Corrections to Tribimaximal Mixing from Nondegenerate Phases

Y.F. Li\textsuperscript{a,b,*} and Q.Y. Liu\textsuperscript{a†}

\textsuperscript{a}Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China.
\textsuperscript{b}INFN, Sezione di Torino, Via P. Giuria 1, I-10125 Torino, Italy

Abstract

We propose a seesaw scenario that possible corrections to the tribimaximal pattern of lepton mixing are due to the small phase splitting of the right-handed neutrino mass matrix. We show that the small deviations can be expressed analytically in terms of two splitting parameters ($\delta_1$ and $\delta_2$) in the leading order. The solar mixing angle $\theta_{12}$ favors a relatively smaller value compared to zero order value (35.3°), and the Dirac type CP phase $\delta$ chooses a nearly maximal one. The two Majorana type CP phases $\rho$ and $\sigma$ turn out to be a nearly linear dependence. Also a normal hierarchy neutrino mass spectrum is favored due to the stability of perturbation calculations.

PACS numbers: 14.60.Pq, 14.60.St, 11.30Hv
Keywords: neutrino mass and mixing; tribimaximal; CP violation

\textsuperscript{*}E-mail: lyfeng@mail.ustc.edu.cn
\textsuperscript{†}E-mail: qiuyu@ustc.edu.cn
I. INTRODUCTION

Due to the outstanding achievements in the neutrino oscillation experiments [1–18], we have convincing evidences on neutrino mass and lepton mixing. In the weak basis that the charged-lepton mass matrix is diagonal, real and positive, we assume massive neutrinos to be Majorana particles and parametrize [19,20] the lepton mixing $V$ as

$$
V = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} e^{-i\delta} & s_{13} e^{i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13} e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13} e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13} e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13} e^{i\delta} & c_{23}c_{13}
\end{pmatrix}
\begin{pmatrix}
e^{i\rho} & 0 & 0 \\
0 & e^{i\sigma} & 0 \\
0 & 0 & 1
\end{pmatrix}.
$$

(1.1)

Now the global fit [20–22] of current experimental data yields $\sin^2 \theta_{12} \sim 0.304^{+0.022}_{-0.016}$, $\sin^2 \theta_{23} \sim 0.50^{+0.07}_{-0.06}$, $\sin^2 \theta_{13} \leq 0.016$ at 1 $\sigma$. So the so-called tribimaximal mixing [23–26] pattern is an excellent approximation for these physical values, which appears as the form of

$$
V_0 = \begin{pmatrix}
2/\sqrt{6} & 1/\sqrt{3} & 0 \\
-1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\
1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2}
\end{pmatrix}.
$$

(1.2)

It corresponds to $\sin^2 \theta_{12} = 1/3$, $\sin^2 \theta_{23} = 1/2$ and $\sin^2 \theta_{13} = 0$ in standard parametrization. This simple form with small integers motivates neutrino theorists to consider some underlying structures. The most intriguing one is flavor symmetry among generations, especially for discrete groups [27,28] such as $A_4$, $S_3$, $Z_2$, which can give predictable values or/and relations of mixing parameters. In general, nearly tribimaximal mixing is also allowed, so small corrections to the standard form in (1.2) are interesting and necessary. Many sources can give these corrections such as charged lepton sector contributions [29–31] and renormalization group effects [32–34].

On the other way, seesaw mechanism [35–39] can naturally explain the smallness of three left-handed neutrino masses. In the simplest type I framework, the effective Majorana mass matrix of neutrinos $M_\nu$ is related to the Dirac mass matrix $M_D$ and the heavy right-handed Majorana mass matrix $M_R$ by the relation

$$
M_\nu \simeq M_D M_R^{-1} M_D^T.
$$

(1.3)

To explore the structure of the seesaw formula, the effective mixing can generally be derived from both $M_D$ and $M_R$. One of them can be chosen to be diagonal or even identity matrix...
for simplifications. One kind of models in ref. [40–42] takes a diagonal $M_D$ for instance. Meanwhile, another kind of seesaw models [43,44] starts from a unit form of $M_R$.

Now for the small corrections of tribimaximal mixing, another source [43,44] in realistic seesaw models attracts our attentions. The leading order mixing matrix is totally derived from the Dirac mass matrix $M_D$, and $M_R$ has the simplest form of $M_0 I$ in the symmetry limit ($M_0$ being a common mass scale and $I$ the identity matrix). The departure of $M_R$ from the unit matrix gives small corrections to the effective mixing matrix. There are two methods in $M_R$ to break the unit form, one is the nondegenerate masses, another is the undegenerate phases: the phase breaking. In this letter, we want to discuss the second case, we consider small phase splitting for complex matrix $M_R$, and maintain the degeneracy of the right-handed neutrino masses. We find that besides small corrections of the three mixing angles, we get nontrivial CP violation phases.

The remaining part of this paper is organized as follows. In section II, we talk about the realistic models from phase breaking, giving the predicted values of mixing parameters. In section III, we do some numerical analysis, displaying the correlations of these parameters. We conclude our topic and give some remarks in section IV. In the Appendix, we give our calculations from perturbation approximations.

II. REALISTIC MODELS

In our model, we adopt the basis started in [43,44]. we think $M_D$ can be diagonalized by tribimaximal mixing matrix in the manner of $V_0^+ M_D V_0^* = \text{diag}\{x, y, z\}$ which is constrained by a discrete flavor symmetry(such as $S_3$ symmetry). ($x, y, z$) are positive mass parameters. For $M_R$, we take the form of $M_0 I$ in the symmetry limit, obviously it is invariant under the symmetry. Then we have

$$M_\nu = M_D M_R^{-1} M_D^T = V_0 \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} V_0^T \frac{1}{M_0} V_0 \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} V_0^T$$

$$= V_0 \frac{1}{M_0} \begin{pmatrix} x^2 & 0 & 0 \\ 0 & y^2 & 0 \\ 0 & 0 & z^2 \end{pmatrix} V_0^T \equiv V_0 M_\nu^{(0)} V_0^T. \quad (2.1)$$

In the following, we want to give small phase splitting to $M_R$, and derive the light neutrino mass spectra and small corrections for lepton mixing parameters compared with the leading
values of $V_0$ including CP violating phases.

A. Scenario A

In this scenario, we give $M_{R1,1}$ a small phase compared to the others, then $M_R$ take the form of

$$M_R = M_0 \begin{pmatrix} e^{-i\delta_1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$  \hspace{1cm} (2.2)

Similar to (2.1), the effective neutrino mass matrix $M_\nu$ turns out to be

$$M_\nu = M_D M_R^{-1} M_D^T V_0 \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} \frac{1}{M_0} \begin{pmatrix} e^{i\delta_1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_0 \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} V_0^T = V_0 \frac{z^2}{3M_0} \begin{pmatrix} (3 + 2\epsilon_1)^2 \eta^2 & \sqrt{2} \omega \eta^2 & 0 \\ \sqrt{2} \omega \eta^2 & (3 + \epsilon_1)^2 \eta^2 & 0 \\ 0 & 0 & 3 \end{pmatrix} V_0^T \equiv V_0 M_\nu^{(1)} V_0^T, \hspace{1cm} (2.3)$$

with $\omega \equiv x/y$, $\eta \equiv y/z$ and $\epsilon_1 \equiv e^{i\delta_1} - 1$. The corrections to $V_0$ come from the diagonalization of matrix $M_\nu^{(1)}$. Assuming $M_\nu^{(1)} \equiv V_1 M_\nu V_1^T$, where $M_\nu = \text{Diag}(m_1, m_2, m_3)$ with $m_i(i = 1, 2, 3)$ being the Majorana neutrino masses. When we parameterize $V_1$ as

$$V_1 = \begin{pmatrix} \cos \alpha & \sin \alpha e^{-i\beta} & 0 \\ -\sin \alpha e^{i\beta} & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\sigma_1} & 0 & 0 \\ 0 & e^{i\sigma_2} & 0 \\ 0 & 0 & e^{i\sigma_3} \end{pmatrix}, \hspace{1cm} (2.4)$$

we can derive these mixing parameters and masses by the relation of $M_\nu = V_1^+ M_\nu^{(1)} V_1^*$ in terms of $\epsilon_1$, $\omega$, $\eta$, $z$ and $M_0$. In our calculation, we view $\epsilon_1$ (or equivalently $\delta_1$) as a small parameter, and give results with the leading terms of the power series of $\delta_1$, such as

$$\epsilon_1 \equiv e^{i\delta_1} - 1 \simeq -\delta_1^2/2 + i\delta_1.$$  

By the perturbation calculations, we derive the relation of $\cos \beta \simeq 1/6 \sin \beta \delta_1$, immediately we have $\cot \beta \simeq \delta_1/6$. Other parameters are listed below

$$\tan 2\alpha \simeq -\frac{2\sqrt{2}}{3} \frac{\omega}{1 - \omega^2} \delta_1, \hspace{1cm} (2.5)$$

$$m_1 \simeq \frac{z^2 \omega^2 \eta^2}{M_0} \{1 - \frac{1}{9} \delta_1^2 + \frac{2}{9} \frac{1}{1 + \omega^2} \delta_1^2\} \simeq \frac{z^2 \omega^2 \eta^2}{M_0} (1 + \frac{1}{9} \delta_1^2),$$

$$m_2 \simeq \frac{z^2 \eta^2}{M_0} \{1 - \frac{1}{9} \delta_1^2 + \frac{2}{9} \frac{\omega^2}{1 + \omega^2} \delta_1^2\} \simeq \frac{z^2 \eta^2}{M_0} (1 - \frac{1}{9} \delta_1^2), \hspace{1cm} (2.6)$$
\[ m_3 = \frac{z^2}{M_0}, \tan 2\sigma_1 \simeq \frac{2}{3}\delta_1, \tan 2\sigma_2 \simeq \frac{1}{3}\delta_1, \tan 2\sigma_3 = 0. \] (2.7)

For the second expressions of \( m_1 \) and \( m_2 \), we omit the contributions of higher powers of \( \omega \). Combining \( V_1 \) together with \( V_0 \) and making a rephasing transformation, we can get the standard expression of neutrino mixing matrix \( V \) as eq.(1.1)

\[
V \equiv V_0 V_1 = \begin{pmatrix}
\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix}
\begin{pmatrix}
\cos \alpha & \sin \alpha e^{-i\beta} & 0 \\
-\sin \alpha e^{i\beta} & \cos \alpha & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
e^{i\sigma_1} & 0 & 0 \\
0 & e^{i\sigma_2} & 0 \\
0 & 0 & e^{i\sigma_3}
\end{pmatrix}
\equiv \begin{pmatrix}
e^{i\rho_1} & 0 & 0 \\
0 & e^{i\rho_2} & 0 \\
0 & 0 & e^{i\rho_3}
\end{pmatrix}
\begin{pmatrix}
\cos \theta_{12} & \sin \theta_{12} & 0 \\
-\frac{1}{\sqrt{2}} \sin \theta_{12} & \frac{1}{\sqrt{2}} \cos \theta_{12} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} \sin \theta_{12} & -\frac{1}{\sqrt{2}} \cos \theta_{12} & \frac{1}{\sqrt{2}}
\end{pmatrix}
\begin{pmatrix}
e^{i\rho} & 0 & 0 \\
0 & e^{i\sigma} & 0 \\
0 & 0 & 1
\end{pmatrix}, \tag{2.8}
\]

where \( \rho_i (i = 1, 2, 3) \) can be rotated away by redefining the phases of three charged-lepton fields. Then the standard parameters appear as

\[
\sin \theta_{12} \simeq \frac{1}{\sqrt{3}} \left( 1 - \frac{\omega}{1 + \omega^2} \right) \left( 1 - \frac{\omega}{1 + \omega^2} \right) \frac{1}{9} \delta_1^2 \right), \tag{2.9}
\]

and two standard Majorana phases are

\[
\tan 2\rho \simeq \frac{2}{3} \left( 1 - \frac{2\omega}{1 + \omega^2} \right) \delta_1, \\
\tan 2\sigma \simeq \frac{1}{3} \left( 1 - \frac{2\omega}{1 + \omega^2} \right) \delta_1. \tag{2.10}
\]

In this scenario, we get a small deviation from tribimaximal mixing for \( \theta_{12} \), and two Majorana phases, but \( \theta_{13} \) and \( \theta_{23} \) remain unchanged. Because of the vanishing value of \( \theta_{13} \), we cannot obtain the information of Dirac CP phase. Contrarily if we give \( M_{R2,2} \) another small phase compared to \( M_{R3,3} \), we can obtain a non-vanishing \( \theta_{13} \) and then a non-trivial Dirac CP violation phase.

**B. Scenario B**

As discussed in the end of last section, we take a general form of \( M_R \) as

\[
M_R = M_0 \begin{pmatrix}
e^{-i\delta_1} & 0 & 0 \\
0 & e^{-i\delta_2} & 0 \\
0 & 0 & 1
\end{pmatrix}. \tag{2.11}
\]
Then we will repeat the same procedure as before. Firstly the effective neutrino mass matrix are

\[ M_\nu = M_D M^{-1}_R M_D^T V_0 \left( \begin{array}{ccc} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{array} \right) V_0^T \left( \begin{array}{ccc} \epsilon^{i\delta_1} & 0 & 0 \\ 0 & \epsilon^{i\delta_2} & 0 \\ 0 & 0 & 1 \end{array} \right) V_0 \left( \begin{array}{ccc} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{array} \right) V_0^T \]

\[ = V_0 \frac{z^2}{6M_0} \left( \begin{array}{ccc} (6 + 4\epsilon_1 + \epsilon_2)\omega^2\eta^2 & (2\epsilon_1 - \epsilon_2)\sqrt{2}\omega\eta^2 & -\sqrt{3}\epsilon_2\omega\eta \\ (2\epsilon_1 - \epsilon_2)\sqrt{2}\omega\eta^2 & (6 + 2\epsilon_1 + 2\epsilon_2)\eta^2 & \sqrt{6}\epsilon_2\eta \\ -\sqrt{3}\epsilon_2\omega\eta & \sqrt{6}\epsilon_2\eta & 6 + 3\epsilon_2 \end{array} \right) V_0^T \equiv V_0 M_\nu^{(2)} V_0^T . \quad (2.12) \]

Then we define \( M_\nu^{(2)} \equiv V_2 \bar{M}_\nu V_2^T \), with \( V_2 \) is an unitary matrix parameterized as

\[ V_2 = \left( \begin{array}{ccc} D_1 & \alpha_3 & \alpha_2 \\ -\alpha_3^* & D_2 & \alpha_1 \\ -\alpha_2^* & -\alpha_1^* & D_3 \end{array} \right) \left( \begin{array}{ccc} e^{i\sigma_1} & 0 & 0 \\ 0 & e^{i\sigma_2} & 0 \\ 0 & 0 & e^{i\sigma_3} \end{array} \right), \quad (2.13) \]

where \( \alpha_j \equiv \sin \theta_j e^{-i\phi_j} \) (for \( j = 1, 2, 3 \)), \( D_1 = \sqrt{1 - |\alpha_2|^2 - |\alpha_3|^2} \) and similar definitions for \( D_2 \) and \( D_3 \). By using \( \bar{M}_\nu = V_2^+ M_\nu^{(2)} V_* \), we arrive at the approximate expressions for \( \theta_i \) and \( \phi_i \) via perturbation calculations.(we will give a careful calculations of perturbation approximations in the Appendix.)

\[ \sin \theta_3 \sin \phi_3 \simeq -\frac{\sqrt{2}}{6} \omega(2\delta_1 - \delta_2), \sin \theta_2 \sin \phi_2 \simeq \frac{\sqrt{3}}{6} \omega \eta \delta_2, \sin \theta_1 \sin \phi_1 \simeq -\frac{\sqrt{6}}{6} \eta \delta_2; \quad (2.14) \]

and

\[ \sin \theta_3 \cos \phi_3 \simeq -\frac{\sqrt{2}}{18} \delta_1^2 - \delta_1 \delta_2 + \delta_2^2, \]

\[ \sin \theta_2 \cos \phi_2 \simeq \frac{\sqrt{3}}{18} \omega \eta \delta_2 (2\delta_1 - \delta_2), \]

\[ \sin \theta_1 \cos \phi_1 \simeq -\frac{\sqrt{6}}{36} \eta (\omega^2 - \eta^2) \delta_2 (2\delta_1 - \delta_2). \quad (2.15) \]

And the masses are

\[ m_1 \simeq \frac{z^2 \omega^2 \eta^2}{M_0} \left( 1 + \frac{1}{9} \delta_1^2 - \frac{1}{3} \delta_1 \delta_2 + \frac{5}{12} \delta_2^2 \right), \]

\[ m_2 \simeq \frac{z^2 \eta^2}{M_0} \left( 1 - \frac{1}{9} \delta_1^2 + \frac{1}{3} \delta_1 \delta_2 + \frac{1}{18} \delta_2^2 \right), \]

\[ m_3 \simeq \frac{z^2}{M_0} \left( 1 - \frac{1}{8} \delta_2^2 \right). \quad (2.16) \]

Finally we give the phase-angles \( \sigma_i \) in eq.(2.13)

\[ \tan 2\sigma_1 \simeq \frac{1}{6} (4\delta_1 + \delta_2), \tan 2\sigma_2 \simeq \frac{1}{3} (\delta_1 + \delta_2), \tan 2\sigma_3 \simeq \frac{1}{2} \delta_2. \quad (2.17) \]
A normal hierarchy mass spectrum (small values of $\omega$ and $\eta$) is favored in this scenario due to the stability of perturbation calculation (see the Appendix). So for the analytical expressions in (2.14) (2.15) and (2.16), we express them with the leading terms of the power series of $\omega$ and $\eta$. More detailed expressions and discussions can be found in the Appendix.

Now we want to exhibit the mixing matrix $V = V_0 V_2$ with the standard parametrization as in eq.(1.1). By a rephasing transformation, we get

$$V = \left( \begin{array}{ccc} 2 \sqrt{6} & \sqrt{3} & 0 \\ \sqrt{3} & 2 \sqrt{6} & 0 \\ 0 & 0 & 1 \end{array} \right) \left( \begin{array}{ccc} D_1 & \alpha_3 & \alpha_2 \\ -\alpha_3^* & D_2 & \alpha_1 \\ -\alpha_2^* - \alpha_1^* & D_3 \end{array} \right) \left( \begin{array}{ccc} e^{i\sigma_1} & 0 & 0 \\ 0 & e^{i\sigma_2} & 0 \\ 0 & 0 & e^{i\sigma_3} \end{array} \right) \equiv \left( \begin{array}{ccc} e^{i\rho_1} & 0 & 0 \\ 0 & e^{i\rho_2} & 0 \\ 0 & 0 & e^{i\rho_3} \end{array} \right). \quad (2.18)$$

By a lengthy but straightforward calculation, we can get the standard predictions. The Dirac type CP phase is predicted as

$$\cos \delta \simeq \frac{5}{6} \frac{\omega}{1 - \omega} (2\delta_1 - \delta_2), \quad (2.19)$$

and the Majorana CP phases are

$$\tan 2\rho \simeq \frac{1}{3} (1 - 2\omega)(2\delta_1 - \delta_2),$$
$$\tan 2\sigma \simeq \frac{1}{6} (1 - 2\omega)(2\delta_1 - \delta_2). \quad (2.20)$$

About the mixing angles, we can obtain a small departure from the tribimaximal mixing in the manner of

$$\sin \theta_{13} \simeq \frac{\sqrt{2}}{6} \left\{ \frac{\eta}{1 + \eta^2} - \frac{\omega \eta}{1 + \omega^2 \eta^2} \right\} |\delta_2|,$$
$$\tan \theta_{12} \simeq \frac{\sqrt{2}}{2} \left\{ 1 - \frac{1}{6} \omega (\delta_1^2 - \delta_1 \delta_2 + \delta_2^2) \right\},$$
$$\tan \theta_{23} \simeq 1 - \frac{1}{9} \omega \eta |\delta_2| (2\delta_1 - \delta_2). \quad (2.21)$$

Immediately, when $\delta_2 \to 0$, the predicted values return to the corresponding ones in Scenario A as we expect to, and the Dirac phase $\delta$ is undetermined due to zero of $\sin \theta_{13}$. The small parameter $\delta_2$ is responsible for the non-vanishing $\theta_{13}$ and non-maximality of $\theta_{23}$, and the mixing angle $\theta_{12}$ depends on both $\delta_1$ and $\delta_2$. In this scenario, we predict a nearly maximal
Dirac phase, so the rephasing invariant Jarlskog parameter $J$ \cite{45,46} is determined only by the three mixing angles in leading order

$$J \approx \frac{\sqrt{2}}{6} \sin \theta_{13} \approx \frac{1}{18} \eta(1 - \omega)|\delta_2|.$$  

(2.22)

Indeed, the most general Dirac mass matrix $M_D$ can be diagonalized by two distinct unitary matrices $V_L$ and $V_R$, so if we identify $V_L$ as the exact tribimaximal form, the high order corrections come from the contribution of $N_R \equiv V_R^+ M_R^{-1} V_R^*$, which is a unitary symmetric matrix if the mass parameters in $M_R$ are degenerated. This case has been discussed for the (light) effective Majorona mass matrix with exact degenerate masses \cite{47,48}. They found there are only two mixing angles and one CP phase in the parametrization of mixing matrix, but it does not include the case with the same CP parity as a limit. In our scenarios, we want to get higher order corrections from the diagonalization of $N_R$, so we are only interested in the nearly unit form of $N_R$. It is convenient to parameterized as $N_R \equiv V_N^T D \delta_i V_N$ where $V_N$ is a real unitary (orthogonal) matrix and $D \delta_i$ is a diagonal unitary matrix with small $\delta_i$ denoting the small departure from unit form of $N_R$. In the beginning of our calculation, we have identified $V_N$ with the explicit form of $V_0$ for simplicity. Although this is only a special example, but it is enough to reveal the property of our scenarios and correlations of these corrections. In next section, we will show them numerically.

### III. NUMERICAL ANALYSIS

*Scenario B* returns to *Scenario A* when $\delta_2 \to 0$, so we can only concern the general case. To do numerical analysis, we will include higher order contributions of $\omega$ and $\eta$ as in ((5.2), (5.7)–(5.9)). Firstly, the correlations of $\omega$ and $\eta$ come form the ratio of $\Delta m^2_{Sud}$ and $\Delta m^2_{Atm}$ by the relation of

$$\frac{\Delta m^2_{Sud}}{\Delta m^2_{Atm}} \simeq \frac{\eta^4(1 - \omega^4)}{1 - \eta^4}.$$  

(3.1)

Using the numerical results in \cite{21,22} which reveal that $\Delta m^2_{Sud} = 7.65^{+0.23}_{-0.20} \times 10^{-5} eV^2$ and $\Delta m^2_{Atm} = 2.40^{+0.12}_{-0.11} \times 10^{-3} eV^2$ at 1 $\sigma$, we can obtain the allowed region of $\omega$ versus $\eta$ in FIG.1. The mass spectrum changes from hierarchy region to nearly degenerate region as $\omega$ grows. And $\eta$ takes a nearly fixed value in the hierarchy region. As discussed in the
Appendix, only the normal hierarchy mass spectrum is valid in our scenarios, so we restrict the parameter $\eta \leq 0.5$ in our analysis.

The departure of mixing matrix from the standard tribimaximal mixing depends on the two small parameters $\delta_1$ and $\delta_2$. If we change the values of $\delta_i$, we can get the correlation relations between mixing parameters (mixing angles and CP phases) defined in (1.1). Scanning $\delta_i$ within a reasonable range ($[-0.5, 0.5]$), we give five pictures FIG.2–FIG.6 as typical examples. The angles and phases are measured in degrees. Some comments are listed below.

• For the relation of between $\theta_{12}$ and $\theta_{13}$ in FIG.2, we can see that the correction of $\theta_{12}$ is only at the left side, which has been revealed in (2.21) for leading corrections, indicating that a relatively small $\theta_{12}$ is favored, which is in accordance with the best fit point ($33.2^\circ$) of global analysis [?]. The magnitude of $\theta_{13}$ is too small ($< 2.5^\circ$) to be measured in the near future. The sensitivity of the proposed reactor neutrino experiments to $\theta_{13}$ is at the level of $\theta_{13} \sim 3^\circ (\sin^2 2\theta_{13} \sim 0.01)$ [20,57]. Moreover, a larger deviation of $\theta_{12}$ from $35.3^\circ$ ($\sin^{-1} \frac{1}{\sqrt{3}}$) implies a more stringent constraint on $\theta_{13}$, including a lower bound.

• In FIG.3, the deviation of $\theta_{23}$ can extend to both side, but with an unsymmetrical shape. The magnitude of this deviation is less than $1^\circ$. A larger $\theta_{13}$ allows a smaller region of $\theta_{23}$. Oppositely, a larger deviation of $\theta_{23}$ from maximality implies a smaller range for $\theta_{13}$.

• The Dirac type CP phase $\delta$ favors a nearly maximal value which can be understood in (2.19) and in FIG.4. The relatively broad width results from the large range of $(2\delta_1 - \delta_2)$. The figure shows that the more deviation of $\delta$ from maximality, the more stringent upper bound $\theta_{13}$ suffers. The Jarlskog parameter $J$ can reach the order of $10^{-2}$ at most, which is limited by the relatively small value of $\theta_{13}$.

• Just as revealed in FIG.5, the two Majorana CP phases $\rho$ and $\sigma$ have distributions among $[-180^\circ, 180^\circ]$ peaked at zero, which is consistent with leading predictions of (2.20). Also, FIG.6 shows that $\rho$ and $\sigma$ are strongly correlated with each other, having a nearly linear dependence. The slope of $\rho$ versus $\sigma$ defined in (2.20) approximates to 0.5, and the corresponding one in FIG.6 lies between $0.75 \sim 1.0$, where the higher order contributions of $\omega$ and $\eta$ have been included.
IV. CONCLUSIONS

To conclude, we discuss possible corrections to the tribimaximal pattern of lepton mixing in a simple seesaw model, which is the results of phase breaking of right-handed neutrino matrix. We consider small phase splitting for complex matrix $M_R$, but maintain the degeneracy of the right-handed neutrino masses. The breaking of the unit form of $M_R$ in the seesaw model gives small corrections to the zero order form of the mixing matrix. As revealed in (2.21), the corrections of $\theta_{13}$, $\theta_{12}$, and $\theta_{23}$ are of order $O(\delta_i)$, $O(\delta_i^2)$, and $O(\delta_i^2)$ respectively. $\theta_{12}$ favors a smaller value compared to the zero order value of 35.3°. The Dirac type CP phase $\delta$ is likely to have a nearly maximal value. For the two Majorana CP phases $\rho$ and $\sigma$, they have a nearly linear correlation.

A normal hierarchy mass spectrum is favored in our scenarios, leading to insignificant renormalization group effects [32–34,43,44] of the mixing matrix. The exact mass degeneracy of the righthanded neutrinos forbids CP violation in the lepton-number-violating decays [49], so there is no thermal leptogenesis [50]. Generally, we should include both effects mentioned in the introduction: the nondegenerate masses and the nondegenerate phases. But our discussions are valid when the mass degeneracy breaking is much smaller than the latter one. The general cases including both (and the corresponding leptogenesis) are certainly interesting and need further discussions.

There are other interesting aspects on the corrections to tribimaximal mixing based on either different assumptions [51,52] or general parameterizations [53–55]. They can only be distinguished by precision measurements in future reactor and accelerator neutrino experiments [20,56,57].

ACKNOWLEDGMENTS

The authors are grateful to B.L. Chen for the help of numerical analysis. This work is supported in part by the National Natural Science Foundation of China under grant number 90203002. The author (YFL) would like to thank the Department of Theoretical Physics of the University of Torino for hospitality and support.
V. APPENDIX: PERTURBATION CALCULATIONS FOR DIAGONALIZING THE NEUTRINO MASS MATRIX

When we want to diagonalize the matrix of $M^{(2)}_{\nu}$ in (2.12), we refer to the assumption that parameters $\delta_i$ are small enough to do perturbation calculations. In the relation of $M^{(2)}_{\nu} = V^*_2 M^{(2)}_{\nu} V^*_2$, we think equations are realized in each order for the power series of $\delta_i$. Then when we put the approximation to the order of $O(\delta_1, \delta_2)$, we can get three equations by the relations of three off-diagonal elements

\[
i\sqrt{2}(2\delta_1 - \delta_2)\omega\eta^2 + \sin\theta_3(\cos\phi_3 + i\sin\phi_3)6\omega^2\eta^2 - \sin\theta_3(\cos\phi_3 - i\sin\phi_3)6\eta^2 = 0, \\
-i\sqrt{3}\delta_2\omega\eta + \sin\theta_2(\cos\phi_2 + i\sin\phi_2)6\omega^2\eta^2 - \sin\theta_2(\cos\phi_2 - i\sin\phi_2)6 = 0, \\
i\sqrt{6}\delta_2\eta + \sin\theta_1(\cos\phi_1 + i\sin\phi_1)6\eta^2 - \sin\theta_1(\cos\phi_1 - i\sin\phi_1)6 = 0. \tag{5.1}
\]

Then we get the results as

\[
\sin\theta_3\sin\phi_3 \simeq -\frac{\sqrt{2}\omega}{6(1 + \omega^2)}(2\delta_1 - \delta_2), \\
\sin\theta_2\sin\phi_2 \simeq \frac{\sqrt{3}\omega\eta}{6(1 + \omega^2)^2}\delta_2, \\
\sin\theta_1\sin\phi_1 \simeq -\frac{\sqrt{6}\eta}{6(1 + \eta^2)}\delta_2, \tag{5.2}
\]

and

\[
\sin\theta_3\cos\phi_3 = \sin\theta_2\cos\phi_2 = \sin\theta_2\cos\phi_2 = 0. \tag{5.3}
\]

Which indicate maximality for $\phi_i(\cos\phi_i = 0)$. We want to know the departure from maximality in order of $O(\delta_1, \delta_2)$, so terms in order of $O(\delta_1^2, \delta_1\delta_2, \delta_2^2)$ must be considered in equations of (5.1). Up to this order those equations have the form of

\[
i\sqrt{2}(2\delta_1 - \delta_2)\omega\eta^2 + i\sin\theta_3\sin\phi_36\eta^2(1 + \omega^2) + \sin\theta_3\cos\phi_36\eta^2(\omega^2 - 1) \\
+\sqrt{2}(-\delta_1^2 + \frac{\delta_2^2}{2})\omega\eta^2 - \sin\theta_3\sin\phi_3\eta^2[(4\delta_1 + \delta_2)\omega^2 + (2\delta_1 + 2\delta_2)] \\
-6\sin\theta_1\sin\phi_1\sin\phi_2 - \sqrt{6}\sin\theta_2\sin\phi_2\delta_2\eta + \sqrt{3}\sin\theta_1\sin\phi_1\delta_2\eta = 0, \tag{5.4}
\]

\[
-i\sqrt{3}\delta_2\omega\eta + i\sin\theta_2\sin\phi_26(1 + \omega^2\eta^2) + \sin\theta_2\cos\phi_26(\omega^2\eta^2 - 1) + \frac{\sqrt{3}}{2}\delta_2\omega\eta \\
- \sin\theta_2\sin\phi_2[(4\delta_1 + \delta_2)\omega^2\eta^2 + 3\delta_2] - 6\eta^2\sin\theta_1\sin\phi_1\sin\phi_3 \\
-\sqrt{2}\sin\theta_1\sin\phi_1(2\delta_1 - \delta_2)\omega\eta^2 - \sqrt{6}\sin\theta_3\sin\phi_3\delta_2\eta = 0. \tag{5.5}
\]
\[ i \sqrt{6} \delta_2 \eta + i \sin \theta_1 \sin \phi_1 6(1 + \eta^2) + \sin \theta_2 \cos \phi_2 6(\eta^2 - 1) - \frac{\sqrt{6}}{2} \delta_2^2 \eta \]

\[- \sin \theta_1 \sin \phi_1 [(2 \delta_1 + 2 \delta_2) \eta^2 + 3 \delta_2] - 6 \omega^2 \eta^2 \sin \theta_2 \sin \phi_2 \sin \theta_3 \sin \phi_3 \]

\[-\sqrt{2} \sin \theta_2 \sin \phi_2 (2 \delta_1 - \delta_2) \omega \eta^2 + \sqrt{3} \sin \theta_3 \sin \phi_3 \delta_2 \omega \eta = 0. \quad (5.6) \]

Firstly, the imaginary parts of the equations are in order of \(O(\delta_1, \delta_2)\), giving the same values of \(\sin \theta_1 \sin \phi_i\) as in (5.2). Contrarily, from the real parts of the order of \(O(\delta_1^2, \delta_1 \delta_2, \delta_2^2)\), we can obtain the expressions for \(\sin \theta_1 \cos \phi_i\). These results read as

\[
\sin \theta_3 \cos \phi_3 \simeq -\frac{\sqrt{2} \omega}{36(1 - \omega^4)} \{2(1 - \omega^2)\delta_1^2 - 2(1 - \omega^2)\delta_1 \delta_2 - (1 + 2 \omega^2)\delta_2^2 + 3 \frac{(1 + \omega^2)(1 + \eta^2 + \omega^2 \eta^2)}{(1 + \eta^2)(1 + \omega^2 \eta^2)} \delta_2 \} , \quad (5.7)
\]

\[
\sin \theta_2 \cos \phi_2 \simeq -\frac{\sqrt{3} \omega \eta}{18(1 - \omega^4)} \delta_2 (2 \delta_1 - \delta_2) \{- \omega^2 \eta^2 + \frac{(1 + \omega^2 \eta^2)(1 + \eta^2 + \omega^2 \eta^2)}{(1 + \omega^2)(1 + \eta^2)} \} , \quad (5.8)
\]

\[
\sin \theta_1 \cos \phi_1 \simeq -\frac{\sqrt{6} \eta}{36(1 - \eta^4)} \delta_2 (2 \delta_1 - \delta_2) \{- \eta^2 + \omega^2 \frac{(1 + \eta^2)(1 + \eta^2 + \omega^2 \eta^2)}{(1 + \omega^2)(1 + \eta^2)} \} . \quad (5.9)
\]

Following the spirits of perturbation calculation, higher order contributions only give small corrections, meaning that the results are stable. Comparing these results with those of (5.3), we can see that if the parameter \(\omega\) or \(\eta\) (or both) is very close to 1 (quasi-degenerate or inverted hierarchy mass spectrum), the property of perturbation will be ruined. So a normal hierarchy (small \(\omega\) and \(\eta\)) neutrino mass spectrum is favored in this scenario. Then for analytical expressions, we can simplify them with the leading terms for power series of \(\omega\) and \(\eta\). For the results of \(\sin \theta_1 \sin \phi_i\) and \(\sin \theta_1 \cos \phi_i\), we have

\[
\sin \theta_3 \sin \phi_3 \simeq -\frac{\sqrt{2}}{6} \omega (2 \delta_1 - \delta_2) , \quad \sin \theta_2 \sin \phi_2 \simeq \frac{\sqrt{3}}{6} \omega \eta \delta_2 , \quad \sin \theta_1 \sin \phi_1 \simeq -\frac{\sqrt{6}}{6} \eta \delta_2 ; \quad (5.10)
\]

and

\[
\sin \theta_3 \cos \phi_3 \simeq -\frac{\sqrt{2}}{18} \omega (\delta_1^2 - \delta_1 \delta_2 + \delta_2^2) , \quad \sin \theta_2 \cos \phi_2 \simeq \frac{\sqrt{3}}{18} \omega \eta \delta_2 (2 \delta_1 - \delta_2) , \quad \sin \theta_1 \cos \phi_1 \simeq -\frac{\sqrt{6}}{36} \eta (\omega^2 - \eta^2) \delta_2 (2 \delta_1 - \delta_2) . \quad (5.11)
\]
Similarly, for the diagonal elements in $\overline{M}_\nu = V_2^+ M_\nu^{(2)} V_2^*$, we can get the information of $m_i$ and $\sigma_i$ as

\[
m_1 \simeq \frac{z^2 \omega^2 \eta^2}{M_0} \left( 1 + \frac{11 - \omega^2}{9 \left( 1 + \omega^2 \right)} \delta_1 - \frac{11 - \omega^2}{9 \left( 1 + \omega^2 \right)} \delta_2 \right) + \frac{1}{72} \frac{\omega^2 - \omega^2 \eta^2 (1 + 5 \omega^2)}{\left( 1 + \omega^2 \right) (1 + \omega^2 \eta^2)} \delta_2^2,
\]
\[
m_2 \simeq \frac{z^2 \eta^2}{M_0} \left( 1 - \frac{11 + 3 \omega^2}{9 \left( 1 + \omega^2 \right)} \delta_1 + \frac{11 + 3 \omega^2}{9 \left( 1 + \omega^2 \right)} \delta_2 \right) + \frac{1}{18} \frac{1 + 4 \eta^2 + 3 \omega^2 \eta^2}{\left( 1 + \omega^2 \right) (1 + \eta^2)} \delta_2^2,
\]
\[
m_3 \simeq \frac{z^2}{M_0} \left( 1 - \frac{1}{8} \delta_2^2 \right) + \frac{1}{12} \frac{1 + 3 \omega^2 \eta^2}{(1 + \eta^2) (1 + \omega^2 \eta^2)} \delta_2^2. \tag{5.12}
\]

and

\[
\tan 2\sigma_1 \simeq \frac{1}{6} (4 \delta_1 + \delta_2), \quad \tan 2\sigma_2 \simeq \frac{1}{3} (\delta_1 + \delta_2), \quad \tan 2\sigma_3 \simeq \frac{1}{2} \delta_2. \tag{5.13}
\]

For small mass ratios, the three masses are expressed as

\[
m_1 \simeq \frac{z^2 \omega^2 \eta^2}{M_0} \left( 1 + \frac{1}{9} \delta_1^2 - \frac{1}{9} \delta_1 \delta_2 + \frac{5}{72} \delta_2^2 \right),
\]
\[
m_2 \simeq \frac{z^2 \eta^2}{M_0} \left( 1 - \frac{1}{9} \delta_1^2 + \frac{1}{9} \delta_1 \delta_2 + \frac{1}{18} \delta_2^2 \right),
\]
\[
m_3 \simeq \frac{z^2}{M_0} \left( 1 - \frac{1}{8} \delta_2^2 \right). \tag{5.14}
\]
REFERENCES

[1] Q.R. Ahmad et al. [SNO Collaboration], Phys. Rev. Lett. 87, (2001) 071301.
[2] Q.R. Ahmad et al. [SNO Collaboration], Phys. Rev. Lett. 89, (2002) 011301.
[3] Q.R. Ahmad et al. [SNO Collaboration], Phys. Rev. Lett. 89, (2002) 011302.
[4] Q.R. Ahmad et al. [SNO Collaboration], Phys. Rev. Lett. 92, (2004) 181301.
[5] Q.R. Ahmad et al. [SNO Collaboration], Phys. Rev. Lett. 101, (2008) 111301.
[6] K.S. Hirata, et al., Phys. Rev. D 44 (1991) 2241.
[7] K.S. Hirata, et al., Phys. Rev. Lett. 66, (1991) 9.
[8] Y. Fukuda, et al., Phys. Rev. Lett. 77, (1996) 1683.
[9] Y. Fukuda et al., [Super-Kamiokande Collaboration], Phys. Rev. Lett. 87 (1998) 1562.
[10] Y. Ashie et al., [Super-Kamiokande Collaboration], Phys. Rev. Lett. 93 (2004) 101801, [hep-ex/0404034].
[11] Y. Ashie et al. [Super-Kamiokande Collaboration], Phys. Rev. D 71, (2005) 112005, [hep-ex/0501064].
[12] K. Eguchi et al. [KamLAND Collaboration], Phys. Rev. Lett. 90, (2003) 021802.
[13] T. Araki et al. [KamLAND Collaboration], Phys. Rev. Lett. 94, (2005) 081801.
[14] T. Araki et al. [KamLAND Collaboration], Phys. Rev. Lett. 100, (2008) 221803.
[15] M. Apollonio, et al. [CHOOZ Collaboration], Eur. Phys. J. C 27, (2003) 331.
[16] M.H. Alm et al. [K2K Collaboration], Phys. Rev. Lett. 90 (2003) 041801.
[17] M.H. Ahn et al. [K2K Collaboration], Phys. Rev. D 74 (2006) 072003.
[18] P. Adamson et al. [MINOS Collaboration] Phys. Rev. Lett. 101 (2008) 131802.
[19] C. Amsler et al. [Particle Data Group], Phys. Lett. B 667, 1 (2008).
[20] A. Strumia and F. Vissani, arXiv:hep-ph/0606054v2.
[21] T. Schwetz, M. Tortola and J.W.F. Valle, New J. Phys. 10 (2008) 113011 [arXiv:0808.2016].
[22] M. Maltoni and T. Schwetz, arXiv:0812.3161 [hep-ph].
[23] P.F. Harrison, D.H. Perkins and W.G. Scott, Phys. Lett. B 530, 167 (2002).
[24] P.F. Harrison and W.G. Scott, Phys. Lett. B 535, 163 (2002).
[25] Z.Z. Xing, Phys. Lett. B 533, 85 (2002).
[26] X.G. He and A. Zee, Phys. Lett. B 560, 87 (2003).
[27] R.N. Mohapatra and A.Y. Smirnov, Ann. Rev. Nucl. Part. Sci. 56, 569 (2006), [arXiv:hep-ph/0603118].
[28] E. Ma, J. Phys. Conf. Ser. 53, 451 (2006), [arXiv:hep-ph/0606024].
[29] S.F. King, J. High Energy Phys. 05 08 (2005), [arXiv:hep-ph/0506297].
[30] F. Plentinger and W. Rodejohann, Phys. Lett. B 625 264 (2005), [arXiv:hep-ph/0507143].
[31] K.A. Hochmuth, S.T. Petcov and W. Rodejohann, Phys. Lett. B 654 177 (2007),
  arXiv:0706.2975 [hep-ph].
[32] S. Luo and Z.Z. Xing, Phys. Lett. B 632 341 (2006), [arXiv:hep-ph/0509065].
[33] M. Hirsch, E. Ma, J.C. Romao, J.W.F. Valle and A.V.del Moral Phys. Rev. D 75 053006
  (2007), [arXiv:hep-ph/0606082].
[34] A. Dighe, S. Goswami and W. Rodejohann, Phys. Rev. D 75 073023 (2007), [arXiv:hep-
  ph/0612328].
[35] P. Minkowski, Phys. Lett. B 67, 421 (1977).
[36] T. Yanagida, in Proceedings of the Workshop on Unified Theory and the Baryon Number
  of the Universe, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979), p. 95.
[37] M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, edited by F. van Nieuwen-
  huizen and D. Freedman (North Holland, Amsterdam, 1979), p. 315.
[38] S.L. Glashow, in Quarks and Leptons, edited by M. Lévy et al. (Plenum, New York,
  1980), p. 707.
[39] R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).
[40] W. Grimus and L. Lavoura, J. High Energy Phys. 07, 045 (2001).
[41] W. Grimus and L. Lavoura, Phys. Lett. B 572, 189 (2003).
[42] W. Grimus and L. Lavoura, J. High Energy Phys. 08, 013 (2005).
[43] S.K. Kang and C.S. Kim, Phys. Lett. B 634, (2006) 520, arXiv:hep-ph/0511106.
[44] S.K. Kang Z.Z. Xing and S. Zhou, Phys. Rev. D 73 (2006) 013001, arXiv:hep-
  ph/0511157.
[45] C. Jarlskog, Phys. Rev. Lett. 55, 1039 (1985).
[46] D.D. Wu, Phys. Rev. D 33, 860 (1986).
[47] G. C. Branco, M. N. Rebelo, and J. I. Silva-Marcos, Phys. Rev. Lett. 82, 683 (1999).
[48] R. Adhikari, E. Ma, and G. Rajasekaran, Phys. Lett. B 486, 134 (2000).

[49] G.C. Branco, M.N. Rebelo and J.I. Silva-Marcos, Phys. Lett. B 633, 345 (2006), arXiv:hep-ph/0510412v2.

[50] M. Fukugita and T. Yanagida, Phys. Lett. B 174, 45 (1986).

[51] X.G. He and A. Zee, Phys. Lett. B 645, 427 (2007), arXiv:hep-ph/0607163v3.

[52] M. Honda and M. Tanimoto, Prog. Theor. Phys. 119, 583 (2008), arXiv:0801.0181v2 [hep-ph].

[53] N. Li and B.Q. Ma, Phys. Rev. D 71, 017302 (2005), arXiv:hep-ph/0412126v2.

[54] S.F. King, Phys. Lett. B 659, 244 (2008), arXiv:0710.0530v3 [hep-ph].

[55] S. Pakvasa, W. Rodejohann and T. Weiler, Phys. Rev. Lett. 100, 111801 (2008), arXiv:0711.0052v2 [hep-ph].

[56] Double Chooz Collaboration, arXiv:hep-ex/0606025.

[57] Daya Bay Collaboration, arXiv:hep-ex/0701029.
FIG. 1. Allowed parameter space of $\omega$ and $\eta$ from $1\sigma$ region of the two $\Delta m^2$.

FIG. 2. Allowed parameter distribution of $\theta_{12}$ and $\theta_{13}$ for $(\delta_1 \delta_2)$ within $[-0.5, 0.5]$. 
FIG. 3. Allowed parameter distribution of $\theta_{23}$ and $\theta_{13}$ for $(\delta_1 \delta_2)$ within $[-0.5, 0.5]$.

FIG. 4. Allowed parameter distribution of $\delta$ and $\theta_{13}$ for $(\delta_1 \delta_2)$ within $[-0.5, 0.5]$. 
FIG. 5. Allowed parameter distribution of $\rho$ and $\theta_{13}$ for $(\delta_1, \delta_2)$ within $[-0.5, 0.5]$.

FIG. 6. Parameter dependence of $\rho$ and $\sigma$ for $(\delta_1, \delta_2)$ within $[-0.5, 0.5]$. 