INVENTORY MODEL WITH ADVANCE PAYMENT AND BACKLOGGING

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Abstract

Advance payment is the important issue in inventory analysis. Due to advance payment another important issue is the capital money. The main aim of this paper is to introduce advance payment concept in the analysis of inventory problem. Annual demand has been considered to developed this model. Also, we have considered the product deteriorate constantly after the certain time period. Due to advance payment supplier may offer to his retailers’ instalment facility. However, purchasing amount must pay before received the lot size. Shortages are allowed partially with the constant backlogging rate. Due to the high nonlinearity of the corresponding optimization problem, we cannot able to find the closed form solution of the objective function. To overcome this difficulties, we have used LINGO 18 software for solving the proposed inventory problem. To validate the proposed model, one numerical examples have been solved. Finally, the effects of changes of different parameters have been studied graphically of the proposed model and a fruitful conclusion has been drawn.

Keyword: EOQ, price dependent demand, advance payment, partial backlogged shortage.

I. Introduction

Price of the product is also an important issue on demand. If the price of the product is high then the demand for the product is very low and vice versa. So, it is an important issue in inventory management. When a manufacturer launches a product in the market, they are very much aware about the demand of the product as well as the price of the product. In this connection, we may describe some recent works related to the price of the product. Sana [XII] proposed a price sensitive inventory model for the deteriorating item. Maihami and Kamalabadi [IX] introduced a joint effect on pricing and non-instantaneous deteriorating items inventory model under-
price under backlogging. Avinadav et al. [II] studied the deteriorating inventory model under price-sensitive demand. Bhunia and Shaikh [III] proposed an inventory model with the frequency of advertisement and price dependent demand. Ghorieshi et al. [IV] developed trade credit inventory model for the deteriorating item under price-dependent demand. Alfares and Ghaithan [I] introduce all unit discount facility in the inventory system under price-dependent demand. Jaggi et al. [VI] proposed a credit financing inventory model with price dependent demand under two warehouse system.

In the literature of the inventory control, a lot of research works have been reported by several researchers in the related with permissible delay in payments whereas very few research works have been reported on advance payment scheme. Advance payment scheme make sure about payment as well as deliver the goods on time. This concept was introduced by Zhang [XIX]. In his model, he considered a fixed per-payment cost. After a long time, Maiti et al. [X] studied an inventory system with prepayment effect. Gupta [V] investigated an inventory model by taking the inventory parameters as interval-valued with advance payment scheme and solved by genetic algorithm. Thangam [XVIII] studied an advance payment inventory model with a price discount for deteriorating item. Taleizadeh et al. [XVII] investigated an inventory model with multiple prepayments under constant demand. Zhang et al. [XX] studied Inventory models considering both advance payment in first model and advance payment & delayed in payment in second model. Taleizadeh [XIV] introduced an inventory model for evaporating items under an advance payment scheme. Taleizadeh [XVI] modified of Taleizadeh [XIV] by taking multiple prepayments with partially backlogged shortages. Pourmohammad and Zia [XI] developed an inventory model by taking both advance payment and delayed payment in together. Zhang et al. [XXI] proposed inventory model with a two-stage supply chain under the advanced payment scheme. Li et al. [VIII] investigated cash flow analysis for deteriorating inventory model under advance payment scheme. Taleizadeh [XVI] studied the disruption effect in the inventory system with advance payment situations. Khan et al. [VII] proposed two warehouse inventory model with multiple prepayment scheme. Shaikh et al. [XIII] introduced an advance payment inventory model where the inventory costs are interval-valued. Advance payment is the important issue in inventory analysis. Due to advance payment another important issue is the capital money. The main aim of this paper is to introduce advance payment concept in the analysis of inventory problem. Annual demand has been considered to developed this model. Also, we have considered the product deteriorate constantly after the certain time period. Due to advance payment supplier may offer to his retailers’ instalment facility. However, purchasing amount must pay before received the lot size. Shortages are allowed partially with the constant backlogging rate. Due to the high nonlinearity of the corresponding optimization problem, we cannot able to find the closed form solution of the objective...
function. To overcome this difficulties, we have used LINGO 18 software for solving the proposed inventory problem. To validate the proposed model, one numerical examples have been solved. Finally, the effects of changes of different parameters have been studied graphically of the proposed model and a fruitful conclusion has been drawn.

II. Assumptions and Notation

To develop the inventory model, we have considered the following assumptions and notation

II.i. ASSUMPTIONS:

1. To developed of the proposed model, annually demand have been considered and which is denoted by \( D \).
2. The rate of deterioration is constant and which is given by \( \alpha \), \( 0 < \alpha << 1 \)
3. Items are not neither replaceable nor refundable.
4. Inventory planning horizon is infinite lead time is \( M \).
5. The retailers pay a fraction amount of total purchase cost \( k \) (say) with each instalment until fulfil the full payment.
6. Supplier offer advance payment facility with equal instalment
7. Shortages are allowed with a constant backlogging rate during the stock out situation and it is denoted by \( \eta \).

Notation

| Notations | Unit | Descriptions |
|-----------|------|--------------|
| A         | $/order | Ordering cost |
| D         | Constant | Annual demand of the product |
| P         | Rs/unit | Selling price per unit |
| \( C_s \) | Rs/unit | Shortage cost |
| \( \alpha \) | Constant | Deterioration rate |
| \( C_p \) | Unit | Purchase cost per unit |
| M         | Year | Length of the lead time during which the enterprise will pay the payments |
| N         | Constant | Number of equally spaced pre payments during the lead time |
| K         | Constant | Fraction of the purchasing cost must be paid with multiple payment |
### III. Problem Description

Let us assume that an enterprise makes an order of \((S + R)\) units of a product by a fraction \(k\) of the purchasing cost by \(n\) equal multiple instalments at equal intervals within the lead time \(M\) and receives the lot by paying the remaining purchasing cost at time \(t = 0\). Shortly after \(R\) units are utilized to fulfil the backlogged demand partially consequently the onhand inventory level becomes \(S\).

![Fig.1: Pictorial representation of the inventory system](image)

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The governing differential equations are as follows

\[
\frac{dl}{dt} = -D \tag{1}
\]
\[
\frac{dl}{dt} + \alpha l(t) = -D \tag{2}
\]
\[
\frac{dl}{dt} = -D \eta \tag{3}
\]

With initial conditions

\[
I(t) = S \quad \text{when} \quad t = 0,
\]
\[
\text{continuous at} \quad t = t_1
\]
\[
\text{and} \quad I(t) - R \quad \text{at} \quad t = T
\]

From (1) solving and using initial conditions we get,

\[
I(t) = -Dt + S \tag{4}
\]

From (2) solving and using initial condition we get,

\[
I(t) = \frac{D}{\alpha} + \frac{D}{\alpha}e^{\alpha(t_1-t)} \tag{5}
\]

From (3) solving and using initial conditions we get,

\[
I(t) = D \eta (t_1-t) \tag{6}
\]

Using continuity \( t = td \) from (4) and (5) we get,

\[
S = Dt_d - \frac{D}{\alpha} + \frac{D}{\alpha}e^{\alpha(t_1-t)} \tag{7}
\]

Using continuity condition at \( t = t_1 \) from (5) and (6) we get,

\[
R = D\eta (T-t_1) \tag{8}
\]

Ordering cost = \( A \),

\[
\text{Holding cost} = h \int_{0}^{t_{1}} I(t)dt + h \int_{t_{1}}^{t_{d}} I(t)dt
\]
\[
= h \left( St_d - \frac{Dt_d^2}{2} \right) + h \left[ -\frac{(D/\alpha)(t_1-t_d)}{\alpha^2} + \frac{(D/\alpha^2)}{\alpha^2}e^{\alpha(t_1-t)} \right]
\]
Shortage cost = \(-C_b \int_{t_i}^{T} I(t) dt\)

\[ = C_b D \eta (T - t_i)^2 / 2 \]

Capital cost = \(I_C \left[ \frac{KC_P(S + R)}{n} M \frac{1}{n(1 + 2 + \ldots + n)} \right] \)

\[ = \frac{(n+1)}{2n} I_C MKC_P \left[ D t_d + \frac{D}{\alpha} (e^{\alpha(t_\eta - t_d)} - 1) + \frac{D}{\delta} (1 - e^{-\delta(T-t_i)}) \right] \]

Purchase cost = \(C_P (S + R)\)

\[ = C_P D \left[ t_d - \frac{1}{\alpha} (1 - e^{\alpha(t_\eta - t_d)}) + D \eta (T - t_i) \right] \]

Lost sale cost = \(C_1 (1 - \delta) \int_{t_i}^{T} D dt\)

\[ = C_1 D (1 - \eta) (T - t_i) \]

Total cyclic cost

\[ X = \left[ \left\{ \text{Ordering cost} \right\} + \left\{ \text{Holding cost} \right\} + \left\{ \text{Deterioration cost} \right\} + \right. \]

\[ \left. \left\{ \text{Shortage cost} \right\} + \left\{ \text{capital cost} \right\} + \left\{ \text{purchase cost} \right\} + \left\{ \text{lost sale cost} \right\} \right] \]

\[ = [A + h(St_d - D t_d^2 / 2) + h[-(D / \alpha)(t_1 - t_d) + (D / \alpha^2)e^{\alpha(t_\eta - t_d)}] + \]

\[ C_b D \eta (T - t_i)^2 / 2 + \frac{n(n+1)}{2n} I_C MKC_P [D t_d + \frac{D}{\alpha} (e^{\alpha(t_\eta - t_d)} - 1) + \frac{D}{\delta} (1 - e^{-\delta(T-t_i)})] \]

\[ + \frac{C_a D}{\alpha} [e^{\alpha(t_\eta - t_d)} - \alpha(t_1 - t_d) - 1] + C_P D[t_d - \frac{1}{\alpha} (1 - e^{\alpha(t_\eta - t_d)}) + D \eta (T - t_i)] \]

\[ + C_1 D (1 - \eta) (T - t_i) \]

So, the optimization problem is

\[ \text{Minimize } TC = \frac{X}{T} \]
subject to \( T > 0 \)

IV. Numerical Illustration

To study the applicability of the proposed model, we have solved one numerical example with the help of LINGO 18 software.

Example: Model with shortages

Let \( C_o = $300 \), \( D = 200 \), \( C_p = $20 \), \( C_i = $30 \),

\( C_h = $1.5 \), \( \alpha = 0.06 \), \( \eta = 0.9 \), \( M = 0.5 \) years, \( n = 10 \), \( I_c = 0.15 \) /dollar/year, \( k = 0.2 \), \( t_d = 0.5 \) year, \( C_s = $25 \).

Using the above mentioned numerical example, we have obtained the optimal solutions of the proposed problem are \( t_1^* = 1.074507 \) years, \( T^* = 1.135699 \) years,

\( S^* = 216.9048 \) units, \( R^* = 11.01444 \) units, \( Q^* = 143.4863 \) units,

\( TC^*(t_1, T) = $4505.061 \).

V. Sensitivity Analysis

In this section, we have investigated the impact of optimal values of \( t_1, T, S, R \) along with the total profit per unit time due to change the original parameters value of one parameter at a time from -20% to +20% and keeping the values of the rest of the parameters as same, the corresponding results are shown in Table-1 which are self-explanatory.

Table 1. Sensitivity analysis of different parameters
| $\theta$ | 10 | 1.086390 | 1.146198 | 219.0229 | 10.76551 | 4498.838 |
| 20 | 1.099123 | 1.157651 | 221.3344 | 10.53512 | 4493.078 |
| -20 | 1.138787 | 1.195991 | 229.1628 | 10.29684 | 4487.121 |
| -10 | 1.118155 | 1.176041 | 221.3344 | 10.53512 | 4490.188 |
| 10 | 1.080155 | 1.140640 | 217.847 | 10.64431 | 4495.808 |
| 20 | 1.065144 | 1.124853 | 214.6041 | 10.74761 | 4498.330 |
| $\eta$ | 10 | 1.138787 | 1.195991 | 229.1628 | 10.29684 | 4487.121 |
| -20 | 1.111362 | 1.111362 | 223.8575 | 0.000000 | 4500.052 |
| -10 | 1.109698 | 1.132506 | 223.5143 | 3.694926 | 4499.103 |
| 10 | 1.081405 | 1.168338 | 217.6850 | 17.21286 | 4482.991 |
| 20 | 1.065144 | 1.124853 | 214.6041 | 10.74761 | 4498.330 |
| $C_p$ | 10 | 1.167731 | 1.218984 | 235.5022 | 9.225534 | 4460.338 |
| -20 | 1.120047 | 1.169423 | 225.6492 | 8.887654 | 4088.921 |
| -10 | 1.079041 | 1.146605 | 217.1984 | 12.16147 | 4896.707 |
| 10 | 1.059718 | 1.136202 | 213.2234 | 13.76714 | 5299.818 |
| 20 | 1.040773 | 1.106099 | 209.3307 | 11.75860 | 4523.665 |
| $C_h$ | 10 | 1.096233 | 1.168936 | 220.7388 | 13.08660 | 4491.432 |
| -20 | 1.097831 | 1.162681 | 221.0681 | 11.67298 | 4492.342 |
| -10 | 1.131959 | 1.186915 | 228.1079 | 9.891979 | 4476.999 |
| 10 | 1.068834 | 1.130816 | 215.0981 | 11.15680 | 4508.620 |
| 20 | 1.040773 | 1.106099 | 209.3307 | 11.75860 | 4523.665 |
| $C_s$ | 10 | 1.100190 | 1.153520 | 221.5543 | 9.599485 | 4493.686 |
| -20 | 1.096233 | 1.168936 | 220.7388 | 13.08660 | 4491.432 |
| -10 | 1.101086 | 1.150067 | 221.7389 | 8.816538 | 4494.196 |
| 10 | 1.068834 | 1.130816 | 215.0981 | 11.15680 | 4508.620 |
| 20 | 1.040773 | 1.106099 | 209.3307 | 11.75860 | 4523.665 |
| $C_t$ | 10 | 1.103885 | 1.149683 | 222.3159 | 8.243640 | 4495.791 |
| -20 | 1.086256 | 1.169823 | 218.6837 | 15.04208 | 4485.752 |
| -10 | 1.093241 | 1.164358 | 220.1223 | 12.80112 | 4489.728 |
| 10 | 1.103885 | 1.149683 | 222.3159 | 8.243640 | 4495.791 |
| 20 | 1.107505 | 1.140428 | 223.0621 | 5.926139 | 4497.853 |
| $D$ | 10 | 1.206845 | 1.279053 | 194.8845 | 10.39788 | 3643.707 |
| -20 | 1.148418 | 1.213195 | 208.3573 | 10.49390 | 4069.077 |
| -10 | 1.056840 | 1.110023 | 233.8949 | 10.53029 | 4915.927 |
| 10 | 1.020077 | 1.068624 | 246.0998 | 10.48614 | 5337.794 |
| 20 | 1.099458 | 1.157838 | 221.4035 | 10.50833 | 4486.468 |

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VI. Conclusion

In this paper, we have studied an inventory problem with annual demand of the product as well as advance payment facility. Here, we have considered suppliers offer advance payment facility to his retailers with multiple instalments. Retailers received the product after full payment of purchasing amount. Due to not available amount on hand to retailers of purchasing amount, retailers may loan for the said amount from other source like bank. The rate of deterioration is considered as constant. A numerical example is solved to examine the validity of the proposed model. Due to the high nonlinearity of the corresponding optimization problem, we cannot able to find the closed form solution of the objective function. To overcome this difficulties, we have used LINGO 18 software for solving the proposed inventory problem.

For further research, one can extend the proposed models incorporating several realistic features such as non-linear price-dependent demand pattern, time dependent demand, stock-dependent demand, displayed stock-dependent demand, non-linear holding cost or trade credit policy (single level, two level, partial and credit risk customers). Also, by relaxing the zero-ending case by non-ending inventory model would be another interesting extension of the model without shortages.
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