DEPENDENCE OF NUCLEAR BINDING ENERGIES ON THE CUTOFF MOMENTUM OF LOW-MOMENTUM NUCLEON-NUCLEON INTERACTION

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Binding energies of $^3$H, $^4$He, and $^{16}$O are calculated, using low-momentum nucleon-nucleon interactions ($V_{\text{low}−k}$) for a wide range of the cutoff momentum $\Lambda$. In addition, single-particle energies in nuclei around $^{16}$O are computed. The dependence of the binding energies and the single-particle energies in these nuclei on the cutoff momentum $\Lambda$ of the $V_{\text{low}−k}$ is examined. Furthermore, the availability of the $V_{\text{low}−k}$ in nuclear structure calculations is discussed.

1. Introduction

One of the fundamental objectives in nuclear structure calculations is to describe nuclear properties, starting with high-precision nucleon-nucleon interactions. Since this kind of interaction has a repulsive core at a short distance, one has been forced to derive an effective interaction or $G$ matrix in a model space for each nucleus from the realistic interaction, except for precise few-nucleon structure calculations.

Recently, Bogner et al. have constructed low-momentum nucleon-nucleon interactions $V_{\text{low}−k}$ from high-precision nucleon-nucleon interactions to use them as microscopic input to the nuclear many-body problem.\(^1\) The $V_{\text{low}−k}$ can be derived using techniques of conventional effective interaction theory or renormalization group method. They have shown that the $V_{\text{low}−k}$ conserves the properties of the original interaction, such as the half-on-shell $T$ matrix and the phase shift within a cutoff momentum $\Lambda$ which specify the low-momentum region. The $V_{\text{low}−k}$ for the typical cutoff...
\( \Lambda = 2.1 \text{ fm}^{-1} \) corresponding to \( E_{\text{lab}} \approx 350 \text{ MeV} \) are almost the same and are not dependent on the realistic nucleon-nucleon interactions employed. Thus, as a unique low-momentum interaction, the \( V_{\text{low-k}} \) at approximately \( \Lambda \sim 2 \text{ fm}^{-1} \) has been employed directly in nuclear structure calculations, such as the shell-model\(^2\) and the Hartree-Fock calculations.\(^3\) Especially, the calculated excitation spectra in the shell-model calculations show the good agreement with the experimental data and are even better than those using the sophisticated \( G \) matrix. Thus, the application of \( V_{\text{low-k}} \) to nuclear structure calculations has been growing. We should notice, however, that the \( V_{\text{low-k}} \) is derived introducing the cutoff momentum \( \Lambda \), and thus the calculated results using the \( V_{\text{low-k}} \) have the \( \Lambda \) dependence to some extent.

One of the central aims of the present work is to examine the \( \Lambda \) dependence in structure calculations. First, we calculate binding energies for few-nucleon systems for which precise calculations can be performed, and confirm the validity of the \( V_{\text{low-k}} \) in the structure calculation by comparing the obtained results with the exact values. Second, we proceed to heavier systems such as \(^{16}\text{O}\) and investigate not only the total binding energy itself but also the single-particle energy which is defined as the relative energy of neighboring two nuclei such as \(^{16}\text{O}\) and \(^{15}\text{O}\). Through the obtained results, we discuss the applicability of the \( V_{\text{low-k}} \) to nuclear structure calculations.

2. Results and discussion

In the following structure calculations, we use the \( V_{\text{low-k}} \) which is derived from the CD-Bonn potential\(^4\) by means of a unitary transformation theory.\(^5,6\) The details of deriving the \( V_{\text{low-k}} \) and its numerical accuracy can be seen in Ref. 7.

2.1. \(^3\text{H}\) and \(^4\text{He}\)

In order to investigate the sensitivity of \( \Lambda \) to the binding energies of \(^3\text{H}\) and \(^4\text{He}\) precisely, we have performed the Faddeev and the Yakubovsky calculations, respectively.\(^a\) For simplicity, only the neutron-proton interaction is used for all the channels.

Figure 1(a) exhibits the calculated ground-state energies of \(^3\text{H}\) by a 34-channel Faddeev calculation as a function of the cutoff momentum \( \Lambda \). The

\(^a\)The collaboration with E. Epelbaum and W. Glöckle in this part of the present work which has already been done in Ref. 7 is highly appreciated.
Figure 1. Calculated ground-state energies of $^3$H (a) and $^4$He (b) as a function of the cutoff momentum $\Lambda$. The solid lines represent the results using the $V_{\text{low}-k}$ for each $\Lambda$. The short-dashed lines are the results using the original CD-Bonn potential, where the high-momentum components beyond $\Lambda$ are simply truncated in the structure calculation. The exact value using the original CD-Bonn potential on the above assumptions is $-8.25$ MeV. The solid line depicts the results using the $V_{\text{low}-k}$ from the CD-Bonn potential. The short-dashed line represents the results using the original CD-Bonn potential, where the high-momentum components beyond $\Lambda$ are simply truncated in the structure calculation. For the case of the original CD-Bonn potential, we need $\Lambda \geq 8$ fm$^{-1}$ to reach the exact value if the accuracy of 100 keV is required. This situation is largely improved if we use the $V_{\text{low}-k}$. Even if we require the accuracy of 1 keV, we do not need the high-momentum components beyond $\Lambda \sim 8$ fm$^{-1}$. However, it should be noted that the results using the $V_{\text{low}-k}$ for the values smaller than $\Lambda \sim 5$ fm$^{-1}$ vary considerably, and there occurs the energy minimum at around $\Lambda = 1.5$ fm$^{-1}$. The magnitude of the difference between the exact value and the calculated result using the $V_{\text{low}-k}$ for the representative cutoff value $\Lambda = 2.0$ fm$^{-1}$ is about 600 keV for $^3$H. A similar tendency can also be seen in the case of $^4$He. We have performed the $S$-wave (5+5-channel) Yakubovsky calculation for $^4$He without the Coulomb interaction. The exact ground-state energy using the original CD-Bonn potential on the above assumptions is $-27.74$ MeV. In Fig. 1(b), the calculated results for $^4$He are shown. The shape of the energy curve is similar to that for $^3$H within the region $\Lambda \geq 2$ fm$^{-1}$. The calculated
results become more overbound as the value of $\Lambda$ becomes smaller. The magnitude of the difference between the exact value and the calculated result using the $V_{\text{low}-k}$ for $\Lambda = 2.0$ fm$^{-1}$ is about 3 MeV for $^4$He. This amounts to five times larger than the result of $^3$H. In the case of $^4$He, the results for $\Lambda < 2.0$ fm$^{-1}$ are not shown due to the numerical instability in the structure calculation.

Concerning the investigation of the $\Lambda$ dependence of the ground-state energies of the few-nucleon systems, a detailed study with three-nucleon forces has recently been reported by Nogga et al.$^8$

### 2.2. $^{16}$O

In order to examine the $\Lambda$ dependence in heavier systems, we calculate the ground-state energy of $^{16}$O within the framework of the unitary-model-operator approach (UMOA).$^6$ The details of recent calculated results for $^{16}$O and its neighboring nuclei using modern nucleon-nucleon interactions can be seen in Ref. 9. In the present study, we follow the same calculation method in that work except for the determination method of the harmonic-oscillator energy $\hbar\Omega$ and the size of the model space. In Ref. 9, we have searched for the optimal value of $\hbar\Omega$ that leads to the energy minimum point by investigating the $\hbar\Omega$ dependence of the ground-state energy for each modern nucleon-nucleon interaction. Then, we have found that the optimal values are at around 14 MeV of which values are very close to the value determined by empirical formula such as $\hbar\Omega = 45A^{-1/3} - 25A^{-2/3}$ MeV. Since the optimal value was $\hbar\Omega = 15$ MeV for the CD-Bonn potential, we use this value in this work for each $V_{\text{low}-k}$. Furthermore, we employ the same size of the optimal model space which is specified by the quantity $\rho_1$ as $\rho_1 = 2n_a + l_a + 2n_b + l_b = 12$, where $\{n_a, l_a\}$ and $\{n_b, l_b\}$ are the sets of harmonic-oscillator quantum numbers for two-body states. We note that these values of $\hbar\Omega$ and $\rho_1$ are not necessarily the optimal ones for each $V_{\text{low}-k}$ in the present study.

In Fig. 2, the $\Lambda$ dependence using the $V_{\text{low}-k}$ from the CD-Bonn potential of the ground-state energy of $^{16}$O is shown. We have used the neutron-neutron, neutron-proton, and proton-proton interaction of the CD-Bonn potential correctly for the corresponding channels, and included the Coulomb interaction. The partial waves up to $J = 6$ are taken into account in the calculation. The value of the ground-state energy of $^{16}$O in the full calculation given in Ref. 9 using the original CD-Bonn potential is $-115.61$ MeV. Thus, the result for $\Lambda = 5.0$ fm$^{-1}$ almost reproduces this
value. The calculated energy curve shows a similar tendency to the results of $^3$H and $^4$He, but the magnitude of the difference between the result of the full calculation and the value at the energy minimum point is considerably larger than those for $^3$H and $^4$He due to the large difference of the mass number. The magnitude of the difference in $^{16}$O amounts to 55 MeV. Even if we choose the typical cutoff $\Lambda = 2.0$ fm$^{-1}$, we still observe the significant overbinding of which magnitude is about 31 MeV. Thus, we may conclude from the results of $^3$H, $^4$He, and $^{16}$O that the $V_{\text{low}-k}$ for the typical cutoff momentum $\Lambda \sim 2$ fm$^{-1}$ cannot reproduce the exact values, showing the significant overbinding.

It should be noted, however, that this does not necessarily mean that the $V_{\text{low}-k}$ for $\Lambda \sim 2$ fm$^{-1}$ is no longer valid in nuclear structure calculations. In fact, the shell-model calculations have shown that the $V_{\text{low}-k}$ for $\Lambda \sim 2$ fm$^{-1}$ can work as well as the $G$ matrix.\textsuperscript{2}

2.3. $^{15}$O and $^{17}$O

In the previous sections, we have seen the results of the total binding energies. We here examine the $\Lambda$ dependence of single-particle energies of the neutron for hole states in $^{16}$O which correspond to the energy levels in $^{15}$O and of neutron particle states in $^{17}$O. The calculation procedure is essentially the same as in Ref. 9. In the present study, however, we do not search for the optimal values of $\hbar \Omega$ and $\rho_1$ for each single-hole or -particle

![Graph](image-url)
Figure 3. The Λ dependence of the single-particle energies of the neutron for the 0p hole states in $^{16}$O which correspond to the energy levels in $^{15}$O (a) and of the neutron particle states in $^{17}$O (b) using the $V_{\text{low}-k}$ from the CD-Bonn potential.

state for simplicity as in the case of $^{16}$O. In the following calculations, we use the values of $\hbar \Omega = 15$ MeV and $\rho_1 = 12$ which are the same as in the calculation of $^{16}$O in the previous section.

Figure 3(a) shows the Λ dependence of the calculated single-particle energies of the neutron for the 0p hole states in $^{16}$O which correspond to the single-hole energy levels in $^{15}$O. The values of the full calculation of the single-particle energy are $-19.34$ and $-25.37$ MeV for the $0p_{1/2}$ and $0p_{3/2}$ states, respectively. Though the present results for Λ = 5.0 fm$^{-1}$ are fairly close to these values, there remain some discrepancies. These discrepancies may be due to the fact that we do not search for the optimal value of $\hbar \Omega$ for each state in the present study. The search for the optimal values of $\hbar \Omega$ and also $\rho_1$ in the structure calculation with the $V_{\text{low}-k}$ should be done for completeness in future.

It is seen from Fig. 3(a) that the single-particle energies for the 0p states become more attractive as the Λ becomes smaller as in the results of the ground-state energies. However, what is interesting here is that the magnitudes of the spacing between the single-particle levels, namely the spin-orbit splitting, hold their values up to Λ $\sim$ 2 fm$^{-1}$, although the structure of the single-particle levels is broken within the area Λ < 2 fm$^{-1}$.

A similar tendency can also be observed in the results of $^{17}$O. In Fig. 3(b), the calculated results of the single-particle energies of the neu-
tron for the $1s$ and $0d$ states in $^{17}\text{O}$ are shown. The values of the full calculation of the single-particle energy are 2.67, $-2.76$, and $-4.11$ MeV for the $0d_{3/2}$, $1s_{1/2}$, and $0d_{5/2}$ states, respectively. The present results for $\Lambda = 5.0$ fm$^{-1}$ are not so different from these values. The tendency of the $\Lambda$ dependence is essentially the same as in $^{15}\text{O}$. It can be seen again that the magnitudes of the spacings between the single-particle levels do not vary very much within the region $\Lambda \geq 2$ fm$^{-1}$, while those are considerably broken within the area $\Lambda < 2$ fm$^{-1}$. These results may suggest that the $V_{\text{low-k}}$ for $\Lambda \sim 2$ fm$^{-1}$ is valid as far as relative energies from a state such as the ground state are concerned.

3. Conclusions

We investigated the dependence of the ground-state energies of $^3\text{H}$, $^4\text{He}$, and $^{16}\text{O}$ on the cutoff momentum $\Lambda$ of the low-momentum nucleon-nucleon interaction $V_{\text{low-k}}$. In all the cases, there appear the energy minima at around $\Lambda = 1.5$ fm$^{-1}$. We have found that the $V_{\text{low-k}}$ for the typical cutoff momentum $\Lambda \sim 2$ fm$^{-1}$ cannot reproduce the exact values for the original interaction, showing the significant overbinding. If we try to reproduce the exact values, we need $\Lambda \geq 5$ fm$^{-1}$. On the other hand, the magnitudes of the spacings between the single-particle levels in nuclei around $^{16}\text{O}$ do not so vary within the region $\Lambda \geq 2$ fm$^{-1}$. This may suggest that the $V_{\text{low-k}}$ for the typical cutoff $\Lambda \sim 2$ fm$^{-1}$ is valid in nuclear structure calculations as far as relative energies from a state such as the ground state are concerned as in the shell-model calculation.

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