Review

Anomalies in one-dimensional electron transport: quantum point contacts and wires*

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Abstract

Over the last several decades, interest in quasi-one-dimensional charge transport has progressed from the seminal discoveries of Landauer quantization of conductance as a function of carrier density, to significant finer-scale phenomena. Those include: (i) fractional conductance, or ‘0.7 anomaly’; (ii) zero-bias anomaly; (iii) Rashba-effect anomalies; and (iv) apparent violation of the Landauer upper bound on conductance. In this work we present a very short summary of the first three items. The last anomaly, which remained theoretically unexamined until recently, is discussed in detail with emphasis on novel low-dimensional physics.

Keywords: conductance quantisation, conductance anomalies, 0.7 anomaly, Rashba anomaly, quantum point contact, 1D wire

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1. Introduction

The point in understanding any anomaly in physics is to delve deeper into the basic laws. ‘The exception proves the rule’, as the popular saying goes; or as Lev Landau, one of the grand masters of physical theory in the twentieth century, pointed out more specifically: ‘Physics is mainly an experimental science. It consists of more than measurements’. The word more implies a great deal of fundamental understanding. In the present context we begin with some basics of our physical systems.

In the laboratory, electronic transport in quasi-one-dimensional (1D) systems is essentially studied in quantum-point-contact (QPC) and quantum-wire structures, whose common property is that propagation becomes quantum-well confined to one dimension alone. The discontinuously step-like profile of their conductance, a genuinely 1D phenomenon known as Landauer quantization, was first reported in QPCs fabricated on high-electron-mobility GaAs/GaAlAs heterojunctions. As a function of carrier density (modulated by a capacitive arrangement) discrete steps appear in the differential conductance $G$. Their size systematically approaches the universal ‘Landauer quantum’ $G_0 = \frac{2e^2}{h}$.

As one increases the carrier density in a typical micro-etched 1D channel, the conductance rises as $G/G_0 = 0, 1, 2, \ldots$ each time that the Fermi level of the system reaches the next higher sub-band [1, 2]. The existence of conduction sub-bands well separated on the thermal energy scale leads to quasi-1D behaviour (in graphene-related materials such as carbon nanotubes, the separation can be much larger; eV rather than tens of meV). In the clearest experiment so far, near-ideal Landauer quantization has been observed in quasi-1D GaAs/GaAlAs wires fabricated by the cleaved-edge overgrowth technique [3].

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Over some thirty years, various explanations for conductance quantization have been advanced [4–6]. They comprise the phenomenological Landauer-Büttiker model [7, 8], the more rigorous Kubo linear-response formula [9] and finally the so-called nonequilibrium-Green-function technique [10]. We cite two of our relevant papers [11, 12] for detailed appraisals of the physics of quantized conductance.

In the Landauer-Büttiker analysis of 1D conductance, the central dogma is that purely coherent, quantum transmission through individual conductive states fully explains the perfectly flat plateau sequence of Landauer steps in $G$. In one dimension, the density of states in each discrete sub-band scales inversely as the group velocity while the flux, of course, is linear in the velocity. The two cancel and thus the conductance cannot change before the next band starts to be populated.

If the channel hosts a (coherent) scattering potential, this may lower the transmitted flux by back-scattering but it can never raise the step ratio $G_{\text{transmitted}}/G_0$ above unity without violating particle conservation. As the carrier density increases, each successive step continues unchanged as more states are filled in the lower sub-band; an extra step is added to the total conductance only when the chemical potential matches and exceeds the next threshold.

Naïve one-body quantum transmission, however, fails to explain any of the substantial number of further experimental observations now available [13–16]. Why and how it fails is by now known [17–21]. Coherent transmission is restricted to weak field linear response. It does not account for natural resistive dissipation in every ohmic structure. Finally, the approach has no way to address processes beyond the simplest forms of elastic back-scattering.

One essential inelastic physical process omitted by popular models is intra-band scattering by phonon emission, responsible for dissipation. Another example is inter-band transitions, which are purely inelastic two body processes with substantial carrier and energy exchange between discrete sub-bands. Inelasticity implies decoherence, which in turn implicates multi-carrier interactions as an essential ingredient in normal 1D transport. Later we will return in some detail to inter-band phenomena and their experimental consequences.

A type of non-integer conductance anomaly was eventually observed in 1D channels: the fractional or so-called ‘0.7’ anomaly; a much discussed phenomenon [22]. Although this feature manifests primarily in III–V structures, it has been speculated that the effect is in fact generic to all 1D conductors. It has been pointed out [22], that the values actually reported in $G_0$ may lower the transmitted conductance; a much discussed phenomenon was to zero. This anomaly has been well understood in terms of tunnelling enhancement between various states, notably those involving coincident energy levels in the device relaxing to the equilibrium limit. [24–30] cover a good spectrum of activities within ZBA physics.

Returning to the ‘0.7’ anomaly, a novel spin-based explanation may emerge from the Rashba effect; this occurs when there is a momentum-dependent splitting of spin sub-bands. The splitting arises from a combined effect of the spin–orbit interaction and inherent asymmetry of the background crystal potential in appropriate materials. For Rashba in two-dimensional degenerate electron systems we refer to the important work of Bychkov and Rashba [31]. The Rashba interaction in an inhomogeneous electric field has since been used to explain the above-mentioned ‘0.7’ in quantum-point contacts [32]. Apart from this, Rashba physics has many other engaging implications for topological matter and exotic Majorana states [33].

Our abbreviated review of the more widely discussed 1D conductance anomalies ends here. In the rest of this paper we address a wholly different nonlinear anomaly, set completely apart from the first three. In particular it has nothing in common with the ‘0.7’ issue, which is of scant importance here.

2. A puzzling measurement

Our present concern is with a new anomaly completely different from the others. In particular it has nothing in common with the ‘0.7 anomaly’. Figure 1 shows the central experimental results of de Picciotto, Pfeiffer, Baldwin and West [15, 16]. We see a series of differential-conductance traces, normalized to $G_0$, for a nearly ideal ballistic quantum wire at various set driving voltages and plotted as functions of the gate voltage controlling the channel chemical potential and so its carrier density. The greyed region holds a collection of highly variable shoulder structures at the ground-state threshold-the ‘0.7 anomaly’.

Of immediate relevance are the conductance steps covering the larger part of the inter-band region up to the onset of the next sub-band. Unlike the ‘0.7’ features [22], they are robustly regular with a systematic dependence on source-drain voltage. We stress the critical conceptual significance of the remarkable, nonlinear 1D conductance steps as revealed by measurements:

- The anomalous steps are extremely flat; with increasing density they run past the early so-called ‘0.7’ features all the way up to the threshold of the next sub-band. They are completely separate from the structures at ground-state threshold.
- The conductance is voltage dependent and cannot be explained by popular linear-response models.
- The steps span the all the region where the chemical potential fills the 1D carrier states between ground-and first-excited-

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state bands. Within single-particle linear quantum transmission, no such conductance can overstep the ideal maximum.

- But in fact, with increasing source-drain voltage, the steps rise up regularly from the weak-field baseline to overtake the presumed maximum $G_0$.

The 1D test devices of [15] and [16] are of extraordinary purity, even today. We remark on the care taken by the authors to isolate the physics of their samples. So it is remarkable that that no authors since the original team [16] have ventured to study the clear violation of popular conclusion for novel, experimentally testable possibilities.

3. Towards an explanation

3.1. Boundary condition

The active channel of a 1D device cannot be divorced from the large source and drain boundary leads, even in principle. Its operative length is no longer exclusively determined by its physical dimensions or the bulk mean free paths of its substrate. Rather, its length is set by the longest carrier mean free path (MFP) for the channel as embedded in its non-ideal bounding leads. Adopting the estimate suggested in [15], we take an operational channel length $L = 2 \mu m$. Our results, however, do not depend on the absolute MFPs but on their ratios.

At low temperature the inelastic mean free path should set the longer scale, fixing $L$; but the observed low-field $G$ falls about 10% below the ideal, $G_0$. Setting the complementary elastic MFP to 0.769 $L$ matches $G$ at weak field, as reported in [15].

3.2. Transport

Our transport equations describe two sub-band populations, with energy gap $E_g$. With two distributions $f_{gj}$ for the lower band $(j = 1)$ and next-higher $(j = 2)$ as functions of momentum $k$, the equations are of extended Boltzmann-Drude type:

\[
\frac{q e}{h} \frac{\partial f_{gj}}{\partial k} = -R_{in}(f_{gj} - \overline{f}_{gj}) - R_{elj} f_{gj} - R_{0j} \times (e^{-E_g/kT} - 1),
\]

\[
\frac{q e}{h} \frac{\partial f_{g2}}{\partial k} = -R_{in}(f_{g2} - \overline{f}_{g2}) - R_{el2} f_{g2} - R_{02} \times (f_{g2}(1 - \overline{f}_{g2}) - e^{-E_g/kT} \overline{f}_{g2}(1 - f_{g2}));
\]

where the reference distributions $\overline{f}_{gj}$ are treated below. On the left-hand side of the equations $\varepsilon = V_{sd}/L$ is the field from the driving voltage. On the right, $R_{mj}$ and $R_{0j}$ are respective intra-band inelastic and elastic scattering rates and $R_{0j}$ is the inter-band rate coupling the populations. The elastic collision term scales with the odd part of the distributions since elastic scattering randomizes the momentum vector but conserves energy: $f_{gj}^o = f_{gj} - (f_{gj} + f_{gj})/2$. The final parameter is the Boltzmann factor $e^{-E_g/kT}$ to boost a carrier from $j = 1$ to $j = 2$.

Expressing energies in thermal units $k_B T$ and momenta in units $k_B T$/$\hbar$, the effective equilibrium functions have the form

\[
\overline{f}_{gj} \equiv 1/(1 + \exp(k^2 - \mu_j)).
\]

Their role is as follows. As transitions redistribute the electron population between the two sub-bands of the channel, the change in respective densities depresses the effective chemical potential of the lower band so $\mu$ is renormalized to $\mu_1 \equiv \mu - \zeta_1$. The now-augmented population in the upper band, $E_s$ units above lower, follows the rise in its effective chemical potential: $\mu - E_s$ goes to $\mu_2 \equiv \mu + \zeta_2 - E_s$. Both quantities $\zeta_1$ and $\zeta_2$ should be non-negative.

For any choice of the pair of renormalized potentials $\mu_j$ the coupled equation (1) are routinely solved via Green functions. The inputs are the expressions for $\overline{f}_{g1}$ and $\overline{f}_{g2}$ from equation (2).

Computing the solution requires us to tie the intra-band MFPs and the inter-band transition probability to the rates $R_{mj}$, $R_{ejj}$ and $R_{0j}$ parametrizing the terms in the coupled
Conservation is the mandatory relation linking the pair of potential shifts \( \zeta \) so the system’s total density \( n(\mu) \), expressed in units \( n_{th} = k_{th}/\pi \), is invariant:

\[
\int dk (\overline{f}_{1}(\mu) + \overline{f}_{2}(\mu - E_{g})) = n(\mu) \int dk (f_{1}(\mu_{1}^{}; \zeta_{2}^{}) + f_{2}(\mu_{2}^{}; \zeta_{2}^{})). \tag{7}
\]

A single constraint cannot fix a unique pair of \( \zeta \). Another independent relation is needed. In the model’s present version we take the Helmholtz free-energy density of the carriers and remove the energy for assembling the (fermionic) system. At equilibrium this vanishes; in a driving field, the net energy ought to change with the field-induced internal rearrangement of the sub-band populations. Ours is a working Ansatz, open to possible refinement as knowledge of the physics develops.

For each band we extend the usual Helmholtz free-energy density [36]

\[
F[f_{j}(\mu_{j})] \equiv \int dk (k^{2} - \mu_{j}^{2})f_{j}^{k} + \int dk (f_{j}^{k} \ln f_{j}^{k} + (1 - f_{j}^{k}) \ln (1 - f_{j}^{k})) \tag{8}
\]

given in units of \( k_{th}Tn_{th} \). The leading right-hand integral is the total internal energy less the assembly energy. The second integral is the Uehling-Uhlenbeck entropy entering into the \( H \)-theorem for fermions [37]. At global equilibrium, minimizing \( F \) (understood as a functional of \( f_{j}^{k} \)) gives the familiar Fermi–Dirac form for \( f_{j}^{k} \).

The behaviour of \( F \) purely as a function of \( \mu \) is not the same as its behaviour as a functional of \( f \) (recall that \( \mu \) and density - effectively \( f \) are conjugate variables). At high sub-band density, \( F \) has no lower bound. Since the form of the \( f_{j}^{k} \) is strictly determined by solving equation (1) given (7), the behaviour of our free energy (8) is strictly a function of \( \mu_{1} \) and \( \mu_{2} \) and must exhibit maxima in their parameter space.

Figure 2 illustrates the geometrical character of our self-consistent prescription at a typical driving field of 1.5 mV/\( \mu \)m. In the parameter space of the effective potential shifts \( \zeta_{1} \) and \( \zeta_{2} \), the plot shows the superposition of the contours of constant carrier density on the landscape of the system’s free energy in the neighbourhood of its maximum. The solution to the transport problem is found where a given line of constant density intersects the maximum of the free energy.

Our analogy is with a a tap feeding fluid into a finite container until it overflows. Likewise the driven system will allow internally as much of the inflowing excess energy as it can, until resistive loss balances the inflow and the internal buildup stops.

### 4. Results

In figure 3, for the same set of values \( V_{sd} \) in figure 1, we show the conductance of equation (6) computed for the device of figure 1 [15]. Scattering parameters are: operational length \( L = \lambda_{sd} = 2 \mu \), \( \lambda_{sd} = 0.769 \), \( L = 1.538 \), \( \lambda_{sd} = 0.769 \), \( L = 10 \mu \). Figure 4 of [16] evidences the thermal broadening of \( G \)
around the sub-band thresholds, presumably from localized Joule heating. We compute our curves at nominal temperature 4 K. The energy-gap value $E_g = 15$ meV and the AlGaAs effective mass are used. On the horizontal axis of figure 2 we map the global chemical potential $\mu$ to the value of a corresponding gate-control voltage, using the data of [15].

Figure 3 should be compared directly with figure 1 as taken from [16], their figure 3(a). Both in the real data and our calculation, a substantial source-drain voltage driving the channel current leads to elevated conductance plateaux that

(a) are inherently non-linear in origin,
(b) are extremely flat and robust,
(c) rise beyond the ideal upper bound $G_0$ on $G$ and thus
(d) exceed the normally expected limit of linear-response models.

In figure 3(a) of [16] (reproduced in our figure 1) the plots show $\partial I/\partial V_{sd}$: the rate of change of current with driving field, plotted as the density increases. When that slope is essentially flat over a broad range of gate voltage (thus density) as in figure 1, then $I/V_{sd}$, as in figure 2, tracks it closely and vice versa.

The present calculation is more sensitive to $V_{sd}$ than the experiment. While in figure 2 the step in $G$ at $V_{sd} = 1$ mV coincides with figure 1, its height at 3 mV is 1.43 $G_0$ while its measured counterpart is 1.16 $G_0$. The overestimate might be accounted for if the inelastic MFP $\lambda_m$ were shortened by optical-phonon emission (the optical-phonon energy in GaAs is 35 meV, about twice $E_g$). Energy-dependent mean free paths within equation (1) are an obvious aspect to examine. Increased local Joule heating as $V_{sd}$ increases may also lower the enhancement [17].
single-particle approach can handle this; for conservation then acts on the far larger stage of multi-particle interactions. Inelasticity and thermodynamic irreversibility govern the physics. Their explicit presence is a requirement that cannot be met by reversible single-particle Hamiltonian dynamics. Certainly, the boundary conditions need special care to interpret the ballistic nature of mesoscopic conduction.

Yet here, as in all normal transport, the physics still needs elastic and inelastic scattering to act with equivalent theoretical status. Beyond explaining the striking enhancement of ballistic conductance [15, 16], other results on temperature behaviour [17] may allow analysis of thermal characteristics for transition-induced changes of $G$ in quantum wires. One mightprobe comparable effects in high-purity carbon nanotubes, whose energy scale and high-field stability far outstrip structures made with III–V technology.

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