A two-dimensional model for a slide guide system subjected to lateral displacement excitation

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Abstract A two-dimensional model for the vibration of a slide guide system subjected to lateral displacement excitation is proposed in this paper. Simulation results with this two-dimensional model illustrate two different movement states—full sliding state and stick-slip state. The tendencies of the lateral vibration amplitude varying with system parameters are given in this paper through numerical simulations. Finally, results derived from the two-dimensional model are compared with a one-dimensional model which does not include the vibration in sliding direction. The comparisons indicate that the lateral response calculated from the abbreviative one-dimensional model is larger than the result from the two-dimensional model.

1. Introduction
In the last decades, an uncountable number of researchers have made great work in nonlinear dynamics [1,2]. And many methods such as harmonic balance method (HBM) and homotopy perturbation method (HPM) have been well developed and applied to study dynamic behaviors of nonlinear systems [3]. Some are employed to settle friction involved problems which are of strongly nonlinear [4]. However, researches for systems with friction contact still need to be furthered due to the increasingly demanding in accurate motion prediction and control of structures.
In this paper modeling and dynamic analysis for a slide guide system are discussed. Friction always exists in a slide guide system, and can not be ignored in its dynamics analysis. Commonly friction force is often replaced by an equivalent viscous damping in many researches [5-7]. However, results from this method often can not show the real dynamical characteristics of the system with friction contact. For example, the stick-slip phenomenon which often appears in friction contact system is impossible to be simulated with such a model. Some authors modeled friction contact systems from the characters of friction force, and studying the dynamical behaviors of the systems [8,9]. Whereas, most of these studies deal with vibration behaviors in only one direction, and seldom involve in the coupling problem between the lateral vibration and the vibration in traction direction. The two-dimensional vibration problem is more complicated than one dimensional. Literatures [10-12] discussed the modeling and numerical simulation for two dimensional systems with friction contact.
As a primary study about dynamics problems of slide guide systems, the impact due to large amplitude vibration is not included in this paper’s discussion. A two-dimensional model considering

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the vibrations both in lateral and tractive directions of a slide guide system, is proposed in this paper. Through numerical simulations, the influence of the friction force on the vibrations in the two directions, in addition to the coupling problem arising from it are studied. Furthermore how the factors such as tractive velocity affect the vibrations is explored in this paper.

2. A two-dimensional model
In this paper we consider a two-dimensional slide guide system subjected to displacement excitation \( x_a \) in the x direction (shown in Figure 1). The slide guide \( m \) moves along the y direction with tractive velocity \( v_b \). The stiffness and damping between the guide and the main body in x and y direction are \( k_x, c_x \), and \( k_y, c_y \) respectively.

![Figure 1. The schematic of a slide guide system.](image)

The equation of motion of the slide guide system can be written as

\[
mx + c_x (\dot{x} - \dot{x}_a) + k_x (x - x_a) + f_x = 0 ; \\
m\ddot{y} + c_y (\dot{y} - \dot{v}_b) + k_y (y - v_y) + f_y = 0 ,
\]

where \( f_x \) and \( f_y \) are the components of friction force in the x direction and y direction respectively, \( y_o \) is the original deformation of the spring \( k_y \) when no vibration occurs.

Friction force can be described as

\[
\vec{f} = f_x \cdot \vec{e}_x + f_y \cdot \vec{e}_y = f_0 \frac{\ddot{x} + \ddot{y}}{\sqrt{\ddot{x}^2 + \ddot{y}^2}} , \\
\text{and} \quad \ddot{x} + \ddot{y} \neq 0,
\]

where \( f_0 = |\vec{f}| = \mu N \). \( \mu \) is Coulomb friction coefficient, \( N \) is the normal force. From above equation we have

\[
f_x = f_0 \frac{\ddot{x}}{\sqrt{\ddot{x}^2 + \ddot{y}^2}} , \quad \sqrt{\ddot{x}^2 + \ddot{y}^2} \neq 0 ;
\]

\[
f_y = f_0 \frac{\ddot{y}}{\sqrt{\ddot{x}^2 + \ddot{y}^2}} , \quad \sqrt{\ddot{x}^2 + \ddot{y}^2} \neq 0 .
\]

When \( \ddot{x} = \ddot{y} = 0 \) (that is \( \sqrt{\ddot{x}^2 + \ddot{y}^2} = 0 \)), if the elastic force in x direction \( f_{x_e} \) and in y direction \( f_{y_e} \) satisfy the following expression
the slide guide will be in sticking phase, till the composition of elastic forces is greater than friction force. In the sticking phase, it has

\[ f_x = f_{ex}, \]
\[ f_y = f_{ey}. \]

With lateral displacement excitation \( x_a = 0 \), then \( \dot{x} = \ddot{y} = 0 \), \( \dot{x} = 0 \), \( \ddot{y} = v_y \), we can get

\[ f_0 = f_y = k_y y_0. \]

Thus the value of \( y_0 \) is given as

\[ y_0 = f_0/k_y. \]  

Assume the displacement excitation \( x_a \) is harmonic, and has the expression \( A \sin(\omega t) \). The dimensionless form of Eq.(1) and (2) are

\[ \ddot{x} + \lambda_x \dot{x} + x = \sin(\eta_x \tau) + \lambda_x \eta_x \cos(\eta_x \tau) - f_{ex}, \]

\[ \ddot{y} + \lambda_y \dot{y} + \eta^2 y = \lambda_y \eta y + \eta y^2 + f_0 - f_{ey}. \]

In above equations, \( \ddot{\cdot} \) denotes the derivative with respect to the dimensionless time \( \tau \). The dimensionless parameters and variables are

\[ \omega_x = \sqrt{\frac{k_x}{m}}, \quad \omega_y = \sqrt{\frac{k_y}{m}}, \quad \eta = \frac{\omega_y}{\omega_x}, \quad \eta_x = \frac{\omega}{\omega_x}, \quad \tau = t \omega_x, \quad \lambda_x = \frac{c_x}{m \omega_x}, \quad \lambda_y = \frac{c_y}{m \omega_y}, \]
\[ p = \frac{v_y}{A \omega}, \quad b_y = p \eta_x, \quad x = \frac{x}{A}, \quad y = \frac{y}{A}, \quad f_x = \frac{f_0}{k_x A}, \quad f_y = \frac{\dot{x}}{\sqrt{x^2 + y^2}}, \quad (\sqrt{x^2 + y^2} \neq 0), \]
\[ f_{ey} = \frac{\dot{y}}{\sqrt{x^2 + y^2}} \quad (\sqrt{x^2 + y^2} \neq 0). \]

3. Conditions for stick-slip motion

If the elastic forces \( f_{ex} \) and \( f_{ey} \) meet the condition

\[ \sqrt{f_{ex}^2 + f_{ey}^2} > f_0, \]  

motion of the slide guide will change to the slip phase. If the velocity of the slide guide happens to be zero, and at the same time, the composition of the elastic forces is smaller than the Coulomb friction force, that is

\[ \sqrt{f_{ex}^2 + f_{ey}^2} \leq f_0, \]  

the slide guide will in the sticking state. The dimensionless expressions for Eq.(10) and (11) are

\[ (\sin(\eta_x \tau) - x_0)^2 + (\eta^2 (b_y \tau - y_0) + f_0)^2 > f_0^2 \]  

and
\[(\sin(\eta_\omega \tau) - x_\eta) \|^2 + (\eta^2(b_\eta \tau - y_\eta) + f_\eta)^2 \leq f_\eta^2. \] (13)

4. Numerical simulation

4.1. Full sliding state

When no sticking phase appears, motion of the slide guide is in full sliding state. With the parameters \(\lambda_x = \lambda_y = 0.02\), \(\eta = 2\), \(f_\eta = 0.5\), \(\eta_\omega = 0.1\), \(p = 0.2\), the simulation results are given in Figure 2. Figures 2a and 2b are the dimensionless responses in x and y directions respectively. Figure 2c exhibits the movement of the slide guide along the dimensionless time \(\tau\). From these figures, it can be seen that there is no sticking state in the motion of the slide guide.

![Figure 2](image)

(2a) (2b) (2c)

Figure 2. Full sliding state (the parameters in calculation are \(\lambda_x = \lambda_y = 0.02\), \(\eta = 2\), \(f_\eta = 0.5\), \(\eta_\omega = 0.1\), \(p = 0.2\)): 2a, dimensionless response \(x_\eta\); 2b, dimensionless response \(y_\eta\); 2c, movement of slide guide along dimensionless time \(\tau\).

4.2. Stick-slip state

With some proper parameters, stick-slip state can happen in the motion of the slide guide. To demonstrate the movement state, taking parameters \(\lambda_x = \lambda_y = 0.02\), \(\eta = 2\), \(f_\eta = 0.5\), \(\eta_\omega = 0.3\), \(p = 0.1\), we get the numerical simulation results shown in Figure 3. From the dimensionless responses in x direction \(x_\eta\) and y direction \(y_\eta\), we can see, the slide guide is in sticking state during some time. The motion of the slide guide along dimensionless time \(\tau\) (in Figure 3c) demonstrates obviously that at the place of the maximum value of \(x_\eta\), the slide guide is in sticking state. During the time, the motion curve varies only along the axis of dimensionless time \(\tau\).

![Figure 3](image)

(3a) (3b) (3c)

Figure 3. Stick-slip state (simulation parameters are \(\lambda_x = \lambda_y = 0.02\), \(\eta = 2\), \(f_\eta = 0.5\), \(\eta_\omega = 0.3\), \(p = 0.1\)): 3a, dimensionless response \(x_\eta\); 3b, dimensionless response \(y_\eta\); 3c, movement of slide guide along dimensionless time \(\tau\).
4.3. The influences of parameters on the vibration

4.3.1. Dimensionless parameter $p$

The variation tendency of the amplitude of the lateral vibration $x_i$ with the dimensionless parameter $p$ is shown in Figure 4. The other dimensionless parameters are $\bar{\lambda}_i = \lambda_i = 0.02$, $\eta = 2$, $f_0 = 0.5$. From the figure, it can be found, in general the amplitude of $x_i$ increases with the increase of the value $p$, and at last it tends to be a constant value. However, at the region where the value of $p$ is relatively small, the variation curve is complicated. The main reason perhaps is the occurrence of stick-slip.

![Figure 4](image)

**Figure 4.** The amplitude of $x_i$ versus dimensionless parameter $p$.

4.3.2. Dimensionless parameter $\eta_{\omega}$

The influence of the dimensionless parameter $\eta_{\omega}$ on the amplitude of $x_i$ is exhibited in Figure 5. The other parameters are the same with those in section 4.3.1. It can be shown that near the point of $\eta_{\omega}$ equaling 1, the amplitude of $x_i$ increases sharply.

![Figure 5](image)

**Figure 5.** The amplitude of $x_i$ versus dimensionless parameter $\eta_{\omega}$.
4.4. Comparison with a one-dimensional model
If the vibration in y direction is ignored (assuming the slide guide moves in y direction with constant velocity $v_y$), a one-dimensional model for the slide guide system can be built. The amplitude of the dimensionless response $x_s$ in x direction derived from this model is demonstrated in Figure 6 by the broken line. The solid line gives the corresponding result calculated with the two-dimensional model proposed in this paper. The calculation parameters are $\lambda_x = \lambda_y = 0.02$, $\eta = 2$, $f_y = 0.5$, $p = 0.5$.

From the comparison, we can found that the result from the two-dimension model is smaller than that from the one-dimensional model. When the value of $\eta_\omega$ is far away from 1.0, the difference between the two results is not very large. However, when $\eta_\omega$ taking the value near 1.0, the result coming from one-dimensional model is far greater than the two-dimension result. Theoretically because the two-dimensional model includes the vibration in y direction, the vibration energy will distribute in the two directions. Therefore the result is certainly smaller than that from the one-dimensional model which makes the energy concentrates in one direction.

![Figure 6. Comparison of results simulated from one-dimension model and from two-dimensional model.](image)

5. Conclusions
In this paper, a two-dimensional model is presented for the vibration of a slide guide system which is subjected to displacement excitation in x direction, and in y direction drawn by tractive machine with a constant velocity. The two-dimensional model is compared with a relatively simple one-dimensional model. Numerical simulation results demonstrate:

- With some appropriate parameters, stick-slip phenomenon can happen in the movement of the slide guide system.
- In general, the amplitude of lateral vibration increases with the increase of the tractive velocity, and the final tendency is a constant value.
- The vibration amplitude in x direction from reductive one-dimensional model is greater than that from the corresponding two-dimensional model.

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