Performance Analysis of Uplink NOMA-Relevant Strategy Under Statistical Delay QoS Constraints
Mylène Pischella, Arsenia Chorti, Inbar Fijalkow

To cite this version:
Mylène Pischella, Arsenia Chorti, Inbar Fijalkow. Performance Analysis of Uplink NOMA-Relevant Strategy Under Statistical Delay QoS Constraints. IEEE Wireless Communications Letters, 2020, 9 (8), pp.1323-1326. 10.1109/LWC.2020.2990563 . hal-02553369

HAL Id: hal-02553369
https://cnam.hal.science/hal-02553369
Submitted on 15 May 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Performance Analysis of Uplink NOMA-Relevant Strategy Under Statistical Delay QoS Constraints

Mylene Pischella, Senior Member, IEEE, Arsenia Chorti, Member, IEEE and Inbar Fijalkow, Senior Member, IEEE

Abstract—A new multiple access (MA) strategy, referred to as non orthogonal multiple access - Relevant (NOMA-R), allows selecting NOMA when this increases all individual rates, i.e., it is beneficial for both strong(er) and weak(er) individual users. This letter provides a performance analysis of the NOMA-R strategy in uplink networks with statistical delay constraints. Closed-form expressions of the effective capacity (EC) are provided in two-users networks, showing that the strong user always achieves a higher EC with NOMA-R. Regarding the networks sum EC, there are distinctive gains with NOMA-R, particularly under stringent delay constraints.

Index Terms: NOMA, effective capacity, QoS delay constraint.

I. INTRODUCTION

Due to the strict delay requirements of many emerging ultra reliable low latency communication (URLLC) applications in beyond fifth generation (B5G) networks, the investigation of the interplay between statistical delay quality of service (QoS) constraints and wireless propagation conditions is highly timely. In this context, employing the link layer metric of the effective capacity (EC) – which indicates the maximum achievable rate under a target delay-outage probability threshold – emerges as a natural choice.

In parallel, non-orthogonal multiple access (NOMA) has been consistently shown to achieve higher sum spectral efficiencies when compared to OMA or other schemes. Moreover, NOMA may be required in B5G networks for very large users densities. Up to now, EC analyses in NOMA networks have focused primarily on the downlink. With respect to the uplink, in [8] it was shown that in two-user networks NOMA is more efficient than OMA at low signal to noise ratios (SNRs), whereas the opposite conclusion holds for a larger number of users. This letter consequently proves that NOMA-R is a very efficient strategy for delay constrained applications.

II. PROBABILITY OF USING NOMA WITH NOMA-R

A. System model

Let us consider a network with \( K \) users employing either OMA, NOMA or NOMA-R in the uplink. The achievable rate of user \( k \in S_K = \{1, \ldots, K\} \) is denoted in the following by \( R_k \), \( \tilde{R}_k \) and \( \hat{R}_k \) for NOMA, OMA and NOMA-R, respectively. We assume that the independent and identically distributed (i.i.d.) fading channel coefficients in the links to the base station (BS), denoted by \( h_k \), follow unit variance Rayleigh distributions. The channel gains \( x_k = |h_k|^2 \) are assumed ordered in decreasing order, so that \( x_k \leq x_{k+1} \) \( \forall k \in \{1, \ldots, K - 1\} \), and, their distributions can be found by using the theory of order statistics. We denote by \( \rho = \frac{1}{N} \) the transmit SNR with \( N \) the additive white Gaussian noise power in each link (assumed the same for all links for simplicity). The transmit power used by user \( k \) is denoted by \( P_k \) so that its received SNR is \( \rho P_k \). Let us assume that a cluster \( S \) of users in \( S_K \) with cardinality \( |S| \) is chosen for NOMA. The achievable rates (in bits/s/Hz) of the \( k \)-th user in \( S \), assuming perfect successive interference cancellation (SIC) decoding, can be expressed as:

\[
R_k = \frac{|S|}{K} \log_2 \left( 1 + \frac{\rho P_k x_k}{1 + \sum_{j \in S, j < k} \rho P_j x_j} \right), \forall k \in S. \tag{1}
\]

On the other hand, for the \( k \)-th user in \( S_K \setminus S \) the achievable rates with OMA are given as

\[
\hat{R}_k = \frac{1}{K} \log_2 (1 + \rho P_k x_k), \forall k \in S_K \setminus S. \tag{2}
\]

The coefficients \( |S|/K \) and \( 1/K \) account for fair division of the resources between the users employing NOMA and OMA,

\footnote{We note that \( \hat{R}_k \) focused on proportional fairness.}
respectively. Although in a standard NOMA network all users will employ NOMA, i.e., $S$ and $S_K$ coincide, in NOMA-R, $S$ is a subset of $S_K$. The formalization of the NOMA-R criterion stems from the requirement that $S$ includes all users whose achievable rates are greater with NOMA than with OMA, i.e.,

$$1 + \frac{\rho P_k x_k}{1 + \sum_{j \in S} \rho P_j x_j} \geq (1 + \rho P_k x_k)^{\frac{1}{\theta_k}}. \quad (3)$$

Users in $S_K \setminus S$ employ OMA. Consequently, the data rate in NOMA-R is $\hat{R}_k = R_k \forall k \in S$ and $\hat{R}_k = \bar{R}_k \forall k \notin S$. In terms of implementation, the BS identifies the subsets $S$ using its knowledge of the network’s full channel state information (CSI). It then transmits to each user a one-bit feedback indicating whether they should use OMA or NOMA and either the user’s index in the OMA subset or the NOMA cluster index if several disjoint NOMA clusters are selected. Therefore, NOMA-R imposes at most one-bit signalling overhead with respect to OMA.

The EC in bits/s/Hz of user $k \in S_K$ is defined as $[1]$: 

$$E_c^k = -\frac{1}{\theta_k T_f B} \ln \left( E[e^{-\theta_k T_f B x_k}] \right) = \frac{1}{\beta_k} \log_2 \left( E[e^{\beta_k \ln(2) x_k}] \right)$$

where $r_k$ is the achievable rate of user $k$ (equal to $R_k$, $\bar{R}_k$, or $\hat{R}_k$ if the user employs NOMA, OMA or NOMA-R, respectively), $T_f$ is the symbol period and $B$ is the occupied bandwidth. $\theta_k$, known as the QoS exponent $[1]$, is the exponent of the exponential decay of the buffer overflow probability. Under a constant packet arrival rate assumption, the EC is defined as the maximum achievable rate such that a target delay-bound violation probability is met. The more stringent the delay requirement, the larger the delay exponent $\theta_k$. To simplify the notation, we define $\beta_k = -\frac{\theta_k T_f B}{\ln(2)}$ as the negative QoS exponent. Closed form expressions of the EC when OMA and NOMA are employed were derived in $[8]$ for $K = 2$ and are not repeated in the present for compactness.

**C. Probability of using NOMA while in NOMA-R for $K = 2$**

In the following, we consider the two-users case and call user 2 the strong user, and user 1 the weak user. As $R_1 \geq \bar{R}_1$ is always fulfilled, the NOMA-R strategy is used whenever $R_2 \geq \bar{R}_2$. The NOMA-R condition consequently simplifies to $x_2 \geq \frac{\rho^2 x_1^2}{\rho P_2}$ and $\tau_2 = \tau(p) = Pr \left( x_2 \geq \frac{\rho^2 x_1^2}{\rho P_2} \right)$. Using the theory of order statistics, the pdf of $x_1$ is $2e^{-2x_1}$, the pdf of $x_2$ is $2e^{-x_2}(1 - e^{-x_2})$ and the joint pdf of $(x_1, x_2)$ is $2e^{-x_1}e^{-x_2}$. Then $\tau(p)$ is equal to:

$$\tau = \int_{x_1=0}^{p_1+\frac{\rho^2 x_1^2}{\rho P_2}} \int_{x_2=x_1}^{+\infty} 2e^{-x_1}e^{-x_2}dx_2dx_1$$

$$+ \int_{x_1=p_1+\frac{\rho^2 x_1^2}{\rho P_2}}^{+\infty} \int_{x_2=p_2+\frac{\rho^2 x_2^2}{\rho P_2}}^{+\infty} 2e^{-x_1}e^{-x_2}dx_2dx_1$$

$$= f(p) + g(p) \quad (5)$$

where

$$f(p) = 1 - e^{-\frac{p_1+\frac{\rho^2 x_1^2}{\rho P_2}}{\rho P_1}}$$

$$g(p) = \left( 1 - \text{erf} \left( \frac{2p_1+\sqrt{p_1^2+4p_2}}{2p_1+\rho P_1} \right) \right) \sqrt{p_2} \quad (7)$$

and $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-t^2} dt$. The boundary in both integrals in (5) is due to the fact that $x_2$ should always be such that $x_2 \geq x_1$, but $\frac{\rho^2 x_1^2}{\rho P_2}$ is lower than $x_1$ if $x_1 \leq \frac{p_1+\frac{\rho^2 x_1^2}{\rho P_2}}{2p_1+\rho P_1}$. $\tau(p)$ is validated with Monte-Carlo simulations, shown in Fig. 1 assuming that $P_1 + P_2 = 1$. 

For the specific case $k = K$, the joint probability density function (pdf) of $(y_K, z_K)$ can be derived by using the moment generating function (MGF) of the weighted sum of the lowest order statistics, denoted as $M_{z_K}$, as in $[10]$. The pdf of $z_k$ can be obtained by using the following properties: $M_{z_K} = \prod_{j=1}^{K-1} M_{z_j} (\rho P_j x_j)$ and $L^{-1} (M_{z_j} (\rho P_j x_j)) (t) = \frac{1}{\rho P_j x_j} (1 - t)^{-1}$, where $L^{-1}$ is the inverse Lagrange transform. Consequently, the joint pdf $f_{z_K, y_K = y}$ can be derived from that of $z_k$ and $y_K$ in $[10]$ eq. (3.41). However, a closed-form expression of $Pr \left( \frac{(1+y_K) + (1+y_K) \rho P_2}{(1+y_K) \rho P_2} \geq z_K \right)$ cannot be obtained, and similarly to the conclusion in $[11]$ Section VD, it should be calculated with a mathematical software. Moreover when $k < K$, to the best of our knowledge, the joint pdf of $(y_K, z_K)$ is yet unknown. Consequently, when $K > 2$, (3) cannot be evaluated analytically with reasonable effort. For all these reasons, our analytical study is limited to $K = 2$ while we provide numerical results for $K > 2$.

Finally, examining the case non i.i.d. channel coefficients, we note that the case of non-identical exponential distributions can be treated as in $[12]$, while the case of non independent coefficients as in $[13]$. If full CSI is not available at the BS, CSI uncertainties can be inserted in users’ distributions to derive an MA selection strategy $[13]$, and, when $K = 2$, (3) can be formulated as a binary hypothesis testing problem $[14]$.

**B. Probability of using NOMA while in NOMA-R for $K \geq 2$**

When NOMA-R selects NOMA for $k \in S$, the sum rate can easily be shown to be equal to $\frac{1}{\beta_k} \log_2 \left( 1 + \sum_{k \in S} \rho P_k x_k \right)$. Consequently, the subset $S$ of $S_K$ that maximizes the sum rate while satisfying (3) is selected by NOMA-R. Several disjoint clusters may also be selected if they independently verify (3).

The probability of using NOMA when NOMA-R is employed, denoted by $\tau_K$ in the following, is the union of the probabilities to verify (3) for any subset $S \subseteq S_K$ with $|S| \geq 2$ and its analytical derivation is very evolved. As an illustrative example, let us assume $S = \{1, \ldots, k\}$ with $k \leq K$. Let $y_k = \rho P_k x_k$ be the weighted $k$th order statistics and $z_k = \sum_{i=1}^{k-1} \rho P_i x_i$ the weighted sum of the lowest $(k-1)$th order statistics. Then $\tau_k$ is equal to:

$$\tau_k = Pr \left( \bigcap_{i=1}^{k-1} \left( 1 + y_i - (1 + y_i)^{\frac{1}{\theta_i}} \geq z_i \right) \right). \quad (4)$$

For the specific case $k = K$, the joint probability density function (pdf) of $(y_K, z_K)$ can be derived by using the
Lemma 1. \(\tau(\rho)\) tends to 1 when \(\rho\) tends to 0 and \(\tau(\rho)\) tends to 0 when \(\rho \gg 1\).

Proof. When \(\rho \to 0\), \(f(\rho) \to 1\). Moreover, \(\text{erf}(x) \approx 1 - e^{-x^2/(x^2)}\) when \(x \gg 1\), which implies that \(g(\rho) \approx a_1 - b_1/\rho\) with \((a, b)\) two strictly positive constants. Therefore, \(g(\rho) \to 0\) when \(\rho \to 0\). Furthermore, when \(\rho \gg 1\), \(f(\rho) \to 0\). As \(\text{erf}(x) \approx \frac{1}{2}x e^{-x^2/2}\) when \(x \to 0\), \(g(\rho) \approx a_2 + b_2/\rho\) where \((a_1, a_2, b_1, b_2)\) are constants and \(g(\rho) \to 0\).

III. NOMA-R EFFECTIVE RATES

The ECs of user \(k = 1, 2\) when employing NOMA, OMA or NOMA-R are denoted by \(E^k_{c,N}\), \(E^k_{c,O}\) and \(E^k_{c,R}\) respectively.

A. NOMA-R EC of user 1

We hereafter derive closed-form expressions of the EC with NOMA-R. As \(\tau\) corresponds to the proportion of time spent in NOMA and \((1 - \tau)\) is the proportion of time spent in OMA, the EC of user \(k \in \{1, 2\}\) when the NOMA-R strategy is employed is given as:

\[
E^1_{c,R} = \frac{1}{\beta_k} \log_2 \left( \mathbb{E} \left[ e^{\beta_k \tau R_k + \beta_k (1-\tau) R_k} \right] \right). \tag{8}
\]

For user 1, the EC achieved with the NOMA-R strategy is:

\[
E^1_{c,R} = \frac{1}{\beta_1} \log_2 \left( \mathbb{E} \left[ (1 + \rho P_{2} x_1)^{\frac{\beta_1 (\tau + 1)}{\beta_2}} \right] \right)
= \frac{1}{\beta_1} \log_2 \left( \frac{2}{\rho P_1} U \left( 1, 2 + \frac{\beta_1 (\tau + 1)}{\beta_2}, \frac{2}{\rho P_1} \right) \right), \tag{9}
\]

where \(U(a, b, z) = \int_0^\infty e^{-z t (a-1)(1+t) (b-a-1)} dt\) denotes the confluent hypergeometric function.

Lemma 2. \(E^1_{c,R}\) is monotonically increasing with \(\rho\).

Proof. Let us consider \(\rho_1\) and \(\rho_2\) such that \(\rho_1 < \rho_2\). For any value of \(\beta_1\), let us define: \(\beta_{1,a} = \frac{\beta_1 (\tau + 1)}{\beta_2} (1 + \tau (\rho_1))\) and \(\beta_{1,b} = \frac{\beta_1 (\tau + 1)}{\beta_2} (1 + \tau (\rho_2))\). Then from [8, eq.(11)] and [9], \(E^1_{c,R}(\beta_1, \rho_1) = E^1_{c,N}(\beta_{1,a}, \rho_1)\) and \(E^1_{c,R}(\beta_1, \rho_2) = E^1_{c,N}(\beta_{1,b}, \rho_2)\). As \(\tau(\rho)\) is a decreasing function with respect to \(\rho\), and \(\beta\) are negative, \(\beta_{1,a} \leq \beta_{1,b}\). Moreover, \(E^1_{c,R}(\beta_1, \rho_1)\) is increasing both with respect to \(\beta\) and to \(\rho\) according to [8]. Consequently, \(E^1_{c,N}(\beta_{1,a}, \rho_1) \leq E^1_{c,N}(\beta_{1,a}, \rho_2) \leq E^1_{c,N}(\beta_{1,b}, \rho_2)\) and:

\[
E^1_{c,R}(\beta_1, \rho_1) \leq E^1_{c,R}(\beta_1, \rho_2) \quad \forall \rho_1 < \rho_2 \tag{10}
\]

B. NOMA-R EC of user 2

For user 2, the NOMA-R EC is given by

\[
E^2_{c,R} = \frac{1}{\beta_2} \log_2 \left( \mathbb{E} \left[ \left( 1 + \rho P_{2 x_2} \right)^{\frac{\beta_2 \tau}{1 + \rho P_{1 x_1}}} \right] \right) \tag{11}
\]

Lemma 3. When \(\rho\) tends to 0, the EC with the NOMA-R strategy becomes equivalent to that of OMA given in [9].

Proof. When \(\rho \to 0\), \(\tau(\rho) \to 1\) and therefore \((1 + \rho P_{2 x_2})^{\frac{\beta_2 \tau}{1 + \rho P_{1 x_1}}} \to 1\). Consequently, \(E^2_{c,R}\) tends to \(E^2_{c,N}\).

Lemma 4. When \(\rho \gg 1\), the EC with the NOMA-R strategy becomes equivalent to that of OMA and its closed-form expression is given by:

\[
E^2_{c,R} \approx \frac{1}{\beta_2} \log_2 \left( \mathbb{E} \left[ (\rho P_{2 x_2})^{\frac{\beta_2 \tau}{1 + \rho P_{1 x_1}}} \right] \right)\tag{12}
\]

Proof. When \(\rho \gg 1\), \((1 + \rho P_{2 x_2})^{\frac{\beta_2 \tau}{1 + \rho P_{1 x_1}}} \to 1\) because \(\tau \to 0\). Then using \((1 + x)^{\rho} \approx x^{\rho}\), the EC of user 2 becomes:

\[
E^2_{c,R} \approx \frac{1}{\beta_2} \log_2 \left( \mathbb{E} \left[ (\rho P_{2 x_2})^{\beta_2 \tau} \right] \right)
\approx \frac{1}{\beta_2} \log_2 \left( \int_0^\infty 2 (\rho P_{2 x_2})^{\beta_2 \tau} e^{-x^2 (1 - e^{-x^2})dx_2} \right).
\]

The integral’s closed-form expression leads to [12].

Theorem 1. The EC of user 1 is always larger with NOMA than with NOMA-R, while OMA is the worst strategy in terms of EC. Moreover, the EC of user 2 is always larger with the NOMA-R strategy than with OMA.

Proof. The NOMA-R instantaneous rate of user 1 is equal to \(R_1 = \frac{1 + \tau}{2} \log_2 (1 + \rho P_{1 x_1})\), according to [9]. Therefore \(R_1 \leq R_1 \leq R_1\). Then as \(\beta_1\) is negative, \(e^{\beta_1 R_1} \leq e^{\beta_1 R_1} \leq e^{\beta_1 R_1}\) and \(\mathbb{E} \left[ e^{\beta_1 R_1} \right] \leq \mathbb{E} \left[ e^{\beta_1 R_1} \right] \leq \mathbb{E} \left[ e^{\beta_1 R_1} \right]\), so that

\[
E^1_{c,N} \geq E^1_{c,R} \geq E^1_{c,O} \tag{13}
\]

The NOMA-R rate of user 2 is \(R_2 = \max \{R_2, R_2\}\). Following the same steps as for user 1, we conclude that

\[
E^2_{c,R} \geq E^2_{c,N} \quad \text{and} \quad E^2_{c,R} \geq E^2_{c,O} \tag{14}
\]

Remark: \(E^2_{c,R}\) asymptotically tends to either \(E^2_{c,N}\) or \(E^2_{c,O}\), both of which are monotonically increasing with \(\rho\). Moreover, contrary to the NOMA strategy, the NOMA-R strategy does not lead to a saturation of the EC of the strong user because of [14] and because \(E^2_{c,O}\) increases without bound with \(\rho\) [8].
NOMA-R is an advantageous strategy for delay constrained applications in BSG, e.g., URLLC, particularly as the users’ delay-outage probability constraints become more stringent.

REFERENCES

[1] D. Wu and R. Negi, “Effective Capacity: a Wireless Link Model for Support of Quality of Service,” IEEE Trans. Wireless Commun., vol. 2, no. 4, pp. 630–643, July 2003.
[2] M. Amjad, L. Musavian, and M. H. Rehmani, “Effective Capacity in Wireless Networks: A Comprehensive Survey,” IEEE Commun. Surveys Tuts., pp. 1–33 (early access), July 2019.
[3] L. Dai, B. Wang, Z. Ding, Z. Wang, S. Chen, and L. Hanzo, “A Survey of Non-Orthogonal Multiple Access for 5G,” IEEE Commun. Surv. Tut., vol. 20, no. 3, pp. 2294–2323, thirdquarter 2018.
[4] Y. Kanaras, A. Chorti, M. Rodrigues, and I. Darwazeh, “An optimum detection for a spectrally efficient non orthogonal FDM system,” in Proc. 13th Int. OFDM WS, Aug 2008, pp. 65–68.
[5] G. Liu, Z. Ma, X. Chen, Z. Ding, F. R. Yu, and P. Fan, “Cross-Layer Power Allocation in Non-Orthogonal Multiple Access Systems for Statistical QoS Provisioning,” IEEE Trans. Veh. Technol., vol. 66, no. 12, pp. 11388–11393, Dec 2017.
[6] W. Yu, L. Musavian, and Q. Ni, “Link-Layer Capacity of NOMA Under Statistical Delay QoS Guarantees,” IEEE Commun. Lett., vol. 20, no. 10, pp. 4907–4922, Oct 2018.
[7] C. Xiao, J. Zeng, W. Ni, R. P. Liu, X. Su, and J. Wang, “Delay Guarantee and Effective Capacity of Downlink NOMA Fading Channels,” IEEE J. Sel. Topics Signal Process., vol. 13, no. 3, pp. 508–523, June 2019.
[8] B. Mouktar, W. Yu, A. Chorti, and L. Musavian, “Performance Analysis of NOMA Uplink Networks under Statistical QoS Delay Constraints,” in Proc. 14th Int. Conf. Commun. (ICC) 2020, pp. 1–7.
[9] M. Pischella and D. Le Ruyet, “NOMA-Relevant Clustering and Resource Allocation for Proportional Fair Uplink Communications,” IEEE Wireless Commun. Lett., vol. 8, no. 3, pp. 873–876, June 2019.
[10] H.-C. Yang and M.-S. Alouini, “Order Statistics in Wireless Communications: Diversity, Adaptation, and Scheduling in MIMO and OFDM Systems,” Cambridge University Press, 2011.
[11] Y. Ko, H. Yang, S. Eom, and M. Alouini, “Adaptive Modulation with Diversity Combining Based on Output-Threshold MRC,” IEEE Trans. Wire. Commun., vol. 6, no. 10, pp. 3728–3737, October 2007.
[12] N. Balakrishnan, “Order statistics from non-identical exponential random variables and some applications,” Commun. Stat. - Theor. Meth., vol. 18, no. 2, pp. 203–253., 1994.
[13] A. Alsawah and I. Fijalkow, “Practical radio link resource allocation for fair QoS-provision on OFDMA downlink with partial chanel-state information,” EURASIP J. Adv. Signal Proc., pp. 1–16, 2009.
[14] S. W. H. Shah, M. M. U. Rahman, A. N. Mian, A. Imran, S. Mumtaz, and O. A. Dobre, “On the impact of mode selection on effective capacity of device-to-device communication,” IEEE Wireless Communications Letters, vol. 8, no. 3, pp. 945–948, 2019.