1. Introduction

An important parameter influencing the power effect of a wheel on the rail is the size and shape of the contact area as well as the normal stress distribution which has the impact on it. Nowadays various methods are used to find out the size of contact areas and stresses [1, 2]. It is necessary to mention the Hertz method as one of the oldest up to date used methods. It provides acceptable results for a large area in spite of many simplifications. Another computational procedure is the Kalker variation method or the strip method which is, thanks to its results, close to reality and is used in the following calculations [3]. Nowadays, another group of calculation program systems (ANSYS, ADINA) is used in certain situations. The systems work on the base of finite elements method theory.

2. Strip method

The Strip Method presupposes quasi-static rolling [1]. The principal idea of the theory is to take into consideration slim contact areas in the y-direction. In Fig. 2 there are two bodies in contact. In fact the geometrical parameters (of railway wheel and rail) should be similar to the reality. The deformation zones are of the similar shapes too. In spite of this fact the parameters (the displacements \( w_1 \) and \( w_2 \)) in Fig. 1 are rather different for better understanding of the theory.

In fact, the contact area should be a plane (parallel with the xy plane).

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Keywords: Wheel/rail contact, contact stress evaluation, modified Strip method, optimized computation procedure.

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* Tomas Lack – Juraj Gerlici*

Faculty of Mechanical Engineering, University of Zilina, Slovakia, E-mail: tomas.lack@fstroj.uniza.sk
The method presupposes the existence of two rotating bodies 1 and 2 with surfaces \( S_1 \) and \( S_2 \). The bodies touch in the point 0, which is at the same time the beginning of their spatial coordinate systems. The axes \( x \) and \( y \) determine the horizontal base. We will mark the horizontal coordinate as the \( z \) – axis. If there is no influence of a normal force \( Q \), then there exclusively exists geometrical binding between the bodies.

If the bodies are pressed against each other by the normal force \( Q \), a deformation and a contact area \( \Omega \) instead of a contact point arises between the bodies.

- The geometrical profile shape of the first body surface will be marked \( f_1(x,y) \), the geometrical profile shape of the second body surface will be marked \( f_2(x,y) \).
- The elastic displacement in the \( z \)-axis direction caused by the deformation of the first body surface will be marked \( w_1(x,y) \), the displacement in the \( z \)-axis direction caused by the deformation of the second body surface will be marked \( w_2(x,y) \).
- The displacement of bodies centers against each other in the axis-z direction will be marked \( d \).
- The perpendicular distance between the points of the deformed bodies surfaces will be marked \( \delta(x,y) \).

3. Normal stress approximation over a strip

The stress evaluation over the \( k^{th} \) strip (Fig. 2) is approximated in a standard way [4]:

\[
p_{k}(x,y) = p_{0k} \cdot \sqrt{1 - \left( \frac{x}{x_{dk}} \right)^2}
\]

where:
- \( p_{0k} \) normal stress in the \( x \) position,
- \( p_{0k} \) maximum normal stress,
- \( x_{dk} \) half length of the \( k^{th} \) strip.

We aimed our interest at increasing the computational effort. To obtain the requested results, deformations and stresses in the middle of strips, we utilized the procedures introduced in the next text.

4. Time optimized computational procedure for contact stresses assessment

The equation system assembly is necessary for the stress evaluation in the middle of strips [4, 5, 6]:

\[
[M] \begin{bmatrix} p^{(0)} \end{bmatrix} = \begin{bmatrix} w^{(0)} \end{bmatrix}
\]

where:
- \( [M] \) influence coefficients matrix,
- \( \{ p^{(0)} \} \) normal stresses in the middle of strips vector,
- \( \{ w^{(0)} \} \) in the middle of strips strains vector.

The \( [M] \) matrix elements are of values:

\[
M_{kj} = H \cdot I_{kj}^{(0)}
\]

where:
- \( I_{kj}^{(0)} \) deformation in the middle of \( k^{th} \) strip.

The vector elements are:

\[
w_{k}^{(0)} = z_{ik} - z_{i0} - D
\]

where:
- \( w_{k}^{(0)} \) deformation in the middle of \( k^{th} \) strip.

The aim is to compute the value \( I_{kj}^{(0)} \) in accordance with relation below:

\[
I_{kj}^{(0)} = 2 \int_{x_{ik}}^{x_{ik}} \sqrt{1 - \left( \frac{x}{x_{ik}} \right)^2} \, dx
\]

when:
- \( y_{ik} = y_{ik} - y_{ik} \)
- \( y_{ik} = y_{ik} + y_{ik} \)

where:
- \( x_{ik} \) half length of the \( k^{th} \) strip,
- \( y_{ik} \) y-coordinate of \( k^{th} \) strip.
This integral can be modified into one-dimension integral in the form:

$$I_k^{(0)} = -\frac{2}{y_d} \int_{y_d}^{y_s} \sqrt{x_n^2 - x^2} \cdot \ln\{AA\} \, dx$$  \hspace{1cm} (9)

$$AA = \frac{[AB][AC]}{x^2}$$  \hspace{1cm} (10)

$$AB = \sqrt{x^2 + (y - y_d)^2} + y - y_d$$  \hspace{1cm} (11)

$$AC = \sqrt{x^2 + (y - y_s)^2} - y + y_s$$  \hspace{1cm} (12)

It is unavoidable to solve the integral in the relation (9) numerically, because it has no analytical solution. This computation is very time consuming for separate strips in the relationship to other strips.

This is the reason for the new procedure development that is not dependent on a real strip length. We presuppose that a strip is of one unit length. Description of individual parameters in the x, y coordinate system is in Fig. 3 and a description of individual parameters in the u, v coordinate system is in Fig. 4.

For the value $I_k^{(0)}$ the relation is valid:

$$I_k^{(0)} = i_k^{(0)} \cdot x_n$$  \hspace{1cm} (13)

Fig. 3 Description of individual parameters in the coordinate system

We establish:
- the $k^{th}$ strip length in the u, v system has the value of $L$
- the $k^{th}$ strip width in the u, v system has the value of $b_k$

For the value $I_k^{(0)}$ the relation is valid:

$$I_k^{(0)} = i_k^{(0)} \cdot x_n$$  \hspace{1cm} (13)

Fig. 4 Description of individual parameters in the coordinate system

For the value $I_k^{(0)}$ the relation is valid:

$$I_k^{(0)} = i_k^{(0)} \cdot x_n$$  \hspace{1cm} (13)

Fig. 5 The integral value “i” in dependence on $\log(Bk)$ on condition $|s - k| = 0$

The value of an integral decimal logarithm “$E$” can be expressed via the polynomial of 6-th grade:

$$E = \left(\left(\frac{a_6 \cdot L + a_0}{L + a_1}\right) + \frac{a_3 \cdot L + a_1}{L + a_0}\right) \cdot L + a_0$$  \hspace{1cm} (16)

where:

- $a_0, a_1, a_2, a_3, a_6$ are parameters
- $L$ is a constant
Polynomial coefficients from $a_6$ to $a_3$ are in Table 1.

The $i(k,s)$ value can be expressed via this relation:

$$i(k,s) = E_0 \cdot i(k,s)$$  \hspace{1cm} (17)$$

Polynomial coefficients from $a_6$ to $a_3$ are in Table 1.

Polynomial coefficients from $t_n$ to $a_6$ are in Table 2.

The $i(k,s)$ value can be expressed via this relation:

$$i(k,s) \approx 10^k$$  \hspace{1cm} (18)$$
For a strip with the coordinate $x$ deformation computation the following relation is valid:

$$w_k(x) = H \sum_{i=1}^{\infty} p^{(i)}_k \cdot I_k(x).$$  \hspace{1cm} (19)$$

where integral:

$$I_k(x) = \int_{-\infty}^{\infty} \sqrt{\frac{1}{(x-a)^2 + (y-b)^2} - x^2} \, dy \, dx \hspace{1cm} (20)$$

This integral can be modified into the one-dimensional integral in form:

$$I_k(x) = -\frac{1}{4x} \int_{-\infty}^{\infty} \sqrt{\frac{1}{(x-a)^2 + (y-b)^2} - x^2} \, dy \, dx$$

$$\hspace{1cm} (21)$$

This integral has no analytical solution, so it is unavoidable to compute this integral numerically. In relation to other strips, the mentioned solution for strip by strip computation is time consuming.

| $s - k$ | $a_k$ | $a_l$ | $a_m$ | $a_n$ |
|---------|-------|-------|-------|-------|
| 0       | -0.00255 | -0.001497 | 0.022537 | 0.009238 |
| 2       | -0.00756 | -0.026714 | 0.015249 | 0.105083 |
| 4       | 0.003839 | -0.008085 | -0.02931 | 0.068803 |
| 6       | 0.007608 | 0.004252 | -0.03718 | 0.037078 |
| 8       | 0.007902 | 0.009316 | -0.03513 | 0.019712 |
| 10      | 0.007030 | 0.01098 | -0.03057 | 0.010126 |
| 12      | 0.005830 | 0.011066 | -0.02574 | 0.004737 |
| 14      | 0.006428 | 0.010462 | -0.0213 | 0.001656 |
| 16      | 0.003549 | 0.009613 | -0.01744 | -0.00014 |
| 18      | 0.002611 | 0.008678 | -0.01415 | -0.00115 |
| 20      | 0.001806 | 0.007735 | -0.01138 | -0.00165 |
| 22      | 0.001127 | 0.006837 | -0.00907 | -0.00186 |
| 24      | 0.000534 | 0.005942 | -0.00714 | -0.00182 |
| 26      | 0.000064 | 0.005204 | -0.00551 | -0.00174 |
| 28      | -0.000320 | 0.00455 | -0.00417 | -0.00164 |
| 30      | -0.000663 | 0.003985 | -0.00305 | -0.00152 |
| 32      | -0.000837 | 0.003496 | -0.0021 | -0.00139 |
| 34      | -0.000121 | 0.003193 | -0.00132 | -0.00141 |
| 36      | -0.000118 | 0.002579 | -0.00064 | -0.00097 |
| 38      | -0.000114 | 0.002428 | -7.3E-05 | -0.00083 |
| 40      | -0.000151 | 0.001912 | 0.000334 | -0.00069 |

We establish:

- the k-th strip length in the $u, v$ system has the value of 1
- the k-th strip width in the $u, v$ system has the value:

$$b_k = \frac{y_k}{x_k} \hspace{1cm} (22)$$

and

$$u_k = \frac{x_k}{x_k} \hspace{1cm} (23)$$

where:

- $x_k$ - coordinate, in the $x, y$ coordinate system on $k^{th}$ strip.
- $x_{k0}$ length of the $k^{th}$ strip.
- $u_k$ - coordinate, in the $u, v$ coordinate system on $k^{th}$ strip.

$$I_k(x_k) = i_{dx}(u_k) \cdot x_{k0} \hspace{1cm} (24)$$

$$i_{dx}(u_k) = i_{dx}(u_k) + q_{dx}(-u_k) \hspace{1cm} (25)$$

$$q_{dx} = 2 \int_{-\infty}^{\infty} \frac{1}{\sqrt{u^2 + \left( 2 \left( s - k \right) - v \right) \cdot b_k } \, dv \, du \hspace{1cm} (26)$$
The \( q_{k,s}^{(0)}(\chi) \) integral values and \( q_{k,s}(\chi) \) integral are utterly analytical.

Further, we compare the computational precision and computation speed gained by the Kalker’s method with the results gathered by other methods.

4. Computation results obtained by different methods

In the following graphs are maximum stresses in contact courses against lateral shift of the wheel along a rail head movement from a wheel rim (Fig. 11), maximum stresses in contact courses against lateral shift of the wheel along a rail head movement to a wheel rim (Fig. 12) as well as the contact patch values courses against lateral shift of the wheel along a rail head movement from a wheel rim (Fig. 13) and contact patch area courses against lateral shift of the wheel along a rail head movement to a wheel rim (Fig. 14).

The computational time spent on the P5 computer, with 2GB RAMM, 3GHz frequency.

Utilized methods:
- **Strip method** level of tenth of second
- **Kalker method with input of strip method results** – level of second
- **Kalker variation method** – level of more than ten seconds.

7. Conclusions

The Strip Method is often used for rail/wheel contact area and contact normal stress evaluation. It presupposes quasi-static rolling. The principal idea of the theory is to take into consideration slim contact areas in the y-direction. The paper deals with the computational time saving procedure when the computation accuracy is guaranteed.
The introduced method enables contact patches and contact stresses between railway wheel and rail [7] under decreased computational time consumption. Rules and procedures characteristic of the Strip method are preserved. The stress computation acceleration is in this case based on the algorithm for numerical solution of integrals. We included the local coordinate system with a presupposed semi-circular course of the normal stress for the purpose of the integral computation. This integral computation is needed for deformation in the middle of individual strips evaluation. The integral is solvable analytically. The input values for the separate spline parts and separate strips computation are possible to insert after analytical solution of the integral. The procedure application may bring the practical benefit for researchers and computation - analysis experts who are interested in the field of vehicle dynamics simulations, rail/wheel contact analysis, as well as new profiles on the base of geometric characteristics design [8].

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