Disorder-enhanced inelastic scattering in ultrathin NbN films

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We report on experimental study of electronic parameters and inelastic scattering mechanisms in ultrathin superconducting NbN films, which are commonly used in single-photon detectors. The studied NbN films with properties representative for photon detectors, are strongly disordered with respect to the electron transport, the Ioffe-Regel parameter varies in range of $1.6 \leq k_F l \leq 6.3$. As the disorder in the studied 2.5 nm thick NbN films increased, the critical temperature $T_c$ lowers from 11.5 K to 3.4 K and this is consistent with the fermionic scenario of suppression of superconductivity. By measuring magnetoconductance in the range from $T_c$ to $\sim 3T_c$, we observe a clear increase trend for the rate of inelastic scattering of electrons as normal-state resistance grows. We assume that this effect is related to strong sensitivity of the electron-phonon scattering to microscopic quality of NbN films. The obtained values of material parameters of these disorder-modified NbN films can be useful for optimization of performance of NbN-based electronic devices.

I. INTRODUCTION

Physical mechanisms that govern the behavior of superconducting and electronic properties in ultrathin films have been studied extensively for the fundamental understanding of impact of disorder and quantum effects on electron transport\textsuperscript{1}. They have been also viewed from the point of potential applications in superconducting devices, such as photon detectors\textsuperscript{2–5}. To optimize the operation of these thin-film devices, it is essential to know parameters, which control the non-equilibrium response to radiation: for instance electronic and phonon heat capacities, electron diffusivity, rates of inelastic electron-electron (e-e) and electron-phonon (e-ph) scattering processes. Numerous studies of electron transport in disordered metals reveal some signs of disorder on the mechanisms of inelastic scattering. For example, a significant enhancement of the e-e scattering rates is expected due to a strong elastic scattering of quasiparticles in thin disordered films\textsuperscript{6} or due to the presence of a moderate density of magnetic impurities in low-dimensional devices\textsuperscript{7,8}. It is also expected that disorder plays an essential role in the e-ph scattering, and one can expect a weakening or strengthening of the e-ph interaction, depending on the specific properties of disordered systems\textsuperscript{9,10}, or an appearance of additional relaxation channels in the limit of strong disorder\textsuperscript{11}. In samples with reduced dimensions, relaxation processes also depend on samples size\textsuperscript{12–14}, which leads to an even greater variety of effects in inelastic relaxation. Thus, there should be carried out an empirical study to understand the effect of disorder on inelastic scattering in specific low-dimensional devices.

Thin film of niobium nitride (NbN) is a typical material, in which disorder can be tuned from moderate to strong limit\textsuperscript{15,16}. This material has been extensively used for the production of modern electronic devices such as SNSPDs (Superconducting Nanowire Single Photon Detectors)\textsuperscript{17,18}, HEB (Hot Electron Bolometer) mixers\textsuperscript{19}, microwave nanoinductors\textsuperscript{20} and resonators\textsuperscript{21}, quantum phase slip devices\textsuperscript{22–26} etc. The choice of NbN for these technological applications has been justified by its relatively high superconducting critical temperature ($T_c \approx 10 \sim 12$ K in $d \approx 5$ nm thick films), high values of resistivity ($\rho > 100 \mu\Omega$cm) and relatively fast e-ph relaxation ($\tau_{e-ph} \sim 10$ ps at $T_c \approx 10$ K)\textsuperscript{27}. On the fundamental level there is observed an ongoing interest in effects of disorder on superconducting and normal state properties in NbN films\textsuperscript{28–33}. Mechanisms of inelastic relaxation in thin NbN films have been studied with various experimental methods, however significant inconsistencies in the data have appeared in literature over the past decades\textsuperscript{27,34–39}. They are mainly caused by changing material parameters due to different deposition conditions. Nevertheless, a steady progress in production of ultrathin NbN films\textsuperscript{16,40,41} opens up new horizons for further research, in particular for a systematic study of the effect of disorder on electron transport and inelastic scattering mechanisms in NbN films.

In this work we have prepared a set of ultrathin NbN samples with a tunable intrinsic disorder by depositing films in different conditions. Here we change only one deposition parameter at a time and fix the others. We studied evolution of electronic parameters and inelastic scattering mechanisms with the increase of disorder. To study inelastic scattering we applied magnetoconductance measurements, which have been successfully used to study inelastic scattering in some samples of NbN\textsuperscript{38,42} and NbTiN\textsuperscript{43} recently. We have observed a clear trend towards the increase of the rate of inelastic electron scattering as the normal-state resistance grows. A qualitative
analysis of the experimental data indicates that the increase in the electron dephasing rates can be caused by the softening of phonons in strongly disordered samples.

II. SAMPLES AND MEASUREMENT SETUP

Ultrathin NbN films are deposited using magnetron sputtering system (AJA International Inc.) with a background pressure of $9 \times 10^{-8}$ torr. Samples are deposited on r-cut sapphire substrates by sputtering of Nb target in an argon-nitrogen atmosphere (99.998% purity of both gases). Growth rate equal to 0.065 nm/s is controlled via quartz crystal microbalance in each deposition run. Studied films have thickness of $d = 2.5$ nm. The level of disorder in five NbN films (s1-s5 in Table I) is varied by changing a substrate temperature in each deposition process $T_{dep}$: 500, 400, 300, 150, and $25^\circ$C (no additional heating), respectively. The films s1-s5 are also grown at a fixed nitrogen concentration of 22%, while operating pressure is maintained constant at 3.6 mtorr. Heating heating), respectively. The films s1-s5 are also grown at a fixed nitrogen concentration of 22%, while operating pressure is maintained constant at 3.6 mtorr. Heating of substrates was carried out by using the built-in resistive SiC heater with a PID controller. The most disordered samples are grown at the following conditions: $T_{dep} = 500^\circ$C; 27% of nitrogen at 6.5 mtorr for s6 and 23% of nitrogen at 6.8 mtorr for s7. Operating pressure was adjusted using a throttle valve. As a result, we obtained NbN films of the same thickness but of different resistance per square $R_s$ and critical temperature $T_c$ by varying heating of substrates and partial nitrogen pressure. To prevent unintentional oxidation of NbN films in the atmosphere, films are covered with a 5-nm thick passivating silicon layer in situ. Structural characterization of a 2.5-nm thick NbN sample (s1) is performed using X-ray diffraction analysis, X-ray photoelectron spectroscopy (XPS) and atomic force microscopy (AFM). The details of structural characterization are given in Supplemental material.

To study transport properties of NbN, we patterned films into 500-µm wide and 1000-µm long Hall-bars. Electrical transport measurements were carried out with Lake Shore 370 AC Resistance Bridge at a bias current less than 1 µA. Normal-state resistance $R_s$ was measured in a four-probe configuration. The measurements were carried out in a homemade $^4$He cryogenic insert immersed in a dewar and performed in a wide temperature range (from 300 K to 1.7 K). At low temperatures we measured magneoresistance $R_s(B)$, temperature dependencies of the second critical magnetic field $B_{c2}(T)$ and the Hall constant $R_H^{25K}$ at 25 K by applying perpendicular magnetic field $B$ up to 4 T. By measuring $R(T)$-dependencies at different values of $B$ (not shown here) we determined the slope $dB_{c2}/dT$ at $T_c$. The latter allows to estimate the critical magnetic field $B_{c2}(0)$, the electron diffusivity $D$ and the Ginzburg-Landau (GL) coherence length $\xi_{GL}(0)$ using the following expressions $B_{c2}(0) = -0.09T_c (dB_{c2}/dT)$, $D = -4k_B/(\pi\gamma)(dB_{c2}/dT)^{-1}$, and $\xi_{GL}(0) = \pi hD/(8k_B T_c)$. Here, the critical temperature $T_c$ is determined as temperature at $R_s = R_{max}/2$.

III. RESULTS AND DISCUSSION

A. Normal-state properties

Figure 1 shows temperature dependencies of the sheet resistance $R_s$ for all NbN films under study. $R_s$ for all films gradually rises as temperature decreases and drops to zero at vicinity of $T_c$. The temperature trend of $R_s$ can be characterized by the resistance ratio parameter $r_R = R_s^{300K}/R_{max}$, where $R_{max}$ is the maximal sheet resistance just above the resistive transition. Commonly observed $r_R < 1$ is well-known behavior for disordered NbN films. It reflects sensitivity of electron system to quantum corrections, meanwhile contribution of e-ph scattering is considered to be negligible here. We also observed suppression of $T_c$ with increasing disorder, which is consistent with the fermionic mechanism of suppression of superconductivity in disordered films (see Appendix A for details). This $T_c$ suppression scenario indicates that the electron transport in strongly disordered NbN films may be affected by strong electron correlations, which are a precursor to the superconductor-insulator transition.

Table I gives an overview of metallic properties for studied NbN films. The details about the carrier density $n$, the Ioffe-Regel (disorder) parameter $k_F l$, the mean free path $l$, the electron diffusivity $D$, and the density of states $N_0$ estimates are provided in Appendix B. High normal-state resistance $R_s$ and low resistance ratio parameter $r_R$ of the magnetron-sputtered NbN films are also known to be strongly influenced by grain-boundary scattering, which is related to electron transmission through grain boundaries. One should note that the graininess of ultrathin NbN films is not as pronounced as that of thick films (see AFM studies in Supplemental material). Meanwhile, we assume that this mechanism can be considered as one of the main factors limiting the mean free path in NbN films (see Appendix B for details).

B. Magnetoconductance

To study inelastic relaxation in NbN films as a function of disorder, we measured magnetoconductance by varying magnetic field $B$ in the range from 0 to 4 T at a set of fixed temperatures from $T_c$ to 30 K. The NbN films studied represent quasi two-dimensional (2D) system with respect to characteristic length scales, i.e. the thermal coherence length $L_T = (\hbar D/k_BT)^{1/2}$ and the superconducting coherence length $\xi_{GL}$ ($d < \xi_{GL}, L_T$). The dimensionless change in magnetoconductance at the fixed $T$ can be determined from the measured $R_s(B,T)$ using the following expression:

$$\delta G(B,T) = \frac{2\pi^2 \hbar}{e^2} \left[ R_s(B,T)^{-1} - R_s(0,T)^{-1} \right].$$

(1)
TABLE I. Parameters of NbN films

| $N^2$ | $R_s^{300K}$ (Ω/sq) | $r_R$ | $k_{pl}$ | $T_c$ (K) | $D$ (cm²/s) | $\xi_{GL}(0)$ (nm) | $B_{c1}(0)$ (T) | $n \times 10^{29}$ | $\tau$ (fs) | $l$ (Å) | $N_0 \times 10^{28}$ (Ω/sq) | $\tau_{e-ph}(T_c)$ (ps) | $p$ | $\varepsilon_F$ (eV) |
|-------|-----------------|------|--------|----------|-------------|-----------------|-----------------|----------------|----------|-----|-----------------------------|---------------------|----|---------------------|
| s1    | 437             | 0.85 | 6.3    | 11.54    | 0.59        | 3.9             | 14.8            | 1.9             | 0.7       | 3.5 | 4.8                        | 530                 | 6.1 | 4.05                |
| s2    | 509             | 0.81 | 5.5    | 10.76    | 0.57        | 4.0             | 14.3            | 1.8             | 0.6       | 3.16 | 4.3                        | 645                 | 5.5 | 4.0                 |
| s3    | 815             | 0.72 | 3.5    | 8.43     | 0.36        | 3.6             | 17.9            | 1.7             | 0.4       | 2   | 4.3                        | 1210                | 9.5 | 3.75                |
| s4    | 912             | 0.69 | 3.2    | 7.02     | 0.35        | 3.5             | 15.3            | 1.6             | 0.5       | 1.9 | 3.9                        | 1420                | 10.0 | 3.25                |
| s5    | 1025            | 0.67 | 3.8    | 6.03     | 0.34        | 4.1             | 13.5            | 1.6             | 0.27      | 1.65 | 3.6                        | 1678                | 11.0 | 3.0                 |
| s6    | 1574            | 0.38 | 2.1    | 3.40     | 0.27        | 4.9             | 9.7             | 1               | 0.28      | 1.5 | 3.0                        | 5500                | 10.0 | 2.4                 |
| s7    | 1950            | 0.26 | 1.6    | -       | -           | -               | -               | 1.1             | 0.15      | 1.1 | 2.4                        | -                   | -   | 6.9                 |

FIG. 1. Temperature dependences of the sheet resistance $R_s(T)$ for all samples (main panel) and only for superconducting samples (s1-s6) in the narrow temperature range near their superconducting transitions (inset). The dashed lines represent fits according to the theory of superconducting fluctuations (see text for details).

Figure 2 shows typical experimental dependencies of $\delta G(B,T)$ for NbN samples. The data, presented for a representative sample (s5), look similar for other samples. The change in conductance above $T_c$ induced by magnetic field mainly originates from superconducting fluctuations. Applying the fluctuation spectroscopy approach, we fitted the experimental data in Figure 2 with the relative magnetoconductance $G^{SC} = G^{SC}(B,T) - G^{SC}(0,T)$, where $G^{SC}(0,T)$ and $G^{SC}(B,T)$ are the sum of superconducting corrections to conductivity in zero and finite magnetic field (see Eqs. (4a)-(4b) in Appendix C). Using $T_c$ as a fitting parameter, we estimated the anomalous Maki-Thompson term, which contains information about the temperature-dependent electron phase-breaking rate $\tau_{e-ph}(T)$. Note that the choice of $T_c$ in 2D disordered superconducting films is ambiguous due to percolation effects and/or a build-in inhomogeneity of superconducting properties, which can lead to the appearance of a "pseudogap mode" in NbN together with disorder. Nevertheless, in our measurements the choice of $T_c$ barely affects the extracted values of $\tau_{e-ph}(T)$. The best fits of $G(B,T)$ in Figure 2 are shown with black curves with values of $T_c$ varying in under 2% in respect to the values determined at $R_s = R_{cmax}/2$. It is important to note that the phase-breaking length $L_{\phi} = \sqrt{D\tau_{e-ph}}$ is larger than the film thickness $d$ in a considered temperature range. This fact supports validity of using 2D expressions for NbN films under study.

The same SC fluctuation effects can be exploited for fitting of $R_s(T)$-dependencies above $T_c$ (see the inset of Figure 2). Here we use the expression $R_s(T) = 1/(R_{NS}^{-1} + e^2/(2\pi^2 h)G^{SC}(0,T))$, where $R_{NS}$ is a fitting parameter and the values of $\tau_{e-ph}$ in $G^{SC}(0,T)$ are those extracted from the magnetoconductance processing. Note that the values of $R_{NS}$ turn out to be close to $R_{cmax}$.
rather than $R_{s}^{300K}$. The discrepancy between $R_{s}^{300K}$ and $R_{na}$ in the disordered NbN films may be explained by contributions of quantum corrections to conductivity due to e-e interaction, weak localization, and even hopping conductivity at approaching the Anderson-Mott transition\(^{54}\). Nevertheless, contribution of these effects to magnetoconductance is negligible in comparison with superconducting fluctuations (see Supplemental Material\(^{44}\) for details), and we do not take them into consideration.

C. Electron phase-breaking rate

As a next step, we explore the impact of disorder on inelastic scattering in NbN films. Figure 3(a) shows a stronger decrease of $\tau_{\phi}^{-1}$ with lowering temperature. The exact expression for $\tau_{\phi}^{-1}$ is represented by sum of scattering mechanisms due to superconducting fluctuations $\tau_{SC}^{-1}$, the e-e scattering rate $\tau_{e-e}^{-1}$, the spin-flip scattering rate $\tau_{s}^{-1}$, and the e-ph scattering rate $\tau_{e-ph}^{-1}$ as follows\(^{51}\):

$$\tau_{\phi}^{-1}(T) = \tau_{SC}^{-1} + \tau_{e-e}^{-1} + \tau_{s}^{-1} + \tau_{e-ph}^{-1}. \quad (2)$$

The presence of surface magnetic defects can significantly increase $\tau_{\phi}^{-1}$ and lead to $T$-independent behavior at low temperatures\(^{8,13}\). The magnetic disorder in superconductors can lead to a time-reversal symmetry break caused by spin-flip scattering, altering superconducting state and suppressing $T_c$ more strongly than nonmagnetic disorder\(^{55–57}\). Recent studies of Nb and NbN-based devices reveal unintentional surface magnetic disorder due to the unpaired spins in native oxide\(^{38–61}\). Nevertheless, we found that the passivating Si layer on top of NbN films fortunately prevented strong oxidation (see XPS analysis in Supplemental Material\(^{44}\)). In addition, the suppression of $T_c$ in our films can be explained by strong electron interaction (see Appendix A). Thus, we treat the effects of magnetic disorder as negligible in analysis of $\tau_{\phi}^{-1}$-dependencies.

The e-e and superconducting fluctuations phase-breaking rates can be defined as\(^{3,62}\):

$$\tau_{SC}^{-1} = \frac{\pi g k_B T}{h} \frac{2 \ln 2}{\epsilon + \beta}, \quad (2a)$$

$$\tau_{e-e}^{-1} = \frac{\pi g k_B T}{h} \ln \frac{1}{2 \pi g}, \quad (2b)$$

where $\beta = 4 \ln 2 / \left[ \sqrt{\ln^2(1/(2\pi g)) + 64/(\pi^2 g)} + \ln(2\pi g) \right]$, $g = e^2 R_{s}^{300K} / (2\pi^2 h)$ and $\epsilon = \ln(T/T_c)$. The value of $R_{s}^{300K}$ is taken here at the highest temperature of our measurements (300 K), where the effect of e-e interactions is expected to be small\(^{54,63}\). The expression for the e-e scattering rate accounts only for the processes with small energy transfer - the so-called Nyquist quasielastic scattering, which dominates in our experimental temperature range with $T < h/(k_B\tau) \sim 10^3$ K. The e-e scattering rate is expected to enhance greatly with increasing disorder\(^{6}\), but we estimate the increase of $\tau_{e-e}^{-1}$ as two times for the given change of $R_{s}^{300K}$. Since $\tau_{\phi}^{-1}$ in our measurements is characterized by a stronger $T$-dependence than the e-e and SC phase-breaking rates, we believe that the e-ph scattering is the dominant dephasing process. Applying that $\tau_{e-ph} = \alpha_{e-ph}(T/T_c)^p$, where $p$ and $\alpha_{e-ph}$ are fitting constants, we fit the data in Figure 3(a) (see the dashed lines). Table I gives an overview of the best-fit values for the power index $p$ and $\alpha_{e-ph}$, together with other dephasing rates. Our findings for the least disordered sample (s1) are also close to results for the e-ph relaxation in NbN films obtained by magnetoconductance and photoresponse techniques in Ref.\(^{38}\).

![FIG. 3. Temperature dependencies of (a) the electron dephasing rate $\tau_{\phi}^{-1}$ extracted from magnetoconductance measurements and (b) the e-ph scattering time $\tau_{e-ph}$ extracted from $\tau_{\phi}^{-1}$. The data is plotted in symbols on a log-log scale. In (a) the solid curves show the best fits of $\tau_{\phi}^{-1}$ by Eq. (2). The dashed lines illustrate contributions of the e-ph, e-e, and SC dephasing mechanisms for sample s1. In (b) the solid lines demonstrate prediction of Eq. (5) for $\tau_{e-ph}$.](image_url)

Figure 3(b) shows the temperature dependencies of $\tau_{e-ph}$ extracted from the total dephasing rate $\tau_{\phi}^{-1}$ by subtracting $\tau_{e-e}^{-1}$ and $\tau_{s}^{-1}$. We observe that the magnitude of $\tau_{e-ph}$ decreases tenfold as disorder in NbN films increases, and it is accompanied by a change of the power index $p$ from 4 to 2.4 in $\tau_{e-ph}^{-1}(T) \propto T^p$. Below we will discuss potential mechanisms affecting the observed enhancement of the e-ph scattering.
D. Electron-phonon scattering in ultrathin films

In search of potential explanation for the observed $\tau_{\text{e-ph}}^{-1}$ behavior, we turned to the existing models of the e-ph scattering in thin disordered metals\textsuperscript{9,64,65}. In general, the e-ph coupling occurs because the passing phonons distort the local lattice structure and conduction electrons respond to the resulting band distortion. A widely used standard model of the low temperature electron-phonon scattering in bulk metals (i.e. jellium model)\textsuperscript{66,67} assumes (i) a clean three-dimensional (3D) free electron gas with a spherical Fermi surface; (ii) a Debye description of the acoustic phonons; (iii) the scalar deformation potential, expected to be dominant at long-wavelength phonons and (iv) the dimensions of the metal are much longer than the average phonon wavelength (3D phonon spectrum). In this model, the average e-ph scattering rate appropriate for these assumptions is given by\textsuperscript{66,67}:

$$\tau_{\text{e-ph}}^{-1} = [6\zeta(3)(k_B T)^3/(2\pi^2 m^3 v_F^3)\exp(\hbar k_F a_s/\Omega)]/[2\pi \rho_m \hbar^2 u_s^2 v_F],$$

where $\hbar$ is the reduced Planck’s constant, $\rho_m$ the mass density, $\Omega$ the metal volume, $u_s$ the sound speed, $v_F$ the Fermi velocity, and $k_F = 3n/(2\Omega)$ the Fermi energy. In disordered metals, when the relation between the phonon wavelength $\lambda_{\text{ph}}$ and the electron mean free path $l$ is given by $\lambda_{\text{ph}} \gg l$, the theory considers the following modifications of the rate $\tau_{\text{e-ph}}^{-1} \propto T^4$\textsuperscript{9,64,65} and $\tau_{\text{e-ph}}^{-1} \propto T^2 l^{-100}$, depending on the dominant phonon polarization (longitudinal or transverse ones) and type of disorder (vibrating or static types). One should note that numerous studies of the inelastic scattering with magnetooconductance reveal that there does not exist a universal temperature behavior of $\tau_{\text{e-ph}}^{-1}$ in disordered conductors\textsuperscript{13}. In particular, the value of the temperature exponent $p$ might be quite sensitive to the microscopic quality and the intrinsic material properties such as characteristic of the Fermi surface\textsuperscript{68,69}, the dimensionality of the electron and phonon systems\textsuperscript{70}, nontrivial phonon dispersion\textsuperscript{71}, etc.

Study of the inelastic scattering with magnetooconductance reveal a change of $p$ and the magnitude of $\tau_{\text{e-ph}}^{-1}$ with increasing disorder in NbN films. Note that the spread of the estimated values of the Fermi energy (see Table I) is within 10%, so in the further analysis, we assume that the change in the electronic parameters makes a negligible contribution to the change in the e-ph scattering. On the other hand, one would expect that $\tau_{\text{e-ph}}^{-1}$ strongly depends on the phonon properties, which, in samples with reduced dimensions, may differ from the Debye spectrum, accepted in the models. Previous studies of the response of thin NbN films to modulated terahertz radiation revealed a $T^1.6$-dependence of the e-ph relaxation rate\textsuperscript{47}, which has been explained by the renormalization of phonon spectrum in thin films. One should note that these measurements have been carried out at low temperatures than in our study ($T < 5 \text{ K}$), at which two-dimensional phonons can actually affect the scattering. In this work, considering the 3D phonon spectrum, we describe data in Figure 3 by using the Sergeev-Mitin model\textsuperscript{10} and taking lower values of the sound velocity in comparison to bulk (see Appendix D for details). This is in line with the qualitative picture: the smaller the phonon velocity, the smaller the interaction region and the shorter the interaction time. Reduction in the sound velocity in NbN films with disorder can be an outcome of softening phonon modes in ultrathin films due to loss of crystal symmetry, granularity and/or weakening of ion bonds on films surface\textsuperscript{43,72,73}. The effect of a large softening of transverse phonon modes is also expected for amorphous solids compared to crystalline material\textsuperscript{74,75}. Typically, NbN films have polycrystalline structure\textsuperscript{76}, which can be considered as a composite of crystalline grains separated by thin amorphous boundaries\textsuperscript{75}. In our study, we observe a significant increase of the sheet resistance with change of deposition conditions, which may indicate a change in morphology from crystalline to more amorphous phase (see also discussion about the film morphology in Appendix B). Such change in the film morphology can potentially lead to phonon softening in strongly disordered NbN samples.

IV. DISCUSSION

In this paper, we investigate the evolution of electronic parameters and inelastic scattering rates with increasing disorder in ultrathin NbN films. We find a clear increase trend of the e-ph scattering rate as the normal-state resistance grows. Our experimental results in Figure 3 reveal that there is no a universal temperature behavior of $\tau_{\text{e-ph}}^{-1}$ for ultrathin disordered NbN films. We suppose that this effect strongly depends on microscopic quality of NbN films, and even a variation of one deposition parameter leads to significant changes in the properties of NbN films.

One can expect an alternative mechanism of enhancement of the inelastic scattering rate for disordered materials, in which electron transport is close to the superconductor-insulator transition\textsuperscript{77}. In this theory, the e-ph energy exchange is controlled by the correlation function of the local density of electronic states. As shown in\textsuperscript{31}, spatial inhomogeneities of superconducting properties of strongly disordered NbN films (with electronic parameters close to sample s6 in our study) correlate with a depletion in the local density of states around the Fermi level, induced by strong electronic interactions, which in turn are interpreted as local variations of the film resistivity. Meanwhile, the way these inhomogeneities can affect the e-ph scattering in the normal state of NbN films needs further theoretical and empirical study.

One should note that, on the one hand, the observed strong sensitivity of $\tau_{\text{e-ph}}^{-1}$ to disorder in NbN films can be beneficial for practical devices. In detectors, the energy relaxation time due to electron-phonon scattering $\tau_{\text{E-PH}}^{-1}$ differ from $\tau_{\text{e-ph}}^{-1}$ by a numerical coefficient $\tau_{\text{E-PH}}^{-1} = \alpha \tau_{\text{e-ph}}^{-1}$, where $\alpha \approx 0.6 - 0.1$ for $p = 2 - 4$, respectively\textsuperscript{78}. It is worth noting that the e-ph scat-
tering time found at \( T_c \) increases as \( \tau_{e-ph}(T_c) \sim T_c^{-1} \) with a weaker temperature dependence than one would expect for \( \tau_{e-ph}(T) \propto T^{-(3\sqrt{3}-4)} \) from the magnetoconductance measurements. These observations show that NbN films with a higher level of disorder, lower \( T_c \) values and stronger e-ph relaxation can be prospective material for increase of sensitivity and spectral characteristics of HEBs devices\(^7\) as well as for boosting the efficiency of single photon detection of SNSPDs. In the latter case, the enhanced inelastic scattering and the electron diffusivity both affect the hotspot size and the superconducting gap suppression\(^79\). This expectation correlates with a recent breakthrough in detection efficiency in SNSPDs, when one uses NbN films with high normal-state resistance values\(^{16,80,81}\). On the other hand, one should keep in mind that the operation of the detectors is also limited by the phonon escape rate, which is proportional to the sound velocity. As shown in Appendix D, the analysis of \( \tau_{e-ph}^{-1}(T) \)-dependences indicates a decrease in the sound velocity with increasing disorder, which in turn can affect the phonon escape rate and limit the device response. Thus, further experimental study of the features of phonon transport in ultrathin disordered metals is advisable.

In summary, we observe strong sensitivity of the inelastic scattering rates to disorder in ultrathin NbN films. Our results show that there is no universal temperature behavior of the electron-phonon scattering rate in NbN films with different level of disorder. Qualitative analysis of the experimental data, based on the existing models of the e-ph scattering in disordered metals, indicates possible phonon softening in strongly disordered samples. Our results call for further theoretical and experimental studies of the e-ph scattering in the presence of strong disorder.

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APPENDIX A: VARIATION OF \( T_c \) IN ULTRATHIN NBN FILMS

Here we discuss the correlation between \( T_c \) and \( R_{s}^{300K} \), shown in Figure 4. The suppression of \( T_c \) with the increase of \( R_{s}^{300K} \) can be related to two different, but not mutually exclusive effects. The first effect results from a decrease of electronic screening, which enhances e-e repulsion and partially cancels the e-ph mediated attractive interaction. This fermionic effect leads to a mechanism described in Ref.\(^82\). The second effect comes from the decrease in superfluid density\(^83\). However, the experimental values of \( n \) at the normal state indicate that the latter mechanism is less important for the films under study. As shown in Figure 4, the experimental values of \( T_c \) follow predictions of the Finkelstein’s theory\(^82\):

\[
\frac{T_c}{T_{c0}} = e^{\gamma} \left( \frac{1 + \sqrt{g/2} + g/4}{1 + \sqrt{g/2} + g/4} \right)^{1/\sqrt{2g}},
\]

where \( g = e^2 R_{s}^{300K} / (2\pi^2 \hbar) \) and \( \gamma = \ln[h/(k_B T_{c0} \tau)] \), where \( T_{c0} \) is the critical temperature in bulk material. It is instructive to note that this description of suppression of \( T_c \) relates to 2D diffusive nature of electron motion, which relies on the following hierarchy of length scales \( l \ll a \ll \xi \). In this paradigm, the enhancement of disorder is related to increase of the normal-state resistance, which is mainly determined by elastic scattering, \( R_s \propto (k_F l)^{-3}(k_F d)^{-184} \). The experimental data is in good agreement with Eq. 3 at the fitting parameters \( \tau \approx 0.35 \text{ fs}, g \approx 7.29 \) and \( T_{c0} = 15 \text{ K} \), see the black dashed line. These fitting parameters are close to previously reported values for NbN films\(^{86,29,32}\). The dependence of \( T_c \) and \( R_{s}^{300K} \) can be described more accurately if we use the experimental values of \( \tau \), which correspond to increase of \( g \) from 6.6 to 7.8. The value of \( g > 1 \) indicates that NbN undergo comparable strong disorder and strong electronic interactions.

APPENDIX B: ELECTRON TRANSPORT

We calculate all electronic parameters at the highest temperature of our measurements (300 K), where the effect of e-e interactions is expected to be small\(^65\). Firstly, to estimate the Hall coefficient \( R_{H}^{300K} \) we consider the expression \( (R_{H}^{25K} - R_{H}^{300K}) / R_{H}^{300K} = \gamma (R_{s}^{25K} - R_{s}^{300K}) / R_{s}^{300K} \), where \( \gamma = 0.68 \) is an empirical parameter\(^85\). Using the value of \( R_{H}^{300K} \) we derive the carrier density \( n = -B / (\pi e R_{H}^{300K}) \), where \( e \) is the electron charge. It is also important to note that samples s1-s5 are characterized by a moderate change (< 20%) in the carrier density \( n \) at increase of \( R_{s}^{300K} \) in two times. Meanwhile, the most disordered samples (s6 and s7) are characterized by a twofold decrease of \( n \) at three- and fourfold increase of \( R_{s}^{300K} \). The latter change in \( n \) can be a result of the increase of Nb and N vacancies due to the increase of the nitrogen partial pressure in the mixture during deposition of NbN\(^15\).

To characterize electron transport, we estimate the mean free path \( l \), the elastic scattering time \( \tau \) and the single-spin density of states (DOs) at the Fermi level \( N_0 \)
assumining the free electron model and using the following expressions: \( l = \frac{h k_F}{(e^2 n R_{300K})}, \tau = l^2 / 3D, \) and \( N_0 = \frac{1}{(2R_{300K} d e^2 D)}, \)

where \( k_F = (3\pi^2 n)^{1/3} \) is the Fermi wave vector. We believe that this \( \text{ab-initio} \) estimates are well justified, since \( l \approx 1 - 3.5 \AA \), which is in order of the lattice constant in our disordered NbN films, thereby a possible Fermi surface anisotropy can be negligible. The electronic transport is also characterized by the Ioffe-Regel parameter \( k_F l \), which is a common indicator of the disorder in homogeneously disordered material. All estimated parameters are listed in Table I. In addition, Figure 5(a) shows the relationship between the experimental values of \( l \) and \( D \), which can be approximated as \( l = 3D / v_F \), where \( v_F \approx (5.3 \pm 0.3) \times 10^5 \text{m/s} \) is the Fermi velocity determined as a fitting parameter here. The estimate of \( v_F \) is two times lower than the one previously reported, and it corresponds to the effective mass of charge carriers \( m_{eff} = h k_F / v_F \), which decreases from \( 4.1 m_e \) to \( 3.3 m_e \) as \( R_{300K} \) increases (here \( m_e \) is the mass of a free electron). Note that taking into account these values of \( m_{eff} \) allows to correct underestimation of \( n \) determined from optical measurements in NbN.

In an attempt to understand electron transport in NbN films better, we performed a simple analysis of resistivity in terms of the grain boundary model. In general, resistivity of granular films can be qualititatively given in the following way: \( \rho_{eff} = (3\pi^2)^{1/3} h / (n_0^{2/3} e^2 l_0) T^{-l_0 / D}, \)

where \( l_0 \) is the mean free path in absence of granularity, \( D \) is the mean grain size, \( T \) is the transmission probability of electrons through the grain boundary, \( n_0 \) is the carrier density of bulk material without crystal boundaries. Resistance here is increased since the effective mean free path is reduced due to the reflection on the grain boundaries: \( l_{eff} = l_0 T^{l_0 / D}. \)

Figure 5(b-d) demonstrates an attempt to evaluate the effect of granularity on resistivity of the studied NbN films. In Figure 5(b-c) we compare the values of resistivity and the mean free path deduced from transport measurements with \( \rho_{eff} \) and \( l_{eff} \) as a function of disorder \( (k_F l) \). Taking into account \( n_0 \approx 2.29 \times 10^{23} \text{cm}^{-3} \) and \( l_0 \approx 1.1 \text{nm} \) reported for crystalline NbN films, we estimated \( l_{eff} \), which is a function of the coupled parameters \( T \) and \( D \). In contrast to thick NbN films, which exhibit a pronounced polycrystalline structure, ultrathin NbN films can represent a quasiamorphous matrix with the expected grain size smaller than the film thickness but larger than the characteristic unit cell. Taking into account the fixed value \( T \approx 0.15^{17} \), we estimate the effective size of a crystalline grain \( D \) in studied NhN samples (Figure 5(d)), which falls within the range \( a < D < d \), where \( d = 2.5 \text{nm} \) is the film thickness and \( a \approx 0.44 \text{nm} \) is the lattice constant.

As revealed in AFM studies (see Supplemental Material) ultrathin NbN films grown at different substrate temperatures are characterized by different grain packing density: NbN films deposited at high \( T_{dep} \) typically have a more compact and dense microstructure than films deposited at lower \( T_{dep} \). In case of films deposited at different nitrogen partial pressures, this difference is not so obvious. Meanwhile, one can expect a decrease in the effective grain size for NbN films deposited at lower \( T_{dep} \), as well as under excessive nitrogen partial pressure.

Thus, increase of disorder in NbN films may reflect change of microstructure due to emergence of some granular structure as well as an increase in the number of point scatterers, such as vacancies at lattice sites.

### APPENDIX C: MAGNETOCONDUCTANCE

To fit the magnetoconductance data at low temperatures, we appeal the theory of superconducting fluctuations. The dimensionless change in magnetoconductance is given by expression:

\[ \delta G^{SC} = G^{SC}(B,T) - G^{SC}(0,T), \]

where \( G^{SC}(B,T) \) and \( G^{SC}(0,T) \) are a sum of quantum corrections to conductivity at finite and zero magnetic fields. Dominant contributions to magnetoconductance for 2D film are given by the Aslamazov-Larkin (AL) term, the Maki-Thompson regular (MT\(_{reg}\)) and anomaly (MT\(_{an}\)) terms and the term due to suppression in the electronic density of states (DOS)\(^{50,51}\). Note that contribution of the weak localization\(^{53}\) and the term due to renormalization of the one-electron diffusion coefficient\(^{51}\) are relatively small (see Supplemental Material\(^{44}\)), and we treat them as negligible here. The sum of three terms in finite and zero magnetic fields are given by expressions:
In (b) the dependence of the effective scattering rate as function of disorder. (d) The dependence of the effective \( v \) where these expressions are valid is limited with the condition \( 4D \tau \ll k_B T \ln(T/T_c) \), which is satisfied in our measurements.

**APPENDIX D: 3D ELECTRON-PHONON SCATTERING RATE**

In search of potential explanation for the observed \( \tau_0^{-1} \) behavior, we turn to the Sergeev-Mitin (SM) model\(^{10}\), which provides the e-ph scattering time considering the complex impact of strong elastic scattering on the three-dimensional (3D) transverse or longitudinal phonon modes. In disordered metals the e-ph scattering is non-local with a characteristic size of the interaction region about the phonon wavelength \( \lambda_T \). In the diffusive limit, when \( l \ll \lambda_T \), the SM theory predicts for \( \tau_{e-ph}^{-1} \) a complex functional dependence on temperature and disorder, which can be described by the following equation:

\[
\tau_{e-ph}^{-1} = f(T^4, l) + f(T^2, l^{-1}),
\]

where the first term is accounted in

\[
f(T^4) \cdot \tau_{e-ph}^{-1} = \frac{\pi^4}{5} \left( \frac{k_B T}{\hbar^2 p_F^2} \right)^2 \left[ \frac{\beta_l}{u_l} + \kappa - \frac{2\beta_l}{u_l} \right] (1 - \kappa),
\]

and the second term is

\[
f(T^2) : \tau_{e-ph}^{-1} = \frac{3\pi^2}{2} \left( \frac{k_B T}{\hbar^2 p_F^2} \right)^2 \left[ \frac{\beta_l}{u_l} + \kappa - \frac{2\beta_l}{u_l} \right] (1 - \kappa).
\]

Here \( \beta_l(u) = (2\varepsilon_F/3)^2 N_0/(2\rho_m u_{(l)}) \) represents the dimensionless e-ph coupling constant, \( p_F = \hbar k_F \) the Fermi momentum, \( \rho_m \) is the mass density, \( u_{(l)} \) denotes sound velocities for longitudinal and transverse phonon modes, the parameter \( \kappa \) characterizes the prevailing type of disorder, which gives the relative part of the vibrating disorder with respect to the total scattering potential. We compare our experimental data with the SM model using as the fitting parameters \( u_{(l)} \), \( \rho_m \), and \( \kappa \) (see the solid lines in Figure 3(b)). The sound velocity of longitudinal phonons is implemented as \( u_l = 2u_t \), the other parameters such as \( k_F \), \( n \), \( N_0 \), and \( l \) are taken from Table I. Because \( u_l \) for longitudinal phonons is usually larger than \( u_t \) for transverse phonons, only scattering of electrons with transverse phonons is important. The results of the fitting of the experimental data with the SM model (the best parameters are listed in Table II) indicate the complexity of the disorder in studied NbN films. To describe the change \( \tau_{e-ph}^{-1}(T) \) with increase of disorder, we adjust the parameter \( \kappa \) from 1.0 to 0.9, which may indicate a contribution of static defects in addition to standard vibrating ones. The decrease of \( \kappa \) also correlates with the potential decrease of the effective grain size with an increase of disorder (see Appendix B for discussion of the grain boundary model). The fitting values \( \rho_m \approx 5.0 \, g/cm^3 \) and \( u_t \approx 1.65 \times 10^3 \, m/s \) for the least-disordered sample (s1) are thereby smaller than the previously reported ones (\( \rho_m \approx 7.8 \, g/cm^3 \) and \( u_t \approx 2.4 \times 10^3 \, m/s \) for 5-nm NbN film) in Ref.\(^{38}\).

\[ G^{SC}(B, T) = \frac{\pi^2 \varepsilon}{4\hbar^2} \left[ \psi \left( \frac{1}{2} + \frac{\varepsilon}{2\hbar} \right) - \psi \left( 1 + \frac{\varepsilon}{2\hbar} \right) \right] - \frac{56\zeta(3)}{\pi^2} \ln \left( \frac{1}{2\hbar} \right) \left[ \psi \left( \frac{1}{2} + \frac{\varepsilon}{2\hbar} \right) - \psi \left( \frac{1}{2} + \frac{\varepsilon}{2\hbar} \right) \right] + \frac{\pi^2}{4(\varepsilon - \gamma_\phi)} \left[ \psi \left( \frac{1}{2} + \frac{\varepsilon}{2\hbar} \right) - \psi \left( \frac{1}{2} + \frac{\gamma_\phi}{2\hbar} \right) \right] \hspace{1cm} (4a) \]

\[ G^{SC}(0, T) = \frac{\pi^2}{8\varepsilon} + \frac{56\zeta(3)}{\pi^2} \ln(\varepsilon) + \frac{\pi^2}{4(\varepsilon - \gamma_\phi)} \ln(\varepsilon/\gamma_\phi). \hspace{1cm} (4b) \]

where \( \psi(x) \) is the Digamma function, \( h = 0.69 B/B_{c2}(0) \) and \( \varepsilon = \ln(T/T_c) \) are the reduced magnetic field and temperature, respectively, \( \gamma_\phi = \pi \hbar/(8k_B T \tau_0) \) the pair-breaking parameter and \( \tau_0 \) the phase-breaking scattering time. The range of magnetic fields and temperatures where these expressions are valid is limited with the conditions \( 4D \tau \ll k_B T \ln(T/T_c) \), which is satisfied in our measurements.
It worth noting that the SM theory describes the scattering on 3D phonons, for which the phonon wavelength $\lambda_p = h u_\nu/(2.82 k_B T)$ is expected to be smaller than the film thickness $d$ (here 2.82 is a constant given in the dominant phonon approximation). Taking into account the values of $u_\nu$ in Table II, the crossover $\lambda_p \approx d$ is expected at 11.4 K and 7.4 K for samples s1 and s6, respectively. Therefore, at higher temperatures, which corresponds to the most of T-range in this study, the e-ph scattering in ultrathin NbN films can be considered in 3D regime.

We also estimate the Debye temperature in the 3D Debye model as $\theta_D = \hbar \nu (4 \pi^2/3)^{1/3} u_\nu/(k_B a)$, where $u_\nu = [1/(2 u_\nu^3 + 1/\nu^3)]^{1/3}$ is the mean sound velocity, $a \approx 0.44$ nm. The estimated values of $\theta_D$, shown in Table II, are noticeably less than the typical values reported for similar thin NbN films ($\theta_D \sim 172 - 174$ K). It was found that both parameters $\rho_m$ and $u_\nu$, as well as the estimated Debye temperature, decrease with increasing disorder (see Table II), which is usually denoted as the phonon softening effect.

| TABLE II. Best-fit values of parameters in the SM model$^{10}$. |
|-----------------|----------|----------|----------|
| Sample | $\rho_m$ (g/cm$^3$) | $u_\nu$ (m/s) | $\kappa$ (K) |
| s1    | 5.0    | $1.67 \times 10^3$ | 1.0 | 127 |
| s2    | 5.0    | $1.50 \times 10^4$ | 1.0 | 114 |
| s3    | 4.0    | $1.40 \times 10^2$ | 0.98 | 106 |
| s4    | 4.0    | $1.40 \times 10^2$ | 0.97 | 106 |
| s5    | 4.0    | $1.33 \times 10^2$ | 0.95 | 101 |
| s6    | 3.5    | $1.1 \times 10^2$ | 0.90 | 84 |

1. B. Sacépé, M. Feigel’man, and T. M. Klapwijk, Nature Physics 16, 734 (2020).
2. A. D. Semenov, Superconductor Science and Technology 34, 054002 (2021).
3. A. Shurakov, Y. Lobanov, and G. Goltsman, Superconductor Science and Technology 29, 023001 (2015).
4. J. Zmuidzinas, Annual Review of Condensed Matter Physics 3, 169 (2012), https://doi.org/10.1146/annurev-conmatphys-020911-125022.
5. P. J. de Visser, S. A. H. de Rooij, V. Murugesan, D. J. Thoen, and J. J. A. Baselmans, Phys. Rev. Applied 16, 034051 (2021).
6. B. Altshuler and A. Aronov, in Electron–Electron Interactions in Disordered Systems (Elsevier, 1985) pp. 1–153.
7. A. Anthore, F. Pierre, H. Pothier, and D. Esteve, Phys. Rev. Lett. 90, 076806 (2003).
8. B. Huard, A. Anthore, N. O. Birge, H. Pothier, and D. Esteve, Phys. Rev. Lett. 95, 036802 (2005).
9. A. Schmid, Z. Physik 259, 421 (1973).
10. A. Sergeev and V. Mitin, Phys. Rev. B 61, 6041 (2000).
11. A. Shytk and M. Feigel’man, Phys. Rev. B 92, 195101 (2015).
12. M. G. Gershenson, D. Gong, T. Sato, B. S. Karasik, and A. V. Sergeev, Applied Physics Letters 79, 2049 (2001), https://doi.org/10.1063/1.1407302.
13. J. J. Lin and J. P. Bird, Journal of Physics: Condensed Matter 14, R501 (2002).
14. J. T. Karvonen, L. J. Taskinen, and I. J. Maasilita, Phys. Rev. B 72, 012302 (2005).
15. S. P. Chochalingam, M. Chand, J. Jesudasan, V. Tripathi, and P. Raychaudhuri, Phys. Rev. B 77, 214503 (2008).
16. P. I. Zolotov, A. V. Semenov, A. V. Divovich, G. N. Goltsman, N. R. Romanov, and T. M. Klapwijk, IEEE Transactions on Applied Superconductivity 31, 1 (2021).
17. G. N. Goltsman, O. Okunev, G. Chulkova, A. Lipatov, A. Semenov, K. Smirnov, B. Voronov, A. Dzardanov, C. Williams, and R. Sobolewski, Applied Physics Letters 79, 705 (2001), https://doi.org/10.1063/1.1388608.
18. A. Divovich, M. Misiaszek, Y. Vakhtomin, P. Morozov, K. Smirnov, P. Zolotov, and P. Kolenderski, Optics Letters 43, 6085 (2018).
19. I. Tretyakov, S. Ryabchun, M. Finkel, A. Maslennikova, N. Kaurowa, A. Lobastova, B. Voronov, and G. Goltsman, Applied Physics Letters 98, 033507 (2011), https://doi.org/10.1063/1.3544050.
20. A. J. Annunziata, D. F. Santavicca, L. Frunzio, G. Cateiani, M. J. Rooks, A. Frydman, and D. E. Prober, Nanotechnology 21, 445202 (2010).
21. D. Niepce, J. Burnett, and J. Bylander, Phys. Rev. Applied 11, 044014 (2019).
22. J. T. Peltonen, O. V. Astafiev, Y. P. Korneev, B. M. Voronov, A. A. Korneev, I. M. Chareva, A. V. Semenov, G. N. Gol’tsman, L. B. Ioffe, T. M. Klapwijk, and J. S. Tsai, Phys. Rev. B 88, 220506(R) (2013).
23. K. Y. Arutyunov, A. Ramos-Alvarez, A. V. Semenov, Y. P. Korneeva, P. P. An, A. A. Korneev, A. Murphy, A. Bezryadin, and G. N. Gol’tsman, Nanotechnology 27, 47LT02 (2016).
24. J. T. Peltonen, Z. H. Peng, Y. P. Korneeva, B. M. Voronov, A. A. Korneev, A. V. Semenov, G. N. Gol’tsman, J. S. Tsai, and O. V. Astafiev, Phys. Rev. B 94, 180508(R) (2016).
25. N. G. N. Constantino, M. S. Anwar, O. W. Kennedy, M. Dang, P. A. Warburton, and J. C. Fenton, Nanomaterials 8 (2018), 10.3390/nano8060442.
26. S. E. de Graaf, S. T. Skacel, T. Hönigl-Decinris, R. Shaikhfoldarov, H. Rotzinger, S. Linzen, M. Ziegler, U. Hübner, H.-G. Meyer, V. Antonov, E. Il’ichev, A. V. Ustinov, A. Y. Tsalenchuk, and O. V. Astafiev, Nature Physics 14, 590 (2018).
27. Y. P. Gousev, G. N. Goltsman, A. D. Semenov, E. M. Gershenson, R. S. Nebosis, M. A. Heusinger, and K. F. Renk, Journal of Applied Physics 75, 3695 (1994), https://doi.org/10.1063/1.356060.
28. M. Mondal, S. Kumar, M. Chand, A. Kamalpure, G. Saraswat, G. Seibold, L. Benfatto, and P. Raychaudhuri, Phys. Rev. Lett. 107, 217003 (2011).
29. M. Chand, G. Saraswat, A. Kamalpure, M. Mondal, S. Kumar, J. Jesudasan, V. Bagwe, L. Benfatto, V. Tripathi, and P. Raychaudhuri, Phys. Rev. B 85, 014508 (2012).
30. C. Delacour, B. Pannetier, J.-C. Villegier, and V. Bouchiat, Nano Letters 12, 3501 (2012), pMID: 22694480.
77 M. V. Feigel’man and V. E. Kravtsov, Phys. Rev. B 99, 125415 (2019).
78 K. S. Il’in, N. G. Ptitsina, A. V. Sergeev, G. N. Gol’tsman, E. M. Gershenzon, B. S. Karasik, E. V. Pechen, and S. I. Krasnosvobodtsev, Phys. Rev. B 57, 15623 (1998).
79 D. Y. Vodolazov, Phys. Rev. Applied 7, 034014 (2017).
80 M. Hofherr, D. Rall, K. Ilin, M. Siegel, A. Semenov, H.-W. Hbers, and N. A. Gippius, Journal of Applied Physics 108, 014507 (2010), https://doi.org/10.1063/1.3437043.
81 Y. P. Korneeva, N. N. Manova, M. A. Dryazgov, N. O. Simonov, P. I. Zolotov, and A. A. Korneev, Superconductor Science and Technology 34, 084001 (2021).
82 A. Finkel’stein, Physica B: Condensed Matter 197, 636 (1994).
83 T. M., Introduction to Superconductivity (McGraw-Hill, 1996).
84 D. S. Antonenko and M. Skvortsov, JEPT Letters 112, 428 (2020).
85 M. Chand, A. Mishra, Y. M. Xiong, A. Kamnapure, S. P. Chockalingam, J. Jesudasan, V. Bagwe, M. Mondal, P. W. Adams, V. Tripathi, and P. Raychaudhuri, Phys. Rev. B 80, 134514 (2009).
86 C. Kittel, Introduction to Solid State Physics, 8th ed. (Wiley, 2004).
87 A. Semenov, B. Günther, U. Böttger, H.-W. Hübbers, H. Bartolf, A. Engel, A. Schilling, K. Ilin, M. Siegel, R. Schneider, D. Gerthsen, and N. A. Gippius, Phys. Rev. B 80, 054510 (2009).
88 R. Yan, G. Khalsa, S. Vishwanath, Y. Han, J. Wright, S. Rouvimov, D. S. Katzer, N. Nepal, B. P. Downey, D. A. Muller, H. G. Xing, D. J. Meyer, and D. Jena, Nature 555, 183 (2018).
89 M. D. Soldatenkova, A. D. Triznova, E. M. Baeva, P. I. Zolotov, A. I. Lomakin, A. I. Kardakova, and G. N. Goltsman, Journal of Physics: Conference Series 2086, 012212 (2021).
90 G. Oya and Y. Onodera, Journal of Applied Physics 45, 1389 (1974), https://doi.org/10.1063/1.1663418.
91 K. R. Babu and G.-Y. Guo, Phys. Rev. B 99, 104508 (2019).
92 A. H. Farha, Investigation of NbNx Thin Films and Nanoparticles Grown by Pulsed Laser Deposition and Thermal Diffusion, Ph.D. thesis (2013).
93 R. Rosenbaum, Phys. Rev. B 32, 2190 (1985).
94 A. Leuchten, Phys. Rev. B 79, 212511 (2009).