Combination of WVD with TRM on Detection of LFM Signal in Inhomogeneous Medium

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Abstract—On the basis of time-reversal focusing theory, time frequency analysis combining with time reversal mirror (TRM) on detection of linear frequency modulation (LFM) signal in inhomogeneous medium is studied. The simulation experiment on time frequency analysis is carried out in a lab water tank. Time reversal is manifested by taking the time-domain scattered fields received on an array, reversing them, and then propagating them back into the same medium. The re-transmitted signal is received near the source and Wigner-Ville Distribution (WVD) analysis is performed. The time varying spectrum of time frequency analysis gives more useful information to recognize different types of target signals. The simulation results show that the new method can suppress multi-scattering. The approach is effective and feasible.

Key words—time frequency analysis, time reversal mirror, underwater acoustic signal processing, inhomogeneous medium

I INTRODUCTION

How to detect linear frequency modulation (LFM) signal in underwater multiple-scattering medium is a challenging problem[1]. Time frequency analysis which provides localized information on the time and frequency domain has important application in the detection of non-stationary signals. The effect of WVD has been studied in other contexts in signals embedded in noise, but not in connection with time-reversal, as it is presented and analyzed here.

As time-reversal mirror (TRM) is useful tool for the analysis of stationary signals by the ability of refocusing[2-5], here, we first propose an approach that combines TRM with WVD to detect LFM signal in underwater inhomogeneous medium. Time reversal is manifested by taking the time-domain scattered fields received on an array, reversing them, and then propagating them back into the same medium[6-8]. The re-transmitted signal is received near the source and WVD analysis is performed. The time varying spectrum of time frequency analysis gives more useful information to discriminate different types of target signals.

In this paper, we explore analytically and experimentally the new method in a regime of parameters where the effects of random medium are fully developed. The experimental results demonstrated that this scheme suppresses multi-scattering.

This paper is organized as follows: First, WVD is analyzed theoretically based on WVD. Then, a series of simulation experiments about detection of LFM signal is conducted using TRM combined with WVD in water tank, followed by corresponding analysis. Finally, we give a summary about the advantages of the approach.

II TIME-REVERSAL MIRROR THEORY

The setup of TRM is showed in Fig.1. VLA is the vertical receive array and SRA is the receive/transmit array, PS is the point source, the horizontal distance between PS and SRA is R. The pressure field for the jth element of the SRA is

\[ G_{\omega}(R; z_j, z_{ps}) \],

where \( z_{ps} \) and \( z_j \) is the depth of the PS and the jth element of SRA, respectively. The Green’ function \( G_{\omega}(R; z_j, z_{ps}) \) satisfy the Helmholtz equation

\[ \nabla^2 G_{\omega}(R; z_j, z_{ps}) + k^2(z_j)G_{\omega}(R; z_j, z_{ps}) = -\delta(R - r_{ps})\delta(z_j - z_{ps}). \tag{1} \]

Where \( k^2(z) = \omega^2 / c^2(z) \) is the wavenumber, in far field, the Green’ function is expressed as follows:

\[ G_{\omega}(R; z_j, z_{ps}) = \frac{i}{\rho(z_{ps}) (8\pi R)^{1/2}} \exp(-i\pi/4) \times \sum_n \frac{u_n(z_{ps}) u_n(z_j)}{k_n^{1/2}} \exp(ik_n R). \tag{2} \]

Where \( \rho(z_{ps}) \) is the medium density at \( z_{ps} \), \( u_n(z) \) corresponds to the modal shape as a function of depth and \( k_n \) is the wavenumber. They are eigenfunction and eigenvalue of the equation

\[ \frac{d^2 u_n(z)}{dz^2} + [k^2(z) - k_n^2] u_n(z) = 0. \tag{3} \]

where \( u_n(z) \) satisfy the equation

\[ \int_0^\infty \frac{u_m(z) u_n(z)}{\rho(z)} dz = \delta_{nm}. \tag{4} \]

Where \( \delta_{nm} \) is impulse response function. After SRA time-reversed the incident signal, the field could be expressed with \( G_{\omega}^r(R; z_j, z_{ps}) \), then the field at observation (r, z) satisfies the wave equation

\[ \nabla^2 P_{ps}(r, z) + k^2(z)P_{ps}(r, z) = \]
\[
\sum_{j=1}^{M} \delta(z - z_j) G^*_\omega(R; z_j, z_{ps}).
\]

Where \( M \) is the number of the array element. For an incident field coming from a point source located at depth \( z_{ps} \), the phase conjugation from a vertical array of \( M \) discrete sources leads to the following time-reversed pressure field at observation point \((r,z)\):

\[
P_{pc}(r,z; \omega) = \sum_{j=1}^{M} G_{\omega}^*(r; z_j, z) G^*_\omega(R; z_j, z_{ps}).
\]

where \( r \) is a horizontal distance between the SRA and the observation point. We simply substitute (2) into (6), which specifies that we sum over all modes and array sources:

\[
P_{pc}(r,z; \omega) = \sum_{m,n} \sum_{j} u_m(z) u_n(z_j) u_n(z_{ps}) \rho(z_j) \rho(z_{ps}) k_m k_n r R \exp(i k_m r - k_n R).
\]

The sum over \( j \)-selects mode \( m = n \), and (7) becomes

\[
P_{pc}(r,z; \omega) \approx \sum_{m} u_m(z) u_m(z_{ps}) \rho(z_{ps}) k_m \sqrt{r R} \exp(i k_m (r - R)).
\]

The press field \((r \neq R)\) change obviously; The press field \((r = R)\) becomes

\[
P_{pc}(r,z; \omega) = \sum_{m} u_m(z) u_m(z_{ps}) \rho(z_{ps}) k_m \sqrt{r R}.
\]

In the plane of the source, the closure relation which defines the modes as a complete set

\[
\sum_{n=0}^{\infty} u_n(z) u_n(z_{ps}) = \delta(z - z_{ps}).
\]

could be applied under the assumption that \( k_m \) are nearly constant over the interval of the contributing modes. This leads (9) to:

\[
P_{pc}(r,z; \omega) \approx \delta(z - z_{ps}).
\]

which proves that the phase-conjugated field focuses back at the source.

### WIGNER-VILLE DISTRIBUTION

WVD of a LFM signal shows the optimal energy concentration in the time-frequency plane. It is this ideal time-frequency resolution which makes WVD useful for LFM signal detection. In this paper, we show how to design a new method which combines TRM with WHT to detect LFM signal in underwater inhomogeneous medium. To explain how this is achieved, one needs to first look closely at the definition of WVD. The WVD of an analysis signal \( z(t) \) is given by

\[
W_z(t, \omega) = \int_{-\infty}^{\infty} z(t + \frac{\tau}{2}) z^*(t - \frac{\tau}{2}) e^{-i\omega \tau} d\tau.
\]

*denotes complex conjugation. WVD maps the one-dimension time signal onto two-dimension time-frequency plane \( W_z(t, \omega) \). WVD of the ideal LFM signal is a line in time-frequency plane.

### IV SIMULATION

To explore the effect of the new approach which combined WVD with TRM, a series of simulation experiments are carried out in water tank. The linear vertical array is made up of 64 transducers. The width of every element is 16mm and the space between elements is 0.18 mm. A receive transducer B is near the point source S. The source S and the center of the array are in the same horizontal plane. The horizontal distance between the array and source S is 3000mm, 200 scattering points whose diameter is 0.8mm were distributed randomly in the water rank. In order to simulate real underwater environment, we set a target T whose diameter is 8mm between the source S and the array. The source S transmits LFM signal to target T, the array receives the signal at the same time. Original double component LFM signal and its power spectrum, WVD image of the signal are shown in Fig.2.

The frequency range and the length of the signal is 0.05MHz-4MHz and 128 points, respectively. After the WVD of the 120 \( \mu \)s time window of the received signal by the array is computed, we get the time-domain signal, power spectrum and time-frequency 2D image (Fig.3(a)). In Fig. 3(a) we can see that the time-domain signal is expanded seriously and there are many sidelobes distributed in the time domain. In time-frequency plane, the time-frequency image of the scattering of the target T appears near 60 \( \mu \)s, whose frequency range is between 1MHz and 4MHz. Because there are many scattering signals caused by random scattering points and their distorted signal, especially several times scattering images of the real target, it is difficult to detect and recognize the LFM signal. Fig. 3(b) is time-frequency 3D image corresponding to Fig. 3(a). As can be seen in Fig. 3(a), there are time-frequency scattering signals in the middle of the time-frequency plane. It affects the detection of the LFM signal seriously.

In the second step of the experiment, time-reverse the signals received by the array A and retransmit it by the every element of the array A in the same medium. The time-reversed time-domain signal received by transducer B near the source point and its power spectrum and WVD time-frequency image are shown in Fig. 4(a). It’s clearly seen that time-domain signal focusing at 60 \( \mu \)s, double component signal remain and many sidelobes caused by multi-scattering disappear. In Fig. 4(a) we also see that the power spectrum corresponding to multi-scattering signal disappears with the two spectrum line caused by double component signal remaining. It’s known that WVD of the LFM signal corresponds a line in the time-frequency plane, therefore, we can detect double component LFM signal preliminarily in Fig. 4(a). In order to better observe the parameter of the time-reversed signal received by the transducer B, we time-reverse it again and get the 3D image of the WVD time-frequency of focusing signal in 160 \( \mu \)s window in Fig.4(b). Clearly we can see the two crossed lines. One line crossed following time linearly whose estimation of the frequency range of the signal is 1.1MHz~2.8MHz, the others decreases following time linearly whose estimation of the frequency range of the signal is 3.8MHz~2.1MHz. The energy of the signal concentrate in the range of 2 \( \mu \)s ~14 \( \mu \)s, especially it has maximum at
There is the cross-term of the double component LFM signal distributed at low frequency and high frequency segments. Although the cross-term of the double component LFM signal exists, the multi-scattering in homogeneous medium can be effectively suppressed by TRM, and the distorted signal can be revised. In this way, we can better detect double component LFM signal and estimate its parameter.

V CONCLUSION

In conclusion, we propose the new approach that combines TRM with WVD time-frequency analysis to detect LFM signal in underwater inhomogeneous medium. The experimental results demonstrate that this scheme not only suppresses multi-scattering, but also reduces cross-term of the multi-component LFM signal. The approach provides new insight into the development of a new application in underwater non-stationary signal detection and target recognition, which can capture important information in underwater acoustic signal processing.

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Fig. 1. Scheme of TRM

Fig. 2. Time frequency diagram of original signal

- a. Diagram of WVD time frequency before time-reversal
- b. Three-dimension diagram of WVD time frequency before time-reversal

Fig. 3. Time frequency diagram before time-reversal

- a. Diagram of WVD time frequency after time-reversal
- b. Diagram of WVD in 16 μs window after time-reversal twice

Fig. 4. Time frequency diagram after time-reversal

- a. Diagram of WVD time frequency before time-reversal
- b. Three-dimension diagram of WVD time frequency before time-reversal