This method is not very complex and is accurate. Describing Functions (EDF) is proposed in literature [26-30]. approximate the non-linear terms, called the Extended analysis using the multivariable describing functions to on combined time-domain analysis and frequency-domain or not accurate enough [21-25]. A modeling technique, based 26]. However, these methods are either very complex [19, 20] derive the small-signal model of the resonant converters [19-27]. These worst-case operating conditions occur at a switching frequency close to the series resonant frequencies. Experimental results will also be included to verify the design methodology proposed.

II. FUNDAMENTAL ANALYSIS AND CONVERTER MODELING

A. Converter Power Stages

Fig. 1 shows the circuit topology for the resonant converter capable of transferring power in both directions. Switches in the primary side form a high-frequency full-bridge inverter converting the input dc voltage into a quasi-square wave ac while the switches in the secondary side serve as a rectifier converting the high-frequency ac into a dc output voltage. The magnetizing inductance and the leakage inductances of the transformer are part of the power stage.

B. The Steady-State Model and the Small-Signal Model

The equivalent circuit of the resonant converter shown in Fig. 1 operating in the BCM is shown in Fig. 2. This equivalent circuit is similar to the equivalent circuit in [29], which allows the dynamic analysis in [29] to be valid here where the steady state and dynamic models derived will be utilized here as well. With the frequency control approach (rather than phase shift control) of the primary side, ZVS can be maintained at all load and line conditions as long as the primary-side current lags behind the primary-side bridge voltage.
Resonant used to design the current loop controller and the voltage loop controller for a 3.5 kW bidirectional resonant converter closed-loop system might be unstable at some operating points.

Dynamics change. If the controller is not properly designed, the moves the converter operating point, and thus the plant.

Battery voltage change during battery charging and discharging designed in [18]. The circuit parameters are listed in Table (II).

| Table I: Resonant Converter Circuit Parameters |
|-----------------------------------------------|
| Resonant Inductor (L₁, L₂) | Magnetizing Inductance (Lₚ) | Resonant Cap. (C₁) | Resonant Cap. (C₂) | Transformer Turns Ratio (n) | Filter Cap. (C₃) |
| 20 μH | 100 μH | 136 nF | 200 nF | 1 | 30 μF |

The gain of the converter is unity at the primary side series resonant frequency, fₛ, of 100 kHz, more than unity for frequencies lower than fₛ, and less than unity for frequencies greater than fₛ.

In [15, 29, 31], SIMPLIS was used to plot the open-loop control-to-output voltage transfer function for a Series-Series compensated IPT system. To validate the small signal model in [29], the open-loop bode-plot by SIMPLIS will be compared to the model from [29]. Figs. 4-6 show the bode-plots for the control-to-output voltage transfer function at different operating points.

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C. Verification of the Small-Signal Model

Fig. 3 shows the voltage gain versus the operating switching frequency curve for the converter with the load resistance, Rₗ, equal to 45 Ω. The rest of the circuit parameters are listed in table I:

So, it is very important to find the worst-case operating conditions for the controller design.

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III. Step-By-Step Procedure to Find the Worst-Case Conditions for Controller Design

In this section, the small-signal model in (12) – (13) will be used to design the current loop controller and the voltage loop controller for a 3.5 kW bidirectional resonant converter designed in [18]. The circuit parameters are listed in Table (II). Battery voltage change during battery charging and discharging moves the converter operating point, and thus the plant dynamics change. If the controller is not properly designed, the closed-loop system might be unstable at some operating points.
The first step in designing a controller would be to find the equivalent load resistance. Then, the operating switching frequency range needs to be determined. With this information, the worst-case for the controller can be determined.

### Output load resistance range

![Equivalent output load resistance for converter operating in the BCM and in the RM](image)

\[ R_{\text{BCM}} = \frac{V_{\text{Batt}}}{I_{\text{Batt}}} \]

\[ R_{\text{RM}} = \frac{V_{\text{DC}}}{I_{\text{DC}}} \]

(a) \( R_L \) during the BCM  
(b) \( R_L \) during the RM

Fig. 7 shows the equivalent load resistance during the BCM and in the RM.

### Determining the Worst-case Operating Conditions for Controller Design

The low-frequency gain and the zero-crossing of the open-loop bode-plot changes as the operating point changes. To find the worst-case, the open-loop bode-plot is plotted at all load conditions and operating switching frequencies.

The range of equivalent output resistance, \( R_L \), depends upon the charging profile. The output charging current versus the battery voltage is shown in Fig. 8. During the constant current mode, the output current is limited to 10 A for the lower battery voltages, while for higher battery voltages, it is reduced to keep the output power at 3.5 kW. So, during the constant current mode,

\[ R_{L,\text{Imin}} = \frac{V_{\text{Batt}}}{I_{\text{Batt}}} = \frac{250 \text{ V}}{10 \text{ A}} = 25 \Omega \]

\[ R_{L,\text{Imax}} = \frac{V_{\text{Batt}}}{I_{\text{Batt}}} = \frac{450 \text{ V}}{7.5 \text{ A}} = 60 \Omega \]

The equivalent resistance under nominal operating condition is:

\[ R_{L,\text{nom}} = \frac{V_{\text{Batt,nom}}}{I_{\text{Batt,nom}}} = \frac{350 \text{ V}}{10 \text{ A}} = 35 \Omega \]

To get the operating frequency range, the output current under steady-state condition is plotted against the converter operating switching frequency under different load conditions. In Fig. 9, the steady-state output current versus the operating switching frequency is plotted for \( R_{L,\text{Imin}} \), \( R_{L,\text{Imax}} \) and \( R_{L,\text{nom}} \).

These curves give a good amount of information about the design of the converter, and the operating frequency range during the closed-loop operation. It can be seen that the operating switching frequency range for the converter operating in the BCM is,

\[ f_{\text{SW,BCM}} = 80 \text{ kHz} \sim 140 \text{ kHz} \]

The battery voltage versus the operating switching frequency for the whole range of load resistance is plotted in Fig. 10. The operating surface is enclosed by the yellow polygon. It can be seen that the enclosed surface is monotonically decreasing, so a PID controller can be used to close the current loop.
The next step is to find the worst-case conditions for the current loop controller design. Fig. 11 shows the open-loop control-to-output current, $G_{i(s)}$, for a load resistance of 60 Ω. The operating switching frequency is varied from 80 kHz to 200 kHz. For a switching frequency of 90 kHz, the open-loop bode-plot zero-crossing frequency is maximum. So, for a fixed load resistance, the worst-case occurs at 90 kHz.

Fig. 12 shows the open-loop control-to-output current, $G_{i(s)}$, for converter operating at a switching frequency of 90 kHz. The output load resistance is varied from 25 Ω to 60 Ω. It can be seen that the zero-crossing frequency of the open-loop bode-plot remains almost the same.

Fig. 9 shows that the converter will operate at a switching frequency of 90 kHz only for an equivalent load resistance 35 Ω or more. So, the worst-case conditions for the current loop controller design are when the output load resistance is between 35 Ω and 60 Ω.

Now, for voltage loop controller, in this mode, the charging current drops to a very small value. So, the equivalent output resistance, $R_o$, becomes very large. In this example, the battery reference voltage is set to 450 V. So, if the minimum charging current is set to 100 mA, then during the constant voltage mode, the charging current drops from 7.5 A to 100 mA. So, $R_{L.V} = \frac{V_{\text{Batt}}}{i_{\text{Batt}}} = 60 \Omega \sim 4.5 \, k\Omega$.

To determine the operating frequency range in the constant voltage mode, the steady-state output battery voltage versus the operating switching frequency curves are plotted for $R_e$. Fig. 13 shows output voltage versus operating switching frequency under different load conditions for converter operating in the BCM.

Fig. 14. Open-loop bode-plot for $G_{v(s)}$, for converter operating at switching frequency of 80 kHz under different load conditions.

IV. OPERATION OF CONVERTER NEAR THE SERIES RESONANT FREQUENCIES

The power stage efficiency of the converter is maximum at the primary-side series resonant frequency [18]. The converter is designed to operate at this point under nominal operating conditions. However, in the last section, it was shown that the worst-case operating conditions for the current and voltage loop controllers occur at this frequency because of the high quality-factor (Q) peak in the open-loop bode. To investigate this high Q peak in the bode-plots, the poles and the zeros in the converter transfer function must be observed when the converter is operating at a switching frequency close to the resonant frequencies. There are two series resonant frequencies in this converter. These are;
\[ f_{ser1} = \frac{1}{2\pi \sqrt{L_I C_I}} = 96.5 \text{ kHz}, \quad f_{ser2} = \frac{1}{2\pi \sqrt{L_R C_R}} = 79.58 \text{ kHz} \]

The small-signal model derived in [29] show that the resonant converter is a 9th order system. So there are nine poles in each transfer function. Most of the poles and zeros are at very high frequency. So, these high-frequency poles and zeros will be ignored, and only the low frequency dominant poles will be examined.

Fig. 20 shows the dominant poles at different operating switching frequencies when the switching frequency is varied from 120 kHz to 80 kHz. At higher switching frequencies, the dominant poles are on real axis, and there is no resonant peaking in the bode-plot. As the converter operating switching decreases, the two poles come close on the real axis and break away at some point from the real axis. In this case, the resonant peaking starts to appear in the open-loop bode-plot. At the midpoint between the two resonant frequencies, the damping is minimum (and Q is maximum), and the magnitude of the resonant peak is maximum.

![Pole-Zero Map](image1)

It can be seen in Fig. 21 that if the converter operating switching frequency is varied from 80 kHz to 60 kHz.

![Pole-Zero Map](image2)

In this section, controllers will be designed for these worst-case conditions. The block diagram for the closed loop system is shown in Fig. 22.

Where, \( X = \) Output current or voltage to be regulated, \( \bar{X} = \) Averaged output current or voltage, \( X^* = \) Output current or voltage reference, \( H_s = \) sensor gain, \( G_{LPF}(s) = \) Low-pass filter implemented with the hardware, \( H_{ADC} = \) Analog-to-digital converter (ADC) gain, \( H_{DSP,X} = \) Digital Signal Processor (DSP) gain, \( G_{LPF}(s) = \) Low-pass filter implemented in DSP, \( G_{cx}(s) = \) Designed controller, \( e^{-Ts} = \) Single sampling period delay

The loop gain of the closed-loop system is,

\[ T_s(s) = G_{xf}(s) \cdot H_s \cdot G_{LPF}(s) \cdot H_{ADC} \cdot H_{DSP,X} \cdot G_{LPF}(s) \cdot G_{cx}(s) \cdot e^{-Ts} \]

The DSP gain is chosen such that the product of all gains is equal to unity, i.e.

\[ H_s \cdot H_{DSP,X} \cdot H_{ADC} = 1 \]

So, the loop gain becomes,

\[ T_s(s) = G_{xf}(s) \cdot G_{LPF}(s) \cdot G_{LPF}(s) \cdot G_{cx}(s) \cdot e^{-Ts} \]

The low-pass filter implemented with hardware is a sallen-key 2nd order filter with a cut-off frequency of 10 kHz. The transfer function for it is,

\[ G_{LPF}(s) = \frac{1}{1 + \frac{s}{2\pi \times 10000} + \left(\frac{s}{2\pi \times 10000}\right)^2} \]

And, the low-pass filter implemented with a DSP is,

\[ G_{LPF}(s) = \frac{1}{1 + \frac{s}{2\pi \times 10000} + \left(\frac{s}{2\pi \times 10000}\right)^2} \]

The controller will be implemented with a Texas Instruments TMS320F28335 DSP with a sampling frequency of 30 kHz, so a 33.33-\(\mu\)s delay is added in the loop.

\[ e^{-Ts} = \frac{1 - \frac{T_s}{2} s + \frac{T_s^2}{2} s^2}{1 + \frac{T_s}{2} s + \frac{T_s^2}{2} s^2} \]

It can be seen in Figures 9, 13 and 16 that the converter is designed to have a negative slope of the current and voltage curves. Because of this negative slope, the phase angle is -180°
at the lower frequencies. So, instead of negative feedback there will be positive feedback in the closed loop as shown in Fig. 22.

For experiments, a programmable electronic load, BK Precision 8526 5000W, is used. In the BCM, the e-load is programmed as a battery load whose voltage is varied from 250V to 450V. In the RM, the e-load is programmed as a current source whose current is varied from 0.1A to 8.5A to emulate the ac-dc stage.

A. Designing a Current Loop Controller for the Converter Operating in the BCM

The loop-gain of the converter operating in the constant current mode in the BCM is,

\[ T_{BCM}(s) = G_{lpf}(s) \cdot G_{lpf2}(s) \cdot G_{avl}(s) \cdot e^{-st} \]

The following controller is designed to have a bandwidth of 515 Hz, and a phase margin of 52.8° at the worst-case operating conditions.

\[ G_{cl}(s) = \frac{10}{s} \]

The worst-case open-loop bode-plot and the closed-loop gain for the current loop with the designed controller are shown in Fig. 23. To check the stability of the closed-loop system, a step change was applied to the output battery voltage from 250 V to 325 V and back to 250 V. The system is found to be stable, with no overshoot or undershoot for both step increase and step decrease in the battery voltage. Fig. 24 and Fig. 25 show the system response to a step change in the battery voltage.

B. Designing a Voltage Loop Controller for Converter Operating in the BCM

The loop-gain of the converter operating in the constant voltage mode in the BCM is,

\[ T_{VBCM}(s) = G_{lpf}(s) \cdot G_{lpf2}(s) \cdot G_{cv}(s) \cdot e^{-st} \]

The controller \( G_{cv}(s) = \frac{0.3}{s} \) is designed to have a bandwidth of 103 Hz, and a phase margin of 82.2°.

The worst-case open-loop bode-plot and the closed-loop gain for the current loop with the designed controller are shown in Fig. 26. To check the stability of the closed-loop system, a step change was applied to the output voltage reference from 315 V to 420 V and back to 315 V for a resistive load of 57 \( \Omega \).

The system is found to be stable, with no overshoot or undershoot for both step increase and step decrease in the reference voltage. Fig. 27 and Fig. 28 show the system response to a step change in the battery voltage.

VI. CONCLUSIONS

The \( CLLLC \)-type resonant converter is similar to an \( LLC \)-type resonant converter with an extra inductor and capacitor in the secondary side. In this paper, a controller design methodology is proposed that can guarantee a stable operation during the entire operating frequency range.

First, the worst-case operating conditions are found for the controller design. If the controller is designed for this worst-
case, it will ensure stable operation at all other operating conditions.

Fig. 27. Step change in the battery voltage from 315 V to 420 V for converter operating in the constant current mode at $R_L = 57 \Omega$ in the BCM.

Fig. 28. Step change in the battery voltage from 420 V to 315 V for converter operating in the constant current mode at $R_L = 57 \Omega$ in the BCM.

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