Probabilistic vortex crossing criterion for superconducting nanowire single-photon detectors

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Superconducting nanowire single-photon detectors have emerged as a promising technology for quantum metrology from the mid-infrared to ultra-violet frequencies. Despite the recent experimental successes, a predictive model to describe the detection event in these detectors is needed to optimize the detection metrics. Here, we propose a probabilistic criterion for single-photon detection based on single-vortex (flux quanta) crossing the width of the nanowire. Our finite-difference calculations demonstrate that a change in the bias current distribution as a result of the photon absorption significantly increases the probability of single-vortex crossing even if the vortex potential barrier has not vanished completely. We estimate the instrument response function and show that the timing uncertainty of this vortex tunneling process corresponds to a fundamental limit in timing jitter of the click event. We demonstrate a trade-space between the timing jitter, quantum efficiency, and dark count rate in TaN, WSi, and NbN superconducting nanowires at different experimental conditions. Our detection model can also explain the experimental observation of exponential decrease in the quantum efficiency of SNSPDs at lower energies. This leads to a pulse-width dependency in the quantum efficiency, and it can be further used as an experimental test to compare across different detection models.

I. INTRODUCTION

Advancements in quantum technologies strongly depends on improvement in the detection of light at the single-photon level. This requires near-unity quantum efficiency, sub-picosecond timing uncertainty (timing jitter), sub-milihertz dark count rate, large bandwidth, and fast reset time [1]. Superconducting nanowire single photon detectors (SSPDs or SNSPDs) are highly promising detectors in a broad range of frequencies from mid-infrared to ultraviolet [2–7] with near unity quantum efficiency [8], picosecond-scale timing jitter [9–12], fast reset time [13], and milihertz dark count rate [14, 15]. They are composed of a thin superconducting nanowire which is biased slightly below the superconducting critical current. Photon absorption triggers a phase transition giving rise to generation of a voltage pulse which is measured by a readout circuit connected to the nanowire.

Owing to the experimental progress on reducing the amplification noises and the uncertainty of the photon absorption location, recent breakthrough results have shown timing jitter below 10 ps [9–11, 16]. Hence, the response function of SNSPDs to a single-photon has approached its intrinsic response limit which only depends on the microscopic mechanism of light-matter interaction in nanowires. To further improve the performance of these detectors, it is required to understand the microscopic mechanism and the trade-space of the photon detection event in these detectors.

Over the past two decades, several important detection models have been proposed to explain the microscopic mechanism of the formation of the first resistive region in SNSPDs [2, 3, 17–20]. In the simplest model, it is assumed that the energy of the absorbed photon increases

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the temperature at the absorption site leading to the nucleation of a hot-spot which causes the current to be directed to the sides [2, 21, 22]. This may cause the current at the edge to surpass the critical depairing current causing formation of a normal conducting region across the width. In another model, the depletion of superconducting electrons around the absorption site is responsible for the formation of the resistive region [23]. Recently, some models have suggested that the motion of vortices or vortex-antivortex pairs can also induce a phase transition at a lower applied bias current [17, 20, 24–26]. Although each of these models explain some macroscopic behaviors of SNSPDs, none of the current models can explain or predict all experimental observations. Hence, a robust model is needed to address the fundamental limits of SNSPDs and the trade-off between the quantum efficiency, timing jitter, and dark counts.

In this paper, we propose a probabilistic criterion for single-photon detection corresponding to the single-vortex crossing from one edge of the nanowire to the other edge. First, we numerically calculate the time-dependent current distribution after the photon absorption and its effect on the vortex potential. We propose that due to the change in the distribution of the superconducting electrons, the probability of the vortex crossing is significantly enhanced even if the vortex potential barrier has not vanished completely. Then, we define the detection probability based on the probability of the single-vortex crossing, because the energy released by one vortex moving across the width is enough to induce a phase transition in the superconducting nanowire. We show that the probabilistic behavior of the single-vortex crossing results in an intrinsic timing jitter on the click event. This timing jitter cannot be eliminated even if the position of photon absorption is known, however, it can be reduced by engineering the structure and the experimental conditions at the cost of a degradation of the quantum efficiency and/or an increase in the dark count rate. Finally, we calculate the quantum efficiency spectrum and show that the quantum efficiency does not suddenly drop to zero when the photon energy is below a threshold. We propose that the response of the detector to the photon pulse-width can be different for the various detection models. Moreover, the quantum efficiency predicted by our model is strongly dependent on the pulse-width. This effect has not been predicted by the previous detection models. Our work unifies previously known ideas of vortex crossing phenomenon with the POVM approach of quantum optics to propose a probabilistic detection criterion for SNSPDs. We propose some observable quantities which can be used to experimentally verify the validity of our probabilistic model.

II. DETECTION MECHANISM

Detection mechanism in SNSPDs consists of three steps: (a) photon absorption and breaking the superconducting electron pairs (known as Cooper pairs) to quasi-particles (QPs) leading to formation of a hot-spot; (b) as a result of the depletion of the Cooper pairs, the superconducting order parameter is suppressed. This causes the current density at the absorption location to be reduced and directed to the sides as illustrated in Fig. 1a; (c) the change in the Cooper pairs and current density reduces the vortex potential barrier and vortices can move across the nanowire and release a measurable voltage pulse (Fig. 1b).

These three steps have been quantitatively described in the appendix. Our finite-difference calculations of QPs...
distribution based on the diffusion model [20] for a TaN SNSPD is illustrated in Figs. 2a and 2b at \( t = 1 \) ps and \( t = 5 \) ps, respectively. We assume a photon with the energy of \( h\nu = 1.5 \text{ eV} \) falls at the center of the SNSPD at \( t = 0 \). The width and the length of the nanowire are \( W = 100 \text{ nm} \) and \( L = 1000 \text{ nm} \), respectively. Figures 2c and 2d display the numerical calculation of the current density normalized to the bias current. It is seen that due to the hot-spot formation at the center, the current is directed to the side-walls of the nanowire, and as a result, the vortex potential barrier is reduced as shown in Fig. 3. If the bias current or the photon energy are high enough, the potential barrier can be vanished completely.

After the single-photon transduction, several processes compete with each other to form the initial normal conducting cross-section. Depending on which one occurs first, different detection models have been proposed. In hot-spot model, it is assumed that the formation of hot-spot is responsible for the phase transition [2, 22]. Nucleation of the hot-spot causes the bias current to be directed to the side-walls. If the current density at the edge surpasses the despairing critical current \( I_{\text{edge}} \geq I_{\text{c,dep}} \), it induces a phase transition to the normal conducting state at the edge and the normal conducting region expands across the width.

In QP model, there is no need to destroy the superconductivity by surpassing the critical current [23]. If the Cooper pairs are depleted inside a volume with a thickness of at least one coherence length \( (\xi \text{-slab}) \), the phase coherence is destroyed which results in a phase transition. This requires the number of QPs inside the \( \xi \text{-slab} \) \( (N_{\text{QP}}^{\xi \text{-slab}}) \) to exceed the number of the superconducting electrons inside the \( \xi \text{-slab} \): \( N_{\text{QP}}^{\xi \text{-slab}} \geq \frac{1}{I_b/I_{\text{c,dep}}} N_{\text{sc}} \). where \( N_{\text{sc}} \) is the initial superconducting-electron number inside the slab before applying the bias current \( (I_b) \).

Vortices can also be responsible for the trigger of a single-photon induced phase transition in SNSPDs. If the photon transduction causes the vortex potential barrier \( (U_v) \) to vanish, vortices move across the width and induce a phase transition [19, 20]. In the next section, we show that even if the barrier has not completely vanished and the kinetic energy of the vortices is not enough to surmount the barrier, there is a considerable probability of single-vortex crossing. This quantum tunneling process causes a new source of timing jitter for the detection event.

### III. QUANTUM TIMING JITTER

According to the most accepted quantum measurement theory, positive-operator-valued measure (POVM), a quantum detector can be regarded as a black box. Each of its outcomes is represented by a positive Hermitian operator \( \hat{\Pi}_m \) with non-negative real eigenvalues. The probability that the \( m \)th outcome occurs in experiments is given by \( P_m = \text{Tr}[\rho \hat{\Pi}_m] \), where \( \rho \) is the initial state of the quantum object to be detected, such as the state of the incident single-photon pulse. The completeness condition, \( \sum_m \hat{\Pi}_m = I \) \((I \text{ is the identity operator})\), expresses the fact that the probabilities sum to one: \( \sum_m P_m = 1 \). For a non-photon-number-resolving photon detector, there are only two possible outcomes: clicking and non-clicking, characterized by \( \hat{\Pi}_c \) and \( \hat{\Pi}_n \) (thus \( m = c, n \)), respectively. The clicking probability, \( P_c = \text{Tr}[\rho \hat{\Pi}_c] = P_1 + P_0 \), contains two parts: the single-photon induced clicking probability, \( P_1 \), characterizing the quantum efficiency of the detector and the dark counting part, \( P_0 \). Recently, the figures of merit and time-dependent spectrum of a single photon in terms of POVMs have been exploited [28, 29]. In the following, we present our microscopic calculation of \( P_1 \) and \( P_0 \) based on the single-vortex crossing model. Especially, we introduce the quantum timing jitter in the amplification process, which has not been incorporated into current
Figure 4. Vortex crossing rate and probability. a, Single-vortex crossing rate as a function of time for NbN, TaN, and WSi SNSPDs. The bias current has been set to achieve a single photon detection probability of 0.5 for a photon energy of $h\nu = 1.5$ eV; $T = 0.6$ °K and $W = 100$ nm. Material parameters are derived from ref. [27]. The enhancement in the vortex crossing rate is as a result of the suppression of the potential barrier. The probability of vortex crossing at the maximum rate is higher. However, there is considerable uncertainty in the vortex crossing time which results in a timing jitter in detection event. b, The evolution of vortex crossing probability as the vortex crossing rate changes. There is a steep change in the probability as the crossing rate goes up.

POVM theory.

Even if there is no photon and the bias current is below the vortex critical current $I_{v,c}$, a vortex can be thermally excited and escape the potential barrier saddle point to form a normal conducting belt [30]. This false-count rate is known as dark-count rate which deteriorates the performance of a single-photon detector [1]. The time-dependent rate of the vortex crossing can be described as:

$$\Gamma_v(t) = \alpha_v I_b \exp(-U_{v,max}(t)/k_B T),$$

(1)

where $k_B$ is the Boltzmann constant and $\alpha_v$ is a constant which is measured experimentally [30]. $U_{v,max}(t)$ is the maximum of the potential barrier for vortex crossing which changes with time after the single photon absorption event. As a result of the change in the vortex potential barrier after the photon absorption, the vortex crossing rate increases exponentially. Figure 4a shows the vortex crossing rate as a function of time after the photon absorption for three different materials. The rate at

Figure 5. Instrument response function. Estimated distribution of number of counts registered on the detector as a function of the delay after the photon absorption when the incoming single-photon energy is $h\nu = 1.5$ eV (blue) and $h\nu = 0.75$ eV (red); $T = 0.6$ °K and $W = 100$ nm. There is a latency between the photon absorption and the click registration and the uncertainty in the latency causes a timing jitter ($t_j$) in the detector. It is seen that the latency and the timing jitter can be reduced if the bias current becomes very close to the switching current ($I_{SW}$) or the photon energy increases. $I_{SW}$ is defined as the minimum bias current at which the detector clicks in the time-bin of the single-photon arrival even if the photon is not absorbed.
Characterizing the probability of \( n_v \) vortex crossing the nanowire during the time interval \([t_0, t]\). Here, the time-dependent function \( \bar{n}_v(t) = \int_{t_0}^{t} \Gamma_v(t') dt' \) is the mean number of vortices crossing the nanowire. Hence, we can define the single-photon detection probability \( P_1 \) after the single-photon absorption \((t_0 = 0)\) and before time \( t \) as the probability of crossing of at least one vortex as:

\[
P_1(t) = 1 - p(0,t) = 1 - \exp \left[-\int_{t_0=0}^{t} \Gamma_v(t') dt' \right]. \tag{3}\]

As seen in Fig. 4b, the detection probability increases rapidly around the crossing rate maximum. The time derivative of \( P_1(t) \) is proportional to the single-photon count rate (also known as instrument response function) measured in experiments \([9]\). If the detection efficiency is low, the photon count rate is approximately the same as \( \Gamma_v(t) \). The rise time of the quantum efficiency is not instantaneous due to the finite diffusion speed of the QPs and the hot-spot formation. Hence, there is a fundamental latency and uncertainty between the time of photon absorption and the quantum vortex tunneling process. This causes a quantum timing jitter \((t_j)\) on photon detection event as illustrated in Fig. 5.

This type of timing jitter is because of the probabilistic crossing of vortices \([34, 35]\) and unlike the geometrical and spatial sources of timing jitter, cannot be eliminated if the uncertainty in the position of the transduction event is reduced \([36–41]\). However, it can be controlled by engineering the structure and controlling the experimental conditions. It is seen in Eq. (1) that as the temperature decreases, the change in the vortex crossing rate becomes sharper. This causes the intrinsic timing jitter to reduce as shown in Fig. 6a. However, the temperature cannot go to zero because QPs do not diffuse at zero temperature. Note that the vortex potential is proportional to the characteristic vortex energy, \( \varepsilon_0 \). Thus, smaller vortex energy in causes a slower change in the vortex crossing rate, similar to the effect of rising the temperature. Hence, although the hot-spot formation and relaxation happens faster in WSi due to the smaller bandgap and faster QP diffusion \([27]\), the timing jitter in WSi is comparable with that in TaN because of the smaller \( \varepsilon_0 \) in WSi nanowires.

Reducing the width of the nanowire results in a faster distribution of the hot-spot across the nanowire width. This leads to a sudden change in the vortex potential barrier, and as a result, the timing jitter decreases considerably as shown in Fig. 6b. Reducing the width helps reducing the geometrical timing jitter as well \([36]\), however, at the cost of a decrease in the transduction efficiency of the device.

Increasing the bias current reduces the vortex potential and increases the vortex crossing rate. This causes not only an increase in the quantum efficiency \((P_1)\) \([42, 43]\), but also an increase in the dark count probability \((P_0)\) as shown in Fig. 7. \( P_0 \) is defined as the probability of the click while there is no interaction between the photon and

\[
p(n_v, t) = \frac{\bar{n}_v(t)}{n_v!} e^{-\bar{n}_v(t)}, \tag{2}\]

Figure 6. **Timing jitter corresponding to vortex crossing.** Timing jitter in NbN, TaN, and WSi SNSPDs as a function of (a) temperature and (b) nanowire width. \( h\nu = 1.5 \text{ eV} \). Decreasing the temperature results in a sharper change in the vortex crossing rate. Hence, the uncertainty of vortex crossing reduces. Reducing the width of the nanowire causes the QPs to distribute faster across the width of the nanowire, and as a result, the potential barrier reduces rapidly.

\( t = 0 \) corresponds to the dark-count rate \([30–33]\). However, the rate is enhanced several orders of magnitude when the potential barrier reaches its minimum. This enhancement during the multiplication and recombination of QPs might be enough to significantly change the probability of vortex escaping the barrier. It is seen that the shape of the vortex crossing rate is different in different superconducting materials depending on the number of QPs generated and how fast they get multiplied, diffused across the width, and recombined. Since the crossing of vortices occurs independent of the other vortices, the crossing events can be regarded as a Poisson process with distribution function,
the detector in the time bin of the photon arrival. $I_{SW}$ is defined as the minimum bias current which is required for at least one vortex to escape the barrier in the time bin of the photon arrival. Note that $I_{SW}$ is smaller than $I_{c,v}$. Increasing the bias current helps to improve the detection probability and the timing jitter but at the cost of an increase in the dark count probability. $T = 0.6$ °K, $W = 100$ nm, and $h\nu = 1.5$ eV.

IV. SPECTRAL QUANTUM EFFICIENCY

The non-deterministic behavior of vortices in the case when the potential barrier has not vanished completely allows us to estimate the quantum efficiency probability even for low energy photons. Figure 8 shows the quantum efficiency based on the single-vortex crossing model as a function of the energy of the absorbed single-photon at different temperatures. The bias current is set to have near unity detection probability when the photon energy is larger than 1 eV. $W = 100$ nm. For high energy photons, the vortex potential barrier drops to zero. Hence, the click event which is as a result of the vortex crossing happens certainly. However, if the photon energy is not high enough to suppress the potential barrier completely, the vortex crossing event becomes non-deterministic, and it drops exponentially as the photon energy is reduced.

Figure 7. The effect of bias current on TaN SNSPD performance. a, Timing jitter, dark count probability ($P_0$), quantum efficiency ($P_1$) versus the bias current ($I_b$). $I_b$ is normalized to the switching current. Note that $I_{SW}$ is smaller than $I_{c,v}$. Increasing the bias current helps to improve the detection probability and the timing jitter but at the cost of an increase in the dark count probability. $T = 0.6$ °K, $W = 100$ nm, and $h\nu = 1.5$ eV.

Figure 8. Single-photon detection probability (quantum efficiency) as a function of single-photon energy and temperature in TaN SNSPDs. The bias current is set to have near unity detection probability when the photon energy is larger than 1 eV. $W = 100$ nm. For high energy photons, the vortex potential barrier drops to zero. Hence, the click event which is as a result of the vortex crossing happens certainly. However, if the photon energy is not high enough to suppress the potential barrier completely, the vortex crossing event becomes non-deterministic, and it drops exponentially as the photon energy is reduced.
performance is not very sensitive when the photon energy is slightly changed around the central frequency of the photon. However, as shown in Fig. 9c, a small perturbation in the photon energy can make a considerable change in the single-vortex crossing rate since the rate exponentially changes with the vortex potential energy. Figure 9d displays the effect of pulse-width on the quantum efficiency in our model. It is seen that there is a remarkable change in the quantum efficiency for ultrashort single-photon pulses. This effect arises due to the exponential tail of the quantum efficiency and clearly differentiates the proposed vortex model from the existing detection mechanisms. A controlled experiment can verify whether our model is correct or not.

V. CONCLUSION

In summary, we have proposed a probabilistic detection criterion in SNSPDs based on a single-vortex moving across the width of the detector. We have shown that even for a non-vanishing vortex potential barrier, there is a significant enhancement in the rate of the vortex crossing after the photon absorption leading to an increase in the click probability. This non-deterministic process insets a considerable intrinsic timing jitter to the detection event. We have shown the trade-space of the timing jitter, quantum efficiency, and dark counts for different superconducting materials and different nanowire structures. We have presented the quantum efficiency spectrum based on our model, and based on that, we can predict a pulse-shaped dependent quantum efficiency in
SNSPDs. This effect is negligible in other proposed models. Our model can predict some observables illustrated in Table I which can be verified experimentally to confirm or reject our model.

Table I. Experimental tests and observables to verify the validity of a detection model.

| Experimental test                  | Observables                                      |
|------------------------------------|--------------------------------------------------|
| Quantum efficiency spectrum        | Exponential decrease at low energies             |
| Response function                   | Latency vs. bias current and photon energy       |
| Timing jitter vs. bias current      | Shoulder at threshold                            |
| Broadband single photon            | Pulse-width dependency of quantum efficiency     |

Appendix A: Detection mechanism formalism

To find the time-dependent current and QP distributions, we use a modified semi-classical diffusion model which has been originally proposed by Semenov et al [56] and developed by Engel and Schilling [20, 27].

1. Quasi-particle multiplication

We assume the photon energy \((h\nu)\) is considerably larger than the superconducting bandgap \((\Delta)\), yet not large enough to make a phase transition and form a normal conducting core at the position of the photon absorption. Hence, when the photon is absorbed, a hot electron with a probability density of \(C_e(\vec{r},t)\) is created. Since the photon energy is usually orders of magnitude larger than the bandgap, when the hot electron diffuses, it breaks a large number of Cooper pairs (> 100 in the visible range) to QPs with a distribution density of \(C_{qp}(\vec{r},t)\). This causes the hot electrons to lose their energy, and as a result, the multiplication process slows down with a life-time of \(\tau_{qp}\) due to electron-phonon interaction [20]:

\[
\frac{\partial C_e(\vec{r},t)}{\partial t} = D_e \nabla^2 C_e(\vec{r},t),
\]

\[
\frac{\partial C_{qp}(\vec{r},t)}{\partial t} = D_{qp} \nabla^2 C_{qp}(\vec{r},t) - \frac{C_{qp}(\vec{r},t)}{\tau_r} + \frac{\hbar \nu}{\Delta \tau_{qp}} \left( n_{se,0} - C_{qp}(\vec{r},t) \right) \frac{\Delta}{n_{se,0}} e^{-t/\tau_p} C_e(\vec{r},t),
\]

where \(D_e, D_{qp}, \tau_r, \) and \(n_{se,0}\) are the hot-electron diffusion coefficient, quasi-particle diffusion coefficient, recombination time, and density of superconducting electrons before the photon absorption, respectively. \(\varsigma\) is the QP conversion efficiency which has been assumed constant. We add the term \((n_{se,0} - C_{qp}(\vec{r},t)) / n_{se,0}\) to include the saturation of QP multiplication. The exact solution of the above equation in a general form is not easy to derive. Hence, to find the solution numerically, we have used a Finite-Difference Crank-Nicolson method. Since, the hot-electrons diffuse quickly \((D_e \gg D_{qp})\), to speed-up the simulations, we have used the analytical solution of eqn. (A1) for the case of an infinite 2D superconductor [20]. A grid size of \(\Delta x = \Delta y = 1 - 3\) nm and a time step of \(D_{qp} \Delta t / \Delta x^2 = 0.01\) is used in our simulations. Neumann boundary condition for the side-walls and zero-flux at the two ends of the nanowire have been considered. The material parameters can be derived from experimental measurements [27, 30].

2. Current redistribution

The current distribution can be calculated by combining superconducting phase coherence condition and continuity equation [57]:

\[
\nabla . (\vec{j}(\vec{r},t)) = \nabla . \left( \frac{\hbar}{m} n_{se}(\vec{r},t) \nabla \varphi(\vec{r},t) \right) = 0,
\]

where \(n_{se}(\vec{r},t) = n_{se0} - C_{qp}(\vec{r},t)\) is the density of superconducting electrons after the photon absorption, \(\varphi\) is the phase of the superconducting order parameter, \(m\) and \(\hbar\) are the electron mass and reduced Planck constant, respectively.

3. Single vortex crossing

Vortices and antivortices are the topological defects in thin superconducting films which exist even if there is no applied magnetic field [57]. Vortices are usually nucleated and enter into the nanowire from the edge where the superconducting order parameter is suppressed. London equation in the presence of a static vortex in a superconducting thin film in \(xy\) plane can be written as [57, 58]:

\[
\vec{H}(r) + 2\pi \frac{\varphi}{c} \nabla \times \vec{j}(r) = \frac{\hbar}{c} \frac{\partial}{\partial t} (\vec{r} - \vec{r}_v),
\]

where \(\Delta = 2\lambda^2 / d\) is the Pearl length [59], \(\lambda\) is the London penetration depth, \(d\) is the film thickness, \(\Phi_0 = hc/2e\) is the magnetic flux quantum due to the presence of a single-vortex at the position \(\vec{r}_v\), \(\vec{H}\) is the magnetic field, \(\vec{j}\) is the current density ignoring the effect of the vortex on the current, and \(c\) is the speed of light in vacuum. Since the thickness of the nanowire is significantly smaller than \(\lambda\), we have averaged the field and the current in the \(z\) direction. For nanoscale SNSPDs \((L \ll \lambda)\), the first term can be neglected [58], and because of the current continuity \((\nabla . \vec{j} = 0)\), we can write the current density in the form of a scaler function as \(j(r) = \nabla \times G(r) \hat{z}\). Thus, eqn. (A4) is reduced to [58]:

\[
\nabla^2 G(r) = -\frac{\epsilon \Phi_0}{2\pi \Lambda} \delta(\vec{r} - \vec{r}_v),
\]
which is equivalent to the 2D Poisson’s equation for a charged particle. For an infinite superconducting film case, the interaction energy between vortices and antivortices for distances shorter than the Pearl length is logarithmic. This allows Berezinski-Thouless-Kosterlitz (BKT) transition and the formation of vortex-antivortex pairs below the BKT critical temperature [58, 60]. However, for a thin superconductor with finite width \((-W/2 < x < W/2)\), the long range interaction between vortices and antivortices is eliminated and single vortices can be found. For a single vortex, eqn. (A5) is reduced to the equation for a charge sandwiched between two parallel grounded plates. The problem is well-known in electrostatics and can be solved using conformal mapping with \(z' = e^{i\pi z/W}\) transformation and using image theory [58]:

\[
G(x, y) = \frac{\Phi_0}{8\pi \Lambda} \ln \frac{\cosh (y\pi/W) + \cos ((x + x_v)\pi/W)}{\cosh (y\pi/W) - \cos ((x - x_v)\pi/W)},
\]

where we have assumed the vortex is placed at \(x = x_v\) and \(y = 0\). The phase of the order parameter, \(\varphi\), can also be derived from \(G\) since the gradient of \(G\) is also proportional to the current [25]:

\[
\varphi (\vec{r}, \vec{r}_v) = \tan^{-1} \frac{\cos \left( \frac{\pi y}{W} \right) \sinh \left( \frac{\pi x - y}{2W} \right)}{\sin \left( \frac{\pi y}{W} \right) - \cos \left( \frac{\pi y + y}{2W} \right) \sin \left( \frac{\pi x}{W} \right)}.
\]

The free energy in presence of a vortex consists of the field energy and the kinetic energy inside the nanowire and the field energy outside [57, 58]. If we assume the vortex core radius is \(\xi\) and we neglect the core energy of the vortex, the self-energy of the vortex can be written as [19]:

\[
U_v^0(x_v) = \frac{\Phi_0}{2c} G(|x - x_v| \to \xi, 0) = \frac{\Phi_0^2}{8\pi^2 \Lambda} \ln \left( \frac{2W}{\pi \xi} \cos \left( \frac{\pi x_v}{W} \right) \right),
\]

If we include the work done by the bias current on a single vortex due to the Magnus force (dual of the Lorentz force on a magnetic flux), the total energy of a single vortex is expressed as [25]:

\[
U_v(x_v) = \frac{\Phi_0^2}{8\pi^2 \Lambda} \ln \left( \frac{2W}{\pi \xi} \cos \left( \frac{\pi x_v}{W} \right) \right) - \frac{\Phi_0}{c} J_y(x_v)x_v.
\]

The Magnus force tries to move the vortex in the direction perpendicular to the direction of the applied bias current, but it cannot overcome the self-energy of the vortex if the bias current is not high enough. Increasing the bias current at the edges \(J_y(x_v, t)\) due to the photon absorption reduces the potential barrier and eases vortex crossing. This barrier finally turns to zero at the vortex critical current which is:

\[
I_{c,v} = \frac{\Phi_0}{4\pi^2 \exp(1) \Lambda \xi W}.
\]

As seen in eqn. (A7), the phase of the order parameter depends on the position of the vortex, \(x_v\), and the phase difference at the two ends of a long nanowire \((L \gg W)\) away from the vortex position can be approximated as:

\[
\varphi(L/2) - \varphi(-L/2) = 2\pi x_v/W.
\]

Hence, as the vortex moves across the width of the nanowire, it applies a time-dependent phase difference between the two terminals of the detector. If the vortex crosses from one edge at \(x_v = -W/2\) to another edge at \(x_v = W/2\), it causes a 2\(\pi\) phase-slip at the two ends of the nanowire. This phase evolution generates a voltage pulse which can be described by the Josephson effect [61]:

\[
V(t) = \frac{\Phi_0}{2\pi c} \frac{d}{dt}(\varphi(L/2) - \varphi(-L/2)) = \frac{\Phi_0}{c} \frac{dx_v}{dt}.
\]

This voltage pulse propagates to the two ends [6, 62] and is dissipated in the presence of the bias current. If the bias current is high enough, the released energy is enough to induce a phase transition in the nanowire from the superconducting state to the normal conducting state. Hence, the current \(I_{c,v}\), which makes the vortex tunneling barrier reduce to zero, is also the critical current for phase transition in a thin superconducting nanowire. This critical current is less than the depairing critical current in a bulk superconductor, \(I_{c,dep}\) [18, 20].

Even if the applied bias current is below \(I_{c,v}\), breaking of the Cooper pairs and redistribution of the bias current due to the photon absorption can also change the potential barrier [20]:

\[
\frac{U_v(x_v, t)}{\varepsilon_0} = \frac{\pi}{W} \int_{x_v}^{x_v - \varepsilon_0} \frac{n_{sc}(x'_v, t)}{n_{sc,0}} \tan \left( \frac{\pi x'_v}{W} \right) dx' - \frac{2W}{I_{c,v} \exp(1) \xi} \int_{x_v}^{x_v - \varepsilon_0} \frac{n_{sc}(x'_v, t)}{n_{sc,0}} J_y (x'_v, t) dx',
\]

where \(\varepsilon_0 = \Phi_0^2/8\pi^2 \Lambda\) is the characteristic vortex energy.

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[1] R. H. Hadfield, *Nature Photonics* 3, 696 (2009).
[2] G. N. Goltsman, O. Okunev, G. Chulkova, A. Lipatov, A. Semenov, K. Smirnov, B. Voronov, A. Dzardanov,
[48] A. Vetter, S. Ferrari, P. Rath, R. Alaee, O. Kahl, V. Kovalyuk, S. Diewald, G. N. Goltsman, A. Korneev, C. Rockstuhl, and W. H. P. Pernice, Nano letters 16, 7085 (2016).
[49] J. Münzberg, A. Vetter, F. Beutel, W. Hartmann, S. Ferrari, W. H. Pernice, and C. Rockstuhl, Optica 5, 658 (2018).
[50] K. Erotokritou, R. Heath, G. Taylor, C. Tian, A. Banerjee, A. Casaburi, C. Natarajan, S. Miki, H. Terai, and R. H. Hadfield, Superconductor Science and Technology (2018).
[51] S. Ferrari, C. Schuck, and W. Pernice, Nanophotonics .
[52] F. Najafi, J. Mower, N. C. Harris, F. Bellei, A. Dane, C. Lee, X. Hu, P. Kharel, F. Marsili, S. Assefa, K. K. Berggren, and D. Englund, Nature Communications 6, 5873 (2015).
[53] S. Jahani and Z. Jacob, Optica 1, 96 (2014).
[54] S. Jahani, S. Kim, J. Atkinson, J. C. Wirth, F. Kalhor, A. Al Noman, W. D. Newman, P. Shekhar, K. Han, V. Van, R. G. DeCorby, L. Chrostowski, M. Qi, and Z. Jacob, Nature Communications 9, 1893 (2018).
[55] M. K. Akhlaghi, E. Schelew, and J. F. Young, Nature Communications 6, 8233 (2015).
[56] A. D. Semenov, G. N. Goltsman, and A. A. Korneev, Physica C: Superconductivity 351, 349 (2001).
[57] M. Tinkham, Introduction to superconductivity (Courier Corporation, 1996).
[58] V. G. Kogan, Physical Review B 75, 064514 (2007).
[59] J. Pearl, Applied Physics Letters 5, 65 (1964).
[60] A. M. Kadin, M. Leung, and A. D. Smith, Physical Review Letters 65, 3193 (1990).
[61] J. R. Clem, Journal of Low Temperature Physics 42, 363 (1981).
[62] D. F. Santavicca, J. K. Adams, L. E. Grant, A. N. McCaughan, and K. K. Berggren, Journal of Applied Physics 119, 234302 (2016).