Period–colour and amplitude–colour relations in classical Cepheid variables – VI. New challenges for pulsation models

Shashi M. Kanbur,1⋆ M. Marconi,2 C. Ngeow,3 I. Musella,2 M. Turner,4 A. James,5 S. Magin1 and J. Halsey1

1Department of Physics, State University of New York at Oswego, Oswego, NY 13126, USA
2Osservatorio Astronomico di Capodimonte, Via Moiariello 16, 80131 Napoli, Italy
3Graduate Institute of Astronomy, National Central University, Jhongli City 32001, Taiwan
4Department of Physics, Rice University, Houston, Texas 77005-1827, USA
5Department of Physics, State University of New York at Geneseo, Geneseo, NY 14454, USA

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ABSTRACT

We present multiphase period–colour/amplitude–colour/period–luminosity relations using the Optical Gravitational Lensing Experiment III (OGLE III) and Galactic Cepheid data and compare with state of the art theoretical pulsation models. Using this new way to compare models and observations, we find convincing evidence that both period–colour and period–luminosity relations as a function of phase are dynamic and highly non-linear at certain pulsation phases. We extend this to a multiphase Wesenheit function and find the same result. Hence our results cannot be due to reddening errors. We present statistical tests and movies depicting the period–colour/period–luminosity and Wesenheit relations as a function of phase for the Large Magellanic Cloud (LMC) OGLE III Cepheid data: these tests and movies clearly demonstrate non-linearity as a function of phase and offer a new window towards a deeper understanding of stellar pulsation. When comparing with models, we find that the models also predict this non-linearity in both period–colour and period–luminosity planes. The models with (Z = 0.004, Y = 0.25) fare better in mimicking the LMC Cepheid relations, particularly at longer periods, though the models predict systematically higher amplitudes than the observations.

Key words: stars: variables: Cepheids – distance scale.

1 INTRODUCTION

Classical Cepheids are the most important primary distance indicators within the Local Group thanks to their characteristic period–luminosity (PL) and period–luminosity–colour (PLC) relations. They are also currently used to calibrate secondary distance indicators and in turn to evaluate the Hubble constant (H0; see e.g. Freedman et al. 2001; Saha et al. 2001). Any systematic effect on the Cepheid PL relations is expected to propagate on secondary distance indicators and, in turn, on the final evaluation of H0. During the last decade there has been a very lively debate on the possible non-universality of Cepheid PL relations traditionally assumed to have the slope of the one derived in the Large Magellanic Cloud (LMC; see e.g. Freedman et al. 2001; Bono et al. 2008; Marconi 2009, and references therein). Unfortunately, in spite of many observational and theoretical efforts, there is not a conclusive answer to this question: non-linear pulsation models predict a significant dependence both on metallicity and the helium content with final effects on the distance scale that can be higher than 10 per cent (Bono et al. 2008; Marconi 2009, and references therein) and empirical tests that either find no metallicity effect or predict a metallicity dependence with an opposite sign with respect to the theoretically predicted one. However, spectroscopically based analyses by Romaniello et al. (2005, 2008) support the trend suggested by the pulsation model results.

Beyond the debated dependence on chemical composition there are both observational (e.g. Kanbur & Ngeow 2004; Ngeow et al. 2005, 2009) and theoretical (e.g. Caputo, Marconi & Musella 2000; Kanbur, Ngeow & Buchler 2004; Marconi, Musella & Fiorentino 2005) indications that the Cepheid PL relation is not always linear. Ngeow & Kanbur (2006a) have also shown that the effect on the inferred H0 of neglecting this possible non-linearity can be of 1–2 per cent, that combined with the predicted dependence on chemical composition can be important in the light of current efforts to push down errors on the H0 estimate to a few per cent.

The Cepheid PL relation used for distance determination is evaluated at mean light: the mean over all pulsation phases. On the other hand, recent studies by Kanbur & Ngeow (2004), Ngeow et al. (2005) and Ngeow & Kanbur (2006b) have suggested that an

*E-mail: shashi.kanbur@oswego.edu
innovative and promising approach to Cepheid pulsation is investigating the period–colour (PC) and the PL relations as a function of phase. Indeed modern data on Cepheids are characterized by excellent phase coverage, particularly in the Magellanic Clouds. This offers a unique opportunity for deriving empirical multiphase PC and PL relations. This approach has already been adopted by Ngeow & Kanbur (2006b) on the basis of data from the Optical Gravitational Lensing Experiment II (OGLE-II) project for Magellanic Cepheids and from various literature sources for Galactic pulsators. These authors have demonstrated that the study of multiphase relations can provide new insights into Cepheid pulsation. They find strong evidence that these observed PC/PL relations vary significantly with phase, are highly dynamic and strongly non-linear at various pulsation phases usually located near minimum light. These relations offer a new way to constrain models and provide deeper insights into pulsation physics by comparing these observed multiphase PC/PL relations with those from theoretical models. In this paper, we update our multiphase PC/PL relations using OGLE III LMC data and present a detailed comparison between theoretical and empirical multiphase PC and PL relations for the Milky Way and the LMC.

Simon, Kanbur & Mihalas (1993), Kanbur & Ngeow (2004) and Ngeow & Kanbur (2006b) also showed the importance of amplitude–colour (AC) relations. A flat PC relation at maximum/minimum light implies a relation between amplitude and colour at minimum/maximum light. Kanbur & Ngeow (2004) found evidence of a change of slope in the PC relation at maximum light at a period of about 10 d and a corresponding change in the AC relation. Hence this paper also presents AC relations as a function of phase.

2 THE OBSERVED MULTIPHASE PC/PL RELATIONS

The data used in this study come primarily from OGLE III and were corrected for reddening using the methods described in Ngeow et al. (2009). The data were fitted with a Fourier series like

\[ V = A_0 + \sum_{k=1}^{N} A_k \sin(k \omega t + \phi_k), \]

where \( A_0 \) is the mean magnitude, \( \omega = 2\pi/P \), where \( P \) is the period in days, \( t \) is the time of observation and \( N \) is the order of the fit. Because the phase coverage and the number of stars is large, the order of the Fourier fit was eight. The resulting Fourier fit was then used to infer properties at maximum/minimum light. We performed a number of tests which checked that the observed multiphase PC/AC/PL relations were independent of the order of the Fourier fit. The Fourier fits were rephased so that maximum light occurs at phase 0 in the \( V \) band, as described in Ngeow & Kanbur (2006b).

In what follows, short- and long-period Cepheids are demarcated by a period of 10 d. Our results are presented in Table 1 and Figs 1–8 using OGLE III data for the LMC results together with results for Galactic Cepheids using data from Ngeow & Kanbur (2006b). Following an anonymous referee’s suggestions we present only Figs 1 and 2 in the published version. Figs 3–8 are in the online version of the paper (see Supporting Information). However in the text that follows we do refer to all these figures.

In all these figures, short-/long-period observed data are represented by black and blue crosses, respectively. Theoretical results are usually represented by coloured symbols related to the mass of the models in question. These are noted on the figures. Some of our results pertain solely to the observed behaviour, some solely to the theoretical relations and some to the comparison of both. However, to save space, we have superimposed the theoretical relations to the observed ones.

The results presented in Figs 1–8 depict PC/AC/PL relations at maximum and minimum light. These figures broadly support the work of Ngeow & Kanbur (2006b) and clearly demonstrate the dynamic nature of the PC/PL relations as a function of phase. These figures, and the movies (in the online version of the paper – see Supporting Information), provide clear evidence of a non-linearity at phases close to minimum light – statistical tests are not needed. This non-linearity is seen in PC/PL/AC relations: changes in one relation are reflected in the other. In particular, we see that at minimum light, the non-linearity is marked, and moreover, it is sharp; see also Ngeow & Kanbur (2006b) and Figs 5 and 6 here. I.e. the data strongly imply a significant change in the slope of both PC/AC/PL relations at a period of about 10 d.

Table 1 presents the results of \( F \) tests as outlined in Ngeow & Kanbur (2006b) and references therein for the PC and PL relations at maximum and minimum light for the LMC using OGLE III Cepheids. In this table, we present the slope and zero-points for the long- and short-period regression lines, the dispersion, the number of data points used and the value of the \( F \) statistic. For the number of data points used here, an \( F \) value greater than about 3 indicates that the data are more consistent with the alternative hypothesis at the 95 per cent confidence interval. We note that certain phases such as minimum light are strongly non-linear according to the \( F \) test whilst other phases (close to maximum light) are marginally linear. The combination of these phases again results in a dilution of the non-linearity at mean light and suggests why this effect has not been observed before. Note that Ngeow et al. (2009) only presented the results of such statistical tests at mean light.

Our results support and extend the conclusions of Ngeow & Kanbur (2006b).

(i) There is a marked non-linearity at a period of \( \log P \approx 1 \) in the PC/PL/AC plots. The PL relation at maximum light is linear and is strongly non-linear at minimum light.

(ii) The PC plots show a great deal of structure with, possibly, changes of slope at other periods as well as at \( \log P \approx 1 \).

(iii) There is a variation in dispersion of both PC/PL relations as a function of phase. The PC and PL relation at maximum light has a greater dispersion than at minimum light. Moreover, the dispersion for a given phase also decreases at periods close to 10 d – at least for the LMC.

(iv) Both Galactic and LMC PC relations at maximum light are flat but the OGLE III results again suggest that the Galactic PC relation is flat for \( \log P \geq 0.8 \), whilst the LMC PC relation is flat for \( \log P \geq 1 \).

(v) We see that at minimum light, higher amplitude Cepheids are driven to redder colours when the PC relation is flat or flatter for those periods – as originally proposed by Simon et al. (1993). For example, the AC(min) relation for Galactic Cepheids is one relation across the entire period range. However, the AC(max) relation is clearly two separate relations, demarcated by a sharp change at a period of 10 d with the longer period relation having a non-zero slope (Ngeow & Kanbur 2006b). This corresponds to a change in the slope of the LMC PC(max and min) relation at a period of 10 d. This is important because a number of authors (Keller & Wood 2006) have noted correlations between mean colour and amplitude. Since mean light relations are simply the average of the same relations as a function of phase, then a correlation between

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Table 1. F-test results for PC/PL/PW relations at various phases.

| Period         | Slope            | ZP       | σ        | N  |
|----------------|------------------|----------|----------|----|
| LMC PC(max)    |                  |          |          |    |
| All            | 0.241 ± 0.018    | 0.296 ± 0.012 | 0.144 | 1625|
| log(P) < 1.0   | 0.319 ± 0.026    | 0.251 ± 0.016 | 0.142 | 1517|
| log(P) > 1.0   | −0.246 ± 0.134   | 0.830 ± 0.156 | 0.159 | 108 |
| F statistic is 14.87 (non-linear) |
| LMC PC(min)    |                  |          |          |    |
| All            | 0.281 ± 0.015    | 0.573 ± 0.01 | 0.117 | 1602|
| log(P) < 1.0   | 0.182 ± −0.021   | 0.629 ± 0.013 | 0.114 | 1494|
| log(P) > 1.0   | 0.510 ± 0.113    | 0.352 ± 0.131 | 0.134 | 108 |
| F statistic is 22.255 (non-linear) |
| LMC PL_v(max)  |                  |          |          |    |
| All            | −2.672 ± 0.044   | 16.691 ± 0.030 | 0.351 | 1624|
| log(P) < 1.0   | −2.574 ± 0.064   | 16.635 ± 0.039 | 0.351 | 1516|
| log(P) > 1.0   | −3.032 ± 0.291   | 17.066 ± 0.337 | 0.344 | 108 |
| F statistic is 2.71 (linear)  |
| LMC PL_v(min)  |                  |          |          |    |
| All            | −2.639 ± 0.038   | 17.395 ± 0.025 | 0.298 | 1621|
| log(P) < 1.0   | −2.883 ± 0.053   | 17.535 ± 0.033 | 0.293 | 1513|
| log(P) > 1.0   | −1.658 ± 0.260   | 16.365 ± 0.301 | 0.307 | 108 |
| F statistic is 24.87 (non-linear) |
| LMC PW(phase = 0.0) |           |          |          |    |
| All            | −3.279 ± 0.020   | 15.921 ± 0.013 | 0.160 | 1629|
| log(P) < 1.0   | −3.368 ± 0.027   | 15.973 ± 0.017 | 0.151 | 1522|
| log(P) > 1.0   | −2.437 ± 0.195   | 14.976 ± 0.226 | 0.229 | 107 |
| F statistic is 26.16 (non-linear) |
| LMC PW(phase = 0.25) |            |          |          |    |
| All            | −3.321 ± 0.027   | 15.819 ± 0.013 | 0.152 | 1629|
| log(P) < 1.0   | −3.252 ± 0.027   | 15.780 ± 0.016 | 0.146 | 1522|
| log(P) > 1.0   | −3.231 ± 0.178   | 15.677 ± 0.207 | 0.209 | 107 |
| F statistic is 6.97 (non-linear) |
| LMC PW(phase = 0.5) |          |          |          |    |
| All            | −3.275 ± 0.0233  | 15.793 ± 0.014 | 0.172 | 1629|
| log(P) < 1.0   | −3.199 ± 0.030   | 15.750 ± 0.019 | 0.166 | 1522|
| log(P) > 1.0   | −3.465 ± 0.194   | 15.977 ± 0.225 | 0.228 | 107 |
| F statistic is 6.28 (non-linear) |
| LMC PW(phase = 0.75) |          |          |          |    |
| All            | −3.329 ± 0.028   | 15.921 ± 0.019 | 0.223 | 1629|
| log(P) < 1.0   | −3.391 ± 0.040   | 15.956 ± 0.025 | 0.222 | 1522|
| log(P) > 1.0   | −3.121 ± 0.199   | 15.708 ± 0.230 | 0.233 | 107 |
| F statistic is 2.61 (linear)  |

Mean colour and amplitude has to be due to the correlation between amplitude and pulsation phases around minimum light. This, in turn, is caused by the interaction of the hydrogen ionization front (HIF) and photosphere (Simon et al. 1993; Kanbur et al. 2004; Kanbur & Ngeow 2006). This again demonstrates why such a multiphase analysis is useful and demanded by the high-quality data now available.

(vi) Movies of the multiphase PC/PL relations are viewable in the online version of the paper (see Supporting Information). They provide further strong evidence of the non-linear nature of the PC/PL relation at phases around minimum light. Further studies of such movies are warranted since they seem to indicate the presence of a number of shocks and different behaviour for Cepheids in different period ranges. These movies also suggest that a group of Cepheids mostly with periods around 10 d (perhaps the bump Cepheids) is the first to brighten subsequent to initial dimming after maximum light. While this may be due to the Hertzsprung progression, this offers an opportunity to study the Hertzsprung progression from a different perspective.

3 THE THEORETICAL SCENARIO

We selected various sets of non-linear convective pulsation models computed with the code originally developed by Stellingwerf (1982) and Bono & Stellingwerf (1994) and adapted to Cepheid pulsators by Bono, Marconi & Stellingwerf (1999). The selected
Figure 1. Multiphase PC/AC relations for reddening-corrected OGLE III LMC Cepheid data and theoretical models with \(Z = 0.02, Y = 0.28\).

Figure 2. Multiphase PC/AC relations for reddening-corrected Galactic Cepheid data and theoretical models with \(Z = 0.02, Y = 0.28\).

In looking at just the theoretical PC relations, we see that some mass sequences are monotonic, some suffer a gradual change of slope whilst others are distinctly non-monotonic, and further, this behaviour is sometimes different (for a given mass sequence) at maximum and minimum light: for example, see Fig. 7 – the 9\(M_\odot\), \(Y = 0.25, Z = 0.004\) mass sequence.

Examples of the non-monotonic behaviour typically bracket a period of 10 d, but in some cases this behaviour also occurs at periods greater than 10 d.

4 THE MULTIPHASE PC/PL RELATIONS: THEORY VERSUS OBSERVATIONS

Inspection of Figs 1–8 suggests the following.

(i) The models with \(Z = 0.004, Y = 0.25\) fare the best in reproducing the LMC PC/PL/AC relations, particularly at long
periods – see e.g. Figs 6 and 7, which compare the PC/PL relations at maximum/minimum light with model results. In particular note the comparison for PC relations at maximum/minimum light for periods in the range between 1.2 < \log P < 1.5.

(ii) The observed LMC AC relation displays a group of stars across a wide period range which are distinct from the main group e.g. Fig. 7. These are modelled quite well by the 9–11 M⊙ models with (Z = 0.004, Y = 0.25) composition. So it could be that the scatter in these plots is real and these stars are high-mass stars with increasingly lower amplitude: perhaps because they are close to the edge of the instability strip. Certainly, the non-monotonic nature of the purely theoretical AC relations at maximum/minimum light would indicate the presence of such stars.

(iii) It is also interesting to note that the LMC data are also quite nicely reproduced by models with Z = 0.02, Y = 0.31, whereas they disagree with models at Z = 0.02, Y = 0.28. This occurrence confirms the result that helium and metallicity effects tend to compensate each other.

(iv) V_{max} and V_{min} as a function of amplitude are systematically higher for models than for observations. This might be due to the sensitivity of model amplitudes to residual uncertainties in the treatment of turbulent convection.

(v) Figs 2–4, particularly the PC relation at maximum light, indicate that models with (Z = 0.02, Y = 0.28) fare better at reproducing the Galactic observations than models with (Z = 0.01, Y = 0.26).

(vi) For Galactic Cepheids, the amplitude range covered by models is much wider than the observed one.

(vii) For a given mass and periods shorter than 10 d, the theoretical PC/PL relation at maximum light has a greater positive slope than at minimum light (where the relations are almost flat). If the models can be regarded as doing a reasonable job of mimicking the observations, this shows why, in part, the observed PC/PL relations have a greater dispersion at maximum rather than at minimum light – see e.g. Fig. 7.

5 THE MULTIPHASE WESENHEIT FUNCTION

A number of different formulations of the Wesenheit function exist in the literature. Madore & Freedman (1991) define it to be

\[ W_V = V - 2.45(V - I) \]

whilst Udalski et al. (1999), who adopt a slightly different extinction law, use the following definition:

\[ W_I = I - 1.55(V - I) \]

Here the quantities V and I are the mean observed magnitudes in these bands. W_V and W_I can be shown to be reddening independent and for this reason the Wesenheit function is the preferred way to use the Cepheid PL relation to estimate distances. It is of interest here to consider the linearity/non-linearity of the Wesenheit function as defined by Udalski et al. (1999).

If we phase both V and I observations relative to a common time origin and then, following a Fourier decomposition, rephase such that maximum V-band brightness is called phase 0, then we can formulate a multiphase ‘Wesenheit-type’ function as

\[ W_{Vph} = I_{ph} - 1.55(V_{ph} - I_{ph}) \]

where V_{ph} and I_{ph} denote these quantities at the phase value. The results of the F test for non-linearity applied to the Wesenheit function are presented in Table 1 and Fig. 9. We see clearly that certain phases of the Wesenheit function, for example at maximum light, are strongly non-linear. Since the reddening along the line of sight cannot vary due to pulsation phase, this result provides strong evidence against the hypothesis that previous statistical tests indicating non-linearity are due to reddening errors. Previous work such as Koen, Kanbur & Choong (2007) suggested that linear Wesenheit functions were due to non-linearities in the PC and PL relations cancelling out: the results presented here support this view.

Another nice demonstration of the non-linearity of the Wesenheit function at certain phases is presented in a movie in the online version of the paper (see Supporting Information), where we have 100 phase points, the y-axis is the Wesenheit function and the x-axis is log P. Again we clearly see a dynamic W_f function which changes as a function of phase, not as much as the V and I band PL relations but nevertheless the non-linearity at certain phases is clear and unambiguous and cannot be due to lack of data in certain period ranges. We get similar results when we use the Madore & Freedman (1998) formulation.

It is our contention that the F-test results presented in Table 1 provide very strong evidence for a change in the slope of the PL, PC and PW relations at a period of 10 d and provide evidence of the pulsation phases which are the most non-linear.

6 CONCLUSIONS

Comparing observations and theory on the multiphase PC/PL/AC plane as described here is a powerful new way to constrain models and gain deeper insights into pulsation physics. The dynamic nature of the multiphase plots provides a new window into the inner workings of Cepheids and deserves more attention both observationally and theoretically. This can best be seen by viewing movies of these multiphase PC/PL relations available – see Supporting Information. The Wesenheit movie depicts the variation with phase of the Wesenheit function, defined by \[ W = I - 1.55(V - I) \]. A number of authors have suggested that because previous work testing for possible non-linearities in the Wesenheit function has yielded negative results, indicating non-linearities in the PL/PC relations at mean light may be due to reddening errors (the Wesenheit function is reddening independent). Our movies indicate clear non-linearities in the multiphase Wesenheit function implying that the cause of the
non-linearity cannot be due to reddening. The effect of this non-linearity on the extragalactic distance scale and cosmic microwave background (CMB) independent estimates of \( H_0 \) is a matter for debate.

It may be argued that a comparison of observations and theory in the multiphase PC/PL planes is another projection of the comparison of observed and theoretical light curves: even if the PL/PC/PW relations at mean light were linear, there is no similar expectation for linearity of the multiphase relations because of, for example, the Hertzsprung progression. This broad argument may be true but it still needs to be demonstrated and the demonstration of the dynamic non-linear behaviour of the PL and PW relations is one of the main results of this paper. The average of the multiphase relations does surely yield information about the mean light relations.

Does such a comparison provide additional insight over and above a comparison of light curves? We would argue that it does. For example, flat PC relations at maximum light yield information about the interaction of the photosphere and hydrogen ionization front – something which would be very difficult to probe by just a comparison of observed and theoretical light curves. Our multiphase comparisons suggest that the greatest non-linearity occurs at minimum light: this information, when investigated in greater detail, can provide a deeper understanding of Cepheid pulsation over and above a comparison of observed and theoretical light curves. This will be the topic of future work.

In any case the results presented in this paper represent an important challenge for theoreticians seeking a deeper understanding of Cepheid pulsation.

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SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article:

Fig. 3. Multiphase PC/AC relations for reddening-corrected OGLE III LMC Cepheid data and theoretical models with \( Z = 0.01, Y = 0.26 \).

Fig. 4. Multiphase PC/AC relations for reddening-corrected Galactic Cepheid data and theoretical models with \( Z = 0.01, Y = 0.26 \).

Fig. 5. Multiphase PC/AC relations for reddening-corrected OGLE III LMC Cepheid data against theoretical models with \( Z = 0.02, Y = 0.31 \).

Fig. 6. Multiphase PC/AC relations for reddening-corrected OGLE III LMC Cepheid data against theoretical models with \( Z = 0.008, Y = 0.25 \).

Fig. 7. Multiphase PC/AC relations for reddening-corrected OGLE III LMC Cepheid data against theoretical models with \( Z = 0.004, Y = 0.25 \).

Fig. 8. Multiphase PL relations for reddening-corrected OGLE III LMC Cepheid data against theoretical models with \( Z = 0.02, Y = 0.28 \), \( Z = 0.004, Y = 0.25 \), \( Z = 0.02, Y = 0.31 \).

Movie 1. Extinction-corrected PC \(( V - I )\) relations as a function of phase for OGLE III LMC Cepheids. The \( x \)-axis is log \( P \), \( y \)-axis is extinction-corrected \(( V - I )\) and the individual frames are at various phases with phase 0 being maximum \( V \)-band light.

Movie 2. Reddening-corrected PL relations in the \( V \) band as a function of phase for OGLE III LMC Cepheids. The \( x \)-axis is log \( P \), \( y \)-axis is dereddened \( V \) band. The individual frames are at various phases with phase 0 being maximum \( V \)-band light.

Movie 3. The LMC OGLE III Cepheid Wesenheit function \( W = I - 1.55(V - I) \), plotted against log \( P \). The different frames represent the Wesenheit function as the star goes through its pulsation cycle with phase 0 being maximum \( V \)-band light.

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