A lattice study of the strangeness content of the nucleon

G. S. Bali, S. Collins, M. Göckeler, R. Horsley, Y. Nakamura, A. Nobile, D. Pleiter, P. E. L. Rakow, A. Sternbeck, G. Schierholz, J. M. Zanotti
(QCDSF Collaboration)

1Institut für Theoretische Physik, Universität Regensburg, 93040 Regensburg, Germany
2School of Physics, University of Edinburgh, Edinburgh EH9 3JZ, UK
3RIKEN Advanced Institute for Computational Science, Kobe, Hyogo 650-0047, Japan
4JSC, Research Center Jülich, 52425 Jülich, Germany
5Theoretical Physics Division, Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK
6Deutsches Elektronen-Synchrotron DESY, 22603 Hamburg, Germany
7CSSM, School of Chemistry & Physics, University of Adelaide, South Australia 5005, Australia

November 19, 2018

Abstract

We determine the quark contributions to the nucleon spin \( \Delta s \), \( \Delta u \) and \( \Delta d \) as well as their contributions to the nucleon mass, the \( \sigma \)-terms. This is done by computing both, the quark line connected and disconnected contributions to the respective matrix elements, using the non-perturbatively improved Sheikholeslami-Wohlert Wilson Fermionic action. We simulate \( n_F = 2 \) mass degenerate sea quarks with a pion mass of about 285 MeV and a lattice spacing \( a \approx 0.073 \) fm. The renormalization of the matrix elements involves mixing between contributions from different quark flavours. The pion-nucleon \( \sigma \)-term is extrapolated to physical quark masses exploiting the sea quark mass dependence of the nucleon mass. We obtain the renormalized value \( \sigma_{\pi N} = (38 \pm 12) \) MeV at the physical point and the strangeness fraction \( f_{T_s} = \sigma_s/m_N = 0.012(14)^{+10}_{-3} \) at our larger than physical sea quark mass. For the strangeness contribution to the nucleon spin we obtain \( \Delta_{sMS}^{MS}(\sqrt{7.4}\, \text{GeV}) = -0.020(10)(2) \).

1 Introduction

Most of the nucleon’s mass is generated by the spontaneous breaking of chiral symmetry and only a small part can be attributed directly to the masses of its valence and sea quarks. The quantities

\[
 f_{T_q} = m_q \langle N | \bar{q} q | N \rangle / m_N = \sigma_q / m_N
\]
parameterize the fractions of the nucleon mass $m_N$ that are carried by quarks of flavour $q$. These scalar matrix elements also determine the coupling strength of the Standard Model Higgs boson (or of any similar scalar particle) at zero recoil to the nucleon. This then might couple to heavy particles, some of which are dark matter candidates \cite{[1]}. The combination $m_N \sum_q f_{T_q}, q \in \{u, d, s\}$, appears quadratically in the cross section that is proportional to $|f_N|^2$, where

$$f_N = m_N \left( \sum_{q \in \{u,d,s\}} \frac{\alpha_q}{m_q} + \frac{2}{9n_h} f_{T_G} \sum_{q \in \{c,b,t,\ldots\}} \frac{\alpha_q}{m_q} \right),$$

with the couplings $\alpha_q \propto m_q/m_W$. $n_h$ denotes the number of heavy quark flavours. Note that due to the trace anomaly of the energy momentum tensor one obtains

$$f_{T_G} = 1 - \sum_{q \in \{u,d,s\}} f_{T_q},$$

so that the coupling $f_N$ only depends mildly on the number and properties of (discovered and not yet discovered) heavy quark flavours \cite{[2]}.

The light quark contribution, the pion-nucleon $\sigma$-term, is defined as

$$\sigma_{\pi N} = \sigma_u + \sigma_d = m_u \frac{\partial m_N}{\partial m_u} + m_d \frac{\partial m_N}{\partial m_d} \approx m_{PS} \left. \frac{\partial m_N}{\partial m_{PS}} \right|_{m_{PS}=m_{\pi}}.$$  

From dispersive analyses of pion-nucleon scattering data, the values $\sigma_{\pi N} = 45(8)$ MeV and $\sigma_{\pi N} = 64(7)$ MeV were obtained while a recent covariant baryon chiral perturbation theory (B$\chi$PT) analysis of the available scattering and pionic atom data \cite{[5]} resulted in the estimate $\sigma_{\pi N} = 59(7)$ MeV.

Not only do the quarks contribute a tiny fraction to the nucleon’s mass, even the nucleon spin is mostly carried by the gluons. This spin can be factorized into a quark spin contribution $\Delta \Sigma$, a quark angular momentum contribution $L_q$ and gluonic contributions $\Delta G$ and $L_G$ (for spin and angular momentum):

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_q + \Delta G + L_G.$$  

In the naïve non-relativistic SU(6) quark model, $\Delta \Sigma = 1$, with vanishing angular momentum and gluon contributions. In this case there will be no strangeness contribution $\Delta s$ to $\Delta \Sigma = \Delta u + \Delta d + \Delta s + \cdots$ where, in our notation, $\Delta q$ contains both, the spin of the quarks $q$ and of the antiquarks $\bar{q}$. Experimentally, $\Delta s$ is obtained by integrating the strangeness contribution to the spin structure function $g_1$ over the momentum fraction $x$. The integral over the range in which data exists usually agrees with zero, see e.g. new COMPASS data \cite{[6]} for $x \geq 0.004$, while global analyses tend to obtain values $\Delta s \approx -0.12$ \cite{[7],[8]}.

Here, we directly compute the quark line disconnected (and connected) contributions to the scalar and axial matrix elements that appear in the above quantities \cite{[9],[10]}. Other recent direct lattice determinations of these quantities include Refs. \cite{[11],[12],[13],[14],[15],[16]}. We will first describe the methods used, before we present results on the quark spin contributions and the scalar matrix elements.

## 2 Methods and simulation parameters

We simulate $n_F = 2$ non-perturbatively improved Sheikholeslami-Wohlert Fermions, using the Wilson gauge action, at $\beta = 5.29$ and $\kappa = \kappa_{ud} = 0.13632$. Setting the scale from the chirally extrapolated nucleon mass \cite{[17]}, we obtain the lattice spacing $a^{-1} = 2.71(2)(7)$ GeV, where the errors are statistical and from the extrapolation, respectively.
We realize two additional valence $\kappa$ values, $\kappa_m = 0.13609$ and $\kappa_s = 0.13550$. The corresponding pseudoscalar masses are $m_{PS, ud} = 285(3)(7)$ MeV, $m_{PS, m} = 449(3)(11)$ MeV and $m_{PS, s} = 720(5)(18)$ MeV. The strange quark mass was fixed so that the $m_{PS, s}$ value is close to the mass of a hypothetical strange-antistrange pseudoscalar meson: $(m_K^2 + m_{\pi}^2 - m_{\pi}^2)^{1/2} \approx 686.9$ MeV. We investigate volumes of $32^3 64$ and $40^3 64$ lattice points, i.e., $L_{PS} = 3.36$ and $4.20$, respectively, where the largest spatial lattice extent is $L \approx 2.91$ fm.

The matrix elements of interest are extracted from the large time behaviour of ratios of three-point functions over two-point functions where we create a proton at a time $t_0 = 0$ and destroy it at zero momentum at a time $t_f$. At an intermediate time $0 < t < t_f$ the current of interest is inserted. The three-point function contains both, a quark line connected and a quark line disconnected contribution. For the example of the axial current the ratios to be calculated read

$$R^{con}(t_f, t) = \frac{\langle \Gamma^\beta_\alpha C^\beta_\alpha_{3pt}(t_f, t) \rangle}{\langle \Gamma^\beta_\alpha_{unpol} C^\beta_\alpha_{2pt}(t_f) \rangle}, \quad R^{dis}(t_f, t) = -\frac{\langle \Gamma^\beta_\alpha C^\beta_\alpha_{2pt}(t_f) \rangle \sum_x \text{Tr}(\gamma_j \gamma_5 M^{-1}(x, t; x, t))}{\langle \Gamma^\beta_\alpha_{unpol} C^\beta_\alpha_{2pt}(t_f) \rangle}, \quad (6)$$

where $M$ is the lattice Dirac operator, $\Gamma_{unpol} = (1 + \gamma_4)/2$ a parity projector and $\Gamma_{pol} = i\gamma_j \gamma_5 \Gamma_{unpol}$ projects out the difference between the two polarizations (in direction $\hat{j}$). We average over $j = 1, 2, 3$ to increase statistics. For the scalar case we have to replace $\gamma_j \gamma_5 \mapsto 1$, $\Gamma_{pol} \mapsto \Gamma_{unpol}$ and add the vacuum condensate $\langle \sum_x \text{Tr} M^{-1}(x, t; x, t) \rangle$ to $R^{dis}$ above. For the up and down quark matrix elements we compute the sum of connected and disconnected terms while only $R^{dis}$ contributes to $\Delta s$ and $\sigma_s$.

The disconnected contribution is computed with the methods described in [18, 9, 10] where we fix $t = 4a \approx 0.29$ fm and vary $t_f$. Employing optimized sink and source smearing we find the asymptotic limit to be effectively reached for $t_f \geq 5a$ and fit the ratios to a constant for $t_f \geq 6a \approx 0.44$ fm, see Fig. 1 for an example.

3 The quark contributions to the proton spin

Non-singlet axial currents renormalize with a renormalization factor $Z_{A}^{\alpha}(a)$ that only depends on the lattice spacing. This was determined non-perturbatively for the action and lattice spacing in use [19].
Figure 2: Volume and (light) valence quark mass dependence of the unrenormalized $\Delta s^{\text{lat}}$.

Table 1: The connected and disconnected contributions to $\Delta q^{\text{lat}}$ of the proton on the $40^364$ volume ($L \approx 2.91$ fm) as well as the renormalized spin content at a scale $\mu \approx \sqrt{7.4}$ GeV.

| $q$ | $\Delta q_{\text{con}}^{\text{lat}}$ | $\Delta q_{\text{dis}}^{\text{lat}}$ | $\Delta q_{\text{MS}}^{\text{MS}}(\mu)$ |
|-----|----------------------------------|----------------------------------|---------------------------------|
| $u$ | 1.071(15)                        | -0.049(17)                       | 0.787(18)(2)                   |
| $d$ | -0.369(9)                        | -0.049(17)                       | -0.319(15)(2)                  |
| $s$ | 0                                | -0.027(12)                       | -0.020(10)(2)                  |

$Z_A^{n^s} = 0.76485(64)(73)$. However, due to the axial anomaly, the renormalization constant of singlet currents, $Z_A^s(\mu, a)$, acquires an anomalous dimension [20] and will depend on the scheme and scale used in the continuum. This has been determined perturbatively [21] and the result for the conversion to the $\overline{\text{MS}}$ scheme reads

$$z(\mu, a) := Z_A^s(\mu, a) - Z_A^{n^s}(a) = C_F n_F \left[15.8380(8) - 6 \ln(a^2 \mu^2)\right] \left(\frac{\alpha_s}{4\pi}\right)^2 + \mathcal{O}(\alpha_s^3).$$  \hspace{1cm} (7)

We extract an improved coupling from the measured plaquette, set $\mu = a^{-1}$ and allow for a 50 % systematic error on $z(\sqrt{7.4}$ GeV) = 0.0055(1)(27). As can be seen from the small anomalous dimension, the scale dependence of $z(\mu)$ is quite mild. Perturbative $\mathcal{O}(a)$ improvement is implemented [22] to $\mathcal{O}(\alpha)$, where again we allow for a 50 % systematic error.

In the $n_F = 1 + 1 + 1$ theory the matrix elements renormalize as follows.

$$\Delta \Sigma^{\overline{\text{MS}}}(\mu) = (\Delta u + \Delta d + \Delta s)^{\overline{\text{MS}}}(\mu) = Z_A^s(\mu, a)(\Delta u + \Delta d + \Delta s)^{\text{lat}}(a),$$  \hspace{1cm} (8)

$$a_8 = \Delta T_8 = (\Delta u + \Delta d - 2\Delta s)^{\overline{\text{MS}}} = Z_A^{n^s}(a)(\Delta u + \Delta d - 2\Delta s)^{\text{lat}}(a),$$  \hspace{1cm} (9)

$$g_A = \Delta T_3 = (\Delta u - \Delta d)^{\overline{\text{MS}}} = Z_A^{n^s}(a)(\Delta u - \Delta d)^{\text{lat}}(a).$$  \hspace{1cm} (10)

We employ $n_F = 2$ mass-degenerate sea quarks so that our singlet current is $\Delta u + \Delta d$ instead. This modifies the renormalization pattern [10]

$$\begin{pmatrix} \Delta u(\mu) \\ \Delta d(\mu) \\ \Delta s(\mu) \end{pmatrix}^{\overline{\text{MS}}} = \begin{pmatrix} Z_A^{n^s}(a) + \frac{z(\mu,a)}{2} & \frac{z(\mu,a)}{2} & 0 \\ \frac{z(\mu,a)}{2} & Z_A^{n^s}(a) + \frac{z(\mu,a)}{2} & 0 \\ 0 & 0 & Z_A^{n^s}(a) \end{pmatrix} \begin{pmatrix} \Delta u(a) \\ \Delta d(a) \\ \Delta s(a) \end{pmatrix}^{\text{lat}},$$  \hspace{1cm} (11)
where $\Delta s^{MS}$ receives light quark contributions but the $\Delta u^{MS}$ and $\Delta d^{MS}$ remain unaffected by the (quenched) strange quark. We remark that unitarity is violated, due to the partial quenching.

In Fig. 2 we display the volume and (light) valence quark mass dependence of our unrenormalized $\Delta s^{\text{lat.}}$. There are no significant finite size effects and we take the independence on the valence quark mass as an indication that our result may also approximately apply to physical light quark masses. We display our $O(a)$ improved results with statistical and systematic errors in Table 1. The $\Delta u^{MS}$ and $\Delta d^{MS}$ values are reduced by about 0.035, due to the sea quark contributions while $\Delta s^{MS}$ increases by 0.002 (< 10%), due to the mixing with light quark flavours.

Note that we find $g_A \approx 1.11(2)$. This underestimation of the value $g_A = 1.267(4)$ from neutron $\beta$-decays can probably be explained by our twice as heavy as physical pseudoscalar meson. Our main finding is a small negative $\Delta s^{MS}(\sqrt{7.4} \text{GeV}) = -0.020(11)$ that is unlikely to decrease significantly if the sea quark mass is reduced: the mixing effects on the renormalization are small, in spite of the comparatively large $\Delta u$ and $\Delta d$ values, and so is the dependence on the light valence quark mass, see Fig. 2.

### 4 The light and strange $\sigma$-terms

In massless schemes that preserve the chiral symmetry the combinations $m_q \bar{q}q$ are invariant under scale transformations (up to lattice artefacts). This operator identity holds for expectation values, independent of the external state. The Wilson action however explicitly breaks the chiral symmetry so that singlet and non-singlet flavour combinations will renormalize differently. It turns out that at our lattice spacing the ratio between the two renormalization factors deviates from unity by as much as 40%. Consequently, the renormalized strangeness matrix element receives large subtractions from light quark contributions. For details, see [9] and references therein. Moreover, without considering mixing with the gluonic operator $aGG$ (that we have not determined), we are not able to implement $O(a)$ improvement. Note that due to the trace anomaly of the energy-momentum tensor such gluonic operators also become relevant for heavy quark masses in the continuum theory [2]. We plan to take this effect into account in the future.

Again we find no significant finite size effects and obtain $\sigma_{PSN}(m_{PS} \approx 285 \text{MeV}) = 106(11)(3) \text{MeV}$. Recently, the ETM Collaboration [11] reported the value $\sigma_{PSN}(m_{PS} \approx 380 \text{MeV}) = 150(1)(10) \text{MeV}$, using a different lattice action. If we assume the leading order chiral behaviour $dm_N/dm_{PS}^2 = \text{const}$ then Eq. (11) suggests to rescale the results according to the ratio of the squared pseudoscalar masses $\approx 1.78$. However, 1.42(19) < 1.78: higher order chiral corrections are relevant. Using additional nucleon mass data we extrapolate our value to the physical point [17] and obtain [9]

$$\sigma_{\pi N}^{\text{phys}} = (38 \pm 12) \text{MeV}, \quad (12)$$

where the dominant error is from the chiral extrapolation, see Fig. 3.

We are also able to compute the strangeness and gluon contributions to the nucleon mass, see Eqs. (1) and (3):

$$f_{Ts} = 0.012(14)^{+10}_{-3}, \quad f_{Tg} = 0.951^{+20}_{-27}. \quad (13)$$

The light and strange quarks contribute a fraction between 3% and 8% to the nucleon mass. The large uncertainty on $f_{Ts}$ or, equivalently, $\sigma_s = 12^{+23}_{-16}$ MeV is due to large cancellations in the renormalization. To reduce these, with Wilson Fermions, one will need to simulate at finer lattice spacings where differences between singlet and non-singlet renormalization constants become smaller. We are able to state a 95% confidence level upper limit on the phenomenologically relevant $y$-ratio: $y < 0.14$. 

5
Figure 3: Extrapolation of $\sigma_{\text{PSN}}/m_{\text{PS}}^2$ to the physical point $[17]$ using covariant BχPT for the 40$^3$64 volume (solid symbol). The broad error band is obtained from nucleon mass data alone. The horizontal line is the leading order expectation and the open symbol our result for the 32$^3$64 volume.

5 Conclusion

We calculated disconnected contributions to the proton structure. We find a pion-nucleon $\sigma$-term $\sigma_{\pi N} = 38(12)$ MeV at the physical point and an upper limit on the ratio of scalar strangeness over light quark content of $y < 0.14$. The renormalized light sea quark contributions amount to less than 10% of the total matrix elements, both for the spin content and for the $\sigma$-term. However, at our lattice spacing, prior to the renormalization, these account for 30% of the bare scalar lattice matrix elements $[9]$ and need to be taken into account for Wilson Fermions.

Our small negative value $\Delta s^{MS}(\sqrt{7.4}\text{GeV}) = -0.020(11)$ indicates stronger than naïvely expected violations of SU(3)$_F$ symmetry in weak decays. This impacts on determinations of polarized parton distribution functions $[23, 7, 8]$ where the constraint on the integrals by the (assumed) proton tensor charge value $a_s = 3F - D$ should probably be relaxed. A small value of $\Delta s$ was also reported in $[13]$, albeit without renormalization. In view of these findings, lattice studies of the spin content of hyperons and of their weak transition matrix elements seem particularly interesting.

Acknowledgements

This work was supported by the European Union (grant 238353, ITN STRONGnet) and by the DFG SFB/Transregio 55. S.C. is supported by the Claussen-Simon-Foundation (Stifterverband für die Deutsche Wissenschaft), A.St. by the EU IRG grant 256594 and J.Z. by the Australian Research Council grant FT100100005. Computations were performed on the SFB/TR55 QPACE supercomputers, the BlueGene/P (JuGene) and the Nehalem cluster (JuRoPA) of the JSC (Jülich), the IBM BlueGene/L at the EPCC (Edinburgh), the SGI Altix ICE machines at HLRN (Berlin/Hannover) and Regensburg’s Athene HPC cluster. The Chroma software suite $[24]$ was used extensively in this work.
References

[1] J. Ellis, K. A. Olive and P. Sandick, New J. Phys. 11 (2009) 105015 [arXiv:0905.0107 [hep-ph]].
[2] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Phys. Lett. B 78 (1978) 443.
[3] J. Gasser, H. Leutwyler and M. E. Sainio, Phys. Lett. B 253 (1991) 252.
[4] M. M. Pavan, I. I. Strakovsky, R. L. Workman and R. A. Arndt, PiN Newslett. 16 (2002) 110 [arXiv:hep-ph/0111066].
[5] J. M. Alarcón, J. Martin Camalich and J. A. Oller, preprints arXiv:1110.3797 [hep-ph]; arXiv:1111.4934 [hep-ph].
[6] M. G. Alekseev et al. [COMPASS Collaboration], Phys. Lett. B 690 (2010) 466 [arXiv:1001.4654 [hep-ex]]; Phys. Lett. B 693 (2010) 227 [arXiv:1007.4061 [hep-ex]].
[7] E. Leader, A. V. Sidorov and D. B. Stamenov, Phys. Rev. D 82 (2010) 114018 [arXiv:1010.0574 [hep-ph]].
[8] D. de Florian, R. Sassot, M. Stratmann and W. Vogelsang, Phys. Rev. D 80 (2009) 034030 [arXiv:0904.3821 [hep-ph]].
[9] G. S. Bali et al. [QCDSF Collaboration], preprint arXiv:1111.1600 [hep-lat].
[10] S. Collins et al. [QCDSF Collaboration], PoS LATTICE 2010 (2010) 134 [arXiv:1011.2194 [hep-lat]] and in preparation.
[11] S. Dinter, V. Drach and K. Jansen [ETM Collaboration], preprint arXiv:1111.5426 [hep-lat].
[12] C. Alexandrou, K. Hadjiyiannakou, G. Koutsou, A. 'OCais and A. Strelchenko, preprint arXiv:1108.2473 [hep-lat].
[13] R. Babich, R. C. Brower, M. A. Clark, G. T. Fleming, J. C. Osborn, C. Rebbi and D. Schaich, preprint arXiv:1012.0562 [hep-lat].
[14] M. Engelhardt, PoS LATTICE 2010 (2010) 137 [arXiv:1011.6058 [hep-lat]].
[15] K. Takeda et al. [JLQCD Collaboration], Phys. Rev. D 83 (2011) 114506 [arXiv:1011.1964 [hep-lat]].
[16] T. Doi, M. Deka, S.-J. Dong, T. Draper, K.-F. Liu, D. Mankame, N. Mathur and T. Streuer, Phys. Rev. D 80 (2009) 094503 [arXiv:0903.3232 [hep-ph]].
[17] A. Sternbeck, G. Schierholz et al. [QCDSF Collaboration], in preparation.
[18] G. S. Bali, S. Collins and A. Schäfer, Comput. Phys. Commun. 181 (2010) 1570 [arXiv:0910.3970 [hep-lat]].
[19] M. Gökceller et al. [QCDSF Collaboration], Phys. Rev. D 82 (2010) 114511 [arXiv:1003.5756 [hep-lat]] and re-analysis by M. Gökceller, private communication.
[20] J. Kodaira, Nucl. Phys. B 165 (1980) 129.
[21] A. Skouroupathis and H. Panagopoulos, Phys. Rev. D 79 (2009) 094508 [arXiv:0811.4264 [hep-lat]].
[22] S. Capitani, M. Gökceller, R. Horsley, H. Perlt, P. E. L. Rakow, G. Schierholz and A. Schiller [QCDSF Collaboration], Nucl. Phys. B 593 (2001) 183 [arXiv:hep-lat/0007004].
[23] R. D. Ball, L. Del Debbio, S. Forte, A. Guffanti, J. I. Latorre, J. Rojo and M. Ubiali [NNPDF Collaboration], Nucl. Phys. B 838 (2010) 136 [arXiv:1002.4407 [hep-ph]].
[24] R. G. Edwards and B. Joó [SciDAC, LHP and UKQCD Collaborations], Nucl. Phys. Proc. Suppl. 140 (2005) 832 [arXiv:hep-lat/0409003]; C. McClendon, Jlab preprint JLAB-THY-01-29 (2001); P. A. Boyle, http://www.ph.ed.ac.uk/~paboyle/bagel/Bagel.html.