Calculation of Passive Earth Pressure on Rigid Retaining Wall Affected by Seepage and Soil Arching

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Abstract: Seepage and soil arching are commonly encountered in the soil behind a rigid retaining wall, so the influence of these factors should be considered when calculating the earth pressure of the retaining wall. In this paper, behind a rigid retaining wall the steady seepage field was analyzed, the soil arching was also presented assuming the parabola soil arching, then improved formula of passive earth pressure on rigid retaining wall is obtained based on the horizontal differential element method. The result showed that the calculation of the passive earth pressure by the horizontal differential element method is consistent with the Coulomb’s earth pressure theory. Considering the seepage effect will reduce the passive earth pressure, while considering the soil arching effect will not change the passive earth pressure. The height of application of passive force from bottom is less than 1/3 of the height of the retaining wall. Considering the soil arching, the height of application of passive force from bottom is moved down, and the height of application of passive force from bottom is moved up by considering the seepage.

1. Introduction  
Earth pressure on retaining walls is an ancient subject in civil engineering, since the 20th century, a large number of scholars have studied the earth pressure of retaining walls. Numerical methods such as the finite difference method, finite element method, and boundary element method are normally employed. Passive earth pressure on rigid retaining wall affected by seepage and soil arching has been highlighted in engineering practice and academic research. The governing equation for seepage can be simplified as a two-dimensional Laplace equation with prescribed boundary conditions. The active earth pressure increases and the passive earth pressure reduces on retaining wall by stable seepage is studied by Barros (2006) [1], Blake (2013) [2], and Hu et al (2017,2018) [3-4]. Some scholars have performed research on the pressure of a retaining wall under the influence of the soil arching. The direction of the large principal stresses during the soil arching is deflected [5]. The trajectory of the large principal stresses is observed or inferred as a catenary [6-7], an arc [8-9], or is parabolic [10]. Peng et al (2012) [11] obtained a formula for the passive pressure with linear characteristics. Zhu and Zhao (2014) [12] deduced the unified expressions of active and passive pressure on the basis of the existing soil arching effect. However, considering the coaction of seepage and soil arching, the passive pressure of rigid retaining wall is not presented.

In this paper, the stable seepage and soil arching behind rigid retaining wall are analyzed, and then considering their coaction the calculation of earth pressure on retaining wall is put forward using the horizontal differential element method. And the results were compared with that of the existing
calculation methods for the passive earth pressure.

2. Seepage behind rigid retaining wall

The Cartesian coordinate system xOy is established which take the top point of rigid retaining wall as coordinate origin and is used to analyze the seepage behind the retaining wall, as is shown in Fig.1. When it rains, seepage in the soil behind rigid retaining wall can be formed. The seepage velocity is commonly low and its Reynolds number is less than 1, so the seepage field with water head \( h(x,y) \) can be defined as laminar flow and obeys Darcy’s law. After a continuous raining, the seepage field changes from unsteady status to steady status and the differential equation of two-dimensional steady seepage is the two-dimensional Laplace differential equation as follow

\[
\nabla^2 h(x,y) = 0
\]

The boundary conditions of Laplace differential equation are defined as

\[
h(x,0) = H, \quad \frac{\partial h}{\partial x}(x, H) = 0, \quad h(\infty, y) < \infty, \quad h(0, y) = H - y
\]

Using method of separation of variables, the water head of seepage field is obtained as:

\[
h(x,y) = H - \sum_{n=0}^{\infty} (-1)^n \frac{2H}{m^2} \sin \left( \frac{my}{H} \right) e^{-\frac{mx}{H}}, \quad m = \frac{2n+1}{2} \pi
\]

the pore-water pressure resulting from the steady seepage field is written as

\[
u(x,y) = \gamma_w [h(x,y) - (H - y)] = \gamma_w [y - \sum_{n=0}^{\infty} (-1)^n \frac{2H}{m^2} \sin \left( \frac{my}{H} \right) e^{-\frac{mx}{H}}]
\]

And the pore-water pressure along the slip surface is as follows

\[
u(y) = \gamma_w [y - \sum_{n=0}^{\infty} (-1)^n \frac{2H}{m^2} \sin \left( \frac{my}{H} \right) e^{-\frac{m(y-x)cot\beta}{H}}]
\]

Where \( \beta \) is the dip angle of slip surface, and is determined by equation (6).

![Fig. 1 Diagram of steady seepage field analysis](image)

3. Soil arching behind rigid retaining wall

According to Coulomb's theory, when the rigid wall produces a large enough displacement in the direction of the soil behind the wall, the soil behind the retaining wall reaches the ultimate equilibrium state and forms a sliding wedge.

\[
\beta = \arctan \{tan\varphi [\sqrt{\cot\varphi cot(\varphi + \delta) + 1} - 1]\}
\]

where \( \varphi \) is the internal friction angle of soil, \( \delta \) is the soil-wall interface friction angle.

As illustrated in reference [13-14], in the sliding wedge the large principal stress \( \sigma_1 \) rotates from horizontal direction in a passive limit state and forms a new trace, as shown in Fig.2. Then the normal line of any point on the new trace represents the direction of the minor principal \( \sigma_3 \) stress after
rotating, and the angle between its normal line and the horizontal line is \( \theta \), and the angle between its tangent line and the horizontal line is \( \pi/2 - \theta \). The horizontal normal stress \( \sigma_h \), vertical normal stress \( \sigma_v \) and horizontal shear stress \( \tau_v \) are calculated respectively as

\[
\sigma_h = \sigma_1 \sin^2 \theta + \sigma_3 \cos^2 \theta \tag{7}
\]
\[
\sigma_v = \sigma_1 \cos^2 \theta + \sigma_3 \sin^2 \theta \tag{8}
\]
\[
\tau_v = (\sigma_1 - \sigma_3) \sin \theta \cos \theta \tag{9}
\]

In Fig.2, for point D on retaining wall, \( \theta \) is equal to \( \theta_1 \), and \( \tau_v = \sigma_h \tan \delta \), we can get

\[
\theta_1 = \arctan \left( \frac{(Kp_0 - 1)^2 - 4Kp_0 \tan \delta}{2Kp_0 \tan \delta} \right), \quad Kp_0 = \frac{\sigma_1}{\sigma_3} = \frac{1 + \sin \varphi}{1 - \sin \varphi} \tag{10}
\]

Where \( Kp_0 \) is the coefficient of passive force by Rankine theory.

Fig.2 Analysis sketch of major principal stress arch behind wall

On slip surface, the angle between the normal line and the horizontal line \( \theta \) is equal to \( \theta_2 \) calculated by

\[
\theta_2 = \frac{\pi}{4} + \frac{\varphi}{2} + \beta \tag{11}
\]

The soil arching curve is defined as new trace of a major principal stress trace assumed to be a parabola in reference \([13-14]\) and is described by

\[
y = \frac{B_0}{2} x^2 + B_1 x + B_2 \tag{12}
\]

Where \( B_0, B_1, B_2 \) are the parameters of soil arching curve.

Then the boundary conditions are defined as

\[
x = 0, y = y_0, y' = \frac{1}{\tan \theta_1} \tag{13}
\]
\[
x = (H - y) \cos \beta, y' = \frac{1}{\tan \theta_2} \tag{13}
\]

From the boundary condition equations (13), the soil arching curve is obtained as

\[
y = \frac{1}{2L \tan \theta_2} \left( \frac{1}{\tan \theta_1} - \frac{1}{\tan \theta_2} \right) x^2 + \frac{1}{\tan \theta_1} x + y_0 \quad 0 \leq x \leq L, L = (H - y_0) \cot \beta \tag{14}
\]

The average vertical stress \( \sigma_{pv} \) and the average horizontal shear stress \( \tau_{pv} \) along the soil arching
curve is calculated respectively as
\[ \sigma_{pv} = \frac{1}{L} \int_0^L \sigma_x \, dx \quad \tau_{pv} = \frac{1}{L} \int_0^L \tau_x \, dx \] (15)

Referring the description in reference [13], the horizontal coefficient of passive pressure \( K_{pw} \) is defined as the ratio of the horizontal normal stress \( \sigma_h \) on the retaining wall and the average vertical stress \( \sigma_{pv} \), and is written as follow
\[ K_{pw} = \frac{\sigma_h}{\sigma_{pv}} = \frac{\cos^2 \theta_1 + K_{p0} \sin^2 \theta_1}{1 + (K_{p0}-1)(1 + \cot \theta_2 - \cot \theta_1)} \] (16)

Then the frictional coefficient \( \tan \phi' \) between two adjacent horizontal soil layers is defined as follow
\[ \tan \phi' = \frac{\tau_{pv}}{\sigma_{pv}} = \frac{(K_{p0}-1)\frac{\sin \theta}{\sin \theta' \sin \theta''}}{1 + (K_{p0}-1)(1 + \cot \theta_2 - \cot \theta_1)} \] (17)

When \( \theta_1 = \theta_2 = \pi/2 \), this means that the minor principal stress does not rotate and soil arching curve is a horizontal line. There is no soil arching effect behind the retaining wall while the horizontal coefficient of passive pressure \( K_{pw} \) is equal to \( K_{p0} \) and \( \tan \phi' = 0 \).

4. Calculation model of passive pressure under the seepage and soil arching effect
Considering seepage and soil arching effect, a differential flat element model of passive pressure on retaining wall is obtained, as shown in Fig.3. The differential flat element is located at any depth of \( y \) in the slip wedge behind retaining wall and has a thickness of \( dy \). Fig.3 shows that under the passive limited state, the horizontal flat element is subjected to vertical gravity \( dW \), average vertical stresses \( \sigma_{pv} \) and \( \sigma_{pv} + d\sigma_{pv} \), the average horizontal shear stress \( \tau_{pv} \) and \( \tau_{pv} + d\tau_{pv} \) acting on the upper and lower surfaces of the element, and the horizontal shear stress \( \tau_{hx} \) and the horizontal stress \( \sigma_{hx} \) acting on left face of the element, the shear stress \( \tau_\beta \), the normal stress \( \sigma_\beta \) and pore water pressure \( u(y) \) induced by seepage acting on slip face of the element.

The horizontal equilibrium of differential flat element indicates that
\[ \sigma_{hx} \, dy + d\left[ \tau_{pv}(H-y)\cot \beta \right] - \tau_\beta \frac{dy}{\sin \beta} \cos \beta - \sigma_\beta \frac{dy}{\sin \beta} \sin \beta - u(y) \frac{dy}{\sin \beta} \sin \beta = 0 \] (19)

The vertical equilibrium of differential element indicates that
\[ \tau_{hx} \, dy - d[\sigma_{pv}(H - y) \cot \beta] + \gamma(H - y) \cot \beta \, dy + \tau_{\beta} \frac{dy}{\sin \beta} \sin \beta - \sigma_{pv} \frac{dy}{\sin \beta} \cos \beta - u(y) \, dy = \frac{30}{0} \]

When \( \tau_{\beta} \) is eliminated using Equations (19-20), and then

\[ \frac{d\sigma_{pv}}{dy} = \frac{1 - C_{1}}{H - y} \sigma_{pv} + C_{2} \gamma_{sat} - C_{3} \frac{u(y)}{H - y} \]

\[ C_{1} = \frac{\tan \beta_{K_{pw}}}{\tan(\beta + \varphi_{\prime}) + \tan \beta_{K_{pw}}} \]

\[ C_{2} = \frac{\tan(\beta + \varphi_{\prime}) + \tan \beta_{K_{pw}}}{\tan(\beta + \varphi_{\prime})} \]

\[ C_{3} = \frac{\tan(\beta + \varphi_{\prime}) + \tan \beta_{K_{pw}}}{\tan(\beta + \varphi_{\prime})} \]

Therefore, the average vertical stresses is obtained as

\[ \sigma_{pv} = \gamma_{sat} H \frac{c_{2}}{2 - c_{1}} \left[ \left( \frac{H - y}{H} \right) c_{1} - \frac{H - y}{H} \right] - C_{3} \int_{0}^{y} u(y) \left( 1 - \frac{y}{H} \right) \frac{c_{1}}{2} \, dy \]

Then the horizontal passive earth pressure \( p_{px} \) is presented as follow

\[ p_{px} = K_{pw} \left[ \gamma_{sat} H \frac{c_{2}}{2 - c_{1}} \left[ \left( \frac{H - y}{H} \right) c_{1} - \frac{H - y}{H} \right] - C_{3} \int_{0}^{y} u(y) \left( 1 - \frac{y}{H} \right) \frac{c_{1}}{2} \, dy \right] \]

The passive earth force \( P_{p} \) is presented as follow

\[ P_{p} = \int_{0}^{H} p_{px} \, dy = K_{pw} \left[ \gamma_{sat} H \frac{c_{2}}{2 - c_{1}} \left[ \left( \frac{H - y}{H} \right) c_{1} - \frac{H - y}{H} \right] - C_{3} \int_{0}^{y} u(y) \left( 1 - \frac{y}{H} \right) \frac{c_{1}}{2} \, dy \right] \]

The height of application of passive earth force from bottom \( H_{0} \) is presented as follow

\[ H_{0} = \frac{\int_{0}^{H} p_{px}(H - y) \, dy}{P_{p}} = H \frac{2c_{1} c_{2} \gamma_{sat} - c_{3} U_{1} y_{w}}{3 c_{1} + 1} \]

Where \( U_{1} = 1 - \sum_{n=0}^{m} \frac{4 \tan \beta_{\prime}(1 - \tan \beta_{\prime})}{(1 + \tan \beta_{\prime})^{2}} \)

The coefficient of passive earth force \( K_{p} \) is presented as follow

\[ K_{p} = \frac{\int_{0}^{H} p_{px} \, dy}{\gamma_{sat} H^{2}/2} = C_{2} - C_{3} U_{1} y_{w} \]

Where \( U_{2} = 1 - \sum_{n=0}^{m} \frac{12 \tan^{2} \beta_{\prime}(1 - \tan \beta_{\prime})}{(1 + \tan \beta_{\prime})^{2}} \)

In Equation (24-27), \( y_{w}, K_{pw} \) and \( \tan \varphi' \) are important parameters, and indicates whether seepage and soil arching are considered. When \( y_{w} = 0 \) and \( K_{pw} = K_{p0} \) & \( \tan \varphi' = 0 \), this model does not consider the soil arching and seepage. When \( y_{w} = 0 \) and \( \tan \varphi' = 0 \), this model only considers the seepage. When \( y_{w} = 0 \) and \( \tan \varphi' = 0 \), this model only considers the soil arching. When \( y_{w} = 0 \) and \( \tan \varphi' = 0 \), this model considers the seepage and soil arching.

5. Comparative analyses and discussions

Fig. 4 plots the passive earth pressure distribution around the top of the rigid retaining wall. In Fig. 4, \( \varphi = 30^\circ, \delta = 15^\circ, \gamma_{sat} = 20 \text{kN/m}^{2} \). It is known that the passive earth pressure distribution along the retaining wall by the horizontal differential element method is non-linear. Considering seepage and soil arching effects will make the non-linear distribution of passive earth pressure more prominent, which is more consistent with the actual situation.
To further assess the effects of seepage and soil arching on passive earth pressure, Table 1 presents the comparisons of $K_p$ values between the present study and the Coulomb’s theory. In Table 1, the coefficient of passive earth force without seepage by this paper proposed method is the same as that of coulomb theory, which verifies the rationality of this method. For this present study, considers the effect of soil arching does not change the $K_p$ values. The seepage will reduce the $K_p$ values, and the $K_p$ values difference between the horizontal differential element method considering the seepage and the horizontal differential element without considering the seepage increases with the increase of $\phi$ and $\delta$.

| $\phi$ | $\delta/\phi$ | The present study | Coulomb |
|-------|--------------|------------------|---------|
|      |              | seepage | soil arching | without seepage and soil arching | |
| 20    | 1/4          | 2.01    | 2.31        | 2.31 | 2.31 |
|       | 1/2          | 2.25    | 2.64        | 2.64 | 2.64 |
|       | 3/4          | 2.55    | 3.03        | 3.03 | 3.03 |
| 25    | 1/4          | 2.46    | 2.94        | 2.94 | 2.94 |
|       | 1/2          | 2.91    | 3.55        | 3.55 | 3.55 |
|       | 3/4          | 3.50    | 4.39        | 4.39 | 4.39 |
| 30    | 1/4          | 3.06    | 3.80        | 3.80 | 3.80 |
|       | 1/2          | 3.88    | 4.98        | 4.98 | 4.98 |
|       | 3/4          | 5.13    | 6.83        | 6.83 | 6.83 |
| 35    | 1/4          | 3.90    | 5.05        | 5.05 | 5.05 |
|       | 1/2          | 5.43    | 7.36        | 7.36 | 7.36 |
|       | 3/4          | 8.30    | 11.86       | 11.86 | 11.86 |
| 40    | 1/4          | 5.12    | 6.95        | 6.95 | 6.95 |
|       | 1/2          | 8.19    | 11.77       | 11.77 | 11.77 |
|       | 3/4          | 16.11   | 24.93       | 24.93 | 24.93 |

The influence of seepage and soil arching on the height of application of passive earth force from bottom $H_0$ by the proposed method is further analyzed by comparing with the method of Cao et al (2019), as shown in Table 2. Cao et al (2019) inferred the trajectory of the large principal stresses as a circular arc. As seen in Table 2, the $H_0$ by the present study is less than that by the method of Cao et
It can be seen that the \( H_0 \) under the soil parabola arching is less than that under the circular arc arching. Whether it is this present study or Cao’s method, the position of \( H_0 \) is always lower than 1/3 of the height of the retaining wall. In this present study, considering the soil arching effect, the \( H_0 \) is moved down, and the \( H_0 \) is moved up by considering the seepage effect.

**Table 2 Comparison of the height of application of passive force**

| \( \varphi \) | \( \delta/\varphi \) | The present study | Cao et al. (2019) |
|---|---|---|---|
| | | seepage | soil arching | without seepage and soil arching |
| 20 | 1/4 | 0.3262 | 0.3004 | 0.3130 | 0.3268 |
| | 1/2 | 0.3109 | 0.2641 | 0.2934 | 0.3191 |
| | 3/4 | 0.2926 | 0.2239 | 0.2738 | 0.3094 |
| 25 | 1/4 | 0.3238 | 0.2893 | 0.3051 | 0.3243 |
| | 1/2 | 0.2992 | 0.2417 | 0.2769 | 0.3130 |
| | 3/4 | 0.2716 | 0.1910 | 0.2481 | 0.2983 |
| 30 | 1/4 | 0.3210 | 0.2763 | 0.2953 | 0.3213 |
| | 1/2 | 0.2832 | 0.2152 | 0.2562 | 0.3050 |
| | 3/4 | 0.2424 | 0.1535 | 0.2148 | 0.2826 |
| 35 | 1/4 | 0.3140 | 0.2605 | 0.2832 | 0.3174 |
| | 1/2 | 0.2610 | 0.1839 | 0.2298 | 0.2941 |
| | 3/4 | 0.2013 | 0.1114 | 0.1718 | 0.2599 |
| 40 | 1/4 | 0.3039 | 0.2412 | 0.2680 | 0.3122 |
| | 1/2 | 0.2298 | 0.1467 | 0.1958 | 0.2785 |
| | 3/4 | 0.1443 | 0.0667 | 0.1171 | 0.2250 |

**6. Conclusions**

This paper analyzed the pore-water pressure in soil induced by stable seepage field and the soil arching action using the assumption of parabola arching trace of large principal stress, then on this basis, a new calculation method of passive earth pressure on rigid retaining wall is put forward using the horizontal differential element method. The conclusions are drawn as follows.

The height of application of passive force from bottom is less than 1/3 of the height of the retaining wall, and the closer to the bottom of the wall with the increase of \( \varphi \) and \( \delta \). Considering the soil arching, the height of application of passive force from bottom is moved down, and the height of application of passive force from bottom is moved up by considering the seepage.

The passive earth force calculated by the horizontal differential element method considering the soil arching, the horizontal differential element method without considering the soil arching and the Coulomb’s theory are equal. The soil arching has no influence on the magnitude of passive earth force.

Considering the seepage will reduce the coefficient of passive earth force, and the ultimate capacity of the retaining structure will be reduced, leading to the potential for instability problems during the retaining wall’s design. In such cases, it is of vital importance to account for the effects of seepage during design calculations.

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