Microcanonical studies concerning the recent experimental evaluations of the nuclear caloric curve

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Abstract

The microcanonical multifragmentation model from [Al. H. Raduta and Ad. R. Raduta, Phys. Rev. C 55, 1344 (1997); 56, 2059 (1997); 59, 323 (1999)] is refined and improved by taking into account the experimental discrete levels for fragments with \( A \leq 6 \) and by including the stage of sequential decay of the primary excited fragments. The caloric curve is reevaluated and the heat capacity at constant volume curve is represented as a function of excitation energy and temperature. The sequence of equilibrated sources formed in the reactions studied by the ALADIN group (\(^{197}\text{Au} + ^{197}\text{Au}\) at 600, 800 and 1000 MeV/nucleon bombarding energy) is deduced by fitting simultaneously the model predicted mean multiplicity of intermediate mass fragments (\( M_{IMF} \)) and charge asymmetry of the two largest fragments (\( a_{12} \)) versus bound charge (\( Z_{\text{bound}} \)) on the corresponding experimental data. Calculated HeLi isotopic temperature curves as a function of the bound charge are compared with the experimentally deduced ones.
I. INTRODUCTION

Nuclear multifragmentation is presently intensely studied both theoretically and experimentally. Due to the similitude existent between the nucleon-nucleon interaction and the van der Waals forces, signs of a liquid-gas phase transition in nuclear matter are searched. While the theoretical calculations concerning this problem started at the beginning of 1980 \[1\], the first experimental evaluation of the nuclear caloric curve was reported in 1995 by the ALADIN group \[2\]. A wide plateau situated at around 5 MeV temperature lasting from 3 to 10 MeV/nucleon excitation energy was identified. The fact was obviously associated with the possible existence of a liquid-gas phase transition in nuclear matter and generated new motivations for further theoretical and experimental work. Similar experiments of EOS \[3\] and INDRA \[4\] followed shortly. Using different reactions they obtained slightly different caloric curves, the plateau-like region being absent in the majority of cases. Factors contributing to these discrepancies are both the precision of the experimental measurements and the finite-size effects of the caloric curve manifested through the dependency of the equilibrated sources \[E^*(A)\] sequence on the reaction type.

Concerning the first point of view, recent reevaluations of the ALADIN group concerning the kinetic energies of the emitted neutrons brought corrections of about 10 % (in the case of the reaction \(^{197}\text{Au}+^{197}\text{Au}, 600 \text{MeV/nucleon}\)). More importantly however it was proven that the energies of the spectator parts are growing with approximately 30 % in the bombarding energy interval 600 to 1000 MeV/nucleon. On the other side, the universality of the quantity \(M_{IMF}(Z_{\text{bound}})\) subject to the bombarding energy variation (which was theoretically proven \[5,6\] to be a signature of statistical equilibrium) suggests that for the above-mentioned reactions the equilibrated sources sequence \([E^*(A)]\) should be the same. Consequently, we deal with an important nonequilibrium part included in the measured source excitation energies which may belong to both pre-equilibrium or pre-break-up stages \[7\]. The SMM calculations suggest a significant quantity of nonequilibrium energy even in the case of the 600 MeV/nucleon bombarding energy reaction \[7–9\].
Thus, the necessity of accurate theoretical descriptions of the break-up stage and of the sequential secondary particle emission appears to be imperative in order to distinguish between the equilibrium and nonequilibrium parts of the measured excitation energies. These approaches should strictly obey the constrains of the physical system which, in the case of nuclear multifragmentation, are purely microcanonic. As we previously underlined [10,11], in spite of their success in reproducing some experimental data, the two widely used statistical multifragmentation models (SMM [12] and MMMC [13]) are not strictly satisfying the microcanonical rules.

The present paper describes some refinements and improvements brought to the sharp microcanonical multifragmentation model proposed in [14,15] and also the employment of the model in its new version in the interpretation of the recent experimental data of the ALADIN group [7,8].

The improvements brought to the model [14,15] are presented in Section II. Section III presents the new evaluations of temperature curves and the first evaluations (performed with this model) of heat capacities at constant volume ($C_V$) represented as a function of system excitation energy and temperature and also the comparison between the model predictions and the recent experimental HeLi isotopic temperature curve [$T_{HeLi}(Z_{bound})$] [7,8]. Conclusions are drawn in Section IV.

II. IMPROVEMENTS BROUGHT TO THE MICROCANONICAL MULTIFRAGMENTATION MODEL

The improvements brought to the microcanonical multifragmentation model concerns both the break-up stage and the secondary particle emission stage.

(i) Primary break-up refinements

Comparing to the version of Ref. [14,15,10] the present model has the following new features:

(a) The experimental discrete energy levels are replacing the level density for fragments with $A \leq 6$ (in the previous version of the model a Thomas Fermi type level density formula was
used for all particle excited states). In this respect, in the statistical weight of a configuration and the correction factor formulas [14,15] the level density functions are replaced by the degeneracies of the discrete levels, $(2S_i + 1)$ (here $S_i$ denotes the spin of the $i$th excited level). As a criterion for level selection (i.e. the level life-time must be greater than the typical time of a fragmentation event) we used $\Gamma \leq 1$ MeV, where $\Gamma$ is the width of the energy level.

(b) In the case of the fragments with $A > 6$ the level density formula is modified as to take into account the strong decrease of the fragments excited states life-time (reported to the standard duration of a fragmentation event) with the increase of their excitation energy. To this aim the Thomas Fermi type formula [14] is completed with the factor $\exp(-\epsilon/\tau)$ (see Ref. [16]):

$$\rho(\epsilon) = \frac{1}{\epsilon \sqrt{48}} \exp(2\sqrt{a\epsilon}) \exp(-\epsilon/\tau), \quad (2.1)$$

where $a = A/\alpha$, $\alpha = 4.7(1.625 + \epsilon/B(A,Z))$ and $\tau = 9$.

(ii) Inclusion of the secondary decay stage

For the $A > 6$ nuclei it was observed that the fragments excitation energies are sufficiently small such as the sequential evaporation scheme is perfectly applicable. According to Weisskopf theory [17] (extended as to account for particles larger than $\alpha$), the probability of emitting a particle $j$ from an excited nucleus is proportional to the quantity:

$$W_j = \sum_{i=0}^{n} \int_{0}^{E^* - B_j - \epsilon_i} \frac{g_i^j \mu_j \sigma_j(E)}{\pi^2 \hbar^3} \frac{\rho_j(E^* - B_j - \epsilon_i - E)}{\rho(E^*)} E dE, \quad (2.2)$$

where $\epsilon_i$ are the stable excited states of the fragment $j$ subject to particle emission (their upper limit is generally around 7 - 8 MeV), $E$ is the kinetic energy of the formed pair in the center of mass (c.m.) frame, $g_i^j = 2S_i + 1$ is the degeneracy of the level $i$, $\mu_j$ and $B_j$ are respectively the reduced mass of the pair and the separation energy of the particle $j$ and finally $\sigma_j$ is the inverse reaction cross-section. Due to the specificity of the multifragmentation calculations we considered the range of the emitted fragments $j$ up to the $A = 16$ limit. For the inverse reaction cross-section we have used the optical model
based parametrization from Ref. [18]. The sequential evaporation process is simulated by means of standard Monte Carlo (see for example [19]).

For nuclei with $4 \leq A \leq 6$ (the only excited states of $A = 4$ nuclei taken into consideration are few states higher than 20 MeV belonging to the $\alpha$ particle) depending on their amount of excitation we consider secondary break-up for $\epsilon > B(A, Z)/3$ and Weisskopf evaporation otherwise (here $\epsilon$ is the excitation energy of the fragment $(A, Z)$ and $B(A, Z)$ is its binding energy). The microcanonical weight formulas have the usual form [14] excepting the level density functions which are here replaced by the discrete levels degeneracies. Due to the reduced dimensions of the $A < 6$ systems, the break-up channels are countable (and a classical Monte Carlo simulation is appropriate) when a mean field approach is used for the Coulomb interaction energy. In this respect, the Wigner-Seitz approach [12] is employed for the Coulomb interaction:

$$U_C = \frac{3}{5} \frac{Z_0^2 e^2}{R} - \sum_i \frac{3}{5} \frac{Z_i^2 e^2}{R_{A_i Z_i}^C},$$

(2.3)

where $A$ and $Z$ denotes the mass and the charge of the source nucleus, the resulting fragments have the index $i$, $R_{A_i Z_i}^C / R_{A_i Z_i} = (1 + \kappa)^{1/3} [(Z_i/A_i)/(Z/A)]^{1/3}$ and $V = (1 + \kappa)V_0$. Here $V$ denotes the break-up volume and $V_0$ the volume of the nucleus at normal density. It should be added that $R$ is the radius of the source nucleus at break-up and $R_{A_i Z_i}$ is the radius of fragment $i$ at normal density.

For each event of the primary break-up simulation, the entire chain of evaporation and secondary break-up events is Monte Carlo simulated.

III. RESULTS

Using the improved version of the microcanonical multifragmentation model, the caloric curves corresponding to two freeze-out radii ($R=2.25$ A$^{1/3}$ and $R=2.50$ A$^{1/3}$ fm) are reevaluated for the case of the source nucleus (70, 32) (the microcanonical caloric curves evaluated with the initial version of the model are given in Ref. [10,11,20]). These are presented in
Fig. 1 (a). One can observe that the main features of the caloric curve from Refs. [10,11,20] are reobtained. Thus, one can recognize the liquid-like region at the beginning of the caloric curve, then a large plateau-like region and finally the linearly increasing gas-like region. One may also notice that the caloric curve behavior at the freeze-out radius variation is maintained: The decrease of the freeze-out radius leads to a global lifting of the caloric curve.

As it is well known, the curves of the constant volume heat capacity \( C_V \) as a function of system excitation energy \( E^* \) and as a function of temperature \( T \) may provide important information concerning the transition region and the transition order. For this reason the curves \( C_V(E^*) \) and \( C_V(T) \) have been evaluated (see Fig. 1 (a) and Fig. 1 (b)). We remind that the constant volume heat capacity \( C_V \) is calculable in the present model using the formula [11]:

\[
C_V^{-1} = 1 - T^2 \left\langle \left( \frac{3}{2} N_C - \frac{5}{2} \right) \frac{1}{K} \right\rangle^2 + T^2 \left\langle \left( \frac{3}{2} N_C - \frac{5}{2} \right) \frac{1}{K^2} \right\rangle.
\] (3.1)

It can be observed that the \( C_V(E^*) \) curve has a sharp maximum around 4.5 MeV/nucleon excitation energy for both considered freeze-out radii. This suggests that a phase transition exists in that region. The transition temperatures can be very well distinguished by analyzing the \( C_V(T) \). One can observe two sharp-peaked maxima pointing the transition temperatures corresponding to the two considered freeze-out radii.

In order to make a direct comparison between the calculated HeLi isotopic temperature and the recent experimental results [7] one has to deduce the sequence of excitation energy as a function of the system dimension \( E^*(A) \). This is done as in Refs. [7,8] using as matching criterion the simultaneously reproduction of the \( \langle M_{IMF} \rangle \) (\( \langle Z_{bound} \rangle \)) and \( \langle a_{12} \rangle \) (\( \langle Z_{bound} \rangle \)) curves. This couple of curves can fairly well identify the dimension and the excitation of the equilibrated nuclear source [7,8]. Here \( M_{IMF} \) stands for the multiplicity of intermediate mass fragments and is defined as the number of fragments with \( 3 \leq Z \leq 30 \) from a fragmentation event while \( a_{12} \) denotes the charge asymmetry of the two largest fragments and, for one fragmentation event is defined as \( a_{12} = (Z_{max} - Z_2)/(Z_{max} + Z_2) \) with \( Z_{max} \leq Z_2 \leq 2 \).
where $Z_{\text{max}}$ is the maximum charge of a fragment and $Z_2$ is the second largest charge of a fragment in the respective event. $Z_{\text{bound}}$ represents the bound charge in one fragmentation event and is defined as the sum of the charges of all fragments with $Z \geq 2$.

The simultaneous fit of the calculated curves $\langle M_{IMF} \rangle (\langle Z_{\text{bound}} \rangle)$ and $\langle a_{12} \rangle (\langle Z_{\text{bound}} \rangle)$ on the corresponding experimental data ($^{197}\text{Au}+^{197}\text{Au}$ at 1000 MeV/nucleon) is given in Fig. 2. The agreement is very good. The equilibrated source sequence $[E^*(A)]$ we used for this purpose is given in Fig. 3 together with the experimental evaluations of the excitation energies as a function of source dimension for the reaction $^{197}\text{Au}+^{197}\text{Au}$ at 600, 800 and 1000 MeV/nucleon. The theoretically obtained sequence is relatively close to the experimental line corresponding to 600 MeV/nucleon bombarding energy. The deviations between the calculated equilibrated source sequence and the three experimental lines suggest that the experimental evaluations contain a quantity of non-equilibrium energy which grows with increasing the bombarding energy. As suggested in Ref. [7,8], its origin may be situated in both the pre-equilibrium and pre-break-up stage. These deviations are exclusively due to the neutron kinetic energies which, reevaluated [7,8] from the 1995 data [2], are much larger.

It should also be pointed that apart from the SMM predictions [7–9], the quantity of non-equilibrium energy predicted by the present model is smaller and thus the model predicted equilibrated source sequence is closer to the experimental line of the 600 MeV/nucleon bombarding energy reaction.

After evaluating the sequence of the equilibrated sources a direct comparison the HeLi calculated isotopic temperature curve with the ones recently evaluated by the ALADIN group [7,8] is performed. To this purpose the uncorrected Albergo temperature is used: $T_{\text{HeLi}} = 13.33/\ln[2.18 \ (Y_{6\text{Li}}/Y_{7\text{Li}}) / (Y_{3\text{He}}/Y_{4\text{He}})]$, the experimental predictions being divided by $f_T = 1.2$ (which is the factor used in the ALADIN evaluation of the HeLi caloric curve chosen as to average the QSM, GEMINI and MMMC models predictions). The result is represented in Fig. 4 as a function of $Z_{\text{bound}}$. It can be observed that the agreement between the calculated $T_{\text{HeLi}}(Z_{\text{bound}})$ and the experimental data corresponding to the $^{197}\text{Au}+^{197}\text{Au}$ reaction at 600 and 1000 MeV/nucleon bombarding energy is excellent on the entire range.
of \( Z_{\text{bound}} \). In comparison, the SMM model predicts in the region \( Z_{\text{bound}} \leq 25 \) a curve steeper than the experimental data.

IV. CONCLUSIONS

Summarizing, the microcanonical multifragmentation model from Ref. [14,10] is improved by refining the primary break-up part and by including the secondary particle emission. The caloric curve rededuced with the new version of the model preserves its general aspect manifesting an important plateau-like region. The transition regions are clearly indicated by the sharp maxima of the \( C_V(E^*) \) and \( C_V(T) \) curves. The model proves the ability of simultaneously fitting the "definitory" characteristics of the nuclear multifragmentation phenomenon \( \langle M_{\text{IMF}} \rangle (\langle Z_{\text{bound}} \rangle) \) and \( \langle a_{12} \rangle (\langle Z_{\text{bound}} \rangle) \). Evaluating the equilibrated source sequence \( E^*(A) \) [by using the criterion of reproducing both \( M_{\text{IMF}} \) and \( a_{12} \) versus \( \langle Z_{\text{bound}} \rangle \)], a nonequilibrium part of the experimentally evaluated excitation energy growing with the increase of the bombarding energy is identified. The direct comparison of the calculated HeLi caloric curve shows an excellent agreement with the experimental HeLi curves recently evaluated by the ALADIN group.
Fig. 1. Microcanonical temperature as a function of source excitation energy (a), heat capacity at constant volume as a function of source excitation energy (b) heat capacity at constant volume as a function of microcanonical temperature (c). Calculations have been performed for the source nucleus (70, 32) with two values of the freeze-out radius.
FIG. 2. $\langle M_{IMF} \rangle (\langle Z_{\text{bound}} \rangle)$ and $\langle a_{12} \rangle (\langle Z_{\text{bound}} \rangle)$ evaluated by means of the microcanonical model in comparison with the experimental evaluations corresponding to the reaction $^{197}\text{Au} + ^{197}\text{Au}$ at 1000 MeV/nucleon bombarding energy. (The deviation of calculated $\langle a_{12} \rangle$ from experimental data for $Z_{\text{bound}} \geq 65$ is, as explained in [7], due to some detection problems.)
FIG. 3. The sequence of equilibrated sources evaluated by means of the microcanonical model (open circles) following the criterion of simultaneous fitting of the calculated $M_{IMF}(Z_{\text{bound}})$ and $a_{12}(Z_{\text{bound}})$ on the corresponding experimental data. The close symbols represent the sequence of excitation energy experimentally measured for the reaction $^{197}\text{Au}+^{197}\text{Au}$ at 600, 700, 1000 MeV/nucleon bombarding energy [7].

FIG. 4. HeLi temperature curves evaluated with the microcanonical model (open circles) in comparison with the experimental HeLi temperatures [7,8] corresponding to the reactions $^{197}\text{Au}+^{197}\text{Au}$ at 600 and 1000 MeV/nucleon bombarding energy.
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