WWγ and WWZ Production in e−e− Collisions in the Left-Right Model

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Abstract

We have investigated the three vector boson production in electron-electron collisions in the framework of the left-right symmetric electroweak model. The process occurs due to lepton number violating interactions mediated by Majorana neutrinos and triplet Higgs scalars. We find that only the reactions with a heavy gauge boson pair in the final state, e−e− → W−W−γ and e−e− → W−W−Z1, are phenomenologically interesting from the point of view of the Next Linear Collider. If the mass of W− is 0.5 TeV and the mass of the heavy Majorana neutrino of the order of 1 TeV, the cross section of both reactions is in the range 1 to 10 fb at \( \sqrt{s} = 2 \) TeV, depending on the mass of the doubly charged triplet Higgs \( \delta^{--} \), yielding a 1% background for the pair production of W−. At the \( \delta^{--} \) resonance the cross section can be as large as 1 pb.
1 Introduction

According to existing plans the next linear collider (NLC) will operate at the center of mass energy range of 0.5 – 2 TeV and deliver 10 fb$^{-1}$ of annual integrated luminosity $[1]$. Such a collider would provide sensitive probes for phenomena beyond the Standard Model (SM), which are expected to manifest themselves at the TeV energy scale and whose cross sections often are in the femtobarn range.

In addition to the $e^+e^-$ reactions, also the $e^-e^-, e^-\gamma$ and $\gamma\gamma$ collision modes are possible in NLC. These will be useful in studying the possible lepton number non-conservation. One much studied lepton number violating reaction is $e^-e^- \rightarrow W^-W^-$. It was first discussed by Rizzo $[2]$ in the context of the “classic” electroweak left-right symmetric model (LR-model) of Pati, Salam, Mohapatra, and Senjanovic $[3]$. Recently this process has been explored in more detail in the framework of the same model by London and Ng $[4]$, the present authors with their collaborators $[5]$, $[6]$, and Rizzo $[7]$, as well as Dicus et al. $[8]$, and Heusch and Minkowski $[9]$.

In the present paper we shall study the next order processes

$$e^-e^- \rightarrow W^-W^-\gamma,$$

(1)

$$e^-e^- \rightarrow W^-W^-Z^0$$

(2)

in the framework of the LR-model. Here $W$ can be the ordinary charged weak boson (to be denoted by $W_1$) or the heavy charged boson ($W_2$) predicted by LR-model, and $Z$ stands for either the ordinary $Z$-boson ($Z_1$) or the heavy neutral boson ($Z_2$) predicted by the model. The reactions (1) and (2) could provide us useful information about several basic interactions of LR-model: gauge boson - lepton, Higgs - lepton, Higgs - gauge boson -couplings, and gauge boson self-couplings. Some of the couplings involved do not appear in the pure $W$ pair production. The lowest order Feynman graphs for the processes $e^-e^- \rightarrow W^-W^-Z$ are presented in
Fig 1. The graphs for the process $e^-e^- \rightarrow W^-W^-\gamma$ are the same with $Z$ replaced by $\gamma$ except that there are no photon counterparts for the graphs 1f, 1g and 1h.

A general feature of the reactions (1) and (2) is that their cross section is highly suppressed unless the $W$ pair of the final state is the heavy one, $W_2W_2$. This fact, true also for the reaction $e^-e^- \rightarrow W^-W^-$, is connected to the masses and mixings of the Majorana neutrinos. In the case of the $W_1$ production, the lepton number violation strength is set mainly by the small mixing between the light and the heavy neutrino, and if this mixing is extremely small or vanishes, by the mass of the light neutrino, whereas in the case of the $W_2$ the mass of the heavy neutrino is relevant.

The processes $e^-e^- \rightarrow W_1^-W_1^-Z_1(Z_2,\gamma)$ and $e^-e^- \rightarrow W_1^-W_2^-Z_1(Z_2,\gamma)$, although kinematically favoured, have the cross section clearly below the femtobarn range (unless there exists a s-channel Higgs resonance at the relevant energy range) and are hence phenomenologically uninteresting.

The reaction $e^-e^- \rightarrow W_2^-W_2^-\gamma$ has an advantage, compared with the processes $e^-e^- \rightarrow W_2^-W_2^-Z_1(Z_2)$, of having a lower production threshold and a larger cross section due to soft photons. It also forms an important background for the pure $W$ pair production.

The organization of the paper is as follows. In Section 2 we describe the basic features of the LR-model. To be self-contained and to fix our notation we give explicitly all relevant interaction terms. The cross sections of the reactions (1) and (2) are derived in Section 3. The numerical results are presented in Section 4. Section 5 is devoted to summary and conclusions.
2 Description of the model

The matter fields are in the LR-model are set into left-handed and right-handed doublets of the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [3]. Here we are only considering leptons, which are accommodated as follows:

$$
\Psi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L = (2, 0, -1), \quad \Psi_R = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_R = (0, 2, -1),
$$

and similarly for the muon and tau families.

The breaking of the gauge symmetry, following the chain $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$, can be arranged by introducing a bidoublet Higgs field

$$
\Phi = \begin{pmatrix} \phi_0^0 & \phi_1^+ \\ \phi_2^- & \phi_0^0 \end{pmatrix} = (2, 2, 0),
$$

with the vev given by

$$
\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix},
$$

and a "right-handed" triplet field $\Delta_R$

$$
\Delta_R = \frac{\vec{\delta}_R \cdot \vec{\tau}}{\sqrt{2}} = \begin{pmatrix} \delta_R^+ / \sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+ / \sqrt{2} \end{pmatrix} = (1, 3, 2)
$$

with the vev given by

$$
\langle \Delta_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}.
$$

The triplet Higgs having both $SU(2)_R$ and $U(1)_{B-L}$ charge takes care of the first step of the symmetry breaking. Its vacuum expectation value sets the mass scale of the "right-handed" gauge bosons $W_2$ and $Z_2$. The experimental lower bounds are
$M_{W_2} \gtrsim 0.65$ TeV and $M_{Z_2} \gtrsim 0.45$ TeV \[1\]. Instead of the triplet one could also use a field transforming as a doublet under $SU(2)_R$ and having a non-vanishing $B - L$. The triplet field has, however, a virtue which makes it a more natural choice. It couples to $|\Delta L| = 2$ lepton currents through the Yukawa coupling $i h_R \Psi_R^T C \tau_2 \Delta_R \Psi_R$ giving rise to Majorana mass terms for right-handed neutrinos. This leads to the see-saw mechanism of neutrino masses \[11\] according to which there are in each fermion family two Majorana neutrinos, one very light ($\nu_1$) and another very heavy ($\nu_2$). The left-handed and the right-handed neutrinos are related to these mass eigenstate Majorana neutrinos as follows (assuming no interfamily mixing):

$$
\nu_L = \frac{1}{2} (1 - \gamma_5) (\nu_1 \cos \eta - \nu_2 \sin \eta),
$$

$$
\nu_R = \frac{1}{2} (1 + \gamma_5) (\nu_1 \sin \eta + \nu_2 \cos \eta).
$$

In addition to the bidoublet and the right-handed triplet Higgses one often introduces also a "left-handed" triplet

$$
\Delta_L = \frac{\vec{\delta}_L \cdot \vec{\tau}}{\sqrt{2}} = \begin{pmatrix}
\delta^+_L / \sqrt{2} & \delta^{++}_L \\
\delta^0_L & -\delta^+_L / \sqrt{2}
\end{pmatrix} = (3, 1, 2),
$$

and gives it a vev

$$
< \Delta_L > = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\
v_L & 0 \end{pmatrix}.
$$

Phenomenologically it is not, however, necessary, since it is not needed for the symmetry breaking or the see-saw mechanism. It will appear in our general expressions for the interactions, but since its effects in the processes we are considering would in any case be small, we will omit it in our later analysis.

There are altogether seven gauge bosons in the model: $W^+_L, W^+_R, W^3_L, W^3_R$, and $B$. They are related to the physical massive vector bosons $W_{1,2}$ and $Z_{1,2}$, and to photon $\gamma \equiv Z_3$ as follows:
\[
\left( \begin{array}{c}
W_L^+ \\
W_R^+
\end{array} \right) = \left( \begin{array}{cc}
\cos \zeta & -\sin \zeta \\
\sin \zeta & \cos \zeta
\end{array} \right) \left( \begin{array}{c}
W_1^+ \\
W_2^+
\end{array} \right)
\]
(11)

and
\[
\left( \begin{array}{c}
W_L^3 \\
W_R^3 \\
B
\end{array} \right) = (R_{ij}) \left( \begin{array}{c}
Z_1 \\
Z_2 \\
Z_3 = \gamma
\end{array} \right) \quad (i = L, R, B, \ j = 1, 2, 3).
\]
(12)

The general form of the matrix \( R \), as well as the expressions for the vector boson masses in terms of coupling constants and the vev's of the Higgs fields, can be found e.g. in [12]. In the limit, where LR-model reproduces the results of the Standard Model, the mixing matrix \( R \) takes the form
\[
R = \left( \begin{array}{ccc}
\cos \theta_w & 0 & \sin \theta_w \\
-\tan \theta_w \sin \theta_w & \sqrt{\cos 2\theta_w / \cos \theta_w} & \sin \theta_w \\
-\sqrt{\cos 2\theta_w} \tan \theta_w & -\tan \theta_w & \sqrt{\cos 2\theta_w}
\end{array} \right),
\]
(13)
where \( \theta_w \) is the counterpart of the Weinberg angle. This corresponds to the case where \( M_{W_2}, M_{Z_2} \to \infty \) and \( \zeta \to 0 \).

Let us now consider the interactions of the left-right symmetric model. The following parts of the Lagrangian are involved in the processes we are interested in:

**Neutral current interactions of the electron:**
\[
\mathcal{L}_{nc}^e = -\frac{1}{2} \left[ g_L \bar{e}_L \gamma^\mu e_L R_{Li} + g_R \bar{e}_R \gamma^\mu e_R R_{Ri} + g' \bar{e}_R \gamma^\mu e_R R_{A} \right] Z_{\mu} \\
= \sum_l \bar{\tau}_l \gamma^\mu \left( G_{eeZ_l}^L \frac{1 - \gamma_5}{2} + G_{eeZ_l}^R \frac{1 + \gamma_5}{2} \right) e Z_{\mu} = \sum_l \bar{\tau}_l \Gamma_{ee}^\mu e Z_{\mu},
\]
(14)
where \( l = 1, 2, 3 \).

**Neutral current interactions of neutrinos:**
\[
\mathcal{L}_{nc}^\nu = \frac{1}{2} \left[ g_L \bar{\nu}_L \gamma^\mu \nu_L R_{Li} + g_R \bar{\nu}_R \gamma^\mu \nu_R R_{Ri} + g' \bar{\nu}_R \gamma^\mu \nu_R R_{A} \right] Z_{\mu} \\
= \sum_l \bar{\nu}_l \gamma^\mu \left( G_{eeZ_l}^L \frac{1 - \gamma_5}{2} + G_{eeZ_l}^R \frac{1 + \gamma_5}{2} \right) \nu Z_{\mu} = \sum_l \bar{\nu}_l \Gamma_{ee}^\mu \nu Z_{\mu},
\]
\[ \sum_{jj'} \nu_{j'} Z_{l \mu}, \]  
(15)

where \( l = 1, 2, 3 \) and \( j, j' = 1, 2 \).

**Charged current interactions:**

\[ \mathcal{L}_{cc} = \sum_{jl} \nu_j (G^L_{lj} \gamma^\mu \frac{1 - \gamma_5}{2} + G^R_{lj} \gamma^\mu \frac{1 + \gamma_5}{2}) e W_{l\mu}^\dagger + h.c. \]

\[ = \sum_{jl} \nu_j e W_{l\mu}^\dagger + h.c. \]

(16)

**ZWW interaction:**

\[ \mathcal{L}_{ZWW} = i \sum_{\mu, \nu} e_{l\nu l} \left( W^+_{\mu\nu} W^{-\mu}_{l\nu} Z_l^\nu - W^+_{\nu\mu} W^{-\mu}_{l\nu} Z_l^\nu + W^+_{\mu\nu} W^{-\mu}_{l\nu} Z_l^\mu \right), \]

where we have used the notation \( W^\mu_{\nu\mu} \).  

**Yukawa interactions:**

\[ \mathcal{L}_{\nu} = f \phi \bar{\nu}_R \Phi \psi_L + g_\phi \bar{\nu}_R \tilde{\Phi} \psi_L + i h_L \psi_L^T C \tau_2 \Delta_L \psi_L + i h_R \psi_R^T C \tau_2 \Delta_R \psi_R + h.c. \]

\[ = \{ (\bar{\nu}_L \nu_L) [f_\phi \phi^+_1 - g_\phi \phi^+_2] + (\bar{\nu}_L \nu_L) [f_\phi \phi^-_2 - g_\phi \phi^-_1] + h.c \} \]

\[ + \ h_R \{ (\nu_R^T C \nu_R \delta^0) (\nu_R^T C \nu_R \delta^+) - \sqrt{2} \nu_R^T C \nu_R \delta^+ + h.c \} + \cdots, \]

(18)

where \( \tilde{\Phi} = \sigma^2 \Phi^* \sigma^2 \).

**Kinetic term of the Higgs triplet:**

\[ \mathcal{L}_{\Delta}^{\text{kin}} = \{(\partial^\mu - ig' B^\mu + g W^\mu_{\mu} \times) \delta \} \cdot \{(\partial^\mu - ig' B^\mu + g W^\mu_{\mu} \times) \delta \} \]

\[ = g^2 \{(W^\mu_{\mu} \cdot W^\mu_{\mu}) (\vec{\delta} \cdot \vec{\delta}) - (W^\mu_{\mu} \cdot \vec{\delta}) (\vec{W}^\mu_{\mu} \cdot \vec{\delta}) \} + \cdots; \]

(19)

**Kinetic term of the Higgs doublet:**

\[ \mathcal{L}_{\phi}^{\text{kin}} = \text{Tr}\{(D^\mu \Phi)^\dagger (D^\mu \Phi)\} \]

(20)

where

\[ D^\mu \Phi = \partial^\mu \Phi - \frac{i}{2} (g_L \vec{\tau} \cdot \vec{V}_L \Phi - g_R \Phi \vec{\tau} \cdot \vec{V}_R) \]

(21)
yielding

\[- \frac{g_L g_R}{\sqrt{2}} \cos \zeta \{ R_{Ll} W^\pm_2 - R_{Rl} W^\pm_1 \} \} Z_l + \cdots . \tag{22} \]

In eqs. (19) and (21) we have used the Cartesian components of the gauge bosons

\( W^\pm_{L,R} = (W^x_{L,R} \mp i W^y_{L,R}) / \sqrt{2}, \ W^0_{L,R} = W^z_{L,R} \) and of the triplet Higgses.

The various coupling constants appearing in the formulas are defined as follows.

The relations between the gauge coupling constants and the charge \( e \) of the positron

are given by (the matrix \( R \) is defined in eq. (12))

\[ g_L = \frac{e}{R_{L3}}, \quad g_R = \frac{e}{R_{R3}}, \quad g' = \frac{e}{R_{B3}}. \tag{23} \]

The neutral current couplings of the electron are described by

\[ \Gamma^\mu_{eeZ_l} = G^L_{eeZ_l} \gamma^\mu \frac{1 - \gamma^5}{2} + G^R_{eeZ_l} \gamma^\mu \frac{1 + \gamma^5}{2}, \tag{24} \]

where

\[ G^L_{eeZ_l} = - \frac{1}{2} (g_L R_{Li} + g'R_{Bi}), \tag{25} \]

\[ G^R_{eeZ_l} = - \frac{1}{2} (g_R R_{Ri} + g'R_{Bi}), \]

and those of neutrinos by

\[ \Gamma^\mu_{jj'Z_l} = \frac{1}{4} \gamma^5 \gamma^\mu, \begin{cases} 
g_L \cos^2 \eta R_{Li} - g_R \sin^2 \eta R_{Ri} - g'(\cos^2 \eta - \sin^2 \eta)R_{Bi}, & j = j' = 1 \\
- (g_L R_{Li} + g_R R_{Ri} - 2 g'R_{Bi}) \cos \eta \sin \eta, & j \neq j' \\
g_L \sin^2 \eta R_{Li} - g_R \cos^2 \eta R_{Ri} + g'(\cos^2 \eta - \sin^2 \eta)R_{Bi}, & j = j' = 2. \end{cases} \tag{26} \]

The charged current vertices are given by

\[ \Gamma^\mu_{ij} = C^L_{ij} \gamma^\mu \frac{1 - \gamma^5}{2} + C^R_{ij} \gamma^\mu \frac{1 + \gamma^5}{2}, \tag{27} \]

\[ \text{8} \]
where
\begin{align*}
G^L_{ij} &= \frac{1}{\sqrt{2}} g_L \begin{pmatrix}
\cos \eta \cos \zeta & \sin \eta \cos \zeta \\
-\cos \eta \sin \zeta & -\sin \eta \sin \zeta
\end{pmatrix}, \\
G^R_{ij} &= \frac{1}{\sqrt{2}} g_R \begin{pmatrix}
-\sin \eta \sin \zeta & \cos \eta \sin \zeta \\
-\sin \eta \cos \zeta & \cos \eta \cos \zeta
\end{pmatrix},
\end{align*}
(28)
and finally the WWZ couplings are
\begin{align*}
-e_{Wl} &= \begin{cases}
g_L \cos^2 \zeta R_{ll'} + g_R \sin^2 \zeta R_{ll'}, \ l = l' = 1 \\
(g_R R_{ll'} - g_L R_{ll'}) \sin \zeta \cos \zeta, \ l \neq l' \\
g_L \sin^2 \zeta R_{ll'} + g_R \cos^2 \zeta R_{ll'}, \ l = l' = 2.
\end{cases}
(29)
\end{align*}

The above equations are given for a general case. We know, however, that in practice the mixing between charged gauge bosons and between left- and right-handed neutrinos are small. If we neglect these mixings altogether by setting \( \eta = \zeta = 0 \) we find for the couplings involved in the phenomenologically interesting reactions \( e^- e^- \to W^- W^- \gamma \) and \( e^- e^- \to W^- W^- Z \) the following simple expressions:
\begin{align*}
\Gamma_{22}^\mu &= \frac{1}{2} G_{22}^\mu \gamma^\mu (1 + \gamma_5) = \frac{1}{2\sqrt{2}} g \gamma^\mu (1 + \gamma_5) \equiv G_{enW} \gamma^\mu (1 + \gamma_5), \\
2 \Gamma_{22l}^\mu &= \frac{1}{2} (g R_{ll'} - g'R_{ll'}) \gamma^\mu \gamma^5 \equiv -G_{nnZ} \gamma^\mu \gamma^5, \\
e_{22l} &= -g_R R_{ll} \equiv G_{WWZ}.
\end{align*}
These obey the relation
\begin{equation}
G_{WWZ} = G_{eeZ}^R + G_{nnZ}.
\end{equation}

A complete analysis of spontaneous symmetry breaking for the LR-model with one bidoublet, one left-triplet and one right-triplet Higgs field was presented in ref. [13]. It was shown that the phenomenology of the Higgs sector is quite restricted and depends crucially on three constants, called \( \beta_i, \ i = 1, 2, 3 \), appearing in the general Higgs potential. If one wants to have neutrinos as Majorana particles and preserve the see-saw mechanism for their masses, as well as at the same time keep the extra
Higgs particles and gauge bosons light enough to be accessible for the TEV-scale accelerators, the couplings $\beta_i$ should be fine-tuned at least to the order of $10^{-7}$. To avoid this unnatural situation one could constrain $\beta$’s to vanish, e.g. by introducing a suitable extra symmetry beyond the ordinary LR-model, in which case the mass scales $v_L$ and $v_R$ are disconnected and there remains a remnant see-saw relation, which is most naturally satisfied by the condition $v_L = 0$.

In the case that the vev $v_L$ of the left-handed triplet $\Delta_L$ vanishes (or if $\Delta_L$ does not exist at all), the linear combination $K_1 \phi_2^- - K_2 \phi_1^-$ is the Goldstone field corresponding to the longitudinal component of the light weak boson $W_1^-$. The Goldstone field corresponding the longitudinal component of the heavy gauge boson $W_2^-$ is in turn the superposition $\delta_R^- - (K_1^2 - K_2^2)\phi_1^- / \sqrt{2}K_1 v_R$. Hence when the left-handed triplet is neglected the only physical singly charged Higgs field $h^-$ is

$$h^- = \frac{1}{\sqrt{1 + K_1^2/2v_R^2(K_1^2 - K_2^2)^2}} (\phi_1^- + \frac{K_1}{\sqrt{2}v_R} \frac{K_1^2 - K_2^2}{K_1^2 + K_2^2} \delta_R^-). \tag{32}$$

The doubly charged Higgs $\delta_R^{--}$ is, of course, a physical field as it does not mix with any other field and is not eaten during the spontaneous symmetry breaking. Both $h^-$ and $\delta^{--}$ have lepton number violating interactions and they contribute in the processes we are considering.

The mass of the electron $m_e$ and the Dirac mass of the neutrino $m_D$ are given by the relations

$$m_e = (f_\phi K_2 + g_\phi K_1) / \sqrt{2}, \tag{33}$$
$$m_D = (f_\phi K_1 + g_\phi K_2) / \sqrt{2}.$$ 

Both of them are very small compared with the considered particle energies and can be safely neglected in our calculations. According to the see-saw mechanism $m_D$ is related to the masses $m_{\nu_1}$ and $m_{\nu_2}$ and to the mixing angle $\eta$ of the Majorana neutrinos $\nu_1$ and $\nu_2$, assuming no interfamily mixing, in the following way:
\[ m_D = \frac{1}{2}(m_{\nu_2} - m_{\nu_1}) \sin 2\eta, \quad (34) \]

where the mixing angle \( \eta \) is given by

\[ \tan \eta = \sqrt{\frac{m_{\nu_1}}{m_{\nu_2}}}. \quad (35) \]

The heavy neutrino mass is most naturally in the TeV range and hence the mixing angle is of the order of \( 10^{-6} \). This indicates that \( m_D \) would be in the MeV scale and its omission is justified.

The Feynman rules for the vertices needed for the calculation of the lowest order amplitudes of the reactions (1) and (2) are collected in Fig. 2.

3 Amplitudes

In this section we shall present the interaction amplitudes of the processes (1) and (2). The Feynman diagrams corresponding the latter reaction are given in Fig. 1. The graphs for the former reaction are the same with the Z-boson replaced by the photon, except that there are no counterparts of the diagrams 1f, 1g and 1h in the photon case.

As was mentioned in Introduction, the cross sections are in a phenomenologically interesting range only if both of the final state W-bosons are the heavy ones, \( W_2 \). It is easy to convince oneself about this by considering, for example, the diagram 1b. If the final state W-boson denoted by \( W(2) \) is the light boson \( W_1 \), which couples mainly in V–A currents, the lower \( e\nu W \) vertex is suppressed in the case of the heavy neutrino \( \nu_2 \), whose interactions are mainly of V+A -type. The vertex favours the light neutrino \( \nu_1 \), which would mean that in the upper \( e\nu W \) vertex the production of a virtual \( W_1 \) is favoured over that of a virtual \( W_2 \). The final state \( W \)-boson denoted
by $W(1)$ would then in the favourable case be $W_1$. Nevertheless, this situation of unsuppressed couplings would yield a negligible cross section because the amplitude is proportional to the mass of the light neutrino. This is because of the chirality matching in the $e\nu W$ vertices. Otherwise two (or one) light $W$’s in the final state only follows from the mixing of the neutrinos or/and the mixing of the $W$ bosons; every light $W$ of the final state then yields either the factor $\sin \eta$ or $\sin \zeta$ in the amplitude. As mentioned earlier, according to the see-saw mechanism the neutrino mixing angle obeys $\sin \eta \lesssim 10^{-5}$, and the experimental data gives the constraint for the mixing angle of charged weak bosons of $\sin \zeta \lesssim 10^{-3}$ \cite{14}. Thus in any case the cross sections of the reactions with one or two $W_1$’s in the final state are very small.

In the following we will thus consider only the reactions with two $W_2$’s in the final state. The amplitudes for these are insensitive to the mixing angles $\eta$ and $\zeta$. We shall let them to vanish, which simplifies our expressions considerably. All terms left in the amplitude are then proportional to the heavy neutrino mass $m_2$.

For Majorana neutrino fields appearing in the amplitudes we apply the Feynman rules given in ref. \cite{15}. From the graph 1a and the corresponding crossed graphs we obtain the amplitudes

$$T_1 = c_R \ G_{eeZ}^{IR} \ \tau_2^{\mu_1 \mu_2 \mu_3} (p_1 - k_3) / [t_{a3}(u_2 - m_{\nu_2}^2)],$$

$$T_2 = c_R \ G_{eeZ}^{IR} \ \tau_1^{\mu_3 \mu_1 \mu_2} (p_2 - k_3) / [t_{b3}(u_1 - m_{\nu_2}^2)],$$

$$T_3 = c_R \ G_{eeZ}^{IR} \ \tau_2^{\mu_2 \mu_1 \mu_3} (p_1 - k_3) / [t_{a3}(t_2 - m_{\nu_2}^2)],$$

$$T_4 = c_R \ G_{eeZ}^{IR} \ \tau_1^{\mu_3 \mu_2 \mu_1} (p_2 - k_3) / [t_{b3}(t_1 - m_{\nu_2}^2)].$$
We have used here the notations
\[ c_R = - 2 m_{\nu_2} G_{enW}^2 (1 + \gamma_5) = - \frac{i}{4} m_{\nu_2} g_R^2 (1 + \gamma_5), \]
\[ \tau_1^{\lambda_1 \lambda_2 \lambda_3} (q_1) = \gamma^{\lambda_1} \not{q_1} \gamma^{\lambda_2} \gamma^{\lambda_3}, \]
\[ \tau_2^{\lambda_1 \lambda_2 \lambda_3} (q_2) = \gamma^{\lambda_1} \gamma^{\lambda_2} \not{q_2} \gamma^{\lambda_3}. \]  
(37)

The diagrams 1b and 1d and their crossed counterparts, all with a virtual \( W \) boson decaying into a \( WZ \) pair, lead to the amplitudes

\[ T_5 = - c_R G_{WWZ} D_{\mu \nu}^W (k_1 + k_3) F^{\mu_1 \nu_1 \mu_3} (k_1, k_1 - k_3, k_3) \gamma^{\mu_2} \gamma^{\mu_3} / (u_1 - m_{\nu_2}^2), \]
\[ T_6 = - c_R G_{WWZ} D_{\mu \nu}^W (k_2 + k_3) F^{\mu_1 \nu_1 \mu_3} (k_2, k_2 - k_3, k_3) \gamma^{\mu_2} \gamma^{\mu_3} / (t_1 - m_{\nu_2}^2), \]
\[ T_7 = - c_R G_{WWZ} D_{\mu \nu}^W (k_1 + k_3) F^{\mu_1 \nu_1 \mu_3} (k_1, k_1 - k_3, k_3) \gamma^{\mu_2} \gamma^{\mu_3} / (t_2 - m_{\nu_2}^2), \]
\[ T_8 = - c_R G_{WWZ} D_{\mu \nu}^W (k_2 + k_3) F^{\mu_2 \nu_2 \mu_3} (k_2, k_2 - k_3, k_3) \gamma^{\mu_1} \gamma^{\mu_3} / (u_2 - m_{\nu_2}^2), \]
\[ T_9 = - 4 c_R G_{WWZ} D_{\mu \nu}^W (k_1 + k_3) F^{\mu_1 \nu_1 \mu_3} (k_1, k_1 - k_3, k_3) g^{\mu_2} / (s - M_W^2), \]
\[ T_{10} = - 4 c_R G_{WWZ} D_{\mu \nu}^W (k_2 + k_3) F^{\mu_2 \nu_2 \mu_3} (k_2, k_2 - k_3, k_3) g^{\mu_1} / (s - M_W^2). \]

Here we have defined
\[ F^{\lambda_1 \lambda_2 \lambda_3} (q_1, q_2, q_3) = 2 q_3^{\lambda_1} g^{\lambda_2 \lambda_3} + q_2^{\lambda_2} g^{\lambda_1 \lambda_3} - 2 q_1^{\lambda_3} g^{\lambda_1 \lambda_2}, \]  
(39)

and
\[ D_{\mu \nu}^W (k) = - \left( g_{\mu \nu} - \frac{k_{\mu} k_{\nu}}{M_W^2} \right) / \left( k^2 - M_W^2 \right). \]  
(40)

From the graphs 1c and 1e and the corresponding crossed graphs, which correspond to production of a \( Z\delta^{-+} \) pair followed by a \( \delta^{-+} \) decay into a \( WW \) pair, one obtains the amplitudes
\[ T_{11} = 4c_R G^{R}_{eeZ} \, g^{\mu_1 \mu_2} \, \gamma^{\mu_3} (p_2 - k_3)/[t_{b3}(s_3 - M_\delta^2)], \]
\[ T_{12} = 4c_R G^{R}_{eeZ} \, g^{\mu_1 \mu_2} (p_1 - k_3)\gamma^{\mu_3}/[t_{a3}(s_3 - M_\delta^2)], \quad (41) \]
\[ T_{13} = 16c_R G^{R}_{eeZ} \, g^{\mu_1 \mu_2} (p_1 + p_2)\gamma^{\mu_3}/[(s_3 - M_\delta^2)(s - M_\delta^2)]. \]

In the photon case one obtains similar amplitudes with \( G_{WWZ} \) and \( G_{eeZ} \) replaced with \(-e\). The graph 1f and its crossed graph, where Z is produced in the virtual neutrino line, leads to the amplitudes

\[ T_{14} = c_R \, G_{nnZ}[\tau_1^{\mu_2 \mu_3 \mu_1}(p_2 - k_2) + \tau_2^{\mu_1 \mu_3 \mu_1}(p_1 - k_1)]/[(t_1 - m_{\nu_2}^2)(t_2 - m_{\nu_2}^2)], \]
\[ T_{15} = c_R \, G_{nnZ}[\tau_1^{\mu_2 \mu_3 \mu_2}(p_2 - k_2) + \tau_2^{\mu_1 \mu_3 \mu_2}(p_1 - k_2)]/[(u_1 - m_{\nu_2}^2)(u_2 - m_{\nu_2}^2)], \quad (42) \]

and the graphs 1g and 1h and their crossed graphs, involving a virtual Higgs \( h^- \), lead to

\[ T_{16} = 2c_R \, G_{hZ} \, g^{\mu_2 \mu_3} \, \gamma^{\mu_1} (p_2 - k_1)/[(s_2 - M_h^2)(u_2 - m_{\nu_2}^2)], \]
\[ T_{17} = 2c_R \, G_{hZ} \, g^{\mu_2 \mu_3} (p_1 - k_1)\gamma^{\mu_1}/[(s_2 - M_h^2)(t_1 - m_{\nu_2}^2)], \]
\[ T_{18} = 8c_R \, G_{hZ} \, g^{\mu_2 \mu_3} (p_1 + p_2)\mu_1/[(s_2 - M_h^2)(s - M_\delta^2)], \]
\[ T_{19} = 2c_R \, G_{hZ} \, g^{\mu_1 \mu_3} \, \gamma^{\mu_2} (p_2 - k_2)/[(s_1 - M_h^2)(t_2 - m_{\nu_2}^2)], \]
\[ T_{20} = 2c_R \, G_{hZ} \, g^{\mu_1 \mu_3} (p_1 - k_2)\gamma^{\mu_2}/[(s_1 - M_h^2)(u_1 - m_{\nu_2}^2)], \]
\[ T_{21} = 8c_R \, G_{hZ} \, g^{\mu_1 \mu_3} (p_1 + p_2)\mu_2/[(s_1 - M_h^2)(s - M_\delta^2)]. \quad (43) \]
In these equations we have used the notation

$$G_{hZ} = \frac{(K_1/\sqrt{2}v_R)^2}{1 + (K_1/\sqrt{2}v_R)^2} (g_L R_{ll} + g_R R_{rl} - 2g'_R R_{bl}).$$  \hfill (44)

Let us note incidentally that the amplitudes $T_{16}, ..., T_{21}$ are of the same form as the terms arising from the longitudinal part of the W-propagator in the amplitudes $T_7, ..., T_{10}$, because, e.g., $D^{W2}_{\mu\nu}(k_1 + k_3)F^{\mu_1\nu_2\mu_3}(k_1, k_1 - k_3, k_3)\gamma^{\mu_2}\gamma^{\mu}$ is equivalent to $F^{\mu_1\nu_2\mu_3}(k_1, (p_2 - k_2)(M_3/M_2)^2 - 2k_3, k_3)\gamma^{\mu_2}\gamma^{\mu}$ when multiplied by the relevant polarization vectors. In other words, they just modify the expressions $(M_3/M_2)^2/(s_1 - M_2^2)$ and $(M_3/M_2)^2/(s_2 - M_2^2)$ appearing in $T_5, ..., T_{10}$.

In eqs. (3), (41), and (43) the coupling $h_R$ has been eliminated by using the relation

$$m_2 \approx 2 h_R \langle \delta^{0}_{R} \rangle = \sqrt{2} h_R v_R.$$ \hfill (45)

The kinematical variables $s_1$, $s_2$, $t_1$, $t_2$ appearing in the above formulas are defined by

$$s = (p_1 + p_2)^2,$$
$$s_1 = (k_1 + k_3)^2,$$
$$s_2 = (k_2 + k_3)^2,$$
$$t_1 = (p_1 - k_1)^2,$$
$$t_2 = (p_2 - k_2)^2.$$

They are the same variables as given by Byckling and Kajantie in [17] for a general $2 \rightarrow 3$ reaction, except that we have renamed their momenta $p_a, p_b, p_1, p_2, p_3$ as $p_1, p_2, p_3, p_2, p_3$, respectively.
where \( M_1, M_2, \) and \( M_3 \) denote generally the masses of the gauge bosons \( W_{l_1}, W_{l_2}, \) and \( Z_{l_3}, \) respectively.

The complete scattering amplitude is of the form

\[
M = i^5 \bar{v}(e_2) \, T^{\mu_1 \mu_2 \mu_3} u(e_1) \, \varepsilon^*_\mu_1 (W_{l_1}) \, \varepsilon^*_\mu_2 (W_{l_2}) \, \varepsilon^*_\mu_3 (Z_{l_3}),
\]

where \( T \) is the sum of the amplitudes \( T_i \) and \( \varepsilon \)'s are the polarization vectors of the weak bosons. The unpolarized total cross section is then given by the formula

\[
\sigma = \frac{1}{(2\pi)^6} \int \prod_{i=1}^3 \frac{d^4k_i}{2E_i} \, \delta^4(p_1 + p_2 - \sum k_i) \langle |M|^2 \rangle \quad \text{(49)}
\]

with

\[
\langle |M|^2 \rangle = \frac{1}{4} \sum_{\text{spins}} |M|^2. \quad \text{(50)}
\]

In the case of the reaction \( e^- e^- \rightarrow W^- W^- Z \) one has

\[
\langle |M|^2 \rangle = -\frac{1}{4} \left( g^{\mu_1 \nu_1} - k_1^{\mu_1} k_1^{\nu_1} M_1^2 \right) \left( g^{\mu_2 \nu_2} - k_2^{\mu_2} k_2^{\nu_2} M_2^2 \right) \left( g^{\mu_3 \nu_3} - k_3^{\mu_3} k_3^{\nu_3} M_3^2 \right) \\
\times \text{Tr} \left\{ T_{\mu_1 \mu_2 \mu_3} \, \not{p}_1 \gamma^0 (T_{\nu_1 \nu_2 \nu_3})^\dagger \gamma^0 \not{p}_2 \right\}, \quad \text{(51)}
\]

in the case of the reaction \( e^- e^- \rightarrow W^- W^- \gamma \) one replaces \( -(g^{\mu_3 \nu_3} - k_3^{\mu_3} k_3^{\nu_3} / M_3^2) \) with \(-g^{\mu_3 \nu_3}\). If there are two identical bosons in the final state, the expression (51) should be multiplied by one half.
A typical feature of the gauge theories is a delicate cancellations among the different partial amplitudes which guarantee a good high-energy behaviour of the total cross section. This offers a good check of the calculation. Another cross check is gauge invariance of the total amplitude, according to which the amplitude $M$ in the case of a photon should vanish when one replaces the polarization vector $\varepsilon^*_{\mu_3}(\gamma)$ with the photon momentum $k^\mu_3$ and performs the contraction. Our results pass this check when we take into account that we have neglected the electron mass.

We have given our formulas in a form where the width of $\Delta^{-}-$ is neglected. Including the width is, of course, straightforward and would not remarkably complicate our computations. The width can be evaluated by using the formulas

$$\Gamma_{\Delta^{-}\rightarrow l^{-}l^{-}} = g_R^2 M_\Delta (M^2_{W_1} + M^2_{W_3} + M^2_{W_2})/(32\pi M^2_{W_2}),$$

$$\Gamma_{\Delta^{-}\rightarrow W^{-}_2 W^{-}_2} = \frac{g_R^2 M_\Delta}{4\pi} \sqrt{1 - 4(M_{W_2}/M_\Delta)^2} \left[1 - (M_{W_1}/M_{W_2})^2\right] \times [3(M_{W_2}/M_\Delta)^2 + (M_\Delta/2M_{W_2})^2 - 1],$$

$$\Gamma_{\Delta^{-}\rightarrow W^{-}_2 h^{-}} = \frac{g_R^2 M_\Delta}{4\pi} (M_{W_1}M_\Delta/M^2_{W_2})^2 \lambda^{3/2}[1, (M_{W_2}/M_\Delta)^2, (M_h/M_\Delta)^2].$$

The two first formulas can be inferred from [16], which gives the corresponding results for the left-handed triplet. The third formula can be deduced from their result by expressing the triplet field $\Delta^{-}$ in terms of the mass eigenstate $h^{-}$ and the appropriate Goldstone field. We have assumed $h$ so heavy that the contribution of the third channel to the total width can be neglected [13].

4 Numerical results

Let us now describe our computations. The trace manipulations in the squared matrix element (51) were carried out by using the symbolic manipulation program REDUCE and they were checked with MATHEMATICA. The phase space integration in (19) was performed by using the formula [17].
\[
\int_{\text{phase space}} |M|^2 = \frac{\pi}{16s} \int < |M|^2 > \frac{d s_2 d s_1 d t_1 d t_2}{\sqrt{-\Delta_4}},
\]
where

\[
16 \cdot \Delta_4 = \begin{vmatrix}
0 & s & M_1^2 - t_1 & t_2 + s - s_1 \\
 s & 0 & t_1 + s - s_2 & M_2^2 - t_2 \\
 M_1^2 - t_1 & t_1 + s - s_2 & 2M_1^2 & s - s_1 - s_2 + M_3^2 \\
 t_2 + s - s_1 & M_2^2 - t_2 & s - s_1 - s_2 + M_3^2 & 2M_2^2
\end{vmatrix}.
\]

The inner integration in \(t_1t_2\)-plane is over an ellipse defined by the condition \(\Delta_4 = 0\). (We performed this integration also analytically for a check.) The 4-fold integrals were performed numerically by using the Monte Carlo program VEGAS [18].

The symmetry relations

\[
\sigma_{ij} = \sigma_{ji}^*,
\]

\[
\sigma_{ij} = \sigma_{icjc}
\]

turned out to be of help in checking the code. Here the \(\sigma_{ij}\) is the cross section contribution coming from the interference term of the amplitudes \(T_i\) and \(T_j\), and \(i_c\) refers to the amplitude obtained from the amplitude of the index \(i\) by charge conjugation:

\[
T_{ic}(p_2, p_1) = C T_i^T(p_1, p_2) C^{-1}.
\]

We found out that the symmetry relations (55) are in general numerically satisfied to several digits although the values of \(\sigma\)'s themselves vary somewhat with the Monte Carlo parameters used.

The input of our computation consists of the particle masses, the gauge coupling constants, and various mixing angles. Assuming that there is no mixing between different neutrino flavours, only the neutrino mixing angle \(\eta\) between the left-handed and the right-handed electron neutrino enters our calculations. The mixing between \(W_L\) and \(W_R\) is described with the angle \(\zeta\) as presented in eq. (11). The mixing
between the neutral gauge bosons is described in terms of three mixing angles, out of which only two are independent in the case that \( g_L = g_R \) as we shall assume. The mixing matrix, which we have denoted \( R \) in the foregoing section, can then be determined by using the experimental data for the two neutral current parameters, e.g. the vector and axial vector \( Z_1 \) couplings of quarks and leptons. In our calculations we have used three different forms for it: the SM limit of \( R \) given by (13) where the Weinberg-Salam angle \( \theta_{WS} \) has been estimated from the experimental value of \( M_{W_1}/M_{Z_1} \), the numerical form obtained as in [5], and the numerical form calculated by using the matrix given in [13] and fixing the \( M_{W_1} \) and \( M_{Z_1} \) to their experimental values.

For the mass of the heavy W-boson we use the value 0.5 TeV (sometimes also the value 0.7 TeV for comparison), which is close to its experimental lower bound. For the mass of \( \delta^- \) Higgs we have used two values, one below the reaction threshold \( (M_{\delta} = 0.8 \text{ TeV}) \), another one well above the threshold \( (M_{\delta} = 10 \text{ TeV}) \).

The cross section is proportional to the square of the mass of the heavy neutrino for which we used the value \( m_2 = 1 \) TeV. This is a natural choice in the sense that the mass originates in the breaking of the \( SU(2)_R \) symmetry, which we have assumed to occur at the TeV-scale. Letting the Yukawa coupling \( h_R \) differ from the value \( O(1) \), also much lighter neutrino mass were possible, but then also the cross section were much smaller. At the energies considerably above the neutrino mass the cross section roughly scales with \( m_2^2 \).

As mentioned, there are delicate cancellations among the amplitudes contributing to the processes we are considering. We demonstrate this in Fig 3 where we have plotted separately cross sections for the reaction \( e^- e^- \rightarrow W_2 W_2 Z_1 \) as a function of the collision energy \( E_{cm} = \sqrt{s} \) corresponding to the subsets of the amplitudes given in eqs. (33), (3), and (41) and their interferences. The contribution from the am-
plitudes (42) is negligibly small, and it was, like the suppressed amplitudes (43), omitted in the Fig 3. Here, and in what follows if not otherwise stated, we have taken the mass of $W_2$ equal to 0.5 TeV and the mass of the heavy neutrino equal to 1 TeV. The triplet Higgs mass is in this figure equal to 10 TeV. The total cross-section, invisible in the scale of the figure, is some thousands of the partial contributions plotted in the figure. At $E_{\text{cm}} = 2$ TeV we obtain $\sigma_{\text{tot}} = 2.4$ fb. The $\sigma_{\text{tot}}$ increases with the collision energy up to $E_{\text{cm}} = M_\delta$, after which it starts to decrease because of a destructive interference of the amplitudes involving virtual triplet Higgses.

In Fig. 4 we present $\sigma_{\text{tot}}$ for $e^-e^- \rightarrow W_2W_2Z_1$ as a function of $E_{\text{cm}}$ for the case of a light $\delta^{--}$, $M_\delta = 0.8$ TeV and for the case of a heavy $\delta^{--}$, $M_\delta = 10$ TeV by using the SM limit of R (solid line) and in the latter case also by using the R given in [3] (dashed line) and the R which is the most consistent with the model [19] (dashdot line). In the case of $M_\delta = 0.8$ TeV the cross section shows a maximum of 4.5 fb at around 2 TeV. With the anticipated luminosity 10 fb$^{-1}$ the peak value would correspond to 45 events per year. The results obtained by using the three different choices of the R matrix are very similar at low energy region (all presented by the divide line in the figure), but they start to differ at high energies. The cross-section corresponding to the SM limit of R and to R of [19] has the expected good high-energy behaviour, while in the case of our third R the cancellations among various amplitudes are less complete.

In Fig. 5 we present the total cross section for the reaction $e^-e^- \rightarrow W_2W_2\gamma$ (here $M_\delta = 0.8$ TeV). A new feature compared with the WWZ case is the singularity caused by soft-photons. We handle the singularity following the standard method of evading it by applying cuts for the minimum energy $E_\gamma$ and the minimum scattering angle $\theta_\gamma$ of the photon, which is a natural procedure from the experimental point of view. In Fig. 5 we have used the cuts $\cos \theta_\gamma \leq 0.8$ and $E_\gamma/E_{\text{cm}} \geq 0.01$. 

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Also here we have demonstrated the effect of cancellations. We have separated the amplitudes mediated by the neutrinos from those mediated by $\delta^{-\nu}$ Higgses. Both contributions are separately physical in the sense that their amplitudes are gauge invariant by themselves. However, they both increase with the energy contradicting eventually with unitarity. Their destructive interference takes care of the good high energy behaviour of the total cross section.

The sensitivity of the total cross section in the applied cuts is illustrated in Fig. 6. We present the total cross section for three sets of the photon energy and scattering angle cuts. Near the threshold the cross the effect of changing the cuts is quite dramatic. The peak value of the total cross section is of the order of 10 fb, that is, about two times the cross section in the case of the $W_2W_2Z_1$ final state.

Fig. 7 presents the cross section as a function of energy for a slightly heavier $W_2$, $M_{W_2} = 0.7$ TeV. One notes a large decrease of the maximum value of the cross section, to some 20th part of its value in the case of $M_{W_2} = 0.5$ TeV. With the increasing energy the cross section approaches to the value expected according to the scaling with the $W_2$ mass at high energies.

We note that the finite width of $\delta$ does not essentially change the results presented above, because the dominant decay channel of $\delta$ is into two leptons and this is still relatively small for the $\delta^{-\nu}$ mass of 0.8 TeV.

The Fig. 8 gives the cross section for $e^-e^- \rightarrow W_2W_2\gamma$ in the case of $M_\delta = 10$ TeV. It should be compared with Fig. 5 where $M_\delta = 0.8$ TeV. We have plotted again separately the contributions from neutrino mediated and the triple Higgs mediated amplitudes. In contrast with the case of Fig. 5, these contributions interfere now constructively.

In the case of heavy triplet Higgs, the zero width approximation applied above is not any more reliable. In fact, for a mass so high as 10 TeV the decay $\delta^{-\nu} \rightarrow W_2^-W_2^-$
alone gives rise to a width which is greater than the mass of the Higgs, making the particle interpretation of the Higgs questionable. If we nevertheless trust on our perturbational calculations the total cross section at the experimentally relevant energy of 2 TeV and below is in a good approximation given by the pure neutrino contribution, i.e., by the curve $\sigma_{n+n}$ in Fig. 8.

In Fig. 9 we present the total cross section for two sets of the photon cuts assuming a $\delta$-pole at 2 TeV with a width of $\Gamma = 0.14M_\delta$. The width has been estimated by using eq. (53) assuming the electron-, muon- and tau-type heavy neutrinos and $W_2$ to have mass equal to 1 TeV and the Higgs $h^-$ so heavy that it can be neglected.

In the calculation we made some simplifying assumptions. Firstly, we have neglected possible family mixing among neutrinos. Such a mixing might increase the production rates of the final states consisting of a light $W$ pair or a light and heavy $W$ pair, as discussed in [9]. Secondly, we have assumed that the gauge couplings associated with the subgroups $SU(2)_L$ and $SU(2)_R$ are equal, $g_L = g_R$. Combined with the assumption of identical Cabibbo-Kobayashi-Maskawa matrices for the left- and right-handed quarks, this is known to lead to the upper bound of 1.4 - 1.6 TeV for $M_{W_2}$ [14]. This would push the production of the $W_2$ pair out of the reach of NLC. The possible deviations of the ratio $K = g_R/g_L$ from the value one has been discussed in [21] and [20], where the allowed region 0.55 to 1.5 was found. The GUT extensions of LR-model favours the value 1 for the upper limit REF. The cross sections we have calculated have an overall factor $g_R^4$, but this hardly leads to the scaling by this factor, because changing of $K$ change the couplings $G_{WWZ}$, $G_{eeZ}$, and $G_{nnZ}$, and also indirectly the matrix $R$. We do not expect the value of $K$ essentially change our numerical results at the energy range of NLC, but at higher energies the effect may be substantial.
5 Summary

The left-right symmetric electroweak model predicts the existence of heavy gauge bosons $W_2^\pm$ and $Z_0^0$, as well as lepton number violating interactions mediated by a heavy Majorana neutrino and a right-handed triplet scalars. We have calculated the total cross section for the lepton number violating processes $e^-e^- \rightarrow W_2^+W_2^-\gamma$ and $e^-e^- \rightarrow W_2^+W_2^-Z_1$, which could be experimentally measured at NLC provided that $W_2$ is not much heavier than the present lower limit of its mass. The reactions are background processes for the heavy vector boson pair production $e^-e^- \rightarrow W_2W_2$ investigated earlier \[4\],\[5\],\[8\], \[9\]. The cross sections of the reactions where one or both of the charged bosons in the final state is the ordinary weak boson $W_1$ is found to be too small to have any phenomenological interest as far as NLC in concerned.

If the mass of the heavy electron neutrino is of order of 1 TeV and the $W_2$ mass near to 0.5 TeV, we find the total cross section of $e^-e^- \rightarrow W_2^-W_2^-Z_1$ to be in the collision energy region of a few TeV is in the range of 1 to 10 fb depending on the mass of the double charged triplet Higgs $\delta^{--}$. The cross section of $e^-e^- \rightarrow W_2^-W_2^-\gamma$ under the same assumptions and with reasonable cuts on the photon energy and scattering angle is of the same order. Both cross sections increase with increasing heavy neutrino mass. At the $\delta^{--}$ pole the cross sections can reach the value 1 pb. From these results one can conclude that $e^-e^- \rightarrow W_2^-W_2^-\gamma$ yields of the order of 1 \% background for the pair production $e^-e^- \rightarrow W_2W_2$.

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Figure captions

Figure 1. The lowest-order Feynman graphs for the process $e^-e^- \rightarrow W^-W^-Z$.

Figure 2. The Feynman rules for vertices appearing in the amplitudes. Here $R = (1 + \gamma_5)/2$ and $L = (1 - \gamma_5)/2$, and the other quantities are defined in the text.

Figure 3. The contributions of various subsets of amplitudes and their interferences to the cross section of the reaction $e^-e^- \rightarrow W^-_2W^-_2Z_1$ as functions of the CM-energy $E = \sqrt{s}$ for the masses $M_{W_2} = 0.5$ TeV, $M_\delta = 10$ TeV and $M_{\nu_2} = 1$ TeV.

Figure 4. The total cross section of the reaction $e^-e^- \rightarrow W^-_2W^-_2Z_1$ as functions of the CM-energy $E = \sqrt{s}$ for the masses $M_{W_2} = 0.5$ TeV, $M_{\nu_2} = 1$ TeV and for two values of the triplet Higgs mass, $M_\delta = 0.8$, $10$ TeV for the different neutral current mixing matrices (see the text).

Figure 5. The total cross section of the reaction $e^-e^- \rightarrow W^-_2W^-_2\gamma$ as functions of the CM-energy $E = \sqrt{s}$ for the masses $M_{W_2} = 0.5$ TeV, $M_\delta = 0.8$ TeV and $M_{\nu_2} = 1$ TeV (solid line). The cuts $\cos\theta_\gamma \leq 0.8$, $E_\gamma/E \geq 0.01$ have been applied for the scattering angle and the energy of the photon. Shown are separately also the contributions from the neutrino exchange and the Higgs exchange contributions and their interference.

Figure 6. The total cross section of the reaction $e^-e^- \rightarrow W^-_2W^-_2\gamma$ as functions of the CM-energy $E = \sqrt{s}$ for the masses $M_{W_2} = 0.5$ TeV, $M_\delta = 0.8$ TeV and $M_{\nu_2} = 1$ TeV and for three set of the photon cuts: $\cos\theta_\gamma \leq 0.9$, $E_\gamma/E \geq 0.01$, $\cos\theta_\gamma \leq 0.8$, $E_\gamma/E \geq 0.01$, $\cos\theta_\gamma \leq 0.8$, $E_\gamma/E \geq 0.05$ have been applied for the scattering angle and the energy of the photon.

Figure 7. The total cross section of the reaction $e^-e^- \rightarrow W^-_2W^-_2\gamma$ as functions of the CM-energy $E = \sqrt{s}$ for the masses $M_{W_2} = 0.7$ TeV, $M_\delta = 0.8$ TeV and $M_{\nu_2} = 1$ TeV and for two sets of the photon cuts: $\cos\theta_\gamma \leq 0.9$, $E_\gamma/E \geq 0.01$, $\cos\theta_\gamma \leq 0.8$, $E_\gamma/E \geq 0.05$. 

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Figure 8. The total cross section of the reaction $e^- e^- \rightarrow W^-_2 W^-_2 \gamma$ as functions of the CM-energy $E = \sqrt{s}$ for the masses $M_{W_2} = 0.5$ TeV, $M_\delta = 10$ TeV and $M_{\nu_2} = 1$ TeV and for the photon cuts $\cos \theta_\gamma \leq 0.9$, $E_\gamma/E \geq 0.01$. Shown are separately also the contributions from the neutrino exchange and the Higgs exchange contributions and their interference.

Figure 9. The total cross section of the reaction $e^- e^- \rightarrow W^-_2 W^-_2 \gamma$ as functions of the CM-energy $E = \sqrt{s}$ for the masses $M_{W_2} = 0.5$ TeV, $M_\delta = 2.0$ TeV and $M_{\nu_2} = 1$ TeV and for two set of the photon cuts: $\cos \theta_\gamma \leq 0.9$, $E_\gamma/E \geq 0.01$, $\cos \theta_\gamma \leq 0.8$, $E_\gamma/E \geq 0.05$. 


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