Calculation of pure annihilation type decay $B^+ \rightarrow D_s^+ \phi$

Cai-Dian Lü*

CCAST (World Laboratory),
P.O. Box 8730, Beijing 100080, China,

and

Institute of High Energy Physics, CAS,
P.O.Box 918(4) Beijing 100039, China†

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Abstract

The rare decay $B^+ \rightarrow D_s^+ \phi$ can occur only via annihilation type diagrams in the standard model. We calculate this decay in perturbative QCD approach with Sudakov resummation. We found that the branching ratio of $B^+ \rightarrow D_s^+ \phi$ is of order $10^{-7}$ which may be measured in the near future by KEK and SLAC $B$ factories. The small branching ratio predicted in standard model makes this channel sensitive to new physics contributions.

* e-mail: lucd@ihep.ac.cn
† Mailing address.
I. INTRODUCTION

The rare $B$ decays are useful for the test of standard model (SM). They are sensitive to new physics contributions, since their branching ratios in SM are small. Some of them have already been measured by CLEO and $B$ factories in KEK and SLAC. Most of them are still on the way of study by both experimental and theoretical studies. Among them, the inclusive or semi-inclusive decays are clean in theory, but with more uncertainty in experimental study. On the other hand, the exclusive decays are difficult for precise theoretical prediction but easier for experimental measurement. The study of exclusive rare $B$ decays require the knowledge of hadronization, which is non-perturbative. The generalized factorization approach has been applied to the theoretical treatment of non-leptonic $B$ decays for some years [1]. It is a great success in explaining many decay branching ratios [2,3]. The factorization approach is a rather simple method. Some efforts have been made to improve their theoretical application [4] and to understand the reason why the factorization approach has gone well [5,6]. One of these method is the perturbative QCD approach (PQCD), where we can calculate the annihilation diagrams as well as the factorizable and non-factorizable diagrams.

The rare decay $B^+ \to D_s^+ \phi$ is pure annihilation type decay. The four valence quarks in the final states $D_s$ and $\phi$ are different from the ones in $B$ meson, i.e. there is no spectator quark in this decay. In the usual factorization approach, this decay picture is described as $\bar{b}$ and $u$ quark in $B$ meson annihilating into vacuum and the $D_s$ and $\phi$ meson produced from vacuum then afterwards. To calculate this decay in the factorization approach, one needs the $D_s \to \phi$ form factor at very large time like momentum transfer $O(M_B)$. However the form factor at such a large momentum transfer is not known in factorization approach. This makes the factorization approach calculation of these decays unreliable.

In this paper, we will try to use the PQCD approach to calculate this decay. The $W$ boson exchange causes the four quark operator transition $\bar{b}u \to \bar{s}c$, the additional $\bar{s}s$ quarks included in $D_s\phi$ are produced from a gluon. This gluon attaches to any one of the quarks participating in $W$ boson exchange. This is shown in Figure 1. In the rest frame of $B$ meson, both $s$ and $\bar{s}$ quarks included in $D_s\phi$ have $O(M_B)$ momenta, and the gluon producing them also has momentum $q^2 \sim O(M_B^2)$. This is a hard gluon. One can perturbatively treat the process where the four quark operator exchanges a hard gluon with $s\bar{s}$ quark pair. It is just
the picture of perturbative QCD approach [5, 6]. Furthermore, the final state of this decay is isospin singlet. It is proportional to the $V_{ub}$ transition. Any $V_{cb}$ transition cannot contribute to it. Therefore there will not be any dominant soft final state interaction contribution. Unlike the $B \to KK$ decays (which may have a large contribution from final state interaction contribution) [7], the decay $B \to D_s \phi$ is a very clean channel for a test of annihilation type contribution.

In the next section, we will show the framework of PQCD briefly. In section III, we give the analytic formulas for the decay amplitude of $B^+ \to D_s^+ \phi$ decays. In section IV, we give the numerical results of branching ratio from the analytic formulas and discuss the theoretical errors. Finally, we conclude this study in section V.

II. FRAMEWORK

PQCD approach has been developed and applied in non-leptonic $B$ meson decays [5, 6, 7, 8, 9, 10, 11] for some time. In this approach, the decay amplitude is described by three scale dynamics, soft($\Phi$), hard($H$), and harder($C$) dynamics. It is conceptually written as the convolution,

$$\text{Amplitude} \sim \int d^4k_1 d^4k_2 d^4k_3 \text{Tr}[C(t)\Phi_B(k_1)\Phi_{D_s}(k_2)\Phi_\phi(k_3)H(k_1, k_2, k_3, t)],$$

where $k_i$'s are momenta of light quarks included in each mesons, and Tr denotes the trace over Dirac and color indices. $C(t)$ is Wilson coefficient of the four quark operator with the QCD radiative corrections. $C(t)$ includes the harder dynamics at a larger scale than $M_B$ scale and describes the evolution of local 4-Fermi operators from $M_W$, $W$ boson mass, down to $t \sim \mathcal{O}(M_B)$ scale, which results in large logarithms $\ln(M_W/t)$. $H$ describes the four quark operator and the quark pair from sea connected by a hard gluon whose scale is at the order of $M_B$, and includes the $\mathcal{O}(M_B)$ hard dynamics. Therefore, this hard part $H$ can be perturbatively calculated. $t$ is chosen as the largest energy scale in $H$, in order to lower the $\alpha_s^2$ corrections to hard part $H$. $\Phi_M$ is the wave function which describes hadronization of the quark and anti-quark into the meson $M$. While $H$ depends on the processes considered, $\Phi_M$ is independent on the specific processes. Determining $\Phi_M$ in some other decays, we can make quantitative predictions here.
We consider the $B$ meson at rest for simplicity. It is convenient to use light-cone coordinate $(p^+, p^-, p_T)$ to describe the meson’s momenta, where $p^\pm = (p^0 \pm p^3)/\sqrt{2}$ and $p_T = (p^1, p^2)$. On this coordinate we can take the $B$, $D_s$, and $\phi$ mesons momenta as $P_1 = M_B/\sqrt{2}(1, 1, 0, T)$, $P_2 = M_B/\sqrt{2}(0, 1 - r^2, 0, T)$, and $P_3 = M_B/\sqrt{2}(0, 1 - r^2, 0, T)$, respectively, where $r = M_{D_s}/M_B$ and we neglect the square terms of $\phi$ meson’s mass $M_\phi^2$. Putting the light spectator quark momenta for $B$, $D_s$ and $\phi$ mesons as $k_1$, $k_2$, and $k_3$, respectively, we can choose $k_1 = (0, x_1 P^-, k_{1T})$, $k_2 = (x_2 P^+_2, 0, k_{2T})$ and $k_3 = (0, x_3 P^-_3, k_{3T})$. Then, integration over $k_2^-, k_3^+$ and $k_1^-$ in eq.(1) leads to

$$\text{Amplitude} \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3$$

$$\text{Tr}[C(t)\Phi_B(x_1, b_1)\Phi_{D_s}(x_2, b_2)\Phi_{\phi}(x_3, b_3)H(x_i, b_i, t)e^{-S(t)}],$$

where $b_i$ is the conjugate space coordinate of $k_{iT}$. The last term, $e^{-S}$, contains two kinds of logarithms. One of the large logarithms is due to the renormalization of ultra-violet divergence $\ln tb$, which describes the QCD running between scale $t$ and $1/b$. The other is from double logarithm due to soft gluon corrections. This double logarithm called Sudakov form factor suppresses the soft dynamics effectively [12]. Thus it makes perturbative calculation of the hard part $H$ applicable at intermediate scale, i.e., $M_B$ scale. We calculate the $H$ for $B^+ \to D_s^+\phi$ decay in the first order in $\alpha_s$ expansion and give the convoluted amplitudes in next section.

In order to calculate analytic formulas of the decay amplitude, we use the wave functions $\Phi_{M, \alpha\beta}$ decomposed in terms of spin structure. As a heavy meson, $B$ meson wave function is not well defined. It is also pointed out by the recent discussion of $B$ meson wave function [13] that, there is no constraint on $B$ meson wave function, if three-parton wave functions are considered. To be consistent with previous calculations [5, 6, 11], we follow the same argument that the structure $(\gamma^\mu \gamma_5)_{\alpha\beta}$ and $\gamma_5\alpha\beta$ components make the dominant contribution in $B$ meson wave function. Then, $\Phi_{M, \alpha\beta}$ is written by

$$\Phi_{M, \alpha\beta} = \frac{i}{\sqrt{2N_c}} \left\{ (P_M \gamma_5)_{\alpha\beta} \phi^A_M + \gamma_5\alpha\beta \phi^P_M \right\},$$

where $N_c = 3$ is color’s degree of freedom, $P_M$ is the corresponding meson’s momentum, and $\phi^A_P$ are Lorentz scalar wave functions. As heavy quark effective theory leads to $\phi^P_B \simeq M_B \phi^A_B$,
then $B$ meson’s wave function can be expressed by

$$
\Phi_{B,\alpha\beta}(x, b) = \frac{i}{\sqrt{2N_c}} [P_1 + M_B] \gamma_{5\alpha\beta} \phi_B(x, b).
$$

(4)

According to Ref.[14], a pseudo-scalar meson moving fast is parameterized by Lorentz scalar wave functions, $\phi$, $\phi_p$, and $\phi_\sigma$ as

$$
\langle D_s^- (P) | \bar{s}(z) \gamma_5 c(0) | 0 \rangle \simeq -f_{D_s} P_\mu \int_0^1 dx \ e^{ixPz} \phi(x),
$$

(5)

$$
\langle D_s^- (P) | \bar{s}(z) \gamma_5 c(0) | 0 \rangle = -i f_{D_s} m_{0D_s} \int_0^1 dx \ e^{ixPz} \phi_p(x),
$$

(6)

$$
\langle D_s^- (P) | \bar{s}(z) \gamma_5 \sigma_{\mu\nu} c(0) | 0 \rangle = \frac{i}{6} f_{D_s} m_{0D_s} \left( 1 - \frac{M_{D_s}^2}{m_{0D_s}^2} \right) (P_\mu z_\nu - P_\nu z_\mu) \int_0^1 dx \ e^{ixPz} \phi_\sigma(x),
$$

(7)

where $m_{0D_s} = M_{D_s}^2 / (m_c + m_s)$. We ignore the difference between $c$ quark’s mass and $D_s$ meson’s mass in the perturbative calculation. This means, $M_{D_s} = m_{0D_s}$. In this approximation, the contributions of eq.(4) are negligible. With the equation of motion eq.(5), eq.(6), lead to

$$
\phi_p(x) = \phi(x) + O \left( \frac{\bar{\Lambda}}{M_{D_s}} \right).
$$

(8)

Therefore the $D_s$ meson’s wave function can be expressed by one Lorentz scalar wave function,

$$
\Phi_{D_s,\alpha\beta}(x, b) = \frac{i}{\sqrt{2N_c}} [\gamma_5 \ P_2]_{\alpha\beta} + M_{D_s} \gamma_{5\alpha\beta} ] \Phi_{D_s}(x, b).
$$

(9)

The wave function $\Phi_M$ for $M = B, D_s$ meson is normalized by its decay constant $f_M$

$$
\int_0^1 dx \ \Phi_M(x, b = 0) = \frac{f_M}{2\sqrt{2N_c}}.
$$

(10)

In contrast to the $B$ and $D_s$ meson, for the $\phi$ meson, being light, the $\sigma_{\alpha\beta}^{\mu\nu}$ component remains. In $B^+ \rightarrow D_s^+ \phi$ decay, the $\phi$ meson is longitudinally polarized. Then, $\phi$ meson’s wave function is parameterized by three Lorentz structures

$$
\frac{M_\phi}{\sqrt{2N_c}} \Phi_\phi(x_3), \quad \frac{k \ P_3}{\sqrt{2N_c}} \Phi_\phi^t(x_3), \quad \frac{M_\phi}{\sqrt{2N_c}} \Phi_\phi^s(x_3).
$$

(11)

In the numerical analysis we will use $\Phi_\phi$, $\Phi_\phi^t$ and $\Phi_\phi^s$ which were calculated from QCD sum rule [16]. They will be shown in section IV.
FIG. 1: Diagrams for $B^+ \rightarrow D_s^+ \phi$ decay. The factorizable diagrams (a),(b) contribute to $F_a$, and non-factorizable (c), (d) do to $M_a$.

III. PERTURBATIVE CALCULATIONS

The effective Hamiltonian related to $B^+ \rightarrow D_s^+ \phi$ decay is given as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs} [C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)]$$

(12)

$$O_1 = (\bar{b}\gamma_\mu P_L s)(\bar{c}\gamma_\mu P_L u), \quad O_2 = (\bar{b}\gamma_\mu P_L u)(\bar{c}\gamma_\mu P_L s).$$

(13)

where $C_{1,2}(\mu)$ are Wilson coefficients at renormalization scale $\mu$. The projection operator is defined as $P_L = 1 - \gamma_5$. The lowest order diagrams contributing to $B^+ \rightarrow D_s^+ \phi$ are drawn in Fig.1 according to this effective Hamiltonian. As stated above, $B^+ \rightarrow D_s^+ \phi$ decay only has annihilation diagrams.

We get the following analytic formulas by calculating the hard part $H$ at first order in $\alpha_s$. Together with the meson wave functions, the amplitude for the factorizable annihilation diagram in Fig.1(a) and (b) results in,

$$F_a = 16\pi C_F f_B M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \Phi_{D_s}(x_2, b_2) \times \left\{ x_3 \Phi_\phi(x_3, b_3) + r(2x_3 - 1) r_\phi \Phi_\phi^t(x_3, b_3) + r(1 + 2x_3) r_\phi \Phi_\phi^s(x_3, b_3) \right\} E_f(t_1^1) h_a(x_2, x_3, b_2, b_3)$$

$$- \left\{ x_2 \Phi_\phi(x_3, b_3) + 2r(1 + x_2) r_\phi \Phi_\phi^s(x_3, b_3) \right\} E_f(t_1^2) h_a(x_3, x_2, b_3, b_2)$$

(14)

where $C_F = 4/3$ is the group factor of SU(3)$_c$ gauge group, and $r_\phi = m_\phi/M_B$. The function $E_f$, $t_1^{1,2}$, $h_a$ are given in the Appendix. Since we only include twist 2 and twist 3 contributions in our PQCD approach, all the $r^2$ and $r_\phi^2$ terms in the calculation are neglected for consistence. The explicit form for the wave functions, $\Phi_M$, is given in the next section. From eq.(14), one can see that the factorizable contribution $F_a$ is independent of the $B$ meson wave function, but proportional to the $B$ meson decay constant $f_B$. 

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The amplitude for the non-factorizable annihilation diagram in Fig. 1(c) and (d) is given as

\[
M_a = \frac{1}{\sqrt{2}N_c} 64\pi C_F M_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \Phi_{D_s}(x_2, b_2) \\
\times \left\{ x_2 \Phi_\phi(x_3, b_2) + r(x_2 - x_3) r_\phi \Phi_\phi^*(x_3, b_2) \right\} E_m(t_m^1) h_a^{(1)}(x_1, x_2, x_3, b_1, b_2) \\
+ r(x_2 + x_3) r_\phi \Phi_\phi^*(x_3, b_2) \right\} E_m(t_m^2) h_a^{(2)}(x_1, x_2, x_3, b_1, b_2) \right].
\]

Unlike the factorizable contribution \(F_a\), the non-factorizable annihilation diagram involve all three meson wave functions.

Thus, the total decay amplitude \(A\) and decay width \(\Gamma\) for \(B^+ \to D_s^+ \phi\) decay are given as

\[
A = F_a + M_a, \quad (16)
\]

\[
\Gamma(B^+ \to D_s^+ \phi) = \frac{G_F^2 M_B^3}{128\pi} |V_{ub}^* V_{cs}|^2, \quad (17)
\]

where the overall factor is included in the decay width with the kinematics factor.

The decay amplitude for CP conjugated mode, \(B^- \to D_s^- \phi\), is the same expression as \(B^+ \to D_s^+ \phi\), just replacing \(V_{ub}^* V_{cs}\) with \(V_{ub} V_{cs}^*\). Since there is only one kind of CKM phase involved in the decay, there is no CP violation in the standard model for this channel. We thus have \(Br(B^+ \to D_s^+ \phi) = Br(B^- \to D_s^- \phi)\).

### IV. NUMERICAL RESULTS

In this section we show numerical results obtained from the previous formulas. At the beginning, we give the branching ratios predicted from the same parameters and wave functions that are adopted in other works. Secondly, we discuss the theoretical errors due to uncertainty of some parameters.

For the \(B\) meson’s wave function, there is a sharp peak at the small \(x\) region, we use

\[
\Phi_B(x, b) = N_B x^2 (1 - x)^2 \exp \left[ \frac{-M_B^2 x^2}{2\omega_b^2} - \frac{1}{2} (\omega_b b)^2 \right], \quad (18)
\]

which is adopted in ref. \(5, 6, 11\). This choice of \(B\) meson’s wave function is almost a best fit from the \(B \to K\pi, \pi\pi, \pi\rho\) and \(\pi\omega\) decays. For the \(D_s\) meson’s wave function, we assume
the form as the following, leaving $a_{Ds}$ a free parameter

$$
\Phi_{Ds}(x, b) = \frac{3}{\sqrt{2N_c}} f_{Ds} x (1-x) \{ 1 + a_{Ds} (1-2x) \} \exp \left[ -\frac{1}{2} (\omega_D b)^2 \right].
$$

(19)

This is a rather flat distribution function. Since $c$ quark is heavier than $s$ quark, this function is peaked at $c$ quark side, i.e. small $x$ region. The wave functions of the $\phi$ meson are derived by QCD sum rules [16]

$$
\Phi_\phi(x) = \frac{f_\phi}{2\sqrt{2N_c}} 6x(1-x),
$$

(20)

$$
\Phi_D^T(x) = \frac{f_D^T}{2\sqrt{2N_c}} \left\{ 3\xi^2 + 0.21 (3 - 30\xi^2 + 35\xi^4) + 0.69 \left( 1 + \xi \ln \frac{x}{1-x} \right) \right\},
$$

(21)

$$
\Phi_s^T(x) = \frac{f_T^s}{4\sqrt{2N_c}} \left\{ 3\xi \left( 4.5 - 11.2x + 11.2x^2 \right) + 1.38 \ln \frac{x}{1-x} \right\},
$$

(22)

where $\xi = 1 - 2x$. In addition, we use the following input parameters:

$$
M_B = 5.279 \text{ GeV}, \quad M_{Ds} = 1.969 \text{ GeV}, \quad m_\phi = 1.02 \text{ GeV},
$$

(23)

$$
f_B = 190 \text{ MeV}, \quad f_\phi = 237 \text{ MeV}, \quad f_D^T = 220 \text{ MeV}, \quad f_{Ds} = 241 \text{ MeV},
$$

(24)

$$
\omega_b = 0.4 \text{ GeV}, \quad a_{Ds} = 0.3, \quad \omega_D = 0.2 \text{ GeV}.
$$

(25)

With these values and eq.(10) we get the normalization factor $N_B = 91.745 \text{ GeV}$. Using the above fixed parameters, we find that the factorizable annihilation diagram contribution is dominant over the non-factorizable contribution. The reason is that the Wilson coefficient in non-factorizable contribution $M_a$ is $C_1(t)$, which is smaller than the one in factorizable contribution $F_a$, $a_1 = C_1/3 + C_2$. Although the real part of $M_a$ is negligible, the imaginary part of $M_a$ is comparable with the imaginary part of $F_a$, it is about 30% of the real part of $F_a$.

The propagators of inner quark and gluon in Figure 1 are usually proportional to $1/x_i$. One may suspect that these amplitudes are enhanced by the endpoint singularity around $x_i \sim 0$. This can be explicitly found in eq.(A8, A9), where the Bessel function $Y_0$ diverges at $x_i \sim 0$ or 1. However this is not the truth in our calculation. First we introduce the transverse momentum of quark, such that the propagators become $1/(x_ix_j + k_T^2)$. There is no divergent at endpoint region. Secondly, the Sudakov form factor Exp$[-S]$ suppresses the region of small $k_T^2$. Therefore there is no singularity in our calculation. We also include the threshold resummation in our calculation of factorizable diagrams, which further suppress
the endpoint region contribution [17]. The dominant contribution is not from the endpoint region of the wave function. As a prove, in our numerical calculations, for example, an expectation value of $\alpha_s$ in the integration for $F_a$ and $M_a$ results in $\langle \alpha_s/\pi \rangle \simeq 0.1$. Therefore, the perturbative calculations are self-consistent.

Now we can calculate the branching ratio according to eqs. (16, 17). Here we use CKM matrix elements [18]

$$|V_{ub}| = 0.0036 \pm 0.0010, \quad |V_{cs}| = 0.9891 \pm 0.016, \quad (26)$$

and the life time for $B^{\pm}$ meson is $\tau_{B^{\pm}} = 1.65 \times 10^{-12}$ s. The predicted branching ratio is

$$\text{Br}(B^+ \rightarrow D_s^+ \phi) = 3.0 \times 10^{-7}. \quad (27)$$

This is still far from the current experimental upper limit [18]

$$\text{Br}(B^+ \rightarrow D_s^+ \phi) < 3.2 \times 10^{-4}. \quad (28)$$

The branching ratios obtained from the analytic formulas may be sensitive to various parameters, such as parameters in eqs. (25). Uncertainty of the predictions on PQCD is mainly due to the meson wave functions. Therefore it is important to give the limits of the branching ratio when we choose the parameters to appropriate extent. The appropriate extent of $\omega_b$ can be obtained from calculation of semi-leptonic decays [19] and other $B \rightarrow \pi \pi$, $B \rightarrow K \pi$ and $B \rightarrow \rho \pi$, $\omega \pi$ decays [5, 6, 11],

$$0.35 \text{ GeV} \leq \omega_b \leq 0.45 \text{ GeV}. \quad (29)$$

The change of value of $\omega_b$ will not alter the result of $F_a$, which is independent of $B$ meson wave function, but will affect the value of $M_a$. We did not find any strict constraints for the $D_s$ meson wave function in the literature. In fact, a future study of $B \rightarrow D_s \pi$ will do this job. At present, $a_{D_s}$ in $D_s$ meson wave function is a free parameter, and we take $0 \leq a_{D_s} \leq 1$. Here we check the sensitivity of our predictions on $\omega_b$ and $a_{D_s}$ within the ranges stated above. The branching ratios normalized by the decay constants and the CKM matrix elements can result in

$$\text{Br}(B^+ \rightarrow D_s^+ \phi) = (3.0^{+2.4}_{-1.0}) \times 10^{-7} \left( \frac{f_B f_{D_s}}{190 \text{ MeV} \cdot 241 \text{ MeV}} \right)^2 \left( \frac{|V_{ub}^* V_{cs}|}{0.0036 \cdot 0.9891} \right)^2. \quad (30)$$

Considering the uncertainty of $f_B$, $f_{D_s}$ and $|V_{ub}^* V_{cs}|$ etc., the branching ratio of $B^+ \rightarrow D_s^+ \phi$ decay is at the order of $10^{-7}$. This may be measured by the current $B$ factory experiments in KEK and SLAC.
V. CONCLUSION

In two-body hadronic $B$ meson decays, the final state mesons are moving very fast, since each of them carry more than 2 GeV energy. There is not enough time for them to exchange soft gluons. The soft final state interaction is not important in the two-body $B$ decays. This is consistent with the argument based on color-transparency [20]. The PQCD with Sudakov form factor is a self-consistent approach to describe the two-body $B$ meson decays. Although the annihilation diagrams are suppressed comparing to other spectator diagrams, but their contributions are not negligible in PQCD approach [5, 6].

In this paper, we calculate the $B^+ \rightarrow D^{+}_s \phi$ decay in the PQCD approach. Since neither of the bottom quark nor the up quark in the initial $B$ meson appeared in the final mesons, this process occurs purely via annihilation type diagrams. It is a charm quark (not an anti-charm quark) in the final states, therefore the usual $V_{cb}$ transition does not contribute to this process. The final states are isospin singlet. There should be no dominant final state interactions through other channels contribute. From our PQCD study, the branching ratio of $B^+ \rightarrow D^{+}_s \phi$ decay is still sizable with a branching ratio around $10^{-7}$, which may be measured in the current running $B$ factories Belle, BABAR or in LHC-B in the future. This may be one of the channels to be measured in $B$ decays via annihilation type diagram. Whether the PQCD predicted branching ratio is good enough to account for the $B^+ \rightarrow D^{+}_s \phi$ decay will soon be tested in the current or future experiments.

The small branching ratio (comparing to the already measured other $B$ decays) predicted in the SM, makes this channel sensitive to any new physics contributions. Since the CP asymmetry predicted for this channel in SM is zero, any non-zero measurement of CP asymmetry will be a definite signal of new physics. We also notice that the supersymmetric contribution will not enhance the decay branching ratio significantly, but it may contribute to a non zero CP asymmetry in this channel, since the supersymmetry couplings can introduce new phases.

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**APPENDIX A: SOME FUNCTIONS**

The definitions of some functions used in the text are presented in this appendix. In the numerical analysis we use one loop expression for strong coupling constant,

\[ \alpha_s(\mu) = \frac{4\pi}{\beta_0 \log(\mu^2/\Lambda^2)}, \quad (A1) \]

where \( \beta_0 = (33 - 2n_f)/3 \) and \( n_f \) is number of active quark flavor at appropriate scale. \( \Lambda \) is QCD scale, which we use as 250 MeV at \( n_f = 4 \). We also use leading logarithms expressions for Wilson coefficients \( C_{1,2} \) presented in ref.\[13\]. Then, we put \( m_t = 170 \) GeV, \( m_W = 80.2 \) GeV, \( m_b = 4.8 \) GeV, and \( m_c = 1.3 \) GeV in the Wilson coefficients calculation.

The function \( E_f \) and \( E_m \) are defined as

\[ E_f(t) = [C_1(t)/3 + C_2(t)] \alpha_s(t) e^{-S_D(t) - S_\phi(t)}, \quad (A2) \]
\[ E_m(t) = C_1(t)\alpha_s(t) e^{-S_B(t) - S_D(t) - S_\phi(t)}. \quad (A3) \]

The above \( S_{B,D,\phi} \) are defined as

\[ S_B(t) = s(x_1 P_1^+, b_1) + 2 \int_{1/b_1}^{t} \frac{d\mu'}{\mu'} \gamma_q(\mu'), \quad (A4) \]
\[ S_D(t) = s(x_2 P_2^+, b_3) + 2 \int_{1/b_2}^{t} \frac{d\mu'}{\mu'} \gamma_q(\mu'), \quad (A5) \]
\[ S_\phi(t) = s(x_3 P_3^+, b_3) + s((1 - x_3) P_3^+, b_3) + 2 \int_{1/b_3}^{t} \frac{d\mu'}{\mu'} \gamma_q(\mu'), \quad (A6) \]

where the last terms of the above formulas are logarithms from the renormalization of ultraviolet divergence. The term \( s(Q, b) \), the so-called Sudakov factor, result from summing up double logarithms caused by collinear divergence and soft divergence. The expression is given as \[10\]

\[ s(Q, b) = \int_{1/b}^{Q} \frac{d\mu'}{\mu'} \left[ \left\{ \frac{2}{3} (2\gamma_E - 1 - \log 2) + C_F \log \frac{Q}{\mu'} \right\} \frac{\alpha_s(\mu')}{\pi} \right. \]
\[ + \left\{ \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{2}{3} \beta_0 \log \frac{\gamma_E}{2} \right\} \left( \frac{\alpha_s(\mu')}{\pi} \right)^2 \log \frac{Q}{\mu'} \right], \quad (A7) \]

\( \gamma_E = 0.57722 \cdots \) is Euler constant, and \( \gamma_q = \alpha_s/\pi \) is the quark anomalous dimension.
The $h$'s in the decay amplitudes are given by performing Fourier transformation on the transverse momenta $k_{iT}$ for propagators of virtual quark and gluon in the hard part calculation, they result in

$$h_a(x_2, x_3, b_2, b_3) = \left(\frac{\pi i}{2}\right)^2 H_0^1(M_B \sqrt{x_2 x_3} b_2) S_t(x_3)$$

(A8)

$$h_a^{(j)}(x_1, x_2, x_3, b_1, b_2) = \left\{\begin{array}{ll}
K_0(M_B \sqrt{x_3} b_2), & \text{for } F_j \geq 0 \\
\frac{\pi i}{2} H_0^{(1)}(M_B \sqrt{-F_j} b_1), & \text{for } F_j < 0
\end{array}\right\} \times$$

(A9)

$$\times \left\{\begin{array}{ll}
\frac{\pi i}{2} H_0^{(1)}(M_B \sqrt{x_2 x_3} b_2) J_0(M_B \sqrt{x_2 x_3 b_2}) \theta(b_2 - b_3) + (b_2 \leftrightarrow b_3)
\end{array}\right\},$$

with the variables $F_1 = x_2(x_1 - x_3)$, $F_2 = x_2 + (1 - x_2)(x_1 + x_3)$. And $H_0^{(1)}(z) = J_0(z) + \imath Y_0(z)$.

The threshold resummation form factor $S_t(x_i)$ is adopted from ref.[19]

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2 + c)}{\sqrt{\pi}\Gamma(1 + c)} [x(1 - x)]^c,$$

(A10)

where the parameter $c = 0.3$. This function is normalized to unity. The hard scale $t$'s in the amplitudes are taken as the largest energy scale in $H$ to diminish the higher order $\alpha_s^2$ corrections:

$$t_1^a = \max(M_B \sqrt{x_3}, 1/b_2, 1/b_3),$$

(A11)

$$t_2^a = \max(M_B \sqrt{x_2}, 1/b_2, 1/b_3),$$

(A12)

$$t_1^m = \max(M_B \sqrt{|F_1|}, M_B \sqrt{x_2 x_3}, 1/b_1, 1/b_2),$$

(A13)

$$t_2^m = \max(M_B \sqrt{F_2}, M_B \sqrt{x_2 x_3}, 1/b_1, 1/b_2).$$

(A14)

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