Positron Excess, Luminous-Dark Matter Unification and Family Structure

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Abstract

It is commonly assumed that dark matter may be composed of one or at most a few elementary particles. PAMELA data present a window of opportunity into a possible relationship between luminous and dark matter. Along with ATIC data the two positron excesses are interpreted as a reflection of dark matter family structure. In a unified model it is predicted that at least a third enhancement might show up at a different energy. The strength of the enhancements however depends on interfamily mixing angles.
Introduction

Since we are observing the energy density of dark matter (DM) necessarily from the standpoint of luminous matter, it may be worth imagining that there exist large DM molecules which lead to intelligent life including scientists who are studying what we call luminous matter. Would such hypothetical DM scientists be sufficiently Ptolemaic to assume that luminous matter is made up of only one or at most a few elementary particles and what could they possibly assume about our knowledge of mathematics and biology?

Here we attempt a small step to address this question which is potentially of broad interest to mathematicians and biologists as well as to theoretical physicists and observational astrophysicists.

From observational data of galaxies and clusters of galaxies it has been established that there exists DM which interacts gravitationally [1]. There is more DM than luminous matter energy density (a.k.a. baryonic matter) by a factor of about five or six, i.e. \( \Omega_{DM}/\Omega_B \sim 5 - 6 \).

Because only gravitational interaction of DM has been observed there is incompleteness in understanding of the nature of the DM. Candidates for its constituents have ranged in mass by many orders of magnitudes all the way from a microelectronvolt to significant fractions of a solar mass.

As always only observational data can provide an arbiter and recent data from PAMELA [2] and ATIC [3] provide a window of opportunity on how dark matter is unified with luminous matter in a LUDUT (Luminous-Dark Unified Theory) containing a unification scale, with subscript truncated for convenience, \( M_{DUT} \). Taken at face value, the positron excess reported by PAMELA and the lepton flux excess reported by ATIC appeared to occur at two different energy scales. There might even be two “bumps” in the ATIC data-for a total of three enhancements?- although the data are not sufficiently significant at the present time.

The aforementioned excesses could have at least three possible origins assuming the data are correct. The excesses can arise from (i) a nearby astrophysical object such as a pulsar and involve no new physics; (ii) annihilation of dark matter with dark antimatter; or (iii) decay of dark matter.

In the present Letter we assume dark matter-antimatter annihilation is absent and that the positron excess arises entirely from source (iii).

\( M_{DUT} \) will be generated by luminous-dark unification involving the electroweak sector of the standard model with color \( SU(3) \) regarded for simplicity as a spectator. Already with this simplification we see that the dark
matter must have a three-family structure like the luminous matter. It is perhaps simpler to attribute the “bumps” in the PAMELA and ATIC data to two different sources rather than one. If our interpretation is correct, there could be at least one more enhancement at a different energy. The amount of the excesses will depend, in our model, on the mixing angles involved in the decay of the dark into luminous matter.

\textsuperscript{#3}If there exists a fourth luminous matter family [4] the same argument predicts a fourth dark matter family.
Necessity for unification

In order for dark matter to decay into positrons there must be a non-gravitational interaction between the relevant dark matter particle \( \chi \) and leptons.

We denote the spin-1/2 dark matter field by \( \chi \) bound into a \( (\bar{\chi}\chi) \) by a strongly-coupled gauge interaction \( SU(n)_{DM} \) similar to technicolor. The annihilation of \( (\bar{\chi}\chi) \) into electron-positron takes place by exchange of a gauge boson of \( SU(n)_{DM} \) which we denote by \( W_{DM} \).

It is immediate to see that \( W_{DM} \) must transform as a doublet under electroweak \( SU(2)_L \) which already hints that unification of \( SU(n)_{DM} \) with \( SU(2)_L \) underlies the non-gravitational interaction of dark with luminous matter.

To accommodate the PAMELA [2] and ATIC [3] data requires that we study the necessary lifetime \( \tau_{\bar{\chi}\chi} \) for a dark-luminous unification scale \( M_{DUT} \).

Assuming strong coupling \( \alpha_{DM} = O(1) \) the lifetime is

\[
\tau_{\bar{\chi}\chi} \sim \left( \frac{M_{DUT}^4}{M_{\bar{\chi}\chi}^2} \right) = \eta^4 (M_{\bar{\chi}\chi})^{-1} \tag{1}
\]

where \( M_{DUT} = \eta M_{\bar{\chi}\chi} \). Given the data it is natural assume \( M_{\bar{\chi}\chi} \simeq 2 \text{ TeV} \) or \( M_{\bar{\chi}\chi} \simeq 10^{-26} \text{ s} \) and so

\[
\tau_{\bar{\chi}\chi} \sim (10^{-26} \text{ s})\eta^4 \tag{2}
\]

To estimate \( \tau_{\bar{\chi}\chi} \) one can use the flux of \( e^+ \) reported by the observations [2,3] to conclude that \( \eta \sim 10^{13} \) [5,6].
Unified Model

The most economical way to unify the SM $SU(2)_L \times U(1)_Y$ with a general dark matter group $SU(n)_{DM}$ is by embedding these two groups as follows.

$$SU(n + 2) \times U(1)_S \rightarrow SU(n)_{DM} \times SU(2)_L \times U(1)_{DM} \times U(1)_S$$
$$\rightarrow SU(n)_{DM} \times SU(2)_L \times U(1)_Y$$
$$\rightarrow SU(n)_{DM} \times U(1)_{em}. \quad (3)$$

With color, the unifying group is $SU(3)_C \times SU(n + 2) \times U(1)_S$. For the first family the assignments under $(SU(3)_C, SU(n + 2), Y_S)$ are

$$(3, n + 2, Y_{c,s})_L + (1, n + 2, Y_S)_L$$
$$+(3, n + 2, Y_{c,s})_R + (1, n + 2, Y_S)_R$$
$$+(3, 1, \{Y^u_S \ Y^d_S\})_L + (3, 1, \{Y^u_S \ Y^d_S\})_R$$
$$+(1, 1, Y_S)_L + (1, 1, Y_S)_R \quad (4)$$

and for the second and third families this pattern is repeated. This feature that the dark matter has family structure like the luminous matter will be central in our interpretation of the positron data from PAMELA and ATIC. Notice that the $SU(n + 2)$ nonsinglet fermions transform under the subgroup $SU(3)_C \times SU(n)_{DM} \times SU(2)_L \times U(1)_{DM} \times U(1)_S$ as

$$(3, n, 1, Y_{c, dm}, Y_{c,s})_{L,R} + (3, 1, 2, Y_q, Y_{c,s})_{L,R}$$
$$(1, n, 1, Y_{dm}, Y_S)_{L,R} + (1, 1, 2, Y_l, Y_S)_{L,R} \quad (5)$$

Three important remarks are in order concerning the assignments in (4).

First, the simplest way to have an anomaly-free model in this case and to avoid fermions with unconventional charges is to have both left and right-handed fermions with identical transformations under the gauge group.

Second, if the dark matter strongly-coupled gauge group were to be a QCD-like model, it is natural for it to be vector-like as it is with $SU(3)_C$.

Third, the above assignments involve right-handed $SU(2)_L$ doublets for the “luminous” (non-dark) matter which are absent in the SM. These “mirror” fermions have been used to construct a model of electroweak-scale right-handed neutrinos with wide implications for the seesaw mechanism at the LHC [7].
In this work, our principal requirement is for dark matter to be EW singlet. In view of the assignments in Eq. (5), this translates into the requirement that the SM \( U(1)_Y \) quantum numbers of the dark matter particles should vanish. (The color-nonsinglet dark matter particles quickly annihilate each other with their density being drastically reduced by the Boltzmann factor when the temperature drops below their masses (of order of a TeV).)

To implement the above requirement, we now work out the generator of \( U(1)_{DM} \). There are \( n + 1 \) diagonal generators in \( SU(n + 2) \), one of which, \( \lambda_3/2 \), can be taken to be \( T_3^L \) of \( SU(2)_L \). One combination of the remaining \( n \) diagonal generators \( \lambda_8/2, \ldots, \lambda_{n^2+4n+3}/2 \) becomes the \( U(1)_{DM} \) generator. It is convenient to use the following generators without the normalizing factors in front: \( T_8 = \text{diag}(1,1,-2,0,\ldots,0) \); \( T_{15} = \text{diag}(1,1,1,-3,0,\ldots,0) \); \( \ldots, T_{n^2+4n+3} = \text{diag}(1,1,\ldots, -(n+1)) \). Let us denote by \( T_{DM} \) the generator of \( U(1)_{DM} \) which is given by

\[
T_{DM} = \alpha_1 T_8 + \alpha_2 T_{15} + \ldots + \alpha_n T_{n^2+4n+3}. \tag{6}
\]

The hypercharge generator can most simply be written as

\[
\frac{Y}{2} = y_{DM} T_{DM} + y_S I, \tag{7}
\]

where \( I \) is the unit matrix and generator of \( U(1)_S \). In order for the dark matter to have vanishing hypercharge, i.e. \( Y/2 \propto \text{diag}(1,1,0,\ldots,0) \) where the first two elements \( (1) \) refer to the SM doublet and the zeros refer to the DM, the coefficients \( \alpha_i \) in (6) are found to be

\[
\alpha_{n-k} = \left( \frac{y_S}{y_{DM}} \right) \frac{n+2}{(n-k+1)(n-k+2)}, \tag{8}
\]

with \( k = 0 \ldots n-1 \). From (7) and (8), one can immediately find the first two nonvanishing diagonal elements of \( Y/2 \), namely \( y_S \frac{n+2}{2} \) and consequently

\[
\frac{Y}{2} = y_S \text{ diag}(\frac{n+2}{2}, \frac{n+2}{2}, 0, \ldots, 0). \tag{9}
\]

From Eq. (9) one obtains the following conditions on the \( U(1)_S \) quantum numbers for the color singlets, \( y_l \), and for the color triplets, \( y_q \):

\[
\frac{y_l}{2} \frac{n+2}{2} = -\frac{1}{2}; \quad \frac{y_q}{2} \frac{n+2}{2} = \frac{1}{6}. \tag{10}
\]

The electric charge is now \( Q = T_{3L} + Y/2 \) where \( T_{3L} = \lambda_3/2 \).
The next step is to determine the scale $M_{DUT}$ where the DM gauge group $SU(n)_{DM}$ becomes unified with the SM $SU(2)_L$. Let us denote by $M_{DM}$ the scale where the $SU(n)_{DM}$ gauge group becomes strongly coupled. At $M_{DUT}$, one has, by definition, $\alpha_2(M_{DUT}) = \alpha_{DM}(M_{DUT}) = \alpha_{DUT}$. The solution to the one-loop evolution equations gives

$$\alpha_{2}^{-1}(M_{DM}) = \alpha_{DUT}^{-1} + 8 \pi b_2 \ln \frac{M_{DM}}{M_{DUT}},$$

$$\alpha_{TC}^{-1}(M_{DM}) = \alpha_{DUT}^{-1} + 8 \pi b_{DM} \ln \frac{M_{DM}}{M_{DUT}},$$  \hspace{1cm} (11)

where $b_2 = (22 - 8n_G - n_{S,2})/48\pi^2$ and $b_{DM} = (11n - 8n_G - n_{S,DM})/48\pi^2$ with the factor of 8 reflecting the fact that one has both left and right-handed fermions and $n_G$ stands for the number of families. The scalar contributions are denoted by the generic notations $n_{S,2}$ and $n_{S,DM}$ which do not reflect their group representations. The unification scale $M_{DUT}$ can be related to the DM scale $M_{DM}$ by

$$M_{DUT} = M_{DM} \exp\{6 \pi \frac{(\alpha_{2}^{-1}(M_{DM}) - \alpha_{DM}^{-1}(M_{DM}))}{(11n - 22) - (n_{S,DM} - n_{S,2})}\}.$$ \hspace{1cm} (12)

Notice that the expression in Eq. (12) is independent of the number of families as expected. As mentioned above, the mass of the dark matter particle is expected to be in the TeV range which will be taken also to be the strongly-coupled DM scale. Hence $M_{DM} = O(\text{TeV})$.

Next, in order to determine the appropriate $n$ and hence $SU(n)_{DM}$, two more inputs are needed. We assume $\alpha_{DM}(M_{DM}) = 1$. Extrapolating $\alpha_{2}^{-1}(M_Z) = 29.44$ to $M_{DM} = O(\text{TeV})$, one obtains $\alpha_{2}^{-1}(1\text{TeV}) = 29.19$ and $\alpha_{2}^{-1}(1\text{TeV}) = 28.17$ for $n_{G} = 3$ and $n_{G} = 4$ respectively.

Finally, we will require that $M_{DM} < M_{DUT} < m_{Pl}$ with $M_{DUT} \sim 10^{15} - 10^{16}$ GeV in order to obtain a reasonable lifetime for the dark matter as discussed above.

From (12) several results concerning a viable $SU(n)_{DM}$ are obtained.

- $n = 2$ i.e. $SU(2)_{DM}$:

It is easy to see that this is an \textit{unviable} scenario since one obtains either $M_{DUT} \gg m_{Pl}$ or $M_{DUT} \ll M_{DM}$. Conversely, one can see from Eq. (11) that for three families $b_{DM} < 0$ and, if $\alpha_{DUT} < 1$ (i.e. perturbative), $\alpha_{DM}(M_{DM}) \ll \alpha_{DUT} < 1$ implying that $SU(2)_{DM}$ is not confining at $M_{DM} = O(\text{TeV})$. 

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• $n = 3$ i.e. $SU(3)_{DM}$:

Again, one can deduce from (12) that $M_{DUT} \gg m_{Pl}$. This is also not viable.

• $n = 4$ i.e. $SU(4)_{DM}$:

Let us denote the scalar contribution in Eq. (12) by $n_{S,DM} - n_{S,2} \equiv x$.

One obtains:

$$M_{DUT} \approx 2.6 \times 10^{13} \text{GeV}; x = 0$$
$$\approx 5.4 \times 10^{15} \text{GeV}; x = 4$$
$$\approx 3 \times 10^{16} \text{GeV}; x = 5$$

(13)

A LUDUT based on $SU(3)_C \times SU(6)_{DUT} \times U(1)$ appears to be the best choice since, for $n \geq 5$, $M_{DUT}$ turns out to be much smaller than the above values.

From hereon, we shall take the DM gauge group to be $SU(4)_{DM}$ and the full gauge group at the scale $M_{DUT}$ is $SU(3)_C \times SU(6) \times U(1)_S$. At the scale $M_{DM}$, one has $SU(4)_{DM} \times SU(3)_C \times SU(2)_L \times U(1)_Y$. The DM particles denoted by $\chi$ above come in two kinds and under $SU(4)_{DM} \times SU(3)_C \times SU(2)_L \times U(1)_Y$ they transform as

$$\chi_I = (4,1,1,0)_{L,R},$$
$$\chi_q = (4,3,1,0)_{L,R}.$$  

(14)

As we have mentioned above, the $SU(4)_{DM}$ singlets come in two chiralities: the SM particles with left-handed doublets and right-handed singlets, and the mirror fermions which carry opposite chiralities. It is beyond the scope of the paper to discuss the physical implications of the mirror fermions. Such discussions can be found in [7]. We will concentrate instead on the DM particles.

When $SU(4)_{DM}$ becomes strongly coupled, there are two types of DM “hadrons”: those coming from the bound states of $\chi_I$ and those coming from the bound states of $\chi_q$.

I) $\chi_I$ bound states:

• DM “mesons”: $\bar{\chi}_I \chi_I$
• DM “baryons”: $4 \chi_I$

II) $\chi_q$ bound states:
• DM “mesons”: $\bar{\chi} q \chi$
• DM “baryons”: $12 \chi q$ (both $SU(4)_{DM}$ and $SU(3)_C$ singlet).

All of these DM “hadrons” have masses in the TeV range. However, as the temperature dropped below the color DM particles $\chi_q$, the Boltzman factor $\exp(-M_{DM}/T)$ drastically reduced their number density since they were still in thermal equilibrium with the SM quarks through QCD interactions.

This leaves the DM “hadrons” coming from the color-singlet $\chi_l$’s. We shall make some comments at the end of the paper concerning their relic density.

Let us for the moment examine what these DM “hadrons”, which we will denote by $M_{\chi_l}$ and $B_{\chi_l}$ from hereon, can do. However, since $B_{\chi_l}$ contains $4 \chi_l$'s, it is twice as heavy as $M_{\chi_l}$ whose mass will be taken to be of order TeV. For the positron excess consideration, only the decay of $M_{\chi_l}$ plays a role while in the minimal model considered here $B_{\chi_l}$ is stable.

**Decay of DM “hadrons”**

The decays of the DM “hadrons” $M_{\chi_l}$ into the SM leptons and quarks proceed through the couplings with the gauge bosons which belong to the $SU(6)/(SU(2)_L \times SU(4)_{DM} \times U(1)_{DM})$ coset group and which we denote by $W_{DM}$. These gauge bosons are assumed to have masses of the order of $M_{DUT}$. The relevant basic interactions are $g_{DUT} W_{DM}^{2,\mu} \bar{l} \gamma_\mu \chi_l$ and $g_{DUT} W_{DM}^{1,\mu} (\bar{\chi}_l \gamma_\mu \chi_l + \bar{l} \gamma_\mu l + \bar{q} \gamma_\mu q)$ where $W_{DM}^{2,1}$ with masses $M_{2,1}$ refers to gauge bosons which are doublets and singlets of $SU(2)_L$ respectively and where, for notational purposes, chiralities are omitted.

One obtains the following 4-fermion operators

$$\mathcal{L}_{\chi f} = \frac{g_{DUT}^2}{M_2^2} (\bar{l} \gamma_\mu \chi_l) (\bar{\chi}_l \gamma^\mu l) + \frac{g_{DUT}^2}{M_1^2} (\bar{\chi}_l \gamma^\mu \chi_l) (\bar{l} \gamma_\mu l + \bar{q} \gamma_\mu q).$$

From (15), one expects the following decay modes $M_{\chi_l} \rightarrow \bar{l} l, \bar{q} q$ with the leptonic modes being expected to dominate, in particular if $M_2 < M_1$. One can estimate the lifetime of these DM “hadrons” to be approximately

$$\tau \sim \frac{\alpha_{DUT}^{-2} M_{DM}^4}{M_{DUT}^5}.$$  \hspace{1cm} (16)

The lifetime $\sim 10^{26} \text{sec}$ is obtained from the above formula with $\alpha_{DUT}$ estimated to be $\sim 26$, $M_{DUT} \sim 10^{16}$ GeV and $M_{DM} = O(\text{TeV})$. Furthermore
the DM “mesons” decay principally into $e^+e^-, \mu^+\mu^-, \tau^+\tau^-$ since the branching ratio into quarks is expected to be smaller. This could possibly fit into the observation that the $\bar{p}$ flux is in agreement with background [2].

One important remark is in order at this point. For simplicity, mixing angles between different families are omitted in (15). However, it goes without saying that DM of different families decay into luminous matter with different strengths because of these mixing angles. As a result, the enhancements at different energies (i.e. different DM masses) are not expected to be uniform: there might well be more enhancement at one energy as compared with another.

**Summary and Discussion**

The most striking prediction of our model comes from the family structure of the DM which follows inevitably from the LUDUT theory unifying at a large scale $M_{DUT}$.

We predict excesses at three mass locations (for three families of DM): one at the PAMELA peak, another at the (yet-to-be confirmed) ATIC peak and a third one which is yet to be observed and which is expected to be located at a mass not too far from the PAMELA and ATIC peaks. The amount of enhancement is expected to vary for different energies because of interfamily mixing angles. In our model, this might be the reason why the excess at PAMELA is larger than that at ATIC. There might already be a third excess between the two “bumps” of PAMELA and ATIC, albeit a suppressed one because of small mixing angles; it is not yet statistically significant.

Such observations could provide a smoking gun for a LUDUT theory which unifies luminous and dark matter and greatly expedite further understanding of the dark sector.

Specific candidates for DM particles have appeared in model building [8]; also, part of the DM energy density can arise from intermediate mass black holes [9]. Other work on the observed positron excess, none of which mentions family structure, is exemplified in [10].

Last but not least, our model contains DM particles which carry color but whose number density is greatly suppressed by the Boltzman factor as we have mentioned above. However, these particles can be produced at the LHC through the gluon fusion process, albeit with a small cross section due to the high mass (TeV’s). The signatures of these particles are under investigation.
More generally we believe that there is every reason that Nature be equally as imaginative in the dark side of the Universe as the luminous side. Thus, while our DM family structure may appear as speculation, the truth about DM could be even more complicated. Pauli said after reading one paper "these ideas are crazy but are they crazy enough?" and the remark may be germane also to the present one.

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