FAST TRACK COMMUNICATION

Late inspiral and merger of binary black holes in scalar–tensor theories of gravity

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Received 13 June 2012, in final form 9 October 2012
Published 22 October 2012
Online at stacks.iop.org/CQG/29/232002

Abstract

Gravitational wave observations will probe nonlinear gravitational interactions and thus enable strong tests of Einstein’s theory of general relativity. We present a numerical relativity study of the late inspiral and merger of binary black holes in scalar–tensor theories of gravity. We consider binaries inside a scalar field bubble, including in some cases a potential. We demonstrate how an evolving scalar field is able to trigger detectable differences between gravitational waves in scalar–tensor gravity and the corresponding waves in general relativity.

PACS numbers: 04.25.D−, 04.30.−w, 04.50.Kd

(Some figures may appear in colour only in the online journal)

Experimental tests on the gravitational redshift, light deflection, Shapiro time delay and perihelion advance have increased our confidence that general relativity (GR) is the correct classical theory of gravity [1]. More compelling evidence is provided by binary pulsar observations [2], where the hardening of the binary is accounted for in exquisite detail by one of the fundamental predictions of GR: gravitational wave (GW) emission. Modified gravity theories are nonetheless still a possibility [1]. To verify that indeed the podium only belongs to Einstein’s theory, tests in the nonlinear regime are needed. The new astronomy of GW observations will soon deliver such opportunity, probing gravity at its strongest grip.

In anticipation of GW observations by LIGO, Virgo and other interferometric detectors, exploring what to expect from alternative theories is crucial in assisting data analysis efforts. This is particularly important when nonlinear effects dominate, a regime only accessible with the tools of numerical relativity. Among the competing alternatives to GR, scalar–tensor (ST) gravity [3, 4] is one of the most popular due to its simplicity, and because of the motivations provided by string theory scenarios and explanations to dark energy [5]. ST gravity in its
simplest form was proposed about a half century ago and is commonly known as Brans–Dicke theory [6].

Studies on observational consequences of ST theories have mostly focused on compact object binaries in the post-Newtonian or extreme-mass-ratio regimes [7–16]. A critical prediction of these studies is the emission of dipole radiation. This radiation depends on the sensitivity of the compact objects [1], a measure of how responsive the mass of an object is to variations in the local value of the gravitational constant. Since the sensitivity of black holes (BHs) is the same regardless of their masses or spins, binary black holes (BBHs) in ST theories have been shown to be equivalent to those in GR to all post-Newtonian orders [7, 16]. If the scalar field is not initially stationary, however, Horbatsch and Burgess [17] recently suggested that BBHs could retain scalar hair [18] and emit dipole radiation, provided the holes have unequal masses.

The goal of the present study is to investigate whether the conclusions from previous studies regarding the BBH indistinguishability between ST and GR theories carries over to the nonlinear regime, i.e., during the late inspiral and merger of the binary. Our results show that this is not the case if the scalar field evolves. In such cases, the scalar field triggers dipole energy loss that leads to detectable differences in the GW polarizations. We induce dynamics in the scalar field by placing a BBH inside a scalar field bubble, which in some cases includes a scalar field potential. The collapse of the bubble has a dramatic influence on the BBH. The dominant effect is the increase of the mass of the BHs from the accretion of the scalar field. As a consequence, the GW emission is quite different from the case of GR gravity. While not astrophysical, the setup of a BBH inside a bubble allow us to make meaningful comparisons and to explore the conditions under which inhomogeneities in the scalar field yield detectable manifestations of ST gravity.

We restrict attention to ST theories in vacuum with a (Jordan frame) action of the form:

\[
S = \int \frac{d^4x}{2\kappa} \sqrt{-\tilde{g}}[F(\varphi) \tilde{R} - \zeta(\varphi) \tilde{\nabla}_\mu \varphi \tilde{\nabla}^\mu \varphi - 2U(\varphi)]
\]

with \(\kappa = 8\pi G\). Under a conformal transformation \(\tilde{g}_{\mu\nu} = F(\varphi) \tilde{g}_{\mu\nu}\), or equivalently \(\tilde{g}_{\mu\nu} = A^2(\varphi)g_{\mu\nu}\), the action (in the Einstein frame) reads [3, 4]

\[
S = \int d^4x \sqrt{-g} \left[ R - \nabla_\mu \phi \nabla^\mu \phi / 2 - V(\varphi) \right]
\]

where \((d\varphi/d\varphi)^2 = [(\zeta/F) + (3/2)(F_x/F)^2]/(4\kappa)\) and \(V = U/(\kappa F^2)\). We set \(A(\phi) = e^{\alpha - b\phi^2/2}\) [3, 4]. Thus, \(F = \varphi\) and \(\zeta = \omega/\varphi\) with \(\omega = -3/2 + \kappa/(a - b\phi^2)\). Brans–Dicke theory is recovered when \(b = 0\), and GR when \(a = b = 0\). We focus here on the case \(a = 0\), and consider different values of \(b\).

The Einstein frame is convenient because the action in (2) yields the same equations as those of GR, namely \(G_{\mu\nu} = \kappa T_{\mu\nu}\) with \(T_{\mu\nu} = \nabla_\mu \varphi \nabla_\nu \varphi - g_{\mu\nu}(\varphi)^2 / 2 + V\) and \(\Box \phi = V_\varphi\). Therefore, in the absence of a potential, \(\phi = \phi_0 = \) constant yields \(G_{\mu\nu} = 0\). Thus, vacuum spacetimes in ST theories will be equivalent to their corresponding spacetimes in GR, independent of the choice of conformal factor \(A(\phi)\). This is also the case in the presence of a potential if one arranges for \(V(\phi_0) = 0\) and \(V_\varphi(\phi_0) = 0\). We carried out BBH simulations that verify this exact equivalence. The BH parameters, radiated energy and waveforms between ST and GR simulations differed only at the level of round-off errors. The main objective of those simulations was to confirm that our computational code does not introduce spurious differences between ST and GR theories.

Therefore, the only avenue to trigger differences between ST theories and GR is by inducing dynamics in \(\phi\). One possibility is with \(\partial_\tau \phi \neq 0\), e.g. \(\partial_\tau \phi = \epsilon \tau\) [17]. Another possibility is by introducing curvature couplings to the action such as \(\phi R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}\) [14, 19]
Table 1. Parameters, masses and spin.

| Case | $\phi_0$ | $4\pi \lambda /M^2$ | $M_0/M$ | $M_1/M$ | $a/M_h$ |
|------|----------|----------------------|---------|---------|---------|
| A    | 0        | 0.990                | 0.952   | 0.686   |
| B    | 1/80     | 1.179                | 0.963   | 0.688   |
| C    | 1/40     | 1.747                | 0.999   | 0.706   |
| D    | 1/80     | 1.217                | 0.983   | 0.685   |

or $\phi e^{\alpha \beta \gamma \delta} R_{\alpha \beta \gamma \delta}$ [14, 20, 21], that anchor $\phi$ to the spacetime curvature. Yet another alternative is by introducing inhomogeneities in $\phi$, which is the focus of the present study.

We consider a BBH inside a scalar field bubble with an initial profile $\phi(r) = \phi_0 \tanh [(r - r_0)/\sigma]$ and an inflationary-inspired potential $V(\phi) = \lambda(\phi^2 - \phi_0^2)^2/8$ [22]. The bubble has radius $r_0$, thickness $\sigma$, and in the exterior (interior) $\phi \approx \phi_0(-\phi_0)$. We shift the conformal factor as $A = e^{-b(\phi^2-\phi_0^2)/2}$, so asymptotically the Einstein and Jordan frames are both Minkowskian and in the same coordinates [3, 4]. A crucial property of our initial setup is that only the wall of the bubble contributes to the energy density in $\phi$. In the interior and exterior of the bubble, the $\phi$ field is to a good approximation constant. Furthermore, in the cases when a potential is present, $V(\pm \phi_0) \approx 0$. Thus, the bubble’s interior is locally equivalent in all the cases we considered. This allows us to make ‘apples with apples’ comparisons between BBHs in ST and GR theories, i.e. to have BBHs with the same BH masses, spins, separation and eccentricity.

We discuss results from a set of representative simulations. The simulations were obtained with the MAYA numerical relativity code of our group [23, 24]. The code was modified to include $\phi$ as a matter source and its corresponding evolution equations [25]. The rest of the code was preserved, namely moving puncture gauges [26, 27] and unconstrained BSSN evolution [28]. As with the addition of hydrodynamics and Maxwell fields, adding a scalar field source to MAYA did not affect its convergence properties nor the degree to which the constraints are satisfied in the course of evolutions. No modifications to our code were needed in the wave-extraction infrastructure, i.e. Weyl scalar $\Psi_4$ computation, since the extraction was performed in the Einstein frame. The implementation of modules to extract the breathing mode characteristic of ST theories was straightforward since it involved contractions of the Riemann tensor with a null-tetrad similar to those used for $\Psi_4$.

In all cases, the binary has non-spinning, equal-mass BHs in quasi-circular orbit, initially separated by $11 M$, with $M$ the mass of the binary. The bubble surrounding the BBH has a radius $r_0 = 120 M$ and thickness $\sigma = 8 M$. The simulations in the Einstein frame differ only in the parameters $\phi_0$ and $\lambda$, as given in table 1. Their particular values were chosen to make the differences between GR and ST more evident. Also in table 1 are the values in the Einstein frame of the ADM mass $M_0$, the mass $M_h$ and spin $a/M_h$ of the final BH. Case A is the reference GR simulation, and D is the only case with non-vanishing potential. Extraction of GWs is carried out in the Jordan frame. For each case, we extract GWs with values $b = 5$ and 10, values consistent with observational constraints [29]. We also do extraction of GWs with $b = 0$, in which the Jordan and Einstein frames are equivalent.

In all cases, the bubble shell collapses. For case D, the thickness of the collapsing shell remains roughly constant since the potential effectively makes the bubble a topologically stable domain wall. This is not the case for B and C. The shell disperses while collapsing. By the time the shell reaches the BBH, it becomes a cloud encompassing the binary. Independently of the details of the initial configuration, the field $\phi$ has a dramatic effect on the BBH dynamics. The merging BHs increase their masses as they accrete $\phi$. Figure 1 shows the evolution of the
mass (in the Einstein frame) of the individual BHs before they merge. We have chosen the Einstein frame representation to isolate the effects of the conformal transformation from those due to the accretion of $\phi$ by the holes. The differences in the initial BH masses among all cases reflect the difficulty of setting up identical binaries because, in the construction of the initial data, $\phi$ contributes to the ADM mass and it is difficult to cleanly separate this contribution from those of the binary. Notice that for cases B and C the mass increases in two stages. The first increase occurs when the bubble is imploding and passes through the binary. The second increase is after the bubble bounces back and expands. The mass increase in C is larger because the bubble is more massive. With the choice of parameters, the most massive shell is C, followed by D and B (see the ADM mass $M_0$ in table 1). For case D, the BH mass has multiple step-like increases. This is because the bubble does not just bounce back and dissipate, but instead it lingers near the binary, bouncing multiple times. The multiple bouncing of the shell is due to the presence of the potential $V$. Notice also in figure 1 that the merging BHs eventually reach a constant mass. The BHs gain 1%, 6% and 3% of their original mass for cases B, C and D, respectively. This gain in the mass of the merging BHs is correlated with the mass $M_h$ of the final BH. That is, in the GR case A, the final BH is $M_h = 0.952 M$, while in all other cases and to a good approximation, the final BH mass is $M_h \approx 0.952 (M + \delta M)$ with $\delta M$ the mass accreted by the merging BHs, i.e. $0.01 M$ for B, $0.06 M$ for C, and $0.03 M$ for D (see e.g. table 1).

The $\phi$-accretion by the BHs has a dramatic effect on the binary dynamics. Figure 2 shows the trajectories (in the Einstein frame) for all cases. Recall that case A is the GR reference case for a BBH in quasi-circular orbit. In all the other cases, the $\phi$-bubble induces eccentricity and accelerates the merger. The more massive the shell the larger the induced eccentricity. In particular, for C the influence of $\phi$ is such that the binary basically plunges.

Not surprisingly, the effect that the bubble has on the binary is also reflected on the emission of gravitational radiation. In figure 3, we plot the $(2,2)$ mode of the Weyl scalar $\Psi_4$, real part as a dashed line and its amplitude as a solid line. The time $t = 0 M$ denotes merger time. The waveforms plotted in panels B, C and D correspond to $b = 0$. Notice the obvious differences among the $\Psi_4$ profiles, and in particular when matched against the GR case A.

Within a given case, the differences between a waveform with $b = 0$ and one with $b = 5$ or 10 are undetectable. We calculated mismatches for advanced LIGO, and they were found to
be \( \lesssim 10^{-3} \). In this regard, a detection of these GWs will not be able to set a constraint on \( b \), but nonetheless a putative observation should be able to identify that the binary is not a BBH in GR gravity. The low mismatches can be understood by studying how the Weyl scalar \( \Psi_4 = -C_{\text{in in}} \) conformally transforms, where the indices denotes contractions with the null tetrad. The tetrad vectors transform as \( \tilde{v}^a = A^{-1} v^a \), and the Weyl tensor as \( \tilde{C}_{abcd} = A^2 C_{abcd} \). Therefore, the Weyl scalar will transform as \( \tilde{\Psi}_4 = A^{-2} \Psi_4 = e^{b(\phi^2 - \phi_0^2)} \Psi_4 \) (tilde quantities correspond to the Jordan frame). At the location of the GW detector, one expects that \( \phi = \phi_0 + \delta \phi \) with \( \delta \phi \ll \phi_0 \) since the scalar field dynamics is such that with time \( \phi \to \phi_0 \). Therefore, \( \tilde{\Psi}_4 \approx (1 + 2b \delta \phi \phi_0) \Psi_4 \approx \Psi_4 \). For instance, at the GW extraction radius in our simulations, we observed that \( \delta \phi \lesssim 10^{-2} \phi_0 \). With our choice of parameters \( b \phi_0^2 \lesssim 10^{-2} \), then \( \tilde{\Psi}_4 \approx \Psi_4 \) to one part in \( 10^4 \). Notice that the differences between \( \tilde{\Psi}_4 \) and \( \Psi_4 \) is independent of the extraction radius. They depend only on the ’residual’ dynamics of \( \phi \).

Figure 2. Trajectories of the BHs when viewed from the Einstein frame.

In figure 3, cases B and D show the characteristic modulation in \( \Psi_4 \) observed in eccentric BBHs. The waveform in case C, on the other hand, is basically a burst, since here the binary essentially plunges. Notice also that the amplitudes in B, C and D show a small blip at \( t \sim -830M, -75M \) and \(-375M \) before merger, respectively. These bumps are due to the shell as it goes through the binary. The bumps are therefore correlated with the mass jumps in figure 1.

As expected, in all cases the final BH relaxes emitting the characteristic quasi-normal ringing observed in figure 3 for times \( t > 0M \). In the Einstein frame, the connection between the mass and spin of the final BH with the quasi-normal frequencies and decay times is the same as in GR [30]. However, in the Jordan frame this is no longer the case since the \( \phi \) modifies the mass of the BH as computed from its apparent horizon [18]. The modifications are nonetheless small since they enter mostly through the conformal factor \( e^{-b(\phi^2 - \phi_0^2)} \) with \( \phi \) quickly approaching \( \phi_0 \). We have extended the evolution of the final BH, and there is no indication of remnant scalar hair.
Figure 3. Mode $(2,2)$ of the Weyl scalar $\Psi_4$, real part as a dashed line and its amplitude as a solid line. The time $t = 0M$ denotes merger time.

ST theories predict dipole energy losses as well as a new GW polarization, sometimes called a breathing mode, given by the traceless Ricci tensor scalar, $\Phi_{22} = -R_{\mu\nu} = -R_{\mu\nu}/2$ [31, 32]. This dipole radiation modifies the evolution of the binary and thus the GW frequency [33, 34]. Figures 4 and 5 show respectively the modes $(0,0)$ and $(2,2)$ of $\Phi_{22}$ in the Jordan frame. These are the strongest modes, with the others at least an order of magnitude weaker. Case A is not included because $\Phi_{22}$ is absent, which was verified numerically by our wave-extraction infrastructure. In each panel, $b = 0$, 5 and 10 are solid, dashed, and dotted lines, respectively. The time range in each panel was chosen to highlight when $\phi$ interacts most strongly with the BHs, with $t = 0M$ denoting merger time. The evident $b$-dependence of $\Phi_{22}$ can be explained by how $\Phi_{22}$ conformally transforms: $\tilde{\Phi}_{2,2} = A^{-2}[\Phi_{2,2} + bD(\partial_t \phi, \ldots)]$, with $D$ a function that vanishes when $\partial_t \phi = 0$. Therefore, at times when $\phi$ undergoes significant evolution, $\tilde{\Phi}_{2,2}$ depends linearly on $b$, as observed in figures 4 and 5. Overlaps of $\tilde{\Phi}_{2,2}$ with different $b$ in cases B and C correspond to moments when $\partial_t \phi \approx 0$ at the wave extraction.
Figure 4. Mode (0,0) of the breathing mode, $rM \Phi_{22}$. In each panel, $b = 0, 5$ and 10 are solid, dashed, and dotted lines, respectively. $t = 0M$ denotes merger time.

location. Then, as with $\Psi_4$, one has $\tilde{\Phi}_{22} \approx \Phi_{22}$. For case D, this does not happen, i.e. $\tilde{\Phi}_{22}$ with different $b$ values do not overlap because the potential induces longer lived dynamics in $\phi$, persisting long after the binary has merged. In principle, an appropriate configuration of interferometers could directly detect the breathing mode $\Phi_{22}$ [34]. Unfortunately, given the amplitudes depicted in figures 4 and 5, such detection will be challenging. Nonetheless, the dramatic effects on $\Psi_4$ (see figure 3) should be sufficient to infer that these GWs are not from the late inspiral and merger of BBHs in vacuum under the influence GR gravity.

This study is a first step towards investigating detectable observational signatures in the GW emission from the late inspiral and merger of a BBH in ST gravity. Our results supports the view that, in order to ‘defeat’ the no-hair constraint of BHs and thus trigger detectable effects on the gravitational radiation, an evolving scalar field is required. Our study shows that inhomogeneities in the scalar field could provide such mechanism. We considered a BBH in a scalar field bubble, but our conclusions can be carry over to more generic scenarios. The only requirement is that the inhomogeneities are such that the merging BHs accrete scalar field and change their masses enough to modify the binary evolution. In a subsequent study, we will explore a broader range of parameters and find the minimum inhomogeneity strength that would yield detectable ST gravity effects.

With the caveat of having used a rather artificial initial setup, we observed differences in the waveforms, computed in ST and GR, that are potentially detectable by interferometers such as advanced LIGO. As pointed out before, the GW emitted after $\phi$ has dissipated away or reached a constant value are identical to what one would find in GR. However, while the
scalar field is dynamically evolving, differences arise that cannot be mimicked by a GR binary with different physical parameters. The reasons for this are the following:

1. While $\phi$ is evolving, the BH masses increase, making in turn the orbit eccentric. Such mass increase and the subsequent increase in orbital eccentricity is not possible for quasi-circular binaries in vacuum GR.

2. While $\phi$ is active, the rate of change of the binding energy is increased because both GWs and scalar field waves remove energy from the system. The scalar field energy flux is dipolar in nature and to leading-order $\propto (v/c)^8$. On the other hand, the GW energy flux is quadrupolar and $\propto (v/c)^{10}$, one post-Newtonian order higher. Therefore, a scalar field modification to the energy flux affects the rate of change of the binary’s binding energy by the balance law, which in turn modifies the binary’s equation of motion with a new (frequency-dependent) term that is absent in GR.

No constant (frequency-independent) rescaling of the masses, spins or eccentricity in GR GWs can be carried out that will lead to the same waveforms computed in STs. One might expect weak correlations with these parameters in GR waveforms, just like the chirp mass is weakly correlated with the spin parameters in a GR parameter estimation analysis (because they enter at different post-Newtonian orders). In the end, whether such effects are distinguishable or not will depend on the signal-to-noise ratio and the strength of the scalar field modification. A more detailed Bayesian parameter estimation and hypothesis testing study would be required to address this topic, and this is beyond the scope of this paper, although interesting as a follow-up study.

Figure 5. Same as in figure 4 but for the mode (2,2).
Acknowledgments

We thank Cliff Will for fruitful discussions. Work supported by NSF grants 0653443, 0855892, 0914553, 0941417, 0903973, 0955825, 1114374 and NASA grant NNX11AI49G, under sub-award 00001944. Computations at Teragrid TG-MCA08X009 and Georgia Tech FoRCE cluster. RH gratefully acknowledges support by the Natural Sciences and Engineering Council of Canada.

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