Status of Neutrino Oscillations

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Abstract. Solar and atmospheric neutrino data require physics beyond the Standard Model of particle physics. The simplest, most generic, but not yet unique, interpretation of the data is in terms of neutrino oscillations. I summarize the results of the latest three-neutrino oscillation global fit of the data, in particular the bounds on the angle $\theta_{13}$ probed in reactor experiments. Even though not implied by the data, bi-maximal neutrino mixing emerges as an attractive possibility either in hierarchical or quasi-degenerate neutrino scenarios.

1. Introduction

Undoubtedly the solar (Suzuki 2000) and atmospheric (Sobel 2000, Becker-Szendy 1992) neutrino problems provide the two most important milestones indicating physics beyond the Standard Model (SM). Of particular importance has been the confirmation in 1998 by the Super-Kamiokande (SK, for short) collaboration of the zenith-angle-dependent deficit of atmospheric neutrinos. Altogether the data provide a strong evidence for $\nu_e$ and $\nu_\mu$ conversions, respectively. Neutrino conversions are a natural consequence of theories beyond the Standard Model (Valle 1991). The first example is oscillations of low-mass neutrinos. While the theoretical understanding of the origin of neutrino masses is still lacking, there is a variety of attractive options available. Most likely, the exceptional nature of neutrinos as the only electrically neutral fermions in the SM underlies the smallness of their mass, as it would be associated with the violation of lepton number. Indeed in gauge theories one expects, on fundamental grounds, neutrinos to be Majorana fermions (Schechter 1980a). This is the generic situation in actual models. It will be surprising indeed if massive neutrinos turn out to be Dirac particles, like the quarks. Lepton number violation would imply processes such as neutrino-less double beta decay (Schechter 1982), novel CP violation effects (Schechter 1980a and 1981a), and/or neutrino electromagnetic properties (Schechter 1981b), so far unobserved. Present data and theoretical considerations suggest either hierarchical or quasi-degenerate neutrino masses. While solar neutrino rates favour the small mixing angle MSW oscillations (Wolfenstein 1978, Smirnov 1986), present data on the recoil-electron spectrum prefer the large mixing solutions. When interpreted in terms of neutrino oscillations, the observed atmospheric neutrino zenith-angle-dependent deficit clearly indicates that the mixing involved is maximal (Gonzalez-Garcia 2001).
Adding information from reactor experiments one concludes that the third angle amongst the three neutrinos is small (Apollonio 1999). Thus, altogether, we have the intriguing possibility that, unlike the case of quarks, neutrino mixing is bi-maximal (Barger 1998, Davidson 1998, de Gouvea 2000, Chankowski 2000fp, Hirsch 2000) which could be tested at the upcoming long-baseline experiments or at a neutrino factory experiment (Quigg 1999) or at the proposed KamLAND experiment (De Braekeleer 2000).

In addition to the above, there is also a long history of searches for neutrino oscillations at accelerators. Except for the unconfirmed hint provided by the LSND experiment (Athanassopoulos 1998, Smith 2000), accelerator searches have so–far been negative. The resulting limits, however, are not very restrictive on the scale of the indications from underground experiments and I will not discuss them any further. Barring exotic neutrino conversion mechanisms the hint of the LSND experiment together with the solar and atmospheric data require three mass scales, hence the need for a fourth light neutrino, which must be sterile (Peltoniemi 1993, Caldwell 1993, Liu 1998, Hirsch 2000, Giunti 2000). The most attractive possibility is to have, out of the four neutrinos, two of them lie at the solar neutrino scale, with the other two maximally-mixed neutrinos at the LSND scale (Peltoniemi 1993, Caldwell 1993, Hirsch 2000). These schemes have distinct implications at future solar & atmospheric neutrino experiments with sensitivity to neutral current neutrino interactions such as SNO. Cosmology can also place restrictions on these four-neutrino schemes (Raffelt 1999).

2. Indications for New Physics

The most solid hints in favour of new physics in the neutrino sector come from underground experiments on solar (Suzuki 2000) and atmospheric (Sobel 2000, Becker-Szendy 1992) neutrinos. The most recent SK data correspond to 1117–day solar and 1144–day (71 kton-yr) atmospheric data samples, respectively. There are also new data from Soudan-2 (5.1 kton-yr) and MACRO.

2.1. Solar Neutrinos

Our sun produces $\nu_e$’s through various nuclear reactions which take place in its interior. The predicted spectrum of solar neutrinos is illustrated in Fig. (1), taken from (Bahcall 1998). I will refer to this model as “the” SSM. Solar neutrinos are detected either with geochemical methods (the $\nu_e +^{37}Cl \rightarrow^{37}Ar + e^-$ reaction at the Homestake experiment and the $\nu_e +^{71}Ga \rightarrow^{71}Ge + e^-$ reaction at the Gallex, Sage and GNO experiments) or through $\nu_e e^-$ scattering on water, using Cerenkov techniques at Kamiokande and Super-Kamiokande. As summarized in Fig. (2) all experiments observe a deficit of 30 to 60 % whose energy dependence follows mainly from the lower Chlorine rate. Note that Fig. (2) includes the latest results from SK, SAGE & GNO presented at $\nu$2000, but not the first results from SNO. It is convenient to present the predictions of various standard solar models in terms of the $^7$Be and $^8$B neutrino fluxes, normalized to the SSM predictions (Bahcall 1998), as seen in Fig. (3), which includes most of the existing solar models.
On the other hand the values of the fluxes indicated by measured neutrino event rates are shown by the contours in the lower-left part of the figure, with a negative best-fit $^7$Be neutrino flux! This discrepancy strongly suggests the need for new particle physics (Bahcall 1994). Since possible non-standard astrophysical solutions are rather constrained by helioseismology studies (Bahcall 1998) one is led to assume the existence of neutrino conversions, such as those induced by very small neutrino masses.

The high statistics of SK after 1117 days of data-taking also provides very useful information on the recoil electron energy spectrum with event rates given for 18 bins starting at 5.5 MeV $^\ast$. The spectrum in Fig. (4) is well described by the flat hypothesis $\chi^2_{flat} = 13/(17\text{dof})$, in contrast with hints from previous 825-day sample. Moreover, SK measures the zenith angle distribution (day/night effect) which is sensitive to the effect of the Earth matter in the neutrino propagation. One sees a slight excess of events at night, but the corresponding day versus night asymmetry $A_{D/N} = \frac{D-N}{D+N} = -0.034 \pm 0.022 \pm 0.013$ is only 1.3$\sigma$ away from zero. In order to combine this day–night information with the spectral data, SK has also presented separately the measured recoil energy spectrum during the day and during the night. This will be referred in the following as the day–night spectra data and contains $2 \times 18$ data bins.

Note that the absence of clear hints of spectral distortion, day-night or seasonal variation implies that, per se, they do not give any clear indication for physics beyond the standard model. From this point of view, despite the increasing weight of such rate-independent observables, the solar neutrino problem rests heavily on the rate discrepancy. Nevertheless, as we will see, rate-independent

$^\ast$They have also reported results of a lower energy bin $5 \text{MeV} < E_e < 5.5 \text{MeV}$, but due to systematic errors this is not yet included in the analysis.
observables are already playing an important rôle in selecting amongst different solutions of the solar neutrino problem.

2.2. Atmospheric Neutrinos

Neutrinos produced as decay products in hadronic showers from cosmic ray collisions with nuclei in the upper atmosphere have been observed in several experiments (Sobel 2000, Becker-Szendy 1992). Although individual $\nu_\mu$ or $\nu_e$ fluxes are only known to within 30% accuracy, their ratio is predicted to 5% over energies that vary from 0.1 GeV to 100 GeV (Gaisser 1998). The long-standing discrepancy between the predicted and measured $\mu/e$ ratio of the muon ($\nu_\mu + \bar{\nu}_\mu$) over the electron atmospheric neutrino flux ($\nu_e + \bar{\nu}_e$) (Gaisser 1995) found both in water Cerenkov experiments (Kamiokande, SK and IMB) as well as in the iron calorimeter Soudan2 experiment is illustrated in Fig. (6). This evidence has now been strengthened by the fact that it exhibits a strong zenith-angle dependence (Sobel 2000) as can be seen from Fig. (7). The zenith-angle distributions for the Super-Kamiokande $e$–like are shown in the left panels, both in the sub-GeV (upper panels) and multi-GeV (lower panels) energy range. The thick solid line is the expected distribution in the SM. It is consistent with the SM expectations. In contrast $\mu$–like events displayed in the right-panels show a clear deficit of neutrinos coming from below, which is very suggestive indeed of $\nu_\mu$ oscillations. In Fig. (8) we also give the predicted best-fit distributions obtained in a global 3-neutrino oscillation description of the data (Gonzalez-Garcia 2001). The thin full line is the prediction for the overall best-fit point of the contained plus up-going atmospheric data sample, with $\tan^2 \theta_{13} = 0.025$, $\Delta m^2_{32} = 3.3 \times 10^{-3}$ eV$^2$ and $\tan^2 \theta_{23} = 1.6$.

Zenith-angle distributions have also been recorded for upward-going muon events in Super-Kamiokande and MACRO, as illustrated in Fig. (9). The thick solid line is the expected distribution in the SM, while the thin full line corresponds, as in Fig. (8), to the prediction for the overall global best-fit point.
3. Three–neutrino fits

The most economical joint description of solar and atmospheric anomalies involves oscillations amongst all three known types of neutrinos. Here I summarize the results of the recent global analysis (Gonzalez-Garcia 2001) of the solar (Suzuki 2000), atmospheric (Sobel 2000, Becker-Szendy 1992) and reactor (Apollonio 1999) neutrino data in terms of three–neutrino oscillations. The present discussion goes beyond previous three–neutrino oscillation analyses including only solar (Fogli 2000) or only atmospheric neutrino data (Fogli 1999). It also updates some joint studies (Barger 1980) which do not take into account the latest SK data. As we saw the most recent solar neutrino rates include the 1117 day SK data sample on the recoil electron energy spectra for day and night periods. On the other hand the atmospheric data sample includes not only contained events but also the upward-going $\nu$-induced muon fluxes. In addition
The pattern of neutrino oscillations expected in any fundamental (gauge) theory of neutrino mass is determined by the structure of the lepton mixing matrix (Valle 1991, Bilenkii 1999). For the simplest three-neutrino theories this is in general characterized by three mixing angles and three CP violating phases (Schechter 1980a). The latter include, in addition to the Dirac-type phase analogous to that of the quark sector, two extra physical (Schechter 1981a) phases associated to the Majorana character of neutrinos. Conservation of CP implies that Dirac phases are zero modulo $\pi$, while Majorana phases are zero modulo $\pi/2$ (Wolfenstein 1981, Schechter 1981b). For our following discussion all three phases are set to zero. In this case the mixing matrix can be conveniently
Figure 7. SK zenith-angle distributions for contained events versus theoretical expectations in the SM and within the oscillation hypothesis chosen in the form (Schechter 1980a)

\[
\begin{pmatrix}
  c_{13}c_{12} & s_{12}c_{13} & s_{13} \\
  -s_{12}c_{23} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\
  s_{23}s_{12} - s_{13}c_{23}c_{12} & -s_{23}c_{12} - s_{13}s_{12}c_{23} & c_{23}c_{13}
\end{pmatrix}
\]  

(1)

The joint study of solar and atmospheric neutrino oscillations is characterized by a five-dimensional parameter space

\[
\begin{align*}
\Delta m^2_\odot &\equiv \Delta m^2_{12} = m^2_2 - m^2_1 \\
\Delta m^2_{\text{atm}} &\equiv \Delta m^2_{32} = m^2_3 - m^2_2 \\
\theta_\odot &\equiv \theta_{12} \\
\theta_{\text{atm}} &\equiv \theta_{23} \\
\theta_{\text{reactor}} &\equiv \theta_{13}
\end{align*}
\]  

(2)

where all mixing angles are assumed to lie in the full range from \([0, \pi/2]\).

From the required hierarchy in the splittings \(\Delta m^2_{\text{atm}} \gg \Delta m^2_\odot\) indicated by the solutions to the solar and atmospheric neutrino anomalies (see below) it follows that the analyses of solar data constrain three of the five independent oscillation parameters, namely, \(\Delta m^2_{21}, \theta_{12}\) and \(\theta_{13}\) since for most cases oscillations over the atmospheric scale average out. Conversely, from the point of view of the atmospheric data analysis one can effectively assume that the lighter neutrinos become degenerate so that one can rotate away the corresponding angle...
Figure 8. Zenith-angle distributions for upward-going muon events in Super-Kamiokande and MACRO.

$\theta_{12}$. The leptonic mixing matrix takes on the simplified form (Schechter 1980b)

$$
\mathbf{R} = \begin{pmatrix}
c_{13} & 0 & s_{13} \\
-s_{23}s_{13} & c_{23} & s_{23}c_{13} \\
-s_{13}c_{23} & -s_{23} & c_{23}c_{13}
\end{pmatrix};
$$

As a result only three oscillation parameters: $\Delta m_{32}^2$, $\theta_{23}$ and $\theta_{13}$ are necessary to describe the 3-neutrino propagation of atmospheric neutrinos.

It follows from the above discussion that $\theta_{13}$ is the only parameter common to both analyses and may potentially allow for some “cross-talk” between the two sectors. It is known that for $\Delta m_{32}^2 \gg \Delta m_{12}^2$ and $\theta_{13} = 0$ the atmospheric and solar neutrino oscillations decouple in two 2-$\nu$ oscillation scenarios. In this respect our results also contain as limiting cases the pure two-neutrino oscillation scenarios and update previous analyses on atmospheric neutrinos (Fornengo 2000, Foot 1998) and solar neutrinos (Gonzalez-Garcia 2000a, Bahcall 1998).

In order to compute the solar neutrinos survival probabilities for any value of the neutrino mass and mixing the full expression for the survival probability has been used, without appealing to the usual approximations whose validity defines the MSW (Smirnov 1986) or the "just-so" (Glashow 1987) regime. The treatment of neutrino oscillations is therefore unified, with MSW and vacuum oscillations considered on the same footing. Likewise, we include in our description conversions with $\theta_{12} > \pi/4$ (Gonzalez-Garcia 2000b). Results are found by numerically solving the Schrödinger neutrino evolution equation in the Sun and
Figure 9. Excluded region in $\Delta m_{32}^2$ and $\sin^2(2\theta_{13})$ from the non-observation of oscillations by the CHOOZ reactor.

the Earth matter, using the electron number density of BP2000 model (Bahcall 2000) and the Earth density profile given in the Preliminary Reference Earth Model (PREM) (Dziewonski, 1981).

**Reactor limits**

To start the summary of the global 3-neutrino fits we first note that, of all laboratory searches for neutrino oscillation, reactors provide the most sensitive one when it comes to comparing with the strong hints from underground experiments. Thus we will first consider the region of oscillation parameters which can be excluded from reactor experiments. The restrictions on $\Delta m_{32}^2$ and $\sin^2(2\theta_{13})$ that follow from the non-observation of oscillations at the CHOOZ reactor experiment are shown in Fig. (9), taken from (Gonzalez-Garcia 2001). The curves represent the 90, 95 and 99% CL excluded region defined with 2 d. o. f. for comparison with the CHOOZ published results. In what follows we will compare this direct bound with those obtained from a global analysis of solar and atmospheric data, as well as consider its effect in combination with the latter.

**Solar data fit**

We first present the allowed regions of solar oscillation parameters $\theta_{12}$, $\Delta m_{21}^2$ as a function of $\theta_{13}$. All plots are taken from (Gonzalez-Garcia 2001) where all details can be found. In Fig. (10) we give the allowed three-neutrino oscillation regions in $\Delta m_{21}^2$ and $\tan^2\theta_{12}$ from the measurements of the total event rates at the Chlorine, Gallium, Kamiokande and Super-Kamiokande experiments. The different panels represent the **allowed** regions at 99% (darker) and 90% CL.
Figure 10. 3-ν oscillation regions allowed at 99% (darker) and 90% CL (lighter) by the latest measurements of the total solar neutrino event rates. The best-fit point is denoted as a star.
Figure 11. 3-ν oscillation solar oscillation regions excluded at 99% CL by the day–night spectra measurement.

Figure 12. 3-ν oscillation regions allowed by all of the solar neutrino data. The best–fit point is denoted as a star.
Figure 13. Relative quality of three–neutrino solutions to the solar neutrino problem.

gets effectively a two-neutrino scheme) the best solution is the LMA solution is the best, while SMA is the worst. One notices also that rate-independent observables, such as the electron recoil energy day-night spectra, are playing an increasing role in discriminating between different solutions to the solar neutrino problem, pushing the best–fit point towards the LMA solution (Gonzalez-Garcia 2001), a trend already noted in earlier 2-neutrino analyses due to the same reason (Gonzalez-Garcia 2000a).

Another issue which could play a more significant role in future investigations is that of seasonal variations, expected in the just-so regime and, more subtly, also in the MSW large mixing solutions, LMA and LOW. The first would result from the eccentricity of the Earth’s orbit around the Sun and could be tested by searching for seasonal variations in the $^7$Be neutrino flux at the Borexino experiment, and possibly at KamLAND (de Gouvea 1999). The latter would result from the regeneration effect at the Earth (day-night effect) and might be tested through time variations of event rates at GNO and Borexino (de Holanda 1999).

An interesting theoretical issue is the possible effect of random fluctuations in the solar matter density (Balantekin 1996, Nunokawa 1996, Bamert 1998) on the solar neutrino event rates. The existence of such noise fluctuations at a few percent level is not excluded by present helioseismology studies. The correlation length $L_0$ associated with the scale of the fluctuation can be assumed to lie between the mean free path of the electrons in the solar medium, $l_{\text{free}} \sim 10$ cm, and the neutrino oscillation length in matter, $\lambda_m$, e. g. $l_{\text{free}} \ll L_0 \ll \lambda_m$. Even small fluctuations can have an important effect on averaged solar neutrino survival probabilities, especially for small solar mixing angles (Balantekin 1996,
Figure 14. 90, 95 and 99% CL regions in \((\tan^2 \theta_{23}, \Delta m^2_{32})\) allowed by the combination of all atmospheric data.

Nunokawa 1996). The fluctuations can affect the \(^7\text{Be}\) neutrino component of the solar neutrino spectrum, implying that Borexino can probe, at some level of precision, the magnitude of solar matter density fluctuations, thus an additional motivation for the experiment (Arpesella 1991).

**Atmospheric and reactor data fit**

Here I present the allowed regions of atmospheric oscillation parameters \(\theta_{23}, \Delta m^2_{32}\) for different values of \(\theta_{13}\), common to solar and atmospheric analyses. All plots are taken from (Gonzalez-Garcia 2001) where details can be found. In Fig. (14) we display the allowed \((\tan^2 \theta_{23}, \Delta m^2_{32})\) regions for different \(\tan^2 \theta_{13}\) values, that follow from the combination of Super-Kamiokande atmospheric neutrino events. The regions refer to 90, 95 and 99% CL. The best-fit point is denoted as a star and corresponds to \(\tan^2 \theta_{13} = 0.026\). In Fig. (15) we present the three–neutrino regions in \((\tan^2 \theta_{23}, \Delta m^2_{32})\) allowed by the combination of all atmospheric neutrino data plus Chooz. The best–fit point is denoted as a star. Comparing Fig. (14) and Fig. (15) one can see the weight of the reactor neutrino bound on the global analysis. The best global fit has \(\tan^2 \theta_{13} = 0.005\) and for \(\tan^2 \theta_{13} = 0.075\) even the 99% CL allowed disappears. Notice the important complementarity between atmospheric data and the reactor limits on \(\theta_{13}\) since the latter apply only for \(\Delta m^2_{32} \gtrsim 10^{-3} \text{eV}^2\).
Figure 15. 90, 95 and 99% CL 3–ν regions in \((\tan^2 \theta_{23}, \Delta m_{32}^2)\) allowed by the combination of all neutrino data.

Combining solar, atmospheric and reactor data

We also obtain the allowed ranges of parameters from the full five–dimensional combined analysis of all of the above neutrino data. In Fig. (16) we give the regions in \(\Delta m_{21}^2\) and \(\tan^2 \theta_{12}\) allowed by the global analysis of solar, atmospheric and reactor neutrino data. The left panel gives the regions for the unconstrained analysis defined in terms of the increases of \(\Delta \chi^2\) for 5 d.o.f. from the global best fit point denoted as a star. The right panel shows the values of \(\tan^2 \theta_{13}\) beyond which the 99% CL region starts to disappear.

4. Putting the pieces together

Together with the solar neutrino data, the angle-dependent atmospheric neutrino deficits provide a strong evidence for physics beyond the Standard Model. Small neutrino masses provide the simplest and most generic explanation of the data. Theoretical neutrino mass models fall into two classes, which I call bottom-up and top-bottom. They can lead to either hierarchical (de Gouvea 2000, Hirsch 2000) or quasi-degenerate (Chankowski 2000) neutrinos. Top-bottom approaches inspired on the idea of Unification typically employ either a see-saw mechanism, or high dimension operators. As examples of minimalistic models I mention two we have recently investigated, both of which require a large mixing solution to the solar neutrino problem (de Gouvea 2000, Chankowski 2000).

Supersymmetry with bilinear breaking of R–parity provides a simple bottom-up-type model which allows for the possibility of probing the neutrino mixing,
as indicated by the underground experiments, within the context of high-energy collider experiments such as the LHC (Hirsch 2000). For additional models and for models involving specific Yukawa textures see (Davidson 1998, Lola 1998, Altarelli 1998).

Last, but not least, I mention that it is impossible to reconcile solar, atmospheric and reactor data with the LSND hint in terms of neutrino oscillations without a light sterile neutrino (Peltoniemi 1993, Caldwell 1993) and in ref. (Hirsch 2000) we give an updated discussion in the context of an interesting model. For more references see (Liu 1998) and for a more complete discussion of limits on 4-neutrino models see (Giunti 2000). If light sterile neutrinos exist they should be probed at neutral-current-sensitive solar & atmospheric neutrino experiments such as SNO (Gonzalez-Garcia 2000c).

5. Conclusion

Within the neutrino oscillation framework present solar and atmospheric data suggest the intriguing possibility of bi-maximal neutrino mixing, which explicitly illustrates the sharp contrast between the lepton and quark sectors of the theory. With good luck this could be checked on the one hand at the upcoming long-baseline experiments or at a neutrino factory (Quigg 1999) and, on the other, via the search of seasonal effects in solar neutrinos, e.g. at the proposed KamLAND experiment (De Braekeleer 2000). In certain models (Hirsch 2000) one may test the neutrino mixing angles involved in the explanation of
the neutrino anomalies at high–energy collider experiments (Porod 2000), illustrating an amusing synergy between accelerator and underground experiments. On the other hand if the LSND result stands the test of time, this would be a strong indication for the existence of a light sterile neutrino which would be another radical difference between leptons and quarks. Let me mention however that the neutrino oscillation interpretation of solar neutrino anomalies is still far from unique. For example, resonant spin flavor precession (Akhmedov 1988) induced by transition magnetic moments (Schechter 1981b). For a recent fit of solar data see ref. (Miranda 2001). From this point of view, it is still too early for a precision determination of neutrino properties from underground experiments. Last, but not least, let me mention that in fifty years of weak interaction physics an answer to the most fundamental question about the nature - Dirac versus Majorana - of neutrinos has so far defied all experimental attempts.

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