Inventory Management for Irregular Shipment of Goods in Distribution Centre

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Abstract. The shipping amount of commodity goods (Foods, confectionery, dairy products, such as public cosmetic pharmaceutical products) changes irregularly at the distribution center dealing with the general consumer goods. Because the shipment time and the amount of the shipment are irregular, the demand forecast becomes very difficult. For this, the inventory control becomes difficult, too. It cannot be applied to the shipment of the commodity by the conventional inventory control methods. This paper proposes the method for inventory control by cumulative flow curve method. It proposed the method of deciding the order quantity of the inventory control by the cumulative flow curve. Here, it proposes three methods. 1) Power method, 2) Polynomial method and 3) Revised Holt’s linear method that forecasts data with trends that is a kind of exponential smoothing method. This paper compares the economics of the conventional method, which is managed by the experienced and three new proposed methods. And, the effectiveness of the proposal method is verified from the numerical calculations.

1. Introduction
The distribution center only ship merchandise after they receive direct orders from customers, who may be wholesalers, supermarkets or large-scale stores. The wholesalers then ship merchandise to retailers, who in turn sell the merchandise to consumers. The supermarkets and other large-scale stores, on the other hand, sell the merchandise directly to consumers. Generally, the sale of merchandise directly to ultimate consumers fluctuates widely according to the characteristics of the merchandise. For this reason, the amounts shipped from distribution center to their customers and the times of shipment also vary greatly. The commodity goods (Foods, confectionery, dairy products, such as public cosmetic pharmaceutical products) does an irregular changes at the distribution center dealing with the general consumer goods. Therefore, the forecast of the shipment time and the shipment volume are difficult. It can not be applied to the shipment of the commodity by the...
conventional inventory management techniques. For this reason, we have proposed a method of performing with the cumulative flow graph [1][2]. The irregular shipment characteristics of goods, we have proposed a theoretical formula that determines the order timing and order quantity. The cumulative inflow corresponds to the order quantity in inventory management. Here, it proposes three methods. 1) Power method, 2) Polynomial method[3] and 3) Revised Holt’s linear method that forecasts data with trends that is a kind of exponential smoothing method. This paper compares the economics of the conventional method, which is managed by the experienced and three new proposed methods. And, the effectiveness of the proposal method is verified for the real distribution center.

2. Cumulative flow curve
Distribution center ship merchandise in direct response to orders from customers, whether wholesalers, supermarkets or other large-scale stores. The merchandise shipped from the distribution centers to wholesalers is then shipped by the wholesalers to retailers, as shown in Figure 1. Then the merchandise is sold by retailers to consumers. The merchandise shipped from distribution centers to supermarkets and other large-scale stores is sold directly to consumers at the supermarkets and those large-scale stores. Movement of warehouses and distribution center can be expressed by the cumulative flow curve. The cumulative flow curve is composed of the inflow, the outflow, and the processing line of the distribution center. Movement of the product can be represented by the inflow and outflow as shown in Figure 2[4][5]. Cumulative flow curve is a graph obtained by the cumulative value of the inflow and outflow of goods. Vertical interval of the cumulative line becomes the inventory volume and horizontal line becomes the lead time[6].

![Figure 1. Relation between inflow and outflow in distribution center](image1)

![Figure 2. Cumulative flow curve at distribution center](image2)

![Figure 3. Amount of inventory to be changed every month](image3)

Figure 2 shows a state in which store them in stock products in distribution centers there. Figure 2 shows cumulative flow curve by orders from customers, and are shipping the goods from distribution centers. Figure 2 also shows that the inventory quantity $Q_a$ are increasingly July and August. And, inventory quantity $Q_b$ has become less in October. Figure 3 shows the inventory quantity calculated from the inflow and outflow of the cumulative flow curve. Inventory levels in Figure 3 is changed to ship the order from the customer on a monthly. In the distribution center, processing time of machine transport number of forklift, man-hours of loading and unloading work, such as conveyor come determined by the amount of stock[7].
3. Characteristic of shipment data
The amount and the time of the products shipped from the distribution center to the supermarket, the large-scale retail stores, and the wholesale make a big change. The reason for change is because the direct product to ship is intended for consumer food, confectionery, dairy products, cosmetics, and pharmaceuticals. In the distribution center, product management, inventory management, shipment planning, delivery vehicle planning, labor management is difficult as compared to the production plant for the change of the product sales of shipping to large. Production planning and inventory control in the production plant can easily control for planning and management since the fluctuation is little[3]. There are various patterns in shipping time by customer. In one pattern, the time of shipments is concentrated at the beginning of month, whereas shipment time focuses at the end of a month in another pattern. In still another type of patterns, shipments are made almost every day. The shipment amount patterns also vary in a similar way to the shipping time patterns: large volume may be concentrated at the beginning of a month or at the end of month, or they can be almost same every day or differ every day. The shipment patterns The shipping pattern can be classified as follows. 1)There are shipments every day, shipments to the normal distribution. 2)Shipment shall be concentrated at the beginning of a month. 3)There is a ship at the end of the month. 4)Those to be shipped to day irregular. 5)There is a shipping amount on the day of at the beginning of a month, shipment shall disappear from the end of the month. 6)There are shipments amount at the end of the month, that there is no ship of a beginning month. 7)The various combinations of shipping these patterns exist. Inventory management of goods with shipping such properties is difficult and is not available, such as reorder point method in the past. Therefore, the method for inventory control, shipment management, storage management using the cumulative flow curves in the distribution center is proposed.

4. Relationship of cumulative inflow and cumulative outflow
Cumulative flow curve is made from the cumulative outflow and cumulative inflow as shown in Figure 4. In Figure 4, the top line of the graph represents the cumulative inflow, the lower line indicates the cumulative outflow. The vertical axis in figure 4 shows the amount of stock. The horizontal axis shows the date of arrival of goods. R1, R2, and R3 are the volume of inventories. Q1, Q2, and Q3 are the amounts of arrival of goods that the distribution center ordered the supplier. The t1 of the horizontal axis represents the day of order quantity Q1 is in-stock.

![Figure 4](image.png)

**Figure 4.** The cumulative inflow and outflow curve and the inventory
The following relations are approved between volume of inventories R and amount Q of the order.

\[
\begin{align*}
R_2 - R_1 &= Q_1 + R_3 - R_2 \\
R_4 - R_3 &= Q_3 + R_4 - Q_2
\end{align*}
\]

(1)

The difference between the two graphs of the upper cumulative inflow and lower cumulative outflow is to the volume of inventories. The inventory control in the distribution center becomes a problem of deciding amount Q of the order.
5. Cumulative flow curve of shipping characteristics

5.1. Cumulative shipments characteristics shipment characteristics

The data must be cumulative in order to draw the cumulative flow curves of the product for the irregular shipment characteristics. Cumulative flow curve can be controlled by the amount of quantity and timing of arrival[8]. However, it is not possible to control the shipment quantity and shipment time. Because this is an order from the customer, it is not possible to control freely. It is necessary to examine the pattern at the shipment amount and the shipment time according to the product. Three shipments of the kind of product and patterns at the shipment time are shown here as a typical example.

1) Figure 5 is a product it ships at the beginning of the month, and without the shipment until the end of the month. The shipment of the cumulative flow curve is shown in figure 6[8][9][10].

2) Figure 7 is a product it ships at the end of the month, and without the shipment until the end of the month. The shipment of the cumulative flow curve is shown in figure 8.

3) Figure 9 is a product regularly shipped. The shipment of the cumulative flow curve is shown in figure 10.

Figure 5. Shipping is concentrated in the beginning of the month. (Item 1)

Figure 6. Cumulative flow curve for figure 3

Figure 7. Shipping is concentrated in the end of the month. (Item 2)

Figure 8. Cumulative flow curve for figure 5

Figure 9. The product is evenly shipped (Item 3)

Figure 10. Cumulative flow curve for fig 7
5.2. Inventory control that uses cumulative flow curve in distribution center
The current state of the inventory control that uses cumulative flow curve at the distribution center is examined here. Figure 11 is cumulative flow curve of item I4. The volume of inventories at this time is shown in figure 12, and the total of the volume of inventories becomes 12549. Cumulative flow curve of item 5 is similarly shown to figure 13. The volume of inventories is shown in figure 14. The total of the volume of inventories becomes 16507. The judgment whether this volume of inventories is a lot or is little cannot be judged because there is no technique of a scientific inventory management. However, because the shipment time and the shipment are irregular. It becomes possible to stock a lot of inventory in the distribution center [8][9][10].

5.3. Decision of cumulative inflow curve
The inventory control is carried at the distribution center is a method by a past experience and intuition. In Figure 11 is the cumulative flow curve for item I4 and the Figure 12 is the volume of inventories. Figure 13 is the cumulative flow curve for item I5 and the Figure 14 is the volume of inventories. It is necessary to ship the amount (Outflow) of cumulative outflow by the order from the customer. Then, we can control at the distribution center is only an amount of the cumulative inflow from the vendor. The cumulative inflow (Inflow) Q shown in the upper row of figure 4 shows the amount of received of goods from the vendor. The problem is the method of deciding amount (Inflow) of the cumulative inflow curve in figure 4. It is necessary to decide the method how the time of received of goods and the amount of cumulative inflow. The volume of inventories can be reduced by efficiently doing the amount of received of goods and time of received of goods. The relation of the cumulative inflow amount > amount of the cumulative outflow is approved from figure 4. The volume of inventories becomes the difference between the amount of cumulative inflow and the amount of the cumulative outflow. It proposes a new method of controlling the amount of the cumulative inflow to reduce the volume of inventories. When the difference is reduced, the volume of inventories is minimized. To reduce the volume of inventories, it proposes a new method here about time and inlet

Figure 11. Cumulative flow curve of Item I4
Figure 12. Volume of inventories of Item I4.

Figure 13. Cumulative flow curve of Item I5
Figure 14. Inventory volume of Item I5

6. Cumulative inflow curve and volume of inventories
flow $Q$ of the order to the amount of the accumulation inflow, that is, the vendor. When this difference is reduced, the volume of inventories is minimized. To reduce the volume of inventories, it proposes a new method here about time and inlet flow $Q$ of the order to the amount of the accumulation inflow, that is, the vendor[11].

6.2. Inventory control by cumulative flow graph.
Cumulative flow graph puts out the order with $t01$ as shown in Figure 15. The commodity arrives to $tR1$. The lead time at this time becomes $tR1-t01$. However, the commodity handled here is a general consumption material. Therefore, it will be delivered the next day when ordering. Therefore, it is thought that there is no lead time. Amount $Q$ of the order is assumed to take the periodic reordering method. The reorder cycle is assumed to be $TC$. Amount $Q$ of the order will be decided with forecasting of the actual data. The forecast period becomes $TC$. The forecasting value is calculated for actual data of Figure 15. At this time, the safety stock $SS$ is put in the forecasting value or the safety stock is separately installed.

In this research paper, the method of demand forecasting to validate the two methods according to the Holts exponential smoothing method and power approximation, polynomial approximation [11]

6.3. Order quantity of cumulative inflow by power and polynomial method
The forecast period is assumed to be $TC$. The forecast shows period p cycle $TC$ by $n$. The predictive value becomes amount $Q$ of the order. Amount $Q$ of the order uses the forecast value by the exponential approximation and the polynomial approximation. The forecast time is assumed to be $x$. Stock shortage is caused when the forecast value is small. Therefore, the safety value $Po$ of the upper bound is given to the forecast value. The expression of the forecast is calculated by using the past data. It forecasts $p$ period ahead by the expression of the forecast of the past data of $n$ period. As a result, the forecast by the power and the polynomial forecast becomes the next expression [8][9][10].

$$\hat{x}(t + p) = A(t + p)^B + \rho \sigma$$
$$\hat{x}(t + p) = A_1 + A_2(t + p) + A_3(t + p)^2 + \rho \sigma$$ (2)

![Figure 16. Forecast for order quantity](image)
It applies to the commodity of Item3 shown in figure 9 to verify the proposed forecast method. The cumulative curve of Item3 becomes figure 10.

6.3.1 Numerical example 1

1) Power forecast method

It forecast to the shipment data of eight days in the past by the Power method. The expression of the forecast becomes as follows.

$$\hat{x}(t) = 15.843 \cdot t^{0.9282} + p\sigma$$

(3)

Order quantity adds and calculates safety stock $P\sigma$ in the expression of the forecast. As a result, order quantity became 82. Next, the prediction error occurs between a predictive value and actual value as shown in figure 17. Table 1 has shown the volume of inventories. The total of the volume of inventories becomes 158.

Table 1. Inventory and order quantity with power method

| week | Inventory | 8 | 9 | 10 |
|------|-----------|---|---|----|
| week |           | 82| 40| 16 |
| week |           | 11| 12|    |
| Inventory |       | 16| 4 | 158|

Figure 17. Power forecast of demand and order quantity

2) Polynomial forecast method

In the same way as the power method to calculate the forecast by polynomial to the shipment data of the past eight days. The expression of the forecast becomes as follows.

$$\hat{x}(t) = 8.5893 + 11.196 \cdot t + 0.1726 \cdot t^2 + p\sigma$$

(4)

Order quantity is calculated by adding the safety stock $P\sigma$ in the equation of prediction. As a result, order quantity became 108. Figure 17 has shown an expression of the forecast and actual data. Figure 17 has shown the relation between the safety stock and the volume of inventories when Order quantity is 100. Table 4 has shown the volume of inventories. The total volume of inventories becomes 218.

Table 2. Inventory and order quantity with polynomial method

| week | Inventory | 8 | 9 | 10 |
|------|-----------|---|---|----|
| week |           | 94| 52| 28 |
| week |           | 11| 12|    |
| Inventory |       | 28| 16| 218|
6.4. Order quantity of cumulative inflow by Improved Holt’s linear method

Linear exponential smoothing is found using two constants, $\alpha$ and $\beta$, and three equations. The equation is given as follow [8][9][10].

\[
\begin{align*}
L_t &= \alpha \cdot Y_t + (1 - \alpha) (L_{t-1} + b_{t-1}) \\
b_t &= \beta (L_t - L_{t-1}) + (1 - \beta) b_{t-1} \\
Q_{t+k} &= L_t + b_t k + Z_t k
\end{align*}
\]

$L_t$ denotes an estimate of the level and $b_t$ denotes a estimate of slope of the time series at time $t$. $L_t$ adjusts for the trend of previous period. $b_{t-1}$ by adding it to the last smoothed value, $L_{t-1}$. $b_t$ the updates the trend, which is expressed as the difference between the last two smoothed values. The trend is modified by smoothing with $\beta$ the trend in the last period ($L_t - L_{t-1}$), and adding that to the previous estimate of the trend multiplied by $(1 - \beta)$. Finally $Q_{t+k}$ is used to forecast ahead. This equation different for Holt’s linear method. The trend $b_t$ is multiplied by the number of periods ahead to forecast $k$, and added to the base value of $L_t$ [11].

6.4.1 numerical example 2

The predictor equation of Holt was calculated by using data here for eight weeks. Figure 13 shows the procedure for requesting the amount of the accumulation inflow from the forecast type of Holt of Item I1. The forecast value of $t=8$ and $k=4$ is calculated as $\alpha=0.8$ and $\beta=0.08$. The forecast value of the seventh week becomes $Q=67$. In graph, triangle indicates the predicted value round is in the actual value. Table 7 shows the volume of inventories at this time.

| Table 3. Inventory and order quantity with improved Holt’s method |
|-----------------|---|---|---|
| week | 8 | 9 | 10 |
| Inventory | 78 | 36 | 12 |
| week | 11 | 12 | Total |
| Inventory | 12 | 0 | 138 |
7. Comparison between current method and improvement method

The volume of inventories of three proposed methods and current methods are compared. Table 4 is a method by the experience of the current method. The volume of inventories is examined for five weeks. The volume of inventories in the stock control managed in the distribution center experiencing becomes 1180. Compared to other methods of Table 5, the current management methods is clear that the amount of inventory is large. The improved Holt’s method \([7][8]\) is the most excellent clearly from Table 5. Because other forecasting methods are used power and straight line, the demand that fluctuates not adapted. For this, the prediction error grows. Power method is more excellent than the polynomial method because there are a lot of quantity safety stocks. The volume of inventories changes by setting the safety stock. Therefore, it is difficult to judge which of these two methods is excellent.

Table 4. Inventory quantity for current method

| week | 8   | 9   | 10  |
|------|-----|-----|-----|
| Inventory | 275 | 233 | 209 |
| week | 11  | 12  | Total |
| Inventory | 239 | 227 | 1183 |

Table 5. Comparison between current method and proposed method

| Inventory quantity | Current method | Power method | Polynomial method | Improved Holt’s method |
|--------------------|----------------|--------------|-------------------|------------------------|
| 1183               | 158            | 218          | 138               |

8. Conclusions

Up to now, the method of theoretically computing the order quantity in the cumulative flow curve has not been proposed. This paper proposed the method of deciding the order quantity of the inventory control by the cumulative flow curve. Three proposed methods are shown in the calculation of the order quantity. The time and volumes of the shipments from the distribution center to the customers are irregular. With the cumulative flow curve, it is possible to inventory control of products that the irregular shipping. To provide useful forecasts of such irregular shipping times and volumes, a method utilizing the power, polynomial method and improved Holt’s method are presented. The volume of inventories of three proposed methods and current methods are compared. By applying the cumulative flow curves, it is clarified that the inventory control is possible for irregular shipment products.
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