MAGNETIC FIELD INTENSIFICATION BY THE THREE-DIMENSIONAL “EXPLOSION” PROCESS

H. Hotta1, M. Rempel2, and T. Yokoyama1

1 Department of Earth and Planetary Science, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan; hotta.h@eps.s.u-tokyo.ac.jp
2 High Altitude Observatory, National Center for Atmospheric Research, Boulder, CO 80307, USA

Received 2012 August 20; accepted 2012 October 2; published 2012 October 17

ABSTRACT

We investigate an intensification mechanism for the magnetic field near the base of the solar convection zone that does not rely on differential rotation. Such mechanism in addition to differential rotation has been suggested by studies of flux emergence, which typically require field strength in excess of those provided by differential rotation alone. We study here a process in which potential energy of the superadiabatically stratified convection zone is converted into magnetic energy. This mechanism, known as the “explosion of magnetic flux tubes,” has been previously studied in thin flux tube approximation as well as two-dimensional magnetohydrodynamic (MHD) simulations; here we expand the investigation to three-dimensional MHD simulations. Our main result is that enough intensification can be achieved in a three-dimensional magnetic flux sheet as long as the spatial scale of the imposed perturbation normal to the magnetic field is sufficiently large. When this spatial scale is small, the flux sheet tends to rise toward the surface, resulting in a significant decrease of the magnetic field amplification.

Key words: stars: interiors – Sun: dynamo – Sun: interior

Online-only material: color figures

1. INTRODUCTION

The solar magnetic field is thought to be generated by dynamo action (Parker 1955). In most solar dynamo models differential rotation is the key process that generates strong toroidal field near the base of the convection zone (Choudhuri et al. 1995; Dikpati & Charbonneau 1999; Hotta & Yokoyama 2010a, 2010b). The mean energy density of a dynamo-generated magnetic field is determined by the energy density of differential rotation as

$$\frac{B^2}{8\pi} \sim \frac{1}{2} \rho (\Delta v)^2, \quad (1)$$

where $B$, $\rho$, and $\Delta v$ denote the strength of magnetic field, density, and typical difference of the fluid velocity at the tachocline. At the tachocline these values are $\rho = 0.2 \; g \; cm^{-3}$, $\Delta v = 2\pi R_{bc} \Delta \Omega = 2\pi \times 5 \times 10^{10} \; cm \times 50 \; nHz = 15,700 \; cm \; s^{-1}$ (see the results of helioseismology in Thompson et al. 2003), where $R_{bc}$ and $\Delta \Omega$ are the radius at the tachocline and the difference of angular velocity between the top and the bottom of the tachocline, respectively. In addition, if we assume a linear velocity profile and formally integrate over the width of the shear layer, we have a drop of another factor of three in energy. Using these values the dynamo-generated magnetic field is estimated to be approximately $B \sim 1.4 \times 10^4 \; G$. This estimate is similar to the values found in mean-field models with Lorentz force feedback by Rempel (2006). Note that in Rempel (2006) latitudinal shear mostly generates toroidal magnetic field and that differential rotation is continuously replenished through the $\Lambda$-effect (parameterized turbulent angular momentum transport), which explains why $10^4 \; G$ can be reached with only small feedback on differential rotation.

On the other hand, theoretical studies of flux emergence and their comparison with observed sunspot properties (Joy’s law, low latitude emergence) imply substantially stronger field near the base of the convection zone. Weber et al. (2011) suggest that a toroidal field strength of about 50 kG is required at the base of the convection zone in order to reproduce the statistical properties of tilted sunspot pairs in rising flux tube simulations (see also review by Fan 2009).

Above results suggest that additional mechanisms for the amplification other than rotational shear are required. One candidate of such a mechanism is the “explosion” of rising magnetic flux tubes (Parker 1994; Moreno-Insertis et al. 1995; Rempel & Schüssler 2001). It is summarized as follows: suppose that there is a superadiabatically stratified atmosphere like the solar convection zone and a dynamo-generated toroidal magnetic field. When the gravity has the profile of $g(r) \propto r^{-2}$, where $r$ denotes the radius, the distribution of gas pressure $p_e(r)$ is expressed as

$$p_e(r) = p_{base} \left[ 1 + \nabla \frac{r_{base}}{H_{base}} \left( \frac{r_{base}}{r} - 1 \right) \right]^{1/\mathcal{V}}, \quad (2)$$

using a constant temperature gradient as

$$\nabla \equiv \frac{d \log T}{d \log p}, \quad (3)$$

$p_{base}$ and $H_{base}$ denote the gas pressure and the pressure scale height at $r = r_{base}$, respectively, where $r_{base}$ is the location of the base of the convection zone. As the toroidal field buoyantly rises as a flux tube, the thermal property in the tube varies adiabatically while the surrounding gas in the convection zone is stratified in a superadiabatic manner. The thermal exchange is mainly by radiative diffusion in the solar convection zone and the timescale of it is at least several tens of years for a 1000 km flux sheet, which is much longer than the timescale of flux emergence on the order of months. Therefore the internal distribution of the gas pressure is expressed as

$$p_i(r) = p_{base} \left[ 1 + \nabla_{ad} \frac{r_{base}}{H_{base}} \left( \frac{r_{base}}{r} - 1 \right) \right]^{1/\mathcal{V}_{ad}}, \quad (4)$$

where $\nabla_{ad}$ denotes the adiabatic temperature gradient. We note that $\nabla > \nabla_{ad}$ and the pressure of the internal medium of
magnetic flux decreases with $r$ slower than that of the external medium. Suppose that the pressure is in balance at the base of the convection zone,

$$
p_i(r_{\text{base}}) + \frac{B^2(r_{\text{base}})}{8\pi} = p_e(r_{\text{base}}). \quad (5)
$$

As the decrease of the pressure in the tube is slower than that of the surrounding plasma, these pressures become the same at a certain height $r = r_{\text{expl}}$, i.e., $p_i(r_{\text{expl}}) = p_e(r_{\text{expl}})$. This height is called the explosion height. Even after passing the explosion height, the flux tube continues to rise up to the surface due to the imbalance between the pressure gradient and the gravitational force along the magnetic field (see Figure 2 of Rempel & Schüssler 2001). We specify the gas pressure distribution $p_i(r)$, after this sweeping of mass. Since the gas pressure is balanced at the surface, i.e., $p_i(r_i) = p_e(r_i)$, the pressure distribution is expressed as

$$p_i(r) = p_e(r_i) \left[ 1 + \nabla \cdot \frac{r_i}{H_i} \left( \frac{r_i}{r} - 1 \right) \right]^{1/\nabla}, \quad (6)
$$

where the subscript “i” denotes the value at the surface. If both ends of the rising flux tube remain at the initial position ($r = r_{\text{base}}$), the amplified magnetic field has the energy density of

$$\frac{B_{\text{amp}}^2}{8\pi} = p_e(r_{\text{base}}) - p_i(r_{\text{base}}). \quad (7)
$$

Since the pressure is in balance at the surface with the amplified field, the magnetic field is amplified up to the strength for which the explosion height approaches the surface. The asymptotic field strength is independent of the initial magnetic field and given by the superadiabaticity of the convection zone.

Moreno-Insertis et al. (1995) investigate explosion using one-dimensional thin flux tube approximation. Rempel & Schüssler (2001) show by using two-dimensional magnetohydrodynamic (MHD) calculation that the magnetic field can be amplified up to $10^5$ G in the solar convection zone. The timescale of amplification is determined by the initial magnetic field and is <0.5 year when the initial plasma beta is $\beta < 10^7$.

In this study, we investigate the possibility of an explosion of a magnetic flux and its consequential intensification by MHD simulations in three dimensions, i.e., in more realistic situations, which have not yet been investigated in previous studies.

2. MODEL

The three-dimensional MHD equations are solved in Cartesian coordinates $(x, y, z)$, where $x$ and $y$ denote the horizontal directions and $z$ denotes the vertical direction. Equations are expressed as

$$\frac{\partial \rho}{\partial t} = - \nabla \cdot (\rho \mathbf{v}), \quad (8)$$

$$\frac{\partial}{\partial t} (\rho \mathbf{v}) = - \nabla \cdot \left[ \rho \mathbf{v} \mathbf{v} + \left( p + \frac{B^2}{8\pi} \right) \mathbf{I} + \frac{\mathbf{BB}}{4\pi} \right] - \rho g \mathbf{e}_z, \quad (9)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad (10)$$

$$\frac{\partial e}{\partial t} = \nabla \cdot \left[ \left( e + p + \frac{B^2}{8\pi} \right) \mathbf{v} - (\mathbf{B} \cdot \mathbf{v}) \frac{\mathbf{B}}{4\pi} \right] - \rho g v_z, \quad (11)$$

$$e = \frac{\rho}{\gamma - 1} + \frac{1}{2} \rho \mathbf{v}^2 + \frac{B^2}{8\pi}, \quad (12)$$

where $\rho, p, \mathbf{v}, \mathbf{B}, \gamma$, and $e$ denote density, pressure, fluid velocity, magnetic field, gravitational acceleration, and total energy per unit volume. The value of a specific heat ration is set as $\gamma = 5/3$. Note that since we use the equation of state for the perfect gas, the internal energy per unit volume is expressed as $p/(\gamma - 1)$ in Equation (12).

The divergence-free condition, i.e., $\nabla \cdot \mathbf{B} = 0$, is maintained using the method introduced in Dedner et al. (2002). The simulation domain is $(-7.5, -20, -1) < (x/H_b, y/H_b, z/H_b) < (7.5, 20, 3)$ and the total grid number is $256 \times 128 \times 384$. We use a fourth-order centered finite difference scheme for spatial derivatives in combination with a fourth-order Runge–Kutta integration in time. We do not use any explicit viscosity or magnetic diffusivity, but apply artificial diffusivities to maintain numerical stability (Rempel et al. 2009). We use periodic boundary conditions for all variables in horizontal directions. At the top and bottom boundary, stress-free and non-penetrating boundary conditions are adopted, i.e., $\partial v_x/\partial z = \partial v_y/\partial z = 0$ and $v_z = 0$. The values of density and pressure at these boundaries are derived from solving Equations (19)–(22). At the top boundary, the magnetic field is set as $B_x = B_y = 0$, and $\partial B_z/\partial z = 0$. At the bottom boundary, the magnetic field is set as $\partial B_x/\partial z = \partial B_y/\partial z = \partial B_z/\partial z = 0$.

The profile of gravitational acceleration is assumed to be

$$g(z) = g_b \left( \frac{z + r_b}{r_b} \right)^2, \quad (13)$$

where $r_b = 5 H_b$ is the location of $z = 0$ from the center of the gravity source. We note that the subscript “b” means the value at $z = 0$. Here $H_b$ is used instead of $H_{\text{base}}$ since the setup of the simulations here is similar to but not identical with the model in Section 1. The initial magnetic field is set as a flux sheet with a perturbation as

$$B_z = B_{0} f \exp \left[ - \frac{(z - z_c)^2}{a^2} \right], \quad (14)$$

$$a = a_0 + a_1 \exp \left[ - \left( \frac{x}{d_x} \right)^2 - \left( \frac{y}{d_y} \right)^2 \right], \quad (15)$$

$$z_c = z_0 + z_1 \exp \left[ - \left( \frac{x}{d_x} \right)^2 - \left( \frac{y}{d_y} \right)^2 \right], \quad (16)$$

$$f = \frac{a_0}{a}, \quad (17)$$

$$B_0 = \sqrt{\frac{8\pi \rho_b}{\beta}}, \quad (18)$$

where $a$ and $z_c$ denote the thickness and the center of a flux sheet, respectively. $d_x$ and $d_y$ are the range of perturbation in the $x$- and $y$-directions, respectively. We specify $a_0 = 0.2 H_b$, $a_1 = 0.25 H_b$, $z_0 = 0.1 H_b$, $z_1 = 0.25 H_b$, $\beta = 300$, and $d_x = 2 H_b, d_y = 1$ is a free parameter from $2 H_b$ to $8 H_b$. As shown above, we are considering a toroidal flux sheet as a source of the magnetic flux prior to the intensification because the magnetic field stretched by the $\Omega$-effect at the bottom of the convection zone is suggested to have a sheet structure according to Rempel (2006). Then $B_z$ is calculated to satisfy the divergence-free condition, i.e., $\partial B_x/\partial x + \partial B_y/\partial y = 0$, with $B_z = 0$ at all boundaries and $B_z = 0$ in the entire domain.

In order to mimic the situation of the solar convection zone, we adopt a similar approach to Rempel & Schüssler (2001).
The upper layer ($z > 0$) is adiabatically stratified (convection zone: $\delta = 0$) and the lower layer ($z < 0$) is subadiabatically stratified (radiative zone: $\delta = -0.2$), where $\delta$ is the superadiabaticity. The magnetic layer is placed at the interface between both regions. Since our convection zone is adiabatically stratified, we add additional entropy within the flux sheet to allow for a buoyant rise and subsequent explosion. Using this setting, the pressure scale height in the flux sheet is longer than that of the external medium and the explosion occurs around $z_{\text{expl}} \sim H_b c_p / \beta \Delta s$, where $\Delta s$ is the added entropy. Using an adiabatic stratification in the upper layer allows us to study the explosion process in a relatively simple situation without thermal convection; this leads, however, to limitations also since some key processes are not captured in our setup. Besides not capturing the influence from convection, our magnetic flux sheet is not stored in a stable layer having neutral magnetic buoyancy, i.e., the overshoot region at the base of the solar convection zone. The flux sheet is however stabilized against buoyancy to some degree by lying on top of our stably stratified lower layer. We will investigate the explosion process in a superadiabatic convection zone with fully developed thermal convection in a future study.

The initial conditions for pressure and density are derived from solving the equations:

\[
\frac{\partial s}{\partial z} = -\frac{c_p \delta}{H_p} - \frac{2 \Delta s (z - z_c)}{a^2} \exp \left[ -\frac{(z - z_c)^2}{a^2} \right],
\]

\[
\frac{\partial p_{\text{tot}}}{\partial z} = -\rho g,
\]

\[
p_{\text{tot}} = p + \frac{B^2}{8\pi},
\]

\[
\rho = \left[ \frac{p}{\exp(s/c_v)} \right]^{1/\gamma}.
\]

We specify $\Delta s = 0.03 c_v$. Using this parameter the explosion height is around $z_{\text{expl}} = 0.18 H_b$. We do not yet aim at a realistic simulation with solar parameters but rather consider a model that allows us to study the explosion and the following intensification in (artificial) isolation. If there is a superadiabatically stratified convection zone, turbulent thermal convection is generated and the situation becomes more complicated.

3. RESULT

In this study, we take the $y$-component of the perturbation on the initial magnetic field ($d_y$) as a free parameter. Figure 1 shows the strength and the field lines of the magnetic field for simulations with $d_y = 2 H_b$ and $d_y = 6 H_b$. We present here only the solutions in the center region cut at $y = 0$ to enhance the clarity of the presentation. The magnetic layer the horizontal directions fills in the whole domain outlined by the white lines. In our three-dimensional situation a value of $d_y = 6 H_b$ (Figure 1(d)) leads to a substantial intensification.
of the magnetic field, which is similar to the two-dimensional calculation reported by Rempel & Schüssler (2001). At the footpoints the magnetic field lines remain almost parallel to the x-axis for \( d_y = 6H_b \). On the other hand, the amplification of magnetic field is significantly less with \( d_y = 2H_b \). In the result with \( d_y = 2H_b \), the footpoint magnetic field rises upward and the magnetic field lines at the back (\( y < 0 \)) are bent to the center (\( y \sim 0 \)) of the calculation domain. At this stage, it is suggested that the intensification of the magnetic field by the explosion process is more likely to occur with a longer perturbation, which is perpendicular to the main magnetic field.

Figure 2(a) shows the temporal evolution of the magnetic energy averaged over the y = 0 plane. Around \( t = 120H_b/c_b \), where \( c_b \) is the speed of sound at \( z = 0 \), the initial intensification of the magnetic field with the explosion process is completed in all cases, i.e., the top of the rising flux is reaching the upper boundary. Figure 2(b) shows the dependence on \( d_y \) of the mean magnetic energy at \( t = 120H_b/c_b \). It is qualitatively clear that a larger scale \( d_y \) causes a more effective intensification.

In the following discussion, we compare the results with different \( d_y \) at the time when the apex of the rising magnetic flux reaches \( z = 2H_b \) in each case. Figure 2(c) shows the profile of the y-component of the fluid velocity averaged over \(-0.1H_b < x < 0.1H_b\) and \( 0 < z < 2H_b \) at \( y = 0 \). Figure 2(d) shows the dependence of the maximum value of the averaged y-component velocity on \( d_y \). The dashed line denotes the dependence of \( v_y = 0.1/d_y \).

(A color version of this figure is available in the online journal.)

The reason is that the center region is already filled with mass, resulting in a smaller \( \Delta p \). In addition, a small value of \( \Delta p \) causes the deviation. Figure 3(a) shows the value of \( v_{\text{perp}} \) along the field lines, where \( v_{\text{perp}} \) is the fluid velocity perpendicular to the magnetic field in the x-z plane. A positive (negative) value of \( v_{\text{perp}} \) indicates a flow which moves the magnetic flux upward (downward). Using small (large) \( d_y \), the footpoint magnetic flux moves upward (downward). Figure 3(b) shows the velocity parallel to the magnetic field along magnetic field lines. The larger velocity is generated with larger \( d_y \). These correspond to the amplification of the magnetic field.

4. DISCUSSION

Our results show that the efficiency of the magnetic intensification is dependent on the spatial scale of the perturbations

![Figure 2](image-url)
Figure 3. (a) Averaged $v_{\text{perp}}$ along magnetic field lines, where $v_{\text{perp}}$ is the fluid speed perpendicular to the magnetic field. We choose the magnetic field lines which start $x = y = 0$ and $z = 0, 0.25H_b$. (b) Averaged velocity parallel to magnetic field $v_{\text{para}}$ is shown. The same magnetic field lines for the panel (a) are chosen.

(A color version of this figure is available in the online journal.)

Figure 4. Schematic picture of the explosion process in three-dimensional situation with (a) large and (b) small $d_y$.

Figure 5. Dependence of the amplified magnetic energy by the explosion process on the location of footpoint.

5. SUMMARY

We investigate an intensification mechanism of the magnetic field near the base of the solar convection zone following the “explosion” process using three-dimensional MHD simulation. Such a mechanism in addition to differential rotation has been suggested by studies of flux emergence, which typically require field strength in excess of those provided by differential rotation alone. Our main result is that enough intensification can be achieved even in a three-dimensional situation as long as the spatial scale of the imposed perturbation in the direction normal to the magnetic flux sheet is sufficiently large. When this spatial scale is small, the efficiency of intensification of the magnetic

$(d_y)$. The reason is understood as follows: in a two-dimensional situation (Rempel & Schüssler 2001) or using large value of $d_y(> 6H_b)$, the pressure gradient $\partial p/\partial x$ leads to strong converging flows toward the center of the rising flux sheet within the $y = 0$ plane. The flow component parallel to the field leads to amplification; the flow component perpendicular to the field prevents the buoyant motion around the legs of the rising flux and consequently plays a role for pinning down the footpoints (Figure 4(a)). On the other hand, using small $d_y(< 2H_b)$, a converging motion in the $y$-direction is promoted by the large pressure gradient $\partial p/\partial y$ and the center of the explosion region is filled with the mass. Thus the pressure around the footpoint becomes large and the flow $v_x$ is suppressed. Consequently magnetic flux moves upward and is less amplified (Figure 4(b)). If the footpoint magnetic field is fixed at the initial position, the amplification of magnetic energy is estimated with Equation (7). In a three-dimensional situation, however, there is a possibility that the footpoint magnetic flux moves upward, thus the amplified magnetic energy is estimated using Equation (7) at $r = r_f$ instead of $r = r_{\text{base}}$, where $r_f$ denotes the location of the footpoint magnetic flux. Figure 5 shows the dependence of the amplified magnetic energy by the explosion process on the location of footpoint with the parameters of our calculation. If the footpoint magnetic field is fixed in the initial position, the magnetic energy of $\sim 2 \times 10^{-2} p_b$ can be generated by the explosion process. The result with large $d_y(= 6H_b)$ reproduces this estimation (see Figure 1(d)) well. Note that Figure 2(a) shows the mean magnetic energy in the $y = 0$ plane, leading to a lower values of $\sim 6 \times 10^{-4} p_b$ at maximum. Figure 5 shows that the efficiency of the intensification significantly decreases as the footpoint magnetic field rises. It is confirmed that even using larger $\beta(=600)$, we can obtain similar results, i.e., strong amplification is achieved with larger $d_y(> 6H_b)$.
field significantly decreases. Here a strong converging flow in
the direction perpendicular to the field (y-direction) is driven,
which leads to weaker flows in the x-direction. The latter are
relevant for the field amplification as well as stabilization of the
footpoints in our setup.

In this study, we used a similar setup to Rempel & Schüssler
(2001), which allows us to study the explosion process in
an adiabatically stratified atmosphere, avoiding the complex
situation caused by presence of turbulent thermal convection.
Using a more realistic setup with a superadiabatic convection
zone and storage of magnetic flux in a subadiabatic region
beneath could influence our results in the following way.

If the initial magnetic flux is stored in a subadiabatic stratification
the footpoint magnetic field is more stable and the criterion
for the length scale of the perturbation might be relaxed, i.e.,
also shorter perturbations could lead to stronger amplification.
However, even with a more realistic storage in a subadiabatic
region, there is no force that could stop the horizontal motion of
the footpoints (buoyancy only works in the vertical direction).
As a consequence also here a fundamental difference between
tube-like and sheet-like behavior will persist.

The presence of turbulent thermal convection could sig-
ificantly influence our results since during the explosion
process the strength of the magnetic field near the top of
the rising flux drops significantly. Thus, it is possible that
the connection between the top and footpoint magnetic field
is disturbed by turbulence, resulting in a reduction of field
intensification.

Therefore, more realistic simulations including a stably strati-
ﬁed overshoot region/radiation zone and a superadiabatic con-
vection zone are required in order to investigate the detailed
conditions under which the explosion process leads to substan-
tial intensification of magnetic field.

The authors are grateful to Y. Fan for her helpful comments. Numerical computations were, in part, carried out on a Cray XT4 at the Center for Computational Astrophysics,
CfCA, of the National Astronomical Observatory of Japan.
This work was supported by Grant-in-Aid for JSPS Fellows.
This work was supported by the JSPS Institutional Program for
Young Researcher Overseas Visits and the Research Fellowship
from the JSPS for Young Scientists. The National Center for
Atmospheric Research is sponsored by the National Science
Foundation. We have greatly benefited from the proofreading/
editing assistance from the GCOE program.

REFERENCES

Choudhuri, A. R., Schüssler, M., & Dikpati, M. 1995, A&A, 303, L29
Clyne, J., Mininni, P., Norton, A., & Rast, M. 2007, New J. Phys., 9, 301
Clyne, J., & Rast, M. 2005, in Proc. SPIE, 5669, 284
Dedner, A., Kemm, F., Kröner, D., et al. 2002, J. Comput. Phys., 175, 645
Dikpati, M., & Charbonneau, P. 1999, ApJ, 518, 508
Fan, Y. 2009, Living Rev. Solar Phys., 6, 4
Hotta, H., & Yokoyama, T. 2010a, ApJ, 709, 1009
Hotta, H., & Yokoyama, T. 2010b, ApJ, 714, L308
Moreno-Insertis, F., Caligari, P., & Schüssler, M. 1995, ApJ, 452, 894
Parker, E. N. 1955, ApJ, 122, 293
Parker, E. N. 1994, ApJ, 433, 867
Rempel, M. 2006, ApJ, 647, 662
Rempel, M., & Schüssler, M. 2001, ApJ, 552, L171
Rempel, M., Schüssler, M., & Knöllker, M. 2009, ApJ, 691, 640
Thompson, M. J., Christensen-Dalsgaard, J., Miesch, M. S., & Toomre, J.
2003, ARA&A, 41, 599
Weber, M. A., Fan, Y., & Miesch, M. S. 2011, ApJ, 741, 11