Demand-Side Power Paradigm-Oriented Analysis of Reactive Electric Spring Stabilization Capabilities

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ABSTRACT Electric Spring (ES) technique is a user-level solution developed to stabilize the supply voltage of a user under variations of the grid voltage. This paper analyzes the stabilization capabilities of a reactive ES that operates according to the demand-side power paradigm. By help of a convenient ES modeling, the extreme values of the active power that a user can draw under the ES action are first determined. Then, it is demonstrated that the demand-side power paradigm is fulfilled only if the distribution line impedance has a resistive component while its reactive component weakens such fulfillment. Lastly, the variations of the grid voltage that ES is able to cope with are worked out. All findings are formulated in terms of normalized quantities and consequently are of general validity. Computer-aided simulation of a case study exemplifies the theoretical findings.

INDEX TERMS Electric spring, voltage stabilization, smart grid, distribution network, distribution generation.

I. INTRODUCTION
The increased grid proliferation of renewable energy sources (RESs) is liable to impair the quality of the electric service. Indeed, the fluctuations of the grid power due to the intermittency of RESs cause the variation of the grid voltage feeding the distribution lines and this, in turn, deviates the supply voltage of a user from the nominal value. Various solutions have been developed to stabilize such a voltage [1]. They can be deployed at two levels: distribution-level and user-level [2].

Electric Spring (ES) technique is a recently developed user-level solution. Conceptualized by Hui et al. in [3], the ES technique is the electric version of the Hooke’s law for a mechanical system [4]. The concept behind it can be briefly explicated as follows. The user loads are separated into two sets, namely the set that tolerates a certain deviation of the supply voltage, termed as Non-Critical Load (NCL), and the set that needs a nearly constant supply voltage, termed as Critical Load (CL). An ES circuitry, hereafter simply termed as ES, modifies the voltage across and -then- the current through NCL so as to keep constant, i.e. to stabilize, the CL voltage at the nominal value under grid voltage variations. This entails that the active power drawn by CL remains constant while that one drawn by NCL changes.

Fig.1 illustrates the schematic of a single-phase user equipped with ES and connected to a distribution line at supply point S. CL is directly connected to S whilst NCL is connected to S through ES. The circuitry of ES is comprising of a PWM voltage source inverter (VSI), filtering inductance $L_f$ and capacitor $C_{ES}$. The branch formed by ES and NCL is called Smart Load (SL) and its terminal voltage, given by the sum of voltage $v_{ES}$ across the capacitor and voltage $v_{NCL}$ across NCL, is just the user voltage $v_S$. By properly commanding VSI, voltage $v_{ES}$ is driven to stabilize the magnitude of $v_S$ despite the variations of the grid voltage. Therefore, CL is supplied at the nominal voltage whilst NCL, due to the presence of $v_{ES}$, is no more.

Two topologies have been originally devised to implement ES, namely reactive and active [5]. In the reactive ES, a capacitor is utilized at the DC input of VSI and the user voltage stabilization is achieved thanks to the reactive power made available by ESs [6], [7]. Instead, in the active ES, a battery is utilized at the DC input of VSI and the user voltage...
stabilization is achieved thanks to both active and reactive powers made available by ES [8]. The reactive ES is cost effective and dispenses of most maintenance tasks. The active one allows the execution of additional services in favor of the grid such as correction of the power factor [9]–[11].

In [12], the two above-mentioned topologies are respectively referred to as ES-1 and ES-2. In addition to them, ESs with other active topologies have been proposed during the last years, like the back-to-back (B2B) ES, where a back-to-back converter is incorporated into the ES circuitry to avoid the use of any battery in the DC voltage link [13], and the RES-integrated ES, where a photovoltaic (PV) panel supplies the use of any battery in the DC voltage link [13], and the RES-integrated ES, where a photovoltaic (PV) panel supplies ES [14]. This paper is concerned with the ES-1 topology, i.e. with the reactive ES.

Application of ES for the stabilization of the user voltage enables to fulfill the demand-side power paradigm in an electric system. As know, in contrast to the traditional supply-side power paradigm, the demand-side one provides for adapting the demand of power to the generation. Notwithstanding this potential of ES application, the existing literature [15]–[17] has not carried out yet an analysis of the user voltage stabilization capabilities of ES that is operated according to the demand-side power paradigm.

The purpose of this paper is to perform such an analysis for a user equipped with a reactive ES (hereafter designated with RESE user) by giving for grant that an excess of power generation produces an overvoltage of the grid feeding the distribution lines whilst a shortage of power generation produces an undervoltage of it. Videlicet, the paper examines the stabilizing action of a reactive ES under the constraint that the active power drawn by NCL tracks the fluctuations of the power generation; this means that, under an excess of power generation, the ES action must be consistent with NCL that draws an active power greater than its nominal value and vice versa.

From the grid point of view, the fulfillment of the demand-side power paradigm would facilitate the integration of distributed RESs since it modifies the power consumption according to the supply conditions. Several papers have recently analyzed the possible impact of ESs on the distribution grid, but the demand-side power paradigm concept has never been addressed. For example, in [18] it is demonstrated that, for different lengths and R/X ratios of a distribution line, the ES installation at different locations from a substation is not an issue for the network stability. In [19] and [20], an optimal ES allocation is suggested with the objective of minimizing the user voltage deviation, but no mention is done about any demand-side power behavior for the optimal operation of the electric system. In [21], ES is inserted in series to the whole user load and its action is intended to limit the deviation of the user load voltage within ±5%; as a consequence, the CL voltage can also deviate within this range, ignoring the distinct voltage needs of CL and NCL. The objective is to provide a load-side virtual inertia through the power reserve that comes from the ±5% deviation of the nominal load voltage. This means that ES is no more utilized to stabilize the CL voltage, but to improve the primary frequency response by the increase of the power system inertia.

Paper [22] compares the stabilization capabilities of a reactive ES with those of the active ESs, in particular of B2B ES, finding out that the equivalent R/X ratio of the distribution line plays an important role in the user voltage regulation exerted by SLs. It is shown that the voltage variations in grids with high PV and Electric Vehicle (EV) penetration increase for higher R/X ratio since both PVs and EVs exchange only active power with the grid; however, it is not considered the effect of the R/X ratio on the demand-side power response.

This paper is organized in VII Sections. Section II explains the modus operandi pursued in the analysis of the operation of a RESE user and conceives an ES model useful to describe its action. Section III calculates the extreme values of the active power range that NCL draws under the ES action. Section IV investigates the voltage drop along a distribution line. Section V determines the voltage stabilization range achievable by a RESE user against the grid voltage variations while fulfilling the demand-side power paradigm; moreover, it highlights that the stabilization range strongly depends on the resistive component of the distribution line impedance. The theoretical findings are expressed analytically by help of normalized quantities and, thus, constitute a general tool for a user to evaluate the stabilization capabilities offered by the ES installation. Section VI presents a case study to exemplify the theoretical findings. Section VII concludes the paper.

Throughout the paper, the following conventions are used: nominal conditions refer to the situation with no variations of the grid voltage; the relevant quantities are denoted by subscript N. Upper-case letters overmarked with a bar denote phasors; those overmarked with a point denote impedance whilst those without any overmark denote magnitude of the respective quantities. Subscripts Max and Min denote respectively maximum and minimum magnitudes of the quantities under the ES action. The normalized quantities are expressed as a fraction of the respective magnitudes in nominal conditions.

II. ES DESCRIPTION

The electric scheme of a single-phase RESE user is illustrated in Fig. 2, together with the supplying distribution line. In the figure, voltage $v_G$ and current $i_G$ are quantities related to
the grid whilst resistance $R_G$ and inductance $L_G$ are related to the distribution line. Voltages $v_{CL}$ and $v_{NCL}$, currents $i_{CL}$ and $i_{NCL}$, resistances $R_{CL}$ and $R_{NCL}$, and inductances $L_{CL}$ and $L_{NCL}$ are quantities related to CL and NCL, respectively. Capacitor $C$ and voltage $V_{DC}$ are quantities related to the DC input stage of ES whilst capacitor $C_{ES}$, inductance $L_I$, voltage $v_I$, current $i_I$ and voltage $v_{ES}$ are related to the AC output stage of ES. Note that current $i_C$ entering in $C_{ES}$ is the sum of $i_{NCL}$ and $i_I$ whilst the current entering in SL is just $i_{NCL}$.

**A. ES OPERATION**

The analysis presented in this paper relies on the behavior of the circuit in Fig. 2 at sinusoidal steady state; therefore, the fundamental components are taken for the VSI output voltage and current. In nominal conditions, current $i_I$ equates $i_{NCL}$ in magnitude and opposes to it in phase; then both $i_C$ and $v_{ES}$ are zero and ES does not exert any action on the user supply.

Further to a grid voltage variation, ES acts by adjusting $v_{ES}$ to the distribution line. Voltages $R$ and $C$ input stage of ES whilst capacitor $C_{ES}$, inductance $L_I$, voltage $v_I$, current $i_I$ and voltage $v_{ES}$ are related to the AC output stage of ES. Note that current $i_C$ entering in $C_{ES}$ is the sum of $i_{NCL}$ and $i_I$ whilst the current entering in SL is just $i_{NCL}$.

By (1), the voltage across $i_{NCL}$ and $a$ in (1) is a positive number that goes from values ranging from 1 to 0, being equal to 1 in nominal conditions. The adjustment of $v_{ES}$ modifies both magnitude and phase of $v_{NCL}$ with respect to $v_S$ and the same occurs for the active and reactive powers drawn by NCL; for a reactive ES, the active power drawn by NCL is equal to that drawn by SL whilst this does not hold for the reactive power; specifically, for $i_{NCL}$ lagging $v_{ES}$, the reactive power absorbed by $C_{ES}$ adds to that of NCL; for $i_{NCL}$ leading $v_{ES}$, it subtracts from that of NCL and, if greater than that of NCL, makes the reactive power absorbed by SL of capacitive type.

**B. MODUS OPERANDI**

Analysis of the scheme of Fig. 2 is performed by assuming that ES keeps $V_S$ steadily at $V_{S,N}$ under grid voltage variations.

**C. ES MODELING**

By (1), the voltage across $C_{ES}$ can be written as

$$jX_{CES}i_C = jX_{CES} (1 - a) i_{NCL}$$

where $X_{CES}$ in (2) is equal to

$$X_{CES} = - \frac{1}{\omega C_{ES}}$$

and $\omega$ in (3) is the grid angular frequency. The term on the right-hand side of (2) shows that ES can be modeled with the equivalent reactance

$$X_{eq} = X_{CES} (1 - a)$$

flowed by current $i_{NCL}$ and, hence, placed in series to NCL. Nature (inductive or capacitive) and value of $X_{eq}$ depend on the ratio $a$ in (1). Eq. (4) substantiates what anticipated before: for $a = 1$, $X_{eq}$ is 0, signifying that ES action does not interfere in the NCL operation; for $a$ ranging from 1 to 0, $X_{eq}$ is less than 0 and the ES action is of capacitive type; for $a$ greater than 1, $X_{eq}$ is greater than 0 and the ES action is of inductive type.

By (4), the scheme of Fig. 2 can be represented as in Fig. 3 (a), where $\bar{Z}_G$, $\bar{Z}_CL$, and $\bar{Z}_{NCL}$ are the impedances of the distribution line, CL and NCL, respectively. The SL branch can also be represented in more detail as in Fig. 3 (b), where $\bar{V}_{X,SL}$ is the voltage across the total SL reactance, given by the sum of $X_{eq}$ and $X_{NCL}$. Inspection of Fig. 3 (b), the reactive nature of $X_{eq}$ and the change of its value with $a$ yield that

i. phase $\varphi_{SL}$ of the SL impedance changes further to the ES action, being equal to that of NCL, i.e. to $\varphi_{NCL}$, in nominal conditions,

ii. the power factor of SL increases for $a$ ranging from 1 to 0, and decreases for $a$ greater than 1; incidentally, the latter inconvenience is circumvented by resorting to active ESs.

**III. NCL-DRAWN ACTIVE POWER EXTREME VALUES**

Power $P_{NCL}$ drawn by NCL in nominal conditions is equal to

$$P_{NCL,N} = \frac{(V_{S,N}\cos\varphi_{NCL})^2}{R_{NCL}}$$

Maximum power $P_{NCL,Max}$ drawn by NCL is reached when voltage $V_{R,NCL}$ across $R_{NCL}$ is equal to $V_{S,N}$. As shown by the diagram with red phasors in Fig. 4, in this circumstance it occurs that i) $V_{ES}$ exactly opposes to the voltage drop across $X_{NCL}$ so that voltage $\bar{V}_{X,SL}$ is zero, and ii) the phase of the SL impedance is also zero, which means that SL works at unity power factor. Note that an additional increase of $V_{ES}$ would reduce $V_{R,NCL}$ and, with it, the active power drawn by
NCL. In correspondence to $P_{NCL,\text{Max}}$, both voltage $V_{R,\text{NCL}}$ and current $I_{\text{NCL}}$ get their maximum magnitudes, denoted as $V_{R,\text{NCL,Max}}$ and $I_{\text{NCL,Max}}$.

The value of $P_{NCL,\text{Max}}$ normalized to $P_{NCL,N}$ is

$$P_{NCL,\text{Max}} = \frac{1}{\cos\phi_{\text{NCL}}}^2$$

The values of $V_{R,\text{NCL,Max}}$ and $I_{\text{NCL,Max}}$, normalized to the respective magnitudes in nominal conditions, are equal to $1/\cos\phi_{\text{NCL}}$. These outcomes are collected in Tab. 1 for $\cos\phi_{\text{NCL}} = 0.9$.

In principle, the minimum power that NCL could draw is zero and is reached when $\bar{V}_{\text{ES}} = \bar{V}_{S,N}$. Nevertheless, it is commonly requested to energize NCL -even if at reduced power- also under an undervoltage, as shown by the diagram with blue phasors in Fig. 5. Let the minimum power $P_{NCL,\text{Min}}$ drawn by NCL be set at a percentage $A$ of $P_{NCL,N}$; the resultant power factor of SL is

$$\left(\cos\phi_{\text{SL,Min}}\right)^2 = A \left(\cos\phi_{\text{NCL}}\right)^2$$

In correspondence to (7), also $V_{R,\text{NCL}}$ and $I_{\text{NCL}}$ get minimum magnitudes, denoted as $V_{R,\text{NCL,Min}}$ and $I_{\text{NCL,Min}}$; their normalized values are equal to $\cos\phi_{\text{SL,Min}}/\cos\phi_{\text{NCL}} = \sqrt{A}$. These outcomes are collected in Tab. 1 for $A = 62\%$ and $\cos\phi_{\text{NCL}} = 0.9$. Note that, for the chosen value of $A$, the power factor of SL when drawing $P_{NCL,\text{Min}}$ is $1/\sqrt{2}$.

### IV. LINE VOLTAGE DROP STUDY

#### A. LINE VOLTAGE DROP CALCULATION

Magnitude $V_{DL}$ of the voltage drop along a distribution line is given by

$$V_G = V_{DL} + V_S$$

As per the IEC standards [23], it is well approximated by

$$V_{DL} = R_G I_{G,a} + X_G I_{G,r}$$

where $I_{G,a}$ and $I_{G,r}$ are the projections of $\bar{I}_G$ respectively along the direction of $\bar{V}_S$ and that of one of an axis rotated of $-\pi/2$ out of $\bar{V}_S$. The two projections are the so-called active and reactive components of $I_G$ and are equal to the sum of the respective components of the CL and NCL currents

$$I_{G,a} = I_{\text{NCL,a}} + I_{\text{CL,a}} \quad I_{G,r} = I_{\text{NCL,r}} + I_{\text{CL,r}}$$

By (8), the user voltage stabilization is attained as long as the variations of $V_G$ are counterbalanced by equal variations of $V_{DL}$. For $V_S$ steadily at $\bar{V}_{S,N}$, the active and reactive components of $I_{CL}$ remain unaltered and the stabilization of $V_S$ can be obtained only by modifying the active and reactive components of $I_{\text{NCL}}$. Rewriting (8) and (9) under $V_G$ variations leads to

$$\Delta V_G = \Delta V_{DL}$$

$$\Delta V_{DL} = R_G \Delta I_{\text{NCL,a}} + X_G \Delta I_{\text{NCL,r}}$$

where the variational quantities in (11) and (12) are taken with respect to the relevant nominal values. By (11), the range of $\Delta V_G$ that can be stabilized depends on the achievable values of $\Delta V_{DL}$; by (12), the latter values depend on the components of the distribution line impedance and the achievable variations, sign included, of the active and reactive components of $I_{\text{NCL}}$.

For the demand-side power paradigm to be fulfilled, the sign of $\Delta I_{\text{NCL,a}}$ must be the same as $\Delta V_G$, i.e. it must be excluded the circumstance by which $\Delta I_{\text{NCL,a}}$ varies in opposition to $\Delta V_G$ and the balance in (12) is reached thanks to the variations of $\Delta I_{\text{NCL,r}}$. In other words, let $\Delta V_G$ be positive; the counterbalancing value of $\Delta V_{DL}$ must not be obtained with a negative one of $\Delta I_{\text{NCL,a}}$ and a positive value of $\Delta I_{\text{NCL,r}}$ and the concurrent prevalence of the absolute value of the voltage drop across the line reactance with respect to the line resistance. This represents the constraint to be imposed for the fulfillment of the demand-side power paradigm when stabilizing the user voltage and is taken on in the next Section.

#### B. NCL CURRENT COMPONENTS

The SL voltage is comprised of two phasors, namely resistive phasor $\bar{V}_{R,\text{NCL}}$ and reactive phasor $\bar{V}_{X,\text{SL}}$. The two phasors are orthogonal, and their sum gives $\bar{V}_{S,N}$. Changing of the reactive phasor ensuing from the adjustment of $X_{eq}$ makes the tip of $\bar{V}_{R,\text{NCL}}$ to describe a semi-circumference whose diameter is $\bar{V}_{S,N}$, as traced in Fig. 6. The same is done by the tip of $\bar{I}_{\text{NCL}}$, the diameter of its semi-circumference being $\bar{V}_{S,N}/R_{\text{NCL}}$. 

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**FIGURE 4.** SL phasor diagram for the maximum active power drawn by NCL. SL phase $\phi_{\text{SL,Max}}$, not reported in the diagram, is zero.

**FIGURE 5.** SL phasor diagram for the set value of the minimum active power drawn by NCL.
When the tip of \( \bar{V}_{NCL} \) moves from NV to PV, the tip of \( I_{NCL} \) moves from NI to PI. This motion is outlined by the SL phasors traced with solid red arrows in Fig. 6, where NV and NI are the tips of \( V_{S,N} \) and \( I_{NCL} \) in nominal conditions and PV and PI are their tips under the situation of maximum active power drawn by NCL. While \( I_{NCL} \) moves towards PI, \( I_{NCL,a} \) increases whilst \( I_{NCL,r} \) decreases; hence \( \Delta I_{NCL,a} \) takes a positive value whilst \( \Delta I_{NCL,r} \) takes a negative value.

In the dual way, when the tip of \( \bar{V}_{NCL} \) moves from NV to QV, the tip of \( I_{NCL} \) moves from NI to QI, where QV and QI are the tips of \( V_{R,NCL} \) and \( I_{NCL} \) under the situation of minimum power drawn by NCL as defined in the previous Section. This motion is outlined by the SL phasors traced with solid blue arrows in Fig. 6. While \( I_{NCL} \) moves towards QI, \( I_{NCL,a} \) decreases whilst \( I_{NCL,r} \) increases; hence, \( \Delta I_{NCL,a} \) takes a negative value whilst \( \Delta I_{NCL,r} \) takes a positive value.

The variations \( \Delta I_{NCL,a} \) and \( \Delta I_{NCL,r} \) as a function of the variation \( \Delta P_{NCL} \) of the active power drawn by NCL are plotted in Fig. 7, where all the quantities are normalized. The plot of \( \Delta I_{NCL,a} \) reveals that:

i. it has the same sign as \( \Delta P_{NCL} \),

ii. it varies proportionally to \( \Delta P_{NCL} \) as one readily deduces from (13) by reminding that \( V_{S,N} \) is constant.

\[
I_{NCL,a} = \frac{1}{V_{S,N}} P_{NCL} \tag{13}
\]

In normalized form, \( \Delta I_{NCL,a} \) as a function of \( \Delta P_{NCL} \) is given by

\[
\frac{\Delta I_{NCL,a}}{I_{NCL,a}} = \frac{\Delta P_{NCL}}{P_{NCL}} \tag{14}
\]

The plot of \( \Delta I_{NCL,r} \) reveals that:

i. it is positive when \( \Delta P_{NCL} \) is negative and vice versa,

ii. it varies inversely to \( \Delta P_{NCL} \) as one readily deduces from (15), being the rate of change of \( \tan \phi_{SL} \) opposite to that of \( P_{NCL} \)

\[
I_{NCL,r} = \frac{\tan \phi_{SL}}{V_{S,N}} P_{NCL} \tag{15}
\]

In normalized form, \( \Delta I_{NCL,r} \) as a function of \( \Delta P_{NCL} \) is given by

\[
\frac{\Delta I_{NCL,r}}{I_{NCL,r}} = \frac{\tan \phi_{SL} P_{NCL}}{\tan \phi_{NCL} P_{NCL}} - 1 \tag{16}
\]

whereas angle \( \phi_{SL} \) can be expressed as

\[
\phi_{SL} = \arccos \left( \frac{\cos^2 \phi_{NCL} + \frac{R_{NCL}}{V_{S,N}^2} \Delta P_{NCL}}{1} \right) \tag{17}
\]

Putting together (11), (14) and (16) yields the two following leading outcomes:

i. only a distribution line with a resistive component of its impedance may allow the fulfillment of the demand-side power paradigm. If the distribution line is purely resistive (R-line), the paradigm is surely fulfilled while it is never fulfilled for a purely reactive line,

ii. a distribution line with mixed resistive-inductive impedance (RL-line) allows the fulfillment of the demand-side power paradigm only if the following constraint is met:

\[
R_G \left| \Delta I_{NCL,a} \right| > X_G \left| \Delta I_{NCL,r} \right| \tag{18}
\]

Noting that

\[
I_{NCL} = \frac{V_{S,N}}{R_{NCL}} \cos \phi_{SL} \tag{19}
\]

and that

\[
I_{NCL,a} = I_{NCL} \cos \phi_{SL}
\]

\[
I_{NCL,r} = I_{NCL} \sin \phi_{SL} \tag{20}
\]

it follows that:

\[
\Delta I_{NCL,a} = \frac{V_{S,N}}{R_{NCL}} \left( \cos^2 \phi_{SL} - \cos^2 \phi_{NCL} \right)
\]

\[
\Delta I_{NCL,r} = \frac{V_{S,N}}{2R_{NCL}} \left[ \sin (2\phi_{SL}) - \sin (2\phi_{NCL}) \right] \tag{21}
\]

By (21), constraint (18) can be reformulated as

\[
\frac{1}{\tan \phi_{G}} > \frac{\left| \cos^2 \phi_{SL} - \cos^2 \phi_{NCL} \right|}{\left| \sin (2\phi_{SL}) - \sin (2\phi_{NCL}) \right|} \tag{22}
\]
where $\varphi_G$ is the phase of the distribution line impedance

$$\tan \varphi_G = \frac{X_G}{R_G} \quad (23)$$

The term at the left-hand side of (22) depends on the ES action while that on the right-hand side is a quantity characteristic of the distribution line supplying the RESE user. Eq. (22) represents the constraint to be met for ES to fulfill the demand-side power paradigm. For small excursions of $\varphi_{SL}$ around the nominal conditions, eq. (22) simplifies

$$|\tan \left(2\varphi_{NCL}\right)| > \tan \varphi_G \quad (24)$$

which permits the a-priori check of the appropriateness of the ES action since it is independent of $\varphi_{SL}$.

V. ES STABILIZATION CAPABILITIES

By exploiting the equations in the previous Section, this Section determines the range of variations of the grid voltage which ES is able to cope with while fulfilling the demand-side power paradigm. Let us analyze first an R-line and then an RL-line.

A. R-LINE

The voltage drop in an R-line as a fraction of the nominal voltage drop is

$$\frac{\Delta V_{DL}}{V_{DL,N}}|_{R-line} = \frac{\Delta I_{NCL,a}}{I_{G,a,N}} \quad (25)$$

By (10), (13), (14) and the twin equation of (13) for CL, eq. (25) can be rewritten as

$$\frac{\Delta V_{DL}}{V_{DL,N}}|_{R-line} = \frac{1}{1 + \frac{P_{CL,N}}{P_{NCL,N}}} \Delta P_{NCL} \quad (26)$$

Eq. (26) discloses that, for an R-line, the normalized value of $\Delta V_{DL}$ is equal to that of $\Delta P_{NCL}$ apart from a coefficient that is less and depends on the ratio $P_{CL,N}/P_{NCL,N}$. When $P_{CL,N}$ is much smaller than $P_{NCL,N}$, the plot of $\Delta V_{DL}$ coincides with that of $\Delta I_{NCL,a}$ in Fig. 7 and the ES stabilization capabilities, expressed in terms of the minimum and maximum values taken by $\Delta V_{DL}$ while $\Delta P_{NCL}$ ranges from $\Delta P_{NCL,Min}$ to $\Delta P_{NCL,Max}$, are exactly equal to $-38\%$ and to $23\%$. Eq. (26) also discloses that the ES stabilization capabilities are notably deteriorated when $P_{CL,N}$ is comparable to $P_{NCL,N}$, as illustrated in Fig. 8 where (26) is plotted for different values of the ratio $P_{CL,N}/P_{NCL,N}$. For instance, for a ratio equal to 1, $\Delta V_{DL}$ ranges only from $-20\%$ to $11\%$.

B. RL-LINE

The voltage drop in an RL-line as a fraction of the nominal voltage drop is

$$\frac{\Delta V_{DL}}{V_{DL,N}}|_{RL-line} = \frac{R_G \Delta I_{NCL,a} + X_G \Delta I_{NCL,r}}{R_G I_{G,a,N} + X_G I_{G,r,N}} \quad (27)$$

By (10) and (21), eq. (27) can be rewritten as

$$\frac{\Delta V_{DL}}{V_{DL,N}}|_{RL-line} = \frac{1 - \frac{\tan \varphi_G}{\tan \left(2\varphi_{NCL}\right)}}{1 + \tan \varphi_G} \cdot \frac{\Delta P_{NCL}}{P_{NCL,N}} \quad (28)$$

where $\varphi_U$ is the phase of the total user impedance, given by the parallel of CL and NCL. Eq. (28) discloses that, for an R-line, the normalized value of $\Delta V_{DL}$ depends on that of $\Delta P_{NCL}$ in a somewhat complex manner. Indeed, in addition to ratio $P_{CL,N}/P_{NCL,N}$ like for an R-line, it is affected by phase $\varphi_U$, which is a fixed quantity for a given user, by phase $\varphi_{SL}$, which is a quantity that changes with $\Delta P_{NCL}$, and by phase $\varphi_G$, which is also a fixed quantity for a given distribution line.

Let i) CL and NCL have the same phase so that it is $\varphi_U = \varphi_{NCL}$, and ii) $P_{CL,N}$ be much smaller than $P_{NCL,N}$. For low values of $\Delta P_{NCL}$, eq. (28) becomes independent on phase $\varphi_{SL}$, as given by

$$\frac{\Delta V_{DL}}{V_{DL,N}}|_{RL-line} = \frac{1 - \tan \varphi_G}{1 + \tan \varphi_G} \cdot \frac{\Delta P_{NCL}}{P_{NCL,N}} \quad (29)$$

This expression confirms that the ES stabilization fulfills the demand-side power paradigm as long as (24) is met because only then do $\Delta V_{DL}$ and $\Delta P_{NCL}$ have the same sign.

C. RL-LINE STABILIZATION CAPABILITIES

For an RL-line, the key equation governing the user voltage stability is (28). To discuss the impact of the phase of the distribution line impedance on a stabilization fulfilling the demand-side power paradigm, reference is made to a user with $\cos \varphi = 0.9$, for both CL and NCL, and $P_{CL,N} \ll P_{NCL,N}$; moreover, RL-lines with different $\tan \varphi_G$ are considered. Note that, for $\cos \varphi = 0.9$, the constraint in (24) becomes $1.26 > \tan \varphi_G$.

Fig. 9(a) plots (28) for some values of $\tan \varphi_G$ less than 1.26; they stand out for the substantial prevalence of the resistive component in the distribution line impedance. The curves demonstrate that ES is able to stabilize the user voltage and
The voltage can be stabilized even for small-section line conductors. The line impedance is limited in magnitude and the interval is located at the border of the negative range of $\Delta P_{NCL}$. This stabilization capability is of poor practical interest. Lastly, by increasing $\tan \varphi_G$ beyond 3 there is no stabilization chance under the demand-side power paradigm even for negative values of $\Delta P_{NCL}$.

It is worth to note that the user voltage can still be stabilized for strongly inductive lines, i.e. with $\tan \varphi_G = 3$ or greater. The fundamental achievement of the analysis above is that the stabilization is here reached with NCL drawing an active power that varies exactly in the opposite direction to that required by the demand-side power paradigm; specifically, by decreasing the active power drawn by NCL under a grid overvoltage and by its increasing under a grid undervoltage. This behavior is at odds with the rule of adapting the load power demand to the generation, which is preferable paradigm for a grid with a large power generation from RESs.

VI. CASE STUDY EXEMPLIFICATION AND TESTING

Theoretical findings of the paper are exemplified and tested for a case study that consists of a single-phase user supplied by a low-voltage (LV) distribution line. Data of the case study are reported in Tab. 2 and are representative of a typical domestic European utility. Inspection of Tab. 2 points out that both CL and NCL are resistive-inductive loads with a power factor of 0.9 and that they together draw an overall power of 6 kW with a power share between NCL and CL of 5 to 1. The LV distribution line has $\tan \varphi_G = 0.3$, which is a common value for small-section line conductors. The line impedance is 1 $\Omega$, meaning that the grid is somewhat weak and justifies the adoption of ES for the user supply voltage stabilization.

Exemplification is carried out by using (28) to calculate the variations ($V_G - V_{G,N}$) of the grid voltage that ES is able to stabilize as a function of the variation of the active power drawn by NCL. The calculated results are plotted with solid blue line in Fig. 10 and show that the user voltage is stabilized against grid voltage variations ranging from $-7$ to $2.1$ V; at the two extremities of the range, there are respectively a lack of 1800 W and a surplus of about 1000 W in the power drawn by NCL, with the lack that is about 5.5% greater and the surplus that is about 15% less than the respective minimum/maximum values calculated from Tab. 1.

Testing is executed by simulating the circuit of Fig. 2 with the Power toolbox in the Matlab/Simulink environment. Two operating limits have been set for the ES operation, namely i) under grid undervoltage, the minimum power drawn by NCL is set at 3.2 kW in accordance with Tab. 1, and ii) under grid overvoltage, the ES action is stopped once its action is no more able to stabilize the user supply voltage.

The simulation results are shown in Fig. 10 by means of red crosses and agree very well with the calculated ones. As an additional exemplification and test, calculated and simulation results are obtained for a LV distribution line with the same impedance but supposing that its $\tan \varphi_G$ is equal to 0.7. The results are shown in Fig. 10 with the dashed blue line and the...
VII. CONCLUSION

Stabilization of the user voltage against the grid voltage variations by means of ES is extensively studied in the literature, but without paying attention to the ensuing variations in the active power drawn by NCL. This in spite of the fact that the grid voltage variations are normally caused by the fluctuations in the power generation, especially in the grids fed by RESs. Indeed, an excess in the power generation produces a grid overvoltage and, vice versa, a shortage in the power generation produces a grid undervoltage. Thus, an ES action that would force the power drawn by NCL in the opposite direction with respect to the power generation stands in the way of the correct functioning of an electric system.

To face this issue, operation and stabilization capabilities of a reactive ES has been analyzed on the basis of the demand-side power paradigm, by imposing that the active power drawn by NCL tracks the power generation. The analysis has been conducted in terms of normalized quantities so that the theoretical findings cover any RESE user, irrespective of the application context.

The significant outcomes of the analysis can be summarized as follows: i) only a distribution line with a resistive component of its impedance is able to give ES the capabilities of stabilizing the user voltage while fulfilling the demand-side power paradigm, ii) the presence of a reactive component in the distribution line impairs such capabilities, until to cancel them out when the reactive component dominates. The outcomes are supported by the formulation of the constraint for the paradigm to be fulfilled and the investigation of the impact of the user and distribution line parameters on the constraint.

Lastly, a numerical exemplification of the theoretical findings has been given for the case study of a typical RESE user connected to a LV distribution line. The calculated results have been validated by simulating the case study in the Matlab/Simulink environment.

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