Deconstructing double-barred galaxies in 2D and 3D. I.
Classical nature of the dominant bulges

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ABSTRACT
We present here a thorough photometric analysis of double-barred galaxies, consisting of i) two-dimensional photometric decompositions including a bulge, inner bar, outer bar, and (truncated) disc; and ii) three-dimensional statistical deprojections to derive the intrinsic shape of bulges, inner bars, and outer bars. This is the first time the combination of both techniques is applied to a sample of double-barred galaxies. It represents a step forward with respect to previous works, which are based on properties of the integrated light through ellipse fitting and unsharp masking. In this first paper of a series of two, we analyse the nature of the dominant bulges within double-barred systems by using several photometric diagnostics, namely Sérsic index, Kormendy relation, colours, and the better suited intrinsic flattening. Our results indicate that almost all bulges in our sample are classical, whereas only 2 out of the 17 galaxies under study appear as potential candidates to host secularly-formed disc-like bulges. Such result poses the possibility that having a central hot structure may be a requirement for inner bar formation.

Key words: galaxies: photometry – galaxies: structure – galaxies: evolution – galaxies: stellar content

1 INTRODUCTION
Double-barred galaxies are disc-like galaxies hosting two stellar structures in the shape of a bar: an outer bar alike to the single bars observed in ~60-70% of all disc galaxies (Aguerri et al. 2009; Erwin 2018), and the so-called inner bar that refers to an additional, smaller barred structure that has been found in at least ~30% of all barred galaxies (Laine et al. 2002; Erwin 2004).

Observations of galaxies with two nested stellar bars date back to the second half of the twentieth century (see de Vaucouleurs 1973, for a first identification of NGC 1291 as a double-barred galaxy). After serendipitous detections, such as those of Sandage & Brucato (1979) and Kormendy (1979), Erwin (2004) was the first to make a thorough compilation of photometric data from various facilities in a catalog of 50 double-barred galaxies. More individuals have been later detected by large photometric surveys such as the Spitzer Survey of Stellar Structures in Galaxies (S4G; Sheth et al. 2010). Moreover, inner bars have been observed up to a redshift ~0.15 (Lisker et al. 2009), thus supporting an scenario in which they are not occasional but common stellar structures. This is a surprising result as theoretically it is not possible to have two sets of closed barred orbits coexisting within a rotating disc; the problem of the orbital support of double-barred systems requires the complex definition of loops introduced by Maciejewski & Sparke (1997) and Maciejewski & Sparke (2000). Simulating the formation and evolution of double-barred galaxies has indeed been proved a hard task (e.g. Du et al. 2015; Wozniak 2015).

After almost 40 years of studies, many important questions remain open about double-barred galaxies. This work aims at providing photometric constraints to some of them through the most complete photometric study of double-barred galaxies ever performed. Our analysis combines two-dimensional (2D) photometric decompositions of double-barred galaxies in their multiple structural components (bulges, bars, and discs), and a three-dimensional (3D) statistical deprojection of their bulges and (inner and outer) bars. We remark this is the first time either 2D decomposi-
tions or 3D deprojections of a sample of double-barred galaxies are presented.

In a series of two papers, we explore the following unanswered questions about double-barred galaxies: i) whether there exists a major incidence of disc-like bulges within double-barred galaxies, where secular evolution is assumed to take place in a very efficient way; ii) whether inner bars form secularly out of disc-like bulges already present in barred galaxies; iii) whether inner bars are transient or long-lived structures; and iv) whether all barred galaxies will develop an inner bar at some stage of their lives. This first paper is devoted to the presentation of the analysis and the photometric properties of bulges within double-barred systems (questions i and ii), while the properties of inner and outer bars (questions iii and iv) are studied in de Lorenzo-Cáceres et al. (2018c, in preparation; hereafter Paper II).

1.1 Frequency of disc-like bulges in double-barred galaxies

Galactic bulges are properly defined as the central excess of light found in the surface brightness distribution of a disc galaxy, i.e., they imply a central concentration of stars. According to their photometric and kinematic properties, these structures are classified into two main groups: classical and disc-like bulges. Both types are believed to be linked to the different evolutionary processes forming them. Classical bulges are considered the result of fast actions, such as monolithic collapse or mergers (Eggen et al. 1962; Aguerri et al. 2001; Bournaud et al. 2007; Honkins et al. 2013). These are pressure-supported systems rather analogous to elliptical galaxies. On the other hand, disc-like bulges show more ordered motions dominated by rotation. Sometimes referred to as pseudobulges, they are thought to be formed through secular evolution mainly driven by bars (Kormendy & Kennicutt 2004, although see Elche-Moral et al. 2004 for a numerical evidence of minor mergers causing similar bulge growth). Indeed, bars promote an angular momentum exchange between dark and baryonic matter or even between baryons and baryons. As a consequence, stars are dragged towards the outer galactic regions or gas is brought into the centre, where it concentrates and may eventually form new stars and stellar structures such as bulges (Combes et al. 1990; Friedli & Benz 1993; Muñoz-Tuñón et al. 2004; Boone et al. 2007).

Shlosman et al. (1983, 1990) show that a double-barred system may theoretically be more efficient than a single bar in the transportation of gas towards the central galactic regions, being even able to reach the sphere of influence of the central black hole. This is the reason why double bars have been proposed as the gas channel for triggering active galactic nuclei, although no conclusive observational evidence has been found so far (Márquez et al. 2000). It is therefore sensible to state that a major incidence of secularly-formed structures should be found in double-barred galaxies, where gas inflow is very efficient. Disc-like bulges are commonly considered the closest consequence of secular evolution, and inner bars themselves are considered as evidence for the presence of secularly-promoted bulges by many authors (Fisher & Drory 2011). Whether disc-like bulges are frequent in double-barred galaxies has not been purposely studied in the literature so far, with the exception of de Lorenzo-Cáceres et al. (2012), who present evidence of the existence of a disc-like bulge in the centre of the double-barred galaxy NGC 357.

During the last decades, great effort has been put into the search of photometric diagnostics that allow to discern the nature of bulges (i.e. classical vs. disc-like). The Sérsic index, $n$, appeared at first to be the best candidate. For example, Fisher & Drory (2008) found that classical bulges tend to have $n > 2$, whereas disc-like bulges show $n < 2$. The Sérsic index is a mathematical parameter describing the shape of the surface-brightness profile of a bulge, once isolated from the rest of galaxy components through a photometric decomposition. Another diagnostic involves the projection of the fundamental plane onto the integrated surface brightness of the bulge within one effective radius versus the bulge effective radius, also known as the Kormendy (1977) relation. This has been argued to be a good discriminator between classical and disc-like bulges, as it shows a correlation that creates a top sequence mostly populated by classical bulges, while the existence of bulges with lower effective surface brightness for a given radius (i.e., outside the classical relation) is explained by their disc-like nature (e.g. Fisher & Drory 2008; Gadotti 2009; Fisher & Drory 2010; Neumann et al. 2017).

More recently, Costantin et al. (2018) tested the photometric diagnostics commonly used to discern between classical and disc-like bulges, and compared them with some kinematic diagnostics. Their results indicate that projected quantities such as the Sérsic index alone do not provide conclusive results and they propose the use of the intrinsic 3D shape of bulges as a better photometric discriminator, based on the intrinsic flattening of the bulge component with respect to the other galaxy structures (see also Méndez-Abreu et al. 2018). Indeed, classical bulges are expected to be pretty spherical structures. On the other hand, rotating disc-like bulges should appear flatter, also depending on the disc thickening.

1.2 2D photometric decompositions

All published photometric studies about double-barred galaxies so far rely on analyses of the integrated galaxy light (e.g. Erwin 2004). Inner bars are usually detected through isophotal analysis as bumps in the ellipticity and position angle profiles (Friedli & Martinet 1993; Laine et al. 2002), and/or through a careful visual inspection of unsharp-masked images (Erwin & Sparke 2003). Better suited photometric decompositions, which model the galaxy light as a combination of individual structures instead of studying the integrated properties, have not been systematically applied for double-barred cases. The only notable exceptions are the recent works by Méndez-Abreu et al. (2014), who performed 2D photometric decompositions of all galaxies of the Calar Alto Legacy Integral-Field Area survey (CALIFA; Sánchez et al. 2012), including two double-barred cases, and de Lorenzo-Cáceres et al. (2018a, submitted; see also Méndez-Abreu et al. 2019), who decomposed the S4G images of the two double-barred galaxies present in the spectroscopic TIMER project (Gadotti et al. 2018).
Deconstructing double-barred galaxies: bulges

2 THE SAMPLE OF DOUBLE-BARRED GALAXIES

The sample of double-barred galaxies is extracted from a catalog of 67 barred galaxies with inner structures presented in Erwin (2004). Among these, 50 galaxies are double-barred, as classified by two photometric diagnostics, namely ellipse fitting and unsharp masking. Bar properties such as sizes and position angles (both measured from ellipse fitting) are also provided by Erwin (2004), who uses a compilation of results from the literature and new measurements made on a variety of images in the optical and near infrared.

For the present analysis, we first selected all double-barred galaxies from the catalog of Erwin (2004) with available Sloan Digital Sky Survey (SDSS: York et al. 2000) imaging, thus obtaining a preliminary sample of 23 objects. SDSS provides a homogeneous set of $g'$-, $r'$-, and $i'$-band images with medium spatial resolution, suitable for inner bar detection at the redshift of our galaxies ($z < 0.015$), and a field-of-view large enough to reach the outermost regions of the galaxy, as required for a proper modeling of the disc component. In particular, we use the images from the SDSS Data Release 9 (Ahn et al. 2012). All images are already soft-bias subtracted, flat-field corrected, sky subtracted, and flux calibrated using the standard SDSS pipelines. Some further treatment of the data is necessary to both re-calibrate the images from nanomaggies to counts (required for the fitting procedure with GASP2D) and to refine the sky subtraction. Details on this process are described in Pagotto et al. (2017) and Costantin et al. (2018). The point spread function (PSF) is measured on each image using a circular Moffat function (Moffat 1969). The mean values of the Full Width at Half Maximum (FWHM) for the $g'$-, $r'$-, and $i'$-images are 1.32, 1.15, and 1.17 arcsec, respectively.

Notwithstanding the careful inspection made by Erwin (2004), our 2D multi-component photometric decompositions reveal that some of these galaxies had been either misclassified as double barred or the SDSS spatial resolution is not enough to readily distinguish the inner bar. In the former case, we found this is mainly due to the presence of other components (e.g., stellar inner rings or complex dust structures). Six galaxies were finally removed from our preliminary sample, namely Mrk 573, UGC 524, NGC 1068, NGC 4303, NGC 4321, and NGC 4736. Our definitive sample is therefore composed of 17 double-barred galaxies. Table I shows the galaxy sample together with some relevant parameters.

All our double-barred galaxies are nearby, with $z < 0.015$ as indicated in Table I (Fisher & Drory 2016) establish that, given the SDSS spatial resolution and a typical bulge effective radius of 2 kpc, photometric decompositions aiming at accurately deriving bulge properties with SDSS data should restrict to galaxies up to $z=0.03$. Costantin et al. (2017) explore the possible errors on the bulge structural parameters when their angular sizes are close to the size of the image PSF. They find that even for bulges with effective radius $1.2\times\sigma$ of the PSF ($\sigma \sim$ FWHM/2.35), the bulge parameters can be recovered within a 10% error. We conclude that our photometric decomposition analysis is therefore not hampered by resolution effects affecting small bulges or inner bars.

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Table 1. DOUBLE-BARRED GALAXIES SAMPLE.

| Name      | Morphological type | Distance (Mpc) | z         |
|-----------|--------------------|---------------|-----------|
| NGC 357   | SB(r)0/a           | 31.6          | 0.008     |
| NGC 718   | SAB(s)             | 22.6          | 0.006     |
| NGC 2642  | SB(r)bc            | 56.8          | 0.014     |
| NGC 2859  | (R)SAB(rs)0/a      | 17.2          | 0.002     |
| NGC 2950  | (R)SB(r)0          | 24.3          | 0.006     |
| NGC 2962  | (R)SAB(rs)0        | 14.9          | 0.004     |
| NGC 3568  | SAB(rs)ab          | 30.0          | 0.007     |
| NGC 3941  | SB(r)0             | 10.5          | 0.003     |
| NGC 4344  | SB(r)0             | 19.3          | 0.004     |
| NGC 4344  | SB(r)0             | 12.0          | 0.003     |
| NGC 4340  | SB(r)0             | 15.3          | 0.003     |
| NGC 4503  | SB0                | 15.3          | 0.004     |
| NGC 4725  | SAB(r)ab           | 12.4          | 0.004     |
| NGC 5850  | SB(r)b             | 35.2          | 0.009     |
| NGC 7280  | SAB(r)0            | 24.3          | 0.006     |
| NGC 7716  | SAB(r)b            | 34.1          | 0.009     |

Notes. (1) Galaxy name; (2) and (3) morphological types and luminosity distances as extracted from the catalog of Erwin (2004). Distances are corrected from Virgo-centric motion; (4) redshifts from NED.

3 TWO-DIMENSIONAL MULTI-COMPONENT PHOTOMETRIC DECOMPOSITIONS WITH GASP2D

The 2D multi-component photometric decompositions are performed with the code GASP2D (Méndez-Abreu et al. 2008, 2014, 2017). GASP2D fits the galaxy 2D surface-brightness distribution with a combination of structural components, parameterised by known mathematical functions. A Levenberg-Marquardt algorithm based on a χ2 minimisation is used to find the most suitable set of parameters describing the galaxy light.

GASP2D has already been tested in a number of works to find the structural composition of a variety of objects, such as AGN hosts (Benítez et al. 2013), galaxies with decoupled polar bulges (Corsini et al. 2012), and isolated galaxies (Morelli et al. 2016). For the present study, the ability of simultaneously fitting two bar structures was added to the code, as already introduced in Méndez-Abreu et al. (2017). The available components for the fitting are therefore a bulge, up to two bars, and a disc that might show none, positive, or negative bending (e.g. Erwin et al. 2007, Pohlen & Trujillo 2006).

For the sake of completeness, in the following we will present the analytical functions describing each component. More details on the fitting procedure can be found in Méndez-Abreu et al. (2017).

The surface brightness distribution of the bulge component is parameterised with a Sérsic profile (Sérsic 1968) of the form

\[ I_b(r_b) = I_{b0} \left( \frac{r_b}{h_b} \right)^{n_b} \exp \left( -\frac{r_b}{h_b} \right) \]

where \( r_b \) is the radius measured in the reference system of the bulge, \( R_b \), \( I_b \), and \( n_b \) are the effective (or half-light) radius, the surface brightness at \( R_b \), and the Sérsic index describing the curvature of the profile, respectively, and \( h_b \approx 0.868 \, n - 0.142 \) (Caon et al. 1993).

The surface brightness distribution of a galaxy disc is allowed to take three different shapes, namely: (i) Type I profile, a single exponential profile, (ii) Type II profile, a double exponential law with a down-bending beyond the so-called break radius, and (iii) Type III profile, a double exponential law with an up-bending in the outer parts of the disc. To account for these possibilities we adopt the following parameterisation:

\[ I_d(r_d) = I_d \left[ e^{-\frac{r_d}{h_d}} \theta + e^{-\frac{r_d - \text{break}}{h_{\text{break}}}} \right] e^{-\frac{r_d}{h_d}} (1 - \theta) \]

where

\[ \theta = \begin{cases} 0 & \text{if } r_d > r_{\text{break}} \\ 1 & \text{if } r_d < r_{\text{break}} \end{cases} \]

and \( r_d \) is the radius measured in the reference system of the disc. \( I_d \), \( h_d \), and \( r_{\text{break}} \) are the central surface brightness, inner scale-length, outer scale-length, and break radius of the disc, respectively.

The projected surface density of a 3D Ferrers ellipsoid (Ferrers 1877, see also Aguerri et al. 2009) is used to de-
scribe the surface-brightness distribution of both the inner and outer bar components:

$$I_{\text{bar}}(r_{\text{bar}}) = I_{0,\text{bar}} \left( 1 - \left( \frac{r_{\text{bar}}}{a_{\text{bar}}} \right)^2 \right)^{n_{\text{bar}}+0.5} \quad ; \quad \text{for } r_{\text{bar}} \leq a_{\text{bar}},$$

where $r_{\text{bar}}$ is the radius in the reference system of each bar. The inner and outer bars are allowed to have different ellipticities and position angles. $I_{0,\text{bar}}$, $a_{\text{bar}}$, and $n_{\text{bar}}$ represent the central surface brightness, length, and shape parameter of the bar, respectively. It is worth noting that $a_{\text{bar}}$ is not an effective radius, but the radius where the bar intensity drops to zero.

The bar surface-brightness distribution is assumed to be axially symmetric with respect to a generalised ellipse (Athanassoula 1990). Therefore, the radial coordinate is defined as

$$r = \left( \left| x \right|^c + \frac{y^2}{(1 - \epsilon_{\text{bar}})} \right)^{1/c},$$

where $\epsilon_{\text{bar}}$ is the bar ellipticity and $c$ controls the shape of the isophotes. A bar with pure elliptical isophotes has $c=2$. It is $c > 2$ if the isophotes are boxy, and $c < 2$ if they are discy. The parameters $\epsilon_{\text{bar}}$ and $c$, as well as the position angle, are assumed to be constant as a function of radius.

### 3.1 Selection of the $n$ and $c$ bar parameters

Recovering all possible bar parameters previously described as free variables in the fitting process is rather difficult even for single-barred galaxies, due to the high number of degeneracies among the parameters. The most commonly used procedure in photometric decompositions involving Ferrers profiles is to keep fixed the two shape parameters to their default $n_{\text{bar}}=2$ and $c=2$ values (Laursen et al. 2003; Méndez-Abreu et al. 2017).

Since we are particularly interested in studying the structure of inner and outer bars with great accuracy, we have investigated which values of $n_{\text{bar}}$ and $c$ provide the best fits. For this purpose, we first perform the $r'$-band double-barred fit in the usual way, i.e., with fixed values of $n_{\text{bar}}=2$ and $c=2$. Variations of the profile with $n_{\text{bar}}$ are explored first: the results from the usual fits are introduced as fixed initial conditions for all the inner and outer bar parameters ($I_{0,\text{bar}}$, $a_{\text{bar}}$, $b/a$, and $P\,\text{A}$) except for the outer bar length $a_{\text{OB}}$, which is allowed to vary. We remark that the correlation between bar length and $n_{\text{bar}}$ makes it mandatory to keep the bar length as a free variable when studying variations of $n_{\text{bar}}$. GASP2D is then run again with fixed integer values for $n_{\text{OB}} \in [1, 4]$. The trends $n_{\text{OB}}$ vs $\chi^2$ are inspected so the minimum providing the best $n_{\text{OB}}$ parameter for the outer bar is found. The process is then repeated for the inner bar case, fixing both $n_{\text{OB}}$ and $a_{\text{OB}}$ parameters to the newly recovered values.

A similar procedure is carried out to derive the best $c$ values, varying the integer values of $c$ within the range $c \in [1, 5]$. For this case, bar length as well as the rest of bar parameters (including the updated $n_{\text{bar}}$ values just obtained in the previous step) can be kept fixed and therefore this procedure is just a $\chi^2$ computation rather than a fitting. Again, the outer bar $c$ is explored first.

### 3.2 Double- and single-bar fits

All the sample galaxies were found to host a bulge component, two bars, and a disc. The $r'$-band images are used as benchmarks to perform the first fit. Figure 2 shows an example of the final $r'$-band fit for the galaxy NGC 357, which is composed of a Type-II disc, a bulge, and the inner and outer bars.

Following the prescription given in Méndez-Abreu et al. (2017), the final parameters for the $r'$-band are used as initial guesses for fitting the $g'$- and $i'$-band images, fixing the $n_{\text{bar}}$ and $c$ values to the best estimates obtained as described in Sect. 3.1. Although very similar, slight band-dependent differences in the measured parameters are expected. Best-fitting parameters for all the structures in each galaxy and band are shown in Tables A1 to A8.

The results from the double-barred fits in the $r'$-band are also used as initial guesses for single-barred $r'$-band fits. The outcomes are found in Tables A1 and A8. These single-barred fits are done for comparison purposes as discussed in Sect. 5.

We remark here that all parameters included in Tables A1 to A8 correspond to direct measurements from the images, i.e., lengths, radii, and scalelengths are provided in arcsec and projected onto the plane of the sky. For the analysis presented throughout the paper, bar parameters (lengths, ellipticities, and position angles) have been deprojected following the recipes given by Gadotti et al. (2007) and angular sizes have been transformed into physical scales. As explained in Zou et al. (2014), bar parameters for galaxies with inclinations $i > 60^\circ$ have not been deprojected due to the high uncertainties introduced in the process.

### 3.3 Error computation

The errors on the individual parameters involved in the fit were computed using a set of tailor-made mock galaxies in a Monte Carlo fashion. The full description of the methodology is presented in Méndez-Abreu et al. (2017) and we refer
the reader to this paper for a complete description. Here we provide a brief summary for the sake of clarity.

A sample of 500 mock double-barred galaxies was simulated using a combination of structural parameters constrained within the limits of our real galaxy sample in the $r'$-band (see Tables A1 and A2). Each model was built up using the equations provided in Sect. 3 for each distinct structure on a 2D grid with the SDSS pixel scale (0.396 arcsec/px). The total galaxy model was then convolved with a circular Moffat PSF with the typical FWHM of our SDSS images in $r'$-band (Sect. 3) in order to reproduce the observed spatial resolution. We also adopted the typical values of CCD gain ($4.86 \, \text{e}^{-}$/ADU) and read-out noise ($5.76 \, \text{e}^{-}$), and added the background and photon noise from the galaxy to yield a signal-to-noise ratio similar to that of the observations.

Finally, the mock images were analysed using GAS2D in the same way as real images. The difference between the input and output values provides us with a systematic (mean value) and statistical (standard deviation) error on the individual parameters. Both errors are added in quadrature to obtain the final values shown in Tables A1 to A8.

4 THREE-DIMENSIONAL SHAPES WITH galaXYZ

We derive the intrinsic 3D shape for the bulges, outer bars, and inner bars of our sample using the galaXYZ code, which follows the procedure described in Méndez-Abreu et al. (2010) and Costantin et al. (2018). This method has been previously applied to different samples of galactic bulges and outer bars, but it is used here for the first time in inner bars. Méndez-Abreu et al. (2018) and Costantin et al. (2018) have already demonstrated that the statistical approach to derive the intrinsic 3D shape is applicable to any galactic structure if the initial assumptions are fulfilled, namely: i) all structures under study can be modelled by a triaxial ellipsoid in the same equilibrium plane as the disc; ii) the galaxy disc is considered to be an oblate spheroid. We allow for the disc to have a intrinsic thickness according to a normal distribution with mean intrinsic axial ratio $(q_0,d)=0.267$ and standard deviation $\sigma_{q_0,d}=0.102$ (Rodríguez & Padilla 2013); and iii) all structures share the same centre, which is adopted as the centre of the galaxy.

All previous conditions are met by the three structures studied here. A caveat is that stellar bars can develop vertically-extended components during the so-called buckling phase. During this time, the bar creates what is called a box/peanut (B/P) structure in its central regions (Combes & Sanders 1981; Athanassoula et al. 1983; Martinez-Valpuesta et al. 2006). These structures do not comply with our first hypothesis. However, in Méndez-Abreu et al. (2018) it is demonstrated that the parameters obtained from our photometric decomposition represent the 'thin' part of the bar. Possible B/P structures present in a bar therefore have a small impact in the derived intrinsic shape.

The galaXYZ code needs as input the measured values of the projected geometric parameters (ellipticity and position angle) of the disc (representative of the galaxy inclination and position of the lines-of-nodes), and the structures under study, i.e. bulges, outer bars, and inner bars in our case. All these parameters, and their corresponding errors, are provided by the 2D photometric decomposition carried out in Sect. 3. Then, galaXYZ randomly generates 1000 geometric configurations by adopting for each parameter a Gaussian distribution centred on its measured value and with a standard deviation equal to its uncertainty. For each geometric configuration, the code evaluates equations 5 and 6 in Costantin et al. (2018). This is carried out in a Monte Carlo fashion with 5000 simulations of the intrinsic semiaxis ratios $B/A$ and $C/A$.

The result of this analysis is a joined probability distribution function (PDF) of $B/A$ and $C/A$, i.e., the PDF of the intrinsic 3D shape of each structure. An example is shown in Fig. 4 for the inner bar of NGC 2859. Our approach is...
Figure 2. 2D multi-component photometric decomposition of the SDSS-DR9 $r'$-band image of the double-barred galaxy NGC 357 performed with GASP2D. The top panels show the original image (left), the 2D best-fitting model (middle), and the residuals (right). The bottom left panel shows the original surface-brightness radial profile (black points) as derived with an isophotal fitting. Blue dashed, green dash-dotted, yellow dash-dotted, and red dotted lines show the bulge, inner bar, outer bar, and disc components, respectively. The disc is truncated showing a down-bending. Residuals are included in the lower subpanel while the inset zooms into the very central region. Bottom middle and bottom right panels show the ellipticity and position angle profiles from the isophotal fitting of the original image (black dots) and the same measurements for the 2D model (green lines).

entirely based on the projected geometric properties of each structure and the PDF is calculated independently for each structure in a galaxy. The most probable $B/A$ and $C/A$ values and their corresponding $1\sigma$ uncertainties are shown in Table 2. We notice here that the width of the PDF in either $B/A$ or $C/A$ does not only depend on the photometric decomposition errors, but mostly on the lack of knowledge of the Euler angle ($\phi$), i.e., the angle describing the position of the intrinsic major axis of the structure with respect to the line-of-nodes in the plane of the disc. Therefore, there are some projected configurations (combination of the disc and structure geometry) that are less suited to derive the 3D shape and they provide large uncertainties. Values with $1\sigma$ errors in any intrinsic semiaxis ratio larger than 0.5 are not included in the final analysis.

On the other hand, galaXYZ performs a statistical recovery of the size of each structure in both the galaxy plane and perpendicular to it. Since galaXYZ uses the outcomes from GASP2D as input values, the results from both analyses are not fully independent. However, the PDF delivered by galaXYZ include statistical as well as methodology-intrinsic uncertainties, and we select as final value the one single point with the maximum likelihood. A good agreement between the in-plane sizes retrieved by GASP2D and galaXYZ therefore indicates the goodness of our combined analysis. Such analysis is shown in Fig. 4. In this case, all 17 galaxies from the sample are included in the plot regardless of their error bars: those finally removed from the analysis with error bars greater than 0.5 are shown in grey, while the coloured symbols correspond to the remaining 8 galaxies. The excellent correspondence between the measurements from both techniques, even for the dismissed galaxies, argues in favor of this analysis and shows how conservative the uncertainties provided by the PDF are.
Figure 3. 3D statistical derivation of the intrinsic shape for the inner bar of the double-barred galaxy NGC 2859. The joined probability distribution functions (PDF) of the in-plane and off-plane axis ratios, $B/A$ and $C/A$, are plotted in green colours. The value with the highest probability is shown with a yellow diamond. The regions corresponding to different shapes are indicated (prolate, oblate, and triaxial; see Méndez-Abreu et al. 2018). Such analysis has been performed for all bulges, inner bars, and outer bars of the sample.

Figure 4. Comparison between the semi-major axis ratios in the galaxy plane for the outer (triangles; left) and inner (squares; right) bars as measured in 2D with GASp2D and then deprojected (horizontal axes), and measured in 3D with galaXYZ (which provides already deprojected values; vertical axes). Coloured symbols correspond to galaxies with uncertainties in any semi-major axis ($B/A$ or $C/A$ - not shown in this plot- for either the inner or outer bar) less than 0.5, while the grey symbols represent galaxies with larger uncertainties. The good agreement between both techniques is indicated by a relationship close to the yellow-dashed 1:1 line in each plot.

5 INFLUENCE OF INNER BARS IN BULGE PARAMETERS

Photometric decompositions have been widely used in the literature for studying the central regions of galaxies and, particularly, the nature of bulges (e.g., Gadotti 2009). Current estimates establish that ~20% of disc galaxies are double-barred (e.g., Erwin & Sparke 2002, Laine et al. 2002, Erwin 2011), and this is most likely a lower limit due to the small sizes of inner bars (~23% of the outer bar size as shown in Paper II). Despite their frequent presence, inner bars have never been taken into account when decomposing galaxies and not including bars in the photometric decompositions may significantly affect the derived bulge parameters (e.g., Laurikainen et al. 2006, Méndez-Abreu et al. 2017), hampering the conclusions obtained in many works. In Fig. 5 we compare the double- and single-barred $r'$-band fits of the double-barred galaxies in order to quantify how
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Figure 5. Bulge structural parameters as measured from the photometric decompositions in $r'$-band including two bars (vertical axes) and the most common method of considering only one single bar (horizontal axes). Top left: bulge effective radii; top right: Sérsic indices; bottom left: ellipticities; bottom right: position angles. The grey dashed line marks the 1:1 relationship in each panel. Values are colour-coded by length of the inner bar as obtained from the double-barred fitting and indicated by the right-hand side colour bar. Except for the Sérsic indices of the most extended bulges ($n < 1$) and the ellipticities of few galaxies with long inner bars, no major bias is introduced by disregarding inner bars.

much bulge measurements (effective radius, Sérsic index, ellipticity, and position angle) are affected when inner bars are not included in the analysis.

The effective radii measurements are in good agreement, with a mean difference of 0.07 kpc between the single- and double-bars fits. This value is lower than the typical errors introduced by the adopted methodology (see Tables A7 and A8) for all the galaxies, and therefore negligible. The agreement between Sérsic indices is also remarkably good except for the two galaxies with $n < 1$, one of which hosts an extremely small bulge (NGC 2681). For the remaining galaxies we obtain a mean difference of 0.2, slightly higher than the errors from the decompositions. The ellipticity shows the largest discrepancy among all parameters, with a mean difference of 0.05, i.e., larger than 10% of the typical values. We note however that such measurement is of the order or only slightly larger than the typical errors derived from GASP2D. Finally, the position angle is the best behaved measurement, with only one discrepant galaxy corresponding to a very spherical bulge ($\epsilon = 0.03$), where the position angle is irrelevant.

We can therefore conclude that the bulge parameters are rather insensitive to disregarding the inner bar in the fits. However, small differences are observed. With the aim of identifying the main contributor to such discrepancies, measurements in Fig. 5 are colour-coded attending to the length of the (dismissed) inner bars. A subtle trend pointing at largest inner bars causing the largest differences in the derived ellipticity and effective radius of the bulges is observed, although inner bar size does not account for discrepancies.
in the Sérsic indices. The same test has been performed by colour-coding the values with other quantities such as Sérsic index, bulge effective radius, and $R_e/a_{IB}$ ratio. No clear correlations have been found, apart from the expected fact that galaxies with large inner bars with respect to the bulge size (i.e., small $R_e/a_{IB}$ ratios) tend to compensate the dismissed inner bar by increasing the size of the bulge ($R_e$). We therefore conclude that most likely a combination of all those parameters (inner bar parameters with respect to bulge parameters) is responsible for the differences found between the single- and double-barred fits. We emphasise again that deviations are small and generally within the error bars.

We note that the inner bar contribution to the total galaxy light can be as low as 0.5% with a mean value of 4%, while the outer bar accounts for [4%, 28%] of the total light. It is therefore reasonable to find that bulge measurements do not vary in a significant way when inner bars are not accounted for and therefore their light is included as bulge light.

For the sake of completeness, we have also investigated whether dismissing the inner bar has any effect over the outer bar parameters. While the length and position angle of the outer bar are absolutely independent from including the inner bar or not, very subtle differences are found for the outer bar ellipticity. However, such differences do not depend on the inner bar size and they are well within the error bars, thus supporting the idea that outer bar parameters are not affected by the inner bar.

6 THE NATURE OF BULGES WITHIN DOUBLE-BARRED GALAXIES

6.1 Traditional photometric diagnostics

Figure 4 shows the distribution of Sérsic indices for the bulges of our double-barred sample. Following the traditional demarcation $n=2$, we find that 6 out of the 17 double-barred galaxies host bulges with $n > 2$ and they can therefore be considered classical bulges. The remaining 11 galaxies, i.e. the majority (65%) of the sample, host disc-like bulges attending to this pure Sérsic index diagnostics. Since our conclusions might be hampered by the limited size of the sample, we compare with the 162 single-barred galaxies of Méndez-Abreu et al. (2017), also shown in Fig. 6. This comparison sample does show a higher incidence of bulges with $n < 2$ (77%), thus complying with the expectation that single/double-barred galaxies should show a large fraction of disc-like bulges. In particular, the two double-barred galaxies in Méndez-Abreu et al. (2017) host bulges with $n < 2$.

For the sake of completeness, Fig. 4 shows the results for the 287 barred galaxies included in the sample of Gadotti (2009) as well. We remind the reader that this represents a more distant sample where bars are modelled with a Sérsic profile instead of a Ferrers profile; these differences may account for the higher abundance of bulges with $n > 2$ found in this case (59%).

The Kormendy (1977) relation is drawn in Fig. 7. As in the previous plot, the samples of bulges within barred galaxies from Gadotti (2009) and Méndez-Abreu et al. (2017) are also shown. The dashed line represents the demarcation found by Gadotti (2009) to separate between the classical and disc-like nature of the bulges. We remark that the
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6.2 3D intrinsic shape of bulges

In Costantini et al. (2018), different diagnostics to constrain the nature of bulges are put at test. The main conclusion of that work is that the traditional pure-photometric methods based on projected quantities such as the Sérsic index do not provide clear separations between classical and disc-like bulges (see also Méndez-Abreu et al. 2018). The probabilistic retrieval of the 3D intrinsic shape of bulges represents a better suited diagnostic, as it allows the study of the intrinsic flattening of these structures, once isolated from the remaining components of the galaxy.

Disc-like bulges are expected to be flattened structures with a close-to-circular projection in the galaxy plane, due to their rotating nature. Only two bulges in our sample comply with these requirements, as indicated by the results shown in Table 2: NGC 3368 (B/A = 0.91 and C/A = 0.41) and NGC 7280 (B/A = 0.89 and C/A = 0.31). They therefore represent the best candidates to host disc-like bulges. Note that NGC 3368 does show a bulge Sérsic index $n < 2$, while NGC 7280 would be classified as a classical-bulge host according to the Sérsic index discriminator.

Méndez-Abreu et al. (2018) discuss why bulges which are intrinsically flatter than their coexistent single bars represent even stronger candidates to have a disc-like nature. Such conclusion is based on the fact that the vertical extension of bars is closely related to that of galaxy discs, so if bulges are as flats as bars, they are also at least as flat as discs, a result which is hard to reconcile with a classical nature. The left panel of Fig. 8 shows the intrinsic semiaxis ratios of bulges and outer bars. Only the 10 out of 17 double-barred galaxies with uncertainties lower than 0.5 in the derived 3D parameters of bulges and outer bars are plotted. Four galaxies show close values of the flattening between bulges and outer bars. These are NGC 3368, NGC 4340, NGC 7280, and NGC 7716, being NGC 3368 the only one
with an intrinsic flattening for the bulge ($C/A = 0.41$) strictly lower than that for the outer bar ($C/A = 0.46$). NGC 3368 and NGC 7280 therefore remain as the two best candidates for hosting disc-like bulges. Note that the bulge of NGC 7716 has an axis ratio $B/A = 0.64$, which does not fulfill the axisymmetric projection requirement.

For the sake of completeness, the right panel of Fig. 8 shows similar results but comparing the intrinsic flattening of bulges and inner bars. A different set of 10 galaxies with low uncertainties affecting bulge and inner bar measurements are plotted in this case. NGC 3368 and NGC 7280 are included in this right panel: they host thicker bulges than the corresponding inner bars, while NGC 4340 hosts a flatter and almost circular bulge, and NGC 3941 and NGC 7716 show similar intrinsic flattenings for bulges and inner bars. The intrinsic shape of inner bars has never been studied in the literature and no relationship between it and the nature of bulges is a-priori expected. A detailed comparison between the 3D shapes of inner and outer bars is presented in Paper II.

### 7 DISCUSSION

We discuss here our results within the context of the two major questions brought up in Sect. 1 about bulges within double-barred systems: i) do double-barred galaxies host a larger fraction of disc-like bulges than non-barred galaxies due to the efficient secular evolution that is expected to take place in them?; and ii) are bars within bars formed due to the presence of a disc-like component resulting from secular evolution due to the large-scale bar?

Following both the Kormendy relation (Fig. 7) and the 3D intrinsic shape (Fig. 8) as diagnostics for the bulge nature, our results agree with double-barred galaxies mostly hosting classical bulges. Only the bulge Sérsic indices point towards a larger fraction of disc-like bulges.

Two galaxies among the sample of 17 individuals, NGC 3368 and NGC 7280, stand out as the best candidates for hosting disc-like bulges as suggested by our preferred diagnostics of intrinsic flattening. With the aim of understanding better the origin of bulges within double-barred galaxies, we compute colours for the isolated bulges by using the results from the 2D photometric decompositions in the different bandpasses (Table 3). Results among galaxies can be directly compared as we use colours integrated within one bulge effective radius, so no bias due to different galaxy sizes affects the results. All our bulges show rather red colours, with median values of $(g'-i') = 1.28$ and $(r'-i') = 0.41$. Note these values agree with the results for classical bulges in, e.g.,

| Galaxy   | $(g'-i')$ | $(r'-i')$ | $\log(M_{*, \text{bulge}} (M_\odot))$ | $\log(M_{\text{gal}} (M_\odot))$ |
|----------|-----------|-----------|-----------------------------------|-----------------------------------|
| NGC 357  | 1.52      | 0.49      | 10.27                             | 10.88                             |
| NGC 718  | 1.07      | 0.34      | 9.83                              | 10.33                             |
| NGC 2642 | 1.53      | 0.46      | 10.45                             | 10.62                             |
| NGC 2681 | -0.04     | -0.10     | 7.91                              | 9.51                              |
| NGC 2859 | 1.20      | 0.41      | 10.40                             | 10.64                             |
| NGC 2950 | 1.18      | 0.36      | 9.97                              | 10.38                             |
| NGC 2962 | 1.46      | 0.47      | 10.00                             | 10.61                             |
| NGC 3941 | 1.47      | 0.50      | 10.44                             | **                                |
| NGC 3945 | 1.20      | 0.39      | 9.80                              | 10.41                             |
| NGC 4314 | 1.10      | 0.39      | 9.76                              | 10.19                             |
| NGC 4340 | 1.14      | 0.40      | 9.38                              | 9.99                              |
| NGC 4503 | 1.29      | 0.43      | 9.92                              | 10.47                             |
| NGC 4725 | 1.28      | 0.49      | 10.24                             | **                                |
| NGC 5850 | 1.38      | 0.47      | 10.66                             | 11.02                             |
| NGC 7716 | 1.38      | 0.46      | 10.53                             | 10.67                             |

** Absolute magnitudes for the galaxies are obtained from SDSS. No values are provided for these galaxies and therefore masses cannot be computed.
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Figure 3 in Fisher & Drory (2011) shows that disc-like bulges are more frequent in less massive galaxies, and therefore any mass bias in our sample needs to be considered as well (see also Fig. 12 in Gadotti 2009). We calculate the galaxy masses by using the colours and the recipe given by Zibetti et al (2009). Results are shown in Table 3. Our galaxies span a wide range of masses between $3 \times 10^9$ and $1 \times 10^{11} M_{\odot}$, with a median value of $2.6 \times 10^{10} M_{\odot}$. Actually, a larger fraction of disc-like bulges is expected in galaxies at these masses, so no mass bias appears to be affecting our analysis.

In summary, most of the diagnostics indicate our double-barred galaxies host classical bulges. At first glance, this finding may look like against the expectations: secular processes should have played a major role in these kind of galaxies and therefore a significant presence of disc-like bulges is expected. However, we must note here that the traditional paradigm of a single, either classical or disc-like, bulge in a galaxy has recently been challenged by many authors: the coexistence of several bulges of different nature has been observationally demonstrated by Méndez-Abreu et al. (2014) and Erwin et al. (2014). An exhaustive analysis combining photometric with kinematic information is claimed to be required in order to clearly distinguish the nature and possible presence of more than one bulge within a galaxy. If our double-barred galaxies were hosting composite bulges, the analysis presented here would correspond to the dominant structure in light, which would be the one revealed in the images and the photometric decompositions. A previously-formed classical bulge could therefore be hiding the presence of a secularly-formed disc-like bulge in our galaxies.

In de Lorenzo-Cáceres et al. (2018a, submitted), we investigate the kinematics and stellar populations of two double-barred galaxies for which MUSE TIMER integral-field spectroscopic data is available: NGC 1291 and NGC 5850 (also included in this sample). For those galaxies we perform 2D photometric decompositions of $3.6 \mu m$ images including a bulge, two bars, (truncated) disc and also an inner disc. Whether we can call these inner discs as disc-like bulges (and therefore composite bulges as both galaxies do host an additional bulge component as well) is a matter of semantics out of the scope of this discussion. However, we do find a strong connection between the size of the inner discs and the inner bars, suggestive of a dynamical origin of those inner bars from instabilities in the small-scale discs. The goal of the photometric study of double-barred galaxies presented here is to study the nature of the dominant bulges, as we do not have the support of high resolution spectroscopic data which may reveal the presence of rotating inner discs. Therefore we cannot discard the possibility of having a faint inner disc coexisting with the classical bulges in several (if not all) double-barred galaxies, as it is indeed the case for NGC 5850 (and also NGC 357, NGC 4725 and NGC 2859, see the kinematic evidences in de Lorenzo-Cáceres et al. 2004, de Lorenzo-Cáceres et al. 2014 and de Lorenzo-Cáceres et al. 2013).

The B/P structures developed at the inner regions of large stellar bars are considered also as ’bulges’ by several authors, attending to the fact that they are central components which cause an excess of light with respect to the pure disc and bar. Although we emphasise that B/P are not isolated structures but they rather belong to the bars, we have performed here a visual inspection of our double-barred galaxies with the aim at looking for signatures of the presence of a B/P component coexisting with the dominant bulge. In particular, we search for a bar lens: an oval component seen in moderately inclined galaxies which is supposed to be the face-on counterpart of the X-shaped B/P structure seen edge-on (Athanassoula et al. 2013; Laurikainen & Salo 2017). Other indicators of the presence of a B/P are isophotal twists in the innermost bar regions, which tend to show boxy isophotes accompanied of narrow outermost isophotes (the so-called spurs, see Erwin & Debattista 2013, 2017). Note that the bulge component found in our analysis and studied throughout this paper does not correspond to the possible B/P revealed by this visual inspection. 7 out of 17 galaxies do not show signatures of the presence of a B/P (NGC 718, NGC 2681, NGC 2962, NGC 3368, NGC 4503, NGC 7280, and NGC 7716), while 4 out of 17 galaxies host clear bar lenses (NGC 2950, NGC 3945, NGC 4314, NGC 4430). For the remaining 6 galaxies, the classification of B/P structures is unclear: NGC 357, NGC 2642, NGC 2859, NGC 3941, and NGC 4725 show weak elliptical structures, spurs, or isophotal twists that may be due to B/P. Finally, NGC 5850 shows a component resembling a bar lens but the kinematic study presented in de Lorenzo-Cáceres et al. (2018a, submitted) reveals that it is indeed an inner disc.

In order to constrain whether there is any further relation between our dominant bulges and the double bars, correlations between the bar parameters (ellipticity and length) and bulge parameters (Sérsic index and effective radius) have been searched for, with no particular results found. As a final note, the fact that all our galaxies are better fit by including a bulge component, i.e., all double-barred galaxies do host a bulge, might be relevant for understanding their formation. Our sample includes some late Hubble types up to Sbc but no bulgeless galaxy is found. This may point towards a connection between the presence of inner bars and formation of bulges, and it even poses the question of whether the presence of a classical bulge is required for forming an inner bar through, e.g., dynamical stabilization of the central regions.

8 CONCLUSIONS

We present a complete photometric study of a sample of 17 double-barred galaxies consisting of two analyses which are for the first time applied to these kind of objects: i) 2D photometric decompositions including a bulge, inner bar, outer
bar, and (truncated) disc, and ii) 3D deprojections of bulges, inner bars, and outer bars thus retrieving their actual intrinsic shape. While the photometric properties of bars are explored in the companion Paper II, the current work focuses on the properties of bulges. In particular, we constrain the classical vs. disc-like nature of bulges in double-barred systems. Our galaxy sample spans a wide range in galaxy masses. The main results are:

- All double-barred galaxies under study host a bulge component.
- The bulge properties derived through photometric decompositions are not significantly affected by dismissing the presence of the inner bar.
- 65% of the galaxies host bulges with Sérsic index $n < 2$.
- All bulges lay at the top sequence of the Kormendy (1979) relation, in the region supposedly populated by classical bulges.
- No correlations are found between inner bar properties (length and ellipticity) and bulge properties (Sérsic index and effective radius).
- Only 2 out of 17 bulges show an intrinsic shape compatible with a disc-like nature, i.e., almost circular in-plane projection and flattened off-plane profile: NGC 3368 and NGC 7280.
- 3 out of 17 bulges (including NGC 3368 and NGC 7280) are flatter than their corresponding outer bars. This has been argued to be an indication of their disc-like nature.
- Inner bars are either flatter or with similar flattening than their coexisting bulges.
- All bulges show rather red colours.

Most previous results support a classical nature for bulges within double-barred galaxies. A major incidence of secularly-formed structures such as disc-like bulges is expected in these galaxies, where two non-axisymmetric structures may help to transport gas to the central regions. We note that composite bulges are not studied in this work and therefore we cannot rule out the possibility of a faint disc-like component, or even other kind of secularly-formed substructure, coexisting within these galaxies; we refer to the dominant bulge in this analysis. The presence of a dominant classical bulge in all double-barred galaxies under study suggests that hosting a hot component may be necessary for the dynamical development of an inner bar within a barred galaxy.

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APPENDIX A: GASP2D STRUCTURAL PARAMETERS OF 18 DOUBLE-BARRED GALAXIES
### Table A1. ALL PARAMETERS FOR THE DOUBLE-BARRED FITS in r'-BAND. First set of 9/17 galaxies.

|       | NGC 337 | NGC 718 | NGC 2642 | NGC 2681 | NGC 2659 | NGC 2950 | NGC 2962 | NGC 3308 | NGC 9341 |
|-------|---------|---------|----------|----------|----------|----------|----------|----------|----------|
| $\mu_e$ | 18.52 ± 0.26 | 18.42 ± 0.15 | 19.58 ± 0.33 | 15.96 ± 0.13 | 18.52 ± 0.16 | 16.86 ± 0.16 | 18.32 ± 0.15 | 18.91 ± 0.16 | 17.53 ± 0.13 |
| $R_e$ | 2.68 ± 0.43 | 3.15 ± 0.32 | 2.65 ± 0.53 | 1.65 ± 0.17 | 5.63 ± 0.47 | 2.65 ± 0.22 | 2.22 ± 0.23 | 14.59 ± 1.22 | 3.84 ± 0.38 |
| $n$ | 1.62 ± 0.18 | 2.24 ± 0.27 | 1.57 ± 0.29 | 0.69 ± 0.05 | 1.66 ± 0.13 | 1.88 ± 0.15 | 1.63 ± 0.20 | 1.44 ± 0.11 | 2.28 ± 0.17 |
| $b/a$ | 0.90 ± 0.04 | 0.84 ± 0.04 | 0.84 ± 0.05 | 0.86 ± 0.02 | 0.95 ± 0.02 | 0.69 ± 0.02 | 0.97 ± 0.04 | 0.69 ± 0.02 | 0.78 ± 0.02 |
| PA | 38.76 ± 5.05 | 163.89 ± 5.20 | 127.90 ± 7.76 | 25.71 ± 3.31 | 103.53 ± 2.97 | 97.36 ± 2.97 | 176.86 ± 5.20 | 154.56 ± 2.97 | 18.42 ± 3.31 |
| $B/T$ | 0.170 | 0.214 | 0.092 | 0.125 | 0.259 | 0.257 | 0.135 | 0.214 | 0.224 |

### Bulge

|       | $\mu_e$ | $R_e$ | $n$ | $b/a$ | PA | $B/T$ |
|-------|---------|-------|-----|------|-----|-------|
| $\mu_e$ | 20.81 ± 0.07 | 20.52 ± 0.07 | 21.11 ± 0.14 | 20.15 ± 0.14 | 21.92 ± 0.14 | 20.17 ± 0.14 | 21.12 ± 0.07 | 20.71 ± 0.07 | 19.29 ± 0.07 |
| $R_e$ | 29.56 ± 1.38 | 22.38 ± 0.85 | 27.61 ± 1.88 | 33.38 ± 0.81 | 63.41 ± 1.05 | 26.33 ± 0.4 | 33.73 ± 1.28 | 90.36 ± 1.33 | 31.17 ± 0.74 |
| $n$ | 0.82 ± 0.01 | 0.95 ± 0.01 | 0.97 ± 0.02 | 0.89 ± 0.01 | 0.84 ± 0.01 | 0.64 ± 0.01 | 0.57 ± 0.01 | 0.65 ± 0.01 | 0.65 ± 0.01 |
| $b/a$ | 0.90 ± 0.04 | 0.84 ± 0.04 | 0.84 ± 0.05 | 0.86 ± 0.02 | 0.95 ± 0.02 | 0.69 ± 0.02 | 0.97 ± 0.04 | 0.69 ± 0.02 | 0.78 ± 0.02 |
| PA | 18.78 ± 0.37 | 19.70 ± 0.58 | 151.41 ± 0.30 | 124.20 ± 0.25 | 84.73 ± 0.45 | 123.80 ± 0.45 | 2.46 ± 0.58 | 160.7 ± 0.45 | 9.15 ± 0.25 |
| $B/T$ | 0.667 | 0.645 | 0.806 | 0.594 | 0.537 | 0.444 | 0.587 | 0.527 | 0.614 |

### Disc

|       | $\mu_e$ | $R_e$ | $n$ | $b/a$ | PA | $B/T$ |
|-------|---------|-------|-----|------|-----|-------|
| $\mu_e$ | 20.73 ± 0.30 | 20.42 ± 0.32 | 21.37 ± 0.45 | 19.43 ± 0.18 | 20.70 ± 0.20 | 19.63 ± 0.20 | 20.69 ± 0.32 | 20.16 ± 0.20 | 19.03 ± 0.18 |
| $R_e$ | 38.42 ± 0.63 | 46.90 ± 0.78 | 53.08 ± 1.53 | 39.64 ± 0.39 | 72.93 ± 0.79 | 45.59 ± 0.49 | 49.52 ± 0.82 | 135.14 ± 1.46 | 34.98 ± 0.34 |
| $n$ | 3 | 4 | 3 | 3 | 3 | 2 | 3 |
| $b/a$ | 0.44 ± 0.04 | 0.42 ± 0.02 | 0.24 ± 0.06 | 0.73 ± 0.02 | 0.63 ± 0.03 | 0.53 ± 0.03 | 0.47 ± 0.02 | 0.59 ± 0.03 | 0.44 ± 0.02 |
| PA | 119.74 ± 0.28 | 150.89 ± 0.33 | 117.11 ± 0.58 | 83.57 ± 0.16 | 159.32 ± 0.13 | 156.05 ± 0.13 | 174.39 ± 0.33 | 128.19 ± 0.13 | 171.29 ± 0.16 |
| $B/T$ | 0.106 | 0.127 | 0.097 | 0.170 | 0.182 | 0.204 | 0.224 | 0.247 | 0.136 |

### Intensities are in magnitudes/arcsec$^2$.

Effective radii, disc scalelengths and break radii, and bar lengths are provided in arcsec.

Position angles are given in degrees from North to East.

$b/a$ is the minor-to-major axis of the projected ellipsoid (ellipticity is $\epsilon = 1-b/a$).

The analytical functions describing the structures are explained in Sect. 3.
Table A2. ALL PARAMETERS FOR THE DOUBLE-BARRED FITS IN r'-BAND. Last set of 8/17 galaxies

| NGC 3945 | NGC 4314 | NGC 4430 | NGC 4593 | NGC 4725 | NGC 5850 | NGC 7280 | NGC 7716 |
|----------|----------|----------|----------|----------|----------|----------|----------|
| $\mu_0$ | 18.39 ± 0.16 | 19.57 ± 0.13 | 19.16 ± 0.16 | 18.93 ± 0.16 | 18.50 ± 0.16 | 20.00 ± 0.16 | 18.93 ± 0.26 | 17.97 ± 0.15 |
| $R_b$ | 5.72 ± 0.48 | 10.94 ± 1.09 | 6.01 ± 0.50 | 5.05 ± 0.42 | 6.80 ± 0.57 | 7.77 ± 0.65 | 3.71 ± 0.59 | 2.04 ± 0.21 |
| n | 1.57 ± 0.12 | 0.94 ± 0.07 | 2.15 ± 0.17 | 2.64 ± 0.20 | 1.68 ± 0.13 | 2.14 ± 0.17 | 3.37 ± 0.36 | 1.70 ± 0.21 |
| b/a | 0.83 ± 0.02 | 0.81 ± 0.02 | 0.76 ± 0.02 | 0.75 ± 0.03 | 0.83 ± 0.03 | 0.97 ± 0.03 | 0.65 ± 0.04 | 0.60 ± 0.04 |
| PA | 150.63 ± 2.97 | 129.41 ± 3.31 | 98.85 ± 2.97 | 18.90 ± 2.97 | 29.97 ± 2.97 | 158.06 ± 2.97 | 66.56 ± 5.05 | 55.52 ± 5.20 |
| B/T | 0.238 | 0.214 | 0.256 | 0.205 | 0.094 | 0.182 | 0.280 | 0.144 |

Deconstructing double-barred galaxies: bulges

Table A3. ALL PARAMETERS FOR THE DOUBLE-BARRED FITS IN g'-BAND. First set of 9/17 galaxies

| NGC 357 | NGC 2642 | NGC 2681 | NGC 2859 | NGC 2950 | NGC 2962 | NGC 3368 | NGC 3941 |
|----------|----------|----------|----------|----------|----------|----------|----------|
| $\mu_0$ | 19.19 ± 0.33 | 22.91 ± 0.93 | 20.70 ± 0.69 | 20.51 ± 0.58 | 20.89 ± 0.51 | 21.97 ± 0.91 | 21.01 ± 0.91 | 21.01 ± 0.91 |
| $R_b$ | 2.76 ± 0.55 | 3.02 ± 0.53 | 2.62 ± 0.52 | 1.48 ± 0.15 | 2.59 ± 0.54 | 2.77 ± 0.28 | 2.28 ± 0.40 | 15.36 ± 1.29 | 3.97 ± 0.40 |
| n | 1.60 ± 0.29 | 2.02 ± 0.40 | 1.19 ± 0.22 | 2.02 ± 0.15 | 1.90 ± 0.18 | 1.67 ± 0.20 | 1.46 ± 0.29 | 1.37 ± 0.11 | 2.59 ± 0.19 |
| b/a | 0.83 ± 0.05 | 0.83 ± 0.05 | 0.80 ± 0.04 | 0.96 ± 0.02 | 0.92 ± 0.04 | 0.71 ± 0.04 | 0.93 ± 0.05 | 0.64 ± 0.03 | 0.78 ± 0.02 |
| PA | 33.72 ± 7.76 | 164.48 ± 7.79 | 131.33 ± 7.76 | 29.70 ± 3.31 | 92.96 ± 5.20 | 100.99 ± 5.20 | 7.11 ± 7.79 | 152.84 ± 2.97 | 19.79 ± 3.31 |
| B/T | 0.164 | 0.203 | 0.053 | 0.166 | 0.204 | 0.224 | 0.110 | 0.173 | 0.214 |

Deconstructing double-barred galaxies: bulges
| Parameter | Value 1 | Value 2 | Value 3 | Value 4 | Value 5 | Value 6 | Value 7 | Value 8 | Value 9 |
|-----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $\mu_0$   | 20.71 ± 0.15 | 20.16 ± 0.26 | 20.05 ± 0.15 | 19.81 ± 0.15 | 19.22 ± 0.16 | 20.85 ± 0.15 | 19.87 ± 0.33 | 18.08 ± 0.31 |
| $R_e$     | 11.50 ± 1.17 | 10.32 ± 0.65 | 6.32 ± 0.64 | 5.14 ± 0.52 | 6.14 ± 0.51 | 7.45 ± 0.76 | 4.26 ± 0.85 | 1.37 ± 0.24 |
| $n$       | 3.41 ± 0.42 | 0.80 ± 0.09 | 2.58 ± 0.32 | 2.60 ± 0.82 | 1.86 ± 0.14 | 3.12 ± 0.26 | 2.16 ± 0.40 | 1.93 ± 0.38 |
| $b/a$     | 0.88 ± 0.04 | 0.82 ± 0.04 | 0.75 ± 0.04 | 0.75 ± 0.04 | 0.85 ± 0.03 | 0.97 ± 0.04 | 0.60 ± 0.05 | 0.45 ± 0.05 |
| PA        | 149.56 ± 5.20 | 123.02 ± 5.05 | 98.14 ± 5.20 | 20.03 ± 5.20 | 34.43 ± 2.97 | 148.30 ± 5.20 | 67.54 ± 7.76 | 53.90 ± 7.79 |
| $B/T$     | 0.340 ± 0.140 | 0.297 ± 0.256 | 0.197 ± 0.070 | 0.138 ± 0.023 | 0.082 ± 0.062 |
| $\mu_0$   | 22.67 ± 0.07 | 22.17 ± 0.07 | 21.52 ± 0.07 | 20.27 ± 0.07 | 20.93 ± 0.07 | 22.37 ± 0.07 | 21.31 ± 0.14 | 20.88 ± 0.09 |
| $b_{bar}$ | 69.07 ± 2.65 | 59.84 ± 2.79 | 34.96 ± 1.33 | 26.45 ± 1.04 | 99.59 ± 1.65 | 65.99 ± 2.51 | 26.80 ± 1.84 | 18.18 ± 1.42 |
| $b/a$     | 0.67 ± 0.01 | 0.92 ± 0.01 | 0.67 ± 0.01 | 0.46 ± 0.01 | 0.50 ± 0.01 | 0.74 ± 0.01 | 0.65 ± 0.02 | 0.82 ± 0.01 |
| PA        | 166.07 ± 0.58 | 137.31 ± 0.37 | 91.68 ± 0.58 | 9.97 ± 0.58 | 42.99 ± 0.45 | 138.64 ± 0.58 | 74.48 ± 0.90 | 37.85 ± 0.73 |
| $B/T$     | 0.458 ± 0.050 | 0.502 ± 0.060 | 0.721 ± 0.089 | 0.762 ± 0.064 | 0.781 ± 0.050 |
| $\mu_0$   | 21.42 ± 0.32 | 21.04 ± 0.30 | 22.47 ± 0.32 | 21.22 ± 0.32 | 21.09 ± 0.20 | 22.40 ± 0.32 | 21.50 ± 0.45 | 21.92 ± 0.42 |
| $a$       | 58.16 ± 0.96 | 125.89 ± 2.05 | 62.42 ± 1.03 | 32.01 ± 0.53 | 54.26 ± 0.59 | 135.51 ± 2.24 | 21.19 ± 0.61 | 30.48 ± 0.91 |
| $n$       | 2              | 3              | 1              | 2              | 3              | 4              | 2              | 2              |
| Outer bar | $b/a$     | 0.63 ± 0.02 | 0.26 ± 0.04 | 0.44 ± 0.02 | 0.48 ± 0.02 | 0.66 ± 0.03 | 0.24 ± 0.02 | 0.51 ± 0.06 | 0.48 ± 0.09 |
| PA        | 74.99 ± 0.33 | 147.29 ± 0.28 | 30.81 ± 0.33 | 176.52 ± 0.33 | 36.25 ± 0.13 | 114.98 ± 0.33 | 55.43 ± 0.57 | 17.19 ± 0.50 |
| $c$       | 2              | 2              | 4              | 2              | 2              | 2              | 3              | 3              |
| $B/T$     | 0.067 ± 0.018 | 0.016 ± 0.017 | 0.017 ± 0.004 | 0.009 ± 0.009 | 0.064 ± 0.006 |

**Table A4. ALL PARAMETERS FOR THE DOUBLE-BARRED FITS IN g'-BAND. Last set of 8/17 galaxies.**

| Parameter | Value 1 | Value 2 | Value 3 | Value 4 | Value 5 | Value 6 | Value 7 | Value 8 | Value 9 |
|-----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $\mu_0$   | 18.03 ± 0.26 | 18.21 ± 0.16 | 19.12 ± 0.26 | 16.68 ± 0.13 | 18.11 ± 0.16 | 16.66 ± 0.16 | 17.93 ± 0.15 | 18.30 ± 0.16 | 17.26 ± 0.13 |
| $R_e$     | 2.70 ± 0.43 | 3.38 ± 0.28 | 2.52 ± 0.40 | 2.59 ± 0.26 | 5.59 ± 0.47 | 2.90 ± 0.24 | 2.35 ± 0.24 | 13.75 ± 1.15 | 3.98 ± 0.40 |
| $n$       | 1.63 ± 0.18 | 2.36 ± 0.18 | 1.62 ± 0.18 | 0.69 ± 0.05 | 1.66 ± 0.13 | 1.84 ± 0.14 | 1.61 ± 0.20 | 1.29 ± 0.10 | 2.62 ± 0.19 |
| $b/a$     | 0.89 ± 0.04 | 0.84 ± 0.03 | 0.87 ± 0.04 | 0.96 ± 0.02 | 0.95 ± 0.03 | 0.72 ± 0.03 | 0.98 ± 0.04 | 0.70 ± 0.03 | 0.78 ± 0.02 |
| PA        | 41.14 ± 5.05 | 165.64 ± 2.97 | 125.17 ± 5.05 | 0.00 ± 3.31 | 97.41 ± 2.97 | 97.76 ± 2.97 | 6.46 ± 2.20 | 156.86 ± 2.97 | 16.72 ± 3.31 |
| $B/T$     | 0.164 ± 0.020 | 0.085 ± 0.129 | 0.237 ± 0.123 | 0.267 ± 0.135 | 0.197 ± 0.226 |

**Table A5. ALL PARAMETERS FOR THE DOUBLE-BARRED FITS IN i'-BAND. First set of 9/17 galaxies.**
Table A6. ALL PARAMETERS FOR THE SINGLE-BARRED FITS IN $r^\prime$-BAND. First set of 9/17 galaxies.

| NGC 3945 | NGC 4314 | NGC 4340 | NGC 4503 | NGC 4725 | NGC 5850 | NGC 7280 | NGC 7716 |
|----------|----------|----------|----------|----------|----------|----------|----------|
| $\mu_0$  | 19.23 ± 0.16 | 19.23 ± 0.13 | 18.90 ± 0.16 | 18.62 ± 0.16 | 18.25 ± 0.16 | 19.77 ± 0.16 | 18.69 ± 0.26 | 17.54 ± 0.15 |
| $R_0$    | 10.06 ± 0.84 | 11.25 ± 1.13 | 6.45 ± 0.51 | 5.39 ± 0.45 | 7.22 ± 0.61 | 8.79 ± 0.74 | 4.04 ± 0.65 | 1.97 ± 0.20 |
| $n$      | 3.30 ± 0.26 | 1.00 ± 0.07 | 2.34 ± 0.17 | 2.70 ± 0.21 | 2.27 ± 0.18 | 2.46 ± 0.19 | 3.07 ± 0.31 | 2.19 ± 0.27 |
| $b/a$    | 0.87 ± 0.03 | 0.61 ± 0.02 | 0.76 ± 0.03 | 0.76 ± 0.03 | 0.84 ± 0.03 | 0.97 ± 0.03 | 0.66 ± 0.04 | 0.57 ± 0.04 |
| PA       | 147.98 ± 2.97 | 132.45 ± 3.31 | 98.75 ± 2.97 | 19.61 ± 2.97 | 28.52 ± 2.97 | 160.29 ± 2.97 | 68.06 ± 5.05 | 56.16 ± 5.20 |
| B/T      | 0.329        | 0.210       | 0.241       | 0.207       | 0.104       | 0.195       | 0.269       | 0.154       |

Table A7. ALL PARAMETERS FOR THE SINGLE-BARRED FITS IN $r^\prime$-BAND. First set of 9/17 galaxies.

| NGC 3945 | NGC 4314 | NGC 4340 | NGC 4503 | NGC 4725 | NGC 5850 | NGC 7280 | NGC 7716 |
|----------|----------|----------|----------|----------|----------|----------|----------|
| $\mu_0$  | 21.12 ± 0.07 | 21.05 ± 0.07 | 20.33 ± 0.07 | 19.62 ± 0.07 | 19.59 ± 0.07 | 21.40 ± 0.07 | 20.19 ± 0.07 | 19.73 ± 0.07 |
| $h_{\text{Bar}}$ | 57.57 ± 0.95 | 63.47 ± 1.53 | 33.24 ± 0.58 | 26.62 ± 0.44 | 85.69 ± 1.42 | 66.96 ± 1.11 | 27.34 ± 1.27 | 15.95 ± 0.61 |
| $h_{\text{Disc}}$ | 0.68 ± 0.01 | 0.92 ± 0.01 | 0.69 ± 0.01 | 0.46 ± 0.01 | 0.47 ± 0.01 | 0.75 ± 0.01 | 0.66 ± 0.01 | 0.83 ± 0.01 |
| n        | 3          | 2          | 4          | 2          | 2          | 2          | 2          | 2          |
| $\beta$  | 147.98 ± 2.97 | 132.45 ± 3.31 | 98.75 ± 2.97 | 19.61 ± 2.97 | 28.52 ± 2.97 | 160.29 ± 2.97 | 68.06 ± 5.05 | 56.16 ± 5.20 |
| $\Psi_{\text{bar}}$ | 0.103 | 0.138 | 0.097 | 0.149 | 0.195 | 0.159 | 0.205 | 0.233 |
| $\Psi_{\text{Disc}}$ | 0.329 | 0.210 | 0.241 | 0.207 | 0.104 | 0.195 | 0.269 | 0.154 |
Table A8. ALL PARAMETERS FOR THE SINGLE-BARRED FITS IN r'-BAND. Last set of 8/17 galaxies

|      | NGC 3945  | NGC 4314  | NGC 4540  | NGC 4561  | NGC 4725  | NGC 5350  | NGC 7280  | NGC 7716  |
|------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Bulge |           |           |           |           |           |           |           |           |
| $\mu_e$ | 18.88 ± 0.09 | 19.72 ± 0.07 | 18.81 ± 0.09 | 18.64 ± 0.09 | 18.12 ± 0.09 | 19.61 ± 0.09 | 18.64 ± 0.10 | 18.07 ± 0.17 |
| $R_e$  | 6.33 ± 0.33  | 10.95 ± 0.48 | 4.97 ± 0.26  | 4.64 ± 0.24  | 5.43 ± 0.28  | 6.82 ± 0.35  | 3.19 ± 0.21  | 2.45 ± 0.31  |
| $n$   | 1.28 ± 0.06  | 2.97 ± 0.07  | 2.04 ± 0.09  | 2.23 ± 0.10  | 1.54 ± 0.07  | 1.99 ± 0.09  | 3.49 ± 0.19  | 1.52 ± 0.11  |
| $b/a$ | 0.69 ± 0.02  | 0.81 ± 0.01  | 0.86 ± 0.02  | 0.79 ± 0.02  | 0.92 ± 0.02  | 0.89 ± 0.02  | 0.67 ± 0.02  | 0.67 ± 0.03  |
| PA    | 156.67 ± 3.05 | 134.37 ± 1.87 | 91.60 ± 3.05 | 12.92 ± 3.05 | 5.76 ± 3.05 | 41.03 ± 3.05 | 72.10 ± 3.06 | 55.53 ± 2.80 |
| B/T   | 0.317       | 0.261       | 0.259       | 0.219       | 0.088       | 0.179       | 0.288       | 0.201       |
| Disc  |           |           |           |           |           |           |           |           |
| $\mu_o$ | 21.12 ± 0.05 | 21.49 ± 0.05 | 20.68 ± 0.05 | 19.43 ± 0.05 | 20.05 ± 0.05 | 21.58 ± 0.05 | 20.59 ± 0.05 | 20.13 ± 0.05 |
| $h_{inner}$ | 44.60 ± 0.66 | 62.51 ± 1.27 | 33.76 ± 0.50 | 26.20 ± 0.39 | 85.96 ± 1.25 | 59.17 ± 0.87 | 26.95 ± 0.94 | 16.44 ± 0.64 |
| $b/a$ | 0.72 ± 0.01  | 0.92 ± 0.01  | 0.66 ± 0.01  | 0.46 ± 0.01  | 0.50 ± 0.03  | 0.74 ± 0.01  | 0.65 ± 0.01  | 0.82 ± 0.02  |
| PA    | 156.77 ± 1.98 | 137.39 ± 1.31 | 92.96 ± 1.98 | 9.99 ± 1.98 | 43.07 ± 1.98 | 136.37 ± 1.98 | 73.83 ± 3.60 | 41.36 ± 3.92 |
| $h_{outer}$ | N/A       | 25.72 ± 1.29 | N/A       | N/A       | N/A       | N/A       | N/A       | N/A       |
| $R_{break}$ | N/A    | 105.84 ± 3.03 | N/A    | N/A    | N/A    | N/A    | N/A    | N/A    |
| D/T   | 0.468       | 0.480       | 0.628       | 0.722       | 0.864       | 0.796       | 0.639       | 0.732       |
| Outer bar |         |           |           |           |           |           |           |           |
| $\mu_o$ | 20.25 ± 0.16 | 20.25 ± 0.24 | 21.75 ± 0.16 | 20.51 ± 0.16 | 20.11 ± 0.16 | 21.41 ± 0.16 | 20.49 ± 0.31 | 21.43 ± 0.25 |
| $a$   | 57.65 ± 0.65 | 123.78 ± 1.40 | 63.33 ± 0.73 | 33.19 ± 0.38 | 56.17 ± 0.64 | 123.71 ± 1.42 | 20.90 ± 0.42 | 32.07 ± 0.79 |
| $n$   | 2           | 3           | 1           | 2           | 3           | 4           | 2           | 2           |
| $c$   | 0.63 ± 0.03  | 0.24 ± 0.02  | 0.45 ± 0.03  | 0.45 ± 0.03  | 0.60 ± 0.03  | 0.26 ± 0.03  | 0.48 ± 0.03  | 0.46 ± 0.06  |
| PA    | 73.25 ± 1.89 | 146.78 ± 1.64 | 31.14 ± 1.89 | 177.80 ± 1.89 | 36.86 ± 1.89 | 116.54 ± 1.89 | 55.13 ± 3.02 | 16.95 ± 2.00 |
| Bar/T | 0.215       | 0.258       | 0.112       | 0.060       | 0.048       | 0.114       | 0.073       | 0.067       |