Neutrinos from the terrestrial passage of supersymmetric dark-matter Q-balls

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Supersymmetry implies that stable non-topological solitons, Q-balls, could form in the early universe and can now exist as a form of dark matter \cite{4-37}. The cosmology of supersymmetric Q-balls has been studied extensively \cite{4-37, 2, 14, 24}. A relic Q-ball symmetric Q-balls has been studied extensively \cite{4-37}. The primordial generation of stable Q-balls requires the formation of an Affleck-Dine scalar condensate \cite{22, 23, 40} which forms along one of the many flat directions of the MSSM. The effects of supersymmetry breaking determine the potential along the flat directions, which we will assume to be

\[ V(\phi) = M_s^4 \log \left( 1 + \frac{|\phi|^2}{M_s^2} \right), \]  

(1)

as in the case of gauge-mediated supersymmetry breaking scenario. The scale \( M_s \) is determined by supersymmetry breaking. The Q-ball radius \( R(Q_B) \) and mass \( M(Q_B) \) in this potential are described by the following relations \cite{24}:

\[ R(Q_B) \approx M_s^{-1} Q_B^{1/4}, \quad M(Q_B) \approx M_s Q_B^{3/4} \]  

(2)

The Q-ball is rendered stable against decay into nucleons as long as the mass of the scalar baryons is less than a nucleon mass: \( \omega = M(Q_B) / Q_B < 1 \text{ GeV} \), which corresponds to \( Q_B \gtrsim 10^{24} \, (M_s / \text{TeV})^{3/4} \).

If Q-balls make up all of the cosmological dark matter, the number of Q-balls found inside Earth at any given time is

\[ N_{\oplus} \sim \frac{\rho_{\text{DM}} V_{\oplus}}{M(Q_B)} \sim 3 \times 10^5 \left( \frac{10^{24}}{Q_B} \right)^{3/4} \left( \frac{\text{TeV}}{M_s} \right), \]  

(3)

where \( \rho_{\text{DM}} \sim 0.3 \text{ GeV/cm}^3 \) is the local dark matter density, and \( V_{\oplus} \) is the Earth’s volume. Since the existing experimental limits \cite{14, 38, 39} imply \( Q_B \gtrsim 10^{24} \), and since cosmological scenarios are consistent with such large Q-balls, we choose \( Q_B \sim 10^{24} \) as a representative reference point. The number of neutrinos generated by the Q-balls passing through Earth is

\[ \frac{dN_\nu}{dt} \sim 10 \left( \pi R(Q_B) \right)^2 n_n v_0 N_{\oplus} \approx 5 \times 10^{17} \left( \frac{10^{24}}{Q_B} \right)^{1/4} \left( \frac{\text{TeV}}{M_s} \right)^3 \text{s}^{-1}, \]  

(4)

where \( n_n \) is the average nucleon density, \( v_0 \sim 10^{-3} \) is the average velocity of the dark matter particles, and we have taken into account that \( \sim 10 \) neutrinos are produced in every \( pp \) annihilation.

The flux at the Earth’s surface is, therefore,

\[ F_{\nu, \oplus} \sim \frac{1}{4 \pi R_{\oplus}^2} \frac{dN_\nu}{dt} \sim 0.1 \left( \frac{10^{24}}{Q_B} \right)^{1/4} \left( \frac{\text{TeV}}{M_s} \right)^3 \text{cm}^{-2} \text{s}^{-1}, \]  

(5)

Although more neutrinos are produced by Q-balls going through the interior of Sun, the flux on Earth is dominated by terrestrial neutrinos because of the \( 1/r^2 \) suppression for the solar flux.
The atmospheric neutrino flux has a peak at 0.1 GeV reaching \( \sim 1 \text{ cm}^{-2} \text{s}^{-1} \) \([41–45]\). The flux of solar neutrinos dominates at low energies, below 10 MeV, as shown in Fig. 1 by a dark vertical band. The flux from Q-balls gives a non-negligible contribution at energies 0.01 \(-\) 0.3 GeV.

Since the Q-ball interaction rate is proportional to the density, this flux is enhanced by the higher density in the core. For simplicity we treat the inner and outer cores as one, and we approximate the Earth density profile by a single step function:

\[
\rho_{\oplus}(r) = \begin{cases} 
\rho_c = 12 \text{ g/cm}^3, & r \leq R_c \\
\rho_m = 3 \text{ g/cm}^3, & R_c < r \leq R_{\oplus},
\end{cases}
\]

(6)

where \( R_c = 3.4 \times 10^3 \text{ km} \) and \( R_{\oplus} = 6.4 \times 10^3 \text{ km} \).

The neutrinos are produced in \( p\bar{p} \) and \( n\bar{n} \) annihilations, which, on average generate \( \langle n_\pi \rangle \approx 5 \) pions per nucleon \([46]\). The subsequent decays of the pions produce neutrinos with energies \( E_\nu \approx 0.1 \text{ GeV} \). We will see that this simple estimate is born out in the more detailed calculation of the neutrino spectrum to follow.

The primary background to the Q-ball induced neutrino effect is the atmospheric background. Extensive Monte Carlo work has been devoted to the subject of calculating this flux, as it is important is being able to observe neutrino oscillations. We use the fluxes from Refs. \([41–44]\). The inclusion of geomagnetic effects is important in a more detailed analysis.

The neutrino spectrum from Q-ball catalyzed antiprotons will depend on the pion spectrum from \( p\bar{p} \) annihilations. For simplicity we focus on the sum over neutrino flavors, ignoring the neutrino oscillations:

\[
dN_\nu / dE_\nu = \frac{\partial N_\nu}{\partial N_\pi} \frac{dN_\pi}{dk_\pi},
\]

(7)

where the pion spectrum is found from the data on \( p\bar{p} \) annihilation at rest \([47]\):

\[
dN_\pi / dk_\pi = \frac{c k^2 e^{-\omega}}{\sqrt{2} \pi}.
\]

(8)

Here \( \omega \equiv \sqrt{m_\pi^2 + k^2} \), and \( \alpha \approx 7 \text{ GeV}^{-1} \), and \( c \approx 90 \text{ GeV}^{-3} \) normalizes the spectrum so that \( N_\pi \approx 5 \). All momenta and energies are measured in the laboratory frame. In the cascade \( \pi^\pm \to \mu^\pm \nu_\mu, \mu \to e + \nu_e + \bar{\nu}_\mu \), the pion energy is split approximately uniformly among the four final decay products, so we have \( E_\pi = \frac{1}{4} \sqrt{k^2 + m_\pi^2} \). In Fig. 1 we show the expected spectrum \( dN_\nu / dE_\nu \) normalized to give the correct total flux from Q-balls, \( F_Q = \int (dN_\nu / dE_\nu) dE_\nu \). To find the evidence of Q-balls, one must be able to discriminate between the ordinary atmospheric spectrum \( F_{\text{atm}} \) and \( F_{\text{tot}} \equiv F_{\text{atm}} + F_Q \). The Q-ball induced flux peaks at \( E_\nu \approx 80 \text{ MeV} \).

The isotropy of the Q-ball flux on the Earth implies that the zenith angle dependence of \( F_Q \) is determined entirely by the Earth geometry and density distribution:

\[
F_\nu(\theta_z) \propto \begin{cases} 
0, & \theta_z < \pi/2 \\
\rho_m \cos \theta_z, & \pi/2 \leq \theta_z \leq \theta_c \\
\rho_m \cos \theta_c + (\rho_c - \rho_m) f_c(\theta_z), & \theta_c \leq \theta_z \leq \pi,
\end{cases}
\]

(9)

where \( f_c(\theta_z) = \arcsin((R_c/R_\oplus)^2 - \sin^2 \theta_z) \), and \( \theta_c = \arcsin(R_c/R_\oplus) = 32^\circ \) is defined as the nadir angle that grazes the core. We show this zenith angle dependence in Fig. 2.

The additional upgoing neutrino flux due to Q-balls changes the downward-upward asymmetry observed by Super-Kamiokande and other experiments, which is usually characterized in terms of \( R = N_{\text{up}}/N_{\text{down}} \) and \( A = N_{\text{up}} - N_{\text{down}} / N_{\text{up}} + N_{\text{down}} \), where upward (downward) going events are those with \(-1 < \cos \theta_z < -0.2 \) \((0.2 < \cos \theta_z < 1)\). Though we have ignored neutrino oscillations and geomagnetic effects, both of these introduce additional zenith angle dependence. For example, Monte Carlo simulations \([44, 48]\) indicate that Soudan should see an asymmetry \( A_\mu = -0.4 \) at \( E_\nu = 0.1 \text{ GeV} \), whereas Super-Kamiokande sees \( A_\mu = +0.4 \) at the same energy. Given equal other conditions, it is probably easier to observe

![FIG. 1: Neutrino flux spectra averaged over zenith angles, in the presence of relic Q-balls (the upper line) and in the absence of relic Q-balls (the lower line), for two values of the baryon number, \( Q_B = 10^{24} \) and \( Q_B = 10^{25} \). The atmospheric neutrino flux shown by the lower curve is based on Ref. [43].](image-url)
the neutrino flux from dark matter Q-balls in a location where the geomagnetic effects suppress the upward going neutrinos relative to downward going neutrinos.

Annual modulation of the dark-matter particle flux due to the Earth motion implies a corresponding modulation of the neutrino flux from Q-balls of the order of 10%.

To summarize, we propose a new experimental probe on dark matter in the form of supersymmetric Q-balls. The relic Q-balls passing through Earth convert nucleons into antinucleons. The $p\bar{p}$ and $n\bar{n}$ annihilations in the wake of the passing Q-balls generate a neutrino flux with a peculiar zenith angle dependence and a small annual modulation. A detailed analysis of the present and future neutrino data can set limits on the relic Q-balls, or it can lead to a discovery of this form of supersymmetric dark matter.

The authors thank John M. Cornwall for helpful discussions. This work was supported in part by DOE grant DE-FG03-91ER40662 and by the NASA ATPF grant NNX08AL48G.

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