On the Spontaneous Identity of Chiral and Super Symmetry Breaking in Pure Super Yang Mills Theories

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Abstract
We show that in supersymmetric pure Yang Mills theories with arbitrary simple gauge group, the spontaneous breaking of chiral fermionic and bosonic charge by the associated gaugino and gauge boson condensates implies the spontaneous breaking of supersymmetry by the condensate of the underlying Lagrangian density. The explicit breaking of the restricted fermionic charge through the chiral anomaly is deferred to a secondary stage in the elimination of infrared singularities or long range forces.

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1 Introduction

The question whether supersymmetry is spontaneously broken or not is of fundamental importance. Many results concerning this problem have been derived in the literature. We know that perturbative corrections do not break supersymmetry. What happens nonperturbatively is not yet clear since there is no theory available to describe this regime.

A particular question is, whether or not there is a relation between the breaking of chiral symmetry and the breaking of supersymmetry. In the case of $N = 1$ pure SYM theories this question has, besides other things, been addressed in [1]. The cited work concludes, that supersymmetry remains unbroken after the breakdown of chiral symmetry. This conclusion is incomplete, since the effective action derived in this analysis does not have a definite chiral weight. Potential $\theta$ angles do not relax, in which case chiral symmetry breaking entails spontaneous CP violation.

We propose to use an appropriate thermodynamical limit of the pure SYM theory. To explore the response of the theory to perturbations we introduce a multiplet of external fields. This is analogous to measuring the hysteresis lines of a ferromagnet exposed to an external magnetic field. In particular one is forced to explicitly break susy by introducing an arbitrary complex mass for the gaugino, corresponding to breaking the rotational invariance of the ferromagnet by the external magnetic field. Note that every potential spontaneous phenomenon necessarily entails a thermodynamical external field extension. Whether the spontaneous phenomenon actually occurs is a dynamical question and is tantamount to follow the relaxation of the static external symmetry breaking fields. The situation in QCD with $n$ massless quark flavors [2] is identical in the sense that the intermediary external field extension necessarily incorporates nonvanishing complex quark masses.

In the thermodynamic limit the $\theta$ angles relax and chiral symmetry [3] is restored. Therefore the different terms in the effective action must have a definite weight under chiral rotations. This knowledge together with the supersymmetric form of the effective action extended to include the external sources suffices to derive consistency conditions for the formation of gaugino and gauge boson condensates. It then follows that in the limit of vanishing gaugino mass, the gaugino condensate can not exist without a corresponding condensate of the gauge bosons and vice versa. Since the gauge boson condensate breaks susy, there is an intimate relation between chiral and supersymmetry breaking. Assuming a confining mechanism similar to the one in QCD, we conclude that susy must be broken. This result does not agree with the conclusion obtained in [4]. However our derivation of the thermodynamical limit involves the use of nontrivial boundary conditions, whereas in [4] trivial boundary conditions are assumed.

This paper is organized in the following manner. In section 2 we derive
the thermodynamical limit of the pure SYM theory. The chiral anomaly
together with the relaxation of the $\theta$ angles is discussed in section 3. Section 4 contains the final link between chiral and susy breaking. We summarize
our results in section 5 and give the derivation of the path measure in the appendix.

2 The thermodynamic limit of the $N = 1$ SYM theory

In this section we discuss the thermodynamic limit of a pure super Yang
Mills theory with simple gauge group $G$.

In order to explore the hysteresis lines of the SYM theory in the thermo-
dynamic limit, we use a chiral multiplet of sources to drive the operators in
the SYM Lagrangian. It is convenient to write the action of the theory as
bilinear in the chiral external source superfield $J$ and the chiral superfield

$$\Phi = \frac{1}{4} \frac{1}{C_2(G)} \text{tr} W_\alpha W_\alpha$$

where $W_\alpha$ denotes the superspace field strength $W_\alpha = -\bar{D}^2 e^{-V} D_\alpha e^V$ of the
vector superfield $V$. In the Wess-Zumino gauge its components are

$$W_\alpha(y, \theta) = \lambda_\alpha(y) + \theta_\alpha D(y) + \theta^\beta i F_{\alpha\beta}(y) + \theta^2 i D_{\alpha\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}}(y)$$

The component fields $\lambda_\alpha, F_{\alpha\beta} = \frac{1}{4} (\sigma^{[\mu} \bar{\sigma}^{\nu]})_{\alpha\beta} F_{\mu\nu}$ and $D$ in eq. (2) depend on
$\theta^\mu = x^\mu - \frac{i}{2} \partial \sigma^{\mu} \theta$ and are in the adjoint representation of the gauge group $G$,

$$\lambda = \lambda^a T^a, \quad F_{\mu\nu} = F_{a\mu\nu} T^a, \quad D = D^a T^a, \quad \text{tr} T^a T^b = C_2(G) \delta^{ab}$$

The action of the covariant derivative on the gaugino is

$$D_{\alpha\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}} = \partial_{\alpha\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}} + i [A_{\alpha\dot{\alpha}}, \bar{\lambda}^{\dot{\alpha}}]$$

where $A_{\alpha\dot{\alpha}} = \sigma^{\mu\dot{\alpha}} A_\mu$ denotes the gauge field. After rescaling the gaugino to
its conventional normalization $\lambda = \sqrt{2} \Lambda$, the chiral superfield $\Phi$ defined in
eq. (1) contains the $x$ dependent component fields

$$\Phi = \partial^2 \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \Lambda^a D \bar{\lambda}^b + i \frac{1}{4} F_{a\mu\nu} \tilde{F}^{a\mu\nu} + i \frac{1}{4} \partial_\mu (\Lambda^a \sigma^\mu \bar{\Lambda}^a) \right)$$

$$+ \partial^\alpha \left( -\frac{1}{\sqrt{2}} \Lambda^\alpha \bar{F}_{\beta} \right) + \frac{1}{2} \Lambda^\alpha \Lambda^a \Lambda_\alpha$$

In the sequel we will use the reparametrization

$$\Phi = \partial^2 L(x) + \partial^\alpha \psi_\alpha(x) + \frac{1}{2} z(x)$$
The source multiplet $J$ is a full chiral superfield,

$$J = -\vartheta^2 m(x) + \vartheta^\alpha \eta_\alpha(x) + \frac{1}{2} j(x)$$  \hspace{1cm} (7)

All the variables $L, \psi$ and $z$ as well as their sources $j, \eta_\alpha$ and $m$ are a priori complex.

A generic bilinear in $\Phi$ and $J$ is

$$\Phi \cdot J = \vartheta^2 \left( \frac{1}{2} (jL - mz) - \psi^\alpha \eta_\alpha \right) + \frac{1}{2} \vartheta^\alpha (j\psi_\alpha + z\eta_\alpha) + \frac{1}{4} jz$$  \hspace{1cm} (8)

Since we are only interested in gaugino and gauge boson condensates, we set the fermionic source $\eta_\alpha$ to zero in what follows. Projecting onto the highest $\vartheta$-component leads to the generic external field Lagrangian density

$$L_J = \int d^2 \vartheta (\Phi \cdot J + \text{h.c.}) = \text{Re}(jL - mz)$$  \hspace{1cm} (9)

Choosing

$$j(x) = \frac{1}{g^2(x)} - i \frac{\theta(x)}{8\pi^2}, \quad m(x) = m_1(x) - i m_2(x)$$  \hspace{1cm} (10)

and rescaling the component fields according to

$$A \to gA, \quad \Lambda \to g\Lambda, \quad D \to gD$$  \hspace{1cm} (11)

we find the component fields of the generic Lagrangian

$$L_J = - \frac{1}{4g^2(x)} F_{\mu\nu}^a F^{a\mu\nu} + \frac{\theta(x)}{32\pi^2} F_{\mu\nu}^a \tilde{F}^{a\mu\nu}$$

$$+ \frac{1}{2} \Lambda^a_i \partial^i \Lambda^b + \frac{\theta(x)}{16\pi^2} \partial_\mu (\Lambda^a_{\sigma \mu} \Lambda^a)$$

$$- \frac{1}{2} m_1(x)(\Lambda^{ao} \Lambda_a^o + \Lambda_a^o \Lambda^{ao}) - i m_2(x)(\Lambda^{ao} \Lambda_a^o - \Lambda_a^o \Lambda^{ao})$$  \hspace{1cm} (12)

The SYM Lagrangian with restored coupling constants is represented in the analogous way

$$L_0 = \int d^2 \vartheta (\Phi \cdot J_0 + \text{h.c.}), \quad J_0 = \frac{1}{2} \left( \frac{1}{g_0^2} - i \frac{\theta_0}{8\pi^2} \right) = \text{const.}$$  \hspace{1cm} (13)

The generating functional for the Green functions in the presence of general $x$ dependent external sources $J$ is

$$Z[J] = e^{iW[J]} = \int \mathcal{D}\mu e^{iS_0 + iS_J}$$  \hspace{1cm} (14)
Remember that $J_0$ and $J$ are sources for the same operators. The separation into $S_0$ and $S_J$ is enforced by imposing the boundary conditions eq. (13) on $J_0$ as well as

$$\lim_{x \to \infty} J(x) = 0$$  \hspace{1cm} (15)$$

Functional integration in the susy environment demands control over the complete set of susy variables [5]. Since finding the path measure $D\mu$ does not directly interfere with what follows, we discuss its derivation together with BRS gauge fixing in the appendix. We note that the procedure adopted eliminates ab initio the auxiliary components of the vector superfield $V$ and imposes after the elimination of all non propagating components the persistent use of of the Wess - Zumino gauge. This is in contrast to full susy chiral gauge fixing [6].

In the context of eq. (14) the notion of susy demands a twofold interpretation. By extrinsic susy we mean the full supersymmetry the action $S_0 + S_J$ has by construction. Intrinsic susy is the invariance of the highest component $L$ of the chiral superfield $\Phi$. Choosing appropriate sources in $J$ will break intrinsic susy because the algebra no longer closes into $L$. Extrinsic susy however remains unbroken for arbitrary external fields $J$.

The generating functional for the connected Green functions $W[J]$ obeys the usual relations

$$\frac{\delta W}{\delta j(x)} = \frac{1}{2} \langle L(x) \rangle = \frac{1}{2} L_{cl}(x), \quad \frac{\delta W}{\delta m(x)} = -\frac{1}{2} \langle z(x) \rangle = -\frac{1}{2} z_{cl}(x)$$ \hspace{1cm} (16)$$

The effective action $\Gamma [\Phi]$ is given by the Legendre transform of $W[J]$\hfill

$$\Gamma [\Phi] = \text{Re} \int d^4 x \left( L_{cl}(x)j(x) - z_{cl}(x)m(x) \right) - W[J]$$ \hspace{1cm} (17)$$

The sources that create prescribed classical fields $L_{cl}$ and $z_{cl}$ are given by the functional derivatives of the effective action with respect to the fields

$$\frac{\delta \Gamma}{\delta L_{cl}(x)} = \frac{1}{2} j(x), \quad \frac{\delta \Gamma}{\delta z_{cl}(x)} = -\frac{1}{2} m(x)$$ \hspace{1cm} (18)$$

The relations in eqs. (16) and (18) are equilibrium conditions for an infinitesimal volume located at $x$. In other words, the system is in equilibrium only if both sets of equations are valid simultaneously for all $x$. To go to the static limit we choose the external fields $j, m$ to be almost constant inside a small sub-volume $V_{\text{sub}}$ of the spacetime $V$ and vanishing on the complement $V \setminus V_{\text{sub}}$ according to eq. (15). Then we take the infinite volume limit $V_{\text{sub}} \subset V \to \infty$. Technically this is equivalent to choosing new boundary conditions

$$\lim_{x \to \infty} J(x) = -g^2 m_{\infty} + \frac{1}{g_{\infty}^2} - i \frac{\theta_{\infty}}{8\pi^2}$$ \hspace{1cm} (19)$$

4
Comparing with eqs. (13) and (15) we see, that the boundary values $g_{\infty}^2, \theta_{\infty}$ and $m_{\infty}$ can be absorbed into $S_0$ by the redefinition

$$J_0 \rightarrow -\vartheta^2 m_{\infty} + \frac{1}{2} \left( \frac{1}{g_0^2} + \frac{1}{g_{\infty}^2} - i \frac{\theta_0 + \theta_{\infty}}{8\pi^2} \right)$$

(20)

Transferring the boundary values from the source $J$ to the quantum mechanical Lagrangian modifies the definition of the latter relative to eq. (13). Together with renormalization this amounts to the substitutions

$$J_0 \rightarrow -\vartheta^2 m_R + \frac{1}{2} \left( \frac{1}{g_R^2} - i \frac{\theta_R}{8\pi^2} \right)$$

$$J \rightarrow -\vartheta^2 (m(x) - m_R) + \frac{1}{2} \left( \frac{1}{g_{R}^2} - \frac{1}{g_R^2} - i \frac{\theta(x) - \theta_R}{8\pi^2} \right)$$

(21)

The subscript $R$ refers to the renormalization group invariant couplings. Since $g_R, \theta_R$ are constants we drop the surface term proportional to $\partial_\mu (\Lambda^a \sigma^\mu \bar{\Lambda}^a)$ and find the redefined Lagrangian

$$\mathcal{L}'_0 = -\frac{1}{4g_R^2} F_{\mu\nu}^a F^{a\mu\nu} + \frac{\theta_R}{32\pi^2} F_{\mu\nu}^a \tilde{F}^{a\mu\nu} + \frac{1}{2} \Lambda^a i D^b \bar{\Lambda}^b$$

$$- \frac{1}{2} m_{1R} (\Lambda^{a\alpha} \Lambda^a_{\alpha} + \bar{\Lambda}^a_{\dot{\alpha}} \bar{\Lambda}^{a\dot{\alpha}}) - i \frac{1}{2} m_{2R} (\Lambda^{a\alpha} \Lambda^a_{\alpha} - \bar{\Lambda}^a_{\dot{\alpha}} \bar{\Lambda}^{a\dot{\alpha}})$$

(22)

The above equation shows, that extrinsic susy incorporates in the quest of a minimal set of equilibrium conditions an ensemble of quantum mechanical Lagrangians with arbitrary complex external gaugino masses $m_{1R}, m_{2R}$. These mass terms break intrinsic susy. The question to answer is, whether or not there remains a trace of this breaking in the limit $m_{1R}, m_{2R} \rightarrow 0$.

In order to construct an effective action simultaneously satisfying the relations eqs. (16, 17, 18) we extend the above procedure to arbitrary combinations of sources $J$ inside $V_{\text{sub}}$ and $J'$ on the complement $V \setminus V_{\text{sub}}$. Taking the infinite volume limit for all possible combinations of sources and sub-volumes finally leads to the effective action via the relations

$$\frac{\partial}{\partial \Phi} \Gamma(\Phi; J'; J) = J \leftrightarrow J = J' \quad \text{and} \quad J = J' \rightarrow 0$$

(23)

These operations are to be performed from left to right and in all permuted sequences. Demanding $J = J'$ in the last step incorporates the equilibrium condition for the two volumes in contact and involves the transfer of the boundary conditions to the quantum mechanical Lagrangian according to eqs. (24 - 22).

Performing the infinite volume limit in all conceivable ways serves to explore the effective action in the thermodynamic limit, which is in principle reached herewith. Of course we cannot analytically go to this limit. Nevertheless it turns out to be a useful gedanken-experiment, since extrinsic susy
together with the equilibrium conditions found are strong enough to imply
relations between different kinds of condensates in the above limit.

Note that in the thermodynamical limit the effective action plays the
role of the inner potential of the theory. Therefore we use the two notions
interchangeable in what follows.

3 The chiral anomaly and the effective action

Up to now the form of the effective action or inner potential in the thermo-
dynamical limit is only restricted by the validity of extrinsic susy. In this
section we will show, that in this limit the chiral invariance is restored, lead-
ing to further restrictions on the Kähler and superpotential that describe
the inner potential.

To show that the renormalized quantities defined in eq. (22) have a phys-
ical meaning beyond perturbation theory we first have to show the existence
of a renormalization scheme that leaves the chiral and trace anomalies in
the same multiplet to all orders in perturbation theory.

The structure of the axial current anomaly to one loop order is

$$
\partial_{\mu}(\Lambda^a \sigma^\mu \bar{\Lambda}^a) = 2C_2(G) \frac{1}{8\pi^2} \left( \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} \right) = 2C_2(G) \text{ch}_2(G)
$$

In parallel the trace of the energy momentum tensor satisfies the anomalous
Ward identity to all orders [7]

$$
\partial^\mu \phi_{\mu} = -3C_2(G) \frac{1}{8\pi^2} \frac{\beta(g)}{\beta_{\text{1 loop}}(g)} \left( \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)
$$

$$
\frac{\beta(g)}{\beta_{\text{1 loop}}(g)} = 1 + \frac{1}{b_1} \sum_{n=2}^{\infty} b_n \kappa^n = 1 + 2C_2(G) \kappa + \cdots
$$

where the rationalized coupling is defined to be $\kappa = \left( \frac{\alpha}{4\pi} \right)^2$. Provided
$\beta_{\text{1 loop}} \neq 0$ the higher loop corrections simply rescale the the renormal-
ization group non-invariant operator $\frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu}$ normalized to its off shell,
$\mu$ scaled two gauge boson matrix element being unity, such as to render the
product renormalization group invariant. At two and higher loop level the
renormalized operators $\Lambda^a \sigma^\mu \bar{\Lambda}^a$ and $\text{ch}_2(F)$ develop anomalous dimension
functions $\gamma(\kappa)$ [8], identical by the Adler Bardeen non-renormalization the-
orem. This leads to a modification of the anomalous chiral Ward identity in
eq. (24) analogous to the higher loop modifications of the trace anomaly.
Keeping the chiral current renormalization group invariant and defining
the non-invariant operator $\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu}$ through the analogous normaliza-
tion procedure to its parity partner $\frac{1}{4\pi^2} F_{\mu\nu} F^{\alpha\mu\nu}$,

$$\partial_\mu (\Lambda^\alpha \sigma^\mu \bar{\Lambda}^\alpha) = 2C_2(G) \frac{1}{8\pi^2} \varepsilon(\kappa) \left( \frac{1}{4} F_{\mu\nu} \tilde{F}^{\alpha\mu\nu} \right),$$

$$\varepsilon(\kappa) = \exp \int_0^\kappa \frac{\gamma(k)}{b(k)}, b(\kappa) = -\beta(g) g$$

it follows [3] that the two field strength bilinears, related by susy, are identically renormalized to two loop order. This fact has been discussed in [10] and using superspace integration techniques in conjunction with dimensional reduction in [11]. Thus susy covariance extended to the full set of superconformal, partially anomalous Ward identities ensures the existence of a renormalization scheme with the all order ultraviolet property

$$\frac{\beta(g)}{\beta_{1 \text{ loop}}(g)} = \varepsilon(\kappa) \quad \text{to all orders in } g \quad \quad (27)$$

Since in this framework $b_1 > 0$, the renormalization effects beyond one loop simply lead to a redefinition of perturbatively renormalization group invariant composite operators. For the right hand side of the chiral Ward identity eq. (26) the all order framework is clearly insufficient due to the exact quantization condition pertinent to the Chern character $\frac{1}{8\pi^2} \varepsilon(\kappa) \left( \frac{1}{4} F_{\mu\nu} \tilde{F}^{\alpha\mu\nu} \right)$ when continued to Euclidean space.

The existence of a renormalization scheme with the property eq. (27) guarantees that the renormalized quantities defined in eq. (22) have a physical meaning beyond perturbation theory.

Using the renormalization group invariant quantities defined in eq. (22) the chiral anomaly of the pure SYM theory in the thermodynamic limit is given by

$$\partial_\mu (\Lambda^\alpha \sigma^\mu \bar{\Lambda}^\alpha) = 2C_2(G) \frac{1}{8\pi^2} \left( \frac{1}{4} F_{\mu\nu} \tilde{F}^{\alpha\mu\nu} \right) - i(m_R \Lambda^\alpha \bar{\Lambda}^\alpha - \text{h.c}) \quad \quad (28)$$

Thus the generating functional eq. (14) satisfies the relation

$$2 \left( \nu \frac{\partial}{\partial \theta} + \frac{\partial}{\partial \text{arg } m} \right) Z = 0 \quad \rightarrow \quad Z = Z \left( \frac{\theta}{\nu} - \text{arg } m \right) \quad \quad (29)$$

where $\nu = C_2(G)$. The discrete symmetry $Z_\nu$,

$$\theta \rightarrow \theta + 2\pi \frac{r}{\nu} \quad \text{for } r = 0, 1, 2, \ldots, \nu - 1 \quad \quad (30)$$

of the generating functional $Z$ is related to fixed time large gauge transformations. Therefore we do not allow the $Z_\nu$ symmetry to be broken spontaneously. This point has been discussed for $\nu \rightarrow \infty$ in [12]. In the thermodynamic limit our analysis agrees.
For simplicity we drop the subscript \( R \) on the components of \( J_0 \) with the understanding that \( J_0 \) always implies the use of the renormalization group invariant quantities.

Using the composite variables \( \chi, \bar{\chi} \) defined through the relation

\[
\chi = e^{-8\pi^2 j_0} = |\chi| e^{i\theta}, \quad |\chi| = e^{-\frac{8\pi^2}{9\nu}}
\]

\[
\chi_{1/\nu} = |\chi_{1/\nu}| e^{i\frac{\theta}{\nu}} \mod Z_\nu, \quad |\chi_{1/\nu}| = e^{-\frac{8\pi^2}{9\nu^2}}
\]

we see, that the phase dependence of \( \Gamma[\Phi] \) is through the products \( \bar{\chi}_{1/\nu} m, \chi_{1/\nu} \bar{m}, \chi_{1/\nu} z \) and \( \bar{\chi}_{1/\nu} \bar{z} \) (33)

The equilibrium conditions in eq. (29) with respect to the imaginary part of the source \( j, j' \) are tantamount to determine the minimum of \( \Gamma[\Phi] \) with respect to the boundary values \( \theta, \theta' \), as discussed in [2]. This minimum condition relaxes \( \theta, \theta' \) to the values

\[
\theta - \theta' = \nu \arg m \mod Z_\nu
\]

(34)
guaranteed by CPT invariance. To exclude nontrivial minima in the relative phase \( \Delta \theta = \theta - \theta' - \nu \arg m \) we remark that in the Euclidean case

\[
e^{-\Gamma_{\text{eucl.}} \sim e^{+\cos (\Delta \theta |z_{\text{cl}}|)}}
\]

(35)

Therefore the minima of \( \Gamma \) are at \( \Delta \theta = 0 \mod 2\pi \) and are absolute minima. Thus the \( \theta, \theta' \) angles always relax in the thermodynamic limit, allowing us to eliminate the variables dual to \( j, j' \) first and to restrict the discussion to the remaining variables related to \( m \).

The relaxation of the \( \theta \) angles restores the CP invariance when all equilibrium conditions are met. Equivalently this can be interpreted as restoration of the anomalous chiral symmetry in the thermodynamical limit.

4 Linking spontaneous breaking of susy and restored chiral symmetry

In the last step we collect the knowledge of the form of the inner potential restricted by extrinsic susy together with the restrictions from the restored chiral invariance and the equilibrium conditions to derive relations between the formation of the gaugino and gauge boson condensates.

As a consequence of the relaxation of the external variable \( j, j' \) discussed at the end of the previous section, the equilibrium conditions in eqs. (16) and (18) become

\[
\frac{\partial}{\partial L_{\text{cl}}(x)} \Gamma(\Phi; J') = \frac{1}{2} j(x) \rightarrow 0
\]

(36)
The above relation is the appropriate one for $L_{\text{cl}}$ and $\bar{L}_{\text{cl}}$ being auxiliary fields of respective chiral multiplets and the associated potential $\Gamma$ being minimized in the process of eliminating them.

As discussed earlier extrinsic susy is strictly valid all the way and restricts the effective action to be of the form

$$\Gamma(L_{\text{cl}}, z_{\text{cl}}, \bar{L}_{\text{cl}}, \tilde{z}_{\text{cl}}; m', \bar{m}') = -\bar{L}_{\text{cl}}K_{zz}L_{\text{cl}} + (L_{\text{cl}}W + \text{h.c.})$$ (37)

where the factors $\chi_{1/\nu}$ and $\bar{\chi}_{1/\nu}$ now are sub-summed in the variables $z_{\text{cl}}$ and $\tilde{z}_{\text{cl}}$. The functions $K(z_{\text{cl}}, \tilde{z}_{\text{cl}}; m', \bar{m}')$ and $W(z_{\text{cl}}; m', \bar{m}')$ denote the Kähler and super potential respectively. The Kähler metric is given by

$$K_{zz} = \frac{\partial^2 K}{\partial z_{\text{cl}} \partial \bar{z}_{\text{cl}}}$$ whereas $W_z = \frac{\partial W}{\partial z_{\text{cl}}}$. After transferring the boundary values of $m'$ to the quantum Lagrangian the equilibrium value of $m'$ is zero and the equilibrium condition becomes

$$\frac{\partial \Gamma}{\partial L_{\text{cl}}} = 0 \quad \text{with} \quad \Gamma = \Gamma(L_{\text{cl}}, z_{\text{cl}}, \bar{L}_{\text{cl}}, \tilde{z}_{\text{cl}}; m, \bar{m})$$ (38)

where now the effective action is to be calculated with fixed Lagrangian masses $m$ and for prescribed constant values of $z_{\text{cl}}$.

Eliminating the auxiliary fields $L_{\text{cl}}$ and $\bar{L}_{\text{cl}}$ according to eq. (38) we obtain

$$\bar{L}_{\text{cl}} = \frac{W_z}{K_{zz}} \rightarrow \Gamma = \frac{|W_z|^2}{K_{zz}}$$ (39)

The superpotential is an analytic function of its natural variable $z_{\text{cl}}$ and parametrically depends on both $m$ and $\bar{m}$. The induced Kähler metric must be positive to ensure stability of large volume fluctuations. This is the positivity property of the internal potential that is well known from semi classical approximations.

The relaxation of $\theta$ angles discussed at the end of section 3 implies the restored chiral invariance of the inner potential under the phase rotations

$$z_{\text{cl}} \rightarrow e^{-i\xi}z_{\text{cl}}, \quad m \rightarrow e^{+i\xi}m \quad \text{and c.c.}$$ (40)

assigning chiral weights 1 and -1 to $z_{\text{cl}}$ and $m$. Hence the superpotential must have a well defined chiral charge whereas the Kähler potential must be neutral.

Finally we relax the gaugino masses $m \rightarrow 0$, 

$$\Gamma(|z_{\text{cl}}|^2; zm, \bar{z}m, |m|^2) \rightarrow \Gamma(|z_{\text{cl}}|^2; 0, 0, 0)$$ (41)

It follows from eq. (41) that the potential $\Gamma$ attains its minimum along a circle in the complex $z_{\text{cl}}$ plane. However the only circle where $L_{\text{cl}} \propto W_z$ can vanish is at the origin $z_{\text{cl}} = 0$. Therefore the only consistent solution with no spontaneous breaking of susy does not admit any condensates. Then
neither gauginos nor gauge bosons can be confined, since a 'wall' to reverse their chirality upon reflection is absent.

The remaining alternative of nontrivial condensate formation — short of spontaneous breaking of gauge invariance — links the gaugino condensates \( z_{cl}, \bar{z}_{cl} \) with their gauge boson partners \( L_{cl}, \bar{L}_{cl} \). The latter spontaneously break susy at a positive value of \( \Gamma \).

Let us discuss the form of the Kähler and super potential for this last case. The inner potential is nonzero and positive at its minimum. Thus \( W_z \) is nonzero, in which case the function \( W = W(z_{cl}) \) is invertible. After the change of coordinates \( z_{cl} = z_{cl}(W) \) the Kähler potential eq. (39) becomes

\[
K_{WW} = \frac{1}{\Gamma}
\]

where \( K \) and \( \Gamma \) depend on \( W \) and \( \bar{W} \). With the substitution \( r = \sqrt{WW} \) eq. (12) is the radial part of the inhomogeneous 2-dimensional Laplace equation

\[
\left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right) K(r) = \frac{4}{\Gamma(r)}
\]

Notice that the source term \( \frac{4}{\Gamma(r)} \) has a finite maximum at the minimum of \( \Gamma \) and goes to finite boundary values at zero and infinity. Given \( \Gamma \) we can solve eq. (43) for \( K \),

\[
K(r) = k + \int_0^r \frac{dr'}{r'} \int_0^{r'} \frac{dr''}{r''} \frac{4r''}{\Gamma(r'')}
\]

The equations derived so far are all structural equations coming from symmetries. The actual form of the effective potential can not be derived from symmetries. It represents the genuine dynamics of the system.

Finally we emphasize that the conclusions reached are not without consequences for more general supersymmetric theories.

5 Summary and conclusions

In the thermodynamical limit of a pure SYM theory the form of the effective action is restricted by the validity of extrinsic susy. The restored chiral invariance valid in this limit further restricts the Kähler metric and superpotential describing the effective action. The equilibrium conditions for the SYM system in contact with external sources driving the operators in the Lagrangian as well as gaugino mass terms link the condensate of the gaugino to the condensate of the Lagrangian and vice versa. The remaining alternatives are: The gaugino condensate is zero and there are no gauge boson condensates, or the gaugino condensate is linked to the gauge boson condensate. Assuming confinement susy must be broken.
This result is in contradiction to the conclusion in [4] since we do not meet the assumption of trivial boundary condition for the various fields while deriving the thermodynamical limit.

A Gauge fixing and extended functional measure

To derive the functional measure eq. (14) we start from the complexified superfield $V$, containing unconstrained components. The extension of $V$ to unconstrained complex components allows us to extend the Lorentz group, i.e. its covering group $SL(2, C)$ to its complex form $SL(2, C) \times SL(2, C)$ accompanied by the extension of configuration space coordinates $x^\mu$ to complex values $z^\mu$. In addition we also extend the fermionic coordinates $\vartheta^{\dot{\alpha}}, \bar{\vartheta}^{\dot{\alpha}}$ of $N = 1$ superspace to $\vartheta^{\alpha}, \bar{\vartheta}^{\dot{\alpha}}$, where $\vartheta$ no longer is related to $\bar{\vartheta}$ by complex conjugation.

The now unrelated susy covariant derivatives become

$$D_\alpha = \partial_\alpha - \frac{1}{2} i \partial_\alphadot \bar{\vartheta}^\beta, \quad \bar{D}_\beta = - i \bar{\vartheta}^\alphadot \partial_\beta + \frac{1}{2} \vartheta^{\alpha} i \partial_\alphadot \beta \quad (45)$$

We can now perform gauge fixing à la BRS using throughout complexified fields chiral with respect to the extended Grassmann variables

$$\Theta^A = (\vartheta^\alpha, \bar{\vartheta}^{\dot{\alpha}}), \quad \partial_A = (\partial_\alpha, \bar{\partial}^{\dot{\alpha}}), \quad \{\partial_A, \Theta^B\} = \delta^B_A \quad (46)$$

To this end we use the complexified connections

$$w_\alpha = e^{-V} D_\alpha e^V, \quad w'_\dot{\alpha} = e^{-V} \bar{D}_{\dot{\alpha}} e^V \quad (47)$$

The connections in eq. (47) only involve the restricted pairs $\vartheta^\alpha$ and $\bar{\vartheta}^{\dot{\alpha}}$ separately and satisfy the relations

$$D_\alpha w_\beta + D_\beta w_\alpha + \{w_\alpha, w_\beta\} = 0$$
$$\bar{D}_{\dot{\alpha}} w'_{\dot{\beta}} + \bar{D}_{\dot{\beta}} w'_{\dot{\alpha}} + \{w'_{\dot{\alpha}}, w'_{\dot{\beta}}\} = 0 \quad (48)$$

Next we set the specific constraints

$$N = D^\alpha w_\alpha, \quad N' = \bar{D}_{\dot{\beta}} w'_{\dot{\beta}}, \quad N = N + N' \quad (49)$$

The combined chiral field $N$ is a hermitian field for restricted hermitian values of $V$ and real space time variables $z^\mu$ in physical space time.

The gauge is fixed by choosing a fermionic vector field $c$ with components in the same basis as $V$ but with opposite fermion parity relative to those in $V$. The field $c$ is to be interpreted in complexified extension.

$$\nabla_\alpha c = D_\alpha c + \{w_\alpha, c\}$$
$$\bar{\nabla}_{\dot{\beta}} c = \bar{D}_{\dot{\beta}} c + \{w'_{\dot{\beta}}, c\} \quad (50)$$
The fermionic BRS operator $S$ is defined as operating on $w_\alpha$, $w_\beta'$ and $c$ like

$$S \left( \begin{array}{c} w_\alpha \\ w_\beta' \end{array} \right) = \left( \begin{array}{c} \nabla_\alpha \\ \nabla_\beta' \end{array} \right) c, \quad Sc = c^2, \quad S^2 = 0 \quad (51)$$

The nilpotency of $S$ follows from its fermionic properties

$$SD_\alpha = -D_\alpha S, \quad S\tilde{D}_\beta = -\tilde{D}_\beta S$$

$$S(f_1f_2) = (Sf_1)f_2 - f_1(Sf_2) \quad (52)$$

with respect to a pair of fermionic superfields $f_1, f_2$.

The BRS operation is completed extending the nilpotent action of $S$ to the fermionic and bosonic vector superfields $\tilde{c}$ and $\tilde{b}$ respectively by the relations

$$S\tilde{c} = \tilde{b}, \quad S\tilde{b} = 0 \quad (53)$$

In our complexified environment the fields $c$, $\tilde{c}$ and $\tilde{b}$ are independent before all BRS operations are performed. Subsequently they are subjected to reality constraints as shown below.

We form the gauge fixing superfields $f$ (fermi) and $g = Sf$ from the constraint superfields $N$ in eq. (49) by means of the Lie frame independent traces

$$f = \text{tr} \tilde{c} \left( \eta \tilde{b} + N \right), \quad g = \text{tr} S \left[ \tilde{c} \left( \eta \tilde{b} + N \right) \right] \equiv Sf \quad (54)$$

In eq. (54) $\eta$ denotes a real parameter, defining Fermi gauges for gauge bosons together with the gauge invariant part of the action.

Finally we form the complex densities $F$ and $G$ associated with the superfields $f$ and $g$ respectively

$$\left( \begin{array}{c} F \\ G \end{array} \right) = \int d^2\theta d^2\bar{\theta} \left( \begin{array}{c} F \\ G \end{array} \right), \quad G \equiv SF \quad (55)$$

The following steps yield the gauge extended functional measure, which we shall denote $D\mu$ :

1. Setting reality constraints.

We project back on physical space time and impose reality (hermiticity) constraints on the fields $c$, $\tilde{c}$, $\tilde{b}$

$$x^\mu \to x^\mu = (t, \vec{x}), \quad \tilde{b} \to b = b^*, \quad (c, \tilde{c}) \to (c, c^*) \quad V \to \frac{1}{i}V, \quad V = V^*, \quad (G, F) \to \text{Re} (G, F) \quad (56)$$
2. Eliminating non-propagating fields.

We define the gauge fixing action

\[ S_{g.f.} = \int d^4x \text{ Re}\, G, \quad S_{g.f.} = S_{g.f.}(b, c^*, c, V) \] (57)

All components of \( b \) and some components of the superfields \( c^*, c, V \) are non-propagating. These components shall be eliminated choosing an extremum of \( S_{g.f.} \). Hereby we choose the Wess Zumino gauge for \( V \), which at this stage is a preselection of non-propagating components.

3. Constructing the extended functional measure.

Having eliminated non-propagating fields, we define the extended functional measure in physical space time

\[ \mathcal{D}\mu = \prod_y (\mathcal{D}V_y) (\mathcal{D}c^*_y) (\mathcal{D}c_y) \exp i (S_{g.f.})_{\text{extr.}} \] (58)

In eq. (58) the functional measure for the gauge field \( V \) extends over all four hermitian components of the gauge boson potentials and over the chiral gaugino fields \( \lambda^a \)

\[ \mathcal{D}V = \prod \mathcal{D}V^a_\mu \mathcal{D}\lambda_\beta^{*a} \mathcal{D}\lambda^a_\alpha \] (59)

The extended functional measure \( \mathcal{D}\mu \) defined in eq. (58) through steps 1. - 3. above is independent of the gauge invariant part of the action. The latter is of course quite unique. Together they form the basis of ultraviolet regularization and renormalization of the theory, which is hereby separated from the infrared or thermodynamic limit.

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