The string tensions of the SU(3) representations

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I report on the status of a computation of fundamental and some higher representation string tensions in pure gauge SU(3). An $O(a^2)$ tadpole improved action and an anisotropic lattice are used. At present, the static quark potentials and the string tensions are calculated by measuring Wilson loops on an $8^3 \times 24$ lattice. Wilson loops for higher representations are measured in terms of Wilson loops of the fundamental representation. At the small and intermediate distances available, rough agreement with Casimir scaling is observed, and no color screening for the 8 representation is seen.

1. INTRODUCTION

According to the theory of confinement, the potential energy of a pair of heavy static quarks should increase linearly as a string or tube of flux is formed between them. For confined representations, one expects to see a Coulombic plus a linear term for the potential, $V(r) \approx -A/r + Kr + C$, where $K$ is the string tension and $r$ is the quark separation. For screened representations, $V(r)$ should level off at very large $r$, but one still expects the confined form for intermediate $r$ with approximate Casimir scaling of the string tension. Color screening must occur for adjoint quarks at sufficiently large separation, but it is very difficult to observe in numerical simulations, at least for zero temperature. The string tensions in various representations are basic properties of the pure non-abelian gauge theory. To be viable, any theory of confinement (monopole condensation, center vortices,...) should thus be expected to reproduce the string tension in all representations. This point was emphasized by Greensite at Lat96[1].

The formation of a flux tube and the establishment of a linear potential between adjoint quarks at non-zero temperature have been shown by many numerical experiments in SU(2)[2]. Also there are some studies for the existence of the string tension for the adjoint and a few other non-fundamental representations in SU(3) at $T \neq 0$[3]. N. A. Campbell et al.[4] reported the adjoint string tension for SU(3) at low temperature as well. In this work with an improved action I attempt to determine precisely the force between static quarks in various representations. So far I have been able to show that the string tension at intermediate distances is representation dependent and roughly proportion to the quadratic Casimir number of the representation.

2. CALCULATIONS

The string tension may be found by measuring the Wilson loops and looking for the area law fall-off for large $t$, $W(r,t) \approx \exp[-V(r)t]$ where $W(r,t)$ is the Wilson loop as a function of $r$, the spatial separation of the quark, and the propagation time $t$, and $V(r)$ is the gauge field energy associated with static quark-antiquark source. The interquark energy for large separation grows linearly, so one can write $V(r) \approx -A/r + Kr + C$ for large $r$, with $K$ the string tension. In my calculations, I have measured the Wilson loops and found the string tension by fitting the data to the above equation.

Direct measuring of Wilson loops for higher representations by multiplying the large matrices is not feasible considering the computer memory and the running time. One may expand the trace of Wilson loops for higher representations in terms of Wilson loops, $U$, in the fundamental representation. The higher representation states are defined by the tensor product method. Let $W$ be the higher representation counterpart to fundamental $U$ for
6, 8, 10, 15s (symmetric), 15a (antisymmetric), or 27, then:

\begin{align}
6 &: \quad tr(W) = 1/2 \left[ (trU)^2 + trU^2 \right] \\
8 &: \quad tr(W) = trU^*trU - 1 \\
10 &: \quad tr(W) = 1/6 \left[ (trU)^3 + 2(trU^3) + 3trUtrU^2 \right] \\
15s &: \quad tr(W) = 1/24 \left[ (trU)^4 + 6(trU)^2trU^2 + 3(trU^2)^2 + 6trU^4 \right] \\
15a &: \quad tr(W) = 1/2trU^* \left[ (trU)^2 + trU^2 \right] - trU \\
27 &: \quad tr(W) = 1/4[trU^2 + (trU^2) \left[ (trU^*)^2 \right] + (trU^*)^2] - trUtrU^* 
\end{align}

3. SIMULATIONS

The measurements have been done on an \(8^3\times 24\) anisotropic lattice with \(a_s/a_t = 3\), where \(a_s\) and \(a_t\) are the spatial and temporal spacing, respectively. The improved action used for the calculations has the form \(\mathfrak{b}\):

\[ S = \beta \left\{ \frac{5}{3} \frac{\Omega_{sp}}{\xi u_s^3} + \frac{4}{3} \frac{\Omega_{zp}}{\xi u_t^3} - \frac{1}{12} \frac{\Omega_{sr}}{\xi u_s^3} - \frac{1}{12} \frac{\xi \Omega_{str}}{u_s^2 u_t^2} \right\} \]

where \(\beta = 6/g^2\), \(g\) is the QCD coupling, and \(\xi\) is the aspect ratio \((\xi = a_s/a_t\) at tree level in perturbation theory). \(\Omega_{sp}\) and \(\Omega_{zp}\) include the sum over spatial and temporal plaquettes; \(\Omega_{sr}\) and \(\Omega_{str}\) include the sum over \(2 \times 1\) spatial rectangular and short temporal rectangular (one temporal and two spatial links), respectively. For \(a_t \ll a_s\), the discretisation error of this action is \(O(a_s^4, a_t^2, a_s a_t^2)\). The coefficients are determined using tree level perturbation theory and tadpole improvement \(\mathfrak{b}\). (The spatial mean link, \(u_s\) is given by \(\mathfrak{b}\left(\frac{1}{2}ReTrP_{ss'}\right)^\dagger\), where \(P_{ss'}\) denotes the spatial plaquette. When \(a_t \ll a_s\), \(u_s\), the temporal mean link can be fixed to \(u_t = 1\), since its value in perturbation theory differs by unity by \(O\left(\frac{u_s^2}{u_t}\right)\).)

To minimize the excited state contamination in the correlation functions, the spatial links are smeared. In the smearing procedure each spatial link is replaced by itself plus a sum of its four neighboring spatial staples times a smearing factor \(\lambda\). Projection back to SU(3) after smearing or averaging over different paths in Wilson loops is necessary, since I want to use eqns. 1 – 6 in which \(U^s\) should be SU(3) matrices. For the same reason I am not able to do thermal averaging, since again it would take links out of SU(3). (Thermal averaging is the replacement of a time-like link by its average with fixed neighbors, which is normally useful to increase statistics.)

I have used MILC Code as a platform for the simulations and my code has been run on a Dec Alpha and the Origin2000 supercomputer at NCSA (single node jobs). The potential for on- and off-axis points is calculated by measuring Wilson loops. Fig 1 shows a typical plot for representation 8. \(Q\), the confidence level of the fit, is calculated by measuring the covariance matrix evaluated by the jack knife method. Fig. 2 shows \(a_t\) versus \(r\) for the fundamental, 6 and 8 representations. The data have been fitted to a Coulombic plus linear term. Only the on-axis points are used in the fits. Without considering systematic errors (in particular, the small violation of rotational invariance), the confidence levels of the fits are not very good. Results are shown in Table 1. There is rough agreement between \(\frac{K}{\xi}\) or \(\frac{1}{\xi^2}\), with \(\frac{K}{\xi}\) as predicted \(\mathfrak{b}\). Also \(\frac{\lambda}{\xi}\) calculated by this work is in agreement with 2.2(4) reported by N. A. Campbell et al. \(\mathfrak{b}\). For \(\beta = 2.4\), I have found \(a_s \approx .28f\) by comparing the fundamental string tension with the phenomenological value \(K = 420\) MeV.

4. CONCLUSIONS

By measuring the Wilson loops for the fundamental and some higher representations (6 and 8), I have shown that the string tensions exist and their values are different for each representation and are in rough agreement with Casimir scaling. At the small and intermediate distances available, no color screening is observed. In the future, I plan to finish the analysis for representa-
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Figure 1. Effective mass plot for representation 8. $T_{\text{min}}$ and $T_{\text{max}}$ show the range of the fit. $\lambda$, the smearing factor, and the smearing levels are determined by some low statistics runs for short, intermediate and long distances.

Table 1
Parameters of the potentials as a function of representation. K is the string tension, $A$ is the Coulombic coefficient term, $C$ is the Casimir number and “f” stands for the fundamental representation. The errors shown are statistical only.

| Repn | $K$    | $\frac{A}{N_f}$ | $\frac{C}{N_f}$ | $C_f$ |
|------|--------|----------------|-----------------|-------|
| 3    | 0.3480(6) | -              | -               | -     |
| 6    | 0.7688(9)  | 2.209(5)       | 2.65(2)         | 2.5   |
| 8    | 0.710(1)   | 2.040(5)       | 2.35(2)         | 2.25  |

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