Experimental Study on Vortex Shedding and Sound Radiation from a Rectangular Cylinder at Low Mach Numbers*

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Sound radiation from a rectangular cylinder, the cross-sectional aspect ratio (AR) of which varied from 0.3 to 4, was studied experimentally at Reynolds numbers of $7.5 \times 10^3$ to $1.4 \times 10^5$ and Mach numbers below 0.16. In addition to measurements of the lift-dipole sound, detailed flow fields around the rectangular cylinder were examined by means of PIV to better understand the dependence of sound radiation on the AR and Reynolds number as well as the vortex-cylinder interaction associated with sound generation. The mean square sound pressure was proportional to $U_\infty^6 S^2$, with almost the same factor of proportionality for aspect ratios larger than 0.5 in the Reynolds number range examined in spite of the fact that the spanwise correlation length and magnitude of the lift fluctuations were highly dependent on the AR. When vortex shedding at the trailing-edge was locked in phase with the leading-edge-generated vortices, which occurred for $AR > 3$, the shed vortices were quite two-dimensional, and correspondingly, the radiated sound exhibited a very sharp spectral peak like a line spectrum; the SPL was as strong as that for $AR = 0.75$, for which the maximum SPL was observed. Reynolds number effects on both SPL and the Strouhal number were weak except around the aspect ratio where reattachment of the separated leading-edge shear layer occurred.

Key Words: Aeroacoustics, Aeolian Tone, Vortex Shedding, Wake

Nomenclature

- $a$: speed of sound
- $f$: frequency
- $h$: transverse length of cylinder
- $l$: spanwise length of cylinder
- $l_s$: spanwise correlation length
- $p$: pressure fluctuation
- $r$: radial distance
- $u$: streamwise velocity fluctuation
- $u'$: rms value of $u$
- $w$: streamwise length of cylinder
- $x$: streamwise distance
- $y$: transverse distance
- $z$: spanwise distance
- $AR$: cross-sectional aspect ratio
- $M$: Mach number
- $R_c$: correlation coefficient
- $Re$: Reynolds number
- $St$: Strouhal number
- $U$: streamwise velocity
- $C_l'$: rms value of fluctuating lift coefficient
- $\lambda$: wavelength of instability wave
- $\nu$: kinematic viscosity
- $\omega$: spanwise vorticity
- $\theta$: angle

Subscripts

- $0$: reference
- $\infty$: freestream

Key Words: Aeroacoustics, Aeolian Tone, Vortex Shedding, Wake

1. Introduction

When a cylindrical body is placed in a uniform flow, von Kármán vortex shedding occurs at a wide range of Reynolds numbers, and in turn, this vortex induces unsteady aerodynamic forces (lift and drag) on the body. In particular, the von Karman vortex-induced fluctuating lift forms a dipole sound source that dominates the acoustic far-field in low subsonic flows.

Many researchers have studied the flow-induced dipole sound in the wake of a circular cylinder since the theoretical work on the acoustic analogy by Lighthill.1) Curle2) extended Lighthill’s acoustic analogy to include solid boundaries on the sound field. Phillips3) estimated the fluctuating lift acting on a circular cylinder using Kovasznay’s experimental data4) at Reynolds numbers of less than 160, and derived the formula of the sound field using Curle’s acoustic analogy. Keefe5) investigated the fluctuating lift and associated sound pressure of a circular cylinder with various ratios of diameter to axial length over Reynolds numbers of $3 \times 10^3$ to $1 \times 10^5$. Fujita et al.6) proposed a modification of the Phillips’ formula by taking the near-field effect into account when the observation location was not far enough from the sound source; see also the review by Fujita.7) The validity of Curle’s acoustic analogy in low-Mach-number flows was also confirmed in the direct numerical simulation of two-dimensional compressible flows over a circular cylinder by Inoue and Hatakeyama.8) Their computational study also showed that Doppler effects play an important role in the sound propagation at finite Mach numbers.

In the case of a rectangular cylinder, the Strouhal number of shed vortices and the aerodynamic forces are highly dependent on the aspect ratio (AR) of the cylinder’s cross-sec-
tion, as demonstrated by Okajima.\textsuperscript{9) His experiments showed variations in the Strouhal number with an $AR$ over a wide range of Reynolds numbers, $Re = 70 - 2 \times 10^4$. The results included an abrupt change in the Strouhal number between $AR = 2$ and $3$ at Reynolds numbers of $5 \times 10^2$ and $10^3$. Sakamoto et al.\textsuperscript{10) examined the fluctuating lift and drag of a rectangular cylinder at $Re = 5.5 \times 10^4$ and showed that the magnitudes of both reached a maximum at around $AR = 0.7$. Parker and Welsh\textsuperscript{11) and Stokes and Welsh\textsuperscript{12) also demonstrated that the characteristic of vortex shedding drastically changed around $AR = 3.2$. Nakamura et al.\textsuperscript{13) examined vortex shedding for $AR$s ranging from 3 to 15, and found that the Strouhal number based on the streamwise (chord) length took a constant value of about 0.6 for $AR$s of 3–5, and then increased stepwise to values that were nearly equal to integral multiples of 0.6 as $AR$ increased. The dependency of the Strouhal number and aerodynamic forces on $AR$ was also demonstrated in numerical simulations by Sohankar et al.\textsuperscript{14) and Sohankar.\textsuperscript{15) }

On the other hand, only a few experiments have been conducted on sound radiation from a rectangular cylinder of a shape other than a square.\textsuperscript{16) The related computational studies on sound generation are also limited to direct simulations of two-dimensional compressible flows at low Reynolds numbers\textsuperscript{17,18) or calculations based on Curle’s acoustic analogy together with large eddy simulations (LES) of incompressible three-dimensional flows.\textsuperscript{19) When the Reynolds number increased, wake vortices lost two-dimensionality and the spanwise coherency of vortex shedding became significant for accurately estimating the sound pressure level. Therefore, a sufficiently large spanwise computational domain is needed for actual prediction of the far-field sound in numerical simulations, even for the application of acoustic analogy in high-Reynolds-number engineering applications. In the present paper, in order to further clarify how the characteristics of the radiated sound depend on the details of vortex shedding at high Reynolds numbers, we experimentally investigated the flow field around a rectangular cylinder and the radiated sound for $AR = 0.3$ to 4.0 at $Re = 7.5 \times 10^3$ to $1.4 \times 10^5$ and Mach numbers $M_{\infty} = 0.03$ to 0.155 in a low-noise wind tunnel. The Reynolds number effects on the flow and sound characteristics were also investigated.

2. Experimental Setup and Procedure

The whole experiment was conducted in an open-return low-turbulence, low-noise wind tunnel. The duct of the wind tunnel was lined with glass wool and punched steel plates. Splinter-type silencers 200-mm-thick were installed upstream and downstream of the axial fan to reduce direct propagation of acoustic noise from the fan. The flow was directed to contraction after six damping screens and a honeycomb in a settling chamber with a 1800 mm $\times$ 1800 mm cross-section. The area ratio of the settling chamber to the nozzle exit with a 500 mm $\times$ 500 mm cross-section was 13. A 1.6-m-long test section was placed in an anechoic chamber that was 4.5 m long, 3.4 m wide and 3 m high. A schematic diagram of the test section is illustrated in Fig. 1. This wind tunnel facility was the same as that used in the experiments by Kobayashi et al.\textsuperscript{20) and Octavianty and Asai.\textsuperscript{21) The maximum flow velocity at the nozzle exit (or inlet of the test section) was 58 m/s and the non-uniformity in the freestream velocity $U_{\infty}$ was less than 1% (in peak-to-peak value). The A-weighted overall sound pressure level (SPL) of the background noise in the frequency range between 20 Hz and 20 kHz was 65.5 dB at $U_{\infty} = 50$ m/s. The residual turbulence (1–200 Hz) was about 0.1% of $U_{\infty}$ at $U_{\infty} = 50$ m/s, and mainly consisted of low-frequency fluctuations below 40 Hz. The magnitude of residual turbulence was almost the same over $U_{\infty} = 30–50$ m/s. During the experiment, the variation in flow temperature was kept within $\pm 0.5^\circ$C by a heat exchanger installed downstream of the axial fan.

A rectangular aluminum cylinder was installed perpendicular to the freestream in the test section. Two large end walls made of wood and Plexiglas, respectively, were used to maintain the two-dimensionality of the main stream. The cylinder was carefully installed between the two end walls such that the cylinder had a zero angle of incidence. Here, small end plates have often been used to control the obliqueness of vortex shedding in experiments for von Kármán vortex street at low Reynolds numbers of $\mathcal{O}(10^3)$; see the review paper by Williamson.\textsuperscript{22) In the present experiment on sound generation in a high Reynolds number turbulent flow past a rectangular cylinder, we focused on natural vortex shedding and did not attach any control device to the cylinder in order to avoid additional (undesirable) noise generation. The streamwise and transverse lengths of the cylinder’s cross-section are denoted by $w$ and $h$, and the aspect ratio was defined as $AR = w/h$ (see Fig. 1(b)). The $AR$ was

![Fig. 1.](image-url)
changed from 0.3 to 4.0 for each transverse length \((h = 10, 20 \text{ and } 40 \text{mm})\); therefore, we could examine the flows at three different Reynolds numbers for each freestream velocity (or each Mach number). The Reynolds number \(Re\) was defined as \(U_\infty h/\nu\), where \(\nu\) is the kinematic viscosity. As for the coordinate system, \(x, y\) and \(z\) are the streamwise, transverse and spanwise directions, respectively, and the origin is located at the center of the cylinder’s cross-section at the mid-span of the cylinder.

A microphone (RION NL-31) was set perpendicular to both the cylinder axis and the mainstream at the mid-span of the cylinder, at a distance of 1 m from the cylinder, to measure the dipole sound. The frequency response of the microphone ranged from 20 Hz to 20 kHz; its output signal was recorded in a computer through a 16-bit analog-to-digital converter with a sampling frequency of 50 kHz.

In order to measure the instantaneous velocity field and to obtain the spanwise vorticity of shed vortices, PIV measurements were performed by illuminating smoke particles 1 \(\mu\)m in diameter with a 1-mm-thick Nd:YAG laser sheet. The pulse duration time of the Nd:YAG laser was 20 ns. The double-pulsed images were captured using a CCD camera with 2048-pixel \(\times\) 2048-pixel resolution (DANTEC FlowSenseEO). The smoke particles were seeded uniformly in the flow from the collector. To examine three-dimensional flow structures around a rectangular cylinder, PIV measurements were conducted for flow fields in the \((x, y)\) and \((x, z)\) planes, with image areas of 200 mm \(\times\) 200 mm and 240 mm \(\times\) 240 mm, respectively. The double-frame images were analyzed using the adaptive correlation algorithm (DANTEC Dynamic Studio) with a final interrogation window of 32 pixels \(\times\) 16 pixels. The interrogation cells were overlapped by 50% in each direction, giving 115 \(\times\) 255 velocity vectors with a spatial resolution of 1.62 mm \(\times\) 0.8 mm in the \((x, y)\) plane and 2.02 mm \(\times\) 1.1 mm in the \((x, z)\) plane. For each PIV measurement, 500 pairs of double-frame images were captured at a sampling interval of 0.25 s.

A constant-temperature hot-wire anemometer was also used to measure the spanwise coherency of vortices shed in the wake. The hot-wire (5 \(\mu\)m tungsten wire) had a sensitive length of 1 mm. A fourth-order polynomial curve was used for hot-wire calibration. Because the flow temperature was well controlled in the current wind tunnel, no temperature correction was necessary. The velocity range from 5 m/s to 50 m/s, the calibration error was less than 0.5% of the maximum velocity. Two I-type probes were used: one was fixed at a certain location and the other was installed on the \(x-z\) traversing mechanism. Hot-wire signals were recorded in a computer through a 16-bit analog-to-digital converter with a sampling frequency of 5 kHz.

3. Sound Radiation from a Rectangular Cylinder

First we confirmed the directivity of dipole sound (Aeolian tone). As shown in the numerical simulation,\(^8\) the Doppler effect shifts the propagation angle of dipole sound by \(\theta_p = \cos^{-1} M\) for low Mach numbers. If the cylinder is in the infinite flow region, the directivity could change by about 9° (\(\theta_p = 81°\)) at the maximum freestream velocity \((M_\infty = 0.155)\). Furthermore, in wind tunnel experiments, refraction of sound waves at the shear layer between the main flow of the wind tunnel and the ambient still air can influence the amplitude and direction of propagating sound. When we measured the sound pressure level (SPL) at 1 m from the cylinder, the (uniform) flow region produced by the wind tunnel (500 \(\times\) 500 mm\(^2\)) in the exit cross-section was limited to only one-fourth the distance between the observation point and the cylinder (sound source). The shear layer thickness 500 mm downstream from the nozzle exit was about 75 mm for \(U_\infty = 52.5\text{ m/s}\). According to an analysis by Amiet,\(^{23}\) the shear layer refraction can change the amplitude and propagation direction of sound waves. However, the amplitude correction is negligible even when the sound wavelength is much smaller than the shear layer thickness; the amplitude correction is at most \(-0.003\) dB even at \(M_\infty = 0.155\). Figure 2 displays the SPLs of sound radiated from a square cylinder (\(h = 10\text{ mm}\)) against the angle \(\theta\) at three different freestream velocities (\(U_\infty = 30, 45\) and 52.5 m/s; \(M_\infty = 0.089, 0.133\) and 0.155, respectively), compared to the directivity of lift-dipole accounting for the Doppler and refraction effects. Here, measurements were taken at a distance of 1 m from the cylinder, and \(\theta\) is the angle between the upstream direction and microphone direction. We see that the experimental data lied on the theoretical curve for all of the freestream velocities. In addition, the SPL was maximum at and around \(\theta = 90°\), even at \(M_\infty = 0.155\). So, for the following sound measurements, the comparisons of SPL for various aspect ratios were made at \(\theta = 90°\).

Figure 3 shows the power spectra of sound pressure for a square cylinder (\(AR = 1.0\)) at \(U_\infty = 52.5\text{ m/s}\) and \(\theta = 90°\). At this freestream condition, the Reynolds numbers were \(3.5 \times 10^4, 7 \times 10^4\) and \(1.4 \times 10^5\) for \(h = 10, 20\) and 40 mm, respectively. The frequency resolution for the power spectral analyses was 0.1 Hz. The figure also includes the background (wind tunnel) noise spectrum without the cylinder for comparison. A prominent narrow-band spectrum was observed at Strouhal numbers \(St = 0.128, 0.126\) and...
0.119 for cylinders of \( h = 10, 20 \) and 40 mm, respectively. Here, \( St \) was defined as \( f h/U_\infty \), and the frequency \( f \) was 670, 332 and 156 Hz for \( h = 10, 20 \) and 40 mm, respectively. Such a prominent spectral peak is no doubt due to the Kármán vortex shedding from the cylinder. It should be noted that the peak spectrum became slightly broader with \( Re \). The Strouhal number was comparable to those in previous experiments.\(^9,24,25\)

Figure 4 illustrates the Strouhal number (\( St \)) against the aspect ratio (\( AR \)) at \( U_\infty = 52.5 \text{ m/s} \) (\( M_\infty = 0.155 \)). For each aspect ratio, measurements were carried out for three different-scale cylinders of \( h = 10, 20 \) and 40 mm. Thus, we could compare the results at three different Reynolds numbers for a fixed freestream velocity (and therefore a fixed Mach number); \( Re = 3.5 \times 10^4, 7 \times 10^4 \) and \( 1.4 \times 10^5 \) for \( h = 10, 20 \) and 40 mm, respectively. As \( AR \) increased, the \( St \) monotonically decreased to 0.08 at \( AR = 2.0 \) for \( Re = 3.5 \times 10^4 \), to 0.06 at \( AR = 2.5 \) for \( Re = 7 \times 10^4 \), and to 0.045 at \( AR = 3.0 \) for \( Re = 1.4 \times 10^5 \). When the \( AR \) was slightly increased for each Reynolds number, the \( St \) discontinuously increased; for \( Re = 3.5 \times 10^4 \), \( St \) increased to more than 0.16 at \( AR = 2.5 \), and decreased with \( AR \) again for \( AR > 3.0 \).

As will be shown later, the shear layers separating at the front corners of the cylinder underwent transition to turbulence and subsequently reattached on the side surfaces of the cylinder between \( AR = 2.5 \) and 3.0. Therefore, the vortex shedding phenomenon may depend on the Reynolds number for these aspect ratios. Except for these particular aspect ratios (\( AR = 2.5 \) and 3.0), the Reynolds number dependence of \( St \) was very weak, at least in the Reynolds number range of \( Re = 7.5 \times 10^3 \) to \( 1.4 \times 10^5 \). Regarding this, Schewe\(^26\) showed experimentally that for both a small aspect ratio \( AR = 0.2 \) and a large aspect ratio \( AR = 5 \), \( St \) was constant over a wide range of Reynolds numbers (\( 2.7 \times 10^4 < Re < 6.4 \times 10^5 \) for \( AR = 0.2 \) and \( 4.0 \times 10^3 < Re < 4.0 \times 10^5 \) for \( AR = 5 \)).

We next focus on how the SPL depends on the aspect ratio. Figure 5 displays the SPLs that correspond to the dipole sound for various \( ARs \) at \( U_\infty = 52.5 \text{ m/s} \) (\( M_\infty = 0.155 \)). Here, the SPL was obtained over the frequency range \( f > 50 \text{ Hz} \). The measurement was conducted using a cylinder of \( h = 20 \text{ mm} \) and \( h = 40 \text{ mm} \) at \( U_\infty = 52.5 \text{ m/s} \), and the Reynolds numbers were \( Re = 7 \times 10^4 \) and \( 1.4 \times 10^5 \), respectively. The SPL peaked at \( AR = 0.75 \), and then monotonically decreased as \( AR \) increased, for \( AR < 2.5 \) at \( Re = 7 \times 10^4 \) and \( AR \leq 3.0 \) at \( Re = 1.4 \times 10^5 \). When \( AR \) increased to 2.5 at \( Re = 7 \times 10^4 \) and \( AR = 3.0 \) at \( Re = 1.4 \times 10^5 \), SPL discontinuously increased to almost the same value as that for \( AR = 0.75 \). Thus, the dependence of \( AR \) on SPL was well correlated with that of \( St \). We examined the SPL for other freestream conditions over \( U_\infty = 11.3 \text{ m/s} \) to 52.5 m/s and obtained similar results.

We expect that the SPL in the far-field can be expressed by Curle’s equation.\(^21\) According to Phillips\(^31\) and Fujita et al.,\(^6\) Curle’s equation for the Aeolian tone from a two-dimensional circular cylinder can be expressed using the Strouhal number and the magnitude (rms value) of lift fluctuations as follows:

\[
\rho^2 = \frac{l_i^2 l_h U_\infty^6 S h^2 L_c}{a_i^2 L_c^2},
\]

where \( \rho^2 \) is the mean square value of the sound pressure, \( \rho_\infty \) is the density, \( l \) is the spanwise length of the cylinder, \( l_i (=m h) \) is the spanwise correlation length (\( m \) is the non-dimensional correlation length), \( C_L' \) is the rms value of the fluctuating lift coefficient, and \( a_\infty \) is the speed of sound.

Figure 6 shows the relation between \( \rho^2 a_i^2 / \rho_\infty^2 \) and \( U_\infty^6 h S h^2 L_c^2 / \rho^2 \) for rectangular cylinders of different aspect ratios, \( AR = 0.3, 0.5, 0.75, 1.0, 2.0, 2.5 \) and 3.0, in a way sim-
ilar to Phillips in the case of a circular cylinder. Here, the sound measurements were done at \( r = 1 \) m for a cylinder of \( h = 20 \) mm. We can see a linear relationship between \( \overline{p^2} \) and \( U_{\infty}^2 \hat{h} S t^2 / r^2 \) for all aspect ratios. In addition, except for small aspect ratios \( AR \leq 0.5 \), the gradients of the curves were almost the same, suggesting that the value of \( I_x C_L^2 \) would be nearly constant for \( AR \geq 0.75 \). These gradients were also much higher than those in the circular cylinder cases presented in the paper by Phillips. The difference between the rectangular and circular cylinders was probably due to larger spanwise coherency and/or larger rms amplitude of lift fluctuation for rectangular cylinders than that for circular cylinders; regarding the corresponding data of \( C_L' \), see Sakamoto et al., Vickery and Lee. For a rectangular cylinder with \( AR = 1.0 \), the experimental result by Vickery at \( Re = 10^5 \) showed that the rms value of fluctuation lift coefficient was three times larger than that in the case of a circular cylinder, which would generate a mean square sound pressure \( \overline{p^2} \) nine times larger than that in the circular cylinder case according to Eq. (1). This surely coincided with our measurement data in Fig. 6, where the sound pressure was plotted in terms of \( \overline{p^2} / \overline{U_{\infty}^2} \). Looking at the comparison between circular and rectangular cylinders, the linear relationship between \( \overline{p^2} \sigma_z^2 / U_{\infty}^2 \) and \( U_{\infty}^2 \hat{h} S t^2 / r^2 \) is still valid for all of the rectangular cylinders.

4. Flow Past a Rectangular Cylinder

In order to understand the strong dependency of the aspect ratio of the cylinder’s cross-section on sound radiation, we examined the flow around the cylinder and the vortex shedding phenomenon using PIV. The observation was conducted for rectangular cylinders with \( h = 20 \) mm at \( U_{\infty} = 52.5 \) m/s \( (Re = 7 \times 10^4) \). Figures 7(a) to (h) compare the distributions of mean velocity and root mean square (rms) value of streamwise velocity fluctuations in the \((x, y)\) plane for \( AR = 0.75, 1.0, 2.5 \) and 3.0. For each aspect ratio, more than 500 PIV data were used. Differences in the reversed flow region and wake width were clearly observed. The reversed flow region was limited to the region of \((x - x_0) / h < 0.5\) for \( AR = 0.75, 1.0 \) and 3.0, while it extended downstream to \((x - x_0) / h \approx 2.5\) for \( AR = 2.5\). The wake width (defined, for example, as the half-value width) in the near wake region also exhibited a distinct difference between the case of \( AR = 2.5 \) and the others. Differences in disturbance development can be understood by comparing the rms distributions. For \( AR = 0.75 \) and 1.0, the maximum rms intensity occurred very close to the cylinder’s rear corners, which no doubt induced high-intensity pressure fluctuations on the cylinder surface. For \( AR = 2.5 \), the intensity of disturbance was maximum at and around \((x - x_0) / h \approx 2.5\), suggesting that the vortex formation occurred away from the cylinder. This is consistent with the weak sound radiation shown in Fig. 5. The disturbance development for \( AR = 3.0 \) seems to be quite different from that of other cases; that is, the disturbance was weak in the wake compared to the other cases. Nevertheless, the SPL was found to be very high. As explained later, the instability of the shear layer originating from the front corners played an important role in sound generation in this case.

Figures 8(a) to (d) illustrate vortex shedding from a rectangular cylinder with \( AR = 0.75, 1.0, 2.5 \) and 3.0, respectively, in terms of instantaneous iso-vorticity contours (the spanwise vorticity) in the \((x, y)\) plane at \( z = 0 \). Here, spanwise vorticity was calculated by means of the finite difference method with second-order accuracy. In all cases, we see that, due to the high Reynolds number, the shed vortices...
were soon diffused by small-scale turbulent eddies. For $AR = 0.75$ where the maximum SPL occurred, the vortex formed very close to the rear surface of the rectangular cylinder. Therefore, it could induce strong anti-symmetric pressure fluctuations on the cylinder surfaces to form a dipole sound source of high intensity. This was also the case for $AR = 1.0$ in Fig. 8(b). In contrast, for $AR = 2.5$, where the SPL was the lowest, the shear layers separating at the front corners extended downstream without reattaching to the side (upper and lower) surfaces of the cylinder, so the wake became wider. More importantly, the Kármán vortex street developed far from the cylinder, as shown in Fig. 8(c). This explains why the lowest-frequency, weakest-intensity wake became wider. More importantly, the Kármán vortex street developed far from the cylinder, so the shear layers separated at the leading-edge reattached on the side surfaces. The intensity of lift-dipole sound also depends on the spanwise coherency of the surface pressure fluctuations associated with vortex shedding. High spanwise coherency of shed vortices over a distance of several reference lengths of cylinder cross-section is responsible for the occurrence of a strong dipole sound in the far-field. Needless to say, we would expect that two-dimensional vortex shedding would give rise to the strongest sound.

To see the behavior of the separated shear layers for the cases $AR = 2.5$ and $3.0$ in more detail, close-up views obtained by PIV are displayed in Figs. 9(a) to (f). For both aspect ratios, the shear layer separating at the front corner (leading-edge) underwent transition to turbulence soon, at a distance of $h$ from the leading-edge. For $AR = 3.0$, interestingly, the shear layer rolled up into large-scale vortices and simultaneously reattached to the cylinder surface. The vortices generated passed adjacent to the rear corner of the cylinder and thus governed the vortex shedding in the wake, contributing to the generation of a strong Aeolian tone. Regarding this phenomenon, Parker and Welsh\textsuperscript{12} and Stokes and Welsh\textsuperscript{12} proposed the phase-locked generation of vortices in the separated shear layer as well as those vortices shed from the rear corner (or wake vortices). Furthermore, Nakamura et al.\textsuperscript{13} reported that the phase-locked phenomenon occurred for a wide range of $AR$s from 3 to 16 at $Re = 1 \times 10^3 - 3 \times 10^4$, and that the Strouhal number based on the streamwise length of the cylinder (i.e., $fw/U_\infty$) was almost constant with the value of about 0.6 for $AR = 3 - 5$. Our experiment also gave an almost constant value of $fw/U_\infty \sim 0.5$, close to Nakamura’s result, for $AR = 3$, 3.5 and 4.0 at $Re = 7 \times 10^4$. When phase-locking occurs, the wake oscillation has to excite the instability of the shear layer separated at the leading-edge, whose wavelength coincides with the streamwise length of the cylinder ($w$). That is, $fw/U_\infty = f\lambda/U_\infty$. Then, because $f\lambda$ denotes the phase velocity of excited instability waves, $fw/U_\infty = 0.5 - 0.6$ gives the phase velocity of about $50 - 60\%$ of $U_\infty$, which is typical for the instability of a free shear layer.

5. Effect of Spanwise Coherency on Sound

The intensity of lift-dipole sound also depends on the spanwise coherency of the surface pressure fluctuations associated with vortex shedding. High spanwise coherency of shed vortices over a distance of several reference lengths of cylinder cross-section is responsible for the occurrence of a strong dipole sound in the far-field. Needless to say, we would expect that two-dimensional vortex shedding would give rise to the strongest sound.

In order to estimate the spanwise coherency quantitatively, the correlation coefficient between streamwise velocity fluctuations at two different $z$ positions, which was a measure of the two-dimensionality of the shed vortices, were examined using two hot-wires. The first hot-wire, fixed at $(x/h, y/h, z/h) = (x_0/h + 1.25, 1.0, 6.5)$, denoted by $A$; the second, denoted by $B$, was traversed in the spanwise direction with the same $x/h$ and $y/h$ coordinates. In other words, the coordinates of $B$ were $B = (x_0/h + 1.25, 1.0, 6.5 + \Delta z)$. The
correlation coefficient \( R_{u_A,u_B} \) is defined as:

\[
R_{u_A,u_B} = \frac{r_{u_A u_B}}{\sqrt{\sigma_{u_A}^2 \sigma_{u_B}^2}}
\]

(2)

where \( u_A \) and \( u_B \) are the streamwise velocity fluctuations at \( A \) and \( B \), respectively. Figure 10 illustrates the correlation coefficients for four aspect ratios, \( AR = 0.75, 1.0, 2.5 \) and 3.0. Here, the measurements were conducted at \( U_\infty = 52.5 \text{ m/s} \) (\( Re = 7 \times 10^3 \)) for the cylinder model with \( h = 20 \text{ mm} \).

We see that the correlation length was rather short for the case of \( AR = 1.0 \) compared to those for the other three cases. The correlation coefficient was larger than 0.7 at \( \Delta z/h < 3.5 \) and 0.5 at \( \Delta z/h < 6 \) for \( AR = 0.75 \), while it was less than 0.5 at \( \Delta z/h > 2 \) for \( AR = 1.0 \). According to the experiment by Fujita,\(^7\) the correlation coefficient took a value larger than 0.7 over the spanwise distance of three diameters for the circular cylinder. Thus, vortex shedding from a square cylinder (\( AR = 1.0 \)) was less coherent than that from a circular cylinder in the spanwise direction. In the cases of \( AR = 2.5 \) and 3.0, the correlation became much stronger. In particular, for \( AR = 3.0 \), the vortex shedding occurred almost two-dimensionally. Consequently, a high correlation coefficient of \( R_{u_A,u_B} \geq 0.9 \) was obtained over the whole span. The high spanwise correlation (for \( AR = 3.0 \)) may have been the result of the onset of phase-locking of the shear layer vortices and wake vortices discussed above. Note that we also confirmed the spanwise coherency of shed vortices by PIV measurements for the cases of \( AR = 0.75, 1.0, 2.5 \) and 3.0.

The three-dimensionality of vortex shedding also influenced the spectra of sound pressure. Figure 11 compares the spectra of sound pressure for \( AR = 0.75, 1.0, 2.5 \) and 3.0 at \( U_\infty = 52.5 \text{ m/s} \) (\( Re = 7 \times 10^3 \)). In the case of \( AR = 1.0 \), where the oblique shedding was distinctly observed, the peak spectrum (corresponding to the vortex shedding frequency) was the broadest among all. The peak spectrum became narrower as \( AR \) increased, and it became like a line spectrum for \( AR = 2.5 \) and 3.0. Thus, the sharpening of the Aeolian tone spectrum really corresponded to the increase in correlation length (shown in Fig. 10).

**6. Conclusion**

Sound radiation from a rectangular cylinder with a cross-sectional aspect ratio (\( AR \)) of 0.3 to 4.0 was examined experimentally at Reynolds numbers ranging from \( 7.5 \times 10^3 \) to \( 1.4 \times 10^5 \) and low Mach numbers below 0.16. Three different-scale cylinders were used for each aspect ratio, which enabled us to see Reynolds number effects under a fixed free-stream velocity (or a fixed Mach number). The main results are summarized below.

1. The mean square sound pressure was proportional to \( U_\infty^6 \cdot St^2 \), with almost the same factor of proportionality for \( AR \)s over 0.75 to 3.0, in spite of the fact that both the spanwise correlation length and the magnitude of lift fluctuations, which were also important quantities to determine the SPL according to Phillips’ formula, were highly dependent on \( AR \).

2. The Reynolds number effect on both SPL and \( St \) was weak except around the critical aspect ratio (\( AR = 2.5 \sim 3.0 \)), at which the reattachment of the shear layer generated at the leading-edge occurred for the Reynolds numbers examined (from \( 7.5 \times 10^3 \) to \( 1.4 \times 10^5 \)).

3. The variation of SPL with \( AR \) was strongly correlated to the variation in Strouhal number, except for \( AR < 0.75 \). After reaching a maximum at \( AR = 0.75 \), the SPL of the Aeolian tone decrease monotonically with decreasing \( AR \) below 0.75, in spite of \( St \) continuing to increase (i.e., even for \( AR < 0.75 \)) as \( AR \) decreased. This is because decreasing the area of the cylinder’s upper and lower surfaces weakened the intensity of the lift dipole.

4. The spanwise coherency of shed vortices was well correlated with the sharpness (broadness) of the sound-pressure spectra. When vortex shedding at the trailing-edge was locked in phase with the vortices generated at the leading-edge, the shed vortices were quite two-dimensional, and cor-
respondingly, the radiated sound exhibited a very sharp spectral peak.

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