Exclusive Reactions in QCD

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1 Introduction

We review the theory of hard exclusive scattering in Quantum Chromodynamics. After recalling the classical counting rules which describe the leading scale dependence of form factors and exclusive cross-sections at fixed angle, the pedagogical example of the pion form factor is developed in some detail in order to show explicitly what factorization means in the QCD framework. The picture generalizes to many hard reactions which are at the heart of the ELFE project. We briefly present the concepts of color transparency.

2 Space-time picture and Counting Rules

We consider here exclusive processes, that is interactions resulting in a final state where all particles are identified. Using a perturbative expansion to study these reactions may a priori be foreseen if a large momentum transfer appears: this is what is called a hard reaction.

Before going to a QCD calculation, it is very instructive to develop first a space-time picture of these reactions.

The simplest exclusive quantity is the pion form factor, as measured in the process $e^- \pi^+ \rightarrow e^- \pi^+$. It measures the ability of the pion to stay itself when being collided by an electron. It is thus a quantity much sensitive

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1 Lecture given at the Les Houches Summer School: Trends in Nuclear Physics Hundred Years Later, July 30 to August 30 1996, to be published in the Proceedings, H. Niffenecker ed. (Elsevier)

2 Unité propre 14 du Centre National de la Recherche Scientifique.
to confinement mechanisms. The physics deals with the restoration of the meson integrity after the violent shock of a high-energy electron with one of the quarks. At the limit $Q^2 = 0$, the meson structure is not resolved, and $F_\pi(0) = 1$.

Let us first derive in a heuristic way the $Q^2$ dependence of the pion form factor by a careful examination of the way this process takes place. In its rest frame, the pion is represented as a collection of partons, quarks and gluons, approximately uniformly spreaded in a sphere of radius $R$ (typically the pion charge radius, around 0.5 fm). In the reaction center of mass frame, where the final electron emerges at an angle of $180^\circ$ with respect to the initial electron, the longitudinal dimension is Lorentz-contracted to $R/\gamma$ with $\gamma = Q^2/2M$. The transverse dimensions are on the other hand not affected by Lorentz-contraction. At time 0, the electron hits one of the partons, the so-called active parton, and both change directions. For the whole process to be elastic, all other partons must be alerted before the moment $t \approx 1/Q$ to form the emerging pion (also contracted in this frame). The motion of the active parton after the collision is $z(t) = -t$, $x(t) = 0$, $y(t) = 0$ whereas the motion of a spectator parton is $z(t) = t + z_0$, $x(t) = x_0$, $y(t) = y_0$ (one has $-1/Q < z_0 < 1/Q$ and $-R \leq x_0, y_0 \leq R$). Between the moments 0 and $1/Q$ a spectator parton can receive and respond to a physical signal emitted by the active parton at time 0 only if the interval $\Delta = t^2 - (t + z_0)^2 - x_0^2 - y_0^2$ is positive, that is if the spectator is at a distance $\sqrt{x_0^2 + y_0^2} < 1/Q$ in the transverse plane. One thus counts the probability to find spectator partons in a transverse disc of radius $1/Q$, in the initial as well as in the final state.

One gets\(^2\)

$$F_\pi^2 \propto \left(\frac{\pi Q^{-2}}{\pi R_\pi^2}\right)^{n_{\text{in}}-1+n_{\text{out}}-1}. \quad (1)$$

Since a pion contains at least a valence quark and antiquark, we get a minimal contribution scaling like $1/Q^2$. Adding for instance one gluon to the valence in the initial state, without changing the final state, yields a contribution scaling like $1/Q^3 \ldots$ These contributions diminish relatively to the valence state contribution as energy increases.

This most important feature of the study of form factors at large transfer may be generalized to other exclusive reactions\(^3\): when the interaction is at short distance, the valence contributes in a dominant way in terms of scaling with the important exception of hadron hadron exclusive reactions for which a separate
Moreover, and this will be crucial for the phenomenon of color transparency, the hadron configurations which contribute have small \( O(1/Q) \) transverse sizes.

Let us summarize: asymptotically, one predicts for the energy dependence of pion and nucleon form factors \(^1\) a power-law fall off:

\[
F_\pi(Q^2) \propto \frac{1}{Q^2}, \quad F_N(Q^2) \propto \frac{1}{Q^4},
\]

and for exclusive reactions differential cross sections at fixed angle \( i.e. \) at large \( s, -t \) and \( -u \) \(^2\):

\[
\frac{d\sigma}{dt} = \frac{1}{s^{N-2}} f_\frac{t}{s},
\]

where \( N \) is the total number of elementary fields participating in the reaction. One thus gets that the transition between valence states dominates hard amplitudes, \( i.e. \) we get a leading contribution with \( N=8 \) for Compton scattering, with \( N=9 \) for meson photoproduction, etc. The QCD analysis presented in section 3 will strengthen the argument presented here and develop a consistent way of calculating the leading contribution. It will however be important to phenomenologically verify that the scaling laws, and thus the dominance of valence states, apply at accessible energies, and this for each physical process under study.

3 Calculating a hard exclusive amplitude: the example of the pion form factor

One now wants to really calculate from QCD a hard amplitude \(^3\); we will here detail the procedure in one of the simplest example, namely the electromagnetic pion form factor at large spacelike transfer \( Q^2 \). This leads us to study has been developed\(^4\) with the important result of valence selection but with different transverse size and scaling behaviours.

\(^4\) In the proton case, there are two form factors and the reasoning developed here does not allow to distinguish them. In fact, if one separates the form factors with respect to their degree of helicity conservation, one shows that the above counting rule applies only for helicity conserving processes (and thus for the magnetic form factor \( G_M \), but that an additional power suppression affects \( F_2 \).
precise first the hadron wave function and the Born hard amplitudes, then
the radiative corrections to see if a sensible picture emerges where a non
perturbative object sensitive to confinement dynamics factorizes from a hard
scattering amplitude controlled by a perturbative expansion in \( \alpha_s(Q^2) \) which
is renormalization group improved. This factorization which is crucial for a
consistent understanding of future experimental data turns out to be valid
for many hard exclusive reactions\(^5\). It may be pictorially described as in
Fig.1.

\[ \text{Figure 1: Factorization of a hard exclusive process: } X \ast T_H \ast X' \]

### 3.1 Description of the pion

Let us specify the kinematics. In the Breit frame the momenta are written
as:

\[
q = \begin{pmatrix} 0 \\ 0 \\ 0 \\ Q \end{pmatrix}, \quad p = \begin{pmatrix} Q/2 \\ 0 \\ 0 \\ -Q/2 \end{pmatrix}, \quad p' = \begin{pmatrix} Q/2 \\ 0 \\ 0 \\ Q/2 \end{pmatrix};
\]

\[ (4) \]

where the pion mass is neglected in front of \( Q \).

To describe the pion in its valence state, one introduces the Bethe-Salpeter (BS)
amplitude\(^7\)

\[
\langle 0 | T (q \overline{u}\partial \lambda(y) P_{ij}(y, 0) \overline{d}\partial \beta(0)) | \pi^+(p) \rangle,
\]

\[ (5) \]

where \( u \) and \( \overline{d} \) are the flavours of the valence quarks of \( \pi^+ \), \( \alpha \) and \( \beta \) are Dirac
indices and \( i, j \) are color indices. The \( P_{ij} \) operator is necessary to have an
amplitude invariant under local gauge transformations; when \( q(y) \) transforms

\[ ^5 \text{indeed factorization also holds for hadron-hadron reactions}\[5\] \]

4
to \( U(y) q(y) \), \( P(y,0) \) transforms to \( U^{-1}(y) P(y,0) U(0) \), compensating the quark and antiquark variations. The BS amplitude is the relativistic generalisation of the Schrödinger wave function describing the bound state of a quark antiquark pair. One may interpret it as the probability amplitude of finding in a \( \pi^+ \) a \( u \) quark at point \( y \) and a \( \bar{d} \) antiquark at the origin.

One often prefers to work in momentum space and defines the Fourier transform of the BS amplitude as

\[
\int d^4y e^{ik.y} \langle 0| T (q_{\alpha i}(y) \ P_{ij}(y,0) \ \bar{q}_{\beta j}(0)) |\pi^+(p)\rangle = X_{\alpha\beta}(k; p - k) \tag{6}
\]

where \( k \) is the quark momentum and, by momentum conservation, \( p - k \) is the antiquark momentum.

To discuss the properties of this amplitude, it is convenient to introduce light-cone coordinates defined as

\[
\begin{align*}
k^+ &= \frac{1}{\sqrt{2}} (k^0 - k^3) \\
k^- &= \frac{1}{\sqrt{2}} (k^0 + k^3)
\end{align*}
\tag{7}
\]

We thus have (listing \( p = [p^+, p^-, p_1, p_2] \))

\[
p = [Q/\sqrt{2}, 0, 0, 0], \quad p' = [0, Q/\sqrt{2}, 0, 0],
\tag{8}
\]

and we parametrize the internal momenta as \( k = [xQ/\sqrt{2}, k^-, \mathbf{k}_\perp] \), where \( x \) is the light-cone fraction carried by the quark inside the pion. The antiquark then carries the fraction \( 1 - x = \bar{x} \). The final pion is treated similarly, with + and – components exchanged: \( k' = [k'^+, x'Q/\sqrt{2}, \mathbf{k}'_\perp] \).

In terms of these variables, the \( k^\mu \) regions favored by the amplitude \( X(k, p - k) \) are simply written as:

\[
k^2_\perp < M^2, \quad |k^-| < M^2/Q.
\tag{9}
\]

### 3.2 The hard scattering at the Born level

The matrix element of Figure 1 is written as the convolution

\[
\int \frac{d^4k \ d^4k'}{(2\pi)^4 (2\pi)^4} \ X(k) \ T^\mu_H(k, k') \ X^\dagger(k'). \tag{10}
\]
At the lowest order in the QCD coupling constant, $g$, one finds 4 Feynman diagrams. One is drawn on Figure 2 and the 3 others are easily deduced by attaching successively the photon to the points 2, 3 and 4.

Let us first evaluate the gluon squared momentum, which is in Feynman gauge, the denominator of the gluon propagator. We have

$$(p' - k' - p + k)^2 = -\bar{x}x'Q^2 - \sqrt{2}Q(k^-\bar{x} + k'^-\bar{x}) - 2k^-k'^+ - (k_\perp - k_\perp')^2$$

$$O(Q^2) \quad O(M^2) \quad O(M^4/Q^2) \quad O(M^2)$$

where typical orders of magnitude indicated refer to the momentum regions favored by the amplitudes $X(k)$ and $X(k')$. Restricting to leading terms in $Q$, we may forget terms of order $M^2$. So, in particular, we write

$$(p' - k' - p + k)^2 \approx -\bar{x}x'Q^2.$$  \hspace{1cm} (12)$$

Figure 2: Born Graph for the pion form factor; the 3 other graphs are deduced by attaching the photon to the points 2, 3 and 4. Propagators joining Bethe-Salpeter amplitudes to the vertices are absorbed, by definition, in these amplitudes.

The same analysis may be repeated for the other quantities present in the hard amplitude $T_H^\mu$, leading to

$$T_H^\mu(k, k') \approx T_H^\mu\left(x\frac{Q}{\sqrt{2}}, x'\frac{Q}{\sqrt{2}}\right).$$  \hspace{1cm} (13)$$
We may then express the convolution of equation (3.7) under the form

\[
\int dx dx' \left( \frac{Q}{2\sqrt{2}\pi} \int \frac{dk^- d\mathbf{k}_\perp}{(2\pi)^3} X(k) \right) T^\mu_H(x, x') \left( \frac{Q}{2\sqrt{2}\pi} \int \frac{dk'^+ d\mathbf{k}'_\perp}{(2\pi)^3} \tilde{X}^\dagger(k') \right),
\]

and the object needed to describe the pion in this reaction is in fact much simpler than the amplitude \( X \) since one may integrate over three components of the internal momentum.

A first simplification comes from the integration over \( k^- \) (for the outgoing pion over the \( k'^+ \)). In terms of the conjugated variable \( y^+ \), this means that one only needs the Bethe-Salpeter amplitude at \( y^+ = 0 \), which is called the light cone wave function, usually noted as \( \psi(x, \mathbf{k}_\perp) \) [8]. A useful property of this wave function is that the support in the \( x \) fraction is limited, as \( 0 \leq x \leq 1 \).

The Dirac structure of the amplitude \( X(k) \) integrated over \( k^- \) and \( \mathbf{k}_\perp \) is easy to extract [9] and one finds

\[
M_{\alpha\beta}(x, p) = \frac{1}{4} \gamma^5 p / \phi(x) |_{\alpha\beta}.
\]

This Dirac structure corresponds to the combination of spinors (\( \uparrow \) and \( \downarrow \) denote respectively the helicity states + and -)

\[
\frac{1}{4} \gamma^5 p |_{\alpha\beta} = \frac{1}{2\sqrt{2}x \bar{x}} \frac{1}{\sqrt{2}} \left( u_\alpha(xp, \uparrow) \bar{v}(\bar{x}p, \downarrow) - u_\alpha(xp, \downarrow) \bar{v}(\bar{x}p, \uparrow) \right),
\]

i.e. one recovers the pion spin wave function in the quark model

\[
\frac{1}{\sqrt{2}} \left( | \uparrow\downarrow \rangle - | \downarrow\uparrow \rangle \right).
\]

The function \( \phi(x) \) is called the distribution amplitude; it “measures” how the pion momentum is distributed between the valence quark and antiquark when their transverse separation vanishes. This is the non perturbative amplitude connecting long distance physics of strong interaction to short distance hard processes.

Let us now precise a little bit the color algebra involved here. A useful way to simplify this matter is to choose for a pion of momentum along the + direction, axial gauges with axis along the - direction (fixing \( A^+ = 0 \)). In these gauges, one has \( P_{ij}(y, 0) = \delta_{ij} \) and the color component for the quark-antiquark pair is simply \( \delta_{ij}/3 \). This fact partly explains the interest of light-cone gauges in the study of hard processes. For another gauge choice,
an explicit form of \( P_{ij}(y, 0) \) is necessary, but \( P_{ij}(y, 0) \) may be perturbatively analyzed and gauge invariance preserved order by order in the perturbative expansion. At zeroth order, one has

\[
P_{ij}(y, 0) = \delta_{ij} + O(g).
\]  

(17)

We are now able to calculate the graph of Figure 2 with a new Feynman rule for the pion

\[
\frac{1}{3} \delta_{ij} \frac{1}{4} \gamma^5 p/|_{\alpha\beta} \phi(x),
\]  

(18)

and a loop integral \( \int_0^1 dx \). The amplitude of the process may thus be written as

\[
\int_0^1 dx \int_0^1 dx' \phi(x) \langle T_{ji}^\mu(x, x') \rangle \phi^*(x')
\]  

(19)

where the hard process is evaluated on the spin and color components written above. Color algebra leads to the trace

\[
\frac{1}{3} \delta_{ij} T^a_{jk} \frac{1}{3} \delta_{kl} T^a_{li} = C_F = \frac{4}{9},
\]  

(20)

and the amplitude neglecting quark masses is

\[
\int_0^1 dx \int_0^1 dx' \left( -\frac{C_F}{3} Tr \left\{ e_u \gamma^\mu \frac{1}{4} \gamma^5 p/ g \gamma^\alpha \frac{1}{4} \gamma^5 p'/ g \gamma^\beta p' - \bar{x}p \right\} \right) \frac{-\eta_{\alpha\beta}}{-\bar{x}'x'Q^2} \phi(x) \phi^*(x') = e_u p^\mu \frac{C_F g^2}{6Q^2} \left| \int_0^1 dx \frac{\phi(x)}{\bar{x}} \right|^2.
\]  

(21)

The graph with the photon attached to point 2 leads to the same expression replacing \( p^\mu \) by \( p'^\mu \). The two other graphs are identical to the two first ones after exchanging \( e_u \leftrightarrow -e_d \) and \( \bar{x} \leftrightarrow x \) in the integrand denominator. Charge conjugation invariance and isospin symmetry lead to the relation \( \phi(x) = \phi(\bar{x}) \), so that one can factorize the term \( (e_u - e_d)(p + p')^\mu \) expected in Eq. (8) and isolate the form factor expression

\[
F_\pi(Q^2) = \frac{C_F g^2}{6Q^2} \left| \int_0^1 dx \frac{\phi(x)}{\bar{x}} \right|^2.
\]  

(22)

Let us stress that we recover the scaling law in \( Q^{-2} \) predicted by the counting rules.
The pion lifetime fixes a constraint on the valence wave function of the pion since one may isolate the weak transition at the quark level under the form of the matrix element of the electroweak current. One gets

\[ \langle 0 | \bar{q}_u(0) \gamma^\mu (1 - \gamma^5) q_u(0) | \pi^+(p) \rangle = \sqrt{2} f_\pi p^\mu, \]  

(23)

where the decay constant, \( f_\pi \), is approximately equal to 92 MeV. This leads to

\[ \int_0^1 dx \phi(x) = \sqrt{2} f_\pi, \]  

(24)

which fixes the normalization of the distribution amplitude. Let us stress that there is no such normalization condition for the proton distribution amplitude, unless one measures proton decay...

### 3.3 Radiative corrections

It is important, when calculating a quantity in any field theory, and in particular in perturbative QCD, to keep track of radiative corrections and control them so that the picture obtained at lowest order survives their inclusion. The ultraviolet regimes does not a priori cause much problem since the theory is known to be renormalizable. In fact, the subtractions to be taken into account are automatically taken care of when correctly treating quark and gluon propagators on the one hand, and the running coupling constant on the other hand.

The infrared regions in the loop calculations must be very carefully scrutinized. In the specific process studied here, one finds in a \( n \) loops diagram corrections of order

\[ \frac{\alpha_s(Q^2)}{Q^2} \left[ \frac{\alpha_s(Q^2) \ln \frac{Q^2}{M^2}}{\alpha_s(Q^2) \ln \frac{Q^2}{\Lambda^2}} \right]^n, \]  

(25)

which, since \( \alpha_s(Q^2) \propto (\ln Q^2/\Lambda^2)^{-1} \) is of the same order as the tree level process! One has to resum these large logarithms in the distribution amplitude to recover the predictibility of the formalism. This is factorization since then the process may be written as the convolution illustrated by Figure 2:

\[ F_\pi = \phi \ast T \ast \phi^* \]  

(26)

where:
– $T$ is a hard amplitude that one can evaluate within perturbative QCD; namely, higher order corrections to $T$ are of order $\alpha_S^n(Q)$, and thus sufficiently small at sufficiently large transfer;

– all large logarithms are absorbed in $\phi$; the distribution $\phi$, which represents the wave function evolves with the scale $Q$ characteristic of the virtual photon probe. This stays an essentially non perturbative quantity expressing the way confined valence quarks share the hadron momentum when they interact at small distance in an exclusive process.

Let us now examine how leading logarithms are resummed in the distribution $\phi$. It turns out that it is most interesting to choose to work in a gauge which is different from the Feynman gauge, namely an axial gauge, with axis $n^\mu$, fixing the condition on gluon fields $A^\mu$ as: $n.A = 0$. The leading corrections have then the form illustrated on Figure 3.

One may show that the graph summation yields

$$\phi(x,Q) = \phi_0(x) + \kappa \int_0^1 du V_{q\bar{q} \to q\bar{q}}(u,x) \phi_0(u) + \frac{\kappa^2}{2!} \int_0^1 du V_{q\bar{q} \to q\bar{q}}(u,x) \int_0^1 du' V_{q\bar{q} \to q\bar{q}}(u',u) \phi_0(u') + \ldots \tag{27}$$

where $\kappa$ contains large collinear logarithms under the form

$$\kappa = \frac{4}{\left(11 - \frac{2}{3}n_f\right)} \ln \frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)}; \tag{28}$$

and $V_{q\bar{q} \to q\bar{q}}$ is a characteristic kernel describing the splitting of the valence distribution of the pion

$$V_{q\bar{q} \to q\bar{q}}(u,x) = \frac{2}{3} \left\{ \frac{x}{u} \left( \frac{1}{u-x} \right)_+ \theta(u-x) + \frac{x}{u} \left( \frac{1}{x-u} \right)_+ \theta(x-u) \right\}, \tag{29}$$

The $(\cdot)_+$ distribution comes from the compensation of infrared divergences (here in the limit $u \to x$) between graphs b and c of Figure 3. This is a
consequence of the colour neutrality of a hadron.

Figure 3: Leading corrections in axial gauge

The equation on \( \phi \) may be rewritten under the integro-differential form

\[
\left( \frac{\partial \phi}{\partial \kappa} \right)_x = \int_0^1 du V(u, x) \phi(u, Q),
\]

the general solution of which is known as

\[
\phi(x, Q) = x(1 - x) \sum_n \phi_n(Q) C_n^{(3/2)}(2x - 1);
\]

with Gegenbauer polynomials \( C_n^{(3/2)} \) (only even \( n \) survive) and the \( Q \)-dependence is known as

\[
\phi_n(Q) = \phi_n(\mu) e^{\lambda_n \kappa} = \phi_n(\mu) \left( \frac{\alpha_S(\mu^2)}{\alpha_S(Q^2)} \right)^{\lambda_n},
\]

where the exponents in the expansion are ordered as

\[
\lambda_0 = 0 > \lambda_2 = -0.62 > \lambda_4 \ldots
\]

Calculating the integral

\[
\int_0^1 dx \phi(x, Q) = \phi_0(Q) \int_0^1 dx x(1 - x) = \frac{\phi_0}{6} = \sqrt{2}f_\pi
\]

one can write down the beginning of the expansion:

\[
\phi(x, Q) = 6\sqrt{2}f_\pi x(1 - x) + (\ln Q^2)^{-0.62} \Phi_2 x(1 - x)[5(2x - 1)^2 - 1] + \ldots
\]
The pion asymptotic distribution, when $Q \to \infty$, is then
\[
\phi(x, Q \to \infty) \sim 6\sqrt{2} f_\pi x (1 - x).
\] (36)

This however does not tell us much on the realistic distribution amplitude at accessible energies: the constants $\Phi_2, \ldots, \Phi_n$ are unknown.

This is how far perturbative QCD can lead us about the distribution amplitude $\phi$; i.e. to understand how strong interactions build a hadron from its valence quarks. To go further, one needs other methods, which are non perturbative by nature. Experiments can guide us to develop new ways since exclusive scattering data may be processed to extract distribution amplitudes. The existing methods, like lattice calculations or QCD sum rules, are still too primitive and rely on too many unchecked hypotheses to be trusted. They however lead to useful rate estimates. They generally evaluate moments of the distribution amplitude defined as:
\[
\int_0^1 dx (2x - 1)^2 \phi(x, \mu), \ldots
\] (37)

Such a study lead Chernyak and Zhitnitsky \[11\] to propose the distribution
\[
\phi_{cz}(x, Q^2) = 6\sqrt{2} f_\pi x (1 - x) \left\{ 1 + [5(2x - 1)^2 - 1] \left( \frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2} \right)^{-0.62} \right\},
\] (38)

with $Q_0 \approx 500\text{MeV}$.

### 3.4 Transverse Degrees of Freedom

A study of one loop corrections \[12\] leads to propose that the scale relevant for the running coupling constant $\alpha_S$ is more likely to be the exchanged gluon virtuality $xx'Q^2$ than the photon virtuality $Q^2$. The whole treatment would then be correct only when the gluon is far off mass shell, that is as far as $x$ or $x'$ do not approach 0. However, for intermediate transfers, it turns out that an important part of the amplitude comes from these regions. One should thus reexamin the whole story in the region where gluons may become soft. In this region transverse momentum (or transverse distance) degrees of freedom become important and invalidate the collinear approximation \[13\].

Let us qualitatively explain the expected modifications.
The elastic interaction of a coloured object (a quark for instance) is suppressed by a Sudakov form factor \([14]\) which quantifies the difficulty of preventing an accelerated charge from radiating. Similarly the elastic interaction of a dipole of transverse size \(b\) is strongly suppressed unless \(b\) approaches \(Q^{-1}\) \([5]\). The approximation where transverse degrees of freedom are neglected leads to consider the region \(b^2 \leq (xx'|q^2|)^{-1}\), which is an unsuppressed region when \(xx'\) is of order 1. When \(xx' \to 0\), this approximation becomes illegitimate, and one should envisage to compute the hard scattering without freezing the transverse degrees of freedom and use the \(b\)-dependence of the wave function \([13]\). One gets (with some technically justified approximations)

\[
T(-xx'q^2, b) \approx \frac{2}{3}\pi\alpha_s(t)C_F K_0(\sqrt{-xx'q^2}b),
\]

where \(K_0\) is a modified Bessel function.

The interest of this improved approach is that taking radiative corrections grouped in the wave function into account, and analyzing through the renormalization group the pertinent scale for the coupling \(\alpha_s\) in the above expression of \(T\), one gets

\[
t = \max(1/b, \sqrt{xx'|q^2|}).
\]

The Sudakov suppression of large transverse sizes enforces the form factor to receive sizeable contributions at large transfer (> 5 GeV\(^2\)), only from the region where \(b\) is sufficiently small. The scale \(t\) of the perturbative approach then remains large enough in the whole relevant integration domain.

### 3.5 Experimental review

Precise experiments on various exclusive amplitudes are needed to determine the distribution amplitudes. Data for the electromagnetic form factor of the pion are yet too poor to even choose between the asymptotic and the CZ forms. This is due to the difficulty of handling a pion target, which is only available through a model-dependent extraction of some quasi-real pion pole exchange in forward pion electroproduction. The case for the \(\pi\gamma\) transition form factor is far better. A quite complete analysis \([13, 16]\) indicates that the good data obtained at \(e^+e^-\) colliders in \(\gamma\gamma\) collisions are well described
by the asymptotic distribution for $Q^2$ in the range $2 - 8GeV^2$, as shown in Figure 4.

Figure 4: The $\pi\gamma$ transition form factor

4 Other hard scattering processes.

The results obtained above for the electromagnetic form factor may be generalized to other hard exclusive processes, with an important difference in the case of hadron - hadron collisions. One thus defines a distribution amplitude for the proton and analyzes the magnetic form factor $G_M$ very similarly. One can then consider sharper reactions as real or virtual Compton scattering, which still only depend on the proton structure but where one can vary dimensionless ratio such as angles.

4.1 The proton distribution amplitude

As for the pion case, the valence nucleon wave function can be written as a combination of definite tensors of colour, flavour and spinor indices with a (unique) proton distribution amplitude $\phi(x, y, z)$. This distribution amplitude may be written as an expansion quite similar to what was derived above for the pion case but on a different basis of polynomials, and without the help of weak decay processes to fix the normalization:

$$\phi(x_i, Q) = 120x_1x_2x_3 \delta(x_1 + x_2 + x_3 - 1) \times$$
$$\left[ \frac{\alpha_S(Q^2)}{\alpha_S(Q_0^2)} \right]^{\lambda_0} A_0 + \frac{21}{2} \left( \frac{\alpha_S(Q^2)}{\alpha_S(Q_0^2)} \right)^{\lambda_1} A_1 P_1(x_i) + \frac{7}{2} \left( \frac{\alpha_S(Q^2)}{\alpha_S(Q_0^2)} \right)^{\lambda_2} A_2 P_2(x_i) + \ldots \right] .$$
where the slow $Q^2$ evolution comes entirely from the terms $\alpha_s(Q^2)^{\lambda_i}$, and the $\lambda_i$’s are decreasing numbers:

$$\lambda_0 = \frac{2}{27} < \lambda_1 = \frac{20}{81} < \lambda_2 = \frac{24}{81} \ldots, \quad (42)$$

and the $P_i(x_j)$’s are Appell polynomials:

$$P_1(x_i) = x_1 - x_3 \quad , \quad P_2(x_i) = 1 - 3x_2, \ldots \quad (43)$$

### 4.2 The proton magnetic form factor

One describes the elastic interaction of a proton and an electron with two form factors $F_1$, which is helicity conserving, and $F_2$, which is not:

$$\langle p', h' | J^\mu(0) | p, h \rangle = e\bar{u}(p', h') \left[ F_1(Q^2)\gamma^\mu + i\frac{\kappa}{2M}F_2(Q^2)\sigma^\mu\nu(p' - p)_\nu \right] u(p, h); \quad (44)$$

or their linear combinations, $G_M$ and $G_E$ called the Sachs form factors. $h$ and $h'$ are respectively the incoming and outgoing proton helicities, $u$ and $\bar{u}$ their spinors and $M$ the proton mass. In this decomposition, $e$ is the proton charge and $\kappa = 1.79$ is its anomalous magnetic moment. In the formalism we are presenting here, only the helicity conserving form factor $F_1$ is easily accessible. There is much discussion on the phenomenological analysis of experimental data, as shown on Figure 5. On the one hand, the $Q^{-4}$ scaling behaviour is clearly seen in the $Q^2$ range $5 - 30 GeV^2$. The slight decrease of $Q^4G_M(Q^2)$ may even be understood as a manifestation of radiative corrections on top of the counting rules. On the other hand, the proposed understanding of the normalization of $Q^4G_M(Q^2)$ through a much asymmetric distribution amplitude based on a QCD sum rule analysis [11], leads to many problems. Does-it mean that $F_1$ is still dominated by long distance soft physics [17]? Progress on this question may come from the computation of QCD corrections to the present leading logarithm analysis,
or from the study of color transparency observables.

Figure 5: $Q^2$ evolution of the proton magnetic form factor

4.3 Compton scattering

The perturbative part of the analysis of real [19, 20] Compton scattering consists in evaluating the 336 topologically distinct diagrams obtained when coupling two photons to the three valence quarks of the proton, two gluons being exchanged. Moreover, there are 42 diagrams with a three-gluon coupling but it turns out that their color factor vanishes.

There are few existing data for real Compton scattering on the proton with $-t > 1GeV^2$ [21]. The higher energy and transfer data seem to approach the dimensional scaling law (in $s^{-6}$) but more data at higher energies and various angles around $90^\circ_{CM}$ are clearly needed before one can use them to extract the proton distribution amplitude. To do this, one should write the cross section as a sum of terms

$$A_i T_H^{ij}(\theta) A_j$$

where the decomposition of the distribution amplitude on the Appell polynomials (Eq. (42)) has been used and where $T_H^{ij}$ are integrals over $x$ and $y$ variables of the product of the hard amplitude at a given scattering angle $\theta$ by the two Appell polynomials $A_i(x)$ and $A_j(y)$. The $T_H^{ij}$ are ugly long expressions but they can be numerically handled.

Determining the proton distribution amplitude from experimental data boils down then to the extraction by a maximum of likelyhood method of the $A_i$ parameters, amputating the series of Eq. (42) to its first $n$ terms, by taking advantage of the (admittedly mild) $Q^2$ dependence of the $A_i$ parameters.
which indicates that they decrease quicker for large $Q^2$ at larger $n$. One should of course verify afterwards that including the term $n + 1$ does not drastically modify the conclusion. One can then explore other reactions, virtual Compton scattering for instance, which must be well described by the same series of $A_i$'s.

4.4 Virtual Compton Scattering

An intense electron accelerator such as ELFE would enable us to go further and study virtual Compton scattering in the large angle region. At lowest order in $\alpha \sim \frac{1}{137}$, Virtual Compton Scattering (VCS) is described as the coherent sum of two amplitudes namely the Bethe Heitler (BH) process where the final photon is radiated from the electron and the genuine VCS process where it comes from the proton. The perturbative calculation \[22\] of the VCS amplitude is not much harder than the real case. As the BH amplitude is calculable from the elastic form factors $G_{M_p}(Q^2)$ and $G_{E_p}(Q^2)$, its interference with the VCS amplitude is an interesting source of information, different from what real Compton scattering yields. The VCS amplitude depends on three invariants; one usually chooses $Q^2, s, t$ or $s, Q^2/s, \theta_{CM}$.

The forward region is also interesting and is currently subject to intense theoretical investigation; the counting rates are expected to be larger but the Bethe Heitler process constitutes there a large background.

4.5 Other processes

Photo- and electro-production of mesons at large angle will allow to probe distribution amplitudes of $\pi$ and $\rho$ mesons in the same way. The production of the $K\Lambda$ final state selects a few hard scattering diagrams. The analysis of these reactions is still to be done if one excepts some works done in the simplifying framework of the diquark model\[7\].

\[7\] The physical idea behind the diquark model is not to consider a new field theory where diquarks would be fundamental. It is mostly to phenomenologically take into account of quark correlations inside baryons which seem to be indicated by experimental data, and thus modelize the proton wave function as a quark-diquark composite, while adopting the general framework indicated by QCD for exclusive reactions, that is assume factorization holds and calculate the hard subprocess with some new Feynman rules for
5 Color transparency

5.1 The physical idea

Hard exclusive scattering (with a typical large $Q^2$ scale) selects a very special quark configuration: the minimal valence state where all quarks are close together, a small size color neutral configuration sometimes referred to as a *mini hadron*.

Such a color singlet system cannot emit or absorb soft gluons which carry energy or momentum smaller than $Q$. This is because gluon radiation — like photon radiation in QED — is a coherent process and there is thus destructive interference between gluon emission amplitudes by quarks with “opposite” color. Even without knowing exactly how exchanges of soft gluons and other constituents create strong interactions, we know that these interactions must be turned off for small color singlet objects. This is color transparency [24].

An exclusive hard reaction will thus probe the structure of a *mini hadron*, i.e. the short distance part of a minimal Fock state component in the hadron wave function. This is of primordial interest for the understanding of the difficult physics of confinement. First, selecting the simplest Fock state amounts to the study of the confining forces in a colorless object which is quite reminiscent of the “quenched approximation” much used in lattice QCD simulations, where quark-antiquark pair creation from the vacuum is forbidden. Secondly, letting the mini-state travel through different nuclei of various sizes allows an indirect but unique way to test how the squeezed mini-state interacts with hadrons, opening a new chapter of strong interaction physics.

To the extent that the electromagnetic form factors are understood as a function of $Q^2$,

\[ e + A \rightarrow e + (A - 1) + p \]

\text{(46)}

diquark-gluon or diquark-photon vertices. It is not obvious at all, to say the least, that such a procedure is consistent. One thus should take calculations in this type of approach as useful exercises which may turn out to explain part of the physical reality is indeed quark-quark correlations are playing a major role in some exclusive processes at medium transfers. A number of calculations have been performed within the diquark picture and we cannot discuss all of them here. It turns out that treating baryons as quark-diquark objects render computations much easier, in particular because the number of Feynman diagrams is quite reduced. This is particularly true if only scalar diquarks are considered. We refer the interested reader to the existing litterature [23].
experiments will measure the color screening properties of QCD. The quantity to be measured is the transparency ratio $T_r$ which is defined as:

$$T_r = \frac{\sigma_{\text{Nucleus}}}{Z\sigma_{\text{Nucleon}}}$$ (47)

At asymptotically large values of $Q^2$, $T_r$ should approach unity. The approach to the scaling behavior as well as the value of $T_r$ as a function of the scaling variable determine the evolution from the pointlike configuration to the complete hadron. We will not present here the many ideas which have recently emerged in this new field [25].

### 5.2 An instructive calculation

Let us now present a somewhat academic but instructive calculation of the high energy limit of a forward amplitude in perturbative field theory [26]. Although such a computation would be more reliable in QED, let us pretend it is going to describe the strong interactions of hadrons. The idea is to use the optical theorem to relate the total cross section to the imaginary part of the forward amplitude.

The first step is to calculate in the region $-t << s \sim -u$ the scattering amplitude of quark-quark scattering through one gluon exchange (Fig.6a); the result is (omitting a factor specifying helicity conservation):

$$\mathcal{M} = 2Cg^2s\frac{1}{t - \lambda^2}$$ (48)

which may be rewritten, but this is more a mathematical trick than a physical idea at this stage, as

$$\mathcal{M} = -2Cs\int d^2b e^{iqa.b}g^2K_0(\lambda b)\frac{2\pi}{2\pi}$$ (49)

where a 2–dimensional Fourier transform in transverse space has been performed, writing $t = -q^2$ with $q$ a 2–dimensional transverse vector. $C$ is a color factor, $g$ the quark gluon coupling and $\lambda$ an effective gluon mass.

Let us now consider two gluon exchange processes. From the 18 diagrams which may be drawn, 14 are vertex or self energy corrections to the one-gluon diagrams, and as such are real. Two of the remaining diagrams dominate at small ($-t$) and are drawn on Fig.6b. Denoting as $q$ the 4–vector of
the first gluon emitted by the bottom fermion line, one finds that the two diagrams have identical boson propagators and bottom fermion expression. From the upper fermion line, one gets adding the two diagrams, after some simple algebra:

$$A \propto \frac{1}{q^- + i\epsilon} - \frac{1}{q^- - i\epsilon}$$  (50)

leading to a $\delta(q^-)$ constraint. Moreover the bottom fermion line leads to a $\delta(q^+)$ constraint, so that the $d^4q$ integration boils down to a 2–dimensional transverse space integration. The resulting amplitude is dominantly imaginary:

$$M = i Cs \int \frac{d^2 q_T}{(2\pi)^2} \frac{1}{(q_T^2 + \lambda^2)((q_T - \Delta_T)^2 + \lambda^2)}$$  (51)

which may be rewritten as

$$M = i Cs \int d^2 b e^{i\Delta_T \cdot b} \left(\frac{g^2 K_0(\lambda b)}{2\pi}\right)^2$$  (52)

where $\Delta_T$ is the (small) 2–dimensional transfer between the initial and final fermion.

The important features of this result is the 2–dimensional nature of the integral and the squared factor reminiscent of the eikonal nature of forward amplitude in the high energy limit.

Figure 6: quark-quark and meson-meson scattering

Let us now go to the almost physical case of meson-meson scattering, that is consider a color singlet formed by a quark–antiquark pair scattering
on another pair (Fig. 6c). A straightforward computation shows that the color factors are the same for all 2-gluon exchange graphs connecting the upper hadron to the lower one, but that an extra minus sign is attached for an antiquark line. The result is then

\[ M = iC_s \int d^2 b e^{i \Delta r \cdot b} \int d^2 r_1 d^2 r_2 \psi^\ast(r_1) \psi^\ast(r_2) \]
\[ \{V(x_1 - x_3) - V(x_2 - x_3) - V(x_1 - x_4) + V(x_2 - x_4)\}^2 \psi(r_1) \psi(r_2) \] 

with

\[ V(x) = \frac{1}{(2\pi)^2} \int \frac{d^2 k e^{ik \cdot x}}{k^2 + \lambda^2} \] 

Suppose now that \( r_1 = x_1 - x_2 \) is small (a mini-hadron); then one may approximate \( V(x_1 - x_3) - V(x_2 - x_3) \sim r_1 \cdot \nabla(x_1 - x_3) \) and similarly for \( V(x_1 - x_4) - V(x_2 - x_4) \), and get:

\[ M \propto r_1^2. \]

leading to a total cross section for the mini hadron scattering on a hadron:

\[ \sigma \sim \frac{\text{Im}M}{s} \propto r_1^2. \]

This is color transparency. Note that if the \( q\bar{q} \) was in a color octet state, the color factor would still be factorized but the resulting amplitude would contain an additional term

\[ M \propto \ldots + \{V(x_1 - x_3) - V(x_1 - x_4)\} \cdot \{V(x_2 - x_3) - V(x_2 - x_4)\} \] 

which does not vanish in the limit \( x_1 \to x_2 \). The color singlet nature of the quark-antiquark pair is thus essential for Eq.56 to be valid.

5.3 Present Data and future prospects

Experimental data [27] on color transparency are very scarce and, although they have been discussed at length [22], not yet conclusive. Color transparency is indeed just an emerging field of study and one should devote much attention to get more information on this physics in the near future.

A second round of proton experiments at Brookhaven has been approved and a new detector named Eva [28] with much higher acceptance has been
taking data for about one year. Along with other improvements and increased beam type, this should increase the amount of data taken by a quite large factor allowing a wider energy range and an analysis at different scattering angles. It would also be very interesting to analyze meson-nucleus scattering and their spin properties in similar conditions [29].

The Hermes detector [30] at HERA is beginning operation. It should enable a confirmation of FNAL data on \( \rho \) meson diffractive production at moderate \( Q^2 \) values and quite smaller values of energies \( 10 \leq \nu \leq 22 \) GeV.

The use of nuclear targets to test color transparency and use nuclear filtering is one of the major goals of the 15–30 GeV continuous electron beam ELFE project [1]. The \((e, e'p)\) reaction should in particular be studied in a wide range of \( Q^2 \) up to 21 GeV\(^2\), thus allowing to connect to SLAC data (and better resolution but similar low \( Q^2 \) data from CEBAF) and hopefully clearly establish this phenomenon in the simplest occurence.

Using light nuclei such as deuteron or helium puts the emphasis on the reduction of final state interaction effects which are quite well under control in this case. The measure of the recoil momentum spectrum of a spectator neutron when a proton has experienced a hard scattering, for instance, is a direct test of the decrease of the subsequent mini-proton neutron elastic scattering cross section.

The measurement of the transparency ratio for photo- and electroproduction of heavy vector mesons, in particular of \( \psi \) and \( \psi' \) will open a new regime where the mass of the quark enters as a new scale controlling the formation length of the produced meson.

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