Definition and Identification of Information Storage and Processing Capabilities as Possible Markers for Turing-universality in Cellular Automata

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To identify potential universal cellular automata, a method is developed to measure information processing capacity of elementary cellular automata. We consider two features of cellular automata: Ability to store information, and ability to process information. We define local collections of cells as particles of cellular automata and consider information contained by particles. By using this method, information channels and channels' intersections can be shown. By observing these two features, potential universal cellular automata are classified into a certain class, and all elementary cellular automata can be classified into four groups, which correspond to S. Wolfram's four classes: 1) Homogeneous; 2) Regular; 3) Chaotic and 4) Complex. This result shows that using abilities of store and processing information to characterize complex systems is effective and succinct. And it is found that these abilities are capable of quantifying the complexity of systems.

1. Introduction

A universal system is a system that can execute any computer program. In other words, it is feasible for it to execute any algorithm [1]. It is found that some systems with simple rules can be a universal system, such as rule 110 in elementary cellular automata [2] [3]. Some tag systems and cyclic tag systems are also proved to be universal, which are also systems with simple rules [4] [5] [10]. Glider system, which is an idealized system to simulate particle process of real physics system, was also proved to be a universal system [2]. And particle machines in periodic backgrounds was proved to be universal [7].

The widespread existence of universal systems implies that some process with simple rules in the real world may be able to execute some algorithms or any algorithm. Because of the significant amount of algorithms, these systems' behaviors can be changeful and complex, which was considered as a potential origin of complexity in [3] [8].

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For cellular automata can show the wide variety of complex phenomena in the real world, and cellular automata are also sufficient generality for a wide variety of physical, chemical, biological, and other systems [11]. Identifying universal cellular automata will help people understand origins of cellular automata’s behaviors and find key dynamics of computation.

In this study, a method is developed to identify potential universal elementary cellular automata. Two abilities of a system are considered: 1) Ability to store information and 2) Ability to process information. We found these two features can identify potential universal cellular automata and quantify the complexity of systems.

1.1 Elementary Cellular Automata

Cellular Automata (CA for singular, CAs for plural) are ideal models for physical systems in which space and time are discrete. And elementary cellular automata (ECA for singular, ECAs for plural) is one of the simplest kind of CAs.

ECAs are dynamic systems defined by deterministic rules, working on a 1-dimension list \( \{c_n\} \) with \( n \) cells. Rules can be expressed by function \( F \):

\[
c_n(t + 1) = F[c_{n-1}(t), c_n(t), c_{n+1}(t)],
\]

where \( n \in \mathbb{Z} \).

Therefore, \( c_n(t + 1) \) is the function of itself \( c_n(t) \) and its two immediate neighbors: \( c_{n-1}(t) \) and \( c_{n+1}(t) \). Each \( c_n(t) \) has two possible states, 0 or 1. So there should be a \( 2^3 = 8 \) length list \( R \) to define a rule, and there will be \( 2^8 = 256 \) different rules. When \( R \) is equal to \( \{0, 0, 0, 1, 1, 1, 1, 0\} \), by considering it as a binary code, it will equal to 30 in decimal base, which is the ECA rule 30.

With a given initial list \( L_0 \), an ECA will apply the function \( F \) to all cells parallelly to update \( L_t \) to \( L_{t+1} \). i.e.,

\[
L_t \xrightarrow{F} L_{t+1}.
\]

By doing this process repeatedly, a matrix \( M^{(\text{rule})} = (L_0, L_1, \ldots, L_t) \) will be generated, which is the “space–time evolution”. Figure 1 shows two space–time evolutions generated by ECA rule 30 and ECA rule 110, started with the same \( L_0 \).

256 different ECAs can be classified. In this paper, we compare our work with Wolfram’s classification, which are class 1-4 in [3, 11]. The classes are: 1) Homogeneous; 2) Regular; 3) Chaotic; 4) Complex. Some typical space–time evolutions are shown in Figure 1. There are also some other classifications, see [3, 5, 6].
Figure 1. Evolution of 4 typical rules from class 1–4. Rule 8 is in class 1, rule 4 is in class 2, rule 30 is in class 3, and rule 110 is in class 4.

2. Methodology

We consider two abilities of ECA rules: Ability to store information, and ability to process information. The ability to store information will make the system stable enough and do not have too much noise. Only when information can be stored, information can move stably in a system, so that the whole system can be related. The ability to process information means interactions between information should be found in a system.

We define a system can store information when its current local states can be used to infer previous states at some location. It’s true that some reversible systems can store all information at the whole system, but this information can hardly be used to infer the previous states because many of them are computational irreducible. Thus, the particle systems can cover the definition.

We identify potential universal ECAs based on a theorem proposed in [7], which considers particle-like structures and their behavior in systems to identify Turing machines and UTM.

A method was developed to extract particle patterns from ECAs to
build “particle machines”, and to measure their computation ability by taking into account their features. First, it is necessary to introduce particles machines and define particles in ECAs.

### 2.1 Particle machines

A particles machine (PM), is a system in which particles can move, collide, annihilate and generate in a homogeneous medium. Figure 2 shows a typical PM. Data and configurations are injected from left in the form of particles, and by executing this system, particles will have interactions. Lines and dotted lines in this figure represent the paths of particles. After time $t$, the system will generate an output. The identity of a particle includes position, phase, and velocity. During collisions, particles can alter their identities, or be generated or annihilated. These changes of particles can be considered as a function of particles that participate in the collision, which is the collision function. Some particles machines are proved to be Turing machines or universal Turing machine (UTM) in [7]. A PM is at least a Turing machine when: 1) Identity of particles can change during collisions; 2) Collision function is depending on identities of particles. For the first requirement, the identity of particles can change during collisions, also means new particles can be generated during collisions. And the second requirement means the result of a collision should depend on types of particles that participate in the collision. If no particles can be generated or annihilated in collisions in a PM, then the PM is not a UTM.

### 2.2 Particles in ECAs

We define a local grid of cells in $M^{(\text{rule})}$ as a particle in ECAs. Here we consider one kind of particles: Their sequence may change periodically or not change through time. We call them “elementary particles”. It will be practical if we start with these simple kind of particles.

Particles contain information, so that information can move in space,
Figure 3. A). An illustration of how it takes a “target particle” $P$ from a matrix generated by ECA rule 110. The rectangle with black frame is the target particle $P$, and gray rectangles mean there has a similar sequence as $P$, which are linear particles $L$. In this figure, the $P$ and $L$s are found, and found 54 same sequences. B). A figure of matrix $M^{(110)}$. $M^{(110)}_{t,x} = p_0$ when there is a $L$ at $\{t,x\}$, or $L_{t,x} = 0$. Dots at $\{t,x\}$ means $M^{(110)}_{t,x} = p_0$.

and have interaction with other information, which is a kind of computation $\mathcal{F}$. All identities of particles: Location, velocity, and sequence, can be computed by collisions. And all of these identities can be preserved if there are no collisions.

To extract particles’ identities from ECAs’ space–time evolutions, a certain sequence should be chosen for the research. We need to choose a sequence as a particle to study, which is the “target particle”. As Figure 3.A shows, we choose target particle $P$ at the center-bottom of a space–time evolution and mark the same sequences as “linear particles” $L$s, which are the dots in Figure 3.B.

$P$ can be explained by the equation $P = L_{t_{\text{max}}} (p_L, p_R)$, where $L_t$ is the $t$–th row of the space–time evolution. $p_L$ and $p_R$ is the start-index and the end-index for $P$.

A particle $P$ at $(t; x)$, its location may be $(t'; x')$ at time $t'$ ($t' < t$). We call the particle at $(t'; x')$ as $P'$s father-particle $P_f$. If let $P$ be $P$, the $P_f$ will be one of the $L$s.

All $L$s in $P'$s light cone are possible to be the father-particle of $P$ (i.e. $P_f$), we assume that there is one and only one $L$ is the $P_f$, and each $L$ has probability $p$ to be the $P_f$. So when there are $n$ $L$s, the probability (i.e. $p_0$) for a $L_i$ to be a father-particle is:

$$p_0(p, n) = p (1 - p)^{n-1}.$$  \hspace{1cm} (3)

All the $L_i$ are drawn on a matrix $M^{(\text{rule})}$, such as Figure 3.B. $M^{(\text{rule})}_{t,x} = p_0$ when there is a $L$ at $\{t,x\}$, or $L_{t,x} = 0$. We call $M^{(\text{rule})}$ “probability matrix”. The positions with black points will add a num-
Figure 4. The average matrix \( \mathcal{M}^{(110)} \), generated with \( 10^6 \) probability matrices, with \( p \) equal to 0.01 for Equation (1).

Each black point means a linear particle \( \mathcal{L} \) of \( \mathcal{V} \) at \((t,x)\), \((t,x)\) is the location of the black point. \( \mathcal{M}_{t,x} \) equals to \( p_0 \) \((p,n)\).

The average \( \mathcal{M}^{(\text{rule})} \) that generated with random initial lists:

\[
\mathcal{M}^{(\text{rule})} = \frac{1}{N} \sum_{i=1}^{N} \mathcal{M}^{(\text{rule})}_{\text{random}},
\]

will show some patterns that represent particles and particles’ behavior. We call \( \mathcal{M}^{(\text{rule})} \) “average matrix”. Figure 4 shows how an average matrix was generated.

The meaning of an average matrix is, if a particle is found at the center-bottom of a space–time evolution, it may come from position \((t,x)\) with probability \( \mathcal{M}_{t,x} / \sum_{t,x} \mathcal{M}_{t,x} \). So the pattern in an average matrix represents traces of particles. We calculate the average matrix with \( N = 10^6 \) for each rule.

2.3 Extracting particle’s identity from an average matrix

By observing patterns of average matrices, the identity of particles can be extracted. A typical average matrix is shown in Figure 4. If particles can emerge, there will be some lines in the average matrix. Each line represents at least one particle, and their variations show interactions between particles.

The change of a line’s intensity with time represents interactions between particles. Because if a particle is moving straight without any interactions, the lines’ intensity will not change through time. But if the particle can be generated by other particles, it will not be found before it was created, so that the intensity will change through time, mostly, the intensity will get higher when \( t \) is getting higher.
3. Result

We get $M$ for all rules, some typical $M$ shown in Figure 5.

We get numbers of particles’ traces for each ECA rules, which correspond to the number of particles. All traces are straight lines with various angles. For rules shown in Figure 5, rule 54 has 3 traces, rule 62 has 2 traces, rule 110 has more than 6 traces, and rule 18 has a smooth trace. We use $T$(rule) to represent the count of traces, such as $T(54) = 3$, which can be used as a parameter to classify ECAs.

The intensity of traces may change through time. The result shows that they have two kind behaviors: 1) Constant, 2) Variational (mostly, the intensity getting higher when $t$ is getting higher). We use $C$(rule) to represent the existence of variation, such as $C(54) = 1$ (1 is variational, 0 is constant ). These two behaviors can be used as a parameter to classify ECAs. In Figure 5, traces in rule 54, 62 and 110 are getting more obvious when time $t$ gets higher. Figure 5 shows how the intensity of particle traces variation with time, where $D(t) = \max(L_t)$.

Power law show in some rules, where $D(t) \sim (t_{\text{max}} - t)^{-\alpha}$, such as Rule 146 and Rule 18, such power law also found by [12] (see Figure 10).

3.1 Identifying Turing Machines and Potential UTM

To identify Turing machines and potential UTM, the two parameters we mentioned above will be used to classify ECA rules into four classes. According to the theorem of particle machines [2], when $T$(rule) $\geq 2$ and $C$(rule) $= 1$, then this ECA rule behave as a Turing machine and potentially be a UTM. A particle machine that is a Turing machine should have at least 2 particle traces so that it is possible to have interactions between particles. And traces’ intensity should change,
The classification of ECAs, divided by the number of paths and intensity variation. The number of paths associated with the ability to store information, and intensity of variation associated with the ability to process information. In each phase, rules will have similar behaviors. In the phase-A, all rules have both a high number of traces ($T \geq 2$) and interactions that can generate particles, so that it is possible for these rules to have complex behaviors. The shape of a point represents its class in Wolfram’s classification. Each point in this figure represents a rule, and their positions were moved randomly ($\sim 0.3$) so that they can be seen clearly without too many overlaps.

which represents that new particles can be generated during collisions. So all rules can be classified into four classes: A). $T \geq 2$ and $C = 1$; B). $T < 2$ and $C = 1$; C). $T < 2$ and $C = 0$; D). $T \geq 2$ and $C = 0$.

Figure 6 shows the final classification for all rules of elementary cellular automata. Each point represents a rule for an elementary cellular automaton. The $x$-axis is “number of traces”, and $y$-axis represent the existence of information traces’ changes, where 0 means constant, 1 means variational. The shape of a point represents its class in Wolfram’s classification.
In class A, rules have complex behaviors, and many particles with plentiful interactions can be found. The information here will be stored and processed. Then they can be considered as a Turing machine with enough complexity and computation ability, which was considered to have connections with Turing universality [8, 14]. In class B, rules will generate some random patterns, particles have too many interactions with the background, so that information traces are dissipated. The information here cannot be stored: In class C, rules will generate continuous or random structures without any complex behavior. Rules in this class do not have particles or have particles but no interactions. In class D, rules will generate some structures that do not have enough interactions, which will not have any complex behavior either. New particles cannot be generated during collisions.

Class C can be divided into two subclasses, as shown in Figure 6, separated by a dotted line. We use “Rule x” to express the subclasses. C0 means the subclass of class C with T equal to 0. C1 means a subclass of class C with T equal to 1. In C0, rules do not have any particles, the information here cannot be stored or processed. In C1, rules have particles but do not have interactions between particles. The information here can only be stored but cannot be processed.

When going through the dark curve in Figure 6 (anticlockwise), the frequency of finding interactions is continually growing. And when the frequency is higher than it in class A, it will generate too much noise, so particles and information will be scattered. When it is lower than the frequency in class A, the number of interactions is not enough to do computation or universal computation, so the behavior is too simple to get complex behaviors.

Some typical rules in these 4 classes show in Table 1. All rules’ classification are shown in Figure 7.

The relation between this classification and Wolfram’s classification was also studied. According to Figure 8, Class C1 and D have a strong correlation to a certain Wolfram class, which is Class 2. While class A, B, and C0 contain some different Wolfram classes. Here the reduced entropy is used to measure the relation between the two classifications because the Wolfram classification does not have an order. The reduced entropy is defined as \( h = H / H_{\text{max}} = H / \log n \) where \( H_{\text{max}} \) is the maximum that entropy \( H \) could be.

### 4. Discussions

In this study, we consider two abilities as key dynamics for computation:

1. Ability to store information;
2. Ability to process information.
Figure 7. This is the final classification of all ECA rules with the method introduced in this paper. In this matrix, each kind of texture or color represents a class defined by this paper (see the column at right side), and the numbers over each square are the rule indexes. Each texture is associated with the ability of processing and storing information, which corresponding to the computation ability. The class A, which have both high information store and process ability, is considered having high computation ability. Rule 110, which is a UTM, is classified into this class.

The ability to store information means there should be particles emerge in a system so that information can move in the system. And in this way, the whole system can be connected and linked to be an entirety, which was considered as a common feature of complex systems. Ability to process information means the system can compute information and execute algorithms.

By using the coarse-grained method, robust patterns can be found, rules with different computation abilities are classified into a particular class (class A, shown in Figure 6).

All ECA rules can be classified into four classes, which correspond to Wolfram’s classification. All rules in class 1 and most rules in class 2 (Wolfram’s classification), was found do not have interactions that can generate new particles. Most rules in class 3 are found do not have enough particles to perform the universal computation. All rules in class 4 are found classified into class A in this study. For rule 146, 183, 18 and 22, which are classified into class 3 (chaotic) by Wolfram, are classified into class A in this study, which means these rules are capable of doing complex computations. This result corresponds to
Figure 8. This figure shows the relation between Wolfram’s classification and the classification in this paper. The orange line with dot markers is the average of $W_i$, which is Wolfram’s classes of the rules in class $i$ of this paper. The average numbers only make sense when all rule in a certain class (of this paper) have a same Wolfram class because the Wolfram class do not have an order. The blue line with square markers is the reduced entropy of Wolfram classes of rules in certain class in this paper, which can measure the correlation between these two kinds of classification. The reduced entropy is defined as $h = H / H_{\text{max}} = H / \log n$, where $H$ is the entropy of $W_i$, and $n$ is the length of $W_i$.

the research [12]. Particles and interactions are found in rule 146, and it is shown that the intensity of traces in the average matrix is corresponding to [12]. The differences of the classifications between this paper’s and Wolfram’s come from the different criterions. For example, in Wolfram class 2, some rules shows particle interactions and others not, which were classified into different classes in this paper.

Since the problems of storing and processing information can be found in various fields, such as chemical systems [13] and hydrodynamics [15, 16], and this method is not based on ECAs’ specific features, so it is potentially to be applied to other systems, such as birds flock [17], traffic flow [18], chaotic behaviors [15, 16], and complex networks [19]. This method can also be used to quantify the complexity of systems, for UTM was considered having the highest complexity by [20], which will make people have a deeper understanding of complex behaviors.

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Appendix

.1 Particles in ECAs

I define a local grid of cells in $M$ as a particle in ECAs. Backgrounds are also particles, which do not have any interactions with other particles or themselves. According to the definition of particles in ECAs:

$$\mathcal{P} = L_{\text{max}}(p_L, p_R).$$

(.1)

In this study, the size of a space–time evolution is $(200, 200)$. The target particle

$$\mathcal{P} = L_{200}(100 - 2, 100 + 2).$$

(.2)

For the formula

$$p_0(p, n) = p(1 - p)^{n-1}.$$  

(.3)

The number of $p$ is a priori hypothesis, choosing a proper $p$ will make images clear. Figure 9 shows that the formula with different $p$ will not change its whole behavior. Experiments show that choosing $p = 0.01$ will make average matrices clear enough.

.2 Particles in Rule 146

Figure 10 show particles in the space-time for rule 146. These particles are also introduced by [12].
.3 Changes of lines’ intensity

The change of a line’s intensity with time represents interactions between particles. Because if a particle moves straight without any interactions, the lines’ intensity will remain unchanged through time. But if the particle can be generated by other particles, it will not be found before it was generated, so that the intensity of lines will change through time, mostly, the intensity will get higher when \( t \) is getting higher. To get particles’ changes of time, we define a function \( D(t) \) to get paths’ intensity:

\[
D(t) = \max(L_i).
\] (4)

Figure 11 shows the procedure of extracting growth pattern of particles and three examples for rule 149, rule 2 and rule 26.

.3.1 The growth of particle traces’ intensity for rule 146

Particles were found in Rule 146 (shown in Figure 10), while also founded earlier in 2010 [12]. In that study, the intensity of particles in rule 146 has a power-law of the form

\[
n_0(t) \sim t^{-\alpha},
\] (5)

with \( \alpha = 0.4789 \pm 0.0006 \) [12]. When this formula with this number was applied to the data in this study (shown in Figure 12), it shows a good fit result.

.4 Typical Rules for Four Classes

Some space–time evolutions of typical rules in each class shown in Table 1.
Figure 10. Particles are found in rule 146.

Figure 11. Extracting growth pattern of particles. A) The growth of particles’ intensity represents interactions of particles. B) An example of particle’s intensity, generated with rule 149, which has a growth pattern. C) Generated with rule 2, which do not has growth pattern. D) Generated with rule 26, with multiple particles and they all do not have growth pattern.
The points are the data for the growth of particles’ traces. And the line is the figure of function $y = (t_{\text{max}} - t)^{-0.4789}$, which has the same form as $n(t) \sim t^{-\alpha}$. It shows that the power–law also is shown in this kind of measurement, and it has a good fit when using the number of $\alpha$ from [12].

Table 1. Typical Rules of Four Classes

|   | A          | B          | C          | D          |
|---|------------|------------|------------|------------|
|   | Rule 22    | Rule 54    | Rule 62    | Rule 110   |
|   | Rule 41    | Rule 30    | Rule 106   | Rule 120   |
|   | Rule 40    | Rule 90    | Rule 1    | Rule 24    |
|   | Rule 9     | Rule 25    | Rule 43    | Rule 57    |

Figure 12. The points are the data for the growth of particles’ traces. And the line is the figure of function $y = (t_{\text{max}} - t)^{-0.4789}$, which has the same form as $n(t) \sim t^{-\alpha}$. It shows that the power–law also is shown in this kind of measurement, and it has a good fit when using the number of $\alpha$ from [12].