Popper’s experiment, Copenhagen Interpretation and Nonlocality

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A thought experiment, proposed by Karl Popper, which has been experimentally realized recently, is critically examined. A basic flaw in Popper’s argument which has also been prevailing in subsequent debates, is pointed out. It is shown that Popper’s experiment can be understood easily within the Copenhagen interpretation of quantum mechanics. An alternate experiment, based on discrete variables, is proposed, which constitutes Popper’s test in a clearer way. It refutes the argument of absence of nonlocality in quantum mechanics.

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I. INTRODUCTION

Despite the tremendous success of quantum mechanics, since its inception, it is one theory which has been a subject of constant debate regarding its interpretation. One might venture to say that it is the most successful, but the least understood theory. “I look upon quantum mechanics with admiration and suspicion”, wrote Albert Einstein in a letter to P. Ehrenfest. The so called EPR argument, first posed by Einstein, Podolsky and Rosen (EPR) in 1930s[1], sums up the discomfort with the picture of the physical world that the quantum theory suggests. The EPR thought experiment, later reformulated in terms of spin-1/2 particles by Bohm, has been interpreted as showing, what Einstein called, “spooky action at a distance”. Bohr[2] tried to counter this argument saying that various definitions are tied to the experimental setup used and cannot be decoupled from it. The very act of measurement can influence the physical reality. The essence of Bohr’s argument constitutes the Copenhagen interpretation of quantum mechanics where the wavefunction of a multi-particle system is regarded as one, and disturbing any part of it, can disturb the whole system. In this view, a measurement on one particle can have a non-local influence on a spatially separated particle, even in the absence of any physical interaction. In the quantum mechanics lore, this has come to be known as nonlocality.

Twentieth century philosopher of science, Karl Popper believed that quantum formalism could be interpreted realistically. He proposed an experiment to demonstrate that a particle could have a precise position and momentum at the same time. For some reason, Popper’s thought experiment did not attract much attention from the physics community, but there has been a recent resurgence of debate on it[3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. New interest has also been generated by its actual realization by Kim and Shih[14], and also by claims that it proves the absence of quantum nonlocality[15].

In this paper, we critically analyze Popper’s thought experiment and its realization, and point out the flaws in the argument. We also propose, what we think is, a discrete version of Popper’s experiment. We believe, this thought experiment captures the essence of “Popper’s test”, and serves to clarify the issues regarding the Copenhagen interpretation and quantum nonlocality.

II. POPPER’S THOUGHT EXPERIMENT

Let us start by describing the thought experiment Popper proposed. Basically it consists of a source $S$ which can generate pairs of particles traveling to the left and to the right, which are entangled in the momentum space. This is to say that momentum along the $y$-direction of the two particles is entangled in such a way, so as to conserve the initial momentum at the source, which is zero. There are two slits, one each in the paths of the two particles. Behind the slits are sitting arrays of detectors which can detect the particles after they pass through the slits (see Fig. 1a).

Being entangled in the momentum space implies that in the absence of the two slits, if particle on the left is measured to have a momentum $p$, the particle on the right will necessarily be found to have a momentum $-p$. 

![FIG. 1: Schematic diagram of Popper’s thought experiment. (a) With both the slits, the particles are expected to show scatter in momentum. (b) Removing slit B, Popper believed, one could test the Copenhagen interpretation.](image-url)
One can imagine a state similar to the original EPR state \[ |\psi\rangle = \int e^{i y_1 p_1 / \hbar} e^{-i y_2 p_2 / \hbar} dp. \] As one can see, this state also implies that if particle on the left is detected at a distance \(y\) from the horizontal line, the particle on the right will necessarily be found at the same distance \(y\) from the horizontal line. In the presence of the slits, Popper argued, when the particles pass through the slits, they experience a large uncertainty in momentum. This results in a larger spread in the momentum, which will be show up as particle being detected even at positions which are away from the line connecting the source and the slit. This spread, because of a real slit is expected. A tacit assumption in Popper’s setup is that the initial spread in momentum of the two particles is not very large.

Popper then suggests that slit B be removed. In this situation, Popper argues that when particle 1 passes through slit A, it is localized in space, to within the width of the slit. If one believes in the Copenhagen interpretation of quantum mechanics, then one would think that when particle 1 is localized in space, particle 2 should also get localized in space. In fact, if we do this experiment without the slits, the correlation in the detected positions of particles 1 and 2, implies just this. This is the collapse postulate of quantum mechanics, for which no mechanism is given. And an act of measurement on particle 1, seems to have a spooky action on the particle 2. But Popper had something else in mind. He was not convinced by the correlation in the detected positions of the particles. He argued that if particle 2 actually experiences a localization in position, its subsequent evolution should show a larger spread in momentum. To be precise, just as much as there was when the real slit B was present. Popper had his own argument to suggest that if such an experiment is actually performed, no extra spread in momentum will be observed. This, he said, showed that Copenhagen interpretation doesn’t work.

### III. IS NONLOCALITY ABSENT?

Based on Popper’s thought experiment, an argument has been put forward by Unnikrishnan \[16\], which claims that there is no nonlocality in quantum mechanics. The argument is as follows. If there were an actual reduction of the state when the particle 1 went through slit A, particle 2 would get localized in a narrow region of space, and in the subsequent evolution, experience a greater spread in momentum. If no extra spread in the momentum of particle 2 is observed, it implies that there is no nonlocal effect of the measurement of particle 1 on particle 2. The tacit assumption here is that the correlation observed in the detected positions of particles 1 and 2, in the absence of the slits, could be explained in some other way, without invoking a nonlocal state reduction.

### IV. REALIZATION OF POPPER’S EXPERIMENT

Popper’s thought experiment was recently realized by Kim and Shih using an entangled two-photon source \[14\]. They used a modified geometry as demanded by the experimental arrangement. When both the slits A and B are present, they observed a significant spread in the momentum, seen as a scatter in the detected positions of the photons. When slit B is removed, particle 1 shows a spread in momentum, but particle 2 doesn’t show any spread. This is in agreement with what Popper had predicted. In fact, particle 2 is observed to lie in a region which is much narrower than the initial spread of the beam. But the question is, does it indicate that Copenhagen interpretation is flawed, or that nonlocality is absent? It should be mentioned here that there have been objections against Kim and Shih’s experiment, pointing out that due to the finite size of the source, the localization of the second particle is not perfect \[10\]. We will come back to this point at the end of the discussion.

### V. ANALYSIS OF THE EXPERIMENT

One problem with the original proposal is that the source is assumed to have a sharp momentum value, which would imply that the position of the source is uncertain by an amount dictated by the uncertainty principle. This was pointed out by Collet and Loudon \[6\] who concluded that due to this uncertainty, the experiment would not be able to test quantum mechanics. Similar arguments have been put forward by others, like Redhead \[8\]. This is a valid criticism, but it turns out that precisely this kind of setup is not crucial for Popper’s experiment. Now spontaneous parametric down conversion (SPDC) in nonlinear optical crystals can yield particles which are entangled in precisely the way required by Popper’s experiment \[14\]. The measured positions of such particles are correlated with nearly unit probability \[10\]. Although the use of finite size source has been a source of controversy \[10\], we address the question that if such particle pairs can be produced (maybe with a more extended source, or by some other means), what will be its consequence for Popper’s experiment. In order to follow the language of Popper’s original proposal, we will continue to use a point source in the subsequent discussion, but it should be kept in mind that the argument can be easily translated to the case of SPDC particle pairs coming from an extended source.

Let us try to analyze the experiment carefully and see what result one would expect within the Copenhagen interpretation of quantum mechanics. Popper argued that according to Copenhagen interpretation, when particle 1 passes through slit A, the wavefunction should get reduced to something which is localized within the width of the slit. But let us ask the question, when do we acquire the knowledge that the particle has passed through the
slit. The answer is, not until particle 1 has been detected by one of the detectors. To reinforce this point, let us assume that we had put an array of detectors next to slit A. In that situation, particle would either get detected by one of the detectors near slit A, or pass through the slit and get detected by the detectors behind the slit. So, we can have knowledge that the particle passed through the slit or not if, either it is detected by the detectors behind the slit, or the ones next to it. Now, in Popper’s experiment, we are only interested in the particles which have passed through slit A, and not in the ones that could not pass through, and are lost somewhere else. In this situation, we can only know that particle 1 passed through the slit, when one of the detectors behind the slit detects it. This is the point at which one has to invoke a measurement, and a reduction of the state, and not at the point when the particle reaches the slit. The detector behind slit A causes a reduction of the state.

If one agrees with this argument, then the analysis of both Kim and Shih [14] and Short [10], which talk of “localization” of the wavefunction at the slit, are questionable.

This is a fundamental flaw in Popper’s argument, which leads him to believe that Copenhagen interpretation doesn’t work. Let us now find out what happens on the right, that is, what does particle 2 do in this situation. Remember that particles 1 and 2 are entangled in momentum states. If particle 1 is found to have momentum \( p' \), then particle 2 will necessarily be found to have a momentum \(-p'\). EPR like states have a property that detected positions of particles are correlated. Although, the momentum spread is assumed to be not very large, we will assume that such a correlation is possible, and will explore its consequence. One can see that with this kind of entanglement, the angular directions of the two particles get correlated, which can also be experimentally verified.

Now for particle 1, only directions which lie approximately within a small angle (see Figure 2a) will allow it to pass through slit A. Other parts of the wavefunction will be blocked by the slit wall. This is a direct consequence of the entanglement in momentum, as detecting particle 2 on the right hand side, gives us the “which path” information about particle 1. For the same reason, if one were to carry out a double-slit interference experiment on particle 1, no interference would be seen.

Now, the interesting thing is that, because of entanglement, this part of the wavefunction also describes particle 2. If particle 1 passes through slit A, whatever evolution it goes through subsequently, does not affect the entangled part of the particle 2 wavefunction. Particle 1 may experience a spread in the wavefunction because of slit A, but the entanglement with particle 2 remains intact. As a result, particle 2 continues to evolve as it would have, if slit A were absent. When particle 1 is detected behind slit A, particle 2 will be detected at a position which lies within a narrow angle which corresponds to particle 1 passing through the region of slit A, but in the absence of the slit (see Fig 2b). This is so, because the part of the particle 1 wavefunction which passes through slit A, is entangled to only that particular part of the particle 2 wavefunction. This all happens with a certain probability. There is also a probability that particle 1 doesn’t enter slit A. In that case, particle 2 will be detected at other positions. As mentioned before, Popper’s experiment doesn’t consider this case, and we will not discuss it here.

This argument can be easily applied to the case of an extended source, as used in Kim and Shih’s experiment. Figure 2(c) gives a schematic representation of what happens in such the case where a combination of a BBO crystal and a converging lens is used. In the language of photons, only some \( k \) values in different regions of the BBO crystal will contribute to the photons passing through the regions of the slits. These directions, indicated by red lines, for the two particles are correlated. Of course there are lot of other \( k \) values, indicated in green, which correspond to particles not passing through the slits. The end result is that the parts of the wavefunctions of the particles indicated in pink, on both sides, are correlated with each other.

So, the conclusion of the preceding discussion is that if particle 1 is detected by any detector behind slit A, par-

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FIG. 2: Schematic diagram to understand what happen’s in Popper’s thought experiment. Due to entanglement in momentum, each direction of particle 1 is correlated to a direction of particle 2. (a) In the absence of slit A, there is a region which corresponds to particle 1 passing within a region where slit A could be present. (b) In the presence of slit A, particle 1 is scattered, but particle 2 travels unaffected. (c) With a broad source, like a BBO crystal, some directions (indicated in red) contribute to the particles passing through the region of the slits. Other directions (indicated in green) contribute to particle going to regions outside the slits.
ticle 2 will be found to have a position which lies within a narrow angle which corresponds to particle 1 passing through the region of slit A, in the absence of the slit. This is the conclusion of the Copenhagen interpretation. As one can see, this is in sharp contrast to what Popper had concluded regarding the Copenhagen interpretation.

This also appears to be in agreement with the experimental result of Kim and Shih[14]. However, if the finite size of the source indeed led to unsatisfactory correlation between the photon pairs, it might be an interesting exercise to repeat the experiment with a better source. In that case, Short’s work will predict a larger momentum spread in the second particle. On the other hand, we predict that a better correlation would not lead to an increase in the momentum spread of the second particle, as argued in preceding discussion.

VI. A DISCRETE VERSION OF POPPER’S TEST

One reason for which Popper’s experiment has been criticized is that it uses continuous variables, and it is not clear at what stage is invoking the uncertainty principle justified. As we saw in the preceding discussion, Popper’s experiment fails to achieve what Popper aimed at. The essence of Popper’s argument, at least as far as nonlocality and the Copenhagen interpretation are concerned, is not based on the precise variables he chose to study, namely position and momentum. Any two variables which do not commute with each other should serve the purpose, as localizing one would lead to spread in the other. This point has also been emphasized by Unnikirshnan[15]. In the following, we present a discrete model which, we believe, captures the essence of Popper’s test.

A. The model

Consider two spin-1 particles A and B, emitted from a source S such that A travels along negative y direction, and B travels along positive y direction. The particles start from a spin state which is entangled in such a way that if z-component of the A spin is found to have value +1, the z-component of B will necessarily have value −1. The initial spin state of the combined system can be written as

$$|\psi\rangle = \alpha|A_2; +1\rangle|B_2; -1\rangle + \beta|A_2; 0\rangle|B_2; 0\rangle + \alpha|A_2; -1\rangle|B_2; +1\rangle$$

(1)

where $|A_2; m\rangle$ and $|B_2; m\rangle$ represent the eigenstates of the z-component of the spins A and B respectively, with eigenvalue $m$. Also, the state $|\psi\rangle$ is normalized, so that $2\alpha^2 + \beta^2 = 1$. Here, the z-components of the spins can be thought as playing the role of momenta in the y direction of the two particles in Popper’s experiment. In that case, the x-component of the spin here can play the role of position of the two particles along y axis, in Popper’s experiment. The two components of the spin do not commute with each other, so localizing one in its eigenvalues, will necessarily cause a spread in the eigenvalues of the other. Thus, this spin system is completely analogous, in spirit, to the system of entangled particles, considered by Popper.

Next, we have to have a mechanism which is equivalent to localizing the particle 1, in Popper’s experiment, in space (what he wanted to achieve by putting a slit). To achieve this, we put a Stern-Gerlach field in the path of particle A, pointing along the z-axis, but inhomogeneous along the (say) x-axis. This will split the particle A into a superposition of three wave packets, spatially separated in the z direction, entangled with the three spin states $|A_2; +1\rangle$, $|A_2; 0\rangle$ and $|A_2; -1\rangle$. Then we put a detector $D_1$ in the path of this particle such that, it detects the central wave packet and localizes the x-component of spin A to the state $|A_2; 0\rangle$. This achieves, what slit A was supposed to achieve in Popper’s experiment, but actually never did, namely localizing the particle in position.

On the other side of the source, we can have a Stern-Gerlach field, in the path of particle B, pointing along the z-direction. This will split particle B into a superposition of three wave-packets, entangled with the three spin states $|B_2; +1\rangle$, $|B_2; 0\rangle$ and $|B_2; -1\rangle$. We have three detectors, $D_+\,\,D_0\,\,D_–$, to detect one component each of the z-component of spin B.

B. What do we expect?

Now, the z-components of spins A and B are entangled. So, it is indisputable that if one finds A in $|A_2; +1\rangle$ state, B would be found in $|B_2; -1\rangle$ state, and if one finds A in $|A_2; -1\rangle$ state, B would be found in $|B_2; +1\rangle$ state, and so on. Also, one can easily verify that if one measures the x component of spin A and finds it in the state $|A_2; 0\rangle$, one would find the x-component of spin B in the state $|B_2; 0\rangle$. But, as operators $B_x$ and $B_2$ do not commute, if one finds spin A in the state $|A_2; 0\rangle$, there should be a spread in the eigenstates of $B_z$. In Popper’s experiment, this would be equivalent to saying, that if particle 1 is localized in position, there should be a spread seen in the momentum of particle 2. This is what the Copenhagen interpretation predicts. At this stage, the equivalence of this experiment with Popper’s experiment is complete.
In addition, if one applies Unnikrishnan’s argument to the present model, detecting particle $A$ in the detector $D1$ leading to observation of a spread in the counts of particle $B$ in the three detectors, amounts to a nonlocal action at a distance.

C. “Doing” the thought experiment

Let us now carry out this thought experiment and see what we get. To start with, we first remove the detector and the Stern-Gerlach field from the path of particle $A$. We start from a spin state $|\psi\rangle$ where $\beta = \sqrt{0.9}$ and $\alpha = \sqrt{0.05}$, which has the following form:

$$|\psi\rangle = \sqrt{0.05}|A_z; +1\rangle|B_z; -1\rangle + \sqrt{0.9}|A_z; 0\rangle|B_z; 0\rangle$$

$$+ \sqrt{0.05}|A_z; -1\rangle|B_z; +1\rangle$$

(2)

It is trivial to see that the three detectors on the right will click in the following manner. The detector $D0$ will show 90 percent counts and the other two will have 5 percent each (see Fig. 4a).

Next we put the Stern-Gerlach field and the detector in the path of particle $A$. As in Popper’s experiment, we have to do coincident count between the detector on the left, and the detectors on the right. As we are measuring the $x$-component of the spin $A$ on the left, it would be natural to write the state $|\psi\rangle$ in terms of the eigenstates $|A_x; m\rangle$. In this form, the state $|\psi\rangle$ looks like

$$|\psi\rangle = |A_z; +1\rangle\left(\sqrt{0.05}\frac{|B_z; +1\rangle + \sqrt{0.9}\sqrt{2}|B_z; 0\rangle}{\sqrt{2}}\right)$$

$$+ \sqrt{0.05}\frac{|B_z; -1\rangle}{\sqrt{2}})$$

$$-\sqrt{0.05}|A_z; 0\rangle\left(\frac{1}{\sqrt{2}}|B_z; +1\rangle - \frac{1}{\sqrt{2}}|B_z; -1\rangle\right)$$

$$+ |A_z; -1\rangle\left(\sqrt{0.05}\frac{|B_z; +1\rangle - \sqrt{0.9}\sqrt{2}|B_z; 0\rangle}{\sqrt{2}}\right)$$

$$+ \sqrt{0.05}\frac{|B_z; -1\rangle}{\sqrt{2}})$$

(3)

It is clear from (3), that in a coincident count between the detector on the left and the detectors on the right, spin $A$ is found in state $|A_z; 0\rangle$ by choice, and spin $B$ ends up in the state $\frac{1}{\sqrt{2}}(|B_z; +1\rangle - |B_z; -1\rangle)$. This means that the detectors on the right will have 50 percent count each in the detectors $D+$ and $D-$, and no count in the detector $D0$! (see Fig. 4b) To start with, the $z$-component of spin $B$ was predominantly localized in the state $|B_z; 0\rangle$, as seen in the experiment without the detector and the field for particle $A$. Localizing the spin $A$ in the state $|A_z; 0\rangle$, results in a large scatter in the $z$-component of spin $B$.

In Popper’s experiment, this will be equivalent to saying that localizing particle 1 in space, leads to a scatter in the momentum of particle 2. Thus we reach the same conclusion that Popper said, Copenhagen interpretation would lead to. But the difference here is that, looking at 4 nobody would say that in actually doing this experiment, one would not see the result obtained here. This comes out just from the mathematics of quantum mechanics, without any interpretational difficulties, as in Popper’s original experiment.

VII. AFTERTHOUGHTS

Now that we are through with discussing both the models, let us find out why this model gives us something which Popper’s thought experiment was unable to. As discussed before, the main flaw in Popper’s argument was in assuming that the Copenhagen interpretation implies that the slit causes a reduction of the state immediately. The discrete model introduced here, relies on an actual detection of the particle, which causes a reduction...
of the state. This results in a spread in the z-component of spin \( B \). So, our conclusion is that the Copenhagen interpretation passes Popper’s test, only that, in our view, this discrete model is the right way to implement it.

Regarding the issue of nonlocality, we can pose the question whether this discrete model really shows “spooky action at a distance”. On the face of it, Fig. 4 seems to suggest that. Without the detector \( D \) 1, the detectors \( D_+, D_0 \) and \( D_- \) show counts primarily concentrated at \( D_0 \), and with coincident counting with \( D_1 \), the detector \( D_0 \) shows no counts and \( D_+ \) and \( D_- \) show 50 percent counts each. Knowledge of particle \( A \) being at \( D_1 \) seems to increase the dispersion in particle \( B \). But a careful look reveals that the overall count distribution for particle \( B \) is still the same as before. We are only choosing those counts that are coincident with \( D_1 \), and are throwing away the rest. So, a defender of locality can argue that we are not affecting particle \( B \) by doing something to particle \( A \). We just see a correlation in various events on the left and the right. In a sense, that is true - the nonlocality that we observe in this discrete model, is only at the level of observing correlations between the spatially separated particles, as is the case with any experiment done till date. Some people like to believe that just the presence of correlations does not prove nonlocality - correlations could be explained in some other way, like by introducing hidden variables, and that it only shows the “incompleteness” of the quantum formalism. This work doesn’t throw any new light on that debate. However, from Popper’s argument, and that of several others, it appears that they would have accepted nonlocality had they found a positive result of Popper’s test. In this light, the result of the present work can be considered a signature of nonlocality.

So, we have concluded that because the slit does not reduce the wavefunction, we should not expect any increase in the momentum spread of the second particle in Popper’s experiment. An immediate thought that comes to mind is that if slit \( A \) were replaced by a real narrow detector, do we expect to see an increase in the momentum spread of particle 2, and if so, what would that indicate? By analysing the discrete model, we have gained some clarity regarding Popper’s experiment. In the discrete model, we saw that the two peaks which appear in the coincident counting, were already present in the initial state as the two smaller peaks which were overshadowed by the bigger central peak. So, the “spread” we see in the coincident counts, was already present in the original state. In Popper’s experiment, if we expect to see a spread in momentum of particle 2 in the coincident counts with particle 1 behind slit \( A \), it has to be already present in the wavefunction. But what about Popper’s original argument that if particle 2 gets localized in a narrow region of space, it should have an increased momentum spread? The catch is that particle 2 can be localized in a narrow region of space only if the wavefunction is of the form \( \psi(y_1, y_2) = \int_{-\infty}^{\infty} e^{ipy_1}/\hbar e^{-ipy_2}/\hbar dp \), which means that momentum spread is infinite! If the momentum spread is already infinite, particle 2 cannot show any extra momentum spread. If, on the other hand, the momentum spread is finite, it will not be possible to precisely localize particle 2 in a narrow region of space, and subsequently one should not expect large momentum spread. Short’s criticism of Kim and Shih’s experiment correctly points out that due to the finite size of the source, the localization of particle 2 will not be precise, but doesn’t say what happens if the source is improved [10]. Others, including Sudbury [5] have recognized the problem with infinite momentum spread being already present in the initial state. So, what is fundamentally wrong in Popper’s proposal is the assumption that according to the Copenhagen interpretation, using entangled particles with a finite momentum spread, particle 2 can be localized precisely in position. With this knowledge one can be sure that in Popper’s experiment, no extra spread in momentum, which is not already present in the initial state, can ever be seen. This is independent of the way in which Popper’s experiment is realized.

With this understanding, we believe, the controversy regarding Popper’s experiment is fully resolved.

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