Robust Power Allocation for Multi-Homing Heterogeneous Networks with Energy Harvesting

MD Tanim Hossain¹, Yongjun Xu¹,²*, Yang Yang, Bowen Gu and Guoquan Li¹

¹School of Communication and Information Engineering, Chongqing University of Posts and Telecommunications, Chongqing, China
²CQUPT-BUL Innovation Institute, Chongqing University of Posts and Telecommunications, Chongqing 400065, China

*Corresponding author email: xuyj@cqupt.edu.cn

Abstract. To improve user's data rate and network coverage, multi-homing heterogeneous network has been considered as a key technology in recent years. However, traditional power allocation schemes cannot provide good robustness and suppress the impact of parameter perturbation due to the random interference and channel delays. A robust power allocation algorithm is proposed for an orthogonal frequency division multiple access based multi-homing heterogeneous network under energy harvesting. Considering the channel uncertainties from base stations to users, the total data rate of all mobile terminals (MTs) is maximized under the rate outage constraint of MT. Based on the uniform distribution of uncertainty, the originally robust counterpart problem is transformed to a deterministic and convex one. The Lagrange dual decomposition theory is used to solve it. Simulation results demonstrate that the proposed scheme has good convergence and robustness.

1. Introduction
Multi-homing heterogeneous network (HetNet), as a promising technology, has been proposed to extend network coverage and improve overall data rates[1]. Mobile terminals (MTs) in multi-homing HetNets are geared up with processing and displaying skills to achieve voice and video. MTs are successful of setting up simultaneous verbal exchange with base stations (BSs) of distinct networks by multiple radio interface exchange. Furthermore, in energy harvesting (EH) based HetNets, each BS is powered by its EH module to prolong the network life [2]. Thus, multi-homing HetNets has been considered a captivating candidate for future communications.

Power allocation is a key technology for multi-homing HetNets to satisfy the quality of service (QoS) of each MT and reduce the energy consumption. In [3], a power and bandwidth allocation algorithm was proposed in cognitive heterogeneous multi-homing networks. In [4], an adaptive cross-layer power allocation algorithm was proposed for network utility maximization according to the current state information. In [5], the energy efficiency (EE) maximization problem of jointly optimizing bandwidth and power allocation was investigated. In [6], a capacity maximization-based power allocation problem with network selection, bandwidth and power allocation was studied. In [7], a bandwidth and power control algorithm was investigated to maximize the overall EE. In [8], an EE-based bandwidth and power control scheme was studied for heterogeneous LET-WiFi multihoming networks. In [9], the EE-based power allocation problem was converted into a convex one by using a fractional programming theory. In [10], an energy management strategy was studied to pursue a good video transmission. In [11], an EE-based power allocation problem was studied for multi-homing HetNets, where a two-phase
optimization method was proposed to reduce the computational complexity. However, most of resource allocation algorithms assume perfect channel state information (CSI), parameter uncertainties are not considered. For practical multi-homing HetNets, it is hard to get exact channel gains because of link delays and quantization errors, which may cause user’s outage. Therefore, it is very important to design the robust power allocation in this network scenario.

In this paper, a robust power allocation problem is formulated in a multi-homing HetNet under imperfect CSI. Specifically, a problem of rate maximization under imperfect CSI is considered subject to MT’s rate constraint, sum transmission power constraint and the harvested energy constraint. With the uniform distribution of channel uncertainty, the outage probability constraint is transformed into a convex one. The transformed problem is effectively resolved by using Lagrange method. Simulation results demonstrate the effectiveness of the proposed algorithm.

![Figure 1. A multiuser multi-homing HetNet.](image)

### 2. System Model and Problem Formulation

Consider a downlink orthogonal frequency division multiple access (OFDMA) based multi-homing HetNet, as shown in Figure 1. In each tier, BS serves multiple MTs by the OFDMA way, and assume each BS can harvest the energy from the surrounding radio environment to prolong the life cycle. Assume that there are $K$ tiers in the multi-homing HetNet, the set is defined as $\forall k \in \{1, 2, ..., K\}$. Define $n$ as the index for the $n$-th MT, the set is defined as $n \in \{1, 2, ..., N\}$. There are $M$ BSs, the set is defined as $m \in \{1, 2, ..., M\}$. To achieve green communication and prolong network lifetime, each BS is equipped with the function of EH.

Considering that each BS has a maximum power level, the sum power of each BS is constrained

$$\sum_{j=1}^{N} p_{ji} \leq P_i^k$$

(1)

where $p_{ji}$ is the transmit power from BS $i$ to MT $j$. $P_i^k$ is the maximum power threshold of BS $i$ in tier $k$.

Since each BS is powered by the EH units, the transmission power of the BS is

$$\sum_{j=1}^{N} p_{ji} \leq E_i$$

(2)

where $E_i$ is the harvested energy.

According to Shannon capacity theorem, the date rate of each MT is

$$r_{ji} = \log_2 (1 + p_{ji} h_{ji})$$

(3)

where $h_{ji}$ denotes the channel gain between BS $i$ to MT $j$. To support the QoS of each MT, we have the minimum rate requirement, i.e.,

$$\sum_{j=1}^{M} r_{ji} \geq R_j^{\min}$$

(4)

where $R_j^{\min}$ denotes the minimum rate threshold of MT $j$. 


Accordingly, the resource allocation problem with sum rate maximization becomes

\[
\begin{align*}
\max_{\mathbf{p}_j} & \quad \sum_{i=1}^{M} \sum_{j=1}^{N} r_{ij} \\
\text{s.t.} & \quad C_1 : \sum_{j=1}^{N} p_{ij} \geq R_{ij}^{\min} \\
& \quad C_2 : P_{ij}^{\min} = \min \{ P_{ij}^{\max}, E_i \} \\
& \quad C_3 : \sum_{j=1}^{N} p_{ij} \leq P_{ij}^{\min}
\end{align*}
\]

(5)

Problem (5) is a non-robust power allocation problem without CSI errors.

3. Robust Power Allocation Model

3.1. Model of Channel Uncertainties

In actual scenarios, it is difficult to accurately obtain channel gains due to the dynamic nature of radio environment. Thus, it is necessary to consider channel uncertainties in the algorithm design. Based on the modelling method of additive uncertainties [12], the channel uncertainty of \( h_{ij} \) can be given as

\[
\hat{h}_{ij} = h_{ij} + \Delta h_{ij}
\]

(6)

where \( \hat{h}_{ij} \) is the estimated channel gain. \( \Delta h_{ij} \) is the corresponding estimation error, which is assumed to obey the uniform distribution \( \Delta h_{ij} \in [-\sigma_{ij}, \sigma_{ij}] \). Where \( \sigma_{ij} \in [0, 1] \) is the upper bound of \( \Delta h_{ij} \).

Hence, the MT’s rate with imperfect CSI becomes

\[
\tilde{r}_{ij} = \log_2(1 + p_{ij}(\hat{h}_{ij} + \Delta h_{ij})).
\]

3.2. Robust Power Allocation Problem

With the consideration of MT’s outage probability, problem (5) can be reformulated as the following robust power allocation problem

\[
\begin{align*}
\max_{\mathbf{p}_j} & \quad \sum_{i=1}^{M} \sum_{j=1}^{N} r_{ij} \\
\text{s.t.} & \quad C_1 : \sum_{j=1}^{N} p_{ij} \geq R_{ij}^{\min} \\
& \quad C_2 : P_{ij}^{\min} = \min \{ P_{ij}^{\max}, E_i \} \\
& \quad C_3 : \sum_{j=1}^{N} p_{ij} \leq P_{ij}^{\min}
\end{align*}
\]

(7)

where \( \phi_j \in [0, 1] \) denotes the outage probability threshold of MT \( j \). \( \tilde{C}_i \) is the outage probability constraint which can ensure the transmission quality of the MT under channel uncertainties. Problem (7) belongs to a non-convex optimization problem and difficult to solve.

4. Robust Power Allocation Algorithm

4.1. Transformation of Outage Probability Constraint

In order to solve problem (7), the outage probability \( \tilde{C}_i \) needs to be converted into a convex constraint. Based on (6), \( \tilde{C}_i \) can be rewritten as

\[
\begin{align*}
\Pr & \left\{ \sum_{i=1}^{M} \log_2(1 + p_{ij}(\hat{h}_{ij} + \Delta h_{ij})) \leq R_{ij}^{\min} \right\} \leq \Pr \left\{ M \log_2(1 + p_{ij}(\hat{h}_{ij} + \Delta h_{ij})) \leq R_{ij}^{\min} \right\} \\
\Leftrightarrow & \Pr \left\{ \Delta h_{ij} \leq (2^{R_{ij}^{\max}/M} - 1)/ p_{ij} \right\} \leq \phi_j
\end{align*}
\]

(8)

Based on the probability density function of uniform distribution, we have
According to probability theory and the probability density function, the distribution function becomes

\[ F(\Delta h_j) = \begin{cases} 
\int_{-\sigma_j}^{-\sigma_j} d\Delta h_j, & \Delta h_j < -\sigma_j \\
\int_{-\sigma_j}^{\Delta h_j} \frac{1}{2\sigma_j} d\Delta h_j, & -\sigma_j \leq \Delta h_j \leq \sigma_j \\
\int_{\sigma_j}^{\infty} d\Delta h_j, & \Delta h_j > \sigma_j 
\end{cases} \]  

(10)

and we have

\[ F(\Delta h_j) = \begin{cases} 
0, & \Delta h_j < -\sigma_j \\
\frac{\Delta h_j + \sigma_j}{2\sigma_j}, & -\sigma_j \leq \Delta h_j \leq \sigma_j \\
1, & \Delta h_j > \sigma_j 
\end{cases} \]  

(11)

Combining (8) and (11), we have

\[ \frac{(2^{R_j^{\min}(M-1)})}{2\sigma_j} p_{j\beta} - \hat{h}_j \leq \phi_j \]  

(12)

and

\[ 1 + p_{j\beta}(\hat{h}_j + (2\phi_j - 1)\sigma_j) \geq 2^{\frac{R_j^{\min}}{\sigma_j}} \Leftrightarrow \log_2 \left[ 1 + p_{j\beta}(\hat{h}_j + (2\phi_j - 1)\sigma_j) \right] \geq R_j^{\min} \]  

(13)

Thus, \( C_1 \) becomes a deterministic one.

4.2. Robust Power Allocation

Combining (8) with (13), we have

\[ \max_{p_{j\beta}} \sum_{i=1}^{M} \sum_{j=1}^{N} \tilde{F}_{j\beta}(\hat{h}_j) \]  

s.t. \( C_1, C_2, C_j : \hat{p}_j \geq R_j^{\min} \)  

(14)

where \( \tilde{F}_{j\beta} = \log_2 (1 + p_{j\beta} \hat{h}_j) \) and \( \hat{p}_j = \hat{h}_j + (2\phi_j - 1)\sigma_j \). The Lagrange function of problem (14) is given by

\[ L(p_{j\beta}, \lambda_j, \mu_j) = \sum_{i=1}^{M} \sum_{j=1}^{N} \tilde{F}_{j\beta}(\hat{h}_j) + \sum_{i=1}^{M} \lambda_j \left( R_j^{\min} - \sum_{j=1}^{N} p_{j\beta} \right) + \sum_{i=1}^{M} \mu_j (p_{j\beta} - R_j^{\min}) \]  

(15)

where \( \lambda_j \) and \( \mu_j \) are Lagrange multipliers for \( C_1 \) and \( C_j \). The function (15) becomes

\[ L(p_{j\beta}, \lambda_j, \mu_j) = \sum_{j=1}^{N} L_j(p_{j\beta}, \lambda_j, \mu_j) - \sum_{j=1}^{N} \lambda_j R_j^{\min} + \sum_{i=1}^{M} \mu_j p_{j\beta}^{\min} \]  

(16)

where

\[ L_j(p_{j\beta}, \lambda_j, \mu_j) = (1 + \lambda_j) \tilde{F}_{j\beta} - p_{j\beta} \mu_j \]  

(17)

Hence, the dual problem of (14) is
\[
\begin{align*}
\min_{\lambda_j, \mu_i} & \quad D(\lambda_j, \mu_i) \\
\text{s.t.} & \quad \lambda_j > 0, \mu_i > 0
\end{align*}
\] (18)

where dual function can be expressed as \(D(\lambda_j, \mu_i) = \max_{p_i} L(p_i, \lambda_j, \mu_i)\).

According to Karush-Kuhn-Tucker (KKT) conditions [12], the transmit power is

\[
p_j^* = \left[\frac{1 + \lambda_j}{\ln 2 \mu_i - \frac{1}{\bar{h}_j}}\right]^+ \tag{19}
\]

where \([x]^+ = \max(0, x)\).

According to the sub-gradient descent method, Lagrangian multipliers are

\[
\begin{align*}
\lambda_j(t + 1) &= \left[\lambda_j(t) - \gamma_1 \left(\sum_{i=1}^M \tilde{r}_p(t) - R_j^{\text{min}}\right)\right]^+ \\
\mu_i(t + 1) &= \left[\mu_i(t) - \gamma_2 \left(P_i^{\text{min}} - \sum_{j=1}^N p_j(t)\right)\right]^+
\end{align*}
\] (20) (21)

where \(\gamma_1\) and \(\gamma_2\) are step sizes, \(t\) is the iteration time. By choosing the appropriate step sizes, the convergence of the algorithm is guaranteed [12].

5. Simulation Results

The effectiveness of the proposed algorithm is verified in this part. There are two BSs in the simulation, the coverage radius of each BS are 100 m. Assuming the path loss model is \(PL = \kappa d^{-\alpha}\), where \(d\) is the distance from the BS to the user and the path-loss exponent is \(\alpha = 4\). The small-scale fading is \(\kappa \sim \mathcal{CN}(0,1)\). The maximum transmit power is \(p_{\text{max}} = 1\ W\), the minimum rate is \(R_j^{\text{min}} = 6\ \text{bps}\), the outage probability threshold is \(\phi_j = 0.1\).

Figure 2 shows the convergence of transmit power under different MTs, where \(p_{11}\) and \(p_{21}\) denote the transmit power from BS 1 to MT 1 and MT 2, and \(p_{12}\) and \(p_{22}\) denote the transmit power of MT 1 and MT 2. It is obvious that when each BS serves two users, the transmission power will converge in about 5 iterations, which has good convergence. For each BS, the transmit power does not exceed the maximum transmit power of the BS, which means it satisfies the transmit power constraint at the BS.

Figure 3 shows the effect of channel uncertainties on the sum rate of MTs. When the channel uncertainties increase, the sum rate of MTs increase. At the same time, when the harvested energy increases, the sum rate of MTs also increases.
Figure 4 demonstrates the actual outage probability of the MT versus the upper bound of channel uncertainties. It can be clearly seen that when the channel uncertainty increases, the outage probability of the MT increases, the reason is that a large channel estimation error may cause the estimated channel parameters to differ significantly from the actual channel parameters, which will result in the communication interruption. However, by comparing with the traditionally non-robust power allocation algorithms, the outage probability of the proposed algorithm is smaller.

Figure 4. Outage probability vs channel uncertainties.

6. Conclusions
A robust power allocation problem imperfect CSI is studied in a multi-homing HetNet with EH to improve the sum data rate and robustness of MTs. The formulated problem is a non-convex one because of random errors. By introducing the uniform distribution model of channel uncertainty, we transformed the original problem with outage probability constraints into a convex one. Lagrange dual method is used to get the distributed solution. Numerical results have shown the efficiency of the algorithm.

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