Development and application of a dynamic stall model for rotating wind turbine blades

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Abstract: In unsteady conditions of wind turbines, both the dynamic stall phenomenon and the three-dimensional (3D) rotational effect affect the rotor aerodynamics. The dynamic stall mechanism for rotating wind turbine blades is first investigated. Through the comparison of the aerodynamic data between the rotating blade and the two-dimensional (2D) airfoil, the normal force slope in the attached flow and the separation point expression in the separated flow are modified in the Beddoes-Leishman (B-L) dynamic stall model for rotating NREL wind turbine blades. The modified model is validated by the comparison between the calculation results and the experimental results of the lift and drag coefficients at different radial positions. Both the hysteresis loop shapes and the calculation values are closer to the experiment than the 2D dynamic stall model. The present dynamic stall model is then coupled to a free vortex wake model. The coupled model is used to calculate the unsteady blade aerodynamic loads and the low speed shaft torque of the NREL wind turbine in a yawed condition. The accuracy is greatly improved by the corrections presented in the paper.

1. Introduction

The unsteady aerodynamic characteristics prediction of wind turbines has been one of the most crucial and most difficult problems in wind power generation system [1]. Factors such as atmospheric turbulence, wind shear, skewed flow, and the turbine tower shadow, etc., all have significant effects on turbine blade inflow conditions. An airfoil section undergoes dynamic stall when it is subjected to any form of unsteady angle of attack motion. Therefore, the dynamic stall is common for wind turbine. Because of the blades rotation, the 3D rotational effect also exists in the rotor aerodynamics. Many dynamic stall models and 3D rotational effect models were built separately by different researchers. Whether via experiments or numerical calculations, the process and mechanism of 2D dynamic stall were well understood. Many mathematical models for its engineering applications were provided, e.g., the well-known B-L model [2] and the ONERA model [3]. Among these models, the B-L model was widely applied to predict the blade unsteady loads by, e.g., Pierce [4], Wang [5] and Gupta [6]. For 3D wings, the investigations [7,8] of 3D dynamic stalls made some progress and some 3D dynamic stall models [9,10] were developed. If the unsteady prediction of wind turbine only considers the dynamic stall characteristic, it will not be accurate because of the existence of the 3D rotational effect.
The rotational effect is one of the typical differences between rotor and wing. It is the cause of the stall delay phenomenon, which is characterized by significantly increased lift coefficient compared to the corresponding 2D case, and by a delay of the occurrence of flow separation to higher angles of attack. Many researchers, both in the helicopter and wind turbine fields, focused on it via computational theory and experimental investigations. Different engineering models, such as Du-Selig model [11], Chaviaropoulos and Hansen model [12], Bak model [13], Lindenburg model [14] and so on, were developed to represent the stall delay phenomenon. A comparison of six models using a prescribed wake vortex scheme [15] was made to illustrate the significance of stall delay models and some defects in these models.

To predict the rotor aerodynamic performance, a free vortex wake (FVW) code, which finds induction directly at the lifting line from the effect of the modeled wake, will be used in this paper. The FVW methods were successfully applied in the helicopter field [16,17,18], of which the experience can be introduced into wind turbine aerodynamic studies due to the similarity between helicopters and wind turbines. Garrel [19] introduced a non-linear vortex-line strength model into the FVW method to represent the rotor wake rollup and the non-linear aerodynamic characteristics of blades. Sant et al. used a FVW code to estimate the angles of attack from blade pressure measurements respectively in the axial conditions [20] and the yawed conditions [21], and provided us new insights into how circulation evolves at the blades. Gupta developed a PC2B (Predictor-Corrector 2nd Backward) scheme [6] for wind turbine application and studied its stability in the reference [22]. He also validated the FVW model [23,24] in steady and unsteady conditions. These above works illustrated that the FVW method is efficient and reliable for wind turbine aerodynamic prediction and wake analysis. In the FVW model of this paper, a time-accurate algorithm which is called D3PC (Three-step and Third-order Predictor-Corrector) algorithm [25] was used to approximate the derivatives on the left hand side of the wake governing equation.

In this study, the dynamic stall mechanism for rotating wind turbine blades is first investigated using the NREL phase VI wind turbine [26], of which the airfoil is S809. From the result of this investigation, a dynamic stall model for rotating NREL wind turbine blades is developed based on the B-L dynamic stall model to correct the normal force slope in the attached flow and the separation point expression in the separated flow. The correctional model is then validated and coupled with a FVW model to predict the unsteady blade aerodynamic loads and performance of NREL phase VI wind turbine. Comparisons between the present dynamic stall model and the 2D dynamic stall model are made in the validation and the coupled calculation.

2. Mechanism of dynamic stall for rotating wind turbine blades
When there are changes, periodic or not, in the inflow conditions and the angle of attack of an airfoil, the phenomenon of dynamic stalls will occur. The delay in the flow field and the dynamic stall vortex, the shedding of a strong vertical disturbance from its leading edge, are two important characteristics of dynamic stalls. In the case of a rotating wind turbine, there are some other characteristics besides those in the 2D dynamic stall due to the 3D effect and the rotational effect. On a 3D rectangular wing, the dynamic stall vortex system is formed and then starts to convect, moving faster at the mid-span than on outboard locations, forming the so-called “Ω” structure [7]. Due to the rotational effect, the centrifugal radial flow results in a thinner boundary layer, and a Coriolis acceleration acts as a favourable chordwise pressure gradient. These two factors delay the boundary layer separation and increase the maximum lift coefficient.

When wind turbines operate in an unsteady flow, the aerodynamic characteristics are affected by the dynamic stall and the 3D rotational effect at the same time, and differ largely from that of a 2D airfoil. Fig. 1 is the aerodynamic forces comparison between the rotating blade section (0.3R) of NREL phase VI wind turbine [26] and the 2D S809 airfoil. The rotating blade data is the experimental results operating at 10 m/s wind speed, 72 rpm rotor speed and 30 degrees yaw angle. The 2D S809 airfoil data is calculated by the B-L dynamic stall model and the change of the angle of attack is the same as 30% radius position of the rotating blade. In Fig. 1(a), the hysteresis loop of lift coefficient at 30%
radial position is flatter than that of the 2D airfoil. It is reasonably explained that the dynamic stall vortex has not been shed from the blade trailing edge at the high angle of attack resulting from the rotational effect. Moreover, this delayed shedding results in a greater drag than the 2D airfoil (see Fig. 1(b)).

![Graphs showing lift and drag coefficients comparison](image)

**Fig.1.** Aerodynamic forces comparison between the rotating blade section (0.3R) and the 2D airfoil

3. **Modeling of dynamic stall for rotating wind turbine blades**

In this paper, similar to the B-L model, the unsteady flow of the rotating wind turbine blades undergoing dynamic stall is divided into three parts, the attached flow, the separated flow, and the dynamic stall vortex.

3.1. Attached flow

The unsteady attached flow is represented in the B-L model for the 2D airfoil by an indicial response. The indicial response can conveniently be split into two components, one to solve for the circulatory loading which builds up quickly and asymptotes to the appropriate steady-state loading, and the other for the initial loading which is impulsive (non-circulatory) and decays rapidly with time. The normal force coefficient corresponding to the circulatory response is

$$C_n^\alpha(N) = C_n^\alpha \left( \alpha_e - \alpha_0 \right)$$

where $N$ is the denotation of the current sample, $\alpha_e$ is the effective angle of attack under the unsteady attached flow condition and $\alpha_0$ is the zero-lift angle of attack.

The blade can be seen as an elliptic wing with finite aspect ratio. Generally, the lift slope of wings is smaller than that of the 2D airfoil because of the 3D effect. From a large number of experiments, it is validated that the lift slopes of the rotating blades are indeed smaller than that of the 2D airfoil (see in Fig.2). Considering the 3D effect, the normal force slope $C_n^\alpha$ should be corrected as

$$C_n^\alpha = \frac{C_{n\infty}^\alpha}{1 + \frac{C_{n\infty}^\alpha}{\pi \gamma}}$$

where $C_{n\infty}^\alpha$ is the normal force slope of the 2D airfoil, and $\gamma$ is the aspect ratio of the blade.

3.2. Separated flow

A phenomenon that appears to be involved in most types of stall is the progressive trailing edge flow separation. The determination of separation point is very important for the accurate modelling of separated flow. For general airfoils with the single stall characteristic, the separation point position $f$ which is the distance to the leading edge with respect to the chord can be generalised in a fitted exponential format as
where the coefficients $S_1$ and $S_2$ define the static stall characteristic, while $\alpha_1$ defines the break point corresponding to $f = 0.7$. $S_1$, $S_2$ and $\alpha_1$ can easily be determined from the wind tunnel static lift data. For the S809 airfoil, presenting the characteristic of double stalls, the separation point position can be expressed by three exponential curves as

$$ f = \begin{cases} 
1 - 0.3 \exp \left( \frac{\alpha - \alpha_1}{S_1} \right) & \alpha \leq \alpha_1 \\
0.04 + 0.66 \exp \left( \frac{\alpha - \alpha_1}{S_2} \right) & \alpha > \alpha_1
\end{cases} \quad (3) $$

where $\alpha_2$ is the angle of attack corresponding to the second rebound in lift curve. $\alpha_1$ and $\alpha_2$ play a significant role in the representations of the separated flow. When $\alpha_1$ and $\alpha_2$ are determined from the 2D static lift data, equation (3) and equation (4) are only suitable for the 2D airfoil. For the rotating blades, the values of $\alpha_1$ and $\alpha_2$ have large differences because of the 3D rotational effect.

$$ f = \begin{cases} 
1 - 0.5 \exp \left( \frac{\alpha - \alpha_1}{S_1} \right) & \alpha \leq \alpha_1 \\
0.7 - 0.68 \exp \left( \frac{\alpha - \alpha_2}{S_2} \right) & \alpha_1 < \alpha \leq \alpha_2 \\
0.006 \exp \left( \frac{\alpha - \alpha_2}{S_3} \right) & \alpha > \alpha_2
\end{cases} \quad (4) $$

where $\alpha_1$ and $\alpha_2$ are obtained from the 2D airfoil characteristics and $\alpha_1 = 10.2^\circ$, $\alpha_2 = 20.2^\circ$. These corrections are suitable for the S809 airfoil. Further investigations on other airfoils need to be done based on more measurements data.

**Fig. 2.** Measured static lift curves in different sections of the rotating NREL blade and 2D airfoil

**Fig. 3.** Values of $\alpha_1$ and $\alpha_2$ along the radial position of the rotating NREL blade
For dynamic conditions, the leading edge pressure response and the unsteady boundary layer response should be considered [5]. The effects of the responses above on the trailing edge separated point may be represented by applying a second-order lag to the value of $f$. Then the value of $f''$ with respect to the changes of angle of attack is obtained and the normal force coefficient under separated flow becomes

$$C_n(N) = C_n^C(N) \left(1 + \frac{\sqrt{f''(N)}}{2}\right)^2$$  \hspace{1cm} (7)

3.3. Dynamic stall vortex
From analysis of static airfoil data, a critical static normal force coefficient, $C_{n1}$, may be obtained which corresponds to the critical pressure for the leading edge separation onset. When $C_n > C_{n1}$, the leading edge separation is initiated and the dynamic stall vortex which is generated near the leading edge subsequently separates from the upper surface and is transported downstream over the chord. Furthermore, when $C_n < C_{n1}$, the flow re-attaches. The total accumulated vortex normal force coefficient $C_n^v$ is allowed to decay exponentially with time and is also updated by a new increment and is represented as in the B-L model [2].

4. Validation for the correctional model
To validate the separation point expression equation (4), the simulation of the nonlinear static characteristics of the S809 airfoil, which was used in the NREL wind turbine blade, was made. $S_1=0.04$, $S_2=0.23$ and $S_3=0.19$ based on measurements were used in equation (4). The modeling results, including the present model and Gupta’s model [6], and the experimental results of the S809 airfoil are shown in Fig. 4. It is shown that the results obtained by the present model are generally quite fine, especially in the attach flow region and the deep stall region. In the separation region, the normal force coefficient agrees better with the experimental result than Gupta’s model. The characteristic of double stalls, which is not modeled by Gupta’s model, is described by the present model. The tangential force coefficient in the separation region is slightly over-predicted, but is smoother than that from Gupta’s model.

In the following use of the correctional dynamic stall model to calculate the unsteady lift and drag coefficients at different radial positions of the NREL wind turbine blade is shown. The operating condition is 10 m/s wind speed, 72 rpm rotor speed and 30 degrees yaw angle.

Fig. 5 shows lift coefficient curves from the experiment and the calculation at 30%, 63% and 95% radial positions. Because of the variation of the tangential speeds along the blade span, the mean value...
and the amplitude of angle of attack increase from the blade tip to root. At the three sections, the results from the correctional model all agree well with the experimental data. However, the 2D model gives worse results especially at 30% radius sections (see Fig. 5(a)). Resulting from the rotational effect, the dynamic stall vortex has not been shed from the blade trailing edge at the high angle of attack, so the hysteresis loop of lift coefficient of the rotating blade is flatter than that of the 2D airfoil. The correctional model can reflect the above characteristic via the $\alpha_1$ and $\alpha_2$ corrections. The lift coefficients at 95% radius section (see Fig. 5(c)) are smaller than the 2D airfoil resulting from the 3D effect. The present calculation values are decreased by the normal force slope correction.

![Graph](image)

(a) Section of 30%R  (b) Section of 63%R  (c) Section of 95%R

Fig. 5. The lift coefficient with angle of attack at different blade sections for a yaw angle of 30° and a wind speed of 10 m/s

Fig. 6 shows drag coefficient curves of the experimental results and the calculation results at 30% and 63% radius positions. The results of the present model are closer to the experimental results than that of the 2D model. As we all know, it is difficult to accurately predict the drag coefficient for both 2D airfoils and 3D wings, so it is understood that highly accurate prediction of the drag coefficient for the 3D rotating blade couldn’t be achieved.

![Graph](image)

(a) Section of 30%R  (b) Section of 63%R

Fig. 6. The drag coefficient with angle of attack at different blade sections for a yaw angle of 30° and a wind speed of 10 m/s

5. Application for the prediction of unsteady blade aerodynamic loads and performance

The correctional dynamic stall model is coupled with the FVW model to predict the unsteady blade aerodynamic loads and performance of the wind turbine from NREL unsteady aerodynamic phase VI experiment. Unlike the blade element momentum (BEM) method where annular average induction is found, the FVW method used here could find induction directly at the lifting line from the effect of the modeled wake. In this section, firstly the FVW model and the coupling procedure are briefly introduced, and then the calculation results are discussed.

5.1. Description of the FVW model

The FVW model assumes that the flow field is incompressible and potential. The blade is modeled as a series of straight constant strength vortex segments lying along the blade quarter chord line. The vortex filaments, extending downstream from the trailing edge of the blade element boundary, are allowed to freely distort under the influence of local velocity field.
To solve the convection equation of the vortex filaments numerically, finite difference approximations are used to approximate the derivatives. For the unsteady calculation, the accuracy of the temporal derivative approximation is a significant part in the FVW model. In this study, the D3PC (Three-step and Third-order Predictor-Corrector) algorithm [25] was used for the temporal derivative, and the five-point central difference approximation [27] was used for the spatial derivative.

5.2. Model coupling

The FVW method is used to calculate the convergent wake at the initial time. When the operating time marches and achieves next time step, the new wake geometry is obtained. The induction at the blade elements can be found from the new vortex wake. The information required by the correctional dynamic stall model, such as mean angle of attack, change amplitude in angle of attack and pitching frequency, can be obtained. Then the present correctional dynamic stall model is coupled in this procedure to calculate the real blade aerodynamic loads. If the time course is complete, the modeling is over. It is noticed that the shed vorticity is not considered in the free vortex wake model because its effect is modeled by the dynamic stall model.

The process of the calculation is shown in Fig. 7.

5.3. Prediction of unsteady blade aerodynamic loads

Fig. 8 shows the variation of the normal force coefficient along the azimuth angle at different blade sections for a yaw angle of 30° and a wind speed of 10 m/s. The three sections are at 30%, 63% and 95% radial positions, respectively indicating the root, the mid-part and the tip. At a blade azimuth of 0°, the blade is located at the 12 o’clock position.

Fig. 9 shows the Angle of attack distribution along the azimuth angle at three blade sections predicted by the FVW model. When the blade is at 0° azimuth angle, the angle of attack reaches a maximum value. Because the rotor is operating in yaw, the angle of attack varies with blade azimuth in approximate sinusoidal fashion at all span locations. In Fig. 8, the trends of the calculation results, whether using the present correctional model or the 2D model, are well matched with the experimental results. However, the values of the correctional model are closer to the experimental data than that of the 2D model. Because of the stall delay resulting from the 3D and the rotational effects, the normal...
The force coefficient is larger than that predicted by the FVW model coupled with the 2D dynamic stall model in the blade root and mid-part. At the blade tip, the airfoil is in the fully attached flow because of the low angles of attack, so the correctional model predicted better via correcting the normal force slope. Around 0° azimuth angle, the loading predictions are poor because of the difficulties in predictions of the unsteady high angle of attack especially around the blade root. Also, the interferences of tower and nacelle, which are not considered in the FVW model, exist in the wind tunnel test.

Fig. 9. Angle of attack along the azimuth angle at different blade sections predicted by the FVW model for a yaw angle of 30° and a wind speed of 10 m/s

Fig. 10 shows the variation of the tangential force coefficient along the azimuth angle at different blade sections for the above operating condition. Like the normal force coefficient, the tangential force coefficient predicted by the correctional model agrees with the experimental data better than the 2D model. However, it is understood that around 0° azimuth angle the predicted tangential force coefficient is not accurate on inboard sections because of the difficult prediction for the drag coefficient of large angles of attack.

Fig. 10. Tangential force coefficient at different blade sections for a yaw angle of 30° and a wind speed of 10 m/s computed with the FVW model

5.4. Prediction of unsteady low speed shaft torque
Fig. 11 shows the low speed shaft torque along azimuth angle for the above operating condition. Because the rotor is operating in yaw, the period of blade aerodynamic load is 360 degrees. However, because of two blades, the period of the low speed shaft torque is 180 degrees. The trends of the calculation results from the two coupled models all agree with the experimental data. The mean value 1205 Nm from the present coupled model is very close to the experimental mean value 1182 Nm. The mean value 1025 Nm from the 2D coupled model is under-prediction because of no considerations of the 3D and the rotational

Fig. 11. The low speed shaft torque along azimuth angle for a yaw angle of 30° and a wind speed of 10 m/s computed with the FVW model
effects. However, it is complex to couple the FVW model with these two different models. Therefore the correctional dynamic stall model, which considers both the 3D and the rotational effects, has a definite advantage. In the wind tunnel test, the tower in the wake can be seen as a potential flow element, so the influence of the tower changes the flow at the rotor. When the blade was at about 180° azimuth angle, the aerodynamic loads of the blade would be influenced by the tower. In this paper, we calculated the aerodynamic performance of the rotor without consideration of the tower. In the future work, the influence of the tower may be calculated using the panel method.

6. Conclusions
A study of the dynamic stall mechanism for the rotating NREL wind turbine blades was performed. The lift slope of the blade is smaller than that of the 2D airfoil because of the 3D effect. Resulting from the rotational effect, it was concluded that the dynamic stall vortex was not shed from the blade trailing edge at the high angle of attack, so the hysteresis loops of the lift coefficient on inboard sections of the rotating blade are flatter than that of the 2D airfoil. From the result of the mechanism investigation, the B-L dynamic stall model is modified to consider the 3D and the rotational effects. In the B-L model, the normal force slope in the attached flow and the separation point expression in the separated flow are modified based on the NREL measurement data. For the rotating NREL blades, the correctional model can greatly improve the accuracy especially around the blade root. Moreover, the flat hysteresis loops of the lift coefficient agree well with the experimental data.

A FVW model is used to couple with the correctional model to calculate unsteady blade aerodynamic loads and the low speed shaft torque of the NREL phase VI wind turbine in a yawed condition. The results along the azimuth angle are closer to the experimental results than the one obtained from the model coupled with a 2D dynamic stall model. The accuracy is also greatly improved especially around the blade root. Although the parameters are modified based on the experimental data of the NREL phase VI wind turbine, the results indicate that the modifications in the dynamic stall model for rotating wind turbine blades are very important.

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