Event-triggered fixed-time adaptive fuzzy control for state-constrained stochastic nonlinear systems without feasibility conditions

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Abstract The problem of event-triggered fixed-time control for state-constrained stochastic nonlinear systems is discussed in this article. Different from the barrier Lyapunov function (BLF)-based and Integral BLF-based schemes that rely on feasibility conditions (FCs), by introducing the nonlinear state-dependent functions, the asymmetric time-varying state constraints are handled without FCs. Combined with the fixed-time stability theory and the dynamic surface control technique with fixed-time filter, the fixed-time stability in probability of the closed-loop system is ensured and the problems of “explosion of complexity” and “singularity” are overcome. Furthermore, the novel fixed-time error compensation signals are designed to compensate the filtering errors, and the event-triggered control technique is used to save network resources. Simulations also illustrate the effectiveness of the proposed method.

Keywords Fixed-time fuzzy control · Event-triggered control · Full state constraint · Stochastic nonlinear systems

1 Introduction

As the requirements of control performance, physical limitations or security considerations, the research of adaptive control for state-constrained systems has attracted a lot of attention, and many significative results have been presented. For instance, model predictive control [1], reference governors [2,3], and set invariance notions [4–6]. The authors of [7] first proposed an integrator backstepping scheme based on BLF to deal with multi-state constraints. Then, many BLF- or IBLF-based schemes for state-constrained systems with different forms were presented, see [8–14] and references therein. However, one drawback of the BLF- or IBLF-based schemes is that the virtual control function needs to satisfy the FCs, i.e., the virtual control function needs to be limited a pre-given constrained region. The existing solution is to find a set of design parameters to satisfy the FCs. Obviously, this is a very complicated process. Moreover, as stated in [15,16], the parameters that satisfy the FCs may not exist when the states are limited to a small set. To this end, the new schemes based on NSDF were presented [17,18], under which the FCs can be completely removed.

As is known to all that the stability time of the system is also an important index to evaluate the controller.
Nevertheless, most of the above schemes can only ensure asymptotic stability. To obtain a finite convergence time, the finite-time stability theory was first proposed [19]. On the basis of this theory, many finite-time control schemes were proposed for state-constrained systems [20–23]. One disadvantage of these methods is that the setting time is always dependent on the initial value of the system. It means that the convergence time will be longer as the initial state deviates from the equilibrium position. Then, the authors of [24] first presented the fixed-time stability theory, under which the convergence time can be predicted in prior and independent of the initial state. Then, many adaptive fixed-time control (FTC) schemes were proposed [25–27]. The authors of [28] extend the fixed-time stability theory to stochastic systems.

As the order of the system goes up, the problems of “explosion of complexity” and “singularity” caused by repeated differentiations of virtual controllers frequently appear in backstepping design procedure. To solve these problems, the DSC technique with a first-order linear filter was first presented in [29]. Then, many DSC schemes with different filters were proposed. To name a list, the DSC schemes with linear filter [30], nonlinear filter [31,32], finite-time command filter [33,34] and fixed-time filter [35] were proposed for nonlinear systems with different forms.

In addition, most of the control schemes for state-constrained systems were constructed under the time-triggered control (TTC) framework, under which the output of the controller is applied to the system continuously, whether it’s needed or not. Clearly, this will produce a lot of redundant data and increase the communication burden. To this end, the ETC technology was proposed in [36–38]. Nevertheless, a limitation of the above works is that the assumption of input-to-state stability is needed. To relax this restriction, an effective ETC method with relative threshold technology was presented in [39]. Then, the method was applied to the time-delay systems [40] and stochastic systems [41,42]. On basis of the finite-time stability theory and the fixed-time stability theory, two event-triggered finite-time controllers [43,44] and an event-triggered fixed-time controller [45] were designed.

Inspired by the previous work, it can be seen that most of the control schemes for state-constrained stochastic systems were constructed by the conventional backstepping technology under the TTC framework. There are two drawbacks in the above methods: (1) As the order of the system goes up, the problems of “explosion of complexity” and “singularity” caused by repeated differentiations of virtual controllers frequently appear in backstepping design procedure; (2) The TTC schemes will produce a lot of redundant data and cause the waste of network resources. Therefore, a significant problem arose naturally: for asymmetric time-varying state-constrained stochastic systems, can we design an event-triggered fixed-time controller without the FCs? As far as we know, this problem has not been solved until now. The advantages of this note are given as follows

(i) Compared with the BLF- or Integral BLF-based schemes that rely on the FCs in [20–23], under which the virtual controllers need to be limited a pre-given constrained region. In this note, by introducing the NSDFs that purely rely on the constrained states, the state constraints are handled directly and the FCs are removed. Moreover, note that the state constraint functions considered in this paper are time-varying asymmetric, but the ones in [20–23] are limited to symmetric constants.

(ii) Unlike the finite-time command filtered control methods [23,33,34], by combining with the fixed-time stability theory and the DSC technology with fixed-time filter, the fixed-time stability in probability of the closed-loop system can be ensured. Meanwhile, the problems of “explosion of complexity” and “singularity” are avoided. In addition, ETC strategy is used to save network resources. Compared with the scheme in [45], the novel FTECSs are designed to compensate the filtering errors. Moreover, the considered systems contain stochastic disturbances and asymmetric time-varying state constraints, which are more general and can enlarge the practical application range.

The remainder of this note is addressed as follows. Section 2 gives the relevant preliminaries. The design procedure and analysis are shown in Sect. 3. The Simulations are shown in Sect. 4. In Sect. 5, a conclude is given.
2 Problem formulation and preliminaries

2.1 Key definition and lemmas

**Definition 1** [28] Given a stochastic system as follows
\[ d\xi = f(\xi)dt + g(\xi)d\omega, \]  
where \( f(\xi) \) is a positive definite function, the differential operator \( L \) of \( V(\xi) \) is defined as
\[ LV(\xi) = \frac{\partial V(\xi)}{\partial \xi} f(\xi) + \frac{1}{2} Tr(g^T(\xi) \frac{\partial^2 V(\xi)}{\partial \xi^2} g(\xi)). \]  

**Lemma 1** [28] Let \( V(\xi) : R^n \to R^+ \) be a positive definite function, if \( \exists \mu_1, \mu_2 > 0, 0 < \gamma_1 < 1, \gamma_2 > 1 \) such that
\[ LV(\xi) \leq -\mu_1 V^{\gamma_1}(\xi) - \mu_2 V^{\gamma_2}(\xi), \]  
then, the solution of system (1) is fixed-time stable in probability (FTSP), and the setting time \( T \) satisfies
\[ E[T] \leq \frac{1}{\mu_1(1-\gamma_1)} + \frac{1}{\mu_2(\gamma_2-1)}. \]  

**Lemma 2** Let \( V(\xi) : R^n \to R^+ \) be a positive definite function, if \( \exists \mu_1, \mu_2, \varrho > 0, 0 < \gamma_1 < 1, \gamma_2 > 1 \) such that
\[ LV(\xi) \leq -\mu_1 V^{\gamma_1}(\xi) - \mu_2 V^{\gamma_2}(\xi) + \varrho, \]  
then, the solution of system (1) is FTSP, and the setting time \( T \) satisfies
\[ E[T] \leq \frac{1}{\mu_1(1-\gamma_1)} + \frac{1}{\mu_2(\gamma_2-1)}. \]  

**Proof** By introducing a constant \( \eta \) \( \in (0, 1) \), then inequality (4) can be rewritten as
\[ LV(\xi) \leq -\eta \mu_1 V^{\gamma_1}(\xi) -(1-\eta)\mu_1 V^{\gamma_1}(\xi) - \mu_2 V^{\gamma_2}(\xi) + \varrho, \]  
which is equivalent to
\[ LV(\xi) \leq -(1-\eta)\mu_1 V^{\gamma_1}(\xi) - \mu_2 V^{\gamma_2}(\xi) + \varrho. \]  

**Case 1** If \( V(\xi) > \left( \frac{\varrho}{(1-\eta)\mu_1} \right)^{\frac{1}{\gamma_1}} \), from (5), one has
\[ LV(\xi) \leq -\eta \mu_1 V^{\gamma_1}(\xi) - \mu_2 V^{\gamma_2}(\xi). \]  

Based on Lemma 1, one can obtain that the solution of system (1) is FTSP, and the setting time satisfies
\[ E[T] \leq \frac{1}{\eta \mu_1(1-\gamma_1)} + \frac{1}{\mu_2(\gamma_2-1)}. \]  

**Case 2** If \( V(\xi) \leq \left( \frac{\varrho}{(1-\eta)\mu_1} \right)^{\frac{1}{\gamma_1}} \), from (6), one has
\[ LV(\xi) \leq -\mu_1 V^{\gamma_1}(\xi) - \eta_2 V^{\gamma_2}(\xi). \]  

Based on Lemma 1, one can obtain that the solution of system (1) is FTSP, and the setting time satisfies
\[ E[T] \leq \frac{1}{\mu_1(1-\gamma_1)} + \frac{1}{\eta_2(\gamma_2-1)}. \]  

**Remark 1** For convenience, in this note, let \( \gamma_1 = \frac{3}{4}, \gamma_2 = 2 \).

**Lemma 3** [13] \( H(\chi) \) denotes an unknown nonlinear function, for a given accuracy \( \epsilon > 0 \), there exists a fuzzy logic system (FLS) \( W^T S(\chi) \) such that
\[ H(\chi) = W^T S(\chi) + \epsilon(\chi), \]  
where \( \chi, W \) denote input vector and weight vector, respectively. \( \epsilon(\chi) \) denotes a bounded approximation error, i.e., \( |\epsilon(\chi)| \leq \epsilon \), \( S(\chi) = \{s_1(\chi), \ldots, s_l(\chi)\}^T \) is a basis function vector with \( l > 1 \) being the number of the fuzzy rules, and \( s_i(\chi) \) denotes a Gaussian function of the following form
\[ s_i(\chi) = \exp\left[ -\frac{(\chi - \mu_i)^T (\chi - \mu_i)}{\eta_i^2} \right], i = 1, \ldots, l \]  
where \( \mu_i \) and \( \eta_i \) denote the center vector and spreads of the Gaussian function, respectively.

**Lemma 4** [25] For \( a \in R \) and \( m \in R^+ \), one has
\[ 0 \leq |a| - a \tanh \left( \frac{a}{m} \right) \leq 0.2785m, \]  
\[ 0 \leq |a| < m + \frac{a^2}{\sqrt{a^2 + m^2}}. \]  

**Lemma 5** [25] For \( \kappa \in [0, 1] \) and \( a_i \in R \) with \( i = 1, \ldots, n \), one has
\[ \left( \sum_{i=1}^{n} |a_i| \right)^\kappa \leq \sum_{i=1}^{n} |a_i|^\kappa, \]  
\[ \left( \sum_{i=1}^{n} |a_i| \right)^2 \leq n \sum_{i=1}^{n} a_i^2. \]  

**Lemma 6** [25] For \( a, b \in R \) and \( p_1, p_2, p_3 \in R^+ \), one has
\[ |a|^{p_1} |b|^{p_2} \leq \frac{p_1}{p_1 + p_2} p_3 |a|^{p_1+p_2} + \frac{p_2}{p_1 + p_2} p_3^{\frac{p_1}{p_2}} |b|^{p_1+p_2}. \]
2.2 Problem statement

The considered system is expressed as

\[
\begin{align*}
\dot{\zeta}_i &= [f_i(\zeta_i) + g_i(\zeta_i)\xi_i + h_i^T(\zeta_i)]\,dt + h_i^T(\zeta_i)\,d\omega \\
\dot{\zeta}_n &= [f_n(\zeta_n) + g_n(\zeta_n)u(t)]\,dt + h_n^T(\zeta_n)\,d\omega \\
y &= \zeta_i,
\end{align*}
\]  

(16)

where \(\bar{\zeta}_i = [\zeta_1, \zeta_2, \ldots, \zeta_i], u(t), y\) denote system state, system input and system output, respectively. \(f_i(\cdot)\) and \(h_i(\cdot)\) are unknown nonlinear functions in function vector, respectively. \(g_i(\cdot)\) denotes a known continuous function. \(\omega \in \mathbb{R}^r\) is an independent standard Brownian motion. All the state variables \(\zeta_i\) need to satisfy \(-\mathcal{F}_{11}(t) \leq \zeta_i \leq \mathcal{F}_{12}(t)\), where \(\mathcal{F}_{11}(t), \mathcal{F}_{12}(t)\) are positive time-varying constraint functions.

The aim of this article is to construct an effective controller \(u(t)\) to ensure that the whole variables of system (16) are fixed-time bounded in probability (FTBIP), and that the output \(y\) tracks the desired trajectory \(y_r\) within a fixed-time interval. In the meantime, the time-varying asymmetric state constraints are not violated.

**Assumption 1** \([41]\) The desired signal \(y_r\) and its derivative \(\dot{y}_r\) are continuous, and \(y_r\) satisfies \(-\mathcal{F}_{11}(t) \leq y_r \leq \mathcal{F}_{12}(t)\).

**Assumption 2** \([43]\) The sign of \(g_i(\cdot)\) is known, and there exist two unknown constants \(\underline{g}_0, \bar{g}_0 \in \mathbb{R}^+\) such that \(0 < \underline{g}_0 \leq |g_i(\cdot)| \leq \bar{g}_0\). In this article, let \(g_i(\cdot) > 0\).

3 Design procedure and main results

3.1 Controller design

To cope with the state constraints, the NSDF \([17]\) is introduced as follows

\[ s_i = \frac{\zeta_i}{(\mathcal{F}_{11}(t) + \zeta_i)(\mathcal{F}_{12}(t) - \zeta_i)}, \quad i = 1, \ldots, n. \]  

(17)

From (17), one has

\[
\dot{s}_i = \mu_i \dot{\zeta}_i + v_i \, dt,
\]

(18)

where

\[
\mu_i = \frac{\mathcal{F}_{11}(t)\mathcal{F}_{12}(t) + \zeta_i^2}{(\mathcal{F}_{11}(t) + \zeta_i)(\mathcal{F}_{12}(t) - \zeta_i)},
\]

(19)

\[
v_i = -\frac{[\mathcal{F}_{11}(t)\mathcal{F}_{12}(t) + \mathcal{F}_{11}(t)\dot{\mathcal{F}}_{12}(t)]\zeta_i}{(\mathcal{F}_{11}(t) + \zeta_i)(\mathcal{F}_{12}(t) - \zeta_i)^2}.
\]

Then system (16) can be rewritten as

\[
\begin{align*}
\dot{s}_i &= [F_i + G_i s_i] \, dt + H_i^T \, d\omega, \\
\dot{\zeta}_n &= [F_n + G_n u(t)] \, dt + H_n^T \, d\omega,
\end{align*}
\]

(20)

where \(F_i = \mu_i f_i(\bar{\zeta}_i) + v_i\) with \(i = 1, \ldots, n\), \(G_i = \mu_i g_i(\bar{\zeta}_i)\) with \(i = 1, \ldots, n - 1\), \(G_n = \mu_n g_n(\bar{\zeta}_n)\), \(H_i = \mu_i h_i(\bar{\zeta}_i)\), \(h_i = (\mathcal{F}_{11+1,1}(t) + \zeta_i)(\mathcal{F}_{1+1,2}(t) - \zeta_{i+1})\) with \(i = 1, \ldots, n\).

**Remark 2** Compared with the BLF- or Integral BLF-based schemes that rely on the FCs in \([20–23]\), under which the virtual controller needs to satisfy \(-\mathcal{F}_{11} \leq \alpha_{i-1} \leq \mathcal{F}_{12}, (i = 2, \ldots, n)\). By introducing the NSDF, the FCs are removed. Moreover, the state constraint functions in \([20–23]\) are the simpler case of symmetric constants, while the case of asymmetrical time-varying state constraints are considered in this paper.

**Remark 3** Note that the NSDF is also used in \([17]\), but the method in \([17]\) only ensures that the system is stable as time tends to infinity. In this note, by combining with the fixed-time stability theory and the DSC technology with fixed-time filter, the fixed-time stability in probability of the closed-loop system can be ensured. At the same time, the novel FTECSs are designed to compensate the filtering errors. In addition, the systems considered in \([17]\) are limited to deterministic strict-feedback nonlinear systems.

Let \(s_{i,c} = \frac{y_r}{(\mathcal{F}_{11}(t) + \zeta_i)(\mathcal{F}_{12}(t) - \zeta_i)}\). Define the following coordinate transformation:

\[
\begin{align*}
\tilde{z}_1 &= s_1 - s_{1,c}, \\
\tilde{z}_i &= s_i - s_{i,c}, \quad i = 2, \ldots, n,
\end{align*}
\]

(22)

where \(s_{i,c}\) is the output of the fixed-time filter with \(\alpha_{i-1}\) as the input, \(\alpha_i\) is the virtual control function. The fixed-time filter \([35]\) is designed as follows

\[
\begin{align*}
\dot{\xi}_{i,1} &= \xi_{i,2} - \phi_{11}\Gamma_1^2 \xi_{i,1} + \alpha_i |1|^\frac{1}{2} \text{sign}(\xi_{i,1} - \alpha_i), \\
\dot{\xi}_{i,2} &= -\phi_{21}\Gamma_1^2 |\xi_{i,1} - \alpha_i|^{|1|} \text{sign}(\xi_{i,2} - \bar{\xi}_{i,1}),
\end{align*}
\]

(23)

where \(\xi_{i,1} = s_{i+1,c}, \xi_{i,2} = \dot{s}_{i+1,c}\), \(\phi_{11}, \phi_{12}, \phi_{21}, \phi_{22}\) denote design parameters, \(\Gamma_1\) and \(\Gamma_2\) denote positive
design constants, \( v_i > 1, v_i = i v - (i - 1) \) and \( v = v_1 \). From [35], with the above filter, \(|\dot{s}_{i+1,c} - \alpha_i| \leq k_i\) in a fixed time.

**Remark 4** In general, there is no systematic theoretical method to design parameters \( \phi_{11}, \phi_{12}, \phi_{21}, \phi_{22} \). As a rule of thumb, the design of parameters generally follows some guidelines, such as the parameters \( \phi_{11}, \phi_{12}, \phi_{21}, \phi_{22} \) are selected such that the matrices \([-\phi_{11} I\] and \([-\phi_{21} 1\]) are Hurwitz. More details on choosing these parameters can be found in [35, 46].

To reduce the influence of filtering errors, the novel FTECSs are designed as follows

\[
\begin{align*}
\dot{\hat{P}}_1 &= -l_{11} \Pi_1^2 + G_1 \Pi_2 \\
&\quad + G_1 (s_{2,c} - \alpha_1) - l_{12} \text{sign}(\Pi_1), \\
\dot{\hat{P}}_i &= -l_{11} \Pi_i^2 + G_1 \Pi_{i+1} - G_{i-1} \Pi_{i-1} \\
&\quad + G_i (s_{i+1,c} - \alpha_i) - l_{12} \text{sign}(\Pi_i), \\
\dot{\hat{P}}_n &= -l_{11} \Pi_n^2 - G_{n-1} \Pi_{n-1} - l_{12} \text{sign}(\Pi_n),
\end{align*}
\]

where \( l_{11}, l_{12} \) denote positive design parameters.

**Remark 5** Compared with the existing DSC methods with finite-time command filter in [33, 34], by introducing the fixed-time filter, the proposed method can not only eliminate the problems of “explosion of complexity” and “singularity”, but also ensure the fixed-time stability in probability of the closed-loop system. Moreover, the influence of filtering errors is reduced effectively by introducing the novel FTECSs.

Let the compensated tracking error be \( e_i = z_i - \dot{\hat{P}}_i \). \( \hat{O}_i = \Theta_i - \dot{\hat{O}}_i \), \( \dot{\hat{O}}_i \) is the estimation of \( \Theta_i \), \( \Theta_i = ||W_i||^2, i = 1, \ldots, n, \alpha_i \) and \( \hat{O}_i \) are constructed as:

\[
\alpha_i = -\frac{e_i^3 \Lambda_i^2}{\mu_i h_i g_0 \sqrt{e_i^6 \Lambda_i^2 + \bar{e}_i^2}}, i = 1, \ldots, n - 1, \tag{25}
\]

\[
\alpha_n = -\frac{e_n^3 \Lambda_n^2}{\mu_n g_0 \sqrt{e_n^6 \Lambda_n^2 + \bar{e}_n^2}}, \tag{26}
\]

\[
\Lambda_i = k_{11} \left( \frac{1}{4} e_i^3 \frac{(e_i^3)^{\frac{3}{2}}}{e_i^3} + k_{12} \left( \frac{1}{4} e_i^2 \right)^2 e_i^5 \right) \\
&\quad + \frac{1}{2d_i} e_i^3 \hat{O}_i ||S_i(\chi_i)||^2, i = 1, \ldots, n, \tag{27}
\]

\[
\dot{\hat{O}}_i = \frac{r_i}{2d_i} e_i^6 ||S_i(\chi_i)||^2 - \sigma_i \hat{O}_i - \frac{c_i}{r_i} \hat{O}_i^3, i = 1, \ldots, n. \tag{28}
\]

where \( \epsilon_i, k_{11}, k_{12}, r_i, \sigma_i, c_i \) are positive design parameters.

The event-triggered controller is defined as follows:

\[
\beta(t) = -(1 + \lambda) \left( \frac{e_i^3 \alpha_i}{\rho} + \frac{\bar{h} \alpha_n^2}{\rho} \right), \tag{29}
\]

\[
u(t) = \beta(t_k), t_k \leq t < t_{k+1}, \quad t_{k+1} = \inf\{ t > t_k ||\beta(t) - u(t)| \geq \lambda|u(t)| + q \}, \tag{30}
\]

where \( \bar{t_k} \) denotes the event-triggered time with \( k \in Z^+, \rho, q \in R^+, \lambda \in (0, 1) \) and \( \bar{h} > q/(1 - \lambda) \).

**Remark 6** From (29)-(31), we can see that \( \nu(t) \) is a constant when \( t \in [t_k, t_{k+1}) \). Compared with the TTC schemes, the ETC technology can save communication resources effectively. In addition, the relative threshold strategy is introduced, which is more flexible than the fixed threshold strategy.

### 3.2 Main results

The main results are concluded by the following Theorem.

**Theorem** For system (16) with Assumptions 1–2, let the actual control input be designed as (30), the virtual control functions be designed as (25), (26), and the update law be designed as (28). The proposed scheme guarantees that the whole variables of system (16) are FTBIP, and that the output tracks the desired trajectory \( z_r \) within a fixed-time interval. In the meantime, the time-varying asymmetric full-state constraints are not violated without involving the FCs.

**Proof** Step 1 From (21) and (22), one has

\[
dz_1 = [F_1 + G_1 s_z - \hat{s}_1 c] dt + H_1^T d\omega = [F_1 + G_1 (\xi_z + s_{2,c}) - \hat{s}_1 c] dt + H_1^T d\omega. \tag{32}
\]

From (24) and (32), one has

\[
de_1 = dz_1 - d\Pi_1 \\
= [F_1 + G_1 \hat{s}_2 + G_1 \hat{s}_1 - \hat{s}_1 c + l_{11} \Pi_1^3] + l_{12} \text{sign}(\Pi_1) dt + H_1^T d\omega. \tag{33}
\]
Choose the Lyapunov function candidate (LFC) \( V_1 \) as
\[
V_1 = \frac{1}{4} e_1^4 + \frac{1}{2r_1^2} \hat{\theta}_1^2. \tag{34}
\]

Then, we have
\[
\mathcal{L} V_1 = e_1^2 \left[ F_1 + G_1 e_2 + G_1 \alpha_1 - \dot{s}_{1,c} + l_{11} \Pi_3^3 \right.
+ l_{12} \text{sign}(\Pi_1)] + \frac{3}{2} e_1^2 H_1^T H_1 - \frac{1}{r_1} \hat{\theta}_1 \dot{\hat{\theta}}_1.
\tag{35}
\]

By using Young’s inequality, one has
\[
\frac{3}{2} e_1^2 H_1^T H_1 \leq \frac{3}{4} e_1^4 ||H_1||^4 + \frac{3}{4} e_1^4 \tag{36}
\]
\[
G_1 e_1^3 e_2 \leq \frac{3}{4} G_1 e_1^4 + \frac{1}{4} G_1 e_2^4. \tag{37}
\]

Substituting (36) and (37) into (35), one has
\[
\mathcal{L} V_1 \leq e_1^2 \left[ F_1 + \frac{3}{4} G_1 e_1 + \frac{3}{4} e_1 ||H_1||^4 + G_1 \alpha_1 + l_{11} \Pi_3^3 \right.
- \dot{s}_{1,c} + l_{12} \text{sign}(\Pi_1)] + \frac{1}{4} G_1 e_2^4 + \frac{3}{4} e_1^2 \tag{38}
- \frac{1}{r_1} \hat{\theta}_1 \dot{\hat{\theta}}_1
\]
\[
\leq e_1^2 \left[ \mathcal{H}_1(\chi_1) + G_1 \alpha_1 \right] - \frac{3}{4} e_1^4
+ \frac{1}{4} G_1 e_2^4 + \frac{3}{4} e_1^2 - \frac{1}{r_1} \hat{\theta}_1 \dot{\hat{\theta}}_1,
\tag{39}
\]

where \( \mathcal{H}_1(\chi_1) = F_1 + \frac{3}{4} G_1 e_1 + \frac{3}{4} e_1 ||H_1||^4 - \dot{s}_{1,c} + l_{11} \Pi_3^3 + l_{12} \text{sign}(\Pi_1) + \frac{3}{2} e_1^4. \)

On the basis of Lemma 3, \( \mathcal{H}_1(\chi_1) \) can be approximated by a FLS, i.e., \( \mathcal{H}_1(\chi_1) = W_i^T S_i(\chi_1) + \epsilon_1(\chi_1), \) where \( |\epsilon_1(\chi_1)| \leq \epsilon_1 \) with \( \epsilon_1 \in R^p. \)

By using Young’s inequality, one has
\[
e_1^3 \mathcal{H}_1(\chi_1) = e_1^3 \left[ W_i^T S_i(\chi_1) + \epsilon_1(\chi_1) \right]
\leq \frac{1}{2a_1} e_1^6 ||W_i||^2 ||S_i(\chi_1)||^2 + \frac{a_1}{2} + \frac{3e_1^4}{4} + e_1^4 \tag{39}
\]
\[
\leq \frac{1}{2a_1} e_1^6 \Theta_1 ||S_i(\chi_1)||^2 + \frac{a_1}{2} + \frac{3e_1^4}{4} + e_1^4. \tag{40}
\]

where \( a_1 \in R^+ \) denotes a design parameter.

According to Lemma 4, \( (25) \) and \( (27) \), we have
\[
e_1^3 G_1 \alpha_1 = -\frac{g_1 e_1^6 \Lambda_1^2}{g_0^2 \sqrt{\epsilon_1^4 \Lambda_1^2 + \epsilon_1^2}}
\leq -\frac{e_1^6 \Lambda_1^2}{\sqrt{\epsilon_1^4 \Lambda_1^2 + \epsilon_1^2}}
\leq \epsilon_1 - e_1^3 \Lambda_1
\leq \epsilon_1 - k_{12} \left( \frac{e_1^4}{4} \right)^2 - k_{12} \left( \frac{e_1^4}{4} \right)^2
- \frac{1}{2a_1} e_1^6 \hat{\theta}_1 ||S_i(\chi_1)||^2. \tag{41}
\]

Substituting (28), (39) and (40) into (38), one has
\[
\mathcal{L} V_1 \leq -k_{11} \left( \frac{e_1^4}{4} \right)^2 - k_{12} \left( \frac{e_1^4}{4} \right)^2 + G_1 e_4^2 \tag{42}
+ \Delta_1 + \frac{\sigma_1}{r_i} \hat{\theta}_1 \dot{\hat{\theta}}_1 + \frac{c_1}{r_i} \hat{\theta}_1 \dot{\hat{\theta}}_1.
\tag{43}
\]

where \( \Delta_1 = \frac{a_1}{2} + \frac{e_4^4}{4} + \epsilon_1 + \frac{3}{4}. \)

Step i = 2, \ldots, n - 1.

Similar to (33), we have
\[
de_i = [F_i + G_i e_{i+1} + G_i \alpha_i - \dot{s}_{i,c} + G_{i-1} \Pi_{i-1} + l_{i1} \Pi_{i} + l_{i2} \text{sign}(\Pi_{i})] dt + \Phi_i^T dw_i.
\]

where \( \Phi_i = \Pi_i - \sum_{j=1}^{i-1} \frac{d\alpha_i}{dx_j} H_j. \)

Consider the LFC \( V_i \) as
\[
V_i = V_{i-1} + \frac{1}{4} e_i^4 + \frac{1}{2r_i} \hat{\theta}_i^2.
\]

Then, we can obtain
\[
\mathcal{L} V_i \leq \mathcal{L} V_{i-1} + e_1^3 \left[ F_i + G_i e_{i+1} + G_i \alpha_i - \dot{s}_{i,c}
+ G_{i-1} \Pi_{i-1}
+ l_{i1} \Pi_{i} + l_{i2} \text{sign}(\Pi_{i}) \right] + \frac{3}{2} e_i^2 \Phi_i^T \Phi_i
- \frac{1}{r_i} \hat{\theta}_i \dot{\hat{\theta}}_i
\leq \mathcal{L} V_{i-1} + e_1^3 \left[ \mathcal{H}_i(\chi_i) + G_i \alpha_i \right] + \frac{3}{4}
- \frac{3e_i^4}{4} - \frac{G_{i-1} e_i^4}{4} + \frac{G_i e_{i+1}^4}{4} - \frac{1}{r_i} \hat{\theta}_i \dot{\hat{\theta}}_i.
\tag{44}
\]
where $\mathcal{H}_i(\chi_i) = F_i - \dot{s}_{i,c} + G_i - \Pi - l_i \Pi^2 + \frac{3G_i e_i}{4} + l_i \dot{s}_i \Pi + l_i \Pi^2 G_i e_i + \frac{3G_i e_i}{4}$.

By using a FLS to approximate $\mathcal{H}_i(\chi_i)$, i.e., $\mathcal{H}_i(\chi_i) = W_i^T S_i(\chi_i) + e_i(\chi_i)$. Similar to (39) and (40), one has

$$e_i^3 \mathcal{H}_i(\chi_i) \leq \frac{1}{2a_i} e_i^6 \Theta_i S_i(\chi_i) + \frac{3e_i^4}{4} + \frac{e_i^4}{4},$$

(45)

$$e_i^3 G_i a_i \leq e_i - k_i \left( \frac{e_i^4}{4} \right)^{1/3} - k_i \left( \frac{e_i^4}{4} \right)^{2} - \frac{1}{2a_i} \frac{\Theta_i S_i(\chi_i)}{e_i^4},$$

(46)

where $a_i \in R^+$ is a design parameter, $|e_i(\chi_i)| \leq e_i$, and $e_i > 0$ is an any given constant.

Substituting (28), (45) and (46) into (44), one has

$$\mathcal{L} V_i \leq \sum_{j=1}^{i} k_{j} \left( e_i^4 \right)^{1/8} \sum_{j=1}^{i} k_{j} \left( e_i^4 \right)^{2} + \Delta_i$$

$$+ \sum_{j=1}^{i} \frac{\Theta_j \Theta_j}{\rho_j} + \sum_{j=1}^{i} \frac{c_j \Theta_j \Theta_j}{\rho_j} + \frac{G_i e_i^4}{4},$$

(47)

where $\Delta_i = \Delta_i - \frac{a_i}{4} + \frac{e_i^4}{4} + e_i + \frac{3}{4}$.

Step n
From (31), we have

$$|\beta(t) - u(t)| < \lambda |u(t)| + q, \forall t \in [t_k, t_{k+1}).$$

(48)

From (48), one has

$$-\lambda |u(t)| - q < \beta(t) - u(t)$$

$$< \lambda |u(t)| + q, \forall t \in [t_k, t_{k+1}).$$

(49)

Then, (49) can be rewritten as

$$\beta(t) = (1 + \omega_1(t) \lambda) u(t) - \omega_2(t) q, \forall t \in [t_k, t_{k+1}),$$

(50)

where $-1 < \omega_1(t), \omega_2(t) < 1$. Then one has

$$u(t) = \frac{\beta(t)}{1 + \omega_1(t) \lambda} - \frac{\omega_2(t) q}{1 + \omega_1(t) \lambda}.$$
\[
\begin{align*}
&\frac{a_n}{2} + \frac{3}{4} - \frac{G_n^{-1}e_n^4}{4} + \frac{e_n^4}{4} \\
&+ \frac{G_n e_n^3 \rho(t)}{1 + \lambda} + |G_n e_n^3 q|.
\end{align*}
\] (58)

Substituting (29) into (58), one has

\[
\mathcal{L} V_n \leq \mathcal{L} V_{n-1} - \frac{1}{r_n} \tilde{\Theta}_n \tilde{\Theta}_n + \frac{1}{2a_n} e_n^6 \Theta_n ||S_n(\chi_n)||^2 \\
+ \frac{a_n}{2} + \frac{3}{4} + \frac{e_n^4}{4} - \frac{G_n^{-1}e_n^4}{4} + \frac{G_n e_n^3 q}{1 - \lambda} \\
- \left| e_n^3 G_n a_n \right| + \tilde{h} \tan \left( \frac{e_n^3 h}{\rho} \right) + \tilde{h} \tan \left( \frac{e_n^3 h}{\rho} \right).
\] (59)

By using Lemma 4 and \( \tilde{h} > q/(1 - \lambda) \), we have

\[
\mathcal{L} V_n \leq \mathcal{L} V_{n-1} - \frac{1}{r_n} \tilde{\Theta}_n \tilde{\Theta}_n + \frac{1}{2a_n} e_n^6 \Theta_n ||S_n(\chi_n)||^2 \\
+ \frac{a_n}{2} + \frac{3}{4} + \frac{e_n^4}{4} - \frac{G_n^{-1}e_n^4}{4} + \frac{G_n e_n^3 q}{1 - \lambda} \\
- \left| e_n^3 G_n a_n \right| + \tilde{h} \tan \left( \frac{e_n^3 h}{\rho} \right) + \tilde{h} \tan \left( \frac{e_n^3 h}{\rho} \right) \\
\leq \mathcal{L} V_{n-1} - \frac{1}{r_n} \tilde{\Theta}_n \tilde{\Theta}_n + \frac{1}{2a_n} e_n^6 \Theta_n ||S_n(\chi_n)||^2 \\
+ \frac{a_n}{2} + \frac{3}{4} + \frac{e_n^4}{4} + \frac{3}{4} + \frac{e_n^4}{4} \\
+ e_n^3 G_n a_n + 0.557 G_n \rho.
\] (60)

Similar to (40) and (46), we have

\[
e_n^3 G_n a_n \leq e_n - k_n \left( \frac{e_n^4}{4} \right)^{\frac{3}{4}} - k_{n2} \left( \frac{e_n^4}{4} \right)^{\frac{3}{4}} \\
- \frac{1}{2a_n} e_n^6 \Theta_n ||S_n(\chi_n)||^2.
\] (61)

Substituting (28) and (61) into (60), one has

\[
\mathcal{L} V_n \leq - \sum_{j=1}^{n} k_{j1} \left( \frac{e_j^4}{4} \right)^{\frac{3}{4}} - \sum_{j=1}^{n} k_{j2} \left( \frac{e_j^4}{4} \right)^{\frac{3}{4}} + \Delta_n \\
+ \sum_{j=1}^{n} \sigma_j \tilde{\Theta}_j \tilde{\Theta}_j + \sum_{j=1}^{n} \frac{c_j}{2 r_j} \tilde{\Theta}_j \tilde{\Theta}_j \tilde{\Theta}_j.
\] (62)

where \( \Delta_n = \Delta_{n-1} + \frac{a_n}{2} + \frac{3}{4} + \frac{e_n^4}{4} + e_n + 0.557 G_n \rho. \)

Let \( \tilde{K}_1 = \min\{k_{11}, \ldots, k_{n1}\}, \tilde{K}_2 = \min\{k_{12}, \ldots, k_{n2}\} \), based on Lemma 5, one has

\[
- \sum_{j=1}^{n} k_{j1} \left( \frac{e_j^4}{4} \right)^{\frac{3}{4}} \leq - \tilde{K}_1 \sum_{j=1}^{n} e_j^4 \left( \frac{e_j^4}{4} \right)^{\frac{3}{4}}
\] (63)

\[
- \sum_{j=1}^{n} k_{j2} \left( \frac{e_j^4}{4} \right)^{\frac{3}{4}} \leq - \tilde{K}_2 \sum_{j=1}^{n} e_j^4 \left( \frac{e_j^4}{4} \right)^{\frac{3}{4}}
\] (64)

Since \( \tilde{\Theta}_j \tilde{\Theta}_j \leq - \tilde{\Theta}_j^2 + \tilde{\Theta}_j^2 \), then we have

\[
\sum_{j=1}^{n} \frac{\sigma_j}{2 r_j} \tilde{\Theta}_j \tilde{\Theta}_j \leq - \sum_{j=1}^{n} \frac{\sigma_j \tilde{\Theta}_j^2}{2 r_j} + \sum_{j=1}^{n} \frac{\sigma_j \tilde{\Theta}_j^2}{2 r_j}.
\] (65)

Substituting (63)-(65) into (62), one has

\[
\mathcal{L} V_n \leq - \tilde{K}_1 \left( \sum_{j=1}^{n} \frac{e_j^4}{4} \right)^{\frac{3}{4}} - \left( \sum_{j=1}^{n} \frac{\sigma_j \tilde{\Theta}_j^2}{2 r_j} \right)^{\frac{3}{4}} \\
- \tilde{K}_2 \left( \sum_{j=1}^{n} e_j^4 \frac{e_j^4}{4} \right)^{\frac{3}{4}} \\
+ \left( \sum_{j=1}^{n} \frac{\sigma_j \tilde{\Theta}_j^2}{2 r_j} \right)^{\frac{3}{4}} + \Delta_n - \sum_{j=1}^{n} \frac{\sigma_j \tilde{\Theta}_j^2}{2 r_j} \\
+ \sum_{j=1}^{n} \frac{\sigma_j \tilde{\Theta}_j^2}{2 r_j} + \sum_{j=1}^{n} \frac{c_j}{2 r_j} \tilde{\Theta}_j \tilde{\Theta}_j \tilde{\Theta}_j.
\] (66)

By using Lemma 6, let \( a = 1, b = \sum_{j=1}^{n} \frac{\sigma_j \tilde{\Theta}_j^2}{2 r_j} \), \( p_1 = \frac{1}{4} , p_2 = \frac{3}{4} , p_3 = \left( \frac{3}{4} \right)^3 \), we have

\[
\left( \sum_{j=1}^{n} \frac{\sigma_j \tilde{\Theta}_j^2}{2 r_j} \right)^{\frac{3}{4}} \leq \frac{27}{256} + \sum_{j=1}^{n} \frac{\sigma_j \tilde{\Theta}_j^2}{2 r_j}.
\] (67)
Substituting (67) into (66), one has

\[
\mathcal{L} V_n \leq -\bar{\kappa}_1 \left( \sum_{j=1}^{n} \frac{e_j^4}{4} \right)^{\frac{3}{4}} - \left( \sum_{j=1}^{n} \frac{\sigma_j \tilde{\Theta}_j \tilde{\Theta}_j}{2r_j} \right)^{\frac{3}{4}} - \frac{\bar{\kappa}_2}{n} \left( \sum_{j=1}^{n} \frac{e_j^4}{4} \right)^{\frac{1}{2}} + \sum_{j=1}^{n} \frac{c_j \Theta_j^3}{r_j^2} + \sum_{j=1}^{n} \frac{c_j \tilde{\Theta}_j \tilde{\Theta}_j}{r_j^2} + \bar{\Delta}_n, 
\]

where \( \bar{\Delta}_n = \Delta_n + \sum_{j=1}^{n} \frac{\sigma_j \Theta_j^2}{2r_j} + \frac{27}{256}. \)

Since \( \Theta_j \Theta_j^3 = \tilde{\Theta}_j \left( \Theta_j^3 - 3\Theta_j^2 \tilde{\Theta}_j + 3\Theta_j \tilde{\Theta}_j^2 - \tilde{\Theta}_j^3 \right), \)
(68) can be reexpressed as

\[
\mathcal{L} V_n \leq -\bar{\kappa}_1 \left( \sum_{j=1}^{n} \frac{e_j^4}{4} \right)^{\frac{3}{4}} - \left( \sum_{j=1}^{n} \frac{\sigma_j \tilde{\Theta}_j \tilde{\Theta}_j}{2r_j} \right)^{\frac{3}{4}} - \frac{\bar{\kappa}_2}{n} \left( \sum_{j=1}^{n} \frac{e_j^4}{4} \right)^{\frac{1}{2}} + \sum_{j=1}^{n} \frac{c_j \tilde{\Theta}_j \tilde{\Theta}_j}{r_j^2} + \bar{\Delta}_n. 
\]

On the basis of Young’s inequality, we can obtain

\[
\sum_{j=1}^{n} \frac{3c_j \tilde{\Theta}_j \Theta_j}{r_j^2} \leq \sum_{j=1}^{n} \frac{9c_j \xi^{\frac{3}{4}} \tilde{\Theta}_j^4}{4r_j^2} + \sum_{j=1}^{n} \frac{3c_j \Theta_j^4}{4r_j^2}, \tag{70}
\]

\[
\sum_{j=1}^{n} \frac{c_j \tilde{\Theta}_j \Theta_j^3}{r_j^2} \leq \sum_{j=1}^{n} \frac{3c_j \tilde{\Theta}_j \Theta_j^2}{r_j^2} + \sum_{j=1}^{n} \frac{c_j \Theta_j^4}{12r_j^2}. \tag{71}
\]

Substituting (70) and (71) into (69), one has

\[
\mathcal{L} V_n \leq -\bar{\kappa}_1 \left( \sum_{j=1}^{n} \frac{e_j^4}{4} \right)^{\frac{3}{4}} - \left( \sum_{j=1}^{n} \frac{\sigma_j \tilde{\Theta}_j \tilde{\Theta}_j}{2r_j} \right)^{\frac{3}{4}} - \frac{\bar{\kappa}_2}{n} \left( \sum_{j=1}^{n} \frac{e_j^4}{4} \right)^{\frac{1}{2}} + \sum_{j=1}^{n} \frac{9c_j \xi^{\frac{3}{4}} \tilde{\Theta}_j^4}{4r_j^2} + \sum_{j=1}^{n} \frac{3c_j \Theta_j^4}{4r_j^2} + \sum_{j=1}^{n} \frac{c_j \Theta_j^4}{12r_j^2} \]

where \( \varrho = \sum_{j=1}^{n} \frac{3c_j \Theta_j^4}{4r_j^2} + \sum_{j=1}^{n} \frac{c_j \Theta_j^4}{12r_j^2} + \bar{\Delta}_n. \)

Let \( \hat{\bar{\kappa}}_1 = \min\{\sigma_j^3\}, \hat{\bar{\kappa}}_2 = \min\{(4 - 9\xi^{\frac{3}{4}})c_j\}, \) then one has

\[
\mathcal{L} V_n \leq -\hat{\bar{\kappa}}_1 \left( \sum_{j=1}^{n} \frac{e_j^4}{4} \right)^{\frac{3}{4}} - \hat{\bar{\kappa}}_1 \left( \sum_{j=1}^{n} \frac{\tilde{\Theta}_j^2}{2r_j} \right)^{\frac{3}{4}} - \frac{\hat{\bar{\kappa}}_2}{n} \left( \sum_{j=1}^{n} \frac{e_j^4}{4} \right)^{\frac{1}{2}} + \sum_{j=1}^{n} \frac{9c_j \xi^{\frac{3}{4}} \tilde{\Theta}_j^4}{4r_j^2} + \sum_{j=1}^{n} \frac{3c_j \Theta_j^4}{4r_j^2} + \sum_{j=1}^{n} \frac{c_j \Theta_j^4}{12r_j^2} + \varrho. \tag{73}
\]

Let \( K_1 = \min\{\hat{\bar{\kappa}}_1, \hat{\bar{\kappa}}_2\}, \hat{K}_2 = \min\{\frac{\hat{\bar{\kappa}}_2}{n}, \hat{\bar{\kappa}}_2\}, \) one has

\[
\mathcal{L} V_n \leq -K_1 \left[ \left( \sum_{j=1}^{n} \frac{e_j^4}{4} \right)^{\frac{3}{4}} + \left( \sum_{j=1}^{n} \frac{\tilde{\Theta}_j^2}{2r_j} \right)^{\frac{3}{4}} \right] - \hat{K}_2 \left[ \left( \sum_{j=1}^{n} \frac{e_j^4}{4} \right)^{\frac{1}{2}} + \left( \sum_{j=1}^{n} \frac{\tilde{\Theta}_j^2}{2r_j} \right)^{\frac{1}{2}} \right] + \varrho. \tag{74}
\]

According to Lemma 5, we have

\[
V_n^{\frac{3}{4}} = \left[ \sum_{j=1}^{n} \left( \frac{e_j^4}{4} + \frac{\tilde{\Theta}_j^2}{2r_j} \right) \right]^{\frac{3}{4}} \leq \left( \sum_{j=1}^{n} \frac{e_j^4}{4} \right)^{\frac{3}{4}} + \left( \sum_{j=1}^{n} \frac{\tilde{\Theta}_j^2}{2r_j} \right)^{\frac{3}{4}}, \tag{75}
\]
Based on (74)–(76), we have
\[
\mathcal{L} V_n \leq -K_1 V_n^2 - K_2 V_n^2 + \varrho,
\]
(77)

where \( K_2 = \frac{k_2}{\eta_n} \).

According to Lemma 2 and the definition of LFCs, we can obtain that \( e_i \) and \( \Theta_i \) are FTBIP, and the setting time satisfies: \( T_1 \leq \frac{4}{K_1} + \frac{1}{K_2} \). From (25)–(27), one can obtain that \( \alpha_i \) and \( \Delta_i \) are also FIBIP.

Since \( e_i = z_i - \Pi_i \), if \( \Pi_i \) is FTBIP, then so is \( z_i \). In what follows, we will show that \( \Pi_i \) is FTBIP.

**Step n + 1** Select the following LFC
\[
V_{n+1} = \sum_{i=1}^{n} \frac{1}{2} \Pi_i^2,
\]
(80)

Then, one has
\[
\mathcal{L} V_{n+1} = \sum_{i=1}^{n} \Pi_i \dot{\Pi}_i
= -l_{11} \Pi_1^4 + \Pi_1(s_2,c - \alpha_1) + \Pi_1 \Pi_2
- l_{22} \Pi_1 \text{sign}(\Pi_1) + \cdots
- l_{11} \Pi_1^4 + \Pi_i(s_{i+1,c} - \alpha_i) - \Pi_{i-1} \Pi_i
+ \Pi_i \Pi_{i+1} - l_{12} \Pi_1 \text{sign}(\Pi_i) + \cdots
- l_{11} \Pi_n^4 - \Pi_{n-1} \Pi_n - l_{n2} \Pi_n \text{sign}(\Pi_n)
= -\sum_{i=1}^{n} l_{11} \Pi_i^4 - \sum_{i=1}^{n} l_{12} \Pi_i \Pi_i
+ \sum_{i=1}^{n-1} \Pi_i(s_{i+1,c} - \alpha_i).
\]
(81)

According to [35], there exists a given positive constant \( \kappa_i \) such that \( s_{i+1,c} - \alpha_i \leq \kappa_i \) is hold in a fixed time \( T_{i2} \). Then, one has
\[
\mathcal{L} V_{n+1} \leq -\sum_{i=1}^{n} l_{11} \Pi_i^4 - \sum_{i=1}^{n} (l_{12} - \kappa_i) |\Pi_i|
\leq -L_1 V_{n+1}^2 - L_2 V_{n+1}^{\frac{1}{2}},
\]
(82)

where \( L_1 = \frac{4}{n} \min\{l_{11}\}, L_2 = \sqrt{2} \min\{l_{12} - \kappa_i\} \).

According to Lemma 1 and the definition of \( V_{n+1} \), it is obtained that \( \Pi_i \) is FTBIP, and the setting time satisfies: \( T_3 = \frac{L_2}{L_1} + \frac{1}{L_2} \). Since \( e_i = z_i - \Pi_i \), it is easy to obtain that \( z_i \) is FTBIP, and the setting time satisfies: \( T = \max\{\max\{T_{i2}\} + T_3, T_1\} \). Then \( s_i \) is FTBIP, from (17), we can obtain that \( z_i \) is FTBIP. According to the above analysis, the conclusion that the solution of system (16) is FTSP can be obtained. From (17), one has \(-\mathcal{F}_{i1}(t) \leq z_i \leq \mathcal{F}_{i2}(t)\), i.e., the time-varying asymmetric full-state constraints are not violated. Furthermore, as \( z_1 \) satisfies
\[
z_1 = \frac{(\xi_1 - v_y)\mathcal{F}_{i1}(t)(\mathcal{F}_{i2}(t) - \eta_y)\mathcal{F}_{i2}(t) - \xi_1 v_y}{\mathcal{F}_{i1}(t)(\mathcal{F}_{i2}(t) - \eta_y)}
\]
(83)

thus, the tracking error \( z_1^* = z_1 - y_r \) can be limited to a small residual set within a fixed-time interval by choosing the appropriate parameters.

### 4 Simulation results

**Example 1** The following nonlinear system is considered
\[
\begin{align*}
\dot{\xi}_1 &= [\sin(\xi_1)e^{-0.5} + (1 + \xi_1^2)]\zeta_2 + h_1^T \omega \\
\dot{\zeta}_2 &= [\xi_2^2 + 2.5u(t)]\zeta_2 + h_2^T \omega \\
y &= \xi_1,
\end{align*}
\]
(84)

where \( h_1 = 1 - \cos(\xi_1), h_2 = \xi_1 \sin(\xi_2) \). Let the desired tracking signal be \( v_r = \sin(0.5t), \mathcal{F}_{i1}(t) = 0.5 + 2^{-0.4t} - 0.5 \sin(t), \mathcal{F}_{i2}(t) = 0.5 + 2^{-0.3t} + 0.5 \sin(t), \mathcal{F}_{i21}(t) = 1 + 0.2 \sin(t + 5), \mathcal{F}_{i22}(t) = 1.5 - 2^{-0.3t} \). The design parameters are set as \( k_{11} = 6, k_{21} = k_{22} = 2, \varepsilon_1 = \varepsilon_2 = 0.001, r_1 = 10, r_2 = 30, \sigma_1 = \sigma_2 = 1, a_1 = a_2 = 0.01, \lambda = 0.2, q = 0.1, \bar{h} = 4, \rho = 8 \). Let the initial values be \( \xi_1(0) = 0.2, \zeta_2(0) = 0.1, \hat{\theta}_1 = 4, \hat{\theta}_2 = 3 \).

From Figs. 1 and 2, it can be seen that the proposed scheme can achieve a good tracking performance, and the asymmetric time-varying state constraints are not violated. Figures 3, 4, and 5 express the profile of the tracking error \( z_1^* \), the estimated parameters \( \hat{\theta}_1, \hat{\theta}_2 \) and the control signal \( u(t) \), respectively. The time interval of triggering events is shown in Fig. 6.

**Example 2** The simulation of the Spring-Mass-Damper system [32] is given as follows.
The model is built as follows

\[
\begin{align*}
\dot{s} &= v \\
\dot{v} &= -k_1 s - k_2 v + F,
\end{align*}
\]  

where \( s, v \) and \( F \) denote position, velocity, and force applied to the object, respectively. Let \( \zeta_1 = s, \zeta_2 = v, u = F, y_r = \sin(0.5t) + 0.5 \sin(t), \mathcal{F}_{11}(t) = 0.5 + 2^{-0.4t} - 0.5 \sin(t), \mathcal{F}_{12}(t) = 0.5 + 2^{-0.3t} + 0.5 \sin(t), \mathcal{F}_{21}(t) = 1+0.2 \sin(t+5), \mathcal{F}_{22}(t) = 1.5 - 2^{-0.3t}. \) Let’s assume that there exist the stochastic disturbances, let \( g_1 = (1 - \cos(\zeta_1)), g_2 = \sin(\zeta_2). \) Then (82) can be
rewritten as
\[
\begin{aligned}
\dot{\zeta}_1 &= \dot{\zeta}_2 \, dt + (1 - \cos(\zeta_1)) \, d\omega \\
\dot{\zeta}_2 &= [\frac{-k_1}{m} \zeta_1 - \frac{k_2}{m} \zeta_2 + \frac{1}{m} u(t)] \, dt + \sin(\zeta_2) \, d\omega \\
y &= \zeta_1,
\end{aligned}
\tag{83}
\]

The parameters are set as \(k_1 = 3, m = 1, k_2 = 0.5, k_{11} = k_{12} = 6, k_{21} = k_{22} = 2, \varepsilon_1 = \varepsilon_2 = 0.001, r_1 = 10, r_2 = 30, \sigma_1 = \sigma_2 = 1, a_1 = a_2 = 0.01, \lambda = 0.2, q = 1.5, \bar{h} = 4, \rho = 8.\) Let \(\zeta_1(0) = 0.2, \zeta_2(0) = 0.1, \hat{\theta}_1 = 4, \hat{\theta}_2 = 3.\) The simulation results are shown in Figs. 7, 8, 9, 10, 11 and 12.

### 5 Conclusion

In this article, the problem of event-triggered fixed-time control is studied for state-constrained stochastic systems. By introducing the NSDFs, the state constraints are handled without FCs. Combined with the fixed-time stability theory and the DSC technique with fixed-time filter, a newly controller is constructed, under which the fixed time stability of the closed-loop system can be ensured, and the problems of “explosion of complexity” and “singularity” are overcome. Furthermore, the
Conflict of interest The authors declare that they have no conflict of interest.

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