B → η’Xs in the Standard Model

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We study B → η’Xs within the framework of the Standard Model. Several mechanisms such as b → η’q from the QCD anomaly, and b → η’s and B → η’sq from four-quark operators are treated simultaneously. We find that the first mechanism give a significant contribution to the branching ratio for B → η’Xs, while the other two mechanisms account for about 15% of the experimental value. The Standard Model prediction for B → η’Xs is consistent with the CLEO data.

The recent observation of B → η’K and B → η’Xs decays with high momentum η’ has stimulated many theoretical activities. One of the mechanisms proposed to account for this decay is b → sq* → sqgη’ where the η’ meson is produced via the anomalous η’ − g − g coupling. According to a previous analysis, this mechanism within the Standard Model (SM) can only account for 1/3 of the measured branching ratio. B(B → η’Xs) = (62 ± 16 ± 13) × 10^{-5}. There are also other calculations of B → η’Xs based on four-quark operators of the effective weak-Hamiltonian. These contributions to the branching ratio, typically 10^{-4}, are too small to account for B → η’Xs, although the four-quark-operator contribution is capable of explaining the branching ratio for the exclusive B → η’K decays. These results have inspired proposals for an enhanced b → sq and other mechanisms arising from physics beyond the Standard Model. In the following we report our recent analysis using next-to-leading effective Hamiltonian and consider several mechanisms simultaneously. We conclude that the standard model is consistent with experimental data from CLEO.

The quark level effective Hamiltonian for the B → η’Xs decay is given by:

\[ H_{\text{eff}}(\Delta B = 1) = \frac{G_F}{\sqrt{2}} \sum_{f=u,c} V_{fb} V_{fs}^* (C_1(\mu)O_1^f(\mu) + C_2(\mu)O_2^f(\mu)) \]

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\[ V_{tb}^* V_{tb} \sum_{i=3}^{6} (C_i(\mu)O_i(\mu) + C_8(\mu)O_8(\mu))], \]  

(1)

The operators are defined in Ref.[13,14]. For numerical analyses, we use the scheme-independent Wilson coefficients obtained in Ref.[14]. For \( m_t = 175 \) GeV, \( \alpha_s(m_Z^2) = 0.118 \) and \( \mu = m_b = 5 \) GeV, we have

\[
C_1 = -0.313, \quad C_2 = 1.150, \quad C_3 = 0.017, \\
C_4 = -0.037, \quad C_5 = 0.010, \quad C_6 = -0.045,
\]

(2)

When the one-loop corrections to the matrix elements are taken into account, the coefficients are modified to \( C_i(\mu) + \tilde{C}_i(q^2, \mu) \) with

\[
\tilde{C}_4(q^2, \mu) = \tilde{C}_6(q^2, \mu) = -3\tilde{C}_3(q^2, \mu) = -3\tilde{C}_5(q^2, \mu) = -P_s(q^2, \mu), \tag{3}
\]

where

\[
P_s(q^2, \mu) = \frac{\alpha_s}{8\pi} C_2(\mu) \left( \frac{10}{9} + G(m_c^2, q^2, \mu) \right), \tag{4}
\]

with

\[
G(m_c^2, q^2, \mu) = 4 \int x(1-x) \log \left( \frac{m_c^2-x(1-x)q^2}{\mu^2} \right) dx. \tag{5}
\]

The coefficient \( C_8 \) is equal to \(-0.144\) at \( \mu = 5 \) GeV and \( m_c \) is taken to be 1.4 GeV.

Let us first work out the four-quark-operator contribution to \( B \rightarrow \eta' X_s \). We follow the approach of Ref.[5,15] which uses factorization approximation to estimate various hadronic matrix elements. The four-quark operators can induce three types of processes represented by 1) \( < \eta' | q \Gamma_i | B > < X_s | s \Gamma_i | q > \), 2) \( \eta' | q \Gamma_2 q | B > < X_s | s \Gamma_2 q | B > \), and 3) \( \eta' | X_s | s \Gamma_3 q | 0 > < 0 | q \Gamma_3 | B > \). Here \( \Gamma_i \) denotes appropriate gamma matrices. The contribution from 1) gives a “three-body” type of decay, \( B \rightarrow \eta' s \bar{q} q \). The contribution from 2) gives a “two-body” type of decay \( b \rightarrow s \bar{q} q \). And the contribution from 3) is the annihilation type which is relatively suppressed and will be neglected. Several decay constants and form factors needed in the calculations are listed below:

\[
\begin{align*}
< 0 | \bar{s} \gamma_\mu \gamma_5 u | \eta' > & = < 0 | \bar{d} \gamma_\mu \gamma_5 d | \eta' > = i f_{\eta'}^u p_{\mu}^u, \\
< 0 | \bar{s} \gamma_\mu \gamma_5 s | \eta' > & = i f_{\eta'}^s p_{\mu}^s, \quad < 0 | \bar{s} \gamma_\mu s | \eta' > = i (f_{\eta'}^u - f_{\eta'}^s) m_{\eta'}^2, \\
f_{\eta'}^u = \frac{1}{\sqrt{3}} (f_1 \cos \theta_1 + \frac{1}{\sqrt{2}} f_8 \sin \theta_8), \quad f_{\eta'}^s = \frac{1}{\sqrt{3}} (f_1 \cos \theta_1 - \sqrt{2} f_8 \sin \theta_8),
\end{align*}
\]

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\[ <\eta'|\bar{q}\gamma_\mu b|B> = F_1^{B\eta}(p_\mu^B + p_\eta'^\mu) + (F_0^{B\eta} - F_1^{B\eta}) \frac{m_B^2 - m_\eta'^2}{q^2} q_\mu, \]

\[ F_{1,0}^{\eta} = \frac{1}{\sqrt{3}} \left( \frac{1}{\sqrt{2}} \sin \theta F_{1,0}^{\eta s} + \cos \theta F_{1,0}^{\eta s} \right). \] (6)

For the \( \eta' - \eta \) mixing associated with decay constants above, we have used the two-angle parametrization. The numerical values of various parameters are obtained from Ref. [16] with \( f_1 = 157 \) MeV, \( f_8 = 168 \) MeV, and the mixing angles \( \theta_1 = -9.1^\circ \), \( \theta_8 = -22.1^\circ \). For the mixing angle associated with form factors, we used the one-angle parametrization with \( \theta = -15.4^\circ \) since these form factors were calculated in that formulation. In the latter discussion of \( b \to s\eta' \), we shall use the same parametrization in order to compare our results with those of earlier works. For form factors, we assume that \( F_1^{B\eta} = F_1^{B\eta s} = F_1^{B\pi} \) with dipole and monopole \( q^2 \) dependence for \( F_1^{B\eta} \) and \( F_0^{B\eta} \), respectively. We used the running mass \( m_s \approx 120 \) MeV at \( \mu = 2.5 \) GeV and \( F_1^{B\pi} = 0.33 \) following Ref. [9].

Using \( V_{ts} = 0.038 \), \( \gamma = 64^\circ \) and \( \mu = 5 \) GeV, we find that the branching ratios in the signal region \( p_\eta' > 2.2 \) GeV (\( m_X < 2.35 \) GeV) are given by

\[ B(b \to \eta's) = 0.9 \times 10^{-4}, \quad B(B \to \eta's\bar{q}) = 0.1 \times 10^{-4} \] (7)

The branching ratio can reach \( 2 \times 10^{-4} \) if all parameters take values in favour of \( B \to \eta'X_s \). Clearly the mechanism by four-quark operator is not sufficient to explain the observed \( B \to \eta'X_s \) branching ratio.

We now turn to \( b \to \eta'\bar{s}q \) through the QCD anomaly. To see how the effective Hamiltonian in Eq. (1) can be applied to calculate this process, we rearrange the effective Hamiltonian such that

\[ \sum_{i=3}^{10} C_i O_i = (C_3 + C_4/N_c)O_3 + (C_5 + C_6/N_c)O_5 - 2(C_4 - C_6)O_A + 2(C_4 + C_6)O_V + C_8 O_8, \] (8)

where

\[ O_A = \bar{s}\gamma_\mu(1 - \gamma_5)T^a b \sum_q \bar{q}\gamma^\mu T^a q, \quad O_V = \bar{s}\gamma_\mu(1 - \gamma_5)T^a b \sum_q \bar{q}\gamma^\mu T^a q. \] (9)

Since the light-quark bilinear in \( O_V \) carries the quantum number of a gluon, one expects \( O_V \) give contribution to the \( b \to s\bar{q}^* \) form factors. In fact, by applying the QCD equation of motion : \( D_\mu G_\mu^{\alpha\nu} = g_5 \sum_q \bar{q}\gamma^\mu T^\alpha q \), we have
In the above, we have defined the form factors $\Delta F$ at $\mu$ and shall result in a destructive interference of the real part also becomes maximal at this threshold. From Eqs. (3), (4) and (5), we find $\text{Re}(\Delta s)(q^2,\gamma_\mu - \gamma_\mu q_\mu)LT^{a\mu}b - iF_2m_b\bar{s}\sigma_{\mu\nu}q^\nu RT^{a\nu}b$. (10)

In the above, we have defined the form factors $\Delta F_1$ and $F_2$ according to the convention in Ref. [4]. We have

$$\Delta F_1 = \frac{4\pi}{\alpha_s}(C_4(\mu) + C_6(\mu)), \quad F_2 = -2C_6(\mu)$$ (11)

We note that our relative signs of $\Delta F_1$ and $F_2$ agree with those in Refs. [4,6] and [17], and shall result in a destructive interference. At the NLL level, $\Delta F_1$ is corrected by $\Delta F_1 = \frac{4\pi}{\alpha_s}(\tilde{C}_4(q^2,\mu) + C_6(q^2,\mu))$.

To obtain the branching ratio for $b \rightarrow s\gamma\eta'$ from $b \rightarrow s\gamma^* v$ vertex, we use the anomalous $\eta' - g - g$ coupling given by: $a_g(\mu)\cos\theta c_{\mu\nu\alpha\beta}q^\alpha k^\beta$ with $a_g(\mu) = \sqrt{NF\alpha_s(\mu)/\pi f_{\eta'}}$, $q$ and $k$ the momenta of the two gluons.

In previous one-loop calculations without QCD corrections, it was found that $\Delta F_1 \approx -5$ and $F_2 \approx 0.2$. In our approach, we obtain $\Delta F_1 = -4.86$ and $F_2 = 0.288$ from Eqs. (2) and (11). However, $\Delta F_1$ is enhanced significantly by the matrix-element correction $\Delta F_1(q^2,\mu)$. The latter quantity develops an imaginary part as $q^2$ passes the charm-pair threshold, and the magnitude of its real part also becomes maximal at this threshold. From Eqs. (3), (4) and (5), one finds $\text{Re}(\Delta F_1(4m_c^2,\mu)) = -2.58$ at $\mu = 5$ GeV. Including the contribution by $\Delta F_1(q^2,\mu)$ with $\mu = 5$ GeV, we find $B(b \rightarrow s\gamma\eta') = 5.6 \times 10^{-4}$ with a cut on $m_X = \sqrt{(k+p)^2} \leq 2.35$ GeV. We also obtain the spectrum $dB(b \rightarrow s\gamma\eta')/dm_X$ as depicted in Fig. 1. The peak of the spectrum corresponds to $m_X \approx 2.4$ GeV. The destructive interference of between $F_1$ and $F_2$ lowers down the branching ratio by about 14\% which is quite different from the results obtained in Refs.[3,4] because our $\Delta F_1$ is larger than theirs.

In our calculation, $a_g(\mu)$ of the $\eta' - g - g$ vertex is treated as a constant independent of invariant-masses of the gluons, and $\mu$ is set to be 5 GeV. In practice, $a_g(\mu)$ should behave like a form-factor which becomes suppressed as the gluons attached to it go farther off-shell. It is possible that the branching ratio we just obtained gets reduced significantly by the form-factor effect in the $\eta' - g - g$ vertex. Should a large form-factor suppression occur, the additional contribution from $b \rightarrow \eta' s$ and $B \rightarrow \eta' s q$ discussed earlier will become crucial. We however like to stress that our estimate of $b \rightarrow s\gamma\eta'$ with $\alpha_s$ evaluated at $\mu = 5$ GeV is conservative. To illustrate this, let us compare branching

\[\text{We thank A. Kagan for discussions which clarified this point.}\]
ratios for $b \to sg\eta'$ obtained at $\mu = 5$ GeV and $\mu = 2.5$ GeV respectively. The branching ratios at the above two scales with the kinematical cut on $m_X$ are $4.9 \times 10^{-4}$ and $8.5 \times 10^{-4}$ respectively. One can clearly see the significant scale-dependence! With the enhancement resulting from lowering the renormalization scale, there seems to be some room for the form-factor suppression in the attempt of explaining $B \to \eta'X_s$ by $b \to sg\eta'$. We do notice that $B(b \to sg\eta')$ is suppressed by more than one order of magnitude if $a_g(\mu)$ is replaced by $a_g(m_{\eta'}) \cdot m_{\eta'}^2/(m_{\eta'}^2 - q^2)$ according to Ref.[6]. However, as pointed out in Ref.[4], the validity of such a prescription remains controversial.

Before closing we would like to comment on the branching ratio for $B \to \eta X_s$. It is interesting to note that the width of $b \to \eta sg$ is suppressed by $\tan^2 \theta$ compared to that of $b \to \eta' sg$. Taking $\theta = -15.4^\circ$, we obtain $B(B \to \eta X_s) \approx 4 \times 10^{-5}$. The contribution from four-quark operator can be larger. Depending on the choice of parameters, we find that $B(B \to \eta X_s)$ is in the range of $(6 \sim 10) \times 10^{-5}$.

In conclusion, we have calculated the branching ratio of $b \to sg\eta'$ by including the NLL correction to the $b \to sg^*$ vertex. By assuming a low-energy $\eta' - g - g$ vertex, we obtain $B(b \to sg\eta') = (5 - 9) \times 10^{-4}$ depending on the choice of the QCD renormalization-scale. Although the form-factor suppression in the $\eta' - g - g$ vertex is anticipated, it remains possible that the anomaly-induced process $b \to sg\eta'$ could account for the CLEO measurement on the $B \to \eta'X_s$ decay. For the four-quark operator contribution, we obtain $B \to \eta'X_s \approx 1 \times 10^{-4}$. This accounts for roughly 15% of the experimental central-value and can reach 30% if favourable parameters are used.

![Figure 1](image.png)

**Figure 1.** The distribution of $B(b \to s + g + \eta')$ as a function of the recoil mass $m_X$.  

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Acknowledgments

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