Modelling background charge rearrangements near single-electron transistors as a Poisson process

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Abstract. – Background charge rearrangements in metallic single-electron transistors are modelled in two-level tunnelling systems as a Poisson process with a scale parameter as only variable. The model explains the recent observation of asymmetric Coulomb blockade peak spacing distributions in metallic single-electron transistors. These distributions are consistent with charge trapping processes within impurities located between transistor island and gate. From the scale parameter determined, we estimate the average size of the tunnelling systems, their density of states, and the height of their energy barrier.

Introduction. – The metallic single-electron transistor (SET) [1,2] is a possible building block of future electronics, based upon the controlled transfer of individual charges onto and off small isolated electrodes, called islands. However, due to the extreme sensitivity of SETs to charges close to these islands, a static background charge arrangement is essential for proper device operation [3,4]. On the other hand, this very high sensitivity makes the SET an excellent device to investigate background charge rearrangements, \textit{e.g.}, in nearby two-level tunnelling systems (TLTS) [5,6].

In recent experiments [7,8] on Al/AlO\textsubscript{x}/Al SETs it was shown that the distribution of nearest-neighbour spacings (NNS) between Coulomb blockade oscillation peaks of the conductance as derived from \textit{I-V} and gate modulation measurements is influenced by such charge rearrangements: they generate a pronounced tail in the NNS distribution towards smaller spacings (see inset of fig. 1). Here, we propose a quantitative model for this tail. We argue that background charge rearrangements can be interpreted in terms of a Poisson process. This way, we can fit the shape of the NNS distribution tail, with a single scale parameter. From this fit, we find an average hopping distance in the TLTS of about 4 nm. We can furthermore estimate the density of states of TLTSs and the barrier height between the two TLTS states.

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**Experimental observations and statistical model.** – The NNS distribution \( p(\Delta V_g) \) in fig. 1 is strongly peaked at \( \Delta V_g = e/C_g \), but displays a remarkable tail towards lower \( \Delta V_g \) as well, which contains up to one third of the total data points. Here, \( V_g \) and \( C_g \) denote the gate voltage and the gate capacitance, respectively. While the regular main peak and its thermal broadening are well understood from orthodox theory, the tail is a peculiar feature and subject of discussion. The spacings observed in the low-\( \Delta V_g \) tail are stable: repeated and reversed gate voltage sweeps reveal the same statistical distribution of events. The tail has been attributed to the rearrangement of background charges in TLTSs, fig. 2a). The general idea of a TLTS can be illustrated by the tunnelling process in semiconducting tunnel diodes [9]. A further important experimental result is that the NNS distribution does not depend on \( V_g \); it was found [8] that the standard deviation of the \( n \)-th neighbour Coulomb peak spacings is proportional to \( \sqrt{n} \).

The clearly observed hysteresis of the Coulomb peak pattern when reversing the gate scan direction [8] suggests the existence of metastable configurations, which explains the high stability and reproducibility of the experimental results. Neither the Coulomb blockade peak positions nor the underlying smooth \( 1/f \) noise spectrum depend on \( V_g \) or the scan speed, which
is very low (< 1 measurement point per second), excluding any time dependence of the TLTS transitions. Hence, other excitations which could spontaneously flip a TLTS (like thermal activation) are much weaker compared to the parameters which we control in the experiment. Note that these experiments are different from dynamic and frequency-dependent background charge noise measurements [5, 6, 10–12].

The stability of the Coulomb peak pattern with regard to the gate voltage sweep direction and the repetition of these sweeps suggests a location of the TLTS outside the tunnel barriers, preferably in the region between the gate and the island electrode. While the distribution of tunnelling events is independent of \( V_g \), the pattern stability means that each single event is triggered at a specific gate voltage. This voltage is communicated to the TLTS site via the gate-induced local electric field. The largest values of this field are found in the region between gate and island electrode, thus the most likely location of the observed TLTSs. In contrast, the field in the tunnel barriers is partially shielded by the overlapping electrodes and is dominated by the source-drain bias \( V_b \) of the transistor. In our case, the TLTS events and Coulomb peak statistics showed absolutely no \( V_b \) dependence [7], which further excludes the existence of charged fluctuators inside the tunnel barriers.

The focus of our model lies on the \( p(\Delta V_g) \) distribution generated by TLTS fluctuations. Since the \( \Delta V_g \) distribution is stable, a single sweep of \( V_g \) already reveals it completely. Therefore, our study is restricted to a single gate sweep. During this sweep, the electric field around the island electrode retains its direction thus causing all background charges in TLTSs to “hop” either towards or away from the island electrode. As a result the TLTS events contribute a monotonic term \( Q_i \) to the charge on the central island. Owing to this monotony \( Q_i \) can be mapped onto the time variable of the common notation of a stochastic process. Its state is monitored by the position of the Coulomb peak, and its increment is indicated by \( \Delta V_g \). The measurement resolution is limited by the width of the main peak in fig. 1, which is given by \( k_B T \) in turn, and not by the method of the experiment.

For regular Coulomb peak spacing, i.e. the main peak of the \( p(\Delta V_g) \) distribution, \( Q_i \) does not change and time stands still in the conventional picture. Thus, the main peak does not occur in our discussion. Sometimes, however, a TLTS will flip and influence an additional charge \( \Delta Q_i \) on the central island. In this case, the experiment shows a smaller Coulomb peak spacing and a stochastic event occurs in the tail of the \( p(\Delta V_g) \) distribution. Because we operate on the \( Q_i \) axis, the value of the stochastic event \( \Delta Q_i \) can be identified conveniently with the waiting time in the common notation of a stochastic process.

Assuming statistic independence of the events, which is the only requirement (see below), the process can be identified as Poisson process [13]. This process is described by a single scale parameter \( 1/\gamma \). The probability that \( n \) jumps occur before the charge \( Q_i \) is accumulated on the island is given by [13]

\[
p_n(Q_i) = \frac{(\gamma Q_i)^n}{n!} \exp[-\gamma Q_i].
\]  

(1)

The Poisson process is non-stationary, which in our context corresponds to an average number of jumps \( \langle n \rangle = \gamma Q_i \), which depends on \( Q_i \). \( \gamma Q_i \) also corresponds to the square standard deviation, i.e. \( \langle (n)^2 \rangle = \langle n \rangle^2 = \langle n \rangle - \langle n \rangle^2 = \gamma Q_i \).

In the framework of the Poisson process it is possible to describe the tail of the \( p(\Delta V_g) \) distribution function. For that purpose we transform \( p(\Delta V_g) \) into \( p(\Delta Q_i) \), which follows from

\[
\Delta Q_i = e - \Delta V_g C_g.
\]  

(2)

\( p(\Delta Q_i) \) is peaked at \( \Delta Q_i = 0 \) and displays a tail into the positive, see right inset in fig. 1. The probability distribution of the charge difference \( \Delta Q_i \) between two jumps corresponds to
the waiting time distribution of the conventional Poisson process. Therefore

\[ p(\Delta Q_i) = - \frac{dp_0(\Delta Q_i)}{d\Delta Q_i} = \gamma \exp[-\gamma \Delta Q_i] \]  

(3)
is the tail of the distribution function \( p(\Delta Q_i) \) with \( p_0(\Delta Q_i) \) as defined in eq. (1). Finally, the mean value and square standard deviation of \( \Delta Q_i \) also follow from the distribution function \( p(\Delta Q_i) \):

\[ \langle \Delta Q_i \rangle = \sqrt{\langle \langle \Delta Q_i \rangle \rangle} = \frac{1}{\gamma}. \]  

(4)

Discussion. – As mentioned above, the presented stochastic model describes only the tail of the \( p(\Delta Q_i) \) distribution, but not its maximum. While the tail is due to events arising from randomly distributed TLTSs, for which the presumptions of the Poisson process will be proven to hold, the maximum reflects the regularly spaced Coulomb blockade peaks and is deterministic in nature. Fortunately, these two ranges can easily be distinguished in our experimental data (see fig. 1). In order to separate both domains, a phenomenological cut is made at \( \Delta Q_{i,\text{min}} = 0.01 e \), which corresponds via (2) to \( \Delta V_{g,\text{max}} = 0.99 e/C_g \). An analysis including the thermally broadened main peak was performed as well, but did not prove to be more instructive.

Our model provides us with three independent methods of determining the value of its sole parameter \( 1/\gamma \): i) a fit to eq. (3), ii) the average \( \langle \Delta Q_i \rangle \), and iii) the deviation \( \langle \langle \Delta Q_i \rangle \rangle \) according to eq. (4). For the data presented in fig. 1 we find the respective values: i) \( 1/\gamma = (0.046 \pm 0.002) e \), ii) \( 1/\gamma = 0.057 e \), and iii) \( 1/\gamma = 0.043 e \). The discrepancy between the average obtained from ii) and the other values can be shown to be due to the \( \Delta Q_{i,\text{min}} \) cut-off. However, the correspondence between i) and iii) is sufficient to demonstrate that the TLTS events are statistically independent because it reflects a unique feature of the Poisson process. This process provides an adequate description of \( p(\Delta Q_i) \) and in turn \( p(\Delta V_g) \). Measurements on five different SET devices of equal geometry and on the same substrate result in a mean \( 1/\gamma = (0.047 \pm 0.003) e \).

The value of \( 1/\gamma \) provides a charge scale of the influence of TLTSs on the island charge \( Q_i \). Thus, it describes the material properties and consequences of the sample fabrication. A small value of \( 1/\gamma \) corresponds to a short tail of the distribution functions \( p(\Delta Q_i) \) and \( p(\Delta V_g) \), which in turn correlates with few TLTS transitions per \( e/C_g \) interval and, thus, rather stable device operation. \( 1/\gamma \) also scales with device dimensions, as outlined below.

A simple geometric argument allows us to deduce a mean tunnelling distance \( \delta \) of the TLTS from \( 1/\gamma \). It is based on the “constant capacitance model” [14]: charge rearrangements within TLTSs induce potential shifts only, but they do not alter any capacitances. This is appropriate here, since only metal electrodes with high electron density and individual charges at the TLTS sites are considered. The assumption of constant capacitances is also supported by the experimental data, since the peak position of \( p(\Delta V_g) \) is independent of \( V_g \) and the bias voltage (see figs. 1d) and 3b) in ref. [7], respectively). In addition, screening of the single TLTS charges is neglected, which is reasonable considering the low concentration of quasi-free electrons around them, i.e. in silicon oxide or aluminum oxide. The treatment is simplified by assuming a constant field between gate and island, and by assuming that only one electron is rearranged in a TLTS switching event.

In this case, Green’s reciprocation theorem [15] can be applied to the setup displayed in fig. 2a),

\[ \sum_{j=1}^{n} Q_j V'_j = \sum_{j=1}^{n} Q'_j V_j, \]

(5)
where \( \{Q_j\} \) and \( \{Q'_j\} \) are two charge configurations in a system of conductors and \( \{V_j\} \) and \( \{V'_j\} \) are the respective voltage configurations.

\( \{Q_j\} \) shall be the charge configuration before and \( \{Q'_j\} \) that after charge transfer. The influence on the gate electrode is negligible, i.e. \( Q_1 = Q'_1 \) and \( V_1 = V'_1 = V_g \). Furthermore, we use a single charged TLTS,

\[
Q_2 = e, \quad Q'_2 = 0, \quad Q_3 = 0, \quad Q'_3 = e.
\]

In our model the charge on the island is conserved, \( Q_4 = Q'_4 = Q_i \). The island potential, however, shows a slight increase, \( V'_4 = V_4 + \Delta V_4 \). This yields with eq. (5)

\[
\Delta V_4 = \frac{e}{Q_i} (V_3 - V'_2).
\]

For symmetry reasons, the mutual contributions to the potentials \( V_3 \) and \( V'_2 \) cancel and \( (V_3 - V'_2) \) is solely determined by \( V_g \),

\[
V_3 - V'_2 = \frac{\delta}{D} V_g,
\]

using the gate-island separation \( D \) and the TLTS size \( \delta \) (see fig. 2). In general, eq. (7) looks different for a different electrode geometry, but a characteristic length scale corresponding to \( D \) can be found and a similar formula will hold in most cases. The equation gives an upper estimate of the voltage difference since it assumes the TLTS to live in the high-field region. This region experiences the widest energy range of TLTS transitions and thus contributes most events to the statistics.

Finally, \( Q_i = C_g V_g \) together with (6) and (7) yields

\[
\Delta V_4 = \frac{\delta}{D} \frac{e}{C_g}.
\]

This value is independent of \( V_g \), as found in the experiment, due to the cancellation of the linear \( V_g \) dependence of \( Q_i \) and \( (V_3 - V'_2) \).

Fixed capacitances allow us to relate \( \Delta V_4 \) of eq. (8) to \( \Delta V_g \) of eq. (2) and to establish a link between \( 1/\gamma \) of (3) and \( \delta \) of (8). Given the regular Coulomb peak separation \( e/C_g \), the link is provided by

\[
\frac{\Delta V_4}{e/C_g} = \frac{1/\gamma}{e}, \quad \delta = \frac{D}{e \gamma}.
\]

Hence, \( \gamma \) scales linearly with \( D \), i.e. the NNS tail is experimentally observable only for sufficiently small \( D \). However, the result (9) is independent of the distance between the TLTS and the island. This is due to the assumption that the charge of the TLTS influences an equal amount of charge on the island, no matter what the capacitance (and thus the distance) between them is. In other words, this result follows directly from using the “constant capacitance model” and neglecting screening. Although ref. [8] discusses the TLTS influence in terms of a general model where the potential rather than the charge of a TLTS is kept fixed, producing both a distance and a size dependence of the TLTSs, it is not in contradiction to the present treatment. We rather consider the range of small charge displacement and small distance from the SET island here (top left part of fig. 5b) in ref. [8]). These conditions are reasonable and have already been concluded in [7,8]. Both methods yield corresponding results in case of short TLTS-island distance.
The values obtained for $1/\gamma$ easily translate into a characteristic tunnelling distance $\delta$ within TLTSs, using eq. (9). With $D = 70-100$ nm, depending on the device, we obtain $\delta = (3.94 \pm 0.19)$ nm, taking into account eight experiments with geometrically different SETs measured at 10 mK. This value agrees well with other experiments [6–8, 16]. It exceeds the thickness of the SET tunnel barriers, which is typically 2 nm or less.

The measurements [7,8] suggest the tunnelling processes themselves not to be field-assisted, since the probability of a TLTS event does not depend on $V_g$. Under these circumstances $\delta \approx 4$ nm is a rather large value. In the case of tunnelling between trap states, however, the larger tunnelling distance can be compensated by lower barrier height, which can in fact be well below 1 eV and still warrant electric field independence of the tunnelling process. Thus, the height of the energy barrier between the two TLTS sites is limited towards low energies by the maximum potential difference ($\approx 80$ meV) and towards high energies by the necessary barrier transparency ($\approx 0.7$ eV)(1). The neglect of multiple electron transitions within one TLTS in our model might be another reason for the large value of $\delta$.

Information on TLTS switching events at elevated temperatures is hard to access due to the increasing overlap between the main peak of the $p(\Delta Q_i)$ distribution and its tail, as the temperature increases. Measurements performed at temperatures up to 200 mK did not reveal any significant variation of $1/\gamma$.

The determination of $1/\gamma$ (and $\delta$) also provides an estimate of the density of states of the TLTSs. The data presented in fig. 1 contain 572 Coulomb oscillation periods, 177 of which belong to the tail of $p(\Delta V_g)$. While the total energy range scanned by $V_g$ is $572 e/C_g \approx 1.8$ V, the potential shift experienced between the two states of a TLTS is only $\delta/D$ times this value, see fig 2b). Hence, the average density of states is

$$D_{\text{TLTS}} = \frac{177}{572} \frac{D}{\delta} \frac{C_g}{e^2} \approx 2.1 \frac{1}{\text{meV}}.$$ 

Owing to lacking information on the energy level of the whole TLTS and its inner structure (which might allow, for instance, more than one TLTS transition), $D_{\text{TLTS}}$ does not easily translate into a density of states of the traps accommodating the TLTS. However, using typical SiO$_x$ trap densities [17] of $10^{23} - 10^{25}$ eV$^{-1}$ m$^{-3}$ yields an estimate of the TLTS-TLTS distance on the order of 0.2 $\mu$m and below. Although this average trap distance seems rather large compared to the device geometry and the number of trapping events measured, it should be noted that our analysis accounts for traps with sufficiently large energy separation compared to $k_B T$, neglecting, e.g., dynamic fluctuators generating excess $1/f$ noise and therefore probing a different mean energy interval. The trapping processes may as well take place in the unavoidable AlO$_x$ layer of possibly lower quality covering the device surface.

Defects with internal degrees of freedom can switch between metastable configurations also due to their interaction with a thermal bath or by photon excitations. Low-frequency excess noise even in high-quality devices is significantly generated by dynamic fluctuations of such defects [16]. While a single fluctuator, also known as random telegraph fluctuator (RTF), shows a Lorentzian spectrum, an ensemble of RTF results in the typical $1/f$ noise [18]. It is reasonable and tempting to assume that the TLTS effects investigated in our studies are of the same origin as RTF generated noise. Due to the metastable nature of the TLTS, the dynamics of a fluctuator crucially depend on its energy threshold separation. However, this

\[(1)\]The lower limit results from the maximum potential difference between the TLTS sites. Because the probability of tunnelling events is independent of the gate voltage value, Fowler-Nordheim tunnelling can be excluded, i.e. the field-induced barrier tilt is less than the barrier height, see fig. 2b). The upper limit follows from a simple WKB argument, which links the barrier transparency to the product of its height and width. In comparison to AlO$_x$ barriers ($\approx 1.5$ eV, $\approx 2$ nm), $\delta \approx 4$ nm yields the given value.
is only vaguely known from our experiments via the observation of Coulomb peak pattern hysteresis [8]. Furthermore, it is not clear how to extrapolate the data, where we can resolve TLTS events with a large hysteresis, towards a small energy interval of \( k_B T \), i.e. for fluctuators of small time constants contributing to non-zero frequency noise. Therefore, a decisive conclusion on the consequences of our results for dynamic fluctuations and noise is too speculative at the moment. However, this subject is under further investigation.

**Conclusion.** – We have investigated charge transfer in two-level tunnelling systems (TLTSs). The study is based on measurements performed on metallic SETs. The charge transfer can be described in terms of a Poisson process, where the influenced charge \( Q_i \) replaces the usual time variable. Thus, the prominent tail of the probability distribution function \( p(\Delta V_g) \) is governed by a single scale parameter \( 1/\gamma \). We can associate this parameter with a length scale \( \delta \) of the TLTS. We find \( \delta = (3.94 \pm 0.19) \) nm for the samples of different geometries under consideration. The height of the energy barrier separating two TLTS states is between 80 meV and 0.7 eV and the density of states of TLTS is found to be \( 2.1 \) (meV)\(^{-1}\).

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