Polarized Deep Inelastic Scattering Off the “Neutron”
From Gauge/String Duality

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Abstract

We investigate deep inelastic scattering off the polarized “neutron” using gauge/string duality. The “neutron” corresponds to a supergravity mode of the neutral dilatino. Through introducing the Pauli interaction term into the action in AdS$_5$ space, we calculate the polarized deep inelastic structure functions of the “neutron” in supergravity approximation at large ’t Hooft coupling $\lambda$ and finite $x$ with $\lambda^{-1/2} \ll x < 1$. In comparison with the charged dilatino “proton,” which has been obtained in the previous work by Gao and Xiao, we find the structure functions of the “neutron” are power suppressed at the same order as the ones of the “proton.” Especially, we find the Burkhardt-Cottingham-like sum rule, which is satisfied in the work by Gao and Xiao, is broken due to the Pauli interaction term. We also illustrate how such a Pauli interaction term can arise naturally from higher dimensional fermion-graviton coupling through the usual Kaluza-Klein reduction.

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I. INTRODUCTION

Gauge/string duality provides us with new insights into gauge theories in a strong coupling regime. There have been substantial progresses in studying strong coupling gauge theories by using such gauge/string duality. A few years ago, Polchinski and Strassler studied the deep inelastic scattering on hadrons by using gauge/string duality, in which the spinless hadron and spin-$\frac{1}{2}$ hadron correspond to supergravity modes of dilaton and dilatino, respectively. The usual structure functions $F_1$ and $F_2$ are calculated for both spinless and spin-$\frac{1}{2}$ hadrons when Bjorken-$x$ is finite ($\lambda^{-1/2} \ll x < 1$) where supergravity approximation is valid. Furthermore, they also investigated the case at small-$x$ where the Pomeron contribution with a trajectory of $2 - O\left(\frac{1}{\sqrt{\lambda}}\right)$ was found. In their work, since an infrared cutoff $\Lambda$ is introduced in order to generate confinement, the model is then called the hard wall model. There are also some earlier studies on high energy scattering in gauge/string duality. There have been a lot of further developments along this direction. A saturation picture based on deep inelastic scattering in AdS/CFT is developed afterwards and recently reviewed in Ref. [18]. In addition, the deep inelastic scattering off the finite temperature plasma in gauge/string duality is studied in Refs. [19–23].

Recently, the above deep inelastic scattering (DIS) calculation in gauge/gravity duality has been extended to the case of polarized DIS off the charged dilatino in Ref. [24] and obtained the spin-dependent structure functions $g_1$ and $g_2$ for a spin-$\frac{1}{2}$ hadron at finite $x$. In Ref. [25], the small $x$ behavior of such spin-dependent structure functions at large coupling limit was analyzed. Furthermore, the nonforward Compton scattering has been also investigated in Refs. [26, 27]. Other recent relevant work can be found in Refs. [28–30].

In Ref. [5] and Ref. [24], both the unpolarized and polarized structure functions when $x$ is finite ($\lambda^{-1/2} \ll x < 1$) are calculated with only minimal interaction. It is found that all the structure functions are power suppressed and vanish in the large $q^2$ limit. So it is worthwhile to investigate how the structure functions look from other possible interactions. Also, since the minimal interaction is proportional to the charge that the dilatino carries, such interaction does not contribute to the structure functions of the neutral dilatino. One of our main objects in this paper is to extend the calculation of the structure functions of the charged spin-$\frac{1}{2}$ hadron —just call it “proton”—in polarized DIS in Ref. [24] to the neutral spin-$\frac{1}{2}$ hadron —just call it “neutron”—through introducing a new interaction —
Pauli interaction term in AdS$_5$ space. Besides, in Ref. [24], the authors have obtained an interesting Burkhardt-Cottingham-like sum rule $\int_0^1 dx g_2(x, q^2) = 0$, which is completely independent of $\tau$ and $q^2$, hence another main object of our paper is to investigate whether such sum rule still holds from other possible interaction terms such as Pauli interaction here. Since there has been extensive study [31–43] of the elastic form factors, we will not take this subject into account in our present work. For simplicity and also consistency with the previous work, we still work in the hard wall model.

This paper is organized as follows. In Sec.II we recall the definitions for various structure functions as well as kinematic variables. In Sec.III we calculate the structure functions of the “neutron” at finite $x$ from the Pauli interaction term. Section IV is devoted to the discussions and comments on these structure functions and compare them with the structure functions of the charged dilatino which have been calculated in the previous work where only minimal interaction was considered. We summarize our results in Sec.V. Finally, in the appendix we will illustrate how a Pauli interaction term in 5D can arise naturally from 6D fermion-graviton coupling through the usual Kaluza-Klein reduction.

II. POLARIZED DEEP INELASTIC SCATTERING

![Diagram](image)

FIG. 1: The lepton interacts with the hadron target through the exchange of a virtual photon; the hadron absorbs the virtual photon and fragments into the final state $X$.

Deep inelastic scattering has played an important role in the history of investigating the internal structure of hadrons. It is the study of lepton-hadron scattering in the limit that $x$ is fixed, and $q^2 \to \infty$. The basic diagram for such process is illustrated schematically in
The structure of the hadron can be completely characterized by the hadronic tensor $W_{\mu\nu}$, which is defined as

$$W_{\mu\nu} = \int d^4\xi \, e^{i\mathbf{q}\cdot\mathbf{\xi}} \langle P, S | [J_\mu(\xi), J_\nu(0)] | P, S \rangle,$$

with $J_\mu$ being the incident current. In our present work, we will specify the hadron as the spin-\(\frac{1}{2}\) hadron. The hadronic tensor $W_{\mu\nu}$ can be split as

$$W_{\mu\nu} = W_{\mu\nu}^{(S)}(q, P) + i W_{\mu\nu}^{(A)}(q, P, S).$$

According to Lorentz and $CP$ invariance, the symmetrical and antisymmetrical parts can be expressed in terms of 8 independent structure functions as\cite{44, 45}

$$W_{\mu\nu}^{(S)} = \left( \eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \left[ F_1(x, q^2) + \frac{MS \cdot q}{2P \cdot q} g_5(x, q^2) \right]$$

- \(\frac{1}{P \cdot q} \left( P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left( P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) \left[ F_2(x, q^2) + \frac{MS \cdot q}{P \cdot q} g_4(x, q^2) \right]$$

- \(\frac{M}{2P \cdot q} \left( P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left( S_\nu - \frac{S \cdot q}{P \cdot q} P_\nu \right) + \left( P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) \left( S_\mu - \frac{S \cdot q}{P \cdot q} P_\mu \right) \right] g_3(x, q^2),$$

$$W_{\mu\nu}^{(A)} = -\frac{M}{P \cdot q} \frac{q^\rho}{P \cdot q} \left\{ S^\sigma g_1(x, q^2) + \left[ S^\sigma - \frac{S \cdot q}{P \cdot q} P^\sigma \right] g_2(x, q^2) \right\} - \frac{\epsilon_{\mu\nu\rho\sigma} q^\rho P^\sigma}{2P \cdot q} F_3(x, q^2),$$

where $M$ is the mass of the hadron, $S$ is its polarization vector, $q$ is the momentum carried by the current and $P$ is the initial momentum of the hadron (See Fig. I). In deep inelastic scattering, we define the kinematic variables as the following

$$x = -\frac{q^2}{2P \cdot q} \quad \text{and} \quad P_X^2 = (P + q)^2.$$

The mass of the intermediate state after the scattering is defined as $M_X^2 = s = -P_X^2$. All the structure functions are only functions of $x$ and $q^2$.

We will use the most plus metric throughout this paper instead of the usual most minus metric in particle physics, so there are some sign changes in our definitions comparing to the usual definitions in\cite{44, 45}.

### III. POLARIZED STRUCTURE FUNCTIONS IN THE HARD WALL MODEL

According to the conjecture of AdS/CFT, at large 't Hooft parameter, the gauge theories have a dual string description, which can make accurate analytic calculations possible. For
the 3 + 1 dimensional conformal gauge theories, the dual string theory lives in the space $\text{AdS}_5 \times W$. The metric in $\text{AdS}_5 \times W$ space can be written as
\[ ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dy^\mu dy^\nu + dz^2) + R^2 ds_W^2. \]
where $y^\mu$ are identified with the space-time coordinates in the gauge theory and $W$ denotes a five-dimensional compact space. We will specify such compact space as $S^5$ in our work. The conformal invariance can be broken through introducing a sharp cut-off $0 \leq z \leq z_0 \equiv 1/\Lambda$, leading to the mass gap of hadrons. This simple model is the so-called hard wall model.

Following the formalism proposed by Polchinski and Strassler in Ref. [5], the incident current is chosen to be the $\mathcal{R}$-current which couples to the hadron as an isometry of $S^5$. According to the AdS/CFT correspondence, the current excites a nonnormalizable mode of a Kaluza-Klein gauge field at the Minkowski boundary of the AdS$_5$ space
\[ \delta G_{ma} = A_m(y, z) v_a(\Omega), \]
where $v_a(\Omega)$ denotes a Killing vector on $S^5$ with $\Omega$ being the angular coordinates on $S^5$. $A_m(y, z)$ is the external potential in the gauge theory corresponding to the operator insertion $n_\mu J^\mu(q)$ on the boundary of the fifth dimension of the AdS$_5$ space with the boundary condition
\[ A_\mu(y, 0) = A_\mu(y)|_{4d} = n_\mu e^{iq\cdot y}. \]
This gauge field fluctuation $A_m(y, r)$ can be viewed as a vector boson field which couples to the $\mathcal{R}$-current $J^\mu$ on the Minkowski boundary, and then propagates into the bulk as a gravitational wave, and eventually interacts with the supergravity modes of the dilatino or dilaton. The gauge field satisfies Maxwell’s equation in the bulk, $D_m F^{mn} = 0$, which can be explicitly written as
\[ \frac{1}{\sqrt{-g}} \partial_m \left[ \sqrt{-g} g^{nk} g^{ml} (\partial_k A_l - \partial_l A_k) \right] = 0, \]
where $m, n, ...$ are indices on AdS$_5$. In the Lorentz-like gauge
\[ \partial_\mu A^\mu + z \partial_z \left( \frac{A_z}{z} \right) = 0, \]
the Maxwell equation can be written as
\[ -q^2 A_\mu + z \partial_z \left( \frac{1}{z} \partial_z A_\mu \right) = 0, \]
\[ -q^2 A_z + \partial_z \left( z \partial_z \left( \frac{1}{z} A_z \right) \right) = 0. \]
The solutions to the above equations with the proper boundary conditions \( F_{z\mu}(y, z_0) = 0 \) are given by \(^1\)

\[
A_\mu = n_\mu e^{iq_y qz} [K_1(qz) + cI_1(qz)], \\
A_z = i n \cdot q e^{iq_y z} [K_0(qz) - cI_0(qz)],
\]

(11)

where

\[
c = K_0(qz_0) / I_0(qz_0).
\]

(12)

In the large \( q^2 \) regime, we can just simply identify \( c \) as 0. However, in the small \( q^2 \) regime, such a term contributes as much as the others. This is just the reason the form factor gives rise to logarithmic divergent charge radii for the charged dilatino in the work \(^{[24]}\) by Gao and Xiao.

Spin-\( \frac{1}{2} \) hadrons corresponds to supergravity modes of the dilatino. In the conformal region the dilatino field can be written as

\[
\lambda = \Psi(y, z) \otimes \eta(\Omega),
\]

(13)

where \( \Psi(y, z) \) is an \( SO(4, 1) \) spinor on AdS\(_5\) and \( \eta(\Omega) \) is an \( SO(5) \) spinor on \( S^5 \). The wavefunction \( \Psi \) satisfies a five-dimensional Dirac equation in AdS\(_5\) space. Let us first review how to derive this five-dimensional Dirac equation in the following.

A convenient choice of vielbein is given by

\[
\epsilon^a_m = \frac{R}{z} \delta^a_m, \quad \varepsilon^m_a = \frac{z}{R} \eta^{ma}, \quad \varepsilon^m_a = \frac{z}{R} \delta^m_a
\]

(14)

where \( m = 0, 1, 2, 3, 5 \). The Levi-Civita connection is given by

\[
\Gamma^p_{mn} = \frac{1}{2} g^{pq}(\partial_n g_{mq} + \partial_m g_{nq} - \partial_q g_{mn})
\]

(15)

Here we use \( a, b, c \) to denote indices in flat space, and \( m, n, p, q \) to denote indices in curved space (AdS\(_5\) space). In addition, the Greek indices \( \mu, \nu \) are defined in Minkowski space. From the metric, one knows

\[
g_{mn} = \frac{R^2}{z^4} \eta_{mn}.
\]

(16)

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\(^1\) If we choose another alternative gauge invariant boundary conditions \( F_{\mu\nu}(y, z_0) = 0 \), we will see it leads to unreasonable constraint \( n^\mu \propto q^\mu \) on the boundary condition (7).
It is straightforward to work out the Levi-Civita connection in AdS\(_5\) space

\[
\Gamma^5_{\mu\nu} = \frac{1}{z} \eta_{\mu\nu}, \quad \Gamma^5_{55} = -\frac{1}{z}, \quad \Gamma^\mu_{\nu5} = -\frac{1}{z} \delta^\mu_\nu .
\]  

(17)

From vielbein and Levi-Civita connection, we can have the spin connection

\[
\omega_{a b}^m = e^a_n \partial_m e^{nb} + e^a_n e^{pb} \Gamma^n_{pm}.
\]

(18)

The only nonvanishing spin connections are

\[
\omega^5_{\nu\mu} = -\omega^5_{\nu\mu} = \frac{1}{z} \delta^\nu_\mu .
\]

(19)

Using above results, the operator \(\slashed{D}\) can be cast into

\[
\slashed{D} = g^{mn} e^a_n \gamma^a \left( \partial_m + \frac{1}{2} \omega_{bc} \Sigma_{bc} \right) = \frac{z}{R} \left( \gamma^5 \partial_z + \gamma^\mu \partial_\mu - \frac{2}{z} \gamma^5 \right) ,
\]

(20)

with \(\Sigma_{\mu5} = \frac{1}{4} [\gamma_\mu, \gamma_5]\). The free dilatino field in AdS\(_5\) space satisfies the Dirac equation

\[
(\slashed{D} - m) \Psi = \frac{z}{R} \left( \gamma^5 \partial_z + \gamma^\mu \partial_\mu - \frac{2}{z} \gamma^5 - \frac{mR}{z} \right) \Psi = 0.
\]

(21)

Its normalizable solution is given by \(\Psi(z,y)\)

\[
\Psi(z,y) = C e^{ip \cdot y \frac{2}{z} \gamma^5} \left[ J_{mR-1/2}(Mz) P_+ + J_{mR+1/2}(Mz) P_- \right] u_\sigma \]

(22)

where

\[
\gamma^\mu \partial_\mu = -iMu_\sigma(\sigma = 1, 2), \quad M^2 = -p^2, \quad P_\pm = \frac{1}{2} (1 \pm \gamma^5) \]

(23)

For the initial hadron, by assuming \(Mz \ll 1\) in the interaction region and expanding the Bessel functions up to linear term in \(M\), one gets

\[
\psi_1 \approx e^{iP \cdot y \frac{2}{z} \gamma^5} \left[ \frac{1}{R^{9/2} z_0^{3/2}} \right] \left( \frac{z}{z_0} \right)^{mR+2} \left[ P_+ u_\sigma + \frac{Mz}{2(mR + 1/2)} P_- u_\sigma \right] .
\]

(24)

\[
\bar{\psi}_1 \approx e^{-iP \cdot y \frac{2}{z} \gamma^5} \left[ \frac{1}{R^{9/2} z_0^{3/2}} \right] \left( \frac{z}{z_0} \right)^{mR+2} \left[ \bar{u}_\sigma P_- + \frac{Mz}{2(mR + 1/2)} \bar{u}_\sigma P_+ \right] .
\]

(25)

For the intermediate hadron, \(M_X \gg 1/z_0\) and

\[
\psi_X \approx e^{i(P+q) \cdot y \frac{2}{z} \gamma^5} \left[ \frac{1}{R^{9/2} z_0^{3/2}} \right] \left( \frac{z}{z_0} \right)^{mR+2} \left[ J_{mR-1/2}(M_X z) P_+ + J_{mR+1/2}(M_X z) P_- \right] u_{X\sigma} .
\]

(26)

\[
\bar{\psi}_X \approx e^{-i(P+q) \cdot y \frac{2}{z} \gamma^5} \left[ \frac{1}{R^{9/2} z_0^{3/2}} \right] \left( \frac{z}{z_0} \right)^{mR+2} \left[ \bar{u}_{X\sigma} P_- J_{mR-1/2}(M_X z) + P_+ J_{mR+1/2}(M_X z) \right] .
\]

(27)
In Ref. [5] and Ref. [24], the interaction between the Kaluza-Klein gauge field and charged dilatino is given by the minimal coupling

\[ S_{int}^M = iR^5 \int d^5x \sqrt{-g} \bar{Q}A_m e^m_a \gamma^a \Psi, \]  

(28)

However, since we are now interested in the effect of the other possible interactions and the neutral dilatino in such \( \mathcal{R} \) current, where the above minimal interaction will not contribute, we need introduce a new interaction term —the Pauli interaction term such that

\[ S_{int}^P = \kappa R^6 \int d^5x \sqrt{-g} F_{mn} e^m_a e^n_b \gamma^a \gamma^b \Psi. \]  

(29)

Actually such term can be derived very naturally from Kaluza-Klein reduction of higher dimensional fermion-graviton coupling. We put such derivation into the final appendix in our present work.

With the Pauli interaction action at hand, following the same line in Ref. [24], we can compute the matrix element

\[ \mathcal{M}^\mu = \langle P_X, \sigma' | J^\mu(0) | P, \sigma \rangle \]

\[ = \frac{1}{2\pi} (M_X/z_0)^{1/2} \left\{ c_1 q \bar{u}_{f\sigma'} [\not{q}, \gamma^\mu] P_- u_{i\sigma} + c_2 q \bar{u}_{f\sigma'} [\not{q}, \gamma^\mu] P_+ u_{i\sigma} \right. \]

\[ + c_3 u_{f\sigma'} (q^\mu \not{q} - q^2 \gamma^\mu) P_- u_{i\sigma} + c_4 u_{f\sigma'} (q^\mu \not{q} - q^2 \gamma^\mu) P_+ u_{i\sigma} \right\}, \]  

(30)

and its complex conjugate

\[ \mathcal{M}^{*\mu} = \langle P, \sigma | J^{\mu}(0) | P_X, \sigma' \rangle \]

\[ = \frac{1}{2\pi} (M_X/z_0)^{1/2} \left\{ -c_1 q \bar{u}_{i\sigma} [\not{q}, \gamma^\mu] P_+ u_{f\sigma'} - c_2 q \bar{u}_{i\sigma} [\not{q}, \gamma^\mu] P_- u_{f\sigma'} \right. \]

\[ + c_3 \bar{u}_{i\sigma} (q^\mu \not{q} - q^2 \gamma^\mu) P_- u_{f\sigma'} + c_4 \bar{u}_{i\sigma} (q^\mu \not{q} - q^2 \gamma^\mu) P_+ u_{f\sigma'} \right\}. \]  

(31)

The coefficients \( c_i (i = 1, 2, 3, 4) \) are given by
\[ c_1 = \frac{c_0 M}{2(\tau - 1)} \int dzz^{\tau+2} K_1(qz)J_{\tau-2}(M_X z) \]
\[ = c_0 2^\tau M_X^{-2} (M_X^2 + q^2)^{-\tau-2} Mq \tau \left[ q^2(\tau - 1) - 2M_X^2 \right] \Gamma(\tau - 1), \quad (32) \]
\[ c_2 = c_0 \int dzz^{\tau+1} K_1(qz)J_{\tau-1}(M_X z) \]
\[ = c_0 2^\tau M_X^{-1} (M_X^2 + q^2)^{-\tau-1} q \Gamma(\tau + 1), \quad (33) \]
\[ c_3 = \frac{c_0 M}{2(\tau - 1)} \int dzz^{\tau+2} K_0(qz)J_{\tau-1}(M_X z) \]
\[ = c_0 2^\tau M_X^{-1} (M_X^2 + q^2)^{-\tau-2} M \tau \left[ q^2 - M_X^2 \right] \Gamma(\tau - 1), \quad (34) \]
\[ c_4 = c_0 \int dzz^{\tau+1} K_0(qz)J_{\tau-2}(M_X z) \]
\[ = c_0 2^\tau M_X^{-2} (M_X^2 + q^2)^{-\tau-1} \left[ q^2(\tau - 1) - M_X^2 \right] \Gamma(\tau), \quad (35) \]

where \( \tau \equiv mR + \frac{3}{2} \) and \( c_0 = 2\pi c'_e x z_0^{-\tau+1} \). In the above calculation, we have relaxed the upper limit of integration from \( z_0 = 1/\Lambda \) to \( \infty \). Now let us continue to calculate the hadronic tensor

\[ W_{\mu\nu} = W_{\mu\nu}^{(S)} + iW_{\mu\nu}^{(A)} = (2\pi)^3 \sum_X \delta \left( M_X^2 + (P + q)^2 \right) \mathcal{M}_\mu^X \mathcal{M}^\nu_\eta \quad (36) \]

In large \( q^2 \) limit, we can make the approximation

\[ \sum_X \delta \left( M_X^2 + (p + q)^2 \right) \simeq \frac{1}{2\pi M_X \Lambda}. \quad (37) \]

Together with \( M_X^2 + q^2 = q^2/x \) and \( M_X = q\sqrt{(1-x)/x} \), we can write the symmetric \( W_{\mu\nu}^{(S)} \) and antisymmetric \( W_{\mu\nu}^{(A)} \) as, respectively,

\[ W_{\mu\nu}^{(S)} = \frac{4q^6}{x} \left[ 2c_2 \left( 1 - \frac{x}{x} \right)^{1/2} + c_4 \right] \left[ 1 + \frac{2M(S \cdot q)}{2P \cdot q} \right] \left( \eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \]
\[ - \frac{8q^6}{x} (4c_2^2 + c_4^2) \left[ 1 + \frac{M(S \cdot q)}{P \cdot q} \right] \frac{1}{P \cdot q} \left( P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left( P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) \]
\[ - \frac{8q^6}{x} \left( 4c_2^2 + c_4^2 \right) \left[ (c_2c_3 - c_1c_4) \frac{2q}{xM} \right] \frac{M}{2P \cdot q} \]
\[ \times \left[ \left( P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left( S_\nu - \frac{S \cdot q}{P \cdot q} P_\nu \right) + \left( P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) \left( S_\mu - \frac{S \cdot q}{P \cdot q} P_\mu \right) \right] \quad (38) \]
\[ W_{\mu\nu}^{(A)} = -\frac{8q^6}{x} \left[ 2c_2 \left( \frac{1-x}{x} \right)^{1/2} + c_4 \right]^2 \frac{\epsilon_{\mu\nu\alpha\beta} q^\alpha P^\beta}{2P \cdot q} - \frac{4q^6}{x} \left[ 2c_2 \left( \frac{1-x}{x} \right)^{1/2} + c_4 \right]^2 \frac{M \epsilon_{\mu\nu\alpha\beta} q^\alpha S^\beta}{P \cdot q} \]
\[ + \frac{2q^6}{x^2} \left[ (4c_2^2 + c_3^2) + 2(4c_1c_2 - c_3c_4) \frac{q}{M} \left( \frac{1-x}{x} \right)^{1/2} + 2(c_1c_4 + c_2c_3) \frac{q}{M} \frac{2x-1}{x} \right] \times \frac{M \epsilon_{\mu\nu\alpha\beta} q^\alpha}{P \cdot q} \left( S^\beta - \frac{S \cdot q}{P \cdot q} P^\beta \right) \]

(39)

To obtain the above final results, we have used the identity

\[ \epsilon_{\mu\nu\alpha\beta} q_\alpha [(q \cdot S) P_\beta - (P \cdot q) S_\beta] = q^\mu \epsilon^{\nu\alpha\beta\gamma} P_\alpha q_\beta S_\gamma - q^\nu \epsilon^{\mu\alpha\beta\gamma} P_\alpha q_\beta S_\gamma - q^2 \epsilon^{\mu\nu\alpha\beta} P_\alpha S_\beta \]

(40)

Comparing with Eq. (3), we can obtain all the structure functions of the “neutron,”

\[ F_1^n = g_1^n = \frac{F_3^n}{2} = \frac{g_5^n}{2} = 16\pi\kappa^2 A' \left( \frac{\Lambda^2}{q^2} \right)^{\tau-1} x^{\tau+1} (1-x)^{\tau-2} \left[ 1 - \tau(2-x) \right]^2, \]

(41)

\[ F_2^n = g_4^n = 32\pi\kappa^2 A' \left( \frac{\Lambda^2}{q^2} \right)^{\tau-1} x^{\tau+1} (1-x)^{\tau-2} \left[ 1 - \tau x(2-4\tau + 3\tau x) \right], \]

(42)

\[ g_2^n = -8\pi\kappa^2 A' \left( \frac{\Lambda^2}{q^2} \right)^{\tau-1} x^{\tau}(1-x)^{\tau-2} \frac{1}{\tau - 1} \]
\[ \times \left[ \tau(2\tau - 5) + 2\tau x(\tau^2 - 10\tau + 8) + \tau x^2(7\tau^2 + 17\tau - 6) - 6\tau^2 x^3(\tau + 1) - 1 \right], \]

(43)

\[ g_3^n = 32\pi\kappa^2 A' \left( \frac{\Lambda^2}{q^2} \right)^{\tau-1} x^{\tau+1}(1-x)^{\tau-2} \frac{1}{\tau - 1} \]
\[ \times \left[ \tau(4x - 3) - \tau^2(x^2 + 7\tau x^2 - 4x - 6\tau x + 2) - 1 \right]. \]

(44)

It is straightforward to compute the moments of all the structure functions when the contributions from \( x \ll \lambda^{-1/2} \) are negligible. Typically there are four different kinds of moments,
e.g.,

\[
\int_0^1 g_1^p(x, q^2) x^{n-1} dx = 16\pi \kappa^2 A'(\Lambda^2/q^2)^{\tau-1} \left[n^2(\tau - 1) + n(\tau - 1)(6\tau + 1)
+ \tau(9\tau^2 - 4\tau - 4) \right] \frac{\Gamma(\tau)\Gamma(n + \tau + 1)}{\Gamma(2\tau + n + 2)} \tag{45}
\]

\[
\int_0^1 g_2^p(x, q^2) x^{n-1} dx = -8\pi \kappa^2 A'(\Lambda^2/q^2)^{\tau-1} \left[n^3(3\tau^2 - 4\tau + 1) + 2n^2\tau(10\tau^2 - 10\tau + 1)
+ n(33\tau^4 - 31\tau^3 - 9\tau^2 - 1) + \tau(16\tau^4 - 13\tau^3 - 13\tau^2 - 6\tau - 2) \right]
\times \frac{\Gamma(\tau)\Gamma(n + \tau + 1)}{(\tau - 1)\Gamma(2\tau + n + 2)} \tag{46}
\]

\[
\int_0^1 g_3^p(x, q^2) x^{n-1} dx = 32\pi \kappa^2 A'(\Lambda^2/q^2)^{\tau-1} \left[n^2(\tau^2 - 1) + n(3\tau^3 + 3\tau^2 - 1) + 2\tau(\tau + 1)^2 \right]
\times \frac{\Gamma(\tau)\Gamma(n + \tau + 1)}{(\tau - 1)\Gamma(2\tau + n + 2)} \tag{47}
\]

\[
\int_0^1 F_2^p(x, q^2) x^{n-1} dx = 32\pi \kappa^2 A'(\Lambda^2/q^2)^{\tau-1} \left[n^2(\tau^2 - 1) + n(6\tau^2 - \tau - 1) + 5\tau^2 \right]
\times \frac{\Gamma(\tau)\Gamma(n + \tau + 1)}{\Gamma(2\tau + n + 2)} \tag{48}
\]

In order to discuss these results further and make comparisons with the previous work by Gao and Xiao in Ref. [24], it is helpful to rewrite the structure functions of the “proton” that have been obtained in Ref. [24],

\[
F_1^p = \frac{F_2^p}{2} = \frac{F_3^p}{2} = g_1^p = \frac{g_3^p}{2} = \frac{g_4^p}{2} = \frac{g_5^p}{2} = \frac{\pi}{2} A'\mathcal{Q}^2(\Lambda^2/q^2)^{\tau-1} x^{\tau+1} (1 - x)^{\tau-2}, \tag{49}
\]

\[
g_2^p = \left( \frac{1}{2\tau - 1} - \frac{x\tau}{\tau - 1} \right) \frac{\pi}{2} A'\mathcal{Q}^2(\Lambda^2/q^2)^{\tau-1} x^{\tau} (1 - x)^{\tau-2}. \tag{50}
\]

Typically there are just two different kinds of moments, e.g.,

\[
\int_0^1 2g_1^p(x, q^2) x^{n-1} dx = \pi A'\mathcal{Q}^2(\Lambda^2/q^2)^{\tau-1} \frac{\Gamma(\tau - 1) \Gamma(n + \tau + 1)}{\Gamma(2\tau + n)} \tag{51}
\]

\[
\int_0^1 2g_2^p(x, q^2) x^{n-1} dx = \pi A'\mathcal{Q}^2(\Lambda^2/q^2)^{\tau-1} \frac{\Gamma(\tau - 1) \Gamma(n + \tau + 1)}{\Gamma(2\tau + n)} \frac{1 - n}{2}. \tag{52}
\]

In all the above expressions, we have defined \( A' = \pi c_i^2 e_X^2 2^{2\tau} \mathcal{Q}^2(\tau) \).

### IV. DISCUSSIONS

In this section, we focus on the interpretation of the structure functions of the “neutron” from Pauli interaction that we obtained from last section using gauge/string duality
and compare these results with the ones of the “proton” in Ref. [24], where only minimal interaction was included.

- Just like what we did in Ref. [24], only the linear term in $M$ is kept in the initial wavefunction and throughout the calculation, the results shown above, whether the “neutron” or the “proton,” are from leading order calculations. The corrections are of order $\frac{M^2}{q^2}$ or $\frac{\Lambda^2}{q^2}$.

- The power order $\Lambda/q$ of the structure functions of the “neutron” from only the Pauli interaction are the same as that from only minimal interaction. Ignoring the relative magnitude of $\kappa$ and $Q$, but only from the naive dimensional analysis, one might expect that the Pauli interaction will lead to less power order $\Lambda/q$ by one than that from the minimal interaction. However, when the warp factor $e^m_\alpha$ is taken into account, the extra $q$ will combine with $z$ from $e^m_\alpha$ and lead to $q z_{int} \sim 1$, which results in the same power suppression as the minimal interaction.

- The relations $F_1^n = \frac{F_1^p}{2} = \frac{g_1^n}{2}$ and $F_2^n = g_2^n$ still hold from the Pauli interaction, but the relation $F_1^p = \frac{F_2^p}{2} = \frac{g_2^p}{2}$ from the minimal interaction is broken. The differences of the
dependence on $x$ of all the structure functions are illustrated in Figs. 2–5, where the coefficient $C_p = \frac{1}{2} \pi A' Q^2 (\Lambda^2/q^2)^{\tau - 1}$ and $C_n = 200 \pi A' \kappa^2 (\Lambda^2/q^2)^{\tau - 1}$.

- In QCD, there is an interesting inequality $F_1 \geq g_1$ [47]. In Ref. [24], we have found that $F_1 = g_1$, i.e. the bound is saturated. Here we see that the saturation condition $F_1 = g_1$ still holds, which indicates that initial hadron is completely polarized. This implies that the struck dilatino just tunnels or shrinks to smaller size of order the inverse momentum transfer during the scattering. As a result, the structure function exhibits a power law behavior in terms of the $q^2$ dependence which comes from the tunneling probability [4, 5].

- For all the moments Eq. (45)–Eq. (48), Eq. (51) and Eq. (46), we expect that the moments are correct at least for $n > 2$ where the low-$x$ contributions are negligible. When one sets $n = 1$ for $g_2^p$, there is an interesting sum rule

$$\int_0^1 dx g_2^p (x, q^2) = 0,$$

which is completely independent of $\tau$ and $q^2$. In QCD, this sum rule is known as the Burkhardt-Cottingham sum rule [48] in large $Q^2$ limit. However, this sum rule can be

FIG. 3: Comparison between the structure functions of $g_2^p$ and $g_2^n$. 
FIG. 4: Comparison between the structure functions of $g_3^{p}$ and $g_3^{n}$.

invalidated by non-Regge divergence at low-$x$. Now let us set $n = 1$ for $g_2^{n}$, we can have

$$\int_0^1 dx g_2^n (x, q^2) = -32\pi \kappa^2 A' (\Lambda^2 / q^2)^{\tau - 1} (4\tau^3 + \tau^2 - 7\tau - 1) \frac{\Gamma(\tau + 1)\Gamma(\tau + 2)}{(\tau - 1)\Gamma(2\tau + 3)}. \quad (54)$$

It is obvious that such sum rule which holds for minimal interaction in the classic supergravity approximation is broken due to introducing the Pauli interaction term. In this place, it is a good opportunity to compare the above conclusion with QED but with an extra nonrenormalizable Pauli interaction term introduced, i.e.

$$S^{QED}_{int} = \int d^4 y \left( i Q \bar{\psi} A \psi + \kappa F_{\mu\nu} \bar{\psi} [\gamma^\mu, \gamma^\nu] \psi \right). \quad (55)$$

where we just specify $\psi$ as a quark field $^2$. It is easy to verify that, in the tree diagram level of $Q$ or $\kappa$ and twist-3 level of $m/q$ ($m$ denotes the mass of the quark), the pure minimal interaction results in

$$F_1(x, q^2) = 2F_2(x, q^2) = a_m^F Q^2 \delta(x - 1), \quad (56)$$

$$g_1(x, q^2) = a_m^g Q^2 \delta(x - 1), \quad g_2(x, q^2) = 0. \quad (57)$$

$^2$ In the realistic QED, from the viewpoint of effective field theory, the Pauli interaction contribution is suppressed by $q/M_P$, where $q$ is the energy scale in which we are working and $M_P$ is Planck energy scale. Hence such contribution is highly suppressed when $q$ is in the scale of GeV or TeV. Since we just want to show the pure effect of such Pauli interaction term, we will neglect such realistic issues.
while the pure Pauli interaction results in

\[ F_1(x, q^2) = a_P^F \kappa^2 \delta(x - 1), \quad F_2(x, q^2) = 0, \]  
\[ g_1(x, q^2) = 0, \quad g_2(x, q^2) = -a_P^g \kappa^2 \delta(x - 1), \]

where \( a_m^F, a_m^g, a_P^F, \) and \( a_P^g \) are all positive coefficients, which is irrelevant with our current problem. Hence, it is obvious that, similar to the AdS\(_5\) space, the Pauli interaction term always makes the Burkhardt-Cottingham sum rule invalidated. It should be clarified that the effective Pauli interaction can be produced from high order contributions in usual QED or instantons \([49]\) in usual QCD without the Pauli interaction term in the original Lagrangian. Actually, from the spirit of the conjecture of AdS/CFT, the Pauli interaction introduced in our present work in AdS space is equivalent to summing over all the loop contributions in CFT side.

- It is obvious that the moments of all the structure functions are power suppressed, for sufficiently large \( q^2 \to \infty \), all these integrals vanishes. Actually in all our calculations, the results are only valid at large \( \Lambda' \) Hooft coupling \( \lambda \) and finite \( x \) with \( \lambda^{-1/2} \ll x < 1 \). In order to take into account the moments of the structure functions completely, we need to consider the very small \( x \) case. For example, the missing contributions from
the Pomeron exchanges to \( F_1 \) and \( F_2 \) peaks around \( x = 0 \).

\[
xF_1 \sim F_2 \propto x^{-1+O(1/\sqrt{\lambda})}
\]  

(60)

where the correction to the Pomeron intercept arises from the curvature of AdS\(_5\). Such Pomeron contribution will survive in the large \( q^2 \) limit and give us a nonvanishing second moment of \( F_1 \) \cite{5,17}, which make energy momentum conserved. There is a similar contribution to \( g_1 \) at small-\( x \) which yields a singular \cite{25}

\[
g_1 \sim \frac{1}{x^{\alpha_{R1}}}.
\]

(61)

with \( \alpha_{R1} = 1 - O(\frac{1}{\sqrt{\lambda}}) \) when \( x \) is extremely small. This contribution will also survive in large \( q^2 \) limit and yield a finite first moment. This may indicate that most of the hadron spin is carried by the small-\( x \) constituents inside the hadron. The detailed discussions on the small-\( x \) limit of the \( g_1 \) structure function can be available in Ref. \cite{25}.

• Just repeat the arguments in Ref. \cite{24} on the parity violating structure functions \( F_3, g_3, g_4, \) and \( g_5 \). These parity violating structure functions are as large as the \( F_2 \) structure function due to the reason that the dilatino is right-handed fermion in massless limit. They are tightly related to the peculiar wavefunction of the dilatino. However, we expect that \( g_1 \) and \( g_2 \) may exhibit some common features of the polarized structure functions of spin-\( \frac{1}{2} \) hadrons in the nonperturbative region when the coupling is large.

• Phenomenologically, we can just regard \( \kappa \) as a free parameter, which can be fixed by the experimental values of the “neutron” magnetic moments. The detailed discussion and fitting results can be found in Ref. \cite{43}.

V. CONCLUSION

Through introducing the Pauli interaction term in the action in the AdS\(_5\) space, using gauge/string duality, we have calculated the structure functions of the “neutron” which is dual to a spin-\( \frac{1}{2} \) dilatino which is neutral corresponding to the \( U(1) \) current we are considering. We obtain both the unpolarized and polarized structure functions. We find the structure functions of the “neutron” purely from Pauli interactions are power suppressed at the same order as the ones of the “proton” purely from minimal interactions. We also find
that the Burkhardt-Cottingham sum rule for $g_2$ which is satisfied independent of $\tau$ and $q^2$ in the minimal interaction is broken due to such a Pauli interaction term.

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**Appendix A: Pauli term from Kaluza-Klein reduction**

In this appendix, we will illustrate how a Pauli interaction term can be produced from Kaluza-Klein reduction of higher dimensional fermion-graviton coupling. Since it is only an illustration, for simplicity, let us just consider the reduction from 6D to 5D. We start with the simple example of a left chiral fermion field in 6D with the sixth dimension $\xi$ compactified. The action reads

$$S = \int d^5x d\xi \sqrt{-G} \left\{ \alpha \Psi_L E^{M,A} \Gamma_A \left( \partial_M + \frac{1}{2} \Omega_{BC}^M \Sigma_{BC} \right) \Psi_L + \text{H.C.} \right\} \quad (A1)$$

where we have suppressed the pure graviton self-interaction and

$$\Psi_L = \frac{1}{2} (1 + \Gamma_7) \Psi, \quad G_{MN} = \eta_{AB} E^A_M E^B_N, \quad \Sigma_{AB} = \frac{1}{4} (\Gamma_A \Gamma_B - \Gamma_B \Gamma_A). \quad (A2)$$

It should be noted that the coefficient $\alpha$ can be generally complex. In the following, we use the capital $G, E, \Omega$ in the six-dimension space and the lower $g, e, \omega$ in the five-dimensional space. We will use indices $M, N, ...$ to denote all six general curved spacetime dimensions, and $A, B, ...$ refer to all six local inertial spacetime dimensions, while $m, n, ...$ denote five dimensions in AdS$_5$ and $a, b, ...$ refer to five dimensions in the local flat five-dimensional spacetime.

Following the formalism of Kaluza-Klein, we write the vielbein field as

$$E^A_N = \begin{pmatrix} e^a_n & A_n \\ 0 & 1 \end{pmatrix}, \quad E^N_A = \begin{pmatrix} e^n_m - A_a \\ 0 & 1 \end{pmatrix}, \quad E^{NA} = \begin{pmatrix} e^{na} \quad 0 \\ -A^a & 1 \end{pmatrix}, \quad (A3)$$
where $A_b = e^a_b A_a$, $A^a = \eta^{ab} A_b$ and we have set the scalar field simply as 1 and neglected the dimension of the field. All the components in Eq. (A3) do not depend on the sixth dimension coordinate $\xi$.

We make the split of $\Gamma$-matrices \[51\] as

\[\Gamma_A = (\gamma_a \otimes \sigma_3, 1 \otimes \sigma_i) , \ a = 0, 1, 2, 3, 5 \text{ and } i = 1, 2\] (A4)

where $\gamma_a$ and $\sigma_i(i = 1, 2, 3)$ are the D=5 $\gamma$-matrices and usual Pauli matrices, respectively. We also decompose the spinor field as

\[\Psi(x, \xi) = \psi(x) \otimes \chi(\xi)\] (A5)

where $\psi(x)$ is the spinor in D=5, and $\chi(\xi)$ is the spinor in D=2.

Using the identity

\[\Gamma_A \Sigma_{BC} = \Sigma_{AB} \Gamma_C + \frac{1}{2} \eta_{AB} \Gamma_C - \frac{1}{2} \eta_{BC} \Gamma_A,\] (A6)

\[\Omega^C_M = E^C_N \nabla_M E^{NB} = -E^{NB} \nabla_M E^C,\] (A7)

and the specific expressions in Eq. (A3), we can have

\[E^M A \Omega^C_M \Gamma_A \Sigma_{BC} = e^{ma} \omega^b_m \gamma_a \Sigma_{bc} - e^{ma} e^{nb} \partial_m A_n \Sigma_{ab} \Gamma_6.\] (A8)

Inserting it into the Lagrangian in Eq. (A1), we can obtain

\[\bar{\psi} e^{ma} \gamma_a \left( \partial_m - i Q A_m + \frac{1}{2} \omega^b_m \Sigma_{bc} \right) \psi + i \bar{\psi} \left( i Q + \frac{1}{4} e^{ma} e^{nb} F_{mn} \Sigma_{ab} \right) \psi + H.C.\]

where we have taken the fermion to be in a charge eigenstate,

\[\partial_6 \chi(\xi) = \frac{\partial}{\partial \xi} \chi(\xi) = i Q \chi(\xi).\] (A10)

In such a way, the action can be rewritten as,

\[S = \int d^5 x \sqrt{-g} \left\{ \alpha \bar{\psi} e^{ma} \gamma_a \left( \partial_m - i Q A_m + \frac{1}{2} \omega^b_m \Sigma_{bc} \right) \psi + H.C. \right\}
+ \int d^5 x \sqrt{-g} \left\{ i \alpha \bar{\psi} \left( i Q + \frac{1}{4} e^{mb} e^{nd} F_{mn} \Sigma_{bd} \right) \psi + H.C. \right\}\] (A11)
where we have normalized

\[
\int d\xi \, \chi(y)^\dagger (1 + \sigma_2) \chi(y) = 1. \tag{A12}
\]

Now we can see that the Pauli interaction term in AdS\(_5\) space has been produced from higher 6D fermion-graviton interaction by using Kaluza-Klein reduction. For the uncharged fermion where \(Q = 0\), only the Pauli interaction term will contribute. The reduction from 10D to 5D will be similar except for more possible complications involved dealing with more extra dimensions, which is beyond the scope of our present work.

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