Different genomic relationship matrices for single-step analysis using phenotypic, pedigree and genomic information

Selma Forni1*, Ignacio Aguilar2,3, Ignacy Misztal3

Abstract

Background: The incorporation of genomic coefficients into the numerator relationship matrix allows estimation of breeding values using all phenotypic, pedigree and genomic information simultaneously. In such a single-step procedure, genomic and pedigree-based relationships have to be compatible. As there are many options to create genomic relationships, there is a question of which is optimal and what the effects of deviations from optimality are.

Methods: Data of litter size (total number born per litter) for 338,346 sows were analyzed. Illumina PorcineSNP60 BeadChip genotypes were available for 1,989. Analyses were carried out with the complete data set and with a subset of genotyped animals and three generations pedigree (5,090 animals). A single-trait animal model was used to estimate variance components and breeding values. Genomic relationship matrices were constructed using allele frequencies equal to 0.5 (G05), equal to the average minor allele frequency (GMF), or equal to observed frequencies (GOF). A genomic matrix considering random ascertainment of allele frequencies was also used (GOF*). A normalized matrix (GN) was obtained to have average diagonal coefficients equal to 1. The genomic matrices were combined with the numerator relationship matrix creating H matrices.

Results: In G05 and GMF, both diagonal and off-diagonal elements were on average greater than the pedigree-based coefficients. In GOF and GOF*, the average diagonal elements were smaller than pedigree-based coefficients. The mean of off-diagonal coefficients was zero in GOF and GOF*. Choices of G with average diagonal coefficients different from 1 led to greater estimates of additive variance in the smaller data set. The correlation between EBV and genomic EBV (n = 1,989) were: 0.79 using G05, 0.79 using GMF, 0.78 using GOF, 0.79 using GOF*, and 0.78 using GN. Accuracies calculated by inversion increased with all genomic matrices. The accuracies of genomic-assisted EBV were inflated in all cases except when GN was used.

Conclusions: Parameter estimates may be biased if the genomic relationship coefficients are in a different scale than pedigree-based coefficients. A reasonable scaling may be obtained by using observed allele frequencies and re-scaling the genomic relationship matrix to obtain average diagonal elements of 1.

Background

Traditional genetic evaluation of livestock combines only phenotypic data and probabilities that genes are identical by descent using the pedigree information. Genetic markers for many loci across the genome can be used to measure genetic similarity and may be more precise than pedigree information [1]. Genomic relationships can better estimate the proportion of chromosomes segments shared by individuals because high-density genotyping identifies genes identical in state that may be shared through common ancestors not recorded in the pedigree. A genomic relationship matrix (G) can be calculated by different methods [1,2].

As an entire population is unlikely to be genotyped in livestock species, Legarra et al. [3] and Misztal et al. [4] have proposed the integration of the numerator relationship matrix (A) and G into a single matrix (H). A BLUP evaluation using H called single-step genomic evaluation...
has been successfully applied in dairy cattle [5]. Besides the computation of H, no further modifications in the standard mixed model equations used in animal breeding have been needed [4].

The formula for H includes the expression G - A, which is the difference between genomic and pedigree-based relationships. If G is inflated, deflated or in some other way incompatible with A, the weighting of the pedigree and genomic information will be incorrect. Various G used in a genetic evaluation by Aguilar et al. [5] have resulted in different scaling and accuracies of EBV. Estimates of the additive variance using G may be much larger than those using A [6]. Different G can lead to different accuracies of EBV [5]. These differences could be due to an incorrect scaling of G relative to A.

The first objective of this study was to apply different genomic matrices to analyses of litter size in a swine population and evaluate the impact of those G on EBV and estimates of variance components. The second objective was to develop a strategy to create an optimal G that is easy to create and yields reasonably accurate EBV and estimates of the additive variance.

Methods

Data

Data of litter size (total number born per litter) for 338,346 sows, of which 1,919 were genotyped using the Illumina PorcineSNP60 BeadChip, were analyzed. Genotypes of their 70 sires were also available. Genotyped females were crosses of two pure lines derived from the same breeds, and they were born in a two-year span. After quality control procedures, 44,298 markers remained and were used to estimate genomic relationship coefficients. In the quality control analysis, SNP were excluded if: the minor allele frequency was smaller than 0.05, the marker mapped to the sex chromosomes, or 2 if the genotype was homozygous 22. Frequencies should be those from the unselected base population, or 2 if the genotype was homozygous 22. Frequencies were obtained following VanRaden [1]:

\[
G = \frac{(M - P)(M - P)^{\prime}}{2 \sum_{j=1}^{m} p_j(1 - p_j)},
\]

where M is an allele-sharing matrix with m columns (m = total number of markers) and n rows (n = total number of genotyped individuals), and P is a matrix containing the frequency of the second allele \(p_j\), expressed as \(2p_j\), \(M_{ij}\) was 0 if the genotype of individual \(i\) for SNP \(j\) was homozygous 11, was 1 if heterozygous, or 2 if the genotype was homozygous 22. Frequencies should be those from the unselected base population, but this information was not available. Instead the frequencies used were: 0.5 for all markers \((G05)\), the average minor allele frequency \((GMF)\), and the observed allele frequency of each SNP \((GOF)\). The last option assured that the average off-diagonal element was close to 0. For GMF only, the second allele was the one with smaller frequency.

A different matrix with observed frequencies \((GOF^*)\) was obtained by modification of the denominator as in Gianola et al. [7]:

\[
GOF^* = \frac{(M - P)(M - P)^{\prime}}{(p_0 - q_0)^2 + 2 \sum_{j=1}^{m} p_j(1 - p_j) \left(\frac{\alpha + \beta + 2}{\alpha + \beta}\right) m},
\]

where \(p_0\) and \(q_0\) are expectations of allele frequencies following a Beta distribution with hyperparameters \(\alpha\) and \(\beta\). The values for the hyperparameters were the same as observed in the genotyped animals.
A normalized matrix was obtained to have average diagonal coefficients equal to 1:

$$GN = \frac{(M - P)(M - P)^{\prime}}{\text{trace}[(M - P)(M - P)^{\prime}]/n}.$$  \hspace{1cm} (4)

The denominator should assure compatibility with A when either the average inbreeding is low or the number of generations low. Higher levels of inbreeding in the genotyped population can be accommodated by substituting “n” in the denominator of GN by the sum of (1 + F) across genotyped animals, where F are individual inbreeding coefficients derived from pedigree. Different from the numerator relationship matrix, values on the diagonal of GN can be smaller than 1. An average diagonal of 1 can also be obtained by multiplying (4) by a constant. A similar relationship matrix with sample variance of 1 can also be obtained by multiplying (4) by a constant. A normalized matrix was obtained to have average diagonal elements equal to 1:

$$\sigma^2_a = \frac{\sum \sigma^2_a}{n}.$$  \hspace{1cm} (5)

The joint distribution of breeding values of genotyped (a1) and non-genotyped animals (a2) is:

$$\left( \begin{array}{c} a_1 \\ a_2 \end{array} \right) \sim \left( \begin{array}{c} 0 \\ A_{11} + A_{12}A_{22}^{-1}(G - A_{22})A_{22}^{-1}A_{21} \end{array} \right) \left( \begin{array}{c} \sigma^2_a \\ \sigma^2_a \end{array} \right) \left( \begin{array}{c} a_1 \\ a_2 \end{array} \right) + G \sigma^2_a.$$  \hspace{1cm} (5)

and the variances of the conditional posterior distributions are:

$$\text{var} \left( a_1 \mid a_2, \sigma^2_a, \sigma^2_e, y \right) =$$

$$\left[ A_{11} + A_{12}A_{22}^{-1}(G - A_{22})A_{22}^{-1}A_{21} \right]^{-1} \sigma^2_a,$$

$$\text{var} \left( a_2 \mid a_1, \sigma^2_a, \sigma^2_e, y \right) = G^{-1} \sigma^2_a.$$  \hspace{1cm} (6a)

The additive variance is on average the same for the entire population, and coefficients of A and G need to be compatible in scale. Variance components were estimated by restricted maximum likelihood (REML) using the EM algorithm [10].

### Results

Pedigree-based and genomic relationship coefficients

Statistics of pedigree-based and genomic relationship coefficients for genotyped animals (A22 or G) are in Table 1. In G05 and GMF, the same allele frequency was used for all markers, and the average of both diagonal and off-diagonal elements was greater than the coefficients in A22. The average minor allele frequency was 0.26. The distribution of frequencies of the second allele was nearly flat (Figure 1). For GOF and GOF*, the average diagonal coefficients were smaller than the pedigree-based coefficients. The average off-diagonal coefficients were zero in both matrices, similar to A22. This allowed obtaining a matrix with average diagonal elements equal to 1 (GN) and average off-diagonal elements equal to zero. For all genomic matrices, diagonal coefficients had greater variance than the pedigree-based coefficients. Off-diagonal genomic coefficients had a greater variance only for GOF and GN. Greater variance was expected between the elements of G than A because genomic relationships reflect the actual fraction of genes shared whereas pedigree-based coefficients are predictions. Predictions have smaller variance than the variable predicted when the prediction error is not zero.

### Table 1 Statistics of relationship coefficients estimated using pedigree and genomic information

| Diagonal elements | Mean   | Minimum | Maximum | Variance |
|-------------------|--------|---------|---------|---------|
| A                 | 1.000  | 1.000   | 1.075   | 0.0003  |
| G05               | 1.253  | 1.178   | 1.462   | 0.00083 |
| GMF               | 1.697  | 1.632   | 1.894   | 0.00073 |
| GOF               | 0.936  | 0.837   | 1.228   | 0.00176 |
| GOF*              | 0.505  | 0.436   | 0.663   | 0.00051 |
| GN                | 1.002  | 0.895   | 1.314   | 0.00201 |

| Off-diagonal elements | Mean   | Minimum | Maximum | Variance |
|-----------------------|--------|---------|---------|---------|
| A                     | 0.032  | 0.000   | 0.600   | 0.00172 |
| G05                   | 0.595  | 0.387   | 1.231   | 0.00160 |
| GMF                   | 1.022  | 0.822   | 1.654   | 0.00155 |
| GOF                   | 0.000  | -0.198  | 1.000   | 0.00241 |
| GOF*                  | 0.000  | -0.105  | 0.540   | 0.00070 |
| GN                    | 0.000  | -0.212  | 1.070   | 0.00275 |

Relationships between genotyped animals (1,989 diagonal and 3,954,132 off-diagonal elements).
Variance components

Estimates of variance components obtained with the full data set are in Table 2, and estimates from the subset are in Table 3. The differences observed in the complete data set were negligible, most likely because genomic relationships were a small fraction of all relationships. Compared to estimates obtained with A, most of the additive variance estimates using the genomic relationships in the smaller dataset were inflated. The inflation was approximately inversely proportional to the difference between the average diagonal and the off-diagonal elements of G. The highest inflation was with GOF*, for which this difference was only 0.51. The additive variance estimates were the same for G05 and GMF despite different averages but with similar differences between average diagonal and off-diagonal elements, 0.66 and 0.68, respectively. Estimates in the smaller data were similar using A and GN, which had very similar diagonal and off-diagonal element averages. Legarra et al. [3] have demonstrated that a normalized genomic matrix, as GN = G/trace(G), allows the same expectation of variance for breeding values of genotyped and non-genotyped animals. Assuming that a genomic relationship matrix standardized such as GN produces realistic estimates of additive variance, the use of genomic information resulted in smaller standard errors (0.30) than only pedigree information (0.44).

Breeding values and accuracies

Estimates of breeding values for genotyped animals were on average similar regardless the choice of G. Table 4 presents correlations between breeding values obtained with different relationship matrices. Small differences were observed in the ranks obtained with different genomic matrices. However, these differences have direct implications on selection decisions and genetic progress. For instance, if 597 animals (top 30%) were selected using GN, 456 animals among the 597 would also be

Table 2 Variance components estimates for litter size using pedigree and genomic relationship coefficients

|          | Additive Variance (ste$^1$) | Residual Variance (ste$^1$) |
|----------|-----------------------------|-----------------------------|
| A        | 1.26 (± 0.03)               | 6.66 (± 0.02)               |
| G05      | 1.28 (± 0.03)               | 6.65 (± 0.03)               |
| GMF      | 1.28 (± 0.03)               | 6.65 (± 0.03)               |
| GOF      | 1.27 (± 0.03)               | 6.65 (± 0.03)               |
| GOF*     | 1.30 (± 0.03)               | 6.64 (± 0.03)               |
| GN       | 1.27 (± 0.03)               | 6.65 (± 0.03)               |

Full data set (n = 338,346), $^1$ste = standard error.

Table 3 Variance components estimates for litter size using pedigree and genomic relationship coefficients

|          | Additive Variance (ste$^1$) | Residual Variance (ste$^1$) |
|----------|-----------------------------|-----------------------------|
| A        | 2.27 (± 0.52)               | 5.30 (± 0.44)               |
| G05      | 3.43 (± 0.56)               | 5.25 (± 0.29)               |
| GMF      | 3.43 (± 0.56)               | 5.25 (± 0.30)               |
| GOF      | 2.41 (± 0.39)               | 5.29 (± 0.30)               |
| GOF*     | 4.46 (± 0.73)               | 5.22 (± 0.30)               |
| GN       | 2.25 (± 0.36)               | 5.30 (± 0.30)               |

Subset of genotyped animals (n = 1,919), $^1$ste = standard error.
selected using A. For other genomic matrices, the number of animals selected in common with GN was: 567 for G05, 568 for GMF, 593 for GOF, and 554 for GOF*.

Correlations between pedigree-based EBV and EBV obtained using either G05 or GN were similar. When applied to dairy data, Aguilar et al. [5] have found substantially higher accuracies for G with allele frequencies equal to 0.5 than with either current or estimated base allele frequencies. When the allele frequency is p, the relative contribution to the diagonal of G is (2p)^2 for the first homozygote, (1-2p)^2 for a heterozygote, and (2-2p)^2 for the second homozygote. With p = 0.5, these contributions are 1, 0, and 1, respectively. When the allele frequencies are assumed different from 0.5, these contributions are different for each homozygote. For example, contributions with p = 0.2 would be 0.16 for the first homozygote, 0.36 for the heterozygote, and 2.56 for the second homozygote. Consequently, rare alleles contribute more to the variance than common alleles. It would be interesting to compare the results with a normalized matrix from G05 by multiplying and deducting a constant as in VanRaden [1]. However, in our experience such matrices were not positive definite. Subtracting of a constant from G might be helpful if this does not create a negative eigenvalue.

Statistics on computed breeding values with various relationship matrices are in Table 5. The means can be clustered in two groups, one for matrices based on the observed allele frequencies where the average off-diagonal is 0, and another for the remaining matrices. When the average off-diagonals were larger than zero, all genotyped animals were related with positive coefficients. The assumption that all animals are related may create biases especially when animals of interest have both phenotypes and genotypes. The exact impact of large off-diagonals is a topic for future research.

Estimates of accuracy obtained using PEV with different genomic matrices are in Table 6. On average, the increase of accuracy from genomic information was for genotyped animals only. The increases were higher for females because of their lower initial accuracy. The accuracies varied depending on the genomic matrix used. Assuming that additive variance and accuracy estimates are most realistic with GN, the accuracies using non-normalized G were inflated. VanRaden et al. [11] have presented computed and realized genomic accuracies for a number of traits, and found the computed accuracies to be inflated.

### Discussion

Pedigrees may include many generations into the history of the population but must end eventually. In standard genetic evaluations, founder animals are the earliest generation recorded and the assumption is that they do not share genes from older ancestors. Relationship and inbreeding coefficients from later generations are estimated as deviations from the founders’ relatedness. Genomic analysis typically reveals that founder animals actually share genes identical by descent, which shift relationship and inbreeding coefficients up or down. Genomic and pedigree-based matrices should be compatible in scale to be integrated. Ideally, genomic relationships should be estimated using the allele frequencies.

### Table 4: Correlations between estimated breeding values using different relationship matrices

|       | A    | G05  | GMF  | GOF  | GOF* | GN   |
|-------|------|------|------|------|------|------|
| A     | 0.798| 0.798| 0.793| 0.799| 0.791|
| G05   | 0.891| 1.000| 0.995| 0.997| 0.993|
| GMF   | 0.891| 1.000| 0.995| 0.997| 0.994|
| GOF   | 0.891| 0.997| 0.997| 0.989| 0.999|
| GOF*  | 0.891| 0.996| 0.996| 0.999| 0.996|
| GN    | 0.888| 0.998| 0.998| 0.997| 0.986|

Genotyped females above diagonal (n = 1,919).
Genotyped males below diagonal (n = 70).

### Table 5: Statistics of estimated breeding values using pedigree and genomic information

|       | Genotyped females (n = 1,919) | Genotyped males (n = 70) |
|-------|-------------------------------|--------------------------|
|       | Mean  | Minimum | Maximum | Variance | Mean  | Minimum | Maximum | Variance |
| A     | 0.359 | -2.755  | 2.282   | 0.467    | 0.159 | -4.097  | 2.847   | 1.185    |
| G05   | 0.372 | -2.898  | 2.501   | 0.443    | 0.135 | -3.717  | 2.525   | 0.996    |
| GMF   | 0.372 | -2.904  | 2.505   | 0.444    | 0.135 | -3.722  | 2.524   | 0.998    |
| GOF   | 0.165 | -3.623  | 2.660   | 0.566    | 0.165 | -2.829  | 2.110   | 0.376    |
| GOF*  | 0.165 | -3.697  | 2.707   | 0.589    | 0.165 | -3.688  | 2.180   | 1.178    |
| GN    | 0.165 | -4.502  | 2.522   | 0.905    | -0.074| -4.502  | 2.522   | 0.905    |

### Table 6: Average accuracy estimates for breeding values using pedigree and genomic relationship coefficients

|       | Full pedigree (n = 382,988) | Genotyped females (n = 1,919) | Genotyped sires (n = 70) |
|-------|-----------------------------|-------------------------------|--------------------------|
| A     | 0.21                        | 0.22                         | 0.62                     |
| G05   | 0.21                        | 0.37                         | 0.63                     |
| GMF   | 0.21                        | 0.43                         | 0.64                     |
| GOF   | 0.21                        | 0.49                         | 0.63                     |
| GOF*  | 0.21                        | 0.43                         | 0.66                     |
| GN    | 0.21                        | 0.28                         | 0.63                     |
from the unselected base population. This information can be rarely extracted from historical data and approximations must be used. Errors in the allele frequency estimates may result in biased relationships and consequently biased GEBVs, especially for young animals [5]. Yang et al. [12] have proposed a genomic relationship matrix that uses the genotyped animals as the base population. They have presented a slightly different formulation than used here for the diagonal elements of $G$. Using the genotyped population as base, $A$ would have to be re-scaled according to $G$ but allele frequencies in the base population would not have to be estimated.

Coefficients of $GN$ had greater variance than the corresponding elements of $A_{22}$. The variance was greater because individuals equally related in the pedigree have more or less alleles in common than expected. Genomic analysis achieved higher accuracies probably because genomic information improved prediction of the Mendelian sampling terms. More differentiation within families and reduction of co-selection of sibs are expected with genomic-assisted selection because Mendelian sampling can be better estimated. As a result, inbreeding across generations is expected to increase more slowly than it would increase with standard evaluations [13].

We considered only phenotypes of crossbred animals. The performance of crossbred animals is considered a different trait than the performance of purebred animals in routine evaluations of this population. Using a multi-trait model, one can predict EBV for elite animals as parents at the nucleus and commercial level simultaneously. However, only additive inheritance is considered in this model and differences in allele frequencies between pure lines are ignored. Cantet and Fernando [14] have shown that ignoring segregation variance could lead to unbiased predictions that do not have the minimum variance. More suitable models should be used to account for heterosis when the objective is to rank crossbred animals [15,16].

Estimates of additive variance were sensitive to the choices of $G$ when a greater part of the pedigree was genotyped. An entire genotyped population is rarely found in livestock species, and pedigree and genomic information have to be combined. Estimates of relationships are always relative to an arbitrary base population in which the average relationship is zero. Genomic and pedigree-based relationships must be relative to the same base to be combined in the $H$ matrix. We chose to use the animals with unknown parents in $A$ as the base, and we modified $G$ accordingly. Because there were no changes in the genetic base, the same additive variance is expected when including the genomics coefficients. A practical solution to avoid inflation of the additive variance is to re-scale $G$ to obtain average diagonal elements equal to 1, when off-diagonal elements are already on average zero. In the data set analyzed, average off-diagonal elements equal to zero were obtained using the observed allele frequencies.

Several studies have indicated accuracy gains with the inclusion of genomic information in genetic evaluations via marker regression or identical-by-descent matrices [11,17,18]. However, some experiences in the dairy industry, however, have indicated that actual improvement may differ from expected because of inflation of genomic breeding values and reliabilities [5,11]. Biases in genomic predictions can be related to incorrect weighting of polygenic and genomic components. The combined pedigree-genomic relationship matrix provides a natural way to weight both components for optimal predictions. In addition, a single-step genomic evaluation eliminates a number of assumptions and parameters required in multiple-step methods, and possibly delivers more accurate evaluations for young animals. The single-step procedure can be easily extended for multiple-trait analysis, and can handle large amounts of genomic information. Extensions to account for other distributions of marker effects, i.e., large QTL or major genes, are also possible [19,20]. Nevertheless, computational efforts may be an issue long-term because the genomic matrix needs to be created and inverted.

**Conclusions**

Estimates of the additive genetic variance with pedigree or joint pedigree-genomic relationships are similar when the differences between the average diagonal and the average off-diagonal elements in $G$ are similar to those in $A$. Adding the genomic information to $A$ results in lower standard errors of additive variance estimates. Accuracies of EBV with the pedigree-genomic matrix are a function not only of the average of diagonal and off-diagonal elements of $G$, but also of the difference between these averages. The accuracy estimates may be inflated with non-normalized $G$. Matrix compatibility can be obtained by using observed allele frequencies and re-scaling the genomic relationship matrix to obtain average diagonal elements equal to 1. If allele frequencies in the base population are different from 0.5, rare alleles contribute more to the genetic resemblance between individuals than common alleles.

**Acknowledgements**

The authors appreciate the efforts of Dr. David McLaren that made possible the partnership between Genus Plc and the University of Georgia.

**Author details**

1. Genus Plc, Hendersonville, TN, USA. 2. Instituto Nacional de Investigación Agropecuaria, La Brusca, Uruguay. 3. Department of Animal and Dairy Science, University of Georgia, Athens, GA, USA.
Authors’ contributions
SF performed data edition, statistical analysis and drafted the manuscript. IA developed scripts for genomic computations and helped in statistical analysis. WM provided core software, mentored statistical analysis and made substantial contributions for the results interpretation. All authors have been involved in drafting the manuscript, revising it critically and approved the final version.

Competing interests
The authors declare that they have no competing interests.

Received: 3 June 2010 Accepted: 5 January 2011
Published: 5 January 2011

References
1. VanRaden PM. Efficient methods to compute genomic predictions. J Dairy Sci 2008, 91:4414-4423.
2. Gianola D, van Kaam BCHM. Reproducing kernel Hilbert spaces regression methods for genomic prediction of quantitative traits. Genetics 2008, 178:2289-2303.
3. Legarra A, Aguilar I, Misztal I. A relationship matrix including full pedigree and genomic information. J Dairy Sci 2009, 92:4656-4663.
4. Misztal I, Legarra A, Aguilar I. Computing procedures for genetic evaluation including phenotypic, full pedigree, and genomic information. J Dairy Sci 2009, 92:4648-4655.
5. Aguilar I, Misztal I, Johnson D, Legarra A, Tsutsuia T, Lavello T. A unified approach to utilize phenotypic, full pedigree, and genomic information for genetic evaluation of Holstein final score. J Dairy Sci 2010, 93:743-752.
6. Snelling WM, Kuehn LA, Thallman RM, Keele JW, Bennett GL. Genomic heritability of beef cattle growth. Proceedings of the Annual Meeting of the American Society of Animal Science 12-16 July 2009, Montreal [http://adsa.asas.org/meetings/2009/abstracts/093.PDF].
7. Gianola D, de los Campos G, Hill WG, Manfredi E, Fernando R. Additive genetic variability and the Bayesian alphabet. Genetics 2009, 183:347-363.
8. Kang MH, Sul JH, Service SK, Zaitlen NA, Kong S, Freimer NB, Sabatti C, Eskin E. Variance component model to account for sample structure in genome-wide association studies. Nature Genetics 2010, 42:348-354.
9. Christensen OF, Lund MS. Genomic prediction when some animals are not genotyped. Genet Sel Evol 2010, 42:2 [http://www.gsejournal.org/content/42/1/2].
10. Dempster AP, Laird NM, Rubin DB. Maximum likelihood from incomplete data via the EM algorithm. J R Soc Stat B 1977, 39:1-38.
11. VanRaden PM, Van Tassell CP, Wiggans GR, Sonstegard TS, Schnabel RD, Taylor JF, Schenkel FS. Invited review: Reliability of genomic predictions for North American Holstein bulls. J Dairy Sci 2009, 92:16-24.
12. Yang J, Benyamin B, McEvoy BP, Gordon S, Henders AK, Maniourque A, Cierget-Darpoux F, Thompson EA. Estimation of the inbreeding coefficient through use of genomic data. Amer J Hum Genet 2008, 91:516-523.
13. VanRaden PM, Van Tassell CP, Wiggans GR, Sonstegard TS, Schnabel RD, Taylor JF, Schenkel FS. Invited review: Reliability of genomic predictions for North American Holstein bulls. J Dairy Sci 2009, 92:16-24.
14. Cantet RJC, Fernando RL. Prediction of breeding values with additive animal models for crosses from 2 populations. Genet Sel Evol 1995, 27:323-334.
15. Quaas RL. Additive models with groups and relationships. J Dairy Sci 1988, 71:1338-1349.
16. Lo LL, Fernando RL, Cantet RJC, Grossman MG. Theory for modeling means and covariances in a 2 breed population with dominance inheritance. Theor Appl Genet 1995, 90:49-62.
17. Villanueva B, Pong-Wong R, Fernandez J, Toro MA. Benefits from marker-assisted selection under an additive polygenic genetic model. J Anim Sci 2005, 83:1747-1752.
18. Hayes B, Visscher P, Goddard M. Increased accuracy of artificial selection by using the realized relationship matrix. Genet Res 2009, 91:47-60.
19. Leutenegger AL, Prum B, Grin E, Verny C, Lemaire A, Clerget-Darpoux F, Thompson E. Estimation of the inbreeding coefficient through use of genomic data. Amer J Hum Genet 2008, 91:516-523.
20. Amin N, van Duijn CM, Aulchenko YS. A genomic background based method for association analysis in related individuals. PLoS ONE 2007, 2(12):e1274.