Spin phase diagram of the interacting quantum Hall liquid

H. M. Yoo¹, K. W. Baldwin², K. West², L. Pfeiffer² and R. C. Ashoori¹

¹Department of Physics, Massachusetts Institute of Technology, Cambridge, MA, USA. ²Department of Electrical Engineering, Princeton University, Princeton, NJ, USA. *e-mail: ashoori@mit.edu
Supplementary information

This file includes additional discussion on:

I. Floating-gate memory device

II. Detailed comparison between SRPT and other techniques

III. 2D - QH ferromagnet tunneling model

IV. Landau level index selection rule in 2D-2D tunneling

V. Possible effects of $B_\parallel$ in the $N = 1$ Landau level

VI. FQH skyrmion near $v = 1/3$

VII. Polarization at $v = 1/3$, 2/5, and 1
I. Floating-gate memory device

In this work, we employed a floating-gate memory device. The floating-gate structure has two advantages. First, the floating-gate structure does not require a direct contact to the 2DES and therefore allows simple device fabrication. In order to tune the total carrier density, we apply a large gate voltage to trap or to leak the charges residing in the double QWs. We then apply a smaller voltage at the same gate to partition the charges between the two QWs. Second, the heavily-doped electrodes on the top and bottom surface eliminate the long-range Coulomb disorders. Previous work\textsuperscript{1,2} demonstrated that the stability of the even-denominator FQH states weakly depends on the transport mobility. Instead, the long-range fluctuations arising from the unscreened ionized impurities in the doped layers are the dominant factor limiting the energy gap of the non-abelian ground states. In our floating-gate structure, any electric charge perturbations from the doped layers are screened within few angstroms in the AlGaAs barrier (see Supplementary Fig. 6). The long-range correlations in the floating-gate device are thus expected to be more stable against the Coulomb disorders in the doped layers.

In the bilayer device, there are several possible effects of placing the second 2DES nearby the system under study. A previous experiment\textsuperscript{3} showed that the second 2DES screens the electron-electron interactions and softens the FQH energy gap. However, in the SRPT measurements, the second 2DES is always fixed at the incompressible $\nu = 1$ state. Thus, such screening effect would likely be greatly suppressed because the immobile charges in the $\nu = 1$ insulator cannot screen the interactions in the system under study. Interlayer excitonic interactions\textsuperscript{4} can also modify the intra-layer interactions in the system under study. Although the excitonic interactions cannot completely be ruled out at extremely low magnetic fields, such effects are very weak when, as in our case, the two layers are widely separated and the sum of the LL filling factors of the two 2DESs differs from one.
II. Detailed comparison between SRPT and other techniques

Despite the progress of NMR, optical, and transport measurements, there have been substantial experimental challenges that hampered investigation of the spin-dependent phenomena in the QHE regime. In optical techniques, photoexcited holes favor the formation of skyrmions that depolarize the ground-state spin in the $N = 1$ LL$^{5-7}$. Transport methods cannot measure the spin-polarization of skyrmion ground-state nor the partially filled $\Lambda$-levels, where our work now demonstrates significant deviations from the non-interacting CF model. Also, some transport measurements rely on tilting the applied field, and this may drive spin-independent phase transitions that confound the results$^8$. Resistively-detected NMR (RD-NMR) spectroscopy$^9$ involves RF-heating and substantial changes to the carrier density that can alter the measured spin state$^{10}$. The nonuniform response also makes the analysis of RD-NMR spectra difficult$^{11}$. Finally, the weak hyperfine interactions limit the applications of RD-NMR to other materials, such as graphene and $p$-type GaAs.

SRPT provides an alternative direct approach to probe the spin-properties of a QH system. The short duty cycles of the pulsed tunneling measurement prevent the undesirable perturbations such as RF-heating or photoexcited defects that commonly occur with prior methods. A tunneling pulse width is also set much shorter than the RC charging time to minimize other possible effects that can alter the ground-state (see Supplementary Fig. 7). In addition, the simple integration of the $I$-$V$ curve gives a quantity directly proportional to the spin-polarized carrier density, whereas the deduced value of the spin-polarization in RD-NMR depends on a fit that demands a knowledge of the bound-state wavefunction and the spatial electronic structure$^9$.

Furthermore, SRPT involves bulk electron tunneling and is sensitive to a bulk property. The detection scheme in RD-NMR, on the other hand, involves a longitudinal transport measurement, which is sensitive to a portion (mostly along the edge) that contributes the electrical conductivity. Although bulk measurements
are unaffected by edge reconstruction\textsuperscript{12}, they are more prone to sample inhomogeneity. However, we note that this is not a technical problem of SRPT because the inhomogeneity effect can be mediated by reducing the sample size or improving the sample quality.

In SRPT, the use of the $\nu = 1$ ferromagnet (the strongest ferromagnetic state in a QH system) allows the precise measurements at low magnetic fields. The lower $B_\perp$ limit is set by the polarization of the $\nu = 1$ ferromagnet. At a very small $B_\perp$, the strength of the Coulomb exchange interactions is significantly reduced, and thermally excited spin-waves weaken the polarization at $\nu = 1$. Supplementary figure 8b shows the ratio ($\gamma$) of depolarized electrons in the $\nu = 1$ QH ferromagnet as a function of $B_\perp$. We deduce $\gamma$ from the integrated tunnel current when both the 2DES layers are at $\nu = 1$. The $B_\perp$ dependence of $\gamma$ reveals an Arrhenius behavior, where a decrease in the exchange Coulomb energy leads to an exponential growth of the spin-flips. Below $B_\perp = 1.5$ T, $\gamma$ rapidly increases and limits the SRPT measurement.

III. 2D - QH ferromagnet tunneling model

Tunneling current between two parallel layers are expressed as follows

\[
I(V) = \frac{4\pi e}{h} \sum_{k,k'} \int dE \int dE' |M_{k,k'}^2 D(E)D'(E')\delta_{E+E_V,E'}^\perp \times \left[ f(E,T)(1 - f'(E',T)) - f'(E',T)(1 - f(E,T)) \right],
\]

where $D(E)$ and $D'(E)$ are the density of states of each 2DES.

The Fermi functions for each layer are given by $f(E,T) = \left\{ 1 + e^{E-\mu/k_BT} \right\}^{-1}$ and $f'(E,T) = \left\{ 1 + e^{E'-\mu/k_BT} \right\}^{-1}$. The tunneling matrix element $M_{k,k'}$ is defined by

\[
M_{k,k'} = -\frac{\hbar^2}{2m_e} \left\{ \psi(z)^* \frac{\partial \psi(z)}{\partial z} - \psi'(z)^* \frac{\partial \psi(z)}{\partial z} \right\} \delta_{k,k'} \int \varphi_{n,x}(y)\varphi_{n',x'}(y)dy,
\]
where $\psi(z)$ is the out-of-plane wavefunction perpendicular to the 2DES and $\varphi_{N,x}(y)$ is the in-plane $N^{th}$ LL wavefunction centered at position $x = X$.

The first term in Eq. S2 is the matrix element perpendicular to the 2DES. At a fixed electron density, the perpendicular component is nearly constant over the tunneling energy range of the measurements\textsuperscript{13}. The last integral term is the in-plane component of the matrix element. The in-plane component is governed by the overlap integral between the two LL wavefunctions and stays constant at a fixed $B_{||}$. Detailed description of the wavefunction overlap integral can be found in Supplementary Section IV.

In the SRPT measurements, one layer is always fixed at the ferromagnetic $\nu = 1$ state. For the 2D-QH ferromagnet tunnel junction, only the spin-down electrons are allowed to tunnel into the $\nu = 1$ QH ferromagnet. Also, the following condition holds at low temperatures:

$$f(E,T)(1-f'(E',T))\delta_{E+E',E'} \gg f'(E',T)(1-f(E,T))\delta_{E+E',E'}.$$ 

Thus, the tunneling I-V characteristics of 2D-QH ferromagnet is given by the following expression:

$$I(V) = \frac{4\pi e}{h} \sum_{k,k'} |M_{k,k'}|^2 \int dE \int dE' D_{\downarrow}(E) D'_{\downarrow}(E') \delta_{E+E',E'} \left[ f(E,T)(1-f'(E',T)) \right],$$

(S3)

where $D(E)$ and $D'(E)$ are the spin-down density of states of the system under study and the $\nu = 1$ QH ferromagnet, respectively. Integrating the tunneling current in Eq. S3 yields

$$\int I(V) dV = \frac{4\pi e}{h} \sum_{k,k'} |M_{k,k'}|^2 \int dED_{\downarrow}(E)f(E,T)\int dVD'_{\downarrow}(E+eV)(1-f'(E+eV,T))$$

(S4)

The last two integral terms are equal to the occupied spin-down density in the system under study ($n_{\downarrow}$) and the unoccupied spin-down density in the $\nu = 1$ QH ferromagnet, respectively. The unoccupied spin-down density at $\nu = 1$ always equals to LL degeneracy $n_D$.

The integrated tunneling current in Eq. S4 can be simplified as follows:
\[ \int I(V)dV = \frac{4\pi e}{\hbar} \sum_{k,k'} \left| M_{k,k'} \right|^2 n_{k'} n_D \] (S5)

where \( \sum_{k,k'} \left| M_{k,k'} \right|^2 \) is the tunneling matrix element. It follows from Eq. S5 that the integrated tunneling current in 2D-QH tunnel junction gives a quantity proportional to the occupied spin-down density \( (n_\downarrow) \) in the system under study. The bare spin-polarization 

\[ P = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow} = 1 - \frac{2n_\downarrow}{\nu n_D} \]

can be therefore directly determined from the integrated tunneling current. In the \( N = 1 \) Landau level, the bare spin-polarization is expressed as follows:

\[ P = 1 - \frac{2n_\downarrow}{(\nu - 2)n_D}, \]

where \( (\nu - 2)n_D \) is equal to the electron density in the top most occupied \( (N = 1) \) LL.

**IV. Landau level index selection rule in 2D-2D tunneling**

Tunneling allows measurements of the electronic density of states (DOS). In a 3D-2D tunnel junction employed in our previous work\(^{13}\), the differential conductance \( dI/dV \) spectra measures the DOS of a 2DES in a magnetic field. Spectra taken in the presence of \( B_\perp \) show multiple staircase patterns of the equidistant LLs as a function of \( \nu \) (Supplementary Fig. 1b). In moving to a 2D-2D tunnel junction, the selection rule imposed by in-plane momentum conservation substantially modifies the tunneling spectrum. In contrast to 3D-2D spectra, the 2D-2D tunneling spectra in Supplementary Fig. 1c shows a single staircase pattern. This single staircase pattern indicates that there is only one energy that allows tunneling for any given \( \nu \). This condition occurs when the LLs with the same orbital index \( N \) are aligned at the same energy. Otherwise, tunneling into the LLs with different \( N \) is forbidden.

The \( N \) selection rule in 2D-2D tunneling can be understood as follows. In the semiclassical picture, free electrons residing in the QWs move on cyclotron orbits under \( B_\perp \). These semiclassical orbits can be understood as quantized harmonic oscillator in which electrons are confined in parabolic magnetic potentials (see Supplementary Fig. 2a). The transition probability between the LLs is proportional to the square of the
spatial overlap between the initial and final state wavefunctions. In the absence of $B_\parallel$, a transition that does not conserve $N$ is forbidden because the wavefunctions are mutually orthogonal. The selection rule of conserving $N$ is, however, broken in a tilted magnetic field due to a “momentum boost” $\hbar \Delta k_y = eB_\parallel d$ obtained by tunneling electrons, where $d$ is the physical distance between the two QWs. The additional momentum displaces the centers of the cyclotron orbits in the two QWs relative to each other by $\Delta X = B_\parallel d/B_\perp$. Non-zero overlap integrals between the two displaced harmonic oscillator wavefunctions allow tunneling with non-conservation of $N$. Therefore, an idealized 2D-2D tunneling device shows a single peak in its $I-V$ characteristic at $B_\parallel = 0$. Multiple current peaks that arise from inter-LL tunneling appear at $B_\parallel > 0$ (see Supplementary Fig. 2b and c).

V. Possible effects of $B_\parallel$ in the $N = 1$ Landau level

Due to the LL index selection rule discussed in Supplementary Section IV, we tilt the sample by $20^\circ$ to allow inter-LL tunneling between the spin-down $N = 0$ LL in the QH ferromagnet and the $N = 1$ LL in the system under study. In this section, we discuss other possible effects of an in-plane magnetic field.

First, prior transport studies$^{14,15}$ suggest that the states at exact fractional quantum Hall states at $\nu = 7/3$ or $\nu = 5/2$ are unchanged up to large tilt angles. Thus, the effect of an in-plane magnetic field on these FQH states is likely to be insignificant when the tilt angle is small.

The remaining question is whether depolarization observed between $11/5 \leq \nu \leq 7/3$ can be driven by a small in-plane magnetic field. The possibility of the anisotropic states, known as the stripe and bubble phases, cannot be ruled out since the transitions into these phases depend on the various sample parameters. However, this tilt induced effect observed in the transport studies appears symmetrically at $11/5 \leq \nu \leq 7/3$ and $8/3 \leq \nu \leq 14/5$$^{16}$. So, if this effect is responsible for depolarization at $11/5 \leq \nu \leq 7/3$, then similar depolarization should be also observed at $8/3 \leq \nu \leq 14/5$. Nevertheless, we observe strong asymmetry of $P^*$
between $11/5 \leq \nu \leq 7/3$ and $8/3 \leq \nu \leq 14/5$. Therefore, the tilt effect is unlikely to be responsible for depolarization observed at $11/5 \leq \nu \leq 7/3$.

Finally, we note that an addition of an in-plane magnetic field, in general, strengthens the spin-polarization of an electronic system. An exception can be a diamagnetic state, where an additional in-plane magnetic field polarizes the electronic spins antiparallel to the probe quantum Hall ferromagnet. Although such a radical scenario (diamagnetism) cannot be 100% ruled out, it still supports our argument that the free composite fermion model cannot explain the measured spin polarization in the $N = 1$ Landau level.

VI. FQH skyrmion near $\nu = 1/3$

In the vicinity of the integer ferromagnetic QH state at $\nu = 1$, the skyrmion formation is energetically favored due to the strong exchange interactions between electrons. In a similar manner, the fractional skyrmions are predicted near the ferromagnetic FQH state at $\nu = 1/3$. However, a small applied magnetic field ($B_\perp > 5$ T in our study) can suppress the fractional skyrmions, whereas the integer quantum Hall skyrmions can survive at a largest magnetic field ($B_\perp \sim 10$ T) we applied. This contrasting behavior of the integer and fractional skyrmions arises from the fact that the inter-CF interactions are much weaker than the inter-electron interactions in the integer quantum Hall states. Furthermore, the sizes of the fractional skyrmion and anti-skyrmion are not equal because the energies of creating the fractional skyrmions at $\nu > 1/3$ and anti-skyrmions $\nu < 1/3$ are not equal. In a finite magnetic field, the anti-skyrmions at $\nu < 1/3$ are predicted to be more stabilized than the skyrmion at $\nu > 1/3$. Figure 3d supports such asymmetric behavior. Between $B_\perp = 2$ T and $B_\perp = 5$ T, the free CF model predicts full polarization at $\nu < 1/3$. In the same magnetic field range, the CF states at $\nu > 1/3$ are partially polarized. However, the measured $P^*$ shows very strong depolarization at $\nu < 1/3$ while the depolarization observed at $\nu > 1/3$ is not significantly large compared to the phase diagram calculated from the free CF model.
VII. Polarization at $\nu = 1/3$, 2/5, and 1

Figure 2a shows that the measured $P^*$ values at $\nu = 1/3$, 2/5, and 1 are smaller than one at $B_\perp > 2$T, whereas the theory predicts the full polarization at these quantum Hall states. This deviation from the full polarization is expected when a small inhomogeneity is taken into account. For example, according to the theoretical CF spin phase diagram (see Fig. 3b), a very small deviation (a few percent change in the electron density) at $\nu = 2/5$ can create a partially polarized state even at a high magnetic field. Therefore, in a real sample, where the electron density is not perfectly uniform, the electrons at the exact FQH states are unlikely to be fully-polarized, but instead, $P^*$ attains an intermediate value depending on the spin-states near the phase boundary. We note that other measurements, including the NMR study\textsuperscript{9}, shows partially polarized $\nu = 2/5$ and $\nu = 1$ states, which deviate from the theoretically predicted fully polarized states.
Supplementary Fig. 1: Comparison between 3D-2D and 2D-2D tunnel junctions. a, Schematic views of 3D-2D and 2D-2D tunnel devices. b,c, Tunneling spectra measured at a fixed $B_\perp = 2$ T. Insets show the schematic energy diagram for the 3D-2D (b) and 2D-2D (c) tunnel junctions. Vertical axis is the energy $E$ referenced to the Fermi energy $E_F$. The horizontal axis is $\nu$. The color scale in the 3D-2D spectra is proportional to $dI/dV$. In the 2D-2D spectra, the tunneling current ($I$) is scaled by the topmost occupied LL index ($N + 1$) because the spectra at high energies encompass tunneling of electrons ejected from all the occupied LLs below $E_F$. The scaling factor ($N + 1$) is constant within the same LL (e.g. $0 \leq \nu < 2$, $2 \leq \nu < 4$, and $4 \leq \nu < 6$). b, In the 3D-2D tunnel junction, the $dI/dV$ measures the DOS of the 2DES under study. The $dI/dV$ spectra show multiple staircase patterns of LLs as a function of $\nu$. c, In the 2D-2D tunnel junction, quantized energy levels develop in both 2D systems. The 2D-2D tunneling spectra show a single staircase pattern. Due to conservation of in-plane momentum, there is only one energy that allows tunneling for any given $\nu$. This condition occurs when the LLs with the same orbital index $N$ are aligned at the same energy (see Supplementary Fig. 2 and Supplementary Section IV for a detailed explanation).
**Supplementary Fig. 2: The Landau level index selection rule for 2D-2D tunneling.**

*a,* A simple cartoon model describing the effect of $N$ selection rule. In the presence of $B_\perp$, 2D electrons move on cyclotron orbits. These semiclassical orbits can be understood as a quantized harmonic oscillator. An additional in-plane field $B_\parallel$ displaces the centers of the cyclotron orbits as well as the harmonic oscillators (see Supplementary Section IV). The inter-LL transitions depend on the overlap integral of the two displaced harmonic oscillator wavefunctions.

*b,*c,* Tunneling $I$-$V$ characteristics in both $B_\perp$ and $B_\parallel$. At $B_\parallel = 0$ (blue), there is a single tunneling peak. Inter-LL tunneling that does not conserve $N$ is forbidden because the wavefunctions are orthogonal. On the other hand, at $B_\parallel > 0$, tunneling with non-conservation of $N$ is allowed as a result of the non-zero overlap integral between the two displaced wavefunctions (black).
Supplementary Fig. 3: The $P^*$ phase diagram measured as functions of $B_\parallel$ and $\nu$ at a fixed $B_\perp = 5.2$ T. Color scale is adjusted to highlight the contrast. Dashed lines are guides to the eye. Note that the IQH skyrmion is suppressed at the largest applied $B_\parallel$, whereas the FQH skyrmion near $\nu = 1/3$ is already suppressed at $B_\parallel = 0$ T.
Supplementary Fig. 4: A possible explanation for partially polarized \( \nu = 4/5 \) and \( 6/5 \) states. 

a, The fully polarized \( \nu = 1/5 \) and \( \nu = 9/5 \) FQH states correspond to the \( \nu^* = 1 \) IQH state of the four-flux CFs. When the LLs are spin-polarized at a large \( B \), the particle-hole conjugate states at \( \nu = 4/5 \) and \( \nu = 6/5 \) are expected in the spin-up and spin-down branches of the \( N = 0 \) LL. 

b, A possible candidate for the observed partially polarized \( \nu = 4/5 \) and \( \nu = 6/5 \) states. These behaviors cannot be understood in terms of the non-interacting model. One possible alternative model\(^{21,22} \) is interacting two-flux CFs at \( 2/3 < \nu \leq 4/5 \) and \( 6/5 \leq \nu < 5/3 \). For instance, the \( \nu = 4/5 \) and \( \nu = 6/5 \) states correspond to the \( \nu^* = -4/3 \) states of the two-flux CFs (the negative sign of \( \nu^* \) represents the negative effective magnetic field). When the CFs form the \( \nu^* = -4/3 \)
FQH state, the partially spin-depolarized FQH states are allowed at $\nu = 4/5$ and $\nu = 6/5$. c, The selected region of the $P^*$ phase diagram in Fig. 3a. d,e, Spin-polarization of the two-flux CF phases is calculated using the same effective mass $m^*_{\text{CF}} = 0.66m_e\sqrt{B_\perp}$. For the four-flux CF phases, an effective mass $m^*_{\text{CF}} = 0.34m_e\sqrt{B_\perp}$ deduced from a prior transport measurement$^{23}$ is assumed. The measured $P^*$ shows a similar trend to the two-flux CF model, where the $\nu = 4/5$ and the nearby states are spin-unpolarized over a broad range of $B_\perp$ (see also the $B_\parallel$ dependence in Supplementary Fig. 3). The agreement between the measured $P^*$ and the two-flux CF model suggests that the spin-unpolarized FQH states at $2/3 < \nu \leq 4/5$ and $6/5 \leq \nu \leq 4/3$ are likely to be the FQH states of interacting two-flux CFs.

Supplementary Fig. 5: Integrated tunneling current in the $N = 1$ LL at different tilt angles. Due to the $N$ selection rule, the tunneling current and the signal-to-noise ratio are substantially reduced at $\theta = 0^\circ$. Although determination of $P^*$ becomes challenging near $\theta = 0^\circ$, the small integrated current near $\nu = 5/2$ and $\nu = 8/3$ indicates full spin-polarization and is consistent with the result at $\theta = 20^\circ$. Theory also predicts that the effect of $B_\parallel$ on the electron interactions is not significant at a small $\theta^{24}$. 
Supplementary Fig. 6: Comparison between the conventional modulation doped (left) and the floating-gate memory (right) structure. The conventional structure requires the Si doping layers that induce the long-range fluctuations in the QW. In the floating-gate structure, the charge fluctuations in the doped layer exponentially decay over the Thomas-Fermi screening length and cannot influence the system under study.
Supplementary Fig. 7: Comparison between SRPT (left) and resistively-detected NMR (right) measurement sequence. SRPT uses a short pulse sequence that consists of a 100 ns tunneling pulse, an opposite discharge pulse, and a 50 μs delay. Resistively-detected (RD-NMR) measurement employs a longer process of nuclear polarization, rf radiation, and readout. In the RD-NMR, the change of longitudinal resistance is measured as a function of RF frequency. In SRPT, spin-polarized tunnel current ($I$) is measured as a function of tunneling pulse heights ($V$).
Supplementary Fig. 8: Tunneling matrix element calibration. **a,** Due to the finite thickness of the QW, the wavefunction moves away from a tunnel barrier as the electron density increases. The tunneling matrix element gradually changes as $\nu$ increases at a fixed $B_{\perp}$. **b,** Comparison between uncorrected (black) and corrected (red) $P$ curves. The matrix element correction compensates for the out-of-plane wavefunction shift in a QW. Note that the application of matrix element correction does not change the qualitative features such as the peaks (maxima) and dips (minima).

Supplementary Fig. 9: The lower $B_{\perp}$ limit of SRPT measurements. **a,** A cartoon illustrating spin-flips in the $\nu = 1$ QH ferromagnet at small $B_{\perp}$. The spin energy gap $\Delta_{\nu=1}$ is proportional to the exchange interaction that grows as $e^2/l_B$ and $\sqrt{B_{\perp}}$. When the spin gap decreases at low $B_{\perp}$, spin waves are thermally excited and reduce the spin-polarization.  
**b,** The ratio ($\gamma$) of depolarized electrons in the QH ferromagnet as a function of $B_{\perp}$ at a fixed $T = 30$ mK. At $B_{\perp}$ below 1.5 T, $\gamma$ rapidly increases and limits the SRPT measurements.
Supplementary Fig. 10: Low frequency magnetocapacitance in the $N = 1$ LL. a, $B_{\perp}$ dependence of sample capacitance measured at a fixed total density $n_{\text{tot}} = 3.61 \times 10^{11}$ cm$^{-2}$. A low frequency ac excitation ($f = 17$ Hz and $V_{\text{ac}} = 265 \mu$V) is applied. Dashed lines are guides to the eye showing the filling factors of the system under study (red) and the second 2DES (green). b, $n_{\text{tot}}$ dependence of low frequency ($f = 28$ Hz) capacitance measured at a fixed $B_{\perp} = 3.75$ T. Horizontal axis is the dc gate voltage $V_{\text{dc}}$ that partitions the total carriers ($n_{\text{tot}}$) into the two layers. c, Temperature dependence at a fixed $B_{\perp} = 3.75$ T and a fixed total density $n_{\text{tot}} = 3.38 \times 10^{11}$ cm$^{-2}$.

Reference

1. Pan, W. et al. Impact of Disorder on the 5/2 Fractional Quantum Hall State. Phys. Rev. Lett. 106, 206806 (2011).
2. Deng, N. et al. $\nu = 5/2$ Fractional Quantum Hall State in the Presence of Alloy Disorder. Phys. Rev. Lett. 112, 116804 (2014).
3. Eisenstein, J. P., Pfeiffer, L. N. & West, K. W. Compressibility of the two-dimensional electron gas: Measurements of the zero-field exchange energy and fractional quantum Hall gap. Phys Rev B 50, 1760–1778 (1994).
4. Eisenstein, J. P., Pfeiffer, L. N. & West, K. W. Interlayer interactions and the Fermi energy of bilayer composite-fermion metals. Phys. Rev. B 98, 201406 (2018).
5. Kukushkin, I. V., Klitzing, K. v. & Eberl, K. Spin polarization of two-dimensional electrons in different fractional states and around filling factor $\nu = 1$. *Phys. Rev. B* **55**, 10607–10612 (1997).

6. Wójs, A., Möller, G., Simon, S. H. & Cooper, N. R. Skyrmions in the Moore-Read State at $\nu = 5/2$. *Phys. Rev. Lett.* **104**, 086801 (2010).

7. Stern, M. *et al.* Optical Probing of the Spin Polarization of the $\nu = 5/2$ Quantum Hall State. *Phys. Rev. Lett.* **105**, 096801 (2010).

8. Eisenstein, J. P. *et al.* Collapse of the Even-Denominator Fractional Quantum Hall Effect in Tilted Fields. *Phys. Rev. Lett.* **61**, 997–1000 (1988).

9. Tiemann, L., Gamez, G., Kumada, N. & Muraki, K. Unraveling the Spin Polarization of the $\nu = 5/2$ Fractional Quantum Hall State. *Science* **335**, 828–831 (2012).

10. Willett, R. L. The quantum Hall effect at 5/2 filling factor. *Rep. Prog. Phys.* **76**, 076501 (2013).

11. Stern, O. *et al.* NMR study of the electron spin polarization in the fractional quantum Hall effect of a single quantum well: Spectroscopic evidence for domain formation. *Phys. Rev. B* **70**, 075318 (2004).

12. Marguerite, A. *et al.* Imaging work and dissipation in the quantum Hall state in graphene. *Nature* **575**, 628–633 (2019).

13. Dial, O. E., Ashoori, R. C., Pfeiffer, L. N. & West, K. W. High-resolution spectroscopy of two-dimensional electron systems. *Nature* **448**, 176–179 (2007).

14. Xia, J., Eisenstein, J. P., Pfeiffer, L. N. & West, K. W. Evidence for a fractionally quantized Hall state with anisotropic longitudinal transport. *Nat. Phys.* **7**, 845–848 (2011).

15. Dean, C. R. *et al.* Contrasting Behavior of the 5 2 and 7 3 Fractional Quantum Hall Effect in a Tilted Field. *Phys. Rev. Lett.* **101**, 186806 (2008).

16. Csáthy, G. A. *et al.* Tilt-Induced Localization and Delocalization in the Second Landau Level. *Phys. Rev. Lett.* **94**, 146801 (2005).

17. Brey, L., Fertig, H. A., Côté, R. & MacDonald, A. H. Skyrme Crystal in a Two-Dimensional Electron Gas. *Phys. Rev. Lett.* **75**, 2562 (1995).
18. Kamilla, R. K., Wu, X. G. & Jain, J. K. Skyrmions in the fractional quantum Hall effect. *Solid State Commun.* **99**, 289–293 (1996).

19. Balram, A. C., Wurstbauer, U., Wójs, A., Pinczuk, A. & Jain, J. K. Fractionally charged skyrmions in fractional quantum Hall effect. *Nat. Commun.* **6**, 8981 (2015).

20. Wójs, A. & Quinn, J. J. Skyrmions in integral and fractional quantum Hall systems. *Solid State Commun.* **122**, 407–411 (2002).

21. Balram, A. C., Tőke, C., Wójs, A. & Jain, J. K. Phase diagram of fractional quantum Hall effect of composite fermions in multicomponent systems. *Phys. Rev. B* **91**, 045109 (2015).

22. Liu, Y. *et al.* Fractional Quantum Hall Effect and Wigner Crystal of Interacting Composite Fermions. *Phys. Rev. Lett.* **113**, 246803 (2014).

23. Yeh, A. S. *et al.* Effective Mass and g Factor of Four-Flux-Quanta Composite Fermions. *Phys. Rev. Lett.* **82**, 592–595 (1999).

24. Peterson, M. R., Jolicoeur, Th. & Das Sarma, S. Orbital Landau level dependence of the fractional quantum Hall effect in quasi-two-dimensional electron layers: Finite-thickness effects. *Phys. Rev. B* **78**, 155308 (2008).