Lattice spacing dependence of phase transition temperature in the classical linear sigma model

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We have investigated the phase transition properties of classical linear sigma model. The fields were kept in contact with a heat bath for sufficiently long time such that the fields are equilibrated at the temperature of the heat bath. It was shown that the sigma model fields undergoes phase transition, but the transition temperature depend crucially on the lattice spacing. In the continuum limit, the transition temperature tends to zero or at least to a very low value.

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The possibility of forming disoriented chiral condensate (DCC) in relativistic heavy ion collisions has generated considerable research activities in recent years. Anslem and Ryskin [1] first considered the possibility of producing classical pion fields in heavy ion collisions. The idea became popular after Rajagopal and Wilczek [2] proposed the quench scenario and Bjorken et al. [3] proposed the Baked-Alaska model for producing DCC. Rajagopal and Wilczek argued that for a second order chiral phase transition, the chiral condensate can become temporarily disoriented in the nonequilibrium conditions encountered in heavy ion collisions. As the temperature drops below $T_c$, the chiral symmetry begins to break by developing domains in which the chiral field is misaligned from its true vacuum value. The misaligned condensate has the same quark content and quantum numbers as do pions and essentially constitute a classical pion field. The system will finally relaxes to the true vacuum and in the process can emit coherent pions. Since the disoriented domains have well defined isospin orientation, the associated pions can exhibit novel centauro-like fluctuations of neutral and charged pions.

Most dynamical studies of DCC have been based on the linear sigma model [4] in which the chiral degrees of freedom are described by the real O(4) field $\Phi = (\sigma, \vec{\pi})$, with the Lagrangian,

$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} \Phi)^2 - \frac{\lambda}{4} (\Phi^2 - f_\pi^2)^2,$$

where $\lambda$ is a positive coupling constant and $f_\pi$ is the pion decay constant. In the vacuum the symmetry is spontaneously broken and the sigma field acquires a non zero vacuum expectation value $\langle 0 | \sigma | 0 \rangle = f_\pi$. Symmetry is restored by a second order phase transition. Transition temperature can be calculated in mean field level as, $T_{c}^2 = 12 f_\pi^2 / (N + 2)$, with $N = 4$ [5]. In this model pions are massless. To be realistic, one introduces a small symmetry breaking term $H_\sigma$ to the lagrangian and pion become massive. The parameters of the model can be fixed by defining the meson masses and pion decay constant.

It is well known that classical fields suffers from divergence problem. Thermodynamic of classical fields are only defined if an ultraviolet cut off is imposed on the momentum. In lattice simulations, finite lattice spacing provides the cut off. Some thermodynamic quantities such as the diffusion rate of the topological charge [18,19] in non-abelian gauge theories, or the Lyapunov exponent [20] which is equivalent to the damping rate of soft thermal excitations are found to be insensitive to the cut off. However, it is not known, whether the critical temperature of phase transition the linear sigma model depend on the momentum cut off or not. With the possibility of experiments being performed at RHIC in search of Quark-gluon Plasma, where one expects to find signal of DCC, this investigation becomes important.

Aim of the present letter is to show that critical temperature of linear sigma model do indeed depend on the lattice spacing or the momentum cut off and in the continuum limit, the phase transition temperature approaches zero. To see phase transition we have used the old idea of finite temperature field theory, that is if you attach the fields with a heat bath, their equilibrium configuration will corresponds to that of the heat bath. We thus solve the equation of motion of sigma model fields in contact with heat bath which we represent as a white noise source. To be consistent with fluctuation-dissipation theorem, we also include a friction term in the equation of motion. We thus propose to study following Langevin equation for O(4) fields,

$$[\square + \eta \partial_t + \lambda (\Phi^2 - f_\pi^2)] \Phi = H n_\sigma + \zeta$$

where $\eta$ is the friction. The heat bath $\zeta$ is represented by a white noise source with zero average and correlation as demanded by fluctuation-dissipation theorem.

$$< \zeta (t, x, y, z) \zeta (t', x', y', z') > = 0 \quad (3a)$$

$$\int < \zeta_0 (t_1, x_1, y_1, z_1) \zeta_0 (t_2, x_2, y_2, z_2) > d^3 x = 2 T \eta \delta_{ab} \quad (3b)$$

Recently it was shown that in $\phi^4$ theory hard modes can be integrated out on the two loop level leading to dissipation and noise in the quasi classical limit for the propagation of the long wavelength fields [21]. This also justify our approach.
If the fields are allowed to evolve in contact with the heat bath at temperature $T$ for a long time, they will be equilibrated at that temperature. Above a certain critical temperature, the fields will undergo symmetry restoring phase transition, if the model allows for such a transition. For sigma model, the condensate value of the sigma field provide a convenient way to probe the phase transition. In the symmetric phase, the condensate is zero, while in the symmetry broken phase it is non-zero.

In the present paper we will consider linear sigma model without the symmetry breaking term. It is well known that there will not be an exact phase transition if the symmetry breaking term is included. We solve the eq.s on a $32^3$ lattice with periodic boundary condition. Solving eq.s require initial conditions ($\phi$ and $\dot{\phi}$). We distribute the initial fields according to a random Gaussian with

$$<\sigma> = f(r) f_\pi \tag{4a}$$

$$<\pi_i> = 0 \tag{4b}$$

$$<\sigma^2> = <\pi_i^2> = v^2 / f(r) \tag{4c}$$

$$<\dot{\sigma}> = <\dot{\pi_i}> = 0 \tag{4d}$$

$$<\dot{\sigma}^2> = <\dot{\pi}>^2 = v^2 \tag{4e}$$

The interpolation function

$$f(r) = \left[1 + e^{r - r_0}/\Gamma\right]^{-1} \tag{5}$$

separates the central region from the rest of the system. We have used $r_0 = 11a$ where $a$ is the lattice spacing and $\Gamma = 0.5$ fm. The initial field configuration corresponds to zero temperature, $<\sigma> \sim f_\pi$, $<\pi_i> \sim 0$. Initial zero temperature fields will thermalise to the temperature of the heat bath if kept in contact for sufficient time. The other parameter of the model is the friction ($\eta$). In the present paper, we use $\eta = \eta_\pi + \eta_\sigma$ and for $\eta_\pi$ and $\eta_\sigma$ use values as calculated by Rischke [3] but its precise value is not of importance here, as we are looking for fields at equilibrium. Friction determines the rate of approach to equilibrium. This aspect of equilibration was verified.

We define volume averaged sigma condensate $<\sigma>$ as the order parameter,

$$<\sigma> = 1/V \int d^3x \sigma \tag{6}$$

In fig.1, we have shown the equilibrium value of the order parameter as a function of the temperature. Results for 4 different lattice spacing, $a=0.5,1.0,2.0$ and 2.5 are shown. For all the lattice spacings, the order parameter decreases from $\sim f_\pi$ at very low temperature to exact zero at some critical temperature ($T_c$) and then remain so beyond the critical temperature. Phase transition in the model is evident. However it is also evident that phase transition temperature (the temperature at which $\sigma$ condensate vanishes) depend strongly on the lattice spacing used. It goes to smaller and smaller values as the lattice spacing is reduced. It seems that in the continuum limit, $T_c \rightarrow 0$.

The present result indicate that classical sigma model fields donot show correct equilibrium behaviour. Phase transition temperature depend strongly on the lattice spacing, and in the continuum limit, it goes to zero or atleast to a very low temperature. Strong dependence of the the transition temperature on lattice spacings indicate that the simulation studies of disoriented chiral condensate using classical sigma model fields are highly questionable. The results obtained from those simulations can not be believed to represent physical systems.

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![FIG. 1. Order parameter as a function of temperature for four different lattice spacing, a=0.5, 1.0, 2.0 and 2.5. The outer ones are for higher lattice spacing](image)

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