Charges of a single grain and the grain in a cloud: Theory and experiments

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Abstract. The effect of cloud density on the grain charging is of great importance in complex plasma physics. The quasi-neutrality condition brings about changing of ratio of a spatially averaged ion density to electron density in the dust cloud. Strong interaction between ions and highly charged grain complicates the analysis of the effect of ion density increasing on ion flux on the grain. The theoretical approach to ion flux correction on the grain space charge by the use of effective ion density instead of spatially averaged density is discussed. The experimental measurements of charges of solitary grains and the grains in the cloud under similar plasma parameters are used to prove the proposed approach.

The complex plasma, i.e., plasma containing macroscopic charged dust grains, is the example of non-ideal plasmas. Energies of the grain–grain and ion–grain interactions are comparable or grater then energies of thermal motions. So, correlations of grain–grain and ion–grain are strong. There are many papers where charging and shielding of the isolated particle are under consideration [1–4]. Usually, the results obtained for isolated particle are applied to particle in the dense cloud by the use space-average ion and electron densities [5, 6]. This approach does not take into account strong correlation between ions and grains.

Similar problem is known for charge colloidal suspensions contained polyions and the small mobile ions. There is an approach based on the consideration of a single Wigner–Seitz cell and linearization of the Poisson–Boltzmann equation near the cell boundary [7, 8]. The effective concentration of small ions and effective charge of the polyion are used for description of the thermodynamic properties. The effective concentration of the small ions corresponds to Wigner–Seitz cell boundary and determines osmotic pressure. Renormalization of the charge in colloidal systems is necessary because of non-linear shielding near the polyion surface. In the case of dusty plasmas, the ion losses on the grain surface prevent overshielding the grain charge, so the charge renormalization for small enough grain is not necessary. But, it would be reasonable to use the effective concentration of ions for calculation of the screening length and plasma flux on the dust grain.

Let us consider the uniform plasma with hot electrons and cold ions, so only ions take part in screening. We assume that at zero potential (ϕ = 0) the ion density $n_i^{\text{eff}}$ which is not equal to the electron density $n_e$. Then the linearized Poisson–Boltzmann equation in this region is

$$\varepsilon_0 \Delta \varphi = \frac{e^2 n_i^{\text{eff}}}{k_B T_i} \varphi - e (n_i^{\text{eff}} - n_e).$$

(1)
where \( \varepsilon_0 \) is the electric constant, \( e \) is the elementary charge, \( k_B \) is the Boltzmann constant and \( T_i \) is the ion temperature. The conditions on the boundary of the spherical Wigner–Seitz cell \((r = R)\) are \( \varphi = 0 \) and \( d\varphi/dr = 0 \). The solution of equation (1) satisfying second condition is
\[
\varphi(r) = \varphi_1 + \frac{Q}{4\pi\varepsilon_0 r} \left( \frac{(kR - 1) \exp(-kr)}{kR - 1 + (kR + 1) \exp(-2kR)} + \frac{(kR + 1) \exp(kr)}{kR + 1 + (kR - 1) \exp(2kR)} \right),
\]
where \( R = (4\pi n_d/3)^{-1/3} \), and \( n_d \) is the dust number density,
\[
k = \left( \frac{e^2 n_{\text{eff}}^{1/2}}{\varepsilon_0 k_B T_i} \right) = k_0 \left( \frac{n_{\text{eff}}^{1/2}}{n_e} \right),
\]
\[
\varphi_1 = k_B T_i \left( 1 - \frac{n_e}{n_{\text{eff}}} \right),
\]
and \( Q \) is the dust grain charge (assumed to be negative). The condition of \( \varphi(R) = 0 \) led to the equation
\[
1 - \frac{n_{\text{eff}}}{n_e} = \frac{Q}{2\pi e n_e R^3} \left( \frac{k^3 R^3}{(kR + 1) \exp(-kR) + (kR - 1) \exp(kR)} \right).
\]

Equation (5) jointly with equation (3) defines the effective ion density as function of the space dust charge density \( Q n_d = 3Q/(4\pi R^3) \) and the unperturbed coupling parameter
\[
\kappa = k_0 R \left( \frac{e^2 n_e}{\varepsilon_0 k_B T_i} \right)^{1/2}.
\]

The dependences of the effective ion density on the spatially averaged ion density for different unperturbed coupling parameters are presented in figure 1. For \( \kappa < 1 \) the effective ion density is close to the spatially averaged density because the Wigner–Seitz cell is smaller then screening length, but for large coupling parameters the effective ion density decreases and tends to the electron density when \( \kappa \gg 1 \).

The decrease in the charge of dust grain in a cloud can be experimentally investigated by comparing the charge of individual particles and the charges of particles in a cloud under the same conditions in plasma. Such experiments were performed by measurements of dust particle drift velocities in the uniform positive column of direct current discharge. At the ground condition the monodisperse particles were dropped one by one though discharge tube (inner diameter of 3 cm) placed in upright position. The drift velocity was determined as half of difference of falling velocities for two directions of the discharge current. At the same discharge conditions the drift of the clouds were observed in the Plasmakristall-4 experiments [9] on board of the International Space Station under microgravity conditions. Only data for small clouds could be used for the comparison because large clouds perturb the discharge conditions and increase electric field end electron temperature. The results are as follows:

- discharge in neon at the pressure of 60 Pa and current of 1 mA, grain diameter is 3.38 \( \mu \)m, drift velocity of the single particle is \( 1.32 \pm 0.08 \) cm/s, drift velocity of the cloud of 4.2 mm in diameter (dust number density is \( 7.7 \pm 1 \times 10^4 \) cm\(^{-3}\)) is \( 1.27 \pm 0.1 \) cm/s;
- discharge in argon with the same grain size, pressure and current, drift velocity of the single particle is \( 0.98 \pm 0.05 \) cm/s, the drift velocity of the cloud of 12.8 mm in diameter (dust number density is \( 17 \pm 5 \times 10^4 \) cm\(^{-3}\)) is \( 1.1 \pm 0.1 \) cm/s, of the cloud of 5.9 mm in diameter (dust number density is \( 6.5 \pm 3 \times 10^4 \) cm\(^{-3}\)) is \( 0.98 \) cm/s;
- discharge in neon at 40 Pa, the current is 0.5 mA, the grain diameter is 6.86 \( \mu \)m, drift velocity of single particle is \( 2.39 \pm 0.15 \) cm/s, drift velocity of the cloud of 6.3 mm in diameter (dust number density is \( 2.8 \pm 0.6 \times 10^4 \) cm\(^{-3}\)) is \( 1.67 \pm 0.1 \) cm/s, of the cloud of 9 mm in diameter (dust number density is \( 4.0 \pm 1 \times 10^4 \) cm\(^{-3}\)) is \( 1.98 \pm 0.1 \) cm/s.
Figure 1. Normalized effective ion density as a function of normalized spatially averaged ion density \( n_{i}^{\text{avr}}/n_e = 1 + Zn_d/n_e \), where \( Z = -Q/e \) for different coupling parameters (6).

The unperturbed coupling parameters for the experimental conditions were in the range from 3.3 to 4. The charge of the dust grain can be determined from drift velocity using experimentally measured axial electric field and electron concentration in the discharge and taking into account neutral drag and ion drag forces [10, 11]. While probe measurements contains significant errors (up to 20%), but that uncertainty give a small effect to the ratio of the charges for similar discharge conditions. The cloud diameters in the experiments were much smaller then the tube diameter, so the influence of the cloud on the axial field and the electron density should be small.

The charges can be calculated using theoretical electron distribution function in the positive column and approximations for ion flux on the grain. The absolute values of the charge depends essentially on the electron distribution function and reflectivity of electrons from grain surface, but the dependences of relative charge decreasing on ratio of the ion density to the electron density lies in the narrow strip. The dependences of the particle charge in cloud normalized on the charge of single particle are presented in figure 2. The strips in figure 2 correspond to simulations of the grain charging in the direct current discharge with longitudinal electric fields from 2 to 3 V/cm in neon and from 3 to 5 V/cm in argon and the electron reflectivity from 0 to 70%. The electron distribution function was calculated by solving Boltzmann’s kinetic equation. The ion flux on the grain was calculated using approximation from [4] with the space-averaged ion density (strip 1) and the effective ion density (strip 2).

The experimental data better correspond to calculations using the effective ion density, but large experimental errors do not allow us to unequivocally prove the proposed approach. The charge measurements in more dense dust clouds are necessary.
Figure 2. Normalized charge as a function of normalized spatially averaged ion density: 1—the charge calculations using space-average ion densities; 2—the calculations using effective ion density; labels A, B and C correspond to the experimental data for 3.4 (A, B) and 6.8 µm (C) grains in argon (A) and neon discharge (B, C).

Thus, the charging and screening of grain in a dust cloud is determined by the density of ions at the boundary of the Wigner–Seitz cell boundary. This ion density generally differs from space-averaged ion density. Only when coupling parameter is smaller then unity, the space-averaged ion density can be used for calculation of the grain charging and the grain–grain interaction.

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