Probe a family non-universal $Z'$ boson effects

in $\bar{B}_s \rightarrow \phi \mu^+ \mu^-$ decay

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Abstract

Motivated by the recent measurement on $\mathcal{B}(\bar{B}_s \rightarrow \phi \mu^+ \mu^-)$ by CDF collaboration, we study the effects of a family non-universal $Z'$ boson on rare semileptonic $\bar{B}_s \rightarrow \phi \mu^+ \mu^-$ decay. In our evaluations, we analyze the dependences of the dimuon invariant mass spectrum and normalized forward-backward asymmetry on $Z'$ couplings and show that these observables are highly sensitive to new $Z'$ contributions. Three limiting scenarios are presented in the detailed analyses. Numerically, within the allowed ranges of $Z'$ couplings under the constraints from $\bar{B}_s - B_s$ mixing, $B \rightarrow \pi K$, $\bar{B}_d \rightarrow (X_s, K, K^*) \mu^+ \mu^-$ decays and so on, $\mathcal{B}(\bar{B}_s \rightarrow \phi \mu^+ \mu^-)$ and $A_{FB}^{(L)}(\bar{B}_s \rightarrow \phi \mu^+ \mu^-)$ could be enhanced by about 96% and 17% (133%) respectively at most by $Z'$ contributions. However, $\mathcal{B}(\bar{B}_s \rightarrow \phi \mu^+ \mu^-)$ is hardly to be reduced. Furthermore, the zero crossing in $A_{FB}(\bar{B}_s \rightarrow \phi \mu^+ \mu^-)$ spectrum at low dimuon mass always exists.

Keywords: B-physics, Rare decays, Beyond Standard Model

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1 Introduction

Rare B decays induced by the flavor-changing neutral current (FCNC) occur at loop level in the Standard Model (SM) and thus proceed at a low rate. They can provide useful information on the parameters of the SM and test its predictions. Meanwhile, they offer a valuable possibility of an indirect search of new physics (NP) for their sensitivity to the gauge structure and new contributions. Experimentally, the fruitful running of BABAR, Belle and Tevatron in the past decade provides a very fertile ground for testing SM and probing possible NP effects. As particle physics is entering the era of LHC, $B_s$ physics has attracted much more attention.

Recently, CDF collaboration has reported the first observation of the rare semileptonic $\bar{B}_s \rightarrow \phi \mu^+\mu^-$ decay and measured its branching fraction to be $[1]$

$$B(\bar{B}_s \rightarrow \phi \mu^+\mu^-) = [1.44 \pm 0.33\text{(stat.)} \pm 0.46\text{(syst.)}] \times 10^{-6} \quad \text{CDF}. \quad (1)$$

Theoretically, many evaluations for $\bar{B}_s \rightarrow \phi \mu^+\mu^-$ decay have been done within both SM and various NP scenarios (for example, Refs. [2, 3]). The SM prediction for $B^{SM}(\bar{B}_s \rightarrow \phi \mu^+\mu^-)$ ($\sim 1.65 \times 10^{-6}$ (QCDSR) [2], for example) agrees well with CDF measurement ($1.44 \pm 0.57 \times 10^{-6}$) for large experimental error. If more exact measurement on $\bar{B}_s \rightarrow \phi \mu^+\mu^-$ is gotten by the running LHC-b and future super-B, the possible NP space will be strongly constrained or excluded. So, it is worth evaluating the effects of the possible NP, such as a family non-universal $Z'$ boson, on $\bar{B}_s \rightarrow \phi \mu^+\mu^-$ decay.

A new family non-universal $Z'$ boson could be naturally derived in certain string constructions [4], $E_6$ models [5] and so on. Searching for such an extra $Z'$ boson is an important mission in the experimental programs of Tevatron [6] and LHC [7]. The general framework for non-universal $Z'$ model has been developed in Ref. [8]. Within such model, FCNC in $b \rightarrow s$ and $d$ transitions could be induced by family non-universal $U(1)'$ gauge symmetries at tree level. Its effects on $b \rightarrow s$ transition have attracted much more attention and been widely studied. Interestingly, the behavior of a family non-universal $Z'$ boson is helpful to resolve many puzzles in $B_{(u,d,s)}$ decays, such as “$\pi K$ puzzle” [9, 10], anomalous $\bar{B}_s - B_s$ mixing phase [11, 12] and mismatch in $A_{FB}(B \rightarrow K^* \mu^+\mu^-)$ spectrum at low $q^2$ region [13, 14].

Within a family non-universal $Z'$ model, $\bar{B}_s \rightarrow \phi \mu^+\mu^-$ decay involves $b - s - Z'$ and $\mu - \mu - Z'$ couplings, which have been strictly bounded by the constraints from $\bar{B}_s - B_s$ mixing,
\[ B \rightarrow \pi K^{(*)}, \ \rho K, \ \bar{B}_d \rightarrow X_s \mu \mu, \ K^{(*)}\mu \mu \text{ decays and so on [10, 12, 13]. So, it is worth evaluating} \]
\[ \text{the effects of a non-universal } Z' \text{ boson on } \bar{B}_s \rightarrow \phi \mu^+ \mu^- \text{ decay and checking whether such settled} \]
\[ \text{values of } Z' \text{ couplings are permitted by CDF measurement on } \mathcal{B}(\bar{B}_s \rightarrow \phi \mu^+ \mu^-). \]

Our paper is organized as follows. In Section 2, we briefly review the theoretical framework for \( b \rightarrow sl^+l^- \) decay within both SM and a family non-universal \( Z' \) model. In Section 3, the effects of a non-universal \( Z' \) boson on \( \bar{B}_s \rightarrow \phi \mu^+ \mu^- \) decay are investigated in detail. Our conclusions are summarized in Section 4. Appendix A and B include all of the theoretical input parameters.

## 2 The theoretical framework for \( b \rightarrow sl^+l^- \) decays

In the SM, neglecting the doubly Cabibbo-suppressed contributions, the effective Hamiltonian governing semileptonic \( b \rightarrow sl^+l^- \) transition is given by [15, 16]
\[ \mathcal{H}_{\text{eff}} = -\frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu). \] (2)

Here we choose the operator basis given by Ref. [15], in which
\[ O_9 = \frac{e^2}{g_s^2}(\bar{d} \gamma_\mu P_L b)(\bar{l} \gamma_\mu l), \quad O_{10} = \frac{e^2}{g_s^2}(\bar{d} \gamma_\mu P_L b)(\bar{l} \gamma_\mu \gamma_5 l). \] (3)

Wilson coefficients \( C_i \) can be calculated perturbatively [17, 18, 19, 20], with the numerical results listed in Table 1. The effective coefficients \( C_{7,9}^{\text{eff}}, C_9^{\text{eff}}, C_{10}^{\text{eff}} \), which are particular combinations of \( C_{7,9} \) with the other \( C_i \), are defined as [15]
\[ C_7^{\text{eff}} = \frac{4\pi}{\alpha_s} C_7 - \frac{1}{3} C_3 - \frac{4}{9} C_4 - \frac{20}{3} C_5 - \frac{80}{9} C_6, \]
\[ C_9^{\text{eff}} = \frac{4\pi}{\alpha_s} C_9 + Y(q^2), \quad C_{10}^{\text{eff}} = \frac{4\pi}{\alpha_s} C_{10}, \] (4)

in which \( Y(q^2) \) denotes the matrix element of four-quark operators and given by
\[ Y(q^2) = h(q^2, m_c)(\frac{4}{3} C_1 + C_2 + 6C_3 + 60C_5) - \frac{1}{2} h(q^2, m_b)(7C_3 + \frac{4}{3} C_4 + 76C_5 + \frac{64}{3} C_6) \]
\[ -\frac{1}{2} h(q^2, 0)(C_3 + \frac{4}{3} C_4 + 16C_5 + \frac{64}{3} C_6) + \frac{4}{3} C_3 + \frac{64}{9} C_5 + \frac{64}{27} C_6. \] (5)

We have neglected the long-distance contribution mainly due to \( J/\Psi \) and \( \Psi' \) in the decay chain \( \bar{B}_s \rightarrow \phi \Psi^{(*)} \rightarrow \phi l^+l^- \), which could be vetoed experimentally [1]. For the recent detailed discussion of such resonance effects, we refer to Ref. [21].
Table 1: The SM Wilson coefficients at the scale $\mu = m_b$.  

| Coefficient       | Value     |
|-------------------|-----------|
| $C_1(m_b)$        | $-0.284$  |
| $C_2(m_b)$        | $1.007$   |
| $C_3(m_b)$        | $-0.004$  |
| $C_4(m_b)$        | $-0.078$  |
| $C_5(m_b)$        | $0.000$   |
| $C_6(m_b)$        | $0.001$   |
| $C_7^\text{eff}(m_b)$ | $-0.303$ |
| $C_9^\text{eff}(m_b)$ | $4.095$  |
| $C_{10}^\text{eff}(m_b)$ | $-4.153$ |

Although there are quite a lot of interesting observables in semileptonic $b \to s \ell^+ \ell^-$ decay, we shall focus only on the dilepton invariant mass spectrum and the forward-backward asymmetry in this paper. Adopting the same convention and notation as [22], the dilepton invariant mass spectrum and the forward-backward asymmetry for $\bar{B}_s \to \phi \ell^+ \ell^-$ decay is given as

$$
\frac{d\Gamma^\phi}{ds} = \frac{G_F^2 \alpha^2 m_{B_s}^5}{2^{10} \pi^5} |V_{ts}^*V_{tb}|^2 \hat{s} \hat{u}(\hat{s}) \left\{ \frac{|A|^2}{3} \hat{s} \lambda(1 + 2 \hat{m}_\phi^2) + |E|^2 \hat{s} \hat{u}(\hat{s})^2 \right. \\
\left. + \frac{1}{4 \hat{m}_\phi^2} \left[ |B|^2 (\lambda - \hat{u}(\hat{s})^2) + 8 \hat{m}_\phi^2 (\hat{s} + 2 \hat{m}_\phi^2) \right] + |F|^2 (\lambda - \hat{u}(\hat{s})^2) + 8 \hat{m}_\phi^2 (\hat{s} - 4 \hat{m}_\phi^2) \right] \right. \\
\left. + \frac{\lambda}{4 \hat{m}_\phi^2} \left[ |C|^2 (\lambda - \hat{u}(\hat{s})^2) + |G|^2 (\lambda - \hat{u}(\hat{s})^2) + 4 \hat{m}_\phi^2 (2 + 2 \hat{m}_\phi^2 - \hat{s}) \right] \right. \\
\left. - \frac{1}{2 \hat{m}_\phi^2} \left[ \text{Re}(B^* C) (\lambda - \hat{u}(\hat{s})^2) (1 - \hat{m}_\phi^2 - \hat{s}) \right. \\
\left. + \text{Re}(F G^*) ((\lambda - \hat{u}(\hat{s})^2) (1 - \hat{m}_\phi^2 - \hat{s}) + 4 \hat{m}_\phi^2 \lambda) \right] \right. \\
\left. - \frac{2 \hat{m}_\phi^2 \lambda}{\hat{m}_\phi^2} \left[ \text{Re}(F H^*) - \text{Re}(G H^*) (1 - \hat{m}_\phi^2) \right] + \frac{\hat{m}_\phi^2 \hat{s}}{\hat{m}_\phi^2} \lambda |H|^2 \right\}; 
$$

(6)

$$
\frac{dA_{FB}^\phi}{ds} = \frac{G_F^2 \alpha^2 m_{B_s}^5}{2^{10} \pi^5} |V_{ts}^*V_{tb}|^2 \hat{s} \hat{u}(\hat{s})^2 \times \left[ \text{Re}(C_9^\text{eff} C_7^\text{eff}^* V A_1 + \frac{\hat{m}_h}{\hat{s}} \text{Re}(C_9^\text{eff} C_7^\text{eff}^* C_{10}^\text{eff}) \left( V T_2 (1 - \hat{m}_\phi) + A_1 T_1 (1 + \hat{m}_\phi) \right) \right], 
$$

(7)

with $s = q^2$ and $\hat{s} = s/m_{B_s}^2$. Here the auxiliary functions $A, B, C, E, F$ and $G$, with the explicit expressions given in Ref. [22], are combinations of the effective Wilson coefficients in Eq. (4) and the $B_s \to \phi$ transition form factors, which are calculated with light-cone QCD sum rule approach in Ref. [23] and given in Appendix B. From the experimental point of view, the normalized forward-backward asymmetry is more useful, which is defined as [22]

$$
\frac{d\bar{A}_{FB}}{ds} = \frac{dA_{FB}}{ds} / \frac{d\Gamma}{ds}; 
$$

(8)
A new family non-universal $Z'$ boson could be naturally derived in many extension of SM. One of the possible way to get such non-universal $Z'$ boson is to include an addition $U'(1)$ gauge symmetry, which has been formulated in detail by Langacker and Plüümacher [8]. Under the assumption that the couplings of right-handed quark flavors with $Z'$ boson are diagonal, the $Z'$ part of the effective Hamiltonian for $b \to s l^+ l^-$ transition can be written as [11]

$$
\mathcal{H}_{\text{eff}}^{Z'}(b \to s l^+ l^-) = \frac{-2 G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ -\frac{B_{sb} B_{tb}^L}{V_{tb} V_{ts}} (\bar{s}b)_{V-A}(\bar{l}l')_{V-A} - \frac{B_{sb} B_{tb}^R}{V_{tb} V_{ts}} (\bar{s}b)_{V-A}(\bar{l}l')_{V+A} \right] + \text{h.c.}. \quad (9)
$$

With the assumption that no significant RG running effect between $M_{Z'}$ and $M_W$ scales, $Z'$ contributions could be treated as modification to wilson coefficients, i.e. $C_{9,10}'(M_W) = C_{9,10}^{SM}(M_W) + \triangle C_{9,10}'(M_W)$. As a result, Eq. (9) could also be reformulated as

$$
\mathcal{H}_{\text{eff}}^{Z'}(b \to s l^+ l^-) = \frac{-4 G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \triangle C_9 O_9 + \triangle C_{10}' O_{10} \right] + \text{h.c.}, \quad (10)
$$

with

$$
\triangle C_9(M_W) = -\frac{g_s^2}{e^2} \frac{B_{sb}^L}{V_{tb} V_{ts}} S_{ll}^{LR}, \quad S_{ll}^{LR} = (B_{ll}^L + B_{ll}^R), \quad \triangle C_{10}'(M_W) = \frac{g_s^2}{e^2} \frac{B_{sb}^L}{V_{tb} V_{ts}} D_{ll}^{LR}, \quad D_{ll}^{LR} = (B_{ll}^L - B_{ll}^R). \quad (11)
$$

$B_{sb}^L$ and $B_{ll}^{LR}$ denote the effective chiral $Z'$ couplings to quarks and leptons, in which the off-diagonal element $B_{sb}^L$ can contain a new weak phase and could be written as $|B_{sb}^L| e^{i\phi_{B_s}^L}$.

To include $Z'$ contributions, one just needs to make the replacements

$$
C_{9,10}^{\text{eff}} \to \bar{C}_{9,10}^{\text{eff}} = \frac{4 \pi}{\alpha_s} C_{9,10}' + Y(q^2),
$$

in the formalisms relevant to $B_s \to \phi \ell^+ \ell^-$.

### 3 Numerical analyses and discussions

With the relevant theoretical formulas collected in Section 2 and the input parameters summarized in the Appendix, we now proceed to present our numerical analyses and discussions.

In Table 2, we present our theoretical predictions for integrated branching fraction and forward-backward asymmetry of $B_s \to \phi \mu^+ \mu^-$ decay. Within the SM, we again find our prediction $B^{SM}(B_s \to \phi \mu^+ \mu^-) = 1.46 \times 10^{-6}$ is perfectly consistent with CDF measurement
Table 2: Predictions for $\mathcal{B}(\bar{B}_s \to \phi \mu^+ \mu^-)\times 10^{-6}$ and $A_{FB}(\bar{B}_s \to \phi \mu^+ \mu^-)\times 10^{-2}$ within the SM and the non-universal $Z'$ model.

|                | Exp. [24] | SM     | S1     | S2     | Scen. I | Scen. II | Scen. III |
|----------------|-----------|--------|--------|--------|---------|----------|-----------|
| $\mathcal{B}$  | $1.44 \pm 0.57$ | $1.46 \pm 0.10$ | $2.47 \pm 1.18$ | $1.40 \pm 0.27$ | $2.86$ | $1.26$ | $1.92$   |
| $\mathcal{B}^L$ | —        | $0.34 \pm 0.04$ | $0.56 \pm 0.27$ | $2.61 \pm 0.19$ | $0.64$ | $0.28$ | $0.44$   |
| $\mathcal{B}^H$ | —        | $0.29 \pm 0.02$ | $0.51 \pm 0.25$ | $1.26 \pm 0.08$ | $0.59$ | $0.26$ | $0.39$   |
| $A_{FB}$       | —        | $25.6 \pm 1.2$ | $19.4 \pm 10.9$ | $24 \pm 0.03$ | $29.9$ | $26.6$ | $8.9$    |
| $A_{FB}^I$     | —        | $5.7 \pm 0.6$ | $6.0 \pm 7.4$ | $0.09 \pm 0.02$ | $13.3$ | $6.9$ | $1.4$    |
| $A_{FB}^H$     | —        | $34.1 \pm 0.2$ | $22.5 \pm 12.9$ | $-0.07 \pm 0.01$ | $35.0$ | $34.8$ | $13.1$   |

Figure 1: Dimuon invariant mass distribution and normalized forward-backward asymmetry of the $\bar{B}_s \to \phi \mu^+ \mu^-$ decay within SM and three limiting scenarios.

$(1.44 \pm 0.57) \times 10^{-6}$. The forward-backward asymmetry for $\bar{B}_s \to \phi \mu^+ \mu^-$ decay is evaluated at $\sim 25\%$, which hasn’t be measured by the experiment. In addition, in Table 2, we also calculate their results $\mathcal{B}^{L,H}$ and $A_{FB}^{L,H}$ at both low ($1 GeV^2 < s < 6 GeV^2$) and high ($14.4 GeV^2 < s < 25 GeV^2$) integration regions, which are sufficiently below and above the threshold for charmonium resonances $J/\psi, \psi'$ respectively. The dimuon invariant mass distribution and forward-backward asymmetry spectrum are shown in Fig. 1. As Fig. 1(b) shows, similar to the situation in $\bar{B}^0 \to K^* \mu^+ \mu^-$ decay, the zero crossing exist in $A_{FB}$ spectrum at $s_0 \sim 3 GeV^2$, whose position is well-determined and free from hadronic uncertainties at the leading order in $\alpha_s$ [17, 22, 25]. In $\bar{B}^0 \to K^* \mu^+ \mu^-$ decay, the $A_{FB}$ spectrum measured by Belle collaboration [26] indicates that there might be no zero crossing, which presents a challenge to
Table 3: The inputs parameters for the $Z'$ couplings [12, 13].

|                  | $|B_{s|b}^L| (\times 10^{-3})$ | $\phi_s^L [^\circ]$ | $S_{\mu\mu}^{LR} (\times 10^{-2})$ | $D_{\mu\mu}^{LR} (\times 10^{-2})$ |
|------------------|-------------------------------|---------------------|----------------------------------|----------------------------------|
| S1               | $1.09 \pm 0.22$               | $-72 \pm 7$         | $-2.8 \pm 3.9$                  | $-6.7 \pm 2.6$                  |
| S2               | $2.20 \pm 0.15$               | $-82 \pm 4$         | $-1.2 \pm 1.4$                  | $-2.5 \pm 0.9$                  |

the SM in low $s$ region. If the future measurement on $A_{FB}(\bar{B}_s \to \phi \mu^+\mu^-)$ spectrum presents a similar result as the one in $B^0 \to K^*\mu^+\mu^-$ decay, it will be a significant NP signal.

Within a family non-universal $Z'$ model, the $Z'$ contributions to $\bar{B}_s \to \phi \mu^+\mu^-$ decay involve four new $Z'$ parameters $|B_{s|b}^L|$, $\phi_s^L$, $S_{\mu\mu}^{LR}$ and $D_{\mu\mu}^{LR}$. Combining the constraints from $B_s - B_s$ mixing, $B \to \pi K^{(*)}$ and $\rho K$ decays, $|B_{s|b}^L|$ and $\phi_s^L$ have been strictly constrained [10, 12]. After having included the constraints from $B_d \to X_s \mu\mu$, $K\mu\mu$ and $K^*\mu\mu$, as well as $B_s \to \mu\mu$ decays, we have also gotten the allowed ranges for $S_{\mu\mu}^{LR}$ and $D_{\mu\mu}^{LR}$ in Ref. [13]. For convenience, we recollect their numerical results in Table 3, in which S1 and S2 correspond to UTfit collaboration’s two fitting results for $\bar{B}_s - B_s$ mixing [27]. Our following evaluations and discussions are based on these given ranges for $Z'$ couplings. With the values of $Z'$ parameters listed in Table 3 as inputs, we present our predictions for the observables in the third and fourth columns of Table 2.

As illustrated in Fig. 2, integrated branching fraction for $\bar{B}_s \to \phi \mu^+\mu^-$ is sensitive to the $Z'$ contributions. Obviously, $\bar{B}_s \to \phi \mu^+\mu^-$ is enhanced by the $Z'$ contributions with large negative $S_{\mu\mu}^{LR}$, $D_{\mu\mu}^{LR}$ and $\phi_s^L$. Moreover, compared Fig. 2 (a,b) with (c,d), we find the effects of solution S1 is more significant than the one of S2. So, for simplicity, we just pay our attention to the solution S1 in the following. As Fig. 2 shows, the $Z'$ contributions with a small negative weak phase $\phi_s^L$ are helpful to reduce $B(\bar{B}_s \to \phi \mu^+\mu^-)$. However, because the range $\phi_s^L > -65^\circ$ is excluded by the constraints from $\bar{B}_s - B_s$ mixing and $B \to \pi K$ decays [10, 12], $B(\bar{B}_s \to \phi \mu^+\mu^-)$ is hardly to be reduced so much by $Z'$ contributions.

In order to see the $Z'$ effect on $A_{FB}(\bar{B}_s \to \phi \mu^+\mu^-)$ explicitly, with $Y(q^2)$ being excluded, we can rewrite $\text{Re}(\tilde{C}_9^{\text{eff}} \tilde{C}_{10}^{\text{eff}}^*)$ and $\text{Re}(\tilde{C}_7^{\text{eff}} \tilde{C}_{10}^{\text{eff}}^*)$ in Eq. (7), which dominates $A_{FB}(\bar{B}_s \to \phi \mu^+\mu^-)$.
Figure 2: The dependence of $\mathcal{B}(\bar{B}_s \to \phi \mu^+ \mu^-)$ on $S_{\mu\mu}^{LR}$ and $D_{\mu\mu}^{LR}$ within their allowed ranges in S1 and S2 with different $\phi_s^L$ values. The black dashed line corresponds to the SM result.

Figure 3: The dependence of $A_{FB}(\bar{B}_s \to \phi \mu^+ \mu^-)$ on $S_{ud}^{L,R}$ and $D_{ud}^{L,R}$ at $s = 1.5 \text{GeV}^2$ (a) and $s = 15 \text{GeV}^2$ (b) with $|B_{db}^L| = 1.09 \times 10^{-3}$, $\phi_s^L = 72^\circ$ (S1) and the central values of the other theoretical input parameters. The blue planes correspond to SM results.

in high and low $s$ regions respectively, as

\[
\begin{align*}
\text{Re}(\tilde{C}_9^{\text{eff}} \tilde{C}_{10}^{\text{eff}*}) & \approx \text{Re}(\tilde{C}_9^{\text{eff}}) \text{Re}(\tilde{C}_{10}^{\text{eff}*}) + \left(\frac{4\pi}{\alpha_s}\right)^2 \text{Im}(\Delta C_9') \text{Im}(\Delta C_{10}'), \\
\text{Re}(\tilde{C}_7^{\text{eff}} \tilde{C}_{10}^{\text{eff}*}) & \approx \text{Re}(\tilde{C}_7^{\text{eff}}) \text{Re}(\tilde{C}_{10}^{\text{eff}*}).
\end{align*}
\]
Combining Eq. (11) and Eq. (14), due to the tiny $Z'$ contribution to $C_{7}^{\text{eff}}$, the only solution to enhance $A_{FB}$ in low $s$ region is a larger negative $D_{\mu\mu}^{LR}$, which also can be found in Fig. 3(a). In high $s$ region, as Fig. 3 (b) shows, $A_{FB}$ could be reduced significantly and enhanced a bit by $Z'$ contributions.

Based on the analyses above, in order to evaluate the exact strength of $Z'$ effects, our following analyses can be divided into three limiting scenarios:

**Scenario I**

In order to get the maximum $\mathcal{B}(\bar{B}_{s} \rightarrow \phi \mu^{+}\mu^{-})$, within the allowed ranges for $Z'$ couplings listed in Table 3, we choose a set of extreme values.

$$|B_{s}\bar{b}| = 1.31 \times 10^{-3}, \phi_{s} = -79^\circ, S_{\mu\mu}^{LR} = -6.7 \times 10^{-2}, D_{\mu\mu}^{LR} = -9.3 \times 10^{-2}$$

named Scenario I. With the central values of the other theoretical input parameters, we get $\mathcal{B}(\bar{B}_{s} \rightarrow \phi \mu^{+}\mu^{-}) = 2.86 \times 10^{-6}$, which is 2.5$\sigma$ larger than CDF result $(1.44 \pm 0.57) \times 10^{-6}$. Compared with the SM prediction $1.46 \times 10^{-6}$, we find $\mathcal{B}(\bar{B}_{s} \rightarrow \phi \mu^{+}\mu^{-})$ could be enhanced by about 96% at most by $Z'$ contributions.

This scenario is the most helpful solution to moderate the discrepancy for $A_{FB}(\bar{B}_{s} \rightarrow K^{*}\mu^{+}\mu^{-})$ between SM prediction and experimental data in low $s$ region [13, 14]. As Fig. 3 (a) shows, we find Scenario I also provides the most helpful solution to enhance $A_{FB}(\bar{B}_{s} \rightarrow \phi \mu^{+}\mu^{-})$ in low $s$ region. Compared with the SM results, we find $A_{FB}^{(L)}(\bar{B}_{s} \rightarrow \phi \mu^{+}\mu^{-})$ could be enhanced by about 17%(133.3%) at most. However, in the high $s$ region, the effect of Scenario I on $A_{FB}(\bar{B}_{s} \rightarrow \phi \mu^{+}\mu^{-})$, as Fig. 3 (b) shows, is not significant.

In addition, due to the strong constraints on $D_{\mu\mu}^{LR}$ from $\bar{B}_{d} \rightarrow X_{s}\mu\mu$ decay, the much larger value $|D_{\mu\mu}^{LR}| > 9.3 \times 10^{-2}$ is forbidden [12], which means the sign of Re($C_{7}^{\text{eff}}C_{10}^{\text{eff}}$*) can hardly be flipped by $Z'$ contributions [13]. So, as Fig. 1 (b) shows, the zero crossing in $A_{FB}$ spectrum also exists and moves to $s_{0} \sim 1GeV^{2}$ point in this scenario.

**Scenario II**

From Fig. 2, one may find that $\mathcal{B}(\bar{B}_{s} \rightarrow \phi \mu^{+}\mu^{-})$ can hardly be reduced by $Z'$ contributions so much within the allowed $Z'$ parameters’ ranges. The most minimal value of $\mathcal{B}(\bar{B}_{s} \rightarrow \phi \mu^{+}\mu^{-})$
appears at

\[ |B_{sb}^L| = 1.31 \times 10^{-3}, \phi_s^L = -65^\circ, S_{\mu\mu}^{LR} = -2 \times 10^{-2}, D_{\mu\mu}^{LR} = -4 \times 10^{-2} \quad \text{Scen. II}, \]

named Scenario II. In this scenario, compared with SM prediction, we find \( B(B_s \rightarrow \phi\mu^+\mu^-) \) could be reduced just by about 14% at most by \( Z' \) contributions. Due to the small \( Z' \) contributions, its effect on \( A_{FB}(B_s \rightarrow \phi\mu^+\mu^-) \) is also tiny.

**Scenario III**

As Fig. 3 (b) shows, \( A_{FB}(\bar{B}_s \rightarrow \phi\mu^+\mu^-) \) would be reduced rapidly in high \( s \) region when \( S_{\mu\mu}^{LR} \) is enlarged. So, we present a limiting scenario for the minimal \( A_{FB}(\bar{B}_s \rightarrow \phi\mu^+\mu^-) \),

\[ |B_{sb}^L| = 1.31 \times 10^{-3}, \phi_s^L = -65^\circ, S_{\mu\mu}^{LR} = 1.1 \times 10^{-2}, D_{\mu\mu}^{LR} = -9.3 \times 10^{-2} \quad \text{Scen. III}, \]

named Scenario III. Compared with SM prediction, \( A_{FB}^{(H)}(\bar{B}_s \rightarrow \phi\mu^+\mu^-) \) is reduced by about 62% (62%). However, as Fig. 3 (b) shows, in the low \( s \) region, \( A_{FB} \) is just enhanced a bit. So, this scenario also leads to the minimal \( A_{FB}(\bar{B}_s \rightarrow \phi\mu^+\mu^-) \sim 8.9\% \), which is 65% smaller than SM prediction. While, in this scenario, our prediction \( B(\bar{B}_s \rightarrow \phi\mu^+\mu^-) = 1.92 \times 10^{-6} \) also agrees with CDF measurement within 1\( \sigma \). So, although Scenario III presents a strange effects on \( A_{FB} \) spectrum, it is not excluded by current measurement either. Moreover, different from Scenario I, zero crossing in \( A_{FB} \) spectrum moves to positive side in this scenario.

## 4 Conclusion

In conclusion, motivated by recent measurement on \( B(\bar{B}_s \rightarrow \phi\mu^+\mu^-) \) by CDF Collaboration, after revisiting \( \bar{B}_s \rightarrow \phi\mu^+\mu^- \) decay within SM, we have investigated the effects of a family non-universal \( Z' \) boson with the given \( Z' \) couplings. Our conclusions can be summarized as:

- Branching fraction and forward-backward asymmetry for \( \bar{B}_s \rightarrow \phi\mu^+\mu^- \) decay are sensitive to \( Z' \) contributions. All of the \( Z' \) couplings listed in Table 3 survive under the constraint from \( B(\bar{B}_s \rightarrow \phi\mu^+\mu^-) \) measured by CDF within errors.

- We present three limiting scenarios: \( B(\bar{B}_s \rightarrow \pi^-K^+) \) and \( A_{FB}^{(L)}(\bar{B}_s \rightarrow \phi\mu^+\mu^-) \) could be enhanced by about 96% and 17% (133%) at most by \( Z' \) contributions (Scenario I);
However, $\mathcal{B}(B_s \to \pi^- K^+)$ is hardly to be reduced (reduced by 14% at most in Scenario II) by $Z'$ contributions; Moreover, in Scenario III, $A^{(H)}_{FB}(\bar{B}_s \to \phi \mu^+ \mu^-)$ reaches its minimal value, which is 65%(62%) lower than SM prediction.

- The zero crossing in $A_{FB}(\bar{B}_s \to \phi \mu^+ \mu^-)$ spectrum always exists in the three scenarios.

The refined measurements for the $B_s$ leptonic decay $\bar{B}_s \to \phi \mu^+ \mu^-$ in the upcoming LHCb and proposed super-B will provide a powerful testing ground for the SM and possible NP scenarios. Our analyses of the $Z'$ effects on the observables for $\bar{B}_s \to \phi \mu^+ \mu^-$ decay are useful for probing or refuting the effects of a family non-universal $Z'$ boson.

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**Appendix A: Theoretical input parameters**

For the CKM matrix elements, we adopt the UTfit collaboration’s fitting results [28]

\[
\begin{align*}
\rho &= 0.132 \pm 0.02 (0.135 \pm 0.04), \quad \eta = 0.367 \pm 0.013 (0.374 \pm 0.026), \\
A &= 0.8095 \pm 0.0095 (0.804 \pm 0.01), \quad \lambda = 0.22545 \pm 0.00065 (0.22535 \pm 0.00065). 
\end{align*}
\] (18)

As for the quark masses, we take [29, 30]

\[
\begin{align*}
m_u = m_d = m_s &= 0, \quad m_c = 1.61^{+0.08}_{-0.12} \text{ GeV}, \\
m_b &= 4.79^{+0.19}_{-0.08} \text{ GeV}, \quad m_t = 172.4 \pm 1.22 \text{ GeV}. 
\end{align*}
\] (19)

**Appendix B: Transition form factors from light-cone QCD sum rule**

In order to calculate the $\bar{B}_s \to \phi \ell^+ \ell^-$ decay amplitude, we have to evaluate the $\bar{B}_s \to \phi$ matrix elements of quark bilinear currents. They can be expressed in terms of ten form factors, which depend on the momentum transfer $q^2$ between the $B_s$ and the $\phi$ mesons ($q = p - k$) [23]:
Table 4: Fit parameters for $B_s \rightarrow \phi$ transition form factors [23].

|        | $F(0)$ | $r_1$ | $m^2_R$ | $r_2$ | $m^2_{fit}$ |        |
|--------|--------|-------|---------|-------|-------------|--------|
| $V^{B_s \rightarrow \phi}$ | 0.434  | 1.484 | 5.32$^2$ | -1.049 | 39.52       | Eq. (22) |
| $A_0^{B_s \rightarrow \phi}$ | 0.474  | 3.310 | 5.28$^2$ | -2.835 | 31.57       | Eq. (22) |
| $A_1^{B_s \rightarrow \phi}$ | 0.311  | —     | —       | 0.308  | 36.54       | Eq. (24) |
| $A_2^{B_s \rightarrow \phi}$ | 0.234  | -0.054| —       | 0.288  | 48.94       | Eq. (23) |
| $T_1^{B_s \rightarrow \phi}$ | 0.349  | 1.303 | 5.32$^2$ | -0.954 | 38.28       | Eq. (22) |
| $T_2^{B_s \rightarrow \phi}$ | 0.349  | —     | —       | 0.349  | 37.21       | Eq. (24) |
| $\tilde{T}_3^{B_s \rightarrow \phi}$ | 0.349  | 0.027 | —       | 0.321  | 45.56       | Eq. (23) |

\[
\langle \phi(k)|\bar{d}\gamma_\mu(1 - \gamma_5)b|\bar{B}_s(p)\rangle = -ie_\mu(m_{B_s} + m_\phi)A_1(q^2) + i(2p - q)_\mu(\epsilon^* \cdot q) \frac{A_2(q^2)}{m_{B_s} + m_\phi} + iq_\mu(\epsilon^* \cdot q) \left[ \frac{2m_\phi}{q^2} - A_3(q^2) - A_0(q^2) \right] + \epsilon_{\mu\nu\rho\sigma}\epsilon^{\nu\rho} p^\sigma k^\rho \frac{2V(q^2)}{m_{B_s} + m_\phi},
\]

(20)

with $A_3(q^2) = \frac{m_{B_s} + m_\phi}{2m_\phi} A_1(q^2) - \frac{m_{B_s} - m_\phi}{2m_\phi} A_2(q^2)$ and $A_0(0) = A_3(0)$,

\[
\langle \phi(k)|\bar{s}\sigma_{\mu\nu} q^\nu(1 + \gamma_5)b|\bar{B}_s(p)\rangle = i\epsilon_{\mu\nu\rho\sigma}\epsilon^{\nu\rho} p^\sigma k^\rho 2T_1(q^2) + T_2(q^2)\left[ \epsilon_\mu(m_{B_s} - m_\phi^2) - (\epsilon^* \cdot q)(2p - q)_\mu \right] + T_3(q^2)(\epsilon^* \cdot q) \left[ q_\mu - \frac{q^2}{m_{B_s}^2 - m_\phi^2} (2p - q)_\mu \right],
\]

(21)

with $T_1(0) = T_2(0)$. $\epsilon_\mu$ is the polarization vector of the $\phi$ meson. The physical range in $s = q^2$ extends from $s_{\text{min}} = 0$ to $s_{\text{max}} = (m_{B_s} - m_\phi)^2$.

These transition form factors have been updated recently within the light-cone QCD sum rule approach [23]. For the $q^2$ dependence of the form factors, they can be parameterized in terms of simple formulae with two or three parameters. The form factors $V$, $A_0$ and $T_1$ are parameterized by

\[
F(s) = \frac{r_1}{1 - s/m^2_R} + \frac{r_2}{1 - s/m^2_{fit}}.
\]

(22)
For the form factors $A_2$ and $\tilde{T}_3$, it is more appropriate to expand to the second order around the pole, yielding

$$F(s) = \frac{r_1}{1 - s/m^2} + \frac{r_2}{(1 - s/m^2)^2}, \quad (23)$$

where $m = m_{\text{fit}}$ for $A_2$ and $\tilde{T}_3$. The fit formula for $A_1$ and $T_2$ is

$$F(s) = \frac{r_2}{1 - s/m_{\text{fit}}^2}. \quad (24)$$

The form factor $T_3$ can be obtained through the relation $T_3(s) = \frac{m^2_B - m^2_\phi}{s} [\tilde{T}_3(s) - T_2(s)]$. All the relevant fitting parameters for these form factors are taken from Ref. [23] and are recollected in Table 4.

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