GRAVITATIONAL ENERGY AS DARK ENERGY: CONCORDANCE OF COSMOLOGICAL TESTS

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ABSTRACT

We provide preliminary quantitative evidence that a new solution to averaging the observed inhomogeneous structure of matter in the universe may lead to an observationally viable cosmology without exotic dark energy. We find parameters which simultaneously satisfy three independent tests: the match to the angular scale of the sound horizon detected in the cosmic microwave background anisotropy spectrum; the effective comoving baryon acoustic oscillation scale detected in galaxy clustering statistics; and Type Ia supernova luminosity distances. Independently of the supernova data, concordance is obtained for a value of the Hubble constant which agrees with the measurement of the Hubble Key team of Sandage and coworkers. Best-fit parameters include a global average Hubble constant $H_0 = 61.7^{+1.2}_{-1.1}$ km s$^{-1}$ Mpc$^{-1}$, a present epoch void volume fraction of $f_v = 0.76^{+0.12}_{-0.09}$ and an age of the universe of $14.7^{+0.5}_{-0.6}$ billion years as measured by observers in galaxies. The mass ratio of nonbaryonic dark matter to baryonic matter is $3.1^{+1.4}_{-0.5}$, computed with a baryon-to-photon ratio that is in concordance with primordial lithium abundances.

Subject headings: cosmological parameters — cosmology: observations — cosmology: theory — dark matter — large-scale structure of universe

Online material: color figures

The apparent acceleration in present cosmic expansion is usually attributed to a smooth “dark energy,” whose nature poses a foundational mystery to physics. Our standard $\Lambda$CDM cosmology, with a cosmological constant $\Lambda$ as dark energy, fits three independent observational tests: Type Ia supernovae (SNe Ia) luminosity distances; the angular scale of the Doppler peaks in the spectrum of cosmic microwave background (CMB) temperature anisotropies; and the baryon acoustic oscillation scale detected in galaxy clustering statistics. In this Letter we provide preliminary evidence that these same tests can all be satisfied in ordinary general relativity without exotic dark energy, within a model (Wiltshire 2007a, 2007b) which takes a new approach to averaging the observed structure of the universe, presently dominated by voids.

Recently a number of cosmologists have questioned whether cosmic acceleration might in fact be an artifact of the actual observed structure of the universe by a smooth featureless dust fluid in Einstein’s equations (for a review see Buchert 2007). The specific solution to the averaging problem we investigate here (Wiltshire 2007a) realizes cosmic acceleration as an apparent effect that arises in the decoupling of bound systems from the global expansion of the universe. In particular, gradients in the kinetic energy of expansion, and more importantly, in the quasi-local energy associated with spatial curvature gradients between bound systems and a volume-average position in freely expanding space, can manifest themselves in a significant difference in clock rates between the two locations. This difference is negligible in the early universe when the assumption of homogeneity is valid, but becomes important after the transition to void dominance, making apparent acceleration a phenomenon registered by observers in galaxies at relatively late epochs.

Galaxies and other objects dense enough to be observed at cosmological distances are bound systems, leading to a selection bias in our sampling of cosmic clocks. Since the clock rates within bound systems are closely tied to a universal finite infinity scale (Ellis 1984; Wiltshire 2007a), gross variations in cosmic clock rates are not directly observable in any observational test yet devised. However, relative to observers in bound systems an ideal comoving observer within a void would measure an older age of the universe, and an isotropic CMB with a lower mean temperature and an angular anisotropy scale shifted to smaller angles.

A systematic variation in clock rates between bound systems and the volume average, which we will find to be 38% at the present epoch, seems implausible given the familiarity of large gravitational time dilation effects occurring only for extreme density contrasts, such as with black holes. However, cosmology presents a circumstance in which conventional intuition based on static Newtonian potentials can fail, because space-time itself is dynamical and the definition of gravitational energy is extremely subtle. The normalization of clock rates in bound systems relative to expanding regions can accumulate significant differences, given that the entire age of the universe has been available for this to occur.

In this Letter we find best-fit parameters for the two-scale fractal bubble (FB) model (Wiltshire 2007a, 2007b). The two scales represent voids, and the filaments and bubble walls which surround them, within which clusters of galaxies are located. The geometry within finite infinity regions in the bubble walls is assumed to be spatially flat, but the geometry beyond these regions is not spatially flat. The relationship between the geometry in galaxies and the volume-average geometry within our present horizon volume is fixed by the assumption that the regionally “locally” measured expansion is uniform despite variations in spatial curvature and clock rates. This provides an implicit resolution of the Sandage–de Vaucouleurs paradox (Wiltshire 2007a): the “locally” measured or “bare” Hubble flow is uniform, but since clock rates vary it will appear that voids expand faster than walls when referred to any single set of clocks.

As observers in galaxies, our local average geometry at the
boundary of a finite infinity region is spatially flat, with the metric

$$ds^2_{\gamma} = -d\tau^2 + a^2_{\gamma}(\tau)[d\eta^2 + \eta^2_{\gamma}d\Omega^2].$$  \hspace{1cm} (1)

Finite infinity regions are contained within filaments and bubble walls. These walls surround voids, where the metric is not given by equation (1) but is negatively curved, with local scale factor $a_{\gamma}$. The average geometry is determined by a solution of the Buchert equations (Buchert 2000), with average scale factor $\bar{a} = f_{gw}a_{gw} + f_wa_w$, where $f_{gw} \ll 1$ and $f_w = 1 - f_{gw}$ are the respective initial void and wall volume fractions at last scattering, when the assumption of homogeneity is justified by the evidence of the CMB and the Copernican principle. It takes the form

$$ds^2 = -d\tau^2 + \bar{\alpha}(t)d\bar{\eta}^2 + A(\bar{\eta}, t)d\Omega^2,$$  \hspace{1cm} (2)

where the area function $A$ is defined by a horizon-volume average (Wiltshire 2007a). The time-parameter $t$ differs from the wall time $\tau$ of equation (1) by the mean lapse function $dt = \bar{\gamma}(\tau)d\tau$. The geometry (2) does not match the local geometry in either the walls or void centers.

When the geometry (1) is related to the average geometry (2) by conformal matching of radial null geodesics it may be rewritten

$$ds^2 = -d\tau^2 + \frac{\bar{\alpha}^2(\tau)}{\bar{\gamma}(\tau)}[d\bar{\eta}^2 + r_{\gamma}^2(\bar{\eta}, \tau)d\Omega^2],$$  \hspace{1cm} (3)

where $r_{\gamma} \equiv \bar{\gamma}^{-1}(1 - f_{ gw})^{-1/3} f_{ gw}^{-1/3} \bar{\eta}(\bar{\eta}, \tau)$. Two sets of cosmological parameters are relevant: those relative to an ideal observer at the volume-average position in freely expanding space using the metric (2), and conventional dressed parameters using the metric (3). The conventional metric (3) arises in our attempt to fit a single global metric (1) to the universe with the assumption that average spatial curvature and local clock rates everywhere are identical to our own, which is no longer true. One consequence is that the dressed matter density parameter $\Omega_m$ differs from the bare volume-average density parameter $\Omega_m$ according to $\Omega_m = \bar{\gamma}^{-1} \bar{\Omega}_m$.

The conventional dressed Hubble parameter $H$ of metric (3) differs from the bare Hubble parameter $H$ of metric (2) according to

$$H = \bar{\gamma} \bar{H} - \frac{d}{dt} \bar{\gamma} = \bar{\gamma} \bar{H} - \bar{\gamma}^{-1} \frac{d}{d\tau} \bar{\gamma}. \hspace{1cm} (4)$$

Since the bare Hubble parameter characterizes the uniform “locally measured” Hubble flow, its present value coincides with the value of the Hubble constant that observers in galaxies would obtain for measurements averaged solely within the plane of an ideal local bubble wall, on scales dominated by finite infinity regions. The numerical value of $H$ is smaller than the global average $\bar{H}$, which includes both voids and bubble walls. Equation (4) thus also quantifies the apparent variance in the Hubble flow below the scale of homogeneity. Local measurements across single voids of the dominant size, diameter $30 \ h^{-1} \ Mpc$ (Hoyle & Vogeley 2004), should give a Hubble “constant” which exceeds the global average $H_0$ by an amount commensurate with $H_n - H_0$. A more robust approach to determine $H_n$ is to consider the census of the CMB and the Copernican principle. It is based on three independent cosmological tests:

1. The first test is the effective angular diameter of the sound horizon, which very closely correlates with the angular scale of the first Doppler peak in the CMB anisotropy spectrum. This is often stated that the angular position of the first peak is a measure of the spatial curvature of the universe.
on the assumption that the spatial curvature is the same everywhere, appropriate for the FLRW models. In the present model there are spatial curvature gradients, and we must revisit the calculation from first principles. Volume-average negative spatial curvature, which accords with tests of ellipticity in the CMB anisotropies (Gurzadyan et al. 2005, 2007), can nonetheless be consistent with our local observation of the angular scale of the first peak (Wiltshire 2007a).

Ideally we should recompute the spectrum of Doppler peaks for the FB model. However, this requires considerable effort, as the standard numerical codes have been written solely for FLRW models, and every step has to be carefully reconsidered. This task is left for future work. The test that we apply here is to ask whether parameters exist for which the effective angular scale of the baryon acoustic oscillation (BAO) matches the angular scale of the sound horizon, \( \delta = 0.01 \) rad deduced for WMAP (Bennett et al. 2003; Spergel et al. 2007), to within 2%, 4%, and 6% (contours running top left to bottom right), and to parameters which fit the effective comoving BAO scale of 104 \( h^{-1} \) Mpc observed in galaxy clustering statistics (Cole et al. 2005; Eisenstein et al. 2005), to within 2%, 4%, and 6% (contours running bottom left to middle right). [See the electronic edition of the Journal for a color version of this figure.]

In Figure 2 we plot parameter ranges which match the \( \delta = 0.01 \) rad sound horizon scale to within 2%, 4%, and 6%, from that of an empty coasting Milne universe, with the same value of \( H_0 \). The R07 gold data set of 182 SNe Ia is binned using the criterion \( n \Delta z_i = 5.8 \), where \( n \) is the number of data points, and \( \Delta z \), the width of the \( i \)th bin. The first bin boundary is set at \( z = 0.023 \), as "Hubble bubble" points with \( z \leq 0.023 \) are excluded. Our bins differ very slightly from those used in Fig. 6 of R07: the single outlier point at \( z = 1.755 \) falls in its own bin. This point, which falls below the theoretical curve, is not shown here, but is included in the \( \chi^2 \) analysis. We use the original distance moduli reported at http://braeburn.pha.jhu.edu/~ariess/R06/sn_sample, without the suggested systematic subtraction of 0.32 mag, as we follow the Cepheid calibration of Sandage et al. (2006). The boxes show the standard statistical errors for the binned data using the reported uncertainties, which already account for luminosity corrections in the MLCSS2k2 reduction (Jha et al. 2007). The whiskers indicate how the residuals move relative to the horizontal axis for the 2 \( \sigma \) limits on \( H_0 \), with the overlap in these two regions has been colored black. [See the electronic edition of the Journal for a color version of this figure.]

The 1, 2, and 3 \( \sigma \) confidence limits (oval contours) for fits of luminosity distances of SNe Ia in the R07 gold data set are compared to parameters within the \( (\Omega_m, H_0) \) plane which fit the angular scale of the sound horizon \( \delta = 0.01 \) rad deduced for WMAP (Bennett et al. 2003; Spergel et al. 2007), to within 2%, 4%, and 6% (contours running bottom left to middle right), and to parameters which fit the effective comoving BAO scale of 104 \( h^{-1} \) Mpc observed in galaxy clustering statistics (Cole et al. 2005; Eisenstein et al. 2005), to within 2%, 4%, and 6% (contours running bottom left to middle right). [See the electronic edition of the Journal for a color version of this figure.]

\[ \eta_a = (4.6–5.6) \times 10^{-10} \] adopted by Tytler et al. (2000) prior to the release of WMAP1. With this range it is possible to achieve concordance with lithium abundances, while also better fitting helium abundances. This potentially resolves an anomaly. With the 2003 WMAP1 release (Bennett et al. 2003), the baryon-to-photon ratio was increased to the very upper range of values that had previously been considered, largely due to the consequence for the ratio of the heights of the first two Doppler peaks. This ratio of peak heights is sensitive to the mass ratio of baryons to nonbaryonic dark matter—rather than directly to the baryon-to-photon ratio—as it depends physically on baryon drag in the primordial plasma. The fit to the Doppler peaks required more baryons than the range of Tytler et al. (2000) admitted, when calibrated with the FLRW model. In the FB calibration, on account of the difference between the bare and dressed density parameters, a bare value of \( \Omega_m = 0.03 \) nonetheless corresponds to a conventional dressed value \( \Omega_m \approx 0.08 \), and an overall mass ratio of baryonic matter to nonbaryonic dark matter of about 1 : 3, which is larger than for \( \Lambda \)CDM. This would certainly indicate sufficient baryon drag to accommodate the ratio of the first two peak heights.

The final set of contours plotted in Figure 2 relate to the independent test of the effective comoving scale of the baryon acoustic oscillation (BAO), as detected in galaxy clustering statistics (Cole et al. 2005; Eisenstein et al. 2005). Similarly to the case of the angular scale of the sound horizon, given that we do not have the resources to analyze the galaxy clustering data directly, we begin here with a simple but effective check. In particular, since the dressed geometry (3) does provide an effective almost-FLRW metric adapted to our clocks and rods in spatially flat regions, the effective comoving scale in this dressed geometry should match the corresponding observed BAO scale of 104 \( h^{-1} \)Mpc. We therefore plot parameter values which match this scale to within 2%, 4%, or 6%.

The best-fit cosmological parameters, using SNe Ia only, are \( H_0 = 61.7^{+1.2}_{-1.1} \) km s\(^{-1}\) Mpc\(^{-1}\) and \( f_{\sigma} = 0.76^{+0.12}_{-0.09} \), with 1 \( \sigma \) uncertainties. Other cosmological parameters derived from these
arises is particularly interesting. The numerical value of present sound horizon and the BAO scale, this leads to different cos-
parameters involves model assumptions. We have removed the
mass ratio of nonbaryonic dark matter to baryonic matter 
Conventional dressed density parameter 
Mass ratio of nonbaryonic dark matter to baryonic matter 
Bare Hubble constant 
Effective dressed deceleration parameter 
Age of universe measured in a galaxy 

Note.—The 1 σ statistical uncertainties from SNe Ia are shown.

are shown in Table 1. Statistical uncertainties from the sound horizon and BAO tests cannot yet be given, but should sign-
ificantly reduce the bounds on $f_{\text{v0}}$, $\Omega_{\text{m0}}$, etc.

One striking feature of Figure 2 is that even if SNe Ia are disregarded, the parameters which fit the two independent tests
relating to the sound horizon and the BAO scale agree with each other, to the accuracy shown, for values of the Hubble constant which include the value of Sandage et al. (2006). However, they do not agree for the values of $H_0$ larger than 70 km s^{-1} Mpc^{-1} which best fit the WMAP data (Bennett et al. 2003; Spergel et al. 2007) with the FLRW model.

The value of the Hubble constant quoted by Sandage et al. (2006) has been controversial, given the 14% difference from
values which best fit the WMAP data with the ΛCDM model (Bennett et al. 2003; Spergel et al. 2007). However, the WMAP analysis only constitutes a direct measurement of CMB temperature anisotropies; the determination of cosmological parameters involves model assumptions. We have removed the assumptions of the FLRW model, in an attempt to model the universe in terms of the distribution of galaxies that we actually observe, with an alternative proposal to averaging consistent with general relativity. Applied to the angular diameter of the sound horizon and the BAO scale, this leads to different cos-

The combination of best-fit cosmological parameters that arises is particularly interesting. The numerical value of present void volume fraction $f_{\text{v0}}$ is identical to that of the dark-energy density fraction $\Omega_{\text{DE}}$ in the ΛCDM model with WMAP (Spergel et al. 2007). If the FB model is closer to the correct description of the actual universe, then in trying to fit a FLRW model, we appear to be led to parameters in which the cosmological constant is mimicking the effect of voids as far as the WMAP normalization to FLRW models is concerned. This it does im-

perfectly, since for a flat ΛCDM model $\Omega_{\text{m}} = 1 - \Omega_{\Lambda0}$, with the result that the best-fit value of $\Omega_{\Lambda0}$ normalized to the CMB does not match the best-fit value of $\Omega_{\Lambda0}$ for SNe Ia with the FLRW model, nor for other tests which directly probe $\Omega_{\Lambda0}$. For example, it has been recently noted that the values of the normali-

zation of the primordial spectrum $n_s$ ∼ 0.76 and matter content $\Omega_{\text{m}}$ ∼ 0.24 implied by WMAP3 are barely compatible with the abundances of massive clusters determined from X-ray measurements (Yepes et al. 2007). For the FB model, by contrast, the dressed density parameter $\Omega_{\Lambda0}$ includes the range preferred in direct estimations of the conventional matter density parameter.

The integrated Sachs-Wolfe effect provides a further interesting test to be determined. Since the observed signal is based on a correlation to clumped structure (Boughn & Crittenden 2004), for large-scale averages any difference from the ΛCDM expectation would largely depend on the difference in expansion history of the two models. However, we might expect foreground voids to give anisotropies below the scale of homogeneity, for which evidence is seen (Rudnick et al. 2007).

In this Letter we have offered preliminary quantitative evidence, via agreement of independent cosmological tests, that the problem of “dark energy” might be resolved within general relativity. The differences in cosmological parameters inferred in the ΛCDM and FB models—including the average Hubble parameter and its variance, the expansion age, dressed matter density, baryon-to-photon ratio, baryon–to–dark matter ratio, CMB ellipticity—are such that the question as to which provides the better concordance model can be answered by future observations and new cosmological tests.

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