Lepton-nucleus scattering in the impulse approximation regime

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We discuss theoretical calculations of electron- and neutrino-nucleus scattering, carried out using realistic nuclear spectral functions and including the effect of final state interactions. Comparison between electron scattering data and the calculated inclusive cross sections off oxygen shows that the Fermi gas model fails to provide a satisfactory description of the measured cross sections, and inclusion of nuclear dynamics is needed. The role of Pauli blocking in charged-current neutrino induced reactions at low $Q^2$ is also analyzed.

1. INTRODUCTION

The field of neutrino physics is rapidly developing after atmospheric neutrino oscillations and solar neutrino oscillations have been established \textsuperscript{123}. Most neutrino experiments measure energy and angle of muons produced in neutrino-nucleus interactions and reconstruct the incident neutrino energy, which determines the neutrino oscillations. To reduce the systematic uncertainty of these experiments it is vital that the nuclear response to weak interactions be under control at a quantitative level. A number of theoretical approaches aimed at providing accurate predictions of neutrino-nucleus scattering observables are discussed in Refs. \textsuperscript{[4,5]}

At $E_\nu=3$ GeV or less, quasi-elastic scattering and quasi-free $\Delta$ production are the dominant neutrino-nucleus processes. However, reactions in this energy regime are associated with a wide range of momentum transfer, thus involving different aspects of nuclear structure.

In this short note we discuss results obtained using the many-body theory of electron-nucleus scattering in the impulse approximation (IA) regime (see, e.g., Ref. \textsuperscript{[6]} and references therein), as well as its extension to charged-current neutrino-induced reactions. We focus on the energy range $0.7 – 1.2$ GeV, and analyze inclusive scattering of both electrons and neutrinos off oxygen, the main target nucleus in SK, K2K and other experiments.

2. RESULTS

Within the IA, the cross section of the process $e + A \to e' + X$ can be written in the form (see, e.g. Ref. \textsuperscript{[7]})

$$\left( \frac{d\sigma_A}{d\Omega_{e'}dv} \right)_{IA} = \int d^3p \ dE P(p, E) \frac{d\sigma_N}{d\Omega_{e'}dv}, \quad (1)$$

where $\nu = E_e - E_{e'}$ is the electron energy loss, the spectral function $P(p, E)$ yields the probability of finding a nucleon with momentum $p$ and removal energy $E$ in the nuclear target and the differential cross section $d\sigma_N/d\Omega_{e'}dv$ describes the elementary electron-nucleon scattering process.

The cross section of Eq. (1) is obtained under the assumption that final state interactions (FSI) between the struck nucleon and the specta-
tor particles be negligible. However, coincidence 
(e, e'p) data unequivocally show that FSI play a
significant role, leading to a sizable reduction of
the outgoing proton flux (see, e.g. Ref. [8]).

A theoretical approach to include the effect of
FSI in inclusive processes has been developed in
Ref. [7]. The resulting cross section can be writ-
ten in the convolution form

\[ \frac{d\sigma}{d\Omega d\nu} = \int d\nu' f_q(\nu - \nu') \left( \frac{d\sigma}{d\Omega d\nu'} \right)_{IA}. \tag{2} \]

The folding function \( f_q(\nu) \) appearing in the above
equation, that reduces to a \( \delta \)-function in absence
of FSI, is simply related to the propagator of the
struck nucleon, evaluated within the eikonal ap-
proximation treating the spectator particles as
fixed scattering centers [7].

In Fig. 1 the (e, e') cross section off oxygen
calculated from Eqs. (1) and (2) using a spectral
function obtained within the Local Density Ap-
proximation [9] is compared to the data of Ref.
[10]. The theoretical calculation, involving no ad-
justable parameters, provides a fairly accurate ac-
count of the measured cross sections in the region
of the quasi-elastic peak. The effect of FSI, lead-
ing to a shift and a quenching of the peak, is
clearly visible. For reference, the figure also shows
the results of the Fermi gas (FG) model, corre-
sponding to Fermi momentum \( p_F = 225 \text{ MeV} \)
and nucleon removal energy \( \epsilon = 25 \text{ MeV} \), which
appear to largely overestimate the data. The fail-
ure of the theoretical calculations to reproduce
the measured cross section in the region of the \( \Delta \)-production peak is likely to be due to deficien-
cies in the description of the elementary electron-
nucleon cross section [11].

In addition to dynamical FSI, arising from by
nuclear interactions, statistical correlations, lead-
ing to Pauli blocking of the phase space available
to the knocked-out particle, must be also taken
into account. A rather crude prescription to es-

timate the effect of Pauli blocking amounts to
modifying the spectral function through the re-
placement

\[ P(\mathbf{p}, E) \rightarrow P(\mathbf{p}, E)\theta(|\mathbf{p} + \mathbf{q}| - \overline{p}_F) \tag{3} \]

where \( \overline{p}_F \) is the average nuclear Fermi momen-
tum.

The effect of Pauli blocking is hardly visible in
the differential cross section shown in Fig. 1 as
the kinematic setup corresponds to \( Q^2 > 0.2 \)
GeV\(^2\) at the quasi-elastic peak. On the other
hand, this effect becomes very large at lower \( Q^2 \).

This feature is illustrated in Fig. 2 showing the
calculated differential cross section \( d\sigma/dQ^2 \)
of the process \( \nu_e + {}^{16}O \rightarrow e + X \), for neutrino
energy \( E_\nu = 1 \text{ GeV} \). The dashed and dot-dash
lines correspond to the IA results with and with-
out inclusion of Pauli blocking, respectively. It
clearly appears that the effect of Fermi statistic
in suppressing scattering shows up at \( Q^2 < 0.2 \)
GeV\(^2\) and becomes very large at lower \( Q^2 \). The
results of the full calculation, in which dynamical
FSI are also included, are displayed as a full line.
The results of Fig. 2 suggest that Pauli blocking
and FSI may explain the deficit of the measured
cross section at low \( Q^2 \) with respect to the pre-
dictions of Monte Carlo simulations [12].

Figure 2. Differential cross section $d\sigma/dQ^2$ for neutrino energy $E_\nu = 1$ GeV. The dot-dash line shows the IA results, while the solid and dashed lines have been obtained using the modified spectral function of Eq. (3), with and without inclusion of FSI, respectively.

3. SUMMARY

We have employed an approach based on nuclear many-body theory to compute inclusive electron- and neutrino-nucleus scattering cross sections in the kinematical region corresponding to beam energy $\sim 1$ GeV, relevant to many neutrino oscillation experiments.

In the region of the quasi-elastic peak, the results of our calculations account for the measured $^{16}O(e, e')$ cross sections at beam energies between 700 MeV and 1200 MeV and scattering angle $32^\circ$ with an accuracy better than 10% [11]. It must be emphasized that the ability to yield quantitative predictions over a wide range of beam energies is critical to the analysis of neutrino experiments, in which the energy of the incident neutrino is not known, and must be reconstructed from the kinematics of the outgoing lepton.

In the region of quasi-free $\Delta$ production theoretical predictions significantly underestimate the data. In view of the fact that the calculated cross sections are in close agreement with the data at higher energies [11], i.e. in the region in which inelastic contributions largely dominate, this problem appears to be mainly ascribable to uncertainties in the description of the nucleon structure functions at low $Q^2$.

The effect of Pauli blocking, not included in the IA picture, while being hardly visible in the lepton energy loss spectra, produces a large effect on the $Q^2$ distributions at $Q^2 < 0.2$ GeV$^2$, and must therefore be taken into account.

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