Out of plane optical conductivity in d-wave superconductors

Balázs Dóra\textsuperscript{1}, Kazumi Maki\textsuperscript{2} and Attila Virosztek\textsuperscript{1,3}

\textsuperscript{1}Department of Physics, Technical University of Budapest, H-1521 Budapest, Hungary
\textsuperscript{2}Department of Physics and Astronomy, University of Southern California, Los Angeles CA 90089-0484, USA
\textsuperscript{3}Research Institute for Solid State Physics and Optics, P.O.Box 49, H-1525 Budapest, Hungary

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Abstract. – We study theoretically the out of plane optical conductivity of d-wave superconductors in the presence of impurities at $T = 0K$. Unlike the usual approach, we assume that the interlayer quasi-particle transport is due to coherent tunneling. The present model describes the $T^2$ dependence of the out of plane superfluid density observed in $YBCO$ and $Tl2201$ for example. In the optical conductivity there is no Drude peak in agreement with experiment, and the interlayer Josephson tunneling is also assured in this model. In the unitary limit we predict a step like behaviour around $\omega \sim \Delta$ in both the real and imaginary part of the optical conductivity.

Introduction. – It is now well established that both the thermodynamics and transport properties of high $T_c$ cuprate superconductors are well described by the BCS theory of d-wave superconductors \cite{1,2}. More recently it is shown that the electron-doped high $T_c$ superconductors also belong to d-wave \cite{3,4}. Also after a long controversy \cite{5}, d-wave superconductivity in $\kappa-(ET)_2$ salts is emerging \cite{6,7}. Therefore single crystals of $YBCO$, $Bi2212$ and $\kappa-(ET)_2$ salts will provide the excellent testing ground of d-wave superconductors. In particular the planar transport in high $T_c$ cuprate superconductors is well described in terms of the BCS like quasi-particles \cite{8,9}. For example the $T$-linear dependence of the in plane superfluid density is established for the first time in $YBCO$ \cite{10}. In contrast the out of plane transport in high $T_c$ cuprates is rather controversial. First of all the absence of the Drude peak in the optical conductivity of $Bi2212$ and $YBCO$ is very difficult to interpret in terms of the usual quasi-particle transport \cite{10,11,12,13}. As an alternative, many authors consider the incoherent tunneling \cite{14,15}. In this way it is easy to obtain a flat optical conductivity as $\omega$ goes to zero. On the other hand there will be no interlayer Josephson coupling \cite{10}, contrary to the observation of the Josephson coupling seen experimentally in $Bi2212$ \cite{16}. Further Josephson plasmons are observed in $Bi2212$, $YBCO$ \cite{17} and in $\kappa-(ET)_2$ salts \cite{18}.
Therefore in the following we assume that the interlayer transport is described in terms of coherent tunneling \[10, 23\]. Let us consider the quasi-particle transport from one layer to another. If it is usual quasi-particle transport \(k = k'\), where \(k\) and \(k'\) are the planar quasi-particle momentum in these two layers. In this model the out of plane transport is essentially the same as the in plane transport. So there should be a Drude tail in the optical conductivity. In the incoherent tunneling model \(k \neq k'\); \(k\) and \(k'\) are uncorrelated. So there will be no Josephson coupling between two layers contrary to the experiment. In the coherent tunneling model, we assume \(k \neq k'\) but \(k \parallel k'\), or at least some directional correlation. This is the model considered by Ambegaokar and Baratoff \[24, 25\] and they call this "specular transmission". We believe that this is the only model which gives the interlayer Josephson coupling in unconventional superconductors.

More generally some mixture of the coherent and the incoherent tunneling is possible. But we limit ourselves to the simplest case.

The first important consequence of this choice is the temperature dependence of the out of plane superfluid density which is different from the in plane one. In particular both in the clean limit and in the presence of impurities the out of plane superfluid density exhibits the \(T^2\) dependence as seen in \(Y\) \(\text{BCO} [26]\), \(Tl2201 [27]\) and \(\kappa-(ET)_2\) salts \[10\].

**Superfluid density at \(T = 0K\).** – As in \[16\] we consider \(d\)-wave superconductivity in the presence of impurities. The effect of impurity scattering is incorporated by renormalizing the frequency in the quasi-particle Green's function \[12–16\]:

\[
\frac{\omega}{\Delta} = u + \alpha \frac{\pi}{2} \frac{\sqrt{1-u^2}}{u K \left(\frac{1}{\sqrt{1-u^2}}\right)}
\]

(1)

\[
\frac{\omega}{\Delta} = u - \alpha \frac{\pi}{2} \frac{u}{\sqrt{1-u^2}} K \left(\frac{1}{\sqrt{1-u^2}}\right)
\]

(2)

for the unitary and the Born limit, respectively, where \(u = \tilde{\omega}/\Delta\), \(\alpha = \Gamma/\Delta\), \(K(z)\) is the complete elliptic integral of the first kind, \(\tilde{\omega}\) is the renormalized frequency and \(\Gamma\) is the scattering rate. We have summarized some properties of these systems in \[16\]. Then the superfluid density at \(T = 0K\) is obtained from the integral:

\[
I_s = 2\pi T \sum_n \left( \frac{\Delta^2 f^2}{\Delta_n^2 + \Delta^2 f^2} \right) = \Delta \int_0^\infty dx \left(1 - \frac{u}{\sqrt{1+u^2}}\right) = \Delta \left(\frac{1}{\sqrt{1+C_0^2}} - \frac{\pi \alpha}{2} \int_{C_0}^{\infty} \frac{du}{u(1+u^2)} K \left(\frac{1}{\sqrt{1+u^2}}\right)\right) = \Delta \left(\frac{1}{\sqrt{1+C_0^2}} - \frac{2 \alpha}{\pi} \int_{C_0}^{\infty} \frac{duu}{(1+u^2)^2} K \left(\frac{1}{\sqrt{1+u^2}}\right)\right)
\]

(3)

for the unitary and the Born limit, respectively, where \(f = \cos(2\phi)\), \(\langle \ldots \rangle\) means average of \(\phi\) and \(C_0\) is given by

\[
C_0 = \frac{\pi \alpha}{2} \sqrt{1+C_0^2} \left(\frac{1}{\sqrt{1+C_0^2}}\right)^{-1}
\]

(4)
and

$$\sqrt{1 + C_0^2} = \frac{2}{\pi} \alpha K \left( \frac{1}{\sqrt{1 + C_0^2}} \right)$$  \hspace{1cm} (6)$$

for the unitary limit and the Born limit, respectively. The out of plane superfluid density is obtained from

$$\rho_s(\Gamma, 0) = I_s / \Delta_{00},$$  \hspace{1cm} (7)

where $\Delta_{00}$ is the superconducting order parameter at $T = 0$K and $\Gamma = 0$. The superfluid density at $T = 0$K in the presence of impurities is shown versus $\Gamma / \Gamma_c$ in fig. 1, where $\Gamma_c = 0.8819 T_{c0}$. In the unitary limit the decrease of $\rho_s(\Gamma, 0)$ is much steeper, because stronger impurity effect destroys superconductivity with more efficacy. At small temperatures $\rho_s, \perp$ decreases with $T^2$ as well as the penetration depth [10], contrary to the linear change of the in plane superfluid density [11]. As a comparison, we show also the in plane superfluid density at $T = 0$K versus $\Gamma / \Gamma_c$ in fig. 2. It is obtained as

$$\rho_{s,ab} = 2\pi T \sum_{n=0}^{\infty} \frac{\Delta^2 f^2}{(C_0^2 + \Delta^2 f^2)^2} =$$

$$= 1 - \frac{\alpha}{C_0} + \frac{\alpha}{C_0} \int \frac{du}{u^2} \left[ 1 - \frac{E \left( \frac{1}{\sqrt{1+u^2}} \right)}{K \left( \frac{1}{\sqrt{1+u^2}} \right)} \right]^2$$  \hspace{1cm} (8)

$$= 1 - \frac{C_0}{\alpha} - \frac{4}{\pi^2} \alpha \int \frac{du}{1 + u^2} \left[ K \left( \frac{1}{\sqrt{1+u^2}} \right) - E \left( \frac{1}{\sqrt{1+u^2}} \right) \right]^2$$  \hspace{1cm} (9)

in the unitary and Born limit, respectively. Here the two limits are farther from each other.

**Optical conductivity at $T = 0$K.** The out of plane optical conductivity is given by $\sigma(\omega) = \sigma_1(\omega) + i \sigma_2(\omega)$, where

$$\sigma_1(\omega) = -\frac{\sigma_n}{2\omega} \text{Re} I(\omega),$$  \hspace{1cm} (10)

$$\omega \sigma_2(\omega) = -\frac{\sigma_n}{2} \text{Im} \left( I(\omega) + 2 \int_{-\infty}^{\infty} \frac{1}{e^{\beta x} + 1} F(u(x), u(x - \omega)) dx \right),$$  \hspace{1cm} (11)

where

$$I(\omega) = \int_{-\infty}^{\infty} \frac{1}{2} \left( \tanh \left( \frac{\beta x}{2} \right) - \tanh \left( \frac{\beta(x + \omega)}{2} \right) \right) \times$$

$$\times \left( F(u(x + \omega), \bar{u}(x)) - F(u(x + \omega), u(x)) \right) dx$$  \hspace{1cm} (12)

and

$$F(u, u') = \left( \frac{uu' + f^2}{\sqrt{f^2 - u'^2} \sqrt{f^2 - u'^2}} \right).$$  \hspace{1cm} (13)

Note that the above expression would be the same, if we use the Mattis-Bardeen approximation [28] commonly used for s-wave superconductors. But we emphasize that we don’t assume
Fig. 1 – The out of plane superfluid density is plotted as a function of $\Gamma/\Gamma_c$ in the unitary (solid line) and Born (dashed line) limit.

Fig. 2 – The in plane superfluid density is plotted as a function of $\Gamma/\Gamma_c$ in the unitary (solid line) and Born (dashed line) limit.

$\alpha \gg 1$ which would be disastrous for unconventional superconductors but we obtain eqs. \[13, 14, 15\] from the coherent tunneling model, since all high $T_c$ cuprates are inherently in the clean limit (i.e. $\alpha \ll 1$). Also eq. \[10\] in the $\alpha \to 0$ limit agrees with $\sigma_1(\omega)$ given in \[11\]. Also now it is clear why $\sigma_1(\omega)$ in \[11\] describes the frequency dependence of $\sigma_1(\omega)$ determined from the transmission experiment on $\text{Bi}_2\text{Sr}_2\text{CuO}_4$ thin film \[29\].

We show in fig. 3 the dc limit of $\sigma_1(\omega)$ versus $\Gamma/\Gamma_c$ for the unitary and the Born limit. In terms of $C_0$ it is given as

$$\frac{\sigma_1(0)}{\sigma_n} = \frac{C_0}{\sqrt{1 + C_0^2}},$$

(14)

where $C_0$ has been defined in eqs. \[3, 4\]. In particular in the limit of $\alpha \ll 1$, $C_0$ is given by

$$C_0 \approx \left(\frac{\pi}{2\alpha}\right) \left[\ln \left(4\sqrt{\frac{2}{\pi\alpha}}\right)\right]^{-1} \frac{1}{\alpha},$$

(15)

$$C_0 \approx 4 \exp \left(-\frac{\pi}{2\alpha}\right)$$

(16)

for the unitary and Born limit, respectively. Therefore $\sigma_1(0)$ in the Born limit is practically zero for $\Gamma/\Gamma_c \leq 0.4$.

We show in fig. 4 and 5 $\sigma_1(\omega)$ versus $\omega/\Delta_{00}$ for the unitary and the Born limit, respectively. In the unitary limit $\sigma_1(\omega)$ rises with a step like feature around $\omega = \Delta$, while in the Born limit it increases monotonically with $\omega$. 

As 

\[ \omega \sigma \sigma_2(\omega) \] is shown in fig. 3 and 4 as a function of \( \omega / \Delta_{00} \) for the unitary and the Born limit. Also \( \omega \sigma_2(\omega) \) approaches zero like \( \omega^{-1} \) as \( \omega \) goes to infinity. So in this sense the Tinkham-Ferrell sum rule should be obeyed in the present model. Of course \( \omega \sigma_2(\omega) \) decays only slowly.

Fig. 3 – The dc conductivity is plotted as a function of \( \Gamma / \Gamma_c \) in the unitary (solid line) and Born (dashed line) limit.

Fig. 4 – The real part of the optical conductivity is shown in the unitary limit for \( \alpha = 0.01, 0.05, 0.1, 0.2 \) and 1 from bottom to top.

Fig. 5 – The real part of the optical conductivity is shown in the Born limit for \( \alpha = 0.01, 0.05, 0.1, 0.2 \) and 1 from bottom to top.
Fig. 6 – $\omega \sigma_2(\omega)$ is shown in the unitary limit for $\alpha = 0.01, 0.05, 0.1, 0.2$ and 1 from top to bottom.

Fig. 7 – $\omega \sigma_2(\omega)$ is shown in the Born limit for $\alpha = 0.01, 0.05, 0.1, 0.2$ and 1 from top to bottom.

with $\omega$. Therefore the energy cut off may be the problem. Also in the $\omega \to 0$ limit, we obtain $\omega \sigma_2(\omega) = 2\sigma_n \Delta_{00} \rho_{s\perp}$ as for the imaginary part of the in plane conductivity.

Concluding remarks. – We have calculated for the first time the out of plane optical conductivity in d-wave superconductors at $T = 0K$ within the coherent tunneling model. This simple model can describe both the absence of the Drude tail and the presence of the interlayer Josephson coupling. On the other hand the present model cannot describe the deviation from the Ferrell-Tinkham sum rule reported in Bi2212 [30]. Therefore a further refinement of the present model may be required. In a future paper we explore the temperature dependence of the out of plane optical conductivity.

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