Experimental Test of Two-way Quantum Key Distribution in Presence of Controlled Noise

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In principle, since the seminal works by Bennett and Brassard [1] and Ekert [2], QKD developed into a promising field of research for near-future technology ([3]; for a review see [4]), and both theoretical [5] and experimental [6] work has been done in order to prove its security and feasibility. A feature of QKD is that none of the users knows in advance the final form of the generated key. On the contrary the secure transmission of a predetermined key has been recently investigated through schemes that exploit [7, 8, 9] or do not exploit [10, 11] entanglement, and that are usually cited as deterministic, with reference to Bob’s in principle possibility of knowing with certainty the information encoded by Alice [12].

In this Letter we present the experimental test of a QKD realized with the two-way protocol described in Ref. [11] and termed LM05. We simulate the noise related to a class of attacks by Eve, and by accordingly varying it we measure all the quantities relevant to an effective transmission of information.

The Protocol—In LM05 Bob prepares a qubit in one of the four states |0⟩, |1⟩ (the Pauli Z eigenstates), |+⟩, |−⟩ (Pauli X eigenstates), and sends it to his counterpart Alice. With probability c Alice uses the qubit to test the channel noise (control mode, CM) or, with probability 1−c, she uses it to encode a bit of information (encoding mode, EM). The CM consists in a projective measurement of the qubit along a basis randomly chosen between Z and X, followed by the preparation of a new qubit in the same state as the outcome of the measurement. The EM is the modification of the qubit state according to one of the following transformations: the identity operation I, which leaves the qubit unchanged and encodes the logical ‘0’, or i Y ≡ ZX, which flips the qubit and encodes the logical ‘1’. Alice can now send the qubit back to Bob who measures it in the same basis he prepared it; in case of an EM run this feature allows Bob to deterministically infer Alice’s operation. After the whole transmission Alice declares the CM and the EM runs. Comparing the data collected during the CM the users estimate the Quantum Bit Error Rates (QBERs) on the forward and backward channels; we call these two ‘partial’ QBERs respectively q1 and q2. Comparing a part of the data collected during the EM the users estimate also a third, ‘total’, QBER QAB. This quantity is not necessary for the security of the protocol but proves useful for estimating the mutual information between Alice and Bob. As usual for a QKD the exchange of the raw key is then followed by the procedures of error correction [15] and privacy amplification [14].

The communication is realized exploiting linear polarization states of near infrared photons. In Fig.1 is reported a sketch of the experimental setup. The photons are generated by a type II down-conversion process: a UV pump beam from a diode laser (λ = 406.5nm, power 25mW) impinges on a 1.5mm thick BBO crystal cut at an angle of 43°. We select the intersection of the ordinary propagating light cone and the extraordinary one at an angular aperture of the two beams around ∼ 3.5° [17]. The wide spectral width of the pump beam (Δλ = 0.9nm) affects the entanglement of the two-photon, as in the case of pulsed pump [16]: to obtain a totally symmetrical spectrum we used an interferometric technique [17]. The two output modes of PBS1 are launched into single mode fibers [18] at 810nm through a pair of interference filters centered at 810nm and a bandwidth of 40nm, which are used to reduce the background light. The fibers terminate onto the sensible area of two APD modules with quantum efficiency ∼ 70% at 810nm. The coincidence rate is around 1000cps for single count rate of around 12000cps. The polarization state after the PBS1 can be expressed as |ψ⟩ = (|00⟩ − |11⟩)/√2. Two Glenn-Laser polarizer (GL) were inserted after the PBS1 to verify the quality of the polarization entanglement of the state. The state purity has been tested by a tomographic reconstruction [19]. We measured an entanglement of formation of 0.989 ± 0.005 [20] and a violation of the CHSH Bell inequality of 98 standard deviations [21]. Bob prepares the qubits measuring one of the output mode of the interferometer with a λ/2 waveplate (WP1), a GL and detector T. The detection of this photon projects the other one on the desired polarization state and serves also as trigger for the whole communication system. We note that entanglement is not necessary for the protocol itself. This specific preparation proce-
dure was chosen because it can be easily extended for a true random passive choice of the initial state using only linear optics components. The prepared photon is then launched into a 5m long single mode fiber at 810nm towards Alice.

Alice passively switches between CM and EM via a 50/50 BS. Control mode: when Alice’s measuring basis is the same as Bob’s the coincidence clicks between the two detectors A₀ and A₁ and the trigger detector T permit to estimate q₁ through the usual formula: 

\[ q₁ = C_{err}/(C₀ + C₁), \]

where \( C₀, C₁ \) is the coincidence counts and \( C_{err} = C₀₁ \) depending on Bob’s state preparation.

To complete the control mode, Alice injects an attenuated light pulse of definite polarization with a wavelength of 810nm in the BS. This pulse is generated by a pulsed diode laser (not shown in figure) with a repetition rate of 80MHz, pulse width 88ps FWHM, attenuated to an average number of photons per pulse of \( \mu = (1.20 ± 0.05) \times 10^{-3} \). Encoding mode: for encoding the message is necessary to realize the I and the iY Pauli operators. A couple of λ/2 waveplates (WP₂,₃) allows to span all the equator of the Bloch sphere (an eventual phase has no importance for the rest of the protocol). As last step, the photon travels back to Bob through a different fiber, 5m long too. The photons sent by Alice are eventually polarization-analyzed at Bob’s side by PBS₃ and a λ/2 waveplate (WP₄) set so that the photons are measured in the same basis as they were prepared. The photons are collected after the PBS₃ into two multimode fibers and then detected by two APD modules, B₀ and B₁. During CM runs a measure of q₂ is obtained in the same way as q₁ with the difference that the photons come from the pulsed laser that also supplies a trigger signal. Coincidence counts between B₀ or B₁ and the trigger T can be associated to logical values ‘0’ or ‘1’ corresponding to Alice encoding in the EM runs. In our experimental tests we used all the coincidence counts to estimate \( Q_{AB} \).

In the inset of Fig. 1 is reported a typical communication test. It consists in a direct measurement of \( Q_{AB} \) for different state preparations performed by Bob and different encodings by Alice. All the eight configurations of interest are reported. The best value we obtained for \( Q_{AB} \) is \( (4.05 ± 0.22) \times 10^{-2} \). Before every test the fibers were aligned using two polarization control pads, one for each fiber. The pads are set so that any polarization input state exits almost unchanged. The usual fidelity for the polarization state after the alignment is ~96%. The fibers proved to remain stable for quite long periods (~ 4h), enough for several runs after the alignment.

Eavesdropping: The CM of LM05 comprises the same security test of BB84 [1], repeated twice. This gives to the two-way LM05 at least the same security level of the one-way BB84. Nevertheless there are indications that the security threshold of two-way schemes can overcome that of one-way schemes. In particular LM05 results secure against individual incoherent attack (IIA) regardless of the noise introduced on the channel by an eavesdropper (Eve) [11]; on the contrary BB84 results secure against individual attacks only if the noise threshold is lower than ~15% [23, 4 Sec.VI.E]. In the optimal IIA Eve prepares two sets of ancillae \( ε, η \) and makes them interact with the qubit: the \( ε \)'s on the forward path and the \( η \)'s on the backward one, after Alice’s encoding stage. By proper measure of her two sets of ancillae, Eve can gain information about the key. There are two mutually exclusive interactions that minimize Eve’s noise on the channel while maximizing her gain [11]:

Z-attack:

\[
\begin{align*}
|0⟩_ε & \rightarrow |0⟩_{εZ} \\
|+⟩_ε & \rightarrow |+⟩_{+Z} + |−⟩_{−Z} \\
|1⟩_ε & \rightarrow |1⟩_{+Z} + |−⟩_{−Z}
\end{align*}
\]

X-attack:

\[
\begin{align*}
|+⟩_ε & \rightarrow |+⟩_{X} \\
|0⟩_ε & \rightarrow |0⟩_{X} + |1⟩_{+X} \\
|−⟩_ε & \rightarrow |−⟩_{X} + |1⟩_{−X}
\end{align*}
\]

where we have introduced the states \( |εZ⟩_{0,1} \), \( |εX⟩_{+−} \), \( |Z⟩_{+−} = |εZ⟩_{0} ± |εZ⟩_{1}/2 \), and \( |X⟩_{+−} = |εX⟩_{0} ± |εX⟩_{1}/2 \). The \( Z \) and \( X \) indicate the basis whose eigenstates remain unchanged under Eve’s action. The states \( |εZ⟩_{0} \) and \( |εX⟩_{+−} \) are normalized and non orthogonal: \( \langle ε_{0,1}|ε_{1,0}⟩ = \cos φ_{X} \) and \( \langle ε_{+−}|ε_{−+}⟩ = \cos φ_{Z}, \) with \( φ_{X, Z} \in [0, π/2] \). Analogous expressions hold for backward propagation with \( η \)'s ancillae in the place of \( ε \)'s. If Eve chooses the Z-attack, Eqs. 1, she introduces no noise on the channel when Bob prepares the eigenstates of the Z-basis but creates disturbance in the conjugate basis X; the same argument applies for the X-attack, Eqs. 2. The absence of a public basis revelation in the LM05 protocol prevents Eve from always choosing the best attack strategy between the \( Z \) and the \( X \)-attack. In the frame of the IIA attack an explicit functional relation among the three QBERs, \( q₁, q₂ \) and \( Q_{AB} \) can be found. Let us consider the expression of the forward QBER for the \( Z \) and \( X \)-state preparation, \( q_{1z} \) and \( q_{1x} \) respectively, as functions of the angles \( φ_{Z} \) and \( φ_{X} \).
\(\phi_x^Z\) chosen by Eve on the forward path:

\[
q_{1z} = (1 - \cos \phi_x^Z)/2 \quad q_{1z} = (1 - \cos \phi_y^Z)/2 \tag{3}
\]

Through these relations Alice and Bob can guess Eve’s angles \(\phi_x^Z, \phi_y^Z\) from the measured quantities \(q_{1z}, q_{1x}\), respectively. Analogous results (with angles \(\phi_x^Z, \phi_y^Z\)) hold for the partial QBERs \(q_{2z}\) and \(q_{2x}\) of the backward channel. The expression of the third QBER \(Q_{AB}\) can be derived as the average probability that Alice and Bob find an error on the total two-way channel in the EM:

\[
Q_{ABi} = q_{1i} + q_{2i} - 2q_{1i}q_{2i} \quad i = (x, z) \tag{4}
\]

This relation is suitable for direct verification since the QBERs on the left side and those on the right side are measured through independent processes, i.e. respectively during EM and CM.

To simulate the presence of an eavesdropper we must control the noise on the channels to generate the same effect caused by Eve’s action described in Eqs.(1) and (2). Consider the following unitary transformation:

\[
U_{\phi_x^Z}^{Z_{\phi_y^Z}} = \cos \phi_x^Z \mathbf{I} + i \sin \phi_y^Z \mathbf{Z}, \tag{5}
\]

where \(\phi_x^Z\) is the angle defined for the Eve’s Z-attack. Following the action of \(U_{\phi_x^Z}^{Z_{\phi_y^Z}}\) on the input states of the X-basis, we find that they are flipped with probability \(\sin^2(\phi_x^Z/2) = (1 - \cos \phi_x^Z)/2\), equal to the expression of \(q_{1z}\) in Eq.(4). Therefore the unitary transformation \(U_{\phi_x^Z}^{Z_{\phi_y^Z}}\) determines on the forward channel the same effect as Eve’s attack. An analogous result is true for the backward path. To evaluate the total QBER \(Q_{AB}\), we must consider the transformations on the X-states for the forward and backward channel, \(U_{\phi_x^Z}^{Z_{\phi_y^Z}}\) and \(U_{\phi_x^Z}^{Z_{\phi_y^Z}}\), and both Alice’s encoding operations, \(\mathbf{I}\) and \(i \mathbf{Y}\). In this way we find the two expression: \(Q_{ABx}^Z(I) = \sin^2(\phi_x^Z/2 + \phi_y^Z/2)\) and \(Q_{ABx}^Z(iY) = \sin^2(\phi_x^Z/2 - \phi_y^Z/2)\). These quantities depend on Alice’s transformation, but if we take the average between them we find the expression \(Q_{ABx}^Z = (1 - \cos \phi_x^Z \cos \phi_y^Z)/2\), exactly equal to Eq.(4) after expressing the partial QBERs through Eqs. (6).

The simulated eavesdropping described by Eq. (6) is realized via the two polarization controlling pads on the fibers connecting Alice and Bob. The Z-attack is achieved aligning the pads so that the Z-states remain almost undisturbed during the propagation. The X-attack is obtained in a similar fashion. The presence of undesired contributions due to \(\mathbf{Y}, \mathbf{X}\) operators during the simulated Z-attack are taken into account through a parameter \(\Delta\), which quantifies the distance from a perfect realization of the unitary transformation of Eq. (6). Once the two fibers are aligned, we measured the three QBERs \(q_1, q_2\) and \(Q_{AB}\). A second parameter \(\xi\) has been introduced to account for the background noise in the detection. In Fig.2 we plotted \(Q_{AB}^m\) vs \(Q_{AB}^x\), i.e. the averages of the quantities respectively present on the left and on the right side of Eq.(4) over all the state preparations.

![Image](https://via.placeholder.com/150)

**FIG. 2:** Plot of \(Q_{AB}^m\) vs \(Q_{AB}^x\), as defined in the text. The experimental points along with their statistical errors are reported as diamonds. The dashed-line represents the relation given in Eq.(1). The solid-line is drawn by setting the parameters, introduced in the text, \(\Delta = 0.05\) and \(\xi = 0.03\). Results for a uniform random distribution of these parameters in the ranges \([0, 0.03]\) and \([0, 0.06]\), respectively, are reported as a contour plot for \(N_t = 5 \times 10^3\) simulated trials. The contour plot represents the frequency of the trials in the bins, equally spaced with area \(0.005 \times 0.005\) normalized to the maximum.

The dashed-line represents the linear relation given by Eq.(6). The slightly sloped band is the result of a numerical simulation with values for the parameters \(\Delta\) and \(\xi\) reported in caption of Fig. 2.

In order to prove the security of our setup we must compare the information shared by Alice and Bob with the one possessed by Eve. It is known [24] that a secret key can be safely distilled with unidirectional classical communication if the condition \(I_{AE} \geq \min[I_{AE}, I_{BE}]\) is accomplished. The average Alice-Bob mutual information is given by:

\[
\bar{I}_{AB} = (I_{ABx} + I_{ABz})/2 \tag{6}
\]

where \(I_{ABi} = 1 - h(Q_{ABi})\) with \(i = (x, z)\), and \(h(x) = x \log_2(x) + (1-x) \log_2(1-x)\) is the binary entropy. To evaluate \(I_{AE}\) we need an estimate of \(Q_{AEZ}\), defined as the error rate between Eve’s guesses on Alice’s encoding and Alice’s real encoding. For Eve’s Z-attack, Eq.(4), \(Q_{AEZ}\) reads:

\[
Q_{AEZ} = \frac{1}{2} - 2\sqrt{q_{1z}q_{2z}(1 - q_{1z})(1 - q_{2z})}. \tag{7}
\]

A similar result holds for \(Q_{AEX}\). We note that these two quantities are independent of initial basis preparation, and depend only on non-orthogonality of Eve’s ancillae. The average Alice-Eve mutual information is then

\[
\bar{I}_{AE} = (I_{AEZ} + I_{AEX})/2, \tag{8}
\]

where \(I_{AEi} = 1 - h(Q_{AEi})\). An expression for the mutual QBER between Bob and Eve, \(Q_{BE}^m\), can be derived in terms of \(Q_{AB}^m\) and \(Q_{AE}^m\) using the relation \(Q_{BE}^m = Q_{AB}^m + Q_{AE}^m - 2Q_{AB}^m \cdot Q_{AE}^m\) [11]. The average mutual information
The mutual information with $\Delta = 0.015$ and $\xi = 0.03$. Results of $N_i = 5 \times 10^6$ simulated trials with the same parameters used for Fig.2 are reported. The asymmetric imperfection of the two channels cause the gray area below the lines $I_{AE}, I_{BE}$.

between Bob and Eve is then expressed as:

$$T_{BE} = \langle I_{BE}^Z + I_{BE}^X \rangle / 4,$$  

where $I_{BEi}^{(Z,X)} = 1 - \hat{h}(Q_{BEi}^{Z,X})$, with $i = (x, z)$.

In Fig.4 are plotted the curves of mutual information $I$ according to Eqs. (6), (8) and (9) as function of the quantity $Q_{AB}^s$, already defined. We report in the same figure experimental, numerical and theoretical values. The solid lines represent our best fit of the experimental data. It is worthy of note the almost perfect intersection of these lines for $I_{AB}, I_{AE}$ and the theoretical (dashed) line at $Q_{AB}^s \approx 19\%$, corresponding to the \(\approx 23\%\) of detection probability in Ref. [11]. Furthermore the curve for $I_{BE}$ is always below $I_{AB}$, implying the security of the scheme against IIA regardless of the noise on the channel.

In conclusion we reported on the experimental test of the two-way deterministic protocol for quantum communication LM05. We modulated the noise on the channel in such a way as to simulate the disturbance introduced by Eve’s IIA. By means of independent measurements of the various involved QBERs we proved the soundness of Eq. (11) and rated the quality of our setup. With a subsequent measure we estimated the mutual information between Alice, Bob and Eve. Although we did not perform a direct, contextual, transmission of a string of bits we believe that the good agreement between experimental data and theoretical predictions presented in this work witnesses its potential feasibility.

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