Implementation of the thermodynamic and phase transition equations of superfluid helium in CFD software

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Abstract. The cryogenic design of the next generation of superconducting accelerator magnets depends on our ability to simulate the helium heat and mass transfer in the internal structure of these magnets. For that matter accurate tools must be developed such as numerical codes integrating the thermodynamic behavior and phase transition in superfluid helium. We have implemented in 2D and 3D, the He II conservation equations in Fluent® CFD software corresponding to a simplified two-fluid model. It consists of a conventional continuity equation, a modified momentum equation for the total fluid and an energy equation including the Gorter-Mellink internal convection term modeling the turbulence regime. The code is mainly suited to simulate transient and steady-state flow configurations. In addition, a new method has been developed to simulate the He II / He I transition in 2D based on a modified Volume Of Fluid method (VOF). The interface between the two states of liquid helium has been locally recreated in the corresponding cells to properly mimic the second order phase transition (no latent heat). Both steady and unsteady numerical simulation have been performed and compared with different experimental results.

1. Introduction
The next generation of superconducting magnet must be able to reach extremely high energy (up to 14 TeV) at low temperature (1.6 K - 2.1 K) in confined space (accelerator magnet). With an high conductivity (10^3 to 10^4 W/(m.K)) at low temperature, superfluid helium is used as refrigerant for these applications. The comprehension and simulation of heat transfer phenomena is fundamental to design superconducting magnets to control their thermal behavior and avoid the magnet quench. In previous papers, simulation of the thermal behavior of the superfluid helium using the Kitamura simplified model in a CFD software has been presented [1]. The maximum temperature tested on these simulations reaches 2.1 K. In case of quench of the magnet, the temperature may exceed the transition temperature \( T_\lambda \) and could be responsible for the appearance of helium gas at high pressure in the cryostat. Few calculations have been performed to estimate the maximum pressure the helium reached in long and narrow tubes [2] and during a quench of the 45-T hybrid magnet at NHMFL [3]. None of these studies have implemented a helium phase change that is necessary to model the real behavior of the helium in case of large transients. This paper focus on modelling the helium superfluid transition using the CFD software Fluent® to simulate thermal and dynamic behavior of this multiphasic mixture.

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The He II behavior is simulated using the simplified two-fluid model and results are compared, in the
first part, with steady-state analytic simulation and transient experiments. In a second part, for the
superfluid transition, the interface identification between fluids is performed using VOF (Volume of
Fluid) method [4] implemented in Fluent®. A specific function has been created to simulate the mass
transfer that occurs during the He II / He I second order transition. The appearance of the helium gas is
performed using the evaporation/condensation model [5] implemented in the CFD software.

2. Numerical model

2.1. The simplified two fluid model in superfluid helium

According to the theory of Landau, superfluid helium is viewed as a mixture of two components; an
inviscid superfluid component and a dissipative normal fluid component [6, 7]. This is called the
“two-fluid” model. And the simplified “two-fluid” model is based on the assumption that the
thermomechanical effect term and the Gorter-Mellink mutual friction term are the dominant terms in
the momentum equation for the superfluid component. These equations are discretized on a three-
dimensional space with a hexahedral mesh. Calculations are performed in Fluent® software based on
an element-based finite volume method (FVM) and additional components (in red in the equations 4
and 5) are implemented as source terms in the Navier-Stokes equations in C language. Then the
conservative and momentum equations for superfluid component lead to the following equations
system,

\[ \rho = \rho_s + \rho_n \]  

\[ \rho u = \rho_s u_s + \rho_n u_n \]  

\[ \frac{\partial \rho}{\partial t} + \nabla (\rho u) = 0 \]  

\[ \rho \frac{\partial u}{\partial t} = -\rho(u \cdot \nabla)u - \nabla p - \rho_s \left( \frac{s}{\rho} \left( \frac{\rho}{\rho_s} \right) \nabla T^2 \right) + \eta \left( \nabla^2 u + \frac{1}{3} \nabla (\nabla \cdot u) \right) - \left( \frac{\rho_s}{\rho} \right) \left( \frac{s}{\rho_s} \right) \left( \frac{\rho}{\rho_s} \right) \nabla \left( \frac{f(T)}{\nabla T} \right)^\frac{1}{3} \nabla T \right) + \rho g \]  

and the energy equation with the equivalent thermal conductivity represented by the Gorter-Mellink
term is written as:

\[ \rho \frac{\partial}{\partial t} (C_p T) = -\rho C_p (u \cdot \nabla T) - \nabla \cdot \left( \frac{f(T)}{\nabla T} \right)^\frac{1}{3} \nabla T \right), \]  

where the indices \( n \) and \( s \) are referring to the normal and superfluid component respectively.

This simplification change the transport equation for the superfluid component from a hyperbolic to a
parabolic equation, thus the so-called second-sound effect is not modeled. The influence of the
second-sound on the heat and fluid flow is only important during very short initial periods but it is not
significant in steady-state condition or very slow process where the temperature relaxation time is considerably long. To avoid numerical instability caused by the small temperature gradients at the beginning of the calculation, the equivalent conductivity is initially forced to be 1000 W/(m.K).

2.2. He II-He I transition
A tool to compute the phase transition, implemented in numerical code, is a way to improve the thermal design in estimating the effects of quenches on the cryogenic system.

2.2.1. VOF model
The VOF method [3] implemented in Fluent® is a surface-tracking technique applied to a fixed mesh that can model two or more immiscible fluids by solving a single set of momentum equations and tracking the volume fraction of each of the fluids throughout the domain (Fig. 1). This method represents a good candidate to preserve the momentum and energy equations during superfluid helium phase transition. Its formulation relies on the fact that two or more fluids (or phases) are not interpenetrating. In each control volume, the sum of the volume fractions of all phases equals unity. Physical properties and conservation equations for each phase are described by the equation (6) and (7):

\[ \rho = \sum \alpha_q \rho_q \]  
\[ \frac{1}{\rho_q} \left[ \frac{\partial}{\partial t} (\alpha_q \rho_q) + \nabla \cdot (\alpha_q \rho_q \mathbf{u}_q) \right] = \sum_{p=1}^{n_m} (\dot{m}_{pq} - \dot{m}_{pq}) \]  

2.2.2. He II-He I Mass transfer
A new sub-code has been implemented in Fluent® to simulate the mass transfer during the second order superfluid helium transition. The temperature field varies within the cell and a certain mass should pass from one phase to the other. The cell is divided in two regions representative of each phase which are proportional to the surface they occupy. When the temperature field varies after the resolution of the energy equation, a new distribution region among the phases is computed with the position of the lambda line (Fig. 2). To compute the lambda line during the phase transition, a first-order Taylor approximation is performed in every cell given by equation (8),

\[ T_A = T_C + \nabla T (x_i - x_c) \]  

![Figure 1. VOF scheme.](image1)

![Figure 2. Lambda line model.](image2)

![Figure 3. Mass transfer model during transition.](image3)
where $T_C$ is the temperature at the center of the cell, $x_C$ is the position vector of cell center and $x_\lambda$ is the position (vector) of the lambda line we are looking for. The fraction between the surface where $T < T_\lambda$ and the cell’s surface gives the actual volume fraction and the mass transfer from He II to He I in the cell (Fig. 3).

2.3. Evaporation/Condensation model
For the He I - gas transition, an evaporation-condensation model implemented in Fluent® has been used [5]. Based on the following temperature regimes, the liquid-vapor mass transfer are described by the equations (9) and (10),

If $T_1 > T_{sat}$

$$m_{l\rightarrow v} = \text{coeff} \cdot \alpha \cdot \rho_\lambda \frac{(T_1 - T_{sat})}{T_{sat}} ,$$

(9)

If $T_v < T_{sat}$

$$m_{v\rightarrow l} = \text{coeff} \cdot \alpha \cdot \rho_\lambda \frac{(T_v - T_{sat})}{T_{sat}} ,$$

(10)

where $\text{coeff}$ is a coefficient that can be interpreted as a relaxation time. Considering the rate of mass transfer mainly governed by the latent heat ($L_v$), this coefficient can be expressed by equations (11) and (12),

$$m = \frac{(T - T_{sat}) \cdot \rho_\lambda \cdot C_p \cdot \alpha_v}{L_v}$$

(11)

$$\text{coeff} = \frac{C_p \cdot T_{sat}}{L_v} .$$

(12)

The surface tension model proposed by Brackbill et al. [8] and implemented in Fluent® has been used to model the interaction between these two phases. Furthermore, an adaptive mesh method is added to properly simulate the phase interface and keep the calculation time small.

3. Simplified two-fluid model validation
The validation of simplified two-fluid model implemented in Fluent against an analytical solution is done in a 2D domain and our method is also compared in 3D to a transient superfluid experiment.

3.1. Analytic solution validation

3.1.1. Numerical conditions
The calculation was performed for a heat flux values $q=18000 \text{ W/m}^2$ and for a bath temperature $T_b=1.8$ K and an initial pressure of 101.3 kPa. The helium properties, the density, viscosity, specific entropy, specific heat capacity and the He II heat conductivity function, were directly imported to the solver from HEPAK® data base. The geometry is modelled by 2D rectangular 0.1x0.05 m$^2$ with a heating plate on the bottom, two lateral adiabatic walls and a wall at fixed temperature $T_b$ on the top of the domain (cf. Fig. 4). This kind of geometry can be compared with a mono-dimensional analytical solution because the thermal transfer takes place only in the longitudinal direction. In addition, the gravity effect is not taken in account.

The corresponding mesh, created on ICEM (CFD mesh builder), is shown in Fig. 5. It is composed by 8206 cells with a refinement near the walls necessary to approximate the velocity gradient near the walls. The characteristic cells dimensions vary from a minimal value of $10^{-4} \times 10^{-1} \text{ m}^2$ and a maximal one of $10^{-3} \times 10^{-3} \text{ m}^2$. The time step for these calculations is $10^{-4} \text{ s}$. 
3.1.2. Results
The results presented in Fig. 7 show the temporal evolution of temperature in the domain. The transient behavior is in good agreement with the analytical behavior shown in Fig. 6. The thermal perturbation propagates from the inlet side to the one at fixed temperature. The analytical steady-state solution, shown as black squares in Fig. 7, is good agreement with the results of simulation (less than 0.1% difference). The steady-state regime is reached after 1 s.

3.2. Transient comparison with experiment

3.2.1. Numerical conditions
The only experiment on the transient regime has been done by Van Sciver [9]. The aim of the experiment was to measure the transient heat transfer to He II in a one-dimensional system. The apparatus, shown in Fig. 8, is composed by a helical coil (10 m long and 9 mm internal diameter) with a heater at the bottom and a He II bath on top ($T_b=1.8$ K) at saturation conditions. Considering that the effect of the gravity is negligible in the experiment we simulated his experiment with a 3D horizontal cylinder of the same length. The mesh, generated on ICEM, counts 62379 cells (Fig. 9).
3.2.2. Results
The results of this simulation are compared with the experiment data on Fig. 10. The evolution of the temperature in the tube is faster than in the experiment (about 0.2 s) mostly due to the fact that the thermal resistivity of the solid part of the tube and the Kapitsa resistance haven’t been modelled in this simulation. Nevertheless, the temperature evolution in the tube behaves like the experiment and seems to evolve to the same steady state value.

4. Superfluid Transition
Even if any simulation of the superfluid transition cannot be compared with any experiments for the moment, we performed calculations on 2D geometries to study the behavior of the physical properties involved in these simulations. For the sake of simplicity, the densities of superfluid and normal liquid helium have been set to be only temperature dependent. The helium gas is modelled as an ideal gas.

4.1. He I - He II transition
4.1.1. Numerical conditions
The purpose of these tests is to analyze the He I / He II interface made by the code and the thermal behavior during the phase transition. A rectangular 2D geometry (150 mm long and 1 mm large) is used with the heater on the left side and the bath on the right side. The adaptive mesh refine 5 times the mesh, from $10^{-5}$ m to $3 \times 10^{-7}$ m, in the region where $2.16 < T < 4.2$ K, where the new phase appear. The purpose is to dispose of finer discretization in the phase transition zone. It coarsens the mesh to its
original size when $T_C>4.2$ K. The simulation parameters are $q=5000$ W/m$^2$ and $T_b=2.155$ K (near the lambda temperature) at 103.5 kPa and the time step is $10^{-6}$ s. Larger time step have been tested but the simulations become unstable and diverge few iterations after the transition appears in the area. Calculations are stopped when temperature of the cell reaches 4.2 K.

4.1.2. Results
The code is capable to manage the interface appearance and its propagation as it is shown in Fig. 11. The interface appears and the front smoothly develops perpendicularly to the heater as expected. Due to the sudden change of thermal conductivity, from about 300 W/(m.K) in He II to 0.014 W/(m.K) in He I, the temperature strongly increase to 4.2 K before the transition. The thickness of the liquid He I phase is about $5 \times 10^{-6}$ m (Fig. 12). The behavior of the equivalent thermal conductivity in He II is stable close to the transition (Fig. 13). This simulation works only if the mesh refinement occurs far from the lambda line position. In some tests where the transition is too close to the mesh refinement, a high increase of the thermal conductivity has been observed (10 times higher than for the successful simulations) and affect the rest of the area.

Figure 11. Volume fraction of He I during the simulation.

Figure 12. Temperature profile near the heater.

Figure 13. Thermal conductivity profile.
4.2. Evaporation/condensation
On the second part of these test, the evaporation/condensation model have been added to the simulation. In this part, parameters of the simulation are $T_b=1.8$ K and $q=100$ kW/m² to increase the apparition of helium gas near the wall of the heater. The geometry is the same as the previous section. The thermal conductivity is still stable close to the He II / He I transition (Fig. 14) and helium gas slowly appear at the surface of the heater (Fig. 15). Despite the high heat fluxes, the computation are very slow; i.e. it takes 2 weeks of calculation to have a $10^{-6}$ m thickness of helium gas near the heater. It is mostly due to the complexity of the models used in these simulations.

![Figure 14. Thermal conductivity profile.](image)

![Figure 15. Volume fraction of He I.](image)

5. Conclusion
This paper has presented the implementation of the simplified “two-fluid” model for superfluid helium and a new specific method to simulate it transition in the CFD solver Fluent®. In superfluid helium, simulations present good agreements with analytic solutions and transient experiments. The superfluid transition has also been simulated with the apparition of both liquid and gas helium at the lambda line. Nevertheless, transition simulations are too slow due to the complexity of the phenomena to solve and this part has to be further studied to improve the speed of these calculations.

6. References
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