The Relationship Between the Volume and Pressure in Ideal Gas

Zhouhao Bao
Wuxi No.1 Senior High School, Wuxi, Jiangsu, 214031, PR. China
chunchenxiao@cas-harbour.org

Abstract. The ideal gas is a physical model for studying the properties of gases. As a lot of students feel difficult in learning this part, I will introduce an interesting way to prove the relationship between the volume and pressure in ideal gas, by the assistance of computer.

Keyword: Ideal gas, volume and pressure, computer, data analysis.

1. Introduction:
We are trying to find out the relationship between the Volume and Pressure in the gas. In order to find out this relationship, we expect an ideal gas. An ideal gas is a gas which doesn’t have any Potential Energy among the gas molecules, which means that there are only Kinetic Energy among them.

![Figure 1 Movement of the gas particles](image)

In Figure 1, the gas particles are moving freely and randomly. The collisions between the particles are perfectly elastic, indicating the total momentum of the particles is unchanged.

The pressure exerted by the gas particles can be deduced from the following equation:

Newton’s Second Law:

\[ \dot{\mathbf{p}} = \frac{\Delta \mathbf{p}}{t} \]

Applied this equation to the pressure law:

\[ \mathbf{p} = \frac{F}{A} = \frac{\Delta p}{At} \]
In the ideal gas the pressure exerted to the wall of the container is increasing while the volume of the gas decreases.

\[ PV = k \]  
(k is a constant number)

We design an experiment by setting up digital apparatus and measure the pressure at volume from 2cm$^3$ to 20cm$^3$ at an interval of 2cm$^3$ in order to know The relationship between the volume and pressure in ideal gas.

2. The Research Process

2.1. Research Question:
How does the pressure in an idea gas related to the volume of the gas.

2.2. Hypothesis:
The pressure of an ideal gas will increase as the volume of the gas decreases.

2.3. Variables:
1) Independent Variable:
   a) The volume of the gas
2) Dependent Variable:
   a) The pressure exerted to the wall
3) Controlled Variables:
   a) Temperature
   b) Syringe
   c) Mass of the gas

Safety:
1) We should pay more attention to the syringes because they are easily broken and it may damage your hand.
2) We should try our best to keep the syringes stable otherwise it will broken if there is a force, which is too much, caused on the surface of the syringe.

2.4. Apparatus

![Figure 2 Required instrument.](image)

2.5. Method and Steps:
1) Connect the gas pressure sensor to the computer
2) Set the volume inside the syringe 10ml and plug the soft tube into the syringe and wait to see if the syringe is not broken.
3) Push the syringe and compress the gas to a volume of 2ml.
4) Wait for the value on the computer monitor become stable and record it.
5) Repeat the step 4 for 5 times for each value.
6) Calculate the average.

2.6. Data Analysis
1) Collecting the data. See the data in Table.1 as following:

| Volume | 1/Volume | Pressure Pa x 10^3 |
|--------|----------|--------------------|
| 2      | 0.50     | 455.0              |
| 4      | 0.25     | 219.9              |
| 6      | 0.17     | 162.1              |
| 8      | 0.13     | 122.0              |
| 10     | 0.10     | 103.5              |
| 12     | 0.09     | 93.8               |
| 14     | 0.07     | 73.3               |
| 16     | 0.06     | 64.7               |
| 18     | 0.06     | 55.5               |
| 20     | 0.05     | 53.5               |

2) Processing the data
i. With the value of pressure above, the average value of pressure for each volume. This can be done by adding all 3 values for a specific volume and divide it by three.

Table.2 The average value of pressure for each volum

| Volume | average Pa x 10^3 |
|--------|------------------|
| 2      | 450.7            |
| 4      | 323.97           |
| 6      | 167.53           |
| 8      | 126.00           |
| 10     | 103.63           |
| 12     | 86.47            |
| 14     | 75.23            |
| 16     | 65.57            |
| 18     | 58.97            |
| 20     | 53.50            |

\[ P_{avg} = \frac{P_1 + P_2 + P_3}{3} \]

E.g. \[ P = \frac{455 + 219.9 + 162.1}{3} = 450.7 \]

ii. The standard deviation for a group of three pressure volume is calculated by using following formula:

\[ \sigma = \sqrt{\frac{(P_1 - \mu)^2 + (P_2 - \mu)^2 + (P_3 - \mu)^2}{3}} \]
Table 3 The average value of pressure for volum

| Volume $m^3 \times 10^{-6}$ | P (Pa) |
|-----------------------------|--------|
| 2                           | 16.0   |
| 4                           | 11.4   |
| 6                           | 4.7    |
| 8                           | 3.6    |
| 10                          | 0.2    |
| 12                          | 2.4    |
| 14                          | 1.7    |
| 16                          | 0.8    |
| 18                          | 0.5    |
| 20                          | 0.3    |

Then we choose the largest standard deviation and use it for constructing the vertical error bar. Slope uncertainty is calculated from the error bar. And finally, to plot a straight line for further investigation, we divide 1 by each volume to get $\frac{1}{V}$.

Table 4 Straight line for further investigation

| Volume $m^3 \times 10^{-6}$ | 1/Volume $m^{-3} \times 10^6$ |
|-----------------------------|-------------------------------|
| 2                           | 0.50                          |
| 4                           | 0.25                          |
| 6                           | 0.17                          |
| 8                           | 0.13                          |
| 10                          | 0.10                          |
| 12                          | 0.08                          |
| 14                          | 0.07                          |
| 16                          | 0.06                          |
| 18                          | 0.06                          |
| 20                          | 0.05                          |

iii. With the data collected and calculated above, $P$ can be plotted against $\frac{1}{V}$.

Figure 3 Relationship between $V$ and $k$

As $1/Volume$ is proportional to the pressure, the y-interception can be derived by substitute another value:

$$b = 450.7 \text{Pa} \times 10^3 - \frac{0.9 \text{Pa} \text{m}^3}{2 \text{m}^3 \times 10^{-6}} = 1.7 \times 10^{-10}$$
3. Evaluation

Table 5 Evaluation result

| Error                          | Significance & Evidence | Improvements                               |
|--------------------------------|-------------------------|--------------------------------------------|
| Systematic Error               |                         |                                            |
| Remaining gas: The gas inside connecting tube will have increase the volume of the gas inside the syringe | Insignificant. The extra gas in the connecting tube decrease the expected pressure. So the line will move upward a little bit. | Use a shorter tube will reduce the effect. |
| Gas leaking: leaking of gas will makek smaller. | Significant. When we added water to the contact of the tubes. The k for each volume increases. | Use water to conceal the contacts |
| Too small range: when the pressure is above 460, the sensor will not show the value correctly. | Significant: Cannot take smaller value to increase the reliability of the best-fit line | Use a better sensor |
| Random Error                   |                         |                                            |
| Eyesight on syringe: when we look at the mark on the syringe, the eyesight cannot read the exact value of the syringe | Insignificant. The little change in value cannot influence the whole trend. | Try to get more readings to decrease the impact. |
| Hard to control the syringe steadily. To be more detailed, especially when the scale that we should achieve with the piston is very low, which means high force is needed to push the syringe and keep the piston at a constant scale, such as 4ml, and it is hard for us to keep the piston at a stable scale. | Insignificant. It will change the volume little and effect the result we get little. | Make the one who has more power to control the syringe. Because the one who has more power means he can easily push the piston and keep it at a place(scale) steadily. |

4. Conclusion
The data collected from the experiment is good for the previous hypothesis. The relationship between the 1/volume and pressure is proportional. Although the line doesn’t go through zero, the y intercept is very small so we can consider it as an error.

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