On the steady motions control of viscoelastic joined robotic manipulators

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Abstract. In this paper, the control problem of the serial multi-link robotic manipulators with rigid and viscoelastic joints is considered. The manipulators structure is such that only the rigid joints are controlled by DC motors. It is shown that such robotic manipulators can make so called stationary motions wherein the part of the generalized coordinates doesn’t change in time and the others change according to a linear law. The robust output feedback controllers are obtained for the robotic manipulators such that the stationary motion is locally, semi-globally, and globally asymptotically stable.

1. Introduction

The paper considers the control problem of a serial robotic manipulator taking into account the viscosity and elasticity forces acting in the connecting elements of its links. Such forces arise from the use of cables, wave drives, etc [1]. Recently, variable viscoelastic joints consisting of pneumatic artificial muscles and magneto rheological brakes have been proposed to use in exoskeletons [2], [3]. The dynamic models of robotic manipulators with elastic or viscoelastic joints have the twice number of independent variables compared to the robots with absolutely rigid joints [4], [5]. The analysis of the numerous studies on the position stabilization and trajectory tracking control problems for robot manipulators with elastic joints was carried out in the papers [6], [7], [8].

The dynamics of a fairly large class of mechanical systems is described by the generalized coordinates which are divided into positional and cyclic ones [9]. Such mechanical systems can perform the motions called the stationary ones [10]. To construct a control structure which provides a stationary motion stabilization of the mechanical systems with cyclic coordinates, the well-known results on the stationary motion stability are widely used [10], [11]. Note that a mandatory condition of constructing the controller for the mechanical systems with cyclic coordinates is the necessity of the position and velocity measurements. The solution to the steady motion stabilization problem for holonomic mechanical systems with cyclic coordinates has been proposed in [12], [13] by using the controllers without velocity measurements.

The solution to the motion control problem for elastic-joint robotic manipulators without velocity and accelerator measurements is quite important. In recent years, a large number of output position feedback control schemes for trajectory tracking of robotic manipulator with elastic joints have been developed. The control problems of elastic joined robotic manipulators were studied in [14], [15], [16], [17], [18] using only position measurements.
The aim of this paper is to obtain the output position feedback control structure to solve the stationary motion stabilization problem for a robotic manipulator with viscoelastic joints modeled as a mechanical system with cyclic coordinates using the positions measurements only.

Throughout this paper the following mathematical notations are used. Denote by \((\cdot)^t\) the transposition operation. Let \(\|x\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_l^2}\) be the vector norm of \(x \in \mathbb{R}^l\). We also introduce the class of Hahn functions \(\mathcal{K} = \{a : \mathbb{R}^+ \to \mathbb{R}^+\}\). Note that \(a \in \mathcal{K}\) if \(a(0) = 0\) and \(a(y)\) is strictly increasing for all \(y \in \mathbb{R}^+\).

2. Mathematical Model of the Robot Manipulator with Rigid and Viscoelastic Joints and Control Problem Formulation

A large class of robotic manipulators with rigid and viscoelastic joints is modeled as mechanical systems with cyclic coordinates. The kinetic energy of such a mechanical system can be as follows

\[
T = T_1 + T_2,
\]

\[
T_1(\dot{s}, \dot{r}, s) = \frac{1}{2} s' A_{11}(s) \dot{s} + s' A_{12}(s) \dot{r} + \dot{r}' A_{21}(s) \dot{s} + \dot{r}' A_{22}(s) \dot{r}, \quad T_2(\dot{s}) = \frac{1}{2} s' J \dot{S},
\]

where \(s \in \mathbb{R}^k\) and \(r \in \mathbb{R}^m\) are the generalized coordinates of the manipulator links. Moreover, the first \(s\) coordinates are the rotation angles of the part of the links controlled by electric drives with the rotation angles \(S \in \mathbb{R}^k\) of their output shafts relative to the corresponding links, \(A_{11} \in \mathbb{R}^{k \times k}, A_{12} = A_{21} \in \mathbb{R}^{k \times m}\), \(A_{22} \in \mathbb{R}^{m \times m}\) are the components of the positive-definite inertia matrix of the links \(A, J = \text{diag}(j_1, j_2, \ldots, j_k)\) is the output shaft inertia matrix, \((j_i > 0, i = 1, 2, \ldots, k)\).

Potential energy of a mechanical system in an uniform gravity field is given by

\[
\Pi(s, S) = \Pi^{(0)}(s) + \frac{1}{2}(s - S)^t C(s - S),
\]

where \(C\) is the shaft stiffness matrix.

We also assume that the mechanical system is also acted upon by viscous friction with generalized forces having the following form in accordance with the chosen coordinates

\[
Q_s = -F_1(s, \dot{s}) - F_2(\dot{s} - \dot{S}), \quad Q_r = -F_3 \dot{r}, \quad Q_S = -F_2(\dot{S} - \dot{s}),
\]

\[
F_1 \in C(\mathbb{R}^k \times \mathbb{R}^k \to \mathbb{R}^k), \quad F_1(s, 0) = 0, \quad s' F_1(s, \dot{s}) \leq 0,
\]

\[
F_2 = \text{diag}(f_1, f_2, \ldots, f_k), \quad F_3 = \text{diag}(f_{k+1}, f_{k+2}, \ldots, f_{k+m})
\]

\[(f_j \geq 0, j = 1, 2, \ldots, k + m).\]

Let us obtain the Lagrange equations for such a mechanical system in the coordinates \(r\) and \(S\)

\[
\frac{d}{dt} \left( \frac{\partial T_1}{\partial \dot{r}} \right) - \frac{\partial T_1}{\partial r} = -\frac{\partial \Pi^{(0)}(s)}{\partial s} - C(s - S) - F_1 - F_2(\dot{s} - \dot{S}),
\]

\[
\frac{d}{dt} \left( \frac{\partial T_1}{\partial \dot{s}} \right) = -F_3 \dot{r} + U_r,
\]

\[
J \ddot{S} = -C(S - s) - F_2(\dot{S} - \dot{s}) + U_S.
\]

Note that if the following holds

\[
U_S = U_S^{(0)} = C(S_0 - s_0) = \left. \frac{\partial \Pi^{(0)}(s)}{\partial s} \right|_{s=s_0} \frac{\partial T_1(0, \dot{r}, s)}{\partial s} \bigg|_{s=s_0, \dot{r} = \dot{r}_0},
\]

\[
U_r = U_r^{(0)} = F_3 \dot{r}_0.
\]
then the manipulator has a steady motion such as
\[
\dot{s} = 0, \quad s = s_0 = \text{constant}, \quad \dot{S} = 0, \quad S = S_0 = \text{constant},
\]
\[
\dot{r} = \dot{r}_0 = \text{constant}, \quad r = r_0(t) = \dot{r}_0 t.
\]

Let us introduce the following notions
\[
\Pi_s^{(0)} = \frac{\partial \Pi^{(0)}(s)}{\partial s} \bigg|_{s=s_0}, \quad T_t^{(0)} = \frac{\partial T_t(0, \dot{r}, s)}{\partial s} \bigg|_{s=s_0, \dot{r}=\dot{r}_0}.
\]

Let us consider the problem of finding the control actions
\[
U^{(1)} = U_S - U_S^{(0)}, \quad U^{(2)} = U_r - U_r^{(0)}
\]
stabilizing the motion (6) based on measuring only the coordinates \( S \) and \( r \).

3. Stabilization Problem Solution
Introduce the state errors
\[
x = s - s_0, \quad y = S - S_0, \quad z = r - r_0(t).
\]

Using (8), from (4) we find the error dynamics equations
\[
\frac{d}{dt} \left( \frac{\partial T_1}{\partial \dot{x}} \right) - \frac{\partial T_1}{\partial x} = -\frac{\partial \Pi^{(0)}}{\partial x} + \Pi_s^{(0)} - T_t^{(0)} + C(y - x) + F_2(y - \dot{x}) - F^{(1)},
\]
\[
\dot{\varphi} = C(x - y) + F_2(\dot{x} - \dot{y}) + U^{(1)},
\]

\[
\frac{d}{dt} \left( \frac{\partial T_1}{\partial \dot{z}} \right) = -F_3 \dot{z} + U^{(2)},
\]

where
\[
T_1 = T_1(\dot{x}, \dot{z}, s_0 + x), \quad \Pi^{(0)} = \Pi^{(0)}(s_0 + x), \quad F^{(1)} = F^{(1)}(s_0 + x, \dot{x}).
\]

Using [19], we introduce the following functions: scalar functions \( \Pi^{(1)} \in C^1(R^k \to R) \), \( \Pi^{(2)} \in C^1(R^m \to R^m) \), \( \Pi^{(j)}(0) = 0 \), \( j = 1, 2 \); vector functions \( f^{(1)} \in C^1(R^k \to R^k) \), \( f^{(2)} \in C^1(R^m \to R^m) \), the components of which \( f_1^{(1)}, f_1^{(2)} \) have a limited number of zeros in a bounded area \( \Gamma_1 \subset R^k \) and \( \Gamma_2 \subset R^m \), matrix functions \( P^{(1)} \in C^1(R^- \to R^{k \times k}) \), \( P^{(2)} \in C^1(R^- \to R^{m \times m}) \) such as
\[
0 \leq \beta^j P^{(j)}(\nu) \beta \leq \alpha_1(\nu) ||\beta||^2, \quad j = 1, 2,
\]
\[
\alpha_2(\nu) ||\beta||^2 \leq \frac{dP^{(j)}(\nu)}{d\nu} \leq \alpha_3(\nu) ||\beta||^2, \quad j = 1, 2,
\]
\[
\alpha_k(\nu) > 0, \int_{-\infty}^{0} \alpha_k(\nu) d\nu = \gamma_k < \infty, \quad k = 1, 2, 3.
\]

Let us show that the problem of motion stabilization is solved by the controller
\[
U^{(1)} = -\frac{\partial \Pi^{(1)}}{\partial y} - \left( \frac{\partial f^{(1)}}{\partial y} \right) \int_0^t P^{(1)}(\tau - t)(f^{(1)}((y(t)) - f^{(1)}(y(\tau))) d\tau,
\]
\[
U^{(2)} = -\frac{\partial \Pi^{(2)}}{\partial z} - \left( \frac{\partial f^{(2)}}{\partial z} \right) \int_0^t P^{(2)}(\tau - t)(f^{(2)}((z(t)) - f^{(2)}(z(\tau))) d\tau.
\]
Let us prove the following statement.

**Statement 1.** Let the control action (12) be such that:

\[
\begin{align*}
\Pi^*(x, y) &= \Pi^{(0)}(s_0 + x) - \Pi^{(0)}(s_0) - x'\Pi_{s}^{(0)} + \frac{1}{2}(x-y)'K(x-y) + \Pi^{(1)}(y) \\
-\frac{1}{2}r_0^2A_{22}(s_0 + x)r_0 + \frac{1}{2}\nu_0^2A_{22}(s_0)r_0 + x'T_{1a}^{(0)} &\geq 0 > a_1(||x|| + ||y||), \\
\Pi^{(2)}(z) &\geq a_2(||z||), \quad \left| \left| \frac{\partial\Pi^*(x, y)}{\partial x} \right| \right| + \left| \left| \frac{\partial\Pi^*(x, y)}{\partial y} \right| \right| \geq a_3(||x|| + ||y||), \quad (13)
\end{align*}
\]

Then, the controller \( U' = (U_S^{(0)} + U^{(1)}, U_{r}^{(0)} + U^{(2)}) \) solves the motion stabilization problem of (6) while ensuring its nonlocal uniform asymptotic stability.

**Proof.**

Let \((x(t), y(t), z(t))\) be a solution of (9). Let us choose the Lyapunov functional candidate such as

\[
V = V_1 + V_2,
\]

\[
V_1(\dot{x}(t), \dot{y}(t), \dot{z}(t), x(t), y(t), z(t)) = \frac{1}{2}(\dot{x}'A_{11}(s_0 + x)\dot{x} + \dot{z}'A_{12}(s_0 + x)\dot{z} + \dot{z}'A_{21}(s_0 + x)\dot{x} + \dot{z}'A_{22}(s_0 + x)\dot{z}) + \frac{1}{2}\nu_0^2J\dot{y} + \Pi^*(x(t), y(t)) + \Pi_1(z(t)),
\]

\[
V_2(y(t), z(t)) = \frac{1}{2} \int_0^t (f_1(y(t)) - f_1(y(\nu)))'P_1(\nu - t)(f_1(y(t)) - f_1(y(\nu)))d\nu
\]

\[
+ \frac{1}{2} \int_0^t (f_2(z(t)) - f_2(z(\nu)))'P_2(\nu - t)(f_2(z(t)) - f_2(z(\nu)))d\nu.
\]

(14)

For the time derivative of the functional (14) we find the estimate

\[
\dot{V} = W_1 + W_2,
\]

\[
W_1 = -\dot{x}F_1(s_0 + x, \dot{x}) - \dot{z}F_3 - (\dot{x} - \dot{y})'F_2(\dot{x} - \dot{y}) \leq 0,
\]

\[
W_2 = \frac{1}{2} \int_0^t (f_1(y(t)) - f_1(y(\nu)))'\frac{\partial P_1(\nu - t)}{\partial \nu}(f_1(y(t)) - f_1(y(\nu)))d\nu
\]

\[
+ \frac{1}{2} \int_0^t (f_2(z(t)) - f_2(z(\nu)))'\frac{\partial P_2(\nu - t)}{\partial \nu}(f_2(z(t)) - f_2(z(\nu)))d\nu
\]

\[
\leq -\frac{1}{2} \int_0^t \alpha_2(\nu - t)||f_1(y(t)) - f_1(y(\nu))||^2d\nu - \frac{1}{2} \int_0^t \alpha_3(\nu - t)||f_2(z(t)) - f_2(z(\nu))||^2d\nu \leq 0.
\]

(15)

The set

\[
\{\dot{V} = 0\} \subset \{f_1(y(\nu)) = f_1(y(t)), f_2(z(\nu)) = f_2(z(t))\}
\]

\[
\subset \{y(\nu) = y(t), z(\nu) = z(t), 0 \leq \nu \leq t, t \in R^+\}
\]

(16)

can contain only the solutions of (9), (12) such as \( y(t) = constant, z(t) = constant. \)

But from the equations (16) we find that by virtue of the conditions (13) such solutions can only be the zero ones

\[
x(t) = 0, \quad y(t) = 0, \quad z(t) = 0.
\]

(17)

Based on the theorems from [19], [20] we obtain the end of the proof.
Remark 1. Statement 1 allows us to obtain the semi-global stabilization of motion (6). The region of uniform attraction of this motion $M_1 \subset M_2 \subset \mathbb{R}^n$ is predetermined by the inequality

$$\sup(V_1(\dot{x}, \dot{y}, \dot{z}, x,y,z) : (x,y,z,\dot{x}, \dot{y}, \dot{z}) \in M_1) < \inf(V_1(\dot{x}, \dot{y}, \dot{z}, x,y,z) : (x,y,z,\dot{x}, \dot{y}, \dot{z}) \in M_2).$$

(18)

4. The motion stabilization problem of a plane three-link manipulator

Using Fantoni and Lozano [21], consider the control problem of a manipulator on a fixed base. The manipulator consists of three absolutely rigid links. Structural elements of the mechanical system are connected by three cylindrical joints $O_1$, $O_2$, and $O_3$ allowing the manipulator to rotate in the horizontal plane. The position of the mechanical system is described by three angular coordinates such as: $r$ is the rotation angle of the first link about the vertical axis $O_1x$; $s_1$ and $s_2$ are the rotation angles of the second and third links relative to the previous one.

We use the same notation as Fantoni and Lozano [21]: $m_k$, $l_k$, $l_{kx}$, and $J_k$ are the mass, length, distance from $k$-th joint to the mass center of $k$-th link and the moment of inertia of the $k$-th link relative to its center of mass ($i = 1, 2, 3$) correspondingly.

We assume that the control actions in the joints $O_2$ and $O_3$ are carried out by the motors, and the rotation of the rotors of these motors is described by the equations

$$J_1\ddot{s}_1 = c_1(s_1 - S_1) + f_1(\dot{s}_1 - \dot{S}_1) + U_{S_1},$$

$$J_2\ddot{s}_2 = c_2(s_2 - S_2) + f_2(\dot{s}_2 - \dot{S}_2) + U_{S_2},$$

(19)

where $S_1$ and $S_2$ are the angles of rotors rotation, $J_1 > 0$ and $J_2 > 0$ are the moments of inertia of the rotors.

The torques of viscous friction act in the joints $O_2$ and $O_3$ such as

$$M_1 = F_1(s, \dot{s}) + f_1(\dot{S}_1 - \dot{s}_1), \quad M_2 = F_2(s, \dot{s}) + f_2(\dot{S}_2 - \dot{s}_2),$$

(20)

where $F_k(s,0) = 0$, $F_k(s,\dot{s})\dot{s} \leq 0 \ (k = 1, 2)$.

The joint rotation of $O_1$ wherein the frictional torque acts $F_3 = -f\dot{r}$, is driven by a rotor creating the control torque $U_3$.

Let us make the following notations

$$y_1 = S_1 - S_{1}^0, \quad y_2 = S_2 - S_{2}^0, \quad z_1 = r(t) - r_0 t.$$

(21)

$$U_1^{(1)} = -u_1(y_1(t)) - \frac{\partial f_1(y_1(t))}{\partial y_1} \int_0^t p_1(\nu - t)(f_1(y_1(t)) - f_1(y_1(\nu)))d\nu,$$

$$U_2^{(1)} = -u_2(y_2(t)) - \frac{\partial f_2(y_2(t))}{\partial y_2} \int_0^t p_2(\nu - t)(f_2(y_2(t)) - f_2(y_2(\nu)))d\nu,$$

$$U^{(2)} = -u_3(z(t)) - \frac{\partial f_3(z(t))}{\partial z} \int_0^t p_3(\nu - t)(f_3(z(t)) - f_3(z(\nu)))d\nu,$$

(22)

where

$$u_j(\xi) = k_j \sin \frac{\xi}{2}, \quad k_1 > 2\theta_3 + 2\theta_4 + 4\theta_5, \quad k_2 > 4\theta_2 + 2\theta_6, \quad k_3 > 0,$$

$$f_j(\xi) = \sin \xi, \quad p_j(\nu) = \beta_j^{-\alpha_j\nu}, \quad \beta_j > 0, \quad \alpha_j > 0, \quad j = 1, 2, 3,$$

(23)

the parameters $\theta_i \ (i = 2, \ldots, 7)$ were introduced in [21].
Based on Statement 1, we can see that for the manipulator (19) under the action of the control torques

\[ U_{S1} = U^{(0)}_1 + U^{(1)}_1, \quad U_{S2} = U^{(0)}_2 + U^{(1)}_2, \quad U_r = U^{(2)} \]  

the stabilizability of the stationary motion is achieved, and each perturbed motion of the mechanical system approaches arbitrarily close to one of the motions

\[ \dot{r} = \dot{r}_0 = \text{constant}, \quad \dot{s}_1 = \dot{s}_2 = 0, \quad \hat{S}_1 = \hat{S}_2 = 0, \]
\[ r(t) = \dot{r}_0 t, \quad s_1^{(j)} = s_1^{10} + 2\pi j, \]
\[ s_2^{(l)} = s_2^{20} + 2\pi l, \quad j, l \in \mathbb{Z} \]

as \( t \to \infty \).

For the purpose of numerical simulation, the stationary motion is chosen as

\[ r_0(t) = 10t \, \text{rad}, \quad s_{10} = 2 \, \text{rad}, \quad s_{20} = 3 \, \text{rad}. \]  

The control gain parameters are chosen such as

\[ k_1 = k_2 = k_3 = 5.5, \quad \alpha_1 = \alpha_2 = \alpha_3 = 2, \quad \beta_1 = \beta_2 = \beta_3 = e. \]  

We consider the simulations results using the initial conditions for the robot manipulator such as

\[ r(0) = 2.5 \, \text{rad}, \quad s_1(0) = s_{10} + 1.5 \, \text{rad}, \quad s_2(0) = s_{20} + 3.5 \, \text{rad}, \]
\[ \dot{r}(0) = \dot{s}_1(0) = \dot{s}_2(0) = -1.5 \, \text{rad/sec}. \]  

In Figures 1 – 3 we show the link trajectories as well as the references for the manipulator. From these results, it can be seen that the controller (24) provides the asymptotic convergence of the real trajectory to the stationary motion.
5. Conclusion

The paper deals with the motion control problem of robotic manipulators with viscoelastic joints. Models of manipulators are considered in which some of the generalized coordinates are cyclic and the rest are positional. Such models can have steady-state motions in which cyclical velocities and positional coordinates are constant throughout the motion. The output feedback control problem is solved without velocity measurements to stabilize a given steady-state motion of a robot manipulator.

Acknowledgments

This work was supported by Russian Foundation for Basic Research, grant No. 19-01-00791.

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