Application of Improved Perturbation Theory to Bridge Damage Identification

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Abstract. This paper introduces two new methods—William B.B. perturbation method and Rayleigh quotient method—into damage identification of local unit of bridges. Taking the structure of a truss bridge as the practical example, the author made detailed analysis of these two methods’ function in damage identification by programming the corresponding calculation process and adding vibration noise to the identification process. The results showed that William B.B. method could more easily generate the perturbation accuracy values and that Rayleigh quotient method had the advantage of higher identification accuracy, simpler calculation and better resistance to noise.

1. Introduction
The bridge health-monitoring-system is achieved by applying sensors and intelligent integration system to collect, record and analyze the current condition and the damage of the bridge structure in real time [1-2]. In recent years, the condition assessment and long-term monitoring of long-span bridge in the operational phase has been drawing increasingly more attention and the real damage of the bridge through inversion calculation based on the field monitoring data has been one of the core problems of the structural damage evaluation system by far [3-4]. By calculating the change of small parameters before and after the structural damage is by far the general method of damage identification to reflect the structural damage degree. And many scholars at home and abroad have done in-depth research on this issue [5-6].

Perturbation theory originated from the quantum mechanics, mainly studies the variation of the inherent characteristics and response characteristics when the small parameter of the structure changes and is widely used in aerospace engineering and intelligent optimization algorithms [7]. But presently it is scarcely applied to structural damage research, particularly to bridge damage research. Combining perturbation theory and vibration theory, Du S. Y., et al. deduced the first and second order damage identification formulations of structure vibration eigenvalue and verified the effectiveness of perturbation method in the small damage stage through investigating an actual example of a continuous girder bridge [8]. Chen H., et al. combined the perturbation theory and Riccati’s transfer matrix method, established the probability identification method of the structural damage according to the principle of probability statistics, inversely deduced the structural damage location and degree by using the optimization method, and verified that this method was able to perform effective identification job when the structural damage degree was up to about 20% [9]. Therefore such conclusions can be drawn that in the case of small damage conditions the traditional perturbation
theory could identify damage satisfactorily, but the identification process was relatively complicated. For this, the present paper introduces two new methods of damage identification aiming to on the one hand simplify the identification process, on the other hand tentatively investigate large damage identification.

2. Low order perturbation theory of bridge damage identification
Rayleigh first introduced structural parameter modification analysis into structural dynamics. Later, Plaut, Huseyin and Rogers and other scholars also made in-depth research on the perturbation theory. In the domestic, Wang and Hu and Chen also made systematic research about this theory[10-11].

At present, the damage of large span structures like bridges usually includes structural cracks, local components falling off, structural deformation, steel bar corrosion, loose connections, etc. Such damage has relatively little influence on the mass of structural components, but has substantial effect on the structure’s stiffness, even to the extent that the damage is proportional to the amount of the stiffness loss[12]. Therefore, concerning the bridge structural damage, the change of the stiffness matrix should be taken into consideration, while the change of the mass matrix can be neglected to some extent.

Suppose $\varepsilon$ is a small random parameter causing bridge structural damage reflected by the change of mass and stiffness matrix, which further inevitably causes the change of dynamic characteristics[13].

$$
[M] = [M_0] + \varepsilon [M_1]
$$

(1)

$$
[K] = [K_0] + \varepsilon [K_1]
$$

(2)

Here $\varepsilon$ represents the bridge’s perturbation, $[M_0]$ and $[K_0]$ represent respectively the initial mass matrix and stiffness matrix before the bridge damage, $[M]$ and $[K]$ represent respectively the initial mass matrix and stiffness matrix after the bridge damage, $[M_1]$ and $[K_1]$ represent respectively mass matrix and stiffness matrix of the damaged local units.

3. Damage identification based on refined William B.B. method
The William B.B.’s method [14] is to calculate inner product by the eigenvector before bridge damage and the mass matrix after bridge structure damage instead of the mass matrix before bridge damage, so that this method can improve the precision of the first-order perturbation method.

Considering the eigenvalue and eigenvector of the bridge vibration, we would obtain:

$$
K_0 \chi_i = \lambda_i M_0 \chi_i
$$

(3)

When there was small bridge structure damage, its stiffness matrix and mass matrix also changed to a certain degree, and thus the corresponding eigenvalue was:

$$
K_1 \chi_i = \lambda_i M_1 \chi_i
$$

(4)

Mass matrix and stiffness matrix changes of the bridge structure itself are shown in Eqs (1)-(2), so the variables of eigenvalues and eigenvectors can be expressed as:

$$
\lambda_i = \lambda_{i0} + \varepsilon \lambda_{i1}
$$

(5)

Then the refined computational formula of William B B method is available:

$$
\hat{\lambda}_{i3} = \lambda_{i0} + \varepsilon \lambda_{i1}
$$

(6)

$$
\varepsilon \hat{\lambda}_{i3} = \frac{\chi_i^T ([\varepsilon K_1] - \lambda_i [\varepsilon M_1]) \chi_i}{\chi_i^T [M] \chi_i}
$$

(7)

So the computational formula of eigenvalue is available,
4. Damage identification based on refined Rayleigh Quotient method

The existing literature which employed perturbation theory to identify damage only considered the small parameters’ first order and second order perturbation and neglected the influence of the higher order perturbation, resulting in inaccurate damage identification of medium and high damage degree. By contrast, Rayleigh quotient method is able to improve identification effectiveness.

Combine Eq. (5) and Eq. (7) with Eq. (6), we would obtain:

$$\lambda_{ii} = \frac{\chi_i^T [K] \chi_i}{\chi_i^T [M] \chi_i}$$ (8)

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Combine Eq. (5) and Eq. (7) with Eq. (6), we would obtain:

$$\left( [K] - \lambda [M] \right) \epsilon \chi + \left( eK - \lambda eM \right) \left( \chi + \epsilon \chi \right) = \epsilon \lambda \left( [M] + eM \right) \left( \chi + \epsilon \chi \right)$$ (9)

Suppose $\epsilon \chi = \epsilon \chi_i$, the left side is multiplied by $\chi_i^T = (\chi_i + \epsilon \chi_i)^T$ and after simplifying the equation, we would obtain the following equation,

$$\epsilon \lambda_{ii} = \frac{\chi_i^T \left( [K] - \lambda [M] \right) \chi_i}{\chi_i^T [M] \chi_i}$$ (10)

Thus in this way, the calculation formula of eigenvalue after damage would be available,

$$\lambda_{ii} = \lambda_i + \epsilon \lambda_{ii} = \frac{\chi_i^T [K] \chi_i}{\chi_i^T [M] \chi_i}$$ (11)

At the right side of Eq. (11), $\chi_i$ is the Rayleigh quotient of stiffness $[K]$ and mass $[M]$ after the bridge structure damage.

5. Example

This article adopted perturbation theory to make the corresponding computing program and investigated the truss bridge structure presented in Figure 1. This finite element model of bar system was divided into 8 nodes and 13 units. The cross section area of each bar is $A = 5.0 \times 10^{-4} \text{m}^2$, the elastic modulus of each bar is $E = 2.1 \times 10^9 \text{Pa}$ and the mass density of each bar is $\rho = 7300 \text{kg/m}^3$.

![Figure 1. Truss bridge structure](image_url)

To simplify the calculation, this paper only selected semi structure to investigate the following working conditions by adopting different perturbation methods.

(1) Shall the bars are damaged at multiple-cells: the third bar and the fifth bar are damaged simultaneously; their stiffness decrease by 20%, as it is shown in Figure 2.
The first and second working conditions indicated that the perturbation method was able to perform damage identification job satisfactorily. Moreover, this method could identify both single-cell damage and multiple-cell damage fair enough.

(2) Shall the third bar is damaged to the degree of 5% ~ 30%: Table 1 summarizes the damage identification results obtained by different perturbation methods.

Table 1. Different methods’ identification results of the 3rd unit with various damage degrees

| Actual damage degree | First-order perturbation method | Second-order perturbation method | William B.B. perturbation method | Rayleigh quotient method |
|----------------------|---------------------------------|----------------------------------|--------------------------------|-------------------------|
| 5%                   | 5.06%                           | 4.83%                            | 5.06%                          | 4.83%                   |
| 10%                  | 11%                             | 10%                              | 11%                            | 10%                     |
| 15%                  | 18.03%                          | 15.60%                           | 18.02%                         | 15.57%                  |
| 20%                  | 26.47%                          | 21.74%                           | 26.45%                         | 21.67%                  |
| 25%                  | 36.75%                          | 28.58%                           | 36.72%                         | 28.40%                  |
| 30%                  | 49.46%                          | 36.29%                           | 49.42%                         | 35.94%                  |

As Table 1 shows, within 5% to 30% damage degree, the four perturbation methods could produce good identification of the structural damage. The first-order perturbation method and the William B.B. perturbation method could produce satisfactory results in identifying damage degree ranging from 5% to 15%. But when the damage went beyond 15%, the identification accuracy would be spoiled by great nuance. The second-order perturbation method and Rayleigh quotient perturbation method could identify the damage degree less than 30% very well, so they could meet the requirements of engineering precision.
Figure 3. Identification error comparison of different methods

Figure 3 represents the error comparison among the four methods. In case of small damage, all the methods could perform identification job well, but with the increase of the damage degree, the identification results had certain deflection. The first-order perturbation result and the William B.B. perturbation result were quite similar, and comparatively speaking, the later one’s accuracy was higher. Similarly, the second-order perturbation result and the Rayleigh quotient perturbation result were also quite similar, and the later one’s accuracy was relatively higher. In case of medium damage, only the second-order perturbation result and the Rayleigh quotient perturbation result were able to achieve high accuracy. Therefore, perturbation method was favorable in identifying small bridge structure damage.

Table 2. Anti-noise performance of Rayleigh quotient method under varying degrees of damage

| Damaged unit | Actual damage degree | Noise level 0% | Noise level 1% | Noise level 5% | Noise level 10% | Noise level 15% |
|--------------|----------------------|----------------|----------------|----------------|----------------|----------------|
| The third unit | 20%                  | 21.67%         | 21.67%         | 21.67%         | 21.70%         | 21.64%         |
|               | 25%                  | 28.40%         | 28.40%         | 28.41%         | 28.39%         | 28.44%         |
|               | 30%                  | 35.94%         | 35.93%         | 35.97%         | 35.94%         | 36%            |

Table 2 summarizes the identification results of Rayleigh quotient method at different noise levels and in different damage cases. It shows that although adding noise had certain influence on the identification results, it was not enough to influence the identification of the damaged location and damage degree. Namely, the Rayleigh quotient method was, to a certain extent, immune to noise in the process of the bridge damage detection.

6. Conclusion

The present paper employed the perturbation methods to analyze the damage identification of a truss bridge under different working conditions and made in-length accuracy comparison of damage identification among the traditional first-order perturbation method, second order perturbation method, William B.B. perturbation method and Rayleigh quotient method. The results show that the first-order perturbation method and William B.B. perturbation method could identify the damage under the degree of 10% and comparing the two methods, the latter was more efficient, and that the second-order perturbation method and Rayleigh quotient method could identify the damage within the degree from 10% to 25% and comparing the two methods, the latter was more efficient with less calculating amount and better accuracy rate.

Therefore, it is safe to draw a conclusion that the refined perturbation methods proposed in this paper is a simplified, convenient and efficient way to initially detect the damage in the local bridge units. It should be noted that such methods was not so efficient in damage identification of the complicated structures. For such limitations, it is necessary for researchers concerned to make in-depth
research on damage identification of the complicated structures to improve the overall level of structure health-monitoring.

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