Online Trajectory Optimization for Rotary-Wing UAVs in Wireless Networks
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Abstract—This paper studies the trajectory optimization problem in an online setting for a single rotary-wing UAV as the source of data for random downlink transmission requests by two ground nodes (GNs) in a wireless network. The goal is to optimize the UAV trajectory in order to minimize the expected average communication delay of requests to transmit a fixed payload to the GNs. It is shown that the problem can be cast as a semi-Markov decision process (SMDP), and the resulting minimization problem is solved via multi-chain policy iteration. It is proved that the optimal trajectory in the communication phase greedily minimizes the communication delay of the current request while moving between target start and end positions, with the end positions selected to minimize the expected average long-term delay in the SMDP. Numerical simulations show that the expected average delay is minimized when the UAV moves towards the geometric center of the GNs during phases in which it is not actively servicing transmission requests, and demonstrate significant improvements over sensible heuristics. Finally, it is revealed that the optimal end positions of communication phases become increasingly independent of the payload, for large payload values.

Index Terms—Rotary-wing UAVs, wireless communication networks, online trajectory optimization, delay minimization

I. INTRODUCTION

Recently, much research has gone into UAVs operating in wireless networks [1–5]. The drive for this is due to the unique benefits that UAVs acting as flying base stations, mobile relays, etc., provide in enhancing the overall network performance, thanks to their unique advantages over terrestrial counterparts in terms of mobility, maneuverability, and higher line-of-sight (LoS) link probability [1]. However, the design of UAV deployment strategies comes with challenges, namely the determination of optimal positioning or trajectories in the face of constraints imposed on UAV energy consumption, network throughput, and/or delay requirements [1–4].

Some research has focused on the optimization of trajectory under energy constraints, as in [2] and [3]. In [6], the fine-grained structure of LoS conditions is exploited to position UAVs optimally as to maximize throughput. In [4], a model-free Q-learning approach was taken in the trajectory design so as to maximize the transmission sum-rate.

All of these efforts consider situations that are solved in the offline case, i.e., the pattern of transmission requests is known in advance, so that the trajectory may be pre-planned accordingly. However, this may be impractical as transmission requests are often random and cannot be determined in advance. In these cases, trajectory design is much more challenging, since it must be continuously adjusted based on the realization of these random processes, and incorporate the uncertainty in the future evolution of the system dynamics. In this paper, we investigate this problem and develop online policies, which adapt the trajectory based on the random realization of downlink transmission requests by two GNs.

In this context, the minimum communication delay to serve one particular GN is achieved by flying as close as possible to the GN, but this design in turn may incur a higher average communication delay if the UAV is to also service other GNs farther away in the network which may request downlink transmission in the future, according to a random process. The UAV may need to travel a long distance to serve the next GN, and thus incur a large communication delay. Therefore, we need to incorporate this uncertainty in the trajectory design.

To address this question, we consider a scenario in which a UAV is serving two GNs far apart, and receives transmission requests according to a Poisson random process. We formulate the problem as that of designing an online trajectory, so as to minimize the average long-term communication delay incurred to serve the requests of both GNs. We prove that the optimal trajectory in the communication phase operates as follows: first, the UAV selects a target end position, which optimizes the trade-off between minimizing the delay of the current request, and minimizing the expected average long-term delay; then, the UAV travels to the selected end point while communicating, following the trajectory that minimizes the communication delay for the current request, provided in closed form. We utilize a multi-chain policy iteration algorithm to optimize the selection of the end position in the communication phase and the trajectory during the waiting phase, in which the UAV is not actively servicing downlink transmission requests. Our numerical results reveal that the UAV should always move towards the geometric center of the two GNs during the waiting phase, and that the optimal trajectory during communication phases becomes independent of the payload and only determined by system parameters as the payload value becomes sufficiently large.

The rest of the paper is organized as follows. In Sec. [II] we introduce the system model and state the optimization problem; in Sec. [III] we formalize the problem as a semi-Markov decision process (SMDP); in Sec. [IV] we provide numerical results; lastly, in Sec. [V] we conclude the paper with some final remarks.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

Consider the scenario where one rotary-wing UAV services two ground nodes (GNs) with random downlink transmission requests of $L$ bits, as depicted in Fig. [I]. The two ground units
Fig. 1: System model depicting downlink transmission request from GN1; the request from GN2 is dropped during the active communication interval. 

GN1 and GN2 are located at positions $x_1 = -a$ and $x_2 = a$ along the x-axis, respectively, both at ground level (height 0). The UAV moves along the line segment connecting the two GNs, at height $H$ from the ground. We let $q(t) \in [-a, a]$ be the UAV’s position along the x-axis at time $t$, and we assume that it is either hovering or moving at speed $V$, hence $|q'(t)| \in \{0, V\}$, where $j'$ denotes derivative of $f$ over time.

We assume that the communication intervals experience LoS links, that the communication power of the UAV is fixed and equal to $P_c$, and that the channel faces no probabilistic elements. This is motivated by the fact that UAVs in low-altitude platforms generally tend to have a much higher occurrence of LoS links [2]. We model the instantaneous communication rate between the UAV in position $q(t)$ and GNs, $r \in \{1, 2\}$ in position $x_r$ as

$$R_s(q(t)) = B \log_2 \left(1 + \frac{\gamma}{H^2 + (q(t) - x_r)^2}\right),$$

where $H^2 + (q(t) - x_r)^2$ is the squared distance between the UAV and GN$r$, $B$ is the channel bandwidth, and $\gamma$ is the SNR referenced at 1 meter (see [3]).

When the UAV has no active transmission requests, future requests arrive according to a Poisson process with mean $\lambda / 2$ requests/second, independently at each GN. Each request requires the transmission of $L$ bits to the corresponding destination. Upon receiving a request from GN$r$, the UAV enters the communication phase, where it services it by transmitting the $L$ bits to GN$r$; any additional requests received during this communication interval are dropped (see also Fig. 1). After the data transmission is completed, the UAV enters the waiting phase, where it awaits for new requests (with rate $\lambda / 2$ for each GN), and the process is repeated indefinitely. During this periodic process of communication and waiting for new requests, the UAV follows a trajectory, part of our design, with the goal to minimize the average long-term communication delay, as discussed next.

B. Problem Formulation

In this work, we consider the unconstrained delay minimization and neglect the propulsion energy consumption from our problem. In fact, it has been shown that a rotary-wing UAV exhibits comparable energy consumption when both moving and hovering [3]; in the special case when the moving and hovering powers are equal (for instance, based on the model in [3], this occurs at speed $V = 38$ m/s), the UAV energy constraint is equivalent to a constraint on the total service time of the UAV, independent of trajectory.

The goal is to define the optimal policy (UAV trajectory) so as to minimize the average communication delay. To this end, let $\Delta_l$ be the delay incurred to complete the transmission of the $l$th request serviced by the UAV. Let $M_l$ be the total number of requests served and completed up to time $t$. Then, we define the expected average delay under a given trajectory policy $\mu$ (to be defined), starting from $q(0) = 0$ at $t = 0$,

$$D_\mu = \lim_{t \to \infty} E \left[ \frac{\sum_{l=0}^{M_l-1} \Delta_l}{M_l} \right].$$

We then seek to determine $\mu^*$ to minimize $D_\mu$, i.e.,

$$\mu^* = \arg\min_\mu D_\mu.$$  \hspace{1cm} (3)

Note that this is a non-trivial optimization problem. While the minimum delay to serve a request, say from GN1, is achieved by flying towards GN1 at maximum speed to improve the link quality, this strategy may not be optimal in an average delay sense: if the UAV receives a new request from GN2 immediately after completing the request to GN1, the delay to serve this second request may be large due to the large distance that must be covered by the UAV.

C. Semi-Markov Decision Process (SMDP) formulation

In general, a solution to [3] would involve the optimization of an intractable number of variables over time (i.e., all possible trajectories followed by the UAV at any given time), over a continuous state space (the interval $[-a, a]$). Therefore, it is advantageous to approximate the system model through discretization and reformulate [3] as an average-cost SMDP.

We define the state space as $S = \mathcal{I} \times \mathcal{R}$, where $\mathcal{R} = \{0, 1, 2\}$ denotes the request status, i.e., no active request (0), a request is received from GN1 (1), and a request is received from GN2 (2), respectively, and

$$\mathcal{I} \triangleq \{-N, N + 1, \ldots, N - 1, N\}.$$  \hspace{1cm} (4)

is the set of $2N + 1$ indices corresponding to discretized positions $Q \triangleq \{q_i = \frac{i}{N}a, \forall i \in \mathcal{I}\}$ along the interval $q(t) \in [-a, a]$. This is a good approximation for sufficiently large $N$, as $\lambda \ll 1$, making the expected number of requests received over the travel time between two adjacent discretized positions much smaller than one. It is also useful to further partition the state space into waiting states, $S_{\text{wait}} = \mathcal{I} \times \{0\}$, and communication states, $S_{\text{comm}} = \mathcal{I} \times \{1, 2\}$.

We now define the actions in each state, the transition probabilities, and duration of each state visit. To define this SMDP, we sample the continuous time interval to define a

\footnote{While in practice the operation time of the UAV is constrained by the amount of energy stored in its battery, and the policy should depend on the amount of time left, the asymptotic case $t \to \infty$ is convenient since it gives rise to stationary policies (i.e., time-independent); this is a good approximation when the dynamics of the waiting and communication phases occur at much faster time scales than the total travel time, i.e., when $M_l$ in [3] is large for practical values of the travel time $t$. For perspective, [3] places typical rotary-wing hovering endurance times in the 15-30 minute range.}
discretized position $(m, r)$. Then the actions available are, $m = \{-1, 0, 1\}$, i.e., move right $(m = 1)$ to position $q_{i+1}$, hover $(m = 0)$, or move left by one discretized position $(m = -1)$ to $q_{i-1})$. The amount of time required to take this action, i.e., to fly between two adjacent discretized positions, is

$$\Delta_0 = \frac{a}{NV}. \quad (5)$$

The new state is then sampled at time $t + \Delta_0$, and is given by $s_{n+1} = (i + m, r_{n+1})$, where the transition probability from state $s_n = (i, 0)$ under action $m \in \{-1, 0, 1\}$ is defined as

$$P(s_{n+1} = (i + m, 0)|s_n = (i, 0), m) = e^{-\lambda_{\Delta_0}}, \quad (6)$$

$$P(s_{n+1} = (i + m, r)|s_n = (i, 0), m) = \frac{1 - e^{-\lambda_{\Delta_0}}}{2}, \quad \forall r \in \{1, 2\},$$

depending on whether no request is received during this time interval ($r_{n+1} = 0$), with probability $e^{-\lambda_{\Delta_0}}$, or a request is received from GN$_r$ ($r_{n+1} = r \in \{1, 2\}$, with probability $[1 - e^{-\lambda_{\Delta_0}}]/2$ for each GN). Upon reaching state $s_n = (i, r) \in S_{\text{comm}}$, with $r \in \{1, 2\}$, at time $t$, the UAV has received a request to serve $L$ bits to GN$_r$. The actions available to the UAV at this point are all trajectories that start from $q_{i}$ and allow the UAV to transmit the entire payload of $L$ bits. Assuming a move and transmit strategy (see (3)), the selected trajectory $q(\cdot)\{−\}$ must satisfy

$$\int_0^\Delta R_c(q(\tau))d\tau \geq L, \quad (7)$$

since all bits need to be transmitted during this phase, and its duration, defining the communication delay, is thus $\Delta$. We define the action space in state $(i, r) \in S_{\text{comm}}$ as the set of all feasible trajectories, $T_r(i) = \{\mathcal{J}_r, \mathcal{J}_r(i \rightarrow j)\}$, where we have defined $T_r(i \rightarrow j)$ as the set of feasible trajectories starting at $q_{i}$, ending in $q_{j}$, and serving GN$_r$, i.e.,

$$T_r(i \rightarrow j) = \{q : [0, \Delta] \rightarrow [-a, a] : \int_0^\Delta R_c(q(\tau))d\tau \geq L,$$

$$|q(\cdot)| \leq V, q(0) = q_i, q(\Delta) = q_j, \exists \Delta > 0\}.$$ \quad (8)

Upon completing the communication phase, the UAV enters the waiting phase again; the new state is then sampled at time $t + \Delta$ (the amount of time elapsed to complete the selected trajectory), and is given by $s_{n+1} = (j, 0) \in S_{\text{wait}}$, where $q_{j}$ is the position reached at the end of the communication phase. Thus, we have defined the transition probability in the SMDP from state $s_n = (i, r)$ under action $q \in T_r(i \rightarrow j)$ as

$$P(s_{n+1} = (j, 0)|s_n = (i, 0), q) = 1, \forall q \in T_r(i \rightarrow j).$$ \quad (9)

In other words, the trajectory selection process in the communication phase can be described as follows: 1) given $(i, r)$, i.e., the current position $q_i$ of the UAV and the request received from GN$_r$, the UAV first selects some $j \in I$, which defines the target position $q_j$ reached at the end of the communication phase; 2) the UAV selects a feasible trajectory $q$ from $T_r(i \rightarrow j)$, executes the trajectory while communicating to GN$_r$, and terminates the communication phase in the new position $q_j$, corresponding to state $(j, 0)$. After this point, the UAV is in the waiting phase again.

With the states and actions defined, we can define a policy $\mu$. Specifically, for states $(i, 0) \in S_{\text{wait}}$, $\mu(i, 0) \in \{-1, 0, 1\}$. Likewise, for states $(i, r) \in S_{\text{comm}}$, $\mu(i, r) = (j, q(\cdot))$, where $j \in I$ (position reached at the end of the communication phase) and $q(\cdot) \in T_r(i \rightarrow j)$ (feasible trajectory starting in $q_i$, ending in $q_j$, to serve GN$_r$).

The communication delay cost during the waiting phase is zero, i.e. $\Delta_{i,0}(m) = 0$, for all states $(i, 0) \in S_{\text{wait}}$ and actions $m \in \{-1, 0, 1\}$. When the UAV is in a communicating phase, we denote the communication delay incurred in state $(i, r)$ under action $(j, q(\cdot))$ as $\Delta_{i, r}(j, q(\cdot))$. Compacty, we write $\Delta_s(\mu(s))$ to denote the delay incurred in state $s \in S$ under the action $\mu(s)$ dictated by policy $\mu$.

With this notation, and having now defined a stationary policy $\mu$, we can rewrite the average delay $\tilde{D}_\mu$ in (2) in the context of the SMDP as

$$\tilde{D}_\mu = \lim_{K \rightarrow \infty} \frac{1}{K} \sum_{n=0}^{K-1} \mathbb{1}_{\Delta_n(s_n(\mu(s)))} = 0, \quad (10)$$

where $\mathbb{1}_A$ is the indicator function of the event $A$. In fact, the numerator in (2) counts the sample average delay incurred in the communication phases up to slot $K$ of the SMDP, whereas the denominator in (2) counts the sample average number of communication slots in the SMDP up to slot $K$. Now, using Little’s Theorem (9), we can rewrite (10) as

$$\tilde{D}_\mu = \frac{\sum_{s \in S} \Pi_{\mu}(s) \Delta_s(\mu(s))}{\sum_{s \in S} \Pi_{\mu}(s)} = \frac{\sum_{s \in S_{\text{comm}}} \Pi_{\mu}(s) \Delta_s(\mu(s))}{\sum_{s \in S_{\text{comm}}} \Pi_{\mu}(s)}, \quad (11)$$

where $\Pi_{\mu}(s)$ is the steady-state probability in the SMDP of the UAV being in state $s$ under policy $\mu$, and the second equality holds since $\Delta_s(\mu(s)) = 0$ and $\chi(s \in S_{\text{comm}}) = 0$ for $s \in S_{\text{wait}}$.

III. ANALYSIS OF POLICY OPTIMIZATION

In this section, we tackle the solution to the optimization problem (4), with $\tilde{D}_\mu$ given by (11). However, (4) cannot be directly solved using dynamic programming techniques, due to the presence of the denominator in (11), which depends on the policy selected $\mu$, hence it affects the optimization. The next lemma demonstrates that the denominator of (11) can be expressed as a positive constant, independent from policy $\mu$ and only dependent on system parameters. In doing so, the optimization of $\mu$ only needs to focus on the minimization of $\sum_{s \in S} \Pi_{\mu}(s) \Delta_s(\mu(s))$, so that (4) can be cast as an average cost per stage problem, solvable with standard dynamic programming techniques.

**Lemma 1.** Let $\pi_{\text{wait}}$ and $\pi_{\text{comm}}$ be the steady-state probabilities that the UAV is in the waiting and communication phases,
\[ \pi_{\text{comm}} = \sum_{s \in S_{\text{comm}}} \Pi_{\theta}(s) \text{ and } \pi_{\text{wait}} = 1 - \pi_{\text{comm}}. \]

We have that
\[ \pi_{\text{wait}} = \frac{1}{2 - e^{-\lambda \Delta_0}}, \quad \pi_{\text{comm}} = \frac{1}{2} - \lambda e^{-\lambda \Delta_0}. \]

\[ \text{(12)} \]

**Proof.** Let \( p_{uw}, p_{uc}, p_{cw}, \) and \( p_{cc} \) be the probabilities of a state request status, \( r \in \mathcal{R} = \{0, 1, 2\} \), transitioning in the SMDP as \( 0 \rightarrow 0 \), \( 0 \rightarrow \{1, 2\} \), \( \{1, 2\} \rightarrow 0 \), and \( \{1, 2\} \rightarrow \{1, 2\} \), respectively. Then, \( p_{uw} = e^{-\lambda \Delta_0} \) (if no request is received, the SMDP remains in the waiting state), \( p_{uc} = 1 - p_{uw} \), \( p_{cc} = 0 \) (if the SMDP is in the communication state, the next state of the SMDP will be a waiting state, see (9)). Therefore, the steady-state probabilities of being in the waiting and communication states, \( \pi_{\text{wait}} \) and \( \pi_{\text{comm}} \), satisfy
\[
\begin{align*}
\pi_{\text{wait}} &= p_{uw} \pi_{\text{wait}} + p_{cw} \pi_{\text{comm}} = e^{-\lambda \Delta_0} \pi_{\text{wait}} + \pi_{\text{comm}}, \\
\pi_{\text{comm}} &= p_{uc} \pi_{\text{wait}} + p_{cc} \pi_{\text{comm}} = (1 - e^{-\lambda \Delta_0}) \pi_{\text{wait}}, \\
\pi_{\text{wait}} + \pi_{\text{comm}} &= 1,
\end{align*}
\]

whence solution is given as in the statement of the lemma. \( \blacksquare \)

When we refer to the denominator of (11), it is evident that it is equal to the steady-state probability that the UAV is in a communication state while following policy \( \mu \), \( \pi_{\text{comm}} \). However, with the result of Lemma 1 \( \pi_{\text{comm}} \) is simply a positive constant determined by system parameters, yielding

\[ \bar{D}_\mu = \sum_{s \in S_a} \Pi_{\theta}(s) \Delta_s(\mu(s)), \]

which we now aim to minimize with respect to policy \( \mu \).

As the problem stands now, the communication phase selects an action from \( T_r(i) \), which is a set containing an uncountable number of trajectories. We now demonstrate, by exploiting a decomposition of policy \( \mu \) and the structure of the problem, that only a finite set of trajectories from \( T_r(i) \) are eligible to be optimal, for each state \((i, r) \in S_{\text{comm}} \), hence making the problem a finite state and action SMDP.

**A. Decomposition of Policy \( \mu \)**

Note from (9) that the transition probability from a communication state \( s_n = (i, r) \) under action \((j, q(\cdot))\) is only affected by the selection of \( j \) and not the particular trajectory \( q(\cdot) \in T_r(i \rightarrow j) \) that leads from \( q_i \) to \( q_j \) during the communication phase. From this independence, it follows that the steady-state probability \( \Pi_{\theta} \) under \( \mu(i, r) = (j, q(\cdot)) \) is only affected by the selection of \( j \) and not the specific trajectory within \( T_r(i \rightarrow j) \).

By establishing this property, we decompose the policy \( \mu \) into the waiting policy \( \theta(i) = m \in \{-1, 0, 1\} \), which defines the optimal action in state \((i, 0) \in S_{\text{wait}} \) of the waiting phase; the end position policy \( J(i, r) \), which selects the end position \( q_j \) with \( j = J(i, r) \) to be reached at the end of the communication phase; and the trajectory policy \( \rho(i, r, j) \), which, given \( j = J(i, j) \), selects a trajectory \( q(\cdot) = \rho(i, r, j) \) from \( T_r(i \rightarrow j) \). Owing to the independence of \( \Pi_{\theta} \) on the trajectory policy \( \rho \), the delay minimization problem can then be rewritten as

\[ \bar{D}_\mu = \min_{\theta, J} \sum_{s \in S_{\text{comm}}} \Pi_{\theta, J}(s) \min_{\rho(s, J(s))} \Delta_s(J(s), \rho(s, J(s))), \]

\[ \text{(13)} \]

Letting
\[ \Delta_\star(i, j, r) \triangleq \min_{q(\cdot) \in T_r(i \rightarrow j)} \Delta(i, r, j, q), \quad \forall (i, r) \in S_{\text{comm}}, \forall j \in \mathcal{J}, \]

we can finally write
\[
\bar{D}_\mu = \min_{\theta, J} \sum_{(i, r) \in S_{\text{comm}}} \frac{\Pi_{\theta, J}(i, r) \Delta_\star(i, J(i, r))}{\pi_{\text{comm}}},
\]

\[ \text{(16)} \]

Note that \( \Delta_\star(i, r, j, q) \) yields the trajectory that minimizes the communication delay when starting from state \((i, r) \), ending in position \( q_j \) while serving GN_r. This result proves that, for any communication state \((i, r) \), there exist only \( 2N + 1 \) trajectories that are eligible to be optimal, one for each possible ending position \( q_j \in Q \). Hence, the problem is finally reduced to that of finding the optimal waiting policy \( \theta \) and end position policy \( J \), which can be solved efficiently via dynamic programming (Algorithm 2). In the next section, we provide a closed form expression of the delay-minimizing trajectories of \( \Delta_\star(i, j) \).

**B. Closed-form Delay Minimizing Trajectory**

With the complete independence of the steady-state probabilities from \( \rho \), we can proceed to solve (15) and then provide the dynamic programming algorithm to solve for \( \theta^* \) and \( J^* \) in (16). By definition of the set \( T_r(i \rightarrow j) \) in (12), \( \Delta_\star(i, j) \) can also be written as

\[
\Delta_\star(i, j, r) = \min_{q(\cdot) \in T_r(i \rightarrow j)} \Delta(i, r, j, q), \quad \forall (i, r) \in S_{\text{comm}}, \forall j \in \mathcal{J}.
\]

The minimizing trajectory \( q^* \) is the one that the UAV should follow when receiving a request from GN_r starting in position \( q_i \) and ending in position \( q_j \).

In defining the optimal trajectory, the following definitions will be useful. Let \( \tau_{p_1, p_2} \triangleq \int_{p_1}^{p_2} \frac{1}{s} \) be the amount of time needed to fly at maximum speed from \( p_1 \) to \( p_2 \in [-a, a] \). Along this trajectory, let

\[
\ell_{p_1, p_2}^{(k)} \triangleq \int_{p_1}^{p_2} \frac{p_2 - p_1}{p_2 - p_1} \) dr,
\]

\[ \text{(18)} \]

be the amount of bits transmitted when moving at maximum speed from \( p_1 \) to \( p_2 \), when serving GN_r.

Clearly, \( \ell_{p_1, p_1}^{(r)} = 0 \) (\( p_1, p_1 = 0 \)), \( \ell_{p_1, p_2}^{(r)} = \ell_{p_1, p_2}^{(r)} (p_1, p_2 = r_{p_1, p_2}) \), and \( \ell_{p_1, p_2}^{(r)} = \ell_{p_1, p_2}^{(r)} (p_1, p_2 = r_{p_1, p_2}) \). The integral \( \ell_{p_1, p_2}^{(r)} \) can be determined in closed form and is found in [2], for example. We also define the trajectory \( v(p_1, \tau_{p_1, p_2}, \tau_{p_1, p_3}) \), \( \tau \in [0, \tau_{p_1, p_2} + \tau_{p_1, p_3}] \), as the one in which the UAV starts at position \( p_1 \), flies at maximum speed to \( p_2 \), hovers at \( p_2 \) for \( \delta \) amount of time, and finally flies at maximum speed from \( p_2 \) to \( p_3 \). Mathematically,

\[
v(p_1, \tau_{p_1, p_2}, \tau_{p_1, p_3}) \triangleq \begin{cases} 
\frac{p_1 + \tau_{p_1, p_2}}{p_2 - p_1}, & \tau \in [0, \tau_{p_1, p_2}] \\
p_2, & \tau \in [\tau_{p_1, p_2}, \tau_{p_1, p_3}] \\
p_2 + \frac{\tau_{p_1, p_3} - \delta}{\tau_{p_1, p_3}}, & \tau \in [\tau_{p_1, p_3} + \delta, \tau_{p_1, p_2} + \tau_{p_1, p_3} + \delta].
\end{cases}
\]

\[ \text{(19)} \]

Clearly, the payload delivered to GN_r when following this trajectory is \( \ell_{p_1, p_2}^{(r)} + \Delta R_{p_2} \epsilon_{p_2, p_3}^{(r)} \), with delay
Theorem 1. Let $q^*(\cdot) \in \mathcal{T}_r(i \rightarrow j)$ be the trajectory that minimizes the communication delay $\Delta^*_r(i,j)$. If $\ell^r_{q_i,q_j} \geq L$, then
\[ q^*(\cdot) = \{q_i \rightarrow (p^*,0) \rightarrow q_j\}(\cdot), \quad \Delta^*_r(i,j) = \tau_{q_i,q_j}, \]
i.e., the UAV flies at maximum speed from $q_i$ to $q_j$ without interruption; otherwise, if $\ell^r_{q_i,q_j} + \ell^r_{x_i,aj} \leq L$, then
\[ q^*(\cdot) = \{q_i \rightarrow (x_r, \delta^*) \rightarrow q_j\}(\cdot), \quad \Delta^*_r(i,j) = \tau_{q_i,x_r} + \tau_{x_r,q_j} + \delta^*, \]
where
\[ \delta^* = \frac{L - \ell^r_{q_i,x_r} + \ell^r_{x_r,q_j}}{R_r(x_r)}, \]
i.e., the UAV flies at maximum speed from $q_i$ to $x_r$, hovers over $x_r$ for $\delta^*$ amount of time, and then flies to $q_j$; finally, if $\ell^r_{q_i,x_r} + \ell^r_{x_r,q_j} > L$, but $\ell^r_{q_i,q_j} < L$, then
\[ q^*(\cdot) = \{q_i \rightarrow (p^*,0) \rightarrow q_j\}(\cdot), \quad \Delta^*_r(i,j) = \tau_{q_i,p^*} + \tau_{p^*,q_j}, \]
where $p^*$ is the unique solution in $[x_r, \min\{q_i, q_j\}]$ (if $r=1$) or $[\max\{q_i, q_j\}, x_r]$ (if $r=2$) of $\ell^r_{p^*,p^*} + \ell^r_{p^*,q_j} = L$; i.e., the UAV flies at maximum speed towards $x_r$ to the farthest point $p^*$ and then back to $q_j$ with $p^*$ uniquely defined in such a way as to transmit exactly the payload.

Proof. Due to space limitations, we provide an outline of the proof. Assume $r=2$ (a similar argument applies to $r=1$ by symmetry). 1) for any trajectory $q(.) \in \mathcal{T}_2(i \rightarrow j)$ of duration $\Delta$, one can construct another trajectory $\tilde{q}(\cdot) \in \mathcal{T}_2(i \rightarrow j)$ of same duration $\Delta$, and such that $|q(t) - x_r| \leq |\tilde{q}(t) - x_r|$, $\forall t \in [0,\Delta]$; such trajectory is obtained by flying at maximum speed towards GN$_2$, possibly hovering on top of GN$_2$ for $\delta$ amount of time (if time allows), and then returning to $q_j$, yielding $\tilde{q}(\cdot) = \{q_i \rightarrow (p^*,\delta^*) \rightarrow q_j\}(\cdot)$, for a proper choice of $p^*$ and $\delta^*$ such that $\tau_{q_i,p^*} + \tau_{p^*,q_j} + \delta^* = \Delta$; 2) note that the UAV is always closer to GN$_2$ under $\tilde{q}(\cdot)$ than is under $q(\cdot)$, hence it delivers a larger payload than $q(\cdot)$ while incurring the same delay; therefore, $q(\cdot)$ is suboptimal; 3) $\tilde{q}(\cdot)$ can be further improved by minimizing the delay (by optimizing $(p^*,\delta^*)$), yielding the three cases provided in the statement of the theorem.

C. Multi-chain Policy Iteration Algorithm

We opt to use a multi-chain PI algorithm to solve (16), as there exist some policies whose induced Markov chain structures are multi-chain. For example, if the waiting policy is $\theta(i) = 0$, and the end position policy is $J(i,r) = i$, then the induced Markov chain has $2N+1$ recurrent classes (hence multi-chain). To accommodate this structure, the pseudocode that follows is based upon the multi-chain PI methods of [10] and succinctly describes how to solve for $\mu^*$. In Algorithm 1, we use a vector notation for $\bar{D}_k$ and $\bar{h}_k$, which denote the average delay and relative value for all states, respectively, following the $k$th policy iterate $\mu^{(k)}$. Likewise, $c_{\mu}$ is the vector notation for the delay cost function under policy $\mu$, supplemented by the optimal minimized trajectory times described by (15) and (11-B) and $P_{\mu}$ is the transition matrix under policy $\mu$.  

\begin{algorithm}[H]
\caption{Multi-chain PI to solve (16)}
1: Initialize $k = -1$, arbitrary policy $\mu^{(0)}$, and loop termination variable;
2: repeat
3: $k \leftarrow k + 1$
4: \textbf{Evaluation}: Solve for gain $\bar{D}_k$ and relative value $\bar{h}_k$ by gain-relative value optimality equations [10];
5: \textbf{Improvement}: Find $\mu^{(k+1)} = \arg\min_{\mu} \{\bar{D}_k \cdot \mu \}$; choose $\mu^{(k+1)} = \mu^{(k)}$ if $\min \{\bar{D}_k \cdot \mu^{(k)} \} = \bar{D}_k \cdot \mu^{(k)}$;
6: if $\mu^{(k+1)} = \mu^{(k)}$ then
7: Find $\mu^{(k+1)} = \arg\min_{\mu} \{c_{\mu} + P_{\mu} \cdot h_k \}$; choose $\mu^{(k+1)} = \mu^{(k)}$ if $\min \{c_{\mu} + P_{\mu} \cdot h_k \} = c_{\mu} + P_{\mu} \cdot h_k$;
8: end if
9: until $\mu^{(k+1)} = \mu^{(k)}$; return $\mu^* = \mu^{(k+1)}$.
\end{algorithm}

IV. NUMERICAL RESULTS

We use the following system parameters, unless specified otherwise: number of states $2N+1=101$; channel bandwidth $B=1$ MHz; 1-meter reference SNR $\gamma_{DB}=40$ dB; UAV height $H=100$m; GN locations $x_1 = -400$m, $x_2 = 400$m; UAV speed $V = 20$m/s; and request arrival rate $\lambda = 0.4$ requests/second.

We vary the payload $L$ across a range of values and find numerically that, regardless, the optimal policy optimized with Algorithm 1 for states $(i,0) \in S_{\text{wait}}$ of the waiting phase is
\begin{equation}
\mu^*(i,0) = \begin{cases} 
m = 1, & i \in \{\{-N,-N-1, \ldots, -1\} 
m = 0, & i = 0 
m = -1, & i \in \{1,2, \ldots, N\} \end{cases}
\end{equation}

In other words, it is optimal for the UAV while in the waiting phase to move towards the geometric center of the two GNs along the line segment connecting the two. Intuitively, under this policy the UAV can more readily service a request that is originated equally likely from GN$_1$ or GN$_2$, when it is located in the geometric center between the two.

In Fig.2 we plot the optimal end position policy $J^*(i,2)$ for
differently loads. We note that, for a sufficiently large payload value, $L$, the optimal end position in the communication phase becomes independent of the initial position $i$ (in this case, $J^*(i,2) \approx 336m$, irrespective of $i$ for $L \gg 1$). This is due to the fact that, for large payload $L$, the UAV hovers over the receiver for a significant amount of time during the communication phase (see the case $\ell_{q_1,r} + \ell_{x,r} \leq L$ in Theorem 1), hence the final part of the trajectory from $x_r$ to the selected end position $q_j$ becomes irrespective of the actual payload value. However, $J^*(i,2)$ does depend on other system parameters, such as the request rate $\lambda$ and UAV height $H$, as seen in Fig. 3. Interestingly, as the request rate increases (the inter-arrival request time $1/\lambda$ decreases) the end position is closer to the geometric center (i.e., farther away from the receiver); this is because requests arrive more often, hence it is desirable for the UAV to terminate the communication phase closer to the center, in order to more readily serve future requests.

Next, we illustrate how the optimal expected average delay $\bar{D}^*_{\mu_1}$ across the same set of payload values, fares against the following heuristic policy: hover until receiving a request; when a request is received, fly at maximum speed towards the receiver until completion; after completion, hover again while waiting for the next request; and repeat this process. The comparison between the optimal policy $\mu^*$ and the heuristic policy is shown for the span of payload values in Fig. 4. Note that the slope of the line for both the optimal and heuristic policies saturates to $[B \log_2(1+\gamma/H^2)]^{-1}$. In fact, when $L \gg 1$, the UAV spends most of the communication time hovering above the receiver (case $\ell_{q_1,r} + \ell_{x,r} \leq L$ in Theorem 1), hence $\Delta^*(i,j) \approx L/(\ell_{q_1,r})$ in (16), yielding

$$\bar{D}^*_{\mu_1} \approx \min_{0,j} \sum_{s(1,r) \in \mathcal{S}_{\text{comm}}} \Pi_{\theta,j}(i,r)L \frac{L}{B \log_2(1+\gamma/H^2)}.$$ 

Overall, the heuristic scheme performs worse, roughly by 2 seconds for large $L$. In fact, when hovering during the waiting phase instead of moving towards the center, the UAV incurs a larger delay to serve a request generated by the more distant GN, due to the longer distance that needs to be covered.

V. CONCLUSIONS

In this paper, we studied the online trajectory optimization problem of one UAV servicing random downlink transmission requests by two GNs, to minimize the expected communication delay. We formulated the problem as an SMDP, exploited the structure of the problem to simplify the trajectory design in the communication phase, and showed that the problem can be solved efficiently via dynamic programming. Numerical evaluations demonstrate an interesting structure in the optimal trajectory and consistent improvements in the delay performance over a sensible heuristic, for a variety of payload values.

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