Electrically Tunable Collective Modes in a MEMS Resonator Array

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Abstract

Using optical diffraction, we study the mechanical vibrations of an array of micromechanical resonators. Implementing tunable electrostatic coupling between the suspended, doubly-clamped Au beams leads to the formation of a band of collective vibrational modes within these devices. The evolution of these modes with coupling strength is clearly manifested in the optical diffraction pattern of light transmitted through the array. The experimental results are analyzed using a simple model for one-dimensional phonons. These structures offer unique prospects for spectral analysis of complex mechanical stimuli.

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Interaction between light and mechanically vibrating systems was first discussed by Einstein and Bohr. A gedanken experiment of optical diffraction by a vibrating two slit structure was employed to demonstrate the complementarity principle of quantum mechanics [1]. These ideas were later elaborated in order to formulate the theory of x-rays and neutrons diffraction by crystals [2] [3]. In these systems mechanical vibrations due to both thermal and quantum fluctuations strongly modify the diffraction pattern. This allows experimental study of the crystal’s normal modes of vibration (phonons) and their dispersion relation. Micro electro-mechanical systems (MEMS) technology allows studying similar phenomena using artificial mechanical systems at mesoscopic length scales. Such studies are motivated not only by scientific interest but rather also by the prospects of developing new micro opto-mechanical devices. This rapidly growing field of research employs MEMS technology to realize a variety of on-chip fully integrated optical devices (for a review see [4]). A particular example related to the present work is the movable diffraction grating. In this device the efficiency of diffraction is controlled by moving the grating with respect to the substrate beneath it, allowing thus optical modulation [5] [6] [7]. Operation of such devices at relatively high frequencies however requires a good understanding of the dynamics and mechanical properties of the system.

In the present paper we use optical diffraction to study the mechanical properties of a periodic array (grating) of suspended doubly-clamped beams made of Au. What is novel and especially interesting about the present work is that our devices allow application of mutual electrostatic forces between the beams. This coupling gives rise to the formation of a band of collective modes of vibration (phonons). We excite these collective modes parametrically and employ optical diffraction to study the response. A simple model describing our system is presented and compared with experiment. We conclude by briefly describing some unique device applications of this system.

The bulk micromachining process employed for sample fabrication is described in Fig. 1. In this process the substrate beneath the grating is completely etched away, thus allowing optical access to the grating from both sides. In the first step chemical vapor deposition is
employed to deposit a 70 nm thick layer of low-stress silicon nitride on the front and back sides of a Si wafer. A square window is opened in the silicon nitride on the back side using photolithography and wet etching (Fig. 1(a)). The structure shown in Fig. 1(b) is realized through a highly selective, anisotropic KOH etch for the backside of the Si wafer. This occurs within the patterned region and yields a 270 µm square silicon nitride membrane on the front side of the wafer. The grating beams and adjacent electrodes are fabricated on top of this membrane using electron beam lithography, followed by thermal evaporation of Au and liftoff (Fig. 1(c)). Each resulting beam has length $b = 270$ µm, width $1$ µm and thickness $0.25$ µm (measured using an atomic force microscope) and the grating period is $a = 4$ µm. In the final step the membrane is removed using electron cyclotron resonance (ECR) plasma etching from the back side of the sample. This process step employs an Ar/NF$_3$ gas mixture, and results in suspension of the Au beam array (Fig. 1(d)). Figure 1(e) shows a side view micrograph of the device. The electrodes form two interdigitated combs; with fingers alternately connected to the two base electrodes. This design allows application of electrostatic forces between the beams.

To characterize uniformity within the device we measure the fundamental resonance frequency of each suspended beam in the array. This is done in-situ, using the output from a commercial scanning electron microscope’s imaging system to detect mechanical displacement. We have employed this technique previously to study the mechanical properties of individual, similarly-fabricated Au beams [9] [10]. Fig. 2(a) shows a typical response peak from an individual beam. By measuring all 67 suspended beams in the array we find that the distribution of resonance frequencies has an average of 179.3 kHz and a standard deviation of 0.53 kHz (see Fig. 2(b)). Individual mechanical quality factors $Q$ range from 2,000 to 10,000. Note that no correlation is found between the location of the beam within the device and its specific resonance frequency or $Q$; the small beam-to-beam variations appear to be random.

What is expected when a voltage $V$ is applied between the two combs? We employ a simple one-dimensional model for an $N$-element array of coupled pendulums [3] to describe
our system (see Fig. 3(a)). While the first and last pendulums in the array are clamped and stationary, all others \((n = 2, 3, ..., N - 1)\) are free to oscillate about their equilibrium positions \(na\). Here \(a\) represents the equilibrium spacing between neighboring pendulums. In the absence of any coupling, the angular frequency for small oscillations of each (identical) pendulum is \(\omega_0\). The displacement of the system is described by a set of coordinates \(x_n\) \((n = 1, 2, ..., N)\) (see Fig. 3(a)). Applying a voltage \(V\) gives rise to an attractive interaction between each pendulum and its nearest neighbors \(\varphi(s) = -C(s)V^2/2\), where \(s\) is the distance between the interacting pendulums and \(C\) is the capacitance. Neglecting coupling between non-neighboring pendulums, and assuming small oscillations, we find the following set of equations of motion:

\[
m \ddot{x}_n = -m\omega_0^2 x_n + u \left(2x_n - x_{n-1} - x_{n+1}\right),
\]

(1)

where \(n = 2, 3, ..., N - 1\), and \(u = -\varphi''(a)\) (here dots represents time derivatives and primes represent spatial derivatives). Note that \(u > 0\) due to the attractive nature of the interaction between nearest neighbors. These equations can be greatly simplified by employing a transformation to the eigenmodes (phonons) of the system:

\[
x_n = \sum_{m=2}^{N-1} \chi_n^{(m)} v_m,
\]

(2a)

where \(\chi_n^{(m)} = \sqrt{2/(N-1)} \sin(k_m(n-1)a)\) is the spatial shape of mode number \(m\) \((m = 2, 3, ..., N - 1)\), \(k_m = (m-1)\pi/L\) is the wavevector, and \(L = (N-1)a\) is the length of the system. Figure 3(b) shows the shape of the three lowest modes \(m = 2, 3, 4\). Substituting Eq. (2a) in Eq. (1) leads to a set of decoupled equations of motion:

\[
\ddot{v}_m = -\omega_m^2 v_m,
\]

(3)

where \(\omega_m^2(V) = \omega_0^2 - \left(2C''(a)/m\right) V^2 \sin^2(k_ma/2)\). A stationary voltage \(V = V_{dc}\) thus gives rise to the formation of a band of collective modes between frequencies \(\omega_0\) and \(\omega_b = \sqrt{\omega_0^2 - (2C''(a)/m) V_{dc}^2}\). The associated wavevectors, \(k_m\), vary from zero to \(\pi/a\) (see Figure 3(c)).
Each mode can be selectively excited by adding an AC voltage to the DC bias, namely $V = V_{dc} + V_{ac} \cos(\gamma t)$. Assuming $V_{ac} << V_{dc}$ we find from Eq. (3):

$$\ddot{v}_m = -\omega_m^2 (V_{dc}) (1 - h_m \cos(\gamma t)) v_m,$$

(4)

where $h_m = 2 (V_{ac}/V_{dc}) \left[ (\omega_0/\omega_m)^2 - 1 \right]$. Thus an AC voltage component gives rise to parametric excitation of each mode with amplitude $h_m$ \[1\] \[12\] \[13\]. Parametric resonance occurs when the frequency of the AC voltage $\gamma$ is close to $2\omega_m/l$, where $l$ is an integer. Near these values the system may exhibit unstable behavior in which the amplitude of oscillations grows as a function of time. In the linear theory of parametric resonance this growth is exponential, and occurs when the amplitude of parametric excitation $h$ exceeds a critical value dependent upon the damping in the system. Most systems, however, possess some degree of non-linearity which comes into play as soon as the amplitude of the motion becomes appreciable. Thus, while the linear theory is useful in determining the conditions for the occurrence of parametric resonance, it is inadequate for determining the steady-state response of the system. Unfortunately, the nonlinear coefficients of our system are not known and therefore its steady-state response cannot be predicted. However, we expect that the response of modes with high index $m$ will be relatively large because $h_m$ increases in magnitude with $m$.

We detect the collective mechanical vibrations of the array by diffraction measurements. With spatially-uniform light incident upon the array, the intensity of diffraction is proportional to the form factor $|\phi(q_x)|^2$ where:

$$\phi(q_x) = \sum_{n=1}^{N} \exp [iq_x (na + x_n(t))].$$

(5)

Here $q_x$ is the $x$ component of the change in wavevector between incoming and outgoing waves (the $x$ direction lies in the plane of the sample and is perpendicular to the long axis of the beams) \[14\]. Consider the case where the system is tuned to a diffraction peak, namely $q_x a = 2\pi l$ with $l$ an integer. As shown below, for this particular case the interpretation of the experimental results becomes greatly simplified. We calculate the form factor for
the case where mode \( m \) oscillates with amplitude \( A_m \) and all other modes are stationary. Assuming small oscillations, namely \( q_x x << 1 \), we find:

\[
|\phi(q_x)|^2 = N^2 + [q_x A_m \cot(k_m a/2) \cos(\omega_m t)]^2 \frac{(-1)^m + 1}{N - 1}. \tag{6}
\]

Thus the collective mechanical oscillations give rise to an oscillatory component in the diffraction signal at angular frequency \( 2\omega_m \). Note that with uniform illumination any mode with \( m \) odd will not contribute to diffraction. This occurs due to the exact cancelation of the response from different portions of the entire device. Similar cancelation gives rise to the \( \cot^2(k_m a/2) \) factor in Eq. (5).

The optical setup utilized for the diffraction measurements is schematically depicted in Fig. 4. A polarization maintaining single mode fiber (numerical aperture NA = 0.15 and core diameter \( d = 9 \mu m \)) delivers infrared light from a laser to the sample. We use a tunable wavelength diode laser operating in the range 1535-1635 nm. A spherical lens (focal length \( f = 1 \) mm) collimates the beam, which illuminates the back side of the grating at normal incidence with respect to the array plane. At the array plane, located a distance \( s_1 = 3 \) cm from the collimator, we estimate the diameter of the beam to be \( 2fNA + s_1 d/f = 570 \mu m \).

For the results presented, the polarization of the incident light is TE, i.e. the electric field is parallel to the grating lines. We note, however, that similar results (not presented here) were obtained with perpendicular polarization. For the present case there are four diffraction peaks with angles \( \theta_n = \sin^{-1}(n\lambda/a) \) \( (n = \pm 1, \pm 2) \) with respect to the normal incidence. Here \( \lambda \) is the wavelength of incident light. Transmitted light is collected by a cylindrical lens and focused into a second, single-mode fiber. The lens enables the intensity of collected light to be maximized without degrading the spectral resolution. The fiber delivers light collected from the first order \( (n = 1) \) diffraction peak to a photodiode detector. The distance between the sample and the lens, \( s_2 = 1 \) cm, was chosen as a compromise between two conflicting considerations, namely, simultaneously maximizing spectral resolution and light intensity. While the former consideration favors a large distance, the later favors a small one. For the distance chosen, the spectral resolution is \( \delta \lambda = a \delta \theta \cos \theta_1 = a (d/s_2) \cos \theta_1 \approx 3 \) nm. All
measurements are done at room temperature in vacuum of \(\simeq 10^{-3}\) torr.

We first study diffraction from the array in the absence of any interelectrode bias voltage. The inset of Fig. 4 shows the intensity detected by the photo diode as a function of \(\lambda\). The full width at half maximum (FWHM) of the diffraction peak is estimated using the Fraunhofer diffraction formula \(\delta \lambda \simeq \lambda/N = 23\) nm. We find good agreement between this estimate and the measured value. No effect is resolved from the thermally-driven mechanical vibrations of the beams. Their effect upon diffraction can be characterized by multiplying the diffracted intensity by a Debye-Waller factor, \(\exp (-2W)\). In the present case we estimate that \(2W \simeq 10^{-8}\), hence thermal fluctuations are not expected to affect the diffraction from our micromechanical array significantly. This is confirmed by the experimental data.

In order to excite the modes of vibrations of the system externally, we apply a voltage \(V = V_{dc} + V_{ac} \cos (\gamma t)\) between either side of the interdigitated electrode arrays. We tune the laser wavelength to the diffraction peak (\(\lambda = 1582\) nm) and measure the photodetector response using a lock-in amplifier operating in 2\(f\) mode. This allows us to detect the Fourier component of the array response at angular frequency \(2\gamma\). Figure 5 is a color map plot showing this second harmonic response, \(R\), as a function of both \(V_{dc}\) and \(f = \gamma/2\pi\). The amplitude of the AC voltage is \(V_{ac} = 50\) mV for these measurements.

With \(V_{dc} = 0\) we find a peak in \(R\) at \(f = 179.3\) kHz, associated with the fundamental frequency of the decoupled beams. The FWHM of the peak is 0.6 kHz, close to the standard deviation found in the distribution of the measured fundamental frequencies. This leads us to the conclusion that the width of the response peak at \(V_{dc} = 0\) is dominated by inhomogeneous broadening caused by the non-uniformity of the array.

As we increase \(V_{dc}\) we observe a gradual increase in the frequency range where relatively large response is observed; we associate this with the formation of a band of collective modes. The lower frequency bound of this range \(f_b = \omega_b/2\pi\) (the bottom of the band) for relatively small \(V_{dc}\) is given theoretically by \(f_b = (\omega_0/2\pi) \left( 1 - C''(a) V_{dc}^2/m\omega_0^2 \right)\). A least squares fit to the measured data (see dashed line in Fig. 5) yields \(C''(a)/m\omega_0^2 = 2.7 \times 10^{-4}\) V\(^{-2}\).
For comparison we derive below a rough order of magnitude estimate of this factor. We substitute $m$, which should represent an effective mass of each beam, by the actual mass and we use the approximation $C''(a) \simeq \varepsilon_0 b/g^2$, where $\varepsilon_0$ is permittivity of free space and $g$ is the gap between neighboring beams. These crude approximations yield a value of $1.6 \times 10^{-4}$ V$^{-2}$ which is quite close to the value deduced from the experimental data. The upper frequency bound of the band, on the other hand, shows some discrepancy with theory. While the measured value depends on $V_{dc}$, our simple model predicts a upper value that is fixed. We obtained similar behavior of this upper frequency with orthogonal polarization, and with another sample of similar design.

The rich and detailed structure of the frequency-dependent response $R$ observed in our experiments is not fully understood. Experimentally, Figure 5 shows that $R$ is oscillatory as a function of frequency, and has a relatively large magnitude close to the lower limit of the band. Theoretically this frequency dependence can be found from Eq. (1) and the density of states (which is determined by the dispersion relation). This leads to an approximated proportionality relation $R \sim A_m^2 \cos (k_m a/2) / \sin^3 (k_m a/2)$ for even $m$. As discussed above, however, our simple linear theory is not adequate for predicting the steady-state response $A_m$. Comparison with experiment leads to the following conclusions: (a) For small $m$ $A_m$ goes to zero faster than $m^{3/2}$. (b) Near the lower bound of the band ($k_m a \rightarrow \pi$) $A_m$ grows faster than $\sqrt{1/\cos (k_m a/2)}$. (c) The spiky behavior indicates that $A_m$ is not a monotonic function of $m$. These intriguing details of the device response warrant further investigation.

Electrically tunable arrays offer unique prospects for opto-mechanical signal processing devices such as tunable filters and optical modulators. A particularly intriguing example is opto-mechanical spectral analysis of electrical waveforms. Consider an arbitrary electrical signal applied between the two comb electrodes of our device. Its Fourier components falling within the vibrational band (formed by the DC electrostatic coupling as described above) will parametrically drive the collective modes of the array. Each of these excited modes will result in a diffracted order with strength directly proportional to the respective Fourier component. Since each order is diffracted at a characteristic angle, continuous,
real-time spectral analysis of the applied waveform can be realized using a photodetector array. In principle, by scaling the size of the resonant beams downward into the realm of NEMS (nano electromechanical systems), the operating frequency of such a device can be extended to very high frequencies [15].

In conclusion, we have demonstrated the ability to induce and control collective modes of mechanical vibration within an artificial mesoscopic lattice. Further experimental and theoretical work will elucidate what appear to be rich dynamics in such systems, and will allow their optimization for novel micro opto-mechanical device applications.
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FIGURES

FIG. 1. The device is fabricated using bulk micromachining techniques. In steps (a) and (b) a suspended membrane of silicon nitride is formed. A gold beam is fabricated on top of the membrane (c) and the membrane is etched, leaving the beam suspended (d). A side view micrograph of the device is seen in (e).

FIG. 2. (a) Response peak of an individual beam with a resonance frequency of 179.28 KHz. (b) Normalized resonance frequencies of all beams in the array \( (\omega_0 - < \omega_0 >)/ < \omega_0 > \), where \(< \omega_0 >\) is the average value.

FIG. 3. (a) A model of \( N \) coupled pendulums. (b) The shape of the three lowest modes. (c) The dispersion relation between the frequency \( \omega_m \) of each mode and the wave vector \( k_m \).

FIG. 4. The optical setup for measurements of the 1st order diffraction peak of the grating. Optical fibers are employed to deliver light to and from the vacuum chamber where the sample is mounted. The DC voltage \( V_{dc} \) introduces coupling between the beams and the AC voltage \( V_{ac} \) is used to parametrically excite the modes of vibration. A lock-in amplifier is employed to measure the response. The inset shows the DC signal of the photo detector as a function of wavelength (for \( V_{dc} = V_{ac} = 0 \)).

FIG. 5. A color map showing relative signal intensities measured at the lock-in amplifier operating in a 2\( f \) mode. The dependence upon both the voltage \( V_{dc} \) and the frequency of the AC voltage \( f \) are shown. The wavelength is tuned to the diffraction peak, namely \( \lambda = 1582 \) nm. The dashed white line shows a fit to the measured lower bound frequency \( f_b \) of the band.