Modeling of Bending and Radial Hydroelastic Oscillations for a Sandwich Circular Plate Resting on an Inertial Elastic Foundation

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Abstract. The paper deals with the development and analysis of a mathematical model for a circular sandwich plate resting an inertial elastic foundation and interacting with pulsating viscous liquid layer. The sandwich plate is the bottom wall of a channel containing a thin layer of viscous liquid. The pressure in the viscous liquid layer changes due to a predetermined pressure pulsation law at the channel contour and its squeeze between the upper channel wall and the vibrating circular sandwich plate. The coupled hydroelasticity problem consisting of the Navier-Stokes equations, the continuity equation, and the dynamics equations for the circular sandwich plate with corresponding boundary conditions was formulated and solved. We studied the viscous fluid motion inside the channel as a creeping one. The elastic foundation was considered in the framework of inertial Winkler foundation model. To write the sandwich plate dynamics equations, we used the kinematic hypothesis of the broken normal. The hydrodynamic parameters of the liquid layer, including its stresses acting on the sandwich plate, were found. The final mathematical model is the system of partial differential equations for studying bending and radial hydroelastic oscillations of the sandwich plate. Its investigation was carried out by the Fourier method. We studied plate dynamic behaviour in the main vibration mode. In particular, the frequency response of the circular sandwich plate were constructed and studied.

Keywords Hydroelasticity, Vibrations, Sandwich plate, Viscous liquid, Elastic foundation.

1. Introduction
Multilayer materials and structures made of them are increasingly used in various industries. Therefore, the statics and dynamics of multilayer structures, as well as the study of their interaction with continuous media, is an urgent problem both from a theoretical and practical point of view. The development review of kinematic theories for studying the stress-strain state of multilayer structural elements is given in [1]. Reference [2] was presented equilibrium and dynamics equations for three-layered beams and plates obtained in the framework of the kinematic hypothesis of the broken normal. On the other hand, the problems of hydroelastic vibrations for homogeneous plates in various formulations are widely studied. For example, we can cite the following sources here. Hydroelastic vibrations of a circular plate were studied in [3] using the approximate Rayleigh energy method. Reference [4] considered vibrations of a circular plate interacting with an ideal fluid based on the formulation and solution of the coupled hydroelasticity problem. The fluid viscosity under hydroelastic vibrations of a circular plate is taken into account in [5]. In reference [6], the frequency
response of two parallel elastic sheets with thin incompressible viscous liquid between them under pressure waves over its upper surface is considered. The stability of a plate interacting with a viscous incompressible fluid was studied in [7]. Reference [8] is devoted to the study of the vibration behavior of a Kirchhoff nano-plate interacting with the surrounding viscous fluid. The experimental study results of natural frequencies and the corresponding decrements of harmonic vibrations for rectangular plates in air or on the fluid free surface are presented in [9]. Vibrations of a homogeneous circular plate interacting with a viscous liquid layer at the one its side and resting on an elastic foundation at the opposite side were studied in [10]. Hydroelastic bending vibrations of a circular plate resting on Pasternak foundation and interacting with an inviscid sloshing liquid are studied in [11]. We note the references [12-15] dealing with the bending vibrations of three-layered beams and plates resting on an elastic foundation. In particular, the free vibrations of a sandwich beam with stiff and compressible core and resting on Winkler foundation were investigated in [12]. In reference [13], the behaviour of circular sandwich plate resting on an elastic foundation under thermal impact was considered. The axisymmetric bending vibrations of a composite circular plate resting on Winkler foundation under local surface loads of different forms are investigated in [14]. Reference [15] was devoted to free vibrations of a three-layered circular plate resting on an elastic foundation under the temperature field influence, taking into account the foundation inertial properties. However, there is much less research on the interaction of multilayer plates with liquid. We can point to references [16-20] which studied the interaction of composite plates and beams with a liquid. However, the above-mentioned papers did not study the radial and bending oscillations of a circular sandwich plate resting on an elastic foundation caused by the action of viscous fluid stresses in the radial and normal directions.

2. Statement of the Problem

Let us consider two parallel coaxial disks forming a narrow channel completely filled with a viscous incompressible liquid. The channel bottom rests on an elastic foundation. We assume the upper disk is absolutely rigid. The bottom disk is a sandwich structure consisting of two face sheets of different thicknesses and a core between them. In other words, the bottom disk is a circular sandwich plate resting on the elastic foundation. We consider the core of the sandwich plate to be lightweight and incompressible, and we also neglect its work in the tangential direction. In this case according to [2], the sandwich plate is deformed as a single package, and its stress-strain state (SSS) is fully described by the radial displacement and deflection of the coordinate surface (the middle surface of the plate core), as well as by the rotation angle of the deformed normal in the core. The elastic foundation will be considered within the framework of inertial Winkler foundation model [15]. Let us introduce a cylindrical coordinate system with a pole located in the core center and restrict ourselves to considering the axisymmetric problem. The channel scheme and its geometric dimensions are shown in Fig. 1.

![Figure 1](image)

Figure 1. The scheme of the channel formed by two disks:
1 is an absolutely rigid disk, 2 is a circular sandwich plate resting on an elastic foundation, 3 is a viscous liquid layer.

We have introduced in Fig. 1 the following notation: $R$ is the radius of the upper and bottom disks, $h_0$ is the gap between the disk and the sandwich plate in an undisturbed state, $2c$ is the thickness of the sandwich plate core, $h_1$ is the upper face sheet thickness of the sandwich plate, $h_2$ is the bottom face sheet thickness of the sandwich plate. Further, when studying the problem, we will assume that $h_0 \ll R$ and $w_m \ll h_0$, where $w_m$ is the sandwich plate bending amplitude. In addition, we will believe that the
pressure pulsation law at the channel contour is given, and the sandwich plate is clamped along its contour. The liquid outflow from the channel will be taken as a free liquid outflow into the same liquid with a given pressure pulsation law, i.e. we assume there is the edge cavity along the channel contour (for simplicity, it is not shown in the figure). In other words, we suppose that when the liquid flows out at the edge, the pressure in the channel edge cross section coincides with the predetermined pressure pulsation law at the channel contour.

The equilibrium equations for the considered circular sandwich plate were obtained in [2, 21], within the framework of the kinematic hypothesis of the broken normal proposed by the authors. Following [2, 21] and taking into account the plate inertial forces in the radial and normal directions, we have written down the dynamics equations for the circular sandwich plate resting on inertial Winkler foundation in the form:

$$
L_2 \left( a_1 u + a_2 \varphi - a_3 \frac{\partial w}{\partial r} \right) - M_0 \frac{\partial^2 u}{\partial t^2} = -q_{sr},
$$

$$
L_2 \left( a_2 u + a_4 \varphi - a_5 \frac{\partial w}{\partial r} \right) = 0,
$$

$$
L_3 \left( a_3 u + a_5 \varphi - a_6 \frac{\partial w}{\partial r} \right) - M_0 \frac{\partial^2 w}{\partial t^2} = -q_{sz} + \kappa w + m f \frac{\partial^2 w}{\partial t^2},
$$

$$
L_3(g) = \frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} (rg) \right], \quad L_3(g) = \frac{1}{r} \frac{\partial}{\partial r} \left[ rL_4(g) \right], \quad M_0 = \rho_1 h_1 + \rho_2 h_2 + \rho_3 2\epsilon,
$$

$$
q_{sr} = \rho v \left( \frac{\partial V_r}{\partial r} + \frac{\partial V_r}{\partial z} \right) \text{ at } z = c + h_1,
$$

$$
q_{sz} = -p + 2 \rho v \frac{\partial V_z}{\partial z} \text{ at } z = c + h_1.
$$

Here $u$ is the radial displacement of the circular sandwich plate; $w$ is the normal displacement of the circular sandwich plate, i.e. deflection; $\varphi$ is the rotation angle of the deformed normal in the plate core; $q_{sr}$ and $q_{sz}$ are the shear and normal stresses of the fluid acting on the plate, respectively; $\kappa$ is the stiffness coefficient of the inertial elastic foundation; $m f$ is the inertial coefficient of the inertial elastic foundation; $V_r$ and $V_z$ are the fluid velocity projections on the axis of the introduced coordinate system; $\rho_1$ is the material density of the $k$-th layer of the circular sandwich plate. The notation for the coefficients $a_1, \ldots, a_6$ is presented in [2].

According to [22], due to the narrowness of the channel, we can assume that the fluid movement between the channel wall is creeping one. In this case we have the following equations for viscous liquid dynamics:

$$
\frac{1}{\rho} \frac{\partial p}{\partial r} = v \left( \frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r} \frac{\partial V_r}{\partial r} + \frac{\partial^2 V_r}{\partial z^2} - V_r \right),
$$

$$
\frac{1}{\rho} \frac{\partial p}{\partial z} = v \left( \frac{\partial^2 V_z}{\partial r^2} + \frac{1}{r} \frac{\partial V_z}{\partial r} + \frac{\partial^2 V_z}{\partial z^2} \right),
$$

$$
\frac{\partial V_r}{\partial r} + \frac{1}{r} \frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial z} = 0,
$$

Here $p$ is the liquid pressure, $v$ is the coefficient of the kinematic viscosity for the liquid, and $\rho$ is the liquid density.

According to the above, as the boundary conditions for Eqs. (1) we consider the clamp conditions at the sandwich plate contour and the conditions for limited sandwich plate deflection at the symmetry axis.
and the boundary conditions for Eqs. (2) are no-slip conditions at the sandwich plate and upper disk, as well as conditions for pressure at the sandwich plate contour and at the symmetry axis

\[ V_r = 0, \ V_z = 0 \text{ at } z = h_0 + c + h_1, \]

\[ V_r = \frac{\partial u}{\partial t}, \ V_z = \frac{\partial w}{\partial t} \text{ at } z = c + h_1, \]

\[ p = p^* \text{ at } r = R, \]

\[ r \frac{\partial p}{\partial r} = 0 \text{ at } r = 0. \]

Here \( p^* = p_0 + p_m \sin(\omega t) \) is the pressure pulsation law at the channel contour, \( p_0 \) is the pressure constant level in the liquid.

3. Determining of Elastic Displacements for the Circular Sandwich Plate

Taking into account the damping properties of the thin viscous liquid layer in the channel gap, further we consider steady-state harmonic oscillations, since transients decay quickly. Moreover, the following relations take place in the problem statement under consideration

\[ h_0 << R, \ w_m << h_0, \ V_z << V_r, \]

\[ \frac{\partial^2 V_r}{\partial z^2} >> \frac{\partial^2 V_r}{\partial r^2}, \ \frac{\partial^2 V_z}{\partial z^2} >> \frac{\partial^2 V_r}{\partial r^2} \frac{1}{r}, \ \frac{\partial^3 V_r}{\partial z^3} >> \frac{\partial^3 V_r}{\partial r^3} \frac{1}{r^2}, \]

\[ \frac{\partial^2 V_r}{\partial z^2} >> \frac{\partial^2 V_z}{\partial r^2} \frac{1}{r}, \ \frac{\partial^2 V_z}{\partial r^2} >> \frac{\partial^2 V_r}{\partial z^2} \frac{1}{r}, \]

\[ \frac{\partial^2 V_r}{\partial z^2} >> \frac{\partial^2 V_z}{\partial r^2} \frac{1}{r}, \]

\[ \frac{\partial^2 V_r}{\partial z^2} >> \frac{\partial^2 V_z}{\partial r^2} \frac{1}{r}. \]

Taking into account the above relations (5) in Eqs. (2) we obtain dynamics equations for the thin layer of the viscous incompressible liquid

\[ \frac{1}{\nu \rho} \frac{\partial p}{\partial r} = \frac{\partial^3 V_z}{\partial z^3}, \]

\[ \frac{\partial p}{\partial z} = 0, \]

\[ \frac{\partial V_r}{\partial r} + \frac{1}{r} \frac{\partial V_r}{\partial r} + \frac{\partial V_z}{\partial z} = 0. \]

After that, we solve Eqs. (6) with boundary conditions (4) and obtain the following expressions for viscous fluid stresses in the radial and normal directions

\[ q_{zz} = -p_0 - 12 \frac{D}{h_0^3} \int_0^\xi \left[ \frac{1}{\xi} \int_0^\xi \frac{\partial w}{\partial t} \, d\xi \right] d\xi, \]

\[ \text{(7)} \]
Substituting Eqs. (7) into Eqs. (1), we obtain a mathematical model describing the radial and bending hydroelastic oscillations for the circular sandwich plate resting on the inertial Winkler foundation. This model consists of Eqs. (1) with expressions for viscous fluid stresses (6) and boundary conditions (3). We apply the Fourier method to study and resolve the resulting mathematical model. In particular, according to the boundary conditions (3) the elastic displacements $u$, $w$, and rotation angle $\phi$ can be represented as a series of eigenfunctions for the Sturm-Liouville problem:

$$w = w_m \sum_{k=1}^{\infty} R_k(r) \left[ \frac{J_0(\beta_k \xi)}{J_0(\beta_k)} - I_0(\beta_k) \right],$$

$$u = -u_m \sum_{k=1}^{\infty} \beta_k \mathcal{Q}_k(r) \left[ \frac{J_1(\beta_k \xi)}{J_0(\beta_k)} + \frac{I_1(\beta_k \xi)}{I_0(\beta_k)} \right],$$

$$\phi = -\phi_m \sum_{k=1}^{\infty} \beta_k T_k(r) \left[ \frac{J_1(\beta_k \xi)}{J_0(\beta_k)} + \frac{I_1(\beta_k \xi)}{I_0(\beta_k)} \right],$$

where $J_0$ is the zero-order Bessel function, $J_1$ is the first-order Bessel function; $I_0$ is the modified zero-order Bessel function; $I_1$ is the modified first-order Bessel function; $\beta_k$ is the root of the transcendental equation $I_1(\beta_k)/I_0(\beta_k) = -J_1(\beta_k)/J_0(\beta_k)$ [2].

We set the number of retained terms in series (5), and then substituted them into the mathematical model of hydroelastic oscillations for the circular sandwich plate resting on inertial Winkler foundation. As a result, we obtain the system of ordinary differential equations for determining the laws of radial displacement, plate deflection, and the rotation angle of the normal in the circular sandwich plate core. Next, we restrict ourselves to considering the main oscillations mode (i.e., we consider the case for $k=1$) and write the system contains three equations, one of which is homogeneous. This allows us to reduce the system of three equations to the system of two equations with respect to radial displacement and deflection, which we have solved. Here we give the final expressions $u$ and $w$ for the main oscillations mode for the circular sandwich plate

$$u = \left[ \frac{J_1(\beta_1 \xi)}{J_0(\beta_1)} + \frac{I_1(\beta_1 \xi)}{I_0(\beta_1)} \right] - \frac{2}{J_0(\beta_1) I_0(\beta_1)} \left( \frac{p_0 R^3}{b_{21} (b_{11} b_{21})} - b_{12} \right) + \frac{p_m R^3}{b_{21}} A_u(\omega) \sin(\omega t + \phi_u(\omega)),$$

$$w = -\left[ \frac{J_0(\beta_1 \xi)}{J_0(\beta_1)} - \frac{I_0(\beta_1 \xi)}{I_0(\beta_1)} \right] - \frac{2}{J_0(\beta_1) I_0(\beta_1)} \left( \frac{p_0 R^3}{b_{21} (b_{11} b_{21})} - b_{12} \right) - \frac{p_m R^3}{b_{21}} A_u(\omega) \sin(\omega t + \phi_u(\omega)).$$

Here we have introduced the following notation

$$A_u(\omega) = \frac{\sqrt{b_{21}^2 (b_{12})^2 + (K_{11} \omega)^2}}{b_{11} (b_{11} b_{21})^2 + (b_{11} K_{21} \omega - b_{21} K_{11} \omega)^2},$$

$$A_u(\omega) = \frac{b_{11}^2 (b_{12})^2}{b_{11} (b_{11} b_{21})^2 + (b_{11} K_{21} \omega - b_{21} K_{11} \omega)^2},$$

$$b_{11} = a_1 - \frac{a_1^2}{a_4} \frac{(M_0 + m_0 \omega^2) R^2}{\beta_1^2}, b_{12} = \frac{a_2 m_0}{a_4 R}, K_{11} = \frac{6 R^3 \rho \nu}{b_{11}^3 h_0^4}, K_{12} = \frac{6 R^3 \rho \nu}{b_{11}^4 d_{11}^4 h_0^6},$$

$$d_{11} = 6 R^3 \rho \nu,$$
\[ \begin{align*}
    b_{21} &= \frac{a_{22}}{a_4} - a_3, \quad b_{22} = \left( \frac{x R}{\beta_5} + 1 \right) \frac{a_6^2}{R} - \frac{a_6^2}{a_4 R} \frac{(M_0 + m_I) \omega^2 R^3}{\beta_i^4}, \\
    K_{21} &= \frac{12 \rho \nu R^5}{\rho_0 \beta_i^4}, \quad d_{31} = \frac{12 R \rho \nu}{\beta_i^4} d_{11}^i = \frac{J_2^2(\beta_i)}{J_0^2(\beta_i)} - \frac{4}{\beta_i} J_1(\beta_i), \\
    d_{11}^i &= \frac{1}{\beta_i^2} \left( \frac{J_2^2(\beta_i)}{J_0^2(\beta_i)} - \frac{4}{\beta_i} J_1(\beta_i) \right). 
\end{align*} \]

4. Results and Discussion

The developed hydroelastic vibrations mathematical model for the circular sandwich plate resting on inertial Winkler foundation and its solution based on the Fourier method allows us to obtain analytical expressions for frequency-dependent distribution laws of the plate's radial displacements and deflections. In this paper, these laws are defined for the main mode of hydroelastic oscillations; they are represented by formulas (9). Furthermore, in Eqs. (9), we distinguished the expressions of \( A_u(\omega) \) and \( A_w(\omega) \), which can be considered as the hydroelastic frequency responses for the circular sandwich plate cross section. Indeed, by setting the value for radial coordinate \( r \) in Eqs. (9), we will obtain expressions for the static radial displacement and deflection due to the constant pressure level \( p_0 \), as well as expressions for the frequency-dependent radial displacement and deflection due to liquid pressure pulsation at the sandwich plate cross section corresponding to the specified radial coordinate. Using \( A_u(\omega) \) and \( A_w(\omega) \), we can find the resonant frequencies of plate vibrations that correspond to the maximum elastic displacements of the circular sandwich plate. The above makes it possible to study the plate's SSS at resonances and determine the distribution laws for fluid pressure inside the channel.

From the analysis of the mathematical model and the obtained laws for the sandwich plate elastic displacements, we concluded that there is a cross-influence of radial displacements and deflections of the sandwich plate, because hydroelastic vibrations are described by a system of two differential equations with respect to radial displacement and deflection. In particular, calculations show that the hydroelastic frequency responses \( A_u(\omega) \) and \( A_w(\omega) \) have two maxima for the main oscillations mode, i.e. there is a cross-action of stiffness and inertia forces in the radial and normal directions. When considering homogeneous plates [10], this effect was not detected. Also, calculations have shown that an increase in the elastic foundation stiffness leads to an increase in resonant frequencies, and an increase in the elastic foundation inertia coefficient leads to a shift of resonances to the low-frequency region.

5. Summary and Conclusion

Thus, the mathematical model has been formulated for the study of hydroelastic oscillations of the circular sandwich plate due to the liquid pressure pulsation at the channel contour. The frequency responses for the radial and bending oscillations of the circular sandwich plate in the main mode are obtained. Our calculations showed the mutual influence of plate stiffness and inertia in the radial and normal directions, as well as the importance of taking into account shear stresses from the liquid layer. This conclusion is based on the appearance of resonant frequencies determined by stiffness and inertia in the radial and normal directions on both the frequency responses \( A_u(\omega) \) and \( A_w(\omega) \). Taking into account the inertial and rigid properties of the elastic foundation leads to a significant change in the values for the resonant frequencies. Therefore, we came to the conclusion that when studying hydroelastic oscillations of sandwich plates, in contrast to homogeneous plates [3-5, 10], it is necessary to take into account both radial and normal inertia forces, as well as shear and normal stresses of the viscous fluid layer, inertial and rigid properties of the elastic foundation.

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