ON THE EFFICIENCY OF INTERNAL SHOCKS IN GAMMA-RAY BURSTS

Andrei M. Beloborodov

Stockholm Observatory, SE-133 36, Saltsjöbaden, Sweden; andrei@astro.su.se

Received 2000 April 26; accepted 2000 June 28; published 2000 August 4

ABSTRACT

The fraction of a fireball kinetic energy that is radiated by internal shocks is sensitive to the amplitude of initial fluctuations in the fireball. We give a simple analytical description for the dissipation of modest-amplitude fluctuations and confirm it with direct numerical simulations. At high amplitudes, the dissipation occurs in a nonlinear regime with efficiency approaching 100%. Most of the fireball energy can then be radiated away by the prompt gamma-ray burst, and only a fraction remains for the afterglow.

Subject heading: gamma rays: bursts

1. INTRODUCTION

Cosmological gamma-ray bursts (GRBs) are huge explosions with energy $\sim 10^{53}$ ergs, which may be triggered, e.g., by the coalescence of two neutron stars or the collapse of a massive star core. The fireball that is created expands relativistically, with a Lorentz factor $\Gamma \sim 10^3$. In a likely scenario, the observed gamma rays are produced at times $t \sim 10^{-2} - 10^3$ s after the explosion, when the relativistic outflow gets optically thin and before it is decelerated by the surrounding medium (see, e.g., Piran 1999 for a review). The outflow has a fluctuating velocity profile, and the gamma rays are generated by internal shocks that develop when faster shells try to overtake slower ones (Rees & Meszaros 1994).

The internal dissipation in relativistic outflows was discussed and simulated numerically in a number of works (e.g., Kobayashi, Sari, & Piran 1997, hereafter KPS; Daigne & Mochkovitch 1998; Lazzati, Ghisellini, & Celotti 1999; Panaitescu, Spada, & Meszaros 1999; Panaitescu & Meszaros 2000; Kumar 1999). It was found that only a small fraction, $\epsilon \ll 0.1$, of the total kinetic energy can be dissipated (e.g., Panaitescu et al. 1999; Kumar 1999; but see also KPS where the maximum $\epsilon \sim 0.4$). The efficiency is further reduced if not all the dissipated energy is radiated. Low $\epsilon$ implies a problem for the model: the prompt GRB should then be dominated by the early afterglow associated with an external shock. Observations show the opposite. Besides, the low efficiency requires the huge total energy of the explosion.

In this Letter, we again address the issue of internal shock efficiency. First, we note two assumptions often adopted in previous works: (1) A uniform probability distribution was assumed for the outflow Lorentz factor, which implies that the rms of the fluctuations is less than $1/\sqrt{3}$ (see eq. [11]). In general, the rms can be much higher. (2) Only a fraction of the dissipated energy was assumed to be radiated because Coulomb $e^+e^-$ collisions could not transfer energy from heated protons to radiating electrons. In fact, the radiative capability of the shocked plasma depends on poorly understood collective processes. A high radiative capability is favored by the observed high luminosities of prompt GRBs.

In § 2, we develop an analytical model for low-rms fluctuations that illuminates previous numerical results. The dissipation proceeds as a self-similar hierarchical coalescence of shells, and the resulting dissipation efficiency $\epsilon \propto A^2$, where $A$ is the amplitude of initial fluctuations. In § 3, we extend the analysis to large amplitudes and find that $\epsilon$ approaches 100%.

2. LOW-AMPLITUDE FLUCTUATIONS

The size of the central engine, $r_0 \sim 10^{17} - 10^{18}$ cm, corresponds to a timescale $t_0 = r_0/c \sim 1$ ms. The outflow generated by the engine can have highly correlated parameters on this timescale, so that portions $\Delta t \leq t_0$ can be considered as individual shells in the continuous outflow. The outflow is then discretized as a sequence of such shells, $i = 1, \ldots, N$, with mass $m_i$ and energy input $e_i$. One after another, the shells are accelerated to Lorentz factors $\Gamma_i = e_i/m_i$ ($e_i$ gets converted into bulk kinetic energy) on a timescale $\Gamma_{i-1} t_0 \ll 1$ s (see Piran 1999). The subsequent evolution of the outflow proceeds in the coasting regime until the shells begin to collide.

Suppose $\Gamma_0$ is white noise with an average value $\Gamma_\alpha$, and an initial rms $\Gamma_{\text{rms}} \ll \Gamma_\alpha$. The masses of the shells can be taken to be equal, $m_i = m_\alpha$, or fluctuating—the results will be the same. The initial scale of the fluctuations is $\lambda_\alpha \approx r_\alpha$. The collision process starts at

$$t_0 \sim \frac{\Gamma_\alpha}{\Gamma_{\text{rms}}} \frac{\lambda_\alpha}{c}$$

After time $t \sim t_0$, the first generation of shell coalescence has been done (the typical mass of a shell increases by a factor of 2), then the second generation occurs, and so on. In the process of hierarchical coalescence, the scale of the fluctuations increases, $\lambda = \lambda(t)$, and so does the average mass of an individual shell, $m(t)/m_\alpha = m(t)/m_\alpha \equiv K(t)$.

The fluctuations look especially simple when viewed from the frame moving with Lorentz factor $\Gamma_\alpha$. We will denote quantities measured in this frame by symbols with a tilde. The fluctuation velocity is $\tilde{v}/c = (\Gamma_\alpha^2 - \Gamma_{\text{rms}}^2)/(\Gamma_\alpha^2 + \Gamma_{\text{rms}}^2)$. Given $\Gamma_{\text{rms}} < \Gamma_\alpha$, we have $\tilde{v}/c \approx (\Gamma_\alpha - \Gamma_{\text{rms}})/\Gamma_\alpha$ with the average $\bar{v}_{\text{rms}} = 0$ and the rms,

$$\frac{\bar{v}_{\text{rms}}}{c} \approx \frac{\Gamma_{\text{rms}}}{\Gamma_\alpha}.$$

The outflow motion is thus decomposed into two parts: the relativistic motion of the center of momentum (CM) with a Lorentz factor $\Gamma_{\text{CM}} \approx \Gamma_\alpha$ and superposed nonrelativistic fluctuations.

The hierarchical coalescence of shells can be expected to
occur in a self-similar regime, so that the collisional timescale for one generation is about the time passed since the explosion,

\[ \frac{\bar{\lambda}}{\bar{t}_{\text{rms}}} \approx \tilde{t} = \frac{\Gamma}{\Gamma_{\text{CM}}}. \]  

Here \( \bar{\lambda} = \Gamma_{\text{CM}} \lambda \) is the scale of the fluctuations in the CM frame. After the coalescence of \( K \) shells with initial momentum \( \tilde{p}_0 \sim m \bar{v}_{\text{rms}} \), the new big shell has a momentum \( \tilde{p} \sim \sqrt{K} \tilde{p}_0 \) (this is the random walk formula: \( K \) momenta \( \sim p_0 \) are summed with random signs). We thus have

\[ m \bar{v}_{\text{rms}} \sim \sqrt{\bar{K} m \bar{v}_{\text{rms}} \tilde{p}_0} = \sqrt{\frac{m}{m_0}} m_0 \bar{v}_{\text{rms}} \tilde{p}_0. \]

Combining equations (2), (3), and (4), we get the self-similar solution describing the hierarchical coalescence at \( t > t_0 \),

\[ \frac{m}{m_0} = \frac{\lambda}{\lambda_0} = \left( \frac{t}{t_0} \right)^{2/3}, \quad \frac{\Gamma_{\text{rms}}}{\Gamma_{\text{rms0}}} = \left( \frac{t}{t_0} \right)^{-1/3}. \]

In Figure 1, we illustrate the solution in equation (5) by direct numerical simulations. Note that the collisions establish a Gaussian distribution of \( \tilde{v} \) with a temperature \( T = \tilde{m} \bar{v}_{\text{rms}} \). From equation (5), it follows that \( T = \text{const.} \) The dissipation can thus be described as the isothermal sticking of shells.

The free energy of the outflow is given by

\[ U_{\text{free}} = \Gamma_{\text{CM}} M \frac{T_{\text{rms}}^2}{2} - \frac{M c^2}{2} \Gamma_{\text{rms}}^2, \]

where \( M \) is the total mass of the outflow. This energy is available for dissipation. From equation (5), one gets

\[ U_{\text{free}} = U_{\text{free}}^0 \left( \frac{t}{t_0} \right)^{-2/3}. \]

Most of the free energy is dissipated at \( t \sim t_0 \). It should be compared with the time at which the outflow becomes optically thin to Thomson scattering,

\[ t \approx \frac{M_{\text{out}}}{8 \pi m_p c^2 \Gamma_{\text{rms}}^2} = \frac{L_{\text{out}}}{8 \pi m_p c^3 \Gamma_{\text{rms}}^3} \approx 2 \times 10^3 L_{52} \Gamma_{-3}^{-3} \text{ s}, \]

where \( M \) is the mass outflow rate and \( L = M c^3 \Gamma_{\text{rms}} \) is the kinetic luminosity (assuming a spherical outflow).

If \( \Gamma_{\text{CM}} < \Gamma \approx 180 \Gamma_{\text{rms0}}/\Gamma_{\text{CM}} \Gamma_{\text{rms0}}^{10/3} \lambda_0 (3 \times 10^3)^{-1/3} \), dissipation starts at \( t_0 < t_c \). The free energy remaining until \( t_c \) is

\[ U_{\text{free}} = U_{\text{free}}^0 (t_0/t_c)^{-2/3}, \]

and most of the energy is dissipated at the optically thick stage. The produced radiation is then trapped in the outflow whose volume increases as \( t^2 \). As a result of adiabatic cooling, the volume-integrated radiation energy decreases as \( t^{-2/3} \). The energy conservation law implies that the adiabatic cooling is accompanied by the regular radial acceleration of the outflow. The bulk of the dissipated energy is thus spent to accelerate the outflow CM, and it is lost as free energy. A fraction \( (t/t_c)^{-2/3} \) of the trapped radiation survives until \( t_c \), and contributes to the observed luminosity,

\[ L_\ast \approx \int_0^{t_c} \left( -\frac{dU_{\text{free}}}{dt} \right) \left( \frac{t}{t_c} \right)^{-2/3} dt + U_{\text{free}}^0. \]

The efficiency (the ratio of \( L_\ast \) to the explosion energy) is

\[ \epsilon \approx \frac{L_\ast}{\Gamma_{\text{CM}} M c^3} \approx \frac{A^2}{2} \left( \frac{t_c}{t_f} \right)^{-2/3} \left( \frac{2}{3} \ln \frac{t_c}{t_f} + 1 \right). \]

Here \( A = \Gamma_{\text{rms0}}/\Gamma_{\text{CM}} \) is the initial fluctuation amplitude.

In the case of \( \Gamma_{\text{CM}} > \Gamma_\ast \), we have \( t_f > t_c \), and all the radiated free energy will escape. Then

\[ \epsilon \approx \frac{U_{\text{free}}^0}{\Gamma_{\text{CM}} M c^3} = \frac{A^2}{2}. \]

If the initial Lorentz factor takes random values between \( \Gamma_{\text{min}} \) and \( \Gamma_{\text{max}} \) (as assumed in, e.g., KPS and Kumar 1999), then

\[ A^2 = \frac{1}{3} - \frac{4 \psi}{3(1 + \psi)} \equiv \frac{\Gamma_{\text{min}}}{\Gamma_{\text{max}}}. \]

Since \( A^2 < \frac{1}{3} \) in equation (11), equation (10) gives \( \epsilon < \frac{1}{3} \). Note that equations (9) and (10) assume that the dissipated energy is converted into radiation on a timescale shorter than the expansion time. Alternatively, a fraction \( \chi \) of this energy may be stored as heat and subject to adiabatic cooling. This fraction is spent to accelerate the CM, and \( \epsilon \) is then reduced by a factor of \( 1 - \chi \). For example, Kumar (1999) assumed that \( \chi = \frac{2}{3} \), which led to \( \epsilon \leq 5\% \).

Numerical simulations of KPS (see their Table 1) are in
It easy to show that at high amplitudes, \( \eta \gg \Gamma_{\text{CM}} \); i.e., the chaotic (free) component of the kinetic energy is much larger than the regular component. If the dissipation occurs in the optically thin regime and if the emitted radiation is approximately isotropic in the CM frame, then the momentum conservation law implies that \( \Gamma_{\text{CM}} \approx \text{const} \). When the dissipation is done, the final kinetic energy is about \( (\Gamma_{\text{CM}} - 1)Mc^2 \ll \eta Mc^2 \); i.e., almost all the energy of the outflow has been radiated to infinity.

In fact, in the high-amplitude case, the fluctuations start to dissipate very early, much before the time of transparency \( \tau_c \). The optically thick dissipation accelerates the CM, as discussed in § 2: the free energy gets transformed into the bulk motion of the outflow as a whole. The process is nonlinear: the CM is substantially accelerated, in contrast to the low-amplitude case. It tends to reduce the initially big difference between \( \Gamma_{\text{CM}} \) and \( \eta \) by increasing \( \Gamma_{\text{CM}} \) while \( \eta \) stays at the initial value \( \eta_0 \).

If, at the time of transparency, the ratio \( \eta/\Gamma_{\text{CM}} \) is still high, then a high efficiency can be expected: the dissipation converts the difference between \( \eta Mc^2 \) and \( (\Gamma_{\text{CM}} - 1)Mc^2 \) into the observed radiation. Note also that the radiation produced during the optically thick stage is not completely lost. Even being cooled adiabatically, it contributes substantially to the outgoing luminosity.

We now illustrate with numerical simulations. We generate a sequence of \( 3 \times 10^4 \) thin shells with an initial separation of 1 ms. Their Lorentz factors fluctuate according to equation (12). The shells have equal mass \( m_0 \). Note that the results depend on the initial mass distribution (in contrast to the linear case), and \( m_i \approx m_0 \) is taken as a simple example. The duration of the central engine activity is 3 s. The results do not change if one assumes longer activity: each 1 s portion of the outflow is causally disconnected from the other portions during the main emission time, \( t \ll 10^3 \) s.

The moment of transparency \( \tau_c \) is roughly estimated by equation (8) with \( \Gamma \sim \Gamma_{\text{CM}} \). The transition from the optically thick to optically thin regime is treated in the simplest way: (1) If two shells merge at \( t < \tau_c \), we assume that no radiation is emitted. Instead, the radiation is trapped and contributes to the kinetic energy of the new big shell. In other words, shell coalescence at \( t < \tau_c \) proceeds with conservation of energy, \( \eta = \text{const} \). A fraction \( (\tau_c/\tau)^{-2/3} \) of the radiation trapped at moment \( \tau \) survives until \( \tau_c \) and contributes to the luminosity \( L_c \).

(2) At \( t > \tau_c \), only specific momentum is conserved in the coalescence events. The energy released in the inelastic collision is radiated away isotropically in the rest frame of the newly formed shell.

We consider two illustrative models. In model 1, \( \tau_c = 200 \) s, and \( \Gamma_0 \) in equation (12) is adjusted in such a way that \( \Gamma_{\text{CM}} \approx 100 \) after the dissipation is finished. In model 2, \( \tau_c = 7 \) s, and \( \Gamma_0 \) is adjusted to get the final \( \Gamma_{\text{CM}} \approx 300 \). In both models, the isotropic kinetic luminosity \( L \sim 10^{50} \) ergs s\(^{-1}\).

The outflow forms by \( t = 3 \) s, and thereafter it has a well-defined CM with a velocity \( \beta_{\text{CM}} = \gamma e/E \), where \( \gamma = (1 + 1/2M^2) \) is the total energy of the outflow. (Note that \( \gamma_0 \) depends on the specific statistical realization \( \Gamma \) and that it varies around the average expected value given by eq. [14].) In Figure 2, we show examples of the nonlinear evolution of \( \eta \) and \( \Gamma_{\text{CM}} \) in models 1 and 2. At high \( \Gamma \), the radiation is mostly produced by shells with \( \Gamma > \Gamma_{\text{CM}} \), resulting in the CM deceleration at \( t > \tau_c \).

Then we compute a sequence of models and find the efficiency, \( \epsilon \equiv L/\eta Mc^2 \), as a function of \( A \) (Fig. 3). At \( A < \), we are in the linear regime of § 2. Shells begin to collide at \( \tau_c \approx A^{-1} \Gamma_{\text{CM}} \lambda_0/c \) and then evolve according to equation (5). At
Fig. 3.—Efficiency \( \epsilon \) as a function of the fluctuation amplitude \( A \). The open circles show the results in model 1 (final \( \Gamma_{\text{CM}} \approx 100 \)), and the filled circles show the results in model 2 (final \( \Gamma_{\text{CM}} \approx 300 \)). The error bars show the standard deviation (where it is larger than the symbol size). At \( A < 1 \), model 2 follows the relation \( \epsilon \approx A^{7/2} \) (solid line) expected in the optically thin linear regime (see eq. [10]). The efficiency of model 1 is reduced as a result of adiabatic losses during the optically thick stage.

A > 1, the efficiency increases up to \( \sim 83\% \) in model 1 and 96\% in model 2. The global parameters of the outflow then display substantial variations depending on the particular realization of the initial \( \Gamma_{\text{CM}} \). Figure 3 shows \( \epsilon \) averaged for many realizations and its standard deviation.

Figure 4 shows examples of the generated light curves. Each collision event produces a pulse of a standard shape corresponding to a thin instantaneously radiating shell. The observed pulse has a width \( \Delta t_{\text{obs}} \sim t / \Gamma^2 \), where \( t \) is the time at which the radiation leaves the shell. Although the initial fluctuations are Gaussian, the nonlinear dissipation generates a highly correlated intermittent signal, reminiscent of observed GRBs.

In the nonlinear regime, the motion of shells in the CM frame is relativistic, and an additional emission mechanism appears: inverse Compton (IC) scattering on the bulk motions (see also Lazzati et al. 1999 and Gruzinov & Mészáros 2000). When the outflow gets optically thin, the radiation can propagate and promote momentum exchange between the shells without direct collisions. This effect should be accounted for in the future. We expect that it will not crucially change the efficiency because (1) the free energy is radiated anyway and (2) photon exchange does not crucially enhance the dissipation rate because direct collisions also occur with relativistic velocities in the CM frame. IC scattering by bulk motions can explain the high-energy tails sometimes observed in GRB spectra: photons emitted by internal shocks are boosted in energy by a factor of \( \Gamma_{\text{CM}}^2 / \Gamma_{\text{CM}}^2 \), resulting in hard gamma-ray emission.

The study of fluctuating fireballs in this Letter was limited to the case of white fluctuations. Fluctuations with an arbitrary spectrum are discussed in A. M. Beloborodov (2000, in preparation). Also, the effects of pair production will be addressed in a future paper.

I thank A. F. Illarionov, J. Poutanen, R. Svensson, and the anonymous referee for comments. This work was supported by the Swedish Natural Science Research Council and RFBR grant 00-02-16135.

REFERENCES

Daigne, F., & Mochkovitch, R. 1998, MNRAS, 296, 275
Gruzinov, A. V., & Mészáros, P. 2000, ApJ, in press
Kobayashi, S., Piran, T., & Sari, R. 1997, ApJ, 490, 92 (KPS)
Kumar, P. 1999, ApJ, 523, L113
Lazzati, D., Ghisellini, G., & Celotti, A. 1999, MNRAS, 309, L13
Panaitescu, A., Spada, M., & Mészáros, P. 1999, ApJ, 522, L105
Piran, T. 1999, Phys. Rep., 314, 575
Rees, M. J., & Mészáros, P. 1994, ApJ, 430, L93
Spada, M., Panaitescu, A., & Mészáros, P. 2000, ApJ, 537, 824
Panaitescu, A., Spada, M., & Mészáros, P. 1999, ApJ, 522, L105
Piran, T. 1999, Phys. Rep., 314, 575
Rees, M. J., & Mészáros, P. 1994, ApJ, 430, L93
Spada, M., Panaitescu, A., & Mészáros, P. 2000, ApJ, 537, 824