Abstract
The major problems in today’s wireless communications are time dispersion and inter symbol interference. To combat these issues various adaptive equalization techniques are used. In literature state of art algorithms of adaptive equalization will be studied in detail, however the utmost goal of these algorithms is to attain high convergence rate, less complexity and least error. In this paper a comparison between least mean square algorithm and fractional least mean square algorithm is presented and experimentally proved that the rate of convergence is high in case of fractional least mean square algorithm. Also it is observed that LMS algorithm has better performance for random signals and fractional LMS has proved very efficiently in case of deterministic signals. These both algorithms are implemented in MATLAB and eight channels with different tap weights are used but in this paper only selected are presented.

Keywords: Adaptive Equalizer, Bit Error Rate, Fractional Least Mean Square, Inter Symbol Interference, Least Mean Square

1. Introduction
As compared to analog transmission system digital transmission systems are very efficient and more reliable for all the multimedia applications. Due to the advancement in wireless technology in few years the demand of wireless communication has rapidly increased. It has left the wired communication behind. This made a tremendous distribution in telecommunication. Due to ease of deployment and mobility, wireless communication is more popular technology than wire based communication. Predominantly, it should be believed that wireless technology proved that it is most vital and fastest emergent fields of telecommunication. The miscellaneous wireless, techniques, principles, applications and protocols are acting an imperative task in today’s telecommunications globe.

To attain trustworthy communication, a wireless coordination is designed to investigate its act and to get better its trustworthiness and functionality. For this motivation, widespread study is being made in wireless systems to design it more perfectly, to get enhanced results.

Channel impairments in telecommunication reduce the quality of the signal to be transmitted. In general in wireless surroundings, the signal to be transmitted arrived at receiver through different path called as multipath propagation. Thus we can say that this multi path propagation fading results in Inter Symbol Interference, which ultimately increases the communication Bit Error Rate (BER). Owing to unusual multipath propagation delays, various delayed copies of the transmitted signal are expected which lead to Inter Symbol Interference. Thus we can say that the transmitted signals are corrupted

*Author for correspondence
so that it cannot be separated lead to ISI. The major reasons for ISI are dependent on transmission media:

- wired transmission – for band limited signal;
- wireless communications – multipath propagation

The utmost demand for the better result and high capacity in wireless communication has led to the improvement of copious techniques of signal processing. The high efficiency of spectrum can be obtained by using the novel techniques of signal processing and to make wireless links more reliable through the implementation of these techniques. Multipath fading is often observed in wireless communication which causes ISI and the non-orthogonality causes Inter Antenna Interference (IAI). To cater the effect of ISI and IAI adaptive equalizers are used.

In 3G communication system, the channels frequently include several interferers and multipath. Therefore we have to have a method or procedure to overcome ISI and to deal with new wireless channel impairments. It is remarkable that every wireless communication channel commonly has memory and relationship between input and output information. Channels having associated errors contain fading wireless channels. Thus the estimation of wireless communication systems deals explicitly with channel impairments via a variety of simulation models is a main concern since last two decades.

2. Theoretical Background

In telecommunication, channel impairments are the foremost barrier in broad band wireless applications. For this purpose the execution of a filter explicitly adaptive in nature is required in order to model the unidentified wireless channel and to carry out inverse modeling like adaptive equalization. The fundamental initiative of equalization is merely to balance for non-ideal features in wireless channels by stir up supplementary filtering. An adaptive filter can be defined as a filter whose features can be customized to attain several goals and is frequently understood to achieve this change (or “adaptation”) without human intervention. The concept of adaptive filter will be explained in further chapters of this research thesis. The requirement of adaptive filtering is an obvious and more famous in order to compare it with non-adaptive filtering for the reason that the non-adaptive need enough awareness related to input signal and the different characteristics of the channel.

On the other hand, an adaptive filter can be modeled as a filter having no knowledge about its input signal and the channel characteristics. In fact it can learn the varying signal characteristics; it can adjust itself by means of the channel parameters, as a result adapting itself through the transmission surroundings with additional efficiency. So this is main reason that adaptive filters are preferred by the engineers that often model this filter in wireless communication, particularly for the purpose when channel impairments are the consideration. Normally adaptive equalization is implemented in order to attain error less and broad band communication but, for this a forceful filtering algorithm is required, in order to make it possible that small introduced error might not lead to a bulky equalization errors.

The implementation of static equalizer is very simple and less complexity has been observed but it is not efficient and reliable as compared to adaptive equalizer. It is difficult to design an equalizer unless and until if we don’t know the transfer function of transmission system and impulse response of channel. That is why adaptive equalizers are used to remove the deficiencies of static equalizer. The filter that without human intervention adapts time-varying characteristics of the transmission channel is known as adaptive equalizer. In this filter the transfer function is adjusted automatically according to implemented algorithm.

This remaining paper is arranged as: section 2 consists of background study of literature, MATLAB implementation in which the two algorithms i.e. LMS and FLMS are discussed in detail in section 3. In section 4 experimental work and its analysis is thoroughly studied. Section 5 describes conclusion and future work.
2.1 Channel Equalization

Inverse modeling is also an application of adaptive filters in different fields of engineering and computer science. It is also referred to by the name of Deconvolution often used in signal processing and digital communication. Equalizer is the foremost famous application of the inverse modeling and is implemented in order to resolve the issues of impairments in channel. The focal importance of this section is on the purpose of Inverse Modeling.

![Figure 2. Equalization System Block diagram.](image)

Figure 2. Equalization System Block diagram.

In Figure 3 a transmission system prepared with a channel Equalizer. The unidentified wireless channel is shown by $H(k)$. Additive White Gaussian Noise here is represented by $n_n(k)$ that mixes with the input signal from the outdoor interferers. Input signal to be transmitted is shown by $u(k)$, become visible in the shape of amplitude or phase modulated waves, is corrupted when passing through the channel. In signal processing the channel distortion is often known as pulse spreading effect, which results in Inter Symbol Interference. The performance of receiver degrades adversely by the addition of noise.

The signal being distorted when passing through a wireless channel can be filter out by implementation of equalizer which we have studied that it eliminate the ISI and reduce the AWGN up to negligible amount so that it is such that the quality of communication signal is desired one.

Mathematically the transfer function for the unidentified channel $H(k)$ of the equalizer can be related to the output of the transmission system as:

$$W(k) = \frac{1}{H(k)} \quad (1)$$

This equation can be further modified in term of transfer function as below:

$$H(k)W(k) = 1 \quad (2)$$

From the above equation it can be concluded that input signal $u(k)$, on comparing with the equalizer output must have the similar result having no distortion or error.

3. Implementation in MATLAB

The project was developed in MATLAB, which is the most powerful and interactive tool that suits such kind of simulations. Its multidimensional characteristics enable developers including engineers and scientists to solve their technical problems. Simulations are developed in MATLAB using its own specific code that does not require use of any other traditional languages. Its graphical representations and user friendly programming environment reduces the complexity of results analysis.

The MATLAB simulations can be developed either by using MATLAB M-file or by the use of Simulink interface. The Experimental work for the thesis is performed by MATLAB M-file. The results obtained are then observed and analyzed and are mentioned in the following articles of this paper.
3.1 LMS Implementation Diagram
The simulation effort essentially proved the extensively acknowledged reputation of channel equalization. By way of previously stated, the adaptive algorithm used here is the LMS algorithm depends on FIR filters.\(^{19}\)

![LMS Implementation Diagram](image)

**Figure 4.** LMS implementation diagram.

In this block diagram it evidently explains the processes involved in the execution of LMS algorithm. The \( u(k) \) here is the input signal whereas \( e(k) \) is the error in this case. The resultant signal obtained by the difference of the filter's output signal and desired signal is known as error signal. While the step size of the weight is denoted by \( \mu \). Vector \( U \) can be obtained by multiplying step size with input signal. The resultant obtained is then mixed with delayed version of the tap weight\(^{20,21}\).

3.2 FLMS Implementation Diagram
The FLMS block diagram is given in Figure 5. In which the input signal is represented by the \( x(n) \), \( n \) indicated the noise of channel, the noise signal is passed through equalizer, mixed with delta function and then fed into LMS algorithm\(^{22,23}\).

![FLMS Implementation Diagram](image)

**Figure 5.** Block diagram of FLMS algorithm.

3.3 Methodology
It is necessary to describe the methodology of the program which is designed for the implementation of LMS algorithm in MATLAB. In this simulation the implementation is done in a simple and easy way. The major steps of the program are as under:

i. First of all stream of bits are produced then it is over sampled. The procedure of oversampling was executed in order to get perfect and particular values.

ii. Then the length of channel is defined.

iii. In order to get the received data the impulse response of channel is convolved with desired data.

iv. The coefficients of equalization adjusted then.

v. After this filter buffers are adjusted.

vi. Then stream of bits are passed through filter.

vii. The filter buffer is convolved with coefficient of equalization in order to get the filter's estimated output. Which can be mathematically expressed as\(^{22}\):

\[
 y(n) = w^H[n]x[n] \tag{3}
\]

viii. Now the error in channel can also be calculated by the following equation:

\[
 e[n] = d[n] - y[n] \tag{4}
\]

ix. The final step is to update the weights or the coming weight can be found by the formula\(^{20}\):

\[
 w[n+1] = w[n] + \mu x[n] e^*[n] \tag{5}
\]

Though, it is essential to state that by convolution process, the length of output filter grow into greater value as that of input length, which is obviously not realistic in certain scenarios. This is why in second simulation of this research a MATLAB filter function is used.

4. Experimental Work and Analysis
For investigating the LMS algorithm, the utmost aspect is to define the circumstances for the convergence and stability of this algorithm. It is essential that by what means and under whatever definite limitations, its convergence level rises and for in what way the algorithm rests con-
stant under such situations. The supreme vital parameter here in this simulation is to note the step size $\mu$.

The simulation is based on two different algorithms and their result by using different step sizes. Total eight channels are used for the implementation of LMS and FLMS algorithms. The impulse response and frequency responses for each channel is calculated. Three results of LMS and three for FLMS algorithm is obtained for 100, 200 and 300 TS.

Furthermore appraisals between the graphical outcomes are obtained which illustrates the consequences on varying parameters both for LMS and FLMS.

### 4.1 Result

Ch\_coeff=1, SNR=20, Eq\_lngth=3, 4QAM

![Figure 6](image6.png)

**Figure 6.** Comparison of LMS and FLMS algorithms with impulse response, frequency response and rate of convergence.

In this figure it is clear that the results obtained from the LMS algorithm compared to FLMS algorithm are not clearer. However the results obtained of FLMS are clearer as the tap size increases.

From the above results it is found that in all the eights channels the simulations of FLMS is very good compared to LMS. As we increase the iterations from 100 to 300 the simulations gives the clearer results.

Ch\_coeff=1, SNR=20, Eq\_lngth=3, 4QAM

![Figure 7](image7.png)

**Figure 7.** Comparison of LMS and FLMS algorithms with impulse response, frequency response and rate of convergence.

### 5. Conclusion and Future Work

This research specifically describes the requirement of Channel Equalization in Wireless communication and also give the in depth acquaintance of Adaptive Filtering focusing on Least Mean Square (LMS) Algorithm and Fractional Least Mean Algorithm. The LMS algorithm remained the most used research algorithm throughout the last some years due to its modest structural design, minor computational complication and rationally good enactment. Study material in shape of together published and unpublished effort was effortlessly.

The foremost share of the research covers the hypothetical acquaintance about the essentials, process and arrangement of the LMS and FLMS Algorithm. Given that related understanding of Channel Equalization and Adaptive filters reinforced this. The researcher senses it is compulsory to remark here the significance of Simulation established for implementation LMS and FLMS Algorithm in MATLAB which managed to a complete learning of the its stability, which illustrate the comparison of the LMS
and FLMS Algorithm. It is experimentally proved that FLMS algorithm is more efficient in terms of convergence rate and fewer amounts of errors occur in it.

Overwhelmingly, it can be quantified that Fractional Least Mean Square when combined with FIR filters produces decent outcomes. But, it does have some limitations and inefficiencies when its performance criterion is considered.

6. References

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