A design method of model error compensator for systems with polytopic-type uncertainty and disturbances

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ABSTRACT
Control systems achieve the desired performance with the model-based controller if the dynamical model of the actual plant is given with sufficient accuracy. However, if there exists a difference between the actual plant and its model dynamics, the model-based controller does not work well and does not achieve the intended desired performance. A model error compensator (MEC) is proposed for overcoming the model error in our previous study. Attaching the compensator for the model error to the actual plant, the output trajectory of the actual plant is made close to that of its model. Then, from the controller, the apparent difference in the dynamics can be smaller, and performance degradation is drastically reduced. MEC is useful for various control systems such as non-linear systems and the control systems with delay, and so on. In this paper, we propose an original design method of the filter parameters in MEC for systems with polytopic-type uncertainty and disturbances. First, we show an analysis method about the robust performance of MEC for the system with the polytopic type uncertainty based on an linear matrix inequality problem. The gain parameters in MEC is designed using particle swarm optimization and the presented analysis method. The effectiveness of the design method for the system with polytopic-type uncertainty and disturbance is evaluated using numerical examples.

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1. Introduction
Many control systems are designed by model-based control. First, we obtain a dynamical model of an actual plant by using system identification or physical modelling. Then, we design a controller for the nominal model. Finally, we obtain the desired control performance by applying the controller to the actual plant. Control systems achieve the intended output response with the model-based controller if the dynamical model of the actual plant is given with sufficient accuracy. On the other hand, the controller designed for the nominal model does not work if there are differences between actual plant dynamics and its model dynamics, such as modelling error, ageing, and so on. In other words, the model-based control is weak against modelling errors and disturbances.

Robust control is a kind of control method that considers modelling errors and disturbances. However, it is difficult to come down to a mathematical problem from a complex design problem. Besides, the design results of robust control often become conservative performance because of robust control such as $H_{\infty}$ control designs for a set of models.

A model error compensator (MEC) is proposed for overcoming the model error in the previous study [1]. The conventional robust control attaches robustness to the system by designing a controller that works well for all models in a set of models. On the other hand, MEC attaches robustness by attaching a compensator for the model error to the actual plant. The MEC makes the output trajectory of the actual plant close to that of its nominal model. Then, from the controller, the apparent difference in the dynamics can be smaller, and performance degradation is reduced. As a result, it is expected that the system with MEC is more robust and can achieve better control performance where the controller is assumed to be designed by the existing design method. In other words, the systems with MEC can be together with various existing designed controllers.

In the previous studies [2–6], application examples about welfare vehicles with MEC are presented. MEC is applied in the closed-loop system and has high accuracy compared to the system without MEC.

It is well known that disturbance observer [7–9] is one of the useful methods to make a part of the system robust, and it is well used and effective. Its conceptual function is similar to MEC. If the given plant is a minimum phase and proper, we can make a disturbance observer with a modified inverse model, and the error of estimated disturbances approaches zero by designing the filter appropriately. However, the disturbance observer cannot consist appropriately for the
non-minimum phase systems because the inverse system of the plant is unstable. In such a case, it is difficult to achieve good robust performance by using the disturbance observer. Moreover, it is also difficult to make an inverse system for many kinds of non-linear systems. On the other hand, MEC can be applied for non-minimum phase systems [10] and non-linear systems [11] without a complicated extension of the system.

The MEC is unique in that for simple systems such as the single-input single-output (SISO) system, the effect of modelling errors and disturbances are reduced by setting the gain with a little trial and error. However, we have to design the gain appropriately for many types of systems such as the multi-input multi-output (MIMO) system, the non-minimum phase system, and so on. The design method in the previous studies [1,10] for these systems are mostly based on additive uncertainty, multiplicative uncertainty, and so on. However, the appropriate representation of uncertainty is different for each plant. Therefore, for the systems represented by various uncertainties, for example, the polytopic type uncertainty [12–15] in state-space representation treated in this paper, a different framework to design MEC from previous studies is required. In addition, the design methods proposed in previous studies are based on frequency domain such as \( \mu \) synthesis. However, when we evaluate the designed MEC in other evaluation indexes, such as the peak value of impulse response, the design method proposed in this paper is useful.

Therefore, in this paper, we propose a design method of parameters using particle swarm optimization (PSO) [16], which is one of the meta-heuristics methods. One of the advantages of meta-heuristics is that they can alter the evaluation function flexibly. This means that we can design the MEC with an arbitrary evaluation index by setting the evaluation function according to the purpose. In this paper, the analysis method about the robust performance of MEC based on linear matrix inequalities (LMIs), which is shown in [17] is used as an evaluation function.

This paper is organized as follows: in Section 2, the research outline of MEC is described. In Section 3, an analysis method about \( H_\infty \) performance of MEC based on LMIs, which is used as an evaluation function in PSO, is shown. In Section 4, we propose a design method combined with PSO and the analysis method. Finally, in Section 5, we offer a numerical example of the proposed method. In this paper, we assume that the actual plant is a polytopic-type uncertain continuous LTI system.

Note that \( H_\infty \) norm of system \( G \) is given by the following equation:

\[
\|G\|_\infty := \sup_{\omega \in \mathbb{R}} \sigma_{\text{max}}(G(j\omega)) \tag{1}
\]

where \( \sigma(\cdot) \) is the maximum singular value.

2. Model error compensator

In model-based control, we obtain the nominal model of an actual plant by system identification. Then, we design a controller for the nominal model, which achieves the desired control performance. However, if there is a modelling error, it is not able to achieve the desired control performance because it is designed for the nominal model. A MEC is proposed for overcoming the model error effect in previous studies [1,10]. MEC \( H \) is attached to plant \( P \) as shown in Figure 1.

By designing MEC \( H \) appropriately, the influence of modelling error and disturbance is reduced, and the dynamics of \( P' \) is close to that of the nominal model. Therefore, from the controller, the apparent dynamics are close to the nominal model and, thus, the controller can achieve control performance as we had designed. The controller is designed with various existing design methods. Hence, the system which had applied MEC is expected to achieve superior robustness and control performance.

Figure 2 shows the basic structure of MEC, where \( P, P_m, \) and \( D \) are the actual plant, nominal model, and differential compensator, respectively. MEC \( H \) (represented by dashed dotted line in Figure 2) includes nominal model \( P_m \) inside and feedbacks the output difference of the actual plant \( P \) and the nominal model \( P_m \). Here, we can make the dynamics of \( P' \) which is represented by the dashed line in Figure 2 close to \( P_m \) if appropriate differential compensator \( D \) is given.
Also, the MEC is unique in that it can be used in conjunction with conventional control systems. Figure 3 are some application examples of MEC. Figure 3(a) shows the feedback system with MEC, Figure 3(b) shows the state feedback system with MEC, and Figure 3(c) shows MPC with MEC. Like Figure 3, we can design the controller by various existing design methods. MEC manages the removal of the effects of modelling errors and disturbances, and the controller can be designed without considering them. This is effective when we compose a complex system, design a controller with MPC, and so on.

Now, we assume that the plant \( P \) is given as SISO and linear time invariant system. The transfer function from input \( u \) to output \( y \) is given by the following equation:

\[
P'(s) = \frac{1 + P_m(s)D(s)}{1 + P(s)D(s)} P(s)
\]  

(2)

where consider the case that there is no modelling error, that is, \( P(s) = P_m(s) \) holds. The dynamics of \( P'(s) \) is given as following equation:

\[
P'(s)|_{p(s)=P_m(s)} = \frac{1 + P_m(s)D(s)}{1 + P_m(s)D(s)} P_m(s) = P_m(s)
\]  

(3)

From Equation (3), it is clear that if \( P(s) = P_m(s) \) holds, differential compensator \( D \) does not affect the system.

If there is modelling error between \( P \) and \( P_m \), it is desirable that the dynamics of system \( P' \) include differential compensator \( D \) which is close to the nominal model \( P_m \). The difference of the dynamics between \( P' \) and \( P_m \) is given by the following equation:

\[
P'(s) - P_m(s) = \frac{1 + P_m(s)D(s)}{1 + P(s)D(s)} (P(s) - P_m(s)) = \frac{1}{1 + P(s)D(s)} (P(s) - P_m(s))
\]  

(4)

As shown in Equation (4), we can reduce the difference of the dynamics if differential compensator \( D \) is set to high gain. On the other hand, we have to design gain appropriately if the plant is MIMO, non-minimum phase, and so on. Most of the previous studies [1,10] proposed the design method of MEC based on frequency domain uncertainty, such as additive uncertainty, multiplicative uncertainty, and so on. Another framework is required if we design MEC for the systems with various uncertainty.

3. Analysis method about the robust performance of MEC

In this section, we explain the system representation of components of MEC and show the generalized plant to be analysed for robust performance. Then, we describe an analysis method of \( H_{\infty} \) performance of MEC. This analysis method is used for PSO as an evaluation function in this paper.

3.1. System representation of MEC

In this paper, we assume a continuous-time linear time invariant system. The equation of state, which represents the dynamics of the plant \( P \), is given by the following equation:

\[
\dot{x}(t) = Ax(t) + B\tilde{u}(t) + B_w w_d(t)
\]  

(5)

\[
y(t) = Cx(t) + D_w w_y(t)
\]  

(6)

where \( t \) is the current time, \( A \) is a square matrix which represents dynamics of plant, also \( B, B_w, C, \) and \( D_w \) are appropriate matrices which are given depending on the number of the input/output. \( A, B, \) and \( C \) are assumed to have polytopic-type uncertainties and these are given as follows using \( \lambda = [\lambda_i] \) and endpoint matrices \( A_i, B_i, \)\( \)
and $C_i$ with appropriate dimensions.

$$A = \sum_{i=1}^{N} \lambda_i A_i, \quad B = \sum_{i=1}^{N} \lambda_i B_i, \quad C = \sum_{i=1}^{N} \lambda_i C_i$$

(7)

where $\lambda$ is a time invariant parameter that belongs to the following set $\varepsilon$:

$$\varepsilon := \left\{ \lambda \in \mathbb{R}^N : \lambda_i \geq 0, \sum_{i=0}^{N} \lambda_i = 1 \right\}$$

(8)

The degree of the plant of Equation (5) is $m_x$, and $x(t) \in \mathbb{R}^{m_x}$, $u(t) \in \mathbb{R}^{m_u}$, $y(t) \in \mathbb{R}^{m_y}$, $w_u(t) \in \mathbb{R}^{m_u}$ and $w_y(t) \in \mathbb{R}^{m_y}$ are the state, control input, plant output, disturbance input and observation noise, respectively. In this paper, it is assumed that the plant is controllable and observable about arbitrary $\lambda \in \varepsilon$.

Also, dynamics of the nominal model $P_m$ which uses for MEC is given as continuous-time linear time invariant system by the following equation:

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t)$$

(9)

$$y_m(t) = C_m x_m(t)$$

(10)

where $A_m$ is a square matrix which represents dynamics of the plant, also $B_m$ and $C_m$ are appropriate matrices which are given depending on the number of the input/output. $x_m(t) \in \mathbb{R}^{m_x}$, $u_m(t) \in \mathbb{R}^{m_u}$ and $y_m(t) \in \mathbb{R}^{m_y}$ are the state of the nominal model, control input, and nominal model output, respectively.

Now, we consider $\Delta A = A - A_m$, $\Delta B = B - B_m$, and $\Delta C = C - C_m$ that are the errors between the model and the actual plant. $\Delta A$, $\Delta B$, and $\Delta C$ are represented as follows using Equations (7):

$$\Delta A = \sum_{i=1}^{N} \lambda_i \Delta A_i, \quad \Delta B = \sum_{i=1}^{N} \lambda_i \Delta B_i$$

$$\Delta C = \sum_{i=1}^{N} \lambda_i \Delta C_i$$

where, $\Delta A_i = A_i - A_m$, $\Delta B_i = B_i - B_m$ and $\Delta C_i = C_i - C_m$.

When we consider that MEC is applied to the above plant $P$ and the nominal model $P_m$, the differential compensator $D$ is given by the following equations:

$$\dot{x}_d(t) = A_d x_d(t) + B_d(y(t) - y_m(t))$$

(11)

$$y_d(t) = C_d x_d(t) + D_d(y(t) - y_m(t))$$

(12)

When we apply MEC to the plant, the input to the nominal model is given as $u_m(t) = u(t)$, and the input to the actual plant is given as $\tilde{u}(t) = u(t) - y_d(t)$, where $u(t)$ is the output of the controller $C$, and $y_d(t)$ is the compensation input.

### 3.2. Equation of state and evaluation output of the generalized plant

This section derives a generalized plant for the plant and the nominal model described in previous section. Figure 4 shows the generalized plant including MEC.

By defining $e(t) = x(t) - x_m(t)$, $\xi(t) = [e(t)^T, x_d(t)^T, x_m(t)^T]^T$ as the state and $v(t) = [w_u(t)^T, w_y(t)^T, u(t)^T]^T$ as the input, we obtain the following equations:

$$\dot{\xi}(t) = \tilde{A}\xi(t) + \tilde{B}v(t)$$

(13)

$$\tilde{A} = \begin{bmatrix} A - BD_d C & -BC_d & \Delta A - BD_d \Delta C \\ B_dC & A_d & B_d \Delta C \\ 0 & 0 & A_m \end{bmatrix}$$

(14)

$$\tilde{B} = \begin{bmatrix} B_w & -BD_d D_w & \Delta B \\ 0 & B_d D_w & 0 \\ 0 & 0 & B_m \end{bmatrix}$$

(15)

where the objective of MEC is making smaller the gap between $y$ and $y_m$, hence, we consider following the evaluation output:

$$e_y(t) = Cx(t) - C_m x_m(t) = \tilde{E} \xi(t)$$

(16)

where $\tilde{E} = [C, 0, \Delta C]$. Note that evaluation output $e_y(t)$ is not $y(t) - y_m(t)$ but excluded the observation noise from $y(t) - y_m(t)$.

Besides, to analyse based on the polytopic-type uncertainty, we define the following matrices:

$$\tilde{A}_i = \begin{bmatrix} A_i - B_i D_d C_i & -B_i C_i & \Delta A_i - B_i D_d \Delta C_i \\ B_d C_i & A_d & B_d \Delta C_i \\ 0 & 0 & A_m \end{bmatrix}$$

(17)

$$\tilde{B}_i = \begin{bmatrix} B_w & -B_i D_d D_w & \Delta B_i \\ 0 & B_d D_w & 0 \\ 0 & 0 & B_m \end{bmatrix}$$

$$\tilde{E}_i = [C_i, 0, \Delta C_i], \quad (i = 1, \ldots, N)$$

where 1,1 element of $\tilde{A}_i$ includes the bilinear term of $B_i$ and $C_i$, and 1,3 element of $\tilde{A}_i$ includes the bilinear
term of $B_t$ and $\Delta C_i$. Therefore, it cannot be a matrix polytope, but if $\Delta B = 0$, $\Delta C = 0$ or $D_d = 0$ is satisfied, then we can obtain the following equation which represents the dynamics of the generalized plant as the matrix polytope using $A_i$, $B_i$, $E_i$ and $\lambda \in \varepsilon$:

$$
\bar{A} = \sum_{i=1}^{N} \lambda_i A_i, \bar{B} = \sum_{i=1}^{N} \lambda_i B_i, \bar{C} = \sum_{i=1}^{N} \lambda_i C_i
$$

The output of systems often represents the state as it is, and these systems satisfy $\Delta C = 0$. Also, $D_d = 0$, which means there is no direct term in the differential compensator, and is often used in many previous control systems design theories. Therefore, satisfying the assumption is not difficult.

Also, the input–output system from $v(t)$ to $e_y(t)$ is expressed as $G_e$. system $G_e$ is affected by not only the disturbance input and the observation noise but also input $u$.

### 3.3. Analysis about $H_\infty$ performance using LMI's

This section describes the analysis method about $H_\infty$ performance about system $G_e$ obtained in the previous section. In this section, we assume that $\Delta B = 0$, $\Delta C = 0$, or $D_d = 0$ holds about system $G_e$ given by Equation (13). As described before, under this assumption, the input and output of the system $G_e$ can be represented by polytopic matrices, and we can easily analyse the system.

First, for the endpoint matrices $(A_i, B_i, E_i)$, the analysis problem about $H_\infty$ performance is obtained as the following LMI's using given $\gamma_\infty > 0$:

$$
X > 0, \begin{bmatrix}
A_i X + X A_i^T & X E_i^T & B_i \\
E_i^T X & -\gamma_\infty^2 I & 0 \\
B_i^T & 0 & -I
\end{bmatrix} < 0
$$

where let LMI constraints of Equations (18) be denoted as $\Psi_t < 0$, and if we find $X > 0$ which satisfies $\Psi_t < 0$ for all $i$, note that $\lambda_i \geq 0$, then the following inequality holds:

$$
\left( \sum_{i=1}^{N} \lambda_i \Psi_t \right) < 0
$$

From $\sum_{i=1}^{N} \lambda_i = 1$, Equations (17), (18) and (19), following LMIs hold:

$$
X > 0, \begin{bmatrix}
A X + X A^T & X E^T & B \\
E^T X & -\gamma_\infty^2 I & 0 \\
B^T & 0 & -I
\end{bmatrix} < 0
$$

If $X$ and $\gamma_\infty$, which satisfy Equation (20) are found, for system $G_e$ the following inequality holds:

$$
\| G_e \|_\infty \leq \gamma_\infty
$$

Therefore, by finding the minimum $\gamma_\infty$ satisfying Equation (20), we can analyse system $G_e$ about $H_\infty$ performance. In other words, the analysis problem about $H_\infty$ is coming down to the problem to find a minimum $\gamma_\infty$ which satisfy Equation (20). This problem can be solved easily with a numerical calculations software such as MATLAB.

$H_\infty$ norm is equal to $L_2$ induced norm, so the following equation is established:

$$
\| G_e \|_\infty = \sup_{v \in L_2, \| v \|_2 \neq 0} \frac{\| e_y \|_2}{\| v \|_2}
$$

In this way, we give the analysis method of MEC about $H_\infty$ performance.

In this paper, we described the analysis method about only $H_\infty$ performance. However, also, LMI constraints of $H_2$ performance, the peak value of impulse response, and pole placement are described in [18]. Therefore, we can obtain these evaluation indexes by setting these constraints and design MEC using these indexes.

### 4. Design method of MEC using PSO

As we described in the previous section, if the parameters of differential compensator $D$ are given, we can analyse the $H_\infty$ performance of generalized plants $G_e$. Hence, we provide the parameters with meta-heuristics such as particle swarm optimization (PSO), and the results of the analysis are used for evaluation of the value in the proposed method. This section describes the design method of MEC combining PSO with the analysis method about the $H_\infty$ performance of MEC.

PSO is a multi-point search algorithm that simulates search behaviour such as fishes and birds. It is well known that PSO is useful for control system design. PSO has the following features: the concept is easy to understand, few parameters to set by the user, and suitable for searching real number variable, which has a continuous value.

Also, meta-heuristics can be changed for the evaluation function flexibly. That is, meta-heuristics can evaluate various indexes; thus, we can design according to the purpose.

Figure 5 shows the flowchart of a procedure to obtain parameters of the differential compensator $D$. Where the evaluation value is set to $\gamma_\infty$ obtained by the analysis method described in the previous section, which is $H_\infty$ performance. Note that as described before, we can set the evaluation value as not only $H_\infty$ norm but also $H_2$ norm, the peak value of impulse response, and so on. The position and velocity of the $i$th particle ($i = 1 \ldots m$) are denoted by $z_i(k) = (z_{i1}(k), \ldots, z_{in}(k))$ and $v_i(k) = (v_{i1}(k), \ldots, v_{in}(k))$, respectively. Where $n$ is the number of parameters to be designed, $m$ is the number of particles, and $k$ is the update count. The position means a set of the design parameters in the differential compensator $D$, and we obtain the evaluation value by analysing MEC using the parameters. The best solution found by whole particles, which is the position
that the best evaluation value is achieved, is denoted by \( g = (g_1, g_2, \ldots, g_n) \). Also, the best solution found by the \( i \)th particle, which means the position that the best evaluation value is obtained by the \( i \)th particle, is denoted by \( p^i = (p^i_1, p^i_2, \ldots, p^i_n) \). The maximum update count is set as \( k_{\text{max}} \). Details of each step are described below.

1. This process initializes variables: Set update count \( k \) to 0. Also set the position \( z^i_i(0) = (z^i_{x1}(0), \ldots, z^i_{xn}(0)) \) and velocity \( z^i_v(0) = (z^i_{v1}(0), \ldots, z^i_{vn}(0)) \) of the \( i \)th particle to a random value. Then, we solve LMIs given by Equation (20) and obtain an evaluation value. Sometimes the evaluation value cannot obtain because the LMIs have no solution depending on the initialized value. In that case, set \( z^i_x \) and \( z^i_v \) to a random value again until the LMIs are solvable. \( p^i \) is set to the \( i \)th particle position and \( g \) is set to \( p^i \) so that the best evaluation value is achieved from whole particles.

2. Evaluate each particle using an analysis method about \( H_{\infty} \) performance of MEC described in the previous section. If the LMIs have no solution, it means that the generalized plant is unstable, therefore the evaluation value of the particle sets to a large value as a penalty.

3. Determine completion: If update count \( k \) does not reach maximum update count \( k_{\text{max}} \), do \( k = k + 1 \) then proceed to parameters’ update. If update count \( k \) reaches a maximum update count \( k_{\text{max}} \), output \( g \) as the optimal parameters of differential compensator \( D \), then end the search.

4. Update position and velocity parameters with the following equation:

\[
z^i_x(k + 1) = z^i_x(k) + z^i_v(k)
\]

where \( \rho \) is a weighting factor for the velocity vector before the update. \( c_1 \) and \( c_2 \) are the weighting factors for each term. \( r_{1,i} \) and \( r_{2,i} \) are the random numbers from 0 to 1 and they are generated for each particle and update count.

It is not so difficult to obtain the initialized values in (1) when the number of the design parameters is not so large. To decrease the number of the parameters, it is useful to design the differential compensator \( D \) as a controllable canonical form described later in Section 5.1. Also, the differential compensator \( D \) can be designed as a PID compensator, and like these, we can decrease the number of parameters to design. In the process of PSO, sometimes a part of the solutions becomes infeasible when the position \( z^i_x(k) \) is updated. At that time, the evaluation value of the infeasible solution is set to a very large value as a penalty. Therefore, infeasible solutions do not affect other candidates of solutions.

5. Simulation

This section shows a numerical example of the design of MEC for the MIMO plant.

5.1. Conditions

The dynamics of actual plant represented by Equations (5) and (6) is given by the following matrices as \( N = 4 \):

\[
A_1 = \begin{bmatrix} 0 & -1 & 1 \\ 0 & -1.6 & 2 \\ 0 & 1 & -4.8 \end{bmatrix},
A_2 = \begin{bmatrix} -0.4 & -1 & 1 \\ 0 & -2.4 & 2 \\ 0 & 1 & -5.2 \end{bmatrix},
A_3 = \begin{bmatrix} -0.2 & -1.2 & 1 \\ 0 & -2 & 2.2 \\ 0.2 & 1 & -5 \end{bmatrix},
A_4 = \begin{bmatrix} -0.2 & -0.8 & 1 \\ 0 & -2 & 1.8 \\ -0.2 & 1 & -5 \end{bmatrix},
B_1 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 2.6 & 5 \end{bmatrix},
B_2 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 1.4 & 5 \end{bmatrix},
B_3 = \begin{bmatrix} 1.2 & 0 \\ -0.8 & 1 \\ 2 & 5 \end{bmatrix},
B_4 = \begin{bmatrix} 0.8 & 0 \\ -1.2 & 1 \\ 2 & 5 \end{bmatrix}
\]

C_1 = C_2 = C_3 = C_4 = \begin{bmatrix} 1 & 0.5 & 0 \\ 1 & 1 & 1 \end{bmatrix}

Figure 5. Flowchart of MEC design using PSO.

\[
z^i_x(k + 1) = \rho z^i_x(k) + r_{1,i} c_1 (p^i - z^i_x(k)) + r_{2,i} c_2 (g - z^i_x(k))
\]
The nominal model matrices $A_m$, $B_m$ and $C_m$ are given as follows:

$$A_m = \begin{bmatrix} -0.2 & -1 & 1 \\ 0 & -2 & 2 \\ 0 & 1 & -5 \end{bmatrix}, \quad B_m = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$C_m = \begin{bmatrix} 1 & 0.5 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

where, the nominal model is the centre of endpoint matrices of the plant, which means $\lambda_i = 0.25$ ($i = 1, \ldots, 4$).

Also, differential compensator $D$ is designed with the controllable canonical form like the following matrices:

$$A_d = \begin{bmatrix} 0 & 1 & 0 \\ z_x & z_x & z_x \\ z_x & z_x & z_x \end{bmatrix}, \quad B_d = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$C_d = \begin{bmatrix} z_x & z_x & z_x & z_x \\ z_x & z_x & z_x & z_x \\ z_x & z_x & z_x & z_x \\ z_x & z_x & z_x & z_x \end{bmatrix}, \quad D_d = \begin{bmatrix} z_x \\ z_x \\ z_x \end{bmatrix}$$

By designing with a controllable canonical form, the number of variables to design decreases, therefore the number of dimensions to search decreases, and an efficient search is expected.

Maximum update count $k_{\text{max}}$ is set to 100 and the number of particles is set to 50. The weighting factor $\rho, c_1, c_2$ are set to 0.8, 1, and 1, respectively.

The initial values of the position and the velocity are randomly selected in the ranges we set. These are set to $[-10,000, 10,000]$ and $[-10, 10]$, respectively. Considering that MEC reduces the influences of disturbances and modelling errors by high gain feedback, the range of the initial value of the position is set to large compared with the velocity.

### 5.2. Design results of MEC

The following is the design results of differential compensator $D$ for the plant described in the previous section:

$$A_d = \begin{bmatrix} 0 & 1 & 0 \\ -6876.04 & -2314.40 & 5421.53 \\ 9503.05 & -1406.48 & -8845.41 \end{bmatrix}$$

$$B_d = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_d = \begin{bmatrix} 237.54 & -7261.59 & 4367.41 \\ -6771.48 & -7585.91 & -4548.58 \end{bmatrix}$$

$$D_d = \begin{bmatrix} 3536.44 & -342.39 \\ -1868.22 & 7496.95 \end{bmatrix}$$

where we apply the above differential compensator $D$ to plant and analyse the generalized plant $G_e$. Then we obtain $\gamma_\infty = 0.0167$, therefore the following formula holds:

$$\|G_e\|_\infty \leq 0.0167$$

Note that the design result may have large negative poles or have high gain. However, there is no problem for practical usage. If these have to be small due to some constraints, we can impose these easily, and this is one of the advantages of using the meta-heuristics design method.

### 5.3. Verify the design results

We compose MEC shown in Figure 2 using differential compensator $D$ designed in the previous section to verify the effectiveness of the design method. The actual plants are given randomly by selecting multiple points in the polytope, and simulations are performed for these plants.

The control input $u$ is given as 0.2 from $t = 0$ to 20. Disturbance input and observation noise are given as random noise, which inputs from $t = 0$ to 20 and follow the normal distribution with average set to 0 and standard deviation set to 1. Figure 6 shows the response of the systems with MEC, each plant only, and ideal. As shown in Figure 6, for the plants in the polytope, designed MEC reduces the influence of the random noise. Also, Figure 7 shows the evaluation outputs of the plant in the polytope with the noise. As shown in Figure 7, designed MEC improves the evaluation output compared to the system without MEC. The ratio of $L_2$ norm from the random noise to the evaluation output is obtained as $\|e_y\|_2/\|v\|_2 = 0.0070$ and it satisfies the analysis result Equation (30).
In addition, the control input $u$ is given as 0.2 from $t = 0$ to 20. Disturbance input and observation noise are given as 0.2 from $t = 10$ to 20. Figure 8 shows the response of the systems with MEC, each plant only, and ideal. From Figure 8, for the plants in the polytope, designed MEC reduces the influence of the step type disturbance. Also, Figure 9 shows the evaluation outputs of a plant in the polytope to the step type disturbance and the designed MEC improves the evaluation output. The ratio of $L_2$ norm of the evaluation output to the step type disturbance is obtained as $\frac{\|e_y\|_2}{\|v\|_2} = 0.0049$ and this satisfies the analysis result Equation (30).

It is shown that MEC designed by the proposed method works well and reduces the influence of the disturbances and modelling errors.

### 6. Conclusions

In this paper, we proposed a design method of MEC combining particle swarm optimization with an analysis method about $H_\infty$ performance of MEC. First, a system representation of components of MEC is given, and generalized plants $G_e$ are derived. Then, we described the analysis method about $H_\infty$ performance of generalized plants based on LMIs. The analysis method with LMIs is used as an evaluation scheme in the proposed method. By altering the evaluation function, we can also design MEC according to the purpose. For example, analysis conditions of $H_2$ performance, the peak value of impulse response, and pole placement are derived as LMIs [18]; thus, we can set the evaluation function to the evaluation index. Finally, a MEC design example is shown in numerical simulation. We compose MEC using the design result and show response waveform to the random and step disturbances. It shows that the designed MEC can reduce the influence of them. Also, we confirmed that the system with MEC satisfies the analysis result.

Future work will apply the proposed method to the non-minimum phase system and unstable system. In previous studies, the compensation structure of MEC was suggested, which uses a parallel feed-forward compensator for the non-minimum phase system. For unstable systems, we can design MEC by designing a controller, which makes the plant stable in advance. Thus, it is expected that the proposed method in this paper is applied to the non-minimum phase system and unstable system by deriving the generalized plant, including a parallel feed-forward compensator and the MEC, respectively.

### Disclosure statement

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