One abstract method for the study of network transportation and its application in the traffic problem in the process of city expansion

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Abstract

One abstract method for the study of network transportation is proposed in this paper. By interpolating the properties of the edges that constitute network into the two leading parameters of the nodes, this method can abstract the configuration of real-life transportation networks without losing the influence of the distance and the capability of the physical facilities, which are essential parts of the transportation networks. With this method we can study network transportation from the local point of view and describe network congestion in detail, especially the dispersing of congestion. After applying this method, as an example, to the traffic problem in the process of city expansion, simulation results show that: the intuitions that with the same rate of particle generation and the same capacity and power, the smaller the network is the harder for the network to become congested; and the network will never be congested if the power of each node equals or is bigger than its capacity are incorrect. And the distance from one crossroad to another and the width of the roads can be designed properly, when designing a city, so as to enlarge the tolerance size of the city in the process of city expansion. This method may provide more appropriate guide for real-life traffic design of network transportation.

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1 Introduction

The research on the efficiency of transportation on complex networks is significantly important in different aspects of natural science and more generally in its practical application. Many models of traffic flow on complex networks \[1,2,3,4,5,6,7,8,9,10,11,12\] can be used to gain intuition about dynamics on complex networks and to explain the characteristics of traffic dynamics on networks so as to determine the leading parameters of network transportation. Currently, the study of network transportation is mainly focused on network congestion.

There are two ingredients, the local or global topological properties of the network and the microscopic dynamical process involved in the network transportation processes, which are believed to affect transportation processes \[13,14\]. In review of the high cost of changing the underlying structure, researchers prefer to find out optimal strategies for traffic routing. Nevertheless, researchers prefer to study network transportation and congestion from the global point of view without considering the real-life characteristic details of network transportation.

For simplicity, we will take all kinds of information packets or cars moving on networks as particles in this paper. In previous studies \[10,11,12,13,14\], network congestion was set to be equal to particle accumulation on networks. An order parameter:
\[
\eta(R) = \lim_{t \to \infty} \langle \Delta L \rangle / R \Delta t
\]  
(1)

or
\[
\xi = \lim_{t \to \infty} W(t) / \lambda N t
\]  
(2)

was used as the criterion parameter to judge whether the network is congested or not. There are several shortages with these previous considerations:

(i) All nodes are given an unlimited capacity \[14\] which is unrealistic.

(ii) Simulations are required a runtime long enough which may beyond the CPU capability or user-confined simulation steps since the congestion may need a long waiting-time to occur.

(iii) The congestion of nodes could not be reflected while the congestion in real-life is node congestion generally.

(iv) The dispersing of congestion in real-life network transportation systems could not be reflected as well.

(v) The real configuration of real-life transport network, such as the influence of the distance and the capability of the physical facilities, also could not be reflected.
Consequently, previous consideration could not reflect the real characteristics of network transportation very well, thus it is difficult to apply previous consideration to solve real transportation problems.

One abstract method for the study of network transportation from the local point of view has been proposed in this paper. By interpolating the real-life configuration properties of the edges of transportation networks into the two leading parameters of the nodes: capacity and power, we can abstract transportation network more reasonably and reflect real-life network transportation in detail more realistically, especially the network congestion. After applying this method, as an example, to the traffic problem in the process of city expansion, simulation results show that: the intuitions that with the same rate of particle generation and the same capacity and power, the smaller the network is the harder for the network to become congested; and the network will never be congested if the power of each node equals or is bigger than its capacity \[16\] are incorrect. And the distance from one crossroad to another and the width of the roads can be designed properly, when designing a city, so as to enlarge the tolerance size of the city in the process of city expansion. This method may provide more appropriate guide for real-life traffic design of network transportation. This method can also be used to study different kinds of network transportation without losing the influence of the distance and capability of the physical facility via which the transportation come into being.

In the next section we will propose our method. In section 3, we will give out the results of its application in the process of city expansion and finally in section 4, discussion.

2 The Method

![Fig. 1. The scheme of crossroads.](image)

To interpolate the configuration of real-life transportation network and make the study of transportation on complex network more appropriate for real-life transportation, we abstract networks by including the properties of the edges connected to a node into the two leading parameters of it: capacity \((C)\) and power \((P)\). This method abstracts real-life transportation network to
topological structure without losing the real configuration of transportation network by regarding the transportation length and the physical facility’s power of delivering/handling particles. The method is implemented in the following way. Every node has two characteristics: capacity ($C$) and power ($P$): the capacity of a node is defined as the maximal number of the incoming particles that can possibly appear in its incoming edges per unit time, while the power is the maximal outgoing particles that can be transported from it via its outgoing edges per unit time. The unit time could be decided according to real criterion demanding. In order to describe this abstract method exactly, we will firstly explain these two characteristics through an example of city traffic before giving out the generalized meaning of them.

Take two crossroads depicted in Fig.1 for example, the capacity of a node is the maximal number of the incoming cars which can possibly appear in the roads connected to it in one periodical time of traffic lights. Then every node has its power of delivering/handling particles which is the maximal number of the cars that could leave from the roads connected to it in one periodical time of traffic lights. Here the period of traffic lights is the unit time. As for the nodes described in Fig.1, for $A$, the incoming flows 1, 3, 6 contribute to its capacity while the out flows 2, 4, 5, 7 contribute to its power; for $B$, the incoming flows 7, 8, 12 contribute to its capacity while the out flows 9, 10, 11 contribute to its power. One more thing, the capacity has relationship with the length and the width of the roads while the power has relationship with the width of the roads, thus

$$C(A) = C(1) + C(3) + C(6)$$
$$P(A) = P(2) + P(4) + P(5) + P(7),$$

$$C(B) = C(7) + C(8) + C(12)$$
$$P(B) = P(9) + P(10) + P(11).$$

All of the roads connected to crossroads have their chances to transport cars in a periodical time of traffic lights. The smallest power of a node in one periodical time of traffic light is its outgoing degree, and this smallest power occurs when all the connected outgoing roads are narrowest with only one driveway. With the widening of the roads the delivering/handling power of nodes will increase. What’s more, one can easily draw a conclusion that it is meaningless to have $P > C$ since it is meaningless to try to deliver/handle more particles than the maximal number of particles that the node can hold.

In previous studies [10][11][12][13][14], network congestion was defined to be equal to the accumulation of particles on the network. But in real-life transportation problems, congestion only occurs when some particles on the network have nowhere to go. Take city traffic for example, one rode became congested if
there is no road for the cars in it to go to, that means the congestion of nodes is caused by the capacity saturation of other nodes but not that of itself. Here we use a different parameter, congestion time $T_c$, as the criterion parameter of the transportation capability of networks. The congestion time $T_c$ is defined as the time when the network run into a congestion state that could not be dispersed anymore. Whereafter, all the nodes in the network will be congested gradually. With the same rate of particle generation, $T_c$ can reflect the transportation capability of networks: small $T_c$ means the network will be congested quickly while infinite $T_c$ means the network will never be congested.

This abstract method can be generalized to all kinds of social, biological, and electronic transportation networks, such as traffic, internet, WWW, etc., since the efficiency of network transportation relies on the quality of its physical facilities, which can affect the maximal number of particles it can hold and deliver/handle. After interpolating the properties of the edges that constitute networks into the two leading parameters of nodes $C$ and $P$, what we need when studying network transportation is its directed topological structure which determines the incoming and outgoing degree distribution of the network, and adds in the network with the intrinsic property of the nodes. And when the incoming and outgoing by edges could not be distinct exactly, the capacity was added with a renormalization condition.

3 Results and Analysis

Below are the simulation results when we apply this abstract method to the traffic problem in the process of city expansion.

![Fig. 2. The number of congested nodes versus time.](image)

When simulation, we adopt a linear relationship between the capacity/power and the incoming degree/outgoing degree: $C = a \times k_{in}$, $P = b \times k_{out}$, where $k_{in}$ is the incoming degree and $k_{out}$ is the outgoing degree, $a$ and $b$ are two tunable parameters which can reflect the length and the width of the roads. And we base our simulation on a most simple two-dimensional lattice by regarding that
two-dimensional lattice is close to the real structure of city traffic network and the roads are mostly two-way roads, namely $k_{in} = k_{out}$. Particles can incoming or outgoing via all of the roads along the shortest path, as well as steer clear of the congested nodes. The number of particles generated at every time step $n = N/2$, $N$ is the number of nodes in the network.

In Fig.2, the dispersing of congested nodes can be observed. The network can only revert to free flow when the number of congested nodes is very small because of the big rate of particle generation at every time step, and the congestion time $T_c = 31$.

![Fig. 3. The congestion time versus the size of the network.](image)

The size of the networks in Fig.3 goes from $2 \times 2$ to $30 \times 30$. As explained above, the smallest value of $b = 1$, and the gap in Fig.3(d-j) means the congestion time exceeds our simulation time steps. Fig.3 shows that:

(i) With the same rate of particle generation and the same capacity and power, the intuition that the smaller the network is the harder for the network to become congested is proved to be incorrect. In fact this situation occurs only when the transportation network has the smallest capacity and power. Otherwise, with the increase of capacity or power, certain values of $a$ and $b$ correspond to networks with certain size that has the most outstanding transportation capability but not the smaller or bigger ones.

(ii) Another intuition that the network will never be congested if the power of each node equals or is bigger than its capacity \[16\], namely $P \geq C$, is also proved to be incorrect (Fig.3 (j)). This account for the fact that congestion is
caused by the situation that there is nowhere for some particles to go but not the saturation of capacity, as explained above.

(iii) The width of the gap in Fig.3 (e-j) is increasing, that means the values of $a$ and $b$ can be set properly by considering the distance from one crossroad to another and the width of the roads connected to them, when designing city traffic, so as to enlarge the tolerance size of the city in the process of city expansion, consequently avoid frequent reconstruction or over-estimated building of city roads which is a waste of natural resource and human power.

What’s more, the simulation results show that the variation of power is more efficient than the variation of capacity to change the transportation capability.

4 Discussion

In summary, one abstract method for the study of network transportation from the local point of view has been proposed in this paper. By interpolating the real-life configuration properties of the edges of transportation networks into the two leading parameters of the nodes: capacity and power, we can abstract transportation network more reasonably and reflect real-life network transportation in detail more realistically, especially the network congestion. You can increase the rate of particle generation per time step, thus avoid the missing of congestion because of the limited simulation time or CPU time, without losing the intrinsic property of the network transportation. After applying this method to the traffic problem in the process of city expansion, we get some significant results that have not been reflected in previous studies, and may provide more instructive guide for the traffic design of network transportation. This method can also be used to study different kinds of network transportation without losing the influence of the distance and capability of the physical facility via which the transportation come into being.

What’s more, we will consider nonlinear or more complex relationship between the capacity/power and the directed degree, as well as changeable rate of particle generation in our future work.

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