Preparation of $n$-qubit Greenberger-Horne-Zeilinger entangled states in cavity QED: An approach with tolerance to nonidentical qubit-cavity coupling constants

Chui-Ping Yang$^{1,2}$

$^{1}$Department of Physics, Hangzhou Normal University, Hangzhou, Zhejiang 310036, China and
$^{2}$State Key Laboratory of Precision Spectroscopy, Department of Physics, East China Normal University, Shanghai 200062, China

(Dated: January 20, 2013)

We propose a way for generating $n$-qubit Greenberger-Horne-Zeilinger (GHZ) entangled states with a three-level qubit system and $(n-1)$ four-level qubit systems in a cavity. This proposal does not require identical qubit-cavity coupling constants, and thus is tolerant to qubit-system parameter nonuniformity and nonexact placement of qubits in a cavity. The proposal does not require adjustment of the qubit-system level spacings during the entire operation. Moreover, it is shown that entanglement can be deterministically generated using this method and the operation time is independent of the number of qubits. The present proposal is quite general, which can be applied to physical systems such as various types of superconducting devices coupled to a resonator or atoms trapped in a cavity.

PACS numbers: 03.67.Lx, 42.50.Dv

I. INTRODUCTION

Quantum entanglement plays a crucial role in quantum information processing. Entanglement of ten photons [1], eight ions [2], three spins [3], two atoms in microwave cavity QED [4], or two excitons in a single quantum dot [5] has been demonstrated in experiments. In addition, experimental preparation of three-qubit entanglement with superconducting qubits or a superconducting qubit coupled to two microscopic two-level systems has been reported recently [6-8]. Although multi-particle entanglement was experimentally created in photons and trapped ions, it is still greatly challenging to create multi-qubit entanglement in other important physical systems.

As is well known, multi-qubit GHZ (Greenberger-Horne-Zeilinger) entangled states are of great interest in the foundations of quantum mechanics and measurement theory, and significant in quantum information processing [9], quantum communication [10-12], error correction protocols [13], and high-precision spectroscopy [14]. Over the past ten years, based on cavity QED technique, many different theoretical methods for creating multi-qubit GHZ entangled states with atoms and superconducting qubits have been presented [15-25]. For instances, (i) the proposals in [15-17] for implementing a GHZ entangled state are based on identical qubit-cavity coupling constants; (ii) the approaches in [18-20] are aimed at probabilistic generation of a GHZ state; (iii) the method presented in [17] is based on the use of an auxiliary qubit, measurement on the qubit states, and adjustment of the qubit level spacings during the entire operation; (iv) the proposals in [21,22] are based on photon detection outside the cavity; and (v) the proposals in [23-25] are based on sending qubits (i.e., atoms) through a cavity. These proposals are important because they opened new avenues for creating multi-particle entanglement.

In this paper, we will focus on a situation that the qubit-cavity coupling constants are nonidentical. This situation often exists in superconducting qubits coupled to a cavity or a resonator. For solid-state devices, the device parameter nonuniformity is often a problem, which results in nonidentical qubit-cavity coupling constants though qubits are placed at locations of a cavity where the magnetic fields or the electric fields of the cavity mode are the same. In addition, the nonidentical qubit-cavity coupling constants may also result from nonexact placement of qubits in a cavity. In the following, our goal is wish to present a way for deterministic preparation of a $n$-qubit GHZ state with tolerance to the qubit-system parameter nonuniformity and nonexact placement of qubits in a cavity. As shown below, this proposal also has these advantages: (i) The operation time is independent of the number of qubits in the cavity; (ii) When compared with the approach in [17], no auxiliary qubits, no measurement on the qubit states, and no adjustment of the qubit level spacings during the entire operation is needed (note that adjustment of the level spacings of the qubits during the operation is not desired in experiments and may cause extra errors); (iii) No adjustment of the cavity mode frequency is required during the entire operation; and (iv) There is no need of photon detection. This proposal is quite general, which can be applied to various types of superconducting qubits and atoms trapped in a cavity.

This paper is organized as follows. In Sec. II, we briefly review the basic theory of a three-level quantum system or four-level quantum systems coupled to a single-mode cavity and/or driven by classical pulses. In Sec. III, we show how to generate an $n$-qubit GHZ state with one three-level quantum system and $(n-1)$ four-level quantum systems in a cavity. In Sec. IV, we compare our proposal with the previous ones. In Sec. V, we give a brief discussion of the
FIG. 1: (Color online) (a) and (c) System-pulse resonant interaction for qubit system 1. In (a), the pulse is resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition; while in (c) the pulse is resonant with the $|0\rangle \leftrightarrow |1\rangle$ transition. (b) System-cavity resonant interaction for qubit system 1. The pulse is resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition. (d) System-cavity-pulse off-resonant Raman coupling for $n-1$ qubit systems $(2, 3, ..., n)$ in a cavity. For simplicity, we here only draw a figure for qubit system $j$ interacting with the cavity mode and a classical pulse $(j = 2, 3, ..., n)$. \( \delta_j = \Delta_{c,j} - \Delta_j \) is the detuning of the cavity mode with the pulse, \( \Delta_j = \omega_{31}^j - \omega_j \) is the detuning between the pulse frequency \( \omega_j \) and the $|1\rangle \leftrightarrow |3\rangle$ transition frequency \( \omega_{31}^j \) of qubit system $j$. Note that $g_j$, $\delta_j$, $\Delta_{c,j}$, and $\Delta_j$ may be different for qubit systems $(2, 3, ..., n)$ due to nonidentical level spacings of the qubit systems which are caused by the nonuniformity of the qubit system parameters. The coupling constant $g_j$ may also vary with qubits due to nonexact placement of qubits in a cavity. The Rabi frequency of the pulse applied to qubit system $j$ is denoted by $\Omega_j$.

experimental issues and possible experimental implementation with superconducting qubits coupled to a resonator. A concluding summary is given in Sec. VI.

II. BASIC THEORY

In this section, we will introduce three types of interaction of qubit systems with the cavity mode and/or the pulse. The results presented below will be employed for generation of a multi-qubit GHZ state discussed in next section.

A. System-pulse resonant interaction

Consider a three-level qubit system (say, qubit system 1) driven by a classical pulse. Suppose that the pulse is resonant with the transition $|1\rangle \leftrightarrow |2\rangle$ of the qubit system 1 but decoupled from the transition between any two other
levels [Fig. 1(a)]. The interaction Hamiltonian in the interaction picture is given by

$$H_I = \hbar \left( \Omega_r e^{i\phi} |1\rangle \langle 2| + \text{H.c.} \right),$$

where $\Omega_r$ and $\phi$ are the Rabi frequency and the initial phase of the pulse, respectively. Based on the Hamiltonian (1), it is straightforward to show that a pulse of duration $t$ results in the following rotation

$$
\begin{align*}
|1\rangle & \rightarrow \cos \Omega_r t |1\rangle - ie^{-i\phi} \sin \Omega_r t |2\rangle, \\
|2\rangle & \rightarrow -ie^{i\phi} \sin \Omega_r t |1\rangle + \cos \Omega_r t |2\rangle.
\end{align*}
$$

Note that the resonant interaction can be done within a very short time by increasing the pulse Rabi frequency $\Omega_r$ (i.e., via increasing the pulse intensity).

In the following, we also need the resonant interaction of the pulse with the $|0\rangle \leftrightarrow |1\rangle$ transition of the qubit system 1 [Fig. 1(c)]. In this case, we have

$$
\begin{align*}
|0\rangle & \rightarrow \cos \tilde{\Omega}_r t |0\rangle - ie^{-i\phi} \sin \tilde{\Omega}_r t |1\rangle, \\
|1\rangle & \rightarrow -ie^{i\phi} \sin \tilde{\Omega}_r t |0\rangle + \cos \tilde{\Omega}_r t |1\rangle,
\end{align*}
$$

where $\tilde{\Omega}_r$ is the Rabi frequency of the pulse.

**B. System-cavity resonant interaction**

Consider qubit system 1 coupled to a single-mode cavity field. Suppose that the cavity mode is resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition while decoupled (highly detuned) from the $|0\rangle \leftrightarrow |2\rangle$ transition and the $|0\rangle \leftrightarrow |1\rangle$ transition of the qubit system 1 [Fig. 1(b)]. The interaction Hamiltonian in the interaction picture, after the rotating-wave approximation, is described by

$$H_I = \hbar (ga^+ |1\rangle \langle 2| + \text{h.c.}),$$

where $a^+$ and $a$ are the creation and annihilation operators of the cavity mode, and $g$ is the coupling constant between the cavity mode and the $|1\rangle \leftrightarrow |2\rangle$ transition of the qubit system. Under the Hamiltonian (4), the time evolution of the states $|2\rangle |0\rangle_c$ and $|1\rangle |1\rangle_c$ of the whole system are as follows

$$
\begin{align*}
|2\rangle |0\rangle_c & \rightarrow \cos gt |2\rangle |0\rangle_c - i \sin gt |1\rangle |1\rangle_c, \\
|1\rangle |1\rangle_c & \rightarrow -i \sin gt |2\rangle |0\rangle_c + \cos gt |1\rangle |1\rangle_c,
\end{align*}
$$

where the states $|0\rangle_c$ and $|1\rangle_c$ are the cavity-mode vacuum state and single-photon state, respectively.

**C. System-cavity-pulse off-resonant Raman coupling**

Consider $(n-1)$ four-level qubit systems $(2, 3, ..., n)$. The cavity mode is coupled to the $|1\rangle \leftrightarrow |3\rangle$ transition of each qubit system, but decoupled (highly detuned) from the transition between any other two levels [Fig. 1(d)]. In addition, a classical pulse is applied to each one of qubit systems $(2, 3, ..., n)$, which is coupled to the $|2\rangle \leftrightarrow |3\rangle$ transition but decoupled from the transition between any other two levels [Fig. 1(d)]. In the interaction picture, the Hamiltonian for the whole system is

$$H = \sum_{j=2}^{n} \left[ \hbar g_j (e^{-i\Delta_{c,j} t} a^+ \sigma^-_{13,j} + \text{H.c.}) + \hbar \Omega_j (e^{-i\Delta_{j} t} \sigma^-_{23,j} + \text{H.c.}) \right],$$

where the subscript $j$ represents the $j$th qubit system, $\Omega_j$ is the Rabi frequency of the pulse applied to the $j$th qubit system, $\sigma^-_{13,j} = |1\rangle_j \langle 3|$, $\sigma^-_{23,j} = |2\rangle_j \langle 3|$, and $g_j$ is the coupling constant between the cavity mode and the $|1\rangle \leftrightarrow |3\rangle$ transition of the $j$th qubit system.

The detuning between the $|2\rangle \leftrightarrow |3\rangle$ transition frequency $\omega_{32}'$ of the $j$th qubit system and the frequency $\omega_j$ of the pulse applied to the $j$th qubit system is $\Delta_j = \omega_{32}' - \omega_j$ ($j = 2, 3, ..., n$) [Fig. 1(d)]. In addition, the detuning between the $|1\rangle \leftrightarrow |3\rangle$ transition frequency $\omega_{13}'$ of the $j$th qubit system and the cavity-mode frequency $\omega_c$ is $\Delta_{c,j} = \omega_{13}' - \omega_c$ [Fig. 1(d)]. Note that the detuning $\Delta_{c,j}$ is not the same for each qubit system (i.e., dependent of $j$) in the case when the level spacings are nonidentical for each qubit system. Under the condition $\Delta_{c,j} \gg g_j$ and $\Delta_j \gg \Omega_j$, the level $|3\rangle$ can be adiabatically eliminated [26] and the effective Hamiltonian is thus given by [27-29].
\[ H_{\text{eff}} = -\hbar \sum_{j=2}^{n+1} \left[ \Omega_j^2 |2\rangle_j \langle 2| + \frac{g_j^2}{\Delta_{c,j}} a^+ a |1\rangle_j \langle 1| \right. \\
\left. + \chi_j (e^{-i \delta t} a^+ \sigma_{12,j}^- + \text{H.c.}) \right], \]

where \( \sigma_{12,j}^- = |1\rangle_j \langle 2| \), \( \chi_j = \frac{\Omega_j g_j}{2} (1/\Delta_j + 1/\Delta_{c,j}) \), and

\[ \delta_j = \Delta_{c,j} - \Delta_j = \omega_{31}^j - \omega_{42}^j - \omega_c + \omega_j. \] (8)

From Eq. (8), it can be seen that the detuning \( \delta_j \) is adjustable by changing the pulse frequency \( \omega_j \). With suitable choice of the pulse frequencies, we can satisfy

\[ \delta_2 = \delta_3 = \ldots = \delta_n = \delta, \]

which will apply below.

For \( \delta \gg g_j^2/\Delta_{c,j}, \Omega_j^2/\Delta_j, \chi_j \), there is no energy exchange between the qubit systems and the cavity mode. Thus, under the condition (9), the effective Hamiltonian (7) can be written as [25,30,31]

\[ H_{\text{eff}} = -\hbar \sum_{j=2}^{n} \left[ \Omega_j^2 |2\rangle_j \langle 2| + \frac{g_j^2}{\Delta_{c,j}} a^+ a |1\rangle_j \langle 1| \right] \\
- \hbar \sum_{j=2}^{n} \left[ \frac{\chi_j^2}{\delta} \left( a^+ a |1\rangle_j \langle 1| - aa^+ |2\rangle_j \langle 2| \right) \right] \\
+ \hbar \sum_{j \neq j'}^{n} \frac{\chi_j \chi_j'}{\delta} \left( \sigma_{12,j}^- \sigma_{12,j'}^- + \sigma_{12,j}^+ \sigma_{12,j'}^+ \right), \]

where the two terms in the second line above describe the photon-number dependent Stark shifts induced by the off-resonant Raman coupling, and the two terms in the last parentheses describe the “dipole” coupling between the two qubit systems \( (j, j') \) mediated by the cavity mode and the classical pulses. In the case when the level \( |2\rangle \) of each qubit system is not populated, the Hamiltonian (10) reduces to

\[ H_{\text{eff}} = -\hbar \sum_{j=2}^{n} \left( \frac{g_j^2}{\Delta_{c,j}} + \frac{\chi_j^2}{\delta} \right) a^+ a |1\rangle_j \langle 1|. \]

It is easy to see that the states \(|0\rangle_j \langle 0|_c, |1\rangle_j \langle 1|_c \) and \(|0\rangle_j \langle 1|_c \) remain unchanged under the Hamiltonian (11). However, if the cavity mode is initially in the photon state \(|1\rangle_c \), the time evolution of the state \(|1\rangle_j \) of the \( j \)th qubit system under the Hamiltonian (11) is given by

\[ |1\rangle_j \langle 1|_c \rightarrow e^{i \lambda_j t} |1\rangle_j \langle 1|_c, \]

where \( \lambda_j = \frac{g_j^2}{\Delta_{c,j}} + \frac{\chi_j^2}{\delta} \), which can be further written as

\[ \lambda_j = \frac{g_j^2}{\Delta_{c,j}} + \frac{\Omega_j^2 g_j^2}{4 \delta} \left( \frac{1}{\Delta_j} + \frac{1}{\Delta_{c,j}} \right)^2. \]

Note that the parameters \( g_j \) is not adjustable once the qubit systems (e.g., solid-state devices) are designed and built in a cavity, and the detuning \( \Delta_{c,j} = \omega_{31}^j - \omega_c \) is fixed when the cavity mode frequency is chosen. As can be seen in Eq. (13), the parameter \( \lambda_j \) here is adjustable by changing the pulse Rabi frequency \( \Omega_j \).

**III. GENERATION OF MULTI-QUBIT GHZ STATES**

Let us consider \( n \) qubit systems \( (1, 2, \ldots, n) \) in a single-mode cavity. The qubit system 1 has three levels shown in Fig. 1(a,b,c) while the four levels of the qubit systems \( (2, 3, \ldots, n) \) are depicted in Fig. 1(d). For qubit system 1,
the cavity mode is resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition but highly detuned from the transition between any two other levels [Fig. 1(b)]. In contrast, for qubit systems $(2, 3, ..., n)$, the cavity mode is off-resonant with the $|1\rangle \leftrightarrow |3\rangle$ transition but highly detuned from the transition between any other two levels [Fig. 1(d)]. These requirements can be achieved by an appropriate choice of qubit systems (e.g., atoms), prior adjustment of the level spacings of the qubit systems (e.g., superconducting devices), or prior adjustment of the cavity mode frequency before the operation. Note that the cavity mode frequency for both optical cavities and microwave cavities can be changed in various experiments (e.g., see, [32-36]). And, for superconducting qubit systems, the level spacings can be readily adjusted by varying the external parameters (e.g., the external magnetic flux and gate voltage for superconducting charge-qubit systems, the current bias or flux bias in the case of superconducting phase-qubit systems and flux-qubit systems, see e.g. [37-39]).

Suppose that the cavity mode and each of qubit systems $(1, 2, ..., n)$ are initially in $|0\rangle_c$ and $(|0\rangle + |1\rangle)/\sqrt{2}$, respectively. The initial state for each of the qubit systems here can be easily prepared by the application of classical pulses. The whole procedure for preparing qubit systems $(1, 2, ..., n)$ in a GHZ state is shown as follows:

Step (i): Apply a classical pulse (with a frequency $\omega = \omega_{21}$ and $\phi = -\pi/2$) to qubit system 1 for a duration $t_{1a} = \pi/(2\Omega_r)$ [Fig. 1(a)], wait to have the cavity mode resonantly interacting with the $|1\rangle \leftrightarrow |2\rangle$ transition of the qubit system 1 for a time interval $t_{1b} = \pi/(2g)$ [Fig. 1(b)], and then apply a pulse (with a frequency $\omega = \omega_{10}$ and $\phi = -\pi/2$) to qubit system 1 for a duration $t_{1c} = \pi/(2\Omega_r)$ [Fig. 1(c)]. According to Eqs. (2), (3), and (5), it can be seen that after the operation of this step, the following transformation is obtained:

$$
\begin{align*}
|0\rangle_1 |0\rangle_c & \quad \rightarrow \quad |0\rangle_1 |0\rangle_c \\
|1\rangle_1 |0\rangle_c & \quad \rightarrow \quad i|0\rangle_1 |1\rangle_c ,
\end{align*}
$$

which leads the initial state $\prod_{j=1}^n (|0\rangle_j + |1\rangle_j) |0\rangle_c$ of the whole system to the following state:

$$
\prod_{j=2}^n (|0\rangle_j + |1\rangle_j) (|1\rangle_1 |0\rangle_c + i|0\rangle_1 |1\rangle_c) .
$$

Here and below, the normalization factor $1/2^{n/2}$ is omitted for simplicity. Eq. (15) shows that the levels $|1\rangle$ and $|2\rangle$ of qubit system 1 are not populated in the case when the cavity mode is in the single-photon state $|1\rangle_c$. Hence, after the operation of this step, the qubit system 1 is decoupled from the cavity mode during the operation of next step.

Step (ii): Apply a classical pulse (with a duration $t_2$) to each of qubit systems $(2, 3, ..., n)$ to induce the off-resonant Raman coupling described in Sec. II C [Fig. 1(d)]. By adjusting the pulse frequencies, we set $\delta_2 = \delta_3 = \ldots = \delta_n = \delta$. Since the level $|2\rangle$ for each of qubit systems $(2, 3, ..., n)$ is not populated during the operation of step (i) above, the effective Hamiltonian describing this step of operation is given by Eq. (11). Accordingly, when the cavity mode is in the single-photon state $|1\rangle_c$, the time evolution of the state $|1\rangle_j$ for qubit system $j$ is then given by Eq. (12). Set $\lambda_2 = \lambda_3 = \ldots = \lambda_n = \lambda$, which can be readily achieved by adjusting the Rabi frequencies $\Omega_2, \Omega_3, \ldots, \Omega_n$ of the pulses applied to qubit systems $(2, 3, ..., n)$. From Eq. (12), one can see that for $t_2 = \pi/\lambda$, we have the transformation $|1\rangle_j |1\rangle_c \rightarrow -|1\rangle_j |1\rangle_c$ ($j = 2, 3, ..., n$). Thus, the state (15) becomes

$$
\prod_{j=2}^n (|0\rangle_j + |1\rangle_j) |1\rangle_1 |0\rangle_c + i \prod_{j=2}^n (|0\rangle_j - |1\rangle_j) |0\rangle_1 |1\rangle_c ,
$$

which shows that the $n$ qubit systems $(1, 2, ..., n)$ have been entangled to each other after the above operations. Since the qubit systems $(1, 2, ..., n)$ are also entangled with the cavity mode, we will need to perform the following operation to disentangle qubit systems $(1, 2, ..., n)$ from the cavity mode.

Step (iii): Perform a reverse operation described in step (i). Namely, apply a classical pulse (with a frequency $\omega = \omega_{10}$ and an initial phase $\phi = \pi/2$) to qubit system 1 for a duration $t_{1a} = \pi/(2\Omega_r)$ [Fig. 1(c)], wait to have the cavity mode resonantly interacting with the $|1\rangle \leftrightarrow |2\rangle$ transition of the qubit system 1 for a time interval $t_{1b} = \pi/(2g)$ [Fig. 1(b)], and then apply a pulse (with a frequency $\omega = \omega_{21}$ and $\phi = \pi/2$) to qubit system 1 for a duration $t_{1c} = \pi/(2\Omega_r)$ [Fig. 1(a)]. According to Eqs. (2), (3), and (5), it can be seen that after the operation of this step, we have:

$$
\begin{align*}
|1\rangle_1 |0\rangle_c & \quad \rightarrow \quad |1\rangle_1 |0\rangle_c \\
0\rangle_1 |1\rangle_c & \quad \rightarrow \quad i|2\rangle_1 |0\rangle_c \\
|1\rangle_1 |0\rangle_c & \quad \rightarrow \quad i|0\rangle_1 |1\rangle_c ,
\end{align*}
$$

which leads the state (16) to the following state

$$
\left[ \prod_{j=2}^n (|0\rangle_j + |1\rangle_j) |0\rangle_1 - \prod_{j=2}^n (|0\rangle_j - |1\rangle_j) |1\rangle_1 \right] \otimes |0\rangle_c .
$$
Eq. (18) demonstrates that the cavity mode returns to its original vacuum state and the qubit systems \( (1, 2, \ldots, n) \) have been disentangled from the cavity mode after the above operations.

The left part of the product in the state (18) can be rewritten as
\[
|0\rangle_1 |+\rangle_2 \cdots |+\rangle_n - |1\rangle_1 |-\rangle_2 \cdots |-\rangle_n,
\]
where \(|+\rangle_j = |0\rangle_j + |1\rangle_j, \) and \(|-\rangle_j = |0\rangle_j - |1\rangle_j (j = 2, 3, \ldots, n)\). Since \(|+\rangle_j \) is orthogonal to \(|-\rangle_j \), the state (19) is a GHZ entangled state of \( n \) qubits.

To reduce the operation errors, the level-spacing inhomogeneity in each four-level system needs to be larger than the bandwidth of the applied pulse, such that the overlapping of pulse spectra or the transition between any two irrelevant levels is negligible. This requirement can be achieved by prior adjustment of the qubit level spacings. For superconducting qubit systems, the level spacings can be readily adjusted by varying the external parameters [37-39].

Several additional points need to be made, which are as follows:

(i) Since the cavity mode is off-resonant with the \( |1\rangle \leftrightarrow |3\rangle \) transition of qubit system \( j (j = 2, 3, \ldots, n) \) during steps (i) and (iii), a phase shift \( \exp(i\varphi_j) \) happens to the state \(|1\rangle \) of qubit system \( j \) when the cavity mode is in the single photon state \(|1\rangle_c \) for either of steps (i) and (iii). Here, \( \varphi_j = g_j^2 (t_{1b} + t_{1c}) / \Delta_{c,j} \). It is easy to find that if this unwanted phase shift for qubit system \( j \) is considered, the fidelity of the prepared GHZ state is given by
\[
F \simeq \frac{1}{4} \left[ 1 + \prod_{j=2}^{n} \left( 1 + \frac{e^{-i2\varphi_j}}{2} \right) \right] \left[ 1 + \prod_{j=2}^{n} \left( 1 + \frac{e^{i2\varphi_j}}{2} \right) \right],
\]
which shows that for \( \varphi_j \sim 0 \), i.e., when the condition
\[
g_j^2 (t_{1b} + t_{1c}) / \Delta_{c,j} \ll 1
\]
is met, we have \( F \sim 1 \). The condition (21) can be reached by increasing \( g \) and \( \tilde{\Omega}_r \) to shorten the operation time \( t_{1b} + t_{1c} \) or increasing the ratio \( \Delta_{c,j} / g_j^2 \).

(ii) For steps (i) and (iii), the pulse process depicted in Fig. 1(a) and (c) must be much faster than the process of the cavity mode resonantly interacting with the \( |1\rangle \leftrightarrow |2\rangle \) transition of the qubit system 1 [Fig. 1(b)], i.e.,
\[
t_{1a}, t_{1c} \ll t_{1b},
\]

such that the internal transition between the two levels \(|1\rangle \) and \(|2\rangle \) of qubit system 1 during the pulses, induced by the resonant interaction of the cavity mode with the \( |1\rangle \leftrightarrow |2\rangle \) transition of qubit system 1, is negligible. Note that the condition (22) can be readily achieved by increasing the pulse Rabi frequencies \( \Omega_r \) and \( \tilde{\Omega}_r \) such that \( \Omega_r, \tilde{\Omega}_r \gg g \).

(iii) The level \(|2\rangle \) of qubit system 1 is populated for a time interval \( t_{1a} + t_{1b} \) during step (i) or step (iii), thus the condition
\[
t_{1a} + t_{1b} \ll \gamma_{2r}^{-1}, \gamma_{2p}^{-1}
\]
needs to be satisfied in order to reduce decoherence caused due to spontaneous emission and dephasing of the level \(|2\rangle \) of qubit system 1. Here, \( \gamma_{2r}^{-1} \) and \( \gamma_{2p}^{-1} \) are the energy relaxation time and dephasing time of the level \(|2\rangle \) of qubit system 1, respectively.

From the above description, one can see that this proposal has the following advantages:

(i) No identical qubit-cavity coupling constants for the \( (n - 1) \) qubit systems \( (2, 3, \ldots, n) \) are required, thus, this proposal is tolerant to the qubit-system parameter nonuniformity and nonexact placement of qubits in a cavity;

(ii) No adjustment of the level spacings of the qubit systems or adjustment of the cavity mode frequency during the entire operation is needed;

(iii) Neither auxiliary qubit systems nor measurement on the qubit states is needed;

(iv) No photon detection is needed;

(v) The entanglement preparation is deterministic;

(vi) The entire operation time is given by
\[
\tau = \pi / g + \pi / \Omega_r + \pi / \tilde{\Omega}_r + \pi / \lambda,
\]
which is independent of the number of qubits in the cavity.
In addition, for the description given above, it can be seen that:

(vi) During the entire operation, the levels $|2\rangle$ and $|3\rangle$ of qubit systems $(2, 3, ..., n)$ are unpopulated and thus decoherence due to spontaneous emission and dephasing from these levels are greatly reduced.

(vii) The level $|2\rangle$ of qubit system 1 is only populated for a very short time $t_{1a} + t_{1b}$ during step (i) or step (iii); and

(viii) The level $|2\rangle$ of qubit system 1 is not occupied during step (ii) [note that the operation of this step (ii) requires much longer time $t_2$ than both step (i) and step (iii)].

Above we have discussed how to prepare a multi-qubit GHZ state with a three-level qubit system and $(n - 1)$ four-level qubit systems in a cavity. The discussion given above is based on qubit systems for which the level spacings become narrower as the levels go up [see Fig. 1(a,b,c) for qubit system 1 and Fig. 1(d) for qubit systems $(2, 3, ..., j)$]. Note that this limitation is unnecessary. Namely, this proposal is also applicable to: (i) the qubit system 1 with such three levels, for which the level spacing between the two lowest levels $|0\rangle$ and $|1\rangle$ is smaller than that between the two upper levels $|1\rangle$ and $|2\rangle$; and (ii) the $(n - 1)$ qubit systems $(2, 3, ..., n)$ with these four levels, for which the level spacing between the two levels $|i\rangle$ and $|i + 1\rangle$ is larger or smaller than that between the two levels $|i + 1\rangle$ and $|i + 2\rangle$ (here, $i = 0, 1$). Furthermore, since the level $|0\rangle$ of each of the qubit systems $(2, 3, ..., n)$ was not involved during the entire operation, one can choose the qubit systems $(2, 3, ..., n)$ for which the transition between the two lowest levels $|0\rangle$ and $|1\rangle$ is forbidden or weak to avoid or reduce decoherence caused by the spontaneous emission from the level $|1\rangle$.

The three-level or four-level qubit systems here are widely available in natural atoms and also in artificial atoms such as superconducting charge-qubit systems [37], phase-qubit systems [38,39], and flux-qubit systems [37,40]. For a detailed discussion, see Ref. [27].

**IV. COMPARING WITH PREVIOUS PROPOSALS**

In this section, we will give a comparison between our proposal and previous ones. For simplicity, we will compare our proposal with the ones in [17,25]. To the best of our knowledge, the proposals in [17,25] are most closely related to our work.

**A. Comparison with the proposal in [25]**

An $n$-qubit GHZ state can be prepared without real excitation of the cavity mode, by using the method introduced in [25]. To see this, consider $n$ three-level qubit systems $(1, 2, ..., n)$ in a cavity. Assume that the cavity mode is coupled to the $|0\rangle \leftrightarrow |2\rangle$ transition and the pulses are coupled to the $|1\rangle \leftrightarrow |2\rangle$ transition, to establish the system-cavity-pulse off-resonant Raman coupling [Fig. 2(a)]. In this case, we can obtain an effective Hamiltonian, i.e., the Hamiltonian (10) with a replacement of $|1\rangle$, $|2\rangle$, and $\sum_{j=2}^{n}$ by $|0\rangle$, $|1\rangle$, and $\sum_{j=1}$ respectively. When the cavity mode is initially
in a vacuum state $|0\rangle_c$, this effective Hamiltonian reduces to

$$H_{\text{eff}} = -\hbar \sum_{j=1}^{n} \frac{\Omega_j^2}{\Delta_j} |1\rangle_j \langle 1| + \hbar \sum_{j=1}^{n} \frac{\chi_j^2}{\delta} |1\rangle_j \langle 1|$$

$$+ \hbar \sum_{j \neq j'}^{n} \frac{\chi_j \chi_{j'}}{\delta} \left( \sigma_{01,j}^{+} \sigma_{01,j'}^{-} + \sigma_{01,j}^{-} \sigma_{01,j'}^{+} \right).$$

(25)

where $\Delta_j = \omega_{2j} - \omega_j$, $\Delta_{c,j} = \omega_{2j0} - \omega_c$, and $\delta = \delta_j = \Delta_{c,j} - \Delta_j$ (which can be reached via adjustment of the pulse frequencies). The notation of $\chi_j$ is the same as that given in Sec. III. To prepare the qubit systems $(1, 2, ..., n)$ in a GHZ state, one will need to apply a second pulse to each of the qubit systems $(1, 2, ..., n)$ to cancel the Stark shifts, i.e., the first term of Eq. (25). After that, the Hamiltonian (25) becomes

$$H_{\text{eff}} = \hbar \sum_{j=1}^{n} \frac{\chi_j^2}{\delta} |1\rangle_j \langle 1| + \hbar \sum_{j \neq j'}^{n} \frac{\chi_j \chi_{j'}}{\delta} \left( \sigma_{01,j}^{+} \sigma_{01,j'}^{-} + \sigma_{01,j}^{-} \sigma_{01,j'}^{+} \right).$$

(26)

For the case of $\chi_1 = \chi_2 = ... = \chi_n = \chi$ (achievable by changing the pulse Rabi frequencies), this Hamiltonian is the same as the Hamiltonian (6) presented in Ref. [25] for $n$ two-level atoms interacting dispersively with a single-mode cavity. According to the discussion in Ref. [25], an $n$-qubit GHZ state can be prepared based on the Hamiltonian (26).

From the description here, one can see that to achieve identical Raman transition strengths $\chi_1$, $\chi_2$, ..., and $\chi_n$, the pulses applied to different qubit systems should have different Rabi frequencies if the qubit-cavity coupling constants are nonidentical. And, to obtain identical detuning $\delta$ for each qubit system, adjustment of the pulse frequencies would be needed. Furthermore, to generate GHZ states, a second off-resonant pulse should be applied to each qubit system to cancel the Stark shift induced by the first pulse. Namely, for generation of an $n$-qubit GHZ state, a total of $2n$ pulses would be required. In contrast, as shown above, our present proposal, which employs four-level qubit systems, needs only $(n - 1)$ pulses applied to the qubit systems $(2, 3, ..., n)$, i.e., one pulse for each of qubit systems $(2, 3, ..., n)$. Hence, the use of the pulses is significantly reduced in the present proposal.

B. Comparison with the proposal in [17]

In Ref. [17], an auxiliary qubit system $a$ was used to create a single photon in the cavity mode, which was then employed to prepare the $n$ qubit systems $(1, 2, ..., n)$ in a GHZ state. As discussed there, measurement on the states of the auxiliary qubit system $a$ and adjustment of the level spacings of each qubit system were both needed during the entire operation. In addition, qubit systems $(1, 2, ..., n)$ were required to have identical three-level structures and the qubit-cavity coupling constants for qubit systems $(1, 2, ..., n)$ were needed to be the same.

The cavity mode in [17] was set to be off-resonant resonant with the $|0\rangle \leftrightarrow |2\rangle$ transition of each of qubit systems $(1, 2, ..., n)$, with a detuning $\delta = \omega_{20} - \omega_c$ [Fig. 2(b)]. The coupling constant between the cavity mode and the $|0\rangle \leftrightarrow |2\rangle$ transition for each of qubit systems $(1, 2, ..., n)$ was denoted as $g'$. As shown in [17], a multiqubit GHZ state was generated by simultaneously performing a common phase shift $e^{i\lambda t}$ on the state $|0\rangle$ of each of qubit systems $(1, 2, ..., n)$ with assistance of the cavity photon, by having $e^{i\lambda t} = e^{i\pi}$ (i.e., $\lambda t = \pi$). Here, $\lambda = (g')^2/\delta$, which was originally defined in [17]. For a detailed discussion, see Ref. [17].

It is noted that the method in [17] can not work in the case when the qubit-cavity coupling constants are nonidentical. The reason is that a common phase shift $e^{i\lambda t}$ can not be simultaneously performed on the state $|0\rangle$ of each of qubit systems $(1, 2, ..., n)$. For instances, when the qubit-cavity coupling constants $g'_j$ and $g'_k$ for qubits $j$ and $k$ are not the same (resulting in different $\lambda_j = (g'_j)^2/\delta$ and $\lambda_k = (g'_k)^2/\delta$), one can not have both of $\lambda_j t$ and $\lambda_k t$ to be equal to $\pi$ for a given time $t$. The discussion here applies to identical atoms in a cavity, for which the detuning $\delta$ of each atom with the cavity mode is the same but the qubit-cavity couplings are nonidentical due to nonexact placement of atoms in the cavity.

Let us now see if the method in [17] can work for a situation that the detuning of each qubit system with the cavity mode and the qubit-cavity coupling constants are both nonidentical. This situation applies to superconducting qubit systems in a cavity. As discussed above, to prepare a multiqubit GHZ state, the condition $\lambda_j = \lambda_k$ needs to be satisfied for arbitrary two superconducting qubit systems $j$ and $k$. Here, $\lambda_j = (g'_j)^2/\delta_j$ and $\lambda_k = (g'_k)^2/\delta_k$. Note that once the superconducting qubit systems are designed and the cavity mode frequency is chosen, the detunings $\delta_j$ and $\delta_k$ are fixed. Thus, in order to obtain $\lambda_j = \lambda_k$, i.e., $(g'_j)^2/\delta_j = (g'_k)^2/\delta_k$, it would be required to exactly place the qubit systems in the cavity to have desired qubit-cavity coupling constants $g'_j$ and $g'_k$, which is not easy to achieve in
experiments due to the fabrication error. This problem becomes more apparent especially when the number of qubit systems in a cavity is large. In contrast, as shown above, this difficulty is avoided in our present proposal, because the effective coupling strength $\lambda_j$ in Eq. (13) can be adjusted by changing the intensity of the pulses, instead of exactly placing qubit systems in a cavity to have desired qubit-cavity coupling constants.

From the discussion here, it can be concluded that our present proposal is quite different from the previous ones [17,25]. Due to the use of a four-level structure, the present proposal can be used to prepare a multiqubit GHZ state for nonidentical qubit-cavity coupling constants (compared with the proposal in [17]), and requires less application of pulses (compared with the proposal in [25]).

V. DISCUSSION

In this section, we will give a brief discussion on the experimental issues. For the method to work:

(i) The occupation probability $p_j$ of the level $|3\rangle$ for qubit system $j$ ($j = 2, 3, ..., n$) during step (ii) is given by [28]

$$p_j \approx \frac{2}{4 + (\Delta_j/\Omega_j)^2} + \frac{2}{4 + (\Delta_{c,j}/g_j)^2},$$

which needs to be negligibly small in order to reduce the operation error.

(ii) As discussed above, the conditions (21), (22), and (23) need to be satisfied;

(iii) The total operation time $\tau$ given in Eq. (24) should be much shorter than the energy relaxation time $\gamma_1^{-1}$ and dephasing time $\gamma_1$ of the level $|1\rangle$ and the lifetime of the cavity mode $\kappa = Q/2\pi\nu_c$, where $Q$ is the (loaded) quality factor of the cavity.

The above requirements can in principle be realized, since one can: (i) reduce $p_j$ by increasing the ratio of $\Delta_j/\Omega_j$ and $\Delta_{c,j}/g_j$; (ii) shorten $t_{1a}, t_{1b}, t_{1c}$ by increasing the pulse Rabi frequency $\Omega_j$, $\Omega_r$, and the coupling constant $g$; (iii) design qubit systems (e.g., superconducting devices) to have sufficiently long energy relaxation times $\gamma_1^{-1}$ and $\gamma_2^{-1}$ and dephasing times $\gamma_1^{-1}$ and $\gamma_2^{-1}$ or choose qubit systems (e.g., atoms) with long decoherence times of the levels $|1\rangle$ and $|2\rangle$; (iii) increase $\kappa$ by employing a high-$Q$ cavity so that the cavity dissipation is negligible during the operation.

For the sake of definitiveness, let us consider the experimental possibility of generation of a six-qubit GHZ state with six superconducting qubits coupled to a resonator (Fig. 3). With the choice of $\Omega_j$, $\Omega_r \sim 10g$, we have $t_{1a}, t_{1b} \sim 0.05\pi/g$ and $t_{1b} \sim 0.5\pi/g$. By setting $\Delta_j \sim 10\Omega_j$, $\Delta_{c,j} \sim 10g_j$, and $\Omega_j \sim 0.9g_j$, we have $\delta = \delta_j \sim 10g_j^2/\Delta_{c,j} \sim 10\Omega_j^2/\Delta_j \sim 10\chi_j$, leading to $\lambda_j \sim 0.1\pi g_j$. For $g_j \sim 0.2g$, we can set $\lambda_2 = \lambda_3 = ... = \lambda_6 = \lambda = 0.022g$, which can be achieved by adjusting the Rabi frequencies $\Omega_2, \Omega_5, ..., \Omega_6$ of the pulses applied to the qubit systems (2, 3, ..., 6) as discussed above. Thus, we have $\tau \sim 46.6\pi/g$. As a rough estimate, assume $g/2\pi \sim 220$ MHz, which could be reached for a superconducting qubit system coupled to a one-dimensional standing-wave CPW (coplanar waveguide) transmission line resonator [42]. For the $g$ chosen here, we have: (i) $t_{1a}, t_{1c} \sim 0.1$ ns and $t_{1b} \sim 1.1$ ns, which shows that the condition (22) is satisfied; (ii) $t_{1a} + t_{1b} \sim 1.2$ ns, showing that the condition (23) is met for $\min\{\gamma_1^{-1} - 1\} \sim 0.5\mu s$ [38,43]; and (iii) $\tau \sim 0.1\mu s$, much shorter than $\min\{\gamma_1^{-1} - 1\} \sim 1\mu s$ [38,43]. In addition, consider a resonator with frequency $\nu_c \sim 3$ GHz (e.g., Ref. [44]) and $Q \sim 5 \times 10^4$, we have $\kappa^{-1} \sim 2.5\mu s$, which is much longer than the total operation time $\tau$. Note that superconducting coplanar waveguide resonators with a quality factor $Q > 10^6$ have been experimentally demonstrated [45].

For the choice of $\Delta_j \sim 10\Omega_j$ and $\Delta_{c,j} \sim 10g_j$ here, we have $p_j \sim 0.04$, which can be further reduced by increasing the ratio of $\Delta_j/\Omega_j$, and $\Delta_{c,j}/g_j$. In addition, for the parameters chosen here, when the unwanted phase shifts mentioned above are considered, a simple calculation shows that the fidelity $F$ is $\sim 0.992$. We should mention that further investigation is needed for each particular experimental set-up. However, this requires a rather lengthy and complex analysis, which is beyond the scope of this theoretical work.

As shown in Sec. III, this proposal requires simultaneous application of pulses with different frequencies and intensities during the operation of step (ii). For the set up in Fig. 3(a), simultaneous application of several pulses of different frequencies and intensities to qubit systems 2, 3, 4, and 5 is feasible in experiments [46]. According to Ref. [46], for a qubit system containing a superconducting loop, one can place an AC current loop on the qubit superconducting loop to create an AC flux $\Phi_c$ (i.e., a classical magnetic pulse) threading the qubit loop [Fig. 3(b,c,d)]; and the frequencies and intensities of pulses applied to qubit systems 2, 3, 4, and 5 can be readily and simultaneously changed by varying the frequencies and intensities of the AC loop currents of the qubit systems 2, 3, 4, and 5 at the same time.

VI. CONCLUSION

In summary, we have proposed a way for creating $n$-qubit GHZ entangled states with a three-level qubit system and $(n - 1)$ four-level qubit systems in a cavity or coupled to a resonator. The main advantage of this proposal is that no identical qubit-cavity coupling constants are required. As a result, qubit systems (e.g., solid-state qubits),
FIG. 3: (Color online) (a) Sketch of the setup for six superconducting qubit systems and a (grey) standing-wave quasi-one-dimensional coplanar waveguide resonator. The two blue curved lines represent the standing wave magnetic field, which is in the $z$-direction. The green dot represents the qubit system 1, which is placed at an antinode of the standing wave magnetic field to achieve the maximal qubit-cavity coupling constant $g$. The red dots represent the qubit systems 2, 3, ..., and $n$, which are placed at locations where the magnetic fields are the same. Each qubit system could be a superconducting charge-qubit system shown in (b), flux-biased phase-qubit system in (c), and flux-qubit system in (d). The superconducting loop of each qubit system, which is a large square for (b) and (d) while a large circle for (c), is located in the plane of the resonator between the two lateral ground planes (i.e., the $x$-$y$ plane). Each qubit system is coupled to the cavity mode through a magnetic flux $\Phi_c$ threading the superconducting loop, which is created by the magnetic field of the cavity mode. A classical magnetic pulse is applied to each qubit system through an AC flux $\Phi_e$ threading the qubit superconducting loop, which is created by an AC current loop (i.e., the red dashed-line loop) placed on the qubit loop. The frequency and intensity of the pulse can be adjusted by changing the frequency and intensity of the AC loop current. $E_J$ is the Josephson junction energy ($0.6 < \alpha < 0.8$) and $V_g$ is the gate voltage. $\lambda$ is the wavelength of the resonator mode, and $L$ is the length of the resonator.

which often have parameter nonuniformity, can be used; and no exact placement of qubits in a cavity is needed. Note that for solid-state qubits (e.g., superconducting qubits), it is difficult to design qubits with identical device parameters; and it is experimentally challenging to place many qubits at different locations where the magnetic fields or electric fields are exactly the same. These hardships are avoided in this proposal. Another interesting property of this proposal is that during the entire operation, there is no need of adjusting the level spacings of the qubit systems or the cavity mode frequency. In experiments, adjustment of the qubit level spacings or the cavity mode frequency during the operation is not desired; and extra errors may be induced by adjustment of the qubit level spacings or the cavity mode frequency. Furthermore, as shown above, this proposal has these additional advantages: (i) The $n$-qubit GHZ state is prepared deterministically and the operation time is independent of the number of qubits in the cavity; (ii) Neither auxiliary qubit systems nor measurement on the qubit states is needed; and (iii) No photon detection is needed. This proposal is quite general, which can be applied to various types of superconducting qubits and atoms trapped in a cavity.

ACKNOWLEDGMENTS

C.P.Y. is grateful to Shi-Biao Zheng for very useful comments. This work is supported in part by the National Natural Science Foundation of China under Grant No. 11074062, the Zhejiang Natural Science Foundation under
Grant No. Y6100098, funds from Hangzhou Normal University, and the Open Fund from the SKLPS of ECNU.

[1] W. B. Gao, C. Y. Lu, X. C. Yao, P. Xu, O. Ghne, A. Goebel, Y. A. Chen, C. Z. Peng, Z. B. Chen, and J. W. Pan, Nature Physics 6, 331 (2010).
[2] H. Hffner, W. Huel, C. F. Roos, J. Benhelm, D. Chek-al-kar, M. Chwalla, T. Krber, U. D. Rapol, M. Riebe, P. O. Schmidt, C. Becher, O. Ghne, W. Dr and R. Blatt, Nature (London) 438, 643 (2005).
[3] D. M. Greenberger, M. A. Horne, and A. Zeilinger, Going beyond Bells theorem. In Kafatos, M. (ed.) Bells theorem, quantum theory and conceptions of the universe (Kluwer Academic, Dordrecht, 1989).
[4] E. Hagley, X. Matre, G. Nogues, C. Wunderlich, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. 79, 1 (1997); S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J. M. Raimond, and S. Haroche, ibid. 87, 037902 (2001).
[5] G. Chen, N. H. Bonadeo, D. G. Steel, D. Gammon, D. S. Katzer, D. Park, and L. J. Sham, Science 289, 1906 (2000).
[6] M. Neeley, R. C. Bialczak, M. Lenander, E. Lucero, M. Mariani, A. D. O'Connell, D. Sank, H. Wang, M. Weides, J. Wenner, Y. Yin, T. Yamamoto, A. N. Cleland, and J. M. Martinis, Nature (London) 467, 570 (2010).
[7] L. DiCarlo, M. D. Reed, L. Sun, B. R. Johnson, J. M. Gambetta, L. Frunzio, S. M. Girvin, M. H. Devoret, and R. J. Schoelkopf, Nature (London) 467, 574 (2010).
[8] G. Sun, X. Wen, B. Mao, J. Chen, Y. Yu, P. Wu, and S. Han, Nature Communications 1, 51 (2010).
[9] M. Hillery, V. Buzek, and A. Berthiaume, Phys. Rev. A 59, 1829 (1999).
[10] S. Bose, V. Vedral, and P. L. Knight, Phys. Rev. A 77, 022326 (2010).
[11] J. J. Bollinger, W. M. Itano, D. J. Wineland, and D. J. Heinzen, Phys. Rev. A 54, 4649 (1996); S. F. Huelga, C. Macchiavello, T. Pellizzari, A. K. Ekert, M. B. Plenio, and J. I. Cirac, Phys. Rev. Lett. 79, 3865 (1997).
[12] L. S. Bishop et al., New J. Phys. 11, 073040 (2009).
[13] L. M. Duan and H. J. Kimble, Phys. Rev. Lett. 90, 253601 (2003).
[14] S. Branca et al., J. Phys. A 41, 404204 (2008).
[15] M. Sandberg, C. M. Wilson, F. Persson, T. Bauch, G. Johansson, V. Shumeiko, T. Duty, and P. Delsing, Appl. Phys. Lett. 92, 203501 (2008).
[16] A. Palacios-Laloy, F. Nguyen, F. Mallet, P. Bertet, D. Vion, and D. Esteve, J. Low Temp. Phys. 151, 1034 (2008).
[17] J. R. Johansson, G. Johansson, C. M. Wilson, and F. Nori, Phys. Rev. Lett. 103, 147003 (2009).
[18] J. Q. Liao, Z. R. Gong, L. Zhou, Y. X. Liu, C. P. Sun, and F. Nori, Phys. Rev. A 81, 042304 (2010).
[19] J. Q. You and F. Nori, Phys. Today 58 (11), 42 (2005).
[20] J. Clarke and F. K. Wilhelm, Nature (London) 453, 1031 (2008).
[21] M. Neeley, M. Ansmann, R. C. Bialczak, M. Hofheinz, N. Katz, E. Lucero, A. O'Connell, H. Wang, A. N. Cleland, and J. M. Martinis, Nature Physics 4, 523 (2008); A. M. Zagoskin, S. Ashhab, J. R. Johansson, and F. Nori, Phys. Rev. Lett. 97, 077001 (2006).
[22] Y. X. Liu, J. Q. You, L. F. Wei, C. P. Sun, and F. Nori, Phys. Rev. Lett. 95, 087001 (2005).
[23] C. P. Yang, Y. X. Liu, and F. Nori, Phys. Rev. A 81, 062323 (2010).
[24] L. DiCarlo, M. D. Reed, L. Sun, B. R. Johnson, J. M. Chow, J. M. Gambetta, L. Frunzio, S. M. Girvin, M. H. Devoret, and R. J. Schoelkopf, Nature (London) 467, 574 (2010).
[25] L. DiCarlo, J. M. Chow, J. M. Gambetta, Lev S. Bishop, B. R. Johnson, D. I. Schuster, J. Majer, A. Blais, L. Frunzio, S.
M. Girvin, and R. J. Schoelkopf, Nature (London) 460, 240 (2009).

[44] P. J. Leek, S. Filipp, P. Maurer, M. Baur, R. Bianchetti, J. M. Fink, M. Goppl, L. Steffen, and A. Wallraff, Phys. Rev. B 79, 180511(R) (2009).

[45] P. K. Day, H. G. LeDuc, B. A. Mazin, A. Vayonakis, and J. Zmuidzinas, Nature (London) 425, 817 (2003).

[46] Y. Yu, Private Communication (2011).