Stochastic optimization model for integrated energy system under uncertainty based on chance-constrained programming

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Abstract. To solve the day-ahead optimal dispatching of the integrated energy system (IES), the influence of uncertainties in the operation is taken into consideration, and the application of combined heat and power (CHP) and Power to Gas (P2G) equipment can improve the system’s ability to accommodate renewable energy and reduce system operating costs. In order to minimize the operating cost of IES, a mixed-integer linear programming (MILP) optimization model based on chance-constrained is proposed in this paper. A scenario-based simulation method is proposed to convert the chance-constrained programming (CCP) model into a deterministic one. The model in this paper can effectively reduce the risk of system operation in uncertain environments and improve the stability of system operation.

1. Introduction

The overuse of fossil fuels has become one of the key problems restricting the development of China caused by environmental pollution and energy shortage. However, the nature of renewable energy is stochastic, which brings security challenges to the system. Integrated Energy System (IES) is becoming significant because it can counterbalance the drawbacks[1]. There are a variety of energy sources with different operating characteristics in the IES, which can effectively promote the complementary and coordinated utilization of energy sources, reduce environmental pollution, and operation cost[2].

The dispatching of IES is different from the traditional power grids and micro-grid[3]. The day-ahead scheduling of IES has been discussed in many research to ensure reliable and economic operations. In Ref[4], they showed that CHP micro-grid can solve the problems between energy demand and the environment. In Ref [5], they proposed a unified model, which defined all kinds of energy buses and their constraints. The IES with uncertainties bring lots of challenges in the day-ahead scheduling. Different algorithms such as stochastic optimization (SO) and robust optimization (RO), can be considered for the IES operation optimization in uncertain environments[6]. The model is an NP-hard problem[7]. A mixed-integer linear programming (MILP) model was proposed for energy adequacy of IES short-term operation[8]. In reference [9], a stochastic day-ahead scheduling model considering uncertainties of wind energy was proposed. The scenario-based SO required sampling.
scenarios according to the distribution of uncertain variables, which was hard to obtain, and the computational time increased as scenario size growing[10].

Considering the above issue, this paper presents a linear stochastic optimization model for the operation of the IES system based on chance-constrained programming (CCP). The forecast errors of renewable energy output are modelled using the scenario analysis technique, and the risks caused by the errors are considered through the CCP model. A scenario-based simulation method is used to convert the CCP model into a deterministic model. The deterministic MILP model can be effectively obtained by plenty of solvers.

2. The IES model

The IES includes many types of energy, such as electricity, heat, and gas. The conversion of different energy can improve the efficiency of energy utilization and the economy and environmental protection of system operation. In IES, the balance and coupled relationship among energy are in consideration. Besides, more attention should be paid to the operation status of each piece of equipment.

2.1. CHP system operation constraints

The system mainly consists of CHP units, distributed generation, heating electricity conversion equipment, and various types of loads. The CHP unit is mainly composed of a gas turbine and exhaust heat boiler, which burns gas to drive gas turbine generating electricity. The passed out gas with high temperature is recovered by exhaust heat boiler and converted into heat energy. The formulas as follows,

\[
\begin{align*}
\Phi_{\text{CHP}} &= c_{\text{CHP}} V_{\text{CHP}} \\
\bar P_{\text{CHP}} &= \eta_{\text{CHP}} V_{\text{CHP}}
\end{align*}
\]

where \(\Phi_{\text{CHP}}\) and \(P_{\text{CHP}}\) are heat release power and discharge power of CHP unit respectively; \(V_{\text{CHP}}\) represents the gas consumption power; \(c_{\text{CHP}}\) and \( \eta_{\text{CHP}}\) indicate the heat conversion efficiency and electricity conversion efficiency respectively.

The P2G technology refers to the technique of converting electric power into gas fuel. The relationship between the gas generated by the unit and the electricity consumption is as follows:

\[
V_{\text{PG}} = \mu_{\text{PG}} P_{\text{PG}}
\]

where \(V_{\text{PG}}\) is the output power of gas; \(P_{\text{PG}}\) represents the power of P2G; \( \mu_{\text{PG}}\) indicates the conversion efficiency.

The electric boiler (EB) can convert electricity into heat through heating elements, and the relationship between the electricity consumed and the heat power generated is as follows:

\[
\Phi_{\text{EB}} = \eta_{\text{EB}} P_{\text{EB}}
\]

where \(\Phi_{\text{EB}}\) is the output heat power of EB; \(P_{\text{EB}}\) represents the electric power consumption of electric boiler; \( \eta_{\text{EB}}\) represents the electric heat conversion efficiency of an electric heat boiler.

2.2. Other constraints on power units

According to the energy form, the energy storage device can be divided into four types: electric, heat, cooling, and gas. The relationship between the state of charge (SOC) and the charging and discharging power is as follows:

\[
SOC(t + 1) = (1 - \sigma)SOC(t) + \frac{\eta_{\text{ch}} P_{\text{ch}}(t) \Delta t}{E_{\text{es}} \eta_{\text{dis}}} - \frac{P_{\text{dis}}(t) \Delta t}{E_{\text{es}} \eta_{\text{dis}}}
\]

where \(P_{\text{ch}}(t)\) and \(P_{\text{dis}}(t)\) are the charge and discharge power respectively; \( \eta_{\text{ch}}\) and \( \eta_{\text{dis}}\) represent the charge and discharge efficiency respectively; \(E_{\text{es}}\) indicates the capacity of the energy storage. The binary variable is added to constrain the charge and discharge power of the energy storage as follows:
\[ \begin{align*}
K_{\text{ess}P_{ch}}^\text{min} & \leq P_{ch} \leq K_{\text{ess}P_{ch}}^\text{max} \\
(1 - K_{\text{ess}P_{dis}}^\text{min}) & \leq P_{dis} \leq (1 - K_{\text{ess}P_{dis}}^\text{max})
\end{align*} \]  

(5)

where \( K_{\text{ess}} \) is the state of storage, 0 indicates discharging and 1 indicates charging; \( P_{ch}^\text{max} \) and \( P_{ch}^\text{min} \) represent the maximum and minimum charging power respectively; \( P_{dis}^\text{max} \) and \( P_{dis}^\text{min} \) represent the maximum and minimum discharge power respectively.

To meet the power balance, the IES interacts with main power grid. The devices limits are as follows

\[ P_{all}^\text{min} \leq P(t) \leq P_{all}^\text{max} \]

(6)

where \( P_{all} \) includes power of all the devices like SOC, interaction power and CHP; \( P_{all}^\text{min} \) and \( P_{all}^\text{max} \) denote the minimum and maximum values of them respectively. The online or offline time of controllable DG in the grid is decided day-ahead, and the constraints as follows

\[ \begin{align*}
x^{t-1} - x' + u' & \geq 0 \\
x' - x^{t-1} + v' & \geq 0 \\
x' P_{all}^\text{min} & \leq P(t) \leq x' P_{all}^\text{max}
\end{align*} \]  

(7)

where \( x', u', v' \) are binary variables; if the DG is on, \( x'=1 \); if the DG is turned on, \( u'=1 \); if the DG is turned down, \( v'=1 \); \( P_{all}^\text{min} \) and \( P_{all}^\text{max} \) denote the minimum and maximum power of DG. Power flow constraints can be added to the model as security constraints. The constraints are as follows

\[ \begin{align*}
P_q &= g_q \left( V_i - V_j \right) - b_q \left( \theta_i - \theta_j \right) \\
Q_q &= -b_q \left( V_i - V_j \right) - g_q \left( \theta_i - \theta_j \right) \\
Pr\{f_i \leq \bar{f}_i\} & \geq \alpha_1, Pr\{f_i \geq \underline{f}_i\} \geq \alpha_1
\end{align*} \]  

(8)

where \( P_q \) and \( Q_q \) represent the active power and reactive power flow from bus \( i \) to bus \( j \); \( V_i \) is the voltage magnitude and \( \theta_i \) is the phase angle; \( g_q \) and \( b_q \) indicate the real and imaginary part of branch admittance. Eq(8) is the linear power flow based on Ref [11]. \( f_i \) includes branch power flow and bus voltage, Eq (9) is the chance-constraint form of their bounds, \( \bar{f}_i \) and \( \underline{f}_i \) indicate their upper and lower limits respectively; \( \alpha_1 \) is the confidence level at which the value does not exceed their bounds.

3. The optimization model

The task of the optimization model for the day-ahead operation of the IES is to determine the power output of each piece of equipment with minimum operation cost based on forecasted wind power and demands. However, wind power output cannot be predicted accurately. A CCP based optimization model considers the uncertainty of wind power output introduced confidence parameters as probabilistic constraints. This model required the objectives and constraints can be satisfied to a certain probability under an operation decision.

As mentioned above, the goal of the proposed model is to make decisions on the schedule of equipment output and satisfy other operational constraints under minimum operating cost. The objective function is formulated as follows

\[ \min F = \sum_{t=1}^{T} \left( C_{\text{price}}(t) P_{\text{grid}}(t) \Delta t + C_{\text{gas}} V_{\text{gas}}(t) \Delta t + C_{\text{dis}}(t) \right) \]

(10)

where \( T \) is the operational period; \( C_{\text{price}}(t) \) is the time-of-day tariff; \( \Delta t \) is the time step; \( C_{\text{gas}} \) is the price of gas; \( V_{\text{gas}} \) indicates the gas purchasing; \( C_{\text{dis}}(t) \) indicates the operating cost of DG, \( C_{\text{dis}}(t) = a \cdot P_{\text{dis}}(t) + c \cdot u' \), \( a \) and \( c \) are the coefficient and start-up costs of DG.
An effective way to solve the CCP model is converting the constraints into equivalent deterministic ones\[12\]. In this paper, we use the scenario-based simulation method. Eq(9) can be transformed into the following formulations:

\[
\begin{align*}
    f_i - \bar{f}_i & \leq L \cdot (1 - \gamma_n) \\
    \bar{f}_i - f_i & \leq L \cdot (1 - \lambda_n) \\
    \sum_{n=1}^{N} P_n \cdot \gamma_n & \geq \alpha_i \\
    \sum_{n=1}^{N} P_n \cdot \lambda_n & \geq \alpha_i
\end{align*}
\]

(11)

where \( \gamma_n \) and \( \lambda_n \) are binary variables used to determine whether the values do not exceed their upper and lower bounds in scenario \( n \) (equal to 1 if true); \( L \) is a very large positive number; \( P_n \) is the probability of scenario \( n \). \( \sum_{n=1}^{N} P_n \cdot \gamma_n \geq \alpha_i \) guarantees the probability that the values do not exceed their upper and lower bounds is over \( \alpha_i \). The formulations (11) is equivalent to Eq(9).

Wind power has certain randomness, and forecast errors are inevitable due to the limited forecasting techniques. The scenario technique is introduced in this paper to deal with uncertainty. Here is the step:

1) The errors of the forecast wind power output sequence \( \{p_T^{WT}, \ldots, p_T^{WT}\} \) are assumed to obey the normal distribution \( N(\mu, \sigma^2) \) in any period, where \( \mu = 0, \sigma = 0.15 p_T^{WT} \).

2) The Latin Hypercube Sampling (LHS)[13] can reflect the overall distribution of random variables with fewer sampling scenarios. So LHS has been used to improve the efficiency of the scenario-based stochastic simulation.

3) To reduce the burden of calculation, the k-means[14] clustering technique is introduced to reduce the number of scenarios after LHS.

4. Case study

The proposed model was applied to the day-ahead operation of the IES. The specific network is shown in Figure 1. The network includes one electricity load, one heat load, one gas load, one wind power DG. The prediction data is shown in Figure 2. There are three electricity rates applied to the system when electricity is purchased from the main grid as peak, normal, and valley level. During the peak hours (12:00-15:00 and 18:00-20:00), the price is $1.25/kWh; in the normal period (7:00-11:00, 16:00-17:00, 21:00), the price is $0.75/kWh; in the valley period (22:00-6:00), the price is $0.35/kWh. The price of the gas is $0.4/kWh.
4.1. Base case study
In this case, $\alpha$ is set to 0.9. The number of scenarios of the forecast error of WT output by the LHS was 500 and these scenarios were reduced to 50 through the k-means clustering technique. The day-ahead scheduling results are shown below. In the electricity grid, the grid combines CHP units, power storage, DG, WT, and interaction power from the main grid to meet the demand. In the heat network, CHP units, storage, and EB are used to meet the demand of heat load. The gas network meets the gas load through P2G, storage, and purchasing gas. As shown in Figures, in the peak time, the DG turned on because of the lower costs and the IES sells electricity to the main grid. In the heat network, due to the high electricity price, the operation economy of EB is poor, and CHP units are mainly used for heating, followed by EB. In the gas network, the gas load is satisfied by purchasing directly, instead of using P2G to convert. In the valley time, the electricity price is lower than the cost of DG, so it turned down. In addition, the storages discharge during the peak period and charge during the valley period.

4.2. The effect of the confidence levels of the CCP model
To highlight the superiority of the proposed model, a deterministic model which does not consider the forecast error of the wind power output was also built for comparison. We assumed the wind power output was predicted accurately in the deterministic model and other conditions kept the same. The results are shown in Table 1. The operating cost of the deterministic model is a little bit lower than the CCP model. This is because the deterministic model only considers the most likely wind power scenario, however, the CCP model considers the likely forecast errors that the decision could satisfy most of the wind power scenario. And with the increase of confidence level, the operating cost increases. This is because a larger confidence level requires a larger regulation capacity of the system. With the increase of confidence level, more and more scenarios can be fully considered to improve the reliability of the system operation.

Table 1 Comparisons of the optimization results obtained from the stochastic with different confidence level and deterministic models

| Model(Confidence level) | Operating cost($) |
|-------------------------|-------------------|
| Deterministic           | 3209.64           |
| 0.9                     | 3252.67           |
| 0.8                     | 3078.02           |
| 0.7                     | 3005.18           |

5. Conclusion
This paper proposed a CCP model to determine day-ahead scheduling decisions of the IES, aiming at finding an economic and safe dispatching plan. In this model, a scenario analysis technique that combines the LHS and k-means clustering is introduced to cope with renewable energy power output forecast error. The CCP model is converted into a scenario-based deterministic MILP model. The decrease of confidence level will reduce the safe and stable operation level of the system. Hence the
dispatching center should set an appropriate confidence level to balance the tradeoff between economic performance and safety of the system.

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