Shakedown analysis of thick cylinders with multiple radial openings

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Abstract. Shakedown and ratcheting behaviour of thick cylinder with multiple radial crossholes is investigated in this paper by inelastic finite element analysis. Effect of opening radius \( r_i/R \), thickness of the vessel i.e. thickness ratios \( t/R \) and availability of ligament between two openings \( L_t/t \) is studied. In order to normalise the applied pressure, limit load for thick cylindrical vessel without any opening was considered and it was found to be highly conservative. Later, interaction diagram normalised with respect to the limit load for thick cylinder taking into account the effects of opening was plotted. In order to distinguish between plastic shakedown and ratcheting, thickness advantage is taken into consideration. A more practical assessment technique is also discussed where the maximum range of thermal stress is computed through finite element analysis and the results were found to be in accordance with the Bree diagram. However the elastic-plastic shakedown boundary was found to be affected significantly with the incorporation of discontinuities and with changes in the parameters.

1. Introduction

Engineering design and life assessment of pressure-retaining components, subjected to cyclic loads, is one of the most challenging issues in the area of pressure vessel research. In the nuclear industry, ASME Boiler and Pressure Vessel Code (section III, Div.1, NB-3000) has been widely employed to design safety class-I components [1]. The simplified design assessment methods in ASME code like the Bree diagram were developed for a thin cylinder subjected to constant internal pressure and a cyclic linear through-the-wall temperature distribution [2]. One dimensional analysis with an elastic-perfectly plastic material model was used to arrive at the limits for shakedown and ratcheting. However in practical situations, thick cylindrical pressure vessels find many applications in several industries ranging from chemical, petroleum to nuclear power plants. These cylindrical vessels are required to have openings in order to accommodate various instruments, for tee fittings, compressor heads, to facilitate fluid transfer, and so on.

Various modifications over the Bree diagram used by the ASME code have been studied so far. The different loading combinations like; constant secondary and cyclic primary load, cyclic secondary and primary out-of-phase and cyclic secondary and primary in-phase have been investigated [3] and the Bree diagram for these loading sequences were found to be different from that used in the ASME code. An interesting result was observed for the case of constant secondary and cyclic primary stress, where ratcheting was absent irrespective of the magnitudes of secondary (thermal) stresses. Nayebi used Armstrong-Frederick non-linear kinematic hardening material model to examine Bree’s problem [4]. Two dimensional problem and the Poisson’s effect were also considered by the author and it was
concluded that the ASME design code is conservative. Bradford et al. carried the studies over Bree diagram with different values of yield stress during on-load and off-load [5]. The temperature dependence of yield stress was taken into account and off-load (no thermal load) yield stress was assumed to be greater than the on-load yield stress. Bradford also carried the investigation over the Bree problem with primary load cycling-strictly in-phase with the secondary load and concluded that this case was substantially more resistant to ratcheting [6].

However all the above mentioned work has been carried out over the same Bree geometric model i.e. thin cylindrical vessels. For the case of thick cylindrical vessels with radial crossholes, many researchers have carried out investigations. Faupel et al. in his studies concluded that incorporating openings in cylindrical vessels leads to stress concentration in the shell around the openings, which reduces the fatigue life of the vessel [7]. Circular holes were found to have more deteriorating effect in comparison to the standard elliptical side holes. Makulsawatudom et al. performed a finite element investigation of the shakedown behaviour of thick walled cylindrical vessels with a radial and offset crosshole subjected to cyclic internal pressure [8]. Various intersection types between the crosshole and the main bore were considered and was concluded that crosshole significantly altered the shakedown behaviour of thick cylindrical vessels. The form of intersection between the crosshole and main bore did not significantly affect the shakedown behaviour. However, above study did not consider the effect of thermal stresses. Makulsawatudom et al. in their another research work presented the SCF for internally pressurised thick cylindrical vessels with radial and offset circular and elliptical crossholes [9]. Again various intersection forms were considered and minimum SCF was observed to occur for the plain intersection configuration. Introducing a chamfer at the intersection reduced the stress concentration at the main bore, however it introduced higher peak stress elsewhere in chamfer region.

Duncan et al. studied the shakedown behaviour of thick cylindrical vessel with radial crosshole under the combined effect of constant primary and cyclic secondary stress [10]. An important conclusion was that the presence of crosshole shifts the elastic-plastic shakedown boundary to 1.2 and no significant effect was observed over the shakedown ratcheting boundary. However the study was carried on only a single thick cylindrical vessel having $r_i/R_i = 0.1$ and $R_o/R_i = 1.5$. This study was further extended by Zheng et al. to investigate the effects of relative thickness $R_o/R_i$, effects of opening radius ratios $r_i/R_i$ and the effect of end conditions on the shakedown and ratcheting behaviour of perforated cylinders [11].

However no work has been carried on so far for the case of multiple openings as per the knowledge of the authors. It is quite evident that crossholes are the source of discontinuities [7] and adjacent crossholes are bound to effect the stress pattern and thus impact the shakedown and ratcheting behaviour of the component. Hence this study has been carried out to investigate the effect of available ligament between two adjacent crossholes $L/t$, effect of opening radius $r_i/R_i$ and the effect of relative thickness $R_o/R_i$ on the shakedown behaviour of thick cylindrical vessels with adjacent openings. Effect of taking into account the change of limit load due to the presence of crossholes while normalising the applied pressure is also studied. Later on the maximum thermal stress range computed on an elastic basis through FE analysis is used to obtain the modified Bree diagram and its impact is also investigated. The perpendicular intersection type between the main bore and the crosshole is addressed in this study as intersection type was found to have no influence over the shakedown behaviour [8].

2. Method validation and finite element modelling

In order to distinguish between shakedown and ratcheting, the variation of plastic strain with the number of cycles was studied. If the trend is increasing i.e. the plastic strain keeps on accumulating continuously with the number of cycles, it is recognized as ratcheting. However if the value of plastic strain is accumulating at a rate lower than the value define by the C-TDF (Committee of Three Dimensional Finite Element Stress Evaluation) method or it is alternating between higher and lower values after few cycles, it could be termed as shakedown. C-TDF method mentions the criteria to
follow in order to judge the presence of Ratcheting or shakedown. “Variations in equivalent plastic strain at the end of each cycle should have a decreasing trend and should become lower than the allowable limit of $10^{-4}$/cycle in the 10th cycle”, is one criterion called “evaluating variation in plastic strain increments” to verify shakedown in the C-TDF method [12].

In order to distinguish between elastic and plastic shakedown, again the variation of plastic strain with number of cycles/time can be studied. If the variation of plastic strain is observed to be constant with number of cycles then it is showing elastic shakedown. However if plastic strain is either increasing {at a rate lower than the permitted limit for it to be called ratcheting} or is alternating between higher and lower values then it can be termed as plastic shakedown.

In order to validate the above mentioned method for distinguishing between different boundaries, a thin cylindrical shell with closed end conditions subject to constant internal pressure and a cyclic through wall thermal gradient is considered. The thermal gradient applied is linear throughout the thickness with outer surface always being at room temperature and inside surface varying between higher and room temperature. Here the thermal loads were applied for 25 cycles. The results obtained were benchmarked with the standard Bree solution to assess the accuracy of this method. Inelastic Finite Element Analysis was performed. The plain thin cylinder was modelled using plane 42 axisymmetric elements through the thickness. The cylinder is subjected to internal pressure in the main cylinder bore. The cylinder is assumed to have closed ends, represented by applying an equivalent axial thrust. Bilinear kinematic hardening material model was used with plastic modulus of zero, as was assumed by Bree in his analysis. The material properties are shown in table 1. The previously mentioned method was able to accurately predict the Bree diagram thus ensuring its correctness.

![Figure 1. Geometric model](image1)

![Figure 2. Finite element model](image2)

To study the shakedown behaviour of thick vessels with cross holes 3D model was generated for 1/8th of the cylindrical vessel using 8 noded solid brick elements. Symmetric boundary conditions are applied in longitudinal and hoop directions. The main cylinder and the associated radial bore were subjected to constant internal pressure. In order to simulate the closed end conditions, an equivalent axial thrust is applied at ends of the cylindrical vessel. The thermal gradient applied is linear throughout the thickness. In each thermo mechanical loading combination, it was assumed that the mechanical load (internal pressure) is a constant value while the temperature gradient changes from zero to the maximum magnitude cyclically. This is done by maintaining the outside temperature at a constant value of 40°C and the inside temperature is allowed to alternate between higher temperature and 40 °C. The thermal load was applied for 50 cycles. The material model used was similar to that used in thin cylinder analysis i.e. elastic perfectly plastic. The mesh size with less than 2% error in comparison to the finest mesh density was chosen. A typical finite element model considered in this study is shown in figure 2. The material properties have been taken directly from [10] and presented in table 1.
Table 1. Material Properties

| Property                        | Value       |
|---------------------------------|-------------|
| Young’s Modulus, E             | 184 GPa     |
| Poisson’s Ratio, $\nu$         | 0.3         |
| Coefficient of expansion, $\alpha$ | $13.35 \times 10^{-6} \text{C}^{-1}$ |
| Design Stress, $S_m$            | 310MPa      |
| Yield Stress, $\sigma_y$        | 465MPa      |

Duncan et al. also emphasised over the need of introducing a Tresca factor. This is because the inelastic analysis are based on the von Mises yield criterion however the ASME Code is based on the Tresca yield criterion. Thus, in a conservative manner, by factoring the value of yield stress by $\sqrt{3}/2$ we can ensure that the factored von Mises yield locus lies inside the Tresca yield locus. Thus applying the $\sqrt{3}/2$ Tresca factor therefore gives a modified FEA yield stress $\sigma_y^m = 402.7$MPa.

3. Calculated results

3.1. Effect of hole diameter
To address the influence of opening radius on the shakedown behaviour, FE models with three different radius ratios for $r_i/R_i = 1/15, 1/10, 2/15$ are considered, keeping $L/t = 1$ as constant in all the models. In all the models inside radius of the vessel, $R_i$ is kept constant as 0.300m. Hence the values of opening radius varies from 0.02m to 0.04m. Different results are obtained for various loading combinations. The results are presented in the normalized form which makes it independent of specific material properties as in figures (2-4). The applied pressure is normalized with respect to Von Mises limit pressure of thick cylinder without any radial opening in X-axis while the maximum of elastic thermal stress $\sigma^T = E\alpha\Delta T/2(1-\nu)$ through the wall is normalized with respect to the modified Von Mises yield stress $\sigma_y^m$. The Bree solution normalized with respect to the corresponding limit load is also superposed in the modified Bree diagram. The limit pressure of the cylinder will decrease in proportion to the amount of area reduced [13]. This causes a change in the limit pressure for the thick cylindrical vessel having radial crossholes and the use of limit pressure of thick cylinder without any radial opening would yield a highly conservative result. Thus curve, with applied pressure being normalized with respect to the limit pressure for perforated vessels, is also plotted. The limit pressure for such cases are calculated through FE analysis. The shakedown boundaries are analysed according to the trend of plastic strain history as discussed in the previous section. In all the interaction diagram {figures 3,4,5}, the solid line has been plotted with X-axis being normalized with respect to limit pressure of thick vessel without any radial opening and dotted line has been plotted with X-axis being normalized with respect to limit pressure of thick vessel taking into account the presence of radial crosshole.

Figure 3. Interaction Diagram of Thick Cylinder with radial openings ($r_i/R_i = 1/15, R_o/R_i = 1.5L/t = 1$)
An important observation to be noted is that when the Bree diagram is being plotted on to the interaction diagram for thick cylinders, it suffers a slight shift in X-axis as also discussed by Camilleri et al., [10]. This is because the Tresca/thick cylinder limit load yields higher a pressure when compared to the thin cylinder theory. Through thick cylinder limit load theory we get the value of limit load as \( P_{L}^{TM} = 402.7 \ln \frac{450}{300} = 163.3 \text{MPa} \) and through thin cylinder limit load theory we get the value of limit load as \( P_{L}^{T} = \frac{\sigma_y t}{r_m} = \frac{402.7 \times 150}{375} = 161.08 \text{MPa} \). This results in a shift of the Bree solution such that at zero thermal stress ratio the pressure ratio is 0.9864.

**Figure 4.** Interaction Diagram of Thick Cylinder with Radial Openings \( \frac{r_i}{R_i} = 1/10, \frac{R_o}{R_i} = 1.5, \frac{L}{t} = 1 \)

In order to analyse the calculated results conveniently, all the shakedown/ratcheting boundaries for different opening radius ratios are presented in figure 6. The results show that the elastic shakedown boundaries reduce significantly due to the presence of radial hole and becomes equal to 1.0 approximately when more than one cross holes are present. It is of great interest that the boundary between the plastic and elastic shakedown zones increase slightly with increasing the opening radius and comes upto 1.15 approximately when the radius ratio \( r_i/R_i \) is equal to 1/10 and then decreases upon further increasing the radius ratio. Moreover, the shakedown/ratcheting boundaries decrease prominently with increasing the opening radius, as shown in figure 6. However, the boundary of elastic shakedown and ratcheting completely agrees with the results in Bree diagram when the opening radius ratio \( r_i/R_i \) is 1/15.

**Figure 5.** Interaction Diagram of Thick Cylinder with Radial Openings \( \frac{r_i}{R_i} = 2/15, \frac{R_o}{R_i} = 1.5, \frac{L}{t} = 1 \)
3.2. Effect of vessel thickness

In order to address the effect of vessel thickness on the shakedown behaviour, two different relative thickness for $R_o/R_i$ equaling 1.25 and 1.5, with the same value of L and same value of opening radius ($r_i = 0.03m$) of both the holes are analysed. The inner radius is kept as $R_i = 0.3m$ in both the models for the sake of comparison. Thus $r_i/R_i = 1/10$ for both the models. The outer radius varies from 0.375m to 0.450m. In order to identify the shakedown region in Bree diagram, more than 70 combinations of thermo mechanical loadings are computed by elastoplastic finite element analysis.

The interaction diagram normalised with corresponding values are presented in figure 7. Here also, the solid line has been plotted with X-axis being normalized with respect to limit pressure of thick vessel without any radial opening and dotted line has been plotted with X-axis being normalized with respect to limit pressure of thick vessel taking into account the presence of radial crosshole. For the sake of comparison, the interaction diagram of both the cases of vessel thickness i.e. $R_o/R_i = 1.5$ , from figure 4 and $R_o/R_i = 1.25$ from figure 7 are presented in figure 8. It can be seen that the vessel thickness has a prominent effect over the shakedown/ratcheting boundaries.
Decrease in the vessel thickness causes the shakedown/ratcheting boundaries to decrease, thus lowering the values of combination of pressure ratio and thermal stress ratio for the onset of ratcheting. Figure 8 also indicates that the variation in elastic/plastic shakedown boundary with the change in thickness is not so significant, but the whole shakedown region decreases prominently due to decrease in the vessel thickness, which was also observed by Zheng et al. [11] for the case of single radial crosshole. Here, the elastic-plastic shakedown boundary becomes equal to 1.05 approximately.

3.3. Effect of available ligament
In order to investigate the effect of available ligament i.e. distance between the center of adjacent radial crossholes, on the shakedown behavior, FE models with two different crossholes spacing \( L/t = 1 \) and 2 are considered. In both the models the radius ratio (thickness) is kept constant as \( R_o/R_i = 1.5 \) with inside radius \( R_i = 0.3m \). Hence the value of ligament \( L \) varies from 0.15m to 0.3m. The size of the radial opening is kept constant as 0.03m. In order to identify the shakedown region in Bree diagram, more than 60 combinations of thermo-mechanical loadings are computed by elastoplastic finite element analysis. The results in the normalized form is presented in figure 9. The Bree solution normalized with respect to the corresponding limit load is also superposed in the modified Bree diagram. The applied pressure is also normalized with respect to the limit load for thick vessels taking into account the effect of crossholes. Again the solid line has been plotted with X-axis being normalized with respect to limit pressure of thick vessel without any radial opening and dotted line has been plotted with X-axis being normalized with respect to the limit pressure of perforated vessel.
For the sake of comparison, the interaction diagram for both the cases of available ligament i.e. $L/t = 1$, from figure 4 and $L/t = 2$ from figure 9 are presented in figure 10 where the applied pressure has been normalized with respect to the corresponding limit load taking into account the effect of openings. It can be seen from figure 10 that increasing the distance between adjacent crossholes improve the shakedown/ratcheting boundary slightly. However no major variation is seen in the elastic/plastic shakedown boundary with changing the available ligament. An important observation to be made is that the limit load for both the cases are approximately the same, which is consistent with the previously mentioned reasoning that the change in limit load is dependent upon the loss of wall cross-section. Since both the cases involved the same amount of material removal, hence they have approximately similar limit pressures.

**Figure 10.** Modified Bree Interaction Diagram for Sidehole Cylinders Considering different Available Ligament

### 4. Discussions

The plastic shakedown-ratcheting boundaries are computed through C-TDF method as mentioned previously. For determining this boundary, trend of plastic strain with respect to time of a node is studied for all the elements present in radial direction of the component. However for the case of thick cylindrical vessels with openings, conditions are reached when partial thickness of the vessel is showcasing plastic shakedown and remaining part is showcasing ratcheting. In this situation it is very difficult to declare with certainty whether the component for the given combination of constant primary and cyclic secondary stress is undergoing plastic shakedown or ratcheting. Although by increasing the number of cycles the component would finally tend to show a particular behaviour throughout the thickness, it cannot be predicted beforehand as to how many cycles would be required in order to achieve a stable behaviour. Thus in order to overcome this situation thickness advantage is taken and the component is declared to undergo that particular behaviour which it is showcasing for majority of the thickness.

**Figure 11.** Modified Bree Interaction Diagram considering different opening radius (*maximum thermal stress range calculated from finite element analysis*)
ASME B&PV Code Section III NB allows only elastic shakedown and plastic shakedown subjected to appropriate low cycle fatigue considerations, and incremental plasticity i.e. ratcheting is not permitted [1]. Different limits have been provided for linear temperature variation and for parabolic variation of temperature through the wall. Now the allowable value of secondary stresses is provided by the code as the limit over the thermal stress ratios. Thermal stress ratios can be defined as the ratio of maximum allowable range of thermal stress computed on an elastic basis and the yield strength. Also the designer is expected to compute the maximum range of thermal stress by performing elastic finite element analysis. Therefore maximum range of thermal stress calculated from the highly simplified thin cylinder formula \[ \sigma_T = \frac{E \alpha \Delta T}{2(1 - \vartheta)} \] is bound to provide overly conservative results. Figures 11, 12, 13 show the interaction diagram which is plotted with the maximum range of thermal stress computed from FEA by performing stress categorisation across the thickness and considering the membrane plus bending part. Thus if we assume that the primary stress limit is obeyed as per the ASME code, then the pressure ratio would be less than 0.67 and it can be seen from the figures that the Bree diagram is sufficiently conservative in the shakedown-ratcheting boundary for perforated vessels. However decreasing the thickness of the cylindrical vessel, causes a severe reduction in the shakedown-ratcheting boundary, as shown in figure 12.

**Figure 12.** Modified Bree Interaction Diagram considering different thickness of vessels (maximum thermal stress range calculated from finite element analysis)

**Figure 13.** Modified Bree Interaction Diagram considering different available ligament (maximum thermal stress range calculated from finite element analysis)
The designer needs to take care while analyzing the Y-axis (thermal stress ratios) of the two sets of modified Bree diagram i.e. figures 11,12,13 and figures 6,8,10 as they have been calculated through different techniques. The Y-axis in figures 6,8,10 is directly proportional to the thermal gradient, $\Delta T$ while the Y-axis in figures 11,12,13 includes the effect of discontinuity stresses as well. For example through figure 10 it can be concluded that the transition from elastic to plastic behaviour occurs at approximately the same $\Delta T$ for both the L/t ratios. However figure 13 shows that even for similar $\Delta T$ the value of maximum thermal stress range is higher for L/t = 1 as the openings are located closer to each other in comparison with the case for L/t = 2 and thus the crossholes would raise the magnitude of stresses. Hence the elastic-plastic shakedown boundary is highly geometry dependent.

5. Conclusion
- Elastic-plastic shakedown boundary is reduced significantly owing to the presence of radial openings. However calculating maximum elastic thermal stress as $\sigma_T = E\alpha \Delta T / 2(1 - \vartheta)$ leads to highly conservative results, as this formula fails to capture the correct state of stress in the component accurately near to the discontinuity. Hence the range of maximum elastic thermal stress should be calculated through FE analysis when discontinuities are present.
- The effect of openings for limit load calculation should be accounted for, or else the results would again be highly conservative.
- Taking thickness advantage into consideration, it is seen that the Bree diagram is conservative and sufficient to predict the shakedown-ratcheting boundary for perforated vessels. Although the elastic-plastic shakedown boundaries varies with parameters and geometries, it is practical and conservative to define it at 1.5.

6. Reference
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