Geometric model of the gas turbine combustion chamber

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Abstract. Gas turbine plants operated on a mixture of natural gas and air are widely used for self-contained power supply of buildings and structures both in urban conditions and in temporary settlements. When designing perspective gas turbine plants, we need to develop a non-toxic, environmentally friendly combustion chamber with the minimal emissions of harmful substances in a wide range of changes in operating parameters. It is a complicated engineering challenge to meet this condition: it is necessary to ensure geometrically accurate profiling of special holes in the cylindrical wall of the combustion chamber to insert fuel nozzles. As opposed to the known methods, the article proposes a profiling method, which combines the advantages of the known methods of descriptive geometry and precise methods of three-dimensional computer simulation. The real profile of a hole in the wall of the combustion chamber is a fourth-order spatial algebraic curve. The article proposes an algorithm for profile approximation using second-order curves by means of three-dimensional computer graphics, which allows us to streamline the preparation of a program for a water jet NC machine.

1. Introduction
Self-contained gas turbine plants operating on a mixture of natural gas and air can supply thermal and electrical energy not only to individual buildings and villages, but also to urban communities. One of the main problems in the design of modern gas turbine plants is the development of low-toxic combustion chambers that have minimal emissions of harmful substances in a wide range of changes in operating parameters [1]. A major role in solving this problem is assigned to geometric modeling of the shape and individual structural elements of the combustion chamber. In particular, it is necessary to ensure accurate profiling of irregularly shaped holes (windows) in the cylindrical wall of the combustion chamber to insert gas-air nozzles, which ensure mixing and subsequent complete combustion of the gas-air mixture inside the combustion chamber.

2. Problem statement
When the gas turbine unit is turned on or off, there appear pressure pulsations (claps) caused by a rapidly moving combustion front inside the combustion chamber. These claps lead to the distortion of the thin-walled shell of the combustion chamber due to mechanical buckling [2]. To eliminate this phenomenon, it is not only necessary to eliminate pressure pulsations, but also to ensure a minimum gap between the shell of the combustion chamber and the refractory liner of the gas nozzle. The gap is minimized by accurate geometric profiling of the windows in the wall of the combustion chamber, into
which the refractory liners of gas nozzles are inserted (Figure 1). The hole profile is a biquadratic curve breaking into two closed spatial curves of a complex-shaped line [3-6].

Holes are cut on a water jet NC machine. To develop a control program, we need to approximate the real profile of a window with a compound curved line of the second order of smoothness [7-12]. The task is complicated by the need to take into account the wall thickness of the cylindrical combustion chamber.

2.1. Three-dimensional computer model of the combustion chamber
The fuel mixture is supplied to the combustion chamber through several symmetrical gas nozzles. The nozzle has a cylindrical body with a diameter of 35...45 millimeters. The diameter of the cylindrical combustion chamber is 400 millimeters. The combustion chamber is made of metal sheet with a thickness of 2...3 millimeters. To obtain a three-dimensional computer model of the window profile, it is enough to form electronic solid-state models of the combustion chamber and cylindrical nozzles, and then to use the "subtract" command [13, 14]. The "subtract" computer command simulates the procedure of window cutting in the wall of the combustion chamber. The window precisely corresponds to the diameter and slope angle of the cylindrical body of the gas nozzle (Figure 2).

2.2. Engineering drawing of the window profile
The three-dimensional electronic model of the combustion chamber does not satisfy the basic production requirements, since it cannot help to organize accurate profiling of windows on a water jet machine. To build an engineering drawing, we used a three-dimensional electronic model and the "flat shot" computer command, which allows us to obtain orthogonal projections of the combustion chamber with a window cut in it (Figure 3).

2.3. Evolvent of the combustion chamber
The combustion chamber is made of rectangular flat steel sheet, in which a specified number of irregularly shaped windows are cut.

To obtain a flat window profile, we have to make an evolvent of the theoretical spatial profile shown in Figure 3. The evolvent is made on the basis of the known invariant of the theory of surfaces: angles, distances and areas of the figures remain equal on the surface and on its evolvent [15, 16]. Reference data of the AutoCAD graphic suite is used to determine the distances on the curvilinear surface of the combustion chamber. There are accurate and approximate evolvents of surfaces. In descriptive geometry, the surface is set using a drawing, and the evolvent is built on the basis of the
drawing using graphical ways. Therefore, the evolvent of any developable curvilinear surface (conical, cylindrical, torso), which is built graphically, is approximate. The use of computer graphics together with the methods of descriptive geometry can significantly improve the accuracy of the evolvent [17].

![Figure 3. Window profile](image1)

![Figure 4. Evolvent of the window profile](image2)

Let us consider an algorithm for building an evolvent of the combustion chamber with irregularly shaped holes cut in it. For definiteness, let us mark point 1 \((1_1, 1_2)\) belonging to the theoretical spatial window profile. Let us build point \(l_0\) on the evolvent (Figure 4). The algorithm consists of three steps.

1. We mark point \(l(1_2, 1_1)\).
2. Using the “properties” command, we determine the arc length \(m=(A-1)\).
3. Laying off the distance \(m\) on the evolvent, we determine the position of point \(10, l_0\) using the transmission lines.

The discrete set of evolvent points is determined according to the considered algorithm. The algorithm of forming the profile points allows us to combine the advantages of visual projection building methods known from descriptive geometry and precise 3D computer-aided modeling methods.

To compile a control program of the water jet machine, we need to draw a continuous curve of the second order of smoothness passing through the discrete set of points. This problem can be solved in two ways: based on computer spline approximation [18], or using sections of second-order curves. The first method satisfies the geometric requirements, but does not allow us to obtain the equation of a curve. The second method does not only allow us to draw a continuous curve of the second order of smoothness, but also to obtain its mathematical description in the form of a system of algebraic equations [19, 20].

3. Approximation of the window profile using a compound curve of the second order of smoothness

The method of engineering discriminant known from descriptive geometry allows us to draw a curved line of the first order of smoothness consisting of sections of second-order curves passing through a given series of points [21].

Let us consider the problem of building a curve of the second order of smoothness. Through the given set of points, we need to draw a line of the second order of smoothness formed by the arcs of the second-order curves. The constructed line should not contain inflection points. This means that any triangles formed by three consecutive points of a given set must have the same sense of rotation. If this condition is satisfied, through these points we can draw a compound curve of the second order of smoothness formed by the arcs of the second-order curves [22, 23].

Assume that the first section is set by points 1, 2, 3 and tangents \(t_1, t_3\). The second section is set by point 4 and tangent \(t_4\) (Figure 5). At point 3, we need to ensure the connection of the section with the total radius of curvature [24]. The only conic \(a^2\) satisfying the conditions of tangency \(t_1, t_3\) passes
through points 1, 2, 3. We find its circle of curvature \( r \) at point 3. The second bypass section \( b_2 \) is set by a circle of curvature \( r \) and point 4 with the tangent \( t_4 \) indicated in it. We find the conic section \( b_2 \) satisfying these conditions. At point 3, we obtain a three-point contact of curves \( a_2 \), \( b_2 \) (the contact of the second order of smoothness). The curve \( a_2 \) intersects the circle of curvature \( r \) at triple point 3 and at point \( A \). The ellipse \( b_2 \) intersects \( r \) at the same triple point 3 and at point \( B \) [25].

The considered algorithm allows us to build a convex compound curve of the second order of smoothness formed by arcs of ellipses (Figure 6). The compound curve passes through the previously found points 1...26 and has common circles of curvature at abutting points 1, 5, 10, R, 17, 26. The second part of the profile not shown conventionally in Fig. 6 is formed by mirroring the curve of line 1...6 relative to the axis of symmetry.

Thus, the theoretical window profile consists of six sections of interconnected ellipses of the second order of smoothness. The coefficients of symmetric equation of each of the six ellipses are determined using the “properties” command of the AutoCAD graphics suite. Based on the obtained information, we compile a control program for the water jet machine and cut out windows on the evolvent of the combustion chamber. A photograph of one of the windows is shown in Figure 7.

4. Taking into account the shell thickness
If the shell thickness is more than 1 percent of its diameter, the actual window profile may differ significantly from its evolvent (Figure 8). In this case, we have to take into account the elastic-plastic deformation that occurs when the workpiece is bent. Let us consider a geometrical model of bending a
plate, which thickness is commensurate with the radius of curvature. Two main hypotheses are used in the theory of calculating plates and shells [26].

1. The median plane of the plate is bent isometrically, i.e., the distances between the points are maintained during bending.

2. When the shell and the plane are aligned, the radial directions become parallel.

According to the first hypothesis, the length of the arcs AB and AC on the shell surface is equal to the length of the sections AB and AC on the evolvent (Figure 9). According to the second hypothesis, the straightness of the radial directions is preserved during bending. For example, points A A, A', A" remain collinear after bending the plate. In the general case, the straightness is not preserved. For example, collinear points B', A, C'' after bending are no longer located on one straight line.

![Figure 9. Shell bending model.](image)

5. **Graphic experiment**

We will assume that the base line does not compress or expand when the plate is bent. Let us divide the plate into several layers. We drill an inclined hole in the plate with the axis m (Figure 10). We mark points 10..90, in which the axis of the hole intersects the boundaries of the layers. We project points 10...90 from the center R onto the base line k. We receive projections 1...9. When the plate is bent, the base line k is straightened. According to the second hypothesis, during bending the center of curvature R is gradually removed to infinity. The radial straight lines become parallel, which allows us to build an evolvent of the plate with a hole drilled in it. The graphic experiment has shown that the hole axis takes the shape of a curve of line m2, which is approximated to a high accuracy by a second-order curve. The approximation error is less than one percent. To make the graphic experiment, specialized software product is used, which precisely builds the second-order curve passing through the specified points [27, 28].

Thus, the algorithm of building an evolvent of a thick-walled shell with a hole drilled in it is based on two hypotheses of the theory of plates and shells, and contains only five steps.

Step 1. We choose several layers of the shell.

Step 2. We mark representative points 10, 20, 30, ... on the borders of the layers.

Step 3. We project the representative points from the center of curvature onto a preselected base line. As a rule, the base line belongs to the median plane of the bendable shell.

Step 4. We transfer the base line to the evolvent. According to the first hypothesis, the length of the arc connecting any two points of the base line is equal to the distance between these points on the evolvent.
Step 5. We transfer the points from the base line to the corresponding layers of the evolvent. According to the second hypothesis, to this end, it is sufficient to conduct a beam of parallel projecting rays.

![Figure 10. Bend of the straight line (graphic experiment).](image)

6. Conclusion
The development of low- (up to 50 kilowatts) and medium-power (up to 1 megawatt) self-contained gas turbine plants for the cold and northern territories of the Russian Federation is a relevant technical task. The requirements of high reliability and minimization of harmful emissions are posed to such plants. In particular, if we want the combustion chamber of a gas turbine plant to meet these requirements, we have to ensure a minimum clearance between the gas nozzle body and the shell of the combustion chamber. The combined use of computer graphics and descriptive geometry tools and methods allows us to develop a geometrically precise algorithm for profiling nozzle windows. The algorithm is based on a three-dimensional computer simulation of the combustion chamber and building of its evolvent. The window profile on the evolvent of the combustion chamber is approximated by a compound curve of the second order of smoothness. The sections of the compound curve are formed by arcs of conic sections, which allows us to describe the window profile with second-order algebraic equations. Based on the obtained mathematical model, we compile a control program for the water jet machine. We proposed a graphic algorithm based on the theory of plates and shells, which allows us to take into account the actual thickness of the shell. It is shown that, when evolved, the axis of the hole in the thick-walled shell is converted into a curved line, which can be approximated by an arc of conic section to a high accuracy.

7. References
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