EMBEDDED VORTICES AND THEIR INTERACTIONS
AT ELECTROWEAK CROSSOVER

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Abstract. We study properties of $Z$–vortices in the crossover region of
the 3D $SU(2)$ Higgs model. Correlators of the vortex currents with gauge
field energy and Higgs field squared (“quantum vortex profile”) reveal a
structure that can be compared with a classical vortex. We define a core
size and a penetration depth from the vortex profile. $Z$–vortices are found
to interact with each other analogously to Abrikosov vortices in a type–I
superconductor.

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1. Introduction

Although the standard model does not possess topologically stable monopo-
le– and vortex–like defects, one can define so-called embedded topological
defects [1, 2]: Nambu monopoles [3] and $Z$–vortex strings [3, 4]. In our nu-
merical simulations of the electroweak theory [5] we have found that the
vortices undergo a percolation transition which, when there exists a dis-
continuous phase transition at small Higgs masses, accompanies the latter.
The percolation transition persists at realistic (large) Higgs mass [6] when
the electroweak theory, instead of a transition, possesses a smooth crossover
around some “crossover temperature” (see Refs. [7]).

We worked in the 3D formulation of the $SU(2)$ Higgs model. This report
is restricted to results obtained in the crossover regime (assuming a Higgs
The defect operators on the lattice have been defined in [9]. A nonvanishing integer value of the vortex operator $\sigma_P$ on some plaquette $P$ signals the presence of a vortex. The lattice gauge coupling $\beta_G$ is related to the 3D continuum gauge coupling $g_3^2$ and controls the continuum limit $\beta_G = 4/(a g_3^2)$ ($g_3^2 \approx g_4^2 T$). The hopping parameter $\beta_H$ is related to the temperature $T$ (with the higher temperature, symmetric side at $\beta_H < \beta_H^\text{cross}$).

2. Vortex profile

Our vortex defect operator $\sigma_P$ is constructed to localize a line-like object (in 3D space–time) with non-zero vorticity on the dual lattice. Within a given gauge field–Higgs configuration, a profile around that vortex “soul” would be hidden among quantum fluctuations. However, an average over all vortices in a quantum ensemble clearly reveals a structure that can be compared with a classical vortex [3, 2]. We have studied correlators of $\sigma_P$ with various operators constructed on the lattice (“quantum vortex profiles”).

Classically, in the center of a vortex the Higgs field modulus turns to zero and the energy density becomes maximal [3, 2]. What can be expected in a thermal ensemble is, that along the vortex soul the (squared) modulus of the Higgs field and the gauge field energy density, $E_g^2 = 1 - \frac{1}{2} \text{Tr} U_P$, substantially differ from the bulk averages characterizing the corresponding homogeneous phase.\(^1\) Indeed, in our lattice study they were found lower (or higher, respectively), with the difference growing entering deeper into the “broken phase” side of the crossover [6] (see Figure 1).

To proceed we have studied, among others, the vortex–gluon energy correlator for plaquettes $P_0$ and $P_R$ located in the same plane (perpendicular

\(^1\)Just on the “broken” side of the crossover, for instance, one would expect to find a core of “symmetric” matter inside the vortex.
Figure 2. (a) The effective mass $m_E$ vs. hopping parameter $\beta_H$ at $\beta_G = 16$ on the lattice $32^3$; (b) Extrapolation of the mass $m_E$ fitted at crossover to the limit $a \to 0$.

to a segment of the vortex path

$$C_E(R) = \langle \sigma_{P_0}^2 \sigma_{P_R}^2 \rangle,$$

as function of the distance $R$ between the plaquettes. To parametrize the vortex shape we fit the correlator data (1) by an ansatz $C_E^\text{fit}(R) = C_E + B_E G(R;m_E)$ with constants $C_E$ and $B_E$ and an inverse penetration depth (effective mass $m_E$). The function $G(R;m)$ is the 3D scalar lattice propagator with mass $2 \sinh(m/2)$ which, instead of a pure exponential, has been proposed to fit point–point correlators in Ref. [11].

If the quantum vortex profile should interpolate between the interior of the vortex and the asymptotic approach to the vacuum, we can only expect to describe the profile by such an ansatz for distances $R > R_{\text{min}}$. The distance $R_{\text{min}}$ (core size) should be fixed in physical units. Therefore we choose (in lattice units) $R_{\text{min}}(\beta_G) = \beta_G/8$ for $\beta_G = 8, 16, 24$ which corresponds to $R_{\text{core}} = a R_{\text{min}} = (2 g^2)^{-1}$. How successful this is to define the vortex core can be assessed studying $\chi^2/d.o.f.$ vs. $R_{\text{min}}$ (to be reported elsewhere).

An example of the behaviour of the effective mass $m_E$ is shown in Figure 2(a). The mass reaches its minimum at the crossover point $\beta_H^{\text{cross}}$. Deeper on the symmetric side the quantum vortex profiles are squeezed compared to the classical ones due to Debye screening leading to a smaller coherence length. Approaching the crossover from this side the density of the vortices decreases thereby diminishing this effect. The extrapolation of the mass $m_E$ (as defined at the crossover temperature) towards the continuum limit is shown in Figure 2(b).

\footnote{A similar method has been used to study the physical properties of Abelian monopoles in SU(2) gluodynamics, Ref. [10].}
3. Inter–vortex interactions and the type of the vortex medium

In the case of a superconductor, the inter–vortex interactions define the type of superconductivity. If two parallel static vortices with the same sense of vorticity attract (repel) each other, the substance is said to be a type–I (type–II) superconductor. To investigate the vortex–vortex interactions we have measured two–point functions of the vortex currents:

\[ \langle |\sigma_{P_0}| |\sigma_{P_R}| \rangle = 2(g^{++} + g^{+-}), \quad \langle \sigma_{P_0} \sigma_{P_R} \rangle = 2(g^{++} - g^{+-}) \]

(2)

where \( g^{\pm \pm}(R) \) stands for contributions to the correlation functions from parallel/anti–parallel vortices piercing a plane in plaquettes \( P_0 \) and \( P_R \). Properly normalized, the correlators \( g^{\pm \pm}(R) \) can be interpreted as the average density of vortices (anti–vortices), relative to the bulk density, at distance \( R \) from a given vortex.

Hence the long range tail of the function \( g^{++} \) is crucial for the type of the vortex medium: in the case of attraction (repulsion) between same sign vortices \( g^{++} \) exponentially approaches unity from above (below) while \( g^{+-} \) is always attractive, independently on the type of superconductivity.

We have seen in our calculations [13] that the tail of \( g^{++} \) belongs to the attraction case (with minimal slope at the crossover). Therefore, electroweak matter in the crossover regime belongs to the type–I vortex vacuum class.

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References

1. T. Vachaspati and M. Barriola, Phys. Rev. Lett. 69 (1992) 1867.
2. M. Barriola, T. Vachaspati, M. Bucher, Phys. Rev. D50 (1994) 2819.
3. Y. Nambu, Nucl. Phys. B130 (1977) 505.
4. N. S. Manton, Phys. Rev. D28 (1983) 2019.
5. M. N. Chernodub et al., Phys. Lett. B434 (1998) 83.
6. M. N. Chernodub et al., Phys. Lett. B443 (1998) 244.
7. K. Kajantie et al., Phys. Rev. Lett. 77 (1996) 2887; M. Gürtler, E.–M. Ilgenfritz, A. Schiller, Phys. Rev. D56 (1997) 3888.
8. K. Kajantie et al., Nucl. Phys. B458 (1996) 90; M. Gürtler et al., ibid. B483 (1997) 383.
9. M. N. Chernodub, F. V. Gubarev, E.–M. Ilgenfritz, Phys. Lett. B424 (1998) 106.
10. S. Thurner et al., Phys. Rev. D 54 (1996) 3457; M. Feurstein, H. Markum and S. Thurner, Phys.Lett. B396 (1997) 203; B. L. G. Bakker, M. N. Chernodub, M. I. Polikarpov, Phys. Rev. Lett. 80 (1998) 30.
11. J. Engels, V. K. Mitryushkin, T. Neuhaus, Nucl. Phys. B440 (1995) 555.
12. E.–M. Ilgenfritz, A. Schiller and C. Strecha, Eur. Phys. J. C8 (1999) 135.
13. M. N. Chernodub, E.–M. Ilgenfritz and A. Schiller, heplat/9909001.