Effect of the Texture and the Orientation Relation between Neighboring Grains on the Mechanical Behavior of Titanium Sheets*

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Uniaxial tensile tests and hydraulic bulge tests were carried out for two differently textured sheets of pure titanium. The sheets were previously examined for the orientation distribution of grains, containing the orientation relation between neighboring grains. The deformation behavior of the titanium sheets was examined in connection with the effect of the orientation difference between neighboring grains on the deformation mechanism of grains.

In the case of the uniaxial tensile test, the correlation in the 0.2% proof stress among differently textured sheets and different test directions can be explained by the preferred orientation of the sheets. However, the plastic flow curves are influenced by the orientation difference between neighboring grains. In the sheets rolled unidirectionally at 1173 K, the orientation difference of the c-axis between neighboring grains is distributed broadly in the whole range from 0 to π/2. The anisotropy of the flow stress is smaller than that of the 0.2% proof stress, and the slope of the stress to the strain is larger than that of the sheets cross rolled at 873 K.

Stress-strain curves and effective stress-effective strain curves of two differently textured titanium sheets, which were estimated from the hydraulic bulge test using the Hill's theory for the anisotropy, were examined. Since the preferred orientation in the sheets cross-rolled at 873 K is (0001)[1010] and the orientation difference of the c-axis between neighboring grains is small, the deformation mechanism of grains under the biaxial tension is extremely different from that under the uniaxial tension. Therefore, as pointed out previously, the overestimation of the anisotropy by the r value takes place. Though the r values of the sheets rolled unidirectionally at 1173 K are relatively high (from 2 to 5), the difference in the deformation mechanism of grains between different loading conditions is small because of the plastic constraint among neighboring grains. As a result, the estimation of the anisotropy by the r value is not so irrelevant.

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I. Introduction

Pure titanium has an hcp structure at room temperature and slip directions of slip systems are all on the basal plane(α). Therefore, the mechanical behavior varies remarkably depending on crystallographic textures. The correlation between the deformation behavior and the texture of titanium sheets has very frequently been studied in connection with the deformability of sheets(β-γ). In such investigations, the attention has been fixed mostly on the plastic anisotropy normal to the sheet plane, which is represented as the ratio of the width to thickness strains, r value, and satisfactory correlation has been recognized between the r value and the drawability or the stretchability. However, it is questionable that the r value, which indicates in a qualitative way the relative extent of normal anisotropy under the uniaxial tension, is an appropriate parameter for the deformation behavior of textured titanium under multiaxial stresses. In fact, plastic anisotropy of titanium sheets was suggested to depend on the stress state and the amount of deformation(δ).

Anisotropic plastic properties may depend not only on the preferred orientations but also...
on the orientation relation between neighboring grains. However, the pole figure, which is a conventional manner to present the texture, contains no knowledge of the orientation relation between neighboring grains. The investigations, which give attention to the effect of the orientation relation between neighboring grains on the deformation behaviors, are few. Only Wilson et al. studied the effect of the orientation distribution on the surface roughness during stretch forming\(^6\). So it is significant that the anisotropic deformation behaviors of textured sheets are reinvestigated in consideration of the aggregate states of grains, the deformation characteristics of individual grains and the stress state.

In the present experiment, uniaxial tensile tests and hydraulic bulge tests were carried out on two differently textured sheets of pure titanium. The sheets were previously examined for the orientation distribution of grains, containing the orientation relation between neighboring grains. The deformation behavior of titanium sheets were studied in connection with the effect of the orientation difference between neighboring grains on the deformation characteristics of grains. Consequently, it was shown evidently that the frequency of occurrence of deformation twins and so the deformation behavior of titanium sheets were influenced by both of the orientation distribution between neighboring grains and the stress state.

II. Materials

In this experiment, two differently textured titanium sheets were used\(^7\). They were hot rolled sheets 5 mm thick. One was rolled unidirectionally at 1173 K and the other was cross rolled at 873 K. The total reduction was about 77\%. The rolled sheets were pickled and annealed in vacuum for 7.2 ks at 1073 K. The average grain size was 50 \(\mu\)m for the unidirectionally rolled sheet and 46 \(\mu\)m for the cross rolled sheet. The thickness of specimen for tensile tests was 1.2 mm.

In the case of the cross rolled sheet, 61\% of the grains had the c-axis normal to the sheet plane, and the ratio of grains of the (0001) [10\(\bar{1}\)0] orientation to the total were 23.1\% as reported previously\(^7\). In the case of the unidirectionally rolled sheet, 50\% of the grains had the c-axis parallel to the sheet plane, and the ratio of grains having the orientation ranging from (10\(\bar{1}\)0)[1125] to (10\(\bar{1}\)0)[112 17] were 12.5\%. Figure 1 shows the angle, \(\theta\), between the c-axis of the grains having preferred orientation and that of their neighboring grains. The frequency of the angle, \(\theta\), in the cross rolled sheet was 93.2\% in the range from 0 to \(\pi/18\). In contrast to it, the angle, \(\theta\), was distributed in the whole range from 0 to \(\pi/2\) in the unidirectionally rolled one, although there was a slight increase in the range from 0 to \(\pi/18\).

III. Procedure

Uniaxial tensile tests and hydraulic bulge tests were carried out for the two differently textured titanium sheets.

The width of the uniaxial tensile test specimen was 3 mm and the length of the parallel portion was 9 mm. The tensile speed was \(8.3 \times 10^{-3}\) mm/s. Every time the specimen extended by a constant elongation of 0.5 mm between the chucks, the strains of the longitudinal and transverse directions and the cross-sectional area were measured. Then, the \(r\) value and the flow stress were calculated. The tensile directions are the rolling direction, the
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Transverse direction and the direction making $\pi/4$ to the rolling direction.

A circular die, of which the internal diameter was 50 mm, was used in the hydraulic bulge test. The bulging pressure and the bulge height were measured by a pressure transducer with wire resistance strain guages and a displacement gauge of a differential transformer, respectively. The bulging pressure and the bulge height were recorded on an X-Y recorder. The test speed was 0.05 mm/s for the bulge height.

The radius of curvature at the top of a bulged sheet was measured for both the rolling and transverse directions. As shown in Fig. 2, the origin was fined at a point which was regarded as the top. For example, to measure the radius of curvature for the rolling direction, the rolling direction and the perpendicular one were designated as the abscissa and the ordinate, respectively. At distances of 2 mm, 3 mm, 4 mm, 5 mm, 7 mm and 9 mm from the origin to the positive and negative directions of rolling, the differences of height, $h$, and $h_r$, were measured. Then, the radii of the curvature were calculated by the circular approximation for three points, $(0, -r')$, $(r, h_r-r')$ and $(-r, h_r-r')$. The radius of curvature at the true top was determined by the extrapolation for the radii of curvature at 2 mm, 3 mm, 4 mm, 5 mm, 7 mm and 9 mm. In the case of the transverse direction, this direction was designated as the abscissa. In such a manner, the radii of curvature at the top for any bulge height were determined in both the directions of rolling and transverse. The measurement accuracy was 0.001 mm.

IV. Results and Discussion

1. Uniaxial tensile tests

Figure 3 shows the results of uniaxial tensile tests. The flow stress of the unidirectionally rolled sheet is much higher than that of the cross rolled sheet at every tensile direction. In

Fig. 2 The procedure to obtain the radius of curvature, \( \rho \). \( \rho \) is calculated from the coordinates of the point, 0. The coordinates of the point, 0, are calculated by the circular approximation of three points, \((0, -r')\), \((-r, h_r-r')\) and \((r, h_r-r')\).

Fig. 3 Stress-strain curves under uniaxial tension.
the unidirectionally rolled sheet, the work hardening rate is also higher than that in the cross rolled sheet till the tensile strain of 0.05.

In the case of the unidirectionally rolled sheet, the 0.2% proof stress in the rolling direction is smaller than in the others. However, the work hardening rate is higher than and the flow stress exceeds that in the others at tensile strains over 0.15. The profiles of the stress-strain curves for the specimens tested in the transverse direction and the /\(\pi/4\) direction to rolling are similar, although the flow stress in the transverse direction is higher as a whole.

In the case of the cross rolled sheet, the 0.2% proof stress has no directionality. Moreover, the profiles of the stress-strain curves are the same in any tensile direction, except that the tensile stress in the rolling direction is slightly higher as a whole.

As a result, the 0.2% proof stress seems to depend mainly on the preferred orientation. In the cross rolled sheet, the ratio of grains having the c-axis normal to the sheet plane is 61%, and then the slip directions of any slip system are parallel to the sheet plane. The slip direction, [1120], is distributed uniformly in every direction in the sheet plane, since (0001)[10\(\bar{1}0\)] is 23.1%, (0001)[1120] is 9.3%, (0001)[41\(\bar{5}0\)] is 8.2%, (0001)[1230] is 7.7%, (0001)[2130] is 5.0% and (0001)[1450] is 7.7%. Therefore, the deformation by slip is easy and the anisotropy in the sheet plane is very small in the cross rolled sheet.

In the case of the unidirectionally rolled sheet, the ratio of grains with the c-axis parallel to the sheet plane is 50%, and their pole of the c-axis is distributed in the range from \(-5\pi/18\) to \(\pi/6\) around the transverse direction. The preferred orientation of this sheet is ranging from (10\(\bar{1}0\))[1125] to (10\(\bar{1}0\) [112 17], and it's ratio is 12.5%. Therefore, the tensile direction which is favourable to the slip deformation is the rolling direction, followed by the \(\pi/4\) direction to the rolling direction and next by the transverse direction. If this sheet has a high proportion of grains with their basal poles parallel to the transverse direction, the 0.2% proof stress in the rolling direction will be close to that of the cross rolled sheet. However, the actual texture of the unidirectionally rolled sheet has a large degree of scatter about the preferred orientation, and the angle, \(\theta\), is distributed broadly. Therefore the proof stress of this sheet is higher than that of the cross rolled one.

In the range of the tensile strains from 0.002 to 0.05, the flow stress of the unidirectionally rolled sheet increases remarkably over that of the cross rolled one. The flow stress of the unidirectionally rolled sheet at the tensile strains over 0.05 tends to have a smaller dependence on the tensile direction than the 0.2% proof stress.

In the practical deformation, individual grains are deformed interfering with each other between neighboring grains, and also the orientation may change momentarily by the slip and the deformation twin. Therefore, the anisotropy represented by the \(r\) value also changes with the amount of deformation. Figure 4 shows the correlation between the \(r\) and the deformation.

\[
r = \ln \left( \frac{w}{w_0} \right) / \ln (t/t_0) = - \ln \left( \frac{w}{w_0} \right) / (\ln (t/t_0) + \ln (L/L_0)).
\]

Fig. 4 Variation of \(r\) values with deformation.
A19: conventional rolling 77% at 1173 K.
A26: cross rolling 77% at 873 K.
value and the tensile strain. The $r$ value was calculated by the following equation.

$$ r = -\varepsilon_w / (\varepsilon_w + \varepsilon_t), \quad (1) $$

where $\varepsilon_t$ is the tensile strain and $\varepsilon_w$ the lateral strain. The line in the figure shows the variation of $r$ values approximated by representing the lateral strain, $\varepsilon_w$, by the following equation.

$$ \varepsilon_w = a_1 \varepsilon_t + a_2 \varepsilon_t^2. \quad (2) $$

As shown in Fig. 4, the $r$ value decreases with the deformation. In the case of the cross rolled sheet, the $r$ value is extremely high in every direction and so they scatter largely. Although the $r$ value decreases with the strain, it is over 7 even at tensile strain of 0.2. It is suggested that the orientation distribution of grains giving high $r$ value will be maintained during the deformation.

The $r$ value of the cross rolled sheet is over 10 at small strains in every tensile direction. It shows a tendency to decrease with the tensile direction in the order of the rolling, the $\pi/4$ to the rolling and the transverse direction. In the case of the unidirectionally rolled sheet, the $r$ value is highest in the transverse direction, next in the $\pi/4$ to the rolling and lowest in the rolling direction. Although the $r$ value in the rolling direction is expected to be small in view of the preferred orientation, it is a fairly high value of about 2.0.

2. Hydraulic bulge tests

For an isotropic sheet, the principle stress in the sheet plane under the balanced biaxial stress condition in the hydraulic bulge test is presented by the following equation(5).

$$ \sigma = \bar{\rho} P / 2t, \quad (3) $$

where $\bar{\rho}$ is the radius of the curvature at the top and constant in any direction, $P$ is the bulging pressure and $t$ is the sheet thickness at the top.

For an anisotropic sheet, the principle stresses in the plane, $\sigma_r$ and $\sigma_t$, can be calculated by the equations from (4) to (8) proposed by Goto(8), using the Hill's theory(9) for an anisotropic material.

$$ m = \sigma_r / \sigma_t. \quad (4) $$

$$ \rho = \rho_r / \rho_t. \quad (5) $$

$$ m = [1 + (1/r)] \rho / [1 + (1/r) + \rho]. \quad (6) $$

$$ \sigma_r = (P/t) \rho_r [1 + (1 + \rho)] / [1 + (1/r)] \rho^2 + 2\rho. \quad (7) $$

$$ \sigma_t = (P/t) \rho_t [1 + (1 + r)] + \rho / [1 + (1/r)] \rho^2 + 2\rho. \quad (8) $$

Here, the rolling direction ($r$) and the transverse direction ($t$) are regarded as the principle axis of anisotropy, and $\sigma_r$, $\sigma_t$, $\rho_r$, and $\rho_t$ are principle stresses, radii of the curvature at the top and $r$ values in the rolling and transverse directions, respectively.

Figure 5 shows the stress-strain curves in the hydraulic bulging test. The results of the uniaxial tensile test are represented together in comparison with each other. The strain on the abscissa is calculated from the thickness strain using a constant rule for volume on the assumption that the strains in the rolling and transverse directions at the top are equal.

The principle stress, $\sigma$, obtained from the eq. (3), for the cross rolled sheet is not presented, but it lies between $\sigma_r$ and $\sigma_t$ in the
same manner as in the case of the unidirectionally rolled sheet. The principle stress assuming the isotropy, $\sigma$, is taken as an average value of the principle stresses in the sheet plane. The levels of the stress-strain curves are higher under biaxial tension than under uniaxial tension for both textured sheets.

It is well-known that such textured sheets having the c-axis parallel to the sheet normal direction as the cross rolled sheet are most suitable for texture hardening of titanium alloys$^{(10)}$. However, in pure titanium sheets the flow stress under the biaxial tensile condition does not reflect the difference in texture. Probably, the occurrence of deformation twinning under biaxial tension is expected to weaken the effect of textures on the flow stress. Especially, in the cross rolled sheet, the $\{11\bar{2}2\}$ deformation twin is likely to act under biaxial tension$^{(11)}$ because of the absence of suitably oriented slip systems, although the slip deformation is preferential under uniaxial tension.

In the cross rolled sheet, the difference between the principle stresses in the plane is small under biaxial tension as well as under uniaxial tension. On the other hand, the difference between $\sigma_n$ and $\sigma_t$ under biaxial tension is large in the unidirectionally rolled sheet. In both textured sheets, the slopes of the stress-strain curves in the biaxial tensile test are larger than in the uniaxial tensile test. In the biaxial tensile test, the slopes of the cross rolled sheet are slightly larger than in the unidirectionally rolled sheet.

Figure 6 shows the variation of the stress ratio, $m$, during the bulge deformation. In the cross rolled sheets, $m$ is close to 1, and a nearly balanced biaxial stress condition is maintained during deformation. However, in the unidirectionally rolled sheet, $m$ is about 0.87 during the deformation, and the stress condition is a non-balanced biaxial one, as expected from Fig. 5. In the hydraulic bulge test, the stress condition is different between the cross rolled sheet and the unidirectionally rolled sheet.

Figure 7 shows the equivalent stress-equivalent strain curves. Yoshida et al. indicated that the estimation of the anisotropy under the plane stress condition by $r$ values is erroneous with the exception of mild steels$^{(10)}$.

In the cross rolled sheet, the equivalent stress-equivalent strain curve is considerably lower in the biaxial tensile test than in the uniaxial tensile test. It is apparent as indicated by Yoshida et al. that the anisotropic plasticity under biaxial tension is overestimated by using $r$ values. However, in the case of the unidirectionally rolled sheet, of which the $r$ values are high ranging from 2 to 5, the equivalent stress-equivalent strain under the biaxial tension is comparable to the one under uniaxial tension.
The change in the deformation characteristics of crystal grains with the loading condition seems to have an effect on the anisotropic plasticity. In the unidirectionally rolled sheet, the angle $\theta$ is distributed broadly, as shown in Fig. 1. Therefore, the plastic constraint among neighboring grains is large even in the uniaxial tensile deformation, and then the change in the deformation characteristic of grains with the loading condition is not so large. In such a case, the $r$ value can be used as a parameter representing the plastic anisotropy. However, it is not applicable to a sheet of which the plastic behaviors vary fairly depending on the loading condition.

3. Results of observation of structure

Figure 8 shows microphotographs of the unidirectionally rolled sheet under the polarized light. Figure 8-a shows a microstructure of the annealed sheet. Figure 8-b is a micrograph of the specimen extended uniaxially parallel to the rolling direction, and Fig. 8-c the one of the top of the bulged specimen. The equivalent strain is about 0.15. Many grains reveal the deformation twins in both specimens extended uni- and bi-axially. In the case of the unidirectionally rolled sheet, the orientation difference in the c-axis between neighboring grains is distributed broadly in the range from 0 to $\pi/2$ as shown in Fig. 1, so that the plastic constraint by neighboring grains is large and introduces deformation twinning even into the uniaxial deformation.

Figure 9 shows microstructures in the cross rolled sheet. Figure 9-a represents a nondeformed structure. Figure 9-b is a micrograph of the specimen elongated uniaxially parallel to the rolling direction. Figure 9-c is a micrograph of the top of the hydraulically bulged specimen. Grains with the deformation twins are few in the uniaxially extended specimen (Fig. 9-b), and the width of such a deformation twin is narrow. On the other hand, deformation twins are observed in all grains in the biaxially stretched specimen (Fig. 9-c), and also the width of a deformation twin is wide. Moreover, deformation twins seem to develop through several grains. Thus, the deformation characteristics of grains change extremely with the loading condition in the cross rolled sheet. On the one hand, the texture change during the deformation would be relatively small in the uniaxial tensile deformation, while it would be large due to the occurrence of deformation twinning in the biaxial tensile deformation.

Figure 10 shows the deformation structures.
in both textured sheets elongated parallel to the transverse direction. In the unidirectionally rolled sheet, the amount of deformation twinning reduces as compared with the specimen extended uniaxially parallel to the rolling direction or biaxially. In the cross rolled sheet, the deformation structure is almost similar in-

Okubo et al.\cite{12} and Mae et al.\cite{13} showed that deformation twins were observed more in the specimen extended uniaxially parallel to the rolling direction than parallel to the transverse one, using two differently textured sheets. Both sheets had a peak of the density of the basal pole at about $\pi/6$ from the sheet normal direction toward the transverse direction, but the degree of the accumulation was different. Fishburn indicated that the deformation twins were able to be observed in the specimen elongated parallel to the transverse direction but not to the rolling one, using the textured sheets with the high peak of the density of the basal pole parallel to the transverse direction\cite{11}.

A peak of the density of the basal pole of the preferred orientation of the unidirectionally rolled sheet used in the present experiment is

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig9.png}
\caption{Micrographs of the section parallel to the sheet plane. The sheets are vacuum annealed for 7.2 ks at 1073 K after cross rolling 77\% at 873 K.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig10.png}
\caption{Micrographs of the deformed uniaxial tensile test pieces in the section parallel to the sheet plane. The tensile direction is $\pi/2$ to the rolling direction.}
\end{figure}
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close to the transverse direction, but the deformation twins are observed in any tensile direction, and the number of them is larger in the specimen elongated parallel to the rolling direction than to the transverse one.

Figure 11 shows micrographs of the unidirectionally rolled sheet, which extended uniaxially up to the tensile strain of about 0.07. Before deformation, etch pits were revealed on the specimen and the orientations of grains were previously measured. In the tensile test parallel to the rolling direction, the deformation twins occurred frequently in grains having the c-axis close to the sheet normal direction. Figure 11-a and c are such examples. Since a grain having the c-axis close to the transverse direction elongates parallel to the tensile direction and shrinks parallel to the thickness direction by slip, the neighboring grain having the c-axis close to the sheet normal direction must conform itself to such deformation by the deformation twin which permits thinning parallel to the c-axis direction. In the tensile deformation parallel to the transverse direction, few deformation twins were observed in the grains having the c-axis close to the sheet plane normal. The amount of the strain parallel to the thickness direction of many grains with the c-axis close to the transverse direction would be relatively small as expected from the r value, so that the constraint in grains having the c-axis close to the sheet plane normal would be small. In the tensile test parallel to the transverse direction, examples in which the deformation twins occur in grains having the c-axis close to the tensile direction are most (Fig. 11-b and d), because of the absence of suitably oriented slip systems. In the case of Fishburn et al., the degree of accumulation of the textured sheets seems to be so high that they can be treated as single crystals.

In the biaxial tensile test, it is easily guessed that the grains are subjected to tension or compression parallel to the c-axis direction owing to the loading condition and so the deforma-
tion twins occur in many grains. However, also in this case, it is regarded that the deformation mechanism of individual grains is constrained not only by the preferred orientation but also by the orientation relation between neighboring grains.

V. Conclusion

Uniaxial tensile tests and hydraulic bulge tests were carried out for two differently textured sheets of pure titanium. The sheets were previously examined for the orientation distribution of grains, containing the orientation relation between neighboring grains. The deformation behavior of titanium sheets were examined in connection with the effect of the orientation difference between neighboring grains on the deformation characteristics of grains.

In the case of the uniaxial tensile test, the difference in the yield stress among the differently textured sheets and the different test directions can be explained mainly by the preferred orientation of sheets. However, the plastic flow curves are influenced by the orientation difference between neighboring grains. In the sheets rolled unidirectionally at 1173 K, the orientation difference of the c-axis between neighboring grains is distributed broadly in the whole range from 0 to \(\pi/2\). The anisotropy of the flow stress is smaller than that of the 0.2% proof stress, and the slope of the stress to the strain is larger than that of the sheets cross-rolled at 873 K.

Stress-strain curves and effective stress-effective strain curves of two differently textured titanium sheets, which were estimated from the hydraulic bulge test using the Hill's theory for anisotropy, were examined. Since the preferred orientation in the sheets cross-rolled at 873 K is (0001)[1010] and the orientation difference of the c-axis between neighboring grains is small, the deformation characteristics of grains under the biaxial tension are extremely different from that under the uniaxial tension. Therefore, as pointed out previously, the overestimation of the anisotropy by the \(r\) value takes place. Though the \(r\) value of the sheets rolled unidirectionally at 1173 K is high ranging from 2 to 5, the change in the deformation behavior of grains with the loading condition is small because of the comparatively large plastic constraint among neighboring grains. As a result, the estimation of the anisotropy by the \(r\) value is not so irrelevant.

Moreover, the present experiment indicates that the occurrence of the deformation twin depends not only on the macroscopic stress condition but also on the microscopic plastic constraint depending on the orientation relation between neighboring grains.

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