A new model for the quark mass matrices

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Abstract

I present a new model for the quark mass matrices, which uses four scalar doublets together with a horizontal symmetry $S_3 \times Z_3$. The model is inspired on a suggestion made a few years ago by Ma, but it is different. The predictions $|V_{ts}| \approx m_s/m_b$ and $|V_{ub}/V_{cb}| > 0.085$ are obtained. Flavour-changing neutral Yukawa interactions do not exist in the down-type-quark sector.

A few years ago, Ernest Ma put forward a model for the quark mass matrices based on the discrete symmetry $S_3 \times Z_3$. His model is characterized by the absence of flavour-changing neutral Yukawa interactions (FCNYI) from the charge-2/3-quark sector. Ma’s model predicts rather low values for both $|V_{ub}/V_{cb}|$ and $m_s/m_d$: those values are moreover correlated, with a larger $|V_{ub}/V_{cb}|$ implying a lower $m_s/m_d$, and vice-versa. In spite of this problem, Ma’s mass matrices should be praised: they are not postulated as an Ansatz, a “scheme”, or a “texture”, rather they follow from a complete model, with a well-defined field content and a well-defined internal symmetry. As a consequence, Ma’s model—which needs to be supplemented by soft symmetry breaking in the scalar potential—is self-contained and consistent from the point of view of quantum field theory. This situation contrasts with the one typical of many, sometimes otherwise quite successful, schemes or textures that have been proposed for the quark mass matrices. Most of those schemes cannot be justified in terms of a complete theory.

In this Brief Report I remark that the role of the up-type and down-type quarks in Ma’s model may be interchanged, one then obtaining another viable model for the mass matrices. The new model enjoys the same features of self-containedness and consistency as Ma’s one. It leads to quite distinct predictions for the quark masses and mixings. A very clear-cut prediction is $|V_{ts}| \approx m_s/m_b$: as a consequence of this relation and of the measured value of $|V_{cb}| \approx |V_{ts}|$, the strange-quark mass must be in the upper part of its allowed range. It also predicts $|V_{ub}/V_{cb}| > 0.085$. FCNYI are absent from the charge−1/3-quark sector, impeding tree-level contributions to the mass differences in the $K^0–\bar{K}^0$ and $B_d^0–\bar{B}_d^0$ systems, and to the CP-violating parameter $\epsilon$.

In my model there are four Higgs doublets $\phi_a$ ($a = 1, 2, 3, 4$) and two $Z_3$ symmetries. I denote $p_{R_i}$ ($i = 1, 2, 3$) the right-handed charge-2/3 quarks, $n_{R_i}$ the right-handed charge−1/3 quarks, and $q_{L_i} = (p_{L_i}, n_{L_i})^T$ the doublets of left-handed quarks. The quantum numbers of the various fields under $Z_3^{(1)}$ and $Z_3^{(2)}$ are given in Table 1.

Besides $Z_3^{(1)}$ and $Z_3^{(2)}$, there is one further horizontal symmetry, which effects the interchanges

$$\phi_1 \leftrightarrow \phi_2, \quad q_{L_1} \leftrightarrow q_{L_3}, \quad p_{R_2} \leftrightarrow p_{R_3}, \quad n_{R_2} \leftrightarrow n_{R_3},$$

Equation (1)

Table 1: Quantum numbers of the fields under the two $Z_3$ symmetries, with $\omega \equiv \exp(2i\pi/3)$.

|          | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\phi_4$ | $q_{L_1}$ | $q_{L_2}$ | $q_{L_3}$ | $p_{R_1}$ | $p_{R_2}$ | $p_{R_3}$ | $n_{R_1}$ | $n_{R_2}$ | $n_{R_3}$ |
|----------|----------|----------|----------|----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $Z_3^{(1)}$ | $\omega^2$ | $\omega$ | 1 | 1 | 1 | $\omega$ | 1 | $\omega$ | 1 | $\omega$ | 1 | $\omega^2$ | |
| $Z_3^{(2)}$ | 1 | 1 | $\omega$ | $\omega^2$ | 1 | 1 | 1 | $\omega^2$ | $\omega$ | 1 | 1 | |

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|----------|----------|----------|----------|----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $Z_3^{(1)}$ | $\omega^2$ | $\omega$ | 1 | 1 | 1 | $\omega$ | 1 | $\omega$ | 1 | $\omega$ | 1 | $\omega^2$ | |
| $Z_3^{(2)}$ | 1 | 1 | $\omega$ | $\omega^2$ | 1 | 1 | 1 | $\omega^2$ | $\omega$ | 1 | 1 | |
and leaves all other fields invariant. This symmetry commutes with $Z_3^{(2)}$ but it does not commute with $Z_3^{(1)}$. Hence, the internal-symmetry group of the model is $S_3 \times Z_3^{(2)}$.

As a consequence of this internal symmetry, the quark mass matrices take the form

$$M_p = \begin{pmatrix}
  y_1 v_3^2 & y_2 v_2^2 & y_2 v_1^* \\
y_3 v_2^2 & y_4 v_4^* & 0 \\
y_3 v_2 & y_4 v_4 & 0
\end{pmatrix}, \quad M_n = \begin{pmatrix}
  y_5 v_3 & 0 & 0 \\
0 & y_6 v_2 & 0 \\
0 & 0 & y_6 v_1
\end{pmatrix},$$  \hspace{1cm} (2)

where the Yukawa coupling constants $y_1$ to $y_6$ and the vacuum expectation values (VEVs) $v_6 = \langle 0 | \phi_0^6 | 0 \rangle = |v_6| \exp(i\theta_6)$ are in general complex. One identifies $|y_6 v_2| = m_d$, the mass of the down quark, while $|y_6 v_2| = m_s$ and $|y_6 v_1| = m_b$ are the masses of the strange quark and of the bottom quark, respectively. Thus, $|v_2/v_1| = m_s/m_b = r$. This ratio of VEVs being different from 1, the internal symmetry of Eq. (2) is spontaneously broken.

One may eliminate most of the phases in the mass matrices by means of rephasings of the quark fields, obtaining

$$M_p = \begin{pmatrix}
f & r g e^{i\psi} & g \\
r h e^{i\psi} & a & 0 \\
h & a & 0
\end{pmatrix}, \quad M_n = \begin{pmatrix}
m_d & 0 & 0 \\
0 & m_s & 0 \\
0 & 0 & m_b
\end{pmatrix},$$  \hspace{1cm} (3)

where $a, f, g$, and $h$ are real and non-negative. $M_p$ is bi-diagonalized by the Cabibbo–Kobayashi–Maskawa matrix $V$ and another unitary matrix, $U_R^p$:

$$V M_p U_R^p = \text{diag} \left( m_u, m_c, m_t \right).$$  \hspace{1cm} (4)

Thus,

$$H \equiv M_p M_p^\dagger = \begin{pmatrix}
f^2 + g^2 (1 + r^2) & ag + rf h e^{-i\psi} & fh + rage^{i\psi} \\
ag + rf h e^{i\psi} & a^2 + r^2 h^2 & rh^2 e^{i\psi} \\
fh + rage^{-i\psi} & rh^2 e^{-i\psi} & a^2 + h^2
\end{pmatrix} = V^\dagger \begin{pmatrix}
m_u^2 & 0 & 0 \\
0 & m_c^2 & 0 \\
0 & 0 & m_t^2
\end{pmatrix} V,$$  \hspace{1cm} (5)

and one immediately sees that

$$\frac{H_{33} - H_{22}}{|H_{23}|} = \frac{1}{r - r} = \frac{m_b - m_s}{m_s},$$  \hspace{1cm} (6)

Using $H_{22} \ll H_{33} \approx m_t^2$ and $|H_{23}| \approx m_s^2 |V_{ts}|$, one finds the main prediction of this model,

$$|V_{ts}| \approx \frac{m_b m_s}{m_b - m_s} \approx \frac{m_s}{m_b}.$$  \hspace{1cm} (7)

Equation (7) is almost exact. In practice, we may write it with $|V_{ts}|$ substituted by the more interesting parameter $|V_{cb}|$, obtaining the slightly worse approximation

$$|V_{cb}| \approx \frac{m_s}{m_b}.$$  \hspace{1cm} (8)

I use the quark masses renormalized at 1 GeV \footnote{The spontaneous breaking of the interchange symmetry of Eq. (2) follows from its soft breaking in the scalar potential \footnote{The spontaneous breaking of the interchange symmetry of Eq. (2) follows from its soft breaking in the scalar potential which is achieved through the introduction of a term $\mu (\phi_1^2 \phi_2^2 - \phi_3^2 \phi_2^2)$.},

$$m_s = (175 \pm 25) \text{ MeV}, \quad m_b = (5.3 \pm 0.1) \text{ GeV}.$$  \hspace{1cm} (9)

The scale uncertainty on the light-quark masses is substantial, while their ratios are relatively well known; in particular \footnote{The spontaneous breaking of the interchange symmetry of Eq. (2) follows from its soft breaking in the scalar potential which is achieved through the introduction of a term $\mu (\phi_1^2 \phi_2^2 - \phi_3^2 \phi_2^2)$.},

$$\frac{m_s}{m_u} = 34.4 \pm 3.7.$$  \hspace{1cm} (10)

Comparing Eqs. (8) and (9) with the experimental value \footnote{The spontaneous breaking of the interchange symmetry of Eq. (2) follows from its soft breaking in the scalar potential which is achieved through the introduction of a term $\mu (\phi_1^2 \phi_2^2 - \phi_3^2 \phi_2^2)$.},

$$|V_{cb}| = 0.0395 \pm 0.0017,$$  \hspace{1cm} (11)
one sees that the main prediction of the model is quite well verified; as a matter of fact, the smallness of the error bar in Eq. (11) allows us to constrain $m_s$ to be in the highest part of its allowed range:

$$m_s (1 \text{ GeV}) \gtrsim 190 \text{ MeV}. \quad (12)$$

In order to find out other predictions of the model one must treat it numerically. One easily concludes that $h \approx m_t$ and the phase $\psi$ is very close to zero. Contrary to what happens in most Ansätze and textures, $|V_{us}|$ is not related to quark-mass ratios, rather it must be fitted to its experimental value 0.22. The exact value of the top-quark mass $m_t$, being quite high, is practically irrelevant. One finds

$$|V_{ub}/V_{cb}| > 0.085, \quad (13)$$

to be compared with the experimental value $|V_{ub}/V_{cb}| = 0.08 \pm 0.02. \quad (14)$$

My model easily accommodates $|V_{ub}/V_{cb}|$ as large as 0.3.\footnote{The error bar in Eq. (14) is probably under-estimated, as there is a substantial uncertainty in the theoretical modelling of $b \rightarrow s \gamma$ decays. Maybe one should not exclude the possibility that $|V_{ub}/V_{cb}|$ is substantially higher than 0.1.} On the other hand, $|V_{ub}/V_{cb}| \lesssim 0.09$ is only marginally possible.

The CP-violating invariant $J \approx (10, 3)$ is small because $\psi$ is so close to zero. One obtains $|J| < 3 \times 10^{-5}$, but usually $J$ is barely enough to account for the observed value of $\epsilon$. This is not a problem, since there are in the model extra CP-violating box diagrams, in particular those with virtual charged scalars.

In order to work out the Yukawa interactions, one should first expand the scalar doublets as

$$\phi_a = e^{i\theta_a} \left( |v_a| + \frac{\rho_a + i\eta_a}{\sqrt{2}} \right), \quad (15)$$

where $\rho_a$ and $\eta_a$ are Hermitian fields. Their Yukawa interactions are given by

$$\mathcal{L}_Y^{(q)} = \cdots - \frac{(\rho_1 + i\eta_1) m_b b_L b_R}{\sqrt{2} |v_1|} - \frac{(\rho_2 + i\eta_2) m_s s_L s_R}{\sqrt{2} |v_2|} - \frac{(\rho_3 + i\eta_3) m_d d_L d_R}{\sqrt{2} |v_3|}$$

$$- \left( \overline{t_L} \right) V \begin{pmatrix} \rho_2 - i\eta_2 & 0 & \rho_1 - i\eta_1 \\ \rho_3 - i\eta_3 & \rho_1 - i\eta_1 & 0 \\ \rho_4 - i\eta_4 & 0 & \rho_4 - i\eta_4 \end{pmatrix} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}. \quad (16)$$

In the first line of Eq. (16) one observes the absence of FCNYI with the down-type quarks. Unfortunately, the physical neutral scalars result from an unspecified orthogonal rotation of the $\rho_a$ and $\eta_a$—wherein $\sum_{a=1}^4 |v_a| \eta_a$ is a Goldstone boson. This is equivalent to saying that the symmetries of the model do not determine the masses and mixings of the neutral scalars. Moreover, the constraints $|v_2/v_1| = m_s/m_b$ and $\sum_{a=1}^4 |v_a|^2 = (2\sqrt{2} G_F)^{-1}$ are insufficient to determine the four $|v_a|$. Under these conditions, any attempt at an evaluation of the effects of the neutral Yukawa interactions—in particular enhanced $D^0 - \bar{D}^0$ mixing, which might be an interesting consequence of the present model—cannot be rigorous and has little genuine value.\footnote{The studies of the FCNYI in Ma’s model suffer from the same limitation: many assumptions about the values of the $|v_a|$ and the neutral-scalar mixings had to be done.} The same may of course be said about an evaluation of the effects of the charged Yukawa interactions, including their contribution to $\epsilon$.

In conclusion, I have shown that, in Ma’s model for the quark mass matrices, the roles of the up-type and down-type quarks may be interchanged, one then obtaining a different viable model. The new model makes the sharp prediction $|V_{cb}| \approx m_s/m_b$ and forces $m_s$ and $|V_{ub}/V_{cb}|$ to be close to the upper end of their allowed ranges. Flavour-changing neutral Yukawa interactions are absent from the charge–$-1/3$-quark sector. The model has the distinctive advantages of having a well-defined field content and horizontal symmetry, and of not containing any poorly justified assumptions.
References

[1] E. Ma, Phys. Rev. D 43, R2761 (1991).

[2] L. Lavoura, Phys. Rev. D 44, 1610 (1991).

[3] L. Lavoura, Phys. Rev. D 46, 4101 (1992).

[4] Recent instances include: H. Fritzsch and J. Plankl, Phys. Lett. B 237, 451 (1990); G. C. Branco and L. Lavoura, Phys. Rev. D 44, R582 (1991); G. F. Giudice, Mod. Phys. Lett. A 7, 2429 (1992); P. Ramond, R. G. Roberts, and G. G. Ross, Nucl. Phys. 406B, 19 (1993); G. C. Branco and J. I. Silva-Marcos, Phys. Lett. B 331, 390 (1994); H. Harayama, N. Okamura, A. I. Sanda, and Z.-Z. Xing, Prog. Theor. Phys. 97, 781 (1997).

[5] J. Gasser and H. Leutwyler, Phys. Rep. 87, 77 (1982).

[6] H. Leutwyler, Phys. Lett. B 378, 313 (1996).

[7] Particle Data Group (C. Caso et al.), Eur. Phys. J. C 3, 1 (1998).

[8] O. Pène, private communication.

[9] C. Jarlskog, Phys. Rev. Lett. 55, 1039 (1995); I. Dunietz, O. W. Greenberg, and D.-D. Wu, Phys. Rev. Lett. 55, 2935 (1995).

[10] N. G. Deshpande, M. Gupta, and P. B. Pal, Phys. Rev. D 45, 953 (1992).