Simulation of pressure induced length change of an optical cavity used for optical pressure standard

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Abstract. The National Institute of Metrology (NIM) has constructed a dual optical cavity for the optical pressure standard project. The elastic deformation induced by pressure is investigated by finite element analysis (FEA) simulation. The distortion coefficient, fractional length change per unit pressure, is calculated to be (9.936 ± 0.014)×10⁻¹²/Pa for the measurement channel and (1.49 ~ 1.54)×10⁻¹¹/Pa for the reference channel. The gravity-induced deformation is also modelled to evaluate the sensitivity to the low-frequency vibration. The cavity is modelled to be supported symmetrically on four points. With optimized support position, the sensitivity to vertical acceleration reduces to 19 kHz/(m/s²) at 633 nm.

1. Introduction
A Fabry-Perot cavity (optical resonator) can be used to measure the refractive index of gases with very high resolution [1]. It’s very promising to establish a so-called optical pressure standard based on refractive index measurements, which allows for a new realization of the Pascal as suggested [2-5]. To determine the pressure a laser is locked to the optical cavity while the laser frequency is interrogated. The frequency will change according to the pressure of the filling gas. The change of frequency is not only due to the refractive index of gas, but also due to the distortion of cavity induced by the pressure. A dual optical cavity using an evacuated reference channel within the same spacer material has the advantage that the length changes are ideally the same for both channels and thus cancelled out [6]. In fact, there is still a residual difference of length change between the two channels i.e. because of mirror bending. A dual optical cavity has been constructed for the optical pressure standard project at the National Institute of Metrology (NIM). In this paper, finite element analysis (FEA) is used to simulate the cavity deformation under pressure and gravity.

2. Methods of simulation
Figure 1 (a) shows the model of the cavity, which consists of a cuboid spacer, two pairs of mirrors and a cylindrical vacuum port. All parts are made from ultra-low expansion (ULE) glass. The spacer is 100 mm long, 50 mm wide and 60 mm high. There are two lengthwise through-holes in the spacer. The holes are 10 mm in diameter, and are vertically distributed on the central line of the spacer cross section. A pair of concave and plano mirrors is bonded at each end of the through-holes in the spacer. The mirrors are 25.4 mm in diameter and 6.35 mm thick. The concave mirror has a 500 mm radius of curvature (ROC) on the inner side and a 4 mm planar annulus region contacting to the spacer. Because of the 10 mm hole, the plano mirror has a annulus contact region of 7.7 mm. The
mirrors have high reflection coatings on the inner side and high transmission coatings on the outer side for the laser wavelength around 633 nm. The top through-hole in the spacer is the so-called “measurement” channel and communicates with the outside of the cavity through a 2 mm lengthwise slot at the top of the spacer. The cylindrical vacuum port is 60 mm long and 30 mm in diameter with a 10 mm through-hole, and is bonded to the spacer at the bottom also by optical contact method. The bottom through-hole in the spacer is the “reference” channel, and is maintained at vacuum through the vacuum port. The whole cavity will be placed in a pressure chamber and immersed in the helium, argon or nitrogen ambience.

Figure 1. (a) 3D view of the cavity model. (b) A quarter of the cavity model with symmetry planes. (c) The quartered model after meshing by tetrahedral elements. (d) FEA results under bulk modulus compression.

The static structural package of the ANSYS workbench is used to simulate the elastic deformation of the cavity under the influence of pressure and gravity. Considering the symmetry of the cavity, only a quarter of the cavity is modelled as shown in figure 1 (b). The model is meshed by tetrahedral elements with a size of 2 mm, and has a refinement of mesh at the mirrors, as shown in figure 1 (c). For the properties of ULE glass, we use the mass density of 2210 kg/m³, Young’s modulus $E=67.6 \text{ GPa}$, and Poisson’s ratio $\nu=0.17$ [7]. Valid boundary conditions and constraints are important to obtain accurate FEA results. To verify these settings used in our simulation, we first consider a simple problem, that is the pressure is loaded on all the surfaces of the cavity, and the gravity is not applied. Figure 1 (d) is the FEA result, which shows the undeformed and deformed profile of the cavity, and the x-axis displacements expressed by the colour scale. As expected, the cavity is homogeneously compressed with a fractional length change of $9.76 \times 10^{-12}/\text{Pa}$, which is the same as the theoretical calculation $1/(3K)$ using the bulk modulus $K=E/[3(1-2\nu)]=34.14 \text{ GPa}$.

3. Numerical results

We first numerically simulate the deformation of the cavity in an ambience of gas pressure while the reference channel is maintained at vacuum, and the length changes of the optical paths in the two channels are evaluated. Then the influence of gravity is simulated. The elastic deformation caused by low-frequency vibration can be modelled by applying a gravitylike force on the cavity [7]. Hence, the length changes under the gravity are used to evaluate the vibration sensitivity. To minimize the vibration sensitivity, we model several support configurations.

3.1. Deformation under pressure

A pressure of 100 kPa is applied to the measurement channel and the outer surface of the cavity, and the cavity is supported by the bottom surface of the spacer. Figure 2 shows the displacements of the
inner surfaces of the mirrors in the direction of optical path. The probe points are on the central line of the mirrors and along the vertical direction.

For the measurement channel, besides the contraction of the optical length, a tilt of both mirrors is observed as shown in figure 2 (a) and (b), forming a capital letter “V”. The tilt angle is 30.2 nrad for the concave mirror and 30.7 nrad for the plano mirror, thus the total angle is about 61 nrad. The length change caused by the tilt is proportional to the square of the tilt angle [7], and can be neglected. From the displacements of the mirrors, the fractional length change caused by bulk modulus compression, i.e. the distortion coefficient is calculated to be \((9.936 \pm 0.014) \times 10^{-12}/\text{Pa}\) in the 5 mm central region.

For the reference channel, large bending of mirrors is expected as shown in figure 2 (c) and (d). Because the concave mirror has a smaller contact area with the spacer, the bending is larger than that of the plano mirror. The distortion coefficient in the reference channel is \((1.49 \sim 1.54) \times 10^{-11}/\text{Pa}\) in the 5 mm central region.

Figure 2. The x-axis displacements of the concave mirror (a) and plano mirror (b) for the measurement channel, and (c) (d) for the reference channel.

3.2. Deformation under gravity

With the bottom of the spacer supported and only downward gravity force applied on the cavity, the elastic deformation of the cavity is calculated and the results are shown in figure 3. The maximum horizontal displacements occur at the mirrors for the reference channel, and are about 0.17 nm. This gravity-induced displacement does not affect the pressure distortion coefficients as calculated in Sec. 3.1, because the gravity always exists and the pressure distortion coefficients represent the relative change induced by pressure. However, the gravity-induced displacement reveals the sensitivity to the low-frequency vibrational perturbations. As shown in figure 3 (b) and (c), the sensitivity to vertical acceleration is about \(1.6 \times 10^9/\text{g}\) for the measurement channel and \(3.2 \times 10^9/\text{g}\) for the reference channel. This corresponds to a laser frequency sensitivity of 77 kHz/(m/s²) and 155 kHz/(m/s²) at 633 nm respectively.

To minimize the vibration sensitivity, we employ an idea of arranging the support to be in the symmetry plane [8]. The cavity is modelled to be supported symmetrically on four points through the blind holes as shown in figure 4 (a). Two parameters \(d\) and \(z\) are modified to find the optimized position. Figure 4 (b) shows the fractional length changes with different configurations of \(d\) and \(z\). At \(d=18\) mm and \(z=21\) mm, the sensitivity to vertical acceleration reduces to about \(4 \times 10^{-10}/\text{g}\), i.e. 19 kHz/(m/s²) for both channels.
4. Conclusions
We have simulated the pressure and gravity induced elastic deformation of a dual Fabry-Perot cavity using FEA. The pressure distortion coefficients are determined for the measurement and reference channels. For optical pressure standard, helium absorption into ULE glass may be a problem [9]. An alternative material Zerodur could be used, and it has a higher Young’s modulus of 90.3 GPa. Also the cavity could be improved by using thicker mirrors and smaller holes to reduce the mirror bending. In the future, the distortion coefficients will be measured experimentally to obtain lower uncertainties. The gravity-induced deformation is calculated to evaluate the vibration sensitivity. With the cavity supported symmetrically on four points and optimized support position, the sensitivity to vertical acceleration could reduce to about 19 kHz/(m/s²) at the laser wavelength of 633 nm.

References
[1] Stone J A and Stejskal A 2004 Metrologia 41 189
[2] Egan P F, Stone J A, Ricker J E and Hendricks J H 2016 Rev. Sci. Instrum. 87 053113
[3] Egan P F, Stone J A, Ricker J E, Hendricks J H and Strouse G F 2017 Opt. Lett. 42 2944
[4] Jousten K, Hendricks J, Barker D, Douglas K, Eckel S, Egan P, Fedchak J, Flügge J, Gaiser C, Olson D, Ricker J, Rubin T, Sabuga W, Scherschligt J, Schödel R, Sterr U, Stone J and Strouse G 2017 Metrologia 54 S146
[5] Hendricks J 2018 Nat. Phys. 14 100
[6] Egan P F, Stone J A, Hendricks J H, Ricker J E, Scace G E and Strouse G F 2015 Opt. Lett. 40 3945
[7] Nazarova T, Riehle F and Sterr U 2006 Appl. Phys. B 83 531
[8] Notcutt M, Ma L-S, Ye J and Hall J L 2005 Opt. Lett. 30 1815
[9] Avdiaj S, Yang Y, Jousten K and Rubin T 2018 J. Chem. Phys. 148 116101