Fidelity of the quantum $\delta$-kicked accelerator

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The sensitivity of the fidelity in the kicked rotor to an acceleration is experimentally and theoretically investigated. We used a Bose-Einstein condensate exposed to a sequence of pulses from a standing light wave followed by a single reversal pulse in which the standing wave was shifted by half a wavelength. The features of the fidelity “spectrum” as a function of acceleration are presented. This work may find applications in the measurement of temperature of an ultracold atomic sample.

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in a freely falling frame using a gauge transformation. The Hamiltonian then becomes
\[
\hat{H}(\vec{N}, \vec{r}, t) = \frac{1}{2} (\vec{N} + \beta + \eta \vec{r}^2) + \phi_d \cos(\theta) \sum_{g=1}^{t} \delta(t' - q \tau),
\]
(2)

In the current fidelity experiments, the initial state $|\psi(0)\rangle$ is kicked $t$ times, each kick having a strength $\phi_d$. At the end of the $t$th kick, a single pulse with strength $t \phi_d$ is applied. We will refer to this as the “reversal kick”, and it can be implemented by shifting the standing wave by $\lambda_G/2$. Thus, the fidelity for a particular $\beta$ rotor is $F(\eta, t) = |\langle \psi(0)| U^\dagger_{\beta, \eta, t} (U_{\beta, \eta, t} |\psi(0)\rangle|^2$. Following the technique introduced in Ref. [17], the final expression for the fidelity is then given by,
\[
F(\eta, t) = \left| e^{-i\phi(\beta, \eta, t - i n_0 \pi / \Delta 1 f / G)} \right|^2,
\]
(3)
\[
J_{0}(\sqrt{(t \phi_d)^2 + \phi_d^2 \left| W(0) \right|^2 - 2 t \phi_d \Re \{W(0)\}})^2,
\]
where $p_0 = n_0 + \beta$ is the initial momentum of the plane wave, $\phi(\beta, \eta, t) = \pi \sum_{q=-\infty}^{\infty} (\beta + q \eta + \eta / 2)^2$ is the global phase, and $W(\beta, \eta) = \sum_{q=-\infty}^{\infty} e^{-i\pi(2\eta + 1 - q \eta) \pi / \Delta 1 f / G}$. In the limit $\eta \to 0$ for $t = 2$ and $\beta = 0$, the general result in Eq. (3) reduces to the special case considered in Ref. [15]. Equation (3) allows for consideration of situations in which the initial state is a mixture of plane waves. Here, this state is assumed to have a Gaussian quasimomentum distribution with a FWHM $= \Delta \beta$. For a given distribution $\rho(\beta)$ of the quasimomentum, the formula for fidelity is generalized as $F(\eta, t) = \int_{0}^{1} \rho(\beta) \langle \psi(0)| U^\dagger_{\beta, \eta, t} (U_{\beta, \eta, t} |\psi(0)\rangle|^2$, where the average is computed numerically based on Eq. (3) [17,18]. From the global phase term, $\phi(\beta, \eta, t)$, it can be seen that when $\beta \neq 0$, the phase induced by different values of $\eta$ depends not only on the magnitude of $\eta$ but also on its sign.

Our experiments to investigate this system were performed using a similar set up to that described in Refs. [15,19]. A Bose-Einstein condensate (BEC) of about 40 000 $^{87}$Rb atoms was created in the $5S_{1/2}, F = 1$ level using an all-optical trap technique. Approximately 5 ms after being released from the trap, the condensate was exposed to a pulsed horizontal standing wave. This was formed by two laser beams of wavelength $\lambda = 780$ nm, detuned 6.8 GHz to the red of the atomic transition. The direction of each beam was aligned at 53° to the vertical. With these parameters, the primary QR (half-Talbot time [20,21], $\tau = 2\pi$) occurred at multiples of $51.5 \pm 0.05$ µs. Each laser beam passed through an acoustooptic modulator driven by an arbitrary waveform generator. This enabled control of the phase, intensity, and pulse length as well as the relative frequency between the kicking beams. Adding two counterpropagating waves differing in frequency by $\Delta f$ resulted in a standing wave that moved with a velocity $v = 2\pi \Delta f / G$. Since the quasimomentum $\beta$ of the BEC relative to the standing wave is proportional to $v$, changing $\Delta f$ enabled $\beta$ to be systematically controlled.

The kicking pulse sequence is similar to that described in Ref. [15]. The atoms were exposed to a set of $t$ periodic pulses (forward pulses) each of length 1.08 µs and a kicking strength $\phi_d$ followed by the reversal pulse (standing wave displaced by $\lambda_G/2$) with a strength $t \phi_d$. We varied the intensity rather than the pulse length to change the kicking strength $\phi_d$. This was done by adjusting the amplitudes of the RF waveforms driving the kicking pulses. This ensured that the experiments were always performed in the Raman-Nath regime (the distance an atom travels during the pulse is much smaller than the spatial period of the potential). Finally, the kicked atoms were absorption imaged in a time-of-flight experiment, and the fraction of atoms that returned to the initial momentum state was determined. Experimentally, the fidelity was defined as $F = \rho_0 / \rho_0$, where $\rho_0$ is the number of atoms in the $n$th momentum order. The value of $\Delta \beta$ was varied by changing the power of the CO$_2$ laser beam, which formed the dipole trap used to realize evaporative cooling in the experiment. By changing the power of the laser for the final step in the evaporative sequence, we were able to change $\Delta \beta$.

Figure 1 shows the experimentally measured fidelity as a function of acceleration for $\ell = 1$ and initial momentum $\beta = 0.5$ due to four kicks each of strength $\phi_d \approx 0.6$ followed by a reversal kick of strength $\phi_d \approx 2.4$. Numerical simulations were performed with these experimental parameters under two different conditions. First, the black solid line is a simulation in which the reversal pulse is perfect in amplitude (amplitude $= t \phi_d$), and there are no random phase fluctuations in the standing wave that could be caused by vibrations of the optics used to form it. In order to attempt to explain the large deviation of this simulation from the experiment, we also carried out a simulation in which the above experimental imperfections were included (red dashed line). Here, we used experimentally realistic values of strength of the reversal kick ($\pm 7\%$ from

FIG. 1. (Color online) Fidelity as a function of the scaled acceleration, $\eta$, due to four kicks of strength $\phi_d \approx 0.6$ followed by a reversal kick of strength $\approx 4 \phi_d$. The black solid (red dashed) line is a numerical simulation with $\tau = 2\pi$ (i.e., $\ell = 1$), $\beta = 0.5$, and initial momentum width $\Delta \beta = 0.06 \mu m G$ without (with) effects such as vibrations and reversal phase imperfections (see more in the text). Circles are experimental data. Note that the fidelity has a rich structure with multiple resonant peaks. All fidelity measurements are $\pm 0.01$.
FIG. 2. (Color online) Plot showing the fidelity as a function of acceleration. Experimentally measured fidelity for \( \ell = 1 \) (blue diamonds), \( \ell = 2 \) (black circles), and \( \ell = 3 \) (red stars) due to four kicks of strength \( \phi_d \approx 0.6 \) followed by a reversal kick of strength \( \approx 4\phi_d \). The lines are the corresponding fidelity from numerical simulations with \( \Delta \beta = 0.06\hbar G \). Note that the horizontal axis is the real acceleration in order to show the reduction in the peak width as \( \ell \) increases.

The ideal kick strength (and a random phase variation due to vibrations of 0.02\( \pi \) per pulse. As can be seen, the fit to the experiment is quite good, leading us to believe that these effects are the most likely reason for the black curve’s poor match to the experiment at the \( \eta = 0 \) resonance. In the simulations that follow, we will employ the method used to generate the red dashed curve (with the same parameters for the experimental imperfections).

Unlike in previous work, where only the central resonance was observed [14,15], it is now possible to see that the fidelity has a more complex structure with many resonances away from \( \eta = 0 \). The validity of the theory for higher resonances at \( \ell = 2 \) and 3 was also tested, the results of which are presented in Fig. 2. Due to the longer time available for momentum state phases to evolve at the larger \( \ell \), the peaks become narrower as \( \ell \) is increased. Note that the fidelity is presented as a function of real acceleration in order to show this effect.

We also examined the dependence of the fidelity to the sign of \( \eta \) (positive and negative acceleration). Asymmetry as predicted by the above theory after Eq. (3) was observed when the \( \beta \) rotor distribution was centered at \( \beta = 0.5 \). It became more prominent as \( \Delta \beta \) was increased as shown in Fig. 3. Note that the results correspond to pulse periods, \( \tau = 4\pi \) (\( \ell = 2 \)).

The origin of the asymmetry is the different phases \( \phi(\beta, \eta, t) \) induced by the negative and positive values of acceleration. Figure 3 shows the development of the asymmetry, both in the experiment and simulations, as \( \Delta \beta \) is increased. The dashed lines are the plot of the simulations with \( \Delta \beta = 0.06\hbar G \) and \( 0.07\hbar G \) (Figs. 3(a) and 3(b), respectively). An “asymmetry visibility” defined as \([F(\eta_-) - F(\eta_+)]/[F(\eta_-) + F(\eta_+)]\) shows an almost linear scaling with the momentum width (\( \Delta \beta \leq 0.08\hbar G \)) of the cloud (see inset). Thus, measurement of the asymmetry may provide a means of determining small \( \Delta \beta \) and, hence, the temperature of ultracold atomic clouds. Interestingly, the asymmetry goes away if the initial \( \beta \) distribution is chosen centered at \( \beta = 0 \) as is possible for

FIG. 3. (Color online) Fidelity as a function of \( \eta \) for \( \tau = 4\pi \) and \( \beta = 0.5 \). Red circles and black stars represent experimental fidelity with negative and positive accelerations, respectively. Panels (a) and (b) correspond to different \( \Delta \beta \) [panel (b) with higher \( \Delta \beta \)]. The measurements were done with four kicks of strength \( \phi_d \approx 0.6 \) followed by a reversal kick of strength \( \approx 4\phi_d \). The dashed lines are the simulations for (a) \( \Delta \beta = 0.06\hbar G \) and (b) \( \Delta \beta = 0.07\hbar G \). The inset shows the asymmetry visibility (see text) as a function of \( \Delta \beta \).

FIG. 4. (Color online) Same as described in the legend of Fig. 3 but for the center of the quasimomentum distribution at \( \beta = 0 \). Note that in contrast to Fig. 3 there is no asymmetry between the positive and negative \( \eta \)’s.
In conclusion, we performed an experimental investigation on the sensitivity of the fidelity to the acceleration by exposing a BEC to a set of δ-kicked rotor optical pulses followed by a stronger reversal pulse. The experimental results and analytical theory were in good agreement with both showing the presence of multiple fidelity resonances. The width of the central fidelity resonance was found to become narrower as the pulse period increased. The importance of the position of the center of the initial momentum distribution was also explored. When the distribution was centered at some values other than zero, an asymmetry between the fidelity at positive and negative values of acceleration was observed, which became more prominent with increasing Δβ. The asymmetry was optimum for a distribution centered at β = 0.5, disappearing almost completely when the distribution was centered at β = 0. These findings can be used to determine the temperature of ultracold atoms, based on the scaling of the asymmetry with Δβ (inset in Fig. 3).

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