MINIMIZING TOTAL COMPLETION TIME IN A TWO-MACHINE NO-WAIT FLOWSHOP WITH UNCERTAIN AND BOUNDED SETUP TIMES

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ABSTRACT. We address a two-machine no-wait flowshop scheduling problem with respect to the performance measure of total completion time. Minimizing total completion time is important when inventory cost is of concern. Setup times are treated separately from processing times. Furthermore, setup times are uncertain with unknown distributions and are within some lower and upper bounds. We develop a dominance relation and propose eight algorithms to solve the problem. The proposed algorithms, which assign different weights to the processing and setup times on both machines, convert the two-machine problem into a single-machine one for which an optimal solution is known. We conduct computational experiments to evaluate the proposed algorithms. Computational experiments reveal that one of the proposed algorithms, which assigns the same weight to setup and processing times, is superior to the rest of the algorithms. The results are statistically verified by constructing confidence intervals and test of hypothesis.

1. Introduction. In some manufacturing environments, an operation must immediately follow a preceding operation as a result of some characteristics of the material, e.g., temperature. In other words, successive processes of a job must be processed without any breaks (Allahverdi [2]). Such manufacturing flowshops are known as no-wait flowshops. Macchiaroli et al. [25] reported that one of the factors requiring the no-wait in process is the effort to reduce the work-in-process. Allahverdi [2] and Hall and Sriskandarajah [17] stated that no-wait flowshops are common in industries such as plastic, pharmaceutical, and chemical. Aircraft landing, patient scheduling, bakery production, and train scheduling problems (Allahverdi [2]) are just few other scheduling problems which can be modeled as a no-wait flowshop scheduling problem.

Hall and Sriskandarajah [17] and Allahverdi [2] provided survey papers related to no-wait flowshop scheduling problems. For example, Engin and Günyaydin [15]
proposed an adaptive learning approach for minimizing makespan for no-wait flowshops. After the survey of Allahverdi [2], more research has been conducted in this area. For example, Ying and Lin [40] proposed a matheuristic for the problem with respect to makespan performance measure. Engin and Güçlü [14] presented an effective new hybrid ant colony algorithm for the no-wait flowshops to minimize makespan while Wang et al. [39] proposed an iterated greedy heuristic for the mixed no-wait flowshop problem with the same makespan performance measure. Li et al. [24] presented a heuristic to minimize total flow time. On the other hand, Riahi and Kazemi [31] proposed a new hybrid ant colony algorithm for no-wait flowshops with respect to makespan and flowtime performance measures.

Inconsistency in job characteristics (processing times, due dates, setup times) is common in practice (Soroush [33, 34]), leading to the study of stochastic manufacturing environments. For example, Seo et al. [32] considered the scheduling problem for normally distributed processing times with the objective of minimizing the expected number of tardy jobs. Pinedo [28] provided an optimal policy for minimizing the expected weighted number of tardy jobs when job processing times are exponentially distributed. On the other hand, Cunningham and Dutta [13] and Ku and Niu [22] also considered the problem where jobs have exponentially distributed processing times. Kalczynski and Kamburowski [18] studied the problem for job processing times with the Weibull distribution, and Portougal and Trietsch [30] proposed heuristics when processing times are random variables with known cumulative distribution functions.

Machines need to be reset before a new job begins in manufacturing environments. The time to set a machine to perform a new job is called setup time. Setup times may be ignored if they are very small compared to processing times. However, it is not possible to ignore them for certain manufacturing environments (Allahverdi [1]). Allahverdi [1] provided a recent survey paper on scheduling problems considering setup times and stated that setup times need to be studied as separate from processing times for eliminating waste, meeting deadlines, increasing productivity, and improving resource utilization. Kopanos et al. [21] stated that setup times are common in a plethora of manufacturing environments, while Allahverdi [1] observed that less than 10% of the existing scheduling literature considers setup times.

In general, scheduling literature studying setup times assume that setup times are deterministic. This assumption is not valid for some scheduling environments since setup times change stochastically for some real-life manufacturing environments. Kim and Bobrowski [20] reported that this is due to breakdowns of tools, shortage of equipment, and crew skills. In fact, Gonzalez-Neira et al. [16] and Wang and Choi [38] stated that some manufacturing settings have a wide range of uncertainties. Furthermore, Aydilek et al. [9] stated that uncertain changes in setup times is a practical problem in manufacturing settings, and Aydilek et al. [10] reported that shop floor managers are encountered with substantial uncertainty in jobs’ attributes, including setup times. They further reported that using imprecise information to make scheduling decisions may yield a quite poor performance and that assuming certain setup time probability distributions is not appropriate for some manufacturing settings. Therefore, they suggested modelling setup times as uncertain variables between some bounds.

Uncertain setup times satisfy the inequality $L_{s_{i,k}} \leq s_{i,k} \leq U_{s_{i,k}}$ where $s_{i,k}$ symbolizes the setup time of job $i$ on machine $k$ while $L_{s_{i,k}}$ and $U_{s_{i,k}}$ symbolize the lower and upper bounds, respectively. Existing literature indicates that the flowshop
scheduling problem has been considered for uncertain setup times. For example, Allahverdi et al. [6] established dominance rules for a two-machine flowshop scheduling problem with respect to makespan and total completion time performance measures, i.e., the problem of \( F2 \) with respect to makespan and total completion time performance. Moreover, Allahverdi [3], Allahverdi [4], and Allahverdi [5] established dominance rules for the \( F2 \) problem with respect to makespan and total completion time problems, respectively. When processing times are also modelled as uncertain variables, Aydilek et al. [9] provided some heuristics for the problem to minimize makespan. Researchers have studied different scheduling problems (for uncertain environments) by using this approach, e.g., Sotskov et al. [36], Lai et al. [23], Braun et al. [12], Matsveichuk et al. [26, 27], Sotskov and Matsveichuk [35], and Sotskov and Lai [37]. The \( F2 \) problem, to the best of our knowledge, was only addressed by Allahverdi and Allahverdi [7].

In this paper, we address the problem of minimizing makespan, \( \text{min} \{ C_{\text{max}} \} \), for the \( F2 \) problem, and \( \text{max} \{ L_s \} \) for the \( F2 \) problem. Moreover, in this paper, the criterion is to minimize the total completion time. Therefore, the main contribution of the paper is to address the problem of minimizing \( \text{min} \{ C_{\text{max}} \} \) and \( \text{max} \{ L_s \} \) for the \( F2 \) problem.

2. A dominance relation. A dominance relation is established in this section. For \( j = 1, \ldots, n \) and \( k = 1, 2 \), let \( s_{j,k} \): the setup time of job \( j \) on machine \( k \), \( Ls_{j,k} \): lower bound of the setup time for job \( j \) on machine \( k \), \( Us_{j,k} \): upper bound of the setup time for job \( j \) on machine \( k \), \( t_{j,k} \): processing time of job \( j \) on machine \( k \), \( s_{j,k} \): setup time of the job in position \( j \) on machine \( k \), \( t_{j,k} \): processing time of the job in position \( j \) on machine \( k \), \( C_{[j]} \): completion time of the job in position \( j \) on the second machine.

Note that \( s_{j,k} \) satisfies

\[
Ls_{j,k} \leq s_{j,k} \leq Us_{j,k} \tag{1}
\]

That is, setup time is an uncertain value within the lower and upper bounds. Allahverdi and Allahverdi (2018) showed that Equation (1) can be written as

\[
C_{[j]} = \sum_{z=1}^{j} \max \{ s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]} \} + t_{[j,2]} \tag{2}
\]

The objective is to minimize total completion time, i.e.,

\[
TCT = \sum_{i=1}^{n} C_{[i]} \tag{3}
\]

Consider a job sequence \( \sigma_1 \) which has job \( g \) in an arbitrary position \( \beta \) and job \( h \) in position \( \beta + 1 \). Let the sequence \( \sigma_2 \) be derived from \( \sigma_1 \) by interchanging the jobs in these two positions \( \beta \) and \( \beta + 1 \) so that job \( h \) is in position \( \beta \) and job \( g \) in position \( \beta + 1 \) in the sequence \( \sigma_2 \). Let \( \gamma_1 \) denote a subsequence comprising the jobs in positions of \( 1, \ldots, \beta - 1 \). Moreover, let \( \gamma_2 \) denote a subsequence comprising the jobs in positions of \( \beta + 2, \ldots, n \). Both \( \sigma_1 \) and \( \sigma_2 \) can be written as \( \sigma_1 = \{ \gamma_1, g, h, \gamma_2 \} \) and \( \sigma_2 = \{ \gamma_1, h, g, \gamma_2 \} \).

Lemma 1. \( C_{[r]}(\sigma_2) = C_{[r]}(\sigma_1) \) for \( r = 1, \ldots, \beta - 1 \).
Proof. It follows from Equation (2) and the fact that both $\sigma_1$ and $\sigma_2$ have the same jobs in positions 1, $\ldots$, $\beta - 1$.

**Theorem 1.** For a two-machine no-wait flowshop scheduling problem to minimize total completion time, suppose that setup times are uncertain within some bounds. If jobs $g$ and $h$ are adjacent, then job $h$ should precede job $g$ when

1. $t_{h,2} \leq t_{g,2}$,
2. $U_{sh,2} \leq L_{sh,2}$,
3. $U_{sh,2} + t_{h,1} - L_{sh,2} \leq L_{s_{g,1}} + t_{g,1} - U_{s_{g,2}}$,
4. $U_{sh,2} + t_{g,2} + \max\{U_{s_{g,2}} + t_{h,2}; U_{s_{g,1}} + t_{g,1}\} \leq L_{s_{g,2}} + t_{h,2} + \max\{L_{sh,2} + t_{g,2}, L_{sh,1} + t_{h,1}\}$

Proof. It should be noted that as long as $U_{sh,2} \leq L_{s_{g,2}}$, it follows that $s_{h,2} \leq s_{g,2}$ by Equation (1). By the same equation, as long as the inequality $U_{s_{h,1}} + t_{h,1} - L_{sh,2} \leq L_{s_{g,1}} + t_{g,1} - U_{s_{g,2}}$ is satisfied, it is true that $s_{h,1} + t_{h,1} - s_{h,2} \leq s_{g,1} + t_{g,1} - s_{g,2}$. Finally, the inequality $U_{sh,2} + t_{g,2} + \max\{U_{s_{g,2}} + t_{h,2}; U_{s_{g,1}} + t_{g,1}\} \leq L_{s_{g,2}} + t_{h,2} + \max\{L_{sh,2} + t_{g,2}, L_{sh,1} + t_{h,1}\}$ implies $s_{h,2} + t_{g,2} + \max\{s_{g,2} + t_{h,2}, s_{g,1} + t_{g,1}\} \leq s_{g,2} + t_{h,2} + \max\{s_{h,2} + t_{g,2}, s_{h,1} + t_{h,1}\}$.

Lateness for jobs in positions $\beta$, $\beta + 1$, and $\beta + 2$ for the two sequences $\sigma_1$ and $\sigma_2$ are computed as:

$$C_{[\beta]}(\sigma_1) = \sum_{z=1}^{\beta-1} \max\{s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]} + \max\{s_{g,2} + t_{[\beta-1,2]}, s_{g,1} + t_{g,1}\} + t_{g,2},$$

$$C_{[\beta]}(\sigma_2) = \sum_{z=1}^{\beta-1} \max\{s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]} + \max\{s_{h,2} + t_{[\beta-1,2]}, s_{h,1} + t_{h,1}\} + t_{h,2},$$

$$C_{[\beta+1]}(\sigma_1) = \sum_{z=1}^{\beta-1} \max\{s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]} + \max\{s_{g,2} + t_{[\beta-1,2]}, s_{g,1} + t_{g,1}\} + \max\{s_{h,2} + t_{g,2}, s_{h,1} + t_{h,1}\} + t_{h,2},$$

$$C_{[\beta+1]}(\sigma_2) = \sum_{z=1}^{\beta-1} \max\{s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]} + \max\{s_{h,2} + t_{[\beta-1,2]}, s_{h,1} + t_{h,1}\} + \max\{s_{g,2} + t_{h,2}, s_{g,1} + t_{g,1}\} + t_{g,2},$$

$$C_{[\beta+2]}(\sigma_1) = \sum_{z=1}^{\beta-1} \max\{s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]} + \max\{s_{g,2} + t_{[\beta-1,2]}, s_{g,1} + t_{g,1}\} + \max\{s_{h,2} + t_{g,2}, s_{h,1} + t_{h,1}\} + \max\{s_{[\beta+2,2]} + t_{h,2}, s_{[\beta+2,1]} + t_{[\beta+2,1]}\} + t_{[\beta+2,2]},$$

where $\beta = 2, \ldots, N$, and $N$ is the total number of jobs.
Given that $\max z \leq s_{j,k} \leq U s_{j,k}$,

It follows from Equations (4-9) that

$$C_{\beta+2}(\sigma_2) = \sum_{z=1}^{\beta+1} \max \{ s_{z,2} + t_{z-1,2}, s_{z,1} + t_{z,1} \}$$
$$+ \max \{ s_{h,2} + t_{\beta-1,2}, s_{h,1} + t_{h,1} \}$$
$$+ \max \{ s_{g,2} + t_{h,2}, s_{g,1} + t_{g,1} \}$$
$$+ \max \{ s_{\beta+2,2} + t_{g,2}, s_{\beta+2,1} + t_{[\beta+2,1]} \} + t_{[\beta+2,2]}, \tag{9}$$

where $s_{j,k}$ satisfies $L s_{j,k} \leq s_{j,k} \leq U s_{j,k}$.

Given that $\max \{ s_{h,2} + t_{\beta-1,2}, s_{h,1} + t_{h,1} \} = s_{h,2} + \max \{ t_{[\beta-1,2]}, s_{h,1} + t_{h,1} - s_{h,2} \}$, and $\max \{ s_{g,2} + t_{\beta-1,2}, s_{g,1} + t_{g,1} \} = s_{g,2} + \max \{ t_{[\beta-1,2]}, s_{g,1} + t_{g,1} - s_{g,2} \}$, we have

$$\sum_{z=\beta}^{\beta+2} C_{[z]}(\sigma_2) - \sum_{z=\beta}^{\beta+2} C_{[z]}(\sigma_1) = \max \{ s_{\beta+2,2} + t_{g,2}, s_{\beta+2,1} + t_{[\beta+2,1]} \}$$
$$+ 2 \max \{ s_{g,2} + t_{h,2}, s_{g,1} + t_{g,1} \}$$
$$+ 3 \max \{ s_{h,2} + t_{h,2}, s_{h,1} + t_{h,1} \}$$
$$- \max \{ s_{[\beta+2,2]} + t_{h,2}, s_{[\beta+2,1]} + t_{[\beta+2,1]} \}$$
$$- 3 \max \{ s_{g,2} + t_{[\beta-1,2]}, s_{g,1} + t_{g,1} - s_{g,2} \}$$

If conditions (i-iv) of the theorem are satisfied, then

$$\sum_{z=\beta}^{\beta+2} C_{[z]}(\sigma_2) \leq \sum_{z=\beta}^{\beta+2} C_{[z]}(\sigma_1). \tag{10}$$

For $r = \beta + 3, \ldots, n$,

$$C_{[r]}(\sigma_1) = \sum_{z=1}^{\beta-1} \max \{ s_{z,2} + t_{z-1,2}, s_{z,1} + t_{z,1} \}$$
$$+ \max \{ s_{h,2} + t_{[\beta-1,2]}, s_{h,1} + t_{h,1} \} + \max \{ s_{g,2} + t_{g,2}, s_{h,1} + t_{h,1} \}$$
$$+ \max \{ s_{[\beta+2,2]} + t_{h,2}, s_{[\beta+2,1]} + t_{[\beta+2,1]} \}$$
$$+ \sum_{z=\beta+3}^{r} \max \{ s_{z,2} + t_{z-1,2}, s_{z,1} + t_{z,1} \} + t_{[r,2]}, \tag{11}$$
Equation (11) and (12) imply

\[ C_{\tau}(\sigma_1) = \sum_{z=1}^{\beta-1} \max\{s_{z,2} + t_{z-1,2}, s_{z,1} + t_{z,1}\} \]

\[ + \max\{s_{h,2} + t_{\beta-1,2}, s_{h,1} + t_{h,1}\} + \max\{s_{g,2} + t_{h,2}, s_{g,1} + t_{g,1}\} \]

\[ + \max\{s_{[\beta+2,2]} + t_{g,2}, s_{[\beta+2,1]} + t_{[\beta+2,1]}\} \]

\[ + \sum_{z=2}^{\tau} \max\{s_{z,2} + t_{z-1,2}, s_{z,1} + t_{z,1}\} + t_{r,2}. \]  

Equation (11) and (12) imply

\[ C_{\tau}(\sigma_1) - C_{\tau}(\sigma_2) = \max\{s_{h,2} + t_{\beta-1,2}, s_{h,1} + t_{h,1}\} + \max\{s_{g,2} + t_{h,2}, s_{g,1} + t_{g,1}\} \]

\[ + \max\{s_{[\beta+2,2]} + t_{g,2}, s_{[\beta+2,1]} + t_{[\beta+2,1]}\} \]

\[ - \max\{s_{g,2} + t_{\beta-1,2}, s_{g,1} + t_{g,1}\} - \max\{s_{h,2} + t_{h,2}, s_{h,1} + t_{h,1}\} \]

\[ - \max\{s_{[\beta+2,2]} + t_{h,2}, s_{[\beta+2,1]} + t_{[\beta+2,1]}\} \]  

(13)

Given that \( \max\{s_{h,2} + t_{\beta-1,2}, s_{h,1} + t_{h,1}\} = s_{h,2} + \max\{t_{\beta-1,2}, s_{h,1} + t_{h,1} - s_{h,2}\} \), and \( \max\{s_{[\beta+2,2]} + t_{g,2}, s_{[\beta+2,1]} + t_{[\beta+2,1]}\} = t_{g,2} + \max\{s_{[\beta+2,2]}, s_{[\beta+2,1]} + t_{[\beta+2,1]} - t_{g,2}\} \). Moreover, since \( \max\{s_{g,2} + t_{\beta-1,2}, s_{g,1} + t_{g,1}\} = s_{g,2} + \max\{t_{\beta-1,2}, s_{g,1} + t_{g,1} - s_{g,2}\} \), and \( \max\{s_{[\beta+2,2]} + t_{h,2}, s_{[\beta+2,1]} + t_{[\beta+2,1]}\} = t_{h,2} + \max\{s_{[\beta+2,2]}, s_{[\beta+2,1]} + t_{[\beta+2,1]} - t_{h,2}\} \).

Equation (13) can be written as

\[ C_{\tau}(\sigma_2) - C_{\tau}(\sigma_1) = s_{h,2} + t_{g,2} + \max\{s_{g,2} + t_{h,2}, s_{g,1} + t_{g,1}\} \]

\[ + \max\{t_{\beta-1,2}, s_{h,1} + t_{h,1} - s_{h,2}\} \]

\[ + \max\{s_{[\beta+2,2]}, s_{[\beta+2,1]} + t_{[\beta+2,1]} - t_{g,2}\} \]

\[ - s_{g,2} - t_{h,2} - \max\{s_{h,2} + t_{g,2}, s_{h,1} + t_{h,1}\} \]

\[ - \max\{t_{\beta-1,2}, s_{g,1} + t_{g,1} - s_{g,2}\} \]

\[ - \max\{s_{[\beta+2,2]}, s_{[\beta+2,1]} + t_{[\beta+2,1]} - t_{h,2}\} \]  

(14)

It follows by conditions (i),(iii) and (iv) specified in the theorem that for \( r(r = \beta + 3, \ldots, n) \)

\[ C_{\tau}(\sigma_2) \leq C_{\tau}(\sigma_1). \]  

(15)

Then, by Inequalities 10 and 15 and Lemma 1

\[ TCT(\sigma_2) \leq TCT(\sigma_1). \]

**Theorem 2.** Suppose that for a two-machine no-wait flowshop scheduling problem, jobs \( g \) and \( h \) are adjacent and setup times are deterministic. To minimize total completion time, job \( h \) should precede job \( g \) when

1. \( t_{h,2} \leq t_{g,2} \),
2. \( s_{h,2} \leq s_{g,2} \),
3. \( s_{h,1} + t_{h,1} - s_{h,2} \leq s_{g,1} + t_{g,1} - s_{g,2} \)
4. \( s_{h,2} + t_{g,2} + \max\{s_{g,2} + t_{h,2}, s_{g,1} + t_{g,1}\} \leq s_{g,2} + t_{h,2} + \max\{s_{h,2} + t_{g,2}, s_{h,1} + t_{h,1}\} \)

**Proof.** The proof follows from that of the previous theorem since \( Ls_{j,k} = s_{j,k} = Us_{j,k} \) for the deterministic setup times.
3. Algorithms (ALG − 1 to ALG − 8). The two-machine no-wait scheduling problem to minimize total completion time is strongly NP-hard even for the case of zero setup times. In other words, the problem for $k = 2$ and $Us_{i,k} = Ls_{i,k} = 0$ for all $i = 1, 2, \ldots, n$, is strongly NP-hard (Hall and Sriskandarajah [17]). In this paper, we consider the problem with uncertain setup times, hence, it is highly unlikely that the addressed problem has a polynomial time solution. Therefore, we propose approximate algorithms to solve the problem.

Advanced algorithms or meta-heuristics such as particle swarm optimization (PSO) are based on the exact values of $s_{j,k}$, but small changes in setup times significantly affect the quality of the solution. Therefore, they cannot provide efficient results (Allahverdi and Aydilek [8]). On the other hand, the value of $s_{j,k}$ is within $Ls_{i,k}$ and $Us_{i,k}$. Hence, a decision on constructing a schedule should be based only on the setup time bounds ($Ls_{i,k}$ and $Us_{i,k}$). That is, it cannot be based on $s_{j,k}$’s as they are not known until the jobs have been completed.

Ordering the jobs based on the Shortest Processing Time (SPT) rule yields the optimal solution for a single-machine scheduling problem when setup times are zero with performance measure the total completion time (Pinedo [29]). We propose some algorithms by converting the two-machine no-wait flowshop problem to a single-machine problem. Steps of the algorithms are given next.

Steps of Algorithm ALG − 1 to ALG − 8

Step 1: Select $n$(given)
Step 2: Select values $t_{i,k}$ for $i = 1, \ldots, n$ and $k = 1, 2$.
Step 3: Select values $Ls_{i,k}$ and $Us_{i,k}$ for $i = 1, \ldots, n$ and $k = 1, 2$.
Step 4: Let $r = 1$.
Step 5: $t_r(1) = t_{r,1}$
Step 6: $t_r(2) = t_{r,2}$
Step 7: $t_r(3) = t_{r,1} + t_{r,2}$
Step 8: $t_r(4) = 0.3t_{r,1} + 0.7t_{r,2}$
Step 9: $t_r(5) = 0.7t_{r,1} + 0.3t_{r,2}$
Step 10: $t_r(6) = t_{r,1} + t_{r,2} + \left(\frac{Ls_{r,1} + Us_{r,1}}{2}\right) + \left(\frac{Ls_{r,2} + Us_{r,2}}{2}\right)$
Step 11: $t_r(7) = 0.3t_{r,1} + 0.7t_{r,2} + 0.3 \left(\frac{Ls_{r,1} + Us_{r,1}}{2}\right) + 0.7 \left(\frac{Ls_{r,2} + Us_{r,2}}{2}\right)$
Step 12: $t_r(8) = 0.7t_{r,1} + 0.3t_{r,2} + 0.7 \left(\frac{Ls_{r,1} + Us_{r,1}}{2}\right) + 0.3 \left(\frac{Ls_{r,2} + Us_{r,2}}{2}\right)$
Step 13: Let $r = r + 1$
Step 14: If $r < n$, go to Step 5, else go to Step 15
Step 15: Let $s = 1$
Step 16: Sequence the jobs according to SPT based on $t_r(s)$ and let the resulting sequence be called $\pi s$
Step 17: Let $s = s + 1$
Step 18: If $s < 8$, go to Step 16, else go to Step 19
Step 19: Set $c = 1$ and $x = 1$
Step 20: Set $t_{h,k} = t_{[c,k]}(\pi x)$ for $k = 1, 2$ and $t_{g,k} = t_{[c+1,k]}(\pi x)$ for $k = 1, 2$
Step 21: If $t_{h,k}$ and $t_{g,k}$ satisfy any of the conditions of Theorem 1, exchange the jobs in $c$ and $c + 1$ positions of the sequence $\pi x$
Step 22: Set $c = c + 1$
Step 23: If $c < n$, go to Step 20
Step 24: Let $x = x + 1$
Step 25: If \( x < 8 \), go to Step 20, else go to Step 26
Step 26: The sequence \( \pi \) is the solution of the algorithm \( ALG - s \)

It should be noted that the main difference among the algorithms is related to the weights assigned to the processing times and setup times on the first and second machines. For example, in algorithm \( ALG - 6 \), equal weights are assigned to the four components: processing time on the first machine, processing time on the second machine, setup time on the first machine, and setup time on the second machine. However, since setup times are unknown, and we only know the upper and lower bounds, an average of the upper and lower bounds is taken. The flowchart of the above algorithms is provided in Figure 1.

4. Computational experiments. The proposed algorithms of \( ALG - 1 \) to \( ALG - 8 \) are compared with each other based on randomly generated data. The processing time \( t_{i,k} (i = 1, \ldots, n; \text{and} k = 1, 2) \) is generated from the uniform distribution of \( U(1, 100) \). Since this distribution’s variance is large, it is usually used in the scheduling literature for generating processing times. Upper bounds on the setup times \( U_{s_{j,k}} \) are also generated from the uniform distribution of \( U(1, 100) \). However, \( Ls_{j,k} \) is generated from the uniform distribution \( U(\max(1, U_{s_{j,k}} - H), U_{j,m}) \), where \( H \) is set at three values of 20, 30, and 40. Generating \( Ls_{j,k} \) and \( U_{s_{j,k}} \) in this way is commonly used in the literature for uncertain times, e.g., Aydilek et al. [11].

Once \( Ls_{j,k} \) and \( U_{s_{j,k}} \) are generated, \( s_{j,k} (L_{s_{j,k}} \leq s_{j,k} \leq U_{s_{j,k}}) \) needs to be generated for an instance of setup time within the lower and upper bounds so that the performance of the proposed algorithms can be evaluated. Considering a specific distribution for generating setup times (within the lower and upper bounds) is not suitable since we do not know its distribution. Hence, we investigate four different distributions: normal, uniform, positive linear, and negative linear. The considered distributions represent symmetric, positively skewed, and negatively skewed. Aydilek et al. [11] also used these distributions to evaluate the performance of their algorithms.

For the uniform distribution, \( s_{i,k} \) is generated from \( U(L_{s_{i,k}}, U_{s_{i,k}}) \). For the normal distribution, the mean \( \mu \) and standard deviation \( \sigma \) are set to \( (L_{s_{i,k}} + U_{s_{i,k}})/2 \) and \( (U_{s_{i,k}} - L_{s_{i,k}})/6 \), respectively. However, there is a small possibility that the generated \( s_{i,k} \) may result in a value which is outside the range of \( L_{s_{i,k}} \) and \( U_{s_{i,k}} \), in which case we regenerate \( s_{i,k} \) so that it falls within the range. For positive linear and negative linear distributions, the probability density functions are \( f_p(s_{i,k}) = 2(s_{i,k} - L_{s_{i,k}})/x(U_{s_{i,k}} - L_{s_{i,k}})^2 \) and \( f_n(s_{i,k}) = 2(U_{s_{i,k}} - t_{i,k})/x(U_{s_{i,k}} - L_{s_{i,k}})^2 \) for \( s_{i,k} \in (L_{s_{i,k}}, U_{s_{i,k}}) \), respectively. Figure 2 shows all four distributions for generating setup times.

The proposed algorithms \( ALG - 1 \) to \( ALG - 8 \) are compared with each other based on two performance measures: average percentage error (Error) and standard deviation (Std). The percentage error is defined as

\[
Error(ALG - i) = 100 \times \frac{TCT(ALG - i) - \min\{TCT(ALG - 1), \ldots, TCT(ALG - 8)\}}{\min\{TCT(ALG - 1), \ldots, TCT(ALG - 8)\}}
\]

We consider five different values for \( n: 100, 200, 300, 400, \) and 500. Furthermore, we consider four distributions (positive, negative, uniform, and normal) for generating \( s_{i,k} \) and three values of 20, 30, and 40 for \( H \). A total of 60 (5*4*3) combinations are considered. For each combination, one thousand replications are generated. Hence, a total of 60,000 problems are generated.
Figure 1. Flowchart of the algorithms

The computational results are summarized in Tables 1, 2, 3, and 4, for positive linear, negative linear, uniform, and normal distributions, respectively. In the tables, the first column denotes the algorithm, the second column indicates $H$, columns three through seven show the $n$ value, while the last column shows the average value. The overall average errors of algorithms $ALG - 1 - ALG - 8$ are $9.42$, $9.59$, $3.04$, $5.02$, $4.94$, $0.02$, $4.14$, and $2.52$, respectively. This is shown in Figure 3. Figure 4 indicates the average standard deviations of the algorithms. Figures 3 and 4 are in general consistent due to a large number of replications; it is clear that for both figures the best performing algorithm is $ALG - 6$.

Figure 5, which is based on Table 5, shows the average errors of the algorithms with respect to the number of jobs, $n$. It is clear from the figure that, in general,
the performances of algorithms $ALG-1$ to $ALG-8$ are not dependent to the number of jobs except for $ALG-6$. For algorithm $ALG-6$, the error decreases as $n$ increases, which is an advantage for the algorithm.

The performance of the average errors of the algorithms with respect to $H$, the gap between the lower and upper bounds of setup times, is illustrated in Figure 6, which is based on Table 6. In general, the performance of algorithms slightly gets worse as $H$ increases, which is expected. However, as can vividly be seen in Figure 7, the performance of $ALG-6$ improves as $H$ increases which is another advantage for algorithm $ALG-6$.

Figure 8 shows the algorithms’ performance with respect to the four distributions. As seen from the figure, performances of the algorithms in general do not change from one distribution to another.

In order to show that the algorithm $ALG-6$ is the best statistically, confidence intervals and test of hypothesis are constructed. 95% confidence intervals of the average errors for the algorithms are given in Table 7.

It is clear from Table 7 that $ALG-6$ performs better than all the algorithms statistically. The second well performing algorithm is $ALG-8$, followed by $ALG-3$. The worst are $ALG-1$ and $ALG-2$. Since the second-best performing algorithm is $ALG-8$, the following null and alternative hypotheses testing are conducted for comparing the performance of $ALG-6$ and $ALG-8$ statistically.

$$H_0 : \mu(\text{ALG-6}) = \mu(\text{ALG-8})$$

$$H_1 : \mu(\text{ALG-6}) < \mu(\text{ALG-8})$$

The null hypothesis is rejected at a significance level of $\alpha = 0.01$.

Finally, the effect of the developed dominance relation (Theorem 1) in Steps 20-23 of the algorithms is also explored. It is observed that the improvement, on average, was less than 10%.

5. Conclusion. The two-machine no-wait flowshop scheduling problem is investigated with the objective of minimizing total completion time when setup times are uncertain. The investigated problem is strongly NP-hard. Therefore, a dominance relation is established, and different algorithms are proposed. The algorithms convert the two-machine no-wait flowshop problem into a single-machine problem for which an optimal solution exists, i.e., the $SPT$ sequence. Then, the algorithms are constructed based on the modified $SPT$ sequences which utilize different weights to the processing times on the first machine, processing times on the second machine, setup time on the first machine, and setup time on the second machine. This is due to the fact that the flowshop has a no-wait constraint.

Finally, the effect of the developed dominance relation (Theorem 1) in Steps 20-23 of the algorithms is also explored. It is observed that the improvement, on average, was less than 10%.
Figure 2. The distributions used for generating $s_{i,k}$ within $Ls_{i,k}$ and $Us_{i,k}$

$ALG - 6$ statistically performs better than the rest. The excellent performance of $ALG - 6$ is not surprising since the algorithm $ALG - 6$ assigns the same weight to processing times on the first and second machines, as well as the setup times on the first and second machines. It should be noted that this characteristic seems to be valid for no-wait flowshops and not for regular flowshops.

The addressed problem can be extended with respect to other objective functions such as total tardiness. Moreover, setup times are treated as sequence-independent in this paper. This is common in many environments. Nevertheless, some environments have setup times as sequence-dependent, e.g., Karacizmeli and Ogulata
Table 1. Error of Algorithms for Positive Linear Distribution

| Algorithm | n | 100 | 200 | 300 | 400 | 500 | Avg. |
|-----------|---|-----|-----|-----|-----|-----|------|
| ALG – 1   | 20| 9.08| 9.11| 9.17| 9.17| 9.27| 9.16 |
| ALG – 2   |   | 9.38| 9.37| 9.28| 9.34| 9.3 | 9.33 |
| ALG – 3   |   | 3.05| 2.98| 2.95| 2.96| 2.97| 2.98 |
| ALG – 4   |   | 4.99| 4.93| 4.88| 4.9  | 4.92|      |
| ALG – 5   |   | 4.85| 4.81| 4.76| 4.81| 4.83| 4.79 |
| ALG – 6   |   | 0.1 | 0.02| 0   | 0   | 0   | 0.02 |
| ALG – 7   |   | 4.23| 4.1 | 4.02| 4   | 3.95| 4.06 |
| ALG – 8   |   | 2.44| 2.35| 2.44| 2.44| 2.44| 2.42 |
| ALG – 1   | 30| 9.17| 9.33| 9.42| 9.4  | 9.4  | 9.34 |
| ALG – 2   |   | 9.5 | 9.56| 9.48| 9.47| 9.47| 9.5  |
| ALG – 3   |   | 3.04| 3.1 | 3.03| 3.03| 3   | 3.04 |
| ALG – 4   |   | 5.04| 5.07| 4.97| 4.98| 4.98| 5.01 |
| ALG – 5   |   | 4.81| 4.93| 4.95| 4.95| 4.95| 4.92 |
| ALG – 6   |   | 0.08| 0.01| 0   | 0   | 0   | 0.02 |
| ALG – 7   |   | 4.2 | 4.12| 4.02| 4.04| 3.96| 4.07 |
| ALG – 8   |   | 2.48| 2.45| 2.46| 2.49| 2.49| 2.47 |
| ALG – 1   | 40| 9.37| 9.47| 9.49| 9.54| 9.56| 9.49 |
| ALG – 2   |   | 9.65| 9.69| 9.64| 9.63| 9.67| 9.66 |
| ALG – 3   |   | 3.01| 3.07| 3.06| 3.05| 3.08| 3.05 |
| ALG – 4   |   | 5.06| 5.06| 5.06| 5.05| 5.06| 5.06 |
| ALG – 5   |   | 4.93| 5.01| 4.96| 5   | 5.05| 4.99 |
| ALG – 6   |   | 0.09| 0.02| 0   | 0   | 0   | 0.02 |
| ALG – 7   |   | 4.24| 4.17| 4.03| 4.1  | 4.12| 4.13 |
| ALG – 8   |   | 2.47| 2.51| 2.53| 2.51| 2.57| 2.52 |
| Avg.      |   | 4.80| 4.80| 4.78| 4.79| 4.79|      |

[19]. Thus, the problem addressed in this paper can be extended to the case of sequence-dependent setup times.

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Table 2. Error of Algorithms for Negative Linear Distribution

| Algorithm | $H$ | 100 | 200 | 300 | 400 | 500 | Avg. |
|-----------|-----|-----|-----|-----|-----|-----|------|
| $ALG - 1$ | 20  | 9.08| 9.13| 9.22| 9.26| 9.27| 9.19 |
| $ALG - 2$ |     | 9.38| 9.26| 9.33| 9.35| 9.27| 9.32 |
| $ALG - 3$ |     | 3.05| 2.97| 3.04| 3.02| 2.99| 3.01 |
| $ALG - 4$ |     | 4.99| 4.85| 4.92| 4.9 | 4.87| 4.91 |
| $ALG - 5$ |     | 4.75| 4.73| 4.89| 4.92| 4.86| 4.83 |
| $ALG - 6$ |     | 0.1 | 0.02| 0   | 0   | 0   | 0.02 |
| $ALG - 7$ |     | 4.23| 4.1 | 4.06| 4.07| 3.95| 4.08 |
| $ALG - 8$ |     | 2.44| 2.41| 2.44| 2.5 | 2.43| 2.44 |
| $ALG - 1$ | 30  | 9.17| 9.37| 9.37| 9.37| 9.41| 9.34 |
| $ALG - 2$ |     | 9.5 | 9.51| 9.53| 9.5 | 9.51| 9.51 |
| $ALG - 3$ |     | 3.04| 3.05| 3.04| 3.04| 3.05| 3.04 |
| $ALG - 4$ |     | 5.04| 5   | 5.03| 5   | 4.99| 5.01 |
| $ALG - 5$ |     | 4.81| 4.92| 4.9 | 4.94| 4.93| 4.9 |
| $ALG - 6$ |     | 0.08| 0.01| 0   | 0   | 0   | 0.02 |
| $ALG - 7$ |     | 4.2 | 4.15| 4.1 | 4.05| 4.01| 4.1 |
| $ALG - 8$ |     | 2.48| 2.54| 2.46| 2.55| 2.53| 2.5 |
| $ALG - 1$ | 40  | 9.37| 9.37| 9.5 | 9.51| 9.55| 9.46 |
| $ALG - 2$ |     | 9.65| 9.54| 9.59| 9.64| 9.62| 9.61 |
| $ALG - 3$ |     | 3.01| 2.96| 3.04| 3.04| 3.07| 3.03 |
| $ALG - 4$ |     | 5.06| 4.97| 5.01| 5.06| 5.05| 5.03 |
| $ALG - 5$ |     | 4.93| 4.85| 4.95| 4.97| 5.01| 4.94 |
| $ALG - 6$ |     | 0.09| 0.01| 0   | 0   | 0   | 0.02 |
| $ALG - 7$ |     | 4.24| 4.12| 4.08| 4.07| 4.08| 4.12 |
| $ALG - 8$ |     | 2.47| 2.44| 2.58| 2.54| 2.57| 2.5 |

Avg. | 4.80| 4.76| 4.80| 4.81| 4.79 |

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Table 3. Error of Algorithm for Uniform Distribution

| Algorithm | $n$ | 100   | 200   | 300   | 400   | 500   | Avg.  |
|-----------|-----|-------|-------|-------|-------|-------|-------|
| ALG − 1   | 20  | 9.05  | 9.32  | 9.34  | 9.37  | 9.41  | 9.30  |
| ALG − 2   | 9.39 | 9.45  | 9.43  | 9.49  | 9.43  | 9.44  |
| ALG − 3   | 3.09 | 3.02  | 2.97  | 3.04  | 3.04  | 3.03  |
| ALG − 4   | 4.98 | 4.96  | 4.92  | 4.98  | 4.96  | 4.96  |
| ALG − 5   | 4.86 | 4.88  | 4.87  | 4.94  | 4.95  | 4.90  |
| ALG − 6   | 0.1  | 0.02  | 0     | 0     | 0     | 0.02  |
| ALG − 7   | 4.24 | 4.08  | 4.09  | 4.09  | 4.05  | 4.11  |
| ALG − 8   | 2.42 | 4.08  | 4.09  | 4.09  | 4.05  | 4.11  |
| ALG − 1   | 30  | 9.09  | 9.47  | 9.52  | 9.64  | 9.56  | 9.46  |
| ALG − 2   | 9.59 | 9.61  | 9.63  | 9.67  | 9.59  | 9.63  |
| ALG − 3   | 3.1  | 3.04  | 3.04  | 3.09  | 3.03  | 3.06  |
| ALG − 4   | 5.13 | 5.03  | 5.02  | 5.06  | 5     | 5.05  |
| ALG − 5   | 4.86 | 4.94  | 4.96  | 5.05  | 5.02  | 4.97  |
| ALG − 6   | 0.1  | 0.02  | 0     | 0     | 0     | 0.02  |
| ALG − 7   | 4.17 | 4.17  | 4.08  | 4.13  | 4.06  | 4.12  |
| ALG − 8   | 2.44 | 2.49  | 2.53  | 2.59  | 2.56  | 2.52  |
| ALG − 1   | 40  | 9.32  | 9.68  | 9.68  | 9.7   | 9.79  | 9.63  |
| ALG − 2   | 9.67 | 9.89  | 9.85  | 9.87  | 9.89  | 9.83  |
| ALG − 3   | 3.05 | 3.06  | 3.06  | 3.08  | 3.06  | 3.07  |
| ALG − 4   | 5.07 | 5.14  | 5.13  | 5.13  | 5.13  | 5.12  |
| ALG − 5   | 4.86 | 5.09  | 5.04  | 5.07  | 5.09  | 5.03  |
| ALG − 6   | 0.09 | 0.01  | 0     | 0     | 0     | 0.02  |
| ALG − 7   | 4.31 | 4.28  | 4.23  | 4.19  | 4.21  | 4.24  |
| ALG − 8   | 2.43 | 2.61  | 2.64  | 2.68  | 2.68  | 2.61  |
| Avg.      |     | 4.81  | 4.86  | 4.86  | 4.89  | 4.88  |

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Table 4. Error of Algorithm for Normal Distribution

| Algorithm | $H$ | 100  | 200  | 300  | 400  | 500  | Avg.  |
|-----------|-----|------|------|------|------|------|-------|
| $ALG - 1$ | 20  | 9.04 | 9.3  | 9.32 | 9.35 | 9.38 | 9.28  |
| $ALG - 2$ | 9.5 | 9.46 | 9.45 | 9.46 | 9.5  | 9.47 |       |
| $ALG - 3$ | 3.03| 3    | 3    | 3.03 | 3.06 | 3.02 |       |
| $ALG - 4$ | 5.01| 4.97 | 4.92 | 4.96 | 4.95 | 4.90 |       |
| $ALG - 5$ | 4.81| 4.88 | 4.9  | 4.94 | 4.95 | 4.90 |       |
| $ALG - 6$ | 0.08| 0.01 | 0    | 0    | 0    | 0.02 | 0.02  |
| $ALG - 7$ | 4.27| 4.09 | 4.13 | 4.08 | 4.06 | 4.13 |       |
| $ALG - 8$ | 2.47| 2.46 | 2.45 | 2.5  | 2.55 | 2.49 |       |

| Algorithm | $H$ | 30   | 100  | 200  | 300  | 400  | 500  | Avg.  |
|-----------|-----|------|------|------|------|------|------|-------|
| $ALG - 1$ | 30  | 9.43 | 9.43 | 9.54 | 9.57 | 9.71 | 9.54 |       |
| $ALG - 2$ | 9.8 | 9.71 | 9.66 | 9.7  | 9.72 | 9.72 |       |
| $ALG - 3$ | 3.11| 3    | 3.09 | 3.08 | 3.12 | 3.08 |       |
| $ALG - 4$ | 5.16| 5.06 | 5.06 | 5.05 | 5.11 | 5.09 |       |
| $ALG - 5$ | 4.99| 4.92 | 5    | 4.99 | 5.08 | 5.00 |       |
| $ALG - 6$ | 0.1 | 0.02 | 0    | 0    | 0    | 0.02 |       |
| $ALG - 7$ | 4.38| 4.15 | 4.17 | 4.17 | 4.17 | 4.21 |       |
| $ALG - 8$ | 2.52| 2.52 | 2.59 | 2.57 | 2.62 | 2.56 |       |

| Algorithm | $H$ | 40   | 100  | 200  | 300  | 400  | 500  | Avg.  |
|-----------|-----|------|------|------|------|------|------|-------|
| $ALG - 1$ | 40  | 9.57 | 9.7  | 9.77 | 9.78 | 9.86 | 9.74 |       |
| $ALG - 2$ | 10.04| 9.9  | 9.86 | 9.86 | 9.92 |       |
| $ALG - 3$ | 3.12| 3.03 | 3.05 | 3.07 | 3.05 | 3.06 |       |
| $ALG - 4$ | 5.25| 5.17 | 5.14 | 5.11 | 5.12 | 5.16 |       |
| $ALG - 5$ | 5.01| 5.03 | 5.05 | 5.11 | 5.1  | 5.06 |       |
| $ALG - 6$ | 0.08| 0.01 | 0    | 0    | 0    | 0.02 |       |
| $ALG - 7$ | 4.47| 4.2  | 4.21 | 4.24 | 4.19 | 4.26 |       |
| $ALG - 8$ | 2.69| 2.6  | 2.73 | 2.71 | 2.7  | 2.69 |       |

Table 5. Error of Algorithms with respect to $n$

| Algorithm | $n$ | 100  | 200  | 300  | 400  | 500  | Avg.  |
|-----------|-----|------|------|------|------|------|-------|
| $ALG - 1$ | 9.27| 9.39 | 9.45 | 9.47 | 9.51 | 9.42 |       |
| $ALG - 2$ | 9.64| 9.58 | 9.56 | 9.58 | 9.57 | 9.59 |       |
| $ALG - 3$ | 3.07| 3.02 | 3.03 | 3.05 | 3.05 | 3.04 |       |
| $ALG - 4$ | 5.09| 5.02 | 5.01 | 5.02 | 5.01 | 5.03 |       |
| $ALG - 5$ | 4.88| 4.92 | 4.94 | 4.97 | 4.99 | 4.94 |       |
| $ALG - 6$ | 0.09| 0.02 | 0.00 | 0.00 | 0.00 | 0.02 |       |
| $ALG - 7$ | 4.28| 4.14 | 4.10 | 4.10 | 4.07 | 4.14 |       |
| $ALG - 8$ | 2.49| 2.49 | 2.53 | 2.55 | 2.56 | 2.52 |       |

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Table 6. Error of Algorithms with respect to $H$

| Algorithm | 20  | 30  | 40  | Avg. |
|-----------|-----|-----|-----|------|
| $ALG - 1$ | 9.24 | 9.42 | 9.59 | 9.42 |
| $ALG - 2$ | 9.40 | 9.60 | 9.77 | 9.59 |
| $ALG - 3$ | 3.01 | 3.06 | 3.06 | 3.04 |
| $ALG - 4$ | 4.94 | 5.04 | 5.10 | 5.03 |
| $ALG - 5$ | 4.86 | 4.94 | 5.01 | 4.94 |
| $ALG - 6$ | 0.02 | 0.02 | 0.02 | 0.02 |
| $ALG - 7$ | 4.10 | 4.13 | 4.19 | 4.14 |
| $ALG - 8$ | 2.46 | 2.52 | 2.59 | 2.52 |

Table 7. Confidence Intervals for Average Errors

| Algorithm | 95% Confidence Interval on the Avg. Error |
|-----------|------------------------------------------|
| $ALG - 1$ | (0.933-9.51) |
| $ALG - 2$ | (9.50-9.68) |
| $ALG - 3$ | (2.98-3.11) |
| $ALG - 4$ | (4.95-5.10) |
| $ALG - 5$ | (4.86-5.01) |
| $ALG - 6$ | (0.02-0.03) |
| $ALG - 7$ | (4.07-4.21) |
| $ALG - 8$ | (2.45-2.59) |

Figure 3. Overall Avg. Error of Algorithms

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Figure 4. Avg. Std. of Algorithms

Figure 5. Overall Avg. Error of Algorithms with respect to $H$

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Figure 6. Overall Avg. Error of Algorithms with respect to $H$

Figure 7. Overall Avg. Error of Algorithm $ALG-6$ with respect to $H$

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