1 INTRODUCTION

Black hole accretion is the main energy source in different types of the objects, such as, X-ray binaries, and active galactic nuclei (AGN). In the past several decades, the measurements of masses of black holes in AGN with the reverberation mapping method have been widely used in the community (e.g., Peterson 1993; Wandel, Peterson, & Malkan 1999; Kaspi et al. 2000; Peterson et al. 2004). The Eddington ratio distributions of large AGN samples show that the bolometric luminosity of luminous AGN is roughly Eddington limited (e.g., Abramowicz et al. 1988). The slim disc models have been extensively studied either analytically or with numerical simulations (e.g., Szuszkiewicz, Malkan, & Abramowicz 1996; Wang et al. 1999; Wang & Netzer 2003; Gu & Lu 2007; Li, Yuan, & Cao 2010; Sadowski et al. 2011; Eggum, Coroniti, & Katz 1988; Ohsuga et al. 2005; Ohsuga & Mineshige 2007, 2011; Yang et al. 2014). The viscous timescale in the slim disc is shorter than the cooling timescale in slim discs (Kawaguchi 2003). A fraction of photons are unable to escape from the disc, and therefore they are trapped in the disc (Beckman 1978). The radiation efficiencies of slim discs are lower than those for standard thin discs due to the radial energy advection in the disc (Shakura & Sunyaev 1973; Laor & Netzer 1989). For more luminous objects, the discs are no longer thin due to strong radiation pressure, and slim accretion discs are suggested to be in these objects (Abramowicz et al. 1988). The slim disc models have been extensively studied either analytically or with numerical simulations (e.g., Szuszkiewicz, Malkan, & Abramowicz 1996; Wang et al. 1999; Wang & Netzer 2003; Gu & Lu 2007; Li, Yuan, & Cao 2010; Sadowski et al. 2011; Eggum, Coroniti, & Katz 1988; Ohsuga et al. 2005; Ohsuga & Mineshige 2007, 2011; Yang et al. 2014). The viscous timescale in the slim disc is shorter than the cooling timescale in slim discs (Kawaguchi 2003). A fraction of photons are unable to escape from the disc, and therefore they are trapped in the disc (Beckman 1978). The radiation efficiencies of slim discs are lower than those for standard thin discs due to the radial energy advection in the disc (Shakura & Sunyaev 1973; Laor & Netzer 1989).
Watarai 2006). The detailed studies show that the luminosity of a slim disc increases slowly with mass accretion rate as, \( L/L_{\text{Edd}} \propto \ln \dot{m} \) (\( \dot{m} = M/M_{\text{Edd}} \), \( M \) is the mass accretion rate, and \( M_{\text{Edd}} = L_{\text{Edd}}/0.1c^3 \), at high accretion rates, and the luminosity of the slim disc is almost saturated at around 10\( L_{\text{Edd}} \) (Watarai et al. 2003; Watarai 2006).

Unlike most analytical works on slim discs, outflows are always present in the numerical simulations of super-critical accretion flows (Ohsuga et al. 2005; Ohsuga & Mineshige 2010; Yang et al. 2014). This implies that only a small fraction of gas ways present in the numerical simulations of super-critical accretion flows (Watarai et al. 2000; Watarai 2006). The detailed studies show that the luminosity of the black hole increases from the mid-plane of the disc along \( z \), and reaches the maximum at \( z/R = \sqrt{2}/2 \) (see equation 7 and the discussion in their paper), in which an exact expression of the vertical component of gravity is adopted instead of the approximation \( GMz/R^3 \) adopted in most literature, which is only valid in the case of \( H/R \ll 1 \). A series of works on the vertical structure of the slim disc indeed show that outflows are inevitably driven from the the disc surface provided the luminosity of the disc reaches the Eddington value (Gu & Lu 2007; Gu et al. 2009; Gu & Lu 2012). Indeed, outflows have been observed in many luminous quasars (e.g., Pounds et al. 2003; Wang et al. 2005; Jiao et al. 2009; Gu 2012). Indeed, outflows have been observed in many luminous quasars (e.g., Pounds et al. 2003; Wang et al. 2005; Jiao et al. 2009; Gu 2012). Indeed, outflows have been observed in many luminous quasars (e.g., Pounds et al. 2003; Wang et al. 2005; Jiao et al. 2009; Gu 2012).

In this work, we investigate the structure of super-critical accretion disc with outflows driven by the radiation force of the disc. We describe our model calculations and results in Sects. 2 and 3. The last section contains the discussion of the results.

2 THE CONDITION FOR OUTFLOWS DRIVEN BY RADIATION FORCE

It was suggested that the outflows are driven by the radiation force at the disc surface when the relative disc thickness \( H/R \) is greater than a certain value (McClintock et al. 2006; Gu & Lu 2007). In this section, we extend the analysis on this issue in McClintock et al. (2006).

The equilibrium in the vertical direction of an accretion disc gives

\[
\frac{dp}{dz} = -\frac{GM\rho z}{(R^2 + z^2)^{3/2}},
\]

at radius \( R \). For a radiation pressure dominated accretion disc,

\[
p = \frac{\varepsilon_r}{3},
\]

where \( \varepsilon_r \) is the energy density of radiation. Substituting equation (2) into (1), we have

\[
\frac{1}{3} \frac{d\varepsilon_r}{dz} = -\frac{GM\rho z}{(R^2 + z^2)^{3/2}} = -\frac{q(z)\kappa_T \rho}{c},
\]

where \( \kappa_T \) is the electron scattering opacity, and \( q \) is the radiation energy flux in \( z \)-direction. We obtain

\[
q(z) = \frac{GMz}{(R^2 + z^2)^{3/2}} \frac{c}{\kappa_T}.
\]

The half-thickness \( H \) of the disc is related to the outgoing radiation flux \( f_{\text{rad}} \) from the unit surface area of the disc at \( z = H \) by

\[
f_{\text{rad}} = q(H) = \frac{GMH}{(R^2 + H^2)^{3/2}} \frac{c}{\kappa_T},
\]

which can be re-written in dimensionless form,

\[
\tilde{f}_{\text{rad}} = \frac{H}{(1 + H^2)^{3/2}}.
\]

The dimensionless quantities are defined as

\[
\tilde{f}_{\text{rad}} = f_{\text{rad}} \frac{R^2 \kappa_T}{GMc}, \quad \text{and} \quad \tilde{H} = \frac{H}{R}.
\]

The vertical component of gravity of the black hole increases from the mid-plane of the disc along \( z \), and reaches the maximum at \( z/R = \sqrt{2}/2 \) (see equation 7 and Fig. 1). The balance between the radiation force and the vertical gravity is broken down if the radiation flux is sufficiently large. This implies that outflows may probably be driven from the disc if the radiation flux of the disc is greater than a critical value \( \tilde{f}_{\text{rad}}^{\text{max}} \) corresponding to the maximal vertical gravity, which is given by

\[
\tilde{f}_{\text{rad}}^{\text{max}} = \left( \frac{\tilde{H}^{\text{max}}}{1 + \tilde{H}^{\text{max}}^2} \right)^{3/2} = \frac{2\sqrt{3}}{9},
\]

where \( \tilde{H}^{\text{max}} = \sqrt{2}/2 \) is used.

Both the vertical gravity and radiation force exerted on the gas in the disc is proportional to gas density \( \rho \), and therefore the density \( \rho \) is canceled in the vertical force balance equation (4). We do not need to assume a specific function of \( z \) for gas density in the above analysis. This means our conclusion does not depend on the vertical density distribution of the disc. Turner (2004) found that the energy dissipation is roughly constant vertically in the radiation-MHD simulation for a radiation-dominated accretion disc. If this is the case, the radiation energy flux in the disc is

\[
q(z) = f_{\text{rad}} \frac{z}{H}.
\]

The radiation force exerted on the unit of mass in \( z \)-direction is

\[
F(z) = q(z) \frac{\kappa_T c}{c} = \tilde{f}_{\text{rad}} \frac{z}{R^2} \frac{GM}{H^2}.
\]

We plot the radiation force in Fig. 1 to compare with the vertical component of gravity. It is found that the radiation force is almost in equilibrium with gravity in \( z \)-direction for the low-\( f_{\text{rad}} \) (i.e., low-\( H \)) case. This is the case considered in the radiation MHD simulation given in Turner (2004). The vertical gravity \( -\rho H\kappa_T z \) is adopted in Turner (2004)'s work, which is a good approximation for \( z/R \ll 1 \). When \( z/R \) is sufficiently large (\( \gtrsim 0.2 \)), the vertical gravity deviates obviously from the linear relation of \( z \) (see equation 7). If the energy dissipation can still remain constant vertically in this case, the radiation force may be dominant over the vertical gravity at the upper lay of the disc (see Fig. 1). It should be cautious on this conclusion, because the disc thickness adopted here is derived on the assumption of the vertical gravity \( -\rho\kappa_T z \). This issue is to be resolved by the future numerical simulation adopting an exact function of vertical gravity. In the case of \( \tilde{f}_{\text{rad}} > \tilde{f}_{\text{rad}}^{\text{max}} \), the radiation force always dominates over the vertical gravity in the region of \( z/R > \tilde{H}_{\text{max}} \) for arbitrary \( z \)-dependent energy dissipation rate even if the disc thickness is not well determined. This can be verified by shifting \( \tilde{f}_{\text{rad}} (\tilde{f}_{\text{rad}} > \tilde{f}_{\text{rad}}^{\text{max}}) \) in the horizontal direction in Fig. 1 for arbitrary value of \( \tilde{H} \).

The condition of outflows driven by radiation force is derived above for a radiation pressure dominated accretion disc, which is probably the case for a disc accreting at a high rate. We should
point out that the condition is valid even if the disc is not radiation pressure dominant. The condition for radiation pressure driving outflows is the radiation force being greater than the maximum of the vertical gravity, i.e.,

\[
\frac{f_{rad}\rho c_T}{c} > \rho F_z^{\max},
\]

(11)

where \( F_z \) is the vertical gravity for unit mass of gas,

\[
F_z(R, z) = \frac{GMz}{(R^2 + z^2)^{3/2}}.
\]

(12)

of which the maximum is \( F_z^{\max} = 2\sqrt{3}GM/9R^2 \). In dimensionless form, it becomes

\[
\tilde{f}_{rad} > 2\sqrt{3}/9,
\]

(13)

where equation (10) is used. This is exactly the same as the condition derived in the first part of this section (see equation [8]). The self-gravity of the disc may dominate over the gravity of the black hole in the outer region of the disc, which is not considered in this work. This may alter the condition for outflows driven by radiation force, and the self-gravity may hamper the disc to launch outflows.

3 THE ACCRETION DISC WITH OUTFLOWS

For a disc accreting at a high rate, the outgoing radiation flux \( f_{rad} \) is \( \tilde{f}_{rad} > \tilde{f}_{rad}^{\max} \). In this case, a fraction of gas in the disc will be channeled into the outflows by the radiation force. The mass accretion rate in the disc will be a function of radius. Following the standard accretion disc model (Shakura & Sunyaev 1973, Frank, King, & Raine 2002), we derive the dissipation power in unit surface area of the disc with a \( R \)-dependent mass accretion rate \( \dot{M}(R) \),

\[
D(R) = \frac{3GM\dot{M}(R)}{8\pi R^3} \left[ 1 - \frac{\dot{M}_\text{in}}{\dot{M}(R)} \left( \frac{R_{\text{in}}}{R} \right)^{1/2} \right],
\]

(14)

where \( \dot{M}_\text{in} \) is the mass accretion rate at the inner radius of the disc \( R_{\text{in}} \). For a non-rotating black hole, \( R_{\text{in}} = 6GM/c^2 \). The dimensionless form of equation (14) is

\[
\tilde{D}(r) = \frac{15\tilde{m}(r)}{r} \left[ 1 - \frac{\tilde{m}_\text{in}}{\tilde{m}(r)} \left( \frac{\tilde{r}_{\text{in}}}{r} \right)^{1/2} \right],
\]

(15)

where

\[
\tilde{D}(r) = D(R) \frac{R^2 c_T}{GMc}, \quad \tilde{m} = \frac{\dot{M}}{\dot{M}_{\text{Edd}}}, \quad \dot{M}_{\text{Edd}} = \frac{L_{\text{Edd}}}{0.1c^2}
\]

\[
L_{\text{Edd}} = 4\pi GMc, \quad r = \frac{R}{R_g}, \quad R_g = \frac{GM}{c^2}.
\]

(16)

In this work, we avoid being involved in the complexity of outflow physics. In the region of the radiation flux being greater than the maximal value given in equation [3], the radiation force dominates over the vertical gravity for the gas in the upper layer of the disc (\( z/R > H^{\max} \)), and it is blown away by the radiation force. This makes the mass accretion rate decrease, and therefore the energy dissipation rate decreases due to the decrease of mass accretion rate in the disc. We assume that this process continues till the radiation flux \( f_{rad} \) becomes \( \tilde{f}_{rad}^{\max} \). We neglect the radial energy advection, and assume \( f_{rad}(R) = D(R) \) in the disc, which is to be justified later in this section.

There is a critical mass accretion rate, \( \tilde{m}_{\text{crit}} \), below which no outflow is driven by the radiation force. The critical rate is calculated with

\[
\tilde{D}(r) = \frac{15\tilde{m}_{\text{crit}}(r)}{r} \left[ 1 - \left( \frac{\tilde{r}_{\text{in}}}{r} \right)^{1/2} \right] = f_{rad}^{\max},
\]

(17)

at \( r = 9r_{\text{in}}/4 \), because \( \tilde{D}(r) \) reaches the maximal value at this radius for a given \( \tilde{m} \). Using equation (3), we obtain

\[
\tilde{m}_{\text{crit}} = \frac{3\sqrt{3}}{5} \approx 1.04.
\]

(18)

The radiation flux \( f_{rad}(R) \) varying with mass accretion rate \( \tilde{m} \) is plotted in Fig. 2.

For an accretion disc accreting at a rate \( \tilde{m} > \tilde{m}_{\text{crit}} \), the outflows are present in the region of the disc between \( r_{\text{w}^0} \) and \( r_{\text{w}^{\max}} \).
The gas at the disc surface starts to be removed into the outflows from the disc at $r_w = \max$, where $D(r_w) = \max$. The outflows are switched off at $r_w = 9r_{in}/4$, because $D(r)$ will decrease within this radius (see Fig. 2). This means that the mass accretion rate at the inner radius of the disc $\dot{m}_{in} \equiv \dot{m}_{\text{crit}}$ if the mass accretion rate at the outer radius of the disc $\dot{m}_{out} > \dot{m}_{\text{crit}}$. The outer radius $r_w = \max$ of the disc with outflows can be determined by solving the equation $D(r_w) = \max$, i.e.,

$$
\frac{15\dot{m}_{out}}{r_w \max} \left[ 1 - \frac{\dot{m}_{in}}{\dot{m}_{out}} \left( \frac{r_{in}}{r_w} \right)^{1/2} \right] = \frac{2\sqrt{3}}{9}, \tag{19}
$$

with a given value of $\dot{m}_{out}$. Using equation (19), we have

$$
r_w \max \approx \frac{45\sqrt{3}}{2} \frac{\dot{m}_{out}}, \tag{20}
$$

for the case of $\dot{m}_{out} \gg \dot{m}_{\text{crit}}$. The outer radii of the region with outflows varying with mass accretion rate $\dot{m}_{out}$ calculated with equations (19) and (20) are plotted in Fig. 3. It is found that the results almost converge when mass accretion rate $\dot{m}_{out} \gg 5$.

The luminosity of an accretion disc with outflows ($\dot{m} > \dot{m}_{\text{crit}}$) can be calculated with

$$
\frac{L}{L_{\text{Edd}}} = \frac{1}{L_{\text{Edd}}} \int_{r_{in}}^{\infty} 4\pi R(R) dR = \int_{r_{in}}^{\max} \frac{\dot{f}_{\text{rad}}(r)}{r} dr, \tag{21}
$$

where

$$
\dot{f}_{\text{rad}}(r) = \begin{cases} 
\frac{15\dot{m}_{in}}{r_{in} \max}, & r \leq r_{\text{min}}, \\
\frac{\dot{m}_{\text{out}}}{r_w \max} \left[ 1 - \frac{\dot{m}_{out}}{\dot{m}_{\text{out}}} \left( \frac{r_{in}}{r_w} \right)^{1/2} \right], & r_w \max < r < r_{\text{max}}, \\
\frac{15\dot{m}_{out}}{r_{w} \max}, & r \geq r_{\text{max}}.
\end{cases} \tag{22}
$$

The integral in equation (21) can be carried out analytically as

$$
\lambda = \frac{L}{L_{\text{Edd}}} = \frac{35\dot{m}_{in}}{27r_{in}} + \frac{15\dot{m}_{out}}{r_w \max} \left[ 1 - \frac{2\dot{m}_{in}}{3\dot{m}_{out}} \left( \frac{r_{in}}{r_w} \right)^{1/2} \right] + \frac{2\sqrt{3}}{9} \ln \left( \frac{r_w \max}{r_{w} \max} \right). \tag{23}
$$

Substituting equation (20) into (23), we obtain an approximation for $\lambda$,

$$
\lambda \approx \frac{35}{162} \dot{m}_{in} + \frac{2\sqrt{3}}{9} \left[ 1 - \frac{2\dot{m}_{in}}{3\dot{m}_{out}} \left( \frac{4\sqrt{3}}{15\dot{m}_{out}} \right)^{1/2} \right] + \frac{2\sqrt{3}}{9} \ln \frac{5\dot{m}_{out}}{3}, \tag{24}
$$

in high-$\dot{m}_{out}$ cases. We plot the Eddington ratio of the disc changing with mass accretion rate $\dot{m}_{out}$ in Fig. 4 in which the analytical approximation (24) agrees with the numerical result quite well if $\dot{m}_{out} \gtrsim 1.5$. It is found that the Eddington ratio $\lambda \propto \ln \dot{m}_{out}$ when $\dot{m}_{out}$ is high.

As discussed above, the outflows are present when accretion rate is higher than a critical value, which means that the mass accretion rate is a function of radius. The outflows are driven from the disc region between $r_{\text{min}}$ and $r_{\text{max}}$, and the mass accretion rate in this region can be calculated by

$$
\dot{D}(r) = \frac{15\dot{m}(r)}{r} \left[ 1 - \frac{\dot{m}_{in}}{\dot{m}(r)} \left( \frac{r_{in}}{r} \right)^{1/2} \right] = \frac{\dot{m}_{\text{out}}}{r}, \tag{25}
$$

which reduces to

$$
\dot{m}(r) = \frac{\dot{m}_{\text{out}}}{r_{\text{max}}} + \frac{\dot{m}_{in} \left( \frac{r_{in}}{r} \right)^{1/2}}{15} = \frac{2\sqrt{3}}{9} r + \frac{9\sqrt{3}}{5} r^{-1/2}. \tag{26}
$$

The mass accretion rate remains constant radially, i.e., $\dot{m} = \dot{m}_{\text{out}}$ in the region of $r \geq r_{\text{max}}$, and $\dot{m} = \dot{m}_{\text{inf}} = 3\sqrt{3}/5$ for $r \leq r_{\text{min}} = 9r_{in}/4 = 27/2$. In Fig. 5 we plot the $r$-dependent mass accretion rate $\dot{m}(r)$ with different values of $\dot{m}_{\text{out}}$. The mass accretion rate in the inner edge of the discs remains constant, $\dot{m}_{\text{inf}} = 1.04$, due to the presence of outflows.
is sufficiently large. It implies that outflows can be driven from the disc if the radiation flux dominates over the vertical gravity in the disc if the radiation flux is large (see equation [19]). For a disc of mass accretion rate $\dot{m}$ when the radiation flux is large as $\dot{m} \sim 3\dot{m}$, the outflows are driven from the disc surface in the disc region between $r_{\text{w}}^\text{min}$ and $r_{\text{w}}^\text{max}$, where $r_{\text{w}}^\text{min} \equiv 9r_{\text{in}}/4$, and $r_{\text{w}}^\text{max}$ is a function of mass accretion rate $\dot{m}$ (see equation [19]). The outer radius of the disc region with outflows $r_{\text{w}}^\text{max} \propto \dot{m}$ when $\dot{m}$ is large (see equation [20]).

4 SUMMARY AND DISCUSSION

4.1 Structure of the disc

The vertical component gravity increases with $z$, and reaches the maximum at $z/R = \sqrt{3}/2$ (see Fig. 1). The radiation force will dominate over the vertical gravity in the disc if the radiation flux is sufficiently large. It implies that outflows can be driven from the disc if the radiation flux of the disc is greater than a critical value. This has already been discussed in the previous works (McClintock et al. 2006; Gu & Lu 2007).

The relative disc thickness of $H/R$ corresponds to a dimensionless radiative flux $f_{\text{rad}}$ at the disc surface for a radiation pressure dominated disc. We derive the critical mass accretion rate $\dot{m}_{\text{crit}} = 3\sqrt{3}/5 \sim 1.04$, below which the relative thickness $H/R$ of the disc is always less than $\sqrt{2}/2$, i.e., no outflow is present (see Fig. 1). In the case of $\dot{m} > \dot{m}_{\text{crit}}$, the outflows are driven from the disc surface in the disc region between $r_{\text{w}}^\text{min}$ and $r_{\text{w}}^\text{max}$, where $r_{\text{w}}^\text{min} \equiv 9r_{\text{in}}/4$, and $r_{\text{w}}^\text{max}$ is a function of mass accretion rate $\dot{m}$ (see equation [19]). The outer radius of the disc region with outflows $r_{\text{w}}^\text{max} \propto \dot{m}$ when $\dot{m}$ is large (see equation [20]). For a disc of mass accretion rate $\dot{m}$, the relative thickness $H/R$ is always less than $\sqrt{2}/2$, i.e., no outflow is present (see Fig. 1). In the case of $\dot{m} > \dot{m}_{\text{crit}}$, the outflows are driven from the disc surface in the disc region between $r_{\text{w}}^\text{min}$ and $r_{\text{w}}^\text{max}$, where $r_{\text{w}}^\text{min} \equiv 9r_{\text{in}}/4$, and $r_{\text{w}}^\text{max}$ is a function of mass accretion rate $\dot{m}$ (see equation [19]). The outer radius of the disc region with outflows $r_{\text{w}}^\text{max} \propto \dot{m}$ when $\dot{m}$ is large (see equation [20]).

Figure 5. The mass accretion rates $\dot{m}(r)$ as functions of radius with different values of $\dot{m}_{\text{out}} = 2$, 10 and 100, respectively. The red line is the outer radius of the advection dominated region in the disc (see equation [22]).

Figure 6. The effective temperatures of the discs as functions of radius with different values of $\dot{m}_{\text{out}} = 0.2$ (black), 1 (red), 2 (green), 10 (yellow) and 100 (blue), respectively. The solid lines indicates the results calculated with black hole mass $M = 10^8 M_\odot$, while the dashed lines are for $M = 10^9 M_\odot$.

Figure 7. The spectra of the discs surrounding a black hole as functions of radius with different values of $\dot{m}_{\text{out}} = 0.2$ (black), 1 (red), 2 (green), 10 (yellow) and 100 (blue), respectively. The solid lines indicates the results calculated with black hole mass $M = 10^8 M_\odot$, while the dashed lines are for $M = 10^9 M_\odot$. 

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Watarai et al. (2000)’s calculations on the slim discs show that radial energy advection is important in the inner region of the discs, and the region of the disc with

$$r \leq r_{\text{adv}} \simeq 3\dot{m},$$

is advection dominated (see equation 16 in Watarai et al. 2000, and note that their $\dot{m}$ should be divided by ten to be consistent with our definition of $\dot{m}$). The advection dominated region disappears in the disc when $\dot{m} \lesssim 2$, which means that the advection is negligible in this case (see equation 19 in Watarai et al. 2000). In Fig. 5, we find that the mass accretion rate $\dot{m}$ derived in this work is always much lower than that required for advection dominated case at any radius in the disc. This justifies the assumption of $D(R) = f_{\text{rad}}$ adopted in our calculations.

The effective temperature of the disc as a function of radius can be calculated with $T_e(R) = [f_{\text{rad}}(R)/c]^1/4$ by using the derived radiation flux (see equation [22]), when the black hole mass $M$ is specified. The spectrum of the disc is therefore available with the derived effective temperature of the disc. We plot the effective temperatures of the discs surrounding massive black holes with different values of mass accretion rate $\dot{m}_{\text{out}}$ in Fig. 6. The temperatures of the inner regions of the discs are almost the same for any values of $\dot{m}_{\text{out}} > \dot{m}_{\text{crit}}$. In Fig. 7, we plot the radiation spectra of the accretion discs varying with mass accretion rates. The spectra of the discs surrounding massive black holes are saturated in UV/soft X-ray wavebands ($\nu \gtrsim 10^{15}$ Hz) when accretion rate is as high as $\dot{m}_{\text{out}} \sim 2$. 

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accreting at an extreme rate $\dot{m} \sim 100$, the disc region with outflows can extend to thousands of gravitational radii (see Fig. 3). Due to the presence of outflows, the mass accretion rate is not a constant radially in the disc. A fraction of gas at the disc surface is blown away by the radiation force, so we assume that mass accretion rate is self-adjusted to let the dissipation power in unit surface area of the disc $D(R) = f_{\text{rad}}^{\max}$. This means that we have assumed the radial inflow timescale at a certain radius to be longer than the timescale for an outflow to change the accretion rate of the whole disc. The outflow timescale can be estimated only if the dynamics of the outflow is well studied, which is beyond the scope of this work. The mass accretion rate as a function of radius is derived in this work (see equation 1 and Fig. 5). At large radii, the mass accretion rate $\dot{m} \propto r$, which is the same as the result given in Gu & Lu (2007). This is qualitatively consistent with the hydrodynamical simulations on super-critical accretion flows (see, e.g., Fig. 6 in Ohsuga et al. 2005). Their numerical simulations indicate that the outflow can be so strong to carry sufficient gas from the disc, though the detailed study of the outflow dynamics is needed to clarify this issue.

We note that the relative disc thickness $H/R$ is larger than that derived in the previous works for the same mass accretion rate $\dot{m}$ (e.g., Kawaguchi 2003). In most of the previous works, an approximation of the vertical component of gravity, $z\Omega_K^2$, is adopted, which over-estimates the vertical gravity compared with the exact one used in this work when $z/R$ is large (see Fig. 1 in Gu & Lu 2007). This approximation leads to underestimate of $H/R$ in the previous works. For relative disc thickness $H/R \lesssim 0.2$, the vertical gravity $z\Omega_K^2$ is a good approximation (see Fig. 1). Our calculations show that the disc thickness $H$ is systematically larger than that calculated in Kawaguchi (2003). The relative disc thickness $H/R \rightarrow \sqrt{2}/2$ in the disc with $R \lesssim 400 R_g$ when $\dot{m} = 10$ (see Fig. 5), which is about twice of that given in Kawaguchi (2003) for the same accretion rate (see Fig. 5 in that paper). In that work, the scale height of the disc is estimated with

$$H' = \frac{c_s}{\Omega_K}, \quad (28)$$

where the sound speed at the mid-plane of the disc $c_s = (\rho_{\text{mid}}/\rho_{\text{mid}}^{1/2})$, which is different from ours even in the limit of $H \ll R$. In this work, the disc thickness can be derived from equation 5.

$$H \simeq \frac{f_{\text{rad}}K}{\Omega_K c}, \quad (29)$$

when $H/R \ll 1$. The outgoing radiation flux is

$$f_{\text{rad}} = \frac{4\pi c T_{\text{mid}}^4}{3hc \rho_{\text{mid}} K H}. \quad (30)$$

Substitute equation 30 into 29, we have

$$H \simeq \frac{(4\pi)^{1/2}}{\rho_{\text{mid}}K^{1/2}}, \quad (31)$$

where $\rho_{\text{mid}} = n T_{\text{mid}}^4/3$ is used for a radiation pressure dominated disc. This can explain the systematic difference of disc thickness between Kawaguchi (2003)’s paper and this work.

Our calculations show that outflows can be driven from the outer region of the disc, which may be truncated at $R_{\text{out}}$, where the disc becomes gravitationally unstable (e.g., Hur 1998; Kawaguchi, Pierens, & Hur 2004). The outer radius of the disc, $R_{\text{out}}$, is a function of black hole mass, accretion rate, and the viscosity parameter $\alpha$, which can be calculated with derived disc structure. However, the detailed disc structure has not been derived in this work, and the constraint on the size of a disc with outflows will be reported in our future work. Our present work cannot tell whether the outflow is damped by the vertical self-gravity or not.

### 4.2 Luminosity of the disc

The luminosity of the disc with outflows can be derived with equation 24. We find that, $L/L_{\text{Edd}} \propto \ln \dot{m}$, if $\dot{m}$ is large (equation 24). This looks similar to the result for the slim disc (Watarai et al. 2000; Watarai 2006), however, the physics is completely different. In their works, no outflow is considered, and the mass accretion rate remains constant radially in the disc.

The cooling timescale is longer than the viscous timescale of the slim disc. Therefore, only a fraction of the dissipated power is radiated away, and the remainder is advected inwards. The radiation of the slim disc is limited by the photon trapping effect within a certain radius, which is roughly proportional to mass accretion rate $\dot{m}$. Thus, the luminosity of the slim disc $L/L_{\text{Edd}} \propto \ln \dot{m}$ (Watarai et al. 2004; Watarai 2006). In our present work, the mass accretion rate at the inner radius of the disc $m_{\text{in}} = m_{\text{crit}} \simeq 1.04$ if $m_{\text{out}} > m_{\text{crit}}$, because most of the gas in the disc is carried away in the outflows. The luminosity of the disc is therefore limited by the outflows (see Sect. 3). We find that the luminosity of the disc with outflows $L/L_{\text{Edd}}$ increases more slowly than that for the slim disc (see Fig. 4). In this work, the disc luminosity $L/L_{\text{Edd}} \sim 3$ provided $\dot{m} \sim 100$, which is lower than $L/L_{\text{Edd}} \sim 10$ for the same accretion rate in Watarai et al. (2000). Our results are comparable with $L/L_{\text{Edd}} \sim 2.6$ for $\dot{m} \sim 100$, as given in Kawaguchi (2003).

Our calculations are carried out based on the balance between the radiation force and the gravity in the vertical direction of the disc, which means that the main conclusions on the maximal disc luminosity in this work will not be altered whether the energy advection is important or not. We find that the radial energy advection is always negligible in an accretion disc with outflows (see Fig. 5) and the detailed discussion in Sect. 5.

The UV/soft X-ray spectrum of the disc surrounding a massive black hole is determined by the effective temperature profile and electron scattering effects (Kawaguchi 2003). In this work, the spectral calculations are carried out based on the local blackbody assumption without including the electron scattering effects. Therefore, the spectral shape of the disc in the high frequency end is predominantly determined by the structure of the inner region of the disc. In Fig. 6 we find that the structure of inner region $(\lesssim 100 R_g)$ of the disc remains unchanged when $m_{\text{out}} \gtrsim 2$, from which most of the accretion power is radiated. The effective temperatures of the discs with outflows in the inner regions almost converge for different values of $m_{\text{out}}$ (see Fig. 6). Therefore, it is found that the emission in the high frequency end of the spectrum $(\nu \gtrsim 10^{19} \text{ Hz})$ is saturated when $m_{\text{out}} \gtrsim 2$ (see Fig. 7). The further calculations of the spectrum including the electron scattering effects, as done by Kawaguchi (2003), are necessary for detailed astrophysical applications. As the structure of the inner region of the disc with outflows accreting at a high rate is insensitive to the mass accretion rate at the outer radius, we believe that the feature of the emission saturation should still be present, though the final results may be quantitatively different from our present results. Such calculations are beyond the scope of this work, which will be reported in our future work.
4.3 Implications of the model

The saturation of the emission in the high frequency end of the disc spectra can be used for estimating the masses of the black holes accreting at high rates, such as, narrow-line Seyfert galaxies. It implies that luminous quasars with $m_{\text{out}} > 2$ can be used as a type of “cosmological candles”, and one does not need to search massive black holes accreting at extremely high rates (Wang et al. 2013).

The relation of $m_{\text{out}} - L/L_{\text{Edd}}$ derived in this work shows that the radiation of the disc can only be several times of Eddington luminosity even if the mass accretion rate $m_{\text{out}} \gg 1$. This seems to be in contradiction with the observations of some ultra-luminous X-ray (ULX) sources, of which the luminosity can be highly super Eddington if normal stellar mass black holes reside in the ULX (e.g., Feng & Soria 2011; Sutton, Roberts, & Middleton 2013; Soria et al. 2014, and the references therein). As suggested by King et al. (2001), this issue can be alleviated if the X-ray emission from the ULXs is mildly beamed to the observer, and the observed X-ray emission is predominantly from the outflows (Begelman, King, & Pringle 2006), which are qualitatively consistent with our results that outflows are inevitably driven from the radiation dominated accretion disc will be reported in our future work, which is beyond the scope of this paper. An alternative possibility is that these ULX may have intermediate mass black holes, of which the luminosity is therefore lower or close to Eddington luminosity (see Feng & Soria 2011 for a review, and references therein).

Luminous quasars are discovered at redshifts $z \geq 6$ (see Fan et al. 2003; Mortlock et al. 2011; Venemans et al. 2013, and the references therein), corresponding to the age of the universe less than 1 Gyr. To grow a black hole to masses that luminous quasars with $m_{\text{crit}} \sim 10^9 M_\odot$ can only be several times of Eddington luminosity even if the mass accretion rate $m_{\text{out}} \gg 1$, even if the mass accretion rate at the outer radius of the disc $m_{\text{out}} \gg 1$, because most of the gas in the disc is removed in the outflows from the disc. Our results seems to be consistent with the continuous accretion scenario for massive black hole growth at high redshifts (e.g., Shapiro 2005; Tanaka & Haiman 2009, 2014).

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