The energies and ANCs for $^5$Li resonances deduced from experimental $p$-$\alpha$ scattering phase shifts using the effective-range and $\Delta$ methods

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Recently a new $\Delta$ method for deducing the energy and asymptotic normalization coefficient (ANC) from phase-shift data has been formulated and applied to resonance states. This differs from the conventional effective-range function (ERF) method by fitting only the nuclear part of the ERF. It also differs from the method which was proposed for bound states by Ramirez Suárez and Sparenberg (see Ref. below) which also named the $\Delta$ method where a pole condition defines by the Eq. $\Delta_l=0$ ($\Delta_l$ is the function in the ERF determined only by the scattering phase shift). Here the standard pole condition, including the Coulomb part into the relate equation, is used for a resonant state. It has been shown that the ERF method does not work for large-charge colliding nuclei. Moreover, even for lower charges it is not clear that the results of the ERF method are accurately enough. The Coulomb part forms a background, which smooths an ERF energy dependence. Therefore, one needs to find when the ERF method becomes inaccurate and this requires recalculating some published results by the $\Delta$ method. This project has already been started in a recent paper for resonances in the $\alpha$-$\alpha$ scattering. Here this method is applied using the $\Delta_l$-function fittings to the experimental $p$-$^4$He scattering phase-shift data in the $P_{3/2}$ and $P_{1/2}$ resonance states. The calculation results are compared with those obtained earlier by the ERF method. The main changes concern resonance energy and width.

Keywords: $^5$Li states $l=1, J=1/2, 3/2$; $p$-$^4$He scattering phase-shift input data; Low-energy resonance energies; Asymptotic normalization coefficient (ANC); Effective-range and $\Delta$ methods calculation comparison

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1. Introduction

It is known that many reactions in supernovae explosions proceed through bound states and low-energy resonance states. To calculate the rate of such reactions, one needs to find the ANC of the radial wave function for bound and resonance states, which can be used to calculate radiative capture cross sections at low energies. This process is one of the main sources of new element creation.

Resonance states are described by the so-called Gamov wave functions that contain only the outgoing waves asymptotically, which exponentially increase due to the complex momenta. In the past (see [1] and references therein), the analytical continuation onto the unphysical energy sheet of the Lippmann–Schwinger as well as the momentum space Faddeev integral equations were used to find the resonance properties. The normalization formula for the bound state vertex function in the momentum space was generalized in [2] for the resonance and virtual states. Some different techniques to determine the resonance energy, width, and resonance wave function based on the solution of the Schrodinger equation have been previously suggested. Instead of the Zel’довich’s normalization procedure [3], which is valid when $|\text{Im } k_r|< \text{Re } k_r$ and difficult to realize in practice due to slow convergence of the integrals, in these methods the normalization of the resonant wave function is achieved using the rotation of the integration contour over $r$ from $R_{\text{max}}$ to the complex plane, where the nuclear potential is cut to zero. Here $k_r = \sqrt{2\mu E_r}$, $k_r = k_0 - i \kappa$, $E_r = E_0 - i \Gamma/2$, $\mu$ is a reduced mass.

The method of the radial Schrodinger equation solution to determine resonances was suggested in Ref. [4] (GAMOW code). In this method the complex eigenvalue and the Gamow wave function can be found by integration of the Schrodinger equation imposing the boundary conditions in the origin and the asymptotic region. An improved version of the GAMOW code, which uses the piecewise perturbation method for integration of the radial equation numerically, is presented in Ref. [5] (code ANTI), which was designed to determine the virtual states, virtual resonances, and the broad resonance by introducing complex Woods-Saxon potential.
A potential S-matrix pole (PSMP) method was developed in Ref. [6] for obtaining the S-matrix pole parameters for bound, virtual and resonant states based on numerical solutions of the Schrodinger equation. In [7] a similar method was proposed for finding the resonance parameters of a nuclear system, obtained from the phase-shift analysis of elastic scattering data, by means of a pole representation of an S-matrix in complex momentum space.

In Ref. [8] the virtual states are calculated using the Berggren representation. In Ref. [9] the Berggren basis is used to calculate the isobaric analog states. Different recent methods of the ANC estimation for bound and resonance states are discussed in Ref. [10].

In the present paper the \( \Delta \) method is applied using the \( \Delta_l \) function fittings to the experimental \( p-^4\text{He} \) scattering phase-shift data in the \( P_{3/2} \) and \( P_{1/2} \) resonance states. We will use as well (see below) designations \( \Delta_r=\Delta(J)\equiv \Delta_r(E) \) where \( J \) is the total and \( l \) is the orbital angular momentum. The name \( \Delta \) method is used here because the fit to experimental data energy dependence of the phase-shift is made for the \( \Delta_r \) function which definition is given below. In the literature the same name is used for the method proposed by Ramírez Suárez and Sparenberg in Ref. [11] where a bound state pole position is defined by the equation \( \Delta_r=0 \). In the Ref. [12] and in the present paper, the standard pole condition including the Coulomb term is used for a resonance (see Eqs. (16-17) below). The Coulomb term is known in an analytical form and does not need fitting.

The present article is organized as follows. A finding a \( \Delta_l \) function at a resonance pole is realized by its analytical continuation from the physical region to the point situated in the fourth quadrant of the complex momentum or in the second energy unphysical sheet. It means that \( \Delta_r \)-function fitting for a concrete resonance should be especially good in the region around the real part \( E_0 \) of the resonance energy \( E_r=E_0-i\Gamma/2 \).

In Sec. 2.1 previously published results are shortly discussed. In Sec. 2.2 the main formulas of the \( \Delta \) method for resonances are given. The calculation results are presented in Sec. 2.3. Two \( \Delta(J) \) function curves for the experimental Coulomb-nuclear phase-shift fitting in the \( P_{3/2} \) and \( P_{1/2} \) states are shown in Fig. 1. Both show a very good fit. In Fig. 1 the \( \Delta(J) \) fittings are compared with each other and \( \text{Re} h(\eta) \) (see below a definition of the Coulomb term \( h(\eta) \)). The related functions are more complicated than the ERF when only three parameters are enough for ERF fitting. The polynomials up to the fourth degree of \( E \) are needed for a good \( \Delta \) fitting. In equations below usual designation \( \Delta_l^{(c)} \) is also used. A table is given which includes the experimental and calculated resonant energy \( E_0 \) and width \( \Gamma \) for the \( \Delta \) method as well as the resulting absolute values of the renormalized amplitude residue \( W_l \), the nuclear vertex constant NVC and ANC.

To check results in Ref. [12] and in the present work a simple formula, approximated ANC dependence on \( E_0 \) and \( \Gamma \) for a narrow resonance, borrowed from Dolinsky and Mukhamedzhanov paper [13], which we call the DM method, is used. The main differences between the results obtained by the ERF and \( \Delta \) methods are seen in the resonance energy and width.

In Sec. 3 the main results of this comparison are summarized.

The unit system \( \hbar=c=1 \) is used.

2. The \( \Delta \) method for deducing the \( ^5\text{Li} \) low-energy resonant state properties

2.1. Previous published studies of the low-energy \( p-^4\text{He} \) scattering resonances

The \( p-\alpha \) resonant scattering is studied earlier in Ref. [14]. The well-known expression (see, for example, Ref. [15]) for the ERF in the \( P \) wave is used in Ref. [14] for finding energy poles of the \( S \)-matrix in the \( N-\alpha \) scattering near the elastic-scattering threshold. However, the re-normalized nuclear vertex constants (NVC) and ANC of the Gamow wave functions are not considered in [14]. The first ERF method application for the ANC finding is made in Ref. [16]. In Ref. [17] the ERF method is generalized to resonance properties calculations.

The N/D method is applied in Refs. [18], [19] for calculating the values of the parameters of the resonances in \( N-\alpha \) scatterings.
In Ref. [20] an analytical approximation is considered in the form of a series for the nonresonant part of the phase shift which can be analytically continued to the point of an isolated resonance pole in the complex plane of the momentum. This method is called S-matrix pole method (SMP). In [20] the parameters for \( p^4 \text{He} \) scattering resonances are calculated using the ERF and SMP methods. These results are compared with those obtained by different methods including the N/D method and the resonant energies calculated in [21].

An alternative derivation of DM formula by the SMP method is given in Ref. [20]. It is shown in [20] that in the state with a total angular momentum \( J = 1/2 \) the resonant energy \( E_{\text{res}} \) for the SMP method decreases slightly while the width \( \Gamma \) for \( J = 3/2 \) increases somewhat compared to the ERF method results obtained in Ref. [14].

In a recent paper, Ref. [12], the new \( \Delta \) method for resonances, where an experimental \( \Delta_l \) function data is fitted, is applied to the ANC calculation for low-energy states of different nuclear systems. This method avoids problems arising when charges of colliding particles increase. The effective-range expansion (ERE) and Padé approximations for finding the ANC are limited by the values of the colliding particle charges. These approaches do not work for large charges when the nuclear term of the ERF is too small, compared with the Coulomb term. The \( \alpha^{12} \text{C} \) system is a good example, when the nuclear part is on average three orders of magnitude smaller than the Coulomb part (see Ref. [12]). This problem of the ERF method appears in [22], where it was found that the ERF fittings for \( \alpha^{12} \text{C} \) scattering behave similarly in the different \( ^{16} \text{O} \) states.

A reasonable way to find when the ERF does not work is to compare the resonance energies and ANCs calculated by the ERF method with those obtained by the \( \Delta \) method for the same input. It is necessary to consider systems lighter than oxygen. This begins in Ref. [12], where some results of the ERF method for the \( \alpha-\alpha \) scattering are compared with the \( \Delta \) method results. In Ref. [12] the \( \Delta \) method is successfully applied to the \( ^7 \text{Be} \), \( ^8 \text{Be} \), and \( ^{16} \text{O} \) resonances using a model with the configurations \( \alpha^3 \text{He}+\alpha, \alpha+\alpha, \) and \( \alpha+^{12} \text{C} \).

An outstanding example of the important role of resonances is known for the unstable \( ^8 \text{Be} \) ground state. It presents a very narrow resonance with the pole at the cms (center-of-mass system) energy (see the review [23] and the references therein, \( E_{\alpha\alpha} \) in eV) \( E_{\alpha\alpha}=E_0-i\Gamma/2=0.9184 \cdot 10^4-i \cdot 2.8 \). From the indeterminacy principle, the lifetime of the \( ^8 \text{Be} \) is \( \tau=\hbar/\Gamma \approx 10^{-16} \) s. The lifetime of \( ^8 \text{Be} \) and the value \( Q = E_0 \) make the creation of \( ^{12} \text{C} \) possible.

The chief protagonists of this discovery are the astronomer Fred Hoyle, the astrophysicist Nobel laureate William Fowler. Hoyle predicted the existence of the resonance state of the \( ^{12} \text{C} \) nucleus with an excitation energy of 7.68 MeV even before of 7.65 MeV was observed in experiments. Hoyle reasoned this from the natural occurrence of \( ^{12} \text{C} \) in the universe (possibly ‘the anthropic principle’, see, for example, [24]). Salpeter theoretically considered the \( ^{12} \text{C} \) creation mechanism as the result of a three \( \alpha \) particles fusion with the intermediate creation of the ground state \( ^8 \text{Be} \) resonance. Fowler et al. carried out experiments, which confirmed Hoyle’s prediction.

At the end of the life of red giant stars compressed by gravitation, their temperature increases up to values \( T > 10^8 \) K. At such temperatures, carbon creation is actual due to the two consecutive processes: \( \alpha+\alpha \rightarrow ^8 \text{Be} (0^+, \text{ground state}) \) and \( \alpha+^8 \text{Be} \rightarrow ^{12} \text{C}^* (0^+, 7.65 \) MeV). A small difference (\( \approx 0.28 \) MeV) between the \( ^{12} \text{C}^* \) energy level and that of the system \( \alpha+^8 \text{Be} \) is especially important for this transition.

2.2. The \( \Delta \) method main formulas and their applications to the low-energy \( ^5 \text{Li} \) resonances

In the present paper, the \( ^5 \text{Li} \) resonance properties are studied by applying the \( \Delta \) method to the low-lying \( P_{3/2} \) and \( P_{1/2} \) resonance states in the \( p-\alpha \) scattering. They are compared with the ERF method results obtained in [20], Table II. The absolute values of the resonance constants are included in Table I below.

The nucleus \( ^5 \text{Li} \) is interesting in that the ground and first excited states are resonance states, which can be treated as single-channel systems. The phase shift of the elastic \( p-\alpha \) scattering for
total angular momentum and parity $J^P = 3/2^-$ passes through $\pi/2$, and therefore leads to a narrow resonance. However, the phase shift of the elastic $p$-$\alpha$ scattering for $J^P = 1/2^-$ does not pass through $\pi/2$, so the corresponding resonance is quite wide. The $p$-$\alpha$ phase shift in Ref. [25] is used as input data to calculate the residue $W_l$ of the re-normalized scattering amplitude, the nuclear vertex constant NVC and the ANC.

The re-normalized scattering amplitude, taking into account the Coulomb interaction, is derived in [26] to enable the analytic continuation of this amplitude to negative energies. The following notations are used below:

\[ \eta = \xi/k \] is the Sommerfeld parameter,
\[ \xi = Z_1 Z_2 \mu a_0 = 1/a_B, \] \[ k = \sqrt{2 \mu E_c} \] is the relative momentum; \( \mu \) and \( E_c \) are the reduced mass and the cms energy of the colliding nuclei with the charge numbers \( Z_1 \) and \( Z_2 \) respectively; \( a_B \) is the Bohr radius and \( \alpha \) is the fine-structure constant.

It is shown in [26] that in the physical energy sheet, the ERF is a real analytic function with the possible exception of single poles. This means that the ERF can be described by the ERE or Padé approximations (see Refs. [12, 22]), whose coefficients can be found by fitting the experimental phase shifts. The same is valid for the \( \Delta_l \) function because the nuclear part of the ERF, including the \( \Delta_l \), is also a meromorphic function of energy in the physical area.

The partial amplitude of the nuclear scattering in the presence of the Coulomb interaction is

\[ f(k) = \exp(2i\sigma)[\exp(2i\delta_l^{(cs)}) - 1]/2ik, \] (2)

where \( \delta_l^{(cs)} \) is the Coulomb-nucleus phase shift,

\[ \exp(2i\sigma) = \Gamma(l + 1 + i\eta)\Gamma(l + 1 - i\eta), \] (3)

As given in Ref. [26], the partial amplitude of the elastic scattering is re-normalized by multiplying it by the function

\[ CF_l(k) = (l!)^2 \exp(\pi\eta)\Gamma(l + 1 + i\eta)^2. \] (4)

Thus, the expression for the re-normalized amplitude of the elastic scattering can be written as

\[ \tilde{f}_l = 1/[k(\cot \delta_l^{(cs)} - i) \rho_l(k)], \] (5)

where the function \( \rho_l \) is defined by the equation

\[ \rho_l(k) = 2\eta C\dot{\sigma}(\eta) \prod_{n=1}^{l} (1 + \eta^2/n^2). \] (6)

Here the following notation is used:

\[ C\dot{\sigma}(\eta) = \pi/\exp(2\pi\eta) - 1, \] (7)

It is easy to recast (5) as

\[ \tilde{f}_l = k^{2l}/[2\xi D_l(k^2) C\dot{\sigma}(\eta)(\cot \delta_l^{(cs)} - i)]. \] (8)

\[ D_l(k^2) = \prod_{n=1}^{l} (k^2 + \xi^2/n^2), \quad D_0(k^2) = 1. \] (9)

We define the \( \Delta_l \) function as

\[ \Delta_l = \pi \cot \delta_l^{(cs)}/\exp(2\pi\eta) - 1 \] (10)

in the positive energy semi-axis.
Writing the expression \( \cot \delta_l \) in the non-physical energy region in Eq. (5) and elsewhere means its analytical continuation, since the phase shift is defined only in the positive energy region. Here and further down we omit the upper indexes in \( \delta_l^{(c)} \) for brevity.

The function \( C_0^2(\eta) \), having the analytical form (8), does not need fitting. This function clearly depends on momentum \( k \) through \( \eta(k) \) which leads to the square root cut of the re-normalized amplitude in the complex energy plane. As mentioned above, the Coulomb part of the ERF leads to a much smoother energy dependence \( K(k^2) \) compared with \( \Delta(k) \).

\[
K(k^2) = 2 \xi D_l(k^2) [C^2_0(\eta)(\cot \delta_l - i) + h(\eta)], \quad (11)
\]

\[
h(\eta) = \Psi(\eta) + (2i\eta)^{-1} - \ln(i\eta) \quad (12)
\]

In [12] the ERF fitting for the \( N\alpha \) scattering is given by the standard equation

\[
k_l(k^2) = -1/a_l + (r_l/2)k^2 - P_l r_l^3 k^4 \quad (13)
\]

which is equivalent to the following equation, where \( E_c \) is the energy of colliding nuclei in the cms system:

\[
k_l(E_c) = b_0 + b_1 E_c + b_2 E_c^2. \quad (14)
\]

The experimental points \( \Delta_l(E_c) \) have a more sophisticated distribution compared with the smooth dependence of \( K_l(E_c) \). \( \Delta_l(E_c) \) increases when energy decreases. In the region \( E_c > 6.5 \text{ MeV} \), the decrease in functions \( \Delta_l(E_c) \) for \( J=3/2, 1/2 \) is replaced by an increase. Due to this, good \( \Delta_l(E_c) \) fittings are obtained using polynomials in powers of \( E_c \) up to \( E_c^4 \) including five fitting parameters instead of three as in (14):

\[
\Delta_l(E_c) = a_0 + a_1 E_c + a_2 E_c^2 + a_3 E_c^3 + a_4 E_c^4. \quad (15)
\]

In (15) and below \( J \) is used instead of \( l \) to indicate the state because of the fixed \( l=1 \) and the different \( J = 1/2, 3/2 \). To find the resonance energy position the standard equation is used:

\[
\cot \delta_l - i = 0. \quad (16)
\]

With the fitted parameters of \( \Delta_l(E_c) \) in (15), the resonance complex energy can be found from the equation

\[
\Delta_l(E_c) - i C_0^2(\eta) = 0 \quad (17)
\]

which is equivalent to Eq. (16), and then the listed values of constants calculated. Our new results are presented in Table 1 in the \( \Delta_l \) lines.

The values of the residue of \( \tilde{f}_l(k) \) at the resonance energy \( E_r \) for the \( \Delta_l \) fitting can be written as

\[
\tilde{W}_l(k_r) = \lim_{k \to k_r} \frac{1}{(2\xi l_1)} \left[ 2 \xi D_l(k_r^2) \lim_{k \to k_r} \frac{\partial}{\partial k} \left[ \Delta_l(k^2) - i C_0^2(\eta) \right] \right]. \quad (18)
\]

(see [12]). Here \( k_r = \sqrt{2\mu E_r} \), \( k_r = k_0 - ik_\eta \). According to the known relations between the NVC \( \tilde{G}_l \), ANC \( (C_l) \) and the residue \( \tilde{W}_l \) one can write

\[
\tilde{G}_l^2 = -(2\pi k_r/\mu^2) \tilde{W}_l, \quad (19)
\]

\[
C_l = (\mu/\sqrt{\pi}) [\Gamma(l + 1 + i\eta) / l!] \exp(-\pi\eta^2/2) \tilde{G}_l \quad (20)
\]

where \( \eta_r = \xi/k_r \). The simple DM relation for the ANC derived in Ref. [13]
\[ |C_i^p| = \sqrt{\mu \Gamma / k_0} \] (21)

is used to check the calculations for the narrow resonance in the ground state \( J^p = 3/2^- \).

3. The calculation results

The \( \Delta(J) \equiv \Delta(E) \) fitting results (15) are shown in Fig. 1.

![Diagram](image)

Fig. 1. Dependence of the \( \Delta(J) \) functions for \( J^p = 1/2^- \) (top line) and for \( J^p = 3/2^- \) (bottom line) vs the cms energy \( E = E_c \). Polynomials up to the fourth degree of \( E_c \) are used for fitting. The solid line without symbols is the Coulomb \( \text{Re}h \) function which can be compared with the \( \Delta(J) \) functions in the ERF.

Table I.
The \( p-α \) scattering resonances. Methods: ERF [20] and \( \Delta \) (present paper); \( J^p \); resonance energy \( E_0 \) in the center-of-mass system (in MeV); corresponding width \( \Gamma \); the residue \( |W| \); NVC \( |\tilde{G}_l^2| \); and ANCs denoted as \( |C_l| \) and \( |C_l^a| \) (for narrow resonances [13]). The fitting models are described in the text. The phase shifts are borrowed from Ref. [25].

| Method | \( J^p \) | \( E_0 \) [MeV] | \( \Gamma \) [MeV] | |\( G_l^2| [fm] | |\( C_l| [fm^{-1/2}] \| |\( C_l^a| [fm^{-1/2}] \|
|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| ERF    | 3/2^-  | 1.481  | 1.041  | 0.295  | 0.0314 | 0.260  | 0.288  |
| \Delta | 3/2^-  | 1.390  | 1.301  | 0.393  | 0.0416 | 0.297  | 0.325  |
| ERF    | 1/2^-  | 2.213  | 4.640  | 0.305  | 0.0459 | 0.357  | 0.526  |
| \Delta | 1/2^-  | 2.611  | 4.534  | 0.223  | 0.0355 | 0.314  | 0.505  |

The following conclusions can be drawn from Table I.

For the state \( J = 3/2^- \), the ANC \( (C_l) \) does not change much, despite the fact that the resonance energy changes significantly. For the \( \Delta \) fitting, \( \Gamma \) increases compared with the width in the ERF method. This means that \( \Gamma \) is more sensitive to the used model than \( E_0 \). This agrees with Eq. (21) (see Ref. [12]).

For the wide resonance state \( J = 1/2^- \), however, there is a decrease in \( E_0 \) and an increase in \( \Gamma \) for the \( \Delta \) fitting, compared with those in the ERF method. Again, one can see that \( \Gamma \) is more sensitive to the fitting model. The large disagreement between \( |C_l| \) and \( |C_l^a| \) which is seen in Table I is understandable for this wide resonance. Using the \( \Delta \) fittings instead of the ERF method seriously changes the resonance energy. In our opinion, this means that the ERF method results should be recalculated even for systems with small charges.
4. Conclusion

In the present paper the properties of the ground $3/2^-$ and first excited $1/2^-$ resonances in $p$-$\alpha$ scattering are studied, using the recent phase-shift input data from Ref. [25]. Recalculation of the published results by the $\Delta$ method is recommended even for systems with smaller charges than $^{16}$O. The $\Delta$ method must be applied when the calculations using the ERF fitting are invalid due to large charges.

Declaration of competing interest

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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