Skyrmion-number dependence of spin-transfer torque on magnetic bubbles

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We study spin transfer torque acting on magnetic bubbles based on collective-coordinate model and micromagnetic simulations. Magnetic bubbles are quantified by the so-called skyrmion number, whose absolute value takes an integer and can be greater than the unity. We find that the transverse velocity of the bubble with respect to the charge current changes its sign as does the bubble’s skyrmion number, and its magnitude is greatly suppressed as the absolute value of skyrmion number increases.

Introduction.— In recent years, attention has been focusing on topologically nontrivial magnetic textures, such as magnetic vortices in soft ferromagnetic nanodiscs and skyrmion lattices in chiral magnetic thin films. They exhibit rich physics stemming from their characteristic structures, which can be advantageous for technological applications. Another interesting example among such nontrivial topological textures is magnetic bubbles. Magnetic bubbles are observed in ferromagnetic films with out-of-plane anisotropy as spot-like closed domains, where the magnetization is oriented in the opposite direction to the one outside the bubbles.

Vortices, skyrmions and bubbles are quantified by a common topological quantity $N_S$, the so-called skyrmion number, which is defined by $N_S = (1/4\pi) \int dz dy (\mathbf{m} \cdot \partial_z \mathbf{m} \times \partial_y \mathbf{m})$, where $\mathbf{m}$ is the unit vector in the direction of the local magnetization, and the integral is taken over the film sample. A vortex and a skyrmion possess, respectively, $N_S = \pm 1/2$ and $\pm 1$, their sign being determined by their polarity. As for a bubble, it carries an integer skyrmion number, but its absolute value, unlike the skyrmions, varies depending on the magnetic profile on its circumference. Systems that involve magnetic bubbles hence provide us with a good ground for investigation of relation between the dynamics of magnetic textures and their topology, especially to find skyrmion-number dependent phenomena.

Since their first observation in the 1960s, the statics and dynamics of magnetic bubbles have been investigated intensively, where it has been learned that a bubble is stabilized in thin films by applying a magnetic field in the film-normal direction with appropriate magnitude, and the motion of the bubble can be driven by magnetic field gradient. More recently, experimental and theoretical studies on bubbles in nanostructures have discovered new phenomena and unveiled interesting features of bubbles; in nanodots a bubble can be stabilized without external field, showing massive behavior, systematic studies on the spin wave excitation spectrum of bubbles show their variety of dynamical reaction, and the electric voltage due to motion of bubble array was predicted. However, we still lack satisfactory theoretical framework to deal with the reaction of bubbles to charge current.

In this work, the dependence of current-driven bubble motion on its topology is studied theoretically and numerically. The collective-coordinate model, where the expression for the velocity of the bubble is derived assuming its steady motion, indicates that the transverse motion of the bubble with respect to the current significantly depends on $N_S$, while the longitudinal motion does not; the transverse velocity changes its sign as does $N_S$, and its magnitude decreases as $|N_S|$ increases. Micromagnetic simulations complement the collective-coordinate model quantitatively, confirming the predicted trend in the bubble dynamics. Our work unveils a new aspect of how the topology of magnetic textures affects their dynamics when the skyrmion number can take an integer value greater than the unity.

Collective-coordinate model.— We consider a cylindrical bubble domain with radius $R$ in a thin film. The distribution of the magnetization direction $\mathbf{m} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ is assumed, as schematically shown in Fig. 1, to be two dimensional and in a static state described by

$$\theta(r, \chi, z) = \pm 2 \tan^{-1} \exp \left[ \frac{Q(r - R)}{\Delta} \right], \quad (1)$$

$$\phi(r, \chi, z) = S \chi + \phi_0, \quad (2)$$

where $(r, \chi, z)$ is the cylindrical coordinate measured from the bubble center, $\Delta$ is the wall width parameter, $\phi_0$ is a constant. Two topological quantities, the polarity $Q$ and the winding number $S$, have been introduced by

$$Q = \frac{1}{\pi} \int_0^\infty \frac{\partial \theta}{\partial r} dr = \pm 1, \quad (3)$$

and

$$S = \frac{1}{2\pi} \int_{\chi=0}^{2\pi} d\phi = \frac{1}{2\pi} \oint ds, \quad (4)$$

$$\int_0^\infty \frac{\partial \theta}{\partial r} dr = \pm 1, \quad (3)$$

$$S = \frac{1}{2\pi} \int_{\chi=0}^{2\pi} d\phi = \frac{1}{2\pi} \oint ds, \quad (4)$$
where \( ds \) is the contour integral taken counterclockwise around the circumference. The above mentioned configuration ensures that the skyrmion number \( N_S \) of the bubble takes an integer value as

\[
N_S \equiv \frac{1}{4\pi} \int \mathbf{m} \cdot \left( \frac{\partial \mathbf{m}}{\partial x} \times \frac{\partial \mathbf{m}}{\partial y} \right) r dr d\chi = QS. \tag{5}
\]

The sign of \( N_S \) depends on both \( Q \) and \( S \), while the absolute value is determined by \( S \). The domain wall separating the inside and outside of the bubble, in general, can contain vertical Bloch lines i.e., there are many possible ways of distributing the azimuthal angle \( \phi \) along the perimeter. The linear dependence of \( \phi \) on \( \chi \) in Eq. (2) implicitly indicates that we are focusing on the following two cases. First, when the magnetization rotates one full turn around the wall of the bubble with no Bloch line, \( \phi = \pm \chi + \phi_0 \), that is, \( S = \pm 1 \). (When \( S = +1 \), the bubble prefers the closure magnetization along the perimeter, i.e., \( \phi_0 = \pm \pi / 2 \), to avoid costing the magnetostatic energy by inducing surface magnetic charges, see Fig. 1 (b) and (d).) In this case \( N_S \) is either 1 or \(-1\) depending on \( Q \) [Fig. 1 (b)-(d)]. As the second case, Eq. (2) is also a good approximation when the Bloch lines are packed so closely that the distance between the adjacent Bloch lines is comparable to the wall width, i.e., \(|S| \cong R / \Delta \) [Fig. 1 (e)]. If our bubble contains a small number of Bloch lines, the distribution of \( \phi \) would be no longer as simple as Eq. (2).

In the presence of a uniform charge current, the magnetization is assumed to obey the following form of Landau-Lifshitz-Gilbert equation:

\[
\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{H} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} - \mathbf{u} \frac{\partial \mathbf{m}}{\partial x} + \beta \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial x} \tag{6}
\]

Here \( \gamma \) is the gyromagnetic ratio, \( \alpha \) is the Gilbert damping constant, and \( \mathbf{H} \) is the effective magnetic field defined by \( \mathbf{H} \equiv - (1 / \mu_0 M_S) \partial \mathbf{w} / \partial \mathbf{m} \), with \( \mathbf{w} \) the magnetic energy density and \( M_S \) the saturation magnetization. The last two terms in Eq. (6) describe the spin transfer torques, where the \( x \) axis is the direction of current flow, \( \beta \) is a dimensionless parameter, and \( \mathbf{u} = g \mu_B P / 2 e M_S \), with \( g \) the g-factor, \( \mu_B \) the Bohr magneton, \( P \) the spin polarization of the conduction electrons, \( e \) the elementary charge, and \( j \) the charge current density.

When the magnetization dynamics can be parametrized by a small number of collective coordinates \( \{ \xi_i(t) \} \), the spatial-dependent LLG equation \( \mathbf{m} = \mathbf{m}(r,t) \) can be rewritten into the equations of motion for \( \{ \xi_i(t) \} \). In the following, we adopt the center position of bubble \( \mathbf{X}(t) = (X(t), Y(t)) \) as the collective coordinate assuming the steady motion of a bubble, i.e., during its motion with constant velocity \( \mathbf{V} \) the bubble stays rigidly cylindrical with constant radius \( R \) and the \( \phi \)-distribution does not change with respect to the coordinate frame moving with the bubble:

\[
\theta(r, t) = \theta(\mathbf{r} - \mathbf{V} t), \quad \phi(r, t) = \phi(\mathbf{r} - \mathbf{V} t). \tag{7}
\]

Therefore, any variation of the magnetization in time occurs through that of \( \mathbf{X} \), i.e., \( \partial \mathbf{m} / \partial t = (\partial \mathbf{m} / \partial X)/(dX/dt) + (\partial \mathbf{m} / \partial Y)/(dY/dt) \). Hereafter, we focus on cases with \( S > 0 \), because a bubble with \( S = -1 \) inevitably produces magnetic charges on its perimeter, which is energetically unfavourable compared to one with \( S = 1 \), and thus can be easily distorted; the present collective-coordinate model is not a good approximation for the case with \( S = -1 \). (As for a bubble with \( S = 6 \), despite even larger magnetic energies they are rather stable or “hard” against external stimuli, because when the domain walls along the perimeter get such packed, the neighboring exchange coupling prevents the magnetization from precession.) By integrating \( dX/dt \) over the sample volume with the help of the assumptions we have made, one obtains the equation of motion for \( \mathbf{X} \)

\[
\frac{d\mathbf{X}}{dt} = \frac{u}{G^2 + (\alpha \Gamma)^2} \left[ G^2 + \alpha \beta \Gamma^2 \right], \tag{8}
\]

where

\[
\begin{align*}
G & = \int \mathbf{m} \cdot \frac{\partial \mathbf{m}}{\partial X} - \frac{\partial \mathbf{m}}{\partial Y} r dr d\chi = 4\pi N_S, \tag{9} \\
\Gamma & = \int |\frac{\partial \mathbf{m}}{\partial X}|^2 r dr d\chi = \frac{2\pi R}{\Delta} \left( 1 + \frac{N_S^2 \Delta^2}{R^2} \right). \tag{10}
\end{align*}
\]

Here, \( \mathbf{H} = 0 \) has been assumed. The torques related to dissipation, i.e., the second and forth terms in the right-hand-side of Eq. (6), contribute \( \Gamma \) with different coefficients, while the other two terms \( G \). The bubble velocity is proportional to the charge current density \( j \). Eq. (8) has the same form with the equation for the steady motion of skyrmions, but now \(|G|\) can be greater than \(4\pi\) and the explicit expressions of \( \Gamma \) are different.

Eq. (8) is the main result of the collective-coordinate model. In Fig. 2, the solid lines plot \( V_\parallel \equiv dX/dt \) and \( V_\perp \equiv dY/dt \) according to Eq. (8) as functions of \( j \) for four different skyrmion numbers, \( N_S = \pm 1 \) and \( \pm 6 \). There the parameters are chosen to be typical for Co/Ni alloys: \( M_S = 6.8 \times 10^5 \) A/m, \( g = 2 \), \( P = 0.5 \), \( \alpha = 0.03 \), \( \beta = 0.01 \), and \( \Delta = 5 \) nm. The bubble radius is assumed to be \( R = 103 \) nm for the bubbles with \(|N_S| = 1 \), and \( 109 \) nm for \(|N_S| = 6 \), to be consistent with the simulations we will show later. \(|N_S| = 6 \) is smaller than \( R / \Delta = 21 \), apparently contradicting to the condition for Eq. (2) to be justified. However, as will be shown soon by simulations, \(|N_S| = 6 \) can be practically large enough to realize the linear \( \chi \)-dependence of \( \phi \). It is seen from Fig. 2 that the longitudinal velocity is approximately \( V_\parallel \simeq u \), having only a little dependence on \( N_S \). Indeed, it does not depend on the sign of \( N_S \) at all. This is reasonable because at any point on the circumference of the bubble, the motion of domain wall in the direction of the electron flow is dictated by the total spin conservation of the whole system, as long as the dissipation parameters are small, i.e., \( \alpha \sim \beta \ll 1 \). On the other hand, the transverse motion is closely related to the topology of the bubble: \( V_\perp \) changes its sign as \( N_S \) does, and its magnitude decreases as \(|N_S| \) increases.
Let us compare the bubbles with $|N_S| = 1$ to the skyrmions. With the parameters assumed above, the collective-coordinate model predicts about five times larger transverse velocity for the bubble compared to the skyrmion, under the same charge current density and in the absence of impurities. This is because of smaller $\Gamma$ of the skyrmion. For both the bubble and skyrmion, the sign of $V_\perp$ is determined by that of $N_S$ in the same manner, and the longitudinal velocities are almost the same.

Below we present our micromagnetic simulations to discuss the above predictions in more quantitative fashion.

**Micromagnetic simulations.**— Here we numerically observe the current-driven motion of magnetic bubbles with four different skyrmion numbers: $N_S = \pm 1$ and $\pm 6$ (Fig. 3). The magnetic bubbles are prepared in equilibrium at the center of the same square thin films, and the in-plane charge current is applied (Fig. 4). The LLG equation \((8)\) is solved numerically by the Object-Oriented Micromagnetic Framework (OOMMF) simulation. In the simulations, we divide the sample of dimensions $600 \times 600 \times 8$ nm$^3$ into $3 \times 3 \times 8$ nm$^3$ unit cells, and again employ the typical material parameters for Co/Ni alloys; $\gamma = 1.76 \times 10^{11}$ Hz/T, the uniaxial perpendicular anisotropy constant $K = 4 \times 10^5$ J/m$^3$, and the exchange stiffness $A = 10^{-11}$ $\text{J/m}$. The other parameters are set to be the same with the ones used in the analytical calculation before. In the absence of charge current, the bubbles with $N_S = \pm 1$ and radius $\approx 103$ nm are realized under the magnetic fields $\mu_0 H_z = \pm 1.1$ mT applied in out-of-plane, while the ones with $N_S = \pm 6$ and radius $\approx 109$ nm are stabilized even in the absence of external field. As seen in Fig. 3, the linear $\chi$-dependence of the azimuthal magnetization angle along the circumference assumed in Eq. \((6)\) is well satisfied in both cases of $N_S = 1$ and 6. We estimate the center position and the radius by circular fitting to the calculated bubbles \(X = \sum_i (1-m^i_z) x f_i(m^i_z) / \sum_i (1-m^i_z) f_i(m^i_z), Y = \sum_i (1-m^i_z) y f_i(m^i_z) / \sum_i (1-m^i_z) f_i(m^i_z), \) and \(R = \sum_i (1-m^i_z) \sqrt{(x^i-X)^2 + (y^i-Y)^2} / \sum_i (1-m^i_z) f_i(m^i_z), \) where $i$ is the index for the unit cells, the weighing function $f_i(m^i_z) = 1$ when $m^i_z < 0.99$ but otherwise zero, and similarly $f_2(m^i_z) = 1$ when $m^i_z < 0.1$ but otherwise zero.

Fig. 4 (a) shows snapshots of time evolution of the four bubbles over 1 ns in the presence of charge current $j \approx 2.35 \times 10^{12}$ A/m$^2$. We don't observe appreciable deformation in the dynamical structures compared to the equilibrium ones; the magnetic profiles shown in Fig. 3 hold well under the present charge current density. In Fig. 4 (b), the trajectories of center positions $(X, Y)$ of each bubble during the same period as in the Fig. 4 (a) are plotted. It is clear that the bubbles with $N_S = 1$ and $-1$ are appreciably displaced perpendicular to the current-flow, and their directions are opposite with each other. On the other hand, the bubbles with $|N_S| = 6$ hardly move to the transverse direction. In the current-flow direction, the bubbles with $|N_S| = 6$ has traveled further than a couple of ten nanometers from the ones with $|N_S| = 1$.

In Fig. 2, the averaged velocities $V_\parallel$ and $V_\perp$ of the four bubbles obtained by the micromagnetic simulations are shown by open symbols. In order not to take into account the transient dynamics at the beginning of the motion, we evaluate the velocities of a bubble from the distance that the bubble travels from $t = 0.4$ to 1 ns. The differences in $V_\parallel$ between the bubbles with $N_S = 1$ and $-1$, and between the ones with $N_S = 6$ and $-6$, are smaller than the size of the symbols. Fig. 2 indicates the overall good agreement between the prediction by collective-coordinate model and the result of micromagnetic simulations; $V_\parallel$ changes its sign as does $N_S$, and its magnitude decreases as $|N_S|$ increases, while $V_\perp$ only shows the small dependence on $N_S$. The deviation of the theory from the simulation develops as the charge current density increases. The biggest deviation comes in $V_\perp$ for the bubbles with $|N_S| = 6$; the transverse motion of these bubbles turns out to be almost entirely suppressed [Fig. 4(b)]. The deviation may be attributed to i) the fact that the assumption of perfect cylinder with constant domain wall width isn’t well satisfied by the bubbles with $|N_S| = 6$, as seen in Fig. 3 (b), and ii) the magnetostatic effect due to the existence of the boundary while Eq. \((6)\) implicitly assumes an infinitely large thin film. We leave the systematic and complete investigation of these effects for the future work.

We also performed the same simulation (not shown) employing bubbles with $S = -1$, $Q = \pm 1$, $R \approx 109$ nm, and $|\mu_0 H_z| = 17$ mT, where appreciable dynamical distortions are observed; they move changing their shape into deforming ellipses. For $V_\parallel$ the simulation in these cases shows a quantitative agreement with the collective-coordinate model, but not for $V_\perp$. (e.g., $|V_\perp| \approx 4$ m/s when $j = 2.35 \times 10^{12}$ A/m$^2$.) However, the direction of the transverse motion is still consistent with the prediction by the analytical model, i.e., both a bubble with $S = 1$ and $Q = 1$ and one with $S = 1$ and $Q = -1$ get negative transverse velocities. This confirms that the sense of the transverse motion of a bubble is determined by $N_S$, not only by $Q$.

Lastly, let us make a short discussion regarding the comparison between the current- and field-induced bubble motions. When the bubble is driven by a magnetic field gradient, the deflection of the bubble propagation from the field gradient originates from the first term in Eq. \((6)\) while the third and fourth terms are relevant in the current-driven transverse motion. Despite this difference in the physical origins, the field-induced transverse velocity also decreases as $\propto |N_S|^{-1}$ with $|N_S|$ increasing. This fact indicates that the skyrmion number plays an essential role in the transverse dynamics of magnetic bubbles.

**Conclusion.**— We have presented analytical and numerical study on spin transfer torque acting on magnetic bubbles, and revealed its dependence on the bubble's
skyrmion number. Because magnetic bubbles, in contrast to vortices and skyrmions, can carry a skyrmion number whose absolute value being greater than the unity, they offer a ground for the study to find new connections between the dynamics and topology of magnetic textures. We have found that the transverse velocity of the bubble with respect to the current is closely related to the bubble’s topology, changing its sign and magnitude as those of the skyrmion number do. We hope this work will open a new route getting to the full understanding of the dynamics of topologically nontrivial magnetic structures.

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FIG. 1: Schematic of magnetic bubble structures in a thin film. Black arrows depict the direction of magnetization. In the bottom figure bubbles with four different skyrmion numbers are shown [(b) $N_S = 1$ with $S = 1$ and $Q = 1$, (c) $N_S = -1$ with $S = 1$ and $Q = 1$, (d) $N_S = -1$ with $S = 1$ and $Q = -1$, and (e) $N \sim R/\Delta$ with $S \sim R/\Delta$ and $Q = 1$]. See the main text for the definitions of the symbols.
FIG. 2: Longitudinal and transverse velocities of bubbles with four different skyrmion numbers $N_S = \pm 1$ and $\pm 6$. Solid lines plot Eq. (8), and the open symbols represent the averaged velocities obtained by micromagnetic simulations.
FIG. 3: Bubbles with (a) $N_S = 1$ and (b) $N_S = 6$, which are numerically stabilized in the absence of charge current. The magnetization points up inside the bubble, while points down outside it. We keep the distribution of the in-plane magnetization around the circumference the same when the polarity of the bubbles are reversed, i.e., when $N_S = -1$ and $-6$. 

(a) 

$N_S = 1$

$R \simeq 103$ nm

$\mu_0 H_z = 1.1$ mT

(b) 

$N_S = 6$

$R \simeq 109$ nm

$\mu_0 H_z = 0$
FIG. 4: (a) Snapshots of the simulated magnetization profiles at equilibrium ($t = 0$) and 1 ns after the charge current $j = 2.35 \times 10^{12}$ A/m$^2$ is applied, for four bubbles that carry different skyrmion number $N_S$. (b) Trajectories of the center position $(X, Y)$ of the four bubbles over the 1 ns. At $t = 0$, all the bubbles have its center located at $(X, Y) = (0, 0)$. 