On KKLT/CFT and LVS/CFT Dualities

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Abstract: We present a general discussion of the properties of three dimensional CFT duals to the AdS string theory vacua coming from type IIB Calabi-Yau flux compactifications. Both KKLT and Large Volume Scenario (LVS) minima are considered. In both cases we identify the large ‘central charge’, find a separation of scales between the radius of AdS and the size of the extra dimensions and show that the dual CFT has only a limited number of operators with small conformal dimension. Differences between the two sets of duals are identified. Besides a different amount of supersymmetry ($\mathcal{N} = 1$ for KKLT and $\mathcal{N} = 0$ for LVS) we find that the LVS CFT dual has only one scalar operator with $\mathcal{O}(1)$ conformal dimension, corresponding to the volume modulus, whereas in KKLT the whole set of $h^{1,1}$ Kähler moduli have this property. Also, the maximal number of degrees of freedom is estimated to be much larger in LVS than in KKLT duals. In both cases we explicitly compute the coefficient of the logarithmic contribution to the one-loop vacuum energy which should be invariant under duality and therefore provides a non-trivial prediction for the dual CFT. This coefficient takes a particularly simple form in the KKLT case.
1 Introduction

Flux compactifications of type IIB string theory have given rise to two major developments within string theory: AdS/CFT duality [1, 2] (see [3] for a review) and the string landscape [4–15] of moduli stabilised four dimensional (4D) string vacua. In the simplest cases, these four dimensional minima have a negative cosmological constant and hence are AdS$_4$ vacua.
It is then natural to inquire if these Anti de Sitter (AdS) vacua of the string landscape have Conformal Field Theory (CFT) duals and if so what the properties of these theories are.

Identifying CFT duals of the AdS (and dS) vacua of the string landscape would be a way to provide a proper non perturbative description of these vacua and put the string landscape on firmer ground. This is the subject of the present article. For previous discussions of this issue see [17–20].

By now there are two main scenarios of moduli stabilisation in type IIB string compactifications on Calabi-Yau (CY) manifolds: KKLT [9] and the Large Volume Scenario (LVS) [22]. Contrary to the original AdS$^5 \times S^5$ background where the flux was enough to stabilise the geometric modulus of $S^5$, in KKLT and LVS scenarios the fluxes fix only part of the geometric moduli (this can be read from the ten dimensional equation of motions [7, 8], like for AdS$^5 \times S^5$) leaving some flat directions. A key ingredient to stabilise the remaining geometric moduli (in a AdS$_4$ vacuum) is the presence of non-perturbative effects in the 4D effective field theory (EFT) obtained after compactification. This makes a full ten dimensional (10D) analysis of these vacua very difficult and we can only rely on the EFT results. Black-brane solutions that were at the origin of the AdS$^5 \times S^5$/CFT$_4$ duality are not available and therefore there is less control on the potential duality in the KKLT and LVS cases. This explains the relative shortage of efforts to study the CFT duals of these vacua during the past ten years. Another difference with the AdS$_5 \times S^5$ is that in both KKLT and LVS scenarios there is a hierarchy between the size of the internal dimensions and the AdS radius. This is important in order to have a separation of scales and be able to study the AdS$_4$/CFT$_3$ duality without having to include in the analysis the full string theory and Kaluza-Klein tower of states.

Even though both KKLT and LVS are based on Calabi-Yau flux compactifications of type IIB string theory down to 4D, they have important differences that should reflect on the dual CFTs.

- The two scenarios realise the separation of scales that allow the neglect of part of the spectrum in different ways. In KKLT this happens because of the small value of the flux superpotential, while in LVS because of the hierarchically large value of the volume of the compactification manifold. In fact, KKLT relies on the possibility of tuning the flux superpotential $W_{\text{flux}}$ to very small values (of the same order of the non-perturbative superpotential), while LVS is based on a generically order one $W_{\text{flux}}$.

\footnote{AdS$_{d+1}$/CFT$_d$ duality has also been used in Calabi-Yau flux compactifications in a different context that should not be confused with our target in this article. In those cases, conifold geometries such as the Klebanov-Strassler warped throat are embedded in compact Calabi-Yau manifolds and provide a stringy realisation of the Randall-Sundrum set-up with the tip of the throat providing the IR brane and the compact Calabi-Yau at the beginning of the throat providing the UV Planck brane [21]. In these cases AdS$_{d+1}$/CFT$_d$ duality is used in the sense that 4D field theories are dual to 5D gravity theories in which locally the five dimensions are the 4D spacetime dimensions plus the direction along the throat, i.e. $d = 4$. On the other hand, in this paper we are concentrating on three-dimensional field theories dual to four-dimensional gravity theories, i.e. $d = 3$.}
• The KKLT AdS$_4$ vacuum preserves $\mathcal{N} = 1$ supersymmetry, whereas the LVS AdS$_4$ vacuum breaks supersymmetry spontaneously, with the breaking being induced by generic fluxes. The fact that the LVS vacuum is not supersymmetric may raise concerns regarding its stability and the existence of a CFT dual. It was shown in [23] that as long as the effective field theory is valid the corresponding vacua are stable under bubble nucleation and therefore a dual CFT is expected to exist. Moreover, the fact that supersymmetry is spontaneously broken on the AdS side raises the question of how this breaking manifests itself on the CFT side.

• Both scenarios allow the possibility to extend the AdS compactifications to include dS. However, they are usually realised in different ways in both scenarios. Addressing the possibility of duals to these dS vacua is very relevant, but since these vacua are more model dependent and the dS/CFT duality is less understood we will not address this issue here. Our discussion here may be relevant for a future approach to this question.

Besides the motivation to find a proper non-perturbative formulation of the string landscape, the study of this duality may be relevant for other applications of the gauge/gravity duality such as for condensed matter systems in which the relevant ingredients are precisely CFT$_3$ and AdS$_4$ (for a review, see [28]). In particular the LVS AdS$_4$ minimum, being non-supersymmetric, is in the appropriate situation to be dual to a condensed matter system.

In this article we make a general discussion of this potential duality with the intention to learn as much as possible about the properties of the CFT$_3$ duals. We are aware of the difficulty of the task and attempt only to extract general properties of the CFT$_3$. We also identify one particular quantity that can be explicitly computed on the AdS side and should be exact across the duality. For this we borrow techniques recently developed for black hole solutions in which the one-loop logarithmic corrections to black hole entropy provide exact quantities which should agree on both sides of the duality. In the present case, we know only the gravity side of the duality. Hence, these calculations correspond to predictions that should be satisfied by any candidate CFT dual.

We organise this paper as follows. In Section 2 we will present a detailed comparison between AdS$_5 \times S^5$ background and the Calabi-Yau flux compactifications. In Section 3 we describe some properties of the three dimensional CFT dual to KKLT and LVS flux compactification. In particular we identify the amount of supersymmetry, the central charge, the conformal dimension of the various operators dual to fields on the gravity side and the baryonic operator/vertex in the dual CFT. In Section 4 we discuss the one loop corrections to the partition function in supergravity. These corrections will correspond to $\frac{1}{N}$ effects in the partition function of the dual CFT. In this computation we look for the universal contribution to the partition function of the dual CFT and discuss the limit in which we perform the computation. In Section 5 we explicitly compute this term in the KKLT and

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[2] See [9, 24–27] for explicit dS minima in the type IIB context considered in this paper.
LVS cases. This gives a prediction for the universal contribution to the partition function of the dual CFTs.

2 AdS backgrounds from flux compactifications

The bosonic part of the 10D supergravity effective action for type IIB string theory in the Einstein frame is

\[
S = \frac{1}{(2\pi)^4\alpha'^2} \int d^{10}x \sqrt{-g} \left\{ R - \frac{\partial_M S \partial^M S}{2(\text{Re} S)^2} - \frac{G_3 \cdot \bar{G}_3}{12 \text{Re} S} - \frac{F_5^2}{4 \cdot 5!} \right\} + S_{\text{CS}} + S_{\text{loc}}. \tag{2.1}
\]

Here \( S = e^{-\phi} + iC_0 \) is the axiodilaton field, \( G_3 = F_3 - iS H_3 \) the complex combination of RR \((F_3 = dC_2)\) and NS \((H_3 = dB_2)\) three-form field strengths and \( F_5 = dC_4 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3 \) the self-dual five-form field strength (for which this action is only a short way of writing the origin of its field equations). The Chern-Simons term is \( S_{\text{CS}} \propto \int C_4 \wedge G_3 \wedge \bar{G}_3 \).

Finally \( S_{\text{loc}} \) is the contribution from local sources such as D-branes and orientifold planes.

2.1 Basics of AdS\(_5\) × S\(^5\)/CFT\(_4\) duality

Let us start recalling some of the relevant results on AdS\(_5\) × S\(^5\)/CFT\(_4\) duality that will be useful to compare with the cases of interest in this article. The original discussion started with the solitonic black brane solutions of the 10D effective action, that has \( N \) units of D3-charge; by taking the near horizon limit one extracts the AdS geometry that in the low energy limit can be connected with the world-volume CFT on D3-branes, which is \( N = 4 \) Yang-Mills in 4D.

For our purposes, it is more illustrative to approach the AdS\(_5\) vacuum from the perspective of flux compactifications of type IIB string theory on S\(^5\), since that is the more natural way to compare this background with the KKLT and LVS ones. One starts in this case from the Freund-Rubin ansatz in which the metric is maximally symmetric, \( G_3 = 0 \), the axiodilaton \( S \) constant and \((F_3)_{\mu
\nu\rho\sigma} \propto \epsilon_{\mu
\nu\rho\sigma}\) (with indices running along the compact dimensions; a similar expression holds for the non-compact dimensions from self-duality of \( F_5 \)). In this way the spacetime is naturally separated in a product of two five-dimensional components. In particular the flux on the compact component, \( S^5 \) is quantised as:

\[
\frac{1}{(2\pi)^4\alpha'^2} \int_{S^5} F_5 = N. \tag{2.2}
\]

One could try to compactify the 10D theory with a background flux given by (2.2): Plugging the \( F_5 \) value back into the 10D action and integrating over the five compact extra dimensions and Weyl rescaling to the 5D Einstein frame gives the 5D Einstein-Hilbert term plus a scalar potential for the \( S^5 \) radius modulus \( R_{S^5} \) of the form:

\[
V(R_{S^5}) = R_{S^5}^{-16/3} \left(-a + bN^2 R_{S^5}^{-8}\right). \tag{2.3}
\]

The first term comes from the \( S^5 \) curvature dominating at small \( R_{S^5} \) and the second term, dominating at large \( R_{S^5} \), comes from the \( F_5^2 \) term in the action; \( a, b \) are \( O(1) \) positive constants. Minimising this potential fixes the value of the radius modulus to \( R_{S^5} \propto N^{1/4} \).
The effective cosmological constant of the non-compact 5D component of the spacetime is given by the value of the potential at the minimum ($\Lambda = V|_{\text{min}}$). In this case, it is negative giving rise to AdS$_5$ with AdS radius equal to the radius of the compact manifold, i.e. $R_{\text{AdS}} = R_{S^5}$. This implies that there is no trustable limit in which we can decouple the KK modes. Anyway, this analysis turns out to give the right answer for the background geometry generated by turning on $F_5$ fluxes, as it can be seen by comparing with the solutions of the 10D equations of motion. Notice also that the combination of fluxes and curvature of the extra dimensions were enough to fix the overall size of the extra dimensions but there is still a flat direction corresponding to the dilaton which is completely arbitrary.

To trust the 10D supergravity analysis, one needs to have the AdS radius larger than the string and the 10D Planck scale. This implies that these solutions are valid in the large $N$ and large $g_s N$ limits since

$$R_{\text{AdS}}/\ell_p^{10d} \sim N^{1/4}, \quad \frac{R_{\text{AdS}}}{\sqrt{\alpha'}} = \frac{R_{\text{AdS}}}{\ell_s} \sim (4\pi g_s N)^{1/4} \equiv \lambda^{1/4}. \quad (2.4)$$

At large $N$ and large 't Hooft coupling $\lambda$ the gravity description is well defined whereas for small 't Hooft coupling the perturbative CFT description is well defined.

A study of the Coulomb branch can naively be used to generate the duality [29]. As one can infer from equation (2.2), the flux $N$ contributes to the D3-brane charge. One can then start on the gravity side by trading the $F_5$ flux for $N$ D3-brane domain walls spanning four of the five non-compact dimensions and being at a fix point along the radial AdS direction. Between two domain walls $D3_k$ and $D3_{k+1}$ ($k = 1, ..., N$) the space is AdS$_5 \times S^5$ with flux $k$. On the left of the first D3-brane domain wall we have the flat 10D spacetime with no flux. Let us put all the branes on top of each other: on the left of the domain wall we have the flat 10D spacetime, while on the right we have the AdS$_5 \times S^5$ background with radius $R_{\text{AdS}} = R_{S^5} \sim N^{1/4}$. The theory living on the domain wall is the dual CFT $\mathcal{N} = 4$ super Yang-Mills theory with gauge group $U(N)$. On the gravity side the Coulomb branch corresponds to moving the branes away from each other. On the CFT side, it corresponds to give non-vanishing vevs to the scalar fields in the adjoint of $\mathcal{N} = 4$ breaking the $U(N)$ groups and the conformal symmetry.

The symmetries on both sides of the duality match in the sense that local symmetries on the AdS side map to global symmetries on the CFT side. Besides the $\mathcal{N} = 4$ supersymmetry, the $SO(4,2) \times SO(6)$ symmetries of the AdS$_5 \times S^5$ map to the $SO(4,2)$ 4D conformal symmetry and $SO(6)$ $R$-symmetry of $\mathcal{N} = 4$ supersymmetry. The number of degrees of freedom is measured by the ‘central charge’, which is given by $c \sim N^2$. This should be large in order for the duality to work. Also the conformal dimension of different operators has a nontrivial structure. In general, for a scalar particle of mass $m$ the dual CFT$_d$ operator has conformal dimension [3]

$$\Delta = \frac{d}{2} \pm \frac{1}{2} \sqrt{d^2 + 4(mR_{\text{AdS}})^2}. \quad (2.5)$$

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3Notice that from the second relation we can see that for fixed 't Hooft coupling $\lambda$ the $g_s$ expansion is equivalent to a $1/N$ expansion. Also for fixed $R_{\text{AdS}}$ the $\alpha'$ expansion is equivalent to an expansion in $1/\lambda$. 

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- 5 -
As we discussed before there is no separation of field theoretical scales since the radius of $S^5$ is the same as $R_{AdS}$. Hence, all Kaluza-Klein (KK) modes have masses of order $m \sim 1/R_{AdS}$ and therefore there are many operators with conformal dimension of order $O(1)$.

2.2 Calabi-Yau flux compactifications

We turn now to phenomenologically interesting Calabi-Yau (CY) flux compactifications that have been shown to be suitable for a controllable moduli stabilisation. Without the introduction of extra ingredients, such as background values of p-form potentials, the simple compactification of type II string theory on such manifolds has plenty of unobserved massless scalars at the 4D EFT level. These scalars are related to the geometric moduli of the Calabi-Yau compact manifold. In type IIB string theory, the relevant ingredients to stabilise the moduli without distorting too much the compact geometry (controlled backreaction) are known: non-zero background values of $G_3$ (three-form fluxes) stabilise the axio-dilaton $S$ and a subset of the geometric moduli, the complex structure moduli $U_\alpha$ ($\alpha = 1, ..., h^{1,2}$).

At lower scales, the rest of the geometric moduli, the Kähler moduli $T_i$ ($i = 1, ..., h^{1,1}$), are stabilised by additional terms in the scalar potential coming from perturbative and non-perturbative $g_s$ and $\alpha'$ corrections. In this section, we will review the two steps: the first one (GKP) is the same in KKLT and LVS, while they are distinguished by the second one.

Axiodilaton and complex structure moduli stabilisation (GKP)

Let us give a short review of the relevant features of the Giddings, Kachru, Polchinski (GKP) scenario, in which both complex structure moduli and dilaton are stabilised by switching on three-form fluxes [8]. This is at the basis of both KKLT and LVS scenarios that we will discuss in the rest of the article.

Compactifying type IIB string theory on a Calabi-Yau orientifold leads to an effective $\mathcal{N} = 1$ supergravity theory in 4D. The low energy action is partially determined by the tree-level Kähler potential:

$$K(S, U_\alpha, T_i) = -2 \ln V - \ln i \int \Omega \wedge \Omega^* - \ln (S + S^*)$$

(2.6)

with $V$ the volume of the Calabi-Yau manifold as a function of the Kähler moduli, $\Omega$ the unique $(3,0)$ form as a function of the complex structure moduli and $S = e^{-\phi} + iC_0$ the axiodilaton as before.

The complex structure moduli can be stabilised by turning on RR and NS fluxes $F_3$ and $H_3$, which obey the following quantisation conditions:

$$\frac{1}{(2\pi)^2 \alpha'} \int_{\Sigma_A} F_3 = M_A \quad \frac{1}{(2\pi)^2 \alpha'} \int_{\Sigma_A} H_3 = -K_A \quad \text{with } M_A, K_A \in \mathbb{Z}$$

(2.7)

for any three-cycles $\Sigma_A \in H_3(X_3)$ of the compact Calabi-Yau three-fold $X_3$. At the level of the 4D effective action they induce a superpotential [30]:

$$W_{flux} = \int G_3 \wedge \Omega, \quad \text{with } G_3 = F_3 - iS H_3.$$
This superpotential is a function of the complex structure moduli $U_\alpha$ and dilaton $S$. The supersymmetry conditions $D_\alpha W = D_S W = 0$ stabilise their values in terms of the flux numbers $M_A$ and $K_A$ in (2.7) (the value of the potential at the minimum is zero, since it is of the no-scale form and hence positive definite). These conditions are satisfied when the complex structure alligns such that the three-form $G_3$ is imaginary self-dual, i.e. $iG_3 = *G_3$. The metric and the five-form $F_5$ are also constrained to depend on a warp factor $e^A$. In particular, the metric on the compact manifold is only conformally equivalent to a Calabi-Yau metric and the compact manifold is called a conformal Calabi-Yau. Since both $F_3$ and $H_3$ enter into the definition of $F_5$, they contribute to the effective D3 brane charge. The vanishing of the total D3 brane charge, needed for D3-tadpole cancellation, implies the condition

$$\frac{1}{(2\pi)^4\alpha'^2} \int F_3 \wedge H_3 + Q_{D3}^{\text{loc}} = 0,$$  

(2.9)

where $Q_{D3}^{\text{loc}}$ is the contribution coming from the localised sources: D3-branes and supersymmetric gauge fluxes on D7-branes will contribute positively, while O3-planes and curvature of D7-brane and O7-planes contribute negatively (see [8]).

The complexified Kähler moduli are

$$T_i = \tau_i + \vartheta_i \quad \text{with} \quad \tau_i = \frac{1}{2} \int_{D_i} J^2 \quad \text{and} \quad \vartheta_i = \int_{D_i} C_4.$$  

(2.10)

Here, $\{D_i\} (i = 1, \ldots, h^{1,1}(X_3))$ is a basis of divisors of the compact manifold $X_3$, $J$ is the Kähler form of $X_3$, $\tau_i$ are the geometric Kähler moduli, i.e. the volumes of the divisors $D_i$, and $\vartheta_i$ the periods of the RR four-form potential $C_4$. The moduli $T_i$ do not appear in the tree-level superpotential $W_{\text{flux}}$ because of the Peccei-Quinn symmetries associated to their axionic component $\vartheta_i$. Hence, they can only appear at the non-perturbative level. In GKP these effects are neglected and then the supersymmetry condition $D_i W = 0$ could be satisfied only if $W_0 = W_{\text{flux}}|_{\text{min}} = 0$. This would impose extra constraints on the fluxes so that the $(3, 0)$ component of $G_3$ would vanish. However, generically this will not be the case, i.e. $D_i W \neq 0$, and still the cosmological constant will vanish as in no-scale models since $\langle K^\alpha \bar{K}^\beta K^\gamma \rangle = 3$. As a consequence, the tree-level scalar potential is positive definite and vanishes when $D_\alpha W = D_S W = 0$.

This situation is similar to the AdS$_5 \times S^5$ case in the sense that fluxes stabilise some of the moduli and leave flat directions. In this case the flat directions will naturally be lifted by perturbative and non-perturbative effects in KKLT and LVS.

Varying the values of the integers $K_A, M_A$ generate many different vacua. We may conceive trading the fluxes for D-brane configurations that carry the same information, like described for AdS$_5 \times S^5$ at the end of Section 2.1. In this case the configuration would be made up of $(p,q)$ 5-branes wrapping the corresponding three-cycles and being domain walls in the non-compact dimensions. The D3-charge of $F_3, H_3$ would be generated by D3-branes stretched between the $(p,q)$ 5-branes. This immediately suggests a ‘Coulomb branch’ approach towards duality. Notice however that at this stage the spacetime is still Minkowski and not AdS.
KKLT Scenario

The KKLT scenario extends the GKP one, adding corrections that allow one to stabilise the Kähler moduli. It is assumed that the relevant correction to the scalar potential is a non-perturbative superpotential \( W_{np} \) which in general depends on the Kähler moduli [31]:

\[
W_{np} = \sum_i A_i e^{-a_i T_j} \tag{2.11}
\]

with \( A_i \) functions of \( S, U_\alpha \). Natural sources of \( W_{np} \) are instantonic E3-branes and gaugino condensation effects on the worldvolume of D7-branes, both wrapping four-cycles of the Calabi-Yau manifold. The assumption of KKLT is that the fluxes can be tuned in such a way that the vacuum expectation value of \( W_{flux} \) is \( W_{flux}|_{min} \equiv W_0 \sim W_{np} \). Thus the contributions to \( W \) can compete to generate a supersymmetric minimum for the Kähler moduli \( T_i \), i.e with \( D_i W = 0 \). Consequently, \( V \propto -\frac{3}{2} |W|^2 < 0 \) and so the minimum is AdS. The vacuum energy gives the value of the cosmological constant, \( V|_{min} = \Lambda \). In KKLT we then have (in four dimensional Planck mass \( M_p \) units):

\[
\Lambda_{KKLT} \sim -R_{AdS}^{-2} \sim -\frac{g_s |W_0|^2}{\sqrt{2}} e^{K_{cs}}. \tag{2.12}
\]

The \( g_s \) factor comes from \( e^{K_S} \) with \( K_S = -\ln(S + \bar{S}) \). The flux dependent constant \( e^{K_{cs}} \) comes from the VEV of the complex structure moduli Kähler potential \( K_{cs} = -\ln i \int \Omega \wedge \bar{\Omega} \) (where the VEVs depend on the flux numbers). In the following we neglect this factor as its value is not relevant for our considerations.

Also other scales are fixed (in terms of \( M_p \)) once we fix all the geometric moduli. The string scale \( M_s \sim g_s M_p / \sqrt{\mathcal{V}} \) is larger than the KK scale \( M_{KK} \sim g_s M_p / \sqrt{\mathcal{V}}^{2/3} \) for volume \( \mathcal{V} \) large in string units. This is similar to the Freund-Rubin cases. However the moduli masses are hierarchically smaller. The complex structure and dilaton masses are of order \( m_{S,a} \sim 1/\mathcal{V} \). The Kähler moduli are even lighter: their masses \( m_i \sim |W_0|/\mathcal{V}^{1/3} \) are highly suppressed by the exponentially small \( |W_0| \) factor (with typical values of order \( |W_0| \sim 10^{-10} \)), even if the volume factor is larger than the KK scale (the volume is only parametrically large in KKLT).

We see that the \( |W_0| \) factor appears also in (2.12). This implies that there is a hierarchy between the size of the extra dimensions \( R_{CY} \sim 1/M_{KK} \) and the AdS radius with ratio:

\[
\frac{R_{CY}}{R_{AdS}} \sim \frac{1/M_{KK}}{R_{AdS}} \sim \frac{\sqrt{2} / g_s^{-1}}{g_s^{-1/2} |W_0|^{-1}} \sim \frac{|W_0|}{\sqrt{2} g_s^{-1/2}} \ll 1 \tag{2.13}
\]

This is clearly different from the AdS_5 \( \times S^5 \) case in which both scales are the same. This is important in order to be able to consistently neglect the KK modes in the effective field theory.

Having now a class of supersymmetric AdS compactifications labelled by fluxes makes this case suitable for the study of AdS/CFT duality. For instance the analog of the 'tHooft coupling and the number of colors \( N \) in that case can be identified easily by calculating the following quantities:

\[
\frac{R_{AdS}}{\ell_{pD}} \sim \frac{\sqrt{2} / g_s^{-1/2}}{|W_0|}, \quad \frac{R_{AdS}}{\ell_s} \sim \frac{g_s^{-1/2} |W_0|^{1/2}}{|W_0|}. \tag{2.14}
\]
The different powers of $g_s$ comes from the ratio $\ell_p^{10d}/\ell_s$ between the 10D Planck length and the string length. Again these quantities are large for small enough $W_0$. So, even if we do not know the CFT dual we may claim that it will be suitable to do perturbative expansions as identified by the above parameters. In particular, the large $N$ limit is realized here by taking $|W_0|$ small.

Uplifting to de Sitter including supersymmetry breaking was also proposed in KKLT by adding anti-D3-branes. This effect is under less control and not relevant for the present article. Also the proposed $dS/CFT$ duality is not that well understood.

LVS Scenario

The large volume scenario (LVS), also extends GKP but it includes not only the non-perturbative corrections to $W$ (2.11) but also the perturbative corrections to the Kähler potential $K$. In the simplest case the most relevant perturbative contribution is the leading order $\alpha'$ correction which modifies the Kähler potential in the following way:

$$-2 \ln V \rightarrow -2 \ln \left( V + \xi (S + S^*)^{3/2} \right)$$  \hspace{1cm} (2.15)

with $\xi$ a constant proportional to the Euler characteristic of the CY. For the generic case of several Kähler moduli and $O(1)$ flux superpotential the Kähler moduli are stabilised in such a way that the volume $V$ is exponentially large. The form of the scalar potential is:

$$V_F \propto \left( K^{SS} |D_S W|^2 + K^{\alpha\beta} D_{\alpha} W D_{\beta} \bar{W} \right) + \left( \frac{A e^{-2a\tau}}{V^2} - \frac{B e^{-a\tau} W_0}{V^2} + \frac{C |W_0|^2}{V^3} \right)$$  \hspace{1cm} (2.16)

In an expansion in $1/V$ the terms in the first bracket dominate and are positive definite so they minimise the potential at $D_S W = D_{\alpha} W = 0$ as in the GKP scenario. The terms in the next bracket compete with each other to stabilise the volume $V$ and another Kähler modulus $\tau$ such that

$$\tau \sim 1/g_s > 1 \quad \text{and} \quad V \sim e^{a\tau} \gg 1.$$  \hspace{1cm} (2.17)

Besides the larger value of the volume and the untuned choice of flux superpotential this scenario differs from the KKLT one in several other ways. The moduli are stabilised at an $AdS_4$ minimum with spontaneously broken supersymmetry. The source of supersymmetry breaking is the same as in GKP, i.e. the three-form fluxes: the perturbative and non-perturbative corrections generate only a subleading contribution to the non-zero $D_i W$, where $i$ runs on the Kähler moduli. The vacuum energy at the minimum goes like

$$\Lambda_{LVS} \sim -\frac{|W_0|^2}{V^3} g_s^{1/2} e^{K_{cs}}.$$  \hspace{1cm} (2.18)

As for KKLT we will neglect the complex structure moduli factor $e^{K_{cs}}$.

In LVS also there is a hierarchy of scales but it is different from that in KKLT. Still $M_s \sim g_s M_p/\sqrt{V^{1/2}} \gg M_{KK} \sim g_s M_p/\sqrt{V^{2/3}}$ and both are much larger than the gravitino mass $m_{3/2} \sim g_s^{1/2} |W_0| M_p/\sqrt{V}$ since the volume is very large $V \gg 1$. Most moduli masses scale with the volume $V$ like the gravitino mass, $m_{S,cs,\tau} \sim M_p/\sqrt{V}$, except for the overall volume
modulus itself which has a mass of order \( m_V \sim M_p/\sqrt{\lambda} \ll m_{3/2} \) and its axion partner which is essentially massless.\(^5\)

Like in KKLT, also in LVS there is a hierarchy between the CY size and the AdS scale. This hierarchy comes now from having a large volume \( V \) rather than a small flux superpotential \( W_0 \).

\[
\frac{R_{CY}}{R_{AdS}} \sim \frac{1/M_{KK}}{R_{AdS}} \sim \frac{V^{2/3}/g_s}{g_s^{1/4}M_3^{3/2}/|W_0|} \sim \frac{|W_0|}{g_s^{5/4}V^{5/6}} \ll 1. \quad (2.19)
\]

The calculation of the analogues of the expansion parameters, \( R_{AdS}/l_p \) and \( R_{AdS}/\ell_s \), is as in KKLT, but using the different expression for \( R_{AdS} \) and again these quantities are large because the volume is large.

\[
\frac{R_{AdS}}{l_p^{\text{norm}}} \sim \frac{g_s V}{|W_0|}, \quad \frac{R_{AdS}}{\ell_s} \sim \frac{g_s^{3/4}V}{|W_0|}. \quad (2.20)
\]

These expansion parameters reflect the known expansions in \( g_s \) and \( 1/V \). Notice that if we keep \( g_s \) and \( V \) fixed, the large \( N \) limit can be realised by taking \( |W_0| \) small, like in KKLT.

Notice that in both KKLT and LVS cases the expansion parameters cannot be made arbitrarily small. This is a due to the fact that the flux numbers \( (2.7) \) are bounded from above \([5, 13, 14]\) by the D3 tadpole cancellation conditions \( (2.9) \). This implies on one side that there is a finite number of flux vacua and on the other side that there is a bound on the value of \( g_s \) and therefore also on the volume in LVS since \( V \sim e^{a/g_s} \).\(^6\) This contrasts with the large \( N \) expansion in which \( 1/N \) can be made arbitrarily small.\(^7\)

### 3 Properties of the CFT\(_3\) duals

Having a precise description of the AdS\(_4\) type IIB flux vacua, it is natural to search for the CFT\(_3\) duals. The situation is much less clear than in the AdS\(_5\) × \( S^5 \)/CFT\(_4\) case. The main obstacle is that there is no clean 10D string theory formulation of the KKLT and LVS scenarios, and most of the results are obtained only through an effective field theory approach. In particular, the description of the non-perturbative effects is valid only within

\(^5\)In the most general cases there may be fields, like those corresponding to K3 fibrations, that get masses only after string loop effects are included and their masses can be smaller than the volume mass \( m_f \sim |W_0| M_p/\sqrt{3} < m_V \) \([32]\).

\(^6\)In \([5]\) a simple example of a rigid CY is presented. For illustration we use this case to show that \( g_s \) will be bounded from below by the tadpole cancellation condition. For a rigid CY, the flux superpotential is \( W_{\text{flux}} = (f_1 + \Pi f_2) - iS(h_1 + \Pi h_2) \equiv F - iSH \), where \( \Pi \) is a complex number determined by the geometry. Let us take \( \Pi = i \) for simplicity. The susy equation \( D_5 W_{\text{flux}} = 0 \) gives \( S = i \frac{F}{\Pi} \). The tadpole cancellation condition is \( \text{Im} H F \leq \mathcal{L} \), where we have separated the D3-brane contribution by the negative contribution coming from O3-planes, D7-branes and O7-planes: \( Q_{\text{D3}}^{\text{bos}} \equiv N_{\text{D3}} - \mathcal{L} \). Fixing the S-duality symmetry, the flux vacua satisfying the tadpole cancellation condition are given by \( h_2 = 0, 0 \leq f_1 < h_1 \) and \( h_1 f_2 \leq \mathcal{L} \). Thus we have \( \frac{1}{g_s} \sim \frac{h_1}{h_2} = \frac{h_1 f_2}{q_1^2} \leq \mathcal{L} \), and hence \( g_s^{\text{min}} \sim \frac{q_1}{\mathcal{L}} \). (In this computation we are excluding the vacua \( h_1 = h_2 = 0 \) that would give \( g_s = 0 \), i.e. non-interacting strings.)

\(^7\)We thank N. Seiberg for emphasising this point.
the effective field theory approximation. Contrary to the $\text{AdS}_5 \times S^5$ case there are no known black-brane solutions in which the AdS factor can be achieved by a near horizon limit. On the other hand we should be able to extract some partial information based on the effective field theory results and by analogy with known cases.

In particular the study of the Coulomb branch motivated \cite{17} to come-up with a concrete proposal for the duals of KKLT compactifications. As anticipated before, the main idea is to consider $(p,q)$ 5-branes that are domain walls separating AdS vacua corresponding to different fluxes. These 4D domain walls are 5-branes wrapping the same 3-cycles threaded by the fluxes and located at different points in the radial direction of AdS.\footnote{Notice that these are precisely the same brane configurations that can nucleate the potential decay of metastable minima as discussed in \cite{23}.} D3-branes must be introduced in order to satisfy the total D3 charge constraint (2.9). These D3 branes will be stretched between the 5-branes. As for the $\text{AdS}_5 \times S^5$ case, the domain wall configurations should represent the dual CFT in its Coulomb branch, i.e. when the fields representing the location of the corresponding branes get a non-zero VEV. This is an interesting proposal that is analogous to the $\text{AdS}_5 \times S^5$ case: it implements a brane/flux duality that seems to be at the core of the gauge/gravity correspondence.

In general, the understanding of the CFT side is very limited. Hence, rather than concentrating on tests of the duality, we will focus on extracting properties that these CFTs will have in order to be dual to the KKLT or LVS AdS$_4$ minima. In reference \cite{33} a set of conditions were spelled out in order for a CFT to have a gravity dual: (i) Having a large central charge $c$; (ii) A small set of operators of conformal dimension of $O(1)$ and (iii) approximate (in an $1/\sqrt{c}$ expansion) factorisation of their correlation functions. In the following we will see that if a CFT dual exists that is dual to KKLT or LVS AdS minima, then it will satisfy the properties just mentioned.

3.1 Supersymmetry

As we have mentioned above, the KKLT $\text{AdS}_4$ minimum is supersymmetric. Therefore, we expect its CFT$_3$ dual to have $\mathcal{N} = 1$ supersymmetry in three dimensions. On the other hand the LVS minimum is not supersymmetric. The supersymmetry is spontaneously broken by fluxes. On the CFT side supersymmetry cannot be spontaneously broken since a spontaneous breaking of supersymmetry would introduce a scale, breaking conformal invariance. Therefore on the CFT side of LVS, supersymmetry should appear as explicitly broken.

If we accept the proposal of \cite{17}, the dual CFT in both cases should live on the world-volume of a bunch of branes, compactified to three dimensions.\footnote{In \cite{17}, the proposal was for the KKLT minimum, but the same considerations can be done for LVS.} The brane configuration is determined by the flux choice. In particular, since the fluxes break supersymmetry (at the GKP level), the brane configuration will be non-supersymmetric with respect to the geometric background, meaning that the brane configuration and the CY metric preserve incompatible supersymmetries. In LVS, the supersymmetry breaking scale is basically the...
same as in GKP:
\[ DT_b W = K T_b W_{\text{flux}} + \partial T_b W_{\text{np}} \sim K T_b W_{\text{flux}} \neq 0 , \] (3.1)
where \( T_b = \tau_b + i \vartheta_b \) and \( \tau_b \) is the volume of the so-called ‘large four-cycle’, i.e. the one that determines the volume of the CY in LVS (i.e. \( V \sim \tau_b^{3/2} \)) as we will see more in detail later.

We see that in LVS the susy breaking by fluxes is not modified by the subleading corrections to the scalar potential that stabilise the Kähler moduli at an AdS_4 minimum. Hence, the couplings generated by the metric on the worldvolume of the branes will explicitly break supersymmetry, i.e. the dual CFT will be non-supersymmetric.

In KKLT, the non-perturbative effect generating the AdS_4 minimum restores supersymmetry, i.e.
\[ DT_i W = K T_i W_{\text{flux}} + \partial T_i W_{\text{np}} = 0 . \] (3.2)
Hence we expect that the non-perturbative effect will generate couplings in the dual CFT that restore supersymmetry. However this entire discussion cannot really be clarified before a 10D understanding of the non-perturbative effects is established.

Finally, notice that local symmetries on the AdS side are usually mapped to global symmetries on the CFT side. Still the SO(3,2) isometry of AdS_4 maps the 3D conformal symmetry. Since the Calabi-Yau manifolds do not carry isometries there are no global symmetries associated to isometries (unlike the R-symmetries in the AdS_5 \times S^5 case). This is consistent with the fact that the 3D CFT has no continuous R symmetries.

### 3.2 Central charge and number of degrees of freedom

In 2+1 dimensional CFTs the central charge (\( c \sim N_{\text{dof}} \)) can be defined at least in two ways [34]: from the two point function of the energy momentum tensor or from the ‘entropy/temperature relation’. Both definitions were proven to be equivalent for theories with AdS duals [35] and to be proportional to \( R^2_{\text{AdS}} \) in 4D Planck units. So we can write:
\[ N_{\text{dof}} \sim R^2_{\text{AdS}} \begin{cases} \frac{\Lambda^2}{g_s |W_0|^2} & \text{KKLT} \\ \frac{V^3}{g_s^{3/2} |W_0|^2} & \text{LVS} \end{cases} \] (3.3)
We see that in both cases, KKLT and LVS, the CFT has a very large central charge, as expected for a CFT that has a gravity dual. This should be interpreted as the analogue of large N.

The number of degrees of freedom should match with the one computed in the dual CFT. If one consider the ensemble of flux vacua, there will be a vacuum with the smallest cosmological constant, i.e. the vacuum with the maximum number of degrees of freedom \( N_{\text{dof}}^{\text{max}} \). If one knows the distribution of \( \Lambda \) over the Landscape of flux vacua and the total number of vacua \( N_{\text{vac}} \), one can estimate what is the minimum value that the cosmological constant will take in the Landscape. For KKLT this problem was studied in [17]: Expressing the volume in terms of the flux dependent parameters \( g_s, W_0, A \) and knowing that the distributions of such quantities are roughly uniform, one obtains a roughly uniform distribution of \( \Lambda \) [13, 14]. This means that \( \Lambda_{\text{min}}^{KKLT} \sim \frac{1}{N_{\text{vac}}} \).
We repeat the estimate for the LVS case. The value of $\Lambda$ at the minimum is given by $\Lambda \sim A_3 e^{-3a/g_s |W_0|}$ (where we have used $V \sim W_0 A e^{a/g_s}$). Because of the exponential factor, the distribution will be extremely peaked at small values of $\Lambda$ (see [16] for a recent discussion of this point). Thanks to the uniform distribution of $g_s$, one can estimate the minimum value of the cosmological constant to be very roughly $\Lambda_{\min}^{LVS} \sim e^{-N_{\text{vac}}}$. This means that the maximal number of degrees of freedom in the LVS case is much larger than the one in KKLT. Summarising:

$$N_{\text{dof}}^{\text{max}} \sim \begin{cases} N_{\text{vac}} & \text{KKLT} \\ e^{N_{\text{vac}}} & \text{LVS} \end{cases} \quad (3.4)$$

For KKLT, in [17] an explanation of $N_{\text{dof}}^{\text{max}} \sim N_{\text{vac}}$ (referred to there as the maximal entropy) was given by counting the degrees of freedom in the proposed CFT dual that we mentioned before, i.e. the three dimensional theory living on a bunch of 5-branes and D3-branes. The degrees of freedom are given by string junctions between the branes and possibly KK modes with mass up to $M_p$. For KKLT, $M_{KK}$ is close to $M_p$, as the volume of the compactification manifolds (and hence of the three-cycles wrapped by the 5-branes) is not very large. In fact, counting the number of string junctions in the most symmetric brane configurations (the one that should have the largest entropy) one finds agreement with $N_{\text{dof,KKLT}}^{\text{max}} \sim N_{\text{vac}}$. For LVS, one should include also the KK modes. For each three-cycle, one has around $M_p/M_{KK} \sim \sqrt[3]{V_2}$ KK modes to consider. Since the volume is exponentially large, this would imply including in the counting a number of modes much larger than in the KKLT case. This justifies the large difference between the values of $N_{\text{dof}}^{\text{max}}$.

### 3.3 Conformal dimensions

The relation between the mass $(m)$ of the various fields on the gravity side and the conformal dimension $(\Delta)$ of the operator in the dual CFT is given in (2.5) for scalar fields. In our case $(d = 3)$:

$$m^2 R_{AdS}^2 = \Delta(\Delta - 3). \quad (3.5)$$

- **KKLT**: Since there is a hierarchy of scales we know that the conformal dimensions of string and KK modes will be hierarchically large. The relevant fields are the moduli. The complex structure moduli and dilaton have masses of order $\sim 1/V$ whereas the Kähler moduli have masses of order the gravitino mass $m \sim m_{3/2} \sim |W_0|/V$. Therefore, from (3.5) we have

$$\Delta_{\text{moduli}} \sim \begin{cases} O(1) & \tau_i, \vartheta_i, U_{\alpha}, S, \\ \frac{1}{|W_0|} \gg 1 & \tau_i, \vartheta_i, U_{\alpha}, S, \end{cases} \quad (3.6)$$

where $T_i = \tau_i + i\vartheta_i$ are the Kähler moduli, $U_\alpha$ the complex structure moduli and $S$ the axiodilaton. For a typical CY there is a relatively large but finite number ($h^{1,1} \sim O(1 - 100)$) of fields with $O(1)$ conformal dimension. Since there is a gravity dual we expect approximate factorisation of the correlation functions for these operators.
• LVS: The masses of the various moduli go as

\[ m_{\tau_s} \sim m_{\alpha_s} \sim m_{3/2} \sim \frac{|W_0|}{\sqrt{V}}, \]
\[ m_U \sim m_S \sim \frac{1}{\sqrt{V}}, \]
\[ m_{\tau_b} \sim \frac{|W_0|}{\sqrt{3/2}}, \]
\[ m_{\vartheta_b} \sim 0, \]

where we have omitted the irrelevant \( g_s \) factors and we are taking a model with one large \( (\tau_b) \) and one small \( (\tau_s) \) Kähler modulus. From these expressions we get that

\[ m^2_{\tau_s} R_{AdS}^2 \sim m^2_{\vartheta_b} R_{AdS}^2 \sim V \gg 1, \]
\[ m^2_U R_{AdS}^2 \sim m^2_S R_{AdS}^2 \sim V \gg 1, \]
\[ m^2_{\tau_b} R_{AdS}^2 \sim O(1), \]
\[ m^2_{\vartheta_b} R_{AdS}^2 \sim 0. \] (3.7)

The above equations suggest that the conformal dimension of the operators dual to complex structure and small Kähler moduli is very large whereas for the operators dual to volume modulus \((V \sim \tau_b^{3/2})\) and its axionic partner it is \(O(1)\):

\[ \Delta_{\text{moduli}} \sim \begin{cases} O(1) & \tau_b, \vartheta_b, \\
V^{1/2} & \tau_s, \vartheta_s, U_{\alpha}, S. \end{cases} \] (3.8)

Since there are only few operators with \(O(1)\) conformal dimension, it suggests that the dual field theory is very strongly coupled. Again correlation functions should approximately factorise.

We find this result particularly interesting since the CFT seems to have only one scalar operator (and its axionic partner) with conformal dimension of \(O(1)\). This is related to the fact that the volume modulus mass is hierarchically smaller than the gravitino mass, despite supersymmetry being broken. A standard concern about this result is if quantum effects, after supersymmetry breaking, will naturally raise the value of this mass to the supersymmetry breaking scale. This issue was discussed in [36] in which the loop corrections to the modulus masses were found to be proportional to \(\delta m^2 \propto g \cdot \Delta m^2_{\text{bos-form}} \sim \frac{m^2_{KK}}{M^2_p} m^2_{3/2} \sim \frac{M^2_p}{\sqrt{V}} m^2_{KK}/m^2_{3/2}\). We see that for very large volume \(V\), \(\delta m \ll m_{\tau_b} \ll m_{3/2}\). It is then expected that in the corresponding CFT quantum corrections will not substantially change the conformal dimension and keep this hierarchy. Having a CFT with such a simple structure of low-lying operators is intriguing and may be interesting to search for.

### 3.4 Wrapped branes and their dual

There are some operators in the dual field theory whose existence depends on the given choice of flux vacuum. This allows us to distinguish two different flux vacua that have the same value of \(W_0, g_s\) and \(A\). One such class of operator we consider here is the baryon like
operator/vertex. These operators/vertices in the field theory are dual to the configuration of Dp-brane wrapping p-cycle in compact directions. They have provided non trivial checks of AdS/CFT duality [37, 38]. In our case it is very natural to consider a configuration of D3-branes wrapping a three-cycle Σ of the CY manifold. This will correspond to a massive particle in AdS4 whose mass is determined in terms of the volume of the three-cycle. Assuming that the particle is stable, we want to find the operator or vertex in the CFT dual.

On the D3-brane world volume there is a gauge field $A_\mu$. In the presence of the gauge field, the world volume theory of the D3-brane contains Chern-Simons coupling involving the background fluxes and RR scalar field,

$$(2\pi\alpha')\mu_3\int_{\Sigma \times \mathbb{R}} A \wedge [F_{(3)} + C_0 H_{(3)}].$$

(3.9)

Here $\mu_3 = \frac{1}{(2\pi)^3\alpha'^2}$ is the D3-brane charge. Now, using (2.7) we find that the background fluxes contribute to the charge of the particle under the $U(1)$ gauge field $A_\mu$ which is given by

$$[M_\Sigma - C_0 K_\Sigma] \int_R A.$$

(3.10)

Here $C_0$ is fixed by $W_{\text{flux}}$ to a constant value depending on fluxes as well. Therefore for a given value of the superpotential, the AdS vacuum for which $[M_\Sigma - C_0 K_\Sigma] \in \mathbb{Z}$, will have operators or vertices in the dual field theory which probe the individual fluxes.

### 3.5 Other properties

Let us collect some further considerations on the dual CFTs.

- In [39] an interesting description was made for CFTs that allow tuning of some couplings (such as a dual description of the cosmological constant tuning on the gravity side). Three conditions were identified for these CFTs: (i) That the CFT allows an AdS dual (implying large central charge, few light operators and approximate factorisation of their correlation functions), (ii) Absence of supersymmetry and (iii) A cut off scale such that $\Delta_{\text{cutoff}} \gg c^{1/D}$. In the LVS case the first two conditions are automatically satisfied. As regards the third condition, we have $\Delta_{\text{cutoff}} \sim \mathcal{V}^{5/2}/\mathcal{V}^{2/3} = \mathcal{V}^{5/6}$ (if we use the KK scale as the cut-off) and $c^{1/D} \sim \mathcal{V}^{3/4}$; since $5/6 > 3/4$, the condition (iii) is satisfied if the volume is very large. It was pointed out in [39] that there is no known CFT satisfying the three conditions. For what is worth the dual of LVS may provide such an example.

- Notice that the generic realisation of KKLT and LVS into a fully fledged compact Calabi-Yau compactification, including chiral matter in visible and hidden sectors, is usually richer than the minimum set-up described here. This would shed further light on fields charged under gauge groups located at D7 or D3 branes. These would have a representation in terms of low-lying operators in the dual theory. Moreover, some of the $U(1)$ groups tend to be anomalous with the anomaly cancelled by the standard Green-Schwarz mechanism and the presence of chiral matter induces D-terms that can...
modify the structure of the minima of the scalar potential. In particular in concrete compact CY realisations of LVS [24, 26, 27], it was found that $D$-terms and $F$-terms of hidden matter fields tend to uplift the value of the minima of the scalar potential allowing even the possibility to have dS vacua. The amount of uplift depends on explicit values of order $\mathcal{O}(1)$ parameters and the tunable fluxes\footnote{A similar situation would happen in the standard KKLT uplift in terms of anti-branes at long throats although in that case supersymmetry is broken explicitly.}. In the AdS cases this would imply an almost smooth increase on the value of the central charge that can be as large as allowed by the parameters of the given model tending towards infinite values. The ‘jump’ to dS would then seem to require a drastic change on the CFT since it would require passing through a singularity in the central charge. This is a manifestation of the difficulties expected to find an extension of the AdS/CFT duality to a potential dS/CFT duality.

- Speculating further, we may add that being the LVS minimum non-supersymmetric, it might be dual to some condensed matter systems such as the quantum critical points of holographic superconductors [28]. For this, extra fields have to be considered, such as gauge fields on the AdS side. These can come for instance from D7 branes wrapping the large four-cycle with gauge coupling $1/g^2 \sim V^{2/3}$, which would correspond to the conductivity $\sigma_{xx}$ in the dual theory. It may be interesting to further study this case searching for Reissner-Nordstrom-like black hole solutions exhibiting instabilities on charge scalar fields to describe phase transitions on the quantum critical point on the CFT side as it was done for other examples in [20].

### 4 Effective potential and quantum logarithmic effects

In AdS/CFT duality, the partition function of the theory of gravity on AdS space is equal to the partition function of the CFT living at its boundary [2, 40]. There have been several checks for this duality but the majority of works are in the infinite $N$ limit which corresponds to studying classical gravity in the bulk.

In this section we briefly review how one-loop corrections to the partition function in gravity systems have been used to learn and test the gauge/gravity duality. Loop corrections to the partition function on the gravity side correspond to going beyond planar limit on the dual field theory side. This provides a test of AdS/CFT duality beyond planar limit which is very non-trivial, as it involves string loop computations on the AdS side. However in the $\alpha' \rightarrow 0$ limit, this reduces to the computation in supergravity. In these procedure it is always worth looking for the quantity which does not depend on the details of the UV theory. Such quantities are universal in the sense that they can be calculated in the low energy effective field theory. One quantity of this type is the logarithmic correction, $\ln(R_{AdS})$, in the partition function of effective field theory on the gravity side. This object has been used quite successfully in studying the entropy of black holes [41–45]: the logarithmic corrections calculated on the supergravity side matches with those computed on the string theory side. A similar comparison has been made in [46] where the supergravity calculation in $\text{AdS}_4 \times X_7$,
where $X_7$ is a compact seven dimensional manifold, reproduces the correct coefficient of the logarithmic correction present in the $1 \over N$-expansion of the partition function of the three dimensional CFT.

Motivated by this success, we will do a similar computation in the KKLT and the LVS cases where we have supersymmetric and non supersymmetric AdS$_4$ minima respectively. Assuming the validity of the AdS$_{d+1}$/CFT$_d$ duality, these vacua will have a dual description in terms of a (unknown) three-dimensional CFT. The computation on the AdS side will give a non trivial prediction for the CFT partition function. As we will explain in detail below, the logarithmic correction, $\ln(R_{\text{AdS}} \epsilon)$, arises at one loop when a particle whose mass scales with some power of $R_{\text{AdS}}$ runs inside the loop. Calculating such logarithmic corrections in KKLT and LVS requires the knowledge of the explicit form of masses of all the moduli fields. These are not available at the moment for all the scalar fields. In particular, for the compactifications we have considered, the Kähler moduli masses are known as functions of few parameters (depending on the flux numbers (2.7)), while the complex structure moduli masses are unknown functions of the fluxes. Since all the masses of the Kähler moduli and gravity multiplets scale with some power of $W_0$ (a function of the flux numbers), we will calculate a similar logarithmic correction, $\ln |W_0|^2$, that does not requires the knowledge of the complex structure moduli masses (that do not scale with $W_0$). We claim that this is a universal prediction for the dual CFT, once one identifies what $W_0$ parametrises in the dual theory.

4.1 The limit $|W_0| \to 0$

The effective field theory in KKLT and LVS (after integrating out the axiodilaton and the complex structure moduli) are basically labelled by three parameters, that are functions of the flux numbers (2.7): the super potential $W_0$, string coupling $g_s$ and the prefactor $A$ of the non-perturbative contribution to the superpotential (in case there is only one non-negligible non-perturbative effect). After stabilising the Kähler moduli, these fields are also function of these parameters. In particular, this happens for the volume of the compactification manifold $\mathcal{V} = \mathcal{V}(W_0, g_s, A)$. Inverting this relation, we can express $A$ in terms of $\mathcal{V}$ and use this last one as the third parameter. The radius of the AdS is given by

$$\frac{1}{R_{\text{AdS}}^2} \sim g_s^2 W_0^2 \sqrt{\mathcal{V}^\beta}. \quad (4.1)$$

Here $\alpha = 1, \beta = 2$ for KKLT and $\alpha = {1 \over 2}, \beta = 3$ for LVS.

Now, in order for the supergravity approximation to work, $R_{\text{AdS}}$ needs to be arbitrarily large. This limit can be achieved in various ways. However in our case we will work in the limit,

$$W_0 \to 0, \quad g_s = \text{fixed}, \quad \mathcal{V} = \text{large but fixed}. \quad (4.2)$$

We motivate this as follows: if we are interested in the coefficient of logarithmic correction $\ln R_{\text{AdS}}$, which is the general quantity of interest in standard AdS/CFT duality, then we need to include all the fields whose mass scales with some power of $R_{\text{AdS}}$. Therefore in order to calculate logarithmic correction $\ln R_{\text{AdS}}$, we need to know the masses of all the moduli fields including the KK modes. This is a rather harder problem at present, due to
the unknown expression for the complex structure moduli masses. An important point to observe is that the masses of KK modes and complex structure moduli do not scale with $W_0$, while the masses of the Kähler moduli, gravitino mass and the cosmological constant scale do scale with $W_0$. Hence only Kähler moduli and the gravity multiplet contribute to the coefficient of $\ln |W_0|^2$, and we can single this out by considering the limit (4.2). This is the reason why we look for the coefficient of $\ln |W_0|^2$.

### 4.2 Effective potential

In this section we will describe the computation of the one loop effective action in supergravity coupled to matter fields. The one loop calculation involves the computation of determinants of the various operators which appear at the quadratic order in the fluctuations of the fields in the Lagrangian about the background fields. The determinants are then expressed in terms of the heat kernel of the operator. The UV divergences of the effective action is captured by the asymptotic expansion of the heat kernel. In this expansion we will look for the logarithmic divergence.

The heat kernel expression for the one-loop effective action is:

$$
\Gamma(1) = -\frac{1}{2} \int_\epsilon^\infty \frac{d\tau}{\tau} \text{Str} \exp[-\tau(\nabla^2 + X + M^2)]
$$

$$
= -\frac{1}{2} \int_\epsilon^\infty \frac{d\tau}{\tau} \text{Str}\{\exp[-\tau(\nabla^2 + X)]e^{-\tau M^2}\}.
$$

(4.3)

Here $\nabla^2 = -ig^{\mu\nu}\nabla_\mu\nabla_\nu$ where $I$ is the unit matrix in the space of fields and $X$ is a spin dependent matrix that is linear in the Riemann tensor \cite{47} (the gauge field background in 4D has been taken to be zero) and $M$ is a field dependent mass matrix. In the second line we have dropped space time derivatives of $M$ since we are just considering the effective potential. Now we use the adiabatic expansion for the heat kernel to write

$$
\Gamma(1) = -\frac{1}{2} \int_\epsilon^\infty \frac{d\tau}{\tau} \frac{1}{16\pi^2\tau^2} \text{STr}\{[a_0^{(s)} I + a_2^{(s)} \tau + a_4^{(s)} \tau^2 + \ldots]e^{-\tau M^2}\}
$$

$$
= -\frac{1}{32\pi^2} \text{STr}\{a_0^{(s)} I_0 + a_2^{(s)} I_2 + a_4^{(s)} I_4 + \ldots\}.
$$

(4.4)

Note that in the first line above the trace includes an integral over the space time. Also the prefix ‘S’ on the trace simply implies tracing over the physical degrees of freedom with a factor $(-1)^{2s}$, $s$ being the spin. The coefficients $a_n^{(s)}$ are integrals over the De Witt coefficients and are given below \cite{47, 48}:

$$
a_0^{(s)} = \int d^4x \sqrt{g} Tr I,
$$

$$
a_2^{(s)} = \frac{1}{6} \int d^4x \sqrt{g} Tr(R + 6X^s),
$$

$$
a_4^{(s)} = \frac{1}{180} \int d^4x \sqrt{g}\{\alpha^s C_{\mu\rho\sigma} C^{\mu\rho\sigma} + \beta^s (R_{\mu\nu} R^{\mu\nu} - \frac{1}{4} R^2) + \gamma^s \Box R + d^s R^2\}.
$$

(4.5)

Here $Tr$ indicates the trace over the various indices of the field like space time indices and internal indices.
In (4.4) $I$ is a unit matrix and $I_{0,2,4}$ are matrix valued integrals, whose entries are of the form

\[ I_0 = \int_\epsilon^\infty \frac{d\tau}{\tau^2} e^{-\tau m^2}, \quad I_2 = \int_\epsilon^\infty \frac{d\tau}{\tau^2} e^{-\tau m^2}, \quad I_4 = \int_\epsilon^\infty \frac{d\tau}{\tau} e^{-\tau m^2}. \]

These integrals satisfy the conditions

\[ \frac{dI_2}{dm^2} = -I_4, \quad \frac{dI_0}{dm^2} = -I_2. \]

Finally we have (substituting $t = m^2 \tau$)

\[ I_4 = \int_{\epsilon m^2}^\infty \frac{dt}{t} e^{-t} = \Gamma(0, \epsilon m^2), \]

where

\[ \Gamma(z, x) \equiv \int_\epsilon^\infty t^{z-1} e^{-t} \]

is the incomplete Gamma function for which we have the expansion (for $z = 0$),

\[ \Gamma(0, x) = -\gamma - \ln x - \sum_{k=1}^{\infty} \frac{(-x)^k}{k(k!)}. \]

Thus $I_4 = -\ln(\epsilon m^2) + f(\epsilon m^2), I_2 = m^2 \ln(\epsilon m^2) + m^2 g(\epsilon m^2), I_0 = -1/2m^4 \ln(\epsilon m^2) + m^4 h(\epsilon m^2)$ where $f$ is an analytic function and $g, h$ are meromorphic functions with poles of order 1 and 2 respectively. Putting these results into (4.4), we obtain

\[ \Gamma_{(1)} = \frac{1}{32\pi^2} \text{Str} \left[ \frac{1}{2} a_0^{(s)} M^4 - a_2^{(s)} M^2 + a_4^{(s)} \right] \ln(\epsilon M^2) \]

\[ + \int d^4x \sqrt{g} \left[ \frac{1}{\epsilon} \text{Str} M^2 + \frac{1}{\epsilon^2} \text{Str} I + V_{(1)}(\epsilon M^2) \right]. \]  

(4.6)

with the last integrand $V_{(1)}$ being an analytic function. In a theory with equal numbers of fermionic and bosonic degrees of freedom such as a supersymmetric theory the $\epsilon^{-2}$ term will vanish. In a supersymmetric theory with zero cosmological constant and unbroken supersymmetry the $O(\epsilon^{-1})$ will also vanish. In a flat background we will only have the first term in the factor multiplying $\ln(\epsilon M^2)$ which of course gives the usual Coleman-Weinberg formula. In the following we will focus on the log divergence term, the first line in (4.6), since the coefficients are independent of the UV regulator and we can find a universal quantity that is just proportional to the log of the flux superpotential.\footnote{Due to the UV divergence, we need to use a cutoff $\epsilon$. In string theory $\epsilon$ is a physical cutoff, effectively $\epsilon = l_s^2$ or $l_{KK}^2$.}

\footnote{Note that modes with masses close to the cutoff, like KK modes and string states, give a suppressed contribution to the first line of (4.6). In any case as noted earlier these will not contribute to the $\ln W_0$ terms.}
4.3 Effective potential $\Gamma_1$ about AdS background

We now compute the De Witt coefficients $a_i$ appearing in the logarithmic divergence for the fields with spin $\leq 2$ about the AdS$_4$ background. In the next section we will use these coefficients to compute the $\ln |W_0|$ term for the cases of KKLT and LVS flux compactifications.

The AdS$_4$ metric is given by

$$ds^2 = R_{AdS}^2 \left( d\eta^2 + \sinh^2 \eta d\Omega_3 \right),$$

(4.7)

where $d\Omega_3$ is the metric of three-sphere. In this background the curvature has the form

$$R_{\mu \nu \rho \sigma} = -\frac{1}{3} L^{-2} (g_{\mu \nu} g_{\rho \sigma} - g_{\mu \rho} g_{\nu \sigma}), \quad R_{\mu \nu} = -L^{-2} g_{\mu \nu}, \quad R = -4L^{-2},$$

(4.8)

where $R_{AdS}^2 = 3L^2$ and $-L^{-2} \equiv -|\Lambda| < 0$ is the AdS cosmological constant (CC).

Let us evaluate the coefficient $a_4^{(s)}, a_2^{(s)}$ and $a_0^{(s)}$ in this background. From (4.8) we have

$$R_{\mu \nu} R^{\mu \nu} = \frac{4}{L^2} g_{\mu \nu}, \quad R_{\mu \nu} - \frac{1}{4} R^2 = 0, \quad C_{\mu \rho \nu \sigma} C^{\mu \rho \nu \sigma} = 0.$$

(4.9)

We parametrize the De Witt coefficients (4.5) as follows,

$$a_4^{(s)} = \frac{d^s}{180} \int d^4x \sqrt{g} R^2, \quad a_2^{(s)} = \frac{c^s}{6} \int d^4x \sqrt{g} R,$$

$$a_0^{(s)} = f^s \int d^4x \sqrt{g}.$$

(4.10)

The coefficients ($d^s, c^s, f^s$) are given in Table 1 (for details see Appendix B). Suppose the theory has neutral chiral supermultiplets (moduli/ini) along with the graviton and the gravitino. Then we have the effective potential

$$\Gamma_1 \sim \frac{1}{32\pi^2} \sum_{s=0}^{2} (-1)^{2s} \left[ \frac{1}{2} a_0^{(s)} m_s^4 - a_2^{(s)} m_s^2 + a_4^{(s)} \right] \ln(\varepsilon m_s^2)$$

$$\sim \frac{1}{32\pi^2} \int \sqrt{g} d^4x \sum_{s=0}^{2} (-1)^{2s} \left[ \frac{1}{2} f^s m_s^4 - \frac{c^s}{6} R m_s^2 + \frac{d^s}{180} R^2 \right] \ln(\varepsilon m_s^2),$$

(4.11)

where we have used (4.10).

The volume of AdS$_4$ is infinite, however in AdS/CFT there is a well defined prescription to extract the finite part [49],

$$\int d^4x \sqrt{g} = \frac{4\pi^2 R_{AdS}^4}{3} = 12\pi^2 L^4.$$

(4.12)

13The metric is given in Poincare coordinates by $ds^2 = R_{AdS}^2 \left( dz^2 + \sum_{i=1}^{3} (dx_i)^2 \right)$. This presentation shows that AdS is conformally flat so that its Weyl tensor is manifestly equal to zero, $C_{\mu \rho \nu \sigma} = 0$.

14Note that we have suppressed for simplicity an additional sum over chiral scalar multiplets - this will be remedied later.
Thus we get,

$$
\Gamma^{(1)} \sim \frac{3L^4}{8} \sum_{s=0}^{2} (-1)^{2s} \left[ \frac{1}{2} f^s m_s^4 + \frac{2c^s}{3L^2} m_s^2 + \frac{4d^s}{45L^4} \right] \ln(\varepsilon m_s^2) .
$$

(4.13)

While carrying out the above computations, we also need to include the contributions of the various ghost fields for the spin $\frac{3}{2}$, $\frac{3}{2}$, and 2 fields. We list in the table the coefficients $d^s, c^s, f^s$, taking into account the contributions of the various ghost fields.$^{15}$

| $s$ | $d^s$ | $c^s$ | $f^s$ |
|-----|-------|-------|-------|
| 0   | 29/12 | 1     | 1     |
| 1/2 | 11/24 | -1    | 2     |
| 1   | -31/6 | -4    | 2     |
| 3/2 | 251/24| 8     | -88   |
| 2   | 1139/6| -22   | 2     |

Table 1. Coefficients appearing in (4.13) for spin $s$ particles.

5 Coefficient of $\ln |W_0|^2$ in type IIB flux compactifications

5.1 KKLT vacua

As we have seen, in the KKLT scenario the Kähler moduli are fixed by non-perturbative contribution to the superpotential. In this section we consider a Calabi-Yau with one Kähler modulus (i.e. $h^{1,1} = 1$). The volume $V$ of the CY will be given in terms of the Kähler modulus $\tau$ by $V = \tau^{3/2}$. We assume that there is a four-cycle $D$ with volume $\tau$ that supports a non-perturbative effect, generating a superpotential of the form $W_{np} = A e^{-aT}$.

The $N = 1$ supergravity potential is determined by the Kähler potential $K$ and the superpotential $W$ of the effective theory. These are functions of the Kähler coordinate $T = \tau + i\vartheta$, where $\tau = \frac{1}{2} \int_D J^2$ is the Kähler modulus and $\vartheta = \int_D C_4$ is the axion coming from the RR four-form potential. After integrating out the complex structure moduli and the axiodilaton, the scalar potential is

$$
V = e^K \left( K^{TT} D_T W D_T W - 3|W|^2 \right) .
$$

(5.1)

In the KKLT case, we have

$$
K = -2 \ln \mathcal{V}(T, \bar{T}) = -3 \ln (T + \bar{T}) , \quad W = W_0 + A e^{-aT} .
$$

(5.2)

$^{15}$ Note that: 1) in the table we have presented the coefficients for Weyl (Majorana) fermion, which we obtained by considering a Dirac fermion and divide the result by half; 2) the coefficients $f^s$ for gravitino is different from 2; this happens because the contribution of the ghosts, with mass $2m_{3/2}$, is included. For more details see Appendix B.
The supersymmetric minimum of this potential is at $D_i W = 0$, i.e. at $\vartheta = 0$ and

$$W_0 = -A e^{-a \tau} \left(1 + \frac{2}{3} a \tau \right). \quad (5.3)$$

The value of the potential at the minimum is

$$V|_{\text{min}} = -\frac{3 W_0^2 a^2}{2 \tau (3 + 2 a \tau)^2} \quad (5.4)$$

where $\tau$ satisfies the relation (5.3). From this we read the cosmological constant, i.e. $\Lambda = V|_{\text{min}}$.

**Scalar masses**

At the minimum, the Hessian of the potential is

$$\partial_i \partial_j V|_{\text{min}} = \begin{pmatrix} \frac{3 W_0^2 a^3}{2 \tau (3 + 2 a \tau)} & 0 \\ 0 & \frac{3 W_0^2 a^2 (2 + a \tau)(1 + 2 a \tau)}{2 \tau (3 + 2 a \tau)^2} \end{pmatrix}, \quad (5.5)$$

with $i, j = \vartheta, \tau$.

We need to calculate the masses of the canonically normalised fields. These are obtained by multiplying the matrix $\partial^2 V$ by $\frac{1}{2} K^{-1}_{TT} = \frac{2 e^{2K/3}}{3}$. The masses of the two scalar fields are then

$$m_{\vartheta}^2 = \frac{W_0^2 a^3}{3 + 2 a \tau}, \quad m_\tau^2 = \frac{W_0^2 a^2 (2 + a \tau)(1 + 2 a \tau)}{\tau (3 + 2 a \tau)^2}. \quad (5.6)$$

**Fermion mass**

In $N = 1$ four dimensional supergravity, the mass matrix for fermion is given by

$$m_{ij}^f = m_{3/2} \left( \nabla_i G_j + \frac{1}{3} G_i G_j \right), \quad G = K + \ln W + \ln \bar{W} \quad (5.7)$$

where $m_{3/2} = e^{K/2}|W|$ is the gravitino mass and

$$\nabla_i G_j = \partial_i G_j - \Gamma^k_{ij} G_k,$$

with $\Gamma^k_{ij}$ given in (A.3). In the case under study, $i = T$. Moreover, since we have a susy vacuum, $D_i W = 0$. Therefore the fermion mass is

$$m_f = m_{3/2} \left[ \frac{W_{TT}}{W} + K_{TT} - K_T K_T \right]. \quad (5.9)$$

Using (5.2), we get

$$m_f = -\frac{3 W_0 a (1 + a \tau)}{2 \sqrt{2} \tau^{3/2} (3 + 2 a \tau)}. \quad (5.10)$$

The canonically normalised mass is

$$m_\psi = -\frac{\sqrt{2} W_0 a (1 + a \tau)}{\tau^{1/2} (3 + 2 a \tau)}. \quad (5.11)$$
Now we can calculate the contribution to the logarithmic corrections due to Kähler moduli. The contribution due to two scalar fields is

\[
\Gamma^s(1) = \left[ \frac{149}{180} + 3a\tau + \frac{25a^2\tau^2}{6} + \frac{8a^3\tau^3}{3} + \frac{2a^4\tau^4}{3} \right] \ln |W_0|^2.
\] (5.12)

The corresponding contribution of the fermion is

\[
\Gamma^f(1) = \left[ \frac{251}{720} + 2a\tau + \frac{11a^2\tau^2}{3} + \frac{8a^3\tau^3}{3} + \frac{2a^4\tau^4}{3} \right] \ln |W_0|^2.
\] (5.13)

Putting the two results together, the contribution due to a single Kähler multiplet is

\[
\Gamma^s(1) - \Gamma^f(1) = \left( -\frac{1}{48} + \frac{1}{2}(1 + a\tau)^2 \right) \ln |W_0|^2 = \left( -\frac{1}{48} + \frac{1}{8}m^2_{\psi}R^2_{AdS} \right) \ln |W_0|^2,
\] (5.14)

where we remind that \( R^2_{AdS} = 3L^2 = \frac{2}{\Lambda} \). This is the result one expects for a supersymmetric AdS\(_4\) minimum, where the scalar masses \( m_{s1,s2} \) are determined in terms of the fermion mass\(^{16} \) \( m_{\psi} \) and the radius of AdS \( R_{AdS} \) [50]:

\[
m^2_{s1,s2} = m^2_{\psi} - \frac{2}{R^2_{AdS}} \pm \frac{m^2_{\psi}}{R_{AdS}}.
\] (5.15)

If one plugs these expressions in (4.13), the resulting contribution to \( \ln |W_0|^2 \) matches with (5.14). One can also verify that (5.15) is fulfilled in the present example.

The contribution coming from the gravity multiplet is a constant, due to supersymmetry. The cosmological constant effectively acts as the mass of the graviton, \( M^2_{(2)} = -2\Lambda = 2L^{-2} \), while \( m_{3/2} = e^{K/2}|W| \) is the mass of the gravitino that in the supersymmetric case is \( M^2_{(3/2)} = \frac{1}{3L^2} \). In this case the contribution to \( \ln |W_0|^2 \) is given by

\[
\Gamma^m(1) - \Gamma^g(1) = \frac{113}{48} \ln |W_0|^2
\] (5.16)

Notice that in the above derivation, the \( e^{K_S+K_{cs}} \) factor in the mass cancels the similar contribution present in \( R_{AdS} \).

Summing up all contributions, we obtain

\[
\Gamma^{W_0}(1) = \frac{1}{8} \left( -19 + m^2_{\psi}R^2_{AdS} \right) \ln |W_0|^2.
\] (5.17)

Notice that \( m^2_{\psi}R^2_{AdS} \) is the combination that appears in the relations between masses and conformal dimensions of the dual operators. For the fermion fields, we have \( R_{AdS}m_{\psi} = \Delta_{\psi} - \frac{d}{2} \) [3]. Hence, in the dual CFT\(_3\) the result (5.17) can also be written as

\[
\Gamma^{W_0}(1) = \frac{1}{8} \left( -19 + \left( \Delta_{\psi} - \frac{3}{2} \right)^2 \right) \ln |W_0|^2.
\]

Due to the fact the KKLT is supersymmetric, we can immediately write the contribution to \( \ln |W_0|^2 \) in the case that the Calabi-Yau three-fold \( X_3 \) has \( h^{1,1} \) Kähler moduli. Each

\(^{16}\)We refer here to the fermion mass in the canonically normalised Lagrangian.
chiral multiplet associated to a Kähler modulus will have a mass scaling like $W_0$ and will give a contribution to $\ln |W_0|^2$ equal to (5.14). Hence the final result is

$$
\Gamma^{W_0}_{(1), h^{1,1}, K, \text{ind}} = \left( -\frac{113 + h^{1,1}}{48} + \frac{R_{\text{AdS}}^2}{8} \sum_{i=1}^{h^{1,1}} m_{\psi_i}^2 \right) \ln |W_0|^2 .
$$

(5.18)

### 5.2 LVS vacua

We consider type IIB compactified on a Calabi-Yau (CY) three-fold $X_3$ and take the simplest LVS example, i.e. we take $X_3$ to have two Kähler moduli $\tau_b$ and $\tau_a$ and a volume form of swiss cheese type:

$$
\mathcal{V} = \tau_b^{3/2} - \tau_a^{3/2} .
$$

(5.19)

Again the flux superpotential $W_{\text{flux}}$ is generated by switching on three-form fluxes $G_3$. This fixes the complex structure moduli and the axiodilaton at high energies, leaving a constant superpotential $W_0$ at lower energies (depending on the flux numbers). We also assume that the divisor $D_s$ with volume $\tau_s$ supports a non-perturbative effect (like an E3-insanton or a D7-brane stack with a condensing gauge group) generating a contribution to the superpotential like in KKLT. The total superpotential is then

$$
W = W_0 + A_s e^{-a_s T_s} .
$$

(5.20)

Here $T_s = \tau_s + i \vartheta_s$ is one of the Kähler variables of type IIB orientifold compactifications ($T_i = \int_{D_i}(J \wedge J + i C_4)$, with $C_4$ the RR four-form potential).

After integrating out the complex structure moduli and the dilaton, the remaining moduli are the deformations of the Kähler form. Their Kähler potential (including the leading $\alpha'$-corrections) is

$$
K(T_s, T_b) = -2 \log \left( \mathcal{V}(T_s, T_b) + \frac{\xi}{g_s^{3/2}} \right) ,
$$

(5.21)

where $\xi = -\frac{\zeta(3)s(X_3)}{4(2\pi)^3}$.

The scalar potential for the Kähler moduli $T_s = \tau_s + i \vartheta_s$ and $T_b = \tau_b + i \vartheta_b$ has a minimum where the volume of $X_3$ is stabilised to be exponentially large. In particular, in the region where $\mathcal{V} \gg 1$ (i.e. $\tau_b \gg \tau_s$) the potential has the form (after minimizing with respect to the axion $\vartheta_s$ and taking $W_0 \in \mathbb{R}^+$ without loss of generality)

$$
V = \frac{8 A_s^2 a_s^2 \sqrt{\tau_s} e^{-a_s \tau_s}}{3 \tau_b^{3/2}} + \cos(a_s \vartheta_s) \frac{4 A_s a_s W_0 \tau_s e^{-a_s \tau_s}}{\tau_b^3} + \frac{3 W_0^2 \xi}{2 g_s^{3/2} \tau_b^{9/2}} .
$$

(5.22)

We see that at this level of approximation, the axion $\vartheta_b$ is a flat direction of the potential. Minimising the potential (5.22) with respect to $\vartheta_s$, $\tau_s$ and $\tau_b$, one obtain the two equations:

$$
\partial_{\vartheta_s} V = 0 \iff \vartheta_s = \frac{\pi}{a_s}
$$

(5.23)

$$
\partial_{\tau_s} V = 0 \iff \tau_s^{3/2} = \frac{3e^{a_s \tau_s} W_0 \sqrt{\tau_s}(a_s \tau_s - 1)}{A_s a_s (4a_s \tau_s - 1)}
$$

(5.24)

$$
\partial_{\tau_b} V = 0 \iff \frac{g_s^{3/2}}{\xi} = \frac{(4a_s \tau_s - 1)^2}{16 a_s \tau_s^{5/2}(a_s \tau_s - 1)}
$$

(5.25)
By restricting to the region in the moduli space where we can trust the supergravity approximation, i.e. \( \tau_s \) large, the two minimising equations (5.24) and (5.25) have the approximated solutions:

\[
V \sim \frac{3e^{a_s \tau_s} \sqrt{a_s} W_0}{4 A_s a_s} \quad \text{and} \quad \tau_s \sim \frac{\xi^{2/3}}{g_s}
\]  

(5.26)

We see that the volume is stabilized at exponentially large values, as required by the approximation we took at the beginning of the computations. Remember that we are keeping only the leading terms in \( 1/\tau_b \) expansion. This will hold in the following as well.

By using (5.23), (5.24) and (5.25), we can compute the value of the potential at the minimum:

\[
V_{\text{min}} = -\frac{12 W_0^2}{\tau_b^3} \frac{(a_s \tau_s - 1)}{(4 a_s \tau_s - 1)^2}.
\]  

(5.27)

### Scalar masses

We are now ready to compute the masses of the four real scalar fields \( \tau_s, \tau_b, \vartheta_s, \vartheta_b \). The masses of the fields are derived by the matrix \( \partial_\vartheta \partial_\vartheta V_{\text{min}} \). In our case this matrix is block-diagonal. The block relative to the axions \( \vartheta_b, \vartheta_s \) is (at leading order in the \( 1/\tau_b \) expansion)

\[
\partial_\vartheta \partial_\vartheta V_{\text{min}} = \frac{6W_0^2}{\tau_b^3} \frac{(a_s \tau_s - 1)}{(4 a_s \tau_s - 1)} \begin{pmatrix} 0 & 0 \\
0 & 2 a_s^2 \tau_s^{3/2} \end{pmatrix}.
\]  

(5.28)

while the block relative to \( \tau_b, \tau_s \) is

\[
\partial_\tau \partial_\tau V_{\text{min}} = \frac{6W_0^2}{\tau_b^3} \frac{(a_s \tau_s - 1)}{(4 a_s \tau_s - 1)} \begin{pmatrix}
9 \tau_s^{3/2} (2a_s \tau_s + 1) & -3 \tau_s^{3/2} (a_s \tau_s - 1) \\
-3 \tau_s^{3/2} (a_s \tau_s - 1) & \tau_s (4a_s \tau_s - 1)
\end{pmatrix}.
\]  

(5.29)

We are interested in the canonically normalised fields. The masses are the eigenvalues of the matrices \( \frac{1}{2} K^{ij} \partial_\vartheta \partial_\vartheta V_{\text{min}} \) and \( \frac{1}{2} K^{ij} \partial_\tau \partial_\tau V_{\text{min}} \). The inverse of the Kähler metric is (at leading order in the \( 1/\tau_b \) expansion)

\[
K^{ij}_{\text{min}} = \begin{pmatrix} 4 \tau_s^2 & 4 \tau_b \tau_s \\
4 \tau_b \tau_s & 8 \tau_s^{3/2} \tau_b^{1/2}
\end{pmatrix}.
\]  

(5.30)

The eigenvalues of the matrix \( K^{ij} \partial_\vartheta \partial_\vartheta V_{\text{min}} \) gives the physical masses of the canonically normalised fields:

\[
m^2_\vartheta = 0
\]  

(5.31)

\[
m^2_\vartheta = 16W_0^2 a_s^2 \tau_s^2 (a_s \tau_s - 1) / \tau_b^3 (4a_s \tau_s - 1)
\]  

(5.32)

\[
m^2_\Phi = 108 W_0^2 a_s^5 \frac{\tau_s^{5/2} (a_s \tau_s - 1) (5 - 11a_s \tau_s + 12a_s^2 \tau_s^2)}{\tau_b^9 (4a_s \tau_s - 1)^2 (1 + 3a_s \tau_s - 6a_s^2 \tau_s^2 + 8a_s^3 \tau_s^3)}
\]  

(5.33)

\[
m^2_\Phi = 8W_0^2 (a_s \tau_s - 1) (1 + 3a_s \tau_s - 6a_s^2 \tau_s^2 + 8a_s^3 \tau_s^3) / \tau_b^4 (4a_s \tau_s - 1)^2
\]  

(5.34)
We immediately realise that $m_{\phi}^2, m_{\bar{\phi}}^2 \gg 1/t^2$, while $m_{\phi}^2$ is of the same order as $1/t^2$.

We can approximate the values of $1/t^2, m_{\phi}^2$ and $m_{\bar{\phi}}^2$ in the limit $a_s \tau_a \gg 1$. This is a valid approximation. In fact $a_s \sim 1$, while $\tau_s \sim \xi^{2/3} g_s$: to be in a controlled regime $g_s \ll 1$ (in the explicit example presented below, $\xi \sim 2.08$). In this approximation

\[
\frac{1}{t^2} = \frac{3W_0^2 \tau_{s}^{1/2}}{4\tau_b^{9/2} a_s} \left( 1 + \frac{1}{2a_s \tau_s} + \ldots \right) \tag{(5.35)}
\]

\[
m_2^2 = \frac{4W_0^2 a_s^2 \tau_{s}^{2/3}}{\tau_b^3} \left( 1 - \frac{3}{4a_s \tau_s} + \ldots \right) \tag{(5.36)}
\]

\[
m_{\phi}^2 = \frac{81W_0^2 \tau_{s}^{1/2}}{8\tau_b^{9/2} a_s} \left( 1 - \frac{2}{3a_s \tau_s} + \ldots \right) \tag{(5.37)}
\]

\[
m_{\bar{\phi}}^2 = \frac{4W_0^2 a_s^2 \tau_{s}^{2/3}}{\tau_b^3} \left( 1 - \frac{5}{4a_s \tau_s} + \ldots \right) \tag{(5.38)}
\]

We see that at leading order in this approximation, we have $m_{\phi}^2 = \frac{27}{2t^2}$ and $m_{\bar{\phi}} = m_{\theta}$.

**Fermion masses**

Let us now compute the masses for the (canonically normalised) moduli. We start from the fermion mass matrix in the sugra sigma model:

\[
m_{ij} = m_{3/2} \left( \nabla_i G_j + \frac{1}{3} G_i G_j \right), \quad G = K + \ln W + \ln \bar{W} \tag{(5.39)}
\]

where $m_{3/2} = e^{K/2|W|}$ is the gravitino mass and $\nabla_i G_j = \partial_i G_j - \Gamma_{ij}^k G_k$. For the present case, this matrix reads

\[
m_{ij}^{3/2} = \frac{3W_0}{8\tau_b^3} \left( \frac{\tau_{s}^{1/2}(2a_s \tau_s + 7)}{\tau_{s}^{3/2}(4a_s \tau_s - 1)^2} - \frac{3\tau_{s}^{1/2}(2a_s \tau_s - 1)}{\tau_{s}^{3/2}(4a_s \tau_s - 1)} \right) \tag{(5.40)}
\]

As for the scalars, we compute the canonically normalised masses for the two mass eigenstates:

\[
m_{\psi} = -\frac{W_0(2a_s \tau_s - 1)}{\tau_{s}^{3/2}} = -\frac{2W_0 a_s \tau_s}{\tau_{s}^{3/2}} \left( 1 - \frac{1}{2a_s \tau_s} + \ldots \right) \tag{(5.41)}
\]

\[
m_{\bar{\psi}} = \frac{8W_0 \tau_{s}^{3/2}(a_s \tau_s - 1)}{\tau_b^3(4a_s \tau_s - 1)^2} = \frac{W_0 \tau_{s}^{1/2}}{2a_s \tau_b^3} \left( 1 - \frac{1}{2a_s \tau_s} + \ldots \right) \tag{(5.42)}
\]

**Contribution to $\ln |W_0|^2$**

We can now compute the contribution to $\ln |W_0|^2$ coming from the Kähler moduli spectrum. Like in KKLT, we assume that there are no further massless fields remaining.

The scalar contribution coming from the four scalars is at leading order in the $1/\tau_b$ expansion:

\[
\Gamma_{(1)}^{s} = \frac{\tau_{b}^3(1 + 6a_s \tau_s - 3a_s^2 \tau_s^2 - 20a_s^3 \tau_s^3 + 88a_s^4 \tau_s^4 - 128a_s^5 \tau_s^5 + 128a_s^6 \tau_s^6)}{12\tau_s^3} \ln |W_0|^2 \tag{(5.43)}
\]
The leading contribution in the $\tau_b$ expansion is basically given by the $m^4$ term relative to the fields $\theta$ and $\phi$. In fact, their masses scales with powers of $\tau_b$ with respect to the $1/L$, i.e. $L \cdot m_{\theta,\phi} \sim \tau_b^{3/4}$.

The fermion contribution is basically given at leading order in $\tau_b$ by the $m^4$ term:
\[
\Gamma^f_{(1)} = \frac{\tau_b^3 (4 a_s \tau_s - 1)^4 (2 a_s \tau_s - 1)^4}{384 \tau_s^3 (a_s \tau_s - 1)^2} \ln |W_0|^2.
\]

(5.44)

Considering both contribution, we obtain
\[
\Gamma^s_{(1)} - \Gamma^f_{(1)} = \frac{\tau_b^3 (31 + 152 a_s \tau_s - 696 a_s^2 \tau_s^2 + 1184 a_s^3 \tau_s^3 - 1136 a_s^4 \tau_s^4 + 1152 a_s^5 \tau_s^5 - 768 a_s^6 \tau_s^6)}{384 \tau_s^3 (a_s \tau_s - 1)^2} \ln |W_0|^2.
\]

(5.45)

The gravity multiplet contributes differently with respect to the KKLT. Since the minimum is not supersymmetric, the gravitino contribution is not determined by the graviton one. In this case the contribution to $\ln |W_0|^2$ is given by
\[
\Gamma^g_{(1)} = \frac{11 \tau_b^3 (4 a_s \tau_s - 1)^4}{96 \tau_s^3 (a_s \tau_s - 1)^2} \ln |W_0|^2.
\]

(5.46)

The $\tau_b$ dependence comes from the gravitino mass, whose $\tau_b$ scaling is different from the one of $1/L$. This is a difference with respect to what happens in the KKLT case.

If we sum up all the contribution, we obtain
\[
\Gamma^{W_0}_{(1)} = \frac{\tau_b^3 (25 - 184 a_s \tau_s + 1176 a_s^2 \tau_s^2 - 3360 a_s^3 \tau_s^3 + 3376 a_s^4 \tau_s^4 + 384 a_s^5 \tau_s^5 - 256 a_s^6 \tau_s^6)}{128 \tau_s^3 (a_s \tau_s - 1)^2} \ln |W_0|^2.
\]

(5.47)

Taking the leading term in the $\tau_s \gg 1$ limit, we obtain
\[
\Gamma^{W_0}_{(1)} \sim -2 a_s^4 \tau_b^3 \tau_s \ln |W_0|^2.
\]

(5.48)

This leading contribution comes from $\Gamma^s_{(1)} - \Gamma^f_{(1)}$, as the gravity contribution is subleading for $\tau_s \gg 1$.

**A simple global model**

We present an explicit global model for a LVS minimum, i.e. we consider an explicit Calabi-Yau threefold and an orientifold projection, with a setup of branes that satisfies all the string theory consistency conditions (like tadpole cancellation, proper quantisation of fluxes, etc...). The compactification manifold $X_3$ is the famous CY $\mathbb{P}^4_{11169}$[18]. More precisely, it is a hypersurface described by the vanishing locus of a polynomial of degrees $(18,6)$ in the toric ambient variety defined by the following weights
\[
\begin{array}{cccccc}
1 & 1 & 1 & 6 & 9 & 0 \\
0 & 0 & 0 & 2 & 3 & 1
\end{array}
\]

and with SR-ideal given by $\{ u_1 u_2 u_3, x y z \}$. This Calabi-Yau manifold has Hodge numbers $h^{1,1} = 2$ and $h^{1,2} = 272$, with Euler characteristic $\chi(X_3) = -540$. 

- 27 -
An integral basis of divisor is given by $D_1, D_z$ (with $D_1 = \{u_1 = 0\}$ and $D_z = \{z = 0\}$), with intersection numbers

\[ D_1^3 = 0 \quad D_1^2 D_z = 1 \quad D_1 D_z^2 = -3 \quad D_z^3 = 9. \] (5.50)

We expand the Kähler form in the basis of Poincaré dual two forms $\hat{D}_1, \hat{D}_z$: $J = t_1 \hat{D}_1 + t_z \hat{D}_z$. The volumes of the divisors $D_z$ and $D_y = 9D_1 + 3D_z$ are

\[ \tau_z = \frac{1}{2} \int_{D_z} J^2 = \frac{1}{2} (t_1 - 3t_z)^2 \quad \tau_y = \frac{1}{2} \int_{D_y} J^2 = \frac{3}{2} t_1^2, \] (5.51)

while the volume of the CY is

\[ V = \frac{1}{6} \int_{X_3} J^3 = \frac{1}{18} (t_1^3 - (t_1 - 3t_z)^3) = \frac{\sqrt{2}}{9} \left( \left( \frac{\tau_y}{3} \right)^{3/2} - \tau_z^{3/2} \right). \] (5.52)

In the following we will use the variables $\tau_b \equiv \tau_y/3$ and $\tau_s \equiv \tau_z$. The volume of $X_3$ takes then the form

\[ V = \frac{\sqrt{2}}{9} \left( \tau_b^{3/2} - \tau_s^{3/2} \right). \] (5.53)

We note that this is equal to (5.19), up to the overall factor. This can be absorbed into a rescaling of $W_0, A_s, \xi$. In detail, this model is equivalent to the one described by the volume form (5.19), if $W_0 \mapsto \frac{\sqrt{2}}{9} W_0$ and $A_s \mapsto \frac{9}{\sqrt{2}} A_s$ and the definition of $\xi$ is also rescaled $\xi \mapsto \frac{9}{2\sqrt{2}} \xi$. The new $\xi$ is equal to $\xi \sim 2.08$ in this model (where we have used $\chi(X_3) = -540$).

The only other (non-flux dependent) parameter in the scalar potential that remains to be determined is $a_s$. It depends on the non-perturbative effects that lives on the four-cycle $D_s = D_z$. We consider two cases, corresponding to two different orientifold involutions. These lead to a different spectrum and different nature of the non-perturbative effect.

1) The orientifold involution is given by

\[ \sigma : \quad z \mapsto -z. \] (5.54)

The fixed point locus is made up of two O7-planes at $z = 0$ and $y = 0$. They do not intersect each other. The orientifold-plane D7-tadpole is cancelled by taking four D7-branes (plus their four images) on top of $z = 0$ and a fully recombined D7-brane wrapping a 4-cycle in the homology class $8D_y$ (called in litterature 'Whitney brane' for its characteristic shape) [51]. The stack on $z = 0$ gives an $SO(8)$ gauge group, while the Whitney brane does not support any gauge symmetry. We choose a background value for the bulk B-field equal to $B = \frac{D_y}{2}$. In this way there is a choice of gauge flux on the D7-branes such that the gauge invariant flux $F = F - t^*B$ can be set to zero. In fact, Freed-Witten anomaly cancellation requires the gauge flux on the branes on $z = 0$ to be half-integrally quantized ($F + \frac{\alpha_1(D_s)}{2} \in H^2(D_s, \mathbb{Z})$). With techniques described in [52, 53] one can compute the D3-charge of this configuration. We make a choice of the flux on the Whitney brane that maximize the absolute value of the charge, obtaining $Q_{D_3}^{D7} = 1491$. This large negative contribution to $Q_{D3}$ allows to switch on positively contributing three-form fluxes on the bulk and two-form fluxes.
on the Whitney brane; these stabilise at large scale the complex structure moduli, the axiodilaton and the open string moduli describing the deformations of the Whitney brane [51].

By using proper index theorems, one can compute (see for example [27]) the number of even and odd (1,2)-forms on $X_3$. With the chosen orientifold involution, we have $h^{1,2}_+ = 0$ and hence $h^{1,2}_- = h^{1,2} = 272$. This means that we have no massless gauge multiplet coming from $C_4$ expanded on even three-forms.

The divisor $D_x$ is a rigid $\mathbb{C}P^2$ and hence it has $h^{1,0} = h^{2,0} = 0$. This means that the theory living on the corresponding D7-brane stack is a pure $SO(8)$ SYM. It undergoes gaugino condensation, generating a superpotential

$$W_{np} = A_s e^{-a_s T_s}.$$  \hfill (5.55)

with $a_s = \pi / 3$.

2) The orientifold involution is given by

$$\sigma : \quad x \mapsto -x.$$  \hfill (5.56)

The fixed point locus is made up of one O7-plane at $x = 0$. The orientifold-plane D7-tadpole is cancelled by a Whitney brane wrapping a four-cycle in the homology class $8D_x$. Hence we do not have any massless gauge multiplet coming from the D7-brane worldvolume. The D3-charge of the D7-brane and the O7-plane (considering zero flux on the D7-brane) is $Q_{D7}^{D3} = 498$.

By using the index theorems, we compute $h^{1,2}_+ = 69$ and $h^{1,2}_- = 203$. This means that we have $n_{\text{gauge}} = 69$ massless gauge multiplets. These will contribute to the coefficient of $\ln |W_0|^2$ with a constant term that is subleading with respect to the $(5.45)$.

The rigid divisor $D_x$ is not wrapped by any D7-brane. On the other hand, an invariant E3-instanton is wrapped on $D_x$ when $B = \frac{D_x}{2}$. This will contribute to the non-perturbative superpotential $W_{np} = A_s e^{-a_s \tau_s}$, with $a_s = 2\pi$. If $B = 0$, the leading contribution will be given by E3-instantons with higher rank, as described in [54].

Inserting the model-dependent value of $a_s$ into (5.47) (or (5.48)), one obtains the coefficient of $\ln |W_0|^2$ in terms of $V$ and $g_s$.

6 Discussion

In this paper we have made some progress in describing the properties of the CFT duals of AdS vacua of KKLT and LVS type. Our main technical result is the identification of a concrete calculation, that we performed, of a duality independent quantity. This is the coefficient of the logarithmic term of the one-loop vacuum energy. For the KKLT case the result is quite simple and depends only on the conformal dimension of the involved Kähler moduli and on $h^{1,1}$. For the LVS case it is a model dependent quantity depending on the
values of the moduli at the minimum. The difference relies on the fact that the KKLT AdS vacua preserve supersymmetry whereas in the LVS case supersymmetry is spontaneously broken. In both cases we present then a concrete prediction that in principle should be computable once a CFT dual candidate is identified. Performing the equivalent calculation on the CFT side is left as an outstanding open question since we still have very limited information on the CFT duals. For example, one would need to know, among other features, the parameters (or the combinations of the parameters) of the CFT that corresponds to $W_0$, $g_s$ and $A$ (or $\tau_s$ and $\tau_b$ in LVS). Only after that can one select the $\ln |W_0|^2$ term in the partition function and check the coefficient.

Our results are a small step towards identifying the CFT duals of the landscape of AdS vacua and therefore towards its proper non-perturbative formulation. They could also lead to applications. The three dimensional CFT duals that we have tried to uncover could provide good candidates for some of the applications of AdS/CFT duality. In particular the non-supersymmetric LVS vacua could be relevant for studies of condensed matter applications. The fact that these non-supersymmetric CFTs are particularly simple with only one scalar operator with $O(1)$ conformal dimension may give rise to interesting implications.

There are many questions left open. A typical chiral model with moduli stabilised has many ingredients that should have a counterpart on the CFT side. Besides string, Kaluza-Klein and moduli states, chiral visible and hidden sectors are present with a diversity of gauge and matter fields which are model dependent but have to manifest themselves in the dual theory. In general essentially all the compact models have anomalous $U(1)$s with anomaly cancelled by the Green-Schwarz mechanism. These gauge fields get a mass by the Stuckelberg mechanism. It may be interesting to find the dual realisation of this mechanism which is generic in string compactifications. A proper understanding of supersymmetry breaking on the CFT side would also be desirable.

Besides the AdS vacua studied here, the string landscape also includes de Sitter solutions. A typical potential will have minima with both signs of the cosmological constant and transitions between them should be approached from the dual side. These dS solutions are less understood but would be interesting to explore, extending some of the discussions in this article.

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A \ \mathcal{N} = 1 \text{ supergravity Lagrangian}

The supergravity Lagrangian in our conventions (MTW) is
\[
\mathcal{L} = \frac{1}{2} R - g_{ij} \partial_{\mu} \phi^i \partial^{\mu} \phi^j - ig_{ij} \bar{\chi}^j \sigma^m \partial_{\mu} \chi^i + \epsilon^{klm} \bar{\psi}_k \bar{\sigma}_l \partial_{\mu} \psi_n \\
- \frac{1}{\sqrt{2}} g_{ij} \partial_{\mu} \phi^i \partial^{\mu} \bar{\phi}^j - \frac{1}{\sqrt{2}} g_{ij} \partial_{\mu} \bar{\phi}^i \partial^{\mu} \phi^j - e^{G/2} \left\{ \psi_a \sigma^{ab} \psi_b + \bar{\psi}_a \bar{\sigma}^{ab} \bar{\psi}_b \\
+ \frac{i}{\sqrt{2}} G_{i \bar{j}} \bar{\chi}^j \sigma^a \bar{\psi}_a + \frac{1}{2} [G_{ij} + G_{i \bar{j}} - \Gamma^{k}_{ij} G_k] \chi^i \chi^j \right\} \\
- e^{G/2} \left\{ [g_{ij} G_{ij} - \frac{1}{3}] \right\} \chi^i \chi^j \right) \right\}.
\] (A.1)

In the above we have
\[
G = K + \ln W + \ln \bar{W} \tag{A.2}
\]
Also in the above Christoffel connection is defined as
\[
\partial_k g_{ij} = g_{mj} \Gamma^m_{ik}. \tag{A.3}
\]

B \ One loop computation

The calculations below are based on the deWitt coefficients given in [47, 48].

B.1 Scalar field

For a scalar field we have the Lagrangian \(^{17}\),
\[
\mathcal{L}_{\text{scalar}} = \frac{1}{2} \left[ -\Box + m_s^2 \right] \phi. \tag{B.1}
\]

For a massless scalar field we have the following deWitt coefficients
\[
a_0 = 1, \quad a_2 = \frac{1}{6} R, \quad a_4 = \frac{1}{180} \left[ C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \left( R_{\mu\nu} R^{\mu\nu} - \frac{1}{4} R^2 \right) + \frac{29}{12} R^2 \right]. \tag{B.2}
\]

Therefore for a massive scalar field, we have
\[
a_4(\text{total}) = \frac{1}{2} a_0 m_s^4 - a_2 m_s^2 + a_4 \\
= \frac{1}{2} m_s^4 - \frac{1}{6} m_s^2 R + \frac{1}{180} \left[ C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \left( R_{\mu\nu} R^{\mu\nu} - \frac{1}{4} R^2 \right) + \frac{29}{12} R^2 \right]. \tag{B.3}
\]

B.2 Vector field

Let us first consider a \(U(1)\) gauge field with Lagrangian
\[
\mathcal{L}_{\text{vector}} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \tag{B.4}
\]

We need to add a gauge fixing term
\[
\mathcal{L}_{\text{g.f.}} = \frac{1}{2} (\nabla_{\mu} A^{\mu})^2. \tag{B.5}
\]

\(^{17}\)Note that all calculations are done in a Euclidean metric.
The total Lagrangian is
\[
L = \frac{1}{2} A_\mu \left( -\Box g^{\mu\nu} + R^{\mu\nu} \right) A_\nu. \tag{B.6}
\]
We also need to include the contribution of two ghost field. Thus the total contribution to deWitt coefficients are given by
\[
a_0 = 2, \quad a_2 = -\frac{4}{6} R, \quad a_4 = \frac{1}{180} \left[ -13 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + 62 \left( R_{\mu\nu} R^{\mu\nu} - \frac{1}{4} R^2 \right) - \frac{31}{6} R^2 \right]. \tag{B.7}
\]

\section*{B.3 Graviton}

We consider the Lagrangian of the form
\[
L = \frac{1}{2} \left( R - 2 \Lambda \right). \tag{B.8}
\]
In this section we will follow the calculation presented in \cite{55}. Since graviton has gauge degree of freedom, we need to add gauge fixing term and also ghost term in the Lagrangian.

We use harmonic gauge in which we
\[
\nabla^\mu \phi_{\mu\nu} = 0, \quad \phi_{\mu\nu} = h_{\mu\nu} - \frac{1}{4} g_{\mu\nu} h_{\mu\mu}. \tag{B.9}
\]
Also the ghost is the grassmann valued vector field $\phi_\mu$ and its Lagrangian is
\[
L_{\text{ghost}} = \phi_\mu^* \left( -g^{\mu\nu} \Box - R^{\mu\nu} \right) \phi_\nu. \tag{B.10}
\]
At the quadratic order the complete action is given by
\[
S = -\int d^4 x \sqrt{g} \left[ \frac{1}{2} \phi^{\mu\nu} \Delta^A(1,1) \phi_{\mu\nu} - \frac{1}{2} \phi^2 \Delta^A(0,0) \phi + \phi_\mu^* \Delta^A \left( \frac{1}{2}, 1 \right) \phi_\mu \right], \tag{B.11}
\]
where
\[
\Delta^A(1,1) \phi_{\mu\nu} = -\nabla^\rho \nabla_\rho \phi_{\mu\nu} - 2 R_{\mu\rho\nu\sigma} \phi^{\rho\sigma},
\]
\[
\Delta^A \left( \frac{1}{2}, \frac{1}{2} \right) \phi_\mu = -\nabla^\rho \nabla_\rho \phi_\mu - \Lambda \phi_\mu \tag{B.12},
\]
\[
\Delta^A(0,0) \phi = -\nabla^\rho \nabla_\rho \phi - 2 \Lambda \phi.
\]
In the above $\phi_{\mu\nu}$ is the traceless part of $h_{\mu\nu}$ and $\phi$ is the trace part. Thus including the contribution of ghost field, we get the following deWitt coefficients
\[
a_0 = 2, \quad a_2 = -\frac{22}{6} R,
\]
\[
a_4 = \frac{1}{180} \left( 212 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \frac{1139}{6} R^2 \right) \tag{B.13}
\]
Since the cosmological constant effectively acts as the mass for the graviton, the total $a_4$ including the contribution of the effective mass is given by
\[
a_4^{(\text{total})} = a_4 + 2 \Lambda a_2 + 2 \Lambda^2 a_0 \tag{B.14}
\]
\[-32\]
B.4 Dirac fermion

The fermionic Lagrangian is

\[ \mathcal{L}_{\text{fermion}} = -i \bar{\psi} \sigma^\mu D_\mu \psi - \frac{1}{2} m \bar{\psi} \psi - \frac{1}{2} m \bar{\psi} \psi \]  
\hspace{1cm} (B.15)

In the above action \( \psi \) is a chiral fermion, \( \bar{\sigma} = (I, -\vec{\sigma}) \), \( \vec{\sigma} \) are Pauli matrices. Now the above can be further written as

\[ \mathcal{L}_{\text{fermion}} = -i \frac{1}{2} \bar{\psi} \sigma^\mu D_\mu \psi - i \frac{1}{2} \psi \sigma^\mu D_\mu \bar{\psi} - \frac{1}{2} m \bar{\psi} \psi - \frac{1}{2} m \bar{\psi} \psi \]  
\hspace{1cm} = -\frac{1}{2} \bar{\Psi} (i \Gamma^\mu D_\mu + m) \Psi \]  
\hspace{1cm} (B.16)

In the above

\[ \Psi = \begin{pmatrix} \psi \\ \bar{\psi} \end{pmatrix}, \quad \Gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}, \quad D_\mu \Psi = \partial_\mu \Psi + \frac{1}{8} \omega_{\mu}^{ab} [\Gamma_a, \Gamma_b] \Psi \]  
\hspace{1cm} (B.17)

Here \( \Gamma \) matrices satisfy the Clifford algebra

\[ \{ \Gamma^a, \Gamma^b \} = -2 \eta^{ab}, \quad \eta^{ab} = (-1, +1, +1) \]  
\hspace{1cm} (B.18)

The above gamma matrix satisfy

\[ \Gamma^a \Gamma^0 = \Gamma^0 \Gamma^a \]  
\hspace{1cm} (B.19)

Defining the gamma matrix \( \gamma^\mu \) as

\[ \gamma^a = i \Gamma^a, \quad \{ \gamma^a, \gamma^b \} = 2 \eta^{ab}, \quad \gamma^a \gamma^0 = -\gamma^0 \gamma^a \]  
\hspace{1cm} (B.20)

We can rewrite the above Lagrangian as

\[ \mathcal{L}_{\text{fermion}} = -\frac{1}{2} \bar{\Psi} (\gamma^\mu D_\mu + m) \Psi \]  
\hspace{1cm} (B.21)

Now we do analytic continuation to Euclidean space. In this case we assume that \( \bar{\psi} \) is indep. of \( \psi \) and hence \( \Psi \) is a Dirac spinor. We calculate the one loop determinant and divide the result by half as we are doubling the number of degrees of freedom. We note that in Euclidean space \( \gamma^\mu \gamma^\nu = \gamma^\mu \). Then the one loop determinant is

\[ \ln Z_{\text{fermion}} \sim \ln \det(\gamma^\mu D_\mu + m) \sim \frac{1}{2} \ln \det(\gamma^\mu D_\mu + m) \det(-\gamma^\mu D_\mu + m) \]  
\hspace{1cm} \sim \frac{1}{2} \ln \det(-\Box + m^2 - \gamma^\mu \gamma^\nu D_\mu D_\nu) \]  
\hspace{1cm} (B.22)

Now

\[ \gamma^\mu \gamma^\nu D_\mu D_\nu \psi = -\frac{1}{4} R \psi, \quad D_\mu D_\nu = \frac{1}{2} [D_\mu D_\nu - D_\nu D_\mu] \]  
\hspace{1cm} (B.23)

Here \( R \) is the Ricci scalar. Thus for the massless Dirac fermion, we get

\[ a_0 = 4, \quad a_2 = -\frac{2}{6} R, \]  
\[ a_4 = \frac{2}{180} \left[ \frac{11}{24} R^2 - \frac{11}{2} \left( R_{\mu\nu} R^{\mu\nu} - \frac{1}{4} R^2 \right) - \frac{7}{4} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right] \]  
\hspace{1cm} (B.24)
In this case for massive Dirac fermion we have

\[
a_4(\text{total}) = \frac{1}{2} a_0 m^4 - a_2 m^2 + a_4
\]

\[
= \frac{1}{90} \left[ 180 \left( m^2 + \frac{R}{4} \right) - 101 \frac{2}{24} R^2 - 11 \frac{1}{2} \left( R_{\mu \nu} R^{\mu \nu} - \frac{1}{4} R^2 \right) - \frac{7}{4} C_{\mu \nu \rho \sigma} C^{\mu \nu \rho \sigma} \right]
\]

(B.25)

Since we have computed the determinant for Dirac fermion, in order to get the result for Weyl/Majorana fermion we have to divide the above result by half.

B.5 Gravitino

Next we consider the Lagrangian for gravitino

\[
\mathcal{L}_{\text{gravitino}} = \epsilon_{klmn} \bar{\psi}_k \tilde{\sigma}_l \tilde{D}_m \psi_n - m_\psi \left[ \psi_a \sigma^{ab} \psi_b + \bar{\psi}_a \tilde{\sigma}^{ab} \bar{\psi}_b \right]
\]

(B.26)

Here

\[
\tilde{D}_m \psi_n = \partial_m \psi_n + \frac{1}{8} \omega_{ab}^m [\Gamma_a, \Gamma_b] \psi_n - \frac{1}{4} \left( K_j \partial_m \phi_j - K_j \partial_m \tilde{\phi}_j \right) \psi_n
\]

(B.27)

For our background, the last term is zero as the scalar fields are constant. The above Lagrangian can also be written as

\[
\mathcal{L}_{\text{gravitino}} = \frac{1}{2} \epsilon_{klmn} \bar{\Psi}_k \Gamma_l \tilde{D}_m \Psi_n + \frac{1}{2} \epsilon_{klmn} \psi_n \sigma_l \tilde{D}_m \bar{\psi}_k - m_\psi \psi_a \sigma^{ab} \psi_b + \bar{\psi}_a \tilde{\sigma}^{ab} \bar{\psi}_b
\]

(B.28)

Now we define a Dirac spinor and \( \Gamma^{\mu \nu} \) as

\[
\Psi_m = \left( \begin{array}{c} \psi_m \\ \bar{\psi}_m \end{array} \right), \quad \Gamma^{\mu \nu} = \frac{1}{2} \left[ \Gamma^\mu, \Gamma^\nu \right]
\]

(B.29)

Then the above Lagrangian can be written as

\[
\mathcal{L}_{\text{gravitino}} = \frac{1}{2} \epsilon_{klmn} \bar{\Psi}_k \Gamma_l \Gamma_5 \tilde{D}_m \Psi_n + \frac{1}{2} m_\psi \psi_a \Gamma^{ab} \psi_b
\]

(B.30)

In the above

\[
\Gamma_5 = i \Gamma_0 \Gamma_1 \Gamma_2 \Gamma_3 = \left( \begin{array}{cc} 1_{2 \times 2} & 0 \\ 0 & -1_{2 \times 2} \end{array} \right)
\]

(B.31)

The gauge transformation can be written as

\[
\delta \Psi_\mu = 2 D_\mu \epsilon + i m_\psi \Gamma_\mu \epsilon, \quad \hat{\epsilon} = \left( \begin{array}{c} \epsilon \\ \bar{\epsilon} \end{array} \right)
\]

(B.32)

Using the relation

\[
\Gamma^{\mu \nu \rho \sigma} = -i \epsilon^{\mu \nu \rho \sigma} \Gamma_5 \Gamma_5
\]

(B.33)

the Lagrangian becomes

\[
\mathcal{L}_{\text{gravitino}} = \frac{i}{2} \bar{\Psi}_k \Gamma^{klm} \tilde{D}_l \Psi_m + \frac{1}{2} m_\psi \bar{\Psi}_a \Gamma^{ab} \Psi_b
\]

(B.34)
Writing in terms of $\gamma$-matrix ($\gamma_\mu = i \Gamma_\mu$), we get
\[ \Gamma^{\mu\nu\rho} = i \gamma^{\mu\nu\rho} \] (B.35)

and
\[ \mathcal{L}_{\text{gravitino}} = -\frac{1}{2} \bar{\Psi}_\mu \tilde{\gamma}^{\mu\nu\rho} \tilde{D}_\rho \Psi_\nu - \frac{1}{2} m_\psi \bar{\Psi}_\mu \gamma^{\mu\nu} \Psi_\nu \] (B.36)

The susy transformation becomes
\[ \delta \Psi_\mu = 2 D_\mu \hat{\epsilon} + m_\psi \gamma_\mu \hat{\epsilon} \] (B.37)

To calculate the gravitino partition function we will follow appendix A of [56]. We consider the following field redefinition. The motivation for this will be clear later.
\[ \Psi_\mu = \eta_\mu + A \gamma_\mu \eta, \quad \eta = \gamma^\mu \eta_\mu, \quad \bar{\eta} = \bar{\eta}_\mu \gamma^\mu \] (B.38)

A is a real constant to be determined later. It is easy to see that the above field redefinitions have trivial Jacobian. Now
\[ \gamma^\mu \Psi_\mu = (1 + 4A) \eta, \quad \bar{\Psi}_\mu = \bar{\eta}_\mu + A \bar{\eta} \gamma_\mu \Rightarrow \bar{\Psi}_\mu \gamma^\mu = (1 + 4A) \bar{\eta} \] (B.39)

We find that
\[ \bar{\Psi}_\mu \gamma^{\mu\rho} \tilde{D}_\rho \Psi_\nu = \bar{\eta} \tilde{D}_\eta \left[ (1 + 4A)^2 - 2A (1 + 4A) - 2A^2 - 2A \right] - (1 + 2A) \bar{\eta} \tilde{D}_\mu \eta_\mu \]
\[ - (1 + 2A) \bar{\eta}_\mu \tilde{D}^\mu \eta - g^{\mu\nu} \bar{\eta}_\mu \gamma^\kappa \tilde{D}_\kappa \eta_\nu \] (B.40)

Therefore choosing $A = -\frac{1}{2}$, the cross terms disappear and we get
\[ \bar{\Psi}_\mu \Gamma^{\mu\nu\rho} \tilde{D}_\rho \Psi_\nu = \frac{1}{2} \bar{\eta} \tilde{D}_\eta + g^{\mu\nu} \bar{\eta}_\mu \gamma^\kappa \tilde{D}_\kappa \eta_\nu \] (B.41)

Also
\[ \bar{\Psi}_\mu \Gamma^{\mu\nu} \Psi_\nu = \bar{\eta} \eta - g^{\mu\nu} \bar{\eta}_\mu \eta_\nu \] (B.42)

The gravitino Lagrangian becomes
\[ \mathcal{L}_{\text{gravitino}} = -\frac{1}{4} \bar{\eta} \left( \tilde{D} + 2m_\psi \right) \eta - \frac{1}{2} g^{\mu\nu} \bar{\eta}_\mu \left( \gamma^\kappa \tilde{D}_\kappa - m_\psi \right) \eta_\nu \] (B.43)

We also need to add gauge fixing condition. We put gauge condition $\eta = 0$ and gauge fixing Lagrangian
\[ \mathcal{L}_{\text{g.f.}} = \frac{1}{4} \bar{\eta} \left( \tilde{D} + 2m_\psi \right) \eta \] (B.44)

This choice of gauge fixing Lagrangian introduces a determinant $det^{-1} (\tilde{D} + 2m_\psi)$. The Lagrangian becomes
\[ \mathcal{L}_{\text{gravitino}} + \mathcal{L}_{\text{g.f.}} = -\frac{1}{2} g^{\mu\nu} \bar{\eta}_\mu \left( \gamma^\kappa \tilde{D}_\kappa - m_\psi \right) \eta_\nu \] (B.45)

The corresponding supersymmetry transformation is
\[ \delta \eta = -\gamma^\mu \delta \Psi_\mu = -2 (\tilde{D} + 2m_\psi) \epsilon \] (B.46)
which will give Fadeev Popov determinant $\sim det^{-2}(\not{D} + 2m_\psi)$.

Therefore the complete partition function of Dirac gravitino is

$$Z_{\text{Dirac gravitino}} \sim \left| \det \left( \gamma^\kappa \tilde{D}_\kappa - m_\psi \right) \right|_{\eta_m} / det^3 (\not{D} + 2m_\psi) |_\eta$$  \hspace{1cm} (B.47)

We have already calculated the coefficient of log correction from Dirac fermion. We here calculate the contribution from numerator. Now

$$\left( \gamma^\kappa \tilde{D}_\kappa - m_\psi \right) \left( -\gamma^\mu \tilde{D}_\mu - m_\psi \right) \eta_\rho = -\Box \eta_\rho + \frac{1}{4} R \eta_\rho - \frac{1}{2} \gamma^\mu \gamma^\nu R_{\mu\nu\rho\sigma} \eta^\rho + m_\psi^2 \eta_\rho$$  \hspace{1cm} (B.48)

Thus we get the deWitt coefficient,

$$a_{4(\text{gravitino})} = \frac{1}{360} \left[ -960 R \left( \frac{1}{4} R + m_\psi^2 \right) + 2880 \left( \frac{1}{4} R + m_\psi^2 \right)^2 + 212 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + 80 R^2 - 32 R_{\mu\nu} R^{\mu\nu} \right]$$  \hspace{1cm} (B.49)

We also need to include ghost contribution. The contribution from ghost is thrice the contribution of a massive Dirac fermion. The deWitt coefficient including the mass term for the ghost $a_{4(\text{ghost})}$ is given by

$$a_{4(\text{ghost})} = \frac{12}{360} \left[ -60 R \left( \frac{1}{4} R + 4m_\psi^2 \right) + 180 \left( \frac{1}{4} R + 4m_\psi^2 \right)^2 - \frac{7}{4} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + 5 R^2 - 2 R_{\mu\nu} R^{\mu\nu} \right]$$  \hspace{1cm} (B.50)

Thus deWitt coefficient including the mass term for the physical gravitino is given by

$$a_{4(\text{total})} = a_{4(\text{gravitino})} - a_{4(\text{ghost})} = \frac{1}{360} \left[ 5 R^2 - 960 R m_\psi^2 - 31680 m_\psi^4 + 233 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 8 R_{\mu\nu} R^{\mu\nu} \right]$$  \hspace{1cm} (B.51)

From the above expression for $a_{4(\text{total})}$, we can extract the coefficients in (4.10) for the physical gravitino,

$$a_0 = -166, \quad a_2 = \frac{16}{6} R$$

$$a_4 = \frac{1}{360} \left[ \frac{251}{6} R^2 + 233 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + 458 \left( R_{\mu\nu} R^{\mu\nu} - \frac{1}{4} R^2 \right) \right]$$  \hspace{1cm} (B.52)

In the above we have calculated for Dirac gravitino, so to extract the contribution for Weyl/Majorana gravitino, we will divide the above results by half.

References

[1] J. M. Maldacena, “The Large N limit of superconformal field theories and supergravity,” Int. J. Theor. Phys. 38 (1999) 1113 [Adv. Theor. Math. Phys. 2 (1998) 231] [hep-th/9711200].
[2] E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2 (1998) 253 [hep-th/9802150].

[3] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, “Large N field theories, string theory and gravity,” Phys. Rept. 323 (2000) 183 [hep-th/9905111]; J. Polchinski, “Introduction to Gauge/Gravity Duality,” arXiv:1010.6134 [hep-th].

[4] M. Grana, “Flux compactifications in string theory: A Comprehensive review,” Phys. Rept. 423 (2006) 91 [hep-th/0509003].

[5] M. R. Douglas and S. Kachru, “Flux compactification,” Rev. Mod. Phys. 79 (2007) 733 [hep-th/0610102].

[6] R. Blumenhagen, B. Kors, D. Lust and S. Stieberger, “Four-dimensional String Compactifications with D-Branes, Orientifolds and Fluxes,” Phys. Rept. 445 (2007) 1 [hep-th/0610327].

[7] K. Dasgupta, G. Rajesh and S. Sethi, “M theory, orientifolds and G - flux,” JHEP 9908 (1999) 023 [hep-th/9908088].

[8] S. B. Giddings, S. Kachru and J. Polchinski, “Hierarchies from fluxes in string compactifications,” Phys. Rev. D 66 (2002) 106006 [hep-th/0105097].

[9] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, “De Sitter vacua in string theory,” Phys. Rev. D 68 (2003) 046005 [hep-th/0301240].

[10] M. R. Douglas, “The Statistics of string / M theory vacua,” JHEP 0305 (2003) 046 [hep-th/0303194].

[11] S. Ashok and M. R. Douglas, “Counting flux vacua,” JHEP 0401 (2004) 060 [hep-th/0307049].

[12] F. Denef, M. R. Douglas and B. Florea, “Building a better racetrack,” JHEP 0406 (2004) 034 [hep-th/0404257].

[13] F. Denef and M. R. Douglas, “Distributions of flux vacua,” JHEP 0405 (2004) 072 [hep-th/0404116].

[14] F. Denef and M. R. Douglas, “Distributions of nonsupersymmetric flux vacua,” JHEP 0503 (2005) 061 [hep-th/0411183].

[15] B. S. Acharya, F. Denef and R. Valandro, “Statistics of M theory vacua,” JHEP 0506 (2005) 056 [hep-th/0502060].

[16] M. Cicoli, J. P. Conlon, A. Maharana and F. Quevedo, “A Note on the Magnitude of the Flux Superpotential,” JHEP 1401 (2014) 027 [arXiv:1310.6694 [hep-th], arXiv:1310.6694].

[17] E. Silverstein, “AdS and dS entropy from string junctions: or, The Function of junction conjunctions,” In *Shifman, M. (ed.) et al.: From fields to strings, vol. 3* 1848-1863 [hep-th/0308175]; Contribution to Strings 2003.

[18] J. Polchinski and E. Silverstein, “Dual Purpose Landscaping Tools: Small Extra Dimensions in AdS/CFT,” arXiv:0908.0756 [hep-th].
[19] O. Aharony, Y. E. Antebi and M. Berkooz, “On the Conformal Field Theory Duals of type IIA AdS(4) Flux Compactifications,” JHEP 0802 (2008) 093 [arXiv:0801.3326 [hep-th]].

[20] F. Denef and S. A. Hartnoll, “Landscape of superconducting membranes,” Phys. Rev. D 79 (2009) 126008 [arXiv:0901.1160 [hep-th]].

[21] H. L. Verlinde, “Holography and compactification,” Nucl. Phys. B 580 (2000) 264 [hep-th/9906182].

[22] V. Balasubramanian, P. Berglund, J. P. Conlon and F. Quevedo, “Systematics of moduli stabilisation in Calabi-Yau flux compactifications,” JHEP 0503 (2005) 007 [hep-th/0502058];
J. P. Conlon, F. Quevedo and K. Suruliz, “Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking”, JHEP 0508 (2005) 007 [hep-th/0505076].

[23] S. de Alwis, R. Gupta, E. Hatefi and F. Quevedo, “Stability, Tunneling and Flux Changing de Sitter Transitions in the Large Volume String Scenario,” JHEP 1311 (2013) 179 [arXiv:1308.1222 [hep-th]], arXiv:1308.1222.

[24] M. Cicoli, S. Krippendorf, C. Mayrhofer, F. Quevedo and R. Valandro, “D-Branes at del Pezzo Singularities: Global Embedding and Moduli Stabilisation,” JHEP 1209 (2012) 019 [arXiv:1206.5237 [hep-th]]; J. Louis, M. Rummel, R. Valandro and A. Westphal, “Building an explicit de Sitter,” JHEP 1210 (2012) 163 [arXiv:1208.3208 [hep-th]].

[25] M. Cicoli, S. Krippendorf, C. Mayrhofer, F. Quevedo and R. Valandro, “D3/D7 Branes at Singularities: Constraints from Global Embedding and Moduli Stabilisation,” JHEP 1307 (2013) 150 [arXiv:1304.0022 [hep-th]].

[26] M. Cicoli, D. Klevers, S. Krippendorf, C. Mayrhofer, F. Quevedo and R. Valandro, “Explicit de Sitter Flux Vacua for Global String Models with Chiral Matter,” JHEP 1405 (2014) 001 [arXiv:1312.0014 [hep-th]].

[27] S. A. Hartnoll, “Lectures on holographic methods for condensed matter physics,” Class. Quant. Grav. 26 (2009) 224002 [arXiv:0903.3246 [hep-th]].

[28] P. Kraus, F. Larsen and S. P. Trivedi, “The Coulomb branch of gauge theory from rotating branes,” JHEP 9903 (1999) 003 [hep-th/9811120].

[29] S. Gukov, C. Vafa and E. Witten, “CFT’s from Calabi-Yau four folds,” Nucl. Phys. B 584 (2000) 69 [Erratum-ibid. B 608 (2001) 477] [hep-th/9906070].

[30] E. Witten, “Nonperturbative superpotentials in string theory,” Nucl. Phys. B 474 (1996) 343 [hep-th/9604030].

[31] M. Cicoli, J. P. Conlon and F. Quevedo, “Systematics of String Loop Corrections in Type IIB Calabi-Yau Flux Compactifications,” JHEP 0801 (2008) 052 [arXiv:0708.1873 [hep-th]]; “General Analysis of LARGE Volume Scenarios with String Loop Moduli Stabilisation,” JHEP 0810 (2008) 105 [arXiv:0805.1029 [hep-th]].
[33] S. El-Showk and K. Papadodimas, “Emergent Spacetime and Holographic CFTs,” JHEP 1210 (2012) 106 [arXiv:1101.4163 [hep-th]].

[34] J. L. Cardy, “Anisotropic Corrections to Correlation Functions in Finite Size Systems,” Nucl. Phys. B 290 (1987) 355.

[35] P. Kovtun and A. Ritz, “Black holes and universality classes of critical points,” Phys. Rev. Lett. 100 (2008) 171606 [arXiv:0801.2785 [hep-th]].

[36] C. P. Burgess, A. Maharana and F. Quevedo, “Uber-naturalness: unexpectedly light scalars from supersymmetric extra dimensions,” JHEP 1105 (2011) 010 [arXiv:1005.1199 [hep-th]].

[37] E. Witten, “Baryons and branes in anti-de Sitter space,” JHEP 9807. 006 (1998) [hep-th/9805112].

[38] S. S. Gubser and I. R. Klebanov, “Baryons and domain walls in an N=1 superconformal gauge theory,” Phys. Rev. D 58, 125025 (1998) [hep-th/9808075].

[39] K. Papadodimas, “AdS/CFT and the cosmological constant problem,” arXiv:1106.3556 [hep-th].

[40] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from noncritical string theory,” Phys. Lett. B 428 (1998) 105 [hep-th/9802109].

[41] S. Banerjee, R. K. Gupta and A. Sen, “Logarithmic Corrections to Extremal Black Hole Entropy from Quantum Entropy Function,” JHEP 1103, 147 (2011) [arXiv:1005.3044 [hep-th]].

[42] S. Banerjee, R. K. Gupta, I. Mandal and A. Sen, “Logarithmic Corrections to N=4 and N=8 Black Hole Entropy: A One Loop Test of Quantum Gravity,” JHEP 1111, 143 (2011) [arXiv:1106.0080 [hep-th]].

[43] A. Sen, “Logarithmic Corrections to N=2 Black Hole Entropy: An Infrared Window into the Microstates,” arXiv:1108.3842 [hep-th].

[44] R. K. Gupta, S. Lal and S. Thakur, “Logarithmic corrections to extremal black hole entropy in $\mathcal{N} = 2$, 4 and 8 supergravity,” JHEP 1411, 072 (2014) [arXiv:1402.2441 [hep-th]].

[45] A. Chowdhury, R. K. Gupta, S. Lal, M. Shyani and S. Thakur, “Logarithmic Corrections to Twisted Indices from the Quantum Entropy Function,” JHEP 1411, 002 (2014) [arXiv:1404.6363 [hep-th]].

[46] S. Bhattacharyya, A. Grassi, M. Marino and A. Sen, “A One-Loop Test of Quantum Supergravity,” Class. Quant. Grav. 31, 015012 (2014) [arXiv:1210.6057 [hep-th]].

[47] S. M. Christensen and M. J. Duff, “New Gravitational Index Theorems and Supertheorems,” Nucl. Phys. B 154, 301 (1979).

[48] D. V. Vassilevich, “Heat kernel expansion: User’s manual,” Phys. Rept. 388, 279 (2003) [hep-th/0306138].
[49] D. E. Diaz and H. Dorn, “Partition functions and double-trace deformations in AdS/CFT,” JHEP 0705 (2007) 046 [hep-th/0702163 [HEP-TH]].

[50] B. de Wit and I. Herger, “Anti-de Sitter supersymmetry,” Lect. Notes Phys. 541, 79 (2000) [hep-th/9908005].

[51] A. Collinucci, F. Denef and M. Esole, “D-brane Deconstructions in IIB Orientifolds,” JHEP 0902 (2009) 005 [arXiv:0805.1573 [hep-th]].

[52] A. Collinucci, M. Kreuzer, C. Mayrhofer and N. O. Walliser, “Four-modulus ‘Swiss Cheese’ chiral models,” JHEP 0907 (2009) 074 [arXiv:0811.4599 [hep-th]].

[53] M. Cicoli, C. Mayrhofer and R. Valandro, “Moduli Stabilisation for Chiral Global Models,” JHEP 1202 (2012) 062 [arXiv:1110.3333 [hep-th]].

[54] P. Berglund and I. Garcia-Etxebarria, “D-brane instantons on non-Spin cycles,” JHEP 1301 (2013) 056 [arXiv:1210.1221 [hep-th]].

[55] S. M. Christensen and M. J. Duff, “Quantizing Gravity with a Cosmological Constant,” Nucl. Phys. B 170, 480 (1980).

[56] D. Hoover and C. P. Burgess, “Ultraviolet sensitivity in higher dimensions,” JHEP 0601, 058 (2006) [hep-th/0507293].