Quasifree pion photoproduction on the deuteron in the
\( \Delta \) region \[ \]

R. Schmidt, H. Arenhövel and P. Wilhelm
Institut für Kernphysik
Johannes Gutenberg – Universität
D–55099 Mainz, Germany
March 8, 1996

Abstract

Photo production of pions on the deuteron is studied in the spectator nucleon model. The Born terms of the elementary production amplitude are determined in pseudovector \( \pi N \) coupling and supplied with a form factor. The \( \Delta \) resonance is considered both in the \( s \) and the \( u \) channel. The parameters of the \( \Delta \) resonance and the cutoff of the form factors are fixed on the leading photoproduction multipoles. Results for total and differential cross sections are compared with experimental data. Particular attention is paid to the role of Pauli correlations of the final state nucleons in the quasifree case. The results are compared with those for pion photoproduction on the nucleon.

---

1Supported by the Deutsche Forschungsgemeinschaft (SFB 201)
1 Introduction

A very interesting topic in medium energy nuclear physics is the electromagnetic production of mesons on light nuclei in order to study possible changes of the elementary reaction in a nuclear medium. Of particular interest is the reaction on the deuteron for the following reasons. The first one is that the structure of the deuteron is well understood in comparison to heavier nuclei. Thus, the low binding energy of the deuteron allows to compare the contributions of its constituents to the electromagnetic and hadronic reactions to those from free nucleons in order to estimate off-shell effects. Secondly, the deuteron may be viewed as a neutron target in order to extract under quasifree conditions the elementary neutron amplitude with the tacit assumption that binding and final state interaction effects are small. To assess the validity of this method it is necessary to estimate the effects of mechanisms beyond this approximation quantitatively.

First investigations on pion photoproduction on the deuteron go back to the early fifties, e.g. [1] and [2], with view on the general structure of spin flip and no spin flip amplitudes. Later, a more systematic calculation was done by Laget [3,4] and Blomqvist and Laget [5]. In this work the influence of pion rescattering effects and NN final state interaction is approximatly included within a diagrammatic ansatz. However, they used different $\Delta$-parametrizations for neutral and charged pion production.

The aim of the present paper is to study pion photoproduction on the deuteron within the spectator nucleon model using a unified elementary production operator. We have taken the full on-shell amplitude of the elementary process but have neglected all kind of final state interactions and furthermore two-body operators. In view of the extraction of neutron data from experiments on the deuteron we put our emphasis on the special kinematical situation corresponding to the quasifree pion production on one nucleon. However, this kinematical condition does not necessarily imply that the pion has been produced on the active nucleon and not on the spectator. Therefore, we have also studied for this kinematical situation the influence of the second nucleon on the differential cross section due to the antisymmetrization of the final NN state.

In Sect. 2, the elementary process of pion photoproduction on the deuteron is briefly reviewed. The nonresonant amplitudes and the contribution of the $\Delta$ resonance are given. The model for the process on the deuteron is outlined in Sect. 3. Finally we present and discuss our results in Sect. 4.
2 Pion photoproduction on the nucleon

We will briefly review pion photoproduction on the nucleon \( \gamma(\vec{k}) + N(\vec{p}) \rightarrow N(\vec{p}') + \pi(\vec{q}) \) because we need the elementary production amplitude for its implementation into two-nucleon space. In this section we will first consider the elementary process in the center of mass frame \((\vec{p} = -\vec{k}, \vec{p}' = -\vec{q})\) and generalize later to an arbitrary frame for the calculations on the deuteron.

The amplitude contains besides nonresonant terms a resonance contribution from the \( \Delta(1232) \) excitation which is described in the framework of an isobar model including its \( s \) and \( u \) channel contributions. For the \( \Delta N\pi \)-vertex we use \[\text{[6, 7]}\]

\[
v_\Delta^i = -\frac{i}{m_\pi} f_{\Delta N\pi} F_\Delta(q)(-)^\mu \tau_{\Delta,\mu} \sigma_{\Delta N\pi} \cdot \vec{q}, \tag{1}\]

where \( \sigma_{\Delta N} \) and \( \tau_{\Delta N} \) are spin and isospin transition operators, respectively. We have introduced a hadronic monopole form factor

\[
F_\Delta(q) = \frac{\Lambda_\Delta^2 + q_\Delta^2}{\Lambda_\Delta^2 + q_\Delta^2}, \tag{2}\]

where \( q_\Delta \) is the c.m. pion momentum on the top of the resonance, i.e., when the invariant mass \( W_{\pi N} \) of the \( \pi N \) state equals the mass of the \( \Delta \) resonance

\[
W_{\pi N}^2 = \left( \sqrt{m_\pi^2 + q_\Delta^2} + \sqrt{M_N^2 + q_\Delta^2} \right)^2 = M_\Delta^2. \tag{3}\]

For the coupling constant, the cutoff and the \( \Delta \) mass we have used

\[
\frac{f_{\Delta N\pi}^2}{4\pi} = 0.31, \quad \Lambda_\Delta = 315\text{MeV}, \quad M_\Delta = 1232\text{MeV}. \tag{4}\]

In the description of the \( \gamma N\Delta \) vertex we follow \[\text{[8]}\]. The nonrelativistic limit of the \( N\Delta \) current matrix element contains contributions of the magnetic dipole and electric quadrupole. Since the strength of the electric quadrupole excitation is much smaller than the magnetic dipole one (see e.g. \[\text{[8]}\]) we will neglect it here. Then the \( \gamma N\Delta \) vertex has the form

\[
v_{\gamma N\Delta} = e \frac{G_{\Delta N}^{M1}}{2M_N} i\sigma_{\Delta N} \cdot \vec{k} \times \epsilon \tau_{\Delta N,0}. \tag{5}\]

Here \( \epsilon \) is the photon polarization vector. The energy dependent and complex coupling \( G_{\Delta N}^{M1} \) is taken from \[\text{[8]}\]. Using the nonrelativistic form of the \( \Delta \) propagator, one finally gets for the resonance part of the production operator

\[
t_{\gamma N} = \frac{F_\Delta(q)}{m_\pi} \left( \frac{e f_{\Delta N\pi}}{2\sqrt{E_p E_p'}} \right) \left( \frac{\tau_{\mu,\tau_0}}{3} - \frac{1}{2} \frac{[\tau_{\mu,\tau_0}]}{3} \right) \frac{\sigma_{\Delta N} \cdot \vec{q}}{E_{p'} - \omega_\gamma - E_\Delta}, \tag{6}\]

\[
+ \frac{F_\Delta(0)}{m_\pi} \left( \frac{e f_{\Delta N\pi}}{2\sqrt{E_p E_p'}} \right) \left( \frac{\tau_{\mu,\tau_0}}{3} + \frac{1}{2} \frac{[\tau_{\mu,\tau_0}]}{3} \right) \frac{\sigma_{\Delta N} \cdot \vec{q}}{E_{p'} - \omega_\gamma - E_\Delta}, \tag{6}\]

\[
W_{\pi N} - M_\Delta + \frac{i}{2} \Gamma_\Delta (W_{\pi N}), \tag{6}\]

\[
E_{p'} - \omega_\gamma - E_\Delta, \tag{6}\]

\[
E_{p'} - \omega_\gamma - E_\Delta, \tag{6}\]

\[
E_{p'} - \omega_\gamma - E_\Delta, \tag{6}\]
where $\Gamma_\Delta$ is the energy dependent width of the $\Delta$ resonance above pion threshold

$$\Gamma_\Delta(q) = \frac{1}{6\pi} \frac{M_N}{\omega q + M_N} \frac{q^3}{m^2} f_N q^2 F_\Delta^2(q). \quad (7)$$

In the $u$ channel contribution given by the second term in (6), we take the values of the form factor $F_\Delta$, the electromagnetic coupling $G_{\Delta N}$ and the width of the resonance $\Gamma_\Delta$ at pion threshold.

For the nonresonant background we used the Born termes in pseudovector $\pi N$ coupling with the coupling constant $\frac{f_{\pi N}}{4\pi} = 0.0735$ from [10]. In order to obtain a better description of the leading multipoles ($E_1^{1/2}$, $E_3^{3/2}$ and $M_3^{3/2}$) it was necessary to introduce a form factor

$$f(q) = \frac{\Lambda^2}{\Lambda^2 + q^2} \quad (8)$$

with the cutoff $\Lambda = 800$ MeV.

The parameters of the $\Delta$ resonance are fixed on the $M_1^{3/2}$ multipole. Due to the use of a constant $\Delta$ mass in the $\Delta$ propagator and a different cutoff $\Lambda_\Delta$ we had to increase $G_{\Delta N}$ from [6] by a factor of 1.15 to fit the experimental multipole. As shown in Fig. 1 the agreement of our results with the data from [9] is very good for both the real and imaginary part of the $M_1^{3/2}$ multipole up to a photon energy of about 400 MeV. We would like to note that for a better description of the real part of the $M_1^{3/2}$ multipole the suppression of the nonresonant background by the form factor was essential. The inclusion of the $u$ channel of the $\Delta$ resonance improved the description of the $M_1^{1/2}$ multipole.

In Fig. 2 the results for the other leading multipoles in the three isospin channels are also shown. The description of these multipoles is in general quite satisfactory. Due to the neglect of the $E_1^+$ excitation of the $\Delta$ resonance we fail to describe the behaviour of the $E_1^{3/2}$ multipole. Furthermore the imaginary part of all multipoles apart from the $M_1^{3/2}$ vanishes in our model because we have not included other resonances. In the upper part of Fig. 2 we show the total cross sections for pion photoproduction on the nucleon compared to experimental data. In contrast to [3] we used in our calculation the same parametrization of the $\Delta$ resonance both for neutral and charged pion photoproduction. The price we have to pay in order to achieve a good description of all physical channels is the introduction of the above mentioned form factor into the nonresonant background amplitude. The agreement with the experimental data is good up to an invariant mass of about 1300 MeV ($\omega^{lab} = 430$ Mev) in the case of $\pi^+$ production. The $\pi^-$ data are slightly overestimated in the resonance region by our calculation.
3 Pion photoproduction on the deuteron

It is reasonable to expect that the dominant process will be the quasifree reaction on one nucleon while the other acts merely as a spectator remaining at rest in the laboratory system so that the final state interaction between the nucleons and pion rescattering on the spectator nucleon may safely be neglected. In this spectator nucleon ansatz we will use the elementary amplitude discussed in Sect. 2. Due to the Fermi motion in the initial state we have to use this amplitude in a form which is not restricted to a special frame of reference. For the nonresonant background this is achieved by evaluation of the corresponding Feynman diagrams in an arbitrary frame. This leads to the following expression for the nonresonant production operator

$$\hat{t}^{\text{nonres}}_{\gamma\pi} = \frac{i f_{\pi N} f(q_{c.m.})}{m_{\pi}} \left\{ \left[ \tilde{\sigma} \cdot \tilde{e} + \frac{\tilde{q} \cdot \tilde{e} \tilde{\sigma} \cdot (\tilde{q} - \tilde{k})}{\omega_{\tilde{q} - \tilde{k}}} \left( \frac{1}{\omega_{\tilde{q} - \tilde{k}} - \omega_{\gamma}} + \frac{1}{\omega_{\gamma} - \omega_{\tilde{q} - \tilde{k}}} \right) \right] [\hat{e}, \tau^+_{\mu}] \right. $$

$$- \frac{\tau^+_{\mu} \tilde{\sigma} \cdot \tilde{q}}{2 E_{\tilde{p}' + \tilde{q}} (\omega_{\tilde{q}} + E_{\tilde{p}'(\tilde{p}' + \tilde{q})})} \left( 2 \tilde{q}' \cdot \tilde{e} + i \tilde{\sigma} \cdot \tilde{k} \times \tilde{e} (\hat{e} + \hat{k}) \right) - \frac{\tau^+_{\mu} \tilde{\sigma} \cdot \tilde{q}}{2 E_{\tilde{p}' - \tilde{k}} (E_{\tilde{p}'(\tilde{p}' - \tilde{k})} - \omega_{\gamma})} \right. $$

$$+ \frac{M_{\pi} \omega_{\tilde{q}} \tilde{\sigma} \cdot \tilde{e}}{E_{\tilde{p}' + \tilde{q}} (E_{\tilde{p}' + \tilde{q}} + E_{\tilde{p}' + \tilde{q}})} + \frac{E_{\tilde{p}' - \tilde{k}} (E_{\tilde{p}' + \tilde{k}} - (\tilde{p}' + \tilde{q} - \omega_{\gamma}))}{M_{\pi}} \right\}, \quad (9)$$

where $\hat{e}$ and $\hat{k}$ denote nucleon charge and anomalous magnetic momentum. In the resonance amplitude $t^\Delta_{\gamma\pi}$ in (1), the photon and pion momenta have to be replaced by the relative photon-nucleon momentum

$$\tilde{k}_{\gamma N} = M_{\pi} \tilde{k} - (M_{\Delta} - M_{N}) \tilde{p} $$

and respective pion-nucleon momentum

$$\tilde{q}_{\pi N} = \frac{M_{\pi} \tilde{q} - \omega_{\tilde{q}} \tilde{p}'}{M_{\pi} + \omega_{\tilde{q}}} \quad (10)$$

In the expressions of the form factors, the coupling $G_{\Delta N}^{M_1}$ and the width of the $\Delta$ resonance we use the c.m. pion momentum as given by the invariant mass of the $\pi N$ subsystem

$$W^2_{\pi N} = \left( \sqrt{M_{\pi}^2 + \tilde{p}'^2} + \sqrt{m_{\pi}^2 + \tilde{q}^2} \right)^2 - (\tilde{p}' + \tilde{q})^2 \quad \left( \sqrt{M_{\pi}^2 + \tilde{q}^2_{c.m.}} + \sqrt{m_{\pi}^2 + \tilde{q}^2_{c.m.}} \right)^2 \quad (12)$$

With $t_{\gamma\pi} = t^\Delta_{\gamma\pi} + t^{\text{nonres}}_{\gamma\pi}$ we use as pion photoproduction operator in the two-nucleon system

$$t^{NN}_{\gamma\pi} (1, 2) = t_{\gamma\pi} (1) + t_{\gamma\pi} (2) \quad (13)$$
Since the final state interaction is neglected we take plane waves for the pion and the NN-final states. For the spin ($|s_{m_s}\rangle$) and isospin ($|t_{m_t}\rangle$) part of the two nucleon wave functions we use a coupled basis. The complete antisymmetric final NN wave functions is

$$|\vec{p}_1, \vec{p}_2, s_{m_s}, t_{m_t}\rangle = \frac{1}{\sqrt{2}} \left( |\vec{p}_1\rangle^{(1)}|\vec{p}_2\rangle^{(2)} + (-)^{1+s+t} |\vec{p}_2\rangle^{(1)}|\vec{p}_1\rangle^{(2)} \right) |s_{m_s}\rangle|t_{m_t}\rangle.$$  

(14)

Only the $t = 1$ channel contributes in the case of charged pions whereas for $\pi^0$ production both $t = 0$ and $t = 1$ channels have to be taken into account. Now one easily writes down the matrix element for pion photoproduction on the deuteron

$$\langle \vec{p}_1', \vec{p}_2', s_{m_s}', t_{m_t}'|t_{NN}^{\gamma}\rangle \langle d, m_d|$$

with $t_{NN}^{\gamma}$ from [13], where we have used the covariant normalization

$$\langle \vec{p}'|\vec{p}'\rangle = (2\pi)^3 \frac{E_p}{M_N} \delta^3(\vec{p}' - \vec{p}) ,$$

(16)

$$\langle d'|d'\rangle = (2\pi)^3 2E_d \delta^3(\vec{d}' - \vec{d}) ,$$

(17)

and the deuteron wave function in the form

$$\langle \vec{p}_1', \vec{p}_2', 1m_s'|d, m_d\rangle = (2\pi)^3 \delta^3 \left( \vec{d} - \vec{p}_1' - \vec{p}_2' \right) \frac{\sqrt{E_{\vec{p}_1'} E_{\vec{p}_2'}}}{M_N} \tilde{\Psi}_{m_s, m_d}(\vec{p}) \left( \frac{1}{2} (\vec{p}_1' - \vec{p}_2') \right)$$

(18)

with

$$\tilde{\Psi}_{m_s, m_d}(\vec{p}) = (2\pi)^3 \sqrt{2E_d} \sum_{L=0,2} \sum_{m_L} i^L u_L(p) Y_{LM} Lm_L(p) (Lm_L 1m_s 1m_d) .$$

(19)

In the laboratory frame (deuteron rest frame) one finds for the matrix element the following expression

$$\langle \vec{p}_1, \vec{p}_2, s_{m_s}, t_{m_t}|t_{NN}^{\gamma}\rangle = \sqrt{2} \sum_{m_s'} \langle s_{m_s}|t_{m_t}\rangle \left( |\vec{p}_1\rangle^{(1)}|\gamma\rangle^{(1)} - |\vec{p}_2\rangle^{(1)}|\gamma\rangle^{(2)} \right) \tilde{\Psi}_{m_s', m_d}(\vec{p})$$

$$+ (-)^{1+s+t} \langle \vec{p}_2\rangle^{(1)}|\gamma\rangle^{(2)} - |\vec{p}_1\rangle^{(1)}|\gamma\rangle^{(1)} \tilde{\Psi}_{m_s', m_d}(\vec{p}) \left| 1m_s' \right> \right|00\rangle .$$

(20)

Note that in [21] all spin and isospin operators act on nucleon “1”.

The differential cross section is then given by

$$d\sigma = (2\pi)^{-5} \delta^4 (k + d - p_1 - p_2 - q) \frac{1}{|\vec{v}_\gamma - \vec{d}|} \frac{d^3 q}{2\omega q} \frac{d^3 p_1}{2E_{\vec{p}_1}} \frac{d^3 p_2}{2E_{\vec{p}_2}} \frac{M_N^2}{2\omega_N E_d} \frac{1}{6 \sum_{m_s, m_d} \sum_{m_s} \sum_{t_{m_t}} |t_{NN}^{\gamma}|^2 .}$$

(21)

We would like to mention that by decoupling the spin wave functions in [21] and performing the spin summation in [21] explicitly, we formally reproduce the expression for the differential cross section given in [3] for an uncoupled spin basis.
4 Results and discussions

Now we will present our results on pion photoproduction on the deuteron using the deuteron wave function of the Bonn potential (full model) \[1\]. The discussion is divided into two parts. In the first part, we compare with experimental data of the total cross section $\sigma_{\text{tot}}$ and the semi-exclusive differential cross section $d\sigma/d\Omega_\pi$ as a test of our model. It is obtained from the fully exclusive cross section $d^5\sigma/d\Omega_\pi dq_\pi d\Omega_P$ where $P$ is the total momentum of the two nucleons in the final state. In order to include symmetrically the contributions of both nucleons in the numerical integration, we have chosen $\Omega_P$ to be an independent quantity. In the second part, we have studied the influence of Pauli correlations in the final state on the differential cross section in a special kinematical situation which allows a direct comparison to the free nucleon process.

4.1 Comparison with experimental data

We will concentrate our discussion on $\pi^-$ photoproduction, since no data for $\pi^+$ and $\pi^0$ production in the $\Delta$ region are available. In the lower part of Fig. 2 we show the total cross sections for pion photoproduction on the deuteron which can be compared to the free nucleon case in the upper part. In the case of $\pi^-$ production on the neutron we compare our results with data from the inverse reaction $\pi^- n \rightarrow p\gamma$ \[12, 13\] and with data extracted from experiments on the deuteron \[14, 15\]. For the pion production operator of Sect. 3 we find an overestimation of the total cross section in the resonance peak for $\pi^-$ production both on the neutron and on the deuteron. For this reason we have calculated the cross sections using a nonresonant background for a lower $\pi N$ coupling constant $f_{\pi N}^2/4\pi = 0.069$. In this case the description of the experimental data in the resonance peak is good. However, for photon energies below the resonance up to about 280 MeV the theoretical results underestimate the data significantly.

The differential cross sections $d\sigma/d\Omega_\pi$ are shown in Fig. 3. The use of the pion production operator of Sect. 2 leads to an overestimation of the data for energies above 300 MeV. However, the shape of the curves is in good agreement with the experiment. Using the lower $\pi N$ coupling $f_{\pi N}^2/4\pi = 0.069$ we found a good description of the data for photon energies above 300 MeV.

4.2 Quasifree kinematics

Now we will discuss the effect of the Pauli correlations, i.e., antisymmetrization of the final NN state for quasifree kinematics. In order to compare with pion photoproduction on the nucleon
we did the calculation in the rest frame of the quasifree \( \pi N \) subsystem (see Fig. 4a) so that its contribution to the t-matrix will be the same as that of a free nucleon in the center of mass frame apart from some minor approximations which are discussed in Appendix. For the genuine quasifree case in Fig. 4a the relative momentum of the two nucleons in the initial state vanishes while for the spectator contribution in Fig. 4b, i.e., reaction on the spectator, the relative momentum is \( \vec{q} - \vec{k} \). In particular for pion production at backward angles, the contribution of the spectator nucleon will be largely suppressed by the deuteron wave function because of the large momentum mismatch, but this will not work for small pion emission angles.

In Appendix we show that the neglect of the contribution of the nucleon with momentum \( -\vec{k} \) in the final state leads to a very simple relation between the differential cross sections of the quasifree and the elementary processes, namely

\[
\frac{d^5\sigma_{q.f.}}{d^3P_{\pi N}d\Omega_\pi} = \frac{u_0^2(0)}{4\pi} \frac{d\sigma_{el.}}{d\Omega_\pi},
\]

(22)

where \( u_0(0) \) is the momentum space wave function of the s-wave at zero momentum and \( P_{\pi N} \) is the total momentum of the \( \pi N \) system. The results for the ratio of these cross sections are shown in Fig. 5. For backward pion emission the relation (22) is verified within about 1% justifying the neglect of the second nucleon for larger pion angles. However, for forward pion emission this is not any more the case. Here one finds a decrease of the cross section up to about 10% for small pion angles due to Pauli correlations. This means that even for quasifree kinematics one will get a modification of the observables for certain kinematics which is not negligible. In our opinion, this is a very important point for the extraction of neutron data from experiments on the deuteron.

5 Summary

We have investigated pion photoproduction on the deuteron in the \( \Delta(1232) \) resonance region in the spectator nucleon model neglecting final state interactions and two body processes. Within this framework, the t-matrix is given as a linear combination of the on-shell matrix elements of pion photoproduction on the two nucleons. The elementary amplitude is fitted to the pion photo production multipoles. We have presented results for total and differential cross sections. A comparison with experimental data has been possible only for \( \pi^- \) production, where we found a slight overestimation of the data which largely go back to an overestimation of the elementary reaction on the neutron.
Particular attention was paid to the quasifree case for which we have studied the effect of the antisymmetrisation of the NN-final state. Even for quasifree kinematics where the process is dominated by the reaction on one of the nucleons, we found an effect of about 10% in the differential cross section due to presence of the second nucleon.

The differences between the theoretical results and the experimental data show very clearly that the calculation of pion photoproduction on nuclei in the nucleon spectator model can only be considered as a first step towards a more realistic description of this process. The studies discussed here will serve as the basis for further investigations including the dynamics of the $\pi$NN system in a more satisfactory way.
Appendix  Evaluation of Equation (22)

Here we will give a brief derivation of equation (22) and a discussion of the approximations used. The general expression for the exclusive differential cross section is given in (21). Evaluating the $\delta$-functions with respect to the independent kinematical quantities $\vec{P}_{\pi N}$ and $\Omega_\pi$, one finds in the frame of reference defined in Fig. 4

$$d^5\sigma = \frac{1}{(2\pi)^5} \frac{1}{(E_d + 2\omega_\gamma)(E_{\vec{q}} + \omega_{\vec{q}})} \frac{M_N^2}{E_{\vec{k}}} \frac{d^3 P_{\pi N} d\Omega_\pi}{8\pi} \frac{1}{6} \sum_{m_\gamma, m_d, s, m_s} |t^{NN}_{\pi\gamma}|^2. \quad (A.1)$$

Taking into account only the contribution of the nucleon with momentum $-\vec{q}$ in the final state the corresponding t-matrix $\tilde{t}^{NN}_{\pi\gamma}$ is given by

$$\langle -\vec{q}, -\vec{k}, s_m | \tilde{t}^{NN}_{\pi\gamma} | d = -2\vec{k}, m_d \rangle = \pi \sqrt{2E_d} u_0(0) \sum_{m_1, m'_1} \langle -\vec{q}, m'_1 | t_{\pi\gamma} | -\vec{k}, m_1 \rangle \times (1/2 m'_1, 1/2 m'_1) (1/2 m_1, 1/2 m_1 | 1 m_d). \quad (A.2)$$

Performing the spin summation in (A.1) explicitly one finds

$$\frac{1}{6} \sum_{m_d, s_m} |t^{NN}_{\pi\gamma}|^2 = \pi^2 u_0^2(0) E_d \sum_{m_1 m'_1} |t_{\pi\gamma}|^2, \quad (A.3)$$

where $m_1$ and $m'_1$ are the spin projections of the nucleon in the initial and final states, respectively. Due to the small binding energy of the deuteron we can use the approximation

$$\frac{2\sqrt{M_N^2 + \vec{k}^2}}{\sqrt{M_N^2 + (2\vec{k})^2}} \sim 1. \quad (A.4)$$

In the c.m. frame, the invariant mass of the $\pi N$ system is given by

$$W_{\pi N} = \omega_\gamma + E_{\vec{k}} = \omega_{\vec{q}} + E_{\vec{q}}. \quad (A.5)$$

From (A.1) with the help of (A.3) it is now easy to verify equation (22)

$$\frac{d^5\sigma}{d^3 P_{\pi N} d\Omega_\pi} = \frac{1}{64\pi^2} \frac{|\vec{q}| M_N^2}{\omega_\gamma W_{\pi N}^2} \frac{u_0^2(0)}{4\pi} \sum_{m_i m_f} |t_{\pi\gamma}|^2 \quad (A.6)$$

Here a second approximation is used implicitly because we compare the elementary and the quasifree process for a given invariant mass of the $\pi N$ system. Due to the binding of the nucleons in the deuteron there is a small difference in the corresponding photon energies $\omega_{\vec{q}, f}$.  

10
and $\omega^l$. Expanding the ratio of the photon energies for a given $W_{\pi N}$ with respect to the relative binding energy $\epsilon = (2M_N - M_d)/M_N \sim 0.0024$ yields

$$\frac{\omega^{q.f.}}{\omega^{e.l.}} = 1 + \frac{M_N^4 + 6 M_N^2 W_{\pi N}^2 + W_{\pi N}^4}{4 W_{\pi N}^2 (W_{\pi N}^2 - M_N^2)} \epsilon + O(\epsilon^2). \quad (A.7)$$

The coefficient of the linear term is about 6.3 at pion threshold and decreases with increasing $W_{\pi N}$. Thus this approximation is very well justified.
References

[1] Chew, G. F., Lewis, H. W.: Phys. Rev. **84**, 779 (1951)

[2] Lax, M., Feshbach, H.: Phys. Rev. **88**, 509 (1952)

[3] Laget, J.-M.: Nucl. Phys. **A296**, 388 (1977)

[4] Laget, J.-M.: Phys. Rep. **69**, 1 (1981)

[5] Blomqvist, I., Laget, J. M.: Nucl. Phys. **A280**, 405 (1977)

[6] Wilhelm, P., Arenhövel, H.: Nucl. Phys. **A593**, 435 (1995)

[7] Weber, H. J., Arenhövel, H.: Phys. Rep. **36**, 278 (1978)

[8] Davidson, R. M., Mukhopadhyay, N. C., Wittman, R. S.: Phys. Rev. **D43**, 71 (1991)

[9] Scattering Analysis Interactive Dial-In (SAID), Virginia Polytechnic Institute, Blacksburg, Virginia

[10] Arndt, R. A., Zhujun Li, Roper, L. D., Workman, R. L.: Phys. Rev. Lett. **65**, 157 (1990)

[11] Machleidt, R., Holinde, K., Elster, C.: Phys. Rep. **149**, 1 (1987)

[12] Comiso, J. C., Blasberg, D. J., Haddock, R. P., Nefkens, B. M. K., Truoel, P., Verhey, L. J.: Phys. Rev. **D12**, 719 (1975)

[13] Salomon, M., Measday, D. F., Poutissou, J.-M.: Nucl. Phys. **B414**, 493 (1984)

[14] von Holtey, G., Knopp, G., Stein, H., Stümpfig, J., Wahlen, H.: Nucl. Phys. **B70**, 379 (1974)

[15] Fujii, T., Kondo, T., Takasaki, F., Yamada, S., Homma, S., Huke, K., Kato, S., Okuno, H., Endo, I., Fujii, H.: Nucl. Phys. **B120**, 395 (1977)

[16] Fischer, G., von Holtey, G., Knop, G., Stümpfig, J.: Z. Phys. **253**, 38 (1972)
[17] Benz, P., Braun, O., Butenschön, H., Finger, H., Gall, D., Idschok, U., Kiesling, C., Knies, G., Kowalski, H., Müller, K., Nellen, B., Schiffer, R., Schlamp, P., Schnackers, H. J., Schulz, V., Söding, P., Spitzer, H., Stiewe, J., F.Storim, Weigl, J.: Nucl. Phys. B65, 158 (1973)

[18] Chiefari, G., Drago, E., Napolitano, M., Sciacca, C.: Lett. Nuovo Cim. 13, 129 (1975)

[19] Asai, M., Endo, I., Harada, M., Hasai, H., Ito, H., Iwatani, K., Kasai, S., Kato, S., Maki, T., Maruyama, K., Murata, Y., Muto, M., Niki, K., Rangacharyulu, C., Shimizu, H., Sumi, Y., Wada, Y., Yoshida, K.: Phys. Rev. C42, 837 (1990)
Figure 1: The s- and p-wave multipoles in the isospin 3/2, 1/2 and 0 channels. The full curve shows the real part of the multipoles, the dash-dotted one the imaginary part of the $M_{1+}^{3/2}$ for our parametrization of the $\Delta$ resonance. The dotted curve shows the result for the calculation without form factor in the Born terms and for $M_{1-}^{1/2}$ the dashed one when only the s channel of the $\Delta$ resonance is taken into account. The data are from [3].
Figure 2: Total cross sections for pion photoproduction on the nucleon and on the deuteron. The full curve shows the result of the full calculation, the long dashed the contribution of the $\Delta$ resonance. The dotted curve shows the full result using the rescaled nonresonant background. The experimental data are from [12] (✱), [13] (▵), [14] (+), [15] (◦) and [16] (•) for the reaction on the nucleon and from [17] (◇), [18] (⋆), and [19] (×) for pion photoproduction on the deuteron.
Figure 3: Differential cross section $d\sigma/d\Omega_\pi$ for $\gamma d \rightarrow pp\pi^-$. See caption of Fig. 2 for the meaning of the curves. The experimental data are from [17].

Figure 4: The contributions of the two nucleons in quasifree kinematics: (a) quasifree case, (b) spectator contribution.
Figure 5: The ratio of the differential cross sections of $\gamma d \rightarrow NN\pi$ and $\gamma N \rightarrow N\pi$. The full curve shows the result for the full calculation on the deuteron, the dashed curve only the contribution of the nucleon which is distinguished by the quasifree kinematic (Fig. 4a).