What Charm Can Tell Us About Beauty

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Abstract

A number of ways are reviewed in which the study of charmed particles can answer corresponding questions about particles containing $b$ quarks. Topics include the properties of resonances, the magnitude of decay constants, the size of spin-dependent effects, and the hierarchy of lifetime differences.

1 Introduction

The study of charmed particles is of interest not only in its own right, but for the information it can provide about particles containing $b$ quarks.

Charmed particles are relatively easy to produce. In the standard electroweak picture, their weak decays are unlikely to exhibit detectable CP-violating effects, and are noticeably affected by strong interactions. The good news is that these strong interactions are rich and easily studied.

Particles containing $b$ quarks are much harder to produce. Their weak interactions (again, in the conventional view) are expected to be a rich source of observable CP-violating phenomena, and to be less polluted by the strong interactions. However, these strong interactions are still important (for example, one needs to know $B$ meson decay constants), but their study is hampered by a lack of statistics. Here, charmed particles can be very helpful.

Many questions regarding $B$ hadrons can benefit from the corresponding studies of charmed particles. These include resonances, spin-dependent effects, lifetime differences, and form factors for heavy-to-light weak transitions. Moreover, since weak decays of $B$ hadrons often involve charm, the branching ratios of charmed particles are crucial in determining the corresponding $B$ branching ratios.

This brief article touches upon some of the ways in which information about charmed particles can be applied to the corresponding states containing $b$ quarks. In Section 2 we review the relevant aspects of heavy quark symmetry permitting an extrapolation from charm to beauty. Section 3 is devoted to the open questions facing the study of CP violation in $B$ decays, with emphasis on parallels with charm. Section 4 is devoted to strange $B$'s: their production, masses, and mixings, and the corresponding questions for charm. Heavy meson decay constants, for which we have partial information in the case of charm, are treated in Section 5. Heavy baryon spectra are discussed in Section 6, while Section 7 treats lifetime differences. We summarize in Section 8.

2 Heavy quark symmetry

In a hadron containing a single heavy quark, that quark ($Q = c$ or $b$) plays the role of an atomic nucleus, with the light degrees of freedom (quarks, antiquarks, gluons) analogous to the electron cloud. The properties of hadrons containing $b$ quarks (we shall call them $B$ hadrons) then can calculated from the corresponding
properties of charmed particles by taking account [3] of a few simple “isotope effects.” For example, if \( q \) denotes a light antiquark, the mass of a \( QQ \) meson can be expressed as

\[
MQQ = mQ + \text{const.}[n, \ell] + \frac{\langle p^2 \rangle}{2mQ} + a\frac{\langle \sigma_q \cdot \sigma_Q \rangle}{m_q m_Q} + O(m^{-2}) .
\]

Here the constant depends only on the radial and orbital quantum numbers \( n \) and \( \ell \). The \( \langle p^2 \rangle/2mQ \) term expresses the dependence of the heavy quark’s kinetic energy on \( m_Q \), while the last term is a hyperfine interaction. The expectation value of \( \langle \sigma_q \cdot \sigma_Q \rangle \) is \((+1, -3)\) for \( J^P = (1^-, 0^-) \) mesons. If we define \( \bar{M} \equiv [3M(1^-) + M(0^-)]/4 \), we find

\[
m_b - m_c + \frac{\langle p^2 \rangle}{2m_b} - \frac{\langle p^2 \rangle}{2m_c} = \bar{M}(Bq) - \bar{M}(cq) \approx 3.34 \text{ GeV} .
\]

so \( m_b - m_c > 3.34 \text{ GeV} \), since \( \langle p^2 \rangle > 0 \). Details of this picture which are of interest include (1) the effects of replacing nonstrange quarks with strange ones, (2) the energies associated with orbital excitations, (3) the size of the \( \langle p^2 \rangle \) term, and (4) the magnitude of hyperfine effects. In all cases there exist ways of using information about charmed hadrons to predict the properties of the corresponding \( B \) hadrons.

3 CP violation and \( B \) mesons

3.1 The CKM matrix

3.1.1 Parameters and their values

In a parametrization [3] in which the rows of the CKM [3, 3] matrix are labelled by \( u, c, t \) and the columns by \( d, s, b \), we may write

\[
V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\approx \begin{bmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{bmatrix} .
\]

Note the phases in the elements \( V_{ub} \) and \( V_{td} \). These phases allow the standard \( V - A \) interaction to generate CP violation as a higher-order weak effect.

The parameter \( \lambda \) is measured by a comparison of strange particle decays with muon decay and nuclear beta decay, leading to \( \lambda \approx \sin \theta \approx 0.22 \), where \( \theta \) is just the Cabibbo [3] angle. The dominant decays of \( b \)-flavored hadrons occur via the element \( V_{cb} = A\lambda^2 \). The lifetimes of these hadrons and their semileptonic branching ratios then lead to estimates in the range \( A = 0.7 - 0.9 \). The decays of \( b \)-flavored hadrons to charmless final states allow one to measure the magnitude of the element \( V_{ub} \) and thus to conclude that \( \sqrt{\rho^2 + \eta^2} = 0.2 - 0.5 \). The least certain quantity is the phase of \( V_{ub} \): \( \text{Arg} \ (V_{ub}^*) = \arctan(\eta/\rho) \). We shall mention ways in which information on this quantity may be improved, in part by indirect information associated with contributions of higher-order diagrams involving the top quark.

The unitarity of \( V \) and the fact that \( V_{ud} \) and \( V_{tb} \) are very close to 1 allow us to write \( V_{ub}^* + V_{td} \approx A\lambda^3 \), or, dividing by a common factor of \( A\lambda^3 \), \( \rho + i\eta \approx (1 - \rho - i\eta) = 1 \). The point \( (\rho, \eta) \) thus describes in the complex plane one vertex of a triangle whose other two vertices are \((0, 0)\) and \((0, 1)\). This triangle and conventional definitions of its angles are depicted in Fig. 1.

3.1.2 Indirect information

Indirect information on the CKM matrix comes from \( B^0 - \bar{B}^0 \) mixing and CP-violating \( K^0 - \bar{K}^0 \) mixing, through the contributions of box diagrams involving two charged \( W \) bosons and two quarks of charge \( 2/3 \) (\( u, c, t \)) on the intermediate lines. Evidence for the top quark with a mass of \( m_t = 174 \pm 10^{+13}_{-12} \text{ GeV}/c^2 \) has recently been reported [5], reducing the errors associated with these box diagrams.
The unitarity triangle. (a) Relation obeyed by CKM elements; (b) relation obeyed by (CKM elements)/Λλ^3

The original evidence for $B^0 - \bar{B}^0$ mixing came from the presence of “wrong-sign” leptons in $B$ meson semileptonic decays. The splitting $\Delta m_B$ between mass eigenstates is proportional to $f_B^2 m_t^2 |V_{td}|^2$ times a slowly varying function of $m_t$. Here $f_B$ is the $B$ meson decay constant. The contributions of lighter quarks in the box diagrams, while necessary to cut off the high-energy behavior of the loop integrals, are numerically insignificant.

The CKM element $|V_{td}|$ is proportional to $|1 - \rho - i\eta|$. Thus, exact knowledge of $\Delta m_B$, $f_B$ and $m_t$ would specify a circular arc in the $(\rho, \eta)$ plane with center (1,0). Errors on all these quantities spread this arc out into a band. Present averages give $(\Delta m_B/\Gamma_B) = 0.71 \pm 0.07$. This value (close to 1) is nearly optimal for observing CP-violating asymmetries in $B^0$ decays.

Similar box diagrams govern the parameter $\epsilon$ in CP-violating $K^0 - \bar{K}^0$ mixing. Here the dominant contribution to the imaginary part of the off-diagonal mass matrix element is proportional to $f_K^2 m_t^2 \text{Im}(V_{td}^2)$ times a slowly varying function of $m_t$. Charmed quarks also provide a small contribution.

The kaon decay constant is known: $f_K = 160$ MeV. The imaginary part of $V_{td}$ is proportional to $\eta(1 - \rho)$. Knowledge of $\epsilon$ thus specifies a hyperbola in the $(\rho, \eta)$ plane with focus at (1,0), which is spread out into a band because of uncertainties in hadronic matrix elements.

3.1.3 Allowed $(\rho, \eta)$ region

Information on $|V_{ub}/V_{cb}|$ specifies a circular band in the $(\rho, \eta)$ plane. When this constraint is added to those mentioned above, one obtains the potato-shaped region shown in Fig. 2. Here we have taken $m_t = 174 \pm 17$ GeV/$c^2$, $f_B = 180 \pm 30$ MeV, $(\rho^2 + \eta^2)^{1/2} = 0.36 \pm 0.14$ (corresponding to $|V_{ub}/V_{cb}| = 0.08 \pm 0.03$), and $A = 0.79 \pm 0.09$ (corresponding to $V_{cb} = 0.038 \pm 0.005$). A parameter known as $B_K$ describes the degree to which the box diagrams dominate the CP-violating $K^0 - \bar{K}^0$ mixing. We take $B_K = 0.8 \pm 0.2$, and set the corresponding value for $B$ mesons equal to 1. A QCD correction to the $B^0 - \bar{B}^0$ mixing amplitude has been taken to be $\eta_{QCD} = 0.6 \pm 0.1$. Other parameters and fitting methods are as discussed in more extensive treatments elsewhere. Several parallel analyses reach qualitatively similar conclusions.

The best fit corresponds to $\rho \approx 0$, $\eta \approx 0.36$, while at 90% confidence level the allowed ranges are:

$$\eta \approx 0.3 : -0.4 \leq \rho \leq 0.4 ;$$

$$\rho \approx 0 : \eta \approx 0.3 \times 2^{\pm 1} .$$

A broad range of parameters gives an acceptable description of CP violation in the kaon system. The study of CP violation in $B$ decays could confirm or disprove this picture.

3.2 Modes of studying CP violation in $B$ decays

Any manifestation of CP violation requires some sort of interference. We give two of the main examples under consideration for $B$ decays. We then discuss how charmed particles can provide useful information in
Figure 2: Contours of 68% (inner curve) and 90% (outer curve) confidence levels for regions in the $(\rho,\eta)$ plane. Dotted semicircles denote central value and ±1σ limits implied by $|V_{ub}/V_{cb}| = 0.08 \pm 0.03$. Plotted point corresponds to minimum $\chi^2 = 0.17$, while (dashed, solid) curves correspond to $\Delta\chi^2 = (2.3, 4.6)$ both cases.

3.2.1 Self-tagging decays

Inequality of the rates for a process and its charge conjugate, such as $B^+ \to \pi^0K^+$ and $B^- \to \pi^0K^-$, would signify CP violation. Under charge conjugation, the weak phases change sign while the strong phases do not. A rate difference can arise if both strong and weak phases are different in two channels (here, $I = 1/2$ and $I = 3/2$). Interpretation requires knowing the strong phase shift difference $\delta \equiv \delta_{3/2} - \delta_{1/2}$.

3.2.2 Decays to CP eigenstates

Interference between a decay amplitude and a mixing amplitude can lead to rate differences between decays of $B^0$'s and $\bar{B}^0$'s to CP eigenstates such as $J/\psi K_S$ or $\pi^+\pi^-$. Here, no strong phase shift is needed to generate an observable effect, and decay rate asymmetries can directly probe angles of the unitarity triangle. However, it is necessary to know the flavor of the initial neutral $B$ meson.

3.3 Final-state phases

Several examples involving charmed particles can be instructive in how one obtains final-state phase shift information from decay rates. These examples turn out to have parallels in the case of $B$ mesons, but the cases of real interest for CP violation in the $B$ system turn out to be somewhat more complex.

The decays $D \to \bar{K}\pi$ are characterized by the quark subprocess $c \to s\bar{u}d\bar{\ell}$, which has $\Delta I = \Delta I_3 = 1$, and so there are two final-state amplitudes, one with $I = 1/2$ and one with $I = 3/2$. The amplitudes for decays to specific charge states can be written in terms of isospin amplitudes as $A(D^+ \to \bar{K}^0\pi^+) = A_{3/2}$; $A(D^0 \to K^-\pi^+) = (2/3)A_{1/2} + (1/3)A_{3/2}$; $A(D^0 \to \bar{K}^0\pi^0) = \sqrt{2}(A_{3/2} - A_{1/2})/3$. The amplitudes then obey
a triangle relation, and by considering the observed rates one finds the relative phase of the $I = 1/2$ and $I = 3/2$ amplitudes to be around $90^\circ$ [13]. This is likely to indicate the importance of resonant structure. The $I = 1/2$ channel is “non-exotic” (it can be formed of a quark-antiquark state), while the $I = 3/2$ channel is “exotic,” requiring at least two quarks and two antiquarks. No resonances have been seen in exotic channels, while there is an $I = 1/2$ $K\pi$ resonance just around the mass of the $D$ meson [14].

Triangle constructions similar to that mentioned above indicate that the relative phase of $I = 1/2$ and $I = 3/2$ amplitudes in $D \to K^+\pi$ appears to be about $90^\circ$, while it appears to be about $0$ in $D \to K\rho$. This difference may be due to details of resonance couplings, but could not have been anticipated a priori. It illustrates the importance of actual measurements rather than theoretical prejudices in the evaluation of final-state phase shift differences.

The decays $D \to \pi\pi$ are governed by the subprocess $c \to du\bar{d}$ (or $c \to u$ penguin subprocesses). The $\Delta I = 1/2$ transitions lead to an $I = 0$ $\pi\pi$ final state, while the $\Delta I = 3/2$ transitions lead to $I = 2$ $\pi\pi$ final state. Again, a triangle relation holds between amplitudes, and the $I = 0$ and $I = 2$ amplitudes are found [17] to have a relative phase consistent with $90^\circ$.

The decays $B \to \bar{D}\pi$ involve the quark subprocess $\bar{b} \to \bar{c}ud\bar{d}$ and so their isospin analysis parallels that of $D \to K\pi$. It has recently been concluded [16] that present data are consistent with a relative phase shift of zero between the $I = 1/2$ and $I = 3/2$ amplitudes.

The decays $B \to K\pi$ involve the quark subprocesses $\bar{b} \to \bar{s}u\bar{u}$ and $\bar{b} \to \bar{s}$ (penguin processes), and thus are characterized by both $\Delta I = 0$ and $\Delta I = 1$ transitions. The $\Delta I = 0$ transitions can lead only to an $I = 1/2$ final state, while the $\Delta I = 1$ transitions lead to both $I = 1/2$ and $I = 3/2$ final states. Four $B \to K\pi$ decay amplitudes then can be expressed in terms of two $I = 1/2$ and one $I = 3/2$ reduced amplitude, leading to a quadrangle relation [17]. Suggestions have been made [18] for incorporating information from $B \to \pi\pi$ decays with the help of flavor SU(3) and untangling various final-state phases in the $K\pi$ channel.

### 3.4 Flavor tagging in neutral $B$ decays

As mentioned above, the decays of neutral $B$ mesons to CP eigenstates can provide crisp information on angles in the unitarity triangle if one can “tag” the flavor of the decaying $B$ at the time of its production. One method for doing this [19] relies on the correlation of a neutral $B$ with a charged pion.

This method [20] is already in use for tagging neutral $D$ decays. The charged $D^*$ resonance is far enough above the neutral $D$ that the decays $D^{*+} \to \pi^+ D^0$ and $D^{*-} \to \pi^- D^0$ are kinematically allowed. Here one is interested in whether a given final state has arisen from mixing or from the doubly-suppressed process $c \to du\bar{u}$.

In the case of $B$ mesons, the $B^*$ is only 46 MeV above the $B$, so the decay $B^* \to B\pi$ is kinematically forbidden. Nonetheless, one can expect non-trivial correlations between the flavor of a produced $B$ and a pion nearby in phase space, either as a result of correlations in the fragmentation process or through the decays of resonances above the $B^*$. In both cases, the corresponding physics for charmed particles is easy to study and will provide interesting information.

### 3.5 Pion – $B$ correlations

The pion-$B$ correlation in a fragmentation picture is illustrated in Fig. 3. When incorporated into a neutral $B$ meson, a $\bar{b}$ quark is “dressed” with a $d$, leading to a $B^0$. The next quark down the rapidity chain is a $\bar{d}$, which will appear in a pion of positive charge. Similarly, a $\bar{B}^0$ is more likely to be correlated with a $\pi^-$. The existence of this correlation in CDF data is still a matter of some debate. It would be interesting to see if it exists for charmed particles. One would have to subtract out the contribution of $D^*$ decays, of course.
Figure 3: Quark graphs illustrating pion-\(B\) correlations. Fragmentation of a \(\bar{b}\) quark leads to a \(B^0\) and a nearby \(\pi^+\), while fragmentation of a \(b\) quark leads to a \(\bar{B}^0\) and a nearby \(\pi^-\).

Table 1: P-wave resonances of a \(b\) quark and a light (\(\bar{u}\) or \(\bar{d}\)) antiquark

| \(J^P\) | Mass (GeV/c\(^2\)) | Allowed final state(s) |
|---------|---------------------|-----------------------|
| \(2^+\) | \(\sim 5.77\)       | \(B\pi, B^*\pi\)     |
| \(1^+\) | \(\sim 5.77\)       | \(B^*\pi\)            |
| \(1^+\) | \(< 5.77\)          | \(B^*\pi\)            |
| \(0^+\) | \(< 5.77\)          | \(B\pi\)              |

3.6 \(B^{**}\) resonances and their charmed equivalents

A \(B^0\) or \(B^{*0}\) can resonate with a positive pion, while a \(\bar{B}^0\) or \(\bar{B}^{*0}\) can resonate with a negative pion. The combinations \(B^0\pi^-\) and \(B^{*0}\pi^+\) are exotic, and not expected to be resonant.

The lowest-lying resonances which can decay to \(B\pi\) or \(B^*\pi\) are expected to be the P-wave \(\bar{b}q\) states. We call them \(B^{**}\) (to distinguish them from the \(B^{*}\)'s). The expectations for masses of these states \([19, 21]\), based on extrapolation from the known \(D^{**}\) resonances, are summarized in Table 1.

The known \(D^{**}\) resonances are a \(2^+\) state around 2460 MeV/c\(^2\), decaying to \(D\pi\) and \(D^*\pi\), and a \(1^+\) state around 2420 MeV/c\(^2\), decaying to \(D^*\pi\). These states are relatively narrow, probably because they decay via a D-wave. In addition, there are expected to be much broader (and probably lower) \(D^{**}\) resonances: a \(1^+\) state decaying to \(D^*\pi\) and a \(0^+\) state decaying to \(D\pi\), both via S-waves.

Once the masses of \(D^{**}\) resonances are known, one can estimate those of the corresponding \(B^*\) states by adding about 3.32 GeV (the quark mass difference minus a small binding correction). Adding a strange quark adds about 0.1 GeV to the mass. Partial decay widths of \(D^{**}\) states are also related to those of the \(B^{**}\)'s \([21]\). Thus, the study of excited charmed states can play a crucial role in determining the feasibility of methods for identifying the flavor of neutral \(B\) mesons.

4 Strange \(B\)'s

4.1 Production

It is important to know the ratios of production of different \(B\) hadrons: \(B^+ : B^0 : B_s : \Lambda_b\). These ratios affect signals for mixing and the dilution of flavor-tagging methods. Aside from effects peculiar to the decays \(D^* \rightarrow D\pi\), one should have similar physics in the ratios \(D^+ : D^0 : D_s : \Lambda_c\).
Table 2: Dependence of mixing parameter $x_s$ on top quark mass and $B_s$ decay constant.

| $m_t$ (GeV/c\(^2\)) | 157  | 174  | 191  |
|-----------------------|------|------|------|
| $f_{B_s}$ (MeV)       |      |      |      |
| 150                   | 7.6  | 8.9  | 10.2 |
| 200                   | 13.5 | 15.8 | 18.2 |
| 250                   | 21.1 | 24.7 | 28.4 |

4.2 Masses

It appears that the $B_s$ states are about 90 MeV above the $B$'s. One predicts a similar splitting for the strange and nonstrange vector mesons. The corresponding splittings for charmed particles are about 100 MeV for both pseudoscalar and vector mesons, as well as for the observed P-wave levels. This leads to a more general question: How much mass does a strange quark add? This is an interesting “isotope effect” which in principle could probe binding effects in the interquark force.

4.3 $B_s - \bar{B}_s$ mixing

The box diagrams which lead to $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing also mix strange $B$ mesons with their antiparticles. One expects $(\Delta m)|_{B_s}/(\Delta m)|_{B_d} = (f_{B_s}/f_{B_d})^2(B_{B_s}/B_{B_d})|V_{ts}/V_{td}|^2$, which should be a very large number (of order 20 or more). Thus, strange $B$’s should undergo many particle-antiparticle oscillations before decaying.

The main uncertainty in an estimate of $x_s \equiv (\Delta m/\Gamma)_{B_s}$ is associated with $f_{B_s}$. The CKM elements $V_{ts} \simeq -0.04$ and $V_{tb} \simeq 1$ which govern the dominant (top quark) contribution to the mixing are known fairly well. We show in Table 2 the dependence of $x_s$ on $f_{B_s}$ and $m_t$. To measure $x_s$, one must study the time-dependence of decays to specific final states and their charge-conjugates with resolution much less than the $B_s$ lifetime (about 1.5 ps).

5 Heavy meson decay constants

5.1 The $D_s$

Direct measurements are available so far only for the $D_s$ decay constant. The WA75 collaboration has seen 6 – 7 $D_s \rightarrow \mu\nu$ events, and Fermilab E653 and the BES detector at the Beijing Electron-Positron Collider (BEPC) also have a handful. The CLEO Collaboration has a much larger statistical sample; the main errors arise from background subtraction and overall normalization (which relies on the $D_s \rightarrow \phi\pi$ branching ratio). The actual measurement is $r \equiv B(D_s \rightarrow \mu\nu)/B(D_s \rightarrow \phi\pi) = 0.245 \pm 0.052 \pm 0.074$.

A better measurement of $B(\phi\pi) \equiv B(D_s \rightarrow \phi\pi)$ is sorely needed. One method is to apply factorization to the decay $B \rightarrow D_s D$, where $D_s \rightarrow \phi\pi$, to obtain the combination $f_{D_s}^2 B(\phi\pi)$. Since $r \propto f_{D_s}^2/B(\phi\pi)$, one can extract both the decay constant and the desired branching ratio. Using this and other methods, Muheim and Stone estimate $f_{D_s}^2 = 315 \pm 45$ MeV and $B(\phi\pi) = (3.6 \pm 0.6)\%$.

The large value of $f_{D_s}$ implies a branching ratio of about 9% for $D_s \rightarrow \tau\nu_\tau$. This is good news for experiments contemplating the production of $\nu_\tau$ in beam dumps.
5.2 The charged $D$

By searching for the decay $D \rightarrow \mu \nu$ in the decays of $D$ mesons produced in the reaction $e^+e^- \rightarrow \psi(3770) \rightarrow D^+D^-$, the Mark III collaboration has obtained the upper limit $f_D < 290$ MeV (90% c.l.). The BES detector at Beijing should be able to improve upon this limit, which is not far above theoretical expectations [30, 31, 32].

The CLEO measurement of $f_{D_s}$ mentioned above relied on photon-$D_s$ correlations in the decay $D_{s}^{*} \rightarrow D_s \gamma$. One may be able to search for the decay $D^+ \rightarrow \mu \nu$ by looking for the $\pi^0 - D^+$ correlation in the decay $D_{s}^{*+} \rightarrow D^+\pi^0$ [29].

5.3 $B$ Meson decay constants

If $f_B$ were better known, the indeterminacy in the $(\rho, \eta)$ plane associated with fits to CKM parameters would be reduced considerably. We show in Fig. 4 the variation in $\chi^2$ for the fit described in Sec. 3.1 when $f_B$ is taken to have a fixed value. An acceptable fit is obtained for a wide range of values, with $\chi^2 = 0$ for $f_B = 153$ and 187 MeV.

The reason for the flat behavior of $\chi^2$ with $f_B$ is illustrated in Fig. 5. The dashed line, labeled by values of $f_B$, depicts the $(\rho, \eta)$ value for the solution with minimum $\chi^2$ at each $f_B$. The product $|1 - \rho - i\eta|f_B$ is constrained to be a constant by $B^0 - \bar{B}^0$ mixing. The product $\eta(1 - \rho)$ is constrained to be constant by the value of $\epsilon$. The locus of solutions to these two conditions lies approximately tangent to the circular arc associated with the constraint on $|V_{ub}/V_{cb}|$ for a wide range of values of $f_B$.

The uncertainty in $f_B$ thus becomes a major source of uncertainty in $\rho$, which will not improve much with better information on $|V_{ub}/V_{cb}|$. Fortunately, several estimates of $f_B$ are available, and their reliability should improve.
Figure 5: Locus of points in $(\rho, \eta)$ corresponding to minimum $\chi^2$ for fixed values of $f_B$. Circular arcs depict central value and $\pm 1\sigma$ errors for $|V_{ub}/V_{cb}|$. Solid dots denote points with $\chi^2 = 0$.

Lattice gauge theories have attempted to evaluate decay constants for $D$ and $B$ mesons. A representative set is

\[
\begin{align*}
  f_B &= 187 \pm 10 \pm 34 \pm 15 \text{ MeV}, \\
  f_{B_s} &= 207 \pm 9 \pm 34 \pm 22 \text{ MeV}, \\
  f_D &= 208 \pm 9 \pm 35 \pm 12 \text{ MeV}, \\
  f_{D_s} &= 230 \pm 7 \pm 30 \pm 18 \text{ MeV},
\end{align*}
\]

(5)

where the first errors are statistical, the second are associated with fitting and lattice constant, and the third arise from scaling from the static ($m_Q = \infty$) limit. The spread between these and some other lattice estimates is larger than the errors quoted above, however.

Quark models can provide estimates of decay constants and their ratios. In a non-relativistic model, the decay constant $f_M$ of a heavy meson $M = Q \bar{q}$ with mass $M_M$ is related to the square of the $Q \bar{q}$ wave function at the origin by $f_M^2 = 12|\Psi(0)|^2/M_M$. The ratios of squares of wave functions can be estimated from strong hyperfine splittings between vector and pseudoscalar states, $\Delta M_{hfs} \propto |\Psi(0)|^2/m_Q m_q$. The equality of the $D_s^* - D_s$ and $D^* - D$ splittings then suggests that

\[
  f_D/f_{D_s} \simeq (m_d/m_s)^{1/2} \simeq 0.8 \simeq f_B/f_{B_s},
\]

(6)

where we have assumed that similar dynamics govern the light quarks bound to charmed and $b$ quarks. In lattice estimates these ratios range between 0.8 and 0.9.

An improved measurement of $f_{D_s}$ and a first measurement of $f_D$ could provide a valuable check on predictions of various theories and could help pin down $B$ meson decay constants, since ratios are expected to be more reliably predicted than individual constants.

6 Charmed baryon spectra

The $\Lambda_c$ baryon is a particularly simple object in heavy-quark symmetry, since its light-quark system consists of a $u$ and $d$ quark bound to a state $[ud]$ of zero spin, zero isospin, and color antitriplet. Comparisons with the $\Lambda_b = b[ud]$ and even with the $\Lambda = s[ud]$ are thus particularly easy.
The \([ud]\) diquark in the \(\Lambda\) can be orbitally excited with respect to the strange quark. The \(L = 1\) excitations consist of a fine-structure doublet, the \(\Lambda(1405)\) with spin-parity \(J^P = 1/2^-\) and the \(\Lambda(1520)\) with \(J^P = 3/2^-\). The spin-weighted average of this doublet is 366 MeV above the \(\Lambda\). These states are illustrated on the left-hand side of Fig. 6.

Within the past couple of years candidates have been observed [35] for a corresponding \(L = 1\) doublet of charmed baryons. These are illustrated on the right-hand side of Fig. 6. The lower-lying candidate, 308 MeV above the \(\Lambda_c\), decays to \(\Sigma_c\pi\), while the higher-lying candidate, 342 MeV above the \(\Lambda_c\), does not appear to decay to \(\Sigma_c\pi\), but rather to \(\Lambda_c\pi\pi\). This pattern can be understood [36] if the lower candidate has \(J^P = 1/2^-\) and the higher has \(J^P = 3/2^+\). The lower state can decay to \(\Sigma_c\pi\) via an S-wave, while the higher one would have to decay to \(\Sigma_c\pi\) via a D-wave. It would have no trouble decaying to \(\Sigma_c^*\pi\) via an S-wave, however. The predicted \(\Sigma_c^*\), with \(J^P = 3/2^+\), has not yet been identified.

The spin-weighted average of the excited \(\Lambda_c\) states is 331 MeV above the \(\Lambda_c\), a slightly smaller excitation energy than that in the \(\Lambda\) system. The difference is easily understood in terms of reduced-mass effects. The \(L \cdot S\) splittings appear to scale with the inverse of the heavy quark (\(s\) or \(c\)) mass.

The corresponding excited \(\Lambda_b\) states probably lie 300 to 330 MeV above the \(\Lambda_b(5630)\), with an \(L \cdot S\) splitting of about 10 MeV.

7 Lifetime differences

Charmed particle lifetimes range over a factor of ten, with

\[
\tau(\Xi_c^0) < \tau(\Lambda_c) < \tau(\Xi_c) \simeq \tau(D^0) \simeq \tau(D_s) < \tau(D^+) \quad .
\]
Effects which contribute to these differences \cite{37} include (a) an overall nonleptonic enhancement from QCD \cite{38}, (b) interference when at least two quarks in the final state are the same \cite{39}, (c) exchange and annihilation graphs, e.g. in $\Lambda_c$ and $\Xi_c^0$ decays \cite{40}, and (d) final-state interactions \cite{41}.

In the case of $B$ hadrons, theorists estimate that all these effects shrink in importance to less than ten percent \cite{42}. However, since the measured semileptonic branching ratio for $B$ decays of about 10 or 11\% differs from theoretical calculations of 13\% by some 20\%, one could easily expect such differences among different $b$-flavored hadrons. These could arise, for example, from final-state interaction effects. As mentioned earlier \cite{18}, there are many tests for such effects possible in the study of decays of $B$ mesons to pairs of pseudoscalars.

8 Summary

Charmed particles are a rich source of information about what to expect in the physics of particles containing $b$ quarks, in addition to being interesting in their own right.

Some properties of charmed particles are expected to be very close to those of $B$ hadrons, such as excitation energies. Others are magnified in the case of charm, being proportional to some inverse power of the heavy quark mass.

Charmed particles are easier to produce than $B$ hadrons in a hadronic environment (and in photoproduction), and so are a natural area of study for fixed-target experiments such as those being performed and planned at Fermilab. The high-statistics study of charmed particles could have a broad impact on fundamental questions in particle physics.

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