Multiparameter Brane Solutions
by Boosts, S and T dualities

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ABSTRACT

We show that the multiparameter (intersecting) brane solutions of string/M theories given in the literature can all be obtained by a suitable combination of boosts in eleven dimension, S and T dualities. We also describe a duality property of the $D$ dimensional multiparameter solutions describing branes smeared in compact directions. This duality is analogous to the T duality of the string theory but is valid for any value of $D$. 

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1. Introduction

Recently Zhou and Zhu obtained multiparameter brane solutions [1]. For particular values of the parameters, these solutions reduce to the well known brane solutions [2] of string/M theories. These multiparameter solutions were interpreted in [3] as describing non BPS brane antibrane configurations and were used to study the tachyon condensation and the related dynamics. Since then, they have been generalised and studied extensively [4, 5, 6, 7, 8].

In all these cases, the multiparameter solutions have been obtained by solving the relevant equations of motion directly with varying levels of generality. The most general of such solutions have been obtained by Miao and Ohta (MO) in [8] which also includes intersecting brane configurations. In this paper we show that all of these multiparameter solutions describing intersecting brane configurations in string/M theories can be obtained by a suitable combination of boosts in eleven dimension, S and T dualities [9, 10, 11].

Assuming an ansatz similar to that in [12] we first obtain a generalisation of the solution in [12], now including a requisite number of compact directions also. Taking the spacetime to be eleven dimensional with at least one compact direction, we boost this solution and reduce to ten dimensions obtaining thereby a ten dimensional charged solution. Applying further a suitable combination of S and T dualities, and lift-boost-reduce procedures to generate more brane charges, will then yield a variety of intersecting brane configurations of string/M theories. The calculations are all straightforward and we present a few brane solutions explicitly by way of illustration. We also point out that starting from extremal (intersecting) brane configurations, the corresponding multiparameter solutions can be written down following a few simple rules, similar to those given in [9].

We show that the $D$ dimensional multiparameter solutions describing branes smeared in compact directions have a duality property when the higher form field strength corresponding to the brane couples to the scalar field in a specific way. This duality is analogous to the T duality of the string theory but is valid for any value of $D$. For $D = 10$, and for $D < 10$ also, this is same as T duality of the string theory. For $D > 10$, this duality can be used as a solution generating technique but otherwise its significance, if any, is not clear.
Although the solutions are obtained here assuming an ansatz as in [12],
they turn out to be completely equivalent to the corresponding ones in [8],
and hence also to those in [1, 3, 4, 6, 7], which were obtained by solving the
relevant equations of motion directly. This is shown by writing our solutions
in isotropic coordinates, and then comparing with those given in [1, 3, 4, 8]
after some elaborate algebraic manipulations. \(^1\) It then follows that the
multiparameter brane solutions of string/M theories given in [1, 3, 4, 5, 6, 7, 8]
can all be obtained by a suitable combination of boosts in eleven dimensions,
S and T dualities.

This paper is organised as follows. In section 2 we present the \((D + 1)\)
dimensional uncharged multiparameter solutions. In section 3 we use them
to obtain \(D\) dimensional uncharged solutions, and also the charged ones by
a boost in \((D + 1)\) dimensions. In section 4 we set \(D = 10\) and write down
explicitly a few multiparameter (intersecting) brane solutions of string/M
theories explicitly. We then present a few simple rules for obtaining such
solutions starting from the corresponding extremal ones. In section 5 we
describe a duality property of the \(D\) dimensional multiparameter solutions
describing branes smeared in compact directions. In an Appendix we show
the equivalence of our solutions to the corresponding ones in the literature.
In section 6 we conclude by mentioning a few issues for further studies.

2. \((D + 1)\) dimensional solutions

Consider the Einstein action
\[
\mathcal{S} = \frac{1}{2\kappa^2 L} \int d^{D+1}X \sqrt{-\hat{g}} \mathcal{R}
\]
in a \(D + 1\) dimensional spacetime and its dimensional reduction to \(D\) dimen-
sions along a compact spatial direction of size \(L\). Let the \(D + 1\) dimensional
line element be given by
\[
d\hat{S}^2 = e^{-\frac{2b}{D-2}} g_{\mu\nu} dX^\mu dX^\nu + e^{2b\phi} (dz + A_\mu dX^\mu)^2
\]
where \(\mu, \nu = 0, 1, \cdots, (D - 1)\) and the fields \((g_{\mu\nu}, \phi, A_\mu)\) are all independent
of the coordinate \(z\) along the compact direction. For \(b = \sqrt{\frac{D-2}{2(D-1)}}\), the action

\(^1\)Along the way, it can be seen that of the two parameters, \(\mu\) and \(\nu\), in the solutions of
[8] only one combination is physically relevant; the other one amounts to a diffeomorphism.
\( \hat{S} \) in (1) reduces to the \( D \) dimensional ‘Einstein frame’ action \( S \) for a scalar \( \phi \) and a 2–form field strength \( F_2 = \partial_\mu A_\nu - \partial_\nu A_\mu \) given by

\[
S = \frac{1}{2\kappa^2} \int d^D X \sqrt{-g} \left( \mathcal{R} - \frac{1}{2} (\partial \phi)^2 - \frac{e^{\lambda \phi} F_2^2}{4} \right)
\]

(3)

where \( \lambda = \frac{2(D-1)b}{D-2} = \sqrt{\frac{2(D-1)}{D-2}} \).

The above action is a particular case of the more general \( D \) dimensional Einstein frame action for \( g_{\mu\nu}, \phi \), and a \((p_1+1)\)–form gauge field \( A_{(p_1+1)} \) given by

\[
S_{p_1} = \frac{1}{2\kappa^2} \int d^D X \sqrt{-g} \left( \mathcal{R} - \frac{1}{2} (\partial \phi)^2 - \frac{e^{\lambda_{p_1}} \phi F_{p_1+2}^2}{2(p_1+2)!} \right)
\]

(4)

where \( \lambda_{p_1} \) is an arbitrary constant which may depend on \( p_1 \) and the \((p_1+2)\)–form field strength \( F_{p_1+2} = dA_{(p_1+1)} \). For \( p_1 = 0 \) and the specific value of \( \lambda_0 = \lambda \), the action (4) reduces to action (3) and can therefore be obtained by dimensional reduction of a \((D+1)\) dimensional action (1).

In the following, we will consider electric type \( p_1 \)–brane solutions in the string/M theories. (Magnetic ones can be easily obtained from these solutions.) Then \( D = 10 \) in the actions (1) and (3). In the action (4), \( D = 11 \lambda_0 = 0 \), and the field \( \phi \) is absent for M theory branes; whereas \( D = 10 \) for string theory with \( \lambda_p = -1, +1 \) for fundamental strings and the 5–branes respectively in Neveu-Schwarz (NS) sector, and \( \lambda_p = \frac{3-p}{2} \) for branes in the Ramond sector.

Now, consider \( D + 1 \) dimensional spacetime with \( p + 1 \) compact spatial directions. Consider the time independent case with the line element \( d\hat{S} \) given by

\[
d\hat{S}^2 = -e^{2\alpha_0} dt^2 + \sum_{i=1}^{p+1} e^{2\alpha_i} dX_i^2 + e^{2\gamma} dr^2 + e^{2\omega} d\Omega^2_{n+1}
\]

(5)

where \( n = D - 3 - p \), \( r \) is the radial coordinate in the \((n+2)\) dimensional non compact transverse space, \( d\Omega^2_{n+1} \) is the standard line element on an \((n+1)\) dimensional unit sphere, and the functions \( (\alpha_i, \gamma, \omega) \) with \( i = 0, 1, \ldots, p+1 \) depend on \( r \) only. The independent non zero components of the Riemann tensor \( \mathcal{R}_{LMN}^K \) for the above ansatz are

\[
\mathcal{R}^a_{bcd} = (\delta_c^a \sigma_{bd} - \delta_d^a \sigma_{bc}) \omega_r^2 e^{2(\omega-\gamma)} + \rho^a_{bcd}
\]
\[ \hat{\mathcal{R}}_{ibj} = \delta_{i}^{a} \delta_{j}^{b} (\alpha_{i})_{r} \omega_{r} e^{2(\alpha_{i} - \gamma)} \]
\[ \hat{\mathcal{R}}_{rbr} = \delta_{b}^{a} (\omega_{rr} + \omega_{r}^{2} - \omega_{r} \gamma_{r}) \]
\[ \hat{\mathcal{R}}_{rjr} = \delta_{j}^{i} ((\alpha_{i})_{rr} + (\alpha_{i})_{r}^{2} - (\alpha_{i})_{r} \gamma_{r}) \]
\[ \hat{\mathcal{R}}_{jkl} = (\delta_{k}^{i} \delta_{j}^{l} - \delta_{l}^{i} \delta_{j}^{k}) (\alpha_{i})_{r} (\alpha_{j})_{r} e^{2(\alpha_{j} - \gamma)} \]  
\( \text{(6)} \)

where \( \sigma_{ab} \) with \( a, b = 1, 2, \cdots, n + 1 \) is the metric on the \((n + 1)\) dimensional unit sphere, \( \rho_{abcd} = (\delta_{a}^{c} \sigma_{bd} - \delta_{a}^{d} \sigma_{bc}) \) its Riemann tensor, and \( (\ )_{r} \equiv \frac{d}{dr} (\ ) \). The Ricci tensor \( \hat{\mathcal{R}}_{MN} \) can be obtained from the above expressions.

Action (1) gives the equations of motion \( \hat{\mathcal{R}}_{MN} = 0 \). Using (6), one can then obtain the equations of motion for \( (\alpha_{i}, \gamma, \omega) \). These equations can be solved easily by assuming the ansatz

\[ e^{\alpha_{i}} = Z^{\hat{a}_{i}} , \quad e^{\gamma} = Z^{\hat{b}} , \quad e^{\omega} = r Z^{\hat{c}} , \quad Z \equiv 1 - \frac{r_{0}^{n}}{r^{n}} \]  
\( \text{(7)} \)

where \( r_{0} \) and \( (\hat{a}_{i}, \hat{b}, \hat{c}) \) are constant parameters. Equations \( \hat{\mathcal{R}}_{ab} = 0 \) imply that \( \hat{b} = \hat{c} - \frac{1}{2} \). Thus, the line element in (5) becomes

\[ d\hat{S}^{2} = -Z^{2 \hat{a}_{0}} dt^{2} + \sum_{i=1}^{p+1} Z^{2 \hat{a}_{i}} dX^{i 2} + Z^{2 \hat{c}} ds^{2}_{n+2} \]  
\( \text{(8)} \)

where we have defined

\[ ds^{2}_{n+2} \equiv \frac{dr^{2}}{Z^{2}} + r^{2} d\Omega^{2}_{n+1} . \]  
\( \text{(9)} \)

Equations \( \hat{\mathcal{R}}_{ij} = \hat{\mathcal{R}}_{rr} = 0 \) imply that the constant parameters \( (\hat{a}_{i}, \hat{c}) \) satisfy the constraints

\[ \sum_{i=0}^{p+1} \hat{a}_{i} + n \hat{c} = \frac{1}{2} , \quad \sum_{i=0}^{p+1} \hat{a}_{i}^{2} + n \hat{c}^{2} - \hat{c} = \frac{1}{4} . \]  
\( \text{(10)} \)

Throughout in the following we assume that the parameters \( (\hat{a}_{i}, \hat{c}) \) satisfy the constraints given in (10) and are otherwise arbitrary. Note that besides \( r_{0} \), there are \( p + 3 \) parameters \( (\hat{a}_{i}, \hat{c}) , i = 0, 1, \cdots, p+1 \) satisfying two constraints. Hence, there are \( p + 2 \) independent parameters which can be taken to be \( r_{0} \) and, for example, \( \hat{a}_{i} , i = 1, 2, \cdots, p + 1 \).
3. *D* dimensional solutions

Upon dimensional reduction along \( z \equiv X^{p+1} \) direction, the solution (8) gives the *D* dimensional uncharged solution:

\[
ds^2 = -Z^{2a_0} dt^2 + \sum_{i=1}^{p} Z^{2a_i} dX^{i2} + Z^{2c} ds_{n+2}^2, \quad e^\phi = Z^q, \quad A_\mu = 0
\]

(11)

where we assume, with no loss of generality, that \( \phi \to 0 \) as \( r \to \infty \). The parameters \((a_i, c, q), i = 0, 1, \ldots, p\) are given by

\[
a_i = \hat{a}_i + \frac{\hat{a}_{p+1}}{D-2}, \quad c = \hat{c} + \frac{\hat{a}_{p+1}}{D-2}, \quad q = \frac{\hat{a}_{p+1}}{b}; \quad b = \sqrt{\frac{D-2}{2(D-1)}}
\]

and, as follows from equations (10), satisfy the constraints

\[
\sum_{i=0}^{p} a_i + nc = \frac{1}{2}, \quad \sum_{i=0}^{p} a_i^2 + nc^2 - c + \frac{q^2}{2} = \frac{1}{4}.
\]

(12)

Throughout in the following we assume that the parameters \((a_i, c, q)\) satisfy the constraints given in (12) and are otherwise arbitrary.

Let us note some properties of the solution in (11).

1. Let \((a_0, c) = (\frac{1}{2}, 0)\). Then necessarily \(a_i = q = 0\) for \(i \neq 0\). The corresponding solution describes uncharged \(p\)–branes in \(D\) dimensions with \(p\) dimensional compact space; or, upon a further \(p\) dimensional compactification, a \(D-p\) dimensional Schwarzschild black hole.

2. Besides \(r_0\), there are \(p+3\) parameters \((a_i, c, q), i = 0, 1, \ldots, p\) satisfying two constraints. Hence there are \(p+2\) independent parameters. They can be taken to be \(r_0\) and, for example, \((a_i, q), i = 1, 2, \ldots, p\). For \(p = 0\), we get the two parameter solution obtained in [12].

3. If \(q \neq 0\) then the \(D\) dimensional curvature invariants, for example the Ricci scalar \(R\), diverge at \(r_0\). Hence, there is a curvature singularity at \(r_0\). In \(D+1\) dimensions, however, the Ricci tensor \(\hat{R}_{MN}\), and thus the Ricci Scalar \(\hat{R}\) also, vanishes identically because of the equations of motion. Hence, other curvature invariants need to be studied to determine the singularities of the \(D+1\) dimensional solutions. It is important to study the curvature invariants and the singularities of all the solutions presented here, but such a study is
beyond the scope of the present paper and will not be pursued here. See [3, 5, 6, 12], for a discussion of certain issues related to singularities. The $D$ dimensional charged solution can be obtained by lifting the uncharged one to $D + 1$ dimensions, boosting along $z$ direction, and reducing back to $D$ dimensions. Consider a time-independent $D$ dimensional solution where $A_\mu = 0$, $e^\phi = e^{\phi_0}$, and the line element is given by

$$ds^2 = g_{00}dt^2 + ds_{D-1}^2$$

with $g_{0\mu} = 0$ for $\mu \neq 0$. This level of generalisation will suffice for our purposes here. The $D + 1$ dimensional line element is then given by

$$\hat{d}\hat{S}^2 = e^{-\frac{2\phi_0}{D-2}}ds^2 + e^{2\phi_0}dz^2 + \cdots \equiv \hat{g}_{00}dt^2 + \hat{g}_{zz}dz^2 + \cdots$$

where $b = \sqrt{\frac{D-2}{2(D-1)}}$, see equation (2). Under a boost along $z$ direction

$$t \rightarrow C t + S z, \quad z \rightarrow S t + C z$$

where $C = \text{Cosh} \, \Theta, \quad S = \text{Sinh} \, \Theta$ with $\Theta$ a boost parameter. The boosted line element becomes

$$\hat{d}\hat{S}^2 = H^{-1} \hat{g}_{00}dt^2 + H \hat{g}_{zz}(dz + A_0 dt)^2 + \cdots$$

where $A_0 = \frac{CS(1-F)}{H}$ and we have defined

$$H = C^2 - FS^2, \quad F = -\hat{g}_{00}(\hat{g}_{zz})^{-1}.$$ (13)

Note that $F = -g_{00} e^{-\lambda \phi}$ where $\lambda = \frac{2(D-1)b}{D-2}$. Using equation (2), the boosted line element can be reduced to $D$ dimensions. In the resulting $D$ dimensional charged solution, the non zero component of the gauge field $A_\mu$ can be written as

$$A_0 = \frac{CS(1-F)}{H} \simeq -\frac{S}{C} \frac{F}{H},$$ (14)

where the last two expressions are physically equivalent since they differ only by a constant ($= \frac{S}{C}$). The line element $ds$ and $e^\phi$ in the charged solution are given by

$$ds^2 = H^A g_{00}dt^2 + H^B ds_{D-1}^2, \quad e^\phi = H^C e^{\phi_0}$$ (15)

where $(A, B, C) = \left( -\frac{D-3}{D-2}, \frac{1}{D-2}, \frac{\lambda}{2} \right)$. 

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Henceforth, unless mentioned otherwise, we assume that

\[ H = C^2 - FS^2, \quad F = Z^{2k} \]  

which is the case for our ansatz (7). Then equation (14) implies that the gauge field charge \( Q \propto k \mathcal{C} \mathcal{S} r_0^n \). Also, in various solutions we present below, the non zero component of the respective higher form gauge fields are given by the expression on the right hand side of the equation (14), and hence the corresponding charges are also \( \propto k \mathcal{C} \mathcal{S} r_0^n \) but with differing expressions for the parameter \( k \), see below. Therefore, in all the solutions presented here, we will write the expressions for \( ds \), \( e^\phi \), and \( k \) only.

Thus, the \( D \) dimensional charged solution obtained by applying the boost described above to the solution (11) is given by

\[
\begin{align*}
    ds^2 &= -H^A Z^{2a_0} dt^2 + H^B \left( \sum_{i=1}^{p} Z^{2a_i} dX^{i2} + Z^{2c} ds_{n+2}^2 \right) \\
    e^\phi &= H^C Z^q, \quad k = a_0 - \frac{\lambda q}{2}
\end{align*}
\]  

where \( (A, B, C) \) and \( \lambda \) are as given above. Note that in string theory \( D = 10 \). Then \( n = 7 - p, b = \frac{2}{3}, \lambda = \frac{3}{2}, (A, B, C) = (-\frac{7}{8}, \frac{1}{8}, \frac{3}{4}) \), and the above solution becomes

\[
\begin{align*}
    ds^2 &= -H^A Z^{2a_0} dt^2 + H^B \left( \sum_{i=1}^{p} Z^{2a_i} dX^{i2} + Z^{2c} ds_3^2 \right) \\
    e^\phi &= H^C Z^q, \quad k = a_0 - \frac{3q}{4}.
\end{align*}
\]  

Let us now note some properties of these solutions.

(1) There is now an extra parameter, namely the \( (D+1) \) dimensional boost parameter \( \Theta \), which generates the \( D \) dimensional charge \( Q \). The uncharged solution (11) is obtained when \( \Theta = 0 \).

(2) Let \( (a_0, c) = (\frac{1}{2}, 0) \). Then necessarily \( a_i = q = 0 \) for \( i \neq 0 \). Also, \( k = \frac{1}{2}, \quad F = Z, \quad H = \left( 1 + \frac{r_0^n \mathcal{S}^2}{\rho^2} \right) \). The solution (17) then describes charged 0–brane smeared in \( p \) compact directions. In string theory, \( D = 10 \) and the solution (18) describes Dirichlet 0–brane (D0–brane) smeared in \( p \) compact directions.

(3) In the extremal limit given by

\[
r_0 \to 0, \quad \Theta \to \infty, \quad r_0^n \mathcal{S}^2 \to finite,
\]  

where \( \mathcal{S} \) is a solution of \( \mathcal{S}^2 = \frac{1}{2}(1 + \frac{r_0^n \mathcal{S}^2}{\rho^2}) \).
\[ Z \to 1 \] and hence the values of the parameters \((a_i, c, q)\) are irrelevant, \(H \to \left( 1 + \frac{2k\sqrt{n}S^2}{r_n^2} \right)\), and the charge \(Q \propto kCSr_0^n\) remains finite. The solution (17) then describes extremally charged 0-brane smeared in \(p\) compact directions. In string theory, \(D = 10\) and the solution (18) then describes extremal \(D0\)-brane smeared in \(p\) compact directions.

### 4. A few brane solutions of string/M theories

Starting from the \(D0\)-brane solutions \(^2\) given in (18), one may obtain other brane solutions of string/M theories by repeated use of \(S\) and \(T\) dualities and boosts in \(11^\text{th}\) dimension [9, 10]. We present a few examples below.

The boost in \(11^\text{th}\) dimension is as given before but now with \(D = 10\). The transformation rules for \(S\) and \(T\) dualities are given in [11]. For the cases of interest here, they are simple enough and we will now explain these transformations briefly as applied to \(ds^2\) and \(e^\phi\).

Under \(S\) duality, \((g_{\mu\nu}, \phi) \to (g_{\mu\nu}, -\phi)\). The \(T\) duality rules are given in [11] in string frame where the string metric \(G_{\mu\nu} = e^{\frac{2}{\phi}}g_{\mu\nu}\). One may convert the Einstein frame solutions to the string frame, apply \(T\) duality rules, and convert back to the Einstein frame. The result of a \(T\) duality, along \(e.g.\) \(X^1\) direction, applied to the \(D0\)-brane solution (18) is that the original 1-form gauge field \(A_0\) now becomes a 2-form gauge field \(A_{01}\) and

\[
\begin{align*}
    ds^2 &= H^{-\frac{2}{p}} \left( -Z^{2a_0}dt^2 + Z^{2\tilde{a}_i}dX^{i2} \right) + H^{\frac{2}{p}} \left( \sum_{i=2}^{p} Z^{2\tilde{a}_i}dX^{i2} + Z^{2q}ds_{9-p}^2 \right) \\
    e^{\phi} &= H^\frac{q}{2}Z^q, \quad k = a_0 - \frac{3q}{4} = \tilde{a}_0 + \tilde{a}_1 - \frac{2\tilde{q}}{4}
\end{align*}
\]

where \((\tilde{a}_i, \tilde{c}, \tilde{q}; \tilde{a}_1) = (a_i, c, q; a_1) + \left( \frac{1}{4}, \frac{1}{4}, -1; -\frac{7}{4} \right) \chi\) with \(\chi = a_1 + \frac{q}{4}\) and \(i \neq 1\).

Note that after a \(T\) duality, the parameters \((a_i, c, q)\) have changed to \((\tilde{a}_i, \tilde{c}, \tilde{q})\), \(i = 0, 1, \cdots, p\). Also, as can be verified easily, if \((a_i, c, q)\) satisfy the constraints (12) then so do \((\tilde{a}_i, \tilde{c}, \tilde{q})\). However, \(k\) has not changed. But when expressed in terms of \((\tilde{a}_i, \tilde{c}, \tilde{q})\), it has a different form.

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\(^2\)Strictly speaking, equation (18) is not a \(D0\)-brane solution if \((a_0, c) \neq \left( \frac{1}{2}, 0 \right)\). Nevertheless, even then we will continue to use such phrases to refer to the solutions.
This will be the case in general in the following. However, instead of ornamenting the labels \((a_i, c, q)\) endlessly after each \(S\) or \(T\) duality, we will simply use the same labels \((a_i, c, q)\) always. Consequently, of course, the expression for \(k\) will be different. Similarly, for \(M\) theory, we will always use the labels \((\hat{a}_i, \hat{c})\). Note that \((a_i, c, q)\) will always satisfy the constraints given in (12), and \((\hat{a}_i, \hat{c})\) those given in (10), but are otherwise arbitrary.

Consider now \(T\) duality along \(X^i\) directions, \(i = 1, 2, \cdots, p_1 \leq p\) in the \(D0\)–brane solution given in (18). The calculation is straightforward and the result is that the original 1-form gauge field \(A_0\) now becomes a \((p_1 + 1)\)-form gauge field \(A_{01} \cdots p_1\) and

\[
\begin{align*}
\frac{ds^2}{H} &= H^A \left(-Z^{2a_0} dt^2 + \sum_{i=1}^{p_1} Z^{2a_i} dX^{i2} \right) + H^B \left(\sum_{i=p_1+1}^{p} Z^{2a_i} dX^{i2} + Z^{2c} ds_{9-p}^2 \right) \\
e^\phi &= H^C Z^q, \quad k = \sum_{i=0}^{p_1} a_i - \frac{(3-p_1)q}{4} \tag{21}
\end{align*}
\]

where \((A, B, C) = (-\frac{7-p_1}{8}, \frac{p_1+1}{8}, \frac{3-p_1}{4})\). Equation (21) describes \(Dp\)–branes smeared in the remaining \((p - p_1)\) compact directions. We will now write down explicitly a few brane solutions of string/M theories.

\(Dp\)–branes are obtained by setting \(p_1 = p\) in equation (21):

\[
\begin{align*}
\frac{ds^2}{H} &= H^A \left(-Z^{2a_0} dt^2 + \sum_{i=1}^{p} Z^{2a_i} dX^{i2} \right) + H^B \left(\sum_{i=p+1}^{p} Z^{2a_i} dX^{i2} + Z^{2c} ds_{9-p}^2 \right) \\
e^\phi &= H^C Z^q, \quad k = \sum_{i=0}^{p} a_i - \frac{(3-p)q}{4} \tag{22}
\end{align*}
\]

Fundamental (F) strings are obtained by \(S\) dualising \(D1\)–branes:

\[
\begin{align*}
\frac{ds^2}{H} &= H^{-\frac{1}{2}} \left(-Z^{2a_0} dt^2 + Z^{2a_1} dX^{12} \right) + H^{\frac{1}{2}} Z^{2c} ds_{8}^2 \\
e^\phi &= H^{-\frac{1}{2}} Z^q, \quad k = \sum_{i=0}^{1} a_i + \frac{q}{2} \tag{23}
\end{align*}
\]

NS sector 5–branes are obtained by \(S\) dualising \(D5\)–branes:

\[
\begin{align*}
\frac{ds^2}{H} &= H^{-\frac{1}{4}} \left(-Z^{2a_0} dt^2 + \sum_{i=1}^{5} Z^{2a_i} dX^{i2} \right) + H^{\frac{1}{4}} Z^{2c} ds_{4}^2 \\
e^\phi &= H^{\frac{1}{4}} Z^q, \quad k = \sum_{i=0}^{5} a_i - \frac{q}{2} \tag{24}
\end{align*}
\]
\(M2\)-branes are obtained by lifting F strings to 11 dimensions:

\[
ds^2 = H^{-\frac{3}{4}} \left( -Z^{2a_0} dt^2 + \sum_{i=1}^{2} Z^{2a_i} dX^{i2} \right) + H^\frac{3}{8} Z^{2c} ds^2_8, \quad k = \sum_{i=0}^{2} \hat{a}_i. \tag{25}\]

\(M5\)-branes are obtained by lifting \(D4\)-branes to 11 dimensions:

\[
ds^2 = H^{-\frac{2}{3}} \left( -Z^{2a_0} dt^2 + \sum_{i=1}^{5} Z^{2a_i} dX^{i2} \right) + H^\frac{2}{3} Z^{2c} ds^2_5, \quad k = \sum_{i=0}^{5} \hat{a}_i. \tag{26}\]

Intersecting brane solutions can also be obtained by further boosts. As an example, we now obtain intersecting \(D1-D5\) brane solution. Start with the \(D4\)-brane solution smeared in a compact \(X^5\) direction, \textit{i.e.} the solution (21) with \(p_1 = 4\) and \(p = 5\), lift it to 11 dimension, boost along the 11th direction which will generate a second charge, and reduce back to 10 dimensions. This will now give the \(D0-D4\) branes smeared in the \(X^5\) direction:

\[
ds^2 = -H^{-\frac{3}{8}} h^{-\frac{3}{8}} Z^{2a_0} dt^2 + H^{-\frac{3}{8}} h^\frac{3}{8} \sum_{i=1}^{4} Z^{2a_i} dX^{i2} \]
\[
+ H^\frac{3}{8} h^\frac{3}{8} \left( Z^{2a_{i5}} dX^{52} + Z^{2c} ds^2_4 \right) \]
\[
e^\phi = H^{-\frac{1}{4}} h^\frac{3}{4} Z^q, \quad k = \sum_{i=0}^{4} a_i + \frac{q}{4} \]
\[
h = C_1^2 - f S_1^2, \quad f = Z^{2l}, \quad l = a_0 - \frac{3q}{4} \tag{27}\]

where \(C_1 = Cosh \Theta_1, S_1 = Sinh \Theta_1\) with \(\Theta_1\) a boost parameter that generates the \(D0\)-brane charge. The \(D0\)-brane gauge field is given by a similar expression as in (14) and (16), but with \((H,F,k)\) replaced by \((h,f,l)\).

\(D1-D5\) intersecting branes are obtained by T dualising the \(D0-D4\) branes above along the \(X^5\) direction:

\[
ds^2 = H^{-\frac{3}{8}} h^{-\frac{3}{8}} \left( -Z^{2a_0} dt^2 + Z^{2a_5} dX^{52} \right) \]
\[
+ H^{-\frac{3}{8}} h^\frac{3}{8} \sum_{i=1}^{4} Z^{2a_i} dX^{i2} + H^\frac{3}{8} h^\frac{3}{8} Z^{2c} ds^2_4 \]
\[
e^\phi = H^{-\frac{1}{4}} h^\frac{3}{4} Z^q, \quad k = \sum_{i=0}^{5} a_i + \frac{q}{2}, \quad l = a_0 + a_5 - \frac{q}{2} \tag{28}\]
where \( h \) and \( f \) are given in (27) but now with \( l \) given as above.

Let us now note some properties of these solutions.

(1) The parameters in all the above solutions are \( r_0 \), the boost parameters, and \((a_i, c, q)\) or \((\hat{a}_i, \hat{c})\), \(i = 0, 1, \ldots\) satisfying two constraints.

(2) The extremal limit is given as in (19) for each boost parameter \( \Theta \).

(3) One can further boost the \( D1 - D5 \) solution in (28) along the common isometric direction \( X^5 \). This will give a three charged system which can also be converted to three intersecting brane systems by a chain of S, T, U dualities. The corresponding solutions for this and other intersecting brane configurations in string/M theories can all be obtained straightforwardly by suitable combinations of lifting to 11 dimensions, boosts, S, and T dualities.

(4) The intersecting brane solutions for general values of \( r_0 \), the boost parameters, and \((a_i, c, q)\) or \((\hat{a}_i, \hat{c})\) can be obtained from the corresponding extremal ones by simple rules, similar to those given in [9] for \( a_i = c = q = 0 \), \( i = 1, 2, \ldots \). These rules are applicable here also with minor differences: In the present case, the non compact transverse space has a line element \( ds_{n+2} \) given in (9). One inserts \((Z^{2a_i}, Z^{2\hat{c}}, Z^q)\) factors in the appropriate places. The parameters \((a_i, c, q)\), or \((\hat{a}_i, \hat{c})\), satisfy the constraints given in (12), or (10), but are otherwise arbitrary. The functions \( H \) and \( F \) associated with each brane are of the form (16), with the exponent \( k \) given by

\[
k = \sum_{i \in \text{brane}} a_i - \frac{\lambda_{p} q}{2}
\]

where \( i \in \text{brane} \) means that \( i \) in the summation runs over the world volume indices of the corresponding brane, and \( \lambda_p = 0, -1, +1, \frac{3-p}{2} \) for M theory branes, F strings, NS 5—branes, and \( Dp \)—branes respectively. If there is a common isometric direction along which branes intersect then a further boost can be added in that direction, see (3) above. The corresponding \( H \) and \( F \) can be obtained using a formula similar to (13). See [9] for more details.

5. Brane solutions and a duality property

Let the spacetime be \( D \) dimensional with \( p \) compact directions. Consider the equations of motion that follow from the action given in (4) where \( \lambda_{p_1} \) is an arbitrary constant which may depend on \( p_1 \). Their solutions that describe
electric type $p_1$–branes smeared over $(p - p_1)$ compact directions are given by

$$
 ds^2 = H^A \left( -Z^{2a_0} dt^2 + \sum_{i=1}^{p_1} Z^{2a_i} dX^i \right) + H^B \left( \sum_{i=p_1+1}^{p} Z^{2a_i} dX^i \right) + Z^{2c} ds_{n+2}^2 
$$

$$
 e^\phi = H^C Z^q, \quad k = \sum_{i=0}^{p_1} a_i - \frac{\lambda_{p_1} q}{2} 
$$

$$
 A_{01...p_1} = \frac{CS (1 - F)}{H} \approx - \frac{S}{C} \frac{F}{H} 
$$

where $(A, B, C) = \left( -\frac{2(D-3-p_1)}{\Delta}, \frac{2(p_1+1)}{\Delta}, \frac{(D-2)\lambda_{p_1}}{\Delta} \right)$, other symbols are all as defined before, and

$$
 \Delta = (p_1 + 1)(D - 3 - p_1) + \frac{(D - 2)\lambda_{p_1}^2}{2}. 
$$

We obtained the solutions (29) by applying the rules given in (4), below equation (28), to the extremal solutions in [2, 6] and then verified that the equations of motion are satisfied. Instead of presenting these details of verification, we will now discuss some properties of the solutions (29) and then relate them to the corresponding solutions in [8] which were obtained by solving the equations of motion directly.

The ADM mass $M$ for the solutions, such as those in (29), is defined by [2, 3]

$$
 \lim_{R \to \infty} g_{00} = -1 + \frac{2\kappa^2}{(n+1)\omega_{n+1}V_p R^n} M 
$$

where $V_p$ is the volume of the $p$ dimensional compact space, $\omega_{n+1}$ is the area of the $(n+1)$ dimensional unit sphere, $\kappa^2$ is given in (4), and $R$ is the isotropic coordinate defined by

$$
 ds_{n+2}^2 = \frac{dr^2}{Z} + r^2 d\Omega_{n+1}^2 = Z \left( dR^2 + R^2 d\Omega_{n+1}^2 \right). 
$$

Equation (32) implies that $Z = \frac{r^2}{R^2}$ and $\frac{dR}{R} = \frac{dr}{r\sqrt{Z}}$, from which the functions $R(r)$ and $r(R)$ can be easily obtained up to a constant factor. Fixing this
factor by requiring $R \rightarrow r$ as $r \rightarrow \infty$, we get

$$2R^n = r^n - \frac{r_0^n}{2} + r^n \sqrt{1 - \frac{r_0^n}{r^n}}, \quad r^n = R^n (1 + Y)^2$$

(33)

where we have defined $Y = \frac{R^n}{R_0^n}$ and $R_0 = R(r_0)$; hence $R_0^n = \frac{r_0^n}{4}$. Also, $Z = G^2$ where

$$G = \frac{1 - Y}{1 + Y}.$$  

(34)

Substituting these expressions into the solutions given in (29) and using the expressions for $H$ and $F$ given in (16), it follows that the ADM mass $M$ defined in (31) is given by

$$M = \mathcal{N} \left( a_0 - kA S^2 \right) r_0^n$$

(35)

where $\mathcal{N} = \frac{(n+1)\omega_n + 1}{4k^2}$. Physically, the mass $M$ must be positive which will impose a mild inequality on the parameters of the solution.

The brane charge $Q$ of the solutions (29) can also be defined similarly using the asymptotic behaviour of the corresponding gauge potential. It is clear that $Q \propto kCS r_0^n$. We define the proportionality constant here so that we get $|Q| = M$ in the extremal limit (19). Hence,

$$Q = \pm \mathcal{N} A kCS r_0^n.$$

(36)

We will now discuss a duality property of the solutions (29). First, note that if $D = 10$ and $\lambda_p = \frac{3 - p}{2}$ then $\Delta = 16$ and equations (29) reduce to the $Dp_1$-brane solutions (21) from which other Dirichlet branes can be obtained by $T$ dualities. It turns out that the solutions (29) also have a similar duality property which is valid for any value of $D$ if $\lambda_p$ is a specific function of $p$ to be obtained below. We now describe this duality as applied to the solution (29).

Consider a “w-frame” where the metric $G_{\mu\nu}$ is given by

$$G_{\mu\nu} = e^{w\phi} g_{\mu\nu}$$

(37)

with $w$ a constant. Now, in the context of the $p_1$-brane solutions (29), consider the duality along an isometric direction $X^j, j \in (1, 2, \cdots, p)$. We
denote this duality by $T_j$ and define it in the $w$–frame by the following transformations:

$$e^\phi \to e^{\tilde{\phi}} = G_{jj}^{-w} e^{v \phi}, \quad G_{jj} \to \tilde{G}_{jj} = G_{jj}^{-U} e^{V \phi}$$

(38)

where $(u, v, U, V)$ are constants; the $(p_1 + 1)$ form gauge field becomes a $p_1$ or a $(p_1 + 2)$ form gauge field according to whether $X^j$ is parallel or transverse to the $p_1$–brane worldvolume respectively; and other fields remain unchanged in the $w$–frame.

Let $T_j$ and $T_{j'}$, $j \neq j'$, be the dualities (38) along two isometric directions $X^j$ and $X^{j'}$ respectively. We now impose the following natural requirements on the duality (38), in close analogy with T duality of the string theory.

(I) $T_j^2 = I$, the identity. Then $U = v$ and $uV = v^2 - 1$.

(II) $T_j T_{j'} = T_j T_{j'}$. Then $v = 1$ and $uV = 0$.

(III) Under (38), the $p_1$–brane solutions (29) transform to a $(p_1 - 1)$ or a $(p_1 + 1)$ brane solutions according to whether $X^j$ is parallel or transverse to the $p_1$–brane worldvolume respectively. After the duality, $X^j$ becomes transverse or parallel to the $(p_1 - 1)$ or the $(p_1 + 1)$ brane respectively.

Let the parameters $(\bar{a}_i, \bar{c}, \bar{q})$, $(\bar{A}, \bar{B}, \bar{C})$, and $\bar{A}_j$ defined by $g_{jj} \equiv H^{A_j} Z^{2a_j}$ in the solution (29) transform under the duality (38) to $(\bar{a}_i, \bar{c}, \bar{q})$, $(\bar{A}, \bar{B}, \bar{C})$, and $\bar{A}_j$ respectively. ³ The meaning of (III) and its consequences, obtained after some algebra, can now be stated as follows.

(a) The parameters $(\bar{a}_i, \bar{c}, \bar{q})$ must satisfy the constraints given in (12). This implies, upon using $U = v = 1$ and $uV = 0$ and after some algebra, that

$$\left( u, v, U, V \right) = \left( w, 1, 1, 0 \right), \quad w^2 = \frac{2}{D - 2}.$$  

(39)

The $T_j$ duality transformation (38) is now similar to that of T duality of the string theory:

$$e^{\tilde{\phi}} = G_{jj}^{-w} e^\phi, \quad G_{jj} \to \tilde{G}_{jj} = G_{jj}^{-1}.$$  

(40)

³$(\bar{a}_i, \bar{c}, \bar{q})$ and $(\bar{A}, \bar{B}, \bar{C}; \bar{A}_j)$ can be obtained by converting the solutions (29) to the $w$–frame, applying duality (38), and converting back to the Einstein frame. The resulting expressions, valid for any value of $(u, v, U, V)$, are unwieldy and will not be presented.
under which the transformed parameters \((\bar{a}_i, \bar{c}, \bar{q}; \bar{a}_j)\), \(i \neq j\) and \((\bar{A}, \bar{B}, \bar{C}; \bar{A}_j)\) are given by

\[
(\bar{a}_i, \bar{c}, \bar{q}; \bar{a}_j) = (a_i, c, q; a_j) + (w^2, w^2, -2w; w^2 - 2) \chi
\]

\[
(\bar{A}, \bar{B}, \bar{C}; \bar{A}_j) = (A, B, C; A_j) + (w^2, w^2, -w; w^2 - 2) \psi
\]

(41)

where \(\chi = a_j + \frac{wq}{2}\) and \(\psi = A_j + wC\).

(b) If \(X^j\) is parallel or transverse to the \(p_1\)-brane worldvolume then one must have

\[
A_j = A, \quad \bar{A}_j = B, \quad (\bar{A}, \bar{B}, \bar{C}) = (A, B, C)|_{p_1 - 1}
\]

or

\[
A_j = B, \quad \bar{A}_j = \bar{A}, \quad (\bar{A}, \bar{B}, \bar{C}) = (A, B, C)|_{p_1 + 1}
\]

(42)

respectively where \((\bar{A}, \bar{B}, \bar{C}; \bar{A}_j)\) are given in (41) and \((A, B, C)|_{p_1 \pm 1}\) means that \((A, B, C)\) are as given below equation (29) but with \(p_1\) replaced by \(p_1 \pm 1\).

Equations \(\bar{A}_j = B\) or \(\bar{A}\) imply that \(A + B + 2wC = 0\) which in turn implies that \(\lambda_{p_1}\) must be a specific function of \(p_1\): \(\lambda_{p_1} = \frac{D - 4 - 2p_1}{2} w\). Using the expressions for \((A, B, C), \lambda_p,\) and \(w^2\), we get \(A + wC = -(B + wC) = -\frac{D - 2}{\Delta}\).

Using the transformations in (41), it is now straightforward to show that the remaining equations in (42) are satisfied. Note also that, with \(\lambda_p\) given as above, we now have \(\Delta = \frac{(D - 2)^2}{4}\) as follows from equation (30).

(c) The expression for \(k\) in (29) does not transform. But, it must have the correct form when written in terms of \((\bar{a}_i, \bar{c}, \bar{q})\) given in (41). That is, if we set, with no loss of generality, \(j = p_1\) or \(= p_1 + 1\) when \(X^j\) is parallel or transverse to the brane respectively then the expression \(k = \sum_{i=0}^{p_1-1} a_i - \frac{\lambda_{p_1 - 1}q}{2}\) must also be expressible respectively as

\[
k = \sum_{i=0}^{p_1-1} \bar{a}_i - \frac{\lambda_{p_1 - 1}\bar{q}}{2} \quad \text{or} \quad \sum_{i=0}^{p_1+1} \bar{a}_i - \frac{\lambda_{p_1 + 1}\bar{q}}{2}.
\]

(43)

Using the transformations in (41), the specific function for \(\lambda_p\) obtained above, and \(w^2 = \frac{2}{D - 2}\), it is straightforward to show that equations (43) are satisfied. From now on in the following we assume that

\[
\lambda_{p_1} = \frac{D - 4 - 2p_1}{2} w, \quad w = \sqrt{\frac{2}{D - 2}}
\]

(44)

where we have chosen a positive sign for \(w\) with no loss of generality.
We now make a few remarks about the duality described above.

(1) The $D$ dimensional action (4) written in the $w$–frame (37) becomes

$$S_{p_1} = \frac{1}{2\kappa^2} \int d^D X \sqrt{-G} \left\{ e^{-\frac{\phi}{w}} \left( \mathcal{R}_G + \frac{1}{w^2}(\partial\phi)^2 \right) - \frac{F_{p_1+2}^2}{2(p_1 + 2)!} \right\}. \quad (45)$$

Note that the function $\lambda_{p_1}$ is such that the $p_1$–brane field strength $F_{p_1+2}$ does not couple to $\phi$ in the $w$–frame.

(2) The duality (40) is analogous to T duality of string theory but is valid for any value of $D$. For $D = 10$, and for $D < 10$ also, this is same as T duality of the string theory and all the expressions above reduce to the well known ones. For $D > 10$, this duality can be used as a solution generating technique but otherwise its significance, if any, is not clear.

(3) The 0–brane solutions in (29) can be generated by a $(D+1)$ dimensional boost iff $\lambda_0$ in (44) equals $\lambda$ in (3) which is possible only for $D = 10$. If $D \neq 10$ then it is not clear how to generate the 0–brane solutions without actually solving the relevant equations of motion.

(4) Note that the duality symmetry (40) transforms $p_1$–branes into $(p_1 \pm 1)$–branes and is not a symmetry of the equations of motion following from action (4). With no gauge fields, however, it is indeed a symmetry of the equations of motion. Just as in T duality of the string theory, this is a symmetry between long and short distances in the compact directions since $G_{ij} \rightarrow \frac{1}{G_{ij}}$. For the standard solutions where $(a_0, c) = (\frac{1}{2}, 0)$, and hence $a_i = q = 0$ for $i \neq 0$, the duality transformations are trivial. It may be of interest to consider other solutions presented here and explore the consequences of this duality. See [13] where a similar duality in time dependent solutions is explored in detail.

6. Conclusion

In this paper we have shown that the multiparameter brane solutions of string/M theories given in the literature can all be obtained by a suitable combination of boosts in eleven dimension, S and T dualities. We also described a duality property of the $D$ dimensional multiparameter solutions describing branes smeared in compact directions.

We have not discussed any physical implications of these solutions since they are beyond the scope of this paper. But they are likely to be interesting
and it is important to study them. We conclude now by mentioning a few aspects of the present solutions that may be studied further.

One should understand the brane antibrane interpretations [3, 4, 5, 7] of such solutions in terms of the present parametrisations and see if any insights can be obtained into tachyon condensation and the related dynamics. Multiparameter solutions have also been applied to the study of the so called S branes [14, 7]. It is of interest to study whether the present parametrisation in terms of boosts, S and T dualities applies in that context also.

Also, one should understand the singularities of the multiparameter solutions, their physical relevance and implications, and their resolution if possible. Perhaps, these can be studied along the lines of [3, 5, 6, 12]. It may also be of interest to study the multiparameter solutions using various string/M theory branes as probes and studying the geodesic motions of such probes. Our preliminary calculations, and also the results of [12], indicate that such a study is likely to be fruitful.

**Appendix: Relation to other solutions**

Recently, multiparameter brane solutions have been obtained by solving directly the equations of motion at various levels of generality [1, 3, 4, 6, 7, 8]. These multi parameter solutions, with suitable symmetry such as $SO(p)$ or $ISO(p,1)$, are interpreted to represent non BPS brane antibranes systems [3, 4, 6, 7, 8]. The singularities of such solutions have been discussed in [3, 5, 6]. The most general multiparameter brane solutions are given by Miao and Ohta (MO) in [8] from which all the others in [1, 3, 4, 6, 7] can be obtained.

The (intersecting) brane solutions for $D = 10,11$, namely for string/M theories, given by MO in [8] can all be obtained by the method presented here by repeated use of boosts, T and S dualities on the solution (11). Also, the smeared $p_1$–brane solutions of [8] for any value of $D$ are indeed the same as the present general solutions (29), obtained here by applying the rules given in (4) below equation (28) to the extremal solutions in [2, 6]. This can be shown as follows.

Consider the MO solutions given in [8]. The main equations required are those numbered (6, 26, 27, 29, 33-35, 38, 39, 42-47) in [8]. Their $\hat{d} = n$ and their $x_\mu \left(c_0, c_\alpha, c_\beta, c_\phi\right) = (a_0, a_i, c - \frac{1}{2n}, q)$ where $x = \sqrt{2\nu - 1} \geq 0$ and $\mu, \nu$
are constant parameters in [8]. Then, their constraints (42) and (29) are the same as those in (12) here. Also, their \( \rho = \frac{x-1}{x+1} \). Consider now their \((n+2)\) dimensional transverse line element given by

\[
dS_{n+2}^2|_{MO} = \left( \frac{f X^2}{g^2} \right)^{\frac{1}{n}} \left( \frac{d\tilde{r}^2}{f} + \tilde{r}^2 d\Omega_{n+1}^2 \right)
\]

where the functions \( g, X, \) and \( f \) can be written, after some algebra, as

\[
\tilde{g} = \frac{\sqrt{f} - \rho}{1 - \rho \sqrt{f}}, \quad X = \frac{(\sqrt{f} - \rho)(1 - \rho \sqrt{f})}{(1 - \rho)^2 \sqrt{f}}, \quad f = 1 - \frac{\mu}{\tilde{r}^n}.
\]

Define the isotropic coordinate \( R \) by

\[
\frac{d\tilde{r}^2}{f} + \tilde{r}^2 d\Omega_{n+1}^2 = \mathcal{F} \left( dR^2 + R^2 d\Omega_{n+1}^2 \right).
\]

It then follows that

\[
2R^n = \tilde{r}^n - \frac{\mu}{2} + \tilde{r}^n \sqrt{1 - \frac{\mu}{\tilde{r}^n}}, \quad \mathcal{F} = \frac{\tilde{r}^2}{R^2} = (1 + Y_1)^{\frac{2}{n}}, \quad f = \left( \frac{1 - Y_1}{1 + Y_1} \right)^2 \text{ where } Y_1 = \frac{\mu}{4R^n}.
\]

Using these expressions, we get

\[
dS_{n+2}^2|_{MO} = \left( \frac{f X^2}{g^2} (1 + Y_1)^{\frac{4}{n}} \right)^{\frac{1}{n}} \left( dR^2 + R^2 d\Omega_{n+1}^2 \right)
\]

Now, define \( r_0^n = x \mu, \ R_0^n = \frac{r_0^n}{1}, \) and \( Y = x Y_1 = \frac{R_0^n}{R^n}. \) The expressions for \( \tilde{g} \) and \( X \sqrt{f} \) now simplify considerably and, after some algebra, are given by

\[
\tilde{g} = \frac{1 - Y}{1 + Y}, \quad X \sqrt{f} = \frac{1 - Y^2}{(1 + Y)^2}.
\]

Using these expressions, it follows that

\[
dS_{n+2}^2|_{MO} = (1 + Y)^{\frac{2}{n}} \left( dR^2 + R^2 d\Omega_{n+1}^2 \right).
\]

Also, with \( r_0^n = x \mu, \) one gets \( g^2 = \tilde{g}^2 = \left( \frac{1 - Y}{1 + Y} \right)^2 = 1 - \frac{r_0^n}{r_n^n} = Z. \) Hence, with no loss of generality, we take \( g = G \) given in (34).

Remarkably \( Y_1, \) and hence \( \mu, \) has disappeared completely from \( \tilde{g} \) and the line element \( dS_{n+2}^2|_{MO} \) and only \( Y, \) and hence the combination \( x \mu, \) remains. This is now true for all the solutions given in [8] as can be seen easily. This
shows that MO solutions do not depend on two independent parameters $\mu$ and $\nu$, but only on one combination given by $r_0^n = x\mu = \mu \sqrt{2\nu - 1}$. 4

The expressions for the gauge field in [8] can now be written in terms of $Z$. Putting together various expressions and setting their $\beta = C^2$ here, it can be seen straightforwardly that their gauge field matches the present one given in (14) including the exponent $k$. It can now be seen by using the above expressions in the MO solutions that the (intersecting) brane solutions for $D = 10, 11$, namely for string/M theories, given by MO in [8] can all be obtained by the method presented here by repeated use of boosts, S and T dualities on the solution (11). Similarly, it can also be seen that the smeared $p_1$-brane solutions of [8] for any value of $D$, namely equation (44) with $A = 1$ in [8], are indeed the same as the general solutions (29) here.

In [8] Miao and Ohta also show that the solutions given in [1, 3, 4], which we refer to as ZZ ones, follow from the MO solutions; hence they follow from the present solutions also. Alternately, this can be shown directly by relating (29), with $p_1 = p$, to the ZZ solutions. The steps are straightforward but involve a fair amount of tedious algebra. We now mention relations between a few select quantities in the ZZ solutions and the present ones.

The ZZ solutions have $SO(p)$ symmetry, hence set $a_1 = a_2 = \cdots = a_p \equiv a$ here; $ISO(p, 1)$ symmetry requires further that $a_0 = a$. Also, their $r$ coordinate is our isotropic coordinate $R$, their $e^h = G$ here, and their $Cosh (k_{ZZ} h) - c_2 \ Sinh (k_{ZZ} h) = G^{-k_{ZZ}} H$ where $c_2 = C^2 + S^2, k_{ZZ} = 2k = 2a_0 + 2pa - \lambda_p q$, see equation (29), and $H$ is given in (16) here. After some algebra, one can relate all the ZZ quantities to the present ones. For example,

$$c_1 = \frac{\lambda_p}{2n} \left( 4(D - 2)a - c_3 \right) + 2q , \quad c_3 = 4(a - a_0) .$$

These relations can be inverted to obtain $(a_0, a, q)$ in terms of $(c_1, c_3, k_{ZZ})$. Using these inverse relations and after a long algebra, one can show that the

The relation between the radial coordinate $\tilde{r}$ of MO and the present one $r$ is given by

$$2R^n = \tilde{r}^n - \frac{\mu}{2} + \tilde{r}^n \sqrt{1 - \frac{\mu}{\tilde{r}^n}} = r^n - \frac{\mu}{2} + r^n \sqrt{1 - \frac{\mu}{r^n}}$$

which, indeed, involves both the parameters $\mu$ and $\nu$. But this is just a diffeomorphism. Only the combination $x\mu$ that appear in the solutions is physically relevant.
constraints (12) here, with $c$ eliminated from them, imply that

$$k_{zz}^2 = \frac{(n + 1)\Delta}{n(D - 2)} - \frac{c_1^2\Delta}{2(D - 2)} - \frac{(n + p)\Delta c_3^2}{4(D - 2)^2} + \frac{1}{4} \left(\lambda_p c_1 + \frac{nc_3}{D - 2}\right)^2$$

where $\Delta$ is as given in equation (30) here. It can be checked that the above expression agrees with that given in [3] for $D = 10$, $\lambda_p = \frac{3-p}{2}$, and with that given in [4] for $c_3 = 0$.

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