Distributed Dual Objective Control of A Flywheel Energy Storage Matrix System Under Jointly Connected Communication Network

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text
has been less effort put into the cooperative control of a flywheel energy storage matrix system (FESMS), which consists of a group of flywheel systems to increase the power capacity and lower down the risk of single point failure (Cao et. al. 2016 [Lai et. al. 2018 [Sun et. al. 2020]. Roughly speaking, the control schemes for the cooperation of FESMS fall into two categories. The first one is the centralized control scheme, where the information of the entire FESMS should be known to all the subsystems. In Lai et. al. (2018), a power sharing mechanism was proposed in a way that the flywheel systems which stored the most energy have the priority to be put into use. While, this mechanism relies on the condition that each flywheel system knows the energy level of the entire FESMS and thus is essentially centralized. Recently, motivated by the study on multiagent system, distributed control has been introduced to the power society to deal with various problems (Magdi et. al. 2016 [Vallejo et. al. 2014). In distributed control, the subsystems shall communicate with each other over the communication network and no global information will be needed. As a result, the communication network can be made sparse and thus more economic efficient in contrast to centralized control. Based on the average consensus algorithm, Cao et. al. (2016) proposed a distributed control scheme for the power sharing of a FESMS, where the sharing criteria is selected to be the current charging and discharging capacity. The work of Cao et. al. (2016) was later extended in Sun et. al. (2020) featuring periodic event-triggered and self-triggered control.

In general, there are two basic control objectives for an energy storage system. First, the power output of the entire energy storage system should track its reference. Second, the energy level of each energy storage unit should be balanced to keep the maximum power capacity of the entire energy storage system. For example, for battery energy storage systems, the state-of-charge should be balanced for all the battery packs (Cai & Hu 2016 [Li et. al. 2017], and for a general energy storage system, the state-of-charge can be redefined as state-of-energy (SOE), which is the ratio of the current stored energy and the energy capacity (Cai 2020 [Morstyn et. al. 2015]. In this paper, we further consider the distributed dual objective control problem of a heterogenous FESMS aiming at simultaneous reference power tracking and SOE balancing. To solve this problem, first, it is proven that there exists a common SOE trajectory for all the flywheel systems on which the dual control objectives can be achieved simultaneously. Next, adding this common SOE trajectory to the command generator leads to a cascaded augmented command generator, which lends itself to the idea of converting the distributed dual objective control problem into a double layer distributed tracking problem. Finally, by using the adaptive distributed observer approach (Liu & Huang 2019 [Zhang & Lewis 2018], the double layer distributed tracking problem is solved by a distributed control law. In contrast to the existing results, the main contributions of this paper are summarized as follows.

- In Cai (2020); Cao et. al. (2016); Sun et. al. (2020), no specific dynamics of the energy storage units were considered. While, in this paper, we have considered the specific rotor dynamics of the flywheel systems with heterogenous inertia, friction, and energy capacity parameters. In fact, the common SOE trajectory is depicted by all these parameters which implicitly determine how the reference power is dispatched within the FESMS.
- In most of the existing results, say Cai (2020); Cao et. al. (2016); Lai et. al. (2018); Sun et. al. (2020), the communication network is assumed to be static. In this paper, it is shown that the proposed control scheme is able to work under jointly connected communication network, i.e., the communication network can
be disconnected for all time being as long as, from time to time, the union of these disconnected networks is connected. This feature endows the proposed control scheme with two advantages. First, it presents certain robustness against unreliable communication environment. Second, less information exchange within the FESMS is needed, and thus the communication cost can be significantly reduced.

The rest of this paper is organized as follows. Notation adopted in this paper are summarized in Section 2. Section 3 first introduces the model of the FESMS, and then gives a mathematical problem formulation for the dual objective control problem. The main results of this paper are presented in Section 4 including the seeking of the common SOE trajectory, the design of the distributed control law, and the stability analysis of the closed-loop system. Section 5 validates the effectiveness of the proposed control scheme by a numerical example, and the paper is closed by Section 6.

2. Notation

\( \mathbb{R} \) denotes the set of real numbers. For \( x_i \in \mathbb{R}^n, i = 1, \ldots, m, \) \( \text{col}(x_1, \ldots, x_m) = [x_1^T \cdots x_m^T]^T \). \( 1_n = \text{col}(1, \ldots, 1) \in \mathbb{R}^n \). For a matrix \( A \in \mathbb{R}^{m \times n}, \text{vec}(A) = \text{col}(A_1, \ldots, A_n) \) where \( A_i \) is the \( i \)-th column of \( A \). \( ||x|| \) denotes the Euclidean norm of a vector \( x \in \mathbb{R}^n \) and \( ||A|| \) denotes the Euclidean norm of a matrix \( A \in \mathbb{R}^{m \times n} \). For a function \( f(t) : [0, +\infty) \rightarrow \mathbb{R}^{m \times n} \), if there exists a nonnegative integer \( q \) and \( \gamma_0 \in \mathbb{R} \) such that \( ||f(t)|| \leq \gamma_0 t^q + \cdots + \gamma_1 t + \gamma_0 \) for all \( t \geq 0 \), then we say \( f(t) \) is bounded by a polynomial function. \( \otimes \) denotes the Kronecker product of matrices.

A graph \( \mathcal{G} = (\mathcal{V}, \mathcal{E}) \) consists of a node set \( \mathcal{V} = \{1, \ldots, N\} \) and an edge set \( \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \). For \( i, j = 1, 2, \ldots, N, i \neq j, \) an edge of \( \mathcal{E} \) from node \( i \) to node \( j \) is denoted by \( (i, j) \), and the node \( i \) is called a neighbor of the node \( j \). If the graph \( \mathcal{G} \) contains a sequence of edges of the form \( (i_1, i_2), (i_2, i_3), \ldots, (i_k, i_{k+1}) \), then the set \( \{(i_1, i_2), (i_2, i_3), \ldots, (i_k, i_{k+1})\} \) is called a directed path of \( \mathcal{G} \) from node \( i_1 \) to node \( i_{k+1} \) and node \( i_{k+1} \) is said to be reachable from node \( i_1 \). The graph \( \mathcal{G} \) is said to contain a spanning tree if there exists a node in \( \mathcal{G} \) such that it is reachable from all the other nodes. Given a set of \( r \) graphs \( \mathcal{G}_k = (\mathcal{V}, \mathcal{E}_k), k = 1, \ldots, r, \) the graph \( \mathcal{G} = (\mathcal{V}, \mathcal{E}) \) where \( \mathcal{E} = \bigcup_{k=1}^{r} \mathcal{E}_k \) is called the union of \( \mathcal{G}_k \) and is denoted by \( \sqcup_{k=1}^{r} \mathcal{G}_k \). A matrix \( \mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N} \) is said to be a weighted adjacency matrix of a graph \( \mathcal{G} \) if \( a_{ij} = 0, a_{ij} > 0 \Leftrightarrow (j, i) \in \mathcal{E}, a_{ij} = 0 \Leftrightarrow (j, i) \notin \mathcal{E} \). Let \( \mathcal{L} = [l_{ij}] \in \mathbb{R}^{N \times N} \) be such that \( l_{ii} = \sum_{j=1}^{N} a_{ij} \) and \( l_{ij} = -a_{ij} \) if \( i \neq j \). Then \( \mathcal{L} \) is called the Laplacian of the graph \( \mathcal{G} \). A time signal \( \sigma(t) : [0, +\infty) \rightarrow \mathcal{P} \) where \( \mathcal{P} = \{1, \ldots, n\} \) for some positive integer \( n \) is said to be a piecewise constant switching signal with dwell time \( \tau \) if there exists a sequence \( \{t_i : i = 0, 1, 2, \ldots\} \) satisfying, for all \( i \geq 1, t_i - t_{i-1} \geq \tau \) for some positive constant \( \tau \) such that, over each interval \( [t_i, t_{i+1}) \), \( \sigma(t) = p \) for some integer \( p \in \mathcal{P} \). \( t_0, t_1, t_2, \ldots \) are called switching instants. Given a node set \( \mathcal{V} = \{1, \ldots, N\} \) and a piecewise constant switching signal \( \sigma(t) \), we can define a switching graph \( \mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)}) \) where \( \mathcal{E}_{\sigma(t)} \subseteq \mathcal{V} \times \mathcal{V} \) for all \( t \geq 0 \). Let \( \mathcal{A}_{\sigma(t)} = [a_{ij}(t)] \in \mathbb{R}^{N \times N} \) denote the weighted adjacency matrix of \( \mathcal{G}_{\sigma(t)} \), \( \mathcal{L}_{\sigma(t)} \) denote the Laplacian of \( \mathcal{G}_{\sigma(t)} \), and \( \mathcal{N}_i(t) \) denote the set of all the neighbors of node \( i \) at time instant \( t \).
3. System modeling and problem formulation

In this paper, we consider a FESMS consisting of \( N \) heterogenous flywheel systems, whose configuration is illustrated by Fig. 1. For \( i = 1, \ldots, N \), as validated in Ghanaatian & Lotfifard (2019); Zhang & Yang (2017), the rotor dynamics of the \( i \)th flywheel are given by:

\[
I_i \ddot{\omega}_i = -B_{vi} \omega_i + T_i
\]

(1)

where \( I_i, \omega_i, B_{vi} \) denote the inertia of the flywheel, rotor angular velocity, and friction coefficient, respectively, and \( T_i = T_{ei} - T_{li} \) with \( T_{ei} \) and \( T_{li} \) denoting the electrical and mechanical load torque, respectively. The kinematic energy stored in the \( i \)th flywheel is \( E_i = \frac{1}{2} I_i \omega_i^2 \). Let \( P_i = -E_i \). Then it follows that

\[
P_i = -I_i \omega_i \dot{\omega}_i = B_{vi} \omega_i^2 - T_i \omega_i
\]

(2)

where the first part \( B_{vi} \omega_i^2 \equiv P_{i,\text{loss}} \) denotes the power loss due to friction, and the second part \( -T_i \omega_i \equiv P_{i,\text{out}} \) denotes the net power output of the \( i \)th flywheel system. Let \( \omega_{i,\max} \) denote the maximum admissible angular velocity of the \( i \)th flywheel. Then the energy capacity of the \( i \)th flywheel is given by \( E_{i,\max} = \frac{1}{2} I_i \omega_{i,\max}^2 \). Thus, the SOE of the \( i \)th flywheel is given by

\[
\phi_i = \frac{E_i}{E_{i,\max}} = \frac{\omega_i^2}{\omega_{i,\max}^2} = \gamma_i \omega_i^2
\]

(3)
where $\gamma_i = 1/\omega_{i\text{max}}^2$. By (1), (2) and (3), it follows that

$$\dot{\phi}_i = \frac{2\gamma_i}{I_i} (-B_{vi}\omega_i^2 + T_i\omega_i) = -\frac{2B_{vi}}{I_i} \gamma_i \omega_i^2 + \frac{2\gamma_i}{I_i} T_i \omega_i = -\frac{2B_{vi}}{I_i} \phi_i - \frac{2\gamma_i}{I_i} P_{i,\text{out}}. \quad (4)$$

In this paper, $P_{i,\text{out}}$ is taken as the control input of the $i$th flywheel system. Let $P_{\text{FESMS}} = \sum_{i=1}^N P_{i,\text{out}}$ denote the power output of the entire FESMS, and $P_{\text{REF}}(t)$ denote the reference for $P_{\text{FESMS}}$, which is assumed to be generated by a command generator in the following form

$$\dot{\eta}_0 = S_0 \eta_0 \quad (5a)$$
$$P_{\text{REF}} = C_0 \eta_0 \quad (5b)$$

where $\eta_0 \in \mathbb{R}^q$ is the internal state of the command generator, $S_0 \in \mathbb{R}^{q \times q}$ and $C_0 \in \mathbb{R}^{1 \times q}$ are constant matrices.

The communication network for the FESMS is modeled as a directed switching graph $\mathcal{G}_\sigma(t) = (\mathcal{V}, \mathcal{E}_\sigma(t))$, where $\mathcal{V} = \{1, \ldots, N\}$ and $\mathcal{E}_\sigma(t) \subseteq \{\mathcal{V} \times \mathcal{V}\}$. In here, the node $i$ of $\mathcal{V}$ is associated with the $i$th flywheel of the FESMS. For $i, j = 1, \ldots, N$, $(i, j) \in \mathcal{E}_\sigma(t)$ if and only if the $j$th flywheel can receive the information from the $i$th flywheel at time instant $t$. By further taking the command generator into consideration, we can define an augmented switching digraph $\tilde{\mathcal{G}}_\sigma(t) = (V, \tilde{\mathcal{E}}_\sigma(t))$ where $V = \{0, 1, \ldots, N\}$ and $\tilde{\mathcal{E}}_\sigma(t) = \mathcal{E}_\sigma(t) \cup \{\{0\} \times \mathcal{V}\}$. Here, the node 0 is associated with the command generator. For $i = 1, \ldots, N$, $(0, i) \in \tilde{\mathcal{E}}_\sigma(t)$ if and only if the $i$th flywheel can receive the information from the command generator at time instant $t$. Let $\tilde{\mathcal{A}}_\sigma(t) = [a_{ij}(t)] \in \mathbb{R}^{(N+1) \times (N+1)}$ be the weighted adjacency matrix of $\tilde{\mathcal{G}}_\sigma(t)$, $\tilde{\mathcal{L}}_\sigma(t)$ be the Laplacian of $\tilde{\mathcal{G}}_\sigma(t)$, and $\tilde{H}_\sigma(t) = \tilde{\mathcal{L}}_\sigma(t) + \text{diag}\{a_{10}(t), \ldots, a_{N0}(t)\}$. The following assumption is imposed on the communication network.

**Assumption 1.** There exists a subsequence $\{j_k : k = 0, 1, 2, \ldots\}$ of $\{j = 0, 1, 2, \ldots\}$ satisfying $t_{j_{k+1}} - t_{j_k} < \epsilon$ for some $\epsilon > 0$, such that every node $i, i = 1, \ldots, N$, is reachable from node 0 in the union digraph $\bigcup_{r=j_k}^{j_{k+1}-1} \tilde{\mathcal{G}}_\sigma(t_r)$.

**Remark 1.** In the literature of multiagent system, Assumption 1 is referred to as the “jointly connected” condition in many existing works, say Liu & Huang (2019); Su & Huang (2012). In contrast to the “spanning tree” condition for a static graph (Cai, 2020; Cai & Hu, 2016), i.e., the communication graph is static and contains a spanning tree with node 0 as the root, and the “all time connected” condition for a switching graph (Dong & Hu, 2010; Dong et al., 2017), i.e., the communication graph is switching and for all time being, it should contain a spanning tree with node 0 as the root, the “jointly connected” condition imposes a much less restrictive requirement on the connectivity of the communication network. More specifically, the communication graph can be disconnected for all time being as long as, from time to time, the union of these disconnected graphs contains a spanning tree with node 0 as the root. (See Fig. 5 of Section 3 for an example of “jointly connected” communication network.) If a control scheme can work under Assumption 1, then it is endowed with at least two advantages. First, it will present certain robustness against unreliable communication environment in the case that the communication link might temporarily fail due to malicious attack or instrument fault. Second, less information exchange among the flywheel systems and the command generator is needed to achieve the control objective, and thus the communication cost will be significantly reduced.
Now, the dual objective control problem for the FESMS can be described as follows.

**Problem 1.** Given systems (4), (5) and the communication network $\hat{G}_\sigma(t)$, design a distributed control law for $P_{i,\text{out}}$, such that

$$\lim_{t \to \infty} (P_{\text{FESMS}}(t) - P_{\text{REF}}(t)) = 0,$$

and for $i, j = 1, \ldots, N$, $i \neq j$,

$$\lim_{t \to \infty} (\phi_i(t) - \phi_j(t)) = 0, \quad \lim_{t \to \infty} \left(\dot{\phi}_i(t) - \dot{\phi}_j(t)\right) = 0.$$

The control objective (6) requires that the power output of the entire FESMS shall follow its reference, and the control objective (7) requires SOE balancing of all the flywheel systems.

### 4. Main results

In this section, we first prove that the solution to Problem 1 exists by showing the existence of a common SOE trajectory for all the flywheel systems on which the two control objectives (6) and (7) in Problem 1 can be simultaneously achieved. Next, based on this common SOE trajectory, an augmented command generator is designed and a distributed control law is synthesized. It is then shown that Problem 1 can be solved by the proposed distributed control law under Assumption 1.

#### 4.1. Problem solvability

**Lemma 1.** If $\phi_i(t) = \psi_0(t)$ for $i = 1, \ldots, N$, where

$$\dot{\psi}_0(t) = -\alpha_0 \psi_0(t) - \beta_0 P_{\text{REF}}(t)$$

with

$$\alpha_0 = \frac{2 \sum_{i=1}^{N} B_{vi} I_i}{\sum_{j=1}^{N} I_j}, \quad \beta_0 = \frac{2 \sum_{i=1}^{N} I_i}{\sum_{i=1}^{N} \frac{L_i}{\gamma_i}},$$

then it follows that

$$P_{\text{FESMS}}(t) - P_{\text{REF}}(t) = 0, \quad \phi_i(t) - \phi_j(t) = 0, \quad \dot{\phi}_i(t) - \dot{\phi}_j(t) = 0.$$

**Proof:** Since all $\phi_i$'s are the same as (8), conditions (9b) and (9c) are satisfied immediately. Thus, we only need to show that condition (9a) is also satisfied. For $i = 1, \ldots, N$, it follows that

$$\dot{\phi}_i = -\frac{2B_{vi}}{I_i} \psi_0 - \frac{2\gamma_i}{I_i} P_{i,\text{out}} = -\frac{2}{\sum_{i=1}^{N} \frac{L_i}{\gamma_i}} P_{\text{REF}}(t) - \frac{2 \sum_{i=1}^{N} B_{vi}}{\sum_{i=1}^{N} \frac{L_i}{\gamma_i}} \psi_0.$$
and thus

\[ P_{i,\text{out}} = \gamma_i \left( \sum_{i=1}^{N} \frac{1}{L_i \gamma_i} P_{\text{REF}}(t) + \frac{\sum_{i=1}^{N} B_{vi} \psi_0}{\sum_{i=1}^{N} \frac{I_i}{\gamma_i} \gamma_i} - \frac{B_{vi} \psi_0}{I_i} \right). \]  

(11)

As a result,

\[ P_{\text{FESMS}}(t) = \sum_{i=1}^{N} P_{i,\text{out}}(t) = \sum_{i=1}^{N} \frac{I_i}{\gamma_i} \left( \sum_{i=1}^{N} \frac{1}{L_i \gamma_i} P_{\text{REF}}(t) + \frac{\sum_{i=1}^{N} B_{vi} \psi_0}{\sum_{i=1}^{N} \frac{I_i}{\gamma_i} \gamma_i} - \frac{B_{vi} \psi_0}{I_i} \right) \]

\[ = \sum_{i=1}^{N} \frac{I_i}{\gamma_i} P_{\text{REF}}(t) + \left( \frac{\sum_{i=1}^{N} B_{vi} \psi_0}{\sum_{i=1}^{N} \frac{I_i}{\gamma_i} \gamma_i} \right) \left( \frac{\sum_{i=1}^{N} B_{vi}}{\sum_{i=1}^{N} \frac{I_i}{\gamma_i} \gamma_i} \right) \psi_0 - \frac{\sum_{i=1}^{N} B_{vi} \psi_0}{I_i} \]

\[ = P_{\text{REF}}(t). \]  

(12)

\[ \square \]

Remark 2. In here, an explanation is provided on how the common SOE trajectory (8) is obtained. By equation (4) and (9c), for \( i = 2, \ldots, N \), we have

\[ \frac{2\gamma_1}{I_1} (P_{1,\text{out}} + \frac{B_{v1}}{\gamma_1} \psi_0) = \frac{2\gamma_i}{I_i} (P_{i,\text{out}} + \frac{B_{vi}}{\gamma_i} \psi_0). \]  

(13)

Thus,

\[ P_{i,\text{out}} = \frac{\gamma_1 I_1}{\gamma_i I_1} (P_{1,\text{out}} + \frac{B_{v1}}{\gamma_1} \psi_0) - \frac{B_{vi} \psi_0}{\gamma_i} \]

\[ = \gamma_1 I_1 \frac{\gamma_i I_1}{\gamma_1 I_1} P_{1,\text{out}} + \left( \frac{B_{v1} I_1}{\gamma_i I_1} - \frac{B_{vi}}{\gamma_i} \right) \psi_0 \]

\[ = \gamma_1 I_1 \gamma_i I_1 P_{1,\text{out}} + \frac{B_{v1} I_1 - B_{vi} I_1}{\gamma_i I_1} \psi_0. \]  

(14)

Therefore, by (9a), we have

\[ P_{\text{REF}}(t) = P_{1,\text{out}} + \sum_{i=2}^{N} P_{i,\text{out}} \]

\[ = P_{1,\text{out}} + \sum_{i=2}^{N} \gamma_1 I_1 P_{1,\text{out}} + \frac{B_{v1} I_1 - B_{vi} I_1}{\gamma_i I_1} \psi_0 \]

\[ = \sum_{i=1}^{N} \frac{\gamma_1 I_1}{\gamma_i I_1} P_{1,\text{out}} + \sum_{i=2}^{N} \frac{B_{v1} I_1 - B_{vi} I_1}{\gamma_i I_1} \psi_0. \]  

(15)

If the common SOE trajectory \( \psi_0(t) \) exists, then for \( i = 1, \ldots, N \), by equation (4), we have

\[ \dot{\psi}_0 = -\frac{2\gamma_i}{I_i} (P_{1,\text{out}} + \frac{B_{vi}}{\gamma_i} \psi_0). \]  

(16)
Thus, we have
\[ \dot{\psi}_0 = \frac{-2\gamma_1}{I_1}(P_{1,\text{out}} + \frac{B_{v1}}{\gamma_1}\psi_0). \]  
(17)

Consequentially, substituting (15) into (17) gives
\[
\dot{\psi}_0 = -2\gamma_1 \left( \frac{P_{\text{REF}}(t) - \sum_{i=2}^{N} \frac{B_{v1}I_{i} - B_{v1}I_1}{\gamma_1 I_1} \psi_0}{\sum_{i=1}^{N} \frac{I_i}{\gamma_i}} + \frac{B_{v1}}{\gamma_1} \psi_0 \right)
\]
\[ = -2 \sum_{i=1}^{N} \frac{I_{i}}{\gamma_{i}} P_{\text{REF}}(t) + 2 \sum_{i=2}^{N} \frac{B_{v1}I_{i} - B_{v1}I_1}{\gamma_i I_1} - \sum_{i=1}^{N} \frac{B_{v1}I_i}{I_1 \gamma_i} \psi_0 \]
\[ = -2 \sum_{i=1}^{N} \frac{I_{i}}{\gamma_{i}} P_{\text{REF}}(t) - 2 \sum_{i=1}^{N} \frac{B_{v1}I_{i}}{I_1 \gamma_i} \psi_0. \]  
(18)

Remark 3. It can be seen that the common SOE trajectory (8) is depicted by the parameters $I_i$, $\gamma_i$ and $B_{vi}$ of all the flywheel systems, and Lemma 1 together with (4) implicitly reveals how the reference power shall be dispatched within the FESMS.

4.2. Control design

First, adding the common SOE trajectory (8) to the command generator (5) leads to the following augmented command generator:
\[
\dot{\eta}_0 = S_0 \eta_0 \]  
(19a)
\[ P_{\text{REF}} = C_0 \eta_0 \]  
(19b)
\[ \dot{\psi}_0 = -\alpha_0 \psi_0 - \beta_0 P_{\text{REF}}. \]  
(19c)

Remark 4. The augmented command generator (19) has a cascaded structure, i.e., the output $P_{\text{REF}}$ of the command generator (19a)-(19b) is the input of the common SOE trajectory (19c). This natural cascaded structure lends itself to the idea of converting the distributed dual objective control problem into a double layer distributed tracking problem by the adaptive distributed observer approach (Liu & Huang, 2019; Zhang & Lewis, 2018). To be more specific, for each flywheel system, an adaptive distributed observer, with the same cascaded double layer structure as (19), can be facilitated to recover the common SOE trajectory. Then, by designing a local tracking controller which drives the local SOE to the common SOE, the problem will be solved by Lemma 1.
Then, for \( i = 1, \ldots, N \), the control law for the \( i \)th flywheel system is designed as

\[
\dot{S}_i = \mu_S \sum_{j=0}^{N} a_{ij}(t)(S_j - S_i) \quad (20a)
\]

\[
\dot{C}_i = \mu_C \sum_{j=0}^{N} a_{ij}(t)(C_j - C_i) \quad (20b)
\]

\[
\dot{\eta}_i = S_i \eta_i + \mu_\eta \sum_{j=0}^{N} a_{ij}(t)(\eta_j - \eta_i) \quad (20c)
\]

\[
\dot{P}_{i,\text{REF}} = C_i \eta_i \quad (20d)
\]

\[
\dot{\alpha}_i = \mu_\alpha \sum_{j=0}^{N} a_{ij}(t)(\alpha_j - \alpha_i) \quad (20e)
\]

\[
\dot{\beta}_i = \mu_\beta \sum_{j=0}^{N} a_{ij}(t)(\beta_j - \beta_i) \quad (20f)
\]

\[
\dot{\psi}_i = -\alpha_i \dot{\psi}_i - \beta_i \dot{P}_{i,\text{REF}} + \mu_\psi \sum_{j=0}^{N} a_{ij}(t)(\psi_j - \psi_i) \quad (20g)
\]

\[
P_{i,\text{out}} = -\frac{I_i}{2\gamma_i} \left(-\alpha_i \psi_i(t) - \beta_i \dot{P}_{i,\text{REF}} - \kappa(\phi_i - \psi_i) + \frac{2B_v i}{I_i} \phi_i \right) \quad (20h)
\]

where \( S_i \in \mathbb{R}^{q \times q}, C_i \in \mathbb{R}^{1 \times q}, \eta_i \in \mathbb{R}^q, \dot{P}_{i,\text{REF}}, \alpha_i, \beta_i, \psi_i \in \mathbb{R} \) are the estimates of \( S_0, C_0, \eta_0, P_{\text{REF}}, \alpha_0, \beta_0 \) and \( \psi_0 \), respectively.

**Remark 5.** The block diagram of the control law (20) is shown by Fig. 2. Systems (20a)-(20d) constitute the first layer of the adaptive distributed observer to recover \( P_{\text{REF}} \), and systems (20e)-(20g) constitute the second layer of the adaptive distributed observer to recover \( \psi_0 \). From Fig. 2, it can be seen that these two layers inherit the same cascaded structure as the augmented command generator (19).

### 4.3. Stability analysis

**Theorem 1.** Given systems (1) and (19), under Assumption 1, if none of the eigenvalues of \( S_0 \) has positive real part, then the control law (20) solves Problem 1 for any \( \mu_S, \mu_C, \mu_\eta, \mu_\alpha, \mu_\beta, \mu_\psi, \kappa > 0 \).

**Proof:** For \( i = 1, \ldots, N \), let \( \dot{S}_i = S_i - S_0, \dot{C}_i = C_i - C_0, \dot{\alpha}_i = \alpha_i - \alpha_0, \dot{\beta}_i = \beta_i - \beta_0, \dot{S} = \text{col}(S_1, \ldots, S_N), \dot{C} = \text{col}(C_1, \ldots, C_N), \dot{\alpha} = \text{col}(\dot{\alpha}_1, \ldots, \dot{\alpha}_N), \dot{\beta} = \text{col}(\dot{\beta}_1, \ldots, \dot{\beta}_N) \). Then it follows that

\[
\text{vec}(\dot{S}) = \mu_S (I_q \otimes H_{\sigma(t)} \otimes I_q) \text{vec}(S) \quad (21a)
\]

\[
\text{vec}(\dot{C}) = \mu_C (I_q \otimes H_{\sigma(t)} \otimes I_q) \text{vec}(C) \quad (21b)
\]

\[
\dot{\alpha} = \mu_\alpha H_{\sigma(t)} \dot{\alpha} \quad (21c)
\]

\[
\dot{\beta} = \mu_\beta H_{\sigma(t)} \dot{\beta}. \quad (21d)
\]
By Corollary 4 of Su & Huang (2012), it follows that all $\text{vec}(\tilde{S}(t))$, $\text{vec}(\tilde{C}(t))$, $\tilde{\alpha}(t)$, $\tilde{\beta}(t)$ will decay to zero exponentially as $t \to \infty$, i.e., all $\tilde{S}(t)$, $\tilde{C}(t)$, $\tilde{\alpha}(t)$, $\tilde{\beta}(t)$ will decay to zero exponentially as $t \to \infty$. Meanwhile, all $S_i(t)$, $C_i(t)$, $\alpha_i(t)$, $\beta_i(t)$ will be bounded for all $t \geq 0$.

For $i = 1, \ldots, N$, let $\bar{\eta}_i = \eta_i - \eta_0$, $\bar{P}_{i,REF} = \bar{P}_{i,REF} - P_{REF}$ and $\bar{\psi}_i = \psi_i - \psi_0$. It follows that

$$\dot{\bar{\eta}}_i = S_i \bar{\eta}_i + \mu_{\eta} \sum_{j=0}^{N} a_{ij}(t) (\bar{\eta}_j - \bar{\eta}_i) - S_0 \bar{\eta}_0$$

$$= S_i \bar{\eta}_i + S_0 \bar{\eta}_i - S_0 \bar{\eta}_i + \mu_{\eta} \sum_{j=0}^{N} a_{ij}(t) (\bar{\eta}_j - \bar{\eta}_i) - S_0 \bar{\eta}_0$$

$$= S_0 \bar{\eta}_i + \tilde{S}_i \bar{\eta}_i + \mu_{\eta} \sum_{j=0}^{N} a_{ij}(t) (\bar{\eta}_j - \bar{\eta}_i)$$

$$= S_0 \bar{\eta}_i + \tilde{S}_i \bar{\eta}_i + \tilde{S}_i \eta_0 + \mu_{\eta} \sum_{j=0}^{N} a_{ij}(t) (\bar{\eta}_j - \bar{\eta}_i)$$

(22)
and

\[
\dot{\psi}_i = -\alpha_i \psi_i - \beta_i \dot{P}_{i, \text{REF}} + \mu_\psi \sum_{j=0}^{N} a_{ij}(t)(\psi_j - \psi_i) + \alpha_0 \psi_0 + \beta_0 P_{\text{REF}}
\]

\[
= -\alpha_i \psi_i + \alpha_0 \psi_i - \beta_i \dot{P}_{i, \text{REF}} + \beta_i P_{\text{REF}} - \beta_i P_{\text{REF}}
\]

\[
+ \mu_\psi \sum_{j=0}^{N} a_{ij}(t)(\psi_j - \psi_i) + \alpha_0 \psi_0 + \beta_0 P_{\text{REF}}
\]

\[
= -\alpha_0 \dot{\psi}_i - \alpha_i \psi_i - \beta_i P_{\text{REF}} - \beta_i \dot{P}_{i, \text{REF}} + \mu_\psi \sum_{j=0}^{N} a_{ij}(t)(\dot{\psi}_j - \dot{\psi}_i)
\]

\[
= -\alpha_0 \dot{\psi}_i - \alpha_i \psi_i - \beta_i P_{\text{REF}} - \beta_i \dot{P}_{i, \text{REF}} + \mu_\psi \sum_{j=0}^{N} a_{ij}(t)(\dot{\psi}_j - \dot{\psi}_i).
\]

Let \(\bar{\eta} = \text{col}(\bar{\eta}_1, \ldots, \bar{\eta}_N)\) and \(\bar{S}_d = \text{block diag}\{\bar{S}_1, \ldots, \bar{S}_N\}\). Then (22) can be written into the following compact form

\[
\dot{\eta} = (I_N \otimes S_0 - \mu_\eta(H_{\sigma(t)} \otimes I_q))\bar{\eta} + \bar{S}_d \bar{\eta} + \bar{S}_d(1_N \otimes \eta_0).
\]

By Corollary 1 of Liu & Huang (2019), it follows that \(\lim_{t \to \infty} \bar{\eta}(t) = 0\) exponentially. Since none of the eigenvalues of \(S_0\) has positive real part, \(\eta_0(t)\) and hence \(P_{\text{REF}}(t)\) are bounded by polynomial functions. As a result, \(\psi_0(t)\) is also bounded by a polynomial function since \(\alpha_0 > 0\). Moreover, \(\eta_i = \bar{\eta}_i + \eta_0\) implies that \(\eta_i\) is also bounded by a polynomial function since there exist \(\rho_i, \rho_i > 0\) such that \(\|\bar{\eta}_i(t)\| \leq \rho_i e^{-\rho_i t} \leq \rho_i\) for all \(t \geq 0\). Then, noting that \(\bar{P}_{i, \text{REF}} = C_i \bar{\eta}_i - C_0 \eta_0 = \bar{C}_i \bar{\eta}_i + C_0 \eta_0\) and the fact that \(\bar{C}_i\) decays to zero exponentially gives that \(\bar{P}_{i, \text{REF}}\) decays to zero exponentially. Let \(\bar{\psi} = \text{col}(\bar{\psi}_1, \ldots, \bar{\psi}_N)\), \(\bar{\alpha} = \text{diag}\{\bar{\alpha}_1, \ldots, \bar{\alpha}_N\}\), \(\bar{\beta} = \text{diag}\{\bar{\beta}_1, \ldots, \bar{\beta}_N\}\), and \(\bar{P}_{\text{REF}} = \text{col}(\bar{P}_{1, \text{REF}}, \ldots, \bar{P}_{N, \text{REF}})\). Then (23) can be written into the following compact form

\[
\dot{\bar{\psi}} = (-\alpha_0 I_N - \mu_\psi H_{\sigma(t)})\bar{\psi} - \bar{\alpha} \bar{\psi} - \bar{\alpha}(1_N \otimes \psi_0) - \bar{\beta}(1_N \otimes P_{\text{REF}}) - \beta \bar{P}_{\text{REF}}.
\]

Since \(\bar{\alpha}, \bar{\beta}, \bar{P}_{\text{REF}}\) decay to zero exponentially, \(\beta\) is bounded, and \(\psi_0, P_{\text{REF}}\) are bounded by polynomial functions, all \(\bar{\alpha}(1_N \otimes \psi_0)\), \(\bar{\beta}(1_N \otimes P_{\text{REF}})\) and \(\beta \bar{P}_{\text{REF}}\) decay to zero exponentially. Then, again by Corollary 1 of Liu & Huang (2019), it follows that \(\lim_{t \to \infty} \bar{\psi}(t) = 0\) exponentially. Then, similarly, \(\dot{\psi}_i = \psi_i + \psi_0\) is bounded by a polynomial function.

Substituting (20a) into (4) gives

\[
\dot{\phi}_i = -\frac{2B_{vi}}{I_i} \phi_i - 2\gamma_i \left(\frac{-I_i}{2\gamma_i} \left(-\alpha_i \psi_i - \beta_i \dot{P}_{i, \text{REF}} - \kappa(\phi_i - \psi_i) + \frac{2B_{vi}}{I_i} \phi_i\right)\right)
\]

\[
= -\alpha_i \psi_i - \beta_i \dot{P}_{i, \text{REF}} - \kappa(\phi_i - \psi_i).
\]
Table 1. System parameters.

|   | \( B_{vi} \)  | \( I_i \text{(kg} \cdot \text{m}^2) \) | \( \omega_{i,\text{max}} \text{(rad/s)} \) |
|---|---|---|---|
| 1 | \( 1 \times 10^{-3} \) | 0.8 | 1000 |
| 2 | \( 0.95 \times 10^{-3} \) | 0.9 | 800 |
| 3 | \( 1.05 \times 10^{-3} \) | 1.0 | 900 |
| 4 | \( 0.9 \times 10^{-3} \) | 1.3 | 1200 |

Let \( \bar{\phi}_i = \phi_i - \psi_0 \). Then we have

\[
\dot{\bar{\phi}}_i = -\alpha_i \bar{\psi}_i - \beta_i \bar{P}_{i,\text{REF}} - \kappa (\phi_i - \psi_i) + \alpha_0 \psi_0 + \beta_0 P_{REF} \\
= -\kappa (\phi_i - \psi_0 + \psi_i) - \alpha_0 \bar{\psi}_i - \alpha_i \psi_i - \beta_i \bar{P}_{REF} - \beta_i \bar{P}_{i,\text{REF}} \\
= -\kappa \bar{\phi}_i + \kappa \bar{\psi}_i - \alpha_0 \bar{\psi}_i - \alpha_i \psi_i - \beta_i \bar{P}_{REF} - \beta_i \bar{P}_{i,\text{REF}}. 
\]

(27)

Since \( \bar{\alpha}_i, \bar{\beta}_i, \bar{P}_{i,\text{REF}}, \bar{\psi}_i \) decay to zero exponentially, \( \beta_i \) is bounded, and \( \psi_i, P_{REF} \) are bounded by polynomial functions, all \( \kappa \psi_i, \alpha_0 \bar{\psi}_i, \alpha_i \psi_i, \bar{\psi}_i \bar{P}_{REF}, \beta_i \bar{P}_{i,\text{REF}} \) will decay to zero exponentially. Then, since \( \kappa > 0 \), \( \lim_{t \to \infty} \bar{\phi}_i(t) = 0 \), and thus the proof is completed by invoking Lemma 1.

\[\square\]

Remark 6. Assuming none of the eigenvalues of \( S_0 \) has positive real part merely rules out exponentially increasing signals, which are barely used in practice since the increasing rate of exponential functions is too fast.

5. Numerical example

In this section, we consider a FESMS consisting of four flywheel systems. The communication network is shown by Fig. 3 where \( \bar{G}_\sigma(t) \) switches among four graphs \( \bar{G}_1, \bar{G}_2, \bar{G}_3, \bar{G}_4 \) periodically every 1s. It can be seen that all these four graphs are disconnected, while the union of them contains a spanning tree with node 0 as the root, and thus Assumption 1 is satisfied. The system parameters are given by Table 1. The command generator is designed as

\[
\dot{\eta}_0 = \begin{pmatrix} 0 & 0.1 \\ -0.1 & 0 \end{pmatrix} \eta_0, \ P_{\text{REF}} = \begin{pmatrix} 1 & 0 \end{pmatrix} \eta_0, \ \eta_0(0) = \begin{pmatrix} 0 \\ 2 \times 10^5 \end{pmatrix}
\]

(28)

Thus, \( P_{\text{REF}}(t) = 20 \sin(0.1t) \) kw.

The control gains are selected to be \( \mu_S = \mu_C = \mu_\eta = \mu_\alpha = \mu_\beta = \mu_\psi = 100, \ \kappa = 1 \). The system initial values are given by \( \psi_0(0) = 0.88, \ \phi_1(0) = \psi_1(0) = 0.85, \ \phi_2(0) = \psi_2(0) = 0.9, \ \phi_3(0) = \psi_3(0) = 0.88, \ \phi_4(0) = \psi_4(0) = 0.87, \) and for \( i = 1, 2, 3, 4 \), \( S_i(0) = 0, \ C_i(0) = 0, \ \eta_i(0) = 0, \ \alpha_i(0) = 0, \ \beta_i(0) = 0. \) The system performance is shown by Figs. 4 and 5. It can be seen that both SOE balancing and power tracking have been achieved.
Figure 3. Communication network.

Figure 4. Performance on SOE balancing of the FESMS.
6. Conclusion

For a heterogenous FESMS, a distributed dual objective control problem aiming at simultaneous reference power tracking and state-of-energy balancing has been considered. It is first shown that a common SOE trajectory exists for all the flywheel systems on which the dual control objectives can be achieved simultaneously. Then, the distributed dual objective control problem has been converted into a double layer distributed tracking problem by making use of the common SOE trajectory, and is then solved by a distributed control law.

Funding

This work was supported by the National Natural Science Foundation of China [grant number 61703167, 61803160] and Guangdong Nature Science Foundation [grant number 2020A1515010810] and Science and Technology Planning Project of Guangdong Province [grant number 2017A040405025] and Guangzhou Science Research Programme [grant number 201904010242].

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