Mirror World with Broken Mirror Parity, $E_6$ Unification and Cosmology

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Abstract

In the present paper we have developed a concept of parallel ordinary (O) and mirror (M) worlds. We have shown that in the case of a broken mirror parity (MP), the evolutions of fine structure constants in the O- and M-worlds are not identical. It is assumed that $E_6$-unification inspired by superstring theory restores the broken MP at the scale $\sim 10^{18}$ GeV, what unavoidably leads to the different $E_6$-breakdowns at this scale: $E_6 \rightarrow SO(10) \times U(1)_Z$ - in the O-world, and $E_6' \rightarrow SU(6)' \times SU(2)'_Z$ - in the M-world. Considering only asymptotically free theories, we have presented the running of all the inverse gauge constants $\alpha_i^{-1}$ in the one-loop approximation. Then a ‘quintessence’ scenario suggested in Refs. [56–61] is discussed for our model of accelerating universe. Such a scenario is related with an axion (‘acceleron’) of a new gauge group $SU(2)'_Z$ which has a coupling constant $g_Z$ extremely growing at the scale $\Lambda_Z \sim 10^{-3}$ eV.

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1 Introduction

In the present paper we consider the concept [1,2] (see also reviews [3,4]) that there exists in Nature a ‘mirror’ (M) world – a hidden mirror sector – parallel to our ordinary (O) world. The M-world as a mirror copy of the O-world contains the same particles and their interactions as our visible world. Observable elementary particles of our O-world have left-handed (V-A) weak interactions which violate P-parity. If a hidden mirror M-world exists, then mirror particles participate in the right-handed (V+A) weak interactions and have an opposite chirality. Lee and Yang were first [1] who suggested such a duplication of the worlds which restores the left-right symmetry of Nature. The term ‘Mirror World’ was introduced by Kobzarev, Okun and Pomeranchuk in Ref. [2], where they have investigated a lot of phenomenological implications of such parallel worlds. The development of this theory is given by Refs. [5–22].

The old idea of the existence of visible and mirror worlds became very attractive over the last years in connection with a superstring theory [23–31]. Having a theory described by the product $G \times G'$ of symmetry groups corresponding to the parallel O- and M-worlds, respectively, it is natural to associate it with superstring theory described by $E_8 \times E'_8$ [23,31].

Superstring theory is a paramount candidate for the unification of all fundamental gauge interactions with gravity. Superstrings are free of gravitational and Yang-Mills anomalies if a gauge group of symmetry is $SO(32)$ or $E_8 \times E_8$. The ‘heterotic’ superstring theory $E_8 \times E'_8$ was suggested as a more realistic model for unification [23,31]. This ten-dimensional Yang-Mills theory can undergo spontaneous compactification for which $E_8$ group is broken to $E_6$ in four-dimensional space.

Among hundreds of papers devoted to the $E_6$-unification we should like to single out Refs. [32–42].

If at small distances we have the $E_6$ unification in our ordinary world, then we can expect to have the same $E_6$ unification in the mirror four-dimensional world assuming a restoration of the left-right symmetry of Nature at small distances.

We can consider a minimal symmetry $G_{SM} \times G'_{SM}$, where $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ stands for the Standard Model (SM) of observable particles: three generations of quarks and leptons and the Higgs boson, while $G'_{SM} = SU(3)'_C \times SU(2)'_L \times U(1)'_Y$ is its mirror gauge counterpart having three generations of mirror quarks and leptons and the mirror Higgs boson. The M-particles are singlets of $G_{SM}$ and O-particles are singlets of $G'_{SM}$. These different O- and M-worlds are coupled only by gravity (or maybe other very weak interaction).

In this paper all quantities of the mirror world will be marked by prime ($'$).

A discrete symmetry ‘MP’ of the interchange $G \leftrightarrow G'$ is called ‘Mirror Parity’. If this parity is conserved, then particle content of both sectors are identical and described by the same Lagrangians with the same masses and coupling constants.

The aim of the present paper is to consider a case suggested in Refs. [43–45], when a mirror parity MP is not conserved. Then, as we show, the evolutions of coupling constants in O- and M-worlds are different. The next assumption that mirror parity is restored by the $E_6$-unification at the scale $\sim 10^{18}$ GeV in both O- and M-worlds leads to the significant consequences for cosmology.
In Section 2 we determine a mirror world and give particle contents existing in the ordinary and mirror worlds.

Section 3 is devoted to symmetry groups considering only in the ordinary world. We assume that SM is extended by MSSM (Minimal Supersymmetric Standard Model). Then its extension by left-right symmetry leads to the $SO(10)$-unification, which is ended by the $E_6$-unification at the superGUT scale $M_{SGUT} \sim 10^{18} \text{ GeV}$, according to the following chain:

\[
SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow [SU(3)_C \times SU(2)_L \times U(1)_Y]_{MSSM} \\
\rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \times U(1)_Z \\
\rightarrow SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_Z \\
\rightarrow SO(10) \times U(1)_Z \rightarrow E_6.
\]

All evolutions corresponding to these symmetry groups in the ordinary world are presented in Figs. 1(a,b) and Figs. 2(a,b) for supersymmetry breaking scales 1 TeV and 10 TeV, respectively. Considering only asymptotically free symmetry groups, we have used the one-loop approximations for these evolutions.

In Section 4 we have considered a mirror world with a broken mirror parity (MP). We have shown the difference between evolutions of all fine structure constants in the O- and M-worlds in the case of broken MP. To get the same $E_6$ unification in both worlds, we are forced to consider a quite different chain of the SM’ extension in the M-world. We have assumed the following chain:

\[
[SU(3)'_C \times SU(2)'_L \times U(1)'_Y]_{SM'} \times SU(2)'_Z \\
\rightarrow [SU(3)'_C \times SU(2)'_L \times SU(2)'_Z \times U(1)'_Y]_{SUSY} \\
\rightarrow SU(3)'_C \times SU(2)'_L \times SU(2)'_Z \times U(1)'_X \times U(1)'_Z \\
\rightarrow SU(4)'_C \times SU(2)'_L \times SU(2)'_Z \times U(1)'_Z \\
\rightarrow SU(6)' \times SU(2)'_Z \rightarrow E_6'.
\]

All the evolutions corresponding to these symmetry groups in the mirror world are given in Figs. 3(a,b) and Figs. 4(a,b) (in the one-loop approximation) for the supersymmetry breaking scale $M'_{SUSY}$.

A comparison of the evolutions considered in both worlds is shown in Figs. 5(a,b) and Figs. 6(a,b).

All parameters of the evolutions are presented in Table 1.

A new mysterious gauge group $SU(2)'_Z$ is considered in Section 5. We have chosen such a particle content of this group which leads to the ‘quintessence’ model of our universe. The axion-like potential is investigated.

A new quintessence scenario in cosmology is developed in Section 6, together with consequences for recent models of dark energy and dark matter. The problem of cosmological constant also is briefly discussed in this Section.
2 Particle content in the ordinary and mirror SM

In the Standard Model (SM) fermions are represented by Weyl spinors, however, the left-handed (L) quarks and leptons: \( \psi_L = q_L, l_L \) and right-handed (R) fermions: \( \psi_R = q_R, l_R \) transform differently under \( SU(2) \times U(1) \) symmetry. A global lepton charge is \( L = 1 \) for leptons \( l_L, l_R \), and a baryon charge is \( B = \frac{1}{3} \) for quarks \( q_L, q_R \).

With the same rights we could formulate the SM in terms of antiparticle fields: \( \tilde{\psi}_R = C\gamma_0\psi^*_L \) and \( \tilde{\psi}_L = C\gamma_0\psi^*_R \), where \( C \) is the charge conjugation matrix and \( \gamma_0 \) is the Dirac matrix. These antiparticles have opposite gauge charges, opposite chirality, \( L = -1 \) for antileptons and \( B = -\frac{1}{3} \) for antiquarks.

We can redefine the notion of particles considering L-particles and R-particles:

\[
L - \text{fermions}: \quad \psi_L, \quad \tilde{\psi}_L \quad \text{and} \quad \tilde{R} - \text{fermions}: \quad \tilde{\psi}_R, \quad \psi_R. \tag{1}
\]

Including Higgs bosons \( \phi \) we have the following SM content of the O-world:

\[
L - \text{set}: \quad (u, d, e, \nu, \tilde{u}, \tilde{d}, \tilde{e}, \tilde{\nu}, \tilde{N})_L, \phi_u, \phi_d;
\]

\[
\tilde{R} - \text{set}: \quad (\tilde{u}, \tilde{d}, \tilde{e}, \tilde{\nu}, u, d, e, N)_R, \tilde{\phi}_u, \tilde{\phi}_d
\]

with \( \tilde{\phi}_{u,d} = \phi_{u,d}^* \).

Considering the minimal symmetry \( G_{SM} \times G_{SM}' \) we have the following particle content in the M-sector:

\[
L' - \text{set}: \quad (u', d', e', \nu', \tilde{u}', \tilde{d}', \tilde{e}', \tilde{\nu}', \tilde{N}')_L, \phi'_u, \phi'_d;
\]

\[
\tilde{R}' - \text{set}: \quad (\tilde{u}', \tilde{d}', \tilde{e}', \tilde{\nu}', u', d', e', N')_R, \tilde{\phi}'_u, \tilde{\phi}'_d. \tag{2}
\]

In general, we can consider a supersymmetric theory when \( G \times G' \) contains grand unification groups: \( SU(5) \times SU(5)', SO(10) \times SO(10)', E_6 \times E_6', \) etc.

3 The SM and its extension in the ordinary world

In the present paper we consider the running of all the gauge coupling constants in the SM and its extensions which is well described by the one-loop approximation of the renormalization group equations (RGEs) from the Electroweak (EW) scale up to the Planck scale. For simplicity of this investigation and with aim to demonstrate our idea, we neglect the contributions of higher loops and Higgs bosons belonging to high representations. Also we do not pay an attention to the realistic values of some unknown scales (namely, a supersymmetric scale \( M_{SUSY} \), seesaw scale \( M_R \), etc.) which we have used in our numerical calculations. We give their reasonable values as examples.

For energy scale \( \mu \geq M_{\text{ren}} \), where \( M_{\text{ren}} \) is the renormalization scale, we have the following evolution for the inverse fine structure constants \( \alpha_i^{-1} \) given by RGE in the one-loop approximation:

\[
\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_{\text{ren}}) + \frac{b_i}{2\pi} t, \tag{4}
\]

where \( \alpha_i = g_i^2/4\pi, \) \( g_i \) are gauge coupling constants and \( t = \ln(\mu/M_{\text{ren}}) \).
3.1 Gauge coupling constant evolutions in the SM

We start with the SM in our ordinary world.

In the SM for energy scale $\mu \geq M_t$ (here $M_t$ is the top quark (pole) mass) we have the following evolutions (RGEs) [46–48] for the inverse fine structure constants $\alpha_i^{-1}$ ($i = 1, 2, 3$ correspond to the $U(1)$, $SU(2)_L$ and $SU(3)_C$ groups of the SM), which are revised using updated experimental results [48] (see also Refs. [49, 50]):

$$\alpha_1^{-1}(t) = 58.65 \pm 0.02 - \frac{41}{20\pi} t, \quad (5)$$

$$\alpha_2^{-1}(t) = 29.95 \pm 0.02 + \frac{19}{12\pi} t, \quad (6)$$

$$\alpha_3^{-1}(t) = 9.17 \pm 0.20 + \frac{7}{2\pi} t. \quad (7)$$

Here $M_{\text{ren}} = M_t$ and $t = \ln(\mu/M_t)$. In Eq. (7) the value of $\alpha_3^{-1}(M_t) = 9.17$ essentially depends on the value of $\alpha_3(M_Z) \equiv \alpha_s(M_Z) = 0.118 \pm 0.002$ (see [48]), where $M_Z$ is the mass of $Z$-boson. The value of $\alpha_3^{-1}(M_t)$ is given by the running of $\alpha_3^{-1}(\mu)$ from $M_Z$ up to $M_t$, via the Higgs boson mass $M_H$. We have used the central value of the top quark mass $M_t \approx 174$ GeV and $M_H = 130 \pm 15$ GeV.

Evolutions (5)-(7) are shown from $M_t$ up to the scale $M_{\text{SU}3}$ in Fig. 1(a) and Fig. 2(a), where $x = \log_{10}(\mu(\text{GeV}))$, $t = x \ln 10 - \ln M_t$.

3.2 Running of gauge coupling constants in the MSSM

In this Subsection we consider the Minimal Supersymmetric Standard Model (MSSM) which extends the conventional SM.

MSSM gives the evolutions (4) for $\alpha_i^{-1}$ ($i = 1, 2, 3$) from the supersymmetric scale $M_{\text{SU3}}$ (here $M_{\text{ren}} = M_{\text{SU3}}$) up to the seesaw scale $M_R$.

Figs. 1(a,b), 2(a,b) present also these evolutions which are given by the following MSSM slopes $b_i$ [46,47]:

$$b_1 = -\frac{33}{5} = -6.6, \quad b_2 = -1, \quad b_3 = 3. \quad (8)$$

In Figs. 1(a,b), 2(a,b) we have presented examples with the scales $M_{\text{SU3}} = 1$ and 10 TeV, respectively:

Figs. 1(a,b) are given for SUSY breaking scale $M_{\text{SU3}} = 1$ TeV and seesaw scale $M_R = 1.25 \cdot 10^{15}$ GeV; $M_{\text{GUT}} \approx 2.4 \cdot 10^{17}$ GeV and $\alpha_{\text{GUT}}^{-1} \approx 26.06$.

Figs. 2(a,b) correspond to SUSY breaking scale $M_{\text{SU3}} = 10$ TeV and seesaw scale $M_R = 2.5 \cdot 10^{14}$ GeV; $M_{\text{GUT}} \approx 6.96 \cdot 10^{17}$ GeV and $\alpha_{\text{GUT}}^{-1} \approx 27.64$.

3.3 Left-right symmetry as an extension of the MSSM

At the seesaw scale $M_R$ the heavy right-handed neutrinos appear. We assume that the following supersymmetric left-right symmetry [51–54] originates at the seesaw scale:

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \times U(1)_Z.$$  (9)
Here we see additional groups $SU(2)_R$ and $U(1)_{(B-L)}$ originated at the scale $M_R$. The group $U(1)_{(B-L)}$ is mixed with the gauge group $U(1)_Y$ leading to the product $U(1)_X \times U(1)_Z$ of the two $U(1)$ groups with quantum numbers $X$ and $Z$ linearly combined into the weak hypercharge $Y$ \[ Y = \frac{1}{5}(X - Z) \].\(^\text{(10)}\)

Considering the running \[\text{(4)}\] for the supersymmetric group \[\text{(9)}\] in the region $\mu \geq M_R$ we have $M_{\text{ren}} = M_R$ and the following slopes \[\text{(46, 47)}\]:\[ b_X = b_1 = -6.6, \quad b_Z = -9, \quad b_3 = 3. \]\[\text{(11)}\]

Also the running of $SU(2)_L \times SU(2)_R$ in the same region of $\mu$ is given by the slope:\[ b_{22} = -2, \]\[\text{(12)}\]

and we have the following evolution:\[ \alpha_{22}^{-1}(\mu) = \alpha_{22}^{-1}(M_R) + \frac{1}{\pi} \ln \frac{\mu}{M_R}, \]\[\text{(13)}\]

where\[ \alpha_{22}^{-1}(M_R) = \alpha_2^{-1}(M_R). \]\[\text{(14)}\]

The next step is an assumption that the group $SU(4)_C \times SU(2)_L \times SU(2)_R$ by Pati and Salam \[\text{(51)}\] originates at the scale $M_4$ giving the following extension of the group \[\text{(9)}\]:\[ SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \times U(1)_Z \rightarrow SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_Z. \]\[\text{(15)}\]

The scale $M_4$ is given by the intersection of $SU(3)_C$ with $U(1)_X$:\[ \alpha_3^{-1}(M_4) = \alpha_X^{-1}(M_4). \]\[\text{(16)}\]

In the MSSM we have the following equation for the slope $b_N$ of the $SU(N)$ group (see \[\text{(46, 47)}\] and \[\text{(41)}\]):\[ b_N = 3N - N_f - \frac{1}{2}N_{\text{vector}} - N \cdot N_{\text{adjoint}} - \ldots, \]\[\text{(17)}\]

where $N_f$ is a number of flavors, $N_{\text{vector}}$ is a number of scalar Higgs fields in the fundamental representation and $N_{\text{adjoint}}$ is a number of Higgses in adjoint representation. Considering only the minimal content of scalar Higgs fields, e.g. quartets $4 + \bar{4}$, we have $N_{\text{vector}} = 2$ and obtain from \[\text{(17)}\] the following slope for the running of $\alpha_4^{-1}(\mu)$:\[ b_4 = 3 \cdot 4 - 6 - 1 = 5. \]\[\text{(18)}\]

Now the evolution \[\text{(4)}\] with $M_{\text{ren}} = M_4$ gives:\[ \alpha_4^{-1}(\mu) = \alpha_4^{-1}(M_4) + \frac{5}{2\pi} \ln \frac{\mu}{M_4}. \]\[\text{(19)}\]

This is the running for the symmetry group $SU(4)$. 

5
3.4 From $SO(10)$ to the $E_6$-unification in the ordinary world

The intersection of $\alpha_{14}^{-1}(\mu)$ with the running of $\alpha_{22}^{-1}(\mu)$ leads to the scale $M_{GUT}$ of the $SO(10)$-unification:

$$SU(4)_C \times SU(2)_L \times SU(2)_R \to SO(10),$$  \[(20)\]

and we obtain the value of $M_{GUT}$ from the relation:

$$\alpha_{14}^{-1}(M_{GUT}) = \alpha_{22}^{-1}(M_{GUT}).$$  \[(21)\]

Then we deal with the running \[(1)\] for the $SO(10)$ inverse gauge constant $\alpha_{10}^{-1}(\mu)$, which runs from the scale $M_{GUT}$ up to the scale $M_{SGUT}$ of the super-unification $E_6$:

$$SO(10) \times U(1)_Z \to E_6.$$  \[(22)\]

The slope of this running is $b_{10}$.

In general, for the $SO(N)$ group we have the following slope \[41, 46, 47]\):

$$b_{N_{SO(N)}} = \frac{3}{2}(N - 2) - N_f - \frac{1}{2}N_{\text{vector}} - \frac{1}{2}(N - 2) \cdot N_{\text{adjoint}} - ... .$$  \[(23)\]

Calculating the $SO(10)$-slope we must consider not only vectorial Higgs fields $N_{\text{vector}} = 2$, but also $N_{\text{adjoint}} = 1$, because the appearance of right-handed particles is impossible without adjoint Higgs field (see explanation in Ref. \[50\]). As a result, we obtain from Eq. \[(23)\] the following $SO(10)$-slope:

$$b_{10} = 12 - 6 - 1 - 4 = 1 .$$  \[(24)\]

Then we have the following running of $\alpha_{10}^{-1}(\mu)$:

$$\alpha_{10}^{-1}(\mu) = \alpha_{10}^{-1}(M_{GUT}) + \frac{1}{2\pi} \ln \frac{\mu}{M_{GUT}} ,$$  \[(25)\]

which is valid up to the superGUT scale $M_{SGUT}$ of the $E_6$-unification.

Finally, as a result of our investigation, one can envision the following symmetry breaking chain in the ordinary world:

$$E_6 \to SO(10) \times U(1)_Z \to SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_Z \to$$

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \times U(1)_Z \to SU(3)_C \times SU(2)_L \times U(1)_Y .$$

All evolutions of the corresponding inverse fine structure constants are given in Figs. 1(a,b) and 2(a,b).

4 Mirror world with broken mirror parity and mirror scales

In this Section, as in Refs. \[43-45\] (see also \[3, 4\]), our main assumption is the principle: “The only good parity... is a broken parity”, what means that in general case the mirror
parity MP is not conserved in Nature. However, at the very small distances the mirror parity is restored and super-unifications \( E_6 \) and \( E'_6 \) (inspired by superstring theory) are identical having the same \( M_{SGUT} \sim 10^{17} \) or \( \sim 10^{18} \) GeV. By this reason, the superGUT scale \( M_{SGUT} \) may be fixed by the intersection of the evolutions of gauge coupling constants in both – mirror and ordinary – worlds, which were not identical from the beginning.

Now it is very interesting to discuss what particle physics exists in the mirror world when the mirror parity MP is spontaneously broken.

If O- and M-sectors are described by the minimal SM with the Higgs doublets \( \phi \) and \( \phi' \), respectively, then we can consider the Higgs potentials:

\[
U = -\mu^2 \phi^+ \phi + \frac{\lambda}{4} (\phi^+ \phi)^2, \tag{26}
\]

and

\[
U' = -\mu'^2 \phi'^+ \phi' + \frac{\lambda'}{4} (\phi'^+ \phi')^2. \tag{27}
\]

In the case of non-conserved MP the VEVs of \( \phi \) and \( \phi' \) are not equal:

\[
v = \frac{2\mu}{\lambda} \neq v' = \frac{2\mu'}{\lambda'}. \tag{28}
\]

Following Refs. [43–45], we assume that \( v' \gg v \) and introduce the parameter characterizing the violation of MP:

\[
\zeta = \frac{v'}{v} \gg 1. \tag{29}
\]

As far as Yukawa couplings have the same values in both worlds, the masses of the SM fermions and massive bosons in the mirror world are scaled up by the factor \( \zeta \):

\[
m'_{q,l} = \zeta m_{q,l},
M'_{W,Z,\phi} = \zeta M_{W,Z,\phi}, \tag{30}
\]

but photons and gluons remain massless in both worlds.

Let us consider now the following expressions:

\[
\alpha_i^{-1}(\mu) = \frac{\ln \mu}{2\pi} \ln \frac{\mu}{\Lambda_i},
\]

— in the O-world, and

\[
\alpha'_i^{-1}(\mu) = \frac{\ln \mu}{2\pi} \ln \frac{\mu}{\Lambda'_i}, \tag{31}
\]

— in the M-world.

A big difference between \( v \) and \( v' \) will not cause a big difference between scales \( \Lambda_i \) and \( \Lambda'_i \) (see [4,44]):

\[
\Lambda'_i = \xi \Lambda_i \quad \text{with} \quad \xi > 1. \tag{32}
\]

The values of \( \zeta \) and \( \xi \) were estimated in Refs. [4,43–45]:

\[
\zeta \approx 30 \quad \text{and} \quad \xi \approx 1.5, \tag{33}
\]
as results of astrophysical implications of the mirror world with broken mirror parity. But it is possible to have $\zeta$ in the region:

$$10 \leq \zeta \leq 100.$$  \hfill (34)

As for the neutrino masses, the same authors have shown that the theory with broken mirror parity leads to the following relations:

$$m'_\nu = \zeta^2 m_\nu,$$

and

$$M'_\nu = \zeta^2 M_\nu,$$  \hfill (35)

where $m_\nu$ are light left-handed and $M_\nu$ are heavy right-handed neutrino masses in the O-world, and $m'_\nu, M'_\nu$ are the corresponding neutrino masses in the M-world. These relations are valid for each of three generations.

4.1 Broken mirror parity and the running of gauge coupling constants in the mirror SM

We assume that M-world is not P- and CP- invariant and differs with O-world.

Considering the mirror SM given by symmetry group $G'_{SM} = SU(3)'_C \times SU(2)'_L \times U(1)'_Y$, we deal with the following one-loop approximation RGE for the running of inverse fine structure constants $\alpha'^{-1}(\mu)$ ($i = 1, 2, 3$ correspond to the $U(1)'$, $SU(2)'_L$ and $SU(3)'_C$ groups of the mirror SM with broken MP):

$$\alpha'^{-1}_i(\mu) = \alpha'^{-1}_i(M'_{ren}) + \frac{b'_i}{2\pi} t',$$ \hfill (36)

where $M'_{ren} = \zeta M_{ren}$ is the renormalization scale in the mirror world and $t' = \ln(\mu/M'_{ren})$.

In the M-world we have scales $\Lambda'_i$ which are different with $\Lambda_i$ (they are given by Eq. (32)), but O- and M-slopes are identical:

$$b'_i = b_i.$$ \hfill (37)

Then in the SM of the M-sector we have the following evolutions:

$$\alpha'^{-1}_i(\mu) = \alpha'^{-1}_i(M_i) + \frac{b_i}{2\pi} t' = \frac{b_i}{2\pi} \ln \frac{\mu}{\Lambda'_i},$$ \hfill (38)

where

$$\alpha'^{-1}_i(M_i) = \alpha'^{-1}_i(M_i) - \frac{b_i}{2\pi} \ln \xi,$$ \hfill (39)

or

$$\alpha'^{-1}_i(M'_i) = \alpha'^{-1}_i(M_i).$$ \hfill (40)

Finally, we obtain the following SM running of gauge coupling constants in the mirror world:
\[(\alpha')^{-1}_1(\mu) = 58.65 \pm 0.02 - \frac{41}{20\pi} t', \quad (41)\]

\[(\alpha')^{-1}_2(\mu) = 29.95 \pm 0.02 + \frac{19}{12\pi} t', \quad (42)\]

\[(\alpha')^{-1}_3(\mu) = 9.17 \pm 0.20 + \frac{7}{2\pi} t', \quad (43)\]

where \(t' = \ln (\mu/M'_t)\). According to Eq. (30), the pole mass of the mirror top quark is \(M'_t = \zeta M_t\).

### 4.2 Mirror MSSM and a seesaw scale in the mirror world

If the Minimal Supersymmetric Standard Model (MSSM) extends the mirror SM in the mirror world, then we can assume that mirror sparticle masses \(\tilde{m}'\) obey the relation analogous to Eq. (30):

\[\tilde{m}' = \zeta \tilde{m}. \quad (44)\]

This relation leads to the assumption that the mirror SUSY-breaking scale is larger than \(M'_{SUSY}\):

\[M'_{SUSY} = \zeta M_{SUSY}. \quad (45)\]

The mirror MSSM gives the evolutions (36) for \(\alpha_i^{-1}(\mu) \ (i = 1, 2, 3)\) from the supersymmetric scale \(M'_{SUSY}\) (here \(M'_{ren} = M'_{SUSY}\)) up to the GUT scale \(M'_{GUT}\).

Here it is worth the reader’s attention to observe that if heavy right-handed neutrino masses are given by Eq. (35), then a mirror seesaw scale \(M'_R\) obeys the following relation:

\[M'_R = \zeta^2 M_R. \quad (46)\]

According to the estimate (33) given by Refs. [4, 43–45], we have:

\[M'_R \sim 10^3 M_R. \quad (47)\]

Now if \(M_R \sim 10^{14}\) GeV, then \(M'_R \sim 10^{17}\) GeV, and a seesaw scale is close to the superGUT scale of the \(E_6\)-unification. This means that mirror heavy right-handed neutrinos appear at the scale \(\sim 10^{17}\) GeV.

Figs. 3(a), 4(a) present the mirror MSSM evolutions of \(\alpha^{-1}_i(\mu) \ (i = 1, 2, 3)\), where slopes \(b_i\) are given by the same Eq. (5) as in the O-world [46, 47]. In Figs. 3(a,b) we have presented an example of the mirror MSSM evolution with the scale \(M'_{SUSY} = 10\) TeV, what corresponds to \(M'_{SUSY} = 1\) TeV and \(\zeta = 10\). But in Figs. 4(a,b) we have shown an example of the mirror MSSM evolution with the scale \(M'_{SUSY} = 300\) TeV, what corresponds to \(M'_{SUSY} = 10\) TeV and \(\zeta = 30\).
4.3 From $SU(6)$ to the $E_6$-unification in the mirror world

Let us consider now the extension of the MSSM in the mirror world.

The first step of such an extension is:

$$[SU(3)'_C \times SU(2)'_L \times U(1)'_Y]_{MSSM} \rightarrow SU(4)'_C \times SU(2)'_L.$$  \hspace{1cm} (48)

Assuming that the supersymmetric group $SU(4)'_C \times SU(2)'_L$ originates at the scale $M'_4$, we find the intersection of $SU(3)'_C$ with $U(1)'_Y$:

$$\alpha'^{-1}_3(M'_4) = \alpha^{-1}_Y(M'_4).$$ \hspace{1cm} (49)

The gauge symmetry group $SU(4)'_C$ starts from the scale $M'_4$, and we have the following evolution:

$$(\alpha')^{-1}_4(\mu) = (\alpha')^{-1}_4(M'_4) + \frac{5}{2\pi} \ln \frac{\mu}{M'_4},$$ \hspace{1cm} (50)

which runs up to the intersection with the running $(\alpha')^{-1}_2(\mu)$ for the supersymmetric group $SU(2)'_L$ having $b_2 = -1$. The point of this intersection is the scale $M'_{GUT}$, which is given by the following relation:

$$(\alpha')^{-1}_4(M'_{GUT}) = (\alpha')^{-1}_2(M'_{GUT}).$$ \hspace{1cm} (51)

At the mirror GUT scale $M'_{GUT}$ we obtain the $SU(6)'$-unification:

$$SU(4)'_C \times SU(2)'_L \times U(1)'_Z \rightarrow SU(6)'.$$ \hspace{1cm} (52)

We see that $U(1)'_Z$ also meets $SU(4)'_C$ and $SU(2)'_L$ at the same GUT scale.

For $\mu \geq M'_{GUT}$ we must consider the running of $(\alpha')^{-1}_6(\mu)$ up to the superGUT scale $M'_{SGUT} = M'_{E6}$:

$$(\alpha')^{-1}_6(\mu) = (\alpha')^{-1}_6(M'_{GUT}) + \frac{b_6}{2\pi} \ln \frac{\mu}{M'_{GUT}} = (\alpha')^{-1}_6(M'_{GUT}) + \frac{11}{2\pi} \ln \frac{\mu}{M'_{GUT}},$$ \hspace{1cm} (53)

where we have used the result

$$b_6 = 11,$$ \hspace{1cm} (54)

obtained from Eq. (17) for $N = 6, N_f = 6, N_{vector} = 2$. Here we assumed the existence of only minimal number of the Higgs fields, namely $h + \bar{h}$, belonging to the fundamental representation $\mathbf{6}$ of the $SU(6)'$ group.

Now it is obvious that we must find some unknown in the O-world symmetry group $SU(2)'_Z$, which must help us to reach the desirable $E_6$-unification at the superGUT scale $M'_{SGUT}$:

$$SU(6)' \times SU(2)'_Z \rightarrow E'_6.$$ \hspace{1cm} (55)

In the present investigation we assume that $E_6$-unifications restore the breakdown of the mirror parity MP and $M'_{SGUT} = M_{SGUT} = M_{E6}$. Then the scale $M_{SGUT}$ of the $E_6 \times E'_6$-unification is given by the following intersection:

$$\alpha^{-1}_{10}(M_{SGUT}) = (\alpha')^{-1}_6(M_{SGUT}).$$ \hspace{1cm} (56)
Finally, one can envision the following symmetry breaking chain in the M-world:

\[ E_6' \rightarrow SU(6)' \times SU(2)'_Z \rightarrow SU(4)'_C \times SU(2)'_L \times SU(2)'_Z \times U(1)'_Z \]
\[ \rightarrow SU(3)'_C \times SU(2)'_L \times SU(2)'_Z \times U(1)'_X \times U(1)'_Z \]
\[ \rightarrow SU(3)'_C \times SU(2)'_L \times SU(2)'_Z \times U(1)'_Y . \]

Here it is quite necessary to understand if there exists the group \( SU(2)'_Z \) in the mirror world. What it could be?

5 The mirror \( SU(2)'_Z \) and cosmological constant

A candidate for such a gauge group \( SU(2)'_Z \), which could be unified with \( SU(6)' \) in the mirror world (see Eq. (55)), was suggested in Refs. [56–61].

We shall consider now the possibilities presented by Ref. [56] and recently by Refs. [57–61] (see also [62, 63]), where a new gauge group \( SU(2)'_Z \) was aimed to introduce a new dynamical scale \( \Lambda_Z \sim 10^{-3} \text{ eV} \), which is consistent with present measurements of cosmological constant [64–70]: a total vacuum energy density of our universe (named cosmological constant) is equal to the following value:

\[ \rho_{\text{vac}} \approx (3 \times 10^{-3} \text{ eV})^4 \quad (57) \]

(see also Ref. [71] for details).

In the model [57–61] an axion-like scalar field \( a_Z \) (‘acceleron’) is related with \( SU(2)'_Z \), which exists in the mirror world. The effective potential considered as a function of the norm of this scalar field has a minimum (so called ‘false’ vacuum) just with vacuum energy density [57].

In the present investigation we assume that the analogous gauge group \( SU(2)'_Z \) leads to our \( E_6' \)-unification in the mirror world. Then a low energy symmetry group of the M-sector is:

\[ G' = SU(3)'_C \times SU(2)'_L \times SU(2)'_Z \times U(1)'_Y = G'_{SM} \times SU(2)'_Z . \quad (58) \]

A new asymptotically free gauge group \( SU(2)'_Z \) gives the running of its inverse fine structure constant \( (\alpha')^{-1}_{2Z}(\mu) \), which has to grow from the extremely low energy scale \( \Lambda_Z \sim 10^{-3} \text{ eV} \) up to the supersymmetric scale \( M_{\text{SUSY}} \) and then continue to run (in our model -it does not change, see Figs. 3(a) and 4(a)) up to the superGUT scale \( M'_{\text{SGUT}} = M_{E6} \sim 10^{18} \text{ GeV} \). But this running essentially depends on the particle content of \( SU(2)'_Z \).

5.1 Particle content of \( SU(2)'_Z \) gauge group

For the case \( \zeta = 30 \) we have obtained the content of \( SU(2)'_Z \) particles, which is not identical with the one given by Refs. [56–61]. The reason of our choice was to obtain the correct running of \( (\alpha')^{-1}_{2Z}(\mu) \), which leads to the scale \( \Lambda_Z \sim 10^{-3} \text{ eV} \) and simultaneously is consistent with our specific description of the running of all the inverse gauge coupling constants in the ordinary and mirror worlds with broken mirror parity.
Considering the evolutions (36), we have used the following equations for slopes $b_{2Z}$ [46, 47] (see also [41]):

$$b_{2Z} = \frac{22}{3} - \frac{4}{3} N_g - \frac{8}{3} N_F - \frac{1}{6} N_{vec} - \frac{2}{3} N_{adj} - \ldots$$  (59)

— for non-supersymmetric $SU(2)$, and

$$b_{2Z}^{SUSY} = 6 - 2N_g - 4N_F - \frac{1}{2} N_{SUSY}^{vec} - 2N_{adj} - \ldots$$  (60)

— for supersymmetric $SU(2)$.

In Eqs. (59) and (60) we have: the number of fermion doublets $N_g$, the number of fermion triplets $N_F$, the number of scalar doublets $N_{vec}$ with $N_{SUSY}^{vec} = 2N_{vec}$ and the number of triplet scalars $N_{adj}$.

Only the following slopes are consistent with our description of the O- and M-sectors, which can give the correct scale $\Lambda_Z \sim 10^{-3}$ eV:

$$b_{2Z} = \frac{13}{3} \approx 4.33 \quad \text{and} \quad b_{2Z}^{SUSY} = 0.$$  (61)

Then the particle content of $SU(2)'_Z$ is as follows:

- two doublets of fermions $\psi_i^{(Z)}$ and two doublets of ‘messenger’ scalar fields $\phi_i^{(Z)}$ with $i = 1, 2$ (here $N_g = 2$, $N_F = 0$, $N_{vec} = 2$ and $N_{SUSY}^{vec} = 4$),

or

- one triplet of fermions $\psi_f^{(Z)}$ with $f = 1, 2, 3$, which are singlets under the SM, and two doublets of ‘messenger’ scalar fields $\phi_i^{(Z)}$ with $i = 1, 2$ (in this case $N_g = 0$, $N_F = 1$, $N_{vec} = 2$ and $N_{SUSY}^{vec} = 4$).

- We also consider a complex singlet scalar field $\varphi_Z$:

$$\varphi_Z = (1, 1, 1, 0)$$  (62)

under the symmetry group $G' = SU(3)'_C \times SU(2)'_L \times SU(2)'_Z \times U(1)'_Y$.

The so called ‘messenger’ fields $\phi^{(Z)}$ carry quantum numbers of both the SM' and $SU(2)'_Z$ groups. They have Yukawa couplings with SM' leptons and fermions $\psi^{(Z)}$.

All the SM' particles are assumed to be singlets under $SU(2)'_Z$.

As a result, we obtain the following evolutions:

(i) for the region $\mu \leq M_{SUSY}'$:

$$\alpha_{2Z}^{-1}(\mu) = \alpha_{2Z}^{-1}(M_{t}') + \frac{b_{2Z}}{2\pi} \ln \frac{\mu}{M_{t}'} \approx \frac{b_{2Z}}{2\pi} \ln \frac{\mu}{\Lambda_Z},$$  (63)

(ii) and for the region $M_{SUSY}' \leq \mu \leq M_{SGUT}'$:

$$\alpha_{2Z}^{-1}(\mu) = \alpha_{2Z}^{-1}(M_{SUSY}') + \frac{b_{2Z}^{SUSY}}{2\pi} \ln \frac{\mu}{M_{SUSY}'}.$$  (64)
Also we have the following relation:

\[ \alpha'^{-1}_{2Z}(M'_{SGUT}) = M_{SGUT} = \alpha_{E6}^{-1} \]  \hspace{1cm} (65)

According to Eq. (61), we have:

(i) for the region \( \mu \leq M'_{SUSY} \):

\[ \alpha'^{-1}_{2Z}(\mu) = \alpha'^{-1}_{2Z}(M'_t) + \frac{13}{6\pi} \ln \frac{\mu}{M'_t} \approx \frac{13}{6\pi} \ln \frac{\mu}{\Lambda_Z}, \]  \hspace{1cm} (66)

(ii) and in the region \( M'_{SUSY} \leq \mu \leq M'_{SGUT} \) the evolution \( \alpha'^{-1}_{2Z}(\mu) \) is unchanged:

\[ \alpha'^{-1}_{2Z}(\mu) = \alpha'^{-1}_{2Z}(M_{SGUT}), \]  \hspace{1cm} (67)

or

\[ \alpha'^{-1}_{2Z}(M'_{SUSY}) = \alpha'^{-1}_{2Z}(M_{SGUT}) = \alpha_{E6}^{-1}. \]  \hspace{1cm} (68)

In Figs. 3(a,b) and 4(a,b) we have shown all evolutions in the mirror world:

Figs. 3(a,b) are given for SUSY breaking scale \( M'_{SUSY} = 10 \) TeV and mirror seesaw scale \( M'_R = 1.44 \cdot 10^{17} \) GeV; \( \zeta = 10; \) \( M_{SGUT} \approx 2.4 \cdot 10^{17} \) GeV and \( \alpha_{SGUT}^{-1} \approx 26.06. \)

Figs. 4(a,b) correspond to SUSY breaking scale \( M'_{SUSY} = 300 \) TeV and mirror seesaw scale \( M'_R = 2.25 \cdot 10^{17} \) GeV; \( \zeta = 30; \) \( M_{SGUT} \approx 6.96 \cdot 10^{17} \) GeV and \( \alpha_{SGUT}^{-1} \approx 27.64. \)

The total pictures of the evolutions in the O- and M-worlds simultaneously are presented in Figs. 5(a,b) and 6(a,b) for the cases \( M_{SUSY} = 1 \) and 10 TeV, \( M_R = 1.25 \cdot 10^{15} \) GeV and \( M'_R = 2.5 \cdot 10^{14} \) GeV, \( \zeta = 10 \) and \( \zeta = 30 \), respectively. It is obvious that respectively \( M'_{SUSY} = 10 \) and 300 TeV; \( M'_R = 1.44 \cdot 10^{17} \) GeV and \( M'_R = 2.25 \cdot 10^{17} \) GeV. Here \( M_{SGUT} \approx 2.4 \cdot 10^{17} \) GeV and \( \alpha_{SGUT}^{-1} \approx 26.06 - \) for Figs. 5(a,b), and \( M_{SGUT} \approx 6.96 \cdot 10^{17} \) GeV and \( \alpha_{SGUT}^{-1} \approx 27.64 - \) for Figs. 6(a,b).

All parameters of these evolutions are presented in Table 1.

### 5.2 The axion potential

As it was shown in Refs. [57–61], the Lagrangian corresponding to the group of symmetry \( [58] \) exhibits a \( U(1)^Z_A \) global symmetry.

A singlet complex scalar field \( \varphi_Z \) was introduced in theory with aim to reproduce a model of Peccei-Quinn (PQ) (well-known in QCD) [72]. Then the potential:

\[ V = \frac{\lambda}{4}(\varphi_Z^+ \varphi_Z - v_Z^2)^2 \]  \hspace{1cm} (69)

gives the VEV for \( \varphi_Z \):

\[ \langle \varphi_Z \rangle = v_Z. \]  \hspace{1cm} (70)

Representing the field \( \varphi_Z \) as follows:

\[ \varphi_Z = v_Z \exp(ia_Z/v_Z) + \sigma_Z, \]  \hspace{1cm} (71)
we have:
\[ \langle a_Z \rangle = \langle \sigma_Z \rangle = 0. \]  
(72)

A boson \( a_Z \) (the imaginary part of a singlet scalar field \( \varphi_Z \)) is an axion and could be a massless Nambu-Goldstone (NG) boson if the \( U(1)^{(Z)}_A \) symmetry is not spontaneously broken. However, the spontaneous breakdown of the global \( U(1)^{(Z)}_A \) by \( SU(2)^{(Z)}_Z \) instantons gives masses to fermions \( \psi^{(Z)} \) and inverts \( a_Z \) into a pseudo Nambu-Goldstone boson (PNGB) with a mass squared [57–61]:
\[ m^2_a \sim \Lambda^3_Z/v_Z \sim 10^{-30} \text{ GeV}^2. \]  
(73)

Then the field \( \varphi_Z \) becomes:
\[ \varphi_Z(x) = \exp(ia_Z/v_Z)(v_Z + \sigma(x)) \approx v_Z + \sigma(x) + ia_Z(x). \]  
(74)

Here the field \( \sigma \) is an inflaton.

The axion potential is given by the PQ model [72] and has (for small \( a_Z \)) the following expression:
\[ V_{\text{axion}} \approx \frac{\lambda}{4}(\varphi_Z^+ \varphi_Z - v_Z^2)^2 + K|\varphi| \cos(a_Z/v_Z), \]  
(75)

where \( K \) is a positive constant.

It is well-known that this potential exhibits two degenerate minima at \( \langle a_Z \rangle = 0 \) and at \( \langle a_Z \rangle = 2\pi v_Z \) with the potential barrier existing between them (see Fig. 7).

The minimum of the above-mentioned potential at \( \langle a_Z \rangle = 0 \) corresponds to the ‘true’ vacuum, while the minimum at \( \langle a_Z \rangle = 2\pi v_Z \) is called the ‘false’ vacuum. Such properties of the present axion lead to the ‘quintessence’ model of our expanding universe. By this reason, the axion \( a_Z \) could be called an ‘acceleron’.

6 Quintessence model of the universe with broken mirror parity

Recent models of the Dark Energy (DE) and Dark Matter (DM) are based on measurements in contemporary cosmology [64–70]. Supernovae observations at redshifts \( 1.25 \leq z \leq 1.7 \) by the Supernovae Legacy Survey (SNLS), cosmic microwave background (CMB), cluster data and baryon acoustic oscillations by the Sloan Digital Sky Survey (SDSS) fit the equation of state for DE: \( w = p/\rho \) with constant \( w \), which is given by Ref. [70]: \( w = -1.023 \pm 0.090 \pm 0.054 \). The value \( w \approx -1 \) is consistent with the present quintessence model of accelerating universe [73–78] (see also reviews [79, 80] and recent Ref. [81]), dominated by a tiny cosmological constant and Cold Dark Matter (CDM) – this is a so called \( \Lambda CDM \) scenario [62].

Here we present the quintessence scenario, which was developed in Refs. [57–61] in connection with the existence of a new gauge group \( SU(2)^{(Z)}_Z \). We have an analogous situation in our model, inspired by the broken mirror parity, although the details of our theory are different. However, in both cases the cosmological implications of suggested scenarios lead to the \( \Lambda CDM \) model.
6.1 Dark energy and cosmological constant

For the ratios of densities $\Omega_X = \rho_X/\rho_c$, where $\rho_c$ is the critical energy density, cosmological measurements gave: $\Omega_B \sim 4\%$ for baryons (visible and dark), $\Omega_{DM} \sim 23\%$ for non-baryonic DM, and $\Omega_{DE} \sim 73\%$ for the mysterious DE, which is responsible for the acceleration of our universe.

In Section 5 we have considered that a cosmological constant (CC) is given by the value $CC = \rho_{vac} \approx (3 \times 10^{-3} \text{ eV})^4$ [64–70]. The main assumption of Refs. [60, 61] is the following idea: the universe is trapped in the false vacuum with CC given by (57), but at the end it must decay into the true vacuum with vanishing CC. The true Electroweak vacuum would have its vacuum energy density $CC = \rho_{vac} = 0$. Such a scenario exists in the model with Multiple Point Principle (MPP) [82–86] (see also reviews [87, 88] and references there). A non-zero (nevertheless tiny) CC would be associated only with a false vacuum. Why CC is zero in a true vacuum? It is a non-trivial problem, but Refs. [89, 90] try to give a solution of this problem.

It follows from the last Section 5 that the potential $V(a_Z)$ determines the origin of DE. As it was shown in Ref. [59], we see that when the temperature of the universe $T$ is high: $T \gg \Lambda_Z$, then the axion potential is flat because the effects of the SU(2)$_Z$ instantons are negligible for such temperatures.

When the temperature begins to decrease, the universe gets trapped in the false vacuum. At $T \sim \Lambda_Z$ the true vacuum at $\langle a_Z \rangle = 0$ has zero energy density (cosmological constant CC), and the barrier between vacua is higher. The difference in energy density between the true and false vacua is now $\Lambda_Z^4$. The universe is still trapped in the false vacuum with $CC = \rho_{vac} = \Lambda_Z^4$.

The first order phase transition to the true vacuum is provoked by the bubble nucleation. In fact, the universe lives in the false vacuum for a very long time. When the universe is trapped into the false vacuum at $\langle a_Z \rangle = 2\pi v_Z$, the deceleration stops and acceleration begins at $\dot{a}_Z = 0$, then $\ddot{a}_Z = 0$ and $w(a_Z) = -1$. The total energy density of the universe is dominated by the energy density of the false vacuum, and our universe undergoes an exponential expansion.

The universe trends to get the true vacuum, which has zero $CC$, but will get it only in a very distant future (by our estimate, in $\sim 10$ billion years), when the phase transition is completed.

In Refs. [56–61] (and in our model) $a_Z$ plays the role of the ‘acceleron’, and a scalar boson $\sigma$, partner of the acceleron, plays the role of the ‘inflaton’ in the “low scale inflationary scenario” [62].

6.2 Dark matter

The existence of DM (non-luminous and non-absorbing matter) in the universe is now well established. Candidates for non-baryonic DM must be particles, which are stable on cosmological time scales. They must interact very weakly with electromagnetic radiation. Also they must have the right relic density. These candidates can be black holes, axions, and weakly interacting massive particles (WIMPs). In supersymmetric models WIMP candidates are the lightest superparticles. The most known WIMP is the lightest
neutralino. WIMPs could be photino, higgsino, or bino.

In our model, as in Refs. [56–61], mirror particles could be considered as candidates of DM: mirror world interacts with ordinary one only by gravity and really is a non-luminous and non-absorbing dark matter. Fermions $\psi^{(Z)}_i$ introduced in Subsection 5.1, could be considered as candidates for HDM (hot dark matter), and their composites (‘hadrons’ of $SU(2)_Z$) could play a role of the WIMPs in CDM. Investigating DM, it is possible to search and study various signals such as: $\psi^{(Z)} + e \rightarrow \psi^{(Z)} + e$, or $\psi^{(Z)} + N \rightarrow \psi^{(Z)} + N$, where $e$ is an electron and $N$ is a nucleon.

The detection of mirror particles: mirror quarks, leptons, Higgs bosons, etc., could be performed at future colliders such as LHC. Also the ‘messenger’ scalar boson $\phi^{(Z)}_i$ can be produced at LHC. Then the decay: $\phi^{(Z)}_i \rightarrow \overline{\psi}^{(Z)}_i + l$, where $l$ stands for the SM lepton, can be investigated with $\psi^{(Z)}$ as missing energies.

Leptogenesis and inflationary model also can be considered as implications of our mirror physics. The full investigation is beyond this paper.

7 Conclusions and outlook

We have discussed in this paper cosmological implications of the parallel ordinary and mirror worlds in the case when the mirror parity (MP) is not conserved. In our investigation the breakdown of MP is characterized by the parameter $\zeta = v'/v$, where $v'$ and $v$ are the VEVs of the Higgs bosons – Electroweak scales – in the M- and O-worlds, respectively. During our numerical calculations, we have used the values $\zeta = 10$ and $\zeta = 30$, in accordance with a cosmological estimate obtained in Refs. [43–45]. We have shown that, as a result of the MP-breaking, the evolutions of fine structure constants in O- and M-worlds are not identical, and the extensions of the SM and SM’ are quite different if we want to have the same $E_6$-unification in both worlds, what is predicted by Superstring theory [31]. We have assumed that the following chain of symmetry groups exists in the ordinary world:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \times U(1)_Z$$

$$\rightarrow SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_Z \rightarrow SO(10) \times U(1)_Z \rightarrow E_6.$$  

Here we have chosen a chain, which leads to the asymptotically free $E_6$ unification, what is not always fulfilled (see Ref. [41]).

Then in the mirror M-world a simple logic dictates to consider the following chain:

$$SU(3)'_C \times SU(2)'_L \times SU(2)'_Z \times U(1)'_Y$$

$$\rightarrow SU(3)'_C \times SU(2)'_L \times SU(2)'_Z \times U(1)'_X \times U(1)'_Z$$

$$\rightarrow SU(4)'_C \times SU(2)'_L \times SU(2)'_Z \times U(1)'_Z$$

$$\rightarrow SU(6)' \times SU(2)'_Z \rightarrow E_6'.$$

The comparison of both evolutions in the ordinary and mirror worlds is given in Figs. 5(a,b) and Figs. 6(a,b), where we have presented the running of all fine structure constants.
Here the SM (SM′) is extended by MSSM (MSSM′), and then we see different evolutions. Figs. 5(a,b) correspond to the SUSY breaking scales $M_{\text{SU} \text{SY}} = 1 \text{ TeV}$ and $M'_{\text{SU} \text{SY}} = 10 \text{ TeV}$, while Figs. 6(a,b) are presented for $M_{\text{SU} \text{SY}} = 10 \text{ TeV}$ and $M'_{\text{SU} \text{SY}} = 300 \text{ TeV}$, according to the MP-breaking parameters $\zeta = 10$ and $\zeta = 30$, respectively. We see $M_{\text{SGUT}} \approx 2.4 \cdot 10^{17} \text{ GeV}$ and $\alpha^{-1}_{\text{SGUT}} \approx 26.06$ – for Figs. 5(a,b), and $M_{\text{SGUT}} \approx 6.96 \cdot 10^{17} \text{ GeV}$ and $\alpha^{-1}_{\text{SGUT}} \approx 27.64$ – for Figs. 6(a,b). It is quite significant to emphasize that in these cases mirror right-handed neutrinos appear only at the scale $\sim 10^{17} \text{ GeV}$, close to the $E'_6$ unification.

The (super)grand unification $E'_6$ is based on the group $E'_6 \supset SU(6)' \times SU(2)_Z'$, and the presence of a new unbroken gauge group $SU(2)_Z'$ in the mirror world gives significant consequences for cosmology: it explains the ‘quintessence’ model of our accelerating universe (see Refs. [56–61]). As we have shown in Figs. 3(a,b) and 4(a,b) for the mirror world (and also in Figs. 5(a,b) and 6(a,b) for both worlds), the $SU(2)_Z'$ gauge coupling, presented by $\alpha^{-1}_{22}(\mu)$, takes its initial value at the superGUT scale $\sim 10^{18} \text{ GeV}$: $\alpha^{-1}_{22}(M'_{\text{SGUT}}) = \alpha^{-1}_{E'_6}$, and then runs down to very low energies, leading to the extremely strong coupling constant at the scale $\Lambda_Z \sim 10^{-3} \text{ eV}$. At this scale $SU(2)_Z'$ instantons induce a potential for an axion-like scalar particle $a_Z$, which can be called ‘acceleron’, because it gives the value $w = -1$ and leads to the acceleration of our universe. The existence of the scale $\Lambda_Z \sim 10^{-3} \text{ eV}$ explains the value of cosmological constant: $CC \approx (3 \times 10^{-3} \text{ eV})^4$, which is given by cosmological measurements [64–70].

It was assumed in Refs. [57–61] that at present time our universe exists in the ‘false’ vacuum given by the axion potential. The universe will live there for a long time and its $CC$ (measured in cosmology) is tiny, but nonzero. However, at the end our universe will jump into the ‘true’ vacuum and will get a zero $CC$. But this problem is not trivially solved, and at present we have only a hypothesis.

Now it is worth the reader’s attention to observe that in the mirror world we have three scales (presumably corresponding to the three MPP vacua [87]):

$$\Lambda_1 = \Lambda_Z \sim 10^{-12} \text{ GeV}, \quad \Lambda_2 = \Lambda_{\text{EW}} \sim 10^3 \text{ GeV} \quad \text{and} \quad \Lambda_3 = \Lambda_{\text{SGUT}} \sim 10^{18} \text{ GeV}.$$  

It is not difficult to notice that they obey the following interesting relation:

$$\Lambda_1 \cdot \Lambda_3 \approx \Lambda_2^2.$$  

In our model of the universe with broken mirror parity we have the following particle content of the group $SU(2)_Z'$, which does not coincide with Refs. [56–61]:

- two doublets of fermions $\psi_i^{(Z)}$ ($i = 1, 2$), or a triplet of fermions $\psi_f^{(Z)}$ ($f = 1, 2, 3$);
- two doublets of scalar fields $\phi_i^{(Z)}$ ($i = 1, 2$).

Also as in Refs. [56–61], we have considered a complex singlet scalar field $\varphi_Z$, which produces ‘acceleron’ $a_Z$ and ‘inflaton’ $\sigma_Z$ and gives an analogous ‘quintessence’ model of our universe.

Unfortunately, we cannot predict exactly the scales $M_{SU \text{SY}}$ and $M_R$ presented in our Figs. 1-6. The quantitative description of the model depends on these scales. Nevertheless,
we hope that a qualitative scenario for the evolution of our universe, given in Refs. [56–61]
and in the present paper, is valid.

In Section 6 we have discussed a possibility to consider the fermions $\psi_i^{(Z)}$ as candidates for the WIMP CDM. Searching DM, it is possible to observe and study various signals of these particles.

Also it is quite significant that this investigation opens the possibility to fix a grand unification group ($E_6$?) from cosmology.

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| Sl. No. | $M_{SUSY}$ in TeV | $M_R$ in GeV | $\zeta$ | $M'_{SUSY}$ in TeV | $M'_R$ in GeV | $M_{SGUT}$ in GeV | $\alpha_{SGUT}^{-1}$ |
|--------|------------------|--------------|--------|-------------------|--------------|------------------|-------------------|
| 1      | 1                | $1.25 \cdot 10^{15}$ | 10     | 10                | $1.44 \cdot 10^{17}$ | $2.4 \cdot 10^{17}$ | 26.06             |
| 2      | 10               | $2.5 \cdot 10^{14}$ | 30     | 300               | $2.25 \cdot 10^{17}$ | $6.96 \cdot 10^{17}$ | 27.64             |
Fig. 1: Figure (a) presents the running of the inverse coupling constants $\alpha^{-1}_i(x)$ in the ordinary world from the Standard Model up to the $E_6$ unification for SUSY breaking scale $M_{SUSY} = 1$ TeV and seesaw scale $M_R = 1.25 \cdot 10^{15}$ GeV. This case gives: $M_{SGUT} \approx 2.4 \cdot 10^{17}$ GeV and $\alpha^{-1}_{SGUT} \approx 26.06$. (b) is same as (a), but zoomed in the scale region $10^{15.8}$ GeV up to the $E_6$ unification to show the details.
Fig. 2: Figure (a) presents the running of the inverse coupling constants $\alpha_i^{-1}(x)$ in the ordinary world from the Standard Model up to the $E_6$ unification for SUSY breaking scale $M_{SUSY} = 10$ TeV and seesaw scale $M_R = 2.5 \cdot 10^{14}$ GeV. This case gives: $M_{SGUT} \approx 6.96 \cdot 10^{17}$ GeV and $\alpha_{SGUT}^{-1} \approx 27.64$. (b) is same as (a), but zoomed in the scale region $10^{15}$ GeV up to the $E_6$ unification to show the details.
Fig. 3: Figure (a) presents the running of the inverse coupling constants $\alpha_i^{-1}(x)$ in the mirror world from the Standard Model up to the $E_6$ unification for SUSY breaking scale $M'_{SUSY} = 10 \, \text{TeV}$ and mirror seesaw scale $M'_R = 1.44 \cdot 10^{17} \, \text{GeV}; \ \zeta = 10$. This case gives: $M_{SGUT} \approx 2.4 \cdot 10^{17} \, \text{GeV}$ and $\alpha_{SGUT}^{-1} \approx 26.06$. (b) is same as (a), but zoomed in the scale region $10^{15.8} \, \text{GeV}$ up to the $E_6$ unification to show the details.
Fig. 4: Figure (a) presents the running of the inverse coupling constants $\alpha_i^{-1}(x)$ in the mirror world from the Standard Model up to the $E_6$ unification for SUSY breaking scale $M'_{\text{SUSY}} = 300$ TeV and mirror seesaw scale $M'_{R} = 2.25 \cdot 10^{17}$ GeV; $\zeta = 30$. This case gives: $M_{\text{SGUT}} \approx 6.96 \cdot 10^{17}$ GeV and $\alpha_{\text{SGUT}}^{-1} \approx 27.64$. (b) is same as (a), but zoomed in the scale region $10^{15}$ GeV up to the $E_6$ unification to show the details.
Fig. 5: In (a) the running of the inverse coupling constants $\alpha_{-1}^i(x)$ in both ordinary and mirror worlds with broken mirror parity from the Standard Model up to the $E_6$ unification for SUSY breaking scales $M_{SUSY} = 1$ TeV, $M'_{SUSY} = 10$ TeV and seesaw scales $M_R = 1.25 \cdot 10^{15}$ GeV, $M'_R = 1.44 \cdot 10^{17}$ GeV; $\zeta = 10$. This case gives: $M_{SGUT} \approx 2.4 \cdot 10^{17}$ GeV and $\alpha_{SGUT}^{-1} \approx 26.06$. (b) is same as (a), but zoomed in the scale region $10^{15.8}$ GeV up to the $E_6$ unification to show the details.
Fig. 6: In (a) the running of the inverse coupling constants $\alpha_i^{-1}(x)$ in both ordinary and mirror worlds with broken mirror parity from the Standard Model up to the $E_6$ unification for SUSY breaking scales $M_{SUSY} = 10$ TeV, $M'_{SUSY} = 300$ TeV and seesaw scales $M_R = 2.5 \cdot 10^{14}$ GeV, $M'_R = 2.25 \cdot 10^{17}$ GeV; $\zeta = 30$. This case gives: $M_{SGUT} \approx 6.96 \cdot 10^{17}$ GeV and $\alpha_{SGUT}^{-1} \approx 27.64$. (b) is same as (a), but zoomed in the scale region $10^{15}$ GeV up to the $E_6$ unification to show the details.
Fig. 7: The axion potential $V$ as a function of $a = |a_Z|$. It shows the ‘true’ vacuum at $\langle a_Z \rangle = 0$ and the ‘false’ vacuum at $\langle a_Z \rangle = 2\pi v_Z$. 

\[ V \]

\[ a \]