The 3D MHD code GOEMHD3 for large-Reynolds-number astrophysical plasmas

Code description, validation and computational performance

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ABSTRACT

Context. The numerical simulation of turbulence and flows in almost ideal, large-Reynolds-number astrophysical plasmas motivates the implementation of almost conservative MHD computer codes. They should efficiently calculate, use highly parallelized schemes scaling well with large numbers of CPU cores, allows to obtain a high grid resolution over large simulation domains and which can easily be adapted to new computer architectures as well as to new initial and boundary conditions, allow modular extensions.

Aims. Implementation, optimization and validation of a fast, well parallelized, and easily extensible, almost conservative MHD simulation code for the numerical investigation of the dynamics of large-Reynolds-number astrophysical plasmas in three dimensions (3D).

Methods. The new GOEMHD3 code discretizes the ideal MHD equations using a fast and efficient dissipationless Leap-Frog scheme which is second-order accurate in space and time and whose initial and boundary conditions can, therefore, easily be modified. GOEMHD3 is parallelized based on the hybrid MPI-OpenMP programming paradigm, adopting a standard, two-dimensional approach.

Results. The ideal part of the equation solver is validated by performing numerical tests of the evolution of the well understood Kelvin-Helmholtz instability and of Orszag-Tang vortices. Further it is shown that the computational performance of the code scales very efficiently with the number of processors up to tens of thousands of CPU cores. This excellent scalability of the code was obtained by simulating the 3D evolution of the solar corona above an active region (NOAA AR1249) for which GOEMHD3 revealed the energy distribution in the solar atmosphere in response to the energy influx from the chromosphere through the transition region, taking into account the weak Joule current dissipation and viscosity in the almost dissipationless solar corona.

Conclusions. The new massively parallel simulation code GOEMHD3 enables efficient and fast simulations of almost ideal, large-Reynolds-number astrophysical plasma flows, well resolved and on huge grids covering large domains. Its abilities are validated by major tests of ideal and weakly dissipative plasma phenomena. The high resolution (2048³ grid points) simulation of a large part of the solar corona above an observed active region proved the excellent parallel scalability of the code using more than 30.000 processor cores.

Key words. magnetohydrodynamics MHD – Sun: corona – Sun: magnetic fields

1. Introduction

For most astrophysical plasmas viscosity and current dissipation (resistivity) are negligibly small, i.e., astrophysical plasmas are nearly ideal, almost dissipationless and hence, for relevant processes and scales, the characteristic Reynolds and Lundquist numbers are very large. This requires specific approaches to correctly take into account turbulence and different kinds of ideal and non-ideal interactions in the plasma flows like, e.g., shock waves, dynamo action and magnetic reconnection (Birn & Priest 2007). Fortunately, improvements in computer technology as well as the development of efficient algorithms allow increasingly realistic numerical simulations of the underlying space plasma processes (Büchner et al. 2003). For the proper numerical description of nearly dissipationless astrophysical plasmas, e.g., of magnetic reconnection (Büchner 2007a) and dynamo action one needs to utilize almost conservative schemes for MHD as well as kinetic plasma descriptions (Elkina & Büchner 2006). The schemes should be as simple as possible in order to run quickly and efficiently. Moreover, in order to ensure flexibility concerning the particular physics problem under consideration they should allow an easy modification of initial and boundary conditions as well as the simple addition and adjustment of physics modules. For this sake, e.g., first the serial second order accurate MHD simulation code LINMOD3D was developed. It was successfully applied to study the magnetic coupling between the solar photosphere and corona based on multi-wavelength observations (Büchner et al. 2004a), to investigate the heating of the transition region of the solar atmosphere (Büchner et al. 2004b), and the acceleration of the fast solar wind by magnetic reconnection (Büchner & Nikutowski 2005a). It was also used to physically consistently describe the evolution of the solar chromospheric and coronal magnetic fields (Büchner & Nikutowski 2005b) and for comparing solar reconnection with spacecraft telescope observations (Büchner 2007b), the electric currents around EUV bright points (Santos et al. 2008), the role of magnetic null points in the solar corona (Santos et al. 2011b) and the triggering of flare eruptions (Santos et al. 2011a). Other typical
ical applications of LINMOD3D were the investigation of the relative importance of compressional heating and current dissipation for the formation of coronal X-ray bright points (Javadi et al. 2011) and of the role of the helicity evolution for the dynamics of active regions (Yang et al. 2013). For the investigation of stronger magnetic field gradients in larger regions of the solar atmosphere, however, an enhanced spatial resolution is required. To a certain degree this was possible using the OpenMP parallelized code MPSCORONA3D which can be run on large shared memory parallel computing resources, e.g. for the investigation of the influence of the resistivity model on the solar coronal heating (Adamson et al. 2013).

For the simulation of further challenging problems, like the development and feedback of turbulence, for high resolution simulations of large spatial domains, for the investigation of turbulent astrophysical plasmas with very large Reynolds numbers, for the consideration of subgrid-scale turbulence for large scale plasma phenomena, one needs to be able to utilize, however, a much larger number of CPU cores than shared memory systems can provide. Hence, MPI-parallelized MHD codes like e.g. ATHENA [1], BATS-R-US [2], BIFROST, ENZO [3] or PENCIL [4] have to be used which run on distributed memory computers. PENCIL is a sixth-order spatial and third-order in time accurate code. It uses central space derivatives and a Runge-Kutta time scheme. ENZO is a hybrid (MHD + N-body) code with adaptive mesh refinement which uses a third-order piecewise parabolic method (Colella & Woodward 1984) with a two-shock approximate Riemann solver. ATHENA allows static mesh refinement, implementing a higher order scheme and utilizing a Godunov method on several different grid geometries (Cartesian, cylindrical). It employs third-order cell reconstructions and a Roe solver, Riemann solvers as well as a split corner-transport upwind scheme (Colella 1990, Stone et al. 2008), with a constrained-transport method [Evans & Hawley 1988, Stone & Gardiner 2009]. BIFROST is a code which is sixth-order accurate in space and third-order accurate in time (Gudiksen et al. 2011). BATS-R-US solves the 3D MHD equations in finite-volume form using numerical methods related to Roe’s approximate Riemann Solver. It uses an adaptive grid composed of rectangular blocks arranged in varying degrees of spatial refinement levels.

Note that all these codes are of an accuracy higher than second order. As a result every time step is numerically expensive and changes or modifications, e.g. of initial and boundary conditions require quite some effort. Meanwhile second-order-accurate schemes implement simpler numerics and efficient solvers which are easier to modify inside the code and in their initial and boundary conditions. On modern computer architectures the desired numerical accuracy can rather easily and numerically cheaply be achieved by enhancing the grid resolution. This served as the motivation for our new GOEMHD3 code to be based on a simple second-order-accurate scheme which is relatively straightforward to implement and to parallelize, and which facilitates modification and extension. GOEMHD3 runs quickly and efficiently on different distributed-memory computers from standard PC clusters to high-performance-computing (HPC) systems like the “Hydra” Cluster of the Max-Planck-Society at the Computing Center (RZG) in Garching, Germany. In order to demonstrate the reach and limits of the code, GOEMHD3 was tested on standard problems as well as by simulating the response of the strongly height-stratified solar atmosphere based on photospheric observations using a large number of CPU cores. In section 2 the basic equations solved by the code are described (2.1), together with their discretization and numerical implementation (2.2). In section 3 the hybrid MPI-OpenMP parallelization of GOEMHD3 is described. The performance of the code was tested with respect to different ideal and non-ideal plasma processes (Sect. 3). All tests are carried out using the same three-dimensional code. For quasi-2D simulations the number of grid points in the invariant direction is reduced to four, the minimum value required by the discretization scheme. Section 3.1 presents a test of the hydrodynamic part of the code by simulating the well-posed problem of a Kelvin-Helmholtz velocity shear instability using the methodology developed by McNally et al. (2012) as it was also applied to test the higher-order codes like PENCIL, ATHENA and ENZO. In section 3.2 ideal MHD limit is tested by simulating vortices according to Orszag & Tang (1979). In the past Kelvin-Helmholtz instability and Orszag-Tang vortex tests have been used also to verify total variation diminishing schemes (Ryu et al. 1995). The possibility of numerical oscillations due to the finite difference discretization was investigated as in Wu (2007). In order to validate the explicit consideration of dissipative processes by GOEMHD3 a current decay test was performed suppressing others terms in the equations (3.3). Section 4 reports the parallel performance of GOEMHD3 by a simulation of the solar coronal evolution in response to changing conditions at the lower boundary of the simulation according to the photospheric plasma and magnetic field evolution. The results are summarized and conclusions for the use of GOEMHD3 are drawn in section 5.

2. Basic equations and numerical implementation

2.1. Resistive MHD equations

For a compressible, isotropic plasma the resistive MHD equations in dimensionless form read

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = \rho \nabla \cdot \mathbf{B} = \chi \nabla \cdot \mathbf{B} \] (1)

\[ \frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{uu} + \frac{1}{2}(\rho + B^2)\mathbf{I} - BB) = -\nabla(p(u - u_0)) + \chi \nabla \cdot \rho \mathbf{u} \] (2)

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - (\nabla \eta) \times \mathbf{j} + \eta \nabla \cdot \mathbf{B} \] (3)

\[ \frac{\partial h}{\partial t} + \nabla \cdot \mathbf{hu} = (\gamma - 1) \rho \nabla \cdot \mathbf{u} + \chi \nabla \cdot \mathbf{B} \] (4)

where the symbols \( \rho, \mathbf{u}, h, \) and \( \mathbf{B} \) denote the primary variables, density, velocity and specific entropy of the plasma, and the magnetic field, respectively. The symbol \( \mathbf{I} \) is the \( 3 \times 3 \) identity matrix. The resistivity of the plasma is given by \( \eta \) and the collision coefficient \( \gamma \) accounts for the coupling of the plasma to a neutral gas moved around with a velocity \( u_0 \). The system of equations is closed by an equation of state. The entropy \( h \) is expressed via the scalar pressure \( p = 2h \rho \). Using the entropy as a variable instead of the internal energy (here adiabatic conditions are assumed, i.e. a ratio of the specific heats \( \gamma = 5/3 \)) then eq. 4 shows that in contrast to the internal energy the entropy is conserved in the absence of Joule and viscous heating. Ampère’s law \( \nabla \times \mathbf{B} \) allows to eliminate the current density \( \mathbf{j} \).
The variables are rendered dimensionless by choosing typical values for a length scale $L_0$, a normalizing density $\rho_0$ and a magnetic field strength $B_0$. For the normalization of the remaining variables and parameters the following definitions are used: $p_0 = \frac{\rho_0^2}{2m}$ for a typical (magnetic) pressure, $\rho_0 = \frac{\rho_0}{\rho_0}$ for a typical (Alfvén) velocity, and $\tau_0$ for the Alfvén crossing time over a distance $L_0$, i.e. $\tau_0 = \frac{L_0}{c_\text{A}}$. The current density is normalized by $J_0 = \frac{B_0}{\mu_0}$, the resistivity by $\eta_0 = \mu_0L_0\eta_0$, and the energy by $E_0 = \frac{B_0^2L_0^2}{\mu_0}$. For simulations of the solar atmosphere typical numerical values are $L_0 = 5000\text{ km}, p_0 = 2 \times 10^{15} \text{ m}^{-3}$ and $B_0 = 10^{-3} \text{ T}$, which yields $p_0 = 0.7958 \text{ Pa}, \rho_0 = 487.7 \text{ km} \cdot \text{s}^{-1}$, $\tau_0 = 10.25 \text{ s}, J_0 = 1.59 \times 10^4 \text{ A} \cdot \text{m}^{-2}$, $\eta_0 = 3.06 \times 10^6 \text{\Omega} \cdot \text{m}$ and $W_0 = 1.99 \times 10^{13} \text{ J}$ for the normalizing energy.

### 2.2. Numerical implementation

The resistive MHD equations (Eqs. 1–4) are discretized on a three-dimensional Cartesian grid employing a combination of a time-explicit Leap-Frog, a Lax, and a DuFort-Frankel finite difference schemes (see Press et al. 2007). For the homogeneous part of the MHD equations a second-order accurate Leap-Frog scheme

$$\psi_i^{n+1} - \psi_i^{n-1} = \frac{\psi_i^{n+1} - \psi_i^{n-1}}{\Delta t}$$

is adopted. A first-order Lax method is used to start the integration from initial conditions, i.e. to compute $\psi^n$ from the given initial values $\psi^{n-1}$, or upon a change of the time step $\Delta t$ (see below).

The Leap-Frog scheme is free from numerical dissipation. It allows to simulate magneto-fluids with very high Reynolds number (e.g. $R_\nu \sim 10^{10}$) which is essential for simulating astrophysical plasma in which the electric resistivity is very small. If necessary numerical oscillations are suppressed by damping controlled by a switching on viscosity which also prevents a steepening of gradients beyond the grid resolution. It also prevents mesh drift instabilities of the staggered Leap-Frog scheme which are due to the fact that odd and even mesh points are decoupled (see, e.g. Press et al. 2007 and Yee 1966). This is done via the diffusion terms \( w_i \nabla^2 \psi_i \) and \( \nabla \cdot \psi_i \) in the right hand sides of equations 1, 2, and 4. In order to maintain second-order accuracy also for the dissipative terms, these viscosity terms are discretized according to a DuFort-Frankel scheme as is the diffusion term in the induction equation \( \psi \).

$$\psi_i^{n+1} = \psi_i^{n-1} + 2\Delta t \left[ w_1 \psi_i^{n-1} + w_2 \psi_i^{n+1} + \frac{1}{2} w_2 \left( \psi_i^{n+1} + \psi_i^{n-1} \right) \right]$$

Here, \( w_1 = \frac{\Delta x}{\Delta x + \Delta y} \), \( w_2 = \frac{\Delta y}{\Delta x + \Delta y} \), and \( w_3 = \frac{\Delta z}{\Delta x + \Delta y + \Delta z} \) are the coefficients necessary to calculate the second order derivatives on the non-equidistant mesh, used. The left derivative is denoted by $\Delta x_i = x_i - x_{i-1}$, right $\Delta x_i = x_{i+1} - x_i$ and total $\Delta x = \Delta x + \Delta x_r$. Combining the Leap-Frog (Eq. 5) and the DuFort-Frankel (Eq. 6) discretization schemes one obtains

$$\psi_i^{n+1} = \psi_i^{n-1} + \lambda \left[ S \cdot H_i - d_x(F_{i+1}) \right]$$

with the fluxes $F_i$ and source terms $S_i$:

$$F_i = \left\{ \begin{array}{ll} \rho u_i - BB + \frac{1}{2}(p_i + B_i^2) & \varepsilon_{3,3} \cdot E \\ \varepsilon_{3,3} \cdot E & \end{array} \right\}, \quad S_i = \left\{ \begin{array}{ll} \frac{0}{\varepsilon_{3,3}} & \varepsilon_{3,3} \cdot E \\ \frac{-\chi \nabla \cdot (u_i - u_0)}{\eta_0} & \frac{\eta_0}{\eta_0} \\ \frac{\chi \nabla \cdot (u_i - u_0)}{\eta^2} & \frac{\eta_0}{\eta_0} \end{array} \right\}.$$
a necessary Lax integration step too frequently and hence compromising the overall second-order accuracy of the Leap-Frog scheme. In order to prevent an unlimited decrease of the time step, limiting values like, e.g., at least 10% and 1% of the initial values of the density and the entropy $h$, respectively, and $u < 3u_4$ are enforced. The values at the corresponding grid points are reset to the corresponding cut-off value and the values at the surrounding grid points are smoothed by averaging over the neighboring grid points.

**Divergence cleaning** Due to discretization errors unphysical finite divergences of the magnetic field may arise. In order to remove such finite values of $\nabla \mathbf{B}$ the following cleaning method is applied which solves a Poisson equation for the magnetic potential $\phi$:

$$\Delta \phi = \nabla \cdot \mathbf{B}$$

(11)

$$\mathbf{B} = \mathbf{B} - \nabla \phi$$

(12)

where $\mathbf{B}$ is the magnetic field with a finite divergence and $\mathbf{B}$ is the cleaned magnetic field. With central differences $d_x = 1/(x_{i+1} - x_{i-1})$, and alike for the other coordinate direction which are suppressed here for brevity, the Poisson equation Eq. [11] is discretized as

$$d_x(d_{x-1} + d_{x+1})\phi_{i+1}^k = d_x(B_{x+1} - B_{x-1}) - d_x(d_{x-1}\phi_{i-2}^k + d_x\phi_{i+2}^k)$$

(13)

and solved with a simple fix-point iteration where $k$ denotes the iteration step. For faster convergence a standard relaxation method is utilized,

$$\phi_{i+1}^k = \xi \phi_{i}^{k+1} + (1 - \xi) \phi_i^k$$

(14)

where the relaxation coefficient $\xi$ depends on the iteration $k$ as

$$\xi = \frac{1}{4} \left( \tanh \left( \frac{16k}{k_{\text{max}}} - 2 \right) + 1 \right) + \frac{1}{2}$$

(15)

**2.3. Hybrid MPI-OpenMP parallelization**

The time-explicit discretization scheme described above can be straightforwardly parallelized using a domain decomposition approach and introducing halo regions ("ghost zones") of width 1, corresponding to an effective stencil length of 3 in each of the coordinate directions. Accordingly, only next-neighbor communication and a single global reduction operation (for computing the time step, cf. Eq.[10]) are necessary for exchanging data between the domains. To be specific, GOEMHD3 employs a two-dimensional domain decomposition in the $y-z$ plane with width-1 halo exchange, using the Message Passing Interface (MPI). Within the individual, "pencil"-shaped domains, a shared-memory parallelization is implemented using OpenMP. The hybrid MPI-OpenMP approach firstly integrates smoothly with the existing structure of the serial code and secondly, thanks to a very efficient OpenMP parallelization within the domains, allows utilizing a sufficiently large number of processor cores, given typical sizes of the numerical grid ranging between 256$^3$ and 2048$^3$ points. In addition, the hybrid parallelization helps to maximize the size (i.e. volume in physical space) of the individual MPI domains, and hence to minimize the surface-to-volume ratio. The latter translates into a smaller communication-to-computation ratio and hence relatively smaller communication times, and the former accounts for larger MPI messages and hence decreases communication overhead (latency). Our parallelization assumes the individual MPI domains to be of equal size (but not necessarily with a quadratic cross section in the $y-z$ plane). This greatly facilitates the technical handling of the extrapolations required by so-called line-symmetric side-boundary conditions (Otto et al. 2007) which are often employed in realistic solar corona simulations. As a side effect, this restriction a priori avoids load-imbalance due to an otherwise non-uniform distribution of the processor workload.

Overall, as shall be demonstrated below (cf. Sect. 4.3), GOEMHD3 achieves very good parallel efficiency over a wide range of processor counts and sizes of the numerical grid, with the hybrid parallelization outperforming a plain MPI-based strategy at high core counts.

**3. Test problems**

In order to assess the stability, the convergence properties and the numerical accuracy of the new GOEMHD3 code, we simulate the standard test problems of the Kelvin-Helmholtz instability and the Orszag-Tang vortex, perform a test (Skála & Bárta 2012) of the resistive MHD properties of the code, and compare our results with numerical and analytical reference solutions. All tests are two-dimensional problems in the space coordinates. In order to perform such two-dimensional simulations with our three-dimensional code the $x$-direction is considered invariant and the numerical grid in this direction covers the minimum number of four points.

**3.1. Kelvin-Helmholtz instability**

The hydrodynamic limit of the GOEMHD3 code is validated by simulating the non-linear evolution of the Kelvin-Helmholtz instability (KHI) in two dimensions, which is a well-known standard test problem in numerical hydrodynamics. The initial instability induced by a velocity shear leads to the formation of large-scale vortices, whose size and growth rate can be followed as a function of time and be compared with (numerical) reference solutions. Here, we closely follow McNally et al. (2012), who have established a standard methodology for this numerical test and published fiducial reference solutions using several high-order schemes (such as PENCIL, ATHENA, ENZO) and high-resolution simulations (up to mesh sizes of 4096$^3$). The KHI test setup proposed by McNally et al. (2012) adopts smooth initial conditions (see also Robertson et al. 2010) in order to avoid the conceptual problems of a sharp contact discontinuity, yet has shown to be discriminative concerning the accuracy of different numerical schemes. The initial conditions (in the two space coordinates $0 < y < 1$ and $0 < z < 1$) read

$$\xi = \frac{\zeta_1 - \zeta_0 \exp \left( \frac{z - z_0}{z_1 - z_0} \right)}{\frac{1}{2} \left( \frac{1}{2} \right)^{\frac{1}{2}}} \text{ if } 1/4 > z \geq 0$$

$$\zeta_{\text{res}} \text{ if } 1/2 > z \geq 1/4$$

$$\zeta_{\text{res}} \text{ if } 3/4 > z \geq 1/2$$

$$\xi = \frac{\zeta_1 - \zeta_0 \exp \left( \frac{z - z_0}{z_1 - z_0} \right)}{\frac{1}{2} \left( \frac{1}{2} \right)^{\frac{1}{2}}} \text{ if } 1/4 > z \geq 0$$

$$\zeta_{\text{res}} \text{ if } 1/2 > z \geq 1/4$$

$$\zeta_{\text{res}} \text{ if } 3/4 > z \geq 1/2$$

(16)

where the symbol $\zeta$ denotes either density $\rho$ or velocity $u$, and $\zeta_{\text{res}} = (\zeta_1 + \zeta_2)/2$. The flow is perturbed by a small velocity deviation in z-direction, $u_z = 0.01 \sin(4\pi y)$. The initial pressure is uniform in space, $p = 5$.

We impose periodic boundary conditions and set $\chi = 2 \cdot 10^{-3}$ for the (artificial) diffusion coefficient. We compute the time evolutions of the spatial maximum of the kinetic energy density of motions along the $y$-direction, $\dot{E}_y = \frac{1}{2} \max_{x,z}(pu_{yz})$ as well
Fig. 1. Time evolution of the $y$-velocity mode amplitude, $A_y$ (top plot) and its growth rate, $\dot{A}_y$ (bottom plot) for the Kelvin-Helmholtz test problem as computed with GOEMHD3 at different numerical resolution (coloured lines). The black line corresponds to the solution obtained by the PENCIL code for a mesh resolution of $4096^2$ (McNally et al. 2012). These quantities can be compared with the reference solutions obtained by McNally et al. (2012), who employed the PENCIL code with a mesh resolution of $4096^2$ points. Figure 1 shows that for both, the amplitude $A_y$ of the instability (upper panel) and its growth rate $\dot{A}_y$ (lower panel), the GOEMHD3 results converge with increasing numerical resolution and exhibit a very good overall agreement with the reference solution. A closer look reveals that the initial evolution of $A_y$ closely resembles the reference solution also at lower resolution, but sufficiently high resolution (at least $512^2$) is required at later times. The growth rate, $\dot{A}_y$, and the maximum of the kinetic energy density of motions along the $y$-direction, $E_y$, behave similarly: while GOEMHD3 initially follows the reference solution, the dependence on resolution becomes more prominent at later times, with the converged GOEMHD3 solutions slightly underestimating the PENCIL reference.

Fig. 2. Time evolution of the maximum kinetic energy density $E_y = \frac{1}{2} \rho u_y^2$ for the Kelvin-Helmholtz test, as computed with GOEMHD3 at different resolution (coloured lines). The black line corresponds to the results obtained by the PENCIL code using a mesh resolution of $4096^2$ (McNally et al. 2012).

Fig. 3. Color-coded mass density, $\rho(y,z)$ at time $t = 2.5$ for the Kelvin-Helmholtz test problem. Panels a), b), c) and d) show the GOEMHD3 results for a numerical resolution of $128^2$, $256^2$, $512^2$, and $1024^2$, respectively.
different effective numerical diffusivity of the codes. The LeapFrog scheme implemented by GOEMHD3 has no intrinsic diffusivity and the initially smooth solution of the Kelvin-Helmholtz problem implies that GOEMHD3 does not produce numerical oscillations at early times. Hence, no smoothing is required which leads to very low effective dissipation rates. The secondary billows in the GOEMHD3 simulations develop earlier than with PENCIL (see Fig. 3 and Fig. 12 of McNally et al. 2012) indicating a larger effective diffusivity of GOEMHD3 at later times when steep gradients develop in the turbulent phase leading to the smoothing procedure become effective. In summary, GOEMHD3 initially has a small numerical diffusivity which leads to a same growth of the Kelvin-Helmholtz instability, but later, when smoothing becomes necessary in GOEMHD3, its effective diffusivity increases above the level of the PENCIL code.

3.2. Orszag-Tang test

The ideal-MHD limit of the GOEMHD3 code is tested by simulating a standard Orszag-Tang vortex setup in two dimensions (Orszag & Tang 1979). The test starts with initially periodic fields where the velocity and magnetic fields both contain X-points, but differ in the modal structure along one spatial direction. Such initial conditions are given by

\[ \rho = \frac{25}{36\pi} , \]
\[ \mu_x = \sin(2\pi x) , \quad \mu_y = -\sin(2\pi y) , \]
\[ B_x = \frac{1}{\sqrt{4\pi}} \sin(2\pi x) , \quad B_y = \frac{1}{\sqrt{4\pi}} \sin(4\pi y) , \]
\[ p = \frac{5}{6\pi} . \]

The simulation box size is \(< -0.5, 0.5 > \times < -0.5, 0.5 >\). The boundary conditions are all periodic. As in the Kelvin-Helmholtz test the smoothing coefficient is chosen as \( \chi = 2 \times 10^{-3} \).

The density distribution at \( t = 0.25 \) is depicted for four different grid resolutions in Fig. 4. A number of additional quantities at time \( t = 0.50 \) and for a grid resolution 256\(^2\) are presented in Fig. 5 (upper row). In general, the GOEMHD3 code computes a non-oscillating solution which reproduces sharp gradients and the structures and dynamics closely resemble the results obtained by Ryu et al. (1995) and Dai & Woodward (1998). Fig. 5 shows, that thermal pressure (panel a) and magnetic pressure (panel b) are anticorrelated, except in the post-shock flows.

In order to directly compare the GOEMHD3 results with another code, we have performed the Orszag-Tang test with the parameters used for testing the ATHENA code in its version 4.2. For this comparison ATHENA is taken with a Courant safety constant \( C = 0.5 \) and a resolution by 256\(^2\) grid points.

Fig. 5 compares the structures of the thermal pressure (panels a, e), magnetic pressure (panels b, f), the vorticity (panels c, g), and the current density (panels d, h), as produced by GOEMHD3 (top row) and ATHENA (bottom row). Compared to ATHENA, GOEMHD3 in general produces slightly shallower gradients which is caused by the smoothing scheme which is necessary to keep the GOEMHD3 simulation stable. GOEMHD3 is flux conservative, as we can see in Fig. 5 it holds position of the shock fronts on the same place as ATHENA.

The artifacts of smoothing become manifest also in the evolution of the total energy within the simulation box which is plotted in Fig. 6. The coloured curves show the resolution-dependent amount of energy dissipation of GOEMHD3 in contrast with a (by construction of the numerical scheme) perfect energy conservation in ATHENA. The quality of energy conservation in the GOEMHD3 simulation increases with numerical resolution. At higher resolution the sharp gradients propagating across the grid span a smaller number of grid points (where smoothing becomes effective) which explains the lower energy dissipation rates.

The running time, on a single CPU core, is similar for the both codes, GOEMHD3 takes 298 s and ATHENA 332 s.

3.3. Resistive decay of a cylindrical current

The test setup for simulating the resistive decay of a cylindrical current in two spatial dimensions as described by Skála & Barta (2012) allows us to estimate the numerical resistivity of the new GOEMHD3 code and to evaluate its applicability to simulations of non-ideal (resistive) MHD problems. Initially, at \( t = 0 \), a cylindrical current is set up using a radial magnetic field \( B = (0, B_0, 0) \), which is given by

\[ B_\phi(r, t) = j_0 \frac{r_0}{x_N} J_1(x_N \frac{r}{r_0}) \exp(-\alpha t) \]

in the internal \( (r \leq r_0) \) region and

\[ B_\phi(r, t) = j_0 \frac{r_0}{x_N} J_1(x_N) \]

in the outer space \( (r > r_0) \). Here, \( j_0 = 1 \) is the amplitude of the current density on the axis of the cylinder, and \( r_0 = 1 \) is its radius. Here, and in the following \( J_l(x) \) denotes the Bessel function of order \( l \), and \( x_N \approx 2.40 \) is the first root of \( J_1(x) \). The decrement \( \alpha \) is defined as \( \alpha = \eta (x_N/r_0)^2 \). The pressure is chosen uniformly \( (p = 1) \) in the whole domain and the density is set to a very large uniform value \( (\rho = 10^{32}) \) which effectively sets the plasma at rest, and the system of MHD equations (11-12) reduces to the induction equation (13) which can be solved analytically. The analytic solution (cf. Skála & Barta 2012) predicts an
Fig. 4. Two-dimensional Orszag-Tang test. Mass density at time $t = 0.25$. Panels a), b), c) and d) show resolution $128^2$, $256^2$, $512^2$ and $1024^2$ respectively.

Fig. 5. For simulations of the two-dimensional Orszag-Tang test with GOEMHD3 (top row) and Athena 4.2 (bottom row) the individual panels show the thermal plasma pressure (a, e), the magnetic pressure (b, f), the vorticity $\nabla \times v$ (c, g), and the current density $\nabla \times B$ (d, h) for a grid of resolution $256^2$ at $t = 0.5$.

Exponential current decay while an infinitesimally thin current ring is induced along the resistive boundary. The current density $j(r, t) = (j_i(r, t), 0, 0)$ evolves according to

$$j_i(r, t) = j_0 J_0 \left( \frac{r}{r_0} \right) \exp(-\alpha t) + \frac{j_0}{2 \pi x_N} J_1(x_N) \left[ 1 - \exp(-\alpha t) \right] \delta(r-r_0)$$

(19)

where $\delta(x)$ is the Dirac delta function.

For the simulations with GOEMHD3 the computational domain is chosen as $<-2.5,2.5> \times <-2.5,2.5>$ and open boundary conditions are applied in the $y$ and $z$ directions. Periodic boundary conditions are used in the invariant $x$ direction. A finite width has to be assumed for the current ring which in our case extends over two grid-points. Also, the magnitude of the current in the ring is finite in the simulation.
We attribute this numerical instability to the anisotropic nature of the discretization scheme whose diffusion operator does not take into account diagonal grid-points.

We should note here, that influence of the anisotropic instability for the magnetic diffusion on the simulations of the coronal events is negligible due to the smoothing of resistivity in space and time in the case of use dynamic resistivity model.

4. Three-dimensional simulation of the Solar corona with GOEMHD3

In order to demonstrate the applicability of the GOEMHD3 code to realistic, three-dimensional simulations of weakly collisional astrophysical plasmas at high Reynolds numbers and to assess the computational performance of the code we have performed a simulation of the evolution of the solar corona above an active region. Being able to simulate such scenarios, where a number of important dynamical processes are still not well understood, has in fact been the main motivation for developing GOEMHD3. As shall be shown below, GOEMHD3 allows us to numerically tackle such problems with significantly higher numerical resolution and accuracy as compared with its predecessor codes.

4.1. Physical context

We choose for this demonstration the Solar corona above active region NOAA AR 1429 in March 2012. This active region is well known since it released many prominent phenomena, like strong plasma heating, particle acceleration and even eruptions. Many of them took place during the two weeks between 2nd and 15th, 2012 making AR1429 one of the most active regions during the 24th solar cycle. As a result the morphology of AR 1429 has been thoroughly investigated by a number of researchers so that the activity phenomena of AR 1429 are now well known, as they were observed in very details using, e.g., the AIA instrument on board of NASA's Solar Dynamics Observatory SDO (see, e.g. Inoue et al. 2014), van Driel-Gesztelyi et al. 2014, Möstl et al. 2013). Very sensitive information was obtained, e.g. about MeV energy (relativistic) electron acceleration processes which is provided by 30 THz radio waves. Examining the role of the continuum below the temperature minimum with a new imaging instrument operating at El Leoncito Kaufmann et al. 2013 studied the 30 THz emissions. For the M8 class flare on March 13, 2012, e.g., they found a very clear 30 THz signature, much cleaner than the white-light observations are able to provide. Another important information about the solar activity are the dynamic spectra of solar proton emissions. The PAMELA experiment, e.g., measures the spectra of strongly accelerated protons over a wide energy range. For four eruptions of AR 1429 the observed energetic protons spectra were analyzed by Martucci et al. 2014. They interpreted them as an indication of first order Fermi acceleration, i.e., of a mirroring of the protons between dynamically evolving plasma clouds in the corona above AR 1429. Changes in the chemistry of the Earth’s atmosphere after the impact of the energetic solar protons emitted by AR 1429 were studied by von Clarmann et al. 2013. These authors used the MISAP spectrometer onboard the late European environmental satellite ENVISAT to measure temperature and trace gas profiles in the Earth atmosphere. They found that the amount of produced by energetic Solar protons from AR 1429 were among the 12 largest Solar particle events, i.e. proton storms, in 50 years. These and more observations of AR 1429 in-

Figure 7 shows that initially the decay of the current density in the center closely follows the time evolution of the analytic solution (Eq. 19), while a sharp drop to zero is observed at a later time that depends on the numerical resolution of the grid (cf. Fig. 8). The drop of the central current is steeper and occurs earlier in time for larger grid resolutions. This artifact is caused by a numerical instability which is spreading away from the limb of the resistive disk and is propagating to the center. The growth rate and speed of propagation increases with the grid resolution which is a strong hint for a numerical origin of the instability.
dicate that very efficient energy conversion processes took place in the corona.

4.2. Initial and boundary conditions

We start the simulation with initial conditions derived in accordance with observations of AR 1429 on March 7th when at 00:02 UT a X5.4 flare eruption took place at heliographic coordinates N18E31. In order to describe the evolution of the corona before the eruption, we initialize the simulation using photospheric magnetic field observations on March 6th at 23:35 UT. Fig. [11] shows the line-of-sight (LOS) component of the photospheric magnetic field of the AR 1429 obtained at this time by the HMI instrument onboard the SDO spacecraft in a field of view of 300 × 300 arcsec². This field of view covers an area of 217.5 × 217.5 Mm² which we choose as the lower boundary of the simulation box. The line-of-sight magnetic field is preprocessed by flux balancing, removing small scale structures and fields close to the boundary before it is used for extrapolation into 3D. In particular a spatial 2D Fourier filtering of the magnetic field data is applied to remove short spatial wavelength modes with wave numbers greater than 16, which correspond to structures do not reach out into the corona, above the transition region. The Fourier filtered magnetic fields are flux balanced and extrapolated into the third dimension according to the MHD box boundary conditions derived by Otto et al. (2007). The resulting initial magnetic field is depicted in Fig. [12]. For the height of the simulation box we choose 300 Mm. The simulation grid spacing in the x and y directions is homogeneous with a mesh resolution given by the sampling over 258 grid points. After the filtering out of all modes with wave numbers larger than 16, such grid allows to resolve all magnetic field structures sufficiently well. Though in the height (z-) direction also 258 grid points are used the grid is nonuniformly distributed in order to better resolve the lower part of the corona / transition region and chromosphere. Figure 9 shows the height-dependent grid spacing (dz) used.

Fig. 9. The grid spacing dz in the z-direction in the simulation of the AR 1429. Where z = 0 is the photosphere. The finer spacing at the bottom part sample better transition region with steep gradients in the density and temperature.

The initial density distribution is prescribed such that the chromospheric density is 500 times larger than the density in the corona according to the equation

\[ \rho(z) = \frac{\rho_0}{2} [1 - \tan(2(z - z_0))] + \rho_{co} \]  

(20)

where, \( \rho_{ch} \) and \( \rho_{co} \) are chromospheric and coronal densities, respectively. Note that the normalizing density is \( \rho_0 = 2 \times 10^{15} \text{ m}^{-3} \). The transition region is initially localized around \( z_0 = 3 \), which corresponds to 15 Mm. The initial thermal pressure \( p = 0.01 \rho_0 = 0.7957 \text{ Pa} \) is homogeneous throughout the whole simulation domain, i.e. gravity effects are neglected. According to the ideal gas law \( T = p/(\gamma p) \) this reveals the temperature height profile. The initial density and temperature height profiles are depicted in Fig. 10. As one can see in the Figure, the initial coronal temperature is of the order of 10⁸ K. The initial plasma velocity is zero everywhere in the corona but finite in the chromosphere.

Fig. 10. Initial height profiles of density and temperature in the simulation of the AR 1429. The chromospheric density is 500 times larger than the density in the corona. The transition region is initially localized around \( z_0 = 3 \), which corresponds to 15 Mm.

For the sides of the simulation box, the boundary conditions are set according to the MHD-equation compatible line symmetry conditions derived by Otto et al. (2007). The top boundaries are open, i.e. \( \partial \phi / \partial n = 0 \), except the normal to the boundary component of the magnetic field which is obtained to fulfill the source-freeness condition \( \nabla \cdot \mathbf{B} = 0 \). The bottom boundary of the simulation box is open for entropy and magnetic fluxes.

The coronal plasma is driven via a coupling to the neutral gas below the transition region. The neutral gas is driven in accordance with the observed photospheric motion. First, the plasma flow velocities are inferred from photospheric magnetic field observations according to (Santos et al. 2008). In order to avoid emerging and submerging magnetic fluxes the motion pattern is then modeled by divergence-free vortices given by

\[ u_0 = \nabla \times \left[ \frac{\phi_0}{\cosh \left( \frac{x^2 + y^2 + z_0^2}{b^2} \right) \cosh \left( \frac{x^2 + y^2 + z_0^2}{l_1} \right)} \right] \]  

(21)

The parameters determining strength and localization of the vortex motion are chosen in accordance with observations. In the simulated case the magnetic fluxes rotate around footpoints given by the set of parameters \( \phi_0 = 0.1, c_0 = 9, d_0 = -49, l_0 = 2 \), and \( l_1 = -2 \). The strength of the plasma driving by the neutral gas is decreasing with the height above the photosphere. This decrease is controlled by a height-dependent coupling term in the momentum equation Eq. 2 (or Eq. 9). The height dependent collision coefficient is defined as

\[ \nu(z) = \frac{\nu_0}{2} [1 - \tanh(20(z - z_c))] \]  

(22)

For the simulated case a good approximation for the coupling coefficient is \( \nu_0 = 3 \) with and \( z_c = 0.25 \) (or 1.25 Mm) as the characteristic height, where the coupling (and, therefore, the photospherically caused plasma driving) vanishes.
4.3. Computational performance of GOEMHD3

Employing the physical setup (i.e. initial and boundary conditions) described in the previous subsection, the parallel scalability and efficiency of the GOEMHD3 code was assessed across a wide range of CPU-core counts and for different sizes of the numerical mesh. The benchmarks were performed on the high-performance-computing system of the Max Planck Society, "Hydra", which is operated by its computing centre, RZG. Hydra is an IBM iDataPlex cluster based on Intel Xeon E5-2680v2 "Ivy Bridge" processors (2 CPU sockets per node, 10 cores per CPU socket, operated at 2.8 GHz) and an InfiniBand FDR 14 network. Hydra’s largest partition with a fully nonblocking interconnect comprises 36 000 cores (1800 nodes). For the benchmarks Intel’s FORTRAN compiler (version 13.1) and runtime were used together with the IBM parallel environment (version 1.3) on top of the Linux (SLES11) operating system.

Figure 13 provides an overview of the parallel performance of GOEMHD3, using the execution time for a single timestep as a metric. Four different grid sizes are considered, namely grids with 256$^3$ cells (black colour), 512$^3$ cells (red), 1024$^3$ cells (green) and 2048$^3$ cells (blue). The figure demonstrates a very good overall strong scalability of the code, i.e. the reduction of the computing time for fixed grid size with an increasing number of CPU cores (compare the measured runtimes plotted as circles with the dashed lines of the same colour which indicate ideal scalability). For example, the parallel efficiency is at the 80% level for the 1024$^3$ grid on 2580 cores (128 nodes) when compared to the baseline performance on 160 cores (8 nodes). Simulations with a 2048$^3$ grid can be performed with a parallel efficiency of 80% on more than 10 000 cores.

Increasing the number of grid points by a factor of 8 (from 256$^3$ to 512$^3$, or from 512$^3$ to 1024$^3$) and at the same time using an eightfold number of CPU cores, the computing time remains almost constant (compare the two sets of filled circles with the corresponding horizontal dotted lines in Fig. 13). This demonstrates a very good weak scalability of GOEMHD3, given that the complexity of the algorithm scales linearly with the number of grid points.

The deviations from the ideal (strong) scaling curves which become apparent at high core counts are due to the relatively larger fraction of time spent in the MPI communication (halo exchange) between the domains. For example, for the 1024$^3$ grid, the percentage of communication amounts to 30% for 10 000 cores and increases up to about 50% at 36 000 cores. For a given number of cores, the communication-time share is larger for...
smaller grids (manifest as a larger deviation from ideal scalability in Fig. 13). The latter observation underlines the benefit of making the MPI domains as large as possible which is enabled by our hybrid MPI-OpenMP parallelization approach (cf. Sect. 2.3). Moreover, by comparison with runs where the OpenMP parallelization was switched off and compute nodes were densely populated with MPI tasks (one MPI task per core), the advantages of the hybrid MPI-OpenMP vs. a "plain" parallelization become immediately apparent. The smaller size of the MPI domains in the "plain" MPI runs (diamond symbols in Fig. 13) accounts for a larger communication-to-computation ratio and a larger number of smaller MPI messages. Accordingly, the communication times increase by about 75%, resulting in total runtimes being larger by 15–30% when compared to the hybrid version using the same number of cores. It has to be noted, that it is crucial for the hybrid approach to achieve a close-to-perfect parallel efficiency of the OpenMP parallelization within the MPI domains in order not to jeopardize the aforementioned performance advantages of the more efficient communication. Additional benchmarks have shown that GOEMHD3 indeed achieves OpenMP efficiencies close to 100% up to the maximum number of cores a single CPU socket provides (10 cores on our benchmark platform), but – due to the effects of NUMA6 and limited memory bandwidth – not beyond.

Overall, GOEMHD3 achieves a floating-point performance of about 1 GFlops/core per core which is about 5% of the theoretical peak performance of the Intel Xeon E5-2680v2 CPU. Floating-point efficiencies in this range are commonly considered reasonable for this class of finite-difference schemes.

### 4.4. 3D simulation of the energy distribution in the photospheric driven solar corona

In order to understand the dependence of the energy distribution in the corona on the inflow of mechanical, thermal and magnetic (Poynting flux) energy from below, through the transition region, we calculated the corresponding coronal energy contents and the fluxes through the transition region.

The energies are calculated based on their rates of change as

\[
E_{\text{kin}} = \int \left[ -\frac{1}{2} \int_S \rho u^2 \mathbf{u} \cdot dS \right] dt
\]

\[
E_{\text{mag}} = \int \left[ \int_S \left( -\mathbf{u} \cdot \nabla \mathbf{B}^2 + (\mathbf{u} \cdot \mathbf{B}) \mathbf{B} - \eta \mathbf{j} \times \mathbf{B} \right) \mathbf{dV} \right] dt
\]

\[
E_{\text{th}} = \int \left[ \int_S \left( -\frac{\gamma}{2(\gamma - 1)} \int_S \rho u^2 \mathbf{dS} \right) + \frac{1}{2} \int_S \left( \mathbf{u} \cdot \nabla \rho + \eta \mathbf{j} \times \mathbf{j} \right) \mathbf{dV} \right] dt
\]

Note that the main contributions to the surface integrals (\( \int_S \)) are mainly due to energy fluxes through the surface transition region. \( \int \) is the latter as the lower boundary for the volume integrals (\( \int_v \)). At the same time the energy fluxes through the side boundaries cancel each other due to the symmetric boundary conditions and the fluxes through the upper boundary are negligibly small.

In order to investigate the dependence of the energy distribution on the dissipative properties of the coronal plasma we start imposing the photospheric-photospheric driving on an as usual large-Reynolds-number (weakly dissipative) corona. Hence the simulation is initiated with a very small background resistivity \( \eta = 10^{-10} \), which from the numerical point of view is still well above the numerical resistivity as convergence tests have shown. According to our normalization length the corresponding characteristic Reynolds number based on the normalizing Alfven speed, i.e., the Lundquist number, is of the order of \( 10^{39} \), if the grid resolution scale it is still \( 2 \times 10^9 \). When enhanced activity was observed at the Sun, the background resistivity is enhanced to \( \eta = 10^{-5} \) which corresponds to microturbulence theory predictions \((\text{Sim} \& \text{Buchner} [2003a]), \text{Sim} \& \text{Buchner} [2003b])

\( \text{Figure [14]} \) depicts the temporal evolution of the kinetic, magnetic and thermal energies within the corona above the transition region and the energy fluxes into/from the corona across the transition region. Note that the curves in the figure correspond to the net changes of the energy, i.e., the excess from the initial values at \( t = 0 \). The figure shows that main energy source for the corona is the Poynting flux generated by the footpoint motion of the flux tubes, not the direct transfer of kinetic energy from the chromosphere. Until about \( t = 20 \tau_A \) (about 200 s, a little more than 3 min) the magnetic energy inflow is enhancing mainly the kinetic energy of the corona, i.e. the coronal flux tubes are driven by the photospheric motion. This process lasts as long as the average propagation time of the corresponding Alfvenic perturbation along typical flux tubes. Hence this Alfven transition time is needed to drive, finally, the whole flux tube system. After that the amount of magnetic energy in the corona steadily increases until, at \( t = 100 \tau_A \), i.e. after about 1000 s (i.e. about 17 min) the resistivity is increased by orders of magnitude (see above). Now the enhanced resistivity \((\eta)\) magnetic diffusivity) quickly heats the corona. Already after only \( 80 \tau_A \) (800 s or 13 min) the thermal energy enhancement of the corona due to the imposed Joule heating reaches almost the level of the kinetic energy enhancement due to the footpoint motion. At the same time the increase of the magnetic energy contents of the corona due to the permanent Poynting flux inflow is slowed down only slightly by the heating process.

For a better understanding of the change of the coronal energy distribution figure [15] depicts the temporal evolution of its kinetic, magnetic and thermal energy contents without taking into account the contribution of energy inflows across the transition region. As one can see in the Figure first, after the Alfvenic transition time has passed, in the course of the almost ideal (large Reynolds number) evolution practically only the kinetic energy of the corona grows completely at the expense of the decreasing magnetic field energy. Then, after the magnetic diffusivity is enhanced at \( t = 100 \tau_A \), the magnetic energy drops faster due to resistive dissipation. The latter enhances the thermal energy contents of the corona via Joule heating. After \( t = 170 \tau_A \) the amount of the released energy within the corona is about half of the kinetic energy as one already could see in Figure [14].

The time evolution of the magnetic field is captured by a movie which can be obtained on the WWW7.

The movie shows that until the moment when the resistivity, i.e. the magnetic diffusivity is enhanced (after about 16 minutes solar time) the coronal magnetic fields evolves almost ide-

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6 non-uniform memory access

7 \( \text{http://www.mpg.de/streaming-eu.s3.amazonaws.com/de/institute/mps/magnetic_field_AR11429_buechner.mp4} \)
We have implemented a new, three-dimensional MHD code based on second-order-accurate finite-difference discretization schemes in order to be able to efficiently simulate large-scale weakly-dissipative (large-Reynolds-number) astrophysical plasma systems. In order to avoid numerical dissipation the conservative part of the equations is solved by a Leap-Frog scheme which is second order accurate in time and in space. Viscosity, diffusion and dissipation terms of the equations are discretized by a DuFort-Frankel scheme. Numerically induced grid-scale oscillations are damped away by introducing an artificial viscosity which is switched on locally. In this paper we have documented the main physical, numerical and computational concepts of the new GOEMHD3 code as well as its computational performance. The code was validated and verified by means of a number of appropriate test problems which allowed us to reveal the limits of the applicability of GOEMHD3 and to describe the ways to achieve the goals when solving concrete problems.

First the code was tested by simulating a velocity-shear Kelvin-Helmholtz instability. Owing to the use of an effectively dissipationless Leap-Frog scheme GOEMHD3 obtained the same linear evolution as simulations by the numerically more expensive, higher order PENCIL code. As expected, at later times, during the non-linear evolution of the instability for the same number of grid points, the dissipation is larger than that of the higher-order PENCIL code (McNally et al. [2012]). The reason is locally switched on artificial viscosity which damps spurious grid-scale oscillations inherent to the Leap-Frog scheme. The amount of necessary damping can, however, easily and computationally cheaply be reduced by enhancing the grid resolution of the overall less expensive second-order scheme. GOEMHD3 revealed the same results for Orszag & Tang (1979) vortices as obtained by Ryu et al. [1995] and by Dai & Woodward [1998].

Gradients are well resolved by two grid-points. Since numerical oscillations are smoothed away by locally switching on viscosity, GOEMHD3 dissipates more energy at steep wave fronts as compared to a higher-order code for the same grid-resolution. This dissipation can be easily overcome by (locally) using a larger number of grid points. The solver for the resistive part of the induction equation was tested separately by imposing a homogeneous resistivity on a current column. The results are in good agreement with an analytically predicted current decay. The code fully reproduces the analytic solution until the enhanced numerical errors reach the center of the current system where the current concentration is maximum. Since the spreading of the numerical error depends only on the number of time steps, not on the real physical time, this phenomenon is of purely numerical nature. In order to cope with this effect GOEMHD3 contains a module which smooths an eventually self-regulated resistivity increase around the maximum gradient of the current growth.

The parallel computing performance of the code was demonstrated by obtaining the scaling of the runtime with the number of CPU cores and grid points (i.e. different numerical resolutions) for a realistic application scenario. To this end GOEMHD3 was initialized to simulate the evolution of the solar atmosphere above an observed active region and thus to obtain the distribution of the energy injected from the photosphere through the transition region into the corona. The calculations revealed an almost linear strong scaling of the runtime with the number of CPU cores for meshes with up to 2048^3 grid points. On the HPC system Hydra of the Max-Planck Society GOEMHD3 exhibited an almost ideal scaling even be-

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**5. Discussion and conclusions**

The parallel computing performance of the code was demonstrated by obtaining the scaling of the runtime with the number of CPU cores and grid points (i.e. different numerical resolutions) for a realistic application scenario. To this end GOEMHD3 was initialized to simulate the evolution of the solar atmosphere above an observed active region and thus to obtain the distribution of the energy injected from the photosphere through the transition region into the corona. The calculations revealed an almost linear strong scaling of the runtime with the number of CPU cores for meshes with up to 2048^3 grid points. On the HPC system Hydra of the Max-Planck Society GOEMHD3 exhibited an almost ideal scaling even be-

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**Fig. 14.** Scaling test simulating the solar corona above AR 1429. Shown are the temporal evolution of thermal, kinetic and magnetic energies within corona above the transit region. The energy fluxes of the thermal, kinetic and magnetic energies from the chromosphere are denoted by flux. For the meaning of the different lines see the line form legend. After \( t \sim 16 \) minutes the background resistivity is enhanced causing Joule heating.

**Fig. 15.** Scaling test simulation of the solar corona above AR 1429: temporal evolution of the thermal, kinetic and magnetic volumetric energies in the solar atmosphere above the transition region. For the meaning of the different lines see the line form legend. Note that after the time \( t \sim 16 \) minutes the resistivity and, therefore, Joule heating is essentially enhanced.
Fig. 16. Snapshot of the magnetic field at the time $t = 130 \tau_A$ (≈ 22 Minutes) of the simulated AR 1429. The magnetic field lines are coloured by the magnitude of the current carrier velocity $j/n$. The bottom plane depicts magnetic field $B_z$ component (perpendicular to the plane).

Beyond 30,000 processor cores. In addition, also a very good weak scalability from 20 cores (1 node) for 256$^3$ grid runs to more than 20,000 cores (1000 nodes) for a 2048$^3$ grid, was obtained, thereby maintaining absolute run times of less than a second per time step.

In summary, the tests of GOEMHD3 and its simulation capabilities have shown that the code very efficiently solves the MHD equations of almost ideal plasma systems at a high speed on a non-equidistant grid. Due to its second order accurate discretization the code is conceptually straightforward to implement and to parallelize on distributed-memory computer architectures. It can simply be adjusted to different types of initial and boundary conditions and extended to include additional physics modules. Due to its excellent computational performance and parallel efficiency the relatively low numerical accuracy per grid point and time step can easily be compensated by adopting an enhanced resolution in space and time. Aiming at the same accuracy for the same problem this is computationally still cheaper than running codes using higher-order schemes.

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