Confinement, Crossing Symmetry, and Glueballs

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ABSTRACT

It is suggested that the quark-confining force is related by crossing symmetry to a color-singlet glueball $\mathcal{G}$ which is well described as a loop of one quantum of color magnetic flux. Electron pair annihilation as high as $\approx 2GeV$ above the $\Upsilon$ mass could produce $\Upsilon \rightarrow \ell^+\ell^-$ accompanied by $\mathcal{G}$ or one of its excited states.
I. Introduction

The concept of crossing symmetry has a venerable place in the history of particle physics. At least since Yukawa, forces between particles have been viewed as derived from the (virtual) exchange of other particles, which themselves can be produced as real, observable objects by acceleration of the particles which the forces influence. If the colliding particles carry appropriate charges, then the force-carrying objects also may be produced by annihilation, but the acceleration or ‘bremsstrahlung’ mechanism is the generic exemplar of crossing symmetry. Our goal here is to explore implications of adopting that same viewpoint to describe the confining force between quarks, a force whose existence is indicated both by phenomenological analyses of hadronic (and especially heavy quarkonium) spectra [1] and by lattice calculations in pure QCD without dynamical quarks [2]. Both types of analysis are consistent with a dominantly Lorentz scalar potential whose magnitude grows linearly with separation between heavy quark and antiquark [3].

Before seeking a candidate for the exchanged particle, let us try to classify the properties it would require. First, it need not be coupled to a charge carried by gluons, since the charge of any color octet object can be screened by the creation of gluon pairs. Thus, even in the absence of light quarks, one would not expect at large distances to find a (linear) confining potential between a heavy color octet particle and its antiparticle. Secondly, one expects the force to bind a quark to an antiquark, but also to bind two quarks, as part of the binding of three quarks to make a baryon. Thus it is permitted, and therefore in our view a desirable simplification, to assume that the exchanged object is a color singlet whose coupling to a particle depends at most on the color SU(3) representation to which that particle belongs. Finally, since the force should be attractive, the simplest possible and therefore most appealing spin assignment is spin zero. Of course this might well be only the lowest in a hierarchy of spin states, as in the familiar example of Regge theory [4].

If for the moment we continue to ignore light quarks, then the object must be constructed out of the only available degrees of freedom, i.e., gluons. Such an object is by definition a glueball. There is a great deal of literature on the possible structure
of (color singlet) glueballs \cite{3}, and on whether some experimentally observed meson resonances may be identified as glueballs \cite{4}. Up to the present such studies have been inconclusive. Current estimates of masses for $0^{++}$ glueballs lie above 1.5\,GeV, and for $2^{++}$, above 2\,GeV \cite{5}.

Since confinement and glueballs are two of the most elusive and difficult phenomena conjectured to occur in the strong interactions, it is not obvious that trying to consider them together is advantageous. Our justification for doing so rests on the outcome – suggestions for new laboratory and lattice experiments to be described below.

It is implausible to model a glueball as a collection of some definite number of gluons, since gluon number is not a well-defined quantity at low energy and large length scales. Already at order $g_S(s)$ (where $g_S(s)$ is the strong coupling constant at the appropriate scale $s$), there are Feynman graphs for an initial single gluon to change to a final gluon pair, making it hard to see how a resonant state could be described in terms of a fixed number of gluons.

Given the intrinsic imprecision of gluon number at large length scales, a glueball might best be pictured in first approximation as a spread-out, classical gluon field configuration, which therefore would have a strong form factor suppression of its couplings at large momentum transfers, corresponding to small distance scales. Clearly this would imply a suppression of glueball production by quark-antiquark annihilation, making such objects difficult to observe cleanly in ordinary hadronic processes. In the absence of light quarks the lightest glueball should be completely stable, but in practice there is no easy way to distinguish a glueball from a color and flavor singlet combination of quark and antiquark. Thus the lack of unambiguous experimental evidence for glueballs should not be surprising.

If the object does not carry color, then what couplings might be possible? Since it is supposed to be made of gluons and to confine quarks, its coupling must be a non-trivial function of color. We have speculated above that there may be no confining potential for color octets, which of course would mean that the coupling of our exchanged object to octets would vanish. Assuming this, the only allowed form of coupling would be a function of the set of operators in the center of color $SU(3)$, namely the triality,
with eigenvalues $T = -1, 0, +1$. This is zero for multiplets in the adjoint (octet) representation and representations which can be constructed from it, and $N (mod 3)$ for representations made of $N$ quarks. For a specific form of coupling to an object $X$, we suggest $g_{GXX} = g_0 \sin^2 (\pi \langle T \rangle / 3)$, where $\langle T \rangle$ is the expectation value of $T$ for a volume of size equal to the geometrical dimensions of a glueball. This has the requisite periodicity in $\langle T \rangle$ and is always of one sign, so that it can never lead to repulsive interactions. For an isolated heavy quark, if light quark contributions are ignored, then the expectation value $\langle T \rangle$ should be an eigenvalue, as gluon fields cannot change $T$.

It is readily seen that our proposed glueball exchange gives exactly the same (scalar) attraction between two quarks as between a quark and an antiquark. It also would give the same attraction between a quark and a diquark (whether $\bar{3}$ or 6), when the constituents of the latter were closer together than the characteristic glueball size. Note that this contrasts with the characteristics of the perturbative, Coulombic octet exchange force between quarks, which does differ for different objects with the same triality.

We have arrived at a picture in which the schematics of confinement forces lead by crossing symmetry to properties of a scalar, color-singlet glueball which ought to be hard to produce or destroy in annihilation reactions, and which is weakly coupled to ordinary color-singlet matter. The internal structure of the glueball suggests a likely tower of excitations, all with quite small widths, since any familiar hadron systems light enough to be radiated in transitions from one $G$ state to another should have small coupling to this exotic object. For the same reason, the mass of the lightest $G$ state must lie quite far above the threshold for a system of two $\pi$ mesons, since otherwise there would be a visible sharp state or states in the dipion spectrum. This is a complementary viewpoint to that of Novikov et al. [7], who, on the basis of QCD sum rules, suggested studies of the dipion spectrum perhaps due to a scalar glueball in $\psi \to 2\pi + \gamma$.

II. Perturbative approach

Let us turn to a perturbative approach to make the discussion more specific. In perturbation theory the coupling due to gluon exchange between two quarks which are
off mass shell by some characteristic amount $\Delta M$ may be estimated at small momentum transfer $q$ by focusing on the most singular part of the QCD coupling.

For exchange of the two-gluon color singlet combination, the quark-quark potential in momentum space should be

$$\tilde{V}(q) \approx \frac{(\alpha_s(q))^2 q^4}{(\Delta M)^2 q^4},$$

(1)

where $\alpha_s(q)$ is $g_S^2(q)/4\pi$, the factor of $q^4$ in the numerator comes from the integration over (small) loop momentum, the $q^4$ in the denominator comes from the two gluon propagators, and $(\Delta M)^2$ from the quark propagators. Let us assume the Richardson ansatz for the leading behavior of $\alpha_s$,

$$\alpha_s(q) = \frac{12\pi}{(33 - 2n_f)\ln(1 + q^2/\Lambda^2)},$$

(2)

where $\Lambda$ is of order the QCD scale but not necessarily equal to $\Lambda_{\overline{MS}}$, and the coefficient is determined by the one-loop $\beta$-function for QCD which depends on the number of light quark flavors, $n_f$. This implies a pole in $\alpha_s$ at $q^2 = 0$, giving a double pole in $\tilde{V}(q)$ and hence by Fourier transformation a linearly rising potential $V(r)$ in coordinate space.

It is straightforward to check that if arbitrary numbers of gluons are exchanged they again lead to a linearly rising potential, as the additional powers of $q$ from the propagators and vertices cancel against those from the additional loop integrals. Thus we may consider the glueball exchange as a sum over all numbers of gluon exchange, which is as near as one might hope to approach a nonperturbative result in a perturbation theory framework. If $\Delta M$ is of the same order as $\Lambda$, then all numbers of exchanges should contribute comparably, and the possibility of obtaining a nonpolynomial coupling (such as that to triality suggested above) also becomes comprehensible.

The inclusion of arbitrary numbers of gluon exchanges in principle allows arbitrary spin exchange, as would occur in Regge theory. This raises the possibility of coupling the spins of the quarks. However, that would lead to extra factors of $q$ in the numerator, and so would not contribute to the confining potential.
III. Extrapolations

If in the lowest-order description the exchanged object responsible for a color-singlet linearly rising potential is a digluon, we also should look at the digluon in the crossed channel to get a hint about the properties expected for the glueball. We can see a serious difficulty at once: We have some insight near $q^2 = 0$, but that is a point well below threshold in the heavy quark-antiquark channel. Nevertheless, let us try. From standard parity considerations, we know the quarks must be in a relative P-wave to produce a scalar glueball. Such a state would have nonzero overlap with configurations corresponding to opposite-pointing magnetic moments for the quark and antiquark, which of course are on opposite sides of the center of mass. This suggests a field configuration which is a loop of color magnetic flux.

There is much evidence that the confining effect should be viewed as due not to a mere static scalar potential, but rather to an electric flux string stretching from quark to antiquark [10]. With confinement described this way, it might seem natural to think that glueballs should be described as loops of that electric flux. However, there are reasons to question whether electric loops could play a distinctive role, with some unique signal of their presence. First, imagine an electric loop in isolation. A spontaneous fluctuation breaking the loop by creation of a light quark pair could ‘unzip’ the loop at the speed of light. This is the ‘hybrid’ problem [9]: It is effectively impossible to distinguish an electric flux loop of any size from a collection of one or more mesons.

Difficult as they might be to observe, electric flux loops in principle could be radiated by accelerated quarks. To decide what quanta are exchanged to produce this force, imagine giving a sudden large impulse to the quark at one end of the hypothesized electric flux string. The most likely result will be to break the string in one or more places, producing a number of quark-antiquark mesons. To produce an electric flux loop, the impulse might put the string into an excited state where it has a kink. This can break off leaving an intact string plus a loop, which again would decay quickly to ordinary mesons, leaving no characteristic trace.

To convert a large loop of color \textit{magnetic} flux into quarks would require lining up many quark-antiquark pairs head-to-tail, each with magnetic moment parallel to the
local direction of the loop. Thus such a gluon field configuration would have small overlap with typical quark configurations, and should be minimally affected by the existence of light quarks. Magnetic flux is proportional to magnetic moment per unit length, so the number of pairs would be proportional to the length of the loop. Since such a quark configuration is quite improbable, as it involves many correlated pairs, the coupling of large magnetic flux loops to conventional mesons should be quite weak.

There is a certain appeal in identifying magnetic loops as the true partners of a force pictured in terms of an electric flux string, since crossing symmetry interchanges temporal and spatial directions, and thus ought to interchange electric and magnetic fields. However, our main point is that the magnetic type should be distinguishable from conventional hadrons, unlike the electric type.

Still another reason why one might expect magnetic loops is found in the Copenhagen “flux spaghetti” picture [11]. There the elemental fluctuations of the pure glue QCD vacuum include tubes of magnetic flux, so that one expects the low-lying excitations to include loops of flux and possibly color magnetic monopoles. If we think about the separation of the heavy quarks as analogous to the separation of ordinary magnetic monopoles inside a superconductor, then by the analogy we might expect generation in our problem of a loop of color magnetic current. Whether there is a distinction in this nonabelian system between magnetic current and magnetic flux is unclear to us. In the following we proceed as if there is no difference.

In summary, crossing symmetry almost certainly holds for the long-range force between heavy quarks, but the naive crossing-symmetric partner, a color electric flux loop, would be practically indistinguishable from ordinary meson systems. However, if large color magnetic loops were produced, their coupling to light quarks might be weak enough to allow distinctive signals of their presence. We want now to suggest that exactly such an effect could occur in a simple and natural way.

IV. Experimental and theoretical tests

Consider an $e^+e^-$ reaction which produces a $b\bar{b}$ pair at a center of mass energy near or above the threshold for free $B$ meson pair production. Let us attempt to picture the evolution of the system by naive classical electrodynamics, waiting until
later to impose quantum constraints on the description. Ignoring light quarks for the moment, we know that as the $b$ quarks separate by about $1\text{fm}$ they will increase in their effective color electric charge coupling to a value of order unity. If we assume the relevant 4-current is locally conserved, then it follows that there will be a large spatial current pulse across the midplane between the quarks. This current will induce a color magnetic flux loop with flux also of order unity. However, once the current pulse has died down, the flux loop (if it approaches sufficient mass to be a glueball) should no longer be static and so should begin to execute motions with one or more periods (perhaps like a TE mode in a microwave cavity, but with the roles of electric and magnetic fields interchanged).

Meanwhile, the heavy quarks will have lost energy, perhaps enough to put them below threshold for $B$-meson pair production. Eventually they will be forced by the linear potential to stop separating and then to come back together. Again, as they pass through $1\text{fm}$ separation, an oppositely directed pulse of current will be generated across the midplane. Depending on how the original magnetic loop is configured by this stage, the new loop induced by the second current pulse may tend to cancel the first one, or double its field strength and quadruple its stored energy, or something in between. If the resulting gluon field configuration is metastable (i.e., a long-lived resonant state $\mathcal{G}$), and the leftover quark energy is close to the $\Upsilon$ mass, there should be a high probability of forming the $\Upsilon$ state. This state in turn could be identified reliably by its decay to a lepton pair.

The key idea here is that the slowly moving heavy quarks could generate a coherent, approximately classical gluon field configuration. Because of its weak coupling to ordinary hadron channels, this configuration might not decay before the heavy quarks have settled into a low ‘bottomonium’ state, which has a finite branching fraction for decay into leptons. In those leptonic decay events, contamination of the glueball structure by ordinary mesons would be minimized. The process is a close analogue to photon emission in a positronium atomic transition, and thus justifies the invocation of crossing symmetry to describe it. As far as we are aware, this reaction is unique among those accessible to laboratory observation in offering the possibility of $O(1)$ coupling.
between quarks and ‘solitons’ of the gluon field. The reason is that the heavy quarks move fairly slowly, so that creation of light quark pairs might be suppressed. Note that the loop of flux must contain one quantum (meaning that a quark which circled the flux would suffer an Aharonov-Bohm phase shift $\pm 2\pi/3$) if it is to be a color singlet, able to decouple from the heavy quarks whose motions produced it.

Having suggested an experimental test of our general scheme, we also wish to propose a theoretical test of the particular claim that the linear confining potential may not affect gluons. Consider a net color singlet made up of two heavy octets and one heavy 27 representation of color $SU(3)$, with their locations specifying the vertices of a triangle. The short-range Coulomb interaction between the two octets is repulsive, since together they make a 27, but if they are separated sufficiently their interaction should be screened by gluons, with no dependence of the static energy on further separation. If there were a linear confining force, one might expect the potential to show a minimum at intermediate separation, while without such a force the potential should be monotonically decreasing. By studying the energy of the whole color-singlet system while varying the length scale as well as the two angles which define the triangle, one may be able to check whether an explicit confinement force is necessary for simple fits to the energy dependence.

V. Confronting the hybrid problem

If the total energy of the $b\bar{b}$ pair were well above $B$-meson production threshold, then the conventional expectation would be that conjugate light quarks accompany each of the heavy quarks, suppressing the long range color separation which we propose as the generator of a color magnetic flux loop. Even below threshold, light quark pair production in principle competes with color magnetic loop production. If we accept typical estimates of minimum glueball masses in the 1 GeV region, then $G$ production could not occur except above $B\bar{B}$ threshold, where $B$-meson production competition must be strong. Nevertheless, study of mesonic states accompanying final states with $\Upsilon$ decay to a lepton pair could well give a much cleaner view of the glueball channel than conventional hadron collisions. Our suggestion is that there should be a nonzero probability of generating the magnetic loop instead of the light quark pair, thus taking
away so much energy from the heavy quarks that they collapse to the $\Upsilon$ ground state.

Let us estimate the hadronic background to this signal. The 3S state has an inclusive branching ratio $\leq 0.1$ for producing a 1S $\Upsilon$. Therefore the 4S state, with a width $10^3$ times greater, should have a branching ratio to $\Upsilon$ of $\leq 10^{-4}$, implying a background dilepton branching ratio signature $\leq 10^{-6}$.

Another background is the purely electromagnetic process $e^+e^- \rightarrow \mu^+\mu^- 2\gamma$, which intrinsically is down by order $\alpha^2$ compared to $B\bar{B}$ production, with the final $\mu^+\mu^-$ mass distributed smoothly over a large range, so that the background under the $\Upsilon$ signal should be utterly negligible. A process with one final $\gamma$ can be excluded by a cut on the center of mass momentum of the final lepton pair.

The crucial issue now is the size of the $G$ signal, i.e., the expected rate for production of a $\mu^+$ and $\mu^-$ with pair mass equal to that of an $\Upsilon$. Here our reasoning is quite simple. We argue for an $\mathcal{O}(1)$ effect once the glueball production threshold is surpassed. Consider, for example, a $1/N_c$ argument (where $N_c$ is the number of colors in the strong gauge theory): In the large $N_c$ limit, the glueball production branching ratio should be suppressed by a factor $\mathcal{O}(1/N_c^2)$, compared to the inclusive rate for $b\bar{b} \rightarrow \text{all}$. This implies an order of magnitude suppression of the proposed reaction compared to the overall rate for background processes not involving a glueball (in this setting mainly $B$-meson pair-production). However, these dominant processes would not particularly populate the region of phase space in which the two heavy quarks are found in the $\Upsilon$ ground state. Consequently, conventional hadronic backgrounds should not be expected to obscure a glueball signal as they are also suppressed in this (or any given) region. Thus, while it is easy to envision factors which could reduce the effect by an order of magnitude, if it were down by more than two orders of magnitude (in rate), identification as a strong-coupling phenomenon would not be credible.

Therefore, since the decay $\Upsilon \rightarrow \mu^+\mu^-$ has a branching ratio $\geq 0.01$ for each of the first three ($S$-)states, we expect that the branching ratio (in comparison to the $B\bar{B}$ production rate) for the $\mu^+\mu^-$ signal after $e^+e^-$ collision at the appropriate energy should lie between $10^{-4}$ and $10^{-2}$. This should be compared with the conventional $\Upsilon$ cascade background estimated above, which is at least 100 times smaller. If a
rise in the observed ratio is seen as the initial $e^+e^-$ energy increases beyond some value above $B\bar{B}$ threshold, then it becomes interesting to look for peculiarities in the accompanying hadron configurations. As the initial collision energy rises further, it might become possible for $\mathcal{G}$ emission to populate excited $\Upsilon$ states, which could be observed directly by their own leptonic decay, at the possible risk of contamination by cascade background more serious than for the $\Upsilon$ ground state.

What are the most promising choices of $e^+e^-$ collision energy for this experiment? If we consider the process as analogous to a radiative atomic transition, then the obvious choice is to produce one of the known resonant states of the $b\bar{b}$ system, so that the $\mathcal{G}$ emission channel, once open, can compete with $B\bar{B}$ production. The radiation should have a form factor with characteristic $\mathcal{G}$ momentum width $\Delta p \approx 200 \text{ MeV}/c$. Given the uncertainty in $\mathcal{G}$ mass, $\Upsilon$ states with excitation energies up to and even beyond $2 \text{ GeV}$ should not be excluded from the search.

Finally, having referred to possible narrow excited states of $\mathcal{G}$, we should discuss their expected spacing. Assuming a mass of order $1 \text{ GeV}$ and radius of order $1 \text{ fm}$, one obtains for the inverse moment of inertia of a loop about an axis through its plane the value $I^{-1} = 80 \text{ MeV}$, which by the rule $E = J(J+1)/2I$ gives $80 \text{ MeV}$ for the energy of the first excited state. (If the radius were even larger and so the spacing smaller, detector resolution limitations could lead to the appearance of a broader state, misshapen from a standard Lorentzian.) It would be interesting if narrow states with this rotational-band splitting could be distinguished experimentally, as they have not been detected in lattice calculations $[^5]$.

VI. Conclusions

We have proposed an analogue for the confinement force of Yukawa’s $\pi$ meson for the nuclear force. Our candidate is a glueball state $\mathcal{G}$, a loop of color magnetic flux whose existence might be verified and more detailed properties studied by electron-positron inelastic collisions leading to $\Upsilon \to \mu^+\mu^-$ plus (possibly excited) $\mathcal{G}$. We believe that this reaction is inherently interesting because there is little conventional background, so that with any appreciable rate the residual material accompanying a final $\Upsilon$ is likely to be remarkable in significant ways. Furthermore, this sort of search would be very
much in the spirit of crossing symmetry: Well separated heavy quarks manifestly feel the confinement force. Therefore these quarks are the best candidates to radiate the associated meson, but in processes which involve large distance scales, such as descent from one quarkonium state to another, rather than annihilation, which is intrinsically short-distance in character.

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