On Sensitivity of Neutrino–Helium Ionizing Collisions to Neutrino Magnetic Moments

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Abstract—We consider theoretically ionization of a helium atom by impact of an electron antineutrino. The sensitivity of this process to neutrino magnetic moments is analyzed. In contrast to the recent theoretical prediction, no considerable enhancement of the electromagnetic contribution with respect to the free-electron case is found. The stepping approximation is shown to be well applicable practically down to the ionization threshold.

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1. INTRODUCTION

Electromagnetic properties of neutrinos are of particular interest, for they open a door to “new physics” beyond the Standard Model (SM) (see, for instance, the review articles [1, 2]). Among these nontypical neutrino features the most studied and well understood theoretically are neutrino magnetic moments (NMM). The latter are also being intensively searched in reactor [3, 4], accelerator [5, 6] and solar [7, 8] experiments on low-energy elastic (anti)neutrino-electron scattering. The current best upper limit on the NMM value obtained in such direct laboratory measurements is

$$\mu_\nu \leq 2.9 \times 10^{-11} \mu_B,$$

where $\mu_B = e/(2m_\mu)$ is a Bohr magneton. This bound, which is due to the GEMMA experiment [4] with a HPGe detector at Kalinin nuclear power station, is by an order of magnitude larger than the constraint obtained in astrophysics [9]:

$$\mu_\nu \leq 3 \times 10^{-12} \mu_B.$$

And it by many orders of magnitude exceeds the value derived in the minimally extended SM with right-handed neutrinos [10]

$$\mu_\nu \leq 3 \times 10^{-10} \mu_B\left(\frac{m_\nu}{1\text{ eV}}\right),$$

where $m_\nu$ is a neutrino mass. At the same time, there are different theoretical scenarios beyond SM that predict much higher $\mu_\nu$ values, thus giving hope to observe NMM experimentally in the not too distant future. Therefore, the major task faced by experiments is to enhance their sensitivity to the $\mu_\nu$ value.

The strategy of experiments searching for NMM is as follows. One studies an inclusive cross section for (anti)neutrino-electron scattering which is differential in the energy transfer $T$. In the ultrarelativistic limit $m_\nu \to 0$, it is given by an incoherent sum of the SM contribution, which is due to weak interaction that conserves the neutrino helicity, and the helicity-flipping contribution, which is due to $\mu_\nu$,

$$\frac{d\sigma}{dT} = \frac{d\sigma_{\text{SM}}}{dT} + \frac{d\sigma_{(\mu)}}{dT}. \quad (1)$$

In the case of reactor experiments, where one deals with electron antineutrinos, the SM term is given by

$$\frac{d\sigma_{\text{SM}}}{dT} = \frac{G_F^2 m_\mu}{2\pi} \left[ (g_\nu + g_A)^2 + (g_\nu - g_A)^2 \left(1 - \frac{T}{E_\nu}\right)^2 \right] + \left(g_\nu - g_A^2 \right)\frac{m_\mu^2 T}{E_\nu^2}, \quad (2)$$

where $E_\nu$ is the incident antineutrino energy, $g_A = -1/2$ and $g_\nu = (4\sin^2\theta_W + 1)/2$, with $\theta_W$ being the Weinberg angle. The $\mu_\nu$ cross section is given by [11, 12]

$$\frac{d\sigma_{(\mu)}}{dT} = 4\pi\alpha \mu_\nu^2 \left(\frac{1}{T} - \frac{1}{E_\nu}\right), \quad (3)$$

where $\alpha$ is the fine-structure constant. Thus, the two components of the cross section (1) exhibit quite different dependencies on the recoil-electron kinetic energy $T$. Namely, at low $T$ values the SM cross section is practically constant in $T$, while that due to $\mu_\nu$ behaves as $1/T$. This means that the experimental sensitivity to NMM value critically depends on lowering the energy threshold of the detector employed for measurement of the recoil-electron spectrum.

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The formulas (2) and (3) assume the electron to be free and initially at rest. The energy threshold reached so far in the aforementioned GEMMA experiment with a HPGe detector is 2.8 keV [4]. This value is already much lower than the binding energy of K-electrons in Ge atoms (~10 keV). This fact makes it necessary to take into account the atomic effects beyond the free-electron (FE) approximation. The results of the corresponding treatment performed in [13] suggested that the electron binding in atoms can dramatically increase the μc contribution to the differential cross section (1) as compared with the FE case. However, the careful and detailed theoretical analysis [14–16] has found no evidence of the claimed “atomic ionization effect”. Moreover, it provided general arguments supporting the so-called stepping approximation formulated in [17] on the basis of numerical calculations for various targets. According to the stepping approximation, the cross section dσ/dT for knocking-out an electron from an atomic orbital follows the FE dependence on T all the way down to the ionization threshold Tf for this orbital with a very small (at most a few percent) deviation. And the orbital becomes “inactive” when T < Tf, thus producing a sharp step in the T dependence of dσ/dT summed over all occupied atomic levels.

Recently, the authors of [18] deduced by means of numerical calculations that the μc contribution to ionization of the He target by impact of electron antineutrinos from reactor and tritium sources strongly departs from the stepping approximation, exhibiting large enhancement relative to the FE approximation. According to [18], the effect is maximal when the T value approaches the ionization threshold in helium, Tf = 24.5874 eV, where the relative enhancement is as large as almost eight orders of magnitude. It was thus suggested that this finding might have an impact on searches for μc, provided that its value falls within the range 10^-13–10^-12 μB. The purpose of the present Letter is to show that (i) the result of [18] is erroneous and (ii) the stepping approximation for helium is well applicable, except the energy region T ~ Tf where the differential cross section substantially decreases relative to the FE case.

2. THEORY OF NEUTRINO-ImpACT IONIZATION OF HELIUM

We consider the process where an electron antineutrino with energy Ėv scatters on a He atom at energy and spatial-momentum transfers T and q, respectively. In what follows we focus on the ionization channel of this process in the kinematical regime $T \ll Ėv$, which mimics a typical situation with reactor ($Ēv \sim 1$ MeV) and tritium ($Ēv \sim 10$ keV) antineutrinos when the case $T \rightarrow T_f$ is concerned. The He target is assumed to be in its ground state $|Φ₁\rangle$ with the corresponding energy $Ē_\nu$. Since for helium one has $αZ \ll 1$, where $Z = 2$ is the nuclear charge, the state $|Φ₁\rangle$ can be treated nonrelativistically. As we are interested in the energy region $T - T_f$, the final He state $|Φ₂\rangle$ (with one electron in continuum) can also be treated in the nonrelativistic approximation.

Under the above assumptions, the SM and μc components of the differential cross section for the discussed ionization process can be presented as [16]

$$\frac{dσ_{SM}}{dT} = \frac{G_F^2}{4\pi}(1 + 4\sin^2θ_W + 8\sin^4θ_W)\int S(T, q^2) dq^2, \quad (4)$$

$$\frac{dσ_{(μ)}}{dT} = 4\pi\alpha μ_c \int S(T, q^2) dq^2, \quad (5)$$

where $S(T, q^2)$ is the dynamical structure factor given by

$$S(T, q^2) = \sum_{f} |⟨Φ₁(r_1, r_2)|e^{iqr_1} + e^{iqr_2}|Φ₂(r_1, r_2)⟩|^2 \times δ(T - E_f + E_i). \quad (6)$$

Here the $f$ sum runs over all final He states having one electron ejected in continuum, with $E_f$ being their energies.

For evaluation of the dynamical structure factor (6) we employ the same models of the initial and final He states as in [18]. The initial state is given by a product of two 1s hydrogenlike wave functions with an effective charge $Z_i$,

$$Φ₁(r_1, r_2) = \varphi_{1s}(Z_i, r_1)\varphi_{1s}(Z_i, r_2), \quad (7)$$

$$\varphi_{1s}(Z_i, r) = \sqrt{\frac{2}{πa_0^3}} e^{-Z_i r/a_0},$$

where $a_0 = 1/(αm_c)$ is the Bohr radius. The final state has the form

$$Φ₂(r_1, r_2) = \frac{1}{\sqrt{2}}[\varphi^-(Z_f, r_1)\varphi_{1s}(Z, r_2) + \varphi^+(Z_f, r_2)\varphi_{1s}(Z, r_1)], \quad (8)$$

where $\varphi^-(Z_f, r)$ is an outgoing Coulomb wave for the ejected electron with spatial momentum $k$. $Z_f$ is the effective charge experienced by the ejected electron in the field of the final He⁺ ion. Contributions to the dynamical structure factor from excited He⁺ states are neglected due to their very small overlap with the K-electron states in the He atom.
It is straightforward to perform the further calculation of the dynamical structure factor analytically$^2$ (see, for instance, the textbook [19]).

Finally, the usual choice of the effective charges is $Z_i = 27/16 \approx 1.69$ and $Z_f = 1$ (see, for instance, [20] and references therein). The value $Z_i = 27/16$ follows from the variational procedure that minimizes the ground-state energy $Z_i$, while the value $Z_f = 1$ ensures the correct asymptotic behavior of the final state. However, the authors of [18] utilized in their calculations the values $Z_i = 1.79$ and $Z_f = 1.1$ derived from fitting the photoionization cross-section data on helium with the present model of the He states.

### 3. RESULTS AND DISCUSSION

The departures of the differential cross sections (4) and (5) from the FE approximation are characterized by the respective atomic factors

$$f_{SM} = \frac{d\sigma_{SM}/dT}{d\sigma_{FE}/dT}, \quad f_{NMM} = \frac{d\sigma_{iQ}/dT}{d\sigma_{iQ}/dT},$$

(12)

where $d\sigma_{SM}/dT$ and $d\sigma_{FE}/dT$ are the SM and μ contributions to the differential cross section for scattering of an electron antineutrino on two free electrons. Let us recall that following [18] one should expect the $f_{NMM}$ value to be of about $10^3$ at $T \rightarrow T_f$.

Numerical results for atomic factors (12) are shown in the figure. They correspond to the kinematical regime $T \ll \alpha m_e \ll 2E_{\nu}$, which is typically realized both for reactor and for tritium antineutrinos when $T < 200$ eV. Note that in such a case one can safely set the upper limit of integrals in (4) and (5) to infinity, as the dynamical structure factor $S(T, q^2)$ rapidly falls down when $q \approx \alpha m_e$ and practically vanishes in the region $q \gg \alpha m_e$. It can be seen from the figure that the atomic factors exhibit similar behaviors for both sets of the $Z_i$ and $Z_f$ parameters discussed in the previous section. Namely, their values are minimal (~0.5) at the ionization threshold, $T = T_i$, and tend to unity with increasing $T$. The latter tendency is readily explained by approaching the FE limit. It can be also seen that a more or less serious deviation (>10%) of the present results from the stepping approximation is observed only in the low-energy region $T < 100$ eV.

Thus, the present calculations do not confirm the huge enhancement of the $\mu_\nu$ contribution with respect to the FE approximation. Moreover, in accord with various calculations for other atomic targets [14–17, 21–24], we find that at small energy-transfer values the electron binding in helium leads to the appreciable reduction of the differential cross section relative to the FE case. We attribute the erroneous prediction of [18] to the incorrect dynamical model that draws an analogy between the NMM-induced ionization and

$^2$The resulting expressions are omitted for the sake of brevity.
photoionization. Indeed, as discussed in [14], the virtual photon in the NMM-induced ionization process can be treated as real only when \( q \to T \). However, the integration in (5) involves the \( q \) values ranging from \( T \) up to \( 2E_v \). Since \( E_v \gg T \), the real-photon picture appears to be applicable only in the vicinity of the lower integration limit. When moving away from that momentum region, one encounters a strong departure from the real-photon approximation which treats the integrand as a constant in the whole integration range, assuming it to be equal to its value at \( q = T \), that is,

\[
\frac{1}{q^2} S(T, q^2) = \frac{1}{T^2} S(T, T^2).
\]

Such an approach is manifestly unjustified, and it gives rise to the spurious enhancement of the \( \mu_\nu \) contribution to the differential cross section.

4. SUMMARY

We carried out a theoretical analysis of ionization of helium by electron-antineutrino impact. Our calculations showed no evidence of the enhancement of the electromagnetic contribution as compared with the FE case. In contrast, in line with previous studies on other targets, we found that the magnitudes of the differential cross sections decrease relative to the FE approximation when the energy transfer is close to the ionization threshold. Thus, no sensitivity enhancement can be expected when using the He target in searches for NMM. And the stepping approximation appears to be valid, within a few-percent accuracy, down to the energy-transfer values as low as almost 100 eV.

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