Spiral Structure as a Recurrent Instability

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Abstract. A long-standing controversy in studies of spiral structure has concerned the lifetimes of individual spiral patterns. Much theoretical work has sought quasi-stationary spiral modes while $N$-body simulations have consistently displayed recurrent, short-lived patterns. The simulations manifest a recurrent cycle of true instabilities related to small-scale features in the angular momentum distribution of particles, with the decay of each instability seeding the growth of the next. Data from the recent Hipparcos mission seem to offer support for the recurrent transient picture.

1. Introduction

It is about a century and a half since Lord Rosse first noted the spiral appearance of M51, but a satisfying and robust theory for the general spiral phenomenon in disc galaxies still eludes us.

There has been substantial progress, of course. Early theoretical efforts focused on the gas, no spirals are seen in S0 galaxies that have little or no interstellar matter, and are most striking where gas is abundant. However, most researchers are now convinced that spirals are driven by the stellar disc through some kind of collective gravitational process. The most compelling reason is that spiral arms are smoother in images of galaxies in the near IR (Schweizer 1976; Rix & Zaritsky 1995; Block & Puerari 1999), indicating that the old disc stars participate in the pattern. In addition, we have known that spiral patterns develop spontaneously in $N$-body simulations ever since Lindblad’s pioneering work in the early 1960s (e.g. Lindblad 1960). The problem is thus largely one of classical dynamics – an application of nothing more sophisticated than Newton’s laws of gravity and motion – with gas playing an important, but secondary, role.

Spirals in tidally interacting galaxies could well result from the interaction itself, and some spirals may be driven by bars. But spirals in a substantial fraction of galaxies cannot be ascribed to either of these triggers, and therefore present the most insistent problem. In this article, I describe my recent work

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on this subject, which stems from my collaboration with Franz Kahn in the late 1980s. He was a great help to me then, as he had been much earlier when I began my career as a graduate student in Manchester. It is clear that he had been interested in spiral structure throughout his career, and was present at the seminal meeting (Woltjer 1962) which seems to mark the change of focus in the wider community to gravitational theories for the phenomenon (which B. Lindblad had pioneered for many years before then).

2. Short- or Long-lived Patterns?

While most theorists agree that spirals are density variations in the disc which are organized by gravity, opinions diverge quite quickly from this starting point. There is not even a consensus on something as fundamental as the lifetime of spiral patterns. Our snapshot view of galaxies gives us no direct information, and two separate schools of thought exist on the longevity of the structures we observe.

C. C. Lin and his co-workers (e.g. Bertin & Lin 1996) favour long-lived quasi-stationary spiral patterns. They suggest that spirals are global instabilities which grow rather slowly in a cool disc with a smooth distribution function (DF). Such modes could be quasi-stationary because of mild non-linear effects, such as gas damping. They can achieve low growth rates for spiral modes by invoking a “Q-barrier” where in-going travelling waves reflect off a dynamically hot and largely unresponsive inner disc. The standing wave pattern which makes up the mode consists of short and long trailing waves that are trapped between reflections at the Q-barrier and at co-rotation; it is excited by mild over-reflection at co-rotation. This type of mode can survive only if the inner reflection occurs outside the inner Lindblad resonance (ILR) so that the waves are shielded from the fierce damping which must occur if that resonance were exposed.

Most other workers favour short-lived patterns, with fresh spirals appearing in rapid succession, as indicated by N-body simulations. Such spirals develop through swing-amplification in some form or another, either as shearing waves (Goldreich & Lynden-Bell 1965) or as forced responses (Julian & Toomre 1966). Both these interpretations are manifestations of the same underlying mechanism (see Toomre 1981).

The role of gas in this picture is as follows: All forms of density wave arise through collective motions of the stars and are therefore weaker when stars move more randomly. Thus fluctuating spiral structure is self-limiting, since the transient patterns themselves gradually scatter stars away from near-circular orbits. If no gas were present, the spirals must fade over time – in less than ten galactic rotations (Sellwood & Carlberg 1984) – as stellar random motions rise. Clouds of gas, when present in a galaxy disc, collide and dissipate most of the random motion they acquire, and therefore remain a dynamically cool and responsive component. Moreover, young stars reduce the rms spread of the total stellar distribution, because they possess similar orbits to those of their parent massive gas clouds. Quite a modest star formation rate is needed to preserve the participation of the stellar disc in spiral waves – a few solar masses per year over the disc of a galaxy is enough.
Toomre & Kalnajs (1991) advocate one theory of this type in which spirals are the polarized disc response to random density fluctuations. If the $\sim 10^{10}$ individual stars of the disc were smoothly distributed, density variations would be tiny; but real galaxy discs are much less smooth because they contain a number of massive clumps, such as star clusters and giant molecular clouds (an extra role for gas). Density fluctuations can be decomposed into a spectrum of plane waves of every pitch angle, which shear continuously from the leading to the trailing direction because of differential rotation. The smooth background disc responds enthusiastically to forcing from this shearing spectrum of density fluctuations, and the entire disc develops a transient shearing streakiness.

An equivalent viewpoint is to regard the background disc as polarized, with each of the orbiting disturbance masses inducing a spiral response in the surrounding medium. The spiral responses are much stronger than the forcing density variations, but remain directly proportional in this picture, the proportionality constant depending on the responsiveness of the background disc. The shot noise from the finite number of particles is itself the source of the spiral responses in the $N$-body simulations of Toomre & Kalnajs. While these authors have developed a rather detailed understanding of their local simulations, the amplitudes of spirals in global $N$-body simulations seems to be independent of the particle number (Sellwood 1989), rather than declining as $N^{-1/2}$ as this theory would predict. Further, it is unclear whether the noise amplitude/responsiveness combination are sufficient to give rise to spirals of the amplitude we observe.

I prefer a model in which the spirals are true instabilities, which recur in rapid succession, as a flag flaps repeatedly in a breeze. I describe this idea in the next few sections, but I emphasize here that it differs radically in two further respects from the quasi-steady waves advocated by Bertin & Lin. First, the ILR in my model is not shielded, but plays a central role, and second, I expect the DF to be far from smooth and, in fact, the features in the DF drive the instabilities. I do not claim a fully worked out theory, however; the weakest link in the picture is the manner in which the cycle recurs.

3. **Groove Modes**

My paper with Franz Kahn, which appeared in *Monthly Notices* in 1991, described a new kind of global spiral instability of a stellar disc, which we called a “groove mode.” Our paper presents both $N$-body simulations and a local theoretical description of the instability. I attempt to provide only a physical interpretation here and leave the interested reader to refer back to our paper for a more thorough development.

As its name implies, the instability occurs in a “groove” in the particle distribution. It is fundamentally a groove in angular momentum density which would give rise to a groove in surface density only if the stars were all on circular orbits. As the epicyclic radii of typical stellar orbits are some $10\% – 20\%$ of their mean radius, whereas we invoke a deficiency of stars over a range of $\lesssim 1\%$ of their angular momentum, the surface density of stars is not significantly reduced anywhere. Nevertheless, it is much easier to envisage the mechanism for a cold disc in which the radial density profile has a sharp notch.
Figure 1. The top panel shows the mechanism of the groove mode. The surface density is lower between the two lines, which were originally straight (dotted) and the sense of the shear flow is indicated by the lightly drawn arrows. The shaded areas are regions in which higher density material has moved into the groove owing to the disturbances on each side, and the destabilizing disturbance forces within the groove are marked by the heavy arrows.

The density excesses in the groove are redrawn in the lower panel where the contours show the supporting response of the surrounding disc. The dashed lines mark the Lindblad resonances.

The mechanism for a groove mode is illustrated in Figure 1 which shows a small patch of the disc so far from its centre that curvature is negligible. Wave-like disturbances on the groove edges bring high surface density material into the groove in the dark shaded regions. The changes in density give rise to disturbance forces; those between the density excesses on either side of the groove are marked by the pairs of opposing arrows within the groove. Material displaced upwards into the groove from its lower edge is therefore pulled forward by the density excess from the opposite edge. Material that is pulled forward gains angular momentum, moves to an orbit of larger mean radius, and in a differentially rotating disc, lags behind its original azimuthal motion. It therefore rises further into the groove, moving less rapidly, relative to the groove centre than before. Similarly, material displaced downwards into the groove from its upper edge is pulled backward by the density excess from the opposite edge, loses angular momentum and sinks further into the groove. Thus each density excess causes the other to continue to grow, and the system is unstable.

It is worth mentioning that Franz Kahn did not see the instability this way. Instead, he instinctively saw that the dispersion relation for waves in a groove
with a smooth profile, such as a Lorentzian, would have a solution in the upper half of the complex plane. To my mind, such an insight conveys no intuitive feel for what is happening, but Franz rightfully trusted it more than physical intuition in situations where the dynamics can be quite subtle.

The instability of the groove alone would be of little consequence were it not for the supporting response of the surrounding disc. A polarized response grows with the density excesses in the groove, as shown in the lower part of Figure 1. The groove itself is unstable over a wide range of wavelengths, but the vigorous supporting response from the surrounding disc favours long wavelengths only. The result is a fiercely growing, global, spiral instability for which co-rotation lies near the groove centre. Our local theory estimates of the mode frequencies based on this picture (Sellwood & Kahn 1991) were in tolerable agreement with those found in the global simulations.

These spiral responses extend as far as the Lindblad resonances on either side, which are marked by the dashed lines. Recall that these resonances occur where stars moving relative to the pattern encounter the periodic disturbance at their epicyclic frequency, $\kappa$. If the pattern speed is $\Omega_p$ and the circular frequency at radius $R$ is $\Omega(R)$, a star encounters an $m$-armed wave at frequency $\omega = m(\Omega - \Omega_p)$; the condition for a Lindblad resonance is $|\omega| = \kappa$.

I should stress that this linear, global instability also occurs in discs with random motion, in which case we invoke a deficiency of stars over a narrow angular momentum range. The above arguments carry over to this more realistic situation.

4. Subsequent behaviour

The linear mode saturates, and ceases to grow, about the time that the density excesses meet across the groove. At this point, the mode has generated a periodic density variation around the groove which will disperse only rather slowly, as the stars which comprise it all have similar angular momenta. The amplitude of the polarized response of the surrounding disc tracks the variations of the mass clump which induced it.

4.1. Angular momentum transport

The inclined spirals exert a torque, however, which redistributes angular momentum. The effect can be viewed as a gravitational stress (Lynden-Bell & Kalnajs 1972), or as wave action carried at the group velocity (Toomre 1969; Kalnajs 1971). The group velocity, $\partial \omega / \partial k$ as usual, is directed radially away from co-rotation for all but the most open trailing waves (Binney & Tremaine 1987, §6.2). As the wave is being set up, the stars inside co-rotation do work on the wave and lose angular momentum to it, while those outside gain energy and angular momentum; the net change over the whole disc is zero, as it must be for a self-excited disturbance. Thus the wave action carried at the group velocity is negative angular momentum inside co-rotation and positive angular momentum outside. When combined with the opposite signs of the group velocity, there is an outward flux of angular momentum everywhere – in agreement with the sign of the gravity torque.
Figure 2. The Lindblad diagram for a disc galaxy model. Circular orbits lie along the full-drawn curve and eccentric orbits fill the region above it. Angular momentum and energy changes between a wave and particles move them along lines of slope $\Omega_p$ as shown. See the text for explanations of the other curves.

4.2. Exchanges at resonances

The disturbance produced by the groove mode therefore generates both positive and negative angular momentum, in equal measure, at co-rotation which is then carried away by the spirals towards the Lindblad resonances on either side. Secular exchanges between the wave and the stars, which are possible only at resonances (Lynden-Bell & Kalnajs 1972), lead to the wave action being absorbed at these locations, which is why a mode could not be set up with an exposed ILR.

Stars moving in a non-axisymmetric potential that rotates at a steady rate conserve neither their specific energy, $E$, nor their specific angular momentum, $J$. But the combination

$$I_J \equiv E - \Omega_p J,$$

known as Jacobi’s invariant, is conserved. Thus at a Lindblad resonance, where a star gains angular momentum $\Delta J$ from the wave, it also changes its energy as

$$\Delta E = \Omega_p \Delta J.$$

Figure 2 shows the Lindblad diagram for a differentially rotating disc with an infinitesimal non-axisymmetric perturbation. The full-drawn curve marks the locus of circular orbits in this $(J, E)$ plane; no particle can lie below this curve, but bound particles with $E > E_c$ move on eccentric orbits in this potential. The resonance condition for a non-circular orbit generalizes to

$$m(\Omega_\phi - \Omega_p) + l\Omega_R = 0,$$
where $\Omega_\phi(E, J)$ and $\Omega_R(E, J)$ are the azimuthal and radial frequencies of orbits (Binney & Tremaine 1987, §3.1), and $l = 0$ for co-rotation and $l = \pm 1$ for Lindblad resonances. The loci of these three principal resonances for arbitrarily eccentric orbits are marked by the broken curves in this Figure.

The displacements caused by wave-particle interactions all have slope $\Omega_p$ in this diagram. As this is the slope of the circular orbit curve at co-rotation, stars which exchange energy and angular momentum there do not move further from that curve, to first order. The two vectors show that at the Lindblad resonances, on the other hand, stars are moved onto more eccentric orbits when angular momentum is redistributed outwards.

### 4.3. Results from a simulation

Figure 3 shows a spiral pattern extracted from one of my simulations. This calculation is of a $Q = 1.5$, half-mass $V = V_0 = \text{const}$ (aka Mestel) disc, with an inner taper applied to give the disc a characteristic length scale, $R_0$, and an outer cut off to limit it to a finite radial extent. The properties of this disc are described in Binney & Tremaine (1987, §4.5) and I have previously reported (Sellwood 1991) confirmation of Toomre’s (1981) prediction that this model with a smooth DF has no true linear instabilities. The 1 million particles in this simulation were drawn from the appropriate DF and placed at random azimuths at the start. A number of transient spiral patterns develop from the particle noise over time, and that illustrated in Figure 3 was extracted by fitting
a coherent wave to \( m = 2 \) Fourier coefficients of the mass distribution for the time interval \( 400 \leq tV_0/R_0 \leq 600 \) – i.e. long after the start. The best-fit pattern speed for this wave is \( 2\Omega_p = 0.364V_0/R_0 \) and co-rotation (full drawn circle) and the Lindblad resonances (dotted circles) for this pattern speed are marked.

The distribution of particles in this run at time 600 is shown in Figure 4. The abscissae are the (instantaneous) specific angular momenta of the particles, and the ordinates are the excess of energy over and above that needed for a circular orbit at this \( J \). Overlayed on the lower plot are the expected loci of particles scattered from nearly circular orbits at the Lindblad resonances for this wave, and the loci of the generalized Lindblad resonances defined by equation (3). The very close similarity in this potential between the scattering trajectory and the locus of the generalized ILR is remarkable, and leads to the strong and coherent tongue of particles scattered up this curve. The impressive agreement between the simulation and the prediction, which has no free parameters, is reassuring.

The near coincidence of the scattering trajectory with the generalized ILR is not repeated at the OLR; there the two curves are practically orthogonal. Consequently, scattering at the OLR does not produce striking changes in the particle distribution in this plot.

### 4.4. Recurrent cycle

The resonant scattering of stars by spiral waves, leading to deficiencies in the DF over narrow regions in this plot, opens up the possibility of a recurrent cycle. Similar, though not identical, deficiencies are responsible for the groove modes discovered by Franz and myself, and it is therefore likely that a fresh instability should develop, perhaps with co-rotation near to one of the Lindblad resonances.

A recurrent instability cycle of this nature was observed by Sellwood & Lin (1989) in simulations of a low mass disc in nearly Keplerian rotation about a central mass. Their model, in which perturbation forces were also restricted to a single Fourier component, differs in many important respects from real galaxies, but the present Mestel disc is much more realistic. Scattering caused by the waves raises the level of random motion of disc particles in the vicinity of the resonance. Sellwood & Lin found that only the groove carved in a previously undisturbed part of the disc caused a new mode to grow – the other Lindblad resonance lay in a region which had already been heated by two previous waves, and was unable to support a new instability. Thus the spiral “disease” progresses radially in a single direction. Once the entire disc has been heated, no further spiral waves can be sustained – unless some cooling mechanism is in effect (e.g. Sellwood & Carlberg 1984; Carlberg & Freedman 1986).

### 5. Hipparcos stars

This model for a recurrent spiral wave instability cycle is now rather complex, and rests heavily on results from simulations. While the simulations have been conducted with as much care as possible, and their behaviour seems physically reasonable, the possibility always exists that the results arise through some pernicious, undetected numerical artifact and bear no relation to what actually happens in real galaxies. It therefore seemed appropriate to me to seek some
Figure 4. Top: The locations of one particle in 20 in the simulation in \((E_{ran}, J)\)-space at a time during the decay of the pattern shown in Figure 3. Bottom: The same with added curves to show (full-drawn the loci of the generalized Lindblad resonances (equation 3) and (dashed) to mark the scattering trajectory given by equation (2) for particles which start with \(E = E_c\).
Figure 5. The density of Solar neighbourhood stars in $(E_{\text{tan}}, L)$-space. The figure was constructed by Dehnen using $\sim 6000$ stars selected from the Hipparcos sample. Apart from the skew to lower $L$, which is caused by the asymmetric drift, a smooth DF would have produced a featureless plot, whereas the contours show one or more distinct ridges.

observational confirmation before plunging any further down what could be a blind alley.

A possibility of an observational test presented itself once the data became available from ESA’s Hipparcos satellite. This space mission measured proper motions and parallaxes for many stars. Of these, some 14,000 were selected by Binney & Dehnen (1998) as being a kinematically unbiased subsample within 100 pc of the Sun, so that parallaxes were known to a precision of 10% or better. The satellite measured five out of the six phase-space coordinates of each of these stars – the radial velocity was not measured. However, Dehnen (1998) cleverly realized that the missing information could be obtained, at least in a statistical sense, if it could be assumed that the velocity distribution of these stars were homogeneous throughout this small volume. This reasonable assumption allows one to infer the distribution of missing velocities in one direction on the sky by equating it to the distributions observed in orthogonal directions. In this way, Dehnen was able to construct the full phase space distribution function.
Figure 5, kindly made for me by Dehnen, shows contours of the phase space density of Solar neighbourhood stars in \((E_{\text{ran}}, J)\)-space. The parabolic lower boundary of the contour distribution reflects the fact that stars on close to circular orbits, whose guiding centers are far from the Sun would never visit the solar neighbourhood, and are therefore missing from this sample. The asymmetry between left and right results from the usual asymmetric drift, because the stellar density rises towards the Galactic centre.

But it is clear from this Figure that the underlying distribution function is not smooth. There is a clear hint of a scattering line (maybe two), which is reminiscent of scattering at an ILR. If this interpretation is correct, it would confirm that spiral arms are transient and lend considerable support to the idea of a recurrent transient cycle of instabilities discussed above. The missing radial velocity information is now being obtained (Pont et al. 1999), which will enable this plot to be redrawn without invoking Dehnen’s stratagem and hopefully, confirm the sub-structure.

It should be noted that Dehnen (1999) suggests these data could also be interpreted as scattering at the OLR of the bar (see also Raboud et al. 1998). We are working to try to determine which interpretation is the more plausible, but either way, the assumption that spiral arms in galaxies are formed in a system having a smooth DF looks rather too idealized in the light of these data.

6. Conclusions

Theoretical effort in an area starved of observational input often loses momentum and may be aimed in quite the wrong direction. Spiral structure theory has not been devoid of observational input, since evidence in favour of density waves has been accumulating for many years. However, the near IR intensity variations (Schweizer 1976 and others), or coherent velocity perturbations (Visser 1978 and others), support only the existence of density waves, and do not test ideas for their origin.

Simulations of disc galaxies were strongly motivated by the desire to fill this gap, and have been reporting for decades that spiral arms are transient. As this result has still not been fully understood, the simulators themselves have worried that it could be an artifact. Such healthy scepticism has prompted them to devote many years of effort to refining, testing and cross-checking their codes (Miller 1976; Sellwood 1983; Inagaki et al. 1984; etc.). But new results from simulations cannot, almost by definition, be checked, and the best we can do is to try to show that the behaviour is physically reasonable. Despite all this care and effort, those intent on calculating slowly growing, quasi-stationary, spiral modes have totally discounted the reported behaviour on the grounds that, in their view, spiral structure is simply too “delicate” a problem for simulated results to have any validity!

The Hipparcos data provide the first observational confirmation that it is wrong to assume that the DF of a disc galaxy is smooth. In retrospect, it is hard to see how it could remain smooth, since almost any realistic disturbance in a disc will scatter stars from (or maybe trap them into) resonant regions of phase space. Obviously, the DF could relax back to a smooth state if scattering were
efficient, but in a purely collisionless disc, the feature could be smoothed only by further collective effects, which themselves will have resonances elsewhere.

The Hipparcos result appears to vindicate the simulations, and it seems highly likely that spirals in real galaxies are recurring, transient patterns. They result from true instabilities provoked by narrow features in the DF, and are of a global nature because long wavelength disturbances are supported most vigorously by the swing-amplifier. While the mechanism for these linear instabilities is now reasonably clear, exactly how the DF is affected, and how the instabilities might recur is not. The resonant scattering peaks, if confirmed, suggest at least one of the processes which sculptures the DF, but it is possible it is not the only, or even the dominant, source of inhomogeneities in the DF. The Hipparcos data have provided a much needed pointer to the way forward in this erstwhile stalled area.

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