JET VERTEX IN THE NEXT-TO-LEADING LOG(S) APPROXIMATION

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Abstract. The next-to-leading corrections to the jet vertex which is relevant for the Mueller-Navelet jets production in hadronic collisions and for the forward jet cross section in lepton-hadron collisions are presented in the context of a $k_t$ factorization formula which resums the leading and next-to-leading logarithms of the energy.

1. Introduction

Recently, in the study of QCD in the Regge limit, a novel element has been defined and computed at the next-to-leading order (NLO). It is the jet vertex [1], which represents one of the building blocks in the production of Mueller-Navelet jets at hadron-hadron colliders and of forward jets [2] in deep inelastic electron-proton scattering. Such processes should provide a kinematical environment for which the BFKL Pomeron [3] QCD analysis could apply, provided that the transverse energy of the jet fixes a perturbative scale and the large energy yields a large rapidity interval.

In a strong Regge regime important contributions, or even dominant, come, in the perturbative language, from diagrams beyond NLO and NNLO at fixed order in $\alpha_s$. This is the main reason for considering a resummation of the leading and next-to-leading logarithmic contributions as computed in the BFKL Pomeron framework. The lackness of unitarity, if the related corrections are not taken into account, forces one to consider an upper bound on the energy to suppress them. It is already known that the leading logarithmic (LL) analysis is not accurate enough [4], being the kinematics selected by experimental cuts far from any asymptotic regime. Moreover at this level of accuracy there is a maximal dependence in the dif-
ferent scales involved (renormalization, collinear factorization and energy scales). For the Mueller-Navelet jet production process the only element still not known at the NLO level was the “impact factor”, which describes the hadron emitting one inclusive jet when interacting with the reggeized gluon which belongs to the BFKL ladder, accurate up to NLL [5]. The jet vertex, now computed, is the building block of this interaction. For the so called forward jet production in DIS the extra ingredient necessary is the photon impact factor, whose calculation is currently in progress [6, 7]. Let us also remind that NLL BFKL approach has recently gained more theoretical solidity since the bootstrap condition in its strong form, which is the one necessary for the self-consistency of the assumption of Reggeized form of the production amplitudes, has been stated and formally proved [8]. This relation is a very remarkable property of QCD in the high energy limit.

![High energy process with jet production.](image)

*Figure 1.* High energy process with jet production. $H$ is the incoming hadron providing a parton $a$ (gluon $g$/quark $q$) with distribution density $f_a$ which scatters with the parton $b$ with production of a jet $J$ in the forward direction (w.r.t $H$) and $i$ is the generic label for outgoing particles.

A theoretical challenge, interesting by itself and appearing in the calculation, is related to the special kinematics. The processes to be analyzed is illustrated in Fig.1: the lower parton emitted from the hadron $H$ scatters with the upper parton $q$ and produces the jet $J$. Because of the large transverse momentum of the jet the parton is hard and the collinear factorization allows for a partonic scale dependence described by the DGLAP evolution equations [9]. Above the jet, on the other hand, the kinematics chosen requires a large rapidity gap between the jet and the outgoing parton $q$: such a situation is described by BFKL dynamics. Therefore the jet vertex lies at the interface between DGLAP and BFKL dynamics, a situation which appears for the first time in a non trivial way. As an essential result of our analysis we find that it is possible to separate, inside the jet vertex, the collinear infrared divergences that go into the parton evolution of the incoming gluon/quark from the high energy gluon radiation inside the rapidity gap which belongs to the first rung of the LO BFKL ladder.
2. Jet Vertex: definition and outline of the derivation

Let us consider, for the process illustrated in Fig. 1, the kinematic variables

\[ p_H = \left( \sqrt{s/2}, 0, 0 \right), \quad p_a = x p_H, \quad p_b = \left( 0, \sqrt{s/2}, 0 \right) \]
\[ p_i = E_i \left( e^{y_i}/\sqrt{2}, e^{-y_i}/\sqrt{2}, \phi_i \right), \quad s := (p_H + p_b)^2. \] (1)

We study the partonic subprocess \( a + b \rightarrow X + \text{jet} \) in the high energy limit

\[ \Lambda_{QCD}^2 \ll E_J^2 \sim \sqrt{s} (\text{fixed}) \ll s \rightarrow \infty. \] (2)

Starting from the parton model, we assume the physical cross section to be given by the corresponding partonic cross section \( \widehat{\sigma} \) (computable in perturbation theory) convoluted with the parton distribution densities (PDF) \( f_a \) of the partons \( a \) inside the hadron \( H \). A jet distribution \( S_J \), with the usual infrared safe behaviour, selects the final states contributing to the one jet inclusive cross section that we are considering. We choose, as jet variables, rapidity, transverse energy and azimuthal angle and the one jet inclusive cross section initiated by quarks and gluons in hadron \( H \) is therefore written as

\[ \frac{d\sigma}{dJ} := \frac{d\sigma_{bH}}{dy_J dE_J d\phi_J} = \sum_{a=q,g} \int dx \, d\hat{\sigma}_{ba}(x) S_J(x) f_a^{(0)}(x). \] (3)

At the lowest order the jet cross section, dominated by a \( t \)-channel gluon exchange, can be written as [1]

\[ \frac{d\sigma}{dJ}^{(0)} = \sum_{a=q,g} \int dx \, \int dk \, h_a^{(0)}(k) V_a^{(0)}(k, x) f_a^{(0)}(x) \] (4)

where \( V_a^{(0)}(k, x) = h_a^{(0)}(k) S_J^{(2)}(k, x) \) is the jet vertex induced by parton \( a \), \( h_a^{(0)}(k) \) is the partonic impact factor (generally expressed in \( D = 4 + 2\varepsilon \) dimensions for regularization purposes at the NLO) and \( f_a^{(0)}(x) \) is the parton distribution density (PDF). The jet distribution is in this case trivial

\[ S_J^{(2)}(k, x) := S_J^{(2)}(p_1, p_2; p_g, p_q) = \delta \left( 1 - x_J / x \right) E_J^{1+2\varepsilon} \delta(k - k_J) \] (5)

with \( x_J := E_J e^{y_J} / \sqrt{s} \).

In the NLO approximation virtual and real corrections enter in the calculation of the partonic cross section \( d\hat{\sigma}_{ba} \). The three partons, produced in the real contributions, in the upper, and lower rapidity region are denoted by 2 and 1 respectively, while the third, which can be emitted everywhere
in rapidity, by 3. Moreover we shall call \( k = p_b - p_2 \) and \( k' = p_1 - p_a \) and \( q = k - k' \). A Sudakov parametrization is chosen

\[
k = -\bar{w} p_b + w p_q + k_\perp, \quad k_\perp = (0, 0, k) \quad (6)
\]

\[
k' = -z p_b + \bar{z} p_q + k'_\perp, \quad k'_\perp = (0, 0, k') \quad (7)
\]

with the typical inequalities \( \bar{w} \sim \bar{z} \ll w \sim z \ll 1 \) valid in the multi Regge kinematics (MRK). Dimensional regularization is used, as usual, to trace the infrared and ultraviolet divergences. On taking into account the infrared (IR) properties of the jet distribution \( S_J^{(3)} \), the following structure is matched exactly up to NLO (i.e. \( \alpha_s^3 \)) \[1\]

\[
\frac{d\sigma}{dJ} = \sum_{a=q,g} \int dx \int dk \, dh_a(k) G(xs, k, k') V_a(k', x) f_a(x), \quad (8)
\]

where

\[
h = h^{(0)} + \alpha_s h^{(1)} + \ldots,
\]

\[
V = V^{(0)} + \alpha_s V^{(1)} + \ldots,
\]

\[
f = f^{(0)} + \alpha_s f^{(1)} + \ldots,
\]

\[
G(xs, k, k') := \delta(k - k') + \alpha_s K^{(0)}(k, k') \log \frac{x_s}{s_0} + \ldots \quad (9)
\]

The partonic impact factor correction in forward direction \( h^{(1)} \) is well known \[10\], the PDF’s \( f_a \) are the standard ones satisfying DGLAP evolution equations

\[
\alpha_s f_a^{(1)}(x, \mu_F^2) := \frac{\alpha_s}{2\pi} \frac{1}{\varepsilon} \left( \frac{\mu_F^2}{\mu^2} \right)^\varepsilon \sum_b P_{ab} \otimes f_b^{(0)} \quad (10)
\]

with \( \mu_F \) the collinear factorization scale, and the BFKL Green’s function \( G \) is defined by the LO BFKL kernel \( K^{(0)} \). The new element is the correction to the jet vertex \( V^{(1)} \) whose expressions can be found in \[1\].

Let us note that the previous form in (8) is clearly suggested by the structure of the leading logarithmic part, but at this level it is not a trivial ansatz. Only after a proper treatment of all the IR singular terms one will be able to extract and define the jet vertices initiated by quarks and gluons in the hadron.

The NLO virtual corrections in the high energy limit are relatively simple and they can be summarized in the following expression:

\[
\frac{d\sigma^{(\text{virt})}}{dJ} = \alpha_s \sum_a \int dx \int dk \, h_q^{(0)}(k) \left[ 2\omega^{(1)}(k) \log \frac{x_s}{k^2} + \Pi_b(k) + \Pi_a(k) \right]
\]

\[
\times V_a^{(0)}(k, x) f_a^{(0)}(x), \quad (11)
\]
where $\omega^{(1)}(k)$ is the 1-loop reggeized gluon trajectory and the $\bar{\Pi}_i(k)$ are related to the part constant in energy, after having performed the ultraviolet (UV) $\varepsilon$-pole subtraction occurring in the renormalization of the coupling.

The NLO real corrections, even in the high energy limit, are much more involved since the interplay of different IR soft and collinear singularities follows a more complicated pattern.

One starts from the partonic differential cross sections $d\hat{\sigma}_{ba}(x)$, appearing in (3), which have been also computed in the high energy regime [10], since in such a case all terms suppressed by powers of $s$ may be neglected. The form of the partonic differential cross section turns out to be quite simple when restricted to one of the two halves of the phase space, which are obtained introducing the rapidity $y' = y + \frac{1}{2} \log \frac{1}{x}$ “measured” in the partonic center of mass frame, and splitting the phase space into two semi-spaces defined by $y'_1, y'_3 > 0$ (lower half) and $y'_3 < 0 < y'_1$ (upper half).

In the quark initiated case one obtains for both regions a reasonable simple expression, while for the gluon initiated part it is convenient to consider separately the $q\bar{q}$ and $gg$ final state cases.

The “upper half region”, with $y'_3 < 0$, leads to the rederivation of the partonic $b$-impact factor, already well known at NLL. The “lower half region”, wherein $y'_3 > 0$, which corresponds to $z > z_{\text{cut}} := \frac{E_3}{\sqrt{s}}$ is precisely the one from which we are able to extract the jet vertexes.

A very important ingredient, as previously mentioned, is given by the jet definition $S^{(n)}_{J}$. As usual it selects from a generic $n$-particle final state the configurations contributing to our one jet inclusive observable and it must be IR safe. The last requirement simply corresponds to the fact that emission of a soft particle cannot be distinguished from the analogous state without soft emission. Furthermore, collinear emissions of partons cannot be distinguished from the corresponding state where the collinear partons are replaced by a single parton carrying the sum of their quantum numbers.

The extraction of all the IR singularities from the real contributions and the matching with the virtual ones is done on employing the standard subtraction method. This consists in approximating the amplitudes in any singular region in order to extract the exact analytic singular behavior, and in leaving the remaining finite part in a form suitable for numerical integration, since normally a full analytical computation is technically impossible. The analytic singular terms of both virtual and real contributions are therefore combined. Moreover, with respect to standard NLO jet analysis, one has also to deal with the leading logarithmic terms which are entangled to the other IR singular terms. For the process considered it can be shown [1] that finally one is left with the collinear singularities associated to the impact factor of the parton $b$, to the definition of the distribution function
for the parton $a$ in $H$ and to the BFKL kernel term. The finite remaining parts correspond to the impact factors and jet vertices.

We recall that the jet distribution functions become essential in disentangling the collinear singularities, the soft singularities, and the leading $\log s$ pieces when the particle 3 is a gluon. The same basic mechanism may be used for both the quark and the gluon initiated cases.

- When the outgoing parton 1 is in the collinear region of the incoming parton $a$, i.e., $y_1 \to \infty$, it cannot enter the jet; only gluon 3 can thus be the jet, $y_3$ is fixed and no logarithm of the energy can arise due to the lack of evolution in the gluon rapidity. No other singular configuration is found when $J = \{3\}$.

- In the composite jet configuration, i.e., $J = \{1, 3\}$, the gluon rapidity is bounded within a small range of values, and also in this case no $\log s$ can arise. There could be a singularity for vanishing gluon 3 momentum; even if the $1 \parallel 3$ collinear singularity is absent, we have seen that, at very low $z$, a soft singular integrand arises. However, the divergence is prevented by the jet cone boundary, which causes a shrinkage of the domain of integration $\sim z^2$ for $z \to 0$ and thus compensates the growth of the integrand.

- The jet configuration $J = \{1\}$ corresponds to the situation wherein gluon 3 spans the whole phase space, apart, of course, from the jet region itself. The LL term arises from gluon configurations in the central region. Therefore, it is crucial to understand to what extent the differential cross section provides a leading contribution. It turns out that the coherence of QCD radiation suppresses the emission probability for gluon 3 rapidity $y_3$ being larger than the rapidity $y_1$ of the parton 1, namely an angular ordering prescription holds. This will provide the final form of the leading term, i.e., the appropriate scale of the energy and, as a consequence, a finite and definite expression for the one-loop jet vertex correction.

It is therefore clear that the energy scale $s_0$ associated to the BFKL rapidity evolution plays a crucial role. The calculations show a natural choice, due to angular ordered preferred gluon emission and the presence of the jet defining distribution, which is also crucial to obtain the full collinear singularities which factorize into the PDF’s. In particular, on considering the term giving the real part of the LL contribution, one can see that outside the angular ordered region

$$\frac{E_3}{z} > \frac{E_1}{1-z} \iff \theta_3 > \theta_1 \iff y_3 < y_1,$$

there is a small contribution to the cross section. This ordering sets the natural energy scale which, for example, leads to the expression of the jet
vertex for the case $s_0(k,k') := (|k'| + |q|)(|k| + |q|)$. A mild modification of such a scale can be performed without introducing extra singularities, contrary to what happens for a generic choice. In any case the use of a different scale requires the introduction of modifying terms. For the useful symmetric Regge type energy scale $s_R = |k||k'|$, one has

$$G(x_s, k, k') = (1 + \alpha_s H_L) \left[ 1 + \alpha_s K^{(0)} \log \frac{x_s}{|k||k'|} \right] (1 + \alpha_s H_R),$$

where $H_L(k, k') = -K^{(0)}(k, k') \log (|k| + |q|)/|k|) = H_R(k', k)$.

We do not present the final vertex expressions here, they can be found in [1].

3. Remarks and final formulas

In order to extend the results of the two loop calculations, which have allowed to obtain the full NLL partonic cross sections, to our inclusive jet production case, it is necessary to check the relation between the partonic impact factors and the jet vertices. More precisely one may define the one loop correction to the “jet impact factor” as $[V \ast f]^{(1)} = V^{(1)} \ast f^{(0)} + V^{(0)} \ast f^{(1)}$. This latter object and the corrections to the partonic impact factor $h^{(1)}$ are different in one important aspects, i.e. their collinear singularities. Essentially the IR singularity associated to the LL real term is absent (subtracted) in the partonic impact factors, while this does not happen in the “jet impact factors” due to the constraining presence of the jet. In the “jet impact factors” are therefore hidden the full collinear singularities which should be factorized to obtain the right behavior of the PDF’s satisfying the DGLAP evolution equations.

Apart from this important difference there is a full correspondence, as shown in [1], between these two objects. In particular the same angular ordering mechanism, which sets a reference for the energy scale, is present. Thus any tuning of the BFKL kernel energy scale is done precisely in the same way for both impact factors and jet vertices and, thanks to the Regge factorization property valid up to NLL in QCD, one is allowed to use the known form of the NLL BFKL Green’s function setting the scale to $s_R$, as previously discussed.

From a two loop analysis, up to $\alpha_s^4$ order, one expects to find the contributions

$$\frac{\dd \sigma}{\alpha_s^3 \dd J}^{(2)} = \frac{1}{2} \log^2 \frac{x_s}{s_R} \left[ h^{(0)} K^{(0)} K^{(0)} V^{(0)} f^{(0)} + \log \frac{x_s}{s_R} \left[ h^{(1)} K^{(0)} V^{(0)} f^{(0)} 
+ h^{(0)} (H_L K^{(0)} + K^{(0)} H_R + K^{(1)})) V^{(0)} f^{(0)} + h^{(0)} K^{(0)} V^{(1)} f^{(0)} 
+ h^{(0)} K^{(0)} V^{(0)} f^{(1)} \right] \right],$$

(14)
where the first term on the RHS is LL while the terms collected in the square brackets are NLL.

Consequently, to obtain the jet cross section with accuracy up to NLL terms, one has to consider the NLL BFKL kernel

$$K = \alpha_s K^{(0)} + \alpha_s^2 K^{(1)}$$

which has been computed with the Regge type scale $s_R = |k_1||k_2|$ and leads to a Green’s function, which resums all terms up to NLL in (8),

$$G(x_s, k_1, k_2) = \int \frac{d\omega}{2\pi i} \left(\frac{x_s}{s_R}\right)^{\omega} \langle k_1 | (1 + \alpha_s H_L) | \omega - K \rangle^{-1} (1 + \alpha_s H_R) | k_2 \rangle.$$

(16)

The formula for the Mueller-Navelet jets [1] can be easily derived symmetrizing the formula (8) for the two jet case. More precisely one obtains

$$\frac{d^2\sigma}{dJ_1 dJ_2} = \sum_{a,b} \int dx_1 dx_2 \int dk_1 dk_2 f_a(x_1) V_a(k_1, x_1) G(x_1 x_2 s, k_1, k_2) V_b(k_2, x_2) f_b(x_2)$$

(17)

where $a, b = q, g$, and the subscripts 1 and 2 refer to jet 1 and 2 in hadron 1 and 2, resp. Once the definition of the jets is given, all the elements are known and a definitive computation may be carried on.

Regarding the Green’s function, two more remarks should be made. Since the high energy asymptotic regime is far, a few iterations of the kernel instead of a full resummation, as given in eq. (16), may be of some interest in a phenomenological application. The known large corrections of the BFKL kernel are related to the presence of large logs. This problem has been solved using optimal renormalization scale methods [11] or by considering an improved version of the kernel, equivalent at NLL level, but including correcting higher order terms, following a collinear prescription [12].

4. Conclusions

A brief review of the definitions and calculations at the NLO accuracy of the jet vertex relevant for the Mueller-Navelet jets at hadron-hadron colliders and for forward jets in deep inelastic electron-proton scattering has been given. This object, defined for high energies, due to specific kinematics, lies at the interface between DGLAP and BFKL dynamics. Being known all the ingredients, it is now possible to perform a numerical analysis of the production of Mueller-Navelet jets at NLL. For the forward jet processes further theoretical studies on the NLL photon impact factors are still required.
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