Refractive Index of an anisotropic Quark-Gluon-Plasma medium in an effective description of hot QCD

M. Yousuf Jamal, Sukanya Mitra and Vinod Chandra

Indian Institute of Technology Gandhinagar, Gandhinagar-382355, Gujarat, India

The present investigation involves explorations on the chromo-dielectric properties of the hot QCD medium produced in relativistic heavy-ion collisions in terms of refractive index. The anisotropic aspect of the hot QCD medium are incorporated by introducing the anisotropy in a particular direction. The isotropic/equilibrium modeling is done within an effective quasi-particle model of hot QCD medium. The possibilities of negative refraction in the medium is also explored in terms of the Depine-Lakhtakia index in the isotropic case and defining refractive indices in appropriate directions in the anisotropic case. Interestingly, both anisotropy and medium effects play significant roles in deciding the optical properties of the hot QCD/Quark-Gluon-Plasma (QGP) medium.

Keywords: Quark-Gluon-Plasma; refractive index; effective fugacity; gluon self-energy; negative refraction; anisotropic QGP

PACS: 12.38.Mh, 13.40.-f, 05.20.Dd, 25.75.-q

I. INTRODUCTION

The hot and dense nuclear matter (Quark-Gluon-Plasma (QGP)) in relativistic heavy-ion collisions (RHIC) behaves more like a near perfect fluid rather than a non-interacting ultra-relativistic gas of quarks (anti-quarks) and gluons [1–3]. The liquidity of the QGP is quantified in terms of the collective flow coefficients such as elliptic flow, $v_2$ and others. The nature of these coefficients in RHIC as a function of transverse momentum requires the QGP to be a near perfect fluid with very small value for the shear viscosity to entropy ($\eta/S$) (smallest among almost all the known fluids in nature). Apart from the collective behavior, quarkonia suppression and strong jet quenching [1, 2] are other interesting phenomena associated with the QGP that highlight its plasma aspects (reminisce of color screening and energy loss).

The goal of the present investigations is to understand plasma aspects of the QGP medium while considering it as a dielectric one and capturing the related interesting physics aspects such as collective plasma excitations and the optical properties such as refractive index of the medium. To understand any medium (say dielectric), the medium must be exposed to the external fields (i.e., electric and magnetic fields). Depending upon its response to the fields, we call it isotropic, anisotropic linear or nonlinear. Once the response of the medium is known in terms of the permittivity and the permeability, the propagation of the electromagnetic/ chromo-electromagnetic waves could be explored in terms of refractive/chromo-refractive indices of the medium.

In the present manuscript, the chromo-refractive index for the hot QCD/QGP (isotropic as well as anisotropic) medium have been investigated in terms of the chromo-electric permittivity and the chromomagnetic permeability within semi-classical transport theory. While setting-up linear transport theory, one must have an adequate modeling of the isotropic/equilibrium (global/local) state of the medium. As the isotropic/equilibrium state could be described in terms of interacting QGP equation of state (EoS), either computed from lattice QCD or improved HTL (Hard thermal loop) resummed perturbation theory. To that end, we employ the quasi-particle description of these equations of state (EoSs) that maps the effects of interactions in the quasi-gluon and quasi-quark distribution functions. The starting point is always the computation of gluon polarization tensor in the hot QCD/QGP medium either within transport theory with the above-mentioned distribution functions or within finite temperature field theory and then extract the above-mentioned responses.

As the momentum anisotropy is present in all the stages of the heavy-ion collisions, the inclusion of the anisotropy is inevitable while studying any of the aspects of the QGP. This anisotropy could be included at the level of distribution functions just by extending the isotropic distribution functions obtained from quasi-particle model following [4, 5]. The collective plasma modes within this approach have already been studied extensively with ideal EoS/leading order HTL in Refs. [6–12] and for the interacting EoS within a quasi-particle model in our previous work [13], (for both iso(aniso)trope QGP). The hot QCD/QGP medium effects were seen to induce significant modifications to the collective modes. The prime goal here is to define a chromo-refractive index for the hot QCD medium and search for possibilities of negative refraction (NR) therein for both isotropic and anisotropic cases.

There are a few studies in which the refractive index of weakly as well as strongly coupled plasma have been
investigated that the refractive index may be negative in some of the materials. Later on, it was shown that there is a certain probability of the QGP to have negative refractive index (NRI) for some frequency (ω) range. Afterwards, there have been various attempts to study the refractive index using holographic model for strongly coupled plasma. Juan Liu et al. have studied the refractive index for weakly coupled plasma. They further extended their study of chromo-refractive index using kinetic theory.

In QCD, the basic mathematical quantity, which is needed to understand the response functions, is the gluon polarization tensor. Considering the fact that in the abelian limit QCD possess the similar features as QED, Jiang et al. studied for the first time the refractive index of gluon (chromo-refractive index) in the case of the viscous QGP. They further extended their study of chromo-refractive index using kinetic theory with a Bhatnagar-Gross-Krook (BGK) collisional kernel. It is important to note that, in all the above mentioned approaches, interactions among quarks and gluons in the QGP medium have not been included since the analysis assumes the QGP as an ideal gas (non-interacting, ultra-relativistic) of gluons and quark-antiquarks. Here, we have incorporated two important aspects while investigating the chromo-refractive index of the QGP medium, viz., the EoS effects (via quasi-particle description) and the momentum anisotropy. In the context of the refractive index of the QGP, the former is not yet been included, and the latter is not extensively studied. As the response of the medium to electric/chromo-electric and magnetic/chromo-magnetic part is not similar, based on this asymmetry in the electric and magnetic sector, the plasma can be classified as magnetizable and non-magnetizable. Here, main interest is in the magnetizable case where in the certain frequency range there is a possibility for the refractive index to be negative. This aspect of negative refraction is not seen in the case of non-magnetizable weakly coupled plasma.

The paper is organized as follows. In section II, the basic formalism for finding the chromo-electromagnetic responses and the refractive index has been presented along with the modeling of hot QCD medium for isotropic as well as small anisotropic hot QCD medium. Section III deals with results and discussions. The conclusions and the possible future extensions of the present work are offered in Section IV.

II. QGP AS AN OPTICAL MEDIUM

Propagation of color-electromagnetic waves in the QGP medium treating it as a dielectric medium has recently been studied in Ref. While defining the refractive index for the medium. As mentioed earlier, the present work is the generalisation of for the QGP medium where the interactions are included by considering the realistic hot QCD/QGP EoSs in terms of their effective quasi-particle descriptions. We will calculate the color refractive index for QGP, while considering the effective quasi-particle model for hot QCD/QGP along the lines of , via, the chromo-electric permittivity and the chromo-magnetic permeability (μ). Before, we compute the chromo-refractive index let us first discuss in brief the refractive index and its general aspects in ω - k space.

For a wave propagating in a continuous dielectric medium, its electric field vector (E) connects the displacement vector (D) through the electric permittivity tensor (εij) and the magnetic field vector (B) can be expressed in terms of the magnetic field induction (H) through magnetic permeability tensor (μij) as

\[ D_i = \varepsilon_{ij}(\omega, k)E_j, \quad B_i = \mu_{ij}(\omega, k)H_j, \] (1)

Here, i, j = 1, 2, 3 are spatial indices, ω is the frequency and k = k is the propagation vector. In the context of QGP, a covariant treatment of the theses quantities is required. Therefore, one uses the fluid four-velocity uμ to define the four- fields Eμ and Bμ in the Fourier space as

\[ \tilde{E}^\mu = u_\alpha F^{\alpha\mu}, \quad \tilde{B}^\mu = \frac{1}{2} \epsilon^{\mu\rho\alpha\beta} u_\rho F_{\alpha\beta}, \] (2)

where \( \epsilon^{\mu\rho\alpha\beta} \) is the four-dimensional Levi-Civita symbol and \( \mu, \rho, \alpha \) and \( \beta \) are here Lorentz four-indices (not to confuse with structure functions and chromo-magnetic permeability). Using the above equation, we can immediately write the field tensor \( F^{\mu\nu} \) as

\[ F^{\mu\nu} = \tilde{E}^\mu u^\nu - \tilde{E}^\nu u^\mu - \epsilon^{\mu\rho\alpha\beta} \tilde{B}_\alpha u_\beta. \] (3)

While including the medium effects, the effective action can be expressed as

\[ S_{eff} = S_0 - \frac{1}{2} \int \frac{d^4K}{(2\pi)^4} A^\mu(K)\Pi_{\mu\nu}(K)A^\nu(-K) + ... \] (4)

where \( A^\mu(K) \) is the soft gauge field in momentum space and \( K_\mu = (\omega, k) \). The medium effect is characterised by the polarisation tensor \( \Pi_{\mu\nu}(K) \). Where the free action \( S_0 \) reads,

\[ S_0 = -\frac{1}{2} \int \frac{d^4K}{(2\pi)^4} \left[ \epsilon_0 \tilde{E}^\mu(K)\tilde{E}_\mu(-K) - \frac{\tilde{B}^\mu(K)\tilde{B}_\mu(-K)}{\mu_0} \right]. \] (5)

Here, \( \epsilon_0 \) and \( \mu_0 \) are the permittivity and permeability of the free space respectively. In terms of \( \epsilon_{\mu\nu} \) and \( \mu \), the effective action can be written as

\[ S_{eff} = \frac{1}{2} \int d^4K \left[ \epsilon_{\mu\nu} \tilde{E}^\mu(K)\tilde{E}^\nu(-K) \right. \]

\[ \left. - \frac{1}{\mu} \tilde{B}_\mu(K)\tilde{B}^\mu(-K) \right]. \] (6)
One can then extract the chromo-electric permittivity and the chromo-magnetic permeability from the effective action $S_{eff}$ in terms of polarization tensor $\Pi^{\mu\nu}$, which include all the medium effects (comparing Eq. (6) to Eq. (4)). Let us now proceed to the computation of chromo-refractive index using the response functions $(\epsilon(\omega, k), \mu(\omega, k))$ for the isotropic and anisotropic medium independently.

A. The chromo-refractive index for the hot QCD medium

As stated earlier, the gluon polarization tensor, $\Pi^{\mu\nu}$ is required to define the responses. The formalism of $\Pi^{\mu\nu}$ for interacting EoSs within the quasi-particle description is a straightforward extension of Ref. [22]. This has been presented in detail in Ref. [13], and a brief account of that is presented below before we define the color refractive index (RI) for the QGP medium. To obtain $\Pi^{\mu\nu}$ in QCD medium, we start with an arbitrary particle distribution function, denoting with $f_i(\mathbf{p}, x)$ where index $i$ is labeling the $i$-th particle. In the abelian limit, the space time evolution of the distribution function in the medium is understood from the Boltzmann-Vlasov HTL [29–35] effective theory.

As the gluon polarization tensor, $\Pi^{\mu\nu}$ is defined in terms of the collision term. We are focusing on the near equilibrium case which allows us to neglect the effects from collisions and so $C[f_i] = 0$. The second rank tensor, $F^{\mu\nu}$ is the chromo-electromagnetic strength tensor which either represents an external field applied to the system, or/and is generated self-consistently by the four-currents present in the plasma, as follows,

$$\partial_{\nu} F^{\mu\nu} = 4\pi j^{\mu}, \quad (8)$$

where $q_i$ is the charge of the plasma species $i$, and $C[f_i]$ denotes the collision term. We are focusing on the near equilibrium case which allows us to neglect the effects from collisions and so $C[f_i] = 0$. The second rank tensor, $F^{\mu\nu}$ is the chromo-electromagnetic strength tensor which either represents an external field applied to the system, or/and is generated self-consistently by the four-currents present in the plasma, as follows,

$$j^{\mu}(x) = \Sigma_i q_i \int d\Gamma p^{\mu} f_i(\mathbf{p}, x), \quad (9)$$

and

$$d\Gamma = \frac{d^3 p}{(2\pi)^3 E}. \quad (10)$$

Eq. (7), can be solved in the linear response approximation i.e., the equation can be linearized around the stationary and homogeneous state described by the distribution $f_i^{eq}(\mathbf{p})$ which is assumed to be neutral when there is no current. The distribution function is then decomposed as:

$$f_i(x) = f_i^{eq}(\mathbf{p}) + \delta f_i(\mathbf{p}, x), \quad (11)$$

where

$$f_i^{eq}(\mathbf{p}) \gg \delta f_i(\mathbf{p}, x), \quad (12)$$

So the induced polarization current will be

$$j_{ind}^{\mu}(x) = \Sigma_i q_i \int d\Gamma p^{\mu}\delta f_i(\mathbf{p}, x). \quad (13)$$

A particular energy scale is needed in order to study the collective behavior (plasma behavior) of hot QCD medium. We choose to work on the scale where the collective motion in the hot medium first appears. At that scale, the soft momentum $p \sim gT \ll T$, the magnitude of the field fluctuations is of the order of $A \sim \sqrt{T}T$ and derivatives are $\partial_{\nu} \sim gT$. Note that there have been several attempts to understand the collective behavior of hot QCD medium either within the semi-classical theory or HTL [29–35] effective theory.

The second rank tensor, $F^{\mu\nu}$, induced by a soft gauge field, $A^{\mu}$ can be obtained as follows,

$$j_{ind}^{\mu}(x) = g \int d\Gamma p^{\mu}\delta f(\mathbf{p}, x). \quad (14)$$

Where $\delta f(\mathbf{p}, x)$, contains the fluctuating part and given as

$$\delta f(\mathbf{p}, x) = 2N_c \delta f_q(\mathbf{p}, x) + N_f(\delta f_s(\mathbf{p}, x) - \delta f_{\bar{q}}(\mathbf{p}, x)). \quad (15)$$

Here $\delta f_s(\mathbf{p}, x)$, $\delta f_q(\mathbf{p}, x)$ and $\delta f_{\bar{q}}(\mathbf{p}, x)$ are the fluctuating parts of the gluon, the quark and anti-quark densities respectively. After solving the transport equations for the fluctuations $\delta f_q$ and $\delta f_{\bar{q}}$, we get the induced current in the Fourier space as

$$j_{ind}^{\alpha}(K) = g^2 \int \frac{d^3 p}{(2\pi)^3} \epsilon^{\alpha\beta} \frac{\partial}{\partial p^\beta} f(\mathbf{p}) \left[ g_{\alpha\beta} - \frac{u_{\alpha} K_{\beta}}{K \cdot u + i\epsilon} \right] A^{\alpha}(K), \quad (16)$$

where $\epsilon$ is a very small parameter needed to avoid unwanted infinities and will be sent to zero in the end. The distribution function, $f(\mathbf{p})$, in terms of gluons and quark distribution functions read,

$$f(\mathbf{p}) = 2N_c f_q(\mathbf{p}) + N_f(f_s(\mathbf{p}) + f_{\bar{q}}(\mathbf{p})). \quad (17)$$

Here, $f_q(\mathbf{p})$ and $f_{\bar{q}}(\mathbf{p})$ denote the isotropic/equilibrium quark/antiquark (at zero baryon density, $f_q \equiv f_{\bar{q}}$) and gluon distribution functions respectively.

In the linear approximation, the equation of motion for the gauge field can be obtained in Fourier space as,

$$j_{ind}^{\alpha}(K) = \Pi^{\mu\nu}(K) A^{\nu}(K). \quad (18)$$

Here, $\Pi^{\mu\nu}(K) = \Pi^{\mu\nu}(K)$ and follows the Ward’s identity, $K_{\mu} \Pi^{\mu\nu}(K) = 0$, i.e., the self-energy tensor is symmetric and transverse in nature. From Eq. (16) and Eq. (18) we can obtain $\Pi^{\mu\nu}(K)$ as

$$\Pi^{\mu\nu}(K) = g^2 \int \frac{d^3 p}{(2\pi)^3} u^{\alpha} \frac{\partial f(\mathbf{p})}{\partial p^\beta} \left[ g^{\nu\beta} - \frac{K^{\nu} u^\nu}{K \cdot u + i\epsilon} \right]. \quad (19)$$
The quantity, $u^\mu = (1, k/|k|)$ is a light-like vector describing the propagation of plasma particle in space-time with the signature, $g_{\mu \nu} = diag(1,-1,-1,-1)$.

As mentioned earlier the isotropic modeling of the medium is done within a quasi-particle description of hot QCD medium where the medium effects have been encoded in the effective gluon and effective quark fugacities respectively [3, 7]. These fugacity parameters define the effective gluon and quark/antiquark momentum distribution functions for the isotropic/equilibrated medium. There are several other quasi-particle models where the medium modifications are captured in terms of effective mass for gluons and quarks [30, 31]. Others are NJL and PNJL based effective models [39], and effective fugacity quasi-particle description of hot QCD (EQPM) [8, 9] employed here and recent quasi-particle models based on the Gribov-Zwanziger (GZ) quantization results leading to non-trivial IR-improved dispersion relation in terms of Gribov parameter [10]. The present analysis considers the EQPM for the investigations on the properties of the hot and dense medium in RHIC. These quasi-particle models have shown their utility while studying transport properties of the QGP [11, 12]. In Ref. [13], the ratio of electrical conductivity to shear viscosity has been explored within the framework of the effective mass model. Mitra and Chandra [18] computed the electrical conductivity and charge diffusion coefficients within EQPM. In the context of quarkonia physics [40, 41] and thermal particle production [51, 52] too, the EQPM played an important role. There are issues with the above approaches while comparing the transport coefficients with their phenomenological estimates [53, 54] from experimental observables at RHIC. Nevertheless, these quasi-particle approaches serve the purpose of modeling the equilibrated/isotropic state of the QGP which is crucial for the transport theory computations.

Three hot QCD equations of state (EoSs) have been considered here and described in terms of EQPM. There are, viz., the (2+1)-flavor lattice QCD EoS [57] at physical quark mass (LEoS), the very recent (2+1)-lattice EoS from hot QCD collaboration [58] (LEoSBZ), and the 3-loop HTL perturbative EoS that has recently been computed by N. Haque et al. [50, 60] which agrees reasonably well with the recent lattice results [55, 61]. The EQPM serves as the input in terms of the effective quasi-particle distribution functions (describing of either of the three EoSs), $f_{eq} \equiv \{f_g, f_q\}$ (describing the strong interaction effects in terms of effective fugacities $z_{g,q}$) as [3, 8]:

$$
    f_{g/q} = \frac{z_{g/q} \exp[-\beta E_p]}{1 + z_{g/q} \exp[-\beta E_p]}, 
$$

(20)

here $\beta = 1/T$, $T$ is the temperature in energy units, $E_p = |p|$ for the gluons and $\sqrt{|p|^2 + m_q^2}$ for the quark degrees of freedom ($m_q$ denotes the mass of the quarks).

This leads to the dispersion relation,

$$
    \omega_{g/q} = E_p + T^2 \partial_T \ln(z_{g/q}). 
$$

(21)

One further requires, Debye mass ($m_D$) computed within EQPM and effective QCD coupling (depicting charge renormalization) in the medium. To compute these quantities, we follow our previous works [13, 14] and references therein.

1. Response Functions for isotropic hot QCD medium

To obtain the response functions in the isotropic medium, we first obtain the $\Pi^{\mu \nu}$ for isotropic case by solving the Eq. (19). For that, we need to construct a covariant form of $\Pi^{\mu \nu}$ analytically by making the possible combination of available symmetric tensors, $\{g_{\mu \nu}, K_{\mu} K_{\nu}, u_{\mu} u_{\nu}, K_{\mu} u_{\nu} + K_{\nu} u_{\mu}\}$, following [22]. One of the possible combination to construct $\Pi^{\mu \nu}(\omega, k)$ in isotropic case can be obtained by using,

$$
    P_{\mu \nu} = g_{\mu \nu} - u_{\mu} u_{\nu} + \frac{1}{k^2} (K_{\mu} - \omega u_{\mu}) (K_{\nu} - \omega u_{\nu}), 
$$

$$
    Q_{\mu \nu} = -\frac{1}{k^2 K^2} (\omega K_{\mu} - K^2 u_{\mu}) (\omega K_{\nu} - K^2 u_{\nu}), 
$$

(22)

as

$$
    \Pi_{\mu \nu}(\omega, k) = \Pi_T(\omega, k) P_{\mu \nu} + \Pi_L(\omega, k) Q_{\mu \nu}, 
$$

(23)

where $\Pi_T(\omega, k)$ is the transverse and $\Pi_L(\omega, k)$ is the longitudinal part of the self-energy. Using the following contractions,

$$
    (g_{\mu \nu} - u_{\mu} u_{\nu}) \Pi^{\mu \nu}(\omega, k) = 2\Pi_T(\omega, k) + (1 + \frac{k^2}{K^2}) \Pi_L(\omega, k), 
$$

(24)

and

$$
    u_{\mu} \Pi^{\mu \nu} u_{\nu} = -\Pi_L(\omega, k) \frac{k^2}{K^2}. 
$$

(25)

The form of $\Pi_T(\omega, k)$ and $\Pi_L(\omega, k)$ can be obtained as follows:

$$
    \Pi_T(\omega, k) = \frac{m_D^2}{2} \frac{\omega^2}{k^2} \left[ 1 - \frac{\omega^2 - k^2}{2\omega k} \log \frac{\omega + k}{\omega - k} \right], 
$$

(26)

$$
    \Pi_L(\omega, k) = \frac{m_D^2}{2} \left[ 1 - \frac{\omega^2}{k^2} \right] \left[ 1 - \frac{\omega}{2k} \log \frac{\omega + k}{\omega - k} \right]. 
$$

(27)

For real valued $\omega$, $\Pi_T(\omega, k)$ and $\Pi_L(\omega, k)$ are real for all $\omega > k$ and complex for $\omega < k$. Since for the real valued $\omega$ the quantity inside the log is give as

$$
    \log \left( \frac{\omega + k}{\omega - k} \right) = \log \left( \frac{k + \omega}{k - \omega} \right) - i\pi \Theta(k - \omega). 
$$

(28)
where \( \Theta \) is the heavyside theta function. The \( m^2_T \) is given as \( \text{[13, 52]} \)

\[
m^2_T = -4\pi\alpha_s(T) \left( 2N_c \int \frac{d^3p}{(2\pi)^3} \partial_p f_q(p) \right) + 2N_f \int \frac{d^3p}{(2\pi)^3} \partial_p f_q(p),
\]

(29)

where, \( \alpha_s(T) \) is the QCD running coupling constant at finite temperature \( T \) and is described in detail in the context of EQPM in \( \text{[48]} \).

In the case of isotropic medium, using the gluon self-energy, one can obtain the chromo-electric permittivity and the chromo-magnetic permeability from Eq. (30) as:

\[
e(\omega, k) = (1 - \frac{\Pi_L(\omega, k)}{K^2}) g_{\mu\nu},
\]

(30)

where, \( g_{\mu\nu} \) are defined as:

\[
\frac{1}{\mu(\omega, k)} = 1 + \frac{K^2\Pi_T(\omega, k) - \omega^2\Pi_L(\omega, k)}{k^2K^2}.
\]

(31)

Substituting the self-energy from Eq. (26) and Eq. (27) into Eq. (30) and Eq. (31) we obtain

\[
\epsilon(\omega, k) = 1 + \frac{m^2_T}{k^2} \left( 1 - \frac{\omega}{2k} \log \frac{\omega + k}{\omega - k} \right)
\]

(32)

and

\[
\mu(\omega, k) = \frac{4k^4}{4k^4 + 6m^2_T\omega^2 + m^2_T(k^2 - 3\omega^2)\log \frac{\omega + k}{\omega - k}}.
\]

(33)

For isotropic medium, if the phase velocity of light is much larger than the thermal velocity of plasma particles, the spatial dispersion is small, and \( \epsilon(\omega, k) \) and \( \mu(\omega, k) \) can be assumed to be independent of \( k \) or equivalently we can perform \( \epsilon(\omega, k) \) and \( \mu(\omega, k) \) expansion at \( k \approx 0 \). Performing small \( k \) expansion to Eq. (32) and (33), we get respectively \( \epsilon(\omega) \) and \( \mu(\omega) \) as:

\[
\epsilon(\omega) \approx 1 - \frac{m^2_T}{3\omega^2} + \mathcal{O}(k^2),
\]

(34)

\[
\mu(\omega) \approx \frac{1}{1 - \frac{2m^2_T}{15\omega^2}} + \mathcal{O}(k^2).
\]

(35)

2. Response Functions for anisotropic hot QCD medium

To describe the anisotropic hot QCD medium, we follow the approach of \( \text{[8, 22]} \). In these approaches, the anisotropic momentum distribution functions for the gluons and quark-antiquarks are obtained by rescaling (stretching and squeezing) of only one direction in momentum space function as:

\[
f_\xi(\hat{p}) = f(\sqrt{p^2 + \xi(p \cdot \hat{n})^2})
\]

(36)

This introduces one more degree of freedom, viz., the direction of anisotropy, \( \hat{n} \) with \( \hat{n}^2 = 1 \). The anisotropy parameter \( \xi \) can be adjusted to reflect either squeezing (\( \xi > 0 \)) or stretching (\( -1 < \xi < 0 \)) of the distribution in the \( \hat{n} \) direction. Choosing, \( \xi = 0 \) will bring us back to the case of isotropic medium.

Next, performing the change of variables \( (|\hat{p}| \equiv \sqrt{p^2 + \xi(p \cdot \hat{n})^2}) \) in Eq. (19), we get

\[
\Pi^{\mu\nu}(\omega, k) = -\frac{m^2_T}{4\pi} \int \frac{d\Omega}{[1 + 1]} \left( g^{\mu\nu} - \frac{K^1 u^\nu}{K \cdot u + i\epsilon} \right).
\]

(37)

Again, a tensor decomposition of \( \Pi^{\mu\nu}(\omega, k) \) for an anisotropic system is possible to develop in which there is only one preferred direction i.e., the direction of anisotropy, \( \hat{n} \).

\[
\Pi_{\mu\nu}(\omega, k) = \alpha P_{\mu\nu} + \beta Q_{\mu\nu} + \gamma R_{\mu\nu} + \delta S_{\mu\nu},
\]

(38)

where \( P_{\mu\nu} \) and \( Q_{\mu\nu} \) are same as defined earlier while \( R_{\mu\nu} \) and \( S_{\mu\nu} \) are defined as:

\[
R_{\mu\nu} = \frac{\tilde{n}_{\mu} \tilde{n}_{\nu}}{n^2},
\]

(39)

\[
S_{\mu\nu} = K_{\mu} \tilde{n}_{\nu} + K_{\nu} \tilde{n}_{\mu} - \frac{K^2}{\omega}(u_{\mu} n_{\nu} + u_{\nu} n_{\mu}).
\]

(40)

The scalar quantities (structure functions), \( \alpha, \beta, \gamma \) and \( \delta \) can be obtained using the following contractions of \( \Pi^{\mu\nu} \):

\[
\alpha = (P_{\mu\nu} - R_{\mu\nu}) \Pi^{\mu\nu}, \quad \beta = Q_{\mu\nu} \Pi^{\mu\nu},
\]

(41)

\[
\gamma = (2R_{\mu\nu} - P_{\mu\nu}) \Pi^{\mu\nu}, \quad \delta = -\frac{2K^2}{\omega^2 n^2} S_{\mu\nu} \Pi^{\mu\nu}
\]

(42)

Where \( \hat{k} \cdot \hat{n} = \cos \theta_{n}, \tilde{n}_{\mu} = P_{\mu\nu} n^\nu \) and \( n^\nu = (0, \hat{n}) \). Note that, all of the four structure functions depend on \( m_D, \omega, k \) and \( \xi \). In the limit of vanishing strength of the anisotropy, \( \xi \to 0 \), the structure functions \( \alpha \) and \( \beta \) reduce to the isotropic hard-thermal-loop self-energies while \( \gamma \) and \( \delta \) go to zero.

\[
\alpha_{iso}(\omega, k) = \Pi_T(\omega, k), \quad \beta_{iso}(\omega, k) = \Pi_L(\omega, k),
\]

(43)

\[
\gamma_{iso}(\omega, k) = 0, \quad \delta_{iso}(\omega, k) = 0
\]

(44)

Using the gluon self energy \( \Pi^{\mu\nu}(\omega, k) \), in Eq. (15) for the anisotropic hot QCD/QGP medium, the chromo-electric permittivity and the chromo-magnetic permeability can be obtained as

\[
\epsilon_{\mu\nu}(\omega, k) = (1 - \frac{\beta}{K^2} g_{\mu\nu} - \frac{1}{\omega^2} \left( \gamma \frac{\tilde{n}_{\mu} \tilde{n}_{\nu}}{n^2} + \delta(K_{\mu} \tilde{n}_{\nu} + K_{\nu} \tilde{n}_{\mu}) \right),
\]

(45)

\[
\frac{1}{\mu(\omega, k)} = 1 + \frac{K^2 \alpha - \omega^2 \beta}{k^2 K^2}.
\]

(46)
In the anisotropic medium there are two polarization state of chromo-electric field, denoted as, the left-handed and right-handed polarized state. The chromo-electric permittivity is then separately defined for each state as:

$$
\epsilon_L = 1 - \frac{\beta}{K^2},
$$

$$
\epsilon_R = 1 - \frac{\beta}{K^2} - \frac{\gamma}{\omega^2}.
$$

Whereas in the limit of small anisotropy, it is possible to obtain analytic expressions of the structure functions order by order in $\xi$. In the linear order approximation in $\xi$, the structure functions are given as,

$$
\alpha(\omega, k) = \Pi_T(\omega, k) + \xi \left[ \frac{\omega^2}{12k^2}(3 + 5 \cos 2\theta_n)m_D^2 + \frac{1}{6}(1 + \cos 2\theta_n)m_D^2 + \frac{1}{4}\Pi_T(\omega, k) \left( 1 + 3 \cos 2\theta_n - \frac{\omega^2}{k^2}(3 + 5 \cos 2\theta_n) \right) \right],
$$

$$
\beta(\omega, k) = \Pi_L(\omega, k) + \xi \left[ \frac{1}{6}\frac{\omega^2}{k^2} - 1 \right] \left( 1 + 3 \cos 2\theta_n \right)m_D^2
+ \Pi_L(\omega, k) \left( \cos 2\theta_n - \frac{\omega^2}{2k^2} \left( 1 + 3 \cos 2\theta_n \right) \right),
$$

$$
\gamma(\omega, k) = \frac{\xi}{3}(3\Pi_T(\omega, k) - m_D^2) \left( \frac{\omega^2}{k^2} - 1 \right) \sin^2 \theta_n.
$$

For real valued $\omega$ the structure functions are real for all $\omega > k$ and complex for $\omega < k$. For imaginary values of $\omega$, all four structure functions are real. Since all the structure constants depend on the Debye mass($m_D$), any modification in Debye mass will modify all of them.

In the small-k expansion the response functions are given as,

$$
\epsilon_L \approx 1 - \frac{m_D^2}{3\omega^2} + \xi \frac{m_D^2}{10\omega^2} \left( 1 - \frac{1}{3} \cos 2\theta_n \right) + \mathcal{O}(k^2),
$$

$$
\epsilon_R \approx 1 - \frac{m_D^2}{3\omega^2} + \xi \frac{m_D^2}{15\omega^2} + \mathcal{O}(k^2).
$$

FIG. 1: (color online) Re(\epsilon)(left panel) and Im(\epsilon)(right panel) are plotted for various EoSs at $\xi = 0$, $T_c = 0.17 GeV$ and $T = 0.25 GeV$. 
and
\[
\mu \approx \frac{1}{1 - \frac{2m_p^2}{Tc^2} - \xi \frac{m_p^2}{Tc^2} \cos^2 \theta_n} + \mathcal{O}(k^2).
\] (51)

**B. Description of Negative Refractive Index (NRI)**

The refractive index \( n \) is formally defined by electric permittivity and magnetic permeability as \( n = \sqrt{\varepsilon(\omega, k) \mu(\omega, k)} \), but the quadratic nature of such a definition implies that it is not sensitive to the sign of \( \varepsilon(\omega, k) \) and \( \mu(\omega, k) \). It was proposed by Veselago [14] that the simultaneous change of sign of \( \varepsilon(\omega, k) \) and \( \mu(\omega, k) \) corresponds to a cross over between different branches of the square root, from \( n = \sqrt{\varepsilon(\omega, k) \mu(\omega, k)} \) to \( n = -\sqrt{\varepsilon(\omega, k) \mu(\omega, k)} \), or from the positive refractive index to the negative one. Next, if the medium is dissipative, the \( \varepsilon \) and \( \mu \), and hence the \( n \), are complex quantities. The condition for the NRI both in isotropic and anisotropic QGP is as follows. Whenever, \( \text{Im} (\mu) \) and \( \text{Im} (\varepsilon) \) vanish and both \( \text{Re} (\mu) \) and \( \text{Re} (\varepsilon) \) are negative simultaneously, in a certain frequency range, this case, NRI can be realised in the medium. In the following discussion, we shall see that for the isotropic medium the sign of \( \varepsilon(\omega, k) \) and \( \mu(\omega, k) \) have a significant physical implication.

Note that the phase velocity is defined by,
\[
\mathbf{v}_p = \frac{1}{\text{Re}(n)} \mathbf{k} = v_p \mathbf{k}
\] (52)
whose sign is the same as that of \( \text{Re}(n) \). But the direction of the energy flow or the Poynting vector is not affected by the sign of \( \varepsilon \) and \( \mu \). In a medium with small dissipation, the direction of the energy flow coincides with that of the group velocity,
\[
\mathbf{S} = v_g U \hat{\mathbf{k}}.
\] (53)
where \( U \) is a positive time-averaged energy density and \( v_g = d\omega/dk \). If we have a negative phase velocity, the direction of the phase velocity can be opposite to the energy flow or the group velocity (i.e., the direction of phase velocity can be radially inward and the direction of energy flow can be radially outward or vice-versa).

\[
v_p < 0, \quad v_g > 0.
\] (54)
This condition works well for the isotropic expansion of the medium as there is a spherical symmetry but does not
holds good for the anisotropic case because of the absence of spherical symmetry. Therefore, from the criterion of the anti parallelism of the phase velocity and the energy flow, a better condition for the isotropic medium has been derived for the NRI, called the Depine-Lakhtakia index (n_{DL}) [15],

\[ n_{DL} = |\epsilon(\omega, k)| \text{Re}(\mu(\omega, k)) + |\mu(\omega, k)| \text{Re}(\epsilon(\omega, k)). \] (55)

Whenever \( n_{DL} < 0 \), we have NRI region and also the directions of the phase velocity and the energy flow are opposite. This is seen to be a better indicator for the

FIG. 3: (color online) \( \text{Re}(n) \) (left panel) and \( \text{Im}(n) \) (right panel) are plotted for various EoSs at \( \xi = 0, T_c = 0.17 \text{GeV} \) and \( T = 0.25 \text{GeV} \).

FIG. 4: (color online) \( n_{DL} \) is plotted for different \( k \) at \( \xi = 0, T_c = 0.17 \text{GeV} \) and \( T = 0.25 \text{GeV} \) for various EoSs.
In this section, we shall discuss the results shown in Fig. 5 (lower panel), at $\omega = 0$. For the plots, we can see that there is a pole in Re($\epsilon$) from a positive value to a negative value at $\omega = k$ and then it is negative for a certain range of frequency. The Im($\epsilon$) contributes only for the frequency range $0 < \omega < k$. In order to check the variation due to change in $k$, we have plotted the same quantities also in Fig. 1 (lower panel), for $k = 0.4$. We have observed that the frequency range for the negative Re($\epsilon$) becomes smaller and hence the medium for $k = 0.4$ comparatively is less dissipative than that for $k = 0.2$.

Similarly, in Fig. 6 (upper panel), we have plotted the real(left) and imaginary(right) part of chromo-electric permittivity ($\epsilon(\omega, k)$) using Eq. (42) for $k = 0.2$. One can observe that there is fall in Re($\epsilon$) from a positive value to a negative value at $\omega = k$ and then it is negative for a certain range of frequency. The Im($\epsilon$) contributes only for the frequency range $0 < \omega < k$. In order to check the variation due to change in $k$, we have plotted the same quantities also in Fig. 1 (lower panel), for $k = 0.4$. We have observed that the frequency range for the negative Re($\epsilon$) becomes smaller and hence the medium for $k = 0.4$ comparatively is less dissipative than that for $k = 0.2$.

NRI in the medium that we have also investigated for the isotropic medium.

III. RESULTS AND DISCUSSIONS

In this section, we shall discuss the results shown in different plots. We have plotted various quantities ($\epsilon$, $\mu$, $n$, etc) with respect to $\omega/m_{D}^{LO}(T)$ at different values of $k$ but fixed temperature ($T = 0.25 GeV$). This particular scaling is chosen to visualise the effects of hot QCD medium interactions. For smaller values of $\omega$ (when $\omega \ll k$) the numbers are quite overlapped, but the difference in the numbers can be seen for the intermediate frequencies. One could easily observe that the results are again merging for the higher values of $\omega$ in all the cases. As the medium is dissipative, it is not only real part of chromo-electric permittivity, chromo-magnetic permeability and the chromo-refractive index are contributing, but the imaginary parts are also present. Initially, we shall discuss the results for isotropic cases, and later on, we shall describe the anisotropic medium in the small anisotropic limit, $\xi = 0.3$. Note that, in all the figures legends, we denote LEOS as $2 + 1$, LEOSBZ as LB and 3-loop HTLpt as HTLpt and Ideal EoS as LO.

In Fig. 5 (upper panel), we have plotted the real(left) and imaginary(right) part of chromo-electric permittivity ($\epsilon(\omega, k)$) using Eq. (42) for $k = 0.2$. One can observe that there is fall in Re($\epsilon$) from a positive value to a negative value at $\omega = k$ and then it is negative for a certain range of frequency. The Im($\epsilon$) contributes only for the frequency range $0 < \omega < k$. In order to check the variation due to change in $k$, we have plotted the same quantities also in Fig. 1 (lower panel), for $k = 0.4$. We have observed that the frequency range for the negative Re($\epsilon$) is somewhat reduced now. The values of Im($\epsilon$) became smaller and hence the medium for $k = 0.4$ comparatively is less dissipative than that for $k = 0.2$.

FIG. 5: (color online) Re($n$) and Im($n$) along with $n_{DL}$ are plotted for various EoSs at $\xi = 0$, $T_{c} = 0.17 GeV$ and $T = 0.25 GeV$.

FIG. 6: (color online) Re($n$) and Im($n$) along with $n_{DL}$ are plotted for various EoSs at $\xi = 0$, $T_{c} = 0.17 GeV$ and $T = 0.25 GeV$ in the small $k$ limit.
range in which the real part of chro-mo-magnetic perme-
ability was negative became comparatively narrow and
also the medium became comparatively less dissipative
for the chro-mo-magnetic field.

The chro-mo-refractive index, $n$, is formally defined as
$\sqrt{\epsilon \mu}$. Using $\epsilon$ and $\mu$ discussed above, we have plotted
real and imaginary part of $n$, with respect to $\omega/m_D^3(T)$,
in the left and right panels of Fig. 3. It is easy to observe
that for the frequency range $0 \leq \omega \leq k$, both the $\text{Re}(n)$
as well as $\text{Im}(n)$ are positive. At $\omega = k$, there is a fall
from positive values to the negative values in $\text{Re}(n)$. In
the range $k < \omega \leq \omega_{mp}$, the real part of $n$ is negative,
and the imaginary part of $n$ is zero. In this frequency
range, $n$ is negative while the medium is not dissipative
for the chro-mo-electromagnetic waves. In the region
$\omega_{mp} \leq \omega \leq \omega_p$ (where in the small $k$-expansion(Eq. (34))
$\omega_p = \sqrt{1/3m_D(T)}$ for all $m_D(T)$), $\text{Im}(n)$ is positive
but $\text{Re}(n)$ is zero. In this frequency range the medium is
opaque for chro-mo-electromagnetic waves. For the frequen-
cies $\omega \geq \omega_p$, $\text{Im}(n)$ is zero and $\text{Re}(n)$ is positive and
hence we have normal color refraction which is reaching
towards the unity for higher values of $\omega$. Here one can
easily observe from the comparison of the upper and
lower panel of Fig. 3 that the frequency range where
$\text{Re}(n) \leq 0$, is contracted for $k = 0.4$ as compared to
$k = 0.2$.

In Fig. 4 we have plotted Depine and Lakhtakia index
($n_{DL}$) to get a better overview of negative refraction re-
regions for the isotropic medium. It can be clearly observed
that for the smaller frequencies $\omega \leq k$ the refractive
index is positive. In the frequency range $(k \leq \omega \leq \omega_{mp})$
both $\text{Re}(\epsilon)$ and $\text{Re}(\mu)$ are negative and hence the re-
fractive index, $n_{DL}$, is negative. In the frequency range
$\omega_{mp} \leq \omega \leq \omega_p$, $n_{DL} = 0.0$. For higher values of $\omega$ (i.e.,
$\omega \geq \omega_p$) value it $n_{DL}$ is positive and is approaching
to unity.

In order to have a comparative study of $\text{Re}(n)$, $\text{Im}(n)$
and $n_{DL}$, for the isotropic medium, we have plotted all
three together in Fig. 3. What we have observed is that in
the frequency range when $\omega < k$, both $\text{Re}(n)$ and $\text{Im}(n)$
are positive which means the refraction is positive. One
can verify that in the same range $n_{DL}$ is also positive.
For the frequencies $k \leq \omega \leq \omega_{mp}$, $n_{DL}$ and $\text{Re}(n)$ are
negative while $\text{Im}(n)$ is zero and hence this is the region
for negative refraction. For $\omega_{mp} \leq \omega \leq \omega_p$, $n_{DL} = 0$
and $\text{Re}(n) = 0$ while $\text{Im}(n) > 0$ and medium is highly
dissipative and opaque for the chro-mo-electromagnetic
waves. For $\omega = \omega_p$ and onwards $n_{DL}$ and $\text{Re}(n)$ are

![Fig. 7: (color online) $\text{Re}(\epsilon_L)$ (left panel) and $\text{Im}(\epsilon_L)$ (right panel) are plotted for various EoSs at $\xi = 0.3$,
$T_c = 0.17\text{GeV}$ and $T = 0.25\text{GeV}$.](image)
positive while Im(n) = 0 i.e., we have a normal chromo-refractive index.

In Fig. 8, we have plotted the real and imaginary part of n along with n_DL using expression of \( \epsilon \) and \( \mu \) expanded in small-\( k \) limit, shown in Eq. (84) and in Eq. (85) respectively. Here we found that the Re(n) and n_DL are negative for the frequencies \( \omega \leq \omega_{mp} \) while Im(n) is zero and so the chromo-refractive index is negative. In the frequency range \( \omega_{mp} \leq \omega \leq \omega_p \), only the Im(n) contributes and hence the medium is opaque as well as highly dissipative. For higher \( \omega (i.e., \omega \geq \omega_p) \), we have normal color refraction.

The frequency(\( \omega \)) range for the isotropic hot QCD/QGP medium where the chromo-refractive index is negative is found to be [\( k, \omega_{mp} \)]. As said earlier, this region is opaque for the chromo-electromagnetic waves. The most important point is that this gap is proportional to \( m_D(T) \). Because of this we can see in Fig. 8 that the gap is reduced in the case of other EoSs as compared to LO. In fact, these \( m_D(T) \)'s are the functions of temperature(\( T \)). Since we are taking a constant temperature (\( T = 0.25GeV \)) in all the cases, the variation with respect to temperature is not observed here. Only the interaction effects using different EoSs which are modifying the Debye mass(\( m_D(T) \)) are shown.

To understand the same properties for anisotropic hot QCD/QGP medium, we have repeated the analysis while considering the anisotropy in a particular direction(\( \hat{n} \)) and plotted the results for the anisotropic strength \( \xi = 0.3 \).

In Fig. 9 we have plotted the real part(left) and the imaginary part(right) of \( \epsilon(\omega) \) using Eq. (84) for \( k = 0.2 \) and \( k = 0.4 \), in the upper and lower panels respectively. One can observe that there is fall in Re(\( \epsilon(\omega) \)) from positive value to the negative value at \( \omega \approx k \) and then it is negative for a certain range of frequencies. The Im(\( \epsilon(\omega) \)) of contributes only for the frequency range \( 0.0 \leq \omega \leq k \). In the case when \( k = 0.4 \), the medium is less dissipative and the frequency range for the negative Re(\( \epsilon(\omega) \)) has been reduced as compared to the case when \( k = 0.2 \). Here, though the pattern is similar but the numbers are slightly smaller as compared to the isotropic case.

In Fig. 10, we have plotted \( \epsilon_R \), in the same way, using Eq. (85). Here, the pattern is slightly different as compared to the isotropic case. Both the Re(\( \epsilon_R(\omega) \)) and Im(\( \epsilon_R(\omega) \)) are initially negative. Then there is a small region where they are positive. This region is comparatively wide for \( k = 0.4 \). After that they also follow the similar pattern.
as it was the isotropic case. Similarly, in Fig. 9 we have plotted for anisotropic medium the real and the imaginary part of chromo-magnetic permeability($\mu(\omega, k)$) for $k = 0.2$ (upper panel) and for $k = 0.4$ (lower panel). Here also, we have found the similar pattern as we got in the case of isotropic medium but the numbers are slightly different due to the effect of anisotropy. We have defined the chromo-refractive index, $n_L$ for the left-handed state as $\sqrt{\epsilon_L / \mu}$ and plotted the $\text{Re}(n_L)$ and $\text{Im}(n_L)$ in Fig. 10. Since we got the similar pattern of $\epsilon_L$ and $\mu$, 

**FIG. 9:** (color online) $\text{Re}(\mu)$ (left panel) and $\text{Im}(\mu)$ (right panel) are plotted for various EoSs at $\xi = 0.3$, $T_c = 0.17\text{GeV}$ and $T = 0.25\text{GeV}$.

**FIG. 10:** (color online) Real and imaginary part of $n_L$ are plotted for different $k$ at $\xi = 0.3$, $T_c = 0.17\text{GeV}$ and $T = 0.25\text{GeV}$ for various EoSs.
the results coming from $n_L$ are also following the similar pattern with a slightly different values as compared to the isotropic one. In Fig. 11 we have plotted the Re($n_L$) and Im($n_L$) where we define the chromo-refractive index, $n_R$ for right handed state as $\sqrt{T_R} \mu$. This time we got a slightly different pattern because of the change in pattern of Re($\epsilon_R$) and Im($\epsilon_R$). Here the Re($n_R$), is positive while Im($n_R$), is negative initially for the smaller values of $\omega$. Then for a small range frequency where both are positive. Afterward, it follows the same pattern.
as the isotropic one with slightly different numbers. In Fig. [12] we have plotted the real and imaginary part of anisotropic $n$ using expression of $\epsilon_L$, $\epsilon_R$ and $\mu$ expanded in small-$k$ limit, shown in Eq.[19], [50] and [51] respectively. In the anisotropic case, $\mu$ in small-$k$ expansion is having k-dependence as shown in Eq.[51] but this dependence is not very strong as one can see in the Fig. [12] (left and right panels). Since the anisotropic strength is not very strong, these $n_L$ and $n_R$ are following the similar pattern as it was there in the isotropic case. The effects from EoSs as compared to leading order are quite visible for smaller values of frequency while they are not very different for the larger values.

In the small anisotropic case ($\xi = 0.3$) as compared to the isotropic one, the only dissimilarity in the pattern is found for the right-handed polarized state. While for the left-handed polarized state the patterns are quite similar. Finally, EoSs effects are clearly visible for certain frequency ranges. This helps in distinguishing the various EoSs employed for modeling the QGP through the EQPM.

IV. SUMMARY AND CONCLUSIONS

In conclusion, chromo-electromagnetic response functions of the QGP in the static limit have been investigated and their implication in understanding the dielectric properties of the hot QCD/QGP medium. In our approach, the hot QCD medium interactions are included exploiting the quasi-particle description of the hot QCD equations of state either computed within recent lattice QCD methods or 3-loop HTL perturbation theory. Realization of the hot QCD medium as an optical medium with the refractive index, we observe that in certain regions in frequency, $\omega$, the refractive index could be negative. In this context, we defined Depine-Lakhtakia index for the isotropic medium and studied its behavior in detail to probe the directions of energy flow there. On the other hand for the anisotropic case depending on the direction of the anisotropy, we have two independent refractive indices. The possibility of negative refraction in the anisotropic case turned out to be quite tricky. Defining the Depine-Lakhtakia index is perhaps not a very viable way to understand this very crucial phenomenon. Nevertheless, we have to look at regions where both $\epsilon$ and $\mu$ are negative keeping in mind the strength and direction of the anisotropy.

In all the cases, be it $\epsilon$ or $\mu$ and respective refractive indices, get significant contributions from the hot QCD medium effects as compared to the case when QGP is approximated as the ideal system (noninteracting ultra-relativistic gas of quarks, antiquarks and gluons) and the anisotropy plays crucial role too.

An immediate future extension of the work would be to include the contributions from the collisional kernel and collectivity and near perfect fluidity of the hot QCD medium/QGP and study its impact on collective plasma properties and refractive index. Equally importantly, obtaining an expression for the inter quark potential in this medium and its phenomenological aspect, polarization energy loss in the hot QCD medium will be a few other interesting directions where our future investigations will focus on.

ACKNOWLEDGEMENTS

V.Chandra would like to sincerely acknowledge DST, Govt. of India for Inspire Faculty Award -IFA13/PH-15 and Early Career Research Award(ECRA) Grant 2016. We would like to acknowledge people of INDIA for their generous support for the research in fundamental sciences in the country.

[1] J. Adams et al. (STAR Collaboration), Nucl. Phys. A 757, 102 (2005); K. Adcox et al. PHENIX Collaboration, Nucl. Phys. A 757, 184 (2005); B.B. Back et al. PHOBOS Collaboration, Nucl. Phys. A 757, 28 (2005); I. Arsene et al. BRAHMS Collaboration, Nucl. Phys. A 757, 1 (2005).

[2] K. Aamodt et al. (The Alice Collaboration), Phys. Rev. Lett. 105, 252302 (2010); Phys. Rev. Lett. 105, 252301 (2010); Phys. Rev. Lett. 106, 032301 (2011).

[3] U. W. Heinz, hep-ph/0407360.

[4] Y. Koike, AIP Conf. Proc. 243, 916 (1992).

[5] Y. Koike, AIP Conf. Proc. 243, 916 (1992).

[6] V. Chandra, R. Kumar, V. Ravishankar, Phys. Rev. C 76, 054909 (2007); [Erratum: Phys. Rev. C 76, 069904 (2007)]; V. Chandra, A. Ranjan, V. Ravishankar, Eur. Phys. J. A 40, 109-117 (2009).

[7] V. Chandra, V. Ravishankar, Phys. Rev. D 84, 074013 (2011).

[8] P. Romatschke and M.Strickland, Phys. Rev. D 68, 036004 (2003).

[9] P. Romatschke and M.Strickland Phys. Rev. D 70, 116006 (2004).

[10] B. Schenke, M. Strickland, C. Greiner and M. H. Thoma, Phys. Rev. D 73 125004(2006).

[11] B. Schenke and M. Strickland, Phys. Rev. D 76, 025023 (2007).

[12] M. E. Carrington, K. Deja and S. Mrowczynski, Phys. Rev. C 90, 034913 (2014).

[13] M. Youssuf, S. Mitra and V. Chandra, Phys. Rev. D 95, 094022 (2017).

[14] V.G.Veselago Sov. Phys. Usp. 10, 509 (1968).

[15] R.A. Depine and A.Lakhtakia, Microwave and Optical Technology Letters, 41, 315 (2004).

[16] A. Amariti et al., JHEP 04,036 (2011)

[17] A. Amariti et al., 1107.1240, (2011).

[18] Ge et al., JHEP 03,104 (2011).

[19] X.Gao, H.-b. Zhang, JHEP 08,075 (2010).

[20] F. Bigazzi et al.(2011), JHEP 04, 060 (2011).

[21] A. Amariti et al., JHEP 10, 104 (2011).
