Testing the X-ray variability of active galactic nuclei with the non-linear prediction method

Bożena Czerny1* and Harry J. Lehto2*

1Nicolaus Copernicus Astronomical Center, Bartycka 18, 00-716 Warsaw, Poland
2Turku University Observatory, Tuorla, Väisälän tie 20, FI-21500 Piikkiö, Finland

ABSTRACT
The analysis of eight EXOSAT X-ray light curves of six active galactic nuclei by non-linear prediction methods indicates that the observed variability is truly stochastic and is not caused by deterministic chaos. This result favours X-ray emission models with multiple centres of production of hot, possibly relativistic, electrons.

Key words: chaos – galaxies: active – galaxies: Seyfert – X-rays: galaxies.

1 INTRODUCTION
X-ray emission is clearly one of the key points in understanding the radio-quiet active galactic nuclei (AGNs). It comprises a substantial fraction of the total bolometric luminosity and is strongly variable on timescales as short as hours (see, e.g. Mushotzky, Done & Pounds 1993). Any AGN model must therefore explain both the level of X-ray emission and its rapid variability.

The longest (up to $2 \times 10^5$ s) uninterrupted light curves for short-time-scale variability studies in the 1–8 keV range have been provided by EXOSAT due to its unusual orbit. Longer time-scale variability has been studied by combining data from different satellites over several years.

Several characteristics of X-ray variability of AGN have emerged in these studies. The X-ray flux is strongly variable (e.g. McHardy 1988, Green, McHardy & Lehto 1993) in most sources. Power spectra are featureless and have a power-law shape $S \propto \nu^\alpha$, with $\alpha = -1$ over a broad range of frequencies ($10^{-4}$–$10^{-3}$ Hz) (McHardy & Czerny 1987; Lawrence et al. 1987; Lehto, McHardy & Abraham 1991; for a review see, e.g., McHardy 1989). The only features seen in these spectra are the expected flattening of the spectrum on time-scales of years, which is claimed for NGC 5506, and the signs of quasi-periodicity in NGC 5548 (Papadakis & Lawrence 1993) and NGC 4051 (Papadakis & Lawrence 1995). Note that the claimed periodicity in NGC 6814 (Mittaz & Branduardi-Raymont 1989; Done et al. 1992) was subsequently discovered to be the property of the galactic X-ray source contaminating the X-ray measurements in the direction of this AGN (Madejski et al. 1993).

As the primary emission strongly dominates in the EXOSAT energy range we can assume that the conclusions apply directly to the primary power-law component of the X-ray emission.

An observed featureless power spectrum is expected in two qualitatively different dynamical systems:

(A) a system that consists of a large number of uncorrelated (weakly correlated) elements varying (almost) independently; and
(B) one in which the global evolution of the system is described by three or more non-linear differential equations showing deterministic chaos in their behaviour.

These two interpretations lead to two different physical pictures.

Case (A) can be interpreted, for example, as randomly generated shocks in outflowing (or accreting) gas, or randomly appearing reconnections of large-scale magnetic field lines, in close analogy to phenomena in solar corona. In this case the emitting medium is clearly clumpy and the emission itself can be modelled as shot noise (e.g. Lehto 1989).

Case (B) could correspond to coherent variability of a single cloud of electrons, e.g. some generalization of an optically thin geometrically thick disc, aspherically accreting hot (possibly pair-dominated) plasma or an instability of a single shock at the base of a jet-forming region.

Sugihara & May (1990) suggested that non-linear prediction methods are (in principle) capable of distinguishing between cases (A) and (B).

The methods (Tsonis & Elsner 1992) are based on the study of the decay of short-term correlations in the light curve. Exponential decay of the correlation is characteristic of a chaotic signal with a strange attractor while a power-law decay is expected from shot noise (more precisely, from the so-called fractional Brownian motion representing stochastic phenomena with power-law power spectra).
We apply this new method to a number of long EXOSAT observations of Seyfert galaxies to which the correlation integral method has been applied previously by Lehto, Czerny & McHardy (1993, hereafter LCMH).

## 2 Method

We apply the non-linear prediction methods of Tsonis & Elsner (1992) to investigate the characteristics of X-ray variability of several AGNs that show broadband 1/f

The essence in these methods is to break up the measured series into short data segments of consecutive data points and then to analyze the statistical properties of these shorter data. They are far more efficient than Fourier methods in studying the behaviour of a system with short-term correlations.

The method compares shapes of two data strings of length \(d\). The mathematical expression of this procedure is the calculation of the distance between the two strings, or vectors \(X = (x_1, x_2, \ldots, x_d)\) and \(Y = (y_1, y_2, \ldots, y_d)\). Usually the distance \(r\), or the norm, is defined as the \(L_2\) norm of the difference vector

\[
r = \left[ \sum_{i=1}^{d} (x_i - y_i)^2 \right]^{1/2},
\]

in close analogy to \(\chi^2\) fit of two curves. We adopt this definition in our paper. Other norms can also be used, for example the Chebychev or \(L_\infty\) norm,

\[
r^* = \max_{i=1,d} |x_i - y_i|,
\]

The non-linear prediction method is used in an attempt to predict the position of the point separated by time interval \(T\) from the sequence \(Y\) on the basis of a known library data set \(X_i\).

This consists of four steps: (i) identifying the sequences from the library \(X_i\) most similar to the one being considered \(Y\) (by minimizing the norm between); (ii) identifying values the strings in the library would have after a time-step \(T\); (iii) calculating the predicted value using the values from (ii); and (iv) finally correlating (iii) with the observed values.

We have used three different methods for step (iii).

(i) Nearest point method. Identify the nearest \(X\) as the best predictor sequence. Use that sequence alone to predict \(Y\) after a time interval \(T\).

(ii) Linear interpolation method. Identify the \(d+1\) closest sequences. Use linear interpolation in \(d\)-dimensional space to obtain a predicted value after a time interval \(T\) from the predictor sequences.

(iii) Exponential weight method. Identify \(d+1\) closest sequences. Predict \(Y\) at time interval \(T\) by using the observed values of the \(X\) sequences, but now interpolate them with a weight of \(\exp(-r)\) for each predictor vector, where \(r\) is the distance between the given sequence and \(Y\).

We divide the data sequence into two halves. The first part forms the library and the second is used to test the quality of prediction. Since we actually know the values, we have a set of pairs \((p_i, f_i)\) consisting of the predicted and known values for step (iv).

The quality of prediction can be evaluated by calculating the correlation coefficient \(R\) between the series of predicted \(p\) and known \(f\) values:

\[
R(T) = \frac{\langle p \cdot f \rangle - \langle p \rangle \langle f \rangle}{\sigma_p \sigma_f},
\]

where \(\langle \cdot \rangle\) denote the average of the quantity over the series and \(\sigma\) is the standard deviation of the mean value of the time series.

If the data is dominated by short-term correlations the quality of prediction decreases with time-step \(T\). Tsonis & Elsner (1992) show that for a chaotic signal (in a sense of deterministic chaos)

\[
1 - R(T) \propto \exp(2KT)
\]

and in the case of fractal Brownian motion (shot noise)

\[
1 - R(T) \propto T^{2H}
\]

Here \(K\) is the Kolmogorov entropy (Kolmogorov 1959), which is the sum of the positive Lyapunov exponents (see, e.g., Schuster 1989) and \(H\) is related to the slope \(a\) of the power density spectrum through the relation \(a = 2H + 1\).

By correlating \(\log(1-R)\) against \(T\) and against \(\log(T)\) we obtain \(R_L\) (deterministic) and \(R_S\) (stochastic), two further correlation coefficients which tell us how well each of the two predictive models describes the data. We identify the preferred system by selecting the system with the larger of \(R_L\) and \(R_S\).

## 3 Results

### 3.1 Tests of the method using NGC 4051 ME

We tested technical details such as the choice of string length, \(d\), the binning, the method of prediction and the time range of prediction on the EXOSAT medium energy (ME) X-ray light curve of NGC 4051.

Binning of the data is the standard approach to increasing the signal-to-noise (S/N) ratio. This is done at an expense of the available dynamical range.

We used generally 200-s bins, corresponding to an S/N ratio of 6 in NGC 4051. We made also some further tests with 50-s bins.

The value of the correlation coefficient depends on the assumed length of the string \(d\). Therefore we first calculated this coefficient for \(d\) ranging from 1 to 20 and the prediction time-step \(T = 1\) (one bin). We show this dependence in Fig. 1 using the three different prediction methods. The linear interpolation method was used only up to \(d = 4\), as the search for the smallest simplex containing a given vector (cf. Sugihara & May 1990) is time consuming.

We can see that the third method, i.e. that which uses the exponential weight, gives the highest values of the correlation coefficient and relatively small wiggles around the systematic trend. Therefore, in all the following computations we use the exponential weight method to compute the predicted values.

The value \(d_{\text{max}}\) corresponding to the best prediction is picked up for the purpose of further calculations for larger \(T\).

We also tested the prediction for the same time interval (i.e. 200 s) but with bins of 50 s. The highest values for the correlation coefficient were met for strings that were longer...
by approximately a factor of 4, as expected, but the values of the correlation coefficient were lower. It was also lower for the prediction for a single time-step (i.e. 50 s). Too-strong binning will lead to a loss of information about the correlations in the system. Assuming a time-step of 800 s (binning again by a factor of 4) results also in the decrease of the correlation coefficient of the prediction for a single time-step (i.e. 800 s). Our initial 200-s binning appears, therefore, to be reasonable.

Next we calculate the prediction quality as a function of prediction time. We use 200-s bins and strings of nine points. Fig. 2 shows the result of plotting log (1 - R) versus log T and log (1 - R) versus T.

The distinction between the two cases is not very profound, but a clear tendency is seen to favour the log–log dependence, which corresponds to the fractal Brownian-type shot noise variability. The correlation coefficient $R_c$ is 0.998, and the coefficient $R_d$ is equal to 0.958.

We test the dependence of the measured correlation decay on the choice of the string length. In Fig. 3 we show the same plot as in the lower panel of Fig. 2, but derived using different string length $d_{\text{max}}$. We see that for the string length $d_{\text{max}} = 9$ the plot is very smooth whilst shorter length values introduce more wiggling. However, in all cases the correlation coefficient $R_c$ is greater than $R_d$ and the slopes of the curves are similar. Therefore the use of the string length giving the best prediction for a single time-step seems to be well justified.

We may be tempted to obtain a more significant result by increasing the dynamical range of the study, that is making predictions for even longer time-steps. However, the decay of correlations takes us into the range dominated by the data noise after some 15 steps, i.e. an interval of 3000 s for this source.

Since the shot noise is favoured, the slope, $2H$, of the curve log (1 - R) versus log T should be related to the slope of the power spectrum, $a$, through the relation $a = 2H + 1$. As $2H = 0.42$ and the power spectrum slope determined by Papadakis & Lawrence (1995) for this light curve is $a = 1.46^{+0.10}_{-0.05}$, the two numbers are in agreement.
3.2 Simulated light curves

In order to test the sensitivity of the method we use two computer-generated light curves of equal lengths. The first one is the Lorenz attractor (Lorenz 1963), an example of a source exhibiting deterministic chaos. We use a time-step of 0.02 and the coordinate X to represent the light curve as in our previous paper (LCMH). The correlation decay plot for this light curve is shown in Fig. 4.

As expected, the character of variability is better described by exponential decay of correlations, and the coefficient $R_x$ is smaller than $R_e$ (0.949 and 0.986 respectively, for the length of light curve of 2000 points).

The second computer-generated light curve (Fig. 5) is an example of shot noise (Lehto 1989). Again, the non-linear prediction method unveils correctly the nature of the light curve ($R_e = 0.99$, $R_x = 0.92$).

3.3 EXOSAT background

The effects of background subtraction may, in principle, influence the results of a variability study (see e.g. Fiore & Massaro 1989). The upper panel of Fig. 6 shows the decay of correlations for the NGC 4051 ME light curve (background subtracted) and the background itself. Clearly the behaviour of the two is different. The level of correlations in the background is quite high and it almost does not decay with the increase of the prediction time. The formal fit slightly favours the interpretation of deterministic chaos, as the $R_e$ and $R_x$ coefficients for the ME background during the observation of NGC 4051 are equal to 0.904 and 0.878, respectively. Such long-range correlations may reflect some systematic trends of an unknown nature. It is encouraging, however, that correlations present in the signal are markedly different from those present in the background.

In the case of the low-energy (LE) EXOSAT light curve of NGC 4051 (lower panel of Fig. 6) the decay of correlations for the source is smooth and nicely follows a straight line on the log–log diagram; the behaviour of the background is...
Figure 6. The decay of correlations as a function of prediction time-step for the source light curve (NGC 4051) (squares) and the background (circles) from the two EXOSAT instruments: ME (upper panel) and LE (lower panel). Straight lines show the best fit to the decay of correlations under the assumption of the stochastic nature of the signal.

Figure 7. The decay of correlations as a function of the prediction time-step for sources from class 1: filled squares – 3C 273; filled triangles – NGC 4151 in 1983; open squares – NGC 4151 in 1985; and open triangles – Mrk 335.

Table 1. The correlation coefficients for power-law and exponential decay of correlations in EXOSAT light curves.

| Name       | Obs. time ($10^3$ s) | $R_d$ | $R_s$ | Class |
|------------|----------------------|-------|-------|-------|
| Mkn335     | 67                   | 0.557 | 0.562 | 1     |
| NGC4151(93)| 62                   | 0.580 | 0.753 | 1     |
| NGC4151(95)| 37                   | 0.544 | 0.741 | 1     |
| 3C273      | 142                  | 0.557 | 0.750 | 1     |
| MCG6-30-15 | 178                  | 0.977 | 0.993 | 2     |
| NGC5506    | 239                  | 0.947 | 0.984 | 2     |
| NGC4051LE  | 202                  | 0.843 | 0.959 | 2     |
| NGC4051ME  | 196                  | 0.954 | 0.999 | 2     |

3.4 EXOSAT light curves

The non-linear prediction method was applied to a number of long EXOSAT X-ray light curves. The data were binned in 200-s bins. The exponential weight method was used for prediction. We excluded the light curve of NGC 6814 because it is heavily contaminated by the X-ray emission from a variable galactic source (Madejski et al. 1993).

Seven of the light curves are from the EXOSAT ME instrument which is sensitive to 1–8 keV radiation. One light curve of NGC 4051 is from the EXOSAT LE instrument operating in the 0.25–1.5 keV band.

The correlation coefficients $R_d$, $R_s$ of the light curves are shown in Table 1 with some other relevant information.

The correlation coefficients of the light curves appear to fall into two classes. The same separation is also evident if plotting the decay of prediction quality as a function of adopted time-step.

Class 1 contains Mrk 335, NGC 4151 and 3C 273 (Fig. 7). All of these sources are characterized by rather low values ($|R| \leq 0.2$) of both correlation coefficients for time-steps greater than 1 bin although the prediction for one bin is sometimes significant – the same effect is observed in the behaviour of the background light curve (see Section 3.2). Visually, the four light curves appear to show little variation.
on short time-scales. On time-scales of hours slowly varying trends are apparent.

No conclusion on the character of variability in these sources can be drawn on formally fitted coefficients \( R_d \) and \( R_s \), which are not significant (lower than \(-0.8\)). The ratio of the two coefficients is greater than 1, in all four light curves, which may favour stochastic behaviour. However, the same is true for the background light curve. It appears that the low correlation coefficients can be understood as being caused by the dominance of Poisson noise over the real variability of the source on short time-scales.

Class 2 contains NGC 4051 (both ME and LE), MCG 6-30-15 and NGC 5506 (Fig. 8). All of these sources show a range of prediction time-steps with high prediction quality, and the decay of prediction quality is smooth. The correlation coefficients \( R_d \) and \( R_s \) are highly significant, and the ratio is larger than 1.

This result means that (at least in class 2 sources) the variability is caused by a truly stochastic phenomenon and not by the deterministic chaos.

The non-linear prediction method used in this paper is basically different from the correlation integral method used for these sources in our previous paper (LCMH). Therefore it is interesting to note that the classification of the sources proposed by us has not changed much. Class 1 sources from LCMH (white noise) all found their place in a new class 1 (sources without good description of short-term correlations). A source classified as 2b in LCMH (NGC 4151) moved to a new class 1. All old class 2 sources (shot noise type behaviour) are again in class 2 and the stochastic character of their variability found strong support from the non-linear prediction method.

The key result is that the light curve NGC 4051 LE, suspected of chaotic behaviour on the basis of the correlation integral method, has now revealed itself as just another stochastic phenomenon.

It is also interesting to note that the character of variability of the source NGC 4051 in the LE and ME bands is very similar; the slopes, \( 2H \), for the two light curves are equal to 0.43 and 0.42, respectively, as if the (at least variable fraction of) radiation comes from the same region and the same emission mechanism. This conclusion was by no means obvious, as in many Seyfert galaxies the soft X-ray band is dominated by the soft X-ray excess (see e.g. Czerny, Zycki & Loska 1995) and the presence of an excess, albeit not a strong one, was also invoked in the case of NGC 4051 (e.g. Walter & Fink 1993).

What is more, the slope of the power spectrum of the LE curve calculated by Papadakis & Lawrence (1995) is significantly steeper than that of the ME curve. On the other hand, the conclusion of their paper was that a power law is not a good representation of the power spectrum of the LE curve. Visual inspection of their fig. 10 shows that if a single point at \( \log(v) \sim -4.2 \) is removed the mean slope would be again consistent with the ME slope and with our result. Perhaps there is an additional power at around this frequency as well. An extension of the power spectrum towards longer time-scales based on combined observations on a number of satellites operating recently might give the answer.

4 DISCUSSION

4.1 Deterministic chaos versus stochastic signal

A number of works have been already devoted to the search for low-dimensional chaos in different astronomical objects, e.g. Voges, Atmanspacher & Scheinbergraber (1987), McHardy & Czerny (1987), Canizzo & Goodings (1988), Lochner, Swank & Szymkowiak (1989), Norris & Matilsky (1987), Fiore & Massaro (1989), Canizzo, Goodings & Mattei (1990), Harding, Shinbrot & Cordes (1990), Kollath (1990), Krolik, Done & Madejski (1993), LCMH.

However, the methods used do not provide the unique answer to the question unless they are supplemented by methods sensitive to the difference between low-dimensional deterministic chaos and stochastic phenomenon. One such method (the non-linear prediction test of Tsonis & Elsner 1992) was applied in this paper whilst two other methods (phase randomization method of Theiler et al. 1992 and the signal differentiation method of Provenzale et al. 1992) were used for analysis of the optical light curve of quasar 3C 345 by Provenzale, Vio & Cristiani (1994).

In both cases the tests indicated the stochastic nature of the astronomical phenomenon. This shows clearly the importance of the supplementary tests; any reliable claims of deterministic chaos have to pass at last one such test.

4.2 Consistency of the thermal Comptonization model of X-ray emission with the observed variability

On the basis of their results on the variability of NGC 4051 Papadakis & Lawrence (1995) concluded that thermal Comptonization models are ruled out since they always predict steeper ME spectra than LE spectra whilst the observa-

![Figure 8. The decay of correlations as a function of the prediction time-step for sources from class 2: filled squares - NGC 5506; filled triangles - MGC 6-30-15; open squares - NGC 4051 ME; open triangles - NGC 4051 LE.](https://academic.oup.com/mnras/article-abstract/285/2/365/1151181)
tions give the opposite effect. Since the thermal Comptonization is currently the most popular mechanism (see e.g. Zdziarski et al. 1995) we re-discuss this conclusion within the light of our results.

Since the correlation slope, $2H$, calculated for the ME and the LE curves (see Section 3.4) are the same our conclusion is that the character of variability (and the mean slopes of the power spectra) are actually the same for both light curves. In the light of the Compton scattering model it implies that the multiple travel time within the hot plasma is shorter than the time-scale for variations of the plasma parameters.

As the mean energy of a photon in the LE band is 0.2 keV, and it is 4 keV in the ME band (Papadakis & Lawrence 1995), change in the photon energy by a factor of 20 is required to change an LE photon into an ME one. The relative energy change in a single scattering is approximately given by

$$\frac{\Delta E}{E} = \Theta(1 + \Theta), \quad \Theta = \frac{4kT_e}{m_ec^2},$$

where $T_e$ is the electron temperature of the thermal plasma. The number $n$ of scatterings required to interchange LE photons into ME photons can be therefore estimated from the relation

$$\left(\frac{\Delta E}{E}\right)^n = 20.$$  

This number is about 8 if the temperature of the plasma is $\sim 100$ keV, and only about 2 if it is $\sim 200$ keV.

If the plasma temperature is high, therefore, the measured delay does not have to be much smaller than a factor of a few times the light travel time of the emitting region and the lack of noticeable delay between the LE and the ME light curves of NGC 4051 ($< 60\,\text{s}$, Papadakis & Lawrence 1995) simply translates into the size of the emitting region of the order of a fraction of a Schwarzschild radius adopting the value of the mass of the central black hole equal to $7 \times 10^6 M_{\odot}$.

It may well be consistent with the models of relatively flat corona, the height being a fraction of the radius and the emissivity strongly peaked close to the horizon of the black hole. Exceptionally detailed studies of the Kz iron line in Seyfert galaxy MCG 6-30-15 (Iwasawa et al. 1996) support a similar view, which also strongly suggests that the central black hole is rapidly rotating.

4.3 Constraints for stochastic models from the power spectra

The conclusion from the analysis of the character of X-ray light curves of (predominantly) radio-quiet AGN can be applied to the detailed models of the X-ray emission.

Below we discuss the relevance of our conclusions to one particular class of models that seem to be a good prospect, namely those that involve magnetized coronae.

The stochastic nature of variability of X-ray radiation from active galactic nuclei agrees well with the coronal nature of emission proposed by Galeev, Rosner & Vayana (1979), de Vries & Kuipers (1992), Haardt, Maraschi & Ghisellini (1994), Stern et al. (1995). For example, in the solar corona the energy is stored in the form of magnetic field loops and then released during reconnections, which results in a solar flare event (see e.g. Demoulin et al. 1993). The efficiency of X-ray production is not very high in this case: only some 10 per cent of the flare energy is emitted in the form of X-ray radiation (Confield et al. 1980). However, the strength of the magnetic field in an accretion disc may be considerably higher, as the rotational velocity is very high (Keplerian) and field amplification may operate far more efficiently.

The stochastic nature of the variability does not prove that the magnetic field growing as a result of dynamo action within the disc is the original cause of the variability, as some corona models were assumed to be powered by coronal accretion itself (e.g. Zycki et al. 1995, Witt et al. 1996) and any instabilities growing within such a corona would give a basically similar effect.

Unfortunately, most of the models deal mostly with the stationary solutions (frequently at a single representative radius) and the attention is focused on the radiation spectra, so at present it is impossible to say whether or not these models are consistent with variability studies.

The only model that can be directly tested against the time-dependent properties is the clumpy corona model developed by Haardt, Maraschi & Ghisellini (1994).

In the model of Haardt et al. (1994) the evolution of a single clump consists of two phases. The first phase, invisible and much longer, is the charge time when the energy accumulates within a clump owing to the magnetic field. The second phase is the rapid discharge accompanied by a flash of radiation. The charge time in this model depends on the radial position of the clump formed ($t_c \sim r_c^2$), whilst the discharge time is independent of radius. Therefore the set of shots consist of the shots with a range of amplitudes ($L_{\text{shot}} \sim r_c^2$) and the same decay times. The power spectra of such a shot noise are given by the formula (Lehto 1989; Papoulis 1984)

$$F(\omega) = 2\pi^2 t_c^2 \delta(\omega) + \frac{\lambda}{\omega^2 + l/\omega^2},$$

where $1/\lambda$ is the average time interval between shots. Such a power spectrum has a shape very different from a power law with slope $\sim 1.5$. More complex coupling between the matter and the magnetic field must be invoked to obtain agreement with the observed power spectra.

4.4 Future observations

The method of time-series analysis presented in this paper could be improved by longer data streams (several thousand points) and a low signal-to-noise ratio (S/N $\sim 1$). The continuous sections of the data should last at least $\sim 10-20$ times longer than the separation of two significant data points.

5 CONCLUSIONS

The non-linear prediction method applied to the EXOSAT X-ray light curves of Seyfert galaxies indicates the following.
ACKNOWLEDGMENTS

We are deeply grateful to Ian McHardy for many helpful discussions, as well as for providing us with the extracted background light curves used in Section 3.3. We also want to acknowledge the considerable help of Piotr Zycki in formulating the numerical scheme for the search of the simplices necessary for the use of linear interpolation methods. We thank Andrzej Zdziarski and Andy Lawrence, the referee, for very helpful comments on the original version of the manuscript. This project was partially supported by the grants numbers 2P 304 01004 and 2P 03D 00410 of the Polish State Committee for Scientific Research.

REFERENCES

Canizzo J. K., Goodings D. A., 1988, ApJ, 334, L31
Canizzo J. K., Goodings D. A., Mattei J. A., 1990, ApJ, 357, 235
Clavel J., Wamsteker W., Glass I. S., 1989, ApJ, 337, 236
Confield R. C. et al., 1980, in Sturrock P. A., ed., Solar Flares. Colorado Associated Univ. Press, Boulder, p. 231
Czerny B., Zycki P. T., Loska Z., 1995, in Böhringer H., Morfill G. E., Trümper J. E., eds, Proc. 17th Texas Symp., Relativistic Astrophysics. The New York Academy of Sciences, New York, p. 539
Demoulin P., van Driel-Gesztelyi L., Schmieder B., Henoux I., de Vries M., Kuijpers J., 1992, A&A, 265, 77
Done C. et al., 1992, ApJ, 400, 138
Done C., Pounds K. A., Nandra K., Fabian A. C., 1995, MNRAS, 275, 417
Fiore F., Massaro E., 1989, in Belvedere G., ed., Accretion Disks and Magnetic Fields in Astrophysics. Kluwer, Dordrecht, p. 267
Galeev A. A., Rosner R., Vayana G. S., 1979, ApJ, 228, 298
Green A. R., McHardy I. M., Lehto H. J., 1993, MNRAS, 265, 664
Haardt F., Maraschi L., Ghisellini G., 1994, ApJ, 432, L95
Harding A. K., Shinbrot T., Cordes J. M., 1990, ApJ, 353, 588
Iwasawa K. et al., 1996, MNRAS, 282, 1038
Kollath Z., 1990, MNRAS, 247, 377
Kolmogorov A. N., 1959, Dokl. Acad. SSR, 124, 754
Krolik J., Done C., Madejski G., 1993, ApJ, 402, 432
Lawrence A., Pounds K. A., Watson M. G., Elvis M. S., 1987, Nat, 325, 692
Lehto H. J., 1989, in Hunt J., Battrick B., eds, Proc. 23rd ESLAB Symp., Two Topics in X-ray Astronomy. ESA, Noordwijk, p. 49
Lehto H. J., McHardy I. M., Abraham R. G., 1991, in Miller R., Wiita P. J., eds, Variability of Active Galactic Nuclei. Cambridge Univ. Press, Cambridge, p. 256
Lehto H. J., Czerny B., McHardy I. M., 1993, MNRAS, 261, 123 (LCMH)
Lochner J. P. D., Branduardi-Raymont G., 1989, MNRAS, 238, 1029
McHardy I. M., 1988, Mem. Soc. Astron. Ital., 59, 239
McHardy I. M., 1989, in Hunt J., Battrick B., eds, Proc. 23rd ESLAB Symp., Two Topics in X-ray Astronomy. ESA, Noordwijk, p. 1111
McHardy I. M., Czerny B., 1987, Nat, 325, 696
Mushotzky R. F., Done C., Pounds K. A., 1993, ARA&A, 31, 717
Norris J. P., Matilsky T. A., 1989, ApJ, 346, 912
Papadakis I. E., Lawrence A., 1993, Nat, 361, 233
Papadakis I. E., Lawrence A., 1995, MNRAS, 272, 161
Papoulias A. P., 1984, Probability, Random Variables and Stochastic Processes, 2nd edn. McGraw-Hill, Singapore
Provenzale A., Smith L. A., Vio R., Murante G., 1992, Physica D, 58, 31
Provenzale A., Vio R., Cristiani S., 1994, ApJ, 428, 591
Schuster H. G., 1989, Deterministic Chaos, 2nd edn. Weinheim
Stern B. E., Putanan J., Svensson R., Sikora M., Begelman M. C., 1995, ApJ, 449, L13
Sugihara G., May R., 1990, Nat, 344, 734
Theiler J., Galdrikian B., Longtin A., Eubank S., Farmer J. D., 1992, in Casdagli M., Eubank S., eds, Nonlinear Modeling and Forecasting, Addison Wesley, California, p. 163
Tsonis A. A., Elsner J. B., 1992, Nat, 358, 217
Voges W., Atmanspacher H., Scheingraber H., 1987, ApJ, 320, 794
Walter R., Fink H. H., 1993, A&A, 275, 417
Witt F., Czerny B., Zycki P. T., 1996, MNRAS, in press
Zdziarski A. A., Johnson W. N., Done C., Smith D., McNaron-Brown K., 1995, ApJ, 438, L63
Zycki P. T., Collin-Souffrin S., Czerny B., 1995, MNRAS, 277, 70

APPENDIX A: DATA REQUIREMENTS

Non-linear prediction methods can be used to study the decay of short-term correlations in the data and to discriminate whether X-ray emission is due to global but highly non-linear behaviour of the emission region or is due to multiple flares.

The presence of noise, however, limits the possibilities for determining the type of behaviour. The error of prediction consists of both the error intrinsic to the dynamical character of the system and the statistical error given as an S/N ratio.

The intrinsic error can again be characterized by the rate of decay of correlations, \[ 1 - R(T) \] and depends on the decay law (power law or exponential). The maximum timestep of prediction is therefore limited by the condition
This means that the data allow us to study the correlation up to

\[
T = \frac{1}{2K} \ln \left( \frac{1}{A^2} \left[ 1 - (N/S)^2 \right] \right) \quad (A2)
\]

for exponential decay of correlation and

\[
T = \left\{ \frac{1}{B} \left[ 1 - (N/S)^2 \right] \right\}^{1/2H} \quad (A3)
\]

for power-law decay of correlations. Here N/S stands for noise-to-signal ratio and \(A, B, H\) and \(K\) characterize the intrinsic source properties. These two formulae allow us to estimate the improvement of the data quality with an increase of the S/N ratio.