Modeling of relativistic pulsar wind interaction with the interstellar medium

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Abstract. The consistent interpretation of the multiwavelength pulsar wind nebula observations require studying of interaction of a relativistic pulsar wind with the interstellar medium (ISM). In this paper we present the results of the Monte-Carlo modeling of the test particle propagation in a pulsar wind nebula beyond the pulsar wind termination shock. The non-thermal electron-positron spectra in the pulsar wind nebula and in the vicinity of the pulsar bow shock are derived in case of the supersonic proper motion of the pulsar through the ISM. The synchrotron emission images of the pulsar wind nebula at photon energies in the far ultraviolet and soft X-ray ranges are constructed. We discuss a possible interpretation of the recent HST and Chandra observations of the pulsar wind nebula associated with the PSR J0437-4715 in the frame of the model.

1. Introduction
Multiwavelength observations of the pulsar wind nebulae (PWNe) produced by the relativistic pulsar winds (PWs) with modern orbital and ground-based telescopes make possible investigations of efficient processes of energy conversion in these systems. These studies, of course, need a consistent interpretation of the observational data. In case of the supersonic motion of the pulsar through the ISM it requires modeling of an energy dependent transport of relativistic particles from the PW termination shock to the ISM.

Indeed, in this case a bow shock (BS) occurs, and the bubble of the shocked PW is separated from the shocked ISM matter by a contact discontinuity (CD) as observed by the Hubble Space Telescope and Chandra (see e.g. [1]). The wind flow, which is heated and decelerated down to a subsonic velocity at its termination shock (TS), collides with the flow of the shocked ISM matter driven by the pulsar motion. Thus, a complex picture of the flows occurs (see, e.g., the results of the simulations in [2, 3]). The magnetic fields in the pulsar wind plasma as well as in the heated postshock flow downstream the BS are thought to be strongly turbulent. Thus, electrons and positrons of the PW plasma injected in the PWN at the TS experience multiple scatterings at the stochastic magnetic field fluctuations of different scales frozen into the plasma flows. Therefore, the motion of the PW particles can be described via the transport equation in terms of diffusion and advection. The presence of the shocked ISM matter flow with the turbulent magnetic field enhanced at the bow shock can result into a confinement of the low energy PW particles in the PWN – BS system. Moreover, the colliding flows may provide an acceleration of the PW particles due to their multiple diffusion across the CD, which was found in [4] to be very fast and efficient. The described effects can strongly influence on the distribution of the synchrotron
emission intensity from the source. Hence, the processes related to the PW particles transport through the PWN with the BS (the BSPWN) may affect the observed picture significantly.

The morphologies of the sources associated with the BSPWNe can differ in various energy ranges. An interesting example of such source is the nebula related to the pulsar PSR J0437-4715. A bow-like structure observed in $H\alpha$ and in the far ultraviolet (FUV) range (6—10 eV) is absent in the soft X-ray range (0.5—7 keV) [1].

In the present work the results of the Monte-Carlo modeling of the relativistic PW particles propagation into the ISM are presented. The test particles transport through the system with specified geometry, diffusion parameters and flows is considered. The results of the particle spectra modeling in the vicinity of the PWN and the BS are presented. Possible interpretation of the observational data for the source related to PSR J0437-4715 in the framework of the developed model is discussed.

2. The Monte-Carlo particle transport model

As was pointed in [5], the modeling of the plasma flows in the BSPWNe with an adequate account for the magnetic field structure is a complicated problem (see e.g. [2, 3]) and it is not fully resolved yet. A non-trivial behaviour of the magnetized flows near the CD imposes some complexities on the modeling of the particle transport there. This problem is alleviated for the rather high energy electrons and positrons responsible for the UV and X-ray synchrotron emission given their large mean free paths. Together with the dissipation of the magnetic inhomogeneities during their transport to the CD and the deceleration of the flows this allows a significant simplification of the problem.

The model presented here is discussed in more details in [5]. Its essential feature is taking into account some geometrical properties of the BSPWNe.

2.1. Geometry

The simulation area is presented as the composition of the axisymmetric regions and is bounded by the cylinder with radius $\rho_{sys}$ and height $h_{sys}$ (see the sketch shown in Figure 1). At this cylinder the free escape boundary conditions are imposed. In each region the mean free path (mfp) of a particle is energy-dependent but homogeneous in the coordinate space. The innermost region bounded by the sphere with radius $r_{ts}$ corresponds to the cold PW. Through its surface (the TS) the particles are injected into the system. The outer boundary of the region “1” corresponding to the shocked pulsar wind (the PWN) has the following shape:

$$ r(\theta) = \begin{cases} a_{in}, & \text{if } \theta \leq \pi/2 \\ a_{in}/\sin\theta, & \text{if } \theta > \pi/2. \end{cases} $$ (1)

Here the spherical coordinates $(r, \theta, \phi)$ with the origin at the pulsar and the zenith direction aligned with the pulsar velocity vector are introduced. The scaling factor $a_{in}$ is a parameter of the geometrical model. The flow velocity downstream of the TS is directed radially and has the value $u_w(r) = c_3 r_{ts}^2 / r^2$. The label “3” marks the region corresponding to the postshock flow of the ISM matter. Its boundaries are described by the equation (2) which was derived in [6] for the BS shape in the thin-shell limit:

$$ r(\theta, \sigma) = \sigma r_0 \csc \theta \sqrt{3(1 - \theta \cot\theta)} $$ (2)

The scaling factor $r_0$ is the standoff distance to the CD determined by the pulsar spin-down power, the proper velocity of the pulsar and the ISM mass density (see e.g. [6]). The model parameter $\sigma$ determines the positions of the inner and outer boundaries of the region. Their apexes are denoted as $a_{out}$ and $a_{sys}$, respectively. The region labeled with “4” corresponds to
Figure 1. Sketch of the model geometry: half of the axial section of the system. Region 1 (between the orange curves) – the shocked PW, 2 — the region near the contact discontinuity, 3 (between the red curves) — the postshock flow of the ISM matter, 4 — the ISM. The black squares labeled with letters A – G mark the bins chosen to show the simulation results for the particle spectra. The angles between the symmetry axis and the gray dashed lines are 30° and 90°. The chosen bins are located at the midpoints of the segments between the intersections of these lines with the surfaces separating the regions.

the unperturbed ISM. The velocity of the flow through the BS in the pulsar rest frame is chosen uniform, directed oppositely to the pulsar velocity and equal to \( u_{PSR} = 104 \text{ km/sec} \) in both third and fourth regions. In the zone near the CD (region “2”) the advection is neglected.

According to the Monte-Carlo model of the relativistic particle propagation (see, e.g. [7]), the phase space of the system is divided into bins – small volumes of the phase space used to ‘detect’ the particle spatial and energy distribution. The model we developed allows to get the mean value of the distribution function in each bin. In the simulations discussed below the spatial binning is chosen as follows. The cylinder containing the system in the coordinate space is splitted into \( N_x \) equal pieces along the axis. Due to the axial symmetry of the system, each piece is divided into \( N_\rho \) coaxial cylindrical layers, whose outer radii are \( \rho_n = n \rho_{phys}/N_\rho \), where \( n = 1, 2, \ldots, N_\rho \). To compensate the effect of the growth of the bin volumes with the growth of the distance from the axis on the detected values, the latter are divided by the ratio of the current bin volume to the volume of a bin with \( n = 1 \). In the momentum space the logarithmic binning is used.

2.2. Resonant PW particle confinement at BS

The cosmic ray protons acceleration at the BS can generate the cosmic ray driven current instabilities which provide a growth of the magnetic field fluctuations intensities in some range of the wavenumber. This can produce decreasing of the mfp for particles of the corresponding energy range. It can be shown that the acceleration of 100 GeV protons gives the sizeable effect in the energy range matching the synchrotron emission of electrons and positrons in FUV in the magnetic field of a few units of the mean induction of the turbulent ISM field \( B_{ISM} = 3 \mu\text{G} \). Low mfp near the BS causes a longtime confinement of such particles in the system. This can give rise to the enhancement of the observed structures brightness in the FUV range.
2.3. Magnetic field and mfp

A spatially uniform mean value of the magnetic field is specified in each of the regions “1”–“4”. The mfp in the PWN, $\lambda_1$, is taken proportionally to the gyroradius $r_g$, in the ISM — in the form of (4) and in the regions “2”–“3” near the CD and close to the BS — in the form of (5)

$$
\lambda_1 (B, \eta, \gamma) = \eta r_g (B, \gamma) = \eta \frac{mc^2 \gamma}{eB}
$$

$$
\lambda_{\text{ism}} (\gamma) = 3 \times 10^{18} \left( \frac{E}{1 \text{ GeV}} \right)^{1/3} \text{ cm}
$$

$$
\lambda_{\text{bow}} (B, \eta, \gamma) =
\begin{cases} 
\eta r_g (B, \gamma), & \text{if } \gamma \leq \gamma_1 \\
\eta r_g (B, \gamma) \gamma / \gamma_1, & \text{if } \gamma_1 < \gamma \leq \gamma_2 \\
\lambda_{\text{ism}} (\gamma), & \text{if } \gamma > \gamma_2
\end{cases}
$$

Here $e$, $m$ are the charge and the mass of an electron (positron), $c$ — the speed of light, $E = mc^2 \gamma$ — the particle energy, $\gamma$ — the particle Lorentz-factor; $\eta \geq 1$ and $\gamma_1$ are the free parameters of the model. The value of $\gamma_1$ regulates the energy range in which the particles are confined longer in the system owing to the resonant scattering mechanism. The particles with the Lorentz-factor above $\gamma_1$ are weakly scattered by the short scale waves. The parameter $\gamma_2$ is defined by the energy at which the mfp reaches the mean ISM value, given by (4).

3. Particle spectra modeling

Here the results of the Monte-Carlo modeling of the particle distribution functions are presented. The chosen parameters of the system geometry, the mfp values and the mean magnetic field magnitudes in the regions of the system are summarized in Table 1. All lengths are given in units of the distance from the pulsar to the BS apex, chosen to be equal to $r_{\text{apex}} = 2.3 \times 10^{16}$ cm, which is likely close to the corresponding value for the source related to the PSR J0437-4715. In the simulation we used $\gamma_1 = 2 \times 10^6$.

The energy distribution of the injected particles is a power-law with an exponential cutoff $f_0 (p) \propto p^{-4.2} \exp (- \gamma / \gamma_c)$ in the range $\gamma_{\text{min}} = 4 \times 10^5 \leq \gamma \leq \gamma_{\text{max}} = 1.2 \times 10^8$, where $p = mc\gamma$, $\gamma_c = 3 \times 10^7$. Here the distribution function $f_0$ is normalized in such a way that the mean particle number density $n_0 = \int f_0 d^3 p$.

The numbers of the spatial bins are $N_x = 200$ and $N_r = 100$. The range of the particle Lorentz-factors which could be detected by the system $\gamma_{\text{det}}_{\text{min}} = 4 \times 10^4 \leq \gamma \leq \gamma_{\text{det}}_{\text{max}} = 6 \times 10^8$ is wider than the injected particles energy range; it is logarithmically binned into $N_m = 400$ bins.
Figure 2. The particle spectra detected in the bins shown in Figure 1. **Left:** red, green and blue curves — the dependence of the detected momentum distribution function in arbitrary units, multiplied at $p^4$, on the momentum value $p$ normalized at $m_c$, for bins A, B and C, respectively. Magenta curve — the same for bin D. **Right:** the same for bins E, F, G and D. The dashed black curves show the spectrum at the termination shock, which is close to the injected one.

In Figure 2 the results of the Monte-Carlo particle spectra modeling are presented. One can see that inside of the BSPWN (regions “1” – “3”) the particle spectrum deforms in comparison with the injected one. The behaviour of the particle distribution function reflects the confinement of the low energy particles in the system due to the resonant scattering diffusion coefficient at the BS and the particle re-acceleration in the colliding shock flows (see [5] for detailed discussion). In region “4” (the ISM) the spectrum becomes somewhat closer to the injected one due to a low probability for a low energy particle to pass through the region against the plasma flow.

4. Synchrotron images

To compare the results of the modeling with the observations we constructed the images of the spatial distributions of the synchrotron emission intensity in the modeled source. We used the approximations given in [8] to obtain the intensity of the synchrotron emission averaged over the orientations of the magnetic field. The results of the simulation at the photon energies $E_{\text{ph}} = 8$ eV and 1 keV are presented in Figure 3. In the image for $E_{\text{ph}} = 8$ eV (Figure 3, left panel) one can see a bright bow-like structure, corresponding to the inner part of region “3” in the model. Just a very faint counterpart can be found in the image for $E_{\text{ph}} = 1$ keV (Figure 3, right panel). The ratio of the intensity from the vicinity of the TS to the intensity from the bow-like structure is about 1 for the FUV image and about 10 for the X-ray one.

5. Conclusions

In this paper the Monte-Carlo model of the transport of the relativistic PW particles accelerated at the termination shock through the pulsar wind nebula is presented. The model accounts for the supersonic proper motion of the pulsar through the ISM. The cosmic-ray driven instabilities at the bow shock are supposed to cause the decrease of the particle mfp in the certain energy.
Figure 3. The simulated synchrotron images of a source similar to J0437-4715, for $E_{ph} = 8$ eV (left) and 1 keV (right). The absolute intensity is given in normalized units to represent the contrast change through the image.

range, corresponding to the energies of the particles emitting the synchrotron radiation in the far ultraviolet range. The physical processes that can be responsible for the observed difference in the morphology of the source related to the PSR J0437-4715 at different frequencies are discussed. The results of the particle spectra modeling for the parameters resembling ones for the BSPWN in this source are presented.

The results show that due to mentioned decreasing of the mfp the low energy particles are confined by the system for a longer time providing the reshaping of the particle spectra. The synchrotron emission images are simulated for the modeled source at the photon energies 8 eV (FUV) and 1 keV (soft X-rays). A bright bow-like structure is apparent in the simulated synthetic FUV image, while its counterpart in the X-ray image is very faint. The ratio of the intensity from the area close to the TS to the intensity from the bow-like structure is about 1 for the FUV image and about 10 for the X-ray one.

The presented synchrotron continuum model can qualitatively reproduce most of the observed features of the frequency-dependent morphology of the source similar to one related to the PSR J0437-4715. The detailed analysis of the line spectra associated with the bow shock will be addressed elsewhere.

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