Medium-induced broadening and softening of a parton shower

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The modifications of the angular and transverse momentum distributions of quarks and gluons inside a parton shower due to the presence of a medium are studied within an analytical description that reduces to the modified leading logarithmic approximation (MLLA) of QCD in the absence of medium.

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The combined measurements of hadrons and photons at large transverse momenta performed at the relativistic heavy-ion collider (RHIC) [1] are most naturally interpreted in terms of the energy degradation of high-\(p_T\) partons in the medium formed in Au–Au or Cu–Cu collisions prior to their hadronization. A significant amount of the fast parton energy is radiated away as soft gluons [2, 3, 4]: the net effect of the medium is an enhancement of this radiation over the “vacuum” parton shower due to the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) parton evolution that leads to jets in \(e^+e^-\) or pp/pp collisions. It is thus expected that the characteristics of a “jet” created in an ultrarelativistic heavy-ion collision differ from those of a jet found in collisions of elementary particles.

Possible medium-induced modifications of jets include a broadening of the transverse momentum spectrum [2, 8] and the distortion of the longitudinal distribution inside the jet [9], increasing jet multiplicities [3, 10, 11, 12], or changes in the jet hadrochemistry [13]. Assessing these effects is an experimental challenge [5, 6, 14, 15, 16], though the first numerical implementations of in-medium parton showers are appearing [20, 21], it is still necessary to have analytical calculations—obviously under simplified assumptions—of possible properties of the modified showers, as references against which Monte-Carlo computations can be tested. In this Letter, I shall thus present results on the angular and transverse momentum distributions of partons inside a medium-modified shower. Following the approach proposed in Ref. [9], the influence of the medium is modeled as an enhancement of the singular \(1/z\) parts of the leading-order parton splitting functions by a constant factor \(1 + f_{\text{med}}\) (this factor is denoted \(N_0\) in Refs. [10, 12]):

\[
P_{\text{gg}}(z) = C_F \left[ \frac{2(1 + f_{\text{med}})}{z} - 2 + z \right],
\]

\[
P_{\text{gg}}(z) = 2C_A \left[ \frac{1}{1 - z} + \frac{1 + f_{\text{med}}}{z} - 2 + z(1 - z) \right],
\]

while the other two splitting functions remain unchanged. The modeling is admittedly crude, yet has the advantage of allowing controlled analytical calculations, while mimicking the increased radiation of soft gluons of the approaches [2, 3, 4] that describe RHIC results on leading hadron suppression. Additionally it incorporates the conservation of energy at each step of the parton shower evolution, which plays possibly an important role in shaping the global jet characteristics. A more realistic change of the splitting functions, which accounts for the coherent Landau–Pomeranchuk–Migdal radiation in a thermalized weakly coupled deconfined medium can be found in Ref. [22]. Reference [23] provides a somewhat equivalent description in terms of Sudakov form factors.

Consider a jet with energy \(E\) and opening angle \(\Theta_0\). For any jet particle with 4-momentum \((k_0, \vec{k})\) one defines

\[
\ell \equiv \ln \frac{E}{k_0} = \ln \frac{1}{x}, \quad y \equiv \frac{k_\perp}{Q_0} \approx \ln \frac{k_0}{Q_0} \sin \Theta, \quad (3)
\]

1 I shall hereafter assume that some notion of a jet—whose specification might be driven by the attempts at identifying collimated clusters of energy above a fluctuating background in data [2, 6]—makes sense even in the high-multiplicity environment of a nucleus–nucleus collision at high energy.
where $Q_0$ is the infrared cutoff parameter and $k_\perp \geq Q_0$ resp. $\Theta \leq \Theta_0$ is the transverse momentum resp. angle of the radiated parton with respect to the energy flow direction (“jet axis”). Finally, let $Y_{\Theta_0} \equiv \ln (E \sin \Theta_0 / Q_0)$, corresponding to the maximum transverse momentum of a particle in the jet. While approximating $\sin \Theta \sim \Theta$ is often made for the small opening angles relevant for jets in high-energy collisions, in the following I shall retain the sine. Additionally, I shall take $Q_0 = \Lambda_{QCD} \simeq 250$ MeV, which gives a good agreement with data for longitudinal hadron spectra, and show results for the values $Y_{\Theta_0} = 5.1$ and $Y_{\Theta_0} = 6$, corresponding to jets with energies typical for RHIC (40 GeV) and LHC (100 GeV), respectively.

The starting point to study the angular distribution inside a jet is the distribution $F_{A_0}^h(x, \Theta, E, \Theta_0)$ of the energy fraction $x$ of hadrons within a subjet with an opening angle $\Theta < \Theta_0$, \cite{17}. $F_{A_0}^h$ is the convolution of the probability $D_{A_0}^h(u, E \Theta_0, u E \Theta)$ that the initial parton $A_0$ emit parton $A$ with the energy fraction $u$ and the virtuality $u E \Theta$ and of the fragmentation function $D_{A}^h(x/u, u E \Theta, Q_0)$ for the production of hadron $h$ off $A$ with the energy fraction $x/u$ and a transverse momentum scale $u E \Theta \geq Q_0$:

$$F_{A_0}^h(x, \Theta, E, \Theta_0) = \sum_{A=g,q} \int_1^x \, du \, D_{A_0}^h(u, E \Theta_0, u E \Theta) \, D_{A}^h(x/u, u E \Theta, Q_0). \quad (4)$$

The number of hadrons $h$ inside the opening angle $\Theta$ is given by the integral of this distribution over $x$ \cite{24}:

$$N_{A_0}^h(\Theta, E, \Theta_0) = \sum_{A=g,q} \int_0^{\Theta_0/E \Theta} \, du \, u \, D_{A_0}^h(u, E \Theta_0, u E \Theta) \, N_{A}^h(u E \Theta, Q_0), \quad (5)$$

where $N_{A_0}^h(u E \Theta, Q_0)$ denotes the multiplicity of hadrons inside the “subjet” initiated by parton $A$ with the virtuality $u E \Theta$. The physical picture of Eqs. (4) is that, as a consequence of the angular ordering of the splitting processes, the hadrons that end up in the subjet originate from quarks and gluons emitted in the shower when its typical $k_\perp$ scale was $u E \Theta$, after an evolution from the initial scale down to that intermediate scale.

Since the purpose in this Letter is to establish the modifications of jets induced by a medium compared to their “vacuum” MLLA properties, for the hadronic fragmentation function $D_A^h$ relevant for vacuum jets I shall use the corresponding MLLA (partonic) function $D_A^{\text{MLLA}}$, in the limit $Q_0 \to \Lambda_{QCD}$. The latter is the case of a gluon jet related to the MLLA “limiting spectrum” \cite{17} through $D_{\text{lim}}^{\text{MLLA}}(\ell, y) = x \, D_{g}^{\text{MLLA}}(\ell, y)$. Note that exact energy conservation is spoiled in the approximations leading to $D_{\text{MLLA}}$ and thus $D_{\text{lim}}$; yet the latter still represents a good baseline at sufficiently small $x$. Consequently, the hadron multiplicity inside a gluon jet is given by

$$N_g^h(E \Theta, Q_0) = K_h Y_{\Theta_0}^{-B/2+1/4} \exp \left( \frac{16 N_c}{\beta_0} \frac{Q_0}{Y_{\Theta_0}} \right), \quad (6)$$

and that inside a quark jet by \cite{25}

$$N_q^h(Y_{\Theta_0}) = \frac{C_F}{N_c} \left( 1 + a - 3N_c \right) \left( \frac{4N_c}{\beta_0 Y_{\Theta_0}} \right) N_g^h(Y_{\Theta_0}), \quad (7)$$

where $Y_{\Theta_0} \equiv \ln (E \sin \Theta_0 / Q_0)$, $a = \left( 11N_c + 2N_f / N_c^2 \right) / 3$, $\beta_0 = (11N_c - 2N_f) / 3$ and $B = a / \beta_0$, while $K_h$ is, following the local parton-hadron duality (LPHD) hypothesis, a proportionality constant relating partonic and hadronic properties. The actual value of $K_h$ will be unimportant in the following, since I shall assume that the same value should hold for vacuum and in-medium jets.

The longitudinal $\ln(1/x)$ distribution of partons inside a (gluon) jet within the adopted modeling of medium effects was reported in Ref. \cite{3}, and amounts to a strong distortion of the MLLA hump-backed plateau. This leads to increased (sub)jet multiplicities, $\beta_0$ in Eqs. (6) being replaced by $\beta_0/(1 + f_{\text{med}})$ and $a$ by $a + 4N_c f_{\text{med}}$.

Regarding $D_{A_0}^h$, for vacuum jets it is given by the usual expression following from the Altarelli–Parisi parton splitting functions. Replacing the latter with the modified functions \cite{12} and performing a Mellin transformation as well as an expansion of $N_{A_0}^h$ around $u = 1$ in Eq. (5), one obtains readily the multiplicity $N_{A_0}^h$; which is plotted vs. the opening angle $\Theta$ in Fig. 1 for both MLLA and medium-distorted jets. The value $f_{\text{med}} = 0.8$, which allows to reproduce a nuclear modification factor $R_{AA} \approx 0.2$ \cite{9}, was adopted for the RHIC energy, while $f_{\text{med}} = 1$ was chosen for the higher (LHC) energy. As in Ref. \cite{24}, the LPHD constant was taken to be $K_h = 0.2$. Figure 1 shows that the gradient $dN_g^h / d\Theta$ is larger over a broader angular range for the medium-modified than

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\^2 The detailed calculations will be reported elsewhere.
for the MLLA gluon jet. This behavior, which is also observed on quarks jets (not shown here), is in the medium- 
angular broadening of the parton shower which was observed in Q-PYTHIA [11]. Note that in Fig. [1] the low-Θ 
region has been suppressed, since even the MLLA results 
are not reliable for small angles (see the remark on low 
k⊥ values below).

The hadron multiplicities ¹Nh represents all hadrons down to the infrared cutoff Q₀. Given 
the high-multiplicity environment of heavy-ion collisions, experimental studies will most probably rather investigate 
the modifications of jet properties above some pT (with respect to the beam axis) cutoff which might be quite large. In that case, the angular broadening is significantly less marked.²

Let me turn to the distribution of transverse momenta k⊥ in a jet. Deriving Eq. [4] with respect to lnΘ—or equivalently, at fixed x, ln k⊥—yields the double differential single-particle inclusive distribution of hadrons inside a jet with opening angle Θ₀ and energy E [17]:

\[
\frac{d²N}{dx \, d\ln k_⊥} \simeq \frac{d²N}{x \, d\ln Θ} = \frac{d}{x \, d\ln Θ} F^{h}_{A₀}(x, Θ, E, Θ₀).
\] (8)

Within a small-x approximation, one can expand D_{A} in Eq. [1] around u = 1 in Eq. [1]. This allows one, after some algebra [26], to rewrite F^{h}_{A₀} in terms of the average color current \langle C \rangle_{A₀} inside a jet initiated by parton A₀ and of the distribution in ℓ = ln(1/x) of hadrons inside a gluon jet, D_{g}(ℓ, y) = x D^{h}_{g}(x, EΘ, Q₀). Inserting the result in Eq. [8] yields

\[
\left( \frac{d²N}{dℓ \, dy} \right)_{g,q} = \frac{d}{dy} \left[ \frac{⟨C⟩_{g,q}}{N_c} D_{g}(ℓ, y) \right].
\] (9)

This expression was computed for vacuum jets within MLLA [20], using the limiting spectrum \^D_{lim} for D_{g}, and later including some next-to-MLLA corrections, which yield improved agreement with measurements by the CDF Collaboration for k⊥ > 1 GeV/c [19]. Given the exploratory approach pursued in this Letter, I shall only investigate the influence of the medium modifications in Eqs. [11] on the MLLA k⊥-distributions. The average color currents \langle C \rangle_{g,q} following from the modified splitting functions can be computed² and inserted in Eq. [9] together with the distorted longitudinal spectrum [10]. For values of ℓ where the assumptions behind the MLLA computation are fulfilled (see discussion in Ref. [26]), the resulting double differential single-particle distribution is, with respect to the vacuum one, depleted at low k⊥ (y ≲ 1.5) and significantly larger at higher k⊥ [27]. The growth at high k⊥ obviously reflects the angular broadening already observed in Fig. [1] while the low-k⊥ behavior should probably be considered with care, since it corresponds to a region where MLLA is not to be trusted.

FIG. 2: ln k⊥-distribution inside gluon jets for the same cases as in Fig. [1].

Integrating Eq. [9] over ℓ yields the transverse momentum distribution inside a jet:

\[
\left( \frac{dN}{d\ln k_⊥} \right)_{g,q} = \int dℓ \left( \frac{d²N}{dℓ \, d\ln k_⊥} \right)_{g,q}.
\] (10)

The latter is displayed in Fig. 2 for vacuum and medium-softened gluon jets at RHIC and LHC energies. While the values below y = ln(k⊥/Q₀) = 1.5, should not be taken too literally—the divergence at y → 0, which reflects that of the running QCD coupling constant \(\alpha_s(k⊥)\) when \(k⊥ \rightarrow \Lambda_{QCD}\), hints at the breakdown of the perturbative regime, the overall trend of medium effects seems to be a push of the distribution towards larger transverse momenta: this is a transverse momentum broadening of the jets. Note that the “usual” source of transverse momentum broadening (which was however not seen in the first JEWEL simulations [28]) is the transfer of momentum to the jet in the interactions between the latter and the medium [11]: this is not taken into account in the model. The phenomenon reported here is different, since no momentum is injected into the jet from the outside. The eventual broadening of the jet will result from both effects, which do not necessarily add up constructively. Investigating that issue is beyond the scope of the present study.

Since the effect of the medium is to redistribute partons from high towards low x-values [9], it was not totally obvious that this would at the same time involve a wider k⊥ distribution as seen in Fig. 2. Both modifications can simultaneously happen only if the presence of the medium makes the transverse momentum spectrum steeper. This is illustrated in Fig. 3 which indeed shows a medium-induced depletion of the yield at high k⊥, which was barely visible on Fig. 2 together with the enhancement at low k⊥. All in all, the k⊥ spectrum is softened.
butions with respect to those of “vacuum” MLLA jets, as the medium is, as anticipated, a broadening of the distributions inside a medium-distorted parton shower. The effect of the medium is, as anticipated, a broadening of the distributions inside a medium-distorted parton shower. The effect of the medium is, as anticipated, a broadening of the distributions inside a medium-distorted and vacuum jets, for gluon and quark jets at typical LHC (Y0 = 6) and RHIC (Y0 = 5.1) energies.

which was also seen in Q-PYTHIA simulations.[11]

Integrating the k⊥ spectrum above a given k⊥ cut yields the multiplicity above that cutoff, which might be easier to assess experimentally. One can then form some “medium modification factor” by dividing the multiplicity in a medium-distorted jet by that inside a vacuum jet. This ratio is shown for both gluon and quark jets in Fig. 4; it is larger than 1 up to k⊥ cut ≃ 10 – 15 GeV/c, and is systematically larger for quark than for gluon jets.

Using an analytical formalism, I have computed the angular and transverse momentum distributions of partons inside a medium-distorted parton shower. The effect of the medium is, as anticipated, a broadening of the distributions with respect to those of “vacuum” MLLA jets, as well as a softening of the k⊥ spectrum. The multiplicity over a lower k⊥ cutoff, which is more easily accessible experimentally, shows an enhancement in medium-modified jets up to large cutoff values.

Strictly speaking, the results obtained here hold at the parton level. Using the LPHD hypothesis, which for (N)MLLA jets leads to a good agreement with measurements of ln(1/x) and k⊥ spectra, they would translate into characteristics at the hadronic level. The hadronization process might however possibly totally reshuffle the distributions inside the jet.[20], in which case the predictions could only be checked in Monte-Carlo simulations.

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