Laminar forced convection heat transfer to a single layer of ordered and disordered spheres

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Abstract. We study laminar forced convection heat transfer to single layer arrays of equidistantly and non-equidistantly spaced spheres. We report average Nusselt numbers as a function of geometry and flow conditions, for open frontal area fractions between 0.04 and 0.95, Prandtl numbers between 0.7 and 10, and Reynolds numbers (based on sphere diameter and the free stream velocity) between 0.1 and 100. For equidistantly spaced arrays of spheres we propose a general analytical expression for the average Nusselt number as a function of the Reynolds number, Prandtl number and the open frontal area fraction, as well as asymptotic scaling rules for small and large Reynolds. For all studied Prandtl numbers, equidistant arrays exhibit decreasing average Nusselt numbers for decreasing open frontal area fractions at low Reynolds numbers. For high Reynolds numbers, the Nusselt number approaches that of a single sphere in cross-flow, independent of the open frontal area fraction. For equal open frontal area fractions, the Nusselt number in non-equidistant arrays is lower than in equidistant arrays for intermediate Reynolds numbers. For very low and high Reynolds numbers, non-uniformity does not influence heat transfer.

Keywords: heated spheres, disordered arrays, low Reynolds number, Nusselt number.

1. Introduction
Mass and heat transfer to densely or loosely packed arrays of spheres has been the subject of several studies [1, 2, 3]. The effects of sphere-to-sphere spacing, packing structure, and bed void fraction on the average and local heat and mass transfer have been the main objectives of these studies.

Many studies have been devoted to mass and heat transfer to linear arrays of spheres aligned with the flow direction [4, 5, 6, 7, 8, 9]. These experimental and computational studies cover a range of free-stream, particle based Reynolds numbers from \( Re = 1 - 1700 \), and a range of Schmidt (or Prandtl) numbers from \( Sc = 0.7 \) to 70. The number of particles in the linear array varied from 2 to 8, and the inter-particle distance was varied from 1 – 10 particle diameters. Ramachandran et al. [4] reported how the average Nusselt number \( Nu \) of leading spheres may exceed that of a single sphere, whereas \( Nu \) numbers of subsequent spheres are always less than that of an isolated sphere. Tsai et al. [5] used an embedded grid to simulate heat and momentum transfer to arrays of spheres aligned along the flow direction in a \( Re \) range between 5 and 100. They found that the average \( Nu \) decreases for increasing inter-sphere spacings. By using a finite-element method, Chiang et al. [6] studied steady laminar axisymmetric flow past
closely spaced monodisperse droplets of constant diameter for $Re$ between 10 and 200. It was found that for decreasing distance between droplets, the heat transfer coefficient of each droplet decreases. Experimental and numerical studies conducted by Wang et al. [7] for heat transfer for fluid around heated solid spheres in tandem, showed that the average $Nu$ increases for increasing inter-sphere distances. Lloyd et al. [8] for forced convection heat transfer to a linear array of eight equidistantly spaced spheres, found that $Nu$ decreases for decreasing distances between spheres and decreasing Reynolds numbers. A relaxation technique used by Aminzadeh et al. [9] to solve the conservation of species equation and obtain the mass transfer rates around two equally sized spheres arranged in tandem, showed that the average Sherwood number $Sh$ is always less than that of a single sphere for low Peclet numbers $Pe$. Pei et al. [10] experimentally studied heat transfer between two adjacent spheres aligned perpendicularly to the flow in turbulent flow regimes. It was found that when the spheres are sufficiently close to cause interacting wakes, the combined blockage of the two wakes results in higher heat transfer rates.

Fewer studies have been devoted to the effect of the precise particle packing structure on heat and mass transfer in packed beds of spheres. In their classical experimental study, Thoenes and Kramers [11] studied mass transfer to a single active sphere placed in various regular packing arrangements of similar, but inactive spheres. For $Re$ based on the hydraulic diameter between 1 and $10^4$ and different gases and liquids as working fluids, it was found that $Sh$ increases with increasing inter-sphere distance between active and inert spheres. For $Re$ between 100 and 700 Dixon et al. [12] experimentally investigated the influence of particle to tube diameter fixed ratio on the heat transport process in fixed beds. Spheres of steel, porous ceramic and nylon were tested in several columns of different diameter showing that the $Nu$ at the wall depends on the ratio of particle and tube diameters rather than those diameters themselves. In a computational study of the fluid flow through packed beds of spheres in simple cubic, rhombohedral and face or body-centred structures at particle based $Re$ numbers between 12 and 2000, Gunjal et al. [13] found that the local $Nu$ increases with increasing particle $Re$ and increasing bed voidage.

In comparison to the rather vast amount of literature on single spheres, 1-D linear arrays and 3-D packed beds of spheres, convective heat and mass transfer to 2-D arrays of spheres arranged in a single plane perpendicular to the flow direction has received little attention. This configuration is of great practical importance for applications such as low pressure drop activated carbon filters [14] and Chemical-Biological-Radiological and Nuclear protective clothing [15].

In the present paper, we performed numerical simulations of the (steady-state or oscillating, depending on $Re$) laminar incompressible flow and heat transfer around spheres arranged in an ordered or disordered single layer perpendicular to the flow direction, and we present averaged, steady-state or time-averaged, heat transfer rates from the flow to the spheres. The free stream $Re$, based on the sphere diameter and the free stream velocity, was varied between 0.1 and 100, and the $Pr$ between 0.7 and 10.

2. Numerical approach

2.1. Studied configurations and boundary conditions

Figure 1 shows the computational domains for the studied configurations. The total length of the computational domain along the streamwise direction $x$ is $L_1 + 41d$, out of which $L_1$ upstream, and $40d$ downstream from the layer of spheres. The upstream length $L_1$ was chosen long enough to have a purely conductive heat inflow through the inlet of the domain, i.e. $L_1 > 10a/u_\infty$ where $a$ is the fluid thermal diffusivity and $u_\infty$ the free stream velocity. In the inlet, on the left-hand side of the domain, we imposed a constant temperature $T_\infty$, and a uniform inlet velocity $u_\infty$. The outlet plane was considered to be sufficiently far downstream from the layer of spheres, such
that a constant pressure and uniform streamwise velocity could be assumed constant across the outlet.

The spheres are solid and non-permeable and all have the same, constant diameter \( d \). On the sphere walls a constant temperature \( T_w \) and no-slip conditions for the velocity were imposed. In the case of non-equidistantly spheres the bounding sides of the rectangular computational domain aligned with the streamwise direction \( x \), were specified as periodic boundaries. For layers of equidistantly spaced spheres, the domain size reduces to an \( 1/8 \) th of the domain with symmetry boundary conditions applied at the three bounding sides.

The domain size \( H \) is set by the prescribed open frontal area fraction \( \epsilon \), according to

\[
\epsilon = 1 - \frac{n_T \pi}{4} \left( \frac{d}{H} \right)^2 \tag{1}
\]

where \( n_T \) equals 1 for ordered layers, whilst we used \( n_T = n_L^2 = 16 \) for the disordered layers.

The fluid properties (density \( \rho \), viscosity \( \mu \), thermal conductivity \( \lambda \) and specific heat \( C_p \)) were assumed to be constant and buoyancy effects were neglected. The free stream Reynolds number was defined as \( Re_\infty = \rho u_\infty d/\mu \), the Prandtl number as \( Pr = (\mu/\rho)/a \). We also define a Reynolds number \( Re_L = \rho(u_\infty/\epsilon)d/\mu = Re_\infty/\epsilon \) based on the average velocity \( u_\infty/\epsilon \) in the plane of smallest cross section.

To generate patterns with a controlled level of disorder, the following procedure was applied, see Fig. 2: first, the spheres are placed on a square grid of \( n_L \) columns by \( n_L \) rows. In this configuration the inter-sphere distances are constant and equal to \( \delta_y = \delta_z = \delta = H/n_L - d \).
Then, for each column, we redistribute the spheres such that the standard variation in the inter-sphere distances \( \sigma = \alpha \delta \), where \( \alpha \) is our disorder control variable, which equals zero for ordered arrays of spheres, whereas larger values of \( \alpha \) lead to increased disorder. For given values of \( n_L \), \( \delta \) and \( \alpha \), an infinite number of distributions can be generated that obey the following equations for \( \delta \) and \( \sigma \):

\[
\delta_{1,z} + \delta_{2,z} + \ldots + \delta_{n_L-2,z} + \delta_{n_L-1,z} + \delta_{n_L,z} = n_L \delta
\]  

(2)

\[
\delta_{1,z}^2 + \delta_{2,z}^2 + \ldots + \delta_{n_L-2,z}^2 + \delta_{n_L-1,z}^2 + \delta_{n_L,z}^2 = n_L(\sigma^2 + \delta^2)
\]  

(3)

To obtain \( n_L \) spheres distributions with specified \( \delta \) and \( \sigma = \alpha \delta \), we randomly draw the values of \( \delta_{1,z}, \delta_{2,z}, \ldots, \delta_{n_L-2,z} \) from a normal probability distribution with the desired values of \( \sigma \) and \( \delta \). Subsequently, the values of \( \delta_{n_L-1,z} \) and \( \delta_{n_L,z} \) are obtained by solving Eqns. (2) and (3). Repeating this procedure for different random values of \( \delta_{1,z}, \delta_{2,z}, \ldots, \delta_{n_L-2,z} \) results in different realizations of the configuration with the same \( \delta \) and \( \alpha \). We randomly generated 5 different packings for each combination of \( \delta \) and \( \alpha \), and we will present standard deviations in \( Nu \) within such a group of 5 random packings.

After the spheres in each column have been randomly redistributed according to the above procedure, the same procedure is applied to redistribute the spheres in each row. We only accepted those distributions in which all spheres lay within the \( H \times H \) square.

Meshing of the three-dimensional computational domain was performed in Gambit [16]. The spheres were modelled as spherical exclusions from the computational domain. The structured and unstructured mesh consisted of 550,000 to 1,800,000 grid cells, and local grid refinement was applied in the regions nearby the surfaces of the spheres and in the interstitial spaces between the spheres.

2.2. The solver

The flow was assumed laminar and incompressible, thus the continuity equation is expressed as

\[
\nabla \cdot \vec{u} = 0
\]  

(4)

where \( \vec{u} \) is the velocity field vector.

Assuming gravity effects negligible and no external body forces, the Navier-Stokes (momentum) equations can be written as

\[
\rho \frac{\partial \vec{u}}{\partial t} + \rho (\nabla \cdot \vec{u} \vec{u}) = -\nabla P + \nabla \cdot \vec{\tau}
\]  

(5)

where \( \nabla P \) and \( \vec{\tau} \) are, respectively, the pressure field gradient and the viscous stress tensor for a Newtonian fluid.

Additionally the heat-transfer equation was also solved

\[
\rho C_p \frac{\partial T}{\partial t} + \rho C_p (\vec{u} \cdot \nabla T) = \lambda \nabla^2 T
\]  

(6)

where \( C_p \) is the fluid specific heat capacity.

Equations (4–6) were solved in three-dimensional, steady-state or transient formulation. The latter was needed for cases with free-stream \( Re_\infty \geq 20 \) and small inter-sphere distances, in
which the interaction between the sphere wakes leads to vortex instability behind the spheres, as reported by Kim et al. [17] and Hill et al. [18]. For large inter-sphere distances, however, the wakes are non-interacting and stable for all $Re_{\infty}$ numbers studied.

A second order QUICK scheme [19] was used to discretize the equations in space, whilst a second order implicit time discretization scheme was used. The pressure field was obtained by employing the SIMPLE [20] algorithm. Convergence limits of the sum of the normalized absolute residuals for all the equations were set to $10^{-6}$. For the unsteady simulations, $10^{-6}$ normalized residuals convergence was assured at each time step. Unsteady simulations were continued until a periodic, pseudo steady-state in the downstream velocity and temperature profiles was reached.

2.3. Nusselt number

A steady-state thermal energy balance over the domain sketched in Fig. 1, assuming constant fluid properties, leads to

$$n_T \pi d^2 h (T_\infty - T_w) = u_\infty H^2 \rho C_p (T_\infty - T_{out}) \quad (7)$$

where $T_{out}$ is the mass weighted fluid temperature at the domain outlet, $n_T$ is the number of spheres, $d$ the sphere diameter, $h$ the average heat transfer coefficient at sphere walls. The temperature $T_w$ at the surface of the spheres was set at a constant for all the studied configurations. With Eqn. (1) this leads to the following relation for the average Nusselt number

$$Nu = \frac{h d}{\lambda} = \frac{u_\infty d (T_\infty - T_{out})}{4a(1-\epsilon)(T_\infty - T_w)} \quad (8)$$

where $a = \lambda/(\rho C_p)$ is the fluid thermal diffusivity. For $\epsilon \rightarrow 1$ the $Nu$ values are expected to approach that of a single sphere, which can be evaluated by the correlation proposed by Whitaker [21]

$$Nu_{\text{Whitaker}} = 2 + (0.4Re^{1/2}_\infty + 0.06Re^{2/3}_\infty)Pr^{0.4} \quad (9)$$

3. Results and discussion

In the following two subsections, we present results for, respectively, equidistantly and non-equidistantly spaced spheres arranged in a single layer perpendicular to the flow direction. Nusselt numbers will be correlated with the free-stream and local Reynolds numbers $Re_\infty$ and $Re_L = Re_\infty/\epsilon$ and the Prandtl number, for various values of the open frontal area fraction $\epsilon$ and the non-uniformity parameter $\alpha$. The Prandtl number is varied from 0.7 to 10, and the free stream Reynolds number $Re_\infty$ from 0.1 to 100. For orderly arranged arrays, we varied the open frontal area fraction from $\epsilon = 0.25$ to $\epsilon = 0.95$. For the non-equidistant arrays we studied open frontal area fractions $\epsilon = 0.4$ and 0.8 and non-uniformity parameters $\alpha = 0.5, 1.0$ and 1.5.

3.1. Equidistant arrays

Figure 3 shows the obtained Nusselt numbers for equidistantly spaced spheres as a function of $Re_L^{1/2}Pr^{1/2}$, for different values of the open frontal area fraction $\epsilon$.

From this figure, four main observations can be made:

(i) For low $Re_L^{1/2}Pr^{1/2}$, $Nu$ increases with increasing open frontal area fraction $\epsilon$.

(ii) For high $Re_L^{1/2}Pr^{1/2}$, $Nu$ is within a factor 2 equal to that of a single sphere (here evaluated by use of the empirical equation proposed by Whitaker [21]), and only slightly dependent of the open frontal area fraction $\epsilon$. 

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Figure 3. Nu number as a function of $\epsilon$ and $Re_{L}^{1/2}Pr^{1/2}$ for ordered arrays of spheres. Also shown is the correlation by Whitaker [21] for a single sphere.

(iii) The value of $Re_{L}^{1/2}Pr^{1/2}$ above which $Nu$ is close to that of a single sphere, decreases with increasing $\epsilon$.

(iv) For small $\epsilon$ and large $Re_{L}$, $Nu$ values exceed those reported for a single sphere by up to a factor 2.

The latter observation can be understood from the fact that the interacting wakes at small inter-sphere distances and large $Re_{L}$ lead to vortex instabilities which enhance heat transfer [10].

The first three observations can be understood and more quantitatively analysed by looking at the temperature distributions and temperature boundary layer formation around the spheres, as shown in Fig. 4.

This figure shows contours of the dimensionless temperature $\theta = (T - T_{w})/(T_{\infty} - T_{w})$, which varies from $\theta = 1$ in the inlet to $\theta = 0$ at the sphere walls.

For small $Re$ and $\epsilon$, e.g. $Re_{\infty} = 10$ and $\epsilon = 0.25$ in Fig. 4, all incoming thermal energy $u_{\infty}H^2\rho C_{p}(T_{\infty} - T_{w})$ is transferred by conduction to the sphere surfaces, and the flow has essentially lost all its thermal energy once it has crossed the sphere plane, even for open frontal areas as large as $\epsilon = 0.95$. With $u_{\infty}H^2(T_{\infty} - T_{w}) = Nu(a/d)n_{T}\pi d^2(T_{\infty} - T_{w})$ and $(H/d)^2 = (n_{T}\pi)/(4/(1 - \epsilon))$ this leads to

$$Nu = \frac{1}{4} \frac{Re_{\infty}Pr}{1 - \epsilon} = \frac{1}{4} Re_{L}Pr \frac{\epsilon}{1 - \epsilon}$$

(10)

The low $Re$ behaviour as described above will occur when the convective time scale $d/(u_{\infty}/\epsilon)$ in which the fluid passes the sphere plane is large compared to the transversal conduction time scale $(\frac{1}{2}L)^2/a$, with $L \approx \sqrt{\epsilon H/n_{L}}$ the average distance between the sphere surfaces, this leads to the criterion:

$$Re_{L} \ll Re_{1} = \frac{16}{\pi} \frac{1 - \epsilon}{\epsilon} Pr^{-1}$$

(11)

The validity of Eqn. (11) is shown in Fig. 5, in which all simulation results have been presented together.
Figure 5. Illustration of Eq.(11) for $0.1 \leq Re_\infty \leq 100$, $0.7 \leq Pr \leq 10$ and $0.25 \leq \epsilon \leq 0.95$ for ordered spheres.

Figure 6. Ratio between $Nu$ of ordered spheres and $Nu$ of a single sphere [21] for $0.1 \leq Re_\infty \leq 100$, $0.7 \leq Pr \leq 10$ and $0.25 \leq \epsilon \leq 0.95$.

For large $Re$ and large $\epsilon$, e.g. $Re_\infty = 100$ and $\epsilon = 0.8$ in Fig. 4, the thickness $\delta_C \approx (1/0.4)Re_L^{-1/2}Pr^{-0.4}d$ of the thermal boundary layers that are formed around the spheres (as can be found from Whitaker’s equation (9) for the appropriate range of $Re$) is thin compared to the inter-sphere distance $L$. As a result, the heat transfer to one sphere is not significantly influenced by the presence of the other spheres and therefore approaches that of a single sphere. Requiring that $\delta_C < 0.1L$, we find that this is the case for

$$Re_L \gg Re_2 \approx \frac{2500}{\pi} \frac{1 - \epsilon}{\epsilon} Pr^{-0.8}$$

(12)

Figure 6, in which all simulations results have been combined, demonstrates that indeed the Nusselt numbers approach that of Whitaker’s correlation for a single sphere when $Re_L/(1 - \epsilon)Pr^{0.8} > 2500/\pi$.

In agreement with observation (iii), the critical value $Re_2$ above which the Nusselt number approaches that of a single sphere decreases with increasing $Pr$ and with increasing $\epsilon$.

All results presented in Fig. 3 can be summarized by means of the correlation stated in Eqn. (13) where $Nu$ is described as a function of $Re_L$, $Pr$ and $\epsilon$:

$$Nu = \frac{\frac{1}{4} \frac{\epsilon}{1 - \epsilon} Re_L Pr \sqrt{\frac{\left(\frac{1}{4} \frac{\epsilon}{1 - \epsilon} Re_L Pr \frac{Nu}{Nu_{Whitaker}}\right)^2}{1 + \left(\frac{\frac{1}{4} \frac{\epsilon}{1 - \epsilon} Re_L Pr}{Nu_{Whitaker}}\right)^2}}}$$

(13)

where $Nu_{Whitaker}$ is the Nusselt correlation for a single sphere by Whitaker [21] given by Eqn. (9).

This is of course a simply form of averaged ratios between Eq. (10) for low $Re$ and Whitaker’s equation [21] for high $Re$. The transition between the two occurs when $\frac{1}{4} \frac{\epsilon}{1 - \epsilon} Re_L Pr / Nu_{Whitaker} = 1$ or $\frac{1}{4} \frac{\epsilon}{1 - \epsilon} Re_L Pr \approx 0.4 Re_L^{1/2} Pr^{0.4}$, which leads to $Re_L = \ldots$
1.6(1 - \epsilon)/\epsilon Pr^{-0.6} \approx Re_1.

Nusselt numbers predicted by Eqn. (13) have been included as solid lines in Fig. 3, showing the good agreement with the simulation data. The RMS error of this fit, defined as
\[ \left( \frac{1}{N} \sum_{i=1}^{N} \left( \frac{Nu_{Eq.(8)} - Nu_{Eq.(13)}}{Nu_{Eq.(8)}} \right)^2 \right)^{1/2}, \]
where \( N \) is the total number of points, was found to be 38\%, which should be considered small given the fact that we predict the \( Nu \) numbers that vary by 3 orders of magnitude. Such a small relative error has no major implications for our general findings.

3.2. Disordered arrays
Figures 7 and 8 show \( Nu \) in disordered, non-equidistantly spaced (\( \alpha > 0 \)) arrays of spheres, compared to those in ordered, equidistantly spaced (\( \alpha = 0 \)) arrays. The open frontal area fraction is \( \epsilon = 0.4 \) and 0.8, the non-uniformity parameter \( \alpha = 0, 0.5, 1 \) and 1.5 and \( Pr = 0.7 \) and 10. For each combination of \( \alpha > 0 \) and \( \epsilon \), five different geometrical configurations were realized. The spread in Nusselt numbers between these different realizations is indicated by the vertical error bars.

Several general observation can be made:

(i) For high and low values of \( Re_L \), \( Nu \) for non-equidistant spheres is equal to that for equidistant spheres.
(ii) For non-equidistantly spaced spheres, $Nu$ is lower than for equidistantly spaced spheres at intermediate values of $Re_L$. $Nu$ can be up to a factor of two lower compared to equidistant spheres.

(iii) The range of $Re_L$ values for which $Nu$ is sensitive to the non-uniformity parameter $\alpha$ increases with increasing $\epsilon$ and with increasing $Pr$.

(iv) For low values of $\epsilon$ and $\alpha$, the spread in Nusselt numbers between different geometry realizations is small. For high values of $\epsilon$ and $\alpha$, the spread in Nusselt numbers is larger compared to the difference in Nusselt numbers at different values of $\alpha$.

The above four observations can again be understood and more quantitatively analysed from the earlier analyses for equidistant spheres, and by looking at the temperature distributions and temperature boundary layer formation around non-equidistant spheres, as shown by contours of $\theta = (T - T_w)/(T_\infty - T_w)$ in Fig. 9.

As was the case for equidistant spheres, at very low values of $Re_L$, as in the left-hand column of Fig. 9, virtually all incoming thermal energy is transferred by conduction to the spheres, with little dependence of the precise distribution of the spheres. As a result, the surface averaged $Nu$, as defined by Eq. (8) has little dependence on $\alpha$ for low values of $Re_L$. On the other hand, for large values of $Re_L$ as in the right-hand column of Fig. 9, thin, non-interacting boundary layers are formed around the individual spheres, and heat transfer is again independent of the precise distribution of the spheres. These arguments explain the above observation (i).

For intermediate values of $Re_L$, as in the central column of Fig. 9, highly interacting thermal boundary layers are formed around the spheres, and heat transfer is locally reduced in regions of more dense packing of the spheres. The relation between local Nusselt numbers and local packing densities can be analysed through the relation between $Nu$ and $\epsilon$ for equidistant spheres at intermediate values of $Re_L$, as presented in the previous section. This showed a strong, non-linear, decrease of $Nu$ with decreasing $\epsilon$. In agreement with observation (ii), the result of this non-linear relationship between local $Nu$ and local $\epsilon$ is that the average $Nu$ in a non-equidistant array with given $\epsilon$ is lower than the average $Nu$ in an equidistant array of equal $\epsilon$.

In line with the above, a strong influence of non-uniformity on Nusselt is expected for an intermediate range of $Re_L$, for which interacting thermal boundary layers are formed around the spheres. Similar to the analysis for equidistant spheres, $Nu$ is found to be independent of $\alpha$.
for $\text{Re}_L \ll \text{Re}_1$ and for $\text{Re}_L \gg \text{Re}_2$ (indicated by the vertical dashed lines in Figs. 7 and 8).

Finally, for small values of $\alpha$ and $\epsilon$, different geometry realizations at given $\epsilon$ and $\alpha$ will exhibit small geometric variations, explaining that there is little variation in Nusselt numbers. For large $\epsilon$ and $\alpha$ however, there may be large geometric differences between different geometry realizations, explaining the observed larger variations in $Nu$ numbers between geometrical configurations with the same value of $\alpha$ and $\epsilon$.

4. Conclusions
For values of the local Reynolds number $\text{Re}_L = \text{Re}_\infty/\epsilon$ (i.e. the free stream Reynolds number divided by the open frontal area fraction $\epsilon$) that are much larger than $\text{Re}_2 \approx 2500/\pi(1-\epsilon)/\epsilon\text{Pr}^{-0.8}$, laminar forced convection heat transfer to equidistantly and non-equidistantly spaced spheres in a single layer perpendicular to the flow direction is independent of the sphere packing density and sphere packing configuration, and can be accurately described by established Nusselt correlations for a single sphere when the Reynolds number in these correlations is replaced by $\text{Re}_L$. For values of $\text{Re}_L$ that are small compared to $\text{Re}_1 = 16/\pi(1-\epsilon)/\epsilon\text{Pr}^{-1}$, $Nu$ decreases strongly with increased packing density, and is given by $Nu = \epsilon\text{Re}_L\text{Pr}/(4(1-\epsilon))$, again independent of the sphere packing configuration. For the intermediate $\text{Re}_L$ regime, i.e. $\text{Re}_1 < \text{Re}_L < \text{Re}_2$, $Nu$ in non-equidistantly spaced arrays of spheres can be up to a factor 2 lower as compared to equidistantly spaced arrays with the same open frontal area fraction. These findings can applied to the design and optimization of e.g. activated carbon filters and CBRN protective clothing.

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Nomenclature

\begin{align*}
a & \quad \text{fluid thermal diffusivity} \ [m^2/s] \\
C_p & \quad \text{fluid specific heat capacity} \ [m^2/s^2/K] \\
d & \quad \text{sphere diameter} \ [m] \\
h & \quad \text{heat transfer coefficient} \ [kg/K/s^3] \\
H & \quad \text{domain height} \ [m] \\
n_L & \quad \text{number of spheres per row/column} \\
n_T & \quad \text{total number of spheres} \\
n_y & \quad \text{number of spheres per column} \\
n_z & \quad \text{number of spheres per row} \\
L & \quad \text{average distance between sphere surfaces} \ [m] \\
L_1 & \quad \text{distance between domain inlet and sphere surfaces} \ [m] \\
T_\infty & \quad \text{free stream fluid temperature} \ [K] \\
T_\text{cw} & \quad \text{cylindrical wire wall temperature} \ [K] \\
T_\text{out} & \quad \text{outlet section temperature} \ [K] \\
u_\infty & \quad \text{free stream fluid velocity} \ [m/s] \\
x & \quad \text{streamwise direction} \\
\alpha & \quad \text{dimensionless level of disorder} \\
\delta_y & \quad \text{sphere to sphere wall distance along y direction} \ [m] \\
\delta_z & \quad \text{sphere to sphere wall distance along z direction} \ [m] \\
\bar{\delta} & \quad \text{sphere to sphere wall mean distance} \ [m] \\
\epsilon & \quad \text{open frontal area fraction} \\
\lambda & \quad \text{fluid thermal conductivity} \ [kgm/s^3/K] \\
\theta & \quad \text{fluid dimensionless temperature}
\end{align*}
\( \mu \)  \( \text{dynamic viscosity} \ [kg/m/s] \)
\( \nu \)  \( \text{kinematic viscosity} \ [m^2/s] \)
\( \rho \)  \( \text{fluid density} \ [kg/m^3] \)
\( \sigma \)  \( \text{standard deviation} \)
\( Nu \)  \( \text{Nusselt Number} \)
\( Pr \)  \( \text{Prandtl Number} \)
\( Re_\infty \)  \( \text{Reynolds number} u_\infty d/\nu \)
\( Re_L \)  \( \text{Local Reynolds number} Re_\infty/\epsilon \)
\( Re_1 \)  \( \text{Low critical Reynolds number} \)
\( Re_2 \)  \( \text{High critical Reynolds number} \)

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