Image gradient L₀-norm based PICCS for swinging multi-source CT reconstruction

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Abstract: Dynamic computed tomography (CT) is usually employed to image motion objects, such as beating heart, coronary artery and cerebral perfusion, etc. Recently, to further improve the temporal resolution for aperiodic industrial process imaging, the swinging multi-source CT (SMCT) systems and the corresponding swinging multi-source prior image constrained compressed sensing (SM-PICCS) method were developed. Since the SM-PICCS uses the L₁-norm of image gradient, the edge structures in the reconstructed images are blurred and motion artifacts are still present. Inspired by the advantages in terms of image edge preservation and fine structure recovering, the L₀-norm of image gradient is incorporated into the prior image constrained compressed sensing, leading to an L₀-PICCS algorithm. The experimental results confirm that the L₀-PICCS outperforms the SM-PICCS in both visual inspection and quantitative analysis.

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1. Introduction

Dynamic computed tomography (CT) has been used to image a changeable object with time in industrial applications [1–4]. To improve the temporal resolution with high image quality for the dynamic CT system, one key problem is how to obtain sufficient projections in short time. However, limited by the constraints of the mechanical gantries, the scanning speed of the conventional CT can reach no more than one rotation per 200ms [5]. To further improve the scanning, the multi-source CT system is developed, in which multiple source-detector pairs installed on a gantry simultaneously rotate around the imaging object to acquire projections. For example, a novel technique called real time tomography (RTT) system [6] was developed by adopting a circular array of X-ray source rather than the complicated mechanical scanning. The C-arm CT was developed for dynamic imaging to improve the temporal resolution by reducing the scan angle range. Those systems have achieved good performance in medical applications. However, the complicated structure and expensive cost make them difficult to be applied in industrial applications.

Recently, a swinging multi-source CT (SMCT) scheme was proposed for aperiodic dynamic imaging. This system consists of Q (Q is an odd integer) X-ray source/detector pairs, and they are distributed uniformly along a circumference. Different from other multi-source CT systems in medical applications [1,7–9], the SMCT system adopts the bearing technique rather than the complicated slip ring. The bearing has advantages in terms of concise and precise structure, easy manufacture and low price, and it is suitable for large-scale serial
production. In addition, compared to the medical multi-source CT system equipped a slip ring for transferring current and signals, the SMCT system loads high transmission current and complicated transmission signals by cable. In the SMCT system, each source-detector pair only swings a small angle so that projections can be gathered at one time. Since there are some local dynamic regions within the objects, it is difficult for the SMCT system to collect sufficient projections within a small time period. This may cause data inconsistency and result in motion artifacts. In the case, the conventional filtered back-projection (FBP) and iterative reconstructed (ART) methods are not powerful enough to obtain high quality images.

Guided by the compressed sensing theory, high quality images can be reconstructed from undersampling data sets. For example, the total variation [10] (TV) minimization used as a common prior knowledge was introduced into CT reconstruction. Indeed, the TV minimization based reconstruction methods can improve reconstructed image quality in terms of noise suppression and artifacts reduction. Particularly, incorporating the prior image constraint into the TV-based reconstruction model, the prior image constrained compressed sensing (PICCS) was proposed for 4DCT reconstruction [11]. Then, the PICCS was also applied to dual-source CT [12] and C-arm CT systems [13].

In aperiodic dynamic imaging, only small regions of the object change with time and other part keeps unchanged, and much information within the projections is redundant. Therefore, the PICCS method was modified for the SMCT system, and it generated swinging multi-source PICCS (SM-PICCS) in our previous work [14]. The goal of SM-PICCS method mainly focuses on minimizing image gradient $L_1$-norm. However, the image gradient $L_1$-norm based reconstruction methods may lead to staircase effect with the loss of image details and features. To address the aforementioned disadvantages, lots of non-convex penalties were proposed to constrain the final reconstruction solution. For example, a more general non-convex PICCS (NC-PICCS) method was proposed in [15]. Compared with the state-of-art PICCS, the NC-PICCS has a better ability to preserve image edge structures and suppress noise. Indeed, the $L^p$-norm ($p<1$) constraint based reconstruction methods were proposed to obtain a better solution than the image gradient $L_1$-norm based methods [16–19], especially in the case of fewer projections. A homotopic $L_p$-minimization was proposed, and it achieved huge success in magnetic resonance imaging (MRI) reconstruction in [17]. Recently, the image gradient $L_0$-norm has been widely used to exploit gradient sparsity in CT image reconstruction, such as limited-angle, low-dose spectral CT, etc [20–26]. Compared with image gradient $L_1$-norm, the image gradient $L_0$-norm counts the number of nonzero pixels, which can directly measure image gradient sparsity [18]. Since image gradient $L_0$-norm minimizes the number of pixels that have non-zero gradient magnitudes rather than the image gradient magnitude, it can effectively preserve the image structures.

Encouraged by the advantages of image gradient $L_0$-norm for image reconstruction, we incorporate it into the PICCS model and generate a new reconstruction model for SMCT system, named $L_0$-PICCS in this study. Compared with the previous SM-PICCS method, the advantages of the proposed $L_0$-PICCS are three-fold. (i) It can avoid staircase effect with noise suppression in the edge areas. (ii) It can provide clearer image structure and finer features. (iii) It can reconstruct high-quality images from fewer projections. The contributions of this work are mainly in the following two aspects: (i) Image gradient $L_0$ norm was incorporated into the PICCS to further improve the ability for reconstructing high-quality images from undersampling projections. (ii) Because the minimization of image gradient $L_0$-norm is a non-convex and NP-hard problem, it is hard to be solved. Thus, the split-Bregman method [27] is employed to optimize the $L_0$-PICCS model.

The rest of this paper is organized as follow. In Section II, the SMCT industrial system is briefly introduced and the $L_0$-PICCS model is proposed and optimized. In Section III, both numerical simulations and realistic data set experiments are performed to validate and evaluate the advantages of the proposed $L_0$-PICCS method. In section IV, we discuss some related issues and make a conclusion.
2. Method

2.1 SMCT system

To image the industrial aperiodic process, a swinging multi-source CT (SMCT) system was proposed to improve the temporal resolution [14]. Figures 1 and 2 show a typical SMCT system and the corresponding fan-beam geometry. The system consists of $Q$ ($Q$ is an odd integer) pairs of X-ray source/detector and they are uniformly distributed on a rotating table with a concentric bearing structure. The imaging object is placed on the origin of the rotating table. The SMCT system is driven by a servo motor to swing a certain angle. For such a SMCT system, it can obtain $Q$ uniform circular projections simultaneously. Assuming the object is not changeable with time, the system can obtain full projections when each X-ray source/detector pair rotates an angle of $2\pi / Q$ in one direction. However, since the object is always changeable with time, it is difficult to obtain the uniform full projections, and it results in poor quality of reconstructed images. To improve the reconstructed image quality, the SM-PICCS reconstruction method was proposed.

![Fig. 1. A representative SMCT system ($Q = 5$).](image1)

![Fig. 2. The fan-beam geometry of the SMCT system in Fig. 1.](image2)

The CT imaging model can be expressed as a linear system. For noise-free projections, it can be approximated as:

$$Af = P,$$  \hspace{1cm} (1)
where $A = (a_{mn})$ is the system matrix with size of $M \times N$, $M$ represents the total number of projections, and $N$ denotes the number of image pixels. $P = [p_1, p_2, \ldots, p_M]^T$ represents the measured projections vector and $f = [f_1, f_2, \ldots, f_N]^T$ is the reconstructed image, which can be obtained by solving Eq. (1). Because $M$ is usually smaller than $N$ in our study, Eq. (1) is ill-posed and difficult to be solved.

The regularization method has been developed to deal with the ill-posed inverse problem of CT reconstruction, which can be formulated as the following minimization problem:

$$\arg \min_{f} \| Af - P \|_2^2 + \gamma R(f),$$

where $\| Af - P \|_2^2$ denotes the data fidelity and $R(f)$ represents the regularization term. The regularization term can be formulated by incorporating prior information which plays an important role in obtaining high quality image, especially for undersampling data set, such as limited-angle, sparse-view, etc. $\gamma$ is the regularization parameter to balance the data fidelity term and the regularization term.

The SM-PICCS was proposed for SMCT in our previous work and it can be expressed as following.

$$\arg \min_{f} \kappa TV(f) + (1 - \kappa) TV(f - f^p), \quad \text{s.t.} \quad Af = P,$$

where $\kappa$ is an empirical parameter and $f^p$ is the prior image. $TV$ represents the L1-norm of the image gradient, which can be denoted as

$$TV(f) = \sum_{i=1}^{I} \sum_{j=1}^{J} \sqrt{\left( f_{i,j} - f_{i-1,j} \right)^2 + \left( f_{i,j} - f_{i,j-1} \right)^2},$$

where $I$ and $J$ are the height and width of reconstructed image and their product equals to $N$. $f_{i,j}$ represents the pixel value located at the image grid $(i, j)$. Here, the image gradient values of image boundary are set to 0.

### 2.2 L0-PICCS model

The image gradient L0-norm was proposed to calculate the number of non-zeros in image gradient image. It can be defined as

$$\| \nabla f \|_0 = \sum_{i=1}^{I} \sum_{j=1}^{J} \varphi( | f_{i,j} - f_{i-1,j} | + | f_{i,j} - f_{i,j-1} |),$$

where $\varphi(\cdot)$ is a function and it can be read as

$$\varphi( | f_{i,j} - f_{i-1,j} | + | f_{i,j} - f_{i,j-1} |) = \begin{cases} 1 & | f_{i,j} - f_{i-1,j} | + | f_{i,j} - f_{i,j-1} | \neq 0 \\ 0 & \text{otherwise} \end{cases}.$$

Compared with the TV, the image gradient L0-norm counts the number of non-zeros for gradient image. Consequently, the image edge information and sharp structure can be effectively retained. Thus, the L0-norm of image gradient is employed to replace the L1-norm in the SM-PICCS algorithm. The reconstruction model of L0-PICCS can be formulated as following

$$\arg \min_{f} \kappa \| \nabla f \|_0 + (1 - \kappa) \| \nabla (f - f^p) \|_0 \quad \text{s.t.} \quad Af = P.$$
Equation (7) is an equality constraint problem, which can be converted into an unconstrained optimization problem.

\[
\arg\min_f \frac{1}{2} \| Af - P \|^2 + \lambda \left( \| \nabla f \|_0 + (1 - \lambda) \| \nabla (f - f^p) \|_0 \right).
\]  

(8)

To solve Eq. (8), two auxiliary variables \( u_1 \) and \( u_2 \) are introduced to replace \( f \) and \( f - f^p \) respectively. Then, the Eq. (8) can be rewritten as

\[
\arg\min_f \frac{1}{2} \| Af - P \|^2 + \lambda \left( \| \nabla u_1 \|_0 + (1 - \lambda) \| \nabla u_2 \|_0 \right) \quad \text{s.t.} \quad u_1 = f, u_2 = f - f^p. \tag{9}
\]

Equation (9) can be further evolved into an unconstrained minimization problem

\[
\arg\min_{f, u_1, u_2, t_1, t_2} \left\{ \frac{1}{2} \| Af - P \|^2 + \lambda \left( \| \nabla u_1 \|_0 + (1 - \lambda) \| \nabla u_2 \|_0 \right) \right. \\
\left. + \frac{\delta_1}{2} \| f - u_1 - t_1 \|^2 + \frac{\delta_2}{2} \| f - f^p - u_2 - t_2 \|^2 \right\},
\]  

(10)

where \( t_1 \) and \( t_2 \) represent two error feedback variables respectively. The problem Eq. (10) can be divided into three sub-problems by employing the split-Bregman strategy [29]:

Sub-problem 1:

\[
f^{(k+1)} = \arg\min_f \frac{1}{2} \| Af - P \|^2 + \delta_1 \| f - u_1 - t_1 \|^2 + \delta_2 \| f - f^p - u_2 - t_2 \|^2 . \tag{11a}
\]

Sub-problem 2:

\[
u_1^{(k+1)} = \arg\min_{u_1} \frac{\delta_1}{2} \| f^{(k+1)} - u_1 - t_1 \|^2 + \lambda \| \nabla u_1 \|_0,
\]  

(11b)

\[
u_2^{(k+1)} = \arg\min_{u_2} \frac{\delta_2}{2} \| f^{(k+1)} - f^p - u_2 - t_2 \|^2 + \lambda \| \nabla u_2 \|_0 . \tag{11c}
\]

Sub-problem 3:

\[
t_1^{(k+1)} = t_1^{(k)} - (f^{(k+1)} - u_1^{(k+1)}), \tag{11d}
\]

\[
t_2^{(k+1)} = t_2^{(k)} - (f^{(k+1)} - f^p - u_2^{(k+1)}). \tag{11e}
\]

Because the mathematical model of Eq. (11a) is a strictly convex problem, the sub-problem 1 can be easily solved. To determine the minimization point of Eq. (11a), the derivative of Eq. (11a) should be equivalent to zero. We have,

\[
A^T \left( Af - P \right) + \delta_1 \left( f - u_1^{(k)} - t_1^{(k)} \right) + \delta_2 \left( f - f^p - u_2^{(k)} - t_2^{(k)} \right) = 0 , \tag{12}
\]

which can be simplified as

\[
\left( A^T A + \delta_1 I + \delta_2 I \right) f = A^T P + \delta_1 \left( u_1^{(k)} + t_1^{(k)} \right) + \delta_2 \left( f^p + u_2^{(k)} + t_2^{(k)} \right) . \tag{13}
\]

Equation (13) is equivalent to

\[
\left( A^T A + \delta_1 I + \delta_2 I \right) f = \left( \left( A^T A + \delta_1 I + \delta_2 I \right) f^{(k)} - \left( A^T A + \delta_1 I + \delta_2 I \right) f^{(k)} \right) + A^T P + \delta_1 \left( u_1^{(k)} + t_1^{(k)} \right) + \delta_2 \left( f^p + u_2^{(k)} + t_2^{(k)} \right) , \tag{14}
\]
Therefore, the $f^{(k+1)}$ can be updated by

$$
f^{(k+1)} = f^{(k)} - \left( A^T A + \delta I + \delta_s I \right)^{-1} A^T (Af^{(k)} - P) + \delta_i (f^{(k)} - u_i^{(k)} - t_i^{(k)}) + \delta_s (f^{(k)} - u_2^{(k)} - t_2^{(k)} - f^p) \right].$$

(15)

The sub-problem 2 includes an L₀-norm minimization and it results in non-convex and NP-hard problem. Fortunately, this problem can be solved by utilizing an approximate method [28]. Here, since Eq. (11c) is similar to Eq. (11b), only Eq. (11b) is discussed. Equation (11b) is equivalent to the following problem:

$$u_i^{(k+1)} = \arg \min_{u_i} \| f^{(k+1)} - u_i - t_i^{(k)} \|_2^2 + \frac{2 \lambda \kappa}{\delta_i} \| \nabla u_i \|_0.$$  (16)

First, let $(\partial_{i,j} u_i)$ be defined as $f_{i,j} - f_{i-1,j}$ and $f_{i,j} - f_{i,j-1}$, where $n = (i-1) \times J + j$. Then, two auxiliary variables $(h, v)$ are introduced to replace $(\partial_{i,j} u_i)$. Equation (16) can be further transformed into the following problem:

$$\arg \min_{u_i, [h, v]} \| f^{(k+1)} - u_i - t_i^{(k)} \|_2^2 + \lambda^* \| \nabla u_i \|_0 + \beta \left( \left( \partial_{i,j} u_i - h \right)^2 + \left( \partial_{i,j} u_i - v \right)^2 \right).$$  (17)

where the $\lambda^* = \frac{2 \lambda \kappa}{\delta_i}$ is a smoothing parameter, which is designed to control the image edge and finer structures. $\beta$ represents an automatically adaptive parameter to control the similarity between $(h, v)$ and $(\partial_{i,j} u_i)$.  

According to [22,28], Eq. (17) can be updated by the following two alternative steps:

$$\left\{ (h, v) \right\}^{(i+1)} = \left\{ \begin{array}{c}
(0, 0) \quad \left( \left( \partial_{i,j} (u_i) \right)_n \right)^{(i)} + \left( \partial_{i,j} (u_i) \right)_n^{(i)} \leq \lambda^*/\beta_i \\
\left( \left( \partial_{i,j} (u_i) \right)_n \right)^{(i)} + \left( \partial_{i,j} (u_i) \right)_n^{(i)} \end{array} \right\}$$

(18)

$$u_i^{(i+1)} = F^{-1} \left\{ \frac{F(f^{(i+1)} - d^{(i)}) + \beta_i (F^*(\partial x) F(h^{(i+1)} + F^*(\partial y) F(v^{(i+1)}))}{F(1) + \beta_i (F^*(\partial x) F(\partial x) + F^*(\partial y) F(\partial y))} \right\},$$  (19)

where $F(\cdot)$ is Fast Fourier Transform (FFT), and $F^*(\cdot)$ is the complex conjugate transform of Fourier Transform. The problem Eq. (11c) can be updated with the same steps. The main steps of L₀-PICCS are summarized in Table 1.
Algorithm 1: \textsc{L}_0-\textsc{PICCS}

1. Input: \( f^p \), \( N_{\text{iter}} \), \( \delta_1 \), \( \delta_2 \), \( \lambda_1 \), \( \lambda_2 \), \( \beta_{\text{max}} = 10^3 \), \( k_1 = 1,7 \), \( k_2 = 10 \)

2. Initialization: \( u_1^{(0)} = 0, u_2^{(0)} = 0, t_1^{(0)} = 0, t_2^{(0)} = 0, f^{(0)} = 0, \beta = \beta_0 \)

3. for \( N < N_{\text{iter}} \)

4. Update \( f^{(k+1)} \) by Eq. (15);

5. Repeat:

6. Updating \( \{ h_a \}^{(k+1)}, \{ v_a \}^{(k+1)} \) using Eq. (18);

7. Updating \( u_1^{(k+1)} \) using Eq. (19);

8. \( \beta_1 \leftarrow k, \beta_1 \);

9. \( i \leftarrow i + 1 \);

10. Until: \( \beta_{\text{max}} \leq \beta_1 \);

11. Solving \( u_2^{(k+1)} \) with steps 6-10;

12. Update \( u_1^{(k+1)} : u_1^{(k+1)} = u_1^{(k+1)} \), \( u_2^{(k+1)} = u_2^{(k+1)} \);

13. Update \( f^{(k+1)} : t_1^{(k+1)} = t_1^{(k)} - (f^{(k+1)} - u_1^{(k+1)}), t_2^{(k+1)} = t_2^{(k)} - (f^{(k+1)} - f^p - u_2^{(k+1)}); \)

14. \( k \leftarrow k + 1 \);

15. End for

Output: reconstruction result \( f^{(k+1)} \)

3. Experiments

The more X-ray source/detector pairs mounted on SMCT system, the better reconstructed image quality and the higher the temporal resolution that can be obtained [14]. However, a large number of X-ray source/detector pairs can reduce radius of the field of view (FOV). In addition, the large number of source/detector pairs can increase the manufacturing costs of the system. Meanwhile, the scattering effect of X-ray source will be aggravated from different directions, which will degrade the reconstructed image quality. Therefore, the number of source/detector pairs are set as 7 and 5 and the corresponding sampling views are 700 and 500 in the simulation. In the realistic data set experiment, the sampling views are 450 and 448 respectively.

| Sources \( w \) | \( \delta_1 \times 10^{-2} \) | \( \delta_2 \times 10^{-2} \) | \( \lambda_1 \times 10^{-5} \) | \( \lambda_2 \times 10^{-9} \) |
|----------------|----------------|----------------|----------------|----------------|
| Noise-free     | 5             | 10             | 5              | 2              | 2.00          | 1.5           |
|                | 25            | 5              | 5              | 2              | 2.00          | 1.5           |
|                | 50            | 3              | 500            | 2              | 2.00          | 4             |
|                | 10            | 14.3           | 333            | 2.00           | 5000          |
| Noise          | 7             | 25             | 5              | 2              | 2.00          | 5000          |
|                | 50            | 2              | 2              | 2.00           | 5000          |
|                | 10            | 6              | 200            | 2.50           | 500           |
|                | 5             | 25             | 5              | 450            | 2.50          | 1.5           |
|                | 50            | 5              | 450            | 2.90           | 1.5           |
|                | 10            | 27.3           | 85.9           | 1.58           | 10000         |
|                | 7             | 25             | 8.2            | 85.9           | 1.58          | 500           |
|                | 50            | 5              | 2              | 2.00           | 1.5           |
Table 2. The parameters of L₀-PICCS (δ₁, δ₂, λ₁⁺, λ₂⁺) for realistic simulation.

| Sources | w | δ₁ | δ₂ (10⁻²) | λ₁⁺ (10⁻²) | λ₂⁺ (10⁻²) |
|---------|---|-----|-----------|-------------|-------------|
| 10      | 0.2| 2.4 | 0.5       | 0.11        |
| 5       | 0.2| 2.4 | 0.6       | 0.14        |
| 30      | 0.16| 2.24| 0.6       | 0.11        |
| 8       | 0.2| 3.5 | 1.3       | 743         |
| 7       | 0.2| 2.75| 1.4       | 238         |
| 32      | 0.2| 2.75| 1.4       | 238         |

Table 3. Numerical simulation parameters

| Parameters                                      | Q = 5 | Q = 7 |
|------------------------------------------------|-------|-------|
| Distance from source to rotation center (mm)   | 577   | 640   |
| Distance from source to detector (mm)          | 977   | 1080  |
| Diameter of field of view (mm)                 | 60    | 60    |
| Efficient detector array length (mm)           | 204.8 | 204.8 |
| Detector pixel size (mm)                       | 0.8   | 0.8   |
| Interval angle between two adjacent projection views | π / 250 | π / 350 |
| Reconstructed image size (pixel)               | 256x256 | 256x256 |

There are four parameters in the L₀-PICCS need to be selected, i.e., the Lagrangian multiplier δ₁ and δ₂, smoothing parameters λ₁⁺ and λ₂⁺. The optimized parameters of all reconstruction methods are picked out by comparing the RMSE and visual evaluation. Tables 2 and 3 list the optimal parameters of L₀-PICCS for different cases. The iteration number for all the algorithms is set as 300. The sampling time is set as 0.1s and other numerical simulations parameters are listed in Table 4.

3.1 Numerical simulation

To evaluate the performance of the proposed L₀-PICCS, three different undersampling factors, i.e., w = 10, 25, 50 are set. The undersampling factor 10 means 70 projections are collected at one time frame for the SMCT system equipped with 7 source/detector pairs. In that case, the temporal resolution of the SMCT system is 1.0s. Similarly, the temporal resolution of the SMCT system can be 0.4s and 0.2s for the undersampling factors 25 and 50.

To demonstrate the advantages of the SMCT system, a phantom (see Fig. 3) was designed to simulate the generation and disappear of bubbles within 5.0s for casting cooling in our previous study [14]. More details about the phantom can refer to [14]. In this work, the range of greyscale value is set as [0 1] rather than [0 1000] in [14]. In this study, to evaluate the denoising performance of the L₀-PICCS, a uniformly distributed Gaussian noise, where the standard variance is set as 10% of the maximum value of noise-free projection, is added to the projection data.

Fig. 3. From left to right columns represent the statues of the casting cooling process at 0.1s, 0.6s and 5.0s, respectively.

The high quality prior image plays an important role for both SM-PICCS and L₀-PICCS reconstruction techniques. For each time frame, the SMCT system can only collect few
projections. The previous proposed SM-PICCS algorithm achieved a great success by incorporating the prior image. The feasibility strategy is to utilize the traditional FBP method to reconstruct the object image from complete data set. However, it is impossible to obtain such a complete data set for a dynamic changeable object even with the SMCT. The projections from different time frames usually can be treated as a “quasi-complete” data set within a short time window. Finally, the prior image can be reconstructed using FBP from such “quasi-complete” data set. However, the used “quasi-complete” data set is corrupted by data inconsistency, which further implies the prior image with severe artifacts from the data inconsistency.

The reconstructed images consist of $256 \times 256$ pixels and each of them covers $1 \times 1$ mm$^2$. For simplicity, we only analyze the reconstructed images at 0.6s in the case of 5 X-ray sources from the noisy projections and the results are displayed in Fig. 4. From Fig. 4, it can be seen that the results reconstructed by ART [29] and total variation minimization with the steepest descent search (TVM-SD) [30] are seriously destroyed by shadow and motion artifacts. Compared with the ART and TVM-SD results, both SM-PICCS and L$_0$-PICCS can effectively restore the image information. Furthermore, compared with the SM-PICCS, the L$_0$-PICCS can more effectively reduce shadow artifacts and preserve the image edge. These advantages have been confirmed by the region indicated by red arrows in Fig. 4. It can be observed that the SM-PICCS cannot be able to separate the adjacent small bubbles with the increase of the sampling factors. However, these blurred bubbles in the SM-PICCS results can be easily distinguished by the L$_0$-PICCS. Indeed, L$_0$-PICCS can provide better spatial resolution than the SM-PICCS. To further demonstrate the out-performance of the L$_0$-PICCS, the differences between reconstructed and true images are also illustrated in Fig. 5. From Fig. 5, we can see that the images from the L$_0$-PICCS are closer to true image than those reconstructed by other competitors.

Fig. 4. Reconstructed results of the time frame 6 from the noisy projections acquired with 5 X-ray sources. The 1st –3rd rows represents the reconstructed results of the undersampling factors 10, 25, 50, respectively. 1st-3rd columns are the results reconstructed by ART, TVM-SD, SM-PICCS, L$_0$-PICCS methods, respectively. The display window is [0.04,0.1].
In order to quantitatively compare the reconstructed results, the index values in terms of RMSE, PSNR and SSIM of all reconstruction algorithms with 5 and 7 X-ray sources are listed in Tables 5, 6 and 7, respectively. From Tables 5 and 6, we can see that the L₀-PICCS always can obtain the smallest RMSE and the largest PSNR in the cases of noise-free and noisy projections. Meanwhile, the proposed L₀-PICCS can achieved the highest SSIM values in all cases. This means that the images reconstructed by L₀-PICCS are closer to the ideal phantom than other comparisons. From Tables 5-7, it can be seen that the results reconstructed by ART and TVM-SD from 7 X-ray sources are better than those obtained from 5 X-ray sources. This is because the projections from 7 X-ray sources are more than that achieved from 5 X-ray sources. However, regarding the L₀-PICCS or SM-PICCS results, we can observe that the RMSEs and SSIMs are very close in the cases of 5 and 7 sources. Unexpectedly, the results using the proposed L₀-PICCS in 5 X-ray sources are even better than those reconstructed by SM-PICCS with 7 X-ray sources. In other words, the L₀-PICCS may be has a potential to reduce the number of X-ray source with the same image quality. The conclusion can be also confirmed by the index of PSNRs.

| Table 4. Quantitative results in terms of RMSEs |
|-----------------------------------------------|
| Unsersampling factors | Q = 5 | Q = 7 |
| | 10 | 25 | 50 | 10 | 25 | 50 |
| Noise-free | | | | | | |
| ART | 0.255 | 0.289 | 0.322 | 0.239 | 0.240 | 0.262 |
| TVM-SD | 0.232 | 0.281 | 0.321 | 0.173 | 0.217 | 0.253 |
| SM-PICCS | 0.056 | 0.058 | 0.068 | 0.056 | 0.059 | 0.069 |
| L₀-PICCS | **0.043** | **0.052** | **0.064** | **0.041** | **0.046** | **0.059** |
| ART | 0.259 | 0.290 | 0.324 | 0.254 | 0.244 | 0.264 |
| Noise | | | | | | |
| TVM-SD | 0.232 | 0.282 | 0.322 | 0.175 | 0.217 | 0.254 |
| SM-PICCS | 0.066 | 0.068 | 0.076 | 0.069 | 0.067 | 0.075 |
| L₀-PICCS | **0.055** | **0.058** | **0.072** | **0.059** | **0.062** | **0.070** |

Fig. 5. The difference images between reconstructed images in Fig. 4 and true image. The display window is [0.04,0.1].
Table 5. Quantitative results in terms of PSNRs

| Undersampling factors | Q = 5 | Q = 7 |
|-----------------------|-------|-------|
|                       | 10    | 25    | 50    | 10    | 25    | 50    |
| Noise-free            | ART   | 12.368| 10.908| 9.850 | 14.317| 12.790| 11.709|
|                       | TVM-SD| 12.711| 11.025| 9.862 | 15.258| 13.293| 11.939|
|                       | SM-PICCS| 25.056| 24.694| 23.353| 25.068| 24.743| 23.533|
|                       | L₀-PICCS| 27.217| 25.729| 23.810| 27.788| 26.720| 24.508|
| Noisy                 | ART   | 12.242| 10.869| 9.817 | 13.964| 12.660| 11.647|
|                       | TVM-SD| 12.681| 11.011| 9.843 | 15.129| 13.260| 11.915|
|                       | SM-PICCS| 23.627| 23.396| 22.407| 23.651| 23.592| 22.705|
|                       | L₀-PICCS| 25.112| 24.583| 22.809| 25.454| 24.179| 23.035|

Table 6. Quantitative results in terms of SSIMs

| Undersampling factors | Q = 5 | Q = 7 |
|-----------------------|-------|-------|
|                       | 10    | 25    | 50    | 10    | 25    | 50    |
| Noise-free            | ART   | 0.8448| 0.7568| 0.6586| 0.9100| 0.8606| 0.8044|
|                       | TVM-SD| 0.8555| 0.7615| 0.6567| 0.9272| 0.8747| 0.8125|
|                       | SM-PICCS| 0.9935| 0.9930| 0.9904| 0.9936| 0.9931| 0.9908|
|                       | L₀-PICCS| 0.9960| 0.9943| 0.9913| 0.9965| 0.9955| 0.9925|
| Noisy                 | ART   | 0.8409| 0.7552| 0.6561| 0.9032| 0.8569| 0.8023|
|                       | TVM-SD| 0.8545| 0.7608| 0.6549| 0.9250| 0.8738| 0.8117|
|                       | SM-PICCS| 0.9909| 0.9904| 0.9878| 0.9909| 0.9908| 0.9887|
|                       | L₀-PICCS| 0.9935| 0.9926| 0.9887| 0.9924| 0.9918| 0.9893|

3.2 Realistic data set

To demonstrate the usefulness of the L₀-PICCS algorithm in practical applications, a specimen is scanned by a micro cone-beam CT system with one X-ray source and one flat panel detector (FPD) in Chongqing University. In this study, the tube voltage and current are set as at 60 kV and 200 mAs. The FPD consists of 1024 × 1024 units and each of them covers an area of 0.127 × 0.127 mm². The distance starting from the X-ray source to FPD and object are 502.0 mm and 132.3 mm respectively. 450 projections are uniformly acquired over a full scan range. Figure 6(a) shows the reconstructed 3D structure of the specimen using FDK algorithm and Fig. 6(b) shows the 50 slices around the middle plane. From Fig. 6, it can be seen that the structure of the specimen is similar along rotation axis direction, i.e., most parts of structure have not changed except for some small regions. Since the distance from the source to object far outweigh that between source and object, the circular cone-beam geometry can be treated as a good approximation of 3D parallel-beam geometry for such micro-CT system. In fact, this is true for most of the nano-CT and micro-CT scanners. Hence, the extracted 50 slices of the FPD along the z-axis can be treated as the 2D dynamic object. Some representative slices reconstructed from different FPD rows are shown in Fig. 7.

To imaging the aperiodic dynamic process in industrial applications, we extract 450 and 448 projections for SMCT systems equipped with 5 and 7 source/detector pairs respectively. The undersampling factors are set as 10, 18, 30 for SMCT system with 5 source/detector pairs and result in the number of projections being 45, 25, 15. Similarly, the undersampling factors are set as 8, 16, 32 for SMCT system with 7 source/detector pairs and lead to the number of projections being 56, 28, 14. To clarify the projection extraction, here the undersampling factor is assumed as 10 for SMCT with 5 source/detector pairs. In the case, the interlaced strategy is adopted to extract 45 projections for one row of FPD.
Fig. 6. The 3D structure of the specimen reconstructed by FDK algorithm (left) and extracted 50 reconstructed slices (right).

Fig. 7. 50 reconstructed slices from 50 FPD slices by using FBP and each reconstructed slice consists of $256 \times 256$ pixels. The display window is [0 0.05].

For simplicity, we only analyze the reconstructed results at time frame 6. The reconstructed images from 6th row of the FPD by using FBP is served as a benchmark (see Fig. 8). Figures 8 and 9 show the reconstructed results for different system configuration with different undersampling factors. From Figs. 8 and 9, we can obviously observe that the results of ART and TVM-SD contain too severe artifacts to the image information is completely destroyed. Compared with ART and TVM-SD results, the SM-PICCS technique can recover high quality image to some extent. However, the image edges and structures indicated with the red arrows in Figs. 8 and 9 are blurred so that some small gaps are disappeared. In contrast, the results are constructed by $L_0$-PICCS have clearer image edges and much finer structures. Especially, the gaps between small structures can be distinguished clearly. This is because that $L_0$-PICCS minimizes the $L_0$-norm of image gradient. It penalizes the non-zero number rather than the amplitude, which is good for image edge protection and finer features recovering.
4. Discussion and conclusion

To study the locally aperiodic dynamic imaging in industrial applications, we previously designed a SMCT system. For the SMCT system, the conventional compressed sensing based reconstruction method may be failed to provide high-quality image. To address this problem, we first applied the PICCS method to the SMCT system. In this study, to further improve the temporal resolution of the SMCT system, the $L_0$-norm of image gradient is incorporated into the PICCS method to obtain an $L_0$-PICCS algorithm. Both numerical and realistic data set experiments demonstrate the $L_0$-PICCS method can provide a higher image quality with sharper image edge and finer structures than the SM-PICCS. This is because image gradient...
L1-norm minimization can lead to a global smoothing by punishing image gradients coefficients. In contrast, the image gradient L0-norm minimization penalizes the non-zero number of the small image gradient rather than the amplitude, which can be beneficial for image edge information protection and finer structure recovering.

From the results with different undersampling factors in numerical simulations, it can be seen that the reconstructed image quality is decreased with the undersampling factor increased. Especially, the undersampling factor is degraded to 25, the reconstructed images from L0-PICCS and SM-PICCS are degraded than those obtained from undersampling factor 10. However, the images reconstructed by the L0-PICCS provide a higher image quality than those obtained by the SM-PICCS. As the aforementioned, increasing the undersampling factor will improve the temporal resolution. Thus, L0-PICCS is able to acquire better images quality than SM-PICCS when the temporal resolution increased. Again, compared with the SM-PICCS, the proposed L0-PICCS method can reconstructed a satisfied image with fewer projections. It indicates the developed method has a potential to reduce the number of the X-ray sources with the same temporal resolution. When the undersampling factor is fixed, the results of L0-PICCS from 5 X-ray sources are more accurate than that obtained by SM-PICCS from 7 X-ray sources. The realistic data set experiments demonstrate that L0-PICCS can acquire a better image quality than SM-PICCS. The realistic data set experiments demonstrate that if there are changing region rapidly within the object, both L0-PICCS and SM-PICCS may fail to reconstruct high quality images. For example, the undersampling factor 32 for SMCT system equipped with 7 X-ray source-detector pairs, the projections are contaminated by data inconsistency and result in image quality are further comprised. Fortunately, the proposed L0-PICCS can still achieved better reconstructed image with clear image edge and finer structures.

Although the L0-PICCS algorithm has obtained excellent performance, there are still some issues. First, the L0-PICCS method contains four parameters, which may become the biggest problem in practical applications. In this study, those parameters are optimized based on RMSE and visual evaluation. Figure 10 further demonstrates the results in terms of the RMSE v.s. four parameters in the case of undersampling factor 10 for SMCT with 5 source/detector pairs. The theoretical analyses and furtherly optimizations are still open problems that need to be fully addressed in our future work [31]. Second, the realistic data set is collected from the micro-CT system equipped with one X-ray source/detector pair. The system ignores the forward and backward X-ray scattering effects. Thus, how to obtain a high quality reconstructed image is a challenge considering scattering effect in practical. Third, the L0-PICCS is first proposed for SMCT system in this study. In fact, it can be also applied to cardiac CT (4DCT) [1,32,33], spectral CT [34,35]. Particularly, when the proposed L0-PICCS is applied to spectral CT reconstruction, the averaged image can be considered as the prior image.

In summary, based on the advantages of the image gradient L0-norm minimization in image edge protection and finer features recovering, it was employed to improve SMCT system temporal resolution by combining the prior image. Considering the image gradient L0-norm optimization is a non-convex and NP-hard problem, the split-Bregman strategy was used to solve the L0-PICCS model. The experiment results demonstrate the proposed L0-PICCS can obtain better reconstructed image quality than other comparisons. This will be extremely meaningful for dynamic CT reconstruction.
Fig. 10. RMSEs v.s. different parameters settings. (a)-(d) represent $\delta_1, \delta_1', \delta_2, \delta_2'$ respectively.

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