Web-based supplementary materials for “Bayesian log-Gaussian Cox process regression: with applications to meta-analysis of neuroimaging working memory studies” by P. Samartsidis, C.R. Eickhoff, S.B. Eickhoff, T.D. Wager, L. Feldman Barrett, S. Atzil, T.D. Johnson and T.E. Nichols

1 Gradient expressions for the LGCP

Let \( \beta_k \equiv \mu_k \ (k = 0, \ldots, K) \). The log-posterior, up to a normalising constant is given by:

\[
\ell(\alpha, \beta, \sigma, \rho, \{\gamma_k\}_{k=1}^{K^*} | \cdot) \propto \sum_{i=1}^{I} \left[ -\sum_{j=1}^{V} A_{v_j} \lambda_i(v_j) + \sum_{j \in x_i} \log \lambda_i(v_j) \right] + \log \text{priors}
\]

where \( \alpha = (\alpha_1, \ldots, \alpha_I)^\top \), \( \beta = (\beta_0, \ldots, \beta_K)^\top \), \( \sigma = (\sigma_0, \ldots, \sigma_{K^*})^\top \), \( \rho = (\rho_0, \ldots, \rho_{K^*})^\top \), \( A_{v_j} = 1_{v_j \in B} \) and the intensity function at each voxel \( v_j \) is defined as:

\[
\lambda_i(v_j) = \alpha_i \exp \left( \sum_{k=0}^{K} \beta_k z_{ik} \right) \exp \left( \sum_{k=0}^{K^*} \sigma_k \left( R_k^{1/2} \gamma_k \right)_j z_{ik} \right)
\]

We now calculate the derivatives with respect to the parameters of interest.

1.1 Partial derivatives with respect to \( \beta_l \)

We have that:

\[
\frac{\partial \log \lambda_i(v_j)}{\partial \beta_l} = \frac{\partial}{\partial \beta_l} \log \alpha_i + \frac{\partial}{\partial \beta_l} \sum_{k=0}^{K} \beta_k z_{ik} + \frac{\partial}{\partial \beta_l} \sum_{k=0}^{K^*} \sigma_k \left( R_k^{1/2} \gamma_k \right)_j z_{ik}
\]

\[
= z_{il}.
\]

As a result:

\[
\frac{\partial \ell(\beta_l | \cdot)}{\partial \beta_l} = -\sum_{i=1}^{I} \sum_{j=1}^{V} \left[ A_{v_j} \frac{\partial}{\partial \beta_l} \lambda_i(v_j) - 1_{v_j \in x_i} \frac{\partial}{\partial \beta_l} \log \lambda_i(v_j) \right] + \frac{\partial}{\partial \beta_l} \log \pi(\beta_l)
\]

\[
= -\sum_{i=1}^{I} \sum_{j=1}^{V} \left[ A_{v_j} \lambda_i(v_j) z_{il} - 1_{v_j \in x_i} z_{il} \right] - \frac{\partial}{\partial \beta_l} \frac{\beta_l^2}{2\tau^2}
\]

\[
= -\sum_{j=1}^{V} \sum_{i=1}^{I} A_{v_j} \lambda_i(v_j) z_{il} + \sum_{i=1}^{I} n_i z_{il} - \frac{\beta_l}{\tau^2},
\]

where \( n_i \) is the total number of foci in study \( i \).
1.2 Partial derivatives with respect to $\sigma_l$

We have that:

$$\frac{\partial \log \lambda_i(v_j)}{\partial \sigma_l} = \frac{\partial \log \alpha_i}{\partial \sigma_l} + \frac{\partial}{\partial \sigma_l} \sum_{k=0}^{K} \beta_k z_{ik} + \frac{\partial}{\partial \sigma_l} \sum_{k=0}^{K^*} \sigma_k \left( R_{1/2}^{1/2} \gamma_k \right) z_{ik}$$

$$= \left( R_{1/2}^{1/2} \right)_j z_{il}.$$  \hspace{1cm} (5)

Therefore:

$$\frac{\partial \ell (\sigma_l | \cdot)}{\partial \sigma_l} = - \sum_{i=1}^{I} \sum_{j=1}^{V} \left[ A_{ij} \frac{\partial}{\partial \sigma_l} \lambda_i(v_j) - 1_{v_j \notin \mathbb{R}^3} \frac{\partial}{\partial \sigma_l} \log \lambda_i(v_j) \right] + \frac{\partial}{\partial \sigma_l} \log \pi(\sigma_l)$$

$$= - \sum_{i=1}^{I} \sum_{j=1}^{V} \left[ A_{ij} \lambda_i(v_j) \left( R_{1/2}^{1/2} \right)_j z_{il} - 1_{v_j \notin \mathbb{R}^3} \left( R_{1/2}^{1/2} \gamma_l \right)_j z_{il} \right] - \frac{\partial}{\partial \sigma_l} \frac{\sigma_l^2}{2\tau^2}$$

$$= - \sum_{j=1}^{V} \left[ \left( R_{1/2}^{1/2} \right)_j \sum_{i=1}^{I} \left[ A_{ij} \lambda_i(v_j) z_{il} - 1_{v_j \notin \mathbb{R}^3} z_{il} \right] \right] - \frac{\sigma_l}{\tau^2}.$$  \hspace{1cm} (6)

1.3 Partial derivatives with respect to $\rho_l$

Again:

$$\frac{\partial \log \lambda_i(v_j)}{\partial \rho_l} = \frac{\partial}{\partial \rho_l} \log \alpha_i + \frac{\partial}{\partial \rho_l} \sum_{k=0}^{K^*} \beta_k z_{ik} + \frac{\partial}{\partial \rho_l} \sum_{k=0}^{K} \sigma_k \left( R_{1/2}^{1/2} \gamma_k \right) z_{ik}$$

$$= \sigma_l \frac{\partial}{\partial \rho_l} \left( R_{1/2}^{1/2} \right)_j z_{il}.$$  \hspace{1cm} (6)

For ease of exposition we complete the derivation for the one-dimensional case; however, similar arguments can be used when $\mathcal{B} \subset \mathbb{R}^3$. Matrices $R_l$ are circulant and so, the matrix-vector product $R_{1/2}^{1/2}$ can be found using the discrete Fourier transform as

$$R_{1/2}^{1/2} = F \Phi_l^{1/2} F^H \gamma_l,$$  \hspace{1cm} (7)

where $\Phi_l$ are the diagonal matrices containing the eigenvalues of $R_l$ and $F$ is the matrix of eigenvectors. In Equation (7), the only term depending on $\rho_l$ is $\Phi_l$ and, hence:

$$\frac{\partial}{\partial \rho_l} R_{1/2}^{1/2} = F \frac{\partial}{\partial \rho_l} \Phi_l^{1/2} F^H \gamma_l$$  \hspace{1cm} (8)

We know that $\Phi_l = \text{diag} \{ \phi_{l_0}, \ldots, \phi_{l_{V-1}} \}$, where for $k = 0, \ldots, V - 1$ we have that:

$$\phi_{l_k} = \sum_{j=0}^{V-1} \exp \left( -\rho_l \| v_0, v_j \|^6 \right) \exp \left( -\frac{2\pi ikj}{V} \right),$$  \hspace{1cm} (9)
where \( \iota \) is the imaginary unit. Now it is straightforward to see that for \( k = 0, \ldots, V - 1 \):

\[
\frac{\partial}{\partial \rho_l} \phi_k^{1/2} = \frac{\partial}{\partial \rho_l} \left( \sum_{j=0}^{V-1} \exp(-\rho_l \|v_0, v_j\|_\delta^2) \exp\left(-\frac{2\pi l k j}{V}\right) \right)
\]

\[
= \frac{\partial}{\partial \rho_l} \sum_{j=0}^{V-1} \exp(-\rho_l \|v_0, v_j\|_\delta^2) \exp\left(-\frac{2\pi l k j}{V}\right)
\]

\[
= \frac{\partial}{\partial \rho_l} \sum_{j=0}^{V-1} \exp(-\rho_l \|v_0, v_j\|_\delta^2) \exp\left(-\frac{2\pi l k j}{V}\right)
\]

\[
= \frac{1}{2} \psi_k / \phi_k^{1/2},
\]

where \( \psi_k \) can be viewed as the \( k \)-th eigenvalue of the of a circulant matrix \( S_l \) with base \( s_l = [\|v_0, v_0\|_\delta^2 \exp(-\rho_l \|v_0, v_0\|_\delta^2), \ldots, \|v_0, v_{V-1}\|_\delta^2 \exp(-\rho_l \|v_0, v_{V-1}\|_\delta^2)] \) and \( S_l = F \Psi_l F^\dagger, \Psi_l = \text{diag} \{ \psi_0, \ldots, \psi_{V-1} \} \). Overall we see that:

\[
\frac{\partial}{\partial \rho_l} R_l^{1/2} \gamma_l = F \frac{\partial}{\partial \rho_l} \Phi_l^{1/2} F^\dagger \gamma_l
\]

\[
= -\frac{1}{2} F \left[ \Psi_l \odot \Phi_l^{1/2} \right] F^\dagger \gamma_l
\]

\[
= -\frac{1}{2} Q_l \gamma_l,
\]

where \( \odot \) stands for element wise division. Combining Equations (6) and (11), we find that:

\[
\frac{\partial \log \lambda_i(v_j)}{\partial \rho_l} = -\frac{1}{2} \sigma_l (Q_l \gamma_l)_j z_{il}.
\]

So:

\[
\frac{\partial \ell (\rho_l | \cdot)}{\partial \rho_l} = -\sum_{i=1}^I \sum_{j=1}^V \left[ A_{ij} \frac{\partial}{\partial \rho_l} \lambda_i(v_j) - 1_{v_j \notin x_i} \frac{\partial}{\partial \rho_l} \log \lambda_i(v_j) \right] + \frac{\partial}{\partial \rho_l} \log \pi(\rho_l)
\]

\[
= -\sum_{i=1}^I \sum_{j=1}^V \left[ A_{ij} \lambda_i(v_j) \left( -\frac{1}{2} \right) \sigma_l (Q_l \gamma_l)_j z_{il} - 1_{v_j \notin x_i} \left( -\frac{1}{2} \right) \sigma_l (Q_l \gamma_l)_j z_{il} \right]
\]

\[
- \frac{\partial}{\partial \rho_l} 1_{\rho_l \in [\rho_{\text{low}}, \rho_{\text{upp}}]}
\]

\[
= \frac{\sigma_l}{2} \sum_{j=1}^V \sum_{i=1}^I \left[ (Q_l \gamma_l)_j \left( A_{ij} \lambda_i(v_j) z_{il} - 1_{v_j \notin x_i, z_{il}} \right) \right]
\]

\[
= \frac{\sigma_l}{2} \sum_{j=1}^V \left[ (Q_l \gamma_l)_j \sum_{i=1}^I \left[ A_{ij} \lambda_i(v_j) z_{il} - 1_{v_j \notin x_i, z_{il}} \right] \right].
\]
1.4 Partial derivatives with respect to $\gamma_l$

Finally:

$$\begin{align*}
\frac{\partial \log \lambda_i(v_j)}{\partial \gamma_l} &= \frac{\partial}{\partial \gamma_l} \log \alpha_i + \frac{\partial}{\partial \gamma_l} \sum_{k=0}^{K^*} \beta_k z_{ik} + \frac{\partial}{\partial \gamma_l} \sum_{k=0}^{K} \sigma_k \left( \mathbf{R}_k^{1/2} \gamma_k \right)_j z_{ik} \\
&= \sigma_l r_{lj} z_{il},
\end{align*}$$

where $r_{lj}$ is the $j$-th row of the matrix $\mathbf{R}_l^{1/2}$. Now we can see that:

$$\begin{align*}
\frac{\partial \ell}{\partial \gamma_l} &= -\sum_{i=1}^{I} \sum_{j=1}^{V} \left[ A_{v_j} \frac{\partial}{\partial \gamma_l} \lambda_i(v_j) - 1_{v_j \in \mathbf{X}_i} \frac{\partial}{\partial \gamma_l} \log \lambda_i(v_j) \right] + \frac{\partial}{\partial \gamma_l} \log \pi(\gamma_l) \\
&= -\sum_{i=1}^{I} \sum_{j=1}^{V} \left[ A_{v_j} \lambda_i(v_j) \sigma_l r_{lj} z_{il} - 1_{v_j \in \mathbf{X}_i} \sigma_l r_{lj} z_{il} \right] - \frac{\partial}{\partial \gamma_l} \gamma_l \gamma_l / 2 \\
&= -\sigma_l \sum_{j=1}^{V} \left[ r_{lj} \sum_{i=1}^{I} A_{v_j} \lambda_i(v_j) z_{il} - 1_{v_j \in \mathbf{X}_i} \right] - \gamma_l \\
&= -\sigma_l \left( \mathbf{R}_l^{1/2} \right)^T \mathbf{c}_l - \gamma_l \\
&= -\sigma_l \mathbf{R}_l^{1/2} \mathbf{c}_l - \gamma_l,
\end{align*}$$

(14)

since $\mathbf{R}$ is a nested block circulant matrix, where $\mathbf{c}_l$ are $V$-vectors with elements $c_{lj} = \sum_{i=1}^{I} \left[ A_{v_j} \lambda_i(v_j) z_{il} - 1_{v_j \in \mathbf{X}_i} \right]$.

1.5 Random effects updates

Let $J$ be the total number of publications from which the $I$ studies in the meta-analysis have been retrieved, and let $C_j$ the set of studies retrieved from paper $j$ ($j = 1, \ldots, J$). For all $j = 1, \ldots, J$ we have that

$$\begin{align*}
\pi(\alpha_j | \cdot) &\propto \pi(\alpha_j) \prod_{i \in C_j} \pi(x_i | \lambda_i) \\
&\propto \alpha_j^{\kappa - 1} \exp \left\{ -\alpha_j \kappa \right\} \exp \left\{ -\alpha_j \sum_{i \in C_j} \sum_{v=1}^{K^*} \lambda_{iv}^* \right\} \alpha_j^{\sum_{i \in C_j} n_i} \\
&= \alpha_j^{\sum_{i \in C_j} n_i + \kappa - 1} \exp \left\{ -\alpha_j \left( \sum_{i \in C_j} \sum_{v=1}^{K^*} \lambda_{iv}^* + \kappa \right) \right\}.
\end{align*}$$

Therefore, we draw $\alpha_j$ from a $\mathcal{G} \left( \sum_{i \in C_j} n_i + \kappa, \sum_{i \in C_j} \sum_{v=1}^{K^*} \lambda_{iv}^* + \kappa \right)$ distribution.
2 Simulation studies

In order to evaluate the performance of the proposed HMC algorithm to sample from the posterior distribution of the latent GPs, we consider two simulation setups. In the first we draw samples directly from the log-Gaussian Cox process model, whereas in the second we create synthetic studies based on a different model to assess its robustness to model misspecification. For consistency, all processes are defined on the same brain atlas used in the application of Section 5, consisting of 216,040 2mm\(^3\) cubic voxels. The average number of foci per simulated dataset is kept low (mean number of foci per study is 5) to resemble the sparsity of points observed in real CBMA data. Finally, the total number of studies is fixed to 200 in both analyses, similar to the sample sizes available in real applications (Kang et al., 2011, for example).

2.1 Setup 1

In this setting we simulate 200 studies, with two spatially varying covariates that account for the mean of two groups of studies, and two non-spatially varying covariates. For \(i = 1, \ldots, 200\) we set:

\[
\lambda_{iv} = \exp \left\{ \sum_{k=1}^{2} \left( \mu_k + \sigma_k \left( R_k^{1/2} \gamma_k \right) \right) z_{ik} + \sum_{i=3}^{4} \beta_k z_{ik} \right\},
\]

where \(z_{i1} \sim \text{Bernoulli}(0.5)\), \(z_{i2} = 1 - z_{i1}\), \(z_{i3} \sim \text{Uniform}[-1, 1]\) and \(z_{i4} \sim \text{Bernoulli}(0.5)\). Note that this parametrisation of the covariates implies existence of two types of studies, say type 1 and 2, with different spatially varying means and the effect of one continuous and one categorical covariate. The expected total number of foci is 3.99 and 4.16 for studies of type 1 and 2 respectively. We draw \(\gamma_1, \gamma_2\) from their \(\mathcal{N}(0, I)\) prior and fix the values of the scalar parameters shown in Table 1. We run the HMC algorithm of Section 4 for 10,000 iterations, discarding the first 4,000 as a burn-in and save every 6 iterations for a total of 1,000 saved posterior samples. This took roughly 14 hours on an NVIDIA Tesla K20c GPU card.

Results are summarised in Table 1 and Figure 3. In Table 1 we see that the scalar parameters are estimated accurately despite the sparsity of points in the realisations. The 95% credible intervals contain the true values of all the parameters in the setup. Traceplots for the parameters \(\sigma_1, \sigma_2, \rho_1\) and \(\rho_2\) can be found in Figure 1, whereas trace plots for \(\mu_1, \mu_2, \beta_3\) and \(\beta_4\) can be found in Figure 2. The red lines indicate the true parameter values.

For \(z_{i3} = z_{i4} = 0\), the median expected number of points is 3.97 (95% CI [3.84, 4.10]) for type 1 and 4.61 (95% CI [4.46, 4.78]) for type 2. These values are very similar to the values we observe in the simulated dataset, that is 3.98 for type 1 and 4.53 for type 2. This indicates that our model does a good job fitting the data. The shape of the latent Gaussian processes \(\mu_k + \sigma_k R_k^{1/2} \gamma_k\) is generally captured for both types as can be seen in Figure 3. In particular, we can see that the maxima in the true and estimated images
Figure 1: Posterior traceplots for the scalar parameters of the LGCP model used to fit the data of Section 2.1. Top row: standard deviations. Bottom row: correlation decay parameters (∗100). The true values are indicated by the solid red lines.
Figure 2: Posterior traceplots for the scalar parameters of the LGCP model used to fit the data of Section 2.1. Top row: overall latent process means. Bottom row: regression coefficients for covariates $z_3$ and $z_4$. The true values are indicated by the solid red lines. 

appear roughly in the same locations. The same cannot be said about the other values but this is expected given the dearth of information in regions of low intensity.
Table 1: Posterior summaries of the scalar parameters of the LGCP model, fit to the simulated data of Section 2.1. Results are based on 1,000 posterior draws. The values for the correlation parameters $\rho_1, \rho_2$ are multiplied by 100. The values for $\beta_1$ and $\beta_2$ are multiplied by 10.

| Parameter | True Value | Posterior median | 95% credible interval |
|-----------|------------|-------------------|-----------------------|
| $\mu_1$   | -13.7      | -13.72            | [-13.99, -13.48]      |
| $\mu_2$   | -14.2      | -14.14            | [-14.47, -13.86]      |
| $\sigma_1$| 1.2        | 1.19              | [1.01, 1.38]          |
| $\sigma_2$| 1.6        | 1.61              | [1.43, 1.81]          |
| $\rho_1$  | 1          | 0.93              | [0.69, 1.27]          |
| $\rho_2$  | 2          | 2.30              | [1.69, 3.15]          |
| $\beta_3$ | 2          | 1.44              | [0.22, 2.52]          |
| $\beta_4$ | 1          | 0.95              | [0.32, 1.65]          |

2.2 Setup 2

In this setup we create datasets with a pattern of points that follows brain structures of interest. Again there are two types of studies, say type 1 and type 2. For each study $i, i = 1, \ldots, 200$, we generate the total number of points from a Negative Binomial distribution with mean $\mu = 6 + 2z_{i3} - 1_{\{z_{i4}=0\}} + 1_{\{z_{i4}=1\}}$ and variance $\mu^2/20$. For the covariates, $z_{i3} \sim \text{Uni}[-1,1]$ and $z_{i4} \sim \text{Bernoulli}(0.5)$. Once we know the exact number of foci per study, we assign the study uniformly at random to one of the 2 types and distribute its foci as follows. For type 1, foci appear systematically in the following regions: each focus can be observed in the right amygdala ($B_R$) with probability 55%, the orbitofrontal cortex ($B_C$) with probability 30% or anywhere else in the brain with probability 15%. The configuration for type 2 differs in that most of the points will go to the left amygdala ($B_L$) instead of the right amygdala. If a focus is assigned to one of the three broad regions, the exact location has a uniform distribution over the region.

In the fourth column of Figure 4 the regions in red and blue correspond to the left and right amygdala respectively while the orbitofrontal cortex is coloured in green.

HMC is run for 10,000 iterations, discarding the 4,000 first as a burn-in and saving every 6 to obtain a total of 1,000 samples from the posterior. The run took approximately 15 hours on a Tesla K20c GPU card.

Results are shown in Figure 4 where in the first two columns we see median posterior log-intensities for the two types, in different axial slices. In both cases, we find that the regions with the highest intensities are the amygdalae and that the orbitofrontal cortex is a region of high posterior intensity as well. The median expected number of points is 5.81 for type 1 (95% CI [5.36,6.32]) and 6.45 for type 2 (95% CI [5.97,6.97]). The observed values are 6.27 and 6.73 respectively.

Conditional on there being exactly one focus, we can estimate the probability that this focus appears in any subset $B \subseteq \mathcal{B}$ as $\int_B \lambda(\xi) \, d\xi / \int_{\mathcal{B}} \lambda(\xi) \, d\xi$. Using the posterior
draws obtained from the HMC algorithm, we can obtain the posterior distribution of any such quantity. For our simulated type 1 data we find that the median posterior probability of observing a focus in the right amygdala ($B_R$) is 0.43 (95% CI [0.40,0.48]). For type 2, the probability of observing a focus in the left amygdala ($B_L$) is 0.42 (95% CI [0.39,0.46]). For the orbitofrontal cortex ($B_C$) the median posterior probabilities are 0.25 for type 1 and 0.23 for type 2, with 95% credible intervals [0.22, 0.28] and [0.20, 0.26] respectively. We therefore see that the model underestimates the probabilities for $B_R$, $B_L$ and $B_C$. This bias can be attributed to the smoothness that is imposed by our parameter $\delta$ thus leading to increases intensities just outside these regions as well as regions where noise foci appear.

An interesting question one may ask is which are the regions of the brain that are activated by one type or the other, but not both. To answer this, one can construct the mean standardised posterior difference map computed as the ratio of the posterior mean of the difference ($\beta_1 - \beta_2$), to the posterior standard deviation of that difference: \[
\frac{(\beta_1)_v - (\beta_2)_v}{\text{sd}(\beta_1)_v - (\beta_2)_v}.\]

Extreme negative or positive values are evidence of differences between the two types. We show the difference map in the third column of Figure 4. As we see, the model distinguishes the the two types in the amygdala but the differences are small in the rest of the brain.
Figure 3: Some true (top row) and estimated (bottom row) latent Gaussian processes for type 1 (columns 1 and 2) and type 2 (columns 3 and 4) in the simulation setup 1 of Section 2.1. Columns 1 and 3 correspond to axial slice $z = -22$; columns 2 and 4 correspond to axial slice $z = 4$. While they may appear dissimilar at first, observe that the most intense regions of the true and estimated intensity match up; in this punishingly sparse setting (mean $\approx 4$ foci per 3D image), the less intense regions have too few points to learn the intensity.
Figure 4: Results for simulation setup 2 of Section 2.2. The top row corresponds to axial slice $z = -24$ whereas the bottom row corresponds to axial slice $z = -16$. Columns 1 and 2 are the estimated log-intensities for type 1 and type 2 respectively. The third column is the standardised mean posterior difference between the two latent Gaussian processes in the corresponding slice; bright colours indicate areas mostly activated by type 1 process. The fourth column shows the regions of the brain systematically activated by the two processes; red for type 1, blue for type 2 and green for both.
3 Analysis of the WM dataset: supplementary plots

This section contains MCMC convergence diagnostics and supplemental material for the real data analysis of Section 5 of the main paper. Posterior traceplots are shown in Figures 5, 6, 7 and 8. The posterior traceplots for the marginal standard deviation parameters $\sigma_k$ and the correlation decay parameters $\rho_k$ are shown in Figure 5 (top and bottom row, respectively). Figure 6 shows posterior traceplots for the overall mean parameters $\mu_k$ (top row), the regression coefficient of the sample size covariate (bottom row, first subplot) and the integrated intensities of verbal and non-verbal studies (bottom row, second and third subplot, respectively). Let $v^m_k$ and $v^M_k$ be the voxels where the $k$-th latent GP $\beta_k$ has the minimum and maximum mean posterior values, respectively. Posterior traceplots of $(\beta_0)^v_m$, $(\beta_0)^v_M$, $(\beta_1)^v_m$, $(\beta_1)^v_M$, $(\beta_2)^v_m$ and $(\beta_2)^v_M$ are shown in Figure 7. Finally, in Figure 8 we present $\alpha_m$ and $\alpha_M$, where $m$ and $M$ index the studies with the lowest and largest mean posterior random effects, respectively. All chains are obtained after applying a thinning factor of 15 to the original MCMC chains of length 15,000. Finally, Figure 9 shows the voxelwise posterior variance of $\exp\{\beta_2\}$, the multiplicative age effect on the intensity of both verbal and non-verbal studies.
Figure 5: Posterior traceplots for some of the scalar parameters of the LGCP model of Equation 10 of the main paper that we fit to the working memory dataset presented in Section 2 of the main paper. Top row: marginal standard deviations $\sigma_k$ of the 3 latent GPs. Bottom row: correlation decay parameters $\rho_k$ of the 3 latent GPs. The chains have been obtained after applying a thinning factor of 15 to the original chains of length 15,000.
Figure 6: Posterior traceplots for some of the scalar parameters of the LGCP model of Equation 10 of the main paper that we fit to the working memory dataset presented in Section 2 of the main paper. Top row: overall means $\mu_k$ of the 3 latent GPs. Bottom row, subplot 1: regression coefficient $\beta_3$ for the sample size covariate. Bottom row, subplots 2-3: integrated intensities for verbal and non-verbal studies, respectively. The chains have been obtained after applying a thinning factor of 15 to the original chains of length 15,000.
Figure 7: Posterior traceplots for some of the scalar parameters of the LGCP model of Equation 10 of the main paper that we fit to the working memory dataset presented in Section 2 of the main paper. Subplots 1-6 correspond to \((\beta_0)_{v_m^0}, (\beta_0)_{v_M^0}, (\beta_1)_{v_m^1}, (\beta_1)_{v_M^1}, (\beta_2)_{v_m^2}\) and \((\beta_2)_{v_M^2}\), where \(v_m^k\) and \(v_M^k\) represent the voxels with the minimum and maximum mean posterior values of \(\beta_k\), respectively. The chains have been obtained after applying a thinning factor of 15 to the original chains of length 15,000.
Figure 8: Posterior traceplots for some of the scalar parameters of the LGCP model of Equation 10 of the main paper that we fit to the working memory dataset presented in Section 2 of the main paper. Subplots 1-2 represent $\alpha_m$ and $\alpha_M$, where $m$ and $M$ index the studies with the lowest and largest mean posterior random effects, respectively. The chains have been obtained after applying a thinning factor of 15 to the original chains of length 15,000.
Figure 9: Posterior variance of $\exp \{ \beta_2 \}$, the multiplicative age effect on the intensity of both verbal and non-verbal studies. Top row shows (from left to right) axial slices $z = -50, -42, -34, -26$ and $-18$, respectively. Middle row shows axial slices $z = -10, -2, +6, +14$ and $+22$, respectively. Bottom row shows axial slices $z = +30, +38, +46, +54$ and $+62$, respectively. Voxels for which the mean posterior $\lambda$ is low (below the 75% quantile over the brain) have been set to zero.
4 Full brain analysis

In this section we present full brain results for the working memory CBMA conducted in Section 5 of the main paper. Table 2 presents probabilities of activation and expected number of foci for several ROIs, along with their 95\% credible intervals. In Table 3 we compare the probability of an activation between working memory studies using verbal and non-verbal stimuli, using the same ROIs. The ROIs have been obtained from the Harvard-Oxford atlas (Desikan et al., 2006). All quantities are based on 1,000 MCMC samples which are obtained after applying a thinning factor of 15 to the original MCMC chains of length 15,000.
Table 2: Working memory CBMA results. Table presents posterior summaries for the % probability of at least one activation in a given ROI (rounded to 2 decimal points), as well as the expected number of foci (rounded to 3 decimal points). All quantities have been calculated based on 1,000 MCMC samples.

| ROI                                                   | Vol.  | Mean   | $p_{0.025}$ | $p_{0.975}$ | $\int B \lambda(\xi) d\xi$ $p_{0.025}$ | $p_{0.975}$ |
|-------------------------------------------------------|-------|--------|-------------|-------------|------------------------------------------|-------------|
| Frontal pole                                          | 25900 | 60.24  | 51.05       | 66.42       | 0.926                                    | 0.714       |
| Insular cortex                                         | 3613  | 33.39  | 26.68       | 39.36       | 0.407                                    | 0.310       |
| Superior frontal gyrus                                 | 8861  | 38.27  | 27.74       | 43.98       | 0.484                                    | 0.325       |
| Middle frontal gyrus                                   | 8421  | 60.55  | 48.23       | 66.48       | 0.933                                    | 0.658       |
| Inferior frontal gyrus, pars triangularis              | 2317  | 22.54  | 15.44       | 27.94       | 0.256                                    | 0.168       |
| Inferior frontal gyrus, pars opercularis               | 2335  | 39.88  | 31.06       | 45.96       | 0.510                                    | 0.372       |
| Precentral gyrus                                       | 13967 | 68.47  | 59.96       | 73.72       | 1.158                                    | 0.915       |
| Temporal pole                                          | 8044  | 9.60   | 5.41        | 13.48       | 0.101                                    | 0.056       |
| Superior temporal gyrus, anterior division             | 916   | 1.77   | 0.80        | 2.87        | 0.018                                    | 0.008       |
| Superior temporal gyrus, posterior division            | 2897  | 8.89   | 5.46        | 11.91       | 0.093                                    | 0.056       |
| Middle temporal gyrus, anterior division               | 1425  | 1.64   | 0.55        | 2.88        | 0.017                                    | 0.005       |
| Middle temporal gyrus, posterior division              | 4206  | 9.37   | 5.18        | 12.76       | 0.099                                    | 0.053       |
| Middle temporal gyrus, temporooccipital part           | 3202  | 12.84  | 6.85        | 17.63       | 0.138                                    | 0.071       |
| Inferior temporal gyrus, anterior division             | 1287  | 1.38   | 0.38        | 2.56        | 0.014                                    | 0.004       |
| Inferior temporal gyrus, posterior division            | 4138  | 5.43   | 2.80        | 8.12        | 0.056                                    | 0.028       |
| Inferior temporal gyrus, temporooccipital part         | 2605  | 11.49  | 6.16        | 16.05       | 0.122                                    | 0.064       |
| Postcentral gyrus                                      | 10638 | 24.29  | 17.60       | 29.24       | 0.279                                    | 0.194       |
| Superior parietal lobule                               | 4489  | 36.16  | 26.16       | 42.31       | 0.450                                    | 0.303       |
| Supramarginal gyrus, anterior division                 | 2910  | 13.19  | 7.98        | 17.24       | 0.142                                    | 0.083       |
| Supramarginal gyrus, posterior division                | 4071  | 28.38  | 19.69       | 33.72       | 0.335                                    | 0.219       |
| Angular gyrus                                          | 3703  | 21.69  | 14.39       | 26.34       | 0.245                                    | 0.155       |
| Anatomical Region                                      | Pixel Value (Mean) | Surface Area (%) | Volume (mm³) |
|-------------------------------------------------------|--------------------|------------------|--------------|
| Lateral occipital cortex, superior division           | 14484              | 61.52            | 53.01        |
|                                                       |                    | 67.73            | 0.959        |
| Lateral occipital cortex, inferior division           | 7490               | 27.53            | 18.66        |
|                                                       |                    | 33.57            | 0.323        |
| Intracalcarine cortex                                | 2211               | 8.36             | 4.35         |
|                                                       |                    | 11.76            | 0.087        |
| Frontal medial cortex                                | 1539               | 1.94             | 0.64         |
|                                                       |                    | 3.62             | 0.020        |
| Juxtapositional lobule cortex                         | 2282               | 26.46            | 16.14        |
|                                                       |                    | 32.74            | 0.308        |
| Subcallosal cortex                                   | 2176               | 4.47             | 1.73         |
|                                                       |                    | 7.16             | 0.046        |
| Paracingulate gyrus                                  | 4095               | 46.22            | 35.94        |
|                                                       |                    | 52.89            | 0.622        |
| Cingulate gyrus, anterior division                    | 4144               | 22.61            | 14.76        |
|                                                       |                    | 28.19            | 0.257        |
| Cingulate gyrus, posterior division                   | 4668               | 11.07            | 5.70         |
|                                                       |                    | 15.52            | 0.118        |
| Precuneous cortex                                    | 7844               | 26.46            | 18.08        |
|                                                       |                    | 32.62            | 0.308        |
| Cuneal cortex                                         | 1743               | 7.75             | 4.09         |
|                                                       |                    | 10.95            | 0.081        |
| Frontal orbital cortex                                | 5188               | 36.94            | 27.27        |
|                                                       |                    | 43.06            | 0.462        |
| Parahippocampal gyrus, anterior division              | 3313               | 5.69             | 2.94         |
|                                                       |                    | 8.43             | 0.059        |
| Parahippocampal gyrus, posterior division             | 2014               | 6.01             | 3.23         |
|                                                       |                    | 8.71             | 0.062        |
| Lingual gyrus                                         | 5388               | 15.62            | 9.12         |
|                                                       |                    | 20.15            | 0.170        |
| Temporal fusiform cortex, anterior division           | 1243               | 1.21             | 0.39         |
|                                                       |                    | 2.22             | 0.012        |
| Temporal fusiform cortex, posterior division          | 2951               | 5.72             | 3.10         |
|                                                       |                    | 8.42             | 0.059        |
| Temporal occipital fusiform cortex                    | 2458               | 13.65            | 8.21         |
|                                                       |                    | 17.69            | 0.147        |
| Occipital fusiform gyrus                              | 3587               | 23.35            | 15.64        |
|                                                       |                    | 28.33            | 0.266        |
| Frontal operculum cortex                              | 1062               | 16.48            | 9.97         |
|                                                       |                    | 20.60            | 0.180        |
| Central opercular cortex                              | 2578               | 11.08            | 6.70         |
|                                                       |                    | 14.67            | 0.118        |
| Parietal opercular cortex                             | 1684               | 6.23             | 3.23         |
|                                                       |                    | 8.92             | 0.064        |
| Planum polare                                         | 1210               | 2.64             | 1.29         |
|                                                       |                    | 3.94             | 0.027        |
| Heschl’s gyrus                                        | 786                | 2.51             | 1.08         |
|                                                       |                    | 3.89             | 0.025        |
| Planum temporale                                      | 1442               | 6.05             | 3.26         |
|                                                       |                    | 8.45             | 0.063        |
| Supracalcarine cortex                                 | 424                | 1.52             | 0.66         |
|                                                       |                    | 2.41             | 0.015        |
| Occipital pole                                        | 9658               | 20.42            | 12.04        |
|                                                       |                    | 25.75            | 0.229        |
| Left cerebral white matter                            | 28034              | 81.33            | 75.05        |
|                                                       |                    | 85.13            | 1.684        |
| Left cerebral cortex                                  | 82249              | 99.04            | 98.05        |
|                                                       |                    | 99.44            | 4.679        |
| Left lateral ventricle                                | 1289               | 4.10             | 1.96         |
|                                                       |                    | 6.02             | 0.042        |
| Region                        | Value1 | Value2 | Value3 | Value4 | Value5 | Value6 |
|-------------------------------|--------|--------|--------|--------|--------|--------|
| Left thalamus                 | 1591   | 16.17  | 9.31   | 21.64  | 0.177  | 0.098  | 0.244  |
| Left caudate                  | 572    | 3.60   | 1.75   | 5.44   | 0.037  | 0.018  | 0.056  |
| Left putamen                  | 923    | 7.83   | 3.91   | 10.96  | 0.082  | 0.040  | 0.116  |
| Left pallidum                 | 312    | 5.15   | 2.58   | 7.59   | 0.053  | 0.026  | 0.079  |
| Brain stem                    | 8078   | 13.45  | 7.80   | 18.29  | 0.145  | 0.081  | 0.202  |
| Left hippocampus              | 921    | 3.38   | 1.21   | 5.37   | 0.034  | 0.012  | 0.055  |
| Left amygdala                 | 390    | 1.41   | 0.48   | 2.43   | 0.014  | 0.005  | 0.025  |
| Left accumbens                | 111    | 0.88   | 0.26   | 1.57   | 0.009  | 0.003  | 0.016  |
| Right cerebral white matter   | 31216  | 79.16  | 72.13  | 83.13  | 1.573  | 1.278  | 1.780  |
| Right cerebral cortex         | 86480  | 98.78  | 97.36  | 99.24  | 4.431  | 3.636  | 4.874  |
| Right lateral ventricle       | 1019   | 3.36   | 1.77   | 4.97   | 0.034  | 0.018  | 0.051  |
| Right thalamus                | 1398   | 12.88  | 7.12   | 17.43  | 0.138  | 0.074  | 0.192  |
| Right caudate                 | 515    | 2.68   | 1.02   | 4.30   | 0.027  | 0.010  | 0.044  |
| Right putamen                 | 800    | 6.82   | 3.39   | 9.80   | 0.071  | 0.034  | 0.103  |
| Right pallidum                | 266    | 3.24   | 1.08   | 5.19   | 0.033  | 0.011  | 0.053  |
| Right hippocampus             | 772    | 1.78   | 0.67   | 2.99   | 0.018  | 0.007  | 0.030  |
| Right amygdala                | 399    | 0.86   | 0.19   | 1.73   | 0.009  | 0.002  | 0.017  |
| Right accumbens               | 86     | 0.64   | 0.15   | 1.26   | 0.006  | 0.001  | 0.013  |
Table 3: Working memory CBMA results. Table presents posterior summaries for the % probability of a reported activation in studies using verbal and non-verbal stimuli. All quantities have been calculated based on 1,000 MCMC samples.

| ROI                              | Vol.  | Verbal       | Non-verbal   |
|----------------------------------|-------|--------------|--------------|
|                                  |       | Mean | p0.025 | p0.975 | Mean | p0.025 | p0.975 |
| Frontal pole                     | 25900 | 59.99 | 46.81  | 67.86  | 60.24 | 41.75  | 69.92  |
| Insular cortex                   | 3613  | 32.79 | 21.14  | 40.70  | 33.86 | 20.43  | 43.28  |
| Superior frontal gyrus           | 8861  | 29.45 | 18.36  | 36.70  | 45.87 | 29.60  | 54.29  |
| Middle frontal gyrus             | 8421  | 53.05 | 42.42  | 60.85  | 66.66 | 50.91  | 74.55  |
| Inferior frontal gyrus, pars triangularis | 2317  | 19.43 | 10.84  | 24.95  | 25.44 | 14.64  | 33.90  |
| Inferior frontal gyrus, pars opercularis | 2335  | 43.66 | 31.68  | 51.15  | 35.69 | 21.49  | 44.80  |
| Precentral gyrus                 | 13967 | 64.10 | 51.56  | 71.00  | 72.09 | 57.82  | 79.52  |
| Temporal pole                    | 8044  | 7.54  | 2.91   | 11.50  | 11.58 | 4.28   | 18.15  |
| Superior temporal gyrus, anterior division | 916   | 2.31  | 0.71   | 4.05   | 1.23  | 0.27   | 2.52   |
| Superior temporal gyrus, posterior division | 2897  | 10.76 | 5.31   | 15.14  | 6.95  | 2.25   | 11.32  |
| Middle temporal gyrus, anterior division | 1425  | 2.19  | 0.48   | 4.36   | 1.09  | 0.15   | 2.56   |
| Middle temporal gyrus, posterior division | 4206  | 9.58  | 4.59   | 14.22  | 9.12  | 3.83   | 14.44  |
| Middle temporal gyrus, temporooccipital part | 3202  | 11.88 | 5.65   | 17.75  | 13.73 | 5.73   | 21.06  |
| Inferior temporal gyrus, anterior division | 1287  | 1.76  | 0.31   | 3.69   | 0.99  | 0.10   | 2.53   |
| Inferior temporal gyrus, posterior division | 4138  | 4.85  | 1.59   | 8.09   | 5.98  | 1.47   | 10.31  |
| Inferior temporal gyrus, temporooccipital part | 2605  | 7.45  | 3.00   | 11.83  | 15.31 | 6.95   | 22.66  |
| Postcentral gyrus                | 10638 | 20.37 | 11.48  | 26.10  | 27.95 | 16.64  | 35.68  |
| Superior parietal lobule         | 4489  | 38.81 | 26.05  | 46.48  | 33.24 | 19.67  | 42.96  |
| Supramarginal gyrus, anterior division | 2910  | 12.78 | 5.90   | 17.83  | 13.56 | 6.23   | 19.83  |
| Supramarginal gyrus, posterior division | 4071  | 29.93 | 19.40  | 36.61  | 26.70 | 14.58  | 35.17  |
| Angular gyrus                    | 3703  | 24.30 | 15.51  | 31.09  | 18.91 | 9.32   | 25.61  |
| Lateral occipital cortex, superior division | 14484 | 55.45 | 42.51  | 63.87  | 66.55 | 52.60  | 74.58  |
| Lateral occipital cortex, inferior division | 7490 | 23.69 | 14.54 | 30.51 | 31.06 | 18.71 | 40.58 |
| Intracalcarine cortex | 2211 | 6.51 | 2.61 | 10.12 | 10.15 | 4.37 | 15.19 |
| Frontal medial cortex | 1539 | 1.21 | 0.24 | 2.62 | 2.66 | 0.60 | 5.58 |
| Juxtaositional lobule cortex | 2282 | 21.21 | 12.33 | 28.23 | 31.23 | 18.26 | 41.74 |
| Subcallosal cortex | 2176 | 4.20 | 1.08 | 7.66 | 4.72 | 1.03 | 9.24 |
| Paracingulate gyrus | 4095 | 42.91 | 29.21 | 51.04 | 49.14 | 33.89 | 59.57 |
| Cingulate gyrus, anterior division | 4144 | 17.69 | 10.95 | 24.25 | 27.15 | 14.86 | 36.16 |
| Cingulate gyrus, posterior division | 4668 | 8.06 | 3.60 | 12.42 | 13.93 | 5.98 | 21.66 |
| Precuneous cortex | 7844 | 22.88 | 12.61 | 30.03 | 29.76 | 18.38 | 39.96 |
| Cuneal cortex | 1743 | 6.71 | 1.97 | 10.43 | 8.76 | 3.29 | 14.54 |
| Frontal orbital cortex | 5188 | 37.26 | 25.37 | 45.24 | 36.48 | 24.30 | 45.43 |
| Parahippocampal gyrus, anterior division | 3313 | 5.93 | 2.24 | 9.59 | 5.44 | 1.65 | 9.35 |
| Parahippocampal gyrus, posterior division | 2014 | 5.80 | 2.27 | 9.47 | 6.21 | 2.46 | 10.81 |
| Lingual gyrus | 5388 | 15.61 | 7.56 | 21.30 | 15.56 | 7.52 | 22.72 |
| Temporal fusiform cortex, anterior division | 1243 | 1.52 | 0.33 | 3.13 | 0.90 | 0.18 | 2.11 |
| Temporal fusiform cortex, posterior division | 2951 | 5.26 | 2.06 | 8.46 | 6.17 | 2.13 | 10.49 |
| Temporal occipital fusiform cortex | 2458 | 8.87 | 4.19 | 13.17 | 18.12 | 8.94 | 26.00 |
| Occipital fusiform gyrus | 3587 | 26.68 | 17.03 | 33.74 | 19.78 | 10.47 | 27.65 |
| Frontal operculum cortex | 1062 | 15.27 | 9.08 | 20.31 | 17.64 | 9.10 | 23.87 |
| Central opercular cortex | 2578 | 10.04 | 4.95 | 14.41 | 12.08 | 5.64 | 17.46 |
| Parietal operculum cortex | 1684 | 7.35 | 2.65 | 11.29 | 5.08 | 1.56 | 9.05 |
| Planum polare | 1210 | 2.37 | 0.79 | 4.02 | 2.91 | 0.98 | 5.02 |
| Heschl’s gyrus | 786 | 3.07 | 0.99 | 5.47 | 1.94 | 0.34 | 3.79 |
| Planum temporale | 1442 | 7.38 | 3.50 | 10.90 | 4.69 | 1.55 | 8.26 |
| Supracalcarine cortex | 424 | 0.95 | 0.29 | 1.74 | 2.07 | 0.51 | 3.64 |
| Occipital pole | 9658 | 19.84 | 10.66 | 26.91 | 20.91 | 11.10 | 29.20 |
| Left cerebral white matter | 28034 | 76.33 | 66.31 | 81.39 | 85.13 | 75.35 | 89.94 |
| Left cerebral cortex | 82249 | 98.88 | 97.31 | 99.41 | 99.14 | 97.59 | 99.63 |
| Left lateral ventricle | 1289 | 3.12 | 1.30 | 5.16 | 5.05 | 1.65 | 8.64 |
| Left thalamus | 1591 | 14.56 | 7.44 | 20.87 | 17.69 | 7.35 | 26.38 |
| Region                        | Volume | Mean 1 | Mean 2 | Mean 3 | SD 1   | SD 2   | SD 3   |
|-------------------------------|--------|--------|--------|--------|--------|--------|--------|
| Left caudate                  | 572    | 3.49   | 1.25   | 5.71   | 3.69   | 1.09   | 6.53   |
| Left putamen                  | 923    | 6.97   | 2.01   | 11.15  | 8.67   | 2.25   | 13.88  |
| Left pallidum                 | 312    | 6.26   | 2.38   | 10.29  | 4.01   | 1.24   | 7.29   |
| Brain stem                    | 8078   | 10.42  | 5.26   | 15.72  | 16.33  | 6.90   | 25.06  |
| Left hippocampus              | 921    | 3.46   | 1.08   | 6.28   | 3.29   | 0.74   | 6.39   |
| Left amygdala                 | 390    | 1.21   | 0.16   | 2.59   | 1.60   | 0.25   | 3.37   |
| Left accumbens                | 111    | 1.07   | 0.15   | 2.29   | 0.69   | 0.09   | 1.49   |
| Right cerebral white matter   | 31216  | 73.44  | 65.11  | 78.96  | 83.50  | 74.00  | 88.46  |
| Right cerebral cortex         | 86480  | 98.22  | 96.50  | 98.97  | 99.12  | 97.60  | 99.61  |
| Right lateral ventricle       | 1019   | 2.66   | 0.96   | 4.35   | 4.05   | 1.21   | 6.74   |
| Right thalamus                | 1398   | 15.17  | 8.00   | 21.79  | 10.48  | 3.36   | 16.86  |
| Right caudate                 | 515    | 2.51   | 0.76   | 4.45   | 2.85   | 0.68   | 5.34   |
| Right putamen                 | 800    | 5.91   | 1.94   | 9.49   | 7.69   | 2.56   | 12.98  |
| Right pallidum                | 266    | 2.74   | 0.65   | 4.97   | 3.73   | 0.93   | 6.91   |
| Right hippocampus             | 772    | 1.77   | 0.52   | 3.42   | 1.78   | 0.46   | 3.55   |
| Right amygdala                | 399    | 0.86   | 0.11   | 2.03   | 0.87   | 0.13   | 2.17   |
| Right accumbens               | 86     | 0.78   | 0.13   | 1.84   | 0.50   | 0.08   | 1.30   |
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