Electromagnetic field and numerical analysis in two dimensions and its interaction in biological tissue

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Abstract. This paper consists in transferring Maxwell’s differential equations to a discrete spatial and temporal formulation that allows coding them and implementing them in an algorithm that provides a numerical solution which provides a visual overview of the physical behavior of electromagnetic waves for two dimensions. This technique provides a tool capable of being applied to various disciplines of science with which it is possible to study systems and quantify the phenomena produced by electromagnetic activity successfully.

1. Introduction

The method of finite differences in the time domain (FDTD) is a very useful tool to perform the study by numerical simulation of the behavior of the electromagnetic field at any point of space, the Maxwell equations are expressed in finite differences and boundary conditions must be satisfies, which in turn is restricted by the difficulty that the geometry of the problem can present. The method uses differences (spatiotemporal) centered to approximate the derivatives of different order, in the present article first the vacuum (air) will be considered as means of propagation which is a homogeneous medium that does not present special characteristics related to some type of material [1].

2. Methodology

The boundary conditions absorbent in two dimensions are necessary tools for the design of the FDTD method [1] and with this to be able to obtain reliable results that are not affected once the electromagnetic field reaches the end of the computational space [2]. Absorbing boundary conditions play an important role in computational simulation because the mesh and grid are limited in space and once the field reaches the corners it is prevented from returning to the workspace [3].

For the electric flux density in the domain of ω, Equation (1).

\[ \widehat{D}(\omega) = \varepsilon_0 \ast \varepsilon_r(\omega) \ast \widehat{E}(\omega) \]  (1)

For the electric flux density vector, Equation (2).

\[ \frac{\partial \widehat{D}}{\partial t} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \ast (\nabla \times \widehat{H}) \]  (2)

For the magnetic field vector, Equation (1).
\[
\frac{\partial \vec{H}}{\partial t} = - \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \star (\vec{\nabla} \times \vec{E}) \tag{3}
\]

According to the previous equations we have the mathematical basis to start the development of the equations for the electromagnetic field in two dimensions, starting this study with the magnetic transverse mode (TM) [4].

2.1. Magnetic transverse mode
As previously mentioned, the magnetic TM is composed of the following group of vectors \((E_x, H_y, H_z)\). For which it will be developed for each of these components its respective equation. Component \(H_x\). To make this component we will quote the equation for the magnetic field (Equation (4) to Equation (6)).

\[
\vec{\nabla} \times \vec{E} = \begin{bmatrix}
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
E_x & E_y & E_z
\end{bmatrix} \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} = \begin{bmatrix}
\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \\
\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \\
\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}
\end{bmatrix} \begin{bmatrix} i \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\partial E_z}{\partial y} \\ -\frac{\partial E_y}{\partial x} \end{bmatrix} \begin{bmatrix} 0 \\ i \\ 0 \end{bmatrix} \tag{5}
\]

\[
\frac{\partial H_x}{\partial t} = - \frac{1}{\mu_0} \star \left( \frac{\partial E_z}{\partial y} \right) \tag{6}
\]

Taking into account the Gaussian normalization procedure, which is given for this case as Equation (7).

\[
\vec{E} = \frac{\sqrt{\varepsilon_0}}{\mu_0} \star \vec{E} \tag{7}
\]

The equation for the magnetic field in the component \(x \rightarrow \langle H_x \rangle\) is Equation (8).

\[
\frac{\partial H_x}{\partial t} = - \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \star \left( \frac{\partial E_z}{\partial y} \right) \tag{8}
\]

Now, for the component \(H_x\) we get Equation (9).

\[
\frac{\partial H_x}{\partial t} = - \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \star \left( \frac{\partial E_z}{\partial y} \right) \tag{9}
\]

Finally, applying finite differences we get Equation (10).

\[
\vec{D}_{z(l,i)}^{n+1} = \vec{D}_{z(l,i)}^{n-1} + \frac{1}{2} \star \left( H_y^{n}_{y(i+1,l)} - H_y^{n}_{y(i-1,l)} - H_x^{n}_{x(l,i+1)} + H_x^{n}_{x(l,i-1)} \right) \tag{10}
\]

2.2. Boundary conditions absorbent in two dimensions
The need for which it is useful to implement the absorbing border conditions at the ends of the computational space is given by the waves that propagate towards the limits of the defined space when arriving at the end of this cause reactions into the interior of the region of interest, which is not convenient because they generate incorrect numerical values in the simulation of the propagation of the electromagnetic wave, which is undesired for the purposes of studying and obtaining results in the area
of analysis. In the first place, the application of the absorbing boundary conditions perfectly matched layer (PML) is proposed [5,6].

The expressions obtained in the design of the absorbent layers PML in the directions (x) and (y) for the components in the transverse magnetic propagation mode TM, for this will be exposed this set of expressions in the following expressions in the Table 1.

Table 1. Final expressions components transverse magnetic mode including conditions of absorbing borders PML.

| Component | Expression |
|-----------|------------|
| Dz       | $D_{z(i,j)}^{n+1} = D_{z(i,j)}^{n-1} + (a)(c) + (b)(d) \cdot \left( \frac{1}{2} \right) \cdot \left( H_{y(i+1,j)}^{n} - H_{y(i-1,j)}^{n} - H_{x(i+1,j)}^{n} + H_{x(i-1,j)}^{n} \right) $ |
| Hx       | $H_{x(i,j)}^{n+1} = H_{x(i,j)}^{n-1} + (c) + (d) \cdot \left( \frac{1}{2} \right) \cdot \text{curl} e_y + \left( \frac{\mu_0 - \mu_n}{2\varepsilon_0} \right) $ |
| Hy       | $H_{y(i,j)}^{n+1} = H_{y(i,j)}^{n-1} + (a) + (b) \cdot \left( \frac{1}{2} \right) \cdot \text{curl} e_x + \left( \frac{\mu_0 - \mu_n}{2\varepsilon_0} \right) $ |

In the following table we have the terms (a, b, c, d) according to $K_n$, which has been found empirically according to the design of absorbent layers PML made by Bérenger (Table 2) [3].

Table 2. Terms a, b, c, and d, according to $K_n$.

| Term | Expression |
|------|------------|
| a    | $\left( \frac{1 - K_n}{1 + K_n} \right)$ |
| b    | $\left( \frac{1}{1} \right)$ |
| c    | $\left( \frac{1 + K_n}{1 - K_n} \right)$ |
| d    | $\left( \frac{1 + K_n}{1} \right)$ |

2.3. With absorbing border conditions

Figure 1 shows the time behavior of the electromagnetic pulse for the electric and magnetic field in two dimensions (2D) will be explained below, keeping in mind that absorbing boundary conditions PML are being considered. It will start by exposing the graphs of the electric field ($E_x$) and later the graphs of the magnetic field in the directions ($x=H_x$) and ($y=H_y$) for different instants of time using a sinusoidal field source.

![Figure 1](image)

Figure 1. (a) Field $E_z$ [V/m] $n = 20$ temporary steps, and (b) Field $E_z$ [V/m] $n = 40$ temporary steps. (a) and (b) according to $K_n$.

3. Propagation considering a biological medium with complex relative permittivity

The propagation of the electromagnetic field in human (biological) tissue will be considered, for which the behavior of the electromagnetic field in this medium will be analyzed considering a range of frequencies. The complex relative permittivity is given by Equation (11).
\[ \varepsilon_r(\omega) = \left( \varepsilon_r + \frac{\sigma}{j\omega \varepsilon_0} \right) \]  

(11)

The previous expression denotes the basic form of the relative complex permittivity \( \varepsilon_r(\omega) \), to carry out the study in human tissue implies making changes in this expression to introduce new elements that help us to formulate the numerical solution in a better way. The new form that will take the relative complex permittivity will be the Equation (12), which represents the model exposed by cole-cole [7].

\[ \varepsilon_r(\omega) = \left( \varepsilon_r + \frac{\sigma}{j\omega \varepsilon_0} - \frac{\omega_0^2 \varepsilon_r}{\omega_0^2 + \alpha^2 + 2 \alpha j \omega + \omega^2} \right) \]  

(12)

From the Equation (12) we introduced three new elements \( (\omega_0, \alpha, \omega) \) which will be defined as follows.

Biological tissues are dielectric materials that can be modeled as a set of clusters formed by cells immersed in an ionic medium which is called the extracellular membrane. Due to this configuration, the behavior of dielectric permittivity and electrical conductivity varies in a range of frequencies, showing that the permittivity decreases with the increase of the frequency and the conductivity increases with the increase of this. This dispersion of the values for permittivity and conductivity can be expressed by means of the Equation (13) for the term \( \alpha \) which is found in the equation of the complex permittivity for biological tissue [8,9].

\[ \alpha = \frac{\omega}{\varepsilon_0} \sqrt{\frac{\varepsilon_r}{2}} \left( \sqrt{1 + \left( \frac{\sigma}{\omega_0 \varepsilon_r + \varepsilon_0} \right)^2} - 1 \right) \]  

(13)

In Table 3 and Table 4, the expressions obtained for the fields \( (E_x, H_y, D_x) \) are appended.

**Table 3.** \( E_x, H_y, D_x \) component expressions for biological tissue.

| Component | Final equation |
|-----------|----------------|
| Ex \( E^n \) | \( E^n = \left( D^n - \frac{\sigma * \Delta t}{\varepsilon_0} \sum_{i=0}^{n-1} E^i + \left[ \frac{B}{A} \right] * S^{n-1} \right) \) |
| Hy \( H_y^{n+1} \) | \( H_y^{n+1} = H_y^{n-1} + \left( \frac{1}{2} \right) * (E_x^{n+1} - E_x^{n}) \) |
| Dx \( D_x^{n+1} \) | \( D_x^{n+1} = D_x^{n-1} + \left( \frac{1}{2} \right) * (H_y^n - H_y^{n+1}) \) |

**Table 4.** Terms associated with the expression of the electric field intensity vector \( E \).

| Component | Final expression |
|-----------|-----------------|
| I         | \( I(k) = I(k) + c^*E(k) \) |
| S         | \( S(k) = |e(k)|^2ex(k) - |b(k)|/|a(k)|S(k) \) |

The non-homogeneity of biological tissues composed of millions of cellular units leads to different values of dielectric permittivity and electrical conductivity, in addition to these values being exposed to the frequency with which the electromagnetic field source is applied to the tissue [10,11]. This membrane acts as a dielectric interface due to its molecular components and can be seen and/or considered by analogy as a parallel plate capacitor as illustrated Figure 2.
For the measurement of the effects of the application of the electromagnetic field in biological tissue there are three regions or frequency ranges defined and denominated as: α region, β region and region γ, these three regions are characterized in Table 5 and Table 6 [12].

**Table 5. Regions of biological tissue dispersion.**

| Symbol | Description |
|--------|-------------|
| α      | The α dispersion ranges from a few mHz to 10 KHz and is related to the dielectric losses of the medium, intracellular structures and ionic diffusion. In this region, measurements are not usually made because it provides little information and the high impedance of the electrodes does not make it easy. |
| β      | The β dispersion ranges from 10 KHz to 700 MHz and is related to the capacity of the cell membrane, and the response of the protein molecules. This is where most measurements are made. |
| γ      | The γ dispersion ranges from 700 MHz to 100 GHz and is related to dipolar relaxation mechanisms such as water molecules, salts, etc. |

Next, we will annex the data corresponding to the dielectric material with which the numerical simulation will be carried out by means of the finite difference method in the time domain FDTD.

**Table 6. Properties of the biological tissue dielectric medium.**

| Space (GRID)       | Conductivity (σ) (S/m) | Relative permittivity (ε_r) (Dimensionless) | Permittivity of the vacuum (ε_0) (F/m) |
|--------------------|------------------------|------------------------------------------|---------------------------------------|
| Medium 1.          |                         |                                          |                                       |
| Air or vacuum (0 a 100) | 0                      | 1                                        | 8.85419*10^-12                       |
|                    |                         |                                          |                                       |
| Medium 2.          | 0.04                   | 4                                        | 8.85419*10^-12                       |
|                    |                         |                                          |                                       |
| Medium 3.          | 0                      | 0                                        | 8.85419*10^-12                       |

**4. Conclusions**

The use of the finite difference method in the time domain FDTD presents ease and adaptability for the application to different study cases where it is desired to approximate the derivatives in finite differences and obtain a numerical solution.

The problem in two dimensions presents the need to implement absorbing boundary conditions so that once the pulse reaches the end of the computational space no reflections from the field to the interior...
of the area of interest and cause erroneous results. The implementation of absorbent layers in two dimensions PML require a rigorous mathematical study in order to provide a gradual attenuation of the electromagnetic field once it reaches the end of computational space, this method was proposed by Bérenger.

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