Miracle at the Gepner Point

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A four-point function of $E_6$ singlets, of interest in elucidating the moduli space of (0,2) deformations of the quintic string vacuum, is computed using analytic and numerical methods. The conformal field theory amplitude satisfies the requisite selection rules and monodromy conditions, but the integrated string amplitude vanishes. Together with selection rules coming from the spacetime R-symmetry [1], this demonstrates the flatness of the gauge-singlet spacetime superpotential through fourth order. Relevance to the more general program of determining the (0,2) moduli space and superpotential is discussed.

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1. Introduction and Motivation

The quintic hypersurface in $\mathbb{CP}^4$ is one of the best-studied string vacua. In the large radius (field theory) limit, it is given by a nonlinear sigma model describing strings propagating on a Calabi-Yau manifold $K$ constructed as follows (see [2] for a review). There are five complex coordinates $s^i$ subject to one scaling relation

$$s^i \sim \lambda s^i \quad (1.1)$$

for any complex number $\lambda$. The hypersurface is given by the vanishing of a homogeneous degree five polynomial in the $s^i$:

$$G(s^i) = G_{ijklm} s^i s^j s^k s^l s^m = 0. \quad (1.2)$$

The parameters $r$ and $G_{ijklm}$ determining the size and complex structure of the manifold are true moduli of the $(2,2)$ string vacuum [3]. In addition to varying the complex structure of the manifold, one can deform the tangent bundle $T$ to produce a more general rank 3 vector bundle $V$ over $K$ as follows. A tangent vector $t \in T$ to the quintic is given by five complex numbers $t^i$ subject to the equivalence $t^i \cong t^i + \lambda s^i$ and the constraint $t^i \frac{\partial G}{\partial s^i} = 0$. A vector $v \in V$ will be given by five complex numbers $v^i$ subject to the equivalence $v^i \cong t^i + \lambda s^i$ and the constraint $v^i \left( \frac{\partial G}{\partial s^i} + H^i \right) = 0$, for five quartic polynomials $H^i = H_{i,j1,j2,j3,j4} s^{j1} \ldots s^{j4}$ satisfying $s^i H^i = 0$. After subtracting the 25 linear redefinitions of the $s^i$ one finds 101 parameters in $G_{ijklm}$ and 224 in $H_{i,j1,j2,j3,j4}$.

The quintic can be realized by a linear sigma model which contains the Kahler parameter $r$ and the complex structure and bundle parameters $G_{ijklm}$ and $H_{i,j1,j2,j3,j4}$ as coupling constants [4]. The deformations given by $H_{i,j1,j2,j3,j4}$ break (2,2) worldsheet supersymmetry down to (0,2). For $r >> 0$ one recovers the nonlinear sigma model on the quintic hypersurface in the infrared. For $r << 0$ one obtains a Landau-Ginsburg orbifold. For a special choice of the defining polynomial $G$, namely

$$G_{Gepner}(s^j) = \sum_i \frac{(s^i)^5}{5}, \quad (1.3)$$

there is substantial evidence that the Landau-Ginsburg orbifold is equivalent to a tensor product of $N = 2$ minimal models, otherwise known as a Gepner model [5] (see [6] and references therein).
For each parameter described above one finds a space-time chiral superfield in the massless spectrum of the model. In particular, at large radius, the string vacuum arising from the quintic hypersurface in $\mathbb{CP}^4$ has 326 $E_6$ singlets, which decompose into states arising from 1 Kahler modulus, 101 complex structure deformations, and 224 deformations of the tangent bundle. At very small radius, one finds 301 singlets in the untwisted sector of the Landau-Ginsburg orbifold and 25 singlets with the same quantum numbers as the “missing” singlets in a twisted sector. The trace part of this the $5 \times 5$ matrix of twisted singlets corresponds to the Kahler modulus, while the traceless part gives 24 “twisted singlets”. At infinite radius all 325 singlets are moduli: the space-time superpotential has 325 flat directions corresponding to the complex structure and bundle deformations.

It was observed in [1] that a term in the spacetime superpotential quartic in the twisted singlets would satisfy the spacetime R-symmetry selection rule. Using a global $SU(5) \times SU(5)$ symmetry under which the worldsheet coupling constants as well as fields transform, the contribution can be restricted to one independent amplitude [1]. At the Gepner point, this satisfies the N=2 minimal model selection rules, suggesting a possible obstruction to deforming in the 24 twisted singlet directions at small radius. In this note we explicitly compute this amplitude, relevant to a more general study of the (0,2) landscape, and discover that it vanishes upon integration over the locations of the vertex operators on the worldsheet (to a convincing numerical precision). Thus at least through fourth order we can deform in all 224 extra singlet directions at LG after all, in keeping with more general arguments for flatness of (0,2) moduli forthcoming in [9].

We will begin by setting up the computation in section 2, reviewing how Gepner model amplitudes break up into free boson correlation functions and parafermion amplitudes. For the quintic, the parafermion theory is (a sector of) the $c=4/5$ N=0 minimal model, and in section 3 we solve the differential equations satisfied by that part of the amplitude and find the unique nonzero linear combination of solutions which have trivial monodromies. In section 4 we assemble the pieces and report on the numerical evaluation of the amplitude.

2. Structure of the amplitude

Gepner models are obtained by combining enough N=2 minimal models (each with $c=3k/(k+2)$) to produce a $c=9$ internal conformal field theory which is modular invariant and spacetime supersymmetric when combined with the four spacetime dimensions [3].
The Gepner model for the quintic consists of 5 copies of the c=9/5 N=2 minimal model. The N=2 minimal models each break up into a free boson (which is the bosonization of the U(1) part of the N=2 superconformal algebra) and a c=4/5 parafermion theory \cite{[11]}. The parafermion theory is a sector of the c=4/5 N=0 minimal model \cite{[11]}]. We will recall the precise formulas for these relationships as we need them.

We are interested in a four-point function of “twisted singlets” $S_j^i$. In the Landau-Ginsburg orbifold the worldsheet fields include complex bosons $s^i$ (which at large radius become the homogeneous coordinates on $\mathbb{CP}^4$ described in the introduction) and fermionic partners $\psi^i_\pm, \bar{\psi}^i_\pm$ \cite{[11]}. The model has 10 sectors, labeled by an integer $k = 0, \ldots, 9$. In this description the twisted singlets are given by states of the form

$$A_{ij} \bar{\psi}^i_{-\frac{1}{5}} s^j_{-\frac{1}{10}} |0>$$

for constant matrix $A_{ij}$, where $|0>$ is the vacuum of the $k = 3$ sector and the subscripts indicate the mode of the worldsheet field used to create the state \cite{[11]}. In the Gepner model, there is a discrete symmetry group,

$$(S_5 \ltimes Z_5^5)/Z_5$$

coming from the fact that the defining polynomial takes the simple form \cite{[1.3]}. There is a $Z_5$ acting on each multiplet $(s^i, \psi^i_\pm)$ modulo a common $Z_5$ phase. From the form of the states \cite{[2.1]}, we see that under the Gepner model discrete symmetry group they transform as

$$S_j^i \rightarrow e^{-\frac{2\pi ik_i}{5}} e^{\frac{2\pi ik_i}{5}} S_j^i$$

(2.3)

where $e^{\frac{2\pi ik_i}{5}}$ is an element of the $l$th $Z_5$.

The four-point function of interest is the following:

$$A = \left< e^\omega (V_1^5)^{-\frac{1}{2}} (z_1, \bar{z}_1) (V_2^5)^{-\frac{1}{2}} (z_2, \bar{z}_2) (V_3^5)_0 (z_3, \bar{z}_3) (V_4^5)_0^{-1} (z_4, \bar{z}_4) \right>$$  (2.4)

The vertex operators in the Gepner model are determined their dimensions, $U(1)$ charges, and transformation properties under \cite{[2.2]}.

$$(V_1^5)_{\alpha F}^{-\frac{1}{2}} (z_1, \bar{z}_1) = S_\alpha (\bar{z}_1) e^{-\frac{\delta (z_1)}{2}} \Phi_{-2, -2; 1, 1} \otimes \Phi_{-1, 0; 2, 1} \otimes \Phi_{1, 1; 0, 2, 1} \otimes \Phi_{1, -1; 0, 2, 1} \otimes \Phi_{1, 0; 0, 3, 1} (z_1, \bar{z}_2),$$

$$(V_2^5)_{\beta F}^{-\frac{1}{2}} (z_2, \bar{z}_2) = S_\beta (\bar{z}_2) e^{-\frac{\delta (\bar{z}_2)}{2}} \Phi_{1, 1; 0, 2, 1} \otimes \Phi_{0, -2; 2, 1} \otimes \Phi_{0, 1; 1, 0} \otimes \Phi_{-1, 0; 1, 0} \otimes \Phi_{0, 0; 0, 3, 1} (z_1, \bar{z}_1),$$

$$(V_3^5)_0 (z_3, \bar{z}_3) = \Phi_{-1, 0; 1, 0} \otimes \Phi_{1, 0; 1, 0} \otimes \Phi_{0, -2; 2, 0, 0} \otimes \Phi_{1, -1; 0, 1, 0} \otimes \Phi_{0, 0; 0, 2, 2} (z_3, \bar{z}_3),$$

and
Here \( S_\alpha \) is the spacetime spin field and \( \phi \) is the standard bosonization of the ghost number current. The \( N=2 \) minimal model primary fields \( \Phi^l_{q,s;\bar{q},\bar{s}} \) (in the notation of \([12]\)) break up into parafermion primary fields \( \phi^l_{q-s,\bar{q}-\bar{s}} \) times exponentials of the free \( U(1) \) boson \( H \), \( e^{\alpha_{q,s} H} e^{\alpha_{\bar{q},\bar{s}} \bar{H}} \), where \( \alpha_{q,s} = \frac{1}{\sqrt{15}} (-q + \frac{5}{2}s) \); see \([12]\) for a review. The parafermion primary field \( \phi^l_{k,\bar{k}} \) has dimension

\[
\begin{align*}
  h^l_{k,\bar{k}} &= \frac{l(l+2)}{20} - \frac{k^2}{12}; \\
  \bar{h}^l_{k,\bar{k}} &= \frac{l(l+2)}{20} - \frac{k^2}{12}.
\end{align*}
\] (2.5)

Under the Gepner model discrete symmetry group \((2.2)\) the \( N = 2 \) minimal model primary fields transform as

\[
\Phi^l_{q,s;\bar{q},\bar{s}} \to e^{-i\pi(q+\bar{q})/5} \Phi^l_{q,s;\bar{q},\bar{s}}.
\] (2.6)

From this formula it is easy to check that the above vertex operators have the correct transformation properties \((2.3)\) under \((2.2)\).

Note that the external and superconformal ghost pieces are independent of \((z_3, \bar{z}_3)\). Their correlation functions yield:

\[
A_{\text{ext}} = e^{\alpha_\beta} \left\langle S_\alpha(\bar{z}_1) S_\beta(\bar{z}_2) \right\rangle = \frac{1}{(\bar{z}_1 - \bar{z}_2)^3}
\] (2.7)

\[
A_{\text{ghost}} = \left\langle e^{-\frac{\phi}{2}}(\bar{z}_1) e^{-\frac{\phi}{2}}(\bar{z}_2) e^{-\phi}(\bar{z}_4) \right\rangle = \frac{1}{z_1^{4}} \frac{1}{\bar{z}_2^{4}} \frac{1}{\bar{z}_4^{2}}
\] (2.8)

We use \( \text{SL}(2,\mathbb{C}) \) to set \( z_1, z_2, \) and \( z_4 \) to 0,1, and \( \infty \), remembering to include the Jacobian \( |z_{12}|^2 |z_{14}|^2 |z_{24}|^2 \) which is also independent of \( z_3 \). Fixing \( z_1, z_2, \) and \( z_4 \) this way the above contributions become constants. From the decomposition described above we find that the parts of the free boson amplitude that depend on \((z_3, \bar{z}_3) \equiv (z, \bar{z})\) are:

\[
A_H = |z_{13}|^{-\frac{4}{15}} |z_{23}|^{-\frac{4}{15}} = |z|^{-\frac{4}{15}} |1-z|^{-\frac{4}{15}}.
\] (2.9)

The parafermion amplitudes for the first four minimal model factors are:

\[
P_1 = \left\langle \phi^0_{0,0}(z_1, \bar{z}_1) \phi^1_{-1,1}(z_2, \bar{z}_2) \phi^1_{-1,1}(z_3, \bar{z}_3) \phi^1_{-1,1}(z_4, \bar{z}_4) \right\rangle
\] (2.10)

\[
P_2 = \left\langle \phi^0_{-1,1}(z_1, \bar{z}_1) \phi^0_{0,0}(z_2, \bar{z}_2) \phi^1_{-1,1}(z_3, \bar{z}_3) \phi^1_{-1,1}(z_4, \bar{z}_4) \right\rangle
\] (2.11)
\[ P_3 = \left\langle \phi^\perp_{-1,1}(z_1, \bar{z}_1)\phi^\perp_{-1,1}(z_2, \bar{z}_2)\phi_{0,0}(z_3, \bar{z}_3)\phi^\perp_{-1,1}(z_4, \bar{z}_4) \right\rangle \quad (2.12) \]

\[ P_4 = \left\langle \phi^\perp_{-1,1}(z_1, \bar{z}_1)\phi^\perp_{-1,1}(z_2, \bar{z}_2)\phi^\perp_{-1,1}(z_3, \bar{z}_3)\phi_{0,0}(z_4, \bar{z}_4) \right\rangle \quad (2.13) \]

Because of the presence of the field \( \phi_{0,0}^0 \), the parafermion identity field, each of these correlation functions reduces to a three-point function equivalent by duality to those worked out in \([11]\):

\[ P_1 = C_{1,1} |z_{23}|^{\frac{4}{5}}|z_{34}|^{\frac{4}{5}}|z_{24}|^{\frac{4}{5}} \quad (2.14) \]

where

\[ C_{1,1}^2 = \frac{\Gamma(1/5)\Gamma^3(3/5)}{\Gamma(4/5)\Gamma^3(2/5)} \quad (2.15) \]

and similarly for \( P_2, P_3, \) and \( P_4 \). All together, the parafermion three-point functions contribute a factor

\[ |z|^{-\frac{4}{5}}|1-z|^{-\frac{4}{5}} \quad (2.16) \]

to the \( z, \bar{z} \)-dependent part of the amplitude. What remains is to compute the nontrivial fifth parafermion amplitude, which will be the subject of the next section.

3. The parafermion amplitude

The nontrivial parafermion factor in the amplitude is

\[ \left\langle \phi_{0,2}^2(0)\phi_{0,2}^2(1)\phi_{0,0}^2(z, \bar{z})\phi_{0,2}^2(\infty) \right\rangle \]

Under the \( Z_3 \times Z_3 \) symmetry of the parafermion model, \( \phi^l_{k,k} \rightarrow e^{\frac{2\pi i m}{3}(k-\bar{k})}e^{\frac{2\pi i n}{3}(k+\bar{k})} \phi^l_{k,k} \) \([11]\), so this amplitude satisfies the selection rules.

The parafermion theory is a unitary theory with \( c = 4/5 < 1 \), and so must be a subset of one of the minimal models. The \( N = 0 \) series of minimal models has \( c = 1 - \frac{6}{m(m+1)} \), \( m = 3, 4, \ldots \). The primary fields \( \phi_{p,q} \) are organized in the Kac table with dimensions

\[ h_{p,q} = \frac{(p(m+1) - qm)^2 - 1}{4m(m+1)}. \quad (3.1) \]

In our case, \( c = 4/5 \Rightarrow m = 5 \). The parafermion primary fields, whose dimensions were given in \((2.3)\), form a subset of the minimal model primary fields \((3.1)\) for \( m = 5 \). In particular, our amplitude is equivalent to the \( \text{N}=0 \ c=4/5 \) minimal model amplitude

\[ \left\langle \phi_{2,1}(0), \phi_{2,1}(1), \phi_{2,1}(z), \phi_{2,1}(\infty) \right\rangle \left\langle \phi_{3,3}(0), \phi_{3,3}(1), \phi_{2,1}(\bar{z}), \phi_{3,3}(\infty) \right\rangle \]
The minimal model primary field $\phi_{p,q}$ has a null state at level $pq$ in its Verma module, and therefore satisfies a differential equation of order $pq$. Our four-point amplitude satisfies second-order differential equations in $z$ and $\bar{z}$ by virtue of the presence of the $\phi_{2,1}$ field for each chirality [13]:

$$
\left[ \frac{3}{2(2\delta + 1)} \frac{d^2}{dz^2} + \sum_{i \neq 3} \frac{1}{(z - z_i)} \frac{d}{dz} - \frac{\Delta_i}{(z - z_i)^2} + \sum_{j < i} \frac{\delta + \Delta_{i,j}}{(z - z_i)(z - z_j)} \right] G(z|z_1, z_2, z_4) = 0.
$$

(3.2)

where $\Delta_{1,2} = \Delta_1 + \Delta_2 - \Delta_4$ and $\delta = \text{dim} \phi_{2,1} = 2/5$, and where $z$ is replaced by $\bar{z}$ for the right-movers. We set $z_1 = 0$, $z_2 = 1$, and $z_4 = \infty$ and substitute in the dimensions for the fields given in (3.1) or equivalently (2.5). Then (3.2) can be rewritten as

$$
z(1 - z)G'' + \frac{6}{5}(1 - 2z)G' + \left[ \frac{12}{5} \Delta - \frac{6}{5} \left( \frac{2}{5} + \Delta \right) - \frac{6}{5} \Delta \left( \frac{1}{z} + \frac{1}{1 - z} \right) \right] G = 0
$$

(3.3)

where $\Delta$ is 2/5 for the holomorphic and 1/15 for the antiholomorphic factors in the amplitude (for which $z$ is also replaced by $\bar{z}$). The holomorphic factor has the two independent solutions

$$ U_1 = z^{\frac{4}{5}}(1 - z)^{\frac{1}{5}} {}_2F_1(6/5, 13/5, 12/5; z) 
$$

(3.4)

$$ U_2 = z^{-\frac{2}{5}}(1 - z)^{-\frac{1}{5}} {}_2F_1(-8/5, -1/5, -2/5; z) 
$$

(3.5)

and the antiholomorphic factor has the two independent solutions

$$ \tilde{U}_1 = \bar{z}^{\frac{4}{5}}(1 - \bar{z})^{\frac{1}{5}} {}_2F_1(4/5, 7/5, 8/5; \bar{z}) 
$$

(3.6)

$$ \tilde{U}_2 = \bar{z}^{-\frac{2}{5}}(1 - \bar{z})^{-\frac{1}{5}} {}_2F_1(-2/5, 1/5, 2/5; \bar{z}) 
$$

(3.7)

where ${}_2F_1(a, b, c; z)$ is the standard hypergeometric function.

To obtain the physical correlation function, we must put the two chiralities together in such a way as to leave the amplitude well-defined on the complex plane, i.e. with trivial monodromies. The amplitude will be proportional to

$$ (\tilde{U}_1 \quad \tilde{U}_2) A \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} 
$$

(3.8)

where $A$ is a $2 \times 2$ constant matrix. The hypergeometric functions in (3.4)-(3.7) are given by convergent power series near $z = 0$, so we can read off the monodromies about $z=0$ from the prefactors:
\[
\begin{pmatrix}
U_1 \\
U_2
\end{pmatrix}
\rightarrow
\begin{pmatrix}
e^{\frac{a}{2}2\pi i} & 0 \\
0 & e^{-\frac{a}{2}2\pi i}
\end{pmatrix}
\begin{pmatrix}
U_1 \\
U_2
\end{pmatrix}
\]  \hspace{1cm} (3.9)

and
\[
\begin{pmatrix}
\tilde{U}_1 \\
\tilde{U}_2
\end{pmatrix}
\rightarrow
\begin{pmatrix}
\tilde{U}_1 \\
\tilde{U}_2
\end{pmatrix}
\begin{pmatrix}
e^{-\frac{a}{2}2\pi i} & 0 \\
0 & e^{\frac{a}{2}2\pi i}
\end{pmatrix}
\]  \hspace{1cm} (3.10)

Requiring (3.8) to remain invariant we learn that \(a_{11} = a_{22} = 0\).

To obtain the transformations under monodromies about \(z = \tilde{z} = 1\) one makes use of the standard formula [14]
\[
\binom{2\Gamma(a,b,c)}{a \Gamma(a,b)-(c-a-b)\Gamma(c-b-a)} = \binom{2\Gamma(a,b)+(c-a-b)\Gamma(c-b-a)}{+ 1-a-b\Gamma(c-b-a) \Gamma(a-b)(c-a-b)}
\]
(3.11)

In [15] Dotsenko used (3.11) to write down the monodromy matrices for general hypergeometric solutions. Using his formulas we find the following monodromies about \(z = \tilde{z} = 1\):
\[
\begin{pmatrix}
U_1 \\
U_2
\end{pmatrix}
\rightarrow
\frac{e^{2\pi i(\frac{c}{2})}}{\lambda' - \lambda}
\begin{pmatrix}
\lambda' - \lambda \omega_2 & \omega_2 - 1 \\
\lambda \omega_2(1 - \omega_2) & \lambda \omega_2 - \lambda
\end{pmatrix}
\begin{pmatrix}
U_1 \\
U_2
\end{pmatrix}
\]  \hspace{1cm} (3.12)

\[
\begin{pmatrix}
\tilde{U}_1 \\
\tilde{U}_2
\end{pmatrix}
\rightarrow
\frac{e^{-2\pi i(\frac{c}{2})}}{\lambda' - \lambda}
\begin{pmatrix}
\lambda' - \lambda \tilde{\omega}_2 & \tilde{\lambda} \tilde{\lambda}'(1 - \tilde{\omega}_2) \\
\tilde{\omega}_2 - 1 & \lambda \tilde{\lambda}' - \lambda
\end{pmatrix}
\]  \hspace{1cm} (3.13)

where \(\omega_2 = e^{-2\pi i(\frac{c}{2})}\), \(\lambda = \frac{\Gamma(-\frac{c}{2})\Gamma(\frac{c}{2})}{\Gamma(-\frac{c}{2})\Gamma(\frac{c}{2})}\), and \(\lambda' = \frac{\Gamma(-\frac{c}{2})\Gamma(\frac{1}{2})}{\Gamma(-\frac{c}{2})\Gamma(\frac{1}{2})}\); and where \(\tilde{\omega}_2 = e^{2\pi i(\frac{c}{2})}\), \(\tilde{\lambda} = \frac{\Gamma(\frac{c}{2})\Gamma(\frac{1}{2})}{\Gamma(\frac{c}{2})\Gamma(\frac{1}{2})}\), and \(\tilde{\lambda}' = \frac{\Gamma(\frac{c}{2})\Gamma(\frac{1}{2})}{\Gamma(\frac{c}{2})\Gamma(\frac{1}{2})}\).

One finds that
\[
\begin{pmatrix}
\tilde{U}_1 \\
\tilde{U}_2
\end{pmatrix}
\begin{pmatrix}
0 & a_{12} \\
a_{21} & 0
\end{pmatrix}
\begin{pmatrix}
U_1 \\
U_2
\end{pmatrix}
= \begin{pmatrix}
\tilde{U}_1 \\
\tilde{U}_2
\end{pmatrix}
\begin{pmatrix}
0 & a_{12} \\
a_{21} & 0
\end{pmatrix}
\begin{pmatrix}
U_1 \\
U_2
\end{pmatrix}
\]  \hspace{1cm} (3.14)

(where \(\tilde{M}_1\) and \(M_1\) are the matrices that appear in (3.13) and (3.12)) if and only if
\[
c \equiv \frac{a_{21}}{a_{12}} = -\frac{(\lambda' - \tilde{\lambda}\tilde{\omega}_2)(\lambda \tilde{\lambda}'(1 - \omega_2))}{(\lambda' - \lambda \omega_2)(\lambda \lambda'(1 - \tilde{\omega}_2))}
\]  \hspace{1cm} (3.15)

That is, up to scale, \(\tilde{U}_1 U_2 + c\tilde{U}_2 U_1\) is the unique solution satisfying the differential equations and monodromy conditions.
4. The integral

Assembling the pieces, our amplitude is

\[ \int d^2z |z|^{-\frac{4}{5}} |1 - z|^{-\frac{4}{5}} (\tilde{U}_1 U_2 + c\tilde{U}_2 U_1) \] (4.1)

Note that because of the chiral nature of the internal amplitude the integrand is not positive definite. We computed this using Mathematica’s numerical integration routine \textit{NIntegrate}, producing a vanishing result good to seven digits. In evaluating (4.1) we excised small regions containing the points 0, 1, and \( \infty \) and evaluated their contribution separately.

This result supports the general arguments for flatness of space-time superpotentials for (0, 2) linear sigma models to be explained in [9]. However, it would be more satisfying to obtain a direct analytical understanding of the vanishing of this integrated amplitude. Perhaps this could be attained by considering the integral as an inner product between the left and right moving solutions, and trying to understand why the two are orthogonal analytically\(^1\).

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\(^1\) This hope is somewhat reminiscent of the argument, involving Atkin-Lehner symmetry, for the vanishing of the one-loop cosmological constant in certain non-space-time supersymmetric vacua [16]; I thank J. Distler for pointing this out to me.
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