

Effects of Inhomogeneity on the Causal Entropic prediction of $\Lambda$

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The Causal Entropic Principle aims to predict the unexpectedly small value of the cosmological constant $\Lambda$ using a weighting by entropy increase on causal diamonds. The original work assumed a purely isotropic and homogeneous cosmology. But even the level of inhomogeneity observed in our universe forces reconsideration of certain arguments about entropy production. In particular, we must consider an ensemble of causal diamonds associated with one cosmology, and we can no longer immediately discard entropy production in the far future of the universe. Depending on our choices for a probability measure and our treatment of black hole evaporation, the prediction for $\Lambda$ may be left intact or dramatically altered.

I. INTRODUCTION

A broad line of argument intended to resolve or ameliorate the notorious problem of the apparent smallness of the cosmological constant ($\rho_\Lambda \approx 1.25 \times 10^{-123}$ in Planck units) is to reject the notion of a fundamental value for $\Lambda$ altogether. In this approach, well-known from the string theory landscape as well as other “multiverse” notions, the problem is transformed to the search for a selection principle that may explain why a value as small as observed is probable. In order to make this formulation two broad decisions must be made, both of which can be controversial: the choice of selection principle, and the probability measure. The first tends to be controversial because a choice of selection principle is a choice about how to categorize our imagined experimental sample of universes in which measurements occur, and thus leads to difficult questions about observers. The choice of probability measure has its own well-known difficulties relating to defining probabilities across different infinite spaces. Different choices for either selection principle or probability measure can lead to wildly different probability predictions, easily changing a prediction of likelihood to an exponentially disfavored one.

Bousso et al. [1] suggested a novel combined approach, the so-called “Causal Entropic Principle” (CEP). For flat universes with a positive fundamental cosmological constant, one can define the causal diamond for a particular world line $\lambda(\tau)$ as the intersection of interiors of the future cone at earliest times and the past cone at late times\(^1\). The resulting region is finite in comoving volume in this flat positive-lambda FRW universe, and diamond-shaped when drawn in comoving coordinates and conformal time. (See Fig. 1). If we restrict our probability measure to the finite interior of this diamond, we can avoid the difficulty in defining a probability measure on infinite spaces. Moreover, the proposed selection principle is a simple weighting proportional to the entropy production $\Delta S$ occurring within the causal diamond. Loosely one may interpret this as an assumption that the number of observers is proportional to the entropy increase within the causal diamond, but in the spirit of [1] we may simply take this weight as a hypothesis and remark that $\Delta S$ has several advantages over some other weightings: 1) It is hearteningly generic, allowing at least the theoretical possibility of application to universes with much different low energy physics from ours. 2) It seems less contrived than typical “anthropic” reasoning; though we may contemplate observers in a universe with no galaxies, it is difficult to imagine them without significant entropy increase. 3) As shown in [3], it can actually reproduce and improve upon previous anthropic results.

Even after accepting the program to calculate likelihoods of physical parameters from some a priori theoretical distribution and after fixing a probability measure, a full calculation of the probability distribution for $\Lambda$ is a formidable task. In an ideal case we would have a background theory giving us some set of cosmological parameters and their prior distribution. We would then allow all parameters to vary and make a prediction for $\Lambda$ by marginalizing over the other parameters, in a scheme such as that in [3]. As a first step, Bousso et al. [1] follow the usual simplification of holding all other physical parameters fixed while modifying only the positive value of $\Lambda$ in a flat FRW universe. Other work has discussed aspects of the CEP for $\Lambda \leq 0$ [4, 5], but in this work we keep the same $\Lambda > 0$ assumption used in the first papers on this subject.

A common drawback of varying only $\Lambda$ in such an approach is the possibility that variation in other parameters could significantly affect the prediction for $\Lambda$ itself. The classic instance is that Weinberg’s prediction of $\rho_\Lambda < 10^{-121}$ [6] under the selection principle that galaxies must form is softened by allowing the density contrast $Q$ or the baryon-to-photon ratio to increase from that observed in our universe. Greater early anisotropies or matter densities and reduced radiation pressure could allow structure to form earlier and thus significantly push up the allowable value of $\Lambda$ [7, 8]. Cline et al. [2] have shown that the entropic approach, at least for $\Lambda$, is resilient when varying $Q$. More recently it has been shown that allowing the curvature of the universe to vary along with $\Lambda$ can dramatically change the CEP predictions for

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\(^1\) The CEP has been recently extended to curved FRW universes in [2].
\( \Lambda \), depending on exactly what priors one take on the cosmic curvature. Other authors have suggested potential limitations of the CEP along with related approaches. In section VII we summarize our conclusions. Throughout this paper we briefly review the CEP method. In section III we discuss the increasing inhomogeneity of entropy produced at late times, illustrated by black hole evaporation. In section IV we describe the necessity of replacing a single causal diamond with an ensemble given an inhomogeneous cosmology. Section V discusses the nature of long-term entropy sources that might compete with stellar entropy production for causal diamonds containing collapsed structures. In section VI we discuss effects on the predicted probability distribution for \( \Lambda \), and in section VII we summarize our conclusions. Throughout we use Planck units with \( \hbar = c = G = 1 \).

\[ \tau = 0 \]

FIG. 1: A causal diamond (depicted schematically here) is the region which can causally impact and be causally impacted by a worldline \( \lambda(\tau) \). The finite entropy produced in the resulting spacetime volume is used in the Causal Entropic Principle as a cosmological weighting factor.

II. THE CAUSAL ENTROPIC PREDICTION FOR \( \Lambda \)

Simply stated, the CEP assumes that the probabilistic weighting for cosmological parameters is proportional to the increase in entropy \( \Delta S \) within a causal diamond associated with that cosmology. Given a multiverse populated with different cosmologies, the CEP thus becomes a tool to calculate probability distributions for measurements of the cosmological parameters themselves. Although in principle one could ask the CEP to thus give predictions for a full range of cosmological parameters, following Bousso et al. we will leave all cosmological parameters fixed at their observed values except the cosmological constant \( \Lambda \).

The causal diamond is defined as the volume contained within the future cone of an early event (taken to be reheating following inflation) as well as within the past cone of a late event on the same world line. The causal diamond is thus the region of space in full causal contact with a particular world line. Following the original argument we will also restrict ourselves to purely positive \( \Lambda \), so that all cosmologies will eventually be dominated by the cosmological constant. In every case a de Sitter horizon will thus form and define the past light cone for the causal diamond.

The CEP choice to restrict entropy increase to that within a causal diamond originated from a holography argument: the universe simply does not consist of a region larger than a single causal diamond. We will not try and argue the pros and cons of this point here, but simply take this restriction as one of our input assumptions. There is however an important extra step which we will talk about in greater detail. If an entire cosmology is represented only by a single causal diamond, we need some way to choose this causal diamond, or equivalently define a particular world line associated with a particular set of cosmological parameters. There is no difficulty doing so in a homogeneous, isotropic universe, as all causal diamonds are identical. Such is clearly not the case for an inhomogeneous universe, and we will thus introduce the statistical notion of an ensemble of causal diamonds associated with a particular cosmology. It should be emphasized that this complication is required even with a very strictly holographic interpretation of the causal diamond.

Black holes immediately come to mind in calculations of cosmological entropy. The entropy associated with the formation of a black hole horizon is explicitly excluded in the CEP, as is de Sitter horizon entropy. This exclusion is important as a single supermassive \( (10^9 M_\odot) \) black hole can have an entropy of \( 10^{91} \) [1], exceeding all other non-horizon entropy sources. One might object, as noted in [1], that this entropy cannot be hidden forever, as on very long time scales the black hole will evaporate and return its entropy to the rest of the causal diamond. Bousso et al. argued that a typical late-time causal diamond is empty and thus we may discount this entropy. We will
revisit this issue below.

It is also important to note that the weighting \( w(\rho) \propto \Delta S \) includes only entropy increase occurring within the causal diamond. Therefore various processes which one might imagine to be strong contributors to entropy increase turn out not to be significant. For example, CMB photons represent a large amount of current entropy, but not of entropy increase within the causal diamond surrounding our world line. The causal diamond at recombination enclosed a much smaller amount of matter (and photons) than a Hubble radius today does, so most CMB photons within our horizon must have entered through the bottom cone of the causal diamond; these photons do not contribute to \( \Delta S \). Other events in the early universe such as nucleosynthesis likewise contribute little to this measure of \( \Delta S \) owing to the small size of the causal diamond. So Bousso et al. [1] restricted themselves to processes active during the era of relatively large comoving scale for the causal diamond. One of the purposes of this paper is to examine whether the very long times available for entropy production in the future of a \( \Lambda \)-dominated universe can compensate for the small volume of matter in causal contact with an observer once a de Sitter horizon forms.

Varying the cosmological constant directly affects the size of the causal diamond, with the comoving 4-volume contained proportional to \( \Lambda^{-1} \). Therefore even before accounting for the effects of entropy production, the CEP rewards smaller values of \( \Lambda \) with greater weight, owing to their larger causal diamonds, at least measured in comoving volume. If we found entropy production to be dominated by a process producing a constant entropy rate per comoving volume, such a process would translate an a priori flat distribution of \( \rho \)

\[
\frac{dP}{d\rho} = \text{const}
\]

into a flat distribution in \( \log(\rho) \)

\[
\frac{dP}{d\rho} \propto w(\rho) = \rho^{-1}
\]

\[
\frac{dP}{d\log(\rho)} \propto w(\rho)\rho = \text{const}
\]

The reduction from a flat distribution to one flat in log space is an indication of how much work the causal diamond portion of the CEP is doing on its own. For realistic entropy sources, the total entropy production (and thus probabilistic weight) was calculated via

\[
w(\rho) \propto \Delta S(\rho) = \int_0^\infty dt V_c(\rho, t) \frac{\partial^2 S(\rho, t)}{\partial V_c \partial t}
\]

Here \( V_c \) is the directly calculable comoving 3-volume of the causal diamond as a function of \( \Lambda \) and \( t \) and \( \partial S/\partial V_c \) is the entropy production rate per comoving volume.

During the current cosmological era for cosmologies similar to ours, calculations in [1] revealed stars to be the greatest contributor to \( \Delta S \) due to photons absorbed and reemitted by cool dust. This large contribution may be seen from estimating entropy increase for a process by \( \Delta S = \frac{dE}{k_B T} \) where \( dE \) is the energy released and \( T \) is the typical temperature (\( k_B = 1 \)). For stars, typical energies released in fusion are about 7 MeV/nucleon. While typical stars produce visible light with an effective \( T \sim \epsilon V \), perhaps half of the photons are absorbed and reemitted by cool dust with a \( T \sim 20\,\text{meV} \). It is the combination of high energy per nucleon, ubiquity of stellar burning, and the low effective temperature of much of the reprocessed starlight which gives stellar entropy the edge over other processes.

### III. COMOVING VOLUME OF UNIVERSE

Stellar entropy production per comoving volume reaches a maximum of \( 2.7 \times 10^{63}/\text{Mpc}^3/\text{yr} \) comoving, as shown in Fig. 2. The causal diamond gets as large as \( \sim 10^{13}\text{Mpc}^3 \). With \( \sim 10^{10} \) years that gives an integrated stellar entropy production \( \Delta S_0 \approx 10^{86} \) reached by about 10 billion years.

Under the CEP with stars as the major source of entropy production, one obtains a weighting and hence a predicted probability distribution for \( \rho \). With several different star formation models [1] the predicted 1-\( \sigma \) probability band of roughly \( 10^{-124} \leq \rho \leq 10^{-122} \) easily contains our universe’s observed value.

![FIG. 2: Integrated stellar entropy production per comoving Mpc³, calculated using the Nagamine et al. star formation model [12] considered in [1]. The long tail is produced by low-mass white dwarfs with lifetimes up to \( 10^{13} \) years, but by \( 10^{10} \) years we have already seen a large fraction of stellar entropy production. A comoving volume of a particular scale contains a fixed amount of matter so long as the universe is homogeneous over the scale of consideration. But a flat universe with a cosmological constant will form a horizon of fixed](image-url)
moving volume of about $3 \times 10^3$. In our universe the peak in comoving 3-volume of a peak in entropy production (middle plot) per comoving volume. FIG. 3: A peak in star formation (top plot) is followed by a peak in entropy production (middle plot) per comoving volume. In our universe the peak in comoving 3-volume of the causal diamond (bottom plot) is near the time of maximal stellar entropy production per $V_c$. The 3-volume $V_c(t)$ of the causal diamond is determined by $\Lambda$; a much earlier peak (larger $\Lambda$) would not allow the diamond to capture as much entropy production. Universes with smaller $\Lambda$ would give larger causal diamonds in late times, but would capture little more stellar entropy production, and are less likely owing to our flat prior. All plots are assuming a homogeneous universe.

physical size. Eventually the the comoving radius corresponding to the horizon length will drop below the scale of matter inhomogeneity. In physical terms, for a world line near a gravitationally collapsed halo, the amount of mass enclosed by a causal diamond will eventually approximate a constant value, rather than exponentially emptying out. Comoving coordinates are no longer a particularly good choice within a collapsed halo.

In our universe a large halo might have mass $10^{15} M_\odot$. Today’s density $\rho_m \approx 3.3 \times 10^{10} \frac{M_\odot}{Mpc^3}$ gives a corresponding comoving volume of about $3 \times 10^4 Mpc^3$, which is nearly a factor of a billion smaller than the maximum comoving size. Any late-time entropy source must therefore compensate for effectively having a causal diamond 3-volume approximately $10^{-9}$ of that during peak stellar entropy production. Whether this is possible depends upon details: the lengths of time available and the scale of entropy production. The most dramatic example would be the inclusion of Hawking radiation from a black hole. Release of a $10^7 M_\odot$ black hole’s $10^{91}$ entropy as Hawking radiation would completely swamp entropy produced by stars in the first $10^{10}$ years of cosmic evolution. The time scale is enormous: $\log_{10}(\tau_{BH}) = 83 + 3\log_{10}[M_{BH}/10^6 M_\odot]$, or about $10^{86}$ years in this case, but we cannot ignore the situation out of hand as any worldline which tracks matter has a high chance of ultimately ending up near (or even in) a black hole.

IV. WHY WE CARE ABOUT PARTICULAR WORLD LINES

In the the formulation of Bousso et al. [1] the universe is considered to be exactly FRW homogeneous and isotropic, which makes a distinction between comoving volume and mass unnecessary. Indeed over the size of the causal diamond this assumption is quite accurate at the beginning of our universe and well through the current time, as the universe is homogeneous well below scales approaching the Hubble length or the current size of the causal diamond. As mentioned above it is in the future that differences among causal diamonds may arise.

Because of the assumption of homogeneity in [1], a particular set of cosmological parameters resulted in a unique, representative causal diamond. Thus the probability is given by

$$
\frac{dP}{d\rho_\Lambda} \propto \frac{dp}{d\rho_\Lambda} \frac{dN}{d\rho_\Lambda}
$$

where $d\rho_\Lambda$ represents the density of vacua per value of $\Lambda$. We may take $d\rho_\Lambda$ to be flat if the landscape has values spaced tightly in the region of interest, and if $0$ is not a special value. With these assumptions, the spacings of vacua can be assumed to be uniform for $\Lambda$ near $10^{-123}$. The quantity $\frac{dp}{d\rho_\Lambda}$ is the term representing the theory’s prior probability for $\Lambda$. Following previous work, we assume prior probability is flat; in other words, the background theory is indifferent to vacua, choosing among them with equal probability.

Critically, in [1], the weighting $w(\rho_\Lambda)$ is the weight of a single representative causal diamond with cosmological constant density $\rho_\Lambda$. If a particular set of cosmological parameters does not yield a single causal diamond, we must replace our single calculation of $w(\rho_\Lambda)$ with a probability distribution

$$
w(\rho_\Lambda) = \int_\Lambda w(\rho_\Lambda, \lambda) d\lambda
$$

where the integral over $\Lambda$ is one over all possible world lines (and hence causal diamonds) given a particular set of cosmological parameters from our background theory.

This discussion may seem counter to the spirit of the causal diamond approach in [1]. Yet unless our background theory is itself phrased in terms of causal diamonds, we cannot skip smoothly from a distribution of cosmological parameters to a distribution of results for causal diamonds. Our prior distribution of $\Lambda$ or the
spacing of vacua is phrased in terms of cosmological parameters, not particular world lines. Assuming perfect homogeneity simply means taking the weight function \( w(\rho_0, \lambda) \) to be proportional to a delta function peaked at a particular world line \( \lambda_0 \) that is “typical” of a perfect FRW universe. Given the tremendous variety of world lines for any structure-forming cosmology, this assumption seems unrealistic: an extreme counterexample would be a world line that runs directly into a black hole horizon at an early era. Nonetheless it remains to be seen whether considering an ensemble of world lines for a cosmology rather than a single one makes a difference in predictions for \( \rho_A \).

In order to calculate the entropy production probability distribution over an ensemble of world lines \( \lambda \), we need to describe how the density of a bundle of world lines behaves over time relative to the coordinates in which we wish to measure entropy production. We argue that for an inhomogeneous universe there are multiple ways to parametrize these world lines and that the choice of parametrization directly affects the results of CEP calculation.

It should be noted that even in the case of a perfectly homogeneous FRW universe not all world lines (and hence causal diamonds) are created identically, as one could imagine arbitrary boosts or even accelerated paths relative to a comoving observer. Even with modest boosts, observers on these paths would have a different experience of the universe owing for example to a strong CMB dipole. Since the group of boosts is not compact, one might expect a “typical” boost to be arbitrarily far from the comoving rest frame, with correspondingly anisotropic physics. Given a homogeneous, isotropic universe, the preservation of symmetry afforded by the choice of a comoving observer seems an enticing motivation for picking a comoving causal diamond. But it must be emphasized that this is indeed a choice, and any appeal that comoving coordinates are natural in the sense that they follow typical matter distributions (and perhaps thus observers) has implications for the inhomogeneous case.

When we move to an inhomogeneous universe we cannot even appeal to a notion of preserving symmetry. For the purposes of simplification we will leave out accelerated world lines and describe our collection of world lines as a congruence of timelike geodesics, with each spacetime point lying on a single geodesic. One can construct such a congruence by specifying a spacelike slice and examining geodesics orthogonal to this slice. Different slices, however, typically result in different inherited parameterizations for the world lines. We will describe two such choices in what follows, but there are of course many others.

For a slice picked at a constant cosmic time in the very homogeneous early stages of a universe like ours, there is a natural parametrization: our entropy production can be measured on a per-mass or, equivalently, comoving coordinate basis, and so we can simply imagine a grid of world lines piercing each spacelike surface with constant cosmic time. In the homogeneous limit for comoving coordinates this grid simply remains fixed in time, yielding a fixed world line density, and corresponding to the simple choice made in Bousso et al. 1.

Things are more complicated as the universe becomes less homogeneous. A starting point is to imagine placing test particles in a fixed, constant spatial density at an early cosmic time, and watching the particles trace out geodesics as the universe evolves. Of course, our universe seems to have performed this very experiment, and as \( \Lambda \) dominates we have a picture of most matter eventually residing within isolated gravitationally bound halos, with exponentially emptying space in between. In this picture, at late times the spatial density distribution of geodesics parallels that of matter itself, so at least roughly, a probability distribution for entropy production over world lines would be equivalent to integration over the matter distribution.

There are other choices that yield dramatically different answers, however. If we choose a slice at late cosmic time and parametrize world lines to have a constant density in physical coordinates, the vast majority of world lines at late times will be located in nearly empty regions with almost no entropy production. When we trace back the world lines to the beginning of the universe, they will not be homogeneously distributed relative to matter, but for the purposes of calculating entropy increase at early times there is no significant difference since the entropy production itself is homogeneous in space.

On the other hand, with this second choice, any entropy production at late times will be exponentially suppressed by the rarity of world lines that are located near matter, and so given this choice it is justifiable to discard late-time entropy sources. It is important to observe, however, that the second choice seems at best no better motivated than the first, and indeed that one could imagine many other intermediate choices for parameterizing world lines. For the remainder of the paper we treat this choice as an open question, and will estimate the effect of late-time entropy production where it seems to matter: that is, under the assumption that typical world lines follow matter distribution from an early time. Therefore we begin by asking what astrophysical processes may produce substantial entropy well into the future.

V. LONG-TERM ENTROPY PRODUCTION

A. Black holes

Black holes contain much more entropy than all other astrophysical sources. In 1, black hole horizon entropy as well as that associated with the formation of a de Sitter horizon were explicitly excluded from the tally of entropy increase. Maor et al. 10 raise the possibility that gravitons produced during black hole mergers could by themselves exceed stellar entropy increase. But even
if one does not count a significant early-time increase in entropy from black holes, on the very long time scale of black hole evaporation, this entropy increase can no longer be avoided. Hawking radiation returns entropy to the matter sector, and it will typically dominate the early-time stellar entropy production as estimated in section (III).

**B. Stellar entropy**

Low-mass white dwarfs may continue burning for as long as $10^{13}$ years. Moreover, even though star formation is already dropping dramatically in our universe due to depletion of cool gas, some small but finite star formation rate will likely exist far into the future owing to collisions among sub-stellar masses and white dwarfs. Further, one might wonder about the time behavior of star formation in universes with very different values of $\Lambda$.

Can stars in a collapsed region far into the future ever exceed the $10^{46}$ entropy produced by the stars in the first $10^{10}$ years? We can calculate an upper bound by simply imagining all baryons within a halo are converted into stars and burned. Consider a massive halo ($10^{15} M_\odot$). Baryons make up about 1/6 of the matter content, or $1.6 \times 10^{14} M_\odot = 2 \times 10^{44} \text{kg/6}$ baryons $\approx 1.5 \times 10^{71}$ baryons, or perhaps $10^{71}$ hydrogen atoms.

Each instance of fusion releases about 7 MeV per baryon. At a temperature of 20 MeV for dust-reprocessing, that is about $3 \times 10^8$ entropy per dust-processed baryon. Even if over very long times 100% of baryons are burned to hydrogen, and half are reprocessed by dust (an overestimate as dust is depleted over time), that allows only $\approx 10^{79}$ entropy, 7 orders of magnitude less than is produced by stellar entropy $\Delta S_0$ up to $10^{10}$ years. It would seem that for the observed cosmological parameters future stellar entropy production can not compete with that in the past.

Varying the cosmological constant affects the estimate in two ways: increasing $\Lambda$ leads to earlier vacuum domination and a smaller value of $\Delta S_0$. However, it simultaneously leads to a smaller typical halo size as discussed later. Eventually large $\Lambda$ will lead to a severe drop in star formation rates at both early and later times. Similarly, small values of $\Lambda$ will push vacuum domination later and later, eventually leaving less stellar entropy to be produced in the vacuum-dominated era. Thus it does not appear that stellar entropy in late eras is a strong competitor to $\Delta S_0$, even when the cosmological constant is varied.

**C. Dark Matter annihilation**

To compete with $\Delta S_0$ we need approximately $10^{15}$ entropy per baryon. With the possible exception of Hawking radiation, this appears to be a tall order. We need a process with a combination of high energy released, low effective temperature, and near universal occurrence. One possibility is annihilations of dark matter. Dark matter masses perhaps 6 times baryonic matter, so the total available energy is $\approx 6$ GeV per baryon. If WIMPs have weak scale masses, the typical handful of annihilation products by themselves cannot produce anywhere near enough entropy. So the interesting case is if the annihilation happens in a low-temperature context so that many low-energy products (typically photons) can be produced by a single annihilation.

Adams et al. [13] explore WIMP capture by white dwarfs. Over the long term white dwarfs make up the bulk of collapsed stellar objects, and they have densities great enough to capture massive WIMPs over time. Due to DM annihilations the dwarfs have a very extended period of low luminosity and low temperature. Adams et al. give typical $T \approx 63$ K, or about 5 meV for DM annihilations, which with 6 GeV/baryon energy gives only $10^{12}$ entropy from annihilating all DM.

**D. Proton decay**

One can also ask about proton decay within white dwarfs (88% of final stellar mass). For a typical GUT decay such as $p \rightarrow e^+ + \pi^0$, 1 GeV per nucleon is ultimately released. Typically about 1/3 is lost to neutrinos which freely stream out of even white dwarfs rather than thermalizing. Thus we need a temperature of $T \approx 10^{-6}$ ev or about $10^{-2}$ K. For proton decay in white dwarfs, $T \approx 0.06$ K with proton decay lifetime $\Gamma = 10^{37}$ years. Using the same bounds on proton decay as Adams et al. [13], $32 < \log \Gamma < 41$, but since $T^4 \propto \Gamma$ we can only push that temperature down another order of magnitude with the simplest proton decay models. But a proton decay mechanism originating from a higher order operator could produce much longer lifetimes and correspondingly lower temperatures, perhaps allowing this process to compete with early stellar evolution.

**E. Dynamical effects**

Given the approximations involved, either proton decay or WIMP annihilation might be considered reasonable competitors to stellar entropy production $\Delta S_0$ in the matter-dominated era. In order to calculate the maximum entropy for each we have simply given each process a maximal value assuming complete conversion of a certain large halo. But halo masses themselves may not be stable on the time scales considered ($\tau \approx 10^{24}$ years for WIMP annihilation and $\approx 10^{37}$ years for proton decay). There are two competing dynamic processes within halos over the very long term (13). Interactions between stars lead to dynamic relaxation and ejection of individual stars on a time scale of $\tau_{\text{relax}} = 100 \tau_{\text{evap}}$, $\approx 10^9 \tau_{\text{evap}}$, $\sim 10^{19} - 10^{20}$ years for typical galactic ra-
radius $R$, random velocity $v$, and number of stars $N$. At
the same time, gravitational radiation should cause or-
bits to decay and eventually drop matter into a central
black hole, on a time scale of $\approx 10^{24}$ years. Adams et al.
estimate perhaps 1-10% of matter remains bound to the
central black hole while the remainder is lost from the
galaxy.

Matter ejected from the gravitational bounds of a
galaxy will in general be lost from the de Sitter horizon as
well. Taking the point of view of a world line following
an example white dwarf ejected this way, within a few
Hubble times the former host halo will have redshifted
beyond the horizon and the only continuing source of en-
tropy increase within an observer’s horizon and causal
diamond would be that produced from the single white
dwarf star. Even the complete proton decay of such a star
would produce a completely negligible amount of entropy
compared to $\Delta S_0$ given the small matter content within
the horizon. On the other hand, we may still wonder
about a single large black hole ejected in this fashion,
since Hawking radiation over extremely long times could
compete with early entropy.

If this picture of dynamical effects is correct, for a
world line near the leftover central black hole in a halo,
of the processes considered again it is only Hawking ra-
diation that could compete with $\Delta S_0$, as on time scales
much shorter than proton decay, essentially all matter
will have either been ejected from the halo or have al-
ready collapsed into the central black hole. WIMP anni-
hilation within white dwarfs has a time scale of $\approx 10^{25}$
years, so the story is relatively similar: rather than con-
tribute appreciably to entropy gain within a single halo,
this process will take place mostly within isolated white
dwarfs.

Dynamical effects may also have important impli-
cations for counting entropy from Hawking radiation. We
intentionally made the choice to parametrize world lines
so that they essentially followed typical paths of matter.
We claimed that this choice was in essence arbitrary, if
straightforward. But having made this choice, we must
accept that the long-term dynamical behavior of matter
would also be that for a typical world line. Our picture
has two basic fates for matter on time-scales well before
black hole evaporation: either it is within a black hole, or
in some small chunk of matter with no black holes within
the de Sitter horizon.

With our parametrization choice it then seems that it
is quite common for world lines themselves to intersect
black holes, but that it is not common for world lines to
stay within a Hubble radius of a black hole long enough to
observe black hole evaporation. Classical world lines may
end at the singularity of a black hole, but our approach
of ignoring horizon entropy is not nearly so obvious once
the horizon itself is crossed. In this picture the CEP may
be safe from the need to count Hawking radiation, but
the details are far from immediate.

It should also be mentioned that we might try to tweak
our matter-following geodesic parametrization to be lit-
tle different except that they generally avoid the partic-
ular dynamics that over time seem to eject matter from
the halo or collapse it into black holes. Such an ap-
proach might seem artificial, but it would still be just one
of many parameterizations, and would immediately re-
store to importance Hawking entropy or that from certain
models of proton decay. Subtle changes in parametriza-
tion clearly can yield very different ideas of what consti-
tutes a typical world line, and we now make the argument
that if there is a significant late-time entropy source in
our parametrization, it can drastically affect the CEP
prediction for a cosmological parameter.

VI. EFFECTS ON PREDICTION FOR $\rho_\Lambda$

All of our late-time effects are at approximately fixed
mass within a horizon. Assuming we are examining an
astrophysical process which is independent of halo scale
(which is certainly true for proton decay itself, but should
be considered a simplification for black holes and white
dwarf processes, since larger halos may have different
astrophysics), the only determinant of entropy production
is the mass of the halo. Thus we can frame our long-term
entropy weight as a function of the mass of the halo,
$w(\rho_\Lambda, \lambda) \rightarrow w(\rho_\Lambda, M_{halo})$. If we assume matter fairly
traces worldlines (i.e., split up phase space evenly at ear-
times), then we can take advantage of the Press-Schechter
mass function to estimate the probability for each world
line (and thus causal diamond) to be within a halo
of mass $M$ and hence to have a weight $w(\rho_\Lambda, M)$. If
we are calculating total entropy, to a first approxima-
tion the weight for a worldline within a mass $M$ halo is
proportional to $M$ in the scale invariant approximation.
If $f(M, t) \propto \exp(-\frac{\delta M}{\sqrt{2\delta(M, t)}})$ is the P-S differential mass function,
then the integrated weight function

$$w(\rho_\Lambda) = \int w(\rho_\Lambda, \lambda) d\lambda$$

$$= \int MP(M)w(\rho_\Lambda, M)$$

$$= \int Mf(M, \infty)dM$$

where $f(M, \infty)$ is the P-S mass function evaluated at a
late time. In a $\Lambda$-dominated universe, these mass frac-
tions approach a fixed value on the timescale of $t_\Lambda \approx
16.7$ billion years for our universe, as the cosmological
constant freezes structure formation. The final halo
mass fraction and differential P-S function are given in
Fig. [4]

Thus the weighting for this kind of constant late-time
entropy source turns out to be proportional to the mass
of the halo a typical piece of matter finds itself within.
One expects this typical halo mass to decrease for larger
few orders of magnitude of the observed value, $\Lambda$. Indeed, from Fig. 4, we can see that for $\Lambda$ within a range of cosmological constants relative to the observed value $\rho_\Lambda$, the halo fraction shifts approximately inversely as $\Lambda$ increases, the halo fraction shifts approximately inversely in mass with increasing $\Lambda$,

$$\frac{dP}{d\log(\rho_\Lambda)} \propto w(\rho_\Lambda) \rho_\Lambda \propto \text{const}$$

In the case of late-time entropy it is not the actual size of the causal diamond volume but rather the mass of the typical halo that controls the entropic weight. Smaller cosmological constants result in larger structure, so that the mass-weighted “typical” halo (and hence the weight) scales approximately as $\rho_\Lambda^{-1}$.

The overall weighting for a cosmology will be $\Delta S_0 + \Delta S_{\text{late}}$. Where the late time entropy increase dominates, the flat prior in $\rho_\Lambda$ is thus transformed to a flat distribution for $\log(\rho_\Lambda)$. While this result addresses the cosmological constant problem to some degree (as we need only explain the smallness of $\log(\Lambda)$), compared with earlier work we have lost the peak in the probability distribution associated with a prediction of the actual value of $\Lambda$. Unless we can definitively rule out significant late-time entropy sources, such a result would undermine some of the success of the CEP. Nonetheless the CEP still benefits to an extent from the suppression of structure formation for large values of $\rho_\Lambda$, which shows up in the weighting dropping below the $\rho_\Lambda^{-1}$ form (as can be seen in Fig. 5).

**VII. CONCLUSIONS**

Standard treatments of the Causal Entropic principle consider a one-to-one mapping between cosmological parameters and causal diamonds. The inhomogeneity of a realistic universe introduces additional complexity because different observers can experience very different causal diamonds, even with the same cosmological parameters. One must have some method of picking a typical causal diamond, or of characterizing an ensemble of causal diamonds for a given cosmology. We have shown that with one reasonable choice of parameterizations for the ensemble of causal diamonds, we are forced to consider very slow entropy sources in the far future. Dynamical effects on the typical halo over long times may prevent these slow entropy sources from being important contributors to the overall measure, but it is easy to imagine particular parameterizations where this is not the case. The entropy associated with black hole evaporation or certain models of particle decay could then ruin CEP predictions for the value of the cosmological constant.

It should also be noted that in a universe with enough inhomogeneity and with smaller causal diamond sizes, the effect of the inhomogeneity would be pushed to earlier time scales and we would need to worry about the clumping of stellar entropy production itself rather than merely late-time events. An example would be a universe with much larger $\Lambda$ and also much larger initial fluctuations.
There are methods to parametrize causal diamonds that seem to avoid the late-time entropy production issue discussed here for universes similar to ours. But this ambiguity seems to point at least to an incompleteness in the CEP as currently formulated. One could of course simply make a felicitous choice of parameterizations and add it to the CEP. But for a wide range of cosmological parameters it may be still be difficult to be sure of capturing a “typical” causal diamond in this fashion. The reliance on entropy associated with a single causal diamond makes this issue much more difficult than it would be for (e.g.) a per-baryon measure, and in that sense is a CEP-specific issue. And it is one that must be addressed to be confident of CEP predictions for nonidealized cosmologies.

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