One–neutron transfer reaction: a toy model in one dimension

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Abstract. A simple 1D toy model to study one–neutron transfer reactions is developed. It is based on the solution of the time dependent Schroedinger equation for a particle initially bound by a fixed potential well, perturbed by a second moving potential, which accounts for the second partner of the reaction. At the end of the time evolution it is possible to evaluate the probability of the transfer of the particle from a potential to the other, as well as the transfer to continuum states in the case of weakly–bound systems. Although rather simple, the model accounts for most of the physical characteristics of these kind of reactions: such as the existence of an optimum Q–value and the dependence on the parameters defining the relative motion of the two potentials.

1. Introduction
Nuclear reactions between heavy ions are traditionally and usefully divided in two groups, direct and compound reactions, depending on the parameters of the reaction, such as bombarding energy and impact parameter. Direct reactions go directly from initial to final state in a single step, while in compound reactions the projectile experiences subsequent collisions leading to the formation of an intermediate system. Of course, it may be possible that direct and compound–nucleus processes both contribute to a given reaction, but in general the greater the incident energy in a reaction, the less likely the possibility of multi step processes.

The usual characteristic of a direct reaction found in the experiments is a tendency for only small amounts of momentum to be transferred, thus the outgoing particle will carry most of the incident energy and momentum of the projectile; since the interaction is restricted mainly in the nuclear surface (peripheral collisions), a typical signature is an angular distribution peaked around the grazing angle. Moreover direct processes occur very rapidly in a time of the order of $10^{-22}$ s, while a compound process typically takes much longer, perhaps $10^{-16}$ to $10^{-18}$ seconds. Among direct reaction there are different channels, each one characterized by a particular behavior: inelastic scattering excites collective states; one–neutron transfer probes the single–particle character of states; two–nucleon transfer goes preferentially to states that exhibit strong pairing correlations. It is possible that two–neutron, two–proton transfers may reveal alpha clustering, and so on. Since direct reactions are highly selective in the nuclear levels that they populate, they are useful tools to investigate the nuclear structure.
Transfer reactions are special kind of direct reaction in which one or more nucleons are transferred between the projectile and the target. Some example with light ions could be (d,p) and (t,p)
reactions, extensively studied at radioactive beam facilities as ISOLDE, MSU, GANIL and TRIUMF. A scheme of a transfer reaction is presented in figure 1.

![Figure 1: A transfer reaction scheme: an incoming nucleus picks up a cluster or a particle from a target nucleus.](image)

Here I am particularly interested in describing one–neutron transfer reactions. To study this kind of reactions I am developing a 1D toy model. At variance with the usual couple–channel approach, which may become rather cumbersome and suffers from the inevitable truncation of the basis, I can provide an exact calculation for the time evolution of the system. A comparison between these two methods is presented in [1].

2. The model

I solve the one–particle time dependent Schroedinger equation

\[ i\hbar \frac{d}{dt} \Psi(x, t) = \mathcal{H}(x, t)\Psi(x, t) \]  

(1)

with Hamiltonian

\[ \mathcal{H}(x, t) = \frac{\hbar^2}{2\mu} \frac{d^2}{dx^2} + V_T(x) + V_P(x_P(t)). \]  

(2)

The target mean–field potential is parametrized by a Woods–Saxon well

\[ V_T(x) = \frac{V_T}{1 + e^{\frac{|x| - R_T}{\alpha_T}}}. \]  

(3)

The initial wave function \( \Psi(x, t = 0) \) is a single particle state in the target potential, calculated applying a Numerov algorithm [2]. The projectile potential

\[ V_P(x) = \frac{V_P}{1 + e^{\frac{|x - x_P(t)| - R_P}{\alpha_P}}}. \]  

(4)

moves with the time dependence of

\[ x_P(t) = x_0 + \frac{1}{2} \alpha t^2, \]  

(5)

where \( x_0 \) is the distance of the closest approach (the distance between the potentials centers). The calculation starts from a distance long enough to avoid interactions with the target. The acceleration could be fixed taking into account \( \mu a = -\frac{d}{dr} V_c(r) \) considering the reduced mass of the system, but, since I am interested to the response of the system at the variation of each parameter, I will consider it as an adjustable parameter.

In figure 2 some steps of the time evolution are reported. The upper part of each picture gives the square of the wave function, while the lower one gives the actual position of the two wells.

1 I made use of D02PVF and D02PCF functions provided by NAG library.
Figure 2: Initially the particle stands in the target potential (a). During the time evolution the projectile approaches (b) and at a certain distance it starts to interact with the target and there is the possibility to transfer the particle (c). When the projectile reaches the distance of closest approach (d), it partially overlaps with the target (the two nuclei do not fuse, this is a peripheral collision). Then the ejectile comes back (e) and when it is far enough one can calculate the transfer probability (f).
3. Results

In order to study the different behaviour of bound and weakly–bound nuclei, for the target and projectile potentials I consider here Woods–Saxon wells with same radius, with depths that only admit one bound state. The two depths are chosen in such a way that the first one generates a weakly–bound level, the second a well bound (respectively $V_1$ and $V_2$ in (6)).

\[
V_1 = -5.00 \text{ MeV} \quad \alpha_T = 0.67 \text{ fm} \quad R = 2.00 \text{ fm} \\
V_2 = -12.00 \text{ MeV} \quad \alpha_P = 0.65 \text{ fm} \quad \mu = 0.975 \text{ amu}
\]

(6)

The different combinations of $V_1$ and $V_2$ parameters and the different energy levels obtained (at a distance of 40 fm between the two wells) are presented in figure 3.

Figure 3: Different combination of potential parameters and corresponding eigenvalues and wave functions. In panel (a) both potentials have depth $V_1$, while in panel (b) both have $V_2$. In panel (c) and (d) they have depths $V_1$ and $V_2$ respectively and vice versa.

3.1. Optimum Q–value

The results for the transfer probabilities obtained within the model confirm both theoretical and experimental findings: the optimum Q–value (difference between energy levels) for the transfer of an uncharged particle without change of angular momentum is zero; in fact leaving other parameters unchanged the transfer probability has a maximum when initial and final states in the two wells have the same energy. So for the cases in panels (c) and (d) of figure 3 the transfer probability is highly suppressed, while it is almost maximum in situations like those in panels (a) and (b). In figure 4 the transfer probability is reported as a function of the Q–value.

Figure 4: Transfer probability as a function of the Q–value, that is the energy difference between final and initial single–particle states. Optimum Q–value corresponds to zero, where in fact it is found maximum probability.
3.2. Bound and weakly-bound systems

In the optimum Q-value condition, the region corresponding to peripheral collision presents an high transfer probability. For weakly-bound situation transfer occurs more likely, because the wavefunctions extend over a larger range, but excitation to the continuum (break-up reaction) also comes into play. One can in fact observe that as the system becomes less bound, breakup would be more probable than transfer. In figure 5 the transfer probability is reported with respect to the distance of closest approach; the case of two weakly-bound nuclei (like those in figure 3a) is shown on the left, while the bound case (as in figure 3b) is represented on the right side.

![Figure 5: Transfer probability with respect to distance of closest approach, for both bound and weakly-bound systems.](image)

4. Conclusions and goals for the future

A one dimensional toy model turns out to be a useful tool even to investigate the properties of rather complex systems. Its simplicity does in fact allow to overcome all those mathematical complexities which usually hinder the full solution. The model encompasses many characteristics of transfer reactions, such as optimum Q-value and distance of closest approach dependence. The next goal will be to extend the model to two-particle systems in order to describe pair transfer reactions in connection with pairing interaction.

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References

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