A high-efficiency quantum non-demolition single photon number resolving detector

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We discuss a novel approach to the problem of creating a photon number resolving detector using the giant Kerr non-linearities available in electromagnetically induced transparency. Our scheme can implement a photon number quantum non-demolition measurement with high efficiency (∼99%) using less than 1600 atoms embedded in a dielectric waveguide.

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In recent years we have seen signs of a new technological revolution, caused by a paradigm shift to information processing using the laws of quantum physics. One natural architecture for realising quantum information processing (QIP) technology would be to use states of light as the information processing medium. There have been significant developments in all optical QIP following the recent discovery by Knill, Laflamme and Milburn that passive linear optics, photo-detectors, and single photon sources can be used to create massive reversible nonlinearities. Such nonlinearities are an essential requirement for optical QIP and many communication applications. These nonlinearities allow efficient gate operations to be performed. In principle, fundamental operations such as the nonlinear sign shift and CNOT gates have been demonstrated experimentally. However, such operations are relatively inefficient (they have a probability of success significantly less than 50%) and so are not scalable, due primarily to the current state of the art in single photon sources and detectors. Good progress is being made on the development of single photon sources. Absorptive single photon resolution detection is possible, with efficiencies up to ∼90% (visible spectrum) and ∼30% (infrared, microwaves). However, true universal optical QIP will require significant further improvements in detector efficiencies, which will likely require a drastic change of approach to detection technology.

In this letter, we propose an implementation of the quantum non-demolition (QND) single-photon detection scheme originally described by Imoto, Haus and Yamanoto, with the required optical nonlinearity provided by the giant Kerr effect achievable with ASK Stark-shifted electromagnetically induced transparency (EIT). We show below that the scheme uses about 1600 EIT atoms and a weak pulse in the probe mode to achieve an error probability less than 1%. The effect of the QND measurement in turn means that signal photons are not destroyed and can be reused if required. Furthermore, for a signal mode in a superposition state (such as a weak coherent state) the number-resolving QND measurement projects the signal mode into a definite number state, so the detector can be used as a heralded source of number eigenstates. We focus on EIT as an example since it has already been used successfully to demonstrate cross-Kerr nonlinearities at low light levels in rubidium. From the perspective of realising our detector application, we concentrate on potential condensed-matter mechanisms of EIT. However, clearly any system capable of producing a comparable form and strength of Kerr interaction can be used, including optical fibers, silica whispering-gallery microresonators and cavity QED systems.

Before we begin our detailed discussion of the EIT detection scheme, we first consider the photon number QND measurement using a cross-Kerr nonlinearity. The Kerr Hamiltonian has the canonical form

\[ H_{QND} = \hbar \chi a^\dag a c^\dag c, \]

where the signal (probe) mode has the creation and destruction operators given by \( a^\dag, a \) (\( c^\dag, c \)) respectively, and \( \chi \) is the strength of the nonlinearity. If the signal field contains \( n_s \) photons and the probe field is in an initial coherent state with amplitude \( \alpha_c \), the cross-Kerr optical nonlinearity causes the combined system to evolve as

\[ |\Psi(t)\rangle_{out} = e^{i\chi t a^\dag a c^\dag c} |n_s\rangle |\alpha_c\rangle = |n_s\rangle |\alpha_c e^{i n_s \chi t}\rangle. \]

We observe immediately that the Fock state \( |n_s\rangle \) is unaffected by the interaction, but the coherent state \( |\alpha_c\rangle \) picks up a phase shift directly proportional to the number of photons \( n_s \) in the \( |n_s\rangle \) state. If we measure this phase shift using a homodyne measurement (depicted schematically in Fig. 1), we can infer the number of photons in the signal mode \( a \). The homodyne apparatus allows measurement of the quadrature operator \( \hat{x}(\phi) \equiv c e^{i\phi} + c^\dag e^{-i\phi} \), with an expected result \( \langle \hat{x}(\phi) \rangle = 2 \text{Re} [\alpha_c] \cos \delta + 2 i \text{Im} [\alpha_c] \sin \delta \), where \( \delta = \phi + n_s \chi t \). For a real initial \( \alpha_c \), a highly efficient homodyne measurement of the momentum quadrature \( Y = \hat{x}(\pi/2) \) would yield the signal \( \langle Y \rangle = 2 \alpha_c \sin (n_s \chi t) \) with a variance of one, thus giving a signal-to-noise ratio of \( \text{SNR}_Y = 2 \alpha_c \sin (n_s \chi t) \). If the input in mode \( a \) is either the Fock state \( |0\rangle \) or \( |1\rangle \), the respective output states of the probe...
mode $c$ are the coherent states $|\alpha_c\rangle$ or $|\alpha_c e^{ixt}\rangle$. Using the momentum quadrature measurement, the probability of misidentifying one of these states for one another is then $P_{\text{error}} = \frac{1}{2} \text{erfc}(\text{SNR}/2\sqrt{2})$. A signal-to-noise ratio of $\text{SNR} = 4.6$ would thus give $P_{\text{error}} \approx 10^{-2}$. To achieve the necessary phase shift we require $\alpha_c \sin (\chi t) \approx 2.3$, which can be achieved in a number of ways dependent upon the range of values available for $\alpha_c$ and $\chi t$. For example, we could choose $\alpha_c \gg 2.3$ with $\chi t$ small and satisfy the above inequality; alternatively we could choose $\chi t = \pi/2$ with $\alpha_c = 2.3$. The particular regime chosen depends on the strength of the Kerr nonlinearity achievable in the physical system.

Figure 2 generalizes the detector shown schematically in Fig. 1 to the case where the polarization of the input state is resolved into different paths by a polarizing beamsplitter. In general we may wish to apply different phase shifts to the two distinct polarizations, but in the case shown in Fig. 2 an identical phase shift is applied to each path, so that the detector is insensitive to the polarization of the input state. This is a particularly useful approach when the efficiency of the EIT system and/or the optical propagation path (e.g., as provided by a photonic crystal waveguide optimized for either TE or TM modes) is polarization-dependent.

We now address the generation of the cross-Kerr nonlinearity required to perform the QND measurement. We consider a model (depicted in Fig. 3) of the nonlinear electric dipole interaction between three quantum electromagnetic radiation fields with angular frequencies $\omega_a$, $\omega_b$, $\omega_c$ and a corresponding four-level $\mathcal{N}$ atomic system. Mode $b$ should be thought of as a pump or coupling field: by choosing the correct conditions, we can factor both the coupling field and the atom out of the evolution of the atom-field system, creating an effective cross-Kerr nonlinear interaction between modes $a$ and $c$. The effective vacuum Rabi frequency for each mode is defined as $|\Omega_k|^2 = (\sigma_k/\eta_k A) A_k \Delta \omega_k / 8\pi$, where $\sigma_k \equiv 3\lambda_k^2/2\pi$ is the resonant atomic absorption cross section at wavelength $\lambda_k \equiv 2\pi c/\omega_k$, $\eta_k$ is the refractive index of the waveguide material, $A$ is the effective laser mode cross-sectional area, $A_k \equiv f_k e^2 \omega_k^2 / 2\pi c m_e c^2$ for a transition with oscillator strength $f_k$, and $\Delta \omega_k$ is the bandwidth of the profile function describing the adiabatic interaction of a pulsed laser field with a stationary atom.

It is difficult to achieve a substantial vacuum Rabi frequency using free-space fields, but encapsulating one or more atoms in a waveguide (such as a line defect in a photonic crystal structure) allows field transversality to be maintained at mode cross-sectional areas on the order of $A \approx (\lambda / 3\eta)^2$. Consider, then, a two-dimensional photonic crystal waveguide constructed from diamond thin film ($\eta = 2.4$), with nitrogen-vacancy color centers fabricated in the center of the waveguide channel. The optical transition at 637 nm in NV-diamond has an oscillator strength of approximately 0.12, within a factor of three of rubidium, which has been used successfully to demonstrate cross-Kerr nonlinearities at low light levels. An EIT transmission window of about 8 MHz has been observed experimentally, so a pulse with $\Delta \omega_k / 2\pi \lesssim 5$ MHz should propagate through this window with negligible loss. The corresponding vacuum Rabi frequency is therefore $\Omega \approx 3.6$ MHz.

We consider a number $N$ of $\mathcal{N}$ atoms, fixed and stationary within a cylinder that is narrow but long compared to the optical wavelengths, with the three frequency modes of the system driven by Fock states con-
taining \( n_a, n_b, \) and \( n_c \) photons, respectively. If the durations of the three pulse envelope functions are long compared to the lifetime of atomic level \( |2\), the evolution of the amplitude where all of the atoms are in the ground state \( |1\) is simply given by \( |1, n_a, n_b, n_c\rangle \rightarrow e^{-iWt} |1, n_a, n_b, n_c\rangle \). In general, \( W \) is complex (see Eq. (139) in Ref. [22]); other states contribute to the full evolution and there is photon absorption and loss in the system. However, for our detector we require the probability of even single photon loss from mode \( a \) to be very small, with a real \( W \) preserving the norm of \( |1, n_a, n_b, n_c\rangle \). For this, we assume that the laser frequencies \( \omega_a \) and \( \omega_b \) are both precisely tuned to the corresponding atomic transition frequencies, so that the EIT Raman resonance condition is satisfied. Then, in the weak-signal regime, we assume that the decoherence rate \( \gamma_k \) is dominated by spontaneous emission from atomic level \( |k\), and that \( |\Omega_a\rangle \lesssim \gamma_2 \), so that \( W \) is given by

\[
W = \frac{N |\Omega_a|^2 |\Omega_c|^2 n_a n_c}{\nu_c |\Omega_b|^2 n_b + i \left( \gamma_4 |\Omega_b|^2 n_b + \gamma_2 |\Omega_c|^2 n_c \right)},
\]

where \( \nu_c = \omega_c - \omega_{43} \). In principle, in this regime we must have \( \nu_c |\Omega_b|^2 n_b \gg \gamma_4 |\Omega_b|^2 n_b + \gamma_2 |\Omega_c|^2 n_c \) to obtain a nearly real \( W \) and a low residual absorption. As we shall show below, in practical cases where \(|Wt| \ll 1\), this constraint can be substantially relaxed. For NV-diamond, \( \gamma_2^{-1} = 2 \times 25 \text{ ms} \) [28, 29], and the spin decoherence lifetime is 0.1 ms [30, 31], so for \( N \lesssim 10000 \), dephasing can be neglected and \(|\Omega_a|/\gamma_2 \approx 1 \).

Under these conditions, the state \( |1, n_a, n_b, n_c\rangle \) simply acquires a phase-shift which is the basis for the emergence of the approximate cross-Kerr non-linearity [22]. In general, when the pump and probe fields are intense coherent states (parameterized by \( \alpha_b \) and \( \alpha_c \), respectively), the evolution of the state \( |1, n_a, \alpha_b, \alpha_c\rangle \) can be approximated as \( |1, n_a, \alpha_b, \alpha_c e^{-i n_a (\theta - i \kappa)}\rangle \) [22], where the angle \( \theta \) and the residual absorption \( \kappa \) is defined for the case \( |\Omega_b|^2 = |\Omega_c|^2 \) by

\[
\theta - i \kappa = \frac{N |\Omega_a|^2 t}{\nu_c |\alpha_b|^2 + i \left( \gamma_4 |\alpha_b|^2 + \gamma_2 |\alpha_c|^2 \right)}.
\]

This is equivalent to a damped evolution generated by the cross-Kerr Hamiltonian of [11], with \( \theta = \chi t \).

What values of \( \theta \)—and, therefore, SNR\(_Y\)—are achievable? To establish an estimate we need to make several assumptions about the physical system and its geometry. We assume that the interaction region (where the light and \( N \) atoms interact) is encapsulated within the photonic crystal waveguide described above, and that the pulses have weakly super-Gaussian profiles so that the bandwidth-interaction time product is \( \Delta \omega_i t \approx 3 \pi \), giving \(|\Delta\rangle^2 t \approx 81 \eta \gamma_2 / 8 \pi \). Suppose now that the largest phase shift that can be applied in practice (without significantly distorting the signal pulse) is \( \theta_{\text{max}} \), and that the corresponding value of \( \alpha_c \) needed to obtain a given signal-to-noise ratio is therefore \( \alpha_c = \text{SNR}_Y / 2 \theta_{\text{max}} \) (if \( \theta_{\text{max}} \ll 1 \)). Using Eq. (4), we can calculate the minimum number of atoms and the corresponding minimum detuning \( \nu_c \) needed to generate this phase shift, by taking the real part and solving for \( \nu_c \) explicitly. As \( \nu_c \) is required to be real we find

\[
N_{\text{min}} = \frac{2 \theta_{\text{max}}}{|\Omega_a|^2 t} \left( \gamma_4 |\alpha_b|^2 + \gamma_2 |\alpha_c|^2 \right)
= \frac{2 \text{SNR}_Y^2}{2 \theta_{\text{max}} |\Omega_a|^2 t} \left( \gamma_4 |\alpha_b|^2/2 + 1 \right)
\]

with

\[
\nu_{c_{\text{min}}} = \frac{N_{\text{min}} |\Omega_a|^2 t}{2 |\alpha_b|^2 \theta_{\text{max}}} = \frac{\gamma_4 |\alpha_b|^2 + \gamma_2 |\alpha_c|^2}{|\alpha_b|^2}.
\]

When we choose \( N = N_{\text{min}} \) and \( \nu_c = \nu_{c_{\text{min}}} \), we find that \( \kappa = \theta \), so that the state \( |1, n_a, \alpha_b, \alpha_c\rangle \) evolves according to \( |1, n_a, \alpha_b, \alpha_c e^{-i n_a (\theta - i \kappa)}\rangle \). Therefore, when \( \theta \ll 1 \), the residual absorption of a signal photon in mode \( a \) can also be made intrinsically small, even though the detuning is not large compared to the absorption linewidth [32].

Figure 4 shows the minimum number of NV-diamond color centers as a function of the maximum single-photon phase angle for three different values of the error probability \( P_{\text{error}} \). We note that \( \gamma_4 = \gamma_2 \) for the optical transitions in NV-diamond, and for convenience we have chosen \( \langle n_b \rangle = 10 \langle n_c \rangle \), requiring a minimum detuning \( \nu_{c_{\text{min}}} / \gamma_2 = 1.1 \) in all three cases. Note that the minimum number of atoms needed to obtain a given phase shift decreases as the phase shift increases, because the constraint that the signal-to-noise ratio remain constant allows the values of \( |\alpha_b| \) and \( |\alpha_c| \) to decrease as \( \theta_{\text{max}} \) increases. As an example, we choose \( P_{\text{error}} = 0.01 \) and \( \theta_{\text{max}} = 0.01 \) radians, requiring \( \langle n_c \rangle = 5.6 \times 10^4 \) to maintain \( \text{SNR}_Y = 4.6 \). Therefore, \( N \approx 1600 \) is sufficient to achieve the desired phase shift, resulting in a residual absorption of less than 1%.

There is considerable flexibility in the engineering design parameter space for this implementation of the QND detector. For example, if we choose \( \theta_{\text{max}} = 0.1 \), following the design procedure outlined above for \( P_{\text{error}} = 0.01 \) leads to a residual absorption of almost 10% for \( N \approx 160 \) and \( \langle n_c \rangle \approx 560 \). However, if we increase the number of atoms to 800 and the detuning to \( \nu_c = 11 \gamma_2 \), then the absorption is reduced to 1%. Note also that the detector can perform a QND measurement on a Fock state with \( n_a > 1 \) with single-photon resolution. For example, as \( n_a \) increases from 1 to 2, the phase shift \( n_a \theta \) doubles, and the SNR also increases from 4.6 to 9.2 for constant \( \alpha_c \). The detector sensitivity improves until the phase shift becomes so large that one of two fundamental limits is reached: either the SNR decreases below the 1% error threshold, or the strong nonlinear interaction begins to
significantly distort the pulse profile of the signal Fock state.

In summary, we have presented a scheme for a highly efficient photon number quantum non-demolition detector (with single-photon resolution) based on the EIT condensed matter system with approximately 1600 color centers. We have explored several different operating regimes, and we have examined in detail the performance of the detector for an NV-diamond photonic crystal waveguide system. In particular, we have shown that efficient detection is possible with small phase shifts, which will likely be necessary to ensure that the EIT optical nonlinearity doesn’t distort the pulse envelope of the signal state. Future modeling will address detector performance for pulsed Fock state profile functions, and much experimental work remains to be done to implement such a detector. For example, fabricating EIT atomic or molecular systems into a dielectric waveguide is challenging but feasible. A method for orienting the color center spins uniformly in such a condensed-matter system must be found, and spatial hole-burning techniques will be needed to overcome the effects of inhomogeneous broadening on the transparency of a condensed matter medium [33]. The overall efficiency of the detector is likely to be limited by the efficiency of the homodyne measurement of the phase shift, which will depend on the degree to which the homodyne detector can be spatiotemporally mode-matched to a single-photon signal. Nevertheless, EIT provides us with the best known candidate mechanism for the implementation of the original QND proposal by Imoto, Haus, and Yamamoto [12], and even a weak nonlinearity could allow efficient reuse of resources in linear optics quantum computation schemes.

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