Gravitational-wave bursts from spin-precessing black holes in binary systems

Chen Zhang, Wen-Biao Han, Shu-Cheng Yang

1 Shanghai Astronomical Observatory, Shanghai, 200030, China
2 School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China
3 School of Astronomy and Space Science, University of Chinese Academy of Sciences, Beijing 100049, China
4 Shanghai Frontiers Science Center for Gravitational Wave Detection, 800 Dongchuan Road, Shanghai 200240, China

1 INTRODUCTION

All the 90 GW events released by the Advanced LIGO-Virgo-KAGRA collaboration are compact binary coalescences (CBCs) (Abbott et al. 2021a). As the two compact objects inspiral towards merger due to the emission of gravitational waves (GWs), if one or both bodies are rapidly rotating, the general relativistic spin-orbit and spin-spin couplings (i.e., the “dragging of inertial frames” by the bodies’ spins) cause the orbital plane and the spins to precess about the direction of the total angular momentum (Apostolatos et al. 1994). Thus spin-precession is an important phenomenological feature which relates to both general relativistic dynamics and astrophysical binary formation scenarios.

During the inspiral stage, gravitational waves are radiated from the time-varying quadrupole due to the orbital motion of two masses \( m_1 \) and \( m_2 \), after that, the merger-ringdown waveforms are produced by the perturbation of the dynamically forming black holes (BHs). The inspiral process can be clearly described by the post-Newtonian (PN) approximation for the quasi-circular orbits. A few of these events have been considered to have spin precessions, for example, the event GW190521 (Abbott et al. 2020; Estellés et al. 2022), which has an effective precession spin \( \chi_P \) as large as 0.67, shows evidence for spin-induced orbital precession. In addition, GW190412, GW190512_180714, GW190521, and GW190814 signals also provide sufficient support for spin precession (Gerosa et al. 2021). However, current waveform models only take into account the signal modulation induced by the spin precession on the inspiral waveform (Pan et al. 2014; Hannam et al. 2014; Babak et al. 2017; Khan et al. 2019; Varma et al. 2019).

In this Letter, we propose that the spin-precession of a black hole itself also can produce detectable GWs. This is due to the variation of quadrupole from the precessing Kerr black hole. It is well known that the Kerr black hole has a quadrupole moment \( Q = -M^3\chi^2 \), for an isolated Kerr black hole, the quadrupole is invariant and can not radiate GWs because of axisymmetric spacetime. However, during the inspiral of precessing binary black holes, the quadrupole moment of black hole itself will change with time and radiate GWs. In this Letter, we calculate this precessing GWs and demonstrate the properties of this brand new signal, especially discuss the detectability.

Just a reminder, this precessing GWs are different from the previous researches on the precessing binaries which only take into account the modulation induced by the spin precession on the inspiral waveform (Pan et al. 2014; Hannam et al. 2014; Babak et al. 2017; Khan et al. 2019; Varma et al. 2019). As a consequence of the no-hair theorem—the claim that mass and spin are the only two properties needed to describe black holes in general relativity (Vishveshwara 1970), the quadrupole momentum should be determined entirely in terms of the mass \( M \) and the dimensionless spin magnitude \( \chi \) of the black hole. Present researches provide tests of the no-hair theorem at the ~ 10% level by analyzing ringdown data from the Advanced LIGO - Virgo detection (Isi et al. 2019; Ota & Chirenti 2020). Considering the ringdown signals just reflect the properties at light-ring (Cardoso & Pani 2017), the precessing waves are directly determined by the mass, spin, and quadrupole of the black hole, then this kind of signals could be a unique tool to test the no-hair theorem.

© 2022 The Authors

Key words: Gravitational waves – Black hole physics
2 WAVEFORM EMITTED BY BLACK HOLE’S PRECESSION

There are two main channels for the formation of compact binaries: common envelope evolution (Frugoni 2021; Mandel & Broekgaarden 2022; Belczynski et al. 2022) and dynamical capture (Vitale et al. 2017; Frugoni et al. 2022, 2020). The latter will induce spins that are randomly oriented for binaries. Precessing spins are a key prediction of binary black holes formed in dense clusters, but might also be present in the case of sources formed in isolation because of, for example, supernova kicks (Kalogera 2000; Mandel & O’Shaughnessy 2010; Gerosa et al. 2013; Rodriguez et al. 2016; Gerosa et al. 2018; Steinle & Kesden 2021; Callister et al. 2021; Frugoni et al. 2021).

This spin precession could occur during the merger of binary black holes, if the spins are not aligned with the orbital angular momentum, a so-called geodetic precession is produced, also known as spin-orbit coupling. The spin-spin coupling can also induce precession, but it is expected to be much smaller than the geodesic precession at large separations (Gerosa et al. 2015). From the Eq. (2.4a) in (Kidder 1995), the last two terms are the spin-spin coupling, due to the magnitude of orbital angular momentum is usually several times larger than the spin magnitude (always less than 1 for black holes), and also considering the coefficient of the spin-orbit term, the spin-one will several times less than the spin-orbit coupling. For simplicity in the present work, we only consider the dominant effect (spin-orbit coupling), and this will not affect the result. For a binary of total mass \( M = m_1 + m_2 \), relative separation \( r = |\mathbf{r}| \) and spins \( \mathbf{S}_1 = \chi_1 m_1^2 \mathbf{S}_1 \) and \( \mathbf{S}_2 = \chi_2 m_2^2 \mathbf{S}_2 \), where \( \mathbf{S}_1,2 \) are unit vectors and dimensionless spin parameter \( 0 < \chi_{1,2} < 1 \), the precession equations of the two binary components are given by: (Buonanno et al. 2003)

\[
\dot{\mathbf{S}}_1 = \frac{1}{r^3} \left[ \frac{4m_1 + 3m_2}{2m_1} \left( G \mu \frac{M \Omega}{c^2 r^{1/2}} \right) \mathbf{L} \right] \times \mathbf{S}_1, \tag{1}
\]

just replace 1 with 2, one can get the equation for \( \mathbf{S}_2 \), where \( \mu = m_1 m_2 / M \) is the reduced mass and \( \mathbf{L} = \mathbf{r} \times \mathbf{F} \) gives the direction of the orbital angular momentum. From Eq. (2), we get the precession rates is

\[
\Omega_{S_1} \approx 2 \times 10^5 F_1(q) \frac{\mu}{M} \frac{\dot{r}}{r^{5/2}}, \quad \Omega_{S_2} \approx 2 \times 10^5 F_2(q) \frac{\mu}{M} \frac{\dot{r}}{r^{5/2}}, \tag{2}
\]

where \( \dot{r} = r/(GM/c^2), \) \( q \) is the mass-ratio defined by \( q = m_2/m_1 \), and

\[
F_1(q) = \frac{q(4 + 3q)}{2(1 + q)^2}, \quad F_2(q) = \frac{q(4q + 3)}{2(1 + q)^2}. \tag{3}
\]

It is clear that when \( q \to 0 \), the precession of \( \mathbf{S}_1 \) disappears and the one of \( \mathbf{S}_2 \) just returns to the well-known geodetic precession. For equal-mass binary \( F_1 = F_2 = 7/8 \).

Considering that the Keplerian orbital frequency has a relation \( \omega_{\text{orb}} \approx 2 \times 10^6 (M_\odot / M) r^{-3/2} \), we can get the relation between the frequencies of inspiral’s and precession’s GWs. For the equal-mass case, if the mass of each black hole is 10 (100) \( M_\odot \), when the objects inspiral from 10 to 3M, then the precession frequency of each spin is from 9 (0.9) Hz to 179 (17.9) Hz, which is in the sensitive band of Advanced LIGO, Virgo and KAGRA detectors.

We can then calculate the waveform emitted by precessing black holes. The waveform of a precessing rigid body can be written as (Maggiore 2007)

\[
h_+ = A_{+,1} \cos \Omega t, \quad h_{\times} = A_{\times,1} \cos \Omega t, \quad A_{+,2} = 2h_0' \sin^2 \alpha \left( 1 + \cos^2 \iota \right), \tag{5}
\]

\[
A_{\times,2} = 4h_0' \sin^2 \alpha \cos \iota, \tag{6}
\]

where \( \iota \) is the angle between the line-of-sight and the initial (non-precessing) spin direction, \( \alpha \) is the spin rotation angle and

\[
h_0' = -G \frac{(I_1 - I_2) \Omega^2}{c^4 D}. \tag{7}
\]

We notice that the waveform has two frequencies, one is equal to the precession frequency \( \Omega \), and the other one is the double of \( \Omega \). For sufficiently strong precessing effect \( \alpha \geq \pi/4 \), by Eqs. (5), the \( 2\Omega \) component is several times larger than the \( \Omega \) one, therefore we mainly focus the dominant part.

The amplitude of the waveform is determined by the difference in the moments of inertia, \( I_2 \) is related to the rotation about the direction of the total angular momentum and \( I_1 \) is related to the principal axis perpendicular to this direction. The principal moment of inertia can be obtained from the following equation (Hartle 2003)

\[
J = \frac{Gm}{c^2} = 2 \frac{G^2 m^3}{c^4} (1 + \sqrt{1 - \chi^2}) \Omega_H, \tag{8}
\]

where \( m \) is the mass, \( a \) is the Kerr parameter, \( J \) is the total angular momentum of the Kerr black hole and \( \Omega_H \) is the rotating frequency of horizon. One then can define the main inertia moment by \( J = I\Omega \) as

\[
I_3 = 2 \frac{G^2 m^3}{c^4} (1 + \sqrt{1 - \chi^2}), \tag{9}
\]

which depends on the spin \( \chi \) of the black hole. Therefore, \( I_1 - I_2 \) can be taken as \( \delta^2 G^2 m^3 / c^4 \), where \( \delta \) is a function determined by the BH mass and spin in GR or by several other parameters in alternative gravity theories. Finally, the expression of the GW strain amplitude given by Eq. (7) becomes:

\[
h_{0i} = - \frac{3 \delta^2 G^2 m^3}{c^5} \frac{\Omega_H^2}{D} \approx 4.8 \times 10^{-23} \left( \frac{1 \text{Gpc}}{D} \right) \left( \frac{m_i}{M} \right) \left( \frac{m_j}{M} \right)^2 \delta F_i(q) r_i^{-5}, \tag{10}
\]

where \( i = \{1, 2\} \). From the above equation, the GW strain amplitude of the signal emitted by precessing black holes increases very rapidly when \( r \) decreases. For \( r \) small enough, we will see that this signal can be detected.

To do so we need to derive the frequency-domain waveform of the GWs from spin precession. To the leading order, the orbital evolution due to the GWs radiation for a circular binary is

\[
\frac{G \mu M}{2 r^2} \dot{r} = - \frac{32 \mu m^2 r^4}{5 c^5} \left( \frac{G M}{r^3} \right)^3, \tag{11}
\]

here and after, we omit the subscript \( i \) of \( m \) and \( F \). From Eq. (2) we can get the relation between the precession frequency and the orbital radius

\[
r = c^{-4/5} G^{3/5} M^{3/5} F^{2/5} \Omega_S^{-2/5}. \tag{12}
\]

Using the above expression of \( r \) and the definition of the angular frequency \( f_p \equiv \Omega_S / 2\pi \), the energy balance equation (11) can be rewritten in a form of a differential equation of \( f_p \) with respect to time \( t \), as

\[
f_p = \frac{23/5 \pi^{8/5} G^{3/5}}{c^{3/5} M^{2/5} F^{8/5}} f_p^{13/5}. \tag{13}
\]
while the equation for inspiral orbital frequency is $\dot{f} \propto f^{11/3}$, it will induce the waveform of precession’s GWs have a different power law with respect to frequency. In terms of $\tau = t_c - t$, where $t_c$ is the coalescence time, the solution of Eq.(13) reads

$$f_p(\tau) = \frac{5^{5/8} c^{9/8} F_1 M^1/4}{64\pi G^{3/8} \mu^{5/8}} \tau^{-5/8}.$$  (14)

Using Eq.(14) and the phase of precession’s GWs $\Phi(t) = \int_0^t \Omega_5 \, dt'$, we find (here $d\tau = -dt$ and we use the signature $(-, +, +, +)$ for the metric)

$$\Phi(\tau) = \frac{5^{5/8} c^{9/8} F_1 M^1/4}{128 G^{3/8} \mu^{5/8}} \tau^{-3/8} + \Phi_c,$$  (15)

$$\Phi(\tau) = \frac{5^{13/8} c^{9/8} F_1 M^1/4}{256 G^{3/8} \mu^{5/8}} \tau^{-13/8}.$$  (16)

Equivalently, using the stationary point condition $2\pi f_p = \Phi(\tau)$ and $\tau = t_c - t$, we can rewrite (14) as

$$\tau = \frac{5 c^{9/5} M^{2/5} F_1^{3/5}}{288 \pi^{3/5} G^{3/5} \mu^{1/5}} f_p^{8/5}.$$  (17)

We can now, compute the Fourier transform of the $\Omega$ component of $h_\kappa$ in Eq.(4) using the stationary phase method (Cutler & Flanagan 1994). Finally, the frequency domain waveform from the precession is given by:

$$\tilde{h}_{+1} = \frac{n^{6/5} f_7^{10/7} G^{7/7} m^3 M^{11/5} F_1^{4/5}}{c^{31/10} \mu^{1/2} D^2} e^{i \psi_{+1}} \sin 2\theta \sin 2\tau,$$  (18)

where

$$\psi_{+1} = 2\pi f \left( t_c + D/c \right) - \Phi_\kappa - \frac{\pi}{4} + \frac{25 c^{9/5} M^{2/5} F_1^{3/5}}{768 (2\pi)^{3/5} \mu (G^3/5)}.$$  (19)

By repeating the same procedure for $\tilde{h}_{+2}$ we can derive the dominant $2\Omega$ component ($f_2 = 2f$) waveform

$$\tilde{h}_{+2} = \frac{n^{6/5} f_7^{10/7} G^{7/7} m^3 M^{11/5} F_1^{4/5}}{c^{31/10} \mu^{1/2} D^2} e^{i \psi_{+2}} \sin 2\alpha \cos 4\tau,$$  (20)

where

$$\psi_{+2} = 2\pi f_2 \left( t_c + D/c \right) - \Phi_\kappa - \frac{\pi}{4} + \frac{25 c^{9/5} M^{2/5} F_1^{3/5}}{192 \pi (\pi f_2 G)^{3/5}}.$$  (21)

In the following calculation we will only consider this dominant part which radiates at $f_2$, so $\tilde{h}_{+1} = \tilde{h}_{+2}$. Note that this is only the gravitational wave of the precession of S1, similarly one can also calculate the precession's waveform of the component black hole.

For equal-mass binary black holes with the same spin magnitude, the total GW strain from spin precession is exactly twice that of the S1 component. From Eqs. (18, 20), we can clearly see that the frequency-domain strain is proportional to $f_7^{10/7}$, which distinguishes from the inspiral one $f^{-7/6}$.

Fig. 1 shows five time-domain precession’s waveforms that are cut off at the light ring. This very short signal, called burst, is very different from the inspiral signal. Furthermore, we can observe that binaries of nearly equal mass have a stronger precession’s signal than unequal cases. Faster rotating and more massive binaries can produce stronger signals.

**3 RESULTS AND DETECTABILITY**

To obtain the magnitude of the precession’s waveforms, we need the value of $\delta$ which is determined by the difference between $I_1$ and $I_3$.

**Figure 1. Five burst precession's waveforms.** All the waveforms are cut off at the light-ring.

**Table 1. SNR of some precession's GW signals detected by Advanced LIGO and ET**

| $(m_1, m_2, \chi)$ | 80, 80, 0.9 | 80, 70, 0.9 | 80, 50, 0.9 | 50, 50, 0.9 | 80, 80, 0.5 |
|-------------------|------------|------------|------------|------------|-------------|
| aLIGO             | 17.4       | 11.1       | 3.4        | 9.4        | 0.6         |
| ET                | 167.3      | 106.7      | 32.3       | 98.1       | 6.1         |

The main inertia $I_3$ is well known for the Kerr black hole. In another way, one can get the inertial moment using the radii of gyration of the event horizon. The rotation-offset radii of horizon is $\sigma_+ = \frac{G m}{c^2} \left[ \left( 1 + \sqrt{1 - \chi^2} \right)^2 + \chi^2 \cos^2 \theta \right]^{1/2}$, (22) then the gyration radius is $(2Gm/c^2\sigma_+)^{1/2}$. Taking $\theta = \pi/2$ we get the main gyration radius $R_z = \left[ 2G^2 m^2 / c^4 \left( 1 + \sqrt{1 - \chi^2} \right) \right]^{1/2}$, (23) then the main inertial moment is $m R_z^2$ and just equals to Eq. (9). Due to the axisymmetry, $I_1 = I_2$ and thus, taking $\theta = 0$ we have

$I_1 = \frac{2G^2 m^3}{c^4} \left[ 1 + \sqrt{1 - \chi^2} \right]^{1/2} + \chi^2 \right]^{1/2}$.

Therefore, $I_1 - I_3$ can be taken as $\delta G^2 m^3 / c^4$ with $\delta = 2[2F_p(1)^{1/2} - \tilde{r}_+ \tilde{r}_-] / (G m / c^2)$. If we adopt the above calculation, then for $0 \leq \chi \leq 1$, we get $0 \leq \delta \leq 2(\sqrt{2} - 1)$. For $\chi = 0.9, \delta \approx 0.52$.

We will now discuss the detection potential of such signals. To simplify the calculation, we assume that $\chi_1 = \chi_2 = \chi$. Using Eq.(2) we obtain the peak frequency of GWs radiated from spin precession which cut off at the light ring (Bardeen et al. 1972) (the precession’s GWs after merger are not considered in our present analysis.)

$$f_{\text{peak}} = \frac{c^3}{2\pi G M} F \left( \frac{c^2}{G M} \right)^{-5/2}.$$  (25)

It is clear that the peak frequency increases with $|\chi|$, $F$ and decreases with $M$, and $F$ positively correlated with mass ratio $q$. In addition, from Eq.(20) the strain of precession’s GWs will be largely reduced for relatively small values of mass ratio. Therefore, the SNR will be larger when the spin becomes extreme and the binary black holes have
nearly equal mass. We then calculate the SNR of the precession’s signal measured by Advanced LIGO and ET (Table 1). From the events detected by Advanced LIGO and Virgo, we choose a comparatively reliable parameter space: mass $50 - 80 \, M_{\odot}$ and spin $0.5 - 0.9$ (such as two extreme spin BH binary events reported in the GWTC-2 catalog, GW190517_055101 has $\chi_{1,2} > 0.8$ with 77% credibility, and GW190521 with 58% credibility (Abbott et al. 2021b)). Moreover, since the SNR is inversely proportional to the distance from the source, we set a fiduciary value of 100 Mpc in our analysis. We notice that for Advanced LIGO, the signal is detectable (SNR $\geq 10$) only if the masses of the black holes are nearly equal ($\leq 50 \, M_{\odot}$ each) with an extreme spin ($\chi \geq 0.9$) and a luminosity distance around 100 Mpc. On the other hand, we found that the SNR in ET is almost 10 times higher than that in Advanced LIGO.

Based on our estimation, the precessing GW signal with appropriate parameters should be strong enough to be measurable by the Einstein telescope. Therefore, in order to determine the three main parameters $(m_1, m_2, \chi)$ that contribute to the precessing GW signal, we assume that the distance of the source is 100 Mpc and that it is facing the observer ($i = \pi/2$). Fig. 2 shows the power spectral density (PSD) of five precession’s signals and one IMR (inspiral-merger-ringdown) signal. Obviously, the precessing GW signal is much weaker than the IMR signal for the same source. However, some sources could be detected even by Advanced LIGO if the mass of each black hole is as large as GW190521 and as close as a few hundred Mpc with high spin and precession (see Table 1). For third generation detectors, precessing GW signals could be more easily detected by ET. Furthermore, due to the spin-spin effect just a few times smaller than the one of spin-orbit, in ET era, it is also possible to detect the spin-spin effect. This will be discussed in a future work.

After passing, the light ring, two BHs merge and a final black hole is formed. During this process, we believe that the rotation axis of the dynamical BH continues to precess for some time. The radiation from the precessing GWs will reduce the precession itself to zero eventually. In this Letter, we do not consider the waveform in this final stage, but it should be very interesting in a future work.

4 CONCLUSIONS

The GWs directly from precessing black hole in binary system usually are too weak to be detected by Advanced LIGO. However, if this type of source is within a few hundred Mpc, and if it is the merger of two relatively massive black holes ($m_{1,2} \geq 50 \, M_{\odot}$) with large spin precession and a comparable mass-ratio, then the precession’s signals should be detectable by current ground-based detectors. For example, as shown in Table 1, if the source is coalescence of equal-mass black holes ($m_1 = m_2 = 80 \, M_{\odot}$) with extreme spin ($\chi = 0.9$), we obtain the SNR is 17.4 for Advanced LIGO. For the third generation detectors like ET, this kind of GWs would be detected more easily. Further more, due the significantly different chirp propriety, this kind of GWs can be easily distinguished from the main inspiral signal in the data.

Considering that the nature of this kind of precessing waveforms comes from the inertia moments of spinning black hole, then the inertia and quadrupole of BH will directly decide the strength of waveforms. However, the degeneration still exists, the spin magnitude, precession frequency, angle and so on also influence the waveform strain. Fortunately the degeneration could be broken, if we combine the inspiral signals. Then one can measure the $I_3 - I_1$ which is determined by the structure of the rotating black holes from the precession’s signals.

In GR, the astrophysical BH should be described by the Kerr solution, which only has two parameters: mass and angular momentum. The quadrupole moment will be determined by $M$, $\chi$ uniquely. In principle, the quadrupole decides the form of precession’s waveforms. However, alternative gravity theories may predict hairy black holes, i.e., at least one more parameter will be introduced to give quadrupoles. From precession’s GWs, in principle one can directly measure the quadrupole, in other words, the no-hair theorem will be tested. This means that this kind of signals could be a unique tool to reveal the instinct structure of black holes.

ACKNOWLEDGEMENTS

This work is supported by The National Key R&D Program of China (No. 2021YFC2203002), NSFC No. 11773059 and No. 12173071, and the Strategic Priority Research Program of the CAS under Grants No. XDA15021102. W. H. is supported by CAS Project for Young Scientists in Basic Research YSBR-006. We thank Prof. Y. Chen for his useful comments.

DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author Wen-Biao Han.

REFERENCES

Abbott R., et al., 2020, Physical review letters, 125, 101102
Abbott R., et al., 2021a, arXiv preprint arXiv:2111.03606
Abbott R., et al., 2021b, Physical Review X, 11, 021053
Apostolatos T. A., Cutler C., Sussman G. J., Thorne K. S., 1994, Physical Review D, 49, 6274
Babak S., Taracchini A., Buonanno A., 2017, Physical Review D, 95, 024010
Bardeen J. M., Press W. H., Teukolsky S. A., 1972, The Astrophysical Journal, 178, 347
Belczynski K., Doctor Z., Zevin M., Olejak A., Banerjee S., Chattopadhyay D., 2022, arXiv preprint arXiv:2204.11730
Gravitational-wave bursts from spin-precessing black holes in binary systems

Buonanno A., Chen Y., Vallisneri M., 2003, Physical Review D, 67, 104025
Callister T. A., Farr W. M., Renzo M., 2021, The Astrophysical Journal, 920, 157
Cardoso V., Pani P., 2017, Nature Astronomy, 1, 586
Cutler C., Flanagan E. E., 1994, Physical Review D, 49, 2658
Estellés H., et al., 2022, The Astrophysical Journal, 924, 79
Fragione G., 2021, The Astrophysical Journal Letters, 923, L2
Fragione G., Loeb A., Rasio F. A., 2020, The Astrophysical Journal Letters, 902, L26
Fragione G., Loeb A., Rasio F. A., 2021, The Astrophysical Journal Letters, 918, L38
Fragione G., Kocsis B., Rasio F. A., Silk J., 2022, The Astrophysical Journal, 927, 231
Ger
dos A., Kesden M., Berti E., O’Shaughnessy R., Sperhake U., 2013, Physical Review D, 87, 104028
Ger
dos A., Kesden M., Sperhake U., Berti E., O’Shaughnessy R., 2015, Physical Review D, 92, 064016
Ger
dos A., Berti E., O’Shaughnessy R., Belczynski K., Kesden M., Wysocki D., Gladysz W., 2018, Physical Review D, 98, 084036
Ger
dos A., Mould M., Gangardt D., Schmidt P., Pratten G., Thomas L. M., 2021, Physical Review D, 103, 064067
Hannam M., Schmidt P., Bohé A., Haegel L., Husa S., Ohme F., Pratten G., Pürrer M., 2014, Physical review letters, 113, 151101
Hartle J. B., 2003, Gravity: an introduction to Einstein’s general relativity
Isi M., Giesler M., Farr W. M., Scheel M. A., Teukolsky S. A., 2019, Physical Review Letters, 123, 111102
Kalogera V., 2000, The Astrophysical Journal, 541, 319
Khan S., Chatziioannou K., Hannam M., Ohme F., 2019, Physical Review D, 100, 024059
Kidder L. E., 1995, Physical Review D, 52, 821
Maggiore M., 2007, Gravitational waves: Volume 1: Theory and experiments. OUP Oxford
Mandel I., Broekgaarden F. S., 2022, Living Reviews in Relativity, 25, 1
Mandel I., O’Shaughnessy R., 2010, Classical and Quantum Gravity, 27, 114007
Ota I., Chirenti C., 2020, Physical Review D, 101, 104005
Pan Y., Buonanno A., Taracchini A., Kidder L. E., Mroué A. H., Pfeiffer H. P., Scheel M. A., Szilágyi B., 2014, Physical Review D, 89, 084006
Rodriguez C. L., Zevin M., Pankow C., Kalogera V., Rasio F. A., 2016, The Astrophysical Journal Letters, 832, L2
Steinle N., Kesden M., 2021, Physical Review D, 103, 063032
Varma V., Field S. E., Scheel M. A., Blackman J., Ger
dos A., Stein L. C., Kidder L. E., Pfeiffer H. P., 2019, Physical Review Research, 1, 033015
Vishveshwara C., 1970, Physical Review D, 1, 2870
Vitale S., Lynch R., Sturani R., Graff P., 2017, Classical and Quantum Gravity, 34, 03LT01

This paper has been typeset from a \LaTeX file prepared by the author.