The dark(er) side of inflation

Gabriela Barenboim

Departamento de Fisica Teorica-IFIC, University of Valencia, Spain

We present a new approach to quintessential inflation, in which both dark energy and inflation are explained by the evolution of a single scalar field. We start from a simple scalar potential with both oscillatory and exponential behavior. We employ the conventional reheating mechanism of new inflation, in which the scalar decays to light fermions with a decay width that is proportional to the scalar mass. Because our scalar mass is proportional to the Hubble rate, this gives adequate reheating at early times while shutting off at late times to preserve quintessence and satisfy nucleosynthesis constraints.

1 Introduction

If we assume the correctness of the standard Friedmann equation evolution, the existence of dark energy engenders two profound dilemmas. The first is the cosmological constant problem, and the second is the “why now?” problem. Quintessence models attempt to address the second problem by introducing a very weakly coupled scalar field whose potential and/or kinetic function have special properties. One of the most successful approaches to quintessence is to combine tracking with an oscillating behavior in the quintessence potential. In such models the equation of state parameter $w(z)$ has a periodic component, leading to occasional periods of accelerated expansion during epochs where $w(z) \simeq -1$.

It is natural in this context to ask whether the quintessence scalar could replace the inflaton. The idea of quintessential inflation has been examined by a number of authors. The straightforward approach is to cobble together a scalar potential which has both a flat, large vev portion (for inflation) and a flat, small vev portion (for quintessence). These features are connected by a steep step which corresponds to a period of cosmic kination. As discussed in such models suffer from generic problems. First, they require significant *ad hoc* tuning to simultaneously produce the features of inflation and quintessence. Second, they require a “sterile” inflaton, in order to avoid the decay of the putative quintessence scalar at the end of
inflation. This in turn requires new mechanisms for reheating, such as gravitational de Sitter phase particle production, leading to difficulties in satisfying the constraints of CMB anisotropies and of primordial nucleosynthesis.

Our approach to quintessential inflation\textsuperscript{11,12} is to take advantage of the tracking and oscillatory potential features that work so well in addressing the “why now” problem of quintessence alone.\textsuperscript{13} We will describe a model with a simple scalar lagrangian with exponential and oscillatory features. The model uses the conventional reheating mechanism of new inflation,\textsuperscript{14} in which the scalar decays to light fermions with a decay width that is proportional to the scalar mass. We show that the scalar mass is proportional to the Hubble rate. As a result, the model has adequate reheating at early times while naturally shutting off at later times.

We present a successful model with only three adjustable parameters. One parameter controls the period between inflationary epochs, a second controls the overall decay width, and the third parametrizes our ignorance about the relative fraction of matter versus radiation produced by reheating. These three parameters are adjusted to produce sufficient inflation along with the correct fractions $\Omega_r/\Omega_\Lambda$, $\Omega_m/\Omega_\Lambda$ of radiation, matter, and dark energy, as measured today. Having thus fixed the model we find that we automatically satisfy all constraints of primordial nucleosynthesis, CMB, and large-scale structure.

\section{An oscillatory potential}

We start with a simple model with a single real scalar quintessence field $\theta$. The action is

$$\int d^4x \sqrt{-g} \left[ \frac{1}{2} f(\theta) g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta - V(\theta) \right],$$

(1)
where the kinetic function $f(\theta)$ and potential $V(\theta)$ are given by:

$$f(\theta) = \frac{3M_P^2}{\pi b^2} \sin^2 \theta; \quad (2)$$

$$V(\theta) = \rho_0 \cos^2 \theta \exp \left[ \frac{3}{b} (2\theta - \sin 2\theta) \right], \quad (3)$$

where $M_P$ is the Planck mass: $1.22 \times 10^{19}$ GeV; $\rho_0$ is the dark energy density observed today: $\simeq (10^{-4} \text{ eV})^4$; $b$ is a dimensionless parameter which controls the periodic behavior. A canonical kinetic term can be restored via a field redefinition $\theta(x) \rightarrow \phi(x)$, where

$$\phi(x) = \phi_0 \cos \theta, \quad (4)$$

with $\phi_0 \equiv \sqrt{3M_P^2/\pi b^2}$.

In the approximation where we ignore the energy density of radiation and matter, and where the only friction is from the metric expansion, the evolution of the model can be solved analytically. Energy conservation requires:

$$\dot{\rho}_\theta = -3H(1 + w)\rho_\theta, \quad (5)$$

where $H$ is the Hubble rate, $w$ is the equation of state parameter, and $\rho_\theta$ is the dark energy density. The solution to this equation as a function of the scale factor $a(t)$ is:

$$\rho_\theta(a) = \rho_0 \exp \left[ -3 \int_1^a \frac{da}{a} (1 + w(a)) \right], \quad (6)$$

where $\rho_0$ is the dark energy density at $a = 1$ (today).

We also know that

$$V(\theta) = \frac{1}{2}(1 - w)\rho_\theta, \quad (7)$$

Making the Ansatz

$$w(a) = -\cos 2\theta(a), \quad (8)$$

one immediately gets a solution to the (flat) Friedmann equation combined with the relations (6-7):

$$\theta(a) = -\frac{b}{2} \ln a; \quad (9)$$

$$w(a) = -\cos [b \ln a]. \quad (10)$$

The expectation value of the quintessence field $\theta$ evolves logarithmically with scale factor from a positive initial value to zero today. Accelerated expansion corresponds to epochs (such as today) where $\theta$ is evolving through one of the flat “steps” of the potential. From (2) we see that the kinetic function is simultaneously suppressed in these epochs, slowing the roll of the scalar field evolution.

The equation of state parameter $w(a)$ has the same periodic form assumed in recent phenomenological analyses. The analysis in showed that, for $b = 1$, $w(a)$ is consistent with all current data from observations of the CMB, Type IA supernovae, and large scale structure. It follows a fortiori that our model with any choice of $b$ less than one also agrees with this data.

To complete the model, we will assume that the quintessence field $\phi$ has a weak perturbative coupling to light fermions. This is the standard reheating mechanism of new inflation. To
avoid the strong constraints on long-range forces mediated by quintessence scalars\cite{18} it is simplest to imagine that our scalar only has a direct coupling to a sterile neutrino. This is sufficient to hide the quintessence force from Standard Model nonsinglet particles\cite{19} while still allowing the generation of a radiative thermal bath of Standard Model particles from quintessence decay.

With this assumption for reheating the evolution equations for quintessence, radiation, and matter become:

\[
\dot{\rho}_\theta = -3H(1 + w)\rho_\theta - k_0 m_\phi (1 + w)\rho_\theta ; \\
\dot{\rho}_r = -4H \rho_r + (1 - f_m)k_0 m_\phi (1 + w)\rho_\theta ; \\
\dot{\rho}_m = -3H \rho_m + f_m k_0 m_\phi (1 + w)\rho_\theta ;
\]  

(11)

where \(k_0\) and \(f_m\) are small dimensionless constants. As long as \(\theta\) is not near a multiple of \(\pi/2\), it is a reasonable approximation to make the replacement \(k_0 m_\phi \rightarrow kH\) where \(k\) is another small dimensionless parameter. This replacement decouples the \(\theta\) evolution equation from the Friedmann equation, giving an immediate analytic solution:

\[
\rho_\theta(a) = \rho_0 \exp \left[ \frac{1}{b} (3 + k) (2\theta - \sin 2\theta) \right],
\]  

(12)

We have used this approximation in the solutions quoted below.

3 Results

Figure 2 shows the results obtained from our model with \(b = 1/7\), \(k = 0.06\), and \(f_m = 10^{-11}\). Shown are the relative energy density fractions in dark energy, radiation, and matter, as a function of \(\log a\). We have chosen to integrate the evolution equations starting from \(a = 10^{-42}\), which in our model corresponds to a temperature of slightly less than \(10^{16}\) GeV, and an initial comoving Hubble radius of about 100 Planck lengths. For simplicity we have also chosen the value of \(w(a)\) now to be exactly \(-1\). Neither of these choices corresponds to a necessary tuning.

Our three adjustable parameters have been chosen such that the values of \(\Omega_r/\Omega_\Lambda\), \(\Omega_m/\Omega_\Lambda\) come out to their measured values at \(a = 1\), and such that we have sufficient inflation. The latter is checked by computing the ratio of the fully inflated size of the initial comoving Hubble radius to the current comoving Hubble radius. This ratio is about 3 in our model, indicating that the total amount of inflation is indeed enough to solve the horizon problem. The flatness problem is solved because the total \(\Omega_r + \Omega_m + \Omega_\Lambda = 1\) within errors.

From Figure 2 we see that we are currently beginning the third epoch of accelerated expansion. The first epoch of inflation accumulated about 18 e-foldings. Quantum fluctuations
Table 1: Relative density fractions of dark energy, radiation, and matter, as a function of the scale factor. Also shown are the temperature $T$ in GeV, and the equation of state parameter $w(a)$.

during this epoch produced the spatial inhomogeneities responsible for large scale structure and CMB anisotropies observed today. Constraints on the physics responsible for the primordial power spectrum of these density fluctuations can be set with WMAP and 2dF data, under the assumption that the Hubble rate is dominated by the contribution from $\rho_\phi$ during the observable part of inflation. As can be seen from the figure a second period of accelerated expansion began just before the electroweak phase transition, and ended well before BBN. The temperature history near the EWPT is shown in Figure 3.

Also shown is the Hubble parameter $H$ of the model normalized to the expansion rate $H_{rad}$ for pure radiation. $H_{rad}$ corresponds to what is assumed in the standard paradigm. Notice that for temperatures of a few GeV the expansion rate is actually somewhat larger than normal, but at higher temperatures it is much less than normal.

Such a nonstandard thermal history will impact on electroweak baryogenesis. For a Higgs sector such that the EWPT is first order, the change in the net baryon asymmetry is proportional to $-\log(H/H_{rad})$, where $H$ is the expansion rate during the phase transition, and $H_{rad}$ is the corresponding expansion rate for pure radiation. If the Higgs sector is such that the EWPT is second order, the baryon asymmetry is proportional to the expansion rate. Clearly one should reevaluate the popular scenarios for electroweak baryogenesis in this light.

Such a model will have major implications for predictions of the relic abundance of dark matter particles with Terascale masses. The dominant production mechanism for such particles may be scalar decays, as suggested by Figure 2. Even if the dark matter particles are thermal
relics, their abundance now will be affected by the nonstandard expansion rates at earlier times.

To compare with contraints from primordial nucleosynthesis\cite{25} we note that the second epoch of dark energy domination ended well before $a \simeq 10^{-10}$, the time at which nucleosynthesis occurred. Indeed dark energy reheating effects are completely negligible from $a \simeq 10^{-10}$ until today.

Figure (2) also shows one (very brief) prior epoch of matter domination. This only occurs if there is some suitably heavy and long-lived matter around to go out of thermal equilibrium when $a \sim 10^{-22}$, e.g. a superheavy neutrino.

It is difficult to extract a precise prediction for the spectral indices of this model, since we are never strictly in the slow roll regime. We will be content here with a rough estimate. This is obtained starting from a canonical field redefinition:

$$\phi(x) = 2\sqrt{3} \frac{k}{b} \cos \theta.$$  \hfill (13)

In terms of the canonical scalar $\phi$, the potential $V$ can be written:

$$V(\phi) \propto \phi^2 H^2(\phi),$$  \hfill (14)

where $H(\phi)$ is the Hubble rate we would get ignoring radiation and matter. During inflation, $H$ is approximately constant, but this is not an especially good approximation since we are not in a slow roll regime. This is similar to the oscillatory models of quintessential inflation discussed in\cite{26}. Taking the potential in the inflationary phase to be approximated by $V \propto \phi^2$, we can estimate the scalar spectral index $n_s$:

$$n_s \simeq 1 - \frac{2}{N}.$$  \hfill (15)
with \( N \) the number of e-folds between the Hubble radius exit and the end of the second inflationary period. For our model \( N = 29 \), giving \( n_s = 0.93 \), in good agreement with recent observations.

It is fair to say that, compared to the simple model presented above, the standard new inflation scenario looks rather extreme. In the evolution history portrayed in Figure 2, the interplay between inflationary expansion and reheating is much milder. In fact, apart from a few very brief periods, reheating effects in our model do not actually increase the temperature of the thermal radiation bath. Instead, the temperature is almost always decreasing, but it decreases more slowly than in the standard ΛCDM evolution.

Our particular realization of Slinky inflation must be regarded with some care, since \( \phi_0 > M_P \) (as in all models of chaotic inflation) and we have not invoked any consistent Planck scale framework such as string theory. However the model itself is surprisingly simple, and the physical picture which emerges from it has some compelling features. These are worthy of further investigation. Also, since the target space parametrized by our scalar is \( S^1 \), it would be interesting to extend this scenario to a class of nonlinear sigma models with other compact target spaces.

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