Quantum Effects of P-wave Absorption by Metallic Films

S Sh Suleymanova, A A Yushkanov and N V Zverev
Moscow Region State University,
Very Voloshinoi str. 24, 141014 Mytishchi, Moscow Region, Russian Federation
E-mail: sevda-s@yandex.ru, yushkanov@inbox.ru, zverev_nv@mail.ru

Abstract. An influence of the kinetic and quantum wave properties of the degenerate electron plasma on the P-wave absorption in the metallic films is studied numerically. One has investigated the films having the width of order of the skin depth when the frequencies of radiation are not larger than the plasma frequency. It is found that in case of ordinary electron relaxation time, the power absorptance coefficient of the quantum electron plasma differs from the absorptances of both the classical electron plasma and the classical electron gas. However in case of the large relaxation times, one observes a coincidence of the quantum and the classical absorptances when the frequencies are much smaller than the plasma frequency, and the difference of the absorptances at the frequencies of order of the plasma frequency.

1. Introduction
Currently, the study of the interaction of electromagnetic radiation with conductors of low size is of great interest [1]. Such an interest is enforced both by theoretical investigations of the properties of conductive substances and by development of nanotechnology [2–6] having tasks of creation and utilization of optical devices like waveguides, optical gates [7], sensors [8–10], etc. with a thin bandwidth of radiation.

It the paper, one investigates numerically an influence of both the kinetic and the quantum wave properties of the degenerate electron plasma of carrier electrons on the absorption of P-waves in thin metallic films. Here one studies the absorptance power coefficient and compares the results for quantum degenerate electron plasma with the results for the classical degenerate electron plasma as well as for the classical electron gas. The comparison is performed for two values of the electron relaxation time: an ordinary relaxation time typical for metals, and a large one.

2. The model and the dielectric functions of degenerate electron plasma
We consider flat metallic film of the width $d$ which is localized between two transparent media with the constant permittivities $\varepsilon_1$ and $\varepsilon_2$. Assume that the electromagnetic P-wave (when the $E$ vector of the wave is parallel to the incidence plane) falls on the film from the medium with $\varepsilon_1$ under the incidence angle $\theta$. Then the power absorptance coefficient $A$ is evaluated throw
Here plasma were obtained in [13,14] and look as follows: plasma at 0 K temperature with constant relaxation time where the quantum wave properties of the conduction electrons. We investigate the quantum degenerate electron constant, \( \omega \), \( Z \) values of the dielectric functions from the film borders [12].

Here the values \( U(j), V(j) \) look as

\[
U(j) = \frac{\cos \theta - Z(j) \sqrt{\epsilon_1}}{\cos \theta' + Z(j) \sqrt{\epsilon_2}}, \quad V(j) = \frac{\cos \theta + Z(j) \sqrt{\epsilon_1}}{\cos \theta' + Z(j) \sqrt{\epsilon_2}}.
\]

(2)

The refractive angle \( \theta' \) to the medium with \( \epsilon_2 \) is evaluated according to the refraction law

\[
\sqrt{\epsilon_1} \sin \theta = \sqrt{\epsilon_2} \sin \theta',
\]

(3)

and \( Z(j) \) is dimensionless surface impedance for the conductive film [12]:

\[
Z(j) = \frac{2i\omega}{d} \sum_n \frac{1}{k_n^2} \left( \frac{(\pi n/d)^2}{\omega^2 \epsilon_{tr}(\omega, k_n)} - \frac{(ck_n)^2}{\omega^2 \epsilon_1(\omega, k_n)} \right).
\]

(4)

Here \( k_n = \sqrt{(\pi n/d)^2 + k_x^2} \), \( k_x = \frac{\omega}{c} \sqrt{\epsilon_1} \sin \theta \), and summation by \( n \) is performed over all odd values \( n = \pm 1, \pm 3, \pm 5, \ldots \) at \( j = 1 \), and over all even \( n = 0, \pm 2, \pm 4, \ldots \) at \( j = 2 \). It worth to note that the surface impedance \( Z(j) \) was evaluated in the case of mirror reflections of conduction electrons from the film borders [12].

In the equation (4), \( \epsilon_{tr}(\omega, k) \) and \( \epsilon_i(\omega, k) \) are respectively the transverse and longitudinal dielectric functions of the conduction electrons. We investigate the quantum degenerate electron plasma at 0 K temperature with constant relaxation time where the quantum wave properties of electrons are taken into account. The dielectric functions of the quantum degenerate electron plasma were obtained in [13,14] and look as follows:

\[
\epsilon_{tr}^{(qu)}(\omega, k) = 1 - \frac{1}{\Omega^2} \left( 1 + \frac{\Omega F(\Omega + i\gamma, Q) + i\gamma F(0, Q)}{\Omega + i\gamma} \right),
\]

(5)

\[
\epsilon_i^{(qu)}(\omega, k) = 1 + \frac{3}{4Q^2} \frac{(\Omega + i\gamma)G(\Omega + i\gamma, Q)G(0, Q)}{\Omega + i\gamma G(0, Q) + i\gamma G(0, Q)}.
\]

(6)

Here

\[
F(\Omega + i\gamma, Q) = \frac{3}{16r} |B_2(\Omega_+ + i\gamma, Q) - B_2(\Omega_- + i\gamma, Q)| + \frac{9}{8} \left( \frac{\Omega + i\gamma}{Q} \right)^2 + \frac{3}{32} Q^2 r^2 - \frac{5}{8},
\]

(7)

\[
G(\Omega + i\gamma, Q) = \frac{1}{r} |B_1(\Omega_+ + i\gamma, Q) - B_1(\Omega_- + i\gamma, Q)| + 2,
\]

(8)

\[
B_\alpha(\Omega + i\gamma, Q) = \frac{1}{Q^{2\alpha+1}} [(\Omega + i\gamma)^2 - Q^2]^{\alpha} \ln \frac{Q + i\gamma - Q}{\Omega + i\gamma + Q},
\]

(9)

and the dimensionless values

\[
\Omega = \frac{\omega}{\omega_p}, \quad \Omega_\pm = \Omega \pm \frac{1}{2} Q^2 r, \quad Q = \frac{v_F k}{\omega_p}, \quad \gamma = \frac{1}{\omega_p r}, \quad r = \frac{\hbar \omega_p}{m_e v_F^2}.
\]

(10)

In the equations (4) – (10), \( \omega \) is the wave frequency, \( c \) is the speed of light, \( \hbar \) is the Planck constant, \( \omega_p \) is the plasma frequency, \( v_F \) is the Fermi velocity of the conduction electrons, \( k \)
is the wave number, $m_e$ is the effective mass and $\tau$ is the relaxation time of the conduction electrons.

The dielectric functions (5), (6) reflect both the kinetic and quantum wave properties of the degenerate electron plasma. These functions in the classical limit $\hbar \rightarrow 0$ go over to the corresponding dielectric functions of the classical degenerate electron plasma [15] where the quantum wave properties are disregarded:

$$
\varepsilon_{tr}^{(cd)}(\omega, k) = 1 - \frac{3}{4\Omega} \left( \frac{2(\Omega + i\gamma)}{Q^2} + B_1(\Omega + i\gamma, Q) \right),
$$

$$
\varepsilon_l^{(cd)}(\omega, k) = 1 + \frac{3}{Q^2} \left( 1 + \frac{\Omega + i\gamma}{2Q} \ln \frac{\Omega + i\gamma - Q}{\Omega + i\gamma + Q} \right) \left( 1 + \frac{i\gamma}{2Q} \ln \frac{\Omega + i\gamma - Q}{\Omega + i\gamma + Q} \right)^{-1}.
$$

And in the long wave limit $k \rightarrow 0$ when the kinetic properties are omitted, both the quantum and the classical dielectric functions (5), (6) and (11), (12) go over to the dielectric functions of the classical electron gas in the Drude – Lorentz approach [12,15]:

$$
\varepsilon_{tr}^{(DL)}(\omega) = \varepsilon_l^{(DL)}(\omega) = 1 - \frac{1}{\Omega(\Omega + i\gamma)}.
$$

3. Results and discussion

In numerical simulations, we study the aluminum films lying on the quartz substrate with $\varepsilon_2 = 2$. The first transparent medium is an air or vacuum with $\varepsilon_1 = 1$. The aluminum metal has the following values [12]: $\omega_p = 1.93 \cdot 10^{16}$ s$^{-1}$, $v_F = 1.34 \cdot 10^6$ m/s, $m_e = 1.35 \cdot 10^{-30}$ kg, and an ordinary relaxation time at usual temperature is $\tau = 10^3 \omega_p$. We investigate also the large (high) relaxation time $\tau = 10^6 \omega_p$ having physical sense only at very low temperature. The film width $d$ in these investigations has the order of the skin depth $c/\omega_p$. The frequencies $\omega$ belong to the range $10^{-3} \omega_p \leq \omega \leq \omega_p$.

![Figure 1](image.png)

**Figure 1.** The absorptance coefficient $A$ as function of the frequency $\omega$ at ordinary relaxation time $\tau = 10^3 \omega_p$ (left plot) and at high relaxation time $\tau = 10^6 \omega_p$ (right plot). Values $d = 70$ nm, $\theta = 60^\circ$, $\omega_p = 1.93 \cdot 10^{16}$ s$^{-1}$, $v_F = 1.34 \cdot 10^6$ m/s, $\varepsilon_1 = 1$ (air), $\varepsilon_2 = 2$ (quartz): 1 – quantum degenerate electron plasma (solid line), 2 – classical degenerate electron plasma (dashed line), 3 – classical electron gas (dotted line).

Numerical calculations of the power absorptance coefficient $A$ by use of equations (1) – (13) have shown that at ordinary relaxation time $\tau = 10^3 \omega_p$, the $A$ coefficient in case of the quantum degenerate electron plasma with dielectric functions (5) and (6) differs from those both in the case of the classical electron plasma with dielectric functions (11) and (12) and in
the case of the classical electron gas with dielectric functions (13) (see left plot in figure 1). The value $A$ for classical electron plasma also differs from the value for classical electron gas but such a disagreement is weaker. And one can see that the absorptance coefficient $A$ of the quantum and classical electron plasma have the smooth maximum at the frequency $\omega \cong \pi v_F/d$ having the meaning of the frequency of periodic movement of conduction electrons between film borders [1,12].

The situation becomes different at high relaxation time $\tau = 10^6 \omega_p$. Here at the frequencies $\omega \ll \omega_p$, the power absorptance coefficient $A$ of the quantum electron plasma almost coincides with the coefficient of the classical electron plasma (see the right plot in the figure 1). The $A$ coefficients of both the quantum and classical electron plasma exceed significantly the absorptance of the classical electron gas at the frequencies $\omega \leq 0.9 \omega_p$. Such a behavior is explained by the phenomenon of the Landau attenuation in plasma [1]. Here one can observe the weak resonant behavior of the quantum and classical absorptances near the frequencies $\omega$ proportional to the $\pi v_F/d$ value. And for the frequencies $\omega$ in the range $\omega \sim (0.5 — 1) \omega_p$, the absorptance of the quantum electron plasma exceeds the absorptance of the classical electron plasma.

These results show an influence of both the kinetic and the quantum wave properties of the conduction electrons on the power absorptance coefficient of the P-wave. The obtained results should be taken into account in theoretical physics and in nanotechnology at the development of thin optical instruments.

Acknowledgements
The work is supported by the RBRF Grant No 19-07-00537 and by a scientific Moscow Region Governor grant for post-graduate students (2019).

References
[1] Paredes-Juarez A, Dias-Monge F, Makarov N M and Perez-Rodriguez F 2010 JETP Letters 90(9) 623–27
[2] Chausov D N, Kurilov A D, Kazak A V, Smirnova A I, Belyaev V V, Gevorkyan E V, Usol’tseva N V 2019
J Mol. Liq. 291 (1) 111259. DOI: 10.1016/j.molliq.2019.111259
[3] Chausov D N, Kurilov A D, Kazak A V, Smirnova A I, Velichko V K, Gevorkyan E V, Rozhkova N N,
Usol’tseva N V 2019 Liq. Cryst. 46 (9) 1345. DOI: 10.1080/02678292.2019.1566503
[4] Chausov D N, Kurilov A D, Belyaev V V, Kumar S 2018 Opto-Elect. Rev. 26 (1) 44. DOI:
10.1016/j.opelre.2017.12.001.
[5] Chausov D N 2018 Liq. Cryst. and their Appl. 18 (3) 45 (in Russ.). DOI: 10.18083/LCAppl.2018.3.45
[6] Chausov D N 2018 Journal of Physics: Conf. Series 996 012019. DOI: 10.1088/1742-6596/996/1/012019
[7] V V Belyaev and A S Solomatin and D N Chausov and A D Kurilov and V G Mazea and V M Shoshin
and Y P Bobylev 2014 App. Opt. 53 H51. DOI: 10.1364/OE.53.000H51
[8] V. M. Kozenkov and A. A. Spakhov and V. V. Belyaev and D. N. Chausov and V. G. Chigrinov 2018
Techn. Phys. 63 576. DOI: 10.1134/S1063784421840438
[9] V. M. Kozenkov and A. A. Spakhov and D. N. Chausov and V. V. Belyaev and O. V. Chausova and V. G.
Chigrinov 2017 J. Phys. Conf. Ser. 867 012039. DOI: 10.1088/1742-6596/867/1/01203
[10] Kazak V, Zlukova L N, Kovaleva M I, Chausov D N, Kuznetsov M M and Gorbudsdykova G F 2018
Liq. Cryst. and their Appl. 18 74 DOI 10.18083/LCAppl.2018.3.74.
[11] Yushkanov A A and Zverev N V 2017 Phys. Lett. A 381 679–84
[12] Fuchs R and Kliwer K L 1969 Phys. Rev. 185(3) 905–13
[13] Latyshev A V and Yushkanov A A 2013 Theor. and Math. Phys. 175(1) 559–69
[14] Latyshev A V and Yushkanov A A 2014 Theor. and Math. Phys. 178(1) 130–41
[15] Alexandrov A F, Bogdankevich L S and Rukhadze A A 1984 Principles of Plasma Electrodynamics (Springer-Verlag Berlin Heidelberg) p. XVI, 490

4