig 7.0/Herwig++ 3.0 release note,” Eur. Phys. J., vol. C76, no. 4, p. 196, 2016.

[11] K. Werner, “Core-corona separation in ultra-relativistic heavy ion collisions,” Phys. Rev. Lett., vol. 98, p. 152301, 2007.
[12] T. Pierog, I. Karpenko, J. M. Katzy, E. Yatsenko, and K. Werner, “EPOS LHC: Test of collective hadronization with data measured at the CERN Large Hadron Collider,” *Phys. Rev.*, vol. C92, no. 3, p. 034906, 2015.

[13] N. Fischer and T. Sjöstrand, “Thermodynamical String Fragmentation,” *arXiv:1610.09818*, 2016.

[14] C. Bierlich, G. Gustafson, L. Lönnblad, and A. Tarasov, “Effects of Overlapping Strings in pp Collisions,” *JHEP*, vol. 03, p. 148, 2015.

[15] T. S. Biro, H. B. Nielsen, and J. Knoll, “Color Rope Model for Extreme Relativistic Heavy Ion Collisions,” *Nucl. Phys.*, vol. B245, pp. 449–468, 1984.

[16] C. Flensburg, G. Gustafson, and L. Lönnblad, “Inclusive and Exclusive Observables from Dipoles in High Energy Collisions,” *JHEP*, vol. 08, p. 103, 2011.

[17] B. Andersson and G. Gustafson, “Semiclassical Models for Gluon Jets and Leptoproduction Based on the Massless Relativistic String,” *Z. Phys.*, vol. C3, p. 223, 1980.

[18] G. S. Bali, “Casimir scaling of SU(3) static potentials,” *Phys. Rev.*, vol. D62, p. 114503, 2000.

[19] C. Bierlich and J. R. Christiansen, “Effects of color reconnection on hadron flavor observables,” *Phys. Rev.*, vol. D92, no. 9, p. 094010, 2015.

[20] V. A. Abramovsky, E. V. Gedalin, E. G. Gurvich, and O. V. Kancheli, “Long Range Azimuthal Correlations in Multiple Production Processes at High-energies,” *JETP Lett.*, vol. 47, pp. 337–339, 1988. [Pisma Zh. Eksp. Teor. Fiz.47,281(1988)].

[21] I. Altsybeev, “Mean transverse momenta correlations in hadron-hadron collisions in MC toy model with repulsing strings,” *AIP Conf. Proc.*, vol. 1701, p. 100002, 2016.

[22] P. Cea, L. Cosmai, F. Cuteri, and A. Papa, “Flux tubes in the SU(3) vacuum: London penetration depth and coherence length,” *Phys. Rev.*, vol. D89, no. 9, p. 094505, 2014.

[23] C. Bierlich, G. Gustafson, and L. Lönnblad, “Diffractive and non-diffractive wounded nucleons and final states in pA collisions,” *JHEP*, vol. 10, p. 139, 2016.