On the Equivalence Between Rotation and Gravity: “Gravitational” and “Cosmological” Redshifts in the Laboratory

Christian Corda\textsuperscript{1,2}\thanks{Christian Corda\newline\texttt{cordac.galilei@gmail.com}}

Received: 26 July 2021 / Accepted: 1 March 2022 / Published online: 29 March 2022
© The Author(s), under exclusive license to Springer Science+Business Media, LLC, part of Springer Nature 2022

Abstract
The Mössbauer rotor effect recently gained a renewed interest due to the discovery and explanation of an additional effect of clock synchronization which has been missed for about 50 years, i.e. starting from a famous book of Pauli, till some recent experimental analyses. The theoretical explanation of such an additional effect is due to some recent papers in both the general relativistic and the special relativistic frameworks. In the first case (general relativistic framework) the key point of the approach is the Einstein’s equivalence principle (EEP), which, in the words of the same Einstein, enables “the point of view to interpret the rotating system $K'$ as at rest, and the centrifugal field as a gravitational field”. In this paper, we analyse both the history of the Mössbauer rotor effect and its interpretation from the point of view of Einstein’s general theory of relativity (GTR) by adding some new insight. In particular, it will be shown that, if on one hand the “traditional” effect of redshift has a strong analogy with the gravitational redshift, on the other hand the additional effect of clock synchronization has an intriguing analogy with the cosmological redshift. Finally, we show that a recent claim in the literature that the second effect of clock synchronization does not exist is not correct.

Keywords Mössbauer rotor effect · Clock synchronization · Equivalence principle · “Gravitational redshift” · ”Cosmological redshift”

\textsuperscript{1} International Institute for Applicable Mathematics and Information Sciences, B. M. Birla Science Centre, Adarshnagar, Hyderabad 500063, India
\textsuperscript{2} Istituto Livi, Via Antonio Marini, 9, 59100 Prato, Italy
1 Introduction

Rotational motions always had and currently have an important role in science in general and in physics in particular, starting from Aristotelian metaphysics, passing through Newtonian physics and arriving to the current relativistic paradigm.

In the framework of the theory of relativity, the Sagnac effect, which is due to the French physicist Georges Sagnac [1, 2], has a historical importance as it shows the absolute character of rotation by also enabling a long and interesting debate on the foundations of the theory of relativity [1–10], which involved the same Einstein [4, 5].

In the context of the GTR, from the historical point of view it was during his analysis of the rotating frame that Einstein had the intuition to represent the gravitational field in terms of space-time curvature [11]. Einstein indeed wrote, verbatim [11]:

The following important argument also speaks in favor of a more relativistic interpretation. The centrifugal force which acts under given conditions of a body is determined precisely by the same natural constant that also gives its action in a gravitational field. In fact we have no means to distinguish a centrifugal field from a gravitational field. We thus always measure as the weight of the body on the surface of the earth the superposed action of both fields, named above, and we cannot separate their actions. In this manner the point of view to interpret the rotating system K’ as at rest, and the centrifugal field as a gravitational field, gains justification by all means. This interpretation is reminiscent of the original (more special) relativity where the pondermotively acting force, upon an electrically charged mass which moves in a magnetic field, is the action of the electric field which is found at the location of the mass as seen by the reference system at rest with the moving mass.

This interpretation by Einstein of the rotating system in terms of a gravitational field permitted various general relativistic analysis of Mössbauer rotor experiments [12–17] and Sagnac experiments [18]. The key point of the above highlighted interpretation by Einstein is the EEP which enables the equivalence between gravitation and inertial forces [13–17]. The rotating reference frame in a Mössbauer rotor experiment is included in the EEP [13–17]. On the other hand, one must recall that, despite the importance that gravitational physicists attribute to the EEP, one must be careful when using it. The EEP has indeed local validity and many limitations when applied to extended regions of space. For example, the authors of [19] referring to the work [20] pointed out that, in general, there is no equivalence of inertial and gravitational effects in a practical situation, in the context of solutions of Einstein’s equations. In addition, in [21] it has been shown that it is no longer possible to establish a frame with uniform acceleration as equivalent to the homogeneous Newtonian gravitational field. In the same ref. [21] the authors mention the letter that Einstein wrote to Max Plank, recognizing that the concept of “uniform acceleration” needs further clarification.
In the framework of the GTR, the historical solutions of the Einstein equations for the gravitational field of a rotating source are very important. These are the Kerr [22] and Kerr–Newman [23] solutions. On the other hand, Einstein, Lense and Thirring found an interesting similarity between the gravitational field of a distribution of mass and the electromagnetic field of a distribution of charge [24]. The Einstein–Thirring–Lense effect was the starting point of the today popular framework of the gravitomagnetism see [25–31] and references within.

One must stress that relativistic rotation effects can be important also in everyday life. It is indeed well known the use of the GTR in global positioning systems (GPS) [32]. In that case, the time difference between a frame co-rotating with the Earth geoid and a fixed, locally inertial, Earth centered frame cannot be neglected [17, 32]. In other words, GPS would not have the requested accuracy if one neglects the relativistic effects due to the rotation of the Earth [17, 32]. This issue will be also partially clarified in this paper by confronting the GPS with the Mössbauer rotor effect.

Rotation effects in astrophysics are important for what concerns the famous Dark Matter problem. Such a problem originated in the 30’s of last century [33]. If one observes the Doppler shift of stars which move near the plane of the Milky Way and one calculates the velocities, one finds a large amount of matter inside the Milky Way preventing the stars from escaping out. Such a (supposed and unknown) matter generates a very large gravitational force, that the luminous mass in the Milky Way cannot explain. In order to achieve such a large discrepancy, the sum of all the luminous components of the Milky Way should be two or three times more massive. One can calculate the tangential velocity of stars in orbits around the centre of the Milky Way like a function of distance from the centre. The result will be that stars which are far away from the centre of the Milky Way move with the same velocity independent on their distance out from the centre.

These puzzling issues generate a portion of the Dark Matter problem. In fact, either luminous matter is not able to correctly describe the radial profile of the Milky Way or, alternatively, Newton theory of gravity cannot describe dynamics far from the centre of the Milky Way.

The dynamical description of various self-gravitating astrophysical systems generates other issues of the Dark Matter problem. For example, one can consider stellar clusters, external galaxies, clusters and groups of galaxies. In such cases, the problem is similar. There is more matter arising from dynamical analyses with respect to the total luminous matter.

Zwicky [34] found that in the Coma cluster the luminous mass is too little to generate the total gravitational field which is needed to hold the cluster together.

In any case, despite the Dark Matter problem is today much more complicated than the sole issue of the rotation curves of the galaxies, one can surely state that, historically, it arose from a rotational motion.

In this paper an important rotational relativistic effect, the Mössbauer rotor effect, will be analysed in a general relativistic framework. The key point of the analysis will be the EEP, which, in the words of the same Einstein, enables “the point of view to interpret the rotating system K’ as at rest, and the centrifugal field as a gravitational field”.

Springer
In fact, the Mössbauer rotor effect recently gained a renewed interest due to the discovery and explanation of an additional effect which has been missed for about 50 years, i.e. starting from the book of Pauli [12], till the more recent experimental analyses. The theoretical explanation of such an additional effect is due to the recent papers [13, 14, 17] in the general relativistic framework, and [15] in the special relativistic framework. In this paper, both the history of the Mössbauer rotor effect and its interpretation from the point of view of Einstein’s GTR are analysed by adding some new insight. In particular, it will be shown that, if on one hand the “traditional” effect of redshift has a strong analogy with the gravitational redshift, on the other hand the additional effect of clock synchronization has an intriguing analogy with the cosmological redshift.

In detail in this work we add the following new results to the discussion:

(i) In Sect. 3 of the paper we discuss a “cosmological” interpretation of the time dilation due to relativistic contraction of the disc between the co-rotating and inertial frames. In fact, in previous works [13, 14, 17] we merely derived an additional effect of clock synchronization, but it was not completely clear that such an effect can be physically interpreted in terms of a real blueshift. In this context, the analogy with the cosmological redshift of the Universe strengthens previous results [13, 14, 17], and could, in principle, generate a further interest in the Mössbauer rotor effect.

(ii) In Sect. 5 of the paper we show that a recent claim in the literature that the second effect of clock synchronization does not exist [35] is not correct. This refutation of the claims in [35] is needed for the sake of completeness.

2 History of the Mössbauer Rotor Effect

The Mössbauer effect takes its name by the discovery of the German physicist Mössbauer [36]. Based on its importance for various research fields in physics and chemistry, Mössbauer was awarded the 1961 Nobel Prize in Physics together with the work by Hofstadter [37] on electron scattering in atomic nuclei. The Mössbauer effect consists in resonant and recoil-free emission and absorption of gamma rays, without loss of energy, by atomic nuclei bound in a solid. Previous experiments had shown the emission and absorption of X-rays. Thus, researchers expected that an analogous phenomenon could exist also for gamma rays. Differently from X-rays, which are due to electronic transitions, gamma rays are due to nuclear transitions. Experiment attempting to observe nuclear resonance due to gamma-rays in gases failed because of the loss of energy due to recoil. This indeed prevents resonance. The idea of Mössbauer was to observe resonance in nuclei of solid iridium [36] by raising the issue of why gamma-ray resonance is possible in solids, but fails in gases. Mössbauer showed that, if one considers atoms bound into a solid, with particular assumptions a fraction of the nuclear events occurs with no recoil [36]. Hence, the observed resonance is due to such a recoil-free fraction of nuclear events [36]. For this reason, the Mössbauer effect is also called recoilless nuclear resonance fluorescence.
Here, we will consider the Mössbauer rotor effect, see Fig. 1. In that case, one considers an absorber which orbits around the source of resonant radiation. Assuming that the $z$–axis is perpendicular to the plane of Fig. 1, the apparatus within the circumference rotates around such a $z$–axis with constant angular velocity $\omega$. The final detector in the right of the Figure is at rest instead. The experiment permits to measure the so called *transverse Doppler effect* through the fractional energy shift for a resonant absorber [16]. In fact, through its motion, such a transverse Doppler effect generates a relative energy shift between emission and absorption lines.

A historical, important experiment on the Mössbauer rotor effect was due to Kündig [16]. He measured the shift of the 14.4-keV Mössbauer absorption line of $\text{Fe}^{57}$ as a function of the angular velocity of the rotor [16]. He claimed that the transverse Doppler shift was in agreement with the prevision of the theory of relativity within an experimental error of 1.1%, by also discussing possible sources of systematic errors [16].

![Fig. 1 Scheme of the Mössbauer rotor experiment, adapted from [39]. Assuming that the $z$–axis is perpendicular to the plane of the Figure, the apparatus within the circumference rotates around such a $z$–axis with constant angular velocity $\omega$. The final detector in the right of the Figure is at rest instead](image-url)
A team of researchers recently reanalysed Kündig’s experiment [38, 39]. They started to reanalyse in [38] the original data of Kündig’s paper. Then, they decided to realize their own experiment on the transverse Doppler effect in the Mössbauer rotor framework [39]. In their first work [38], the authors found that the data processing of Kündig’s original analysis was erroneous due to the presence of mistakes. After the correction of such mistakes, the experimental data gave the correct value of the fractional energy shift as [38]

$$\frac{\Delta E}{E} = -k \frac{v^2}{c^2}$$  \hspace{1cm} (1)

Thus, they obtained $k = 0.596 \pm 0.006$ rather than previous result $k = 0.5$ which was considered consistent with the relativity theory by Kündig [16], Pauli [12] and others. This appeared very strange, but we will see that it can be solved by using both a special relativistic and a general relativistic treatment. In any case, from the experimental point of view we stress that the authors of [38] found that

- The deviation of the coefficient $k$ in the above Eq. (1) from 0.5 is almost 20 times higher than the measuring error;
- Such a deviation is not due to the influence of rotor vibrations and/or other kinds of disturbing factors. In fact, it seems that all the potential disturbing factors were excluded by the very good methodology applied by Kündig [16].

Kündig’s methodology worked through a first-order Doppler modulation of the energy of $\gamma$-quanta on the rotor at each fixed frequency of rotation [16]. Hence, the experiment of Kündig must be considered as being very precise and surely more precise than other similar experiments [40–44]. In fact, the experiments analysed in refs. [40–44] measured only the count rate of detected $\gamma$-quanta as a function of rotation frequency. The authors of [38] have also shown that the experiment analysed in [44] is in agreement with the supposition $k > 0.5$. Such an experiment contains much more data than the others discussed in [40–43]. The new results in [38] motivated the authors to realize their own experiment [39]. In this new experiment, the authors followed neither the scheme of the Kündig’s experiment [16], nor the schemes of the other previously cited experiments [40–44]. In that way, they obtained a completely independent information on the value of $k$ in Eq. (1). In fact, the authors of [39] refrained from the first-order Doppler modulation of the energy of $\gamma$-quanta. Thus, the uncertainties in the realization of this method have been excluded [39]. The authors measured the count rate of detected $\gamma$-quanta $N$ as a function of the rotation frequency $v$ [39]. In addition, differently from the experiments [40–44], in [39] the influence of chaotic vibrations on the measured value of $k$ has been taken into due account. The authors of [39] also developed a method involving a joint processing of the data collected for two selected resonant absorbers with the specified difference of resonant line positions in the Mössbauer spectra. The final result of the value of $k$ was $k = 0.68 \pm 0.03$ [39]. Hence, the experiment [39] confirmed that the coefficient $k$ in Eq. (1) substantially exceeds 0.5. The scheme of the new Mössbauer rotor experiment is shown in Fig. 1, see [39] for details.
3 The Mössbauer Rotor Effect in the General Relativistic Framework

3.1 The “Gravitational Redshift”

Following [13, 14, 17], one considers a transformation from an inertial coordinate system, centered in the source in Fig. 1, with the $z$-axis perpendicular to the plane of the Figure, to a second coordinate system, which rotates around the $z$-axis in cylindrical coordinates. In the flat Lorentzian coordinate system, the metric is [13, 14, 17]

\begin{equation}
\text{ds}^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dz^2.
\end{equation}

One performs a transformation to a reference frame \{t', r', \phi' z'\}, which has constant angular velocity $\omega$ around the $z$-axis, obtaining [13, 14, 17]

\begin{align*}
t &= t' \quad r = r' \quad \phi = \phi' + \omega t' \quad z = z'.
\end{align*}

Thus, one gets the well known Langevin line-element for the rotating reference frame [7, 8, 13, 14, 17]

\begin{equation}
\text{ds}^2 = \left(1 - \frac{r'^2 \omega^2}{c^2}\right) c^2 dt'^2 - 2\omega r'^2 d\phi' dr' - dr'^2 - r'^2 d\phi'^2 - dz'^2.
\end{equation}

Through the above discussed EEP, the metric (4) can be interpreted as representing a stationary gravitational field [13, 14, 17]. For a well understanding of technical details of the last statement the reader can see paragraph 89 of [10] and [45]. Then, the EEP permits to consider the inertial force that a rotating observer experiences as if the same observer is subjected to a gravitational “force” [13, 14, 17]. Therefore, a first contribution is due to the “gravitational blueshift” [13, 14, 17]. This contribution can be calculated starting from Eq. (25.26) in [46] as it is written down in the 20th printing of 1997

\begin{equation}
z \equiv \frac{\Delta \lambda}{\lambda} = \frac{\lambda_{\text{received}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}} = |g_{00}(r')|^{-\frac{1}{2}} - 1.
\end{equation}

Equation (5) gives the redshift of a photon emitted by an atom at rest in a gravitational field and received by an observer at rest at infinity. In the current analysis Eq. (5) must be rescaled. In fact, Eq. (5) applies to a gravitational field which decreases with increasing radial coordinate, but here one has a “gravitational field” which increases with increasing radial coordinate $r'$. The result is a blueshift (a negative shift) rather than the typical redshift of a real gravitational field. In addition, in Eq. (5) the zero potential is set at infinity, while in the current analysis the zero potential is set in $r' = 0$. One can also use the proper time $\tau$ instead of the wavelength $\lambda$ by recalling that the delay of the emitted (received) radiation is connected to the emitted (received) wavelength by $c\Delta \tau = \lambda$.

Therefore, from Eq. (4), one obtains [13, 14, 17]
Strictly speaking, Eqs. (5 and 6) are valid only for time-independent metrics with $g_{0j} = 0$ [46]. In general, this is not the case of Eq. (4). On the other hand, resonant radiation propagates from the source in the radial direction in the laboratory frame. For radial motions it is $d\phi = 0$ in the laboratory frame. Therefore, one gets from Eq. (3)

$$d\phi' + \omega dt' = 0 \Rightarrow \omega = -\frac{d\phi'}{dt'}.$$  

Then, for radial motions in the laboratory frame, the Langevin metric of Eq. (4) reduces to

$$ds^2 = \left(1 - \frac{r^2 \omega^2}{c^2}\right) c^2 dt'^2 - dr'^2 + r'^2 d\phi'^2 - dz,$$

which has indeed $g_{0j} = 0$ and enables the using of Eqs. (5 and 6) in the present approach. In Eq. (6) $\Delta \tau_1$ is the delay of the emitted radiation, $\Delta \tau_2$ is the delay of the received radiation, $r_1' = c \tau_1$ is the radial coordinate of the detector, where $\tau_1$ is the proper time that has been measured by the rotating observer during the trajectory of the light, and $v = r'_1 \omega$ is the tangential velocity of the detector [13, 14, 17]. The physical meaning of $\Delta \tau_1 \neq \Delta \tau_2$ is that the proper time between two subsequent photons emissions as recorded by the clock at the source is different from the proper time between the subsequent absorptions of the same two photons as recorded by the identical clock at the absorber. This was the original way of thinking of Einstein [47], adopted in an elegant way by Weyl [48]. In other words, $\Delta \tau_1$ denotes the proper time at the source that elapses between successive light-ray emissions (or, say, the emission of a single wavelength), and $\Delta \tau_2$ denotes the proper time elapsing on the absorber between successive light-ray absorptions. On the other hand, the above analysis has been made in rotating frame. But the final measurement is made in the Lorentzian frame in the laboratory. In such a frame the static observer sees no “gravitational field”. Instead, the Lorentzian observer sees the opposite redshift predicted by special relativity when the final detector sees the absorber as being at its closest approach. Kündig [16] pointed out that the rotating observer comes to the conclusion that his clock is slowed down by the “gravitational potential”. Thus, the clock of the Lorentzian observer is faster than the clock of the rotating observer. Then, the fractional energy shift in the laboratory results

$$\frac{E_2 - E_1}{E_1} = \frac{\triangle E_1}{E_1} \approx -\frac{1}{2} \frac{v^2}{c^2}. $$

(9) Springer
Hence, one gets $k_1 = \frac{1}{2}$ as the contribution to $k$ from the first effect [13, 14, 17]. Following Pauli’s book [12], Kündig [16] obtained the same result of $k_1 = \frac{1}{2}$.

### 3.2 The “Cosmological Redshift”

The necessity of an additional effect can be understood through the following observation. The calculation in Subsection 3.1 has been made in the reference frame of the rotating observer. As the final detector moves with respect to the rotating observer [13, 14, 17], see Fig. 1, the clock of the Lorentzian non-rotating observer is not synchronized with the clock of the rotating observer. Therefore, a second, additional effect contributes to the transverse Doppler effect [13, 14, 17]. This is an effect of clock synchronization which was not present in previous works [16, 38, 39]. This second effect was erroneously calculated in [13, 14] by using Eq. (10) of [32]. Such an equation represents the variation of proper time $d\tau$ on the moving clock having radial coordinate $r'$ for values $v \ll c$. We wrote “erroneously” for the following reasons. Eq. (10) of [32] reads

$$d\tau = dt' - \frac{\omega r'^2 d\phi'}{c^2}. \tag{10}$$

Instead, in [13, 14] it has been used

$$d\tau = dt' \left(1 - \frac{r'^2 \omega^2}{c^2}\right). \tag{11}$$

Thus, one sees that Eqs. (10 and 11) are equal only if $\omega = \frac{d\phi'}{dt'}$. But this is not the case because in [13, 14] light which propagates in the radial direction has been considered. This should imply $d\phi' = dz' = 0$, but this is another mistake because in the rotating frame light does not propagate in the radial direction. Let us perform a correct analysis. Following [17], one observes that $t'$ represents the time coordinate for the rotating observer. But Eq. (3) means also $t' = t$, and $t$ represents both of the coordinate time and the proper time for the inertial observer located in the laboratory. The interpretation of the rotating frame in terms of a gravitational field permits to relate the rate $d\tau$ of the proper time to the rate $dt'$ of the coordinate time as [10, 17]

$$d\tau^2 = g_{00} dt'^2. \tag{12}$$

Equation (12) works for any two infinitesimally separated events occurring at one and same point in space. Now, Eq. (4) gives $g_{00} = \left(1 - \frac{r'^2 \omega^2}{c^2}\right)$. Therefore, Eq. (12) becomes

$$c^2 d\tau^2 = \left(1 - \frac{r'^2 \omega^2}{c^2}\right) c^2 dt'^2. \tag{13}$$

Hence, by using again Eq. (3), one obtains
\[ c^2 dt'^2 = c^2 dr^2 = dr^2 = c^2 dt, \quad \text{(14)} \]

where the equality
\[ c^2 dt'^2 = dr^2 \quad \text{(15)} \]
is obtained because light propagates in the radial direction for the observer in the laboratory (the source is indeed at rest in the laboratory frame). Thus, one can set \( d\phi = dz = 0 \) in Eq. (2) and, by inserting the condition of null geodesics \( ds = 0 \) in the same equation one gets Eq. (15). Therefore, Eq. (13) becomes
\[ c^2 d\tau^2 = \left( 1 - \frac{r'^2 \omega^2}{c^2} \right) dr'^2. \quad \text{(16)} \]

Taking the root square of Eq. (16) one gets
\[ cd\tau = \sqrt{1 - \frac{r'^2 \omega^2}{c^2}} dr'. \quad \text{(17)} \]

Equation (17) coincides with Eq. (10) in [13]. Hence, now one can carefully remake the analysis in [13, 14]. One can approximate Eq. (17) with
\[ cd\tau \approx \left( 1 - \frac{1}{2} \frac{r'^2 \omega^2}{c^2} + \ldots \right) dr'. \quad \text{(18)} \]

Equation (18) takes into account the second effect of order \( \frac{v^2}{c^2} \) to (proper) time dilation
\[ c\Delta \tau_3 = \int_0^{r_1'} \left( 1 - \frac{1}{2} \frac{(r')^2 \omega^2}{c^2} \right) dr' - r_1' = - \frac{1}{6} \frac{(r_1')^3 \omega^2}{c^2} = - \frac{1}{6} r_1' \frac{v^2}{c^2}. \quad \text{(19)} \]

\( \Delta \tau_3 \) in this equation represents the difference between the proper time (distance) that has been measured by the rotating observer and the proper time (distance) that has been measured by the Lorentzian observer during the trajectory of the light which propagates from the source to the final detector. Then, the additional effect of clock synchronization (at order \( \frac{v^2}{c^2} \)) to the blueshift is
\[ z_2 \equiv \frac{\Delta \tau_3}{\tau_1} = -k_2 \frac{v^2}{c^2} = - \frac{1}{6} \frac{v^2}{c^2}. \quad \text{(20)} \]

This means \( k_2 = \frac{1}{6} \). This effect is something similar to the cosmological redshift. In fact, Eq. (19) shows that a variation of the \textit{proper distance (time)} between the source and the detector is present, while the radial distance, in the sense of the difference of the radial coordinates of the source and the detector, remains constant. Thus, in a certain sense, the radial coordinate \( r' \) represents a \textit{comoving coordinate}.

In fact, if one defines the proper time that has been measured by the Lorentzian observer during the trajectory as \( \tau_L \equiv \tau_1 \) and the proper time that has been measured
by the rotating observer during the trajectory as \( \Delta r_3 + \tau_{\text{Lor}} \). Equation (19) can be rewritten as

\[
1 + z_2 = \frac{\tau_R}{\tau_L}, \tag{21}
\]

which is similar to the well known formula of the cosmological redshift [46]

\[
1 + z = \frac{a_r}{a_e},
\]

where \( a_r \) and \( a_e \) are the values of the scale factor of the Universe at the instants of the reception and of the emission of the light, respectively. This has an intuitive and intriguing representation if one recalls that in the rotating coordinate system the ratio of the circumference to its radius in a plane with \( z = \text{constant} \) is larger than \( \frac{2\pi}{\sqrt{1 - \frac{v^2}{c^2}}} \) [10]. In fact, the element of spacial distance for the Langevin metric (4) is [10]

\[
dl^2 = dr^2 + dz^2 + \frac{r^2 d\phi^2}{1 - \frac{r^2 \omega^2}{c^2}}, \tag{22}
\]

which immediately shows that the ratio of the circumference to its radius in a plane with \( z = \text{constant} \) is \( \frac{2\pi}{\sqrt{1 - \frac{v^2}{c^2}}} \). Thus, in a certain sense the rotating observer “lives” in a space which is contracted in its proper distance (time) in the radial direction perpendicular to the axis of rotation. This is the physical meaning of Eq. (19).

The following analysis shows that Eqs. (20 and 21) really represent a blueshift. Let us consider the first peak of wave which arrives at the detector after the activation of the source of light. Then, the Lorentzian observer will count a number of wavelengths

\[
N_L \equiv \frac{c \tau_1}{\lambda_L}, \tag{23}
\]

where \( \lambda_L \) is the wavelength of light. Instead, the rotating observer will count a number of wavelengths

\[
N_R \equiv \frac{c \tau_1 + c \Delta \tau_3}{\lambda_R}, \tag{24}
\]

where now the wavelength of light is \( \lambda_R \). Causality requires \( N_R = N_L \) which implies

\[
\frac{\lambda_R}{\lambda_L} = \frac{c \tau_1 + \Delta \tau_3}{c \tau_1} = \frac{\tau_R}{\tau_L}, \tag{25}
\]

which is equal to Eq. (21).

Also in this case the analysis has been made in the rotating frame. But, again, the final measurement is made in the Lorentzian frame in the laboratory. Thus, the observer in the Lorentzian frame will measure an opposite effect of clock synchronization \(-c \Delta \tau_3\) resulting in a redshift expressed by the inverse wavelengths ratio
which corresponds to a second fractional energy shift

\[
\frac{\Delta E_2}{E_1} \simeq -\frac{1}{6} \frac{v^2}{c^2}.
\]  

Therefore, Eqs. (9 and 27) permit to obtain the total fractional energy shift as

\[
\frac{\Delta E}{E_1} = \frac{\Delta E_1}{E_1} + \frac{\Delta E_2}{E_1} \simeq -\frac{2}{3} \frac{v^2}{c^2}.
\]  

This theoretical result is completely consistent with the experimental result \( k = 0.68 \pm 0.03 \) in [39].

4 Discussion

Let us discuss the physical meaning of the above general relativistic analysis. The additional factor \(-\frac{1}{6}\) in Eq. (27) is due to clock synchronization. This generates a further redshift effect that in a certain sense is analogous to the cosmological redshift, because it is due to the variation of the proper distance (time) between the source and the detector in the rotating frame. Missing its presence in [16, 35, 38, 39, 49, 50] was due to the incorrect comparison of proper time rates between the clock of the rotating observer and the clock of the Lorentzian observer. Consequences of this misunderstanding were claims of invalidity of the GTR and/or some unscientific attempt to explain the Mössbauer rotor through “exotic” effects [49, 50]. Such unscientific effects must be instead completely rejected.

Reference [32] is useful for analysing the Langevin line-element. This is similar to the use of the GTR in GPS and permits to gain an interesting physical meaning [13, 14]: the additional effect giving the correction of \(-\frac{1}{6}\) in Eq. (20) is analogous to the additional effect that one must consider in GPS when one takes into account the difference between the time measured in a frame co-rotating with the Earth geoid and the time measured in a non-rotating (locally inertial) Earth centered frame (and also the difference between the proper time of an observer at the surface of the Earth and at infinity). In fact, one cannot merely consider the gravitational redshift due to the Earth’s gravitational field, without considering the effect of the Earth’s rotation. In that case, GPS cannot work [13, 14]! The insight is that one cannot merely use the time elapsing on the orbiting GPS clocks in order to transfer time from one transmission event to another. Instead, one must be careful to consider important path-dependent effects. This is exactly what happened in the above analysis of clock synchronization [13, 14]. In other words, the additional effect giving the important correction \(-\frac{1}{6}\) in Eq. (20) must be considered neither an obscure mathematical artifact nor a negligible physical detail. Instead, it is a remarkable physical effect that
one must take into due account [13, 14]. Further details on the analogy between the above analysis and the use of the GTR in GPS have been highlighted in [13, 14].

For the sake of completeness, we stress that our results are also confirmed by two recent results on the Mössbauer rotor experiment [15, 51]. In [15] the authors wanted to underline some mathematical aspects in general relativistic frameworks. They neither discuss in detail the different physical interpretations of the experimental results proposed during the recent years nor wanted to propose a new one. Instead, starting from the analysis in [13], they analyzed three different types of time involved in the Mössbauer rotor experiment, linking a term introduced in [13] to the difference between coordinate and physical velocity of light. In [51] the approach has been considered from the viewpoint of arbitrary moving continuum. Continuum characteristics have been determined by a deformation tensor, a stress tensor, a strain velocity tensor, a spin tensor and the first curvature vectors of the world lines of medium particles [51]. It has been indeed shown that these physical values are located on the hypersurfaces orthogonal to the world lines of medium particles and specify the physical space [51]. The time counting from the physical space permits to obtain the blueshift of the frequency at the end of the tube that coincides with the above result [51].

Another recent result in [18] has shown that a general relativistic analysis similar to the one proposed in this paper works also for the Sagnac effect.

5 Clock Synchronization is a Real Effect

In the recent paper [35], the Authors claim that the second effect of clock synchronization does not exist. The Authors of [35] criticize indeed the results in the paper of Iovane and Benedetto, i.e. Ref. [15]. In fact, such results are consistent with our previous results in [13, 14, 17] and with the analysis in Sect. 3.2 of this paper, because in [15] it is shown that the Mössbauer rotor apparatus substantially generates a desynchronization of clocks between the reference frame of laboratory and the reference frame of the rotating observer. Consequently, the two reference frames must be re-synchronized as Sect. 3.2 of this paper and in [13, 14, 17]. The criticism of the Authors of [35] is that, in their paper, verbatim,

following Corda, Iovane and Benedetto considered the propagation of photons in the radial direction of the rotating system, and further defined the proper time of the absorber next to the proper time of the detector, and they finally derived the expected energy shift between emission and absorption lines in Mössbauer rotor experiments involving the so-called effect of the “desynchronization of clocks” that they have introduced.

Then, the Authors of [35] add that, verbatim,

However, the fact remains that in all known Mössbauer experiments in a rotating system implemented up to date (including [15, 18–21, 23–27]), the resonant -quanta propagate over the rotor along the straight line, joining a spinning source and a detector as seen by a laboratory observer.
Consequently, the Authors of [35] conclude that the problem of the correct interpretation of the results of Mössbauer experiments in a rotating system cannot be reduced to an unaccounted-for desynchronization effects between the clocks in a rotating system and in a laboratory frame - so much so that further search of possible approaches to the explanation of these experiments in the framework of general theory of relativity (GTR) is required.

Here we do not want to enter in the details of the paper of Iovane and Benedetto [15] but we must stress that, concerning our analysis, the Authors of [35] make confusion. In fact, one can easily check in Sect. 3.2 of this paper as well as in [17] that we considered light propagation in the radial direction of the observer in the laboratory, not in the radial direction of the rotating observer. Thus, the criticism of the Authors of [35] is completely inconsistent.

On the other hand, the claim in [35] can be easily dismissed with the following analysis. Let us rewrite Eq. (12) as

\[ d\tau^2 = g_{00}dt^2. \]  

(29)

We recall that this equation works for any two infinitesimally separated events occurring at one and the same point in space. In the Lorentzian frame one has \( g_{00} = 1 \). Then

\[ d\tau_L^2 = dt_L^2, \]  

(30)

where we re-labelled with \( \tau_L \) and \( t_L \) the proper time and the coordinate time of the observer in the Lorentzian frame. In the case of the rotating clock one has \( g_{00} = \left( 1 - \frac{r^2\omega^2}{c^2} \right) \). Then

\[ d\tau_R^2 = \left( 1 - \frac{r^2\omega^2}{c^2} \right) dt_R^2, \]  

(31)

where we re-labelled with \( \tau_R \) and \( t_R \) the proper time and the coordinate time of the rotating observer. But, from the Langevin transformation (3) it is

\[ t_R = t_L. \]  

(32)

Then, from the last 3 equations one immediately gets

\[ d\tau_R^2 = \left( 1 - \frac{r^2\omega^2}{c^2} \right) d\tau_L^2, \]  

(33)

which means that clock of the rotating observer and the clock of the Lorentzian observer in the laboratory measure the same proper time if and only if \( r = r' = 0 \), i.e. only in the origin of the two reference frames. Thus, clock synchronization is necessary in all the other points. Hence, Eq. (33) gives the difference of proper time between the rotating observer and the observer at rest in the laboratory frame. Thus,
we have shown that the claim in [35] that the second effect of clock synchronization does not exist is wrong.

6 Conclusion Remarks

Both the history of the Mössbauer rotor effect and its interpretation from the point of view of Einstein’s GTR have been analysed by adding some new insight. In particular, it has been shown that, if on one hand the “traditional” effect of redshift has a strong analogy with the gravitational redshift, the additional effect of redshift due to clock synchronization has an intriguing analogy with the cosmological redshift. The general relativistic interpretation of the Mössbauer rotor experiment is an important consequence of the EEP. In Sect. 5 of this paper it has been shown that the claim in [35] that the second effect of clock synchronization does not exist is not correct.

Acknowledgements The Author thanks an unknown Referee for very useful comments and suggestions.

References

1. Sagnac, G.: L’ether lumineux demontre par l’effet du vent relativ d’ether dans un interferometre en rotation uniforme. C. R. 157, 708 (1913)
2. Sagnac, G.: Sur la preuve de la realite de l’ether lumineux par l’experience de l’interferographe tournant.Par l’effet du vent relativ d’ether dans un interferometre en rotation uniforme. C. R. 157, 1410 (1913)
3. von Laue, M.: Zur Diskussion iiber den starren Korper in der Relativitatstheorie. Phys. Z. 12, 85 (1911)
4. Einstein, A.: Bemerkung zu P. Harzers Abhandlung: Die Mitfuhrung des Lichtes in Glas und die A aberration. Astron. Nachr. 199, 8 (1914)
5. Einstein, A.: Antwort auf eine Replik P. Harzers. Astron. Nachr. 199, 47 (1914)
6. von Laue, M.: Theoretisches liber optische Beobachtungen zur Relativitatstheorie. Phys. Z. 21, 659 (1920)
7. Langevin, P.: Sur la theorie de relativite et l’experience de M. Sagnac. C. R. 173, 831 (1921)
8. Langevin, P.: Relativite—Sur l’experience de Sagnac. C. R. 2015, 304 (1937)
9. Rizzi, G., Ruggiero, M.L.: The relativistic Sagnac Effect: two derivations. In: Rizzi, G., Ruggiero, M.L. (eds.) Relativity in Rotating Frames. Kluwer Academic Publishers, Dordrecht (2003)
10. Landau, L.D., Lifshitz, E.M.: The Classical Theory of Fields, 2nd edn. Pergamon Press, Oxford (1962)
11. Einstein, A.: The collected papers, Volume 6: the Berlin years: writings, 1914–1917 (English translation supplement) pp. 31–32
12. Pauli, W.: Theory of Relativity. Pergamon Press, London (1958)
13. Corda, C.: New proof of general relativity through the correct physical interpretation of the Mössbauer rotor experiment. Int. J. Mod. Phys. D 27, 1847016 (2018)
14. Corda, C.: Interpretation of Mossbauer experiment in a rotating system: a new proof for general relativity. Ann. Phys. 355, 360 (2015)
15. Iovane, G., Benedetto, E.: Coordinate velocity and desynchronization of clocks. Ann. Phys. 403, 106 (2019)
16. Kündig, W.: Measurement of the transverse Doppler effect in an accelerated system. Phys. Rev. 129, 2371 (1963)
17. Corda, C.: Mossbauer rotor experiment as new proof of general relativity: rigorous computation of the additional effect of clock synchronization. Int. J. Mod. Phys. D 28, 1950131 (2019)
18. Benedetto, E., Feleppa, F., Licata, I., Moradpour, H., Corda, C.: On the general relativistic framework of the Sagnac effect. Eur. Phys. J. C 79, 187 (2019)
19. Lichtenegger, H., Mashhoon, B.: In: Iorio, L. (ed.) Second chapter of The Measurement of Gravitomagnetism: A Challenging Enterprise. Nova Science, New York (2007)
20. Brill, D.R., Cohen, J.M.: Rotating masses and their effect on inertial frames. Phys. Rev. 143, 1011 (1966)
21. Schucking, E.L., Surowitz, E.J.: Einstein’s apple: His first principle of equivalence. http://arxiv.org/ gr-qc/0703149
22. Kerr, R.P.: Gravitational field of a spinning mass as an example of algebraically special metrics. Phys. Rev. Lett. 11, 237 (1963)
23. Newman, E.T., Couch, E., Chinnapared, K., Exton, A., Prakash, A., Torrence, R.: Metric of a rotating, charged mass. J. Math. Phys. 6, 918 (1965)
24. Pfister, H.: On the history of the so-called Lense-Thirring effect. Gen. Relat. Gravit. 39, 1735 (2007)
25. Iorio, L., Lichtenegger, H.I.M., Ruggiero, M.L., Corda, C.: Phenomenology of the Lense-Thirring effect in the solar system. Astrophys. Space Sci. 331, 351 (2013)
26. Iorio, L.: The post-Newtonian gravitomagnetic spin-octupole moment of an oblate rotating body and its effects on an orbiting test particle; are they measurable in the Solar System? Mon. Not. R. Astron. Soc. 484, 4811 (2019)
27. Iorio, L.: Measuring general relativistic dragging effects in the Earth’s gravitational field with ELXIS: a proposal. Class. Quant. Gravit. 36, 035002 (2019)
28. Iorio, L., Corda, C.: Gravitomagnetism and Gravitational Waves. Open Astron. J. Suppl. 1–M5, 84 (2011)
29. Ruggiero, M.L., Tartaglia, A.: Test of gravitomagnetism with satellites around the Earth. Eur. Phys. J. Plus 134, 205 (2019)
30. Ruggiero, M.L.: Gravitomagnetic field of rotating rings. Astrophys. Space Sci. 361, 140 (2016)
31. Ruggiero, M.L.: Gravito-electromagnetic effects of massive rings. Int. J. Mod. Phys. D 24, 1550060 (2015)
32. Ashby, N.: Relativity in the global positioning system. Living Rev. Relat. 6, 1 (2003)
33. Oort, J.H.: The force exerted by the stellar system in the direction perpendicular to the galactic plane and some related problems. Bull. Astron. Neth. 6, 249 (1932)
34. Zwecky, F.: Die Rotverschiebung von extragalaktischen Nebeln. Helv. Phys. Acta 6, 110 (1933)
35. Kholmetskii, A.L., Yarman, T., Yarman, O., Arik, M.: Concerning Mössbauer experiments in a rotating system and their physical interpretation. Ann. Phys. 411, 167912 (2019)
36. Mössbauer, R.L.: Kernresonanzfluoreszenz von Gammastrahlung in Ir\(^{191}\). Z. Phys. A (in German) 151, 124 (1958)
37. Hofstadter, R.: The Electron-Scattering Method and Its Application to the Structure of Nuclei and Nucleons. Nobel Lecture. Accessed 11 Dec 1961
38. Kholmetskii, A.L., Yarman, T., Missevitch, O.V.: Kündig’s experiment on the transverse Doppler shift re-analyzed. Phys. Scr. 77, 035302 (2008)
39. Kholmetskii, A.L., Yarman, T., Missevitch, O.V., Rogozev, B.I.: A Mössbauer experiment in a rotating system on the second-order Doppler shift: confirmation of the corrected result by Kündig. Phys. Scr. 79, 065007 (2009)
40. Hay, H.J., et al.: Measurement of the red shift in an accelerated system using the Mössbauer effect in \(^{57}\)Fe\(^{2+}\). Phys. Rev. Lett. 4, 165 (1960)
41. Hay, H. J.: In: Schoen, A., Compton D.M.T. (eds.) Proceedings of the 2nd Conference on the Mössbauer Effect, p. 225. Wiley, New York (1962)
42. Granshaw, T.E., Hay, H.J.: In: Proceedings of the International School of Physics, Enrico Fermi, p. 220. Academic, New York (1963)
43. Champeney, D.C., Moon, P.B.: Absence of Doppler shift for gamma ray source and detector on same circular orbit. Proc. Phys. Soc. 77, 350 (1961)
44. Champeney, D.C., Isaak, G.R., Khan, A.M.: A time dilatation experiment based on the Mössbauer effect. Proc. Phys. Soc. 85, 583 (1965)
45. Rizzi, G., Ruggiero, M.L.: Space geometry of rotating platforms: an operational approach. Found. Phys. 32, 1525 (2002)
46. Misner, C.W., Thorne, K.S., Wheeler, J.A.: Gravitation. Feeman and Company, London (1973)
47. Einstein, A.: Die Grundlage der allgemeinen Relativitatstheorie. Ann. Phys. 354, 769 (1916)
48. Weyl, H.: Raum, Zeit, Materie: Ressource Vorlesungen uber Allgemeine Relativitatstheorie. Springer, Berlin (1923)
49. Yarman, T., Kholmetskii, A.L., Arik, M.: Mössbauer experiments in a rotating system: Recent errors and novel interpretation. Eur. Phys. Jour. Plus 130, 191 (2015)
50. Kholmetskii, A.L., Yarman, T., Yarman, O., Arik, M.: Response to The Mössbauer rotor experiment and the general theory of relativity by C. Corda. Ann. Phys. 374, 247 (2016)

51. Foukzon, J., Men'kova, E.R.: Comment on The Mössbauer rotor experiment and the general theory of relativity [Ann. Physics 368, 258–266 (2016)] Ann. Phys. 413, 168047 (2020)

**Publisher’s Note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.