GLOBAL QUANTIZATION OF VACUUM ANGLE AND MAGNETIC MONOPOLES AS A NEW SOLUTION TO THE STRONG CP PROBLEM

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ABSTRACT

The non-perturbative solution to the strong CP problem with magnetic monopoles as originally proposed by the author is described. It is shown that the gauge orbit space with gauge potentials and gauge tranformations restricted on the space boundary and the globally well-defined gauge subgroup in gauge theories with a $\theta$ term has a monopole structure if there is a magnetic monopole in the ordinary space. The Dirac's quantization condition then ensures that the vacuum angle $\theta$ in the gauge theories must be quantized to have a well-defined physical wave functional. The quantization rule for $\theta$ is derived as $\theta = 0, 2\pi/n$ ($n \neq 0$) with $n$ being the topological charge of the magnetic monopole. Therefore, the strong CP problem is automatically solved with the existence of a magnetic monopole of charge $\pm 1$ with $\theta = \pm 2\pi$. This is also true when the total magnetic charge of monopoles are very large ($|n| \geq 10^9 2\pi$). The fact that the strong CP violation can be only so small or vanishing may be a signal for the existence of magnetic monopoles and the universe is open.

1. Introduction and Summary of the Main Results

Yang-Mills theories$^1$ and their non-perturbative effects have played one of the most important roles in particle physics. It is known that, in non-abelian gauge theories a Pontryagin or $\theta$ term,

$$\mathcal{L}_\theta = \frac{\theta}{32\pi^2} \epsilon^{\mu\nu\lambda} \, \text{sgn} \, F_{\mu \nu} F^a_{\lambda \sigma} \,,$$

(1)
can be added to the Lagrangian density of the system due to instanton$^2$ effects in gauge theories. This term can induce CP violations for an arbitrary value of $\theta$. Especially, such an effective $\theta$ term in QCD may induce CP violations in strong interactions. In our discussions relevant to QCD, $\theta$ is simply used to denote $\theta + \text{arg}(\text{det}M)$ effectively with $M$ being the quark mass matrix, when the effects of electroweak interactions are included. However, the experimental results on the neutron electric dipole moment strongly limit the possible values of the $\theta$ in QCD ($\leq 10^{-9}$, modulo $2\pi$ for example). This is the well-known strong CP problem. One of the most interesting understanding of the strong CP problem has been the assumption of

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an additional Peccei-Quinn $U(1)_{PQ}$ symmetry, but the observation has not given evidence for the axions needed in this approach. Thus the other possible solutions to this problem are of fundamental interest.

Recently, a non-perturbative solution to the Strong CP problem with magnetic monopoles has been proposed originally by the author. In our solution, it is proposed that the vacuum angle with magnetic monopoles must be quantized. Our quantization rule is derived essentially by two different methods. This is given by $\theta = 0$, or $\theta = 2\pi N/n$ ($n \neq 0$) with integer $n$ being the relevant topological charge of the magnetic monopole and $N$ may be fixed as 1 in the method 1 and is an arbitrary integer in the method 2. The first method is to show the existence of a monopole structure in the relevant gauge orbit space in Schrödinger formulation, and using the Dirac quantization rule for having a well-defined wave functional. The second method is to show that there exist well-defined gauge transformations which will ensure the quantization of $\theta$ by the constraints of Gauss’s law due to the non-abelian magnetic charges carried by the magnetic monopoles proportional to $\theta$ as noted in Ref. 21 and generalized in Ref.22 to the non-abelian case for the generalized magnetic monopoles.

Therefore, we conclude that strong CP problem can be solved due to the quantization of $\theta$ in the presence of magnetic monopoles, for example monopoles of topological charge $n = \pm 1$ with $\theta = \pm 2\pi$, or $n \geq 2\pi 10^9$ with $\theta \leq 10^{-9}$. Moreover, the existence of non-vanishing magnetic flux through the space boundary implies that the universe must be open. In this note, we will briefly describe and review our solution to the strong CP problem with magnetic monopoles with the first method.

2. Quantization Condition on $\theta$ and Solution to the Strong CP Problem

The main idea of our discussions is based on the follows. A wave functional in the gauge orbit space corresponds to a cross section of the relevant fiber bundle for the theory. Topologically, if there is a non-vanishing gauge field as the curvature in the gauge orbit space, then the flux of the curvature through a closed surface in the gauge orbit space must be quantized to have a cross section. Physically, this is equivalent to say that the magnetic flux through the closed surface must be quantized according to the Dirac quantization condition in order to have a well-defined wave functional in the quantum theory.

In this method, we will extend the method of Wu and Zee in Ref.7 for the discussions of the effects of the Pontryagin term in pure Yang-Mills theories in the gauge orbit spaces in the Schrödinger formulation. This formalism has also been used with different methods to derive the mass parameter quantization in three-dimensional Yang-Mills theory with Chern-Simons term. It is shown in Ref. 7 that the Pontryagin term induces an abelian background field or an abelian structure in the gauge configuration space of the Yang-Mills theory. In our discussions, we will consider the case with the existence of a magnetic monopole. We will show that magnetic monopoles in space will induce an abelian gauge field with non-
vanishing field strength in gauge configuration space, and magnetic flux through a two-dimensional sphere in the induced gauge orbit space on the space boundary is non-vanishing. Then, Dirac condition\cite{9-10} in the corresponding quantum theories leads to the result that the relevant vacuum angle $\theta$ must be quantized as $\theta = 2\pi/n$ with $n$ being the topological charge of the monopole to be generally defined. Therefore, the strong CP problem can be solved with the existence of magnetic monopoles.

We will now consider the Yang-Mills theory with the existence of a magnetic monopole at the origin. Our derivation applies generally to a gauge theory with an arbitrary simple gauge group or a U(1) group outside the monopole. This gauge group under consideration may be regarded as a factor group in the exact gauge group of a grand unification theory. Note that there can be Higgs field and unification gauge fields confined inside the monopole core, which will be ignored in our discussion outside the monopoles.

As we will see that an interesting feature in our derivation is that we will use the Dirac quantization condition both in the ordinary space and restricted gauge orbit space to be defined. The Lagrangian of the system is given by

$$\mathcal{L} = \int d^4x \left\{-\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{\theta}{32\pi^2} \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu}^a F_{\lambda\sigma}^a \right\}. \quad (2)$$

We will use the Schrödinger formulation and the Weyl gauge $A^0 = 0$. The conjugate momentum corresponding to $A^a_i$ is given by

$$\pi_i^a = \frac{\delta\mathcal{L}}{\delta \dot{A}_i^a} = \dot{A}_i^a + \frac{\theta}{8\pi^2} \epsilon_{ijk} F_{jk}^a. \quad (3)$$

In the Schrödinger formulation, the system is similar to the quantum system of a particle with the coordinate $q_i$ moving in a gauge field $A_i(q)$ with the correspondence\cite{6-7}

$$q_i(t) \rightarrow A^a_i(x, t), \quad (4)$$

$$A_i(q) \rightarrow A^a_i(A(x)), \quad (5)$$

where

$$A^a_i(A(x)) = \frac{\theta}{8\pi^2} \epsilon_{ijk} F_{jk}^a. \quad (6)$$

Thus there is a gauge structure with gauge potential $A$ in this formalism within a gauge theory with the $\theta$ term included. Note that in our discussion with the presence of a magnetic monopole, the gauge potential $A$ outside the monopole generally need to be understood as well defined in each local coordinate region. In the overlapping regions, the separate gauge potentials can only differ by a well-defined gauge transformation\cite{10}. In fact, single-valuedness of the gauge function corresponds to the Dirac quantization condition\cite{10}. For a given $r$, we can choose two extended semi-spheres around the monopole, with $\theta \in [\pi/2 - \delta, \pi/2 + \delta]$ ($0 < \delta < \pi/2$) in the overlapping region, where the $\theta$ denotes the $\theta$ angle in the spherical polar coordinates. For convenience, we will use differential forms\cite{10} in our discussions,
where \( A = A_i dx^i \), \( F = \frac{1}{2} F_{jk} dx^j dx^k \), with \( F = dA + A^2 \) locally. For our purpose to discuss about the effects of the abelian gauge structure on the quantization of the vacuum angle, we will now briefly clarify the relevant topological results needed, then we will realize the topological results explicitly.

With magnetic monopoles, we need to generalize the gauge orbit space of ordinary gauge theories to include the space boundary which is noncontractible with non-vanishing magnetic flux quantized according Dirac quantization condition. With the constraint of Gauss’ law, the quantum theory in the finite space region in this formalism is described in the usual gauge orbit space \( \mathcal{U}/\mathcal{G} \). The \( \mathcal{U} \) is the space of well-defined gauge potentials and \( \mathcal{G} \) denotes the space of continuous gauge transformations with gauge functions mapping the space boundary to a single point in the gauge group. Due to the existence of magnetic monopoles, the gauge transformations on the space boundary \( S^2 \) can be non-trivial, the physical effects of the well-defined gauge transformations need to be considered. As it is known that\(^{12} \), only the gauge transformations generated by the generators commuting with magnetic charges may be well-defined globally. On the space boundary, \( \mathcal{U} \) will also be used to denote the induced gauge configuration space with gauge potentials restricted on the space boundary, and \( \mathcal{G} \) also denotes the continuous gauge transformations restricted on the space boundary and well-defined gauge subgroup. Then we will call corresponding \( \mathcal{U}/\mathcal{G} \) as restricted gauge orbit space. Collectively, they will be called as the usual space for the finite coordinate space region and the restricted space on the space boundary. There should not be confusing for the notations used both for the usual spaces and restricted spaces. As we will see that the magnetic charges up to a conjugate transformation are in a Cartan subalgebra of the gauge group, then on the space boundary \( S^2 \), we need to consider a well-defined gauge subgroup \( G = U(1) \) for the quantization of \( \theta \). Similar to the usual gauge orbit space on the compactified coordinate space by restricting to gauge functions mapping the space boundary to a single point in the gauge group, the restricted gauge orbit space is well-defined since the space boundary \( S^2 \) is compact.

Note that the physical meaning of the restricted gauge orbit space can be understood as follows. Let \( \Psi_{\text{phys}}(\mathcal{A}(x)) \) denote the physical wave functional and \( \Psi_{\text{phys}}(\mathcal{A}(x))|_{S^2} \) be its restriction on the space boundary \( S^2 \) which actually only depends on the direction of \( x \). Then, the \( \Psi_{\text{phys}}|_{S^2} \) must be invariant under the gauge transformations well-defined on the entire space boundary. Namely the \( \Psi_{\text{phys}}|_{S^2} \) is defined in the restricted gauge orbit space. However, in the finite space region, the \( \Psi_{\text{phys}}(\mathcal{A}(x)) \) for finite \( x \) is only required to be invariant under the gauge transformations with gauge function going to the identity at the spatial infinity. Namely, it is defined the usual gauge orbit space. The entire \( \Psi_{\text{phys}} \) is then well-defined in the generalized gauge orbit space as described.

Now consider the following exact homotopy sequence\(^{13} \) both for the usual and restricted spaces:

\[
P_N(\mathcal{U}) \xrightarrow{p^*} P_N(\mathcal{U}/\mathcal{G}) \xrightarrow{\Delta} P_{N-1}(\mathcal{G}) \xrightarrow{i_*} P_{N-1}(\mathcal{U}) \quad (N \geq 1).
\]

(7)
Note that homotopy theory has also been used to study the global gauge anomalies \(^{14-16}\), especially by using extensively the exact homotopy sequences and in terms of James numbers of Stiefel manifolds\(^17\). One can easily see that \(\mathcal{U}\) is topologically trivial, thus \(\Pi_N(\mathcal{U}) = 0\) for any \(N\). Since the interpolation between any two gauge potentials \(A_1\) and \(A_2\)

\[ A_t = tA_1 + (1-t)A_2 \quad (8) \]

for any real \(t\) is in \(\mathcal{U}\) (Theorem 7 in Ref.10, and Ref.7). since \(A_t\) is transformed as a gauge potential in each local coordinate region, and in an overlapping region, both \(A_1\) and \(A_2\) are gauge potentials may be defined up to a gauge transformation, then \(A_t\) is a gauge potential which may be defined up to a gauge transformation, namely, \(A_t \in \mathcal{U}\). Thus, we have

\[ 0 \xrightarrow{P} \Pi_N(\mathcal{U}/G) \xrightarrow{\Delta} \Pi_{N-1}(G) \xrightarrow{\iota} 0 \quad (N \geq 1)\]

This implies that

\[ \Pi_N(\mathcal{U}/G) \cong \Pi_{N-1}(G) \quad (N \geq 1) \]

As we will show that in the presence of a magnetic monopole, the topological properties of the system are drastically different. This will give important consequences in the quantum theory. In fact, the topological properties of the restricted gauge orbit spaces are relevant for our purpose since as we will see that only the integrals on the space boundary \(S^2\) are relevant in the quantization equation for the \(\theta\). Now for the restricted spaces, the main topological result we will use is given by

\[ \Pi_2(\mathcal{U}/G) \cong \Pi_1(G) = \Pi_1(G) \oplus \Pi_3(G) \]

(11)

for a well-defined gauge subgroup \(G\). As we will see that in the relevant case of \(G = U(1)\) for our purpose \(\Pi_3(G) = 0\). The condition \(\Pi_2(\mathcal{U}/G) \neq 0\) corresponds to the existence of a magnetic monopole in the restricted gauge orbit space. We will first show that in this case \(\mathcal{F} \neq 0\), and then demonstrate explicitly that the magnetic flux \(\int_{S^2} \mathcal{F} \neq 0\) can be nonvanishing in the restricted gauge orbit space, where \(\mathcal{F}\) denotes the projection of \(\mathcal{F}\) into the restricted gauge orbit space.

Denote the differentiation with respect to space variable \(x\) by \(d\), and the differentiation with respect to parameters \(\{t_i \mid i = 1, 2, \ldots\}\) which \(A(x)\) may depend on in the gauge configuration space by \(\delta\), and assume \(d\delta + \delta d = 0\). Then, similar to \(A = \alpha_a dx^a\) with \(\mu\) replaced by \(a, i, x\), \(A = A^a L^a dx^i\), \(F = \frac{1}{2} F_{jk}^a L^a dx^j \wedge dx^k\) and \(tr(L^a L^b) = -\frac{1}{2} \delta^{ab}\) for a basis \(\{L^a \mid a = 1, 2, \ldots, rank(G)\}\) of the Lie algebra of the gauge group \(G\), the gauge potential in the gauge configuration space is given by

\[ A = \int d^3 x A^a_i (A(x)) \delta A^a_i (x) \]

(12)

Using Eq.(6), this gives

\[ A = \frac{\theta}{8\pi^2} \int d^3 x \epsilon_{ijk} F_{ijk}^a (x) \delta A^a_i (x) = -\frac{\theta}{2\pi^2} \int_M tr(\delta AF), \]

(13)
with M being the space manifold. With \( \delta F = -D_\delta A = \{-d(\delta A) + A\delta A - \delta AA\} \), we have topologically

\[
\mathcal{F} = \delta A = \frac{\theta}{2\pi^2} \int_M \text{tr}[\delta A D_\delta A(\delta A)] = \frac{\theta}{4\pi^2} \int_M d\text{tr}(\delta A\delta A) = \frac{\theta}{4\pi^2} \int_{\partial M} \text{tr}(\delta A\delta A). \tag{14}
\]

Usually, one may assume \( A \to 0 \) faster than \( 1/r \) as \( x \to 0 \), then this would give \( F = 0 \). However, this is not the case in the presence of a magnetic monopole. Asymptotically, a monopole may generally give a field strength of the form

\[
F_{ij} = \frac{1}{4\pi^2} \varepsilon_{ijk}(\hat{r})k G(\hat{r}), \tag{15}
\]

with \( \hat{r} \) being the unit vector for \( r \), and this gives \( A \to O(1/r) \) as \( x \to 0 \). Thus, one can see easily that a magnetic monopole can give a nonvanishing field strength in the gauge configuration space. To evaluate the \( \mathcal{F} \), one needs to specify the space boundary \( \partial M \) in the presence of a magnetic monopole. We now consider the case that the magnetic monopole does not generate a singularity in the space. In fact, this is so when monopoles appear as a smooth solution of a spontaneously broken gauge theory similar to 't Hooft Polyakov monopole. For example, it is known that there are monopole solutions in the minimal SU(5) model. Then, the space boundary may be regarded as a large 2-sphere \( S^2 \) at spatial infinity. For our purpose, we actually only need to evaluate the projection of \( \mathcal{F} \) into the gauge orbit space.

In the gauge orbit space, a gauge potential can be written in the form of

\[
A = g^{-1}ag + g^{-1}dg, \tag{16}
\]

for an element \( a \in U/G \) and a gauge function \( g \in G \). Then the projection of a form into the gauge orbit space contains only terms proportional to \( (\delta a)^n \) for integers \( n \). We can now write

\[
\delta A = g^{-1}[\delta a - D_a(\delta gg^{-1})]g. \tag{17}
\]

Then we obtain

\[
A = -\frac{\theta}{2\pi^2} \int_M \text{tr}(f\delta a) + \frac{\theta}{2\pi^2} \int_M \text{tr}[fD_a(\delta gg^{-1})], \tag{18}
\]

where \( f = da + a^2 \). With some calculations, this can be simplified as

\[
A = \hat{A} + \frac{\theta}{2\pi^2} \int_{S^2} \text{tr}[f\delta gg^{-1}], \tag{19}
\]

where

\[
\hat{A} = -\frac{\theta}{2\pi^2} \int_M \text{tr}(f\delta a), \tag{20}
\]

is the projection of \( A \) into the gauge orbit space. Similarly, we have

\[
\mathcal{F} = \frac{\theta}{4\pi^2} \int_{S^2} \text{tr}\{[\delta a - D_a(\delta gg^{-1})][\delta a - D_a(\delta gg^{-1})]\} \tag{21}
\]

or

\[
\mathcal{F} = \hat{\mathcal{F}} - \frac{\theta}{4\pi^2} \int_{S^2} \text{tr}\{\delta aD_a(\delta gg^{-1}) + D_a(\delta gg^{-1})\delta a - D_a(\delta gg^{-1})D_a(\delta gg^{-1})\}, \tag{22}
\]
where

$$\hat{F} = \frac{\theta}{4\pi^2} \int_{S^2} \text{tr}(\delta a \delta a).$$

(23)

Now all our discussions will be based on the restricted spaces. To see that the flux of \(\hat{F}\) through a closed surface in the restricted gauge orbit space \(U/G\) can be nonzero, we will construct a 2-sphere in it. Consider an element \(a \in U/G\), and a loop in \(G\). The set of all the gauge potentials obtained by all the gauge transformations on \(a\) with gauge functions on the loop then forms a loop \(C^1\) in the gauge configurations space \(U\). Obviously, the \(a\) is the projection of the loop \(C^1\) into \(U/G\). Now since \(\Pi_1(U) = 0\) is trivial, the loop \(C^1\) can be continuously extented to a two-dimensional disc \(D^2\) in the \(U\) with \(\partial D^2 = C^1\), then obviously, the projection of the \(D^2\) into the gauge orbit space is topologically a 2-sphere \(S^2 \subset U/G\). With the Stokes’ theorem in the gauge configuration space, We now have

$$\int_{D^2} F = \int_{D^2} \delta A = \int_{C^1} A.$$  

(24)

Using Eqs.(19) and (24) with \(\delta a = 0\) on \(C^1\), this gives

$$\int_{C^1} A = \frac{\theta}{2\pi^2} \text{tr} \int_{S^2} [f \delta gg^{-1}].$$

(25)

Thus, the projection of the Eq.(26) to the gauge orbit space gives

$$\int_{S^2} \hat{F} = \frac{\theta}{2\pi^2} \text{tr} \int_{S^2} \{ f \int_{C^1} \delta gg^{-1} \},$$

(26)

where note that in the two \(S^2\) are in the gauge orbit space and the ordinary space respectively. We have also obtained this by verifying that

$$\int_{D^2} \text{tr} \int_{S^2} [\delta a D_a(\delta gg^{-1}) + D_a(\delta gg^{-1}) \delta a - D_a(\delta gg^{-1}) D_a(\delta gg^{-1})] = 0,$$

(27)

or the projection of \(\int_{D^2} F\) gives \(\int_{S^2} \hat{F}\).

In quantum theory, Eq.(26) corresponds to the topological result \(\Pi_2(U/G) \cong \Pi_1(G)\) on the restricted spaces. The discussion about the Hamiltonian equation in the schrodinger formulation will be similar to that in Refs.7 and 8 including the discussions for the three-dimensional Yang-Mills theories with a Chern-Simons term. We need the Dirac quantization condition to have a well-defined wave functional in the formalism. In the gauge orbit space, the Dirac quantization condition gives

$$\int_{S^2} \hat{F} = 2\pi k,$$

(28)

with \(k\) being integers. The Dirac quantization condition in the gauge orbit space will be clarified shortly. Now let \(f\) be the field strength 2-form for the magnetic monopole. The quantization condition is now given by

$$\exp\{\int_{S^2} f\} = \exp\{G_0\} = \exp\{4\pi \sum_{i=1}^r \beta_i H_i\} \in \mathbb{Z}.$$  

(29)
Where $G_0$ is the magnetic charge up to a conjugate transformation by a group element, $H_i$ ($i=1, 2, ..., r=\text{rank}(G)$) form a basis for the Cartan subalgebra of the gauge group with simple roots $\alpha_i$ ($i=1, 2, ..., r$). We need non-zero topological value to obtain quantization condition for $\theta$. As it is known from Ref.12, only the gauge transformations commuting with the magnetic charges can be globally well-defined, only those gauge transformations can be used for determining the global topological quantities. Consider $g(x,t)$ in the well-defined $U(1)$ gauge subgroup commutative with the magnetic charges on the $C^1$

$$g(x,t) |_{x \in S^2} = \exp\{4\pi mt \sum_{i,j} \frac{(\alpha_i)^j H_j}{<\alpha_i, \alpha_i>}\},$$

with $m$ being integers and $t \in [0,1]$. In fact, $m$ should be identical to $k$ according to our topological result $\Pi_2(U/G) \cong \Pi_1(G)$. The $k$ and $m$ are the topological numbers on each side. Thus, we obtain in the case of non-vanishing vacuum angle $\theta$

$$\theta = \frac{2\pi}{n} (n \neq 0).$$

(31)

Where we define generally the topological charge of the magnetic monopole as

$$n = -2 <\delta, \beta'>$$

(32)

which must be an integer$^{17}$. Where

$$\beta' = \sum_i \frac{2\alpha_i}{<\alpha_i, \alpha_i>},$$

(33)

the minus sign is due to our normalization convention for Lie algebra generators. Note that the parameter $t$ of $g(x,t)$ in eq.(30) may be regarded as the time parameter topologically when the time evolution is included, the two end points of the closed loop then correspond to the time infinities. The $g(x,t)$ is not a constant in the entire spacetime, and does not generate a Noether symmetry, its invariance will not eliminate any charged states.

Therefore, we conclude that in the presence of magnetic monopoles with topological charge $\pm 1$, the vacuum angle of non-abelian gauge theories must be $\pm 2\pi$, the existence of such magnetic monopoles gives a solution to the strong CP problem. But CP cannot be exactly conserved in this case since $\theta = \pm 2\pi$ correspond to two different monopole sectors. The existence of many monopoles can ensure $\theta \to 0$, and the strong CP problem may also be solved. In this possible solution to the strong CP problem with $\theta \leq 10^{-9}$, the total magnetic charges present are $|n| \geq 2\pi 10^9$. This may possibly be within the abundance allowed by the ratio of monopoles to the entropy$^{19}$, but with the possible existence of both monopoles and anti-monopoles, the total number of magnetic monopoles may be larger than the total magnetic charges. Generally, one needs to ensure that the total number is consistent with the experimental results on the abundance of monopoles. The $n = \pm 2$ may also possibly solve CP if it is consistent with the experimental observation.
Note that we only considered non-singular magnetic monopole in the space. For ’t Hooft Polyakov monopole, the full gauge group inside the monopole is simply connected, it will not give any boundary contribution to the term in Eq.(26). However, outside the monopole, the gauge symmetry is spontaneously broken, it is known that the unbroken gauge group cannot be simply connected to have monopole solutions. For example, in SU(5) model, inside the monopole, SU(5) is simply connected; outside the monopole the exact gauge group G=SU(3)xU(1) satisfies \( \Pi_1(G) = \mathbb{Z} \). We expect that in general, the GUT monopoles are smooth solutions, and therefore cannot have a mathematical boundary at a given short distance around the monopole relevant to our boundary contribution. Therefore, the realistic world meet the condition to have our solution to the strong CP problem.

The effect of a term proportional to \( \epsilon^{\mu \nu \lambda \sigma} F_{\mu \nu} F_{\lambda \sigma} \) in the presence of magnetic charges was first considered relevant to chiral symmetry. The effect of a similar U(1) term was discussed for the purpose of considering the induced electric charges as quantum excitations of dyons associated with the ’t Hooft Polyakov monopole and generalized magnetic monopoles. Note that since our solution needs non-vanishing magnetic flux through the space boundary, this implies that only an open universe can be consistent with our solution. Note that the relevance to the \( U_A(1) \) problem is discussed in Ref. 23.

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