Urban Rail Transit Network Vulnerability Measurement Based on Complex Network Theory: A Case Study of Chongqing Rail Transit

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Abstract. With the rapid growth of social economy, urban rail transit has become a major mode for public transportation, which puts forward higher requirements for the safety of urban rail transit. It is urgent for rail transit operators to find out the key stations in the network and to take corresponding preventive measures in order to avoid heavy losses. Based on the theory of complex network, the study of urban rail transit network vulnerability was carried out through computer simulation at different attack strategies to find out the influence of failing key stations to entire network. Then, a case study of Chongqing urban rail transit network was carried out. Both random attack strategy and selected attack strategy were applied to the network. By comparing the relative efficiency declines of two attack strategies, it is concluded that Chongqing rail transit network is a typical scale-free network, and is robust under random attacks, and is vulnerable under selected attacks. Furthermore, the results show that most of the important stations are stations with large degree.

Introduction

With the development of the economy, the improvement of people's living standards, and the strong demand for travel demand, the number of car ownership increases by a large number each year, along with various traffic problems, such as traffic congestion, traffic safety issues and so on. In order to meet people's huge travel needs and save road resources, large-capacity public transportation has become the only way to travel because public transportation is characterized by its large transportation capacity, low energy consumption, relatively low cost, and more environmental concern. The government's esteem to tackle traffic problems, coupled with some subsidies encouraged by the national government, make public transport cost-effective[1].

In recent years, rail transit is the most popular method in public transportation. Because of its large capacity, punctuality, and safety, it has become the main mode of public transportation. While the urban rail system is booming, it also puts high demands on the safe operation of the urban rail system. Looking back at history, we have found that the consequences of many security incidents are extremely serious. On the morning of March 20, 1995, a terrorist attacked Tokyo subway, which had caused 12 deaths, 5,500 people were poisoned, 1036 were hospitalized, and the main subway line around the Japanese government had to be closed. 26 subways On July 7, 2005, during the morning rush hour, four Britons took bombs on three subways and committed suicide, killing 52 people, injuring 700, and paralyzing the entire London subway network[2]. Therefore, those stations that are attacked will cause global networks or global efficiency to be abruptly reduced. They are very vulnerable to attacks. If we can find out these critical stations and protect them, and prevent them in advance. Reducing the occurrence of such events can effectively ensure the safe and efficient operation of the rail transit system.
Urban Rail Transit Network Vulnerability

Network Vulnerability Definition

In the context of transportation research, vulnerability is often used to express “the sensitivity of road networks to threats”, i.e. the significant impact that dangers can have on network systems[3]. Some researchers associate vulnerability with risk theory and use two risk assessment indicators to define vulnerability, one is the probability of interruption, and the other is the consequence of the interruption. However, in some areas, the probability of a particular event occurring is particularly low or unrecognizable, showing the limitations of this approach.

Therefore, some researchers have improved the definition of vulnerability[4]. The definition of vulnerability after improvement suggests that vulnerability should be more related to the degree of impact caused by network disruption, rather than the probability of disruption being related to the matter itself.

In this paper, the definition of vulnerability is that Urban rail transit network vulnerability refers to the impact of a station in an urban rail transit network system on the overall rail network after being attacked. This effect is measured by the average connectivity efficiency.

Vulnerability Measurement

The vulnerability of the entire network is an evaluation criterion for changes in the overall network performance when the network is subjected to external shocks, such as rainfall and flood[5]. This performance varying amount can seen as a measure for the overall performance of the network. When the urban rail transit network is affected by internal or external influences, the function of a certain section will be disabled, causing the traffic demand of the section to flow into other sections of the network[6]. If the inflowing traffic volume is greater than the maximum flow that the network section can withstand, the delay or congestion is caused, and the traffic of the segment will flow to other segments, and so on, resulting in chain congestion of the entire network[7]. Therefore, the analysis of the vulnerability of the urban rail transit network is to find out the potential stations that are seriously affected by other stations due to the failure of station.

According to graph theory, an urban rail transit network can be represented as $G(A, L)$, where $A$ is a set of stations and $L$ is a set of transit links between stations.

The efficiency of any two different stations $i$ and $j$ can be noted as $\varepsilon_{ij}$ in network

$$\varepsilon_{ij} = \frac{1}{d_{ij}}. \quad (1)$$

where $d_{ij}$ denotes the distance of transit link between station $i$ and station $j$.

If there is no transit link connecting station $i$ and station $j$, the efficiency of two stations is set to $\varepsilon_{ij}=0$.

The efficiency of entire urban rail transit can be noted as $E$:

$$E = \frac{1}{N(N-1)} \sum_{i \neq j} \varepsilon_{ij} = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d_{ij}}. \quad (2)$$

The relative efficiency decline rate of rail transit network can be denoted as $e$:

$$e = \frac{E_{BeforeAttack} - E_{AfterAttack}}{E_{BeforeAttack}}. \quad (3)$$

where $E_{BeforeAttack}$ is the efficiency of rail transit network before any station is attacked or off-line and $E_{AfterAttack}$ is the efficiency after at least one station is attacked or off-line.
Case Study—Chongqing Rail Transit Network Vulnerability Measurement

Chongqing Rail Transit Network

At present, the Chongqing Rail Network is in operation, with lines 1, 2, 3, 5, 6, and 10 and Guobo. Line 1 starts from Chaotianmen in the east and reaches Jiandingpo in the west. There are 24 stations, 8 of which are elevated and 16 underground. It uses a steel rail subway system. Line 2 starts from Jiaochangkou and reaches Yudong in the west. It is a straddle monorail system. There are 25 stations, including 22 elevated stations and 3 underground stations. Line 3 starts from Yudong to Jiangbei Airport, with 45 stations, including 34 elevated stations and 11 underground stations. The straddle-type monorail system is adopted. Line 5 starts from the Yuanbozhongxin in the north and ends at Dalong Mountain. There are 9 stations in total, all of which are underground stations, using steel wheel rail A-type car system. Line 10 from Wangjiazhuang to Liyuci, a total of 19 stations, including 18 subway stations, 1 elevated station, using steel wheel rail A-type system; Line 6 starts from the Cayuan to Beibei, with 33 stations, including 27 underground stations and 6 elevated stations, which are powered by steel rails. The Guobo line is a branch line of the line 6. There are total 6 stations from Lijia to Wangjiazhuang.

So, there are 141 stations and 139 transit links operating now in Chongqing rail transit network. The network topology is stated in Figure 1.

![Figure 1. Chongqing rail transit network topology.](image)

The probability distribution of degree of station is given in Figure 2. It shows that most of the 141 stations have a degree of 2, and only 12 stations have a large degree of 3 or 4, which proves that the Chongqing rail transit network is a scale-free network.

![Figure 2. Probability distribution of degree of station.](image)

### Attack Strategies

We study the two main types of attack strategies, one is random attack strategy and the other is selected attack strategy. The main consideration of randomness is to randomly select stations to attack them in the network, and then randomly select the next one, and continue to attack the stations until all stations are selected accordingly. The selected attack strategy means that all stations in the network are attacked in order one by one.
In order to obtain more intuitively sense of the difference between the impact of the two attack strategies, we can compare the two attack strategies in the same picture for comparison on the average network efficiency of the network, see Figure. 3.

![Figure 3. Comparison of network efficiency variations of two attack strategies.](image)

By contrast, it can be observed that the selected attack has a greater impact on the network, especially after the first attack. The selected attack strategy reduces the average network efficiency from 0.1991 to 0.1779, while the random attack obviously has less impact, just from 0.1991 to 0.1927. This shows that the impact of selected attacks on the network is very significant. So we should find the key stations for protection. From the above analysis, we can show that the ranking of the key points is as follows: LiangLuKou, HongQiHeGou, NiuJiaoTuo, HongTuDi, DaPing.

From the above, it is easy to see that the slope under selected attack strategy is greater than the random attack strategy. The selected attack strategy is more destructive than the random attack strategy. The random attack strategy has a strong stickiness. This result is typical with the network structure, which testifies that Chongqing rail transit network is a scale-free network.

**Conclusions**

From the above analysis of the two attack strategies, it can be seen that the Chongqing rail transit network is a typical scale-free network. It exhibits a strong stickiness under random attack strategies and shows strong vulnerability under selected attack strategies. In this case, we must strengthen the protection of important stations and take corresponding preventive measures to avoid damage to these important sites which may result in heavy losses in the entire rail transit network. Most of the important stations are stations with large degree, which is caused by the physical structure of the network.

Once the station is built, it is difficult to expand, unless the construction takes into account the expansion to reserve a certain space in advance. Thus, when important stations suffer from large passenger flows, the rail transit operation system will face great challenges. So, it is necessary to design a better operating schedule and to take inbound measures to protect the station, specifically in the phase of preliminary rail transit design and planning.

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