Physics of charge carriers confined to an antiphase domain wall in a two-dimensional antiferromagnet (AF) is one of the most important and least understood ingredients of the stripe-based approach to high-$T_c$ superconductivity. Both the long-distance interactions and the short-distance physics seem to be important for understanding even the basic phenomenology of stripes. Given the existence of metallic stripes in an antiferromagnet, their physics should not be far different from that of a more conventional one-dimensional electron gas (1DEG). However, as suggested by model calculations, metallic stripes in an AF environment may develop a spin gap. (In a conventional spin-rotation-invariant 1DEG a spin gap can only appear in the presence of attractive interactions.) With spin degrees of freedom gapped, coupling between adjacent stripes could lead either to a global superconducting order or to an insulating charge-density wave (CDW) state. In a 1DEG with repulsive interactions, CDW correlations diverge more strongly and one would expect them to dominate superconductivity. A way around this difficulty was suggested by Kivelson, Fradkin and Emery (KFE), who noticed that transverse fluctuations of conducting stripes would induce a phase mismatch between CDWs on neighboring stripes, effectively suppressing the interstripe coupling and the tendency to form a charge-ordered state. In contrast, such fluctuations do not suppress superconducting ordering, since the superconducting phase is not spatially modulated.

Transverse dynamics of insulating and metallic stripes has been previously studied by Zaanen et al. Of a particular interest to us is the example of a metallic stripe with a built-in coupling of charge motion and transverse fluctuations. The model describes an initially insulating stripe with a 4$k_F$ CDW (1/2 electron per unit cell). Assuming that doped holes reside on rows adjacent to the site-centered stripe, Zaanen et al. found charge-1/2 solitons, which shift the domain wall one lattice constant sideways (left/right), thus acquiring a transverse flavor.

In this paper, we construct a similar model to study the effect of transverse fluctuations of a stripe on its charge dynamics. First, we employ Hartree-Fock (HF) numerics and an analysis of fermion zero modes to identify elementary excitations of a weakly doped domain wall in the Hubbard model. Hartree-Fock numerics and analysis of fermion zero modes suggest that for $U \geq 3t$ charged excitations are mobile holons, $Q = 1$, $S = 0$. Each holon resides on a kink in the position of the domain wall. We construct a simple model in which transverse stripe dynamics is induced solely by motion of the holons. In the absence of spin excitations (spinons, $Q = 0$, $S = 1/2$), stripe fluctuations do not suppress a tendency to form a global charge-density order.

To study a possible effect of transverse fluctuations of a stripe on a two-dimensional antiferromagnet on its charge dynamics, we identify elementary excitations of a weakly doped domain wall in the Hubbard model. Hartree-Fock numerics and analysis of fermion zero modes suggest that for $U \geq 3t$ charged excitations are mobile holons, $Q = 1$, $S = 0$. Each holon resides on a kink in the position of the domain wall. We construct a simple model in which transverse stripe dynamics is induced solely by motion of the holons. In the absence of spin excitations (spinons, $Q = 0$, $S = 1/2$), stripe fluctuations do not suppress a tendency to form a global charge-density order.
a 1D band in the middle of the Hubbard gap $\Delta$ (Fig. 2). Fermions confined to a site-centered wall have a Dirac spectrum $E = \pm v |k_x - \pi/2|$ and acquire a small mass (gap) $\Delta < \Delta$ in the case of a bond-centered wall. This gap arises because an electron moving along the wall feels a nonzero $x$-staggered magnetization (vanishing by symmetry for a site-centered wall). At half-filling, the Fermi energy lies between the two midgap bands ($E_F = U/2$).

A bond-centered domain wall with a wiggle is similar to an AF chain with a kink in $x$-staggered magnetization (Fig. 3). As in the model of polyacetylene, [1] a pair of degenerate localized states (zero modes) appears in the middle of the smaller gap. A wall with a kink doped with a single hole has a $S = 0$, $Q = 1$ soliton at the wiggle (Fig. 3). In view of translational invariance, the charged defect can move along the wall. We thus identify it with a holon of the large-J$_z$ limit (Fig. 3). At $U = 2 \ldots 3t$ (rather weak coupling), holons are cigar-shaped like spin bags of Schrieffer et al. [14] a consequence of the Fermi surface nesting. Orientation of a holon along one or the other lattice diagonal is correlated with the direction of the wiggle (the transverse flavor). We have verified that holons are the lowest-energy states of doped charges on an empty wall for $U \geq 3t$ (at the HF level). At a weaker coupling, they bind into bipolarons with $Q = 2$, $S = 0$, and zero isospin.

Continuum Hartree-Fock approximation. HF equations for a linearly polarized Hubbard AF read

$$-t \sum_{(r,r')} \psi_{-s(r')} + U[\delta n(r) - s m(r)] \psi_s(r) = E \psi_s(r),$$

where $m(r) = (-1)^{x+y} \langle s_z(r) \rangle$ is the staggered magnetization and $s$ is the staggered spin index. In what follows, density fluctuations $\delta n(r)$ will be neglected to restore charge-conjugation symmetry. Eq. (1) can be cast in a matrix form using Pauli matrices $\{\sigma_i\}$. Staggered spin $s$ is an eigenvalue of $\sigma_3$, the hopping term is proportional to $\sigma_1$. Most importantly, zero modes of Eq. (1) are invariant under charge conjugation $\psi(r) \rightarrow \sigma_2 \psi^*(r)$. If such modes are present, the system contains solitons such solutions $|k_x| \approx |k_y| \approx \pi/2$ (Fig. 3, right). Introducing smoothly varying amplitudes of a fermion wavefunction,

$$\psi_s(r) \approx \sum_{\alpha=\pm} \sum_{\mu=\pm} \psi_{\alpha \mu s}(r) e^{i \pi(\alpha x + \mu y)/2},$$

adds two more indices, $\alpha = \text{sgn} k_x$ and $\mu = \text{sgn} k_y$. Accordingly, we preserve only those Fourier components of magnetization which connect the Fermi patches:

$$\langle s_z(r) \rangle \approx \sum_{\alpha=0} \sum_{\mu=0} m_{\alpha \mu}(r) e^{i \pi(\alpha x + \mu y)}.$$

With the new indices come two more mutually commuting sets of Pauli matrices, $\{\tau^x\}$ and $\{\tau^y\}$, $i = 1, 2, 3$. In terms of these, $k \approx \frac{1}{2} (\tau^x, \tau^y)$, $(-1)^x = \tau^x$, $(-1)^y = \tau^y$. The HF Hamiltonian becomes

$$H_{HF} = -2it \alpha \tau^x \tau^z \partial_x - 2it \alpha \tau^y \tau^z \partial_y - U \sigma_3 m(r),$$

$$m = m_{11} + m_{01} \tau^x + m_{10} \tau^y + m_{00} \tau^x \tau^y.$$

Only $m_{11}(r)$, staggered magnetization proper, survives in the bulk inducing the Hubbard gap $\Delta = U|m_{11}(\infty)|$.

On a straight domain wall in the $x$ direction, $m_{01} = m_{00} = 0$, while $m_{10} \neq 0$. The spectrum of midgap states depends on the symmetry of the wall. If the stripe is site-centered (bond-centered), wall fermions have a gapless (gapped) spectrum. The absence of a gap can be
solutions with $\sigma$ bond-centered walls arises in the first order in the wall in 2D — see Fig. 2, right. 

To zeroth order in $m_{10}$, solutions of Eq. (4) are eigenstates of $\sigma_2$, $\tau_3^y$ and, e.g., $\tau_3^x$ (each zero mode comes from a single Fermi patch), giving a total of 8 linearly independent zero modes. As usual, only half of these solutions [those with $\sigma_2 \tau_3^y m_{11}(+\infty) < 0$] are localized on the wall, so that there are 4 zero modes. Remarkably, in addition to the usual twofold spin degeneracy, there is another spin-like degree of freedom, which will prove to be the transverse flavor. The origin of holons (at weak coupling) is thus exposed: compared to a 1D chain, there are twice as many “Fermi points” on a straight domain wall in 2D — see Fig. 2, right.

The difference between one-particle spectra of site and bond-centered walls arises in the first order in $m_{10}$. Eq. (4) has four zero modes if $\tau_3^x$ is an odd function of $y$, i.e., for a site-centered wall. On a bond-centered wall, $m_{10}(y)$ is even and fermions have a gap (Fig. 2).

$$E(0) = \pm U \langle m_1 \rangle \equiv \pm U \int m_{10}(y) \psi_1^*(y) \psi(y) dy.$$  (5)

The one-particle spectrum $E(p_x)$ on a straight site-centered stripe can be determined approximately by starting with $p_y = 0$ [Eq. (4)] and treating the first term in Eq. (3) perturbatively. In the limit $p_x \to 0$, states outside the main gap can be neglected, which reduces the Hilbert space to the four zero modes [Eq. (3)]. By using the degenerate perturbation theory, we find a Dirac spectrum (dashed lines in Fig. 2, left):

$$E(p_x) \approx \pm vp_x, \quad v = 2ta\langle \tau_1^y \rangle.$$  (6)

Domain-wall holons at $U \ll t$. As illustrated in Fig. 3, magnetization on a bond-centered wall with a wiggle can be obtained by superimposing $m(r)$ of a straight site-centered wall and that of a spin chain with a kink in $x$-staggered magnetization. Away from the wiggle, $m_0(r) = m_1(r) = 0$. To simplify the discussion, we will neglect these components altogether. Decompose $m(r)$ into an $x$-independent part and the rest:

$$m(r) = m^{(0)}(y) + m^{(1)}(r), \quad m^{(0)}(y) = m^{(0)}(y), \quad m^{(1)}(\pm \infty, -y) = m^{(1)}(\pm \infty, y).$$

The Hamiltonian (3) can now be split in two parts:

$$H_{HF}^{(0)} = -2ita \sigma_1^y \partial_y - U \sigma_3 [m^{(0)}_{11}(y) + m^{(0)}_{10}(y) \tau_1^y], \quad H_{HF}^{(1)} = -2ita \sigma_1^x \partial_x - U \sigma_3 [m^{(1)}_{11}(r) + m^{(1)}_{10}(r) \tau_1^y].$$  (6)

As shown above, the “transverse part” 3 has 4 zero modes for each $p_x$. Within this Hilbert space, $H_{HF}^{(1)}$ describes right and left-moving fermions with spin, which see a staggered magnetization

$$\langle m_1(x) \rangle = \int dy \psi_1^*(y) [m_{10}(r) + m_{11}(r) \tau_1^y] \psi(y),$$

where $u(y)$ is a zero mode (4) of Eq. (4). The midgap fermion band acquires a gap of its own, $\Delta = U|\langle m_{11}(\infty) \rangle| < \Delta$, with two zero modes (one for each spin) inside this smaller gap. “Longitudinal” wavefunctions of the two zero modes satisfy the equation

$$\sigma_2 \frac{d\psi(x)}{dx} = \frac{U}{2ta} \tau_3^y \langle m_1(x) \rangle \psi(x).$$  (7)

The existence of two holon flavors can now be deduced from Eqs. (3) and (7). The zero modes have a finite norm only if

$$\sigma_2 \tau_3^y m^{(0)}_{11}(+\infty) < 0, \quad \sigma_2 \tau_3^y \langle m_{11}(+\infty) \rangle < 0.$$  (8)

It follows then that the product of eigenvalues

$$\tau_3^x \tau_3^y \begin{pmatrix} \sigma \cr \psi \end{pmatrix} = \begin{pmatrix} \sigma \cr \psi \end{pmatrix},$$

can be identified with the holon isospin $2\rho$. This can be seen by extrapolating Eq. (4) to larger values of $U$, which reduces the size of holons. We have $(\tau_1^z) = (-1)^{\rho} \tau_3^z = (-1)^{\rho}$, where $n_0$ is the row number of the chain in Fig. 3. According to Eq. (4), if $\tau_3^x \tau_3^y = +1$, spins on the chain and to the right (left) of the wiggle are an extension of the upper (lower) AF domain, as for the $\rho = +1/2$ wiggle in Fig. 3. Thus, $\rho = \tau_3^x \tau_3^y/2$. This identification is consistent with numerical HF solutions (Fig. 3), where $\tau_3^x \tau_3^y = \text{sgn} \kappa \text{sgn} \kappa'$ can be inferred from the orientation of a holon.

Effective holon Hamiltonian. Large-$J_z$ cartoons (e.g., Figs. 4 and 3) suggest that any configuration of a domain wall is uniquely represented by an interface drawn through integer (if a hole is present) or half-integer (bond, no hole) lattice points. In the absence of overhangs, the imaginary-time interface dynamics due to spin exchanges and hole hops can be described by an SOS-type model. (Locality in time assumes that coupling to the bulk spin waves can be neglected.) In the dilute limit, mobile single-particle excitations are spinons and holons, each equipped with an isospin $1/2$. When spinons are frozen out, the only remaining excitations are holons, and the SOS model reduces to that of (iso)spin-$1/2$ fermions with the short-range Hamiltonian

$$H = \sum_i \left[ \frac{-t \psi_1^\dagger \psi_{i+1}}{\psi_{i+1}} + J_x s_i^+ s_{i+1}^- + \text{h.c.} \right] + \tilde{U} n_i + \tilde{V} n_i + n_{i+1} + \tilde{J}_x s_i^+ s_{i+1}^- + \text{h.c.}.$$  (9)

Here $n_i \equiv n_{i+} + n_{i-}$ is the holon-number operator, $n_{i+} \equiv \psi_1^\dagger \psi_{i+}$, and isospin operators are $s_i^+ \equiv (n_{i+} - n_{i-})/2$ and $s_i^- \equiv \psi_1^\dagger \psi_{i-}$. Taking $\tilde{U} \to +\infty$ excludes double occupations, while isospin exchange $\tilde{J}_x$ allows two holons on one side of the stripe to hop to the other side. By construction, we do not expect isospin-rotation invariance; in general, $\tilde{J}_x \neq \tilde{J}_x$.

Holons can be transported between different stripes without disturbing the AF order only in pairs (with opposite isospins). In contrast to the model of a quarter-filled chain 4 with the same Hamiltonian (10), the field
ψ† describes $Q=1$ particles, and the appropriate pairing operator $\psi^\dagger_{i\uparrow}\psi^\dagger_{i+1\downarrow}$ has charge $Q=2$. This operator is a combination of the (iso)singlet (SP) and an (iso)triplet (TP₀) pairing operators,

$$O_{\text{SP}}, O_{\text{TP₀}} = (\psi^\dagger_{i\uparrow}\psi^\dagger_{i+1\downarrow} + \psi^\dagger_{i\downarrow}\psi^\dagger_{i+1\uparrow})/\sqrt{2},$$

and an instability in either of these channels could lead to global (Cooper-pair) superconductivity, even though isospin is local to a given stripe.

To analyze this scenario, we have performed a standard weak-coupling analysis at a generic density. Because a stripe horizontal on average is invariant under isospin reflection, isospin and charge degrees of freedom separate. The scaling in each sector is determined by the usual constants $K_s$, $K_c$: for repulsive interactions $K_s > K_c^{-1} > 1$. In the absence of the isospin gap, both SP and TP₀ correlation functions have the temperature exponent $\mu_{\text{SP}} = \mu_{\text{TP₀}} = K_c^{-1} + K_s - 2 > 0$, i.e., there is no divergence in either channel. Unlike in a more conventional 1D EG where the spin-rotation invariance requires $J_z = J_x$ and another mechanism (e.g., the “spin-gap proximity effect” [4]) is needed to develop the spin gap, here an isospin gap arises naturally for $J_z > |J_x|$ (an easy-axis anisotropy). At low temperatures, such a system is in the Luther-Emery phase, the TP₀ (an easy-axis anisotropy). At low temperatures, such a system is in the Luther-Emery phase, the TP₀ (an easy-axis anisotropy).

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