Comment on

“Analysis of General Power Counting Rules in Effective Field Theory”

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Abstract

In a recent paper [1] a master formula has been presented for the power counting of a general effective field theory. We first show that this master formula follows immediately from the concept of chiral dimensions (loop counting), together with standard dimensional analysis. Subsequently, [1] has disputed the relevance of chiral counting for chiral Lagrangians, and in particular for the electroweak chiral Lagrangian including a light Higgs boson. As an alternative, a power counting based on ‘primary dimensions’ has been proposed. The difficulties encountered with this scheme led the authors to suggest that even the leading order of the electroweak chiral Lagrangian could not be clearly defined. Here we demonstrate that the concept of primary dimensions is irrelevant for the organization of chiral Lagrangians. We re-emphasize that the correct counting is based on chiral dimensions, or the counting of loop orders, and show how the problems encountered in [1] are resolved.
1 Introduction

The electroweak chiral Lagrangian including a light Higgs boson has been developed as a systematic effective field theory (EFT) for the case of strong dynamics in the Higgs sector in a series of papers [2–5], building on previous work by many authors [6–31]. Phenomenological implications of the formalism, in particular to test anomalous Higgs couplings at the LHC, have been discussed in [32, 33].

In order to define the electroweak chiral Lagrangian as a systematic EFT, it is necessary to specify basic assumptions. These concern the particle content below a mass gap (assumed to be of the order of TeV), the relevant symmetries, and the power counting.

In [2–5] the following assumptions were made:

(i) SM particle content, where (transverse) gauge bosons and fermions are weakly coupled to the Higgs-sector dynamics.

(ii) SM gauge symmetries; conservation of lepton and baryon number; conservation at lowest order of custodial symmetry, CP invariance in the Higgs sector and fermion flavour. The latter symmetries are violated at some level, but this would only affect terms at subleading order. Generalizations may in principle be introduced if necessary.

(iii) Power counting by chiral dimensions [24, 26], equivalent to a loop expansion [4], with the simple assignment of 0 for bosons (gauge fields $X_\mu$, Goldstones $\varphi$ and Higgs $h$) and 1 for each derivative, weak coupling (e.g. gauge or Yukawa), and fermion bilinear:

$$d_X[X_\mu, \varphi, h] = 0, \quad d_X[\partial_\mu, g, y, \bar{\psi}\psi] = 1$$

The total chiral dimension $d_X$ of a term in the Lagrangian determines its loop order $L$ through $d_X = 2L + 2$.

This picture for the electroweak chiral Lagrangian including a light Higgs has recently been questioned in [1], however, without addressing the EFT assumptions in detail (see also [34–36]). In particular, the relevance of chiral dimensions for the EFT power counting has been disputed, even for the standard case of pion chiral perturbation theory ($\chi$PT). In the following we critically examine the results and arguments presented in [1]. We show that the framework of the electroweak chiral Lagrangian as outlined above is consistent and we emphasize that chiral counting is the relevant counting in this case. We also take the opportunity to illustrate the workings of chiral dimensions in a toy scenario of pions, photons and (heavy) leptons.

2 Master formula for power counting

In the first part of [1] the derivation of general power counting rules for effective field theory is reviewed and a master formula is presented. Here we show how this result follows from the counting of chiral dimensions, together with standard dimensional analysis.
We assume an effective theory for light scalars $\phi$, gauge fields $A$ and fermions $\psi$ at a typical scale $f$ much below a cutoff $\Lambda$ with gauge, Yukawa and quartic scalar couplings $g$, $y$ and $\lambda$. We will work in $d = 4$ dimensions. Generalizations to arbitrary dimensions are possible but inessential for our discussion.

A generic term in the EFT Lagrangian can then be written as

$$\partial N^p \phi^N \psi^N g^N g^N y^N \lambda^N$$

(2)

The task of power counting is to determine the (parametric) size of the coefficient of this term in the Lagrangian. Using the results of [4], the loop order $L$ of the coefficient, or equivalently the power of $1/(16\pi^2)$, is given by the chiral dimension $d_{\chi} \equiv 2L + 2$ of the term. The canonical dimension $d_{c}$ of the term and the requirement that the Lagrangian has dimension 4 then fixes the dependence on the dimensionful parameter $f$. Loop factors are thus defined, as usual, with respect to the typical scale of the theory. The coefficient of the term in (2) then reads

$$f^{4-d_{c}}\left(\frac{4\pi}{f}\right)^{d_{\chi}-2}$$

(3)

up to a factor of order unity. The chiral dimension as defined in [4] and the canonical dimension are given by

$$d_{\chi} = N_p + \frac{N_\psi}{2} + N_g + N_y + 2N_\lambda$$

$$d_{c} = N_p + N_\phi + N_A + \frac{3}{2}N_\psi$$

(4)

Assuming $\Lambda \equiv 4\pi f$ [11], the product of the coefficient (3) and the operator (2) can be written as

$$f^{2}A^{2}\left[\frac{\partial}{\Lambda}\right]^{N_p} \left[\psi\right]^{N_\psi} \left[\frac{A}{f}\right]^{N_A} \left[\frac{\psi}{\sqrt{\Lambda}}\right]^{N_\psi} \left[\frac{g}{4\pi}\right]^{N_g} \left[\frac{y}{4\pi}\right]^{N_y} \left[\frac{\lambda}{16\pi^2}\right]^{N_\lambda}$$

(5)

up to factors of order unity. This result is equivalent to the master formula quoted in eq. (28) of [1]. Eliminating $f = \Lambda/4\pi$, (5) takes the form

$$f^{2}A^{2}\left[\frac{\partial}{\Lambda}\right]^{N_p} \left[\frac{4\pi\phi}{\Lambda}\right]^{N_\phi} \left[\frac{4\pi A}{\Lambda}\right]^{N_A} \left[\frac{4\pi\psi}{\Lambda^{3/2}}\right]^{N_\psi} \left[\frac{g}{4\pi}\right]^{N_g} \left[\frac{y}{4\pi}\right]^{N_y} \left[\frac{\lambda}{16\pi^2}\right]^{N_\lambda}$$

(6)

identical to eq. (22) of [1] and equivalent to the result already obtained in [4].

We emphasize that the concept of chiral dimensions ensures that all the terms in the leading-order Lagrangian are homogeneous. The notion of chiral number $N_{\chi} \equiv N_p + N_\psi/2$ used in [1] is different as the number of couplings is not considered.

Beyond the case of (6), the $4\pi$ counting may be generalized, as discussed in [1]. This generalisation consists in an independent rescaling of each of the conserved quantities

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1In some models of EWSB the electroweak scale $v$ differs from the breaking scale $f$. However, for power-counting considerations, both may consistently be taken to be of the same order $v \sim f \ll \Lambda$. 

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g, y, λ by an arbitrary factor of \((4\pi)^\nu\), with the power \(\nu\) not necessarily an integer. Such a rescaling is equivalent to keeping track of the number of each of these couplings separately. It then allows one to expand results in powers of \(g\), \(y\) or \(\lambda\) in addition to the EFT expansion. It is already clear from (5) that this can always be done if desired. Generically, however, the weak couplings \(g\), \(y\), \(\lambda\) can be taken to be of \(\mathcal{O}(1)\), and no separate expansion is required.

It should be emphasized that (5) and (6) encode the topological constraints of a consistent power counting for a generic perturbative theory. However, in addition to this, some dynamical information needs to be provided in order to define the full systematics of the expansion. This systematics is different depending on whether the underlying dynamics is weakly or strongly coupled. To define it, the role of the couplings \(g\), \(y\), \(\lambda\), which enter the master formula (6) in a nontrivial way, has to be specified.

As a first example, assume that all fields are weakly coupled to the heavy sector, such that a weak coupling can be associated with every field in the master formula. Eq. (6) then becomes

\[
\frac{\Lambda^4}{\kappa^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{\kappa \phi}{\Lambda} \right]^{N_\phi} \left[ \frac{\kappa A}{\Lambda^{3/2}} \right]^{N_A} \left[ \frac{\kappa \psi}{4\pi} \right]^{N_\psi} \quad (7)
\]

In order to simplify the notation we have introduced a generic weak coupling \(\kappa\), which may stand for \(g\), \(y\) or \(\sqrt{\lambda}\), as appropriate. To obtain (7) we have simply pulled a factor of \(\kappa/4\pi\) in front of every field, and written an overall factor of \((\kappa/4\pi)^{-\nu}\) to keep the canonical normalization of the kinetic terms. The nontrivial dynamical assumption here is that the residual number of couplings is \(N_\kappa' \geq 0\). Setting for the moment \(N_\kappa' = 0\), (7) is then seen to reduce to the standard power counting by canonical dimensions, where higher-dimensional operators are suppressed by inverse powers of a large energy-scale \(\Lambda\). For a given order in \(1/\Lambda\), \(N_\kappa'\) counts the number of loop corrections.

As a second example, consider the chiral perturbation theory of pions interacting with the photon field (see [4] for a more detailed discussion). In this case the pions are strongly coupled to the heavy sector and cannot be associated with a weak coupling that would multiply each field. The photon, on the other hand, is still weakly coupled \(\sim \varepsilon\) and (6) becomes

\[
\frac{\Lambda^4}{16\pi^2} \left[ \frac{\partial}{\Lambda} \right]^{N_p} \left[ \frac{\phi}{f} \right]^{N_\phi} \left[ \frac{A}{f} \right]^{N_A} \left[ \frac{\varepsilon}{4\pi} \right]^{N_\varepsilon} \quad (8)
\]

Here \(f\) can be identified with the pion decay constant, related to the cutoff \(\Lambda \equiv 4\pi f\) through a (loop) factor \(4\pi\) \([11]\).

The EFT described by (8) is valid at energies of order \(f \ll \Lambda\) and the expansion parameter is \(f^2/\Lambda^2 = 1/16\pi^2\), corresponding to a loop factor. With this identification the EFT can be viewed as being organized in terms of a loop expansion, equivalent to a counting of chiral dimensions. As an example, consider the operator \(e^2 \langle U^\dagger Q U Q \rangle\),

\[\text{In Ref. [1] the distinction between weak and strong coupling refers to the size of the couplings } g, y, \lambda. \text{ However, this only means that the theory is inside or outside the perturbative regime. What really determines the counting is the nature of the underlying dynamics.} \]

\[\text{The important point is that this theory is renormalizable only order by order. The loop expansion guarantees this renormalizability, while an expansion by canonical dimension does not.}\]
where $Q$ is the quark electric charge and $U$ the Goldstone matrix. This term represents a radiatively induced pion potential and contains the electromagnetic correction to the pion mass. According to (8) its coefficient is $(\Lambda^4/16\pi^2)(e/4\pi)^2 = e^2 f^4$, corresponding to a leading-order coefficient for $e = \mathcal{O}(1)$. An identical result is obtained from counting the chiral dimension of this operator, which is $d_\chi = 2$ from the factor $e^2$. Of course, the equivalence of (8) with the counting of chiral dimensions holds in general, as has been demonstrated above.

The following conclusions can be drawn:

- There are essentially two different expansions that may be used to organize a low-energy EFT: The expansion in inverse powers of a high-energy scale and the loop expansion. They are governed by the canonical dimension and the chiral dimension, respectively, of operators in the Lagrangian. In addition, a separate expansion in powers of weak couplings $\kappa$ can be performed. This expansion is a special case of the more general scenario which treats $\kappa$ as a quantity of order one. It is independent of the loop expansion. The above possibilities of organizing an EFT have been discussed in [4].

- Both (7) and (8) are obtained as special cases from the master formula in (6). Nevertheless they represent a different organization of the respective EFT: (7) by canonical dimensions, (8) by chiral counting. The difference is related to the role of (weak) couplings that needs to be specified in addition to (6).

- The power counting of chiral perturbation theory and its extensions, e.g. when coupled to photons, is governed by chiral dimensions (see also Sec. 5 below). The consistency of the framework can be proven by comparison with the explicit determination of the one-loop effective action [26].

- The counting of weak couplings due to their chiral dimension may seem unfamiliar. However, it can be relevant even in the context of the usual standard-model operators of dimension 6 [37, 38]. Consider for example the Higgs-gluon operator $H^\dagger HG_{\mu\nu}G_{\mu\nu}$. Dimensional counting alone only implies that the coefficient is $\sim 1/\Lambda^2$. Assuming now that all fields are weakly coupled implies that the operator comes with a factor of $y^2 g_s^2$ (or equivalent combinations in terms of chiral dimensions). The master formula (6) then yields

$$\frac{1}{16\pi^2} \frac{1}{\Lambda^2} y^2 g_s^2 H^\dagger H G_{\mu\nu} G^{\mu\nu}$$

Therefore, under the assumption of weak coupling to the heavy physics, the coefficient is suppressed by an additional loop factor, as has been discussed in [38, 39]. Since $1/(16\pi^2 \Lambda^2) = 1/(16\pi^2)^2 f^2$, this double suppression amounts (effectively) to a suppression by two loop orders (even though the operator may be induced by a one-loop diagram with internal particles of mass $\Lambda$ [40, 41]). Note that this double suppression is correctly reproduced by the chiral dimension of $y^2 g_s^2 H^\dagger H G_{\mu\nu} G^{\mu\nu}$, which amounts to $d_\chi = 2L + 2 = 6$. 


The electroweak chiral Lagrangian with a light Higgs implies strongly-coupled dynamics and therefore is organized through a loop expansion, with a power counting in terms of chiral dimensions as defined in eq. (4) [4]. A priori, the electroweak vev \( \nu \) and the scale of the strong dynamics \( f \) need not be distinguished for the purpose of power counting. This is equivalent to treating \( \xi = \nu^2/f^2 \) as a quantity of order one. The EFT is then very similar to chiral perturbation theory with pions and photons. However, a further expansion in powers of \( \xi \) can still be performed. This results in an EFT with a double expansion in both loop order and in canonical dimensions, as explained in [5].

3 Cross sections

Section IV of [1] considers the power counting of cross sections. For the case of pion scattering in chiral perturbation theory, the following counting rules are quoted

\[
\sigma_k(\varphi\varphi \to \varphi\varphi) \sim \frac{\pi(4\pi)^2}{E^2} \frac{E^4}{\Lambda^4}
\]

(10)

\[
\sigma_4(\varphi\varphi \to \varphi\varphi) \sim \frac{\pi(4\pi)^2}{E^2} \frac{E^8}{\Lambda^8}
\]

(11)

\[
\sigma_k(\varphi\varphi \to 4\varphi) \sim \frac{\pi(4\pi)^2}{E^2} \frac{E^8}{\Lambda^8}
\]

(12)

The amplitudes for the scattering of pions \( \varphi \) are evaluated at tree level with interactions from the Lagrangian of \( \mathcal{O}(p^2) \) (\( \sigma_k \)) or \( \mathcal{O}(p^4) \) (\( \sigma_4 \)). Factors of energy or momentum are denoted by \( E \), and \( \Lambda = 4\pi f \) is the EFT cutoff.

We agree with the results in (10) – (12), but would like to comment on the interpretation given in [1]. In that paper, a comparison is made between (11) and (12). The fact that their scaling with \( \Lambda \) is identical, despite the different chiral counting of the underlying interaction, is taken as an indication against the relevance of chiral counting in this context. We disagree with such a conclusion. First, (11) and (12) refer to different processes. Since the number of final-state particles is different, the phase-space factors are not the same. It is then unclear what should be inferred from such a comparison. More meaningful is the comparison between (11) and (12), which represent different contributions to the same process. However, the next-to-leading correction \( \Delta \sigma_{k,4} \) to the leading-order cross section (10) is not given by (11), but by the interference of the leading-order amplitude from \( \mathcal{L}_2 \) and the next-to-leading order term from \( \mathcal{L}_4 \), leading to

\[
\Delta \sigma_{k,4}(\varphi\varphi \to \varphi\varphi) \sim \frac{\pi(4\pi)^2}{E^2} \frac{E^6}{\Lambda^6}
\]

(13)

The correction is of order \( E^2/\Lambda^2 \equiv E^2/16\pi^2 f^2 \) relative to (11). This is exactly of the size expected from chiral counting where the next-to-leading terms are suppressed by a loop factor. Since \( \Lambda = 4\pi f \), the presence of inverse powers of \( \Lambda \) in the formulas above is in agreement with the loop expansion and the chiral counting. The statement in the abstract
of [1] that “the size of cross sections is controlled by the Λ power counting of EFT, not by chiral counting, even for chiral perturbation theory” is therefore not justified.

The latter conclusion can be made even more explicit by noting that the formulas in (10) – (12) can indeed be obtained through the counting of chiral dimensions. For this purpose we employ the optical theorem, also considered in [1], which we write schematically as

$$\sigma(i \to f) \sim \frac{1}{E^2} \text{Im}\mathcal{M}(i \to f \to i)$$

(14)

The derivation of (10) may suffice as an example. In this case, the forward amplitude $\mathcal{M}$, with $i = f = \varphi\varphi$, is a one-loop amplitude with two vertices from the $O(p^2)$ Lagrangian, and thus an $O(p^4)$ term, according to chiral counting (chiral counting reduces to momentum counting in pion chiral perturbation theory). We then have ($p \sim E$)

$$\mathcal{M}(i \to f \to i) \sim E^4$$

(15)

Since $\mathcal{M}$ is dimensionless, the energy dependence has to be $(E/f)^4$. The compensating scale can only be $f$, not $\Lambda$, as the loop amplitude in the low-energy EFT depends only on the physics in the IR, not in the UV. In addition, a chiral dimension of $d_\chi \equiv 2L + 2 = 4$ implies a loop order of $L = 1$, and thus a factor of $1/16\pi^2$ for the coefficient in (15). Therefore,

$$\mathcal{M}(i \to f \to i) \sim \frac{E^4}{f^4} \frac{1}{16\pi^2}$$

(16)

Inserting (16) in (14) and using that the imaginary part yields a factor of $\pi$, one finds

$$\sigma_k(\varphi\varphi \to \varphi\varphi) \sim \frac{\pi}{E^2} \frac{E^4}{f^4} \frac{1}{16\pi^2}$$

(17)

which is equivalent to (10). The remaining formulas, and similar ones, can be derived along the same lines.

We finally comment on the counting of $E/f$ factors in chiral perturbation theory. In general, these factors can be taken to be of order unity, corresponding to $E \sim f$, where $f \ll \Lambda$ is the energy scale at which the EFT is valid. Such a counting is fully consistent with the loop expansion and therefore with the order-by-order renormalization of the chiral Lagrangian. If $E$ is assumed to be numerically larger than $f$, $E^2/f^2$ terms may be considered to be enhanced with respect to other terms of the same chiral order such as $m_\pi^2/f^2$. Any result based on chiral counting can then be further approximated exploiting this enhancement. However, this does not invalidate the (more general) results obtained under the assumption $E \sim f$. In any case, the enhancement of $E$ is limited by the requirement that $E \ll \Lambda$.

4 Electroweak chiral Lagrangian

The electroweak chiral Lagrangian including a light Higgs, referred to as HEFT in [1], is discussed in section V of that paper. We disagree with several of the assertions made
there. In the following we will address the most relevant points.

- The authors of [1] introduce the concept of primary dimension $d_p$ as an ordering principle for chiral Lagrangians. The primary dimension of a quantity $R$ is defined as the canonical dimension of the first nonvanishing term in an expansion of $R$ in powers of field variables. For instance, the Goldstone matrix $U = \exp(2i\Pi^a T^a/f)$, $T = U\sigma_3 U^\dagger$ and $V_\mu = (\partial_\mu U)U^\dagger$ have $d_p$ equal to 0, 0 and 2, respectively.

We argue that the primary dimension $d_p$ is irrelevant for the power counting of chiral Lagrangians. This follows immediately from a simple counterexample. Consider the operators

$$-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \quad g'g\langle UT_3 B_{\mu\nu}U^\dagger W^{\mu\nu}\rangle$$

of the (Higgsless) electroweak chiral Lagrangian, where the first is the $B$-field kinetic term, and the second corresponds to the electroweak $S$-parameter. These operators enter the chiral Lagrangian at leading and next-to-leading order, respectively [9, 10]. The concept of primary dimension is unable to reproduce this crucial distinction since both terms are assigned $d_p = 4$. Consequently, counting primary dimensions does not lead to the correct power counting for the electroweak chiral Lagrangian (with or without light Higgs).

Table II of [1] lists operators of the electroweak chiral Lagrangian together with their primary dimension $d_p$, canonical dimension $d$ and $N_\chi$ (the chiral dimension of a term up to the contribution of any weak couplings). Like $d_p$, neither $d$ nor $N_\chi$ can distinguish the order of the terms in (18), yielding $d = 4$ and $N_\chi = 2$ for both.

The problem is resolved with the rules of chiral counting, which imply a chiral dimension of $d_\chi = 2$ for the gauge kinetic term, and $d_\chi = 4$ for the $S$-parameter term, indicating the correct order of the terms in the chiral Lagrangian. As this example shows, keeping track of the chiral dimensions of the weak couplings is essential to get the right counting.

- We disagree with the assertion made in the first column of page 11 in [1] that $U = \exp(2i\Pi^a T^a/f)$, $V_\mu$ or $T$ contain “hidden factors of $\Lambda$”, with $\Lambda = 4\pi f$ the cutoff of the chiral Lagrangian. This view contradicts basic EFT principles, according to which the physics at high energy is fully contained in operator coefficients, whereas EFT field variables, such as $U$, encode the low-energy (IR) dynamics and are independent of the UV.

- In an attempt to construct the leading-order of the electroweak chiral EFT including a light Higgs, eq. (62) of [1] defines a Lagrangian $\mathcal{L}_{d_p \leq 4}$ that collects the terms with $d_p \leq 4$. The resulting expression is incorrect for several reasons.

(i) Unlike the LO gauge-kinetic terms $X_{\mu\nu}^2$, the operators $g^2 X_{\mu\nu}^2 h$ arise only at NLO. The latter operators could only appear at leading order if the gauge
fields were strongly coupled to the heavy sector, which is not the case and would in fact be inconsistent. The expression $\mathcal{L}^{d_p \leq 4}$ fails to account for this important feature, listing the above interaction terms at the same (leading) order.

(ii) The Goldstone kinetic term $-(f^2/4)\langle V_\mu V^\mu \rangle$ (which comes with an incorrect sign in [1]) implies a $W$-boson mass of $M_W = gf/2$, rather than the correct value $M_W = gv/2$. The problem arises since the presentation in [1] does not carefully distinguish the electroweak vev $v$ from the new physics scale $f$.

(iii) The Higgs potential and the Higgs mass scale as $\Lambda^2$, while the size of the coefficient is left unspecified. Chiral counting dictates that the chiral dimension of this coefficient has to be taken into account. Consistency requires that the coefficient carries a chiral dimension of 2, implying an electroweak-scale mass for the light Higgs. One explicit realization of this occurs in composite-Higgs models where the Higgs potential is generated radiatively. It then comes with two powers of weak couplings ($g^2$, $y^2$) and a corresponding loop suppression of the scale $\Lambda^2$. We note that there is no ambiguity here: The chiral dimension of the coefficient cannot be ignored, otherwise the Higgs mass would be of the size of the cutoff $\Lambda$ and it would be inconsistent to include the Higgs as a field in the EFT. The assumptions spelled out in Sec. I reflect a similar consistency requirement for the gauge-boson and fermion sectors.

- The comments following eq. (65) of [1] suggest to associate the operators

$$e^2 \langle F_{R\mu\nu}^2 + F_{L\mu\nu}^2 \rangle, \quad e^2 \langle U^\dagger F_{R\mu\nu} U F_{L\mu\nu} \rangle \quad (19)$$

of chiral perturbation theory with the leading-order photon kinetic term in the power counting. This contradicts the well-known fact that these operators enter the Lagrangian as counterterms at next-to-leading order [24]. Again, the correct classification follows immediately from the chiral dimension $d_\chi = 4$ of these terms, whereas the primary dimension fails to capture the difference between (19) and the leading-order kinetic term ($d_p = 4$ for both).

- At the end of Sec. V in [1] various attempts to define a leading-order Lagrangian are considered, in particular based on the criteria $N_\chi \leq 2$ or $d_p \leq 4$, or else using a simultaneous counting in both $N_\chi$ and $d_p$. As discussed above, none of these options is valid.

5 A toy model

To illustrate how the power counting based on chiral dimensions works in a setting where Goldstone dynamics is coupled to gauge fields and fermions, the following example may be considered. Take the pions of QCD, together with a light lepton $\psi$ of mass $m$ (e.g. of order 100 MeV) and a heavy lepton $\Psi$ of mass $M$, gauged in the usual way under
the electromagnetic $U(1)$. A theory of this type, with quarks that are either electrically charged or neutral, has also been discussed in [1]. Here we assume the realistic case of charged quarks as the difference to neutral quarks is immaterial for our discussion. The pion dynamics has a cutoff $\Lambda = 4\pi f$ of order 1 GeV. The QED coupling is weak, $e = \mathcal{O}(1)$. Whereas in nature $e \approx 0.3$, it will be illuminating to also imagine a toy scenario where $e = 1$ numerically. Perturbativity in QED would still hold but a further expansion in $e$ could no longer be performed.

In addition to $\Lambda$ the theory contains another large scale $M$. It is true, as mentioned in [1], that $\Lambda$ and $M$ are in principle unrelated. Three possibilities may then be distinguished: $M \ll \Lambda$, $M \sim \Lambda$, and $M \gg \Lambda$.

In the first case, the lepton $\Psi$ cannot be integrated out and will simply remain an explicit degree of freedom of the low-energy EFT.

In the second case, the scales $M$ and $\Lambda$ can be identified for the purpose of power counting. While their physical origin might be very different, the ratio $\Lambda/M$ is a number of order unity. When the physics at $M \sim \Lambda$ is integrated out, $\Lambda/M$ will be encoded in the $\mathcal{O}(1)$ coefficients of the EFT Lagrangian. This is the standard case for a bottom-up construction of the EFT, in which the details of the physics at the cutoff are unknown.

In the third case, the effects of $\Psi$ in the EFT coefficients would be suppressed by powers of $\Lambda/M \ll 1$. Some coefficients might tend to zero or other simplifications might occur. However, no information will be lost in the usual formulation of the EFT with general coefficients.

The issue of a possible hierarchy among two (or more) different heavy mass scales is clearly not restricted to the case of a chiral Lagrangian. It could also arise for weakly-coupled theories where the low-energy EFT is organized by canonical dimensions. Suppose such an EFT describes physics at a scale $v \ll \Lambda_1 \ll \Lambda_2$. The essential feature is the mass gap between $v$ and $\Lambda_1$. A general expansion in powers of $1/\Lambda_1$ will provide the most general set of higher-dimensional operators, where $\mathcal{O}(1)$ coefficients encode the physics at energies $\sim \Lambda_1$ and above, irrespective of its details and of the presence of a further scale $\Lambda_2$. What matters is the particle content, the symmetries, and the power counting.

The authors of [1] insist that when a theory contains both strongly-coupled and weakly-coupled dynamics, there is no unified counting. If this were true, it would imply that interactions of pions with photons are beyond an EFT description. The covariant derivative $D_\mu U = \partial_\mu U + ieA_\mu[Q, U]$ unavoidably mixes the strongly-coupled and the weakly-coupled sector. The combined dynamics can still be consistently organized in terms of a loop expansion. As a result, we disagree with the statement in Sec. V of [1] that “it is not helpful to force both sectors into a unified counting with a single expansion parameter”. On the contrary, a well-defined counting is an essential ingredient of any bottom-up EFT.

One can use the previous example to illustrate how the power-counting formula works depending on whether the dynamics is strongly or weakly coupled. At scales $f \ll \Lambda, M$, the heavy fermion $\Psi$ can be integrated out. The EFT Lagrangian will contain (among

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\footnote{Electromagnetism is perturbative not because $e$ is numerically small, but because of the loop expansion, with $e = \mathcal{O}(1)$.}
The first line contains leading-order terms, scaling as $f^4$ at energies of order $f$ (the scale where the EFT is valid). The second line displays the well-known operators from the Euler-Heisenberg (EH) Lagrangian (see e.g. [42]), arising once the heavy lepton $\Psi$ has been integrated out. These $(F_{\mu\nu})^4$ operators scale as $f^8$ from dimensional analysis.

Consider first a theory without pions. The EH operators are then catalogued as $d = 8$ with one loop suppression, where

$$a_1 = -\frac{1}{36(16\pi^2)M^4}, \quad a_2 = \frac{7}{90(16\pi^2)M^4}$$

(21)

One can easily check that both the right dimensional scaling and the number of loop suppressions follow straightforwardly from eq. (7). The coefficients of the local operators can be determined exactly because we know the UV physics that has been integrated out. In other words, this is an example of a top-down EFT.

When pions are present and $M \sim \Lambda = 4\pi f$ one should use instead eq. (8) as the power-counting formula. Additional operators appear, mixing the dynamics of pions and photons (and light fermions $\psi$). According to our previous discussion, this EFT should now be organized with chiral dimensions. This might seem surprising because the operators shown in the second line of (20) do not contain explicit pion fields. However, we note that those operators have now an additional contribution coming from 3-loop pion exchange diagrams, which require $a_{1,2}$ as counterterms. This is precisely what eq. (8) indicates: counting $M \sim \Lambda = 4\pi f$, the operators $e^4(F_{\mu\nu})^4$ are suppressed by three powers of the loop factor $1/(16\pi^2)^3$ with respect to the leading order. Adding the pions has therefore changed the dynamics and the expansion of the EFT. The EFT is now a bottom-up one and the coefficient of $(F_{\mu\nu})^4$ operators contains hadronic dynamics parametrically of the same size as the contribution of $\Psi$.

This result is readily obtained using the counting of chiral dimensions. We are given the operator $e^4(F_{\mu\nu})^4$ within the EFT of the model above. The problem is to find the magnitude of the coefficient. First, we note that $(F_{\mu\nu})^4$ has to come with at least four powers of the coupling $e$. This is because we know that each photon field is weakly coupled $\sim e$ to the heavy sector that has been integrated out. The chiral dimension of this operator is 8 (4 derivatives and 4 couplings). $d_\chi \equiv 2L + 2 = 8$ implies loop order $L = 3$. This gives an estimate for the coefficient of $1/(16\pi^2)^3$, up to factors of order unity and $1/f^4$ from dimensional analysis, in agreement with the explicit result in (20), (21).

It is important to realize the conceptual difference between a bottom-up and a top-down construction of an EFT. In the top-down case the physics in the UV is known and the EFT is constructed as its low-energy approximation. Clearly, all the details of the EFT coefficients are then known explicitly, as seen for instance in (21). The situation
Figure 1: Sample diagrams for $\pi^+\pi^- \to \pi^+\pi^-$ scattering in chiral perturbation theory coupled to photons. The first line displays leading-order, the second line next-to-leading order contributions in the chiral counting.

is different for a bottom-up EFT, where only the particle content, the symmetries and a power counting are specified, but the detailed dynamics in the UV is unknown. This is the case of the electroweak chiral Lagrangian.

For the toy scenario discussed here, the bottom-up perspective would be the construction of the EFT from the pions, the photon and the light lepton. Nothing would be known about the existence of the heavy lepton $\Psi$. Nevertheless, the operators $e^4 (F_{\mu\nu})^4$ would be written at 3-loop order in the EFT. This would be consistent with their appearance in the concrete case of (21), or with any similar case where $\Psi$ would be substituted by other heavy degrees of freedom.

It is also instructive to see how the EFT counting based on chiral dimensions works for pion scattering within the toy model above (where $e = 1$ may be chosen). The model corresponds to pion chiral perturbation theory coupled to photons and is conceptually similar to the electroweak chiral Lagrangian. An interesting aspect here is the interplay of the pions, arising from a strongly coupled sector, with the weakly coupled photons. This situation can still be treated by a loop expansion, which is automatically consistent with the renormalization of the EFT [24]. The resulting systematics is illustrated for $\pi^+\pi^- \to \pi^+\pi^-$ scattering at leading and next-to-leading order in Fig. 1.

The first line shows leading-order amplitudes, which are of order unity since $p \sim f$ and $e = \mathcal{O}(1)$. They have chiral dimension $d_\chi = 2$. The loop diagrams and local counterterms in the second line are suppressed by a loop factor. All of them consistently carry a chiral dimension of $d_\chi = 4$. This scheme provides a consistent counting, irrespective of whether the interactions come from pion dynamics or from photon exchange. In particular, the two leading-order contributions are allowed to be numerically of similar size. This does not preclude, however, the possibility to make further approximations if e.g. $p^2/f^2$ is numerically larger than $e^2$, either because $e \ll 1$, or because $p$ is somewhat larger than $f$. 

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Indeed, a possible approximation, often considered in practice, would be to put $e \to 0$, in which case pure chiral perturbation theory would be recovered. Yet the picture in Fig. 1 based on chiral counting remains a consistent and general starting point.

We remark that the $SU(2)$ gauge coupling $g \approx 0.6$ in the electroweak EFT is numerically not far from 1. For electroweak-scale processes such as Higgs decays typical momentum factors $\sim p/v$ and $g$ are of comparable order, even numerically. It is in particular not necessary, and would in fact be inconvenient, to expand in powers of $g$ or $M_W/v \sim g$. Chiral counting takes this feature automatically into account.

6 Conclusions

We have shown that the topological master formulas derived in [1] for generic effective field theories are equivalent to the ones already discussed by some of us in previous papers [2–4]. In those papers it was already emphasized that a topological analysis amounts to a unique assignment of chiral dimensions to fields and couplings. However, it is important to realize that topological master formulas are not power-counting formulas by themselves: one still needs to specify the nature of the underlying dynamics. As a result, the topological master formulas can lead to expansions in canonical dimensions or chiral dimensions, depending on whether the underlying dynamics is weakly or strongly coupled. In [1] the choice of dynamics is not carefully spelled out and the concept of chiral dimensions incorrectly implemented, since couplings are not given chiral dimensions. This leads the authors of [1] to cast doubts on the role of chiral dimensions in scenarios with strongly-coupled dynamics and eventually prompts them to introduce the notion of primary dimensions. In this comment we have shown that primary dimensions lead to serious inconsistencies and cannot be a valid organizing criterion for an EFT expansion. Instead, all the inconsistencies are dispelled once chiral dimensions are correctly implemented. We have illustrated the workings and the usefulness of chiral dimensions with various examples including the loop-suppression of certain dimension-6 operators in the SM, chiral perturbation theory with pions and photons, the Euler-Heisenberg Lagrangian, and the generic loop counting for amplitudes, cross sections and phase-space factors. In particular, and contrary to what is concluded in [1], chiral dimensions define an unambiguous way how to systematically build an electroweak chiral EFT with a light Higgs.

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