Cosmology and the Unexpected

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In these two lectures I will discuss some outstanding problems in the standard model of cosmology, concentrating on the physics that might be related to the title of this school, “Searching for the totally unexpected in the LHC era.” In particular, I will concentrate on dark energy, dark matter, and inflation.

I. INTRODUCTION

In the last decade or two we have made remarkable progress in measuring cosmological parameters to unprecedented accuracy. Parameters such as the Hubble constant, \( H_0 = 72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1} \), the temperature of the cosmic background radiation (CBR), \( T_0 = 2.728 \pm 0.004 \text{ K} \), and many other parameters are now known to impressive precision. Of course, we don’t invest so much time and effort to determine cosmological parameters because of an interest in numerology, but rather, we do it because the parameters are necessary input for the task of constructing a standard cosmological model.

The precision cosmological measurements have lead to the latest cosmological model, usually called the standard cosmological model, or \( \Lambda \)CDM, where \( \Lambda \) indicates Einstein’s cosmological constant (or more generally, dark energy), and CDM stands for cold dark matter. Aspects of this cosmological standard model relevant for the purposes of these lectures are indicated in Figure 1. The most remarkable feature of the standard cosmological model is that it seems capable of accounting for all cosmological observations; i.e., it seems to work!

There are several interesting features of the \( \Lambda \)CDM model illustrated in Fig. 1. First, while the early universe was radiation dominated, at present the radiation energy density is only 0.005% of the total. The figure also illustrates why chemistry is not very important: the chemical elements (by which I mean elements other than the simplest elements of hydrogen and helium) are only about 0.025% of the total. Neutrinos contribute a much larger fraction of the total mass-energy density, about 0.5%. The neutrino contribution is about as large as the contribution from stars. Most of the baryons in the Universe are not found in stars, but rather they are in the form of a hot intracluster gas of hydrogen and helium. This completes what we “see” (although there is only indirect evidence for the cosmic neutrino background).

The most striking feature of the present composition of the universe in the \( \Lambda \)CDM model is that 95% of the present universe is dark! Of the dark components, roughly 25% of the total mass-energy density is dark matter, associated with galaxies, clusters of galaxies, and other bound structures. The bulk of the mass-energy of the universe, about 70% in the standard \( \Lambda \)CDM model, is dark energy, which drives an accelerated expansion of the Universe.

In the first lecture I will discuss issues associated with dark energy. In the second lecture, I will discuss possibilities for dark matter. Also in the second lecture I will briefly discuss some aspects of cosmic inflation, in particular what we might learn about inflation from present observations.

For the purposes of this school, let me begin by recalling Einstein’s comment to Arnold Sommerfeld on December 9, 1915, “How helpful to us is astronomy’s pedantic accuracy, which I used to secretly ridicule.” While Einstein was referring to precision measurements of the advance of the perihelion of Mercury, now we find precision measurements helpful to us because dark energy, dark matter, inflation, and baryo/leptogenesis seem to point to physics beyond the standard model of particle physics.

II. DARK ENERGY

The most important point regarding dark energy, which I will stress repeatedly, is that all evidence for dark energy is indirect. The only effect of dark energy is on the expansion history of the Universe.

The expansion rate of the Universe is perhaps the most fundamental quantity in cosmology. In the standard cosmological model the expansion rate is determined from the 00-component of the Einstein equations, \( G_{00} = 8\pi G T_{00} \). In the Friedmann-Robertson-Walker-Lemaitre (FLRW) model, \( G_{00} = 3H^2 + k/a^2 \), where \( H = \dot{a}/a \) is the expansion
rate (here, $a$ is the Robertson-Walker scale factor and $k = \pm 1$ or 0 depending on the geometry). With the assumption of a perfect-fluid stress tensor, $T_{00} = \rho$, where $\rho$ is the mass density.

The stress tensor is assumed to consist of several components with different equations of state. For any component $i$, the equation of state is defined as $w_i = p_i/\rho_i$. The energy density in component $i$ evolves with redshift $z \equiv a_0/a - 1$ as $\rho_i(z) = \rho_i(0)(1 + z)^{3(1+w_i)}$, where $a_0$ is the present value of the scale factor. In the event that $w$ is a function of $a$, then $(1 + z)^{3(1+w_i)}$ should be replaced by $\exp \left\{3 \int_{a_0}^a da a^{-1} [1 + w(a)] \right\}$. Then the expansion rate as a function of redshift can be expressed as

$$H^2(z) = H_0^2 \left[ (1 - \Omega_{\text{TOT}}) (1 + z)^2 + \Omega_M (1 + z)^3 + \Omega_R (1 + z)^4 + \Omega_w (1 + z)^{3(1+w)} \right].$$

Here, $\Omega_i$ is the ratio of the present energy density in component $i$ compared to the critical density $\rho_C = 3H_0^2/8\pi G$, and $\Omega_{\text{TOT}} = \sum_i \Omega_i$. The subscript "M" indicates a matter component ($w = 0$) while the subscript "R" indicates a radiation component ($w = 1/3$).

The value of $\Omega_{\text{TOT}}$ is well determined by WMAP to be $1.026^{+0.015}_{-0.016}$ at the 68% confidence level. The value of $\Omega_R$ is also well determined by CBR measurements: It is of order $5 \times 10^{-5}$. Finally, $\Omega_w$ is determined by measuring the expansion history of the Universe, and determining the values of $\Omega_w$ and $w$ necessary to fit the data.

Many cosmological observables depend on the expansion history of the Universe. They often depend on the expansion history through the coordinate distance $r$ of a source of redshift $z$. Recall that the Robertson-Walker metric can be written in the form

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right],$$

where $d\Omega$ is the angular differential and $r$ is the comoving “radial” coordinate. The radial coordinate of a source at redshift $z$ is determined by integrating the null geodesic ($ds^2 = 0$) to obtain

$$r(z) = \frac{1}{\sinh \left[ \int_0^z \frac{dz'}{H(z')} \right]},$$

where $\sin, 1, \sinh$ obtains for $k = +1, 0, 1$, respectively.
TABLE I: The relationship of observables to the time evolution of the expansion rate of the Universe. The coordinate distance \( r(z) \) depends on the time evolution of \( H(z) \) through Eq. (3).

| Observable             | Notation      | Definition                                      | Value                                                                 |
|------------------------|---------------|-------------------------------------------------|-----------------------------------------------------------------------|
| luminosity distance    | \( d_L(z) \)  | \((\text{Luminosity}/4\pi\text{Flux})^{1/2}\)  | \( d_L(z) \propto r(z)(1 + z) \)                                     |
| angular-diameter distance | \( d_A(z) \) | \( \text{Physical size}/\text{Angular size} \)   | \( d_A(z) \propto r(z)/(1 + z) \)                                     |
| volume element         | \( dV(z) \)   | \( \sqrt{H} drd\Omega \)                       | \( dV(z) = \frac{r^2(z)}{\sqrt{1 - kr^2(z)}} drd\Omega \)          |
| age of the Universe    | \( t(z) \)    | time from \( z = \infty \) to \( z \)          | \( t(z) = \int_{z}^{\infty} \frac{dz'}{(1 + z')H(z')} \)          |

FIG. 2: The results of the Supernova Legacy Survey [5]. Shown is the apparent magnitude of supernovae normalized to that expected in the standard ΛCDM model.

Now let us turn to the observables defined in Table I. One can see that they all depend on the time evolution of the expansion rate, hence to the properties of the stress tensor, hence to a new fluid characterized by \( w \).

The best evidence for dark energy comes from luminosity-distance-redshift determinations using Type 1a supernovae as standard candles of known (or at least calibrated) luminosity [4]. A recent large survey is the Supernova Legacy Survey (SNLS) [5]. The results for the observed magnitude of supernovae as a function of redshift is shown in Fig. 2. Without performing a statistical analysis, it is apparent that the data fit the ΛCDM model and the Einstein–de Sitter (EdS) model is “observationally challenged.”

It is important to realize that dark energy experiments do not actually measure dark energy directly. They determine the expansion history of the Universe by measuring cosmological observables sensitive to the expansion history. What is determined is that the EdS model does not fit the data. Any information about the new component to the stress tensor depends on the assumptions you put in. You have to input a cosmological model and compare it to the data. An example of such an analysis is shown in Fig. 3. To construct this figure, it was assumed that the expansion history of the Universe is described by a FLRW model with dark energy described by a fluid with \( w = -1 \), i.e., Einstein’s cosmological constant. Also employed are priors on cosmological parameters such as \( H_0 \).

If one adopts the model used in Fig. 3 then there is a cosmological constant with an equivalent energy density of \( \rho_V \sim 10^{-30} \text{g cm}^{-3} \). Henceforth I will refer to this as the cosmoillogical constant. There are two reasons for referring to this cosmological constant as illogical, rather than logical.

First, the magnitude of the cosmoillogical constant is, well, illogical. Length scales and mass scales of the cosmoillogical constant are given in Table III. Fundamental length scales and mass scales shown in Table III are, as I said, illogical.

A cosmoillogical constant is equivalent to, and indistinguishable from, contributions to the vacuum energy. It is only meaningful to consider them together as a vacuum energy. Among the contributions to vacuum energy is the fact that all fields are harmonic oscillators with a zero point energy. This should contribute a vacuum energy of...
FIG. 3: Values of $\Omega_\Lambda$ and $\Omega_M$ allowed at confidence levels of 68%, 90%, and 95% assuming a cosmological model with $w = -1$. The data used to place the limits are shown in Fig. 2.

TABLE II: The length scales and mass scales implied by a cosmological constant with an energy density of $10^{-30}$ g cm$^{-3}$.

|                | $\rho_V$  | $\Lambda = 8\pi G$ |
|----------------|-----------|---------------------|
| length scale   | $10^{-3}$ cm | $10^{29}$ cm        |
| mass scale     | $10^{-4}$ eV  | $10^{-33}$ eV       |

$\rho_V = \sum_{\text{all particles}} \pm \int d^3 k \sqrt{k^2 + m^2}$, where the sign of the contribution depends on the spin-statistics of the particle. The individual integrals have quartic divergences, so formally the answer is infinity. (Unless for some reason—such as supersymmetry (SUSY)—there are exact cancellations between bosons and fermions.) This need not cause alarm because, after all, this is field theory. One might imagine introducing a cutoff $\Lambda_C$ to the integral so that the integral is finite, and the individual contributions are $\pm \Lambda_C^4$. If you assume $\Lambda_C$ is related to gravity, then this contribution to $\rho_V$ is $M_P^4 = (10^{28}$ eV$)^4 = 10^{112}$ eV$^4$. If you conjecture that $\Lambda_C$ is related to the SUSY breaking scale (assumed here to be 1 TeV), then $\rho_V = M_{\text{SUSY}}^4 = (10^{12}$ eV$)^4 = 10^{48}$ eV$^4$. These scales are far away from the observed value of $\rho_V = (10^{-4}$ eV$)^4 = 10^{-16}$ eV$^4$.

Another contribution to vacuum energy comes from the vacuum energy associated with spontaneous symmetry breaking. Associated with spontaneous symmetry breaking of GUTs, SUSY, electroweak, and chiral symmetries are vacuum energies of $10^{100}$ eV$^4$, $10^{48}$ eV$^4$, $10^{52}$ eV$^4$, and $10^{32}$ eV$^4$, respectively. Once again, these scales are far away from the observed value of $\rho_V$.

A not unrelated issue is the timing of the epoch in the history of the Universe when the dark energy density is comparable to the density of matter and radiation. A graph of the relative contributions of $\Omega_M + \Omega_R$ and $\Omega_\Lambda$ is shown in Fig. 4. It turns out to be very curious that while in the distant past vacuum energy was “in the noise,” and in the distant future matter and radiation densities will be “in the noise,” while today we live at the very convenient epoch.
when they are just about the same value. More than curious, this fact is illogical.

Astronomical evidence for a vacuum energy today would be remarkable. But as the saying goes, remarkable results require remarkable evidence. The evidence from supernova observations has been tested and probed for systematic uncertainties. While there is no indication of anything fishy, it is crucial to find corroborating evidence from other techniques with different astronomical systematic uncertainties. Fortunately, there is other evidence that there is dark energy. (As discussed below, a more exact statement is that there is other evidence that the time evolution of the expansion rate is not described by the EdS model, and well fit by ΛCDM.)

Other evidence comes from 1) cosmic subtraction, 2) baryon acoustic oscillations, 4) weak lensing, 4) galaxy clusters, 5) the age of the Universe, and 6) structure formation.

First let me mention the sophisticated mathematical operation of cosmic subtraction. The idea is illustrated in Fig. 5. The idea is simple. CBR observations imply $\Omega_{\text{TOTAL}}$ is very near unity. A variety of astronomical observations illustrated in Fig. 5 result in $\Omega_M \approx 0.3$. (The observations resulting in this value of $\Omega_M$ include dynamics of galaxies and galaxy clusters, gravitational lensing, X-ray observations of galaxy clusters that measure the depth of the potential well by determining the gas temperature, CBR observations, numerical simulations of the large-scale structure of the Universe, and determination of the “shape factor” of the power spectrum of density fluctuations.)

Now for the promised sophisticated operation of subtraction:

$$1 - 0.3 = 0.7.$$  \hfill (4)

Let me expand on this for the benefit of any string theorists who might be in the audience. $1 - 0.3$ is not equal zero, but in fact, it is 0.7. This profound result holds in any spacetime dimension and is perfectly consistent with the AdS/CFT correspondence. This “missing” component to the total stress energy of the Universe is conveniently attributed to dark energy.

Before turning to other techniques, it is a good time to re-emphasize the important point made previously. The measurements do not tell us that there is dark energy. Rather, the observations only provide indirect evidence for dark energy. To elaborate on this, let me review the procedure.

1. Assume a model cosmology. In this case, the assumptions involve:
   (a) The FLRW model, which results in the Friedmann equation for $H(z)$.
   (b) The energy and pressure content of the Universe and how they scale with redshift.
   (c) Input or integrate over cosmological parameters such as $H_0$, $\Omega_B$, etc.
2. Within the cosmological model, calculate observables such as $d_L(z)$, $d_A(z)$, $H(z)$, etc.
3. Compare to observations.
4. Discover that the model cosmology fits with dark energy, but not without dark energy.
FIG. 5: Cosmic subtraction. CBR observations imply $\Omega_{\text{TOTAL}} \simeq 1$, while a variety of techniques imply $\Omega_B \simeq 0.3$. The remaining 0.7 is readily attributed to dark energy.

So what the observations seem to tell us is that the observed $H(z)$ is not described by $H(z)$ calculated from the EdS model, but is well described by $H(z)$ described by the $\Lambda$CDM model.

Again, all evidence for dark energy is indirect!

So, what should we do? We can no longer just hide from the data. The evidence is overwhelming that $H(z)$ is not described by the EdS model. Again, this is not just from the supernova projects, but from a variety of techniques.

So we don’t know what the answer is, but we do seem to know something very valuable: We know an equation that doesn’t work; namely, $G_{00}(\text{FLRW}) \neq 8\pi G T_{00}(w = 0 \text{ perfect fluid matter})$. \hfill (5)

Let’s exploit this non-equation and imagine how we can fix it.

First, we might try to add something to the right-hand side of the equation; a $\Delta T_{00}$ if you will. This $\Delta T_{00}$ is usually what we mean by dark energy.

Theorists employ two tools in constructing a $\Delta T_{00}$. Now experimentalists believe that theorists don’t know anything about tools, but that is not true. In my own case, I owned a British sports car for over 20 years (a 1965 Austin-Healey Sprite) and an Italian economy car (a 1974 Fiat 128) for more than 5 years. Anyone who has owned either of those marvels of engineering has had to use tools very often. In my experience, you only need two tools to fix anything: duct tape and WD-40. There are only two rules:

1. If something moves and it shouldn’t, use duct tape.
2. If something doesn’t move and it should, squirt it with WD-40.

The equivalent theoretical tools used for dark energy are the anthropic (or Landscape is you speak string) explanation,\(^1\) and scalar fields (known as quintessence for this purpose).

The anthropic/Landscape/duct-tape idea is that there are many sources of vacuum energy. String theory is conjectured to have many (perhaps more than $10^{500}$!) vacua. The different vacua have different values of the total vacuum energy density. Presumably most of them have a vacuum energy much, much larger (say, by a factor of $10^{100}$) than the observed value. But don’t worry, since although exponentially uncommon, vacua with vacuum energy as small as observed are hospitable for life while the more common values result in an inhospitable universe. The anthropic idea is that we should only consider cosmological models with values of the vacuum energy that are hospitable for life. Therefore, there is an anthropic selection effect.

So you see that the anthropic explanation is just like duct tape. There were some loose ends after a calculation. Like duct tape, the anthropic explanation need not be elegant or permanent, but it ties down the loose ends.

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\(^1\) The anthropic explanation is sometimes referred to as the anthropic principle.
Now the quintessence idea borrows a concept from high-energy physics: whenever there is dynamics that is poorly understood, invent a scalar field.\(^2\) One way to visualize the quintessence field is that there are many contributions to \(\rho_V\). Some unknown principle or dynamics results in the minimum value \(\rho_V = 0\), but we are not there yet—we are still evolving to the ground state.

Now let’s turn to ideas for modifying the left-hand side of non-equation (5), adding a \(\Delta G_{00}\). The first class of ideas involves modifying gravity. The first type in this class involves extra dimensions.

1. The braneworld modifies the Friedmann equation (6). In some models with branes the Friedmann equation does not arise from the 00-component of the Einstein equation, but rather from the Israel discontinuity condition as one crosses the brane.

2. In some extra-dimension models the gravitational force law is modified at large distances.\(^7\)

3. In models with branes the wavefunction of gravitons can leak into the bulk.\(^8\)\(^,\)\(^9\). This is a sort of “tired graviton” explanation for dark energy.

4. There are extra-dimension theories where gravity becomes repulsive at \(R \sim Gpc\)\(^10\).

5. All models of extra dimensions have “Kaluza–Klein” (KK) excitations. It could be that the \(n = 1\) KK excitation of the graviton is very light, say with a mass \(m \sim (Gpc)^{-1}\)\(^11\).

Then there are theories (not necessarily based on extra dimensions) of the type that modify the low-energy gravitational action. The idea is that Einstein and Hilbert got it wrong in 1915, and rather than the Einstein-Hilbert action for gravity, \(S = (16\pi G)^{-1} \int d^4x \sqrt{-g}R\), it is more general, say of the form \(S = (16\pi G)^{-1} \int d^4x \sqrt{-g}f(R)\), where \(f(R)\) is some function of, for instance, the Ricci scalar \(R\)\(^12\). While it is very reasonable to expect terms like \(R^2\) or \(R^{m}\) \(R_{\mu\nu}\), which affect the theory in the ultraviolet, what seems to be required for dark energy are terms that affect the theory in the infrared. Terms like \(R^{-1}\) would work, but they lead to problems.

One of the things we have come to understand from all these approaches is how difficult it is to modify gravity on large scales without destroying agreement with observations on solar system scales, or leading to non-linearities, ghosts, or other theoretical unpleasantness.

Finally, there is the idea (very unpopular in most quarters), that the Friedmann equation has large corrections due to inhomogeneities in the Universe\(^13\)\(^,\)\(^14\)\(^,\)\(^15\)\(^,\)\(^16\). This idea is still under development, but it is my favorite idea (since I have worked on it a lot).

Perhaps theoretical physicists should turn to astronomers and use the famous quote of Einstein, “Nothing more can be done by the theorists. In this matter it is only you, the astronomers, who can perform a simply invaluable service to theoretical physics.”\(^3\)

So now let us turn to astronomy and take a look at strategy and techniques for dark energy. Recently in the U.S., the Dark Energy Task Force (DETF)\(^17\) was commissioned by the Department of Energy, NASA, and the NSF to recommend strategy and approaches to study dark energy.

The strategy recommended by the DETF is

1. Determine as well as possible whether the expansion is consistent with being due to a cosmological constant \((w = -1)\).

2. If the expansion is not due to a cosmological constant, probe as well as possible the underlying dynamics by measuring as well as possible the time evolution of dark energy, i.e., determine \(w(a)\).

3. Search for a possible failure of general relativity through comparison of the effect of dark energy on cosmic expansion with the effect of dark energy on the growth of cosmological structures like galaxies or galaxy clusters.

An observation program to accomplish this strategy is illustrated in Fig. 6. The effect of dark energy on \(H(z)\) is probed by observables like \(d_L(z)\), \(d_A(z)\), and \(V(z)\). Measurements of these observables can be done through

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\(^2\) One might think the fact that there is no known fundamental scalar field might temper the enthusiasm for introducing them.

\(^3\) Einstein wrote this in August 1913 to Berlin astronomer Erwin Freundlich, encouraging him to mount a solar-eclipse expedition to measure the bending of starlight as it passed near the sun. The astronomer eagerly accepted the challenge from the theoretical physicist. Unfortunately for the astronomer, the eclipse of 1914 was in the Crimea during the outbreak of the First World War. In an extraordinary rendition, Freundlich was captured, his equipment confiscated, and he was imprisoned as an enemy combatant. Eventually he was released, but of course he missed the eclipse. This is just as well, because in 1914 Einstein’s prediction for the deflection of light by the sun on the basis of his (at the time) incomplete theory of gravity was wrong.
measurements of supernovae, galaxy clusters, baryon acoustic oscillations, and lensing. Observations of the growth of structure is interesting because the expansion rate $H(z)$ enters the equation for the growth of instabilities (in the linear regime the equation is given in the figure). If gravity is modified there is potentially a source term that must be included.

The DETF identified four main techniques for studying dark energy by determining the time evolution of $H(z)$: 1) supernovae, 2) baryon acoustic oscillations, 3) weak lensing, and 4) galaxy cluster surveys. I will say a few words about each technique.

A. Supernovae

As you can see from Fig. 6, we use supernovae to study dark energy by using them as standard candles and measuring the luminosity-distance as a function of redshift. This is a well developed and well practiced technique, so I won’t say very much about it. Unlike the techniques discussed below, with supernovae there is a lot of information per object. Unclear at present is how good a standard candle supernovae can be after calibration. The fact that we do not really know how Type Ia supernovae actually explode might intimidate the timid, but luckily astronomers are not timid. It is unclear how well the supernova technique will prove to be. There is a lot of discussion and debate about whether it is necessary to make space observation, and whether photometric redshifts\(^4\) will be useful.

Since the supernova technique is so well known and familiar to most people, I won’t review it further, but use the time to discuss less familiar techniques.

B. Baryon Acoustic Oscillations

The baryon acoustic oscillations (BAO) technique is less familiar, so I will discuss a little bit of the physics behind it. It might be useful to refer to Fig. 7 during the discussion.

Before recombination, the Universe was ionized. The photons provided an enormous pressure and restoring force preventing baryons from moving. Any perturbations in the baryons would not grow, but would oscillate as sound waves.

After recombination, the universe is neutral and photons can travel freely. The baryon perturbations can grow or fall into dark matter potential wells.

The phenomena of interest for BAO occurs in the transition between the tightly coupled and decoupled regimes. As recombination starts to occur, the photon mean-free-path becomes long. While the baryons are still coupled to the

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\(^4\) Redshifts traditionally have been taken by spectroscopic techniques. Photometric redshifts (photo-z’s) use multicolor photometry as a crude spectrograph to determine the redshift.
FIG. 7: A cartoon describing the transition between photons and baryons tightly coupled before recombination in the early universe, and photons free from baryons after recombination in the late universe.

FIG. 8: The acoustic peaks in the angular power spectrum of CBR fluctuations $^8$ and the BAO feature in the matter two-point correlation function $^{18}$. 
photons, as the photons stream out of overdensities, they drag the baryons along. (This is known as Silk damping.) You can view this by considering an initial overdensity of a mixture of dark matter, photons, neutrinos, and baryons. When recombination begins, the overdensity launches a spherical shock wave in the photon-baryon fluid traveling outward at a velocity of $c/\sqrt{3}$. Eventually, the photons completely decouple from the baryons, the baryons lose their pressure support, and the sound speed of the baryon shock wave plummets. Eventually, the shock stalls after traveling a distance of about 150 Mpc. What is left behind is an overdensity in the baryons on a distance scale of 150 Mpc. Dark matter falls into the baryon overdensities and the net effect is a feature in the two-point correlation function of density perturbations on a scale of 150 Mpc.

This physics behind this feature in the matter distribution is exactly the physics behind the acoustic peaks in the angular power spectrum of CBR fluctuations. This is illustrated in Fig. 8.

So nature provides us with a standard ruler of 150 Mpc. This can be used to probe dark energy through measuring its angular size as a function of redshift, $d_A(z)$, and by measuring its radial size, $H^{-1}\delta(z)$.

This technique has several advantages: The physical size of the feature can be determined by the well measured CBR acoustic peaks. It is purely a geometric technique, largely free of astrophysical systematic errors.

The technique has a couple of disadvantages: The feature in the power spectrum is small (this is because $\Omega_B \ll \Omega_M$). Since the fundamental scale is 150 Mpc, huge survey volumes are required to include many, many 150 Mpc scales. Finally, nonlinear effects are important at smallish $z$, so the signal is cleaner at $z \sim 2$ to 3. However, if dark energy behavior with $z$ is close to a constant, the sweet spot for dark energy is at lower redshifts.

BAO is an emerging technique, largely free of systematic errors, with promise for the future.

### C. Weak Lensing

The weak lensing technique is illustrated in Fig. 9. Suppose there is a source at a distance of $D_{OS}$ and a gravitational lens at a distance from the source of $D_{LS}$. If the impact parameter is $b$, the deflection angle is

$$\delta \theta = \frac{4GM}{b \cdot D_{OS}} D_{LS}.$$  

(6)

Dark energy enters the game because geometric distances $D_{LS}$ and $D_{OS}$ are affected by the time evolution of $H(z)$. Dark energy also affects the growth rate of the lensing mass $M$.

Generally, the deflection angles are so small that the signal for any given galaxy is very small. Weak lensing is based on the idea that gravitational lensing leads to a distortion (or shearing) of the source image. Although the signal per galaxy is small, there are a lot of galaxies! Since huge numbers of galaxies are required, it is likely that photometric redshifts will have to be used.

There are two main sources of systematic errors involved in the weak lensing technique. The first is the uncertainty in photo-$z$’s. The second is related to the point spread function of the telescope. Effects of the atmosphere and imperfections in the optics lead to a distortion of images unrelated to lensing. This leads to a debate about whether weak lensing should be done from space or from ground. Space observatories have the advantage of no atmosphere to distort the image, and the ability to push into the near-infrared where errors in photometric redshifts are expected to be less. Ground observatories have the advantage of a larger aperture and less expensive telescopes.
The weak lensing landscape now has current projects imaging hundreds of square degrees with deep multicolor data and thousands of square degrees in shallow two-color data. In the near future, projects such as the Dark Energy Survey will image thousands of square degrees with deep multicolor data, and eventually LSST will image an entire hemisphere very deep in six colors.

Weak lensing is also an emerging technique, with great promise.

D. Galaxy Clusters

Galaxy cluster surveys measure galaxy cluster masses, redshifts, and spatial clustering. They are sensitive to dark energy through the volume–redshift relation, the angular-diameter–redshift relation, the growth rate of structure, and the amplitude of clustering.

In my opinion, the advantage of the cluster technique is that it is sensitive to dark energy in many ways. There are problems associated with the cluster technique. The cluster selection must be well understood, it is unclear what proxy should be used to determine the cluster mass, and photo-

E. Combining Techniques

In my opinion there is no single technique that should be pursued in exclusion of the others. There are three reasons for this opinion. First, we will soon be at the point where systematic errors will dominate the uncertainties. Different techniques have different systematic uncertainties. Second, we have no idea about the nature of dark energy, and it is important to see its effects in several ways. For instance, if a new gravity theory is the answer, it is possible that the effect on the growth of structures may be different than the effect on luminosity distances. Finally, different techniques have different degeneracies with other cosmological parameters, and great improvements in accuracy can be gained by combining techniques [17].

III. DARK MATTER

Now let me turn to the issue of dark matter. Refer back to Fig. 5. There are several methods to determine \( \Omega_M \). They all point to a value of \( \Omega_M \sim 0.3 \). There are also many reasons to believe that the baryon contribution \( \Omega_B \) is about 0.04. The two best pieces of evidence for \( \Omega_B \) are big-bang nucleosynthesis (BBN) and CBR observations. Now that we have mastered subtraction, we can see that the bulk of the matter density is non-baryonic. Here, I wish to discuss possibilities for dark matter, so I won’t spend a lot of time motivating the fact that there must be dark matter. I will also assume that dark matter is non-baryonic.

The list of non-baryonic dark matter candidates is quite long. An entire lecture could be devoted just to listing them. Rather than do that, I will just concentrate on a few possibilities.

A. Cold Thermal Relic

The first candidate I want to consider is a cold thermal relic. Let’s call the particle \( X \). The idea behind a cold thermal relic is illustrated in Fig. 10. The first assumption of a cold thermal relic is that the particle was in local thermal equilibrium (LTE) when the temperature was greater than the mass of the particle. The second assumption is that there is no asymmetry between \( X \) and \( \bar{X} \). The final assumption is that the particle remains in LTE until temperatures drop below \( M_X \), and the particle becomes “cold.”

With the above assumptions, for \( T > M_X \) the abundance of the particle should be about as abundant as photons. For \( T < M_X \), the abundance of the X relative to photons is Boltzmann suppressed as long as the X remains in LTE.

But eventually the X will “freeze out” of LTE. As T drops below \( M_X \), the particle becomes exponentially rare, so annihilation shuts off. Also, as the temperature drops below \( M_X \) it becomes exponentially unlikely for a collision in the plasma to have enough center-of-momentum energy to create a \( XX \) pair.

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5 In some dark-matter models, the X is a Majorana particle, in which case \( X \equiv \bar{X} \).
The important feature of a cold thermal relic is that the more strongly a particle interacts, the greater will be its annihilation (and creation) cross section, the longer it will remain in equilibrium, and the lower its eventual freeze-out abundance will be. So the weaker the interactions, the larger will be the final abundance.

Detailed calculations of the freeze-out abundance yield the result that the final abundance is proportional to the inverse of the annihilation cross section, $\sigma_A$, and proportional to the inverse of $M_X$. Since the present contribution to $\Omega$ for the $X$ is equal to $M_X$ times the present number density of $X$, $\Omega_X \propto \sigma_A^{-1}$, and independent of $M_X$.\(^6\)

Now if we want $\Omega_X$ to be the dark matter, we know the approximate value of $\Omega_X$ required, so we know the approximate value of $\sigma_A$. Since it turns out to be of the order of the weak scale, the $X$ is described as a weakly

\(^6\) More exactly, to first approximation $\Omega_X$ depends on $M_X$ only through the dependence of $\sigma_A$ on $M_X$. 
interacting massive particle, or WIMP. From now on, I will refer to a cold, thermal relic as a WIMP.

If we know the annihilation cross section, then we know something about the scattering cross section and the production cross section. This is illustrated in Fig. 11 in the case that the WIMP annihilates into $qq$.

So if the WIMP has a weak-scale annihilation cross section, the scattering cross sections should also be weak scale, and the production cross section for direct production should also be weak scale.

A weak-scale cross section is very interesting. A cross section that large should be within reach of direct detection experiments, where the relic WIMPs are detected by the small energy deposited in a sensitive detector. There are presently about a dozen experiments that are underway or planned.

A weak-scale cross section also offers exciting possibilities for indirect detection. The idea is that the WIMPs gather in the center of galaxies, the Sun, or Earth, they annihilate, and produce a signal that can be distinguished from anything produced by ordinary astrophysical processes. For instance, if the annihilation products include high-energy neutrinos, positrons, or antiprotons, that could be a signal for WIMPs.

Finally, if the annihilation cross section is weak scale, then the production cross section should also be weak scale. That should be within the range of production and detection at accelerators with enough center-of-mass energy to produce the WIMPs. If this happens, then for the first time in the last 13.78 billion years, WIMPs would be produced in abundance. This time, they would be produced not as a result of the tremendous temperatures of the big bang, but rather as the result of human curiosity and ingenuity.

The favorite candidate for a WIMP is the neutralino of low-energy SUSY. The neutralino is a linear combination of gauginos (winos and binos) and Higgsinos. The mass and interactions of the lightest neutralino depends on the particular linear combination of the gauginos and Higgsinos, and its mass.

There are over one hundred new parameters in low-energy SUSY. In order to get some sort of handle on the problem, SUSY is usually studied within some sort of “constrained” models. Within these constrained models, generally what is discovered is that SUSY models consistent with accelerator data typically have too small an annihilation cross section, which results in too large a value of $\Omega_X$ (recall $\Omega_X \propto \sigma_A^{-1}$). Therefore some sort of cleverness/chicanery is required to increase the annihilation cross section and give an acceptable $\Omega_X$. The possibilities include $s$-channel resonance through light $H$ and $Z$ poles, co-annihilation with stops or staus, large $\tan \beta$ models\(^7\) where annihilation occurs through a broad $A$ resonance, or high values of the universal scalar mass that makes the neutralino Higgsino-like so it can annihilate easily into $W$ and $Z$ pairs. Or, perhaps the true low-energy SUSY model (if there is one) is unconstrained.

Today, direct detection experiments, indirect detection experiments, and colliders are racing for discovery of the WIMP. Imagine that by 2010 we have credible signals from all three. A question to ask is, how will we know we are seeing the same phenomenon? Let’s hope for this problem!

There are a lot of opinions (i.e., papers) on this subject, let me mention just three that are representative of the spread of opinion. Arnowitt and Dutta\(^{19}\) conclude that we will learn enough about SUSY from the LHC to be confident that the particle discovered in colliders is the same particle seen in direct and indirect detection experiments. Baltz et al.\(^{20}\) conclude that we will not learn enough from the LHC, and we will require an ILC. Finally, Chung et al.\(^{21}\) conclude that the answer depends on where in parameter space the low-energy SUSY model lives.

As mentioned, the neutralino is everyone’s favorite dark matter candidate because of the rich physics possibilities it will provide. It is fair to say that if dark matter is a complex natural phenomenon, the neutralino is a simple, elegant, compelling explanation. However, that doesn’t mean it is right. As Tommy Gold once told me, “For every complex natural phenomenon, there is a simple, elegant, compelling, wrong explanation.” So I plead with you to keep an open mind, and since the title of this course is Searching for the ‘totally unexpected’ in the LHC era, and to many people it would be totally unexpected that the neutralino is not the dark matter, let’s keep an open mind and consider some other possibilities.

First, let’s consider the possibility that the dark matter is a cold thermal relic other than a neutralino. One possibility is that it is a Kaluza–Klein (KK) excitation. In 1984, Dick Slansky\(^{22}\) and I pointed out that since extra-dimensional theories have excited states corresponding to momentum in extra dimensions, if there is some sort of conservation law keeping the extra dimensions stable, the KK excitations would be thermally produced in the early universe and appear as dark matter. We only considered the possibility that the size of extra dimensions are Planckian (size about $G^{-1/2}$), with the result that there would be way too much dark matter. Recently Servant and Tait\(^{23}\), and Cheng, Feng, and Matchev\(^{24}\) independently returned to this scenario. They greatly improved our analysis in two ways: 1) They had a much better explanation of the origin of the parity keeping the first excited mode stable (involving orbifolding the extra dimension—that technology was not around in 1984), and 2) they imagined the extra dimensions could be as large as the weak scale (which would have been considered crazy in 1984).

\(^7\) Here $\tan \beta$ is the ratio of vacuum expectation values of the two Higgs of the model.
In any case, detailed analysis suggests that the lightest KK mode (presumably the KK photon) is an excellent dark matter candidate. Furthermore, the collider signal for the KK excitations could be confused with SUSY. This scenario leads to very interesting possibilities.

B. Solitons

So far I have assumed that the dark matter is an elementary particle. However, there are other possibilities. In particular, the dark matter may be in the form of a soliton configuration known as a $Q$-ball or a non-topological soliton.

If there is a scalar field with a conserved global charge $Q$, then there are scalar field configurations that consist of a lump of coherent scalar condensate of charge $Q$ in which the energy scales as $Q^{3/4}$, so the soliton can not decay to $Q$ single-particle states.

$Q$-balls exist in the minimal supersymmetric standard model \[25\] where the scalar fields are squarks and sleptons, and the conserved charge is baryon (or lepton) number. The mass of the $Q$-ball is $M \sim (1 \text{ TeV}) B^{3/4}$. There is a penalty to pay in the energy, so one has to go to very large $B$ to find a stable soliton; in this case the minimum $B$ is $10^{12}$.

There are several mechanisms for producing solitons \[26\]; in this case, the most promising possibility is a fragmentation of an Affleck-Dine condensate. If this is the correct scenario it is possible that the baryon asymmetry is also generated through the fragmentation, so it is possible to relate the baryon asymmetry (hence $\Omega_B$) to the dark matter density.

Note that solitons have a non-thermal origin.

C. Supermassive relics

Recall that the present mass density for thermal relics is proportional to $\sigma_A^{-1}$. Therefore, there is a minimum annihilation cross section to ensure that $\Omega_X < 1$.

But for a particle of mass $M_X$, there is a maximum cross section for each partial wave based on unitarity; roughly $\sigma_A < M_X^{-2}$.

Since there is a minimum annihilation cross section and a maximum annihilation cross section (which depends on the mass), there is a maximum mass for the thermal WIMP if $\Omega_X < 1$. The mass turns out to be about 240 TeV \[28\].

So if the dark matter has a mass in excess of 240 TeV, it must have a nonthermal origin. Solitons discussed in the previous subsection is an example of a nonthermal relic. My favorite example of a massive nonthermal relic is the WIMPZILLA. The most promising WIMPZILLA production mechanism is gravitational production during inflation. So I will postpone the discussion of WIMPZILLAS until after I have said a few words about inflation.

D. Other possibilities

Because of the lack of time I will not be able to cover other possibilities. I could have equally well discussed neutrinos, gravitinos, axions, axion clusters, axinos, and so forth.

IV. INFLATION

The subject of inflation is well developed, but not well understood. I won’t have time to discuss the basics of inflation: motivation, how inflation solves classical cosmological problems like the age problem, the flatness problem, the monopole problem, whether inflation is eternal or not, transplanckian issues, and so on.

Here I will only say a few words about the origin of perturbations. I choose this because the measurements of the perturbation properties can tell us something about inflation models.

The way we usually picture inflation is shown in Fig. 12. A scalar field, called the inflaton, evolves classically in an inflaton potential. While the inflaton is displaced from its minimum (here conveniently adjusted to have vanishing potential energy at the minimum), there is an effective vacuum energy $\rho_V = V(\phi)$ that dominates the energy density and drives a quasi-de Sitter phase.

However, the classical picture is not the complete picture. As first pointed out by Schrödinger in 1939, the expanding Universe leads to particle creation. I won’t have time to go through the quantum field theory arguments, but I will simply try to give a physical motivation why the expanding universe leads to particle creation.
FIG. 12: A cartoon illustrating the classical dynamics of inflation.

FIG. 13: A strong electromagnetic field affects vacuum fluctuations, and if sufficiently strong, can lead to particle production. The expanding Universe can also rip apart vacuum fluctuations and lead to particle creation.

As illustrated in Fig. 12, it is well known that particle production is possible in a sufficiently strong electromagnetic field. If the energy gained in acceleration of an $e^+e^-$ pair in a distance of an electron Compton wavelength exceeds the electron rest mass, then pair creation can occur. It is as if the strong electromagnetic field rips $e^+e^-$ pairs out of the vacuum fluctuations.

One can picture the expansion of the Universe having the same effect. Particles and antiparticles come out of the vacuum, get caught in the expansion of space, and are ripped out of the virtual sea and turned to real particles.

The potential energy of the inflaton field is the energy of the zero-momentum mode of the field. By definition, the zero-momentum mode is homogeneous and isotropic. As particles of the inflaton field are created due to expansion of the Universe, they are created with non-zero momentum. If there are non-zero momentum modes of the inflaton field, the inflaton field can not be perfectly homogeneous and isotropic.

Since the inflaton field dominates the energy density, the inhomogeneities in the inflaton field lead to metric fluctuations. The metric fluctuations can be divided into fluctuations known as scalar fluctuations, which are what we usually refer to as density fluctuations, and tensor fluctuations, which are equivalent to a gravitational wave background.

The scalar and tensor perturbation spectra are a function of the expansion rate during inflation, and how the expansion rate changes during inflation. Since the inflaton potential determines the expansion rate, the inflaton potential will determine the perturbation spectra.

Some basic classes of inflaton models are illustrated in Fig. 14. Large-field models have positive second derivative, small-field models have negative second derivative, hybrid models can have positive or negative second derivative, but when inflation ends the vacuum energy is not zero, and there must be a second field required to describe the
TABLE III: Parametrization of scalar and tensor perturbation spectra. Here, $k_*$ is some conveniently chosen wavenumber.

|                         | scalar perturbations (density perturbations) | tensor perturbations (gravitational waves) |
|-------------------------|---------------------------------------------|-------------------------------------------|
| amplitude at $k = k_*$  | $P_S(k_*)$                                  | $P_T(k_*)$                                |
| spectral index          | $n_S \equiv \frac{d \ln P_S(k_*)}{d \ln k}$ | $n_T \equiv \frac{d \ln P_T(k_*)}{d \ln k}$ |
| running of the spectral index | $n'_S \equiv \frac{dn_S}{d \ln k}$           | $n'_T \equiv \frac{dn_T}{d \ln k}$         |

non-inflationary evolution to the true ground state.

The scalar and tensor perturbation amplitudes are in general functions of wavelength (or wavenumber $k$). It is usually quite accurate to parametrize the spectra in terms of amplitudes and spectral indices as described in Table III.

There are a few things we have learned from attempts to build inflation models.

1. The inflaton potential must be “flat” in the sense that its derivative must be unusually small.
2. While it is always easy to write a potential that is flat, it must remain flat when one includes radiative corrections. This presumably suggests some (possibly approximate) symmetry.
3. Although it has often been said that SUSY can come to the rescue, in practice it is not so easy to implement [29].
4. Many attractive SUSY models give $V(\phi) \sim A + \ln \phi$ hybrid models.
5. There are no general predictions, but in many models we find:
   (a) Ratio of tensor to scalar amplitudes: $r \sim P_T(k_*)/P_S(k_*) \sim \text{(small)}$.
   (b) Scalar spectral index: $|n_S - 1| \sim \text{(small)}$.
   (c) Tensor spectral index: $|n_T| \sim \text{(small)}$.
   (d) Running of $n_S$: $|n'_S| \sim \text{(small)}^2$. 

FIG. 14: Classes of inflationary models.
(e) Running of $n_T$: $|n'_T| \sim (\text{small})^2$.

6. We need experimental guidance.

For phenomenological guidance, it is now traditional to present the observational results on $n_S$ and the ratio of tensor to scalar perturbations on the same graph \[30\]. The utility of this approach can be seen in Fig. 15. To a first approximation, small-field, large-field, and hybrid models populate different regions of the $n_S$r plane.

The experimental situation is shown in Fig. 16. If one analyses only the WMAP three-year data set, and takes a prior that there is no running of the spectral index, the Harrison–Zel’dovich spectrum ($n_S = 1, n'_S = 0$) is ruled out at approximately the 90% confidence level. The result is little changed if one adds additional data sets. If one allows running of the scalar spectral index, then a slightly blue ($n > 1$) spectrum is preferred. My conclusion is that one cannot make a strong statement about whether the Harrison–Zel’dovich spectrum is ruled out or not. It is also premature to draw any conclusion regarding phenomenology of inflation models.

A. Wimpzillas

Now let us return to the possibility of producing non-thermal dark matter during inflation. Particles of all types (at least those that are not conformally coupled) are produced during inflation. If there is a massive stable particle species, they will also be produced during inflation and be around today. Two groups \[32, 33\] proposed this could be the source of dark matter.

Detailed calculations show that that if there is a stable particle with mass comparable to the inflaton mass, they would be produced in the proper abundance to be the dark matter. Since the mass of the inflaton is expected to be $10^{11}$ or $10^{12}$ GeV, the dark matter particle would be supermassive—much more massive than a WIMP can be—hence, it is known as a WIMPZILLA.

Unlike thermal relics, where the contribution to $\Omega$ depends on the annihilation cross section and independent of the mass, the WIMPZILLA contribution to $\Omega$ is independent of the interactions and depends only on the mass.

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[1] W. L. Freedman et al., Ap. J. 553, 47 (2001).
[2] D. J. Fixen, et al., Ap. J. 473, 576 (1996).
[3] D. N. Spergel et al., Ap. J. Suppl. 170, 377 (2007).
[4] A. Riess et al., Astron. J. 116 1009 (1998); S. Perlmutter et al., Ap. J. 517, 565 (1999).
[5] P. Astier et al. [The SNLS Collaboration], Astron. Astrophys. 447, 31 (2006).
[6] P. Binev, C. Deffayet, U. Ellwanger and D. Langlois, Phys. Lett. B 477, 285 (2000).
[7] C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D 65, 044023 (2002).
[8] R. Gregory, V. A. Rubakov and S. M. Sibiryakov, Phys. Lett. B 489, 203 (2000).
[9] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B 485, 208 (2000).
[10] C. Csaki, J. Erlich, T. J. Hollowood and J. Terning, Phys. Rev. D 63, 065019 (2001).
[11] I. I. Kogan, S. Mouslopoulos, A. Papazoglu, G. G. Ross and J. Santiago, Nucl. Phys. B 584, 313 (2000).
[12] S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, Phys. Rev. D 70, 043528 (2004).
[13] S. Rasanen, JCAP 0402, 003 (2004).
[14] E. W. Kolb, S. Matarrese, A. Notari and A. Riotto, Phys. Rev. D 71, 023524 (2005).
[15] A. Notari, Mod. Phys. Lett. A 21, 2997 (2006).
[16] E. W. Kolb, S. Matarrese and A. Riotto, New J. Phys. 8, 322 (2006).
[17] A. Albrecht et al., arXiv:astro-ph/0609591.
[18] D. J. Eisenstein, et al., Ap. J. 633, 560 (2005).
[19] R. Arnowitt and B. Dutta, arXiv:hep-ph/0210339.
[20] E. A. Baltz, M. Battaglia, M. E. Peskin and T. Wizansky, Phys. Rev. D 74, 103521 (2006).
[21] D. J. Chung, L. L. Everett, K. Kong and K. T. Matchev, arXiv:0706.2375 [hep-ph].
[22] E. W. Kolb and R. Slansky, Phys. Lett. B 135, 378 (1984).
[23] G. Servant and T. M. P. Tait, Nucl. Phys. B 650, 391 (2003).
[24] H. C. Cheng, J. L. Feng, and K. T. Matchev, Phys. Rev. Lett. 89, 231805 (2004).
[25] A. Kusenko, M. E. Shaposhnikov and P. G. Tinyakov, Pisma Zh. Eksp. Teor. Fiz. 67, 229 (1998) [JETP Lett. 67, 247 (1998)].
[26] J. A. Frieman, G. B. Gelmini, M. Gleiser and E. W. Kolb, Phys. Rev. Lett. 60, 2101 (1988).
[27] A. Kusenko and M. E. Shaposhnikov, Phys. Lett. B 418, 46 (1998).
[28] K. Griest and M. Kamionkowski, Phys. Rev. Lett. 64, 615 (1990).
[29] D. H. Lyth and A. Riotto, Phys. Rep. 314, 1 (1999).
[30] S. Dodelson, W. H. Kinney and E. W. Kolb, Phys. Rev. D 56, 3207 (1997).
[31] W. H. Kinney, E. W. Kolb, A. Melchiorri and A. Riotto, Phys. Rev. D 74, 023502 (2006).
[32] D. J. Chung, E. W. Kolb and A. Riotto, Phys. Rev. D 60, 063504 (1999).
[33] V. Kuzmin and I. I. Tkachev, JETP Lett. 68, 271 (1998).