A Class of Weibull Mixture Distributions

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Abstract
We have derived a class of mixture distributions which we call weibull mixtures of distributions. Estimation of unknown parameters along with some properties of these distributions are also prescribed.

Keywords: Mixing distribution; Mixture distribution; Weibull distribution

Introduction
Mixture distribution was first coined in 1894. A number of authors like Pearson, Rider, Blischke, Chahine, Roy, et al., authors [1-13] defined mixtures of two distributions and studied various mixture distributions which they called poisson mixture, binomial mixture, negative binomial mixture, Chi-square mixture, Erlang mixture, Laplace mixture, Rayleigh mixture, F, Dual mixture of distributions. Weibull distribution is widely used in bio-statistics, but weibull mixture distribution has not yet been premeditated. In the present paper, we define first the weibull mixture of distributions and then weibull mixtures of normal, lognormal, gamma, exponential, beta, rectangular, erlang, chi-square, t and F distributions and studied some of their properties.

Preliminaries
A mixture distribution is a weighted average of probability distribution of positive weights that sum to one. The distributions thus mixed are called the components of the mixture. The weights themselves comprise a probability distribution called the mixing distribution. Because of this property of weights, a mixture is in particular again a probability distribution. Mixtures occur most commonly when the parameter of a family of distributions, given by the density function \( f(x; \theta) \), is itself subject to the change variation. The mixing distribution \( g(\theta) \) is then a probability distribution on the parameter of the distributions. The general formula for the finite mixture is \( \sum_{i=1}^{k} f(x; \theta_i)g(\theta_i) \); the infinite analogue, in which \( \theta_i \) is a probability density function, is \( \int f(x; \theta)g(\theta)d\theta \).

Main Results
Here in this paper, we define the weibull mixtures of some well known distributions such as normal, lognormal, gamma, exponential, beta, rectangular, erlang, chi-square, t and F distributions. Then some characteristics of these distributions such as characteristic functions, moments, and shape characteristics are also obtained. The main results of the paper are presented in form of definitions and theorems. Comparison of the probability density functions and the first two moments are prescribed in the tertiary section.

Definition 3.1
A random variable \( X \) is said to have a weibull mixed distribution if its probability density function is defined as

\[
f(x; a, b, \alpha) = \int_0^x abr^{b-1}e^{-r} g(x; \alpha)dr
\]

Where \( g(x; \alpha) \) is a probability density function. The name of weibull mixture distribution comes from the fact that the distribution (3.1) is the weighted average of \( g(x; \alpha) \) with weights equal to the ordinates of weibull distribution.

Definition 3.2
If \( X \) follows a weibull mixture of Normal distribution with parameters \( a \) and \( b \), then the density function is given by

\[
f(x; a, b) = \int_0^x abr^{b-1}e^{-r} dr, \quad \infty < x < \infty
\]

with parameters \( a \) and \( b \) such that

\[
\int_0^\infty f(x; a, b)dx = 1
\]

The characteristic function and moments of the same distribution are presented in the theorem below.

Theorem 3.1
If \( X \) has a weibull mixture of normal distributions with parameters \( a \) and \( b \) then its characteristic function is represented as

\[
\phi(t) = \int_0^x abr^{b-1}e^{-r} \left( \frac{1}{r+1/2} \right)^m dr
\]

\[
\int_0^\infty abr^{b-1}e^{-r} 2^r dr = \frac{1}{r+1/2}
\]

The 2nd moment about origin is zero. Mean = 0,

\[
Varinace = 1 + 2a \left( \frac{1}{1 + \frac{1}{2}} \right) \beta_1 = 0, \beta_2 = \frac{3 + 8a^2}{7}, \frac{1 + 1/2 + 4a^2}{7}, \frac{1 + 2/7}{7}, \frac{1 + 2a^2}{1 + 1/2}
\]

Remark: For \( a = b = 0 \), \( \phi (t) \), \( \mu_2 \), \( \mu_3 \), \( \mu_4 \) are same for Normal distribution with mean zero and variance unity.

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then its characteristic function is obtain as

\[ \phi(t) = \mathbb{E}[e^{itX}] = \sum_{k=0}^{\infty} \frac{(it)^k}{k!} \mathbb{E}[X^k] \]

and the \( k \)th moment about origin is

\[ \mathbb{E}[X^k] = \frac{\Gamma(k+1)}{\Gamma(k+\alpha+\beta)} \]

It is said to have a weibull mixture of Lognormal distribution with parameters \( a, b, \alpha \) where

\[ \phi(t) = \mathbb{E}[e^{itX}] = \sum_{k=0}^{\infty} \frac{(it)^k}{k!} \mathbb{E}[X^k] \]

Theorem 3.2

A random variable \( X \) having the density function

\[ f(x; a, b, \alpha, \beta) = \int_{0}^{\infty} abr^{b-1}e^{-e^{st}} \mathbb{E}[X^k] \]

is defined a weibull mixture of Gamma distribution with parameters \( a, b, \alpha, \beta \) whereas

\[ \int_{0}^{\infty} f(x; a, b, \alpha, \beta) dx = 1. \]

The characteristic function and moments are followed by the next theorem.

Theorem 3.3

If \( X \) denotes a weibull mixture of gamma variate with parameters \( a, b, \alpha, \beta \) then its characteristic function is obtained as

\[ \phi(t) = ab \left( \frac{it}{\beta} \right)^a \int_{0}^{\infty} r^{b-1}e^{-e^{st}} \mathbb{E}[X^k] dr \]

Mean

\[ \mu = \frac{1}{\beta} \left( a + b \right)^2 \int_{0}^{\infty} r^{b-1}e^{-e^{st}} \mathbb{E}[X^k] dr \]

Variance

\[ \sigma^2 = \frac{1}{\beta^2} \left( a + b \right)^4 \int_{0}^{\infty} r^{2b-1}e^{-e^{st}} \mathbb{E}[X^k] dr \]

Remark: \( \phi(t) \), \( \mu \), \( \sigma^2 \), and \( \beta \) are true for Gamma distribution with parameters \( a, b, \alpha, \beta \) when \( a = b = 0 \). For \( a = b = 1 \), weibull mixture of Gamma distribution should be equivalent to weibull mixture of Exponential distribution. As such we also derived the weibull mixture of Exponential distribution.

Estimates of parameters by the method of moments:

Let \( X_1, X_2, \ldots, X_n \) be a random sample from the distribution (3.8). We assume that parameters \( a, b, \alpha, \beta \) are unknown.

\[ \hat{\beta} \left( \hat{\alpha} + \hat{\beta} \right)^2 + \frac{1}{\hat{\beta}^2} \left( \hat{\alpha} + \hat{\beta} \right)^4 \int_{0}^{\infty} r^{2b-1}e^{-e^{st}} \mathbb{E}[X^k] dr \]

\[ \hat{\beta} = \frac{1}{\hat{\alpha}^2} \int_{0}^{\infty} r^{2b-1}e^{-e^{st}} \mathbb{E}[X^k] dr \]

Defnition 3.5

A random variable \( X \) having the density function

\[ f(x; a, b, \alpha, \beta) = \int_{0}^{\infty} abr^{b-1}e^{-e^{st}} \mathbb{E}[X^k] dr, \]

is said to have a weibull mixture of Exponential distribution with parameters \( a, b, \alpha, \beta \) and

\[ \int_{0}^{\infty} f(x; a, b, \alpha, \beta) dx = 1. \]

Theorem 3.4

If \( X \) follows weibull mixture of exponential distribution with parameters \( a, b, \alpha, \beta \) then its characteristic function is given by

\[ \phi(t) = ab \left( \frac{it}{\beta} \right)^a \int_{0}^{\infty} r^{b-1}e^{-e^{st}} \mathbb{E}[X^k] dr, \]

Mean

\[ \mu = \frac{1}{\sigma^2} \int_{0}^{\infty} r^{2b-1}e^{-e^{st}} \mathbb{E}[X^k] dr \]

Variance

\[ \sigma^2 = \frac{1}{\mu^2} \int_{0}^{\infty} r^{4b-1}e^{-e^{st}} \mathbb{E}[X^k] dr \]

\[ \beta = \frac{1}{\sigma^2} \int_{0}^{\infty} r^{4b-1}e^{-e^{st}} \mathbb{E}[X^k] dr \]

\[ \beta = \frac{1}{\sigma^2} \int_{0}^{\infty} r^{4b-1}e^{-e^{st}} \mathbb{E}[X^k] dr \]

\[ \beta = \frac{1}{\sigma^2} \int_{0}^{\infty} r^{4b-1}e^{-e^{st}} \mathbb{E}[X^k] dr \]
Therefore, if \( a = b = 0 \), then all of \( \Pi (1, \alpha, \mu, \mu, \mu, \beta) \) and \( \beta_2 \) are similar to those of Exponential distribution with parameter \( a \).

Parameter estimation: If \( X_1, X_2, X_3, \ldots, X_n \) be a random sample drawn from the distribution (3.12) and parameters \( a, b \) are assumed known, then the distribution contains only one unknown parameter \( a \). So, \( \mu = \text{mean} = \frac{a}{a+b} \), and \( m' = \sum_{m=1}^{m} X \). Therefore, 
\[
\frac{1}{a} \left[ 1 + a^2 \left( \frac{1}{1+b} \right) \right] = m'.
\]
Hence, 
\[
\hat{a} = \frac{1}{m'} \left( \frac{1}{1+b} \right)
\]
(3.15)

Definition 3.6

If a random variable \( X \) has the density function
\[
f(x;a,b,a,\beta) = \int_{0}^{\infty} ab^{b-1} e^{-ax} \left( \frac{a}{a+r} \right) e^{-\frac{a}{a+r} x} dr, x > 0
\]
then it is said to have a weibull mixture of Erlang distribution with parameters \( a, b, a, \beta \) and since
\[
\int_{0}^{\infty} f(x; a, b, a, \beta) dx = 1
\]
The characteristic function as well as the moments is stated in the following theorem.

Theorem 3.5

If \( X \) has weibull mixture of erlang distributions with parameters \( a, b, a, \beta \) and \( \beta \) then its characteristic function is given by
\[
\phi(t) = ab \left[ 1 - \frac{it}{a+b} \right] e^{-ax} \int_{0}^{\infty} e^{-\frac{a}{a+r} x} \left( \frac{a}{a+r} \right) e^{-\frac{a}{a+r} x} dr, x > 0
\]
(3.18)

Mean = \[
\frac{1}{a+b} \left[ a + \frac{a^2}{1+b} \left( 1 - \frac{1}{1+b} \right) \right]
\]
Variance = \[
\frac{a^2}{a+b} \left[ a + \frac{a^2}{1+b} \left( 1 - \frac{1}{1+b} \right) \right] + \frac{3a^2}{a+b} \left( 1 - \frac{1}{1+b} \right) \]
(3.19)

Remark: When \( a = b = 0 \), then all of \( \Pi (1, \alpha, \mu, \mu, \mu, \beta) \) and \( \beta_2 \) are to be true for Erlang distribution with parameters \( a \) and \( b \).

Estimating parameters: For a random sample \( X_1, X_2, X_3, \ldots, X_n \) from the distribution (3.16), we assume that parameters \( a, b, \beta \) are known and \( a \) is an unknown parameter. Here, \( \mu = \frac{1}{ab} \left[ a + b \right] \) and \( m' = \sum_{m=1}^{m} X \). We obtain
\[
\hat{a} = \frac{1}{m'} \left( \frac{1}{1+b} \right)
\]
(3.19)

Definition 3.7

A random variable \( X \) having the density function
\[
f(x;a,b,m) = \int_{0}^{x} ab^{b-1} e^{-ax} \left( \frac{r}{m} \right) e^{-\frac{r}{m} x} dr, 0 < x < m
\]
is said to be weibull mixture of Rectangular distribution with parameters \( a, b \) and \( m \) satisfying
\[
\int_{0}^{m} f(x;a,b,m) dx = 1
\]
(3.21)

Different moments of the above mentioned distribution are expressed below.

Theorem 3.6

If \( X \) follows a weibull mixture of rectangular distribution with parameters \( a, b, \) and \( m \) then its characteristic function is obtained as
\[
\phi(t) = \int_{0}^{m} ab^{b-1} e^{-ax} \sum_{r=0}^{\infty} \left( \frac{t}{m} \right)^{r} e^{-\frac{r}{m} x} dr
\]
and the \( r \)th moment about origin is \( m' = \sum_{r=0}^{m} ab^{b-1} e^{-ax} \left( \frac{r}{m} \right) e^{-\frac{r}{m} x} dr \).

Remark: If \( a = b = 0 \) then all the values of \( \phi(t) \) are true for Rectangular distribution with parameter \( m \).

Definition 3.8

A random variable \( X \) having the density function
\[
f(x;a,b,a,\beta) = \int_{0}^{x} ab^{b-1} e^{-ax} \left( \frac{r}{m} \right) e^{-\frac{r}{m} x} dr, 0 < x < 1
\]
is said to have a weibull mixture of Beta distribution of \( 1^st \) kind with parameters \( a, b, a, \beta \). Here we have
\[ \int_{0}^{\infty} f(x,a,b,\alpha,\beta)dx = 1 \quad (3.24) \]

**Theorem 3.7**

If \( X \) follows weibull mixture of beta distributions of first kind with parameters \( a,b, \alpha \) and \( \beta \), then its \( s \)th moment about origin is given by
\[ \phi_s(t) = \int_{0}^{\infty} x^s f(x,a,b,\alpha,\beta)dx \quad (3.25) \]

**Remark:** For \( a = b = 0 \), all the values of \( \mu_1, \mu_2, \mu_3 \) and \( \mu_4 \) are true for Beta distribution of 1st kind with parameters \( a, b, \alpha \) and \( \beta \).

**Definition 3.9**

A random variable \( X \) having the density function
\[ f(x,a,b,\alpha,\beta,\mu_1,\mu_2,\mu_3,\mu_4) \]

is called a weibull mixture of Beta distribution of second kind with parameters \( a,b, \alpha \) and \( \beta \).

**Theorem 3.8**

If \( X \) follows weibull mixture of beta distribution of second kind with parameters \( a,b, \alpha \) and \( \beta \), then its \( s \)th moment about origin is given by
\[ \int_{0}^{\infty} x^s f(x,a,b,\alpha,\beta)dx \quad (3.26) \]

**Remark:** Putting \( a = b = 0 \) then all the values of \( \mu_1, \mu_2, \mu_3 \) and \( \mu_4 \) are true for Beta distribution of 2nd kind with parameters \( a \) and \( \beta \).

**Definition 3.10**

A random variable \( X \) with the density function
\[ f(x;\alpha,\beta,a,b) = \int_{0}^{\infty} x^s f(x,a,b,\alpha,\beta)dx > 0 \quad (3.27) \]

is said to have a weibull mixture of Chi-square distribution having the parameters \( a,b \) and \( n \) since
\[ \int_{0}^{\infty} f(x;\alpha,\beta,a,b,n)dx = 1. \quad (3.28) \]

**Theorem 3.9**

If \( X \) has weibull mixture chi-square distribution with parameters \( a,b \) and \( n \) then its characteristic function is expressed as
\[ \phi(t) = ab(1-2it)^{-n} \quad (3.29) \]

**Remark:** Setting \( a = b = 0 \) we find all the values of \( \mu_1, \mu_2, \mu_3, \mu_4 \) and \( \beta \) are true for Chi-square distribution with parameters \( n \).

Parameter estimation: Let \( X_1, X_2, X_3, \ldots, X_n \) be a random sample from the distribution (3.28). We assume that the parameters \( a \) and \( b \) are known and \( n \) is unknown. Now, \( \mu_1 = n \sum \frac{X_i}{n} = \bar{x} \).

As such, \( n + 2a \bar{x} + \frac{2}{b} = \bar{x} \). Therefore, \( n = \bar{x} - 2a \bar{x} + \frac{2}{b} \quad (3.30) \)

**Table 1:** Comparison of density functions of different Weibull mixture distributions. \( \chi^2 > 0 \).

| Sl. | Name of the distribution | Probability density function \( f(x) \) | Support | Parameters |
|-----|--------------------------|----------------------------------------|---------|------------|
| 1   | Weibull mixed Normal     | \[ \int_{0}^{\infty} x^s f(x,a,b,\alpha,\beta)dx = 1 \] | \(-\infty < x < \infty\) | \(a,b,\alpha,\beta\) |
| 2   | Weibull mixed Lognormal  | \[ \int_{0}^{\infty} x^s f(x,a,b,\alpha,\beta)dx = 1 \] | \(x > 0\) | \(a,b,\alpha,\beta\) |
| 3   | Weibull mixed Gamma      | \[ \int_{0}^{\infty} x^s f(x,a,b,\alpha,\beta)dx = 1 \] | \(x > 0\) | \(a,b,\alpha,\beta\) |
| 4   | Weibull mixed Exponential| \[ \int_{0}^{\infty} x^s f(x,a,b,\alpha,\beta)dx = 1 \] | \(x > 0\) | \(a,b\) |
| 5   | Weibull mixed Erlang     | \[ \int_{0}^{\infty} x^s f(x,a,b,\alpha,\beta)dx = 1 \] | \(x > 0\) | \(a,b,\alpha,\beta\) |
| 6   | Weibull mixed Rectangular| \[ \int_{0}^{\infty} x^s f(x,a,b,\alpha,\beta)dx = 1 \] | \(0 < x < 1\) | \(a,b,m\) |
| 7   | Weibull mixed Beta 1st kind | \[ \int_{0}^{\infty} x^s f(x,a,b,\alpha,\beta)dx = 1 \] | \(0 < x < 1\) | \(a,b,\alpha,\beta\) |
| 8   | Weibull mixed Beta 2nd kind | \[ \int_{0}^{\infty} x^s f(x,a,b,\alpha,\beta)dx = 1 \] | \(x > 0\) | \(a,b,\alpha,\beta\) |
| 9   | Weibull mixed Chi-square | \[ \int_{0}^{\infty} x^s f(x,a,b,\alpha,\beta)dx = 1 \] | \(x^2 > 0\) | \(a,b,n\) |
| 10  | Weibull mixed t          | \[ \int_{0}^{\infty} x^s f(x,a,b,\alpha,\beta)dx = 1 \] | \(\chi^2 > 0\) | \(a,b,n\) |
| 11  | Weibull mixed F          | \[ \int_{0}^{\infty} x^s f(x,a,b,\alpha,\beta)dx = 1 \] | \(F > 0\) | \(a,b,n,\alpha\) |

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Definition 3.11

If \( t \) as a random variable has the density function

\[
f(t; a, b, n) = \frac{ab^r}{n!} \left( 1 + \frac{t}{b} \right)^{r-1} e^{-\frac{t}{b}} dt, \quad -\infty < t < \infty
\]

then it is said to have a Weibull mixture of \( t \) distribution with parameters \( a, b, n \) if

\[
\int_{-\infty}^{\infty} f(t; a, b, n) dt = 1
\]

The following theorem expresses here some of the properties of the distribution.

**Theorem 3.10**

If \( t \) is Weibull mixture of \( t \) distribution with parameters \( a, b, n \) then the \( 2s^{th} \) moment about origin is given by

\[
n_s \int_{-\infty}^{\infty} ab^r e^{-\frac{t}{b}} \left( 1 + \frac{t}{b} \right)^{r-s} dt \quad \text{and the } (2s+1)^{th} \text{ moment about origin}
\]

### Table 2: Comparison among first two moments of different Weibull mixed distributions.

| SL | Name of the distribution | Mean | Variance |
|----|--------------------------|------|----------|
| 1  | Weibull mixed Normal     | 0    | \( 1 + 2a^{\frac{1}{\beta}} \left( \frac{1}{b} + 1 \right) \) |
| 2  | Weibull mixed lognormal  | can be obtained from equation 3.7 | can be obtained from equation 3.7 |
| 3  | Weibull mixed Gamma      | \( \frac{1}{\beta} \left( a + a^{\frac{1}{\beta}} \left( \frac{1}{b} + 1 \right) \right) \) | \( \frac{1}{\beta} a^{\frac{1}{\beta}} \left( \frac{1}{b} + 1 \right)^{\frac{1}{\beta}} \) |
| 4  | Weibull mixed Exponential| \( \frac{1}{\alpha} \left( 1 + a^{\frac{1}{\beta}} \left( \frac{1}{b} + 1 \right) \right) \) | \( \frac{1}{\alpha} a^{\frac{1}{\beta}} \left( \frac{1}{b} + 1 \right)^{\frac{1}{\beta}} \) |
| 5  | Weibull mixed Erlang     | \( \frac{1}{\alpha} \left( a + a^{\frac{1}{\beta}} \left( \frac{1}{b} + 1 \right) \right) \) | \( \frac{1}{\alpha} a^{\frac{1}{\beta}} \left( \frac{1}{b} + 1 \right)^{\frac{1}{\beta}} \) |
| 6  | Weibull mixed Rectangular| can be achieved from equation 3.22 | can be achieved from equation 3.22 |
| 7  | Weibull mixed Beta 1st kind | equation 3.25 provides | equation 3.25 provides |
| 8  | Weibull mixed Beta 2nd kind | \( \frac{1}{(\beta - 1)\beta a} \left( \frac{1}{b} + 1 \right)^{\frac{1}{\beta}} \) | \( \frac{1}{(\beta - 1)\beta a} \left( \frac{1}{b} + 1 \right)^{\frac{1}{\beta}} \) |
| 9  | Weibull mixed Chi-square | \( n + 2a^{\frac{1}{\beta}} \left( \frac{1}{b} + 1 \right) \) | \( 2n + 4a^{\frac{1}{\beta}} \left( \frac{1}{b} + 1 \right) + 4a^{2\frac{1}{\beta}} \left( \frac{1}{b} + 1 \right)^{2} \) |
| 10 | Weibull mixed \( t \)     | 0    | \( \frac{n}{n - 2} \left( 1 + 2a^{\frac{1}{\beta}} \left( \frac{1}{b} + 1 \right) \right) \) |
| 11 | Weibull mixed \( F \)     | \( \frac{n_2}{n_1(n_2 - 2)} \left( n_1 + 2a^{\frac{1}{\beta}} \left( \frac{1}{b} + 1 \right) \right) \) | \( \frac{n_2}{n_1(n_2 - 2)} \left( n_1 + 2a^{\frac{1}{\beta}} \left( \frac{1}{b} + 1 \right) \right)^{2} \) |
is zero, $\beta = 0, \beta_z = \frac{n}{n - 4} \left\{ \frac{3 + 8a}{n} \right\} \left\{ \frac{1}{1 + b} + 4a^2 \right\} \left\{ \frac{1}{1 + \frac{2}{b}} \right\}

Remark: If $a = b = 0$ then all the values of $\mu_1, \mu_2, \mu_1', \mu_2', \mu_1''$, and $\mu_2''$ are true for $F$ distribution with parameter $n$.

Definition 3.12

A random variable $F$ having the density function

$$f(F; a, b, n_1, n_2) = \int_0^\infty abr^{b-1}e^{-br} \frac{n_1}{n_2} F^{n_1-1} F^{n_2} \frac{n_1 + n_2}{2} dr; F > 0$$

is said to have a Weibull mixture of $F$ distribution with parameters $a, b$, $n_1$ and $n_2$ if

$$\int_0^\infty f(F; a, b, n_1, n_2) dF = 1$$

(3.35)

The following theorem presents the characteristic function and moments of this distribution.

Theorem 3.11

If $F$ follows Weibull mixture of $F$ distribution with parameters $a, b$, $n_1$ and $n_2$ then its characteristic function is given by

$$\phi(t) = \int_0^\infty abr^{b-1}e^{-br} \sum_{i=0}^\infty \frac{n_1}{n_2} \left\{ \frac{n_1}{2} + r + x \right\} \left\{ \frac{n_2}{2} - x \right\} x! dr$$

(3.36)

and the $S^\text{th}$ moment about origin $n_1 \int_0^\infty abr^{b-1}e^{-br} \frac{n_1}{n_2} F^{n_1} \frac{n_1 + n_2}{2} dr$.

Remark: For $a = b = 0$ all the values of $\phi(t)\mu_1, \mu_2, \mu_1', \mu_2', \mu_1''$, and $\mu_2''$ are true for $F$ distribution with parameters $n_1$ and $n_2$.

Comparison

A Comparison among various features of the different Weibull mixture distributions is shown in the following table 1 and table 2.

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