How magnetic field and stellar radiative feedback influences the collapse and the stellar mass spectrum of a massive star forming clump

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ABSTRACT

Context. In spite of decades of theoretical efforts, the physical origin of the stellar initial mass function (IMF) is still debated. Aim. We aim at understanding the influence of various physical processes such as radiative stellar feedback, magnetic field and non-ideal magneto-hydrodynamics on the IMF. Methods. We present a series of numerical simulations of collapsing 1000 M☉ clumps taking into account radiative feedback and magnetic field with spatial resolution down to 1 AU. Both ideal and non-ideal MHD runs are performed and various radiative feedback efficiencies are considered. We also develop analytical models that we confront to the numerical results. Results. The sum of the luminosities produced by the stars in the calculations is computed and it compares well with the bolometric luminosities reported in observations of massive star forming clumps. The temperatures, velocities and densities are also found to be in good agreement with recent observations. The stellar mass spectrum inferred for the simulations is, generally speaking, not strictly universal and in particular varies with magnetic intensity. It is also influenced by the choice of the radiative feedback efficiency. In all simulations, a sharp drop in the stellar distribution is found at about Mₘₐₓ ≈ 0.1 M☉, which is likely a consequence of the adiabatic behaviour induced by dust opacities at high densities. As a consequence, when the combination of magnetic and thermal support is not too large, the mass distribution presents a peak located at 0.3-0.5 M☉. When magnetic and thermal support are large, the mass distribution is better described by a plateau, i.e. dn/dlogM ∝ M⁻³, Γ = 0. At higher masses the mass distributions drop following power-law behaviours with a maximum mass Mₘₐₓ whose value increases with field intensity and radiative feedback efficiency. Between Mₘᵢₙ and Mₘᵪ, the distributions inferred from the simulations agree well with an analytical model inferred from gravoturbulent theory. Due to the density PDF ∝ ρ⁻³/₂ relevant for collapsing clouds, values on the order of Γ = 3/4 are inferred both analytically and numerically. More precisely, after 150 Mₖ of gas have been accreted, the most massive star has a mass of about 8 M☉ when magnetic field is significant, and 3 M☉ only when both radiative feedback efficiency and magnetic field are low, respectively. Conclusions. When both magnetic field and radiative feedback are taken into account, they are found to have a significant influence on the stellar mass spectrum. In particular both reduce fragmentation and lead to the formation of more massive stars.

Key words. ISM: clouds – ISM: structure – Turbulence – gravity – Stars: formation

1. Introduction

Star formation is a topic of fundamental importance in astrophysics. In particular the mass distribution of stars, described by the initial mass function (IMF) [Salpeter 1955; Kroupa 2001; Chabrier 2003; Bastian et al. 2010; Offner et al. 2014; Lee et al. 2020], plays a crucial role in setting the abundances of heavy elements and regulating stellar feedback, which in turn play major roles in the formation and evolution of galaxies and the interstellar medium. In the efforts to find a complete description of the IMF, it is sometimes overlooked that observed stellar masses span more than three orders of magnitudes: from 0.1 M☉ to more than 100 M☉. This facts likely implies the existence of several regimes of dominant physical processes and star formation conditions. Clearly this problem requires a long standing community effort and during the last decades several teams have conducted systematic investigations with the help of numerical simulations, introducing progressively more and more physical processes with increasingly higher numerical resolution.

The first attempts to obtain stellar mass spectra from numerical simulations in isothermal, self-gravitating, supersonic turbulent flows have been made by Klessen et al. (2001) and Bate et al. (2003). Together with several high-resolution studies performed by various authors (e.g. Girichidis et al. 2011a, Bonnell et al. 2011a, Ballesteros-Paredes et al. 2015, Lee & Hennebelle...
they find stellar mass spectra that present similarities with the observationally inferred mass spectra. In particular, at high masses the distributions are comparable to powerlaws, i.e. \(dN/d\log M \propto M^{-\Gamma}\), although in many runs, values of \(\Gamma = 3/4\) to 3, seemingly smaller than the canonical \(\Gamma = 1.3\) value inferred by Salpeter (1955) have been obtained (see the discussion in Lee & Hennebelle 2018a). The inferred distributions also present a peak, which however, when the simulations are strictly isothermal, is due to limited spatial resolution. A robust, numerically converged peak is obtained when an effective equation of state with an adiabatic index larger than 4/3 is taken into account (Lee & Hennebelle 2018a).

The influence of the magnetic field on the stellar mass spectrum has been investigated by Haugbølle et al. (2018), Lee & Hennebelle (2019), Guszejnov et al. (2020) performing high spatial resolution simulations with various magnetisations. The resulting mass spectra have been found to be similar to those inferred from simulations without magnetic field. In particular Guszejnov et al. (2020) stress that magnetic field cannot provide a characteristic mass that may explain the peak of the IMF and that thermal processes have to be considered.

Several attempts have been made to study the IMF using radiative transfer calculations. Urban et al. (2010) considered radiative feedback, i.e. stellar and accretion luminosity, by adding them onto the sink particles. They concluded that isothermal and radiative transfer calculations are significantly different, in particular the stars are much more massive in simulations with radiative feedback. Bate (2009) performed high resolution calculations, introducing the sink particles at very high density the released gravitational energy, i.e. \(n > 10^{19}\) cm\(^{-3}\) but stellar feedback onto the sink particles is not explicitly included, which makes it much weaker than it should. Krumholz et al. (2012) performed adaptive mesh refinement calculations with a resolution of 20-40 AU. Both stellar and accretion luminosity are added to the sinks. A relatively flat mass spectrum that is to say such that \(\Gamma \approx 0\) is inferred when winds are not considered while in the presence of stellar winds the mass spectra present a peak around 0.3 \(M_\odot\) and a power-law with \(\Gamma \approx 0.5 - 1\). Likely enough when winds are present, the radiation escape along the cavities and the heating is reduced. Mathew & Federrath (2020) presented simulations with a spatial resolution of 200 AU and perform calculations which use either a polytropic equation of state or heating from stars. They found that when heating is included more massive stars would form. Hennebelle et al. (2020) conducted adaptive mesh simulations with a spatial resolution of 4 AU and down to 1 AU. Both stellar and accretion luminosity are treated, with various efficiencies, \(f_{\text{acc}}\), ranging from 0 to 50\%, as well as two sets of initial conditions, namely a very compact and more standard clumps have been considered. For the most compact clumps and when \(f_{\text{acc}}\) is high, a flat mass spectrum develops. Otherwise all runs present mass spectra with a peak around 0.3-0.5 \(M_\odot\) and a powerlaw at higher masses, even when radiative feedback is not considered, i.e. \(f_{\text{acc}} = 0\), and when a barotropic equation of state is used instead. High efficiency radiative feedback runs however tend to present a broader distribution, both at the low mass and high mass end, with high mass stars up to 2 - 3 times more massive than in the barotropic and low feedback efficiency runs.

In the present paper we pursue the investigation of the origin of the stellar mass spectrum within a massive star forming clump. In particular, we focus on the role that magnetic field, in conjunction with radiative feedback may have. A number of studies performed calculations with both magnetic field and radiative feedback although most of the time, without predicting mass spectrum. As revealed by previous work (Peters et al. 2010, 2011, Commercon et al. 2011a, Myers et al. 2013) both these physical processes significantly influence the collapse and star formation, particularly by reducing the fragmentation. Moreover, their joint effect is not a mere superposition. These studies, however, did not present sufficient statistics to draw conclusions regarding the stellar mass spectrum. A stellar mass spectrum has been obtained by Li et al. (2018) where a magnetized and radiative calculation is performed with a spatial resolution of about 30 AU. These statistics need to be expanded and various initial conditions must be systematically explored. To do so we perform high resolution simulations of massive star forming clumps where both magnetic field and radiative feedback are accounted for. To get a good description of the small scales which are mandatory to describe the formation of low mass stars, we employ an adaptive mesh refinement with a spatial resolution down to 1 AU. As the magnetic intensity is likely varying from clump to clump and not many constraints from observations are available yet, we explore three magnetisations. Also the radiative feedback efficiency is subject to large uncertainties, so we consider two different values. Importantly, we also perform a simulation in which non-ideal MHD effects, namely ambipolar diffusion (Mestel & Spitzer 1956), are explicitly taken into account. We stress that these runs are the first for which both magnetic field and radiative feedback are taken into account, while considering a configuration which leads to sufficient statistics and spatial resolution to provide a reliable stellar distribution in the range 0.1 to 10 \(M_\odot\).

The paper is structured as follows. The second section presents the equations that are being solved, the relevant physical processes as well as the numerical methods used to solve these equations. It also presents the initial conditions and describes the various runs presented in the paper. In the third section, we look at the evolution of the clump during its collapse and investigate the effect of the magnetisation and radiative feedback. The global properties such as the total accreted mass and radiated energy, the temperature, magnetic field and mass distribution are studied. An analytical model, which is presented in an appendix is developed to understand the temperature distribution in the simulations. Comparisons are made with observational results. The fourth section presents the stellar mass spectrum obtained in the simulations. They are quantitatively compared with an analytical model which gives more insight into the effect of the different physical processes and is also presented in an appendix. In the fifth section a discussion is given while the sixth section concludes the paper.

2. Numerical simulations

2.1. Equations, numerical methods and setup

In this paper we solve the equations of the radiative magnetohydrodynamics. All the radiative quantities are estimated in the co-moving frame and assuming the grey approximation; that is to say the radiative energies are integrated over the entire frequency spectrum (e.g. Commercon et al. 2011a). The equations...
are

\[ \begin{align*}
\partial_t \rho + \nabla \cdot [\rho \mathbf{u}] &= 0, \\
\partial_t \rho \mathbf{u} + \nabla \cdot [\rho \mathbf{u} \otimes \mathbf{u} + \rho \mathbf{P}] &= -\lambda \nabla E_t + \mathbf{F}_L - \rho \nabla \phi, \\
\partial_t E_t + \nabla \cdot [\mathbf{u} (E_t + P + \frac{\rho \epsilon}{\kappa})] &= -\frac{\rho \epsilon}{\kappa} \nabla \cdot \mathbf{u} E_t + \nabla \cdot \left( \frac{\partial \epsilon}{\partial \rho} \nabla E_t \right) + S, \\
-\frac{1}{\kappa} (\mathbf{u} \cdot \mathbf{B}) \mathbf{B} - \mathbf{E}_{\text{AD}} \times \mathbf{B} &= -\frac{\rho \epsilon}{\kappa} \nabla \cdot \mathbf{u} E_t + \nabla \cdot \left( \frac{\partial \epsilon}{\partial \rho} \nabla E_t \right) + S, \\
-\frac{1}{\kappa} \mathbf{B} \times \mathbf{B} &= \nabla \times \left[ \mathbf{u} \times \mathbf{B} + \mathbf{E}_{\text{AD}} \right], \\
\mathbf{E}_{\text{AD}} &= \nabla \phi,
\end{align*} \]

(1)

where \( \rho \) is the material density, \( \mathbf{u} \) is the velocity, \( P \) the thermal pressure, \( \lambda \) is the radiative flux limiter (Minerbo 1978), \( E_t \) is the radiative energy, \( \mathbf{F}_L \) the Lorentz force, \( \phi \) the gravitational potential, \( E_T \) the total energy \( E_T = \rho e + 1/2 \rho u^2 + B^2/(8\pi) + E_t \) (\( e \) is the gas specific internal energy), \( \mathbf{B} \) is the magnetic field, \( \mathbf{E}_{\text{AD}} \) is the ambipolar electromotor field (EMF), \( \kappa_p \) is the Planck mean opacity, \( \kappa_r \) is the Rosseland mean opacity, \( \nabla \cdot \mathbf{u} \) the radiation pressure, \( S \), the luminosity source, and \( T \) is the gas temperature. The ambipolar EMF is given by

\[ \mathbf{E}_{\text{AD}} = \frac{\eta_{\text{AD}}}{B^2} \left[ (\nabla \times \mathbf{B}) \times \mathbf{B} \right], \]

(2)

where \( \eta_{\text{AD}} \) is the ambipolar diffusion resistivity, calculated as a function of the density, temperature, and magnetic field amplitude.

The numerical method is overall very similar to the one used in Hennebelle et al. (2020). The simulations were performed with the adaptive mesh refinement (AMR) magnetohydrodynamics (MHD) code RAMSES (Teyssier 2002) fromng et al. 2006). When non-ideal MHD, i.e. ambipolar diffusion, is included the scheme is the one described in Masson et al. (2012) and used in previous studies (Masson et al. 2016; Hennebelle et al. 2020b; Mignon-Risse et al. 2021; Commerçon et al. 2021; Lebreuilly et al. 2021). The resistivities are the ones calculated in Marchand et al. (2016).

In all simulations presented here, radiative transfer is accounted for using the flux limited diffusion method assuming grey approximation (see Commerçon et al. 2011b, 2014). The flux limited diffusion method is known to present some restrictions for instance it does not treat shadows well due to its isotropic nature. More accurate methods such as the M1 method (González et al. 2007), the hybrid method (Kuiper et al. 2010; Mignon-Risse et al. 2020) or the VETTAM method (Menon et al. 2022) have been developed and deal significantly better with anisotropic radiative transfer (see also Jaura et al. 2018; Peter et al. 2022). However, they tend to be more costly than the flux limited diffusion method employed in this work, and are often limited in their current implementations. This certainly represents line of future improvements.
At high density, the equation of state is taken from Saumon & Chabrier (1992) and Saumon et al. (1995) which takes into account H2, H, H+, He, He+, and He2+ (the He mass concentration is 0.27). The opacities are as described in Vaytet et al. (2013). For the range of temperatures and densities covered in this work, the opacities are the ones calculated in Semenov et al. (2003).

The boundary conditions are periodic. The cloud is initially spherical and has a radius four times lower than the computational domain size. All simulations were run on a regular grid of 256^3 computing cells and 10 AMR levels have been further...
Fig. 3. Radial profiles at several timesteps in HYDROf05 and MHD10f05, respectively. The black dashed line in the density panel shows the singular isothermal sphere density.
The dotted line shows a powerlaw behaviour \( M \propto n^{-3/2} \) as expected for a density PDF \( n^{3/2} \) (see Sect. 4.2.3).

2.2. Sink particles and stellar feedback

The sink particle algorithm is described in Bleuler & Teyssier (2014). Sink particles are formed at the highest refinement level at the peak of clumps whose maximum density is larger than \( n_{\text{acc}} \). The sink particles are created if the parent clump has a density \( n > n_{\text{acc}} \) and if it is sufficiently gravitationally bound (see Bleuler & Teyssier 2014). The value of \( n_{\text{acc}} \) is equal to \( 10^{13} \) cm\(^{-3}\). With this value of \( n_{\text{acc}} \), the computational cells having a density equal to \( n_{\text{acc}} \) possess a mass of roughly 1-2\% of the mass of the first hydrostatic core, \( M_0 = 0.03M_\odot \). At each time step, 10\% of the gas mass inside the sink’s accretion radius and with a density above \( n_{\text{acc}} \) is retrieved from the grid and accreted by the sink.

The sinks are not allowed to merge. The impact of changing the value of \( n_{\text{acc}} \) has been discussed in Hennebelle et al. (2020a). It has been found that both the spatial resolution and the value of \( n_{\text{acc}} \) may influence the peak of the stellar distribution. However once the first hydrostatic core is sufficiently resolved, this should not be the case.

Sink particles are also a source of radiation due to the stellar luminosity and gas accretion. The accretion luminosity is given
Fig. 5. Mass-weighted temperature in density intervals as a function of density at six time steps for the six runs. The dotted lines show a powerlaw behaviour of \( T \propto n^{1/2} \) and \( T \propto n^{3/4} \), respectively.

by

\[ L_{\text{acc}} = \frac{f_{\text{acc}} GM \dot{M}}{R}. \]  

where \( M_* \) and \( R_* \) are respectively the star’s mass and radius while \( \dot{M} \) is the accretion rate. If all the kinetic energy of the infalling gas was radiated away, we would have \( f_{\text{acc}} \approx 1 \). The accretion luminosity has been shown to be the dominant source of gas heating at early time and has important effects on the surrounding gas (e.g. Krumholz et al. 2007; Offner et al. 2009). The stellar luminosity of the protostars, \( L_* \), and \( R_* \) are taken from Kuiper \\& Yorke (2013) (see also Hosokawa \\& Omukai 2009). As discussed in Hennebelle et al. (2020a), the value of \( f_{\text{acc}} \) that should be used is not clearly established. In particular, the radiation is emitted at very small scale, i.e. few stellar radii and is not expected to propagate uniformly because of the highly anisotropic density distribution (e.g. Krumholz et al. 2012). As in Hennebelle et al. (2020a), we perform simulations in which we use an effective accretion luminosity and explore the values \( f_{\text{acc}} = 0.1 \) and 0.5. By considering an effective luminosity smaller than the estimated total luminosity, we envisage that the rest of the energy either escape preferentially along the cavities open by winds and jets or is converted into jet or a wind kinetic energy. This is obviously an important source of uncertainties which requires further investigations.

We start considering accretion and stellar luminosities when the sink has a mass of about 2 \( M_* \), i.e. 0.07 \( M_\odot \). The reason is that due to the limited spatial resolution, when the sink is introduced the protostar is not truly formed yet. Since the size of the sink particles is not very different from the radius of the first hydrostatic core, it seems reasonable to assume that the protostar is formed only when the sink reaches a mass equal to a few
Note that although reasonable, this assumption clearly requires further investigation. For instance, [Bhandare et al. (2020)] who have performed two-dimensional simulations of the second Larson cores, i.e. the young protostar, found that they grow with time far beyond the solar radii. This clearly suggests that at least some of the accretion energy is not fully radiated away but somehow stored in the star for some time. Indeed, the accretion shock at the edge of the second Larson core is subcritical (Vaytet et al. 2013) meaning that most of the accretion energy is advected inside the protostar and not immediately radiated away. This constitutes an important source of uncertainty for calculations such as the ones performed in this work.

### 2.3. Initial conditions and runs performed

Our initial conditions consist in spherical clouds in which a turbulent velocity field has been added. The velocity field has a classical Kolmogorov power-spectrum equal to 11/3 with random phases. A fully self-consistent approach would require to also set up the density and magnetic field fluctuations. This is however not an easy task. In practice it requires running a large scale simulations and zooming-in or at least performing a preliminary run without self-gravity (see for instance Lane et al. 2022). Note that [Lee & Hennebelle (2018a)] have compared various approaches including starting from a previous phase during which the simulation is run without self-gravity and starting directly from a prescribed turbulent field as it is done here. They found very similar results. This suggests that, at least in the context of collapsing clumps, the choice of the initial turbulent field may not be so important, probably because as the collapse proceeds, the fluctuations evolve and the initial perturbations are largely forgotten.

The clump we consider has a mass of $10^3 M_\odot$ and an initial radius of 0.4 pc corresponding to a uniform density of about $8 \times 10^4$ cm$^{-3}$ initially. Observationally, this corresponds to relatively standard massive star forming clumps (e.g. Urquhart et al. 2014; Elia et al. 2017, 2021; Lin et al. 2022). With an initial temperature of 10 K, the ratio of the thermal over gravitational energy is about 0.008. The clump density leads to a freefall time of about 110 kyr. The initial value of the Mach number is equal to 7 leading to a turbulent over gravitational energy ratio of about 0.4, i.e. the clumps are close to be initially virialised.

We set up the simulations with a uniform initial magnetic field through the cloud and intercloud medium. We considered two initial mean-field strengths, with mass-to-flux ratios, $\mu$, of respectively 10 and 100 (corresponding to about 100 $\mu$G and 10 $\mu$G respectively). These values are motivated by the observations of $\mu$ on the order of a few in dense cores (e.g. Crutcher 2012; Myers & Basu 2021). This selection also aims to account for the broad dispersions in the $\mu$ values of the massive clumps identified in the 1-kpc scale simulation presented in [Hennebelle 2018]. In these MHD simulations, which have a spatial resolution down to 400 AU, it has been found that the mass-to-flux ratio, which presents a broad dispersion, is indeed on the order of a few for Solar mass cores but is lower for the more massive ones. More precisely, Fig. 5 of [Hennebelle 2018] shows the distribution of self-gravitating objects with density larger than $10^4$ cm$^{-3}$. It shows a clear trend that the mass-to-flux ratio, $\mu$, is in-

![Fig. 6. Magnetic field as a function of density at several timesteps for the four magnetized runs. The dotted line shows a powerlaw behaviour $B \propto \eta^{1/2}$.

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creasing with mass in spite of a broad distribution. The most massive clumps displayed has a mass of about 100 $M_\odot$ and thus one needs to extrapolate to get a hint on 1000 $M_\odot$ clumps. Based on this figure, we would expect that the typical $\mu$ of a 1000 $M_\odot$ clumps is certainly larger than 10. This may at first sight be surprising because low mass cores have been observed to present values of $\mu$ on the order of a few (Pattle et al. 2022). It should however be remembered that the mass-to-flux ratio is the ratio of a volume over a surface weighted quantity. Thus considering objects with similar mean density, the mass-to-flux is expected to increase with the object size.

Another fundamental aspect is the physics of the magnetic field evolution. Whereas many studies have assumed ideal MHD, it is however clear that this is a poor approximation at high den-

### 3. General clump description

In this section, we look at the global evolution and final properties of the collapsing clump as a whole. We start by describing the general morphology, before proceeding to discuss the star formation and luminosity. We then study the gas density and temperature distribution inside the clump.

#### 3.1. Total accreted mass

Figure 1 displays the total mass accreted (top-left panel) by sink particles, $M_{\text{tot}}$, as a function of time for the six simulations. The total accretion rate is also plotted (top-right panel). Since it is a heavily fluctuating quantity, the latter is calculated by averaging its instantaneous value over 100 uniformly spaced time intervals. As indicated above, all simulations are run until about 150 $M_\odot$ have been accreted, corresponding to a star-formation efficiency of 15%. There are two exceptions, NMHDf05 for which at the end of the simulation 100 $M_\odot$ have been turned into the sinks, and MHD10f05 for which the final total mass of sinks is equal to 200 $M_\odot$. Two groups of simulations are easily distinguished. On one hand the hydrodynamical simulations and the low magnetized one, MHD10f05 and on the other hand the more magnetized ones, i.e. MHD10f01, MHD10f05 and NMHDf05. As expected, due to magnetic support, the latter group of simulations collapses a bit more slowly. Two points are worth mentioning, first the simulations with $f_{\text{acc}} = 0.1$ and with $f_{\text{acc}} = 0.5$ behave very similarly showing that in spite of strong heating, radiation does not significantly alter the large scale dynamics. Similar conclusion is also reached for the ambipolar diffusion. This is because i) thermal support is rather weak and an increase of temperature even by a factor of several does not make thermal support sufficiently strong to provide a significant support at the clump scale; and also because ii) magnetic field is only significantly modified by non-ideal MHD processes, at high density (say $n > 10^7$ cm$^{-3}$). Interestingly, we see that after a fast increase the accretion rate, $\dot{M}_{\text{tot}}$, reaches values, which are nearly identical for all simulations and equal to about $10^{-5} M_\odot$ yr$^{-1}$. This is because the accretion rate is controlled by the largest scale, here the clump, which is globally collapsing. The magnetic intensity considered here is too weak to significantly modify this global dynamics.

#### 3.2. Total luminosity

Bottom-left panel of Fig. 1 portrays the total luminosities, $\Sigma(L_\star + L_{\text{acc}})$ of the sink particles (see also Fig. C.1). As for the accretion rate, after an increase which takes about 0.02 Myr, it reaches, in the case with $f_{\text{acc}} = 0.5$, a plateau at about $1 - 2 \times 10^5 L_\odot$. In the case $f = 0.1$, the total luminosity is 10-20 times lower for run HYDROf01 and 3 times lower for run MHD10f01. Note that Fig. 1 shows that the clumps spend about 0.02 Myr in the protostellar phase with a luminosity several times lower than the peak values.

It is interesting to compare these values with observations. Although what is observationally available is the bolometric luminosities rather than the total source luminosities, they are obviously related and can be compared. In particular, it is expected that the radiation emitted by the star in the visible domain is heavily fluctuating, the latter is calculated by averaging the values seen in Fig. 13 of Elia et al. (2017). It is interesting to compare these values with observations. Although what is observationally available is the bolometric luminosities rather than the total source luminosities, they are obviously related and can be compared. In particular, it is expected that the radiation emitted by the star in the visible domain is heavily fluctuating, the latter is calculated by averaging the values seen in Fig. 13 of Elia et al. (2017). As for the accretion rate, after an increase which takes about 0.02 Myr, it reaches, in the case with $f_{\text{acc}} = 0.5$, a plateau at about $1 - 2 \times 10^5 L_\odot$. In the case $f = 0.1$, the total luminosity is 10-20 times lower for run HYDROf01 and 3 times lower for run MHD10f01. Note that Fig. 1 shows that the clumps spend about 0.02 Myr in the protostellar phase with a luminosity several times lower than the peak values.

Since the mass of the clump is 1000 $M_\odot$, this means that once the luminosity is about 10$^5$ $L_\odot$, the luminosity per solar mass is about 10 – 100 $L_\odot$ / $M_\odot$. Again this is in good agreement with the values seen in Fig. 13 of Elia et al. (2017).

In order to define a reference with which the luminosities can be compared, we define a quantity $L_{\text{glob}} = 0.5 \times GM_{\text{tot}}\dot{M}_{\text{tot}}/R_\odot$, which would correspond to the accretion luminosity of an object of mass $M_{\text{tot}}$ and radius 2 $R_\odot$, accreting at a rate $\dot{M}_{\text{tot}}$ with an efficiency $f_{\text{acc}} = 0.5$. The ratio $\sum L_\star/L_{\text{glob}}$ is expected to be smaller than 1 because the luminosity is a non-linear quantity, which decreases with the number of stars. It gives a sense of how fragmented is the clump and how efficiently is the gravitational energy converted into radiation. Bottom-right panel shows $\sum L_\star/L_{\text{glob}}$ for the six runs as a function of time. As expected, this quantity decreases with time and, at later time, it reaches values as small as 10$^{-3}$ and as high as 2×10$^{-2}$, depending on the run. Interestingly, there is a clear trend for the magnetized
runs to have values 1.5-2 times larger than their hydrodynamical counterpart. As we show later this is because magnetic field tends to reduce fragmentation, therefore building more massive stars which present higher luminosities.

3.3. General morphology

Figure 2 portrays the column density of the whole clump at time \( t = 0.11 \) Myr, which as seen from Fig. 1 corresponds to a time where approximately 100-120 \( M_\odot \) have been accreted. The dark circles show the sink particles, which represent individual stars. The six simulations present a similar pattern. A complex network of interwoven and interconnected filaments have formed and three of them appear to be a little more prominent.

Table 1. Summary of the runs performed. \( R_c \) is the initial clump radius. \( \mu \) is the mass-to-flux over critical mass-to-flux ratio. \( M \) is the initial clump Mach number. \( l_{\text{max}} \) is the maximum level of grid used and \( dx \) corresponds to the maximum resolution which is equal to 1.15 AU in these runs. \( f_{\text{acc}} \) gives the fraction of the accretion luminosity which is taken into account in the calculation. NMHD tells whether non-ideal MHD is accounted for.

| Name         | \( R_c \) (pc) | \( \mu \) | \( M \) | \( l_{\text{max}} \) | \( dx \) (AU) | \( f_{\text{acc}} \) | NMHD |
|--------------|---------------|--------|-------|----------------|-------------|---------------|-------|
| NMHD10f05    | 0.4           | 10     | 7     | 18             | 1.15        | 0.5           | yes   |
| MHD10f05     | 0.4           | 10     | 7     | 18             | 1.15        | 0.5           | no    |
| MHD10f01     | 0.4           | 100    | 7     | 18             | 1.15        | 0.1           | no    |
| MHD100f05    | 0.4           | 10     | 7     | 18             | 1.15        | 0.5           | no    |
| HYDROf05     | 0.4           | \( \infty \) | 7 | 18 | 1.15 | 0.5 | no |
| HYDROf01     | 0.4           | \( \infty \) | 7 | 18 | 1.15 | 0.1 | no |

The first row displays the gas temperature that we remind is often parametrized by the density profile in the clump inner part broadly behaves as \( r^{-1} \) while at 0.1 pc it is typically 50-70 K.

4. Gas and magnetic field distribution

4.1. Density, velocity and temperature profiles

Figure 3 displays radial profiles of various density-weighted quantities for runs MHD10f05 (left) and HYDROf05 (right) and at several timesteps. For the sake of conciseness only 2 simulations are being displayed and discussed here. The adopted center is the position of the most massive sink particle, which is located in the hub at \( y = 0.75 \) pc and \( z = 0.65 \) pc.

The first row displays the gas temperature that we remind is initially uniform and equal to 10 K. As time goes on, temperature increases by 2 to 3 orders of magnitudes in the center and about one order of magnitude in the clump’s outer part. The temperature profile in the clump inner part broadly behaves as \( r^{-1} \) while it is almost flat in the clump outer part. We further note the presence of many temperature peaks associated to sink particles distributed through the clouds. We also stress that the temperature is clearly larger by a factor \( \approx 1.5 \) in run MHD100f05 than in run HYDROf05. We will come back on this particular point later but this effect is similar to what has been reported by Commerçon et al. [2011a], where radiative MHD calculations where also performed and higher temperatures have been reported in the MHD case. This is a consequence of the non-linearity of the accretion luminosity proportional to \( MM \). By reducing fragmentation and extracting angular momentum, magnetic field increases both \( M \) and \( M \) leading to higher accretion luminosity. These temperatures appear to be in good agreement with the ones presented in Fig. 10 of Lin et al. [2022]. For instance, the temperature at few 0.01 pc is about 100-200 K while at 0.1 pc it is typically 50-70 K.

Second row shows the density profiles. The straight line represents the density of the singular isothermal sphere (SIS), i.e. \( \rho = \frac{c_{s,0}^2}{(2\pi G)^{\frac{3}{2}}} / \left( \frac{\pi}{2} \right)^{\frac{3}{2}} \), where \( c_{s,0} \) is the sound speed taken here equal to 0.2 km s\(^{-1}\). As collapse proceeds, the density increases from outside-in and after roughly 0.1 Myr, it presents a power-law-like shape close to, but slightly shallower than, \( r^{-2} \). As we see the values evolve with times and also slightly depend on the radius. We see however that it is nearly 2 orders of magnitude denser than the SIS, which is an expected consequence of the low initial thermal energy and the compactness of the cloud. The density in run HYDROf05 is slightly lower than in run MHD10f05, which is a consequence of the magnetic support. Let us remember that \( r^{-2} \) density profile is the expected density structure of a spherical collapsing cloud (e.g. Larson 1969; Shu 1977) as, together with a uniform radial velocity, it leads to a roughly constant accretion rate through the cloud (E 2018; Gomez et al. 2021). When turbulence is included, it is however common to find profiles slightly shallower, for instance \( \rho \propto r^{-1.5} \) is often
reported (e.g. Murray & Chang 2015, Li et al. 2018), though in the present case this value seems a little too shallow.

The radial velocity through the cloud is presented in the third row. At 0.1 Myr, a constant radial velocity of $\approx 1.5 - 2$ km s$^{-1}$ appears to reasonably represent the cloud radial velocity for radius between 0.03-0.3 pc. The radial velocity increases towards the cloud inner part where it reaches $\approx 10$ km s$^{-1}$. The parallel velocity, which represent both turbulent and rotation (i.e. the non-radial component) is displayed in the fourth row. Due to the chosen initial conditions, it is of the order of $\approx 2$ km s$^{-1}$ in the cloud outer part. As the collapse proceeds and due to the increase of $v_r$, turbulence is further amplified towards the cloud center (Hennebelle 2021) and this behaviour explains the density profile being shallower than $r^{-2}$. These velocity values are in good agreement with the values presented in Fig. 22 of Lin et al. (2023). For instance at 0.1 pc, values of about 3 km s$^{-1}$ are reported.

4.2. Mass distribution

The mass distribution, which we remind is equivalent to the mass weighted density PDF, is displayed in Fig. 4 for the six simulations at several timesteps. The distribution contains several features and going from low to high densities four domains can be identified: the interclump medium, the clump outer part, the collapsing envelopes and the high density material. We stress that the last two do not correspond to a single physical region but rather develop around each individual collapse center.

4.2.1. The interclump medium

At low density the mass distribution presents a roughly log-normal shape which peaks at about 500 cm$^{-3}$ (e.g. Vázquez-Semadeni 1994, Federrath et al. 2008, Kritsuk et al. 2011) and remains stationary throughout time. It is due to the development of turbulence in the cloud outer part. The latter has formed by the turbulent-driven expansion of the cloud external layer. Clearly it contains a small amount of mass.

4.2.2. The clump outer part

At higher density, i.e. $\rho \approx 10^5 - 10^6$ cm$^{-3}$, a second peak of the mass distribution located at the cloud initial mean density, is visible. It contains most of the mass of the cloud and shifts toward higher densities as collapse proceeds. Meanwhile as expected the mass it contains, declines over time. Overall the mass distribution of this density range is similar for the six simulations. We can nevertheless note that the peak is a bit broader for the two hydro runs, than for the more magnetized runs MHD10f01, MHD10f05 and NMHDf05. This is likely a consequence of the magnetic field which is known to reduce the turbulent dispersion of the density distribution (e.g. Molina et al. 2012).

4.2.3. The collapsing envelopes

At densities higher than its peak value, the mass distribution is better described by a powerlaw behaviour up to densities of $10^9$ and even $10^{10}$ cm$^{-3}$. This part of the mass distribution corresponds to the $n \propto r^{-\alpha}$, $\alpha \approx 2$, envelope discussed in Fig. 5.

Let us remember that there is a simple correspondence between $\alpha$ and the index of the mass distribution. Let $dN/d\log n$ be the number of fluid particles located between radius $r$ and $r + dr$. We have $dN \propto r^2 dr$. But since $n \propto r^{-\alpha}$, we have $dN/d\log n \propto n^{-3/\alpha}$ and the mass weighted density PDF is

$$n \propto n^{-3/\alpha + 1}.$$  \hspace{1cm} (4)

For $\alpha \approx 2$, we thus find that $n dN/d\log n \propto n^{-1/2}$, which indeed is close to the observed behaviour of the mass distribution between $10^7$ and $10^9 - 10^{10}$ cm$^{-3}$ as shown by a comparison with the dotted lines.

Several aspects are worth noticing. First at early stages (black and red curves), the mass distributions evolve with time. More mass is gradually accumulated at high densities as collapse proceeds. Once the $n \propto r^{-2}$ envelope is fully developed, the mass distribution is stationary. This is all consistent with the stationarity observed in Fig. 3 illustrating that the accretion rate remains broadly constant with time.

4.2.4. The high density material

At density larger than $10^9 - 10^{10}$ cm$^{-3}$, the mass distribution becomes flatter, meaning that mass is piling up. This is a consequence of rotational and thermal supports. Indeed protoplanetary disks form (see Lebreuilly et al. 2021, for a description of disks in similar simulations). Clearly the amount of mass significantly varies with magnetisation and it is several times higher in the hydro runs than in the significantly magnetized ones (MHD10). This is a clear consequence of magnetic braking, which by extracting angular momentum leads to smaller and less massive disks.

4.3. Temperature vs density distributions

Figure 5 shows the mean temperature as a function of density in the six simulations. In each density bin, the mean temperature is simply the mass weighted temperature. The overall behaviour is as suggested by the temperature profiles shown in § 4.1.

The temperature associated to the high density material is typically larger than $\approx 300$ K and reaches values of few thousands K. As expected the temperature increases with $f$.

For the lower density material (i.e. $n < 10^6$ cm$^{-3}$), we see first that the temperature decreases roughly as $T \propto n^{-0.3-0.5}$ (as indicated by the dotted line) and then at density of about $10^7$ cm$^{-3}$, it reaches a plateau and remains constant, $T = T_{\text{ext}}$, at lower densities. Depending of the runs and the time the temperatures vary between 10 and up to $\approx 30$ K.

To interpret these temperatures, we developed a simple spherical model which is presented in § A. Although we see from Fig. 2 that the clouds are not spherical and that the sources are not clustered in the center as assumed in our model, this nevertheless allows us to get a deeper understanding of these temperatures. The inferred powerlaw behaviours are as described by Eq. (A.4) and Eq. (A.8). More precisely, Eq. (A.4) combined with Eq. (A.1) predicts that for $T > 100$ K, $T \propto n^{-3/2}$ while for $T < 100$ K, $T \propto n^{13/2}$.

To quantitatively estimate the values of $T_{\text{ext}}$, we use Eq. (A.9)

$$T_{\text{ext}} = 33.5K \left( \frac{T_0}{0.5} \right)^{3/(4+2\alpha)} \left( \frac{L_{\text{rot}}}{10^7L_0} \right)^{1/(4+2\alpha)} \left( \frac{\delta}{100} \right)^{-1/(2+\alpha)}$$ \hspace{1cm} (5)

where we remind that $\delta$ is as defined by Eq. (A.1) and $T_0$ is the optical depth at which the radiation is free streaming.
From Fig. 1 and Fig. 5 we see that when $L_{\text{tot}} = 10^5 \, L_\odot$, $T_{\text{ext}} \approx 30 \, \text{K}$, whereas when $L_{\text{tot}} = 10^7 \, L_\odot$, $T_{\text{ext}} \approx 20 \, \text{K}$, which is close to what Eq. (5) predicts. Looking at Fig. 5 of Elia et al. (2017), we see that 20-30 K corresponds to the temperature of the warmest star forming clumps, which agrees well with the relatively high luminosities that we inferred. The HiGAL-based temperature is the average temperature of the cold dust in a clump. They are derived from 160- to 500- (and 870, 1100, when available) $\mu m$ grey-body fit, so that probed temperatures cannot be higher than that. Since the mass in the outer part of the clump dominates, this cold component corresponds to that of the outer layers and of most of the volume of the clump. This should therefore broadly correspond to what $T_{\text{ext}}$ is.

4.4. Magnetic field distributions

Figure 6 portrays the volume weighted magnetic intensity as a function of gas density for the 4 magnetized simulations. Overall we see that, at least for $n$ between $10^7$ and $10^9 \, \text{cm}^{-3}$, the magnetic field scales with density broadly as $B \propto n^{1/2}$, a result observed in previous works (see for instance Hennebelle & Inutsuka[2019] for a review). This is a consequence of the field amplification induced by field lines dragging by collapsing motion. Even more simply, this is likely a consequence of energy equipartition. As seen in Fig. 3, $\rho^2$ depends weakly on $r$ while $n \propto r^{-2}$, therefore the kinetic energy scales as $r^{-2}$ and thus $B \propto r^{-1/2}$. Interestingly, this implies that the Alfvén velocity, $V_{A}$, remains roughly constant in this range of density. We see however that its value is not identical for the four runs. We estimate that for runs MHD10f05 and MHD10f01, $V_{A} = 1 - 1.2$ km s$^{-1}$ while for run MHD100f05, $V_{A}$ is less than half this value. When non-ideal MHD is treated, the Alfvén velocity is reduced by tens of percents at $n \approx 10^9 \, \text{cm}^{-3}$.

At lower densities, the behaviour depends on the field intensity. For run MHD100f05, the dependence of $B$ on $n$, is a bit stiffer. This is expected as when the field is weak, the clump contraction tends to be spherical in which case $B \propto n^{2/3}$ (Lee et al. 2015). This explains why the magnetic field at high density in run MHD100f05 is larger than a tens of the $B$ values in run MHD10f05. Magnetic intensity is more vigorously amplified when it is weaker.

At high densities, i.e. $n > 10^9 \, \text{cm}^{-3}$, the magnetic field is further amplified up to density values on the order of $10^{11} \, \text{cm}^{-3}$. The highest magnetic intensities vary from one run to the other. In the most magnetized runs, MHD10f05 and MHD10f01, it reaches $\approx 100 \, \text{G}$ and about one third of this in run MHD100f05. Run NMHD10f05 presents different behaviour. For $n > 10^{10} \, \text{cm}^{-3}$, the intensity is nearly independent of $n$ and the largest intensities is about $\approx 30 \, \text{G}$. This behaviour, which has been discussed previously (e.g. Masson et al. 2016; Wurster & Li 2018) is a consequence of ambipolar diffusion, that tends to diffuse the field. This implies that the influence of magnetic field on the high density gas is significantly reduced compared to ideal MHD runs.

5. Stellar mass spectrum

5.1. Fragmentation and massive stars

Figure 7 portrays the number of sink particles as a function of accreted mass (left panel) as well as the mass of the most massive star (right panel).

The number of sinks at the end of the simulations is typically between 100 and 300 depending of the runs. As anticipated from the clump images, both magnetic field and radiative feedback reduce fragmentation. Here we see that the differences between runs HYDROf05 and MHD10f05 or between HYDROf01 and MHD10f01 is about a factor of 2, the difference being more pronounced for the two runs with $f_{\text{frac}} = 0.5$. On the other hand the differences between runs MHD10f05 and MHD10f01 is on the order of 50%, showing that whereas radiative feedback contributes to reduce fragmentation, its effect is comparatively lower than magnetic field. Indeed, although the initial magnetisation of run MHD100f05 is quite weak, it nevertheless reduces the fragmentation by tens of percents compared to run HYDROf05. Interestingly run NMHD10f05, that treats ambipolar diffusion and has the same magnetisation than run MHD10f05, presents a number of sinks similar to run MHD100f05.

In all runs but HYDROf01, two phases can be distinguished. When $M_{\text{tot}}$ is smaller than $\approx 3 \, M_\odot$ (\approx 10 for run HYDROf05), the number of sinks increases fast and is nearly proportional to $M_{\text{tot}}$ with $M_{\text{tot}} \approx 0.7 - 1$. Beyond this value, the number of sinks increases much less rapidly, and typically $M_{\text{tot}} \approx 0.2 - 0.3$. For instance for run MHD10f05, the number of sinks has roughly doubled between the time when $M_{\text{tot}} \approx 10 \, M_\odot$ and $M_{\text{tot}} \approx 100 \, M_\odot$. This is most certainly related to radiative feedback and to the global increase of temperature within the clumps. The consequence is obviously that the sink particles, build their masses in this second phase after fast fragmentation has occurred.

At the end of the runs, the mass of the most massive star is between 3 and 10 $M_\odot$. The observed trends are in good agreement with the sink numbers. The mass of the most massive star is higher when magnetic field and radiative feedback are larger and magnetisation is comparatively slightly more efficient than radiative feedback in producing massive stars. Two phases of growth can also be distinguished, typically below and above $M_{\text{tot}} \approx 10 \, M_\odot$, where $M_{\text{max}}$ grows respectively slowly and fast. We observe that $M_{\text{max}} \propto M_{\text{tot}}^{\frac{3}{2}}$ with $M_{\text{max}} \approx 0.5$ when $M_{\text{tot}} < 10 \, M_\odot$ while $M_{\text{max}} \approx 1$ otherwise.

Note an important feature of the stellar mass distribution is that in a group of stars which in total contains about 100-120 $M_\odot$, a star more massive than 8 $M_\odot$, a star more massive than 8 $M_\odot$ is expected. We see that in our simulations only runs MHD10f05 and MHD10f01 have reached this value. Runs HYDROf05 and MHD10f05 are slightly below while run HYDROf01 is almost a factor 3 below. This may constitute a hint that magnetic field is playing a role regarding the building of the massive stars, essentially by reducing the cloud fragmentation.

5.2. The sink mass function

Figure 8 displays the sink mass function, ought to represent the initial mass function, for the six runs and 3 values of $M_{\text{tot}}$. *

5.2.1. Analytical model

Before presenting the stellar mass spectra induced from the simulations, we discuss an analytical model that will be useful to interpret the results. It is in essence the model proposed in Hennebelle & Chabrier (2008) in which the density PDF is the one appropriated to the gravitational collapse and stated by Eq. (4) as proposed in Lee & Hennebelle (2018a). For the sake of completeness, it is described in appendix B. Equations (B.6) and (B.7) are the final equations to be used. Let us remember that the model predicts two asymptotic behaviours. At small mass, when thermal and/or magnetic support dominates, $\Gamma \rightarrow 0$, while at larger mass, when turbulent support dominates, $\Gamma \rightarrow 3/4$. The transition between these two regimes occurs at scales or equival
alently masses (see Eq. [B.6]) for which thermal/magnetic and turbulent supports are comparables.

In order to be compared with the numerical simulations, one needs to specify the values of the sound speed \(c_s\), of the Alfvén speed, \(V_a\), and of the turbulent velocity dispersion, \(V_0\). All these values can be inferred from the results presented in § A. Another important point when comparing simulations with the analytical model is the normalisation. For this purpose, we write

\[
M_{\text{tot,}i} = \int_{M_{\text{min}}}^{M_{\text{max}}} MN(M) dM
\]

\[
= \int_{\log(M_{\text{max}})}^{\log(M_{\text{min}})} MN(M) \frac{M}{\log 10} d\log 10 M. \tag{6}
\]

Thus, \(N_0\) as defined by Eq. [B.7] is determined once \(M_{\text{tot,}i}\), \(M_{\text{min}}\) and \(M_{\text{max}}\) are specified. The various parameters are reported in Table A. Since \(c_s\), \(V_a\) and \(V_0\) are all evolving with time and positions, the reported values are global estimates.

We recall that the model is isothermal in nature. The sound speed may vary for instance over time but remains uniform within the whole cloud. This has an important consequence, which is that the model does not predict a minimum stellar mass. The peak of the distribution however is barely a powerlaw-like behaviour, whose index cannot be reliably determined due to the lack of statistics. A tentative \(M^{-1}\) distribution (dotted line) is represented for comparison. This value is similar to what previous authors have inferred from simulations (Bonnell et al. 2011b; Girichidis et al. 2011b; Ballesteros-Paredes et al. 2015; Lee & Hennebelle 2018a; Padoan et al. 2020). Overall we see that there is a good agreement between the analytical model (red dotted line), and the sink mass distribution, or \(M > 0.1 \, M_\odot\). We stress that the main effect of increasing radiative feedback is to broaden the distribution toward larger masses. From the analytical model, we see that this is compatible with this being a consequence of the mean cloud temperature increasing due to the radiative heating.

The peak of the distribution however is barely affected. This confirms, as claimed in Hennebelle et al. (2020a), that radiative feedback is not responsible of setting the peak of the IMF. In fact, at early time (total accreted mass of 50 \(M_\odot\)), the distribution is clearly peaked toward 0.1-0.2 \(M_\odot\) which is several time the mass of the first hydrostatic core. As time goes on, the mass of the most massive stars increases while the number of low mass objects remains constant or increases moderately. This is entirely compatible with the idea that the stars inherit a minimum mass reservoir equal to a few times the mass of the first hydrostatic core (Hennebelle et al. 2019; Colman & Teyssier 2020), which is mostly accreted. After this, the stars keep accreting from their mass reservoir which likely is set by gravo-turbulence (Padoan et al. 1997; Hennebelle & Chabrier 2008; Hopkins 2012). While this process should largely be deterministic in nature, it is also likely the case that stochastic processes modulate this accretion as well (Bonnell et al. 2001; Basu & Jones 2004; Basu et al. 2015).

5.2.2. The hydrodynamical runs

The two top panels of Fig. 8 show results for run HYDROf05 and HYDROf01. The mass spectra are similar to those obtained in Hennebelle et al. (2020a) with slightly different initial conditions and less spatial resolution. Essentially most of the sinks have their mass between few \(10^{-2}\) and \(1 \, M_\odot\). The distributions present a plateau that ranges between \(\approx 0.1\) and \(\approx 0.5 \, M_\odot\). A relatively sharp drop occurs around 0.1 \(M_\odot\) and we get a small number of objects at lower mass, particularly in run HYDROf05. At mass larger than \(\approx 0.5 \, M_\odot\), the distribution drops following a

5.2.3. The influence of magnetic field on the stellar mass spectrum

The influence of magnetic field can be seen by comparing on one hand runs HYDROf05, MHD100f05 and MHD10f05 and on the other hand run HYDROf01 with run MHD10f01. Clearly, magnetic field has a significant impact on the mass spectrum, that it tends to broaden towards larger masses. In fact, the low mass distribution is almost unchanged. Again this provides further confirmation that radiative feedback has no significant impact on the low mass end of the stellar initial mass function since as discussed above magnetic field leads to stronger radiative feedback. This also obviously shows that magnetic field does not influence
the low mass end of mass spectrum in good agreement with the idea that it is mainly linked to the mass of the hydrostatic core.

Run MHD10f05 presents a plateau that extends from about \(0.1 M_\odot\) to \(\approx 2 M_\odot\). It is reminiscent of run A presented in Fig 6 of Lee & Hennebelle (2018a) and in the run presented in Fig 2 bottom panel of Jones & Bate (2018). These runs have in common to have a high thermal energy initially, or equivalently a low Mach number. The analytical model suggests that when thermal support is high, a collapsing clump would indeed develop a stellar mass spectrum \(dN/d \log M \propto M^0\), while when turbulent support dominates the support of the mass reservoir, \(dN/d \log M \propto M^{-3/4}\) is expected. Likely enough run MHD10f05 falls in the regime where thermal and magnetic field dominates over turbulence at the scale of the mass reservoirs and this explains the flat mass spectrum. This is indeed what the good agreement with the MHD models and the simulations suggests since the broad plateau (where \(dN/d \log M \propto M^0\)) displayed by the analytical models is due to combination of a high Alfvén velocity and a high sound speed.

Compared to run MHD10f05, the mass spectrum of run MHD10f01 presents a plateau that is less broad. This is the case both for the numerical and the analytical models, which are again in good agreement. This clearly is due to the lower temperatures in run MHD10f01, which compared to run MHD10f05, leads to weaker thermal support.

5.2.4. The impact of ambipolar diffusion

The mass spectrum of run NMHD10f05 presents similarities with the one of run MHD10f05 but also significant differences. Overall it is more similar to the mass spectrum of run MHD10f01.
First of all, unlike run MHD10f05, it does not present a plateau that extends up to \( \approx 3 \, M_\odot \) but rather stops at \( 1 \, M_\odot \) and the most massive stars are also less massive. This is in good agreement with the slightly lower magnetic field which is found for run NMHDf05 (see Fig. 6) than for run MHD10f05. The similarity with run MHD10f01 likely comes from the total support due to both thermal and magnetic supports are closer because run MHD10f01 has stronger field but lower temperatures than run NMHDf05.

A more surprising difference comes from the low mass objects. As can be seen there are more sink particles of masses lower than \( 0.1 \, M_\odot \) in run NMHDf05 than in the ideal MHD runs but also more than in the hydrodynamical runs. The reason for this remains to be clarified. The most likely explanation is the relatively weak magnetic field intensity at density above \( 10^{10} \, \text{cm}^{-3} \) in run NMHDf05 compared for instance to run MHD10f05. As seen from Fig. 6, the change of behaviour is relatively sharp, with \( B \) being very comparable in runs NMHDf05 and MHD10f05 below \( 10^{10} \, \text{cm}^{-3} \). Thus while in both runs, high densities may develop due to field support, the field support drops at density above \( 10^{10} \, \text{cm}^{-3} \) for run NMHDf05 and this may favor fragmentation. This may also be due to the difference in the disk populations that form in the various runs and presented in Lebreuilly et al. (2021). The disks formed in non-ideal MHD runs are intermediate in mass and size between the hydrodynamical disks and the the ones which form in ideal MHD runs. While the latter are usually very stable due to the fast growth of a toroidal magnetic component, the former fragments but since more mass is available in bigger disks, they tend to form bigger objects than in non ideal MHD disks.

6. Discussions

6.1. Dependence of the high-mass slope of the stellar mass spectrum

As discussed in the previous section, our numerical results suggest that from a few solar mass to at least \( 7-8 \, M_\odot \), the stellar distribution presents a power law behaviour \( dN/d\log M \propto M^{-\Gamma} \), with \( \Gamma \approx 3/4 \). Analytically, this behaviour is found when at the scale of the individual mass reservoir, \( i) \) the dominant support against gravity is turbulence and \( ii) \) when the density PDF is \( \propto \rho^{-3/2} \), which is a consequence of gravitational collapse. On the other-hand, when the density PDF is close to a lognormal distribution, we do expect \( \Gamma \approx 1.3 \) as discussed in Lee & Hennebelle (2018a). In essence, the density PDF is a direct estimate of how the gas mass is distributed amongst densities and therefore controls the number of density fluctuations at a given density. Typically a log-normal distribution has less dense gas than a PDF \( \propto \rho^{-3/2} \) and therefore less small mass objects are produced with the former than with the latter. The transition between the two exponents, \( \Gamma = 3/4 \) and \( \Gamma = 1.3 \), is expected to occur at the density, \( \rho_{\text{trans}} \), which typically connects the turbulent log-normal PDF to the power law \( \rho^{-3/2} \) gravitational PDF. In the present simulations this occurs around \( \approx 10^6 \, \text{cm}^{-3} \). Combining Eqs. (B.3) and (B.6) we can estimate the mass, \( M_{\text{trans}} \), it corresponds to

\[
M_{\text{trans}} \approx 15 \, M_0 \left( \frac{V_0}{3 \, \text{km s}^{-1}} \right)^2 \left( \frac{R_c}{0.3 \, \text{pc}} \right)^3 \left( \frac{\rho_{\text{trans}}}{10^6 \, \text{cm}^{-3}} \right)^{-2}.
\]

(7)

where for simplicity, we have assumed \( \eta = 0.5 \) and \( V_0 \approx GM/R_c \). Because of the sixth power which appears for \( V_0 \) or \( R_c \), the clump radius, the value of \( M_{\text{trans}} \) is clearly not accurate and likely can abruptly change from one environment to another. Typically we expect a fast transition around \( R_c \approx 0.3 \, \text{pc} \). It is however illustrative and shows that for our simulations, at high mass, the mass spectra are expected to be mostly if not exclusively described by the \( \Gamma = 3/4 \) exponent since our stellar masses are smaller than \( 15 \, M_\odot \). It also shows that in a less dense and compact clump, the transition should occur at smaller masses since the value of \( V_0 \), or equivalently the value of \( R_c \), should be smaller. While most of the studies which have started from massive clumps, comparable to the ones studied here, tend to present \( \Gamma \) lower than the canonical Salpeter exponent (see the discussion in Lee & Hennebelle (2018a)), which works in which the IMF is obtained from larger scale clouds studies which have attempted to obtain the IMF in larger scale simulations with initial conditions that correspond to more standard giant molecular clouds, generally report \( \Gamma \) values that are closer to 1.3. This is the case for instance for the run XL-F presented in Fig. 4 of He et al. (2019) and the run presented in Fig. 3 of Padoan et al. (2020) for masses between 10 and 50 \( M_\odot \), respectively. This is also the case for the runs presented in Ntormousi & Hennebelle (2019) and the core mass function extracted from these simulations (Louvet et al. 2021).

6.2. Observationally inferred mass distribution in actively star forming regions

While it may sound at first surprising not to find \( \Gamma \approx 1.3 \), which is the slope inferred by Salpeter (1955), it should be stressed that recent observations have been inferring that in some actively star forming regions, the IMF may indeed be top-heavy (Zhang et al. 2018, Lee et al. 2020). More precisely, in the Arches cluster Hosek et al. (2019) inferred \( \Gamma \approx 0.8 \). On the other-hand, recent studies of the core mass function also obtained within massive star forming regions, have also inferred power law behaviours with indices \( \Gamma \approx 0.95 \) (Motte et al. 2018, Pouteau et al. 2022).

| Name       | \( R_c \) (pc) | \( c_s \) (km s\(^{-1}\)) | \( V_a \) (km s\(^{-1}\)) | \( V_0 \) (km s\(^{-1}\)) | \( M_{\text{tot}} \) (\( M_\odot \)) | \( M_{\text{max}} \) (\( M_\odot \)) | \( M_{\text{min}} \) (\( M_\odot \)) |
|------------|---------------|-----------------|-----------------|-----------------|--------------------------------|--------------------------------|--------------------------------|
| NMHD10f05  | 0.3           | 0.35            | 1               | 3               | 100                           | 3                              | 0.1                           |
| IMHD10f05  | 0.3           | 0.35            | 1               | 3               | 150                           | 8                              | 0.1                           |
| IMHD10f01  | 0.3           | 0.25            | 1               | 3               | 150                           | 7                              | 0.1                           |
| IMHD100f05 | 0.3           | 0.35            | 0.3             | 3               | 150                           | 7                              | 0.1                           |
| HYDROf05   | 0.3           | 0.35            | 0               | 3               | 150                           | 7                              | 0.1                           |
| HYDROf01   | 0.3           | 0.25            | 0               | 3               | 150                           | 3                              | 0.1                           |

Table 2. Parameters used to confront the stellar initial mass function inferred from the simulations with the analytical model stated in § B. \( R_c \) is the clump radius, \( c_s \) is the typical sound speed, \( V_a \) the Alfvén speed, \( V_0 \) the velocity dispersion, \( M_{\text{tot}} \) is the total accreted mass, \( M_{\text{max}} \) the mass of the most massive stars formed and \( M_{\text{min}} \) the smallest mass for which the comparison is meaningful.
As cores are widely assumed to be the progenitors of stars out of which they build their mass, the inferred $\Gamma$ are compatible with the idea that the shape of the IMF in massive star forming regions is inherited from the shape of the CMF, at least at high masses, although eventually it should be compared with the IMF of the very same region.

While more detailed investigations, including careful comparisons between simulations and observations must be carried out before firm conclusion can be drawn, there is a clear suggestion coming from both observations and theories that systematic variations of the IMF may occur, particularly in very compact star forming regions.

6.3. Limits of the present work and the universality of the IMF

Our work presents several important limits that need to be discussed. Indeed, one of the conclusion is that the combination of magnetic field and radiative transfer possibly leads to more variability that what observational inferences of the IMF may have led to conclude. Admittedly, this question even for our own Galaxy remains difficult to address, particularly because of the relatively limited samples that are often available but it seems nevertheless unavoidable that at least some level of fluctuations should be present (see for instance the comprehensive discussion provided in [Dib2022]).

Determining whether the variations observed in the present work are compatible with the galactic fluctuations of the IMF, is beyond the scope of the present paper but it is worth to remind that an important source of variations is due to the efficiency of the accretion luminosity expressed by the parameter, $\Gamma_{acc}$. Whereas there may be some variability of $\Gamma_{acc}$, likely enough it is not a factor of 5 as we have been exploring here. The other possibly extreme variations we have considered is magnetic intensity since we have explored a factor of 10 (and even go to pure hydrodynamical cases). This is not well constraint yet but a 1000 M$_\odot$ clump is a relatively large ensemble and it is unclear what are the variations of the magnetisation in the galactic populations.

Finally, we stress that in this work a possibly important process has been omitted, namely the protostellar jets. Recently, Guszejnov et al. [2021] have been exploring their impact in simulations comparable to the ones presented here (with a resolution of few tens of AU). They concluded that protostellar jets may be playing a significant role in setting the IMF in particular for the formation of low mass objects in the presence of a significant initial magnetic field. Whether this process may help explaining the universality of the IMF is however not clear yet.

7. Conclusions

With the goal of understanding how magnetic field and radiative feedback influence the collapse and the fragmentation of a massive star forming clump, we have performed high resolution adaptive mesh calculations with a spatial resolution down to about 1 AU. Six runs in which 2 radiative feedback efficiencies, 3 magnetic intensities as well as the impact of non-ideal MHD are explored. We show that the physical characteristics of the simulated star forming clumps compare well with various observations. This is for instance the case for the observational bolometric luminosities that we compared with the total luminosities of the sink particles produced in the simulations as well as for the gas temperatures. For the latter, we develop an analytical model which agrees well with the temperatures inferred from the simulations.

The stellar mass spectra of the six runs are analysed in detail and compared with an analytical model in which thermal, magnetic and turbulent supports are playing a major role. Overall the analytical model reproduces well the numerical mass spectra for masses above $\sim 0.1$ M$_\odot$. At this mass which corresponds to a few times the mass of the first hydrostatic core the underlying gas thermodynamics is nearly adiabatic and specific models should be considered (e.g. [Hennebelle et al. 2019]). The combination between simulations and analytical results allows us to clearly assess the role and influence of each physical process which are as follows:

- in the density range at which the gas is not adiabatic, the density PDF which is $\sim -3/2$ is deeply shaping the stellar mass spectrum and leads to two physical distinct regimes for the mass spectra.
- at masses larger than $\sim 0.1$ M$_\odot$, thermal pressure and magnetic field may lead to a flat mass spectrum, i.e. $dN/d\log M \propto M^{-3}$ with $\Gamma = 0$ if they are strong enough compared to turbulence.
- at larger scales, turbulence dominates and may lead to a mass spectrum with $\Gamma = 3/4$. At even larger scales and lower density, the PDF is expected to be log-normal in shape and stiffer mass spectra, with larger $\Gamma$ are expected.
- the transition between the regime with $\Gamma \approx 0$ and $\Gamma \approx 3/4$ is not universal and depends on the local physical processes such as thermal support, magnetic field and Mach number.

Generally speaking, we find that the main effect of magnetic field and radiative transfer is to reduce the total number of fragments and to increase the mass of the most massive stars. These latter have been found to increase with the magnetic intensity and the radiation feedback efficiency. For instance, in the present work we found that for the hydrodynamical simulation with the lowest efficiency, the most massive star produced after 150 M$_\odot$ have been accreted, is about 3 M$_\odot$. With a higher radiative feedback efficiency or a sufficiently strong initial field, stars of masses 7-8 M$_\odot$ are produced. We therefore conclude that whereas magnetic field and radiative feedback may not be essential to explain the peak or the various slope values of the IMF, they may be essential to reproduce the exact shape (like the transition between the various regimes), the level of fragmentation i.e. the number of stars formed, and the mass of the most massive stars.

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Appendix A: A simple analytical model for the temperature

To better understand the temperature profiles through the clump, we make use of the simple model discussed in Hennebelle et al. (2020a), improving on various aspects. The model assumes that the cloud is spherically symmetric and that all sources are located in the clump center. As seen in §4.1, the radial profile of the density field in collapsing envelopes is given by

\[ \rho(r) = \frac{\delta_c c_s^2}{2\pi Gr^2}, \quad (A.1) \]

where \( \delta_c \) is a dimensionless factor which typically is equal to \( \approx 30-100 \). We assume that gas and dust have the same temperature and are stationary. The grey body approximation that is being used, leads when the medium is optically thick to

\[ -4\pi r^2 \frac{c}{3K(r)} \delta_c (\alpha r^4) = \sum (L_s + L_{acc}), \quad (A.2) \]

where \( \alpha \) is the radial density constant. In this expression we assume that all the emitting sources are located in the clump center. The opacity temperature dependencies (e.g. Semenov et al. 2003), suggest that we can distinguish two regimes of temperature,

\[ \kappa(T) = 0.5 \, \text{cm}^2 \, \text{g}^{-1} \quad \text{for} \quad T > T_{\text{crit}} = 100 \, \text{K}, \quad (A.3) \]

\[ \kappa(T) \approx \kappa_0 \left( \frac{T}{T_{\text{crit}}} \right)^{\alpha} \quad \text{for} \quad T < T_{\text{crit}}, \]

where \( \alpha \) is typically between 1 and 2. In this work we adopted \( \alpha = 1.5 \). Combining Eqs. (A.1), (A.3) and (A.2), we get

\[ T(r) = \left( \frac{T_{\text{crit}}^4 - \frac{4}{5} \left( \frac{1}{r_{\text{crit}}} - \frac{1}{r_{\text{ext}}} \right) \right)^{1/4} \quad \text{for} \quad T > T_{\text{crit}}, \]

\[ T(r) = \left( \frac{KT_{\text{crit}}^4 - \frac{4}{5} \left( \frac{1}{r_{\text{crit}}} - \frac{1}{r_{\text{ext}}} \right) \right)^{1/4} \quad \text{for} \quad T < T_{\text{crit}} \quad \text{and} \quad r < r_{\text{ext}}, \quad (A.4) \]

\[ K = \frac{f_{\text{acc}} \delta_c c_s^2}{24 \pi a^2 G} \sum (L_s + L_{\text{acc}}), \quad (A.5) \]

\[ r_{\text{crit}}^3 = \frac{K^4}{4} \left( \frac{T_{\text{crit}}^4 - \frac{4}{5} \left( \frac{1}{r_{\text{crit}}} - \frac{1}{r_{\text{ext}}} \right) \right)^{4/3}, \quad (A.6) \]

Finally, \( T_{\text{crit}} \) is the temperature where the optical depth is about 1 and \( r_{\text{crit}} \) the corresponding radii. We therefore have

\[ \kappa(T_{\text{crit}}) \rho(T_{\text{crit}}) r_{\text{crit}}^2 = \kappa_0, \quad \text{where} \quad \kappa_0 \text{should be on the order of 1}. \]

Combining Eqs. (A.1), (A.3), (A.4), we obtain for \( r_{\text{crit}} \)

\[ r_{\text{crit}} = \left( \frac{\kappa_0 \delta_c c_s^2}{2\pi \rho(T_{\text{crit}})} \right)^{3/(4-\alpha)} \cdot \left( \frac{K}{4 \pi G} \right)^{\alpha/(4\alpha(2\alpha))}, \quad (A.7) \]

At this point, the radiative flux becomes simply equal to the term \( cE_R \) and Eq. (A.2) becomes invalid. Under the assumption that the temperature remains the one of a blackbody, it then remains constant at larger radii and thus

\[ T(r) = T_{\text{crit}} = \left( KT_{\text{crit}}^4 - \frac{4}{5} \left( \frac{1}{r_{\text{crit}}} - \frac{1}{r_{\text{ext}}} \right) \right)^{1/4} \quad \text{for} \quad r > r_{\text{ext}}, \quad (A.8) \]

The expression for \( T_{\text{crit}} \) is obtained by continuity at \( r_{\text{crit}} \). By combining Eq. (A.8) and Eq. (A.7), we find that

\[ T_{\text{crit}} = \pi(1 - \frac{\alpha}{4} L_{\text{acc}} \frac{2\pi}{ac} \kappa_0 \delta_c c_s^4)^{1/(4\alpha(2\alpha))} \quad (A.9) \]

Appendix B: Analytical model of the mass spectrum

For completeness, we describe here the analytical model developed in Hennebelle & Chabrier (2008) and Lee & Hennebelle (2018a) that we use in the paper to interpret the numerical results.

It is based on the equality of mass of the density fluctuation which are unstable at scale \( R \) (left-hand term) and the mass that ends up into the structures, i.e. the stars.

\[ \frac{M_{\text{ext}}(R)}{V_c} = \int_0^\infty \exp(\delta) \mathcal{P}_R(\delta) d\delta = \int_0^M M' \mathcal{P}(R,M') dM', \quad (B.1) \]

where \( \delta = \ln(\rho/\bar{\rho}) \), \( \mathcal{P}_R \) is the density PDF, \( \mathcal{P}(R,M) \) is the probability of finding a self-gravitating clump of mass \( M' \) embedded into a self-gravitating clump of mass \( M_R \) unstable at scale \( R \). It is assumed to be 1.

Taking the derivative with respect to \( R \), we get

\[ \mathcal{P}_R(\rho) = \mathcal{P}_0 \left( \frac{\rho}{\bar{\rho}} \right)^{-1.5}, \quad (B.2) \]

The mass of the density fluctuations is given by

\[ M_{\text{ext}}(R) = \frac{\bar{\rho}}{M_R} \frac{dR}{dM} \left( -\frac{d\delta}{dR} \right) \exp(\delta) \mathcal{P}(\delta_R), \quad (B.3) \]

Here we assume that the density PDF is given by

\[ \mathcal{P}(\rho) = \mathcal{P}_0 \left( \frac{\rho}{\bar{\rho}} \right)^{-1.5}, \quad (B.4) \]

The gravitational instability criterion for a clump of mass \( M \) at scale \( R \) is

\[ M > M_J = \frac{a_j}{\sqrt{G \rho \exp(\delta)}} \left[ c_s^2 + \frac{v^2}{6} + \frac{V^2}{3} \right] \left( \frac{R}{R_J} \right)^{2a_j - 3}, \quad (B.5) \]

where \( c_s \) is the sound speed, \( V_c \) the Alfvén speed, \( v \) the rms velocity dispersion at the cloud scale, \( R_c \) the cloud radius and \( \eta \) an exponent to describe the turbulent scale dependence. Typically \( \eta = 0.3 - 0.5 \) and in this work the value \( \eta = 0.5 \) is assumed for simplicity. Equation (B.5) is the standard Jeans mass expression in which the support is assumed to be as suggested by the virial theorem. Note that the surface terms are not taken into account, they would typically modify this expression by a factor of 2. Taking the standard definition of the Jeans mass, the mass enclosed in a sphere of diameter equal to the Jeans length, we get \( a_j = 3/2/6 \). With Eq. (B.3), this implies

\[ M_{\text{ext}}(R) = \frac{a_j}{G} \left( \frac{c_s^2}{6} + \frac{v^2}{6} + \frac{V^2}{3} \right) \left( \frac{R}{R_J} \right) \left( \frac{M}{M_R} \right), \quad (B.6) \]

where \( M_R \) is the critical mass at scale \( R \).

With Eq. (B.4), Eq. (B.2) leads to

\[ \mathcal{N}(M_{\text{ext}}(R)) = N_0 \left( \frac{R}{M_R} \right)^{3/2} \frac{dR}{dM} \left( -\frac{1}{M_R} \frac{dM_{\text{ext}}}{dR} + \frac{3}{R} \right), \quad (B.7) \]

Knowing the cloud physical conditions, \( c_s \), \( V_c \), \( V_0 \), together with Eq. (B.6), Eq. (B.7) allow to predict the stellar mass spectrum. The normalisation coefficient \( N_0 \) is determined by specifying the total mass within stars.
It is useful to see that
\[ M \to 0 \Leftrightarrow N \to M^{-1} \Leftrightarrow \frac{dN}{d \log M} \to M^0. \tag{B.8} \]

In this limit, the mass reservoir is thermally supported and the mass spectra present a plateau, i.e. \( \frac{dN}{d \log M} \propto M^0 \).

On the other hand, in the limit
\[ M \to \infty \Leftrightarrow N \to M^{3/(4\eta+2)-5/2} \Leftrightarrow \frac{dN}{d \log M} \to M^{3/(4\eta+2)-3/2}, \tag{B.9} \]

As revealed by Eq. (B.6), in this limit the mass reservoir is dominated by the turbulent dispersion. For \( \eta = 0.5 \), the mass spectrum is \( \frac{dN}{d \log M} \propto M^{-3/4} \).

We recall that at small masses, the asymptotic behaviour will eventually break down when the gas becomes adiabatic due to the dust opacity and the formation of the first hydrostatic core, while at large masses, the assumption of the density PDF being \( \propto \rho^{-3/2} \), is eventually invalid (typically it eventually turns into a log-normal distribution). Therefore while useful, these asymptotic behaviours must be handled with care.

Appendix C: Accretion and stellar luminosities

To get a better understanding of the origins of the luminosities, we investigate the stellar and accretion luminosities separately. The two panels of Fig. C.1 show the sum of the stellar luminosities (left panel) and the sum of the accretion luminosities (right panel). In a first phase, up to time \( \approx 0.1 \) Myr, the accretion luminosity largely dominates. Then as stars of few solar masses have formed, the stellar luminosities increase steeply and then reach values comparable to the accretion luminosities.
Fig. C.1. This figure complements Fig. [1] Left panel displays the total stellar luminosities and right one the total accretion luminosity.