Buoyancy and the Penrose process produce jets from rotating black holes

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Received 10 June 2013, revised 21 October 2013
Accepted for publication 10 February 2014
Published 11 March 2014

Abstract

The exact mechanism by which astrophysical jets are formed is still unknown. It is believed that the necessary elements consist of a rotating (Kerr) black hole and a magnetized accreting plasma. We model the accreting plasma as a collection of magnetic flux tubes/strings. If such a tube falls into a Kerr black hole, then the leading portion loses angular momentum and energy as the string brakes. To compensate for this loss, momentum and energy is redistributed to the trailing portion of the tube. We found that buoyancy creates a pronounced helical magnetic field structure aligned with the spin axis. Along the field lines, the plasma is centrifugally accelerated close to the speed of light. This process leads to unlimited stretching of the flux tube since one part of the tube continues to fall into the black hole and, simultaneously, the other part of the string is pushed outward. Eventually, reconnection cuts the tube. The inner part is filled with new material and the outer part forms a collimated bubble-structured relativistic jet. Each plasmoid can be considered as an outgoing particle in the Penrose mechanism: it carries extracted rotational energy away from the black hole while the falling part, with corresponding negative energy, is left inside the ergosphere.

Keywords: active galaxies, jets, quasars

(Some figures may appear in colour only in the online journal)

Online supplementary data available from stacks.iop.org/PhysScr/89/045003/mmedia

1. Introduction

Astrophysical jet streams can be found all over the universe and are manifestations of violent energetic outbursts from massive cosmic objects. These jets are a mixture of superheated, low density gas, extremely energetic particles and magnetic fields which are ejected from the pole area in the form of narrow columns of gas saturated with elementary particles. On the intergalactic scale, these powerful emissions originate in the core of active galactic nuclei, including exotic astrophysical objects such as quasars and blazars [1–3].

In recent years, analytical work [2, 4–6] and numerical simulations [7–10] reveal the ingredients which are necessary to form a jet: plasma accretion, a magnetic field and a black hole. To underline the physics, it is interesting to look at the interaction of the magnetic field with the accreting matter drawn towards the black hole and its role in the jet formation process. This interaction will be most effective in the presence of a rotating (Kerr) black hole [11]. Two different models of energy extraction from a rotating black hole are often discussed in the literature. On one hand is the Penrose mechanism, where inside the ergosphere a particle splits into two particles. One possibly escapes to infinity with higher mass-energy than the original infalling particle, whilst the other particle falls with negative mass-energy past the event horizon into the hole [12, 13]. This mechanism is based on the frame-dragging of rotating bodies as first derived within general relativity by Lense and Thirring [14] in 1918. On the other hand is the Blandford–Znajek [4] mechanism, where the accreted material has a magnetic field threaded through as it falls into the rotating black hole. Again, frame-dragging coils and twists the magnetic field like a rope. The associated huge electric currents will eventually transfer their energy into the plasma, which is blown away from the black hole in the form of jets [5, 6].

Because of frame-dragging there exists a region around the Kerr black hole called the ergosphere. The outer surface is the static limit boundary. The energy of a particle can assume
both positive and negative values inside the ergosphere, as measured by an observer at infinity [11]. If a particle with zero angular momentum falls into the black hole, an external observer will see the particle start to corotate together with the black hole. The deeper the particle falls into the black hole, the faster it will be seen to rotate from the external observer. Positive and negative angular momentum for a particular particle can be defined if it rotates faster or slower than the zero angular particle as it is seen from the external observer. The rotation induced by frame-dragging tends to zero at infinity and takes a maximal value at the event horizon, so it constitutes a strong differential rotation.

A magnetized plasma can be modelled as a gas-like collection of magnetic flux tubes, each of them behaving as a nonlinear string. Therefore, one can simulate the motion of a flux tube (two-dimensional problem) instead of solving the complete set of magnetohydrodynamics (MHD) equations numerically (four-dimensional problem). This approach has been successfully applied to the solar cycle model [15], the magnetic barrier (or depletion layer) at the dayside magnetopause [16, 17] and magnetic reconnection in the magnetotail [18, 19]. This method has also been used in the problem of relativistic string—Kerr black hole interaction. In [24] the general problem was stated, and the first numerical simulation showed the possibility of energy extraction with the help of a flux tube from a rotating black hole [23]. Finally, an improved numerical scheme was able to simulate the appearance of the jet [25]. In the present paper, the important role of buoyancy and magnetic reconnection is discussed in the process of jet formation as well as the propagation of plasmoids outwards from the black hole.

2. Thin flux tube approximation

In classical ideal MHD, the magnetic field is frozen into the flow, i.e. magnetic flux tubes move together with the flow as sketched in figure 1. If a flux tube connects two elements of plasma, it will do so during the evolution of the plasma flow [20]. From a mathematical point of view, this means that the Lie derivative of the two vector fields (namely the ratio of magnetic field over plasma density $B/\rho$ and plasma velocity $v$) vanishes [18]. One can use this set of vector fields as basis vectors for a coordinate system where the fluid trajectories (parametrized by $\tau$) and the magnetic field lines (parametrized by $\alpha$) serve as coordinate lines [11]. In those coordinates the convective derivative and the magnetic stress term in the momentum equation become simple

$$\frac{\partial^2 r}{\partial \tau^2} - \frac{1}{4\pi} \frac{\partial}{\partial \alpha} \left( \frac{\partial (\rho r)}{\partial \alpha} \right) = -\frac{1}{\rho} \nabla P(r) - \nabla \Phi.$$  \hspace{1cm} (1)

Here, $r(\tau, \alpha)$ is the position of a point on the flux tube, $\rho$ is the plasma density, $P$ is the total pressure (gas plus magnetic) and $\Phi$ is the gravitational potential. Once the function $r(\tau, \alpha)$ is found, the velocity field and magnetic field are obtained from

$$\frac{\partial r}{\partial \tau} = v, \quad \frac{\partial r}{\partial \alpha} = B/\rho.$$  \hspace{1cm} (2)

If one considers the right hand side of equation (1), i.e. the distribution of the total pressure and the gravitational potential as given functions, this equation is similar to the equation of the nonlinear elastic string. It describes the dynamics of a massive infinitely thin flux tube embedded in plasma. Note that the plasma density can be found from the following nonlinear algebraic equation which is just a definition of the total pressure:

$$P(r) = p(\rho) + \frac{\rho^2}{8\pi} \left( \frac{\partial r}{\partial \alpha} \right)^2,$$  \hspace{1cm} (3)

where $p(\rho)$ is the equation of state, for example an adiabatic law.

Equation set (1) governs plasma inertia, magnetic tension forces, the redistribution of energy and buoyancy effects. It is a hyperbolic system which describes slow and Alfènic MHD waves in a given total pressure gradient distribution, i.e. the accumulation and relaxation of Maxwellian stresses. By specifying the total pressure, the fast waves dynamic is not self-consistently described any more. But the essential physics is still kept as the remaining waves are able to redistribute angular momentum and energy along the massive flux tube which, by itself, conserves the line integrated total angular momentum and energy.

The buoyancy of a thin flux tube can be taken into account, as is shown in [21]. For this, let us consider a flux tube embedded in a plasma without a magnetic field under static equilibrium:

$$\frac{1}{\rho} \nabla p(\rho^*) + \nabla \Phi = 0.$$  \hspace{1cm} (4)

The distribution of the gas pressure outside the tube can be used as the total pressure distribution in equation (1), $p(\rho^*) = P(r)$, and this equation takes the form

$$\frac{\partial^2 r}{\partial \tau^2} - \frac{1}{4\pi} \frac{\partial}{\partial \alpha} \left( \frac{\partial (\rho r)}{\partial \alpha} \right) = -\frac{1}{\rho} \nabla P(r),$$  \hspace{1cm} (5)

where $\rho^*$ is the plasma density outside the flux tube. Therefore if $\rho < \rho^*$ the tube will rise whereas for $\rho > \rho^*$ the flux tube will sink. The magnetic pressure inside the tube compensates part of the total pressure in equation (4). This means that the magnetic field stimulates the buoyancy of the tube: the stronger the magnetic field, the faster the tube emerges.

It is interesting to remember that the buoyancy of magnetic tubes is believed to be responsible for the formation of the solar spots [15]. Hence, the buoyancy effect is a necessary element for solar cycle theory.
3. Thin flux tubes in the relativistic MHD

The relativistic MHD equations can be presented in terms of the time-like vector of the four-velocity \( u^t, u^i u_i = 1 \) and the space-like four-vector of the magnetic field

\[
 h^i = *F^{ik} u_k, \tag{6}
\]

where \(*F^{ik}\) is the dual tensor of the electromagnetic field, \( h^i h_i < 0 \) [22].

\[
 \nabla_i \rho u^i = 0, \tag{7}
\]

\[
 \nabla_i T^{ik} = 0, \tag{8}
\]

\[
 \nabla_i (h^i u^j - h^j u^i) = 0. \tag{9}
\]

Here, \( T^{ij} \) is the stress–energy tensor

\[
 T^{ij} = Q u^i u^j - P g^{ij} - \frac{1}{4\pi} h^i h^j, \tag{10}
\]

where

\[
 P \equiv p - \frac{1}{8\pi} h^k h_k, \quad Q \equiv p + \varepsilon - \frac{1}{4\pi} h^k h_k \tag{11}
\]

\( p \) is the pressure, \( P \) is the total (plasma plus magnetic) pressure, \( \varepsilon \) is the internal energy including \( \rho c^2 \) and \( g_{ik} \) is the metric tensor with signature \((1, -1, -1, -1)\).

Equation (7) is the continuity equation, (8) the energy–momentum equation and (9) the Maxwell equation.

Using (7) the Maxwell equation (9) can be rewritten in the form of a Lie derivative

\[
 \frac{h^i}{\rho} \nabla_i u^j + \frac{u^j}{q} \nabla_i h^i = 0, \tag{12}
\]

and one can introduce coordinates \( \tau, \alpha \) such that [11]

\[
 x^i_\tau = \frac{\partial x^i}{\partial \tau} = u^i \quad \text{and} \quad x^i_\alpha = \frac{\partial x^i}{\partial \alpha} = h^i \tag{13}
\]

with new coordinate vectors \( u^i/q, h^i/\rho \) tangent to the trajectory of a fluid element and to the magnetic field line. This allows the definition of a magnetic flux tube as a bundle of the magnetic field lines \( h^i/\rho \). The motion of the flux tube is introduced as a Lie dragging of the vector field \( h^i/\rho \) along the vector field \( u^i/q \). It turns out [23] that the mass coordinate \( \alpha \) along the flux tube has the sense of the mass of the plasma for a tube with unit flux in the proper system of reference. The second coordinate \( \tau \) is not Lagrangian or proper time, but just a time-like parameter which traces the flux tube in the space–time of general relativity.

Using (13), the energy–momentum equation (8) can be rearranged to form a set of string equations

\[
 - \frac{\partial}{\partial \tau} \left( \frac{Q q}{\rho} x^i_\tau \right) - \frac{Q q}{\rho} \Gamma^j_{ik} x^i_\tau x^k_\tau + \frac{\partial}{\partial \alpha} \left( \frac{\rho}{4\pi q} x^i q x^i_\alpha \right) \right)
\]

\[
 + \frac{\rho}{4\pi q} \Gamma^i_{jk} x^i_\alpha x^j_\tau x^k_\tau = - \frac{g^{ij}}{\rho q} \frac{\partial P}{\partial x^i}, \tag{14}
\]

where \( \Gamma^j_{ik} \) are Christoffel symbols.

The string equations (14) for a flux tube embedded in a gravitational field \( g_{ik}(x^j) \) and a pressure field \( P(x^i) \) can also be derived from the action [24]

\[
 S = - \int \frac{Q}{\rho} \sqrt{g_{ik} x^i_\tau x^k_\tau} \, dt \, da. \tag{15}
\]

The action (15) is invariant under \( \tau \)-reparametrization \( \tau \to \tau(\tau) \). Hence, the canonical Hamiltonian vanishes identically and we need a gauge condition to fix the parametrization. The appropriate gauge condition is [25]

\[
 q = \sqrt{g_{ik} x^i_\tau x^k_\tau} = \frac{1}{w}, \tag{16}
\]

where \( w = \sqrt{\varepsilon + p}/\rho \) is the enthalpy of the plasma. Using the gauge condition (16) it can be shown that equations (14) are of a hyperbolic type with relativistic Alfvénic and slow mode characteristics.

The Kerr metric in Boyer–Lindquist coordinates has two cyclic variables, namely the coordinate time \( \tau \) and the azimuth angle \( \phi \). Therefore, there are two conservation laws for the string—energy \( E \) and the angular momentum \( L \):

\[
 E = \int_{a_1}^{a_2} \frac{Q q}{\rho} (g_{ik} l^i_\tau + g_{ik} q \phi_\tau) \, da, \tag{17}
\]

\[
 L = - \int_{a_1}^{a_2} \frac{Q q}{\rho} (g_{ik} l^i_\tau + g_{ik} q \phi_\tau) \, da. \tag{18}
\]

It is supposed that there is no flux of energy and angular momentum through the ends \( a_1, a_2 \) of the flux tube.

The string equations (14) have been solved numerically using the total variation diminishing scheme. The conservation laws (17) and (18) have been used to control the accuracy of the numerical scheme. More details on the method can be found in [25].

Summing up, a relativistic flux tube is characterized by the internal parameters of density, pressure and magnetic field, and it is embedded in the \textit{a priori} specified external gravitational field and pressure field. Once this equation for the evolution of the relativistic flux tube has been found, several properties from classical MHD appear to immediately relate to general relativity: the stretching of field lines leads to a decrease in density and, consequently, to buoyancy forces. The infinite stretching of the field line has to be stopped by some nonideal process like field line reconnection [26].

4. Energy extraction from a rotating black hole

Consider an initially straight infinitely long massive magnetic flux tube with zero angular momentum everywhere along the string as sketched in figure 2(a). If this was just a convected fluid tube (no magnetic field), each part of the tube would start to spin up with the zero angular momentum observer (ZAMO) angular velocity as it falls into the black hole. Due to the inhomogeneous character of the Lense–Thirring torques, the flux tube becomes stretched and twisted (figure 2(b)). The increase of magnetic tension slows down the rotation of the central part of the tube nearest to the black hole, and therefore the latter will rotate locally slower than ZAMO and thus gain negative energy and momentum. After a while, negative
Differential rotation due to the Lense–Thirring effect near the Kerr black hole winds up the massive magnetic flux tube ((a)–(d)). Inside the ergosphere (light grey), magnetic tension slows down the rotation of the leading part of the tube marked in red. This part carries negative energy and angular momentum. The stretching of the flux tube is visualized by markers. The stretched field line has lower density and buoyancy and produces the helical magnetic structure along the spin axis (e). Centrifugal forces accelerate the plasma along the field lines. This is the birth of the jet (f). See supporting online material (available from stacks.iop.org/PhysScr/89/045003/mmedia): Animation 1.

5. Buoyancy

At this stage, another physical process plays an important role. The part of the flux tube with extra positive energy loses more and more plasma, and the flux tube will feel the relativistic analogon of the buoyancy force \[21\]. Unfortunately, it is not possible to find such a distribution of total pressure to balance the gravity of a Kerr black hole, because even without a magnetic field there should be a plasma flow across the event horizon. A simple solution to this problem does not exist, and we cannot use the distribution of the plasma pressure in the right hand side of the string equation \[14\]. But the Kerr black hole is such a powerful object that the actual distribution of the total pressure is not that important. In fact, the buoyancy force first slows down the radial plasma accretion and then eventually pushes some fragment with positive extra energy along the spin axis outside the static limit surface, as shown in figures 2(c)–(e). This is the birth of a jet (see figure 3 at \[\tau \approx 50\]). The buoyancy force generates the pronounced double helical magnetic field structure aligned with the spin axis. Along these field lines, the plasma is centrifugally accelerated to nearly the speed of light.

An important property of this mechanism, including buoyancy, is that it does not depend on the detailed initial magnetic field distribution except that the characteristic size of
Figure 4. Differential rotation twists the magnetic field, which is then expelled by the buoyancy force along the spin axes and forms a double helical structure. On the left, the evolution of a field line initially parallel to the spin axis, on the right, the evolution of a field line initially inclined with respect to the spin axis. Although the structure is asymmetric for the inclined case, the resulting helical structure is still along the spin axis. Magnetic reconnection shown in the enlarged detail (left) creates autonomous magnetic structures (plasmoids) detached completely from the black hole. See supporting online material (available from stacks.iop.org/PhysScr/89/045003/mmedia): Animation 2.

The magnetic field should be of the order of the event horizon. In this case, the Lense–Thirring effect produces the spiral magnetic field while buoyancy creates the outgoing helical structure. Animation 2 of the online supporting material (available from stacks.iop.org/PhysScr/89/045003/mmedia) shows the same as Animation 1, but for an inclined magnetic field line.

This process evidently leads to an unlimited stretching of the flux tube since one part of the string continues to fall into the hole and, simultaneously, another part of the string is pushed outward. An important fact is that this is a continuous process which does not depend sensitively on actual distribution of the total pressure. Apparently at some point this stretching must be limited by some nonideal process. Two such mechanisms are discussed in the literature. The first mechanism is popular in astrophysics and considers a gap with an extremely strong parallel electric field, such that this field can produce electron–positron pairs from the electromagnetic field. Such a gap will produce an electron–positron plasma which will fill up the tube. One can imagine such an even steady state and axisymmetric process. The physics of the steady state process is based on the Blandford–Znajek mechanism further discussed in [2, 5, 6, 27].

A second possibility to limit the stretching process is magnetic reconnection. It consists of a local breakdown of the ideality condition which allows for a slipping of magnetic field lines with respect to the plasma flow. In this nonideal region, the field lines are topologically reordered [3, 25, 28]. This will work most effectively if the stretched flux tube is reconnected outside the static limit boundary to itself (figure 4). A closed double helical structured field line with low density will form a bubble which will freely evolve, and the release of magnetic tension will power the jet stream while the ergospheric part of the tube is supplied with new accreting material. Animations 3 and 4 of the supporting online material (available from stacks.iop.org/PhysScr/89/045003/mmedia) show details of this evolution.

6. Jet relaxation

The structure of the magnetic field inside the outgoing plasma bubble becomes more and more simple due to the relaxation of Maxwellian stresses. The double helical structure will relax into a simpler circular structure. As a consequence, the collimation effect becomes less and the width of the jet slowly increases. At some point, the magnetic field cannot confine the rotating plasma any more, and the jet quickly spreads out (figure 5). In this picture, there is no need for collisions with external plasmas. Each plasmoid can be considered as an outgoing particle in the Penrose mechanism: it carries additional positive energy away from the black hole. Therefore, the Penrose mechanism is used in a two-fold sense in this scenario. Firstly, for the behaviour of the flux tube inside the ergosphere and secondly, for the behaviour of the plasmoid outside the ergosphere.

So far, we have considered only self-reconnection of magnetic flux tubes which produce relatively simple structures of the magnetic field. In reality, the situation is expected to be more complicated and chaotic: some tubes fall down into the black hole and, simultaneously, the other tubes will rise due to buoyancy. Hence, they can collide and reconnect with each other. In other words, the probability of a flux tube reconnecting with some other tube seems to be
higher than self-reconnection. In such a case, the magnetic structures will be very complicated, and during the relaxation stage we can expect additional collisions of the outgoing flux tubes. As a result, further reconnection events are also likely to occur during the next stage of jet propagation. Reconnection converts magnetic energy into kinetic and internal energy of the plasma, and gives rise to energetic particles which are important for the models of extragalactic radio sources [28].

7. Conclusions

The buoyancy effect can play an important role in the course of jet creation. At the beginning, it slows down the motion of the plasma towards the black hole, especially for that part of the tube with low density. Eventually this part starts to move outwards producing a spiral structure needed for centrifugally driven jets. This might resolve an important question on relativistic jets: How does a magnetized plasma falling into the huge gravitational centre of a black hole which, in principle, swallows everything, produce outgoing streams near the speed of light?

A further necessary element in this scenario is magnetic field line reconnection. In this context, the question arises on where the correct place is to reconnect the field lines. One might expect that this place is near the event horizon because of nearly antiparallel field lines there. However, it turns out that gravity in this region is so strong that the release of Maxwellian stresses there by reconnection cannot prevent the plasma from falling into the black hole. The more appropriate site for reconnection is likely to be near the static limit surface, where magnetic reconnection can produce a chain of plasmoids and supply the black hole with new material.

In addition, we propose a new relaxation mechanism. It is well known that jets eventually start to spread out, getting wider and wider. Usually this is connected to the collision with the plasma outside or a Kelvin Helmholtz instability. Our point of view is that Maxwellian relaxation continues all the way along and leads to a simplification of the magnetic jet structure. Roughly speaking, each plasmoid tends to become a circle-like structure which leads to a jet of finite thickness.

In our scenario, the Penrose mechanism has been used twice: locally to redistribute angular momentum and energy along the string and, consequently, to extract energy from the Kerr black hole, and globally in the form of an outgoing plasmoid which transfers additional positive energy and angular momentum away from the black hole. This plasmoid can be considered as a classical Penrose particle which carries away rotational energy from the black hole.

The accretion of magnetized plasma into a Kerr black hole with differential rotation results in a common pattern, i.e. a helical structure of the magnetic field, a spinning plasma flow which eventually leads to jet formation via the frame dragging effect, the Penrose mechanism, relativistic buoyancy, centrifugal acceleration and reconnection. The predictions of the present mechanism results from the application of energy and angular momentum conservation for magnetic flux tubes. There is almost no dependence on the initial configuration of the magnetic field.

Acknowledgments

The authors thank B Punsly for useful discussions. VSS acknowledges financial support from the TU Graz.

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