Spin-charge-family theory is offering next step in understanding elementary particles and fields and correspondingly universe

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Abstract. More than 40 years ago the standard model made a successful new step in understanding properties of fermion and boson fields. Now the next step is needed, which would explain what the standard model and the cosmological models just assume: a. The origin of quantum numbers of massless one family members. b. The origin of families. c. The origin of the vector gauge fields. d. The origin of the Higgses and Yukawa couplings. e. The origin of the dark matter. f. The origin of the matter-antimatter asymmetry. g. The origin of the dark energy. h. And several other open problems. The spin-charge-family theory, a kind of the Kaluza-Klein theories in \((d = (2n - 1) + 1)\)-space-time, with \(d = (13 + 1)\) and the two kinds of the spin connection fields, which are the gauge fields of the two kinds of the Clifford algebra objects anti-commuting with one another, may provide this much needed next step. The talk presents: i. A short presentation of this theory. ii. The review over the achievements of this theory so far, with some not published yet achievements included. iii. Predictions for future experiments.

1. Introduction
The standard model made a great step in understanding properties of fermion and boson fields by: i. Starting with massless fields. ii. Assuming quantum numbers of one family of massless quarks and leptons and relating handedness with charges. iii. Postulating the existence of several families. iv. Postulating the existence of the vector gauge fields of the charges of quarks and leptons. v. Postulating a simple action for fermions and vector bosons under the requirement of the gauge invariance. vi. Postulating the existence of the scalar field, which breaks the weak and the hyper charges of the vacuum making fermions and heavy bosons massive. viii. Postulating the Yukawa couplings.

Properties of fermions and bosons in the standard model are presented and commented on in Appendix B.

Although the assumptions from i.- v. are elegant, in particular the assumption that all the elementary fields are massless gaining masses through the interactions only, as well as the choice of simple actions for massless fermion and vector boson fields, yet these assumptions need the explanation, why has nature "decided" to make this particular choice of fermions and vector gauge fields and what steps to take in the evolution.

The assumption (vi.) that there is the massive scalar field, carrying the charges in the fundamental representations of the groups, while all the other bosons (vector bosons) carry
charges in the adjoint representations of the groups, and the assumption (viii.) that the Yukawa couplings take care of the fermion properties, without explaining the origin of these couplings, do not seem either elegant or simple.

The experiments have confirmed so far the existence of three families of fermions with properties required by the standard model, of the vector fields, which are the gauge fields of the charges $SU(3)$, $SU(2)$ and $U(1)$, and of the Higgs, all in accordance with the standard model assumptions.

To be able to predict the outcome of future experiments the next step beyond the standard model is needed.

Just adding several new fields by repeating ideas of the standard model, without explaining the assumptions of this so far so successful model, has to my understanding a little chance to be the right step.

The spin-charge-family theory [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] does explain all the assumptions of the standard model: i. The charges of the left and of the right handed quarks and leptons of one family - the right handed neutrinos are in this theory regular members of each family - and of their antiquarks and antileptons. ii. The appearance and properties of families. iii. The appearance and properties of the vector gauge fields of the family members charges. iv. The appearance and properties of scalars fields, explaining the properties of the Higgses carrying charges in the fundamental representations of the groups and the Yukawa couplings.

The spin-charge-family theory is offering also the explanation for the existence of the phenomena not explained by the standard model: a. For the dark matter [13]. b. For the (ordinary) matter-antimatter asymmetry [2].

This theory predicts: a. At the low energy regime two decoupled groups of four families; a.i. The fourth [1, 5, 4, 9, 12] to the already observed three families of quarks and leptons will be measured at the LHC [14]; a.ii. The lowest of the upper four families might be considered as constituting the dark matter [13]. b. New scalar fields with the weak and the hyper charges of the Higgs [1, 4], some of them will be measured at the LHC. c. New $SU(2)$ vector gauge fields, explaining the appearance of the hyper charge, and its $U(1)$ gauge field. d. New scalar fields, which are in the fundamental representations of the colour charge (triplets), explaining the ordinary matter-antimatter asymmetry and causing the proton decay.

Within this theory many consequences of the standard model, like the ”miraculous” cancellation of the triangle anomalies, can straightforwardly be explained (Subsect. 4.3).

2. Short presentation of the spin-charge-family theory

This section follows a lot the similar one from Ref. [2]. It briefly presents the spin-charge-family theory ([1, 2, 4, 5], and the references therein). The details of the theory will follow in Sect. (3).

Let me start with the assumptions on which the theory is built. Comments, following the assumptions, will explain the meaning of each of the assumptions.

A i. Fermions ($\psi$) carry in $d = (13 + 1)$ as the internal degrees of freedom only two kinds of spins, no charges, determined by the two kinds of the Clifford objects $^1$, ($\gamma^a$ and $\tilde{\gamma}^a$), and interact correspondingly with the two kinds of the spin connection fields - $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$, (the gauge fields of $S^{ab} = \frac{i}{4} (\gamma^a \gamma^b - \gamma^b \gamma^a)$, the generators of $SO(13, 1)$ and $\tilde{S}^{ab} = \frac{i}{4} (\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)$.

$^1$ There exist only two kinds of the Clifford algebra objects, connected with the left and right multiplication ([1], Sect. IV. Eq.(28)).
the generators of $\widetilde{SO}(13,1)$, and the vielbeins $f^a_a$.

$$\mathcal{A} = \int d^4x \, E \mathcal{L}_f + \int d^4x \, E \left( \alpha R + \tilde{\alpha} \tilde{R} \right) ,$$

$$\mathcal{L}_f = \frac{1}{2} \left( \bar{\psi} \gamma^a p_0 \psi + \text{h.c.} \right) ,$$

$$p_0 = f^a a_0 + \frac{1}{2E} \{ p_0, E f^a \} ,$$

$$p_0 = p_0 - \frac{1}{2} s^a b_0 b_a - \frac{1}{2} \tilde{s}^a b_0 b_a ,$$

$$R = \frac{1}{2} \{ f^a \{ f^b \} (\omega_{a b, \alpha} - \omega_{c a b, \omega^c_{\beta}}) \} + \text{h.c.} ,$$

$$\tilde{R} = \frac{1}{2} \{ f^a \{ f^b \} (\tilde{\omega}_{a b, \alpha} - \tilde{\omega}_{c a b, \tilde{\omega}^c_{\beta}}) \} + \text{h.c.} .$$

(1)

Here $f^a = f^a f^b - f^{ab} f^b$.

$R$ and $\tilde{R}$ are the two scalars ($R$ is a curvature).

**A ii.** The manifold $M^{(13+1)}$ breaks first into $M^{(7+1)} \times M^6$ (which manifests as $SO(7,1) \times SU(3) \times U(1))$, affecting both internal degrees of freedom - the one represented by $\gamma^a$ and the one represented by $\tilde{\gamma}^a$. Since the left handed (with respect to $M^{(7+1)}$) spinors couple differently to scalar (with respect to $M^{(7+1)}$) fields than the right handed ones, the break can leave massless and mass protected 2((7+1)/2−1) families [34]. The rest of families get heavy masses.

**A iii.** There is additional breaking of symmetry: The manifold $M^{(7+1)}$ breaks further into $M^{(3+1)} \times M^4$.

**A iv.** There is a scalar condensate (Table 1) of two right handed neutrinos with the family quantum numbers of the upper four families, bringing masses of the scale $10^{16}$ GeV or higher to all the vector and scalar gauge fields, which interact with the condensate [2].

**A v.** There are the scalar fields with the space index $(7,8)$ carrying the weak ($\tau^1$) and the hyper charges ($Y = \tau^3 + \tau^4$, $\tau^5$ and $\tau^8$ are generators of the subgroups of $SO(4)$, $\tau^4$ and $\tau^8$ are the generators of $U(1)_{\gamma}$ and $SU(3)$, respectively, which are subgroups of $SO(6)$), which with their nonzero vacuum expectation values change the properties of the vacuum and break the weak charge and the hyper charge. Interacting with fermions and with the weak and hyper bosons, they bring masses to heavy bosons and to twice four groups of families. Carrying no electromagnetic ($Q = \tau^3 + Y$) and colour ($\tau^3$) charges and no $SO(3,1)$ spin, the scalar fields leave the electromagnetic, colour and gravity fields in $d = (3 + 1)$ massless.

**Comments (C) on the assumptions (A):**

$^2$ $f^a = f^a f^b - f^{ab} f^b$. $R$ and $\tilde{R}$ are the two scalars ($R$ is a curvature).

$^3$ A toy model [34, 35] was studied in $d = (5+1)$ with the same action as in Eq. (1). The break from $d = (5+1)$ to $d = (3+1) \times S^2$ was studied. For a particular choice of vielbeins and for a class of spin connection fields the manifold $M^{(5+1)}$ breaks into $M^{(3+1)}$ times an almost $S^2$, while $2^{((3+1)/2−1)}$ families remain massless and mass protected. Equivalent assumption, although not yet proved how does it really work, is made in the $d = (13+1)$ case. This study is in progress.
C i. The simple starting action, Eq. (1), of the spin-charge-family theory leads in the low energy regime - after the breaks of the starting symmetry - to the effective action, which is the standard model action with the right handed neutrinos included, what offers the explanation for all the standard model assumptions, as well as for the appearance of the families, of the Higgs and of the Yukawa couplings:

C i.a. One Weyl (massless) representation of $SO(13,1)$ contains $[5,3,4,9,1]$, if analyzed with respect to the subgroups $SO(3,1), SU(2)_I, SU(2)_II, SU(3)$ and $U(1)$ (Eqs. (A1) - (A6) of Ref. [1]), all the family members and anti-members assumed by the standard model, with the right handed neutrinos as the regular members of each family in addition: It contains the left handed weak ($SU(2)_I$) charged and $SU(2)_II$ chargeless colour triplet quarks and colourless leptons and right handed weak chargeless and $SU(2)_II$ charged coloured quarks and colourless leptons, as well as the right handed weak charged and $SU(2)_II$ chargeless anti-coloured (anti-triplet) antiquarks and (anti)colourless antileptons, and left handed weak chargeless and $SU(2)_II$ charged antiquarks and antileptons. (The anti-fermion states are reachable from the fermion states by the application of the discrete symmetry operator $C_N, P_N$, presented in Ref. [36].)

C i.b. Before the electroweak break all observable gauge fields are massless: the gravity, the colour octet vector gauge fields (of the group $SU(3)$ from $SO(6)$), the weak triplet vector gauge fields (of the group $SU(2)_I$ from $SO(4)$), and the hyper singlet vector gauge field (a superposition of $U(1)$ from $SO(6)$ and the third component of $SU(2)_II$ triplet gauge fields). All are the superposition of the $J^a_c \omega_{a\sigma}$ spin connection gauge fields.

C i.c. Before the electroweak break there are two decoupled massless groups of four families of quarks and leptons, in the fundamental representations of $SU(2)_R \times SO(3,1)$ and $SU(2)_II \times SO(4)$ and $\tilde{SU}(2)_L, SO(3,1) \times \tilde{SU}(2)_I, SO(4)$ groups, respectively - the subgroups of $SO(3,1)$ and $SO(4)$ (Table 5). These eight families remain massless up to the electroweak break due to the "mass protection mechanism": Right handed members have no left handed partners with the same charges.

C i.d. There are scalar fields $[2,1]$ with the space index (7,8) and with respect to the space index with the weak charge and the hyper charge of the Higgs's scalar (Eq. (21)). They have additional quantum numbers, belonging either to one of the two groups of two triplets or to three singlets.

One group of the two triplets belong to the family groups $\tilde{SU}(2)_R, SO(3,1)$ and $\tilde{SU}(2)_II, SO(4)$ and couple to the upper four families. The second group of the two triplets belong to the family groups $\tilde{SU}(2)_L, SO(3,1)$ and $\tilde{SU}(2)_I, SO(4)$ and couple to the lower four families. All these scalars are the superposition of $f^a_s \omega_{a\sigma}$. Scalars, belonging to three singlets, are the gauge fields of the charges $(Q, Q', Y')$ and couple to the family members of both groups of families. They are the superposition of $f^a_s \omega_{a\sigma}$. Both kinds of scalar fields determine the fermion masses (Eq. (31)), offering the explanation for the Yukawa couplings and the heavy bosons masses.

C i.e. The starting action contains also additional $SU(2)_II$ (from $SO(4)$, Eq. (A21)) vector gauge fields (one of the components contributes to the hyper charge gauge fields as explained above), as well as the scalar fields with the space index $s \in (5,6)$ and $t \in (9,10, \ldots ,14)$. All these fields gain masses of the scale of the condensate (Table 1) with which they interact. They

$$\begin{align*}
\mathbf{A}_1 &= (\omega_5, -\omega_{17}, \omega_{14}, \omega_{18}, \omega_{16}, -\omega_{12}, \omega_{10}, \omega_{11}), \\
\mathbf{A}_5^L &= (\omega_{23}, i \omega_{51}, \omega_{31}, i \omega_{23}, \omega_{12}, i \omega_{31}), \\
\mathbf{A}_5^R &= (\omega_{23}, \omega_{51}, \omega_{31}, \omega_{23}, \omega_{12}, \omega_{31}), \\
\mathbf{A}_s^N &= (\omega_{23}, \omega_{51}, \omega_{31}, \omega_{23}, \omega_{12}, \omega_{31}), \\
Q &= -\tan^2 \vartheta_1 + \tan^2 \vartheta_1, \\
Q' &= -\tan^2 \vartheta_1 + \tan^2 \vartheta_1, \\
Y &= \tau^2 + \tau^2, \\
Y' &= -\tan^2 \vartheta_1 + \tan^2 \vartheta_1.
\end{align*}$$
all are expressible as superposition of $f_{\mu m} \tilde{\omega}_{a b y}$. In the case of free fields (if no spinor source, carrying their quantum numbers, is present) both $f_{\mu m} \tilde{\omega}_{a b y}$ and $f_{\mu m} \tilde{\omega}_{a b y}$ are expressible with vielbeins, Eq. (C9) in Ref. [1, correspondingly only one kind of the three gauge fields are the propagating fields.

C ii., C iii.: There are many ways of breaking symmetries from $d = (13 + 1)$ to $d = (3 + 1)$. The assumed breaks explain why the weak and the hyper charges are connected with the handedness of spinors, manifesting correspondingly the properties of the family members - the quarks and the leptons, left and right handed (Table 2) - and of the vector gauge fields.

Since the left handed members are weak charged while the right handed ones are weak chargeless 7, the family members remain massless and mass protected up to the electroweak break, when the nonzero vacuum expectation values of the scalar fields with the space index $(7, 8)$ break the weak and the hyper charge symmetry.

Antiparticles are accessible from particles by the application of the operator $\mathcal{C}_A \cdot \mathcal{P}_Y$, as explained in Refs. [36, 37]. This discrete symmetry operator does not contain $\gamma^a$'s degrees of freedom: To each family member there corresponds the anti-member, with the same family quantum number.

C iv.: It is the condensate of two right handed neutrinos with the quantum numbers of the upper four families (Table 1), which makes massive all the scalar gauge fields (with the index $(5, 6, 7, 8)$, as well as those with the index $(9, \ldots, 14)$) and the vector gauge fields, manifesting nonzero $\tau^4, \tau^{23}, \tau^4, \tau^{23}, N^5 \bar{R}$. Only the vector gauge fields of $Y$, $SU(3)$ and $SU(2)_{1}$ remain massless, since they do not interact with the condensate, their corresponding quantum numbers are zero ($Y = 0$, $\tau^3 = 0$ and $\tau^1 = 0$).

C v.: At the electroweak break the scalar fields with the space index $s = (7, 8)$ - originating in $\tilde{\omega}_{a b y}$ 9, as well as some superposition of $\tilde{\omega}_{a b y}'s$ with the quantum numbers $(Q, Q', Y')$ (footnotes in this paper and Ref. [1], Eq. (22)), conserving the electromagnetic charge - change their mutual interaction, and gaining nonzero vacuum expectation values change correspondingly also their masses. They contribute to the mass matrices of twice four families, as well as to the masses of the heavy vector bosons (to the two members of the weak triplet and the superposition of the third member of the weak triplet with the hyper vector field (Ref. [1], Eqs. (17-20)).

All the rest scalar fields keep masses of the scale of the condensate and are correspondingly unobservable in the low energy regime.

The fourth family to the observed three ones is predicted to be observed at the LHC. Its properties are under consideration [14, 15]. The baryons of the stable family of the upper four families offer the explanation for the dark matter [13]. The triplet and anti-triplet scalar fields contribute together with the condensate to the matter/anti-matter asymmetry [2].

7 The left handed anti-members are weak chargeless while the right handed anti-members are weak charged.

8 $\tilde{\varphi}^i := \frac{1}{2}(S^{58} - S^{67}, S^{57} + S^{68}, S^{56} - S^{77}), \tilde{\tau}^i := \frac{1}{2}(S^{58} + S^{67}, S^{57} - S^{68}, S^{56} + S^{77}), \tilde{N}_L := \tilde{N}_{(L,R)} := \frac{1}{2}(S^{58} \pm iS^{67}, S^{56} \pm S^{77}, S^{51} \pm iS^{52}, S^{61} \pm iS^{62}), \tilde{\rho} := \frac{1}{2}(S^{50} - S^{51} + 3S^{12}, S^{59} + S^{50} - 3S^{12}, S^{59} + S^{50} + 3S^{12}, S^{51} + S^{52} + S^{53} + S^{54} + S^{55} + S^{56} + S^{57} + S^{58} + S^{59} + S^{60} + S^{61} + S^{62} + S^{63} + S^{64} + S^{65} + S^{66} + S^{67} + S^{68} + S^{69} + S^{70})$.

9 $\tilde{A}^{N_{L,k}} \tilde{A}^{N_{R,k}} \tilde{A}^{N_{L,k}} \tilde{A}^{N_{R,k}} = \{(\tilde{\omega}_{(7,8), \tilde{A}^{N_{L,k}} \tilde{A}^{N_{R,k}}}) + \frac{i}{2}(\tilde{\omega}_{(7,8), \tilde{A}^{N_{L,k}} \tilde{A}^{N_{R,k}}}) - \frac{i}{2}(\tilde{\omega}_{(7,8), \tilde{A}^{N_{L,k}} \tilde{A}^{N_{R,k}}}) + \frac{i}{2}(\tilde{\omega}_{(7,8), \tilde{A}^{N_{L,k}} \tilde{A}^{N_{R,k}}})\}$, $\tilde{A}^{(L)}_{(7,8)} = (\tilde{\omega}_{(7,8), \tilde{A}^{N_{L,k}} \tilde{A}^{N_{R,k}}})$, $\tilde{A}^{(R)}_{(7,8)} = (\tilde{\omega}_{(7,8), \tilde{A}^{N_{L,k}} \tilde{A}^{N_{R,k}}})$, with $N_{L,k} A^{N_{L,k}} \tilde{A}^{N_{R,k}} = N_{R,k} A^{N_{L,k}} \tilde{A}^{N_{R,k}} = 0$ and $Q A^{Q}_{(7,8)} = 0$ (Ref. [1], Eq. (22)).
I shall formally rewrite the part of the action in Eq.(1), which determines the spinor degrees of freedom, in the way that we can clearly see that the action does manifest in the low energy regime by the standard model required degrees of freedom of the fermions (Table B1), of the vector gauge fields (Table B2) and of the scalar gauge fields (Table B3) [5, 3, 4, 10, 1, 9, 6, 7, 8, 11, 12, 13, 14].

\[ \mathcal{L}_f = \bar{\psi} \gamma^m (p_m - \sum_{A,i} g^{Ai} \tau^{Ai} A_{nm}^i) \psi + \]
\[ \{ \sum_{s=7,8} \bar{\psi} \gamma^s p_{0s} \psi \} + \]
\[ \{ \sum_{t=5,6,9,\ldots,14} \bar{\psi} \gamma^t p_{0t} \psi \}, \]

where \( p_{0s} = p_s - \frac{1}{2} S_{s'} s' \omega_{s s'} - \frac{1}{2} \tilde{S}^{ab} \omega_{abs} \) \( p_{0t} = p_t - \frac{1}{2} S_{t'} t' \omega_{t t'} - \frac{1}{2} \tilde{S}^{ab} \omega_{abt} \), with \( m \in \{ 0, 1, 2, 3 \} \), \( s \in \{ 7, 8 \} \), \( (s', s') \in \{ 5, 6, 7, 8 \} \), \( (a, b) \) (appearing in \( \tilde{S}^{ab} \)) run within either \( \{ 0, 1, 2, 3 \} \) or \( \{ 5, 6, 7, 8 \} \), \( t \) runs in \( \{ 5, \ldots, 14 \} \), \( (t', t') \) run either in \( \{ 5, 6, 7, 8 \} \) or in \( \{ 9, 10, \ldots, 14 \} \). The spinor function \( \psi \) represents all family members of all the \( 2^{2+4+1}=8 \) families.

The first line of Eq. (2) determines (in \( d = 3+1 \)) the kinematics and dynamics of spinor (fermion) fields, coupled to the vector gauge fields. The generators \( \tau^{Ai} \) of the charge groups are expressible in terms of \( S^{ab} \) through the complex coefficients \( c^{Ai}_{ab} \) (definition of \( \tau^{Ai} \) can be found in the footnote 8), the rest in 10,

\[ \tau^{Ai} = \sum_{a, b} c^{Ai}_{ab} S^{ab}, \]

fulfilling the commutation relations

\[ \{ \tau^{Ai}, \tau^{Bj} \} = i \delta^{AB} f^{Aijk} \tau^{Ak} . \]

They represent the colour \( (\tau^{3i}) \), the weak \( (\tau^{1i}) \) and the hyper \( (Y) \) charges, as well as the \( SU(2)_R \) \( (\tau^{2i}) \) and \( U(1)_I \) \( (\tau^{4}) \) charges, the gauge fields of these last two groups gain masses interacting with the condensate, Table 1. The condensate leaves massless, besides the colour and gravity

\[ Y \equiv \tau^{4} + \tau^{23}, \quad Y' = -\tau^{4} \tan^{2} \vartheta_{1} + \tau^{13}, \quad \vartheta_{1} \] is the electroweak angle, breaking the weak \( SU(2) \) and the hyper charge, \( \vartheta_{2} \) is the angle of the break of \( SU(2)_R \) from \( SU(2)_I \times SU(2)_R \). \( Q := \tau^{13} + Y \) remains the conserved charge.

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**Table 1.** This table is taken from [2]. The condensate of the two right handed neutrinos \( \nu_R \), with the \( VIII^{th} \) family quantum numbers, coupled to spin zero and belonging to a triplet with respect to the generators \( \tau^{2i} \), is presented together with its two partners. The right handed neutrino has \( Q = 0 = Y \). The triplet carries \( \tau^{4} = -1, \tau^{23} = 1 \), \( \tau^{13} = -1 \), \( N^{3}_{R} = 1, \tilde{N}^{3}_{L} = 0, \tilde{Y} = 0, \tilde{Q} = 0 \). The family quantum numbers are presented in Table 5.
gauge fields, the weak and the hyper charge vector gauge fields. The corresponding vector gauge fields $A_{m}^{Ai}$ are expressible with the spin connection fields $\omega_{stm}$.

$$A_{m}^{Ai} = \sum_{s,t} c_{st}^{Ai} \omega_{stm}, \quad (5)$$

with $(s,t)$ either in $(5,6,7,8)$ or in $(9,\ldots,14)$, in agreement with the assumptions A ii. and A iii.. I demonstrate [1, 18] in Subsect. 3.2 the equivalence between the usual Kaluza-Klein procedure leading to the vector gauge fields through the vielbeins and the procedure with the spin connections used by the spin-charge-family theory.

All the vector gauge fields, appearing in the first line of Eq. (2), except $A_{m}^{2\pm}$ and $A_{m}^{Y'}$ ($= \sin \theta_{2} A_{m}^{\pm} - \sin \theta_{2} A_{m}^{\pm} Y'$ and $\tau^{4}$ are defined in the footnote 11), are massless before the electroweak break. $A_{m}^{4}$ carries the colour charge $SU(3)$ (originating in $SO(6)$), $\bar{A}_{m}^{1}$ carries the weak charge $SU(2)I$ ($SU(2)I$ and $SU(2)II$ are the subgroups of $SO(4)$) and $A_{m}^{Y}$ ($= \sin \theta_{2} A_{m}^{2\mp} + \cos \theta_{2} A_{m}^{4}$) carries the corresponding $U(1)$ charge ($Y = \tau^{23} + \tau^{4}$, $\tau^{4}$ originates in $SO(6)$ and $\tau^{23}$ is the third component of the second $SU(2)II$ group, $A_{m}^{4}$ and $\bar{A}_{m}^{2}$ are the corresponding vector gauge fields). The fields $A_{m}^{2\pm}$ and $A_{m}^{Y'}$ get masses of the order of the condensate scale through the interaction with the condensate of the two right handed neutrinos with the quantum numbers of one of the groups of four families (the assumption A iv., Table 1). (See Ref. [1].)

### 3.1. Quarks and leptons in the spin-charge-family theory

To offer the explanation for the origin of quantum numbers of one (anyone) family of massless quarks and leptons, assumed by the standard model (Table B1), the spin-charge-family theory must answer the question, where do the standard model charges originate and why are the weak and the hyper charge connecting with the spin of quarks and leptons. The theory must answer also the question: Where do families of quarks and leptons originate?

This section demonstrates that spinors, which carry in $d = (13 + 1)$ nothing but spins of two kinds, determined by the two kinds of the Clifford algebra objects (Sect. IV. and App. B in [1]), explain the origin of spins and charges and of families: One kind of spins manifests in $d = (3 + 1)$ at low energies the spin and all the charges, connecting the spin (the handedness) and the charges [6, 7, 8, 11, 31, 32, 33]. The second kind explains the origin of families.

To explain the appearance of the electroweak break, which causes that all the families become massive, the properties of the scalar fields must be explained. This is done in Subsects. 3.3, where also the appearance of the Yukawa couplings is explained, while masses of the two groups of four families, predicted by the spin-charge-family theory, will be discussed in Subsect. 4.1.

In Table 2 one Weyl representation of spinors in $d = (13 + 1)$ is presented. The technique [6, 7, 31, 32], Appendix A, is used, which makes that the states themselves demonstrate properties of spinors. Besides the states also the quantum numbers of the members are presented with respect to the groups $SO(3,1)$, $SU(2)I$, $SU(2)II$, $U(1)II$ and $SU(3)$, which are the subgroups of the group $SO(13,1)$. One easily sees that the states of one Weyl representation include all the quarks and the leptons, and the antiquarks and the antileptons of one family of quarks and leptons, with just the quantum as assumed by the standard model, Table B1.

Table 2 demonstrates that left handed quarks and leptons carry the weak charge $(SU(2)I$ with $\tau^{11}$ as generators) and the hyper charge $(Y = \tau^{23} + \tau^{4}$, $\tau^{4}$ is a subgroup of $SO(2)II$, which is a subgroup of $SO(4)$, $\tau^{4}$ are generators of $U(1)II$, which is a subgroup of $SO(6)$) just as required by the standard model, Table B1, while the right handed quarks and leptons are weak

11 $Y' := -\tau^{4} \tan^{2} \theta_{2} + \tau^{23}$, $\tau^{4} = -\frac{1}{2}(S^{10} + S^{11,12} + S^{13,14})$. 




Table 2. The left handed ($\Gamma^{(13,1)} = -1$) ($= \Gamma^{(7,1)} \times \Gamma^{(6)}$) multiplet of spinors - the members of the $SO(13,1)$ group representation, manifesting the subgroup $SO(7,1)$ - of the colour charged quarks and antiquarks and the colourless leptons and antileptons, is presented in the massless basis using the technique presented in Appendix A. It contains the left handed ($\Gamma^{(3,1)} = -1$) weak charged ($\tau^{13} = \pm \frac{1}{2}$) and $SU(2)_{II}$ chargeless ($\tau^{23} = 0$) quarks and the right handed weak chargeless and $SU(2)_{II}$ charged ($\tau^{23} = \pm \frac{1}{2}$) quarks of three colours ($e^{a} = (\tau^{53}, \tau^{58})$) with the ”spinor” charge ($\tau^{4} = \frac{1}{2}$) and the colourless left handed weak charged and right handed weak chargeless leptons with the ”spinor” charge ($\tau^{4} = -\frac{1}{2}$). $S^{12}$ defines the ordinary spin $\frac{1}{2}$.

Table contains also the corresponding anti-states with opposite charges, reachable from the particle states by the application of the discrete symmetry operator $C, P$, presented in Refs. [36, 37]. The vacuum state, on which the nilpotents and projectors operate, is not shown. The reader can find this Weyl representation also in Refs. [2, 10, 1]. Table is separated into three parts.

| i | $\Gamma^{(13,1)} \times \Gamma^{(7,1)} \times \Gamma^{(6)}$ | $\Gamma^{(3,1)} \times \Gamma^{(7,1)} \times \Gamma^{(6)}$ | $\Gamma^{(3,1)} \times \Gamma^{(7,1)} \times \Gamma^{(6)}$ | $\Gamma^{(3,1)} \times \Gamma^{(7,1)} \times \Gamma^{(6)}$ |
|---|---|---|---|---|
| 1 | $u_{R}^{3}$ | $\frac{1}{2}$ | $1$ | $0$ | $\bar{C}_{\text{antiquark}}$ | $\bar{S}_{\text{lepton}}$ | $\bar{Y}$ | $\bar{Q}$ |
| 2 | $u_{R}^{5}$ | $\frac{1}{2}$ | $1$ | $0$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $Y^{+}$ | $Q^{+}$ |
| 3 | $d_{L}^{0}$ | $\frac{1}{2}$ | $1$ | $0$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\bar{C}_{\text{antiquark}}$ | $\bar{S}_{\text{lepton}}$ |
| 4 | $d_{L}^{1}$ | $\frac{1}{2}$ | $1$ | $0$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\bar{C}_{\text{antiquark}}$ | $\bar{S}_{\text{lepton}}$ |
| 5 | $d_{L}^{2}$ | $\frac{1}{2}$ | $1$ | $0$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\bar{C}_{\text{antiquark}}$ | $\bar{S}_{\text{lepton}}$ |
| 6 | $d_{L}^{3}$ | $\frac{1}{2}$ | $1$ | $0$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\bar{C}_{\text{antiquark}}$ | $\bar{S}_{\text{lepton}}$ |
| 7 | $u_{R}^{4}$ | $\frac{1}{2}$ | $1$ | $0$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\bar{C}_{\text{antiquark}}$ | $\bar{S}_{\text{lepton}}$ |
| 8 | $u_{R}^{5}$ | $\frac{1}{2}$ | $1$ | $0$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\bar{C}_{\text{antiquark}}$ | $\bar{S}_{\text{lepton}}$ |
| 9 | $u_{R}^{6}$ | $\frac{1}{2}$ | $1$ | $0$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\bar{C}_{\text{antiquark}}$ | $\bar{S}_{\text{lepton}}$ |
| 10 | $u_{R}^{7}$ | $\frac{1}{2}$ | $1$ | $0$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\bar{C}_{\text{antiquark}}$ | $\bar{S}_{\text{lepton}}$ |

chargeless, carrying the $SU(2)_{II}$ charge with $\tau^{2i}$ as generators, determining the hyper charges of quarks and leptons, again in agreement with the standard model.

Quarks carry the colour charge in the fundamental representation and the ”fermion charge” $\tau^{4} = \frac{1}{2}$, leptons are colourless carrying the ”fermion charge” $\tau^{4} = -\frac{1}{2}$. Correspondingly is the hyper charge of either left handed or right handed quarks and leptons in agreement with the standard model assumptions.

Table 2 demonstrates that left handed antiquarks and antileptons are weak chargeless and right handed antiquarks and antileptons are weak charged. Antiquarks carry antitriplet charges, while leptons are anticolourless. Correspondingly fermions and anti-fermions carry opposite colour, hyper, electromagnetic and ”fermion” charges than fermions.

Handedness is in the one Weyl representation of $SO(13,1)$ strongly related to the weak charge and the hyper charge.

Left and right handed neutrinos (carrying nonzero $Y^{C} = -\tau^{4} \tan \beta + \tau^{23}$ quantum number) are the regular members of each family, and so are the antineutrinos.

Let me call attention to the reader that the term $\gamma^{0} \tilde{A}_{(7)}^{Ai} A_{(8)}^{Ai}$, where $\gamma^{0}$ represents the superposition of either $S_{ab}$ or $\tilde{S}_{ab}$ and $A_{(7)}^{Ai}$ represent correspondingly the superposition of either the scalar fields $\omega_{\alpha_{ab}}$ or the scalar fields $\tilde{\omega}_{\alpha_{ab}}$, $s \in (7, 8)$, as presented in Subsect. 3.3, Eq. (20), transforms the right handed $u_{R}^{3}$ quark from the first line of Tables 2–4 into the left handed
Table 3. Continuation of Table 2

| i | \(I^PQ > 0\) | \(I^{PQ} = 0\) | \(I^{PQ} = 1\) | \(I^{PQ} = (1 - 1)\) | \(I^{PQ} = (1 - 1)\) | \(I^{PQ} = -1\) | \(Y\) | \(Q\) |
|---|---|---|---|---|---|---|---|---|
| 17 | \( u_R^3 \) | 0.12 | 0.06 | 0.05 | 0.04 | 0.03 | -0.02 | -0.01 | -1 |
| 18 | \( u_R^3 \) | 0.03 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 25 | \( \psi_R \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 26 | \( \psi_R \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 27 | \( \psi_R \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 28 | \( \psi_R \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 29 | \( \psi_R \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 30 | \( \psi_R \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 31 | \( \psi_R \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 32 | \( \psi_R \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 4. Continuation of Table 2

| i | \(I^PQ > 0\) | \(I^{PQ} = 0\) | \(I^{PQ} = 1\) | \(I^{PQ} = (1 - 1)\) | \(I^{PQ} = (1 - 1)\) | \(I^{PQ} = -1\) | \(Y\) | \(Q\) |
|---|---|---|---|---|---|---|---|---|
| 33 | \( d_R^1 \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 34 | \( d_R^1 \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 35 | \( q_L^1 \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 36 | \( q_L^1 \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 37 | \( d_R^2 \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 38 | \( d_R^2 \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 39 | \( q_R^2 \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 40 | \( q_R^2 \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 41 | \( d_L^2 \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 42 | \( d_L^2 \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 43 | \( q_R^3 \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 44 | \( q_R^3 \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 45 | \( q_R^3 \) | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

\(u_L^4\) quark from the seventh line of the same table \(^{12}\), which can, due to the properties of the scalar fields (Eq. (21)), be interpreted also in the standard model way, namely, that \(A_{18}^{A_i}\) “dress” \(u_L^4\) giving it the weak and the hyper charge of the left handed \(u_L^3\) quark, while \(\gamma^0\) changes handedness. Equivalently happens to \(\nu_R\) from the 25\(^{th}\) line, which transforms under the action of \(\gamma^0 (-) \tau^{Ai} A_{18}^{A_i}\), into \(\nu_L\) from the 31\(^{th}\) line.

\(^{12}\)This transformation of the right handed family members into the corresponding left handed partners can easily be calculated by using Eqs. (A.12, A.10, A.26).
Table 5. Eight families of the right handed $\nu^c_{1R}$ (2–4) quark with the spin $\frac{1}{2}$, colour charge $(\tau^3 = 1/2, \tau^8 = 1/(2\sqrt{3})$, and of the colourless right handed neutrino $\nu_R$ of spin $\frac{1}{2}$ are presented in the left and in the right column, respectively. They belong to two groups of four families, one (I) is a doublet with respect to $(\vec{N}_R$ and $\vec{\tau}^{(1)})$ and a singlet with respect to $(\vec{N}_L$ and $\vec{\tau}^{(2)})$, the other (II) is a singlet with respect to $(\vec{N}_L$ and $\vec{\tau}^{(1)})$ and a doublet with respect to $(\vec{N}_R$ and $\vec{\tau}^{(2)})$. All the families of each of the two groups follow from the starting one by the application of one of the two operators $(\vec{N}_{RL}; \vec{\tau}^{(1,2,3)})$, Eq. (A.26), respectively. The generators $(N_{RL}^\pm, \vec{\tau}^{(2,1,3)})$ (Eq. (A.26)) transform $u_{1R}$ of one family to all the members of the same family of the same colour. The same generators transform equivalently the right handed neutrino $\nu_{1R}$ to all the colourless members of the same family. The table is taken from Ref. [2].

| Family | $N_{RL}^+$ | $N_{RL}^-$ | $\vec{\tau}^{(1)}$ | $\vec{\tau}^{(2)}$ |
|--------|------------|------------|----------------|----------------|
| I      | $u_{1R}^+$ | $u_{1R}^-$ | $\vec{\tau}^{(1)}$ | $\vec{\tau}^{(2)}$ |
| II     | $u_{2R}$  | $u_{2R}$  | $\vec{\tau}^{(1)}$ | $\vec{\tau}^{(2)}$ |

The operator $\gamma^0 (\pm) \tau^{Ai} A_{78}^{Ai}$ transforms $d_{L}^{11}$ from the third line of Tables 2–4 into $d_{L}^{11}$ from the fifth line of this table, or $e_{R}$ from the $2^{7th}$ line into $e_{L}$ from the $29^{th}$ line, where $A_{78}^{Ai}$ belong to the scalar fields from Eq. (19).

The operator $\tau^{Ai}$, if representing the first three operators in Eq. (19), (only) multiplies the right handed family member with its eigenvalue.

The term $\gamma^0 (\mp) \tau^{Ai} A_{78}^{Ai}$ of the action (Eqs. (1, 2)) determines the Yukawa couplings (3.3, 4.1).

Since spinors (fermions) carry besides the family members quantum numbers also the family quantum numbers, determined by $\vec{S}^{ab} = \frac{i}{4}(\vec{\gamma}^{a}\vec{\gamma}^{b} - \vec{\gamma}^{b}\vec{\gamma}^{a})$, there are correspondingly $2^{7+1}/2 - 1 = 8$ families [1], which split into two groups of families, each manifesting the $(SU(2)_{SO(3,1)} \times SU(2)_{SO(4)} \times U(1))$ symmetry.

The eight families of the right handed $u_{1R}$ quark (the first member of the eight-plet of quarks from Tables 2–4) and of the right handed $\nu_{1R}$ leptons (the first member of the eight-plet of leptons from Tables 2–4) are presented as an example in Table 5 [4].

All the other members of any of the eight families of quarks or leptons follow from any member of a particular family by the application of the operators $N_{RL}^+$ and $\tau^{(2,1,3)}$ on this particular member.

The eight families separate into two groups of four families: One group contains doublets with respect to $\vec{N}_R$ and $\vec{\tau}^{(2)}$, these families are singlets with respect to $\vec{N}_L$ and $\vec{\tau}^{(1)}$. Another group of four families contains doublets with respect to $\vec{N}_L$ and $\vec{\tau}^{(2)}$, these families are singlets with respect to $\vec{N}_R$ and $\vec{\tau}^{(1)}$.

If $\tau^{Ai}$ represents the last four operators of Eq. (19) in Subsect. 3.3, the operators $\gamma^0 (\mp) \tau^{Ai} A_{78}^{Ai}$ for $(u_R, \nu_R)$ and $(d_R, e_R)$, respectively) transform the right handed family member of
A kinds of the spin connection fields. After the break of the starting symmetry there are no spinor sources present. Spinors (fermions) interact with the vielbeins and the two connection fields are uniquely determined by the vielbeins (Ref. [1], Eqs. ((30)-(32), (C9))), if gravitational ones: the vielbeins and the spin connections of two kinds. Both kinds of the spin connection gauge fields of both kinds

3.2. Vector gauge fields in the spin-charge-family theory

In the starting action of Eq. (1) of the spin-charge-family theory all the gauge fields are the gravitational ones: the vielbeins and the spin connections of two kinds. Both kinds of the spin connection fields are uniquely determined by the vielbeins (Ref. [1], Eqs. ((30)-(32), (C9))), if there are no spinor sources present. Spinors (fermions) interact with the vielbeins and the two kinds of the spin connection fields. After the break of the starting symmetry SO(13,1) the starting action manifests at low energies the effective action. Eq. (2) represents the effective action for fermions in $d = (3 + 1)$ interacting with the vector gauge fields, which are the superposition of the spin connection gauge fields with the vector index $m$ ($m = (0,1,2,3)$) - $A_{\mu}^{\text{Ai}} = \sum_{s,t} c^{\text{Ai}st} \omega_{stm}$ - and the scalar gauge fields, which are the superposition of the spin connection gauge fields of both kinds $\omega_{abs}$’s and $\tilde{\omega}_{abs}$’s with the scalar index $s (s \geq 5)$ - $A^{(Q,Q',Y')} = \sum_{s,t} c^{(Q,Q',Y')}_{s,t} \omega_{stm}$ (for $(Q,Q',Y')$, respectively), $\tilde{A}_{s}^{\text{Ai}} = \sum_{s,t} c^{\text{Ai}st} \tilde{\omega}_{stm}$. I comment in this section that the vector gauge fields in the spin-charge-family theory appear equivalently either from vielbeins $f^{\mu}_{m}$ - like it is usually proceeded in the Kaluza-Klein-like theories [40, 39] (with which the spin-charge-family theory has many things in common) - or from the spin connection fields $\sum_{s,t} c^{\text{Ai}st} \omega_{stm}$. This is indeed known for a long time [39, 40, 41].

This section reviews Refs. [1, 18, 42], where this equivalence is demonstrated when the spaces of $d \geq 5$ have the metric tensor $g_{\sigma\tau} = \eta_{\sigma\tau} f^{-2}$, where $(x^{\sigma}, x^{\tau})$ determine the coordinates of the (almost [34]) compactified space, $\eta_{\sigma\tau}$ is the diagonal matrix in this space and $f$ is any scalar function of these coordinates.

Let the space with $s \geq 5$ have the symmetry allowing the infinitesimal transformations of the kind

$$x^{\mu} = x^{\mu}, \quad x^{\sigma} = x^{\sigma} - i \sum_{A,i,s,t} c^{\text{Ai}st} M_{stm} x^{\sigma}, \quad (6)$$

where $M_{stm} = S_{stm} + L_{stm}$, $L_{stm} = x^{s} p^{t} - x^{t} p^{s}$, $S_{stm}$ concern internal degrees of freedom of boson and fermion fields, $\{M^{st}, M^{st'}\}_- = i(\eta^{st} M^{ts'} + \eta^{ts'} M^{st} - \eta^{st} M^{t's'} - \eta^{t's'} M^{st'})$. From Eq. (6) it
follows

\[-i \sum_{s,t} c_{Ai}^{st} M_{st} x^\sigma = E^\sigma_{\tau Ai} = \sum_{s,t} c_{Ai}^{st} \left( x_s f^\sigma_t - x_t f^\sigma_s \right),\]

\[\sum_{s,t} c_{Ai}^{st} M_{st} \tau^\sigma = i E^\sigma_{\tau Ai}, \tag{7}\]

and correspondingly: \(\tau_{Ai} = E^\sigma_{\tau Ai} p_{\sigma}^{}\), where \(\tau_{Ai} = \sum_{s,t} c_{Ai}^{st} M_{st}\) with the commutation relations \(\{\tau_{Ai}, \tau_{Bj}\} = i \delta^{AB} f_{ABC} \tau_{Ck}, f_{ABC}^{}\) are the structure constants of the symmetry group \(A\). One derives, when taking into account Eq. (7) and the commutation relations among generators of the infinitesimal transformations \(\tau_{Ai}\) the equation for the Killing vectors \(E^\sigma_{\tau Ai}\) \[39\]

\[E^\sigma_{\tau Ai} p_{\sigma}^{} E^\tau_{\tau Bj} p_{\tau}^{} - E^\sigma_{\tau Bj} p_{\tau}^{} E^\tau_{\tau Ai} p_{\sigma}^{} = i \delta^{AB} f_{ABC} E^\sigma_{\tau Ai} p_{\sigma}^{} E^\tau_{\tau Bj} p_{\tau}^{}, \tag{8}\]

and the Killing equation

\[D_\sigma E_{\tau Ai} + D_\tau E_{\sigma Bj} = 0,\]

\[D_\sigma E_{\tau Ai} = \partial_\sigma E_{\tau Ai} - \Gamma^\tau_{\sigma \alpha} E_{\tau \alpha Ai}. \tag{9}\]

Let the corresponding background field \((g_{\alpha \beta} = e^a_{\alpha} e^a_{\beta})\) be

\[e^a_{\alpha} = \left( \delta^{m}_{\mu}, e^m_{\sigma} = 0 \right), \quad f^a_{\alpha} = \left( \delta^{m}_{\mu}, f^m_{\sigma} = 0 \right), \tag{10}\]

so that the background field in \(d = (3 + 1)\) is flat. From \(e^a_{\mu} f^\sigma_{\alpha} = \delta^a_{\mu} = 0\) it follows

\[e^a_{\mu} = - \delta^m_{\mu} e^a_{\sigma} f^\sigma_{m}. \tag{11}\]

This leads to

\[g_{\alpha \beta} = \left( \eta_{mn} + f^m_{\tau} e^\tau_{\sigma} e^s_{\sigma} - f^m_{\tau} e^s_{\sigma}, - f^m_{\tau} e^s_{\sigma}, \right) \tag{12}\]

and

\[g^{\alpha \beta} = \left( \eta^{mn}, f^m_{\sigma} f^s_{\tau}, f^m_{\sigma} f^s_{\tau} \right). \tag{13}\]

We have: \(\Gamma^\tau_{\sigma \alpha} = \frac{1}{2} g^{\tau \sigma'} (g_{\sigma \alpha} + g_{\sigma \alpha} + g_{\sigma \alpha})\).

Let us make a choice for the vielbein

\[f^\sigma_{m} = \sum_A \tilde{A}^A_{m}, \tag{14}\]

where we expect \(\tilde{A}^A_{m}\) that they manifest in \(d = (3 + 1)\) as the gauge fields of the charges \(\tau_{Ai}\).

To prove this we must compare the gauge fields \(A^A_{m} = c_{Ai}^{st} \omega_{stm}\), appearing in Eq. (2), with the gauge fields \(\tilde{A}^A_{m}\).

If there are no fermions present then the vector gauge fields of the family members charges and of the family charges - \(\omega_{stm}\) and \(\tilde{\omega}_{stm}\), respectively - are uniquely expressible with the vielbeins \([2, 1, 42]\). We are interested in the vector gauge fields in \(d = (3 + 1)\), for which we find

\[\omega_{stm} = \frac{1}{2E} \left\{ f^m_{\sigma} \left[ e_{ln} \partial_\tau (E f^\tau_{ln}) - e_{ln} \partial_\tau (E f^\tau_{ln}) \right] \right. \]

\[+ e_{ln} \partial_\tau [E \left( f^m_{\sigma} f^\tau_{ln} - f^m_{\sigma} f^\tau_{ln} \right)] - e_{ln} \partial_\tau [E \left( f^m_{\sigma} f^\tau_{ln} - f^m_{\sigma} f^\tau_{ln} \right)] \right\}. \tag{15}\]
We must show that if we calculate $c_{Ai} ^{st} \omega_{stm}$ by taking $\omega_{stm}$ from Eq. (15), and if we put on the right hand side of this equation the vielbeins $f^\sigma_m = \sum_A F^A_{m} \bar{A}^A_m$ from Eq. (14), we must end up with the equality relation

$$A^A_m = \bar{A}^A_m. \quad (16)$$

It is not difficult to check that Eq. (16) follows, if we take into account that $e^s_\mu = -\delta^m_\mu$ and make a choice of the symmetry of space $(d-4)$: $f^s_\mu = f \delta^s_\mu$ (Ref. [1], Sect. II).

Calculating from Eqs. (12, 13) the Riemann curvature $R^{(d)}$ in $d$-dimensional space by taking into account that $(d = (3+1))$ space is flat, one obtains

$$R^{(d)} = R^{(d-4)} - \frac{1}{4} g_{\sigma \tau} E_A^{\sigma} E_A^{\sigma'} F^{A}_{mn} F_{A'}^{mn},$$

$$F^{A}_{mn} = \partial_m A^A_n - \partial_n A^A_m - i f^{Aijk} A^A_m A^A_k,$$

$$A^A_m = \sum_{st} c^{A}_{st} \omega^{st}_m,$$

$$\tau^{A} = \sum_{st} c^{A}_{st} M_{st}. \quad (17)$$

The integration of the action $\int E d^4x (d^{-4}) x R^{(d)}$ over $(d-4)$ space (in which it turns out that only even functions of coordinates $x^\sigma$ give nonzero contributions) leads to the well known effective action for the vector gauge fields in $d = (3+1)$ space: $\int d^4x \left\{ \frac{1}{2} F^A_{\mu \nu} F^{A\mu \nu} \right\}.$

The quadratic form for the gauge vector fields, Eq. (17), in $d = (3+1)$, obtained from the curvature $R^{(d)}$ can be found in many text book [39]. In Ref. ([21], Sect. 5.3) the Lagrange function for the gauge vector fields is derived by using the Clifford algebra space. The author allows besides the curvature $R$ also its quadratic form $R^2$ (Eq. (240)).

3.3. Scalar fields in the spin-charge-family theory explain the origin of the Higgs and Yukawa couplings

In the spin-charge-family theory the spin connection fields of both kinds, $\omega_{abs}$ and $\bar{\omega}_{abs}$, carrying the space index $s = (7,8)$, explain the Higgs and the Yukawa couplings of the standard model. They all belong to the weak charge doublets ([1, 2] and references therein), as will be demonstrated in this section.

After gaining nonzero vacuum expectation values these scalar fields break the weak and the hyper charges of the vacuum (the assumption $A$ v. and comments $C$ v.) making all the fermions, massive due to the interaction with the vacuum. Also the heavy bosons gain masses, while interacting with the vacuum.

The gauge scalar fields with the space index $s > 8$ contribute to the matter-antimatter asymmetry in the universe [2].

This section follows mainly the equivalent sections in Refs. [2, 1].

It turns out [2] that all scalars (the gauge fields with the space index $s \geq 5$) of the action (Eq. (1)) carry charges in fundamental representations due to the space index: They are either doublets (Table 6), $s = (5,6,7,8)$, or triplets (Table 9, and Sect. II, Table I in Ref. [2]), $s = (9,10,13,14)$.Scalars with the space indices $s \in (7,8)$ and $s \in (5,6)$ are the $SU(2)$ doublets (Table 6) with respect to this degree of freedom.

All scalars carry additional quantum numbers: Besides the quantum numbers determined by the space index $s$ they carry also the quantum numbers $A_i$, Eq. (3), the states of which belong to the adjoint representations. They originate in $S^{ab}$ or $\bar{S}^{ab}$, Eq. (A.3), $S^{ab} = \frac{1}{4} (\gamma^a \gamma^b - \gamma^b \gamma^a)$, $\bar{S}^{ab} = \frac{1}{4} (\gamma^a \bar{\gamma}^b - \bar{\gamma}^b \gamma^a)$, the gauge fields of which are $\omega_{abs}$ and $\bar{\omega}_{abs}$, respectively. $S^{ab}$ determine family members spin and charges, $\bar{S}^{ab}$ determine family charges.
The infinitesimal generators $S_{ab}$, which apply on the spin connections $\omega_{bde} (= f^{a}_{
abla} \omega_{bde})$ and $\tilde{\omega}_{bde} (= f^{a}_{\nabla} \tilde{\omega}_{bde})$, on either the space index $e$ or any of the indices $(b, d, b, d)$, operates as follows

$$S^{ab} A^{d...e...g} = i (\eta^{ac} A^{d...b...g} - \eta^{be} A^{d...a...g}),$$

(see Section IV. and Appendix B in Ref. [1]).

The expressions for the infinitesimal operators of the subgroups of the starting group (presented in Eq. (3) and the footnote 8 and determined by the coefficients $c^{Ai}_{ab}$ in Eq. (3)) are the same for all three kinds of degrees of freedom, $S^{ab}$, $\tilde{S}^{ab}$ or $S^{ab}$. Correspondingly the commutation relations are also the same.

At the electroweak break all the scalar fields with the space index $(7, 8)$, those which belong to one of twice two triplets carrying the family quantum numbers ($\tilde{\tau}^{Ai}$) and those which belong to one of the three singlets carrying the family members quantum numbers $(Q, Q', Y')$, Eq. (19), start to self interact, gaining nonzero vacuum expectation values and breaking the weak charge, the hyper charge and the family charges.

Let me introduce a common notation $A^{Ai}_{s}$ for all the scalar fields with $s = (7, 8)$, independently of whether they originate in $\omega_{abs}$ - in this case $Ai = (Q, Q', Y')$ - or in $\tilde{\omega}_{abs}$ - in this case all the family quantum numbers of all eight families contribute.

$$A^{Ai}_{s} \text{ represents } (A^{Q}_{s}, A^{Q'}_{s}, A^{Y'}_{s}, \tilde{\tau}^{1}_{s}, \tilde{\tau}^{2}_{s}, \tilde{\tau}^{3}_{s}, \tilde{\tau}^{N}_{L}, \tilde{\tau}^{N}_{R}),$$

$$\tau^{Ai} \text{ represents } (Q, Q', Y', \tilde{\tau}^{1}, \tilde{\tau}^{2}, \tilde{\tau}^{3}, \tilde{\tau}^{N}_{L}, \tilde{\tau}^{N}_{R}).$$

(19)

Here $\tau^{Ai}$ represent all the operators, which apply on the spinor states. These scalars, the gauge scalar fields of the generators $\tau^{Ai}$ and $\tilde{\tau}^{Ai}$, are expressible in terms of the spin connection fields (Ref. [1], Eqs. (10, 22, A8, A9)).

Let me demonstrate [1] that all the scalar fields with the space index $(7, 8)$ carry with respect to this space index the weak and the hyper charge ($\mp \frac{1}{2}$, $\pm \frac{1}{2}$), respectively. This means that all these scalars have properties as required for the Higgs in the standard model.

Let me make a choice of the superposition of the scalar fields so that they are eigenstates of $\tau^{13} = \frac{1}{2}(S^{56} - S^{78})$ (Eq (3) and footnotes at one page before). I rewrite for this purpose the second line of Eq. (2) as follows (the momentum $p_{s}$ is left out $^{13}$)

$$\sum_{s=(7,8),A,i} \bar{\psi} \gamma^{8} (-\tau^{Ai} A^{Ai}_{s}) \psi =$$

$$-\bar{\psi} \{ (\gamma^{7} + i \gamma^{8}) A^{Ai}_{7} + (\gamma^{7} - i \gamma^{8}) A^{Ai}_{8} \} \psi,$$

(20)

with the summation over $A$ and $i$ performed, since $A^{Ai}_{s}$ represent the scalar fields $(A^{Q}_{s}, A^{Q'}_{s}, A^{Y'}_{s}, \tilde{\tau}^{1}_{s}, \tilde{\tau}^{2}_{s}, \tilde{\tau}^{3}_{s}, \tilde{\tau}^{N}_{L}, \tilde{\tau}^{N}_{R})$.

The application of the operators $Y$ ($Y = \tau^{23} + \tau^{4}$, $\tau^{23} = \frac{1}{2}(S^{56} + S^{78})$, $\tau^{4} = -\frac{1}{2}(S^{910} + S^{1112} + S^{1314})$, $Q$ ($Q = \tau^{13} + Y$ and $\tau^{13}$ ($\tau^{13} = \frac{1}{2}(S^{56} - S^{78})$) on the fields $(A^{Ai}_{7} \mp i A^{Ai}_{8})$ gives ($S^{ab}$ is defined in Eq. (18)).

$^{13}$ It is expected that solutions with nonzero momenta lead to higher masses of fermion fields in $d = (3 + 1)$ [34, 35]. We pay correspondingly no attention to the momentum $p_{s}$, $s \in (5, \ldots, 8)$, when having in mind the lowest energy solutions, manifesting at low energies.
Since one sees that the scalar fields besides the quantum numbers determined by the space index also the quantum numbers charge respectively. Keeping transform one member of a doublet from Table 6 into another member of the same doublet, those superposition of fields of the model one replaces \((A_i^Q, A_i^{Q'}, A_i^{Y'})\) with respect to the family quantum numbers, one sees that the scalar fields \(A_i^A\) (\(= (A_i^Q, A_i^Y, A_i^{Y'}, \tilde{A}_i^1, \tilde{A}_i^2, \tilde{A}_i^3, \tilde{A}_i^\nu, \tilde{A}_i^8_n)\)), rewritten as \(A_i^A = (A_i^Q \mp i A_i^{A_8})\), are eigenstates of \(\tau^{13}\) and \(\tilde{Y}\), having the quantum numbers of the standard model Higgs’ scalar.

These superposition of \(A_i^A\) are presented in Table 6 as two doublets with respect to the weak charge \(\tau^{13}\), with the eigenvalue of \(\tau^{23}\) (the second \(SU(2)_{11}\) charge) equal to either \(-\frac{1}{2}\) or \(+\frac{1}{2}\), respectively.

The operators \(\tau^{13} = \tau^{11} \pm i\tau^{12}\)

\[
\tau^{13} (A_i^Q \mp i A_i^{A_8}) = \pm \frac{1}{2} (A_i^Q \mp i A_i^{A_8}), \nonumber
\]

\[
Y (A_i^Q \mp i A_i^{A_8}) = \mp \frac{1}{2} (A_i^Q \mp i A_i^{A_8}), \nonumber
\]

\[
Q (A_i^Q \mp i A_i^{A_8}) = 0. \tag{21}
\]

Since \(\tau^{4}, Y, \tau^{13}\) and \(\tau^{1+}, \tau^{1-}\) give zero if applied on \((A_i^Q, A_i^{Q'}, A_i^{Y'})\) with respect to the quantum numbers \((Q, Q', Y')\), and since \(Y\) and \(\tau^{13}\) commute with the family quantum numbers, one sees that the scalar fields \(A_i^A\) are either triplets as the gauge fields of the family quantum numbers \((\tilde{N}_R, \tilde{N}_L, \tilde{N}_2, \tilde{N}_1)\) or they are singlets as the gauge fields of \(Q = \tau^{13} + Y, Q' = -\tan^2 \vartheta_1 Y \pm \tau^{13}\) and \(Y' = -\tan^2 \vartheta_2 \tau^4 + \tau^{23}\).

One finds

\[
\tilde{N}_L A_i^\nu (\pm) = A_i^\nu (\pm), \quad \tilde{N}_L A_i^{A_8} (\pm) = 0, \nonumber
\]

\[
Q A_i^Q (\pm) = 0. \tag{23}
\]

with \(Q = S^{56} + \tau^4 = S^{56} - \frac{1}{3} (S^{90} + S^{1112} + S^{1314})\), and with \(\tau^4\) defined in the footnote 8 (if one replaces \(S^{ab}\) by \(S^{ab}\) from Eq. (18)).
Similarly one finds properties with respect to the $A_i$ quantum numbers for all the scalar fields $A_i^{Ai}$.

After the appearance of the condensate (Table 1), which breaks the $SU(2)_{II}$ symmetry (bringing masses to all the scalar fields), the weak charge $\tau^2$ and the hyper charge $Y$ remain the conserved charges.

The nonzero vacuum expectation values of the scalar fields of Eq. (19) break the mass protection mechanism of quarks and leptons and determine correspondingly the mass matrices (Eq. (31)) of the two groups of quarks and leptons.

Obviously the scalar fields in the spin-charge-family theory have all the properties of the Higgs. I show below and in Subsect. 4.1 that these scalar fields explain also the Yukawa couplings of the standard model.

All other scalar fields: $A_i^{ Ai}$, $s \in (5, 6)$ and $A_i^{ Ai}$, $t \in (9, \ldots, 14)$ have masses of the order of the condensate scale and contribute to matter-antimatter asymmetry [2].

**Effective action for scalar fields with the space index (7, 8)**

It would be possible, at least in principle, to derive the low energy effective action for scalars from the starting action (Eq. 1) by guessing the boundary conditions, under which the universe evolved, since all the scalar fields, as well as their Lagrange density, are included in the starting action. This is an extremely demanding project.

In what follows the effective Lagrange density for the scalar fields is assumed to be changed from the Lagrange density before the electroweak break $\mathcal{L} = E \{ (p_m A_i^{ Ai}) \dagger (p_m A^{ Ai}) - (m_{Ai}^1)^2 A_i^{ Ai} A_i^{ Ai} \}$ to

$$\mathcal{L}_{sg} = E \sum_{A, i} \{ (p_m A_i^{ Ai}) \dagger (p_m A^{ Ai}) - (-\lambda A_i + (m_{Ai}^1)^2) \} A_i^{ Ai} A_i^{ Ai} + \sum_{B, j} \Lambda_{Bj}^{Ai} A_i^{ Ai} A_i^{ Ai} A_{s_j}^{ Bj} A_{s_j}^{ Bj} \} , (24)$$

where $\lambda A_i + (m_{Ai}^1) = m_{Ai}$ and $m_{Ai}$ manifests as the mass of the $A_i^{ Ai}$ scalar.

The Lagrange density leads to the coupled equations of motion for many scalar fields with, in this assumption, harmonic interactions. It requires a lot of effort to extract the dependence of the eigen modes on the parameters of the Lagrange density to see the influence of the parameters on the properties of fermions. This work has not yet been done. First attempts are in progress.

**Yukawa couplings in the spin-charge-family theory**

Let $\psi^{(\alpha)}_{(L,R)}$ denote massless and $\Psi^{(\alpha)}_{(L,R)}$ massive four vectors for each family member $\alpha = (u_{L,R}, d_{L,R}, \nu_{L,R}, e_{L,R})$, let say for the group of four families among which there are the observed three families, after taking into account loop corrections in all orders.

$$\psi^{(\alpha)}_{(L,R)} = V^{(\alpha)}_{(L,R)} \Psi^{(\alpha)}_{(L,R)}, \quad (25)$$

and let ($\psi^{(\alpha, k)}_{(L,R)}, \Psi^{(\alpha, k)}_{(L,R)}$) be any component of the four vectors, massless and massive, respectively.

On the tree level we have $\psi^{(\alpha)}_{(L,R)} = V^{(\alpha)}_{(L,R)} \Psi^{(\alpha)}_{(L,R)}$ and

$$< \psi^{(\alpha, k)}_L \cdot \mathcal{M}^{(\alpha)}_L \psi^{(\alpha, k)}_R > = < \Psi^{(\alpha)}_{(L,R)} \cdot V^{(\alpha)}_L \mathcal{M}^{(\alpha)}_L V^{(\alpha)}_R \Psi^{(\alpha)}_{(L,R)} >, \quad (26)$$

14 It is $\tau^2$ which determines the hyper charge $Y$ ($Y = S^{23} + \tau^4$) of these scalar fields, since $\tau^4$, if applied on the scalar index of these scalar fields, gives zero, according to equations in the footnote 9.
with $\mathcal{M}_{\alpha}^{\alpha}k k' = \sum_{A,i} (-g_{A}^{\alpha} v_{A}) C_{k k'}^{\alpha}$. Here $g^{\alpha} v_{A}z$ represent the nonzero vacuum expectation values of the scalar fields. In this case the coefficients $C_{k k'}^{\alpha}$ are determined by the mass matrices, Eq.(31), on the tree level. It then follows

$$\Psi^{\alpha} V_{\alpha}^{\alpha \dagger} \mathcal{M}_{\alpha}^{\alpha} V_{\alpha}^{\alpha} \Psi^{\alpha} = \Psi^{\alpha} \text{diag}(m_{\alpha(1)}, \cdots, m_{\alpha(4)}) \Psi^{\alpha},$$

(27)

On the tree level the coupling constants $m_{\alpha(\theta)}^{\alpha}$ (in some units) of the dynamical scalar fields $\Phi_{\Psi(\alpha)k}$ - the superposition of $A_{s}^{\alpha}$ - to the family member $\Psi^{\alpha k}$ belonging to the $k^{th}$ family are equal to

$$\Phi_{\Psi(\alpha)(\theta) k k'}^{\alpha} = \delta_{k k'}^{\alpha} m_{\alpha(\theta) k}^{\alpha} \Psi^{\alpha k}.$$  

(28)

The superposition of scalar fields $(\Phi_{\Psi(\alpha)k}^{\alpha})$, which couple to fermions and depend on the quantum numbers $\alpha$ and $k$, are in general different from the superposition, which are their mass eigenstates. Each family member $\alpha$ of each massive family $k$ couples in general to different superposition of scalar fields.

It turns out that mass matrices of both - quarks and leptons - behave in a very similar way. No additional neutrinos, offering a "sea-saw" mechanism, are needed. All this is already included in the starting action.

### 3.4. The condensate in the spin-charge-family theory

The appearance of the condensate of two right handed neutrinos with properties presented in Table 1 is in this paper assumed so that in the low energy regime the spin-charge-family theory leads to the effective action, explaining the assumptions of the standard model and consequently the observed phenomena. The condensate should appear during the expansion of the universe, due to particular boundary conditions and the conditions in the universe in the time of the appearance of the condensate. This study has not yet been done.

The condensate, presented on Table 1, does not influence the colour, the weak and the hyper charges ($\vec{\tau}^{3}$, $\vec{\tau}^{1}$, $Y$, respectively) of the corresponding gauge fields. Since these vector gauge fields don’t interact with the condensate, the colour, the weak and the hyper charges remain the conserved quantities up to the electroweak phase transition.

The condensate changes the properties of the scalar fields, which are before the appearance of the condensate massless scalar gauge fields. Interaction with the condensate makes all the scalar fields massive.

After the electroweak break, when the scalar fields with the space index $s = (7,8)$ - those with the family quantum numbers $(N^{i}_{(L,R)}, \tilde{\tau}^{i(1,2)})$ and those with the family members quantum numbers $(Q, Q', Y')$ - start to strongly self interact (Eq. (24)), gaining nonzero vacuum expectation values (Eq. (24)) and correspondingly changing their own masses as well as the properties of the vacuum, so that the weak charge and the hyper charge are no longer conserved quantities. The only conserved charges are then the colour and the electromagnetic charges.

### 4. Summary of the spin-charge-family theory achievements so far

To understand better the history of our universe the explanation of the standard model assumptions is certainly needed. It is also needed to know the number of families in the low energy regime and to understand the appearance of phenomena like the existence of the dark matter, the matter-antimatter asymmetry and the dark energy.
I have demonstrated so far, that the spin-charge-family theory, starting with the simple action in $d = (13 + 1)$ for fermions (carrying only two kinds of spins, no charges) and for the gauge fields to which fermions are coupled (vielbeins and two kinds of spin connection fields) offers the explanation for all the assumptions of the standard model:

a. The theory explains all the properties of the family members - quarks and leptons, left and right handed, relating handedness and charges, and their right and left handed antiquarks and antileptons.

b. It explains the appearance and properties of the families of family members.

c. It explains the existence of the gauge vector fields of the family members charges.

d. It explains the appearance and properties of the scalar field (the Higgs) and the Yukawa couplings.

The spin-charge-family theory predicts that there are at the low energy regime two decoupled groups of four families of quarks and leptons, what means that besides the observed three there is the fourth not yet observed family of quarks and leptons.

The existence of two decoupled groups of four families also means that the stable of the upper four families must also be observed. In Subsect. 4.4 [13] the possibility is discussed that the stable of the upper four families constitutes the dark matter.

In this section I overview the spin-charge-family theory achievements, explaining:

i. The properties of the lower four families, the three of which have already been observed, as they follow from the properties of the scalar fields of this theory, Subsect. 4.1,

ii. discussing whether or not present experiments speak or not against the existence of the fourth family (not yet published), in particular I shall comment the contribution of the fourth family to the production of the Higgs in the quark-fusion process, Subsect. 4.2, the topics which seem to speak the most against the existence of the fourth family.

iii. The existence of the dark matter, Subsect. 4.4.

iv. The explanation for the matter-antimatter asymmetry, Subsect. 4.5.

4.1. Masses of the lower four families of quarks and leptons in the spin-charge-family theory [12, 14, 15]

This subsection is a short report of the not yet published results of Ref. [15] (published in the Proceedings).

There are two groups of four families. The mass matrix of each family member of each of the group of four families demonstrates in the massless basis the $U(1) \times SU(2) \times SU(2)$ symmetry (each of the two $SU(2)$ is a subgroup, one of $SO(3,1)$ and the other of $SO(4)$).

The scalars with the family quantum numbers split the eight families into twice four families. To the masses of the lower four families the scalar fields, which are the gauge fields of $\tilde{N}_L$ and $\tilde{\tau}^1_i$ contribute. To the masses of the upper four families the gauge fields of $\tilde{N}_R$ and $\tilde{\tau}^2_i$ contribute. The scalars with the family members quantum numbers $(Q,Q',Y')$ contribute to the masses of the lower and upper four families.

I discuss here properties of quarks and leptons of the lower four families, Eq. (31).

Let $\psi_i, i = (1,2,3,4)$, denote the massless basis for a particular family member $\alpha$. And let us denote the two kinds of the operators, which transform the basis vectors into one another as

$$\tilde{N}_L^i, i = (1,2,3), \quad \tilde{\tau}^1_L^i, i = (1,2,3).$$

(29)
One finds

\[ \tilde{N}_L^3 (\psi_1, \psi_2, \psi_3, \psi_4) = \frac{1}{2} (\psi_2, 0, \psi_4, 0), \]

\[ \tilde{N}_L^+ (\psi_1, \psi_2, \psi_3, \psi_4) = (0, \psi_1, 0, \psi_3), \]

\[ \tilde{\tau}_{13} (\psi_1, \psi_2, \psi_3, \psi_4) = \frac{1}{2} (\psi_1, -\psi_2, \psi_3, \psi_4), \]

\[ \tilde{\tau}_{1+} (\psi_1, \psi_2, \psi_3, \psi_4) = (\psi_3, \psi_4, 0, 0), \]

\[ \tilde{\tau}_{1-} (\psi_1, \psi_2, \psi_3, \psi_4) = (0, 0, \psi_1, \psi_2). \]

This is indeed what the two SU(2) operators in the spin-charge-family theory do on the lower four families. The gauge scalar fields \( A_{A_i}^{A_i} \), \( (A, i) = [\tilde{N}_L^i, i = (\pm, 3), \tilde{\tau}_{1i}^i, i = (\pm, 3)] \), Eqs. (20, 19), of these operators determine the off diagonal and diagonal matrix elements after the electroweak phase transition in which scalar fields gain nonzero vacuum expectation values.

In addition to these two kinds of SU(2) scalars there are three U(1) scalars, which distinguish among the family members, contributing on the tree level the same diagonal matrix elements for all the families.

In loop corrections in all orders the symmetry of mass matrices remains unchanged, while the three U(1) scalars manifest in off diagonal elements as well.

All the scalars, the two triplets and the three singlets, are doublets with respect to the weak charge, contributing to the weak and the hyper charge of the fermions so that they transform the right handed members into the left handed ones with the phases presented in Table 2.

\[
M^\alpha = \begin{pmatrix}
-a_1 - a & e & d & b \\
e & -a_2 - a & b & d \\
d & b & a_2 - a & e \\
b & d & e & a_1 - a
\end{pmatrix}^\alpha.
\]

Although any accurate 3×3 submatrix of the 4×4 unitary matrix determines the 4×4 matrix uniquely, neither the quark nor (in particular) the lepton 3×3 mixing matrix are measured accurately enough that it would be possible to determine three complex phases of the 4×4 mixing matrix as well as the mixing matrix elements of the fourth family members to the lower three.

We therefore assumed in our calculations [12, 14, 15] that the mass matrices are symmetric and real. Correspondingly the mixing matrices are orthogonal. We fitted the 6 free parameters of each quark mass matrix, Eq. (31), to twice three, that is 6, measured quark masses, and to the 6 (from the experimental data extracted) parameters of the corresponding 4×4 mixing matrix.

While the experimental accuracy of the quark masses of the lower three families does not influence the calculated mass matrices considerably, it turned out that the experimental accuracy of the 3×3 quark mixing matrix is not good enough to trustworthy determine the mass intervals for the fourth family quarks.

Taking into account our calculations, in which we fit parameters of Eq. (31) to the experimental data for masses and mixing matrices for quarks and the meson decays evaluations in the literature, as well as our own evaluations, we estimated that the fourth family quarks masses might be above 1 TeV. Choosing the masses of the fourth family quarks we were able not only to calculate the fourth family matrix elements to the lower three families, but also predict towards which values will the matrix elements of the 3×3 submatrix move in more accurate experiments [15].
The two fitted mass matrices, Ref. ([15], Eqs. (23, 27)) lead to masses of Eq. (32) for the choice of $M_{u_d}/\text{MeV}/c^2 = 700\,000 = M_{d_u}/\text{MeV}/c^2$

\[
\begin{align*}
M^u_{\text{MeV}/c^2} &= (1.3, 620.0, 172\,000, 700\,000), \\
M^d_{\text{MeV}/c^2} &= (2.88508, 55.024, 2\,899.99, 700\,000),
\end{align*}
\]  
(32)

and to masses of Eq. (33) for the choice of $M_{u_d}/\text{MeV}/c^2 = 1\,200\,000 = M_{d_u}/\text{MeV}/c^2$

\[
\begin{align*}
M^u_{\text{MeV}/c^2} &= (1.3, 620.0, 172\,000, 1\,200\,000), \\
M^d_{\text{MeV}/c^2} &= (2.88508, 55.024, 2\,899.99, 1\,200\,000).
\end{align*}
\]  
(33)

They lead to the $4 \times 4$ mixing matrix in which we fit two kinds of the experimental - the old data ($\text{exp}_o$) and the new data ($\text{exp}_n$) each used in calculations for the choice $m_{u_d} = m_{d_u} = 700$ GeV ($\text{old}_1, \text{new}_1$) and $m_{u_d} = m_{d_u} = 1\,200$ GeV ($\text{old}_2, \text{new}_2$), Eq. (34)

\[
|V_{ud}| = \begin{pmatrix}
\text{exp}_o & 0.97425 \pm 0.00022 & 0.2252 \pm 0.0009 & 0.00415 \pm 0.00049 \\
\text{exp}_n & 0.97425 \pm 0.00022 & 0.2253 \pm 0.0008 & 0.00413 \pm 0.00049 \\
\text{old}_1 & 0.97423 & 0.22531 & 0.003 \\
\text{old}_2 & 0.97425 & 0.22536 & 0.00301 \\
\text{new}_1 & 0.97423 & 0.22531 & 0.00299 \\
\text{new}_2 & 0.97423 & 0.22538 & 0.00299 \\
\text{exp}_o & 0.230 \pm 0.011 & 1.006 \pm 0.023 & 0.0409 \pm 0.0011 \\
\text{exp}_n & 0.225 \pm 0.008 & 0.986 \pm 0.016 & 0.0411 \pm 0.0013 \\
\text{old}_1 & 0.22526 & 0.97338 & 0.042 \\
\text{old}_2 & 0.22534 & 0.97336 & 0.04239 \\
\text{new}_1 & 0.22534 & 0.97335 & 0.04245 \\
\text{new}_2 & 0.22531 & 0.97336 & 0.04248 \\
\text{exp}_o & 0.0084 \pm 0.0006 & 0.0429 \pm 0.0026 & 0.89 \pm 0.07 \\
\text{exp}_n & 0.0084 \pm 0.0006 & 0.0400 \pm 0.0027 & 1.021 \pm 0.032 \\
\text{old}_1 & 0.00663 & 0.04197 & 0.9991 \\
\text{old}_2 & 0.00663 & 0.04198 & 0.9991 \\
\text{new}_1 & 0.00667 & 0.04203 & 0.99909 \\
\text{new}_2 & 0.00667 & 0.04206 & 0.99909 \\
\end{pmatrix}
\]  
(34)

It was noticed [15] that the mass matrix elements of the $u$ and $d$ quarks matrices change by a factor of $\approx 1.5$ in the average, when the masses of the fourth family members grow from 700 GeV to 1 200 GeV. The two mass matrices become more "democratic" (the matrix elements become closer to each other). Although the mixing matrix elements of the $3 \times 3$ submatrix of the $4 \times 4$ matrix do not change a lot with the masses of the fourth family quarks, they do change so that they agree better with the newer [19] than with the older [20] experimental values.

From the above results it follows:

i. The prediction of the calculated mixing matrix elements, obtained by fitting the symmetry of the mass matrices (Eq. (31)) to the experimental data [20], was confirmed by more accurate experimental data [19]. In all cases are the calculated $3 \times 3$ matrix elements closer to the new experimental values than to the old experimental values.

ii. The fourth family masses change the mass matrices considerably, while their influence on the $3 \times 3$ submatrix of the $4 \times 4$ mixing matrix is much weaker.

iii. We expect that more accurate experiments will bring a slightly smaller values for $(V_{u_d}, V_{u_d\text{,}1}, V_{u_d\text{,}3})$, smaller $(V_{u_d\text{,}2}, V_{u_d\text{,}3})$, $(V_{u_d\text{,}1}, V_{u_d\text{,}2})$, $V_{u_d\text{,}3}$ will slightly grow and $(V_{u_d\text{,}2})$ $V_{u_d\text{,}3}$ will grow.

iv. The matrix elements $V_{u_d\text{,}4}$ and $V_{u_d\text{,}4}$ change considerably with the mass of the fourth family members, and they differ quite a lot also when using new instead of the old experimental data.
for the mixing matrix.

v. Fitting (twice 6) free parameters of the mass matrices to the new experimental data [19] gives smaller uncertainty in fitting procedure than when fitting to the old experimental data [20], while the masses of the fourth family members do not influence the uncertainty of the calculations considerably. Only very accurate mixing matrix elements would allow to determine fourth family quarks masses more accurately.

vi. It is difficult to predict the interval for the masses of the fourth family members since the choice of the fourth family quark masses does not appreciably influence either the fitting procedure or the obtained $3 \times 3$ mixing submatrix, and also not the accuracy of the masses of the three lower families. Other experimental data, like decays of mesons, speaks for masses of the fourth family quarks close or above 1 TeV.

vii. For the masses of the fourth family members above 1 TeV the mass matrices are close to the democratic matrix: The matrix elements are closer to one another the higher is the mass of the fourth family member. In such a case are the fourth family masses mostly determined by the scalars carrying the family quantum numbers. Correspondingly the masses of the $u_4$-quarks are closer to the masses of the $d_4$-quarks.

The complex mass matrices would lead to unitary and not to orthogonal mixing matrices. The more accurate experimental data for quarks mixing matrix would allow us to extract also the phases of the unitary mixing matrix, allowing us to predict the fourth family masses.

4.2. Is the existence of the fourth family in agreement with the present experiments?

This part is following equivalent part in my contribution to the Proceedings to the Conference on New Physics at the Large Hadron Collider, 29 February – 4 March, 2016, Nanyang Executive Centre, NTU, Singapore. The paper, a more elaborated version of this notice, is in preparation.

The spin-charge-family theory predicts the existence of the fourth family to the observed three, while there has been no direct observation of the fourth family quarks with the masses below 1 TeV. The fourth family quarks with masses above 1 TeV contribute according to the standard model (the standard model Yukawa couplings of quarks to the scalar Higgs are proportional to $m^2 \nu$, where $m_4^2$ is the fourth family member ($\alpha = u, d$) mass and $\nu$ the vacuum expectation value of the scalar) to either the quark-gluon fusion production of the scalar fields (the Higgs) or to the scalar field decay into two photons $\approx 10$ times too much in comparison with the observations. Correspondingly the high energy physicists do not expect the existence of the fourth family members at all [26].

I am stressing [30] in this subsection that the $u_i$-quarks and $d_i$-quarks of an $i^{th}$ family, if they couple with the opposite sign (with respect to the "±" degree of freedom) to the scalar fields carrying the family $(\bar{A}, i)$ quantum numbers $^{15}$ and have the same masses, do not contribute to either the quark-gluon fusion production of the scalar fields with the family quantum numbers or to the decay of these scalars into two photons. Since the $u_4$-quarks and $d_4$-quarks might have similar masses (Subsect. 4.1) and since their masses are for $m_{u_4} > 1$ TeV and $m_{d_4} > 1$ TeV mostly determined by the scalars with the family quantum numbers, the observations so far are consequently not in contradiction with the spin-charge-family theory prediction that there exists the fourth family coupled to the observed three.

The couplings of $u_i$ and $d_i$ to the scalars carrying the family members quantum numbers are determined besides by the corresponding couplings also by the eigenvalues of the operators $(Q, Q', Y')$ on the quarks states (which do distinguish between $u_i$ and $d_i$).

The strong influence of the scalar fields carrying the family members quantum numbers to the masses of the lower (observed) three families manifests in the huge differences in the masses

\(^{15}\) The contribution of the scalar fields $\bar{A}_+^{\pm i}$, $(\bar{A}, i) = (\bar{\tau}_1^i, \bar{N}_L^i)$, Eq. 19), are the same for all the family members.
Table 7. The weak, hyper and electromagnetic charges for quarks in their massless basis are presented. The colour charge is not shown. These and other properties of quarks and leptons can be read from Tables 2–4.

| state | $\tau^{13}$ | $Y$ | $Q$ |
|-------|-------------|-----|-----|
| $u_{Ri}$ | 0 | $\frac{1}{2}$ | $\frac{1}{3}$ |
| $u_{Li}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{2}{3}$ |
| $d_{Ri}$ | 0 | $-\frac{1}{2}$ | $-\frac{1}{3}$ |
| $d_{Li}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{2}{3}$ |

of $u_i$ and $d_i$, $i = (1, 2, 3)$, among families $(i)$ and family members $(u, d)$. For the fourth family quarks, which are more and more decoupled from the observed three families the higher are their masses [15, 14], the influence of the scalar fields carrying the family members quantum numbers on their masses is expected to be much weaker. Correspondingly the $u_4$ and $d_4$ masses become closer to each other the higher are their masses and the weaker are their couplings (the mixing matrix elements) to the lower three families.

If the masses of the fourth family quarks are close to each other, then $u_4$ and $d_4$ contribute in the quark-gluon fusion very little to the production of the scalar field - the Higgs - which is mostly superposition of the scalar fields with the family members quantum numbers, what is in agreement with the observation: In the quark-gluon fusion production of the Higgs mostly the top ($u_3$) contributes.

In Tables 2–4 the phases of all the states are chosen to be 1. Here I use different phases, those which enable the usual presentation of fermions under the change of spin and under $CNP$.

In Table 7 the properties of $u$ and $d$ quarks (and their antiquarks) needed in Fig. 1 are presented. Fig. 1 presents the properties of $u$ and $d$ quarks, contributing in the quark-gluon fusion to the production of the scalar fields $\Phi_{Ai}^{\pm}$, Eq.(19), $(A_{Ai}^{\pm} = \Phi_{Ai}^{\pm} + v_{Ai}^{\pm})$. One notices the opposite signs of the couplings of $u_i$ with respect to $d_i$ for either $\Phi_-$ or for $\Phi_+$. Correspondingly the fourth family quarks of almost the same mass contribute very little to the production of any scalar, with the scalar Higgs included, which is in agreement with the observation. Also to the decay of the Higgs can the fourth family quarks contribute very little: It is, as observed, the $u_3$- quark (the top), which contributes the most to either the production or to the decay of the Higgs.

The fourth family quarks can still contribute to the production and to the decay of the scalars of the masses of a few TeV, to which they couple stronger than to the Higgs.

The figures are valid for any $Ai$ and correspondingly also for any superposition of $\Phi_{Ai}^{\pm}$.

4.3. Anomaly cancellation in the spin-charge-family theory

In the standard model the triangle anomalies ”miraculously” disappear due to the fact that the sum of all possible traces $Tr[\tau^{Ai}\tau^{Bi}\tau^{Ck}]$, where $(\tau^{Ai}, \tau^{Bi}, \tau^{Ck})$ are the generators of one, of two or of three of the groups of $SU(3), SU(2)$ and $U(1)$ over the representations of one family of the left handed fermions and their anti-fermions (and separately of the right handed fermions and their anti-fermions), contributing to the triangle currents, are equal to zero [22, 23, 24, 25].

Let me demonstrate that this cancellation of the standard model triangle anomaly follows straightforwardly, if the $SO(3,1), SU(2), U(1)$ and $SU(3)$ are considered as subgroups of the

16 A more elaborated version of this report is in preparation.
orthogonal group $SO(13,1)$.

To the triangle anomaly the right-handed spinors (fermions) and antispinors contribute with the opposite sign than the left handed spinors and their antispinors. Their common contribution to anomalies is proportional to [24]

$$
(\sum_{(A,i,B,j,C,k)_{L,L}} Tr[\tau^{Ai} \tau^{ Bj} \tau^{ Ck}] - \sum_{(A,i,B,j,C,k)_{R,R}} Tr[\tau^{Ai} \tau^{ Bj} \tau^{ Ck}]),
$$

where $\tau^{Ai}$ are in the standard model the generators of the infinitesimal transformation of the groups $SU(3), SU(2)$ and $U(1)$, while in the spin-charge-family theory $\tau^{Ai}$ are irreducible subgroups of the starting orthogonal group $SO(2(2n+1) - 1,1), n = 3$. The indexes $L,L$ (R,R) denote the left (right) handed spinors and their antispinors (right (left)), respectively.

In the first seven columns (up to $||$) of Table 8 the properties of one family members assumed by the standard model, running in the triangle, are presented. The last two columns - taken from Table 2 - describe additional properties which quarks and leptons (and antiquarks and antileptons) would have, if the standard model groups $SO(3,1), SU(2), SU(3)$ and $U(1)$ are embedded into the $SO(13,1)$ group. To demonstrate that the "miraculous" cancellation of the

![Diagram](image-url)
Table 8. Properties of the left handed quarks and leptons and antiquarks and antileptons, and of the right handed quarks and leptons and antiquarks and antileptons, as assumed by the standard model, are presented in the first seven columns. In the last two columns the two quantum numbers are added, which the fermions and anti-fermions would have if the standard model groups $SO(3, 1), SU(2), SU(3)$ and $U(1)$ are embedded into the $SO(13)$ group. The whole quark part appears, due to the colour charges, three times. One can check that the hyper charge is the sum of $\tau^{1}_{ul,R} + \tau^{23}_{ul,R}$ (Table 2). The quantum numbers are the same for all the families.

| $\nu_L$ | name | $SU(2)_{1J}$ charge | $\nu_{ul,R}$ | $\nu_{ul,R}$ | $SU(2)_{1J}$ charge | $\nu_{ul,R}$ | $\nu_{ul,R}$ |
|---------|-------|------------------|--------------|--------------|------------------|--------------|--------------|
| 1       | 1     | 0                | 1            | 1            | 0                | 1            | 1            |
| 2       | 1     | 0                | 1            | 1            | 0                | 1            | 1            |
| 3       | 1     | 0                | 1            | 1            | 0                | 1            | 1            |
| 4       | 1     | 0                | 1            | 1            | 0                | 1            | 1            |
| 5       | 1     | 0                | 1            | 1            | 0                | 1            | 1            |
| 6       | 1     | 0                | 1            | 1            | 0                | 1            | 1            |

triangle anomalies is "trivial", one takes into account that the standard model groups can easily be interpreted (unified) by making the next step beyond the standard model. In the standard model the triangle anomaly for the Feynman triangle diagrams, in which the gauge vector fields of the charges

$$U(1) \times U(1) \times U(1),$$

$$SU(2) \times SU(2) \times U(1),$$

$$SU(3) \times SU(3) \times SU(3),$$

$$SU(3) \times SU(3) \times U(1),$$

$$U(1) \times \text{gravitational}$$

contribute to the triangle anomaly, occurs if the traces in Eq.(35) are not zero for either the left handed quarks and leptons and their left handed antiparticles or the right handed quarks and leptons and their right handed antiparticles.
To see that embedding the standard model groups into the orthogonal group SO(13, 1) makes the cancellation of the triangle anomalies self evident, let us recognize: The subgroups of the SO(13, 1) group are SO(7, 1) and SO(6). The subgroups of SO(6) are the colour group SU(3) with the generators denoted by $\tau^3 i$, $i = (1, \ldots, 8)$ and the $U(1)$ (we shall call it $U(1)_{II}$) group with the generator $\tau^4$. One sees that all the quarks have $\tau^4 = \frac{1}{6}$, all the antiquarks have $\tau^4 = -\frac{1}{6}$, while the leptons have $\tau^4 = -\frac{1}{2}$ and the antileptons have $\tau^4 = \frac{1}{2}$. Correspondingly the trace of $\tau^4$ over all the family members is equal to zero.

The subgroups of the SO(7, 1), as seen in Table 2, have as subgroups SO(3, 1), SU(2)$_I$ and SU$_{2II}$, with the generators $\tau^{11}$ (representing the weak group operators) and $\tau^{2I}$ (representing the generators of the additional SU(2) group), respectively. The left handed spinors are SU(2)$_I$ (weak) doublets and SU(2)$_{II}$ singlets, while the right handed spinors are the SU(2)$_I$ (weak) singlets and SU(2)$_{II}$ doublets. Correspondingly are the left handed antispinors the SU(2)$_I$ (weak) singlets and SU(2)$_{II}$ doublets, while the right handed antispinors are the SU(2)$_I$ (weak) doublets and the SU(2)$_{II}$ singlets.

The hypercharge of the standard model corresponds to the sum of $\tau^4$ and $\tau^{23}$

$$Y = \tau^4 + \tau^{23}. \tag{37}$$

To the triangle Feynman diagram, to which three hyper $U(1)$ boson fields contribute, the sum $\sum_i Tr(Y_i)^3$ runs over all the members ($i$) of the left handed spinors and antispinors, and of the right handed spinors and antispinors separately. Embedding the standard model groups into SO(13, 1) it follows

$$\sum_{i_{L,R}} (Y_{i_{L,R}})^3 = \sum_{i_{L,R}} (\tau^4_{i_{L,R}} + \tau^{23}_{i_{L,R}})^3$$

$$= \sum_{i_{L,R}} (\tau^4_{i_{L,R}})^3 + \sum_{i_{L,R}} (\tau^{23}_{i_{L,R}})^3$$

$$+ \sum_{i_{L,R}} 3 \cdot (\tau^4_{i_{L,R}})^2 \cdot \tau^{23}_{i_{L,R}} + \sum_{i_{L,R}} 3 \cdot \tau^4_{i_{L,R}} \cdot (\tau^{23}_{i_{L,R}})^2; \tag{38}$$

for either the left, $i_L$, or the right, $i_R$, handed members. Table 8 demonstrates clearly (last column) that $\sum_{i_{L,R}} (\tau^4_{i_{L,R}})^3 = 0$, when the contribution of left (right) handed spinors and antispinors are taken into account.

Table 8 also demonstrates (the last but one column) even more trivially that $\sum_{i_{L,R}} 3 \cdot (\tau^4_{i_{L,R}})^2 \cdot \tau^{23}_{i_{L,R}} = 0$, since the contribution of either spinors or antispinors - left or right handed - separately are equal to zero.

The easiest is to evaluate $\sum_{i_{L,R}} (\tau^{23}_{i_{L,R}})^3 = 0$ and $\sum_{i_{L,R}} 3 \cdot (\tau^4_{i_{L,R}})^2 \cdot \tau^{23}_{i_{L,R}} = 0$ since, as seen from Table 8, the summation separately within the quarks and lepton representations give zero.

Since all the members belong to one spinor representation, it is straightforwardly that all the triangle traces are zero, if the standard model groups are the subgroups of the orthogonal group SO(13, 1).

From only the standard model assumptions point of view the cancellation of the triangle anomalies does look miraculously. For our $\sum_{i_{L,R}} (Y_{i_{L,R}})^3$ one obtains for the left handed members: $[3 \cdot 2 \cdot (\frac{1}{6})^3 + 2 \cdot (-\frac{1}{2})^3 + 3 \cdot ((-\frac{2}{3})^3 + (\frac{1}{3})^3) + (\frac{1}{3})]$, and for the right handed members: $[3 \cdot ((\frac{2}{3})^3 + (\frac{1}{3})^3) + (-1)^3] + 3 \cdot 2 \cdot (-\frac{1}{6})^3 + 2 \cdot (\frac{1}{2})^3$.

4.4. Dark matter in the spin-charge-family theory

As discussed in Sect. 2 the spin-charge-family theory [5, 3, 6, 7, 8, 9, 10, 4, 11, 12, 13, 14, 15, 16, 17, 2, 1] predicts in the low energy region two decoupled groups of four families. In Ref. [13] the
possibility that the dark matter consists of clusters of the fifth family - the stable heavy family of quarks and leptons (with (almost) zero Yukawa couplings to the lower group of four families) - is discussed.

I review here briefly the estimation done in Ref. [13].

We used in Ref. [13] the simple hydrogen-like model to evaluate properties of the fifth family heavy baryons, taking into account that for masses of the order of a few TeV or larger the one gluon exchange determines the force among the constituents of the fifth family baryons ([13], Sect. II). We estimated the fifth family neutron as the most stable nucleon.

Due to very large masses of the fifth family baryons "the nuclear interaction" among these baryons has very interesting properties.

We also estimated the behaviour of the neutral clusters when scattering among themselves and with the ordinary matter. We studied possible limitations on the family properties due to the cosmological evidences, the direct experimental evidences ([13], Sect. IV) and all others known properties of the dark matter.

We followed the behaviour of the fifth family quarks and antiquarks in the plasma of the expanding universe, through the freezing out procedure, solving the Boltzmann equations, through the colour phase transition, while forming neutrons, up to the present dark matter ([13], Sect. III).

The cosmological evolution suggested the limits for the masses of the fifth family quarks

$$10 \text{ TeV} < m_{q_5} c^2 < \text{a few} \cdot 10^2 \text{ TeV}$$

and for the scattering cross sections

$$10^{-8} \text{ fm}^2 < \sigma_{c_5} < 10^{-6} \text{ fm}^2,$$}

while the measured density of the dark matter does not put much limitation on the properties of heavy enough clusters.

The direct measurements limited the fifth family quark mass to ([13], Sect. IV.)

$$\text{several} 10 \text{ TeV} < m_{q_5} c^2 < 10^5 \text{ TeV}.$$}

We also find that our fifth family baryons of the mass of a few $10 \text{ TeV}/c^2$ have for a factor more than 100 times too small scattering amplitude with the ordinary matter to cause a measurable heat flux on the Earth’s surface.

4.5. Matter-antimatter in the spin-charge-family theory

I shortly overview in this section the properties, quantum numbers, and discrete symmetries of those scalar and vector gauge fields appearing in the starting action (Eqs. (1, 2) which cause transitions of antileptons into quarks and back, and antiquarks into quarks and back. The appearance of the condensate breaks this symmetry making possible under non thermal conditions the ordinary (mostly made of the first family members) matter-antimatter asymmetry. The reader can find details in Ref. [2].

Scalar gauge fields, contributing to matter-antimatter asymmetry and causing also the proton decay, carry the triplet or antitriplet colour charges (see Table 9) and the fractional hyper and electromagnetic charge.

The Lagrange densities from Eqs. (1, 2) manifest $C_N \cdot \mathcal{P}_N$ invariance [36]. All the vector and the spinor gauge fields are massless before the appearance of the condensate (Subsect. 3.4) and reactions creating particles from antiparticles and back go in both directions equivalently. Correspondingly there is no matter-antimatter asymmetry. It is the condensate, which breaks this symmetry.
Let me analyze the Lagrange density of Eq. (2) before the appearance of the condensate. The term \( \gamma t \frac{1}{2} \hat{S}^i s_i \omega_{s's't'} \) can be rewritten as follows

\[
\gamma t \frac{1}{2} \hat{S}^i s_i \omega_{s's't'} = \sum_{+, -} \left( t' t'' \right) \frac{1}{2} \hat{S}^i s_i \omega_{s's't'} ,
\]

\( \omega_{s's't'} = \omega_{s's't'}(\pm) = (\omega_{s's't'} \mp i \omega_{s's't'}), \)

\( (t t') = (\pm) = \frac{1}{2} \left( \gamma t \pm \gamma t' \right), \)

\( (t t') \in \left\{ (9 10), (11 12), (13 14) \right\}. \) (42)

I introduced the notations \((t t')\) and \(\omega_{s's't'}\) to distinguish among different superposition of states in equations below.

The expression \((t t') \frac{1}{2} \hat{S}^i s_i \omega_{s's't'}\) can be further rewritten as follows

\[
(t t') \frac{1}{2} \hat{S}^i s_i \omega_{s's't'} = \left(t t'\right) \frac{1}{2} \hat{S}^i s_i \omega_{s's't'}(\pm) = \left(t t'\right) \left\{ \tau^{2+} A_{t'}^{2+} + \tau^{2-} A_{t'}^{2-} + \tau^{23} A_{t'}^{23} + \tau^{1+} A_{t'}^{1+} + \tau^{1-} A_{t'}^{1-} + \tau^{13} A_{t'}^{13} \right\} ,
\]

\( A_{t'}^{2+} = (\omega_{t'} + \omega_{58}) \left( i(\omega_{t'} - \omega_{57} \pm \omega_{68} \mp \omega_{78}) \right), \quad A_{t'}^{23} = (\omega_{56} \pm \omega_{78}) , \quad A_{t'}^{13} = (\omega_{56} \pm \omega_{78}). \) (43)

Equivalently one expresses the term \( \gamma t \frac{1}{2} \hat{S}^{ab} \tilde{\omega}_{ab} t' \) in Eq. (2) with \( \hat{S}^{ab} \) as the infinitesimal generators of either \( \hat{SO}(3, 1) \) or \( \hat{SO}(4) \) and \( \tilde{\omega}_{ab} \) belonging to the corresponding gauge fields with \( t = (9, \ldots, 14) \), by using Eqs. (A.23 - A.26), as

\[
\gamma t \frac{1}{2} \hat{S}^{ab} \tilde{\omega}_{ab} t' = \left(t t'\right) \frac{1}{2} \hat{S}^{ab} \tilde{\omega}_{ab} t' = \left(t t'\right) \left\{ \tilde{\tau}^{2+} A_{t'}^{2+} + \tilde{\tau}^{2-} A_{t'}^{2-} + \tilde{\tau}^{23} A_{t'}^{23} + \tilde{\tau}^{1+} A_{t'}^{1+} + \tilde{\tau}^{1-} A_{t'}^{1-} + \tilde{\tau}^{13} A_{t'}^{13} \right\} ,
\]

\( \tilde{\tau}^{2+} A_{t'}^{2+} = (\tilde{\omega}_{t'} - i \tilde{\omega}_{01}) \left( i(\tilde{\omega}_{t'} - i \tilde{\omega}_{01}) \right), \quad \tilde{\tau}^{23} A_{t'}^{23} = (\tilde{\omega}_{12} \mp i \tilde{\omega}_{03}) , \quad \tilde{\tau}^{13} A_{t'}^{13} = (\tilde{\omega}_{12} \pm i \tilde{\omega}_{03}) . \) (44)

The expressions for \( \tilde{A}_{2+}^{\pm} \), \( \tilde{A}_{23}^{\pm} \), \( \tilde{A}_{13}^{\pm} \) can easily be obtained from Eq.(43) by replacing
in expressions for $A^{01}_{t't'}$, $A^{23}_{t't'}$, $A^{12}_{t't'}$, and $A^{13}_{t't'}$, respectively, $\omega_{t't'}$, $\tilde{\omega}_{t't'}$ by $\tilde{\omega}_{t't'}$.

The term $\gamma^{t}_{t'} \frac{1}{2} S^{t'}e_{t'\nu} e_{t'\mu} S^{t'}e_{t'\nu}$ in Eq. (2) can be rewritten with respect to the generators $S^{t'}e_{t'\nu}$ and the corresponding gauge fields $\omega_{t't'}$, $\tilde{\omega}_{t't'}$ as one colour octet scalar field and one $U(1)_{II}$ singlet scalar field (Eq. A.22)

$$\gamma^{t}_{t'} \frac{1}{2} S^{t'}e_{t'\nu} e_{t'\mu} S^{t'}e_{t'\nu} = \sum_{+(t', t')} \sum_{+(t', t')} \{ \tau^{z3} \cdot A^{3}_{t't'} + \tau^{4} \cdot A^{4}_{t't'} \},$$

$$\quad (t', t') \in ((9,10), (11, 12), (13, 14)). \quad (45)$$

Considering all the above equations (42 - 45), and leaving out $p_{(\bar{s})}$ since in the low energy limit the momentum does not play any role, the $L_f^{t'}$ follows

$$L_f^{t'} = \psi^{\dagger} \gamma^{0}(-) \{ \sum_{+(t', t')} \sum_{+(t', t')} \} \psi,$$

$$\quad \left[ \tau^{z2} \cdot A^{2}_{t't'} + \tau^{z2} \cdot A^{2}_{t't'}, \right. \quad (46)$$

$$\left. + \tau^{1} \cdot A^{1}_{t't'} + \tau^{1} \cdot A^{1}_{t't'} \right] \psi,$$

where $(t, t')$ run in pairs over $[(9, 10), \ldots, (13, 14)]$ and the summation must go over $+$ and $-$ of $t'$.\footnote{On Table 9, taken from Ref. [2], the quantum numbers of the scalar and vector gauge fields, appearing in Eq. (2), are presented, where is taken into account that the spin of gauge fields is determined according to Eq. (47),

$$(S^{ab})_{c} e^{d \ldots g} A^{d \ldots g} = i (\eta^{ae} \delta^{b}_{c} - \eta^{bc} \delta^{a}_{c}) A^{d \ldots g}, \quad (47)$$

for each index $(\in (d \ldots g))$ of a bosonic field $A^{d \ldots g}$ separately. We must take into account also the relation among $S^{ab}$ and the charges (the relations are, of course, the same for bosons and fermions), presented in Eqs. (A.20, A.21, A.22)).

The scalar fields with the scalar index $s = (9, 10, \ldots, 14)$, presented in Table 9, carry one of the triplet colour charges and the "spinor" charge equal to twice the quark "spinor" charge, or the antitriplet colour charges and the anti "spinor" charge. They carry in addition the quantum
Table 9. Quantum numbers of the scalar gauge fields carrying the space index $t = (9, 10, \cdots, 14)$, appearing in Eq. (2), are presented. The space degrees of freedom contribute one of the triplets values to the colour charge of all these scalar fields. These scalars are with respect to the two $SU(2)$ charges, ($\tau^{13}$ and $\hat{\tau}^2$), and the two $\bar{SU}(2)$ charges, ($\bar{\tau}^1$ and $\bar{\tau}^2$), triplets (that is in the adjoint representations of the corresponding groups), and they all carry twice the \textquotedblleft spinor\textquotedblright number ($\tau^4$) of the quarks. The quantum numbers of the two vector gauge fields, the colour and the $U(1)_H$ ones, are added.

| field | prop. | $\tau^4$ | $\tau^4$ | $(\tau^4, \tau^4)$ | $Y$ | $Q$ | $\tau^4$ | $\tau^4$ | $N_L$ | $N_R$ |
|-------|-------|---------|---------|-----------------|-----|-----|---------|---------|-------|-------|
| $A_{1/2}^{(0)}$ | scalar | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | ($\bar{\frac{1}{3}}, \frac{1}{3}$) | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $A_{1/2}^{(1)}$ | scalar | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | ($\bar{\frac{1}{3}}, \frac{1}{3}$) | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $A_{1/2}^{(2)}$ | scalar | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | ($\bar{\frac{1}{3}}, \frac{1}{3}$) | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $A_{1/2}^{(3)}$ | scalar | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | ($\bar{\frac{1}{3}}, \frac{1}{3}$) | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $A_{1/2}^{(4)}$ | scalar | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | ($\bar{\frac{1}{3}}, \frac{1}{3}$) | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $A_{1/2}^{(5)}$ | scalar | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | ($\bar{\frac{1}{3}}, \frac{1}{3}$) | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $A_{1/2}^{(6)}$ | scalar | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | ($\bar{\frac{1}{3}}, \frac{1}{3}$) | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $A_{1/2}^{(7)}$ | scalar | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | ($\bar{\frac{1}{3}}, \frac{1}{3}$) | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $A_{1/2}^{(8)}$ | scalar | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | ($\bar{\frac{1}{3}}, \frac{1}{3}$) | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $A_{1/2}^{(9)}$ | scalar | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | ($\bar{\frac{1}{3}}, \frac{1}{3}$) | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $A_{1/2}^{(10)}$ | scalar | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | ($\bar{\frac{1}{3}}, \frac{1}{3}$) | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $A_{1/2}^{(11)}$ | scalar | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | ($\bar{\frac{1}{3}}, \frac{1}{3}$) | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $A_{1/2}^{(12)}$ | scalar | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | ($\bar{\frac{1}{3}}, \frac{1}{3}$) | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $A_{1/2}^{(13)}$ | scalar | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | ($\bar{\frac{1}{3}}, \frac{1}{3}$) | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |
| $A_{1/2}^{(14)}$ | scalar | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | ($\bar{\frac{1}{3}}, \frac{1}{3}$) | $\bar{\frac{1}{3}}$ | $\frac{1}{3}$ | 0 | 0 | 0 | 0 |

numbers of the adjoint representations originating in $S^{ab}$ or in $\tilde{S}^{ab}$.

Let us choose the 57th line of Table 2, which represents in the spinor technique the left handed positron, $\bar{e}^+_L$, to see what do the scalar fields, appearing in Eq. (46) and in Table 9, do when applying on the left handed members of the Weyl representation presented on Table 2, containing quarks and leptons and antiquarks and antileptons [9, 10, 36].

\[ \text{17 Although carrying the colour charge in one of the triplet or antitriplet states, these fields cannot be interpreted as superpartners of the quarks since they do not have quantum numbers as required by, let say, the N = 1 supersymmetry. The hyper charges and the electromagnetic charges are namely not those required by the supersymmetric partners to the family members.} \]
If we make, let say, the choice of the term \( (\gamma^0)^{910}_F \uparrow \uparrow 2\Xi \) \( A^{2\Xi}_{910F} \) (the scalar field \( A^{2\Xi}_{910F} \)) is presented in the 7\( ^{th} \) line in Table 9 and in the second line of Eq. (46)), the family quantum numbers will not be affected and they can be any. The state carries the "spinor" (lepton) number \( \tau^4 = \frac{1}{2} \), the weak charge \( \tau^{13} = 0 \), the second \( SU(2)_{II} \) charge \( \tau^{23} = \frac{-1}{2} \) and the colour charge \( (\tau^{33}, \tau^{38}) = (0, 0) \). Correspondingly, its hyper charge \( (Y = \tau^4 + \tau^{23}) \) is 1 and the electromagnetic charge \( Q(= Y + \tau^{13}) \) is 1.

So, what does the term \( (\gamma^0)^{910}_F \uparrow \uparrow 2\Xi \) \( A^{2\Xi}_{910F} \) make on this spinor \( \varepsilon_L^\uparrow \)? Making use of Eqs. (A.10, A.12, A.26) of Appendix A one easily finds that operator \( \gamma^0 \) \( \uparrow \uparrow 2 \) transforms the left handed positron into \( (+i)(+)[-][-] \) \( \uparrow \uparrow 2 \), which is \( d^R_L \), presented on line 3 of Table 2. Namely, \( \gamma^0 \) transforms \( [-i] \) into \( (+i) \), \( (+) \) transforms \( [-] \) into \( (+) \), while \( \tau^{2-} \) \( (= - (-) (-)) \) transforms \( (+) \) into \( (+) \) in \( [-] \). The state \( d^R_L \) carries the "spinor" (quark) number \( \tau^4 = \frac{1}{2} \), the weak charge \( \tau^{13} = 0 \), the second \( SU(2)_{II} \) charge \( \tau^{23} = \frac{-1}{2} \) and the colour charge \( (\tau^{33}, \tau^{38}) = (1, \frac{1}{2}) \). Correspondingly its hyper charge is \( (Y = \tau^4 + \tau^{23}) = -1 \) and the electromagnetic charge \( (Q = Y + \tau^{13}) = -\frac{2}{3} \). The scalar field \( A^{2\Xi}_{910F} \) carries just the needed quantum numbers as we can see in the 7\( ^{th} \) line of Table 9.

If the antiquark \( \bar{u}^F_L \), from the line 43 (it is not presented, but one can very easily construct it) in Table 2, with the "spinor" charge \( \tau^4 = \frac{1}{3} \), the weak charge \( \tau^{13} = 0 \), the second \( SU(2)_{II} \) charge \( \tau^{23} = \frac{1}{2} \) and the colour charge \( (\tau^{33}, \tau^{38}) = (0, \frac{1}{2}) \), the hyper charge \( Y(= \tau^4 + \tau^{23}) = 1 \) and the electromagnetic charge \( Q(= Y + \tau^{13}) = \frac{2}{3} \). These two quarks, \( d^R_L \) and \( u^F_L \), can bind together with \( \bar{u}^F_R \) from the 9\( ^{th} \) line of the same table (at low enough energy, after the electroweak transition, and if they belong to a superposition with the left handed partners to the first family) into the colour chargeless baryon - a proton. This transition is presented in Figure 2.

The opposite transition at low energies would make the proton decay.

Similar transitions go also with other scalars from Eq. (46) and Table 9. The \( \tilde{A}^F_{\tau' F}, \tilde{A}^F_{\tau' F}, \tilde{A}_N^F_{\tau' F} \) and \( \tilde{A}_N^F_{\tau' F} \) fields cause transitions among the family members, changing a particular member into the antinumber of another colour and of another family. The term \( (\gamma^0)^{910}_F \uparrow \uparrow 2\Xi \) \( A^{2\Xi}_{910F} \) transforms \( \varepsilon_R^\uparrow \) into \( u^F_L \), changing the family quantum numbers.

The action from Eqs. (1, 2) manifests \( C_N \cdot P_N \) invariance. All the vector and the spinor gauge fields are massless.

Since none of the scalar fields from Table 9 have been observed and also no vector gauge fields like \( A^F_\mu, A^F_{\mu' F} \) and other scalar and vector fields, there must exist a mechanism, which makes the non observed scalar and vector gauge fields massive enough.

Scalar fields from Table 9 carry the colour and the electromagnetic charge. Therefore their nonzero vacuum expectation values would not be in agreement with the observed phenomena. One, however, notices that all the scalar gauge fields from Table 9 and several other scalar and

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18 I expect that the condensate (Table 1) appears on the scale of unification - \( \geq 10^{16} \) GeV. Interacting with these vector gauge fields the condensate make them massive.
vector gauge fields couple to the condensate with the nonzero quantum number \( \tau^4 \) and \( \tau^{23} \) and nonzero family quantum numbers.

It is not difficult to recognize that the desired condensate must have spin zero, \( Y = \tau^4 + \tau^{23} = 0, Q = Y + \tau^{13} = 0 \) and \( \tau^1 = 0 \) in order that in the low energy limit the spin-charge-family theory would manifest effectively as the standard model.

I make a choice of the two right handed neutrinos of the \( VIIIth \) family coupled into a scalar, with \( \tau^4 = -1, \tau^{23} = 1 \), correspondingly \( Y = 0, Q = 0 \) and \( \tau^1 = 0 \), and with family quantum numbers (Eqs. (A.24, A.23)) \( \tilde{\tau}^4 = -1, \tilde{\tau}^{23} = 1, \tilde{\tau}^3 = 1 \), and correspondingly with \( \tilde{Y} = \tau^4 + \tau^{23} = 0, \tilde{Q} = Y + \tau^{13} = 0 \), and \( \tilde{\tau}^1 = 0 \). The condensate carries the family quantum numbers of the upper four families, see Subsect. 3.4.

The condensate made out of spinors couples to spinors differently than to antispinors -"anticondensate" would namely carry \( \tau^4 = 1, \) and \( \tau^{23} = -1 \) - breaking correspondingly the \( C\mathcal{N}_\mathcal{V} \) symmetry: The reactions creating particles from antiparticles are not any longer symmetric to those creating antiparticle from particles.

Such a condensate leaves the hyper field \( \Lambda^Y_m = \sin \vartheta_2 A_{23}^m + \cos \vartheta_2 A_4^m \) (for the choice that \( \sin \vartheta_2 = \cos \vartheta_2 \) and \( g^4 = g^2 \), there is no justification for such a choice, \( \Lambda^Y_m = \frac{1}{\sqrt{2}} (A_{23}^m + A_4^m) \) follows) massless, while it gives masses to \( \Lambda_R^{23} \) and \( \Lambda_R^{13} = (\frac{1}{\sqrt{2}} (A_{23}^m - A_4^m) \) for \( \sin \vartheta_2 = \cos \vartheta_2 \) and it gives masses also to all the scalar gauge fields from Table 9, since they all couple to the condensate through \( \tau^4 \).

The weak vector gauge fields, \( A^{\lambda}_{m} \), the hyper charge vector gauge fields, \( \Lambda^{Y}_m \), and the colour vector gauge fields, \( \tilde{A}^1_m \), remain massless.

The scalar fields with the scalar space index \( s = (7, 8) \) (there are three singlets which couple to all eight families, two triplets which couple only to the upper four families and another two triplets which couple only to the lower four families) - carrying the weak and the hyper charges
of the Higgs’s scalar - wait for gaining nonzero vacuum expectation values to change their masses while causing the electroweak break.

The condensate does what is needed so that in the low energy regime the spin-charge-family manifests as an effective theory. This effective theory agrees with the standard model to such an extent that it is in agreement with the observed phenomena, explaining the standard model assumptions and predicting new fermion and boson fields.

It also may hopefully explain the observed matter-antimatter asymmetry if the conditions in the expanding universe would be appropriate, Ref. ([2], Sect. VI.). The work needed to check these conditions in the expanding universe within the spin-charge-family theory is very demanding. Although we do have some experience with following the history of the expanding universe [13], this study needs much more efforts, not only in calculations, but also in understanding the mechanism of the condensate appearance, relations among the velocity of the expansion, the temperature and the dimension of space-time in the period of the appearance of the condensate. This study has not yet been really started.

5. Conclusions

To better understand the history of the universe and also to make next step in understanding the dynamics of the elementary fermion fields and boson (vector and scalar) gauge fields it is needed to explain the assumptions of the standard model, as well as the phenomena like the existence of the dark matter, matter-antimatter asymmetry and dark energy.

We must understand the origin of: A. the family members quantum numbers, B. the family quantum numbers, C. the origin of vector gauge fields, D. the origin of the Higgs and Yukawa couplings.

One of the most urgent questions in the elementary particle physics is: Where do the families originate? Explaining the origin of families would answer the question about the number of families which are possibly observable at the low energy regime, about the origin of the scalar field(s) and the Yukawa couplings (the couplings of fermions to the scalar field(s)), about the differences in the fermions properties - the differences in the masses and mixing matrices among family members – quarks and leptons, as well as about the hierarchy in quark and lepton masses.

I demonstrated in this talk, that the spin-charge-family theory - starting with the simple action in $d = (13 + 1)$ for fermions and bosons - offers the explanation for all the assumptions of the standard model:

a. The theory explains all the properties of the family members - quarks and leptons, left and right handed, and their right and left handed antiquarks and antileptons $^{19}$, explaining why the left handed spinors carry the weak charge while the right handed do not (the right handed neutrino is the regular member of each family).

b. It explains the appearance and the properties of the families of family members.

c. It explains the existence of the gauge vector fields of the family members charges.

d. It explains the appearance and the properties of the scalar field (the Higgs) and the Yukawa couplings.

All the gauge fields, vector and scalar in $d = (3 + 1)$, origin in vielbeins and the two kinds of spin connection fields in $d = (13 + 1)$ - the gravity. The two spin connection fields are uniquely expressible with the vielbeins, if there are no spinors present [42].

$^{19}$ One Weyl representation of $SO(13 + 1)$ contains, if analyzed with respect to the standard model groups, all the members of one family, the coloured quarks and colourless leptons, and the anticoloured antiquarks and (anti)colourless antileptons, with the left handed spinors carrying the weak charge and the right handed ones weak chargeless, while the left handed antispinors are weak chargeless and the right handed ones carry the weak charge.
It also offers the explanation for the phenomena, which are not part of the standard model, like:

e. It explains the existence of the dark matter.
f. It explains the origin of the (ordinary) matter-antimatter asymmetry.

5.1. Predictions for the future experiments

The theory predicts:

g. There are twice two groups of four families of quarks and leptons at low energies.
g.i. The fourth family with masses above 1 TeV, weakly coupled to the observed three families, will be measured at the LHC.
g.ii. The quarks and leptons of the fifth family - that is of the stable one of the upper four families - form the dark matter. The family members, which form the chargeless clusters, manifest, due to their very heavy masses, a "new nuclear force".
h. The predicted scalar fields with the space index (7, 8) manifest in \((d = (3 + 1))\) as the weak and hyper charges doublets (as required by the standard model Higgs) with respect to the space index. These scalars carry in addition:

h.i. Either they carry one of the three family members quantum numbers, \((Q, Q', Y')\) - belonging correspondingly to one of three singlets.
h.ii. Or they carry family quantum numbers - belonging correspondingly to one of the twice two triplets.
h.iii. The three singlets and the two triplets determine mass matrices of the lower four families, contributing to masses of the heavy vector bosons.
h.iv. These scalars determine the observed Higgs and the Yukawa couplings.
i. The predicted scalar fields with the space index \((9, 10, ..., 14)\) are triplets with respect to the space index. They cause the transitions from antileptons into quarks and antiquarks into quarks and back.
i.i. The condensate breaks the matter-antimatter symmetry, causing the asymmetry in the (ordinary) matter with respect to antimatter.
i.ii. These condensate is responsible also for the proton decay.
j. The condensate is a scalar of the two right handed neutrinos with the family quantum numbers of the upper four families.
k. The condensate gives masses to all the gauge fields with which it interacts.
k.i. It gives masses to all scalar fields and to vector fields, leaving massless only the colour, the weak, the hyper vector gauge fields and the gravity in \(((3 + 1))\).
l. There is the \(SU(2)\) (belonging together with the weak \(SU(2)\) to \(SO(4)\) gauge fields included in \(SO(7, 1)\)) vector gauge field, which gain masses of the order of the appearance of the condensate.
m. At the electroweak break the scalar fields with the space index \((7, 8)\) change their mutual interaction, and gaining nonzero vacuum expectation values, break the weak and the hyper charges and correspondingly the mass protection of fermions, making them massive.
n. The symmetry of mass matrices allow, in the case that the experimental data for the mixing submatrix \(3 \times 3\) of the \(4 \times 4\) mixing matrix would be accurate, to determine the mixing matrix and the masses of the fourth family quarks. The accuracy, with which the masses of the six lower families are measured so far, does not influence the results appreciably. Due to uncertainty of the experimental data for the \(3 \times 3\) mixing submatrix we are only able to determine the \(4 \times 4\) quark mixing matrix for a chosen masses of the the fourth family quarks. However, we also predict how will the \(3 \times 3\) submatrix of the mixing matrix change with more accurate measurements.
n.i. The fourth family quarks mass matrices are for masses above 1 TeV closer and closer to the democratic matrices. The less the scalars with the family members quantum numbers contribute to masses of the fourth family quarks, the closer is \(m_{u_4}\) to \(m_{d_4}\).
n.ii. The large contribution of the scalars with the family members quantum numbers \((Q, Q', Y')\) to the masses of the lower four families manifests in the large differences of quarks masses of the lower four families.

n.iii. Although we have done calculations also for leptons, further analyses of their properties must wait for more accurate experimental data.

o. In the case that the \(u_4\) and \(d_4\) quarks have similar masses - determined mostly by the scalar fields carrying the family quantum numbers - they contribute mostly to the production of these scalars, while their contribution to the production of those scalars which carry the family members quantum numbers - to the Higgs in particular - is much weaker, which is in agreement with the experiment 20.

p. All the degrees of freedom discussed in this talk are already a part of the simple starting action Eq.(1).

p.i. The way of breaking symmetries (ordered by the conditions determining the history of our universe) is assumed so that it leads in \(d = (3 + 1)\) to the observable symmetries, although we could in principle derive it from the starting action and boundary conditions.

p.ii. Also the effective interaction among scalar fields is assumed, although we could derive it in principle from the starting action and the boundary conditions.

r. The spin-charge-family theory easily explains what in the standard model seems like a miracle: no triangle anomalies.

s. A lot of efforts has been put in this theory to show that it could work as a next step below the standard model proving like:

s.i. There is possibility in the Kaluza-Klein-like theory that breaking symmetries can leave fermions massless.

s.ii. That vielbeins in the Kaluza-Klein theories and spin connections in the spin-charge-family theory represent the same vector gauge fields in \(d = (3 + 1)\).

5.2. Open questions in the spin-charge-family theory

There are several open problems in the spin-charge-family theory:

t. Since this theory is, except that fermions carry two kinds of spins - one kind taking care of spin and charges, the second one taking care of families - a kind of the Kaluza-Klein theories, it shares at very high energy with these theories the quantization problem.

u. The dimension of space-time, \(d = (13 + 1)\), is in the spin-charge-family theory chosen, since \(SO(13,1)\) contains all the members, assumed in the standard model.

It contains also the right handed neutrino (which carries the \(Y'\) quantum number).

u.i. It should be shown, however, how has nature ”made the decision” in evolution to go through this dimension and what is indeed the dimension of space-time (infinite?).

v. There are many other open question, like:

v.i. What is the reason for the (so small) dark energy?

v.ii. At what energy the electroweak phase transition occurs? (Let me add that there is the answer within the spin-charge-family theory to this open problem.)

v.iii. Why do we have fermions and bosons? [43]

It is encouraging that the more work is done on this theory the more answers to the open questions is found.

\(^{20}\) The coupling constants of the singlet scalar fields differ among themselves and also from the coupling constants of the two triplet scalar fields.
Acknowledgments
The author thanks the organizers of IARD - Martin Land and Matej Pavšič - to give her opportunity to present her work, which might have a real chance to show the next step beyond the standard models.

Appendix A. Short presentation of spinor technique [1, 7, 31, 32]

This appendix is a short review (taken from [4]) of the technique [7, 33, 31, 32], initiated and developed in Ref. [7], while proposing the spin-charge-family theory [5, 3, 6, 7, 8, 9, 10, 4, 11, 12, 13, 14, 15, 16, 17, 2, 1]. All the internal degrees of freedom of spinors, with family quantum numbers included, are describable in the space of $d$-anticommuting (Grassmann) coordinates [7], if the dimension of ordinary space is also $d$. There are two kinds of operators in the Grassmann space fulfilling the Clifford algebra and anticommuting with one another Eq. (A.1). The technique was further developed in the present shape together with H.B. Nielsen [33, 31, 32].

In this last stage we rewrite a spinor basis, written in Ref. [7] as products of polynomials of Grassmann coordinates of odd and even Grassmann character, chosen to be eigenstates of two Lorentz groups, as well as transformation properties of the states under any Clifford algebra technique offers an elegant way to see all the quantum numbers of states with respect to the family quantum numbers included, are describable in the space of $d$-anticommuting (Grassmann) coordinates [7], if the dimension of ordinary space is also $d$. There are two kinds of operators in the Grassmann space fulfilling the Clifford algebra and anticommuting with one another Eq. (A.1). The technique was further developed in the present shape together with H.B. Nielsen [33, 31, 32].

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in order that $\gamma^a$ are compatible with (A.1) and formally unitary, i.e. $\gamma^a \gamma^a = I$.

One finds from Eq. (A.4) that $(S^{ab})^\dagger = \eta^{ab} \eta^{bb} S^{ab}$. Recognizing from Eq. (A.3) that the two Clifford algebra objects $S^{ab}, S^{cd}$ with all indices different commute, and equivalently for $\tilde{S}^{ab}, \tilde{S}^{cd}$, we select the Cartan subalgebra of the algebra of the two groups, which form equivalent representations with respect to one another

\begin{equation}
S^{03}, S^{12}, S^{56}, \ldots, S^{d-1 d}, \quad \text{if} \quad d = 2n \geq 4,
\end{equation}

\begin{equation}
\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \ldots, \tilde{S}^{d-1 d}, \quad \text{if} \quad d = 2n + 1 > 4,
\end{equation}

\begin{equation}
\tilde{S}^{03}, S^{12}, S^{56}, \ldots, \tilde{S}^{d-1 d}, \quad \text{if} \quad d = 2n \geq 4,
\end{equation}

\begin{equation}
\tilde{S}^{03}, S^{12}, S^{56}, \ldots, \tilde{S}^{d-2 d-1}, \quad \text{if} \quad d = 2n + 1 > 4.
\end{equation}

The choice for the Cartan subalgebra in $d < 4$ is straightforward. It is useful to define one of the Casimirs of the Lorentz group - the handedness $\Gamma (\{ \Gamma, S^{ab}\}_- = 0)$ in any $d$

\begin{equation}
\Gamma^{(d)} : = (i)^{d/2} \prod_{a} (\sqrt{\eta^{aa}} \gamma^a), \quad \text{if} \quad d = 2n,
\end{equation}

\begin{equation}
\Gamma^{(d)} : = (i)^{(d-1)/2} \prod_{a} (\sqrt{\eta^{aa}} \gamma^a), \quad \text{if} \quad d = 2n + 1.
\end{equation}

One proceeds equivalently for $\tilde{\Gamma}^{(d)}$, substituting $\gamma^a$’s by $\tilde{\gamma}^a$’s. We understand the product of $\gamma^a$’s in the ascending order with respect to the index $a$: $\gamma^0 \gamma^1 \cdots \gamma^d$. It follows from Eq. (A.4) for any choice of the signature $\eta^{ab}$ that $\Gamma^\dagger = \Gamma$, $\Gamma^2 = I$. We also find that for $d$ even the handedness anticommutes with the Clifford algebra objects $\gamma^a (\{ \gamma^a, \Gamma \}_+ = 0)$, while for $d$ odd it commutes with $\gamma^a (\{ \gamma^a, \Gamma \}_- = 0)$.

To make the technique simple we introduce the graphic presentation as follows

\begin{equation}
\gamma^a (k) = \frac{1}{2} (\gamma^a + \frac{\eta^{aa}}{ik} \gamma^b), \quad [k] := \frac{1}{2} (1 + \frac{i}{k} \gamma^a \gamma^b),
\end{equation}

where $k^2 = \eta^{aa} \eta^{bb}$. It follows then

\begin{equation}
\gamma^a = \frac{ab}{(k) + (\bar{a} k)}, \quad \gamma^b = ik \eta^{aa} (\frac{ab}{(k)} - (\bar{a} k)),
\end{equation}

\begin{equation}
S^{ab} = \frac{k}{2} (\frac{ab}{(k)} - (\bar{a} k)).
\end{equation}

One can easily check by taking into account the Clifford algebra relation (Eq. (A.1)) and the definition of $S^{ab}$ and $\tilde{S}^{ab}$ (Eq. (A.3)) that the nilpotent $(k)$ and the projector $[k]$ are "eigenstates" of $S^{ab}$ and $\tilde{S}^{ab}$

\begin{equation}
S^{ab} (k) = \frac{1}{2} k \frac{ab}{(k)}, \quad S^{ab} [k] = \frac{1}{2} k \frac{ab}{[k]},
\end{equation}

\begin{equation}
\tilde{S}^{ab} (k) = \frac{1}{2} k \frac{ab}{(k)}, \quad \tilde{S}^{ab} [k] = -\frac{1}{2} k \frac{ab}{[k]},
\end{equation}

which means that we get the same objects back multiplied by the constant $\frac{1}{2} k$ in the case of $S^{ab}$, while $\tilde{S}^{ab} (k)$ multiply by $k$ and $[k]$ by $(-k)$ rather than $(k)$. This also means that when $\frac{ab}{(k)}$ and $\frac{ab}{[k]}$ act from the left hand side on a vacuum state $|\psi_0\rangle$ the obtained states are the eigenvectors
of $S^a_{\alpha\beta}$ We further recognize that $\gamma^a$ transform $(k)_{\alpha\beta}$ into $[k]_{\alpha\beta}$, never to $[k]_{\alpha\beta}$, while $\gamma^\alpha$ transform $(k)_{\alpha\beta}$ into $[k]_{\alpha\beta}$, never to $[k]_{\alpha\beta}$

$$\gamma^a_{\alpha\beta} (k) = \eta^a_{\alpha\beta} [k], \quad \gamma^\alpha_{\alpha\beta} (k) = -ik [k], \quad \gamma^a_{\alpha\beta} [k] = (k), \quad \gamma^\alpha_{\alpha\beta} [k] = -ik \eta^a_{\alpha\beta} [k], \quad \gamma^a_{\alpha\beta} [-k] = -i \eta^a_{\alpha\beta} [-k], \quad \gamma^\alpha_{\alpha\beta} [-k] = i (k), \quad \gamma^a_{\alpha\beta} [-k] = -k \eta^a_{\alpha\beta} [k].$$

(A.10)

From Eq. (A.10) it follows

$$S^{ac} \frac{a b}{c d} (k)(k) = \frac{i}{2} \eta^{a c} \frac{a b}{c d} [-k] [k], \quad S^{ac} \frac{a b}{c d} [k][k] = \frac{i}{2} \eta^{a c} \frac{a b}{c d} [k][k],$$

$$S^{ac} \frac{a b}{c d} (k)[k] = \frac{i}{2} \eta^{a c} \frac{a b}{c d} [-k] (k), \quad S^{ac} \frac{a b}{c d} (k)[k] = \frac{i}{2} \eta^{a c} \frac{a b}{c d} [k](k),$$

$$S^{ac} \frac{a b}{c d} (k)[-k] = \frac{i}{2} \eta^{a c} \frac{a b}{c d} [-k] [-k], \quad S^{ac} \frac{a b}{c d} (k)[-k] = \frac{i}{2} \eta^{a c} \frac{a b}{c d} [-k] [-k].$$

(A.11)

From Eq. (A.11) we conclude that $S^{ac}$ generate the equivalent representations with respect to $S_{\alpha\beta}$ and opposite.

Let us deduce some useful relations

$$\frac{a b}{a b} \frac{a b}{a b} (k)(k) = 0, \quad \frac{a b}{a b} \frac{a b}{a b} (k)[-k] = 0, \quad \frac{a b}{a b} \frac{a b}{a b} [k][k] = 0, \quad \frac{a b}{a b} \frac{a b}{a b} [k][-k] = 0.$$

(A.12)

We recognize in Eq. (A.12) the demonstration of the nilpotent and the projector character of the Clifford algebra objects $(k)$ and $[k]$, respectively. Defining

$$\frac{a b}{a b} \frac{a b}{a b} ( \pm i) = \frac{1}{2} (\tilde{\gamma}^a \mp \tilde{\gamma}^b), \quad ( \pm 1) = \frac{1}{2} (\tilde{\gamma}^a \pm i \tilde{\gamma}^b),$$

one recognizes that

$$\frac{a b}{a b} \frac{a b}{a b} (k)(k) = 0, \quad \frac{a b}{a b} \frac{a b}{a b} (k)[-k] = 0, \quad \frac{a b}{a b} \frac{a b}{a b} [k][k] = 0, \quad \frac{a b}{a b} \frac{a b}{a b} [k][-k] = 0.$$

(A.13)

Recognizing that

$$\frac{a b}{a b} \frac{a b}{a b} (k) = \eta^{a a} (k), \quad \frac{a b}{a b} \frac{a b}{a b} [k] = [k],$$

(A.15)

we define a vacuum state $|\psi\rangle > 0$ so that one finds

$$\frac{a b}{a b} \frac{a b}{a b} \frac{a b}{a b} < k)(k) >= 1, \quad \frac{a b}{a b} \frac{a b}{a b} \frac{a b}{a b} < k|k] >= 1.$$
Taking into account the above equations it is easy to find a Weyl spinor irreducible representation for \( d \)-dimensional space, with \( d \) even or odd.

For \( d \) even we simply make a starting state as a product of \( d/2 \), let us say, only nilpotents \( k \), one for each \( S_{ab} \) of the Cartan subalgebra elements (Eq.\( (A.5) \)), applying it on an (unimportant) vacuum state. For \( d \) odd the basic states are products of \( (d-1)/2 \) nilpotents and a factor \((1 \pm \Gamma)\). Then the generators \( S_{ab} \), which do not belong to the Cartan subalgebra, being applied on the starting state from the left, generate all the members of one Weyl spinor.

\[
\begin{align*}
0_d & \ 12 \ 35 \ \cdots \ 0 \ d-1 \ d-2 \ |\psi_0 > \\
(k_{0d})(k_{12})(k_{35}) \cdots (k_{d-1 \ d-2}) & |\psi_0 > \\
[-k_{0d}] & [-k_{12}](k_{35}) \cdots (k_{d-1 \ d-2}) |\psi_0 > \\
[-k_{0d}](k_{12}) & [-k_{35}] \cdots (k_{d-1 \ d-2}) |\psi_0 > \\
n & \cdots \\
[k_{0d}] & [-k_{12}](k_{35}) \cdots (k_{d-1 \ d-2}) |\psi_0 > \\
(k_{0d}) & [-k_{12}](k_{35}) \cdots (k_{d-1 \ d-2}) |\psi_0 > \\
\end{align*}
\]
\( (A.17) \)

All the states have the same handedness \( \Gamma \), since \( \{\Gamma,S_{ab}\}_- = 0 \). States, belonging to one multiplet with respect to the group \( SO(q,d-q) \), that is to one irreducible representation of spinors (one Weyl spinor), can have any phase. We made a choice of the simplest one, taking all phases equal to one.

The above graphic representation demonstrates that for \( d \) even all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of nilpotents \( k_{ab} \), by transforming all possible pairs of \( (k_{ab})(k_{mn}) \) into \( [-k_{ab}][k_{mn}] \). There are \( S^{ab}, S^{ab}, S^{mn}, S^{mn} \), which do this. The procedure gives \( 2^{d/2-1} \) states. A Clifford algebra object \( \gamma^a \) being applied from the left hand side, transforms a Weyl spinor of one handedness into a Weyl spinor of the opposite handedness. Both Weyl spinors form a Dirac spinor.

We shall speak about left handedness when \( \Gamma = -1 \) and about right handedness when \( \Gamma = 1 \) for either \( d \) even or odd.

While \( S_{ab} \) which do not belong to the Cartan subalgebra (Eq.\( (A.5) \)) generate all the states of one representation, \( \tilde{S}_{ab} \) which do not belong to the Cartan subalgebra (Eq.\( (A.5) \)) generate the states of \( 2^{d/2-1} \) equivalent representations.

Making a choice of the Cartan subalgebra set of the algebra \( S_{ab} \) and \( \tilde{S}_{ab} \) \( S_{03}, S_{12}, S_{56}, S_{78}, S_{910}, S_{1112}, S_{1314}, S_{303}, S_{1256}, S_{789}, S_{101112}, S_{1314} \), a left handed \( (\Gamma^{(13,1)} = -1) \) eigenstate of all the members of the Cartan subalgebra, representing a weak chargeless \( u_R \)-quark with spin up, hyper charge \( 2/3 \) and colour \( 1/2, 1/(2\sqrt{3}) \), for example, can be written as

\[
\begin{align*}
0^3 & \ 12 \ 56 \ 78 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \\
(+) & (+) \ || \ (+) (+) \ || \ (+) (-) \ |\psi_0 > \\
\frac{1}{2^7} & (\gamma^9 - \gamma^3)(\gamma^1 + i\gamma^2)(\gamma^5 + i\gamma^6)(\gamma^7 + i\gamma^8) \\
(\gamma^9 + i\gamma^{10})(\gamma^{11} - i\gamma^{12})(\gamma^{13} - i\gamma^{14}) |\psi_0 >. \\
\end{align*}
\]
\( (A.18) \)

This state is an eigenstate of all \( S_{ab} \) and \( \tilde{S}_{ab} \) which are members of the Cartan subalgebra (Eq.\( (A.5) \)).
The operators $\tilde{S}^{ab}$, which do not belong to the Cartan subalgebra (Eq. (A.5)), generate families from the starting $u_R$ quark, transforming the $u_R$ quark from Eq. (A.18) to the $u_R$ of another family, keeping all of the properties with respect to $S^{ab}$ unchanged. In particular, $\tilde{S}^{01}$ applied on a right handed $u_R$-quark from Eq. (A.18) generates a state which is again a right handed $u_R$-quark, weak chargeless, with spin up, hyper charge $(2/3)$ and the colour charge $(1/2, 1/(2\sqrt{3}))$

\[
\tilde{S}^{01} \begin{pmatrix} \begin{pmatrix} 03 & 12 & 56 & 78 & 910 & 11121314 \\
\end{pmatrix} \end{pmatrix} = \frac{i}{2} \begin{pmatrix} \begin{pmatrix} 03 & 12 & 56 & 78 & 910 & 11121314 \\
\end{pmatrix} \end{pmatrix} .
\]  

(A.19)

Below some useful relations [9] are presented

\[
\tilde{N}_\pm (= \tilde{N}_{(L,R)}) : = \frac{1}{2} (S^{23} \pm iS^{01}, S^{31} \pm iS^{02}, S^{12} \pm iS^{03}) ,
\]

(A.20)

\[
\tilde{\tau}^1 : = \frac{1}{2} (S^{58} - S^{67}, S^{57} + S^{68}, S^{56} - S^{78}) ,
\]

\[
\tilde{\tau}^2 : = \frac{1}{2} (S^{58} + S^{67}, S^{57} - S^{68}, S^{56} + S^{78}) ,
\]

(A.21)

\[
\tilde{\tau}^3 : = \frac{1}{2} \{ S^{912} - S^{1011}, S^{911} + S^{1012}, S^{910} - S^{1112}, S^{914} - S^{1013}, S^{913} + S^{1014}, S^{1114} - S^{1213}, S^{1113} + S^{1214}, \frac{1}{\sqrt{3}} (S^{910} + S^{1112} - 2S^{1314}) \},
\]

\[
\tilde{\tau}^4 : = -\frac{1}{3} (S^{910} + S^{1112} + S^{1314}) ,
\]

(A.22)

\[
\tilde{N}_{L,R} : = \frac{1}{2} (\tilde{S}^{23} \pm i\tilde{S}^{01}, \tilde{S}^{31} \pm i\tilde{S}^{02}, \tilde{S}^{12} \pm i\tilde{S}^{03}) ,
\]

(A.23)

\[
\tilde{\tau}^1 : = \frac{1}{2} (\tilde{S}^{58} - \tilde{S}^{67}, \tilde{S}^{57} + \tilde{S}^{68}, \tilde{S}^{56} - \tilde{S}^{78}) ,
\]

\[
\tilde{\tau}^2 : = \frac{1}{2} (\tilde{S}^{58} + \tilde{S}^{67}, \tilde{S}^{57} - \tilde{S}^{68}, \tilde{S}^{56} + \tilde{S}^{78}) ,
\]

(A.24)

\[
\tilde{\tau}^4 : = -\frac{1}{3} (\tilde{S}^{910} + \tilde{S}^{1112} + \tilde{S}^{1314}) .
\]

(A.25)

\[
N^\pm_+ = N^1_+ \pm i N^2_+ = - \begin{pmatrix} \begin{pmatrix} 03 & 12 \\
\end{pmatrix} \end{pmatrix} , \quad N^\pm_- = N^1_- \pm i N^2_- = \begin{pmatrix} \begin{pmatrix} 03 & 12 \\
\end{pmatrix} \end{pmatrix} ,
\]

\[
\tilde{N}^\pm_+ = - (\tilde{\mp}) (\pm) , \quad \tilde{N}^\pm_- = (\tilde{\mp}) (\pm) ,
\]

\[
\tilde{\tau}^{1\pm} = (\mp) (\pm) (\mp) , \quad \tilde{\tau}^{2\mp} = (\pm) (\mp) (\mp) ,
\]

\[
\tilde{\tau}^{1\mp} = (\mp) (\pm) (\mp) , \quad \tilde{\tau}^{2\mp} = (\pm) (\mp) (\mp) .
\]

(A.26)
Appendix B. Standard model assumptions

More than 40 years ago the standard model offered an elegant new step in understanding the origin of fermions and bosons by postulating:

- The existence of massless family members: coloured quarks and colourless leptons, both left and right handed, the left handed members distinguishing from the right handed ones in the weak and hyper charges and correspondingly mass protected, Table B1.
- The existence of the vector gauge fields (massless before the electroweak break) to the observed charges of the family members, Table B2.
- The existence of a massive scalar field carrying the weak charge \((\pm \frac{1}{2})\) and the hyper charge \((\pm \frac{1}{2})\), which by its "nonzero vacuum expectation values" breaks the weak and the hyper charge and correspondingly breaks the mass protection of fermions and those vector bosons which interact with this vacuum, Table B3.
- The existence of the Yukawa couplings of fermions, which together with (the gluons and) the scalar take care of the properties of fermions after the electroweak break.

Table B1. Table represents the standard model assumptions for each of the three so far observed \((i = 1, 2, 3)\) families of quarks and leptons, massless before the electroweak break. Each family contains the left handed weak charged quarks and right handed weak chargeless quarks, each quark belonging to the colour triplet \((1/2, 1/(2\sqrt{3})), (-1/2, 1/(2\sqrt{3})), (0, -1/(\sqrt{3}))\), and the left handed weak charged and right handed weak chargeless colourless leptons. \(\tau^{13}\) defines the third component of the weak charge, \(Y\) is the hyper charge determining the electromagnetic charge \(Q = Y + \tau^{13}\). The standard model assumes to each family member of each family the corresponding anti-fermions.

| \(i\) | Handedness | Weak charge \(Y^{13}\) | Hypercharge \(Y\) | Colour charge | Sign charge | Q   |
|------|------------|-----------------|-------------|--------------|-------------|-----|
| \(u_L\) | -1         | \(\frac{1}{2}\) | \(\frac{1}{6}\) | colour triplet | \(\frac{2}{3}\) |
| \(d_L\) | -1         | \(-\frac{1}{2}\) | \(\frac{1}{6}\) | colour triplet | \(-\frac{1}{3}\) |
| \(e_L\) | -1         | \(\frac{1}{2}\) | \(-\frac{1}{2}\) | colourless | 0 |
| \(\nu_L\) | -1         | \(-\frac{1}{2}\) | \(-\frac{1}{2}\) | colourless | -1 |
| \(u_R\) | 1          | weakless | \(\frac{2}{3}\) | colour triplet | \(\frac{2}{3}\) |
| \(d_R\) | 1          | weakless | \(-\frac{1}{3}\) | colour triplet | \(-\frac{1}{3}\) |
| \(\nu_R\) | 1          | weakless | 0 | colourless | 0 |
| \(e_R\) | 1          | weakless | -1 | colourless | -1 |

The standard model assumptions have been confirmed without offering surprises. The last unobserved field, the Higgs, detected in June 2012, was confirmed in March 2013.
Table B2. Vector fields, the gauge fields of the hyper, weak and colour charges, all massless before the electroweak break. They all are vectors in the adjoint representations with respect to the weak, colour and hyper charges.

| name            | handedness | weak charge | hyper charge | colour charge | elm charge |
|-----------------|------------|-------------|--------------|---------------|------------|
| hyper photon    | 0          | 0           | 0            | colourless    | 0          |
| weak bosons     | 0          | triplet     | 0            | colourless    | triplet    |
| gluons          | 0          | 0           | 0            | colour octet  | 0          |

Table B3. Higgs is the scalar field with the weak charge and the hyper charge $\pm\frac{1}{2}$ and $\mp\frac{1}{2}$, respectively. The 0. Higgs$_d$ and 0. Higgs$_u$ are not assumed. In Table these two are added to manifest the fundamental representation of the charge groups. The two components, $<\text{Higgs}_u>$ and $<\text{Higgs}_d>$, ”dress” in the standard model the right handed u-quarks and d-quarks, respectively, giving them the charges of the left partners.

| name           | handedness | weak charge | hyper charge | colour charge | elm charge |
|----------------|------------|-------------|--------------|---------------|------------|
| 0- Higgs$_u$   | 0          | $\frac{1}{2}$ | $\frac{1}{2}$ | colourless    | 1          |
| $<\text{Higgs}_d>$ | 0          | $-\frac{1}{2}$ | $\frac{1}{2}$ | colourless    | 0          |
| $<\text{Higgs}_u>$ | 0          | $\frac{1}{2}$ | $-\frac{1}{2}$ | colourless    | 0          |
| 0- Higgs$_d$   | 0          | $-\frac{1}{2}$ | $-\frac{1}{2}$ | colourless    | −1         |

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