Decay studies of rare-earth nuclei to superheavy elements and the associated shell effects

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Abstract. The effects of shell closure in nuclei via cluster decay have been investigated. In this context, the Preformed Cluster Model (PCM) based on Quantum Mechanical Fragmentation Theory is used. The key point in the cluster radioactivity is that it involves the interplay of close shell effects of parent and daughter nucleus. Small half life for a parent indicates shell stabilized daughter and long half life indicates the stability of the parent against the decay. Cluster decays of rare-earth nuclei are studied with a view to look for neutron magic shells for the $^{50}$Sn nucleus as the daughter product always, to find the most probable cluster decays and the possibility, if any, of new neutron shells. The recently observed $\alpha$-decay chains $^{293-294}$117 which were produced by the fusion reactions with target $^{249}$Bk and projectile $^{48}$Ca at Dubna, in Russia. The $\alpha$-decay calculations are performed for all the parents of the two alpha decay chains and compared with the experimental data and with other theoretical model.

1. Introduction
Since the discovery of $^{14}$C-decay from $^{223}$Ra by Rose and Jones [1] in 1984, many other $^{14}$C-decays from other radioactive nuclei ($^{221}$Fr, $^{221,222,224,226}$Ra, $^{223,225}$Ac and $^{226}$Th) and some 12 to 13 neutron-rich clusters, such as $^{20}$O, $^{23}$F, $^{22,24-26}$Ne, $^{28,30}$Mg and $^{32,34}$Si, have been observed experimentally for the ground-state decays of translead $^{226}$Th to $^{242}$Cm parents [2, 3, 4, 5], which all decay with the doubly closed shell daughter $^{208}$Pb ($Z=82, N=126$) or its neighboring nuclei. Theoretically, such an exotic natural radioactivity of emitting particles (nuclei) heavier than $\alpha$-particle was already predicted in 1980 by Sandulescu, Poenaru and Greiner [6] on the basis of the Quantum Mechanical Fragmentation Theory (QMFT). Todate, $^{34}$Si is the heaviest cluster observed with the longest decay half-life ever measured ($\log_{10}T_{1/2}(s) = 29.04$) from $^{238}$U parent, and the smallest branching ratio of cluster w.r.t. $\alpha$-decay ($B = \lambda_{\text{cluster}}/\lambda_\alpha \sim 10^{-17}$ for $^{28,30}$Mg decay of $^{238}$Pu [4]. Keeping in mind the doubly magic nature of the $^{208}$Pb daughter, a second island of heavy-cluster radioactivity was predicted by Poenaru et. al. [7] on the basis of analytical supersymmetric fission model (ASAFM), in the decays of some neutron-deficient rare-earth nuclei in to $^{100}$Sn ($Z=N=50$) daughter or a neighboring nucleus. Recently, Sarkar et al., [8] predicted a new shell closure at $N = 90$ for the Sn isotopes on the basis of shell model calculations. Experimentally, several unsuccessful attempts [9, 10, 11, 12] have been made to measure the $^{100}$Sn-daughter radioactivity from the $^{114}$Ba parent nucleus produced in $^{58}$Ni+$^{58}$Ni reaction. Instead, a new phenomenon of intermediate mass fragments (IMFs, with $3 \leq Z \leq 9$), also referred to as 'clusters' or 'complex fragments', emitted from the excited compound nucleus, was observed at
various incident energies [13, 14, 15, 16], with $^{12}$C as one the fragment or cluster emitted. However, the above study of ground-state cluster-decays of rare-earth nuclei still remains of interest from the point of view of associated shell effects.

In this paper, the heavy cluster emissions to many more rare-earth parents (329 cases) with $^{50}$Sn always as the daughter product is considered. Specifically, we have considered the emission of various isotopes of C, O, Ne, Mg, Si, S, Ar, Ca, Ti, Cr, Fe and Ni respectively, from neutron-deficient to neutron-rich Ba, Ce, Nd, Sm, Gd, Dy, Er, Yb, Hf, W, Os and Pt parents, with a view to look for $^{100}$Sn and $^{132}$Sn radioactivities, as well as any other new Sn radioactivity with new shell closures in neutrons. Since the cluster decays are more probable with daughters as magic nuclei, the decay half-lives are expected to drop (be minimum) for the magic daughters. The same idea was utilized earlier for the (spherical) sub-shell closed $^{40}$Zr daughter [17, 18], including also a brief report of the results on $^{50}$Sn daughter [17].

Next, the superheavy mass region extend effectively due to the availability and advancement in the radioactive nuclear beam technology. The $^{48}$Ca beam is most prominent at present for the synthesis of superheavy elements (SHE). The neutron rich superheavy elements $Z=112-116$, and 118 have been produced by (the doubly magic nucleus) $^{48}$Ca induced reactions with actinide targets $^{238}$U, $^{237}$Np, $^{244}$Pu, $^{243}$Am, $^{248}$Cm and $^{248}$ Cf respectively at low excitation energies at FLNR in Dubna, Russia [19, 20, 21]. In addition to the above mentioned SHE, recently two isotope $^{293}$117 and $^{294}$117 are observed of a new superheavy element Z=117, as a result of the fusion reaction between $^{48}$Ca projectiles and radioactive $^{249}$Bk target nuclei [22]. In this reaction, the excitation energy of the compound nucleus $^{297}$117 is reported to be $E^*=39$ MeV and $E^*'=35$ MeV, which as a result de-excite by the emission of four and three neutrons, respectively. Recently observed two isotope of the element Z=117 with mass number 293 and 294 are studied for the alpha decay characteristics. Calculations for the $^{293,294}$117 alpha decay chains are compared with the experimental results [22] and the macroscopic- microscopic (MM) model dependent calculations of A. Sobiczewski [23].

The calculations are based on the preformed cluster model (PCM) [24, 25, 26], described briefly in Sect. 2. The results of calculation are presented in Sect. 3 and a summary of results is given in Sect. 4.

2. The Preformed Cluster Model (PCM)

The preformed cluster model (PCM) [24, 25, 26] uses the dynamical collective coordinates of mass and charge asymmetries

$$\eta = (A_1 - A_2)/(A_1 + A_2)$$

and

$$\eta_Z = (Z_1 - Z_2)/(Z_1 + Z_2),$$

first introduced in the Quantum Mechanical Fragmentation Theory [27, 28, 29]. These are in addition to the usual coordinates of relative separation R and deformations $\beta_i$ ($i=1,2$). Then, in the standard approximation of decoupled R- and $\eta$-motions [24, 25], The decay constant $\lambda$ (the decay half-life $T_{1/2}$) in PCM is defined as

$$\lambda = \frac{ln2}{T_{1/2}} = P_0 \nu_0 P.$$  \hspace{1cm} (1)

Here $P_0$ is the cluster (and daughter) preformation probability and P the barrier penetrability which refer, respectively, to the $\eta$ and R motions. The $\nu_0$ is the barrier assault frequency. The
$P_0$ are the solutions of the stationary Schrödinger equation in $\eta$,

$$
\{-\frac{h^2}{2B_\eta} \frac{\partial}{\partial \eta} \frac{1}{\sqrt{B_\eta}} \frac{\partial}{\partial \eta} + V_R(\eta)\} \psi^{(\nu)}(\eta) = E^{(\nu)} \psi^{(\nu)}(\eta),
$$

which on proper normalization are given as

$$
P_0 = \sqrt{B_\eta} | \psi^{(\nu)}(\eta(A_i)) |^2 \left(\frac{2}{A}\right),
$$

with $i=1$ or 2 and $\nu=0,1,2,3...$  ($\nu=0$ correspond to the ground state and $\nu=1,2,3...$ for the higher excited states.) Eq. (2) is solved for the ground state ($\nu=0$) and at a fixed $R = R_a = C_i (= C_1 + C_2)$. The $C_i$ are Süssmann central radii $C_i = R_i - (1/R_i)$, with the radii $R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{1/3}/fm$. The fragmentation potential $V_R(\eta)$ in (2) is calculated simply as the sum of the Coulomb interaction, the nuclear proximity potential [30] and the ground state binding energies of two nuclei,

$$
V(R_a, \eta) = -\sum_{i=1}^{2} B(A_i, Z_i) + \frac{Z_1 Z_2 e^2}{R_a} + V_P,
$$

with B.E.'s taken from the 2003 experimental compilation of Audi and Wapstra [31] and from the 1995 calculations of Möller et al. [32] whenever not available in [31]. Thus, full shell effects are contained in our calculations that come from the experimental and/or calculated [32] binding energies.

The charges $Z_1$ and $Z_2$ in (4) are fixed by minimizing the potential in $\eta_Z$ coordinate. The Coulomb and proximity potentials in (4) are for spherical nuclei. The mass parameters $B_\eta(\eta)$, representing the kinetic energy part in (2), are the classical hydrodynamical masses [33].

The WKB tunnelling probability, calculated is $P = P_a P_b$ with

$$
P_i = \exp\left[-\frac{2}{\hbar} \int_{R_a}^{R_i} \{2\mu[V(R) - V(R_a)]\}^{1/2} dR\right]
$$

and

$$
P_b = \exp\left[-\frac{2}{\hbar} \int_{R_b}^{R_0} \{2\mu[V(R) - Q]\}^{1/2} dR\right].
$$

These integrals are solved analytically [25] for $R_0$, the second turning point, defined by $V(R_b) = Q$-value for the ground-state decay. The assault frequency $\nu_0$ in (1) is given simply as

$$
\nu_0 = \left(\frac{2E_2}{\mu}\right)^{1/2}/R_0,
$$

with $E_2 = (A_1/A)Q$, the kinetic energy of the lighter fragment, for the $Q$-value shared between the two products as inverse of their masses.

### 3. Results and Discussion

In the present study, however, as other most probable clusters (isotopes of O, Ne, Mg, Si, S, Ar, Ca, Ti, Cr, Fe and Ni) from heavier neutron-deficient and neutron rich rare-earth parents ($^{118-170}$Ce, $^{118-176}$Nd, $^{122-184}$Sm, $^{132-190}$Gd, $^{132-194}$Dy, $^{138-206}$Er, $^{148-200}$Yb, $^{154-208}$Hf, $^{156-208}$W, $^{160-210}$Os and $^{168-210}$Pt) are also considered, interestingly, $^{12}$C remains to be the most favorable cluster-decay from $^{112}$Ba parent with $^{100}$Sn-daughter, but for $^{132}$Sn-daughter the most favorable cluster is $^{78}$Ni from $^{210}$Pt. This is illustrated in Fig. 2 for both the $^{100}$Sn and $^{132}$Sn daughters, where the most probable clusters emitted from Ba to Pt parents are plotted. The
The decay half-lives $T_{1/2}(s)$ and other characteristic quantities like preformation factors $P_0$, Q-values (in MeV), and penetrabilities $P$ of different Carbon clusters emitted with $^{50}$Sn daughters from various isotope of Ba nuclei, plotted as a function of daughter neutron number $N$.

Figure 2. $\log_{10}T_{1/2}(s)$ for the most probable clusters emitted from various Ba to Pt parents with (a) $^{100}$Sn (b) $^{116}$Sn and (c) $^{132}$Sn daughter, calculated on the basis of the PCM, plotted as a function of cluster proton number $Z_2$. Note the different ordinate-scales used in these figures.

fact that the most probable cluster $^{78}$Ni, arising from Pt parents, occur, at $N_1 = 82$ of the $^{50}$Sn daughter, is illustrated in Fig. 3 for $T_{1/2}$ alone. Interesting enough, in addition to the strong minima at daughter neutrons $N_1 = 82$, a minimum is also shown to be present at $N_1 = 66$ for the $^{50}$Sn daughter, with $^{64}$Ni as the emitted cluster from $^{180}$Pt parent. Thus, a new possibility of $^{116}$Sn-daughter radioactivity is indicated.

Finally, Fig. 4 gives a complete histogram of the decay half-lives $\log_{10}T_{1/2}(s)$ as a function of the neutron number $N_1$ of the emitted $^{50}$Sn-daughters with the most probable clusters (minimum $T_{1/2}$ values) from some 329 parents taken from Ba to Pt with mass numbers $A = 110 – 210$. Note that here $^{50}$Sn-daughter is kept fixed and all possible clusters are considered from different parents (total 1617 combinations with $^{50}$Sn-daughter and probable cluster), and then the one with minimum half-life time is plotted. Apparently, the shortest half-life time $\log_{10}T_{1/2}(s) = 2.27$ (with $Q$-value=22.16 MeV) is obtained for $^{12}$C decay of $^{112}$Ba. The role of magic $N_1 = 82$ is also evident with a minimum in the histogram at $^{132}$Sn-daughter due to the emission of $^{78}$Ni cluster from $^{210}$Pt parent. The predicted half-life $\log_{10}T_{1/2}(s) = 34.974$ (with a $Q$-value=119.292 MeV), which is beyond the limit of present day experiments. Thus, as expected, the strongest shell effects occur at $N_1 = 50$ and 82. In addition, another minimum due to $^{64}$Ni cluster emitted from $^{180}$Pt parents could also be of interest for a closed shell (either spherical and/or deformed) at $N_1 = 66$. This minima are comparable to $N_1 = 82$, with a predicted decay half-life of $\log_{10}T_{1/2}(s) = 34.975$ (with a $Q$-value=124.192 MeV), which by all means is very large for experiments.

In the second section of superheavy elements decay study, Fig.5 contains two $\alpha$-decay chains.
of the Z=117 element, $^{294}_{117}$ and $^{293}_{117}$ are shown for the alpha decay half-lives and Q-value calculations. These two decay chains are calculated for the two different Q-values i.e., $Q^{PCM}$ and $Q^{Exp}$ to see the effect of the Q-value on the alpha decay half-lives. The Q-value of $^{293}_{117}$ parents in PCM is more than two order less than the experimental and the MM values, and hence the correspond change in half-life. This value is calculated by using with B.E.’s taken from the 2003 experimental compilation of Audi and Wapstra [31] and from the 1995 calculations of Møller et al. [32] whenever not available in [31]. The experimental alpha decay half-lives are maximum at $^{274}_{107}$ (Z=107, N=167) and $^{286}_{113}$ (Z=113, N=173) parent nuclei, in $^{294}_{117}$ α-decay chain. While in $^{293}_{117}$ α-decay chain it is maximum at $^{285}_{113}$ (Z=113, N=172) parent nucleus.

Maximum half-lives at these nuclei can be attributed either due to the magicity of protons at Z=114 or neutrons at N=172 or due to both. Interesting, the smaller Q-value means larger half-life, which is followed by one $α$-decay chain $^{293}_{117}$ but not completely followed by the second $^{294}_{117}$ α-decay chain. This discrepancy is noticed at $^{290}_{115}$ and $^{282}_{111}$ parent nuclei. Since both the theoretical model (PCM and MM) follow the same trend. In Fig.5 the PCM model calculations for the alpha decays are compared with the experimental results and the macroscopic-microscopic (MM) model calculations. The PCM model calculations for the alpha decay half-lives shows the same trend as the MM calculations and experimental results. The differences in half lives may be either due to the consideration of alpha decays in these decay chains from ground state of the parent to the ground state of the daughter. Whereas the possibility of alpha-decay from the excited state also exists [34, 35, 36]. Also, the penetration and preformation probabilities are calculated for both the alpha decay chains $^{293,294}_{117}$. The preformation probability factor $P_0$ calculated from eq.(3) is larger for the $^{290}_{115}$ and $^{293}_{117}$ parents in the ground state i.e., $ν=0$. Here, decay constant $λ$ defined in eq. (1) is a combined effect of the $P_0$ the cluster (and daughter) preformation probability, $P$ the barrier penetrability and the $ν_0$ which is the barrier assault frequency.
Figure 5. The $\alpha$-decay half-lives calculated on the basis of PCM and comparision with experimental data[22] and those calculated on the basis of the MM [23] plotted as a function of the parent nucleus mass for the $\alpha$-decay chains of $^{294-293}_{117}$.

4. Summary
As already stated in the introduction, the cluster decays of various isotopes of $^{56}$Ba to $^{78}$Pt parents are calculated for the daughter nucleus to be always an isotope of $^{50}$Sn nucleus. For example, for the neutron-deficient $^{110-132}$Ba and neutron-rich $^{144-150}$Ba parents considered here, different isotopes of Carbon cluster would give rise to various isotopes of $^{50}$Sn daughter. This is illustrated in Fig. 1 for the Q-values of various C-decays, together with the logarithms of penetrability P, preformation factor $P_0$, and the decay half-life $T_{1/2}$ as a function of $N_1$, the neutron number of $^{50}$Sn daughter. The impinging frequency $\nu_0$ is nearly constant $\sim 10^{21}$ (s$^{-1}$). All the four quantities Q, P, $P_0$, and $T_{1/2}$ show the shell effects at magic $N_1=50$ and 82; the Q, P and $P_0$ being large and $T_{1/2}$ small at these numbers. Thus, the most favorable decay is $^{12}$C from $^{112}$Ba nucleus in the $48 \leq N_1 \leq 70$ region, leaving behind $^{100}$Sn as the daughter product, and the $^{14}$C cluster from $^{146}$Ba in the $72 \leq N_1 \leq 86$ region with $^{132}$Sn as the daughter product. The calculated alpha decay half-lives are compared with the experimental results and the other theoretical model (MM) calculations. The PCM calculation results shows the same trend as in experimental data and MM calculations. In the alpha decay study, role of the Q-value is also studied. A small change in the Q-value produce a large variation in the half-life.

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6. References

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