Simulation of stress concentration factors in combined discontinuities on flat plates

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Abstract. The objective of this paper was to determine the stress concentration factors for static load conditions (Kc) through the use of a finite elements software. The concentrators originate an increase in the calculated stress to design mechanical elements so that quantify this increase allow to obtain viable designs capable to endure the loads to which they are subjected. Among the most relevant results is validation, through statistical analysis, of use of ANSYS software for the determination of this parameter in flat plates with a central hole subjected to axial load conditions; the graphs of Kc for flat plates with combined discontinuities, central hole – groove and central hole – fillet, under axial load conditions were also obtained; the Kc value for flat plates with central hole – groove has a polynomial relation of degree 5 with the r/h parameter and decreases in so far as that this magnitude increases; additionally is also demonstrated that the concentration factor depends on the H/h relation and there is no defined trend between these two parameters. The Kc value for flat plates with central hole – fillet has a polynomial relation of degree 5 with the r/h parameter and also depends on the relation H/h without showing a defined trend between these two parameters.

1. Introduction
The distribution of stresses through a mechanical element when is subjected to axial loads, torsional or bending moments, is drastically affected by the presence of geometric discontinuities included in the design of parts or present, due to microscopic irregularities proper of manufacturing processes or material defects. The presence of these irregularities results in an increase of stress in the vicinity of the discontinuity such, that in order to obtain pieces capable of supporting the loads applied, these concentration factors must be included in the design process.

The first mathematical study of the stress concentration was published shortly after 1900; in order to work with different simple cases, it has been developed experimental methods to quantify the local stresses and from these data to determine the Kc value as the relation between this magnitude and the nominal stress calculated from simple formulas. In recent years there have begun using computer simulations based on finite elements, which allow to make mathematical models of real systems calculation, in an easiest and most economical way to modify than a prototype [1-5].

The information available in the literature is quite limited and is restricted to simple geometric configurations of stress concentrators, such as holes, fillets or grooves; recently, it has been reported more complex configurations where are involved two or more discontinuities located on different cross sections of flat plates [3, 6]. To validate the use of a finite elements software for obtaining this parameter in more complex conditions as combined concentrator, i.e. several irregularities present simultaneously on the same cross sections, represent a significant contribution to the design process of
mechanical elements, as it will allow to predict, in a better way, the maximum stresses present in the components of a machine due to the action of different loads.

2. Conceptual framework

2.1. Stress concentrators

The stress concentrators are geometrical irregularities that cause an increase in the average stress that should occur in regions near these discontinuities; the relationship between the maximum stress that occurs and the average stress to be presented is defined as stress concentration factor, $K_c$; which is determined by experimental or analytical methods and presented as graphs for easy interpretation.

A typical example of a stress concentrator is a rectangular bar with a fillet, subjected to tensile load, as shown in Figure 1(a). If the bar is cut in the cross section of the fillet, these stresses will be shown in Figure 1(b); the stress distribution along the cut surface is not uniform and reaches a maximum value in the vicinity of the fillet, where stresses increase considerably. The maximum value of the stress by those points is found by multiplying the average stress for a $K_c$ stress concentration factor [7, 8].

![Figure 1. Flat plate with fillet (a) tension load applied and (b) stress distribution in the cross section of the fillet.](image)

2.2. Stress concentration factors, $K_c$

The stress concentration factor for static load can be determined as the relation of the actual maximum stress in the discontinuity and the average stress, and is obtained through Equation (1) [9].

$$K_c = \frac{\text{Actual Maximum Stress}}{\text{Average Stress}}$$  \hspace{1cm} (1)

The average stresses are determined by the elementary equations and are defined by the type of load that is acting on the element. In the case of an axial load that causes tension or compression, this value is calculated through Equation (2) [10].

$$\sigma_{\text{average}} = \frac{\text{Axial Force}}{\text{Area of Cross Section}}$$  \hspace{1cm} (2)

The stress concentration factors are function of the discontinuity type, the geometry of the discontinuity and the type of load experienced.

2.3. Finite elements

The general idea of the finite elements method is the division of a continuum, in a set of small elements interconnected by a series of points called nodes. The equations governing the behaviour of continuum also govern the element. This way is gotten through a continuous system which is governed by a differential equation or a system of differential equations to a system with a finite number of degrees of freedom whose behaviour is modelled by a linear equations system or not [11, 12].

Onto nodes are materialized the fundamental unknowns of the problem; in the case of structural elements, these unknowns are the nodal displacements, as from these can be calculated the remaining unknowns of interest: stresses, strains, etc. The finite elements method as a mathematical formulation is relatively new; although its basic structure has been known for quite some time, in recent years it has undergone a great development due to advances in computer technology. These advances have
3. Methodology

The model used for simulation was a flat plate of AISI 1020 steel with two concentrators located on the same cross section subjected to axial load. In order to fulfill with the objective of the project, first is made a validation of the software to use, reason for which the behavior of a sheet with a central hole and subjected to axial load is simulated and the values obtained from the concentration factor are compared with the theoretical data found in the literature for the same condition of stress concentration. Later simulations for conditions of combined discontinuities were made. The methodology for the development of the Project consisted of three stages:

3.1. Simulation of sheets with central hole

The steps carried out to perform the simulations were the following:

- Definition of the model used for the simulations; in this case a static model was defined.
- Entering the mechanical properties of the material to the database software and define it as the default for simulations; the data provided were: modulus of elasticity \( E = 205 \text{GPa} \), Yield strength \( S_y = 250 \text{MPa} \) and Ultimate strength \( S_u = 460 \text{MPa} \).
- Making the graphical representation of the element to simulate; a rectangular sheet of 0.00635m thick, 0.05080m wide and 0.40000m length was drawn; Additionally grooves or fillets were performed in the middle section of the length of the sheet with the dimensions of the radius of the central hole; the relations used between the largest and smallest width of the sheets were 1.1, 1.5 and 2.0.
- Performing meshing process, in Figure 2(a) and 2(b) is shown the meshing result of the element for combinations of flat plate concentrators with central hole – fillet and central hole – groove.
- Definition of loads and the necessary supports to ensure the static equilibrium of the element.
- Running simulations and determination of equivalent Von Mises maximum stress present.

![Figure 2. Result of meshing process for flat plate with (a) central hole – fillet and (b) central hole – groove.](image)

3.2. Validation of the use of ANSYS software in obtaining stress concentration factors

The stress concentration factors were determined through Equation 1, by dividing the Von Mises equivalent stress obtained from simulations between the average stress determined through the fundamental equations of materials resistance to axial load. The use of ANSYS software in the determination of stress concentration factors is validated if, when you make a hypothesis test with the decision variable \( t \) of Student, no significant differences for a confidence level of 95% occur between the theoretical and simulated data.
3.3. Determination of $K_c$ graphs on flat plates with combined concentrators

The simulations to determine stress concentration factors for static load conditions on flat plates with combined discontinuities (central hole – groove and central hole - fillet) were made to the following relations: wide (H/h) of 1.1 to 1.5 and 2.0; and radius (hole, fillet or groove) and smaller width (r/h) de 0.02, 0.05, 0.10, 0.15, 0.20, 0.25 and 0.30. The axial loads used in the simulations were 20000, 35000 and 50000N; these forces were applied specifically at the centroid of the cross section of the plate.

4. Obtained results

4.1. Validation of ANSYS software

The results of the maximum Von Mises stresses obtained from the simulations were used to determine the stress concentration factors in flat plates with a central hole and subjected to axial load; these data are listed in Figure 3.

![Figure 3](image)

**Figure 3.** Comparison between stress concentration factor $K_c$ theoretical and simulated, for a flat plate with a central hole.

To validate the use of ANSYS software in obtaining concentration factors, it was proceeded to determine the error % between the theoretical data and those obtained through simulation; in addition a statistical analysis provides an experimental decision variable of t Student of 0.237 value which is below the 2.1448 obtained from the table of this variable for a confidence of 95%, which allows to affirm that there are no significant differences between the theoretical and simulated data.

4.2. $K_c$ stress concentration factor for flat plates with combined discontinuities

In Figures 4 and 5 can be observed the behaviour of stress concentration factors for flat plates with central hole – groove and central hole – fillet respectively; in both cases it is observed a polynomial relation of 5 degrees between $K_c$ and the relation radius/smaller width (r/h).

In the combined discontinuity central hole – groove is observed that the concentration factor decreases as the radius of the discontinuities (hole and groove) increases, consistent with the behaviour exhibited in most concentration factors with simple discontinuities; additionally it can be concluded that there is no definite trend of this parameter with respect to the relation between widths (H/h) of the flat plate, which is due to the maximum value of stress is placed in some cases in the central hole and others in the groove.
In the combined discontinuity central hole – fillet is observed that the concentration factor decreases as the radius of discontinuities (hole and fillet) increases, in almost all the relations of (H/h), as for the r/h values lower than 0.05 in H/h equal to 1 has an opposite behaviour. Additionally it is concluded that there is no definite trend of this parameter with respect to the ratio between widths (H/h) of the flat plate, which is due to the maximum value of stress is placed in some cases in the central hole and others in the fillet.

![Figure 4. Stress concentration factor in a flat plate with central hole – Groove for axial load.](image1)

![Figure 5. Stress concentration factor in flat plate with central hole – fillet for axial load.](image2)

5. Conclusions
The finite elements software can be used as a tool to determine graphs of stress concentration factors as for flat plates with a central hole and subjected to axial load, it can be statistically demonstrated that there are no significant differences between the theoretical data and the obtained data through simulation. The maximum error calculated between these data was 5.42%, when the geometrical relation between the hole diameter and width of sheet is 0.04.

In flat plates with a central hole – Groove is observed that the concentration factor decreases as the radius of discontinuities increases and there is no definite trend of this parameter with respect to the
relation between widths (H/h) of the flat plate, which is due to the maximum value of stress in some cases is located in the central hole and others in the Groove.

In flat plates with central hole – fillet is observed that the concentration factor decreases as the radius of discontinuities increases in almost all relations (H/h), as the values r/h smaller than 0.05 in H/h equal to 1 has an opposite behaviour. Additionally it is concluded that there is no definite trend of this parameter with respect to the relation between widths (H/h) of the flat plate.

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