Heavy Quarkonium Dissociation by Thermal Gluons at Next-to-leading Order in the Quark-Gluon Plasma

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Abstract

Using the chromo-electric dipole coupling Hamiltonian from QCD multipole expansion, we derive the dissociation cross sections of heavy quarkonia by thermal gluons at next-to-leading order (NLO, also known as inelastic parton scattering dissociation) in the Quark-Gluon Plasma (QGP) in the framework of second order quantum mechanical perturbation theory. While being divergent (infraredly sensitive) in vacuum, the cross sections thus derived become finite in the QGP as rendered by the finite thermal gluon masses. In contrast to the leading order (LO, also known as gluo-dissociation) counterparts dropping off toward high energies, the NLO cross sections turn out to increase monotonously with the incident gluon energy as a result of new phase space being opened up. We then carry out a full calculation of the pertinent dissociation rates for various charmonia and bottomonia within a non-relativistic in-medium potential model. The NLO process is shown to dominate the dissociation rate toward high temperatures when the binding energies of heavy quarkonia become smaller relative to the Debye screening mass.

Keywords: Heavy Quarkonium, Quark Gluon Plasma, Ultrarelativistic Heavy-Ion Collisions

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1. Introduction

It was first suggested in the seminal work by Matsui and Satz [1] that heavy quarkonium (bound states of heavy quark-antiquark $Q\bar{Q}$) dissociation, as a result of the screening of the binding color forces at short distances in a hot thermal bath, can be used as a probe of the deconfined state of nuclear matter known as Quark-Gluon Plasma (QGP). This picture was supported by the experimental observation of the sequential suppression of the $\Upsilon$ states at the LHC [2], with the less bound excited $2S$ and $3S$ states being more suppressed than the most tightly bound ground $1S$ states.

Besides static screening, however, other mechanisms involving in particular collisions with plasma particles can lead to dynamical dissociation of the bound states. These collisional processes generate inelastic widths that enter the transport equations describing the evolution (yields, momentum spectra) of the bound states in the QGP [3, 4, 5, 6, 7, 8, 9]. In this respect, the one that has been intensively studied in the past decades is the gluo-dissociation of heavy quarkonium $g + \Psi \rightarrow Q + \bar{Q}$ ($\Psi$ denotes a $QQ$ bound state) [10, 11, 12, 13, 14, 15, 16, 17], an analog of photo-dissociation of neutral atoms owing to dipole transition. In this process, an energetic thermal gluon from the medium is absorbed by the bound state, thereby the binding energy of the bound state is overcome, and the latter is dissociated into an unbound color octet ($Q\bar{Q}$). The cross section of this leading order (LO) scattering of thermal gluons off heavy quarkonium, when convoluted with the gluon distribution function, leads to a dissociation rate that decreases with increasing temperature, see e.g., [18], due to the fact that as the binding energy of the bound state gets lower, the peak of the LO cross section shifts toward smaller gluon energy corresponding to very small phase space [19].

It was first noted in [19] that this artifact of the LO approximation could be removed by taking into account of the next-to-leading order (NLO) inelastic scattering $g/q + \Psi \rightarrow g/q + Q + \bar{Q}$, and a “quasi-free” scenario was proposed, in which the incident thermal gluon/quark is not absorbed but instead is scattered off the heavy quark $Q$ (or $\bar{Q}$) inside...
the bound state by exchanging another space-like gluon. The NLO scattering of light quarks/gluons off heavy quarkonium was then computed in the perturbative QCD approach in [20]. However, after various divergences (collinear, soft, soft-collinear; they should be rendered finite in the QGP by thermal parton masses) were carefully handled, the resulting cross section appeared negative in quite a kinematic region [20], obscuring the physical significance of the result. A substantial step toward the understanding of the NLO contribution to the heavy quarkonium dissociation rate was the realization that the effective potential entering the LO contribution to the heavy quarkonium dissociation rate as arising from singlet-to-octet breakup, and the NLO contribution as Landau damping.

In the present work, we revisit the NLO heavy quarkonium dissociation by thermal gluons in the QGP from the dynamical scattering point of view. We calculate the NLO inelastic scattering cross section of heavy quarkonium with thermal gluons in the framework of second-order quantum mechanical perturbation, using a color-electric dipole coupling effective interaction Hamiltonian from the QCD multipole expansion [23, 26]. The simple second-order perturbation techniques adopted here, besides enabling us to systematically include the bound state effects, allow for a crystal clear derivation of the NLO break-up cross section under consideration, without plague from handling various divergences as encountered in typical diagrammatic calculations [21]. In the following Sec. 2 we demonstrate in detail the derivation of the NLO dissociation cross section, using the 1S charmonium state J/ψ as an example. The cross section thus derived turns out to be divergent if the final state outgoing gluon is massless, thus not well defined in vacuum. Assuming an “effective” gluon mass of the constituent value (~ 600 MeV), we find that the NLO cross section increases with the incident gluon energy, in contrast to the LO (gluo-dissociation) counterpart that peaks at gluon energy ~ ϵB (binding energy of the bound state) above the threshold and then quickly drops off [18]. In Sec. 3 we employ a nonrelativistic in-medium potential model and carry out a full calculation of the NLO dissociation cross sections and pertinent dissociation rates for various charmonia and bottomonia by thermal gluons in the QGP: here the NLO cross sections become physical and finite as rendered by the thermal gluon masses. These NLO dissociation rates indeed take over from the LO counterparts toward high temperatures; that is, the artifact inherent in the LO approximation is removed. Finally, we briefly summarize and give an outlook in Sec. 4. In this work, we focus on the thermal gluons as the incident parton scattering off heavy quarkonium, but not touching upon thermal light quarks, as our current formalism is constrained to on-shell gluons only.

2. Deriving the NLO dissociation Cross Section of Heavy Quarkonium by an External Gluon

2.1. The effective interaction Hamiltonian

The color-electric dipole coupling of the QQ system to external soft gluons, being also a core concept of the later developed pNRQCD [27], was first realized by Peskin in a seminal operator-product-expansion analysis of how a heavy quark system interacts with external light degrees of freedom [16] (see also [28] for an early pertinent investigation). A key observation made by Peskin was, to arrive at the dipole coupling, one needs to sum up all possible ways of coupling of external gluon to the QQ system, in particular the one peculiar to QCD in which the external gluon couples to the gluon exchanged between the Q and Q̄ [10], Peskin’s perturbative analysis was promoted to the effective Lagrangian level from the perspective of multipole expansion of QCD, which can further be transcribed into a nonrelativistic effective Hamiltonian [25].
Figure 1: (Color online) Vertices for the $Q\bar{Q}$ singlet to octet transition (upper), and octet to octet transition (lower), due to interaction with an external gluon, with $T_F = 1/2$ and $N = N_c = 3$. Adapted from [22].

$H_{\text{eff}} = H_0 + H_1$,

$H_0 = \frac{\vec{p}^2}{m_Q} + V_1(|\vec{r}|) + \sum_a \frac{\lambda^a}{2} X^a V_2(|\vec{r}|)$,

$H_1 = V_{SO} + V_{GO}$,  

where $V_1$ and $V_2$ are the $Q\bar{Q}$ potentials arising from gluon exchange ($\vec{r}$ being the relative $Q\bar{Q}$ separation) in color singlet and octet configurations, respectively; together with the kinetic energy term, they make up the zeroth order Hamiltonian $H_0$ of the $Q\bar{Q}$ system. The coupling of the $Q\bar{Q}$ system to the external soft gluons $H_1$ consists of also two parts: $V_{SO}$ being the $Q\bar{Q}$ singlet $|\bar{S}\rangle$ to octet $|O\rangle$ transition vertex (through interacting with an external gluon) that corresponds to a matrix element

$$<O, a|V_{SO}|\bar{S}\rangle = \langle O, a|\frac{1}{2}g_\alpha F_\alpha (\frac{\lambda^b}{2} - \frac{\lambda^b}{2}) \cdot E^b|\bar{S}\rangle >$$

(2)

and $V_{GO}$ associated with the octet $|O\rangle$ to octet transition with the matrix element

$$<O, a|V_{GO}|O, b\rangle = \frac{i g_\alpha}{2} d^{abc} E^c <O|\vec{r}|O\rangle > .$$

(3)

These two kinds of vertices are pictorially represented in Fig. 1. In Eqs. (2) and (3), $g_\alpha$ is the strong coupling constant, $N_c = 3$ the number of colors in the fundamental representation of $SU(3)$, and $\lambda^a/2$ and $\lambda^b/2$ the color matrices of $Q$ and $\bar{Q}$, respectively; the $a, b, c$ denote the color indices of the $(Q\bar{Q})_8$ octet or the gluons, $d^{abc} = 2\text{tr}[\lambda^a/2 \lambda^b/2 \lambda^c/2]$ a totally symmetric $SU(3)$ group invariant, and $E^a = \partial_\alpha A^a_\alpha - \partial_\alpha A^a_\beta + g_s f^{abc} A^b_\alpha A^c_\beta$ the electric field of the gluons with color $a$, which reduces to $E^a = -\frac{\partial A^a}{\partial t}$ in the Wyle gauge $A^a_\alpha = 0$ and can be further quantized using

$$\vec{A}^a(t, \vec{x}) = \sum_{\vec{k}, \lambda} N_{\vec{k}, \lambda} \hat{e}_{\vec{k}, \lambda} [a^a_{\vec{k}, \lambda} e^{i\vec{k} \cdot \vec{x} - i\omega t} + h.c.] .$$

(4)

where $\vec{k}$ is the gluon momentum, $\omega$ the energy, and $e_{\vec{k} \lambda = 1, 2}$ the two physical polarization vectors.

With normalization constant $N_{\vec{k}} = \sqrt{2\pi\delta_{\vec{k}}} (V$ being the spatial volume) in the rationalized Gauss unit as used here, the creation and annihilation operators of gluons in Eq. (4) satisfy the commutation relation $[a^a_{\vec{k}, \lambda}, a^b_{\vec{k}^{'}, \lambda^{'}}] = \delta_{\vec{k}, \vec{k}^{'}} \delta_{\lambda, \lambda^{'}} \delta^{ab}$. While the derivation of the LO (gluo-dissocation) cross section involves only the singlet to octet transition vertex $V_{SO}$ in deriving the NLO cross section, the octet to octet transition vertex $V_{GO}$ (similar to the three-gluon vertex but involving massive colored $QQ$ octet) will also be involved when dealing with the transition from intermediate states to the final state in the second order perturbation, which can be seen clearly in a moment.

2.2. Derivation of the NLO cross section

We now use the effective Hamiltonian specified above to derive the NLO dissociation cross section of heavy quarkonium by an external gluon in the framework of the second order quantum-mechanical perturbation. We use the $1S$ charmonium state $J/\psi$ as an example and illustrate the derivation of the cross section of the process: $g + J/\psi \rightarrow g + c + \bar{c}$. The natural units $\hbar = c = 1$ are used throughout.

We start with the standard expression of the transition matrix element in second order perturbation theory

$$T_{fi} = \sum_m \frac{<f|H_i|m><m|H_f|i>}{E_i - E_m + i\epsilon} ,$$

(5)

where the initial state involves a bound state $J/\psi$ plus an incident gluon of momentum $\vec{k}$, polarization $\lambda$ and color $a$: $|i> = |J/\psi, g(\vec{k}, \lambda, a)\rangle >$, and the final state involves an unbound octet $(cc)_8$ of internal relative (between $c$ and $\bar{c}$) momentum $\vec{p}$ and color $b$, plus an outgoing gluon of momentum $\vec{K}$, polarization $\sigma$ and color $c$: $|f> = |(cc)_8(\vec{p}, b), g(\vec{K}, \sigma, c)\rangle >$. Note that we work in the
rest frame of the $J/\psi$ and in the same spirit of calculating the LO (gluon-dissociation) cross section, neglect the three-momentum transferred by the incident gluon to the $J/\psi$. The latter approximation is justified by the fact that the mass of the $Q\bar{Q}$ bound states (e.g., the mass of $J/\psi$ is 3.097 GeV, and of $\Upsilon(1S)$ is 9.460 GeV) is much larger than the typical momentum of thermal gluons in QGP which is of the order of temperature. As a result, the center-of-mass momentum of the final state unbound octet $(c\bar{c})_8$ is neglected; that is, the rest frame of $J/\psi$ is approximately also the rest frame of the $(c\bar{c})_8$, and then one needs only to deal with the internal relative momentum $\vec{q}$ of the $(c\bar{c})_8$. Such an approximation is apparently better for the more massive bottomonium states; we will come back to this point when calculating the pertinent dissociation rates.

Eq. (5) involves a summation over all possible intermediate states $|m\rangle$. Upon an observation of Eqs. (2) and (3), the intermediate states to be summed over can only be of two kinds: a pure one-octet state $|m := |(c\bar{c})_8(q, d)\rangle$ (of internal relative momentum $\vec{q}$ and color $d$), or an one-octet (of internal relative momentum $\vec{q}$ and color $d$) plus two-gluons (of momenta $\vec{k}_1$ and $\vec{k}_2$, polarizations $\lambda_1$ and $\lambda_2$, and colors $d_1$ and $d_2$, respectively) state $|m := |(c\bar{c})_8(q, d), g(\vec{k}_1, \lambda_1, d_1), g(\vec{k}_2, \lambda_2, d_2)\rangle$, in order for neither of the matrix elements $< f|H|/|m \rangle =< f|V_{OO}|m \rangle$ and $< m|H|i > =< m|V_{SO}|i >$ appearing in the numerator of Eq. (5) to vanish. In the following, we discuss the contributions from these two kinds of intermediate states separately.

With the first kind of intermediate states $|m := |(c\bar{c})_8(q, d)\rangle$, the matrix element $< m|V_{SO}|i >$ in the numerator of Eq. (5) can be computed by using Eqs. (2) and (4)

$$< m|V_{SO}|i > = \delta^{ad} g_s \sqrt{\frac{\omega^2}{3V}} \sum_d \delta^{adbdc} \left[ \frac{\omega^2}{6V} \right] \sum_q \int d^3r e^{-i\vec{q}\cdot\vec{r}} J_{10}(r) Y_{00}(\theta, \phi),$$

where $J_{10}(r)$ is the normalized radial wave function of the 1S state $J/\psi$. We have neglected the interaction between $c$ and $\bar{c}$ in the octet state and therefore it is represented by a plane wave. Upon expanding the plane wave into a series of spherical waves $e^{-i\vec{r}\cdot\vec{r}} = 4\pi \sum_m \sum_l (-i)^l j_l(r) Y_{lm}(\theta, \phi) Y_{lm}(\theta', \phi')$ (primed angels for $\vec{p}$, and unprimed for $\vec{r}$) and using the orthogonality relation for the spherical harmonics $\int d\theta d\phi d\phi' Y_{lm}^*(\theta, \phi) Y_{lm}(\theta', \phi') = \delta_{ll'} \delta_{mm'}$

plus a recursion relation for the spherical Bessel functions $\frac{d}{dx} j_0(x) = -j_1(x)$, the matrix element can be further reduced as

$$< m|V_{SO}|i > = \delta^{ad} g_s \sqrt{\frac{\omega^2}{3V}} \sum_d \delta^{adbdc} \left[ \frac{\omega^2}{6V} \right] \int d^3r e^{-i\vec{q}\cdot\vec{r}} J_{10}(r) Y_{00}(\theta, \phi),$$

(6)

In a similar way, the other matrix element in the numerator of Eq. (5) can be computed by using Eqs. (3) and (4)

$$< f|V_{OO}|m > = \frac{g_s}{2V} \left( \begin{array}{c} \omega^2 \frac{\omega^2}{2V} d^{adbdc} \bar{\epsilon}_{\sigma\bar{\sigma}} \epsilon^{c\bar{c}8} \left( (c\bar{c})_8(\vec{p}) \right) \bar{\epsilon}^{(c\bar{c})_8(\vec{q})} \end{array} \right) > = \left( \begin{array}{c} \frac{-ig_s}{2V} \left( \begin{array}{c} (2\pi)^3 d^{adbdc} \frac{\omega^2}{2V} \bar{\epsilon}_{\sigma\bar{\sigma}} \cdot V \Theta^3(\vec{q} - \vec{p}) \end{array} \right) \end{array} \right)$$

which involves no bound state wave function. Plugging these two matrix elements into Eq. (5) and performing the summation over the momentum $\vec{q}$ and color $d$ of the intermediate states $|m >= |(c\bar{c})_8(q, d)\rangle$, one obtains the first contribution to the second order transition matrix element

$$T^{(1)}_{ji} = \frac{-ig_s^2}{2V^2} \left( \begin{array}{c} (2\pi)^3 \frac{\omega^2}{6V} \sum_d \delta^{adbdc} \frac{\omega^2}{V} \int d^3r e^{-i\vec{q}\cdot\vec{r}} J_{10}(r) Y_{00}(\theta, \phi), \epsilon_{\sigma\bar{\sigma}} + (\epsilon_{\bar{\sigma}\sigma} - \vec{p} \cdot \vec{p}) B(p, k), \end{array} \right),$$

(9)

where

$$A(p, k) = \int \frac{d^3r d\vec{r}}{p(\epsilon_B + \omega_k - \frac{\vec{p} \cdot \vec{r}}{m_Q} + i\epsilon)},$$

$$B(p, k) = \frac{g_s^2}{2m_Q} \int \frac{d^3r d\vec{r}}{p(\epsilon_B + \omega_k - \frac{\vec{p} \cdot \vec{r}}{m_Q} + i\epsilon)^2} - \int \frac{d^3r d\vec{r}}{p(\epsilon_B + \omega_k - \frac{\vec{p} \cdot \vec{r}}{m_Q} + i\epsilon)^4}.$$

In Eq. (9), an integration by part with respect to $d^3\vec{r}$ was performed to integrate out the $d^3\vec{q}$ and arrive at the last line. All energies (including the binding energy $\epsilon_B$ of the bound states) are measured relative to the threshold of the $c\bar{c}$ pair. We note that the appearance of the spherical Bessel function of the second order $j_2(pr)$ here is simply ascribed to the selection rule $\Delta l = 1$ of the color-electric dipole transition that now acts twice from the initial $S$-wave ($l = 0$) state.
The second kind of intermediate states \(|m|:=|(cc)_{8}(q,d), g(k_1, \lambda_1, d_1), g(k_2, \lambda_2, d_2)\rangle\) involves not only a \((cc)_{8}\) octet, but another two gluons. Similar but a bit more lengthy (to handle the two-gluon state with creation and annihilation operators) manipulations lead to matrix elements

\[
|m|_{SO} \rangle = \sqrt{2} \frac{n_s}{V} \sqrt{\frac{\pi}{3}} \left| \sqrt{\omega_{k_1} \delta_{k_1 k_2} \delta_{k_2 k_3} \delta_{k_3 k_1}} \cdot \hat{q} \right| r^3 dr j_1(qr) R_{10}(r),
\]

and

\[
< f | VOO | m > = \frac{ig_s}{(2V)^{3/2}} (2\pi)^3 \nabla \delta^3(q - \rho).
\]

\[
[e_{k_1 k_3} \sqrt{\omega_{k_1} \delta_{k_1 k_3}} \delta_{k_2 k_3} \delta_{k_3 k_1} \cdot \hat{q} \delta_{k_2 k_3} \delta_{k_3 k_1} \delta_{k_1 k_3}] \cdot \sqrt{\omega_{k_2} \delta_{k_2 k_3} \delta_{k_3 k_1} \delta_{k_1 k_3}} \delta_{k_1 k_3} \delta_{k_3 k_1} \delta_{k_2 k_3} \delta_{k_3 k_1} \delta_{k_1 k_3}]
\]

Combining these two matrix elements and conducting the corresponding summation over intermediate state indices \(\sum_m \frac{V}{4\pi} \int d^3 q \sum_d \sum_{k_1 \lambda_1 d_1} \sum_{k_2 \lambda_2 d_2}\) (the property of \(d^{abc}\) being totally symmetric is exploited here to simplify the algebra) in Eq. (13), one obtains the second contribution to the second order transition matrix element

\[
T^{(2)}_{f_i} = - d^{abc} \frac{ig_s}{V} \sqrt{\frac{\pi \omega_{k_1} \omega_{k_2}}{6V}} \cdot \hat{q} \cdot [C(p, \kappa) e_{\sigma \sigma} + (e_{\sigma \sigma} \cdot \hat{p}) \bar{p} \rho] D(p, \kappa),
\]

where

\[
C(p, \kappa) = \int r^4 dr j_1(qr) R_{10}(r) \frac{p(-e_B - \omega_{\kappa} - \frac{p^2}{m_Q} + i\epsilon)}{p},
\]

\[
D(p, \kappa) = \int r^4 dr j_2(qr) R_{10}(r) \frac{2p^2}{m_Q} \int r^4 dr j_1(qr) R_{10}(r) \frac{p(-e_B - \omega_{\kappa} - \frac{p^2}{m_Q} + i\epsilon)}{p(-e_B - \omega_{\kappa} - \frac{p^2}{m_Q} + i\epsilon)^2}.
\]

Note that the prefactors of \(T^{(2)}_{f_i}\) and \(T^{(1)}_{f_i}\), besides having opposite signs, differ by a factor 2 arising from the second kind of intermediate states involving two gluons as identical bosons. Again, the appearance of the spherical Bessel function of the second order \(j_2(qr)\) here is due to two successive color-electric dipole transitions from the initial \(S\)-wave \((l = 0)\) state.

The contributions \(T^{(1)}_{f_i}\) and \(T^{(2)}_{f_i}\) from the first and second kind of intermediate states respectively are added coherently (since they involve the same initial/final states) to get the full second order transition rate for \(g + J/\psi \rightarrow g + c + \bar{c}\)

\[
\Gamma_{i \rightarrow f} = \frac{2\pi}{h} \sum_f |T^{(1)}_{f_i} + T^{(2)}_{f_i}|^2 \delta(E_i - E_f),
\]

which, upon dividing by the incident flux \(v_{rel}/V\) \((v_{rel} being the relative velocity between the incident gluon and the target \(J/\psi\)) and averaging (summing) over initial (final) state degeneracies, is converted into the NLO cross section

\[
\sigma(E_g) = \frac{2\pi V}{(2\pi)^3} \int d^3 \rho \sum_b \frac{V}{(2\pi)^3} \int d^3 R \sum_e \int d\Omega_k \frac{1}{4\pi} \left| \sum_{\lambda=1,2} \left| e_{\lambda \rho} \cdot \hat{p} \rho^2 = \frac{1}{3} \bar{p} \rho^2 \right|^2 \sigma(E_g)
\]

where \(E_g = \omega_{Z}\) is the incident gluon energy in the rest frame of \(J/\psi\). The average (summation) over initial (final) state degeneracies can be readily performed by using the identity

\[
\int d\Omega_k \frac{1}{4\pi} \sum_{\lambda=1,2} |e_{\lambda \rho} \cdot \hat{p} \rho^2 = \frac{1}{3} \bar{p} \rho^2 \right|^2
\]

\((\rho\) being an arbitrary vector) twice, the first for the polarization vector of the incident gluon \(e_{\lambda \rho}\), and the second for that of the outgoing gluon \(e_{\lambda \rho}\) in the final state. Further using the identity \(\sum_{abc} d^{abc} d^{bac} = 40/3\), the cross section can be finally computed from

\[
\sigma(E_g) = \frac{5}{216\pi^2} g_s^4 E_g \int dp^2 \int k^2 dk \omega_{Z} \times \{ \cdot \cdot \} \delta(-\epsilon_B + E_g - \frac{p^2}{m_Q} - \omega_\rho),
\]

where

\[
\{ \cdot \cdot \} = [A^2(p, k) + \frac{1}{3} B^2(p, k) + \frac{2}{3} A(p, k) \bar{B}(p, k)] + 4[C^2(p, k) + \frac{1}{3} D^2(p, k) + \frac{2}{3} C(p, k) \bar{D}(p, k)]
\]

\[
- 4[A(p, k) C(p, k) + \frac{1}{3} (A(p, k) \bar{D}(p, k) + B(p, k) \bar{D}(p, k))].
\]

Note that in Eqs. (13) and (14), the zero point of the denominators is not reached and thus the \(i\epsilon\)
factor can be dropped, as ensured by the energy-conserving $\delta$-function. As a result, these functions are all real in Eq. (11). To see how the integrations over final state momenta in Eq. (18) work out with the energy-conserving $\delta$-function, we define $p_c = \sqrt{(E_g - \epsilon_B - m_g)m_Q}$, with $m_g$ being an effective gluon mass and $\omega_Z = \sqrt{\kappa^2 + m_g^2}$. Apparently, if $p > p_c$, one has $-\epsilon_B + E_g - \frac{p^2}{m_g} > \omega_Z$, so that the zero point of the argument of the $\delta$-function can never be reached for any $\kappa > 0$, and therefore the corresponding integrand vanishes; that is to say, $p_c$ serves as a cut-off for the integration over $p$. On the other hand, for $p < p_c$, the $\delta$-function can be integrated out via $\int dk$, which, together with $\int_{p_c}^\infty dp$, yields a finite result. However, if the gluon is massless, the factor $-\epsilon_B + \omega_Z - \frac{p^2}{m_g}$ in the denominator of $A(p,k)$ and $B(p,k)$ (see Eq. (11)), which is exactly $\omega_Z$ (the energy of the outgoing final state gluon), upon using the $\delta$-function, would cause an infrared divergence (from $\kappa \to 0$) that manifests itself in the integration $\int_{p_c}^\infty dp$ as $p \to p_c$. In contrast, no divergence would occur if the initial state incident gluon is massless (the same is true for LO dissociation cross section [18], as the factor $-\epsilon_B - \omega_Z - \frac{p^2}{m_g}$ in the denominator of $C(p,k)$ and $D(p,k)$ (see Eq. (14)), which is nothing but just the incident gluon energy $E_g = \omega_Z$, must be larger than $\epsilon_B$ in order to overcome the bound state binding energy.

To sum up, we’ve demonstrated the derivation of the NLO dissociation cross section for the 1S state $J/\psi$. The same cross section for the $P$-wave state $\chi_c$ has been also derived, which is a bit more complicated because of dependence of the pertinent wave function on the azimuthal angle. The NLO cross section is divergent (actually infraredly sensitive) if the gluons are massless; more exactly, the divergence/infrared sensitivity is due to the masslessness of the final state gluon. The fact that the NLO inelastic scattering cross section cannot be related with a zero-temperature ($i.e., m_g = 0$) process was already pointed out in [24] but in a different context: the NLO cross section in [24] was read off from a dissociation rate formula involving the imaginary potential [21] due to Landau damping which diverges if the space-like gluon exchanged between $Q$ and $\bar{Q}$ is not screened by a screening mass $m_Q \sim g_s T$.

Nevertheless, to make a comparison with the LO (gluon-dissociation) cross section that is well defined even in vacuum, we assume an effective mass of the typical constituent value $m_g = 600$ MeV, in comparison with the corresponding LO (gluon-dissociation) cross section. Vacuum bound state wave functions with Coulomb potential and full Cornell potential are used and compared (see text for more details).

Figure 2: (Color online) NLO inelastic scattering cross sections of $J/\psi$ (upper panel), and $\Upsilon(1S)$ (lower panel) with a gluon of effective mass $m_g = 600$ MeV, in comparison with the corresponding LO (gluon-dissociation) cross section. Vacuum bound state wave functions with Coulomb potential and full Cornell potential are used and compared (see text for more details).
incident gluon now needs to overcome the binding energy plus the final state gluon mass in order to break up the bound state), increases monotonously with the incident gluon energy, therefore taking over from the LO result toward high energy.

3. NLO Dissociation of Various Heavy Quarkonia in an In-medium Potential Model

3.1. In-medium NLO cross sections

We now turn to the calculation of in-medium NLO dissociation cross section of heavy quarkonia by thermal gluons. In QGP, the gluons gain a temperature dependent thermal mass, which is taken to be \( m_g(T) = \sqrt{3/4g_s T} \) with fixed \( g_s = 2.3 \) for \( N_f = 3 \) active light flavors and \( N_c = 3 \) colors [29], rendering the NLO cross section finite. To get the in-medium bound state radial wave functions \( R_{nl}(r) \) that enter the evaluation of the cross sections, we solve the radial Schrödinger equation of the \( Q\bar{Q} \) system

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dR_{nl}}{dr} \right) + \left[ m_Q(E_{n,l} - 2m_Q - V(r,T)) - \frac{l(l+1)}{r^2} \right] R_{nl}(r) = 0,
\]

which is a modification of the vacuum Cornell potential \( V(r,0) = -\alpha/r + \sigma/m_D(T)r \) with the Debye screening mass \( m_D/T = -4.058 + 6.32 \cdot (T/T_c - 0.885)^{0.1055} \) \( T_c = 172.5 \text{ MeV} \) fitted to lattice data [31, 18]. With coupling strength \( \alpha = 4/3\alpha_s = 0.471 \) of the Coulomb part, the string tension \( \sigma = 0.192 \text{ GeV}^2 \) of the confining part, and heavy quark masses \( m_c = 1.320 \text{ GeV}, \quad m_b = 4.746 \text{ GeV} \), the vacuum masses of charmonia and bottomonia below threshold are well reproduced [32, 18]. The temperature-dependent binding energy of the bound state is then obtained via \( \epsilon_B(T) = 2m_Q + \sigma/m_D(T) - E_{n,l}(T) \), whose zero point defines the dissociation temperature of the bound state under consideration [32, 18].

The thus solved temperature-dependent wave functions and binding energies are then used in Eq. (18), to compute the NLO cross sections for various heavy quarkonia. The results for ground states \( J/\psi \) and \( \Upsilon(1S) \) in vacuum but with a “constituent” gluon mass were already shown in Fig. 2; results of in-medium NLO cross sections for various charmonia are displayed in Fig. 8 in comparison with the LO counterparts [18], for several temperatures below the corresponding dissociation temperature of the quarkonium under consideration. Again, unlike the LO (gluo-dissociation) cross sections dropping off toward high energies, the NLO cross sections monotonously increase with the incident gluon energy. The less tightly bound excited states (\( \psi(2S) \) and \( \chi_c \)) possessing a larger radius, exhibit accordingly larger NLO as well as LO cross sections than the ground state \( J/\psi \). As temperature increases, the radius of the bound states expands, and the NLO cross sections grow fast at all gluon energies. We have checked, technically, this is primarily due to the fact that the overlap between the expanding bound state wave function with the octet state wave function \( j_1(pr) \) increases rapidly with temperature, resulting in a fast increasing transition matrix element as spelled out in Eqs. (10) and (14). Naturally such an increase with temperature is more prominent for NLO than LO, since the former involves two successive dipole transitions. Besides, the decreasing binding energies that enter the evaluation of the transition matrix element as a parameter also help the NLO cross sections grow with temperature.

When the temperature rises near to the pertinent dissociation temperature, the bound state radius grows very fast and starts to blow up [32, 18] and certainly overtakes the thermal gluon wave-length, thereby invalidating the dipole coupling mechanism underlying our calculations. Yet the NLO cross sections near the dissociation temperature from our calculations yield dissociation rates on the order of \( \sim \text{GeV} \) (see the next section), which agrees with the empirical value used in phenomenological studies of heavy quarkonium transport in QGP [32]. The latter seems not unreasonable from a conceptual consistency point of view: when the bound state cannot be supported because of the screening of the potential, it better be also quickly dissociated via dynamical inelastic scatterings with thermal partons.

Similar behavior of the in-medium NLO versus LO cross sections for bottomonia has also been found, as shown in Fig. 4. The most tightly bound ground state \( \Upsilon(1S) \) has the smallest size (and accordingly smallest cross section), guaranteeing that
the dipole coupling mechanism is most applicable. Furthermore, the technical approximation made in our calculations that the rest frame of the bound state is considered also the rest frame of the final state octet \((Q\bar{Q})_8\) \(i.e.,\) neglecting the recoil effect on the bound state by the incident gluon, see Sec. 2.2, should be well justified for the much more massive bottomonia. Therefore, we deem that our results of NLO cross sections are quantitatively most reliable for \(\Upsilon(1S)\), especially at temperatures not too close to its dissociation temperature. The excited states \(\Upsilon(2S)\) and \(\chi_b\) have similar sizes and binding energies as those of \(J/\psi\), and therefore, similar NLO (and also LO) cross sections, too.

3.2. NLO dissociation rates in QGP

The dissociation rate, an input of phenomenological studies of heavy quarkonia transport in the QGP, is obtained by folding the pertinent cross section with the thermal gluon distribution. For a bound state sitting at rest in the
the Bose distribution with gluon energy of the temperature (for massless gluons, the value of the order of a couple of times 200 – 400 MeV reached in relativistic heavy-ion collisions at RHIC and the LHC, the typical thermal gluon energy then is on the order of \( \leq 1 \text{ GeV} \), which is much smaller than the mass of charmonia and bottomonia. We thus conclude that in calculating the dissociation rate, the thermal gluons do not quite probe the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high energies, and therefore, the approximation we’ve made in calculating the NLO cross section at very high ener...
mains more effective, rendering the LO dissociation rates dominant over the NLO results. For the most tightly bound Υ(1S), this temperature region is relatively broad, extending up to \( \sim 1.5T_c \). At higher temperatures, the NLO dissociation rates take over from the LO results that quickly drop off, and keep increasing with temperature, reaching the order of \( \sim \text{GeV} \) near the dissociation temperatures for each quarkonium. In the case of \( \Upsilon(1S) \) (upper panel of Fig. 5), the calculated NLO dissociation rate was compared with the quasi-free result [4], whose underlying cross section was obtained by doubling that of the thermal gluon scattering off single bottom quark including appropriate interference effect [6]. Apparently the former increases much faster than the quasi-free result toward high temperatures, which might be due to the fact that the effect of the expanding bound state wave function, while captured in the present NLO calculation, is lacking in the quasi-free scenario.

As remarked before, at temperatures very close to the heavy quarkonium dissociation temperature, the radius of bound state starts to blow up and thus the dipole coupling mechanism underlying our calculations must become invalidated. So the NLO dissociation rates near the dissociation temperatures shown here may be quantitatively questionable. Yet this large dissociation rate seems to be supported by a transport study of bottomonia phenomenology in QGP [32], where, according to the authors, an empirical value of the effective “dissociation” or “decay” rates of order of \( \geq 2 \text{GeV} \) at and above the dissociation temperature for the then unbound states was indeed needed to give a fair description of the bottomonium suppression at the LHC energy. Then if the dissociation rate were to be a continuous function across the dissociation temperature, the NLO dissociation rates of order of \( \sim \text{GeV} \) near the dissociation temperature as calculated here may not be unreasonable.

4. Summary

In this work, we have calculated the NLO inelastic dissociation cross sections of heavy quarkonia by thermal gluons in the QGP in the approach of second-order quantum mechanical perturbation, using a color-electric dipole coupling effective interaction Hamiltonian from the QCD multipole expansion \[ \text{[22 23]} \]. Bound state effects have been systematically included and investigated, and comparisons with the LO counterparts calculated in the same framework \[ \text{[18]} \] have been made and examined. The NLO cross section turns out to be infraredly sensitive to the mass of the outgoing gluon in the final state, and therefore is not well defined in vacuum. When regulated by an effective gluon mass of the constituent value, its magnitude is of the order of several mb at higher energies with vacuum bound state wave functions, comparable to the peak value of the LO counterpart. In QGP, the NLO dissociation cross section becomes physical and finite, as rendered by the thermal gluon mass, and the magnitude at given gluon energy increases with temperature due to the growing radius as well as the decreasing binding energy of the bound states. The NLO dissociation rates obtained by folding the pertinent cross sections with the thermal gluon distribution, increases monotonously with temperature, thus removing the artifact of the LO (gluon-dissociation) approximation in which the dissociation rates decrease toward high temperatures \[ \text{[15]} \]. Our calculations, taking the dynamical scattering point of view, might provide another perspective of understanding the NLO break-up process of heavy quarkonium in QGP, complementing the Landau damping mechanism proposed before \[ \text{[21 23 24]} \].

As an outlook, the approach and techniques adopted here might be generalized to investigate the second order transitions between different quarkonium eigenstates in the QGP, which, together with the NLO (and LO) dissociation rates evaluated in the present work, could be applied not only to the classical transport study of the Boltzmann type of the heavy quarkonium evolution in the QGP \[ \text{[3 4 5 6 7 8 9]} \], but also might be useful in the open quantum system approach to intermediate heavy quarkonium production \[ \text{[33 34 35]} \].

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