Transition Form Factors of $\chi_b^2(1P) \to B_c \bar{l} \nu$ in QCD

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Abstract

The form factors of the semileptonic $\chi_b^2(1P) \to B_c \bar{l} \nu$ decay are calculated using the QCD sum rule approach. The results obtained are then used to estimate the decay widths of this transition in all lepton channels. The orders of decay rates indicate that this transition is accessible at LHC for all lepton channels.

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1 Introduction

Quarkonia are flavorless bound states composed of combinations of quarks and their antiquarks. Since discovery of $J/\psi$ meson in 1974, many new quarkonia states have been detected. The quarkonia systems consist of charmonium and bottomonium. Due to the large mass there are no toponium bound states and no light quark-antiquark states because of the mixture of the light quarks in experiments. The large mass difference between charm and bottom quarks prevents them from mixing. The heavy quark bound states may provide key tools for understanding the interactions between quarks, new hadronic production mechanisms and transitions, the magnitude of the CKM matrix elements and also analyzing the results of heavy-ion experiments.

The $J/\Psi$ suppression in ultra relativistic heavy-ion collisions was first suggested as a signal of the formation of a quark-gluon plasma (QGP). However, most recently, attentions have shifted to the bottomonium states due to that they are more massive than charmonium states. The bottom quarks and anti-quarks are relatively rare within the plasma, so the probability for reproduction of the bottomonium states through recombination is much smaller than for charm quarks. Consequently, it is expected that the bottomonium system to be a cleaner probe of the QGP than the charmonium system. Hence investigations on the properties of bottomonium systems can help us get useful information not only about the nature of the $b\bar{b}$ systems, but also on the existence of QGP.

Since the discovery of quarkonium states, QCD sum rule technique as one of the most powerful nonperturbative tools to hadron physics [1, 2] has played an important role in understanding the quarkonia spectrum. In order to find missing states one should know their physical properties to develop a successful search strategy. Clearly significant progress in understanding of quarkonium production cannot be reached without detailed measurements of the cross sections and fractions of the quarkonia. Thereby, form factors and decay widths of quarkonia become significant for completing quarkonia spectrum. Completing the bottomonium spectrum is a crucial validation of theoretical calculations and a test of our understanding of bottomonium states in the context of the quark model. Bottomonium states are considered as great laboratories to search for the properties of QCD at low energies.

In this connection, we investigate the decay properties of tensor $\chi_{b2}(1P)$ meson as one of the important members of the bottomonia to the heavy $B_c$ meson in the present work. The $B_c$ meson with $J^P = 0^-$ is the only meson consists of two heavy quarks with different flavors. Yet, other possible $B_c$ states (the scalar, vector, axial-vector and tensor) have not been observed, however many new $B_c$ species are expected to be produced at the Large Hadron Collider (LHC) in the near future.

Taking into account the two-gluon condensate corrections, the transition form factors of the semileptonic $\chi_{b2}(1P) \to B_c \ell \nu$ decay channel are calculated within the three-point QCD sum rule. We use the values of transition form factors to estimate the decay rate of the transition under consideration at all lepton channels. The interpolating current of $\chi_{b2}(1P)$ with quantum numbers $I^G(J^{PC}) = 0^+(2^{++})$ contains derivatives with respect to space-time. So, we start our calculations in the coordinate space then we apply the Fourier transformation to go to the momentum space. To suppress the contributions of the higher states and continuum, we apply a double Borel transformation.
The article is organized as follows. We derive the QCD sum rules for the transition form factors in Section II. Last section is devoted to the numerical analysis of the obtained sum rules, estimation of the decay rates at all lepton channels and concluding remarks.

2 QCD Sum Rules for $\chi_{b2} \to B_c \bar{c} \nu$ Transition Form Factors

The semileptonic $\chi_{b2} \to B_c \bar{c} \nu$ decay is based on $b \to c \bar{l} \nu$ transition at quark level whose effective Hamiltonian can be written as

$$H_{\text{eff}} (b \to c \bar{l} \nu) = \frac{G_F}{\sqrt{2}} V_{cb} \gamma_{\mu} (1 - \gamma_5) b l^\mu (1 - \gamma_5) \bar{\nu},$$

where $G_F$ is the Fermi weak coupling constant and $V_{cb}$ is element of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. After sandwiching the effective Hamiltonian between the initial and final states, the amplitude of this transition is obtained in terms of transition matrix elements. These matrix elements will be parameterized in terms of transition form factors later.

In order to start our calculations, we consider the three-point correlation function

$$\Pi_{\alpha \beta} (p, p', q) = i^2 \int d^4 x e^{-ip \cdot x} \int d^4 y e^{-ip' \cdot y} \langle 0 | T | j_{B_c} (y) j_{\mu}^{tr,V-A} (0) j_{\chi_{b2}}^{\alpha \beta} (x) | 0 \rangle,$$

where $T$ is the time-ordering operator and $j_{\mu}^{tr,V-A} (0) = \bar{c}(0) \gamma_{\mu} (1 - \gamma_5) b(0)$ is the transition current. To proceed we also need the interpolating currents of the initial and final mesons in terms of the quark fields, which are given as

$$j_{\chi_{b2}}^{\alpha \beta} (x) = \frac{i}{2} \left[ \bar{b}(x) \gamma_{\alpha} \stackrel{\rightarrow}{D}_\beta (x) b(x) + \bar{b}(x) \gamma_{\beta} \stackrel{\rightarrow}{D}_\alpha (x) b(x) \right],$$

and

$$j_{B_c} (y) = \bar{b}(y) \gamma_5 c(y),$$

where the covariant derivative $\stackrel{\rightarrow}{D}_\alpha (x)$ denotes the four-derivative with respect to $x$ acting on two sides simultaneously and it is defined as

$$\stackrel{\rightarrow}{D}_\alpha (x) = \frac{1}{2} \left[ \stackrel{\rightarrow}{D}_\alpha (x) - \stackrel{\leftarrow}{D}_\alpha (x) \right],$$

with

$$\stackrel{\rightarrow}{D}_\beta (x) = \bar{\nabla}_\beta (x) - \frac{i}{2} \lambda^a A_\beta^a (x)$$

and

$$\stackrel{\leftarrow}{D}_\beta (x) = \bar{\nabla}_\beta (x) + \frac{i}{2} \lambda^a A_\beta^a (x).$$

Here, $\lambda^a$ are the Gell-Mann matrices and $A_\beta^a (x)$ denote the external gluon fields.
According to the method used, the correlation function in Eq.(2) is calculated in two different ways. In physical or phenomenological side we obtain it in terms of hadronic parameters such as masses and decay constants. In QCD or theoretical side we evaluate it in terms of QCD degrees of freedom like quark masses as well as quark and gluon condensates via operator product expansion (OPE). The QCD sum rules for form factors are obtained by equating the above representations to each other. After applying a double Borel transformation, the contributions of the higher states and continuum are suppressed.

The hadronic side of the correlation function is obtained by inserting complete sets of intermediate states into Eq.(2). After performing the four-integrals over $x$ and $y$, we get

\[
\Pi_{\mu\alpha\beta}^{PHYS}(p,p',q) = \frac{\langle 0 | j_{Bc}(0) | B_c(p') \rangle \langle B_c(p') | j_{\mu}^{tr,V} | \chi_{b2}(p,\varepsilon) \rangle \langle \chi_{b2}(p,\varepsilon) | j_{\alpha\beta}^{\chi_{b2}}(0) | 0 \rangle}{(p'^2 - m_{Bc}^2)(p^2 - m_{\chi_{b2}}^2)} + \ldots ,
\]  

(8)

where $\ldots$ denotes the contribution of the higher states and continuum. To go further, we need to know the following matrix elements:

\[
\langle B_c(p') | j_{\mu}^{tr,V} | \chi_{b2}(p,\varepsilon) \rangle = h(q^2)\epsilon_{\mu\nu\theta\eta}\epsilon^{\nu\lambda}P_{\lambda}P^{\theta}q^{\eta},
\]  

(9)

\[
\langle B_c(p') | j_{\mu}^{tr,A} | \chi_{b2}(p,\varepsilon) \rangle = -i\left\{K(q^2)\epsilon_{\mu\nu}P^{\nu} + \epsilon_{\theta\eta}P^{\theta}P^{\eta}[P_{\mu}b_+(q^2) + q_\mu b_-(q^2)]\right\},
\]  

(10)

\[
\langle \chi_{b2}(p,\varepsilon) | j_{\alpha\beta}^{\chi_{b2}} | 0 \rangle = f_{\chi_{b2}}m_{\chi_{b2}}^3\varepsilon_{\alpha\beta}^*,
\]  

(11)

and

\[
\langle 0 | j_{Bc} | B_c(p') \rangle = i\frac{f_{Bc}m_{Bc}^2}{m_c + m_b},
\]  

(12)

where $h(q^2)$, $K(q^2)$, $b_+(q^2)$ and $b_-(q^2)$ are transition form factors, $\epsilon_{\alpha\beta}$ is the polarization tensor associated with the $\chi_{b2}$ tensor meson; $f_{\chi_{b2}}$ and $f_{Bc}$ are leptonic decay constants of $\chi_{b2}$ and $B_c$ mesons, respectively, $P_{\mu} = (p + p')_{\mu}$ and $q_\mu = (p - p')_{\mu}$.

Putting all matrix elements given in Eqs. (9), (10), (11) and (12) into Eq.(8), the final representation of the correlation function on physical side is obtained as

\[
\Pi_{\mu\alpha\beta}^{PHYS}(p,p',q) = \frac{f_{\chi_{b2}}f_{Bc}m_{\chi_{b2}}m_{Bc}^2}{8(m_b + m_c)(p^2 - m_{\chi_{b2}}^2)(p'^2 - m_{Bc}^2)}\left\{\Delta K(q^2)q_\alpha g_{\beta\mu} - \frac{2}{3}\left[\Delta' b_-(q^2) + \Delta K(q^2)\right]q_\mu g_{\alpha\beta} - \frac{2}{3}\left[\Delta' b_+(q^2) + K(q^2)(\Delta + 4m_{\chi_{b2}}^2)\right]P_{\mu}g_{\alpha\beta} - i(\Delta - 4m_{\chi_{b2}}^2)h(q^2)\varepsilon_{\theta\eta\beta\mu}P_{\lambda}P_{\eta}q_{\alpha} + \text{other structures}\right\} + \ldots,
\]  

(13)

where

\[
\Delta = m_{Bc}^2 + 3m_{\chi_{b2}}^2 - q^2
\]  

(14)

and

\[
\Delta' = m_{Bc}^4 - 2m_{Bc}^2(m_{\chi_{b2}}^2 + q^2) + (m_{\chi_{b2}}^2 - q^2)^2.
\]  

(15)
Note that we represented only the structures which we will use to find the corresponding form factors. Meanwhile, we used the following summation over the polarization tensors to obtain Eq. (13)

\[ \sum_{\lambda} \varepsilon_{\mu\nu}^{\lambda} \varepsilon_{\alpha\beta}^{\lambda} = \frac{1}{2} \eta_{\mu\alpha} \eta_{\nu\beta} + \frac{1}{2} \eta_{\mu\beta} \eta_{\nu\alpha} - \frac{1}{3} \eta_{\mu\nu} \eta_{\alpha\beta}, \]  

(16)

where

\[ \eta_{\mu\nu} = -g_{\mu\nu} + \frac{p_{\mu} p_{\nu}}{m_{\lambda i}^2}. \]  

(17)

The next step is to calculate the QCD side of the correlation function in deep Euclidean region, where \( p^2 \to -\infty \) and \( p'^2 \to -\infty \) via OPE. Placing the explicit expressions of the interpolating currents into the correlation function and contracting out all quark pairs via Wick’s theorem, we obtain

\[ \Pi_{QCD}^{ij}(p, p', q) = \frac{-i^3}{2} \int d^4x \int d^4y e^{-ipx} e^{ip'y} \times \left\{ Tr \left[ S_b^{ia}(x-y) \gamma_5 S_c^{cj}(y) \gamma_\mu (1 - \gamma_5) \frac{\gamma^\beta}{2} (x) S_b^{ji}(-x) \gamma_\alpha \right] + [\beta \leftrightarrow \alpha] \right\}, \]  

(18)

where \( S \) is the heavy quark propagator and it is given by [3]

\[ S^{ai}_Q(x) = \frac{i}{(2\pi)^4} \int d^4k e^{-ik(x)} \left\{ \frac{\delta^{ai}_{\mu}}{k - m_Q} - \frac{g_s G_4^{ij}}{4} \frac{\sigma^{ij}_{\nu}(k + m_Q)}{(k^2 - m_Q^2)^2} \right. \]

\[ + \frac{\pi^2}{3} \left( \frac{\alpha_s GG}{\pi} \right) \delta^{ai}_{\mu} m_Q \frac{k^2 + m_Q^2}{(k^2 - m_Q^2)^4} + \cdots \right\}, \]  

(19)

where \( Q = b \) or \( c \) quark. Replacing the explicit expression of the heavy quark propagators in Eq. (18) and applying integrals over \( x \) and \( y \), we find the QCD side as

\[ \Pi_{QCD}^{ij}(p, p', q) = \left( \Pi_1^{pert}(q^2) + \Pi_1^{nonpert}(q^2) \right) q_\alpha g_{\beta \mu} + \left( \Pi_2^{pert}(q^2) + \Pi_2^{nonpert}(q^2) \right) q_\mu g_{\beta \alpha} \]

\[ + \left( \Pi_3^{pert}(q^2) + \Pi_3^{nonpert}(q^2) \right) P_\mu g_{\beta \alpha} + \left( \Pi_4^{pert}(q^2) + \Pi_4^{nonpert}(q^2) \right) \varepsilon_{\lambda \nu \beta \mu} P_\lambda P_\alpha q_\nu \]

+ other structures.  

(20)

Here \( \Pi_i^{pert}(q^2) \) with \( i = 1, 2, 3, 4 \) are the perturbative parts which are expressed in terms of double dispersion integrals as

\[ \Pi_i^{pert}(q^2) = \int ds \int ds' \frac{\rho_i(s, s', q^2)}{(s - p^2)(s' - p'^2)} + \text{subtracted terms}, \]  

(21)

where the spectral densities are defined as \( \rho_i(s, s', q^2) = \frac{1}{\pi} \text{Im}[\Pi_i^{pert}] \). The spectral densities corresponding to four different structures shown in Eq. (20) are obtained as

\[ \rho_1(s, s', q^2) = \int_0^1 dx \int_0^{1-x} dy \frac{3}{16\pi^2} \left[ m_c (-3 + 4x + 2y) + m_b (-5 + 8x + 4y) \right], \]
\[ \rho_2(s, s', q^2) = \int_0^1 dx \int_0^{1-x} dy \frac{3\left[ m_c(3 - 4x - 2y) + m_b(-1 + 4x + 2y) \right]}{8\pi^2}, \]
\[ \rho_3(s, s', q^2) = -\int_0^1 dx \int_0^{1-x} dy \frac{3\left[ m_c(1 - 2y) + m_b(1 + 2y) \right]}{8\pi^2} \] (22)

and
\[ \rho_4(s, s', q^2) = 0. \] (23)

The function \( \Pi^{\text{nonpert}}_i(q^2) \) in nonperturbative parts are calculated from a similar manner and by considering the two-gluon condensates contributes. Having calculated both the physical and OPE sides of the correlation function, now, we match them to find QCD sum rules for form factors. In the Borel scheme we get
\[ K(q^2) = \frac{8(m_b + m_c)}{f_{\chi^02} f_{B \bar{B}_c} m_{\chi^02} \Delta} \frac{e^{m_{\chi^02}/M^2} e^{m_{B_c}/M^2}}{e^{m_{\chi^02}/M^2} e^{m_{B_c}/M^2}} \times \left\{ \int_{4m_b^2}^{s_0} ds \int_{(m_b + m_c)^2}^{s'_0} ds' e^{s'/M^2} e^{s/M^2} \rho_1(s, s', q^2) \Theta[L(s, s', q^2)] + \hat{B}_{M_2} \hat{B}_{M_2} \Pi^{\text{nonpert}}_1(q^2) \right\}, \]
\[ b_-(q^2) = -\frac{12(m_b + m_c)}{f_{\chi^02} f_{B \bar{B}_c} m_{\chi^02} \Delta} e^{m_{\chi^02}/M^2} e^{m_{B_c}/M^2} \times \left\{ \int_{4m_b^2}^{s_0} ds \int_{(m_b + m_c)^2}^{s'_0} ds' e^{s'/M^2} e^{s/M^2} \rho_2(s, s', q^2) \Theta[L(s, s', q^2)] + \hat{B}_{M_2} \hat{B}_{M_2} \Pi^{\text{nonpert}}_2(q^2) \right\}, \]
\[ -\frac{\Delta}{\Delta'} K(q^2) \right\}, \]
\[ b_+(q^2) = -\frac{12(m_b + m_c)}{f_{\chi^02} f_{B \bar{B}_c} m_{\chi^02} \Delta} e^{m_{\chi^02}/M^2} e^{m_{B_c}/M^2} \times \left\{ \int_{4m_b^2}^{s_0} ds \int_{(m_b + m_c)^2}^{s'_0} ds' e^{s'/M^2} e^{s/M^2} \rho_3(s, s', q^2) \Theta[L(s, s', q^2)] + \hat{B}_{M_2} \hat{B}_{M_2} \Pi^{\text{nonpert}}_3(q^2) \right\}, \]
\[ -\frac{\Delta + 4m_{\chi^02}^2 K(q^2)}{\Delta'} \right\} \] (24)

and
\[ h(q^2) = -i \frac{8(m_b + m_c)}{f_{\chi^02} f_{B \bar{B}_c} m_{\chi^02} (4m_{\chi^02}^2 - \Delta)} e^{m_{\chi^02}/M^2} e^{m_{B_c}/M^2} \times \left\{ \int_{4m_b^2}^{s_0} ds \int_{(m_b + m_c)^2}^{s'_0} ds' e^{s'/M^2} e^{s/M^2} \rho_4(s, s', q^2) \Theta[L(s, s', q^2)] + \hat{B}_{M_2} \hat{B}_{M_2} \Pi^{\text{nonpert}}_4(q^2) \right\} \] (25)

where \( M^2 \) and \( M'^2 \) are Borel mass parameters; and \( s_0 \) and \( s'_0 \) are continuum thresholds in the initial and final channels. Here, \( \Theta \) is the step function and \( L(s, s', q^2) \) is given by
\[ L(s, s', q^2) = s'y(1 - x - y) - sxy + m_c^2(x + y - 1) - m_b^2(x + y) + q^2x(1 - x - y). \] (26)
The functions $\hat{B}_{M^2}\hat{B}_{M^2}\Pi_{i}^{\text{nonpert}}(q^2)$ are written as

$$\hat{B}_{M^2}\hat{B}_{M^2}\Pi_{i}^{\text{nonpert}}(q^2) = \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_0^1 dx \, e^{-\frac{m_b^2(1+\frac{M^2}{M^2_x})+\frac{M^2_{K^*}}{M^2_x}(m_b^2-q^2 x)}{M^2_x(1+\frac{M^2}{M^2_x})}} f_i(q^2),$$

(27)

where $f_i(q^2)$ are very lengthy functions and we do not present their explicit expressions here.

3 Numerical results

To numerically analyze the sum rules obtained for the form factors, we use the meson masses from PDG [4]. Considering the fact that the results of sum rules considerably depend on the quark masses, decay constants and gluon condensate, we use the values of these parameters from different sources. For the quark masses we take into account all the pole values and those obtained at $\overline{MS}$ scheme from the Refs. [5–15]. For $f_{B_c}$, we consider all the values predicted using different methods in Refs. [16–19]. In the case of $f_{\chi_b^2}$, we use the only value exists in the literature, i.e. $f_{\chi_b^2} = (0.0122 \pm 0.0072)$ [20]. For the gluon condensate, we also use its value from different sources [1, 21–28] calculated via different approaches.

From the sum rules for the form factors it is also clear that they contain extra four auxiliary parameters, namely the Borel parameters $M^2$ and $M^2_x$ as well as continuum thresholds $s_0$ and $s'_0$. The general criteria is that the physical quantities like form factors should be independent of these parameters. Therefore, we need to determine their working regions such that the form factors weakly depend on these parameters. To find the Borel windows, we require that the higher states and continuum contributions are sufficiently suppressed and the perturbative parts exceed the nonperturbative contributions and the series of OPE converge. As a result we get the windows: $14 \ \text{GeV}^2 \leq M^2 \leq 20 \ \text{GeV}^2$ and $8 \ \text{GeV}^2 \leq M^2_x \leq 12 \ \text{GeV}^2$. The continuum threshold $s_0$ and $s'_0$ are not completely arbitrary but they are related to the energy of the first excited states with the same quantum numbers as the interpolating currents of the initial and final channels. In this work the continuum thresholds are chosen in the intervals $104 \ \text{GeV}^2 \leq s_0 \leq 108 \ \text{GeV}^2$ and $43 \ \text{GeV}^2 \leq s'_0 \leq 45 \ \text{GeV}^2$.

The dependence of form factors $K$ and $b_+$, as examples, on Borel parameters $M^2$ and $M^2_x$ at $q^2 = 0$ are plotted in figures 1 and 2. From these figures we see that form factors show overall weak dependence on the Borel mass parameters. The behaviors of the form factors $K$, $b_+$, $b_-$ and $h$ in terms of $q^2$ are shown in figures (3-6). In these figures, the red triangles show the QCD sum rule predictions, yellow-solid line denotes the prediction of fit function obtained using the central values of the input parameters and the green band shows the uncertainty due to errors of input parameters. Note that to obtain the central values, we consider the average values of input parameters discussed above, however, to calculate the uncertainties we consider all errors of these parameters from different sources previously quoted. As it is seen from these figures the sum rules results are truncated at some points. Hence, to enlarge the region to whole physical region we need to find some fit functions such that their results coincide well with the QCD sum rules predictions at reliable regions. For this reason we show the $q^2$ dependence of form factors including both
the sum rules and fit results in figures (3-6). Our numerical calculations reveal that the following fit function well defines the form factors under consideration:

\[
f(q^2) = f_0 \exp \left[ a \left( \frac{q^2}{m_{\chi_{b2}}^2} \right) + b \left( \frac{q^2}{m_{\chi_{b2}}^2} \right)^2 \right],
\]

where the values of the parameters, \( f_0, a \) and \( b \) obtained at \( M^2 = 17 \, GeV^2 \) and \( M^2 = 10 \, GeV^2 \) for \( \chi_{b2} \to B_c \bar{\nu} \) transition are presented in table 1.

Our final purpose in this section is to obtain the decay width of the \( \chi_{b2} \to B_c \bar{\nu} \) transition at all lepton channels. The differential decay width for this transition is obtained as

\[
d\Gamma dq^2 = \frac{G_F^2 V_{cb}^2}{2^{10} 3^2 m_{\chi_{b2}}^3 \pi^3 q^6} (m_t^2 - q^2)^2 \Delta^{3/2} \left\{ |b_-(q^2)|^2 \Delta' m_t^2 q^4 \\
+ |b_+(q^2)|^2 \Delta' \left[ (m_{B_c}^2 - m_{\chi_{b2}}^2)^2 m_t^2 + (m_{B_c}^2 - m_{\chi_{b2}}^2)^2 q^2 - 2(m_{B_c}^2 + m_{\chi_{b2}}^2)q^4 + q^6 \right] \right\}
\]

Table 1: Parameters appearing in the fit function of the form factors.

|         | \( f_0 \)            | \( a \)            | \( b \)            |
|---------|-----------------------|---------------------|---------------------|
| \( K(q^2) \) | \(-0.871 \pm 0.279\) | \(5.239 \pm 1.677\) | \(-7.588 \pm 2.428\) |
| \( b_-(q^2) \) | \(-0.134 \pm 0.043 \, GeV^{-2}\) | \(8.973 \pm 2.871\) | \(56.462 \pm 18.068\) |
| \( b_+(q^2) \) | \(0.304 \pm 0.097 \, GeV^{-2}\) | \(10.054 \pm 3.217\) | \(53.922 \pm 17.255\) |
| \( h(q^2) \) | \((-2.594 \pm 0.830) \times 10^{-4} \, GeV^{-2}\) | \(5.224 \pm 1.672\) | \(3.891 \pm 1.245\) |
Table 2: Numerical results of decay widths at different lepton channels.

| Decay Channel | Γ(GeV)                  |
|---------------|-------------------------|
| $\chi_{b2} \rightarrow B_c \bar{e} \nu_e$ | $(1.054 \pm 0.506) \times 10^{-13}$ |
| $\chi_{b2} \rightarrow B_c \bar{\mu} \nu_\mu$ | $(1.041 \pm 0.500) \times 10^{-13}$ |
| $\chi_{b2} \rightarrow B_c \bar{\tau} \nu_\tau$ | $(2.398 \pm 1.175) \times 10^{-14}$ |

After performing integration over $q^2$ in Eq.(29) in the interval $m_\ell^2 \leq q^2 \leq (m_{\chi_{b2}} - m_{B_c})^2$, we obtain the total decay widths as presented in table 2 for different leptons. The errors belong to the uncertainties coming from the determination of the working regions for auxiliary parameters as well as those of the other input parameters. The orders of decay rates at all lepton channels show that these transitions are accessible at LHC in near future.
Figure 3: $K(q^2)$ as a function of $q^2$ at $s_0 = 106 \text{ GeV}^2$, $s'_0 = 44 \text{ GeV}^2$, $M^2 = 17 \text{ GeV}^2$ and $M'^2 = 10 \text{ GeV}^2$. The red triangles show the QCD sum rule predictions, yellow-solid line denotes the prediction of fit function obtained using the central values of the input parameters and the green band shows the uncertainty due to errors of input parameters.

In summary, we have calculated the transition form factors for the semileptonic $\chi_{b2} \to B_c \bar{l} \nu$ transition using QCD sum rule technique. We took into account the two-gluon condensate contributions as nonperturbative effects. We used these form factors to estimate the order of decay widths at all lepton channels. The order of decay width reveal that these transitions can be seen at LHC in near future. Any comparison of the experimental results with our predictions can provide us with essential knowledge on the nature of the tensor $\chi_{b2}(1P)$ state.

4 Conflict of Interests

The authors declare that there is no conflict of interest regarding the publication of this paper.

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Figure 4: The same as figure 3 but for $h(q^2)$.

Figure 5: The same as figure 3 but for $b_\pm(q^2)$. 
Figure 6: The same as figure 3 but for $b_-(q^2)$.

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