t − b − τ Yukawa unification for \( \mu < 0 \) with a sub-TeV sparticle spectrum

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We show compatibility with all known experimental constraints of \( t − b − τ \) Yukawa coupling unification in supersymmetric \( SU(4)_c \times SU(2)_L \times SU(2)_R \) which has non-universal gaugino masses and the MSSM parameter \( \mu < 0 \). In particular, the relic neutralino abundance satisfies the WMAP bounds and \( \Delta(g − 2)_\mu \) is in good agreement with the observations. We identify benchmark points for the sparticle spectra which can be tested at the LHC, including those associated with gluino and stau co-annihilation channels, mixed bino-Higgsino state and the \( A \)-funnel region. We also briefly discuss prospects for testing Yukawa unification with the ongoing and planned direct detection experiments.

I. INTRODUCTION

Supersymmetric \( SO(10) \), in contrast to its non-supersymmetric version, yields third family \( (t − b − τ) \) Yukawa unification via the unique renormalizable Yukawa coupling 16-16-10, where the 10-plet is assumed to contain the two minimal supersymmetric standard model (MSSM) Higgs doublets \( H_u \) and \( H_d \). The 16-plet contains the 15 chiral fermions per family of the standard model (SM) as well as right handed neutrino. The implications of this unification have been extensively explored over the years [1, 2]. More recently, it has been argued in [3, 4] that \( SO(10) \) Yukawa unification predicts relatively light (\( \lesssim \) TeV) gluinos, which can be readily tested [5] at the Large Hadron Collider (LHC). The squarks and sleptons turn out to have masses in the multi-TeV range. Moreover, it is argued in [3, 4] that the lightest neutralino is not a viable cold dark matter candidate, at least in the simplest models of \( SO(10) \) Yukawa unification.

Spurred by these developments we have investigated \( t − b − τ \) Yukawa unification [4, 6] in the framework of supersymmetric \( SU(4)_c \times SU(2)_L \times SU(2)_R \) [7] (4-2-2, for short), with a positive supersymmetric bilinear Higgs parameter \( (\mu > 0) \). The 4-2-2 structure allows us to consider non-universal gaugino masses while retaining Yukawa unification. In particular, assuming left-right symmetry [7, 8], or more precisely C-parity [9], we have only one additional free parameter in the soft supersymmetry breaking (SSB) sector compared to the \( SO(10) \) model. In particular, this allows us to distinguish \( M_2 \) from \( M_3 \), where \( M_2 \) (\( M_3 \)) denotes the asymptotic \( SU(2)_L \) (\( SU(3)_c \)) gaugino mass. An important conclusion reached in [4, 6] is that with gaugino non-universality, Yukawa unification in 4-2-2 is compatible with neutralino dark matter, with gluino co-annihilation [4, 6, 10] playing an important role.

The main purpose of this paper to extend the 4-2-2 discussion to the case of \( \mu < 0 \), where \( \mu \) denotes the supersymmetric Higgs mass parameter. Most authors normally do not consider \( \mu < 0 \) because it can create serious disagreement with the measured value of the muon anomalous magnetic moment \( (g − 2)_\mu \) [11]. The new contribution to \( (g − 2)_\mu \) from supersymmetric particles is proportional to \( \mu M_2 \tan \beta \frac{m_\nu}{\tilde{m}^2} \) [12], where \( \tilde{m} \) is the heaviest sparticle mass in the loop. In \( SO(10) \) Yukawa unification, the scalar masses are very heavy (multi-TeV), so that the supersymmetry (SUSY) contribution to \( (g − 2)_\mu \) effectively decouples and one is left with the standard model result [3, 4]. On the contrary, in 4-2-2, with both \( \mu < 0 \) and \( M_2 < M_3 \), the SUSY contributions to \( (g − 2)_\mu \) turn out to be important, as we will see, and thereby yield significantly improved agreement with experimental data.

The outline for the rest of the paper is as follows. In Section II we briefly describe the model and the boundary conditions for SSB parameters which we employ for our scan. In Section III we summarize the scanning procedure and the experimental constraints that we have employed. In Section IV we discuss how \( \mu < 0 \) leads to better Yukawa unification than \( \mu > 0 \). In Section V we present the results from our scan and highlight some of the predictions of the 4-2-2 model. The correlation between the spin-independent and spin-dependent direct detection of dark matter and Yukawa unification condition is presented in Section VI where we also display some benchmark points. Our conclusions are summarized in Section VII.

II. THE 4-2-2 MODEL

In 4-2-2 the 16-plet of \( SO(10) \) matter fields consists of \( \psi (4, 2, 1) \) and \( \psi_c (\bar{4}, 1, 2) \). The third family Yukawa coupling \( \psi_c \psi H \), where \( H(1, 2, 2) \) denotes the bi-doublet (1,2,2), yields the following relation valid at \( M_{\text{GUT}} \).

\[
Y_t = Y_b = Y_\tau = Y_{\nu_\tau}. \tag{1}
\]

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Supplementing 4-2-2 with a discrete left-right (LR) symmetry [7, 8] (more precisely C-parity) [9] reduces the number of independent gauge couplings in 4-2-2 from three to two. This is because C-parity imposes the gauge coupling unification condition \( g_L = g_R \) at \( M_{\text{GUT}} \). We will assume that due to C-parity the SSB mass terms, induced at \( M_{\text{GUT}} \) through gravity mediated supersymmetry breaking [13] are equal in magnitude for the squarks and sleptons of the three families. The tree level asymptotic MSSM gaugino SSB masses, on the other hand, can be non-universal from the following consideration. From C-parity, we can expect that the gaugino masses at \( M_{\text{GUT}} \) associated with \( SU(2)_L \) and \( SU(2)_R \) are the same (\( M_2 \equiv M_2^L = M_2^R \)). However, the asymptotic \( SU(4)_c \) and consequently \( SU(3)_c \), gaugino SSB masses can be different. With the hypercharge generator in 4-2-2 given by \( Y = \sqrt{2/5} (B-L) + \sqrt{3/5} I_3, \) \( B-L \) and \( I_3 \) are the diagonal generators of \( SU(4)_c \) and \( SU(2)_R \), we have the following asymptotic relation between the three MSSM gaugino SSB masses:

\[
M_1 = \frac{3}{5} M_2 + \frac{2}{5} M_3.
\]  

The supersymmetric 4-2-2 model with C-parity thus has two independent parameters (\( M_2 \) and \( M_3 \)) in the gaugino sector. In order to implement Yukawa unification it turns out that the SSB Higgs mass terms must be non-universal at \( M_{\text{GUT}} \). Namely, \( m_{U_2}^2 < m_{U_3}^2 \) at \( M_{\text{GUT}} \), where \( m_{H_u}(m_{H_d}) \) is the up (down) type SSB Higgs mass term. The fundamental parameters of the 4-2-2 model that we consider are as follows:

\[
m_0, m_{H_u}, m_{H_d}, M_2, M_3, A_0, \tan \beta, \text{sign(\( \mu \))}.
\]

Here \( m_0 \) is the universal SSB mass for MSSM sfermions, \( A_0 \) is the universal SSB trilinear scalar interaction (with the corresponding Yukawa coupling factored out), \( \tan \beta \) is the ratio of the vacuum expectation values (VEVs) of the two MSSM Higgs doublets, and the magnitude of \( \mu \), but not its sign, is determined by the radiative electroweak breaking (REWSB) condition. In this paper we mainly focus on \( \mu < 0 \) as well as \( M_2 < 0 \), as explained earlier. Although not required, we will assume that the gauge coupling unification condition \( g_3 = g_1 = g_2 \) holds at \( M_{\text{GUT}} \) in 4-2-2. Such a scenario can arise, for example, from a higher dimensional \( SO(10) \) [14] or \( SU(8) \) [15] model after suitable compactification.

III. PHENOMENOLOGICAL CONSTRAINTS AND SCANNING PROCEDURE

We employ the ISAJET 7.80 package [16] to perform random scans over the parameter space listed in Eq.(3). In this package, the weak scale values of gauge and third generation Yukawa couplings are evolved to \( M_{\text{GUT}} \) via the MSSM renormalization group equations (RGEs) in the \( DR \) regularization scheme. We do not strictly enforce the unification condition \( g_3 = g_1 = g_2 \) at \( M_{\text{GUT}} \), since a few percent deviation from unification can be assigned to unknown GUT-scale threshold corrections [17]. The deviation between \( g_1 = g_2 \) and \( g_3 \) at \( M_{\text{GUT}} \) is no worse than 3.5%, and it is also possible to get perfect gauge coupling unification. Perfect gauge coupling unification is typically not possible with gaugino universality. With nonuniversality in the gaugino sector, one can adjust \( M_3/M_2 \) appropriately to get exact gauge coupling unification. If neutrinos acquire mass via Type I seesaw, the impact of the neutrino Dirac Yukawa coupling in the RGEs of the SSB terms, gauge couplings and the third generation Yukawa couplings is significant only for relatively large values (\( \sim 2 \) or so). In the 4-2-2 model we expect the largest Dirac Yukawa coupling to be comparable to the top Yukawa coupling (\( \sim 0.6 \) at \( M_{\text{GUT}} \)). Therefore, we do not include the Dirac neutrino Yukawa coupling in the RGEs.

The various boundary conditions are imposed at \( M_{\text{GUT}} \) and all the SSB parameters, along with the gauge and Yukawa couplings, are evolved back to the weak scale \( M_Z \). In the evaluation of Yukawa couplings the SUSY threshold corrections [19] are taken into account at the common scale \( M_{\text{SUSY}} = \sqrt{m_{\tilde{g}} m_{\tilde{\nu}}}. \) The entire parameter set is iteratively run between \( M_Z \) and \( M_{\text{GUT}} \) using the full 2-loop RGEs until a stable solution is obtained. To better account for leading-log corrections, one-loop step-beta functions are adopted for gauge and Yukawa couplings, and the SSB parameters \( m_i \) are extracted from RGEs at multiple scales \( m_i = m_i(m_j) \). The RGE-improved 1-loop effective potential is minimized at an optimized scale \( M_{\text{SUSY}} \), which effectively accounts for the leading 2-loop corrections. Full 1-loop radiative corrections are incorporated for all sparticle masses.

The requirement of REWSB [20] puts an important theoretical constraint on the parameter space. Another important constraint comes from limits on the cosmological abundance of stable charged particles [21]. This eliminates regions in the parameter space where charged SUSY particles, such as \( \tilde{\tau}_1 \) or \( \tilde{\ell}_1 \), become the lightest supersymmetric particle (LSP). We accept only those solutions for which one of the neutralinos is the LSP and saturates the WMAP (Wilkinson Microwave Anisotropy Probe) dark matter relic abundance bound.

We have performed random scans for the following parameter range:

...
\begin{align*}
0 & \leq m_0, m_{H_u}, m_{H_d} \leq 20 \text{ TeV} \\
-2 \text{ TeV} & \leq M_2 \leq 0 \\
0 & \leq M_3 \leq 2 \text{ TeV} \\
45 & \leq \tan \beta \leq 55 \\
-3 & \leq A_0/m_0 \leq 3 \\
\mu & < 0,
\end{align*}

(4)

with \( m_t = 173.1 \text{ GeV} \) [22]. The results are not too sensitive to one or two sigma variation in the value of \( m_t \). We use \( m_h(m_Z) = 2.83 \text{ GeV} \) which is hard-coded into ISAJET. This choice of parameters was influenced by our previous experience with the 4-2-2 where we set \( \mu, M_2 > 0 \).

In scanning the parameter space, we employ the Metropolis-Hastings algorithm as described in [23]. All of the collected data points satisfy the requirement of REWSB, with the neutralino in each case being the LSP. Furthermore, all of these points satisfy the constraint \( \Omega_{\text{CDM}} h^2 \leq 10 \). This is done so as to collect more points with a WMAP compatible value of cold dark matter (CDM) relic abundance. For the Metropolis-Hastings algorithm, we only use the value of \( \Omega_{\text{CDM}} h^2 \) to bias our search. Our purpose in using the Metropolis-Hastings algorithm is to be able to search around regions of acceptable \( \Omega_{\text{CDM}} h^2 \) more fully. After collecting the data, we impose the mass bounds on all the particles [24] and use the IsaTools package [25] to implement the following phenomenological constraints:

\begin{align*}
  m_h \text{ (lightest Higgs mass)} & \geq 114.4 \text{ GeV} \quad [26] \\
  BR(B_s \to \mu^+ \mu^-) & < 5.8 \times 10^{-8} \quad [27] \\
  2.85 \times 10^{-4} & \leq BR(b \to s \gamma) \leq 4.24 \times 10^{-4} \quad (2\sigma) \quad [28] \\
  0.15 & \leq \frac{BR(B_d \to \tau \nu \tau \nu)}{BR(B_d \to \tau \nu \tau \nu)_{\text{SM}}} \leq 2.41 \quad (3\sigma) \quad [28] \\
  \Omega_{\text{CDM}} h^2 & = 0.111^{+0.028}_{-0.037} \quad (5\sigma) \quad [29] \\
  3.4 \times 10^{-10} & \leq \Delta(g-2)_{\mu}/2 \leq 55.6 \times 10^{-10} \quad (3\sigma) \quad [11]
\end{align*}

We apply the experimental constraints successively on the data that we acquire from ISAJET.

**IV. SIGN OF \( \mu \) AND YUKAWA UNIFICATION**

To first appreciate the impact of the sign of \( \mu \) on Yukawa coupling unification and to see why \( \mu < 0 \) is preferred over \( \mu > 0 \), in Fig. 1 we show the evolution of the top, bottom and tau Yukawa couplings for a representative Yukawa coupling unification solution. We observe that Yukawa unification requires relatively large threshold corrections to \( y_b \). To quantify the magnitude of the threshold corrections, we scan the parameter space given in Eq.(4) and calculate the finite and logarithmic corrections to \( \delta y_i \), where the index \( i \) refers to top, bottom and tau Yukawa couplings.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{yukawa_couplings.png}
\caption{Evolution of top (red), bottom (blue) and tau (green) Yukawa couplings for \( \mu < 0 \).}
\end{figure}

Following [3], we define the quantity \( R \) as,
Thus, $R$ is a useful indicator for Yukawa unification with $R \leq 1.1$, for instance, corresponding to Yukawa unification within 10%.

Fig. 2 shows a plot of $\delta y_i / y_i$ versus $R$ for both $\mu > 0$ and $\mu < 0$. Points in blue, green and red represent, respectively, $\delta y_b / y_b$, $\delta y_\tau / y_\tau$ and $\delta y_t / y_t$. Note that we choose the sign of $\delta y_i$ from the perspective of evolving $y_i$ from $M_{\text{GUT}}$ to $M_Z$. Fig. 2 confirms the trend seen in Fig. 1 that $y_t$ and $y_\tau$ receive small threshold corrections compared to $y_b$. Therefore, it is reasonable to focus on the threshold corrections to $y_b$ while studying Yukawa unification.

The scale at which Yukawa coupling unification is to occur is set by gauge coupling unification and is $M_{\text{GUT}}$. Let us first consider the case of $y_t(M_{\text{GUT}}) \approx y_\tau(M_{\text{GUT}})$. Because the threshold corrections to $y_t$ are very small, it is convenient to think of $y_\tau$. The SUSY correction to the tau lepton mass $\delta m_\tau$ is given by $\delta m_\tau = v \cos \beta \delta y_\tau$. In order to get the correct $\tau$ mass ($m_\tau$), one has to get an appropriate $\delta y_\tau$. Because of the range of values of $\cos \beta$ for large $\tan \beta$, there is freedom to choose the value of $\delta y_\tau$. It may be possible to trade this freedom in favor of top-tau Yukawa unification $y_t(M_{\text{GUT}}) \approx y_\tau(M_{\text{GUT}})$. One then needs the correct SUSY contribution to $\delta y_b$ in order to achieve Yukawa coupling unification $y_t(M_{\text{GUT}}) \approx y_b(M_{\text{GUT}}) \approx y_\tau(M_{\text{GUT}})$.

In order to understand why sign of $\mu$ is very crucial for Yukawa unification condition lets first analyze the analytical expression of threshold corrections for the bottom Yukawa coupling. The dominant contribution to $\delta y_b$ comes from the gluino and chargino

$$R = \frac{\max(y_t, y_b, y_\tau)}{\min(y_t, y_b, y_\tau)}$$

(5)
loops, and in our sign convention, is given by \[ \delta y_b^{\text{finite}} = 12\pi^2 \frac{\mu m_2 \tan \beta}{m_b^2} + \frac{y_f^2 \mu A_t \tan \beta}{m_f^2}, \] (6)

where \( g_1 \) is the strong gauge coupling constant, \( m_2 \) is the gluino mass, \( m_b \) is the bottom mass, \( m_t \) is the top mass, and \( A_t \) is the top trilinear coupling. One can see from Fig. 2 that in order to achieve Yukawa coupling unification \( R \sim 1 \), the threshold corrections to \( y_b \) have to be negative (in our sign convention for \( \delta y_b \)) and in a somewhat narrow interval \((-0.5 \lesssim \delta y_b/y_b \lesssim -1.5)\) considering the full range of possible values of \( \delta y_b \). The logarithmic corrections to \( y_b \) are in fact positive. This leaves the finite corrections to provide for the correct \( \delta y_b \) to compensate for the ‘wrong’ sign of the logarithmic corrections. If \( \mu > 0 \), the gluino contribution is positive, and so the contribution from the chargino loop must cancel the contribution from the gluino loop and the logarithmic correction, as well as provide the correct (negative) contribution to \( \delta y_b \). This can be achieved only for a large \( m_0 \), as for large \( m_0 \) and for large \( A_t \), the gluino contribution scales as \( M_1/\mu \) while the chargino contribution scales as \( A_t/\mu \). It also should be noted that the numerical factor for the gluino contribution is larger than than the corresponding factor for chargino contribution. Therefore, a sufficiently large value of \( A_t \) and \( m_0 \) is needed. This large required value of \( A_t \) is the reason behind the requirement of \( A_0/m_0 \sim -2.6 \) for \( \mu > 0 \).

The scenario with \( \mu < 0 \) is interesting because the gluino contribution to \( \delta y_b \) has the correct sign to obtain the required b-quark mass. Thus, we should expect that with \( \mu < 0 \), we can realize Yukawa unification for a wider range \( A_0 \) values. With the threshold contribution to \( y_t \) proportional to \( M_2 \tan \beta \), and with \( M_2 \) as a free parameter in the 4-2-2 model, we also should expect Yukawa unification to occur over a broader range of \( \tan \beta \) values.

To proceed further, in Fig. 3 we present \( \delta y_b/y_b \) versus \( m_0 \) for \( \mu > 0 \) and \( \mu < 0 \), by performing a scan over the parameter space given in Eq.(4). Here \( \delta y_b \) includes full one loop finite and logarithmic corrections, and the black points correspond to 10% or better Yukawa unification \( (R \leq 1.1) \). It is clear from Fig. 3 that Yukawa coupling unification with a relatively light \( m_0 \sim 400 \) GeV can be realized for \( \mu < 0 \). The \( \mu > 0 \) scenario typically requires a very large \( m_0 \gtrsim 8 \) TeV.

V. YUKAWA UNIFICATION AND SPARCITELI SPECTROSCOPY

We now present the results of the scans over the parameter space listed in Eq.(4). In Fig. 4 we show the results in the \( R \cdot m_0 \), \( R \cdot \tan \beta \), \( R \cdot A_0/m_0 \) and \( M_3 \cdot M_2 \) planes. The gray points are consistent with REWSB and \( \chi^0_1 \) LSP. The blue points satisfy the WMAP bounds on \( \chi^0_1 \) dark matter abundance, sparticle mass bounds, constraints from \( BR(B_s \to \mu^+\mu^-) \) and \( BR(b \to s\gamma) \). The green points belong to the subset of blue points that satisfy all constraints including \( (g-2)_\mu \). In the \( M_3 \cdot M_2 \) plane, points in red represent the subset of green points that satisfies Yukawa coupling unification to within 10%.

In the \( R \cdot m_0 \) plane of Fig. 4 we see that with both \( \mu < 0 \) and \( M_2 < 0 \), we can realize Yukawa unification consistent with all constraints mentioned in Section III including the one from \( (g-2)_\mu \). This is possible, as previously noted, because we can now implement Yukawa unification for relatively small \( m_0(\sim 400 \) GeV) values because \( \mu < 0 \), and, in turn, \( (g-2)_\mu \) obtains the desired SUSY contribution which is proportional to \( \mu M_2 \). This is more than an order of magnitude improvement on the \( m_0 \) value required for Yukawa unification with \( \mu > 0 \). We also see from the \( R \cdot A_0/m_0 \) plane that, as explained earlier, Yukawa unification for an essentially arbitrary value of \( A_0 \) is obtained for \( \mu < 0 \). Our observation about relaxing the possible range of \( \tan \beta \) that accommodates Yukawa unified models is explicitly shown in the \( R \cdot \tan \beta \) plane. This also suggests that we are perhaps somewhat conservative in limiting the range of \( \tan \beta \). While it is beyond the scope of this paper, it would be nice to systematically search for a lower bound on \( \tan \beta \) that is consistent with third family Yukawa unification.

The \( M_3 \cdot M_2 \) plane of Fig. 4 has some interesting features. The large gray regions appear because the relic density of neutralinos is too high. This may seem peculiar given that we have non-universal Higgs boundary conditions. However, it is readily explained by the fact that \( M_2 < 0 \). The bulk of the gray region has a neutralino that is too light \( (m_{\tilde{\chi}^0_1} \lesssim 30 \) GeV). While it appears possible to have a relic density of a relatively light neutralino \( (\lesssim 30 \) GeV) consistent with WMAP, \( BR(B_s \to \mu^+\mu^-) \) and \( BR(b \to s\gamma) \), so that some of this gray region may turn blue, there will always be regions where some of these constraints are not satisfied.

In Fig. 5 we show the relic density channels consistent with Yukawa unification in the \( m_{\tilde{\chi}^+_1} - m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^-_1} - m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^+_1} - m_{\tilde{\chi}^0_1} \) and \( m_{\tilde{\chi}^-_1} - m_{\tilde{\chi}^0_1} \) planes. All of the points shown in this figure satisfy the requirements of REWSB, \( \chi^0_1 \) LSP, particle mass bounds and constraints from \( BR(B_s \to \mu^+\mu^-) \) and \( BR(b \to s\gamma) \). The light blue points satisfy, in addition the constraint from \( (g-2)_\mu \). The green points form a subset of light blue points that satisfies Yukawa unification to within 10%. The orange points satisfy all the constraints mentioned in Section III, while the red points form a subset of orange points that have \( R \leq 1.1 \). This choice of color coding is influenced from displaying the sparticle spectrum with and without neutralino dark matter, while still focussing on all the other experimental constraints. The idea is to show the myriad of solutions that implement Yukawa unification and are consistent with all known experimental bounds except for the bound on relic dark matter density from WMAP. The appearance of a variety of Yukawa unified solutions with a very rich sparticle spectrum is a characteristic feature of \( \mu < 0 \).
We can see in Fig. 5 that a variety of coannihilation and annihilation scenarios are compatible with Yukawa unification and neutralino dark matter. Included in the $m_A - m_{\tilde{\chi}_0^0}$ plane is the line $m_A = 2m_{\tilde{\chi}_0^0}$ which indicates that the $A$ funnel region is compatible with Yukawa unification. In the remaining planes in Fig. 5, we draw the unit slope line which indicates the presence of gluino, stau and wino coannihilation scenarios. From the $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_0^0}$ plane, it is easy to see the light Higgs ($h$) and $Z$ resonance channels. Our results are focussed on relatively light neutralinos ($m_{\tilde{\chi}_0^0} \lesssim 225$ GeV). We expect, based on the discussion presented, that other coannihilation channels like the stop coannihilation scenario are also consistent with Yukawa unification but we did not find them because of lack of statistics.

VI. YUKAWA UNIFICATION AND DARK MATTER DETECTION

In light of the recent results by the CDMS-II [30] and Xenon100 [31] experiments, it is important to see if Yukawa unification, within the framework presented in this paper, is testable from the perspective of direct and indirect detection experiments. The question of interest is whether $\mu \sim M_1$ is consistent with Yukawa unification, as this is the requirement to get a bino-higgsino admixture for the lightest neutralino which, in turn, enhances both the spin dependent and spin independent neutralino-nucleon scattering cross sections. In Fig. 6 we show the spin independent and spin dependent cross sections as a function of the neutralino mass. In the case of spin independent cross section, we also show the current bounds and expected reach of the CDMS and Xenon experiments. The color coding is the same as in Fig. 5. A small region of the parameter space consistent with Yukawa unification and the experimental constraints discussed in Section III (red points in the figure) is excluded as we can see, by the current CDMS and XENON bounds. This shows that the ongoing and planned direct detection experiments will play a vital role in testing Yukawa unified models.
FIG. 5. Plots in the $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0}$, $m_{\tilde{\chi}_1^0} - m_{\tilde{\chi}_2^0}$, $m_{\tilde{\tau}} - m_{\tilde{\chi}_1^0}$ and $m_A - m_{\tilde{\chi}_1^0}$ planes. All points satisfy the requirements of REWSB, $\tilde{\chi}_1^0$ LSP, particle mass bounds and constraints from $BR(B_s \rightarrow \mu^+\mu^-)$, $BR(B_s \rightarrow \tau\nu\tau)$ and $BR(b \rightarrow s\gamma)$. Light blue points further satisfy the constraint on $(g-2)_\mu$. Green points form a subset of light blue points that satisfies Yukawa unification to within 10%. Orange points satisfy all the constraints mentioned in Section III while red points form a subset of orange points that have $R \leq 1.1$.

FIG. 6. Plots in the $\sigma_{SI} - m_{\tilde{\chi}_1^0}$ and $\sigma_{SD} - m_{\tilde{\chi}_1^0}$ planes. Color coding is the same as in Fig. 5. In the $\sigma_{SI} - m_{\tilde{\chi}_1^0}$ plane we show the current bounds (black lines) and future reaches (red lines) of the CDMS (solid lines) and Xenon (dotted lines) experiments. In the $\sigma_{SD} - m_{\tilde{\chi}_1^0}$ plane we show the current bounds from Super K (black line) and IceCube (dotted red line) and future reach of IceCue DeepCore (red solid line).

In the case of spin dependent cross section, we show in Fig. 6 the current bounds from the Super-K [32] and IceCube [33]
experiments and the projected reach of IceCube DeepCore. It should be noted that IceCube currently is sensitive only to relatively large neutralino masses and therefore does not constrain the parameter space that we have considered. Likewise, while Super-K is sensitive in this region, the bounds are not stringent enough to rule out anything. However, from Fig. 6 we see that the future IceCube DeepCore experiment will be able to constrain a significant region of the parameter space.

It is interesting to comment on the correlation between spin independent and spin dependent cross sections. While both of these cross sections are enhanced by the presence of a larger higgsino component in the neutralino, the spin independent cross section falls off as $1/m_\chi^4$. In Fig. 7 we plot the correlation of the spin independent cross section versus the spin dependent cross section for $m_\chi \gtrsim 50$ GeV. We impose this lower bound on the neutralino mass only because for $50$ GeV $< m_\chi < 400$ GeV, both the spin independent and spin dependent future reach and spin independent current bounds are nearly flat. This provides one with an opportunity to identify regions with simultaneously high spin independent and spin dependent cross section which may have the best hope for being tested in ongoing and future experiments.

In Fig. 7 we also show lines corresponding to $\sigma_{SI} = 4 \times 10^{-8}, 1.5 \times 10^{-9}$ (pb) and $\sigma_{SD} = 1.5 \times 10^{-5}$ (pb). These lines serve as a guide to demonstrate the cross correlation of spin independent and spin dependent cross sections and the plausibility of either discovering a Yukawa unified neutralino dark matter solution or definitively ruling out certain regions of the allowed parameter space. It is to be noted that since ongoing and future experiments require large spin independent and spin dependent cross sections, neutralino coannihilation channels such as stau coannihilation, bino-wino coannihilation and gluino coannihilation may be difficult to test with direct as well as indirect detection experiments such as IceCube DeepCore.

Let us remark on the low mass neutralinos that we have found in this model. Because of the relative negative sign between $M_2$ and $M_3$, it is possible in principle to have $M_1 \sim 0$. This means that within this framework, the neutralino may be as light as we want. The neutralino mass nonetheless is bounded from below because of the relic density bounds on dark matter. The 4-2-2 model as we have presented it has all the ingredients needed to lower the neutralino mass to the lowest possible value allowed by various constraints. We do not focus on finding the lightest neutralino in this model. Of the data that we collected, the lightest neutralino found that is consistent with Yukawa unification has mass $\sim 43$ GeV. If we do not insist on Yukawa unification, we can get a neutralino as light as $\sim 32$ GeV. If we are willing to give up neutralino dark matter, then $M_1 \sim 0$ consistent with Yukawa unification is possible as is evident in Fig. 5. This, of course, requires invoking some other dark matter candidate such as axino.

![Fig. 7. Correlation of $\sigma_{SI}$ and $\sigma_{SD}$. Color coding same as in Fig. 5. Also shown are lines corresponding to $\sigma_{SI} = 4 \times 10^{-8}, 1.5 \times 10^{-9}$ (pb) and $\sigma_{SD} = 1.5 \times 10^{-5}$ (pb).](image)

Finally in Table I we present some benchmark points for the 4-2-2 Yukawa unified model with $\mu < 0$. All of these points are consistent with neutralino dark matter and the constraints mentioned in Section III. Point 1 corresponds to a solution with perfect Yukawa unification ($R = 1.0$). Point 2 represents a solution with minimum neutralino mass (43 GeV) consistent with Yukawa unification. Point 3 has a significant bino-higgsino admixture and, therefore, has relatively large spin independent and spin dependent neutralino-proton scattering cross-sections. Point 4 depicts a solution with a stop mass of only 826 GeV. Finally in point 5 we show an example with bino-gluino coannihilation channel with the gluino as light as 259 GeV. This should be relatively easy to find at the LHC.

### VII. CONCLUSIONS

We have shown that Yukawa coupling unification consistent with known experimental constraints is realized in a SUSY $SU(4)_c \times SU(2)_L \times SU(2)_R$ model. With $\mu < 0$ Yukawa coupling unification is achieved for $m_0 \gtrsim 400$ GeV, as opposed to $m_0 \gtrsim 8$ TeV for $\mu > 0$, by taming the finite corrections to the b-quark mass. By considering $M_2 < 0$ and $M_5 > 0$ gauginos...
and $\mu < 0$, we can obtain the correct sign for the desired contribution to $(g - 2)_\mu$. This enables us to simultaneously satisfy the requirements of $t - b - \tau$ Yukawa unification, neutralino dark matter and $(g - 2)_\mu$, as well as a variety of other known bounds. We have demonstrated the existence of a variety of coannihilation scenarios involving gluino, wino and stau, in addition to the light Higgs, $Z$ and $A$ resonance solutions. The Yukawa unified solutions may also have relatively large spin independent and spin dependent interaction cross sections with nucleons in the case of mixed bino-Higgsino dark matter. Finally, within the 4-2-2 model, it is possible to obtain relatively low neutralino masses $\sim 43$ GeV ($\sim 30$ GeV without Yukawa unification) consistent with neutralino dark matter.

| Point 1 | Point 2 | Point 3 | Point 4 | Point 5 |
|--------|--------|--------|--------|--------|
| $m_0$  | 1027   | 1800   | 1210   | 980    | 1720   |
| $M_1$  | -665   | -81    | -414   | -126   | -538   |
| $M_2$  | -1475  | -543   | -940   | -517   | -943   |
| $M_3$  | 550    | 611    | 374    | 460    | 70     |
| $\tan \beta$ | 49.1 | 52.8  | 50.6   | 47.0   | 47.6   |
| $A_0/m_0$ | 0.26  | 1.06   | -1.15  | -1.08  | -1.25  |
| $m_{H^u}$ | 743   | 1919   | 1231   | 1090   | 295    |
| $m_{H_d}$ | 1505  | 2395   | 1745   | 1869   | 1729   |
| $m_h$  | 141    | 115    | 114    | 115    | 115    |
| $m_H$  | 847    | 573    | 781    | 1100   | 1006   |
| $m_A$  | 841    | 569    | 776    | 1090   | 1000   |
| $m_{H^\pm}$ | 852  | 581    | 787    | 1100   | 1010   |
| $m_{\tilde{\chi}^0_{1,2}}$ | 280,341 | 43,352 | 168,242 | 56,337 | 233,782 |
| $m_{\tilde{\chi}^0_{3,4}}$ | 352,1236 | 380,513 | 246,795 | 371,476 | 1210,1216 |
| $m_{\tilde{\chi}^\pm_{1,2}}$ | 342,1225 | 355,509 | 239,786 | 338,475 | 782,1217 |
| $m_{\tilde{g}}$ | 1321  | 1470   | 955    | 1110   | 270    |
| $m_{\tilde{u}_{L,R}}$ | 1771,1489 | 2170,2130 | 1550,1410 | 1400,1320 | 1818,1697 |
| $m_{\tilde{t}_{1,2}}$ | 1053,1410 | 1400,1440 | 822,1040 | 826,965 | 1070,1248 |
| $m_{\tilde{d}_{L,R}}$ | 1773,1512 | 2180,2160 | 1550,1440 | 1400,1370 | 1820,1730 |
| $m_{\tilde{b}_{1,2}}$ | 954,1399 | 1350,1430 | 774,1020 | 724,906 | 992,1245 |
| $m_{\tilde{\nu}_L}$ | 1391  | 1810   | 1340   | 1000   | 1807   |
| $m_{\tilde{\nu}_R}$ | 1211  | 1420   | 1100   | 759    | 1550   |
| $m_{\tilde{\tau}_{1,2}}$ | 1393,1096 | 1820,1820 | 1340,1250 | 1010,1040 | 1809,1763 |
| $\sigma_{SI}(pb)$ | 4.02 $\times 10^{-8}$ | 4.1 $\times 10^{-9}$ | 4.1 $\times 10^{-8}$ | 9.5 $\times 10^{-10}$ | 1.1 $\times 10^{-10}$ |
| $\sigma_{SD}(pb)$ | 8.4 $\times 10^{-5}$ | 7.5 $\times 10^{-6}$ | 1.7 $\times 10^{-4}$ | 8.2 $\times 10^{-6}$ | 2.9 $\times 10^{-8}$ |
| $\Omega_{CDM}h^2$ | 0.08  | 0.11   | 0.09   | 0.08   | 0.11   |
| $R$    | 1.01   | 1.11   | 1.09   | 1.07   | 1.08   |
| $g_1/g_1(M_{GUT})$ | 0.98  | 0.98   | 0.99   | 0.98   | 1.00   |

TABLE I. Sparticle and Higgs masses (in GeV), with $m_t = 173.1$ GeV. All of these benchmark points satisfy the various constraints mentioned in Section III and are compatible with Yukawa unification. Point 1 exhibits ‘perfect’ Yukawa unification, point 2 has the lightest neutralino, point 3 shows ‘large’ spin independent and spin dependent cross-sections, points 4 and 5 correspond to the lightest stop and gluino respectively. Point 5 also provides an example of bino-gluino coannihilation channel.
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