Use of Transverse polarization to probe R-parity violating supersymmetry at ILC.

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Abstract

In supersymmetric theories with R-parity violation, squarks and sleptons can mediate Standard Model fermion-fermion scattering processes. These scalar exchanges in $e^+e^-$ initiated reactions can give new signals at future linear colliders. We explore use of transverse beam polarization in the study of these signals in the process $e^+e^- \rightarrow b\bar{b}$. We highlight certain asymmetries, which can be constructed due to the existence of the transverse beam polarization, which offer discrimination from the Standard Model (SM) background and provide increased sensitivity to the R-parity violating couplings.

1 Introduction

In the Standard Model (SM), baryon and lepton number conservation is not guaranteed by local gauge invariance. In fact, in the supersymmetric extension of the SM, the most general superpotential respecting the gauge symmetries of the SM contains bilinear and trilinear terms which do not conserve either of baryon ($B$) and lepton ($L$) numbers. Clearly, the simultaneous presence of both lepton- and baryon-number violating operators could lead to very rapid proton decay, especially for TeV scale sparticle masses. The existence of all such terms can be forbidden by postulating a discrete symmetry \cite{1}, called R-parity, which
implies a conserved quantum number \( R_p \equiv (-1)^{3B+L+S} \), where \( S \) stands for the spin of the particle. The very definition implies that all the SM particles have \( R_p = +1 \) while all the superpartners are odd under this symmetry. Thus, apart from suppressing proton decay, it also guarantees the stability of the lightest supersymmetric particle (LSP) thereby offering a ready-made candidate for cold dark matter.

However, while a conserved R-parity seems desirable, it is perhaps too strong a requirement to be imposed. For one, this symmetry is an ad hoc measure and there does not exist an overriding theoretical motivation for imposing it, especially since a suppression of proton decay rate could as well be achieved by ensuring that one of \( B \) and \( L \) is conserved. Indeed, it has been argued [2] that this goal is better served by imposing a generalized baryon parity instead. Unlike R-parity, this latter (\( Z_3 \)) symmetry also serves to eliminate dimension-five operators that could potentially have led to proton decay. Furthermore, non-zero R-parity violating (RPV) couplings provide a means of generating the small neutrino masses, either at tree level or loop level, that the neutrino oscillation experiments seem to call for. It is thus of both theoretical and phenomenological interest to consider violations of R-parity [3].

The most general R-parity violating superpotential is

\[
W \supset \sum_i \kappa_i L_i H_2 + \sum_{i,j,k} \left( \lambda_{ijk} L_i L_j E^c_{k} + \lambda'_{ijk} L_i Q_j D^c_{k} + \lambda''_{ijk} U^c_i D^c_{j} D^c_{k} \right)
\]  

(1)

where \( i, j, k \) are generation indices, \( L \) (\( Q \)) denote the left-handed lepton (quark) superfields, and \( E, D \) and \( U \) respectively are the right-handed superfields for charged leptons, down and up-type quarks. The couplings \( \lambda_{ijk} \) and \( \lambda''_{ijk} \) are antisymmetric in the first two and the last two indices respectively. A conserved baryon number requires that all the \( \lambda''_{ijk} \) vanish identically thereby avoiding rapid proton decay. Neutrino masses, being very small, restrict quite strongly the size of the dimensional couplings \( \kappa_i \) in Eq. (1) and of the vacuum expectation values (vev’s) of the neutral scalar components of the fields \( L_i, v_i \). Note, however, that strictly speaking it is also possible to construct models with \( \kappa_i, i = 1, 3 \) not necessarily small. In this note, we focus on the effect of the trilinear terms in the superpotential.

Written in terms of the component fields these terms lead to the interaction Lagrangians

\[
\mathcal{L}_{LLE} = \frac{1}{2} \lambda_{ijk} \left[ \bar{\nu}_i L \bar{\ell}_{kR} \ell_{jL} + \bar{\ell}_{jL} \bar{\ell}_{kR} \nu_{iL} + (\bar{\ell}_{kR})^c (\nu_{iL})^c \ell_{jL} - (i \leftrightarrow j) \right] + h.c.
\]  

(2)
and

\[ \mathcal{L}_{\text{LQD}} = \lambda_{ijk}^{' \prime} \left[ \bar{\nu}_{iL} \bar{d}_{kR} d_{jL} + \bar{d}_{jL} \bar{d}_{kR} \nu_{iL} + (\bar{d}_{kR})^\ast \nu_{iL} \nu_{jL} \right. \]

\[ \left. - \bar{\nu}_{iL} \bar{d}_{kR} u_{jL} - \bar{u}_{jL} \bar{d}_{kR} \ell_{iL} - (\bar{d}_{kR})^\ast \ell_{iL} \nu_{jL} \right] + h.c. \] (3)

Just like the usual Yukawa couplings, the magnitudes of the couplings \( \lambda_{ijk} \), \( \lambda_{ijk}^{' \prime} \) are entirely arbitrary, and are restricted only from phenomenological considerations. The preservation of a GUT-generated \( B - L \) asymmetry, for example, necessitates the preservation of at least one of the individual lepton numbers over cosmological time scales \([4]\). Nonzero RPV couplings mean a decaying LSP, whose decay may or may not be always prompt, and which is mostly taken to be a neutralino \([5]\), even though non-\( \tilde{\chi}_1^0 \), \( \tilde{\tau}_1 \) candidates for the LSP are also possible \([6]\). In all the cases the decaying LSP gives rise to striking collider signatures \([7]\). However, the failure so far of the various collider experiments \([8, 9]\) to find any evidence of supersymmetry implies constraints on the parameter space. Even if superpartners are too heavy to be produced directly, their effects can still be probed using low-energy observables \([3, 10]\). The remarkable agreement of the measured values with the SM predictions implies strong bounds on these couplings which generally scale with the sfermion mass \( m_f \) \([3, 11]\).

In this work we study processes directly sensitive to the size of such couplings through the modification of SM amplitudes due to sparticles exchange \([12-14]\). The exchange of spin-0 particles in a \( 2 \rightarrow 2 \) scattering process would give a completely different chiral behaviour to the amplitudes as compared to the vectorial exchanges in the SM. The cleanliness of the signal at the next generation International Linear Collider (ILC) \([15]\) and excellent reconstruction of the angular variables would help us study the chiral properties of the amplitudes. The aim of this work is to investigate use of transverse beam polarization to probe such contributions through the measurement of cross-sections and study of kinematical properties of the final states. Specifically, we will see that transverse polarization can probe interference between SM amplitudes and certain RPV mediated amplitudes which are absent with longitudinally polarized or unpolarized beams. As a result, the additional effects can depend quadratically on the RPV couplings rather than quadratically. This can make studies with transversely polarized beams more sensitive to R-parity violating couplings.

We concentrate on the simplest process \( e^+ e^- \rightarrow f \bar{f} \) at the ILC. We discuss the advantages of having transversely polarized beams at ILC in Section 2 and its role in addressing issues pertaining to the chiral nature of interactions. In Section 3 we present the analysis and
2 Transverse polarization at ILC

An $e^+e^-$ linear collider operating at a center-of-mass energy of several hundred GeV will offer an opportunity to make precision measurement of the properties of the electroweak gauge bosons, top quarks, Higgs bosons, and also to constrain new physics [15]. Linear colliders are expected to have the option of longitudinally polarized beams, which could add to the sensitivity of these measurements and reduce background in the search for new physics [16].

It has further been realized that spin rotators can be used to convert the longitudinal beam polarization to transverse polarization. This has inspired studies which investigate the role of transverse polarization in constraining new physics [16–18], though these studies are yet far from being exhaustive.

It was pointed out long ago by Hikasa [19] that transverse polarization can play a unique role in isolating chirality-violating couplings, to which processes with longitudinally polarized beams are not sensitive. This has been demonstrated recently in different situations [18, 20].

Polarization effects are different for chirality-conserving and chirality-violating new interactions. In the limit of vanishing electron mass, there is no interference of the chirality-violating new interactions with the chirality-conserving SM interactions. As a result, in this limit, any contribution from chirality-violating interactions which is polarization independent or dependent on longitudinal polarization also vanishes.

Transverse polarization effects for the two cases are also different. The cross terms of the SM amplitude with the amplitude from chirality-conserving interactions has a part independent of transverse polarization and a part which is bilinear in transverse polarization of the electron and positron, denoted by $P_T^e^-$ and $P_T^e^+$ respectively. For the case of chirality-violating interactions, the cross term has only terms linear in $P_T^e^-$ and $P_T^e^+$, and no contributions independent of these.

The interference of new chirality-violating contributions with the chirality-conserving SM couplings give rise to terms in the angular distribution proportional to $\sin \theta \cos \phi$ and $\sin \theta \sin \phi$, where $\theta$ and $\phi$ are the polar and azimuthal angles of a final-state particle.
Chirality-conserving new couplings, on the other hand, produce interference contributions proportional to \(\sin^2 \theta \cos 2\phi\) and \(\sin^2 \theta \sin 2\phi\). Chirality-violating contributions do not interfere with the chirality-conserving SM contribution with unpolarized or longitudinally polarized beams when the electron mass is neglected. Hence transverse polarization would enable measurement of chirality-violating couplings through the azimuthal distributions.

In what follows, we will study a process which has an \(s\)-channel contribution from scalars which violates chirality, as well as a \(t\)-channel contribution from scalars which conserves chirality. In view of the above remarks, the effects of these two kinds of contributions will be different, and it is possible to study these separately.

3. The process \(e^+e^- \rightarrow f \bar{f}\) at ILC

It is needless to say that ILC will have the ability to make precise tests of the structure of electroweak interactions at very short distances. Looking at the simplest process of \(e^+e^- \rightarrow f \bar{f}\), the SM cross-section prediction can be put in the form

\[
\frac{d\sigma(e^-_L e^+_R \rightarrow f_L \bar{f}_R)}{d \cos \theta} = \frac{\pi \alpha^2}{2s} N_C (1 + \cos \theta)^2 \times \left| Q_f + \left( \frac{1}{2} - \sin^2 \theta_w \right) \left( T^3_f - Q_f \sin^2 \theta_w \right) \frac{s}{s - m_Z^2} \right|^2
\]

where \(N_C = 1\) for leptons and 3 times for quarks, \(T^3_f\) is the weak isospin of \(f_L\), and \(Q_f\) is the electric charge. For \(f_L\) production, the \(Z\) contribution typically interferes with the photon constructively for an \(e^-_L\) beam and destructively for an \(e^-_R\) beam. Thus, initial-state polarization is a useful diagnostic at the ILC. Applied to familiar particles, they would provide a diagnostic of the electroweak exchanges that might reveal new heavy weak bosons or other types of new interactions. We focus on the case when the beams are transversely polarized and look at some specific processes which would be sensitive to couplings as discussed in the previous section.

3.1 Polarization study of \(e^+e^- \rightarrow b \bar{b}\)

We consider the process

\[
e^-(k_1, s_1) + e^+(k_2, s_2) \rightarrow b(p_1) + \bar{b}(p_2).
\]
In SM, this process proceeds via the s-channel exchange of $\gamma$ and $Z$. On including R-parity violation, the process can receive contributions from RPV couplings from the s-channel sneutrino exchange and the t-channel sfermion exchange. The representative Feynman graphs for these latter contributions are shown in Fig. 1(a) and Fig. 1(b) respectively. The s-channel diagrams, for example, involve chirality-conserving couplings for the exchange of a photon and a $Z$ and chirality-violating couplings in an s-channel exchange of a sneutrino. In the absence of any beam polarization, or with just the longitudinal polarization, these two contributions do not interfere, and the RPV couplings appear only at quartic order. With transverse polarization, the interference between the vector and scalar exchanges survive, giving rise to characteristic azimuthal distributions of the type $\cos \phi$ and $\sin \phi$, which enable discrimination of the RPV contribution from the SM contribution, whose azimuthal dependence has the form $\cos 2\phi$ and $\sin 2\phi$. Since this contribution is at quadratic order in the RPV couplings, transverse polarization leads to enhanced sensitivity to these couplings. However, in case of the $b\bar{b}$ final state we consider, the enhancement is unfortunately annulled by the suppression factor of $M_b^2/s$ arising because of the chirality-violating coupling of $b\bar{b}$ to the sneutrino. The characteristically different azimuthal distribution because of the spin-0 sneutrino, does, however, survive.

The $t$-channel sfermion exchange diagrams involving RPV couplings, on the other hand, do interfere with the SM diagrams with longitudinal or no polarization of $e^+$ and $e^-$. With transverse polarization, they give rise to azimuthal distributions of the same kind as the pure SM contributions (with terms proportional to $\cos 2\phi$ and $\sin 2\phi$. However, their contributions to the azimuthal distributions being quadratic rather than quartic in the RPV couplings, they still offer a sensitive test of these couplings.

We first write down the various terms contributing to the transition probability for the process, and then study separately the contributions of the RPV s-channel and $t$-channel exchanges. We choose the following notation for introducing the beam polarizations through the projection operators for electrons and positrons:

$$\sum_{s_1} u(k_1, s_1) \bar{u}(k_1, s_1) = \frac{1}{2}(1 + P_L\gamma_5 + \gamma_5 P_T \gamma_5) \gamma_1,$$

$$\sum_{s_2} \bar{v}(k_2, s_2) \bar{v}(k_2, s_2) = \frac{1}{2}(1 - P_L\gamma_5 + \gamma_5 P_T \gamma_5) \gamma_2,$$

where $t_{1,2}$ are the transverse polarization 4-vectors for the electron and positron beams,
respectively. In the above equation, $P_L$ and $P_T$ represent the degrees of longitudinal and transverse polarizations. For our analysis, we chose $|P_T^e^-| = 0.8$ and $|P_T^{e^+}| = 0.6$. For the transverse beam polarization 4-vectors we assume $t_1^\mu = (0, 1, 0, 0) = -t_2^\mu$.

The process of Eq. 5 is mediated by the $\gamma$ and $Z$-boson propagators in the SM. As can be seen from the RPV Lagrangian given in Eqs. 2 and 3, the sneutrinos contribute through an $s$-channel exchange only when both $\lambda$ and $\lambda'$ couplings are simultaneously non-zero. The amplitude due to the $t$-channel exchange of squarks is non-zero when only $\lambda'$ couplings are non-vanishing. It is straightforward to write down the amplitudes for the above process, and the RPV contributions are given by

\[
M_1 = -i\lambda'_{j33}\lambda_{j11} [\bar{u}(p_1)\mathcal{P}_Lv(p_2)] [\bar{v}(k_2, s_2)\mathcal{P}_Ru(k_1, s_1)] / (s - M_{\tilde{\nu}_j}^2 + i\Gamma M_{\tilde{\nu}_j}),
\]
\[
M_2 = -i\lambda'^2_{i33} [\bar{u}(p_1)\mathcal{P}_Lu(k_1, s_1)] [\bar{v}(k_2, s_2)\mathcal{P}_Rv(p_2)] / (t - M_{\tilde{u}_i}^2),
\]

where $\mathcal{P}_L, \mathcal{P}_R$ are the left and right chirality projection matrices.

The total amplitude may be written as

\[
\mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_1 + \mathcal{M}_2,
\]

where $\mathcal{M}_{\gamma,Z}$ are the amplitudes for $s$-channel $\gamma$ and $Z$ exchanges, and $\mathcal{M}_{1,2}$ given in Eq. 7 are the amplitudes for the RPV diagrams in Fig. 1. The detailed formulae for all interference terms are collected in the Appendix. We find in the interference terms, dependent on the RPV couplings quadratically rather than quartically, the characteristic $\sin \theta \cos \phi / \sin \theta \sin \phi$ dependence of the term linear in $P_T^{e^-/e^+}$. It is also interesting to note that the term $\mathcal{M}_2^*\mathcal{M}_1$
corresponding to the interference of the $s$-channel and $t$-channel RPV amplitudes is not symmetric in the $e^-$ and $e^+$ polarizations. This peculiar structure is the result of the chiral nature of the RPV couplings. Moreover, the term gives a non-zero contribution only when the electron beam has transverse polarization, and the positron beam has longitudinal (or no) polarization. The chiral nature of the RPV couplings is also reflected in the fact that the term vanishes for vanishing final state fermion mass $M_f$.

The above treatment can be very easily generalized to the case of $t\bar{t}$ production too. In that case, the $M_f^2/s$ suppression encountered in the case of the $b\bar{b}$ final state would be considerably reduced. One must however note that we do not have any $s$-channel contribution for up-type quarks in the final state, a fact which is obvious from the structure of the RPV Lagrangian given in Eq. 2 and Eq. 3.

We now focus on the contributions to the $b\bar{b}$ final state, coming from the $s$-/t-channel scalar exchange due to the RPV couplings and compare them with the SM expectations. For simplicity, we have considered the cases of only sneutrino exchange in the $s$ channel or only squark exchange in the $t$ channel. This is sufficient, as normally one considers the case where only the relevant RPV couplings are non-zero. For studying sneutrino exchange we consider only one non-vanishing (or dominant) combination of the $\lambda$ and $\lambda'$ couplings, while for the squark exchange contributions we consider only one non-vanishing (or dominant) $\lambda'$-coupling. Simultaneous presence of more than one coupling could potentially lead to flavour-changing neutral currents and hence is subject to rather stringent constraints. Of course, needless to say that in our numerical studies presented in the following subsections, we choose values of the couplings consistent with these constraints.

We have performed our numerical studies in the context of an ILC operating with a center-of-mass energy ($\sqrt{s}$) of 500 GeV and the choice of transverse polarization for the colliding beams is $(+0.8, +0.6)$. The final state fermions satisfy the kinematic cuts:

- The minimum transverse momenta $p_T^f$ of the fermions should be 20 GeV.
- The fermions should not be close to the beam pipe and must respect the angular cut of $10^\circ < \theta^f < 170^\circ$. 

3.1.1 Sneutrino exchange in the s-channel

In this section we discuss the case where the only non-zero RPV contribution to the process $e^+e^- \rightarrow b\bar{b}$ is via sneutrino exchanges in the s channel (Fig. 1a). This means that the relevant non-zero RPV coupling combination would be $\lambda_{j11}\lambda'_{j33}$, where $j$ corresponds to the sneutrino flavor ($\tilde{\nu}_j$). Due to the antisymmetry property of the $\lambda_{ijk}$ couplings in its first two indices, we know that only $j = 2, 3$ for the sneutrino flavor can contribute in the s-channel. This further ensures that the relevant $\lambda'$ couplings will be $\lambda'_{233}$ or $\lambda'_{333}$. The $\lambda'$ couplings that can contribute in the t-channel squark exchange must have the form $\lambda'_{1k3}$ where $k$ determines the squark flavor. Thus assuming one non-vanishing $\lambda'_{j33}$ and the others to be zero corresponds to the situation that when the sneutrino diagram contributes, the squark exchange diagram would not. Since we would like to restrict ourselves to a single non-zero $\lambda'$ coupling at a time, we have chosen $\lambda_{211}\lambda'_{233} \lesssim 7.2 \times 10^{-4} \left( \frac{M_{\tilde{\nu}_j}}{100 \text{ GeV}} \right)^2$, which is consistent with limits estimated from LEP for the above process.

In Fig. 2 we show the normalized differential cross section dependence on the azimuthal angle $\phi$ for the SM as well as for the excess over the SM for different values of the sneutrino mass. The coupling constants are chosen for each sneutrino mass to saturate the experimental bounds, as discussed in the text. Also shown in solid lines is the SM expectation.

Figure 2: The normalized differential cross-section for the R-parity violating contribution as a function of the azimuthal angle for different values of sneutrino mass. The coupling constants are chosen for each sneutrino mass to saturate the experimental bounds, as discussed in the text. Also shown in solid lines is the SM expectation.
mass, with the combinations of $\lambda$ and $\lambda'$ chosen to saturate the experimental bounds of Eq. 9. Thus, we have used the values $\lambda_{211}\lambda'_{233} = 0.0045, 0.0088, 0.0405$ for $M_{\tilde{\nu}_j} = 250, 350, 750$ GeV, respectively. The azimuthal angle is defined with respect to the direction of $e^-$ as the $z$ axis and the transverse polarization direction of $e^-$ as the $x$ axis. It is clear from Fig. 2 that the distribution for the SM is symmetric about $\phi = \pi$. It can also be checked that pure sneutrino exchange also produces a symmetric distribution. However, there is a marked asymmetry about $\phi = \pi$ for the interference between the SM and the RPV contributions.

We define an asymmetry which isolates the new physics contribution, given by

$$A = \frac{\sigma(0 < \phi \leq \pi) - \sigma(\pi < \phi \leq 2\pi)}{\sigma(0 < \phi \leq 2\pi)}$$  \hspace{1cm} (10)

A quick look at Fig. 2 shows that this asymmetry vanishes for the SM. We note that this azimuthal dependence for the s-channel exchange is proportional to the mass of the final state fermion, which in this case is the mass of the $b$ quark. Thus we would not have expected any azimuthal dependence if the final state had massless fermions. Note also that a sneutrino of mass 500 GeV then would be produced at the peak of a resonance. In this case the asymmetry identically vanishes as the dominant contribution comes from the direct term of sneutrino exchange. This is also highlighted in Fig. 3 where we show the asymmetry $A$ as a function of the sneutrino mass, for two different integrated luminosities, viz., $L = 500$ and 1000 fb$^{-1}$. We allow the coupling product to scale to the maximum value as allowed for that particular mass of the sneutrino, given by Eq. 9.

The figure also shows corresponding to each luminosity the asymmetry values needed to differentiate the RPV model from SM at $1 \sigma$ and $2 \sigma$ levels, and also at the $3 \sigma$ level, in case of Fig. 3(b).

We find that the asymmetry is quite sensitive to the mass of the sneutrino. It has the expected structure of a resonant term interfering with a non-resonant one. Thus it peaks for sneutrino masses which are very close to the center-of-mass energy $\sqrt{s}$ of the machine, and goes through a zero at $\sqrt{s}$ as seen in Fig. 3(a) and (b). In the asymmetry we have defined, and which is shown in Fig. 3, SM contributions appearing in the denominator are large, and hence there is a huge suppression of the asymmetry.
Figure 3: The asymmetry $A$ as defined by Eq. 10 for the signal as a function of the sneutrino mass for integrated luminosities $L = 500, 1000 \, fb^{-1}$ and for the maximum value of the product of RPV couplings $\lambda_{211}\lambda'_{233}$ for that sneutrino mass. Also shown are the SM expectation and discovery limits at 1$\sigma$, 2$\sigma$ and 3$\sigma$ levels.

3.1.2 Squark exchange in the t-channel

In this section we discuss the case where the only non-zero RPV couplings contribute to $e^+e^- \rightarrow b\bar{b}$ via $t$-channel squark exchange. In this case the non-zero RPV couplings would have the form $\lambda'_{1j3}$ where $j$ corresponds to the generation index of the exchanged squark, and the sneutrino exchange term vanishes as all $\lambda$ couplings are assumed to be zero. We have restricted our choice to RPV $\lambda'_{1j3}$ coupling which satisfies [11]

$$\lambda'_{1j3} \leq 0.02 \left( \frac{M_{\tilde{q}_j}}{100 \, GeV} \right).$$

(11)

In Fig. 4 we show by broken lines the dependence on the azimuthal angle $\phi$ of the normalized differential cross section for the excess over the SM, where the squark exchanged in the $t$-channel has a mass of 400 GeV. The solid lines represent the SM expectation, identical to what was shown in Fig. 2. It is clear that the contributions from the new physics to the azimuthal distribution are quite different from that of the SM. Also, the previously defined asymmetry vanishes identically for both the SM as well as the new physics contribution. So, to highlight the RPV contribution, we define a new asymmetry in the azimuthal angle,

$$A_Q = \frac{\sigma(0 < \phi \leq \frac{\pi}{4}) - \sigma(\frac{\pi}{4} < \phi \leq \frac{3\pi}{4}) + \sigma(\frac{3\pi}{4} < \phi \leq \frac{5\pi}{4}) - \sigma(\frac{5\pi}{4} < \phi \leq \frac{7\pi}{4}) + \sigma(\frac{7\pi}{4} < \phi \leq 2\pi)}{\sigma(0 < \phi \leq 2\pi)}$$

(12)
Figure 4: The normalized differential cross-section for the R-parity violating contribution as a function of the azimuthal angle for t-channel exchange of a squark of mass 400 GeV. Also shown in solid lines is the SM expectation.

In Fig. 5 we plot against squark mass the asymmetry $A_Q$ for both SM and the total signal (SM+RPV in the figure) corresponding to two different integrated luminosities, viz., $L = 500$ and $1000 \text{ fb}^{-1}$. The RPV coupling is allowed to scale to its maximum permissible value corresponding to the mass of the squark. The figure also shows corresponding to each luminosity the asymmetry values needed to differentiate the RPV model from SM at $1\sigma$, $2\sigma$ and $3\sigma$ levels, where use has been made of the relation

$$|A_Q - A_{SM}| = n\sqrt{\frac{1 - A_{SM}^2}{L\sigma_{SM}}}$$

for the deviation of the asymmetry $A_Q$ from the SM asymmetry $A_{SM}$ by $n\sigma$ for an SM cross section $\sigma_{SM}$ and integrated luminosity $L$. It can be seen that the defined asymmetry can easily differentiate the RPV contributions from the SM one. With the high luminosity expected to be available at the ILC, it will be able to differentiate even for very small values of RPV couplings, or equivalently, low squark masses, at the $3\sigma$ level.

4 Conclusions

We have thus investigated the special role played by transverse polarization in probing the RPV couplings at an $e^+e^-$ collider. We illustrate this with the process $e^+e^- \rightarrow b\bar{b}$. Trans-
Figure 5: The asymmetry $A_Q$ in the azimuthal distribution for $t$-channel exchange of squark as a function of the squark mass exchanged for the maximum allowed value of $\lambda$ for that squark mass for integrated luminosities $L = 500, 1000$ fb$^{-1}$. Also shown are the SM expectation and discovery limits at $1\sigma$, $2\sigma$ and $3\sigma$ levels.

verse polarization in fact allows us to construct azimuthal asymmetries which can probe contribution coming from the interference terms, dependent on the RPV couplings only quadratically. These asymmetries help us isolate the contribution coming from RPV couplings and thus offer interesting possibilities for probing them. We find, using the currently allowed maximum values of the $\lambda$ couplings, that these help us probe squark masses over a wide range much beyond the energy available at the collider. Alternatively, the increased sensitivity at lower squark masses possible for higher luminosity indicates reach to lower values of the RPV couplings beyond the current limits.

We have also compared the possible sensitivity using total cross section for the process with longitudinal polarization, with the one obtainable using azimuthal asymmetries with transversely polarized beams. We find that the sensitivity of the total cross section in case of $s$-channel sneutrino exchange is larger by a factor varying from about 10 to about 500. This can be attributed to the severe suppression coming from the $b$ mass in the azimuthal asymmetries. Having observed such an excess the next challenge is to identify the new physics responsible for it. In principle, the polar-angle distribution could be used to discriminate RPV theory from the SM and other theories like extra $Z$ models, as was done for example in [21]. However, it was seen in [21] that the sensitivity of polar distribution for a leptonic
final state is rather low, with or without longitudinal polarization. If we were to use the same strategy as that of ref. [21] we see, using the results therein, after correcting them for the values of $s$, luminosity as well as a different ($b\bar{b}$) final state, that the ratio of RPV couplings to sneutrino mass for which the polar distribution can discriminate the model is above that allowed by present experimental limits.

In case of $t$-channel squark exchange, the sensitivity of the total cross section is higher by a factor of order 5, compared to that from the azimuthal asymmetries, even though there is no suppression due to the $b$ mass. Again, extrapolating the results of Ref. [21], which considered $t$-channel sneutrino exchange for a leptonic final state to our case, we expect that the corresponding ratio of coupling to squark mass is ruled out.

The azimuthal asymmetries then show that in the region still allowed by the current data, the effects would be beyond $2\sigma$ or $3\sigma$ fluctuations of the SM expectations and hence can be probed. Thus they have a higher sensitivity to these couplings, i.e., the change in the polar-angle distributions expected with the present limits on the couplings will be not be beyond fluctuations of the SM whereas the azimuthal asymmetries would be.

In principle, if the $b$-mass suppression were not there, then transverse polarization could have provided even higher sensitivity. Nevertheless, azimuthal asymmetries we consider here have a greater reach for RPV couplings than polar distributions. Moreover, the azimuthal asymmetries we use for the $s$-channel sneutrino case are vanishing for chirality-conserving couplings in theories like extra $Z$ theories, or extra-dimensional theories with massive spin-1 or massive graviton exchange in the $s$ channel. Hence their presence would clearly discriminate the RPV theory with its chirality-violating couplings from the rest.

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6 Appendix

The amplitudes with all interference terms for the new physics signal, i.e. the RPV contributions are given explicitly as

\[
\mathcal{M}_1^a \mathcal{M}_1 = \frac{\lambda_{j33}^2 \lambda_{j11}^2}{16} \frac{1}{(s - M_{\ell j}^2)^2 + \Gamma_s^2 M_{\ell j}^2} (1 + P_L^{e+} + P_L^{e-} + P_L^{e+} P_L^{e-})(4s^2 - 5s M_f^2)
\]

\[
\mathcal{M}_2^a \mathcal{M}_1 = \frac{\lambda_{j33}^2 \lambda_{j11}^2}{8} \frac{s \sqrt{(s - M_f^2)} }{(s - M_{\ell j}^2 + i \Gamma_s M_{\ell j})(t - M_{\ell j}^2)} (1 + P_L^{e+}) P_T^{e-} M_f \sin \theta (- \cos \phi + i \sin \phi)
\]

\[
\mathcal{M}_1^a \mathcal{M}_2 = \frac{\pi Q_e Q_f \lambda_{j33}^2 \lambda_{j11}^2}{\alpha s \sqrt{(s - M_f^2)} M_f \sin \theta} \left[ P_T^{e+} (1 + P_L^{e+}) (\cos \phi + i \sin \phi) + P_T^{e-} (1 + P_L^{e-}) (\cos \phi - i \sin \phi) \right]
\]

\[
\mathcal{M}_2^a \mathcal{M}_2 = \frac{\pi \lambda_{j33}^2 \lambda_{j11}^2}{8 \alpha s_{W} c_{W} c_{W} (s - M_Z^2 - i \Gamma_Z M_Z)(s - M_{\ell j}^2 + i \Gamma M_{\ell j})} \times \left[ P_T^{+} (1 + P_L^{e+}) (c_A + c_V) (\cos \phi - i \sin \phi) - P_T^{e+} (1 + P_L^{e+}) (c_A - c_V) (\cos \phi + i \sin \phi) \right]
\]

Note that these contain the full dependence on both longitudinal and transverse polarization. It is a trivial exercise to isolate amplitudes for the individual cases, depending on the choice of beam polarization. In our study we will be interested only in the case where the beams are transversely polarized. In the above expressions, \( M_f \) stands for the fermion mass in the final state, which in this case is the \( b \)-quark mass. For the couplings of quarks and leptons to the \( Z \) boson the following notation is used:

\[
c_V = 2T_3^{e-} - 4Q_e s_W^2, \quad c_A = -2T_3^{e-},
\]
and

\[ f_V = 2T_3^f - 4Q_f s_W^2, \quad f_A = -2T_3^f. \]

For completeness we also present below the direct amplitudes in the SM, coming from the exchange of \( \gamma \) and the \( Z \) boson in the \( s \)-channel. Since the beams would be either longitudinally or transversely polarized, we give the SM amplitudes for both the cases separately.

- **SM amplitudes with longitudinally polarized beams:**

\[
|M_\gamma|^2 = \frac{Q_e^2 Q_f^2 e^4 N_c^f}{s^2} \left(1 - \frac{P_L^e P_L^{e^+}}{s}ight) \left[4u^2 + 4su + 2s^2 + 4M_f^4 - 8uM_f^2\right]
\]

\[
M_{\gamma Z}^* M_Z = \frac{Q_e^2 Q_f^2 e^4 N_c^f}{16s_W^2 c_W^2} \frac{(1 - \frac{P_L^e P_L^{e^+}}{s})c_V f_V + (P_L^e - P_L^{e^+})c_A f_A}{s (s - M_Z^2 + i\Gamma_Z M_Z)} \times \left[(4u^2 + 4su + 2s^2 + 4M_f^4 - 8uM_f^2)\right]
\]

\[
|M_Z|^2 = \left(\frac{e^4 N_c^f}{256s_W^4 c_W^4} \frac{(1 - \frac{P_L^e P_L^{e^+}}{s})}{s (s - M_Z^2)^2 + (\Gamma_Z M_Z)^2} \times \left(2s^2 + 4su - 4sM_f^2\right)\right]
\]

- **SM amplitudes with transversely polarized beams:**

\[
|M_\gamma|^2 = \frac{Q_e^2 Q_f^2 e^4 N_c^f}{s^2} \left[1 - \frac{P_T^e P_T^{e^+}}{s}\right] \left(4u^2 + 4su - 8uM_f^2 + 4M_f^4\right) + (2s^2 + P_T^e P_T^{e^+} (2sM_f^2 - 2s^2) \sin^2 \theta \cos^2 \phi)\]

\[
M_{\gamma Z}^* M_Z = \frac{Q_e^2 Q_f^2 e^4}{16s_W^2 c_W^2} \frac{N_c^f}{s (s - M_Z^2 + i\Gamma_Z M_Z)} \times \left[(1 - \frac{P_T^e P_T^{e^+}}{s})c_V f_V (4u^2 + 4su + 4M_f^4 - 8uM_f^2) - c_A f_A (2s^2 + 4su - 4sM_f^2)\right]

\]

\[
+ 2c_V f_V s^2 + c_V f_V P_T^e P_T^{e^+} (2sM_f^2 - 2s^2) \sin^2 \theta \cos^2 \phi
\]

\[
+ i c_A f_V P_T^e P_T^{e^+} (2sM_f^2 - 2s^2) \sin^2 \theta \sin \phi \cos \phi\]

\[
|M_Z|^2 = \left(\frac{e^4}{256s_W^4 c_W^4} \frac{N_c^f}{s (s - M_Z^2)^2 + (\Gamma_Z M_Z)^2} \times \left[(1 - \frac{P_T^e P_T^{e^+}}{s})(c_V^2 + c_A^2)(f_V^2 + f_A^2) (4u^2 + 4su + 4M_f^4 - 8uM_f^2)\right]\right)
\]
\[
\begin{align*}
&\left\{ f_A^2(c_V^2 + c_A^2)(2s^2 - 8sM_f^2) + 2s^2 f_V^2(c_V^2 + c_A^2) - 8c_V c_A f_V f_A (s^2 + 2su \\
&- 2sM_f^2) + P_T^- P_T^{e+} (c_V^2 + c_A^2)(f_V^2 + f_A^2)(2s^2 - 2sM_f^2) \sin^2 \theta \cos^2 \phi \right\}.
\end{align*}
\]

We have followed a notation similar to that mentioned above for the fermion couplings.

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