Screening enhancement factors for laboratory CNO and rp astrophysical reactions.

Theodore E. Liolios *
Hellenic Naval Academy of Hydra
School of Deck Officers, Department of Science
Hydra Island 18040, Greece

Abstract

Cross sections of laboratory CNO and rp astrophysical reactions are enhanced due to the presence of the multi-electron cloud that surrounds the target nuclei. As a result the relevant astrophysical factors are overestimated unless corrected appropriately. This study gives both an estimate of the error committed if screening effects are not taken into account and a rough profile of the laboratory energy thresholds at which the screening effect appears. The results indicate that, for most practical purposes, screening corrections to past relevant experiments can be disregarded. Regarding future experiments, however, screening corrections to the CNO reactions will certainly be of importance as they are closely related to the solar neutrino fluxes and the rp process. Moreover, according to the present results, screening effects will have to be taken into account particularly by the current and future LUNA experiments, where screened astrophysical factors will be enhanced to a significant degree.

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*www.liolios.info
I. INTRODUCTION

Proton-induced nuclear reactions in stellar plasmas play a crucial role in advanced stellar nucleosynthesis. For example the (rapid) (proton-capture) process \([1,2]\) is the dominant reaction sequence in high-temperature hydrogen burning. Such processes occur when hydrogen fuel is ignited under highly degenerate conditions in explosive events on the surface of white dwarfs, neutron stars and ordinary supermassive stars. The proton-rich nuclei which participate in the rp process are currently under study in various laboratories around the world while much interest has attracted the prospect of studying relevant proton capture reactions for radioactive nuclei in radioactive ion-beam facilities. The beam energy in such reactions is sometimes so low (astrophysical energy) that the proton impinging on the multielectron target will actually experience an acceleration due to the electron cloud of the target.

A recent study \([3]\) derived an analytic screening enhancement factor for binary multi-electron reactions (between atoms \(A_{Z_1}X\) and \(A_{Z_2}X\) at a relative energy \(E\)), which is actually a corrective factor \(f(Z_1, Z_2, A_1, A_2, E)\) defined as

\[
f(Z_1, Z_2, A_1, A_2, E) = \frac{S_{\text{exp}}(E)}{S(E)} \tag{1}\]

where \(S_{\text{exp}}(E)\) is the experimental value of the astrophysical factor while \(S(E)\) is the corresponding one for bare nuclei (disregarding the electron cloud).

Thus experimentalists should actually plot (or tabulate) the values \(S_{\text{exp}}(E) / f\) instead of the value \(S_{\text{exp}}(E)\). However, many such astrophysical factors which have not been corrected for screening have been used directly in the thermonuclear reaction rates \([4]\).

On the other hand, as shown recently \([5]\) the effort to lower the energy beam close to zero (so that the value \(S(0)\) is more accurate) is pointless. The thermonuclear reaction rate is more accurate if the value \(S(E_0)\) is used instead of \(S(0)\) and that is where the experimentalist should focus. Nevertheless, if the uncorrected value of \(S(E_0)\) is used then the relevant reaction rate is of course overestimated by a factor of \(f(Z_1, Z_2, A_1, A_2, E_0)\).

In Ref. \([3]\) the new corrective method was applied to non-resonant reactions of the CNO cycle in typical solar conditions. In the present study we extend that method to more advanced proton-induced reactions (e.g. of the rp process) which, of course, are not expected to be non-resonant ones; on the contrary some of them exhibit very dense patterns of resonances at relatively high energies so that statistical methods such as the Hauser-Feshbach one are needed in order to calculate the respective reaction rate. However, when the compound nucleus exhibits low-level densities then the statistical model breaks down \([6]\). Therefore, especially for proton-rich nuclei near the proton drip line with small Q-values of proton capture reactions, the Maxwellian averaged reaction rate \(N_A < \sigma u >\) is determined by single isolated (narrow) resonances

\[
N_A < \sigma u > = N_A \left(\frac{2\pi}{\mu k}\right)^{3/2} \hbar^2 \omega^2 \gamma \exp \left(-\frac{E_r}{kT}\right) \tag{2}\]

and their non-resonant (tail) contributions.
\[ N_A < \sigma u >_{\text{tail}} = N_A \left( \frac{2}{\mu} \right)^{1/2} f_{pl} \frac{\Delta E_0}{(kT)^{3/2}} S_{\text{eff}} \exp \left( -\frac{3E_0}{kT} \right) \] (3)

so that \( N_A < \sigma u > = N_A < \sigma u >_r + N_A < \sigma u >_{\text{tail}}, \) where we have used the familiar notation of Ref. [7] (i.e. \( N_A \) is the Avogadro number, \( \hbar \) is Planck’s constant, \( (\omega \gamma)_r \) is the usual quantity appearing in the Breit-Wigner single-level formula, \( k \) is Boltzmann’s constant, \( \mu \) is the reduced mass, \( E_0 \) is the most effective energy of interaction, \( \Delta E_0 \) the relevant energy window, \( S_{\text{eff}} \) the effective astrophysical factor, and \( T \) is the temperature).

It should be emphasized that \( f_{pl} \) is the screening enhancement factor due to plasma effects which of course is calculated via entirely different models (see for example Ref. [8] and references therein). The interplay between plasma and laboratory screening enhancement factors has been thoroughly discussed in Ref. [5].

Note that despite the presence of resonances the concept of the most effective energy of interaction \( E_0 \) still serves its purpose which is to point to the energy region where a resonance will have dramatic effects on the reaction rate.

If the quantity \( S_{\text{eff}} = S(0) (S_{\text{eff}} = S(E_0)) \) is to be accurately determined then the beam energy should be as low (as close to \( E_0 \)) as possible. In practice, however, it is not feasible to lower the energy so much which means that either experimentalists obtained \( S_{\text{eff}} \) by extrapolating from higher energy values (definitely higher than \( E_0 \)) or they relied on theoretical models (e.g. [9]). In the first case, which is the most usual, a substantial error is committed sometimes. The higher the energy of the last measurement used in the extrapolation the further the experiment from the astrophysically important region and, of course, the larger the error. Note that by ”shooting” from that far not only can we commit a statistical error in the fitting procedure but, more importantly, one cannot possibly know if there is a resonance close to \( E_0 \) (above or below). If such resonances exist and pass undetected then the implications to critical astrophysical models can be enormous. As an example we refer to the \(^3\)He\(^\left( ^3\right)\)He, \( 2p \)^4\(^4\)He reaction [10,11] and the relevant neutrino fluxes which, until recently, suffered from such large uncertainties.

The need for an accurate \( S_{\text{eff}} \) appearing in the non-resonant component of the reaction rate indicates that it is urgent that we should provide experimentalists with a clear profile of all the laboratory energies beyond which the relevant astrophysical factor must be corrected before used in stellar evolution codes. We will assume that the target is in a neutral atomic state as well as that the energy of proton projectiles is roughly the relative energy of the collision.

The former assumption needs some justification. Admittedly considering the target to be in an atomic state is an undesirable simplification which has been used by other similar studies as well [12,13]. Note that, some times, the projectile itself is in a molecular state (e.g. \( D_2 \)) and then the whole process becomes even more complicated. Actually, for a full dynamical treatment of electronic degrees of freedom, the molecular few-body problem, which takes into account explicitly the electrons and nuclei of the system, should be properly solved. This is a very difficult task and the only relevant study available [14] so far is referring to hydrogenic molecules. The results indicate that the phenomenon is extremely complex where, for example, even the orientation of the target molecule plays a non-negligible role. Unfortunately, taking into account the molecular nature of the targets
involved in CNO and rp experiments is beyond the ambitions of the present study, whose only objectives are: a) to approximately define the energy region where screening effects are important and, b) to provide an estimate of the screening enhancement. However, it is very encouraging that Ref. [14] came to the conclusion that taking into account the molecular nature of their target increases the relevant screening energy. Thus, at first sight, considering the target to be in an atomic state seems to be a conservative approach which adopts the smallest possible screening energy. This is not the case, according to other studies [15,16], which subtract the molecular binding energy and the take-away kinetic energy of the spectator nuclei from the total (atomic) screening energy. In fact the situation is more complicated since the "solid state" effects are not fully understood nowadays and are still under investigation. In view of the inability of all available theoretical models to account for the large experimental screening energies the present study, by fulfilling its objectives (a,b), provides a good point of reference to all future efforts to solve the screening puzzle.

Under the previously defined assumptions the screening enhancement factor of laboratory proton-induced reactions is confined between the sudden limit (SL) and the adiabatic limit (AL) so that \( f_{\text{SL}} < f < f_{\text{AL}} \), where the two limits for neutral targets can be derived if we modify appropriately the respective formulas of Ref. [3]:

Sudden Limit:

\[
 f_{\text{SL}} (E) \simeq \exp \left[ \frac{0.765Z_1^{7/3}A^{1/2}}{E^{3/2}_{(\text{keV})}} \right] \tag{4}
\]

Adiabatic Limit:

\[
 f_{\text{AL}} (E) \simeq \exp \left[ \frac{0.3176A^{1/2}}{E^{5/2}_{(\text{keV})}} Z_1 \left[ (Z_1 + 1)^{7/3} - Z_1^{7/3} \right] \right] \tag{5}
\]

where \( A \) is the reduced mass number \( A = A_1A_2 (A_1 + A_2)^{-1} \).

These formulas have been derived in the framework of the Thomas-Fermi model, taking into account exchange, ionization, thermal, and relativistic effects of the atomic cloud.

It is easy to show that when \( Z_1 > 8 \), as is usually the case for advanced proton-induced astrophysical reactions, the gap between the above two limits narrows considerably thus providing an excellent constraint for the respective screening enhancement factor. If we also take into account that for such massive targets \( A \approx 1 \) then the SEF can be written

\[
 f (E) \simeq \exp \left( 0.765Z_1^{7/3}E^{-3/2}_{(\text{keV})} \right) \tag{6}
\]

and of course describes the relevant screening enhancing effect in the lab in a very accurate way.

In order to accurate measure \( S_{\text{eff}} \) \((S(E_0) \text{ or } S(0))\) nuclear astrophysics experiments have to be carried out (at or lower than) the most effective energy of interaction which for the reactions considered here takes the simple approximate form:

\[
 E_0 \simeq 1.22(Z_1T_6)^{2/3} \text{keV} \tag{7}
\]
In figure 1 we plot the most effective energy of interaction given by Eq. (7) with respect to plasma temperature for various proton-induced thermonuclear reaction. This is the energy at (or below) which the experiment has to be carried out in order to accurately determine the value of $S(E_0)$ (or $S(0)$). In figure 2 we plot the laboratory screening enhancement factor with respect to the relative energy of interaction (center of mass) for various proton-induced thermonuclear reactions with $Z_1 = 6, 10, 15, 20, 25, 30, 35, 40$. We have focused on the atomic effects threshold for each reaction so that we can provide the experimentalist with the limit beyond which the present screening corrections become important.

Now let us investigate the importance of the corrections proposed discussing some particular experiments which although have already been contaminated by the screening effect their results have not been corrected.

1) The $S_{eff}$ value of the reaction $^{14}N (p, \gamma)^{15}O$ adopted in Ref. [4] is the one obtained in Ref. [18], which incorporated data [19] contaminated by the screening effect. At $T_9 < 1$ (i.e. $E_0 < 446$ keV) the reaction is dominated by the $1/2^+$ resonance at $E_r = 259.4 \pm 0.4$ keV which has a resonance strength $(\omega \gamma)_r = 0.014$ eV. If for simplicity we ignore the effects of subthreshold resonances then the relative contribution of the resonant term (Eq. (2)) and the tail one (Eq. (3)) will be

$$N_A < \sigma u >_r = 2389.5 T_9^{-3/2} \exp \left( -3T_9^{-1} \right) \frac{cm^3 mol^{-1} s^{-1}}{MeV \cdot b}$$  \hspace{1cm} (8)

$$N_A < \sigma u >_{tail} = 1.5329 \times 10^{10} S_{eff} T_9^{2/3} \exp \left( -15.193T_9^{-1/3} \right) \frac{cm^3 mol^{-1} s^{-1}}{MeV \cdot b}$$  \hspace{1cm} (9)

Ref. [4] adopts the value $S_{eff} = S(0) = (3.2 \pm 0.8) \times 10^{-3} MeV \cdot b$ admitting a systematic error of 25% in the calculation of $N_A < \sigma u >_{tail}$. However, the screening error committed in Ref. [19] has been neglected thus overestimating $S(0)$ by a certain degree. The implications of neglecting the screening enhancement can be very significant, especially if one is interested in solar reaction rates. In fact, the value $S(0)$ in question relied on measurements taken [19] at energies as low as $E_{cm} = 93$ keV. The astrophysical factor measurements at such energies according to our SEF are overestimated by a factor $f_{14N+p} (93keV) = 1.083$. To assess the significance of such errors let us naively assume that the 8.3% error committed at $E_{cm} = 93$ is the actual error in the evaluation of $S(0)$ (note that the measurements of Ref. [19] suffer more from the fact the its energy is far from the most effective energy of interaction than from lack of screening corrections). This assumption is particularly valid for future experiments (with $E_{cm} < 93$ keV) where the screening error will naturally be much larger than 8% (thus rendering our assumption a conservative one). As regards the solar neutrino problem an +8% error in the value of $S_{14N+p}(0)$ leads to a similar overestimation of the theoretical values of the neutrino fluxes generated by the solar reactions $^{13}N (e^+ \nu_e)^{13}C$ and $^{15}O (e^+ \nu_e)^{15}N$, thus accentuating unnecessarily the discrepancy between theory and experiment. The significance of such corrections as the ones we propose here was underlined in a recent work [20] where it was pointed out that the solar model predictions for the CNO fluxes are not precise because the CNO fusion reactions are not as well studied as pp reactions. In the same work we learn that the current 1σ error in the standard model CNO neutrino fluxes is 17% − 25% that is much larger than the respective error in the pp
reactions. Since an accurate measurement of the CNO neutrino fluxes would constitute a stringent test of stellar evolution theory [20] the corrections proposed in the present paper are particularly relevant.

Moreover, an +8% error in the value of $S_{14N+p}(0)$ causes an underestimation of the mean lifetime of $^{14}N$ in the hot CNO cycle (the generator of the rp process) which translates into an overestimation of the production rate of $^{15}O$ which initiates the rp process via the $^{15}O(\alpha,\gamma)^{19}Ne$ reaction.

The need for a more accurate astrophysical factor for the reaction in question is now obvious. Any future experiment aimed at increasing the accuracy of $S(0)$ by lowering the beam energy will certainly commit errors larger than 10% rendering the present SEF indispensable. According to the most recent NuPECC report [21] the new 400 kV accelerator of the LUNA collaboration at the Gran Sasso is currently being used for the measurement of the cross section of the reaction $^{14}N(p,\gamma)^{15}O$ at 70 keV. According to the present paper the screening corrections at such temperatures will be of the order of 13%.

2) The reaction $^{19}F(p,\alpha\gamma)^{16}O$ has a number of resonances in the astrophysically important energy region, while recently a new one was discovered ([22]) at $E_r = 237$ keV. In the relevant experiment the beam energy had to be lowered below the atomic effects threshold so that some of its astrophysical factor measurements were enhanced and need to be corrected. Thus, the low energy values of $S_{\text{exp}}(E)$ tabulated in Ref. ([22]) should be replaced by $S_{\text{exp}}(E)/f(E)$ so that:

| $E_{cm}$ (keV) | 188.8 | 198.3 | 203.1 | 231.0 | 235.8 | 240.5 | 250.1 |
|----------------|------|------|------|------|------|------|------|
| $S_{\text{exp}}(E)$ ($MeV\cdot b$) | 5±4 | 12±3 | 26±4 | 20±6 | 22±5 | 29±5 | 33±5 |
| $S(E)_{\text{exp}}/f(E)$ ($MeV\cdot b$) | 4.7±4 | 11.4±3 | 24.8±4 | 19.2±6 | 21.2±5 | 28.0±5 | 31.9±5 |

The largest correction occurs at the lowest energy ($E_{cm} = 188.8$ keV) and it is of order 5%. Thus, so far, no significant screening error has been committed in the experiment in question. However as admitted in Ref. [22] the S-factor is still uncertain due to lack of experimental information at lower energies. This uncertainty will naturally lead to experiments where the energy should be much lower. At such low energies the experimental measurements will be considerably enhanced and of course the present screening enhancement factors may be important to the accuracy of the experiment.

3) The reaction $^{86}Sr(p,\gamma)^{87}Y$, which is important both to the p and the rp process, was recently investigated [23] in the astrophysically relevant energy range. In that experiment the beam energy was lowered below the atomic effects threshold causing an enhancement of the respective astrophysical factor. In fact the $S(E)$ value measured at the lowest energy of the experiment $S(1477 keV)$ has been overestimated by 6.7% while all the measurements taken at energies $E_{cm} < 2000 keV$ have been overestimated by at least 4%. A slight screening enhancement has already appeared at $S(1477 keV)$ (see the relevant plot in Ref. [23]) which for most practical purposes can be disregarded. Any attempt to lower the beam energy in order to search for undetected resonances or improve the $S(0)$ value will cause considerable enhancement of the measurements, which can be easily corrected through the screening enhancement factor given by Eq. (6).

In absolute values, the screening errors which will be committed in future CNO and rp astrophysical reactions are indeed important. However, their actual importance can only be assessed when projected against the entire background of experimental errors encoun-
tered in the laboratory. For example, in the reaction $^{19}\text{F} (p, \alpha\gamma)^{16}\text{O}$ the present model gives a screening error of order $5\%$, while the experimental uncertainty is $80\%$. Likewise, the best available astrophysical factor $S(0)$ for the reaction $^{14}\text{N} (p, \gamma)^{15}\text{O}$ carries a systematic error of $25\%$ when the respective screening error given by the present model is of order $8\%$. It seems therefore that screening corrections for the astrophysical reactions in question can play a significant role only if experimentalists manage to reduce other larger experimental errors. This has been the case in the third experiment discussed above (i.e. $^{86}\text{Sr} (p, \gamma)^{87}\text{Y}$) where the experimental uncertainty is of order $8\%$ while the screening error is of order $6.7\%$.

In conclusion, the present study gives a limit of errors in ignoring electron screening. Its results indicate that, for most practical purposes, screening corrections to past CNO and rp experiments can be disregarded. Regarding future experiments, such as the ones conducted by the LUNA collaboration, corrections to the CNO reactions will certainly be of importance as they are closely related to the solar neutrino fluxes (via the $^{13}\text{N} (e^+\nu_e)^{13}\text{C}$ and $^{15}\text{O} (e^+, \nu_{\text{e}})^{15}\text{N}$ reactions) and the rp process (via the HotCNO cycle). However, it is up to experimentalists to judge the actual importance of screening corrections by comparing them with all other experimental errors encountered in the laboratory.

Finally, we should point out that:

a) While we have only investigated proton-induced astrophysical reactions, the formalism of Ref. [3] can be very easily applied to the study of alpha-induced reactions where corrections are expected to be just as important.

b) The importance of the effects of the "solid state" environment on screening has been studied in a recent experiment [17] where the usual puzzling result emerged again: The experimentally obtained screening energy is larger than the one predicted by theoretical models. Due to the importance of screening corrections and the future ambitious plans [21] in nuclear astrophysics experiments a more sophisticated study is needed which should attempt to solve the relevant molecular few-body problem.

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FIGURE CAPTIONS

Figure 1. The most effective energy of interaction $E_0$ with respect to plasma temperature for various proton-induced thermonuclear reactions $(p + \frac{A}{2} X)$ with $Z_1 = 6, 10, 15, 20, 25, 30, 35, 40$

Figure 2. The laboratory screening enhancement factor with respect to the relative energy of interaction (center of mass) for various proton-induced thermonuclear reactions $(p + \frac{A}{2} X)$ with $Z_1 = 6, 10, 15, 20, 25, 30, 35, 40$
