ELASTIC SCATTERING AT THE LHC

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Abstract
The first data from the TOTEM experiment agree well with Regge theory, and demand a hard-pomeron contribution.

1 Introduction
Invented half a century ago\cite{1}, Regge theory is one of the major successes of high energy physics. It describes high-energy scattering at small $t$ in terms of the exchanges of the known mesons and of other objects which are probably associated with glueball exchange. The latter are known as pomerons and the inclusion of one such exchange, that of the so-called soft pomeron, suffices to describe hadron-hadron scattering processes at small $t$. \cite{2}.

At large $t$ it is necessary to introduce an additional term into the $pp, \bar{p}p$ amplitudes, which we have identified \cite{3} as arising from triple-gluon exchange.

The soft pomeron contributes to hadron-hadron total cross sections a term with energy dependence

$$s^{\epsilon_1} \quad \epsilon_1 \approx 0.08$$  \hspace{1cm} (1)

Deep-inelastic lepton scattering data require the introduction of a second pomeron, the so-called hard pomeron \cite{4}. Although hard-pomeron exchange is not needed to describe hadron-hadron total cross sections up to energies $\sqrt{s}$ below 1 TeV, it may nevertheless be present, giving a small contribution with energy dependence

$$s^{\epsilon_0} \quad \epsilon_0 \approx 0.4$$  \hspace{1cm} (2)

We have pointed out \cite{5} that such a hard-pomeron contribution is certainly present if the upper of the two contradictory measurements \cite{6} at the Tevatron should turn out to be correct, and that this leads to a considerable uncertainty in the prediction for the total-cross section at the LHC:

$$\sigma^{TOT} = 125 \pm 25 \text{mb} \quad \sqrt{s} = 14 \text{ TeV}$$  \hspace{1cm} (3)

Without a hard pomeron contribution, the prediction is just over 100 mb at 14 TeV, and close to 91 mb at 7 TeV. The TOTEM collaboration has now found \cite{7}:

$$\sigma^{TOT} = 98.3 \pm 0.2 \text{ (stat)} \pm 2.8 \text{ (syst)mb} \quad \sqrt{s} = 7 \text{ TeV}$$  \hspace{1cm} (4)

The TOTEM collaboration has also measured\cite{7,8} $pp$ elastic scattering. In this paper we use these data, together with those for $pp$ and $\bar{p}p$ total and differential cross sections below 1.8 TeV and for

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the proton structure function $F_2(x, Q^2)$ at small $x$ from HERA, to conclude that a moderate hard-pomeron contribution is indeed present in hadron-hadron scattering. The prediction (3) is now refined to

$$\sigma^\text{TOT} = 113 \pm 5\text{mb} \quad \sqrt{s} = 14 \text{ TeV}$$  \hspace{1cm} (5)$$

We note that there are alternative approaches combining soft and hard pomerons\(^9\).

2 Data fit

Some 20 years ago we fitted\(^10\) all existing $pp$ and $\bar{p}p$ total cross section data at energies $\sqrt{s}$ greater than 10 GeV with a combination of two fixed powers of $s$, one corresponding to $\rho, \omega, f_2, a_2$ exchange and the other to the soft-pomeron exchange of (1). We pointed out that these were to be regarded as effective powers, which were actually a combination of fixed powers arising from single exchanges and more complicated terms from multiple exchanges. When the HERA data at small $x$ appeared it was immediately apparent\(^4\) that the same fixed powers of $1/x$ would fit the data only if another $(1/x)^{\epsilon_0}$ term with $\epsilon_0 \approx 0.4$ were added.

While previously we used the separate data from the two HERA experiments, in this paper we use the combined data\(^11\).

For lepton-induced reactions there are no constraints from unitarity, so fixed powers of $1/x$ are allowed and we will continue to assume that they represent a good approximation. But for hadron-hadron processes unitarity is violated at large $\sqrt{s}$ if the fixed powers are not moderated by the introduction of additional terms in the amplitude. The fixed powers result from the exchange of single particles, and the additional terms correspond to multiple exchanges. Although we know some general analytic properties of these multiple-exchange terms, a full numerical calculation of them is still beyond present knowledge.

One way to take account of these known analytic properties is to write the amplitude as a Fourier transform over the two-dimensional momentum transfer $q$, where $q^2 = -t$,

$$A(s, t) = 2is \int d^2b \ e^{-iq \cdot b} \tilde{A}(s, b)$$ \hspace{1cm} (6a)$$

and write

$$\log(1 - \tilde{A}(s, b)) = -\chi(s, b)$$ \hspace{1cm} (6b)$$

so that

$$A(s, t) = 2is \int d^2b \ e^{-iq \cdot b} (1 - e^{-\chi(s, b)}) = 2is \int d^2b \ e^{-iq \cdot b} \sum_{n=1}^{\infty} \frac{1}{n!} (-\chi(s, b))^n$$ \hspace{1cm} (6c)$$

This is known as the eikonal representation.

If we identify the $n = 1$ term with the contribution from single exchanges

$$A_{\text{SINGLE}}(s, t) = 2is \int d^2b \ e^{-iq \cdot b} \chi_S(s, b)$$ \hspace{1cm} (7)$$

then the further terms have the correct analytic properties to describe the multiple exchanges. One possibility is to insert the function $\chi_S(s, b)$ that this gives into the full expansion, but this is only a model and it has no theoretical foundation. We prefer to take as our model for double exchange

$$2is\lambda \int d^2b e^{-iq \cdot b} \frac{1}{2} (\chi_S(s, b))^2$$ \hspace{1cm} (8)$$
where $\lambda$ is assumed to be a constant, whose value has to be fixed. We shall assume also that we can neglect the contributions from more than double exchange ($n > 2$). We stress that again this is only a model: as yet we do not have the theoretical knowledge to go beyond this.

$pp$ elastic scattering data from the CERN ISR$[^{12}]$ find a dip, which is deepest at $\sqrt{s} = 30.54$ GeV at which energy it is at $|t| = 1.425$ GeV$^2$. We fix the value of $\lambda$ in (8) by requiring that the imaginary part of the amplitude vanishes near the dip: it turns out that $t = 1.4$ GeV$^2$ is a good point to choose.

The dip moves slowly inwards towards $t = 0$ as $\sqrt{s}$ increases. This means that at values of $t$ on either side of the dip the amplitude varies rapidly with energy and so general principles require that it also have non-negligible real part. In order correctly to model the dip we have to ensure that both the real and the imaginary parts become very small at the same value of $t$.

We introduce 4 Regge trajectories:

$$\alpha_i(t) = 1 + \epsilon_i + \alpha'_i t$$

with $i = 0$ referring to the hard pomeron, $i = 1$ to the soft pomeron, $i = 2$ the degenerate trajectory for $f, a_2$ exchange and $i = 3$ for $\omega, \rho$ exchange. We fix $\alpha'_1$ at the value 0.25 GeV$^{-2}$ that has been known$[^{13}]$ for nearly 40 years, and$[^{14}]$ $\alpha'_2 = 0.8$ GeV$^{-2}, \alpha'_3 = 0.92$ GeV$^{-2}$. We try various values for $\alpha'_0$ and find that 0.1 GeV$^{-2}$ works well. We treat the four values of the $\epsilon_i$ as free parameters.

For the high-energy $pp$ elastic amplitudes we use

$$A(s,t) = \sum_{i=0}^{3} Y_i e^{-\frac{1}{2}i\pi\alpha_i(t)} (2\nu\alpha'_i)^{\alpha_i(t)}$$

with

$$2\nu = \frac{1}{2}(s - u) \quad Y_i = -X_i \quad (i = 0, 1, 2) \quad Y_3 = iX_3$$

with $X_0, X_1, X_2, X_3$ real positive. The factor $i$ multiplying $X_3$ is a manifestation of the Regge signature factor$[^2]$ for negative $C$-parity exchange. The amplitude for $\bar{p}p$ scattering is the same, except that $Y_3$ has the opposite sign. The normalisation of the amplitudes is such that $\sigma_{\text{TOTAL}} = s^{-1} \text{Im} A(s,t = 0)$.

At large $t$ triple-gluon exchange contributes to the amplitude$[^3]$

$$C s t^{-4}$$

where the data give

$$C = 3.4 \text{ GeV}^{-4}$$

This form cannot be valid near $t = 0$. We have tried various forms for small $t$ which match smoothly to (11a) at some value $t = t_0$, and find that

$$\frac{C s}{t_0^4} e^{4(1-t/t_0)}$$

works as well as any. Triple-gluon exchange, which is real, is important in obtaining the correct dip structure. We adjust the value of $t_0$ so as to match the shape of the dip as best we can at $\sqrt{s} = 30.54$ GeV. We use $t_0 = 5.4$ GeV$^2$.

For deep inelastic lepton scattering at small $x$ we use an expression which successfully fitted$[^4]$ the separate H1 and ZEUS data:

$$F_2(x, Q^2) = \sum_{i=0}^{2} f_i(Q^2)(1/x)^{\epsilon_i}$$

(12a)
We have shown \cite{15} that this form for $f_0(Q^2)$ gives a variation with $Q^2$ that agrees to very high accuracy with DGLAP evolution, and have explained that this is the only part of $F_2(x, Q^2)$ at small $x$ to which DGLAP may validly be applied.

We use these forms for the $pp, \bar{p}p$ amplitudes and for $F_2(x, Q^2)$, including $Q^2 = 0$, to make a simultaneous fit to the data for $F_2(x, Q^2)$ with $x < 0.001$ and the $pp, \bar{p}p$ total cross sections at energies above 10 GeV and below 1 TeV. Then we adjust the value of $\lambda$ to correctly reproduce the dip in the $pp$ elastic differential cross section at 30.54 GeV to obtain the fit shown in figure 1. As we have said, with no hard-pomeron term in the $pp$ amplitude this gives too low a cross section when extrapolated to 7 TeV. As is shown in figure 1, it also comes below the TOTEM elastic-scattering data.

In order to increase the total cross section so as to agree with the TOTEM measurement (4), we need to include a hard-pomeron contribution to the $pp$ (and also $\bar{p}p$) total cross section. Figure 2 shows the fit with $X_0 = 1.2$ and still $\alpha'_0 = 0.1$. This fit has

$$A_0 = 0.063, \ A_1 = 0.315, \ A_2 = 0.229, \ Q_0^2 = 2.95 \text{ GeV}^2, \ Q_1^2 = 794 \text{ MeV}^2, \ Q_2^2 = 303 \text{ MeV}^2$$
\[ \epsilon_0 = 0.362, \quad \epsilon_1 = 0.093, \quad \epsilon_2 = -0.360, \quad \epsilon_3 = -0.533 \]

\[ X_0 = 1.2, \quad X_1 = 243.5, \quad X_2 = 246.9, \quad X_3 = 136.7, \quad \lambda = 0.440 \]

3 Discussion

We make the following comments:

- The combined HERA data favour a value of the hard-pomeron power \( \epsilon_0 \) some 10–20% smaller than that we obtained from fitting the separate H1 and ZEUS data.
- The differences between the powers \( \epsilon_2 \) and \( \epsilon_3 \) corresponding to the \( C = +1 \) and \( C = -1 \) particle exchanges is in accord with the powers obtained from making a Chew-Frautschi plot\([14]\).
- Our fit in figure 2 to the TOTEM elastic scattering data is not perfect, but we only have a model: there is no known theory. Our expression for the double exchange has the correct analytic properties but its exact form is not known, and it cannot be exactly correct to neglect the triple and higher exchanges.
- In our model, \( \chi(s,b) \) in (6c) is taken to be given by
  \[ 1 - e^{-\chi(s,b)} = \chi_S(s,b) - \frac{1}{2}(\chi_S(s,b))^2 \]  
  Unitarity requires that \( \text{Re} \chi(s,0) > 0 \), or
  \[ |1 - \chi_S(s,0) + \frac{1}{2}(\chi_S(s,0))^2| < 1 \]

We find that this is exceeded by 4.5%, so again our model is not perfect.

References

1. T Regge, Il Nuovo Cimento, 14 (1959) 951; G F Chew and S C Frautschi, Physical Review Letters 8 (1962) 41
2. A Donnachie, H G Dosch, P V Landshoff and O Nachtmann, Pomeron Physics and QCD, Cambridge University Press (2002)
3. A Donnachie and P V Landshoff, Physics Letters B387 (1996) 637
4. A Donnachie and P V Landshoff, Physics Letters B437 (1998) 408
5. P V Landshoff, arXiv:0811.0260v1
6. CDF collaboration: F Abe et al, Physical Review D50 (1994) 5550; E710 Collaboration, N Amos et al: Physics Letters B243 (1990) 158
7. TOTEM collaboration: G Antchev et al, Europhys Lett 96 (2011) 21002
8. TOTEM collaboration: G Antchev et al, Europhys Lett 95 (2011) 41001
9. A D Martin et al, arXiv:1011.0287; E Gotsman et al, arXiv:0903.0247, arXiv:0901.1540
10. A Donnachie and P V Landshoff, Physics Letters B296 (1992) 227
11 HERA data: www-h1.desy.de/psfiles/figures/d09-158.nce+p.txt

12 CHHAV collaboration: E Nagy et al, Nuclear Physics B150 (1979) 221

13 G A Jaroskiewicz, Physical Review D10 (1974) 170

14 Reference [2], figure 2.13

15 A Donnachie and P V Landshoff, Physics Letters B533 (2002) 277