Barrier Lyapunov function based integrated missile guidance and control considering phased array seeker disturbance rejection rate

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ABSTRACT A two-dimensional missile–target engagement geometry is established considering the effect of the disturbance rejection rate (DRR) problem caused by phased array radar beam pointing error and radome refraction error in the guidance and control system. Compared with the traditional two-dimensional missile–target engagement geometry, the established model clearly shows the effect of the disturbance rejection rate problem on the missile line of sight and field-of-view constraint. The integrated missile guidance and control scheme is developed by adding the integral barrier Lyapunov function to dynamic surface control to satisfy the seeker field-of-view constraint while ensuring that the system has strong robustness. The target maneuvering and unmodeled disturbance with the effect of the DRR error slope as uncertain disturbances of the system and use the reduced-order extended state observer for online estimation. Moreover, the actuator deflection angle and angular rate constraints are guaranteed via the command filter. Stability analysis and numerical simulations show that the estimation and compensation of the seeker DRR error allow the proposed scheme to intercept a high-speed target accurately.

INDEX TERMS integrated guidance and control, field-of-view constraint, disturbance rejection rate, barrier Lyapunov function

I. INTRODUCTION

The performance of aircraft is improving rapidly with the continuous optimization of computing equipment and algorithms and the continuous emergence of high-tech materials. The targets of air defense missiles will have the characteristics of high stealth, speed, and maneuverability. The high maneuverability of the target will reduce the adaptability and robustness of existing guidance systems and thus the interception accuracy of the systems [1]. The phased array radar seeker is a new type of high-performance seeker that searches for and tracks a target by controlling the phase of each unit in the antenna to generate the antenna beam pointing direction. A larger antenna diameter increases the operating range of the seeker, enables beam scanning, beam tracking, and other functions in a short time, and improves the ability of air defense missiles to track high-speed targets accurately [2]. When the missile is far from the target, the lag in tracking dynamics (due to long-term signal accumulation and the discrete beam pointing control in the angle tracking loop) results in the seeker having detection error that is too large to track a high-speed highly maneuverable target [3].

The strapdown seeker system removes the need for a complex mechanical servo mechanism, eliminates the cross-coupled disturbances of the pitch and yaw channels, improves the tracking speed, and reduces the overall volume and cost of the weapon system [4]. Although the strapdown seeker system is more reliable than the traditional mechanical platform seeker, the price of such reliability is that the full strapdown seeker introduces large measurement errors into the guidance system that cannot be avoided. At the same time, the traditional mechanical platform seeker provides the guidance system with measurement information relative to the inertial space, and the strapdown seeker measurement component obtains the angle between the missile and line-of-
sight (LOS). Missile body attitude coupling in the guidance signals means that the system more readily loses stability and that there is divergence of the terminal miss distance. For tactical missiles that use proportional navigation and its variants as the terminal guidance law, the full strapdown seeker needs to extract the high-precision LOS angular rate and decouple the missile attitude disturbance. Nesline [5] proposed a method of extracting the angular rate of the LOS relative to the inertial space for wave-controlled seekers, which completes the decoupling of the missile body disturbance. Ehrich [6] proposed the use of a jitter adaptive parameter identification method to measure and correct the LOS angular rate error output by the seeker, so as to decouple the missile attitude disturbance. Furthermore, the phased array radar seeker uses a digital shifter to control the antenna beam pointing but, in principle, this is open-loop control. Even if phantom-bit technology is used to optimally control the beam pointing, it remains impossible to precisely and stably control the beam pointing angle, leading to not only the coupling of the attitude disturbance in the guidance signals but also the coupling with the beam pointing error [7]. Meanwhile, the presence of the radome couples the guidance signals obtained by the seeker with the radome refraction error. The above-mentioned beam pointing error and radome refraction error are collectively called the DRR error. The DRR error greatly affects the acquisition of the guidance signals of the strapdown phased array radar seeker. Gurfil [8] proposed a two-step estimator for estimating and compensating the radome error slope. Zarchan [9], [10] designed the application of the jitter adaptive method to estimate and compensate for a constant radome error slope. The amplitude and frequency of jitter need to be selected repeatedly when adopting this method, which increases the complexity of the algorithm. In engineering nowadays, the method of ground radome error calibration with look-up table compensation is widely used. This method is similar to the scale error calibration [11]. The ground calibration condition and the actual flight environment are different, leading to large fluctuations in the calibration results of the missile during actual flight even after long-term storage. It is impossible to suppress the adverse effects of errors on the guidance and control system. The Kalman filter is a filter technology widely used for guidance disturbance signals because it obtains more accurate guidance signals. William Yueh [12]–[14] combined the multi-model algorithm with the parallel Kalman filter, proposed a method for estimating and compensating the radome error slope, and analyzed the performance of the guidance system during the flight of a skid-to-turn/bank-to-turn controlled missile. For this method, Lin [15] took the radome error slope as the disturbance factor of the measurement and estimated the radome error slope through multi-model weighted summation. Cao [16] established an extended Kalman filter model for the three-dimensional inertial coordinate system and proposed a new multi-model radome error slope estimation and compensation algorithm by calculating the corresponding model probability, thus reducing the effect of the radome error on the missile guidance and control system. Lin [17] designed an extended state Kalman filter method, which combines the extended Kalman filter with the extended state observer (ESO), for the nonlinear radome error slope and nonlinear beam pointing error slope to obtain a more precise LOS angular rate.

In the design of the classic missile guidance and control system, the guidance system and the control system are independent of each other. The guidance system is an outer loop that calculates and outputs guidance commands in real time using the missile–target engagement model. The control system, as the inner loop, allows the missile to fly to the target smoothly and ultimately to destroy the target. Furthermore, in most optimal guidance law designs, the autopilot delay is accounted for using a simple model. No direct control of basic missile parameters, such as the angle-of-attack and body rate, is available. This degrades the strapdown seeker processing accuracy and warhead effectiveness [18]. To reduce the effect of DRR error, many researchers [19]–[21] have regarded the DRR error slope as a constant in the guidance system design. It is difficult for the guidance and control system of the terminal guidance stage to meet the spectrum separation condition, resulting in a large terminal miss distance or a large number of missed targets. The integrated missile guidance and control strategy treats the guidance system and control system as a whole and directly generates missile control commands according to the relative position information of the missile and target. The adoption of this strategy improves the stability of the guidance and control system greatly. Lin [18] built the design models of this system, including the planar intercept kinematics and missile longitudinal dynamics, within a single mathematical framework. Using these models, Lin derived an analytical solution for optimal integrated missile guidance and control system. Additionally, a target acceleration model and a missile radome error compensation scheme are included in the state formulations. Unfortunately, the models and compensation schemes of this optimal system are linearly time varying.

The field-of-view (FOV) constraint of the strapdown seeker is worth considering when designing the guidance and control system. Zhao [22] replaced the traditional missile LOS angular rate obtained by the strapdown seeker with the missile body-LOS angular rate to establish a model and introduced a neural network interference observer to estimate the model disturbance when designing an integrated guidance and control scheme. Tian [23] further considered the thrust situation using the missile body-LOS angular rate model and proposed an integrated missile guidance and control scheme based on dynamic surface control. Guo [24] designed three-dimensional integrated guidance and control considering the seeker’s FOV constraint, making the design of missile integrated guidance and control more practical.

Inspired by the above works, this article focuses on the DRR problem of the full strapdown phased array radar seeker. A two-dimensional missile–target engagement geometric model is first established considering the DRR prob-
lem. An integral-type barrier Lyapunov function (iBLF) dynamic surface control integrated missile guidance and control scheme based on this model is then designed. The designed scheme satisfies the seeker FOV constraint while ensuring the robustness of the system. At the same time, the reduced-order ESO is introduced to estimate target maneuvering and unmodeled disturbance under the effect of the DRR problem online. Stability analysis and numerical simulations are performed to show that because the estimation and compensation of the DRR error slope are considered throughout the design process, the proposed integrated guidance and control (IGC) method accurately intercepts the target without violating the FOV constraint.

The contributions of this article are as follows: 1. The radome error and beam pointing error are considered in the missile–target engagement model.

2) An iBLF is used to satisfy the FOV constraint, and a command filter is added to the dynamic surface control to avoid the effects of a limited actuator deflection angle and angular rate.

3) A reduced-order ESO is adopted to estimate the target maneuvering and unmodeled disturbance, which is affected by the DRR problem, and thus enhance the robustness of the proposed IGC design.

The remainder of this article is organized as follows. Section 2 establishes a two-dimensional planar missile–target relative-motion model. Section 3 derives a novel detailed IGC model. Section 4 introduces the proposed IGC scheme in detail and presents a closed-loop stability analysis. Section 5 verifies the effectiveness of the proposed IGC scheme through simulation experiments. Conclusions are presented in Section 6.

A. MISSILE KINEMATICS MODEL CONSIDERING THE EFFECT OF DRR ERROR

According to the model in the literature, the two-dimensional engagement geometry between a moving target and a roll-stabilized axisymmetric missile mounted on a strapdown seeker considering the error introduced by the DRR problem is shown in Figure 1.

where $T$ is the accurate target position, $r$ is the relative distance along the LOS from the missile to the accurate target position; $T^*$ is the apparent target tracked by the seeker under the influence of beam pointing error and radome error; $r^*$ denotes the relative distance along the LOS from the missile to the apparent target. $a_X$ and $a_Y$ are respectively the missile acceleration components along and perpendicular to the missile velocity $V_M^*$; $a_X^m$ and $a_Y^m$ are respectively the missile acceleration components along and perpendicular to the missile body axis $\vec{MX}_M$, $\dot{a}_X$ and $\dot{a}_Y$ are respectively the target acceleration components along and perpendicular to the target velocity $V_T$, respectively. $\theta_M$, $\varphi$ and $\alpha$ are respectively the missile flight path angle, pitch angle, and angle-of-attack, and $\theta_T$ is the target flight path angle. The LOS makes an angle $\lambda$ with respect to the accurate target position and makes an angle $\hat{\lambda}$ with respect to the apparent target position. $\lambda_R$ denotes the LOS error angle due to the disturbance rejection rate error. The LOS makes an angle with respect to the missile body axis known as the body–LOS angle for the full strapdown seeker. $\lambda_B$ is the body–LOS angle of the missile and accurate target position, $\hat{\lambda}_B$ is the body–LOS angle of the missile and apparent target, and $\psi$ and $\epsilon$ are respectively the phased array radar beam pointing angle and beam pointing error angle.

From Figure 1, the beam pointing error angle $\epsilon$ can be written as

$$\epsilon = \lambda - \varphi - \psi, \quad (1)$$

the LOS angle $\hat{\lambda}$ measured by the phased array radar seeker as

$$\hat{\lambda} = \lambda + \lambda_R = \varphi + \psi + \epsilon + \lambda_R. \quad (2)$$

According to the working principle of the radome, there is a functional relationship between the LOS error angle of the missile body and the beam pointing angle under the influence of DRR errors,

$$\lambda_R = f(\psi). \quad (3)$$

Then,

$$d\lambda_R = \left(\frac{\partial \lambda_R}{\partial \psi}\right) d\psi \quad (4)$$

The differentiation of (2) yields

$$\frac{d\hat{\lambda}}{dt} = \frac{d\lambda}{dt} + \frac{d\lambda_R}{dt} \quad (5)$$

For the sake of simplicity, the effect of the DRR errors can be approximated by an error slope. This error slope is called the DRR error slope and denoted $R$, and it is expressed as

$$R = \frac{\partial \lambda_R}{\partial \psi}. \quad (6)$$

It follows that

$$\frac{d\lambda_R}{dt} = \left(\frac{\partial \lambda_R}{\partial \psi}\right) \left(\frac{d\psi}{dt}\right). \quad (7)$$

It follows from (1) that

$$\psi = (\lambda - \varphi) - \epsilon = \lambda_B. \quad (8)$$

FIGURE 1: The engagement geometry for the strapdown missile considering the DRR errors.
The differentiation of (12b) yields (13) in next page.

After a series of manipulations yields

\[
\begin{align*}
\dot{\lambda} &= -2r^* \dot{\lambda} - \frac{R}{1 + R} r^* \ddot{\varphi} \\
&\quad + \frac{\cos (R\lambda - R\varphi)}{1 + R} \left[ \dot{V}_T \sin (\theta_T - \lambda) - \dot{V}_M \sin (\theta_M - \lambda) + V_T \dot{\theta}_T \cos (\theta_T - \lambda) - V_M \dot{\theta}_M \cos (\theta_M - \lambda) \right] \\
&\quad - \frac{\sin (R\lambda - R\varphi)}{r^* (1 + R)} \left[ \dot{V}_T \cos (\theta_T - \lambda) - \dot{V}_M \cos (\theta_M - \lambda) - V_T \dot{\theta}_T \sin (\theta_T - \lambda) + V_M \dot{\theta}_M \sin (\theta_M - \lambda) \right]
\end{align*}
\]

The target acceleration information cannot be obtained accurately, and the related items of the target acceleration are thus regarded as model disturbance items,

\[
d_1 = \frac{\cos (R\lambda - R\varphi)}{r^* (1 + R)} \left[ \dot{a}_X \sin (\theta_T - \lambda) + a_Y^T \cos (\theta_T - \lambda) \right] - \frac{\sin (R\lambda - R\varphi)}{r^* (1 + R)} \left[ a_X^T \cos (\theta_T - \lambda) - a_Y^T \sin (\theta_T - \lambda) \right].
\]

Define

\[
\begin{align*}
\dot{V}_T &= a_X^T \\
V_T \dot{\theta}_T &= a_Y^T
\end{align*}
\]

The model disturbance term \(d_1\) is then transformed as

\[
d_1 = \frac{\cos (R\lambda - R\varphi)}{r^* (1 + R)} \left[ a_X^T \sin (\theta_T - \lambda) + a_Y^T \cos (\theta_T - \lambda) \right] - \frac{\sin (R\lambda - R\varphi)}{r^* (1 + R)} \left[ a_X^T \cos (\theta_T - \lambda) - a_Y^T \sin (\theta_T - \lambda) \right].
\]

thus having

\[
\begin{align*}
\ddot{\lambda} &= \frac{2r^* \dot{\lambda}}{r^*} - \frac{R}{1 + R} \ddot{\varphi} + d_1 \\
&\quad - \frac{\cos (R\lambda - R\varphi)}{r^* (1 + R)} \left[ \dot{V}_M \sin (\theta_M - \lambda) + V_M \dot{\theta}_M \cos (\theta_M - \lambda) \right] \\
&\quad + \frac{\sin (R\lambda - R\varphi)}{r^* (1 + R)} \left[ \dot{V}_M \cos (\theta_M - \lambda) - V_M \dot{\theta}_M \sin (\theta_M - \lambda) \right]
\end{align*}
\]

Defining

\[
\begin{align*}
\dot{V}_M &= a_X \\
V_M \dot{\theta}_M &= a_Y
\end{align*}
\]
\[
(1 + R) \left( \dot{r}^* \lambda + r^* \dot{\lambda} \right) - R (\dot{r}^* \dot{\phi} + r^* \ddot{\phi})
= -\sin (R\lambda - R\varphi) \left[ R\dot{\lambda} - R\dot{\varphi} \right] [V_T \sin (\theta_T - \lambda) - V_M \sin (\theta_M - \lambda)]
+ \cos (R\lambda - R\varphi) \left[ \dot{V}_T \sin (\theta_T - \lambda) + V_T \cos (\theta_T - \lambda) \left( \dot{\theta}_T - \lambda \right) - \dot{V}_M \sin (\theta_M - \lambda) - V_M \cos (\theta_M - \lambda) \left( \dot{\theta}_M - \lambda \right) \right]
- \cos (R\lambda - R\varphi) \left[ R\dot{\lambda} - R\dot{\varphi} \right] [V_T \cos (\theta_T - \lambda) - V_M \cos (\theta_M - \lambda)]
- \sin (R\lambda - R\varphi) \left[ \dot{V}_T \cos (\theta_T - \lambda) - V_T \sin (\theta_T - \lambda) \left( \dot{\theta}_T - \lambda \right) - \dot{V}_M \cos (\theta_M - \lambda) + V_M \sin (\theta_M - \lambda) \left( \dot{\theta}_M - \lambda \right) \right]
= - \left( R\dot{\lambda} - R\dot{\varphi} \right) [\sin (R\lambda - R\varphi) (V_T \sin (\theta_T - \lambda) - V_M \sin (\theta_M - \lambda))
+ \cos (R\lambda - R\varphi) (V_T \cos (\theta_T - \lambda) - V_M \cos (\theta_M - \lambda))]
\]

Substituting (19) into (18) yields

\[
\dot{\lambda} = \frac{2\dot{r}^* \lambda}{r^*} - \frac{R}{1 + R} \ddot{\varphi} + d_1
= -\frac{\cos (R\lambda - R\varphi)}{r^* (1 + R)} [a_x \sin (\theta_M - \lambda) + a_y \cos (\theta_M - \lambda)]
+ \frac{\sin (R\lambda - R\varphi)}{r^* (1 + R)} [a_x \cos (\theta_M - \lambda) - a_y \sin (\theta_M - \lambda)]
\]

Substituting (21) into (20) yields

\[
\dot{\lambda} = - \frac{2\dot{r}^* \lambda}{r^*} - \frac{R}{1 + R} \ddot{\varphi} + d_1
= \frac{\cos (R\lambda - R\varphi)}{r^* (1 + R)} [a_x \sin (\theta_M - \lambda) + a_y \cos (\theta_M - \lambda) - g \cos \lambda]
+ \frac{\sin (R\lambda - R\varphi)}{r^* (1 + R)} [a_x \cos (\theta_M - \lambda) - a_y \sin (\theta_M - \lambda) - g \sin \lambda]
\]

Considering the effect of gravity during flight, the acceleration signals along and perpendicular to the missile velocity satisfy the equations

\[
a_x = a_x + g \sin \theta_M \quad (21a)
\]
\[
a_y = a_y + g \cos \theta_M \quad (21b)
\]

Figure 1 shows that the missile angle-of-attack \( \alpha \) can be expressed as

\[
\alpha = \varphi - \theta_M \quad (23)
\]
Substituting (23) into (22) yields
\[
\ddot{\lambda} = -\frac{2\dot{r}^*\dot{\lambda}}{r^*} - \frac{R}{1 + R} \dot{\varphi} + d_1
- \frac{\cos (R\lambda - R\varphi)}{r^*(1 + R)} \left[ n_x \sin (\varphi - \lambda) (a_x \cos \alpha + a_y \sin \alpha) \right. \\
+ n_y (\varphi - \lambda) - g \cos \lambda \\
- \frac{\sin (R\lambda - R\varphi)}{r^*(1 + R)} \left[ n_x \cos (\varphi - \lambda) \right. \\
- n_y \sin (\varphi - \lambda) - g \sin \lambda
\]

The full strapdown phased array radar seeker is fixed to the missile body and the main axis of the seeker beam coincides with the missile axis, to ensure the validity of the established model, and therefore information of the missile acceleration along and perpendicular to the missile velocity is required. \(a_x, a_y\) are thus converted to acceleration information along and perpendicular to the missile body axis, \(n_x, n_y\); that is,
\[
n_x = a_x \cos \alpha + a_y \sin \alpha \\
n_y = a_y \cos \alpha - a_x \sin \alpha
\]

Substituting into (22) yields
\[
\ddot{\lambda} = -\frac{2\dot{r}^*\dot{\lambda}}{r^*} - \frac{R}{1 + R} \dot{\varphi} + d_1
- \frac{\cos (R\lambda - R\varphi)}{r^*(1 + R)} \left[ n_x \sin (\varphi - \lambda) \right. \\
+ n_y (\varphi - \lambda) - g \cos \lambda \\
- \frac{\sin (R\lambda - R\varphi)}{r^*(1 + R)} \left[ n_x \cos (\varphi - \lambda) \right. \\
- n_y \sin (\varphi - \lambda) - g \sin \lambda
\]
(24)

Under normal circumstances, when the phased array radar seeker is in a stable tracking state, the DRR error slope \(R << 1\), and the FOV angle of the seeker is then expressed as
\[
\dot{\lambda}_B = \dot{\lambda} - \varphi
= \lambda + \lambda R - \varphi
= (1 + R) (\lambda - \varphi)
\approx \lambda_B
\]
(27)

In the two-dimensional longitudinal plane, the missile pitch attitude dynamics are given by
\[
\dot{\varphi} = \omega_z
\]
\[
\omega_z = \frac{m^2_q S L \alpha}{J_z} + \frac{m^2_s q S L \delta_z}{J_z} + \frac{m^2_s q S L \delta_z}{J_z} + d_2
\]
(28a)
\(28b\)

where \(\omega_z = (\omega_z L) / (V_m)\) denotes the one-dimensional angular velocity, \(q = 0.5 \rho V^2_m\) denotes the head-on dynamic pressure, \(m^2_q\), \(m^2_s\), and \(m^2_s\) are respectively the pitch moment derivatives with respect to the angle-of-attack, dimensionless pitch rate, and actuator deflection angle. \(J_z\) is the moment of inertia around the \(Oz\) axis, and \(d_2\) denotes the disturbance items of the pitch plane dynamics including external atmospheric disturbances, system model parameters, and aerodynamic parameter disturbances and ignoring the coupling effect and the error items existing in the linearization process.

Combining (26–28) yields
\[
\ddot{\lambda}_B = -\frac{2\dot{r}^*\dot{\lambda}}{r^*} + d_1 + d_2 + d_3
+ \frac{\cos (R\lambda_B)}{r^*} \left[ n_x \sin (\lambda_B) - n_y \cos (\lambda_B) + g \cos (\lambda_B + \varphi) \right]
+ \frac{\sin (R\lambda_B)}{r^*} \left[ n_x \cos (\lambda_B) - n_y \sin (\lambda_B) - g \sin (\lambda_B + \varphi) \right]
- \frac{m^2_q S L \alpha}{J_z} - \frac{m^2_s q S L \delta_z}{J_z} - \frac{m^2_s q S L \delta_z}{J_z}
\]
(29)

where \(d_3 = -R \ddot{\varphi}\) is the missile attitude disturbance coupled with the guidance loop.

According to the literature [25], the dynamics of the missile actuator can be regarded as the dynamics of a first-order servo system, that is,
\[
\dot{\delta}_z = -\omega_a \delta_z + \omega_a \delta
\]
(30)

where \(\omega_a\) is the bandwidth of the actuator and \(\delta\) is the control input.

Selecting the states of the guidance and control system as
\[
x_1 = \lambda_B, x_2 = \dot{\lambda}_B \text{ and } x_3 = \delta_z , \text{ and the control input is } u = \delta. \text{ According to (29), the IGC model for the missile equipped with the full strapdown phased array radar seeker considering the DRR effect and the constraints of the seeker FOV can be expressed as}
\[
\dot{x}_1 = x_2 \\
\dot{x}_2 = f_1 + g_1 x_3 + d \\
\dot{x}_3 = f_2 + g_2 u
\]
(31)

where
\[
f_1 = -\frac{2\dot{r}^*\dot{\lambda}}{r^*} + d_1 + d_2 + d_3
+ \frac{\cos (R\lambda_B)}{r^*} \left[ n_x \sin (\lambda_B) - n_y \cos (\lambda_B) + g \cos (\lambda_B + \varphi) \right]
+ \frac{\sin (R\lambda_B)}{r^*} \left[ n_x \cos (\lambda_B) - n_y \sin (\lambda_B) - g \sin (\lambda_B + \varphi) \right]
- \frac{m^2_q S L \alpha}{J_z} - \frac{m^2_s q S L \delta_z}{J_z} - \frac{m^2_s q S L \delta_z}{J_z}
\]
\[
g_1 = -\frac{m^2_s q S L \delta_z}{J_z}, \quad f_2 = -\omega_a \delta_z, \quad g_2 = \omega_a
\]

Assumption 1: Define \(d = d_1 + d_2 + d_3\) as the total disturbance term and suppose that its derivative is bounded and satisfies
\[
|\dot{d}(t)| \leq L_d
\]
(32)

where \(L_d\) is a positive constant.

On the basis of the IGC model (31), the FOV constraint is expressed as
\[
|\lambda_B| = |x_1| \leq \lambda_{B_{max}}
\]
(33)
where $\lambda_{B_{\text{max}}}$ denotes the maximum allowable missile body–LOS angle.

**Assumption 2:** Accurate interception requires the target to be captured and locked by the seeker in the initial stage of the terminal guidance of the intercepting missile. Supposing that the initial body–LOS angle $\lambda_{B_{1}}$ is given by

$$|\lambda_{B_{1}}| < \lambda_{B_{\text{max}}}, \quad (34)$$

**III. IGC SCHEME DESIGN**

Considering the physical limitations of the strapdown phased array seeker, such as the fixed connection with the missile body and FOV constraint, the pure tracking method is selected as the basis for the implementation of accurate interception. In implementing the guidance method of the pure tracking method, the necessary condition for accurate interception is to nullify the leading angle. Therefore, the goal of the IGC scheme is to drive the intercepting missile body–LOS angle $\lambda_{B} = -\alpha$ without violating the seeker FOV constraint.

**Remark 1:** Notably, in actual engineering, the saturation of the actuator needs to be considered when designing the IGC scheme.

In obtaining the accurate differential of the negative angle of attack in real time, the fastest tracking differentiator is used to track the negative angle-of-attack online; that is,

$$\begin{cases}
\dot{\chi}_{1} = \chi_{2} \\
\dot{\chi}_{2} = -Cr \text{sng} \left( \chi_{1} + \alpha + \frac{\lambda_{B_{\text{max}}}^{2} \chi_{2}}{2Cr} \right)
\end{cases} \quad (35)$$

where $\chi_{1}(t)$ will track the negative angle-of-attack signal most rapidly under the acceleration limit $|\dot{\chi}_{1}| \leq Cr - \alpha(t)$, and the tracking speed increases with $Cr$. Obviously, when $|\chi_{1}(t)|$ is sufficiently close to $-\alpha(t)$, $\chi_{2}(t) = \dot{\chi}_{1}(t)$ can be regarded as an approximate differential of $-\alpha(t)$. $Cr > 0$ is the maximum acceleration allowed by the fastest tracking differentiator. The proof is given in the literature [26].

According to the dynamic surface control and iBLF, the IGC scheme can be designed as follows.

**Step 1:** To enable the intercepting missile body–LOS angle to track the reference signal $x_{1d} = -\alpha$ effectively, define the error surface $z_{1}$ as

$$z_{1} = x_{1} - x_{1d}, \quad (36)$$

To ensure that the FOV constraint (33) is satisfied, the following Lyapunov function is designed based on iBLF [27] for the error surface $z_{1}$.

$$V_{1}(z_{1}, \chi_{1}) = \int_{0}^{z_{1}} \frac{\sigma \lambda_{B_{\text{max}}}^{2}}{\lambda_{B_{\text{max}}}^{2} - (\sigma + \chi_{1})^{2}} d\sigma \quad (37)$$

The differentiation of (37) yields

$$\dot{V}_{1} = \frac{\partial V_{1}}{\partial z_{1}} \dot{z}_{1} + \frac{\partial V_{1}}{\partial \chi_{1}} \dot{\chi}_{1} = \frac{\lambda_{B_{\text{max}}}^{2} z_{1}}{\lambda_{B_{\text{max}}}^{2} - x_{1}^{2}} (x_{2} - \chi_{2}) + \frac{\partial V_{1}}{\partial \chi_{1}} \dot{\chi}_{2} \quad (38)$$

Apply the partial integration method and the substitution $\sigma = \beta z_{1}$ to obtain

$$\frac{\partial V_{1}}{\partial \chi_{1}} \dot{\chi}_{1} = z_{1} \left( \frac{\lambda_{B_{\text{max}}}^{2}}{\lambda_{B_{\text{max}}}^{2} - x_{1}^{2}} - \vartheta (z_{1}, \chi_{1}) \right) \quad (39)$$

where

$$\vartheta (z_{1}, \chi_{1}) = \int_{0}^{1} \frac{\lambda_{B_{\text{max}}}^{2}}{\lambda_{B_{\text{max}}}^{2} - (\beta z_{1} + \chi_{1})^{2}} d\beta = \frac{\lambda_{B_{\text{max}}}}{z_{1}} \tanh^{-1} \left( \frac{z_{1} + \chi_{1}}{\lambda_{B_{\text{max}}}^{2} - \chi_{1}} - \tanh^{-1} \left( \frac{\chi_{1}}{\lambda_{B_{\text{max}}}^{2} - \chi_{1}} \right) \right) = \frac{\lambda_{B_{\text{max}}}}{2 z_{1}} \ln \left( \frac{\lambda_{B_{\text{max}}} + z_{1} + \chi_{1}}{\lambda_{B_{\text{max}}} - z_{1} - \chi_{1}} \right) \quad (40)$$

According to L'Hôpital’s rule,

$$\lim_{z_{1} \to 0} \vartheta (z_{1}, \chi_{1}) = \lim_{z_{1} \to 0} \frac{\lambda_{B_{\text{max}}}}{\lambda_{B_{\text{max}}}^{2} - (z_{1} + \chi_{1})^{2}} = \frac{\lambda_{B_{\text{max}}}}{\lambda_{B_{\text{max}}}^{2} - \chi_{1}^{2}}$$

$$|\chi_{1}| < \lambda_{B_{\text{max}}}, \quad \text{and} \quad \vartheta (z_{1}, \chi_{1}) \quad \text{is therefore definite in the neighborhood of} \quad z_{1} = 0.$$ Substituting (39–40) into (38) yields

$$\dot{V}_{1} = \frac{\lambda_{B_{\text{max}}}^{2} z_{1}}{\lambda_{B_{\text{max}}}^{2} - x_{1}^{2}} (x_{2} - \chi_{2}) + z_{1} \left( \frac{\lambda_{B_{\text{max}}}^{2}}{\lambda_{B_{\text{max}}}^{2} - x_{1}^{2}} - \vartheta \right) \chi_{2} \quad (41)$$

Choosing the virtual control input $x_{2d}$ to keep $\dot{V}_{1}$ negative definite, such that $x_{2d}$ is expressed as

$$x_{2d} = -k_{1} z_{1} + \frac{\lambda_{B_{\text{max}}}^{2} - x_{1}^{2}}{2 \lambda_{B_{\text{max}}}} g \chi_{1} - l_{1} |z_{1}| \sigma \text{sgn} (z_{1}) \quad (42)$$

where $k_{1} > 0$, and $l_{1} > 0$.

To avoid the "complexity expansion" problem due to differentiation, let $x_{2d}$ pass the first-order filter with a time constant $\tau_{1}$ to obtain its derivative $\dot{x}_{2d}$:

$$\tau_{1} \dot{x}_{2d} + x_{2c} = x_{2d}, \quad x_{2c}(0) = x_{2d}(0) \quad (43)$$

The tracking error of the first-order filter is defined as

$$e_{f_{1}} = x_{2c} - x_{2d} \quad (44)$$

**Step 2:** Define the error surface $z_{2}$ as

$$z_{2} = x_{2} - x_{2c} \quad (45)$$

Select the Lyapunov function for the error surface $z_{2}$ as

$$V_{2} = \frac{1}{2} z_{2}^{2} \quad (46)$$

The differentiation of (46) yields

$$\dot{V}_{2} = z_{2} \dot{z}_{2} = z_{2} \left( f_{1} + g_{1} x_{3} - \dot{x}_{2c} \right) \quad (47)$$

To suppress the adverse effects of the nonlinear term $f_{1}$ and the total disturbance term $d$ of the system, the reduced-order
ESO is used to estimate the disturbance \( f_1 + d \) online. The reduced-order ESO design is

\[
\begin{align*}
\dot{\xi}_1 &= -\omega_n\xi_1 - \omega_n^2 x_2 - \omega_{10} g_1 x_3 \\
\dot{f}_1 &= \xi_1 + \omega_{10} , \quad \xi_{10} = -\omega_{10} x_2 
\end{align*}
\] (48)

Select the virtual control input \( x_{3d} \) to keep \( \dot{V}_2 \) negative definite, such that \( x_{3d} \) can be expressed as

\[ x_{3d} = \frac{1}{g_1} \left[ -k_2 z_2 - \dot{f}_1 - l_2 |z_2|^2 \text{sgn}(z_2) + \dot{x}_{3c} \right] \] (49)

where \( k_2 > 0, l_2 > 0 \).

To ensure the stable operation of the actuator, the amplitude and speed saturation of the actuator are often considered in engineering practice. A command filter is introduced to obtain the differential of the virtual control command while considering the saturation of the actuator. The command filter is described by

\[
\begin{align*}
\dot{v}_1 &= v_2 \\
\dot{v}_2 &= 2\xi_\omega \left[ S_R \left( \frac{\omega_n^2}{2\xi_\omega} (S_M(U) - v_1) - v_2 \right) \right]
\end{align*}
\] (50)

where \( v_1 = x_{3c}, v_2 = \dot{x}_{3c}, U = x_{3d}, \xi \) is the damping ratio of the command filter, \( \omega_n \) is the bandwidth of the command filter, and \( S_M \) and \( S_R \) are respectively the amplitude limit and rate limit functions.

The structure of the command filter is shown in Figure 2.

![Figure 2: Structure of the command filter](image)

**Remark 2:** According to (50), if the virtual control input \( x_{3d} \) is bounded, then \( x_{3c} \) and \( \dot{x}_{3c} \) are bounded and continuous. Error \( e_{f2} = x_{3c} - x_{3d} \) can be adjusted through the bandwidth \( \omega_n \), and increasing the bandwidth \( \omega_n \) can make \( x_{3c} \) converge to \( x_{3d} \) faster and more accurately. The first-order differential signal obtained using the integration method effectively overcomes the influence of the amplified noise signal due to the differential operation on the desired signal.

**Remark 3:** The command filter will introduce filter errors, which will increase the difficulty of obtaining the smallest tracking error and reduce the dynamic response of the system. To avoid the effect of the filter error, the compensation tracking error is defined as \( \mu_2 = z_2 - \xi_2 \), where \( \xi_2 \) is an error compensation signal expressed as

\[ \dot{\xi}_2 = -k_2 \xi_2 + g_1 (x_{3c} - x_{3d}) \] (51)

**Step 3:** Define the error surface \( z_3 \) as

\[ z_3 = x_3 - x_{3c} \] (52)

and select the Lyapunov function for the error surface \( z_3 \) as

\[ V_3 = \frac{1}{2} z_3^2 \] (53)

The differentiation of (53) yields

\[ \dot{V}_3 = z_3 \dot{z}_3 = z_3 (f_2 + g_2 u) \] (54)

In suppressing the adverse effects of the nonlinear term \( f_2 \) on the system, a reduced-order ESO is used to estimate the disturbance \( f_3 \) online. The reduced-order ESO is designed as

\[
\begin{align*}
\dot{\xi}_2 &= -\omega_2 \xi_2 - \omega_2^2 x_3 - \omega_2 g_2 u \\
\dot{f}_2 &= \xi_2 + \omega_2, \quad \xi_2 = -\omega_2 x_3 
\end{align*}
\] (55)

Select the actual control input \( u \) to keep \( \dot{V}_3 \) negative definite. \( u \) is expressed as

\[ u = \frac{1}{g_2} \left[ -k_3 z_3 - \dot{f}_2 - l_3 |z_3|^2 \text{sgn}(z_3) + \dot{x}_{3c} \right] \] (56)

where \( k_3 > 0, l_3 > 0 \).

The IGC scheme considering the seeker FOV constraint is summarized as

\[
\begin{align*}
\dot{x}_1 &= \chi_2 \\
\dot{\chi}_2 &= -Cr \text{sgn} \left( \chi_1 + \alpha + \frac{\chi_2 |\chi_2|}{2Cr} \right) \\
z_1 &= x_1 - x_{1d} \\
\rho &= \frac{\lambda B_{\max}}{2z_1} \text{ln} \left( \frac{(\lambda B_{\max} + z_1 + \chi_1)(\lambda B_{\max} - \chi_1)}{(\lambda B_{\max} - z_1 + \chi_1)(\lambda B_{\max} + \chi_1)} \right) \\
x_{2d} &= -k_1 z_1 + \frac{(\lambda B_{\max} - x_2^2)}{\lambda B_{\max}} \rho x_2 - l_1 |z_1|^2 \text{sgn}(z_1) \\
x_{2c} &= x_{2d} - \tau_1 \dot{x}_{2c} \\
z_2 &= x_2 - x_{2c} \\
\dot{\xi}_1 &= -\omega_1 \dot{\xi}_1 - \omega_1^2 x_2 - \omega_{10} g_1 x_3 \\
\dot{f}_1 &= \xi_1 + \omega_{10} \\
x_{3d} &= \frac{1}{g_1} \left[ -k_2 z_2 - \dot{f}_1 - l_2 |z_2|^2 \text{sgn}(z_2) + \dot{x}_{3c} \right] \\
U &= x_{3d}, \quad v_1 = x_{3c}, \quad v_2 = \dot{x}_{3c} \\
\dot{v}_1 &= v_2 \\
\dot{v}_2 &= 2\xi_\omega \left[ S_R \left( \frac{\omega_n^2}{2\xi_\omega} (S_M(U) - v_1) - v_2 \right) \right] \\
z_3 &= x_3 - x_{3c} \\
\dot{\xi}_2 &= -\omega_2 \dot{\xi}_2 - \omega_2^2 x_3 - \omega_2 g_2 u \\
\dot{f}_2 &= \xi_2 + \omega_2, \quad \xi_2 = -\omega_2 x_3 \\
u &= \frac{1}{g_2} \left[ -k_3 z_3 - \dot{f}_2 - l_3 |z_3|^2 \text{sgn}(z_3) + \dot{x}_{3c} \right] 
\end{align*}
\] (57)

### A. CLOSED-LOOP SYSTEM STABILITY ANALYSIS

The following lemmas are introduced prior to analyzing the stability of the closed-loop system.
**Lemma 1:** [27] For any constant $k_{a_1}$ and $k_{b_1}$, let $\mathcal{Z}_1 := \{z_1 \in \mathbb{R} : -k_{a_1} < z_1 < k_{b_1}\} \subset \mathbb{R}$ and $\mathcal{N} := \mathbb{R}^l \times \mathcal{Z}_1 \subset \mathbb{R}^{l+1}$ be an open set. Considering the system

$$\dot{\eta} = h(t, \eta)$$  \hspace{1cm} (58)

where $\eta := [\omega, z_1]^T \in \mathcal{N}$, $h: \mathbb{R} \times \mathcal{N} \to \mathbb{R}^{l+1}$ is piecewise continuous in time and locally Lipschitz in $z$. Assuming that there are functions $U: \mathbb{R}^l \to \mathbb{R}_+$ and $V_1: \mathcal{Z}_1 \to \mathbb{R}_+$ that are continuously differentiable and positive definite in each domain. When $z_1 \to -k_{a_1}$ or $z_1 \to k_{b_1}$ such that

$$V_1(z_1) \to \infty$$  \hspace{1cm} (59)

Meanwhile, there exists

$$\gamma_1(\|\omega\|) \leq U(\omega) \leq \gamma_2(\|\omega\|)$$  \hspace{1cm} (60)

where $\gamma_1$ and $\gamma_2$ are class $K_\infty$ function.

Let $V(\eta) := V_1(z_1) + U(\omega)$ and $z_1$ belong to the set $z_1 \in (-k_{a_1}, k_{b_1})$. If the inequation

$$\dot{V} = \frac{\partial V}{\partial \eta} h \leq 0$$  \hspace{1cm} (61)

holds, then any $t \in [0, \infty)$, $z_1(t)$ will always be in the open set $z_1 \in (-k_{a_1}, k_{b_1})$.

Define the estimation error of the reduced-order ESO as

$$e_{re1} = \dot{f}_1 - f_1 - d$$  \hspace{1cm} (62a)

$$e_{re2} = \dot{f}_2 - f_2$$  \hspace{1cm} (62b)

The differentiation of (62) yields

$$\dot{e}_{re1} = -\omega_1 e_{re1} - \dot{f}_1 - d$$  \hspace{1cm} (63a)

$$\dot{e}_{re2} = -\omega_2 e_{re2} - \dot{f}_2$$  \hspace{1cm} (63b)

The quadratic Lyapunov function is defined as

$$V_1 = \frac{1}{2} e_{re1}^2 + \frac{1}{2} e_{re2}^2$$  \hspace{1cm} (64)

The differentiation of $V_1$ yields

$$\dot{V}_1 = e_{re1} \dot{e}_{re1} + e_{re2} \dot{e}_{re2} = -\omega_1 e_{re1}^2 - \dot{f}_1 - d - \omega_2 e_{re2}^2 - \dot{f}_2 \leq -\lambda_{\text{min}} \kappa \|E\|^2 + \|H\| \|E\|$$  \hspace{1cm} (65)

where $E = [e_{re1}, e_{re2}]^T, \kappa = \text{diag}(\omega_1, \omega_2) \text{ and } H = [\dot{f}_1 + d, \dot{f}_2]$. It follows from

$$\|E\|^2 \geq \frac{\|\dot{f}_1 + d\|}{\theta \lambda_{\text{min}} \kappa} \geq \frac{\|H\|}{\theta \lambda_{\text{min}} \kappa}$$  \hspace{1cm} (66)

that

$$\dot{V}_1 \leq -\lambda_{\text{min}} \kappa (1 - \theta) \|E\|^2$$  \hspace{1cm} (67)

where $0 < \theta < 1$.

The tracking error $e_{f1}$ produced by the first-order filter is defined using the Lyapunov function

$$V_2 = \frac{1}{2} e_{f1}^2$$  \hspace{1cm} (68)

The differentiation of (68) and Young’s inequality yield

$$\dot{V}_2 = e_{f1} \dot{e}_{f1} = (\dot{x}_{2c} - \dot{x}_{2d}) (x_{2c} - x_{2d}) = \left(\frac{x_{2d} - x_{2c}}{\tau_1} - \dot{x}_{2d}\right) (x_{2c} - x_{2d})$$  \hspace{1cm} (69)

**Assumption 3:** For the command filter output $x_{3c}$, there is a constant $\kappa_2$ such that $\|x_{3c} - x_{3d}\| \leq \kappa_2$.

For the error compensation signal $\zeta_2$ in the command filter, define the Lyapunov function

$$V_3 = \frac{1}{2} \zeta_2^2$$  \hspace{1cm} (70)

The differentiation of (70) and Young’s inequality yield

$$\dot{V}_3 = \zeta_2 \dot{\zeta}_2 = -k_2 \zeta_2^2 + g_1 (x_{3c} - x_{3d})$$  \hspace{1cm} (71)

$$\leq -k_2 \zeta_2^2 + \sigma_2 \kappa_2 \zeta_2$$

Applying Assumption 3 and Cauchy’s inequality yields

$$\dot{V}_3 \leq -k_2 \zeta_2^2 + \sigma_2 \zeta_2 \|\zeta_2\| (x_{3c} - x_{3d})$$  \hspace{1cm} (72)

According to Young’s inequality,

$$\sigma_2 \zeta_2 \|\zeta_2\| \leq \frac{\sigma_2}{2} (\zeta_2^2 + \kappa_2^2)$$  \hspace{1cm} (73)

Substituting (73) into (72) yields

$$\dot{V}_3 < -k_2 \zeta_2^2 + \frac{\sigma_2}{2} (\zeta_2^2 + \kappa_2^2)$$  \hspace{1cm} (74)

Select $k_2$ to satisfy $k_2 > 0.5 \sigma_2$, where $k_2 = \min (k_2)$. Define

$$a = \min \left(\frac{k_2 - \frac{3}{2} \sigma_2}{\tau_1}\right) \text{ and have}$$

$$\dot{V}_3 \leq -2a V_3 + \frac{1}{2} \sigma_2 \kappa_2$$  \hspace{1cm} (75)

$\zeta_2$ then satisfies the condition of uniform ultimate boundedness:

$$\|\zeta_2\| \leq \sqrt{\frac{\sigma_2 \kappa_2}{2a}} + 2 \left(V_3(0) - \frac{\sigma_2 \kappa_2}{2a}\right) e^{2at}$$  \hspace{1cm} (76)

According to (65), (41) can be rewritten as

$$\dot{V}_1 = \frac{\lambda_2}{\lambda_{B_{\text{max}}}} \la z_{21} \right(2 \lambda_{B_{\text{max}}} > \lambda_{11} - \theta \lambda_{B_{\text{max}}})$$  \hspace{1cm} (77)
According to (69), (47) can be rewritten as
\[
\dot{V}_2 = z_2 (z_3 + e_{r1} - k_2 z_2 - l_2 |z_2|^{a_2} \text{sgn}(z_2)) \\
\leq -k_2 z_2^2 + z_3^2 + e_{r1}^2
\]  
(78)

It follows from (75) that
\[
\dot{V}_3 = z_3 (-k_3 z_3 + e_{r2} + e_{f2} - l_3 |z_3|^{a_3} \text{sgn}(z_3)) \\
\leq -k_3 z_3^2 + e_{r2}^2 + e_{f2}^2
\]  
(79)

In summary, define the Lyapunov function \( V = V_1 + V_2 + V_3 \), and its derivative
\[
\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3
\]  
\[\leq -k_1 \lambda_{B_{\max}} z_1^2 - k_2 z_2^2 - k_3 z_3^2 + e_{f1}^2 + e_{f2}^2 + e_{r1}^2 + e_{r2}^2
\]  
(80)

It is seen that the appropriate selection of parameters ensures that \( V \) converges to a compact set:
\[
\dot{V} = -k_1 \lambda_{B_{\max}} z_1^2 - k_2 z_2^2 - k_3 z_3^2 + C
\]  
(81)

where \( C = e_{f1}^2 + e_{f2}^2 + e_{r1}^2 + e_{r2}^2 \).

IV. NUMERICAL SIMULATIONS AND ANALYSIS
A. VERIFICATION OF THE EFFECT OF DRR ERROR

The nonlinear model of a roll-stabilized missile equipped with a phased array radar seeker is
\[
\begin{align*}
\dot{X}_m &= V_m \cos \theta \\
\dot{Y}_m &= V_m \sin \theta \\
\dot{V}_m &= \frac{P \cos \alpha - D - m g \sin \theta}{m} \\
\dot{\theta} &= \frac{P \sin \alpha + Y - m g \cos \theta}{m V_m} \\
\dot{\alpha} &= \omega_z - \frac{P \sin \alpha + Y - m g \cos \theta}{m V_m} \\
\dot{\omega}_z &= M_Y / I_Y
\end{align*}
\]  
(82)

where \( X_m \) and \( Y_m \) denotes the coordinates of the missile in the inertial coordinate system. The missile thrust \( P \), drag \( D \), lift \( Y \) and pitch moment \( M_z \) are modeled as
\[
\begin{align*}
P &= 300 N \\
D &= c_{xz} qS + c_{xz}^2 qS \alpha^2 \\
Y &= c_{y\alpha} qS\alpha + c_{y\delta} qS \delta_z \\
M_z &= m_c^2 q S \lambda + \frac{m_c^2 q S L^2 \omega_z}{I_Y} + m_c^2 q S \lambda \delta_z
\end{align*}
\]  
(83)

Considering the actual physical limitations, the allowable maximum actuator deflection angle is set as \( \delta_z = 20^\circ \) and the allowable maximum actuator deflection angular rate is set as \( \dot{\delta}_z = 200^\circ/s \). The missile structural parameters and aerodynamic coefficients used in the simulations are given in Table 1.

It is assumed that the velocity of the target is \( V_T = 300 m/s \) without lateral maneuvering. The initial position

\[X_{T0} = 6000 m, \ Y_{T0} = 3000 m, \ \text{and the flight path angle} \ \theta_T = 20^\circ \ \text{throughout the flight.} \]

The initial velocity of the missile is \( V_M = 1500 m/s \), and the initial flight path angle is \( \theta_M = 30^\circ \). Considering the limitation of the phased array radar seeker FOV, we suppose that the maximum FOV constraint is \( \lambda_{B_{\max}} = 60^\circ \). The simulation parameters of the iBLF-IGC scheme are given in Table 2.

| Parameter | Value | Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|-----------|-------|
| \( m \)   | 4 (kg) | \( I_Y \) | 0.85 (kg \cdot m^2) |
| \( S \)   | 0.013 (m^2) | \( c_{x0} \) | 0.18 |
| \( g \)   | 9.8 (m/s^2) | \( c_{y0} \) | 15.63 (rad^-1) |
| \( e_{x1}^2 \) | 7.65 (rad^-2) | \( m_c^2 \) | -2.01 (rad^-1) |
| \( \delta_z \) | -0.05 (rad^-1) | \( m_c^2 \) | -1.57 (rad^-1) |

TABLE 1: Missile structural parameters and aerodynamic coefficients

X

TABLE 2: Design parameters of the iBLF-IGC scheme

The adverse effects of the seeker DRR error slope are reflected by setting \( R = -0.1 \sin \left( \frac{3}{4} \pi t \right) \). The simulation results are presented in Figure 3.

Figure 3(j) shows that the existence of the DRR error slope R of the full strapdown phased array radar seeker indeed affects the nonlinear and error terms of the system. It is thus necessary to consider the error slope produced by the seeker DRR problem comprehensively in the design of the integrated missile guidance and control scheme. Figure 3(b) shows that compensating for the system’s nonlinear and interference terms under the influence of the total error slope as shown in Figure 3(i) effectively reduces the terminal miss distance of the guidance and control system. As shown in Figure 3(d–g), during the interception process, the actuator deflection angle makes the response of the missile body’s angle of attack positive, thereby offsetting the effect of gravity in the simulation model. When the missile is far from the target, the error between the apparent and real target positions is negligible. The seeker LOS angle without compensation for the DRR error slope is near zero but does not converge to zero. As the distance between the missile and target reduces gradually, the seeker cannot point to the real target position owing to the effect of the DRR error slope, resulting in a huge terminal miss distance. In contrast, the LOS angle of the seeker after compensating for the seeker DRR error slope can converge to zero when the missile and target are relatively far away from one another. With a gradual decrease in the
missile–target distance, the missile can still obtain the real target information and ensure that it is always pointed at the real target position to achieve an accurate interception. Additionally, Figure 3(a) shows that although the seeker DRR error slope and its rate of change are small, the slope has a large adverse effect on the terminal miss distance when the distance between the missile and target is small.

B. SIMULATION INVOLVING A LATERAL MANEUVERING TARGET

A missile equipped with a phased array radar seeker is usually launched to intercept a target with high maneuvering ability. It is assumed that the target has a fixed flight speed \( V_T = 3000 \text{m/s} \) and has lateral maneuvering of \( \alpha_T = 30 \cos \left( \frac{2}{\pi} t \right) \text{m/s}^2 \). A distance between the missile and target \( R < 10 \text{ m} \) is regarded as an accurate interception because of the dead zone of the seeker. Considering the dead zone of the seeker when the distance between the missile and target \( R < 10 \text{ m} \) is regarded as an accurate interception.

A control-group TanBLF-IGC [28] scheme is designed using the tangent Lyapunov function to further highlight the characteristics of the iBLF-IGC scheme:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -C_r \text{sgn} \left( \chi_1 + \alpha + \frac{\chi_2 |x_2|}{2C_r} \right) \\
z_1 &= x_1 - x_{1d} \\
\dot{\xi}_{tan} &= \frac{\pi z_1}{2I_{tan}^{max}} \\
x_{2d} &= -k_1 z_1^2 \cos^2 \xi - l_1 |z_1|^{a_1} \text{sgn} (z_1) + \chi_2 \\
x_{2c} &= x_{2d} - \tau_1 \dot{x}_{2c} \\
z_2 &= x_2 - x_{2d} \\
\dot{\xi}_1 &= -\omega_{01} \xi_1 - \omega_{02} x_2 - \omega_{03} |g_1 x_3| \\
\dot{\xi}_2 &= \dot{\xi}_1 + \omega_{01} \\
x_{3d} &= \frac{1}{g_1} \left[ -k_2 z_2 - \dot{f}_1 - l_2 |z_2|^{a_2} \text{sgn} (z_2) + \dot{x}_{2c} - \frac{\sin \xi}{\cos^3 \xi} \right] \\
U &= x_{3d}, \quad v_1 = x_{3c}, \quad v_2 = \dot{x}_{3c} \\
v_1 &= v_2 \\
v_2 &= 2\xi_{\omega_{n}} \left[ S_R \left( \frac{\omega_n^2}{2\xi_{\omega_{n}}} (S_M (U) - v_1) - v_2 \right) \right] \\
z_3 &= x_3 - x_{3c} \\
\dot{\xi}_2 &= -\omega_{02} \xi_2 - \omega_{03} x_3 - \omega_{04} g_2 u \\
\dot{f}_2 &= \xi_2 + \omega_{02} \\
u &= \frac{1}{g_2} \left[ -k_3 z_3 - \dot{f}_2 - \frac{\omega_{03}}{2} |\xi_3|^{a_3} \text{sgn} (z_3) + \dot{x}_{3c} \right] \\
\end{align*}
\]

The initial velocity of the missile is \( V_M = 1500 \text{m/s} \), and the initial flight path angle \( \theta_M = 30^\circ \). It is assumed that the seeker maximum FOV is \( \lambda_{tan}^{max} = 60^\circ \). The seeker DRR error slope setting is \( R = -0.1 \sin \left( \frac{3}{4} \pi t \right) \). The simulation results are presented in Figure 4.

Figure 4(i–j) shows that by virtue of the reduced-order ESO, the integrated guidance and control scheme based on the two different Lyapunov functions can accurately estimate and compensate for the nonlinear and disturbance terms affected by the seeker DRR error slope in the system. Figure 4(g) shows that when the distance between the missile and target is relatively large, the two design schemes ensure that the LOS angle of the seeker converges to zero. When the distance between the missile and target is small, the two design schemes increase the actuator deflection angle response in time to intercept the maneuvering target. Figures 4(c) and 4(k) show that the virtual control command \( \alpha_1 \) in the TanBLF-IGC scheme is designed to ensure that the error surface \( z_1 \) converges to zero while ensuring \( S_1 \) does not violate constraints. The difference is that the iBLF-IGC scheme only needs to consider that the seeker FOV constraint is not violated in the design of virtual control commands whereas in the case of the TanBLF-IGC scheme, \( z_1 \) conflicts with the constraints but the actual seeker FOV angle does not violate the constraints, and the FOV angle cannot be effectively constrained, which increases the miss distance. The reason for this difference is that the TanBLF-IGC scheme uses the tangent Lyapunov function and the bound term is the sliding surface \( z_1 \), and not the body–LOS angle. The iBLF-IGC scheme uses the integral Lyapunov function and the bound term is the body–LOS angle, which must be constrained in the IGC scheme design. Moreover, the TanBLF-IGC scheme requires more control energy when the initial tracking error is small; that is, in the error surface convergence process, the actuator deflection angle response produced by the TanBLF-IGC scheme is greater than that of the iBLF-IGC scheme, which is reflected in Figure 4 (d–e). Furthermore, Figure 4(e) shows that when the actuator deflection angular rate is constrained, the command filter effectively ensures that the constraint is not violated.

V. CONCLUSION

This article proposed an iBLF dynamic surface missile IGC control scheme for strapdown phased array seeker homing missiles considering the adverse effects of the DRR problem, resulting from radome error and radar beam pointing error.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| \( k_1 \) | 80 | \( k_2 \) | 3 |
| \( l_1 \) | 5 | \( l_2 \) | 1 |
| \( \tau \) | 0.001 | \( \omega_{01} \) | 200 |
| \( \omega_{02} \) | 200 | \( \xi_1 \) | 0.7 |
| \( \omega_n \) | 150 | \( a_1 \) | 0.3 |
| \( a_2 \) | 0.8 | \( a_3 \) | 0.8 |

TABLE 3: Parameters of the TanBLF-IGC scheme

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The iBLF dynamic surface control substantially reduces the disturbance introduced by radome error and beam pointing error such that the missile can intercept the target more precisely. The use of iBLF ensures that the seeker FOV constraint is not violated and the command filter guarantees that the requirements of the actuator deflection angle and angular rate are satisfied. Additionally, the use of a reduced-order EOS solves the problem that the effects of the DRR problem and thus the target maneuvering and external disturbances are difficult to measure. The stability analysis and simulations show that the proposed scheme is effective and feasible. In future research, we will further verify the effectiveness of the scheme through practical examples and study the iBLF dynamic surface control strategy for a high-speed target with high maneuverability under huge nonlinear DRR effects.

REFERENCES

[1] H. Zhao, “Research on overseas phased array radar seeker technology development (in Chinese),” Aero Weaponry, no. 3, pp. 10–17, 2018.

[2] M. I. Skolnik, Radar handbook. McGraw-Hill Education, 2008.

[3] Q. WEN, T. LU, Q. XIA, and Z. SUN, “Beam-pointing error compensation method of phased array radar seeker with phantom-bit technology,” Chinese Journal of Aeronautics, vol. 30, no. 3, pp. 1217–1230, 2017.

[4] P. L. Vergez and J. R. McClendon, “Optimal control and estimation for strapdown seeker guidance of tactical missiles.” Journal of Guidance, Control, and Dynamics, vol. 5, no. 3, pp. 225–226, 1982.

[5] F. W. Nesline and P. Zarchan, “Line-of-sight reconstruction for faster homing guidance.” Journal of Guidance, Control, and Dynamics, vol. 8, no. 1, pp. 3–8, 1985.

[6] R. D. Ehrich and P. Vergez, “Strapdown seeker technology for the terminal guidance of tactical weapons.” In AGARD Guidance and Control Aspects of Tactical Air-launched Missiles 15 p (SEE 881–16902 07-15, 1980).

[7] T. Lu, Q. Wen, and J. Yin, “The effect of phantom-bit technology on the performance of phased array seeker detection in the case of the initial beam angle.” Optik, vol. 127, no. 20, pp. 9996–101003, 2016.

[8] P. Gurfil and N. J. Kasdin, “Improving missile guidance performance by in-flight two-step nonlinear estimation of radome aberration.” IEEE transactions on control systems technology, vol. 12, no. 4, pp. 532–541, 2004.

[9] P. Zarchan, “Proportional navigation and weaving targets.” Journal of Guidance, Control, and Dynamics, vol. 18, no. 5, pp. 969–974, 1995.

[10] P. Zarchan and H. Pratt, “Adaptive radome compensation using dither.” Journal of Guidance, Control, and Dynamics, vol. 22, no. 1, pp. 51–57, 1999.

[11] W. W. Willman, “Effects of strapdown seeker scale-factor uncertainty on optimal guidance.” Journal of Guidance, Control, and Dynamics, vol. 11, no. 3, pp. 199–206, 1988.

[12] W. R. Yueh, “Adaptive estimation scheme for radome error calibration.” In The 22nd IEEE Conference on Decision and Control. IEEE, 1983, pp. 546–551.

[13] W. YUEH and C.-F. LIN, “Bank-to-turn guidance performance analysis with in-flight radome error compensation.” In 17th Fluid Dynamics, Plasma Dynamics, and Lasers Conference. American Institute of Aeronautics and Astronautics, 1984, p. 1889.

[14] W. R. Yueh and C.-F. Lin, “Guidance performance analysis with in-flight radome error calibration.” Journal of Guidance, Control, and Dynamics, vol. 8, no. 5, pp. 666–669, 1985.

[15] J.-M. Lin and Y.-F. Chau, “Radome slope compensation using multiple-model kalman filters.” Journal of Guidance, Control, and Dynamics, vol. 18, no. 3, pp. 637–640, 1995.

[16] X. Cao, C. Dong, Q. Wang, and Y. Chen, “Radome slope estimation using multiple model based on ekf (in chinese).” ACTA AERONAUTICA ET ASTRONAUTICA SINICA, no. 8, pp. 1608–1613, 2010.

[17] S.-Y. Lin, D. Lin, and W. Wang, “A novel online estimation and compensation method for strapdown phased array seeker disturbance rejection effect using extended state kalman filter.” IEEE Access, vol. 7, pp. 172330–172340, 2019.

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(a) The trajectory of missile
(b) The relative missile-target distance
(c) Body-LOS angle
(d) Actuator deflection angle
(e) Actuator deflection angular rate
(f) Angle-of-attack
(g) Look angle
(h) Pitch angle
(i) DRR error slope
(j) Error and its estimation
(k) Sliding surfaces

FIGURE 3: Comparison of seeker DRR error slope compensation before and after.
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**FIGURE 4: Comparison of the different barrier Lyapunov functions intercepting maneuvering target**