Axisymmetric stagnation-point flow and heat transfer of a viscous fluid on a moving plate with time-dependent axial velocity and uniform transpiration

M. Ja'fari, A.B. Rahimi
Department of Mechanical Engineering, Faculty of Engineering, Ferdowsi University of Mashhad, Mashhad, P.O. Box 91775-1111, Iran

Received 4 January 2012; revised 2 June 2012; accepted 30 October 2012

Abstract The unsteady viscous flow and heat transfer in the vicinity of an axisymmetric stagnation point of an infinite moving plate with transpiration $W_0$ are investigated when the axial velocity and wall temperature vary arbitrarily with time. The free stream is steady and with a strain rate of $a$. An exact solution of the Navier–Stokes equations and energy equation is derived in this problem. A reduction of these equations is obtained by the use of appropriate transformations for the most general case when the transpiration rate is also time-dependent, but results are presented only for uniform values of this quantity. The general self-similar solution is obtained when the axial velocity of the plate and its wall temperature vary as specified time-dependent functions. For completeness, sample semi-similar solutions of the unsteady Navier–Stokes equations have been obtained numerically using a finite difference scheme. All the solutions above are presented for different values of dimensionless transpiration rate, $S = W_0 / \sqrt{a\nu}$, where $\nu$ is the kinematic viscosity of the fluid. The effects of the sundry parameters, including transpiration rate, Prandtl number, oscillation frequency and accelerating/decelerating parameter, on the velocity and temperature profiles, as well as surface shear stresses and heat transfer coefficient, are investigated and results are shown through graphs.

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1. Introduction

Stagnation regions exist on all blunt bodies moving in a viscous fluid. The results of studies about stagnation flow problems are of great technical importance, for example, in the prediction of skin-friction, as well as heat/mass transfer, near stagnation regions of bodies in high speed flows, and also in the design of thrust bearings and radial diffusers, drag reduction, transpiration cooling and thermal oil recovery. In certain stagnation flow problems, the Navier–Stokes equations reduce to nonlinear ordinary differential equations through a similarity transform. The task of finding exact solutions of the Navier–Stokes equations is a difficult one due to the nonlinearity of these equations. Removing the nonlinearity in these problems is usually accomplished by superposition of fundamental exact solutions that lead to nonlinear coupled ordinary differential equations by separation of coordinate variables. Fundamental studies, in which flows are readily superposed and/or in the axisymmetric case, include the following papers presented in the literature: uniform shear flow over a flat plate, in which the flow is induced by a plate oscillating in its own plane be-neath a quiescent fluid [1], two-dimensional stagnation-point flow [2], the flow induced by a disk rotating in its own plane [3], flow over a flat plate with uniform normal suction [4], three-dimensional stagnation-point flow [5], and axisymmetric stagnation flow on a circular cylinder [6]. Further exact solutions to the Navier–Stokes equations are obtained by superposition of the uniform shear flow and/or stagnation flow on a body oscillating or rotating in its own plane, with or without suction. The examples are as follows: superposition of two-dimensional and three-dimensional stagnation-point flows [7], superposition of uniform suction at the boundary of a rotating disk [8], also, the solution for a fluid oscillating about a nonzero mean flow parallel to a flat plate with uniform suction [9], superposition of stagnation-point flow on a flat plate oscillating in its...
own plate, and also consideration of the case where the plate is stationary and the stagnation stream is made to oscillate \[10\], uniform shear flow aligned with outflowing two-dimensional stagnation-point flow \[11\], uniform flow along a flat plate with time-dependent suction and included periodic oscillations of the external stream \[12\], heat transfer in an axisymmetric stagnation flow on a cylinder \[13\], unsteady laminar axisymmetric stagnation flow over a circular cylinder \[14\], nonsimilar axisymmetric stagnation flow on a moving cylinder \[15\], transient response behavior of an axisymmetric stagnation flow on a circular cylinder, due to time-dependent free stream velocity \[16\], unsteady viscous flow in the vicinity of an axisymmetric stagnation point on a cylinder \[17\], shear flow over a rotating plate \[18\], radial stagnation flow on a rotating cylinder with uniform transpiration \[19\], suppression of turbulence in wall bounded flows by high-frequency span wise oscillations \[20\], axisymmetric stagnation flow toward a moving plate \[21\], axisymmetric stagnation-point flow impinging on a transversely oscillating plate with suction \[22\], axisymmetric stagnation-point flow and heat transfer of a viscous fluid on a moving cylinder, with time-dependent axial velocity and uniform transpiration \[23\], axisymmetric stagnation-point flow and heat transfer of a viscous fluid on a rotating cylinder, with time-dependent angular velocity and uniform transpiration \[24\], similarity solution of non-axisymmetric heat transfer in stagnation-point flow on a cylinder, with simultaneous axial and rotational movements \[25\], non-axisymmetric three-dimensional stagnation-point flow and heat transfer on a flat plate \[26\], and three-dimensional stagnation flow and heat transfer on a flat plate with transpiration \[27\], and investigation of two-dimensional unsteady stagnation-point flow and heat transfer impinging on an accelerated flat plate \[28\].

In this study, the unsteady viscous flow and heat transfer in the vicinity of an axisymmetric stagnation point of an infinite plate with time-dependent axial movement and with uniform transpiration is considered, though reduction of the Navier–Stokes equations and the energy equation is obtained for the most general case of time-dependent transpiration rate. Our motivation in this study is to generalize the stagnation-point flow and heat transfer problem of a viscous fluid on an infinite plate by letting this flat plate move with an arbitrary function of time. Also, by an appropriate viscous length scale, we calculate the Nusselt number in this problem. It is shown that a moving plate without heat transfer (an adiabatic plate) and also without unsteady surface shear stress (reduction of resistance against axial movement of the plate) is achievable, and these are the practical applications of this study. An exact solution of the Navier–Stokes equations and the energy equation is obtained. The general self-similar solution is obtained when the axial velocity of the plate and its surface temperature vary as specified time-dependent functions. In particular, the plate may move with constant speed, with exponentially increasing/decreasing axial velocity, with harmonically varying axial speed, or with accelerating/decelerating oscillatory axial speed. The surface temperature may have the same types of behavior as the plate motion. Sample distributions of shear-stresses, velocity and temperature fields are presented for different forms of plate movement and different values of transpiration parameter, Prandtl number and other parameters of the problem. Particular cases of these results are compared with existing results of Wang \[21\], Weidman and Mahalingam \[22\] and Rahimi and Abbassi \[27\], correspondingly. For completeness, some similar solutions of the unsteady Navier–Stokes equations and energy equation are obtained, and one of these solutions is presented for special cases when the time-dependent axial velocity of the plate is a ramp function.

2. Problem formulation

The flow is considered in Cartesian coordinate, \((x, y, z)\), with corresponding velocity components, \((u, v, w)\) (see Figure 1). We consider the laminar unsteady incompressible flow and heat transfer of a viscous fluid in the neighborhood of an axisymmetric stagnation point of an infinite plate located in the plane, \(z = 0\), when it moves axially with a velocity that varies with time. An external fluid along the \(z\) direction, with strain rate \(\alpha\), impinges on this flat plate and produces a two-dimensional flow on the plate. A uniform normal transpiration, \(W_0\), at the plate surface may occur, where \(W_0 > 0\) corresponds to suction into the plate, though the formulation of the problem is for the more general case of time-dependent transpiration rate. The unsteady Navier–Stokes and energy equations in the Cartesian coordinate governing the flow and heat transfer are given as:
and (8) are inviscid solutions of [1–4], which is valid far above the plate, and \( p_0 \) is stagnation pressure. For the temperature field, we have:

\[
z = 0 : T = T_W(t),
\]

\[
z \to \infty : T \to T_\infty,
\]

\[
t = 0 : T(z, t) = T(z)_{\text{steady-state}}.
\]

A reduction of the Navier–Stokes equations is obtained by applying the following transformation:

\[
u = a f'(\eta, \tau) + H(\eta, \tau), \quad v = a f''(\eta, \tau),
\]

\[
w = -2\sqrt{a}v f(\eta, \tau) - W_0(\tau), \quad \eta = \sqrt{\frac{a}{v}} z, \quad \tau = at.
\]

Transformations (11) satisfy Eq. (1) automatically and their insertion into Eqs. (2)–(4) yields a coupled system of differential equations, in terms of \( f(\eta, \tau) \) and \( H(\eta, \tau) \), and an expression for the pressure:

\[
f'''' + (2f + S)f'' - f'^2 + 1 = \frac{a}{2} \frac{d^2 f}{d \eta^2} = 0.
\]

\[
H'''' + (2f + S)H' - f'H - \frac{dH}{d \eta} = 0.
\]

In these equations, primes indicate differentiation with respect to \( \eta \). Eq. (14) is obtained by integrating Eq. (4) in the \( z \) direction; and by use of the potential flow solution, Eqs. (7) and (8) as boundary conditions, and \( S(\tau) = W_0(\tau)/\sqrt{a}v \) is the dimensionless transpiration rate. From conditions (6) and (7), the boundary conditions for Eqs. (12) and (13) are as follows:

\[
\eta = 0 : f = 0, \quad f' = 0, \quad H = U(\tau),
\]

\[
\eta \to \infty : f' = 1, \quad H = 0.
\]

In this paper, \( S(\tau) \) is considered constant and, also, none of the boundary conditions of Eq. (12) are functions of time. Assuming steady-state conditions for this equation, we have:

\[
\tau = 0 : \frac{d f'}{d \tau} = 0.
\]

Consequently, \( f \) does not change with respect to time and the result of the steady-state solution is the same as the solution for all later times. Therefore, in this case, \( f(\eta, \tau) = f(\eta) \), and Eq. (12) is reduced to the following form:

\[
f'''' + (2f + S)f'' - f'^2 + 1 = 0.
\]

The terms involving \( f(\eta, \tau) = f(\eta) \) in Eqs. (11) comprise the Cartesian similarity form for steady axisymmetric stagnation-point flow. Eq. (17) is the same as the one obtained by Weidman and Mahalingam [22]. The initial condition for Eq. (13) is:

\[
\tau = 0, \quad H(\eta, \tau) = H(\eta)_{\text{steady-state}}.
\]

To transform the energy equation into a nondimensional form for the case of defined wall temperature, we introduce:

\[
\Theta = \frac{T(\eta, \tau) - T_\infty}{T_W(\tau) - T_\infty}.
\]

By using Eqs. (11) and (19), the energy equation may be written as:

\[
\Theta'''' + Pr \left[ (2f + S)\Theta'' - \frac{d \Theta}{d \tau} \frac{d T_W(\tau)}{d \tau} \right] = 0.
\]
where \( Pr = \nu /\alpha \) is the Prandtl number, and primes indicate differentiation, with respect to \( \eta \). The boundary and initial conditions are:

\[
\begin{align*}
\eta &= 0 : \Theta = 1, \\
\eta &\to \infty : \Theta = 0, \\
\tau &= 0 : \Theta(\eta, \tau) = \Theta(\eta)_{\text{steady-state}}.
\end{align*}
\]

(21)

(22)

Here, Eqs. (13) and (20) are solved for different forms of \( U(\tau) \) function and prescribed values of wall temperature. In what follows, first, the self-similar equations and the exact solutions for some particular \( U(\tau) \) and \( T_W(\tau) \) functions are presented. Then, for completeness, the semi-similar equations and their numerical solutions are obtained and a sample result for Eq. (13) is presented.

### 3. Self-similar solutions

Eqs. (13) and (20) can be reduced to a system of ordinary differential equations if we assume that the functions, \( H(\eta, \tau) \) and \( \Theta(\eta, \tau) \) in Eq. (20), are separable as:

\[
H(\eta, \tau) = U(\tau) \cdot h(\eta),
\]

\[
\Theta(\eta, \tau) = \theta(\eta) \cdot \Theta(\tau).
\]

(23)

Substituting these separations of variables into Eqs. (13) and (20), correspondingly, gives:

\[
\frac{h''}{h} + (2f + S)\frac{h'}{h} - (f' + \alpha)h = 0,
\]

(24)

\[
\frac{\theta''}{\theta} + \frac{\theta'}{\theta} \left( 2f + S \right) - \frac{f' + \alpha}{\theta} = 0.
\]

(25)

The general solution to the differential equations in Eqs. (24) and (25), with \( \tau \) as the independent variable, are as follows:

\[
U(\tau) = A \cdot \exp(\alpha \tau),
\]

\[
\Theta(\tau) = \frac{B \cdot \exp(\gamma \tau)}{T_W(\tau) - T_\infty},
\]

(26)

(27)

where \( i = \sqrt{-1} \) and \( A, B, \alpha, \beta, \gamma, \delta \) are constants. Substituting these solutions into the differential equations in Eqs. (24) and (25), with \( \eta \) as an independent variable, results in:

\[
h'' + (2f + S)h' - (f' + \alpha)h = 0,
\]

(28)

\[
\theta'' + \theta' \left( 2f + S \right) - (\gamma + \delta)\theta = 0.
\]

(29)

Boundary conditions for axial velocity are:

\[
\eta = 0 : h = 1, \\
\eta \to \infty : h = 0.
\]

(30)

And, for the above defined wall temperature, we have:

\[
\eta = 0 : \Theta = 1 = \theta(\eta) \to \theta(0) = 1, \\
\psi(\tau) = 1 \to T_W(\tau) - T_\infty = B \cdot \exp((\gamma + \delta)\tau), \\
\eta \to \infty : \Theta = 0 = \theta(\infty) \to \theta(\infty) = 0.
\]

(31)

Note that, in (26), \( A = 0 \) corresponds to the case of a plate with no axial movement. If \( A \neq 0 \) and \( \alpha = \beta = 0 \), Eq. (26) gives the case of a uniformly moving plate with constant axial velocity [21]. \( A \neq 0, \alpha = 0 \) and \( \beta \neq 0 \) correspond to the case of a moving plate with a harmonic velocity in its own plate [22]. The case of \( A \neq 0, \alpha \neq 0 \) and \( \beta \neq 0 \) is the most general case, which is considered in this paper. To obtain the solution of Eqs. (28) and (29), and to work with real and imaginary parts separately, it is assumed that the functions, \( h(\eta) \) and \( \eta(\eta) \), are complex functions as:

\[
\begin{align*}
h(\eta) &= h_1(\eta) + ih_2(\eta), \\
\theta(\eta) &= \theta_1(\eta) + i\theta_2(\eta).
\end{align*}
\]

(32)

(33)

Substituting Eqs. (32) and (33) into Eqs. (28) and (29), the following coupled systems of differential equations are obtained:

\[
\begin{align*}
h_1'' + (2f + S)h_1' - (f' + \alpha)h_1 + \beta h_2 &= 0, \\
h_2'' + (2f + S)h_2' - (f' + \alpha)h_2 &= 0.
\end{align*}
\]

(34)

(35)

\[
\begin{align*}
\theta_1'' + \frac{Pr(2f + S)}{2} - (\gamma + \delta)\theta_1 &= 0, \\
\theta_2'' + \frac{Pr(2f + S)}{2} - (\gamma + \delta)\theta_2 &= 0.
\end{align*}
\]

(36)

(37)

The boundary conditions for functions \( h_1, h_2, \theta_1 \) and \( \theta_2 \) are:

\[
\eta = 0 : h_1 = 1, \\
h_2 = 0, \\
\theta_1 = 1, \\
\theta_2 = 0.
\]

(38)

The \( H(\eta, \tau) \) function from Eq. (23) becomes:

\[
H(\eta, \tau) = U(\tau) \cdot h(\eta) = A \cdot \exp(\alpha \tau) [h_1(\cos(\beta \tau)) - h_2(\sin(\beta \tau)) + i(h_1(\cos(\beta \tau)) + h_2(\cos(\beta \tau))).
\]

(39)

We obtain a numerical solution of the boundary-value problem (15) and (17), (34)-(38) by using an iterative and adaptive step-size algorithm based on a standard shooting method, combined with a fourth-order Runge-Kutta method, employing the function NDSolve in Mathematica [29]. For prescribed values of \( S, Pr, \alpha, \beta, \gamma, \delta \), we treat the equations as initial-value problems, using initial guesses for \( f(0), h_1(0), h_2(0), \theta'_1(0), \theta'_2(0) \) and integrate to a sufficiently large value of \( \eta \).

### 4. Semi-similar solutions

Eqs. (13) and (20) may be solved directly for any chosen \( U(\tau) \) and \( T_W(\tau) \) functions. The solutions obtained this way are called semi-similar solutions. These equations along with boundary conditions (15) and (21) were solved by using a central finite difference method, which leads to a tri-diagonal matrix. Assuming steady state for \( \tau < 0 \), the solution starts from \( U(0), T_W(0) \) and marching through time, time-dependent solutions for \( \tau > 0 \) were obtained. A sample axial velocity profile will be presented in later sections.

### 5. Shear-stress and heat transfer coefficient

The instantaneous shear stress on the surface of the moving plate is calculated from the following equation:

\[
\sigma = \sigma_1 + \sigma_2 = \mu \left[ \frac{\partial u}{\partial z} + \frac{\partial v}{\partial z} \right]_{z=0},
\]

(40)

where \( \mu \) is the fluid viscosity. Using the transformations (11), the shear stress on the surface of the plate becomes:

\[
\sigma = \rho a^{1/2} \nu^{1/2} f'(0) e_r + \rho \nu^{1/2} \left| H'(0, \tau) \right| \cdot \exp(i\eta) i
\]

(41)

where \( \left| H'(0, \tau) \right| = \sqrt{H'(0, \tau)^2 + \left( H_0(0, \tau)^2 \right)_{\text{imaginary}}} \)

\[
= \sqrt{x^2 + y^2} \quad \text{and} \quad \theta = \tan^{-1} \left( \frac{y}{x} \right)
\]

In Eq. (41), \( e_r \) is the unit vector aligned radially on the surface of the plate. Thus, the shear stress is composed of a steady radially-directed component due to the axisymmetric stagnation-point flow and an unsteady component aligned with the direction of the plate movement. Some numerical values of
shear stress will be presented later for a few examples of axial velocities. The local heat transfer coefficient for a defined wall temperature is given by:

\[ h_1 = \frac{q W}{T_W(\tau) - T_\infty} = \frac{-k_1}{\eta} \] for semi-similar case, \( h_1 = \frac{-k_1}{\eta} \) for self-similar case.

In terms of Nusselt number, we have:

\[ Nu = \frac{h_1 l_c}{k} = -(\theta'_1(0) + i\theta''_1(0)) = Nu_{\text{real}} + iNu_{\text{imaginary}}, \]

where \( l_c = \sqrt{v/a} \) is an appropriate viscous length scale.

6. Presentation of results

In this section, the solutions of the self-similar Eqs. (17), (28) and (29) and the semi-similar Eq. (13), along with surface shear stresses, for different functions of axial velocity and prescribed values of transpiration rates are presented. Figure 2 presents the profile of \( f'() \) for selected values of suction and blowing rates. These results are in agreement with the results of Rahimi and Abbassi [27], when the velocity ratio in \( x \) and \( y \) directions in the cited reference converge to 1. The steady surface shear-stress parameter, \( f''(0) \), as a function of transpiration parameter, \( S \), is shown in Figure 3. By increasing transpiration parameter \( S \), the steady surface shear-stress increases. Our results for steady surface shear stress for transpiration rates between \(-2 \) and \( 2 \) are in agreement with the results of Weidman and Mahalingam [22], as shown in Figure 4.

Sample profiles of the \( h(\eta) \) function for \( U(\tau) \) in accelerating and decelerating exponential form are presented in Figures 5–8, for selected values of transpiration rate. Note that \( \alpha = 0 \) indicates the case of a moving plate with constant axial velocity, our results are in agreement with the results of Wang [21]. It is interesting to note that as \( \alpha \) increases, the depth of the diffusion of the fluid velocity field decreases (see Figure 8). For \( \alpha < 0 \) at any rate of transpiration, and for the absolute value of \( \alpha \) greater than a certain value, the fluid velocity in the vicinity of the plate is greater than the plate velocity. It can be seen that as the transpiration rate increases, the depth of the diffusion of the fluid velocity decreases and vice versa. It is obvious that for high suction rates, the boundary layer becomes small and the effect of \( \alpha \) is negligible on the thickness of this layer. On the other hand, a high rate of blowing thickens the boundary layer, and \( \alpha \) has a considerable effect on it. The unsteady shear-stress parameter on the surface of the plate, as a function of transpiration parameter \( S \) for \( U(\tau) \), in accelerating and decelerating exponential form, is presented in Figure 9. The absolute value of this parameter increases for different values of \( \alpha \) with increasing transpiration parameter, \( S \). The practical application of this result is that by providing blowing on the surface of a plate, a reduction in resistance against its axial movement inside a fluid can be achieved.

Also, it can be seen that as \( \alpha \) increases, the absolute value of unsteady shear-stress increases. It is also interesting to note that at any transpiration rate, there is a special value of negative \( \alpha \), for which the value of unsteady shear stress is zero. Consequently, a moving plate without unsteady shear-stress is achievable by a combination of appropriate values of transpiration rate and \( \alpha \). Sample profiles of \( h_1(\eta) \) and \( \theta_1(\eta) \) functions for pure harmonic motion of the plate, with different frequencies at \( S = 1 \) and different transpiration rates at \( \beta = 1 \), are depicted in Figures 10 and 11, respectively. We observe that as transpiration rate increases, the depth of the diffusion of the fluid velocity field decreases. Also, by increasing the frequency parameter, \( \beta \), the depth of diffusion of \( h_1(\eta) \) decreases and the
depth of diffusion of $h_2(\eta)$ increases. Sample profiles of $h_1(\eta)$ and $h_2(\eta)$ functions for an accelerating and decelerating oscillatory motion at $\beta = 2$ and $S = 1$ are shown in Figure 12, for selected values of $\alpha$. Again, it can be seen that as $\alpha$ increases, the depth of the diffusion of the fluid velocity decreases. Figure 13 shows the variations of $|h'(0)|$ with frequency parameter $\beta$ for selected values of $S$. The absolute value of unsteady shear stress increases by increasing transpiration rate and the frequency parameter. Our results for $S = -1, 0, 1$ are in agreement with the results of Weidman and Mahalingam, [22], as shown in Figure 14. Figure 15 shows the semi-similar solution of the fluid flow when the time-dependent axial velocity of the plate is a ramp function, in which the function $H(\eta, \tau)$ is shown in terms of $\eta$ for different values of nondimensional time and $S = 2$. The process of this solution is explained in earlier sections. Figure 15 shows that, as time passes and along with increasing velocity, the thickness of the boundary layer increases.

Sample profiles of the $\theta(\eta)$ function for wall temperature varying exponentially with time are presented in Figures 16–18, for selected values of $\gamma$, transpiration rates and Prandtl number, respectively. From Figure 16, it can be seen that for $\gamma > 0$, the absolute value of the initial slope of the temperature increases...
with increasing values of $\gamma$, and, consequently, the heat transfer coefficient increases. But, for $\gamma < 0$ and any particular value of transpiration rate, as the absolute value of $\gamma$ increases beyond a certain value, the flow temperature in the vicinity of the plate is greater than the wall temperature. It is interesting to note that at any transpiration rate, there is a certain value of negative $\gamma$ for which the slope of the temperature on the surface becomes zero, and, therefore, there is no heat transfer. Consequently, a moving plate without heat transfer (adiabatic plate) is achievable by an appropriate combination of parameters of the problem. From Figures 16 and 19, we observe that the absolute value of the initial slope of the temperature increases, and the depth of the diffusion of temperature field decreases with increasing values of transpiration rates and Prandtl number. Consequently, the heat transfer coefficient increases. Note that $\gamma = \delta = 0$ indicates the case of a moving plate with constant wall temperature. Sample profiles of $\theta_1(\eta)$ and $\theta_2(\eta)$ functions for wall temperature varying harmonically with time at $Pr = 2$ is depicted in Figures 19 and 20, for selected values of frequency parameter and transpiration rate. It can be seen from these figures that by increasing the oscillation frequency and transpiration rates, the depth of diffusion of $\theta_1(\eta)$ decreases and its absolute values of initial slope increase. By decreasing the frequency parameter, the depth of diffusion of $\theta_2(\eta)$ and its absolute values of initial slope decrease. Also, by increasing transpiration rates, the depth of diffusion of $\theta_2(\eta)$ and its absolute values of initial slope decrease. Sample profiles of $\theta_1(\eta)$ and $\theta_2(\eta)$ functions for wall temperature varying with accelerating and decelerating oscillatory functions of time at $Pr = 2$ and $S = \delta = 1$ are depicted in Figure 21, for selected values of $\gamma$. The effects of $\gamma$ on the profiles of $\theta_1(\eta)$ and $\theta_2(\eta)$ functions are similar to the effects of $S$ in the former case, only with this difference, that the effect of $S$ is stronger than $\gamma$. Sample profiles of the local heat transfer coefficient (Nusselt number) in terms of Prandtl number for wall
temperature varying exponentially with time at $S = 2$ are presented in Figure 22, for selected values of $\gamma$. It can be seen that by increasing the Prandtl number and $\gamma$, the local heat transfer coefficient increases. It is obvious that the imaginary part of the Nusselt number is zero in this case. Sample profiles of the real part of the Nusselt number, in terms of transpiration rate for wall temperature varying harmonically with time at $Pr = 2$, are presented in Figure 23, for selected values of frequency parameter. We observe that the local heat transfer coefficient increases with increasing frequency parameter and transpiration rate, and the effect of transpiration rate on the local heat transfer coefficient is greater than the frequency parameter.

7. Conclusions

An exact solution of the Navier–Stokes equations and energy equation is obtained for the problem of stagnation-point flow on a moving plate with uniform suction and blowing rate. However, the formulation of the problem is for the more general case of time-dependent suction and blowing rate. A general self-similar solution is obtained when the plate has different types of time-dependent axial motion including: constant axial velocity, exponential axial velocity, pure harmonic movement, and both accelerating and decelerating oscillatory motion. Results of heat transfer for different time-dependent wall temperature including: constant wall temperature, exponential and harmonically and both accelerating and decelerating oscillatory types of wall temperature, are displayed. Also, one sample semi-similar solution for this problem has been obtained numerically, using a finite difference scheme, for a special case, when the time-dependent axial velocity of the plate is a ramp function. Axial components of the fluid velocity and the steady and unsteady surface shear stresses are obtained for different values of transpiration rate, oscillation frequency and accelerating and decelerating parameter. The absolute values of the steady and unsteady surface shear stresses increase with increasing transpiration rate, oscillation frequency and accelerating and decelerating parameter. The depth of the diffusion
frequency, $\delta$, and accelerating and decelerating parameter, $\gamma$. It is shown a moving plate without heat transfer is achievable by an appropriate combination of parameters of the problem.

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Mohammad Ja'fari was born in Yasuj, Iran, in 1985. He received his B.S. degree in Mechanical Engineering from Yasuj University, Iran, in 2009 and his M.S. degree in Mechanical Engineering from Ferdowsi University of Mashhad, Iran, in 2011.

Asghar Baradaran Rahimi was born in Mashhad, Iran, in 1951. He received his B.S. degree in Mechanical Engineering from Tehran Polytechnic, in 1974, and a Ph.D. degree in Mechanical Engineering from the University of Akron, Ohio, USA, in 1986. He has been Professor in the Department of Mechanical Engineering at Ferdowsi University of Mashhad since 2001. His research and teaching interests include: heat transfer and fluid dynamics, gas dynamics, continuum mechanics, applied mathematics and singular perturbation.
کارگاه‌های آموزشی مرکز اطلاعات علمی جهاد دانشگاهی

مباحث پیشرفته یادگیری عمیق؛ شبکه‌های توجه گرافی (Graph Attention Networks)

کارگاه آنلاین آموزش استفاده از وب‌سایت WEB OF SCIENCE

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