Cracking Modeling In Partially Saturated Media With Phase-Field Method And Application To Rainfall-Induced Landslides

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Abstract. Cracking is the principal mechanics of failure in engineering structures and geological systems. Complex cracking patterns are observed in rock-like materials under compressive stresses. On the other hand, rainfall is one of the main triggering factors of landslides. Therefore, the rainfall induced landslides should be investigated by considering complex relationships between rainfall and rising groundwater table and related reduction of shear strength. In this paper, we present a new phase-field method for modeling the transition from diffuse damage to localized cracks. The damage variable is assumed to be driven by the deviatoric strain and positive (or dilatant) volumetric strain. This allows the description of tensile, shear and mixed cracks. The phase-field method is extended to partially saturated porous media by considering hydromechanical and phase-field coupling mechanisms. The proposed phase-field method is then applied to the analysis of rainfall-induced landslides in partially saturated conditions. The evolutions of pore water pressure, displacement and damage fields are predicted. Different failure mechanisms are discussed.

Keywords: Hydromechanical coupling, porous media, phase-field method, cracking, damage, landslides.

1. Introduction
One important cause of landslides is rainfall, especially in tropical regions with hot and humid climatic conditions [1]. The limit equilibrium method [2], which calculates the safety factor based on the limit equilibrium condition of slopes, has been the most prevalent way to evaluating slope stability thus far. Recently, computer modeling techniques such as the finite element method have been developed to estimate the slope safety factor in conjunction with the strength reduction approach [3]. However, this stability analysis that evaluates the equilibrium of forces on a single, representative slice of the sliding slope in the limit state may overlook the effects of crack evolution on the occurrence of slope failure. With the advancement of computer technology and related numerical analysis tools in recent decades, complex finite element analysis coupling with stress-seepage analyses, unsaturated soil behavior and plasticity provides an excellent framework for not only evaluating slope stability but also modelling progressive movement of slopes under heavy precipitation [4,5].

More recently, a new non-local damage theory, the so-called phase-field method has attracted more and more interests for modeling the initiation and propagation of multiple cracks in engineering materials and structures [6]. This method is initially based on the variational principle for linear fracture mechanics of elastic brittle mate-rials [7], and on the
optimal approximation methods of functionals with jumps [8]. However, in most phase-field models developed so far, only tensile cracks have been considered and their evolutions are generally related to tensile strain energy. The situation is different for rock-like materials subjected to compressive stresses, where both tensile and shear as well as mixed cracks can be induced.

In the present work, we shall develop a new phase-field method for complex cracking patterns and materials degradation mechanisms in rock-like materials, by splitting the strain energy into a spherical part and a deviatoric one. The proposed phase-field method is further extended to partially saturated porous media in order to account for hydromechanical coupling. Then, the new extended phase-field method for partially saturated media shall be applied to analyzing the failure mechanisms of rainfall induced landslides. In particular, the evolutions of rainfall induced displacement and damage field are investigated in terms of water saturation variation.

2. Phase-field method for partially saturated media
In this work, a partially saturated porous medium occupying the volume $\Omega$ is considered as a mixture composed of solid skeleton (noted by index s), pore water (noted by index w) and pore air (noted by index g). It is subjected to the body force $\mathbf{f}_b$ in $\Omega$, the surface force $\mathbf{t}_n$ on the external boundary $\partial \Omega$. During the entire loading history, we will identify the fields of displacement $\mathbf{u}$ (strains and stresses) and pore pressure (both for pore water and air), as well as the initiation and propagation of cracks inside. The current investigation is conducted in isothermal conditions.

2.1. Regularized crack fields
To overcome the difficulties in tracing the evolution of each individual cracks, in a regularized framework [9], it is proposed to approximate the total area of cracks by a crack surface density function as:

$$A_f = \int \mathbf{r} \cdot dA \cong \int_{\Omega} \gamma(d, \nabla d) \, dV \#$$

(1)

The auxiliary scalar $d \in [0, 1]$ is the phase-field variable which denotes an undamaged state by 0 and a fully damaged state by 1. A common form of the crack surface density function was introduced in [10] and is used here:

$$\gamma(d, \nabla d) = \frac{(d)^2}{2l_d} + \frac{l_d}{2} |\nabla d|^2 \#$$

(2)

2.2. Energy functions for damaged partially saturated media

**Stored energy.** The stored energy density of undamaged partially saturated media is assumed to be divided into two parts. One is the effective elastic contribution related of the solid skeleton, and another is the contribution related to the pore fluids response[11]:

$$\psi = \psi^{eff}(\varepsilon) + \psi^{fuid}(\varepsilon, m_w, m_g)$$

$$= \frac{1}{2} \mathbf{\sigma} : \varepsilon + \frac{1}{2} M_{ij} \left[ b_j \varepsilon_{ij} - \left( \frac{m_i}{p} \right)_{ij} \right] \left( b_j \varepsilon_{ij} - \left( \frac{m_i}{p} \right)_{ij} \right) (i,j = w, g)$$

(3)

in which $\mathbf{\sigma}^{b}$ is the Bishop’s effective stress tensor. In order to define physically based criteria for the growth of tensile and shear cracks, the effective elastic stored energy is decomposed into a spheric part and a deviatoric part [12] and degraded by a specific degradation function. Consequently, the effective stored energy density for damaged materials is rewritten as:
The degradation function $g(d) = (1 - d)^2$ proposed in [6] is adopted here. $k$ is the bulk modulus and $\mu$ is the shear modulus of undamaged materials. $\mathbf{e}_D = \mathbf{e} - \frac{1}{2}\text{tr} \left( \mathbf{e} \right) \mathbb{I}$ is the deviatoric part of the strain tensor with $\mathbb{I}$ being the second order identity tensor. The operator $\langle \cdot \rangle \pm$ is defined as: $\langle \cdot \rangle \pm = (\cdot \pm |\cdot|)/2$. Finally, one can obtain the rate of stored energy function for a damaged partially saturated media:

$$
\dot{E} = \int_{\Omega} \left[ \frac{\partial \psi}{\partial e} \nabla \dot{u} + \frac{\partial \psi}{\partial m_w} \dot{m}_w + \frac{\partial \psi}{\partial m_g} \dot{m}_g \right] dV \# 
$$

Potential dissipation. As suggested in [13], the fracture energy is assumed to be fully dissipative. On the other hand, inspired by the previous works of the energetic approach to fracture [14], the Griffith-type critical energy release rate $g_c$ is introduced here. Therefore, combining with the crack surface density Eq.(1), the energy dissipated rate trough crack formation reads:

$$
\dot{D} = \int_{\Omega} g_c \left[ \left\{ \frac{d}{dt} \right\} d + l_d \nabla d \nabla d \right] dV 
$$

External load power. As the basic assumption of phase-field method[13], the fracture phase-field $d$ is driven by the displacement $u$ of the solid. For this reason, there is no prescribed external loading associated with the phase-field. Then, the external applied energy rate is:

$$
\dot{P} = \int_{\Omega} f_b \cdot \dot{u} \ dV + \int_{\partial \Omega} t_n \cdot \dot{u} \ dS + \int_{\Omega} p_w \cdot \dot{m}_w \ dV + \int_{\Omega} p_g \cdot \dot{m}_g \ dV 
$$

2.3. Governing equations

Energy conservation requires a balance between the internal power and the power due to external load. Therefore, With Eq.(5), Eq.(6) and Eq.(7) and the Gauss theorem, one obtains the balance equations for stress, pore pressures and phase-fields. To simplify the problem, we assume that pore air pressure is constantly equal to the atmospheric pressure.

Momentum balance. With the linear poroelasticity conditions[11], one obtains:

$$
\text{div}(\sigma) + f_b = 0, \text{ with } d\sigma = \mathbb{C}^b(\mathbf{d}):\mathbf{e} - S_w b(\mathbf{d}p_w) \mathbb{I} \n$$

Here $S_w$ is the saturation degree of pore water, which is defined by [15] as:

$$
S_w = S_r + S_e (1 - S_r), \quad S_e = [1 + (\beta p_c)^n]^{-m} \n$$

where $S_r$ is the residual degree of saturation, $p_c (= p_g - p_w)$ the capillary pressure, and $\beta (1/\text{kPa})$, $n$ and $m (= 1 - 1/n)$ are curve fitting parameters of the soil water characteristic curve (SWCC).

The diffusivity equation for pore water. The fluid flow in porous media is described by the Darcy’s conduction law and the mass balance equation. Together with the constitutive relations, it gives:

$$
\frac{k_w \delta_{tt}}{\mu_w} \text{div}(\nabla p_w - \rho_w g) = \frac{1}{M} \frac{\delta p_w}{\delta t} + b S_w \frac{\delta e_i}{\delta t} 
$$

where $k_p$ is the saturated permeability, $\mu_w$ is the dynamic water viscosity, $g$ the gravitational acceleration and, $k_r$ is the relative permeability which is related to the saturation degree:
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\[ k_p = \sqrt{S_w} \left[ 1 - (1 - S_w^{1/m})^m \right] \]  

\((11)\)

**Governing equations of phase-field variables.** As \(g_c > 0\), the non-negative fracture dissipation Eq.(6) implies that the evolution rate of crack surface density function does not decrease. Physically, that means that the evolution of cracks is an irreversible process. In order to ensure this irreversibility condition, the concept of energy history function introduced in [3] is here adopted, which reads:

\[ H(t) = \max_{\tau \in [0, t]} W^\tau_+(\tau), \quad \text{with} \quad W^\tau_+(\tau) = \frac{k}{2} (\text{tr}(\varepsilon_\tau))^2 + \mu \varepsilon_D : \varepsilon_D \]  

\((12)\)

Consequently, the governing equation of phase-field variable is modified to:

\[ -2(1 - d)H - g_c \left[ \frac{d}{U_d} - l_d \text{div}(\nabla d) \right] = 0 \]  

\((13)\)

2.4. **Coupling between different fields**

As illustrated above, the phase field depends on the history energy from the mechanical field, and the mechanical field is influenced by the stiffness matrix which depends on the damage values. The pore pressure field and the mechanical field are coupled by the Biot coefficient. On the other hand, we assumed that the phase field influences the saturated permeability, which reads as

\[ k_p(d) = k_p^0 \exp(d) \]  

\((14)\)

3. **Numerical modeling and results**

3.1. **Input data of modeling**

In this study, we consider an ideally slope with an inclination angle of 40° to the horizon and a height of 20m. The mechanical boundary conditions are set as shown in the Fig. 1

![Geometrical domain and boundary conditions of analysis](image)

**Fig. 1** Geometrical domain and boundary conditions of analysis

For the boundary conditions used for hydraulic analysis, the rainfall (20mm/h ≈ 5.56×10^{-6} m/s) is assumed to be a water flux applied to the surface of the slope. The initial pore pressure distribution is assumed to be proportional to the vertical distance from the water table. Maximum negative pore pressure is specified as -60kPa. For many quasi-static geotechnical engineering problems, the atmospheric pressure is set as the reference pressure \((p_a \approx 0)\). We assume that the slope is isotropic and homogeneous and discrete the analysis domain into 20850 rectangular elements with 21161 nodes. In Table 1, the mechanical, hydraulic and damage parameters are presented.
Table 1. Table captions should be placed above the tables

| Parameters          | Symbol | Value       |
|---------------------|--------|-------------|
| Mechanics           | Bulk modulus | $k$     | 3.4 GPa |
|                     | Shear modulus | $\mu$   | 0.7 GPa |
| Hydraulic           | Residual degree of saturation | $S_r$ | 0.8 % |
|                     | Saturated permeability | $k_p$ | $5 \times 10^{-12}$ m² |
|                     | Fitting parameters for SWCC | $\beta$ | $57 \times 10^{-6}$ Pa⁻¹ |
| Phase-field         | Critical energy | $g_c$ | 0.55 N/m |
|                     | Smeared crack length | $l_d$ | 0.25 m |

3.2. Main numerical results

First of all, with the numerical model above, Fig. 2 presents the distribution of pore pressure at the end of rainfall (after 66h) without considering the influence of damage. The underground water table has a light increment because of the rainfall infiltration. Around the toe of the slope, there are some regions changing from unsaturated state into fully saturated state. The change of the saturated condition means the negative pore pressure changes into a positive pore pressure, which could cause a significantly influence on the effective stress. For comparing, the distributions of pore pressure considering the influence of damage when landslides occurs for the slope are presented in Fig. 2. It can be found that the pore pressure is significantly different in the damage zone.

![Fig. 2 Distribution of pore pressure (a) without considering damage; (b) with damage](image)

As an example for evolution of the rainfall induced damage in the analysis do-main, the distribution of global phase-field variable at three selected time steps during the period of rainfall are presented in Fig. 3. One can see that the damage appears firstly around the toe of the slope, which is mainly caused by increment of under-ground water level. With the incremental amount of rainfall infiltration, the damage region propagates from the toe to the top surface of slope along a nearly circular path. As shown in Fig. 3(c), when the damage zone reaches the top of slope, it means the slope is totally collapsed. Fig. 3(d) gives the distribution of displacement vector when the slope failure occurs.
Next, we assume that there is a localized crack embedded in the analysis domain, which is paralleled to the surface of the slope with the depth of 8m in the middle of the slope surface as shown in Fig. 4(a). In Fig. 4, one can find the development path of the global damage for a slope with pre-crack. It is found that the pre-crack has a significant effect on the slope failure pattern. As it can be seen that the pre-crack influenced the growth direction of cracks. Also, because of the existence of pre-crack, the time when the slope failure is earlier than the case without pre-crack.

It is well-known that landslides represent the combined effect of the predisposing factors and triggering factors. In this study, the aim is to demonstrate the ability of the proposed method to describe the initiation and propagation of localized damage zones and cracks due to rainfall in slopes. Consequently, to simplify the problems, some of the predisposing factors, such as intercalations[4], heterogenous of rocks and land use, are ignored. The infiltration of rainfall can also vary due the change of saturation degree. For those reasons, in future works,
we are going to applying the proposed phase-field method into the landslides analysis with more complex conditions.

4. Conclusion
In this paper, we have proposed a new phase-field method for modeling the initiation and propagation of cracks in rock-like materials. In order to described the initiation and propagation of tensile, shear and mixed cracks under different loading conditions, the effective elastic strain energy has been decomposed into a spheric part and a deviatoric part. Further, the coupling between the hydraulic and mechanics for partially saturated media has been taken into account. The proposed phase-field method for partially saturated media has successively applied to the analysis of rainfall induced landslides. The proposed method is able to describe the initiation and propagation of localized damage zones and cracks due to rainfall. The existence of initial weak zones and fractures enhances the failure process and also affects the cracking pattern.

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