STOCHASTIC MODELLING AND ANALYSIS OF HARVESTING MODEL: APPLICATION TO “SUMMER FISHING MORATORIUM” BY INTERMITTENT CONTROL

XIAOLING ZOU
Department of Mathematics, Harbin Institute of Technology(Weihai)
Weihai 264209, China

YUTING ZHENG
Department of Basic Course, Xingtai Polytechnic College
Xingtai 054000, China

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Abstract. As we all know, “summer fishing moratorium” is an internationally recognized management measure of fishery, which can protect stock of fish and promote the balance of marine ecology. In this paper, “intermittent control” is used to simulate this management strategy, which is the first attempt in theoretical analysis and the intermittence fits perfectly the moratorium. As an application, a stochastic two-prey one-predator Lotka-Volterra model with intermittent capture is considered. Modeling ideas and analytical skills in this paper can also be used to other stochastic models. In order to deal with intermittent capture in stochastic model, a new time-averaged objective function is proposed. Besides, the corresponding optimal harvesting strategies are obtained by using the equivalent method (equivalency between time-average and expectation). Theoretical results show that intermittent capture can affect the optimal harvesting effort, but it cannot change the corresponding optimal time-averaged yield, which are accord with observations. Finally, the results are illustrated by practical examples of marine fisheries and numerical simulations.

1. Introduction. Optimal management of resources is directly related to the sustainable development of resources, which has important practical significance and is highly valued by the academic community. Optimal capture policy has been and will continue to be a hot topic in the study of biological populations. Clark [8] obtained optimal harvesting strategies for deterministic population models in prestigious journal Science in 1977. Since then, many scholars have started to study the optimal harvesting problems for deterministic models. With the passage of time and the vigorous development of scientific research, scientists have discovered the existence of environmental noises which have critical effects on population models, hence, many scholars have focused on optimal harvesting strategies for stochastic population systems (see, e.g., [2, 27, 32] and references cited therein).
using Fokker-Planck equation, Beddington and May [6] studied the optimal harvesting problem for a stochastic Logistic model in *Science*. [20] studied the optimal harvesting policy for a generalized stochastic logistic model which extended the conclusion of [6]. [3] considered the optimal harvesting policy with time-discounted value for a stochastic logistic model. It is worth mentioning that these articles are focused on single specie, the reason is that it is difficult to obtain explicit solutions of Fokker-Planck equations for high-dimensional stochastic models. So, the optimal harvesting problem for high-dimensional stochastic systems need some new ideas or new methods. Many authors use different yield functions to study the optimal harvesting problems for various stochastic models, and this paper is also inspired by this reason.

Most of the researchers have devoted themselves to constant capture. However, constant capture is not conducive to the protection and restoration of fishery resources[24]. In order to alleviate the enormous pressure on fishery resources caused by overfishing, summer moratorium of marine fishing is implemented within a specified period of time every year, namely, no fishing operations are allowed in a certain period of time and in certain waters. The purpose is to protect the normal reproduction of aquatic organisms and ensure the sustainable recovery and development of fish resources. So, it is very important to consider the phenomenon of “summer fishing moratorium” during the mathematical modeling, and this is the main characteristic for this paper. Next, we will introduce intermittent control to model the phenomenon of “summer fishing moratorium”.

In recent years, the theory of discontinuous control systems has become hot issues. Intermittent control, which is a more efficient control method in discontinuous control, has attracted wide attentions, such as [7, 26, 30] and references cited therein. As a direct and effective engineering control method, intermittent control has been widely applied to engineering fields such as manufacturing, transmission and communications [14, 15]. Motivated by these, we will use intermittent control to model “summer fishing moratorium” in this paper. As far as we know, this article is the first attempt to model the phenomenon of ‘summer fishing moratorium” with intermittent control. The mathematical expression of intermittent capture $H(t)$ is given as follows:

$$H(t) = \begin{cases} m, & \text{if } nT \leq t < nT + \theta \\ 0, & \text{if } nT + \theta \leq t < (n + 1)T \end{cases}$$

Here, $T$ is a capture cycle, generally in the measure of years. $nT \leq t < nT + \theta$ represents the time allowed for capture in the cycle, and $m$ is the harvesting effort, i.e. catch-per-unit-effort. That is, if $nT \leq t < nT + \theta$, the harvesting effort is a constant capture $m$; if $nT + \theta \leq t < (n + 1)T$, marine fishery enters the closed fishing season.

The way of modeling “summer fishing moratorium” by intermittent control is of great practical significance, and several living examples are given to explain this regard. The summer fishing moratorium mainly focuses on the spring and summer of each year, that is, the breeding period of major marine species and the growing period of larvae. This policy can effectively protect juvenile populations of marine organisms, replenish populations and improve community structure[5, 10]. Furthermore, this policy can also curb the over-exploitation of fishery resources and promote the overall stability of marine fishery production[18, 24]. To sum up, it
is of great practical significance to consider intermittent harvesting in the study of optimal fishing problem.

2. Model description and a new time-averaged objective function. A classical three-species Lotka-Volterra model with two competing populations of prey and one predator can be expressed as follows:

\[
\begin{align*}
\frac{dx(t)}{dt} &= x(t)(r_1 - a_{11}x(t) - a_{12}y(t) - a_{13}z(t))dt \\
\frac{dy(t)}{dt} &= y(t)(r_2 - a_{21}x(t) - a_{22}y(t) - a_{23}z(t))dt \\
\frac{dz(t)}{dt} &= z(t)(r_3 + a_{31}x(t) + a_{32}y(t) - a_{33}z(t))dt,
\end{align*}
\]

where \(x(t)\) and \(y(t)\) are the sizes of two competing populations of prey; \(z(t)\) is the size of the predator; \(r_i\) and \(a_{ij}\) \((i, j = 1, 2, 3)\) are the growth rates and the intra-specific rates for the corresponding populations; \(a_{12}\) and \(a_{21}\) represent the inter-specific competition coefficients; \(a_{13}\) and \(a_{23}\) stand for the capture rates; \(a_{31}\) and \(a_{32}\) are the efficiencies of the food conversion; \(a_{ij} > 0\) and \(r_i > 0\) \(i, j = 1, 2, 3\). However, it is well recognized that all kinds of models are inevitably affected by environmental fluctuations, such as the temperature, the climate, the water resources, etc. For examples, epidemic models \([1, 12]\), chemostat models \([17, 25]\), biological population models \([22, 33]\), complex network models \([13, 29]\) and so on. By these random effects, the parameters are often perturbed by white noises. We will mainly consider that the environmental noise mainly affects the growth rate of the populations, i.e. \(r_1 \rightarrow r_1 + \sigma_1 B_1(t)\). Throughout this paper, unless otherwise specified, we let \((\Omega, F, \{F_t\}_{t \geq 0}, \mathbb{P})\) be a complete probability space with filtration \(\{F_t\}_{t \geq 0}\) satisfying the usual conditions (It is right continuous and \(F_0\) contains all the \(\mathbb{P}\)-null sets). Let \(B_i(t), (i = 1, 2, 3)\) denote the independent and standard Brownian motions defined on this probability space. \(\sigma_i^2, i = 1, 2, 3\) denote the intensities of white noise. Then we can obtain the following stochastic model:

\[
\begin{align*}
\frac{dx(t)}{dt} &= x(t)(r_1 - a_{11}x(t) - a_{12}y(t) - a_{13}z(t))dt + \sigma_1 x(t)dB_1(t) \\
\frac{dy(t)}{dt} &= y(t)(r_2 - a_{21}x(t) - a_{22}y(t) - a_{23}z(t))dt + \sigma_2 y(t)dB_2(t) \\
\frac{dz(t)}{dt} &= z(t)(r_3 + a_{31}x(t) + a_{32}y(t) - a_{33}z(t))dt + \sigma_3 z(t)dB_3(t),
\end{align*}
\]

Taking the “summer fishing moratorium” into account, we obtain the following harvesting model with intermittent control:

\[
\begin{align*}
\frac{dx(t)}{dt} &= x(t)(r_1 - H_1(t) - a_{11}x(t) - a_{12}y(t) - a_{13}z(t))dt + \sigma_1 x(t)dB_1(t) \\
\frac{dy(t)}{dt} &= y(t)(r_2 - H_2(t) - a_{21}x(t) - a_{22}y(t) - a_{23}z(t))dt + \sigma_2 y(t)dB_2(t) \\
\frac{dz(t)}{dt} &= z(t)(r_3 - H_3(t) + a_{31}x(t) + a_{32}y(t) - a_{33}z(t))dt + \sigma_3 z(t)dB_3(t)
\end{align*}
\]

where \(H_i(t), i = 1, 2, 3\) are the intermittent harvesting efforts, and their mathematical expressions are as follows:

\[
H_i(t) = \begin{cases} 
    m_i, & \text{if } nT \leq t < nT + \theta \\
    0, & \text{if } nT + \theta \leq t < (n + 1)T 
\end{cases}
\]

Let’s call \(H(t) = (H_1(t), H_2(t), H_3(t))\) for convenience.

Obviously, the optimal harvesting strategy relies on an optimal objective function. C.W. Clark \([8]\) introduced proportional objective function for deterministic systems based on catch-per-unit-effort hypothesis, i.e.

\[ W = EX^* \]
where $X^*$ is an equilibrium point, i.e. $(x_1^*, x_2^*, x_3^*, \ldots, x_n^*)^T$. $E$ denotes the constant capture, i.e. $(E_1, E_2, E_3, \ldots, E_n)$. For stochastic systems, this deterministic objective function has been extended to the time-averaged objective function

$$W = \lim_{t \to \infty} \frac{1}{t} \int_0^t \mathbf{E}X(s)ds,$$

(6)

However, due to the introduction of intermittent control $H(t)$, it is difficult to study the optimal harvesting problems with objective function (6). So, we need to find a new objective function due to the existence of intermittent control in this paper.

The solution of system (4) is denoted as $X_t = (x(t), y(t), z(t))$ in this paper. We will prove $X_t$ is asymptotically stable in distribution. In particular, $(X_t(\omega))_{t \geq 0}$ converges to a random variable $Q(\omega)$ in distribution, and $\pi(\cdot)$ is the probability measure induced by the random variable $Q(\omega)$. Here, $Q(\omega)$ is a random variable defined on the same complete probability space. Then, we will use the following new time-averaged function as the objective function

$$W = \lim_{t \to \infty} \frac{1}{t} \int_0^t <H(s) \cdot \mathbf{E}Q(\omega)> ds,$$

(7)

where $<a \cdot b>$ is the symbol of inner product and $\mathbf{E}Q(\omega)$ is the expectation of $Q(\omega)$. Our aims are to find the optimal harvesting effort and the corresponding maximum harvesting yield.

3. Global positive solution and asymptotically stable in distribution of system (4).

3.1. Existence and uniqueness of global positive solution. A very natural focus is the nonnegative solution for biological population models. So, we will show the existence and uniqueness of a global positive solution for system (4) in this subsection.

Theorem 3.1. There exists a unique global positive solution for system (4) with any initial value $(x(0), y(0), z(0)) \in \mathbb{R}_+^3$ almost surely, where

$$\mathbb{R}_+^3 = \{(x, y, z) \mid x \geq 0, y \geq 0, z \geq 0\}.$$

Proof. There exists a unique local solution since the coefficients of system (4) satisfy the local Lipschitz conditions on every intervals $[nt, nT]$, $[nT, (n + 1)T]$. Next, we will show this solution is globally. Obviously, $(x(t), y(t), z(t)) \equiv (0, 0, 0)$ is a solution of system (4), so, solution of system (4) with any initial value $(x(0), y(0), z(0)) \in \mathbb{R}_+^3$ satisfies $x(t) \geq 0, y(t) \geq 0, z(t) \geq 0$ almost surely by the stochastic comparison theorem. Accordingly, we can find two auxiliary systems such that $X_1(t) \leq x(t) \leq X_2(t), Y_1(t) \leq y(t) \leq Y_2(t), Z_1(t) \leq z(t) \leq Z_2(t)$ a.s., where

$$\begin{align*}
&dX_1(t) = X_1(t)(r_1 - m_1 - a_{11}X_1(t) - a_{12}Y_1(t) - a_{13}Z_1(t))dt + \sigma_1X_1(t)dB_1(t) \\
&dY_1(t) = Y_1(t)(r_2 - m_2 - a_{21}X_1(t) - a_{22}Y_1(t) - a_{23}Z_1(t))dt + \sigma_2Y_1(t)dB_2(t) \\
&dZ_1(t) = Z_1(t)(r_3 - m_3 + a_{31}X_1(t) + a_{32}Y_1(t) - a_{33}Z_1(t))dt + \sigma_3Z_1(t)dB_3(t)
\end{align*}$$

(8)

and

$$\begin{align*}
&dX_2(t) = X_2(t)(r_1 - a_{11}X_2(t) - a_{12}Y_2(t) - a_{13}Z_2(t))dt + \sigma_1X_2(t)dB_1(t) \\
&dY_2(t) = Y_2(t)(r_2 - a_{21}X_2(t) - a_{22}Y_2(t) - a_{23}Z_2(t))dt + \sigma_2Y_2(t)dB_2(t) \\
&dZ_2(t) = Z_2(t)(r_3 + a_{31}X_2(t) + a_{32}Y_2(t) - a_{33}Z_2(t))dt + \sigma_3Z_2(t)dB_3(t)
\end{align*}$$

(9)
3.2. Asymptotically stable in distribution and ergodic properties. In this section, we will first prove system (4) is asymptotically stable in distribution, then the mean vector of stationary distribution are obtained by using the ergodic theory. This subsection will provide the basis for the latter equivalent method.

Lemma 3.2. [4] Let \( f(t) \) be a nonnegative function defined on \([0, +\infty)\) such that \( f(t) \) is integrable on \([0, +\infty)\) and is uniformly continuous on \([0, +\infty)\), then \( \lim_{t \to \infty} f(t) = 0 \).

Lemma 3.3. Let \( (x(t), y(t), z(t)) \) denotes the solution of system (4) with initial value \( (x(0), y(0), z(0)) \in R^3_+ \), then it has the following property:

\[
\sup_{0 \leq t < \infty} \mathbb{E}|(x(t), y(t), z(t))| < \infty.
\]

The proof of Lemma 3.3 can be found in Appendix A.

Remark 1. As a by-product of Lemma 3.3, we also have

\[
0 \leq \mathbb{E}|x(t)| < \infty, 0 \leq \mathbb{E}|y(t)| < \infty, 0 \leq \mathbb{E}|z(t)| < \infty.
\]

Assumption 1. \( a_{ii} > \sum_{j=1, j \neq i}^n a_{ij} \).

Assumption 1 means that the intra-specific competition coefficients are stronger than inter-specific competition coefficients [9].

Lemma 3.4. Suppose assumption (1). Let \( (x_i(t), y_i(t), z_i(t)), i = 1, 2 \) represent solutions of system (4) with initial values \( (x_1(0), y_1(0), z_1(0)) \in R^3_+ \), then

\[
\lim_{t \to \infty} \mathbb{E} \left\| (x_1(t), y_1(t), z_1(t)) - (x_2(t), y_2(t), z_2(t)) \right\| = 0
\]

uniformly.

The proof of Lemma 3.4 can be found in Appendix B.

Lemma 3.5. {Asymptotically stable in distribution} The transition probability of system (4) converges weakly to a unique probability measure \( \pi(\cdot) \) as \( t \to \infty \).

The proof of Lemma 3.5 can be found in Appendix C.

Remark 2. Lemma 3.5 shows system (4) is asymptotically stable in distribution. That is to say, \( X_t = (x(t), y(t), z(t)) \) converges to a random variable \( Q(\omega) = (X(\omega), Y(\omega), Z(\omega)) \) in distribution, and \( \pi(\cdot) \) is induced by the very same random variable \( Q(\omega) \). Hence, there exist measures \( \pi_1(\cdot), \pi_2(\cdot) \) and \( \pi_3(\cdot) \) which are induced by random variables \( X(\omega), Y(\omega) \) and \( Z(\omega) \) respectively. Moreover, \( \pi_1(\cdot), \pi_2(\cdot) \) and \( \pi_3(\cdot) \) are the stationary distributions of \( x(t), y(t) \) and \( z(t) \) respectively.

Theorem 3.6. {Almost sure equivalency between time-average and expectation}

\[
\lim_{t \to \infty} \frac{1}{t} \int_0^t x(s)ds = \mathbb{E}[X(\omega)], a.s.
\]

\[
\lim_{t \to \infty} \frac{1}{t} \int_0^t y(s)ds = \mathbb{E}[Y(\omega)], a.s.
\]
\[
\lim_{t \to \infty} \frac{1}{t} \int_0^t z(s)ds = \mathbb{E}[Z(\omega)], \ a.s.
\]

where \(X(\omega), Y(\omega)\) and \(Z(\omega)\) are introduced in Remark 2.

**Proof.** On the one hand, Theorem 3.26 in [23] means \(\pi_i(\cdot)\) is strong mixing, so \(\pi_i(\cdot)\) is also ergodic, that is to say,

\[
\lim_{t \to \infty} \frac{1}{t} \int_0^t x(s)ds = \int_{R_+} X \pi_1 (dX), \ a.s.
\]

\[
\lim_{t \to \infty} \frac{1}{t} \int_0^t y(s)ds = \int_{R_+} Y \pi_2 (dY), \ a.s.
\]

\[
\lim_{t \to \infty} \frac{1}{t} \int_0^t z(s)ds = \int_{R_+} Z \pi_3 (dZ), \ a.s.
\]

On the other hand, by the definition of expectation for a random variable (p.9 of [11]), we can conclude

\[
\int_{R_+} X \pi_1 (dX) = \int_{\Omega} X(\omega) d\mathbb{P}(\omega) = \mathbb{E}[X(\omega)],
\]

\[
\int_{R_+} Y \pi_2 (dY) = \int_{\Omega} Y(\omega) d\mathbb{P}(\omega) = \mathbb{E}[Y(\omega)],
\]

\[
\int_{R_+} Z \pi_3 (dZ) = \int_{\Omega} Z(\omega) d\mathbb{P}(\omega) = \mathbb{E}[Z(\omega)].
\]

Therefore, the conclusions are proved. \(\square\)

4. **Equivalent method for optimal harvesting problems.** In the previous section, we discussed the asymptotically stable in distribution of \(X_t\), and obtained the almost sure equivalency between time-average and expectation. In this section, the explicit expression for the new time-averaged objective function proposed in section 2 is obtained by using the almost sure equivalence.

From Theorem 3.6 we know that

\[
W = \lim_{t \to \infty} \frac{1}{t} \int_0^t \langle \mathbf{H}(s) \cdot \mathbf{Q}(\omega) \rangle ds
\]

\[
= \lim_{t \to \infty} \frac{1}{t} \int_0^t \left( H_1(s) \mathbb{E}[X(\omega)] + H_2(s) \mathbb{E}[Y(\omega)] + H_3(s) \mathbb{E}[Z(\omega)] \right) ds
\]

\[
= \lim_{t \to \infty} \frac{1}{t} \int_0^t \left( H_1(s) \lim_{s \to \infty} \frac{1}{s} \int_0^s x(v)dv \right) ds
\]

\[
+ \lim_{t \to \infty} \frac{1}{t} \int_0^t \left( H_2(s) \lim_{s \to \infty} \frac{1}{s} \int_0^s y(v)dv \right) ds
\]

\[
+ \lim_{t \to \infty} \frac{1}{t} \int_0^t \left( H_3(s) \lim_{s \to \infty} \frac{1}{s} \int_0^s z(v)dv \right) ds, a.s.
\]

For simplicity, we introduce the following notations:

\[
b_i = r_i - m_i, \theta_i/T, \phi_1 = a_{22}b_1 - a_{12}b_2, \phi_3 = a_{22}b_1^2/2 - a_{12}b_2^2/2,
\]

\[
\phi_2 = a_{11}b_1 - a_{21}b_1, \phi_3 = a_{11}b_1^2 - a_{21}b_1^2/2,
\]

\[
\phi_4 = a_{22}(r_1 - r_2^2/2) - a_{12}(r_2 - r_2^2/2),
\]

\[
\phi_5 = a_{11}b_1 + a_{31}b_1, \phi_6 = a_{11}b_1^2 + a_{31}b_1^2/2,
\]

\[
\phi_7 = a_{22}(r_1 - r_2^2/2) - a_{12}(r_2 - r_2^2/2),
\]

\[
\phi_8 = a_{11}b_1 - a_{21}b_1, \phi_9 = a_{11}b_1^2 - a_{21}b_1^2/2.
\]

\[
\phi_{10} = a_{22}(r_1 - r_2^2/2) - a_{12}(r_2 - r_2^2/2).
\]
\[ \dot{\phi}_4 = a_{11}(r_2 - \sigma_2^2/2) - a_{21}(r_1 - \sigma_1^2/2), \]

\[ G = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ -a_{31} & -a_{32} & a_{33} \end{bmatrix}, \quad R = \begin{bmatrix} a_{11} & b_1 & \sigma_1^2/2 \\ a_{21} & b_2 & \sigma_2^2/2 \\ -a_{31} & b_3 & \sigma_3^2/2 \end{bmatrix}, \]

\[ G_1 = \begin{bmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & -a_{32} & a_{33} \end{bmatrix}, \quad \tilde{G}_1 = \begin{bmatrix} \sigma_1^2/2 & a_{12} & a_{13} \\ \sigma_2^2/2 & a_{22} & a_{23} \\ \sigma_3^2/2 & -a_{32} & a_{33} \end{bmatrix}, \]

\[ G_2 = \begin{bmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ -a_{31} & b_3 & a_{33} \end{bmatrix}, \quad \tilde{G}_2 = \begin{bmatrix} a_{11} & \sigma_1^2/2 & a_{13} \\ a_{21} & \sigma_2^2/2 & a_{23} \\ -a_{31} & \sigma_3^2/2 & a_{33} \end{bmatrix}, \]

\[ G_3 = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ -a_{31} & -a_{32} & b_3 \end{bmatrix}, \quad \tilde{G}_3 = \begin{bmatrix} a_{11} & a_{12} & \sigma_2^2/2 \\ a_{21} & a_{22} & \sigma_3^2/2 \\ -a_{31} & -a_{32} & \sigma_3^2/2 \end{bmatrix}. \]

Let \( A_{ij} \) be the complement minor of \( a_{ij} \) in the determinant \( G \) and \( \zeta_1 = 2b_1/\sigma_1^2, \zeta_2 = \phi_2/\phi_1, \zeta_3 = G_3/\tilde{G}_3 \), a simple calculation yields \( \zeta_1 > \zeta_2 > \zeta_3 \).

**Lemma 4.1.**

\[ \lim_{t \to \infty} \frac{1}{t} \int_0^t H_i(s)ds = \frac{m_i \theta}{T}. \]

**Proof.** For any \( t > 0 \), there exists a positive integer \( n \) such that \( t = nT + \varphi, \)

\[ 0 \leq \varphi < T. \]

By the fact that \( t \to \infty \) implies \( n \to \infty \), therefore

\[ \lim_{t \to \infty} \frac{1}{t} \int_0^t H_i(s)ds = \lim_{n \to \infty} \frac{\int_0^T H_i(s)ds + \int_T^{2T} H_i(s)ds + \cdots + \int_{nT}^{nT+\varphi} H_i(s)ds}{nT + \varphi} = \lim_{n \to \infty} \frac{nm_i \theta + \int_{nT}^{nT+\varphi} H_i(s)ds}{nT + \varphi} = \lim_{n \to \infty} \frac{m_i \theta + (\int_{nT}^{nT+\varphi} H_i(s)ds)/n}{T + \frac{\varphi}{n}} = \frac{m_i \theta}{T}. \]

\[ \Box \]

**Assumption 2.** \( R > 0 \).

This assumption implies that the survival ability of \( x(t) \) is stronger than \( y(t) \), see literatures ([9]).

**Assumption 3.** \( G > 0, G_i > 0 \) for \( i = 1, 2, 3; A_{33} > 0, \phi_1 > 0, \phi_2 > 0 \).

\( G > 0, G_i > 0 \) for \( i = 1, 2, 3 \), that is to say system (4) has a positive equilibrium if there are no stochastic noises. \( A_{33} > 0, \phi_1 > 0, \phi_2 > 0 \), which means system (4) has a positive equilibrium if both stochastic noise and predator are absent.

**Assumption 4.** \( \tilde{G}_i > 0 \) for \( i = 1, 2, 3; \phi_3 > 0, A_{23} > 0, A_{32} > 0, A_{31} < 0 \).

Under Assumptions 2-4, we can conclude the following lemmas as the same way of Theorem 1 in literature [21].
Lemma 4.2. If $\zeta_1 > 1 > \zeta_2$, then $y(t)$ and $z(t)$ go to extinction a.s., and the time average of $x(t)$ is the solution of the following linear equation:

$$a_{11}(x) = b_1 - \frac{\sigma_1^2}{2}.$$ 

That is to say,

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t x(s)ds = \langle x \rangle = \frac{b_1 - \sigma_1^2/2}{a_{11}}, \text{ a.s.}$$

Lemma 4.3. If $\zeta_2 > 1 > \zeta_3$, then $z(t)$ goes to extinction a.s., and the time averages of $x(t)$ and $y(t)$ are the solutions of the following system of linear equations:

$$\begin{cases}
   a_{11}(x) + a_{12}(y) = b_1 - \frac{\sigma_1^2}{2}, \\
   a_{21}(x) + a_{22}(y) + a_{23}(z) = b_2 - \frac{\sigma_2^2}{2}, \\
   -a_{31}(x) - a_{32}(y) + a_{33}(z) = b_3 - \frac{\sigma_3^2}{2}
\end{cases}$$

i.e. $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\
   a_{21} & a_{22} & a_{23} \\
   -a_{31} & -a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \langle x \rangle \\
   \langle y \rangle \\
   \langle z \rangle \end{pmatrix} = \begin{pmatrix} b_1 - \frac{\sigma_1^2}{2} \\
   b_2 - \frac{\sigma_2^2}{2} \\
   b_3 - \frac{\sigma_3^2}{2} \end{pmatrix}$.

That is to say,

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t x(s)ds = \langle x \rangle = \frac{G_1 - \tilde{G}_1}{G}, \text{ a.s.}$$

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t y(s)ds = \langle y \rangle = \frac{G_2 - \tilde{G}_2}{G}, \text{ a.s.}$$

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t z(s)ds = \langle z \rangle = \frac{G_3 - \tilde{G}_3}{G}, \text{ a.s.}$$

Lemma 4.4. If $\zeta_3 > 1$, then the time averages of $x(t), y(t)$ and $z(t)$ are the solutions of the following system of linear equations:

$$\begin{cases}
   a_{11}(x) + a_{12}(y) + a_{13}(z) = b_1 - \frac{\sigma_1^2}{2}, \\
   a_{21}(x) + a_{22}(y) + a_{23}(z) = b_2 - \frac{\sigma_2^2}{2}, \\
   -a_{31}(x) - a_{32}(y) + a_{33}(z) = b_3 - \frac{\sigma_3^2}{2}
\end{cases}$$

That is to say,

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t x(s)ds = \langle x \rangle = \frac{G_1 - \tilde{G}_1}{G}, \text{ a.s.}$$

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t y(s)ds = \langle y \rangle = \frac{G_2 - \tilde{G}_2}{G}, \text{ a.s.}$$

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t z(s)ds = \langle z \rangle = \frac{G_3 - \tilde{G}_3}{G}, \text{ a.s.}$$

Theorem 4.5. If $\zeta_1 > 1 > \zeta_2$, then the optimal harvesting effort is

$$m_1^* = \frac{T}{2\theta}(r_1 - \frac{\sigma_1^2}{2}),$$

and the maximum yield is

$$W^* = \frac{1}{4a_{11}} (r_1 - \frac{\sigma_1^2}{2})^2.$$
Hence, the time-averaged objective function becomes

\[ \lim_{t \to \infty} \frac{1}{t} \int_0^t x(s)ds = \frac{b_1 - \sigma_1^2/2}{a_{11}}, \lim_{t \to \infty} \frac{1}{t} \int_0^t y(s)ds = 0, \lim_{t \to \infty} \frac{1}{t} \int_0^t z(s)ds = 0, \text{ a.s.} \]

Then the time-averaged objective function becomes

\[ W = \frac{\theta}{T} \frac{m_1}{a_{11}} \frac{b_1 - \sigma_1^2/2}{a_{11}} = \frac{\theta}{T} \frac{r_1 - 2m_1 - \sigma_1^2/2}{a_{11}} \]

\[ = \left( \frac{\theta}{T} \right)^2 \frac{1}{a_{11}} m_1^2 + \left( \frac{\theta}{T} \right) (r_1 - \sigma_1^2/2) m_1. \]

It is easy to calculate

\[ \frac{dW}{dm_1} = -2 \left( \frac{\theta}{T} \right)^2 \frac{1}{a_{11}} m_1 + \left( \frac{\theta}{T} \right) (r_1 - \sigma_1^2/2). \]

and the unique stagnation point is

\[ m_1^* = \frac{T(r_1 - \sigma_1^2/2)}{2\theta}. \]

Consequently,

\[ W^* = -\left( \frac{\theta}{T} \right)^2 \frac{1}{a_{11}} m_1^2 + \left( \frac{\theta}{T} \right) (r_1 - \sigma_1^2/2) m_1^* = \frac{1}{4a_{11}} (r_1 - \frac{\sigma_1^2}{2})^2 \]

is the maximum value since

\[ \frac{d^2W}{dm_1^2} = \left( \frac{\theta}{T} \right)^2 \frac{1}{a_{11}} < 0. \]

The proof of Theorem 4.5 is completed.

**Theorem 4.6.** If \( \zeta_2 > 1 > \zeta_3 \) and \( (a_{12} + a_{21})^2 - 4a_{11}a_{22} < 0 \) are satisfied. Then the optimal harvesting efforts are

\[ m_1 = \frac{T}{\theta} \left[ \frac{2a_{11}\phi_4 + (a_{12} + a_{21})\hat{\phi}_4}{4a_{11}a_{22} - (a_{12} + a_{21})^2} \right] A_{33} := \frac{T}{\theta} m_1^{**}, \]

\[ m_2 = \frac{T}{\theta} \left[ \frac{2a_{22}\hat{\phi}_4 - (a_{12} + a_{21})\phi_4}{4a_{11}a_{22} - (a_{12} + a_{12})^2} \right] A_{33} := \frac{T}{\theta} m_2^{**}. \]

The maximum yield is

\[ W^* = \frac{m_1^{**} \phi_4 + m_2^{**} \hat{\phi}_4 - a_{22}m_1^{**} - a_{11}m_2^{**} + (a_{12} + a_{21}) m_1^{**} m_2^{**}}{A_{33}}. \]

**Proof.** According to Lemma 4.3 we have

\[ \lim_{t \to \infty} \frac{1}{t} \int_0^t x(s)ds = \frac{\phi_1 - \hat{\phi}_1}{A_{33}}, \text{ a.s.} \]

\[ \lim_{t \to \infty} \frac{1}{t} \int_0^t y(s)ds = \frac{\phi_2 - \hat{\phi}_2}{A_{33}}, \text{ a.s.} \]

Hence, the time-averaged objective function becomes

\[ W = \frac{\theta}{T} m_1 \phi_4 - \left( \frac{\theta}{T} \right)^2 (a_{22}m_1^2 - a_{12}m_1m_2) + \frac{\theta}{T} m_2 \hat{\phi}_4 - \left( \frac{\theta}{T} \right)^2 (a_{11}m_2^2 - a_{21}m_1m_2) \]

\[ A_{33}. \]
It is easy to calculate
\[
\frac{\partial W}{\partial m_1} = \frac{\varrho}{\theta^2} \phi_4 - \frac{(\varrho/\theta)^2}{A_{33}} [2a_{22}m_1 - (a_{12} + a_{21})m_2],
\]
\[
\frac{\partial W}{\partial m_2} = \frac{\varrho}{\theta^2} \phi_4 - \frac{(\varrho/\theta)^2}{A_{33}} [2a_{11}m_2 - (a_{12} + a_{21})m_1],
\]
and the unique stagnation point is
\[
m_1^* = \frac{T}{\theta} \left[ 2a_{11} \phi_4 + (a_{12} + a_{21}) \phi_4 \right] / [4a_{11}a_{22} - (a_{12} + a_{21})^2 A_{33}],
\]
\[
m_2^* = \frac{T}{\theta} \left[ 2a_{22} \phi_4 - (a_{12} + a_{21}) \phi_4 \right] / [4a_{11}a_{22} - (a_{12} + a_{21})^2 A_{33}].
\]
Then the corresponding optimal yield is
\[
W^* = \frac{\varrho}{\theta} m_1^* \phi_4 - \frac{(\varrho/\theta)^2}{A_{33}} \left( a_{22}m_1^2 - a_{12}m_1m_2 + \varrho m_2^* \phi_4 - \frac{(\varrho/\theta)^2}{A_{33}} (a_{11}m_2^2 - a_{21}m_1m_2) \right).
\]
\[
= \frac{\varrho}{\theta} m_1^* \phi_4 + \frac{\varrho}{\theta} m_2^* \phi_4 - a_{22}m_1^2 \varrho m_2^* - a_{11}m_2^2 \varrho m_1^* + (a_{12} + a_{21}) m_1^* m_2^*
\]
due to $T = B^2 - AC = (a_{12} + a_{21})^2 - 4a_{11}a_{22} < 0$.

**Theorem 4.7.** Assume $\zeta_3 > 1$ and \(\begin{pmatrix} 2A_{11} & A_{12} + A_{21} & A_{13} + A_{31} \\ A_{12} + A_{21} & 2A_{22} & A_{23} + A_{32} \\ A_{13} + A_{31} & A_{23} + A_{32} & 2A_{33} \end{pmatrix}\) is a positive definite matrix. Then the optimal harvesting efforts are

\[
m_1^* = \frac{T}{\theta} \begin{pmatrix} Q & A_{12} + A_{21} & A_{13} + A_{31} \\ P & 2A_{22} & A_{23} + A_{32} \\ L & A_{23} + A_{32} & 2A_{33} \end{pmatrix} := \frac{T}{\theta} \tilde{m}_1^*,
\]
\[
m_2^* = \frac{T}{\theta} \begin{pmatrix} 2A_{11} & A_{12} + A_{21} & A_{13} + A_{31} \\ A_{12} + A_{21} & P & A_{13} + A_{31} \\ A_{13} + A_{31} & Q & A_{23} + A_{32} \end{pmatrix} := \frac{T}{\theta} \tilde{m}_2^*,
\]
\[
m_3^* = \frac{T}{\theta} \begin{pmatrix} 2A_{11} & A_{12} + A_{21} & P \\ A_{12} + A_{21} & 2A_{22} & Q \\ A_{13} + A_{31} & A_{23} + A_{32} & L \end{pmatrix} := \frac{T}{\theta} \tilde{m}_3^*.
\]
The corresponding optimal time-averaged yield is
\[ W^* = \frac{m_1^* \cdot [Q - (m_1^* A_{11} + m_2^* A_{21} + m_3^* A_{31})]}{G} \]
\[ + \frac{m_2^* \cdot [P - (m_1^* A_{12} + m_2^* A_{22} + m_3^* A_{32})]}{G} \]
\[ + \frac{m_3^* \cdot [L - (m_1^* A_{13} + m_2^* A_{23} + m_3^* A_{33})]}{G}, \]
where
\[ Q = \begin{vmatrix} r_1 - \sigma_1^2 / 2 & a_{12} & a_{13} \\ r_2 - \sigma_2^2 / 2 & a_{22} & a_{23} \\ r_3 - \sigma_3^2 / 2 & -a_{32} & a_{33} \end{vmatrix}, \]
\[ P = \begin{vmatrix} a_{11} & r_1 - \sigma_1^2 / 2 & a_{13} \\ a_{21} & r_2 - \sigma_2^2 / 2 & a_{23} \\ -a_{31} & r_3 - \sigma_3^2 / 2 & a_{33} \end{vmatrix}, \]
\[ L = \begin{vmatrix} a_{11} & a_{12} & r_1 - \sigma_1^2 / 2 \\ a_{21} & a_{22} & r_2 - \sigma_2^2 / 2 \\ -a_{31} & a_{32} & r_3 - \sigma_3^2 / 2 \end{vmatrix}. \]

**Proof.** Based on Lemma 4.4, it holds that
\[ \lim_{t \to \infty} \frac{1}{t} \int_0^t x(s) ds = \frac{G_1 - \tilde{G}_1}{G}, \text{ a.s.} \]
\[ \lim_{t \to \infty} \frac{1}{t} \int_0^t y(s) ds = \frac{G_2 - \tilde{G}_2}{G}, \text{ a.s.} \]
\[ \lim_{t \to \infty} \frac{1}{t} \int_0^t z(s) ds = \frac{G_3 - \tilde{G}_3}{G}, \text{ a.s.} \]
Thus, it is clear that
\[ W = \frac{\theta}{T} m_1 \cdot [Q - \frac{\theta}{T} (m_1 A_{11} + m_2 A_{21} + m_3 A_{31})] \]
\[ + \frac{\theta}{T} m_2 \cdot [P - \frac{\theta}{T} (m_1 A_{12} + m_2 A_{22} + m_3 A_{32})] \]
\[ + \frac{\theta}{T} m_3 \cdot [L - \frac{\theta}{T} (m_1 A_{13} + m_2 A_{23} + m_3 A_{33})]. \]

It is easy to obtain that
\[ \frac{\partial W}{\partial m_1} = \frac{Q \theta}{T} - \left( \frac{\theta}{T} \right)^2 \left[ 2A_{11} m_1 + (A_{12} + A_{21}) m_2 + (A_{13} + A_{31}) m_3 \right] \]
\[ \frac{\partial W}{\partial m_2} = \frac{P \theta}{T} - \left( \frac{\theta}{T} \right)^2 \left[ (A_{12} + A_{21}) m_1 + 2A_{22} m_2 + (A_{23} + A_{32}) m_3 \right] \]
\[ \frac{\partial W}{\partial m_3} = \frac{L \theta}{T} - \left( \frac{\theta}{T} \right)^2 \left[ (A_{13} + A_{31}) m_1 + (A_{23} + A_{32}) m_2 + 2A_{33} m_3 \right]. \]

Hence, the unique stagnation point is
\[ m_1^* = \frac{T}{\theta} m_1^*, m_2^* = \frac{T}{\theta} m_2^*, m_3^* = \frac{T}{\theta} m_3^*, \]
where

\[
m_{1}^{**} = \begin{bmatrix} Q & A_{12} + A_{21} & A_{13} + A_{31} \\ P & 2A_{22} & A_{23} + A_{32} \\ L & A_{23} + A_{32} & 2A_{33} \end{bmatrix}
\]

\[
m_{2}^{**} = \begin{bmatrix} 2A_{11} & A_{12} + A_{21} & A_{13} + A_{31} \\ A_{12} + A_{21} & 2A_{22} & A_{23} + A_{32} \\ A_{13} + A_{31} & A_{23} + A_{32} & 2A_{33} \end{bmatrix}
\]

\[
m_{3}^{**} = \begin{bmatrix} 2A_{11} & A_{12} + A_{21} \\ A_{12} + A_{21} & 2A_{22} \\ A_{13} + A_{31} & A_{23} + A_{32} \end{bmatrix}
\]

Noting that

\[
\begin{bmatrix} 2A_{11} & A_{12} + A_{21} \\ A_{12} + A_{21} & 2A_{22} \\ A_{13} + A_{31} & A_{23} + A_{32} \end{bmatrix}
\]

is a positive definite matrix, so, the corresponding optimal time-averaged yield is:

\[
W^{*} = \frac{\theta}{\tau} m_{1}^{*} \cdot \left[ Q - \frac{\theta}{\tau} (m_{1}^{*} A_{11} + m_{2}^{*} A_{21} + m_{3}^{*} A_{31}) \right] G
+ \frac{\theta}{\tau} m_{2}^{*} \cdot \left[ P - \frac{\theta}{\tau} (m_{1}^{*} A_{12} + m_{2}^{*} A_{22} + m_{3}^{*} A_{32}) \right] G
+ \frac{\theta}{\tau} m_{3}^{*} \cdot \left[ L - \frac{\theta}{\tau} (m_{1}^{*} A_{13} + m_{2}^{*} A_{23} + m_{3}^{*} A_{33}) \right] G
= m_{1}^{*} \cdot \left[ Q - (m_{1}^{*} A_{11} + m_{2}^{*} A_{21} + m_{3}^{*} A_{31}) \right] G
+ m_{2}^{*} \cdot \left[ P - (m_{1}^{*} A_{12} + m_{2}^{*} A_{22} + m_{3}^{*} A_{32}) \right] G
+ m_{3}^{*} \cdot \left[ L - (m_{1}^{*} A_{13} + m_{2}^{*} A_{23} + m_{3}^{*} A_{33}) \right] G.
\]

Now, let’s consider a special case for Theorem 4.5.

**Corollary 1.** Suppose \( \theta = T \). In other words, there is no intermittent capture. Then the optimal harvesting strategy is:

\[
m^{*} = \left( \frac{r_{1} - \sigma_{1}^{2}/2}{2a_{11}} \right).
\]

The optimal harvesting yield is

\[
W^{*} = \left( \frac{r_{1} - \sigma_{1}^{2}/2}{4a_{11}} \right)^{2}.
\]
These conclusions are agree with the classical results obtained by J.R. Beddington and R.M. May [6], which were published in Science in 1977.

Corollary 2. Suppose $\theta = T$, which is a special case for Theorem 4.6. Then the optimal harvesting strategies are:

$$m_1^* = \frac{2a_{11}\phi_4 + (a_{12} + a_{21})\tilde{\phi}_4}{[4a_{11}a_{22} - (a_{12} + a_{21})^2]A_{33}},$$

$$m_2^* = \frac{2a_{22}\tilde{\phi}_4 - (a_{12} + a_{21})\phi_4}{[4a_{11}a_{22} - (a_{12} + a_{12})^2]A_{33}}.$$

The corresponding maximum yield is

$$W^* = m_1^*\phi_4 - (a_{22}m_1^{*2} - a_{12}m_1^*m_2^*) + m_2^*\tilde{\phi}_4 - (a_{11}m_2^{*2} - a_{21}m_1^*m_2^*).$$

These conclusions are consistent with the results given by Zou and Wang in [31].

Corollary 3. Suppose $\sigma_1 = 0$, the model becomes a deterministic model with intermittent capture. As far as we know, even the deterministic conclusion has not been studied so far. Hence, results in this paper are novel.

5. Practical examples and numerical simulations.

5.1. Practical examples.

Remark 3. (Effect of intermittent capture) In view of Theorems 4.5-4.7 and corollaries 1-2, it is easy to find that $\theta/T$ can affect the optimal harvesting efforts, but it cannot change the corresponding optimal time-averaged objective functions. In this section, we will explain these interesting results with some examples in literatures.

Practical example 1: $\theta/T$ can affect the optimal harvesting efforts (See literature [24]). According to the survey data conducted by east China sea region fishery resources dynamic monitoring network, more than 100 thousand fishing boats have been shortened by 2 to 3 months of operation time each year, and the efficiency of fishery production has been improved, i.e. the harvesting efforts have been improved clearly. This practical example shows intermittent capture can affect the optimal harvesting efforts.

Practical example 2: $\theta/T$ cannot change the corresponding optimal time-averaged yield (See literature [28]). The fishing boats do not go out to sea to produce during the summer closed fishing. As a result, this will temporarily reduce the output of fishery resources. However, the output of the loss during the summer closed fishing was compensated by the production of the fishing season. So, the actual economic outputs remained relatively stable.

5.2. Numerical simulations. Finally, we make some numerical simulations for this stochastic two-prey one-predator Lotka-Volterra model, we let $r_1 = 0.5, r_2 = 0.9, r_3 = 0.7, a_{11} = 0.46, a_{12} = 0.15, a_{13} = 0.35, a_{21} = 0.3, a_{22} = 1.4, a_{23} = 0.2, a_{31} = 0.4, a_{32} = 0.5, a_{33} = 2, \sigma_1 = 0.3, \sigma_2 = 0.3, \sigma_3 = 0.4, \theta = 0.7$ and $T = 1$. Based on these parameters, we can obtain that the optimal harvesting efforts are
\( m_1^* = 0.2543, m_2^* = 0.4950, m_3^* = 0.7736. \) The numerical simulations under these parameters can be found in Figure 1. In addition, the time-average for all the populations are given in Figure 2. Furthermore, it is easy to work out that \( \zeta_3 = 4.3372 > 1, \)

![Figure 1. Numerical simulations for sample paths](image1)

![Figure 2. Numerical simulations for time average](image2)
\[
\begin{pmatrix}
2A_{11} & A_{12} + A_{21} & A_{13} + A_{31} \\
A_{12} + A_{21} & 2A_{22} & A_{23} + A_{32} \\
A_{13} + A_{31} & A_{23} + A_{32} & 2A_{33}
\end{pmatrix}
= \begin{pmatrix}
5.8 & -1.155 & -0.05 \\
-1.155 & 2.12 & 0.183 \\
-0.02 & 0.183 & 1.75
\end{pmatrix}
\]
is a positive definite matrix. Moreover, \( R = 0.157, G = 1.3755, G_1 = 0.5980, G_2 = 0.3698, G_3 = 0.3210, A_{33} = 0.5990, \varphi_1 = 0.3678, \varphi_3 = 0.1580, G_1 = 0.0723, G_2 = 0.0181, G_3 = 0.074, \phi_2 = 0.2017, A_{23} = 0.17, A_{32} = 0.013, A_{31} = -0.46. \) We can easily verify that assumptions 2-4 are also satisfied, so we can obtain the maximum yield is \( W^* = 0.3492. \)

Due to the existence of stochastic factors, it is difficult to see the effect of intermittent control from Figure 1. Therefore, in order to clearly reflect the effect of intermittent control through numerical simulations, we give a numerical simulation in a one-dimensional simple case (see Figure 3). As can be seen from Figure 3, the population size has an obvious downward trend in the fishing season, while the population size has an obvious upward trend in the fishing off season.

**Figure 3.** The effects of intermittent control in one-dimensional situation

6. **Conclusions and discussions.** The main contributions of this paper are as follows:

- Intermittent control is introduced to simulate the phenomenon of “summer fishing moratorium”.
- A new objective function is proposed.
- Optimization problems are solved by almost sure equivalence method.

Conclusions in the paper state that

- Stochastic factors affect the optimal harvesting problems through parameters \( \sigma_i^2/2, i = 1, 2, 3, \) which are usually thought of as the intensities of stochastic noises.
- Intermittent controls affect the optimal harvesting problems through parameter \( \theta/T, \) which is the time proportion.
It is worth mentioning that the modeling ideas and problem-solving methods in this paper can be applied to other stochastic models.

Appendix A: Proof of Lemma 3.3.

Proof. Let \( V_1 = a_{31}^t x, V_2 = a_{32}^t y, V_3 = a_{mn}^t z \), where \( a_{mn} = \min\{a_{13}, a_{23}\} \).

\( V = V_1 + V_2 + V_3 \), then \( dV = dV_1 + dV_2 + dV_3 \). Using the generalized Itô formula, we obtain

\[
dV = LV \, dt + a_{31}^t \sigma_1 x(t) dB_1(t) + a_{32}^t \sigma_2 y(t) dB_2(t) + a_{mn}^t \sigma_3 z(t) dB_3(t),
\]

where

\[
LV = a_{31}^t [x(t)(1 + r_1 - H_1(t) \, a_{11} x(t) - a_{12} y(t) - a_{13} z(t))] dt
+ a_{32}^t [y(t)(1 + r_2 - H_2(t) \, a_{21} x(t) - a_{22} y(t) - a_{23} z(t))] dt
+ a_{mn}^t [z(t)(1 + r_3 - H_3(t) + a_{31} x(t) + a_{32} y(t) - a_{33} z(t))] dt
\]

\[
\leq e^{t} [-a_{11} a_{31} x^2(t) + a_{31} (1 + r_1 - H_1(t)) x(t) - a_{31} a_{13} x(t) z(t)] dt
+ e^{t} [-a_{22} a_{32} y^2(t) + a_{32} (1 + r_2 - H_2(t)) y(t) - a_{32} a_{23} y(t) z(t)] dt
+ e^{t} [-a_{33} a_{mn} z^2(t) + a_{mn} (1 + r_3 - H_3(t)) z(t) + a_{31} a_{mn} x(t) z(t)] dt
\]

\[
\leq e^{t} [-a_{11} a_{31} x^2(t) + a_{31} (1 + r_1 - H_1(t)) x(t)] dt
+ e^{t} [-a_{22} a_{32} y^2(t) + a_{32} (1 + r_2 - H_2(t)) y(t)] dt
+ e^{t} [-a_{33} a_{mn} z^2(t) + a_{mn} (1 + r_3 - H_3(t)) z(t)] dt
\]

It is easy to see that there exist three positive constants \( M_1, M_2 \) and \( M_3 \) such that

\[
-a_{11} a_{31} x^2(t) + a_{31} (1 + r_1 - H_1(t)) x(t) \leq M_1,
-a_{22} a_{32} y^2(t) + a_{32} (1 + r_2 - H_2(t)) y(t) \leq M_2,
-a_{33} a_{mn} z^2(t) + a_{mn} (1 + r_3 - H_3(t)) z(t) \leq M_3.
\]

Integrating from 0 to \( t \) on both sides of the inequality (10), we can obtain that

\[
e^{t} (a_{31} x(t) + a_{32} y(t) + a_{mn} z(t))
\leq e^{t} (a_{31} x(0) + a_{32} y(0) + a_{mn} z(0)) + (M_1 + M_2 + M_3) \int_{0}^{t} e^{s} ds
+ \int_{0}^{t} e^{s} a_{31} \sigma_1 x(s) dB_1(s) + \int_{0}^{t} e^{s} a_{32} \sigma_2 y(s) dB_2(s) + \int_{0}^{t} e^{s} a_{mn} \sigma_3 z(s) dB_3(s).
\]

Besides, taking expectations on both sides,

\[
\mathbb{E} [a_{31} x(t) + a_{32} y(t) + a_{mn} z(t)] \leq a_{31} x(0) + a_{32} y(0) + a_{mn} z(0) + M_1 + M_2 + M_3 < \infty.
\]

Then we can easily show that,

\[
\mathbb{E} |x(t), y(t), z(t)| \leq \mathbb{E} |x(t) + y(t) + z(t)|
\leq a_{31} x(0) + a_{32} y(0) + a_{mn} z(0) + M_1 + M_2 + M_3 =: M_4.
\]

To sum up,

\[
\sup_{0 \leq t < \infty} \mathbb{E} |x(t), y(t), z(t)| \leq M_4 < \infty, \quad t > 0
\]

for any initial value \((x(0), y(0), z(0)) \in R^3_+\). The proof is completed. \(\Box\)
Appendix B: Proof of Lemma 3.4.

Proof. Define

\[ V(t) = |\ln x_1(t) - \ln x_2(t)| + |\ln y_1(t) - \ln y_2(t)| + |\ln z_1(t) - \ln z_2(t)|, \]

then

\[ dV(t) = d|\ln x_1(t) - \ln x_2(t)| + d|\ln y_1(t) - \ln y_2(t)| + d|\ln z_1(t) - \ln z_2(t)|, \]

where

\[
\begin{align*}
\text{d} \ln x_1(t) &= (r_1 - H_1(t) - a_{11} x_1(t) - a_{12} y_1(t) - a_{13} z_1(t) - \frac{1}{2} \sigma_1^2) \, dt + \sigma_1 \, dB_1(t) \\
\text{d} \ln y_1(t) &= (r_2 - H_2(t) - a_{21} x_1(t) - a_{22} y_1(t) - a_{23} z_1(t) - \frac{1}{2} \sigma_2^2) \, dt + \sigma_2 \, dB_2(t) \\
\text{d} \ln z_1(t) &= (r_3 - H_3(t) + a_{31} x_1(t) + a_{32} y_1(t) - a_{33} z_1(t) - \frac{1}{2} \sigma_3^2) \, dt + \sigma_3 \, dB_3(t)
\end{align*}
\]

Next, let’s write \( x_i(t) \) as \( x_i, y_i(t) \) as \( y_i \) and \( z_i(t) \) as \( z_i \), consequently,

\[
\begin{align*}
dV &= -\text{sign}(x_1 - x_2)[a_{11}(x_1 - x_2) + a_{12}(y_1 - y_2) + a_{13}(z_1 - z_2)] \, dt \\
&\quad -\text{sign}(y_1 - y_2)[a_{21}(x_1 - x_2) + a_{22}(y_1 - y_2) + a_{23}(z_1 - z_2)] \, dt \\
&\quad -\text{sign}(z_1 - z_2)[-a_{31}(x_1 - x_2) - a_{32}(y_1 - y_2) + a_{33}(z_1 - z_2)] \, dt \\
&\leq \{[a_{11}|x_1 - x_2| + a_{22}|y_1 - y_2| + a_{33}|z_1 - z_2|] + a_{12}|y_1 - y_2| \\
&\quad + a_{13}|z_1 - z_2| + a_{21}|x_1 - x_2| + a_{23}|z_1 - z_2| + a_{31}|x_1 - x_2| \\
&\quad + a_{32}|y_1 - y_2| \} \, dt \\
&\leq \{[(-a_{11} + a_{21} + a_{31})|x_1 - x_2| + (-a_{22} + a_{12} + a_{32})|y_1 - y_2| \\
&\quad + (-a_{33} + a_{13} + a_{23})|z_1 - z_2|] \} \, dt.
\end{align*}
\]

Let \( N = \min\{a_{11} - a_{21} - a_{31}, a_{22} - a_{12} - a_{32}, a_{33} - a_{13} - a_{23}\} \), then the aforementioned inequality (11) becomes:

\[ dV \leq -N \left(|x_1(t) - x_2(t)| + |y_1(t) - y_2(t)| + |z_1(t) - z_2(t)|\right). \] (12)

Integrating from 0 to \( t \) on both sides of the inequality (12) and let \( t \to \infty \), it is not difficult to obtain

\[ V(t) + N \int_0^\infty [|x_1(s) - x_2(s)| + |y_1(s) - y_2(s)| + |z_1(s) - z_2(s)|] \, ds \leq V(0) < \infty. \]

Then

\[ \int_0^\infty |x_1(s) - x_2(s)| + |y_1(s) - y_2(s)| + |z_1(s) - z_2(s)| \, ds < \infty. \] (13)

Taking expectations on both sides of inequality (13),

\[ \mathbb{E} \int_0^\infty \left[|x_1(s) - x_2(s)| + |y_1(s) - y_2(s)| + |z_1(s) - z_2(s)|\right] \, ds < \infty, \]

which result in

\[ \int_0^\infty \mathbb{E} \left[|x_1(s) - x_2(s)| + |y_1(s) - y_2(s)| + |z_1(s) - z_2(s)|\right] \, ds < \infty. \]
Integrating from 0 to $t$ and taking expectations on both sides of (4) yield

$$
\begin{align*}
\mathbb{E}[x(t)] &= x(0) + \int_0^t [(r_1 - H_1(s)) \mathbb{E}[x(s)] \, ds - \int_0^t [a_{11} \mathbb{E}[x^2(s)] + a_{12} \mathbb{E}[x(s)y(s)] + a_{13} \mathbb{E}[x(s)z(s)]] \, ds
\end{align*}
$$

and

$$
\mathbb{E}[y(t)] = y(0) + \int_0^t [(r_2 - H_2(s)) \mathbb{E}[y(s)] \, ds - \int_0^t [a_{21} \mathbb{E}[x(s)y(s)] + a_{22} \mathbb{E}[y^2(s)] + a_{23} \mathbb{E}[y(s)z(s)]] \, ds
\end{align*}
$$

and

$$
\mathbb{E}[z(t)] = z(0) + \int_0^t [(r_3 - H_3(s)) \mathbb{E}[z(s)] \, ds - \int_0^t [a_{31} \mathbb{E}[x(s)z(s)] + a_{32} \mathbb{E}[y(s)z(s)] + a_{33} \mathbb{E}[z^2(s)]] \, ds.
\end{align*}
$$

This implies that $\mathbb{E}[x(t)], \mathbb{E}[y(t)]$ and $\mathbb{E}[z(t)]$ are continuous functions about $t$.

In the proof of Lemma 3.3, we have showed that $\mathbb{E}[x(t)], \mathbb{E}[y(t)]$ and $\mathbb{E}[z(t)]$ are bounded functions, and the boundedness of $\mathbb{E}[x^2(t)], \mathbb{E}[y^2(t)]$ and $\mathbb{E}[z^2(t)]$ can be followed as the same line. So, we have

$$
0 \leq \mathbb{E}[x(t)y(t)] \leq (\mathbb{E}[x^2(t)])^{\frac{1}{2}} (\mathbb{E}[y^2(t)])^{\frac{1}{2}} < \infty,
$$

$$
0 \leq \mathbb{E}[x(t)z(t)] \leq (\mathbb{E}[x^2(t)])^{\frac{1}{2}} (\mathbb{E}[z^2(t)])^{\frac{1}{2}} < \infty,
$$

$$
0 \leq \mathbb{E}[z(t)y(t)] \leq (\mathbb{E}[z^2(t)])^{\frac{1}{2}} (\mathbb{E}[y^2(t)])^{\frac{1}{2}} < \infty,
$$

which imply $\mathbb{E}[x(t)y(t)], \mathbb{E}[x(t)z(t)]$ and $\mathbb{E}[y(t)z(t)]$ are also bounded functions. Thereby, we can obtain that $d\mathbb{E}[x(t)]/dt, d\mathbb{E}[y(t)]/dt$ and $d\mathbb{E}[z(t)]/dt$ are bounded functions, and this implies $\mathbb{E}[x(t)], \mathbb{E}[y(t)]$ and $\mathbb{E}[z(t)]$ are uniformly continuous functions. Hence, Lemma 3.2 yields

$$
\lim_{t \to \infty} \mathbb{E} \left[ \left| (x_1(t), y_1(t), z_1(t)) - (x_2(t), y_2(t), z_2(t)) \right| \right]
$$

$$
\leq \mathbb{E} \left[ |x_1(t) - x_2(t)| + |y_1(t) - y_2(t)| + |z_1(t) - z_2(t)| \right] = 0.
$$

This proof is completed.

\section*{Appendix C: Proof of Lemma 3.5.}

\textbf{Proof.} Let $p(t, X_t, \cdot)$ represent the transition probability of process $X_t$, $\mathcal{B}(\mathbb{R}^n)$ represent all the probability measures on $\mathbb{R}^n$. Let

$$
\mathcal{L} = \{g : \mathbb{R}^n \to \mathbb{R} : |g(x) - g(y)| \leq |x - y|, \text{and } |g(\cdot)| \leq 1\}
$$

for any $\mathcal{B}_1, \mathcal{B}_2 \in \mathcal{B}(\mathbb{R}^n)$, define

$$
d_{\mathcal{L}}(\mathcal{B}_1, \mathcal{B}_2) = \sup_{g \in \mathcal{L}} \left| \int_{\mathbb{R}^n} g(x) \mathcal{B}_1(dx) - \int_{\mathbb{R}^n} g(x) \mathcal{B}_2(dx) \right|.
$$

By Lemma 3.4, we can obtain $\{p(t, X_t, \cdot)\}$ is a Cauchy sequence in space $\mathcal{B}(\mathbb{R}^n)$ with measure $d_{\mathcal{L}}$, so there is a unique probability measure $\pi(\cdot)$ on $\mathbb{R}^n$, such that $\lim_{t \to \infty} p_{\mathcal{L}}(t, X_t, \cdot) = \pi(\cdot) = 0$. Noticing the fact that weak convergence of probability is a concept of metric [19], so, $\lim_{t \to \infty} d_{\mathcal{L}}(p(t, X_t, \cdot), \pi(\cdot)) = 0$ is equivalent to the fact that $p(t, X_t, \cdot) \xrightarrow{\text{weakly}} \pi(\cdot)$ as $t \to \infty$. The proof is completed. \hfill \Box
REFERENCES

[1] L. J. S. Allen and A. M. Burgin, Comparison of deterministic and stochastic SIS and SIR models in discrete time, *Math. Biosci.*, **163** (2000), 1–33.

[2] L. H. R. Alvarez, Optimal harvesting under stochastic fluctuations and critical depensation, *Math. Biosci.*, **152** (1998), 63–85.

[3] L. H. R. Alvarez and L. A. Shepp, Optimal harvesting of stochastically fluctuating populations, *Journal of Mathematical Biology*, **37** (1998), 155–177.

[4] I. Barbálat, Système d’équations différentielles d’oscillations non linéaires, *Rev. Math. Pures Appl.*, **4** (1959), 267–270.

[5] J. Batsleer, A. D. Rijnsdorp, K. G. Hamon, H. M. J. van Overzee and J. J. Poos, Mixed fisheries management: Is the ban on discarding likely to promote more selective and fuel efficient fishing in the dutch flatfish fishery?, *Fish Res.*, **174** (2016), 118–128.

[6] J. R. Beddington and R. M. May, Harvesting natural populations in a randomly fluctuating environment, *Science*, **197** (1977), 463–465.

[7] A. Bottaro, Y. Yasutake, T. Nomura, M. Casadio and P. Morasso, Bounded stability of the quiet standing posture: An intermittent control model, *Hum. Movement Sci.*, **27** (2008), 473–495.

[8] C. W. Clark, *Mathematical Bioeconomics. The Optimal Management of Renewable Resources*, Pure and Applied Mathematics. Wiley-Interscience [John Wiley & Sons], New York-London-Sydney, 1976.

[9] J. H. Connell, On the prevalence and relative importance of interspecific competition: Evidence from field experiments, *Am. Nat.*, **122** (1983), 661–696.

[10] G. Da Prato and J. Zabczyk, *Ergodicity for Infinite Dimensional Systems*, Cambridge University Press, 1996.

[11] J.-M. Ecoutin, M. Simier, J.-J. Albaret, R. Læ, J. Raffray, O. Sadio and L. T. de Morais, Ecological field experiment of short-term effects of fishing ban on fish assemblages in a tropical estuarine mpa, *Ocean Coastal Manage.*, **100** (2014), 74–85.

[12] B. Øksendal, *Stochastic Differential Equations*, Springer-Verlag, 1985.

[13] A. Gray, D. Greenhalgh, L. Hu, X. Mao and J. Pan, A stochastic differential equation sis epidemic model, *SIAM J. Appl. Math.*, **71** (2011), 876–902.

[14] Y. Guo, W. Zhao and X. Ding, Input-to-state stability for stochastic multi-group models with multi-dispersal and time-varying delay, *Appl. Math. Comput.*, **343** (2019), 114–127.

[15] S. Hong and N. Hong, $H^\infty$ switching synchronization for multiple time-delay chaotic systems subject to controller failure and its application to aperiodically intermittent control, *Nonlinear Dyn.*, **92** (2018), 869–883.

[16] C. Hu, J. Yu, H. Jiang and Z. Teng, Exponential stabilization and synchronization of neural networks with time-varying delays via periodically intermittent control, *Nonlinearity*, **23** (2010), 2369–2391.

[17] Z. Y. Huang, A comparison theorem for solutions of stochastic differential equations and its applications, *Proc. Amer. Math. Soc.*, **91** (1984), 611–617.

[18] L. Imhof and S. Walcher, Exclusion and persistence in deterministic and stochastic chemostat models, *J. Differ. Equations*, **217** (2005), 26–53.

[19] B. Johnson, R. Narayananakumar, P. S. Swathilekshmi, R. Geetha and C. Ramachandran, Economic performance of motorised and non-mechanised fishing methods during and after-ban period in ramanathapuram district of tamil nadu, *Indian J. Fish.*, **64** (2017), 160–165.

[20] G. B. Kallianpur, Stochastic differential equations and diffusion processes, *Technometrics*, **25** (1983), 268.

[21] W. Li and K. Wang, Optimal harvesting policy for stochastic logistic population model, *Appl. Math. Comput.*, **218** (2011), 157–162.

[22] M. Liu and K. Wang, Dynamics of a two-prefy one-predator system in random environments, *J. Nonlinear Sci.*, **23** (2013), 751–775.

[23] O. Ovaskainen and B. Meerson, Stochastic models of population extinction, *Trends Ecol. Evol.*, **25** (2010), 643–652.

[24] N.-T. Shih, Y.-H. Cai and I.-H. Ni, A concept to protect fisheries recruits by seasonal closure during spawning periods for commercial fishes off taiwan and the east china sea, *J. Appl. Ichthyol.*, **25** (2009), 676–685.

[25] L. Wang, D. Jiang and G. S. K. Wolkowicz, Global asymptotic behavior of a multi-species stochastic chemostat model with discrete delays, *J. Dyn. Differ. Equ.*, **32** (2020), 849–872.
[26] W. Xia and J. Cao, Pinning synchronization of delayed dynamical networks via periodically intermittent control, Chaos, 19 (2009), 013120, 8pp.
[27] B. Yang, Y. Cai, K. Wang and W. Wang, Optimal harvesting policy of logistic population model in a randomly fluctuating environment, Phys. A, 526 (2019), 120817, 17pp.
[28] Y. Ye, Assessing effects of closed seasons in tropical and subtropical penaeid shrimp fisheries using a length-based yield-per-recruit model, ICES J. Mar. Sci., 55 (1998), 1112–1124.
[29] C. Zhang, W. Li and K. Wang, Graph-theoretic method on exponential synchronization of stochastic coupled networks with Markovian switching, Nonlinear Anal-Hybri., 15 (2015), 37–51.
[30] G. Zhang and Y. Shen, Exponential synchronization of delayed memristor-based chaotic neural networks via periodically intermittent control, Neural Networks, 55 (2014), 1–10.
[31] X. Zou and K. Wang, Optimal harvesting for a stochastic lotka-volterra predator-prey system with jumps and nonselective harvesting hypothesis, Optim. Control Appl. Methods., 37 (2016), 641–662.
[32] X. Zou and K. Wang, Optimal harvesting for a stochastic n-dimensional competitive lotka-volterra model with jumps, Discrete Cont. Dyn-B, 20 (2015), 683–701.
[33] X. Zou, Y. Zheng, L. Zhang and J. Lv, Survivability and stochastic bifurcations for a stochastic Holling type II predator-prey model, Commun. Nonlinear Sci Numer. Simulat., 83 (2020), 105136, 20 pp.

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E-mail address: zouxiaoling1025@126.com
E-mail address: 18766317120163.com