BFKL SIGNATURES AT A LINEAR COLLIDER

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The BFKL dynamics can be successfully tested at the future high energy $e^+e^-$ linear collider. The total $\gamma^*\gamma^*$ cross-section is calculated in the Leading Log QCD dipole picture of BFKL dynamics, and compared with the one from 2-gluon exchange. The rapidity dependence of the cross-section remains a powerful tool to increase the ratio between the BFKL and the 2-gluon cross-sections and is more sensitive to BFKL effects, even in the presence of higher order corrections. Other potential signals like diffractive $J/\Psi$ and 'forward' jet productions will also be discussed.

1 $\gamma^*\gamma^*$ total cross-section

1.1 Differential cross-sections

Here, we want to calculate the total $\gamma^*\gamma^*$ cross-section derived in the Leading Log QCD dipole picture of BFKL dynamics. This could be a good test of the BFKL equation which can be performed at $e^+e^-$ colliders (LEP or linear collider LC). The advantage of this process is that it is a process which does not involve non perturbative couplings.

In this study, we compare the 2-gluon and the BFKL cross-sections. Defining $y_1$ (resp. $y_2$), and $Q_1^2$ (resp. $Q_2^2$) to be the rapidities and the squared transferred energies for both virtual photons, one gets

$$d\sigma_{e^+e^-}(Q_1^2, Q_2^2; y_1, y_2) = \frac{4}{9} \left( \frac{\alpha_s}{16} \right)^2 \alpha_s^2 \pi^2 \sqrt{\frac{\pi}{2}} \frac{dQ_1^2}{Q_1^2} \frac{dQ_2^2}{Q_2^2} \frac{dy_1}{y_1} \frac{dy_2}{y_2} \frac{1}{y_1 y_2/Q_1 Q_2} \times \frac{4\alpha_s N_c}{\pi} Y \ln 2 \sqrt{\frac{\alpha_s}{\pi}} Y \zeta(3) [2l_1 + 9t_1] [2l_2 + 9t_2],$$

for the BFKL-LO cross-section, where $t_1 = \frac{1 + (1 - y_1)^2}{2}$, $t_2 = 1 - y_2$, and an analogous definition for $t_2$ and $l_2$, and $Y = \ln \frac{y_1 y_2}{\sqrt{Q_1^2 Q_2^2}}$. The 2-gluon cross-section has been calculated exactly within the high energy approximation (NNNLO calculation) and reads

$$d\sigma_{e^+e^-}(Q_1^2, Q_2^2; y_1, y_2) = \frac{dQ_1^2}{Q_1^2} \frac{dQ_2^2}{Q_2^2} \frac{dy_1}{y_1} \frac{dy_2}{y_2} \frac{64(\alpha_s^2 e.m)^2}{243\pi^3} \frac{1}{Q_1^2} \left[ t_1 t_2 \ln^3 \frac{Q_1^2}{Q_2^2} + (7t_1 t_2 + 3t_1 l_2 + 3t_2 l_1) \ln^2 \frac{Q_1^2}{Q_2^2} + \left( \frac{119}{12} - 2\pi^2 \right) t_1 t_2 + 5(t_1 l_2 + t_2 l_1) + 6l_1 l_2 \right] \ln \frac{Q_1^2}{Q_2^2}.$$
\[ t_1 t_2 + \left( \frac{1063}{9} - \frac{14}{3} \pi^2 \right) t_1 t_2 + (46 - 2 \pi^2)(t_1 l_2 + t_2 l_1) - 4 l_1 l_2 \].  \hspace{1cm} (2)

Figure 1 shows the differential cross-sections in the BFKL, DGLAP Double Leading Logarithm (DLL) and 2-gluon approximation, as a function of $\ln Q_1^2/Q_2^2$ and for three values of $Y$. The cross-sections on the left hand side are calculated using the exact unintegrated formulae, for respectively the BFKL, DGLAP (in the double Leading Log approximation) and 2-gluon exchange cross-sections. Also the phenomenological HO-BFKL cross-sections, as detailed in section 1.2, are given.

We note that the 2-gluon cross-section is almost always dominating the DGLAP one in the Double Leading Log approximation. The saddle point approximation turns out to be a very good approximation to an accuracy better than 10% for the BFKL cross-section and is not displayed in the figure. We note that the difference between the BFKL and 2-gluon cross-sections increase with $Y$.

On the right side of Figure 1, curves for the exact LO and saddle-point DGLAP calculations are shown, as well as the full NNNLO (eq. 2) result and the LO (eq. 2, $\ln^3 Q_1^2/Q_2^2$ term only) result for the 2-gluon cross-section. Unlike for the BFKL calculation, for the DGLAP case the saddle-point approximation appears to be in worse agreement with the exact calculation, and overestimates the cross-section by one order of magnitude, which is due to the fact that we are far away from the asymptotic regime. The comparison between the DGLAP-DLL and the 2-gluon cross-section in the LO approximation shows that both cross-sections are similar when $Q_1$ and $Q_2$ are not too different (the dashed line describes the value $Q_1^2/Q_2^2 = 2$), so precisely in the kinematical domain where the BFKL cross-section is expected to dominate. However, when $Q_1^2/Q_2^2$ is further away from one, the LO 2-gluon cross-section is lower than the DGLAP one, especially at large $Y$. This suggests that the 2-gluon cross-section could be a good approximation of the DGLAP one if both are calculated at NNNLO and restricted to the region where $Q_1^2/Q_2^2$ is close to one. In this paper we will use the exact NNNLO 2-gluon cross-section in the following to evaluate the effect of the non-BFKL background, since the 2-gluon term appears to constitute the dominant part of the DGLAP cross-section in the region $0.5 < Q_1^2/Q_2^2 < 2$.

1.2 Integrated cross-sections

Results based on the calculations developed above will be given for a future Linear Collider (500 centre-of-mass energy). $\gamma^*\gamma^*$ interactions are selected at $e^+e^-$ colliders by detecting the scattered electrons, which leave the beampipe, in forward calorimeters. For the LC it has been argued\cite{4} that angles as low as 20 mrad should be reached. Presently angles down to 40 mrad are foreseen to be instrumented for a generic detector at the LC.

Let us first specify the region of validity for the parameters controlling the basic assumptions made in the previous chapter. The main constraints are required by the validity of the perturbative calculations. The “perturbative” constraints are imposed by considering only photon virtualities $Q_1^2$, $Q_2^2$ high enough so that the scale $\mu^2$ in $\alpha_S$ is greater than 3 GeV$^2$. $\mu^2$ is defined using the Brodsky Lepage Mackenzie (BLM) scheme\cite{5}: $\mu^2 = \exp(-5/4)\sqrt{Q_1^2 Q_2^2}$. In this case $\alpha_S$ remains always
small enough such that the perturbative calculation is valid. In order that gluon contributions dominates the QED one, $Y$ is required to stay larger than $\ln(\kappa)$ with $\kappa = 100$. (see Ref. 5 for discussion). Furthermore, in order to suppress DGLAP evolution, while maintaining BFKL evolution will constrain $0.5 < Q_1^2/Q_2^2 < 2$ for all nominal calculations. In this paper we will use the exact NNNLO 2-gluon cross-section in the following to evaluate the effect of the non-BFKL background, since the 2-gluon term appears to constitute the dominant part of the DGLAP cross-section in the region $0.5 < Q_1^2/Q_2^2 < 2$.

We will not discuss here all the phenomenological results, and some detail can be found in 6, as well as the detailed calculations. We first study the effect of increasing the LC detector acceptance for electrons scattered under small angles and the ratio of the 2-gluon and the BFKL-LO cross-sections increase by more a factor 3 if the tagging angle varies between 40 and 20 mrad. The effect of lowering the tagging energy is smaller. An important issue on the BFKL cross-section is the importance of the NLO corrections and we adopt a phenomenological approach to estimate the effects of higher orders. First, at Leading Order, the rapidity $Y$ is not uniquely defined, and we can add an additive constant to $Y$. A second effect of NLO corrections is to lower the value of the so called Lipatov exponent in formula 1. In the $F_2$ fit described in Ref. 3, the value of the Lipatov exponent $\alpha_P$ was fitted and found to be 1.282, which gives an effective value of $\alpha_s$ of about 0.11. The same idea can be applied phenomenologically for the $\gamma^*\gamma^*$ cross-section. For this purpose, we modify the scale in $\alpha_s$ so that the effective value of $\alpha_s$ for $Q_1^2 = Q_2^2 = 25$ GeV$^2$ is about 0.11. Finally, the results of the BFKL and 2-gluon cross-sections are given in Table 1 if we assume both effects. The ratio BFKL to 2-gluon cross-sections is reduced to 2.3 if both effects are taken into account together. In the same table, we also give these effects for LEP with the nominal selection and at the LC with a detector with increased angular acceptance.

Another idea to establish the BFKL effects in data is to study the energy or $Y$ dependence of the cross-sections, rather than the comparison with total cross-sections itself. To illustrate this point, we calculated the BFKL-NLO and the 2-gluon cross-sections, as well as their ratio, for given cuts on rapidity $Y$ (see table 2). We note that we can reach up to a factor 5 difference ($Y \geq 8.5$) keeping a cross-section measurable at LC. The cut $Y \geq 9$ would give a cross-section hardly measurable at LC, even with the high luminosity possible at this collider.

### 2 Diffractive $J/\Psi$ production as a probe of the QCD pomeron

Another probe of the QCD pomeron at LC which has been proposed is the double diffractive production of $J/\Psi$ in $\gamma\gamma$ collisions i.e. the process $\gamma\gamma \rightarrow J/\Psi J/\Psi$. It

|       | BFKL$_{LO}$ | BFKL$_{NLO}$ | 2-gluon | ratio |
|-------|-------------|--------------|---------|-------|
| LEP   | 0.57        | 3.1E-2       | 1.35E-2 | 2.3   |
| LC 40 mrad | 6.2E-2   | 6.2E-3       | 2.64E-3 | 2.3   |
| LC 20 mrad | 3.3       | 0.11         | 3.97E-2 | 2.8   |

Table 1: Final cross-sections (pb), for selections described in the text.
Table 2: Final cross-sections (pb), for selections described in the text, after different cuts on $Y$

| $Y$ cut | BFKL$_{NLO}$ | 2-gluon | ratio |
|---------|--------------|---------|-------|
| no cut  | 1.1E-2       | 3.97E-2 | 2.8   |
| $Y \geq 6.$ | 5.34E-2 | 1.63E-2 | 3.3   |
| $Y \geq 7.$ | 2.54E-2 | 6.58E-3 | 3.9   |
| $Y \geq 8.$ | 6.65E-3 | 1.43E-3 | 4.7   |
| $Y \geq 8.5$ | 1.67E-3 | 3.25E-4 | 5.1   |
| $Y \geq 9.$ | 5.36E-5 | 9.25E-6 | 5.8   |

Figure 1: Differential cross-sections for different values of $Y$ (see text).
should be noted that both sides of the diagram are characterized by the same (hard) scale provided in this case by the relatively large charm quark mass. This process has also the advantage that its cross-section can be almost entirely calculated perturbatively. The only non-perturbative element is a parameter determined by the $J/\Psi$ light cone wave function which can however be obtained from the measurement of the leptonic width $\Gamma_{J/\Psi \rightarrow l^+l^-}$ of the $J/\Psi$. The BFKL equation is solved in the non-forward configuration taking into account dominant non-leading effects which come from the requirement that the virtuality of the exchanged gluons along the gluon ladder is controlled by their transverse momentum squared.

The cross-section exhibits an approximate $(W^2)^{2\lambda}$ dependence. The parameter $\lambda$ slowly increases with increasing energy $W$ and changes from $\lambda \approx 0.23$ at $W = 20$ GeV to $\lambda \approx 0.28$ at $W = 500$ GeV i.e. within the energy range which is relevant for possible LC measurements.

The BFKL effects significantly affect the $t$-dependence of the differential cross-section leading to steeper $t$-dependence than that generated by the Born term. Possible energy dependence of the diffractive slope is found to be very weak.

The 2-gluon and BFKL cross-sections after the consistency constraints are still quite different at LC and it would be possible to see clearly some BFKL enhancement of the cross-section. At at center of mass energy of 200 GeV (resp. 500 GeV), the 2-gluon and BFKL cross-sections are 0.03 and 0.13 pb (resp. 0.11 and 0.65 pb). Going to the $e\gamma$ option instead of the $e^+e^-$ one increases sensitively the BFKL cross-section. At a center of mass energy of 400 GeV, the BFKL cross-section is about 6.5 pb whereas the 2-gluon one is about 0.8 leading to a clear distinction between them.

Unfortunately, these cross-sections are purely theoretical as they do not take into account the angular acceptance of the detectors. If it is possible to detect the decayed muon coming from the $J/\Psi$ down to 5 degrees (resp. 15 degrees), it is possible to detect 25% (resp. 4%) of the cross-sections discussed above inside the acceptance of the detector. Thus, about 50, 500 and 5000 such events are expected at LC in the $ee$, $e\gamma$, and $\gamma\gamma$ options for 100 fb$^{-1}$. To be able to study this kind of processes, high luminosity as well as a good coverage of the muon detectors in the forward and backward part down to 5 degrees is necessary.

In the same spirit, another measurement based on diffractive photon production in $\gamma\gamma$ interactions has been proposed in\cite{8}. The typical cross-section is of the order of 1 pb at LC for $t$ between 0 and $10^{-4}$. The study within the detector acceptance is under way.

3 "Forward jets” production

The idea of looking for "forward jets” at LC follows the motivation of the search for effects of BFKL-like parton shower evolutions performed at HERA\cite{10}. The method proposed originally by A. Mueller\cite{9} for looking for BFKL effects at HERA starts from the idea that the BFKL evolution predicts a different momenta ordering in the parton cascade compared to the DGLAP one. The DGLAP evolution predicts a strong ordering in the parton transverse momenta $k_T$ whereas the BFKL one relaxes this ordering. Thus the BFKL evolution predicts additional contributions to the hadronic final state coming from partons with large transverse momenta, and high
transverse momenta partons going forward in the HERA frame. The same idea can easily be applied to LC if one considers on one side a virtual photon of virtuality $Q^2$ interacting with a resolved photon. If one requires the jet close in rapidity to the resolved photon to have a $k_T$ close to $Q^2$, one enhances the BFKL evolution. The DGLAP cross-section will be strongly suppressed because of the $k_T$ ordering whereas the BFKL evolution will predict a non vanishing cross-section.

A detailed simulation of this effect has been performed for LC. To be able to tag the "forward jet", cuts on the jet angle larger than 5 degrees, and on the jet transverse energy greater than 4 GeV have been performed. To enforce the BFKL cross-section compared to the DGLAP one, a cut on the ratio of the transverse energy of the jet over $Q^2$ has been added ($0.5 < k_T^2/Q^2 < 2$). This leads to a LO-BFKL over Born cross-section ratio varying between 0.05 and 0.1. It should also be noticed that the $e\gamma$ collider provides a 10-time higher cross-section than the $ee$ option. In figure 2 is displayed the result for the Born and LO-BFKL cross-section as well as their ratio for an $ee$ (full line) and $e\gamma$ (dashed line) machine.

4 Conclusion

We first discussed the difference between the 2-gluon and BFKL $\gamma^*\gamma^*$ cross-sections both at LEP and LC. The LO BFKL cross-section is much larger than the 2-gluon cross-section. Unfortunately, the higher order corrections of the BFKL equation
(which we estimated phenomenologically) are large, and the 2-gluon and BFKL-NLO cross-sections ratios are reduced to a factor two to four. The $Y$ dependence of the cross-section remains a powerful tool to increase this ratio and is more sensitive to BFKL effects, even in the presence of large higher order corrections. The uncertainty on the BFKL cross-section after higher order corrections is still quite large. We thus think that the measurement performed at LEP or at LC should be compared to the precise calculation of the 2-gluon cross-section after the kinematical cuts described in this paper, and the difference can be interpreted as BFKL effects. A fit of these cross-sections will then be a way to determine the BFKL pomeron intercept after higher order corrections. A possible measurement at LC would then be of great importance provided it is possible to tag electrons at low scattering angles.

The second measurement described in this paper is the double diffractive production of $J/\Psi$ in $\gamma \gamma$ collisions. The measurement of its cross-section (and more specifically of its $t$ dependence) allows a clear distinction between the DGLAP and BFKL evolutions. A good coverage at low angle of the muon detection is then needed. If this is fulfilled, a clear BFKL signal could be shown at LC with this measurement.

One of the golden channels for BFKL searches at HERA, i.e. forward jet production can also been used at LC provided tagging of jets at low angle is feasible. The predicted ratio between the BFKL-LO and 2-gluon cross-sections is then found to be between 0.05 and 0.1. The effects of higher order corrections to the BFKL equation still need to be studied for this process.

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