Investigation of mixed convection magnetized Casson nanomaterial flow with activation energy and gyrotactic microorganisms

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Abstract

Present article addresses mixed convection magnetohydrodynamic Casson nanomaterial flow by stretchable cylinder. The effects of thermal, solutal and motile density stratifications at the boundary of the surface are accounted. Flow governing expressions are acquired considering aspects of permeability, thermal radiation, chemical reaction, viscous dissipation and activation energy. The obtained flow model is made dimensionless through transformations and then tackled by NDsolve code in Mathematica. Physical impacts of sundry variables on nanomaterial velocity, temperature distribution, volume fraction of microorganisms and mass concentration is investigated through plots. Furthermore, quantities of engineering interest like surface drag force, heat transfer rate, density number and Sherwood number are computed and analyzed. We observed that fluid velocity diminishes for higher curvature variable, Casson fluid material variable, Hartmann number and permeability parameter. Fluid temperature has a direct relation with Eckert number, thermophoresis variable, Brownian dispersal parameter, Prandtl number and Hartmann number. Volume fraction of gyrotactic microorganisms is decreasing function of bioconvection Lewis number, stratification parameter and bioconvection Peclet number. Detailed observations are itemized at the end.

Nomenclature

\( (u, w) \) \hspace{1cm} velocity components

\( \beta \) \hspace{1cm} Casson fluid parameter

\( u_0 \) \hspace{1cm} positive constant

\( \rho \) \hspace{1cm} density of fluid

\( C_0 \) \hspace{1cm} reference concentration

\( \mu \) \hspace{1cm} dynamic viscosity

\( \sigma \) \hspace{1cm} electrical conductivity

\( Sc \) \hspace{1cm} Schmidt number

\( C_p \) \hspace{1cm} specific heat

\( T \) \hspace{1cm} nanofluid temperature

\( \varphi(\eta) \) \hspace{1cm} dimensionless concentration

\( \tau \) \hspace{1cm} heat capacity ratio

\( Q_0 \) \hspace{1cm} heat generation coefficient

\( f'(/eta) \) \hspace{1cm} dimensionless velocity

\( (r, x) \) \hspace{1cm} coordinates of cylinder

\( g \) \hspace{1cm} gravitational acceleration
\( C \)  
nanofluid concentration

\( \theta(\eta) \)  
dimensionless temperature

\( B_0 \)  
magnitude of magnetic field

\( q_m \)  
local mass flux

\( q_w \)  
surface heat flux

\( \rho_p \)  
density of nanoparticles

\( \rho_f \)  
density of nanofluid

\( \rho_m \)  
density of microorganism

\( K_r^2 \)  
reaction rate parameter

\( E_a \)  
activation energy

\( W_c \)  
maximum cell swimming speed,

\( \delta_h \)  
heat generation parameter

\( S_2 \)  
mass stratified number

\( K^* \)  
curvature parameter

\( G_c \)  
solutal Grashof number

\( K_l \)  
porosity parameter

\( \gamma \)  
reaction rate parameter

\( Ec \)  
Eckert number

\( Le \)  
Lewis number

\( Pr \)  
Prandtl number

\( Lb \)  
bioconvection Lewis number

\( \nu \)  
kinematic viscosity

\( q_w \)  
is used for surface heat flux

\( \tau_w \)  
surface shear stress

\( T_w \)  
wall temperature

\( C_\infty \)  
ambient fluid concentration

\( T_0 \)  
reference temperature

\( T_\infty \)  
ambient fluid temperature

\( l \)  
characteristic length of stretching cylinder

\( T_0 \)  
reference temperature

\( N_\infty \)  
ambient concentration of microorganisms

\( Ha \)  
Hartmann number

\( N_t \)  
thermophoresis parameter

\( \chi(\eta) \)  
dimensionless microorganisms concentration

\( Nb \)  
Brownian motion parameter

\( C_w \)  
wall concentration

\( \alpha \)  
thermal diffusivity

\( \gamma \)  
chemical reaction parameter

\( D_T \)  
thermophoresis diffusion coefficient

\( q_n \)  
wall motile microorganisms flux

\( N_w \)  
wall concentration of microorganisms

\( \tau_w \)  
surface shear stress

\( K^* \)  
permeability of porous medium

\( \beta_T \)  
thermal expansion coefficient,

\( N \)  
nanofluid concentration of microorganisms

\( (a_1, b_1, c_1, a_2, b_2, c_2) \)  
constants
1. Introduction

Nanoﬂuids are engineered by colloidal suspension of nanometer-sized (1–100 nm) solid particles in carrier liquids. Solid particles of metals, carbides, oxides, or carbon nanotubes are suspended in conventional base liquids such as water, oils and ethylene glycol. Choi [1] was the ﬁrst who introduced the notion of nanoﬂuid and proved that thermal conductivity of carrier liquids can signiﬁcantly improve by addition of man-sized particles. In recent years nanoﬂuids got attention of researchers and scientists due to their practical usages in various ﬁelds of science and engineering such as extraction of geothermal power, nuclear reactors, industrial cooling, automotive, electronics, biomedical and energy sources. Non-Newtonian nanoﬂuids like paint, blood, melted butter, honey, paste, custard and various types of pints which hold relationship of nonlinear stress-strain. Casson model is an important model because it exhibits non-Newtonian behavior below a critical shear stress and above that it behaves like an elastic solid. Examples of Casson ﬂuid include sauces and juices, blood and honey. Due to its dual nature and potential usages in daily life, numerous researchers investigated Casson ﬂuid [2–10].

The least amount of energy required to initiate a chemical reaction is named as activation energy (AE). Svante Arrhenius in 1889 introduces the notion of AE. The AE has vital practical usages in formation of medicine, oil and mixtures. Bestman [11] explores the impact of AE on heat transmission performance with in boundary layer ﬂow by a porous surface. Simultaneous impacts of AE and bioconvection phenomenon on radiative cross nanoﬂuid is analyzed by Azam et al [12]. Ullah et al [13] explored features of nanoﬂuid model and Maxwell nanoﬂuid model is an important model because it exhibits non-Newtonian behavior below a critical shear stress and above that it behaves like an elastic solid. Examples of Casson ﬂuid include sauces and juices, blood and honey. Due to its dual nature and potential usages in daily life, numerous researchers investigated Casson ﬂuid [2–10].

1.1. Nanoﬂuid

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Phenomenon of bioconvection in nanoﬂuid occurs when self-propelled gyrotactic microorganisms are added. Gyrotactic microorganisms are less dense as compared to the base liquids and move vertically upward direction. When density of microorganisms in upper surface of base liquid increases density stratification generates and microorganisms fall down. Return up swimming of these microorganisms retains status quo in process of bioconvection in the liquid. Bioconvection is useful in various ﬁelds of science like bio-microsystems, biomedicine and microbial ﬂuid cells. Stability analysis in process of bioconvection in ﬂuid ﬂow that contains both solid small particles and microorganisms is reported by Kuznetsov and Avramenko [21]. Xun et al [22] explored the features of temperature dependent bioconvection in a rotating frame with variable thermal conductivity. Mahdy and Nabwey [23] examined heat transfer characteristics in bio-nanoﬂuid in presence of...
stagnation point with zero mass flux boundary constraints. Impact of multi slip conditions on bioconvective flow of magnetized Carreau nanomaterial with thermal radiation is inspected by Elayarani et al [24]. Ramzan et al [25] investigated the irreversibility in bioconvective nano fluid together with CNT’s past a permeable vertical cone with heat generation/absorption and Joule heating. Some more studies in this regard can be seen in [26–30].

The main theme of current exploration is to scrutinize the stratified mixed convection Casson fluid flow with gyrotactic microorganisms and solid nanoparticles. The effects of viscous dissipation, heat generation and permeability are accounted. Furthermore, activation energy, and chemical reaction are considered in concentration relation. Ndsolve function [25, 30] in Mathematica software is executed to examine the behavior of flow.

2. Formulation

Here, we investigated flow of magnetized Casson nanofluid by stretched surface of cylinder. Flow contains gyrotactic microorganisms and solid nano-sized particles. Impacts of heat generation, viscous dissipation and mixed convection are considered. Stratification impacts at the boundary are further assimilated. Moreover mass concentration relation is modeled in view of chemical reaction with Arrhenius energy. Characteristics of the flow are inspected in (r, x) system of coordinates (see figure 1). The governing boundary layer model equations for bioconvective Casson nanomaterial are

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{w}{r} + \frac{\partial w}{\partial r} &= 0, \\
\frac{u}{r} \frac{\partial u}{\partial x} + \frac{w}{r} \frac{\partial u}{\partial r} &= \nu \left( 1 + \frac{1}{\beta} \right) \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - \frac{\sigma \beta f u}{\rho} + g \beta f (1 - C_{\infty})(T - T_{\infty}) - \\
&\quad \frac{g}{\rho_f} [\beta_f (\rho_p - \rho_f) (C - C_{\infty}) + \beta_w (\rho_m - \rho_f) (N - N_{\infty})] - \frac{\nu}{K^2} u,
\end{align*}
\]

Figure 1. Flow geometry and coordinate system.
\[
\begin{align*}
\frac{\partial}{\partial t} u + \frac{\partial}{\partial x} u &= \alpha \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) + \tau \left[ D_N \frac{\partial}{\partial r} \left( \frac{\partial U}{\partial r} \right) + \frac{2}{r} \frac{\partial U}{\partial r} \right]^2 \\
- \frac{\partial}{\partial t} \frac{\partial}{\partial x} u^2 - \frac{\nu}{\rho} \left( 1 + \frac{1}{\beta} \right) \left( \frac{\partial u}{\partial x} \right)^2 + \frac{D_N}{C_0} (T - T_\infty),
\end{align*}
\]

\[
\frac{\partial}{\partial t} \phi + \frac{\partial}{\partial x} \phi = D_N \left[ \frac{\partial}{\partial r} \left( \frac{\partial \phi}{\partial r} \right) + \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} \right) + \frac{\partial^2 \phi}{\partial x^2} \right] + \frac{\partial^2}{\partial r^2} \phi^2 + K \phi^3 (C - C_\infty) \left( \frac{T - T_\infty}{T_\infty - T_0} \right),
\]

\[
\begin{align*}
\eta &= \sqrt{\frac{u}{v} \frac{\partial}{\partial x} u - \frac{R}{\partial r} f(\eta)}, & u &= u_0 f(\eta),
\end{align*}
\]

Considering

\[
\begin{align*}
\eta &= \sqrt{\frac{u}{v} \frac{\partial}{\partial x} u - \frac{R}{\partial r} f(\eta)}, & \theta(\eta) &= \frac{T - T_\infty}{T_\infty - T_0}, & \varphi(\eta) &= C - C_\infty, & \chi(\eta) &= \frac{N - N_\infty}{N_0 - N_\infty}.
\end{align*}
\]

In view of (7), continuity expression (1) is identically satisfied whereas (2–6) can be expressed as follows:

\[
\begin{align*}
\frac{f'''}{f''} &= \frac{f''}{f'} + \left( 1 + \frac{1}{\beta} \right) \left[ (1 + 2K\eta) f'' + 2Kf'' \right] \\
- H_a \frac{f''}{f'} + K \frac{f''}{f'} + G_c (\theta - G_c \varphi - R_b \chi) &= 0,
\end{align*}
\]

\[
\begin{align*}
\text{Pr}(f'') &= f''(0) = -S_1 f' - \delta f'' + \left( 1 + \frac{R}{\beta} \right) \left( 1 + 2K\eta \right) \theta'' - \left( 1 + \frac{1}{\beta} \right) \left( 1 + 2K\eta \right) \varphi'' + 2K \chi'' \left( 1 + 2K\eta \right) \theta'' + 2K \varphi'' + 2K \chi'' = 0,
\end{align*}
\]

\[
\begin{align*}
\left( 1 + 2K\eta \right) \varphi'' + 2K \varphi'' + \frac{N_0}{N_b} \left( 1 + 2K\eta \right) \theta'' + 2K \theta'' \right) \\
+ \text{LePr}(f' f'' - \varphi' f'' - S_2 f' f'') - \text{Sc} \gamma (1 + \beta \eta) \frac{\partial}{\partial r} \varphi = 0,
\end{align*}
\]

with

\[
\begin{align*}
f(0) &= 0, & f'(0) &= 1, & \theta(0) &= 1 - S_1, & \varphi(0) &= 1 - S_2, & \chi(0) &= 1 - S_3, & \text{at } r = R, \\
f'(\infty) &\rightarrow 0, & \theta(\infty) &\rightarrow 0, & \varphi(\infty) &\rightarrow 0, & \chi(\infty) &\rightarrow 0, & \text{as } \eta &\rightarrow \infty.
\end{align*}
\]

Mathematically

\[
\begin{align*}
K &= \frac{1}{\alpha} \frac{v}{u} \frac{\partial}{\partial x} u, & R &= \frac{4\alpha \beta \gamma}{K}, & G_c &= \frac{\beta (1 - C_{\infty}) (T_{\infty} - T_0)}{2\alpha \beta \gamma}, & \text{Pr} &= \frac{\mu c}{k}, & S_2 &= \frac{b_2}{b_1},
\end{align*}
\]

\[
\begin{align*}
S_1 &= \frac{u_1}{u_0}, & \gamma &= \frac{K}{u_0}, & N_t &= \frac{\partial \beta \gamma}{\partial \gamma} (T_{\infty} - T_0), & G_c &= \frac{\beta (1 - C_{\infty}) (T_{\infty} - T_0)}{2\alpha \beta \gamma}, & S_3 &= \frac{b_3}{b_1},
\end{align*}
\]

\[
\begin{align*}
N_b &= \frac{\beta (1 - C_{\infty}) (T_{\infty} - T_0)}{2\alpha \beta \gamma}, & R_b &= \frac{\beta (1 - C_{\infty}) (N_\infty - N_0)}{2\alpha \beta \gamma}, & E_c &= \frac{\partial \beta \gamma}{\partial \gamma} (T_{\infty} - T_0), & \text{Le} &= \frac{\alpha}{D_1}, & \delta &= \frac{T_{\infty} - T_0}{T_{\infty} - T_0},
\end{align*}
\]

\[
\begin{align*}
\text{Pr} &= \frac{\mu}{\alpha}, & H_{ad} &= \frac{\beta \gamma}{\rho \beta}, & S_c &= \frac{\mu c}{k}, & E_1 &= \frac{E_c}{T_{\infty}}, & L_e &= \frac{\alpha}{D_1}, & \delta_h &= \frac{\beta (1 - C_{\infty}) (T_{\infty} - T_0)}{2\alpha \beta \gamma}, & \Omega &= \frac{N_\infty}{N_\infty - N_0},
\end{align*}
\]
Table 1. Computational values of $-\theta'(0)$ for different values of $\text{Pr}$ and $b = 1.0$, while other parameters are set equal to zero.

| $\text{Pr}$ | Farooq et al [9] | Chamkha et al [31] | Present study |
|------------|------------------|-------------------|---------------|
| 1.0        | 0.332097         | 0.332173          | 0.332281      |
| 10         | 0.728170         | 0.72831           | 0.72902       |
| 100        | 1.571894         | 1.57218           | 1.57329       |
| 1000       | 3.387265         | 0.38699           | 0.38905       |

Figure 2. Sketches of $f'(\eta)$ for $\beta$.

Figure 3. Sketches of $f'(\eta)$ for $K^*$. 
2.1. Physical quantities
Heat transfer rate ($N\text{u}_x$), surface drag force ($C_f$), density number ($N\text{tt}_x$) and mass transfer rate ($S\text{h}_x$) are addressed as:

\[
\begin{align*}
C_f &= \frac{2\text{r}_x}{\mu_x} \\
N\text{u}_x &= \frac{\text{q}_x}{k(\text{r}_x - \text{r}_0)} \\
S\text{h}_x &= \frac{\text{q}_x}{\rho_x \frac{\partial \text{c}_x}{\partial \xi}(\text{c}_a - \text{c}_\beta)} \\
N\text{tt}_x &= \frac{\text{q}_x}{\rho_x \frac{\partial \text{c}_x}{\partial \xi}(\text{c}_a - \text{c}_\beta)}
\end{align*}
\]

(14)
where

\[
\begin{align*}
\tau_w &= \left(1 + \frac{1}{3}\right) \left(\frac{\partial u}{\partial r}\right)_{r=R}, \quad q_w = -k \left(\frac{\partial T}{\partial r}\right)_{r=R}, \\
q_m &= -D_a \left(\frac{\partial C}{\partial r}\right)_{r=R}, \quad q_n = -D_n \left(\frac{\partial N}{\partial r}\right)_{r=R}. \quad (15)
\end{align*}
\]

Non dimensional forms are

\[
\begin{align*}
\frac{1}{2} \tau_x \text{Re}^+ &= \left(1 + \frac{1}{3}\right) f''(0), \quad Sh_x \text{Re}^+ = -\varphi'(0), \\
Nu_x \text{Re}^+ &= -\left(1 + \frac{1}{3}\right) \theta'(0), \quad Nn \text{Re}^+ = -\chi'(0). \quad (16)
\end{align*}
\]

In which \( \text{Re}_x \left(= \frac{u_x R}{v}\right) \) symbolizes the local Reynolds number.
3. Numerical solution

This section concerns to examine the impact of various sundry variables on Casson fluid velocity\( (f'(\eta)) \), temperature\( (\theta(\eta)) \), mass concentration\( (\phi(\eta)) \) and concentration of microorganisms\( (\chi(\eta)) \). Interesting physical quantities i.e. surface drag force\( (C_f) \), heat transfer rate\( (Nu_e) \), mass transfer rate\( (Sh_e) \) and density number\( (Nn_e) \) are numerically computed. Graphical and numerical computations are acquired with the help of Mathematica taking,\[ \beta = \delta_k = S_1 = S_3 = Lb = \Omega = \delta = 0.1, Nb = 0.3, Sc = 0.7, Pr = 2.5, \\
K^* = E_3 = Le = \gamma = 1.0, Gc = Rb = S_2 = 0.2, Pe = K_i = R = 0.5, m_1 = 2.0, Ha = Nt = Gt = 0.2 \]
and\( Ec = 0.4 \).
4. Validation of numerical solution

Table 1 is constructed to compare the current numerical results with already published results. It is observed from table 1 that the present numerical results are in good agreement with Farooq et al [9] and Chamkha et al[31].

5. Discussion

5.1. Velocity profile

This subsection is arranged to study the behavior of velocity field ($f'(\eta)$) against involved variables like ($\beta$), ($K^*$), ($Ha$), ($K_i$), ($Gc$), ($Gt$) and ($Rb$). Figure 2 discloses the impact of fluid material variable on ($f'(\eta)$). It is
observed here that velocity profile decays for greater values of \((\beta)\). In fact plastic dynamic viscosity of liquid increases via higher Casson fluid parameter values. Due to which internal resistance in the fluid raises, therefore velocity reduces. Figure 3 deliberates the behavior of \((K^*)\) on \((f'(\eta)))\). It is perceived from figure 3 that \((f'(\eta)))\) reduces for rising \((K^*)\). Figure 4 delineated to investigate the behavior of \((Gc)\) on \((f'(\eta)))\), increments in \((Gc)\) decreases velocity graphs. Figure 5 depicts the impact of \((Gt)\) on velocity profile. Increasing values of \((Gt)\) enhances the velocity profile. Figure 6 shows that velocity profile decays in view of higher \((Ha)\) estimation. Since \((Ha)\) is linked with Lorentz force which is basically a resistive force and opposes the fluid flow. Therefore \((f'(\eta)))\) and layer thickness decays for higher \((Ha)\). Physical effect of permeability variable is captured in figure 7. It is clear from figure 7 that \((f'(\eta)))\) has inverse relation with permeability of the surface. In fact for higher \((K_l)\) resistance between fluid and surface increases and consequently fluid velocity diminishes. Figure 8 demonstrates
the characteristics of $Rb$ on ($f'(\eta)$), it noticed here that velocity profile retards when $Rb$ takes higher approximations.

5.2. Temperature profile
Salient characteristics of $(Ec), (\delta_h), (S_i), (R), (Ha), (Nb), (Nt)$ and $(Pr)$ on $(\theta(\eta))$ is examined in figures 9–16. Improvement in $\theta(\eta)$ is observed for higher $(Ec)$ (see figure 9). Physically, enthalpy difference reduces and kinetic energy of nanomaterial improves for rising $(Ec)$, therefore $\theta(\eta)$ decays. Behavior of $\theta(\eta)$ for variable values of heat generation is studied in figure 10, here we examined that temperature curves enhances for higher $(\delta_h)$. Since for higher $(\delta_h)$ more energy is supplied to the fluid, due to which $\theta(\eta)$ enhances. Figure 11 shows that temperature field diminished in view rising $(S_i)$. In fact difference between surface and ambient fluid regularly reduces, so $(\theta(\eta))$ decreases. Figure 12 characterizes the performance of thermal radiation parameter on $(\theta(\eta))$.
As expected \((\theta(\eta))\) and layer thickness boosts up for larger estimation of \((R)\). Temperature profile rises for higher \((Ha)\) (see figure 13). Physically, amplification in \((Ha)\) produces extra Lorentz force and that provides additional heat to the system. Consequently \((\theta(\eta))\) enhances. Figures 14 and 15 shows that \((\theta(\eta))\) accelerates via \((Nb)\) and \((Nt)\). It is due to the fact that higher \((Nb)\) enhances nanoparticles inter collision and higher \((Nt)\) boosts repulsion process from hot surface to ambient, therefore \((\theta(\eta))\) and layer thickness upsurges. Impact of \((Pr)\) on \((\theta(\eta))\) is captured in figure 16, here retardation in \((\theta(\eta))\) for selected values of \((Pr)\) is inspected. Since higher \((Pr)\) decays thermal diffusivity within Casson nanoliquid and thus \((\theta(\eta))\) and relevant layer decays.

5.3. Concentration profile
This segment is devoted to analyze the impact of \((\gamma), (S_2), (Le), (Nb), (Nt), (Sc), (Pr)\) and \((E_1)\) on mass concentration \((\varphi(\eta))\) in figures 17–24. Figure 17 explores that \((\varphi(\eta))\) declines via rising \((\gamma)\). Since reactive
species rapidly spreads with in the fluid, due to which \((\varphi(\eta))\) decays. Figure 18 illustrates the impact of \(S_2\) on concentration profile. Due to greater solutal stratification values concentration of fluid reduces. Basically for greater \(S_2\) concentration difference between the surface and ambient liquid decreases, therefore concentration profile reduces. The impact of Lewis number is captured in figure 19. Mass concentrations curves decays versus rising \((Le)\). Since higher \((Le)\) improves thermal diffusivity but mass diffusivity with in fluid retards and thus \((\varphi(\eta))\) and concentration layer thickness decreases. Physical behavior of \((\varphi(\eta))\) in view of rising \((Nb)\) and \((Nt)\) is depicted in figures 20 and 21. Clearly \((Nb)\) and \((Nt)\) have opposite impact on \((\varphi(\eta))\). Salient features of \((Sc)\) and \((Pr)\) on \((\varphi(\eta))\) is reported in figures 22 and 23. From these figures, we noticed that \((Sc)\) and \((Pr)\) have similar influence on \((\varphi(\eta))\) figure 24 explores that \((\varphi(\eta))\) boosts slightly for selected estimations of \(E_1\). In fact revised Arrhenius function reduces as \(E_1\) grows, which effects the reproductive chemical reaction and thus \((\varphi(\eta))\) and solutal layer boosted.

Figure 18. Sketches of \(\varphi(\eta)\) for \(S_2\)

Figure 19. Sketches of \(\varphi(\eta)\) for \(Le\).
5.4. Motile density

Figures 25–29 are plotted to investigate the physical effect of \(S_j\), \(Lb\), \(Pe\), \(K^*\) and \(\Omega\) on motile density/volume fraction of microorganisms \(\chi(\eta)\). Figure 25 is sketched to examine the behavior of \(\Omega\) on microorganisms concentration. Here we observed that \(\chi(\eta)\) of Casson fluid decays with increasing \(\Omega\). Since volume fraction of microorganisms in ambient liquid groves via rising \(\Omega\) and thus density of microorganisms diminished. Figure 26 is prepared to investigate the features of \(K^*\) on microorganisms concentration in Casson fluid. Here, we observed that \(\chi(\eta)\) retards in view of larger \(K^*\). For higher estimations of \(Lb\), a decay in \(\chi(\eta)\) is observed from figure 27. Physically, higher \(Lb\) causes diffusivity of microorganisms in fluid and consequently \(\chi(\eta)\) retards. From figure 28, one can clearly noticed that \(\chi(\eta)\) reduces in view of larger \(Pe\).
Figure 29 signifies the behavior of S₃ on motile density. Clearly (χ(η)) reduces versus rising S₃, for higher S₃ variation in microorganisms concentration between surface and ambient liquid minimizes. Consequently motile density and thickness of motile density layer decays.

5.5. Physical quantities
In this section salient features of surface drag force (Cfₓ), heat transfer rate (Nuₓ), mass transfer rate (Shₓ) and density number (Nnx) against involved parameters is investigated through tables 2–5. Variation in (Cfₓ) for rising (β), (K'), (Ha), (K₁), (Gr) (Gc) and (Rb) is displayed in table 2. Here, we examined that (Cfₓ) raises for higher estimations of (K'), (Ha), (K₁), (Gc) and (Rb) while reverse impact of (β) and (Gr) on (Cfₓ) is perceived.
Table 3 is constructed to scrutinize the effect of \( Ec \), \( Ha \), \( Nb \), \( Nt \), \( Pr \), \( R \), \( \delta_h \), \( S_i \), \( \beta \) and \( K^* \) on heat transfer rate. From Table 3, one can observe that \( Nux \) upshots for higher \( Pr \), \( R \) and \( \beta \) whereas diminished against \( Ec \), \( Ha \), \( Nb \), \( Nt \), \( R \), \( S_1 \) and \( \delta_h \).

Table 4 illustrates the infiltration in \( Sh_x \) versus sundry variables \( g \), \( K^* \), \( Ec \), \( Nb \), \( Nt \), \( Sc \), \( Le \), \( S_1 \), \( n_1 \) and \( Pr \). Clearly mass transfer rate enhances for greater \( n_1 \), \( Ec \), \( K^* \), \( Nb \), \( Sc \) and \( Le \) whereas surface mass transfer rate reduces when \( Nt \), \( Pr \), \( S_2 \) and \( Ec \) are increased. Variation in \( Nux \) against \( K^* \), \( Lb \), \( Pe \), \( S_i \) and \( \Omega \) is depicted in Table 5. It is noticed here that density number decays for selected values of \( K^* \), \( Lb \), \( Pe \), \( S_i \) and \( \Omega \).
6. Conclusions

The summarized findings of current investigation are as follows

- Velocity field is decreasing function of ($K^*$), ($G_s$), ($\beta$), ($Ha$), ($K_1$) and ($R_b$).
- Fluid temperature improves for higher ($Ec$), ($\delta_h$), ($Ha$), ($Nb$), ($Nt$), ($Pr$) and ($R$) but opposite trend in ($\theta(\eta)$) via ($S_1$) is noticed.

Figure 26. Sketches of $\chi(\eta)$ for $K^*$.

Figure 27. Sketches of $\chi(\eta)$ for $Lb$. 
Mass concentration shows increasing behavior for higher $\text{Nb}$, $\text{Pr}$ and $E_1$ whereas diminished versus $\gamma$, $\text{Le}$, $\text{Nt}$, $\text{Sc}$, and $S_2$.

Motile density is reducing function of $L b$, $P e$, $\text{W}$, $S_3$ and $K^\ast$.

Magnitude of $C f'$ decays for higher estimations of $\beta$ and $G t$ while it boost ups for higher values of $K^\ast$, $H a$, $K_1$, $G_1$ and $R_1$.

Intensity of $N u$ decreases against $E c$, $H a$, $\text{Nb}$, $\text{Nt}$, $\delta_h$, $S_1$ and $\beta$.

$S h_x$ improves with as increasing $n_i$, $\gamma$, $K^\ast$, $\text{Nb}$, $\text{Sc}$ and $\text{Le}$ while decreases with increasing estimations of $\text{Nt}$, $\text{Pr}$, $S_3$ and $E_1$.
Table 2. Numerical simulations for ($C_f$) via ($\beta$, ($Ha$), ($K_e$), ($G_c$) ($G$) and ($Rb$).

| $\beta$ | $K_e$ | $Ha$ | $K_i$ | $G_i$ | $G_c$ | $Rb$ | $\frac{1}{\nu_c R_e^{\frac{1}{2}}}$ |
|---------|-------|------|-------|-------|-------|------|-----------------|
| 0.1     | 1.0   | 0.2  | 0.5   | 0.2   | 0.2   | 0.1  | 1.80287         |
| 0.2     | 1.0   | 0.2  | 0.5   | 0.2   | 0.2   | 0.1  | 2.67304         |
| 0.3     | 2.0   | 1.0  | 1.5   | 0.4   | 0.8   | 0.4  | 3.35782         |
|         | 3.0   |      |       |       |       |      | 9.35148         |

Table 3. Numerical simulations for ($Nu_a$) versus ($E_c$), ($Nb$), ($Nt$), ($Ha$), ($Pr$), ($h_0$), ($S$), ($R$), ($\beta$) and ($K_e$).

| Pr | $R$ | $K_e$ | $Ha$ | $E_c$ | $Nt$ | $Nb$ | $h_0$ | $\beta$ | $Nu_a R_e^{\frac{1}{2}}$ |
|----|-----|-------|------|-------|------|------|-------|---------|-----------------|
| 0.5| 0.5 | 1.0   | 0.2  | 0.4   | 0.2  | 0.3  | 0.1   | 0.1     | 1.80287         |
| 1.0| 1.0 | 1.0   | 0.2  | 0.4   | 0.2  | 0.3  | 0.1   | 0.1     | 2.67304         |
| 1.5| 1.0 | 1.0   | 0.2  | 0.4   | 0.2  | 0.3  | 0.1   | 0.1     | 3.35344         |

Table 4. Numerical simulations for ($Sh_a$) versus $\gamma$, $K_e$, $E_c$, $Nb$, $Nt$, $Sc$, $Le$, $S_2$, $n_1$ and $Pr$.

| $\gamma$ | $K_e$ | $E_c$ | $Nb$ | $Nt$ | $Sc$ | $Le$ | $Pr$ | $S_2$ | $n_1$ | $Sh_a R_e^{\frac{1}{2}}$ |
|----------|-------|-------|------|------|------|------|------|-------|-------|-----------------|
| 1.0      | 1.0   | 1.0   | 0.3  | 0.2  | 0.7  | 1.0  | 0.5  | 0.2   | 2.0   | 0.584953        |
| 2.0      | 2.0   | 2.0   | 0.6  | 0.9  | 0.3  | 0.4  | 0.9  | 1.2   | 2.0   | 0.759621        |
| 3.0      | 3.0   | 3.0   | 0.6  | 0.9  | 0.3  | 0.4  | 0.9  | 1.2   | 2.0   | 0.834331        |
Density number is increasing function of (Lb), (Pe), (Ω) and (K').

Data availability statement

No new data were created or analysed in this study.

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