Electrons and Nuclei: Fundamental Interactions and Structure

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Abstract

Examples are given of the usefulness of electrons in interaction with nuclei for probing fundamental interactions and structure.

I. INTRODUCTION

Electrons offer a great tool for testing the validity of theories of fundamental interactions (e.g., electroweak theory) and structure. The electromagnetic interaction of the electron with nucleons and mesons is known and is sufficiently weak that detailed structural information about nuclei and nucleons can be obtained. The weak interaction is less well pinned down, so that tests of the standard model can be carried out. Where the interaction is known, there is an additional tool for obtaining structural information, such as strangeness matrix elements in a nucleon. Nuclei are very useful targets because it is easy to change the charge and neutron/proton ratio. I will illustrate these remarks with several examples:

1) Beta Decay: Tests of CVC, the unitarity of the CKM matrix within the standard model, 2nd. class currents and time reversal invariance (TRI).
2) Double Beta Decay and neutrino masses
3) Parity Violation and tests of the standard model; measurements of the neutron radii of nuclei; measurements of strangeness in the nucleon; the anapole moment of nuclei
4) Studies of time reversal invariance

I will not be able to cover all of these subjects in detail and will concentrate on the more recent ones, but remind you of some of the others.

II. BETA DECAY

A. The Conserved Vector Current

Nuclear $\beta$ decay has been used to tests the validity of the conserved vector current hypothesis. The first tests, accurate to about 10% were a comparison of the weak magnetism in beta decays of allowed isospin 1 partners $^{12}B$ and $^{12}N$ with the gamma decay of the same multiplet in $^{12}C$ to the ground state of $^{12}C$. More recent tests use polarization measurements, a comparison of spectra, and a measurement of slopes to improve the accuracy of this CVC test to a few %. The analysis must assume the non-existence of 2nd class currents.

What are 2nd class currents? They are weak currents with the opposite G-parity than the normal ones ($G = C \exp^{-i\pi I_2}$, where $C$ is the charge conjugation operator and $I_2$ is the
second component of the isospin operator). An example is a term \(\bar{u}(p')\sigma_{\mu\nu}q^\nu\gamma_5u(p)\), where \(q = p - p'\). Such currents are not expected to arise at the fundamental level of the standard model, but can come from radiative corrections and from \(m_d > m_u\). They have been sought assiduously, but have not yet been found; however at a recent conference Minamisono et al. \(^2\) reported a preliminary sighting of such currents in the beta decays of mass 12 nuclei at a level of a few \%. It would be interesting to compare any findings with theoretical work as a test of the standard electroweak theory; to my knowledge, no such detailed calculations have yet been done.

Another use of electrons emitted in superallowed Fermi beta decays \((0^+ \rightarrow 0^+)\) in an isomultiplet is as a test of the standard model via unitarity of the CKM matrix.\(^1\) Could there be a missing interaction, f.i.? The matrix element \(V_{ud}\) connecting up and down quarks is by far the largest one in the unitarity of

\[
U \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1
\]

Here \(V_{ud}\) and \(V_{ub}\) connect the up quark with the strange and bottom quarks, respectively. The precise measurements of superallowed transitions together with radiative corrections and removal of charge dependent nuclear effects allow one to determine \(V_{ud}\) to better than \(10^{-3}\). In addition, these measurements, (Fig.1), show that CVC holds to \(\sim 4 \times 10^{-4}\).

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**Fig.1.** \(F_t\) values for \(0^+ \rightarrow 0^+\)–decays.

A straightforward analysis of the experiments, including a recent \(^{10}\)C experiment \(\|\), gives \(V_{ud} = .9740 \pm .0006\). Together with the measurements of \(V_{us}\) and \(V_{ub}\), one then obtains \(U = .9972 \pm .0019\). However, it has been proposed that the nuclear charge–dependent corrections should be corrected by a smooth Z-dependence. \(\|\) In that case \(U = .9980 \pm .0019\). Is the discrepancy from unitarity meaningful? Personally, I doubt it. The largest uncertainty may reside in the charge–dependent nuclear corrections and there is now an attempt to push the calculation of these corrections to increased accuracy. \(\|\)

Lastly let me mention tests of time reversal invariance. \(\|\) I will not describe the standard searches for TRI–odd terms \((D\vec{J} \cdot \vec{p}_e \times \vec{p}_\nu)\) in beta decays, particularly \(^{19}\)Ne, where \(\vec{J}\) is the spin direction of the parent nucleus. Recently there have been searches for the R–term, \(R\vec{J} \cdot \vec{j} \times \vec{p}_e\) in the beta decays of \(^8\)B and \(^8\)Li, where \(\vec{j}\) is the spin direction of the electron; the advantage of these nuclei is that they decay to the unstable \(^8\)Be, which breaks up into two alpha particles, which readily can be detected. This experiment is underway. \(\|\) Another
method has been used at the Sherrer Institute by Allet et al; they measure the transverse polarization of the emitted electrons directly and thus find a limit $\text{Im}C_T/C_A < .012$ at the 68% level; here $C_T$ and $C_A$ are the coefficients of the axial tensor and axial vector currents. It would be useful if these experiments could be increased in accuracy so as to at least reach the level where final state interactions spoil TRI tests; here this would be $7 \times 10^{-4}$. The experimenters still have a ways to go.

\section*{B. Double Beta Decay}

The decay $(A, Z) \rightarrow (A, Z + 2) + e^- + e^- + \bar{\nu} + \bar{\nu}$ is expected in the standard model and has been seen in several nuclei ($^{82}\text{Se}$, $^{100}\text{Mo}$, and $^{150}\text{Nd}$) with half lives of about $10^{20}y$, consistent with the standard model. \footnote{10} Searches for the no neutrino decay mode, important for determining whether $\nu$’s are massive and of the Majorana type, have been continuously improved; they use Ge crystals; at present the lower limit on the half life is $3 \times 10^{24}y$ for $^{74}\text{Ge}$. This experiment sets a lower bound on the mass of the most massive Majorana neutrino mass, $m_\nu \sqrt{\text{10}^{24}y/\text{1/2}^{(\text{74}\text{Ge})}}$. Improved experiments hope to push this mass down further by one to two orders of magnitude or find the double beta decay.

\section*{C. Parity Nonconservation Studies}

The first test of the electroweak theory and its neutral currents came from electron scattering on hydrogen and deuterium at SLAC. At lower energies, parity violating (pv) scattering of electrons on nucleons and nuclei allows one to determine all four weak currents of the nucleons: the axial vector and the vector couplings to protons and neutrons. The experiments are difficult and so far only one experiment of polarized electrons on $^{12}\text{C}$ has been carried out at MIT. \footnote{2} The results agree with the theory at a level of 10%.

More recently, in an ongoing experiment (SAMPLE) at MIT, the weak interaction of the electron and proton is being used to investigate strangeness in the nucleon. As in all pv experiments, it is the interference of the weak interaction with the electromagnetic one that is being detected by searching for a parity-odd signal such as $<\vec{j}> \cdot \vec{p}$, where $\vec{p}$ is the incident momentum of the electron and $<\vec{j}>$ is its polarization.

Initial evidence for non-vanishing strangeness matrix elements in the nucleon came from measurements of the spin structure of the proton by polarized electrons on polarized nuclear targets. This indicated that

$$\Delta s \equiv s \uparrow - s \downarrow + \bar{s} \uparrow - \bar{s} \downarrow \approx 0.1 - 0.2.$$  \hspace{1cm} (1)

Further indications of a non-negligible fraction of strangeness came from elastic neutrino scattering on protons. At the present time, the axial vector strangeness matrix element for the proton stands at $0.1 \pm .03$.

For elastic $e^- p$ scattering, the standard model gives

$$J_{\mu}^{em} = \bar{u}(p')\left[\gamma_\mu F_1^{em} + i\sigma_{\mu\nu} \frac{q^\nu}{2M} F_2^{em}\right]u(p),$$  \hspace{1cm} (2)
\[ J^Z_{\mu} = \bar{u}(p')[\gamma_\mu F^Z_1 + i\sigma_{\mu\nu} \frac{q'^\nu}{2M} F^Z_2 + \gamma_\mu F^Z_A \gamma_5]u(p), \] (3)

If strangeness is included, then SU(3) notation is helpful, and we have

\[ F^\text{em}_1 = \frac{1}{2} \left[ F^{(8)}_1 + \tau_3 F^{(3)}_1 \right] \implies 1 \text{ for } p, \] (4)

\[ F^\text{em}_2 = \frac{1}{2} \left[ (\kappa_p + \kappa_n) F^{(8)}_2 + \tau_3 (\kappa_p - \kappa_n) F^{(3)}_2 \right] \implies \kappa_p \text{ for } p, \] (5)

\[ F^{Z}_1 = \frac{1}{2} \left[ -F^{(0)}_1 + \tau_3 y F^{(3)}_1 + y F^{(8)}_1 \right] \implies \frac{1}{2} \left( 1 - 4 \sin^2 \theta_W \right) \text{ for } p, \] (6)

\[ F^{Z}_2 = \frac{1}{2} \left[ -g_2 F^{(0)}_2 + \tau_3 y (\kappa_p - \kappa_n) F^{(3)}_2 + y (\kappa_p + \kappa_n) F^{(8)}_2 \right] \implies \frac{1}{2} \left[ (1 - 4 \sin^2 \theta_W) \kappa_p - \kappa_n - \kappa_s \right] \text{ for } p, \] (7)

\[ F^{Z}_A = \frac{1}{2} \left[ -g_A^{(0)} F^{(0)}_A + 2g_A \tau_3 F^{(3)}_A + (6F - 2D) F^{(8)}_A \right] \implies -\frac{1}{2} g_A^{(0)} + g_A + (3F - D) \text{ for } p, \] (8)

where form factors have been normalized to unity at squared momentum transfers \( Q^2 = 0 \), and the proton values are given at \( Q^2 = 0 \). We also use \( y = (1 - 2 \sin^2 \theta_W) \), \( g_A = 1.26 \), \( g_2 = \kappa_p + \kappa_n + \kappa_s \), \( 6F - 2D \approx 1.1 \) where \( F \) and \( D \) are the fractions that are odd and even, respectively under \( SU(3); \kappa_p \) (\( \kappa_n \)) is the anomalous magnetic moment of the proton (neutron), \( \kappa_s \) is the strange magnetic moment, and the superscripts on the form factors refer to \( SU(3) \) transformation properties. There are two new and unconstrained couplings and form factors, \( g_2 F^{(0)}_2 \) and \( g_A^{(0)} F^{(0)}_A \), where \( F^{(0)}_i = F^{(0)}_i + F^{(8)}_i \).

The Sample experiment at MIT has as its aim the measurement of \( g_2 F^{(0)}_2 (Q^2) \) at small \( Q^2 \) where \( F^{(0)}_2 \approx 1 \). The experimenters measure the cross section of 200 MeV electrons polarized parallel and antiparallel to their momenta on a H target. The asymmetry \( a \) is given by

\[
a = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} = -\frac{G Q^2}{\sqrt{2\pi\alpha}} \left\{ [2\tan^2 \frac{\theta}{2} (F^{\text{em}}_1 + F^{\text{em}}_2) (F^{Z}_1 + F^{Z}_2) + F^{\text{em}}_1 F^{Z}_2 + F^{\text{em}}_2 F^{Z}_1 \tau] \right.

\[
- \frac{E + E'}{2M} \tan^2 \frac{\theta}{2} (1 - 4 \sin^2 \theta_W) F^{Z}_A (F^{\text{em}}_1 + F^{\text{em}}_2) \}

\times \left\{ (F^{\text{em}}_1)^2 + \tau (F^{\text{em}}_2)^2 + 2\tan^2 \frac{\theta}{2} (F^{\text{em}}_1 + F^{\text{em}}_2)^2 \right\}^{-1} \] (9)

where \( E(E') \) is the initial (final) electron energy, \( \theta \) is the scattering angle, and \( \tau \equiv Q^2/4M^2 \). The last term in Eq. (9) is small because \( (1 - 4 \sin^2 \theta_W) \approx 0.1 \), and the second one is small at back angles, where the first term dominates. The experiment is carried out for an average
$Q^2 \approx 0.1 GeV^2 (130^\circ < \theta < 170^\circ)$. At these angles, the asymmetry is sensitive to $F_2^Z$ and thus allows a determination of $\kappa_s$. The results to date [13] are shown in Fig. 2.

Fig. 2. Results for the parity-violating asymmetry measured in the 1995 and 1996 running periods. The hatched region is the asymmetry band (due to the axial radiative correction) for $\mu_s = G_{M}^s = 0$. 
where the hatched band corresponds to $\kappa_s = 0$ (it is a band due to uncertain radiative corrections). At $Q^2 = 0.1 \text{ GeV}^2$, the authors find $\kappa_s F_s = 0.23 \pm 0.37 \pm 0.15 \pm 0.19 \text{n.m.}$, where the first error is statistical, the second one is systematic and the third one is due to axial radiative corrections. At present, $\kappa_s$ is consistent with zero, but its possible positive value is opposite to that predicted by most theorists [14], e.g., by using $N \to \Lambda K \to N$. The SAMPLE experiment will measure the coupling of the $Z$-boson to the proton in the future; also work at MIT and at TJNAF is expected to bring the errors down to a level of $\pm 1 \text{n.m.}$ (nucleon magneton).

Other precision $\text{pv}$ studies of the weak interactions of electrons and nuclei have been carried out with atoms. Despite their being at lower momenta, where the $\text{pv}$ effects are smaller, $\sim 10^{-11}$, these experiments have reached the incredible precision of $1/2\%$, which atomic theory has yet to equal; theoretical errors are at the level of $\sim 1\%$ [15]. At this level of precision the atomic experiments provide meaningful tests of the standard model. The dominant weak interaction term is $a_{\mu} V_{\mu}$ where the lower case $a_{\mu}$ is the axial current of the electron and $V_{\mu}$ is the vector current of the nucleus. This vector current is coherent over the nucleus, giving an effective charge

$$Q_W = (1 - 4 \sin^2 \theta_W) Z - N$$

which is large for heavy atoms. The measurement on $Cs$, a one valence electron atom, at the $1/2\%$ level, give $Q_W = -72.35 \pm 0.27 \text{exp} \pm 0.54 \text{th}$. This results in an $s$-parameter [17] $s = -1.0 \pm (0.3) \text{exp} \pm (1.0) \text{th}$, where $s$ is one of the parameters which measures deviation from the standard model. For instance, this gives a lower limit on the mass of a second $Z$ boson of $\sim 500 \text{ GeV}$. [17]

It has been proposed that $\text{pv}$ measurements on a series of isotopes would allow one to obtain neutron radii for these nuclei. This is because $Q_W$ is primarily sensitive to the neutron distribution [see Eq.(10)] and $\sin^2 \theta_W$ is well known from other measurements. This method may prove to be the most accurate means of measuring differences of neutron and proton radii of nuclei. [18]

The term $v_{\mu} A_{\mu}$ is much smaller than $a_{\mu} V_{\mu}$ because for the electron $v_{\mu} \propto (1 - 4 \sin^2 \theta_W) \sim 0.1$ and only a single nucleon contributes to $A_{\mu} \propto <\vec{\sigma}>$, the nuclear spin. Thus, the asymmetry is reduced by $\ge 500$. The atomic measurements of this term make use of the hyperfine structure, which is due to the nuclear spin. This term has not yet been detected because it is hidden by the stronger nuclear anapole moment. What is this? It is a parity violating moment discovered by Zel’dovich in 1957. [19] (I discovered it independently in 1973, [20] but did not name it, but simply called it an axial coupling of the photon.) The anapole exists only for virtual photons [20] that penetrate the nucleus. It is, in reality, a combination of an electromagnetic interaction and a $\text{pv}$ component of the nuclear wavefunction. This combination gives rise to a current similar to that in a winding on the surface of a doughnut; see Fig.3. [21]
Fig. 3. Surface current ($\vec{J}$) on a doughnut, producing a toroidal magnetic field ($\vec{B}$) and anapole ($\vec{T}$).

The most general form for an axial coupling of the photon to a nucleon or other spin 1/2 particle is

$$A_\mu = \bar{\psi} \frac{i q \gamma^\mu - q^2 \gamma^\mu}{M^2} \gamma_5 \psi a,$$

where $a$ is the magnitude of the anapole moment. Note the need for the $q^2$ dependence to preserve gauge invariance. The first term vanishes for a conserved vector current of the electron. Combined with the $1/q^2$ propagator, the second term is a contact one, unlike the electromagnetic interaction.

The anapole moment can be written as

$$\bar{a} = -\pi \int d^3r r^2 \vec{J}(r) = \frac{1}{e} \sqrt{2} \frac{1}{\ell} \frac{J + \frac{1}{2}}{J(J + 1)} \bar{J}_a \kappa_a,$$

where $J$ is the spin of the nucleus and $\kappa_a$ is a dimensionless constant which is a measure of the anapole moment. The $r^2$ allows $\bar{a}$ to be produced by surface currents; it comes from the $q^2$ dependence of the axial coupling. The anapole is a purely nuclear term which is measured in atoms. The effect is of order $\alpha G_F$, but it is proportional to the square of the nuclear radius and therefore $A^{2/3}$. In Cs and heavier atoms, it is larger by almost an order of magnitude than the asymmetry due to neutral weak currents.

The recent measurement in atomic Cs at the 1/2% level [16] has discovered the anapole moment in the $6S \rightarrow 7S$ transition; the experimenters find it through a difference in the hyperfine $F = 4 \rightarrow 3$ and $F = 3 \rightarrow 4$ transitions of $0.077 \pm 0.011$ mV/cm. This gives $\kappa_a = 0.364 \pm 0.062$. The theoretical value is sensitive to the $\pi$ - nucleon coupling and gives [22]

$$f_\pi = 9.5 \pm 2.1_{\text{exp}} \pm 3.5_{\text{th}} \times 10^{-7}.$$  

By comparison, nucleon pv experiments in $^{18}F$ are sensitive only to $f_\pi$ and give [23]

$$f_\pi \leq 1.5 \times 10^{-7}.$$  

There appears to be a clear discrepancy between these two determinations of $f_\pi$. However, the value determined from the anapole moment depends also on the pv couplings of other mesons, primarily the $\rho$, to the nucleon via the combination $\sim f_\pi + \frac{1}{2} f_\rho$. It could be that the $\rho$-N pv constant, determined from other pv experiments in light nuclei and the N-N system and consistent with each other and theory, is not correct; or perhaps some other effect has been omitted in the anapole calculations. The so-called ”best value” of $f_\pi$, based on a quark model calculation is [23]

$$f_\pi \simeq (4.6 \pm 4.6) \times 10^{-7},$$

where the error is often neglected. Nevertheless, there appears to be an a (experimental?) discrepancy which remains to be resolved.
In addition to all of these studies, the electron has been used in studies of time reversal invariance (TRI). Specifically, searches for an atomic electric dipole moment in $^{199}\text{Hg}$ are sensitive to any simultaneous pv and TRI in this system. E.N. Fortson et al. [24] obtain

$$d_E^{(199\text{Hg})} = -(1 \pm 2.4 \pm 3.6) \times 10^{-28} e - cm,$$

or

$$|d_E^{(199\text{Hg})}| \leq 8.7 \times 10^{-28} e - cm \text{ (95\% confidence level).}$$

This experiment has about the same sensitivity as measurements of the neutron electric dipole moment and gives

$$\theta \leq 10^{-10},$$

where $\theta$ is defined by the term

$$\mathcal{L} = -\theta \alpha T \epsilon^{\mu \nu \alpha \beta} G_{\mu \nu} G_{\alpha \beta},$$

which would be the strong CP problem, except that $\theta \ll 1$. But no one knows why $\theta$ should be so small. At low energies, the pion is the embodiment of QCD and the electric dipole moment comes about from a T-odd $\pi$ - N coupling [25,26]

$$\mathcal{L} = -(\tau \bar{\psi} T \psi \cdot \phi),$$

and $f_\pi^T \leq 10^{11}$ from the upper limits of both the neutron and $^{199}\text{Hg}$ electric dipole moments. I hope that I have convinced you that, despite its age, the electron and its interaction with nucleons and nuclei remains a very useful tool for testing fundamental interactions and learning important information about the structure of hadrons.
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