Adaptive observer of magnetic flux, angular position and velocity for synchronous motor with permanent magnets

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Abstract. An adaptive magnetic flux and position observer has been designed for a permanent magnet synchronous motor. The measurable signals are stator winding currents and voltages only. The observer provides observation error convergence to zero in finite time for measurements without noise. The design method is iteratively applied to obtain the rotor velocity and load torque estimates.

Introduction

The wide application of permanent magnet synchronous motors (PMSM) in aviation and automotive industry, biomedical and household equipment, robotic systems, especially in control systems with a wide speed range and high rate of starts, stops, and reverse [1], has led to an increasing interest of researchers in the related control problems. One of the actively developing fields is sensorless control based on the estimates of the PMSM rotor position and velocity using the measurements of the stator winding currents and voltages only [2], wherein the key element is fast and accurate estimation of the current rotor position and velocity.

In the observer for the nonlinear PMSM model proposed in [3], the phase-locked loop method is used to estimate the rotor angular velocity. This method has a local convergence of the estimation error to zero, and it is hard to show the stability of the closed-loop system. This paper presents the adaptive to external load observer with the global convergence of the estimation errors to zero in finite time. The observer is based on the method proposed in [4] and constructed in two steps. In the first step, an algorithm for the magnetic flux and position estimation is designed. In the second step, the observer for the rotor angular velocity and load torque is obtained using the position estimate.

1. Problem formulation.

The classical, two-phase αβ model of the unsaturated, nonsalient PMSM is considered [5]:

\[\begin{align*}
\dot{\theta} &= \omega, \\
J \dot{\omega} &= n_p i^T \lambda - \tau_L, \\
\lambda &= u - Ri, \\
Li &= \lambda - \lambda_m \begin{bmatrix}
\cos(n_p \theta) \\
\sin(n_p \theta)
\end{bmatrix}
\end{align*}\]  

(1)
where $\lambda \in \mathbb{R}^2$ is the magnetic flux, $i \in \mathbb{R}^2$ is the current vector, $u \in \mathbb{R}^2$ is the voltage vector, $L, R \in \mathbb{R}$ are the inductance and resistance of the stator windings, respectively, $J > 0$ is the rotor moment of inertia, $\theta \in [0, 2\pi]$ is the angular position of the rotor, $\omega \in \mathbb{R}$ is the mechanical angular velocity, $\tau \in \mathbb{R}$ is the load torque, $n_p \in \mathbb{N}_+$ is the number of pole pairs, $\lambda_m > 0$ is the magnetic flux from permanent magnets, $j = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

The only measured signals are the stator winding currents and voltages. The motor parameters, except the number of pole pairs, the stator windings resistance and inductance, are considered unknown. The external load is assumed to be constant. The friction is not taken into account.

The goal is to design an observer that uses the current and voltage measurements and provides estimates of the magnetic flux, angular position, velocity, and load torque in finite time, such that for time $t \geq t_1$ the following conditions hold:

\[
\begin{align*}
|\hat{\lambda}(t) - \lambda(t)| &= 0, \\
|\hat{\theta}(t) - \theta(t)| &= 0, \\
|\hat{\omega}(t) - \omega(t)| &= 0, \\
|\hat{\tau}_L(t) - \tau(t)| &= 0,
\end{align*}
\]

where $\hat{\lambda}(t), \hat{\theta}(t), \hat{\tau}_L(t)$ and $\hat{\omega}(t)$ are estimates of magnetic flux, position, load torque and rotor angular velocity, respectively.

2. The magnetic flux and position observer.

Let us introduce the mapping $\phi(x)$ of the form

\[
\chi := \phi(x) = \begin{bmatrix} \cos(n_p x) \\ \sin(n_p x) \end{bmatrix},
\]

for which we have the inverse mapping

\[
x = \phi^L(\chi) = \frac{1}{n_p} \arctan \frac{X_1}{X_2}.
\]

The derivative of (3) is computed as follows

\[
\dot{\chi} = u - Ri - L\dot{i}.
\]

Using a linear filter of the form $\frac{\alpha}{p + \alpha}$, where $p := \frac{d}{dt}$ and $\alpha > 0$, we obtain the filtered versions of the measured signals: $i_f = \frac{\alpha}{p + \alpha}[i], u_f = \frac{\alpha}{p + \alpha}[u]$.

Lemma 1. The system of equations (3) and (5) verifies linear regression equation

\[
\phi^\top(t, \alpha) \chi = c(t, \alpha)
\]

with the computable, parameterized signals

\[
\phi(t, \alpha) = u_f(t, \alpha) - Ri_f(t, \alpha) - L\dot{i}_f(t, \alpha),
\]

\[
c(t, \alpha) = \zeta_f(t, \alpha) - \frac{L}{\alpha} \phi^\top(t, \alpha) \dot{i}_f(t, \alpha) - \frac{L^2}{2\alpha} \dot{i}_f(t, \alpha)^2,
\]
where

where

\[ \zeta(t) := \frac{1}{\alpha} \left[ u(t) - Ri(t) \right] \varphi(t, \alpha) + \frac{L}{\alpha^2} \left[ \dot{u}_f(t, \alpha) + \left( \frac{\alpha L}{2} - R \right) \dot{i}_f(t, \alpha) \right] \dot{i}_f(t, \alpha). \]

**Remark.** The proof of **Lemma 1** is given in [4].

Following Dynamic Regressor Extension and Mixing (DREM) method [4] we apply two linear filters with transfer function \( W_i(s) = \frac{\alpha_i}{s+\alpha_i} \) and parameters \( \alpha_i > 0, i = 1,2 \), to the regression presented in **Lemma 1** and obtain the extended regression model:

\[ C(t) = \Phi(t) \chi(t), \]

where

\[ C(t) := \begin{bmatrix} c(t, \alpha_1) \\ c(t, \alpha_2) \end{bmatrix}, \quad \Phi(t) := \begin{bmatrix} \varphi(t, \alpha_1) \\ \varphi(t, \alpha_2) \end{bmatrix}. \]

In the next “mixing” step of the DREM method, we decompose the original linear regression model (6) into a system of scalar equations:

\[ Y_x(t) = \Delta(t) \chi(t), \]

where

\[ Y_x(t) := \text{adj} \{ \Phi(t) \} C(t), \quad \Delta(t) := \chi \det \{ \Phi(t) \}. \]

From the expressions (4), (5) and (7), one can obtain the magnetic flux observer equation:

\[ \hat{\lambda}(t) = u - Ri + \gamma \Delta \left[ Y_x - (\lambda(t) - Li) \Delta \right], \]

where \( \gamma > 0 \) is the adaptation gain, \( \lambda(t) \) is the magnetic flux estimate.

Applying the modification described in [6] gives the finite time magnetic flux estimate:

\[ \hat{\lambda}(t) = \frac{1}{1-w_\lambda} (\lambda - \lambda(0)) w_\lambda - \int_0^t (Ri - u + L \frac{di}{dt}) d\tau, \]

where the function \( w_\lambda \) is calculated as follows

\[ w_\lambda = \begin{cases} \rho_\lambda, w_1 \geq \rho_\lambda \\ w_1, w_1 < \rho_\lambda \end{cases}, \]

where \( \rho_\lambda \in (0,1), \quad w_1 = -\gamma \Delta^2 \tilde{w}_1 \) is a computable signal with initial conditions \( w_1(0) = 1 \). The described modification ensures the global convergence of the estimation error to zero in finite time if the following condition holds:

\[ \text{exists } t > 0 : \int_0^t \Delta^2 d\tau \geq -\frac{1}{\gamma_s} \ln(\rho_\lambda). \]

Using the estimate of the magnetic flux (9), we recover the rotor position also in finite time:

\[ \hat{\theta}(t) = \hat{\lambda}(t) - Li, \]

\[ \hat{\theta}(t) = \frac{1}{n_p} \arctan \frac{\hat{\lambda}_2(t)}{\hat{\lambda}_1(t)}. \]

3. **The angular velocity and load torque observer.**

We now iteratively apply same method to design the observer of the rotor velocity and load torque using the position estimate obtained in the previous step (11). From (1) we derive the system of equations:
\[ L \frac{di}{dt} + Ri - u = -\lambda_m n_p jC \omega, \]
\[ \dot{\omega} = -i^T d(t) - \frac{1}{J} \tau_L, \]  
(12)

where \( C = \begin{bmatrix} \cos n_p \hat{\theta} \\ \sin n_p \hat{\theta} \end{bmatrix} \) and \( d(t) = -\lambda_m n_p jC \).

Applying a linear filter of the form \( \frac{\alpha}{p + \alpha} \) and swapping lemma [7] to (12) yields
\[ z = Q \begin{bmatrix} \tau_L \\ \omega \end{bmatrix}, \]
(13)

where \( Q = [q_1 \ q_2] \),
\[ z = L \frac{\alpha p}{p + \alpha} i + Ri_f - u_f - \frac{1}{p + \alpha} \left[ \frac{i^T d(t)}{J} \frac{\alpha}{p + \alpha} d(t) \right], \]
\[ q_1 = \frac{1}{J} \frac{1}{p + \alpha} \left[ \frac{\alpha}{p + \alpha} d(t) \right], \]
\[ q_2 = \frac{\alpha}{p + \alpha} d(t). \]

Using DREM method to (13), we derive scalar regression and construct the load torque and velocity estimates:
\[ \dot{\tau}_L = \gamma_z \Delta q \begin{bmatrix} Z_1 - \tau_L \Delta q \end{bmatrix} \]
\[ \dot{\omega} = -i^T d(t) - \frac{1}{J} \tau_L + \gamma_m q^T (Z_2 - \omega \Delta q), \]  
(14)

where \( Z = \text{adj}(Q) z, \Delta_q = \text{det}(Q), \gamma_m, \gamma_z > 0 \) are adaptation gains, \( \dot{\omega} \) and \( \tau_L \) are the estimates of the velocity and load torque, respectively.

In the next step, we apply the modification [6] similarly to (9) to estimate the load torque in finite time:
\[ \dot{\tau}_L(t) = \frac{1}{1 - w_{\tau_z}} (\tau_L - \tau_L(0) w_{\tau_z}), \]  
(15)

where
\[ w_{\tau_z} = \begin{cases} \rho_{\tau_z}, & w_3 \geq \rho_{\tau_z} \\ w_3, & w_3 < \rho_{\tau_z} \end{cases}, \]

where \( \rho_{\tau_z} \in (0,1), \dot{w}_3 = -\gamma_z \Delta q \dot{w}_3 \) is a computable signal, initial condition is \( w_3(0) = 1 \).

In the last step, combining (14), (15) and the modification [6], we obtain the finite time velocity estimate:
\[ \dot{\omega}(t) = \frac{1}{1 - w_{\omega}} (\omega - \omega(0) w_{\omega} - w_{\omega} \int_0^t -i d(\tau) - \dot{\tau}_L d \tau), \]  
(16)

where
where \( \rho_{w} \in (0,1) \), \( \dot{w}_4 = -\gamma_{\omega} \mathbf{A}_{\mathbf{Q}}^{T} \mathbf{w}_4 \) is calculated with initial conditions \( w_4(0) = 1 \).

One can show that similar to condition (10) for algorithm (9), there are \( t_{\tau_4}, t_{\lambda} \) after which the estimation error of the angular velocity and load torque is equal to zero in the absence of measurement noise. Hence \( \exists t_i : t_i \geq t_4, t_i \geq t_{\tau_4}, t_i \geq t_{\omega}, t_i \geq t_\omega, t_i \geq t_\theta \), and the desired condition (2) is satisfied.

**Conclusions**

The adaptive observer of magnetic flux, angular position, and velocity for a synchronous motor with permanent magnets has been constructed. The estimation errors converge to zero in finite time in the absence of measurement noise. The observer is constructed by iterative application of the method [4]. The disadvantages of the observer are the assumption about the constant load torque and the neglect of friction.

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