The quark condensate at finite temperature

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Abstract

The temperature evolution of the quark condensate is studied using three different methods. In the spirit of a many-body approach we make an expansion in the scalar density up to second order. Our result is consistent with chiral perturbation theory to two-loop order.

1. INTRODUCTION

Chiral symmetry, which is an approximate symmetry of QCD in the light-flavor sector, is spontaneously broken in the QCD vacuum. A topic of interest for nuclear physics is the restoration of chiral symmetry, at high temperature and/or large baryonic density. The approach to restoration shows up as a decrease of the magnitude of the quark condensate, which is the order parameter of the spontaneous breaking. Within a hadronic picture the physical origin of this restoration is very simple in nature. Each hadron contributes a certain amount to the decrease, governed by its sigma commutator \(\Sigma_h\). For an assembly of independent hadrons, their effects add. This leads to the following expression for the evolution with temperature of the condensate

\[
\frac{\langle q\bar{q} \rangle_T}{\langle q\bar{q} \rangle_0} = 1 - \sum_h \frac{\Sigma_h \rho_h^0(T)}{f_\pi^2 m_\pi^2}
\]  

(1)
where the sum extends over all hadron species present in the medium and \( \rho_s^h \) is the corresponding scalar density. In a baryon free regime (zero baryonic chemical potential) the scalar density is produced by thermally excited hadrons. At low temperature \( \text{eq. (1)} \) is the leading contribution. Deviations arise from the interaction between the hadrons and the major problem is to evaluate their effect in a way which is consistent with chiral symmetry. In this work we shall investigate the temperature evolution of several quantities linked to chiral symmetry, namely the quark condensate, the pion mass and the weak pion decay constant. The results that we obtain in the chiral Lagrangian approach have already been given in chiral perturbation theory \[1,2\]. Our approach provides alternative methods suitable for incorporating other hadron species, especially vector mesons, in a consistent way.

For temperatures less than the pion mass the presence of mesons other than pions can be ignored, since their thermal abundance is small. It is also sufficient to expand to second order in the density.

To respect chiral symmetry we start from the standard nonlinear chiral Lagrangian, including the explicit chiral symmetry breaking piece, namely

\[
L = \frac{f_\pi^2}{4} \text{Tr} \partial_\mu U \partial^\mu U^\dagger + \frac{1}{4} f_\pi^2 m_\pi^2 \text{Tr}(U + U^\dagger). \tag{2}
\]

The \( SU(2) \)-matrix \( U \) is usually expressed in terms of the canonical pion field \( \vec{\Phi} \) as

\[
U = \exp \left( i \vec{\tau} \cdot \vec{\Phi} F(X) \right) \tag{3}
\]

where \( F(X) \) is an arbitrary odd polynomial of \( X = (\Phi^2/f_\pi^2)^{1/2} \)

\[
F(X) = X + \alpha X^3 + \beta X^5 + .... \tag{4}
\]

Different choices of the function \( F(X) \) have been introduced in the literature, corresponding to different values of the parameters \( \alpha \) and \( \beta \). For instance, the simple choice \( F(X) = X \) is commonly used in chiral perturbation expansions. The choice of Weinberg is \( F(X) = \arcsin[X/(1 + X^2/4)] \) \[3\]. On the other hand \( F(X) = \arcsin X \) gives the PCAC field, satisfying \( \partial_\mu A_\mu^i = -m_\pi f_\pi^2 \Phi_i \). Physical observables should not depend on the particular
function $F(X)$. In practice, however, a particular choice is made and some approximations are needed. It is thus important to check the independence of the results on $F(X)$.

In the case of a cold baryonic medium the independence on the representation has been established recently by Delorme et al. [4] and by Kirchbach and Wirzba [5]. It is one of the purposes of the present paper to explicitly check this property for the quark condensate in a hot baryon-free environment. A baryon-free zone may possibly be realised experimentally at RHIC and the LHC and is certainly present in the early universe. To derive the temperature dependence of the quark condensate we will use three methods. The first (sect.1) relies on the grand potential method introduced in ref. [6]. In the second method (sect.2) we evaluate the thermal expectation value of the chiral symmetry breaking piece of the Lagrangian. Both methods demonstrate the expected independence of the result on the particular choice of $F(X)$. Once this independence is proven we will introduce a third method (sect. 3) which requires the validity of the PCAC relation, i.e. corresponds to a particular choice of $F(X)$. In this case the condensate can be derived as the scattering amplitude of soft pions in the system. The condensate is then expressed in terms of the self energy for soft pions. In this derivation we will prove the validity of the Gell-Mann-Oakes-Renner relation (GOR) in a hot medium, extending the previous derivation of Chanfray et al. [7] which was restricted to zero temperature.

2. THE GRAND POTENTIAL METHOD

To obtain an expression for the chiral condensate ratio, $\langle \bar{q}q \rangle_T / \langle \bar{q}q \rangle_0$, in terms of pionic degrees of freedom we can use the fact that, according to the Feynman-Hellmann theorem, $\langle \bar{q}q \rangle_T$ is related to the derivative of the free-energy density (the grand potential) $\tilde{\Omega}(T)$ with respect to the bare quark mass $m$ as

$$\langle \bar{q}q \rangle_T = \partial \tilde{\Omega}(T) / \partial m \quad (5)$$

(we ignore the fact that up and down quark masses not the same). Denoting the difference in free energy density as $\Omega(T) = \tilde{\Omega}(T) - \tilde{\Omega}(0)$ and using the Gell-Mann Oakes Renner relation
(GOR) in the vacuum \[ \text{[8]} \]

\[
m^2 f^2 = -2m \langle \bar{q}q \rangle_0 = -m \frac{\partial \tilde{\Omega}(0)}{\partial m}
\] (6)

one immediately derives that

\[
\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = 1 - \frac{1}{f^2} \frac{\partial \Omega(T)}{\partial m^2}
\] (7)

This expression is identical to that of ref. \[ \text{[2]} \] if one ignores small corrections to \( f_\pi \) and \( m_\pi \) of order \( m \). For a free pion gas the grand potential is

\[
\Omega(T) = \frac{3T}{2\pi^2} \int_0^\infty dk \, k^2 \ln \left( 1 - e^{-\omega_k/T} \right); \quad \omega_k = \sqrt{m^2 + k^2}.
\] (8)

Taking the derivative with respect to the pion mass and using (7) leads to

\[
\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = 1 - \frac{m_\pi/2}{f^2 m^2} \rho_\pi^s(T)
\] (9)

with

\[
\rho_\pi^s(T) = 3 \int \frac{dk}{(2\pi)^3} \frac{m_\pi}{\omega_k} \frac{1}{\exp(\omega_k/T) - 1}.
\] (10)

Keeping in mind that the pion sigma commutator is \( \Sigma_\pi = m_\pi/2 \) (with nonrelativistic normalization) and that \( \rho_\pi^s \) is the scalar density of pions, we recover the independent-particle expression of eq. \[ \text{[1]} \] for a gas of free pions.

In the evaluation of the free energy, \( \Omega(T) \), for interacting pions we start from the well-known expression for interacting boson systems at finite temperature \[ \text{[9,10]} \]. In terms of the pion Green’s function

\[
D(\omega, k, T) = (\omega^2 - m^2 - k^2 - \Pi(\omega, k, T))^{-1},
\] (11)

the grand potential is given by

\[
\Omega(T) = \Omega'(T) - \frac{3}{2} \int \frac{dk}{(2\pi)^3} \int \frac{d\omega}{\pi} f(\omega) \text{Im} \left\{ \ln[-D^{-1}(\omega, k, T)] + D(\omega, k, T)\Pi(\omega, k, T) \right\}
\] (12)

where \( f(\omega) = (\exp(\omega/T) - 1)^{-1} \) is the Bose factor and \( \Omega'(T) \) is the sum of all contributions from ‘skeleton diagrams’ representing the perturbation expansion of \( \Omega \). These are evaluated by using full single-particle Green’s functions rather than bare ones. Furthermore
$$\frac{\delta \Omega}{\delta \Pi} = 0$$  \quad (13)$$

and

$$\frac{\delta \Omega'}{\delta D} = \Pi$$  \quad (14)

(see refs. [9–11]). Since the physical pion mass enters the propagator, \(D\), the self energy \(\Pi(\omega, k, T)\) is the difference

$$\Pi(\omega, k, T) = \Pi(\omega, k, T) - \Pi(\omega, k, 0),$$  \quad (15)

whereby the infinite contributions to the mass operator \(\Pi(\omega, k, 0)\) due to vacuum fluctuations are removed. Consequently also \(\Omega(T)\) is finite and no regularization of the loop integrals is needed.

The key quantity in eq. (12) is the pion self energy \(\Pi\). To calculate it we use the nonlinear \(\sigma\)-Model Lagrangian defined in eq. (2). To lowest order, the interaction part takes the form

$$\mathcal{L}_{\text{int}} = \frac{1}{f_{\pi}^2} \left[ -m_{\pi}^2 \left( \alpha - \frac{1}{24} \right) (\bar{\Phi} \Phi)^2 + \left( \alpha - \frac{1}{6} \right) \bar{\Phi} \partial^\mu \Phi \partial_\mu \Phi + \left( 2\alpha + \frac{1}{6} \right) \bar{\Phi} \partial_\mu \Phi \partial^\mu \Phi \right].$$  \quad (16)

The pion self energy \(\Pi\) contains single-pion and three-pion contributions (see Fig. 1a and 1b)

$$\Pi(\omega, k, T) = \Pi(\omega, k, T)^{1\pi} + \Pi(\omega, k, T)^{3\pi}.$$  \quad (17)

In the present approach and for the temperature range of interest \((T \leq m_{\pi})\), only the single-pion contribution is needed. This is equivalent to the Hartree approximation and leads to:

$$\Pi(\omega, k, T) = \lambda \ j(T) \left[ \left( 80\alpha - \frac{10}{3} \right) m_{\pi}^2 - \left( 40\alpha - \frac{8}{3} \right) (\bar{m}_\pi^2 + \omega^2 - k^2) \right]$$  \quad (18)

where \(\lambda \equiv 1/6f_{\pi}^2\) and \(j(T)\) is the thermal loop integral given by

$$j(T) = 3 \int \frac{d \mathbf{k}}{(2\pi)^3} \frac{1}{2\omega_k} \frac{1}{\exp(\omega_k/T) - 1}.$$  \quad (19)
Here $\tilde{\omega}_k = (\tilde{m}_\pi^2 + k^2)^{1/2}$ is the in-medium pion dispersion relation with an effective mass $\tilde{m}_\pi$. Its value is obtained by solving the following equation:

$$\tilde{m}_\pi^2 = m_\pi^2 + \Pi(\tilde{m}_\pi, k = 0, T)$$

which yields

$$\tilde{m}_\pi^2 = 1 + \frac{(80\alpha - 10/3)\lambda j}{1 + (80\alpha - 16/3)\lambda j} m_\pi^2$$

(21)

Since $\tilde{m}_\pi^2$ is proportional to the bare mass it vanishes in the chiral limit $m_\pi \rightarrow 0$, at all temperatures as required by the Goldstone theorem. Notice that, for 'on-shell' pions, the $\alpha$-dependence of the self energy disappears, as it should, and $\Pi_{os} = 2\lambda j$. The pion propagator $D(\omega, k, T)$ can now be written as

$$D(\omega, k, T) = \gamma(\omega^2 - k^2 - \tilde{m}_\pi^2)^{-1},$$

(22)

In addition to the effective mass it also contains a temperature dependent residue $\gamma(T) = \left[1 + \left(40\alpha - \frac{8}{3}\right)\lambda j\right]^{-1}$

(23)

which arises from the energy dependence of the $\pi\pi$-interaction given in (16).

Finally we need to evaluate the interaction part $\Omega'$ which takes the form

$$\Omega' = \frac{\lambda}{2} \int \frac{dk}{(2\pi)^3} \int \frac{dp}{(2\pi)^3} \int d\omega \frac{f(\omega)ImD(\omega, k, T)}{\pi} \int d\eta \frac{f(\eta)ImD(\eta, p, T)}{\pi} \left(80\alpha - \frac{10}{3}\right) m_\pi^2 - \left(40\alpha - \frac{8}{3}\right) \left[(\omega^2 - k^2) + (\eta^2 - p^2)\right].$$

(24)

Once $\Omega(T)$ is determined one can find the chiral condensate ratio from eq. (7). When taking the derivative one should note that $\Omega$ depends on the free pion mass $m_\pi$, both explicitly and through the self energy $\Pi$. Due to the stationarity conditions (13) and (14) the $m_\pi$-dependence of $\Pi$ does not influence $\partial \Omega / \partial m_\pi^2$ and we obtain

$$\frac{\partial \Omega}{\partial m_\pi^2} = d(T) - \frac{\lambda}{2} (80\alpha - 10/3) d^2(T).$$

(25)

where $d(T) \equiv \gamma(T) j(T)$ which is identical to the result of ref. [6] for $\alpha = -1/12$ (the 'Weinberg' choice for $F(X)$ [3]). Hence, with eq. (7),
\[
\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = 1 - 6\lambda d(T) - (240\alpha - 10)(\lambda d(T))^2
\]

(26)

At first sight, this expression has an unwanted \(\alpha\)-dependence. This model dependence disappears, however, if we expand the residue given in eq. (23) to first order in the coupling constant \(\lambda\). This gives the evolution of the condensate to second order in \(\lambda\), independent of \(\alpha\)

\[
\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = 1 - 6\lambda j(T) - 6(\lambda j(T))^2
\]

(27)

where the first-order term \(6\lambda j\) is just the contribution from a free pion gas (see eq. (9)), which dominates. The expression (27) represents, in the spirit of a many-body approach, an expansion in the density. We can check its consistency with chiral perturbation theory. In the chiral limit \((m_\pi \to 0)\) where \(j\) can be calculated analytically \((j = T^2/8)\), we recover the chiral perturbation result to order \(T^4\)

\[
\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = 1 - \frac{T^2}{8f_\pi^2} - \frac{T^4}{384f_\pi^4}.
\]

(28)

of Gasser and Leutwyler [1].

3. EXPECTATION VALUE OF THE CHIRAL SYMMETRY BREAKING HAMILTONIAN

The condensate ratio can also be obtained directly from the in-medium expectation value of the symmetry breaking Hamiltonian of the nonlinear sigma model (eq. (2))

\[
H_{\chi SB} = -\frac{1}{4}m_\pi^2f_\pi^2tr(U + U^\dagger) = -m_\pi^2f_\pi^2\cos F(X).
\]

(29)

Using the GOR in vacuum (eq. (3)) we obtain

\[
\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = \langle \cos (F(X)) \rangle_T
\]

(30)

where \(\langle .. \rangle_T\) on the rhs denotes a thermal expectation value. In order to be consistent with the second-order expansion in expression (27) we have to evaluate the thermal expectation value to fourth order in the pion field:
\[
\frac{\langle qq \rangle_T}{\langle qq \rangle_0} = 1 - \frac{1}{2} \left( \frac{\Phi^2}{f^2} \right)_T - \left( \alpha - \frac{1}{24} \right) \left( \frac{\Phi^2 \Phi^2}{f^2} \right)_T \tag{31}
\]

In order to evaluate the expectation values explicitly, the pion field is expanded in terms of creation and annihilation operators \( B^\dagger_{i,k} \) and \( B_{i,k} \) for the quasi pions, specified by the propagator given in (22)

\[
\Phi_i(r, t) = \frac{1}{\sqrt{2\omega_k}} \int d^3k (2\pi)^{3/2} \gamma \frac{1}{2} \left( B_{i,k}(t) + B^\dagger_{i,-k}(t) \right) e^{i k \cdot r} \tag{32}
\]

The presence of the \( \gamma^{1/2} \) factor ensures that the form (22) of the pion propagator \( D \) is recovered once the time dependence \( B_k(t) = B_k \exp(-i\tilde{\omega}_k t) \) is introduced. We have verified that the field operator (32) obeys canonical commutation relations. The expectation value of \( \langle \Phi^2 \rangle_T \) is calculated by normal ordering with respect to physical vacuum and yields

\[
\langle \Phi^2 \rangle_T = \gamma \int \frac{d^3k}{(2\pi)^3} \frac{3}{2\omega_k} \sum_{i=1}^{3} \langle B^\dagger_{i,k} B_{i,k} \rangle_T = 2\gamma j \tag{33}
\]

Similarly \( \langle \Phi^2 \Phi^2 \rangle_T \) is obtained by using Wick’s theorem and gives

\[
\langle \Phi^2 \Phi^2 \rangle_T = \frac{5}{3} \left( \langle \Phi^2 \rangle_T \right)^2 = \frac{20}{3} \gamma^2 j^2 \tag{34}
\]

Expanding the residue \( \gamma \), defined in (23), for both (33) and (34) to first order in \( \lambda j \) and inserting in (31), we recover the expression (27) of the previous section. In the next section we will introduce a third method which is based on the use of the PCAC pion field.

### 4. SOFT-PION METHOD

In this section, the values of the parameters \( \alpha \) and \( \beta \) in the expansion of \( F(X) \) (eq. (4)) are chosen such that the PCAC relation

\[
\partial_\mu A^\mu_i(x) = -f^2 \delta_{\pi} m_\pi^2 \Phi_i(x) \tag{35}
\]

is fulfilled. This corresponds to the values \( \alpha = 1/6 \) and \( \beta = 3/40 \). In the previous sections, however, it has been shown that the condensate ratio, to second order in the density, is independent on these parameters, which legitimates this third approach. The aim here is to
obtain the condensate ratio from its relation to the self energy for soft pions. In deriving such a relation we will show the validity of the in-medium version of the GOR which relates the effective pion mass, the effective pion decay constant and the in-medium quark condensate. This has been previously shown to hold in a dense but cold hadronic medium by Chanfray et al. [7].

We start from the QCD operator identity

\[ \left[ Q^5 (0), \partial_\mu A^\mu_i (0) \right] = -i 2m \bar{q}q (0). \tag{36} \]

Taking the thermal expectation value of this relation and using closure on the lhs one has

\[ \int d\mathbf{r} \sum_{n,m} \frac{e^{-\beta E_n}}{Z} \left( \langle n| A^0_i (\mathbf{r}, 0)|m\rangle \langle m| \partial_\mu A^\mu_i (0)|n\rangle - \langle n| \partial_\mu A^\mu_i (0)|m\rangle \langle m| A^0_i (\mathbf{r}, 0)|n\rangle \right) \tag{37} \]

\[ = -2im\langle \bar{q}q \rangle_T \tag{38} \]

where \( Z \) is the partition function. After spatial integration, translational invariance requires that \( p_n = p_m \).

The above equation is equivalent to the following energy-weighted sum rule

\[ 2m\langle \bar{q}q \rangle_T = -\sum_{n,m} \frac{e^{-\beta E_n}}{Z} 2(E_m - E_n) |\langle m| A^0_i (0)|n\rangle|^2, \tag{39} \]

It is now possible to derive a generalized GOR relation at finite temperature. For this purpose we assume that the sum rule (39) is saturated by exciting a single quasi pion, \( \tilde{\pi} \), (of mass \( \tilde{m}_\pi \)) from the heat bath, i.e.

\[ 2m\langle \bar{q}q \rangle_T = -\sum_n \frac{e^{-\beta E_n}}{Z} 2 \tilde{m}_\pi |\langle n + \tilde{\pi}| A^0_i (0)|n\rangle|^2 \tag{40} \]

It is natural to introduce a thermally averaged pion decay constant, \( \tilde{f}_\pi \), through

\[ \sum_n \frac{e^{-\beta E_n}}{Z} |\langle n + \tilde{\pi}| A^0_i (0)|n\rangle|^2 = \frac{(\tilde{f}_\pi \tilde{m}_\pi)^2}{2\tilde{m}_\pi} \tag{41} \]

The generalized GOR relation follows immediately

\[ 2m\langle \bar{q}q \rangle_T = -\tilde{f}_\pi^2 \tilde{m}_\pi^2. \tag{42} \]
Notice that Lorentz invariance is broken by the very concept of temperature. Therefore the renormalization of $f_\pi$ from the GOR relation \([12]\) concerns only the time component of the axial current.

Coming back to the energy-weighted sum rule \((39)\), we introduce the PCAC interpolating pion field through $\Phi_i(x) = -\partial_\mu A^\mu_i(x)/f_\pi m_\pi^2$. Taking the matrix element of this PCAC identity between states $|m\rangle$ and $|n\rangle$ yields

$$i(E_n - E_m) \langle n | A^0_i(0) | m \rangle = -f_\pi m_\pi^2 \langle n | \Phi_i(0) | m \rangle$$ \hspace{1cm} (43)

and leads to an alternative form of the sum rule

$$2m \langle \bar{q}q \rangle_T = -2f_\pi^2 m_\pi^4 \sum_n \frac{e^{-\beta E_n}}{Z} \sum_m \frac{\langle m | \Phi_i(0) | n \rangle^2}{E_m - E_n}.$$ \hspace{1cm} (44)

This relation links the condensate to the axial polarizability of the thermal bath. On the other hand it is a simple matter to show that the thermal pion propagator $D(0,0,T)$, calculated at the soft pion point $\omega = 0$, $k = 0$, is

$$D(0,0,T) = -(m_\pi^2 + \Pi(0,0,T))^{-1}$$

$$= \int dt \int d\mathbf{r} \langle (-i)[\Phi_i(\mathbf{r},t), \Phi_i(0)] \rangle_T \theta(t)$$

$$= -2 \sum_{n,m} \frac{e^{-\beta E_n}}{Z} \sum_m \frac{\langle m | \Phi_i(0) | n \rangle^2}{E_m - E_n}.$$ \hspace{1cm} (45)

Comparing eqs. \((14)\) and \((15)\), we obtain

$$\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = \left(1 + \frac{\Pi(0,0,T)}{m_\pi^2} \right)^{-1}.$$ \hspace{1cm} (46)

This is the generalization of a result already obtained for nuclear matter by M. Ericson \([12]\), which incorporates the coherent rescattering of the soft pion in an optical potential coinciding with the soft-pion self energy $\Pi(0,0,T)$.

In order to evaluate the condensate, we need the expression of the self energy to two-loop order, including in particular the contribution from the diagram of Fig. 1c. These were not necessary in our previous approach \((18)\). The two-loop contribution requires to expand the
Lagrangian to sixth order in the pion field. Both parameters $\alpha$ and $\beta$ enter in this case. The result for the self-energy is

$$
\Pi(\omega, k, T) = \left( \frac{5}{6} \langle X^2 \rangle_T + \frac{35}{24} \langle X^2 \rangle_T^2 \right) m^2 \pi - \left( \frac{1}{3} \langle X^2 \rangle_T + \frac{10}{9} \langle X^2 \rangle_T^2 \right) \tilde{m}^2 \pi \\
- \left( \frac{1}{3} \langle X^2 \rangle_T + \frac{5}{9} \langle X^2 \rangle_T^2 \right) (\omega^2 - k^2)
$$

(47)

where $\langle X^2 \rangle_T = 12 \lambda j$ represents the thermal loop integral of quasi pions. In expression (47) we take for consistency the effective mass to first order, i.e. $\tilde{m}^2 \pi = m^2 \pi (1 + 2 \lambda j)$. We obtain the self energy for soft pions from (47) and when inserting its value in (46) we recover, to second-order in $\lambda j$, our previous result of eq. (31). Notice that the expression (47) for $\Pi(\omega, k, T)$ is valid at the level of an extended Hartree approximation. To this order there is an additional contribution corresponding to a pion fluctuating in three pions (see Fig. 1b) which, however vanishes at the soft pion point. Therefore it does not affect the soft-pion self energy and hence the condensate. However it enters in the residue $\gamma$ to second order in the coupling constant $\lambda$. Since we did not calculate this term, our residue and hence our expression (21) for the pion mass are only valid to first order. This is also the case for the pion decay constant $\tilde{f}_\pi$. From the GOR relation we obtain $\tilde{f}_\pi = f_\pi (1 - 4 \lambda j)$, in agreement with ref. [1].

In summary we have studied the evolution with temperature of the quark condensate in the framework of a many-body approach, considering thermal excitations of pions. The $\pi\pi$ interaction is taken from the nonlinear sigma model. Our expansion parameter is the scalar density of the quasi pions and we work up to second order. We have shown in two different approaches that, up to this order, the condensate is independent of the transformation of the pion field as expected. In line with ref. [12] we have used a third method, based on PCAC, to derive the condensate which is related to the self energy of soft pions. In the chiral limit our results are consistent with chiral perturbation theory, to two-loop order. The next stage will be to introduce the rho meson exchange in the $\pi\pi$ interaction in a way which preserves the chiral properties of this interaction.
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Figure Captions

**Fig. 1:** The pion self energy including the tadpole (a) 3-\(\pi\) intermediate states (b) and the two-\(\pi\) loop contribution (c).
