Seepage equation and relative resistance scaling law considering high Reynolds number flow in porous media

Junwen Liu *, Xiaohong Wang and Min Wang

Department of Thermal Science and Energy Engineering, University of Science and Technology of China, Hefei, 230026, China

*Corresponding author e-mail: 596951616@qq.com

Abstract. Based on the volume averaging method, the local linearized macroscopic flow equation in porous media with inertia effect is derived, and the solution of Navier-Stokes equation with large Re can be obtained recursively, thus avoiding the problems of high computational cost and poor computational stability caused by solving Navier-Stokes equation directly. An example of a square periodic array model shows that the average velocity direction is not always consistent with the direction of the macroscopic pressure gradient.

1. Introduction

Darcy's law is the most extensive empirical formula for calculating the flow in porous media. For single-phase incompressible Newtonian fluids, when Re approaches zero, the Darcy's law can be derived from Stokes equation. However, when Re becomes larger, the seepage relation will change from the linear Darcy law to the nonlinear form [2]. The physical mechanism of the nonlinear modification of Darcy's law has not been fully clarified, which may be affected by the curvature of the flow field [3-6], the separation of flow, and the narrowing of the flow channel [4,7]. Forchheimer equation is based on Darcy's law to add a quadratic correction term to consider the nonlinear characteristics of seepage flow [2]. Like Darcy's law, the Forchheimer equation is obtained from a large number of experimental data.

Although the Forchheimer equation is widely used in the fields of petroleum, geology and chemical engineering, its theoretical explanation is still controversial. In the early 1990s, C. Mei et al. used the spatial homogenization method to develop the perturbation expansion series for both velocity and pressure. The theory shows that for isotropic and periodically arranged structures, the first-order revision of Darcy's law has a cubic relationship with Re. That is to say, the seepage scheme with inertia term can be written as follows, and a large number of numerical results also confirm this form. However, Whitaker S. used large-scale averaging theory to study the Navier-Stokes equation at pore scale.

Unlike a large number of experimental results, the numerical simulation results of porous media flow do not show a simple and obvious Forchheimer equation form. On the pore scale, the results of the seepage model with fixed pressure gradient direction show that with the increase of Re, the nonlinear seepage can be divided into three stages: 1. weak inertia stage; 2. strong inertia stage; 3. self-similar stage. Firstly, when Re < 1, the revision term of Darcy's law has a cubic relation with Re, which can be called weak inertia stage.

In summary, although there are a lot of theoretical and numerical studies on the flow in porous media with inertia effect, there is no definite conclusion on the nonlinear characteristics of the seepage equation.
For the case of fixed pressure gradient in the existing numerical examples, the obvious variation law of the nonlinear term of resistance can not be obtained. In this paper, the local linearization form of nonlinear macroscopic seepage equation is deduced based on the volume average method, and the local expansion algorithm is proposed to calculate the relevant parameters. The seepage relation of the square periodic array model under the condition of fixed average velocity direction is calculated, and the variation of pressure drop resistance and friction resistance with Re is studied respectively. The scaling law of pressure difference resistance varies with Re is obtained.

2. Flow model in porous media with inertial terms

The dimensionless dimension can be defined by choosing the correlation dimension of porous media structure at pore scale as the characteristic length $L$.

$$
\frac{r^*}{L}, \frac{A^*}{L^2}, \frac{v^*_p}{L}, \frac{p^*_p}{L^2}, \frac{\mu^*}{L^2}
$$

$A$ represents the area, $A$ represents the density, viscosity, velocity, and pressure of the phase fluid, respectively. The superscript * represents the dimensionless quantity of the relevant physical quantities. The incompressible steady state problem of single phase flow in porous media can be described as:

$$
\nabla \cdot \mathbf{v} + \nabla \cdot (-\mathbf{p}^*_p + \nabla \mathbf{v}^*_p) = 0
$$

(2)

$$
\nabla \cdot \mathbf{v}_p^* = 0
$$

(3)

(B.C.1) $v^*_p = 0$, at $A^*_{\partial A}$

(4)

(B.C.2) $v^*_p = f(r^*)$, at $A^*_{\partial e}$

(5)

It represents the interface of the solid and the boundary. The boundary condition (2.c) is no slip condition for the fluid solid boundary. The speed and pressure are decomposed as follows: [1]:

$$
\mathbf{v}_p^* = \langle \mathbf{v}_p^* \rangle + \mathbf{v}_p^* \\
\mathbf{p}_p^* = \langle \mathbf{p}_p^* \rangle + \mathbf{p}_p^*
$$

(6)

(7)

Among them, the average volume of velocity and pressure, and the perturbation velocity and pressure, respectively. After the volume averaged of the above equations (2) was obtained by Whitaker S., the macroscopic equation of percolation equation was obtained.

$$
-\nabla \cdot \langle \mathbf{p}_p^* \rangle = \frac{1}{V^*_p} \int_{A^*_{\partial e}} \mathbf{n} \cdot \left( -\mathbf{v}_p^* + \nabla \mathbf{v}_p^* \right) dA^*
$$

(8)

From the force balance of steady flow, the magnitude of the macroscopic pressure gradient should be the same as the dimensionless resistance $F$ of a cylinder of unit length. Moreover, (4) indicates that the macroscopic pressure gradient can be made up of two parts: pressure difference force and friction force. For convenience of study, the macroscopic pressure gradient is decomposed into parallel resistance and vertical lift in the direction of velocity. Therefore, the dimensionless pressure drag and frictional resistance are:
Among them, it is the unit vector of mean velocity direction. In fact, the angle between the macroscopic pressure gradient and the average velocity is usually small, and the lift is usually much smaller than the resistance, so the lift is not discussed in this paper.

The equation (2) is replaced by equation (3), and the equations of disturbance speed and disturbance pressure can be obtained: [11, 12]:

\[
\left( v_{j}^{*} + \bar{v}_{j}^{*} \right) \bar{v}_{i,j}^{*} = -\bar{p}_{j}^{*} + \bar{v}_{i,j}^{*} - \frac{1}{V^{\beta}} \int_{\sigma_{i,j}} n_{ij} \left( -I_{ij} \bar{p}^{*} + \bar{v}_{i,j}^{*} \right) dA^{*}
\]

\[\tilde{v}_{i,j}^{*} = 0, \text{ in the } \beta\text{-phase} \] (12)

(B.C.1) \[\tilde{v}_{j}^{*} = -\tilde{v}_{i}^{*}, \text{ at } A_{l}^{\beta} \] (13)

(B.C.2) \[\tilde{v}_{i}^{*}(r^{*} + l_{i}^{*}) = \tilde{v}_{i}^{*}(r^{*}), \quad \bar{p}^{*}(r^{*} + l_{i}^{*}) = \bar{p}^{*}(r^{*}), \quad n=1,2,3 \] (14)

Average: \[
\left\langle \tilde{v}_{i}^{*} \right\rangle^{\beta} = 0
\] (15)

The relevant dimensionless resistance can be calculated after obtaining the solution of the equations.

3. Numerical test

Considering porous media with periodic distribution in two-dimensional space, a periodic element is shown in Fig. 1. The edge length of the element is a central square of the solid phase, and the edge length is taken as the porosity in this paper. The area is subdivided into grids.

![Fig.1 The unit cell with a solid square arranged in the center of the square](image)

Here, the characteristic length of Re is taken as the D of the square of a solid square, that is, the Reynolds number is defined as

\[
Re = \frac{\rho_{s} \left\langle |v_{s}| \right\rangle d}{\mu_{\beta}}
\]

(16)
Figure 2 shows the relationship with Re. Fig. 3.a) shows the variation of Re with fixed and different conditions. The results show that the results of this paper agree with those of Lasseux D. et al.. At that time, relatively small, relative to the linear part can be ignored, the seepage law basically satisfies Darcy's law; with the increase from 0 to 45 degrees, the growth rate of Re increases. For the case, Lasseux D. et al. divides the seepage problem with inertia effect into three regions according to the relation with Re: (1) when the seepage problem with inertia effect is of cubic relation with Re, it can be called weak inertia region; (2) when the seepage problem with Re is of cubic relation, it can be called transitional region; (3) when the seepage problem with Re is of cubic relation.

![Figure 2](image)

**Fig 2.** Variation of $f_c / f^0$ with Re, where a) $\theta_p$ is given (For comparison, Lasseux et al.'s numerical results are also shown, and b) $\theta_s$ is given.

4. Conclusion
In this paper, the relationship between flow resistance and Re in porous media is studied considering inertia effect. Firstly, the solution of Navier-Stokes equation with large Re can be obtained by local linearization of macroscopic flow equation, which avoids the problems of high computational cost and poor computational stability caused by solving Navier-Stokes equation directly. Then, for the square periodic alignment model, the resistance variation law under the condition of fixed average velocity direction and macroscopic pressure gradient direction is compared. At the same time, the variation law of pressure resistance and friction resistance with Re is studied under fixed conditions.

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