SMALL-SCALE GRADIENTS OF CHARGED PARTICLES IN THE HELIOSPHERIC MAGNETIC FIELD

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ABSTRACT

Using numerical simulations of charged-particles propagating in the heliospheric magnetic field, we study small-scale gradients, or “dropouts,” in the intensity of solar energetic particles seen at 1 AU. We use two turbulence models, the foot-point random motion model and the two-component model, to generate fluctuating magnetic fields similar to spacecraft observations at 1 AU. The turbulence models include a Kolmogorov-like magnetic field power spectrum containing a broad range of spatial scales from those that lead to large-scale field-line random walk to small scales leading to resonant pitch-angle scattering of energetic particles. We release energetic protons (20 keV–10 MeV) from a spatially compact and instantaneous source. The trajectories of energetic charged particles in turbulent magnetic fields are numerically integrated. Spacecraft observations are mimicked by collecting particles in small windows when they pass the windows at a distance of 1 AU. We show that small-scale gradients in the intensity of energetic particles and velocity dispersions observed by spacecraft can be reproduced using the foot-point random motion model. However, no dropouts are seen in simulations using the two-component magnetic turbulence model. We also show that particle scattering in the solar wind magnetic field needs to be infrequent for intensity dropouts to form.

Key words: cosmic rays – diffusion – Sun: flares – Sun: magnetic fields – turbulence

Online-only material: color figures

1. INTRODUCTION

The propagation of energetic particles in turbulent magnetic fields is of great importance in space physics and astrophysics. The large-scale transport of charged particles is usually studied by solving the cosmic-ray transport equation first derived by E. N. Parker (1965). Determining diffusion coefficients in the equation is crucial to studying the transport of charged particles. The diffusion coefficients parallel and perpendicular to the background magnetic field ($\kappa_\parallel$ and $\kappa_\perp$) are considered to be quite different, with $\kappa_\parallel$ usually larger than $\kappa_\perp$ (Jokipii 1966; Giacalone & Jokipii 1999).

An unsolved issue in the transport of energetic particles is that the mean-free paths of energetic particles inferred from solar energetic particle (SEP) events are usually much longer than those derived from the quasi-linear theory (Palmer 1982; Bieber et al. 1994). It was proposed that the anisotropy of magnetic turbulence may have influences on the diffusion coefficient parallel to the magnetic field (Bieber et al. 1996; Chandran 2000; Yan & Lazarian 2002), while some numerical simulations found that the diffusion coefficient has a weak dependence on the anisotropy (Giacalone & Jokipii 1999; Qin et al. 2006). The discrepancy between observations and theories is still not well resolved.

The transport of charged particles normal to the magnetic field is also not well understood. Some analyses give a rather small cross-field diffusion width with the ratio of the perpendicular diffusion coefficient to the parallel diffusion coefficient $\kappa_\perp/\kappa_\parallel \sim 10^{-4}$ or smaller (Roelof et al. 1983). Recent numerical simulations and analytical studies find a larger value of $\kappa_\perp/\kappa_\parallel \sim 0.02$–0.05 for energetic particles moving in the heliospheric magnetic field at 1 AU (Giacalone & Jokipii 1999; Qin et al. 2002; Matthaeus et al. 2003). However, some spacecraft measurements suggest that the ratio can reach 0.2 or even larger (Dwyer et al. 1997; Zhang et al. 2003), which is unexpectedly large compared with those obtained from numerical simulations (Giacalone & Jokipii 1999). Newly available data show that impulsive SEP events are occasionally seen by all three spacecraft (STEREO A/B, and ACE), with a separation of more than 100° in heliospheric longitude (Wiedenbeck et al. 2010, 2013; Dresing et al. 2012). Giacalone & Jokipii (2012) and Dresing et al. (2012) have suggested that $\kappa_\perp/\kappa_\parallel$ has to be as large as a few percent to explain these multi-spacecraft observations.

Impulsive SEPs are ideal for studying the transport of charged particles because their acceleration time is much shorter than the propagation time in the heliospheric magnetic field. Recently, observations by spacecraft such as ACE and Wind have revealed new features of particle transport (Mazur et al. 2000; Chollet et al. 2007; Chollet 2008; Chollet & Giacalone 2008, 2011). Mazur et al. (2000) reported that the intensity of impulsive SEP events often shows small-scale sharp gradients, or “dropouts.” These dropouts are commonly seen in impulsive SEP events and the typical convected distance between dropouts is about 0.03 AU, similar to the correlation scale in the solar wind turbulence (e.g., Wicks et al. 2010). The occurrence of dropouts does not seem to be associated with rapid magnetic field changes, meaning that it is more related to some large-scale transport effects. The phenomenon indicates that the diffusion of energetic particles transverse to the local magnetic field is very small (Chollet & Giacalone 2011); the transport of energetic particles in the solar wind is likely dominated by some other effect. It is worth noting that the motions of energetic charged particles transverse to the magnetic field can be considered as two parts: particle motions across the local magnetic field caused by drift or scattering and particle motions along meandering magnetic field lines but normal to the mean magnetic field. The observed SEP dropouts may be interpreted as that the motions of particles across the local magnetic field is small,

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but a large part of perpendicular diffusion can be contributed by field-line random walk. These new observations have provided an excellent opportunity to examine and constrain relative contributions from these two effects to large-scale perpendicular transport.

Using numerical simulations that consider large-scale turbulent magnetic fields, Giacalone et al. (2000) have demonstrated that dropouts can be reproduced when energetic particles are released in a small source region near the Sun. This model is consistent with magnetic turbulence models that allow a large perpendicular diffusion coefficient caused by field-line random walk (Giacalone & Jokipii 1999). The time duration between numerically produced dropouts is several hours, which is similar to that observed in impulsive SEP events (Mazur et al. 2000; Chollet & Giacalone 2008). It also naturally reproduces the feature that the typical convected distance between dropouts is similar to the correlation scale in the solar wind turbulence (Mazur et al. 2000). Ruffolo et al. (2003) and Chuychai et al. (2007) have proposed a different idea on the basis of the two-component turbulence model (Matthaeus et al. 1990; Bieber et al. 1996). They argue that some magnetic field lines in the solar wind can have a restricted transverse displacement. The corresponding magnetic flux tubes connecting to SEP source regions are concentrated by energetic particles. For magnetic field lines that meander in space, energetic particles that are initially confined to the field lines will diffuse away. However, this effect may depend on the two-component magnetic field model they use (a combination of a two-dimensional fluctuation and a one-dimensional fluctuation). Chuychai et al. (2007) studied the trapping of magnetic field lines using the combined field of an idealized flux-tube magnetic field and a one-dimensional fluctuating magnetic field. Tooprakai et al. (2007) studied the trapping of energetic particles using the same magnetic field model. However, the trapping of charged particles in the magnetic field generated by the two-component turbulence model has not been well explored by numerical simulations.

Although previous numerical simulations have successfully reproduced SEP dropouts (Giacalone et al. 2000), they assume ad hoc pitch-angle scattering that is not realistic. Physically, the pitch-angle scattering of charged particles is due to small-scale magnetic fluctuations, which is not present in the Giacalone et al. (2000) model. The purpose of this study is to include the effect of small-scale magnetic turbulence and examine the propagation of SEPs in a turbulent magnetic field that has a power spectrum similar to that derived from observations. In this study, we use two different types of three-dimensional magnetic field turbulence models often used in studying the transport of energetic particles in space, i.e., the foot-point random motion model (Jokipii & Parker 1969; Giacalone et al. 2006) and the two-component model (Matthaeus et al. 1990). The generated fluctuating magnetic field has a Kolmogorov-like power spectrum, with wavelengths from just larger than the correlation scale, leading to large-scale field-line random walk, down through small scales that lead to resonant pitch-angle scattering of the particles. In Section 2, we describe the magnetic turbulence models and numerical methods we use to study the propagation of energetic particles. The parameters used in the simulations are also listed. In Section 3, we use test-particle simulations in the magnetic field generated from the two magnetic turbulence models to study the propagation of SEPs. We show that dropouts during impulsive SEP events can be reproduced using the foot-point random motion model when the source region is small compared with the correlation scale. However, for the two-component model, we find that dropouts are not seen in the simulations for the parameters we use. Our order-of-magnitude estimate shows that for the intensity dropouts to form, particle scattering has to be infrequent, leading to a fairly large mean-free path. We discuss and summarize the results in Section 4.

2. NUMERICAL MODEL AND SIMULATION

In this study, we consider the propagation of energetic particles from a spatially compact and instantaneous source in turbulent magnetic fields. We use two magnetic field turbulence models that capture the main observations of magnetic field fluctuation in the solar wind: the foot-point random motion model (e.g., Jokipii & Parker 1969; Jokipii & Kota 1989; Giacalone et al. 2006) and the two-component model (Matthaeus et al. 1990; Bieber et al. 1996). This section gives a mathematical description of the turbulent magnetic field models and the numerical method for integrating the trajectories of energetic charged particles.

2.1. Turbulent Magnetic Fluctuations

In a three-dimensional Cartesian geometry $(x, y, z)$, a turbulent magnetic field can be expressed as follows:

$$
B = B_0 + \delta B
= B_0 \hat{z} + \delta B_x(x, y, z, t) \hat{x} + \delta B_y(x, y, z, t) \hat{y} + \delta B_z(x, y, z, t) \hat{z}.
$$

(1)

This expression assumes a globally uniform background magnetic field $B_0$ in the $z$ direction and a fluctuating magnetic field component $\delta B$.

The two-component model is a quasi-static model for the wave-vector spectrum of magnetic fluctuation on the basis of observations of the solar wind turbulence (Matthaeus et al. 1990; Bieber et al. 1996). In this model, the fluctuating magnetic field is expressed as the sum of two parts: a slab component, $\delta B' = (B'_x(z), B'_y(z), 0)$, and a two-dimensional component, $\delta B^{2D} = (B^{2D}_x(x, y), B^{2D}_y(x, y), 0)$. The slab component is a one-dimensional fluctuating magnetic field with all wave vectors along the direction of the background magnetic field $\hat{z}$, and the two-dimensional component only consists of magnetic fluctuations with wave vectors along the transverse directions $\hat{x}$ and $\hat{y}$. It has been observed that magnetic field fluctuations have components with wave vectors nearly parallel or perpendicular to the magnetic field, with more wave power concentrated in the perpendicular directions (usually about 80% in the solar wind; Bieber et al. 1996). This model captures the anisotropic characteristic of the solar wind turbulence but neglects the turbulence component that propagates obliquely to the magnetic field $B_0$.

Another often-used model for magnetic turbulence is based on the idea that magnetic fluctuations can be generated by foot-point random motions (Jokipii & Parker 1969; Jokipii & Kota 1989; Giacalone et al. 2006). One can consider a Cartesian geometry with the uniform magnetic field $B_0$ along the $z$ direction and the source surface lying in the $x$–$y$ plane at $z = 0$. Since magnetic field lines are frozen in the surface velocity field, magnetic field fluctuations of the form in Equation (1) can be produced by foot-point motions. We assume that the surface foot-point motion is described by $v_{fp}(x, y, t) = \nabla \times \Psi(x, y, t)$,
where $\Psi$ is an arbitrary stream function. The fluctuating component of the magnetic field anywhere is given as follows:

$$\delta \mathbf{B}^{ip} = \frac{B_0}{U} \mathbf{v}_{fp}(x, y, t - z/U). \quad (2)$$

The magnetic field is assumed to have no dynamical variation but to be continuously dragged outward by a background fluid (the solar wind) with a convection speed $U$. When the magnetic field is evaluated at a certain time, it is fully three-dimensional with dependence on $x$, $y$, and $z$.

In both of these magnetic fluctuation models, magnetic fields are variable in three spatial dimensions. As demonstrated by Jokipii et al. (1993) and Jones et al. (1998), it is important to consider particle transport in a fully three-dimensional magnetic field because, in one- and two-dimensional fields, the particles adhere to the magnetic field lines on which they started their gyromotion because of the presence of at least one ignorable spatial coordinate. The magnetic fluctuations can be constructed using the random phase approximation (e.g., Giacalone & Jokipii 1999) and assuming a power spectrum of magnetic field fluctuations. This power spectrum can be determined from spacecraft observations (Coleman 1968; Bieber et al. 1993). The slab component $\delta \mathbf{B}^{s}$, two-dimensional component $\delta \mathbf{B}^{2D}$, and fluctuating magnetic field produced by the foot-point random motion $\delta \mathbf{B}^{fp}$ can be expressed as follows (Giacalone & Jokipii 1999; Giacalone et al. 2006):

$$\delta \mathbf{B}^{s} = \sum_{n=1}^{N_n} A_n [\cos \alpha_n (\cos \phi_n \hat{x} + \sin \phi_n \hat{y}) + i \sin \alpha_n (-\sin \phi_n \hat{x} + \cos \phi_n \hat{y})] \times \exp(i k_n z + i \beta_n), \quad (3)$$

$$\delta \mathbf{B}^{2D} = \sum_{n=1}^{N_n} A_n [\sin \alpha_n (-\cos \phi_n \hat{x} + \sin \phi_n \hat{y})] \times \exp[i k_n (\cos \phi_n x + \sin \phi_n y) + i \beta_n], \quad (4)$$

$$\delta \mathbf{B}^{fp} = \left( \frac{\partial}{\partial y} - \hat{y} \frac{\partial}{\partial x} \right) \times \left[ \sum_{n=1}^{N_n} \left( -\frac{1}{k_n} \right) A_n e^{ik_n x (\cos \phi_n + \sin \phi_n)} \right] \left( 1 + i \omega_n (t - z/U) + \beta_n \right), \quad (5)$$

where $\beta_n$ is the phase of each wave mode, $A_n$ is its amplitude, $\omega_n$ is its frequency, $\alpha_n$ is the polarization angle, and $\phi_n$ determines the spatial direction of the $k$-vector in the $x$-$y$ plane. $\beta_n$, $\alpha_n$, and $\phi_n$ are random numbers between 0 and $2\pi$. The frequency is taken to be $\omega_n = 0.1U/k_n$. This assumes that the Alfvén speed is 0.1$U$, which is typical in the solar wind. All of the forms of fluctuating magnetic fields satisfy the condition $\nabla \cdot \mathbf{B} = 0$.

The amplitude of the magnetic fluctuation at wave number $k_n$ is assumed to follow a Kolmogorov-like power law:

$$A_n^2 = \frac{\sigma^2}{1 + (k_n L_c)^2} \left[ \sum_{n=1}^{N_n} \frac{\Delta V_n}{1 + (k_n L_c)^2} \right]^{-1}, \quad (6)$$

where $\sigma^2$ is the magnetic wave variance and $\Delta V_n$ is a normalization factor. In one-, two-, and three-dimensional omnidirectional spectra, $\Delta V_r = \Delta k_n$, $2\pi k_n \Delta k_n$, and $4\pi k_n^2 \Delta k_n$, and $\nu = 5/3, 8/3, \text{ and } 11/3$, respectively. A logarithmic spacing in wavenumbers $k_n$ is chosen so that $\Delta k_n/k_n$ is a constant.

Giacalone et al. (2006) has pointed out that these two models are closely related and the two-component model can be reproduced using the foot-point random motion model by choosing a particular set of velocity field fluctuations. Both of these simplified models assume a quasi-static field that may not be appropriate for describing magnetic turbulence. Current sheets, which are known to exist in the solar wind and are possibly related to turbulence, are not included in our kinematic model. However, observations of dropouts show no correlation with current sheets (Mazur et al. 2000; Chollet & Giacalone 2008). Nevertheless, these two models are very useful in studying the transport of energetic charged particles in magnetic turbulence and explaining observations of SEP events. Because our models use a Cartesian coordinate system with a constant solar wind speed and constant average magnetic field, the energetic particles do not undergo adiabatic cooling, nor adiabatic focusing. These effects would arise in a spherical geometry but are not included in our model. Moreover, our models also assume that the magnetic-field turbulence variance does not change with distance.

In the simulations, we generally use parameters similar to what is observed in the solar wind at 1 AU. The minimum and maximum wavelengths $\lambda_{\min}$ and $\lambda_{\max}$ are taken to be $5 \times 10^{-5}$ AU and 1 AU. The minimum wavelength is shorter than the resonant scale of particles with the lowest energy. We choose $\Delta k_n/k_n = 0.02$ and a total number of $N_n = 460$ wave modes are summed in the simulations. The mean magnetic field $B_0$ is taken to be 5 nT. The total variance of magnetic fluctuation $\sigma^2 = 0.3B_0^2$. For the two-component model, the two-dimensional fluctuations contribute 80% of the total magnetic power and the one-dimensional fluctuations contribute 20% of the total magnetic power, as suggested by Bieber et al. (1996). The convection velocity of the solar wind $U$ is set to be 400 km s$^{-1}$. The correlation length is assumed to be $L_c = 0.01$ AU. These parameters, which are taken to not vary with distance from the Sun because of our assumed geometry, are based on observed values at 1 AU. In reality, these parameters likely vary with distance from the Sun. However, we feel that some key parameters that govern the interplanetary scattering of particles may not vary significantly in the inner heliosphere. For example, Helios observations show that the correlation length varies only by a factor of about five from 0.3 AU to 1 AU (e.g., Bavassano et al. 1982b; Bruno & Carbone 2013). The solar wind speed does not change significantly beyond the Alfvén point (McGregor et al. 2011). The ratio between the turbulence variance and the background magnetic field squared $\sigma^2/B_0^2$ only weakly depends on the radial distance (e.g., Bavassano et al. 1982a). Observations of SEP events show that the scattering properties are similar between the inner heliosphere and at 1 AU, indicating that the change of turbulence property does not cause strong variations in particle scattering (Kallenrode et al. 1992; Kallenrode 1993).

In Figure 1, we illustrate 100 turbulent magnetic field lines originated from a surface region within $-L_c < x < L_c$ and $-L_c < y < L_c$ at $z = 0$ at time $t = 0$ produced by foot-point random motion. It is clear that the magnetic field lines meander on scales significantly greater than the correlation length. Field lines that start out from a compact source—smaller than the correlation scale of turbulence—meander considerably with increasing distance from the source in the $z$ direction.
Table 1
List of Simulation Runs

| Run | Model                  | Source Radius ($L_c$) | Minimum Wavelength (AU) | Dropouts? |
|-----|------------------------|-----------------------|-------------------------|-----------|
| 1   | Field line random walk | 5.0 × 10^{-5}        | N                       |
| 2   | Field line random walk | 5.0 × 10^{-5}        | Y                       |
| 3   | Two component          | 0.2 × 10^{-5}        | N                       |
| 4   | Two component          | 0.2 × 10^{-5}        | N                       |
| 5   | Two component          | 2.0 × 10^{-3}        | Y                       |

Note. $L_c$ is taken to be 0.01 AU.

Figure 1. Turbulent magnetic field lines produced by the foot-point random motion model originated from $-L_c < x < L_c$ and $-L_c < y < L_c$ at $t = 0$. See Section 2.1 for description and parameters.

Figure 2. $x$ and $y$ component of the turbulent magnetic field at $z = 1$ AU generated by the two-component model and foot-point random motion model. (A color version of this figure is available in the online journal.)

shows the $x$ and $y$ components of the magnetic fields generated from two models at $z = 1$ AU.

2.2. Test-particle Simulations

To study the propagation of energetic particles in the heliospheric magnetic field, we numerically integrate the trajectories of energetic particles in magnetic fields generated from the previously described magnetic turbulence models. At each time step, the magnetic field vector at the position of each particle is calculated from the magnetic turbulence models. The numerical technique used to integrate the trajectories of energetic particles is the Bulirsh-Stoer method, which Press et al. (1986) describes in detail. It is highly accurate and conserves energy well. The algorithm uses an adjustable time-step method that is based on the evaluation of local truncation error. The time step is increased if the local truncation error is smaller than $10^{-6}$ for several consecutive time steps. In the case of no electric field, the energy of a single particle in the fluctuating magnetic field is conserved to a high degree, with a change of less than 0.01% during the simulation. In our test-particle simulations, the charged particles are released impulsively at $z = 0$ and their trajectories are numerically integrated until they reach boundaries at $z = 1.6$ AU and $z = -0.1$ AU. The spacecraft observations at 1 AU are mimicked by collecting particles in windows of a size of $L_x \times L_y = 0.01$ AU $\times$ 0.01 AU when the particles pass the windows at $z = 1$ AU. The size of the collection regions is about the same size as a correlation length. We have also used smaller collection windows (one fifth of a correlation length) to confirm the results presented in this paper. The collected particle energies and arrival times in each window are analyzed as a spacecraft observation. The source regions are taken to be a circle at the $z = 0$ plane with a radius of 1, much smaller than the correlation scale $R_{source} = 0.2L_c$, and 2, much larger than the correlation scale $R_{source} = 5L_c$. A parameter list for different runs is given in Table 1. The energy for released particles ranges from 20 keV to 10 MeV. The velocity distribution of released particles is prescribed to follow a power law $f = f_0v^{-4}$ with random pitch angles between 0° and 90°. For each run, we release about 60 million particles to get a sufficient number of particles collected in the windows.

3. NUMERICAL SIMULATION OF SMALL-SCALE GRADIENTS IN SEP INTENSITY FLUX

We use turbulent magnetic fields generated from the foot-point random motion model and the two-component model described in Section 2 to study the SEP intensity dropouts.
Figure 3. Example of SEP event simulated using the foot-point random motion model for the case of the large source region. Upper panel: energy-time plot. Middle panel: the inverse velocity $1/v$ vs. the time after the release. Lower panel: the count rate as a function of time in 30 minute bins. The simulated event does not show any SEP dropouts.

Figure 4. Example of SEP dropouts simulated using the foot-point random motion model for the case of the small source region. Upper panel: energy-time plot. Middle panel: the inverse velocity $1/v$ vs. the time after the release. Lower panel: the count rate as a function of time in 30 minute bins. This example clearly shows dropouts.

Figure 5. Example of the observed SEP dropouts that show different path lengths observed by ACE/ULEIS detector. Figure courtesy of Dr. Joseph Mazur, Aerospace Corporation. 
(A color version of this figure is available in the online journal.)

observed by spacecraft such as ACE and Wind (Mazur et al. 2000; Chollet & Giacalone 2008). Table 1 summarizes the simulation runs. In Figure 3, we show a simulated SEP event using the foot-point random motion model for the case of a large source region (Run 1). The upper panel shows the energy-time plot and the middle panel shows the inverse velocity $1/v$ versus the time after the initial release. In the lower panel, we show the count rate as a function of time in 30 minute bins. One can see that, in this case, the simulated SEP event does not show any intensity dropout. The variation of flux is within a factor of two on the timescale of several hours after the initial onset. In the small source region case, dropouts can be frequently seen. An example is given in Figure 4, which shows plots similar to Figure 3, but for the small source region case (Run 2). It is shown that two SEP dropouts can be clearly seen during $t = 3.7–5.4$ hr and $t = 16.4–18.3$ hr. The scatter plots show two clear gaps, and the flux shows a drop of more than one order-of-magnitude in about an hour. For the parameters we use, the convected distances for the dropouts are $2.4 \times 10^6$ km ($1.6 L_c$) and $2.7 \times 10^6$ km ($1.8 L_c$), respectively. These results are consistent with spacecraft observations (Mazur et al. 2000; Chollet & Giacalone 2008). The time intervals of these dropouts are typically several hours, which are similar to that observed in space (Mazur et al. 2000). These results are similar to those of Giacalone et al. (2000), but unlike that study we have made no assumptions of ad hoc scattering. Instead, pitch-angle scattering in the present model is due to small-scale magnetic fluctuations.

SEPs accelerated close to the Sun can exhibit distinct velocity dispersions as they arrive at 1 AU along different paths. Figure 5 shows an example of impulsive SEP events plotted as $1/v$ versus time. The observation was made by ACE/ULEIS detector in 1999. It displays at least two distinct arrival times at 1 AU,
which indicates that particles follow at least two different field-line lengths. In our simulations, we also find that the apparent path lengths can have different values. An example of the simulated events is presented in Figure 6. In this plot, we use blue dashed lines as a reference, which represent particles travel along a field line with a length of 1.1 AU and cosine pitch angle $\mu = v_\parallel/v = 1$. The time differences between the dashed lines are 3 hr. Some particles collected at about $t = 16$ hr and $t = 20$ hr are above the second blue line, indicating that they arrived earlier than did other particles. In addition, the slopes of velocity dispersions between dropouts are different, indicating particles traveling along field lines with distinct lengths.

We have also attempted to use the two-component model to study SEP intensity dropouts. However, we did not find any clear dropout in our simulations for either small or large source region. Figure 7 shows a simulated SEP event using the two-component model when the source region is small (Run 4). The particles in the energy-time plot and $1/v$-time plot show a more broadened distribution, indicating enhanced scattering compared with the cases for the foot-point random motion model. The simulated event has only some variations in flux within a factor of two during several hours. We have increased the number of particles and changed the size of the collection windows in the simulation and confirmed that those factors do not change this result. To further resolve this issue, we have prepared two scatter plots that show the positions of about 400,000 energetic particles projected on the $x$-$z$ plane at $t = 14.4$ hr after the initial release. The results are shown in Figure 8 for the foot-point random motion model (Run 2) and in Figure 9 for the two-component model (Run 4). It can be clearly seen in Figure 8 that the particles follow the braiding magnetic field lines, and therefore they are separated as the field lines meander in space. However, this feature is not clearly seen in Figure 9 for the two-component model.

A possible reason that dropouts are not seen in simulations using the two-component model is that the model contains a slab component that can more efficiently scatter energetic particles in pitch-angle. To demonstrate this, we measure diffusion coefficients of particle transport in the two models by implementing the technique used by Giacalone & Jokipii (1999). We use the definition of diffusion coefficients $\kappa_{\parallel/\perp} = \langle \zeta^2 \rangle /2t$, where $\zeta$ is the spatial displacement at a given time $t$. We calculate the perpendicular and parallel diffusion coefficients for 1 MeV protons in the two turbulence models using the same parameters that were used in the simulation. The results are shown in Figure 10. Blue lines represent the displacements for particles propagating in the two-component model and dashed red lines represent results for foot-point random motion model. For the two-component model, the parallel and perpendicular coefficients are $\kappa_\parallel = 1.3 \times 10^{21}$ cm$^2$ s$^{-1}$ and $\kappa_\perp = 1.3 \times 10^{22}$ cm$^2$ s$^{-1}$, respectively. For the foot-point random motion model, the parallel and perpendicular coefficients are $\kappa_\parallel = 1.07 \times 10^{22}$ cm$^2$ s$^{-1}$ and $\kappa_\perp = 3.4 \times 10^{19}$ cm$^2$ s$^{-1}$, respectively. It is shown that the parallel diffusion coefficient for the two-component model is about one order of magnitude smaller than that for the foot-point random motion model, meaning that particles experience more scattering in the magnetic field generated from the two-component model. Because the inefficient pitch-angle scattering, particles follow field lines longer and the transfer from one field line to the next occurs less frequently. The calculation also shows a smaller ratio of $\kappa_\perp/\kappa_\parallel$ for the foot-point random motion model ($\kappa_\perp/\kappa_\parallel = 0.0032$) compared with that for the two-component model ($\kappa_\perp/\kappa_\parallel = 0.01$). Although drift motions caused by the gradient and curvature of magnetic field can also cause particles travel off a field line, a small value of $\kappa_\perp/\kappa_\parallel$ indicates that the effect is fairly small. If large-scale field-line meandering is the explanation for SEP dropouts, pitch-angle scattering resulting from small-scale scattering should be small so energetic particles can be mostly

![Figure 6](https://example.com/fig6.png) Examples of SEP dropouts produced from numerical simulations. The simulated SEP event shows different apparent path lengths. (A color version of this figure is available in the online journal.)

![Figure 7](https://example.com/fig7.png) Example of SEP event simulated using the two-component model for the case of a small source region. Upper panel: energy-time plot. Middle panel: the inverse velocity $1/v$ vs. the time after the release. Lower panel: the count rate as a function of time in 30 minute bins. For the two-component model, the simulated events does not show any clear SEP dropouts for small or large source region.
confined to their respective field lines. When the pitch-angle scattering is large, particles efficiently scatter off their original field lines, and observers cannot see the intermittent intensity dropouts.

We can quantitatively relate the mean-free path to the existence of intensity dropouts using an approach similar to that of Giacalone et al. (2000). Consider charged particles, initially confined within some flux tube, that gradually leak off of the tube as they undergo pitch-angle scattering, causing them to be displaced normal to the local magnetic field by about one gyroradius each time they scatter. As the field line from which they move initially along advects outward with the solar wind, an observer far from the source will see the particles decay from this field line due to their leakage off of it. The length scale of this decay is given by the following:

$$L_{\text{decay}} = \sqrt{2\kappa_{\text{local}}} T$$

where $\kappa_{\text{local}}$ is the local cross-field diffusion coefficient, and $T = r/V_w$ is the time takes the fluid element—to which the field is frozen—that was initially associated with the injection of particles at the Sun to reach the observer. Here, $r$ is the distance.
between the source and the observer, and $V_w$ is the speed of the solar wind. For intensity dropouts to form, $L_{\text{decay}}$ must be less than the correlation scale $L_e$ of magnetic turbulence in the solar wind. We assume that locally the cross-field diffusion coefficient is given by the following:

$$\kappa_{\perp,\text{local}} = \kappa_{\parallel}(r_g/\lambda_1)^2,$$

where $r_g = v/\Omega$ is the gyroradius of the energetic particle and $\Omega$ is its gyrofrequency, and $\lambda_1$ is the parallel mean-free path. This is the so-called hard-sphere scattering approximation for cross-field diffusion and applies locally. Since $\kappa_{\parallel} = \nu\lambda_1/3$, we find that if

$$\lambda_1 \gg \frac{2v^3r}{3L_e^2V_w\Omega^2},$$

then dropouts can occur. Using the parameters of our simulation, we find that for 1 MeV protons, dropouts can result for $\lambda_{1,1\text{ MeV}} \gg 0.01$ AU. We also note that Chollet & Giacalone (2011) directly measured $L_{\text{decay}}$ for some SEP events seen by ACE. They found $L_{\text{decay}} = 0.001$ AU for 0.3–5 MeV nucleon$^{-1}$ ions. Using the observed value of $L_{\text{decay}}$ in (7) and combining with (8), we solve for $\lambda_1$ and find $\lambda_1 = 1$ AU. This is consistent with our simulation results using the foot-point random motion model but is larger than our results using the two-component model. Thus, we suggest that there are no dropouts in the two-component model because the scattering mean-free path is too short and the dropouts, which may be present closer to the Sun, are filled in by 1 AU.

To further test the effect of pitch-angle scattering, we make one more numerical simulation using the two-component model. In this case, the smallest wavelength is taken to be $2 \times 10^{-3}$ AU, which is larger than the resonant scale of the protons of highest energy (10 MeV) in our simulation. Thus, we essentially remove small-scale resonant scattering, which have the effect of increasing the mean-free path. Figure 11 shows a scatter plot similar to Figure 8 and Figure 9. We find that, in this case, particles follow and are separated by the braiding magnetic field lines and that dropouts can form. This indicates that if pitch-angle scattering is infrequent, small-scale sharp gradients in SEP intensity flux can form. In Figure 12, we show a simulated event using the two-component model with a minimum wavelength of $2 \times 10^{-3}$ AU. The event shows dropouts similar to the case using the foot-point random motion model. Two main drops in flux can be readily seen.

4. DISCUSSION AND SUMMARY

In this paper, we studied small-scale gradients in the intensity of SEPs when energetic particles propagate in the heliospheric magnetic field. We numerically integrated the trajectories of energetic charged particles in the turbulent magnetic field generated from the commonly used magnetic turbulence models, i.e., the foot-point random motion model (Jokipii & Parker 1969; Giacalone et al. 2006) and the two-component model (Matthaeus et al. 1990). The turbulence models include a Kolmogorov-like magnetic field power spectrum containing a broad range of spatial scales from those that lead to large-scale field-line random walk to small scales leading to resonant pitch-angle scattering of energetic particles. The observations of SEP events were simulated by collecting charged particles that reach 1 AU much as a spacecraft detector would.

We have reproduced SEP dropouts in the numerical simulation using the foot-point random motion model, assuming that the radius of the SEP source region is smaller than the correlation scale of magnetic turbulence. The intervals of these dropouts are typically several hours, similar to the timescale of dropouts observed in space. The velocity dispersions of simulated SEP events appear to have distinct path lengths, which indicates that the energetic particles travel along field lines with different lengths. We have also attempted to use the two-component model to simulate dropouts in SEP intensity. However, we did not find any clear evidence of intensity dropouts in the simulation. This is probably because particle scattering is more efficient in the two-component model compared with that in the foot-point random motion model. We have demonstrated this by calculating diffusion coefficients in the simulations. This explanation is also supported by recent observational analysis by Chollet & Giacalone (2011). They inferred the intensity-fall-off lengths at the edges of the dropouts using ACE/ULEIS data and showed that energetic particles rarely scatter off a magnetic field line during the propagation in interplanetary turbulence. We showed that the parallel mean-free path inferred from this observation is consistent with that calculated using the foot-point random motion model, but about one order of magnitude larger than that in the two-component model.
Determining the value of large-scale diffusion coefficients of charged particles propagating in turbulent magnetic fields has been a long-standing issue for many decades. It is well-known that the mean-free paths inferred from SEP observations are usually much longer than those derived from quasi-linear theory (Palmer 1982; Bieber et al. 1994; He & Wan 2012). It has been proposed that the anisotropy of magnetic turbulence can strongly influence the diffusion coefficient parallel to the magnetic field (Bieber et al. 1996). However, it was later found that the diffusion coefficient has a weak dependence only on the anisotropy (Giacalone & Jokipii 1999; Qin et al. 2006).

Here, we showed that the small-scale gradients in the intensity of energetic particles during impulsive SEP events can provide a strong constraint on the mean-free path of charged particles. As energetic particles propagate from the source close to the Sun to 1 AU, pitch-angle scattering caused by small-scale magnetic fluctuations has to be infrequent so that energetic particles can be largely confined to their original field lines. Otherwise, charged particles can effectively move off field lines and dropouts cannot form.

The magnetic turbulence models we used have a fixed set of parameters like magnitude of magnetic field and turbulence variance. This is needed because in our model, magnetic field fluctuations are generated in a Cartesian geometry. The parameters can actually vary from close to the Sun to 1 AU. Although previous studies have shown this variation does not significantly change the scattering of particles (Kallenrode et al. 1992; Kallenrode 1993), this effect may need to be considered in the future.

We did not observe a significant effect of particle trapping on magnetic field lines with a restricted transverse displacement as proposed by Ruffolo et al. (2003) and Chuychai et al. (2007). Tooprakai et al. (2007) have studied numerically the trapping process for charged particles in magnetic fluctuations. The difference between our simulations and their simulations is that we directly used the two-component model as proposed...
by Matthaeus et al. (1990) and Bieber et al. (1996), whereas Tooprakai et al. (2007) used a combined field of an idealized two-dimensional flux-tube magnetic field and a one-dimensional fluctuating magnetic field.

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