Hadronization Corrections to Jet Cross Sections in Deep-Inelastic Scattering

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Abstract: The size of non-perturbative corrections to high $E_T$ jet production in deep-inelastic scattering is reviewed. Based on predictions from fragmentation models, hadronization corrections for different jet definitions are compared and the model dependence as well as the dependence on model parameters is investigated. To test whether these hadronization corrections can be applied to next-to-leading order (NLO) calculations, jet properties and topologies in different parton cascade models are compared to those in NLO. The size of the uncertainties in estimating the hadronization corrections is compared to the uncertainties of perturbative predictions. It is shown that for the inclusive $k_T$ ordered jet clustering algorithm the hadronization corrections are smallest and their uncertainties are of the same size as the uncertainties of perturbative NLO predictions.

1 Introduction

Before the prediction of a perturbative QCD calculation (“parton-level” cross section) can be compared to a measured “hadron-level” jet cross section, the size of non-perturbative contributions (“hadronization corrections”) has to be estimated. Advanced techniques based on “power corrections” \cite{1} are presently only available for the mean values of event shape variables and predict very large hadronization corrections for most of the HERA kinematic range, preventing these observables to be used for stringent tests of perturbative QCD. For such tests observables with small hadronization corrections are needed, for example the production rate of jets with high transverse energies\textsuperscript{1}. Predictions of hadronization corrections to these observables are presently only available in the form of phenomenological fragmentation models such as the Lund string model (as implemented in JETSET \cite{2}) and the HERWIG cluster fragmentation model \cite{3}. These models are implemented in event generators that include leading order matrix elements and a perturbative parton cascade which is matched to the hadronization model.

Based on these models, hadronization corrections are compared for different jet definitions, including a new angular ordered jet clustering algorithm (“Aachen algorithm”). The model

\textsuperscript{1}Throughout the whole paper “transverse energy” always refers to transverse energies in the Breit frame, where “transverse” means the direction perpendicular to the z-axis, which is given by the direction of the incoming proton and the exchanged virtual photon.
dependence and the dependence on model parameters is investigated. These model estimates are usually needed for comparisons of perturbative QCD in next-to-leading order (NLO) to measured data distributions. We therefore also discuss the compatibility of jet topologies in parton cascade models and in NLO. Finally we review the uncertainties of NLO predictions and compare their size to the uncertainties of the estimates of hadronization corrections.

2 Definitions

The present study includes four different jet clustering algorithms which differ in two aspects in how they define jets. The first aspect is the ordering in the clustering of particles. This is either done in the order of smallest relative transverse momenta ("$k_\perp$ ordering") or in the order of smallest angles ("angular ordering") between particles. The second aspect concerns the definition of the jets inside the event. In one case all particles are clustered either to one of the hard jets or to the proton remnant ("exclusive" definitions), while in the other case only some particles are clustered into the hard jets, while other particles remain outside the hard jets ("inclusive" definitions). The following four jet definitions are used, all in the Breit frame:

- **the exclusive $k_\perp$ ordered algorithm** as proposed in [4].
- **the Cambridge algorithm** as proposed in [5] but modified for DIS to consider the proton remnant as a particle of infinite momentum according to the prescription in [4]. This algorithm is similar to the exclusive $k_\perp$ but uses angular ordering.
- **the inclusive $k_\perp$ ordered algorithm** as proposed in [6].
- **the Aachen algorithm** — this is a new jet definition, invented for these comparisons. In analogy to the modification from the exclusive $k_\perp$ algorithm to the Cambridge algorithm, we have modified the inclusive $k_\perp$ algorithm to obtain an inclusive algorithm with angular ordering. The definition is very simple: particles with smallest $R^2_{ij} = \Delta \eta_{ij}^2 + \Delta \phi_{ij}^2$ are successively clustered into jets, until all distances $R_{ij}$ between jets are above some value $R_0$ (as for the inclusive $k_\perp$ algorithm we set $R_0 = 1$). The jets with highest $E_T$ are considered in the analysis. In dijet production in the Breit frame this definition is, at NLO, identical to the inclusive $k_\perp$ algorithm.

For the exclusive jet definitions the recombination of particles is performed in the "$E$-scheme" (addition of four-vectors), while for the inclusive definitions it is done in the "$E_T$-scheme" (the $E_T$ of the jet is the scalar sum of the particle $E_T$s) [3]. To obtain jet cross sections of similar size for all jet definitions the following parameters are used:

- for the inclusive jet definitions jets are required to have $E_{T\text{jet}} > 5 \text{ GeV}$, $E_{T\text{jet1}} + E_{T\text{jet2}} > 17 \text{ GeV}$
- for the exclusive jet definitions, the resolution of jets in the event is defined by the resolution parameter $y_{\text{cut}}$

\[
y_{\text{cut}} < \frac{k^2_{\perp ij}}{100 \text{ GeV}^2} \quad \text{with} \quad k^2_{\perp ij} = 2 \min(E^2_i, E^2_j) \left(1 - \cos \theta_{ij}\right) \quad \text{and} \quad y_{\text{cut}} = 1.
\]

These parameters are identical to those used in a recent dijet analysis by the H1 collaboration [8].
Only events with (at least) two jets in the central region of the detector acceptance ($-1 < \eta_{\text{jet,lab}} < 2.5$) are accepted. The studies are performed in the kinematic range $0.2 < y < 0.6$ and $150 < Q^2 < 5000 \text{ GeV}^2$ (unless stated otherwise).

We define the hadronization corrections to an observable $\mathcal{O}$ as the ratio of its value in a perturbative calculation (“parton-level”: $\mathcal{O}_{\text{parton}}$) and its value in a calculation including perturbative and non-perturbative contributions ( “hadron-level”: $\mathcal{O}_{\text{hadron}}$) $c_{\text{hadr.corr.}} = \mathcal{O}_{\text{parton}} / \mathcal{O}_{\text{hadron}}$.

All predictions have been obtained by the QCD models HERWIG5.9 \cite{HERWIG} (using leading order matrix elements (LO ME), parton shower and cluster fragmentation), LEPTO6.5 \cite{LEPTO} (LO ME, parton shower and string fragmentation) and ARIADNE4.08 \cite{ARIADNE} (LO ME, dipole cascade and string fragmentation). The calculations have been performed for the HERA data-taking in 1997 (820 GeV protons collided with 27.5 GeV positrons) using CTEQ4L parton distributions and the 1-loop formula for the running of $\alpha_s$. The LEPTO predictions are obtained without the soft color interaction model. The NLO calculations are performed using the program DISENT \cite{DIS} in the MS-scheme for CTEQ4M parton distributions and the 2-loop formula for the running of $\alpha_s$. The renormalization scale is set to the average transverse energy of the dijet system $\mu_r = \langle E_T \rangle$, the factorization scale to the mean $E_T$ of the jets $\mu_f = \langle E_T \rangle \simeq 14 \text{ GeV}$.

### 3 Size and Model Dependence of the Predictions

The hadronization corrections as defined above are shown in Fig. \ref{fig:hadcorr} for the HERWIG model as a function of $Q^2$ for the different jet definitions. While at $Q^2 > 1000 \text{ GeV}^2$ all jet definitions have similar and reasonably small corrections (below 10%), at smaller $Q^2$ large differences are seen. In all cases the corrections are smaller for inclusive jet definitions than for exclusive definitions, and smaller for $k_\perp$ ordered algorithms than for angular ordered ones. Only the inclusive $k_\perp$ algorithm shows a small $Q^2$ dependence and acceptably small corrections, even down to very small $Q^2$ values (below 10%). For this definition we will study in more detail differential distributions. In Fig. \ref{fig:hadcorr} the hadronization corrections from different models are shown as a function of the average transverse jet energy $\langle E_T \rangle$ and the reconstructed parton momentum fraction $\xi$ in different regions of $Q^2$. While the corrections for the $\xi$ distribution are

**Figure 1:** The hadronization corrections to the dijet cross section for different jet definitions as a function of $Q^2$ as predicted by the HERWIG cluster fragmentation model.
flat in all $Q^2$ regions, we observe a slight decrease towards higher $E_T$. The predicted corrections agree within 3% between the different models.

The predictions of these models may, of course, depend on parameters that define the perturbative parton cascade, as well as on parameters of the hadronization model. We have investigated the sensitivity of the LEPTO/JETSET model predictions to variations of some parameters as listed in Table 1. Fig. 3 gives an overview on the effects of these variations which are seen to be small in all cases (less than 4% level).

| LEPTO / JETSET model parameters | default      | variation                  |
|---------------------------------|--------------|----------------------------|
| $\Lambda_{QCD}$ in initial state parton shower | 0.25 GeV     | 0.25 – 0.4 GeV             |
| $\Lambda_{QCD}$ in final state parton shower | 0.23 GeV     | 0.23 – 0.4 GeV             |
| $Q_{ISR}^0$ cutoff for initial state parton shower | 1 GeV        | 0.7 – 2.0 GeV              |
| $Q_{FSR}^0$ cutoff for final state parton shower | 1 GeV        | 0.5 – 4.0 GeV              |
| width of Gaussian primordial $k_t$ of partons in the proton | 0.44 GeV     | 0.44 – 0.7 GeV             |
| width of Gaussian distribution in $k_t$ when a non-trivial target remnant is split into a particle and a jet | 0.35 GeV     | 0.35 – 0.7 GeV             |
| Gaussian width of $p_t$ for primary hadrons | 0.36 GeV     | 0.25 – 0.45 GeV            |
| $a$ parameter in the symm. Lund fragmentation function | 0.3          | 0.1 – 1.0                  |
| $b$ parameter in the symm. Lund fragmentation function | 0.58         | 0.44 – 0.7                 |

Table 1: Overview on the LEPTO and JETSET parameters and the ranges in which they are varied for the studies of the uncertainties of hadronization corrections.
Figure 3: The hadronization corrections to the $E_T$ (left) and to the $\xi$ distributions (right) for the dijet cross section defined by the inclusive $k_{\perp}$ algorithm. Shown are the predictions from the LEPTO/JETSET model with default parameter settings (line) and the changes obtained by parameter variations as described in the text (dotted lines).

4 Parton Cascade Models vs. NLO Calculations

There is no unique way to separate perturbative and non-perturbative contributions in theoretical calculations. A consistent treatment requires a well defined matching of both contributions, e.g. by the introduction of an “infrared matching scale” \footnote{This, however, is not (yet) available for high $E_T$ jet cross sections in DIS. The only available predictions are those of the hadronization models mentioned above. So the following question appears: “What are the uncertainties if we nevertheless use these model predictions for estimating the hadronization corrections to be applied to NLO calculations?”}. Our attempt to tackle this problem is to assume that non-perturbative effects alter the production rates of multi-jet events only due to the change of the final state topology. Hadronization effects for example cause particles to migrate out of the phase space considered for a particular jet, leading to a decrease of the jet’s transverse energy. For a fixed $E_{T_{\text{jet}}}$ selection cut the resulting jet cross section will be reduced in this case. The argument is therefore the following: If the final states in the NLO calculation and parton shower models show the same properties, the same influence of hadronization processes is to be expected for both. In this case the model predictions can be used to estimate the hadronization corrections for the NLO calculation. In the following we address this question by comparing the predictions of the parton cascade models and NLO for the distribution of jets inside the event (angular jet distributions), the internal structure of the single jets (subjet multiplicities) and the dependence of the dijet cross section on the $R_0$ parameter in the jet definition.
Figure 4: Higher order corrections to jet pseudorapidity distributions for the forward and the backward jet in the HERA laboratory frame, the forward jet in the Breit frame and the average pseudorapidity of the dijet system. Displayed are the predictions from the leading-order matrix elements (LO) and those including higher order corrections from either the next-to-leading order (NLO) calculation, or as given by parton showers (HERWIG, LEPTO) or the dipole cascade (ARIADNE) for the inclusive $k_\perp$ algorithm. Positive pseudorapidities are towards the proton direction in both the laboratory and the Breit frame.

4.1 Higher Order Corrections to Angular Jet Distributions

The pseudorapidity distributions of jets are shown in Fig. 4 for the forward and the backward jet in the HERA laboratory frame (top), for the forward jet in the Breit frame (bottom left) and for the average jet pseudorapidity of the dijet system (bottom right).

Compared are the predictions from the leading-order matrix elements (LO, necessarily the same for DISENT, HERWIG, LEPTO and ARIADNE) and those including higher order corrections from either the next-to-leading order, or as given by parton showers (HERWIG, LEPTO) or the dipole cascade (ARIADNE) for the inclusive $k_\perp$ algorithm. All angular jet distributions are shifted by the NLO corrections to the forward (i.e. the proton) direction — a feature which is reproduced by all parton cascade models (with the exception of the shift in $\eta_{\text{forward,lab}}$ by ARIADNE). We therefore do not derive any uncertainty on the estimation of hadronization corrections for NLO from the study of the angular jet distributions.
Figure 5: Subjet multiplicities for an inclusive jet sample defined by the inclusive $k_\perp$ algorithm. Compared are the predictions of different parton cascade models to the $O(\alpha_s^2)$ calculation.

4.2 Internal Jet Structure

Another test of the comparability of the different approaches is the internal structure of jets. We decided to compare the average number of subjets that are resolved at a resolution scale $y_{cut}$ which is a fraction of the transverse jet energy (a detailed definition of this observable can be found in [13]). The average number of subjets is shown in Fig. 5 as a function of the resolution parameter $y_{cut}$ in different regions of $E_T$ for an inclusive jet sample in the same $\eta_{lab}$ region where the dijet sample is defined. These subjet multiplicities are sensitive to perturbative processes at larger $y_{cut}$ values, while towards smaller $y_{cut}$ non-perturbative contributions become increasingly important.

At smaller $y_{cut}$ the $O(\alpha_s^2)$ calculation\(^3\) has a very different behavior than the parton cascade models where the number of subjets is limited by the available number of partons due to the cutoff in the parton shower while the $O(\alpha_s^2)$ calculation smoothly approaches the divergence at $y_{cut} \to 0$. These differences become smaller towards higher $E_T$ where both approaches show similar qualitative behavior, although significant differences remain. Especially the dipole cascade in ARIADNE gives a much smaller number of subjets than the $O(\alpha_s^2)$ calculation. The best agreement with the $O(\alpha_s^2)$ calculation is observed for HERWIG at larger values of $y_{cut}$ (which characterize the last steps in the clustering procedure). Larger values of $y_{cut}$ are hence connected to the coarse structure of jets which is the region where parton cascades and NLO can be compared.

\(^3\)While the $O(\alpha_s^2)$ calculation makes next-to-leading order predictions for dijet cross sections, it describes the internal structure of jets only at leading order.
It is important to note that the spread of the models in the relevant region of larger $y_{\text{cut}}$ is of the same order as their difference to the NLO calculation. In the previous section we demonstrated that the predicted hadronization corrections agree well between the different models. We therefore conclude that 1) the hadronization corrections are not sensitive to differences in the subjet multiplicities, and 2) that the observed differences between model predictions and NLO in the subjet multiplicities do not enter as an uncertainty in the estimation of the hadronization corrections to NLO predictions.

4.3 Radius Dependence of the Dijet Cross Section

The definition of the inclusive $k_{\perp}$ algorithm contains a single free parameter $R_0$ which defines the maximal distance within which particles are clustered in each step. It follows that the final jets are all separated by distances above $R_0$.

The dependence of the dijet cross section on the value of $R_0$ in the jet definition is directly correlated to the broadness of the jets. In Fig. 6 we compare the $R_0$ dependence of the dijet cross section for parton jets (left) and for hadron jets (right). Shown is the ratio of the dijet cross section at $R_0$ to the dijet cross section at $R_0 = 1$ in the range $0.6 < R_0 < 1.2$.

For the (broader) hadron jets a large $R_0$ dependence is observed (HERWIG: $\pm 13\%$ for a reasonable variation $0.8 < R_0 < 1.2$ around the preferred value of $R_0 = 1$). This dependence is reduced for the parton jets, but slightly different for NLO ($\pm 4\%$) than for the parton cascade models (HERWIG: $\pm 7\%$).

The different behavior of the NLO dijet cross sections and the model predictions for the same observable as a function of $R_0$ constitutes an uncertainty when applying the model predictions of the hadronization corrections to the NLO calculations. This difference (and the corresponding uncertainty) is however below 5%.

5 Properties of the Perturbative Cross Sections

In this section we give a brief overview on some properties and uncertainties of the perturbative NLO cross sections. Fig. 7 (left) shows the size of the NLO corrections (i.e. the k-factor which
Figure 7: The NLO correction to the dijet cross section as a function of $Q^2$ for the inclusive $k_\perp$ algorithm for two different renormalization and factorization scales: $E_T$ and $Q$ (left). The dependence of the NLO dijet cross section for the inclusive $k_\perp$ algorithm on variations of the renormalization scale (right).

is defined as the ratio of the NLO and the LO cross section) for the inclusive $k_\perp$ algorithm as a function of $Q^2$. To be sensitive to the fraction of the $\mathcal{O}(\alpha_s^2)$ contributions, both the NLO and the LO calculations have been performed using the same (CTEQ4M) parton densities and the 2-loop formula for the running of $\alpha_s$. The k-factor is shown for two different choices of the renormalization scale: $\mu_r = E_T$ and $\mu_r = Q$. For both scales the k-factor shows a strong dependence on $Q^2$. While at large $Q^2$ the NLO corrections are small, they become sizeable for $Q^2 < 100\text{ GeV}^2$. Throughout it is seen that the k-factor is smaller for a renormalization scale of the order of the transverse jet energies.

It is usually assumed that the scale dependence of a cross section is somehow correlated to the possible size of higher order corrections, and therefore a measure of the uncertainty. Fig. 7 (right) shows the relative change of the dijet cross section when the renormalization scale $\mu^2$ is changed by a factor of four up and down. The comparison is made for the scales $\mu_r = E_T$ and for $\mu_r = Q$. The dependence on the renormalization scale becomes large at small $Q^2$. Only for $Q^2 > 100\text{ GeV}^2$ this dependence is reasonably small (below 10%). Over the whole range of $Q^2$ the renormalization scale dependence is smaller for the scale $\mu_r = E_T$ than for the scale $\mu_r = Q$. The same variation has been studied for the factorization scale and yields a negligible dependence (below 2 %) over the whole range.

6 Summary and Conclusions

Hadronization corrections to jet cross sections in deep-inelastic scattering have been investigated based on predictions from the hadronization models HERWIG and JETSET as implemented in the event generators HERWIG, LEPTO and ARIADNE. It is seen that these corrections are smaller for inclusive and $k_\perp$ ordered jet definitions as compared to exclusive and angular ordered algorithms. For reasonably large transverse jet energies, the inclusive $k_\perp$ algorithm has hadronization corrections below 10% over very large regions of phase space. The predictions from different models are in very good agreement and show only a weak dependence on the settings of specific model parameters. For the inclusive $k_\perp$ algorithm the corresponding uncertainties are not larger than 4%.
A consequent and consistent consideration of hadronization corrections for perturbative next-to-leading order (NLO) predictions requires a well-defined matching of a hadronization model to the NLO calculation. This is, however, not (yet) available. Any other approach can only be an approximation and is subject to various uncertainties. Based on the assumption that these uncertainties are directly connected to the differences in the final state topology in parton cascade models and in NLO calculations, we have compared their predictions for various topological variables. It is seen that changes in angular jet distributions w.r.t. leading order calculations are very similar for the parton cascade models and NLO and hence do not contribute to the uncertainty. The subjet multiplicities show differences in their behavior in the NLO calculations and the parton cascade models. These differences can however be shown to have no significant influence on the predictions of the hadronization corrections. The dependence on the radius parameter $R_0$, which is directly correlated to the broadness of the jets, turns out to be different for NLO and the parton cascades, leading to an uncertainty of less than 5%.

We conclude that for the inclusive $k_\perp$ jet algorithm at sufficiently large $Q^2$ and $E_{T\text{jet}}$ the hadronization corrections are under control with uncertainties not larger than those of the perturbative NLO calculations. This will allow meaningful tests of perturbative QCD with a precision of better than 10%.

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