An electronic transition-based bare bones particle swarm optimization algorithm for high dimensional optimization problems

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Abstract

An electronic transition-based bare bones particle swarm optimization (ETBBPSO) algorithm is proposed in this paper. The ETBBPSO is designed to present high precision results for high dimensional single-objective optimization problems. Particles in the ETBBPSO are divided into different orbits. A transition operator is proposed to enhance the global search ability of ETBBPSO. The transition behavior of particles gives the swarm more chance to escape from local minimums. In addition, an orbit merge operator is proposed in this paper. An orbit with low search ability will be merged by an orbit with high search ability. Extensive experiments with CEC2014 and CEC2020 are evaluated with ETBBPSO. Four famous population-based algorithms are also selected in the control group. Experimental results prove that ETBBPSO can present high precision results for high dimensional single-objective optimization problems.

Introduction

An electron transition is essentially an energy change of electrons in the particles that make up matter. According to the principle of conservation of energy, the outer electrons of a particle absorb energy as they move from a lower to a higher energy level, and release energy as they move from a higher to a lower energy level. The energy is the absolute value of the difference between the energies of the two energy levels. In this paper, we use particles to simulate the electronic transition behavior to solve high dimensional optimization problems. Particles are designed to search in different orbits. The particles on the worse-positioned orbits have the opportunity to make a transition to the better-positioned orbits.

Optimization problems appear everywhere in our daily life. Whenever we want to make a choice, we believe the option is better. In numerical optimization problems, the numerous global optimization (GO) problems is often described in Eq 1:

\[ f : X \rightarrow \mathbb{R} \]

\[ x^* \in X \]

\[ f(x^*) \leq f(X) \]
where $X \subset \mathbb{R}^D$ is a nonempty compact set that contains all feasible solutions, $D$ is the dimension of the problem, $f$ is a real valued objective function, $x^\star$ is the theoretical optimal solution [1]. The purpose of an optimization algorithms is finding the $x^\star$, even the objective functions maybe non-convex, multimodal, or badly scaled [2].

To solve numerical optimization problems, population-based methods like genetic algorithms (GA) [3], differential evolution (DE) [4], particle swarm optimization (PSO) [5] ant colony (AC) [6], walf pack algorithm [7] are proposed. Among these methods, PSO is famous for fast convergence and easy applying. PSO algorithms are widely applied in engineering like route planning [8, 9], data clustering [10, 11], feature selection [12, 13], image segmentation [14, 15], power system [16, 17], engineering areas [18–20], and so on.

The class PSO algorithm is inspired by the social behavior of fish and birds. Particles begin searching from random solutions and aim at the solution which can minimize the target problem. In PSO, each particle represents a solution of the target problem. In function optimizations, each particle retains follow attributes: velocity, represents how fast a particle is moving in the search space; current fitness value, represents the function value at current position; current position, represents the coordinate at this generation; personal best value, represents the best function value across all generations; personal best position, represents the coordinate of the personal best value. The behavior of the particles is controlled by many parameters, so to achieve the best performance researchers need to adjust parameters for every specific problem. Also, with the developing of technology, optimization problems become high dimensional and multimodal. Traditional PSO methods sometimes difficult to provide high precision results. For some complicate problems, PSO methods are easily fall into local minimal and leading to unacceptable results.

Researchers tried to improve the performance of PSO by combining different strategies. In 2016, Pornsing [21] proposed a self-adaptive strategy to improve the search ability of PSO. In 2017, Chen proposed a new biogeography-based learning strategy for PSO [22]. In 2018, Xu proposed a novel chaotic PSO for combination optimization problems [23]. In 2018, Tian proposed a chaos-based initialization strategy and robust update mechanisms for PSO [24]. In 2019, Ghasemi proposed a new parameter control strategy to enhance the search ability of PSO [25]. In 2019, Xu [26] combined the quantum behavior with PSO and achieved better search ability. In 2021, Yamanaka tried to improve the performance PSO by introducing the new concept of particle clustering [27].

Bare bones PSO (BBPSO) [28] is a simple type of PSO. With the cancellation of the velocity term, BBPSO can solve different types of optimization problems without any parameters. In 2017, Guo [29] combined a pair-wise strategy with BBPSO (PBBPSO). Particles change information with a pair-particle during iterations. In addition, three particles are placed in one local group in hierarchical BBPSO (HBBPSO) [30]. Three particles form two different spatial structures to handle different optimization problems. On the other hand, Guo [31] proposed a dynamic local search strategy to enhance the local search ability of BBPSO. In 2018, Guo [32] developed a dynamic allocation operator for BBPSO. In 2019, Guo [33] proposed a fission-fusion strategy for BBPSO. In 2020, Guo [34] proposed a novel BBPSO based method for traveling salesman problem (TSP). Proposed method can present high precision for TSPs.

The rest of this paper is organized as follows: Section 2 introduces the proposed method; Section 3 introduces the numerical experiments; Section 4 presents the conclusions of this paper.

**Materials and methods**

The electronic transition-based bare bones particle swarm optimization (ETBBPSO) algorithm is proposed in this section.
Particle swarm explosion

The initialization of ETBBPSO is called particle swarm explosion (PSE). In PSE, particles are randomly dispersed into the search space. Then, the personal best position and the personal best value of every particle will be calculated. The global best position and the global best value of the particle swarm will be recorded.

Dynamic particle grouping

In ETBBPSO, particles will be assigned to different orbits. During the evolutionary process, particles in orbits play two different roles: the core and the satellite. Each orbits contains one core and several satellites. The number of satellites can be zero. Different evolutionary strategies are applied to different roles. A core particle aims at searching around the global best particles and Enhancing the global search capability of the orbit. The candidate position of a core particle is selected by Eq 2.

\[
\begin{align*}
\alpha &= \frac{(pbest(core^t) + Gbest^t)}{2} \\
\beta &= |pbest(core^t) - Gbest^t| \\
pbest(core^{t+1})_{\text{candidate}} &= \text{GauDis}(\alpha, \beta)
\end{align*}
\]

where the \( pbest(core^t) \) is the personal best position of the core particle in \( (t) \)th generation, \( Gbest^t \) is the personal best position of the global best particle in \( (t) \)th generation, \( pbest(core^{t+1}) \), candidate is the candidate new position for main particle in \( (t + 1) \)th generation, \( \text{GauDis}(\alpha, \beta) \) is the Gaussian distribution with a mean \( \alpha \) and a standard deviation \( \beta \).

A satellite particle aims at searching around the core of the orbit and implementing a local search. The candidate position of a satellite particle is selected by Eq 3.

\[
\begin{align*}
\gamma &= \frac{(pbest(core^t) + pbest(satellite^t))}{2} \\
\delta &= |pbest(core^t) - pbest(satellite^t)| \\
pbest(satellite^{t+1})_{\text{candidate}} &= \text{GauDis}(\gamma, \delta)
\end{align*}
\]

where the \( pbest(core^t) \) is the personal best position of the core particle in \( (t) \)th generation, \( pbest(satellite^t) \) is the personal best position of the satellite particle in \( (t) \)th generation, \( pbest(satellite^{t+1}) \) is the personal best position of the global best particle in \( (t) \)th generation, \( pbest(satellite^{t+1})_{\text{candidate}} \) is the candidate new position for the satellite particle in \( (t + 1) \)th generation, \( \text{GauDis}(\gamma, \delta) \) is the Gaussian distribution with a mean \( \gamma \) and a standard deviation \( \delta \).

A dynamic particle grouping (DPG) strategy is used to divide the particle swarm into different orbits. At the beginning of DPG, the particle \( x_1 \) is selected as the core of the first Orbit. Then the next particle is selected to compare with the previous core. If the selected particle is better than the previous core, a new orbit will be created and the selected particle will be the core of the new orbit. Otherwise the particle will be a satellite of the original orbit. Then this process will be repeated until all particles have been assigned to orbits. The pseudo code of DPG is shown in Algorithm 1.

Algorithm 1 Dynamic Particle Grouping

Require: Max generation time, \( MIT \)  
Require: Test function, \( F \)  
Require: Search Space, \( R \)  
Require: Number of particle, \( n \)  
Require: \( Pbest\_value \)
Require: \texttt{Pbest\_position}

Require: \texttt{Gbest\_value}

Require: \texttt{Gbest\_position}

Require: \texttt{NO}

Require: \texttt{t}

1: while \( t < MIT \) do

2: if \( NO == 1 \) then

3: \( t = t + 1 \)

4: Select the first particle \( x_1 \) as the core for Orbit(\texttt{NO})

5: \texttt{CurrentCore = x_1}

6: for \( i \) in range \((2, n)\) do

7: if \( pb_i < pb_{\texttt{CurrentCore}} \) then

8: Select a new position for \( x_i \) according to Eq \texttt{2}

9: Create a new Orbit, \( NO = NO + 1 \)

10: \texttt{CurrentCore = x_i}

11: else

12: Make \( x_i \) a satellite of \texttt{CurrentCore}, belonging to Orbit(\texttt{NO})

13: Select a new position for \( x_i \) according to Eq \texttt{3}

14: end if

15: end for

16: Update \texttt{Pbest\_value}, \texttt{Pbest\_position}, \texttt{Gbest\_value}, \texttt{Gbest\_position}

17: In each Orbit, make the particle with a smallest \texttt{Pbest\_value} be the new core

18: end if

19: end while

Particle transition

To enhance the local search ability of the top orbit, the particle transition operator (PTO) is proposed. All orbits will be ranked according to the personal best value of their cores. Then all particles in the second best orbit will transit to the best orbit. By doing this the best orbit will gather more and more particles to obtain stronger local search ability. Other orbits still have change to do global search and replace the best orbit. The pseudo code of the PTO is shown in Algorithm 2.

Algorithm 2 Particle transition operator

Require: Max generation time, \( MIT \)

Require: Test function, \( F \)

Require: Search Space, \( R \)

Require: Number of particle, \( n \)

Require: \texttt{Pbest\_value}

Require: \texttt{Pbest\_position}

Require: \texttt{Gbest\_value}

Require: \texttt{Gbest\_position}

Require: \texttt{NO, t}

Require: Orbit

1: while \( t < MIT \) do

2: if \( NO > 1 \) then

3: \( t = t + 1 \)

4: Rank all Orbits according the personal best values of their cores

5: A core with a smaller personal best value is a better core, its corresponding Orbit is the better Orbit

6: Merge the first and second best Orbits

7: Select a new position for all cores according to Eq \texttt{2}

8: Select a new position for all satellites according to Eq \texttt{3}

9: \( NO = NO - 1 \)
10: Update Pbest_value, Pbest_position, Gbest_value, Gbest_position
11: end if
12: end while

Console

The DPG and the PTO collaborate to balance the local and global search. The DPG adaptive grouping strategy is applied so that the internal structure of the particle swarm can change with the change of the target problem. The PTO will merge the top two orbits. By doing this, the distribution of orbits enables the particle swarm to take into account the global search capability while enhancing the local search capability in specific regions. The overall process of ETBBPSO is shown in Algorithm 3. The flowchart of the ETBBPSO is shown in Fig 1.

Algorithm 3 Console

Require: Max generation time, MIT
Require: Test function, F

Flowchart of ETBBPSO

Notes:
NLG, number of local groups
DPG, dynamic particle grouping, Algorithm 1
PT, particle transition, Algorithm 2
T, max iteration times
End, output the global best value

Fig 1. The flowchart of ETBBPSO.

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Require: Search Space, $R$
Require: Number of particle, $n$
Require: $P_{best\_value}$, $P_{best\_position}$, $G_{best\_value}$, $G_{best\_position}$
Require: NO, $t$, Orbit

1: Run PSE
2: while $t < MIT$ do
3: if NO == 1 then
4: Run Algorithm 1
5: end if
6: if NO > 1 then
7: Run Algorithm 2
8: end if
9: end while

Results
Experiments with CEC2014

In order to conduct a fair and comprehensive comparison, the CEC2014 benchmark functions are selected in experiments. Four famous population based methods PBBPSO [29], DABBPSO [32], DLS-BBPSO [31], and FBBPSO [33] are selected in the control group. PBBPSO conducts a novel paired evolution strategy and has shown reliable performance in single modal and multi-modal optimization problems. DABBPSO integrates the scattering and ordering of particle swarms. DLS-BBPSO carries out a dynamic local search operator and show powerful ability in single-objective optimization problems. FBBPSO is the state-of-the-art method based on Bare-bones PSO and has shown great performance and stability on CEC2014. To test the extreme optimization capability of ETBBPSO, experiments are conducted in 100-dimension, max generation time is 1.00E+4, population size for all algorithms is 100. Details of benchmark functions can be found in Table 1. All code are written in Matlab R2020b.

Experimental results and discussion. Experimental results are shown in Tables 2 and 3. Mean is the mean calculation error (CE) from 31 independent runs. CE is defined as $|\text{Global\_Best\_Value} - \text{Theoretically\_Minimum}|$. Std is the standard deviation of the 31 independent runs, Rank is the ranking results of the six algorithms. The Wilcoxon rank sum test is also implemented and average rank results are shown in Table 4.

Numerical analyses are listed below: In $f_{19}$, $f_{23}$, $f_{25}$, the result of ETBBPSO is ranked second among the six algorithms. In $f_{1}$, $f_{3}$, $f_{5} - 7$, $f_{12}$, $f_{17}$, $f_{20}$, $f_{27}$, $f_{28}$, $f_{30}$, the result of ETBBPSO is ranked second among the six algorithms. In $f_{2}$, $f_{9}$, $f_{11}$, $f_{13} - 16$, $f_{18}$, $f_{21}$, $f_{24}$, the result of ETBBPSO is ranked third among the six algorithms. It can be seen that ETBBPSO can present the top three results in 24 test functions. This proves that B is able to give an efficient optimization solution for most problems. It also proves that the electronic transition strategy can provide acceptable solutions for different problems. In $f_{10}$ and $f_{22}$, the result of ETBBPSO is ranked fourth among the six algorithms. In $f_{4}$, $f_{8}$, $f_{26}$, $f_{29}$, the result of ETBBPSO is ranked fifth among the six algorithms. These results suggest that ETBBPSO search ability is easily bounded in the face of such problems, and this is a major direction for future research. It is worth noting that ETBBPSO never finished last in the ranking test, which proves that ETBBPSO does not give extremely bad results even when faced with problems that it is not very good at handling. A ranking competition is designed for every test function. The algorithm presents the best results will get 1 point, presents the second-best results will get 2 points, presents the third-best results will get 3 points, presents the fourth-best results will get 4 points, presents the fifth-best results will get 5 points, presents the worst results will get 6 points. The mean ranking results are shown at the bottom of Table 2. ETBBPSO shows the best results in the 100-dimension test. This is mainly because DPG is able to divide the particle swarm into
different orbits. The whole partitioning process is self-controlled by the algorithm and does not require any parameters. Then, PTO enhances the local search ability of particle swarm while taking into account the global search ability, making it more possible for the particle swarm to escape from the local optimum.

Evolutionary curves across iterations are shown in Figs 2 to 31. The horizontal axis represents the number of iterations, vertical axes represents CEs.

**Discussion.** A ranking competition is designed for every test function. The algorithm presents the best results will get 1 point, presents the second-best results will get 2 point, presents the third-best results will get 3 point, presents the forth-best results will get 4 point, presents the fifth-best results will get 5 point, presents the worst results will get 6 point. The mean ranking results are show in the bottom of Table 3. ETBBPSO shows the best results in the 100-dimension test. This is mainly because DPG is able to divide the particle swarm into different orbits. The whole partitioning process is self-controlled by the algorithm and does not

| Types                      | Function Name                                           | Dimension | Search Range   | Theoretically Minimum |
|----------------------------|---------------------------------------------------------|-----------|----------------|-----------------------|
| Unimodal Functions         | \( f_1 = \) Rotated High Conditioned Elliptic Function  | 100       | (-100,100)     | 100                   |
|                            | \( f_2 = \) Rotated Bent Cigar Function                 | 100       | (-100,100)     | 200                   |
|                            | \( f_3 = \) Rotated Discus Function                    | 100       | (-100,100)     | 300                   |
| Simple Multimodal Functions| \( f_4 = \) Shifted and Rotated Rosenbrock’s Function  | 100       | (-100,100)     | 400                   |
|                            | \( f_5 = \) Shifted and Rotated ACKLEY’s Function       | 100       | (-100,100)     | 500                   |
|                            | \( f_6 = \) Shifted and Rotated Weierstrass’s Function | 100       | (-100,100)     | 600                   |
|                            | \( f_7 = \) Shifted and Rotated Griewank’s Function    | 100       | (-100,100)     | 700                   |
|                            | \( f_8 = \) Shifted Rastrigin’s Function               | 100       | (-100,100)     | 800                   |
|                            | \( f_9 = \) Shifted and Rotated Rastrigin’s Function   | 100       | (-100,100)     | 900                   |
|                            | \( f_{10} = \) Shifted Schwefel’s Function            | 100       | (-100,100)     | 1000                  |
|                            | \( f_{11} = \) Shifted and Rotated Schwefel’s Function| 100       | (-100,100)     | 1100                  |
| Hybrid Functions           | \( f_{12} = \) Shifted and Rotated Katsu function      | 100       | (-100,100)     | 1200                  |
|                            | \( f_{13} = \) Shifted and Rotated HappyCat Function  | 100       | (-100,100)     | 1300                  |
|                            | \( f_{14} = \) Shifted and Rotated HGBat Function     | 100       | (-100,100)     | 1400                  |
|                            | \( f_{15} = \) Shifted and Rotated Expanded           | 100       | (-100,100)     | 1500                  |
|                            | Griewank’s plus Rosenbrock’s Function                   | 100       | (-100,100)     |                       |
|                            | \( f_{16} = \) Shifted and Rotated Expanded Scaffer’s F6 Function | 100       | (-100,100)     |                       |
| Composition Functions      | \( f_{17} = \) Hybrid Function 1 (N = 3)              | 100       | (-100,100)     | 1700                  |
|                            | \( f_{18} = \) Hybrid Function 2 (N = 3)              | 100       | (-100,100)     | 1800                  |
|                            | \( f_{19} = \) Hybrid Function 3 (N = 4)              | 100       | (-100,100)     | 1900                  |
|                            | \( f_{20} = \) Hybrid Function 4 (N = 4)              | 100       | (-100,100)     | 2000                  |
|                            | \( f_{21} = \) Hybrid Function 5 (N = 5)              | 100       | (-100,100)     | 2100                  |
|                            | \( f_{22} = \) Hybrid Function 6 (N = 5)              | 100       | (-100,100)     | 2200                  |
|                            | \( f_{23} = \) Composition Function 1 (N = 5)         | 100       | (-100,100)     | 2300                  |
|                            | \( f_{24} = \) Composition Function 2 (N = 3)         | 100       | (-100,100)     | 2400                  |
|                            | \( f_{25} = \) Composition Function 3 (N = 3)         | 100       | (-100,100)     | 2500                  |
|                            | \( f_{26} = \) Composition Function 4 (N = 5)         | 100       | (-100,100)     | 2600                  |
|                            | \( f_{27} = \) Composition Function 5 (N = 5)         | 100       | (-100,100)     | 2700                  |
|                            | \( f_{28} = \) Composition Function 6 (N = 5)         | 100       | (-100,100)     | 2800                  |
|                            | \( f_{29} = \) Composition Function 7 (N = 3)         | 100       | (-100,100)     | 2900                  |
|                            | \( f_{30} = \) Composition Function 8 (N = 3)         | 100       | (-100,100)     | 3000                  |

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Table 2. Experimental results, CE of PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO for \( f_1 \)–\( f_{15} \). Mean is the mean value from 31 independent runs, STD is the standard deviation of the 31 runs, Rank is the rank of 6 algorithms.

| Function Number | Data Type | PSO | PBBPSO | DA-BBPSO | DLS-BBPSO | FBBPSO | ETBBPSO |
|-----------------|-----------|-----|--------|----------|----------|--------|--------|
| \( f_1 \)       | Mean      | 1.454E+08 | 4.725E+07 | 4.253E+07 | 4.872E+07 | 5.172E+07 | 4.339E+07 |
|                 | Std       | 6.297E+07 | 1.608E+07 | 1.514E+07 | 1.505E+07 | 1.934E+07 | 1.797E+07 |
|                 | Rank      | 6     | 1      | 4        | 5        | 2      | 3      |
| \( f_2 \)       | Mean      | 9.363E+06 | 2.879E+04 | 5.013E+04 | 4.562E+04 | 3.781E+04 | 4.392E+04 |
|                 | Std       | 5.200E+07 | 4.923E+04 | 5.615E+04 | 4.438E+04 | 4.272E+04 | 5.973E+04 |
|                 | Rank      | 6     | 1      | 4        | 5        | 2      | 3      |
| \( f_3 \)       | Mean      | 6.772E+03 | 2.103E+04 | 1.736E+04 | 1.647E+04 | 1.893E+04 | 1.631E+04 |
|                 | Std       | 3.541E+03 | 1.666E+04 | 1.462E+04 | 1.391E+04 | 1.216E+04 | 1.052E+04 |
|                 | Rank      | 6     | 1      | 4        | 5        | 2      | 3      |
| \( f_4 \)       | Mean      | 5.351E+02 | 1.356E+02 | 1.470E+02 | 1.282E+02 | 1.551E+02 | 1.624E+02 |
|                 | Std       | 7.200E+02 | 4.452E+01 | 5.656E+01 | 4.246E+01 | 4.468E+01 | 4.725E+01 |
|                 | Rank      | 6     | 2      | 3        | 4        | 5      | 1      |
| \( f_5 \)       | Mean      | 7.908E+01 | 1.564E+02 | 1.517E+02 | 1.233E+02 | 1.055E+02 | 1.036E+02 |
|                 | Std       | 6.381E+00 | 2.104E+04 | 1.765E+01 | 1.647E+01 | 1.764E+01 | 1.928E+01 |
|                 | Rank      | 1     | 6      | 5        | 4        | 3      | 2      |
| \( f_6 \)       | Mean      | 4.081E-03 | 4.133E-03 | 1.987E-03 | 3.259E-03 | 4.606E-03 | 2.780E-03 |
|                 | Std       | 6.490E-03 | 5.486E-03 | 4.380E-03 | 5.736E-03 | 7.290E-03 | 5.494E-03 |
|                 | Rank      | 1     | 6      | 5        | 4        | 3      | 2      |
| \( f_7 \)       | Mean      | 3.705E+00 | 3.205E+02 | 3.704E+02 | 3.254E+02 | 3.281E+02 | 3.407E+02 |
|                 | Std       | 7.200E+00 | 8.020E+03 | 8.226E+03 | 9.828E+03 | 8.886E+03 | 8.886E+03 |
|                 | Rank      | 1     | 2      | 6        | 3        | 4      | 5      |
| \( f_8 \)       | Mean      | 3.573E+02 | 9.789E+02 | 1.006E+03 | 9.273E+02 | 1.059E+03 | 9.322E+02 |
|                 | Std       | 5.343E+01 | 1.442E+02 | 1.404E+02 | 1.586E+02 | 1.539E+02 | 1.740E+02 |
|                 | Rank      | 1     | 6      | 5        | 4        | 3      | 2      |
| \( f_9 \)       | Mean      | 3.705E+00 | 3.414E+03 | 8.020E+03 | 6.390E+03 | 6.642E+03 | 6.543E+03 |
|                 | Std       | 7.200E+00 | 8.816E+02 | 8.226E+02 | 9.828E+02 | 8.886E+02 | 8.886E+02 |
|                 | Rank      | 1     | 2      | 6        | 3        | 4      | 5      |
| \( f_{10} \)    | Mean      | 3.855E+00 | 3.987E+00 | 4.040E+00 | 4.015E+00 | 3.960E+00 | 3.901E+00 |
|                 | Std       | 3.983E-01 | 2.166E-01 | 1.733E-01 | 2.399E-01 | 4.298E-01 | 6.487E-01 |
|                 | Rank      | 1     | 4      | 6        | 5        | 3      | 2      |
| \( f_{11} \)    | Mean      | 6.861E-01 | 7.117E-01 | 7.211E-01 | 7.375E-01 | 6.858E-01 | 7.059E-01 |
|                 | Std       | 5.123E-02 | 8.652E-02 | 1.013E-01 | 9.950E-02 | 8.404E-02 | 8.266E-02 |
|                 | Rank      | 2     | 4      | 5        | 6        | 1      | 3      |
| \( f_{12} \)    | Mean      | 3.855E-01 | 4.972E-02 | 5.907E-01 | 5.608E-01 | 6.102E-01 | 5.452E-01 |
|                 | Std       | 1.500E-01 | 2.573E-01 | 3.229E-01 | 2.760E-01 | 2.894E-01 | 2.632E-01 |
|                 | Rank      | 1     | 2      | 6        | 5        | 1      | 3      |
| \( f_{13} \)    | Mean      | 6.745E+01 | 6.357E+01 | 7.252E+01 | 5.186E+01 | 6.924E+01 | 6.724E+01 |
|                 | Std       | 1.249E+01 | 1.804E+01 | 1.858E+01 | 1.901E+01 | 1.919E+01 | 2.413E+01 |
|                 | Rank      | 4     | 2      | 6        | 5        | 3      | 2      |

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Table 3. Experimental Results, CE of PSO, PBBSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO for \( f_{16}^{\text{to}} f_{30}^{\text{}} \). Mean is the mean value from 31 independent runs, STD is the standard deviation of the 31 runs, Rank is the rank of 6 algorithms. Average rank point is at the bottom of the table.

| Function Number | Data Type | PSO        | PBBSO      | DA-BBPSO   | DLS-BBPSO  | FBBPSO     | ETBBPSO    |
|-----------------|-----------|------------|------------|------------|------------|------------|------------|
|                 |           | Mean 4.574E+01 | 4.741E+01 | 4.715E+01 | 4.712E+01 | 4.665E+01 | 4.678E+01 |
|                 |           | Std 4.751E-01  | 9.261E-01 | 9.833E-01 | 8.539E-01 | 9.872E-01 | 9.423E-01 |
|                 |           | Rank 1       | 6          | 5          | 4          | 2          | 3          |
| \( f_{16} \)    |           | Mean 1.497E+07 | 9.276E+06 | 7.522E+06 | 8.617E+06 | 9.513E+06 | 7.641E+06 |
|                 |           | Std 8.087E+05 | 2.908E+06 | 3.370E+06 | 4.707E+06 | 6.360E+06 | 3.086E+06 |
|                 |           | Rank 6       | 4          | 1          | 3          | 5          | 2          |
| \( f_{18} \)    |           | Mean 1.679E+02 | 1.088E+02 | 1.080E+02 | 1.135E+02 | 1.072E+02 | 1.065E+02 |
|                 |           | Std 1.780E+05 | 9.621E+03 | 1.303E+04 | 7.654E+03 | 1.132E+04 | 1.284E+04 |
|                 |           | Rank 6       | 2          | 5          | 4          | 1          | 3          |
| \( f_{20} \)    |           | Mean 9.281E+03 | 2.410E+05 | 1.920E+05 | 2.112E+05 | 1.614E+05 | 1.443E+05 |
|                 |           | Std 2.844E+03 | 2.549E+05 | 1.207E+05 | 1.428E+05 | 1.337E+05 | 8.422E+04 |
|                 |           | Rank 1       | 6          | 2          | 5          | 4          | 3          |
| \( f_{22} \)    |           | Mean 6.073E+06 | 4.210E+06 | 4.81E+06  | 5.010E+06 | 5.283E+06 | 4.672E+06 |
|                 |           | Std 3.942E+06 | 1.955E+06 | 2.218E+06 | 2.894E+06 | 2.809E+06 | 2.093E+06 |
|                 |           | Rank 6       | 1          | 2          | 4          | 5          | 3          |
| \( f_{24} \)    |           | Mean 2.157E+03 | 5.133E+03 | 3.902E+03 | 5.345E+03 | 3.721E+03 | 4.04E+03   |
|                 |           | Std 5.585E+02 | 1.452E+03 | 1.231E+03 | 1.376E+03 | 6.787E+02 | 1.194E+03 |
|                 |           | Rank 1       | 5          | 3          | 6          | 2          | 4          |
| \( f_{26} \)    |           | Mean 3.531E+02 | 3.472E+02 | 3.450E+02 | 3.450E+02 | 3.450E+02 | 3.450E+02 |
|                 |           | Std 1.536E+00 | 1.215E+01 | 1.036E+05 | 2.707E+05 | 4.445E-05 | 6.985E-06 |
|                 |           | Rank 6       | 5          | 2          | 3          | 4          | 1          |
| \( f_{28} \)    |           | Mean 3.850E+02 | 3.889E+02 | 3.949E+02 | 3.901E+02 | 3.925E+02 | 3.892E+02 |
|                 |           | Std 4.407E+00 | 5.792E+00 | 7.376E+00 | 4.694E+00 | 6.482E+00 | 7.166E+00 |
|                 |           | Rank 1       | 2          | 6          | 4          | 5          | 3          |
| \( f_{30} \)    |           | Mean 2.807E+02 | 2.046E+02 | 2.046E+02 | 2.045E+02 | 2.048E+02 | 2.043E+02 |
|                 |           | Std 1.254E+01 | 1.043E+00 | 8.726E+01 | 8.597E-01 | 1.138E+00 | 9.194E-01 |
|                 |           | Rank 6       | 4          | 3          | 2          | 5          | 1          |

Average Rank 3.40 3.47 4.00 3.53 3.83 2.77

https://doi.org/10.1371/journal.pone.0271925.t003
Table 4. Parameters of the CEC2020 test.

| Dimension | 20 |
|-----------|----|
| Population size | 100 |
| Max iteration times | 10000 |
| Independent runs | 31 |
| Search Range | [-100,100] |
| Control Group | FBBPSO and BBPSO |

https://doi.org/10.1371/journal.pone.0271925.t004

Fig 2. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_1$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

https://doi.org/10.1371/journal.pone.0271925.g002

Fig 3. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_2$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

https://doi.org/10.1371/journal.pone.0271925.g003
require any parameters. Then, PTO enhances the local search ability of particle swarm while taking into account the global search ability, making it more possible for the particle swarm to escape from the local optimum.

**Experiments with CEC2020**

The standard BBPSO and the ETBBPSO are tested with the CEC2020 benchmark functions. Parameters are shown in Table 4. Experimental results are shown in Table 5. CE is defined as $|\text{GlobalBestValue} - \text{TheoreticallyMinimum}|$. The ETBBPSO scored 4 firsts and 4 seconds in 10 test functions, shown great performance in this experiments.
Fig 6. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_6$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.
https://doi.org/10.1371/journal.pone.0271925.g006

Fig 7. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_7$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.
https://doi.org/10.1371/journal.pone.0271925.g007

Fig 8. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_8$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.
https://doi.org/10.1371/journal.pone.0271925.g008
Fig 9. Comparison of convergence speed between PSO, PBBSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_8$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

https://doi.org/10.1371/journal.pone.0271925.g009

Fig 10. Comparison of convergence speed between PSO, PBBSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_9$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

https://doi.org/10.1371/journal.pone.0271925.g010

Fig 11. Comparison of convergence speed between PSO, PBBSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{10}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

https://doi.org/10.1371/journal.pone.0271925.g011
Fig 12. Comparison of convergence speed between PSO, PB-BPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{11}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

https://doi.org/10.1371/journal.pone.0271925.g012

Fig 13. Comparison of convergence speed between PSO, PB-BPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{12}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

https://doi.org/10.1371/journal.pone.0271925.g013

Fig 14. Comparison of convergence speed between PSO, PB-BPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{13}$, (a) iteration 0-6000, (b) iteration 6000-10000, the unit is 100 iteration.

https://doi.org/10.1371/journal.pone.0271925.g014
Fig 15. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{16}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

https://doi.org/10.1371/journal.pone.0271925.g015

Fig 16. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{15}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

https://doi.org/10.1371/journal.pone.0271925.g016

Fig 17. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{16}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

https://doi.org/10.1371/journal.pone.0271925.g017
Fig 18. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{17}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

https://doi.org/10.1371/journal.pone.0271925.g018

Fig 19. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{10}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

https://doi.org/10.1371/journal.pone.0271925.g019

Fig 20. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{10}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

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Fig 21. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{20}$ (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

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Fig 22. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{21}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

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Fig 23. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{23}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

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Fig 24. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{23}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

https://doi.org/10.1371/journal.pone.0271925.g024

Fig 25. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{24}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

https://doi.org/10.1371/journal.pone.0271925.g025

Fig 26. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{25}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

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Fig 27. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{26}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

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Fig 28. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{27}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

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Fig 29. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{28}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

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Conclusions

In this paper, a novel electronic transition-based bare bones particle swarm optimization (ETBBPSO) algorithm is proposed for high dimensional optimization problems. A dynamic particle grouping (DPG) strategy and a particle transition operator (PTO) collaborate to find the global optimal solution for high dimensional optimization problems. The DPG is mainly used to assign particles to different orbits, with a variable number of orbits and a variable number of particles per orbit. The PTO is used to combine the best and the second-best orbits. Particles that belong to the second-best orbits will transit to the best orbit to enhance the local search ability of the best orbit. A set of comprehensive experiments are designed to verify the optimization ability of ETBBPSO. Several famous population-based methods are used in the control group. Experimental results prove that ETBBPSO is able to present high precision results for high dimensional optimization problems.

Fig 30. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{29}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

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Fig 31. Comparison of convergence speed between PSO, PBBPSO, DA-BBPSO, DLS-BBPSO, FBBPSO and ETBBPSO, $f_{30}$, (a) iteration 0-6000, (b) iteration 6000-10000 the unit is 100 iteration.

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Table 5. Experimental Results with CEC2020, CEs of BBPSO and ETBBPSO. Mean is the mean value from 31 independent runs, STD is the standard deviation of the 31 runs.

| Function Number | Data Type | BBPSO | ETBBPSO |
|-----------------|-----------|-------|---------|
| \( f_1 \)       | Mean      | 3.002E+04 | 1.278E+04 | 1.778E+04 |
|                 | Std       | 3.915E+04 | 2.346E+04 | 3.322E+04 |
|                 | Rank      | 3       | 1       | 2       |
| \( f_2 \)       | Mean      | 5.768E+02 | 6.037E+02 | 5.568E+02 |
|                 | Std       | 2.124E+02 | 2.718E+02 | 2.086E+02 |
|                 | Rank      | 2       | 3       | 1       |
| \( f_3 \)       | Mean      | 4.746E+01 | 4.553E+01 | 4.348E+01 |
|                 | Std       | 9.161E+00 | 1.028E+01 | 1.141E+01 |
|                 | Rank      | 3       | 2       | 1       |
| \( f_4 \)       | Mean      | 2.238E+00 | 2.506E+00 | 2.473E+00 |
|                 | Std       | 1.021E-00 | 9.659E-01 | 8.925E-01 |
|                 | Rank      | 1       | 3       | 2       |
| \( f_5 \)       | Mean      | 9.121E+04 | 8.048E+04 | 7.323E+04 |
|                 | Std       | 8.335E+04 | 7.829E+04 | 7.657E+04 |
|                 | Rank      | 3       | 2       | 1       |
| \( f_6 \)       | Mean      | 1.176E+01 | 2.218E+01 | 2.948E+01 |
|                 | Std       | 1.335E+01 | 3.763E+01 | 4.494E+01 |
|                 | Rank      | 1       | 2       | 3       |
| \( f_7 \)       | Mean      | 4.601E+04 | 3.929E+04 | 4.205E+04 |
|                 | Std       | 3.750E+04 | 2.595E+04 | 4.416E+04 |
|                 | Rank      | 3       | 1       | 2       |
| \( f_8 \)       | Mean      | 1.429E+03 | 1.219E+03 | 8.123E+02 |
|                 | Std       | 1.217E+03 | 1.085E+03 | 1.013E+03 |
|                 | Rank      | 3       | 2       | 1       |
| \( f_9 \)       | Mean      | 4.614E+02 | 4.725E+02 | 4.623E+02 |
|                 | Std       | 2.869E+01 | 2.237E+01 | 2.611E+01 |
|                 | Rank      | 1       | 3       | 2       |
| \( f_{10} \)    | Mean      | 4.363E+02 | 4.317E+02 | 4.387E+02 |
|                 | Std       | 3.185E+01 | 3.263E+01 | 3.014E+01 |
|                 | Rank      | 2       | 1       | 3       |

Average Rank 2.2 2 1.8

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