The dynamics of texture-like configurations are briefly reviewed. Emphasis is given to configurations in 2+1 dimensions which are constructed numerically. Confirming previous semi-analytical studies it is shown that they can be stabilized by partial gauging of the vacuum manifold (semilocality) in a finite range of parameter space. When these configurations are extended to 3+1 dimensions (stringy textures) it is shown that they can support persistent currents if a twist (Hopf charge) is introduced in the scalar field sector. The pressure induced by these persistent currents is also studied in closed loops. In the context of a simple model, twist induced pressure is shown to be insufficient to stabilize the loops against collapse due to tension.

I. INTRODUCTION

A key feature of texture-like topological defects is the fact that the topological charge emerges by integrating over the whole physical space (not just the boundary). These defects have played an important role in both particle physics and cosmology. Typical examples are the skyrmion which offers a useful effective model for the description of the nucleon and the global texture where an instability towards collapse of the scalar field configuration has been used to construct an appealing mechanism for structure formation in the universe.

A typical feature of this class of scalar field configurations is instabilities towards field rescalings which usually lead to collapse and subsequent decay to the vacuum via a localized highly energetic event in space-time. The property of collapse is a general feature of global field configurations in 3+1 dimensions and was first described by Derrick. This feature is particularly useful in a cosmological setup because it provides a natural decay mechanism which can prevent the dominance of the energy density of the universe by texture-like defects. At the same time, this decay mechanism leads to a high energy event in space-time that can provide the primordial fluctuations for structure formation.

In the particle physics context where a topological defect predicted by a theory can only be observed in accelerator experiments if it is at least metastable, the above instability is an unwanted feature. A usual approach to remedy this feature has been to consider effective models where non-renormalizable higher powers of scalar field derivatives are put by hand. This has been the case in QCD where chiral symmetry breaking is often described using the low energy 'pion dynamics' model. Texture-like configurations occur here and as Skyrme first pointed out they may be identified with the nucleons (Skyrmions). Here textures are stabilized by non-renormalizable higher derivative terms in the quantum effective action. However no one has ever found such higher derivative terms with the right sign to stabilize the Skyrmion.

An alternative approach to stabilize texture-like configurations is the introduction of gauge fields which can be shown to induce pressure terms in the scalar field Lagrangian thus balancing the effects of Derrick-type collapse. In the case of complete gauging of the vacuum manifold however, it is possible for the texture configuration to relax to the vacuum manifold by a continuous gauge transformation that can remove all the gradient energy (the only source of field energy for textures) from the non-singular texture-like configuration. This mechanism of decay via gauge fields is not realized in singular defects where the topological charge emerges from the boundaries. In these defects, singularities, where the scalar field is 0, can not be removed by continuous gauge transformations.

Recent progress in semilocal defects has indicated that physically interesting models can emerge by a partial gauging of the vacuum manifold of field theories. This partial gauging (semilocality) can lead to new classes of stable defect solutions that can persist as metastable configurations in more realistic models where the gauging of the vacuum is complete but remains non-uniform. A typical example is the semilocal string whose embed-
The simplest texture-like configuration appears in 1+1 dimensions in a field theory with a two-component real scalar field $\Phi = (\Phi_1, \Phi_2)$ which breaks a global $O(2)$ symmetry and its dynamics is determined by the potential $V(\Phi) = \frac{1}{4}(\Phi^2 - \eta^2)^2$. The vacuum manifold $M_0$ of this theory is $\Phi^2 = \eta^2$ i.e. a circle $S^1$. It has a non-trivial first homotopy group $\pi_1(S^1) = \mathbb{Z}$. Therefore there are topologically non-trivial field configurations in 1+1 dimensions such that as one travels along in space, $\Phi$ winds once around $M_0$ (Fig. 1a). As shown in Fig.1a the magnitude of $\Phi$ is close to $\eta$ everywhere and therefore the total energy of the configuration may be approximated as

$$E = \frac{1}{2} \int_{-\infty}^{+\infty} dx \, \Phi'^2 \equiv T$$

$$E = \frac{1}{2} \int_{-\infty}^{+\infty} dx \, \Phi'^2 \equiv T$$

Figure 1: (a) The field configuration of a texture in 1+1 dimensions covers completely the vacuum manifold $S^1$ as the physical space is spanned. (b) In 2+1 dimensions the vacuum manifold $S^2$ is also covered and there is the possibility of field twist along the $z$ axis.

A rescaling of the spatial coordinate $x$ to $\alpha x$ with $\alpha < 1$ leads to a field configuration $\Phi(\alpha x)$ which has expanded in space relatively to $\Phi(x)$. The total energy of $\Phi(\alpha x)$ is clearly $E_\alpha = \alpha E < E$ and therefore the dynamics will favor expansion of the configuration $\Phi(x)$ to infinity. Thus texture-like configurations in 1+1 dimensions are unstable to expansion. This argument does not hold for configurations with appreciable total potential energy (e.g. domain walls). The potential energy scales as $\frac{1}{\alpha}$ with re-scaling and can therefore prevent the expansion triggered by the gradient energy.

The 1+1 dimensional texture can be stabilized forming configurations known as ribbons [1]. This can be achieved in two ways. The first includes the introduction of a potential term $V_1(\Phi) = \mu(\Phi - \eta)^2$ that explicitly breaks the $O(2)$ symmetry and leads to a rescaled energy $E_\alpha = \alpha T + V_1/\alpha$ which can be minimized with respect to $\alpha$. The second is implemented by the introduction of a cutoff $L$ in the one dimensional space which, given the boundary conditions, is equivalent to compactifying it. With this cutoff the energy becomes $E = \int_{-L}^{L} dx \, \Phi'^2/2$ and $E_\alpha = \alpha \int_{-\alpha L}^{\alpha L} dx \, \Phi'^2/2$ which can also be minimized with respect to $\alpha$. 

II. TEXTURE FIELDS AND THEIR DYNAMICS

The simplest texture-like configuration appears in 1+1 dimensions in a field theory with a two-component real scalar field $\Phi = (\Phi_1, \Phi_2)$ which breaks a global $O(2)$ symmetry and its dynamics is determined by the potential $V(\Phi) = \frac{1}{4}(\Phi^2 - \eta^2)^2$. The vacuum manifold $M_0$ of this theory is $\Phi^2 = \eta^2$ i.e. a circle $S^1$. It has a non-trivial first homotopy group $\pi_1(S^1) = \mathbb{Z}$. Therefore there are topologically non-trivial field configurations in 1+1 dimensions such that as one travels along in space, $\Phi$ winds once around $M_0$ (Fig. 1a). As shown in Fig.1a the magnitude of $\Phi$ is close to $\eta$ everywhere and therefore the total energy of the configuration may be approximated as

$$E = \frac{1}{2} \int_{-\infty}^{+\infty} dx \, \Phi'^2 \equiv T$$
The texture configuration in 1+1 dimensions can be generalized to 3+1 dimensions by considering a field theory describing a global symmetry breaking $O(4) \rightarrow O(3)$. Consider for example a four component scalar field $\Phi = (\Phi_1, \Phi_2, \Phi_3, \Phi_4)$ whose dynamics is described by the potential

$$V(\Phi) = \frac{\lambda}{4}(\mathbf{\Phi}^2 - \eta^2)^2 \tag{2}$$

The initial condition ansatz

$$\Phi = (\sin \chi \sin \theta \sin \varphi, \sin \chi \sin \theta \cos \varphi, \sin \chi \cos \theta, \cos \chi) \tag{3}$$

with $\chi(r)$ varying between 0 and $\pi$ as $r$ goes from 0 to infinity and $\theta, \varphi$ spherical polar coordinates, describes a configuration that winds once around $M_0 = S^3$ as the physical space is covered. The energy of this configuration is of the form

$$E = \int_{-\infty}^{+\infty} \frac{1}{2} (\nabla \Phi)^2 + V(\Phi) \equiv T + V \tag{4}$$

where we have allowed for possible small potential energy excitations during time evolution. A rescaling of the spatial coordinates $r \rightarrow \alpha r$ leads to $E_\alpha = \alpha^{-1} T + \alpha^{-3} V$ which is monotonic with $\alpha$ and leads to collapse, highly localized energy and eventual unwinding of the configuration. These highly energetic and localized events in spacetime have provided a physically motivated mechanism for the generation of primordial fluctuations that gave rise to structure in the universe.

The possible stabilization of these collapsing configurations could lead to a cosmological overabundance and a cosmological problem similar to the one of monopoles, requiring inflation to be resolved. At the same time however it could lead to observational effects in particle physics laboratories. There are at least two ways to stabilize a collapsing texture in 3+1 dimensions. The first is well known and includes the introduction of higher powers of derivative terms in the energy functional. These terms scale like $\alpha^p \ (p > 0)$ with a rescaling and can make the energy minimization possible thus leading to stable skyrmions. Stable Hopfions \cite{12} (solitons with non-zero Hopf topological charge) have also been constructed recently by the same method. The second method of stabilization is less known (but see ref. \cite{13}) and can be achieved by introducing gauge fields that partially cover the vacuum manifold. An example is the Lagrangian \cite{14}

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D_\mu \Phi^a D^\mu \Phi^a + \frac{1}{2} (\partial_\mu \Phi^4)^2 - V \tag{5}$$

describing the dynamics of an O(3) gauge field $A^a_\mu$ coupled to the three components ($\alpha = 1, 2, 3$) of the scalar field $\Phi = (\Phi_1, \Phi_2, \Phi_3, \Phi_4)$ whose dynamics is determined by the potential \cite{13}.

The field strength and the covariant derivative are given by $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + ge^{abc}A^b_\mu A^c_\nu$ and $D_\mu \Phi^a = \partial_\mu \Phi^a + ge^{abc}A^b_\mu \Phi^c$, respectively.

The above is a simple extension of the Georgi-Glashow O(3) model with one classically relevant parameter

$$\tilde{\beta} = \frac{g}{\sqrt{\lambda}} \tag{6}$$

as revealed after the rescaling $x^i \rightarrow x^i/(\sqrt{\lambda})$, $\Phi^a \rightarrow v \Phi^a$, $\Phi^4 \rightarrow v \Phi^4$, and $A^a_\mu \rightarrow v A^a_\mu$.

For $\tilde{\beta} = 0$, (3) possesses an O(4) global symmetry. The ansatz describing a semilocal texture in 3+1 dimensions is of the form

$$\Phi = (f(r) \sin \theta \sin \varphi, f(r) \sin \theta \cos \varphi, f(r) \cos \theta, G(r))$$

$$A^a_\mu = \epsilon_{a\nu} \frac{x^j}{r} W(r) \tag{7}$$

with $f(r)$ and $W(r)$ necessarily vanishing at the origin $r = 0$ and at infinity.

It is convenient to define $K(r) \equiv 1 - \tilde{\beta} r W(r)$, in which case the field equations for the three unknown functions of the ansatz take the form:

$$f'' + \frac{2f'}{r} - \frac{2f}{r^2} K^2 + (1 - f^2 - G^2)f = 0$$

$$K'' - \frac{K(K - 1)}{r^2} - \tilde{\beta}^2 f^2 K = 0$$

$$G'' + \frac{2G'}{r} + (1 - f^2 - G^2)G = 0 \tag{8}$$

while the corresponding boundary conditions, dictated by the finiteness of the energy and the field equations at the origin, are

$$f(0) = 0, \quad G(0) = 1, \quad K(0) = 1 \quad f(\infty) = 0, \quad G(\infty) = 1, \quad K(\infty) = 1 \tag{9}$$

As discussed in the introduction, a complete gauging of the vacuum manifold would allow the gauge fields to unwind the topological charge by a continuous gauge transformation and a decay to the vacuum. This can not be achieved by the semilocal gauging of (3). It may also be shown that the gauge fields induce terms that scale as $\alpha^p \ (p > 0)$ in the energy corresponding to (3) thus preventing the collapse. The detailed analysis of this model is currently in progress. Another concrete example of this mechanism in a simple model will be discussed in section III.
Let us now consider textures in 2+1 dimensions which are also known as Belavin-Polyakov vortices [13]. These can form in theories with global symmetry breaking leading to a vacuum manifold $S^2$. Consider for example a field theory describing a global symmetry breaking $O(3) \rightarrow O(2)$ where the vacuum manifold $M_0$ is $S^2$. This is achieved by a three component scalar field $\Phi = (\Phi_1, \Phi_2, \Phi_3)$ whose dynamics is described by the potential (3). The ansatz

$$(\Phi_u, \Phi_3) = [\Phi_u e^{i\varphi}, \Phi_3(\rho)] = [\sin \chi(\rho) e^{i\varphi}, \cos \chi(\rho)]$$

(10)

where $\chi(\rho)$ varies between 0 and $\pi$ as $\rho$ goes from 0 to infinity (Fig. 1b), describes a configuration that winds once around infinity. It should therefore only be trusted for relatively large closed loops ($L \gg \eta^{-1}$).

Allowing for small potential energy excitations during evolution leads to an energy of the form

$$E = \int_{-\infty}^{+\infty} d^2x \frac{1}{2} (\nabla \Phi)^2 + V(\Phi) \equiv T + V$$

(11)

which after coordinate rescaling becomes $E_\alpha = T + \alpha^{-2}V_1$ and the configuration is weakly collapsing if the potential term becomes significant while otherwise it is neutral with respect to rescaling. Stabilization can now be achieved in two steps: First we introduce a potential energy term

$$V_1(\Phi) = \mu(\Phi_3 - \eta)^2$$

(12)

which explicitly breaks $O(3)$ and is therefore non-vanishing at finite distance. Then a rescaling leads to $E_\alpha = T + \alpha^{-2}V_1$ which implies collapse. The collapse may now be halted by the introduction of partial gauging (semlocality) through a $U(1)$ gauge field. As shown in section III, a rescaling now leads to $E_\alpha = T - \alpha^{-1}V_2 + \alpha^{-2}V_1$ where $T, V_1$ and $V_2$ are energy components independent of $\alpha$ and therefore $E_\alpha$ can be minimized with respect to $\alpha$ leading to stability and to a virial theorem relating the terms $V_1$ and $V_2$.

The 2+1 dimensional configuration (10) may easily be extended to 3+1 dimensions as a stringy texture by assuming uniformity along the z-axis (Fig. 2). A more interesting case arises if we allow $\Phi_u$ to twist as we move along the z-axis. This would generalize the ansatz (11) to

$$(\Phi_u, \Phi_3) = [\Phi_u e^{i\varphi} e^{iu(z)}, \Phi_3(\rho)]$$

(13)

This approximation of a closed loop by a finite size cylinder has the advantage of retaining cylindrical symmetry but it fails to distinguish the center of the loop from infinity. It should therefore only be trusted for relatively large closed loops ($L \gg \eta^{-1}$). The identification of the two faces (upper with lower) of the cylinder induces a twist topological charge defined as

$$N_t = \frac{1}{2\pi} \int_0^{2\pi L} \frac{du}{dz} dz$$

(14)

in addition to the usual $N_w$ charge of $\pi_3(S^2)$ of the two dimensional texture.

The product $N_wN_t$ is the Hopf topological charge $Q_H$. It appears due to the non-trivial $\pi_3(S^2)$ which classifies the twisted loops of stringy textures. The presence of a nonzero Hopf charge combined with the presence of gauge fields could prevent loop collapse due to the increase of gradient energy of the twisted component as...
the loop collapses (cylinder shrinks). It will be seen however in section III that this pressure is not sufficient to balance the tension of the closed stringy texture in order to stabilize it.

III. APPLICATION ON A SIMPLE MODEL

In order to make the ideas described in the previous sections more concrete let us consider a simple model described by the Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (D_\mu \Phi)^4 D^\mu \Phi + \partial_\mu \Phi_3 \partial^\mu \Phi_3$$

$$- V(\Phi, \Phi_3) + \frac{1}{2} m^2 A_\mu A^\mu$$ (15)

where $\Phi = \Phi_1 + i \Phi_2$, $F_{\mu\nu} = \partial_\mu \Phi_\nu - \partial_\nu \Phi_\mu$, $D_\mu = \partial_\mu - ie A_\mu$ and

$$V(\Phi, \Phi_3) = \frac{\lambda}{4} (\Phi^4 + \Phi_3^2 - \eta^2)^2 + \frac{k^2}{8} (\Phi_3 - \eta)^4$$ (16)

The energy density of the static configuration is

$$\mathcal{E} = \frac{1}{4} F_{ij}^2 + \frac{1}{2} (D_i \Phi)^2 D^i \Phi + \partial_i \Phi_3 \partial^i \Phi_3$$

$$- V(\Phi, \Phi_3) + \frac{1}{2} m^2 A_i A^i$$ (17)

where $i, j = 1, 2, 3$. We now use the following twisted semilocal stringy texture ansatz

$$\Phi = f(\rho)e^{iN_\omega \varphi}e^{iu(z)}, \quad \Phi_3 = g(\rho)$$

$$\tilde{A} = e\varphi a(\rho) + e_z b(\rho)$$ (18)

where $0 \leq z \leq 2\pi L$. This ansatz describes approximately a large circular loop of SSST in a model with explicitly broken O(3) symmetry and an additional twist $u(z)$ of the $\Phi$ field along the length of the loop. This length is approximated by a large length on the $z$-axis. The assumption that the radius of the loop is $L$ (with $L \gg \eta^{-1}$) imposes the following boundary conditions on the field function $u(z)$

$$u(2\pi L) - u(0) = 2\pi N_i$$ (19)

where $N_i$ is the topological charge due to twist. Assuming in addition that the twist is uniform along the length of the loop we obtain

$$\dot{u} = \frac{du}{dz} = \frac{N_i}{L}$$ (20)

Since $N_i$ is conserved the configuration supports the following superconducting current densities

$$j_z = \frac{\delta \mathcal{L}}{\delta \Phi} = -e(\dot{u} - eA_z)^2$$ (21)

$$j_\varphi = \frac{\delta \mathcal{L}}{\delta \Phi_3} = -e\left(\frac{N_\omega}{\rho} - eA_\varphi\right)^2$$ (22)

The configuration (18) viewed as a loop in three-dimensions represents a mapping from space (a large ‘three sphere’) onto the vacuum manifold (two sphere). The mapping is the non-trivial Hopf fibration $S^3 \rightarrow S^2$ with fiber $S^1$. The corresponding Hopf topological charge is

$$Q_H = \frac{1}{8\pi} \int_{S^2} \epsilon_{abc} \Phi_a d\Phi_b d\Phi_c = N_\omega \cdot N_i$$ (23)

With the ansatz (18) the energy density (17) takes the form

$$\mathcal{E} = \frac{1}{2}(a' + a_0)^2 + \frac{1}{2} b^2 + \frac{1}{2} f^2 + \frac{1}{2} \left(\frac{N_\omega}{\rho} - e\omega\right)^2 f^2$$

$$+ \frac{1}{2} (\dot{u} - e\varphi)^2 f^2 + \frac{1}{2} g^2 + \frac{\lambda}{4} (f^2 + g^2 - \eta^2)^2$$

$$+ \frac{k^2}{8} (g - \eta)^4 + \frac{1}{2} m^2 (a^2 + b^2)$$ (24)

We now impose the following rescaling

$$[\text{field}] \rightarrow [\text{field}] \frac{m}{\sqrt{2\lambda}}, \quad \tilde{x} \rightarrow \frac{\tilde{x}}{m}$$ (25)

where [field] refers to the field functions $f(\rho)$, $g(\rho)$, $a(\rho)$ and $b(\rho)$ while $\tilde{x}$ refers to $\rho$ and $z$. The energy density $\mathcal{E}$ becomes

$$\mathcal{E} = \frac{m^4}{2\lambda} \left(\frac{1}{2} (a' + a_0)^2 + \frac{1}{2} b^2 + \frac{1}{2} f^2 + \frac{1}{2} \left(\frac{N_\omega}{\rho} - e\omega\right)^2 f^2ight.$$

$$+ \frac{1}{2} (\dot{u} - e\varphi)^2 f^2 + \frac{1}{2} g^2 + \frac{\lambda}{4} (f^2 + g^2 - m_H^2)^2$$

$$+ \frac{k^2}{8} (g - m_H)^4 + \frac{1}{2} (a^2 + b^2)]$$ (26)

where $\tilde{e} = e/\sqrt{2\lambda}$, $\tilde{k} = k/\sqrt{2\lambda}$ and $m_H = \sqrt{2\lambda} \eta/m$.

By extremizing the static energy density (26) we obtain the field equations
The gauge field

An other virial condition may be obtained by rescaling of the energy and the field equations at the origin and may be written as

\[ f(0) = 0, \ a(0) = 0, \ \rho b'(0) = 0, \ \rho f'(0) = 0 \]
\[ \rho g'(L) = 0, \ g(L) = \eta, \ b(L) = 0, \ \vec{B}(L) \equiv \vec{\nabla} \times \vec{A}(L) = 0 \]

(28)

We have used a relaxation method with locally variable mesh size to solve this system of equations in order to identify parameter sectors where solutions exist. The validity of the derived solutions was verified by checking that they satisfy to a good approximation virial conditions that can be obtained analytically from the energy functional [24] by appropriate rescalings. In particular by demanding that the solution is an extremum of the total energy in two dimensions and therefore its energy does not change to first order by a rescaling \( \rho \rightarrow \alpha \rho \) we have the condition \( \frac{\delta E}{\delta a} = 0 \) which implies

\[ v_1 = \frac{E_1 - E_2}{E_1 + E_2} = 0 \]

(29)

where

\[ E_1 = \int_0^L d\rho \ N_w \ \hat{e} f^2 a \]
\[ E_2 = \frac{1}{2} \int_0^L d\rho \ \rho [a^2(1 + \hat{e}^2 f^2) + \frac{1}{4}(f^2 + g^2 - m_H^2)^2]
\]
\[ \frac{\hat{k}^2}{4}(g - m_H)^4 + b^2 + \left( \frac{N_t}{L} - \hat{e} b \right)^2 f^2 \]

(30)

An other virial condition may be obtained by rescaling the gauge field \( a(\rho) \) whose boundary conditions are insensitive to rescaling (\( a(0) = a(L) = 0 \)). By rescaling \( a \rightarrow \alpha a \) and demanding \( \frac{\delta E}{\delta a} = 0 \) we obtain

\[ v_2 = \frac{E_1 - E_3}{E_1 + E_3} = 0 \]

(31)

where

\[ E_3 = \int_0^L d\rho \ \rho [a^2(1 + \hat{e}^2 f^2) + (1 + \hat{e}^2 f^2) a^2] \]

(32)

All the solutions obtained by the relaxation method satisfied the above virial conditions to a very good approximation. In all cases we had \( v_1 \approx 10^{-3} \) and \( v_2 \approx 10^{-3} \).

As mentioned above, the model considered here reduces to that of Ref. [3] for zero twist \( (N_t = 0) \). Thus we can test numerically the semi-analytic approximate results of Ref. [3] by finding parameter sectors where stable solutions exist in the limit \( N_t \rightarrow 0 \). In this limit the only relevant parameters for the existence and stability of solutions were shown in Ref. [3] to be

\[ a = \frac{\hat{k}^2}{e^2} \]
\[ b = \frac{2}{e^2 m_H^4} \]

(33)

(34)

Figure 3: For values of the parameters \( a \) and \( b \) below the dashed \( (N_t/L = 0) \) and dotted \( (N_t/L = 0.01) \) lines we have found numerically, classically stable solutions. No solutions were found above these lines. The dashed line is in very good agreement with the approximate semi-analytical results of Ref. [3] obtained for \( N_t = 0 \).

We have first solved numerically the coupled system of field equations [27] for zero twist and found the range of parameters in \( a \)-\( b \) space where solutions exist. The results are shown in Fig. 3 (dashed line) and their agreement with the approximate results of Ref. [3] is very good.

We then introduce a small twist \( (N_t = 10 \text{ with } L = 1000) \)
and solve the field equations again for the same parameter values. The parameter region for solution existence gets reduced as expected and is also shown in Fig. 3 (dotted line) in the a-b plane. The derived solutions have also been varied numerically using a large number of smooth random fluctuations and we verified that the perturbations always tend to increase the total energy. Thus the derived solutions which are obviously extrema of the total energy correspond to minima (not maxima) and are stable solutions of the field equations.

In order to examine the effectiveness of pressure induced by the superconducting currents we have looked for solutions in the parameter sectors corresponding to high current density (twist charge density) just before the solution is lost due to increased energy density. We have found that even though there are solutions with relatively high twist charge density (provided that $m_H$ is also large enough) the effect of the pressure is not nearly enough to stabilise the shrinking loop configuration and produce a total energy local minimum for a finite value of $L$. Thus even though the solutions obtained are local minima of the energy as two dimensional configurations, when considered in three dimensions as loops they are expected to shrink monotonically towards zero loop radius ($L = 0$) due to tension.

![Figure 4](image-url)

Figure 4: The parameter sectors where stable solutions were found in the parameter region of high $N_t$. In all cases we had $N_t = 70$ and $N_w = 1$. The parameter space was scanned by reducing $L$ for each fixed $m_H$ ($300 \leq m_H \leq 1000$).

The parameter sectors where highly twisted stable solutions have been found are shown in Fig. 4 as a function of the loop radius $L$. Notice that the increase of $m_H$ improves the stability of the solution and allows the loop to shrink further before the solution disappears due to very large twist topological charge. Thus for larger $m_H$ the length $L$ of the loop can drop further before the instability sector is reached. We anticipate that this effect is similar to the current quenching which is seen in the case of the usual superconducting strings [13]. As the loop shrinks due to tension the twist topological charge density increases and at a critical density it becomes energetically favorable for the field $f$ to become 0 in order to unwind part of the twist topological charge and therefore reduce the superconducting current. This effect is known as current quenching. The critical size of the loop for current quenching is expected to decrease as the parameter $m_H$ increases because it becomes more costly energetically for $f$ to vanish. This behavior is seen in Fig. 4. At quenching the solution breaks down because in our approximation of uniform large loop we have not allowed dynamics in $\dot{u}(z)$ assuming that the twist charge density remains uniform at all times. This approximation is valid for currents smaller than the quenching currents but in order to describe the actual quenching process when the conservation of $N_t$ is violated one needs a full 3-dimensional simulation of the loop evolution [16]. This is outside the scope of the present study.

![Figure 5](image-url)

Figure 5: The field profile for $L = 950$. The rest of the parameters were: $m_H = 8$, $\tilde{k} = 0.01$, $\tilde{c} = 0.1$, $N_t = 1$ and $N_w = 1$. The virial theorems were satisfied ($v_1 \simeq 6 \times 10^{-4}$, $v_2 \simeq 10^{-4}$).

In Fig. 5 we show the field functions $f(\rho)$ and $g(\rho)$ for some choice of parameters.

An interesting question that needs to be addressed in the context of the model studied is the question of spring formation [15,17]. The total energy of the large loop configuration ($L \gg \eta^{-1}$) may be written as:
\[ E(L) = 2\pi \int_0^{2\pi L} dz \int_0^L dp \rho \mathcal{E}(f, g, a, b) \]  

(35)

where \( \mathcal{E} \) in its rescaled form is given by equation (26). Given the presence of an \( L \)-dependent term (the twist gradient) in \( \mathcal{E} \) it could have been anticipated that \( E(L) \) has a minimum at some \( L = L_{\text{spring}} \). This would imply the formation of a loop stabilized by the pressure induced by the twist charge which acts against the tension of the loop. These objects were anticipated by analytical arguments in the Witten models [13,17] of superconducting strings but no such solution has been found explicitly so far.

Clearly, spring formation could occur only for \( L \) large enough so that the solution exists \( i.e. L_{\text{spring}} > L_{\text{quench}} \).

If this minimum of the energy could be achieved at some \( L_{\text{spring}} > L_{\text{quench}} \) then at the \( L_{\text{quench}}(m_H) \) shown in Fig. 3, \( E(L) \) would have a negative derivative with respect to \( L \) \( i.e. \) the total energy \( E \) would tend to decrease towards its minimum as \( L \) increased from \( L_{\text{quench}} \) towards \( L_{\text{spring}} \). We have checked all points at \( L_{\text{quench}} \) shown in Fig. 3 and found that \( \frac{dE}{dL}_{L=L_{\text{quench}}} > 0 \). Therefore we conclude that for all the parameter sectors we investigated no spring solutions exist.

IV. CONCLUSION - OUTLOOK

The main points of this talk can be summarized as follows

- Semilocality can stabilize textures in 2+1 dimensions.
- Twisted Superconducting Semilocal Stringy Texture (SSST) configurations exist for a finite sector of parameter space.
- No SSST loops stabilized by current pressure (springy textures) were found in the simple model considered.

An alternative way to stabilize SSST loops is the introduction of angular momentum whose conservation can stabilize loops against collapse more effectively than twist pressure. Loops stabilized by angular momentum are known as vortons [13]. Potential extensions of this work include the study of the effects of angular momentum and the embedding of these configurations in realistic two-Higgs electroweak models [13].

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