Amplitude analysis of $D^+_s \to \pi^+\pi^0\eta$ and first observation of the pure $W$-annihilation decays $D_s^- \to a_0(980)^+\pi^0$ and $D^+ \to a_0(980)^+\pi^0$
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We present the first amplitude analysis of the decay $D_s^+ \to \pi^+ \pi^0 \eta$. We use an $e^+e^-$ collision data sample corresponding to an integrated luminosity of 3.19 fb$^{-1}$ collected with the BESIII detector at a center-of-mass energy of 4.178 GeV. We observe for the first time the pure W-annihilation decays $D_s^+ \to a_0(980)^0 \pi^0$ and $D_s^+ \to a_0(980)\pi^+\pi^-$. We measure the absolute branching fractions $B(D_s^+ \to a_0(980)^0\pi^0, a_0(980)^0 \to \pi^+\pi^-\eta) = (1.46 \pm 0.15_{\text{stat.}} \pm 0.23_{\text{sys.}})\%$, which is larger than the branching fractions of other measured pure W-annihilation decays by at least one order of magnitude. In addition, we measure the branching fraction of $D_s^+ \to \pi^+ \pi^0 \eta$ with significantly improved precision.

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The theoretical understanding of the weak decay of charm mesons is challenging because the charm quark mass is not heavy enough to describe exclusive processes with a heavy-quark expansion. The W-annihilation (WA) process may occur as a result of final-state-interactions (FSI) and the WA amplitude may be comparable with the tree-external-emission amplitude [1,3]. However, the theoretical calculation of the WA amplitude is currently difficult. Hence measurements of decays involving a WA contribution provide the best method to investigate this mechanism.

Among the measured decays involving WA contributions, two decays with $VP$ mode, $D_s^+ \to \omega\pi^+$ and $D_s^+ \to \rho^0\pi^+$, only occur through WA, which we refer to as ‘pure WA decay’. Here $V$ and $P$ denote vector and pseudoscalar mesons, respectively. The branching fractions (BFs) of these pure WA decays are at the $\mathcal{O}(0.1\%)$ [5]. These BF measurements allow the determination of two distinct WA amplitudes for $VP$ mode. In addition, they improve our understanding of SU(3)-flavor symmetry and CP violation in the charm sector [3]. However, for $SP$ mode, where $S$ denotes a scalar meson, there are neither experimental measurements nor theoretical calculations of the BFs.

Two decays with $SP$ mode $D_s^+ \to a_0(980)^0\pi^0$ and $D_s^+ \to a_0(980)^0\pi^+$ are pure WA decays if $a_0(980)^0$- $f_0(980)$ mixing is ignored. Their decay diagrams are shown in Fig. 1. In this Letter, we search for them with an amplitude analysis of $D_s^+ \to \pi^+\pi^0\eta$. We also present improved measurements of the BFs of $D_s^+ \to \pi^+\pi^0\eta$ and $D_s^+ \to \rho^0\eta$ decays. Throughout this Letter, charge conjugation and $a_0(980) \to \pi\eta$ are implied unless explicitly stated.

![Diagram](image)

FIG. 1. $D_s^+ \to a_0(980)^0(\pi^0(+)\pi^0(-))$ decay topology diagrams, where the gluon lines can be connected with the quark lines in all possible cases and the contributions from FSI are included.

We use a data sample corresponding to an integrated luminosity of 3.19 fb$^{-1}$, taken at a center-of-mass energy of 4.178 GeV with the BESIII detector located at Beijing Electron Position Collider [7]. The BESIII detector and the upgraded multi-gap resistive plate chambers used in the time-of-flight systems are described in Refs. [8] and [9], respectively. We study the background and determine tagging efficiencies with a generic Monte Carlo (GMC) sample that is simulated with GEANT4 [9]. The GMC sample includes all known open-charm decay processes, which are generated with CONEXC [10] and EVTGEN [11], initial-state radiative decays to the $J/\psi$ or $\psi(3686)$, and continuum processes. We determine signal efficiencies from Monte Carlo (MC) samples of $D_s^+ \to \pi^+\pi^0\eta$ decays that are generated according to the amplitude fit results to data reported in this Letter.

In the data sample, the $D_s$ mesons are mainly produced via the process of $e^+e^- \to D_s^+D_s^-$, $D_s^- \to \gamma D_s^-$; we refer to the $\gamma$ directly produced from the $D_s^-$ decay as $\gamma_{\text{direct}}$. To exploit the dominance of the $e^+e^- \to D_s^+D_s^-$ process, we use the double-tag (DT) method [14]. The single-tag (ST) $D_s^-$ mesons...
are reconstructed using seven hadronic decays: \(D^\pm \rightarrow K^0\bar{K}^-, D^\pm \rightarrow K^+K^-\pi^-, D^\pm \rightarrow K^0_sK^{0}\pi^0, D^\pm \rightarrow K^+K^-\pi^0\), \(D^\pm \rightarrow K^+K^-\pi^-\pi^0\), \(D^\pm \rightarrow \pi^+\pi^-\eta\), and \(D^\pm \rightarrow \eta\pi^0\). A DT is formed by selecting a \(D^+_s \rightarrow \pi^+\pi^0\eta\) decay in the side of the event recoiling against the \(D^-\) tag. Here, \(K^0_s\), \(\pi^0\), \(\eta\) and \(\eta'\) are reconstructed using \(\pi^+\pi^-\gamma\), \(\gamma\gamma\), \(\pi^+\pi^-\eta\) channels, respectively. The selection criteria for charged tracks, photons, \(K^0_s\) and \(\pi^0\) are the same as those reported in Ref. [13].

The \(\eta^{(s)}\) candidate is required to have an invariant mass of \(\gamma\gamma\) (\(\pi^+\pi^-\eta\)) combination in the interval [0.490, 0.580] ([0.938, 0.978]) GeV/c^2. The invariant mass of the tagged (signal) \(D^+_s\) candidates \(M_{\text{tag}}\), \(M_{\text{sig}}\) are required to be in the interval [1.90, 2.03] GeV/c^2 ([1.87, 2.06] GeV/c^2). For the ST \(D^+_s\) mesons, the recoil mass \(M_{\text{rec}}\) is expressed as:

\[
M_{\text{rec}} = \sqrt{(E_{\text{tot}} - \sqrt{\mathbf{p}_{D_s}^2 + m_{D_s}^2} - \sqrt{\mathbf{p}_{D_s}^2 + m_{D_s}^2})^2}
\]

It is required to be within the range [2.05, 2.18] GeV/c^2 to suppress events from non-\(D^+_s\) processes. Here, \(E_{\text{tot}}, \mathbf{p}_{D_s}\) is the four-momentum of the colliding \(e^+e^-\) system, \(\mathbf{p}_{D_s}\) is the three-momentum of the \(D_s\) candidate and \(m_{D_s}\) is the \(D_s\) mass [2]. For events with multiple tag candidates for a single tag mode, the one with a value of \(M_{\text{rec}}\) closest to \(m_{D_s}\) is chosen. If there are multiple signal candidates present against a selected tag candidate, the one with a value of \((M_{\text{tag}} + M_{\text{sig}})/2\) closest to \(m_{D_s}\) is accepted.

We perform a seven-constraint (7C) kinematic fit to the selected DT candidates for two reasons. First, to successfully perform an amplitude analysis, the 7C fit ensures that all events fall within the Dalitz plot. Second, it allows the selection of the \(\gamma_{\text{direct}}\) candidate. In the 7C fit, aside from constraints arising from four-momentum conservation, the invariant masses of the \((\gamma\gamma)\), \((\gamma\eta)\), and \(\pi^+\pi^0\eta\) combinations used to reconstruct the signal \(D^+_s\) candidate are constrained to the nominal \(\pi^0\), \(\eta\) and \(D^+_s\) masses [2], respectively. The \(\gamma_{\text{direct}}\) candidate used in the 7C fit that produces the smallest \(\chi^2\) is selected. We require \(\chi^2\) to be less than 1000 to avoid introducing a broad peak in the \(M_{\text{sig}}\) background distribution arising from events that are inconsistent with the signal hypothesis. To further suppress the background, we perform another 7C kinematic fit, referred to as the ‘7CA fit’, by replacing the signal \(D^+_s\) mass constraint with a \(D^+_s\) mass constraint in which the invariant mass of either the \(D^+_s\) or \(D^-_s\) candidate and the selected \(\gamma_{\text{direct}}\) is constrained to the nominal \(D^+_s\) mass [5]. We require one of the values of the \(\chi^2\) to be less than 500, to ensure reasonable consistency with the signal hypothesis. To suppress the background associated with the fake \(\gamma_{\text{direct}}\) candidates in the signal events, we veto events with cost \(\theta_{\text{c}} < 0.998\), where \(\theta_{\text{c}}\) is the angle between the \(\eta\) momentum vector from a \(\eta\) mass constraint fit and that from the 7CA kinematic fit. After applying these criteria, we further reduce the background, by using a multi-variable analysis method [16] in which a boosted decision tree (BDT) classifier is developed using the GMC sample. The BDT takes three discriminating variables as inputs: the invariant mass of the photon pair used to reconstruct the \(\eta\) candidate, the momentum of the lower-energy photon from the \(\eta\) candidate, and the momentum of the \(\gamma_{\text{direct}}\) candidate. Studies of the GMC sample show that a requirement on the output of the BDT retains 77.8% signal and rejects 73.4% background. Events in which the signal candidate lies within the interval 1.93 < \(M_{\text{sig}}\) < 1.99 GeV/c^2 are retained for the amplitude analysis. The background events in the signal region from the GMC sample are used to model the corresponding background in data. To check the accuracy of the GMC background modeling, we compare the \(M_{\pi^-\pi^+}\), \(M_{\pi^-\eta}\) and \(M_{\pi^-\eta}\) distributions of events outside the selected \(M_{\text{sig}}\) interval between data and the GMC sample; the distributions are found to be compatible within the statistical uncertainties. We retain a sample of 1239 \(D^+_s\) candidates that has a purity of \((97.7 \pm 0.5)\%\).

The amplitude analysis is performed using an unbinned maximum-likelihood fit to the accepted candidate events in data. The background contribution is subtracted in the likelihood calculation by assigning negative weights to the background events. The total amplitude \(\mathcal{M}(p_j)\) is modeled as the coherent sum of the amplitudes of all intermediate processes, \(\mathcal{M}(p_j) = \sum c_n e^{i\phi_n} A_n(p_j)\), where \(c_n\) and \(\phi_n\) are the amplitude and phase of the \(n\)th amplitude, respectively. The \(n\)th amplitude \(A_n(p_j)\) is given by \(A_n(p_j) = P_n S_n F_n^D F_n^\pi\). Here, \(P_n\) is a function that describes the propagator of the intermediate resonance. The resonance \(\rho^+\) is parameterized by a relativistic Breit-Wigner function, while the resonance \(a_0(980)\) is parameterized as a two-channel-coupled Flatté formula \((\pi\eta\text{ and } K\bar{K})\), \(P_{a_0(980)} = 1/(m^2_{a_0(980)} - s_i) - i(m_{a_0(980)} + (K\bar{K}))\). Here, \(\rho_{\pi\pi}\) and \(\rho_{K\bar{K}}\) are the phase space factors: \(2\eta/\sqrt{s_n}\), where \(\eta\) denotes the magnitude of the momentum of the daughter particle in the rest system and \(s_n\) is the invariant mass square of \(a_0(980)\). We use the coupling constants \(g^2_{\eta\pi\pi} = 0.341 \pm 0.004\) GeV^2/c^4 and \(g^2_{K\bar{K}} = 0.892 \pm 0.022 g^2_{\rho\pi\pi}\) reported in Ref. [17]. The function \(S_n\) describes angular-momentum conservation in the decay and is constructed using the covariant tensor formalism [18]. The function \(F_n(D^0)\) is the Blatt-Weisskopf barrier factor of the intermediate state \((D_s\) meson). Further, according to the topology diagrams shown in Fig. [1] the W-annihilation amplitudes of decays \(D^+_s \rightarrow a_0(980)^{0}\pi^0\) and \(D^+_s \rightarrow a_0(980)^0\pi^+\pi^-\) imply the relationship \(A(D^+_s \rightarrow a_0(980)^0\pi^+) = -A(D^+_s \rightarrow a_0(980)^0\pi^-)\).

For each amplitude, the statistical significance is determined from the change in \(2\ln\mathcal{L}\) and the number of degrees of freedom (NDOF) when the fit is performed with and without the amplitude included. In the nominal fit, only amplitudes that have a significance greater
than 5σ are considered, where σ is the standard deviation. In addition to the $D^+_s \rightarrow \rho^+ \eta$ amplitude, both $D^+_s \rightarrow a_0(980)^+ \pi^0$ and $D^+_s \rightarrow a_0(980)\pi^+$ amplitudes are found to be significant. However, the latter two amplitude phases are found to be approximately 90% correlated with one another; their fitted $c_n$ are found to be consistent with each other while a difference in $\phi_n$ is found to be close to π, which indicates there is no significant $a_0(980)^- - f_0(980)$ mixing in $D^+_s \rightarrow a_0(980)^0\pi^+$. Consequently, in the nominal fit, we set the values of $c_n$ of these two amplitudes to be equal with a phase difference of π. We refer to the coherent sum of these two amplitudes as “$D^+_s \rightarrow a_0(980)\pi^+$”. The non-resonant process $D^+_s \rightarrow (\pi^+\pi^0)_{V\eta}$ is also considered, where the subscript $V$ denotes a vector non-resonant state of the $\pi^+\pi^0$ combination. We consider other possible amplitudes that involve $\rho(1450)$, $a_0(1450)$, $\pi_1(1400)$, $a_2(1320)$, or $a_2(1700)$, as well as the non-resonant partners; none of these amplitudes has a statistical significance greater than 2σ, so they are not included in the nominal model. In the fit, the values of $c_n$ and $\phi_n$ for the $D^+_s \rightarrow \rho^+ \eta$ amplitude are fixed to be one and zero, respectively, so that all other amplitudes are measured relative to this amplitude. The masses and widths of the intermediate resonances used in the fit, except for those of the $a_0(980)$, are taken from Ref. [6].

For $D^+_s \rightarrow \rho^+ \eta$, $D^+_s \rightarrow (\pi^+\pi^0)_{V\eta}$, and $D^+_s \rightarrow a_0(980)\pi$, the resulting statistical significances are greater than 20σ, 5.7σ, and 16.2σ, respectively. Their phases and fit fractions (FFs) are listed in Table I. Here the FF for the $n$th intermediate process is defined as $\text{FF}_n = \frac{|A_n|^2 d\Phi_3}{\sum |A_n|^2 d\Phi_3}$, where $d\Phi_3$ is the standard element of the three-body phase space. The Dalitz plot of $M_{\pi^+\pi^0\eta}$ versus $M_{a_0^0\eta}^2$ for data is shown in Fig. 2(a). The projections of the fit on $M_{\pi^+\pi^0\eta}$, $M_{\pi^+\eta}$ and $M_{a_0^0\eta}$ are shown in Figs. 2(b–d). The projections on $M_{\pi^+\pi^0\eta}$ and $M_{a_0^0\eta}$ for events with $M_{\pi^+\pi^0\eta} > 1.0$ GeV/$c^2$ are shown in Figs. 2(e,f), in which $a_0(980)$ peaks are observed. The fit quality is determined by calculating the $\chi^2$ of the fit using an adaptive binning of the $M_{\pi^+\pi^0\eta}$ Dalitz plot that requires each bin contains at least 10 events. The goodness of fit is $\chi^2$/NDOF=82.8/77.

| Amplitude | $\phi_n$ (rad) | FF$_n$ |
|-----------|----------------|--------|
| $D^+_s \rightarrow \rho^+ \eta$ | 0.69 (fixed) | 0.783 ± 0.050 ± 0.021 |
| $D^+_s \rightarrow (\pi^+\pi^0)_{V\eta}$ | 0.612 ± 0.172 ± 0.342 | 0.054 ± 0.021 ± 0.025 |
| $D^+_s \rightarrow a_0(980)\pi$ | 2.794 ± 0.087 ± 0.044 | 0.232 ± 0.023 ± 0.033 |

For the $\rho^+$ resonance, the effects of the non-resonant state and $D_1$ meson within ±2 GeV/$c^2$. In addition, for the $\rho^+$ resonance, the effective radius reported in Ref. [6] is used as an alternative. The uncertainty related to the background level is determined by changing the background fraction within its statistical uncertainty.

The uncertainty related to the assumed background shape is

![Graphs and plots](image-url)
estimated by using an alternative distribution simulated with $D^+_s \rightarrow \pi^+ f_0(980)$, $f_0(980) \rightarrow \pi^0 \pi^0$. To estimate the uncertainty from the experimental effect related to the kinematic fits and BDT classifier, we set the $\chi^2$ requirements for the result of the two kinematic fits to be twice the values used in the nominal selection, alter the $\cos \theta_{\eta}$ requirement to be greater than 0.996, and adjust the BDT requirement such that the purity is approximately equal to the sample used in the nominal fit. The fitter performance is investigated with the same method as reported in Ref. [20]. The biases are small and taken as the systematic uncertainties. The contributions of individual systematic uncertainties are summarized in Table II, and are added in quadrature to obtain the total systematic uncertainty.

Further, we measure the total BF of $D^+_s \rightarrow \pi^+ \pi^0 \eta$ without reconstructing $\gamma_{\text{direct}}$ to improve the statistical precision. The ST yields ($Y_{\text{tag}}$) and DT yield ($Y_{\text{sig}}$) of data are determined by the fits to the resulting $M_{\text{tag}}$ and $M_{\text{sig}}$ distributions, as shown in Figs. (a-g) and Fig. (h), respectively. In each fit, the signal shape is modeled with the MC-simulated shape convoluted with a Gaussian function, which accounts for any difference in resolution between the data and MC, and the background is described with a second-order Chebychev polynomial. These fits give a total ST yield of $Y_{\text{tag}} = 255895 \pm 1358$ and a signal yield of $Y_{\text{sig}} = 2626 \pm 77$. Based on the signal MC sample, generated according to the amplitude analysis results reported in this Letter, the DT efficiencies ($\epsilon_{\text{tag,sig}}$) are determined. With $Y_{\text{tag}}$, $Y_{\text{sig}}$, $\epsilon_{\text{tag,sig}}$ and the ST efficiencies ($\epsilon_{\text{tag}}$), the relationship $B(D^+_s \rightarrow \pi^+ \pi^0 \eta) = \sum_i \frac{Y_{\text{sig}}}{Y_{\text{tag}}} \epsilon_{\text{tag,sig}} \epsilon_{\text{tag}}$, where the index $i$ denotes the $i^{\text{th}}$ tag mode, is used to obtain $B(D^+_s \rightarrow \pi^+ \pi^0 \eta) = (9.50 \pm 0.28_{\text{stat}} \pm 0.41_{\text{sys}})\%$.

For the total BF measurement, the systematic uncertainty related to the signal shape is studied by performing an alternative fit without convolving the Gaussian resolution function. The BF shift of 0.5% is taken as the uncertainty. The systematic uncertainty arising from the assumed background shape and the fit range is studied by replacing our nominal ones with a first-order Chebychev polynomial and a fit range of $[1.88, 2.04]$ GeV/$c^2$, respectively. The largest BF shift of 0.6% is taken as the

### Table II. Systematic uncertainties on the $\phi$ and FFs for different amplitudes in units of the corresponding statistical uncertainties.

| Amplitude | Source | I | II | III | IV | V | Total |
|-----------|--------|---|----|-----|----|---|-------|
| $D^+_s \rightarrow \rho^+ \eta$ | FF | 0.06 | 0.34 | 0.13 | 0.12 | 0.15 | 0.41 |
| $D^+_s \rightarrow (\pi^+ \pi^0) \eta$ | $\phi$ | -0.97 | 0.18 | 0.03 | 0.17 | 1.99 |
| | FF | 0.61 | 1.03 | 0.12 | 0.06 | 0.08 | 1.21 |
| $D^+_s \rightarrow a_0(980)\pi$ | $\phi$ | -0.41 | 0.07 | 0.28 | 0.09 | 0.51 |
| | FF | 0.58 | 1.31 | 0.02 | 0.06 | 0.11 | 1.45 |

FIG. 3. Fits to (a-g) the $M_{\text{tag}}$ distributions of seven tag modes (indicated in each sub-figure) and (h) the $M_{\text{sig}}$ distribution of signal candidates. The dots with error bars are data. The (blue) solid lines are the total fit. The (red) dashed and the (green) long-dashed lines are signal and background, respectively. In (a-g), the $D^+_s$ signal regions are between the arrows.

related uncertainty. The possible bias due to the measurement method is estimated to be 0.2% by comparing the measured BF in the GMC sample, using the same method as in data analysis, to the value assumed in the generation. The uncertainties from particle identification and tracking efficiencies are assigned to be 0.5% and 1.0% [15], respectively. The relative uncertainty in the $\pi^0$ reconstruction efficiency is 2.0% [15], and the uncertainty in $\eta$ reconstruction is assumed to be comparable to that on $\pi^0$ reconstruction and correlated with it. The uncertainty from the Dalitz model of 0.6% is estimated as the change of efficiency when the model parameters are varied by their systematic uncertainties (this term is not considered when calculating the BFs of the intermediate processes). The uncertainties due to MC statistics (0.2%) and the value of $B(\pi^0/\eta \rightarrow \gamma \gamma)$ used [5] (0.5%) are also considered. Adding these uncertainties in quadrature gives a total systematic uncertainty of 4.3%.

We obtain $B(D^+_s \rightarrow \pi^+ \pi^0 \eta)$ to be $(9.50 \pm 0.28_{\text{stat}} \pm 0.41_{\text{sys}})\%$. Using the FFs listed in Table I, the BFs for the intermediate processes $D^+_s \rightarrow \rho^+ \eta$ and $D^+_s \rightarrow (\pi^+ \pi^0) \eta$ are calculated to be $(7.44 \pm 0.52_{\text{stat}} \pm 0.38_{\text{sys}})\%$ and $(0.51 \pm 0.20_{\text{stat}} \pm 0.25_{\text{sys}})\%$, respectively. With the definition of fit fraction, fraction of $D^+_s \rightarrow a_0(980)(0) \pi^0(+)$, $a_0(980)(0) \rightarrow \pi^0(0)$ with respect to the total fraction of $D^+_s \rightarrow a_0(980)\pi$, $a_0(980) \rightarrow \pi \eta$ is evaluated to be 0.66. Multiplying by the FF of $D^+_s \rightarrow a_0(980)\pi$ determined from the nominal fit and $B(D^+_s \rightarrow \pi^+ \pi^0 \eta)$, the BF of $D^+_s \rightarrow a_0(980)(0) \pi^0(+)$, $a_0(980)(0) \rightarrow \pi^0(0)$ is determined to be $(1.46 \pm 0.15_{\text{stat}} \pm 0.13_{\text{sys}})\%$.

In summary, we present the first amplitude analy-
sis of the decay $D^+_s \rightarrow \pi^+ \pi^0 \eta$. The absolute BF of $D^+_s \rightarrow \pi^+ \pi^0 \eta$ is measured with a precision improved by a factor of 2.5 compared with the world average value $^3_3$. We observe the pure WA decays $D^+_s \rightarrow a_0(980)\pi$ for the first time with a statistical significance of $16.2\sigma$. The measured $B(D^+_s \rightarrow a_0(980)^{0}(0)\pi^{0(+)})$ is larger than other measured BFs of pure WA decays $D^+_s \rightarrow \omega \pi^+$ and $D^+_s \rightarrow \rho^0 \pi^+$ by at least one order of magnitude. Furthermore, when the measured $a_0(980)^0 f_0(980)$ mixing rate $^2_1$ is considered, the expected effect of $a_0(980)^0 f_0(980)$ mixing is lower than the WA contribution in $D^+_s \rightarrow a_0(980)^{0(+)\pi^+}$ decay by two orders of magnitude, which is negligible.

With the measured $B(D^+_s \rightarrow a_0(980)^{0(0)\pi^{0(+)}})$, the WA contribution with respect to the tree-exponential emission contribution in $SP$ mode is estimated to be $0.84 \pm 0.23$ $^3_2$, which is significantly greater than that $(0.1 \sim 0.2)$ in $VP$ and $PP$ modes $^4_6$. This measurement sheds light on the FSI effect and non-perturbative strong interaction $^4_1$, and provides a theoretical challenge to understand such a large WA contribution. In addition, the result of this analysis is an essential input to determine the effect from $a_0(980)^0$ on the $K^+K^-S$-wave contribution to the model-dependent amplitude analysis of $D_s^+ \rightarrow K^+K^-\pi^+$$^3_2$ $^2$.

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