The proton spin and the Wigner rotation

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Abstract

It is shown that in both the gluonic and strange sea explanations of the Ellis-Jaffe sum rule violation discovered by the European Muon Collaboration (EMC), the spin of the proton, when viewed in its rest reference frame, could be fully provided by quarks and antiquarks within a simple quark model picture, taken into account the relativistic effect from the Wigner rotation.

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The European Muon Collaboration (EMC) experiment on deep inelastic scattering of polarized leptons from polarized protons has received attention by the particle physics community recently. The reason is that the result of the integrated spin-dependent structure function data is significantly smaller than that expected from the Ellis-Jaffe sum rule, and that in a naive interpretation of this small result one is led to the conclusion that the fraction of the proton spin carried by quarks and antiquarks is smaller than expected. There are essentially two theoretical aspects for understanding this startling EMC result. The first concerns the mechanism for the violation of the Ellis-Jaffe sum rule (EJSR), which is obtained by using the experimental values of $G_A/G_V$ measured in weak semileptonic decays together with flavor SU(3) symmetry results and an additional assumption of a negligible strange sea contribution to the proton spin. Many theoretical speculations have been proposed to explain the EJSR violation, such as to attribute the small EMC result to gluonic contributions due to the U(1) axial anomaly, non-vanishing strange quark contributions, consequences of the QCD anomaly and the spontaneous breaking of the chiral symmetry, the unjustified $x \to 0$ extrapolation of the data, flavor SU(3) symmetry breaking, and non-perturbative QCD effects at low $Q^2$. This paper is not intended to discuss the aspect concerning the EJSR violation. Our attention is focused on the second issue concerning the EMC result: the small data triggered the proton “spin crisis”, i.e., the intriguing question of how the spin of the proton is distributed among its quark spin, gluon spin and orbital angular momentum. At present it is commonly taken for granted that the EMC result implies that there must be some contribution due to gluon polarization or orbital angular momentum to the proton spin. For example, in the gluonic and strange sea explanations of the EJSR breaking, the proton spin carried by the spin of quarks and antiquarks was estimated to be of about 70% in the former and negligible in the latter. We will show, however, that the above understandings are not in contradiction with a simple quark model picture in which the spin of the proton, when viewed in its rest ref-
erence frame, is fully provided by the vector sum of the spin of quarks and antiquarks.

The key points for understanding the proton spin puzzle lie in the facts that the vector sum of the constituent spin for a composite system is not Lorentz invariant by taking into account the relativistic effect from the Wigner rotation [14], and that it is in the infinite momentum frame the small EMC result was interpreted [1, 7] as an indication that quarks and antiquarks carry a small amount of the total spin of the proton. From the first fact we know that the vector spin structure of hadrons could be quite different in different frames from the relativistic viewpoint. We thus can understand the proton “spin crisis”, because there is no need to require that the sum of the spin of quarks and antiquarks be equal to the proton spin in the infinite momentum frame, even if the vector sum of the spin of quarks and antiquarks equals to the proton spin in the rest frame. In fact this idea has already been presented [15] in a light-cone language in which the effect of the Wigner rotation manifests itself as the effect of the Melosh rotation [16]. It was shown that the small EMC result could be naturally reproduced within the SU(6) naive quark model by taking into account the effect of the Melosh rotation without contributions due to gluon spin and orbital angular momentum. The validity of the Bjorken sum rule [17] may be also retained if one imposes some further constraints on the flavor distribution of quarks in the proton. However, the work of Ref. [15] consider neither the weak hyperon decay data used in previous analyses nor the mechanism that breaks the EJSR. The analysis was also presented in a particular (though most convenient) language which might be considered as a model-dependent framework with no universal significance. Therefore we need to re-analyze this issue in conventional languages.

We now explain why the vector sum of the constituent spin for a composite system is not Lorentz invariant. It should be kept in mind that the notion of spin is essentially a relativistic notion associated with the space-time symmetry group of Poincaré [14]. In the relativistic dynamical theory of the quantum system [18], measurable physical quantities are closely related
to the ten generators of the Poincaré group. The explicit representation of

generators relies on the form of dynamics\[18, 19\]. In the conventional instant

dynamics, the generators $P^\mu = (H, \vec{P})$ of the space-time translations

have the physical significance of energy and momentum, and the generators

$J^{\mu\nu}$ of the infinitesimal Lorentz transformations are related to the angular

momentum $J^k$ by $J^k = (1/2)\epsilon_{ijk}J^{ij}$ and the boost generator $K^k$ by $K^k = J^{k0}$.

The conventional 3-vector spin $\vec{s}$ of a moving particle with finite mass $m$ and

4-momentum $p_\mu$ can be defined by transforming its Pauli-Lubanski 4-vector

$w_\mu = (1/2)J_{\rho\sigma}p^\nu\epsilon_{\nu\rho\sigma\mu}$ \[20\] to its rest frame via a rotationless Lorentz boost

$L(p)$, which satisfies $L(p)p = (m, \vec{0})$, by $(0, \vec{s}) = L(p)w/m$ \[21\]. Under an

arbitrary Lorentz transformation, a particle of spin $\vec{s}$ and 4-momentum $p_\mu$

will transform to the state of spin $\vec{s}'$ and 4-momentum $p'_\mu$ by

$$\vec{s}' = \Re_w(\Lambda, p)\vec{s}, \quad p' = \Lambda p,$$

(1)

where $\Re_w(\Lambda, p) = L(p')\Lambda L^{-1}(p)$ is a pure rotation known as the Wigner

rotation. For simplicity we assume that the proton is a composite system of

moving quarks and antiquarks and that the proton spin is fully provided by

the spin of quarks and antiquarks in the proton rest frame. When the proton

is boosted, via a rotationless Lorentz transformation along its spin direction,

from the rest frame to a frame where the proton is moving, each quark

spin will undergo a Wigner rotation, and these spin rotations may produce,

arising from the relativistic effect due to internal quark motions, a significant

change in the vector sum of the spin of quarks and antiquarks. The proton

spin, however, remains the same as that in the rest frame according to the

spin definition. In consequence the vector sum of the spin of quarks and

antiquarks may differ non-trivially from the proton spin in the new frame.

It then becomes necessary to clarify what is meant by the quantity $\Delta q$

defined by $\Delta q \cdot S_\mu = \langle P, S | \bar{q} \gamma_\mu \gamma_5 q | P, S \rangle$, where $S_\mu$

is the proton polarization vector. $\Delta q$ can be calculated from $\Delta q = \langle P, S | \bar{q} \gamma^+ \gamma_5 q | P, S \rangle$, as the

instantaneous fermion lines do not contribute to the $+$ component\[22\]. One

can easily prove, by expressing the quark wave functions in terms of light-
cone Dirac spinors\[23\] (i.e., the quark spin states in the infinite momentum
\[
\Delta q = \int_0^1 dx \left[ q^\uparrow(x) - q^\downarrow(x) \right],
\]

(2)

where \( q^\uparrow(x) \) and \( q^\downarrow(x) \) are probabilities of finding, in the proton infinite momentum frame, a quark or antiquark of flavor \( q \) with fraction \( x \) of the proton longitudinal momentum and with polarization parallel and antiparallel to the proton spin, respectively. However, if one expresses the quark wave functions in terms of conventional instant form Dirac spinors (i.e., the quark spin states in the proton rest frame, with the normalization condition \( \bar{u}\gamma^+ u = -\bar{v}\gamma^+ v = 1 \)), it can be found, that

\[
\Delta q = \int d^3\vec{p} M_q \left[ q^\uparrow(p) - q^\downarrow(p) \right] = \langle M_q \rangle \Delta q_L,
\]

(3)

with

\[
M_q = \left[ (p_0 + p_3 + m)^2 - \vec{p}_\perp^2 \right] / [2(p_0 + p_3)(m + p_0)]
\]

(4)

being the contribution from the relativistic effect due to quark transversal motions, \( q^\uparrow(p) \) and \( q^\downarrow(p) \) being probabilities of finding, in the proton rest frame, a quark or antiquark of flavor \( q \) with rest mass \( m \) and momentum \( p_\mu \) and with spin parallel and antiparallel to the proton spin respectively, and \( \Delta q_L = \int d^3\vec{p} \left[ q^\uparrow(p) - q^\downarrow(p) \right] \) being the net spin vector sum of quark flavor \( q \) parallel to the proton spin in the rest frame. Thus one sees that the quantity \( \Delta q \) should be interpreted as the net spin polarization in the infinite momentum frame if one properly considers the relativistic effect due to internal quark transversal motions.

From the above considerations, one naturally reaches the conclusion that the spin structure of a composite system should be defined in the rest frame of the system with internal constituent motions also taken into account. Thereby we can understand the “spin crisis”, simply because the quantity \( \Delta \Sigma = \Delta u + \Delta d + \Delta s \) represents, in a strict sense, the sum of the quark helicity (or longitudinal-component spin) in the infinite momentum frame rather than the vector sum of the spin carried by quarks and antiquarks in the proton rest frame. It is possible that the value of \( \Delta \Sigma = \Delta u + \Delta d + \Delta s \)
is small whereas the simple spin sum rule

\[ \Delta u_L + \Delta d_L + \Delta s_L = 1 \] (5)

still holds. By eq.(5) we mean a simple quark model picture in which the proton spin is fully provided by quarks and antiquarks in the proton rest frame. The gluon spin and the orbital angular momentum may contribute to the proton spin in the rest frame. However, one may reasonably speculate that the proton, being the most stable hadron in the real world, can be described in the simple quark model picture with the vector sum of the spin of quarks and antiquark being the proton spin. We will show that this philosophy could be apply to both the gluonic [6] and strange sea [7] explanations of the EJSR breaking, though the real situation might be complicated.

Theoretically, the integrated spin-dependent structure function of the proton is related through short-distance expansion to the flavor isotriplet, octet and singlet components of the axial vector matrix elements, \( \Delta q^3 \), \( \Delta q^8 \) and \( \Delta q^0 \), by the relation[2,3]

\[
\int_0^1 dx g_1^p(x) = \frac{1}{12}\Delta q^3 + \frac{1}{36}\Delta q^8 + \frac{1}{9}\Delta q^0.
\] (6)

The two non-singlet axial charges, \( \Delta q^3 \) and \( \Delta q^8 \), can be inferred by [7]

\[
\Delta q^3 = \Delta u - \Delta d = G_A/G_V = 1.261
\] (7)

from neutron decay [24] plus isospin symmetry, and by

\[
\Delta q^8 = \Delta u + \Delta d - 2\Delta s = 0.675
\] (8)

from strangeness-changing hyperon decays [25] plus flavor SU(3) symmetry. Prior to the EMC experiment, the flavor singlet (or the U(1)) axial charge was evaluated by Ellis-Jaffe and Gourdin[3], suggesting \( \Delta s = 0 \), to be \( \Delta q^0 = \Delta \Sigma = \Delta u + \Delta d + \Delta s = \Delta q^8 \). Then one obtains, neglecting small QCD corrections, the EJSR \( \int_0^1 dx g_1^p(x) = 0.198 \), a value which is significantly larger than the revised EMC result \( \int_0^1 dx g_1^p(x) = 0.126 \).

A possible explanation of this discrepancy was proposed [3] by adding to the “naive” U(1) charge, \( \Delta \Sigma = \Delta u + \Delta d + \Delta s \), a gluonic contribution
due to the Adler-Bell-Jackiw anomaly, \( \Delta q^0 = \Delta \Sigma - (\alpha_s/2\pi)n_f \Delta g \), where \( \Delta q^0 \) is the physical U(1) charge used in the Ellis-Jaffe relation eq.(6), \( \alpha_s \) is the QCD coupling constant, and \( n_f \) is the number of excited flavors. It is expected that \( \Delta \Sigma \approx \Delta q^8 \) and that the gluonic contribution, \( (\alpha_s/2\pi)n_f \Delta g \), nearly compensates \( \Delta \Sigma \). Then one obtains, combining eqs.(7),(8) and eq.(6) inferred by the EMC result, that

\[
\Delta u = 0.968, \quad \Delta d = -0.293, \quad \Delta s = 0, \tag{9}
\]

and

\[
\Delta \Sigma = \Delta u + \Delta d + \Delta s = 0.675. \tag{10}
\]

This result was interpreted \([6, 13]\) as an implication that a large fraction \((\Delta \Sigma \approx 70\%)\) of the proton spin is carried by the spin of quarks and antiquarks balanced by a sizable contribution \((1 - \Delta \Sigma \approx 30\%)\) arising from the gluon spin and/or from orbital angular momentum. An alternative explanation of the small EMC result was proposed \([7]\), based on the Skyrme model, by still attributing \( \Delta q^0 = \Delta \Sigma \) but suggesting a large \( \Delta s \). Combining eqs.(7), (8) and eq.(6) inferred by the EMC result, one obtains

\[
\Delta u = 0.750, \quad \Delta d = -0.511, \quad \Delta s = -0.218, \tag{11}
\]

and

\[
\Delta \Sigma = \Delta u + \Delta d + \Delta s = 0.020. \tag{12}
\]

This result was interpreted \([7]\) as an implication that most of the proton spin is due to gluons and/or orbital angular momentum. These interpretations of the proton spin structure triggered some further studies \([5, 26]\) about how the proton spin is distributed among the spin and orbital angular momentum of its quarks, antiquarks and gluons.

We now show that in both of the above explanations of the EJSR violation the spin of the proton could be fully provided by the vector sum of the spin of quarks and antiquarks in the simple quark model picture based on our above definition of the spin structure of a composite system and above clarification of the physical implication of the quantity \( \Delta q \). Since \( <M_q> \), the
average contribution from the relativistic effect due to internal transversal motions of quark flavor $q$, ranges from 0 to 1, and $\Delta q_L$, the net spin vector polarization of quark flavor $q$ parallel to the proton spin in the proton rest frame, is related to the quantity $\Delta q$ by the relation $\Delta q_L = \Delta q/ <M_q>$, we have sufficient freedom to make the simple quark model spin sum rule, i.e., $\Delta u_L + \Delta d_L + \Delta s_L = 1$, satisfied while still preserving the values of $\Delta u$, $\Delta d$ and $\Delta s$, i.e., eqs.(9) and (11), in the two explanations, respectively. In the gluonic explanation, we could choose $\Delta u_L = 4/3$, $\Delta d_L = -1/3$, and $\Delta s_L = 0$ as those in the most simply SU(6) configuration of the naive quark model, then we need $<M_u> \approx <M_d> \approx 0.7$ to preserve eq.(9). It can be found that we could reproduce such $<M_u>$ and $<M_d>$ by using a quark mass $m \approx 330$ MeV and $\sqrt{p_{\perp}^2} \approx 330$ MeV in the constituent quark model framework. In the strange sea explanation, the non-vanishing $\Delta s$ reflects polarizations of sea thus some number of sea quarks should be introduced. Then one has large freedom to choose arbitrary $\Delta u_L$, $\Delta d_L$, and $\Delta s_L$ constrained by the simple quark model spin sum rule eq.(5) while still preserving eq.(11). In both of the above cases the proton spin is fully provided by the vector sum of the spin of quarks and antiquarks.

We need to clarify the seeming contradiction between the statements in this paper and those in much of the literature about the fraction of the proton spin carried by quarks and antiquarks. The difference is arising from the definitions of spin. In this paper we refer the spin of a moving quark to the conventional 3-vector spin defined by transforming its Pauli-Lubánski 4-vector to its rest frame. Thus a quark or antiquark, when simply described by a conventional instant form Dirac spinor (i.e., $u^{\uparrow\downarrow}(p)$ for quark and $v^{\uparrow\downarrow}(p)$ for antiquark), provides 1/2 net spin contribution parallel or antiparallel to the proton spin. Whereas in the literature the spin operator of quarks is referred to $\bar{\psi}\gamma_{\mu}\gamma_5\psi$, which is essentially the non-conserved axial vector current. From this definition a quark state expressed by $u^{\uparrow\downarrow}(p)$ or $v^{\uparrow\downarrow}(p)$ contributes only a value of $(1/2)M_q$ net spin contribution parallel or antiparallel to the proton spin. The reason for this reduction of spin contribution can be ascribed to a negative spin contribution from the lower
component of the Dirac spinor if the quark transversal motions are considered. However, the lower component is considered to have again a positive orbital angular momentum in this situation, and in sum the total contribution (i.e., spin+orbital angular momentum) from this quark state to the proton spin is the same as that of the 3-vector spin defined in our paper. Thus the “spin crisis” could be understood within the simple quark model picture by considering the relativistic effect from internal quark transversal motions.

We have made many simplifications in the above discussions of the proton spin problem. The most important simplification is that we treat the Wigner rotation in a free quark approximation and do not consider any dynamical effect due to quark interactions in the boost from the proton rest frame to the infinite momentum frame. However, the inclusion of dynamical effects can only change the results (e.g., the explicit expression of $M_q$) quantitatively. It should not affect the qualitative conclusions, because the effects from kinematics should be first considered before the introduction of other dynamical effects. We are still far from the answer to how the proton spin is distributed among the spin and orbital angular momentum of quarks, antiquarks and gluons. We only provide two toy cases in a simple quark model picture in which the proton spin is fully provided by quarks and antiquarks with the EMC result of the proton spin-dependent structure function also satisfied. For further understanding of the proton spin structure we need more theoretical and experimental works.

In summary, we discussed the Wigner rotation in the spin structure of a composite system. We showed that the proton spin puzzle caused by the EMC polarized muon proton data could be understood within a simple quark model picture in which the proton spin, when viewed in its rest reference frame, is fully provided by quarks and antiquarks, taking into account the relativistic effect due to internal quark transversal motions.
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