Predicting Glaucoma Visual Field Loss
by Hierarchically Aggregating Clustering-based Predictors

Motohide Higaki¹, Kai Morino¹, Hiroshi Murata², Ryo Asaoka², Kenji Yamanishi¹,³

¹ Graduate School of Information Science and Technology, The University of Tokyo, Tokyo 113-8656, Japan.
² Department of Ophthalmology, The University of Tokyo, Tokyo 113-8655, Japan.
³ CREST, JST, Honcho, Kawaguchi, Saitama 332-0012, Japan.

March 24, 2016

Abstract

This study addresses the issue of predicting the glaucomatous visual field loss from patient disease datasets. Our goal is to accurately predict the progress of the disease in individual patients. As very few measurements are available for each patient, it is difficult to produce good predictors for individuals. A recently proposed clustering-based method enhances the power of prediction using patient data with similar spatiotemporal patterns. Each patient is categorized into a cluster of patients, and a predictive model is constructed using all of the data in the class. Predictions are highly dependent on the quality of clustering, but it is difficult to identify the best clustering method. Thus, we propose a method for aggregating cluster-based predictors to obtain better prediction accuracy than from a single cluster-based prediction. Further, the method shows very high performances by hierarchically aggregating experts generated from several cluster-based methods. We use real datasets to demonstrate that our method performs significantly better than conventional clustering-based and patient-wise regression methods, because the hierarchical aggregating strategy has a mechanism whereby good predictors in a small community can thrive.

keywords: glaucoma, hierarchical aggregating strategy

1 Introduction

1.1 Motivation and Purpose of this Study

In this study, we address the issue of predicting the glaucomatous visual field loss based on patient datasets. Glaucoma is an eye disease that eventually losses the visual field. The goal of the present study is to accurately predict the glaucoma progression using small visual field datasets. Because measurements of visual field is expensive, early prediction of glaucoma progression based on limited measurements is important for real clinical settings. A conventional approach for predicting visual field loss is patient-wise linear regression [6] and clustering-based method [7]. In patient-wise linear regression, we construct a predictor for each individual patient. However, as the number of measurements for each patient is very small, we cannot produce a good predictor for individuals. Liang et al. proposed a clustering-based method for glaucoma progression which utilizes spatiotemporal disease pat-
terns. Each patient is categorized into a cluster based on each spatiotemporal pattern. Then, for each cluster, a linear regression predictor is constructed. A predictor formed from a single cluster is called a **cluster-based predictor**; the clustering-based method selects an appropriate pattern from a pool of cluster-based predictors. The prediction accuracy of this method is highly dependent on the quality of the clustering method. Furthermore, a clustering-based method will not work well if the target patient is not a typical member of clusters.

The aim of the present study is to overcome this weakness in clustering-based methods. We propose a novel framework for aggregating a number of cluster-based predictors. This significantly improves the prediction accuracy over that of a single cluster-based predictor in the case of real glaucoma patient datasets. We present a schematic illustration of our approach in Fig. 1.

### 1.2 Previous Work

In addition to conventional patient-wise linear regression and clustering-based methods [7], various techniques have been developed to predict the visual field loss in glaucoma patients. Bengtsson et al. [1] introduced the **visual field index** (VFI), and showed that only five measurements were needed to produce a correlation coefficient of 0.84 with the VFI obtained using all measurements. Russell et al. [14] used Bayesian linear regression to derive the **mean sensitivity index**. They showed that their approach outperformed the ordinary linear regression method when there were fewer than nine measurements. Murata et al. proposed a variational Bayesian learning method [12]. Maya et al. proposed a multi-task learning method to predict the visual field loss, using matrix decomposition [9]. Maya et al. also proposed a hierarchical MDL-based clustering method to detect progressive patterns of glaucoma [8]. Recently, Tomoda et al. proposed a prediction method utilizing information of eye pressure [15]. Other methods were reviewed by Liang et al. [7], including those reported by Noureddin et al. [13], Fitzke et al. [5], Chan et al. [3], Chan et al. [1], and Mayama et al. [10].

However, the best predictor for each patient will differ according to his/her disease features. In this context, it is crucial to investigate a strategy to aggregate these prediction methods and automatically to generate the best prediction. Thus, the present study addresses the issue of combining several prediction methods to obtain a better prediction than that produced using a single method. Such aggregating strategies have worked well in other applications, for example, the issue of predicting prostate specific antigen from short time series [11].

### 1.3 Novelty and Significance of this Study

We propose a new aggregating strategy based on a previously proposed aggregating algorithm ([2], [16]). In this method, each predictor is treated as an **expert**, and the prediction is obtained by taking a weighted average of all experts’ predictions. In theory, this method performs almost as well as the best expert in terms of the worst-case regret. However, in practice, the aggregating algorithm outperforms all of the experts in terms of the instantaneous prediction loss.

The novelty of the present study is a novel aggregating algorithm which is adapted to glaucoma visual loss prediction: We modify the original aggregating algorithm as follows to enable its practical application to the specialized setting of glaucoma visual loss prediction:

1. **Learning rate adaptation**: The learning rate is estimated using training examples to maximize the improvement rate. This indi-
Figure 1: Schematic illustration of the prediction of visual fields in glaucoma patients. (a) Glaucoma progression. Regions filled with grayscale colors correspond to local portions of a visual field. Darkness corresponds to the degree of loss of the visual field. (b) Schematic of our proposed method. Circles show the normality of a visual field at each time point. The question mark in the dotted circle is the prediction target. Dotted lines denote cluster-based predictors. Our proposed method aggregates the cluster-based predictors and gives a better prediction.

indicates the improvement in prediction loss relative to the patient-wise linear prediction. This learning rate achieves better performance than the theoretically designed learning rate.

2) Batch learning adaptation: The expert weights are determined in a batch process, rather than the online process employed in the original aggregating algorithm. The modified algorithm optimizes the use of all patient data, thereby producing better predictions than the original method.

3) Hierarchical aggregation: We propose an aggregation process, the hierarchical aggregating strategy. This method first aggregates all cluster-based predictors from a single clustering method to obtain intermediate predictors, then aggregates these intermediate predictors (see Fig. 2(c)). When we have several clustering-based methods, we can aggregate several types of experts. A facile idea is aggregating all cluster-based predictors over all the clustering methods at once (see Fig. 2(b)). We call this simple method the flat aggregating strategy. Although the performance of the flat aggregation is better than clustering-based methods, our analysis indicates that this hierarchical aggregating strategy exhibits much better performance than the flat aggregating strategy.

The significance of this study can be summarized as follows:

A) Better use of predictors in the area of glaucoma visual field loss: We demonstrate that our proposed method delivers better prediction accuracy than the flat aggregating strategy, the conventional clustering-based methods, and patient-wise linear regression.

B) A new framework for aggregating visual field loss predictors: There are a number of promising methods for predicting glaucoma visual field loss, and many more are expected to be proposed in the future. Thus, our method provides a general strategy whereby new methods do not have to compete with existing methods. Instead, our approach allows numerous methods to collaborate in the hierarchical aggregation framework. Therefore, by adding new methods to the pool of experts, our hierarchical aggregation approach obtains better predictions than those given by previous techniques.

The remainder of this paper is organized as follows. In Section 2 we describe the prediction of visual fields. Section 3 reviews the aggregating algorithm, and Section 4 discusses the construction of experts from clustering-based methods. In Section 5 we present the details of our modification of the existing aggregating algorithm, and present our experimental results in Section 6. Section 7 contains
the observation is larger. This difference is
when the difference between its prediction and
The weight assigned to an expert decreases
automatically using the loss function
where
expert prediction
ber of experts. The weights
expert’s prediction at time
t\). The upper bound
predictions during each time step, \( \hat{y}_t \). Through a clustering method, we generate a cluster-based predictor from the cluster containing \( y_1, \ldots, y_{t-1} \). These predictors enables us to produce a prediction using the above vector formalism.

3 Review of Existing Aggregating Algorithm

We introduce the basic framework of an existing aggregating algorithm [2]. This method predicts \( \hat{y}_t \) at time \( t \) by combining various expert predictions during each time step, \( \hat{y}_t = (\sum_{i=1}^{N} w_{i,t} f_{i,t})/(\sum_{j=1}^{N} w_{j,t}) \), where \( f_{i,t} \) is the \( i \)th expert’s prediction at time \( t \) and \( N \) is the number of experts. The weights \( w_{i,t} \) are updated automatically using the loss function \( l(\cdot, \cdot) \) of expert prediction \( f_{i,t} \) for outcome \( y_t \) as follows:

\[
w_{i,t+1} = w_{i,t} \exp(-\eta l(y_t, f_{i,t})), \tag{1}
\]

where \( \eta \) is a constant called the learning rate. The weight assigned to an expert decreases when the difference between its prediction and the observation is larger. This difference is evaluated using loss functions, e.g., \( l(x,y) = (x - y)^T(x - y) \).

The performance of this aggregating algorithm is evaluated based on the regret, which is defined as \( \sum_{k=1}^{n} l(y_k, \hat{y}_k) - \min_{1 \leq i \leq N} \sum_{k=1}^{n} l(y_k, f_{i,k}) \). The upper bound of the regret is known to be \( \ln N/\eta + \eta n/8 \) [2]. The optimal learning rate \( \eta_* = \sqrt{8 \ln N/n} \) is derived by minimizing the regret to \( \sqrt{n \ln N/2} \). We call this theoretically optimal learning rate \( \eta_* \) the Regret (RG)-optimal \( \eta \). For more details, see [2].

4 Generating Experts from Clustering Results

In this section, we explain how to construct experts from existing clustering methods. In the process of prediction with a single clustering method, a cluster-based predictor is generated for each cluster. The most suitable predictor then gives the prediction. Such clustering-based methods often produce good predictions, but it is not clear whether the selected predictor is optimal. If the target patient is not adequately represented by a single cluster, none of the cluster-based predictors will produce sufficiently accurate predictions. In this case, combining the contributions from several clusters may improve the prediction accuracy. Therefore, we can think of cluster-based predictors as experts, and aggregate them to produce better predictions. The process of generating experts from clustering-based methods is illustrated in Fig. 2(a).

4.1 Spatiotemporal Clustering

Liang et al. [7] proposed a spatiotemporal clustering algorithm that predicts the progress of glaucoma using past patient information. The algorithm comprises a spatial feature clustering method and a prediction module that
uses the temporal characteristics of each spatial cluster. This scenario is illustrated in Fig. 3.

We employ the spatial feature clustering method (called uv-EM) proposed in [7]. This is an expectation-maximization (EM) algorithm that learns the spatial cluster centers and temporal feature vectors of glaucoma patients’ visual fields.

From the clinical knowledge that TD decreases linearly over time, we assume that the clustering-based predictors can be written as follows [7]:

\[ y(t) = w_1 t + w_2, \]  

(2)

where \( y(t) \) is the predicted TDs at time \( t \), and \( w_1 \) and \( w_2 \) denote the progression rate and initial TD, respectively. These predictors are constructed using the following two methods. The first method is temporal-shift linear regression (TSLR), where the disease progression rate of all patients belonging to the same spatial cluster is assumed to be the same. However, the initial state of the disease can be different in each patient. Thus, we obtain a cluster-based predictor through linear regression by applying optimal time-shifts to the patients within a cluster. The second method is slope clustering (SC), where there are at most \( C \) progression rates for local visual points in each spatial cluster.

These predictions are more accurate than the traditional patient-wise LR predictors [7], which implies that these clustering-based methods represent the characteristic of glaucoma patient datasets. Hereafter, we denote uv-EM + SC as SC and uv-EM + TSLR as TSLR for simplicity (For more details, see [7]).

### 4.2 Construction of Experts from Clustering Methods

Next, we explain how to construct experts using TSLR, SC, and patient-wise LR. Each expert is a linear function of time, as in (2). Under TSLR, \( w_1 \) is as the cluster-based predic-
5 Algorithm for Aggregating Clustering Methods

To utilize the experts introduced above, we modify the original aggregating algorithm as follows: 1) The learning rate \( \eta \) is selected to ensure that it is practical; 2) the weight is determined in a batch process, rather than an online process; 3) a hierarchical aggregation algorithm is applied to the clustering-based methods. The pseudo-code is shown in Alg. 1.

5.1 Empirical Determination of the Optimal Learning Rate \( \eta \)

As described in Section 3, the existing algorithm derives a theoretical RG-optimal \( \eta \). Our goal is to minimize the instantaneous prediction loss at a desired time point; thus, the regret is not appropriate. The root mean squared error (RMSE) was used to measure the prediction accuracy as

\[
\sqrt{\frac{1}{D} \sum_{j=1}^{D} (\hat{y}_j - y_j)^2}
\]

where \( \hat{y} = (\hat{y}_1, \ldots, \hat{y}_D) \) are the predictions and \( y = (y_1, \ldots, y_D) \) are the observations. The performance of the prediction method is measured in terms of the Improvement Rate (IR), which is defined as

\[
IR(n) = \frac{1}{N(n)} \sum_{i=1}^{N(n)} a_i(n),
\]

where \( n \) is the number of observation points \( n \in [2, 10] \) and \( N(n) \) is the number of patients in the test dataset. The value of \( a_i(n) \) is

\[
1 - \frac{\text{RMSE}_f(i)}{\text{RMSE}_{LR}(i)} \quad \text{if } n < L_i,
\]

where \( \text{RMSE}_{LR}(i) \) is the RMSE of the ith patient using patient-wise LR, and \( \text{RMSE}_f(i) \) is the RMSE using prediction method \( f \). The larger the value of IR, the greater the improvement of \( f \) relative to the patient-wise LR. When \( f \) has the same accuracy as the patient-wise LR, \( IR = 0 \). We empirically determine the improvement-rate (IR)-optimal \( \eta \) by max-

---

**Algorithm 1** Aggregation of clustering methods.

**Input**: Observables \( y_1, \ldots, y_n \) for the target patient.

Determine \( w_1 \) based on past patient datasets using TSLR, SC, and LR.

Determine \( w_2 \) from the observations.

Determine RG-optimal and IR-optimal values of \( \eta \) using the experts.

Determine the weights of the experts based on these values of \( \eta \).

**Output**: Make prediction using the aggregating algorithm.

---

Figure 3: Generating clusters using a spatiotemporal clustering algorithm [7].
5.2 Determining Expert Weights Using a Batch Process

In the usual online algorithm, the expert weights are updated after each new observation. For the glaucoma datasets, samples are usually obtained at intervals of a few months. Thus, it is natural to use all available data points to determine the weights of the experts. Our method employs the following batch update rule:

\[ w_{i,t+1} = \exp\left(-\eta \sum_{\tau=1}^{t} l(y_{\tau}, f_{i,\tau})\right), \]

which replaces the traditional rule (1).

5.3 Hierarchical Aggregation

We propose a hierarchical aggregating strategy, where all the predictors given by each single clustering-based method are first aggregated to construct intermediate predictors. These intermediate predictors are then aggregated to construct the final predictor (see Fig. 2(c)).

The purpose of the proposed method is to adequately deal with experts generated from a small expert source. The number of experts generated from each clustering-based method is naturally different. In the flat aggregating strategy, a good expert belonging to a small community may not greatly affect the prediction because a large community gathers considerable expert weights. On the contrary, in the hierarchical aggregating strategy, experts within each community are aggregated to generate intermediate predictors. Then, the final prediction is given by aggregating intermediate predictors that only one intermediate predictor belongs to each community. Therefore, such effects on the biased community size may be canceled at the second aggregation. In the real glaucoma datasets, the number of experts generated from clustering-based methods is widely distributed, therefore, our proposed method showed the best performance among all the methods as shown in the next Section 6.

6 Experimental Application to Glaucoma Datasets

We compare the effectiveness of our proposed hierarchical aggregating strategy against a single cluster-based predictor for glaucoma datasets.

6.1 Problem Setting

The dataset used in this study is the same as that used by Liang et al. [7]. We divided the dataset into two parts, a learning set (90%, 977 patients) and a test set (10%, 109 patients). In the learning period, we constructed experts using clustering methods, and used them to determine RG-optimal and IR-optimal \( \eta \) values. In the test period, we predicted the final observation point for each patient. The prediction errors were evaluated using ten-fold cross-validation.

We set the number of spatial clusters to \( K = 40 \) and the number of slope clusters to \( C = 5 \), as reported in [7]. In our dataset, \( D = 74 \). Clusters were omitted if they contain fewer than three patients. LR, SC, and TSLR gave 977, 977, and 38 experts, respectively. For all calculations, we used the loss function \( l(x,y) := \|x-y\|_2/(30\sqrt{D}) \).

6.2 Determination of the Optimal Learning Rate \( \eta \)

The IR-optimal \( \eta \) is obtained through ten-fold cross-validation using portions of the learning dataset. As shown in Fig. 4, IR attained peaks at different values of \( \eta \). The IR-optimal \( \eta \) generally agrees with the optimal \( \eta \) obtained using the test dataset. However, the RG-optimal \( \eta \) are clearly smaller than the optima.
Table 1: IR versus observation point \( n \) using IR-optimal \( \eta \), RG-optimal \( \eta \), the best expert, and the methods of Liang et al. \[7\] based on LR (upper), TSLR (middle), and SC (lower) experts. IRs corresponding to the best performance are underlines. We denote cases in which our method is statistically-significant better than Liang et al.’s methods with * \((p < 0.05)\) and ** \((p < 0.01)\).

|        | LR          | TSLR         | SC          |
|--------|-------------|--------------|-------------|
|        | \( n = 2 \) | \( n = 2 \)  | \( n = 2 \) |
| IR-optimal | 0.852 | 0.853 | 0.850 |
| RG-optimal | 0.852 | 0.852 | 0.851 |
| Best expert | 0.695 | 0.834 | 0.761 |
| Liang et al. | 0.818 | 0.850 | 0.818 |

6.3 Prediction using Experts Generated by a Clustering Method

We compared our aggregating strategies with IR-optimal \( \eta \) and RG-optimal \( \eta \) with the pre-
Figure 5: Dependence of IR on the number of observations $n$ for experts generated using (a) LR, (b) SC, and (c) TSLR. The performance of the patient-wise LR corresponds to the zero horizontal line. (d) IRs for the five aggregation algorithms with IR-optimal $\eta$ are compared. Error bars show the standard deviations of $a_i(n)$.

Predictions of the best expert (i.e., that with the largest weight at the final observation point) and the methods of Liang et al. \[7\]. As shown in Figs. 5(a)-(c) and Table II, the aggregating strategy with IR-optimal $\eta$ produced a greater IR than that of the other methods in almost all cases. We applied a one-sided binomial statistical test to these results by counting the number of “wins” in prediction accuracy among the patients in the test dataset. The null hypothesis that the prediction ability of our method is the same as that of Liang et al.’s method was rejected in many cases (shown in Table II with symbols * ($p < 0.05$) and ** ($p < 0.01$), where the p-index is denoted by $p$). Thus, aggregating cluster information allows for better prediction performance than using only one cluster.

Our method with IR-optimal $\eta$ also outperformed the best expert. This indicates that the clusters do not adequately represent the patient characteristics. This result also suggests that existing glaucoma prediction methods could be enhanced using our aggregating algorithm.
6.4 Prediction using Hierarchical Aggregating Algorithms

The hierarchical aggregating strategy with IR-optimal $\eta$ achieved better IR values than the flat aggregating strategy and single clustering methods. In addition, a one-sided binomial statistical test showed that the hierarchical approach with IR-optimal $\eta$ was statistically significantly better than the original TSLR and SC in all cases. These results are summarized in Fig. 5(d) and Table 2.

Our proposed aggregating algorithm clearly works very well. Aggregating algorithms with IR-optimal $\eta$ exhibited the best performance in almost all cases, as shown in Tables 1 and 2. Moreover, the hierarchical aggregating strategy with IR-optimal $\eta$ achieved the best performance in terms of IRs and the one-sided binomial test ($p < 0.01$) as shown in Fig. 5(d). This implies that the hierarchical method is the best means of aggregating the experts generated by clustering-based methods for glaucoma datasets.

6.5 Discussion

Hierarchical aggregation outperformed the flat aggregating strategy for the following reasons. If there are a few experts, those with larger weights (i.e., better experts) in a clustering method will lose their influence in the flat aggregating strategy, because the initial weight becomes small during the flat aggregation process over a large number of experts. Such experts will survive in the hierarchical aggregating strategy, because this approach gives equal initial weights to all clustering methods, regardless of how many experts they include. In other words, the hierarchical aggregating strategy has a mechanism whereby good predictors in a small community (i.e., a clustering-based method with a small number of experts) can thrive. This works particularly well when the number of experts varies widely across all of the cluster-based methods. This was the case in our experiments where the three clustering-based methods had 977, 977, and 38 experts for LR, SC, and TSLR, respectively, and TSLR produced a very good predictor (see Table 2).

When the number of experts does not deviate widely, the advantages of the hierarchical strategy are not so pronounced. Such a situation might cause the data to be overfitted through the repeated aggregation. Hence, whether flat or hierarchical aggregation is preferable depends on the distribution of the experts in the clustering-based methods, as well as the nature of the data itself.

7 Conclusion

In the present study, we have developed prediction algorithms that aggregate the experts generated by clustering-based methods. Our aggregation framework has been then applied to the prediction of glaucoma progression. There are three main differences between our proposed method and the existing approach. First, we have used an empirically optimized learning rate. Second, the expert weights are determined by using a batch process. Third, we have examined the hierarchical aggregating strategies for multiple clustering-based methods. We found that the hierarchical aggregating strategy gives consistently better predictions than the traditional patient-wise linear regression, single clustering-based methods, the best expert, and the flat aggregation.

The findings reported here may contribute to knowledge discovery and data mining by connecting clustering methods and predictions through aggregation algorithms. Our method for aggregating experts generated several clustering methods that obtained better prediction results than the original clustering-based method. This suggests that the prediction ac-
Table 2: IR versus observation point \( n \) using IR-optimal \( \eta \) and RG-optimal \( \eta \). IRs corresponding to the best performance are underlined. Statistically-significant results with respect to TSLR [7] are indicated with * (\( p < 0.05 \)) and ** (\( p < 0.01 \)). In all cases, the proposed method was significantly better than original SC [7] with \( p < 0.01 \).

| Strategies          | \( n = 2 \)  | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
|---------------------|--------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Flat (IR)           | *0.851       | **0.734| **0.620| **0.518| **0.420| **0.324| **0.243| **0.184| **0.154|
| Hierarchical (IR)   | **0.856      | **0.740| **0.628| **0.527| **0.433| **0.339| **0.259| **0.202| **0.175|
| Flat (RG)           | 0.851        | **0.731| 0.612 | 0.504 | 0.401 | 0.297 | *0.212| 0.148 | 0.122 |
| Hierarchical (RG)   | **0.853      | **0.733| **0.615| **0.506| 0.403 | 0.298 | *0.212| 0.148 | 0.121 |

Accuracy could be further improved by embedding additional clustering methods.

From a clinical significance viewpoint, our method may help to establish good predictions of glaucoma progression. In addition to its good prediction performance, our framework of aggregating cluster-based predictors may be sufficiently flexible to include other clustering methods and predictors, because the algorithm does not assume any specific constraints. This flexibility will allow clinicians to add novel prediction methods. Therefore, our method will contribute to improving the future performance of glaucoma predictions. Moreover, our proposed method is not limited to glaucoma progress prediction. Therefore, our proposed aggregating strategies for clustering-based methods can be applied in a wide range of areas.

**Acknowledgments.**

This work is partially supported by JST-CREST.

**References**

[1] B. Bengtsson, V. M. Patella, and A. Heijl. Prediction of glaucomatous visual field loss by extrapolation of linear trends. *Archives of Ophthalmology*, 127(12):1610–1615, 2009.

[2] N. Cesa-Bianchi and G. Lugosi. *Prediction, Learning, and Games*. Cambridge University Press, 2006.

[3] K. Chan, T.-W. Lee, P. A. Sample, M. H. Goldbaum, R. N. Weinreb, and T. J. Sejnowski. Comparison of machine learning and traditional classifiers in glaucoma diagnosis. *IEEE Transactions on Biomedical Engineering*, 49(9):963–974, 2002.

[4] K. Chan, T.-W. Lee, and T. J. Sejnowski. Variational learning of clusters of undercomplete nonsymmetric independent components. *The Journal of Machine Learning Research*, 3:99–114, 2003.

[5] F. W. Fitzke, R. A. Hitchings, D. Poinosawmy, A. I. McNaught, and D. P. Crabb. Analysis of visual field progression in glaucoma. *British Journal of Ophthalmology*, 80(1):40–48, 1996.

[6] C. Holmin and C. E. T. Krakau. Regression analysis of the central visual field in chronic glaucoma cases. *Acta Ophthalmologica*, 60:267–274, 1982.

[7] Z. Liang, R. Tomioka, H. Murata, R. Asaoka, and K. Yamanishi. Quantitative prediction of glaucomatous vi-
sual field loss from few measurements. 

*Proc. IEEE International Conference on Data Mining (ICDM’13)*, pages 1121–1126, 2013.

[8] S. Maya, K. Morino, H. Murata, R. Asaoka, and K. Yamanishi. Discovery of glaucoma progressive patterns using hierarchical mdl-based clustering. *the 21th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD2015)*, pages 1979–1988, 2015.

[9] S. Maya, K. Morino, and K. Yamanishi. Predicting glaucoma progression using multi-task learning with heterogeneous features. *Proceedings of IEEE BigData2014*, 2014.

[10] C. Mayama, M. Araie, Y. Suzuki, K. Ishida, T. Yamamoto, Y. Kitazawa, M. Shirakashi, H. Abe, H. Tsukamoto, H. K. Mishima, K. Yoshimura, and Y. Ohishi. Statistical evaluation of the diagnostic accuracy of methods used to determine the progression of visual field defects in glaucoma. *Ophthalmology*, 111(11):2117–2125, 2004.

[11] K. Morino, Y. Hirata, R. Tomioka, H. Kashima, K. Yamanishi, N. Hayashi, S. Egawa, and A. Kazuyuki. Predicting disease progression from short biomarker series using expert advice algorithm. *Scientific Reports*, 5:8953, 2015.

[12] H. Murata, M. Araie, and R. Asaoka. A new approach to measure visual field progression in glaucoma patients using variational bayes linear regression. *Investigative ophthalmology and visual science*, 55(12):8386–8392, 2014.

[13] B. Noureddin, D. Poinoosawmy, F. Fietzke, and R. Hitchings. Regression analysis of visual field progression in low tension glaucoma. *British Journal of Ophthalmology*, 75(8):493–495, 1991.

[14] R. A. Russell, R. Malik, B. C. Chau- man, D. P. Crabb, and D. F. Garway-Heath. Improved estimates of visual field progression using bayesian linear regression to integrate structural information in patients with ocular hypertension. *Investigative Ophthalmology and Visual Science*, 53(6):2760–2769, 2012.

[15] K. Tomoda, K. Morino, H. Murata, R. Asaoka, and K. Yamanishi. Predicting glaucomatous progression with piecewise regression model from heterogeneous medical data. *9th International Conference on Health Informatics (HEALTH-INF 2016)*, (accepted).

[16] V. G. Vovk. Aggregating strategies. *Proceedings of Third Workshop on Computational Learning Theory*, pages 371–386, 1990.