Schwarzschild Black Holes from Brane-Antibrane Pairs

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Abstract

We show that D=4 Schwarzschild black holes can arise from a doublet of Euclidean $D3$-$\overline{D3}$ pairs embedded in D=10 Lorentzian spacetime. By starting from a D=10 type IIB supergravity description for the $D3 - \overline{D3}$ pairs and wrapping one of them over an external 2-sphere, we derive all vacuum solutions compatible with the symmetry of the problem. Analysing under what condition a Euclidean brane configuration embedded in a Lorentzian spacetime can lead to a time-independent spacetime, enables us to single out the embedded D=4 Schwarzschild spacetime as the unique solution generated by the $D3-\overline{D3}$ pairs. In particular we argue on account of energy-conservation that time-independent solutions arising from isolated Euclidean branes require those branes to sit at event horizons. In combination with previous work this self-dual brane-antibrane origin of the black hole allows for a microscopic counting of its Bekenstein-Hawking entropy. Finally we indicate how Hawking-radiation can be understood from the associated tachyon condensation process.

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1 Introduction and Summary

One of the most fascinating problems in fundamental physics today is to find a proper set of microstates for spacetimes possessing an event horizon such that the counting of their number matches the Bekenstein-Hawking (BH) entropy

\[ S_{BH} = \frac{A_H}{4G_4}, \]  

where \( A_H \) is the area of the spacetime’s event horizon. In the framework of String-Theory such a set of entropy-carrying microscopic states could be identified mostly for extremal black holes and near-extremal ones (for some recent reviews see [1]). The basic idea was to count microscopic states in the weakly coupled regime and to export the result to the strongly coupled regime by using supersymmetry. This made it difficult to employ the same strategy for highly non-extreme spacetimes such as the Schwarzschild black hole or the de Sitter universe. One of the approaches to deal with the Schwarzschild black hole was to use certain boost-transformations and/or the T- and S-dualities of String-Theory and thereby to achieve an entropy-preserving mapping to a spacetime whose entropy-counting was under control, e.g. the three-dimensional BTZ black hole [2] or near-extremal brane configurations [3]. During this process one however also had to compactify time which might be justified from the Euclidean approach to black hole thermodynamics. Other approaches within String-Theory to microscopically derive the D=4 Schwarzschild BH-entropy had been undertaken in [4], [5], [6], [7], [8].

Finally, of course, one would like to know directly the microscopic states in the strongly-coupled regime making up the black hole. Are they located at or around the horizon like in the entanglement entropy [9], [10], thermal atmosphere [11] or “shape of the horizon” [12] approaches? Or do they live elsewhere in the interior or exterior region of a black hole? Unfortunately the answer to this question is obscured by the indirect method of counting states as has been pointed out in [13]. Another puzzling aspect related to the BH-entropy is its universality. One would like to know why it universally applies not only to black holes but also to other spacetimes possessing e.g. cosmological event horizons.

In an approach to understand the origin and the universality of the BH-entropy for spacetimes with spherical event horizons from strongly coupled (\( g_s \approx 1 \)) String-/M-Theory, it was proposed in [14] to consider doublets of orthogonal Euclidean dual brane pairs. One brane of each pair had to wrap a sphere \( S^2 \) situated in the D=4 external spacetime while the complete 6- resp. 7-dimensional (for String- resp. M-Theory) internal
part of spacetime was wrapped by the remaining portion of the brane plus its dual brane partner. To connect this brane picture to a D=4 spacetime with spherical event horizon $S^2_H$ whose associated BH-entropy we would like to derive at a microscopic level, it was proposed the following. The brane configuration would act as a supergravity source capable of producing the D=4 spacetime in the external part of its D=10 metric solution with $S^2 \equiv S^2_H$ identified. Through the identification of a Euclidean brane’s tension as the inverse of a fundamental smallest volume unit it was then possible to consider chain-states on the discretized branes’ worldvolumes (see [13] for other indications of a discretized worldvolume at strong coupling) and by counting their number to derive the D=4 BH-entropy plus its logarithmic correction. This mechanism works for all D=4 spacetimes as long as we can identify a doublet of dual Euclidean branes with the D=4 spacetime under investigation in the prescribed manner. The main purpose of this paper is to provide such an identification for the case of the D=4 Schwarzschild black hole.

The organisation of the paper is as follows. In section 2 we will start by analysing which branes and their duals might qualify for a potential description of the D=4 Schwarzschild spacetime. Non-extremality and charge-lessness of the latter leads us to consider an equal amount of branes as antibranes while the property of being a vacuum solution (in the Einstein equations sense) brings us to the non-dilatonic $D3$-$\overline{D3}$ pairs. We then describe the D=10 geometry as appropriate for an exterior solution with the branes-antibranes acting as the gravitational source. In section 3 we derive all D=10 vacuum solutions which respect the imposed symmetry – among them a simple embedding of the D=4 Schwarzschild spacetime into D=10 spacetime. The Euclidean nature of our considered branes and antibranes embedded in a D=10 Lorentzian spacetime implies however a further constraint which we will discuss in section 5. There we will argue that an isolated Euclidean brane (or a finite number of them) in a Lorentzian embedding violates energy-conservation and leads to a time-dependence unless it is placed at an event horizon. On account of an infinite time-dilatation at the horizon, energy is conserved from an outside observers point of view. At the same time the ensuing geometry becomes time-independent. Demanding the existence of an event horizon then singles out uniquely the embedded Schwarzschild solution as representing the exterior geometry of our $D3$-$\overline{D3}$ doublet. Therefore we might now invoke the results of [14] to derive the BH-entropy of the Schwarzschild black hole and its logarithmic corrections from counting chain-states on the branes-antibranes’ worldvolume. We end with section 6 by speculating on the origin of the Hawking-radiation from the tachyon condensation point of view related to the $D3$-$\overline{D3}$ doublet.
2 The Brane-Antibrane Pair Configuration

In the following we will assume a type II String-Theory on a Lorentzian spacetime manifold $\mathcal{M}^{1,3} \times \mathcal{M}^6$ where the internal space $\mathcal{M}^6$ is taken to be compact.

The D=4 Schwarzschild spacetime describes a non-rotating black hole which bears no charges under any long-range gauge-field and is uniquely characterized by its mass $M$. Moreover, it breaks all supersymmetry if included as a background in String-Theory. In order to obtain this spacetime from a doublet of dual brane pairs (the motivation for this comes from the fact that such brane configurations allow for a determination of the BH-entropy of the associated D=4 spacetime without the need for supersymmetry\[14\]) of type II String-Theory, natural candidates are the non-supersymmetric Euclidean brane-antibrane configurations\[2\]

$$(Dp, D(6 - p)) + (\overline{Dp}, D(6 - p)) \quad (2)$$

or the fundamental string - NS5-brane configurations

$$(F1, NS5) + (\overline{F1}, \overline{NS5}) , \quad (NS5, F1) + (\overline{NS5}, \overline{F1}) \quad (3)$$

where the first and second component in brackets are mutually orthogonal. Moreover it is understood that the first component in each bracket wraps an external $S^2$ contained in $\mathcal{M}^{1,3}$ while the remaining worldvolume coordinates of both components cover the internal $\mathcal{M}^6$ completely.\[14\] Because we will drop the F1-NS5 pairs shortly, let us concentrate on the $Dp$-branes subsequently.

Setting all background tensor fields and the world-volume gauge-field strength to zero except for the RR $(p + 1)$-form $C_{p+1}$, the Euclidean $Dp$-brane action in Einstein-frame is

$$S_{Dp} = T_{Dp} \int d^{p+1}x e^{(\frac{p-3}{4})\Phi} \sqrt{\det g_{Dp}} + \mu_{Dp} \int C_{p+1} \quad (4)$$

with

$$T_{Dp} = \mu_{Dp} = \frac{1}{(2\pi)^p \alpha'^{(\frac{p+1}{2})}} \quad (5)$$

the tension and charge of the brane, $\Phi$ the dilaton and $g_{Dp}$ the induced metric on the brane. For the Euclidean antibrane $\overline{Dp}$ the sign of the second term is replaced by a minus.\[2\]See \[16\] for supergravity solutions which interpolate between Lorentzian brane-antibrane pairs and Schwarzschild black holes.
These specify the sources of our system. To keep things simple let us start with a single \(Dp\)-brane and add its dual plus antibrane partners step by step.

Together with the D=10 Einstein-frame supergravity bulk action for this background (with a vanishing NS-NS 2-form potential \(B\), in particular the Chern-Simons terms of type II supergravity will be absent)

\[
S_{SG} = \frac{1}{2\kappa_{10}^2} \int \left( eR - \frac{1}{2} d\Phi \wedge \ast d\Phi - \frac{1}{2} e^{a\Phi} F \wedge \ast F \right),
\]

where \(F = dC_{p+1}\), \(e = \sqrt{-\det g}\) and \(a\) is a constant depending on the rank of \(C_{p+1}\), this results in the following coupled Einstein-, Maxwell- and dilaton equations

\[
R_{AB} - \frac{1}{2} R g_{AB} = \frac{e^{a\Phi}}{2(p+2)!} \left( (p+2) F_{AA_2...A_{p+2}} F_{B_2...B_{p+2}} - \frac{1}{2} g_{AB} F^2 \right) \\
+ \kappa_{10}^2 T_{Dp} e^{\left( \frac{p+1}{4} - p \right) \Phi} \delta^9 - p (\bar{x}_D^p - \bar{x}_{Dp,0}) \frac{\sqrt{\det g_{Dp}}}{\sqrt{-\det g}} g_{ij} \delta^i_j \delta^j_B + \frac{1}{2} \left( \partial_A \Phi \partial_B \Phi - \frac{1}{2} g_{AB} \partial_C \Phi \partial^C \Phi \right),
\]

\[
e^{a\Phi} d^* F = 2\kappa_{10}^2 (-1)^p \ast j_{Dp},
\]

\[
\Box \Phi = \frac{a}{2(p+2)!} e^{a\Phi} F^2 + 2\kappa_{10}^2 \delta^9 - p (\bar{x}_D^p - \bar{x}_{Dp,0}) T_{Dp} e^{\left( \frac{p-1}{4} \right) \Phi} \frac{\sqrt{\det g_{Dp}}}{\sqrt{-\det g}},
\]

where \(i, j\) are indices along the brane, \(A, B, \ldots\) are D=10 bulk indices, \(\bar{x}_D^p\) denote all coordinates transverse to the brane and \(\bar{x}_{Dp,0}\) give the brane localisation. Furthermore, \(F^2 \equiv F_{A_1...A_{p+2}} F^{A_1...A_{p+2}}\) and the brane-current \(j_{Dp}\) is given by

\[
j_{Dp} = \mu_{Dp} \delta^{10-(p+1)} (\bar{x}_D^p - \bar{x}_{Dp,0}) \frac{\sqrt{\det g_{Dp}}}{\sqrt{-\det g}} \omega_{Dp}
\]

with \(\omega_{Dp} = \sqrt{\det g_{Dp}} dx^{i_1} \wedge \ldots \wedge dx^{i_{p+1}}\) the positively oriented metric volume element on the \(Dp\)-brane’s worldvolume.

If we now add an anti-\(\overline{Dp}\)-brane which coincides with the \(Dp\)-brane then due to the opposite RR-charges \(F\) vanishes. This is consistent with the Bianchi-identity which receives a magnetic source term from the dual \(D(6 - p)\)-brane

\[
e^{a\Phi} d F = -2\kappa_{10}^2 \ast j_{D(6-p)}
\]

but which also gets compensated by adding the anti-\(\overline{D(6 - p)}\)-brane. Thus we can neglect the field-strength \(F\) in the above field equations (and similarly the hitherto suppressed dual field-strength associated with the dual brane \(D(6 - p)\)). Moreover, we are interested
In the solution exterior to the sources, i.e. for $\vec{x}^{Dp} \neq \vec{x}^{Dp}_{\perp 0}$, describing the long-range fields. Hence we can also drop the brane source terms which means that we are left with

$$R_{AB} = \frac{1}{2} \partial_A \Phi \partial_B \Phi$$

(12)

$$\Box \Phi = 0$$

(13)

From the Einstein equations we recognize that we have to demand a constant dilaton in order to obtain a vacuum solution like Schwarzschild (there might also be dilatonic brane-antibrane configurations with $\Phi = \Phi(x_4, \ldots, x_9)$ such that $R_{\mu \nu} = 0$ and $R_{mn} \neq 0$ ($\mu, \nu = 0, \ldots, 3$; $m, n = 4, \ldots, 9$) but these will not be studied here). Without loss of generality we can then set $\Phi = 0$ and end up with the D=10 vacuum Einstein equations

$$R_{AB} = 0$$

(14)

It is well-known that the only non-dilatonic $Dp$-brane is the self-dual $D3$-brane. Thus our dual brane pair doublet will consist of two $D3-\overline{D3}$ Euclidean brane-antibrane pairs located at some finite value of $r$, the D=4 radial distance, as depicted in fig.1. This non-dilatonic property also excludes the second possibility (3). One might think that a brane-antibrane pair would annihilate itself within a very short lifetime. How this lifetime gets infinitely extended will be addressed later on in section 4. It might also be of interest to study the effect on the geometry when more background fluxes are switched on (see e.g. [17]) and lower-dimensional branes are induced as suggested by K-Theory [18]. However, this is beyond the scope of the present paper.

In order to solve the vacuum equations in the exterior region of the sources, let us make an Ansatz for the D=10 geometry reflecting the symmetry-properties of the two

| $D3$ | $\overline{D3}$ | $D3$ | $\overline{D3}$ |
|-----|----------------|-----|----------------|
| $t$ | $r$ | $\theta$ | $\phi$ | 4 | 5 | 6 | 7 | 8 | 9 |

Figure 1: The two Euclidean $D3-\overline{D3}$ brane-antibrane pairs are oriented along the directions marked by dots. The coordinates $t, r, \theta, \phi$ describe the D=4 portion with $\theta, \phi$ describing the $S^2$. The whole configuration is located at some common fixed value of $r$. 

In order to solve the vacuum equations in the exterior region of the sources, let us make an Ansatz for the D=10 geometry reflecting the symmetry-properties of the two
Euclidean $D3\overline{D3}$ pairs. Searching for a time-independent static solution (actually time-independence in the context of Euclidean branes poses a further constraint on the location of the brane to be imposed later on) and noting that the only spatial coordinate transverse to all branes is the D=4 radial coordinate $r$, the Ansatz should be

$$ds^2 = -e^{2\lambda(r)}dt^2 + e^{2\nu(r)}dr^2 + (r^2d\Omega^2 + dx_4^2 + dx_5^2) + e^{2\psi(r)}(dx_6^2 + \ldots + dx_{9}^2) ,$$

with $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ the metric of a unit two-sphere. The fact that no further $r$-dependent factor multiplies the $\theta, \phi, x_4, x_5$ worldvolume of the one $D3\overline{D3}$ pair is because such a factor can be set equal to one by a redefinition of $r$ and further noticing that the $D3\overline{D3}$ worldvolume has to be multiplied as a whole by a common factor.

### 3 The Vacuum Solutions

The D=10 Einstein vacuum equations for this metric deliver the following second order ordinary differential equations (ODE’s)

$$\Lambda' + \Lambda^2 - \Lambda N + 2\frac{\Lambda}{r} + 4\Lambda\Psi = 0$$

$$\Lambda' + \Lambda^2 - \Lambda N - 2\frac{N}{r} + 4\Psi' + 4\Psi^2 - 4N\Psi = 0$$

$$\Lambda - N + \frac{1}{r}(1 - e^{2\nu}) + 4\Psi = 0$$

$$\Lambda\Psi + \Psi' + 4\Psi^2 - N\Psi + 2\frac{\Psi}{r} = 0$$

where we have defined for convenience

$$\Lambda = \lambda', \quad N = \nu', \quad \Psi = \psi'.$$

Together with these ODE’s we have to choose an appropriate boundary condition at some value of $r$. We should expect that if we depart sufficiently far from the brane configuration towards larger $r$ that the influence of the gravitational source becomes smaller and smaller until finally the D=10 spacetime should approach flat Minkowski spacetime. Thus our boundary conditions will be asymptotic flatness at $r \to \infty$

$$\lambda \to 0 , \quad \nu \to 0 , \quad \psi \to 0 .$$

Our task in the following will be first to find all solutions to this set of vacuum equations which are compatible with asymptotic flatness and then second to select that
subclass of solutions which actually qualifies as an exterior solution sourced by the two Euclidean $D3 - \overline{D3}$ pairs by examining under what condition Euclidean branes can lead to a stationary solution in coordinates adapted to an asymptotic observer.

To start with, one derives from the linear combination $(16) - (18) \times \Lambda$ that

$$
\Lambda' + \frac{\Lambda}{r}(1 + e^{2\nu}) = 0 \quad (22)
$$

while the combination $(19) - (18) \times \Psi$ leads to

$$
\Psi' + \frac{\Psi}{r}(1 + e^{2\nu}) = 0. \quad (23)
$$

From the formal solution of these two equations for $\Lambda$ and $\Psi$ one deduces that we face four different cases. Either $\Lambda$ or $\Psi$ or both are equal to zero or otherwise they have to be proportional to each other. We will now study these different cases in detail.

### 3.1 The Case $\Lambda = \Psi = 0$

Let us begin with the simplest case of $\Psi = \Lambda = 0$. It is easy to see that here the Einstein vacuum equations produce just the D=10 flat Minkowski metric

$$
ds^2 = -dt^2 + dr^2 + r^2d\Omega^2 + dx_4^2 + dx_5^2 + \ldots + dx_9^2, \quad (24)
$$

with spacetime topology $M^{1,3} \times T^6$ due to compactness of the internal space.

### 3.2 The Case $\Lambda \neq 0, \Psi = 0$

This is the next easiest case which will bring us to the D=4 Schwarzschild solution. Because $\Psi = 0$, $\psi$ will be constant and can be set to zero by an appropriate scaling of the $x_6, \ldots, x_9$ coordinates. By subtracting $(16) - (17)$, we obtain $\Lambda = -N$ which amounts to the relation

$$
\lambda = -\nu \quad (25)
$$

as an additive integration constant again can be absorbed into a redefinition of the coordinates. With this input, $(18)$ leads to the ODE for $\nu$

$$
\nu' = \frac{1}{2r}(1 - e^{2\nu}) \quad (26)
$$
which is solved by

\[ e^{2\nu} = \left(1 \pm \frac{r_0}{r}\right)^{-1} \]  

(27)

with a positive \( r_0 \). Altogether, we end up with an embedding (“lift”) of the D=4 Schwarzschild solution into D=10 spacetime

\[ ds^2 = -(1 \pm \frac{r_0}{r}) dt^2 + (1 \pm \frac{r_0}{r})^{-1} dr^2 + r^2 d\Omega^2 + dx_4^2 + dx_5^2 + dx_6^2 + \ldots + dx_9^2. \]  

(28)

Again the internal six-dimensional space is topologically \( T^6 \). The solution with the plus-sign would correspond (if one could smooth out its naked singularity) to a source with negative mass. It cannot describe the \( D3-\overline{D3} \) geometry as all (anti-)branes involve only positive tensions and will therefore be ruled out.

An alternative characterisation of this spacetime is as a black D6-brane in its ultra non-extreme limit. In this limit the black D6-brane looses its magnetic Ramond-Ramond 2-form charge while the dilaton becomes constant thus giving a non-dilatonic vacuum solution.

### 3.3 The Case \( \Lambda = 0, \Psi \neq 0 \)

In this case \( \lambda \) has to be constant and without loss of generality can be set to \( \lambda = 0 \). It therefore remains to determine \( \nu \) and \( \psi \).

Of the vacuum Einstein equations, \( \text{(13)} \) is identically satisfied. The remaining three equations can be chosen conveniently as \( \text{(17)} - 4 \times \text{(19)}, \text{(18)} \) and \( \text{(19)}/\Psi \)

\[ N = -2\Psi(3\Psi r + 2) \]  

(29)

\[ N = 4\Psi + \frac{1}{r}(1 - e^{2\nu}) \]  

(30)

\[ N = (\ln(r_1\Psi))' + 4\Psi + \frac{2}{r}. \]  

(31)

Here a positive constant \( r_1 \) with dimension of a length has been introduced to keep the function inside the logarithm dimensionless. Clearly we have an overconstrained system with three equations in two unknowns. The last equation can be integrated directly to yield after exponentiation

\[ (e^{4\psi})' = 4 \frac{r_1}{r^2} e^\nu, \]  

(32)
where the exponentiated integration constant can be identified with $r_1$. From this equation it is easy to see that $\Psi \geq 0$. Consequently (29) demands that $N \leq 0$ and therefore from (30) it follows that
\begin{equation}
 e^{2\nu} \geq 1 .
\end{equation}

Unfortunately eliminating $N$ in the above equations and solving for $\Psi$ leads to rather complicated differential equations. Let us therefore proceed by using the first two equations to eliminate $\Psi$ and obtain a differential equation for $\nu$. More specifically we solve (30) for $\Psi$ and with this result eliminate $\Psi$ in (29). The ensuing quadratic equation in $N = \nu'$ possesses the solution
\begin{equation}
 -\frac{3}{8} \nu' r = H(\nu)(H(\nu) \pm 1) ,
\end{equation}
where
\begin{equation}
 H(\nu) \equiv \sqrt{5 + 3e^{2\nu}/\sqrt{8}} .
\end{equation}
Notice that $H \geq 1$ due to (33). In order to resolve the sign ambiguity let us consider its asymptotics. From the asymptotic boundary conditions (21) we infer that $H \to 1$. Thus we obtain from (34) for $\nu$ the asymptotic behaviour $\nu \simeq -\frac{8}{3}(1 \pm 1) \ln r$ which only gives $\nu \to 0$ if we choose the minus-sign in (34).

Employing the chain-rule the above ODE for $\nu$ translates into an ODE for $H(r) \equiv H(\nu(r))$
\begin{equation}
 \frac{H'(r)}{(H(r) - 1)(5 - 8H^2(r))} = \frac{1}{3r} .
\end{equation}
By standard integration techniques this ODE can be shown to possess the following implicit solution
\begin{equation}
 \frac{1}{(H(r) - 1)(H(r) + 5)^{1/2}} = \frac{r}{r_2} ,
\end{equation}
with positive $r_2$. By rewriting (33) as
\begin{equation}
 e^{2\nu} = \frac{1}{3}(8H^2(r) - 5)
\end{equation}
we gain the solution for the radial component of the metric.
Next we have to determine $\psi$ which we will do by starting from (32). Using (38), (36) and applying the chain-rule to rewrite the derivative as a derivative with respect to $H$, we obtain

$$\frac{d(e^{4\psi})}{dH} = -\frac{12r_1}{\sqrt{3r(H-1)}\sqrt{8H^2 - 5}}.$$ (39)

The $r$ dependence which hinders a straightforward integration can be transformed into a $H$ dependence by means of the solution (37), giving the ODE

$$\frac{d(e^{4\psi})}{dH} = -\frac{\sqrt{6r_1}(H + \sqrt{\frac{5}{8}}\sqrt{\frac{7}{5}})}{r_2(H - \sqrt{\frac{5}{8}}\sqrt{\frac{7}{5}})}.$$ (40)

By an auxiliary transformation, $H = \sqrt{\frac{5}{2}}(\frac{1}{(x-1)} + \frac{1}{2})$, of the integration variable $H$ to a new variable $x$, this ODE can trivially be integrated with the result

$$e^{2\psi} = \left(\sqrt{6r_1} - h(1) - h(H(r))\right)^{\frac{1}{2}} + 1, \quad h(y) = \frac{\sqrt{\frac{2}{5}y + \frac{1}{2}} - \sqrt{\frac{7}{5}}}{\sqrt{\frac{2}{5}y - \frac{1}{2}}}.$$ (41)

However, we will now show that the overconstrained system of equations (29),(30),(31) requires a much more stringent condition on $H(r)$. Namely with help of (36) it turns out that (23) or equivalently the difference (31) - (30) amounts to

$$(H(r) - 1)\left(\frac{r_2 + \sqrt{6r_1}h(1)}{r_2 e^{4\psi}}\right) = 0,$$ (42)

which requires setting $H(r) \equiv 1$. This would render the metric trivial and therefore violates our assumption that $\Psi \neq 0$. Hence we can conclude that there is no solution for the case $\Lambda = 0, \Psi \neq 0$.

### 3.4 The Case $\Lambda \neq 0, \Psi \neq 0, \Psi = c\Lambda, c \neq -\frac{1}{4}, -\frac{2}{3}, 0$

Due to the proportionality of $\Psi$ and $\Lambda$ the D=10 Einstein vacuum equations reduce again to a system of three equations in two unknowns

$$\Lambda' + a\Lambda^2 + \Lambda\left(\frac{2}{r} - N\right) = 0$$ (43)

$$a\Lambda' + (1 + 4c^2)\Lambda^2 - a\Lambda N - \frac{2}{r}N = 0$$ (44)

$$\Lambda' + \frac{1}{r}(1 + e^{2\nu})\Lambda = 0.$$ (45)
with the constant coefficients

\[ a = 1 + 4c, \quad b = 4c(2 + 3c). \] (46)

The combination \(2a^2 - b > 0\) which will appear below is positive for any value of \(c\). Moreover, \(c \neq 0\) implies \(a \neq 0\) and \(b \neq 0\) if we exclude the case \(c = -\frac{2}{3}\) which will be treated separately.

These equations can be brought into a more manageable form by taking the linear combinations \(a \times (43) - (44)\) and \((43) - (43))/\Lambda

\[
\begin{align*}
\frac{1}{r}b\Lambda^2 + \frac{2}{r}(a\Lambda + N) &= 0 \\
\frac{1}{r}a\Lambda &= \frac{1}{r}(e^{2\nu} - 1) + N \quad \text{(48)} \\
\Lambda' + \frac{1}{r}(1 + e^{2\nu})\Lambda &= 0. \\
\text{(49)}
\end{align*}
\]

Now we can multiply the first equation by \(a^2\) and eliminate \(a\Lambda\) in it by substituting the second equation. Some minor manipulations then bring (47) into the following ODE for \(\nu\)

\[
(br\nu' + k(\nu))^2 = 2a^2k(\nu), \\
\text{(50)}
\]

where

\[
k(\nu) \equiv 2a^2 + b(e^{2\nu} - 1). \\
\text{(51)}
\]

Due to the square on the lhs also the rhs has to be non-negative. Thus it proves convenient to define a new non-negative function

\[
K(\nu) = \sqrt{k(\nu)} \\
\text{(52)}
\]

such that the ODE for \(\nu\) becomes

\[
br\nu' = -K(\nu)(K(\nu) \pm \sqrt{2}|a|). \\
\text{(53)}
\]

The sign-ambiguity in this equation which results from taking the square-root can be resolved once more by examining its asymptotic behaviour. At \(r \to \infty\) the asymptotic boundary condition demands \(\nu \to 0\) and thus that \(K(\nu)\) approaches

\[
K(\nu) \simeq \sqrt{2}|a|. \\
\text{(54)}
\]

\(^3\text{Notice that the parameter } a \text{ defined here has nothing to do with the constant } a \text{ appearing in (1)–(3).}\)
Hence we can deduce from (53) that in the asymptotic regime $b\nu \simeq 2a^2(1 \pm 1) \ln r$. To satisfy the asymptotic boundary condition for $\nu$ we therefore have to choose the minus sign in (53). We can now rewrite (53) as an ODE for $K(r) \equiv K(\nu(r))$ by employing the chain-rule

$$\frac{K'(r)}{(\sqrt{2}|a| - K(r))(K^2(r) - 2a^2 + b)} = \frac{1}{br}.$$  

(55)

By standard integration techniques this leads to the following implicit solution for $K(r)$

$$\frac{1}{|\sqrt{2}|a| - K(r)|} \frac{|\sqrt{2}a^2 - b - K(r)|^{\frac{|a|}{\sqrt{2}(2a^2 - b)}} + \frac{1}{2}}{(\sqrt{2}a^2 - b + K(r))^{|\frac{|a|}{\sqrt{2}(2a^2 - b)}} - \frac{1}{2}} = \frac{r}{r_3}$$  

(56)

with an integration constant $r_3 > 0$. Knowing $K(r)$ the radial component of the metric is then obtained simply by rewriting the defining equation for $K(r)$ as

$$e^{2\nu} = \frac{K^2(r) - 2a^2 + b}{b}.$$  

(57)

It remains to determine $\lambda$ and $\psi$. For the former we use (48) and substitute $e^{2\nu}$ in it through (57). This gives

$$a\lambda' = \frac{1}{r} \left( \frac{K^2 - 2a^2}{b} \right) + \nu'.$$  

(58)

We will now bring the term in brackets into a form which can be easily integrated. To this aim we use (33) which allows us to bring (38) into the form

$$a\lambda' = \frac{K'}{\sqrt{2}|a| - K} - \frac{1}{r} + \nu'$$  

(59)

which can be integrated directly to give

$$\lambda = \frac{-1}{a} \ln |(\sqrt{2}|a| - K)\frac{r}{r_4}| + \nu \frac{r_4}{a}$$  

(60)

with an integration constant $r_4 > 0$. Once more using (57) to express $\nu$ in terms of $K$, we obtain the time-component of the metric

$$e^{2\lambda} = \left( \left( \frac{r_4}{r} \right)^2 \frac{K^2(r) - 2a^2 + b}{b(K(r) - \sqrt{2}|a|)^2} \right)^{\frac{1}{a}}.$$  

(61)

Finally, since we have $\Psi = c\Lambda$, the internal metric component $e^{2\psi}$ is related to the time-component via

$$e^{2\psi} = (e^{2\lambda})^c$$  

(62)
where we used the freedom of scaling coordinates to drop the integration constant.

It is not hard to check that the results obtained for \( e^{2\lambda}, e^{2\nu}, e^{2\psi} \) indeed satisfy all three equations of the overconstrained system \((17),(18),(19)\). Consequently we have the following class of vacuum solutions in this case

\[
ds^2 = - \left( \frac{r_4}{r} \right)^2 \frac{K^2(r) - 2a^2 + b}{b(K(r) - \sqrt{2|a|})^2} dt^2 + \left( \frac{K^2(r) - 2a^2 + b}{b} \right) dr^2 + (r^2d\Omega^2 + dx_4^2 + dx_5^2) + \left( \frac{r_4}{r} \right)^2 \frac{K^2(r) - 2a^2 + b}{b(K(r) - \sqrt{2|a|})^2} (dx_6^2 + \ldots + dx_9^2) \tag{63}
\]

provided that \( a \neq 0, b \neq 0 \).

We should add that in order to obtain an asymptotically flat vacuum solution one has to relate the two positive constants \( r_3 \) and \( r_4 \) in the following way

\[
r_4 = r_3 \left| \sqrt{2a^2 - b} - \sqrt{2|a|} \right| \frac{\sqrt{\sqrt{2(2a^2 - b)}}}{\sqrt{2(2a^2 - b) - b}} \tag{64}
\]

which can be seen from an explicit derivation of the weak-field limit of \((63)\).

### 3.5 The Case \( \Lambda \neq 0, \Psi \neq 0, \Psi = -\frac{1}{4}\Lambda \)

The case \( c = -\frac{1}{4} \) corresponds to the situation where \( a = 0 \) which we had omitted previously. Here the vacuum Einstein equations reduce to the equations

\[
\Lambda' + \Lambda\left( \frac{2}{r} - N \right) = 0 \tag{65}
\]

\[
\frac{5}{4}\Lambda^2 - \frac{2}{r} N = 0 \tag{66}
\]

\[
\Lambda' + \frac{1}{r}(1 + e^{2\nu})\Lambda = 0 \tag{67}
\]

Subtracting the first from the third equation leads to

\[
\nu' = \frac{1}{r}(1 - e^{2\nu}) \tag{68}
\]

which is solved by

\[
e^{2\nu} = (1 \pm \left( \frac{r_5}{r} \right)^2)^{-1} \tag{69}
\]
with $r_5$ a positive integration constant. Notice that the rhs of (58) is twice as large as in the Schwarzschild case and consequently leads to the quadratic dependence on $r$ instead of a linear dependence as for Schwarzschild.

Next, we determine $\lambda$ from (66) by using the solution for $\nu$

$$\lambda = \pm 2\sqrt{\frac{2}{5}} \ln \left( \frac{r}{r_5 + \sqrt{r_5^2 + r^2}} \right). \quad (70)$$

Notice that the sign-ambiguity in front is unrelated to the one under the square-root which coincides with the one of (69). By appealing to a scaling of $t$, we have suppressed the integration constant. Thus we obtain four different solutions. However those with a minus-sign under the square-root have a restricted range $r \leq r_5$ and can therefore (in these coordinates) not reach spatial infinity. Because there is no reason why without any further gravitational sources spacetime outside the brane-antibrane configuration should abruptly come to an end, we will discard these as exterior solutions.

Finally, we have $\psi = -\frac{1}{4} \lambda$, once more suppressing the integration constant and therefore arrive at the following vacuum solution (one can check that it satisfies the complete set of overconstrained equations (65),(66),(67))

$$ds^2 = -\left( \frac{r}{r_5 + \sqrt{r_5^2 + r^2}} \right)^{\pm 4\sqrt{\frac{2}{5}}} dt^2 + (1 + (\frac{r_5}{r})^2)^{-1} dr^2 + (r^2 d\Omega^2 + dx_4^2 + dx_5^2)
+ \left( \frac{r}{r_5 + \sqrt{r_5^2 + r^2}} \right)^{\mp \sqrt{\frac{2}{5}}} (dx_6^2 + \ldots + dx_9^2). \quad (71)$$

### 3.6 The Case $\Lambda \neq 0$, $\Psi \neq 0$, $\Psi = -\frac{2}{3} \Lambda$

This is the second case which we had left out before and it corresponds to setting $b = 0$. In this case the Einstein equations (47),(48),(49) amount to

$$\frac{5}{3} \Lambda = N \quad (72)$$
$$-\frac{5}{3} \Lambda = N + \frac{1}{r} (e^{2\nu} - 1) \quad (73)$$
$$\Lambda' + \frac{1}{r} (1 + e^{2\nu}) \Lambda = 0. \quad (74)$$

The first equation leads straight to $\frac{5}{3} \lambda = \nu$ while adding (72) and (73) gives

$$\nu' = \frac{1}{2r} (1 - e^{2\nu}). \quad (75)$$
This is the same ODE as in the Schwarzschild case and gets solved by

\[ e^{2\nu} = \left(1 \pm \frac{r_6}{r}\right)^{-1}, \tag{76} \]

where \( r_6 > 0 \). With \( \lambda = \frac{2}{3} \nu \) and \( \psi = -\frac{2}{3} \lambda \) one can check that also (74) is satisfied and one finds the vacuum solution

\[
\begin{align*}
\begin{aligned}
\frac{ds^2}{r^2} &= -(1 \pm \frac{r_6}{r})^{-\frac{3}{5}} dt^2 + (1 \pm \frac{r_6}{r})^{-1} dr^2 + (r^2 d\Omega^2 + dx_4^2 + dx_5^2) \\
&+ (1 \pm \frac{r_6}{r})^\frac{2}{5} (dx_6^2 + \ldots + dx_9^2). 
\end{aligned}
\end{align*}
\tag{77}
\]

\section{4 Time-Dependence, Energy-Conservation and Euclidean Branes at the Horizon}

In order to discriminate which one of the solutions obtained in the previous section actually represents the exterior geometry of the Euclidean D3-D3 pair doublet, we will now examine under what condition such a doublet can lead to a stationary, i.e. time-independent geometry.

In as much as a conventional D-brane breaks translational symmetry orthogonal to its worldvolume a Euclidean D-brane embedded in a Lorentzian spacetime breaks furthermore time-translation symmetry because time is now a transverse coordinate. A breaking of space-translation symmetry results in a violation of momentum conservation for those momentum components orthogonal to the D-brane’s worldvolume. Hence, in the case of a Euclidean D-brane also energy-conservation would be violated. This is most obvious in the weakly coupled \((g_s \ll 1)\) regime where a Euclidean D-brane satisfies a Dirichlet-boundary condition in time, i.e. it exists only for a snapshot at a moment in time – before or after it is non-existent. This sudden creation out of nothing and instantaneous destruction afterwards clearly violates energy-conservation (see fig.2a).

In the case of a Euclidean brane-antibrane pair in flat spacetime there are two notions of lifetime. One is related to its Euclidean nature as explained before, the other is related to its decay via tachyon condensation (see [19] for the latter). Presumably both notions coincide and are of order the string-time \(t_s = \sqrt{\alpha'}\). For the brane-antibrane pair energy is conserved when it decays into radiation, however energy-conservation becomes a problem when the pair becomes created out of nothing due to its Euclidean nature. This might be cured by some finely tuned incoming radiation [19] but there may be doubts whether the
entropy in this process “radiation → brane-antibrane pair” is decreasing thus violating the second law of thermodynamics.

In the presence of gravitational fields we are used to the phenomenon of time-dilatation. Accordingly the lifetime of a Euclidean brane could be enhanced (from the point of view of an exterior observer) in a strong gravitational field (see fig.2b). Though the two incidents of energy-conservation violation (the creation and later annihilation of the brane) become more separated in an exterior observer’s time it is still present.

There is however one unique possibility of obtaining a consistent theory with Euclidean branes and at the same time obtaining a stationary geometry. This is when the Euclidean branes are located precisely at a spacetime event horizon (see fig.2c). At an event horizon the gravitational redshift becomes infinitely large thus rendering a Euclidean brane’s lifetime infinitely large. The troublesome energy-violating incidences become removed to $t \to \pm \infty$ which means they do not occur. From an exterior (to the brane) observer’s point of view time on the Euclidean brane stands still and so the configuration becomes time-independent, i.e. stationary. In the same manner the short lifetime of a brane-antibrane due to annihilation gets infinitely enhanced from the an exterior observer’s view and thus its exterior geometry becomes actually time-independent at a classical level.

Due to this reasoning which suggests that isolated Euclidean branes embedded in a
Lorentzian spacetime should sit at event horizons we will impose on our vacuum solution the requirement that it should possess such an event horizon where we can locate the Euclidean brane-antibrane pairs (notice that all our vacuum solutions are given in asymptotically flat coordinates as appropriate for an external observer). Due to the location of the $D3 - D\bar{3}$ pairs, the horizon must appear at some finite value of $r$. As an aside we want to remark that one could also think of an array of Euclidean branes along the time direction with the effect to arrive at a smeared out brane with no time-dependence any more and thus also a stationary geometry. This however requires an infinity of Euclidean branes which is not what we have here.

In order to examine which of our solutions exhibits the required event horizon let us mention two necessary criteria for such a horizon. Consider radial null curves for which $ds^2 = 0$ and all coordinates are constant except for $t$ and $r$. This leads to

$$\frac{dt}{dr} = \pm \sqrt{-\frac{g_{rr}}{g_{tt}}} .$$

(78)

Their usage lies in the fact that they give the slope of the light-cones. One of the characteristics of an event horizon is that the light-cones fold up. Thus we demand that

$$\frac{g_{rr}(r_H)}{g_{tt}(r_H)} \to \infty$$

(79)

at a horizon $r = r_H$. A further useful characterization of an event horizon is its infinite redshift from an exterior observer’s point of view. Energies $E_1$ and $E_2$ belonging to the same physical process but measured at radial positions $r_1$ and $r_2$ will differ by an amount

$$\frac{E_1}{E_2} = \sqrt{\frac{g_{tt}(r_2)}{g_{tt}(r_1)}} .$$

(80)

If we place $r_2 = r_H$ at an event horizon and $r_1 > r_H$ outside, an infinite redshift at the horizon implies that

$$g_{tt}(r_H) \to 0 .$$

(81)

This will serve as our second criterion. Let us now see which solutions satisfy these two criteria and thus qualify as representing the exterior stationary geometry of a horizon-located $D3-D\bar{3}$ brane configuration.

The first case ($\Lambda = \Psi = 0$) which gave the flat Minkowski spacetime clearly has no horizon and moreover does not describe a localized gravitational source as its ADM-mass vanishes. Thus we can rule it out.
The second case \((\Lambda \neq 0, \Psi = 0)\) of the embedded Schwarzschild solution possesses a well-known event horizon at \(r = r_0\) and thus fulfills the above criteria.

The third case \((\Lambda = 0, \Psi \neq 0)\) gave no solutions.

In the fourth case \((\Lambda \neq 0, \Psi \neq 0, \Psi = c\Lambda)\) the first criterion \(\Box\) can only be satisfied if either \(K'(r) \to \infty\) and \(b > 0\) or \(K'(r) \to \sqrt{2a^2 - b}\). For the first choice one learns from \(\Box\) that \(r \to r_3\) and thereby \(g_{tt}\) does not vanish but approaches a positive value in contradiction to \(\Box\). For the latter choice \(\Box\) says that this is only possible at \(r \to 0\). But to get a regular horizon with finite area the brane set-up should be placed at some finite value of \(r_H > 0\). Therefore this class of solutions has to be dismissed.

For the fifth solution \((\Psi = -\frac{1}{4}\Lambda)\) \(g_{tt}\) can only vanish at \(r \to 0\) which eliminates this solution as a candidate for the same reason as for the second choice in the previous case.

As to the sixth solution \((\Psi = -\frac{2}{3}\Lambda)\) the first criterion \(\Box\) demands that \(1 \pm \frac{a}{r} \to 0\) while the second criterion \(\Box\) would demand that \(1 \pm \frac{a}{r} \to \infty\). This is impossible to fulfill simultaneously which leads to the rejection also of this solution.

Since the Euclidean brane-antibrane pairs must give rise to some vacuum solution we can now conclude that this will be the embedded D=4 Schwarzschild solution with the \(D3-D\bar{3}\) doublet placed at its horizon. As has been stressed already in the introduction the main importance of this identification lies in the fact that now we can straightforwardly follow the general framework presented in \[14\] to derive the BH-entropy and its corrections for the D=4 Schwarzschild solution by counting chain-states on the worldvolume of the \(D3-D\bar{3}\) doublet configuration.

### 5 The Tachyon and Hawking-Radiation

Let us finally speculate about the origin of the black hole’s Hawking radiation. It is well known that classically a black hole does not radiate - it does so only when quantum effects are taken into account \[20\]. Let us try to understand this from our picture of the black hole in terms of the Euclidean brane-antibrane pairs.

Coinciding brane-antibrane pairs suffer from an instability related to the open string tachyon on their joint worldvolume. The tachyon possesses a Mexican-hat like potential - be it quartic or more complicated involving the error function \[21\]. Thus tiny perturbations, e.g. quantum fluctuations would initiate a rolling of the tachyon down the hill if
placed initially on top of the hill to describe the brane-antibrane pair. According to Sen’s conjecture \[22\] the brane-antibrane pair will have annihilated itself and produced just the closed string vacuum by the time the tachyon reaches its potential minimum by virtue of the equality

\[ V(T_{\text{min}}) = -2 \frac{T_{Dp}}{g_s} \]  

where \( V(T_{\text{min}}) \) is the negative value of the tachyon potential at its minimum. In this tachyon-condensation process the relative U(1) gauge-group under which the tachyon is charged gets spontaneously broken by a Higgs-mechanism while the fate of the overall U(1) under which the tachyon is neutral poses a problem \[23\]. An interesting proposal for its resolution is the idea that the overall U(1) gets confined \[24\], leaving behind confined electric flux tubes which can be identified with the fundamental closed strings \[25\] (see however \[26\] for criticism of this proposal).

Since the system starts with positive energy \( 2T_{Dp}/g_s \) (the tachyon potential is zero on top of the hill) and ends up with zero energy due to (82), it is commonly believed that the surplus of energy will be radiated off into the bulk. For our concern the interesting aspect lies in the fact that *classically* no such radiation will be produced as shown recently in \[27\] but quantum mechanically there is no obstruction and thus it should occur. This might be understood from the fact that the process of radiation production has to transform open strings on the brane-antibrane into closed strings which can escape towards the bulk. However such an interaction arises only at the *quantum level* (1-loop from the open string point of view) and thus forbids a classical radiation into the bulk. Due to this coincidence with the quantum appearance of Hawking radiation which is likewise classically forbidden, it is natural to conjecture that Hawking radiation in the black hole’s brane-antibrane pair picture might be understood as closed string modes send into the bulk when the tachyon rolls down the hill.

It would be nice to make this more precise and also get a qualitative understanding of the negative specific heat of the Schwarzschild black hole from the brane description (see \[28\] for an approach involving a multitude of brane-antibrane pairs to explain the negative specific heat for black 3-branes). It seems, however, that we have to acquire first a better understanding of the dynamics of the tachyon condensation process itself. For progress in this direction see e.g. \[27\], \[29\] or the interesting recent application of brane-antibrane pairs to cosmology (see e.g. \[30\]). We hope to report on progress on these issues and on a related embedding of de Sitter spacetime in the future. Here, it would be interesting to find a qualitatively different mechanism than suppression in warped geometries \[31\] to
approach the problem of a finite but small vacuum energy.

Acknowledgements

The author wants to thank Allen Hatcher and Ashoke Sen for correspondence. A.K. associated to the Aristotle University of Thessaloniki acknowledges financial support provided through the European Community’s Human Potential Program under contract HPRN-CT-2000-00148 “Physics Across the Present Energy Frontier” and support through the German-Greek bilateral program IKYDA-2001-22.

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