Vibration of transformer sheets using non-linear characteristics

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Abstract. Intensive research has been carried out over the years to reduce acoustic noise resulting from the vibration of transformer plates. The noise of transformer sheets excited by electromagnetic field can be mostly attributed to induced forces between the plates. The vibration generated from transformer structures causes abnormal vibration, breakage of the machine, and noise. Vibration measurement and analysis of the structure is important to prevent this phenomenon. This chapter presents the research resulted by measurement and Finite Element Method of the resonance of the transformer sheets.

The goal of this research is to introduce the coupling of the developed nonlinear hysteresis model and the Finite Element Method. The COMSOL and MATLAB based finite element simulation method combines the electromagnetic, structural mechanics and acoustic models. The moving mesh (ALE) geometry of the transformer sheets leads on a valid model and assures the calculations of the electromagnetic properties in different deformed shapes. The measurement of the natural frequencies of the plates yields the relation between the excitation frequency and the resonance of the system.

The goals of the developed model are:

• to derive the magnetic field intensity $H$ and with hysteresis nonlinearity the magnetic flux density $B$ properties inside the transformer sheets with $d = 0.35$ mm and $l = 150$ mm;
• to compute the induced current density $J_i$ inside the plates;
• to calculate the Lorentz force $F$ distribution along the sheets;
• to derive the plane stresses the horizontal and vertical displacements along the sheets are calculated;
• applying the ALE moving mesh the deformation of plates can be simulated.

For modelling the nonlinear characteristic of the magnetic material $M = \mathcal{H}(H)$ a novel developed hysteresis model is used. The advantages of the hysteresis model are its easy identification, past memory representation and numerical simplicity [1]. So, the hysteresis characteristic can be determined as

$$B = \mathcal{H}(H, \partial B/\partial H, \psi_i(H_i, B_i)) .$$

1. The developed scalar hysteresis model
In this section a novel scalar hysteresis model is constructed on the basis of a special modification field, determined from the permeability vectors. The model properties are determined from measured data points. The aim of this research is to develop a new hysteresis model, based on finite number of measured data points and on the introduced special vector field that is identified as modification vector field.
The hysteresis model presented below is introduced by the aid of a modification vector field $\Psi_i$, that can be derived from the permeability vectors belonging to the ascending and the descending measured loops. The vector field $\Psi_i$ contains spatial vector components. Introducing this modification vector field the next point, $y(t_i + \Delta t)$ on the hysteresis curve can easily be determined from the actual value of the input signal $s(t_i)$ and from the differential permeability $\mu_{diff}(s, y)$ as the past local memory representation of the model. So, the hysteresis characterist can be determined as

$$y = \mathcal{H}\{s, \frac{\partial y}{\partial s}, \Psi_i(s, y)\},$$

where $\frac{\partial y}{\partial s} = \mu_{diff}$ is the local differential permeability and a vector component $\Psi_i(s, y)$ of the vector field $\Psi_i$ can be determined from the measured ascending and descending first order reversal curves, where $i$ represents the actual measured point, $s$ is the input and $y$ is the output signal. Without accommodation any measured data points $\rho(s, y)$ on the hysteresis loop can be achieved from the major loop only from the direction, defined by the local differential permeability vector

$$\mu_{diff,i} = \frac{\partial y}{\partial s}\bigg|_{s=s_i, y=y_i},$$

which one represents the memory of the hysteresis in this model, because applying different local differential permeability vectors as the local memory of the hysteresis model, different output can be achieved.

1.1. Identification of the scalar model

Denote $\hat{\mu}_{diff}(s, y) : G \to \mathbb{R}^2 \mathbb{G} \subset \mathbb{R}^2 = \{(s, y), i = 1, \cdots, Z\}$ where $Z$ is the number of measured points and $\rho(s, y)$ represents the measured data. From the measured points of the FORC the local differential permeability vectors $\hat{\mu}_{diff}(s_{xx}, \cdot)$ can be calculated along $s_{xx} = \text{const}$ lines as

$$\hat{\mu}_{diff}(s_{xx}, y) = (\Delta s_{xx}, \Delta y),$$

where $\Delta f_i = f_i - f_{i-1}$ specifies the segment of validity. The $s_{xx}$ means an equidistant inner point along the $s$ axis, so $s_{xx} = k(s_{\text{max}} - s_{\text{min}})/K|k, K \in \mathbb{N}$. If $\hat{\mu}_{diff}(s, y)$ and $\mu_{diff}(d_{i+1}, y_{i+1})$ values are known the relation between the calculated modification vectors $\Psi_i$ and the local differential permeability vectors are $\mu_{i+1} = (\bar{\Psi}_i + \mu_i)/2$, where from the modification vector $\bar{\Psi}_i$ can be determined as

$$\bar{\Psi}_i(H_i, B_i) = (2\Delta H_{i+1} - \Delta H_i, 2\Delta B_{i+1} - \Delta B_i).$$

In case if the measured points $\rho(s, y)$ are equidistant in $s$ then the vector

$$s \in \mathbb{R}, \quad s_k = s_{\text{min}} + k(s_{\text{max}} - s_{\text{min}})/K$$

is defined, where $K$ the number of the equidistant points in $s$. The limiting points are specified as $s_0 = -s_{\text{sat}}, \quad s_K = s_{\text{sat}}$. Let introduce matrix form to denote the measured set of the output signal as

$$y \in \mathbb{R}^2, y_{k,n} = y_i : \exists \rho(s_i, y_i),$$

$$s_i \in \mathbb{s}, \quad k \in [0, K] \subset \mathbb{Z}^+, \quad n \in [1, N] \subset \mathbb{N},$$

where $N$ is the number of the FORC. Applying this nomination the $k$ -th rows in $y$ means the $k$ -th assignment in the matrix form $s$ and the $n$ -th column means the $n$ -th ascending or
descending curve. All modification vectors \( \Psi^*_k \) must also be stored in the same sequence as above. For the reversal curves from descending ones \( s^-, y^- \), \( \Psi^-_k \), to the positive loops \( s^+, y^+ \), \( \Psi^+_k \) can be defined. To the simulation of the hysteresis loop at any \( s_j, y_j \) point any \( s^*_k \in s, \ y^*_k \in y \) measured points can be found as \( |s^*_k - s^*_j| = \min_k, |y^*_k - y^*_j| = \min_n \), and then \( \Psi^*_k(s_k, y_k) \) can be determined for \((k, n)\) pairs with \(* \in [+, -] \). So the sign ‘\( * \)’ has been defined for positive and negative reversal curves

\[
\Psi^*: \mathbb{R}^2 \rightarrow \mathbb{R}^2, \ \Psi^*(s_j, y_j) = \Psi^*_k(s_k, y_k),
\]

\[
|s^*_k - s^*_j| = \min_k, \ |y^*_k - y^*_j| = \min_n.
\]  

The direction of the change in \( s \) can be determined as \( * = \text{sign} (\Delta s_j) \), while \( \Psi^*(s_j, y_j) \) can be interpolated if \( s_j \notin s^* \).

If \( s^* \) contains equidistant elements, \( \Delta s_{i+1} - \Delta s_i = 0 \) , then from the measured values the modification vector at point of \((s_i, y_i)\) is

\[
\Psi^*_k(s_i, y_i) = (\Delta s_{i+1}, 2\Delta y_{i+1} - \Delta y_i).
\]

Calculating the required \( y_i \) from the modification vector field \( \Psi \)

\[
\Delta y_{i+1} = (\Psi^*(s_i, y_i)[2] + \Delta y_i) / 2,
\]

so

\[
y_{i+1} = (\Psi^*(s_i, y_i)[2] + \Delta y_i) / 2 + y_i.
\]

By definition \( \Psi^*(s_i, y_i)[2] \) means the second, \( \Psi^*(s_i, y_i)[1] \) the first scalar component of the modification field vector \( \Psi^*(s_i, y_i) \). The prehistory of the material is represented by the local \( \hat{\mu} \text{diff}(s_i, y_i) \) permeability vector through the tangent of the curve. Figure 1 shows the vector representation of the scalar hysteresis model. The modification field vector \( \Psi^*(H_i, B_i) \) is calculated from \( \hat{\mu} \text{diff}(s_i, y_i) \) and \( \hat{\mu} \text{diff}(s_{i+1}, y_{i+1}) \) vectors according to (11).

![Figure 1. Representation of the model.](image)

1.2. Application of the scalar model

In this section the introduced hysteresis model is applied in two cases. First, if the simulation data set, \( s \) has equidistant difference, coincident with the measured data, and second, if the steps in \( s \) are not equidistant and different from the measured values.
Generally, if the measured points in \( s^* \) have equidistant difference, \( s_j \in s^* \), any \( y_{j+1} \) nonlinear system output value can be calculated from \( y_j \), and from the local measured \( \mu^*_\text{diff} (s_j, y_j) \) and from the determined \( \Psi^* (s_j, y_j) \)

\[
y_{j+1} = (\Psi^* (s_j, y_j) [2] + \Delta y_j) / 2 + y_j
\]

Simulating a hysteresis loop if \( s_j \notin s^* \) and \( \Delta s_j \) is different from the measured \( \Delta s_i \) data then \( \Psi^* (s_j, y_j) [2] \) must be scaled with a scalar as

\[
\eta \Psi^* (s_j, y_j) [2],
\]

\[
y_{j+1} = (\eta \Psi^* (s_j, y_j) [2] + \Delta y_j) / 2 + y_j .
\]

The scalar \( \eta \) can be determined from the simulated \( \Delta s_{j+1}, s_j, y_j \) and from the measured \( \Psi^* (s_j, y_j) [1] \) values as

\[
\eta = \frac{\Delta s_{j+1}}{\Psi^* (s_j, y_j) [1]},
\]

where \( \Psi^* (s_j, y_j) [1] \) can be determined from (8) according to the measured data.

### 1.3. Turning points

In the last section the model has been specified when the local permeability vectors \( \mu^*_\text{diff} (s_j, y_j) \) are known. This should be different when the direction of the excitation signal \( s \) is changing, it means that increasing excitation values turn to decreasing excitation values and opposite. This is the point on the major loop where the reversal curves start and called as turning point. In these points the modification value \( \Psi^* (s_j, y_j) \) is different from the previous vectors, the modification vectors has to be specified at the turning point. Otherwise the modification vectors \( \Psi^* (s_{\text{sat}}, y_{\text{sat}}) \) must be calculated according to (13) and when the excitation is over saturation \( s_{j+1} > s_{\text{sat}} \), the vector field must keep the saturation condition of the hysteresis curve, so the direction of \( \Psi^* (s_{\text{sat}}, y_{\text{sat}}) \) must be the same as the local differential permeability vector. For the exact simulation the turning point has to be specified. If \( s_{j+1} \notin s^* \) and \( s_{j+1} < -s_{\text{max}} \), \( s_{j+1} > s_{\text{max}} \) the hysteresis loop is in saturation condition,

\[
\left. \frac{\partial^2 y}{\partial s^2} \right|_{s=\pm s_{\text{max}}} = 0,
\]

and

\[
\left. \frac{\partial y}{\partial s} \right|_{s=\pm s_{\text{max}}} = \mu_0 .
\]
the hysteresis loop \( s_j, y_j \) must be inside the ascending and descending major loops. Otherwise when \( s_j \notin s^*_\ast \) and \( s_j < -s_{\text{max}} \) or \( s_j > s_{\text{max}} \) these turning point conditions ensure that the model remain in between saturation condition [1].

\[ \begin{array}{c|c|c|c|c|c}
 s(t)/s_{\text{max}} & y(t)/y_{\text{max}} \\
 \hline
 -1.1 & -0.89 & -0.67 & -0.45 & -0.23 & -0.016 \\
 0.2 & 0.42 & 0.64 & 0.86 & 1.1 \\
 \end{array} \]

Figure 2. Modification vectors \( \Psi^\ast(s^\ast, y^\ast) \) on the major loop.

Figure 3. Turning points of \( \Psi^\ast(s^\ast, y^\ast) \) vector fields.

After the \( \Psi^\ast \) modification vector fields are derived from the ascending and descending curves for the resulted simulation the accuracy of the model is investigated. Figure 4 shows the simulated and measured data points under decreasing sinusoidal excitation process. It can be seen that the simulated results and the measurement points are very close. The measured values are for checking the simulation validity. The accommodation property of the hysteresis curve can be ensued from the property of the real material and the construction philosophy of the simulation based on measured data, as it can be seen in figure 5.

\[ \begin{array}{c|c|c|c|c|c}
 s(t)/s_{\text{max}} & y(t)/y_{\text{max}} \\
 \hline
 -1.1 & -0.75 & -0.39 & -0.034 & 0.32 & 0.68 \\
 0.2 & 0.43 & 0.64 & 0.86 & 1.1 \\
 \end{array} \]

Figure 4. Simulation result and measurement data.

Figure 5. Representation of the accommodation property.
2. Simulation of the transformer sheets

Electrical machines and transformers have a laminated core from ferromagnetic materials. A part of the vibration and noise arising from the magnetic core that caused by the magnetic forces and magnetostriction. The magnetostriction can be defined as the deformation of the ferromagnetic material in the presence of magnetic field. The magnetostrictive behaviour is so complex and varies for different materials. It also depends on the the applied magnetic field.

The unwanted noise and the vibrations in electrical machines and transformers in mainly classified into three groups: mechanical noise, aerodynamic noise and magnetic noise. The laminated core of the transformers is a stack of laminations of ferromagnetic material. Due to their high permeability these materials can produce high magnetic induction with weak magnetic fields.

The magnetic noise of the transformers is caused by magnetic forces and magnetostriction. The magnetic forces are tending to deform the geometry of the magnetic core. Combined with periodic magnetic field excitation it leads to vibration.

This is widely accepted that vibration and the sound from transformer come from the following items [3]:

- Magnetostriction of the core. It means that the specimen extends along the magnetic flux density $B$ direction but shorten toward that direction. The magnetostriction has direct and close connection with the magnetic flux density $B$ and the winding voltage;
- Electrodynamic of the specimen and the windings due to leak magnetic flux. This is an important property but negligible for little dry-type transformers that for the measurement were used;
- Oil pumps and transformer cooling fan vibration. The frequency range of those cooling equipments are usually below than 100 Hz, that is why those frequencies easily can be isolated from other vibration signals. In the measurement it was not needed to use cooling equipment that is why we can ignore those vibration sources.

Figure 6 shows the representation of the computational problem. Applying the above field equations to the two infinite large plates in $z$-direction with sinusoidal excitation of the magnetic field is investigated. The deformation of $H_y$, $M_y$, $B_y$ waves can be calculated along direction $z$. The magnetic forces $F'$ and $F''$ acting on the plates separately are induced along in direction $z$. In the cross-section of the plates the current density vectors $J'$ and $J''$ are inside of the material.

![Figure 6. The schematic of the infinite plates.](image)

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2.1. Equations of the electromagnetic field

The well-known Maxwell’s equation system defines relations between electromagnetic field quantities, such as electric field intensity $E$, magnetic field intensity $H$, electric flux density $D$ and the magnetic flux density $B$. The electric current density $J$ and the electric charge density $\rho$ are the source of the electromagnetic field.

In the studied eddy current field problem the electric and magnetic field are coupled, so the field quantities are depending on the time variation. So it is supposed that $\frac{\partial}{\partial t} \neq 0$. In the eddy-current field the region $\Omega_c$ of the ferromagnetic specimen with nonlinear hysteresis characteristic is surrounded by the nonmagnetic and non-conducting region (e.g. air) $\Omega_n$. The equations are discussed in eddy-current and eddy-current free regions [4],[5],[6],[7],[8].

The classical differential form of the Maxwell’s equations are the following in the eddy-current free region $\Omega_n$:

$$\nabla \times H = J_0 \text{ in } \Omega_n,$$

(17)

$$\nabla \cdot B = 0 \text{ in } \Omega_n,$$

(18)

where $J_0$ is the source current density of the excitation coils.

In the eddy current region $\Omega_c$ the Maxwell’s equations can be written as:

$$\nabla \times H = J \text{ in } \Omega_c,$$

(19)

$$\nabla \times E = -\frac{\partial B}{\partial t} \text{ in } \Omega_c,$$

(20)

$$\nabla \cdot B = 0 \text{ in } \Omega_c,$$

(21)

$$\nabla \cdot J = 0 \text{ in } \Omega_c,$$

(22)

where $J$ is the eddy current density.

2.2. The constitutive relations

The constitutive relations are the follows:

$$B = \begin{cases} \mu_0 H & \text{in air } \Omega_n, \\ \mu_0\mu_r H & \text{in magnetically linear material } \Omega_c, \\ \mathcal{H}\{H\} & \text{in magnetically non-linear material } \Omega_c, \end{cases}$$

(23)

and

$$J = \sigma E \text{ in air } \Omega_c.$$

(24)

$$H = \begin{cases} \nu_0 B & \text{in air } \Omega_n, \\ \nu_0\nu_r B & \text{in magnetically linear material } \Omega_c, \\ \mathcal{H}^{-1}\{B\} & \text{in magnetically non-linear material } \Omega_c. \end{cases}$$

(25)

The inverse form of the relation (24) is the following:

$$E = \rho J \text{ in air } \Omega_c.$$

(26)

Here $\mu$ is the permeability, $\nu$ is the reluctivity, $\sigma = 7.47 \cdot 10^5$ Sm$^{-1}$ is the conductivity and $\rho$ is the resistivity of the material, $\mu_0 = 4\pi10^{-7}$ H/m is the permeability of vacuum [2], [8].
2.3. The boundaries of the regions

The two regions \( \Omega_c \) and \( \Omega_n \) are coupled at the interface \( \Gamma_{nc} \). The \( \Gamma_B \) signs the closing boundary of the eddy current free region, where the normal component of the magnetic flux density \( b \) is disappearing.

These conditions can be formulated as

\[
B \cdot n = -b \quad \text{or} \quad B \cdot n = 0 \quad \text{on} \quad \Gamma_B,
\]

(27)

where \( n \) is the outer normal unit vector of the region. The interface between the conducting and non-conducting regions \( \Gamma_{nc} \) can be described by the following terms

\[
H_c \times n_c + H_n \times n_n = 0 \quad \text{on} \quad \Gamma_{nc},
\]

(28)

and

\[
B_c \times n_c + B_n \times n_n = 0 \quad \text{on} \quad \Gamma_{nc},
\]

(29)

and

\[
J \times n_c = 0 \quad \text{on} \quad \Gamma_{nc},
\]

(30)

where \( n_c, n_n, H_c, H_n, B_c, B_N \) are the outer normal unit vector of the conducting and non-conducting regions. The magnetic field intensity and the magnetic flux density vectors in the corresponding region on the boundary, where \( n_n = -n_c \) along \( \Gamma_{nc} \).

Furthermore the initial conditions of the magnetic field must be initialized as

\[
B_n(t = 0) = B_{n,0} \quad \text{on} \quad \Omega_N \quad \text{and} \quad B_c(t = 0) = B_{c,0} \quad \text{on} \quad \Omega_c.
\]

(31)

To solve an electromagnetic field problem it is usual that the partial differential equation system of the studied phenomenon is simplified to a potential formulation. This section shows the determination of the potential formulations. In the investigated electromagnetic field problem the specimens carrying the eddy currents and surrounded by a non-conducting medium which is free of eddy currents. The solutions of the Maxwell’s equations are handled by scalar and vector potentials \([4],[5],[6],[7],[8],[9]\).

2.4. Eddy current magnetic fields

As it was written the eddy current field is defined by Maxwell’s equations (19),(20),(21), constitutive relations (23),(24) and inverse relations (25),(26). In this work the eddy current field is described by magnetic vector potential \( A \) that can be coupled with an electric scalar potential denoted by \( V \) \([4],[5],[6],[7],[8],[9]\).

In the eddy current region the field quantities are depending on the time variation, so \( \partial / \partial t \neq 0 \).

The magnetic flux density vector \( B \) can be described by the curl of the magnetic vector potential \( A \) as:

\[
B = \nabla \times A.
\]

(32)

Basically the vibration of the transformer sheets is a two dimensional problem the electric scalar potential can be selected as \( V = 0 \) in the \( A, V \) — potential formulation. In this case the electric field intensity is the following in two dimensional case:

\[
E = -\frac{\partial A}{\partial t}.
\]

(33)
Substituting (32) and (33) into (19) and using the constitutive relations (26) and (25) the partial differential equation of the $A, V$ – potential formulation in two dimensional case is the following:

$$\nabla \times (\nu \nabla \times A) + \sigma \frac{\partial A}{\partial t} = 0 \text{ in } \Omega_c.$$  \hfill (34)

2.5. Coupling static magnetic field and eddy current field formulations

In the presented problem the specimens carrying the eddy currents and surrounded by non-conducting medium where a static magnetic field is present (figure 7). The potential formulations of the static and eddy current fields must be coupled because the static field is induced by the eddy currents and source currents of coils.

![Figure 7. Representation of the eddy current field problem.](image)

In this work the static magnetic field $\Gamma_n$ is described by magnetic vector potential $A$, this is a more usual way than applying reduced magnetic scalar potential $\Phi$.

The eddy current field in $\Gamma_c$ the magnetic vector potential $A$ is coupled with electric scalar potential $V$, so it called $A, V – A$ formulation.

The magnetic vector potential $A$ is used in this formulation in all of the the regions $\Omega_c$ and $\Omega_n$. The electric scalar potential $V$ is used only in the region $\Omega_c$. The interface conditions are defined as well.

The summarized equations are the following in two dimensional assignment:

$$\nabla \times (\nu_0 \nabla \times A) + \sigma \frac{\partial A}{\partial t} = -\nabla \times I \text{ in } \Omega_c,$$  \hfill (35)

$$\nabla \times (\nu \nabla \times A) = J_0 \text{ in } \Omega_n,$$  \hfill (36)

$$\mathbf{n}_c \times A + \mathbf{n}_n \times A = 0 \text{ on } \Gamma_{nc},$$  \hfill (37)

$$(\nu_0 \nabla \times A + I) \times \mathbf{n}_c + (\nu \nabla \times A) \times \mathbf{n}_n = 0 \text{ on } \Gamma_{nc}.$$  \hfill (38)
2.6. The weak formulation

The finite element simulation is calculated by using the weak form of the presented potential formulations. The weak form of a partial differential equation is obtained by using the weighted residual method. The Galerkins method is a particular form of residual method and it is widely used in electromagnetism. The weighted residual method is applied to minimize the residual of a partial differential equation and based on the inner product of the partial differential equation and a weighting function \( W \).

The weak formulations of \( A \), \( V - A \) formulation are based on the partial differential equations (35) and (36) and on the interface condition (37):

\[
\int_{\Omega_c} W_k \cdot \left[ \nabla \times (\nu_0 \nabla \times A) \right] d\Omega \\
+ \int_{\Omega_c} W_k \cdot \left( \sigma \frac{\partial A}{\partial t} \right) d\Omega \\
+ \int_{\Omega_n} W_k \cdot \left[ \nabla \times (\nu \nabla \times A) \right] d\Omega \\
+ \int_{\Gamma_{nc}} W_k \cdot \left[ (\nu_0 \nabla \times A + I) \times n_c + (\nu \nabla \times A) \times n_n \right] d\Gamma \\
= \int_{\Omega_n} W_k \cdot J_0 d\Omega - \int_{\Omega_c} W_k \cdot (\nabla \times I) d\Omega. \tag{39}
\]

2.7. Numerical modelling of electromagnetic field computation

The analysis of electromagnetic field problems requires a solid technique for the treatment of the nonlinearity. The system of nonlinear equations is solved by fixed point (FP) iteration technique that shows great advantages with respect to other methods.

In principle the polarization method that is used to solve the nonlinear equations. It based on the decomposition of the nonlinear system into a linear and a nonlinear part. The decomposition of the hysteretic relationship \( B = \mathcal{H}\{H\} \) is linking the magnetic field intensity \( H \) and the magnetic flux density \( B \) into a liner term and a residual nonlinearity \( I \) as

\[
H = \nu_{FP} B + I, \tag{40}
\]

where the residual \( I \) is computed by iteratively and starting from any trial value. The constant \( \nu_{FP} = 1/\mu_{FP} \) controls the convergence of the process. The \( \mu_{FP} \) can be estimated from the minimum and maximum slopes of the hysteresis curve as:

\[
\mu_{FP} = \frac{\mu_{\text{max}} + \mu_{\text{min}}}{2}. \tag{41}
\]

This scheme requires that the field computation gives as the magnetic flux density \( B \) as result. In this computation the magnetic vector potential \( B = \nabla \times A \) was used, where from the magnetic flux density \( B \) can be calculated and involves the magnetic field intensity as unknown.

The new residual \( I \) is computed making use the direct relationship between \( H \) and \( B \) as

\[
I = H - \nu_{FP} B = H - \nu_{FP} \mathcal{H}\{H\}. \tag{42}
\]

The nonlinear iteration steps can be summarized as follows:

(i) the iteration is started by an arbitrary value of \( I^{(0)} \);
(ii) in the \( n^{th} \) iteration step (\( n > 0 \)) the magnetic flux density \( B^{(n)} \) is calculated from the Maxwell’s equations using \( I^{(n)} \);
(iii) the magnetic field intensity $H$ can be calculated as

$$H^{(n+1)} = \nu_{FP} B^{(n)} + I^{(n)}; \quad (43)$$

(iv) the nonlinear term must be updated by using the magnetic field intensity and the direct hysteresis model $B = \mathcal{H}\{H\}$ as

$$I^{(n+1)} = H^{(n+1)} - \nu_{FP} \mathcal{H}\{H^{(n+1)}\}; \quad (44)$$

(v) steps (1)-(4) must be repeated until the convergence that criterion can be defined as

$$\| I^{(n+1)} - I^{(n)} \| < \varepsilon, \quad (45)$$

where $\varepsilon$ is a small positive number. In this work $\varepsilon = 10^{-4}$ has been chosen.

2.8. Solution of the problem

The electromagnetic problem has been solved by finite element method. The $A$, $V - A$ potential formulations are used for the simulation in the time domain. The commercial finite element code, the COMSOL Multiphysics was used to building up the finite element model. The COMSOL Multiphysics package is a finite element analysis and solver package for various physics and engineering applications. It has an extensive and well-managed interface to MATLAB where the programming and the postprocessing calculations were done. The COMSOL Multiphysics also allows to couple different systems of partial differential equations using the weak form. It has many modules which from the AC/DC Module, Structural Mechanics Module, Moving Mesh module. The AC/DC module is used to simulate the model in static analysis type, when $\partial / \partial t = 0$. The transient model, when $\partial / \partial t \neq 0$ has used the static model and computed in MATLAB using the fixed point technique. The finite element mesh contains 75088 elements and 37655 mesh points, number of boundary elements is 3232, the number of degrees of freedom is 482867.

The simulation result using the polarization method can be seen in figure 8. The results corresponds to different time points applying $f_e = 50$ Hz excitation frequency. The changes of the Lorenz force in time leads to the vibration of the plates. The calculated displacement of the upper transformer sheet can be seen in figure 9. It can be seen that the maximum distance of the displacement is $6 \cdot 10^{-9}$ m.

3. Conclusion and future work

In this paper the calculation method of the transformer sheet vibration was discussed. Applying the developed hysteresis model and the moving mesh property in the finite element calculation it leads reliable results in the electromagnetic field calculation.

The postprocessing of the nonlinear field problem has been yielded to the calculation of the magnetic force between the plates. It is planned as a future goal that the simulated vibration results are compared with and without the moving mesh property. The difference between the two calculation methods verifies the necessity to taking into account the deformation of the sheets in the electromagnetic FEM calculations.

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Figure 8. Lorenz force distribution of the upper transformer sheet applying moving mesh (ALE) geometry at different times.

Figure 9. Displacement of the upper transformer sheet.

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