Effects of an $H - \mu - \tau$ coupling in quarkonium lepton flavor violation decays

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In this work we study the consistency of a possible non-vanishing coupling $H\mu\tau$ of the order of $3.6 \times 10^{-3}$ as pointed recently by the CMS and ATLAS collaborations [1, 2], with measured lepton flavor violation processes involving quarkonium. We show that the most promising channel to confirm this excess is to look for the lepton flavor tau decay into a $f_0$ and $\mu$ where the experimental limit could strongly improved with the new B factories as Belle II.

Recently CMS collaboration has observed a slight excess of signal events with a significance of 2.4 standard deviations which can be interpreted as a Higgs particles decaying into a muon and tau leptons:

$$Br(H \to \mu\tau) = \left\{ \begin{array}{c}
0.84^{+0.39}_{-0.37} \% \ [1] \\
0.77 \pm 0.62 \% \ [2]
\end{array} \right.$$  

Using the reported value of Higgs mass to be 125 GeV [3], this requires the coupling $H\mu\tau$ to be of the order of $3.6 \times 10^{-3}$. Even if this observation is very challenging to be explained in new physics models [4], this value of the lepton flavor violating coupling is not in contradiction with the experimental upper limits for $\tau \to \mu\gamma$ and $\tau \to 3\mu$. Indeed, using the formalism in Refs. [5, 6] we obtain the values listed in Table I for the corresponding branching ratios. The expected value from $H \to \mu\tau$ is done assuming that the Higgs couplings to charged leptons are given by SM values [7, 8].

The computation of Higgs-induced lepton flavor violation (LFV) in channel decays involving only charged leptons are even more intricated as the smallness of the lepton Yukawa couplings imply that higher loop contributions can be bigger than tree-level one [5, 6]. To avoid this problem, we shall study the effect of the LFV $H \to \tau\mu$ coupling in processes involving quarkonium. In Ref. [10] the effects of heavy sterile Majorana neutrinos in LFV decays of vector quarkonia has been studied. Sterile Majorana neutrinos induces $\gamma\ell_i\ell_j$, $Z\ell_i\ell_j$ LFV couplings at one loop and $WWl_i\ell_j$ LFV couplings at tree level. These couplings produce LFV effects in quarkonia decay studied in Ref. [10]. Another effect not studied there, is to produce a non-vanishing $H\ell_i\ell_j$ LFV coupling at one level, whose effects in quarkonium decay were no analyzed in Ref. [10].

In this paper, we assume no specific models for new physics behind the $H \tau\mu$ coupling and we systematically study the LFV decays of quarkonia involving the $H\tau\mu$ coupling, considering the phenomenological value of $3.6 \times 10^{-3}$ pointed by the CMS and ATLAS collaborations. We shall show that even if the expected branching ratio are still below the experimental limit, some of them could be accessible to next generation of $B$ factories as Belle II.

The description of processes involving the annihilation or creation of heavy quarkonia can be systematically done in the framework of non-relativistic quantum chromodynamics (NRQCD) [11]. This is a systematic expansion in terms of $\alpha_s$ and the quarks relative velocity $v$ with a clear separation of the perturbative phenomena occurring at the scale $m_Q$ and the non-perturbative ones occurring at the scale $m_Q v$. The non-perturbative effects are encoded in universal matrix elements with a well defined hierarchy in the $v$ expansion. The novelty of this systematic approach is that, for some processes, color-octet configurations of the created or annihilated quark-antiquark pair yield contributions of the same order as the old color-singlet contributions to a given order in the $\alpha_s$ and $v$ expansion.

In this work we are interested in the order of magnitude of the branching ratios of the considered processes and will focus on the color singlet contributions which can be calculated using the old quarkonium techniques described in [12, 14]. A more refined analysis can be done in the most promising channels but this is beyond the scope of the

| $\tau \to \mu\gamma$ | $4.4 \times 10^{-8}$ | $1.3 \times 10^{-9}$ |
|---------------------|---------------------|---------------------|
| $\tau \to 3\mu$     | $2.1 \times 10^{-8}$ | $1 \times 10^{-10}$ |

**TABLE I**: LFV in $\tau$ LFV decays involving charged leptons
The present work.

The invariant amplitude for the annihilation of color-singlet quarkonium in a $^{2S+1}L_J$ angular momentum configuration $\overline{Q}Q^{[2S+1]L_J} \rightarrow X$ is given by [13, 14]

$$M[\overline{Q}Q^{[2S+1]L_J} \rightarrow X] = \int \frac{d^2q}{(2\pi)^2} Tr[\mathcal{O}(Q,q)\chi(Q,q)],$$

(2)

where $\mathcal{O}(Q,q)$ is the operator entering amplitude for the corresponding free quarks transition

$$M[\overline{Q}(\frac{Q}{2} - q, s_2), Q(\frac{Q}{2} + q, s_1) \rightarrow X] = \pi(\frac{Q}{2} - q, s_2)\mathcal{O}(Q,q)u(\frac{Q}{2} + q, s_1),$$

(3)

and $\chi(Q,q)$ denotes the wave function for the $^{2S+1}L_J$ bound state

$$\chi(Q,q) = \sum_{M,S_z} 2\pi\delta(q^0 - \frac{Q^2}{2m_Q})\psi_{LM}(q)P_{S,S_z}(Q,q)\langle LM; SS_z|JJ_z\rangle.$$

(4)

Here, $P_{S,S_z}$ stands for the spin projectors

$$P_{S,S_z}(Q,q) = \frac{N_c}{m_Q} \sum_{s_1,s_2} u(\frac{Q}{2} + q, s_1)\pi(\frac{Q}{2} - q, s_2)\frac{1}{2}$$

(5)

$$\sum_{s_1,s_2} \frac{1}{2S+1} S \langle \frac{Q}{2} + q + m_Q \rangle \left\{ \gamma^5 \right\} \left\{ \gamma^5 \right\} \langle \frac{Q}{2} - q - m_Q \rangle \text{ for } \left\{ \begin{array}{c} S = 0 \\ S = 1 \end{array} \right\},$$

(6)

where $\varepsilon(Q,S_z)$ denotes the polarization vector of the spin one system.

For $s$-wave quarkonium the wave function is rapidly damped in the relative momentum $q$ and the leading terms are given by $P_{S,S_z}(Q,0)$ and $\mathcal{O}(Q,0)$. In the zero-binding approximation the quarkonium mass $M$ is given by $M \approx 2m_Q$ and the amplitude reads

$$M[\overline{Q}Q^{[2S+1]J} \rightarrow X] = \frac{R(0)}{\sqrt{4\pi}} \sqrt{\frac{3}{4M}} Tr[\mathcal{O}(Q,0) \left\{ \gamma^5 \right\} \langle Q - M \rangle \text{ for } \left\{ \begin{array}{c} S = 0 \\ S = 1 \end{array} \right\},$$

(7)

with $M$ denoting the quarkonium physical mass and

$$\int \frac{d^2q}{(2\pi)^2} \psi_{00}(q) = \frac{R(0)}{\sqrt{4\pi}}.$$

(8)

A similar calculation of the invariant amplitude for the production of color singlet quarkonium, $X \rightarrow \overline{Q}Q^{[2S+1]J} + Y$ yields

$$M(X \rightarrow \overline{Q}Q^{[2S+1]J} + Y) = -\frac{R(0)}{\sqrt{4\pi}} \sqrt{\frac{3}{4M}} Tr[\mathcal{O}(Q,0) \left\{ \gamma^5 \right\} \langle Q + M \rangle \text{ for } \left\{ \begin{array}{c} S = 0 \\ S = 1 \end{array} \right\},$$

(9)

For $p$-wave quarkonium, the wave function at the origin vanishes and the leading term for the annihilation amplitude is given by the first term in the expansion in $q$ of Eq. (2). A straightforward calculation yields

$$M[\overline{Q}Q^{[2S+1]P_J} \rightarrow X] = -i \sum_{M,S_z} \langle 1M; SS_z|JJ_z\rangle \varepsilon_\alpha(M) \sqrt{\frac{3}{4\pi}} R'(0) Tr[\mathcal{O}_\alpha(Q,0)P_{S,S_z}(Q,0) + \mathcal{O}(Q,0)P_{\alpha\beta}(Q,0)],$$

(10)

where

$$A^\alpha(Q,q) = \frac{\partial A(Q,q)}{\partial q_\alpha},$$

(11)

and in this case

$$\int \frac{d^3q}{(2\pi)^3} q^\alpha \psi_{1M}(q) = -i \sqrt{\frac{3}{4\pi}} R'(0) \varepsilon_\alpha(M).$$

(12)
The polarization vector $\varepsilon_{\alpha}(M)$ satisfies the following relations

$$
\sum_{M,S_z}(1M; 1S_z|00)\varepsilon_{\alpha}(M)\varepsilon_{\beta}(S_z) = -g_{\alpha\beta} + \frac{Q_{\alpha}Q_{\beta}}{M^2},
$$

(13)

$$
\sum_{M,S_z}(1M; 1S_z|Jz)\varepsilon_{\alpha}(M)\varepsilon_{\beta}(S_z) = -i\frac{1}{M}g^{\mu\nu}\varepsilon_{\alpha\beta}(J_z),
$$

(14)

$$
\sum_{M,S_z}(1M; 1S_z|2Jz)\varepsilon_{\alpha}(M)\varepsilon_{\beta}(S_z) = \varepsilon_{\alpha\beta}(J_z).
$$

(15)

The Higgs to $\overline{Q}Q[2S+1L_J]$ quarkonium coupling is obtained from the diagram in Fig. 1 which yields the following operator

$$
\mathcal{O}(Q,q) = i\frac{m_Q}{v},
$$

(16)

where $v$ stands for the Higgs vacuum expectation value. Using this operator in the previous formulae it is easy to show that the only non-vanishing coupling of the Higgs to quarkonium is to $S = 1$, $J = 0$ $p$-wave quarkonium, in which case we obtain

$$
\mathcal{M}[\overline{Q}Q[3P_0] \to H] = \frac{3R'(0)}{v} \sqrt{3M/\pi}.
$$

(17)

Notice that this coupling is proportional to the derivative of the wave function at the origin, which according to the NRQCD rules is suppressed by a $v^2$ factor with respect to the wave function at the origin. This makes the radiative transitions involving $s$-wave quarkonium configurations of the same order as the non-radiative ones involving $p$-wave quarkonium configurations. The radiation changes the quarkonium quantum numbers allowing the corresponding quarkonium to couple to the Higgs. The calculation of Higgs-mediated lepton flavor violating radiative transitions involving $s$-wave quarkonium requires to work out Higgs-quarkonium-photon coupling. This transition is induced by the diagrams in Fig. 2.

From these diagrams, we identify the transition operator as

$$
\mathcal{O}(Q,q) = ieeQ\frac{m_Q}{v} \left[ \frac{\gamma(k)(\frac{Q}{2} + \bar{k} + m_Q - \frac{Q}{2} - \bar{k} - m_Q)}{(\frac{Q}{2} + q + k)^2 - m_Q^2} - \frac{\gamma(k)(\frac{Q}{2} + \bar{k}) - \frac{\gamma(k)}{(\frac{Q}{2} + k)^2 - m_Q^2}}{\gamma^5(Q - M)} \right].
$$

where $e_Q$ stands for the heavy quark charge in units of $e$. For $s$-wave $J = 0$ from Eq. (17) we obtain

$$
\mathcal{M}[H \to \overline{Q}Q[1S_0]\gamma] = -\frac{ee_QM^2R(0)}{4v} \sqrt{3/4\pi M} \left[ \frac{\gamma(k)(\frac{Q}{2} + \bar{k}) - \frac{\gamma(k)}{(\frac{Q}{2} + k)^2 - m_Q^2}}{\gamma^5(Q - M)} \right] = 0.
$$

(19)
Similarly for s-wave $J = 1$ we get
\[
\mathcal{M}[H \to \overline{Q}Q[^3S_1] \gamma] = \frac{ie_Q m_Q R(0)}{2v} \sqrt{\frac{3}{4\pi M}} T_r \left[ \left( \frac{\epsilon(k)(\overline{Q} + \overline{k}) - (\overline{Q} + \overline{k})\epsilon(k)}{(\overline{Q} + k)^2 - m_Q^2} \right) \eta(Q)(Q - M) \right],
\]  
where $\eta(Q)$ stands for the polarization vector of the quarkonium. A straightforward calculation yields
\[
\mathcal{M}[H \to \overline{Q}Q[^3S_1] \gamma] = \frac{e e_Q R(0)}{v} \sqrt{\frac{3M}{\pi}} T_{\mu\nu} \epsilon^\mu \eta' \eta''
\]  
with
\[
T_{\mu\nu} = g_{\mu\nu} - \frac{Q_{\mu} k_{\nu}}{Q \cdot k}.
\]  

Now we focus on the Higgs mediated LFV processes. We start with the $\overline{Q}Q[^3P_0] \rightarrow \mu\tau$ decay through the diagram in Fig. 3. A direct calculation yields the following decay width
\[
\Gamma[^3P_0 \rightarrow \mu^- \tau^+] = \frac{27 y^2 |R'(0)|^2 M^2}{8\pi^2 v^2 m_H^4} \left(1 - \frac{m_\tau^2}{M^2}\right)
\]  
where we neglected the muon mass and $y$ stands for the $H_{\mu\tau}$ coupling.

Next we go through the corresponding radiative process $\overline{Q}Q[^3S_1] \rightarrow \mu\tau\gamma$. This decay proceeds through the diagrams in Fig. 4.

Neglecting the muon mass we obtain the following decay width
\[
\Gamma[^3S_1 \rightarrow \mu^- \tau^+\gamma] = \frac{\alpha c_Q^2 y^2 |R'(0)|^2 M^4}{12\pi^3 v^2 m_H^4} f \left(\frac{m_\tau^2}{M^2}\right),
\]  
where
\[
f(x) = 1 - 6x + 3x^2 + 2x^3 - 6 \ln(x).
\]  
The $H_{\mu\tau}$ coupling can also mediate LFV decays of the tau meson involving light quarkonium. Although this is beyond the scope of the systematic NRQCD expansion due to the light quark mass, we still can use the quarkonium techniques taking care of extracting the corresponding non-perturbative pieces from the appropriate experimental data. The first possible decay is $\tau \rightarrow \mu \overline{Q}Q[^3P_0]$ which goes through the diagram shown in Fig. 5. The decay width is given by
\[
\Gamma(\tau^- \rightarrow \mu^- \overline{Q}Q[^3P_0]) = \frac{27 y^2 |R'(0)|^2 m_\tau M}{16\pi^2 v^2} m_H^4 \left(1 - \frac{M^2}{m_\tau^2}\right),
\]
\[ 5 \] Feynman diagram for the \( \tau \to \mu Q Q^3 P_0 \) decay.

\[ 6 \] Feynman diagram for the \( \tau \to \mu Q Q^3 S_1 \gamma \) decay.

where we neglected the muon mass.

The corresponding radiative decay is \( \tau \to \mu Q Q^3 S_1 \gamma \). The Feynman diagrams for this process are given in Fig. 6. Neglecting the muon mass we obtain the following decay width

\[ \Gamma(\tau^- \to \mu^- Q Q^3 S_1 \gamma) = \frac{\alpha e g^2 |R(0)|^2 m_\tau^2 M m_H^4}{32\pi^3 v^2} h \left( \frac{M^2}{m_\tau^2} \right), \tag{27} \]

where

\[ h(x) = (1 - x)^3 + \frac{3}{2} x [(1 - x)(3 - x) + 2 \ln(x)]. \tag{28} \]

Finally, although the calculation does not require to use the quarkonium techniques, it is interesting to estimate the effects of the \( H\mu\tau \) coupling in LFV decay of gauge bosons. As a sample we calculate \( W \to \mu \tau \pi \). The non-perturbative piece of this decay is related to the pion decay constant. This decay is induced by the diagram in Fig. 7.

\[ \text{FIG. 7: Feynman diagram for the } W \to \tau \mu \pi \text{ decay.} \]

The amplitude for \( W(Q, \varepsilon) \to \mu(p_1)\tau(p_2)\pi(p_3) \) is

\[ M = \frac{yg^2 V_{ud} f_\pi}{2\sqrt{2}m_W} \frac{p_3 \cdot \varepsilon(Q)}{(p_1 + p_2)^2 - m_H^2} \pi(p_2) v(p_1), \]

where \( g \) is the weak coupling constant. The resulting decay width is

\[ \Gamma(W^- \to \mu^- \tau^+ \pi^-) = \frac{f_\pi^2 g^2 \lambda^2 V_{us}^2}{73728\pi^3 m_W} f(a, b), \tag{29} \]

where

\[ f(a, b) = \frac{1}{a} \left[ a^2 (b^2 - 1) (24a^6 - 6a^4 (4b^2 + 7) + a^2 (b^2 + 17) (2b^2 + 1) - 6b^2) - 6 (a^2 - 1)^2 (4a^6 - a^4 (6b^2 + 1) + 2a^2 b^4 + b^4) \ln \left( \frac{a^2 - 1}{a^2 - b^2} \right) - 12b^4 \ln(b) \right], \tag{30} \]
with $a = \frac{m_c}{m_W}$, $b = \frac{m_s}{m_W}$ and we neglected the pion and muon masses.

The results for the studied decays depend on the color-singlet matrix elements $R(0)$ for $^3S_1$ quarkonium and $R'(0)$ for $^3P_0$ quarkonium. The same matrix elements appear in the leptonic decay of the first and two photon decays of the latter. We use the available experimental results on these decays to extract the phenomenological value of the matrix elements. The only matrix element that cannot be calculated this way is $R'(0)$ for the $^3P_0$ $b\bar{b}$ quarkonium $\chi_{b0}$. There is no available experimental data on the $\chi_{b0} \to \gamma\gamma$ transition, but $R'(0)$ has been calculated in several potential models summarized in Ref. [15] yielding $R'(0) \approx 1$ GeV$^5$ and we will use this value in our calculations.

The leptonic decay of vector quarkonia is induced by the diagram in Fig. (8). The corresponding decay width is

$$\Gamma(\overline{Q}Q[^3S_1] \to l^+ l^-) = \frac{4\alpha^2 e_Q^2 |R(0)|^2}{M^2}$$  \hspace{1cm} (31)

where we neglected the lepton mass. The two photon decay of $^3P_0$ quarkonium proceeds through the diagrams in Fig. (9). The decay width is

$$\Gamma(^3P_0 \to \gamma\gamma) = \frac{432\alpha^2 e_Q^2 |R'(0)|^2}{M^4}$$  \hspace{1cm} (32)

In Table II we give the results for the matrix elements of the different quarkonia. As a first approximation we use an $s\bar{s}$ configuration for the $f_0(980)$. There is no available information on the total width of the $\chi_{b0}$, thus we report the decay width when we use $R'_{\chi_{b0}}(0) = 1$ GeV$^5$ from quark model calculations [15];

$$\Gamma(\chi_{b0} \to \mu\tau) = 5.5 \times 10^{-17} \text{ GeV}.$$  \hspace{1cm} (33)

The branching ratios of the remaining decays are calculated using the estimates for the non-perturbative matrix elements in Table II. We list in Table III the so obtained branching ratios. Here, the branching ratios include a factor 2 to account for the two charge states where appropriate, e.g. $BR(\chi_{c0} \to \mu\tau) = BR(\chi_{c0} \to \mu^- \tau^+) + BR(\chi_{c0} \to \mu^+ \tau^-)$.

In general these branching ratios are small. The most promising decay is the $\tau \to \mu f_0$. We recall that we assumed an $s\bar{s}$ configuration for this meson. The nature of the low lying scalar mesons is an old problem (see [16] and references therein) and it would be desirable to have a closer approximation to the non-perturbative effects in this decay.

| Process | $\Gamma_{exp}(\text{GeV})$ | $|R(0)|^2(\text{GeV}^2)$ | $|R'(0)|^2(\text{GeV}^2)$ |
|---------|-----------------|-----------------|-----------------|
| $\Upsilon \to e^+ e^-$ | $1.28 \times 10^{-6}$ | 4.856 | - |
| $J/\psi \to e^+ e^-$ | $5.54 \times 10^{-6}$ | 0.560 | - |
| $\phi \to e^+ e^-$ | $1.26 \times 10^{-6}$ | $5.53 \times 10^{-2}$ | - |
| $\chi^0 \to \gamma\gamma$ | $2.34 \times 10^{-6}$ | - | $3.10 \times 10^{-2}$ |
| $f_0 \to \gamma\gamma$ | $0.29 \times 10^{-6}$ | - | $1.08 \times 10^{-4}$ |

TABLE II: Numerical values of the non-perturbative matrix elements extracted from the leptonic and two photon decays of quarkonia.
TABLE III: Branching ratios for lepton flavor violation decays involving the $H\mu\tau$ coupling.

| Process | Branching Ratio | Exp. bound |
|---------|-----------------|------------|
| $\chi_{c0}\rightarrow \mu\tau$ | $1.5 \times 10^{-17}$ | |
| $\Upsilon\rightarrow \mu\tau\gamma$ | $5.7 \times 10^{-14}$ | |
| $J/\psi\rightarrow \mu\tau\gamma$ | $5.1 \times 10^{-17}$ | |
| $\tau\rightarrow \mu f_0(980)$ | $8.4 \times 10^{-12}$ | $< 3.4 \times 10^{-8}$ |
| $\tau\rightarrow \mu f_0$ | $1.7 \times 10^{-14}$ | |
| $W\rightarrow \mu\tau\pi$ | $3.2 \times 10^{-17}$ | |

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