Prime Cordial Labeling of Some Graphs
K. GAYATHRI, 
Research Scholar,
Periyar Maniammai Institute of Science & Technology,
Vallam, Thanjavur.
email: vcdulaganathan@gmail.com

Dr. A. SASIKALA, 
Associate Professor and Head,
Department of Mathematics,
Periyar Maniammai Institute of Science & Technology,
Vallam, Thanjavur.
email: sasikala@pmu.edu

Dr. C. SEKAR,
Associate Professor of Mathematics (Retd.),
Aditanar College of Arts & Science,
Tiruchendur, Tamilnadu.
email: sekar.acas@gmail.com

Abstract
A bijection \( f \) from the vertex set \( V \) of a graph \( G \) to \( \{1,2,...,|V|\} \) is called a prime cordial labeling of \( G \) if each edge \( uv \) is assigned the label 1 if \( \gcd(f(u),f(v)) = 1 \) and 0 if \( \gcd(f(u),f(v))>1 \), where the number of edges labeled with 0 and the number of edges labeled with 1 differ at most by one. In this paper we prove that 8- polygonal snake containing \( n \) number of 8- polygon, Splitting graph of \( C_n \) for \( n \geq 5 \) and Armed Crown \( C_{2k} \odot P_m \) for all \( k \geq 3 \) and \( m \geq 2 \) admit prime cordial labeling.

Key words: Graph labeling, prime labeling, cordial labeling, prime cordial labeling, 8- polygonal snake, splitting graph, armed crown.

Subject classification code:05C78

1. Introduction
Graph labeling is a strong relation between numbers and structure of graphs. Graph labeling was first introduced in the late 90’s. Many studies in graph labeling refer to Rosa’s research in 1967 [4]. A dynamic survey on different graph labeling problems with an extensive bibliography can be found in J.A. Gallian [3]. The concept of cordial labeling was introduced by Cahit [2]. Sundaram et al [6] introduced the concept of prime cordial labeling.

Graph labeling is an active area of research in graph theory which has mainly evolved through its many applications in coding theory, communication networks, mobile telecommunication system, optimal circuit layouts and graph decomposition problems. A systematic study of various theories in graph labeling were carried out by Bloom and Golomb [1].

2. Preliminaries
The following theorems are proved by S.K. Vaidya and P.L. Vihol [8]

Theorem 2.1
The graph obtained by duplicating each edge by a vertex in a cycle \( C_n \) admits prime cordial labeling except for \( n = 4 \).

Theorem 2.2
The graph obtained by duplicating a vertex by an edge in cycle \( C_n \) is a prime cordial graph.

Theorem 2.3
The path union of \( m \) copies of cycle \( C_n \) is a prime cordial graph.

Theorem 2.4
The friendship graph \( f_n \) is a prime cordial graph for \( n \geq 3 \)

The following theorems are proved by M. A. Seoud, A. T. M. Matar, R.A. Al-Zuraiqi [5]

Theorem 2.5
If \( G \) is not a prime cordial graph of order \( m \) then \( G \cup K_1 \) is a prime cordial graph if \( E(G) = m-1 \), \( n \) or \( n+1 \).
Theorem 2.6
The graph obtained by duplicating a vertex $v_k$ in the rim of the helm $H_n$ is a prime cordial graph.

Theorem 2.7
The graph obtained by fusing the vertex $u_1$ with $u_2$ in a helm graph $H_n$ is a prime cordial graph.

Theorem 2.8
The graph $G$ which is obtained by attaching central vertex of a star $K_{1,n}$ at one of the vertices $C_3$ is a prime cordial graph.

The following theorems are proved by A. Sugumaran and V. Mohan [7]

Theorem 2.9
Cycle butterfly graph $B_{n,m}$ admits prime cordial labeling for $n = 3$ and for all integers $m > 2$ and $m = 1$.

Theorem 2.10
The cycle butterfly graph $B_{m,n}$ admits prime cordial labeling, where $n \geq 4$ and $m \geq 1$.

Theorem 2.11
The $W$-graph $W_{2n+1}$ is a prime cordial graph.

Theorem 2.12
The $H$-graph admits prime cordial labeling.

3. Main result

Definition 3.1
Consider $k$ copies of path $P_n$. An $n$-polygonal snake containing $k$ number of $n$-polygons is obtained from a path $v_0, v_1, \ldots, v_k$ by identifying the pendant vertices of the $i^{th}$ copy of path $P_n$ with $v_i$ and $v_i$ for $i = 1, 2, \ldots, k$.

Theorem 3.2
The $8$-polygonal snake containing $n$ number of $8$-polygons admits a prime cordial labeling.

Proof
The $8$-polygonal snake containing $n$ number of $8$-polygons has $7n+1$ vertices and $8n$ edges. Let $v_1^{(1)}, v_2^{(1)}, \ldots, v_8^{(1)}$ be the vertices of the $1^{st}$ copy of the $8$-polygon where $v_i^{(1)} = v_0, v_i$ for $i = 1, 2, \ldots, n$.

Define $f: V(G) \rightarrow \{1, 2, 3, \ldots, 7n+1\}$ as follows:

- $f(v_1^{(i)}) = 7(i-1) + 1$ for $i = 1, 2, \ldots, n$
- $f(v_2^{(i)}) = 7n + 1$
- $f(v_3^{(i)}) = 7i + 5$ for $i \equiv 1 (\text{mod} 3)$
- $f(v_3^{(i)}) = 7i + 3$ for $i \equiv 0, 2 (\text{mod} 3)$
- $f(v_4^{(i)}) = 7i$ for $i \equiv 1, 2 (\text{mod} 3)$
- $f(v_4^{(i)}) = 7i - 2$ for $i \equiv 0 (\text{mod} 3)$
- $f(v_5^{(i)}) = 7i + 3$ for $i \equiv 1 (\text{mod} 3)$
- $f(v_5^{(i)}) = 7i + 5$ for $i \equiv 2 (\text{mod} 3)$
- $f(v_5^{(i)}) = 7i$ for $i \equiv 0 (\text{mod} 3)$
- $f(v_6^{(i)}) = 7i + 6$ for $i \equiv 1 (\text{mod} 3)$
- $f(v_6^{(i)}) = 7i + 4$ for $i \equiv 2 (\text{mod} 3)$
- $f(v_6^{(i)}) = 7i$ for $i \equiv 0 (\text{mod} 3)$
- $f(v_7^{(i)}) = 7i + 2$ for $i \equiv 0, 2 (\text{mod} 3)$
- $f(v_7^{(i)}) = 7i$ for $i \equiv 1 (\text{mod} 3)$
- $f(v_7^{(i)}) = 7i + 6$ for $i \equiv 0, 2 (\text{mod} 3)$
Clearly, $f$ is one-one.

It is clear that $|e_f(0)-e_f(1)| = 0 \leq 1$,

where $e_f(0)$ denotes number of edges having the label 0 and $e_f(1)$ denotes number of edges having the label 1.

This shows that the 8-polygonal snake containing $n$ number of 8-polygon has a prime cordial labeling.

**Example 3.3**

8-polygonal snake containing 12 number of 8-polygon.

Splitting Graph

**Definition 3.4**

Let $G$ be a graph. For each point $v$ of a graph $G$, take a new point $v'$. Join $v'$ to those points of $G$ adjacent to $v$. The graph thus obtained is called the splitting graph of $G$. We denote it by $S'(G)$.

**Theorem 3.5**

Splitting graph of $C_n$ has prime cordial labeling for $n \geq 5$

**Proof:**

We name the vertices of the graph $S'(C_n)$ as follows:

This graph has $2n$ vertices and $3n$ edges. Define $f(v) = 2i$ for $i = 1, 2, 3$

- $f(v_1) = 10$
- $f(v_6) = 8$

For $i = 6, 7, 8, \ldots, n$, define

- $f(v_i) = 2i+2$ if $i \equiv 2$ (mod 6)
- $2i-2$ if $i \equiv 3$ (mod 6)
- $2i$ if $i \equiv 2, 3$ (mod 6)

For $i = 6, 7, 8, \ldots, n$

- $f(v'_i) = 2i+1$ if $i \equiv 1$ (mod 6)
- $2i-3$ if $i \equiv 2$ (mod 6)
- $2i-1$ if $i \equiv 1, 2$ (mod 6)

Clearly, $f$ is one-one.

It is clear that $|e_f(0)-e_f(1)| = 0 \leq 1$

Thus $S'(C_n)$ has prime cordial labeling for $n \geq 5$. 

3
Example 3.6
Prime cordial labeling of the graph $S'(C_{30})$

Armed crown

Definition 3.7
Armed crown is cycle with paths of equal length attached at each vertex of the cycle. We denote an armed crown by $C_n \otimes P_m$ where $P_m$ is a path of length $m-1$.

Theorem 3.8
The Armed Crown $C_{2k} \otimes P_m$ admits prime cordial labeling for all $k \geq 3$ and $m \geq 2$.

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices of the cycle $C_n (n = 2k)$. Let $v_{j(1)}, v_{j(2)}, \ldots, v_{j(m)}$ be the vertices of the path $P_m$ of length $m-1$ attached with $u_j (1 \leq j \leq n)$ by identifying $u_j$ with $v_{j(m)}$.

Case 1: $C_{2k} \otimes P_m$ where $m \equiv 0 \pmod{3}$,

For $j = 1, 2, \ldots, k$, define
$$f(v_{j(i)}) = 2i + 2m(j - 1), \quad i = 1, 2, \ldots, m$$
For $j = k + 1, k + 2, \ldots, 2k - 4$, define
$$f(v_{j(i)}) = 2i - 1 + 2m(2k - j), \quad i = 1, 2, \ldots, m$$
For $j = 2k - 3$ and $2k - 2$ define
$$f(v_{j(i)}) = 2i - 1 + 2m(2k - j), \quad i = 1, 2, \ldots, m - 2$$
$$f(v_{j(m-1)}) = 2m - 1 + 2m(2k - j)$$
$$f(v_{j(m)}) = 2m - 3 + 2m(2k - j)$$
For $j = 2k - 1$ and $2k$ define
$$f(v_{j(i)}) = 2i - 1 + 2m(2k - j), \quad i = 1, 2, \ldots, m$$
Case 2: $C_{2k} \oplus P_m$ where $m \equiv 1,2 \pmod{3}$
For $j=1,2,\ldots,k$, define
$f(j) = 2i + 2m(j-1)$, $i = 1,2,\ldots,m$
For $j = k+1, k+2, \ldots, 2k-3$, define
$f(j) = 2i - 1 + 2m(2k-j)$, $i = 1,2,\ldots,m$
For $j = 2k-2$, define
$f(j) = 2i - 1 + 2m(2k-j)$, $i = 1,2,\ldots,m - 2$
$f(j) = 6m - 1$
$f(j) = 6m - 3$
For $j = 2k-1$ and $2k$ define
$f(j) = 2i - 1 + 2m(2k-j)$, $i = 1,2,\ldots,m$
Clearly $f$ is one-one.
It is clear that $|e_f(0) - e_f(1)| = 0 \leq 1$
Thus the armed crown $C_{2k} \oplus P_m$ has prime cordial labeling for all $k \geq 3$ and $m \geq 2$.

Example 3.9
Prime cordial labeling of the graph $C_{12} \oplus P_6$

Example 3.10
Prime cordial labeling of the graph $C_{10} \oplus P_5$
Conclusion and Scope

In this paper, prime cordial labeling for the splitting graph of $C_n$ for $n \geq 5$, 8-polygonal snake and armed crown $C_{2k} \Theta P_m$ for $k \geq 3$ and $m \geq 3$ have been investigated. Further discussion will be performed in this context for the special graphs namely Desargues graph and Heawood graph, extending the graphs by the operation duplication will also be discussed.

References

[1] G.S. Bloom and S.N. Golomb, (1977), Applications of numbered undirected graphs, Proc. IEEE, Vol. 65, pp. 562-570.
[2] I. Cahit, Cordial graph A weaker version of graceful and harmonious graphs, Ars Combinatoria, 23(1987), 201-207.
[3] J.A. Gallian, A dynamic survey of graph labeling. The Electronics Journal of Combinatorics, 19(2018), # Ds6, 1-260.
[4] A. Rosa, (1967) On Certain Valuations of the vertices of a graph, in Theory of Graphs, International Symposium, Rome, July 1966, Gordon and Breach, New York and Dunod, Paris, pp. 349-355.
[5] M. A. Seoud, A. T. M. Matar, R.A. Al-Zuraiqi, Prime cordial labeling, circulation in Computer Science Vol. 2, No. 4, pp. (1-10), May 2017.
[6] M. Sundaram, R. Ponraj and S. Somasundaram, Prime cordial labeling of graphs, Journal of Indian Academy of Mathematics, 27(2005) 373-390.
[7] A. Sugumaran and V. Mohan, Further results on prime cordial labeling Annals of Pure and Applied Mathematics, Vol. 14, No. 3, 2017, 489-496.
[8] S.K. Vaidya and P.L. Vihol Prime cordial labeling for some Cycle Related Graphs, Int. J. Open Problems Compt., Maths, Vol-3, No-5, December 2010, ISSN 1998-6262;.