The effects of non-universal extra dimensions on the fermion electric dipole moments in the two Higgs doublet model.

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Abstract

We study the effects of non-universal extra dimensions on the electric dipole moments of fermions in the two Higgs doublet model. We observe that the $t$ quark and $b$ quark electric dipole moments are sensitive to the extra dimensions, however, in the case of charged lepton electric dipole moments, this sensitivity is relatively weak.

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1 Introduction

The electric dipole moments (EDMs) of fermions are worthwhile to study since their origin is the CP violating interaction, which is weak in the standard model and it pushes one to investigate new models beyond. There are number of experimental results on the fermion EDMs in the literature. The electron, muon and tau EDMs have been measured experimentally as $d_e = (1.8 \pm 1.2 \pm 1.0) \times 10^{-27} e \text{ cm}$ [1], $d_\mu = (3.7 \pm 3.4) \times 10^{-19} e \text{ cm}$ [2] and $d_\tau = (3.1) \times 10^{-16} e \text{ cm}$ [3] respectively and the experimental upper bound of neutron EDM has been found as $d_N < 1.1 \times 10^{-25} e \text{ cm}$ [4].

There is an extensive theoretical work done on the EDMs of fermions. The calculations of quark EDMs in the framework of the standard model (SM) [5] has shown that the non-zero contribution existed at the three loop level [6] and they were estimated as $\sim 10^{-30} (e - \text{ cm})$, which is a negligible quantity. In this case, the complex phase, which is the source of CP violation, is coming from the complex Cabbibo-Kobayashi-Maskawa (CKM) matrix elements. Since the numerical results of fermion EDMs are tiny in the SM, there is enough reason to analyze these physical quantities in the framework of new physics beyond. There are many sources of CP violation in the models beyond the SM, such as multi Higgs doublet models (MHDM), supersymmetric model (SUSY), extra dimensions [7],..., etc.

The EDMs of quarks was calculated in the multi Higgs doublet models [8, 9, 10], including the two Higgs doublet model (2HDM). In these calculations $b$-quark and $t$-quark EDMs were obtained as $10^{-21} - 10^{-20} e - cm$ and $10^{-20} e - cm$. In [11], it was observed that the new contributions due to the $H^\pm$ particles vanished at the two loop order with the assumption that the CP violating effects came from only the CKM matrix elements and $H^\pm$ particles also mediated CP violation besides $W^\pm$ bosons. In [12], it was concluded that the enhancement of three orders of magnitude in the electric dipole form factor of the $b$ quark with respect to the prediction of 2HDM I and II was possible. [13] is devoted to the calculation of leading contribution to the EDM of the top quark in Higgs-boson-exchange models of CP nonconservation and the dipole moments were estimated of the order $10^{-20} (e - cm)$. In [14] lepton electric dipole moments in the supersymmetric seesaw model has been studied. In [15], the EDM of the electron has been predicted as $d_e$ as $10^{-32} e - cm$ using the experimental result of $d_\mu$ and the upper limit of $BR(\mu \rightarrow e\gamma)$. The work [16] is devoted to quark and lepton EDM moments in the framework of the SM with the inclusion of non-commutative geometry. In recent works, the EDMs of nuclei, deuteron, neutron and some atoms have been studied extensively [17].

Our work is devoted to the investigation of the fermion EDMs in the case that the CP
violating interactions are carried by complex Yukawa couplings appearing in the flavor changing (FC) neutral current vertices, in the model III version of the 2HDM, with the inclusion of the extra dimensions. There is an extensive work on the extra dimensions in the literature [18]-[28]. The main motivation of such dimensions is to find a solution to the gauge hierarchy problem of the SM. The effects of each extra dimension is felt with the production of Kaluza-Klein (KK) states of the fields, which are obtained after the compactification on a circle of radius R. The number $\frac{1}{R}$ is known as the compactification scale and the its size have been estimated in the range $200 – 500 \text{GeV}$, using electroweak precision measurements [18], the $B \to \bar{B}$-mixing [19], [20] and the flavor changing process $b \to s\gamma$ [21]. Furthermore, this size has been obtained as large as few hundred of GeV in several works [23, 24, 25, 26, 27]. If all the fields live in higher dimensions [18, 26], such extra dimensions are called as 'universal extra dimensions' (UED's), and in this case the extra dimensional momentum, and therefore the KK number at each vertex, is conserved. As a result of KK number conservation, the KK modes enter into the calculations as loop corrections. However, if some of the particles do not feel the extra dimensions and others do, namely non-universal extra dimension case, the coupling of two zero modes with the KK mode is switched on and the contributions of extra dimensions to the tree level processes become non-zero.

In the present work, we consider the effects of non-universal extra dimensions on the EDMs of fermions by assuming that the new Higgs doublet and the gauge sector feel the extra dimensions, however, the other SM fields do not feel and are confined on 4D brane. Notice that in the case of UED, where all the fields are accessible to the extra dimensions, there does not exist any new contribution to the fermion EDMs due to the KK modes of Higgs fields and fermions, at least in the one loop level. The higher dimensional effects on the EDMs of fermions are carried by the intermediate charged, $H^\pm$, and neutral Higgs, $h^0$ and $A^0$, fields with the vertices including ”two zero modes-KK mode”. We study those additional effects for one and two spatial extra dimensions.

In the numerical calculations, we observe that the $t$ quark and $b$ quark EDMs are sensitive to the extra dimension, especially the double one, however, in the case of charged lepton EDMs, this sensitivity is relatively weak. Therefore, the future accurate measurements of EDMs of fermions may be an effective tool to check the existence and the number of extra dimensions and the restriction of the compactification scale.

The paper is organized as follows: In Section 2, we present EDMs of fermions, t-quark, b-quark and charged leptons, in the model III version of the 2HDM with the inclusion of the
non-universal extra dimensions. Section 3 is devoted to discussion and our conclusions.

2 Electric dipole moments of fermions in the two Higgs doublet model with the inclusion of extra dimensions

The existence of the fermion EDM depends on the CP violating fermion-fermion-photon interaction. In the framework of the SM, the CP violation is carried by the complexity of the Cabbibo Cobayashi Maskawa (CKM) matrix elements and the estimated numerical values of EDMs of fermions are extremely small. This makes it charming to investigate new complex phases by considering the physics beyond the SM. The model III version of the 2HDM is one of the candidate since the FC neutral currents (FCNC) are permitted at tree level and the new Yukawa couplings can be complex in general. With the addition of spatial extra dimensions which are felt by the new Higgs doublet there appear additional contributions sensitive to the compactification scale $1/R$ where $R$ is the radius of the compactification. The Yukawa Lagrangian responsible for the EDM fermion in such a single extra dimension reads:

$$
\mathcal{L}_Y = \eta_{ij}^U \bar{Q}_{iL} \tilde{\phi}_1 U_{jR} + \eta_{ij}^D \bar{Q}_{iL} \phi_1 D_{jR} + \xi_{5ij}^U \bar{Q}_{iL} \phi_2 \left( \phi_2 |_{y=0} \right) U_{jR} + \xi_{5ij}^D \bar{Q}_{iL} \phi_2 \left( \phi_2 |_{y=0} \right) D_{jR} + \eta_{ij}^E \bar{l}_{iL} \phi_1 E_{jR} + \xi_{5ij}^E \bar{l}_{iL} \phi_2 \left( \phi_2 |_{y=0} \right) E_{jR} + \text{h.c. ,}
$$

where $y$ represents the extra dimension, $L$ and $R$ denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, $\phi_i$ for $i = 1, 2$, are the two scalar doublets, $\bar{Q}_{iL}$ are left handed quark doublets, $U_{jR}(D_{jR})$ are right handed up (down) quark singlets, $l_{iL}$ ($E_{jR}$) are lepton doublets (singlets), with family indices $i, j$. The Yukawa couplings $\xi_{5ij}^{U,D,E}$, complex in general, are dimensionful and rescaled to the ones in 4-dimension as $\xi_{5ij}^{U,D,E} = \sqrt{2} \pi \xi_{ij}^{U,D,E}$. Here we choose the Higgs doublets $\phi_1$ and $\phi_2$ as

$$
\phi_1 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v + H^0 \end{array} \right) \left( \sqrt{2} \chi^+ \\ i \chi^0 \right) ; \phi_2 = \frac{1}{\sqrt{2}} \left( \sqrt{2} H^+ \\ H_1 + i H_2 \right) .
$$

with the vacuum expectation values,

$$
< \phi_1 > = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v \end{array} \right) ; < \phi_2 > = 0 .
$$

and collect SM (new) particles in the first (second) doublet. Notice that $H_1$ and $H_2$ are the mass eigenstates $h^0$ and $A^0$ respectively since no mixing occurs between two CP-even neutral bosons $H^0$ and $h^0$ at tree level in our case.

In the following we use the dimensionful coupling $\xi_{N}^{U,D,E}$ with the definition $\xi_{N,ij}^{U,D,E} = \sqrt{\frac{G_F}{\sqrt{2}}} \xi_{N,ij}^{U,D,E}$ where $N$ denotes the word "neutral".
Since the new Higgs field $\phi_2$ is accessible to extra dimension, the compactification on a circle of radius $R$ results in the expansion of $\phi_2$ into its KK modes as
\[
\phi_2(x, y) = \frac{1}{\sqrt{2\pi R}} \left\{ \phi_2^{(0)}(x) + \sqrt{2} \sum_{n=1}^{\infty} \phi_2^{(n)}(x) \cos(ny/R) \right\}
\]
where $\phi_2^{(0)}(x)$ the 4-dimensional Higgs doublet which contains the charged Higgs boson $H^+$, the neutral CP even (odd) $H^1$ ($H^2$) Higgs bosons. The non-zero KK mode of Higgs doublet $\phi_2$ includes a charged Higgs of mass $\sqrt{m_{H^\pm}^2 + m_n^2}$, a neutral CP even Higgs of mass $\sqrt{m_{h_0}^2 + m_n^2}$, a neutral CP odd Higgs of mass $\sqrt{m_{A_0}^2 + m_n^2}$ where $m_n = n/R$ is the mass of $n$’th level KK particle. In addition to the new Higgs field the gauge fields feel the extra dimensions and therefore there exist their KK modes after the compactification, however they do not bring new contributions to the EDM of fermions.

The effective EDM interaction for a fermion $f$ is given by
\[
\mathcal{L}_{EDM} = i e d_f \bar{f} \gamma_5 \sigma^{\mu\nu} F_{\mu\nu},
\]
where $F_{\mu\nu}$ is the electromagnetic field tensor, ’$d_f$’ is EDM of the fermion and it is a real number by hermiticity. In Figs. 1 and 2 we present the 1-loop diagrams which contribute to the EDMs of fermions with the help of the complex Yukawa couplings. Here, the charged Higgs contributions play the important role in the case of the quarks. However, for charged leptons we assume that there is no CKM type lepton mixing matrix and, therefore, only the neutral Higgs part gives a contribution to their EDMs.

Now, we would like to present the EDMs of fermions ($t$ quark, $b$ quark and charged leptons) with the addition of a single non-universal extra dimension where the second Higgs doublet and gauge sector feels the extra dimensions and all other SM particles are restricted to the 4D brane. The top quark EDM reads:
\[
d_t = d_t^0 + 2 \sum_{n=1}^{\infty} \left( \int_0^1 dx \, d_t^{H^\pm} + d_t^{h_0} + d_t^{A_0} \right),
\]
where
\[
d_t^{H^\pm} = \frac{4 G_F}{\sqrt{2}} \frac{1}{48 \pi^2} \frac{m_b}{m_t^2} |V_{tb}|^2 \left\{ \frac{m_b}{r_b y_t + x^2 y_t - x (1 + y_t + r_b y_t)} \right\},
\]
\[
d_t^{h_0} = \frac{4 G_F}{\sqrt{2}} \frac{1}{16 \pi^2} \frac{1}{m_t} |V_{c_t}|^2 \frac{r_1}{\sqrt{r_1 (r_1 - 4)}} \left\{ \arctan \left( \frac{r_1}{\sqrt{r_1 (r_1 - 4)}} \right) - \arctan \left( \frac{r_1 - 2}{\sqrt{r_1 (r_1 - 4)}} \right) \right\} , \text{ for } r_1 < 4,
\]


d_t^0 = -\frac{4G_F}{\sqrt{2}} \frac{1}{16\pi^2} \frac{1}{m_t} Im(\xi_{N,tt}^U) Re(\xi_{N,tt}^U) Q_t \left\{ 1 - \frac{r_1 (r_1 - 2)}{\sqrt{r_1 (r_1 - 4)}} \ln \frac{\sqrt{r_1} - \sqrt{r_1 - 4}}{2} \right\} , \text{ for } r_1 > 4 , \\
d_t^{A0} = -d_t^{h0} (r_1 \rightarrow r_2) , \tag{7}

d_t^0 \text{ is the contribution without the extra dimension, namely } n = 0 \text{ case, and } r_b = \frac{m_b^2}{m_t^2} , r_1 = \frac{m_0^2 + n^2/R^2}{m_t^2} , r_2 = \frac{m_0^2 + n^2/R^2}{m_t^2} , y_t = \frac{m_0^2}{m_t^2 + n^2/R^2} , Q_b \text{ and } Q_t \text{ are charges of } b \text{ and } t \text{ quarks respectively.}

In eqs. (7), we take only internal } b \text{ (t)-quark contribution for charged (neutral) Higgs interactions. Here, we consider that the Yukawa couplings } \xi_{N,tt}^U, i = u, c, \text{ and } \xi_{N,bj}^D, j = d, s \text{ are negligible compared to } \bar{\xi}_{N,tt}^U \text{ and } \bar{\xi}_{N,bb}^D \text{ (see [15]). Furthermore we use the parametrization}

\xi_{N,tt}^U = |\bar{\xi}_{N,tt}^U| e^{i\theta_t} , \tag{8}
\xi_{N,bb}^D = |\bar{\xi}_{N,bb}^D| e^{i\theta_b} ,

\text{for the complex Yukawa couplings } \bar{\xi}_{N,tt}^U \text{ and } \bar{\xi}_{N,bb}^D. \text{ Notice that the neutral Higgs contributions to the } d_t \text{ are switched on if we choose the coupling } \bar{\xi}_{N,tt}^U \text{ complex. In our numerical calculations for } d_t \text{ we choose this coupling complex to determine the strength of the neutral Higgs contributions and observe that they are small compared to the charged Higgs ones.}

\text{Similarly the } b \text{ quark EDM [9] reads:}

d_b = d_b^0 + 2 \sum_{n=1}^{\infty} (d_b^{H\pm} + d_b^{h0} + d_b^{A0}) , \tag{9}

\text{where } d_b^{H\pm} , d_b^{h0} \text{ and } d_b^{A0} \text{ are}

d_b^{H\pm} = -\frac{4G_F}{\sqrt{2}} \frac{1}{32\pi^2} \frac{1}{m_t} \bar{\xi}_{N,tt}^U \xi_{N,tt}^D |V_{tb}|^2 y_t ((-1 + Q_t (-3 + y_t) - y_t) (y_t - 1) + 2 (Q_t + y_t) \ln y_t) , \\
d_b^{h0} = -\frac{4G_F}{\sqrt{2}} \frac{1}{16\pi^2} \frac{Q_b}{m_b} \bar{\xi}_{N,bb}^D \xi_{N,bb}^D (1 - \frac{r_1 (r_1 - 2)}{\sqrt{r_1 (r_1 - 4)}} \ln \frac{\sqrt{r_1} - \sqrt{r_1 - 4}}{2} - \frac{1}{2} r_1 \ln r_1) , \\
d_b^{A0} = -d_b^{h0} (r_1 \rightarrow r_2) . \tag{10}

\text{where } d_b^0 \text{ is the contribution without the extra dimension. In eq. [10], we take into account only internal } t \text{-quark contribution for charged Higgs boson and internal } b \text{-quark contribution for neutral Higgs interactions by following the previous assumption. Furthermore, we choose } \xi_{N,tt}^U \text{ real and } \xi_{N,bb}^D \text{ complex since the } d_b \text{ is sensitive (not sensitive) to the imaginary part of the coupling } \bar{\xi}_{N,bb}^D (\xi_{N,tt}^U).
Finally, we will present the charged lepton EDMs with the addition of non-universal extra dimensions. Since there is no CKM type lepton mixing matrix according to our assumption only the neutral Higgs part gives a contribution to their EDMs and l-lepton EDM \(d_l\) \((l = e, \mu, \tau)\) can be calculated as a sum of contributions coming from neutral Higgs bosons \(h_0\) and \(A_0\) \cite{13},

\[
d_l = -\frac{iG_F}{\sqrt{2}} \frac{e}{32\pi^2} \frac{Q_l}{m_l} \left(\left(\bar{\xi}_{N,lr}^D\right)^2 - (\bar{\xi}_{N,rl}^D)^2\right) \left((F_1(y_{h_0}) - F_1(y_{A_0})) + 2 \sum_{n=1}^{\infty} (F_1(y_{h_0}^n) - F_1(y_{A_0}^n))\right),
\]

for \(l = e, \mu\) and

\[
d_\tau = -\frac{iG_F}{\sqrt{2}} \frac{e}{32\pi^2} \frac{Q_\tau}{m_\tau} \left(\left(\bar{\xi}_{N,\tau r}^D\right)^2 - (\bar{\xi}_{N,\tau l}^D)^2\right) \left((F_2(r_{h_0}) - F_2(r_{A_0})) + 2 \sum_{n=1}^{\infty} (F_2(r_{h_0}^n) - F_2(r_{A_0}^n))\right)
- \frac{Q_\mu}{m_\tau} \left(\left(\bar{\xi}_{N,\mu r}^D\right)^2 - (\bar{\xi}_{N,\mu l}^D)^2\right) \left((r_{h_0} ln(z_{h_0}) - r_{A_0} ln(z_{A_0}))
+ 2 \sum_{n=1}^{\infty} (r_{h_0}^n ln(z_{h_0}^n) - r_{A_0}^n ln(z_{A_0}^n))\right),
\]

(12)

where the functions \(F_1(w)\), \(F_2(w)\) and \(F_3(w)\) are

\[
F_1(w) = \frac{w(3 - 4w + w^2 + 2lnw)}{(-1 + w)^3},
F_2(w) = \frac{wlnw + \frac{2(-2 + w)ln(\frac{\sqrt{w} - \sqrt{w - 4}}{\sqrt{w}})}{\sqrt{w}(w - 4)}}{wlnw + \frac{2(-2 + w)ln(\frac{\sqrt{w} - \sqrt{w - 4}}{\sqrt{w}})}{\sqrt{w}(w - 4)}}.
\]

(13)

Here \(y_H^n = \frac{m_H}{m_H + n^2/R^2}\), \(r_H^n = \frac{1}{y_H^n}\) and \(z_H^n = \frac{m_H}{m_H + n^2/R^2}\), \(y_H = y_H^0\), \(r_H = r_H^0\) and \(z_H = z_H^0\), \(Q_\tau\) and \(Q_\mu\) are charges of \(\tau\) and \(\mu\) leptons respectively. In eq. (11) we take into account only internal \(\tau\)-lepton contribution respecting our assumption that the Yukawa couplings \(\bar{\xi}_{N,i,j}^D\), \(i, j = e, \mu, \tau\) are small compared to \(\bar{\xi}_{N,\tau i}^D i = e, \mu, \tau\) due to the possible proportionality of the Yukawa couplings to the masses of leptons in the vertices. In eq. (12) we present also the internal \(\mu\)-lepton contribution, which can be neglected numerically. Notice that, we make our calculations in arbitrary photon four momentum square \(q^2\) and take \(q^2 = 0\) at the end. We used the parametrization

\[
\bar{\xi}_{N,\tau l}^D = \bar{\xi}_{N,\tau l}^D e^{i\theta_l},
\]

(14)

and the Yukawa factors in eqs. (11) and (12) can be written as

\[
\left((\bar{\xi}_{N,lr}^D)^2 - (\bar{\xi}_{N,rl}^D)^2\right) = -2isin2\theta_l |\bar{\xi}_{N,rl}^D|^2
\]

(15)

where \(l = e, \mu, \tau\). Here \(\theta_l\) is CP violating parameters which is the source of the lepton EDM.
In the case two extra spatial dimensions which are felt by the second Higgs doublet $\phi_2$ the compactification of the extra dimensions is done on a torus $S^1 \times S^1$ and the doublet $\phi_2$ can be expanded into its KK modes as

$$
\phi_2(x, y, z) = \frac{1}{2\pi R} \left\{ \phi_2^{(0,0)}(x) + 2 \sum_{n,r} \phi_2^{(n,r)}(x) \cos(ny/R + rz/R) \right\}.
$$

(16)

Here the indices $n$ and $r$ are positive integers including zero but both are not zero at the same time and each circle is considered the same radius $R$. Furthermore, $\phi_2^{(0,0)}(x)$ is the 4-dimensional Higgs doublet including the charged Higgs boson $H^\pm$, the neutral CP even (odd) $h^0$ ($A^0$) Higgs bosons. The non-zero KK mode $\phi_2^{(n,r)}(x)$ ($n$ or $r$ is different than 0) of Higgs doublet $\phi_2$ contains a charged Higgs of mass $\sqrt{m_{H^\pm}^2 + m_n^2 + m_r^2}$, a neutral CP even Higgs of mass $\sqrt{m_{h^0}^2 + m_n^2 + m_r^2}$, a neutral CP odd Higgs of mass $\sqrt{m_{A^0}^2 + m_n^2 + m_r^2}$ where the mass terms $m_n = n/R$ and $m_r = r/R$ exist due to the compactification.

3 Discussion

In this section we analyze the effects non-universal extra dimensions on the EDMs of fermions, including top quark, bottom quark and charged leptons. Since the EDM interaction is CP violating, one needs a CP violating phase and we consider the complex Yukawa couplings appearing in the FCNC at tree level in the framework of the model III.

For the top quark EDM, the CP violating parameters are $\sin\theta_b$ and $\sin\theta_t$ play the main role (see eq. (8)). In this case, the neutral Higgs boson contribution exists if the parameter $\sin\theta_t$ is non-zero, namely, the Yukawa coupling drives $t - t - h^0(A^0)$ interaction has a complex phase. On the other hand the bottom quark EDM appears with the non-zero parameter $\sin\theta_b$, which is the imaginary part of the Yukawa coupling responsible for the $b - b - h^0(A^0)$ interaction. These CP violating parameters, the Yukawa couplings, $\tilde{\xi}_N^{U(D)}$, and the masses of new Higgs bosons, $H^\pm$, $h^0$ and $A^0$ are the free parameters of the model used and they should be restricted using the experimental measurements.

In our numerical calculations we neglect all the quark sector Yukawa couplings except $\tilde{\xi}_N^{U(D)}$ and $\tilde{\xi}_N^{U(D)}$ since they are negligible due to their light flavor contents, by our assumption, similar to the Cheng-Sher scenario [29]. Furthermore, for the couplings $\tilde{\xi}_N^{U(D)}$, $\tilde{\xi}_N^{U(D)}$ and the CP violating parameters $\sin\theta_t$ and $\sin\theta_b$, we use the constraint region which is obtained by restricting the Wilson coefficient $C_t^{ij}$ in the region $0.257 \leq |C_t^{ij}| \leq 0.439$. Here upper and lower limits were calculated using the CLEO measurement [30]

$$
Br(B \rightarrow X_s\gamma) = (3.15 \pm 0.35 \pm 0.32) \times 10^{-4}.
$$

(17)
and all possible uncertainties in the calculation of $C_{7}^{eff}$ [31]. Notice that $C_{7}^{eff}$ is the effective coefficient of the operator $O_{7} = \frac{e}{16\pi^{2}} s_{a}^{\alpha} (m_{b} R + m_{s} L) b_{a} F^{\mu\nu}$ (see [31] and references therein). We respect also the additional constraint for the angle $\theta_{t}$ and $\theta_{b}$, comes from the experimental upper limit of neutron electric dipole moment, $d_{n} < 10^{-25} \text{e}\cdot\text{cm}$, which leads to $\frac{1}{m_{t} m_{b}} \text{Im} (\bar{\xi}_{U}^{N,tt} \bar{\xi}_{D}^{N,bb} < 1.0$ for $M_{H^{\pm}} \approx 200 \text{GeV}$ [32]. In our numerical calculations we choose the upper limit for $C_{7}^{eff} > 0$, fix $\bar{\xi}_{D}^{N,bb} = 30 m_{b}$ and take $\bar{\xi}_{U}^{N,tt} \sim 35 \text{GeV}$, respecting the constraints mentioned.

The charged lepton EDM depends on the leptonic complex Yukawa couplings $\bar{\xi}_{N,ij}^{D, i, j = e, \mu, \tau}$ which are another set of free parameters in the model III. Similar to the previous assumptions, we consider the Yukawa couplings $\bar{\xi}_{N,ij}^{D, i, j = e, \mu}$, as smaller compared to $\bar{\xi}_{N,\tau i}^{D, i = e, \mu, \tau}$ and we assume that $\bar{\xi}_{N,ij}^{D}$ is symmetric with respect to the indices $i$ and $j$. For the coupling $\bar{\xi}_{N,\tau\mu}^{E}$ we use the restriction (see [33]) coming from the experimental uncertainty, $10^{-9}$, in the measurement of the muon anomalous magnetic moment [34]. Notice that the predicted upper limit for the coupling $\bar{\xi}_{N,\tau\mu}^{E}$ is $30 \text{GeV}$ in [33]. For the coupling $\bar{\xi}_{N,\tau\tau}^{E}$ there is no stringent prediction and we take an intermediate value which is greater than the coupling $\bar{\xi}_{N,\tau\mu}^{E}$.

The inclusion of the spatial extra dimensions that are felt by the new Higgs doublet results in the new contributions, emerging from the KK excitations of the new charged and neutral Higgs fields, to the EDMs of fermions. The new vertices appearing in the calculation of EDMs are coming from the fermion-fermion-KK Higgs interaction where the KK number is not conserved in contrast to the case of UED [18, 26]. The compactification of extra dimensions to the torus brings new parameter, called the compactification radius $R$. Here $R$ is the size of the extra dimension and it needs to be restricted. The lower bound for inverse of the compactification radius is estimated as $\sim 300 \text{GeV}$ [18].

In Fig. 3, we plot EDM $d_{t}$ with respect to compactification scale $1/R$ for $m_{H^{\pm}} = 400 \text{GeV}$, in Fig. 3 we plot EDM $d_{t}$ with respect to compactification scale $1/R$ for $m_{H^{\pm}} = 400 \text{GeV}$,
$m_{h^0} = 85\,GeV, \ m_{A^0} = 90\,GeV$, the small value of $\sin \theta_t = 0.1$ and the intermediate value of $\sin \theta_b = 0.5$. Here the solid (dashed) line represents charged Higgs (total) contribution to the $t$-quark EDM without extra dimension, solid (dashed) curve represents charged Higgs (total) contribution of a single extra dimension to the $t$-quark EDM and dotted (double dashed) curve represents charged Higgs (total) total contribution to the $t$-quark EDM. This figure shows that the contribution of the extra dimensions are, especially for the charged Higgs part, larger than the one without the extra dimension, for $1/R \leq 600\,GeV$. In fact, in the case of vanishing $\sin \theta_t$ the neutral Higgs bosons $h^0$ and $A^0$ and their KK modes do not have any contribution to the EDM of top quark and for nonzero $\sin \theta_t$, they are suppressed even with the addition of extra dimensions. The EDM of top quark reaches the numerical value $3.0 \times 10^{-21} \,(e - cm)$ for $1/R \sim 300\,GeV$ with the inclusion of a single extra dimension and it is almost six times larger than the one obtained without extra dimensions. This is an interesting result since the EDM of top quark is sensitive to a single extra dimension and it may ensure a powerful information about the existence of extra dimensions and the restriction of the compactification scale, with the forthcoming experimental measurements.

Fig. 4 is devoted to the compactification scale $1/R$ dependence of the EDM $d_b$ for $m_{H^\pm} = 400\,GeV, \ m_{h^0} = 85\,GeV, \ m_{A^0} = 90\,GeV$, and the intermediate value of $\sin \theta_b = 0.5$. Here the solid (dashed) line represents charged Higgs (total) contribution to the $b$ quark EDM without extra dimension, solid (dashed) curve represents charged Higgs (total) single extra dimension contribution to the $b$ quark EDM and dotted (double dashed) curve represents charged Higgs (total) total contribution to the $b$ quark EDM. We see that the contribution of the extra dimensions is larger than the one without the extra dimension, for $1/R \leq 600\,GeV$. Here the neutral Higgs contribution is suppressed similar to the top quark EDM and we do not present in this figure. The EDM of $b$ quark is $5.0 \times 10^{-20} \,(e - cm)$ for $1/R \sim 300\,GeV$ in the case of the inclusion of a single extra dimension and it is more than one order larger compared to the one without extra dimensions. It is shown that the EDM of $b$ quark is sensitive to a single extra dimension and this physical quantity is a candidate to test the extra dimensions and to obtain stringent restrictions for the compactification scale, with the accurate future experimental measurements.

Fig. 5 shows the compactification scale $1/R$ dependence of $d_t$ and $d_b$ for $m_{H^\pm} = 400\,GeV, \ m_{h^0} = 85\,GeV, \ m_{A^0} = 90\,GeV$, and the intermediate value of $\sin \theta_b = 0.5$ in the case of non-universal two extra spatial dimensions. Here the solid (dashed) line represents $d_t \ (d_b)$ without extra dimension, solid (dashed) curve represents $d_t \ (d_b)$ with the inclusion of two extra
dimensions. It is realized that the EDM of $t$ ($b$) quark is much more sensitive to two extra dimensions and it is two orders (almost three orders) larger compared to the one without extra dimensions, for $1/R \sim 300 \text{GeV}$. This is a valuable information to test the compactification scale and even the number of extra dimensions.

At this stage, we start to analyze the effects of the extra dimensions on the charged lepton EDMs. Here we make analysis for $\mu$ and $\tau$ lepton and choose the numerical values $\xi^{E}_{N,\mu} = 10 \text{GeV}$, $\xi^{E}_{N,\tau} = 50 \text{GeV}$, $\sin\theta_{\tau} = \sin\theta_{\mu} = 0.5$. The assumption that there is no CKM type matrix in the leptonic sector leads to the fact that there exist only neutral Higgs contributions in the expression of EDM of charged leptons.

In Fig. 6 (7) we present the compactification scale $1/R$ dependence of $d_{\mu}$ ($d_{\tau}$) for $m_{h^0} = 85 \text{GeV}$, $m_{A^0} = 90 \text{GeV}$. Here the solid-dashed-small dashed lines represent $d_{\mu}$ ($d_{\tau}$) without-with a single-with two extra dimensions. It is observed that $d_{\mu}$ ($d_{\tau}$) is weakly sensitive to the extra dimensions for $1/R \geq 300 \text{GeV}$ and the contribution due to the two extra spatial dimension is almost 10% for $1/R \sim 300 \text{GeV}$.

Now we would like to summarize our results:

- The EDM of top quark is sensitive to the extra dimensions, especially to two extra dimensions. The inclusion of a single (double) extra dimension causes the EDM to enhance to the numerical value $3.0 \times 10^{-21} (e - cm)$ ($10^{-19} (e - cm)$) for $1/R \sim 300 \text{GeV}$ and it is almost six times (two orders) larger than the one without extra dimensions. This is informative to check the existence of extra dimensions and the restriction of the compactification scale, with the help of the forthcoming experimental measurements.

- The EDM of $b$ quark is also sensitive to the extra dimensions and it reaches the numerical value $5.0 \times 10^{-20} (e - cm)$ ($5.0 \times 10^{-18} (e - cm)$) for $1/R \sim 300 \text{GeV}$ in the case of the inclusion of a single (double) extra dimension. The addition of extra dimensions results in the enhancement of EDM of one order (three orders) compared to the one without extra dimensions, for $1/R \sim 300 \text{GeV}$. This is a strong sensitivity and it can be used even to test the number of extra dimensions, besides the restrictions for the compactification scale.

- $d_{\mu}$ ($d_{\tau}$) is weakly sensitive to the extra dimensions for $1/R \geq 300 \text{GeV}$ and the contribution due to the two extra spatial dimension is almost 10% for $1/R \sim 300 \text{GeV}$.

Therefore, the experimental investigation of the EDMs of the top quark and bottom quark can give powerful information about the existence of extra dimensions.
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Figure 1: One loop diagrams contribute to EDM of top and bottom quarks due to charged Higgs boson $H^\pm$ (a and b) and neutral Higgs bosons $h^0$, $A^0$ (c) in the 2HDM, including KK modes. Wavy lines represent the electromagnetic field and dashed lines the Higgs field.
Figure 2: One loop diagrams contribute to EDM of charged leptons due to neutral Higgs bosons $h^0$, $A^0$ in the 2HDM, including KK modes. Wavy lines represent the electromagnetic field and dashed lines the Higgs field where $l_{1(i)} = e, \mu, \tau$.

Figure 3: $d_t$ with respect to compactification scale $1/R$ for $m_{H^\pm} = 400 \text{GeV}$, $m_{h^0} = 85 \text{GeV}$, $m_{A^0} = 90 \text{GeV}$, $\sin \theta_t = 0.1$ and $\sin \theta_b = 0.5$, in the case of a single extra dimension. Here the solid (dashed) line represents charged Higgs (total) contribution to $d_t$ without extra dimension, solid (dashed) curve represents charged Higgs (total) single extra dimension contribution to $d_t$ and dotted (double dashed) curve represents charged Higgs (total) total contribution to the $d_t$. 

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Figure 4: The same as Fig. 3 but for $d_b$ and $\sin \theta_b = 0.5$.

Figure 5: The compactification scale $1/R$ dependence of $d_t$ and $d_b$ for $m_{H^\pm} = 400 \, GeV$, $m_{h^0} = 85 \, GeV$, $m_{A^0} = 90 \, GeV$, and the intermediate value of $\sin \theta_b = 0.5$ in the case of non-universal two extra spatial dimensions. Here the solid (dashed) line represents $d_t$ ($d_b$) without extra dimension, solid (dashed) curve represents $d_t$ ($d_b$) with the inclusion of two extra dimensions.
Figure 6: The compactification scale $1/R$ dependence of $d_{\mu}$ for $m_{h^0} = 85\,\text{GeV}$, $m_{A^0} = 90\,\text{GeV}$. Here the solid-dashed-small dashed lines represent $d_{\mu}$ without-with a single-with two extra dimensions.

Figure 7: The same as Fig. 6 but for $d_{\tau}$.  
