$D^* \to D\pi$ and $D^* \to D\gamma$ decays:

Axial coupling and Magnetic moment of $D^*$ meson

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Abstract

The axial coupling and the magnetic moment of $D^*$-meson or, more specifically, the couplings $g_{D^*D\pi}$ and $g_{D^*D\gamma}$, encode the non-perturbative QCD effects describing the decays $D^* \to D\pi$ and $D^* \to D\gamma$. We compute these quantities by means of lattice QCD with $N_f = 2$ dynamical quarks, by employing the Wilson ("clover") action. On our finer lattice ($a \approx 0.065$ fm) we obtain: $g_{D^*D\pi^+} = 20 \pm 2$, and $g_{D^*D\rho/\gamma} = 2.0 \pm 0.6 \text{ GeV}^{-1}$. This is the first determination of $g_{D^*D\rho/\gamma}$ on the lattice. We also provide a short phenomenological discussion and the comparison of our result with experiment and with the results quoted in the literature.

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1 Introduction

Measuring the width of the nearest resonances in the charmed meson spectrum is a challenging experimental task. The methods developed by the CLEO-collaboration allowed them to make the first measurement of the charged vector meson width, $\Gamma(D^{*+}) = 96 \pm 22 \text{ keV}$ \cite{1}. Unfortunately no similar attempt has been made in the experiments at $B$-factories or at CLEO-c. The experimental value for $\Gamma(D^{*+})$ turned out to be very interesting because it provided us with the quantity allowing to check on various theoretical tools that are being used to compute the phenomenologically interesting hadronic matrix elements such as those needed for the decay constants, form factors, bag parameters and so on. The coupling $g_c$, that can be extracted from $\Gamma(D^{*+})$, as discussed in later sections of this paper, appeared to be much larger than predicted by many quark models and by all the techniques of QCD sum rules (see ref. \cite{2} for a discussion). The CLEO result was however consistent with the old fashioned Adler-Weisberger sum rule combined with phenomenological observations made with baryons \cite{2}.

The first lattice calculation of this coupling in the charmed sector was made in ref. \cite{3}, where it was possible to extract this coupling ($g_c$) in the soft pion limit without running into notorious difficulties of dealing with the final state interactions in non-leptonic decays on the euclidean lattice. In this paper we provide an update to that result. We use the same action and the same methodology as in ref. \cite{3} but the major qualitative difference with respect to ref. \cite{3} is that here we get rid of the quenched approximation. The gauge field configurations used in this work contain the fluctuations of the $N_f = 2$ mass-degenerate dynamical light quarks. Besides we also implement several technical improvements to make the extraction of the form factors from the correlation functions computed on the lattice cleaner (double ratios of correlation functions, twisted boundary conditions). On the basis of our results we conclude that the lattice evaluation of $D^{*} \rightarrow D\pi$ decay is consistent with experiment, and points towards the upper end of the measured $\Gamma(D^{*+})$ \cite{1}.

Beside that quantity, in this paper we report on the first lattice determination of the soft photon coupling to the lowest $D$-meson states, $g_{D^{*}D\gamma}$. In particular we obtain $\Gamma(D^{*0} \rightarrow D^{0\gamma}) = 25 \pm 13 \text{ keV}$, in good agreement with experiment, although our error bars are still too large to make any stronger statement. The two quantities computed here are obviously phenomenologically interesting since the experimental information about these decays already exists. Therefore they can be used to check on various quark model calculations and to perhaps refine the QCD sum rule analyses. Moreover the quantities computed here are particularly interesting to the lattice practitioners. $g_c$-coupling is a parameter which appears in the expressions obtained in heavy meson chiral perturbation theory (HMChPT) that are being used to guide the chiral extrapolations of the phenomenologically interesting quantities computed on the lattice, such as the $D$-meson decay constant and its semileptonic decay form factors. Instead, $g_{D^{*}D\gamma}$ is important in controlling the impact of the structure dependent term in the radiative leptonic $D$-meson decays in the region of phase space in which the photon is soft. Finally, the radiative decay $D^{*} \rightarrow D\gamma$ can be used as a benchmark calculation to compare the lattice approaches among themselves and with experiment. The obvious advantage is that in this case one goes beyond the hadronic spectrum and computes the hadronic matrix element but there
is no need for a CKM parameter to make the precision test of lattice QCD vs. experiment.

The present paper is organized as follows: we first define the hadronic couplings which are the subject of this paper, and relate them to the form factors that are to be computed non-perturbatively; in Sec. 3 we explain the strategy to compute the relevant hadronic matrix elements on the lattice, the strategy which we then implement in Sec. 4 where we also present the results directly accessible from our lattices; in Sec. 5 we provide a phenomenological discussion and compare our results with experiment and with various model calculations; we finally conclude in Sec. 6.

2 Definitions

In this section we define the couplings $g_{D^*D\pi}$ and $g_{D^*D\gamma}$, and relate them to the matrix elements that can be computed on the lattice.

2.1 $g_{D^*D\pi}$

Generically, the coupling of the pseudoscalar ($P$) and vector ($V$) mesons to a soft pion is defined as

$$\langle P(k)\pi(q)|V(p,\lambda)\rangle = (e_\lambda \cdot q) g_{VP\pi},$$

where $q = p - k$ is the pion momentum and $\lambda$ labels the polarization state of the vector meson. Physically this matrix element describes the amplitude of the soft pion emission process, $V \to P\pi$, such as $K^* \to K\pi$ and $D^* \to D\pi$, or the kinematically forbidden, but nonetheless very interesting processes, $B^* \to B\pi$ and $\omega \to \rho\pi$. In this paper we will focus onto $g_{D^*D\pi}$, and follow a rather standard procedure, first proposed in ref. [4] and then implemented in refs. [3, 4, 5]. To describe it in just a few lines we first define the matrix element of the axial current

$$\langle D(k)|A^\mu|D^*(p,\lambda)\rangle = 2m_V A_0(q^2) \frac{e_\lambda \cdot q}{q^2} q^\mu + (m_D + m_{D^*}) A_1(q^2) \left( \frac{e_\mu}{q^2} - \frac{e_\lambda \cdot q}{q^2} q^\mu \right)$$

$$+ A_2(q^2) \frac{e_\lambda \cdot q}{m_D + m_{D^*}} \left( p^\mu + k^\mu - \frac{m_D^2 + m_{D^*}^2}{q^2} q^\mu \right),$$

conveniently parameterized in terms of three form factors, $A_{0,1,2}(q^2)$. We restrain our discussion to the charged pion case, so that no anomalous term appears in the axial current. At $q^2$ close to $m_\pi^2$ we can use the reduction formula and write

$$\frac{f_\pi m_\pi^2}{m_\pi^2 - q^2} \langle D(k)\pi(q)|D^*(p,\lambda)\rangle = \langle D(k)|\partial_\mu A^\mu|D^*(p,\lambda)\rangle,$$

which at $q^2 = 0$ leads to

$$g_{D^*D\pi} = \frac{2m_{D^*}}{f_\pi} A_0(0).$$

(4)
This formula is not useful for lattice QCD because the form factor $A_0(q^2)$ is dominated by the pion ($J^P = 0^-$-state in the $t$-channel), so that both the $q^2$ dependence of $A_0(q^2)$ and the mass dependence of $A_0(0)$ are extremely steep. However, from the fact that no massless state can couple to the axial current, we can benefit from the condition that

$$2m_D A_0(0) - (m_D^* + m_D) A_1(0) - (m_D^* - m_D) A_2(0) = 0,$$

and eventually arrive at

$$g_{d^*Dπ} = \frac{m_D^* + m_D}{f_π} A_1(0) \left[ 1 + \frac{m_D^* - m_D}{m_D^* + m_D} A_2(0) \right].$$

In other words the problem of computing the coupling $g_{d^*Dπ}$ is reduced to the problem of computing the form factor $A_1(0)$ and the ratio $A_2(0)/A_1(0)$. We stress again that in the above definitions the soft pion is charged $g_{d^*Dπ} \equiv g_{d^*Dπ^+}$. The coupling to the neutral pion is related to the one we compute here via isospin, i.e., $g_{d^*Dπ^0}^2 = 2g_{d^*Dπ^+}^2$.

### 2.2 $g_{D^*Dγ}$

The coupling of the pseudoscalar ($P$) and vector ($V$) mesons to a photon is defined as

$$\langle \gamma(q, \eta_\lambda') P(k) | V(p, \epsilon_\lambda) \rangle = e \epsilon_{\mu \alpha \beta} \eta_\lambda' \epsilon_\lambda \epsilon_\nu k^\alpha k^\beta g_{VP\gamma},$$

which we will accede by computing the matrix element of the electromagnetic current, namely

$$\langle P(k) | J_{\mu}^{em} | V(p, \epsilon_\lambda) \rangle = e \epsilon_{\mu \alpha \beta} \epsilon_\lambda \epsilon_\nu p^\alpha k^\beta \frac{2 V(q^2)}{m_V + m_P},$$

where $J_{\mu}^{em} = Q_Q Q_{\gamma} Q_{\mu} Q + Q_{\bar{q}} q_{\gamma} q$, and the matrix element is expressed in a rather standard way, i.e. in terms of the form factor $V(q^2)$. In the above expressions $e = \sqrt{4\pi \alpha_{em}}$, and $Q_{Q, \bar{q}}$ is the charge of the heavy quark and the light antiquark (or vice versa). It is very important to keep track of the relative sign difference between the two terms in $J_{\mu}^{em}$. In the case of $D$-mesons we have

$$g_{D^{+*}D^{+\gamma}} = \frac{2}{m_D + m_{D^*}} F_d(0), \quad F_d(0) = \frac{2}{3} V^{qq}(0) \left[ -\frac{1}{2} + \frac{V^{cc}(0)}{V^{qq}(0)} \right],$$

$$g_{D^{\gamma*}D^{0\gamma}} = \frac{2}{m_D + m_{D^*}} F_u(0), \quad F_u(0) = \frac{2}{3} V^{qq}(0) \left[ 1 + \frac{V^{cc}(0)}{V^{qq}(0)} \right].$$

We evidently separated the ‘charm’ ($\bar{c}\gamma_\mu c$) from the ‘up/down’ ($\bar{q}\gamma_\mu q$) contributions to the electromagnetic current. The matrix elements of which will be computed separately. We should emphasize that we do not consider the isospin violating effects and we consistently take $m_u = m_d \equiv m_q$. Notice that, contrary to the pionic coupling ($g_{d^*Dπ}$), the coupling to the soft photon ($g_{D^*Dγ}$) is dimension-full, and it is a measure of the magnetic moment of the $D^*$-meson. To make the lattice computation more straightforward our target will be to compute the dimensionless form factor $F_{u,d}(0)$.  

4
3 Correlation functions computed on the lattice

In this section we list the correlation functions that are to be computed on the lattice, define the convenient double ratios of correlation functions which lead to the desired form factors. We then discuss the method that helps exploring the kinematical configurations in which \( \vec{q} \neq 0 \).

3.1 Axial form factors

In order to reach the matrix element \( \langle \rangle \) we need to compute the following three-point correlation functions

\[
C^{(1)}_{\mu
u}(\vec{q}; t) = \sum_{\vec{x}, \vec{y}} \langle \mathcal{V}_\mu(\vec{0}, 0) A_\nu(\vec{x}, t) \mathcal{P}^\dagger(\vec{y}, t_S) e^{-i\vec{q}\vec{x}} \rangle,
\]

\[
C^{(2)}_{\mu
u}(\vec{q}; t) = \sum_{\vec{x}, \vec{y}} \langle \mathcal{P}(\vec{0}, 0) A_\mu(\vec{x}, t) \mathcal{V}_\nu^\dagger(\vec{y}, t_S) e^{-i\vec{q}\vec{x}} \rangle,
\]

\[
E^{(1)}(\vec{q}; t) = \sum_{\vec{x}, \vec{y}} \langle \mathcal{P}(\vec{0}, 0) V_0^\dagger(\vec{x}, t) \mathcal{P}^\dagger(\vec{y}, t_S) e^{-i\vec{q}\vec{y}} \rangle,
\]

\[
E^{(2)}(t) = \frac{1}{3} \sum_{i, \vec{x}, \vec{y}} \langle \mathcal{V}_i(\vec{0}, 0) V_0^\dagger(\vec{x}, t) \mathcal{V}_i^\dagger(\vec{y}, t_S) \rangle,
\]

where \( \mathcal{V}_\mu = \bar{c} \gamma_\mu q \), and \( \mathcal{P} = \bar{c} \gamma_5 q \) are the interpolating operators for the vector and the pseudoscalar meson respectively, while \( V_0^\dagger = \bar{q} \gamma_\mu q \) and \( A_\mu = \bar{q} \gamma_\mu \gamma_5 q \). The sources are fixed at \( t = 0 \) and \( t_S = 24 \), and the matrix element is extracted from the usual time dependence of the correlation functions. In recent years it became clear that a more reliable information can be extracted if one cancels the exponential time dependencies in the correlation functions by combining them into suitable double ratios \([6, 7, 8]\). This is why in eq. (10) we also defined the elastic correlation functions, \( E^{(1,2)} \). In the case in which both \( D^- \) and \( D^* \)-mesons are at rest the only contributing form factor is \( A_1(q_{max}^2) \), with \( q_{max}^2 = (m_{D^*} - m_D)^2 \). In that case, the convenient double ratio to consider is

\[
R_0(t) = \frac{C^{(1)}_{01}(\vec{0}; t) C^{(2)}_{01}(\vec{0}; t)}{E^{(1)}(\vec{0}; t) E^{(2)}(\vec{0}; t)} \rightarrow \frac{\langle D(\vec{0})|A_0|D^*(\vec{0})\rangle \langle D^*(\vec{0})|A_0|D(\vec{0})\rangle}{\langle D(\vec{0})|V_0|D(\vec{0})\rangle \langle D^*(\vec{0})|V_0|D^*(\vec{0})\rangle} = \frac{[\langle m_{D^*} + m_D \rangle A_1(q_{max}^2)]^2}{4m_D m_{D^*}}.
\]

Since in eq. (6) we need the form factors at \( q^2 = 0 \), an extra step is needed. As it is well known, on the periodic cubic lattice of size \( L \), the smallest momentum that can be given to a particle is \( q_{min} = \frac{2\pi}{L} \), which on the currently accessible lattices would be far too large, and would eventually push \( q^2 \) to relatively large negative values. It has been shown in refs. [9] that by imposing the twisted boundary conditions on one of the quark propagators in the correlation function, one can explore the arbitrary small momenta of the hadron associated to that “twisted” quark. A simple recipe to implement this idea in...
practical calculations consists in rephasing the gauge links as

\[ U_\mu(x) \rightarrow U_\mu^\theta(x) = e^{i\theta_\mu/L} U_\mu(x), \quad \text{where} \quad \theta_\mu = (0, \vec{\theta}), \tag{11} \]

and then inverting the propagator on the lattice with periodic boundary conditions, i.e. by inverting the Wilson-Dirac operator on such a rephased gauge field configuration, \( S_q(x, 0; U^\theta) \equiv \langle q(x)\bar{q}(0) \rangle \). Finally the twisted propagator is obtained as

\[ S^\theta_q(x, 0; U) = e^{i\vec{\theta}/L} S_q(x, 0; U^\theta). \tag{12} \]

The net effect is that the resulting ground state extracted at large time separations between the sources in two-point correlation functions, e.g.

\[ \sum_{\vec{x}} \langle \text{Tr} \left[ S^\theta_q(0, x; U) \gamma_5 S_q(x, 0; U) \gamma_5 \right] \rangle \rightarrow \frac{1}{2E_\pi} |\langle 0|\bar{q}\gamma_5 q|\pi \rangle|^2 e^{-E_\pi t}, \tag{13} \]

satisfies the following dispersion relation [9, 10]

\[ E_\pi^2 = m_\pi^2 + |\vec{\theta}|^2 / L^2. \tag{14} \]

Therefore we can give as small a momentum to a daughter pseudoscalar meson in eqs. (2,8) as we want. An important discussion and description of the same physics by means of a low-energy effective theory in ref. [11] assert that adding a small twisted angle to the boundary condition on the valence quark, but not on the sea quarks, has a negligible impact on the low energy QCD observables, such as \( f_{K,\pi} \) and \( m_{K,\pi} \), and we shall neglected it in what follows. For further reference, when using the twisted quark propagator, we take vector \( \vec{\theta} \) to be of the form \( \vec{\theta} = (\theta_0, \theta_0, \theta_0) \). We will also always keep the decaying vector meson at rest. More specifically, we compute the correlation functions (10) as follows

\[
\begin{align*}
C^{(1)}_{\mu\nu}(\vec{q}; t) &= \langle \sum_{x, \vec{y}} \text{Tr} \left[ S_c(y, 0; U) \gamma_\mu S_q(0, x; U) \gamma_\nu \gamma_5 S^\theta_q(x, y; U) \gamma_5 \right] \rangle,
C^{(2)}_{\mu\nu}(\vec{q}; t) &= \langle \sum_{x, \vec{y}} \text{Tr} \left[ S_c(y, 0; U) \gamma_5 S^\theta_q(0, x; U) \gamma_\mu \gamma_5 S_q(x, y; U) \gamma_\nu \right] \rangle,
E^{(1)}(\vec{q}; t) &= \langle \sum_{x, \vec{y}} \text{Tr} \left[ S_c(y, 0; U) \gamma_5 S^\theta_q(0, x; U) \gamma_0 S_q(x, y; U) \gamma_5 \right] \rangle,
E^{(2)}(t) &= \frac{1}{3} \langle \sum_{i, x, \vec{y}} \text{Tr} \left[ S_c(y, 0; U) \gamma_i \gamma_0 S_q(0, x; U) \gamma_0 S_q(x, y; U) \gamma_i \right] \rangle,
\end{align*}
\tag{15}\]

where \( \vec{q} = \vec{\theta}/L \), and \( \langle \ldots \rangle \) denotes the average over gauge field configurations, \( U \). To extract the form factor at \( |\vec{q}| \neq 0 \), we first note that for large time separations one has

\[ \tilde{C}_{ij}^{(2)}(\vec{q}; t) \equiv \frac{1}{3} \sum_{i=1}^{3} C_{ii}^{(2)}(\vec{q}; t) - \frac{1}{6} \sum_{i,j=1}^{3} C_{ij}^{(2)}(\vec{q}; t) \big|_{i \neq j} \rightarrow \]

6
Figure 1: The Feynman diagrams used to compute the ratios $R_1^V(t)$ and $R_2^V(t)$ in eqs (22) and (23) respectively. The full dot symbols are used to denote the transition operator, the matrix element of which we are interested in. The symbol “$\vec{\theta}$” with a given quark line indicates that the twisted boundary conditions were imposed in the propagator inversion of that particular quark. The single- and double-line distinguish between the light and the heavy quark propagator.

\[
\frac{\langle D(\vec{q})|P|0 \rangle}{2E_D} e^{-E_D t} \times (m_D + m_{D^*}) A_1(q^2) \times \frac{\langle 0|\gamma_i|D^*(\vec{0}) \rangle}{2m_{D^*}} e^{-m_{D^*}(t_S - t)},
\]

and therefore the convenient double ratio to extract $A_1(q^2)$ looks very similar to $R_0(t)$ in eq. (10), and reads

\[
R_1(t) = \frac{\tilde{C}_{ij}^{(1)}(\vec{q}; t) \tilde{C}_{ij}^{(2)}(\vec{q}; t)}{E^{(1)}(\vec{q}; t) E^{(2)}(\vec{0}; t)} \rightarrow \frac{[(m_{D^*} + m_D) A_1(q^2)]^2}{4E_D m_{D^*}}. 
\]

With our $D^*$-meson at rest the point $q^2 = 0$ is reached when the twisting angle

\[
\theta_0' = \frac{L}{\sqrt{3}} \frac{m_{D^*}^2 - m_D^2}{2m_{D^*}}.
\]

In practice, we give a few values of $\theta_0$ around $\theta_0'$ so that we can interpolate the form factors to $q^2 = 0$. As for the correcting factor $A_2(q^2)/A_1(q^2)$ we extract it from the following ratio of the correlation functions

\[
R_2(t) = -\frac{2 q_i m_{D^*}}{\tilde{C}_{ij}^{(2)}(\vec{q}; t)} A_2(q^2) \rightarrow \frac{2 q_i m_{D^*}}{(m_{D^*} + m_D)^2} A_2(q^2) A_1(q^2),
\]

where, with the definition $q_i = \theta_0/L$. This concludes our strategy for computing the axial coupling, $g_{D^*D\pi}$, on the lattice.


### 3.2 Vector form factors

Concerning the coupling to a soft photon, we should first emphasize that our vector meson is at rest so that the matrix element \(\langle 0 | \bar{c} \gamma_{\mu} c | \bar{c} \gamma_{\mu} c \rangle\) simply becomes

\[
\langle D(\vec{k}) | J^0_{\text{em}} | D^*(0, \vec{e}^\lambda) \rangle = 0 , \quad \text{and} \quad \langle D(\vec{k}) | \bar{J}_{\text{em}} | D^*(0, \vec{e}^\lambda) \rangle = (\vec{e}_\lambda \times \vec{k}) \frac{2e m_D^*}{m_{D^*} + m_D} \langle \bar{V} q^2 \rangle .
\]

The correlation functions needed in this case are

\[
C^{(1qq)}_{ij}(\vec{q}; t) = \langle \sum_{\vec{x}, \vec{y}} \text{Tr} \left[ S_{\text{e}}(y, 0; U) \gamma_i S_q(0, x; U) \gamma_j S_q^\dagger(x, y; U) \gamma_5 \right] \rangle ,
\]

\[
C^{(1cc)}_{ij}(\vec{q}; t) = -\langle \sum_{\vec{x}, \vec{y}} \text{Tr} \left[ S_{\text{e}}(x, 0; U) \gamma_i S_q(0, y; U) \gamma_5 S_q^\dagger(y, x; U) \gamma_j \right] \rangle ,
\]

\[
C^{(2qq)}_{ij}(\vec{q}; t) = \langle \sum_{\vec{x}, \vec{y}} \text{Tr} \left[ S_{\text{e}}(y, 0; U) \gamma_5 S_q^\dagger(0, x; U) \gamma_i S_q(x, y; U) \gamma_j \right] \rangle .
\]

Notice in particular the sign difference of “\(\bar{c} \gamma_{\mu} c\)” with respect to the “\(\bar{q} \gamma_{\mu} q\)” part of the electromagnetic current, which comes from the Wick contraction. Physically that accounts for the spin difference between the \(D\) and \(D^*\) states. Finally, that minus-sign compensates the relative sign difference between the electric charge of the light and heavy quark/antiquark in the heavy-light meson. That sign-flip has been properly taken into account when writing the expressions (20). Finally from the double ratio

\[
R^V_1(t) = -\frac{C^{(1qq)}_{ij}(\vec{q}; t) C^{(2qq)}_{ij}(\vec{q}; t)}{E^{(1)}(\vec{q}; t) E^{(2)}(0; t)} \rightarrow \frac{|\vec{q}|^2 m_{D^*}}{3 E_D} \frac{|V^{qq}(q^2)|^2}{(m_{D^*} + m_D)^2} ,
\]

we extract the desired form factor. In the above expression we use the symmetry of the problem and average over the equivalent indices, e.g.

\[
C^{(2qq)}_{ij}(\vec{q}; t) = \frac{1}{6} \left[ C^{(2qq)}_{12}(\vec{q}; t) + C^{(2qq)}_{23}(\vec{q}; t) + C^{(2qq)}_{31}(\vec{q}; t) - C^{(2qq)}_{21}(\vec{q}; t) - C^{(2qq)}_{32}(\vec{q}; t) - C^{(2qq)}_{13}(\vec{q}; t) \right] .
\]

Notice yet another “−” sign in eq. (22) which is meant to compensate the sign difference between the two matrix elements accessed from \(C^{(2qq)}_{ij}(\vec{q}; t)\) (photon emission, \(D^* \rightarrow D\gamma\)) and from \(C^{(1qq)}_{ij}(\vec{q}; t)\) (photon absorption, \(\gamma D \rightarrow D^*\)). Finally, the ratio of the two form factors is computed through

\[
R^V_2(t) = -\frac{C^{(2cc)}_{ij}(\vec{q}; t) C^{(2qq)}_{ij}(\vec{q}; t)}{C^{(2qq)}_{ij}(\vec{q}; t)} \rightarrow \frac{V^{cc}(q^2)}{V^{qq}(q^2)} ,
\]

where \(C^{(2cc)}_{ij}(\vec{q}; t)\) is of the same form as \(C^{(2qq)}_{ij}(\vec{q}; t)\) after replacing \(q \leftrightarrow c\) in eq. (21). The Feynman diagrams of the double ratios leading to \(R^V_1(t)\) and \(R^V_2(t)\) are shown in fig. 1a and 1b respectively.
In short, the subtraction of the sea quarks are mass degenerate. To fully respect the spacing, we consider only the fully unquenched situations in which the light valence and the sea quarks are not modified by the subtraction of $O(a)$ effects.

### 3.3 Improvement and renormalization

The gauge field configurations that are being used in this work are obtained with the Wilson plaquette gauge action, while the effects of dynamical quarks are incorporated by the Wilson plaquette gauge action, while the effects of dynamical quarks are incorporated by

\begin{align}
A^{\text{impr.}}_\mu(x) &= Z_A(g_0^2) \left( 1 + b_A(g_0^2)(am_q) \right) \left[ A_\mu(x) + c_A \partial_\mu P(x) \right], \\
V^{\text{impr.}}_\mu(x) &= Z_V(g_0^2) \left( 1 + b_V(g_0^2)(am_q) \right) \left[ V_\mu(x) + c_V \partial_\mu T_{\mu\nu}(x) \right],
\end{align}

(25)

where $T_{\mu\nu}(x) = i\bar{q}(x)\sigma_{\mu\nu}q(x)$, and $\sigma_{\mu\nu} = (i/2)[\gamma_\mu, \gamma_\nu]$. When sandwiched between the external $V$ and $P$ states, or more specifically $D^*$ and $D$, the piece proportional to $c_A(g_0^2)$ does not modify the form factors $A_{1,2}(q^2)$. It only changes $A_0(q^2)$ as

\begin{equation}
A^{\text{impr.}}_0(q^2) = \left( 1 - c_A(g_0^2) \frac{q^2}{2m_q^{\text{AWI}}} \right),
\end{equation}

(26)

where $m_q^{\text{AWI}}$ stands for the bare quark mass obtained on the lattice via the axial Ward identity (AWI). In other words, our determination of the coupling $g_{D^*D}$ does not feel the effect of improvement of the bare axial current. Instead, the vector form factor does get improved as

\begin{equation}
V^{\text{impr.}}(q^2) = V(q^2) + c_V(g_0^2)(m_D + m_{D^*}) T_1(q^2),
\end{equation}

(27)

where we used the standard parameterization of the tensor density matrix element,

\begin{align}
\langle D(k)|T^{\mu\nu}|D^*(p, e_A)\rangle &= \left\{ e^{\mu\nu\alpha\beta} \left[ \left( p_\beta + k_\beta - \frac{m_D^2}{q^2} - \frac{m_{D^*}^2}{q^2} \right) T_1(q^2) + \frac{m_{D^*}^2 - m_D^2}{q^2} q_3 T_2(q^2) \right] \\
&+ \frac{2k_\alpha}{q^2} e^{\mu\nu\sigma\rho} k_\beta p'_\sigma \left( T_2(q^2) - T_1(q^2) + \frac{q^2}{m_{D^*}^2 - m_D^2} T_3(q^2) \right) \right\} e_A^\lambda(p).
\end{align}

(28)

We implemented this improvement in our calculation and the results presented in the next section are improved and renormalized. The values of the improvement ($c_V(g_0^2)$) and renormalization constants ($Z_{V,A}(g_0^2)$, $b_{V,A}(g_0^2)$) used in this work, with the appropriate list of references, are listed in table [I].

| $\beta$ | $c_{SW}$ | $c_V$ | $Z_V$ | $Z_A$ | $b_V$ | $b_A$ |
|---|---|---|---|---|---|---|
| 5.29 | 1.919 | -0.034 | 0.743 | 0.772(4) | 1.91 | 1.31 |
| 5.40 | 1.823 | -0.032 | 0.757 | 0.783(4) | 1.79 | 1.30 |

Table 1: Renormalization and improvement constants used in this work. Apart from $c_V$ and $b_A$, which are estimated by using the 1-loop (boosted) perturbation theory, all the constants are determined non-perturbatively.
4 Numerical results

4.1 Lattice details

The correlation functions needed to complete this work are computed on the gauge field configurations produced by the QCDSF collaboration [19]. They were obtained by using the Wilson plaquette gauge field action, and the improved Wilson fermions with $N_f = 2$ mass-degenerate dynamical light quarks at $\beta = 5.29$ ($a \approx 0.075 \text{ fm}$) and $\beta = 5.40$ ($a \approx 0.065 \text{ fm}$) on the lattices of volume $24^3 \times 48$. From the ensemble of available configurations we chose the ones corresponding to the lighter sea quarks, the hopping parameters of which are listed in table 2. The number of configurations used in this study is also given in the same table. We chose to use the configurations that are separated by 20 molecular dynamics trajectories of the unit length in the hybrid Monte-Carlo algorithm (HMC). In computing the quark propagators we attempted using the so called “color dilution” technique [21] but since we did not observe any appreciable effect in improving the statistical quality of our correlation functions, we returned to the standard BiCG-Stab algorithm [22]. With the computed propagators we then calculated the 2- and 3-point correlation functions in a standard way. We first recomputed the masses of light quarks and of the pseudoscalar mesons consisting of the light valence quark and antiquark whose mass is equal to that of the sea quark.

We tried to fix the charm quark mass in several ways, of which we describe one. Since the charm quark is not propagating in the sea, its phenomenology is quenched and its hopping parameter can be fixed by tuning its value to make the ratio of the light-strange pseudoscalar and the strange-charmed mesons equal to its physical value, $m_{\eta_{ss}}/m_{D_s} = 0.35$, where $m_{\eta_{ss}} = \sqrt{2m_K^2 - m_\pi^2} = 0.684 \text{ GeV}$. On the very same configurations the strange quark mass has been computed in ref. [18], quoting $r_0m_{s}^\text{MS}(2 \text{ GeV}) = 0.2545(47)$ at $\beta = 5.29$, and $r_0m_{s}^\text{MS}(2 \text{ GeV}) = 0.2671(48)$ at $\beta = 5.40$. By using these values and the numerical constants provided in ref. [18], as well as the force parameter [20], $r_0/a = 6.25(10)\beta=5.29$,

| $\beta$ | $\kappa_{\text{sea}} = \kappa_{\text{val}}$ | # conf. | $m_\pi$ | $m_D$ | $m_{D^*}$ |
|-------|-----------------|--------|---------|--------|------------|
| 5.29  | $\kappa_1 = 0.1355$ | 60     | 0.3271(23) | 0.789(3) | 0.854(4) |
|       | $\kappa_2 = 0.1359$ | 80     | 0.2451(25) | 0.760(5) | 0.805(6) |
|       | $\kappa_3 = 0.1362$ | 100    | 0.1549(24) | 0.731(6) | 0.783(9) |
| 5.40  | $\kappa_1 = 0.1356$ | 130    | 0.3124(18) | 0.725(3) | 0.771(3) |
|       | $\kappa_2 = 0.1361$ | 120    | 0.2166(23) | 0.691(4) | 0.734(5) |
|       | $\kappa_3 = 0.13625$ | 160    | 0.1843(26) | 0.681(2) | 0.721(4) |

Table 2: Masses of the light-light and heavy-light mesons in which the light valence quark has the same mass as the sea quark. All results are given in lattice units and the charm quark hopping parameter is fixed to $\kappa_{\text{charm}}^{[\beta=5.25]} = 0.125$ and $\kappa_{\text{charm}}^{[\beta=5.40]} = 0.126$. The gauge field configurations are separated by 20 unit-length HMC trajectories.
7.39(26)\_\beta=5.40, we extract $\kappa_{\text{strange}}^{[\beta=5.29]} \approx 0.1355$ and $\kappa_{\text{strange}}^{[\beta=5.40]} \approx 0.1359$. After having fixed one valence quark to $\kappa_{\text{strange}}$ and varying $\kappa_{\text{charm}}$, we were able to reproduce $m_{\eta_s}/m_{D^*} = 0.35$, for $\kappa_{\text{charm}}^{[\beta=5.25]} = 0.125$, and $\kappa_{\text{charm}}^{[\beta=5.40]} = 0.126$, after linearly extrapolating the sea quark mass to zero. By adopting other strategies to fix the charm hopping parameter, the resulting $\kappa$ for all of our masses, for which the light quark hopping parameter $\kappa$ differs by at most $5 \times 10^{-4}$. We checked, however, that the results of this paper are insensitive to that variation. In table 2 we list the pseudoscalar ($D$) and vector ($D^*$) meson masses, for which the light quark hopping parameter $\kappa_q = \kappa_{\text{sea}}$ and the heavy quark is fixed to $\kappa_{\text{charm}}$. Notice the labels $\kappa_{1,2,3}$ in table 2, which will be used in the next subsections when reporting our results for the form factors. Every time the light valence quark appears in a correlation function, its mass is always kept equal to the sea quark mass.

4.2 Form factors

In this subsection we present our main numerical results. We first computed the ratios (10) which exhibit the long plateaus and lead to $A_1(q^2)_{\max}$, with $q^2_{\max} = (m_{D^*} - m_D)^2$. Since $q^2_{\max}$ is very small and since the shape of the axial form factor is expected to be driven by a rather heavy $a_1$-state, the result for $A_1(0)$ will differ only slightly with respect to $A_1(q^2_{\max})$. \footnote{From the light-light two point correlation functions, $C_{A_1A_1}(t) = \sum_{i<j}(A_i(0)A_j(x))$ we extract the masses of $a_1$-state and from our data we have:}

$$\langle m_{a_1}/m_\pi \rangle = \{1.9(2), 2.5(3), 3.9(3)\}_{\beta=5.29}, \text{ and } \langle m_{a_1}/m_\pi \rangle = \{2.0(0), 2.6(1), 2.9(1)\}_{\beta=5.40}.$$

On our lattices, $m_{a_1}$ is about 10 times larger than $q^2_{\max}$.

$$g_{c}(q^2, m^2_\pi) = \frac{2\sqrt{m_D m_D^*}}{m_D + m_D^*} A_1(q^2) \left[ 1 + \frac{m_{D^*} - m_D}{m_{D^*} + m_D} A_2(q^2) \right], \quad (29)$$

that will eventually lead us to the $g_{c}$-coupling, defined as,

$$g_{D^*D\pi} = \frac{2\sqrt{m_D m_D^*}}{f_\pi} g_{c}. \quad (30)$$

$$1$$
Concerning the momentum injections corresponding to \( \theta_0 \in \{0.5, 1.0, 1.5\} \), by choosing \( r_0 = 0.467 \) fm, we have \( |\vec{q}|_{\beta=5.29} \in \{96(2), 191(3), 287(5)\} \) MeV, and \( |\vec{q}|_{\beta=5.40} \in \{113(4), 226(8), 339(12)\} \) MeV. These values would be 7% smaller if we made another (also standard) choice, namely \( r_0 = 0.5 \) fm. The results for all our data-points (both \( \beta \), all \( \kappa_{\text{sea}} \), and all \( \theta_0 \)) are collected in table 3. Notice that at \( \beta = 5.40 \) we did not compute the form factors with \( \theta_0 = 1.5 \), as it was clear that for that value \( q^2 \ll 0 \) and would not help us interpolating to \( q^2 = 0 \).

We now turn to the results for the vector form factor. In this case the form factor is not accessible when both mesons are at rest. When \( \theta_0 \neq 0 \) we obtain the results for

Figure 2: In the upper plots we illustrate the time dependence of the ratios \( R_0(t) \), \( R_1(t) \) and \( R_2(t) \), defined in eqs. (10, 17, 19), leading to the dominant form factor \( A_1(q^2) \) and to \( A_2(q^2)/A_1(q^2) \). In the lower plots the similar illustration is provided for the ratios \( R^V_1(t) \) and \( R^V_2(t) \), defined in eqs. (22) and (24), and leading to the form factor \( V^{qq}(q^2) \) and \( V^{cc}(q^2)/V^{qq}(q^2) \), respectively. The fit on the plateau interval \( t \in [10, 15] \) is also shown, for both \( \theta_0 = 0.5 \) and \( \theta_0 = 1.0 \), corresponding to about 110 MeV and 220 MeV respectively.
### Table 3

The form factors $A_1(q^2)$ and $g_c(q^2, m_c^2)$ [c.f. eq. (29)] relevant to the axial coupling, and the form factors $V^{qq}(q^2)$ and $V^{cc}(q^2)/V^{qq}(q^2)$ relevant to the magnetic moment of our $D^*$-meson.
from our calculation an extrapolation to the chiral limit is required. The relevant formula, vector form factors as indicated in eq. (9). Also give the results for the value obtained through the linear interpolation to \(q^2\) cannot distinguish amongst different interpolating formulas, and we choose to quote our masses of \(a\) and the illustration is provided in fig. 3 for the case of \(t\) physical assumption, such as the nearest pole dominance of the particle exchanged in the \(t\)-channel, namely \(a_1\)-meson to \(g_c(q^2, m_{\pi}^2)\), and the \(\rho\)-meson dominance to the \(V^{qq}(q^2)\). The masses of \(a_1\) and \(\rho\) are easily computed on the same lattice. However, since the range of \(q^2\)'s that we are probing is very short and since our results are not sufficiently accurate we cannot distinguish amongst different interpolating formulas, and we choose to quote our value obtained through the linear interpolation to \(q^2 = 0\). All results are listed, in table 4 and the illustration is provided in fig. 3 for the case of \(g_c(q^2, m_{\pi}^2)\). In that same table we also give the results for \(F_{u,d}(0, m_{\pi}^2)\) obtained after combining the interpolated values of the vector form factors as indicated in eq. (9).

Second step is far more delicate. In order to reach the physically interesting result from our calculation an extrapolation to the chiral limit is required. The relevant formula,

|                | \(\beta = 5.29\) |                | \(\beta = 5.40\) |
|----------------|------------------|----------------|------------------|
|                | \(\kappa_1\)    | \(\kappa_2\)  | \(\kappa_3\)    | \(\kappa_1\)    | \(\kappa_2\)  | \(\kappa_3\)    |
| \(V^{qq}(0)\) | 4.56 ± 1.12      | 3.14 ± 1.43    | 5.88 ± 2.88      | 4.98 ± 0.54      | 4.78 ± 0.82    | 4.34 ± 1.08      |
| \(V^{cc}(0)/V^{qq}(0)\) | 0.26 ± 0.13 | 0.76 ± 0.36 | 0.33 ± 0.36 | 0.50 ± 0.08 | 0.43 ± 0.08 | 0.42 ± 0.16 |
| \(F_u(0, m_{\pi}^2)\) | 3.62 ± 1.17 | 3.71 ± 1.57 | 5.19 ± 2.14 | 4.98 ± 0.45 | 4.55 ± 0.81 | 4.10 ± 0.89 |
| \(F_d(0, m_{\pi}^2)\) | −0.95 ± 0.63 | −0.56 ± 0.77 | −0.69 ± 1.55 | −0.00 ± 0.26 | −0.23 ± 0.27 | −0.24 ± 0.52 |
| \(g_c(0, m_{\pi}^2)\) | 1.37 ± 0.11 | 1.02 ± 0.12 | 0.89 ± 0.14 | 1.10 ± 0.04 | 0.99 ± 0.07 | 0.90 ± 0.06 |

Table 4: Results of the linear interpolation of our results from table 3 to \(q^2 = 0\), for each of the light quarks that we have at our disposal \((\kappa_1, \kappa_2, \kappa_3)\). We also show the particular combinations \(F_{u,d}(0, m_{\pi}^2)\) which are needed to arrive at the magnetic moments, i.e. to the couplings \(g_{D^{+}D_{+}\gamma}\) and \(g_{D^{0}D_{0}\gamma}\).
derived in HMChPT, reads [23, 24]

\[
g_{c}(m_{\pi}^2) = g_{c}^{(0)} \left[ 1 - \frac{4(g_{c}^{(0)})^2}{(4\pi f r_0)^2} (m_\pi r_0)^2 \log(m_\pi r_0)^2 + \frac{c_2}{r_0^2} (m_\pi r_0)^2 \right],
\]

\( (31) \)

with the particularly interesting parameter \( g_{c}^{(0)} \) which appears in all the HMChPT formulas that are being used to guide the chiral extrapolations of the phenomenologically interesting quantities in charm physics computed on the lattice. In the expression (31) we multiplied with suitable factors of \( r_0/a \) to make the expression independent on the lattice spacing, \( a \).

Two important underlying assumptions are made when applying this formula: (i) Eq. (31) is obtained in the static heavy quark limit and its use is equivalent to the assumption that the \( 1/m_{c}^{n} \)-corrections do not modify the chiral behavior of this coupling. That assumption is quite reasonable because the chiral dynamics is driven by the light quark vertex (in our case the current \( \bar{q}\gamma_\mu\gamma_5 q \)), while the heavy quark is only a spectator; (ii) The HMChPT formula applies to our data too. Of our three light quarks in each set, one is heavier than the physical strange quark, one is somewhat lighter and our lightest dynamical quark is about a third (half) of the mass of the strange quark when \( \beta = 5.29 \) (5.40). The assumption that such a heavy light quarks obey the behavior driven by the HMChPT formula is strong and the check of its validity is one of the most active research topics in the lattice QCD community. With these two underlying assumption in mind we fit our data to eq. (31) and obtain

\[
g_{c}^{(0)}_{\text{HMChPT}} = \{ 0.63(11)_{\beta=5.29}, 0.68(7)_{\beta=5.40} \},
\]

\( (32) \)
while at the physical pion mass we have, \( g_c(m^\text{phys}_\pi) = \{0.66(11)_{\beta=5.29}, 0.71(7)_{\beta=5.40}\} \). If the logarithmic correction in eq. (31) is set to zero we obtain the simple linear extrapolation which gives \( g_{\text{linear}}(0) = \{0.70(15)_{\beta=5.29}, 0.81(11)_{\beta=5.40}\} \). As a small check, we combine the data of two sets and use only those obtained in the case in which the sea quark mass is lighter than the physical strange quark mass, and we obtain \( g_{\text{HMCChPT}}(0) = 0.67(14) \), and \( g_c(m^\text{phys}_\pi) = 0.69(14) \). These results are to be compared with the quenched value obtained in ref. [3], \( g_c = 0.66(8)(5) \). We conclude that no effect due to unquenching is visible from our results.

On the other hand, the result for the coupling to the soft photon is completely new. It has never been computed even in the quenched approximation. HMCChPT formula for this coupling was obtained in refs. [24, 25] and reads

\[
F_u(m^2_\pi) = \sqrt{m_D m_D} \left[ \frac{2}{3} \beta \left( 1 - \frac{3}{2} \frac{3 + g^2_c}{(4\pi f r_0)^2} (m_\pi r_0)^2 \log(m_\pi r_0)^2 \right) - \frac{g^2_c + d_u}{4\pi f^2} m_\pi \right],
\]

\[
F_d(m^2_\pi) = -\sqrt{m_D m_D} \left[ \frac{1}{3} \beta \left( 1 - \frac{3}{2} \frac{1 + 2 g^2_c}{(4\pi f r_0)^2} (m_\pi r_0)^2 \log(m_\pi r_0)^2 \right) - \frac{g^2_c + d_d}{4\pi f^2} m_\pi \right] ,
\]

with \( \beta \)-being the dimension-full parameter which is a measure of the magnetic moment of the light quark inside the static heavy-light vector meson, and \( d_u,d \) are the counter-term coefficients which are supposed to be obtained from the fit. As it can be seen from table [4] our data for \( F_d(0,m^2_\pi) \) are consistent with zero, whereas the results for \( F_u(0,m^2_\pi) \) change very little when varying the pion mass, contrary to what the HMCChPT would suggest. In other words, the part linear in \( m_\pi \) in eq. (33) is far too large to provide a suitable description of our data, unless one accepts a huge value for the counter-term coefficient \( (d_u,d) \) which would then contradict the perturbative feature of HMCChPT. For that reason we only linearly extrapolate our data and arrive at

\[
F_u(0) \in \{(5.3 \pm 2.3)_{\beta=5.29}, (3.9 \pm 1.1)_{\beta=5.40}\} ,
\]

\[
F_d(0) \in \{-(0.1 \pm 1.4)_{\beta=5.29}, -(0.4 \pm 0.6)_{\beta=5.40}\} .
\]

We hope to revisit this issue when more accurate values for \( F_u(m^2_\pi) \) obtained with ever lighter \( m_\pi \) become available.

## 5 Phenomenology

In this section we discuss our physical results and make a phenomenological discussion based on the currently available experimental information. Furthermore, we compare our results for the magnetic moment with the results obtained by other theoretical approaches such as QCD sum rules and quark models.

### 5.1 Our results

From our results obtained with HMCChPT we conclude

\[
g_c \in \{0.66(11)_{\beta=5.29}, 0.71(7)_{\beta=5.40}\} \Rightarrow g_{D^+ D^+} \in \{(18.6 \pm 3.2)_{\beta=5.29}, (20.1 \pm 2.1)_{\beta=5.40}\} . \tag{35}
\]
The expression for the decay width reads

$$\Gamma(D^* \to D\pi) = \frac{C}{24\pi m^2_{D^*}} g^2_{D^*D\pi} |\vec{k}_\pi|^3,$$  \tag{36}

where $C = 1$ if the outgoing pion is charged, and $C = 1/2$ if it is neutral. For the charged decaying meson we have

$$|\vec{k}_\pi| = \sqrt{\left(\frac{m^2_{D^*} - m^2_D + m^2_{\pi}}{2m_{D^*}}\right)^2 - m^2_\pi} \Rightarrow |\vec{k}_{\pi^-}| = 39.4 \text{ MeV}, \ |\vec{k}_{\pi^0}| = 38.3 \text{ MeV}, \tag{37}$$

and we get

$$\Gamma(D^{*-} \to D\pi) = \begin{cases} (72 \pm 24)_{\pi^-} \text{ keV}, & (33 \pm 11)_{\pi^0} \text{ keV}, & \beta = 5.29, \\ (82 \pm 17)_{\pi^-} \text{ keV}, & (38 \pm 8)_{\pi^0} \text{ keV}, & \beta = 5.40. \end{cases} \tag{38}$$

As for the couplings to the soft photon, our results are

$$g_{D^{*+}D^+\gamma} = \{ -0.1(7)_{\beta=5.29}, -0.2(3)_{\beta=5.40} \} \text{ GeV}^{-1},$$

$$g_{D^{*0}D^0\gamma} = \{ (2.7 \pm 1.2)_{\beta=5.29}, (2.0 \pm 0.6)_{\beta=5.40} \} \text{ GeV}^{-1}. \tag{39}$$

In other words, the decay width for the radiative decay of a charged $D^*$-meson is negligibly small (consistent with zero) whereas for the neutral one we have

$$\Gamma(D^{*0} \to D^0\gamma) = \frac{\alpha_{\text{em}}}{3} g^2_{D^{*0}D^0\gamma} k^3_\gamma = \begin{cases} (48 \pm 31) \text{ keV}, & \beta = 5.29, \\ (27 \pm 14) \text{ keV}, & \beta = 5.40, \end{cases} \tag{40}$$

where $2m_{D^{*0}} k^2_\gamma = m^2_{D^{*0}} - m^2_{D^0}$, i.e. $k_\gamma = 137.1$ MeV.

5.2 Comparison with experiment, other methods and more phenomenology

- Let us remind the reader that from the width of the charged $D^*$-meson measured by CLEO, $\Gamma(D^{*+}) = 96 \pm 22$ keV \[1\], and by using the experimentally established $B(D^{**} \to D^+\gamma) = 0.016(4)$ \[26\], we get

$$\Gamma_{\text{exp.}}(D^{*+}) [1 - 0.016(4)] = \Gamma(D^{*+} \to D^0\pi^+) + \Gamma(D^{*+} \to D^+\pi^0)$$

\footnote{For convenience, in addition to $f_\pi = 131$ MeV, we list the numerical values of the meson masses used in this paper \[24\]:

$$m_{D^*} = 1869.6(2) \text{ MeV}, \quad m_{D^{*+}} = 2010.3(2) \text{ MeV}, \quad m_{\pi^+} = 139.6 \text{ MeV},$$

$$m_{D^0} = 1864.8(2) \text{ MeV}, \quad m_{D^{*0}} = 2007.0(2) \text{ MeV}, \quad m_{\pi^0} = 135.0 \text{ MeV}.$$}
\begin{equation}
\frac{2m_{D^0}|\vec{p}_{\pi^0}|^3 + m_{D^+}|\vec{p}_{\pi^0}|^3}{12\pi m_{D^+} f_{\pi}^2} g_c^2 \Rightarrow g_c = 0.61(7),
\end{equation}
a well known result which is in good agreement with the value obtained in this paper on our finer lattice, \( g_c = 0.71(7) \).

- With the experimental \( g_c \) and by using the isospin symmetry we can evaluate the width of the \( D^{*0} \)-meson as

\begin{equation}
\Gamma(D^{*0}) = \Gamma(D^{*0} \to D^- \pi^+) + \Gamma(D^{*0} \to D^0 \pi^0) + \Gamma(D^{*0} \to D^0 \gamma).
\end{equation}

The first channel turns out to be kinematically forbidden. For the last one, instead, we have the experimental information that \( B(D^{*0} \to D^0 \gamma) = 0.381(29) \) \[26\], so that with the \( g_c \) value extracted from eq. (41) we have

\begin{equation}
\Gamma(D^{*0}) = \frac{1}{1 - (0.381 \pm 0.029)} \frac{m_{D^0}|\vec{p}_{\pi^0}|^3}{12\pi m_{D^{*0}} f_{\pi}^2} g_c^2 = 68 \pm 17 \text{ keV},
\end{equation}

where \( |\vec{p}_{\pi^0}| = 43 \text{ MeV} \). The above result is well below an old—but the only available—experimental bound, \( \Gamma(D^{*0}) < 2.1 \text{ MeV} \) \[27\].

- To be able to compare our results for \( g_{D^*D\gamma} \) couplings with experiment, we first combine \( \Gamma(D^{*+}) = 96 \pm 22 \text{ keV} \) \[1\], and \( B(D^{*+} \to D^+ \gamma) = 0.016(4) \) \[26\], to get

\begin{equation}
B(D^{*+} \to D^+ \gamma) \times \Gamma(D^{*+}) = \frac{\alpha_{em}}{3} g_{D^*D\gamma}^2 k_{\gamma}^2 \Rightarrow g_{D^{*+}D^+\gamma} = 0.50(8) \text{ GeV}^{-1},
\end{equation}

where, in this case, \( k_{\gamma} = 135.6 \text{ MeV} \). Similarly, from the result in eq. (43) and \( B(D^{*0} \to D^0 \gamma) = 0.381(29) \) \[26\], we obtain

\begin{equation}
g_{D^{*0}D^{0}\gamma} = 2.02(26) \text{ GeV}^{-1}.
\end{equation}

We see that these results are in a quite good agreement with values obtained on our finer lattice, namely \( g_{D^{*+}D^+\gamma} = -0.2(3) \text{ GeV}^{-1} \), and \( g_{D^{*0}D^{0}\gamma} = 2.0(6) \text{ GeV}^{-1} \).

- We can also compare to the experimentally measured ratio \[26\] \[28\]

\begin{equation}
R_0 = \frac{\Gamma(D^{*0} \to D^0 \pi^0)}{\Gamma(D^{*0} \to D^0 \gamma)} = 1.74 \pm 0.02 \pm 0.13.
\end{equation}

From our data we have \( R_0 = 1.1^{+1.0}_{-0.7} \) at \( \beta = 5.29 \), and \( R_0 = 2.4^{+1.8}_{-1.3} \) at \( \beta = 5.40 \). Broadly speaking both our lattice results are consistent with experiment. It will be interesting if one can increase the accuracy by which this ratio is computed and either check on the validity of various lattice actions that are currently used in the literature, or to check on the chiral extrapolations.
| Method                     | Ref. | $\Gamma(D^{++} \rightarrow D^{+}\gamma)$ | $\Gamma(D^{*0} \rightarrow D^{0}\gamma)$ | $g_{D^{*+}D^{+}\gamma}$ | $g_{D^{*0}D^{0}\gamma}$ |
|----------------------------|------|----------------------------------------|----------------------------------------|-------------------------|-------------------------|
| Light Front Model          | 29   | 0.90(2)                                | 20.0(3)                                | 0.4                     | 1.8                     |
|                            | 30   | 0.56                                   | 21.7                                   | 0.3                     | 1.9                     |
| Chiral Quark Model         | 31   | 1.0 ± 1.5                              | 38.5 ± 43.5                           | 0.4 ± 0.5               | 2.5 ± 2.6               |
|                            | 32   | 0.15 ± 0.25                            | 11 ± 13                                | 0.16 ± 0.20             | 1.4 ± 1.5               |
| Schrödinger-like Model     | 33   | 0.28                                   | 17.4                                   | 0.2                     | 1.7                     |
| (“S”)                     |      |                                        |                                        |                         |                         |
| (“V”)                     |      |                                        |                                        |                         |                         |
| Salpeter-like Model        | 34   | 0.46                                   | 20.8                                   | 0.3                     | 1.8                     |
| Bag Model                  | 35   | 1.72                                   | 7.18                                   | 0.5                     | 1.1                     |
| Moment 3pt*-Sum Rule       | 36   | 0.03(8)                                | 7.3(2.7)                               | 0.02(11)                | 1.1(2)                  |
| Light Cone Sum Rule (LCSR) | 37   | 1.5                                    | 14.4                                   | 0.5                     | 1.5                     |
| Lattice QCD                | this work | 0.8(7)                               | 27(14)                                | -0.2(3)                 | 2.0(6)                  |
| Experiment                 | 26   | 1.54(52)                               | 26(7)                                  | 0.50(8)                 | 2.02(26)                |

Table 5: The table of results for the radiative $D^*$ meson decays as computed by various quark models, QCD sum rules, lattice QCD and those extracted from experiment in the way discussed in Sec. 5 of this paper. “S” / “V” labels the scalar/vector potential in the model of ref. 33.

The ratio $R_0$, as well as the quantities discussed in this section, are interesting because they do not involve the weak interactions (CKM matrix elements) and the comparison between lattice QCD and experiment requires less assumptions.

Finally, since our result for the soft photon coupling is new, we should also compare it to the existing results in the literature. In table 5 we collect the results obtained by using various quark models and various QCD sum rule techniques and compare them to the values obtained in this paper. In extracting the couplings $g_{D^*D\gamma}$ from various papers, we used eq. (40) and the results for the decay widths quoted in the cited papers. To that list we should add ref. 38 in which the radiative $D^*$-meson decay was used to fix the unknown parameter $\chi(1.3 \text{ GeV})\varphi(1/2)$, the product of susceptibility of the quark condensate and the twist-2 photon distribution amplitude at the middle point. More precisely, the problem of large contributions due to coupling to the first radial excitations 39 was circumvented by applying the QCD sum rule to the branching fraction, rather than the decay width alone.

6 Summary and perspectives

In this paper we report the results of the first lattice QCD study of the $g_{D^*D\gamma}$ coupling, i.e. of the magnetic moment of the charmed vector meson. Our result for the neutral $D^*$ meson is fully consistent with the current experimental decay width $\Gamma(D^* \rightarrow D^{+}\gamma)$. On
the other hand our value for the charged meson suffers from large statistical errors but it
is small and both consistent with zero and with the measured $\Gamma(D^* \to D^+\gamma)$.

We also provide the results of our new computation of the axial coupling $g_{D^*D\pi}$, which
corroborates the previous (quenched) result obtained on the lattice. Our value agrees with
the experimental result for $\Gamma(D^{*+})^{\text{CLEO}}$, with a tendency towards its larger values.

The results presented in this paper are obtained by using the non-perturbatively $\mathcal{O}(a)$-
improved Wilson quarks, both dynamical and valence ones. In spite of the improved
technique to extract the form factors using the twisted boundary conditions and the con-
venient double ratios, the gain is not that obvious because our statistical errors are large.
Besides an obvious goal to reduce the statistical uncertainties one should also try and
probe the smaller light quark masses and thereby arrive at the precision determination of
the couplings $g_{D^*D\pi}$ and $g_{D^*D\gamma}$. We argued that the comparison of the accurate lattice re-
sults for these quantities may be a good testing ground for various lattice QCD approaches
(different quark actions, effective treatments of heavy quark, chiral extrapolations) against
experiment, because they involve the calculation of hadronic matrix element but, being the
strong and electromagnetic processes, no CKM parameter is needed. The approach used
here could also be used to compute the $g_{K^*K\gamma}$ coupling for which the experimental value
is already known to a 5% accuracy.

Finally the unquenching made here is done with $N_f = 2$ dynamical quarks and a
natural step forward would be to include the effects of the ‘strange’ dynamical quark, $N_f = 2 \to 2 + 1$.

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