The equivalence theorem and the Bethe-Salpeter equation

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Abstract

We solve the Bethe-Salpeter equation for two-particle scattering in a field-theoretical model using two lagrangians related by a field transformation. The kernel of the equation consists of the sum of all tree-level diagrams for each lagrangian. The solutions differ even if all four external particles are put on the mass shell, which implies that observables calculated by solving the Bethe-Salpeter equation depend on the representation of the theory. We point out that this violation of the equivalence theorem has a simple explanation and should be expected for any Bethe-Salpeter equation with a tree-level kernel. Implications for dynamical models of hadronic interactions are discussed.

Keywords: Bethe-Salpeter equation, Equivalence theorem, Lagrangian models of hadronic interactions

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I. INTRODUCTION

It has been known for a long time [1] that the same scattering matrix can be obtained using lagrangians related by transformations of interpolating fields. Transformations of this kind were studied in detail in [2]. This independence of the S-matrix on the choice of interpolating fields is called the equivalence theorem. At the root of the equivalence theorem lies the fact that the S-matrix in quantum field theory is defined in terms of free fields whose properties are not changed by the allowed transformations [3].

The relevance of the equivalence theorem to hadronic physics has been reemphasised recently. Utilising lagrangian models for pion-nucleon scattering [4], Compton scattering on the pion [5], pion-pion and nucleon-nucleon bremsstrahlung [6], nucleon-nucleon scattering [7] and 3-body scattering [8], it has been demonstrated that various field transformations relate lagrangians which describe completely different off-shell 3-, 4- and higher-point vertices while leading to identical on-shell scattering amplitudes. Typically, these studies involved
only tree-level or one-loop calculations and dealt with local lagrangians and field transformations. To our knowledge there have been no multi-loop calculations illustrating the equivalence theorem. Yet, many recent dynamical models describing pion-nucleon [9] and photon-nucleon [10] interactions are based on summations of infinite series of loop diagrams. This is usually done by solving the Bethe-Salpeter equation (BSE) [11] or one of its modifications (such as 3-dimensional reductions thereof) with the kernel consisting of a sum of tree-level diagrams.

It is natural to extend the work of [4–6] to such multi-loop approaches and to examine the validity of the equivalence theorem for solutions of the BSE. This is the main objective of the present letter. Due to the complexity of the equation a general study is very difficult, therefore we analyse the problem using an example of scattering of two neutral and spinless particles (represented by scalar fields $\phi$ and $\sigma$). The relative simplicity of this model helps clarify the essential issues of the problem while avoiding many of the technical complications of more realistic approaches. We consider two representations of the model lagrangian which are related through a field transformation. First, we show that the equivalence theorem is satisfied at tree level even though the lagrangian and the field transformation involve form factors, thereby extending conclusions of Refs. [4–6] to such non-local [12] lagrangians and transformations. Next, we solve the BSE for $\phi\sigma$ scattering in the two representations. Following the usual practice, we construct the kernel of the equation as the sum of the tree-level diagrams. We find that the on-shell scattering amplitudes calculated in the two representations are not equal to each other, indicating that the equivalence theorem is not obeyed by the solution of the BSE.

We argue that the principal origin of this representation-dependence is the well-known fact that certain classes of loop graphs are not generated by the BSE with a tree-level kernel. These loop graphs should, however, be included in the full amplitude for which the equivalence theorem is presumed to hold.

**II. TWO REPRESENTATIONS OF A MODEL FIELD THEORY**

We consider a system consisting of two species of spinless neutral particles, described by scalar fields $\phi$ and $\sigma$ whose masses are $M$ and $m$, respectively, and assume that a $\sigma$ couples to two $\phi$’s with a strength $g$. We take the masses close to the nucleon and pion masses, $M = 1000 \, \text{MeV}$, $m = 150 \, \text{MeV}$, and the coupling constant close to the pion-nucleon coupling constant, $g = 13$. The interaction is equipped with a form factor, as is done usually in dynamical models of hadronic interactions [9,10]. The model lagrangian reads\(^1\)

$$\mathcal{L} = \frac{1}{2} (\partial \phi)^2 - \frac{M^2}{2} \phi^2 + \frac{1}{2} (\partial \sigma)^2 - \frac{m^2}{2} \sigma^2 + \frac{g}{2} \left[ H (-\partial^2) \sigma \right] \phi \left[ G (-\partial^2) \phi \right],$$

where $\partial^2 \equiv \partial_{\mu} \partial^{\mu}$. A function of $(-\partial^2)$ can be viewed as a formal series in powers of $(-\partial^2)$. It corresponds to a form factor in momentum space. The 3-point $\phi\phi\sigma$ vertex extracted from lagrangian Eq. (1) has the form

\(^1\)Throughout the calculations, we use the metric and conventions of [13].
\[
\frac{ig}{2} H(q^2) \left[ G(p_1^2) + G(p_2^2) \right],
\]

where \( q, p_1 \) and \( p_2 \) are the 4-momenta of the \( \sigma \), the first and the second \( \phi \)'s, respectively.

The vertex function Eq. (2) defines the structure of the theory in the first representation, which we will call representation (I). Representation (II) is introduced through a transformation of the \( \phi \) field. Similar to lagrangian Eq. (1), we include a form factor \( F \) in the transformation,

\[
\phi \rightarrow \phi + f \phi \left[ F(-\partial^2)\sigma \right], \quad \sigma \rightarrow \sigma,
\]

where \( f \) is a parameter. Thus, Eq. (3) can be regarded as a non-local \( ^{12} \) version of transformations considered in \( ^{2} \). The form factors are normalised so that \( G(M^2) = H(m^2) = F(m^2) = 1 \). In the actual calculation we adopt a traditional functional dependence

\[
G(p^2) = \frac{M^2 - \Lambda_\phi^2}{p^2 - \Lambda_\phi^2}, \quad H(q^2) = \frac{m^2 - \Lambda_\sigma^2}{q^2 - \Lambda_\sigma^2}, \quad F(p^2) = \left( \frac{m^2 - \Lambda_\tau^2}{q^2 - \Lambda_\tau^2} \right)^2,
\]

where \( \Lambda_\phi, \Lambda_\sigma, \Lambda_\tau \) are cut-off parameters and the stronger falloff of \( F \) ensures the convergence of the loop integrals in representation (II).

Under the field transformation Eq. (3) lagrangian Eq. (1) changes as

\[
\mathcal{L} \rightarrow \frac{1}{2}(\partial\phi)^2 - \frac{M^2}{2} \phi^2 + \frac{1}{2}(\partial\sigma)^2 - \frac{m^2}{2} \sigma^2 + \frac{g}{2}[H\sigma]_{\phi}[G\phi] - f[F\sigma]_{\phi} \left[ (\partial^2 + M^2)\phi \right]
+ \frac{f^2}{2} [F\sigma]^2 + \frac{f^2}{2} \phi^2[F\sigma]^2 + \frac{g^2}{2}[F\sigma][\partial F\sigma] - \frac{M^2 f^2}{2} \phi^2[F\sigma]^2
+ \frac{g f}{2}[H\sigma][F\sigma][G\phi] + \frac{g f}{2}[H\sigma][G\phi F\sigma] + \frac{g f^2}{2}[H\sigma][F\sigma][G\phi F\sigma],
\]

where we have omitted a full derivative on the right-hand side and used a shorthand notation in which, e.g., \( [G\phi F\sigma] \equiv \left[ G(-\partial^2)(\phi[F(-\partial^2)\sigma]) \right] \). Lagrangian Eq. (4) contains 3-, 4- and 5-point vertices which define the structure of the theory in representation (II). The 3-point \( \phi\phi\sigma \) vertex reads

\[
\frac{ig}{2} H(q^2) \left[ G(p_1^2) + G(p_2^2) \right] + if F(q^2) \left[ p_1^2 + p_2^2 - 2M^2 \right],
\]

with the same definition of the 4-momenta as in Eq. (2). The 4-point \( \phi\phi\sigma\sigma \) vertex is

\[
if^2 \left[ s + u - 2M^2 \right] F(q^2) F(q^2)
+ \frac{ig}{2} \left[ H(q^2) F(q^2) + H(q^2) F(q^2) \right] \left[ G(p^2) + G(p'^2) + G(s) + G(u) \right],
\]

where the 4-momenta of the incoming (outgoing) \( \sigma \) and \( \phi \) are denoted as \( q \) and \( p \) (\( q' \) and \( p' \)), respectively. The usual Mandelstam variables for \( \sigma\sigma \) scattering are \( s = (p + q)^2 = (p' + q')^2 \), \( u = (p - q')^2 = (p' - q)^2 \). We do not give an explicit expression for the 5-point vertex as it will not be required in the following calculations.
III. THE BETHE-SALPETER EQUATION FOR $\phi\sigma$ SCATTERING

The $\phi\sigma$ scattering amplitude $T(q', p'; q, p)$ can be obtained by solving the integral equation

$$T(q', p'; q, p) = V(q', p'; q, p) + \frac{i}{(2\pi)^4} \int d^4k V(q', p'; k, p + q - k) S_2(p + q - k, k) T(k, p + q - k; q, p),$$

where $V(q', p'; q, p)$ is the kernel (potential) of the equation, $S_2(p + q - k, k)$ is a $\phi\sigma$ propagator and the integration is done over the 4-momentum of an intermediate $\sigma$. The exact scattering amplitude (i.e. the one including all possible 4-point diagrams) obeys Eq. (8) if $S_2$ is the product of fully dressed $\phi$ and $\sigma$ propagators, and $V$ includes all 2-particle irreducible 4-point graphs [1]. In this case, Eq. (8) is sometimes called “the full Bethe-Salpeter equation”. Clearly, obtaining a solution of the full Bethe-Salpeter equation is not feasible as its kernel would contain an infinite number of loop diagrams. In practical calculations the kernel of Eq. (8) is usually chosen as a sum of lowest order diagrams and

$$S_2(p + q - k, k) = D_\phi^{(0)}(p + q - k) D_\sigma^{(0)}(k),$$

where $D_\phi^{(0)}$ and $D_\sigma^{(0)}$ are free propagators with poles at the physical $\phi$ and $\sigma$ masses, respectively. The resulting equation obeys two-body $\phi\sigma$ unitarity and is often referred to as simply “the Bethe-Salpeter equation” (as opposed to the full Bethe-Salpeter equation). We will also adhere to this terminology.

The tree-level amplitude equals the potential in which the “physical” masses and coupling constant are used. In representation (I) it is given by the sum of $s$- and $u$-channel diagrams, $T_{\text{tree}}^{(I)} = T_{\text{tree, s}}^{(I)} + T_{\text{tree, u}}^{(I)}$, where, using Eq. (2),

$$T_{\text{tree, s}}^{(I)}(q', p'; q, p) = \frac{-ig^2}{4(s - M^2)} \left[ G(p'^2) + G(s) \right] H(q'^2) \left[ G(s) + G(p^2) \right] H(q^2),$$

and $T_{\text{tree, u}}^{(I)}(q', p'; q, p)$ can be written by applying the crossing transformation

$$q \leftrightarrow -q' \quad \text{or alternatively} \quad p \leftrightarrow -p' \quad \text{(entailing} \quad s \leftrightarrow u)$$

(11)

to $T_{\text{tree, s}}^{(I)}(q', p'; q, p)$. In addition to $s$- and $u$-channel diagrams, the tree amplitude in representation (II) contains a contact term, $T_{\text{tree}}^{(II)} = T_{\text{tree, s}}^{(II)} + T_{\text{tree, u}}^{(II)} + T_{\text{tree, c}}^{(II)}$, where, using Eqs. (3) and (7),

$$T_{\text{tree, s}}^{(II)}(q', p'; q, p) = \frac{-i}{s - M^2} \left\{ f(p'^2 + s - 2M^2)F(q'^2) + \frac{g}{2}[G(p'^2) + G(s)]H(q'^2) \right\}$$

$$\times \left\{ f(s + p'^2 - 2M^2)F(q'^2) + \frac{g}{2}[G(s) + G(p^2)]H(q^2) \right\},$$

(12)

$T_{\text{tree, u}}^{(II)}(q', p'; q, p) = T_{\text{tree, s}}^{(II)}(-q, p'; -q', p)$ and $T_{\text{tree, c}}^{(II)}(q', p'; q, p)$ is given by Eq. (7). The tree amplitudes in both representations are crossing symmetric, i.e. invariant under the transformation Eq. (11). This is because they comprise all diagrams dictated by the corresponding lagrangians at lowest order, see Fig. (I).
It is straightforward to verify that these lowest-order amplitudes in the two representations coincide if the external particles are put on the mass shell, i.e.,

$$T_{\text{tree}}^{(I)}(q', p'; q, p) = T_{\text{tree}}^{(II)}(q', p'; q, p) \quad \text{if} \quad q^2 = q'^2 = m^2, \quad p^2 = p'^2 = M^2,$$

thus explicitly demonstrating that the equivalence theorem is obeyed at tree level.

The Bethe-Salpeter equation Eq. (8) iterates the tree-level potential \( V \) to all orders. As a result, in the \( s \)-channel pole diagram the \( \phi \) propagator and the \( \phi \phi \sigma \) vertices become dressed whereas the non-pole \( u \)-channel and contact diagrams are not affected [13]. For this reason, in the \( s \)-channel diagram of the potential \( V \) we use a bare mass \( M_0 \) in the \( \phi \) propagator and a bare coupling constant \( g_0 \) to all orders. As a

\begin{align*}
V(q', p'; q, p) = v(q', p'; q, p) + \frac{\Gamma^{(0)}(p', p' + q', q') \Gamma^{(0)}(p + q, p, q)}{s - M_0^2},
\end{align*}

where \( v \) is the \( u \)-channel diagram (or is the sum of the \( u \)-channel and the contact diagrams for the case of representation (II)) and the structure of the \( s \)-channel pole diagram is shown explicitly. The bare \( \phi \phi \sigma \) vertex \( \Gamma^{(0)}(p_1, p_2, q) \) is given by Eqs. (9) or (10) (in representation (I)) or (11) (in representation (II)), with \( g_0 \) used instead of \( g \).

The solution of the BSE has the form

\begin{align*}
T(q', p'; q, p) = t(q', p'; q, p) + \frac{\Gamma(p', p' + q', q') \Gamma(p + q, p, q)}{s - M_0^2 - \Sigma(s)},
\end{align*}

where \( t \) is the solution of the BSE with the non-pole potential \( v \). The dressed \( \phi \phi \sigma \) vertex \( \Gamma \) and the \( \phi \) self-energy \( \Sigma \) can be written in terms of \( t \),

\begin{align*}
\Gamma(p', p, q) &= \Gamma^{(0)}(p', p, q) + \frac{i}{(2\pi)^4} \int d^4 k \Gamma^{(0)}(p + q + k, k) S_2(p + q - k, k) \\
&\quad \times t(k, p + q - k; q, p),
\end{align*}

\begin{align*}
\Sigma(s) &= -\frac{i}{(2\pi)^4} \int d^4 k \Gamma^{(0)}(p + q, p + q - k, k) S_2(p + q - k, k) \Gamma(p + q - k, p + q, k),
\end{align*}

with the \( \phi \sigma \) propagator \( S_2 \) given in Eq. (9).

Renormalisation imposes two requirements on the dressed \( s \)-channel diagram of the on-shell \( T \) matrix: (i) it must have a pole at \( s = M^2 \), and (ii) the residue at this pole must be equal to \( g^2 \). In other words, in terms of Eq. (15) it is required that

\begin{align*}
\lim_{s \to M^2} \frac{\Gamma(p', p' + q', q') \Gamma(p + q, p, q)}{s - M_0^2 - \Sigma(s)} \bigg|_{q^2 = q'^2 = m^2, \ p^2 = p'^2 = M^2} = \frac{g^2}{s - M^2}.
\end{align*}

These two conditions are satisfied by appropriately choosing the bare mass \( M_0 \) and the bare coupling constant \( g_0 \). The particular procedure used to fix \( M_0 \) and \( g_0 \) is immaterial, as long as renormalisation conditions Eq. (18) are fulfilled (for example, one can follow the standard renormalisation procedure described in [13]). The renormalisation conditions Eq. (18) are the same in both representations as they involve the representation-independent physical parameters \( M \) and \( g \).
IV. SOLUTION OF THE BSE IN THE TWO REPRESENTATIONS

We solve the BSE for $\phi\sigma$ scattering in the two introduced representations. Through solving the BSE one sums up a certain class of loop diagrams up to infinite order. A question arises whether this class of loops is sufficient for the solution to obey the equivalence theorem. As we shall see, it is not the case. It is known that iterating the potential according to the BSE does not generate certain loop diagrams which should be included in the full theory. Up to one-loop level, this is illustrated in Figs. (1) and (2). At tree level, shown in Fig. (1), the scattering amplitude $T_{\text{tree}}$ contains all the diagrams dictated by the lagrangians in both representations. As a consequence, the choice of representation does not affect the on-shell tree amplitude. At one-loop level, however, the BSE generates only those diagrams which are shown in column A of Fig. (2) for representation (I) and in columns A and C for representation (II). We note that the diagrams not generated in representation (I) would render the one-loop amplitude crossing symmetric. The set of diagrams missing from the BSE in representation (II) is larger: in addition to the graphs required by the crossing symmetry, also the loop correction to the 4-point vertex and the diagrams formed from the 5-point vertex are not generated.

In Figs. (3) and (4) we compare the S-wave phase shifts for $\phi\sigma$ scattering obtained in representations (I) and (II). To ensure that the lagrangians Eq. (1) and Eq. (5) are indeed related by the field transformation Eq. (3), in both representations we kept the same coupling constant $g$ and the same cut-off parameters $\Lambda_\phi$ and $\Lambda_\sigma$. In the calculations shown we chose the same values for all the cut-offs, $\Lambda_T = \Lambda_\phi = \Lambda_\sigma = 2$ GeV, and the transformation parameter $f$ was varied between 0 and 0.13. The phase shifts obtained in representations (I) and (II) are denoted as $\delta^{(I)}$ and $\delta^{(II)}$. The extent of the representation-dependence of the phase shift is related to the difference $\delta^{(II)} - \delta^{(I)}$. In Fig. (3) we show the results obtained using two different values of $f$. With decreasing energy the representation-dependence of the phase shift gets smaller as the loop contributions become less important. The difference between the amplitudes in the two representations is appreciable even at low energies, as can be seen by comparing the scattering lengths $a^{(I)}$ and $a^{(II)}$ calculated in the two representations. We have checked that it is impossible to find transformation parameters $\Lambda_T$ and $f$ (except for the trivial case $f = 0$) such that the phase shifts are the same in the two representations. The dependence of the difference between the phase shifts in the two representations on the transformation parameter $f$ is shown in Fig. (4) for three different values of the scattering energy.

From the above considerations it follows that the observables which are dominated by the tree-level diagrams should not exhibit a noticeable dependence on the choice of representation in which the BSE is solved. In the present calculation this is exemplified by the P-wave $\sigma\phi$ phase shift (not shown), which is given almost exclusively by the tree-level kernel and is therefore not sensitive to the choice of representation.

One frequently used approximation to the Bethe-Salpeter equation is the K-matrix approach (see, e.g., recent models [13]). Its essential simplifying feature is that the driving

\[ \text{Note that if this amplitude were not symmetric under the crossing transformation Eq. (1), the equivalence theorem would be violated even at tree level.} \]
term, the K-matrix, is needed only with on-shell external particles and the scattering amplitude is obtained by algebraically iterating $K$. The K matrix is traditionally chosen to be equal to a crossing symmetric on-shell tree-level amplitude. Therefore, one should expect that the equivalence theorem holds for the amplitude obtained in the K-matrix approach. We verified that this is indeed the case.

In the calculations presented so far, the regularisation form factors $G$, $H$ and $F$ were incorporated in the lagrangian and in the transformation. In order to check that the reached conclusions are not an artefact of this regularisation procedure, we have solved a BSE for $\phi\sigma$ scattering using the local analogues of the lagrangian Eq. (1) and transformation Eq. (3). The kernel of the equation is obtained in both representations by substituting unity for the form factors $G$, $H$ and $F$. In this calculation, we chose a simple and pragmatic way to regularise the BSE: the measure in Eq. (8) was covariantly deformed as $d^4k \rightarrow d^4k \Lambda_M^4/(k^2 - \Lambda_M^2)^2$, where $\Lambda_M$ is a cut-off mass. The phase shift calculated using this model exhibits a representation-dependence which is qualitatively similar to the one discussed above for the explicitly non-local lagrangians and transformations.

V. CONCLUSIONS

Even though we presented our argument using a particular model, the preceding discussion suggests that the main conclusion of this study – that the on-shell scattering amplitude calculated from the Bethe-Salpeter equation is representation-dependent – has a rather wide validity. Indeed, the BSE does not generate the full set of loop diagrams adequate for the fulfillment of the equivalence theorem. In particular, this certainly is the case for all realistic dynamical models of hadronic interactions based on the BSE. The choice of a convenient representation for constructing a tree-level kernel influences the solution of the BSE even on-shell, and thus should be recognised as an additional model assumption. Our results should, however, not be construed as a claim that the equivalence theorem would not hold in general, i.e. if one were able to calculate the amplitude including all the loop diagrams required by the lagrangian up to infinite order. Formally, such a calculation could be furnished as a solution of the full Bethe-Salpeter equation which contains an infinite number of diagrams in its kernel. The situation we have studied, using the BSE with a tree-level kernel, is more closely related to the type of calculations which are actually carried out.

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FIGURES

FIG. 1. The set of diagrams included in the tree amplitude $T_{\text{tree}}$. The solid and dashed lines denote $\phi$'s and $\sigma$'s, respectively. The contact term is present only in representation (II). Note that the 3-point vertices are different in the two representations, see Eqs. (2) and (6).

FIG. 2. Columns A and B: the one-loop graphs which follow from the lagrangian in representation (I). Columns A, B, C and D: the one-loop graphs which follow from the lagrangian in representation (II). The BSE in representation (I) (representation (II)) generates only the diagrams in column A (columns A and C). Note that the 3-point vertices are different in the two representations, see Eqs. (2) and (6).
FIG. 3. Comparison of the S-wave phase shifts for $\phi \sigma$ scattering obtained from the BSE in representations (I) and (II) for two different values of the transformation parameter $f$. The scattering lengths are given in the units of inverse $\sigma$ mass. The notation is explained in the text.

FIG. 4. The difference between the $\phi \sigma$ S-wave phase shifts obtained in the two representations as a function of the transformation parameter $f$, shown for different values of the $\sigma$ laboratory energy.
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