Constraints on fall-back discs in radio pulsars and anomalous X-ray pulsars

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ABSTRACT
Calculations have been made of fall-back disc heating by the pulsar wind as distinct from the soft X-rays emitted by the neutron-star surface. The relation between these heating rates and measured near-infrared fluxes in the K and Ks bands places severe constraints on the inner radii of any fall-back discs that may be present in radio pulsars and in some anomalous X-ray pulsars. The lower limits found are so large that the discs concerned can have no significant effect on pulsar spin-down.

Key words: accretion, accretion discs – stars: neutron – pulsars: general.

1 INTRODUCTION
Dust and debris discs are a commonplace phenomenon in astrophysics, being a feature of the major planets and of pre-main-sequence stars (see e.g. Beckwith et al. 1990). The properties of dust discs or viscous ionized discs formed as a consequence of fall back in supernovae have been a matter of continuing interest in neutron-star physics. But until recently, there has been no observational evidence that they exist. For a bibliography, we refer to Perna, Hernquist & Narayan (2000), Menou, Perna & Hernquist (2001), Blackman & Perna (2004), Eksi & Alpar (2005), Eksi, Hernquist & Narayan (2005) and to the many earlier papers on the subject cited by these authors.

Further interest has been stimulated by flux measurements on the anomalous X-ray pulsar 4U 0142+61 in the 4.5 and 8.0 μm infrared bands (Wang, Chakrabarty & Kaplan 2006). The energy spectrum derived from these measurements, and from earlier Ks-band and optical fluxes, is consistent with blackbody emission from a dust disc with an inner radius surface temperature of 1200 K, superimposed on a power-law spectrum. The optical emission, which is pulsed at the neutron-star rotation frequency, consists almost entirely of the power-law component, which is assumed to be of magnetospheric origin.

There are also flux limits at 4.5, 8.0 and 24 μm in the mid-infrared for a limited number of isolated neutron stars (Wang, Kaspi & Higdon 2007b) and for some supernova remnants at 4.5 and 8.0 μm (Wang, Kaplan & Chakrabarty 2007a). But in the K and Ks bands, a considerable number of magnitudes have been measured. These have been summarized by Mignani et al. (2007) and include radio pulsars, anomalous X-ray pulsars (AXP) and soft-gamma repeaters (SGR). The purpose of this paper is to see how these measurements constrain possible fall-back disc parameters.

Fall-back disc formation is a detail of the supernova event and no attempt is made to describe it here. Initially, the disc must have been internally ionized and hence viscous, with mass and angular momentum transfer rates intrinsic to its formation. We assume that at some later time, a transition to a passive disc occurs through the thermal ionization instability (see Menou et al. 2001) in which free electron recombination produces a sudden decrease in opacity which propagates rapidly inward. The disc formed has little internal ionization and viscosity, and is gaseous. Its luminosity is then principally a result of interaction with the pulsar wind or with blackbody radiation from the neutron-star surface. The distribution of mass between dust grains and gaseous molecules depends on the various sublimation temperatures concerned. Our assumption is that, at the time of observation, the inner radius of the disc lies outside the light-cylinder radius RLC of the neutron star. An important consequence is that we can treat the disc as a partial termination of the pulsar wind rather than as a boundary condition to be satisfied by the solution for the magnetospheric fields.

A previous paper (Jones 2007) on the interaction of a thin passive disc with the pulsar wind described the ablation processes in some detail and calculated the alignment and precession torques acting on the disc. Section 2 of this paper extends the discussion of disc luminosity. The K- and Ks-band magnitudes listed by Mignani et al. for sets of radio pulsars and AXP or SGR are used, in Section 3, to constrain the inner radii and temperatures of passive fall-back discs that might be present in these objects. Where possible, the predicted mid-infrared fluxes from these objects are compared with the flux limits found by Wang, Kaspi & Higdon.

2 INTERACTION WITH THE PULSAR WIND
The Deutsch vacuum solution for the electromagnetic fields E and B of a rotating neutron star (see Michel & Li 1999; Eksi & Alpar 2005) makes it possible to write down explicit expressions for the...
spherical polar components of the momentum density $p$,

$$p = \frac{1}{4\pi c} D \times B,$$

(1)
in the inertial frame at radii $r > R_{LC}$, the $z$-axis being parallel with the neutron-star spin angular velocity $\Omega$. The momentum density in a physical pulsar wind, in which both fields and relativistic particles are present, may differ from this in detail, but the time-averaged long-range part of the azimuthal component ($p_\phi$) must be finite because it is directly related to the neutron-star spin-down torque (see section 2.1 of Jones 2007). The composition of the wind does not significantly affect its interaction with the disc and it is therefore correct to use this quantity derived from the vacuum solution in calculations of ablation and, in particular, of alignment and precession torques.

The vacuum-solution time-averaged polar component is ($p_\theta = 0$ for all angles, but its unidirectional time-average, taken in the direction of either increasing or decreasing $\theta$, can contribute to heating of the disc surface though not to a torque. With the addition of this component, the power input per unit area, to the one side of a thin disc that is viewed by an observer, can be written down and equated with the blackbody radiation emitted at the local temperature $T(r)$:

$$\sigma T^4(r) = \frac{3L_\nu R_{LC}}{8\pi^3 r^3}\left(\sin \beta + \frac{2}{3} \cot \xi\right),$$

(2)

where $L_\nu$ is the pulsar wind luminosity, $\beta$ is the disc tilt angle and $\xi$ is the angle between the neutron-star magnetic and spin axes. The first term in equation (2) is derived from the long-range part of ($p_\phi$) by means of equation (14) of Jones (2007). The second term, in the approximation of small $\beta$, is from the long-range part of the unidirectional time-average of $p_\phi$ evaluated in the vicinity of $\theta = \pi/2$. The status of the vacuum solution form of this term is much less certain. Apart from an apparent unphysical singularity at $\xi = 0$, which is a consequence of the vacuum-field solution and our use of $\mathcal{L}_\nu \propto \sin^2 \xi$ as a working parameter, the term is linearly dependent on the radial component of the electric displacement $D$ which is undetermined because the total charge of the star and the physical, as opposed to vacuum, magnetosphere is an unknown quantity, as has been emphasized by Michel & Li (1999).

The values that might be found for the tilt angle $\beta$ are, of course, unknown, but there is a case for assuming that its value at formation is non-zero as a consequence of the fact that type II supernovae are generally asymmetric (see Goldreich, Lai & Sahringer 1997). This suggests that the angular momenta of the neutron star, the fall-back disc and the ejected matter may not all be precisely parallel.

The uncertainties in $\beta$ and in the polar term are so considerable that we will replace the vacuum-field quantity $\sin \beta + 2/3 \cot \xi$ in equation (2) by a free parameter $\xi$ and adopt a conservative range of values $0.03 \leq \xi \leq 0.3$. Neglect of the wind albedo is a further uncer-

3 DISC PARAMETERS

Recent observations in the near-infrared have produced $H$-, $K_{\text{s}}$- and $K_{\text{b}}$-band magnitudes for many isolated neutron stars, including both radio pulsars and AXP. A useful listing of these has been published by Mignani et al. (2007) who also include estimates of the distances $d$. We have used the Spitzer Science Centre magnitude to flux-density converter to obtain the observed energy fluxes per unit frequency interval $f_{\nu}$.

The values found for $r_i$ are quite insensitive to the outer radius $r_o$ because, for the temperatures found here, the outer part of the disc contributes little to the $K$-band flux. Therefore, we have adopted a fixed $r_o = 3.4 r_i$ equal to the best-fitting ratio found by Wang et al. in the case of 4U 0142+61. With these assumptions, values of $r_i$ and the inner radius temperature $T_i$ have been computed and are given in the right-hand side six columns of Table 1 for $\zeta = 0.03$, $0.1$, $0.3$.

The discs of Table 1 need not be massive: for $\zeta = 0.03$, the masses are in the interval $10^{-7}$–$10^{-5}$ $M_\odot$, assuming a density $\Sigma = 10^3$ g cm$^{-2}$, but their angular momenta can be of the same order as that of the neutron star. Certainly for the lower temperatures given in Table 1, the discs must consist almost entirely of dust grains and only a small gaseous component. But even a very low concentration of free electrons and ions is sufficient to stop the pulsar wind (see Jones 2007) and there can be no doubt that such concentrations must be present from a variety of sources.

For consistency with our Section 1 definition of a fall-back disc, we require that a satisfactory solution has $r_i > R_{LC}$. The first six rows in the table refer to radio pulsars, and they satisfy this condition by large margins. But it is interesting to see that, in many cases, the AXP and SGR do not. In particular, there is no solution for 4U 0142+61, the object for which evidence of a disc is most compelling. This is not a cause for concern because equation (2) contains only the pulsar wind contribution to disc heating. For the AXP and SGR, the soft X-ray luminosity $L_{\text{sed}}$, possibly of thermal origin, exceeds $L_{\text{rad}}$ by several orders of magnitude so that its power input can be the more important if the disc is not of negligible thickness but subtends a finite solid angle at the neutron star, as in the model adopted by Wang et al. (2006).
gaseous component, it would have a diffuse surface of approximate depth

\[ h = \sqrt{\frac{k_B T}{m_{A,Z} \Omega_k^2}}, \]

where \( m_{A,Z} \) is an atomic (or molecular) mass and \( \Omega_k \) is the Kepler angular frequency. For the 4U0142+61 best-fitting inner radius \( r_1 = 2.0 \times 10^{11} \text{ cm} \) found by Wang et al., the very small Kepler frequency, \( 1.5 \times 10^{-4} \text{ rad s}^{-1} \) leads to a fairly large depth, \( h \approx 2 \times 10^{5} \text{ cm} \), for atomic hydrogen at \( T = 10^7 \text{ K} \). The 4U0142+61 luminosity is \( \mathcal{L}_w \approx 10^{35} \text{--} 10^{36} \), so that although the disc is thin, the X-ray power input, \( (1 - \eta_g) h \mathcal{L}_\text{bol}/r_1 \), exceeds that given by equation (2) even for the large value of the X-ray albedo \( \eta_g \) adopted by Wang et al. (2006). The solid angle subtended by the disc may also be increased by departures from the plane assumption arising from r-dependent alignment torques (Jones 2007) or disc instability (Petterson 1977; Pringle 1996). The X-ray contribution to disc heating is therefore important for the AXP and SGR, but not for the radio pulsars which have \( \mathcal{L}_\text{bol} \ll \mathcal{L}_w \).

### 4 COMPARISON WITH OBSERVATION

The electromagnetic spectra of a limited number of radio pulsars are known over wide frequency intervals. In the case of B0565+14, as an example, apart from the lack of measurements between \( 10^{10} \) and \( 10^{14} \text{ Hz} \), the shape of the spectrum is of blackbody emission from the neutron-star surface superimposed on a background which decreases monotonically with frequency and is possibly of some common magnetospheric origin (see Koptsevich et al. 2001; fig. 6). The K-band flux listed in Table 1 is consistent with this spectrum. It is also known that the optical emission is pulsed at the rotation frequency. For example, Shearer et al. (1997), observing B-band emission, found a 1σ upper limit of only \( 8 \times 10^{-31} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \) on the unpulsed component of the flux. With regard to the AXPs listed in Table 1, the optical emission from 4U0142+61 has a pulsed fraction of 0.3, and the optical and X-ray pulse profiles are in phase and of similar shape (Kern & Martin 2002; Dhillon et al. 2005).

Unambiguous evidence for the presence of a disc would be an unpulsed blackbody excess which, for the temperatures given in Table 1, would probably be best observed in the 4.5 and 8.0 \( \mu \text{m} \) bands. With the possible exception of 4U0142+61, existing spectra provide no such evidence.

Even for the smallest value of \( \zeta \) assumed in Table 1, the radio pulsar disc inner radii that would be compatible with the K-band fluxes are all larger than the best fit \( r_1 = 2.0 \times 10^{11} \text{ cm} \) found by Wang et al. (2006) in the case of 4U0142+61, although they are of a similar order of magnitude. It might be argued that the correct value of \( \zeta \) is much smaller than 0.03, perhaps by one or more orders of magnitude. For this to be so, the neutron-star spin \( \Omega \) would have to be almost exactly normal to the plane of the disc. Also, the star and magnetosphere must be electrically neutral to the extent that the unpulsed blackbody component of the the disc is at \( 1300 \text{ K} \) and has a smaller radius \( r_1 \).

### Table 1

| Neutron star | \( P \) (s) | \( \mathcal{L}_w \) (erg s\(^{-1}\)) | \( f_i \) (erg cm\(^{-2}\) s\(^{-1}\) Hz\(^{-1}\)) | \( r_i \) (kpc) | \( T_i \) (K) | \( r_1 \) (cm) | \( T_1 \) (K) | \( r_1 \) (cm) | \( T_1 \) (K) |
|--------------|------------|----------------|---------------|-------------|--------|-------------|--------|-------------|--------|
| B0531+21     | 0.033      | \( 4.6 \times 10^{36} \) | \( 2.0 \times 10^{-26} \) | 1.73         | 4.6 \times 10^{11} | 1970      | 1.0 \times 10^{12} | 1440   | 2.0 \times 10^{12} | 1180   |
| B0633+17     | 0.237      | \( 3.2 \times 10^{34} \) | \( 2.9 \times 10^{-30} \) | 0.16         | 2.5 \times 10^{11} | 470       | 4.0 \times 10^{11} | 440    | 6.2 \times 10^{11} | 420    |
| B0656+14     | 0.385      | \( 3.8 \times 10^{34} \) | \( 6.1 \times 10^{-30} \) | 0.29         | 2.6 \times 10^{11} | 540       | 4.2 \times 10^{11} | 500    | 6.6 \times 10^{11} | 470    |
| B0833−45     | 0.089      | \( 6.9 \times 10^{36} \) | \( 2.0 \times 10^{-29} \) | 0.29         | 1.0 \times 10^{13} | 480       | 1.7 \times 10^{12} | 450    | 2.6 \times 10^{12} | 420    |
| B1509−58     | 0.151      | \( 1.8 \times 10^{36} \) | \( 1.2 \times 10^{-28} \) | 4.2          | 6.2 \times 10^{10} | 1020      | 1.1 \times 10^{12} | 880    | 1.9 \times 10^{12} | 780    |
| J1119−6127   | 0.408      | \( 2.3 \times 10^{36} \) | \( <1.1 \times 10^{-29} \) | 6.0          | \( >5.5 \times 10^{11} \) | \( <360 \) | \( >9.7 \times 10^{11} \) | \( <760 \) | \( >1.6 \times 10^{12} \) | \( <690 \) |
| J0142+61     | 0.290      | \( 1.2 \times 10^{36} \) | \( 6.1 \times 10^{-29} \) | \( \geq 5 \) | –       | –           | –       | –           | –      |
| J0408−5937   | 0.645      | \( 5.6 \times 10^{35} \) | \( 2.0 \times 10^{-29} \) | 3            | \( 1.3 \times 10^{11} \) | \( 1110 \) | \( 2.4 \times 10^{11} \) | 950    | \( 4.1 \times 10^{11} \) | 840    |
| J0708−4009   | 1.10      | \( 5.8 \times 10^{32} \) | \( 3.2 \times 10^{-28} \) | 5            | –       | –           | –       | –           | –      |
| J1810−197    | 5.54       | \( 2.7 \times 10^{33} \) | \( 3.2 \times 10^{-29} \) | 4            | \( 5.0 \times 10^{10} \) | \( 1840 \) | \( 1.1 \times 10^{11} \) | \( 1380 \) | \( 2.0 \times 10^{11} \) | 1140   |
| IE 2259+586  | 6.98       | \( 5.6 \times 10^{31} \) | \( 1.4 \times 10^{-28} \) | 3.0          | –       | –           | –       | –           | –      |
| SGR 1806−20  | 7.56       | \( 5.0 \times 10^{34} \) | \( 6.1 \times 10^{-29} \) | 15           | –       | \( 1.7 \times 10^{13} \) | \( 2180 \) | \( 4.0 \times 10^{13} \) | 1570   |

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a heated disc, which could be confirmed by a search for a pulsed component.

5 CONCLUSIONS

The present paper considers heating of a thin passive disc by non-radial components of the pulsar wind, and computes inner radii and temperatures that are compatible with the observed K- and Ks-band fluxes for a number of radio pulsars and AXP. In the case of the radio pulsars, the inner radii are two or more orders of magnitude larger than the \( R_{\text{LC}} \) and the discs concerned, if they exist, can have no significant effect on pulsar spin-down. The conservative, and probably incorrect, assumption made is that the observed K and Ks fluxes have no pulsed component and are of thermal origin. It is also assumed that there is no internal disc viscosity and that heating by the radial flux of soft X-rays from the neutron-star surface is negligible except in the case of the AXP.

The possibility that some of the individual neutron stars listed in Table 1 have discs has been investigated by previous authors. K-band fluxes from discs with inner radii near the Alfvén surface have been calculated in the case of B0656+14, and the four AXP J0142+61, J1048−5937, J1708−4009 and IE 2259+586, by Perna et al. (2000). Also, the radio pulsars B0531+21 and B0833−45 were considered by Blackman & Perna (2004). The disc model differs from that of Section 2 in that the sources of heating were restricted to internal viscous dissipation and the soft X-ray spectrum of the neutron-star surface. Even so, the predicted K-band fluxes for the four AXP were \( f_{\nu} \sim 10^{-27} - 10^{-26} \text{ erg cm}^{-2} \text{ s}^{-1} \text{Hz}^{-1} \), considerably higher than the observed values listed in Table 1. In the case of the three radio pulsars, inclusion of heating by non-radial wind components is crucial and leads us to the conclusion that the proposed discs are not consistent with the observed K-band fluxes. This is also true in the general case, treated by Ekşi & Alpar (2005), of discs with inner radii near the light-cylinder radius. This conclusion should perhaps be qualified because we do not consider discs with \( r_i < R_{\text{LC}} \). Thus, a disc satisfying this condition at formation, with \( R_{\text{LC}} \) given by the initial angular frequency \( \Omega_0 \), would have its middle section removed by wind ablation leaving an outer section with the properties given in Table 1 and an inner section completely within the present light-cylinder radius. This inner sector would form part of the closed magnetosphere and would not intersect open magnetic flux lines or interact with the pulsar wind. The properties of such remnant discs, if they exist, are not considered in this paper.

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