Application of the local antimagic total labeling of graphs to optimise scheduling system for an expatriate assignment

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Abstract. An expatriate assignment at the branch company in other country must be optimised due to the short term duty. Accuracy calculation is needed to optimise the expatriate assignment including its scheduling system. There are many ways and methods to optimise the scheduling system. One of the methods is making use of graph theory instruments such as local antimagic total labeling. The local antimagic total labeling of a graph is a map from the set of vertices and edges of the graph to the set of positive integers from 1 to the number of vertices and edges such that the weight of two adjacent vertices are different. The vertex weight is calculated by adding the labels of the vertex and all edges incident to it. In this paper, this labeling is applied to optimise the scheduling system for the expatriate assignment. This system can be modelled by a graph where the assignments are represented by vertices and connection between the assignments are represented by edges. The represented graph is then labelled by the local antimagic total labeling in order to obtain the minimum number of different vertex weights. The such number is equivalent to the optimal time allocated for the expatriate assignment.

1. Introduction

Global economy is influenced by the international business which dominates most of the commercial activities in the world. Interaction of cross border between countries became daily activities of companies around the world. The companies activities imply that multi national companies send human resources to overseas if the companies have branches in other countries.

Employee or human resource who is sent to get global assignment in overseas is called expatriate. McNulty [10] defined expatriate as a person who lives outside native country, and physically is mobile across international border for professional or personal reasons, for short or long periods of time, organizationally sponsored or not, and crossing an ocean or moving across land.

Expatriate assignments at the branch companies in other countries must be optimised due to the short term duty. The research on expatriate assignment has been conducted by several authors such as [3, 8, 14, 16, 4, 5]. Optimisation of calculation is needed to assign the expatriates including the scheduling system. There are many ways and methods to optimise the scheduling
system. One of the methods is making use of graph theory instruments such as vertex coloring of graph.

A \( k \)-coloring of a graph, where \( k \) is a given integer, is a color assignment to the vertices of the graph \( G \), such that every vertex of \( G \) has one of the \( k \) colors [6]. A coloring of graph \( G \) is called proper coloring if any two adjacent vertices have different colors. The chromatic number of graph \( G \), denoted by \( \chi(G) \), is the smallest integer \( k \) such that the graph \( G \) has a proper \( k \)-coloring. Figure 1 illustrates an example of \( k \)-coloring for proper coloring, not proper coloring and an optimal proper 2-coloring.

![Figure 1. Example of vertex coloring of graphs](image)

The vertex coloring of graph can be approached by using other topic in graph theory, namely, the local antimagic labeling of a graph. The local antimagic labeling of the graph \( G \) is defined as an assignment from the edge of \( G \) to the positive integers, \( f_1 : E \rightarrow \{1, 2, \ldots, |E|\} \) such that the weights of two adjacent vertices are distinct. The weight of a vertex \( u \), denoted by \( w_1(u) \), is calculated by the sum of all edge labels incident to \( u \), that is, \( w_1(u) = \sum_{e \in E(u)} f_1(e) \). Thus for two adjacent vertices \( u \) and \( v \), \( w_1(u) \neq w_1(v) \). The notion of local antimagic labeling was introduced by Arumugam, Premalatha, Bača and Semaničová-Feňovčíková [1]. Clearly, this labeling induces a proper vertex coloring of graph \( G \) by assigning color \( w(u) \) to the vertex \( u \) in \( G \). The minimum number of colors, which is the number of different weights, from all coloring induced by the local antimagic labelings of \( G \) is called the local antimagic chromatic number of \( G \), denoted by \( \chi_{la}(G) \).

The local antimagic labeling was then extended by Putri, Dafik, Agustin and Alfarisi [13] by labeling both the vertices and edges of graph \( G \). The local antimagic total labeling on a graph \( G \) is defined to be an assignment from the vertices and edges of graph \( G \) to positive integers, \( f_2 : V \cup E \rightarrow \{1, 2, \ldots, |V| + |E|\} \) such that the weights of two adjacent vertices are different. In this case, the weight of a vertex \( u \), denoted by \( w_2(u) \), is calculated by the sum of all edge labels incident to \( u \) and the label of \( u \) as well, that is, \( w_2(u) = f_2(u) + \sum_{e \in E(u)} f_2(e) \). Thus, for two adjacent vertices \( u \) and \( v \), \( w_2(u) \neq w_2(v) \). As the local antimagic labeling, any local antimagic total labeling also induces a proper vertex coloring of \( G \) where the vertex \( u \) is colored by \( w(u) \). The local antimagic total chromatic number of \( G \), denoted by \( \chi_{lat}(G) \), is defined similarly, that is, the minimum number of colors needed over all colorings that are induced by local antimagic total labelings of \( G \). Clearly, \( \chi_{la}(G) \geq \chi_{lat}(G) \geq \chi(G) \) for any graph \( G \).

Several results on determining both the local antimagic chromatic number and local antimagic total chromatic number of some particular classes of graphs have been discovered by many authors. For example, see [1, 11, 2, 9] for the known results on the local antimagic chromatic number of graphs and [13, 7, 12, 15] for the local antimagic total chromatic number ones.
In this paper, we apply the vertex coloring using local antimagic total labeling of graph to develop a scheduling system for the expatriate assignment.

2. Methods

The research was undertaken according to the following stages. Firstly, the expatriates were assigned to do their tasks in some divisions based on the results of interviews with some agro-industry companies that employ expatriates in the Jember area. Secondly, the expatriate assignment was modeled by a graph where the vertices represented the divisions and the edges between two vertices represented the connection between two divisions sharing the same expatriates. Furthermore, the vertices of the graph that represents the expatriate assignment were colored by using the local antimagic total labeling. Then, the scheduling system was developed using the result of local antimagic vertex coloring of the represented graph. Finally, the scheduling system of the expatriate assignment was simulated using the local antimagic total vertex coloring methods.

3. Main Results

The respondents of this research consisted of 30 expatriates from various countries who work in some agro-industry companies in Jember area. These expatriates were assigned to do their tasks in 7 divisions, namely, Administration and Storage, Finance, Human Resources, General Affairs, Marketing, Production, and Regional. Some expatriates have common tasks in the same division. The expatriate assignment in the 7 divisions can be simulated in Table 1.

| Division                  | Expatriates                  |
|---------------------------|------------------------------|
| Administration and Storage| E2, E5, E7, E10, E15, E22, E28|
| Finance                   | E1, E10, E22, E25, E30       |
| Human Resources           | E4, E8, E9, E11, E16, E20, E25|
| General Affairs           | E3, E5, E6, E9, E12, E17, E22, E25, E29, E30 |
| Marketing                 | E1, E4, E13, E21, E23, E29   |
| Production                | E6, E14, E19, E23, E24, E27, E28 |
| Regional                  | E7, E9, E11, E16, E18, E26, E27 |

The expatriate assignment in the 7 divisions was then modeled by a graph where the divisions are represented by vertices of the graph and two divisions sharing the same expatriates are connected by an edge. The graph which is obtained from the representation of the expatriate assignment is illustrated in Figure 2.

The represented graph of expatriate assignment, say $G$, was then labeled by using local antimagic total to obtain the local antimagic total chromatic number of $G$. Since the represented graph $G$ as shown in Figure 2 contains a subgraph that is isomorphic to the complete graph $K_4$, thus $\chi_{lat}(G) \geq \chi(G) \geq 4$. However, the local antimagic total vertex coloring of the represented graph $G$ gives 4 different colors as shown in Figure 3, that is, $\chi_{lat}(G) \leq 4$. Combining with the lower bound, this implies that $\chi_{lat}(G) = 4$.

The result of the local antimagic total vertex coloring of the represented graph is optimal. This result is implemented to develop a scheduling system for expatriates assignment that is given in Table 1. Thus, the optimal scheduling system to assign 30 expatriates in 7 divisions need 4 allocation times as shown in Table 2 which is equal to the local antimagic total chromatic number of the represented graph $G$.

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**Table 1.** The expatriate assignment in some divisions

| Division                  | Expatriates                  |
|---------------------------|------------------------------|
| Administration and Storage| E2, E5, E7, E10, E15, E22, E28|
| Finance                   | E1, E10, E22, E25, E30       |
| Human Resources           | E4, E8, E9, E11, E16, E20, E25|
| General Affairs           | E3, E5, E6, E9, E12, E17, E22, E25, E29, E30 |
| Marketing                 | E1, E4, E13, E21, E23, E29   |
| Production                | E6, E14, E19, E23, E24, E27, E28 |
| Regional                  | E7, E9, E11, E16, E18, E26, E27 |
The scheduling system for the expatriate assignment using local antimagic total vertex coloring of graph can be generalised to any finite number of expatriates and any finite number of divisions. The generalisation of the scheduling system is even easier if the represented graphs of the expatriate assignment is isomorphic to the particular class of graph. This is because of the local antimagic chromatic number of particular classes of graphs, such as complete graph, star, path, cycle, friendship graph, complete bipartite graph, and wheel, have been discovered by by [1] and the local antimagic total chromatic number for some family of graphs have also been discovered such as prisms and Möbius ladders [7], brooms [12], stars [13], wheels, fans and

**Figure 2.** The graph of the expatriate assignment representation

**Figure 3.** The local antimagic total vertex coloring of the represented graph

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Table 2. The optimal scheduling system for expatriate assignment in 7 divisions

| Time | Division(s)                                      | Color (Weight) |
|------|-------------------------------------------------|----------------|
| 1    | General Affairs                                 | Red (55)       |
| 2    | Finance, Production                             | Green (54)     |
| 3    | Marketing, Regional                             | Yellow (53)    |
| 4    | Administration and Storage, Human Resources     | Blue (52)      |

friendship graphs [15].

4. Conclusion

This scheduling system guarantees that there is no expatriate have the same allocation time in some divisions. Besides, this scheduling system also provides an optimal allocation time. The reasons are that the divisions with common expatriate will have different allocation times which is equivalent to the different colors of the vertices in the represented graph and the number of different colors is minimum.

Acknowledgement

The research was supported by DRPM, Directorate General of Strengthening for Research and Development, Ministry of Research, Technology and Higher Education through Penelitian Dasar research grant year 2019 Decree No. 7/E/KPT/2019 and Contract No. 175/SP2H/LT/DRPM/2019.

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