Instanton interpolating current for $\sigma$–tetraquark

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Abstract

We perform a QCD sum rule analysis for the light scalar meson $\sigma$ ($f_0(600)$) with a tetraquark current related to the instanton picture for QCD vacuum. We demonstrate that instanton current, including equal weights of scalar and pseudoscalar diquark-antidiquarks, leads to a strong cancelation between the contributions of high dimension operators in the operator product expansion (OPE). Furthermore, in the case of this current direct instanton contributions do not spoil the sum rules. Our calculation, obtained from the OPE up to dimension 10 operators, gives the mass of $\sigma$–meson around 780MeV.

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1 Introduction

Nowadays there is a lot of controversy in the interpretation of the scalar mesons with the masses below 1 GeV [1]. In the constituent quark models, they are expected to have the quark content of $q\bar{q}$ as the normal members of flavor $SU(3)_{f}$ nonet with one unit of orbital excitation for positive parity. However, from the fact that the orbital excitation gains energy about 0.5 GeV, it is difficult to explain their light masses as well as their mass spectrum (so called inverted mass spectrum) [2]. Additionally, the two candidates to the members of nonet, the isovector $a_{0}(980)$ and isoscalar $f_{0}(980)$, have a very peculiar properties. Indeed, their masses are degenerated and they have strong coupling to $K\bar{K}$ channel in strong contradiction with expectation of simple $q\bar{q}$ picture of the mesons. This puzzle stimulated alternative interpretations of these mesons as various types of tetraquark states [3], e.g., meson–meson molecule states [4], diquark–antidiquark bound states [2, 3, 6, 7, 8], and as some hybrid states of mixture of $q\bar{q}$ and meson–meson [9] and of diquark–antidiquark and meson–meson [10].

In this paper, we will consider the properties of the lightest scalar meson state $\sigma (f_{0}(600))$ as a diquark–antidiquark bound state within the QCD sum rule [11] with the special choice of the tetraquark current. We should mention that the QCD sum rules (SR) for the light scalar mesons were already considered separately with the interpolating currents of the scalar diquark–antidiquark and of the pseudoscalar diquark–antidiquark. In these calculations only contributions to OPE up to the operators of dimension 6 have been considered [7, 8]. However, it was recently shown in [12, 13, 14] that SR for multi-quark systems might receive a large contributions from the operators of higher dimensions which can lead to strong instability of the obtained results for some types of interpolating currents. In particular, it has been demonstrated that in the case of the scalar diquark–scalar antidiquark interpolating current for light tetraquarks, the contribution of the operators of dimensions 8 violates the requirement of positivity of left-hand side (LHS) and leads to disappearance of the bound state tetraquark signal [13].

In this Letter we suggest to use special type of interpolating tetraquark current for lightest scalar meson $\sigma$ which leads simultaneously to the cancelation of high dimensional condensate contributions to the OPE and some dangerous instanton contribution to SR. The $\sigma$–state has the vacuum quantum numbers and should couple to the QCD vacuum very strongly. Instantons, topological fluctuations of gluon fields, play very important role in structure of QCD vacuum [15] and in spectroscopy of the multiquark hadrons [16, 17, 18]. Therefore, our basic idea is to use the color–spin–flavor structure of the four–quark interaction induced by instantons [19] to fix the possible “good” interpolating current for $\sigma$–state. This specific interaction gives strong correlations between scalar diquark of $3_{c}$ and scalar antidiquark of $3_{c}$, between pseudoscalar diquark of $3_{c}$ and pseudoscalar antidiquark of $3_{c}$, and between tensor diquark of $6_{c}$ and tensor antidiquark of $6_{c}$, where the subscript $c$ means color. With this inspection, we propose that the interpolating current for $\sigma(600)$ consists of the above three types of diquark–antidiquark combinations. Our additional argument in favor of instanton current is based on the phenomena of the cancelation of high dimension operator contributions expected for the case of the OPE in the self–dual vacuum fields [20]. Instanton field is a self–dual field and this self–duality property manifests directly in the color–spin–flavor structure of the four–quarks instanton interaction. Therefore, we can expect a similar cancelation for the case of tetraquark current obtained from the quark–quark instanton induced interaction.
We construct the QCD SR with the OPE up to operators of dimension 10 and show that for the case of instanton current with equal weights for the scalar diquark–antidiquark and the pseudoscalar diquark–antidiquark, the cancelation takes place separately for the high dimension operator contributions and for some dangerous instanton contributions to the SR. Our results for the left–hand side (LHS) of the SR are very stable. For the phenomenological, right–hand side (RHS) of the SR, we apply the two resonance approximation for the spectral representation, which allows to avoid the well known problem of strong dependence of the results for multiquark systems on the value of threshold \[12, 21\].

This paper is organized as follows. In Sec. II, using the instanton induced quark–quark interaction, we fix the general structure of interpolating current for \(\sigma\)–tetraquark. In Sec. III we construct the standard QCD sum rule for \(\sigma\) based on the OPE. In Sec. IV the direct instanton effects to the sum rule are considered. We present the numerical results in Sec. V and discuss them in the Conclusion.

2 The instanton current for \(\sigma\)-tetraquark

The famous ’t Hooft instanton induced interaction between light quarks \[22\] for case of \(N_f = 2\) can be written in following form \[19\]

\[
L = \frac{G}{4(N_c^2 - 1)} \left[ \frac{2N_c - 1}{2N_c} \left( (\bar{\psi}\tau_\mu^- \psi)^2 + (\bar{\psi}\gamma_\tau_\mu^- \psi)^2 \right) + \frac{1}{4N_c} (\bar{\psi}\sigma_{\rho\sigma} \tau_\mu^- \psi)^2 \right]
\]  

where \(\psi\) is two flavors spinor, \(N_c\) is the number of colors and \(\tau_\mu^- = (\bar{\tau}, i)\). This Lagrangian can be transformed to the Lagrangian for the interactions between diquark and antidiquark by a Fierz transformations in the spin, flavor and color spaces:

\[
\mathcal{L} = \frac{-G}{8N_c(N_c - 1)} \left[ (\psi^T C \gamma^5 \tau_2 \lambda^A \psi)(\bar{\psi}\tau_2 \lambda^A C \bar{\psi})^T + (\psi^T \tau_2 \lambda^A C \psi)(\bar{\psi}\tau_2 \lambda^A C \bar{\psi})^T \right] + \frac{G}{16N_c(N_c + 1)} (\psi^T \tau_2 \lambda^S C \sigma_{\rho\sigma} \psi)(\bar{\psi}\tau_2 \lambda^S \sigma_{\rho\sigma} C \bar{\psi})^T ,
\]

where \(\lambda^{A,S}\) are the antisymmetric and symmetric color generators normalized as \(\text{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}\), respectively. The first two terms correspond to the scalar and the pseudoscalar diquarks in antisymmetric \(\bar{3}_c\) representation and the last one to the tensor diquark in symmetric \(\bar{6}_c\) color state.

By introducing the spin matrices,

\[
\Gamma_S = C\gamma^5, \quad \Gamma_{PS} = C, \quad \Gamma_{T,\rho\sigma} = C\sigma_{\rho\sigma},
\]

we can rewrite the Lagrangian in terms of flavors

\[
\mathcal{L} = \frac{G}{2N_c(N_c - 1)} \epsilon_{abc} \epsilon_{ade} \left[ (u_b^T \Gamma_S d_c)(\bar{u}_d \Gamma_S \bar{d}_e^T) - (u_b^T \Gamma_{PS} d_c)(\bar{u}_d \Gamma_{PS} \bar{d}_e^T) \right] + \frac{G}{4N_c(N_c + 1)} (u_b^T \Gamma_{T,\rho\sigma} d_c \Gamma_S)(\bar{u}_d \Gamma_{T,\rho\sigma} \bar{d}_e^T + (\bar{u}_d \Gamma_{T,\rho\sigma} \bar{d}_e^T),
\]

where \(\Gamma_i = \gamma^0\Gamma_i^0\gamma^0\) and the properties

\[
\Gamma_S = -C\gamma^5, \quad \Gamma_{PS} = C, \quad \Gamma_{T,\rho\sigma} = -\sigma_{\rho\sigma} C,
\]
have been used. One can see that only a very restricted set of diquarks can strongly interact with instanton field, namely scalar, pseudoscalar and tensor diquarks.

Therefore we suggest the following interpolating current for the $\sigma$–meson:

$$J_\sigma = \alpha J^S_\sigma + \beta J^P_{PS} + \gamma J^T_\sigma$$

(6)

where each current is defined by

$$J^S_\sigma = \epsilon_{abc} \epsilon_{ade} (u^T_b \Gamma^S d_c)(\bar{u}_d \Gamma^S d^T_e),$$

$$J^P_{PS} = \epsilon_{abc} \epsilon_{ade} (u^T_b \Gamma^P_{PS} d_c)(\bar{u}_d \Gamma^P_{PS} d^T_e),$$

$$J^T_\sigma = (u^T_a \Gamma^T d_a)(\bar{u}_a \Gamma^T d^T_a + \bar{u}_a \Gamma^T d^T_a),$$

(7)

where $\alpha, \beta$ and $\gamma$ are some constants. From the Lagrangian Eq. (4), for $N_c = 3$, it is expected that the ratio of the coefficients

$$\alpha : \beta : \gamma = 1 : -1 : \frac{1}{4}$$

(8)

may provide some specific properties of OPE expansion for the $\sigma$ correlator and may finally lead to the most stable QCD sum rule.

### 3 The OPE contribution to $\sigma$-meson correlator

The $\sigma$–correlator for the case of current Eq. (6) is decomposed into nine parts

$$\Pi_\sigma = i \int d^4x e^{ix} \langle 0 | T J^\sigma(x) J^\sigma(0) | 0 \rangle$$

$$= \alpha^2 \Pi^{S,S} + \beta^2 \Pi^{P_{PS},P_{PS}} + \gamma^2 \Pi^{T,T}$$

$$+ \alpha \beta (\Pi^{S,P_{PS}} + \Pi^{P_{PS},S}) + \alpha \gamma (\Pi^{S,T} + \Pi^{T,S}) + \beta \gamma (\Pi^{P_{PS},T} + \Pi^{T,P_{PS}}).$$

(9)

$\Pi^{A,B}$ means the correlator between $A$–type current and $B$–type current. Full set of the diagrams for the $\sigma$–correlator is presented in Fig. 1. It is evident that we should consider only the last diagram contribution to SR since only that diagram is relevant in the description of $\sigma$ as the tetraquark state. The propagator for massless quarks $q = u, d$ in the fixed point gauge in Fig. 1 up to order of the $g^2$ in strong coupling constant is the following

$$S^q_{ab}(x) = -i \langle 0 | T q_a(x) \bar{q}_b(0) | 0 \rangle$$

$$= \delta_{ab} \left( \frac{\hat{x}}{2\pi^2 x^4} + i \frac{\langle \bar{q} q \rangle}{12} - \frac{x^2}{192} \langle g \bar{q} \sigma \cdot G q \rangle + i \frac{x^4}{2^9 \cdot 3^3} \langle \bar{q} q \rangle \langle g^2 G^2 \rangle \right)$$

$$- i \frac{g}{32 \pi^2} \frac{\epsilon_{ab}^\mu \epsilon_{ab}^\nu}{x^2} \left( \hat{x} \sigma_{\mu \nu} + \sigma_{\mu \nu} \hat{x} \right),$$

(10)

where $a, b$ are the color indices.

The OPE, up to the operators of dimension 10, yields the imaginary part of the $\sigma$–correlator

$$\frac{1}{\pi} \text{Im} \left. \Pi_{OPE}^\sigma(q^2) \right|_{(a)} = (\alpha^2 + \beta^2 + 4\gamma^2) \frac{(q^2)^4}{2^{12} \cdot 5 \cdot 3\pi^6}$$

(10)
Figure 1: Diagrammatic representation of the various correlators Eq (9). Each quark line means the full quark propagator.

\[
+\bigg(\alpha^2 + \beta^2 + 88\gamma^2 + 12\alpha\gamma - 12\beta\gamma\bigg)\frac{\langle g^2 G^2 \rangle}{2^{11} \cdot 3\pi^6} (q^2)^2 \bigg|_{(b)}
\]
\[
+\bigg(\alpha^2 - \beta^2\bigg)\frac{\langle \bar{q}q \rangle^2}{12\pi^2} \bigg|_{(c)} - \bigg(\alpha^2 - \beta^2\bigg)\frac{\langle \bar{q}q \rangle \langle ig\bar{q}\sigma \cdot Gq \rangle}{12\pi^2} \bigg|_{(d)}
\]
\[
+\bigg(\alpha^2 - \beta^2\bigg)\frac{59(\langle ig\bar{q}\sigma \cdot Gq \rangle)^2}{2^9 \cdot 3^2\pi^2} \delta(q^2) \bigg|_{(e)}
\]
\[
+\bigg(\alpha^2 - \beta^2\bigg)\frac{7\langle g^2 G^2 \rangle \langle \bar{q}q \rangle^2}{2^5 \cdot 3^3\pi^2} \delta(q^2) \bigg|_{(f)} ,
\]

(11)

where each term corresponds to each diagram shown in Fig 2 and the factorization hypothesis for high dimension operators has been used.

Figure 2: Diagrammatic representation of Eq. (11).

4 The direct instanton contribution

In addition to contributions of power type from the OPE expansion to the QCD SR, there are exponential contributions coming from direct instantons contributions as shown in Fig. 3. Their contributions can be calculated by using the following formula in Euclidean space for the quark propagator on the instanton background in the regular gauge

\[
S_{ab,\text{inst}}(x, y) = A_q(x, y)\gamma_\mu\gamma_\nu(1 + \gamma_5)(U\tau^-\tau^+U^\dagger)_{ab} ,
\]

(12)

where

\[
A_q(x, y) = -i\frac{\rho^2}{16\pi^2 m^*_q}\phi(x - z_0)\phi(y - z_0)
\]

and

\[
\phi(x - z_0) = \frac{1}{[(x - z_0)^2 + \rho^2]^{3/2}}.
\]

Here \(\rho\) stands for the instanton size, \(z_0\) for the center of the instanton. \(U\) represents the color orientation matrix of the instanton in \(SU(3)_c\) and \(\tau^+_\mu,\tau^-\nu\) are \(SU(2)_c\) matrices. The
effective mass of quark on the instanton vacuum is \( m_q^* = m_q - 2\pi^2 \rho_c^2 \langle \bar{q}q \rangle / 3 \) with current quark mass \( m_q \). At the final stage, we multiply the result by a factor of two to take into account the anti–instanton contribution and integrate over the color orientation and the instanton size.

![Diagram (a)](image1)

![Diagram (b)](image2)

Figure 3: Direct instanton contributions to the correlator.

With the definition \( Q^2 = -q^2 \), the direct instanton contribution to the \( \sigma \)–correlator from the two diagrams in Fig. 3 is given by

\[
\Pi^\sigma_{I+f}(Q) = (\alpha^2 - \beta^2) \frac{32 n_{\text{eff}} \rho_c^4}{\pi^8 m_q^*} f_6(Q) + [19(\alpha^2 + \beta^2) - 6\alpha \beta + 912 \gamma^2 + 72 \alpha \gamma + 72 \beta \gamma] \frac{n_{\text{eff}} \rho_c^4 \langle \bar{q}q \rangle^2}{18 \pi^4 m_q^*} f_0(Q),
\]

(13)

where Shuryak’s instanton liquid model for QCD vacuum with density \( n(\rho) = n_{\text{eff}} \delta(\rho - \rho_c) \) has been used and \( f \) means the contribution from anti–instanton. \( f_6(Q), f_0(Q) \) are the functions defined by

\[
f_6(Q) = \int d^4 z_0 \int d^4 x \frac{\epsilon^{i q \cdot x}}{x^6 [z_0^2 + \rho_c^2]^{3}[(x - z_0)^2 + \rho_c^2]^3},
\]

\[
f_0(Q) = \int d^4 z_0 \int d^4 x \frac{\epsilon^{i q \cdot x}}{[z_0^2 + \rho_c^2]^{3}[(x - z_0)^2 + \rho_c^2]^3}.
\]

(14)

Here \( \rho_c \) is the average instanton size. Moreover, let us also note that direct instanton contribution is possible only for the different quark flavors. Therefore, in case of tetraquark \( \sigma \) meson with pure \( u \)– and \( d \)– quark content, there is no direct instanton contribution in any subsystem of three quarks. On the other hand, three–body instanton contribution in \( \bar{u}d \bar{s}, u \bar{d}s, u \bar{d} \bar{s} \) subsystems might be quite important in the case of \( f_0(980) \) and \( a_0(980) \) tetraquarks. Furthermore, such interaction can also lead to sizeable mixing of \( u \bar{d}u \bar{d} \sigma \)–meson with convenient \( s \bar{s} \) two–quark state.

5 Numerical analysis of QCD sum rule for instanton current

In order to avoid strong dependence of the multiquark mass on the value of threshold \cite{21}, we apply the two resonances approximation to the spectral representation of the correlator as

\[
\text{Im } \Pi^\sigma(s^2) = \pi \sum_n \delta(s^2 - m_n^2) \langle 0 | J^\sigma | n \rangle \langle n | J^{\sigma \dagger} | 0 \rangle
\]

\[
= 2\pi f_1^2 m_1^8 \delta(s^2 - m_1^2) + 2\pi f_2^2 m_2^8 \delta(s^2 - m_2^2) + \theta(s^2 - s_0^2) \text{Im } \Pi^{\text{OP}}(s^2)(15)
\]
with the convention
\[ \langle 0 | J^\nu | S_i \rangle = \sqrt{2} f_i m_i^4. \]  

(16)

In this case the QCD sum rule is the following
\[
(\alpha^2 + \beta^2 + 48 \gamma^2) \frac{M^{10} E_4}{2^9 \cdot 5 \pi^6} + (\alpha^2 + \beta^2 + 88 \gamma^2 + 12 \alpha \gamma - 12 \beta \gamma) \frac{\langle g^2 G^2 \rangle M^6 E_2}{2^{10} \cdot 3 \pi^6}
\]
\[ + (\alpha^2 - \beta^2) \frac{\langle \bar{q} q \rangle^2}{12 \pi^2} M^4 E_1 - (\alpha^2 - \beta^2) \frac{\langle i g \bar{q} \gamma \cdot G q \rangle}{12 \pi^2} M^2 E_0
\]
\[ + (\alpha^2 - \beta^2) \frac{59(ig \bar{q} \gamma \cdot G q)^2}{2^{10} \cdot 3^2 \pi^2} - (\alpha^2 - \beta^2) \frac{7 \langle g^2 G^2 \rangle \langle \bar{q} q \rangle^2}{2^6 \cdot 3^3 \pi^2} + (\alpha^2 - \beta^2) \frac{32 n_{eff} \rho_i^4}{\pi^8 m_i^2} \hat{B}[f_0(Q)]
\]
\[ + [19(\alpha^2 + \beta^2) - 6 \alpha \beta + 912 \gamma^2 + 72 \alpha \gamma + 72 \beta \gamma] \frac{n_{eff} \rho_i^4 \langle \bar{q} q \rangle^2}{18 \pi^4 m_i^2} \hat{B}[f_0(Q)]
\]
\[ = 2 f_1^2 m_1^8 e^{-m_1^2/M^2} + 2 f_2^2 m_2^8 e^{-m_2^2/M^2} \]  

(17)

up to operators of dimension 10, where \( \hat{B}[f_{0,6}(Q)] \) is the Borel transform \( f_{0,6}(Q) \) function given by Eq. (14). The contribution from the continuum is encoded in the functions \( E_n(M) \) defined by
\[
E_n(M) = \frac{1}{\Gamma(n+1) M^{2n+2}} \int_0^{s_0} ds^2 e^{-s^2/M^2} (s^2)^n,
\]

where \( s_0 \) is the threshold of the continuum and \( M \) is the Borel mass. In the calculation of the Borel transformed \( f_0(Q) \) we only include the contribution from the pole at finite distance \( x^2 \sim -\rho^2 \), which corresponds to the direct instantons effect (see discussion in [12]):
\[
\hat{B}[f_0(Q)] = -\frac{\pi^4 M^{12}}{213} \int_0^1 dt \int_0^1 dy \frac{e^{-M^2 \rho_c^2/(4ty(1-y))}}{y^2(1-y)^2} \left( X^2 + 5X^3 + 10X^4 + 10X^5 + 5X^6 + X^7 \right),
\]
\[
\hat{B}[f_0(Q)] = \frac{\pi^4 M^6}{16} e^{-M^2 \rho_c^2/2} \left( K_0(M^2 \rho_c^2/2) + K_1(M^2 \rho_c^2/2) \right),
\]

(19)

where \( X = (1 - t)/t \) and \( K_n(x) \) is the McDonald function.

For the numerical analysis with the massless quarks \( q = u, d \), we use the following condensates at normalization point \( \mu = 1 \)GeV and the average size of instanton,
\[
\langle \bar{q} q \rangle = -(0.23 \text{ GeV})^3, \quad \langle ig \bar{q} \gamma \cdot G q \rangle = 0.8 \text{ GeV}^2 \langle \bar{q} q \rangle, \\
\langle g^2 G^2 \rangle = 0.5 \text{ GeV}^4, \quad \rho_c = 1.6 \text{GeV}^{-1}.
\]

(20)

As we already mentioned, the sum rules constructed with only the scalar diquark–antidiquark, \( \beta = \gamma = 0 \), and with only pseudoscalar diquark–antidiquark, \( \alpha = \gamma = 0 \), are not stable. More precisely, the sum rule with the scalar diquark–antidiquark looses its physical meaning because the LHS of the sum rule has definite negative value [13]. On the other hand, the LHS of the sum rule with the pseudoscalar diquark–antidiquark has a definite positive value but its slope is negative so that it is impossible to fit a physical mass for the resonance. We present this situation in Figs. 4 and 5 with the value \( s_0 = 1.0 \) GeV which
is the threshold taken usually in the single resonance approximation for the RHS of SR [7, 13]. The origin of this behavior of the LHS of the sum rule lies in large contributions from the higher dimensional operators and direct instantons.

Therefore, the pure scalar or pseudoscalar diquark content of $\sigma$–tetraquark is not favored in the QCD sum rule approach and we need to find another interpolating current which will be not so much affected by the higher dimension condensates and the direct instantons. Let us discuss in detail the origin of the specific dependence, of the different terms in the LHS of the SR Eq.(17), on the parameters of current $\alpha$ and $\beta$. We would like to emphasize, that the chirality structure of the current plays an important role in the appearance or disappearance of some OPE and instanton contributions\(^1\). Indeed, the inspection of our general current Eq.(6) shows that there are several possible contributions to the SR with definite chirality flip. Firstly, the chirality conserved diagonal transitions between four terms in Eq.(21) gives the factor $(\alpha^2 + \beta^2)$ in front of chirality conserved OPE terms in the SR. Secondly the contribution is coming from a non–diagonal transition between the first and second line in Eq.(6). In this case a flip of chirality of two quarks in the four quark system happens and the factor $(\alpha^2 - \beta^2)$ in front of the high dimension OPE condensates, and some instanton contributions, appears. There is also the possibility to have a chirality flip for all quarks coming from transitions

\(^1\)We should mention, that from our point of view, the chirality structure of the multiquark currents might be one of the cornerstones of multiquark spectroscopy because it provides a strong restriction on the possible “good” multiquark currents. Particularly, in recent papers [14, 24] it has been shown that the chirality structure of the pentaquark current is very important to explain its small width.
between first (third) and second (fourth) terms. In OPE expansion this contribution is not vanishing only for massive quarks. Indeed, one cannot put all quark lines to zero virtuality in chirality odd condensates due to necessity to have non-zero momentum transfer through the last diagram in Fig.1. This momentum transfer with chirality flip in the corresponding quark line can be produced in the OPE only for a mass dependent term in the perturbative part of the full quark propagator. Therefore, for \( \sigma \)-tetracquark such contribution is not possible. In addition to the chirality arguments above, the spin structure of the tensor current

\[
J_T^\sigma \sim -(u_R C \sigma_{\rho \sigma} d_R + u_L C \sigma_{\rho \sigma} d_L)(\bar{u}_R \sigma^{\rho \sigma} C \bar{d}_R + \bar{u}_L \sigma^{\rho \sigma} C \bar{d}_L)
\]  

(22)

restricts further the high dimension OPE contributions from the tensor current to the sum rule. It is evident from our discussion above that the cases \( \alpha = -\beta \) and \( \alpha = \beta \) are very peculiar cases for the \( \sigma \) SR. With these values of \( \alpha \) and \( \beta \) the contributions from operators with dimension higher than \( d = 6 \) and some of direct the instanton contributions in Eq.(17) vanish. Therefore, for these currents we could expect more stable QCD SR results. The difference between the two cases is again in the chirality structure of the corresponding current. For \( \alpha = -\beta \) each term in the first line in Eq.(21) has four units of chirality, while for \( \alpha = \beta \) each term in second line in Eq.(21) has zero chirality. From the point of view of the instanton model for the QCD vacuum the two cases presented in Fig.6

![Figure 6: Two suitable currents for \( \sigma \)-meson a) single instanton current and b) current induced by instanton–antiinstanton molecules. The symbol \( I \) (\( \bar{I} \)) denote instanton (antiinstanton).](image)

The Fig.6a corresponds to our single instanton current which we discussed in Sec.II. This current could be related to the phase of instanton liquid with spontaneous chiral symmetry breaking \([15]\). The Fig.6b corresponds to the chirality symmetric component of the QCD vacuum. Within the instanton model one can consider it as contribution coming from instanton–antiinstanton molecules. The latter component is expected to give a small contribution at zero temperature but might be important at temperatures above the deconfinement temperature. Therefore, we will choose \( \alpha = -\beta \) as a “good” interpolating current for \( \sigma \)-tetracquark based on the instanton picture of the QCD vacuum and the chirality arguments given above \(^2\). Assigning \( \alpha = -\beta = 1, \gamma = 1/4 \) to the total

\(^2\)We should point out, that if we will choose \( \alpha = \beta \), the analysis of SR gives for the value of the tetracquark mass approximately the same as for the case of “good” current for the threshold \( s_0 = 2 \) GeV. The physical meaning of the small splitting of the tetracquark masses with two completely different currents is difficult to explain for the present.
current based on the structure of the instanton Lagrangian Eq. (8), we have stability in the sum rule thanks to the cancelation in the contributions from the higher dimensional operators and some part of the instanton effects. With the best fit in the two resonance approximation, the fitted masses and the residues are summarized in Tab. 1 and the good quality of the fit is shown in Fig. 4. We interpret the lower mass in our SR as the mass of $\sigma$-tetraquark and the higher mass as the mass of its first radial excitation. According to the PDG [25], the candidates for the excited states are $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. We can see that the lower mass $m_1$ almost does not depend on the value of the threshold and the higher mass $m_2$ looks similar to $f_0(1710)$. Indeed, since the quality of the fit of the masses and the residues is best at $s_0 = 2.0$ GeV, we can interpret the lower mass $\sim 780$ MeV as the mass of the $\sigma$ and the higher mass $\sim 1775$ MeV as the mass of the state $f_0(1710)^3$. In this scheme the $f_0(1370)$ might be treated as a conventional $q\bar{q}$ state and the $f_0(1500)$-meson can be consider as a glueball candidate [9].

| $s_0$ (GeV) | $m_1$ (GeV) | $m_2$ (GeV) | $f_1(10^{-3}\text{GeV})$ | $f_2(10^{-3}\text{GeV})$ |
|-------------|-------------|-------------|---------------------------|---------------------------|
| 2.0         | 0.7822      | 1.7756      | 4.399                     | 0.395                     |
| 2.2         | 0.7964      | 1.9488      | 4.241                     | 0.426                     |
| 2.4         | 0.8102      | 2.1016      | 4.095                     | 0.459                     |
| 2.6         | 0.8261      | 2.2519      | 3.935                     | 0.491                     |

Table 1: Fitted masses and residues in the two resonances approximation with $\alpha = 1$, $\beta = -1$, $\gamma = 1/4$.

Figure 7: The left hand side (solid line) and the right hand side (dashed line) of the QCD sum rule with masses and residues presented in Tab. 1 at $s_0 = 2.0$ GeV.

6 Conclusion

We have discussed a novel interpolating current for scalar meson $\sigma(600)$ treated as a tetraquark state. The color–spin–flavor structure of our current is fixed by the the properties of the instanton induced quark–quark interaction and reflects the topological structure

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$^3$Recently, this meson was reported to have a mass $f_0(1790)$ by BES collaboration [26].
of the QCD vacuum. Our “good” current has very peculiar chirality structure because it includes both type of diquarks, scalar and pseudoscalar, with equal weights. In this connection we would like to point out that the similar improvement of the OPE convergence for some specific currents was previously observed in the case of the usual hadrons. Well known example is so–called Ioffe’s current for the nucleon which is widely using for description of the nucleon properties [27]. This current has also very peculiar chirality structure and includes scalar and pseudoscalar diquarks with equal weights.

We have demonstrated that the definite chirality structure of our “good” current for \( \sigma \) leads to the cancelation of the high dimensional operator contributions to OPE and to the vanishing of some instanton contributions which can spoil the QCD sum rules. As a result, within QCD sum rule approach, we have obtained a very stable result for the mass of the \( \sigma \) meson around 780 MeV. This mass lies inside the range of the PDG \( m_{\sigma(600)} = 400 \div 1200 \) MeV. We should also mention the possible change in the result for the mass if the mixing between the usual two quark states and our tetraquark state is taken into account. An additional shift of the \( \sigma \)-mass might come from the large perturbative QCD corrections in multiquark hadrons [28].

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