Flux $1/f^\alpha$ noise in 2D Heisenberg spin glasses: effects of weak anisotropic interactions

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We study the dynamics of a two-dimensional ensemble of randomly distributed classical Heisenberg spins with isotropic RKKY and weaker anisotropic dipole-dipole couplings. Such ensembles may give rise to the flux noise observed in SQUIDs with a $1/f^\alpha$ power spectrum ($\alpha \lesssim 1$). We solve numerically the Landau-Lifshitz-Gilbert equations of motion in the dissipationless limit. We find that Ising type fluctuators, which arise from spin clustering close to a spin-glass critical behavior with $T_c = 0$, give rise to $1/f^\alpha$ noise. Even weak anisotropic interactions lead to a crossover from the Heisenberg-type criticality to the much stronger Ising-type criticality. The temperature dependent exponent $\alpha(T) \lesssim 1$ grows and approaches unity when the temperature is lowered. This mechanism acts in parallel to the spin diffusion mechanism. Whereas the latter is sensitive to the device geometry, the spin-clustering mechanism is largely geometry independent.

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INTRODUCTION

Excess low-frequency flux noise is a ubiquitous phenomenon observed in superconducting quantum interference devices (SQUIDs) and flux quantum devices (SQUIDs) and phase quantum devices (SQUIDs) with RKKY coupling and found that diffusion of the spin magnetization produces flux noise with a $1/f^\alpha$ power spectrum for frequencies $f \lesssim f_1 \sim D W J^2$, where $D$ is the spin diffusion coefficient and $W$ the SQUID loop width. Without further assumptions, the spectrum is temperature independent. In order to reconcile the model with experimental data, showing an exponent $\alpha(T) \lesssim 1$ which grows as $T$ decreases below $\approx 1$ K, it has been conjectured that the spin diffusion coefficient depends on temperature. Concerning this question for Heisenberg spin glasses (HSGs) above the spin glass (SG) transition theory predicts different properties even below the lower critical dimension $d^c_{\text{Ising}} \approx 2.5$. For SQUIDs, however, remains unexplained.
- Isotropic HSG models have a lower critical dimension of $d_{c}^{\text{HSG}} = 3$ or 4 for RKKY-type or short-range interactions, respectively [29, 30]. Thus, no spin-freezing occurs in these SG models in 2D at nonzero temperatures. However, critical behavior is observed in numerical simulations at low temperatures when the SG correlation length, diverging as $\xi(T) \sim (T - T_c)^{-\nu}$ with critical temperature $T_c = 0$, reaches the system size. The critical exponents are $\nu \sim 1$ for the 2D HSG [29] and $\nu \sim 3$ for the 2D ISG [27]. It is known that even weak anisotropic couplings make the HSG exhibit properties of an ISG [28].

In order to further analyze these questions we performed a numerical study of the Landau-Lifshitz-Gilbert equations of the random spin ensemble in the dissipationless limit. Our main results are as follows:

(i) In a model with pure RKKY interaction ($\sim J_0$) magnetization fluctuations of finite wavelength relax by diffusion at a rate $\Gamma_k = D k^2$. As temperature is lowered, the diffusion coefficient increases weakly, possibly logarithmically, consistent with the prediction of Ref. 24 extrapolated to $d = 2$.

(ii) In the presence of weak dipolar couplings $J_1 \ll J_0$, at high temperatures $T \gtrsim J_{1\max}$ the relaxation rates of magnetization fluctuations are $\Gamma_k = \Gamma_0 + D k^2$, where $\Gamma_0$ is the relaxation rate of the total magnetization, which is no longer conserved. As a consequence the lower cut-off frequency $f_l$ for the $1/f$-like flux noise spectrum arising from spin diffusion [20] is now $f_l \sim \max\{D(2\pi/W)^2, \Gamma_0\}/2\pi$. For realistic system sizes and strength of the dipole-dipole coupling we find $f_l \sim \Gamma_0/2\pi \sim 50 \text{ MHz}$, ruling out spin diffusion as an explanation of $1/f$ noise at lower frequencies.

(iii) The dipolar coupling leads to a new mechanism for $1/f^\alpha$ flux noise with $\alpha \lesssim 1$ at experimentally accessible temperatures. A Griffiths SG phase exists at temperatures $T \lesssim J_{1\max}$ [31, 32]. In this regime clusters of spins with relatively strong dipolar couplings exhibit Ising-like dynamics and magnetization switching. As temperature is lowered the clusters increase in size and switch with lower rates due to higher energy barriers. A $1/f^\alpha$ spectrum in the magnetization fluctuations is found for frequencies $f \lesssim f_u \approx 200 \text{ MHz}$. The exponent $\alpha(T) \leq 1$ approaches unity as the temperature decreases.

Our paper is organized as follows: We first specify the model of the HSG forming the basis of the numerical simulations. In the next two sections we investigate the spin dynamics, first with purely isotropic and then with additional anisotropic couplings. We conclude with a comparison with experiments.

**THE MODEL**

We consider an ensemble of classical Heisenberg spins randomly distributed in a plane, e.g. on the surface of a SQUID loop. We assume the spin dynamics to be described by the Landau-Lifshitz-Gilbert equation [33, 34]

$$\dot{S}_i(t) = -\gamma S_i \times ( - \partial_s \cdot H + \mathbf{h}_i(t) - \eta \dot{S}_i) .$$

Here we use units such that $|S_i| = 1$ and the damping parameter $\eta$ is dimensionless. The noise fields $\mathbf{h}_i(t)$ are assumed to be Gaussian distributed and white with correlators $\langle h_{\nu i}(t) h_{\nu j}(t') \rangle = 2T \eta \delta_{ij} \delta_{\nu\nu'} \delta(t - t')$ and $\nu = x, y, z$. The Hamiltonian is given by

$$H = \sum_{(ij)} J_{0 ij} S_i \cdot S_j - \sum_{(ij)} \left[ 3( S_i \cdot r_{ij}) ( S_j \cdot r_{ij}) r_{ij}^2 - S_i \cdot S_j \right],$$

where $J_{0 ij}$ and $J_{1 ij} > 0$ are the isotropic RKKY and the anisotropic dipole-dipole coupling strengths between spins $i$ and $j$ separated by $r_{ij} = r_i - r_j$. Both interactions are long range, decaying as $1/r_{ij}^3$. In our simulations we actually assume the spins to be placed on a $N \times N$ square lattice with lattice constant $a$ and periodic boundary conditions, and we include only nearest neighbor couplings. In order to model the spin ensemble with random separations, $J_{0 ij}$ and $J_{1 ij}$ will be multiplied by random numbers specified below.

In this work, we concentrate on dissipationless spin dynamics, $\eta = 0$. However, to introduce temperature, we begin the simulations with nonzero $\eta$ and enhanced temperature which we reduce gradually to the chosen value. We then switch off the dissipation.

From the spin trajectories, $S_i(t)$, we obtain the discrete spatial Fourier components of the magnetization $S_{\nu}(k; t) = N^{-1} \sum_i S_i(t) e^{i r_i \cdot k}$ and then the power spectra $S_{\nu}(k; \omega) = \langle |\mathcal{F}_{\nu}(s_{\nu}(k; t))|^2 \rangle$. Here $\mathcal{F}_\omega$ denotes the time-Fourier transform. At high temperatures, we achieve ergodicity within the simulation time $t_{\text{sim}}$, which is chosen to be of the order of 10 times the relevant relaxation times. In these cases the averages $\langle \ldots \rangle$ are determined from sufficiently long time blocks of the numerical data and – for better convergence – several realizations of the random couplings. On the other hand, at low temperatures, when we find the power spectrum to rise as $1/f^\alpha$, we are most interested in the lowest accessible frequencies $f_{\min} \sim t_{\text{sim}}^{-1}$. In these cases the averages are taken without partitioning the available data by averaging over several realizations of the random couplings.

The spins on the surface of the SQUID lead to a total flux threading the SQUID loop [20, 35]

$$\Phi(t) = \sum_{k,\nu} B_\nu(k) s_\nu(k, t),$$

and their fluctuations lead to the flux noise power spectrum $S_\nu(\omega) \approx \sum_{k,\nu} E_{\nu}^2(k) S_\nu(k; \omega)$. The form factor $E_{\nu}(k)$ depends strongly on the geometry. If we consider a planar, square-shaped SQUID with narrow lines of width $W$, the flux noise contribution from one side of the square
along the $y$-direction with narrow width in $x$-direction) is given by \(26\, \Phi_{ij}(t) \approx \sum_{k,i} B_{x}(k_x, k_y = 0) s_{x}(k_x, k_y = 0; t)\) with \(B_{x}(k_x, k_y = 0) \sim |k_x|^{-1/2}\).

For purely isotropic interactions the total magnetization is conserved and the spin dynamics reduce to spin diffusion, \(i.e.\) \(S_{\nu}(k; \omega) \propto 2\Gamma_{k}(\Gamma_{k}^{2} + \omega^{2})^{-1}\), with relaxation rate \(\Gamma_{k} = Dk^{2}\). In combination with the form factor given above a power spectrum \(S_{k}(\omega) \sim \omega^{-1}\) is obtained within the frequency range \(\omega_{i} \leq \omega \leq \omega_{u}\) with \(\omega_{i} \sim D(2\pi/W)^{2}\) and \(\omega_{u} \sim D(2\pi/W)^{2}\).

**ISOTROPIC INTERACTIONS**

We first analyze the conditions for spin diffusion for a 2D HSG model with purely isotropic interactions, \((J_{ij} = 0)\). For simplicity, in this section we assume RKKY-like couplings of the form \(J_{ij} = J_{0}\theta_{ij}\) with independent random numbers \(\theta_{ij}\) chosen from a normal distribution with zero mean. We checked that our results are not sensitive to details of the distribution.

Figure 1 depicts the calculated power spectra \(S_{\nu}(k; \omega)\) for two wave numbers and two temperatures. We note that the spectra can be fitted to Lorentzians of the form \(2C_{\nu}\Gamma_{k}(\Gamma_{k}^{2} + \omega^{2})^{-1}\). The fit is excellent at high temperatures, \(T \gtrsim J_{0}\), while at lower ones, \(T \ll J_{0}\), it is valid only up to a cut-off frequency, which decreases as \(T\) is lowered. The parameter \(C_{k}\) is proportional to the static susceptibility for wavenumber \(k\) and approximately independent of \(k\). As illustrated in Fig. 2, the relaxation rates are well described by \(\Gamma_{k} = Dk^{2}\). At high temperatures \((T \gtrsim J_{0})\) the diffusion coefficient \(D\) is constant, at low temperatures it is weakly temperature-dependent (see inset of Fig. 2). The dependence appears to be logarithmic, consistent with the results of Ref. 23, \(D \propto (T - T_{c})^{2/d - 1}\) \((2 < d < 4)\), extrapolated to \(d = 2\), where \(T_{c} = 0\) \[29\).

Our data also show evidence of critical behavior at low temperatures. For the HSG with nearest neighbor couplings, we find that the SG correlation length \(\xi(T)\) diverges at low temperatures as \(\xi(T) \approx 0.4a[(T - T_{c})J_{0}^{-1}]^{-\nu}\) with exponent \(\nu \approx 1.0\) and \(T_{c} = 0\).

**ANISOTROPIC INTERACTIONS**

We turn next to spin dynamics in the presence of weak anisotropic couplings \((J_{ij} \ll |J_{0ij}|)\). Such couplings break the rotational symmetry of the problem, \(i.e.\), in general we expect different results for different spin directions, \(\nu = x, y, z\). In addition, the total magnetization is no longer conserved.

In the random ensemble there exist spins which are separated by distances smaller than \(r_{\text{typ}}\), and, accordingly, the RKKY and dipolar couplings are much stronger than the typical values of \(J_{0}\) and \(J_{1}\). To account for the distribution of couplings we set \(J_{0ij} = \pm J_{0}(r_{\text{typ}}/r_{ij})^{3}\) and \(J_{ij} = J_{1}(r_{\text{typ}}/r_{ij})^{3}\), and choose the random distances \(r_{ij}\) according to the distribution \(P(r) = 2r r_{\text{typ}}^{-3} e^{-\left(r^{2} - r_{\text{min}}^{2}\right)/r_{\text{typ}}^{2}}\). The exponential accounts for the decreasing probability for spins with separation much larger than \(r_{\text{typ}}\) to be nearest neighbors.

At high temperatures \(T \gtrsim J_{1}\) the noise spectra with small \(k\) still have a Lorentzian shape with relaxation rates \(\Gamma_{\nu}(k) = \Gamma_{\nu,0} + Dk^{2}\), where \(\Gamma_{\nu,0} = A_{\nu} J_{1}^{2} J_{0}^{-1}\) and the numerical coefficients are \(A_{x} \approx 2A_{x,y} \approx 2\) and \(\langle J_{ij}^{2} \rangle \approx J_{1}^{2} r_{\text{typ}}^{4}/2r_{\text{min}}^{4}\). The dynamics in this temperature regime are of the Heisenberg type, \(i.e.\), the spins explore the entire Bloch sphere, except that the out-of-plane magnetization fluctuations relax faster than the in-plane ones. As a result of the relaxation, the low-frequency cut-off \(f_{1}\) for the \(1/f\)-like flux noise spectrum, which arises from the specific geometrical form factor \(2k\), is now given by \(f_{1} \sim \max\{D(2\pi/W)^{2}, \Gamma_{\nu,0}\}/2\pi\). As we will argue below this rules out the diffusion mechanism as origin of the observed \(1/f\) noise at low frequencies.
For temperatures $T \lesssim J_{1 \text{max}}$ the dynamics change from Heisenberg to Ising type. At the crossover $T \approx J_{1 \text{max}}$, the first pairs of closely spaced and thus strongly coupled spins begin to form. Each pair behaves as a two-state fluctuator with the two energy minima corresponding to both spins pointing in the same direction, but either parallel or antiparallel to the vector connecting their positions. In the absence of RKKY interactions the energy barrier between these two minima would be $\Delta U = J_{1ij}$, but in the presence of strong RKKY coupling it is enhanced, $\Delta U = 3J_{1ij}$, provided the coupling happens to be ferromagnetic. Strongly coupled pairs with antiferromagnetic RKKY interaction tend to form singlets. (They do not contribute to the noise flux, but they may contribute to the noise of magnetic susceptibility due to rare thermal jumps to the triplet state [25].)

The distribution of energy barriers, related to the distribution of spin separations $r_{ij}$ discussed above, leads to an Ising-type switching and a $1/f$-like flux noise spectrum up to temperatures of order $T \lesssim J_{1 \text{max}}$. In addition, at lower temperatures, as illustrated in Fig. 3 and Fig. 4(a), more complicated cluster configurations emerge with sizes $N_c(T)$ depending on temperature. The larger clusters turn out to be mostly random (glassy) with magnetization of order $\mu_c \sim \sqrt{N_c}$. The magnetization of the clusters, $\mu(t)$, switches in Ising-type fashion between the values $\pm \mu_c$, as illustrated in Fig. 4(b). The larger the cluster the slower is the switching. For large clusters both ferromagnetic and antiferromagnetic RKKY couplings raise the energy barriers. The slow switching of the clusters is driven by the bath of weakly coupled paramagnetic spins. Indeed, since we assumed vanishing spin dissipation, $\eta = 0$, this coupling is the only mechanism driving the cluster dynamics.

As a result of the switching dynamics the power spectra for the total magnetization, as well as the Fourier modes with non-vanishing $k$, exhibit an $1/f^\alpha$ dependence in a frequency range $f_1(T) \lesssim f \lesssim f_2$, as shown in Fig. 5. The upper cut-off frequency is proportional to the strength of the anisotropic coupling $f_2 \propto J_{1 \text{max}}$. For the distribution and parameters considered here we find from our simulation $f_2 \sim 200$ MHz. Because of limitations of the simulation time we can not precisely determine the lower cut-off frequency $f_1$. It is dominated by large clusters which emerge at low temperatures. Figure 5 also shows that the exponent $\alpha(T) < 1$ tends to increase as the temperature is lowered. At higher frequencies $f_1 \lesssim f \lesssim J_0 \max/2\pi$, the spectral functions corresponding to small wave numbers $k$ decay roughly as $f^{-3}$, whereas those with larger $k$ decay as $f^{-2}$. For even higher frequencies $f \gtrsim J_0 \max/2\pi$, the spectral lines decay rapidly depending on the distribution of the coupling strengths.

We emphasize that the $1/f^\alpha$-like flux noise spectrum arising from the magnetization switching dynamics of the clusters is not significantly affected by the SQUID geometry and form factor because the finite-$k$ Fourier modes already exhibit $1/f^\alpha$-like spectra.

At low temperatures $T \lesssim J_1$ the system shows a tendency towards spin-glass freezing. Since the lower critical dimension of an ISG is higher than 2, one expects $T_c = 0$, and the observed freezing is a consequence of finite size, with $\xi(T) \gtrsim W$, and limited simulation time.

**SUMMARY AND COMPARISON**

We have performed numerical simulations of the Landau-Lifshitz-Gilbert equations in the dissipationless limit to study the dynamics of 2D Heisenberg spin glasses with dominant RKKY-like interactions, first without and then with additional, weaker dipole-dipole couplings.

For the case of purely isotropic interactions of strength
around $J_0$, we find that the spin-diffusion description for the long-wavelength modes can be employed at all temperatures as long as the system size exceeds the HSG correlation length, $W \gg \xi(T) \propto T^{-\nu}$ with $\nu \approx 1$. We further found that the diffusion coefficient, $D \sim J_0^{\nu^2}$, has a weak, possibly logarithmic temperature dependence at low temperatures consistent with Ref. [23].

In the presence of weak dipole-dipole coupling at high temperatures, $J_{J1J} \lesssim T$, we find that the long-wavelength modes relax with rates $\Gamma_{\nu}(k) = \Gamma_{\nu,0} + Dk^2$, where $\Gamma_{\nu,0} = A_{\nu} \langle J^2 \rangle / J_0$ accounts for the relaxation of the total magnetization along the $\nu$-direction ($\nu = x, y, z$) and $A_{\nu} \approx 2A_{x,y} \approx 2$. As a result the low-frequency cut-off $f_l$ for the $1/f$-like flux noise spectrum which arises from the specific geometrical form factor [24], is now given by $f_l \sim \max\{D(2\pi/W)^2, \Gamma_{\nu,0}\}/2\pi$. For the parameters specified above we find $f_l \approx \Gamma_{\nu,0}/2\pi \sim 50$ MHz. Thus, in the presence of sufficiently strong anisotropic couplings, the mechanism of Ref. [20] does not explain the observed low-frequency noise even if the device geometry and size would allow for it.

If the temperature is lower than the strength of the anisotropic coupling, spin clusters develop with an Ising-type dynamics of magnetization switching. The size of the clusters and the effective barriers against switching increase as temperature is lowered. The range of relaxation rates following from this scenario leads to a $1/f^{\alpha(T)}$ power spectrum at temperatures $T \lesssim J_{1 \text{max}}$. As the temperature is lowered the cluster sizes grow, as does the exponent $\alpha(T) \lesssim 1$. We estimated that the upper bound for the anisotropic couplings may be as large as $J_{1 \text{max}} = 70$ mK. Thus, in our model $1/f$ noise would start to appear at temperatures below roughly 100 mK. The experimental observation of $1/f$ noise up to temperatures of order 4.2 K suggests an even stronger anisotropic spin-spin interaction.

In Fig. 5 the frequency range over which we find a $1/f^\alpha$ flux noise spectrum lies between $f_l < 200$ kHz and $f_u \sim 200$ MHz (for $J_0 = 2\pi \times 1$ GHz). While the upper cut-off frequency $f_u$ is readily accessible from our numerical analysis and roughly agrees with experiments [36, 37], the lower temperature-dependent cut-off frequency $f_l(T)$ can not be determined due to the limitations of simulation time. We conjecture that the appearance of very large clusters, larger than we can simulate, would lead to $1/f$ noise down to much lower frequencies.

In order to compare our results with the observed magnitude of the flux noise power spectrum $S_\Phi(\omega)$ for SQUIDs we extrapolate our data. Our simulation is restricted to a square of size $90 \times 90$ nm$^2$ (based on $a = r_{\text{typ}} = 1.5$ nm), while the line width $W$ and outer dimension of the SQUID loop $L \approx 10W$ are typically much larger. Assuming that the fluctuations from different squares of the size considered contribute independently, we find $S_\Phi(\omega) = \rho_s \bar{g} (L/W)(\mu_B \mu_0)^2 N^{-2} S_\Phi(k = 0; \omega)$, where $\bar{g} \approx 3$ is the squared form factor [26] averaged over the SQUID loop width. From Fig. 5 we find that $S_\Phi(f = 100$ kHz) $\approx 10^{-5}(\mu_B \mu_0)^2$ Hz$^{-1}$. Extrapolating this result down to $f = 1$ Hz (assuming an exponent $\alpha = 1$) we find $S_\Phi(f = 1$ Hz) $\approx 1(\mu_B \mu_0)^2$ Hz$^{-1}$, which is the same order of magnitude as the measured flux noise power spectrum.

The fact that the cluster magnetic moment scales as $\mu_c \propto \sqrt{N_c}$ implies that the clusters are glassy or random (as opposed to ferromagnetic or antiferromagnetic). This has important implications [38]. First, we note that the mean square flux noise for $N_c$ independent spins would satisfy $\langle \Phi^2 \rangle \sim N_c \mu_c^2$. For random clusters with $N_c$ spins we have $\mu_c = \mu_B \sqrt{N_c}$ and $\langle \Phi^2 \rangle \sim (N_s/N_c)(\mu_B \sqrt{N_c})^2 = N_s \mu_B^2$. Thus, the total effective number $(N_s)$ of spins is conserved, regardless of the cluster size. This is not the case for ferromagnetic or antiferromagnetic clusters. Second, the observed Curie-law paramagnetism [2] implies a classical unsaturated behavior, that is $\mu_c B < k_B T$, where $B$ is the ambient magnetic field. For $B = 10$ mT and $T = 50$ mK, we find $\mu_c < 7 k_B T$ or $N_c \lesssim 49$. In our simulations virtually all the clusters satisfy this restriction. For larger, experimentally relevant systems and low temperatures beyond the range covered by our simulations we expect larger and slower clusters to appear. These would be responsible for the $1/f^\alpha$ noise at low frequencies. Our estimates show, however, that the contribution to the susceptibility of the large clusters required to extend the frequency range down to say $10^{-4}$Hz is masked by the large number of remaining, smaller clusters.

Finally, we mention that our simulations provide indications of spin-glass freezing at low temperatures $T \lesssim J_1$. Since the lower critical dimension of an ISG is higher than 2, one expects $T_c = 0$, and the observed freezing is a consequence of finite size, with $\xi(T) \approx W$, and limited simulation time. Our parameters translate to a system size $W \sim 90$ nm, time scales $\tau_{\text{sim}} \sim 2 \times 10^{-3} J_1^{-1} \approx 3 \mu$s and freezing temperatures below $T \lesssim 1$ mK. This is too low to be observable. On the other hand, spin freezing has been observed for isolated gold rings [17] at $T \sim 25$ mK, which...
may again suggest a dipolar coupling much stronger than the value assumed in this paper.

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