Why Halley did not discover proper motion and why Cassini did

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Abstract
In 1717 Halley compared contemporaneous measurements of the latitudes of four stars with earlier measurements by ancient Greek astronomers and by Brahe, and from the differences concluded that these four stars showed proper motion. An analysis with modern methods shows that the data used by Halley do not contain significant evidence for proper motion. What Halley found are the measurement errors of Ptolemaios and Brahe. Halley further argued that the occultation of Aldebaran by the Moon on 11 March 509 in Athens confirmed the change in latitude of Aldebaran. In fact, however, the relevant observation was almost certainly made in Alexandria where Aldebaran was not occulted. By carefully considering measurement errors Jacques Cassini showed that Halley’s results from comparison with earlier astronomers were spurious, a conclusion partially confirmed by various later authors. Cassini’s careful study of the measurements of the latitude of Arcturus provides the first significant evidence for proper motion.

Keywords
discovery proper motion – Edmond Halley – Jacques Cassini

Introduction
The possibility of motion of the stars relative to one another was raised by Hipparchos, around 130 BC, as we know from the discussion by Ptolemaios in the first chapter

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of Book 7 of the Almagest. Hipparchos argued that the conclusion that the stars are fixed on the celestial sphere requires proof that they do not move with respect to one another. He investigated this by considering configurations of stars, such as alignments, in which it is easier to detect relative position changes, and by comparing his data with those of Timocharis, who lived one and a half century before. Hipparchos found no evidence for relative motion. Ptolemaios affirms this result over the longer time span of about 280 years separating him from Hipparchos and even more from Timocharis. The configurations and alignments connect stars in the Zodiac to those above or below it, since the main goal was to discover whether stars outside the Zodiac increase their longitudes at the same rate as the stars in the Zodiac. However, Ptolemaios notes explicitly that the study of these alignments can also uncover displacements of stars within the separate parts of the alignments.

There is possibly an echo to this in the statement by Macrobius who lived around 400 AD, that the stars have their ‘own motion’ (suo motu) on top of their motion with the heavenly sphere. Macrobius refers to the incredible number of centuries required for the completion of ‘one circuit’ (una ambitio) implying repeated passages in a closed loop. The main problem in interpreting Macrobius is that he clearly doesn’t know what he is talking about. Because of his reference to a closed loop, it is probable that his statement refers not to proper motion of individual stars, but to precession.

Brahe concluded that the ancient measurements are rather inaccurate. For example, he compares the latitudes of Aldoboram (Aldebaran) that result from the measurements by Timocharis, Hipparchos and Ptolemaios, and ‘cannot but wonder’ about the large differences between them. He warned, correctly, that the latitude from Ptolemaios is erroneous: ‘widest of the mark’.

In 1717 Halley published Considerations on the change of the latitudes of some of the principal fixt stars, in which he ignores the warning by Brahe and compares contemporaneous measurements with those of Hipparchos and Ptolemaios for the latitude of four stars, viz. Palilicium or the Bull’s Eye (i.e. Aldebaran), Sirius, Arcturus and the bright shoulder of Orion (Betelgeuse). He also compares contemporaneous measurements of Sirius with those of Brahe. On the basis of this he suggested that all four stars had shown proper motion.

In the same paper, Halley refers to an occultation of Aldebaran observed on 11 March 509, in or near Athens according to the 17th century French astronomer and polymath Ismaël Boulliau (Bullialdus): ‘when in the beginning of the night the Moon was seen to follow that star very near, and seemed to have eclipsed it’. Boulliau computes that such an occultation could not have happened, from which Halley concluded that an actual occurrence of an occultation was possible only if ‘the latitude of Pallicium were much less than we at this time find it’. As we will see below Halley refers to the absolute value of the latitude, and implies a proper motion in the southern direction.

By the time of Halley the concept that stars are attached to a sphere, or to spheres if they do not all participate in the same precessional motion, had been replaced with the idea of stars moving in three-dimensional space. Because in the Copernican system the daily, yearly and long-term precessional motions of the stars are apparent, merely reflecting rotation, revolution and precession of the axis of the Earth, there was no need to assume that the stars all share the same motions because they are attached to a sphere. With his discovery that planets move in ellipses, Kepler was forced to conclude...
Table 1. Ecliptic latitudes of the four stars discussed by Halley in various catalogues: Ptolemaios / Hipparchos in a modern edition (Toomer\textsuperscript{1}) and in an edition by Hudson\textsuperscript{10}; Brahe in the edition by Kepler\textsuperscript{11}; and Flamsteed\textsuperscript{12}. For each catalogue the one-sigma error $\sigma_\beta$ in the latitude is listed (from Verbunt & van Gent\textsuperscript{13,14}, and Lequeux\textsuperscript{15}). We list the difference $d_\beta$ between the catalogue latitude $\beta_{\text{cat}}$ with the correct latitude $\beta_{\text{HIP}}$ computed for the catalogue equinox from data obtained with the HIPPARCOS satellite, $d_\beta = \beta_{\text{cat}} - \beta_{\text{HIP}}$. We also list the latitudes measured by Richer and Cassini, and from the re-analysis of Brahe’s measurements by Cassini\textsuperscript{6}.

| catalogue        | date  | $\sigma_\beta$ | $\beta$ (Arcturus) | $d_\beta$ | $\beta$ (Sirius) | $d_\beta$ |
|------------------|-------|----------------|--------------------|-----------|------------------|-----------|
|                  |       | (')            |                     |           | (')              |           |
| Ptolemaios ed.   | -128  | 23             | +31 30             | -41.4     | -39 10           | -2.4      |
| ed. Hudson       |       |                | +31 10             | -61.4     | -39 10           | -2.4      |
| Brahe            | 1601  | 2              | +32 02 30          | 1.8       | -39 30           | 0.8       |
| Flamsteed        | 1690  | 0.5            | +30 57 00          | 0.0       | -39 32 08       | -0.1      |
| Brahe/Cassini    | 1584  |                | +31 00 29          | -1.0      | -39 32 10       | -1.5      |
| Richer           | 1672  |                | +30 57 25          | -0.4      | -39 31 55       | -0.1      |
| Cassini          | 1738  |                | +30 55 26          | 0.4       | -39 33 00       | (a)       |

| catalogue        | date  | $\sigma_\beta$ | $\beta$ (Aldebaran) | $d_\beta$ | $\beta$ (Betelgeuze) | $d_\beta$ |
|------------------|-------|----------------|---------------------|-----------|----------------------|-----------|
|                  |       | (')            |                     |           | (')                  |           |
| Ptolemaios ed.   | -128  | 23             | -5 10               | 27.0      | -17 00               | -41.1     |
| ed. Hudson       |       |                | -5 30               | 7.0       | -17 10               | -51.1     |
| Brahe            | 1601  | 2              | -5 31               | -1.3      | -16 06               | -1.2      |
| Flamsteed        | 1690  | 0.5            | -5 29 49            | -0.5      | -16 04 26            | -0.3      |
| Brahe/Cassini    | 1589  |                | -5 30 23            | -0.6      |                      |           |
| Cassini          | 1738  |                | -5 29 34            | -0.4      |                      |           |

(a) the latitude of Sirius is given by Cassini as ‘larger by about a minute’ than found in Flamsteed, Richer and Cassini’s reanalysis of Brahe.

that the planets move freely in space, thereby also undermining the concept of a sphere of the stars. It is important to note, however, that 18th century astronomers could no more determine distances or radial motions of the stars than Hipparchos or Ptolemaios, so that technically the challenge of observationally determining proper motion is the same in the 18th century as in ancient Greece.

How conclusive is the evidence produced by Halley for proper motions? This was questioned already by Jacques Cassini\textsuperscript{6}, the son of Gian Domenico Cassini, who argued that the measurements by Hipparchos / Ptolemaios were too inaccurate to be of use for the determination of proper motion, and who corrected the latitudes given by Brahe. In modern times, van de Kamp\textsuperscript{7} notes that the proper motions of Aldebaran and Betelgeuse are very small, and that Halley’s results for these stars must be considered spurious. Even so, the determination of proper motions of Sirius and Arcturus by Halley is often still considered valid\textsuperscript{8}. With regard to the occultation, Neugebauer\textsuperscript{9}, referring to computations by Stephenson, agrees with Boulliau that no occultation took place! Notwithstanding these problems, Halley is still generally credited with the discovery of proper motion. In this paper we take a close look at the argumentation by Halley, and at the study by Cassini.

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Table 2. Columns 2-5: Difference in latitude found by subtracting from the values given by Ptolemaios (left) or Brahe (right) the values found by converting the positions from Halley’s time to the equinox of the old catalogue. (a) as given by Halley, (b) converting positions from Flamsteed using obliquity values used by Halley, (c) idem using correct values for obliquity (d) the actual difference $\Delta \beta_{\text{HIP}}$ between latitudes computed for both epochs from HIPPARCOS satellite data.

| star      | $\Delta \beta (')$ Ptolemaios to Halley | $\Delta \beta (')$ Brahe to Halley |
|-----------|------------------------------------------|-------------------------------------|
|           | (a) | (b) | (c) | (d) | (a) | (b) | (c) | (d) |
| Aldebaran | 35  | 38.6| 33.4| 5.9 | 1.1 | −0.5| 0.3 |
| Sirius    | 42  | 44.1| 36.2| 38.6| 4.5 | 4.6 | 2.8 | 1.9 |
| Arcturus  | 33  | 28.1| 27.2| 68.6| 4.6 | 5.2 | 3.3 |
| Betelgeuse| −60 | −34.1| −41.1| −0.3| 0.9 | −0.9| 0.0 |

**Latitude differences: Halley**

Precession at constant obliquity leads to an increase of the ecliptic longitude, but does not affect the latitude. The slow change of obliquity leads to slow changes in latitude. In Table 1 we collect ecliptic latitudes of Aldebaran, Sirius, Arcturus and Betelgeuse from the star catalogues of Ptolemaios, Brahe, and Flamsteed. The latitudes given by Ptolemaios (taken from Toomer’s edition\[1\]) differ by much more from those given by 17th century astronomers than can be explained with the change in obliquity. In our analysis we assume that Ptolemaios corrected longitudes determined by Hipparchos, undo his corrections and use the epoch of Hipparchos, $-128\[13\]$. That Ptolemaios corrected measurements by Hipparchos for precession rather than make observations himself, was concluded by Brahe; Halley will have been aware of this. Our results are not affected by this choice.

In his 1712 edition of the star catalogue of Ptolemaios, Hudson\[19\] acknowledges emendations made by Halley. In the same year Halley edited a star catalogue\[20\] based on data from Flamsteed, which he had surreptitiously obtained in collusion with Newton\[18\]. This work may have alerted Halley to large latitude differences. (The positions of the four stars in Table 1 in the 1712 pirate edition are identical to those in the 1725 edition by Flamsteed\[22\] himself.)

The proper motions in the latitude direction cannot be derived simply by dividing the difference between the latitudes for different catalogues by the time interval, but require correction for the change in obliquity between the catalogue equinoxes. Halley remarks that the value for the obliquity used by Brahe is $23^\circ 31' 30''$ larger than in Halley’s time. Since Brahe used $\epsilon = 23^\circ 31' 30''$, this implies that Halley for his own epoch used $\epsilon = 23^\circ 29'$, and (since he mentions that the value at the time of Ptolemaios was $22'$ larger) $\epsilon = 23^\circ 51'$ for 150, the epoch he assumed for Ptolemaios. With these values Halley makes the conversions of the latitude from the equinox of his time to that of Ptolemaios, and derives the remaining differences in latitude. We list his results in Table 2. Halley gives no information on his sources for the star catalogue by Ptolemaios and for the contemporaneous catalogue, but it stands to reason that for the latter he used...
the data of Flamsteed. From Table 1, we see that use of the catalogues of Brahe would have led to very similar values. Halley also does not explain how he computed the precession.

The one-sigma errors \(\sigma_\beta\) in latitude and the deviations \(d_\beta\) in latitude by Ptolemaios (Table 1) are comparable in size to the latitude differences \(\Delta_\beta\) derived by Halley: in modern parlance only the (very wrong) \(\Delta_\beta\) of Betelgeuse is significant at the two-sigma level. The error distribution of the latitudes in the star catalogue of Ptolemaios has more large deviations than a Gaussian, hence the significance of the \(\Delta_\beta\) values is even less than estimated from the Gaussian with \(\sigma_\beta = 23'\). Indeed, three of the four stars in Table 1 have \(|d_\beta| > 23'\). This indicates that the \(\Delta_\beta\) values by Halley are spurious, and that the proximity of the value for Sirius to the correct value is accidental. An interesting twist is given by the positions assigned to Aldebaran and Arcturus by Hudson, who acknowledges Halley, in his edition of the star catalogue of Ptolemaios. These values, which we also list in Table 1, imply much smaller proper motions. Apparently, Halley changed his mind between 1712 and 1717.

The exercise of comparing contemporaneous latitudes with those in an old catalogue was repeated by Halley for the catalogue of Brahe, for Sirius only. Halley gives two values for \(\Delta_\beta\) for Sirius. Taking into account that the catalogue of Brahe has more large errors than described with a Gaussian, even the larger value of 4.5' is not significant at the two-sigma level. In addition, Halley allows the possibility that \(\Delta_\beta = 2'\) (albeit for the somewhat unlikely assumption that Brahe ignored atmospheric refraction).

The uncertainty in the obliquity also adds to the errors in the comparison with Ptolemaios. At the end of his article Halley expresses some doubt that the value for the obliquity at the time of Hipparchos to Ptolemaios was indeed 22' larger than in his own time. This doubt is justified, as the correct difference in obliquity is 15' and 13' for the epochs of Hipparchos and Ptolemaios, respectively (Seidelman). The deviations in latitude due to the uncertainty in \(\epsilon\) are much smaller than the typical uncertainty \(\sigma_\beta\) in the star catalogue of Ptolemaios. We return to this below.

### Latitude differences: modern

To see whether the data available to Halley imply proper motion when analysed with modern methods, we re-analyse them twice, first with the values for the obliquity used by Halley, and then with the correct values for the obliquity \(\epsilon\). This enables us to gauge the effect of using wrong values for \(\epsilon\). We convert the ecliptic coordinates \(\lambda, \beta\) from the Historia Coelestis to equatorial coordinates, precess these with modern equations to the epoch of Ptolemaios or Brahe, convert the result into ecliptic coordinates \(\lambda_2, \beta_2\) at the older epoch, and subtract the latitude in the older catalogue from \(\beta_2\). The equatorial coordinates computed from the ecliptic coordinates given by Flamsteed are identical within rounding errors to the equatorial coordinates in his catalogue, when the value for \(\epsilon = 23^\circ 29'\) is used; after precession and reconversion to ecliptic coordinates with \(\epsilon = 23^\circ 51'\), the \(\Delta_\beta\) listed as (b) in Table 2 may be computed. A modern equation for the obliquity gives \(\epsilon = 23^\circ 28'46.5''\) for 1691 and \(\epsilon = 23^\circ 42'39.6''\) for −128; with these values we compute the \(\Delta_\beta\) listed as (c) in Table 2. We also list as (d) the correct change in latitude, computed from the data from the HIPPARCOS Catalogue. The same calculation provides the deviation in latitude \(d_\beta\) of the position given by...
Figure 1. Left: differences $d_\beta$ between correct and catalogued positions of stars in Ptolemaios. Stars from Table 1 are highlighted. P=Palilicium (Aldebaran). Right: Positions of stars near Aldebaran, no.14 in Taurus in the star catalogue of Ptolemaios (red), and correct positions computed from data of the HIPPARCOS satellite (black). Scale in degrees. The inset indicates the magnitude scale. Ptolemaios puts Aldebaran too far North.

Ptolemaios with the correct one. The values for $d_\beta$ in Table 1 are taken from Table 6 of Verbunt & van Gent with a change in sign, i.e. $d_\beta > 0$ indicates that the position given by Ptolemaios is too far North. As an illustration we show in Figure 1 the differences $d_\beta$ for all stars in the Almagest, and also compare the positions in the Almagest of the stars near Aldebaran with the correct ones.

Table 2 shows that the values we compute with the obliquities given by Halley are close to the values he gives, but not identical. Halley indicates that his $\Delta\beta$ for Betelgeuse is a very rough estimate. For the other stars the difference may arise from a variety of causes: he certainly used different precession equations, and possibly used slightly different star positions and obliquity both for his time and for Ptolemaios, a different epoch for Ptolemaios, and a different overall correction to the longitudes in Ptolemaios. The proximity of the values of $\Delta\beta$ listed as (a) and (b) in Table 2 shows that our effort to replicate Halley is close to what he actually did.

The effect of a wrong value for the obliquity depends on the celestial position, in particular on the ecliptic longitude. It is smallest for Arcturus, and largest for Sirius.

None of the displacements in latitude $\Delta\beta$ computed with correct values for the obliquities at the epochs of Flamsteed and Ptolemaios, listed as (c) in Table 2, is significant, due to the relatively large errors $\sigma_\beta \simeq 23'$ in Ptolemaios. This is true a fortiori when we realize that Halley did not study a random selection of stars but selected some which appeared to have high proper motion. Our computation using positions for both epochs derived from HIPPARCOS data gives the largest change in latitude $\Delta\beta$ between Ptolemaios and Flamsteed for Arcturus, the star with the smallest $\Delta\beta$ in the analysis of Halley. The reason for this is that the error in latitude by Ptolemaios is in the same direction as the proper motion in latitude for Arcturus.
masking the real change. Conversely, the error in latitude by Ptolemaios for Aldebaran is in the opposite direction as the proper motion in latitude, and adds to the real change.

Turning to the comparison of star positions from Flamsteed and Brahe, we see that use of the correct value for the obliquity in 1601, \( \epsilon = 23^\circ 29' 28.1'' \), leads to a rather smaller displacement in latitude \( \Delta \beta \) for Sirius than the value given by Halley. With \( \sigma_\beta = 2' \) for the catalogue of Brahe, this smaller displacement is not significant.

We conclude that according to modern criteria, the latitude differences found by comparing star positions from the epoch of Halley (presumably from Flamsteed) with those in the catalogues of Ptolemaios and Brahe provide no evidence for proper motion. Halley ignored measurements errors, and this is what led to his seemingly positive result.

**Occultation of Aldebaran by the Moon: Halley**

Halley’s second argument is the occultation of Aldebaran by the Moon on 11 March 509, described and analysed by Boulian. The inclination of the lunar orbit to the ecliptic causes a monthly oscillation of the position of the Moon around the ecliptic. The inclination of the rotation axis of the Earth with respect to the ecliptic causes a daily oscillation of the direction to the Moon as seen from a particular location – the daily parallax. The combined effect of these two oscillations is illustrated for Athens in Figure 2. The Figure shows that the daily variation in topocentric latitude of the Moon due to the daily parallax (\( \sim 0.3^\circ \)) is much larger than the actual proper motion of Aldebaran between 509 and 1717 (\( \sim 0.065^\circ \)). The parallax of the Moon depends on its altitude above the horizon, and hence on (the slowing down of) the rotation of the Earth. A detailed knowledge of the Earth’s rotation speed is therefore needed to draw strong conclusions on the proper motion of Aldebaran. The monthly and daily variations in the distance to the Moon also cause a monthly and daily variation in the the apparent size of the Moon.

Whereas Halley was very interested in lunar motion, his systematic observations of the Moon and analysis of his own and earlier (in particular Flamsteeds) data only started after 1720. Before that he computed tables of lunar positions based on Newton’s theory. Whereas these appeared more or less satisfactory for the observations at hand, the theoretical predictions worsened rapidly in the next 18-yr cycle (Cook). This implies that Halley could not compute the position of the Moon on 11 March 509 with sufficient accuracy to decide whether there was an occultation. The brevity of his report indicates that he did not perform such a computation – even if he did, it is clear from modern insight that the resulting uncertainty was too large to allow significant conclusions. We return to this below.

It is more likely that Halley based his conclusion on the text by Boulliou. Boulliou prints a Greek text from the astronomer Heliodorus, and provides a Latin translation. We translate from the Greek (see Appendix):

On 15 to 16 Phamenoth 225 I saw the Moon following the bright [star] from the Hyades, after the lighting of the lamps, by at most half a finger, and it appeared to have occulted it, because the star was next to the
Figure 2. Motion of the Moon along the sky for the period of a month centered on March 11, 509. The dashed line gives the geocentric position, i.e. the direction of the line that connects the center of the Earth to the center of the Moon. The angle between the lunar orbit and the Earth equator causes a monthly oscillation. The solid line gives the topocentric position in Athens of the southernmost point of the Moon, showing the daily parallax. The upper cross indicates the correct latitude of Aldebaran in 509 AD, the lower cross the position in 509 computed from the position in 1690 and zero proper motion. The inset details the motion near 11 March 509.

The Egyptian date corresponds to 11 March 509, a finger in ancient astronomy corresponds to 5', and the lamps were lighted after dusk. After performing the required computations Boulliau concludes that in fact no occultation took place. He took 7:20 pm as the local time for earliest visibility in Athens, and used a time difference between Athens and Hven (Uranienborg) of 45 minutes, close to the correct value of 44.1 minutes. We list his numbers in Table 3. Boulliau does not mention a latitude for Aldebaran, but we may assume that he used the value $-5^\circ31'$ determined in Uranienborg, i.e. the value by Brahe (see Table 1). The southern edge of the Moon was taken by Boulliau to be 15' south of its topocentric center, i.e. at $-5^\circ26'27''$, thus about 4.5' north of Aldebaran. Bouilliau concluded that no occultation could have taken place.

Halley will have noted that the position of Aldebaran according to Flamsteed is 1'11'' further North than that by Brahe (see Table 1), but even then an occultation could only have taken place if Aldebaran was further north in 509 than this, hence if Aldebaran moves south with time.
Table 3. Geocentric and topocentric positions of the center of the Moon on 11 March 509 for an observer in Athens according to Boulliau (Bo) and according to our modern computations for Athens (At, coordinates 23°43′40″ East, 37°59′02″ North) and for Alexandria (Al, 29°55′ E, 31°12′ N). CA moment of closest approach to Aldebaran; EV moment of earliest visibility of Aldebaran. For Athens we also list results for a computation in which the length of the day did not change between 509 and 1700, hence $\Delta t = \Delta t(1700) = 11$ s. We also give the correct position of Aldebaran in 509, and the position obtained by converting its position in Flamsteed’s catalogue to 509 without proper motion.

|        | $\Delta t$ (s) | UT (h) | LT (h) | LST (h) | $\lambda_g$ (°) | $\beta_g$ (°) | $\lambda_t$ (°) | $\beta_t$ (°) |
|--------|----------------|--------|--------|----------|-----------------|----------------|-----------------|----------------|
| Bo EV  | 19.333         | 50.507 | -4.967 | 49.879   | -5.191          |                |                 |                |
| At CA  | 13.627         | 15.209 | 2.589  | 48.975   | -4.934          | 48.980         | -5.343          |                |
| At EV  | 16.733         | 18.315 | 5.703  | 50.552   | -4.982          | 50.025         | -5.271          |                |
| Al CA  | 13.714         | 15.709 | 3.089  | 49.019   | -4.935          | 48.962         | -5.220          |                |
| Al EV  | 16.333         | 18.328 | 5.715  | 50.349   | -4.976          | 49.808         | -5.160          |                |
|        | At CA 11       | 16.122 | 17.704 | 5.090    | 49.451          | -4.949         | 49.009          | -5.250         |
|        | position Aldebaran in 509 computed from HIPPARCOS data | 49.010 | -5.573 |          |                 |                 |                 |
|        | Flamsteed position Aldebaran converted to 509 | 49.014 | -5.641 |          |                 |                 |                 |

Occultation of Aldebaran by the Moon: modern

In our analysis of the possible occultation of Aldebaran by the Moon we discuss three problematic aspects of the computation by Boulliau and – if he made one independently – by Halley. The first is the slow-down of the rotation of the Earth, which was unknown in the 17th century. This implies that the time elapsed since 11 March 509 is shorter than one would estimate based on the length of the day near 1700 AD. Boulliau and Halley were not aware of this. We investigate this aspect by comparing correct calculations with those for a constant rotation speed of the Earth. The second aspect is the question where the occultation was observed. The Greek text does not mention this, and Boulliau and Halley assumed that it was in Athens. Neugebauer argues on the basis of the career of Heliodorus, that the observation of the occultation was made in Alexandria. We investigate this by making calculations both for Athens and for Alexandria. The third aspect is the reliability of the Greek text. We’ll discuss this as we proceed.

We start by computing the relative positions of the Moon and Aldebaran, for observers in Athens and in Alexandria, on 11 March 509, at two moments: the moment of closest approach and the moment of earliest visibility of Aldebaran in the evening. To obtain the dynamical time JDE, we add $\Delta t = 5620$ s to the Julian Day JD (Morrison & Stephenson, the uncertainty in $\Delta t$ is 140 s).

We compute the geocentric position and apparent diameter of the Moon using the fits made to the numerical integration of ELP/MPP02. To find the topocentric position and diameter of the Moon, we compute the Mean Stellar Time at Greenwich from the dynamic time, correct it for nutation, and convert it to the Local Stellar Time at Athens. From this and the geocentric position of the Moon we compute the daily parallax.
Figure 3. Relative positions of the Moon and Aldebaran at the moment of closest approach and at the moment of first visibility of Aldebaran for Athens (top) and for Alexandria (below) on 11 March 509, computed with modern knowledge. The black crescents correspond to the illuminated part of the Moon. The upper position of Aldebaran is the correct one, the lower position is computed assuming no proper motion between 1690 and 509. The dashed line indicates the geocentric position of the center of the Moon and add it to the geocentric position to obtain the topocentric position and apparent diameter. Our results are listed in Table 3 and shown in Figure 3. The visible limiting magnitude at the position of Aldebaran depends on its distance to the horizon, Sun and Moon\cite{23}. We determine the instance when this limiting magnitude equals that of Aldebaran (V ≃ 1) at dusk on 11 March 509 to be 16:44 UT, which converts to a local time in Athens of about 6:20 pm. This is the earliest possible moment at which Aldebaran could theoretically be discerned from the sky background — the actual observation was probably somewhat later, and hence the separation between the Moon and Aldebaran slightly larger. We show two positions for Aldebaran: the correct one computed from HIPPARCOS data\cite{18} and the one computed for epoch 509 from the position at epoch 1690 in Flamsteed’s catalogue assuming zero proper motion. In the absence of proper motion, the ecliptic position of Aldebaran changes due to precession. The position of the Moon at first visibility indicates a problem with the Greek text: its
edge is not within ‘a finger’, i.e. < 2◦5 from Aldebaran, but about 46′ in Athens and 34′ in Alexandria. If we accept the reading ‘at most six fingers’ from Heiberg and Neugebauer, the Greek text is more in agreement with an observer in Alexandria. Even in Alexandria, however, the position of Aldebaran with respect to the illuminated part of the Moon does not match the convoluted description in the text. It is clear that Aldebaran was not occulted on 509 March 11 in Alexandria. Remarkably, however, we find that the conclusion by Halley is correct that an occultation of Aldebaran in Athens was possible only if the star was further North in 509 than in 1690!

It does not follow that Halley proved the proper motion of Aldebaran with this argument. The accuracy required to prove or disprove the occultation of Aldebaran in 509 was well beyond reach for Boulliau, Halley or their contemporaries: at closest approach Aldebaran was just 1′4 within the limb of the Moon. For example, Halley was not aware of the slow-down of the rotation of the Earth. If we repeat our computation for Δt = 11 s, ignoring the slowdown of the rotation of the Earth between 509 and 1700, we find a topocentric latitude in Athens for the Moon about 6′ further North, which implies there is no occultation of Aldebaran.

**Cassini**

Jacques Cassini in 1738 investigated the possible proper motion of stars. He notes that Brahe decided that the comparison of modern measurements of star positions with those made by ancient Greek astronomers does not provide evidence for proper motion, whereas Halley decides it does. Cassini concludes that only comparison between modern observers can be trusted; as we have seen above, this conclusion is correct.

Picard and Jean Dominique Cassini, the father of Jacques Cassini, made accurate measurements of Arcturus in their study of precession. Jacques Cassini compares these, and in particular the measurement of the ecliptic latitude of Arcturus by Richer in 1672 in Cayenne, with his own measurement 86 year later in Paris (see Table 1), and finds a change in latitude of −2′. Cassini remarks that this change is confirmed with the latitude of Arcturus determined by Flamsteed for 1690. For comparison with Brahe’s measurements, Cassini redetermines the latitude of Arcturus from an altitude measurement made by Brahe on 24 February 1584. His better knowledge of refraction, and especially his better value of the obliquity, partially offset by a less accurate latitude of Hven, enables Cassini to obtain a more accurate value than Brahe did (Table 4). The derived change in latitude between 1584 and 1738 is 5′. To decide whether this difference is significant, Cassini converts the meridional altitude 54°36′40″ of η Boo, measured by Brahe on 7 February 1586 into a latitude of 28°07′22″ for η Boo at that date. This is only 3″ higher than the value Cassini measures in 1738, and thus, η Boo shows no significant proper motion.

Cassini proceeds to redetermine from Brahe’s altitude measurements the latitudes of Sirius ‘near the end of the 16th century’ and of Aldebaran in 1589. From his results, shown in Table 4, Cassini concluded that these stars showed no significant proper motion in latitude. He drew the same conclusion for other stars from the differences he found between his own measured latitudes in 1738 and the (redetermined) latitudes from Brahe’s measurements. These differences (and the values according to modern computations) are 20″ for Antares (66″), 8″ for γ Aql (53″), 22″ for Spica (36″), 16″.
Table 4. Ecliptic latitudes used by Cassini to derive the proper motion of Arcturus. $d$ indicates the difference between the listed value $L$ and the correct value $C$: $d = L - C$. Italics indicates values derived by us from other tabulated values. The last column gives values for 1584 from modern computation.

| Arcturus | 1672 | 1584 Brahe | 1584 Brahe-C | 1584 modern |
|----------|------|------------|--------------|-------------|
| $h$      | 55 28 15 | 55 28 15 | 55 29 25 |
| $R$      | 0 | 40 | 42 |
| $\phi$   | 55 54 30 | 55 54 15 | 55 54 28 |
| $\delta$ | 20 54 15 | -0.5 | 21 22 45 | 21 21 50 | 21 23 11 |
| $\epsilon$ | 23 29 00 | 0.1 | 23 31 30 | 23 29 30 | 23 29 36 |
| $\alpha$ | 210 10 45 | 0.0 | 209 11 30 | 209 10 43 |
| $\beta$  | 30 57 25 | -0.4 | 31 00 29 | 31 01 26 |
| $\lambda$ | 199 39 45 | 0.2 | 198 27 28 | 198 25 58 |

Table 5. Ecliptic latitudes for Altair and $\beta$ Aql according to various catalogues and measurements.

|        | Ptolemaios | Brahe/Cassini | Flamsteed | Cassini |
|--------|------------|---------------|-----------|---------|
| $\alpha$ Aql | 29 10 -13.3 | 29 18 11 -0.8 | 29 19 11 0.4 | 29 19 08 0.4 |
| $\beta$ Aql | 27 10 -0.9  | 26 45 08 -0.3 | 26 44 20 0.4 | 26 43 40 0.4 |
| $\Delta \beta$ | 2 00 -12.4 | 2 33 03 -0.5 | 2 34 51 0.0 | 2 35 28 0.0 |

for $\alpha$ CrB ($53''$), $25''$ for $\alpha$ Oph ($94''$), $13''$ for $\alpha$ Her ($59''$), and $< 120''$ for Rigel ($64''$), Betelgeuze ($66''$), Regulus ($18''$) and $\alpha$ Cap ($52''$). (Here we assume that the star ‘preceding Aquila’ is $\gamma$ Aql.) Cassini concludes that the first five have no evidence for proper motion, whereas those with upper limits of 2' are ‘rather less evident’ than the case for Arcturus.

Finally, Cassini considers the pair $\alpha$ Aql – $\beta$ Aql (see Table 5). For $\alpha$ Aql the latitude has increased between Ptolemaios and Brahe and on to Flamsteed and Cassini’s measurements, whereas the latitude of $\beta$ Aql has steadily decreased over the same time interval. As a result the difference in latitude between these two stars has increased by 36' since the time of Ptolemaios.

We can see here that Cassini is not consistent in his trust in numbers from Ptolemaios: where these confirm the trend between his time and (the revised) Brahe, he accepts them, but when they do not, as for Aldebaran, he concludes that this ‘shows that the ancient observations are not sufficient for research of this type’. In the cases of Arcturus, Sirius and $\eta$ Boo he notes that their latitudes in the catalogue of Ptolemaios are compatible with his conclusions from measurements by Brahe and later astronomers.

From the above numbers we see that the accuracy of the latitudes reetermined from Brahe’s data by Cassini is generally better than 1’, and that the latitudes determined by Richer and Cassini are generally more accurate than 0.5’. Thus, the proper motion derived by Cassini for Arcturus is significant, and his conclusion that the data in his
possession do not show significant proper motion for Sirius, Aldebaran, Betelgeuze and the other stars mentioned above is correct. In the case of $\beta$ Aql he is too optimistic: with our knowledge of the typical errors in Ptolemaios, Brahe/Cassini, Flamsteed and Cassini we see from Table 5 that the differences in its latitude are not significant.

Discussion

Halley is not the first astronomer who mistook measurement errors for proper motion. In comparing contemporaneous observations with earlier star catalogues, the eight-century Chinese astronomer I-Hsing found north-south movements for ten asterisms (Needham\textsuperscript{25}). From the magnitude of the displacements, 4 to 5 degrees, it is obvious that, like Halley, I-Hsing discovered position errors rather than proper motion.

Occultations of Aldebaran by the Moon in Alexandria do occur, but not on 11 March 509. An almost central occultation of Aldebaran by the Moon occurred on 12 February 509, about 7 o’clock local time in the morning. The occultation on 7 April 509 occurred well after Aldebaran became visible, after midnight. These occultations do not fit the description by Heliodorus. If his description fits a real occultation in Alexandria, it was not one month before or after the date given. In any case, the text by Heliodorus does not prove proper motion of Aldebaran.

Halley took the latitudes given by ancient astronomers and by Brahe too much at face value, and as a result interpreted their measurement errors as evidence for proper motions. In contrast, Cassini followed Brahe in questioning the reliability of the latitudes given by Ptolemaios, and decided they were too uncertain to be used as evidence for proper motion. By redetermining the latitudes derived from meridional altitude measurements by Brahe, Cassini halved the uncertainty in these latitudes, and thus gave significant proof of the proper motion of Arcturus, while invalidating the results of Halley.

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Appendix

The Greek text about the apparent occultation of Aldebaran by the Moon is printed by Boulliau using many ligatures which ‘more often dismay than enlighten’ the modern reader, to paraphrase Ingram. We therefore give a transcription kindly provided by Dr. Frederik Bakker from the Center for the History of Philosophy and Science at Radboud University.

Here is an emendation necessary for the verb form to be grammatically correct, and the question mark indicates a sign which is difficult to read in Boulliau. This sign is followed by ‘ and is translated by Boulliau as ‘half’, which implies that it represents either β or ι, even though in print it looks mostly like a ς.