MACHINE INTERFERENCE PROBLEM INVOLVING UNSUCCESSFUL SWITCHOVER FOR A GROUP OF REPAIRABLE SERVERS WITH VACATIONS

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Abstract. The purpose of this paper is to explore the multiple-server machine interference problem with standby unsuccessful switchover and Bernoulli vacation schedule. Failure times of operating and standby machines are assumed to have exponential distributions and repair times of the failed machines and vacation times of servers are also assumed to have exponential distributions. After the completion of service, the server either goes for a vacation or may continue serving for the next machine. The vacation policy we considered is a single vacation policy. In practice, the switchover may experience a significant failure. The matrix analytical method and recursive method are employed to obtain the steady-state probability vectors, and closed-form expressions of some important system characteristics are obtained. The problem of cost optimization dealt with a number of numerical examples is provided by the Quasi-Newton method, the pattern search method, and the Nelder-Mead simplex direct search method. Expressions of various system characteristics are derived. Sensitivity analysis is performed numerically for system parameters. This paper presents the first time that machine interference problem with unsuccessful switchover for a group of repairable servers with vacations has been obtained, which is quite useful for the decision makers.

Nomenclature. The notations used in this paper are described as follows.

- \( M \) Number of machines that are operating
- \( W \) Number of machines are available as standbys
- \( S \) Number of repairmen who provide the repair service to the failed machines
- \( \lambda \) Mean failure rate of operating machines
- \( \alpha \) Mean failure rate of standbys
- \( \mu \) Mean repair rate
- \( \delta \) Mean vacation rate
- \( \theta \) Probability of switching rate
- \( \beta \) Probability that after repair completion of the failed machines, the repairman may take a vacation of random length

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The stationary probability that there are \( n \) failed machines in the system and \( i \) vacationing repairman, \( i = 0, 1, \ldots, S, n = 0, 1, \ldots, M + W \)

- \( E[F] \): Expected number of failed machines in the system
- \( E[O] \): Expected number of operating machines in the system
- \( E[W] \): Expected number of standby machines in the system
- \( E[V] \): Expected number of vacationing repairmen in the system
- \( E[I] \): Expected number of idle repairmen in the system
- \( E[B] \): Expected number of busy repairmen in the system

- \( E_{SR} \): Average switching failure rate
- \( E_{MA} \): Machine availability
- \( E_{OU} \): Operating utilization (the fraction of busy servers)
- \( Av \): The steady-state probability that at least \( M \) machines are in operation and function properly (system availability).

- \( R_h \): Cost per unit time per failed machine in the system
- \( R_W \): Cost per unit time when one machine is acting as a standby
- \( R_b \): Cost per unit time for a busy repairman
- \( R_i \): Cost per unit time for an idle repairman
- \( R_v \): Cost per unit time for a vacationing repairman
- \( R_f \): Cost per unit time for a standby machine having switching failure
- \( R_s \): Fixed cost for every repair rate
- \( R_a \): Cost per unit time of the availability shortage
- \( TAC \): The average cost function

1. **Introduction.** The machine interference problems (MIP) represent a group of very important problems used to analyze computer network, telecommunications, and aircraft maintenance. As for vacation policy for MIPs, Gupta [3] first proposed the M/M/1 MIP with spares and server vacations in which the vacation policies involve vacations, single vacation and hybrid vacation scheme. Some researchers who have incorporated this concept in their works include Yue et al. [32], Ke and Wang [14], Chakravarthy [26], Ke et al. [10][11], Wang et al. [27], Maheshwari et al. [22], Ke and Wu [15], Shrivastava and Mishra [25], Choudhury and Ke [2]. Recently, Wu and Ke [30] studied a MIP in which repairmen operate a \((V, R)\) vacation policy. With such policy, if the number of the failed machines is reduced to \( R-V \) at a service completion, these \( V \) idle servers will together take a synchronous vacation. Keren et al. [16] proposed a model for a special case of the MIP where each of machines randomly requests several different service types. Chen and Wang [1] adopting the Laplace transform technique to analyze the system reliability of the retrial MIP with a controllable server. Using the supplementary variable technique, Yang and Chang [31] proposed a MIP with constant retrial policy, wherein if a failed machine finds the repairman busy upon arrival, it enters into an orbit. Wang et al. [29] studied optimization analysis of retrial MIP with sever breakdown and threshold recovery policy. Kumar et al. [17] dealt with MIP with F-policy and a repair facility that has two unreliable servers and the provision of warm standbys. He et al. [4] discussed a MIP with repairman’s single working vacation. They employed Markov process theory and matrix analytical method to obtain various system performance measures.

In daily life, the switchover may not succeed with certain probability due to some reasons. Lewis [20] first introduced the concept of the unsuccessful standby switchover in the reliability with the standby system. Wang et al. [28] compared
the steady-state availability of four different repairable systems with the standby
components, reboot delay and standby unsuccessful switchover. Huang et al. [7]
applied a parametric nonlinear programming approach to investigate a repairable
system with switching failure and fuzzy parameters. Ke et al. [9] explored the
reliability and sensitivity analysis of a system when switching to primary units may
fail. Using the bootstrap method, Liu et al. [21] investigated the availability for a
repairable system with the unsuccessful switchover. Ke et al. [8] studied reliability
measures of a repairable system with standby unsuccessful switchover and reboot
delay. Hsu et al. [5] discussed a standby system with general repair, reboot delay,
unsuccessful switchover, and unreliable repair facility. Hsu et al. [6] examined an
\( M/M/R \) MIP with a switching failure probability, reboot delay, and repair pressure
coefficient. Sadjadi and Soltani [23] have investigated a robust redundancy allo-
cation problem with the choice of redundancy strategy and component type. Kuo
and Ke [18] analyzed the availability of a series system with unsuccessful switchover
and unreliable server under the steady state condition. Lee [19] considered a redund-
cy model with generally distributed repair times, unsuccessful switchover and
interrupted repairs. Ke et al. [12] utilized the supplementary variable technique to
study an \( M/G/1 \) MIP with the unsuccessful switchover. Shekhar et al. [24] studied
the performance modeling and reliability analysis of a redundant machining system
composed of several functional machines. Ke et al. [13] using the method of the
supplementary variable to study an \( M/G/1 \) MIP with an unreliable repairman and
imperfect switchover of standbys.

Our study is motivated by some practical systems. For instance, there is an
application of Cyber-Physical System (CPS) located in computer numerical control
(CNC) of machine tools in industry. Our model is inspired by some behaviors of
CPS. We focused on MIP with unsuccessful switchover activation policy for a group
of repairable servers with vacations. And CPS is a combination of computing,
networking, and physical processes. The CPS benefits CNC Machine to make the
operating more efficient and safer. As well it reduces the cost of building these
machines and allows individual machines to form complex arrangement. In the
systems, some CNC machine tools are active and the others are standby. The
information of the machines’ status can be presented to the repairmen through
CPS. CPS has Information Management System (IMS) that includes machine health
awareness analytics, data mining, and etc. To keep the system functioning well,
CPS provides the following functions: detect, locate, and analyze the information
of the machines’ status and switch the standby machine to be active and notify
repairmen for maintenance and vacation activities. However, the switchover is not
always successful. This may be caused by some issues, such as software or hardware
failure on CPS. When none of machines fail, the repairmen may take a vacation
or deals other work or continuous in the system. Therefore, our model will help
the engineers or managers who design a CPS with CNC to control the system more
stably and effectively.

The aim of this paper contains the following aspects. Firstly, we will use the
computer software MATLAB to solve steady-state solutions for the MIP with standby
unsuccessful switchover and Bernoulli vacation schedule. The results are applied
to get the various system characteristics, such as the expected number of failed
machines, the expected number of operating and standby machines, the expected
number of idle, busy and vacationing repairmen, average switching failure rate, ma-
chine availability and operative utilization, etc. Secondly, we will make the present
system economically viable by constructing a cost function to decide the optimum values of some system parameters at a minimum cost. We adopt three heuristic search techniques to obtain the optimum values and perform a comparative analysis among them. The main contribution of the article lies in two important problems. First of all, as we know, the more system features we take into consideration simultaneously, the more difficult we are in system modeling and equation derivation. The present model has not been investigated in the literature so far. Secondly, we adopt three heuristic search techniques to obtain the optimum values and perform a comparative analysis among them. The results give very useful and helpful information for someone who wants to perform optimization works.

The paper is structured as follows. In the next section, we describe the MIP with standby unsuccessful switchover and Bernoulli vacation schedule in detail. In Section 3, a quasi-birth-and-death (QBD) process and its infinitesimal generator are established. Furthermore, a matrix-geometric approach is utilized to obtain the stationary probabilities. In Section 4, the formulae for some important system characteristics are derived in terms of matrix form. In Section 5, we develop a structure of average cost. Three heuristic search techniques are implemented to optimize an approximate solution at a minimum cost. Furthermore, a comparison is made among three techniques. Finally, in Section 6, we make some concluding remarks.

2. Model description. Initially, we consider a MIP with standby unsuccessful switchover where the servers follow the Bernoulli vacation policy. The detailed description of this model is as below. The system consists of \( L = M + W \) homogeneous machines with \( M \) machines operating and \( W \) machines standbys. The failure times of operating and standby machines are assumed to obey exponential distribution with rates \( \lambda \) and \( \alpha \) \((0 \leq \alpha \leq \lambda)\). When an operating machine fails, a standby machine if available is immediately substituted for it. If the standby machine switches over successfully, its characteristics are the same as those of an operating machine. However, the switchover from a failed operating machine to a standby machine may fail with probability \( \theta \). If a standby machine does not switch successfully, another standby machine if available attempts to switch. This process continues while switching is successful or all standby machines are exhausted. The failed machine (either operating or standby) is immediately sent to the repair facility for repair. There are \( S \) repairmen who provide the repair service to the failed machines. The time to repair a failed machine is according to an exponential distribution with mean \( 1/\mu \). Each repairman can only repair one failed machine at a time. When the repair of the failed machine is completed, the repairman checks the system state and determines whether to leave for a vacation. If the number of the failed machines is less than the available repairmen, the repairman may take a vacation of random length with probability \( \beta \) or continue to repair with probability \( 1 - \beta \). Otherwise, the repairman always waits in the system for repair. The vacation times are assumed to follow exponential distribution with rate \( \delta \). The vacation policy is single vacation policy. That is, after completing a vacation, the repairman backs to repair the failed machines or stays idly in the system.

3. Matrix-geometric Analysis. Now we focus on the stationary analysis by using the matrix-geometric approach. The state of the system can be characterized by the pairs \( E = \{(i, n); i = 0, 1, \ldots, S, n = 0, 1, \ldots, L\} \), where \( i \) indicates the number
of vacationing servers and $n$ represents the number of failed machines in the system. 
The mean failure rate and mean repair rate of the machines are defined as

\[
    \lambda_n = \begin{cases} 
        M\lambda(1 - \theta) + (W - n)\alpha, & 0 \leq n \leq W - 1 \\
        (L - n)\lambda, & W \leq n \leq L 
    \end{cases}
\]

and

\[
    \mu_n = \begin{cases} 
        n\mu, & 1 \leq n \leq S - i - 1 \\
        (S - i)\mu, & S - i \leq n \leq L 
    \end{cases}
\]

Denote $\pi_{i,n}$, $(i,n) \in E$, by the stationary distribution. Then, the infinitesimal generator $P$ can be partitioned as follows:

\[
P = \begin{bmatrix} 
    Y_0 & X_0 & Z_1 & Y_1 & X_1 & \cdots & \cdots & \cdots & Z_{S-1} & Y_{S-1} & X_{S-1} \\
    Z_1 & Y_1 & X_1 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
    \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
    Z_{S-1} & Y_{S-1} & X_{S-1} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
    Z_S & Y_S & X_S & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots 
\end{bmatrix}
\]

(1)

Each entry of the matrix $P$ is a matrix with dimension $(L + 1) \times (L + 1)$ and described as:

\[
    Y_i = \begin{bmatrix} 
        -x_0 & \lambda_0 & a_1 & \cdots & a_{W-1} & M\lambda\theta^W \\
        y_1 & -x_1 & \lambda_1 & \cdots & a_{W-2} & M\lambda\theta^{W-1} \\
        y_2 & -x_2 & \cdots & \cdots & \cdots & \cdots \\
        y_3 & \cdots & a_1 & \cdots & M\lambda\theta^2 \\
        \vdots & \vdots & \vdots & \ddots & \ddots & \ddots \\
        y_W & \cdots & -x_W & -\lambda_W & \cdots & \cdots \\
        y_{W+1} & \cdots & -x_{W+1} & -\lambda_{W+1} & \cdots & \cdots \\
        y_{W+2} & \cdots & \cdots & \cdots & \cdots & \cdots \\
        y_{W+3} & \cdots & \cdots & \cdots & \cdots & \cdots \\
        \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
        y_{L-1} & \cdots & -x_{L-1} & \cdots & \cdots & \cdots \\
        y_L & \cdots & \cdots & \cdots & \cdots & \cdots \\
        \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    \end{bmatrix}
\]

\[
    X_i = \begin{bmatrix} 
        0 & \beta\mu_1 & 0 & \cdots & \cdots \\
        \beta\mu_1 & 0 & \cdots & \cdots & \cdots \\
        \beta\mu_2 & \cdots & 0 & \cdots & \cdots \\
        \cdots & \cdots & \cdots & \cdots & \cdots \\
        \beta\mu_{S-1} & 0 & \cdots & \cdots & \cdots \\
        0 & \cdots & \cdots & \cdots & \cdots \\
        \cdots & \cdots & \cdots & \cdots & \cdots \\
        0 & \cdots & \cdots & \cdots & \cdots \\
        0 & 0 & \cdots & \cdots & \cdots \\
    \end{bmatrix}
\]

\[
    Z_i = i\delta I_{(L+1)}
\]

\[
a_j = M\lambda\theta^j(1 - \theta)
\]

\[
x_0^i = M\lambda + W\alpha + i\delta
\]
\[ x_j^i = \begin{cases} M\lambda_j + (W - j)\alpha_j + \mu_j + i\delta_j, & j = 1, 2, \ldots, W \\ (L - j)\lambda_j + \mu_j + i\delta_j, & j = W + 1, W + 2, \ldots, L \end{cases} \]
\[ y_j^i = \begin{cases} (1 - \beta)\mu_j^i, & j = 1, 2, \ldots, S - i \\ \mu_j^i, & j = S - i + 1, S - i + 2, \ldots, L \end{cases} \]

and \( \mathbf{I}_{(k)} \) is the identity matrix with order \( k \).

Let \( \Pi = [\Pi_0, \Pi_1, \ldots, \Pi_S] \) represent the corresponding steady-state probability vector of \( \mathbf{P} \) where \( \Pi_i = [\pi_{i,0}, \pi_{i,1}, \ldots, \pi_{i,L}], i = 0, 1, \ldots, S \). Thus, the steady-state equations \( \Pi \mathbf{P} = 0 \) yields the following results:

\[ \Pi_0 Y_0 + \Pi_1 Z_1 = 0, \]
\[ \Pi_{i-1} X_{i-1} + \Pi_i Y_i + \Pi_{i+1} Z_{i+1} = 0, \quad 1 \leq i \leq S - 1 \]
\[ \Pi_{S-1} X_{S-1} + \Pi_S Y_S = 0. \]

The above equations can be rewritten as follows:

\[ \Pi_i = \Pi_{i-1} H_i, \quad 1 \leq i \leq S \]
\[ \Pi_0 Y_0 + \Pi_0 H_1 Z_1 = 0 \]

where \( H_i = \begin{cases} -X_{i-1}(Y_i + H_{i+1} Z_{i+1})^{-1}, & 1 \leq i \leq S - i \\ X_{S-1}(-Y_S)^{-1}, & i = S \end{cases} \)

Consequently, \( \Pi_0 \) can be determined by solving Eq. (6) and the following normalizing condition:

\[ \sum_{i=0}^{S} \Pi_i \mathbf{e} = \Pi_0 \left[ \mathbf{I} + \sum_{i=1}^{S} \Omega_i \right] \mathbf{e} = 1 \]

where \( \Omega_i = H_1 \mathbf{H}_2 \cdots \mathbf{H}_i \) and \( \mathbf{e} \) is a column vector with each component equal to one. Once \( \Pi_0 \) being determined, the steady-state probabilities \( \Pi_i (i = 1, 2, \ldots, S) \) can be computed from Eq. (5).

4. **System characteristics.** In order to evaluate the performance of the proposed system, we present the explicit expressions for various system characteristics in the following:

1. Expected number of failed machines in the system is:

\[ E[F] = \sum_{i=0}^{S} \sum_{n=0}^{L} n\pi_{i,n} = \Pi_0 \left[ \mathbf{I} + \sum_{i=1}^{S} \Omega_i \right] [0, 1, \cdots, L]^T. \]

2. Expected number of operating machines in the system is:

\[ E[O] = M - \sum_{i=0}^{S} \sum_{n=W+1}^{L} (n - W)\pi_{i,n} \]
\[ = M - \Pi_0 \left[ \mathbf{I} + \sum_{i=1}^{S} \Omega_i \right] \begin{bmatrix} W+1 \\ 0, \cdots, 0, 1, 2, \cdots, M \end{bmatrix}^T. \]

3. Expected number of standby machines in the system is:

\[ E[W] = \sum_{i=0}^{S} \sum_{n=0}^{W} (W - n)\pi_{i,n} \]
\[ \Pi_0 = \left[ I + \sum_{i=1}^{S} \Omega_i \right] \left[ \begin{array}{c} W, W - 1, \ldots, 0, \ldots, 0 \end{array} \right]^T. \]  

4. Expected number of vacationing repairmen in the system is:
\[ E[V] = \sum_{i=0}^{S} \sum_{n=0}^{S-i} i \pi_{i,n} = \Pi_0 \sum_{i=1}^{S} \Omega_i \left[ \begin{array}{c} \# = S - i + 1, \ldots, \# = L - S + 1 \end{array} \right] \left[ \begin{array}{c} 1, \ldots, 1, 0, \ldots, 0 \end{array} \right]^T. \]

5. Expected number of idle repairmen in the system is:
\[ E[I] = \sum_{i=0}^{S} \sum_{n=0}^{S-i} (S - i - n) \pi_{i,n} = \Pi_0 [S, S - 1, \ldots, 0, \ldots, 0]^T + \Pi_0 \sum_{i=1}^{S} \Omega_i \left[ \max\{S - i, 0\}, \max\{S - i - 1, 0\}, \ldots, 0, \ldots, 0 \right]^T. \]

6. Expected number of busy repairmen in the system is:
\[ E[B] = S - E[V] - E[I]. \]

7. Average switching failure rate of the standby machines in the system is
\[ E_{SR} = \sum_{i=0}^{S} \sum_{n=1}^{W} M \lambda \theta \pi_{i,n-1} = M \lambda \theta \Pi_0 \left[ I + \sum_{i=1}^{S} \Omega_i \right] \left[ \begin{array}{c} \# = W, \ldots, \# = M + 1 \end{array} \right] \left[ \begin{array}{c} 1, \ldots, 1, 0, \ldots, 0 \end{array} \right]^T. \]

8. Machine availability is
\[ E_{MA} = 1 - \frac{E[F]}{L}. \]

9. Operating utilization (the fraction of busy servers) is
\[ E_{OU} = \frac{E[B]}{S}. \]

5. **Cost analysis.** The machines will be automatically controlled the maintenance and repair strategy depending on the degree of the server’s workload and ensure backup capacities to maintain the production. To assure the machines to run stably and effectively is a very important issue to the manufacturing. We simulate some behaviors of CPS with CNC machine tools for intelligent manufacturing and use them in our model in our study. The number of standby machines and the server’s behavior are worth to be considered. The average cost function per machine per unit time is developed in which the number of standby machines, the number of servers, the mean repair rate and the mean vacation rate are decision variables. Following the formation of the average cost function, our main aim is to determine the optimum number of standby machines, the optimum number of servers and the optimum value of repair rate and vacation rate so as to minimize this function. Through the optimum value of the parameters from the system with the repair rate and vacation rate of the servers, the managers or the engineers can design their repair system in the manufacturing and improve the system efficiency. The following cost elements associated with different activities are considered:
\[ R_h \equiv \text{cost per unit time per failed machine in the system}, \]
Based on the above cost elements and the corresponding system characteristics, the average cost function is constructed as follows:

\[
TAC = \frac{1}{2} \{R_h E[F] + R_w E[W] + R_b E[B] + R_i E[I] - R_v E[V] + R_f E_{SR} + R_s \mu + R_a (0.9 - \bar{A})\}
\]

where \(A \bar{v} = \sum_{i=0}^{S} \sum_{n=0}^{W} \pi_{i,n}\) is the steady-state probability that at least \(M\) machines are in operation and function properly (system availability), and the value “0.9” represents an acceptable level of availability.

Due to the fact that this cost function is a nonlinear and highly complex expression, it is difficult to obtain the optimal solution. To overcome this, a two-step search approach is utilized. First, given the discrete decision variables \((W, S)\), the Quasi-Newton (QN) method, the pattern search (PS) method and the Nelder-Mead simplex direct search (NM) method are employed to search the approximate optimum solution of the continuous decision variables. Next, based on this solution, the direct search method is used to obtain the optimal value \((W^*, S^*, \mu^*, \delta^*)\) to minimize the average cost function.

To demonstrate this search procedure, a numerical example is presented in Table 1. The default values of different cost elements are \(R_h = 150, R_w = 125, R_b = 100, R_i = 85, R_v = 60, R_f = 80, R_s = 75, R_a = 150\). Other parameters include \(M = 6, \lambda = 0.5, \alpha = 0.2, \theta = 0.02, \beta = 0.02\). For convince, we assume that the numbers of repairmen and standby machines are bounded by a positive integer \(2M\), respectively. For each given \((W, S)\), we use the Quasi-Newton method to obtain the optimal value \((\mu^*, \delta^*)\) and the corresponding \(TAC\) is presented in Table 1. Then, for each possible \(S\), we find the optimal value \(W\) to minimize \(TAC\). We summarize these results as a set of minimum cost values for \(S = 1, 2, \ldots, 2M\). Finally, we can find \(S^*\) directly from this set. Therefore, one can find the optimal value \((W^*, S^*, \mu^*, \delta^*) = (2, 2, 2.75, 3.07)\) and the corresponding cost function is 90.90.

Also, we vary the values of \(\mu\) and \(\delta\), consider \((W, S) = (2, 2)\) and the values of \(\mu\) and \(\delta\) range from 0.1 to 10. Figure 1 depicts the numerical results of \(TAC\). From Figure 1, we obtain the optimal solution is \((\mu^*, \delta^*) = (4.3, 2.85)\) and the corresponding \(TAC\) is 655.0928.

We now perform a comparison among QN method, PS method and NM method. The default parameters for the numerical results displayed in Table 2 are set as follows:

\[
M = 6, \lambda = 0.2, 0.4, 0.6, \alpha = 0.2, \theta = 0.02, \beta = 0.02.
\]

Table 2 shows that the optimal solutions and the corresponding \(TAC\) obtained by these three methods are almost identical. Moreover, one also can see that the
Table 1. The average cost function for given \((W, S)\) with \(M = 6, \lambda = 0.5, \alpha = 0.2\lambda, \theta = 0.02, \beta = 0.02\).

| \((S, W)\) | (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) | (1,7) | (1,8) |
|---|---|---|---|---|---|---|---|---|
| \(TAC\) | 99.72 | 98.43 | 99.20 | 100.74 | 102.52 | 104.32 | 106.04 | 107.65 |
| \((S, W)\) | (1,9) | (1,10) | (1,11) | (1,12) | (2,1) | (2,2) | (2,3) | (2,4) |
| \(TAC\) | 96.70 | 98.92 | 100.97 | 102.83 | 104.51 | 106.02 | 107.39 | 108.62 |
| \((S, W)\) | (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3,6) | (3,7) | (3,8) |
| \(TAC\) | 97.43 | 94.99 | 96.04 | 97.96 | 100.03 | 102.00 | 103.83 | 105.47 |
| \((S, W)\) | (3,9) | (3,10) | (3,11) | (3,12) | (4,1) | (4,2) | (4,3) | (4,4) |
| \(TAC\) | 106.95 | 108.28 | 109.46 | 110.53 | 98.44 | 97.75 | 99.75 | 99.94 |
| \((S, W)\) | (4,5) | (4,6) | (4,7) | (4,8) | (4,9) | (4,10) | (4,11) | (4,12) |
| \(TAC\) | 102.17 | 104.22 | 106.04 | 107.65 | 109.07 | 111.45 | 112.44 | 113.31 |
| \((S, W)\) | (5,9) | (5,10) | (5,11) | (5,12) | (6,1) | (6,2) | (6,3) | (6,4) |
| \(TAC\) | 109.33 | 110.60 | 111.72 | 112.72 | 95.48 | 94.17 | 96.15 | 98.83 |
| \((S, W)\) | (6,5) | (6,6) | (6,7) | (6,8) | (6,9) | (6,10) | (6,11) | (6,12) |
| \(TAC\) | 101.39 | 103.66 | 105.63 | 107.35 | 108.84 | 110.16 | 111.33 | 112.37 |
| \((S, W)\) | (7,2) | (7,3) | (7,4) | (7,5) | (7,6) | (7,7) | (7,8) | (7,9) |
| \(TAC\) | 91.92 | 94.27 | 97.24 | 100.01 | 102.43 | 104.53 | 106.35 | 107.93 |
| \((S, W)\) | (7,10) | (7,11) | (7,12) | (8,3) | (8,4) | (8,5) | (8,6) | (8,7) |
| \(TAC\) | 109.32 | 110.54 | 111.64 | 91.97 | 95.24 | 98.25 | 100.85 | 103.10 |
| \((S, W)\) | (8,8) | (8,9) | (8,10) | (8,11) | (8,12) | (9,4) | (9,5) | (9,6) |
| \(TAC\) | 105.03 | 106.71 | 108.19 | 109.49 | 110.65 | 92.94 | 96.20 | 99.01 |
| \((S, W)\) | (9,7) | (9,8) | (9,9) | (9,10) | (9,11) | (9,12) | (10,5) | (10,6) |
| \(TAC\) | 101.41 | 103.48 | 105.28 | 106.85 | 108.24 | 109.48 | 93.93 | 96.95 |
| \((S, W)\) | (10,7) | (10,8) | (10,9) | (10,10) | (10,11) | (10,12) | (11,6) | (11,7) |
| \(TAC\) | 99.53 | 101.75 | 103.67 | 105.36 | 106.84 | 108.16 | 94.73 | 97.49 |
| \((S, W)\) | (11,8) | (11,9) | (11,10) | (11,11) | (11,12) | (12,7) | (12,8) | (12,9) |
| \(TAC\) | 99.87 | 101.92 | 103.72 | 105.31 | 106.71 | 95.32 | 97.86 | 100.06 |
| \((S, W)\) | (12,10) | (12,11) | (12,12) |
| \(TAC\) | 101.98 | 103.67 | 105.17 |

6. Conclusions. This study carried out an analysis of a machine interference problem with standby unsuccessful switchover and Bernoulli vacation schedule. Utilizing matrix-geometric methods, it established the stationary distribution of the system. In addition, we present the explicit expressions for various system characteristics in the matrix form. An average cost function per unit time was constructed to optimize the number of standbys, the number of servers, service rate and vacation rate at a minimum cost. Three approaches were introduced to deal with the optimization problem. Numerical performances were also provided to compare these three methods.
Figure 1. Plot of the average cost function versus the mean service rate and mean vacation rate

Table 2. The minimum average cost function for varying values of $\lambda$ with $M = 6$, $\alpha = 0.2\lambda$, $\theta = 0.02$, $\beta = 0.02$.

|          | QN method |          |          | MN method |          |          | PS method |
|----------|-----------|----------|----------|-----------|----------|----------|-----------|
| $\lambda$ | 0.2       | 0.4      | 0.6      | 0.2       | 0.4      | 0.6      | 0.2       |
| $TAC$    | 68.34     | 85.47    | 95.95    | 68.34     | 85.47    | 95.95    | 68.34     |
| $(W^*, S^*)$ | (1,1) | (2,2) | (2,2) | (1,1) | (2,2) | (2,2) | (1,1) |
| $(\mu^*, \delta^*)$ | (2.37, 5.43) | (2.38, 2.14) | (3.09, 4.09) | (2.37, 5.43) | (2.38, 2.14) | (3.09, 4.09) | (2.37, 5.43) |
| Iterations | 1204 | 1427 | 1583 | 6741 | 6074 | 6176 | 11679 |
| CPU Time  | 6.26 | 5.62 | 5.64 | 6.39 | 5.77 | 5.62 | 22.44 |

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