Method for optimizing the allocation of the following car schedule based on minimum running time

Xingyu Wu
School of Electronic and Information Engineering, Beijing Jiaotong University, Beijing, China
13120192804@163.com

Abstract—Rail transit has become one of the most important means of transportation in various countries, which brings huge energy consumption. This paper studies how to allocate the running time of trains between multiple stations to adjust the schedule, to achieve the purpose of energy-saving. Before time allocation between multiple stations, there must be an optimal operation strategy for single-station trains. Studies have shown that under the optimal operation strategy, the longer the time, the smaller the energy consumption, and the remaining redundant time is allocated based on the minimum time. The train operation schedule can reduce energy consumption. In the first phase of this paper, the minimum running time between stations is modeled and solved. Based on this, the existing optimal operation strategy is used to obtain the time and energy consumption relationship curve. In the second phase, the operation redundancy is allocated based on the train energy consumption time relationship model. In terms of time, the optimized energy-saving schedule is obtained to achieve the purpose of energy-saving train.

1. INTRODUCTION
Rail transit has become one of the most important means of transportation in various countries due to its large volume, fast speed, low cost, safety and reliability. However, the rail transit system consumes a lot of energy while providing convenient and efficient services. In 2017, my country's railway energy consumption converted standard coal was as high as 1621,500 tons, an increase of 1.5% over the previous year, and the trend is increasing year by year [1]. In 2018, the operating mileage of urban rail transit in Beijing was 618 kilometers [2], and the converted power consumption was about 1.68 billion kWh. Such a huge amount of energy consumption not only brings energy to the country's energy supply but under the background of the era of promoting energy conservation and emission reduction and building an ecological civilization, exploring how to achieve energy conservation and consumption reduction of the rail transit system is undoubtedly of great practical significance. According to statistics, the energy consumption of train traction accounts for 40~70% of the total energy consumption of the rail transit system [3]. How to reduce the energy consumption of train traction has always been a research hotspot in the field of rail transit.

The purpose of train energy-saving control is to design and optimize a reasonable train operation schedule and select the best train operation control strategy under the actual operating conditions such as safety, timing, complex lines, arbitrary speed limit and maximum traction/braking force boundary constraints so that The reduction of train traction energy consumption is an optimal control problem. When the train is running between multiple stations, due to different line conditions, the same time...
redundancy increment results in different reductions in traction energy consumption. At present, many optimal control models have been established at home and abroad based on different strength constraints, and many optimal control methods with good results have been obtained. Khmelnitsky established a continuous energy-saving maneuvering model, using the Pontryagin maximum principle to analyze and prove that the optimal driving strategy of the train consists of four operating conditions: maximum traction, cruising, idling, and maximum braking [4]; Ghoseiri Et al. established a multi-objective optimization model that considers minimizing passenger travel time and minimizing train energy consumption [5]; Ding Yong et al. proposed a train energy-saving operation for subway train operation rules, considering non-straight ramps and timing constraints Two-stage joint optimization strategy. Based on the genetic algorithm, the optimal number of idle trains and the idle control points for the energy-saving operation of a single station are first solved, and then the train running time between each station is optimized [6]; Su Shuai et al. Aimed at the goal to establish an energy-saving driving strategy and also can be distributed between stations [7]; Huang Youneng et al. proposed a two-stage optimization method for train energy-saving operation, first of all, with timing as a constraint, based on particle swarm algorithm to save energy between single stations The driving strategy is optimized, and then the total running time is constrained, and the running time between the stations is optimized to reduce the total traction energy consumption of the train [8]; Cucala et al. further optimized Sicre's model and extended the single-objective function to a dual-objective function to minimize the total energy consumption and train delay [9]; Niu Hongxia et al. proposed a synchronous optimization model that simultaneously optimizes station redundancy time and interval redundancy time [10];

In summary, domestic and foreign scholars have conducted extensive research on the listed energy-saving operation models, algorithm research and simulation, but most of the research has studied train driving strategies and train schedules as two independent processes. This process is interrelated and will have an impact on train energy consumption. Based on previous research, this paper optimizes energy-saving driving strategies and train schedules. By calculating the minimum running time between stations, the total reserved redundant time is obtained, and the redundant running time is allocated according to the relationship between time and energy consumption under the optimal energy-saving operation strategy. The schedule is optimized to achieve the purpose of energy-saving.

2. THE SOLUTION OF MINIMUM RUNNING TIME BETWEEN STATIONS

2.1. Building the model

The minimum running time between stations refers to the minimum time for trains to run between stations under the conditions of satisfying all line conditions, speed limit constraints, and maximum traction and braking force restrictions. How to solve the minimum running time between stations is essentially a type of optimal control problem. This paper studies the numerical algorithm of this problem based on the control parameterization method and time scale transformation strategy.

The train dynamics differential equations and related constraints are as follows:

\[ \frac{dv_1(t)}{dt} = v_1(t) \]  \hspace{1cm} (1)

\[ \frac{dv_1(t)}{dt} = \frac{1}{mp} \left( F_1(t) + F_2(t) - r_2(v_1(t)) - r_1(y_1(t)) \right) \]  \hspace{1cm} (2)

\[ y_1(0) = 0, v_1(0) = 0 \]  \hspace{1cm} (3)

\[ y_1(t_f) = L, v_1(t_f) = 0 \]  \hspace{1cm} (4)

\[ 0 \leq v_1(t) \leq V_{\text{max}}(y_1(t)) \]  \hspace{1cm} (5)

\[ 0 \leq F_1(t) \leq F_{\text{max}}(v_1(t)) \]  \hspace{1cm} (6)

\[ -F_{2\text{max}}(v_1(t)) \leq F_2(t) \leq 0 \]  \hspace{1cm} (7)
and $v_t(t)$ are the displacement and speed of the train at time $t$; $m$ is the mass of the train; $p$ is the slewing mass coefficient; $F_t(t)$ and $F_z(t)$ are the traction and braking force of the train; $r_s(v_t(t))$ is the basic running resistance of the train and $r_l(v_t(t))$ is the additional resistance of the line ramp, $V_{max}(v_t)$ represents the line speed limit, $F_{1max}(v_t)$ and $F_{2max}(v_t)$ represent the maximum traction and braking force of the train; $L$ is the distance between stations; $t_f$ is the end time of the train running to the end. In order to obtain the minimum running time of the train, the objective function is as follows:

$$J = t_f$$

(8)

Under the given dynamic equations and state equations, satisfying the initial conditions, line conditions, and boundary constraints, the optimal control is obtained to minimize the objective function $J$. Convert the free terminal time to a fixed terminal time and set a new time variable $\phi$ so that:

$$t = \lambda t_f \phi$$

(9)

Where $\lambda > 0$, $\phi \in \left(0, \frac{1}{\lambda}\right)$, so that the problem is transformed into an optimal control problem with fixed terminal time.

2.2. Algorithm steps

Divide Line $[0, L]$ and Time Domain $[0. \frac{1}{\lambda}]$ into N sub-segments, so that the line ramp and speed limit on each sub-segment are constant. Then divide each sub-segment at equal intervals. Find the optimal decision variables $\delta$ and $\theta$ to make the objective function $J$ the smallest. At this time, the problem is transformed into a nonlinear programming problem, which is solved using a gradient algorithm.

Step1: Initialize:

$$0 \rightarrow \delta_1, 0 \rightarrow \delta_{k+1} \rightarrow \theta, \sum_{k=1}^{q} \theta_k \rightarrow t_f, k = 1, 2, ..., q$$

(10)

Step2: Use $h_1(s)$ and $h_2(s)$ to calculate the discrete time points and find the state variables;

Step3: Calculate the function value with route and ramp constraints;

Step4: Calculate gradient information with more sensitivity function;

Step5: Optimize the value of variable $t_f$ with a more nonlinear programming method.

3. TRAIN SCHEDULE OPTIMIZATION

Based on previous research, this paper optimizes the energy-saving driving strategy and train schedule of the train at the same time. The optimal maneuvering strategy between trains in a single station refers to the model of energy-saving driving strategy based on particle swarm algorithm studied by Huang Youneng et al. [10]. Through the minimum operating time and the total redundant reserved time in the previous chapter, the time energy consumption curve of the train between single stations is obtained according to the train energy-saving driving strategy, and the curve is used to allocate the total operating redundancy time to achieve energy-saving timetable optimization.

3.1. Optimization of energy saving between stations based on particle swarm optimization

According to the existing research results, particle swarm optimization is used to obtain the optimal driving strategy between single stations. According to the different running time, respectively obtain the train energy consumption corresponding to different time and then use the fitting method to obtain the corresponding between each station on the line. First, the line between the stations is discretized into several sections to ensure the line in the same section. The slope is the same as the speed limit. At the
same time, after discretizing the line, the target speed at the discrete distance corresponding to each section will also be discretized.

Discretize the lines between stations into N sections. The line ramp and speed limit on each section are the same. The target speed at the discrete distance corresponding to each section will also be discretized. The expression is as follows:

\[
\begin{align*}
L &= \{L_1, L_2, L_3, \ldots, L_{N-1}, L_N\} \\
V &= \{V_1, V_2, V_3, \ldots, V_{N-1}, V_N\}
\end{align*}
\]  

(11)

In this way, each discretized section will have a corresponding target speed. Different target speed sequences correspond to different train operation strategies, and the corresponding energy consumption and time are also different. The final optimization result is to obtain a set of target speed sequences \(V\) with the lowest energy consumption and meeting the running time requirements of the schedule.

Using particle swarm optimization to solve the problem, set the number of particles to \(x\), and use the training target speed sequence as the position information of the particles, then the solution space is \(n\)-dimensional. The target speed sequence with the lowest fitness obtained by optimization is transformed into a train running curve, and the energy-saving driving strategy between the stations is obtained.

Figure 1 shows the energy-saving driving strategy of trains between single stations based on particle swarm optimization.

Figure 1. Optimization algorithm flow of train energy-saving driving strategy

Through the above method, the optimal driving curve and corresponding energy consumption at a certain time can be obtained. To obtain the energy consumption corresponding to different running times, on the premise of meeting the running time specified in the timetable, multiple calculations are performed to obtain the energy consumption of the optimal driving strategy corresponding to different times, and then fitted to obtain the optimal train operation strategy. The curve of energy consumption versus time.
3.2. Train schedule optimization based on redundant time allocation

Based on the minimum operating time and train time-energy consumption curve obtained above, the total redundant operating time is allocated to optimize the schedule to reduce the total energy consumption of the train.

According to the existing research and the time curve of traction energy consumption obtained by the particle swarm optimization algorithm above, the energy consumption of the train within the same station gradually increases with time. Assuming that there is a and b between the two stations, the minimum operating time is \( t_{\text{min1}} \) and \( t_{\text{min2}} \), respectively, and the reserved time \( \Delta t \) is allocated based on the minimum operating time, and the total energy consumption is reduced by \( \Delta W_1 \) and \( \Delta W_2 \), respectively. Comparing the size of the two values, the allocation can be determined Which station should be allocated to the reserved time \( \Delta t \), so that the full name of the traction energy consumption is minimized. The constraints are:

\[
\Delta t = \frac{t_{\text{r}}}{n} \quad (12)
\]
\[
t_{\text{pi}} = t_{\text{pi}} + \Delta t \quad (13)
\]
\[
t_{\text{pi}} \leq t_{\text{maxi}} \quad (14)
\]

\( t_{\text{r}} \) is the total redundant time; \( n \) is the reserved time for \( n \) equal parts; \( t_{\text{pi}} \) is the running time after the redundant time is allocated among the \( i \) stations.

The steps for redundant time allocation are as follows. \( \Delta W_1 \) to \( \Delta W_h \) are the energy consumption reductions after \( \Delta t \) is allocated to \( h \) stations respectively. Iterative loops are made using the relationship between the minimum operating time obtained before and the time energy consumption under the optimal train operation strategy. Iterative loop allocates \( n \) \( \Delta t \) to each station.

- Step1: Obtain the minimum running time and energy consumption curve between each station;
- Step2: Obtain the total reserved time and \( \Delta t \);
- Step3: Bring \( \Delta t \) to the energy consumption curve between each station;
- Step4: Get \( \Delta W_1 \sim \Delta W_h \), judge \( t_{\text{pi}} = t_{\text{pi}} + \Delta t \) and \( t_{\text{pi}} \leq t_{\text{maxi}} \), if it is the next step, otherwise return to Step3;
- Step5: Obtain the optimal time allocation between each station, and achieve the purpose of optimizing the timetable to save energy consumption.

By allocating the total redundant time reserved for train operation through the above method, an optimized energy-saving schedule can be obtained. Under the optimized timetable, the train runs with the optimal maneuvering strategy, achieving total energy saving in traction.

4. Research Conclusion

This paper investigates the problem of generating energy-saving schedules by redundantly configuring the running time of multiple stations with energy-saving as the goal and considering the actual operating conditions. First, The optimal control calculation model of the minimum running time between stations is established. Based on the control parameterization method and time scale transformation strategy, the minimum running time is solved by converting the terminal time free problem into a terminal time fixed problem; then, the existing particles Group algorithm is used to obtain the optimal energy-saving driving strategy of the train. A relationship curve between train traction energy consumption and running time between single stations is established. The curve is used to allocate the redundant running time of the train and the iterative loop method is used to obtain the optimized train energy-saving schedule.

The two-stage optimization method proposed in this paper can organically integrate train operation organization and control, not only in subway train operation organization and control but also in the operation optimization process of long-distance trains between cities. The train operation scenario with
fixed time plays a good role in optimizing energy saving. This article only considers the energy-saving driving of a single car, does not consider the mutual influence between multiple trains running on the same line at the same time, nor does it consider the change of parking time requirements in the station and the impact of passenger flow in different stations on time. This will be the next research direction to be considered.

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