Lepton flavor violating decay $Z \rightarrow \ell_i^{\pm} \ell_j^{\mp}$ in the 331 model

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We study the lepton flavor violating (LFV) decays $Z \rightarrow \ell_i^\pm \ell_j^{\mp}$ ($\ell_{i,j} = e, \mu, \tau$) in the framework of the minimal 331 model. The main contributions arise at the one-loop level via a doubly charged bilepton with general LFV couplings. We obtain an estimate for the corresponding branching ratios by using the bounds on the LFV couplings of the doubly charged bilepton from the current experimental limits on the decays $\ell_i \rightarrow \ell_j \gamma$ and $\ell_i^- \rightarrow \ell_i^+ \ell_i^- \ell_i^-$. A bound on the bilepton mass is also obtained through the current limit on the anomalous magnetic moment of the muon. It is found that the bilepton contributions to LFV $Z$ decays are not expected to be at the reach of experimental detection. In particular, the branching ratio for the $Z \rightarrow \mu^\pm \tau^{\mp}$ decay is below the $10^{-10}$ level for a bilepton mass of the order of 500 GeV.

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1. Introduction

In the standard model (SM), neutrinos are considered massless and thus lepton flavor violation (LFV) is forbidden at any order of perturbation theory. Even if the theory is extended with massive neutrinos, LFV transitions such as $\ell_i \rightarrow \ell_j \gamma$ would be induced up to the one-loop level and would be strongly suppressed due to a GIM-like mechanism: it was found that $\text{BR}(\mu \rightarrow e\gamma) \simeq 10^{-25} - 10^{-45}$ in the SM extended with non-diagonal lepton flavor couplings and massive neutrinos with a mass $m_\nu$ of a few eVs. Any signal of LFV would thus be a hint of new physics. However, recent evidences of neutrino oscillations and thereby a nonzero neutrino mass clearly point to LFV and have thus triggered the interest on the study of LFV decays such as $\ell_i \rightarrow \ell_j \gamma$, $\ell_i^- \rightarrow \ell_i^+ \ell_i^- \ell_i^-$, and $Z \rightarrow \ell_i^\pm \ell_j^\mp$ ($\ell_{i,j} = e, \mu, \tau$). Several theoretical extensions of the SM do predict such LFV transitions with a non-negligible rate. Currently there are stringent experimental constraints on LFV
Muon decays: \( \text{BR}(\mu \rightarrow e\gamma) < 2.4 \times 10^{-12} \) \(^2\), \( \text{BR}(\mu \rightarrow 3e) < 1.0 \times 10^{-12} \) \(^3\), and \( \text{BR}(\mu \text{Ti} \rightarrow e\text{Ti}) < 3.6 \times 10^{-11} \) \(^4\). Even more, the bound on the \( \mu \rightarrow e\gamma \) rate is expected to be improved up the level of \( 10^{-13} \) by the MEG experiment. \(^5\) On the other hand, the current bounds on LFV transitions involving the \( \tau \) lepton are less stringent: \( \text{BR}(\tau \rightarrow e\gamma) < 3.3 \times 10^{-8} \) \(^6\), \( \text{BR}(\tau \rightarrow \mu\gamma) < 4.4 \times 10^{-8} \) \(^6\), \( \text{BR}(\tau \rightarrow 3e) < 3.6 \times 10^{-8} \) \(^7\), and \( \text{BR}(\tau \rightarrow e^-e^+\mu) < 3.7 \times 10^{-8} \) \(^8\). As a matter of fact, the possibility that LFV transitions involving the last two lepton generations may be larger than those involving the electron has been widely conjectured in the literature. As far as LFV \( Z \) decays are concerned, the most stringent experimental bounds were obtained at LEP: \(^9\), \(^10\)

\[
\begin{align*}
\text{BR}(Z \rightarrow \ell^+\ell^-) &< 1.7 \times 10^{-6}, \\
\text{BR}(Z \rightarrow \ell^+\ell^\pm) &< 9.8 \times 10^{-6}, \\
\text{BR}(Z \rightarrow \mu^+\mu^\pm) &< 1.2 \times 10^{-5},
\end{align*}
\]

The study of these LFV \( Z \) decays has been the source of great interest as the future international linear collider with its Giga-\( Z \) option would allow a yearly production of \( 10^9 \) \( Z \) bosons \(^11\) which would open up the possibility for detecting some \( Z \) boson rare decays. Several predictions for the \( Z \rightarrow \ell_i^\pm \ell_j^\mp \) decay width have been obtained in the framework of various SM extensions, such as supersymmetric theories, \(^12\), \(^13\), the two-Higgs doublet model, \(^14\), \(^15\), \(^16\), \(^17\), \(^18\), \(^19\), \(^20\), \(^21\), \(^22\), \(^23\), \(^24\), \(^25\), \(^26\), \(^27\), \(^28\), \(^29\), the Zee-model, \(^19\), the scalar triplet model, \(^13\), the left-right symmetric model, \(^16\), \(^17\), top-color assisted technicolor, \(^22\), the SM with massive neutrinos, \(^18\), \(^19\), effective theories, \(^21\), etc.

The 331 model \(^25\), \(^29\) is an appealing SM extension based on the \( SU(3)_L \times U(1)_X \) gauge group. This model predicts new physics at the TeV scale and it is also attractive due to its peculiar mechanism of anomaly cancellation, which requires that the fermion family number is a multiple of the quark color number, thereby suggesting a solution to the flavor problem. A remarkable feature of the 331 model is the prediction of new exotic particles with masses bounded from above at the TeV scale due to theoretical constraints. Therefore, such a model could be confirmed or ruled out in a near future. Among the new particles predicted by the model, there are singly and doubly charged scalar bosons, exotic quarks of electric charges \(-4/3e\) and \(5/3e\), singly and doubly charged gauge bosons, and an extra neutral gauge boson. The new charged gauge bosons are known as bileptons as they carry two units of lepton number. LFV can be induced at the tree-level in the scalar and gauge sectors. In particular, we will focus on the possibility that the new doubly charged gauge bileptons can give rise to LFV at the tree-level, which in turn can induce LFV \( Z \) decays at the one-loop level. Our aim is to present such a calculation.

\(^a\)All the experimental limits used in this work correspond to the 90\% C.L. limits unless stated otherwise.
and obtain an estimate for the $Z \to \ell_i^\pm \ell_j^\mp$ branching ratios. To constrain the LFV bilepton couplings we will use the current experimental bounds on the LFV decays $\ell_i \to \ell_j \gamma$ and $\ell_i^- \to \ell_j^+ \ell_k^- \ell_k^-$. The anomalous magnetic moment of the muon will be used to constrain the bilepton mass.

The rest of our presentation is organized as follows. In Sec. 2 we present an overview of the minimal 331 model and consider the possibility of LFV mediated by the doubly charged vector bilepton. Section 3 is devoted to the presentation of our calculation, whereas the numerical results and the analysis are presented in Sec. 4. The conclusions and outlook are presented in Sec. 5.

2. The minimal 331 model

The minimal 331 model is based on the $SU_c(3) \times SU(3)_L \times U(1)_X$ gauge group. In this model, neutrinos are massless and the leptons are accommodated in antitriplets of $SU(3)_L$. The most economic scalar sector of the minimal 331 model requires three scalar triplets and one sextet of $SU(3)_L$. One scalar triplet is necessary to break $SU(3)_L \times U(1)_X$ down to the electroweak gauge group, whereas electroweak symmetry breaking (EWSB) requires the two remaining scalar triplets and the sextet. The latter is necessary to provide realistic masses for the leptons. This minimal Higgs sector has the following quantum numbers

$$\phi_Y = \left( \Phi_Y, \phi_Y^0 \right) : (1, 3, 1); \quad \phi_1 = \left( \Phi_1, \Delta^- \right) : (1, 3, 0); \quad \phi_2 = \left( \Phi_2, \rho^- \right) : (1, 3, -1),$$

where $\Phi_i = (\phi_i^+, \phi_i^0)$, with $\Phi_i = i \tau^2 \Phi_i^+$ for $i = 1, 2, 3; \Phi_Y = (\Phi_Y^+, \Phi_Y^+)$ contains the would-be Goldstone bosons associated with the new doubly charged, $Y^+$, and singly charged, $Y^+$, bileptons; the real and imaginary parts of $\phi_Y^0$ correspond to one physical Higgs boson and the would-be Goldstone boson associated with the extra neutral gauge boson, $Z'$. In addition the scalar sextet is given by
\[ H = \left( \begin{array}{c} T \frac{\tilde{\Phi}^3}{\sqrt{2}} \\
\frac{\sqrt{2}}{2} \eta^{--} \end{array} \right) : (1, 6, 0), \] (6)

where \( T \) is a SU(2) triplet

\[ T = \left( \begin{array}{cc} T^{++} & T^+ / \sqrt{2} \\
T^+ / \sqrt{2} & T^0 \end{array} \right), \] (7)

whereas \( \Delta^-, \rho^{--}, \) and \( \eta^{--} \) are singlets of SU(2) with hypercharge \(-2, -4, \) and \(+4, \) respectively.

The covariant derivative in the fundamental representation of SU(3) \( \times \) U(1)\( _X \) can be written as

\[ D_\mu = \partial_\mu - i g \frac{\lambda^a}{2} W^a_\mu - i g_X \frac{X^0}{2} X_\mu, \quad (a = 1 \ldots 8), \] (8)

with \( \lambda^a \) the Gell-man matrices and \( \lambda^9 = \frac{\sqrt{2}}{3} \text{diag}(1, 1, 1) \). The first stage of spontaneous symmetry breaking (SSB) is triggered by the vacuum expectation value (VEV) of \( \phi_Y \), which breaks the SU(3) \( \times \) U(1)\( _X \) gauge group down to SU(2) \( \times \) U(1)\( _Y \). In this stage of SSB, the exotic quarks and the new gauge bosons acquire their masses. The bileptons appear in a SU(2) \( \times \) U(1)\( _Y \) doublet with hypercharge 3 and are mass degenerate. They are defined in terms of the gauge eigenstates as follows

\[ Y^\mu = \left( \begin{array}{c} Y^{++}_\mu \\
Y^{+-}_\mu \end{array} \right) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} W^4_\mu - i W^5_\mu \\
W^6_\mu - i W^7_\mu \end{array} \right). \] (9)

The gauge fields \( W^8_\mu \) and \( X_\mu \) mix to produce the extra neutral gauge boson, \( Z' \), along with a massless gauge boson, \( B_\mu \), which is associated with the U(1)\( _Y \) group. They are given by

\[ Z'_\mu = c_\theta W^8_\mu - s_\theta X_\mu, \] (10)
\[ B_\mu = s_\theta W^8_\mu + c_\theta X_\mu, \] (11)

where \( s_\theta = \sin \theta, c_\theta = \cos \theta \) and \( \tan \theta = g_X / (\sqrt{2} g) \). The coupling constant associated with the hypercharge group is \( g' = g s_\theta / \sqrt{3} \). The remaining fields associated with the unbroken generators of SU(3)\( _L \) are the gauge bosons of the SU(2)\( _L \) group, which are denoted by \( W^i_\mu \) for \( i = 1, 2, 3 \).

After the first stage of SSB we are left with the SM with its particle content plus the new gauge bosons and exotic quarks, together with several scalar multiplets of SU(2)\( _L \): three doublets \( \Phi_i \) (\( i = 1, 2, 3 \)), one triplet \( H \), and various singlets. Electroweak symmetry breaking (EWSB) proceeds at the Fermi scale via the VEV of the SU(2)\( _L \) doublets \( < \Phi^0_i >_0 = v_i / \sqrt{2} \) (\( i = 1, 2 \)). By simplicity it can be assumed
that \(< H >_0 = 0\). In this stage, the SM particles acquire their masses and the bileptons and the \(Z'\) boson receive additional mass contributions. The extra mass terms for the bileptons, which arise from the Higgs kinetic-energy sector, violate the custodial \(SU(2)\) symmetry. Therefore, the bilepton masses split:

\[
m_{Y^{++}} = \frac{g^2}{4} (u^2 + v_2^2), \quad m_{Y^+} = \frac{g^2}{4} (u^2 + v_1^2).
\]

This mass splitting is bounded by the hierarchy of the SSB:

\[
|m_{Y^+}^2 - m_{Y^{++}}^2| \leq 3m_W^2.
\]

On the other hand, by matching the gauge coupling constants at the first stage of SSB, it is found that

\[
\frac{g_X^2}{g^2} = \frac{6s_W^2(m_{Z'})}{1 - 4s_W^2(m_{Z'})},
\]

which means that \(s_W^2(m_{Z'})\) has to be smaller than \(1/4\). It was found that this condition implies that the new \(Z'\) boson cannot be heavier than \(3.1\) TeV.\textsuperscript{31,32}

From this result and the symmetry-breaking hierarchy \(u \gg v_1, v_2, v_3\), it is inferred that the bilepton masses are smaller than \(m_{Z'}/2 \simeq 1500\) GeV.

### 2.1. LFV in the 331 model

The possibility of LFV in the 331 model was first analyzed in Ref.\textsuperscript{30} and more recently in Ref.\textsuperscript{33} in a more general context. In the scalar sector, LFV can be mediated by the neutral and charged scalar bosons. However, these interactions are expected to be very suppressed due to the smallness of the Yukawa couplings. We will thus not consider LFV mediated by scalar bosons in our calculation.

As far as the gauge sector is concerned, the neutral \(Z'\) gauge boson cannot mediate LFV at the tree-level: it turns out that the \(Z'\) boson couplings to the leptons are flavor universal, so this gauge boson cannot mediate LFV at the tree level as the rotation of flavor states to mass eigenstates yields a diagonal coupling matrix. Moreover, it is interesting to note that the \(Z'\) gauge boson has a leptophobic nature as its couplings to a lepton pair are suppressed by the \(\sqrt{1 - 4s_W^2}\) factor.\textsuperscript{34}

On the other hand, the interactions between the bileptons and the leptons can be written in flavor space as

\[
\mathcal{L} = -\frac{g}{\sqrt{2}} \bar{\ell}_R^{'\gamma \mu} \ell_L^{'Y^{++}} + \frac{g}{\sqrt{2}} \bar{\ell}_R^{'\gamma \mu} \nu'_L Y^+ + \text{H.c.}
\]

After a rotation to the physical states is performed (\(\ell'_{L,R} = U_{L,R} \ell_{L,R} \) and \(\nu'_L = U_{L} \nu_L\)) we are left with

\[
\mathcal{L} = -\frac{g}{\sqrt{2}} \bar{\ell}_R^{Y'\mu} Y'_{L} Y^{++} + \frac{g}{\sqrt{2}} \bar{\ell}_R^{Y'\mu} Y'_{L} Y^{+} + \text{H.c.}
\]

where we introduced the unitary flavor mixing matrix \(V' = U^T_R U_L\). The singly charged bilepton effects on the \(Z \to \ell_i^{\pm} \ell_j^{\mp}\) decay will vanish due to the zero mass of
neutrinos. We will thus only need to consider LFV mediated by the doubly charged bilepton.

3. Analysis of the $Z \rightarrow \ell_i^\pm \ell_j^{\mp}$ decay

To calculate the $Z \rightarrow \ell_i^\pm \ell_j^{\mp}$ decay we will neglect the masses of the outgoing leptons but the internal lepton mass will be retained. The transition amplitude will be calculated using the Feynman-t’Hooft gauge. In our approximation, the contributions to the $Z \rightarrow \ell_i^\pm \ell_j^{\mp}$ decay arise from the four Feynman diagrams shown in Fig. 1. Although there is also a set of Feynman diagrams obtained by replacing each bilepton gauge boson by its associated would-be Goldstone boson, $G_Y$, the respective amplitudes contain terms proportional to products of the lepton masses due to the form of the $G_Y \ell_i \ell_j$ coupling, so these contributions can be neglected from the calculation. Apart from the Feynman rule for the coupling of the doubly charged bilepton to a lepton pair, which can be extracted from Eq. (15), we only need the Feynman rule for the $Z Y^{++} Y^{--}$ vertex. The latter is given in the Feynman-t’Hooft gauge by:

$$Z_\alpha(k) Y^{--}_\mu (k_1) Y^{++}_\nu (k_2) = - \frac{ig}{2c_W} g_{ZYY} \left[ (k_2 - k_1)_\alpha g_{\mu \nu} + (k - k_2 - k_1)_\mu g_{\alpha \nu} - (k - k_1 - k_2)_\nu g_{\alpha \mu} \right],$$

(16)

where all the particles are incoming and $g_{ZYY} = 1 - 4s_W^2$, with $s_W = \sin \theta_W$ and $c_W = \cos \theta_W$. 

Fig. 1. Feynman diagrams for the $Z \rightarrow \ell_i^\pm \ell_j^{\mp}$ decay at the one-loop level in the 331 model. In the Feynman-t’Hooft gauge there is an additional set of Feynman diagrams obtained by replacing each bilepton gauge boson by its associated would-be Goldstone boson, but these diagrams can be neglected since their amplitudes are proportional to the lepton masses.
To obtain the transition amplitude for each Feynman diagram we used the method of Ref. [36] which is meant for processes involving vertices with complex conjugate fields, such as our lepton number violating vertices. The Feynman parameters technique was used to solve the loop integrals. In the massless outgoing lepton limit, the \( Z(p) \to \ell^\pm_i(p_i)\ell^\mp_j(p_j) \) decay amplitude can be written as

\[
iM(\bar{Z} \to \ell^\pm_i(p_i)\ell^\mp_j(p_j)) = \sqrt{\alpha} P_L v(p_j)e^\alpha(p), \tag{17}\]

with \( P_L = (1 - \gamma^5)/2 \). The \( F_{ij}^{ij} \) function depends on the internal lepton mass and the bilepton mass. It can be expressed in the form:

\[
F_{ij}^{ij}(x_k, x_Y) = \frac{g^2}{2\pi^2} \sum_{k=1}^{3} V_{ik}^Y V_{jk}^Y \cdot I(x_k, x_Y), \tag{18}\]

with

\[
I(x_k, x_Y) = \sum_{n=1}^{4} f_n(x_k, x_Y). \tag{19}\]

We have introduced the notation \( x_k = m^2_k/m^2_Z \) and \( x_Y = m^2_Y/m^2_Z \), with \( m_k \) the internal lepton mass and \( m_Y \) the bilepton mass. The \( f_n \) function stands for the contribution of the \( n \)th Feynman diagram of Fig. [1] After some lengthy algebra, we obtain:

\[
\begin{align*}
 f_1(x_k, x_Y) &= 2g_{ZY} \left[ (\delta_{Yk} + 2) (B_Y - B_{kY}) - (\delta_{Yk}^2 + 2x_Y - x_k) C_{Yk} \right. \\
 &\quad \left. - \frac{1}{2} B_Y \right], \\
 f_2(x_k, x_Y) &= 2g_R \left[ 1 + (\delta_{Yk} + 2) (B_k - B_{kY}) - \frac{1}{2} B_k + (1 + \delta_{Yk})^2 C_{kY} \right] \\
 &\quad + 2g_L x_k C_{kY}, \\
 (f_3 + f_4)(x_k, x_Y) &= g_L (1 - B_{kY} - dB_{kY}),
\end{align*} \tag{20} \tag{21} \tag{22}\]

where \( g_{L,R} = g_V \pm g_A \), with \( g_V = -1/2 + 2s^2_W \) and \( g_A = -1/2 \) the vector and axial \( Z \)-lepton couplings. Note that the amplitudes of the two bubble diagrams must be combined to obtain the right limit for massless outgoing leptons. In addition, the following functions (the subscript denotes the corresponding dependence) have been introduced:
\[ B_Y = 2 \left( 1 - \tau_Y \arccot \tau_Y - \log(x_Y) \right) + \Delta, \]  
\[ (23) \]

\[ B_k = 2 \left( 1 - \lambda_k \text{arcoth} \lambda_k - \log(x_k) \right) + \Delta, \]  
\[ (24) \]

\[ B_{kY} = 1 + \frac{1}{\delta_{Yk}} \left( x_k \log(x_k) - x_Y \log(x_Y) \right) + \Delta, \]  
\[ (25) \]

\[ dB_{kY} = \frac{1}{2\delta_{Yk}} \left( x_Y^2 - x_k^2 - 2x_kx_Y \log \left( \frac{x_k}{x_Y} \right) \right), \]  
\[ (26) \]

\[ C_{kY} = \log \left( \frac{\delta_{Yk}}{\delta_{Yk} + 1} \right) \log(\delta_{Yk}) + F \left( \frac{x_Y}{\delta_{Yk}} \right) - F(\lambda_k) - F(\lambda_-), \]  
\[ (27) \]

\[ C_{kY} = G(x_k) - G(x_Y) - 4 \arccot (\tau_Y) \arctan \left( \frac{\tau_Y}{2\delta_{Yk} - 1} \right) \]  
\[ \quad - \log \left( \frac{\delta_{Yk}}{\delta_{Yk} - 1} \right) \log(x_Y) + 2 \text{Re} \left[ H(\tau_+) \right], \]  
\[ (28) \]

where \( \delta_{Yk} = x_Y - x_k, \lambda_k = \frac{1}{2} (1 \pm \lambda_k), \lambda_k = \sqrt{1 - 4x_k}, \tau_+ = \frac{1}{2} (1 + i\tau_Y) \) and \( \tau_Y = \sqrt{4x_Y - 1} \). \( \Delta \) stands for the usual ultraviolet singularity in dimensional regularization. Furthermore

\[ F(\lambda) = \log(\lambda) \log \left( \frac{\delta_{Yk}}{\delta_{Yk} + 1} \right) - \log(\lambda - 1) \log \left( \frac{\delta_{Yk} + 1}{\delta_{Yk}} \right) + L_2 \left( \frac{\lambda - 1}{\delta_{Yk} + 1} \right), \]  
\[ (29) \]

\[ G(x) = \log(x) \log \left( \frac{\delta_{Yk}^2 + x_k - x}{\delta_{Yk}^2} \right) + L_2 \left( \frac{x}{\delta_{Yk}^2 + x_k} \right), \]  
\[ (30) \]

\[ H(\tau) = L_2 \left( \frac{\tau}{\tau + \delta_{Yk} - 1} \right) - L_2 \left( \frac{\tau}{\tau - \delta_{Yk}} \right). \]  
\[ (31) \]

Although each Feynman diagram is ultraviolet divergent by itself, all the divergences cancel each other out. This becomes evident when we write \( g_{ZYY} = -2gV = -(gL + g_R) \).

We would like to point out that to cross-check our calculation, we made an alternative evaluation via the Passarino-Veltman method, using the unitary gauge and without any approximation. The resulting amplitudes are rather lengthy to be included here, but numerical evaluation showed a nice agreement between our approximate result and the exact calculation.

The \( Z \to \ell_i^+ \ell_j^- \) decay width is given in the massless outgoing lepton limit by

\[ \Gamma(Z \to \ell_i^+ \ell_j^-) = \frac{m_Z}{24\pi} \left( |F_{ij}^{\ell_l}|^2 + |F_{ij}^{\ell_U}|^2 \right). \]  
\[ (32) \]
To obtain an estimate for the $Z \to \ell_i^+ \ell_j^-$ branching ratios, we need to analyze the bounds on the mixing matrix $V_Y$ and the bilepton mass. Below we will examine the bounds obtained from the $\ell_j^- \to \ell_i^+ \ell_k^-$ and $\ell_j \to \ell_i \gamma$ decays together with the muon anomalous magnetic moment.

4. Numerical analysis

4.1. Bounds on the bilepton LFV couplings

The three-body decay $\ell_i^- \to \ell_j^+ \ell_k^-$ proceeds at the tree-level through the Feynman diagram of Fig. 2. Its decay width can be obtained straightforwardly in the limit of massless outgoing leptons. It is given by:

$$\Gamma(\ell_i^- \to \ell_j^+ \ell_k^-) = \frac{g^4 m_5^2}{321\pi^3 m_Y^4} |V_{kk}^Y|^2 \left(|V_{ij}^Y|^2 + |V_{ji}^Y|^2\right).$$

(33)

The experimental 90% C.L. limits on the LFV decays $\mu^- \to e^+ e^- e^-$ [3] and $\tau^- \to e^+ e^- \mu^-$ translate into the following bounds with 90% C.L.:

$$\frac{|V_{11}^Y|^2 (|V_{12}^Y|^2 + |V_{21}^Y|^2)}{m_Y^4} \leq 2.57 \times 10^{-20} \quad \text{GeV}^{-4},$$

(34)

$$\frac{|V_{11}^Y|^2 (|V_{13}^Y|^2 + |V_{31}^Y|^2)}{m_Y^4} \leq 5.14 \times 10^{-12} \quad \text{GeV}^{-4},$$

(35)

$$\frac{|V_{11}^Y|^2 (|V_{23}^Y|^2 + |V_{32}^Y|^2)}{m_Y^4} \leq 5.29 \times 10^{-12} \quad \text{GeV}^{-4}.\quad (36)$$

We used the $\mu$ and $\tau$ mean lifetimes given in Ref. [37]. Following Ref. [30] we will parametrize the mixing LFV matrix as $V_Y^{ij} \simeq \delta_{ij} + \alpha_{ij} e^{i\theta_{ij}}$, where $\alpha_{ij} = -\alpha_{ji}$ and $\theta_{ij} = -\theta_{ji}$ are mixing angles and CP-violating phases, respectively. Therefore we obtain the following limits with 90% C.L.
As far as the $\ell_i \to \ell_j \gamma$ decay, it proceeds via the triangle Feynman diagrams shown in Fig. 3. The bubble diagrams does not contribute in the limit of massless outgoing lepton. Using the same scheme described above to obtain the $Z \to \ell_i^\pm \ell_j^\mp$ amplitude, we obtain the $\ell_i \to \ell_j \gamma$ amplitude, which can be written as

$$M(\ell_i \to \ell_j \gamma) = \frac{ieq_\mu}{m_i + m_j} \bar{u}(p_j)\sigma^{\alpha\mu} \left( f_{ij}^{A} + f_{ij}^{V} \gamma^5 \right) u(p_i)\epsilon_{\alpha}(p),$$

where $q = p_i - p_j$ and $p_i$ ($p_j$) are the photon and the incoming (outgoing) lepton 4-momenta, whereas

$$f_{ij}^{V,A} = \frac{g^2 m_i}{2 \pi^2 m_Y^2} \int_0^1 \int_0^{1-x} \left( \frac{x}{M_1} + \frac{2(1-x)}{M_2} \right) \sum_{k=1}^3 \left( V_{ik}^Y V_{kj}^{Y*} \pm V_{jk}^Y V_{ki}^{Y*} \right) m_k,$$

with $M_1 = x(1-y\xi_i) + (1-x)\xi_k$, $M_2 = x(\xi_k - y\xi_i) + (1-x)$ and $\xi_a = m_a^2/m_Y^2$. We have dropped any terms independent of the internal lepton mass, which cancel due to the unitarity of the $V^Y$ matrix. Furthermore, if one neglects the lepton masses

$$\frac{|V_{12}^Y|^2}{m_Y^2} < 1.28 \times 10^{-20} \text{ GeV}^{-4},$$

$$\frac{|V_{13}^Y|^2}{m_Y^2} < 2.57 \times 10^{-12} \text{ GeV}^{-4},$$

$$\frac{|V_{23}^Y|^2}{m_Y^2} < 2.64 \times 10^{-12} \text{ GeV}^{-4}.$$
in the $M_1$ and $M_2$ coefficients, it follows that

$$f_{ij}^{V,A} = \frac{3g^2 m_i}{2\pi^2 m_Y^2} \sum_{k=1}^{3} \left( V_{ik}^Y V_{kj}^Y \pm V_{jk}^Y V_{ki}^Y \right) m_k,$$

(42)

which coincides with the result previously found in Ref. [30]. In the massless outgoing lepton limit, the $\ell_i \to \ell_j \gamma$ decay width can be written as

$$\Gamma(\ell_i \to \ell_j \gamma) = \frac{e^2 m_i}{8\pi} \left( |f_{ij}^V|^2 + |f_{ij}^A|^2 \right).$$

(43)

Since the sum in Eq. (42) depends on the internal lepton mass and the $\mu^- \to e^- e^- e^-$ decay gives a strong constraint on $|V_{12}^Y|$, we can neglect the electron and muon terms. It follows that the experimental 90\% C.L. limits on the decays $\mu \to e \gamma$, $\tau \to e \gamma$, and $\tau \to \mu \gamma$ translate into the following bounds with 90\% C.L.

$$|V_{13}^Y|^2 m_Y \leq 2.92 \times 10^{-21} \text{ GeV}^{-4},$$

(44)

$$|V_{13}^Y|^2 m_Y \leq 2.35 \times 10^{-13} \text{ GeV}^{-4},$$

(45)

$$|V_{23}^Y|^2 m_Y \leq 3.14 \times 10^{-13} \text{ GeV}^{-4}.$$  

(46)

As far as the lepton anomalous magnetic moment $a_\ell = (g - 2)/2$ is concerned, it can receive new contributions in the 331 model from the scalar and gauge sectors. The doubly charged bilepton contributes through triangle diagrams analogue to those of Fig. 3. Its contribution is given by

$$a_\ell = \frac{g^2 m_\ell}{32\pi^2 m_Y^2} \sum_{k=1}^{3} |V_{\ell k}^Y|^2 \left( 9m_k \cos(2\theta_{\ell k}) + 7m_\ell \right).$$

(47)

If the flavor mixing matrix $V^Y$ is diagonal, $a_\ell$ becomes

$$a_\ell = \frac{2g^2 m_\ell^2}{3\pi^2 m_Y^2},$$

(48)

which agrees with the result obtained before in Ref. [38]. We also observe that the contribution to $a_\ell$ from the scalar bosons as well as the singly charged bilepton and the extra neutral gauge boson are subdominant, so the 331 model contribution, $a_\ell^{331}$, can be assumed to arise mainly from the doubly charged bilepton. The current experimental limit on $a_\mu$ is not useful to bound the LFV couplings but it can constrain the bilepton mass $m_Y$. For the theoretical and experimental values of $a_\mu$, we will use the most recent data quoted in Ref. [37]. The discrepancy between the theoretical SM contribution, $a_\mu^{SM}$, and the world average, $a_\mu^{Exp}$, of experimental measurements[39] is given by:
\[ \Delta a_\mu = a_\mu^{\text{Exp.}} - a_\mu^{\text{SM}} = 255 \pm 63 \times 10^{-11}, \]  

(49)

where the \( e^- e^+ \rightarrow \pi^- \pi^+ \) data from BABAR were used to evaluate the hadronic contribution to \( a_\mu^{\text{SM}} \). Although this discrepancy is about 3.2 standard deviations, it is not yet conclusive since there is still a considerable discrepancy between the various evaluations of the hadronic contribution. If \( \Delta a_\mu \) is ascribed to the doubly charged bilepton, we get the following bound on \( m_Y \) with 95% C.L.:

\[ m_Y \geq 421 \text{ GeV}. \]  

(50)

Let us assess how this bound compares with other indirect bounds. The very stringent bound \( m_Y > 800 \text{ GeV} \) was obtained from muonium-antimuonium conversion.\(^{41}\) This bound, which would rule out the minimal 331 model, is based on the assumptions that the \( V^Y \) matrix is flavor diagonal and the scalar sector of the model does not contribute significantly to muonium-antimuonium conversion. Another stringent bound, \( m_Y > 750 \text{ GeV} \), arises from fermion pair production and lepton-flavor violating processes.\(^{42}\) These bounds can be evaded if one considers an extended Higgs sector or less restrictive assumptions.\(^{43}\) We will rather consider a bilepton mass of a few hundreds of GeV to obtain an estimate of the \( Z \rightarrow \ell_i^\pm \ell_j^- \) branching ratios.

### 4.2. The \( Z \rightarrow \ell_i^\pm \ell_j^- \) branching ratios

Due to the unitarity of \( V^Y \), i.e. \( \sum_k V^Y_{ik} V^Y_{kj} = \delta_{ij} \), and neglecting imaginary phases, we can write

\[
\text{BR}(Z \rightarrow \ell_i^\pm \ell_j^-) = \lambda |V^Y_{i2} V^Y_{j3} (I(x_2, x_Y) - I(x_1, x_Y))|^2,
\]

(51)

with \( \lambda = \frac{m_Z^2}{12 \pi^2} \left( \frac{g^3}{s \pi c_{\gamma W}} \right)^2 \). Notice that according to Eq. (57), \( |V^Y_{i2}| \) is strongly constrained, so one can write

\[
\text{BR}(Z \rightarrow e^\pm \mu^\pm) \simeq \lambda |V^Y_{13}|^2 |V^Y_{23}|^2 |I(x_3, x_Y) - I_1(x_3, x_Y)|^2,
\]

(52)

\[
\text{BR}(Z \rightarrow e^\pm \tau^\pm) \simeq \lambda |V^Y_{13}|^2 |I(x_3, x_Y) - I_1(x_1, x_Y)|^2,
\]

(53)

\[
\text{BR}(Z \rightarrow \mu^\pm \tau^\pm) \simeq \lambda |V^Y_{23}|^2 |I(x_3, x_Y) - I_3(x_2, x_Y)|^2.
\]

(54)

Numerical evaluation together with the bounds (44)-(46) give the upper bounds on the LFV \( Z \) decays shown in Table 1 for \( m_Y = 100 \text{ GeV} \) and \( m_Y = 500 \text{ GeV} \). While the loop amplitude magnitude decreases for larger values of \( m_Y \), the bounds on the LFV matrix elements \( V^Y_{ij} \) loosen up. This explains the fact that the bounds on the LFV \( Z \) decays are slightly weaker for larger \( m_Y \). However, the bounds obtained for \( m_Y = 1000 \text{ GeV} \) are of similar order of magnitude than those obtained for...
Table 1. Upper limit with 90\%C.L. on the bilepton contribution to LFV $Z$ decays. The bounds given in Eqs. (44)-(46) were employed.

| BR ($Z \rightarrow e^\mp \mu^\pm$) | $m_Y = 100$ GeV | $m_Y = 500$ GeV |
|----------------------------------|----------------|----------------|
| $3.2 \times 10^{-20}$ | $1.1 \times 10^{-19}$ |
| $6.37 \times 10^{-11}$ | $4.57 \times 10^{-10}$ |
| $6.56 \times 10^{-11}$ | $2.26 \times 10^{-10}$ |

$m_Y = 500$ GeV. Our results indicate that the bilepton mediated $Z$ decays would be far from the reach of detection, though the corresponding branching ratios are of the same order of magnitude than in other SM extensions. For instance, in the framework of the Zee-model it was found that $BR(Z \rightarrow e^\mp \mu^\pm) < 4.2 \times 10^{-16}$, $BR(Z \rightarrow e^{\mp}\tau^{\pm}) < 1.1 \times 10^{-8}$, and $BR(Z \rightarrow \mu^{\mp}\tau^{\pm}) < 1.6 \times 10^{-12}$ for typical values of the model parameters.\[19 also, in the SM enlarged with massive neutrinos with masses of the order of a few dozens of GeV, the upper bound $BR(Z \rightarrow \mu^{\mp}\tau^{\pm}) \lesssim 10^{-11}$ was obtained.\[18

5. Final remarks
We have investigated the $Z \rightarrow \ell_i^{\pm} \ell_j^{\mp}$ decay in the framework of the minimal 331 model. We focused on the contributions mediated by the doubly charged bilepton and estimated the bounds on the LFV bilepton couplings from the experimental constraints on the decays $\ell_i \rightarrow \ell_j \gamma$ and $\ell_i^{-} \rightarrow \ell_j^{+} \ell_k^{-} \ell_k^{-}$. Our results indicate that these contributions seem to be far from experimental detection. The smallness of the decay rates can be explained mainly from the fact that the LFV bilepton couplings are strongly constrained by current experimental data.

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