GAUGE DEPENDENCE OF THE RESUMMED THERMAL GLUON SELF ENERGY

R. Baier\textsuperscript{1}, G. Kunstatter\textsuperscript{2} and D. Schiff\textsuperscript{3}

\textsuperscript{1}Fakultät für Physik, Universität Bielefeld, D-4800 Bielefeld 1, Germany
\textsuperscript{2}Winnipeg Institute for Theoretical Physics and Physics Department, University of Winnipeg, Winnipeg, Manitoba, Canada R3B 2E9
\textsuperscript{3}LPTHE\textsuperscript{†}, Université Paris-Sud, Bâtiment 211, F-91405 Orsay, France

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ABSTRACT The gauge dependence of the hot gluon self energy is examined in the context of Pisarski’s method for resumming hard thermal loops. Braaten and Pisarski have used the Ward identities satisfied by the hard corrections to the n-point functions to argue the gauge fixing independence of the leading order resummed QCD plasma damping rate in covariant and strict Coulomb gauges. We extend their analysis to include all linear gauges that preserve rotational invariance and display explicitly the conditions required for gauge fixing independence. It is shown that in covariant gauges the resummed damping constant is gauge fixing independent only if an infrared regulator is explicitly maintained throughout the calculation.

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\textsuperscript{†} Laboratoire associé du Centre National de la Recherche Scientifique
1. INTRODUCTION

There has recently been a great deal of interest in QCD at high temperature, in part due to the hope that a new state of matter, the quark gluon plasma, might soon be observed in relativistic heavy ion collisions[1]. There are, however, very few observable properties of this plasma that can be predicted directly from the fundamental theory. Most experimental properties are derived from phenomenological models that are often ad hoc and contain many parameters. The few physical quantities that can be calculated using perturbative QCD have recently been the subject of much controversy[2], in part due to the apparent gauge fixing dependence of the gluon plasma damping constant at one loop[3]. This gauge fixing dependence has been attributed by some to the supposed breakdown of perturbative QCD[4], and by others to the need for a manifestly gauge covariant or invariant linear response theory[5]. The correct resolution to this puzzle has recently been proven to lie in the inaccuracy of the naive loop expansion as a self-consistent approximation scheme: an accurate, and gauge independent damping constant can only be obtained by resumming an infinite number of loop diagrams. Although the need to re-sum was realized fairly early[6], an explicit prescription for performing the necessary resummation has only recently been provided by Pisarski[7]. The proof that the resummation does indeed yield a gauge independent damping rate was provided using two distinct methods. The first, due to Braaten and Pisarski (BP)[8], involved explicitly performing the resummation required and using Ward identities satisfied by the improved perturbative n-point functions to show that the same resummed damping constant is obtained to leading order in strict Coulomb gauge and in all covariant gauges. A similar result was also obtained in covariant gauges using a partial resummation technique[9]. The second method, due to Kobes, Kunstatter and Rebhan (KKR)[10] used properties of the generating functional for QCD to provide a non-perturbative proof of the gauge fixing independence of the locations of the physical poles of the gluon propagator.

In effect, both the BP and KKR proofs were based on the fact that the gauge dependence of the physical structure functions vanishes “on-shell”. That is, given the structure function $f(k)$ whose zeroes determine the location of the physical poles in the gluon propagator, one can show that

$$\Delta f(k) = f(k)\Delta X(k) ,$$  

where $\Delta X(k)$ can be calculated perturbatively. Eq.(1.1) has the immediate consequence that the physical pole position (and hence also the mass and damping rates, which are essentially the real and imaginary parts of the pole) are gauge fixing independent, providing that $\Delta X(k)$ does not have poles when $f(k) = 0$. KKR provided a non-perturbative proof
of (1.1) for gauges theories in general, and then discussed how to apply the result to a self-consistent approximation for the gluon plasma damping rate. BP on the other hand, provided a perturbative proof of (1.1) using the resummed Feynman rules appropriate for finite temperature QCD. Clearly, the former proof was very general and necessarily quite formal, while the latter proof considered only a restricted family of gauges. Moreover the proof of Braaten and Pisarski was also formal, to the extent that it did not explicitly calculate the particular $\Delta X(k)$ that arose, but instead assumed that it would be well behaved. KKR argued that the pole structure of $\Delta X(k)$ would be essentially determined by the poles of the full ghost propagator, and hence distinct in general from the solutions to the physical dispersion relations, but as will be shown below, their analysis must be re-evaluated in the presence of infrared divergences.

The purpose of the present paper is to extend the Ward identity analysis of Braaten and Pisarski to display explicitly the gauge fixing dependence of all the components of the resummed gluon self energy and to include all possible linear gauges in which rotational symmetry (in the rest frame of the heat bath) is unbroken. Thus we consider all covariant and non-covariant gauges in which the four-velocity of the heat bath is the only additional vector that appears.

The paper is organized as follows: In Section 2, we present the Feynman rules for one loop QCD calculations in the class of gauges considered. This serves to establish conventions and notation and also to remind the reader of the source of the tree level Ward identities obeyed by the bare n-point functions. Section 3 explains briefly the need for considering higher loop diagrams, and describes how a corrected damping rate can be obtained using the resummation technique proposed by Pisarski[7] and implemented by Braaten and Pisarski[8]. Section 4 uses the Ward identities to analyze in detail the gauge dependence of the gluon self energy, and its associated damping rate, in this improved approximation scheme. In Section 5 we examine the integrals that must be evaluated in Covariant and Coulomb gauges in order to prove gauge fixing independence of the damping constant and find the surprising result that in covariant gauges, the imaginary part of $\Delta X(k)$ does develop poles on the physical mass shell and leading to gauge dependent damping rates unless an infrared regulator is maintained throughout the calculation. This extends to gluons a similar result concerning the gauge dependence of the resummed fermion damping rate[11]. Section 6 closes with conclusions.
2. PRELIMINARIES

For simplicity we consider only gluons. The discussion can be generalized to include quarks without much difficulty. The Feynman rules for the theory are derivable from a generating functional of the form:

\[ Z = \int \prod_{a,\mu,x} dA^a_\mu(x) d\bar{C}^a(x) dC^a(x) \exp(I_{cl} + I_{g.f.} + I_{gh}) . \]  

(2.1)

In the above, \( I_{cl} \) is the classical \( SU(N) \) Yang-Mills action:

\[ I_{cl} = -\frac{1}{4} \int d^4 x F^a_{\mu \nu} F^a_{\mu \nu} , \]

(2.2)

where the colour indices \( a, b, c \ldots \) run from 1 to \( N^2 - 1 \) and the Yang-Mills field strength is

\[ F^a_{\mu \nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^{abc} A^b_\mu A^c_\nu . \]

(2.3)

The classical action is invariant under the following infinitesimal gauge transformations of the vector potential:

\[ \delta A^a_\mu(x) = D^{ac}_\mu(x) \lambda^c(x) , \]

(2.4)

where we have defined the covariant derivative operator:

\[ D^{ab}_\mu(x) \equiv \delta^{ab} \partial_\mu - gf^{abc} A^c_\mu(x) \]

(2.5)

and \( \lambda^a(x) \) is an arbitrary, infinitesimal Lie algebra valued field. The fields \( \bar{C}^a \) and \( C^a \) are the Grassman-valued Faddeev-Popov ghosts necessary to make the source-free generating functional independent of the gauge fixing term \( I_{g.f.} \), which is of the form:

\[ I_{g.f.} = -\frac{1}{2\xi} \int d^4 x (\mathcal{F}^a)^2 , \]

(2.6)

where \( \mathcal{F}^a \) is, in principle, an arbitrary function of the vector potential, restricted only by the condition that its associated Faddeev-Popov operator be invertible. We will restrict the consideration to linear gauges

\[ \mathcal{F}^a = \mathcal{F}^\mu(x) A^a_\mu(x) , \]

(2.7)

where \( \mathcal{F}^\mu \) is an arbitrary (possibly non-local) differential operator, in which case the Faddeev-Popov operator takes the form:

\[ Q^{ab} \equiv \frac{\delta \mathcal{F}^b}{\delta A^c_\mu(x)} D^{ac}_\mu(x) = \mathcal{F}^\mu(x) D^{ab}_\mu(x) . \]

(2.8)
The ghost action is then:

\[ I_{gh} = \int d^4x \overline{C}^a \mathcal{F}^\mu \mathcal{D}_\mu^{ab} C^b. \]  

(2.9)

Since we are interested in finite temperature Green’s functions\+[12] using the imaginary time formalism, we will assume that we have already Wick rotated to Euclidean space-time, with metric signature \((++++)\). All functions are therefore periodic in imaginary time with period \(\beta = 1/T\), as required by the Matsubara formalism. This in turn requires the energy, \(k_0\), to be an integer multiple of \(2\pi T\). The retarded real time thermal Green’s functions from which the plasma parameters can be extracted are obtained from the imaginary time Green’s functions by analytically continuing the external momenta back to real time, after all Euclidean sums and integrals over internal momenta have been performed.

At finite temperature, the velocity of the heat bath introduces a preferred direction in space-time which breaks manifest Lorentz invariance. We will denote this velocity by the four vector \(n^\mu\), and assume that we are in the rest frame of the heat bath, so that \(n^\mu = \delta_0^\mu\). If the gauge fixing condition does not break rotational invariance, then the momentum space representation for the most general \(\mathcal{F}^\mu\) is:

\[ \mathcal{F}^\mu = d(k)k^\mu + b(k)n^\mu \]  

(2.10)

for arbitrary functions \(d(k)\) and \(b(k)\) of the four momentum \(k^\mu\).

In order to simplify the following discussion it is useful to introduce some relevant tensors. First of all, we will need the projection operator:

\[ P^\nu_\mu = \delta^\nu_\mu - \frac{k^\nu k_\mu}{k^2}, \]  

(2.11)

where \(k^2 \equiv (k^0)^2 + \vec{k}^2\). This can be used to isolate the component of \(n^\mu\) that is orthogonal to the four momentum, \(k^\mu\), namely:

\[ \tilde{n}^\mu(k) \equiv P^\mu_\nu n^\nu = \delta_0^\mu - k^\mu \frac{n \cdot k}{k^2}. \]  

(2.12)

Note that \(\tilde{n}^2 = \vec{k}^2/k^2\). It will be useful to re-express the momentum space gauge fixing condition in terms of the resulting orthogonal basis:

\[ \mathcal{F}^\mu = a(k)k^\mu + b(k)\tilde{n}^\mu, \]  

(2.13)

where \(a(k) = d(k) + b(k)n \cdot k/k^2\).
Finally, we define:

\[ A^{\mu\nu} \equiv P^{\mu\nu} - \frac{\tilde{n}^{\mu} \tilde{n}^{\nu}}{\tilde{n}^2} = \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} - \frac{k_i k_j}{k^2} \end{pmatrix}, \]  \hspace{1cm} (2.14)

which projects out spatially transverse modes, and is orthogonal to both \( k^\mu \) and \( \tilde{n}^\mu \).

The corresponding Feynman rules are as follows:

The bare 2-point function is

\[ (D^{(0)-1})_{\mu\nu}^{ab} = \delta^{ab} \left( k^2 P_{\mu\nu} + \frac{1}{\xi} (b \tilde{n}_\mu + ak_\mu) (b \tilde{n}_\nu + ak_\nu) \right), \]  \hspace{1cm} (2.15)

with associated propagator:

\[ \delta^{ab} D^{(0)}_{\mu\nu} (k) = \delta^{ab} \left( \frac{1}{k^2} P_{\mu\nu} - \frac{b}{ak^4} (\tilde{n}_\mu k_\nu + \tilde{n}_\nu k_\mu) + \left( \frac{\xi k^2 + \tilde{n}^2 b^2}{a^2 k^4} \right) k_\mu k_\nu \right). \]  \hspace{1cm} (2.16)

The ghost propagator and ghost-ghost-gluon vertex in this family of gauges are, respectively:

\[ D^{ab} = \delta^{ab} \frac{1}{a(k)k^2}, \]  \hspace{1cm} (2.17)

and

\[ gf^{abc} \Gamma^\mu (k) = gf^{abc} (ak^\mu + b\tilde{n}^\mu). \]  \hspace{1cm} (2.18)

Due to the linearity of the gauge fixing condition, the three and four point functions depend only on the classical action, and are, respectively:

\[ -igf^{abc} \Gamma^{\mu\nu\rho} (p, q, k) = -igf^{abc} ((p - q)^\rho \delta^{\mu\nu} + \text{perm}), \]  \hspace{1cm} (2.19)

and

\[ -g^2 N \Gamma^{\mu\nu\rho\tau} (p, q, r, s) = -g^2 N (2\delta^{\mu\nu} \delta^{\rho\tau} - \delta^{\mu\rho} \delta^{\nu\tau} - \delta^{\mu\tau} \delta^{\nu\rho}), \]  \hspace{1cm} (2.20)

where \( p + q + k = 0 \) and we have traced over the colour indices in the four point function.

An important aspect of the above bare n-point functions is that they obey the following Ward identities (suppressing colour indices):

\[ k_\mu \Gamma^{\mu\nu} (k) = 0, \]  \hspace{1cm} (2.21a)

\[ k_\rho \Gamma^{\mu\nu\rho} (p, q, k) = -\Gamma^{\mu\nu} (p) + \Gamma^{\mu\nu} (q), \]  \hspace{1cm} (2.21b)

\[ s_\sigma \Gamma^{\mu\nu\lambda\sigma} (p, q, r, s) = \Gamma^{\mu\nu\lambda} (p + s, q, r) - \Gamma^{\mu\nu\lambda} (p, q + s, r), \]  \hspace{1cm} (2.21c)
where $\Gamma^{\mu\nu} \equiv k^2 P^{\mu\nu}$ refers to the gauge independent part of the tree level 2-point function in (2.15) above. These identities follow directly from the gauge invariance of the classical action from which the n-point functions were derived. In particular, one can express the gauge invariance of the classical action in terms of the following functional differential equation:

$$D_{\mu}^{ab} \frac{\delta I_{cl}}{\delta A_{\mu}^b(x)} = 0,$$

which expresses the fact that the directional derivatives of the action along the gauge orbits are zero. The Ward identities then can be derived by functionally differentiating this equation with respect to the vector potential, and then evaluating the resulting expression at $A_{\mu}^a = 0$. Differentiating once yields (2.21a), twice yields (2.21b) and three times yields (2.21c).
3. RESUMMED PLASMA PARAMETERS

The loop expansion does not provide a self-consistent approximation for the finite temperature QCD damping rate. In general, an infinite number of loop orders contribute to the leading order result[6]. This can be understood by noting that at finite temperature, QCD contains two independent mass scales: $T$ and $gT \ll T$. Momenta that are of order $T$ or greater are called “hard”, while those that are of order $gT$ or smaller are called “soft”. For n-point processes in which all legs are of order $gT$, one loop corrections in which all internal momenta are hard contribute to the process to the same order as tree level[7]. Moreover, these are the only such large “corrections”. This implies, for example, that the dispersion relations can be consistently calculated to order $g^2T^2$ by considering only one loop self energy diagrams with hard internal momenta. No higher loop corrections are required. Explicit calculations show that the hard loop self energy diagram is real. The damping constant therefore appears only at the next order in the resummed perturbative expansion, namely $g^2T$. One loop calculations to this order require summing over diagrams with soft internal legs and as mentioned above, for soft processes (internal or external) one needs to incorporate the hard thermal loop corrections to the vertices and the propagator.

It is worth stressing that this algorithm is only valid for the leading order $(g^2T)$ damping constant, and must be modified in order to calculate corrections either to the dispersion relations, or the damping rate. For example, in order to calculate the real part of the dispersion relations to order $g^2T$, it is necessary to consider higher loop diagrams involving the modified n-point functions[7].

Remarkably, the hard thermal loop corrections to the n-point functions are gauge fixing independent and obey the same Ward identities as the tree-level n-point functions [7,8,13]. \footnote{The gauge invariance of the hard thermal loop contributions to the quark self energy was first shown by Klimov and Weldon[14].}

Let $\delta\Pi^{\mu\nu}$, $\delta\Gamma^{\mu\nu\rho}$, and $\delta\Gamma^{\mu\nu\rho\sigma}$ denote the hard thermal loop corrections to the two, three and four-point functions, respectively. Then it can be shown that[8,13]:

\begin{align}
  k_{\mu}^{\ast} \Gamma^{\mu\nu}(k) &= 0 , \\
  k_{\rho}^{\ast} \Gamma^{\mu\nu\rho}(p,q,k) &= - \Gamma^{\mu\nu}(p) + \Gamma^{\mu\nu}(q) , \\
  s_{\sigma}^{\ast} \Gamma^{\mu\nu\lambda\sigma}(p,q,r,s) &= \Gamma^{\mu\nu\lambda}(p+s,q,r) - \Gamma^{\mu\nu\lambda}(p,q+s,r) ,
\end{align}

where we have defined the corrected n-point functions:

\begin{equation}
  \Gamma^{\mu\nu} = k^2 P^{\mu\nu} + \delta\Pi^{\mu\nu} ,
\end{equation}

\footnote{The gauge invariance of the hard thermal loop contributions to the quark self energy was first shown by Klimov and Weldon[14].}
\[ *\Gamma^{\mu\nu\rho} = \Gamma^{\mu\nu\rho} + \delta\Gamma^{\mu\nu\rho} , \]  
\[ *\Gamma^{\mu\nu\rho\sigma} = \Gamma^{\mu\nu\rho\sigma} + \delta\Gamma^{\mu\nu\rho\sigma} . \]  
\( (3.2b) \quad (3.2c) \)

The tree level Ward identities obeyed by the \( *\Gamma \) imply that the resummed n-point functions can be derived from a gauge invariant (resummed) effective Lagrangian. The existence of this effective Lagrangian, which has explicitly been constructed in Refs.[13,15], plays an important role in understanding the connection between the BP and KKR proofs of gauge fixing independence.

In terms of the \( *\Gamma \) n-point functions, the resummed expression for the leading order imaginary part of the real time gluon self energy is:

\[
Im*\Pi^{\mu\nu}(K_0,|\vec{k}|) = \frac{g^2N}{2}Im\int_{soft} dp *D_{\alpha\beta} *\Gamma^{\mu\nu\alpha\beta}(k, -k, p, -p) \\
- \frac{g^2N}{2}Im\int_{soft} dp *\Gamma^{\alpha\beta\mu}(p, q, k) *D_{\alpha\alpha'}(p) *D_{\beta\beta'}(q) *\Gamma^{\alpha'\beta'\nu}(p, q, k) \\
- g^2NIm\int_{soft} dp \frac{(a(p)p^\mu + b(p)\tilde{n}^\mu)(a(q)q^\nu + b(q)\tilde{n}^\nu)(q))}{a(p)p^2a(q)q^2},
\]

\( (3.3) \)

where \( *D^{\mu\nu} \) is the effective propagator defined as:

\[
*D^{\mu\nu} = \left((D^{(0)})_{\mu\nu}^{-1} + \delta\Pi_{\mu\nu}\right)^{-1} \\
= ( *\Gamma_{\mu\nu} + \frac{1}{\xi}(b\tilde{n}_\mu + ak_\mu)(b\tilde{n}_\nu + ak_\nu))^{-1}.
\]

\( (3.4) \)

In the above and what follows, \( Im \) refers to the imaginary part of the integrals after analytic continuation of the external momenta: \( ik_0 \to K_0 + i\epsilon \), and

\[
\int_{soft} dp \equiv T \sum_{p_0=2\pi nT} \int_{soft} \frac{d^3\vec{p}}{(2\pi)^3},
\]

\( (3.5) \)

where the spatial integration is performed over soft momenta only, and therefore must be cut-off at some value \( \Lambda \) such that \( gT << \Lambda << T \). In the calculation of the imaginary part of the self energy, which is all that we will be concerned with here, the precise value of the cut-off turns out to be irrelevant.

Standard linear response theory[16] implies that the response of the plasma to small external perturbations is determined by the physical poles in the retarded thermal gluon propagator. It is convenient to write the full propagator in terms of the hard thermal loop propagator, \( *D_{\mu\nu} \), (suppressing colour indices) and the resummed self energy \( *\Pi_{\mu\nu} \) (with the imaginary part at order \( g^2T \) given in (3.3)).
\[ D^{\mu\nu} = (D_{\mu\nu}^{-1} + \Pi_{\mu\nu})^{-1} = \frac{1}{T(k)} A^{\mu\nu} + \frac{1}{L(k)} \left[ \tilde{n}^\mu \tilde{n}^\nu + \frac{1}{C(k)} (\tilde{n}^\mu k^\nu + \tilde{n}^\nu k^\mu) + \frac{1}{D(k)} \frac{k^\mu k^\nu}{k^2} \right] . \quad (3.6) \]

For the class of gauges given in Section 2, the most general form of the gluon self energy in momentum space is:

\[ *\Pi^{\mu\nu} = *\Pi_t(k) A^{\mu\nu} + *\Pi_l(k) \frac{\tilde{n}^\mu \tilde{n}^\nu}{\tilde{n}^4} + *\Pi_c(k) (\tilde{n}^\mu k^\nu + \tilde{n}^\nu k^\mu) + *\Pi_d(k) \frac{k^\mu k^\nu}{k^2} . \quad (3.7) \]

Note that it is not transverse in general. The components of the propagator in (3.6) can be related to those of the self energy as follows[17]:

\[ T(k) \equiv k_T^2 + \Pi_t , \quad (3.8a) \]
\[ L(k) \equiv k_L^2 + \Pi_l + \frac{b^2 \tilde{n}^4}{\xi} \left[ 1 - \frac{(1 + \xi *\Pi_c/ab)^2}{(1 + \xi *\Pi_d/a^2 k^2)} \right] , \quad (3.8b) \]
\[ C(k) \equiv -\frac{a}{b \tilde{n}^2} \frac{(k^2 + \xi *\Pi_d/a^2)}{(1 + \xi *\Pi_c/ab)} , \quad (3.8c) \]
\[ D(k) = \frac{a^2 k^2}{\xi} \left( \frac{1 + \xi *\Pi_d/k^2 a^2}{k_L^2 + \Pi_l + \tilde{n}^4 b^2/\xi} \right) , \quad (3.8d) \]

where we have defined

\[ k_T^2 \equiv k^2 + \delta \Pi_T , \quad (3.9) \]

and

\[ k_L^2 \equiv k^2 + \delta \Pi_L . \quad (3.10) \]

In terms of the above decomposition:

\[ *\Gamma^{\mu\nu} = k_T^2 A^{\mu\nu} + k_L^2 \frac{\tilde{n}^\mu \tilde{n}^\nu}{\tilde{n}^4} , \quad (3.11) \]

consistent with the Ward identity (3.1a).

The explicit expressions for the hard thermal loop corrections to the two-point function are, after analytic continuation back to real time \(ik_0 \to K_0 + i\epsilon[8,14]:\)

\[ \delta\Pi_T(K_0, |\vec{k}|) \sim + \frac{3m_s^2 K_0^2}{2k^2} \left[ 1 - \left(1 - \frac{k^2}{K_0^2}\right) \frac{K_0}{2|\vec{k}|} ln \left(\frac{K_0 + |\vec{k}|}{K_0 - |\vec{k}|}\right) \right] , \quad (3.12) \]
and

\[ \delta \Pi_L(K_0, k^2) \sim +3m_g^2 \left[ 1 - \frac{K_0}{2|k|} \ln \left( \frac{K_0 + |k|}{K_0 - |k|} \right) \right], \tag{3.13} \]

where \( m_g^2 \equiv g^2 NT^2/9 \) in the absence of quark fields. The hard thermal loop corrections to the three and four-point functions are not required in the following discussion. In fact, as shown by Braaten and Pisarski[8], and elaborated below, the specific form of the corrected n-point functions is irrelevant to the formal proof of gauge independence of the resummed plasma damping rates. It is necessary and sufficient that the n-point functions satisfy the Ward identities (3.1a)-(3.1c).

The physical modes of the gluon in the plasma are transverse with respect to the momentum four-vector. As can be seen above, the presence of the preferred four-vector \( \tilde{n}^{\mu} \) leads to the existence of two independent physical modes at finite temperature. The first, determined by the coefficient of \( A^{\mu\nu} \) is the usual spatially transverse gluon mode. The second is a spatially longitudinal collective mode which is absent at zero temperature and is usually called the “plasmon mode”. The dispersion relations and damping rates for the gluon and plasmon modes are therefore obtained from the real and imaginary parts, respectively of the solutions to:

\[ \mathcal{T}(k) = 0, \tag{3.14a} \]
\[ \mathcal{L}(k) = 0. \tag{3.14b} \]

Note that in principle, the non-transverse parts of the self energy can contribute to the longitudinal dispersion relation and damping rate. However, in linear gauges, \( *\Pi_c \) and \( *\Pi_d \) are not independent, but are related by the following (exact) Ward identity[18]:

\[ \mathcal{F}_\mu \mathcal{F}_\nu D^{\mu\nu} = \xi, \tag{3.15} \]

which, in terms of the parametrizations given above reduces to:

\[ *\Pi_d (k_L^2 + *\Pi_l) = \tilde{n}^4 k^2 *\Pi_c^2. \tag{3.16} \]

Eq.(3.16) can be used to express the longitudinal structure function in the following form[17]:

\[ \mathcal{L}(k) = \frac{(k_L^2 + *\Pi_l - \frac{\tilde{n}^4}{n^2} *\Pi_c)^2}{(k_L^2 + *\Pi_l + \frac{\tilde{n}^4}{n^2} *\Pi_c^2)}. \tag{3.17} \]

Thus, in general, it appears that \( *\Pi_c \) can not only shift the location of the longitudinal pole, but also change the order of the pole from first order to second order. One of the
results of the present paper, however, is to show that in linear gauges, \( \Pi_c \) does not in fact contribute to the leading order dispersion relations. This has previously been only verified in strict Coulomb and covariant gauges\[8\].

Given the two structure functions \( T \) and \( L \), it is straightforward to derive the dispersion relations \( K_{0,t,l}^2 = \omega_{t,l}(\vec{k}) \), which are determined by the zeros of the structure functions. Normally one assumes that the system is underdamped, so that the imaginary part of the pole is small compared to the real part. That is:

\[
K_0 = \omega_p - i\gamma,
\]

where \( \gamma \ll \omega_p \). In this case, given a general structure function of the form: \( K_{0}^2 - f(K_0, \vec{k}) \), the dispersion relation becomes:

\[
\omega_p^2 = \text{Re} f(\omega_p, \vec{k}),
\]

while the damping rate is:

\[
\gamma = \left. \frac{\text{Im} f(\omega, \vec{k})}{\omega^2 \frac{\partial}{\partial \omega} \left[ \frac{\text{Re} f(\omega, \vec{k})}{\omega^2} \right]} \right|_{\omega=\omega_p}.
\]

For the gluon excitation at rest (\(|\vec{k}| = 0\)), Eqs. (3.12) and (3.13) yield the same mass for the longitudinal and transverse modes, namely

\[
m_{T,L}^2 = m_T^2.
\]

Application of the above techniques in Coulomb gauge yield a damping rate of\[8\]:

\[
\gamma_{T,L} \sim 6.63 \frac{g^2 NT}{24\pi}.
\]

In reference \[8\] the Ward identities were used to prove that the same damping rate would result in covariant gauges, while Ref.\[10\] provided general arguments concerning the gauge independence of \( \gamma \). In the following section we examine explicitly the gauge dependence of the imaginary part of the leading order resummed self energy (3.3) in a general class of linear gauges.
4. GAUGE DEPENDENCE OF THE RESUMMED SELF ENERGY

We will now calculate the gauge dependent part of the imaginary part of the resummed self energy as given by Eq.(3.3) above. For convenience we write the propagator (3.4) in the following form:

\[
\ast D^{\mu\nu} = \frac{1}{k^2_T} A^{\mu\nu} + \frac{1}{k^2_L} \tilde{n}^\mu \tilde{n}^\nu + \frac{\beta(k)}{k^2_L} \left( \tilde{n}^\mu k^\nu + \tilde{n}^\nu k^\mu \right) + \frac{\alpha(k)}{k^2_L} \frac{k^\mu k^\nu}{k^2}, \tag{4.1}
\]

where we have defined

\[
\beta(k) \equiv -\frac{b}{a} \frac{\tilde{n}^2}{k^2}, \tag{4.2}
\]

\[
\alpha(k) \equiv \frac{\xi k^2_L}{a^2 k^2} + k^2 \beta^2. \tag{4.3}
\]

Since the modified 3- and 4-point functions are gauge fixing independent, all the gauge dependence in the self energy will come from the gauge dependent terms in the propagator. This can be isolated by defining:

\[
\ast D^{\mu\nu} = \ast D'^{\mu\nu} + \Delta \ast D^{\mu\nu}, \tag{4.4}
\]

where we consider the gauge variation about an arbitrary “fiducial” gauge with resummed propagator:

\[
\ast D'^{\mu\nu} \equiv \frac{1}{k^2_T} A^{\mu\nu} + \frac{1}{k^2_L} \tilde{n}^\mu \tilde{n}^\nu + \frac{\beta_0(k)}{k^2_L} \left( \tilde{n}^\mu k^\nu + \tilde{n}^\nu k^\mu \right) + \frac{\beta_0^2}{k^2_L} k^\mu k^\nu. \tag{4.5}
\]

Note that the above form of the propagator assumes that \( \xi_0 = 0 \) in the fiducial gauge, so that the gauge fixing condition is enforced by a delta function in the path integral. For example, if the fiducial gauge is strict Coulomb gauge, then \( \beta_0 = k_0^2/k^2 \), while in strict Covariant gauge, \( \beta_0 = 0 \). These examples will be treated in detail below. The gauge dependent part of the propagator can now be written:

\[
\Delta \ast D^{\mu\nu} = \frac{\Delta \beta}{k^2_L} \left( \tilde{n}^\mu k^\nu + \tilde{n}^\nu k^\mu \right) + \frac{\Delta \alpha}{k^2_L} \frac{k^\mu k^\nu}{k^2}, \tag{4.6}
\]

where \( \Delta \beta \equiv \beta - \beta_0 \) and

\[
\Delta \alpha \equiv k^2 (\beta^2 - \beta_0^2) + \frac{\xi k^2_L}{a^2 k^2}. \tag{4.7}
\]

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The expression for the resummed self energy \( *\Pi^{\mu\nu} \) in Eq.(3.3) can easily be separated into its gauge independent (for a specific fiducial gauge) and gauge dependent parts. We consider each term in expression (3.3) in turn. The first term, involving the 4-point function, has the following gauge dependent contribution:

\[
Im < \Delta^* D * \Gamma^{(4)} > = \frac{g^2 N}{2} Im \int_{soft} dp \Delta^* D_{\alpha\beta} * \Gamma^{\mu\nu\alpha\beta} = g^2 N Im \int_{soft} dp \left[ \frac{\Delta \beta(p)}{p^2 L} (\tilde{n}_\alpha(p)p_\beta + \tilde{n}_\beta(p)p_\alpha) + \frac{\Delta \alpha p_\alpha p_\beta}{p^2} \right] * \Gamma^{\mu\nu\alpha\beta}(k, -k, p, -p) \\
= g^2 N Im \int_{soft} dp \frac{\Delta \beta(p)}{p^2 L} \tilde{n}_\alpha(p)( * \Gamma^{\mu\nu\alpha}(k, q, p) + * \Gamma^{\mu\nu\alpha}(k, q, p)) - g^2 N Im \int_{soft} dp \frac{\Delta \alpha}{p^2 L p^2} (2 * \Gamma^{\mu\nu}(k) - * \Gamma^{\mu\nu}(k - p) - * \Gamma^{\mu\nu}(q)) ,
\]

(4.8)

where the last line was obtained by applying the Ward identities (3.1b) and (3.1c) successively. Note that \( \Delta \beta(-p) = -\Delta \beta(p) \) and \( \Delta \alpha(-p) = \Delta \alpha(p) \).

The gauge dependent contribution from the 3-point functions (2nd term in (3.3)) is considerably more complicated. It can, however, be simplified by applying the Ward identities whenever an internal 4-momentum is contracted with one leg of either a three, or two point function. The result can be summarized as follows:

\[
Im \left[ < * \Gamma^{(3)} \Delta^* D * \Gamma^{(3)} > + < * \Gamma^{(3)} * D \Delta^* D * \Gamma^{(3)} > + < * \Gamma^{(3)} \Delta^* D \Delta^* D * \Gamma^{(3)} > \right] = - \left\{ X_\mu^\alpha(k) * \Gamma^{\alpha\nu} + X_\nu^\alpha(k) * \Gamma^{\alpha\mu} + * \Gamma^{\alpha\beta} X_{\alpha\alpha'} * \Gamma^{\alpha'\nu} \right. \\
+ g^2 N Im \int_{soft} dp \left[ \frac{\Delta \beta(p)}{p^2 L} \tilde{n}_\beta(p) * \Gamma^{\nu\beta\mu}(q, p, k) + (\mu \leftrightarrow \nu) \right] \\
+ g^2 N Im \int_{soft} dp \frac{\Delta \alpha(p)}{p^2 L p^2} * \Gamma^{\mu\nu}(q) \\
+ g^2 N Im \int_{soft} dp \left[ \frac{\Delta \beta(p)\tilde{n}_\mu(p)}{\tilde{n}^2(p)} \left( \frac{q'}{q^2} - \frac{\beta_0(q)\tilde{n}'(q)}{\tilde{n}^2(q)} \right) + (\mu \leftrightarrow \nu) \right] \\
\left. - \frac{g^2 N}{2} Im \int_{soft} dp \frac{\Delta \beta(p)\Delta \beta(q)}{\tilde{n}^2(p)\tilde{n}^2(q)} \left[ \tilde{n}_\mu(p)\tilde{n}'(q) + (\mu \leftrightarrow \nu) \right] \right\} ,
\]

(4.9)

where we have used the identity:

\[
\tilde{n}_\mu(k) * \Gamma^{\mu\nu}(k) = \vec{k}_L \tilde{n}'(q) / \tilde{n}^2 ,
\]

(4.10)
and defined:

\[ X^{\mu\alpha} = g^2 N\text{Im} \int_{\text{soft}} dp \frac{\Delta\beta(q)}{q_L^2 q^2} \left( \tilde{n}_{\beta'}(q) \ast \Gamma^{\alpha'\beta'\mu}(p, q, k) \ast D'_{\alpha'}(p) + \tilde{n}_{\alpha}(q) \left[ \frac{p^\mu}{p^2} - \beta_0(p) \frac{\tilde{n}_\mu(p)}{\tilde{n}_\nu(p)} \right] \right) \]

\[ - g^2 N\text{Im} \int_{\text{soft}} dp \frac{\Delta\alpha(q)}{q_L^2 q^2} \left[ p^{\mu\alpha}(p) + \beta_0(p) \frac{p^{\alpha\mu}(p)}{\tilde{n}_\nu(p)} \right] \]

\[ - g^2 N\text{Im} \int_{\text{soft}} dp \frac{\Delta\alpha(q) \Delta\beta(p)}{q_L^2 q^2} p^{\alpha\mu} \frac{\tilde{n}_{\mu}(p)}{\tilde{n}_\nu(p)} \]

\[ - \frac{g^2 N}{2} \text{Im} \int_{\text{soft}} dp \frac{\Delta\beta(p) \Delta\beta(q)}{p_L^2 q_L^2} \left( \tilde{n}_{\alpha}(p) \tilde{n}_{\beta'}(q) \ast \Gamma^{\alpha'\beta'\mu}(p, q, k) p^{\alpha} \right) \]

\[ + \frac{g^2 N}{2} \text{Im} \int_{\text{soft}} dp \Delta\beta(p) \Delta\beta(q) \left[ \frac{\tilde{n}_{\mu}(p) \tilde{n}_{\nu}(q)}{\tilde{n}_\nu(p) q_L^2} + \frac{\tilde{n}_{\mu}(q) \tilde{n}_{\nu}(p)}{\tilde{n}_\nu(q) p_L^2} \right], \quad (4.11) \]

and

\[ Y_{\alpha\alpha'} = g^2 N\text{Im} \int_{\text{soft}} dp \frac{\Delta\alpha(q)}{q_L^2 q^2} \ast D'_{\alpha'}(p) \]

\[ - g^2 N\text{Im} \int_{\text{soft}} dp \frac{\Delta\beta(p) \Delta\beta(q)}{p_L^2 q_L^2} \tilde{n}_{\alpha}(p) \tilde{n}_{\alpha'}(q) \]

\[ + g^2 N\text{Im} \int_{\text{soft}} dp \frac{\Delta\beta(p) \Delta\alpha(q)}{p_L^2 q_L^2 q^2} \left[ \tilde{n}_{\alpha}(p) p_{\alpha'} + \tilde{n}_{\alpha'}(p) p_{\alpha} \right] \]

\[ + \frac{g^2 N}{2} \text{Im} \int_{\text{soft}} dp \frac{\Delta\alpha(p) \Delta\alpha(q) p_{\alpha} p_{\alpha'}}{p_L^2 q_L^2 q^2} \]. \quad (4.12) \]

Finally, the gauge dependent part of the ghost contribution is

\[ \text{Im} < \Delta_{\text{ghost}} > = 2 g^2 N\text{Im} \int_{\text{soft}} dp \frac{\Delta\beta(q)}{\tilde{n}_\nu(q)} \left( \frac{p^\mu}{p^2} - \beta_0(p) \frac{\tilde{n}_\mu(p)}{\tilde{n}_\nu(p)} \right) \]

\[ - g^2 N\text{Im} \int_{\text{soft}} dp \frac{\Delta\beta(p) \Delta\beta(q)}{\tilde{n}_\nu(p) \tilde{n}_\nu(q)} \tilde{n}_\mu(p) \tilde{n}_\nu(q). \quad (4.13) \]

It should be noted that in order to derive the above expressions, extensive use was made of the identity (4.10) and:

\[ \text{Im} \int_{\text{soft}} df(p, q) = \text{Im} \int_{\text{soft}} df(q, p), \quad (4.14) \]

which is valid providing the cut-off is much greater than the external momentum.

After summing (4.8), (4.9) and (4.13), one finds a complete cancellation of all terms that do not contain at least one factor of the gauge invariant two point function \( \ast \Gamma^{\mu\nu}(k) \).
The final expression for the gauge dependent part of the imaginary part of the self energy, therefore takes the simple form:

\[ Im \Delta^* \Pi^{\mu\nu}(k) = Z(k) \ast \Gamma^{\mu\nu}(k) - \ast \Gamma^{\mu\alpha} X^{\alpha\nu} - X^{\mu\alpha} \ast \Gamma^{\alpha\nu} - \ast \Gamma^{\mu\alpha} Y_{\alpha\alpha'} \ast \Gamma^{\alpha'\nu}, \] (4.15)

where we have defined

\[ Z(k) \equiv -g^2 N Im \int_{soft} dp \frac{\Delta \alpha(p)}{\bar{p}^2 L p^2}, \] (4.16)

and \( X^{\mu\alpha} \) and \( Y^{\alpha\nu} \) are given in (4.11) and (4.12) above. It should be stressed that Eq.(4.15) above is valid for finite and/or infinitesimal gauge changes.

Expression (4.15) contains the desired information about the gauge dependence of the resummed self energy. First of all, by contracting with \( k^\mu k^\nu \), using the decomposition (3.7) and the transversality of \( \ast \Gamma^{\mu\nu} \), one can easily verify that the gauge dependent part of \( \ast \Pi_d \) is identically zero to this order. This is consistent with the Ward identity (3.16) which implies that, if \( \ast \Pi_c \sim g^2 T \), then \( \ast \Pi_d \) must vanish to this order.

Secondly, by contracting \( Im \Delta^* \Pi^{\mu\nu} \) in (4.15) with \( \bar{k}_L^{\mu} \bar{n}_\nu \) one obtains:

\[ Im \Delta^* \Pi_c = \frac{1}{\bar{n}^2} \bar{k}^2 L \bar{n}^\alpha \frac{1}{\bar{k}^2} \bar{n}^\alpha \bar{n}^{\alpha\nu} k^\nu. \] (4.17)

To derive (4.17) one needs to use the transversality of \( \ast \Gamma^{\mu\nu} \), and the identity (4.10). Since Braaten and Pisarski verified that \( Im \ast \Pi_c \) vanishes on the longitudinal mass shell in strict Coulomb gauge, we have proven that this will be true in any gauge, providing only that the coefficient of \( \bar{k}^2_L \) in (4.17) does not have a pole at \( \bar{k}^2_L = 0 \). On this basis one can conclude that the non-transverse part of the resummed self energy will not contribute to the damping constant of the longitudinal mode in any gauge.

By contracting (4.15) with the projector \( A_{\mu\nu} \), one finds that:

\[ Im \Delta^* \Pi_t(k) = \frac{1}{\bar{n}^2} \bar{k}^2_L Z(k) - \frac{\bar{k}^2_L}{2} A_{\mu\nu} Y^{\mu\nu}, \] (4.18)

while contracting with \( \bar{n}_\mu \bar{n}_\nu \) yields

\[ Im \Delta^* \Pi_t(k) = \frac{1}{\bar{n}^2} \bar{k}^2_L Z(k) - \frac{\bar{k}^2_L}{\bar{n}^2} \bar{n}_\mu \bar{n}_\nu X^{\mu\nu} \] (4.19)

Thus, the gauge variation of the structure functions vanishes on shell, providing that the corresponding coefficients in the above equations do not develop poles on the mass shell. As discussed in the introduction and in Ref.[10], this implies the gauge fixing independence of the resummed damping constants, and generalizes the formal proof of Braaten and Pisarski[8], who considered only Coulomb and Covariant gauges. In the next section we examine in more detail the integrals that appear in (4.18).
5. DAMPING RATE GAUGE DEPENDENCE FOR COVARIANT AND COULOMB GAUGES

Let us study the gauge variation of the transverse structure function as an example. We will evaluate explicitly the coefficients of $k_T^2$ in (4.18) in order to check their behaviour on mass-shell ($k_T^2 = 0$).

Covariant Gauges

We start by investigating variations around a fiducial covariant gauge, taking

$$
\Delta \beta = 0 \quad (\beta = \beta_0 = 0), \quad \Delta \alpha = \xi \frac{\vec{k}^2}{k^2}
$$

(with a trivial change of notation for the gauge parameter as compared with (4.7)).

Using the expressions for $X^{\mu \nu}$ and $Y^{\mu \nu}$ as given in (4.11) and (4.12) and the following identities:

$$
A^{\mu \nu}(k) P^{\mu \nu}(p) = 2 - \frac{\vec{p}^2}{p^2} + \frac{(\vec{k} \cdot \vec{p})^2}{k^2 p^2}
$$

$$
A^{\mu \nu}(k) A^{\mu \nu}(p) = 1 + \frac{(\vec{k} \cdot \vec{p})^2}{k^2 p^2}
$$

$$
A^{\mu \nu}(k) \tilde{n}^{\mu}(p) \tilde{n}^{\nu}(p) = \frac{p^2}{p^4} \frac{\vec{p}^2}{k^2} \left( 1 - \frac{(\vec{k} \cdot \vec{p})^2}{k^2 p^2} \right)
$$

$$
A^{\mu \nu}(k) p^{\mu} p^{\nu} = \vec{p}^2 - \frac{(\vec{k} \cdot \vec{p})^2}{k^2}
$$

where $p^2 = p_0^2 + \vec{p}^2$, we get

$$
Im \Delta^* \Pi_t = -\xi g^2 N Im \left[ -k_T^2 \int_{soft} \frac{dp}{q^4} \left( 1 - \frac{\vec{p}^2}{p^2} + \frac{(\vec{k} \cdot \vec{p})^2}{k^2 p^2} \right) \right.
$$

$$
+ \frac{1}{2} k_T^4 \int_{soft} \frac{dp}{q^4} \left\{ \frac{1}{p^2} \left( 1 + \frac{(\vec{k} \cdot \vec{p})^2}{k^2 \vec{p}^2} \right) + \frac{p_0^2 \vec{p}^2}{\vec{p}_L^2 p^4} \left( 1 - \frac{(\vec{k} \cdot \vec{p})^2}{k^2 \vec{p}^2} \right) \right\}
$$

$$
+ \frac{\xi}{4} k_T^4 \int_{soft} \frac{dp}{q^4 p^4 \vec{p}^2} \left( 1 - \frac{(\vec{k} \cdot \vec{p})^2}{k^2 \vec{p}^2} \right) \right].
$$

(5.2)

It should be noted that, in the derivation of (5.2) from (4.18), the analytic continuation of the external momentum is implicitly assumed so that $Im \Delta^* \Pi_t(k) \equiv Im \Delta^* \Pi_t(K_0, \vec{k})$. 

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In the following, we evaluate the two integrals in (5.2) that are potentially dangerous on-shell: \( I_1(k) \) and \( I_2(k) \) denoted respectively as "transverse" and "longitudinal" and given by:

\[
I_1(k) = \text{Im} \int_{\text{soft}} \frac{dp}{q^4 p_T^2}, \quad (5.3a)
\]

and

\[
I_2(k) = \text{Im} \int_{\text{soft}} \frac{dp}{q^4 p^4 p_L^2}. \quad (5.3b)
\]

The integrals in (5.2) which do not depend on the plasma excitation properties \( p_T \) and \( p_L \), respectively, are treated in Ref.[19]; indeed they are well behaved on the physical mass shell, \( k_T = 0 \).

(i) **Calculation of the transverse integral**

We restrict ourselves to the calculation for the gluon excitation at rest: \( \vec{k} = 0 \). In order to evaluate the double pole we use the identity

\[
\frac{1}{q^4} = \lim_{m \to 0} \left( -\frac{\partial}{\partial m^2} \right) \frac{1}{q^2 + m^2}. \quad (5.4)
\]

It is easy to derive that on the mass shell, i.e. when \( K_0 = m_g \), the integral \( I_1 \) has no imaginary part except for \( m = 0 \). Therefore we define \( I_1(m_g, \vec{0}) \) by taking the following limits:

\[
I_1(m_g, \vec{0}) = \lim_{K_0 \to m_g} \lim_{m \to 0} \left\{ \left( -\frac{\partial}{\partial m^2} \right) \text{Im} \int_{\text{soft}} \frac{dp}{(q^2 + m^2)p_T^2} \right\}, \quad (5.5)
\]

with \( p = (w, \vec{p}), \; q = (w', -\vec{p}), \; w' = K_0 - w \).

Following the method of Refs.[8, 20], which we already used in a related study of the fermion damping rate [11], we calculate the imaginary part above by using the spectral representation of the effective transverse gluon propagator \( \Delta_t(p) = \frac{1}{p_T^4} \) [14, 21] and of the free one \( \Delta(q) = \frac{1}{q^2 + m^2} \), as

\[
\hat{I} = \text{Im} \int_{\text{soft}} \frac{dp}{(q^2 + m^2)p_T^2}
\]

\[
= \frac{\pi}{n_B(K_0)} \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{+\infty} dw \; dw' \; \delta(K_0 - w - w') \; n_B(w)n_B(w') \rho_t(w, p) \epsilon(w') \delta(w'^2 - E^2),
\]

\[
(5.6)
\]
with \( E^2 = p^2 + m^2 \) and \( \rho_t(w,p) \), the spectral density associated to \( \Delta_t(p) \) given in Ref. [21]. The spectral density associated to \( \Delta(q) \) is \( \epsilon(w')\delta(w'^2 - E^2) \), and \( n_B \) is the Bose distribution.

Integrating with respect to \( w' \) and using the symmetry property:

\[
\rho_t(-w,p) = -\rho_t(w,p),
\]

we get

\[
\hat{I} = \frac{1}{8\pi n_B(K_0)} \int_0^\infty \frac{pdp^2}{E} \int_0^\infty dw \rho_t(w,p)
\cdot \left\{ n_B(E)\left[ n_B(w)\delta(K_0 - w - E) + (1 + n_B(w))\delta(K_0 + w - E)\right] \right. \\
\left. + (1 + n_B(E))\left[ n_B(w)\delta(K_0 - w + E) + (1 + n_B(w))\delta(K_0 + w + E)\right] \right\}.
\]

We first concentrate on the time like pole contribution in \( \rho_t(w,p) \):

\[
\rho_t^{\text{pole}} = \rho_t^{\text{res}}(w,p)\delta(w - w_t(p)),
\]

where \( w_t(p) \) is the transverse mode dispersion relation and is given in Refs. [14, 21], together with \( \rho_t^{\text{res}} \). Taking \( K_0 > m + m_g \), without loss of generality, we obtain

\[
\hat{I}_t^{\text{pole}} = \frac{1}{8\pi n_B(K_0)} \left[ \frac{p_0}{E_0} \rho_t^{\text{res}}(w_t(p_0),p_0)n_B(E_0) \right]
\cdot \left[ n_B(w_t(p_0))\frac{1}{\partial E/\partial p^2 + \partial w_t/\partial p^2} \right]_{p=p_0},
\]

with \( p_0 \) given by the \( \delta \)-constraint:

\[
w_t(p_0) = K_0 - E_0 \equiv K_0 - \sqrt{p_0^2 + m^2}.
\]

In order to perform the interesting limit \( m \to 0 \) and \( K_0 \to m_g \), we notice that the solution of (5.10) vanishes for \( m = 0 \) as

\[
p_0 = K_0 - m_g + O(p_0^2),
\]

using

\[
\frac{w_t(p_0)}{p_0} \to 0 \quad m_g + \frac{3}{5} \frac{p_0^2}{m_g} \pm ... \quad (5.11)
\]
Working out the derivative with respect to $m^2$ and taking $m \to 0$, we find that the most singular term in the limit $p_0 \to 0$ comes from the derivative

$$ \frac{\partial}{\partial m^2} \left( \frac{p_0}{E_0} \right) p_0 \to 0 \rightarrow - \frac{1}{2p_0^2}.$$ 

This allows to derive in the appropriate limit:

$$ I_{1,\text{pole}}^\text{pole}(m_g,0) \simeq \frac{1}{K_0 \to m_g} \frac{T}{8\pi} \frac{1}{2m_g (K_0 - m_g)^2}.$$

(5.12)

In order to conclude about $I_1$, we should now take care about the contribution coming from the branch cut of the effective gluon propagator associated to Landau damping. In this case, from first inspection, it seems safe to work directly on-shell and to calculate

$$ I^\text{disc}_1(m_g,0) = \lim_{m \to 0} \left( - \frac{\partial}{\partial m^2} \right) \hat{I}^\text{disc}_1.$$

(5.13)

Going back to Eq.(5.7), we end up by writing $I^\text{disc}_1$ as the sum of two terms $I^a$ and $I^b$,

$$ I^a \propto \lim_{m \to 0} \left( - \frac{\partial}{\partial m^2} \right) \int_{m^2 - m_g}^{m^2 - m_g} p^2 dp \frac{n_B(E)}{E} n_B(m_g - E) \rho_t^\text{disc}_1(m_g - E, p),$$

$$ I^b \propto \lim_{m \to 0} \left( - \frac{\partial}{\partial m^2} \right) \int_{m^2 - m_g}^{\infty} p^2 dp \frac{n_B(E)}{E} (1 + n_B(E - m_g)) \rho_t^\text{disc}_1(E - m_g, p),$$

(5.14)

neglecting irrelevant numerical factors and with $\rho_t^\text{disc}_1$, the transverse spectral density associated to Landau damping, given in [21].

When taking the derivative $\frac{\partial}{\partial m^2}$ each term yields two contributions:

- the first one is given by the derivative of the integration bounds. In this case, the only source of potential singular behaviour is associated, in both $I^a$ and $I^b$, to the value of the integrand at $p = E = m_g$. Since $\lim_{w \to 0} \rho_t^\text{disc}_1(w, p) \cdot n_B(w)$ is finite, no singularity is encountered.

- the second one is obtained when taking the derivative of the integrand. Writing $\frac{\partial}{\partial m^2} = \frac{1}{2p} \frac{\partial}{\partial E}$ and integrating by parts, we end up as above with a finite contribution.

As a consequence, the contribution of $I_1(k)$ to $\text{Im} \Delta^* \Pi_t(k)$ is, in the appropriate limit - neglecting numerical factors,

$$ \text{Im} \Delta^* \Pi_t^{(I_1)} \propto \xi g^2 N \frac{T}{m_g} \frac{k_t^4}{(K_0 - m_g)^2}.$$

(5.15)
With
\[ k_T^2 = m_g^2 - K_0^2 \sim K_0 \rightarrow m_g (m_g - K_0) , \]
this leads to
\[ \text{Im} \Delta^* \Pi_t^{(I_1)} \propto \xi g^2 N T m_g . \] (5.16)

(ii) Calculation of the longitudinal integral

In order to complete the argument, we need to calculate \( I_2 \). This in fact, using
\[ p_0^2 = p^2 - \vec{p}^2 , \]
amounts to calculating two integrals:
\[ I_{2a} = \text{Im} \int_{\text{soft}} \frac{d p}{q^4} \frac{\vec{p}^2}{p^2 \vec{p}_L^2} , \] (5.17)
and
\[ I_{2b} = \text{Im} \int_{\text{soft}} \frac{d p}{q^4} \frac{\vec{p}^4}{p^4 \vec{p}_L^2} . \] (5.18)
These two integrals may be calculated, following the same line as above, from the basic integral
\[ I(k) = \text{Im} \int \frac{d p}{q^2 + m^2} \frac{1}{p^2 + m^2} \frac{1}{\vec{p}_L^2} , \] (5.19)
for \( \vec{k} = 0 \). This integral, analogous to a vertex integral, is evaluated with the technique of Ref. [8], already used here which starts by introducing “mixed” propagators for which an integral representation in terms of spectral densities is given. To be specific we introduce the mixed propagator \( \tilde{\Delta}(\tau, \vec{q}) \) as:
\[ \Delta(q_0, \vec{q}) = \int_{0}^{\beta=1/T} d\tau e^{iq_0\tau} \tilde{\Delta}(\tau, \vec{q}) = \frac{1}{q^2 + m^2} , \] (5.20)
and similarly for \( \Delta(p_0, \vec{p}) \) and \( \Delta_L(p) = \frac{1}{\vec{p}_L^2} \).

This allows to write \( I(k) \) as
\[ I(k) = \text{Im} \int \frac{d^3 p}{(2\pi)^3} \int_{0}^{\beta} d\tau_1 \int_{0}^{\beta} d\tau_2 e^{i\tau_1 k_0} \tilde{\Delta}(\tau_1 - \tau_2, E_p) \tilde{\Delta}(\tau_1, E_q) \tilde{\Delta}_L(\tau_2, p) , \] (5.21)
with \( E_p = \sqrt{p^2 + m^2} , E_q = \sqrt{p^2 + m'^2} \), and in the following \( p^2 = \vec{p}^2 \).
The spectral representation which is then used in order to calculate the imaginary part,

$$\tilde{\Delta}(\tau, E) = \int_{-\infty}^{+\infty} dw \ e^{-w\tau} \rho(w, E)(1 + n_B(w))$$ (5.22)

is only valid when $0 \leq \tau \leq \beta$ and therefore $\tilde{\Delta}(\tau_1 - \tau_2, E_p)$ should be defined to be periodic with period $\beta$ [8]. This leads to (after analytic continuation: $ik_0 \rightarrow K_0 + i\epsilon$)

$$I(K_0, \vec{k} = 0) =$$

$$\pi(e^{\beta K_0} - 1) \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{+\infty} dw_1 \int_{-\infty}^{+\infty} dw_2 \int_{-\infty}^{+\infty} dw \frac{\rho(w, E_p)\rho(w_1, E_q)\rho_L(w_2, p)}{w - w_2}$$

$$\cdot n_B(w_1)\{n_B(w_2)\delta(K_0 - w_1 - w_2) - n_B(w)\delta(K_0 - w_1 - w)\}.$$ (5.23)

The calculation then follows closely the steps described for $I_1$. Let us first consider $I_{2a}$ and focus on the pole contribution in the spectral function for the longitudinal mode $\rho_L(w, p)$:

$$\rho_{\text{pole}}^L(w, p) = \rho_{\text{res}}^L(w, p)\delta(w - w_L(p)) = -\frac{1}{p^2} \frac{w(w^2 - p^2)}{3m_g^2 - w^2 + p^2}\delta(w - w_L(p)), (5.24)$$

with $w_L(p)$ the dispersion relation given in Refs. [14, 21].

The only interesting configuration in the limit $K_0 \rightarrow m_g$ is due to the term proportional to $\delta(K_0 - E_q - w_L(p))$ in complete analogy with the case of the transverse integral. We find

$$I_{2a}^\text{pole}(m_g, \vec{0}) = \frac{1}{8\pi n_B(K_0)} \left(-\frac{\partial}{\partial m^2}\right) \int_0^{p_p^2} dp \int_0^{\infty} dw_2$$

$$\frac{\rho_{\text{res}}^L(w_2, p)}{E_p^2 - w_2^2} n_B(E_q)n_B(w_2)\delta(K_0 - E_q - w_2). (5.25)$$

It turns out that there is a complete correspondence with the calculation of $I_1$, i.e. replacing:

$$\rho_t^\text{res}(w_t(p_0), p_0) \rightarrow \frac{\rho_{\text{pole}}^L(w_L(p_0), p_0)}{p_0^2 - w_L^2(p_0)}, (5.26)$$

which both equal $\frac{1}{2m_g}$ when $p_0 \rightarrow 0$. We obtain therefore the same singular value for $I_{2a}$ and $I_1$. For $I_{2b}$, the additional differentiation with respect to $m^2$ amounts to replacing the above correspondence by:

$$\rho_t^\text{res}(w_t(p_0), p_0) \rightarrow \frac{p_0^4\rho_{\text{pole}}^L(w_L(p_0), p_0)}{(p_0^2 - w_L^2(p_0))^2}, (5.27)$$
which in the limit \( p_0 \to 0 \) yields \( I_{2b} \simeq -\frac{p_0^2}{m_g^2} I_1 \). The integral \( I_{2b} \) is therefore negligible in the relevant on-shell limit.

Going back to (5.2), we find that the gauge dependence of the transverse structure function is indeed non vanishing on the mass-shell

\[
Im \Delta^* \Pi_t(m_g, \vec{0}) = -\xi g^2 N k_T^4 (I_1(m_g, \vec{0}) + I_a(m_g, \vec{0})) \propto g^2 N T m_g ,
\]

which implies a gauge dependence for the damping rate (cf. Eq.(3.21)):

\[
\delta \gamma_T \propto \frac{Im \Delta^* \Pi_t}{m_g} \propto \xi g^2 N T ,
\]

for the transverse gluon excitation at rest.

**Coulomb gauges**

In this case one may vary the effective propagator (Eq.(4.4)) as

\[
* D^{\mu \nu} \to * D^{\mu \nu} + \xi_c k^\mu k^\nu \frac{k^4}{k^4}
\]

around the strict Coulomb gauge (\( \xi_c = 0 \)). The gauge dependent part of the propagator can be written as in (4.6) with \( \Delta \beta = 0 \) and \( \Delta \alpha = \xi_c \frac{k^2}{k^4} \) so that the gauge variation \( Im \Delta^* \Pi_t \) is obtained from (5.2) with trivial changes:

\[
Im \Delta^* \Pi_t = -\xi_c g^2 N \left[ -k_T^2 \int_{soft} \frac{dp}{q^4} \left( 1 - \frac{\vec{p}^2}{p^2} + \frac{(\vec{k} \cdot \vec{p})^2}{k^2 p^2} \right) \right. \\
+ \frac{1}{2} k_T^4 \int_{soft} \frac{dp}{q^4} \left\{ \frac{1}{p_T^2} \left( 1 + \frac{(\vec{k} \cdot \vec{p})^2}{k^2 p^2} \right) + \frac{\vec{p}_0^2 p^2}{p_T^2 p^4} \left( 1 - \frac{(\vec{k} \cdot \vec{p})^2}{k^2 p^2} \right) \right\} \\
+ \frac{\xi_c}{4} k_T^4 \int_{soft} \frac{dp}{q^4 \vec{p}^2} \left( 1 - \frac{(\vec{k} \cdot \vec{p})^2}{k^2 p^2} \right) \right] .
\]

The only potentially dangerous contribution is proportional to the integral

\[
\int_{soft} \frac{dp}{q^4 \vec{p}^2 p^4} .
\]

It does not depend, however, on \( k_0 \) and therefore this yields - after continuation - a vanishing imaginary part.

We thus conclude that the problem of gauge dependence arises in the general class of covariant gauges but not in Coulomb gauges. The implications of our results are discussed in the next Section.
6. DISCUSSION

In the previous Section we have shown by explicit calculations that the gluonic damping rate for the excitation at rest remains gauge parameter dependent in the class of covariant gauges. This extends the result previously derived for the fermionic rate [11] in the same framework of the resummed effective perturbative expansion [8].

In the course of the calculation in Section 5 it is stated that the integrals in Eq.(5.2) have no imaginary part on the mass-shell $K_0 = m_g$ for all values of the mass parameter $m > 0$. This is most easily seen from the constraint (5.10) which cannot be fulfilled when $w_t = m_g$; a solution is only possible on the mass shell for $m = 0$. One might therefore argue that continuity in $m$ requires the expression of Eq.(5.2) to vanish on shell for $m = 0$ as well; indeed this would be the case if the order of limits (cf. Eq.(5.5)) were interchanged to $\lim_{m \to 0} \lim_{K_0 \to m_g}$. However, the parameter $m$ was introduced as a technical device to treat the double pole $1/(q^2)^2$ in the gauge dependent term in Eq.(5.4), so the order of limits is fixed as in Section 5. Nevertheless this observation suggests that the observed gauge dependence may be intimately related to infrared divergences, despite the fact that the damping rate itself is definitely infrared finite.

The discussion may be clarified by introducing an infrared regulator, as advocated by Rebhan [22] (see also refs.[23] and [24]). In order to demonstrate the problem, we consider in detail the integral (cf. Eq.(5.6)):

$$\hat{I}(k_0, \vec{k} = 0) = \int \frac{dp}{q^2} \frac{1}{p_T^2}, \quad (6.1)$$

which is an intermediate step in deriving a contribution to the self energy of the same form as the first term in the second line of (5.2), namely

$$I(k_0, \vec{k} = 0) \sim k_T^4 \int \frac{dp}{(q^4)} \frac{1}{p_T^2}. \quad (6.2)$$

The discussion for these integrals may be generalized to the more complicated cases corresponding to the integrals in Eqs.(5.3), (5.17) and (5.18), and even to $\Delta^*\Pi_t$ (Eq.(5.2)) itself, before taking the imaginary parts. However, in order to simplify the analysis, the simpler dispersions [9] $p_T^2 = p_0^2 + \vec{p}^2 + m_g^2$, and $\vec{p}_L^2 = \tilde{n}^2 \vec{p}_T^2$ are used, instead of the physical dispersion relations (Eqs.(3.9, 3.10)).

We start in the imaginary time-framework, $k_0 = 2\pi m T$, and later the continuation $ik_0 \to K_0 + i\epsilon$ is performed. In order to regularize possible infrared singularities dimensional regularization is applied with analytically continued spatial dimensions $n = 3 + 2\epsilon$ [23]:

$$\int dp = T \sum_{p_0 = 2\pi m T} \frac{d^n p}{(2\pi)^n}. \quad (6.3)$$

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In the high temperature limit the leading term of (6.1) is:

\[ \hat{I}(k_0, \vec{k} = 0) \propto \frac{T}{\sqrt{k_0^2}} \sqrt{\frac{(k_0^2 + m_g^2)^2}{k_0^2}} \int_0^\infty dp \frac{p^{2\epsilon}}{p^2 + \frac{(k_0^2 + m_g^2)^2}{4k_0^2}} \]

(6.4)

Irrelevant numerical factors are neglected. As long as the energy \( k_0 \) is discrete and euclidean there are no singularities present and we are allowed to take \( \epsilon = 0 \). In the next step the analytic continuation to the Minkowski energy \( K_0 \) is performed, and the integral becomes imaginary:

\[ Im \ \hat{I}(K_0, \vec{k} = 0) \propto \frac{T}{K_0} , \]

(6.5)

and independent of \( m_g \). One may note that at non-zero temperature \( Im \ \hat{I} \) is also defined for \( K_0 < m_g \), in contrast to the \( T = 0 \) contribution, which has only support for \( K_0 > m_g \).

The integral \( I(k_0, \vec{k} = 0) \) of (6.2) contains a term \( \sim k_T^2 T \int_0^\infty dp \ p^{2\epsilon-2} \), which is infrared singular in three spatial dimensions. Therefore a non-vanishing regulator \( \epsilon \) seems appropriate, at least for the real part of the integral (after analytic continuation).

However, it is remarkable that indeed these infrared singularities cancel in \( \Delta^* \Pi_t \) - at least in the described approximation: \( \Delta^* \Pi_t \) has precisely the same structure as exemplified by the integral in (6.4)!

The imaginary parts of the integrals (6.1) and (6.2) are smooth functions in terms of the continued external energy; for \( \epsilon = 0 \) and at the value of \( K_0 = m_g \) we find e.g. for \( \hat{I} \) from (6.4),

\[ Im \ \hat{I}(m_g, \vec{k} = 0) \propto \frac{T}{m_g} , \]

(6.6)

in agreement with the result for \( \hat{I}^{pole} \) in Eq.(5.9) for \( w_t = m_g \) and \( m = 0 \). I.e. a non-vanishing answer is obtained with the following prescription: first the necessary integrations are performed for discrete external euclidean energies, where an infrared regulator may be introduced, which is then removed before the analytic continuation to external Minkowski energies is done. The discontinuity of the integrals has to be evaluated when staying off-shell, and finally the on-shell limit is taken. This prescription is equivalent to the calculation of the imaginary part of the integrals as described in Section 5; it appears to be the appropriate procedure when dealing with the effective euclidean theory at finite temperature as it is implemented by Braaten and Pisarski [8]. However, it leads to a dependence on the choice of the covariant gauge parameter.
The resolution of the gauge dependence problem advocated in Refs. [22-24] follows the steps of taking the on mass-shell limit just after the continuation, but keeping the infrared regulator $\epsilon$ different from zero. As emphasized by Rebhan [22], the correct treatment of mass-shell singularities at zero temperature requires an infrared regulator in the framework of perturbation theory [18]. For the example above it is obvious (cf. Eq. (6.4)) that

$$\hat{I}(K_0 = m_g, \vec{k} = 0) = 0,$$

(6.7)

for $\epsilon > 0$, so there is no imaginary part at $K_0 = m_g$, and gauge independence is restored!

Although this prescription does appear to fix the problem of gauge dependence at this order, it is perhaps appropriate to make the following comments:

1. At zero temperature, the infrared regulator is required to make the wave function renormalization well defined. In order to evaluate the pole position, the regulator is not required. In the present case, we are only interested in the pole position and in the effective (truncated) approximation under consideration for calculating damping rates the necessity of the infrared regulator is not at all obvious: only the imaginary parts of the integrals in (5.2) are physically relevant (since higher loops can contribute to order $g^2 T$ to the real part of the self energy) and these imaginary parts are not infrared divergent. (We repeat that even the real part of $\Delta^* \Pi_t$ is free of infrared singularities in the approximation investigated above.) Therefore the limit $\epsilon \to 0$ can be taken in (6.4) without encountering a singularity. Using the prescription in [22,23], however, a discontinuous behaviour of the function $Im \hat{I}(K_0, \vec{k} = 0)$ (and therefore of $\Delta^* \Pi_t$) is enforced: it vanishes at $K_0 = m_g$, whereas for $K_0 \neq m_g$ it behaves as $T/K_0$ (in the limit that $\epsilon = 0$). In this way, the value of the damping rate, a supposedly physical quantity, is affected by the presence, or absence of such a regulator. Clearly, the question as to whether or not the regulator is necessary in the expressions considered is considerably more subtle than at zero temperature.

2. As long as $\Delta X$ in the gauge dependence identity (1.1) is well-behaved on mass-shell, gauge dependence is guaranteed algebraically order by order in any self-consistent perturbative expansion. In the presence of the mass shell singularities discussed above, the gauge dependent terms in the lowest order damping rate need to be “regulated away”. They do not vanish algebraically, but instead rely on the presence of the regulator to control the value of the integrals in question. This mechanism for ensuring gauge independence is very different in this regard from the algebraic proof, and it is not clear whether it will work at higher orders. (This remains an important question,
in our opinion, even if such higher order contributions are difficult, if not impossible, to calculate in practice.)

3. For gauge independence to hold, the infrared regulator must preserve the Ward identities of the theory. Dimensional regularization[23] does so trivially, but results in a somewhat strange analytic behaviour for the off-shell propagator due to terms of the form \((K_0^2 - m_g^2)^\epsilon\). While a cut-off avoids this particular problem[22], verifying the Ward identities is somewhat more problematic.

To summarize, one appears to be left with the following choice: one can retain an infrared regulator throughout the calculation, with potential consequences as outline above, or one may use the argument of continuity (this time in \(K_0\)) to follow the prescription advocated in Section 5 for calculating the damping rates in the effective theory of Ref.[8]. However, if the latter choice is made, it remains an open problem to determine whether or not there exist missing terms[25] in the resummation which restore (covariant) gauge independence of the damping rate.

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