Improving Classical And Quantum Free-Space Communication By Adaptive optics
With Spatially Separated Beacon And Signal Beams
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(Dated: June 27, 2022)

Optical free-space communications, classical and quantum, are affected by atmospheric turbulence due to fluctuation of air density, pressure, and temperature. This turbulence induces a time-dependent inhomogeneous refractive index in air, distorting the wavefront of electromagnetic waves. Consequently, light beam spreads and wanders at the receiving end causing signal loss. In the case of measurement-device-independent quantum key distribution, random phase fluctuation also reduces the visibility of the Hong-Ou-Mandel interference, further lowering signal fidelity and thus the secure key rate. Adaptive optics (AO) technique has been used to compensate this kind of wavefront distortion in astronomical observations and classical optics communication. Frequency multiplexing AO method is also employed in several recent quantum communication experiments. Here we point out the deficiency of the frequency multiplexing method. More importantly, we introduce a spatial-multiplexing-based AO protocol with time delay between spatially separated pulses. Using phase screen simulation, we show that for low earth orbit satellite to ground communication, our method performs better than systems that only use frequency multiplexing. Actually, our method can used to increase the signal rate in both classical and quantum communication particularly in the case when the source and the receiver move relative to each other.

I. INTRODUCTION

Quantum communication channels are the fundamental blocks of building quantum networks, which can be used in a variety of tasks including distributed quantum computation, quantum-assisted imaging, blind quantum computation and quantum key distribution (QKD). To perform QKD over a distance of, say, at least 100 km, sending photons through free space is the most feasible method to date. A major advantage of using free-space channel is that it has a lower attenuation rate than optical fibers of the same length [1]. In fact, several pioneering demonstrations of long distance free space QKD, including ground-to-ground and satellite-to-ground ones, have been reported [2–4].

One of the major challenges of free-space QKD is atmospheric turbulence that distorts the photon source, thereby lowering the secure key rate [5, 6]. Clearly, this problem is not unique to quantum communication. It is also faced by a variety of tasks in classical optical communication and astronomical imaging. AO is a well-established way to solve this problem in the classical setting. The idea is that through feedback control of deformable mirrors and/or spatial light modulators, one can correct the distorted wavefront and hence increase the fidelity and the signal to noise (SNR) ratio of the signal [7]. No wonder the quantum communication community has begun to use AO technique in a few recent free-space quantum communications experiments [4, 8].

For free-space QKD, AO technologies increase the key rate by reducing the widening effect and spatial noise of the noise so that the system can get a higher yield or coupling efficiency [5]. Specifically, in daytime quantum communication, one has to use spatial filters to block background noise. In the absence of atmospheric turbulence, the size of the spatial filter can be set to be the diffraction limit of the light used. This allows 84% photons to pass through while most of the scattering photons are blocked [8]. However, atmospheric turbulence increases the size of the beam focus spot, reducing the photon transmission efficiency. Fortunately, AO system can compensate the transmission efficiency in daytime quantum communication without altering the filtering size [8]. In this regard, AO could well be an essential tool in high key rate free-space quantum communication systems.

Our discussion on the effects of turbulence on the optical signal so far is also valid for classical communication. We point out here that there is an additional challenge for quantum free-space communication to overcome. Since the light intensity of quantum signal used are generally much weaker than the classical ones, it is not effective to measure part of the quantum signal beam for AO correction. To solve this problem, some groups use the feedback signals generated by a beacon beam whose wavelength is differ from the signal beam [4, 8]. The drawback of this approach is that the wavefront distortion experienced by the two beacon and signal beams may differ. This difference generally increases with communication distance and can be significant when this distance is long. Alternatively, one may use time multiplexing by sending identical frequency beacon and signal beams along the same optical path at

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The main goal of the paper is to combat atmospheric turbulence for satellite-to-ground QKD. In particular, we focus on the case when the quantum source is located at a LEO satellite and the receiver is stationary on the ground. Here we solve this problem by using two set of artificial sources emitting at the same or nearly the same wavelength—a strong (pulsed) beacon source to perform effective AO correction and weak (pulsed) signal source(s) for the actual optical quantum communication. As the light sources are multiplexed spatially, they can be separated effectively by focusing the light beams. In addition, the performance of the AO system increases when the transmission distance increases for satellite-to-ground communications. This is because the overlapping region of the sources increases. The phase distortion information carried by the beacon light can better represent the phase distortion on the weak signal source. To optimize performance, we employ two more tricks. First, we put the pulsed beacon beam in front of the pulsed signal beam along the direction of motion of the satellite relative to the ground. Second, we fire each pulsed beacon beam shortly before firing the corresponding pulsed signal beam. In doing so, the beacon beam acts like a pioneer that probes the wavefront distortion of an optical path that will shortly be used by the signal beam. More importantly, if the two beams move sufficiently rapidly relative to the detector(s), the advanced pulsed beacon beam and the corresponding delayed pulsed signal beam can be made to travel along essentially the same optical path by carefully adjusting the delay time. Consequently, provided that the artificially imposed delay time between the two pulsed beams is shorter than the reflective index fluctuation timescale of the atmosphere, our technique can achieve the same level of AO correction for stationary sources without spatial multiplexing. We remark that this idea is equally effective in classical communication whenever the relative motion between the source and the receiver is non-negligible. Note that the present work is based on our recent patent application [9] and the master thesis of one of the authors [10].

We begin by showing our schematic design and reporting the simulation models used in Sec. II. Then in Sec. III, we determine the feasibility of the scheme for the case of static atmosphere and we compare the effectiveness of our scheme with the wavelength division multiplexing scheme. In Sec. IV, the spatial dependence of turbulence is discussed. The maximum and minimum separation distance are derived. In Sec. V, we study the performance of our scheme in dynamical atmosphere. Finally, we summarize our findings in Sec. VII.

II. METHODOLOGY

A. Design Of The Beacon And Signal Beams

The intuition of our improved method is that two physically nearby light beams of similar frequency pass through more or less the same air column at more or less the same time should be distorted in roughly the same way. Hence, a wavefront correction method based solely on the signal received by a wavefront sensing module that detects the beacon source beam should be able to correct both light beams at the same time with high fidelity. One may ask why we do not put the two beams together as time multiplexing technique should also work as long as the time interval of beam switching is much higher than the change in atmospheric wavefront distortion. Our answer is that although pure wavelength division multiplexing works as demonstrated by recent experiments [4, 8], we show in this paper that our technique can attain a better secret key rate for moving sources. Specifically, by placing the beacon beam ahead of the signal beam along the direction of motion of the sources relative to the receiver, our method can better correct wavefront distortion. In fact, in a lot of cases, it is possible to make the two beams to travel through almost the same optical path by carefully adjusting the delay time between these beams. Consequently, if the atmospheric turbulence fluctuation time scale is much shorter than this delay time, the level of AO correction should be equal to the situation of non-moving sources.

Figure 1 shows the schematic of spatial multiplexing AO system. It consists of two physically nearby sources as well as a wavefront sensing module that detects the beacon source beam plus a nearby signal detection module that detects the signal source beam(s). To reduce photon loss in long distance communication, each of the beam source is placed at the focus of telescope on the satellite so that the emitted light beam close to the source can be well approximated by traveling plane wave. Our hope is that with this spatial configuration the optical paths of the two set of sources with the same or almost the same wavelength should experience more or less the same wavefront distortion. The wavefront correction then goes as follows. The beacon detection module estimates the atmospheric distortion and generates feedback signals to the control system. Then the control system drives the actuators of the deformable mirror or the spatial light modulator in the AO system. This should correct the wavefront distortion of the beacon
beam as well as the possibly much weaker signal source beam simultaneously. Surely, in order to work, the two set of sources must be placed sufficiently close so that their optical paths are similar and at same time sufficiently far apart so that cross-talk between the beacon and signal source(s) due to effects such as diffraction and scattering is negligible.

The spatial configuration of our method is similar to the standard artificial guide star technique used in observational astronomy [11]. Note, however, that there are two major differences. First, all sources we used are artificial. Second, our beacon source is placed physically closed (and not just close in terms of apparent angular separation) to the signal source(s). We remark that this spatial configuration works not just for secure quantum communications. It is directly applicable to classical optical communication in free-space as well. And in this case, the intensity of the signal source(s) need not be low. In addition, our method is applicable to ground-based, air-to-ground as well as satellite-to-ground communications, stationary as well as moving sources relative to the sensing and detecting modules. Furthermore, a nice feature of our method is that the signal transmission rate will then be independent of the beacon source.

We now discuss our temporal configuration. For source(s) moving relative to the detector(s), we may further optimize the performance as follows. For simplicity, we assume that the sources and detector(s) are separately mounted on two rigid bodies. We place the beacon source ahead of the signal source(s) in the sense that \( \vec{v}_t \cdot \vec{L} \geq 0 \) at all time where \( \vec{v}_t \) and \( \vec{L} \) are the instantaneous tangential velocity vector of the source relative to the detector(s) and the instantaneous position vector of the signal source relative to the beacon source, respectively. More importantly, we set a delay time \( T_r \) between the beacon and signal sources and adjust it possibly dynamically so as to reduce the angular separation between the optical paths of the advanced beacon source and the delayed signal source. In particular, suppose \( \vec{v}_t \parallel \vec{L} \), then there is a delay time \( T_r \) after which the advanced beacon source and the delayed signal source will propagate along the same optical path. Consequently, these two sources will have the same wavefront distortion provided that \( T_r \) is much shorter than the atmospheric temporal fluctuation timescale. Under this condition, our method should greatly increase the fidelity of the signal source.

### B. Phase Screen Simulation

We use phase screen simulation to verify the effectiveness of this method. To simplify matter, we ignore the effects due to haze and cloud. We first describe our simulation of a static atmosphere with time-independent reflective index. Specifically, our results are obtained via AO corrections on a random sample of spatially inhomogeneous reflective indices in the atmosphere. (We are going to discuss the time dependence effect of atmospheric turbulence in Sec. V.) We use the PROPER library written in Matlab [12] to simulate the light propagation in this medium. This program
switches between angular spectrum algorithm and Fresnel approximation Fourier algorithm for near-field and far-field light propagation. It also provides routines for telescope and deformable mirror simulation. We also include diffraction effect of the telescope in the detection end in our simulation. Figure 2 shows the setup of the receiving end telescope and the AO system used in our simulation.

For the free-space channel, the setup is similar to that reported in Ref. [6]. We divide the atmosphere into two layers. The upper (lower) layer is modeled by 1 (10) phase screen(s). This is an economical choice on the number and distribution of phase screens for the atmospheric turbulence is much more serious in the lower atmosphere. The satellite altitude, layers division altitude, and receiver altitude used are 400 km, 20 km and 0 km, respectively. We fix the size of the phase screens to $1024 \times 1024$; and we repeat the simulation 1000 times in each scenario. The parameters used in the simulation are tabulated in Table I. Here we emphasize that the telescope parameters in Table I are based on a real telescope in Lulin Observatory [13]. We even include the diffraction effect of the supporting spider vanes in our simulation. As for the wavelength of the photon sources, we pick 780 nm wavelength because this wavelength has better spatial filtering strategies, geometric coupling, and size of focus spot [14].

| AO system parameters |  |
|----------------------|----------------------|
| Signal wavelength    | 780 nm               |
| DM actuator array size | $64 \times 64$      |
| Initial beam diameter | 0.05 m               |
| Primary mirror diameter | 1.03 m             |
| Secondary mirror diameter | 0.36 m           |
| Focal length         | 8 m                  |

TABLE I. AO system parameters used in the simulations based on a real Cassegrain telescope in Lulin Observatory.

We consider the situation that the quantum signal beam detection is triggered by the beacon beam. Each relatively strong beacon pulse is sent followed shortly by a corresponding weak coherent quantum signal pulse. This setting automatically compensates the zero-order distortion of the wavefront for each pulse. More importantly, the phase information of the beacon beam is extracted as the feedback signal. The phase is compared with the ideal situation in which the light beam propagates through a perfect atmosphere without turbulence and a perfect optical detection module. The difference of the profiles is used to correct the phase error of the signal beam, which spatial separated with the beacon beam, by applying the deformable mirror (DM).

![FIG. 2. Simulation setup for the receiving end.](image)

We model the atmospheric phase turbulence by a set of phase screens that changes the phase of the light wave. These phase screens are generated by using FFT on random complex numbers whose distribution follows the Kolmogorov’s turbulence theory. More precisely, we use the modified von Kármán phase noise power spectral density (PSD) [15],

$$\Phi_{\kappa m\nu K} = \frac{0.49r_0^{-5/3}\exp(-\kappa^2/\kappa_m^2)}{(\kappa^2 + \kappa_0^2)^{11/6}},$$  

where $\kappa_0 = 2\pi/L_0$, $\kappa_m = 5.92/l_0$, $\kappa$ is the spatial frequency in rad/m, and $r_0$ in meters is the Fried parameter that characterizes the atmospheric coherence diameter [16]. Here $L_0$ ($l_0$) in meters is the mean size of the largest (smallest)
eddies. Furthermore, we model $L_0$ by the Coulman-Vernin profile [17],

$$L_0(h) = \frac{4}{1 + \left(\frac{h - 8500}{2500}\right)^2},$$  \quad (2)$$

where $h$ is the altitude in meters. The value of $r_0$ varies with the altitude and the zenith angle according to the equation

$$r_0 = 0.423 k^2 \sec(\zeta) \int_0^h C_n^2(h) \, dh \right)^{-3/5},$$  \quad (3)$$

where $\zeta$ is zenith angle, $k$ is the wavenumber of the light. In addition, the refractive index structure parameter $C_n^2(h)$ is assumed to follow the Hufnagel-Valley model [18], namely,

$$C_n^2(h) = 0.00594 \left(\frac{v}{27}\right)^2 \left(\frac{h}{10^5}\right)^{10} \exp\left(-\frac{h}{1000}\right) + 2.7 \times 10^{-16} \exp\left(-\frac{h}{1500}\right) + 1.7 \times 10^{-14} \exp\left(-\frac{h}{100}\right),$$  \quad (4)$$

with wind speed $v = 21$ m/s.

Following Ref. [19], we use Fourier transform method with subharmonics to generate phase screens. That is to say, the phase screen of Fourier transform method can be written as

$$\phi(x,y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} c_{n,m} \exp[i 2\pi (f_{x,n} x + f_{y,m} y)],$$  \quad (5)$$

where $f_x$ and $f_y$ are the spatial frequencies along the $x$ and $y$ directions, respectively. Here, $c_{n,m}$ are random complex coefficients that have circular complex Gaussian distribution with variance given by [20, 21]

$$\langle |c_{n,m}|^2 \rangle = \Phi_{\phi}(f_{x,n}, f_{y,m}) \Delta f_{x,n} \Delta f_{y,m} = \frac{1}{L_x L_y} \Phi_{\phi}(f_{x,n}, f_{y,m}).$$  \quad (6)$$

We use the subharmonic method that proposed by Lane et al. [22] to create a low frequency phase screen. Specifically, a low frequency phase screen is generated using subharmonic and add it to the FT phase screen. The screen $\phi_{LF}(x,y)$ is computed by summing the $N_p$ phase screens, namely,

$$\phi_{LF}(x,y) = \sum_{p=1}^{N_p} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} c_{n,m} \exp[i 2\pi (f_{x,n,p} x + f_{y,m,p} y)].$$  \quad (7)$$

The frequency spacing of the $p^{th}$ screen we use is $\Delta f_p = 1/(3^p L)$.

![FIG. 3. Schematic representation of the communication channel. The colored plates here are the randomly generated (time-independent but spatially inhomogeneous) phase screens. The color of each pixel represents the phase change when light passes through that region.](image-url)
FIG. 4. Both beams are aiming to the receiver, the distance between the beam varies with \( z \).

to the first phase screen is almost zero. Consequently, the performance of our system is unaffected even though the two beams do not overlap on the first phase screen (that is, in the upper atmosphere).

Note that the overlapping area of the two beams increases if either the individual beam size or the transmission distance of the beams increase. As shown in Figure 4, the beams are tilted a small angle to aim at the receiver. We assume that the center of the beams reach the receiving end at the same location. Therefore, when they pass through a phase screen, one beam is offset by the distance \( \Delta x = L(z_{\text{max}} - z)/z_{\text{max}} \). Here \( L \), \( z_{\text{max}} \), and \( z \) are the separation between the beams, distance between the transmitter and the receiver and the propagated distance. Since \( L \ll z_{\text{max}} \), the beams travel the same distance and the relative tilt angle can be ignored.

The coherent efficiency \( \gamma \) is defined by

\[
\gamma = \frac{1}{2} \int \int \left( |E_{\text{ideal}}|^2 |E_{\text{received}}|^2 + |E_{\text{ideal}}^* E_{\text{received}}|^2 \right) ds \right]\right|^2.
\]

Here \( E_{\text{ideal}} \) is the electric field of ideal situation, namely, when the beam passes through the atmosphere without turbulence plus perfect optics in the detector side, and \( E_{\text{received}} \) is the distorted or compensated electric field actually detected. Moreover, the integral is over the receiver surface. Clearly, \( 0 \leq \gamma \leq 1 \). Besides, \( \gamma = 1 \) means that \( E_{\text{ideal}} \) and \( E_{\text{received}} \) align perfectly. Clearly, \( \gamma \) can be used to quantify signal distortion caused by atmospheric turbulence.

### III. SIMULATION RESULTS FOR STATIC ATMOSPHERE

Using the phase screens generated in Sec. II B, we report the performance of our method for static atmosphere in this section. Clearly, our findings also apply to the case of dynamical atmosphere provided that there is no time delay \( T_r \) between the beacon and signal beams. Figure 5 shows our simulation result of the coherent efficiency \( \gamma \) versus separation distance \( L \). Without AO correction, \( \gamma \) is about 0.3 when the zenith angle \( \zeta = 0^\circ \). Moreover, \( \gamma \) decreases as \( \zeta \) increases. In fact, \( d\gamma/d\zeta \) decreases rapidly for large value of \( \zeta \) so that when \( \zeta = 75^\circ \), \( \gamma \) is only about 0.05.

As expected, \( \gamma \) increases by turning on the AO. The smaller the zenith angle \( \zeta \), the higher the improvement of \( \gamma \). For instance, when \( L = 2 \) m, the system can correct the distortion to \( \gamma = 0.958 \) at \( \zeta = 0^\circ \) and \( \gamma = 0.566 \) at \( \zeta = 75^\circ \). Figure 5 also depicts that \( \gamma \) decreases when \( L \) increases. This is because as \( L \) increases, the overlapping area of the two beams is reduced. Thus, the phase distortion of the beacon beam is less correlated with that of the signal beam. For each fixed \( \zeta \), the coherent efficiency suddenly drops as \( L \) increases. Besides, the distance \( L \) for this sudden drop to occur decreases with \( \zeta \). This drop is related to the isoplanatic angle of the turbulence, namely, the minimum angle that two stars can be separated and still have their light pass through the same turbulent region. In other words, the angle between our two sources is smaller than the isoplanatic angle, their phase distortions can be considered as almost the same. Hence, when \( L \) increases so that the angular separation between the two sources increases beyond the isoplanatic angle, the effectiveness of AO correction decreases suddenly.

Last but not least, for a given \( L \), the value of \( \gamma \) decreases as the zenith angle \( \zeta \) increases. This is caused by two factors. First, the light beams have to travel along longer optical paths. Second, the Fried parameter \( r_0 \) in Eq. (3) gets smaller. Figure 5 also shows that for very large zenith angle (\( \zeta \geq 60^\circ \)), the value of \( \gamma \) as a function of \( L \) has more than one sudden drop. We do not have a good explanation. It is instructive to know why although such study is of pure academic nature because in practice it is more reasonable to deploy a network of satellites on the sky and pick...
the one with the smallest zenith angle to perform QKD. Further discussion on the effect of isoplanatic angle can be found in Sec. IV A.

Note that the system cannot perfectly recover the signal even when $L = 0$ because of the limited number of actuators in the DM. Hence, it is not able to completely compensate high order turbulence. Contribution of high order distortion becomes more significant when $\zeta$ increases. Thus, $\gamma < 1$ and it decreases when $\zeta$ increases. This is the general trend we find in Figure 6. Note however that our simulation shows that $\gamma$ is not a truly decreasing function of $\zeta$ especially when $L \gtrsim 4$ m and when $\zeta \lesssim 30^\circ$. This is not due to sampling error; and we do not have a good explanation right now.

A. Comparison Between Wavelength Division Multiplexing And Our Scheme

We compare the coherent efficiency $\gamma$ of our method with systems that use wavelength division multiplexing (WDM) to combine the signal beam and the beacon. Note that the phase deviation is inversely proportional to the wavelength. So we adjust the phase screens in our simulation according to the ratio of the wavelength of the signal beam and the beacon beam. We fix the beacon wavelength to the standard optical communication wavelength of 808 nm. We then compare the values of $\gamma$ between systems using WDM alone and ours with $L = 2$ m. As shown in Figure 7, the coherent efficiency of our spatial separation scheme is always better than that of WDM for all zenith angle $\zeta$. (Again, the fluctuation of the $\gamma$ curves in Figure 7 is not due to insufficient sampling. We do not have a good explanation for
it.) In terms of $\gamma$, our method is at least 11% better than that of WDM. In some cases, such as when $\zeta$ is between $40^\circ$ to $45^\circ$, our improvement is as high as 30%. Actually, chromatic distortion of the equipment has not included in our simulation. Thus, the actual value of $\gamma$ for the WDM method is slightly lower than the red curve shown in in Figure 7. This further illustrates the superiority of our method over WDM.

FIG. 6. Coherent efficiency $\gamma$ versus zenith angle $\zeta$ in degrees for various separation distances between the beacon and signal beams $L$ in meters.

FIG. 7. Coherent efficiency $\gamma$ versus zenith angle $\zeta$ for spatial separation systems and WDM systems. The blue curve is calculated with $L = 2$ m. The red curve is calculated with the wavelength of the beacon is 808 nm.
IV. SPATIAL DEPENDENCE OF TURBULENCE

A. Maximum Physical Distance Between The Beacon And Signal Sources

Clearly, AO technique is effective if the optical paths of the two light sources experience more or less the same optical distortion at all times. This requirement is satisfied when separated distance of the light sources $L$ is less than $z_{\text{max}}\theta_0$, where

$$\theta_0 = \left[2.913k^2 \sec^{8/3}(\zeta) \int_0^{h_{\text{max}}} h^{5/3}C_n^2(h) \, dh\right]^{3/5}$$

is the isoplanatic angle [23], and $h_{\text{max}}$ is the attitude of the source. Our simulation results show that there is a significant drop in coherent efficiency at $L \approx z_{\text{max}}\theta_0$. This is because the separation of the beams is too large compare to the isoplanatic angle.

B. Minimum Physical Distance Between The Beacon And Signal Sources

The minimum possible distance of the beacon and signal sources is determined by both the resolving power of the optics and the level of cross-talk between the two set of sources. Note that upon successful AO correction, the center of the image of the beacon beam should be around the center of the optically sensitive surface of the wavefront sensing module. We put a field stop in the signal detection module to filter the noise spatially. Naturally, we set the radius of the field stop to the diffraction limit of the signal detection module [5]. The diffraction pattern depends on the structure of the telescope. In our simulation, we use a 1.03 m Cassegrain telescope whose parameters are taken from a real telescope in Lulin Observatory [13]. The light intensity of the beacon beam at a distance $x$ away from the center equals

$$I_R(x) \approx I_R(0) \left(\frac{f \lambda}{\pi Dx}\right)^2 \left[J_1 \left(\frac{\pi Dx}{f \lambda}\right) - bJ_1 \left(\frac{b\pi Dx}{f \lambda}\right)\right]^2,$$

where $f$ is the effective local length of the telescope, $b = 0.36/1.03$ is ratio of the diameters of the secondary to primary mirrors of the Cassegrain telescope used, $I_R(0) \approx 2\epsilon_0 c E_R^2 (D/2)^4/R^2$, and $J_1(\cdot)$ is the order one Bessel function of the first kind. Hence, the total light energy flux of the beacon beam imparted on the optically sensitive surface of the signal detection module is $\int_{\text{FS}} I_R(x) \, dA$ where the integral is over the area of the field stop of the signal detection module. For example, when $L = 2$ m, $\int_{\text{FS}} I_R(x) \, dA = 4.36 \times 10^{-12} I_R(0)$. The minimum distance should be set according to the required decay from the beam center. Otherwise, stray beacon beam photons will seriously affect the signal detection statistics.

V. MODELING TIME DEPENDENCE TURBULENCE

All simulation results reported so far assumed a static atmosphere. We now extend our analysis to the more realistic dynamic atmosphere for moving sources. To compare the difference between stationary and moving sources, we use the Greenwood frequency $f_G$, which is an effective way to approximately quantify the rate of change of turbulence [7, 24]. Recall that

$$f_G = \left[0.1022k^2 \sec(\zeta) \int_0^{h_{\text{max}}} C_n^2(h)v^{5/3}(h) \, dh\right]^{3/5},$$

where $v(h) = v_{\text{wind}}(h) + v_{\text{app}}(h)$ is the natural wind speed plus the apparent wind speed due to the movement of the satellite. This assumption of simply adding two scalar speeds is justified because the LEO satellite moves at great angular speed so that $v_{\text{app}} \gg v_{\text{wind}}$. We further assume that the natural wind speed follows the altitude-dependent Bufton wind profile [25]

$$v_{\text{wind}}(h) = v_g + 30 \exp \left[-\left(\frac{h - 9400}{4800}\right)^2\right].$$
Here \( v_g \) is the natural wind speed and is taken to be 5 m/s in our simulation. Adding the apparent wind speed, \( v_{\text{app}}(h) = \omega_s h \), the total wind speed can be written as \([26]\),

\[
v(h) = \omega_s h + v_g + 30 \exp \left[ - \left( \frac{h - 9400}{4800} \right)^2 \right],
\]

where \( \omega_s \) is the angular slewing rate of the satellite. For simplicity, we assume that the satellite is moving in a circular orbit. Thus, the angular slewing rate is equal to

\[
\omega_s = \left[ \frac{GM_\oplus}{h_{\text{max}}^2 (h_{\text{max}} + R_\oplus)} \right]^{1/2} \cos^2(\zeta),
\]

where \( G \) is the universal gravitational constant, \( M_\oplus \) and \( R_\oplus \) are the Earth mass and radius, respectively. Since \( v_{\text{app}} \gg v_{\text{wind}} \), the Greenwood frequency for the LEO satellite tracking case can be much higher than the intrinsic frequency of the atmospheric turbulence. As shown in Figure 8, when the zenith angle is 0°, \( f_G \) intrinsic to the channel is about 64 Hz while \( f_G \approx 380 \text{ Hz} \) when slewing is included.

Recall that we put the beacon beam ahead of the signal beam. We also set a delay time of \( T_r \) between the beacon beam and the signal beam. For simplicity, we assume that \( \vec{v}_t \parallel \vec{L} \) and the response time of the AO system is less than or equal to \( T_r \) in our simulations. In this way, when the system receives the beacon signal at \( t = 0 \), it compensates the signal at \( t = T_r \). Figure 9 shows that if both beams are placed at the same physical location, the angle between the two timestamps is larger then the case which the beams are spatially separated. The apparent wind speed can therefore be reduced by a factor of \( \theta_1/\theta_2 \). The equivalent angular slewing rate is

\[
\omega'_s = \omega_s \frac{\theta_1}{\theta_2} = \omega_s \frac{|\theta_2 - \theta_1|}{\theta_2} = \omega_s \frac{|\omega_s T_r - \theta_1|}{\omega_s T_r} = \omega_s \frac{\theta_s}{T_r},
\]

where \( \theta_s = L/z_{\text{max}} \) is the angular separation between the beacon and the signal beam. Combined with Eqs. (11) and (13), it is clear that one can completely eliminate the effect of apparent wind speed and hence attain optimal performance for the AO system if \( \theta_s/T_r = \omega_s \). Indeed this is what we observed in Figure 8.

Note that for \( \theta_s/T_r < \omega_s \), the performance of our setup is worse than the case of a stationary source because the delay time \( T_r \) is not short enough to allow the pulsed signal and beacon beams to travel through an almost identical optical path. The more interesting case is when \( \theta_s/T_r > \omega_s \). In this case, the reduction in performance as reflected by the value of \( f_G \) is because the delay time \( T_r \) is too fast. Surely, we can artificially lengthening the delay time \( T_r \) between the beacon and signal beams to fix \( f_G \) to its optimal value.

Lastly, in Figure 8, when the zenith angle is not large, the Greenwood frequency curves calculated with \( L = 2 \text{ m} \) are lower than the curve calculated with no spatial separation. Since both \( \omega_s \) and \( \theta_s/T_r \) decrease when the zenith
angle increases, the curves decrease and approach to the intrinsic frequency curve. Also, as $\omega_s$ decreases more rapidly than $\theta_s/T_r$ along $\zeta$, the curves with spatial separation intersect the curve without slewing. This means $\theta_s/T_r = \omega_s$ at that point. For zenith angle larger than this point, $\theta_s/T_r > \omega_s$, the delay time should be decreased to keep the frequency near the intrinsic frequency. Finally, we remark that the condition $\vec{v}_t \parallel \vec{L}$ is essential in tuning $\theta_1$ to 0 and hence attaining the intrinsic Greenwood frequency. Nevertheless, as long as $\vec{v}_t \cdot \vec{L} > 0$, it is possible to improve the system performance by carefully adjusting $T_r$.

![Satellite location and beams’ path at $t = 0$ and $t = T_r$. Here $\theta_1$ is the angle between the signal beam’s paths (red lines at $t = 0$ and $t = T_r$) and $\theta_2$ is the angle between the reference beam’s path (blue lines at $t = 0$ and the signal beam’s path at $t = T_r$).](image)

**VI. SCATTERING NOISE BY THE STRONG BEAM**

The scattering caused by the strong beacon beam will affect the final key rate. Some photons from the beacon may enter the signal receiving module and create errors. Here we estimate the scattering by the strong laser in the clear sky scenario. We use sky-scattering noise to get a rough estimate on the laser scattering noise. The equation for calculating the number of sky-noise photons entering the system is given by [5],

$$N_b = \frac{H_b(\lambda)\Omega_{FOV} \pi D_R^2 \Delta \lambda \Delta t}{4hc}, \quad (16)$$

with $H_b$ in W m$^{-2}$sr$\mu$m is the sky radiance, $\Omega_{FOV} = \pi \Delta \theta^2 / 4$ is the solid-angle field of view with a field stop, $D_R$ is diameter of the receiver primary optic, $\Delta \lambda$ equals to the spectral filter bandpass in $\mu$m, and $\Delta t$ is the photon integration time of the receiver. Furthermore, $\Delta \theta$ is calculated by $D_{FS}/f$ with $D_{FS}$ being the diameter of the field stop. We assume $\Delta \lambda = 1$ as both beams use the same or nearly the same wavelength, the spectral filter is not able to block the photons from the beacon beam.

In astrophotography, a bright star that is close to target can be used as a beacon to probe the channel. Therefore, the brightness of the beacon laser should be similar to a bright star. The sky radiance caused by the laser can be estimated by the sky radiance by the stars. Under moonless clear night condition, the typical sky radiance is about $1.5 \times 10^{-5}$ W m$^{-2}$sr$\mu$m [27]. Using the parameters mentioned above and let $\Delta t = 1$ ns, the probability of receiving a beacon photon will be in the order of $10^{-8}$, which is good enough in practice.

**VII. CONCLUSIONS**

In this paper, we report a method to apply AO technologies to optical communication systems. The main ideas are the spatial separation of the strong beacon and the weak signal beam plus applying a suitable time delay between these two beams. By phase screen simulations, we show that the performance of our scheme is better than the WDM method for the LEO satellite case. Moreover, for fast-moving sources, our design can reduce the apparent wind speed caused by the movement of the object. This can reduce the Greenwood frequency of the turbulence. We estimate the cross-talk caused by the diffraction and the scattering of the beacon. As there is a field stop in the beacon receiving module and the power of the beacon is not high, the cross-talk by the beacon can be neglected. Further analysis of the system performance can be found in Ref. [10]. Lastly, we stress that our method is also applicable to classical optical free-space communication as we do not any quantum property of the signal source.
ACKNOWLEDGMENTS

We would like to thank Hoi-Kwong Lo, Alan Pak Tao Lau, Chengqiu Hu, Wenyuan Wang, and Gai Zhou for the discussion in optics. This work is supported by the RGC grant 17302019 of the Hong Kong SAR Government.

[1] S. Pirandola, U. L. Andersen, L. Banchi, M. Berta, D. Bunandar, R. Colbeck, D. Englund, T. Gehring, C. Lupo, C. Ottaviani, J. L. Pereira, M. Razavi, J. S. Shaari, M. Tomamichel, V. C. Usenko, G. Vallone, P. Villoresi, and P. Wallden, Advances in Optics and Photonics 12, 1012 (2020).

[2] S.-K. Liao, W.-Q. Cai, W.-Y. Liu, L. Zhang, Y. Li, J.-G. Ren, J. Yin, Q. Shen, Y. Cao, Z.-P. Li, F.-Z. Li, X.-W. Chen, L.-H. Sun, J.-J. Jia, J.-C. Wu, X.-J. Jiang, J.-F. Wang, Y.-M. Huang, Q. Wang, Y.-L. Zhou, L. Deng, T. Xi, L. Ma, T. Hu, Q. Zhang, Y.-A. Chen, N.-L. Liu, X.-B. Wang, Z.-C. Zhu, C.-Y. Lu, R. Shu, C.-Z. Peng, J.-Y. Wang, and J.-W. Pan, Nature 549, 43 (2017).

[3] F. Xu, X. Ma, Q. Zhang, H.-K. Lo, and J.-W. Pan, Reviews of Modern Physics 92, 10.1103/revmodphys.92.025002 (2020).

[4] Y. Cao, Y.-H. Li, K.-X. Yang, Y.-F. Jiang, S.-L. Li, X.-L. Hu, M. Abulizi, C.-L. Li, W. Zhang, Q.-C. Sun, W.-Y. Liu, X. Jiang, S.-K. Liao, J.-G. Ren, H. Li, L. You, Z. Wang, J. Yin, C.-Y. Lu, X.-B. Wang, Q. Zhang, C.-Z. Peng, and J.-W. Pan, Physical Review Letters 125, 260503 (2020).

[5] M. T. Gruneisen, B. A. Sickmiller, M. B. Flanagan, J. P. Black, K. E. Stoltenberg, and A. W. Duchane, in Emerging Technologies in Security and Defence II; and Quantum-Physics-based Information Security III, Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 9254, edited by K. L. Lewis, M. T. Gruneisen, M. Dusek, R. C. Hollins, J. G. Rarity, T. J. Merlet, and A. Toet (2014) p. 925404.

[6] E. Villasenor, R. Malaney, K. A. Mudge, and K. J. Grant, GLOBECOM 2020 - 2020 IEEE Global Communications Conference 10.1109/globecon42002.2020.9348086 (2020).

[7] R. K. Tyson, Principles Of Adaptive Optics, 3rd ed. (CRC Press, New York, 2011).

[8] M. T. Gruneisen, M. L. Eickhoff, S. C. Newey, K. E. Stoltenberg, J. F. Morris, M. Bareian, M. A. Harris, D. W. Oesch, M. D. Oliker, M. B. Flanagan, B. T. Kay, J. D. Schiller, and R. N. Lanning, Physical Review Applied 16, 10.1103/physrevapplied.16.014067 (2021).

[9] K. S. Chan and H. F. Chau, Improving classical and quantum free-space communication by adaptive optics and by separating the reference and signal beams with time delay for source(s) moving relative to the detector(s) (2022), patent Application PCT/CN2021/096100.

[10] K. S. Chan, Improving Quantum Key Distribution By Adaptive Optics, Master’s thesis, University of Hong Kong (2022).

[11] D. N. Burrows, ed., The WSPC Handbook Of Astronomical Instrumentation, Vol. 2 & 3 (World Scientific, Singapore, 2020).

[12] J. E. Krist, in Optical Modeling and Performance Predictions III, Vol. 6675, edited by M. A. Kahan, International Society for Optics and Photonics (SPIE, 2007) pp. 250 – 258.

[13] Lulin Observatory website, http://www.lulin.ncu.edu.tw/instrument/LOT/ [Accessed: 28 April 2022].

[14] R. N. Lanning, M. A. Harris, D. W. Oesch, M. D. Oliker, and M. T. Gruneisen, Quantum communication over atmospheric channels: A framework for optimizing wavelength and filtering (2021).

[15] L. Andrews and R. Phillips, Laser beam propagation through random media (SPIE—The International Society for Optical Engineering, 2005).

[16] D. L. Fried, J. Opt. Soc. Am. 55, 1427 (1965).

[17] C. E. Coulman, J. Vernin, Y. Coqueugniot, and J. L. Caccia, Appl. Opt. 27, 155 (1988).

[18] R. E. Hufnagel and N. R. Stanley, J. Opt. Soc. Am. 54, 52 (1964).

[19] J. Schmidt, Numerical simulation of optical wave propagation: With examples in MATLAB (2010) pp. 1–197.

[20] B. M. Welsh, in Propagation and Imaging through the Atmosphere, Vol. 3125, edited by L. R. Bissonnette and C. Dainty, International Society for Optics and Photonics (SPIE, 1997) pp. 327 – 338.

[21] W. A. Coles, J. P. Filice, R. G. Frehlich, and M. Yadlowsky, Appl. Opt. 34, 2089 (1995).

[22] C. M. Harding, R. A. Johnston, and R. G. Lane, Appl. Opt. 38, 2161 (1999).

[23] V. M. Acosta, D. Dequal, M. Schiavon, A. Montmerle-Bonnefois, C. B. Lim, J.-M. Conan, and E. Diamanti, Analysis of satellite-to-ground quantum key distribution with adaptive optics (2021), arXiv:2111.064747.

[24] D. P. Greenwood, J. Opt. Soc. Am. 67, 390 (1977).

[25] R. J. Sasiela, Electromagnetic Wave Propagation in Turbulence (SPIE, Bellingham, 2007).

[26] L. C. Andrews, Field Guide to Atmospheric Optics (SPIE, Bellingham, 2004).

[27] E.-L. Miao, Z.-F. Han, S.-S. Gong, T. Zhang, D.-S. Diao, and G.-C. Guo, New Journal of Physics 7, 215 (2005).