Levitation of Bose-Einstein condensates induced by macroscopic non-adiabatic quantum tunneling

Katsuhiro Nakamura and Akihisa Kohi
Department of Applied Physics, Osaka City University, Osaka 558-8585, Japan

Hisatsugu Yamasaki
Department of Physics, Waseda University, Tokyo 169-8555, Japan

Víctor M. Pérez-García
Departamento de Matemáticas, E. T. S. Ingenieros Industriales, Universidad de Castilla-La Mancha, 13071 Ciudad Real, Spain.
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We study the dynamics of two-component Bose-Einstein condensates trapped in different vertical positions in the presence of an oscillating magnetic field. It is shown here how tuning appropriately the oscillation frequency of the magnetic field leads to the levitation of the system against gravity. This phenomenon is a manifestation of a macroscopic non-adiabatic tunneling in a system with internal degrees of freedom.

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The now classical observation of Bose-Einstein condensation with dilute atomic vapors in a series of experiments [1] has been the stimulus of a great number of experimental and theoretical works in the field of matter waves and macroscopic quantum dynamics.

Shortly after the production of single-component Bose-Einstein condensates (BECs), multicomponent condensates were observed [2, 3] in mixtures of two hyperfine states of $^{87}$Rb, $|F = 1, m_f = -1 \rangle$ and $|F = 2, m_f = 1 \rangle$ with and without “sag” corresponding to equal or different trap centers, respectively. These works, together with other experiments which followed soon [4, 5, 6], stimulated many studies on multi-component BECs which open scenarios different from those found in single component BECs, for instance for their ground state and excitations.

In spinor BECs it is easy to induce Rabi-type transitions optically. Similar phenomena involving coupling of different states occur in tunneling of BECs in optical lattices or double-well potentials [6, 7, 8, 9, 10, 11, 12, 13, 14].

In this paper we describe a striking levitation phenomenon which occurs when both the spatial aspects of the dynamics are put together with the two internal degrees of freedom present in a two-component Bose-Einstein condensate in which transitions between components are driven by an oscillating magnetic field. In this scenario the dynamics splits into a fast complex spatio-temporal oscillation of the condensate wavefunctions together with a slow dynamical levitation of the total center of mass against gravity with neither applied mechanical force nor associated classical trajectories.

In this paper we will study the dynamics of two-component BECs where each component is trapped in different vertical positions as schematically indicated in Fig. 1. An optical dipole trap together with a magnetic field gradient can provide such kind of confinement [10, 11]. To fix ideas we will consider our system to be the $|F = 1, m_f = -1 \rangle$ and $|F = 2, m_f = 1 \rangle$ hyperfine states of $^{87}$Rb coupled by an oscillating magnetic field of frequency $\Omega$ inducing transitions between them. An oscillating magnetic field induces transitions between both states. A Franck-Condon-type vertical transition is indicated by the black arrow. Initially all atoms are in the first component.

Fig. 1. Schematic plot of the setup studied in this paper. Each of the components of a two-component BEC is subject to the action of gravity and different confining potentials $V_1$ and $V_2$ with minima $z_1^*$ and $z_2^*$, respectively. An oscillating magnetic field induces transitions between both states. A Franck-Condon-type vertical transition is indicated by the black arrow. Initially all atoms are in the first component.
magnetically tightly confined along one of the transverse directions to two effective dimensions.

The Hamiltonian density of our system is

$$\mathcal{H} = \sum_{j=1}^{2 \ell} \left( \frac{\hbar^2}{2m} |\nabla \psi_j|^2 + V_j |\psi_j|^2 + \frac{U_{jj}}{2} |\psi_j|^4 \right) + U_{12} |\psi_1|^2 |\psi_2|^2 + B \cos(\Omega t) (\psi_1^* \psi_2 + \psi_2^* \psi_1),$$

(1)

where $V_j$ are the confining potentials acting on each of the species given by

$$V_j(x, z) = \frac{1}{2} m \omega^2 \left[ x^2 + (z - z_j)^2 \right] + \epsilon_j + mgz,$$

(2)

where $\epsilon_j$ stands for the internal electronic energy and $g$ is the gravity constant. $V_j(x, z)$ has the minimum value $m g z - \frac{3\pi}{2} (\delta z)^2 + \epsilon_j$ at $(x, z) = (0, z_j^* = z_j - g/\omega^2)$. $U_{11}$, $U_{22}$, and $U_{12} = U_{21}$ are the effective nonlinear interaction coefficients defined by $U_{ij} = 4\pi \hbar^2 a_{ij} N/m\ell$ in 2-dimensional (2D) space where $N$ is the total number of atoms and $\alpha_{ij}$ are the scattering lengths for binary collisions. Finally, $\ell = \sqrt{\hbar/m\omega}$ is a characteristic length.

The Gross-Pitaevskii equation (GPE) for the mean field dynamics of our system is derived through Lagrange equations with use of the Hamiltonian $H = \iint \mathcal{H} \, dx \, dz$. Additionally, we change to new variables scaled as $\omega t \rightarrow t$, $x/\ell \rightarrow x$, $z/\ell \rightarrow z$, $\ell \psi \rightarrow \psi$ and obtain

$$i\frac{\partial \psi_1}{\partial t} = \left[ -\frac{1}{2} \nabla^2 + V_1' + U_{11}' |\psi_1|^2 + U_{12}' |\psi_2|^2 \right] \psi_1$$

$$+ B' \cos(\Omega t) \psi_2$$

(3a)

$$i\frac{\partial \psi_2}{\partial t} = \left[ -\frac{1}{2} \nabla^2 + V_2' + U_{22}' |\psi_2|^2 + U_{21}' |\psi_1|^2 \right] \psi_2$$

$$+ B' \cos(\Omega t) \psi_1$$

(3b)

where $V_j' = V_j'(x, z) = \frac{1}{2} \left[ x^2 + (z - z_j^*)^2 \right] + g' z + \epsilon_j'$ with $U_{jj}' = 4\pi N a_{jj}/\ell$, $B' = \hbar/\omega$, $\Omega' = \Omega/\omega$, $g' = g/\ell \omega^2$ and $\epsilon_j' = \epsilon_j/\ell \omega$ being dimensionless constants. The normalization condition for $\psi$ is $\iint (|\psi_1|^2 + |\psi_2|^2) \, dx \, dz = 1$.

Initially we prepare a circularly-symmetric Gaussian wavepacket with its center of mass at $r_1 = (0, z_1^*)$,

$$\psi_1 = \frac{1}{\sqrt{\pi (1 + \frac{V_1'}{2\epsilon_j'})^{1/4}}} \exp \left( -\frac{x^2 + (z - z_1^*)^2}{2 \sqrt{1 + \frac{V_1'}{2\epsilon_j'}}} \right)$$

(4)

which minimizes the energy $E = \int d^2r \left( \frac{\hbar^2}{2m} |\nabla \psi_1|^2 + V_1' |\psi_1|^2 + \frac{U_{11}'}{2} |\psi_1|^4 \right)$ over a family of gaussian functions and approximates the ground state when all atoms are in state $|1\rangle$. We then apply an external microwave field that induces a Franck-Condon type transition between the species with frequency $\Omega' = \Delta V_{12}' = V_1'(0, z_1^*) - V_1'(0, z_2^*)$. Without loss of generality, we will consider the case $z_2^* = -z_1^*$. After the oscillating magnetic field is switched on, we study the dynamics according to Eq. \ref{eq:3}. We have integrated them by different numerical methods: an alternating direction implicit method, a Crank-Nicholson’s method, and a split-step method with pseudospectral integration of the spatial derivatives, all of which lead to identical results. The

FIG. 2: [Color online] Time dependence of the population mixing for the case of resonance of Franck-Condon type. Dashed and solid lines stand for $P_1 = \iint |\psi_1|^2$ and $P_2 = \iint |\psi_2|^2$, respectively for $B' = 50$. (a) $\delta z = 1.0$ (b) $\delta z = 5.0$.

FIG. 3: (a) Time evolution of the center of mass $\bar{z} = \iint \psi^1 \psi^3 \, dx \, dz/ \iint (|\psi_1|^2 + |\psi_2|^2) \, dx \, dz$ for $B' = 50$ and $\delta z = 1.0$ (broken line) and $\delta z = 5.0$ (solid line). The dotted line stands for the case far from the standard resonance ($\Omega' = 1, \delta \epsilon = 500$) with $\delta z = 5.0$. (b) and (c) Pseudocolor plots of $|\psi(x, z, t)|^2 = |\psi_1(x, z, t)|^2 + |\psi_2(x, z, t)|^2$ for $t = 0$ and $t = 2.08$ on the region $(x, z) \in [-6.5, 6.5] \times [-9.6, 9.6]$ showing the rising (white broken lines) of the total wavefunction.
typical examples of the time evolution of the center of mass (CM) of the system along the vertical direction. The Rabi oscillation responsible for population mixing becomes apparent more pronounced as \( \delta z \) is increased. It is very interesting to point out that no levitation occurs in the case far from the standard resonance (\( \Omega' \ll \delta \epsilon' \)), which is confirmed in the lowest curve in Fig. 4(a).

In our Franck-Condon type transition (\( \Omega' = \Delta \Omega'_{12} \)), the spatial orbital functions of BEC are coupled with the electronic degrees of freedom, and the Rabi oscillation (with period \( \tau_{RB} = 2\pi/B' \)) becomes modulated sooner or later. In Fig. 2, we show the time dependence of both populations for different values of \( \delta z \). We find that unless \( \delta z \) is sufficiently small, the regular oscillation responsible for population mixing becomes suppressed as time elapses, which is attributed to the energy conversion from electronic to mechanical energy.

We have also studied the motion of the total center of mass (CM) of the system along the vertical direction defined by \( \bar{z} = \int \psi^* x \psi d^2 r = \int z (|\psi_1|^2 + |\psi_2|^2) d^2 r \).

In Fig. 4(a), except for the lowest curve, we show typical examples of the time evolution of the center of mass. In the Franck-Condon resonance situation described above we find a rising of the total center of mass against gravity. This is an interesting phenomenon which happens while the underlying population mixing shows a fast Rabi oscillation. In our numerical simulations we observe that the CM of the condensate exhibits a smooth and upward acceleration, reaching a maximum height \( \sim z_1^* \) for \( t = 2.2 \) corresponding to \( \Omega' \approx 2 \tau_{RB} \). The physical phenomenon is a levitation of the condensate through a macroscopic quantum transition which is not accompanied by either an applied mechanical force nor a classical trajectory. Figs. (b) and (c) are the total probability density \( |\Psi(x, z, t)|^2 = |\psi_1(x, z, t)|^2 + |\psi_2(x, z, t)|^2 \) before and after the levitation, respectively. The levitation effect becomes more pronounced as \( \delta z \) is increased.

Fig. 5 shows the spatiotemporal dynamics of the wavefunction \( |\psi_2(x, z, t)|^2 \) for times located in the three regions which are clearly discriminated in the dynamics for \( \delta z = 5 \). First, in the early stage region \((t = 0 \sim 0.5)\) the wavefunction oscillates near the minimum \( (z_1^*) \) of the lower potential well. As time goes on \((t = 1 \sim 1.5)\), the central position of the solutions moves towards the minimum of the upper well \( (z_2^*) \). Finally, when the CM position reaches its peak value \( \sim z_2^* \) for \( t = 2 \sim 2.2 \), the wavefunction has a double-humped structure with its valley located near the minimum of the upper well. Fig. 4 shows that the two components \( |\psi_1|^2 \) form a domain structure in this time region, a typical feature of the system due to the nonlinear interactions. The nonlinearity plays an important role in the formation of domain structures during the transitions, a feature which is not present in the noninteracting case.

The origin of this transport phenomenon is that the condensate is effectively driven by the oscillating magnetic field to the higher energy states of the upper level, thus leading to a Franck-Condon type vertical transition. After the transition the new position is unstable, and the
atoms relax towards the new minimum (at the position with larger \( z \)), leading to the observed transport.

We have studied the dependence of the levitation phenomenon on the problem parameters \( B' \) and \( \delta z = z_2' - z_1' \). In Fig. 6(a) we show the dependence on \( B' \) of the upward acceleration rate, \( \alpha \), evaluated by using a quadratic function of \( t (\tilde{z} = z_1' + \frac{1}{2} \alpha t^2) \) which is fitted to the CM data in the early stage region. Interestingly, \( \alpha \) has a maximum at some optimal value of \( B' \). This fact indicates that the Rabi oscillation is essential for the energy conversion between electronic and orbital degrees of freedom; however, when the Rabi oscillation is too fast to guarantee the relaxation towards \( z_2' \), there is a reduction of the effective energy conversion.

In Fig. 6(b) we see that under a fixed \( B' \), \( \alpha \) increases monotonically in \( \delta z \), but indicates a slight derivation from the linear law in the case of non-Franck-Condon type transition (e. g. in the case of standard resonance, \( \Omega' = \delta \epsilon' \)).

In conclusion, we have studied the dynamics of two-component Bose-Einstein condensates subject to an a.c. driving magnetic field. In the case of Franck-Condon’s vertical transition in which the driving frequency \( \Omega \) corresponds to the energy difference between the trapping potentials \( V_1 \) and \( V_2 \) at the minimum position \( z_1' \) of \( V_1 \), the condensate initially located around \( z_1' \) is effectively driven to unstable states of \( V_2 \), relaxing towards the potential minimum \( z_2' = z_1' + \delta z \) of \( V_2 \). In this situation the population mixing shows a fast Rabi oscillation, while the mean center-of-mass of the condensate exhibits a smooth and upward acceleration with its rate (\( \alpha \)) showing a maximum at some optimal magnetic-field amplitude and a monotonic increase in \( \delta z \). This phenomenon is a manifestation of macroscopic non-adiabatic quantum tunneling with internal degrees of freedom, which differs essentially from other quantum tunneling processes with Bose-Einstein condensates discussed up to now.

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Electronic address: nakamura@phys.eng.osaka-cu.ac.jp
Electronic address: victor.perezgarcia@uclm.es

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