AUTOMORPHISMS OF HYPERKÄHLER MANIFOLDS IN THE VIEW OF TOPOLOGICAL ENTROPY

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Dedicated to Professor Igor Dolgachev on the occasion of his sixtieth birthday

Abstract. First we show that any group of automorphisms of null-entropy of a projective hyperkähler manifold $M$ is almost abelian of rank at most $\rho(M) - 2$. We then characterize automorphisms of a K3 surface with null-entropy and those with positive entropy in algebro-geometric terms. We also give an example of a group of automorphisms which is not almost abelian in each dimension.

1. Introduction - Background and main results

The aim of this note is to study groups of automorphisms of a hyperkähler manifold from two points of view: topological entropy and how close to (or how far from) abelian groups. Our main results are Theorems (1.3) and (2.1).

1. Let $M$ be a compact Kähler manifold. We denote the biholomorphic automorphism group of $M$ by $\text{Aut}(M)$. By the fundamental work of Yomdin, Gromov and Friedland, the topological entropy $e(g)$ of an automorphism $g \in \text{Aut}(M)$ can be computed in three different ways: topological, differential-geometrical, and cohomological (see [Yo], [Gr], [Fr], also [DS2]). In this note, we employ the cohomological one as its definition:

$$e(g) := \log \delta(g).$$

Here $\delta(g)$ is the spectral radius of the action of $g$ on the cohomology ring $H^*(M, \mathbb{C})$, i.e. the maximum of the absolute values of eigenvalues of the $\mathbb{C}$-linear extension of $g^*|H^*(M, \mathbb{Z})$. One has $e(g) \geq 0$, and $e(g) = 0$ iff the eigenvalues of $g^*$ are on the unit circle $S^1 := \{ z \in \mathbb{C} | |z| = 1 \}$. Furthermore, by Dinh and Sibony [DS], $e(g) > 0$ iff some eigenvalues of $g^*|H^{1,1}(M)$ are outside the unit circle $S^1$. A subgroup $G$ of $\text{Aut}(M)$ is said to be of null-entropy (resp. of positive entropy) if $e(g) = 0$ for $\forall g \in G$ (resp. $e(g) > 0$ for $\exists g \in G$).

2. Next we recall a few facts about hyperkähler manifolds. K3 surfaces are nothing but 2-dimensional hyperkähler manifolds. All what we need is reviewed in [Og2, Section 2]:

Definition 1.1. A hyperkähler manifold is a compact complex simply-connected Kähler manifold $M$ admitting an everywhere non-degenerate global holomorphic 2-form $\omega_M$ such that $H^0(M, \Omega^2_M) = \mathbb{C}\omega_M$.

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Let $M$ be a hyperkähler manifold. Then the second cohomology group $H^2(M, \mathbb{Z})$ admits a natural $\mathbb{Z}$-valued symmetric bilinear form of signature $(3, 0, b_2(M) - 3)$, called Beauville-Bogomolov-Fujiki’s form (BF-form for short). BF-form is exactly the cup-product when $M$ is a K3 surface. The signature of the Néron-Severi group $NS(M)$ w.r.t. BF-form is either $(1, 0, \rho(M) - 1)$, $(0, 1, \rho(M) - 1)$, or $(0, 0, \rho(M))$. Here $\rho(M)$ is the Picard number of $M$. We call these three cases hyperbolic, parabolic, and elliptic respectively. Due to Huybrechts \[Hu\], $M$ is projective iff $NS(M)$ is hyperbolic. Note also that, in dimension 2, $NS(M)$ is hyperbolic, parabolic, elliptic iff the algebraic dimension $a(M)$ is 2, 1, 0, respectively (see eg. [BPV]).

In our previous note, we have shown the following:

**Theorem 1.2.** [Og2] The bimeromorphic automorphism group $\text{Bir } (M)$ of a non-projective hyperkähler manifold $M$ is an almost abelian group of finite rank. More precisely, $\text{Bir } (M)$ is an almost abelian group of rank at most $\rho(M) - 1$ when $NS(M)$ is elliptic (resp. parabolic). We call a group $G$ almost abelian (resp. almost abelian of finite rank $r$) if there are a normal subgroup $G(0)$ of $G$ of finite index, a finite group $K$ and an abelian group $A$ (resp. $A = \mathbb{Z}^r$) which fit in the exact sequence

$$1 \rightarrow K \rightarrow G(0) \rightarrow A \rightarrow 0.$$ 

The rank $r$ is well-defined, and invariant under replacing $G$ by a subgroup $H$ of finite index and by a quotient group $Q$ of $G$ by a finite normal subgroup (cf. [Og2, Section 9]). It is then clear that a subgroup of an almost abelian group of rank $r$ is an almost abelian group of rank at most $r$.

3. Our main result is the following:

**Theorem 1.3.** Let $M$ be a projective hyperkähler manifold. Let $G < \text{Aut } (M)$ (resp. $G < \text{Bir } (M)$). Assume that $G$ is of null-entropy (resp. of null-entropy at $H^2$-level, that is, the eigenvalues of $g^*|H^2(M, \mathbb{Z})$ are on the unit circle $S^1$). Then $G$ is an almost abelian group of rank at most $\rho(M) - 2$. Moreover, this estimate is optimal in $\dim M = 2$.

The key ingredient is Theorem (2.1), a result of linear algebra, whose source has been back to an important observation of Burnside [Bu1], and Lie’s Theorem (cf. [Hm]). We shall prove Theorem (1.3) in Section 3.

4. Next, as an application of Theorem (1.3), we shall reproduce the following algebro-geometric characterization of positivity of entropy of automorphisms of a K3 surface. We should notice that this result is essentially known and can be read from works of Cantat [Ca1, 2]:

**Theorem 1.4.** Let $M$ be a (not necessarily projective) K3 surface, $G < \text{Aut } (M)$, and $g \in \text{Aut } (M)$. Then:

1. $G$ is of null-entropy iff either $G$ is finite or $G$ makes an elliptic fibration $\varphi : M \rightarrow \mathbb{P}^1$ stable. In particular, if $G$ is of null-entropy, then $G$ has no Zariski dense orbit, and conversely, if $G$ makes an elliptic fibration stable, then $G$ is almost abelian of finite rank.
2. $\epsilon(g) > 0$ iff $g$ has a Zariski dense orbit, i.e. there is a point $x \in M$ such that the set $\{g^n(x) | n \in \mathbb{Z}\}$ is Zariski dense in $M$. 
Theorem (1.4) is proved in Sections 3 and 4. This theorem is also motivated by earlier observations of [Sn] and by the following question posed by McMullen:

**Question 1.5.** [Mc] Does a K3 automorphism $g$ have a dense orbit (in the Euclidean topology) when a K3 surface is projective and $e(g) > 0$?

In the same paper [Mc], he constructed a K3 surface $M$ of $\rho(M) = 0$ having an automorphism $g$ with Siegel disk. For this $(M, g)$, one knows that $e(g) > 0$, $g$ has a Zariski dense orbit but no dense orbit in the Euclidean topology, and that $\text{Aut}(M) \simeq \mathbb{Z}$ (See [ibid] and also [Og2]).

5. So far, all groups in consideration are almost abelian. As a sort of counterparts, we shall show the following Theorem in Section 5:

**Theorem 1.6.** (1) Let $M$ be a K3 surface admitting two different Jacobian fibrations of positive Mordell-Weil rank. Then $\text{Aut}(M)$ is not almost abelian (and hence is of positive entropy). This happens, for instance, for a K3 surface $M$ of maximum Picard number $\rho(M) = 20$.

(2) In each dimension $2m$, there is a projective hyperkähler manifold $M$ whose $\text{Aut}(M)$ is not almost abelian (and hence is of positive entropy).

Theorem (1.6)(1) is a part of refinement of a result [Ca2] (see also [CF] for a generalization in foliated case) and also a slight generalization of a result of Shioda and Inose [SI]: $\text{Aut}(M)$ is an infinite group for a K3 surface $M$ with $\rho(M) = 20$. In [ibid], this has been shown by finding a Jacobian fibration of positive Mordell-Weil rank.

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2. Group of isometries of null-entropy of a hyperbolic lattice

The main result of this section is Theorem (2.1).

A lattice is a pair $L := (L, b)$ of a free abelian group $L \simeq \mathbb{Z}^r$ and a non-degenerate symmetric bilinear form

$$b : L \times L \rightarrow \mathbb{Z}.$$ 

A submodule $M$ (resp. an element $v \neq 0$) of $L$ is primitive if and only if $L/M$ (resp. $L/Zv$) is free.

A scalar extension $(L \otimes K, b \otimes id_K)$ of $(L, b)$ by $K$ is written as $(L_K, b_K)$. We often write $b_K(x, y)$, $b_K(x, x)$ ($x, y \in L_K$) simply as $(x, y)$, $(x^2)$.

The non-negative integer $r$ is called the rank of $L$. The signature of $L$ is the signature, i.e. the numbers of positive-, zero-, negative-eigenvalues, of a symmetric matrix associated to $b_R$. It is denoted by $\text{sgn} L$. The lattice $L$ is called hyperbolic (resp. parabolic, elliptic) if $\text{sgn} L$ is $(1, 0, r - 1)$ (resp. $(0, 1, r - 1)$, $(0, 0, r)$).

In what follows, $L$ is assumed to be a hyperbolic lattice.

The positive cone $\mathcal{P}(L)$ of $L$ is one of the two connected components of:
\[ \mathcal{P}'(L) := \{ x \in L \mid (x, x) > 0 \} . \]

The boundary (resp. the closure) of \( \mathcal{P}(L) \) is denoted by \( \partial \mathcal{P}(L) \) (resp. \( \overline{\mathcal{P}}(L) \)). Obviously, \( (x^2) = 0 \) if \( x \in \partial \mathcal{P}(L) \). Let \( x, x' \in \overline{\mathcal{P}}(L) \setminus \{0\} \). Then, by the Schwartz inequality, \( (x, x') \geq 0 \) and the equality holds exactly when \( x \) and \( x' \) are proportional boundary points.

We denote the group of isometries of \( L \) by:

\[ O(L) := \{ g \in \text{Isom}_{\text{group}}(L) \mid (gx, gy) = (x, y) \forall x, y \in L \} . \]

We have an index 2 subgroup:

\[ O(L)' := \{ g \in O(L) \mid g(\mathcal{P}(L)) = \mathcal{P}(L) \} . \]

Let \( g \in O(L)' \). The spectral value \( \delta(g) \) of \( g \) is the maximum of the absolute values of eigenvalues of \( g \mathcal{C} \). By abuse of notation, we call

\[ e(g) := \log \delta(g) \]

the entropy of \( g \). As we shall see in Proposition (2.2), \( e(g) \geq 0 \), and \( e(g) = 0 \) if and only if the eigenvalues of \( g \mathcal{C} \) lie on the unit circle \( S^1 := \{ z \in \mathbb{C} \mid |z| = 1 \} \). An element \( g \) of \( \text{GL}(r, \mathbb{C}) \) is called unipotent if all the eigenvalues are 1.

The aim of this section is to prove the following:

**Theorem 2.1.** Let \( L \) be a hyperbolic lattice of rank \( r \) and \( G \) be a subgroup of \( O(L)' \). Assume that \( G \) is an infinite group of null-entropy, i.e. \( |G| = \infty \) and that \( e(g) = 0 \) for all \( g \in G \). Then:

1. There is \( v \in \partial \mathcal{P}(L) \setminus \{0\} \) such that \( g(v) = v \) for all \( g \in G \). Moreover, the ray \( R_{>0}v \) in \( \partial \mathcal{P}(L) \) is unique and defined over \( \mathbb{Z} \). In other words, one can take unique such \( v \) which is primitive in \( L \).
2. There is a normal subgroup \( G^{(0)} \) of \( G \) such that \( [G : G^{(0)}] < \infty \) and \( G^{(0)} \) is isomorphic to a free abelian group of rank at most \( r - 2 \). In particular, \( G \) is almost abelian of rank at most \( r - 2 \).

We shall prove Theorem (2.1) dividing into several steps. In what follows \( G \) is as in Theorem (2.1).

**Proposition 2.2.** (1) The eigenvalues of \( g \in G \) lie on the unit circle \( S^1 \).

2. There is a positive integer \( n \) such that \( g^n \) is unipotent for all \( g \in G \).

3. There is \( g \in G \) such that \( \text{ord} \ g = \infty \).

**Proof.** Since \( g \in O(L) \) and \( L \) is non-degenerate, we have \( \det(g) = \pm 1 \). Let \( \alpha_i \) (\( 1 \leq i \leq r \)) be the eigenvalues of \( g \mathcal{C} \) (counted with multiplicities). Then

\[ 1 = |\det g| = \Pi_{i=1}^r |\alpha_i| . \]

Thus, \( e(g) \geq 0 \), and \( e(g) = 0 \) if and only if the eigenvalues of \( g \mathcal{C} \) lie on \( S^1 \). This proves (1).

Since \( g \) is defined over \( \mathbb{Z} \), the eigenvalues of \( g \mathcal{C} \) are all algebraic integers. Thus they are all roots of unity by (1) and by the Kronecker Theorem [Ta]. Since the eigen polynomial of \( g \) is of degree \( r \) and it is now the product of cyclotomic polynomials, the eigenvalues of \( g \) lie on:

\[ \cup_{d \mid r} \mu_d < S^1 . \]
Here \( \mu_d = \langle \zeta_d \rangle := \{ z \in \mathbb{C} \mid z^d = 1 \} \) and \( \varphi(d) := |\text{Gal}(\mathbb{Q}(\zeta_d)/\mathbb{Q})| \) is the Euler function. There are finitely many \( d \) with \( \varphi(d) \leq r \). Let \( n \) be their product. Then \( g^n \) is unipotent for all \( g \in G \). This proves (2).

Let us show (3). We have \( G < \text{GL}(L \mathbb{C}) \simeq \text{GL}(r, \mathbb{C}) \). Suppose to the contrary that \( G \) consists of elements of finite order. Then each \( g \in G \) is diagonalizable over \( L \mathbb{C} \). Let \( n \) be as in (2). Then, \( g^n = \text{id} \) for all \( g \in G \), i.e. \( G \) has a finite exponent \( n \). However, then \( |G| < \infty \) by the following theorem due to Burnside [Bu2] (based on [Bu1]), a contradiction:

**Theorem 2.3.** Any subgroup of \( \text{GL}(r, \mathbb{C}) \) of finite exponent is finite, i.e. \( |G| < \infty \).

\[ \square \]

**Remark 2.4.** After Theorem (2.3), Burnside asked if a group of finite exponent is finite or not (the Burnside problem). It is now known to be false in general even for finitely generated groups. The original Burnside problem has been now properly modified and completely solved by Zelmanov. (See for instance [Za] about Burnside problems.)

Set
\[
H := \{ g \in G \mid g \text{ is unipotent} \}.
\]

**Lemma 2.5.** \( H \) is a normal subgroup of \( G \). Moreover, \( H \) has an element of infinite order.

**Proof.** Let \( g \) be as in Proposition (2.2)(3) and \( n \) be as in Proposition (2.2)(1). Then \( g^n \) is an element of \( H \) of infinite order. The only non-trivial part is the closedness of \( H \) under the product, that is, if \( a, b \in H \) then \( ab \in H \). Since \( a^l b^m \in G \), this follows from the next slightly more general result together with Proposition (2.2)(1).

**Proposition 2.6.**

1. Let \( z_i \in S^1 \) (\( 1 \leq i \leq r \)). Then
   \[
   \sum_{i=1}^{r} z_i \leq r ,
   \]
   and
   \[
   \sum_{i=1}^{r} z_i = r \iff z_i = 1 \forall i .
   \]

2. Let \( A, B \in \text{GL}(r, \mathbb{C}) \) such that \( A \) and \( B \) are unipotent and such that the eigenvalues of \( A^l B^m \) lie on \( S^1 \) for all \( l, m \in \mathbb{Z}_{\geq 0} \). Then \( A^l B^m \) is unipotent for all \( l, m \in \mathbb{Z}_{\geq 0} \).

**Proof.** The statement (1) follows from the triangle inequality. Let us show (2). We may assume that \( A \) is of the Jordan form
\[
A := J(r_1, 1) \oplus \cdots \oplus J(r_k, 1) .
\]

We fix \( m \) and decompose \( B^m \) into blocks as:
\[
B^m = (B_{ij}) , \ B_{ij} \in M(r_i, r_j, \mathbb{C}) .
\]

We have
\[
\text{tr} A^l B^m = \sum_{i=1}^{k} \text{tr} J(r_1, 1)^l B_{i} .
\]
Using an explicit form of $J(r_i,1)^l$ and calculating its product with $B_{ii}$, we obtain:

$$\text{tr} A^l B^m = b_s l^s + b_{s-1} l^{s-1} + \cdots + b_1 l + \text{tr} B^m .$$

Here $s = \max \{r_i\} - 1$ and $b_t$ are constant being independent of $l$.

On the other hand, by the assumption and (1), we have

$$|\text{tr} A^l B^m| \leq r ,$$

for all $l$. Thus, by varying $l$ larger and larger, we have

$$b_t = 0 \text{ for } t = s, s-1, \cdots, 1$$

inductively. This implies

$$\text{tr} A^l B^m = \text{tr} B^m ,$$

for all $l$. Since $B$ is unipotent, so is $B^m$. Thus, $\text{tr} B^m = r$ and therefore

$$\text{tr} A^l B^m = r .$$

Since the eigenvalues of $A^l B^m$ lie on $S^1$, this implies the result. \hfill \Box

**Corollary 2.7.** $H < T(r, \mathbb{Q})$ under suitable basis $\langle u_i \rangle^r_{i=1}$ of $L_{\mathbb{Q}}$. Here $T(r, \mathbb{Q})$ is the subgroup of $\text{GL}(r, \mathbb{Q})$ consisting of the upper triangle unipotent matrices.

**Proof.** By Lemma (2.5), $H$ is a unipotent subgroup of $\text{GL}(r, \mathbb{Q})$. Thus, the result follows from Lie’s Theorem (see for instance [Hm, 17.5 Theorem]). Here is a small remark. Lie’s Theorem in [Hm] is formulated and proved over algebraically closed field of characteristic 0. So, precisely speaking, we have that $H < T(r, \mathbb{C})$ under suitable basis $\langle u_i \rangle^r_{i=1}$ of $L_{\mathbb{C}}$. On the other hand, $L_{\mathbb{Q}}$ and all the elements of $H$ are defined over $\mathbb{Q}$. Thus, such basis $\langle u_i \rangle^r_{i=1}$ of $L_{\mathbb{C}}$ are nothing but solutions of a system of linear equations with rational coefficients in the range $\det (u_i) \neq 0$. Thus, existence of such basis over $\mathbb{C}$ implies the existence of the desired basis over $\mathbb{Q}$ as claimed. \hfill \Box

Let $G$ and $H$ be as above. So far, we did not yet use the fact that $G$ and $H$ are subgroups of $O(L)$. From now, we shall use this fact.

**Lemma 2.8.** (1) Let $N$ be a subgroup of $O(L)$. Assume that there is an element $x$ of $L$ such that $(x^2) > 0$ and $a(x) = x$ for all $a \in N$. Then $|N| < \infty$. In particular, there is no such $x$ for $H$.

(2) There is a unique ray $\mathbb{R}_{>0} v (\neq 0)$ in $\partial L$ such that $a(v) = v$ for all $a \in H$.

Moreover, the ray $\mathbb{R}_{>0} v$ is defined over $\mathbb{Z}$, i.e. one can take unique such $v$ which is primitive in $L$.

(3) Let $v$ be as in (2). Then $b(v) = v$ for all $b \in G$.

**Proof.** Let us first show (1). By assumption, $N$ can be naturally embedded into $O(x_L^2)$. Here $x_L^2$ is the orthogonal complement of $x$ in $L$. Since $L$ is hyperbolic and $(x^2) > 0$, the lattice $x_L^2$ is of negative definite. Thus $O(x_L^2)$ is finite. This shows (1).

Let us show (2). By (1) and by $|H| = \infty$, we have $r \geq 2$. First, we find $v \in \partial L \cap L \setminus \{0\}$ such that $a(v) = v$ for all $a \in H$ by the induction on $r \geq 2$.

By Corollary (2.7), there is $u \in L \setminus \{0\}$ such that $a(u) = u$ for all $a \in H$. $H$ is then naturally embedded into $O(u_L^2)$ if $(u^2) \neq 0$. 

By (1), we have \((u^2) \leq 0\). If \((u^2) = 0\), then we are done. If \((u^2) < 0\), then \(u^\perp_L\) is of signature \((1, 0, r - 2)\). If in addition \(r = 2\), then \(u^\perp_L\) is of positive definite and \(\text{O}(u^\perp_L)\) is finite, a contradiction. Hence \((v^2) = 0\) when \(r = 2\) and we are done when \(r = 2\). If \(r > 2\), then by the induction, we can find a desired \(v\) in \(u^\perp_L\) and we are done.

If necessarily, by replacing \(v\) by \(\pm v/m\), we have that \(R_{\geq \alpha}v \subset \partial \mathcal{P}(L)\) and \(v \in L\) is primitive as well.

Next, we shall show the uniqueness of \(R_{\geq \alpha}v\). Suppose to the contrary that there is a ray \((0 \neq) R_{\geq \alpha}u \subset \partial \mathcal{P}(L)\) s.t. \(a(u) = u\) for all \(a \in H\) and \(R_{\geq \alpha}u \neq R_{\geq \alpha}v\). Then the common eigenspace of eigenvalue 1

\[
V := \bigcap_{h \in H} V(h, 1)
\]

is a linear subspace of \(L_C\) of \(\dim V \geq 2\). Since each \(V(h, 1)\) is defined over \(Q\), so is \(V\). Thus, we may find such \(u \in L\). So, from the first, we may assume that \(u \in L\). We have \(((v + u)^2) > 0\) by the Schwartz inequality. We have also \(a(v + u) = v + u\) for all \(a \in H\). However, we would then have \(|H| < \infty\) by (1), a contradiction. Now the proof of (2) is completed.

Finally, we shall show (3). Let \(b \in G\). Put \(b(v) = u\). Let \(a \in H\). Since \(H\) is a normal subgroup of \(G\), there is \(a' \in H\) such that \(ab = ba'\). Then

\[
a(u) = a(b(v)) = b(a'(v)) = b(v) = u.
\]

Verying \(a\) in \(H\) and using (2), we find that \(u \in R_{\geq \alpha}v\), i.e. \(b(v) = \alpha v\) for some \(\alpha > 0\). Since the eigenvalues of \(b\) are on \(S^1\), we have \(\alpha = 1\).

\[
\Box
\]

**Proposition 2.9.** Let \(L\) be a hyperbolic lattice of rank \(r\) and \(N\) be a subgroup of \(O(L)\). Assume that there is a primitive element \(v \in \partial \mathcal{P}(L) \cap L \setminus \{0\}\) such that \(h(v) = v\) for all \(h \in N\). Let

\[
\overline{L} := v^\perp_L / \mathbb{Z}v.
\]

Then \(\overline{L}\) is elliptic, of rank \(r - 2\), and the isometry \(N\) on \(L\) naturally descends to the isometry of \(\overline{L}\), say \(h \mapsto \overline{h}\). Set

\[
N^0 := \ker (N \rightarrow O(\overline{L}) \times \{\pm 1\} : h \mapsto (\overline{h}, \det h)).
\]

Then \(N^0\) is of finite index in \(N\) and \(N^0\) is a free abelian group of rank at most \(r - 2\). Moreover \(N\) is of null-entropy.

**Proof.** The first part of the proposition is clear. We shall now show the last two assertions. Since \(\overline{L}\) is elliptic, the group \(O(\overline{L})\) is finite. Hence \(|N : N^0| \leq 2 \cdot |O(\overline{L})| < \infty\). Choose an integral basis \(\langle \overline{m}_i \rangle_{i=1}^{r-2}\) of \(\overline{L}\). Let \(u_i \in v^\perp_L\) be a lift of \(\overline{m}_i\). Then

\[
\langle v, u_i \rangle_{i=1}^{r-1}
\]

forms an integral basis of \(v^\perp_L\). Since \(v^\perp_L\) is primitive in \(L\), there is an element \(w \in L\) such that

\[
\langle v , u_i \rangle_{1 \leq i \leq r - 2} , w\rangle
\]

forms an integral basis of \(L\).

Let \(h \in N^0\). Using \(h(\overline{m}_i) = \overline{m}_i\), we calculate

\[
h(v) = v , h(u_i) = u_i + \alpha_i(h)v ,
\]
where $\alpha_i(h)$ $(1 \leq i \leq r - 2)$ are integers uniquely determined by $h$. Since $\det h = 1$, it follows that $h(w)$ is of the form:

$$h(w) = w + \beta(h)v + \sum_{i=1}^{r-2} \gamma_i(h)u_i,$$

where $\beta(h)$ and $\gamma_i(h)$ are also integers uniquely determined by $h$.

This already shows that $N^0$ is unipotent. Then $N$ is of null-entropy, because $[N : N^0] < \infty$, as well.

Let us show that $N^0$ is an abelian group of rank at most $r - 2$. Varying $h$ in $N^0$, we can define the map $\varphi$ by:

$$\varphi : N^0 \rightarrow \mathbb{Z}^{r-2} ; \ h \mapsto (\alpha_i(h))_{i=1}^{r-2}.$$

Now the next claim completes the proof of Proposition (2.9).

**Claim 2.10.** $\varphi$ is an injective group homomorphism.

**Proof.** Let $h, h' \in N^0$. Then by the formula above, we calculate:

$$h'h(u_i) = h'(u_i + \alpha_i(h)v) = h'(u_i) + \alpha_i(h)v = u_i + (\alpha_i(h') + \alpha_i(h))v.$$

Thus $\alpha_i(h'h) = \alpha_i(h') + \alpha_i(h)$ and $\varphi$ is a group homomorphism.

Let us show that $\varphi$ is injective. Let $h \in \text{Ker} \varphi$. Then,

$$h(v) = v, \ h(u_i) = u_i, \ h(w) = w + \beta(h)v + \sum_{i=1}^{r-2} \gamma_i(h)u_i.$$

It suffices to show that $h(w) = w$. Using $(v, u_i) = 0$ and $(h(x), h(y)) = (x, y)$, we calculate:

$$(w, u_i) = (h(w), h(u_i)) = (w, u_i) + \sum_{i=1}^{r-2} \gamma_j(h)(u_j, u_i),$$

that is,

$$A(\gamma_j(h))_{j=1}^{r-2} = (0)^{r-2}_{j=1}.$$

Here $A := ((u_i, u_j)) \in \text{M}(r, \mathbb{Z})$. Since $(u_i, u_j) = (\overline{u}_i, \overline{u}_j)$ and $\overline{L}$ is elliptic, we have $\det A \neq 0$. Thus, $(\gamma_j(h))_{j=1}^{r-2} = (0)^{r-2}_{j=1}$, and therefore

$$h(w) = w + \beta(h)v.$$

By using $(v^2) = 0$, we calculate

$$(v^2) = (h(w)^2) = (w^2) + 2\beta(h)(w, v).$$

Thus $\beta(h)(w, v) = 0$. Since $(v, v) = (u_i, v) = 0$ and $\overline{L}$ is non-degenerate, we have $(w, v) \neq 0$. Hence $\beta(h) = 0$, and therefore $h(w) = w$. □

Now Theorem (2.1) follows from Proposition (2.9) applied for $N = G$ and $v \in \partial \mathcal{P}(L) \cap L \setminus \{0\}$ in Lemma (2.8).
In this section, we prove Theorem (1.3) and Theorem (1.4)(1).

Let us show Theorem (1.3). Let $M$ be a projective hyperkähler manifold and $G < \text{Aut}(M)$ (resp. $G < \text{Bir}(M)$) be a subgroup of null-entropy (resp. of null-entropy at $\mathcal{H}^2$-level). Set $H := \text{Im}(r_{NS} : G \to O(NS(M)))$. Since $M$ is projective, $|\text{Ker} r_{NS}| < \infty$ by [Og2, Corollary 2.7]. Thus $G$ is almost abelian of rank, say $s$, iff so is $H$. However, $H$ is almost abelian of rank $\leq \rho(M) - 2$ by Theorem (2.1).

Next we shall show the optimality of the estimate. There is a Jacobian K3 surface $\varphi : M \to \mathbf{P}^1$ s.t. $\rho(M) = 20$ and the Mordell-Weil rank of $\varphi$ is 18 (See e.g. [Co], [Ny], [Og1]). As we shall show in the next Section, the action of a Mordell-Weil group is of null-entropy. This completes the proof of Theorem (1.3).

Remark 3.1. Let $M$ be a non-projective hyperkähler manifold and $G < \text{Bir}(M)$. Then, by [Og2] (the proof of Theorem (1.2) there), we know:

(1) If $NS(M)$ is elliptic, then $G$ is of null-entropy at $\mathcal{H}^2$-level iff $|G| < \infty$. Indeed, if $g \in G$ is of null-entropy at $\mathcal{H}^2$-level, then $g^*T(M) = id$ by [Og2, Theorem 2.4]. Moreover, $|\text{Im}(r_{NS} : G \to O(NS(M)))| < \infty$. Thus, $\text{Im}(r : G \to O(H^2(M, \mathbf{Z})))$ is finite. Then $|G| < \infty$ by [Og2, Theorem 2.3]. The other direction is clear.

(2) If $NS(M)$ is parabolic, then $G$ is always of null-entropy at $\mathcal{H}^2$-level. Indeed, by [Og2, Corollary 2.7], it suffices to show that $N := \text{Im}(r_{NS} : G \to O(NS(M)))$ is of null-entropy (on $NS(M)$). However, in the proof of [Og2, Proposition (5.1)], we find a finite index normal unipotent subgroup $N^{(0)}$ of $N$. Thus, $N$ is of null-entropy.

Let us show Theorem (1.4)(1). Let $a(M)$ be the algebraic dimension of a K3 surface $M$. We argue by dividing into three cases where $NS(M)$ is elliptic, parabolic, and hyperbolic.

If $NS(M)$ is elliptic, then the result follows from Remark (3.1)(1) above.

If $NS(M)$ is parabolic, then a subgroup $G$ of $\text{Aut}(M)$ is always of null-entropy by Remark (3.1)(2) above. On the other hand, when $NS(M)$ is parabolic, the algebraic dimension of $M$ is 1 (by the classification theory of surfaces). The algebraic reduction map gives rise to an elliptic fibration $a : M \to \mathbf{P}^1$ of $M$. This fibration is stable under $\text{Aut}(M)$, and therefore so is under $G$. This completes the proof when $NS(M)$ is parabolic.

Let us consider the case where $NS(M)$ is hyperbolic. In this case $M$ is projective.

First we show "only if" part. As before, set $H := \text{Im}(r_{NS} : G \to O(NS(M)))$. We may assume that $|G| = \infty$. Then so is $H$ by [Og2, Corollary 2.7]. Applying Theorem (2.1) for $H$, we find a primitive element $v \in \partial \mathcal{P}(M) \cap NS(M) \setminus \{0\}$ s.t. $h^*(v) = v$ for all $h \in H$. Here the positive cone $\mathcal{P}(M) := \mathcal{P}(NS(M))$ is taken to be the component which contains an ample class of $M$.

By the Riemann-Roch theorem, $v$ is represented by a non-zero effective divisor, say $D$, with $h^0(\mathcal{O}_M(D)) \geq 2$. Decompose $|D|$ into the movable part and fixed part:

$$|D| = |E| + B,$$

Then $(E^2) \geq 0$ and the class $|E|$ is $H$-stable. Thus $(E^2) \leq 0$ by Lemma (2.8)(1). Therefore $(E^2) = 0$. Hence $|E|$ is free and defines an elliptic fibration

$$\varphi : M \to \mathbf{P}^1$$
on $M$. This is $G$-stable. Moreover, $\varphi$ is the unique elliptic fibration stable under $G$. Otherwise, there is another class $[C] \in \partial P(M) \cap NS(M) \setminus \{0\}$ such that $[C] \not\in R_{>0}[E]$ and $H$-stable, a contradiction to Lemma (2.8)(2).

Next, we shall show "if part". The result is clear if $G$ is finite. So, we may assume that $|G| = \infty$ and the existence of a $G$-stable elliptic fibration $\varphi : M \to \mathbf{P}^1$.

Let $C$ be a fiber of $\varphi$. Then $[C] \in NS(M) \cap \partial P(M) \setminus \{0\}$. $[C]$ is $G$-invariant as well. Thus, the result follows from Proposition (2.9) and [Og2, Corollary 2.7].

Let us finally show the last two statements in Theorem (1.4)(1). The second one follows from the contra-position of Theorem (1.3). Let us show the first one. The result is clear if $|G| < \infty$. So, we may assume that $|G| = \infty$. Then $G$ makes an elliptic fibration $\varphi : M \to \mathbf{P}^1$ stable. Recall that any elliptic fibration on a K3 surface admits at least three singular fibers. (see eg. [Ca2]; This follows from $\chi_{top}(M) = 24$ and $\rho(M) \geq 20$. See also [VZ] for a more general account.) Thus, $\text{Im}(G \to \text{Aut}(\mathbf{P}^1))$ is finite. Therefore, each orbit lies in finitely many fibers. This completes the proof of Theorem (1.4)(1).

4. AUTOMORPHISM OF A PROJECTIVE K3 SURFACE OF POSITIVE ENTROPY

In this section, we shall show Theorem (1.4)(2).

Let $M$ be a K3 surface and $g \in \text{Aut}(M)$. If $\epsilon(g) = 0$, then $\langle g \rangle$ is of null-entropy. Thus $\langle g \rangle$ has no Zariski dense orbit by Theorem (1.4)(1). This shows "if part" of Theorem (1.4)(2). Let us show "only if part" of Theorem 1.6(2). Assuming $\epsilon(g) > 0$, we want to find a point $x \in M$ such that $\{g^n(x) | n \in \mathbf{Z}\}$ is Zariski dense in $M$. Note that $\text{ord} \ g = \infty \ y \epsilon(g) > 0$.

Let $\mathcal{F} = \bigcup_{n \in \mathbf{Z}\setminus\{0\}} \mathcal{F}_n$, where

$$\mathcal{F}_n := \{ y \in M | g^n(y) = y \} ,$$

and

$$C := \bigcup_{C \subset M, C \not\cong \mathbf{P}^1} C .$$

Since $\text{ord} \ g = \infty$, each $\mathcal{F}_n$ is a proper closed analytic subset of $M$. Since $M$ is not uniruled, $\mathcal{C}$ is also at most countable union of $\mathbf{P}^1$. Thus, $\mathcal{F} \cup \mathcal{C}$ is a countable union of proper closed analytic subsets of $M$. Hence

$$M \not= \mathcal{F} \cup \mathcal{C} .$$

Choose $x \in M \setminus (\mathcal{F} \cup \mathcal{C})$ and set $\mathcal{O}(x) := \{g^n(x) | n \in \mathbf{Z}\}$.

The next claim will complete the proof.

Claim 4.1. $\mathcal{O}(x)$ is Zariski dense in $M$.

Proof. Let $S$ be the Zariski closure of $\mathcal{O}(x)$ in $M$. Suppose to the contrary that $S$ is a proper subset of $M$. Since $\mathcal{O}(x)$ is an infinite set by $x \not\in \mathcal{F}$, the set $S$ is decomposed into non-empty finitely many complete irreducible curves and (possibly empty) finite set of closed points. Let $C$ be a 1-dimensional irreducible component of $S$. Then, there is $N$ such that $g^{Nk}(C) = C$ for all $k \in \mathbf{Z}$. Note that $g^n(x) \in C$ for some $n$. (Indeed, otherwise, $S \setminus (C \cap (S \setminus C))$ would be a smaller closed subset containing $\mathcal{O}(x)$.) By the choice of $x$, we have $x \in g^{-n}(C)$ and therefore $C \cong g^{-n}(C) \not\cong \mathbf{P}^1$. Hence $(C^2) \geq 0$. If $(C^2) > 0$, then $M$ is projective and $g^N$, whence $g$, is of finite order on $NS(M)$ by Proposition (2.8)(1). Then ord $g < \infty$ by [Og2, Corollary 2.7], a contradiction. If $(C^2) = 0$, then $C$ defines an elliptic fibration which is stable under $g^N$. Therefore $S = M$. \qed
5. Non-abelian subgroup of automorphisms of a K3 surface

In this section, we shall prove Theorem (1.6).

Theorem (1.6)(2) follows from Theorem 1.6(1) by considering the Hilbert scheme $\text{Hilb}^m(S)$ of 0-dimensional closed subschemes of length $m$ of a K3 surface $S$. Indeed, by [Be], $\text{Hilb}^m(S)$ is a hyperkähler manifold of dimension $2m$ having natural inclusions $H^2(S, \mathbb{Z}) \subset H^2(\text{Hilb}^m(S), \mathbb{Z})$ and $\text{Aut}(S) \subset \text{Aut}(\text{Hilb}^m(S))$. Thus, $\text{Hilb}^m(S)$ for a K3 surface $S$ in Theorem (1.6)(1) satisfies the requirement.

We shall show Theorem (1.6)(1). Let $M$ be a K3 surface and $\varphi_i : M \to \mathbb{P}^1$ ($i = 1, 2$) be two different Jacobian fibrations of positive Mordell-Weil rank on $M$. Note that $M$ is projective under this assumption.

Let $\text{MW}(\varphi_i)$ be the Mordell-Weil group of $\varphi_i$. Choose $f_i \in \text{MW}(\varphi_i)$ s.t. $\text{ord} f_i = \infty$. We may regard $f_i$ as an element of $\text{Aut}(M)$. Let $G := \langle f_1, f_2 \rangle < \text{Aut}(M)$.

First we shall show that $\text{Aut}(M)$ is not almost abelian. Note that any subgroup of an almost abelian group is again almost abelian (by definition). So, it suffices to show that $G$ is not almost abelian. Suppose to the contrary that $G$ is almost abelian. Then, by definition, there are a normal subgroup $H$ of $G$ of finite index, a finite subgroup $N$ of $H$ and an abelian group $A$ which fit in with the exact sequence:

$$1 \to N \to H \to A \to 0.$$ 

Since $|G/H| < \infty$, there is a positive integer $m$ s.t. $f_1^m, f_2^m \in H$. Set $g_i := f_i^m$. Let $n := |N|$. Since $A$ is abelian, one has $g_1^{-1}g_2g_1g_2^{-1} \in N$ for each $j \in \mathbb{Z}$. Then, we have $n + 1$-elements in $N$:

$$g_1^{-1}g_2g_1^{-1}, g_1^{-2}g_2^2g_1^{-2}, \ldots, g_1^{-n}g_2^n, g_2^{-1}, g_1^{-n}g_2^{n+1}g_2^{-1} \in N.$$ 

Thus, at least two of them have to coincide. Hence $g_1^k g_2 = g_2 g_1^k$ for some positive integer $k$. Set $g := g_1^k$ and $h := g_2$. Then $gh = hg$ and $g^*(e_1) = e_1$, $h^*(e_2) = e_2$. Here $e_i$ is the class of a general fiber of $\varphi_i$ ($i = 1, 2$). Set $e_3 := h^*(e_1)$. Then $e_3$ is a (primitive) class of elliptic pencils on $M$. If $e_3 = e_1$, then $e_1$ and $e_2$ are both $h$-stable. Therefore, $e_1 + e_2$ is also $h$-stable. However, since $(e_1 + e_2)^2 > 0$, we have then $\text{ord} h < \infty$ on $N \text{S}(M)$, whence $\text{ord} g_2 < \infty$ by [Og2, Corollary 2.7], a contradiction to $\text{ord} g_2 = \infty$. Consider next the case $e_3 \neq e_1$, i.e. the case where $e_1$ and $e_3$ correspond different elliptic pencils. Using $g^*h^* = h^*g^*$ and $g^*(e_1) = e_1$, one has $g^*(e_3) = e_3$. Then $e_1 + e_3$ is $g$-stable and $(e_1 + e_3)^2 > 0$, a contradiction for the same reason as above. Thus, $G$ is not almost abelian.

Since $G$ is not almost abelian, $G$ is of positive entropy by the contra-position of Theorem (1.3).

Next we shall show that a K3 surface $M$ of Picard number 20 has at least two different Jacobian fibrations of positive Mordell-Weil rank. Note that $M$ is necessarily projective. In what follows, $U$ stands for an even unimodular hyperbolic lattice of rank 2, and $A_1, D_4, E_8$ stands for the negative definite root lattice whose basis is given by the corresponding Dynkin diagram. Since the transcendental lattice $T(M)$ is positive definite and of rank 2, $T(M)$ can be primitively embedded into $U^{\oplus 2}$, and $T(M)$ can be also uniquely primitively embedded into the K3 lattice $\Lambda_{K3} := U^{3} \oplus E_8(-1)^{\oplus 2}$ (See e.g. [Ni]). Note that $H^2(M, \mathbb{Z}) \simeq \Lambda_{K3}$. Thus the Néron-Severi lattice $NS(M)$ of $M$ is of the form:

$$NS(M) = U \oplus E_8(-1)^{\oplus 2} \oplus N,$$

where $N$ is a negative definite lattice of rank 2. Put $d := \text{det} N$. If $d = 3, 4$, then $N = A_2$, $A_1^2$, and if $d \neq 3, 4$, then $N$ is not a root lattice. Recall that
the \( \mathbb{Q} \)-rational hull of the ample cone of a projective K3 surface is the fundamental domain of the group of \((-2)\)-reflections on the rational hull of the positive cone (See e.g. [BPV]). Using this fact and the Riemann-Roch formula, one finds a Jacobian fibration \( \varphi_1 : M \to \mathbb{P}^1 \) whose reducible singular fibers are \( II^* + II^* + I_3 \) or \( II^* + II^* + IV \) if \( d \neq 3, 4 \), \( II^* + II^* + I_3 \) or \( II^* + II^* + I_3 + II^* + A + B \) if \( d = 3 \) and \( II^* + II^* + A + B \) if \( d = 4 \). Let \( \text{MW} (\varphi_1) \) be the Mordell-Weil group of \( \varphi_1 \). Then rank \( \text{MW} (\varphi_1) > 0 \) unless \( d = 3, 4 \) by Shioda’s rank formula (See for instance [Sh]). Note also that this \( \varphi_1 \) is essentially the same Jacobian fibration studied in [SI].

Let us find another Jacobian fibrations of positive Mordell-Weil rank. Join two \( II^* \) singular fibers of \( \varphi_1 \) by the 0-section and throw out the components of multiplicity 2 at the edge of two \( II^* \), say \( C_1 \) in one \( II^* \) and \( C_2 \) in the other \( II^* \). In this way, one obtains a divisor of Kodaira’s type \( I_{12}^* \), say, \( D \), on \( M \). The pencil \( |D| \) gives rise to a Jacobian fibration \( \varphi_2 : M \to \mathbb{P}^1 \) with reducible singular fiber of type \( I_{12}^* \) and with sections \( C_1, C_2 \). Take \( C_1 \) as the 0 and consider \( C_2 \) as an element \( P \) of \( \text{MW} (\varphi_2) \). We have that rank \( \text{MW} (\varphi_2) > 0 \) (regardless of \( d \)). Indeed, otherwise, by using Shioda’s rank formula, we see that the remaining reducible singular fibers of \( \varphi_2 \) are either \( I_3, VI, I_2 + I_2, I_2 + III, \) or \( III + III \). In each case, the value \( \langle P, P \rangle \) of Shioda’s height pairing of \( P \) is positive, as it is checked by his explicit height pairing formula. This, however, implies ord \( P = \infty \), a contradiction to the assumption. (See [Sh] for the definition and basic properties on Shioda’s height pairing on the Mordell-Weil group.)

Thus we obtain two Jacobian fibrations \( \varphi_1, \varphi_2 \) of positive Mordell-Weil rank unless \( d \neq 3, 4 \). When \( d = 3, 4 \), Vinberg [Vi] calculated the full automorphism group \( \text{Aut} (M) \). It contains a subgroup \( H \) isomorphic to the free product \( \mathbb{Z}_2 * \mathbb{Z}_2 * \mathbb{Z}_2 \) of three cyclic groups of order 2. This \( H \) is far from being almost abelian. Thus, by Theorem (1.4)(1), \( H \) does not make the Jacobian fibration \( \varphi_2 \) stable. Now, by transforming \( \varphi_2 \) by \( H \), one can find another Jacobian fibration of positive rank also when \( d = 3, 4 \). This completes the proof of Theorem (1.6)(1).

Remark 5.1. Under the same assumption in Theorem (1.6)(1), a K3 surface \( M \) has a dense \( \text{Aut} (M) \) orbit in the Euclidean topology. This remarkable result has been found by Cantat [Ca2] in a slightly more general situation.

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