Photon polarization in Compton scattering: pulse shape effects

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Photon polarization in Compton scattering: pulse shape effects

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Abstract.
We study in the framework of quantum electrodynamics the scattering of a plane wave electromagnetic field on free electrons in the low intensity limit. We derive analytic formulas describing the polarization properties of the emitted photons. We discuss and illustrate with a numerical example the effects of the electromagnetic pulse duration on their polarization.

1. Introduction
The study of inelastic scattering of radiation on electrons is deeply connected with the progress of quantum physics, starting with A. Compton major contribution (1923). The basis for the theoretical understanding of the process is the QED calculation of the scattering of low intensity monochromatic radiation on free electrons [1]. The availability of the first intense sources of radiation made necessary the distinction between the scattering in the low intensity regime (linear Compton scattering, LC) and the process at high intensity (nonlinear Compton scattering, NLC). The first studies of NLC are already almost fifty years old (see reviews [2], [3], [4]). In this paper, for more clarity, we shall also use the term “monochromatic linear Compton” (MLC) scattering.

The numerous recent studies of NLC have overcome the monochromatic case. Calculations are possible in the case of plane-wave pulses with fixed propagation direction but arbitrary duration and shape [5, 6] using in the Furry picture a Volkov solution for describing the state of the electron dressed by the laser field and treating the interaction between the electron and the quantized field of the emitted photon in first order perturbation theory. The papers published up to now present differential cross sections corresponding either to well defined states of polarization of the emitted photon and initial/final electrons, or summed/averaged over the polarization. Polarization properties are studied very little: in the monochromatic case a detailed analysis of polarization properties of the electrons and photons was presented by Ivanov et al [7], the calculation following the scheme used in the MLC case as developed in the textbook by Berestetskii, Lifshitz and Pitaevskii [8]. Polarization of the radiation emitted in nonlinear Thomson scattering (the classical limit of NLC) was also studied [9, 10].

The purpose of this paper is twofold: on the one hand we present an analytic expression of the transition probability valid in the low intensity limit in the case of a plane wave laser pulse with finite duration, on the other hand we adapt to this case the general formalism for the study of polarization properties of the scattered radiation. Our result is formally similar to
the corresponding MLC expression, and we shall discuss the differences between them. Then, in the same conditions we write the equations for determining the Stokes parameters of the emitted photon and also compare with the case of MLC. The formalism developed here for the low intensity limit can be used also in the case of arbitrary intensity; the corresponding results in this case will published elsewhere.

In Sect. 2 we briefly review general textbook results for MLC in the case of unpolarized electrons in both initial and final state, following closely the formalism of [8], albeit in a slightly simplified version. Next, in Sect. 3, starting from the general formulas of the non-linear Compton scattering [5], valid for the incoming radiation described by an intense plane wave laser pulse of finite duration, we derive their low intensity limit; we shall denote this case by “plane-wave linear Compton” (PWLC), in order to distinguish it from MLC. The results are written such that the effect of the finite duration of the pulse are easily visible by comparison with the monochromatic case. Finally, we derive the Stokes parameters of the emitted photon in the same approximation. The Stokes parameters are defined with respect to a particular system of axes, chosen as in the case of MLC (Sect. 87 of [8]), and in order to simplify the results we shall describe the laser field in a particular convenient gauge. Our conclusions are summarized in Sect. 5. In Appendix is discussed the relation between MLC and PWLC.

2. The linear monochromatic Compton scattering

The analysis of the polarization state of the final particles in the linear Compton scattering in term of Stokes parameters is a textbook problem [8]. For further use, we introduce the notations and review the main equations leading to Eq. (12), a convenient way of presenting the LC transition probability for initially polarized photons as a function of the Stokes parameters of the emitted photon. We denote the initial and final electron momenta by \( p_1 \) and respectively \( p_2 \) and, by \( \hbar k_1 \) and \( \hbar k_2 \) the momenta of the absorbed respectively emitted photon. Using the energy momentum conservation law it follows that the results can be expressed only in terms of two relativistic invariants which shall be chosen here as

\[
X = \frac{p_1 \cdot \hbar k_1}{(mc)^2} = \frac{p_2 \cdot \hbar k_2}{(mc)^2}, \quad Y = \frac{p_2 \cdot \hbar k_1}{(mc)^2} = \frac{p_1 \cdot \hbar k_2}{(mc)^2};
\]

the remaining two invariants are

\[
\hbar k_1 \cdot \hbar k_2 = (mc)^2(X - Y), \quad p_1 \cdot p_2 = (mc)^2(1 + X - Y).
\]

In the case of unpolarized initial/final electrons, the transition probability between states with fixed initial and final photon polarization \( s_1 \) and respectively \( s_2 \) is, up to a factor dependent on the photon energy, but not relevant for the calculation of the Stokes parameters,

\[
|Q|_{s_1,s_2}^2 = \frac{(X - Y)^2}{2XY} - \frac{(X - Y)^2}{2XY}\left|s_1 \cdot s_2 - \frac{(s_1 \cdot p_1)(s_2 \cdot p_2)}{(mc)^2X} + \frac{(s_1 \cdot p_2)(s_2 \cdot p_1)}{(mc)^2Y}\right|^2 + \frac{(X + Y)^2}{2XY}\left|s_1^* \cdot s_2 - \frac{(s_1^* \cdot p_1)(s_2 \cdot p_2)}{(mc)^2X} + \frac{(s_1^* \cdot p_2)(s_2 \cdot p_1)}{(mc)^2Y}\right|^2
\]

\[
\equiv C_{\alpha \beta} s_1^\alpha s_2^\beta.
\]

We mention that the above equation is more general that formula (11.13) in the book by Jauch and Rohrlich [1] because it allows the initial and final polarization 4-vectors \( s_1 \) and \( s_2 \) to be complex. By gauge transformations, one can always make the two polarization vectors to obey the conditions

\[
s_1 \cdot k_2 = s_2 \cdot k_1 = 0,
\]

(4)
with the consequences
\[ p_2 \cdot s_1 = p_1 \cdot s_1, \quad p_2 \cdot s_2 = p_1 \cdot s_2, \]
and then Eq. (3) reduces to the simpler form
\[
| Q |^2_{s_1 s_2} = \frac{(X - Y)^2}{2XY} - s_1 \cdot s_2 - \frac{(s_1 \cdot p_1)(s_2 \cdot p_1)(Y - X)^2}{(mc)^2 XY} + \frac{(X + Y)^2}{2XY} | s_1^* \cdot s_2 | - \frac{(s_1^* \cdot p_1)(s_2 \cdot p_1)(Y - X)^2}{(mc)^2 XY} \\
\equiv \tilde{C}_{\alpha\beta} s_2^\alpha s_2^\beta.
\]

Next we want to express the transition probability to a final state in which the emitted photon is in a mixed polarization state. We choose a pair of reference four-vectors defined by (see [8], Sect. 87)
\[
\rho^{(1)} = N/\sqrt{-X^2}, \quad \rho^{(2)} = -P/\sqrt{-P^2}, \quad N^\mu = \epsilon^{\mu\nu\lambda\rho} k_{1,\nu} k_{2,\lambda} p_{1,\rho}, \quad P^\mu = \epsilon^{\mu\nu\lambda\rho} k_{1,\nu} k_{2,\lambda} e_{1,\rho}
\]
with \( \epsilon^{\mu\nu\lambda\rho} \) the four-dimensional Levi-Civita tensor, with respect to which the Stokes parameters of the final state will be expressed. The two basis vectors obey the condition (4), so the simpler form of the transition probability (6) will be used. We replace \( s_2^\alpha s_2^\nu \) in the polarization matrix by the density matrix written in term of the Stokes vector \( \vec{\xi}^{(2)} \) of the emitted radiation
\[
\rho^{(2)\mu\nu} = (e_1^\mu e_1^\nu + e_2^\mu e_2^\nu) + \xi^{(2)}_1 (e_1^\mu e_2^\nu + e_2^\mu e_1^\nu) - i \xi^{(2)}_2 (e_1^\mu e_2^\nu - e_2^\mu e_1^\nu) + \xi^{(2)}_3 (e_1^\mu e_1^\nu - e_2^\mu e_2^\nu)
\]
\[
\rho^{(2)\mu\nu} = \rho^{(2)\mu\nu}_0 + \sum_{i=1,3} \rho^{(2)\mu\nu}_i \xi^{(2)}_i.
\]

At the same time we decompose the initial polarization vector in the basis formed by \( e_1 \) and \( e_2 \),
\[
s_1 = c_1 e_1 + c_2 e_2, \quad |c_1|^2 + |c_2|^2 = 1;
\]
such an expansion is always possible in the particular gauge (4). Then, after a straight calculation, (6) becomes the desired expression of the transition probability between the initial state of fixed polarization described by the coefficients \( c_1 \) and \( c_2 \) and the final state characterized by the Stokes vector \( \vec{\xi}^{(2)} \);
\[
| Q |^2_{s_1 \xi^{(2)}} = \frac{X}{Y} + \frac{Y}{X} - 2|c_2|^2 \frac{(X - Y)(2XY - X + Y)}{X^2 Y^2} - \xi^{(2)}_1 (c_1^* c_2 + c_2^* c_1) \frac{2(XY - X + Y)}{XY} + i \xi^{(2)}_2 (c_1^* c_2 - c_2^* c_1) \frac{X^2 + Y^2}{X^2 Y^2} (XY - X + Y)
\]
\[
+ 2 \xi^{(2)}_3 \left[ 1 - |c_2|^2 \frac{X^2 Y^2 + (XY - X + Y)^2}{X^2 Y^2} \right]
\equiv \mathcal{R}_0 + \sum_{i=1,3} \mathcal{R}_i \xi^{(2)}_i.
\]

This expression, containing \( c_1 \) and \( c_2 \) will be useful for the comparison of PWLC and MLC results in next section. If written in terms of the Stokes parameters of the incident photon, the expression reduces to a formula equivalent to Eq. (87.11) of [8].

If, instead of the emission probability for a Stokes vector fixed by the detector, one wants to calculate the Stokes vector \( \text{of the photon, before the detection} \), the expressions of its components are (see also Eq. (87.17) of [8])
\[
\xi^{(j)}_i = \frac{\mathcal{R}_i}{\mathcal{R}_0}.
\]
3. The linear plane wave Compton scattering

3.1. The transition amplitude of NLC

In the case of nonlinear Compton scattering, we consider the scattering of a laser pulse with fixed propagation direction $\mathbf{n}_1$, defined by a vector potential which depends on coordinates and time only through the combination $ct - \mathbf{n}_1 \cdot \mathbf{r} = n_1 \cdot x$, with $n_1 = (1, \mathbf{n}_1)$,

$$A^\mu(\phi) = A_0 f(\phi) \left[ \zeta_1 a_1^\mu \cos(k_1 \phi) + \zeta_2 a_2^\mu \sin(k_1 \phi) \right] \equiv A_1(\phi) a_1^\mu + A_2(\phi) a_2^\mu. \quad (14)$$

In the above equation $f(\phi)$ is the laser pulse envelope, chosen such that

$$\lim_{\phi \to \pm \infty} f(\phi) = 0 \quad (15)$$

and the two four vectors $a_1$ and $a_2$ are mutually orthogonal, orthogonal on the four-vector $n_1$, and normalized

$$a_1 \cdot a_2 = a_1 \cdot n_1 = a_2 \cdot n_1 = 0, \quad (a_1)^2 = (a_2)^2 = -1. \quad (16)$$

The parameter $k_1$ in (14) is related to the central frequency $\omega_1$ of the pulse by $k_1 = \omega_1/c$. In the monochromatic limit the associated photons carry the momentum $n_1 \hbar k_1$. The two parameters $\zeta_1$ and $\zeta_2$ obey the condition $\zeta_1^2 + \zeta_2^2 = 1$ and characterizes the laser polarization. The degrees of linear, respectively circular polarization of the laser are $P_L^{(L)} = (\zeta_1^2 - \zeta_2^2)^2$, $P_C^{(L)} = 4\zeta_1^2 \zeta_2^2$, $P_L^{(L)} + P_C^{(L)} = 1$.

In [5] we have calculated the transition amplitude for the spontaneous emission of a photon by an electron dressed by the laser field of arbitrary intensity. In the initial state, at $t \to -\infty$ the electron has the momentum $p_1$ and spin index $i_1$, and the momentum and spin of the electron in the final state are denoted by $p_2$ and respectively $i_2$. The transition amplitude for the emission of a photon of momentum $k_2$ and polarization vector $s_2$, is given by Eqs. (27) and (28) of [5]. We write it here as

$$M_{1 \to 2} = \frac{e}{i\hbar} \sqrt{\frac{\hbar}{2e\omega_2 V V}} \sqrt{\frac{(mc)^2}{p_1^2 p_2^2}} mc(2\pi\hbar)^3 \times \delta(p_{1\perp} - p_{2\perp} - \hbar k_2) \delta(n_1 \cdot (p_1 - p_2 - \hbar k_2)) Q(2,1), \quad (17)$$

displaying the product of a two-dimensional $\delta$ function, expressing the conservation of the component of the total momentum orthogonal on the laser propagation direction, $P_{tot\perp}$, with a one-dimensional $\delta$ function with the argument $n_1 \cdot P_{tot}$. The quantity $Q(2,1)$ is

$$Q(2,1) = \langle \bar{\psi}_{i_2}(p_2) | \left\{ B_0(2,1) \hat{s}_2^+ + (mc)^2 \frac{(n_1 \cdot s_2^+)}{(p_1 \cdot n_1)} B_2(2,1) \right\} | v_{i_1}(p_1) \rangle \quad (18)$$

where the Feynman “slash” was denoted using the symbol ”hat” ($\hat{a} = a_\mu \gamma^\mu$). The previous result is expressed in terms of four integrals $B_0, B_2, A_1$ and $A_2$ defined as

$$B_0(2,1) = \frac{mc}{h} \int_{-\infty}^{\infty} d\phi \, e^{-\frac{i}{\hbar} G(\phi)} \quad B_2(2,1) = \frac{mc}{h} \int_{-\infty}^{\infty} d\phi \, \frac{-e^2 A_2^2(\phi)}{2(mc)^2} e^{-\frac{i}{\hbar} G(\phi)} \quad (19)$$

$$A_1(2,1) = \frac{mc}{h} \int_{-\infty}^{\infty} d\phi \, e^{-\frac{i}{\hbar} A_1(\phi)} \quad A_2(2,1) = \frac{mc}{h} \int_{-\infty}^{\infty} d\phi \, \frac{-e^2 A_1^2(\phi)}{2(mc)^2} e^{-\frac{i}{\hbar} G(\phi)} \quad (20)$$

$^1$ $Q(2,1)$ differs from that in (Eq. (28) of [5] by the use of complex final polarization 4-vector $s_2$ and by the normalization convention of the free Dirac spinors $|v_i(p)\rangle$).

4
expressed. We choose these parameters as consequence, there are three independent invariant in terms of which all the others can be \(\delta\) of only three one-dimensional 

\[ O \text{ is a measure of the external field intensity. In the low intensity limit the terms of the order } \mathcal{O}(\eta^2) \text{ are neglected, so we neglect the integral } \mathcal{B}_0(2, 1) \text{ and the dependence of the field in the exponent } G(\phi) \text{ defined in } (21); \text{ then the integrals } \mathcal{A}_1(2, 1) \text{ and } \mathcal{A}_2(2, 1) \text{ are expressed only in terms of the Fourier transform } \Phi \text{ of the envelope } f(\phi): \]

\[ \mathcal{A}_1(2, 1) \approx \eta\phi_1 \int_{-\infty}^{\infty} d\phi f(\phi) \cos(k_1 \phi) e^{-\frac{i}{2\eta} n_1 \cdot (p_1 - \hbar k_2 - p_2) \phi} = \eta\phi_1 [\Phi(hk_1 - Q) + \Phi(-hk_1 - Q)], \]

\[ \mathcal{A}_2(2, 1) \approx \eta\phi_2 \int_{-\infty}^{\infty} d\phi f(\phi) \sin(k_1 \phi) e^{-\frac{i}{2\eta} n_1 \cdot (p_1 - \hbar k_2 - p_2) \phi} = -i\eta\phi_2 [\Phi(hk_1 - Q) + \Phi(-hk_1 - Q)]. \]
where we defined the dimensionless Fourier transform of \( f(\phi) \) by
\[
\Phi(q) \equiv \frac{(mc)}{2\pi \hbar} \int_{-\infty}^{\infty} d\phi e^{-i q \phi} f(\phi).
\] (28)

We observe here that usually the pulse envelope is real and symmetric, which makes the Fourier transform \( \Phi(q) \) real. We shall write the transition amplitude in the low intensity limit as
\[
Q(2, 1) = \sqrt{2(mc)\pi\eta} \left[ \Phi(hk_1 - Q)Q^{(+)}(2, 1) + \Phi(hk_1 + Q)Q^{(-)}(2, 1) \right]
\] (29)
\[
Q^{(\pm)}(2, 1) = (mc)\langle \vec{n}_2(p_2) \rangle \left\{ \frac{1}{Q} \left( \frac{s^{(\pm)}_1 \cdot p_1}{n_1 \cdot p_1} - \frac{s^{(\pm)}_1 \cdot p_2}{n_1 \cdot p_2} \right) s^{(\pm)}_2 + \right. \\
\left. \frac{1}{2} \left[ \hat{n}_1 \frac{s^{(\pm)}_1 s^{(\pm)}_2}{(n_1 \cdot p_2)} + \frac{s^{(\pm)}_2 s^{(\pm)}_1}{(n_1 \cdot p_1)} \right] \right\} |v_{1i}(p_1)|,
\] (30)

using two complex vectors defined by
\[
s^{(\pm)}_1 = a_1 \zeta_1 \pm i a_2 \zeta_2, \quad s^{(-)}_1 = s^{(+)*}_1.
\] (31)

The vectors \( s^{(+)}_1 \) and \( s^{(-)}_1 \) are normalized but they are not orthogonal, except for the case of circular polarization of the laser field
\[
s^{(+)\ast}_1 \cdot s^{(+)}_1 = s^{(-)\ast}_1 \cdot s^{(-)}_1 = -1, \quad s^{(+)\ast}_1 \cdot s^{(-)}_1 = \zeta^2 - \zeta^2_1.
\] (32)

We mention that Eq. (30) is identical with the transition matrix element of MLC \(^2\), written for the case when the incoming photon has direction \( \mathbf{n}_1 \), momentum \( Q \) and polarization vector \( s^{(\pm)}_1 \). More details are given in the Appendix, where is discussed the monochromatic limit.

Starting from the transition amplitude, we write now the probability of emission of a photon with the fixed polarization \( s_2 \), assuming, as before, that the polarization of the emitted photon is orthogonal on \( n_1 \) and also that the two vectors \( a_1 \) and \( a_2 \) are orthogonal on \( k_2 \), a situation that can be always reached by a gauge transformation:
\[
|Q|^2_{i, s_2} = \frac{(x - y)^2}{2xy} \left[ \Phi^2_+ + \Phi^2_- - 2\Phi_+ \Phi_- \text{Re}\{s^{(+)}_1 \cdot s^{(+)}_1\} \right]
\] (33)
\[
- \frac{(x - y)^2}{xy} \left[ \Phi^2_+ |T(s^{(+)}_1)|^2 + \Phi^2_- |T(s^{(-)}_1)|^2 + 2\Phi_+ \Phi_- \text{Re}\{T(s^{(+)}_1)T(s^{(+)}_1)\} \right]
\]
\[
+ \frac{(x + y)^2}{xy} \left[ \Phi^2_+ |T(s^{(-)}_1)|^2 + \Phi^2_- |T(s^{(+)}_1)|^2 + 2\Phi_+ \Phi_- \text{Re}\{T(s^{(-)}_1)T(s^{(-)}_1)\} \right]
\]
where we used
\[
T(s^{(\pm)}_1) = s^{(\pm)}_1 \cdot s_2 + \frac{s^{(\pm)}_1 \cdot p_1 (s_2 \cdot p_2)(x - y)^2}{(mc)^2xyz}, \quad \Phi_\pm = \Phi(Q \mp \hbar k_1).
\] (34)

Equation (33) is the correspondent of Eq. (3) for the case of a plane wave laser field of very low intensity.

\(^2\) The identity between the form usually written in the literature (see, e.g., R. Greiner, *Quantum electrodynamics*, third edition, Springer, 2003, Sect. 3.7) and the expression given in Eq. (30) can be immediately proven.
The next step in our calculation was to derive the equivalent of (12), i.e., the expression of the transition probability as a function of the Stokes parameters of the final photon. We have followed exactly the same procedure as in MLC using the same two basis vectors \( e_1 \) and \( e_2 \), defined by Eq. (7). We express the vectors \( s_1^{(\pm)} \) in terms if \( e_1 \) and \( e_2 \) as

\[
s_1^{(+)} = d_1 e_1 + d_2 e_2, \quad s_1^{(-)} = d_1^* e_1 + d_2^* e_2, \quad |d_1|^2 + |d_2|^2 = 1.
\]  

(35)

After a straight calculation the final result is

\[
|Q|_{\xi(2)}^2 = \mathcal{R}_0 + \sum_{k=1,3} \mathcal{R}_k \xi_k^{(2)}
\]  

(36)

with

\[
\mathcal{R}_0 = (\Phi_+^2 + \Phi_-^2) \left[ \frac{x}{y} + \frac{y}{x} + 2|d_2|^2 \left( \frac{x - y}{x^2 y^2 z^2} \right) \right] + \frac{x}{y} \left( \frac{x - y}{x^2 y^2 z^2} \right)
\]  

(37)

\[
\mathcal{R}_1 = \left[ (\Phi_+^2 - \Phi_-^2) (d_1^* d_2 + d_2^* d_1) + 4\Phi_+ \Phi_- Re\{d_1 d_2\} \right] \frac{2(x^2 - xy(z + 2) + y^2)}{xyz};
\]  

(38)

\[
\mathcal{R}_2 = i(d_1^* d_2 - d_2^* d_1) (\Phi_+^2 - \Phi_-^2) \frac{(x^2 + y^2)(xy(z + 2) - x^2 - y^2)}{x^2 y^2 z^2};
\]  

(39)

\[
\mathcal{R}_3 = 2(\Phi_+^2 + \Phi_-^2) \left[ 1 - |d_2|^2 \frac{x^2 y^2 z^2 + (x^2 - xy(z + 2) + y^2)^2}{x^2 y^2 z^2} \right] + \frac{2(\Phi_+^2 - \Phi_-^2)}{x^2 y^2 z^2} \left[ Re\{d_1^2\} - Re\{d_2^2\} \frac{(x^2 - xy(z + 2) + y^2)^2}{x^2 y^2 z^2} \right] .
\]  

(40)

The Stokes parameters of the emitted photon are calculated according to the same formula (13), with the components \( \mathcal{R} \) given by the previous equations (37-40). As discussed in the Appendix in the monochromatic limit, when \( \Phi_- \to 0 \) and the relations (26 are valid), the previous expressions reduce to (12); however, in the case of a finite pulse it is not possible to write the formula in terms of the Stokes parameters corresponding to the laser field, because of the terms \( Re\{d_1^2\}, Re\{d_2^2\} \).

4. Numerical examples

We illustrate the previous results with an numerical example. For simplicity we choose to work in the reference frame of the incoming electron, with the \( O_z \) axis taken along the laser propagation direction \( n_1 \) and assume that the laser field is linearly polarized, along the \( Ox \) axis of the reference frame; we take the central frequency of the laser field \( h\omega_1 = 0.8(mc^2) \) (or \( h\kappa_1 = 0.8(mc) \)). We shall only consider the case when the photon is emitted in the plane \( Oyz \), at an angle \( \theta_2 \) with respect to \( O_z \). It is interesting to review the monochromatic predictions for this case: in the monochromatic limit, the frequency of the emitted photon is fixed by its direction; according to the well known Compton formula in the electron rest frame

\[
h\omega_2 = h\omega_1 \frac{mc^2}{mc^2 + h\omega_1 (1 - \cos \theta_2)} ;
\]  

(41)
the polarization state is defined by Eqs. (12) and (13) from which it follows that the degree of circular polarization is 0, and the degree of linear polarization is

\[
\xi_L^{(f)} = \frac{2}{\frac{\omega_2}{\omega_1} + \frac{\omega_1}{\omega_2}}. \tag{42}
\]

In the plane wave case, the main difference is that for a fixed observation direction the frequency \(\omega_2\) of the emitted photon can take any value in the interval

\[
\omega_2 \in \left[0, \frac{(mc^2)x}{1 - \mathbf{n}_1 \cdot \mathbf{n}_2}\right]. \tag{43}
\]

We mention that the upper limit of the allowed interval can be obtained by imposing the condition \(p_1 \cdot p_2 > (mc)^2\), i.e. \(z > 0\) (see also Eq. (49) from [10] and the discussion there).

We have considered a Gaussian laser pulse with the envelope

\[
f(\phi) = e^{-\frac{\omega_1^2 \phi^2}{N^2 c^2}}, \tag{44}
\]

for four values of \(N\) (a parameter proportional to the FWHM of the pulse expressed in cycles): \(N = 1, 2, 4, 10\).

The degree of linear polarization of the emitted photon, calculated according to (37-40), as a function of \(\omega_2, \theta_2\) is represented in Fig. 1, for the four pulse lengths. One can see that in fact for the longer pulses \((N > 2)\) the graphs are practically identical. In the monochromatic limit, due to the one-to-one correspondence between energy and direction, the figure reduces to a curve, represented in red on the last graph.

The representation used in Fig. 1 does not take into account the fact that, although the emission of a photon with any frequency in the allowed interval (43) is in principle possible, in fact the emission probability varies strongly with \(\omega_2\) and it also depends on the pulse shape.

**Figure 1.** (Color online) The degree of linear polarization of the emitted photon as a function of \(\omega_2, \theta_2\) for four pulse lengths. The red line on the last figure is the monochromatic prediction.
Figure 2. (Color online) The scaled degree of linear polarization of the emitted photon as a function of $\omega_2$, $\theta_2$ for four pulse lengths.

Then it could be useful to define a “scaled” polarization vector $\xi^{(f)}$, calculated as the product of the true Stokes vector $\xi^{(f)}$ and the density of probability for emission of a photon with the frequency $\omega_2$ and in the direction $\mathbf{n}_2$. This quantity is represented in Fig. 2 for the same cases as before. One can see that, this time, the modifications due to the laser pulse duration are important. For longer pulses ($N = 4, 10$) the results are already close to the monochromatic limit, i.e. the photon detection probability has non-zero values only in a relatively small interval around the position of the monochromatic curve. On the other hand, in the case of the shorter pulse, since the emission probability is non-negligible for a large part of the allowed domain, the scaled Stokes parameters are non zero and almost constant in a relatively large domain.

5. Conclusions

We have reviewed the formalism used for the calculation of the Stokes parameters of the photon emitted in linear Compton scattering of monochromatic radiation and then we have considered the case of scattering of a plane wave pulse in the low intensity limit. We illustrated our results with a numerical example. In the Appendix we have reobtained the MLC transition amplitude as a limiting case of the formulas valid for plane wave pulses.

Appendix A. The monochromatic limit

The monochromatic limit is obtained as the particular case of a constant envelope $f(\phi) \to 1$, whose Fourier transform is a $\delta$ function $\Phi(q) \to (mc)\delta(q)$. The photons associated to the monochromatic plane wave have the momentum $\hbar k_1 \mathbf{n}_1$, with $k_1$ defined in Eq. (14). Then with Eq. (29), the total transition amplitude (17) becomes

$$\mathcal{M}^{\text{mono}}_{i \rightarrow f} = \frac{1}{\sqrt{2}} \pi \eta (mc) \frac{e}{i\hbar} \sqrt{\frac{\hbar}{2\epsilon_0 \omega_2 V}} \sqrt{\frac{(Mc)^2}{p_1^2 p_2^2}} (2\pi \hbar)^3 \delta(p_{1,\perp} - p_{2,\perp} - \hbar k_{2,\perp}) \times$$

$$\times \delta[n_1 \cdot (p_1 - p_2 - h k_2)] \left[ \delta(Q - h k_1)Q^{(+)}(2, 1) + \delta(Q + h k_1)Q^{(-)}(2, 1) \right] \quad (A.1)$$
One can easily prove the identity
\[
\delta(p_1, \perp - p_2, \perp - \hbar k_2, \perp - \bar{\hbar} k_2) \delta(n_1 \cdot (p_1 - p_2 - \hbar k_1)) = 2\delta(p_1 \pm \hbar k_1 - p_2 - \hbar k_2), \quad (A.2)
\]
and if one notices that the \(\delta\) function of argument \(p_1 - \hbar k_1 - p_2 - \hbar k_2\) is always zero, the transition amplitude reduces to
\[
\mathcal{M}_{i \rightarrow f}^{\text{mono}} = \frac{1}{\sqrt{2}} \eta(mc) \frac{e}{\sqrt{\hbar}} \sqrt{\frac{\hbar}{2e\epsilon_0\omega_1 V}} \frac{1}{V} \frac{(Mc)^2}{p_1 p_2} (2\pi)^4 \delta(p_1 \pm \hbar k_1 - p_2 - \hbar k_2) Q^{(+)}(2, \perp) \quad (A.3)
\]
More than that, in the expression of the matrix element \(Q^{(+)}(2, 1)\), evaluated with the four conservation laws, we recognize easily the matrix element of the linear monochromatic Compton effect, although written in slightly different form.

So, we have obtained the standard expression of the transition amplitude of the linear Compton effect, except for the factor \((mc)\eta/\sqrt{2} \equiv |e|A_0/2\) which replaces another factor \(|e|\sqrt{\hbar/2e\epsilon_0\omega_1 V}\) present in MLC, the difference being a direct consequence of the classical description adopted for the laser field.

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