1. INTRODUCTION

It is believed that hadronic matter undergoes a confinement/deconfinement phase transition at sufficiently high temperatures $T$ and/or densities $n_B$. In lattice calculations the expectation value of the Polyakov loop $\langle L \rangle$ is treated as an order parameter for this phase transition. Physically, however, this quantity has no meaning since $T \ln |\langle L \rangle|$ refers to the free energy of an isolated heavy quark and contains divergent self-energy contributions. Instead of $T \ln |\langle L \rangle|$ a meaningful determination of the free energy of a heavy quark pair is proposed in [1]. We use this determination to construct a physical order parameter for the mentioned phase transition. As a first test of our investigation we refer to lattice studies of the pure $SU(3)$ theory [1].

2. COLOR AVERAGING AT DIFFERENT ENERGY SCALES

We have calculated the color averaged free energy $F_{av}$ of a static heavy quark pair using Polyakov loop correlations [2] and the color singlet free energy $F_s$ using the time-like closed Wilson loops [3] at finite temperature. The color averaged free energy is related to the color singlet ($F_s$) and octet ($F_o$) contributions: $\exp(-F_{av}/T) = 1/9 \exp(-F_s/T) + 8/9 \exp(-F_o/T)$. In a static system the internal energy reduces to the potential $V$ and therefore the free energy $F$ refers to the potential, $F = V - TS$, where we assume that the entropy $S$ may stay $R$-dependent ($R$: distance). In a perturbative treatment of QCD the singlet is of attractive while the octet potential is of repulsive Coulomb form [4]. In the hadronic phase additional linear terms contribute to the potentials and signal confinement.

For $RT << 1$, the repulsive octet contribution to the color averaged free energy is exponentially suppressed and the linear terms become negligible; hence we find

$$\lim_{RT \to 0} \{F_{av}(R,T) - F_s(R,T)\} = T \ln 9.$$  (1)

Due to this relation both free energies coincide at $T = 0$. Moreover (1) predicts a singlet-like behavior of $F_{av}$ at short distances [1]. We argue that the difference $T \ln 9$ is caused by the difference of the entropy contributions; $\ln 9$ has the typical form of entropies if one identifies 9 with the sum over the nine possible color configurations: $S_s - S_{av} = \ln 9$.

For $RT >> 1$ and $T > T_c$, the contributions from the potentials become negligible and the color averaging leads to $S_{av} = \ln \{1/9 \exp(S_s) + 8/9 \exp(S_o)\}$. For $T < T_c$, however, the contributions from the potentials and entropies to $F_{av}$ cannot easily be separated because of the linear confining terms in the potentials.

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We have studied (1) with our lattice data [1] in terms of the difference \((F_{\text{av}} - F_s)/T\) in Fig. 1a. The data approach the value \(\ln 9\) in both phases at distances \(RT \lesssim 0.1\). At these distances the averaged potential behaves like a singlet one. At large \(RT\), however, the difference vanishes although \(F_{\text{av}}\) and \(F_s\) separately approach non-zero constants for \(T > T_c\) and diverge for \(T < T_c\). We note that the deviations of \((F_{\text{av}} - F_s)/T\) from \(\ln 9\) at large \(RT\) indicate that octet contributions to the averaged free energy are important at all temperatures. In fact, Fig. 1a and the above mentioned considerations suggest \(S_{\text{av}} = S_s = S_o\) for large \(R\) and \(T > T_c\).

3. THE RENORMALIZED POLYAKOV LOOP

In [1] the renormalized free energy of a heavy quark pair is obtained by matching the lattice data to the \(T = 0\) potential at short distances. With respect to (1) \(F_{\text{av}}\) and \(F_s\) coincide at large distances and yield the free energy of an infinitely separated quark pair (Fig. 1a). We suggest that \(L_{\text{ren}} \equiv \exp(-F_{\text{av}}(\infty)/2T)\), where \(F_{\text{av}}(\infty)\) has to be taken from normalized data, can be used as an order parameter for the deconfinement phase transition. Our results are shown in Fig. 1b where we have taken the \(F\)-values at \(R_0 = 0.55 fm\) rather than at infinity due to the finite physical size of the lattices used in our simulation. The order parameter nearly vanishes below \(T_c\) and increases rapidly above \(T_c\) indicating the phase transition. We expect \(L_{\text{ren}}\) to have a well defined continuum limit because the short distance normalization to the \(T = 0\) potential is independent of the lattice cut-off [1]. In fact, through this short distance matching we have defined a renormalized Polyakov loop \(L_{\text{ren}}\).

![Figure 1: The relation between the color singlet and color averaged free energy as a function of \(RT\) (a) and the new order parameter as a function of \(T/T_c\) (b).](image)

4. CONCLUSIONS

We have shown that color averaging influences the free energy in both phases. Thus there exists a non-vanishing octet contribution to \(F_{\text{av}}\) in the confinement phase. Our data indicate that the entropy contributions to the free energy is \(R\) and \(T\) dependent, which makes a straightforward extraction of the potential from free energies difficult. We have constructed an order parameter for the deconfinement phase transition which has a well defined continuum limit. It is an interesting question how the discontinuity of the first order SU(3) transition becomes visible in this order parameter.

REFERENCES
1. F. Zantow et al, [hep-lat/0110103](http://arxiv.org/abs/hep-lat/0110103)
2. L. G. McLarren, B. Svetitsky, Phys. Rev. D24 (1981) 450
3. S. Nadkarni, Phys. Rev. D33 (1986) 3738
4. L. S. Brown, W. I. Weisberger, Phys. Rev. D20 (1979) 3239