The High $E_T$ Drop of $J/\psi$ to Drell-Yan Ratio from the Statistical $c\bar{c}$ Coalescence Model

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Abstract

The dependence of the $J/\psi$ yield on the transverse energy $E_T$ in heavy ion collisions is considered within the statistical $c\bar{c}$ coalescence model. The model fits the NA50 data for Pb+Pb collisions at the CERN SPS even in the high-$E_T$ region ($E_T \gtrsim 100$ GeV). Here $E_T$-fluctuations and $E_T$-losses in the dimuon event sample naturally create the celebrated drop in the $J/\psi$ to Drell-Yan ratio.
Measurements of the $J/\psi$ suppression pattern in heavy ion collisions had been proposed as a diagnostic tool for the deconfinement of strongly interacting matter at the early stage of the reaction [1] (see also [2] for a modern review). However, it was found that the $J/\psi$ suppression in proton-nucleus and nucleus-nucleus collisions with light projectiles (up to S+U) can be explained by inelastic collisions with the nucleons of the incident nuclei (the so-called ‘normal nuclear suppression’) [3]. In contrast, the NA50 experiments with a heavy projectile and a heavy target (Pb+Pb) revealed ‘anomalous’ $J/\psi$ suppression [4–6], i.e. an essentially stronger drop of the $J/\psi$ over Drell-Yan ratio $R$ than it could be expected from a simple extrapolation of the light projectile data. The dependence of $R$ on the neutral transverse energy $E_T$, which is used as an estimator of the collision centrality, shows a two step behavior. The onset [5] of the anomalous $J/\psi$ suppression (deviation of the data downwards from the ‘normal nuclear suppression’ curve) supposedly takes place at $E_T \approx 35$ GeV. The second drop has been noted at $E_T \approx 100$ GeV. The initial interpretation [6] attributed these two drops to the successive melting of the charmonium $\chi_c$ states (which contribute about 45% to the measured $J/\psi$ number) and primary $J/\psi$’s in the quark-gluon plasma.

Other interpretations are, however, possible. Because of relatively high statistical errors of the data at low $E_T$, the behavior around $E_T \approx 40$ GeV can be equally well interpreted as smooth decreasing of the measured ratio $R(E_T)$ due to charmonium suppression by comoving hadrons [8]. Discussing the second drop of the $J/\psi$ to Drell-Yan ratio, one has to take into account [9] that the neutral transverse energy $E_T$ estimates different quantities in the regions below and above $E_T \approx 100$ GeV. At $E_T \lesssim 100$ GeV, it is proportional with good accuracy to the number of nucleon participants $N_p$ and provides a reliable measure of the collision centrality. The large $E_T$ ‘tail’, however, is produced mainly due to fluctuations of the transverse energy at almost fixed (nearly maximal) centrality. The influence of these fluctuations on the $J/\psi$ suppression can partially explain the second drop. Better agreement of the co-mover model with the data in the region $E_T \gtrsim 100$ is achieved when a loss of transverse energy in the dimuon data sample with respect to the minimum bias one is also taken into account [10].

The $E_T$ fluctuations appear to be important in the deconfinement scenario as well. It was found [11] that while the first drop can be explained by melting of $\chi_c$’s, the onset of primary $J/\psi$ melting does not produce any visible structure on the curve. The second drop appears due to the fluctuations of $E_T$.

All mentioned approaches [1,7–11], despite of their differences, have one common feature: charmonia are assumed to be created exclusively at the initial stage of the reaction in primary nucleon collisions. Any subsequent interactions (with sweeping nucleons, co-moving hadrons or quark-gluon medium) may only destroy them. Despite of rather successful agreement with the $J/\psi$ data, such a scenario seems to be in trouble explaining the $\psi'$ yield. The recent lattice simulations [12] suggest that the temperature of $\psi'$ dissociation $T_d(\psi')$ lies far below the deconfinement point $T_c$: $T_d(\psi') \approx 0.1–0.2T_c$ [13]. Therefore, not only the quark-gluon plasma, but also a hadronic co-mover medium should completely eliminate $\psi'$ charmonia in

\footnote{Later it was found that even the sweeping nucleons of the colliding nuclei might, under certain assumptions, produce the observed suppression [1].}
central Pb+Pb collisions at SPS. However, the experiment revealed a sizable \( \psi' \) yield (see, for instance, [13,15]). It was observed [16] that \( \psi' \) to \( J/\psi \) ratio decreases with centrality only in peripheral lead-lead collisions, but remains approximately constant and equal to its thermal value at sufficiently large number of participants \( N_p \geq 100 \).

A natural way to explain the \( \psi' \) yield in nucleus-nucleus collisions without running into contradiction with the lattice data is to assume that a charmed quark and an antiquark that have been created at the initial (‘hard’) stage of the reaction can coalesce to form a hidden charm meson at a later stage, close to the chemical freeze-out point. This possibility has been considered in the literature [17–22] within various models.

The statistical \( c\bar{c} \) coalescence model of \( J/\psi \) production suggested in [18,19] will be considered in the present paper. The charmed quark-antiquark (\( c\bar{c} \)) pairs are assumed to be created at the initial stage of the nucleus-nucleus reaction in hard parton collisions. Creation of \( c\bar{c} \) pairs after the hard initial stage as well as their possible annihilation is neglected. Therefore, the number of charmed quark-antiquark pairs remains approximately unchanged during the subsequent evolution. They are then distributed over open and hidden charm particles at the final stage of the reaction in accordance with laws of statistical mechanics. It is postulated that the deconfined medium prevents formation of charmonia at early stage of the reaction. In fact, it was found [23] that \( J/\psi \) becomes unbound slightly above the deconfinement point \( T_d(J/\psi) \approx 1.1T_c \). Therefore, all charmonia observed in central Pb+Pb collisions supposed to be created at the final stage of the reaction.

It was shown in our previous paper [24] that the statistical \( c\bar{c} \) coalescence model is in excellent agreement with the NA50 data for (semi)central \( (N_p \gtrsim 100) \) Pb+Pb collisions. However, the analysis of [24] was restricted to the region \( E_T \lesssim 100 \) GeV, where \( E_T \) estimates the collision centrality. Fluctuations of \( E_T \) were not taken into account. The aim of the present paper is to extend the applicability domain of the model to \( E_T \gtrsim 100 \) GeV and describe the second drop of the anomalous \( J/\psi \) suppression. Two effects appear to be important in this region:

- Influence of fluctuations of thermodynamic parameters of the system on the \( J/\psi \) production.
- A loss of neutral transverse energy, \( E_T \), in the event sample collected with the dimuon trigger relative to the minimum bias events.

As far as charmed (anti)quarks are created exclusively at the initial ‘hard’ stage of the reaction in collisions of primary nucleons, the average number of \( c\bar{c} \) pairs \( \langle c\bar{c} \rangle_{AB(b)} \) created in a nucleus-nucleus (A+B) collision at fixed impact parameter \( b \) is proportional to the number of primary nucleon-nucleon collisions and can be found in Glauber’s approach:

\[
\langle c\bar{c} \rangle_{AB(b)} = AB\sigma^{NN}_{c\bar{c}}T_{AB}(b).
\]

(1)

Here \( T_{AB}(b) \) is the nuclear overlap function calculated assuming a Woods-Saxon distribution of nucleons in nuclei [25] (see e.g. Appendix of [24]). \( \sigma^{NN}_{c\bar{c}} \) is the production cross section of a \( c\bar{c} \) pair in a nucleon-nucleon collision. There are indirect experimental indications [26] and a phenomenological model [27] suggesting an enhancement of the open charm in heavy ion collisions with respect to a direct extrapolation of nucleon-nucleon data. Therefore, \( \sigma^{NN}_{c\bar{c}} \) in nucleus-nucleus collisions is not necessarily equal to its value measured in a proton-proton
experiment. We consider $\sigma_{c\bar{c}}^{NN}$ as a free parameter. Its value is fixed by fitting the NA50 data.

The primary nucleon-nucleon collisions in a nucleus-nucleus reaction are independent and the probability to produce a $c\bar{c}$ pair in a single collision is small. Therefore, event-by-event fluctuations of the number of $c\bar{c}$ pairs in nucleus-nucleus collisions at fixed impact parameter $b$ are Poisson distributed. Assuming exact conservation of this number one obtains the following statistical coalescence model result for the average number of produced $J/\psi$'s per A+B collision [19]:

$$
\langle J/\psi \rangle_{AB(b)}^{(T,\mu_B)} = \langle c\bar{c} \rangle_{AB(b)} \left[ 1 + \langle c\bar{c} \rangle_{AB(b)} \right] \frac{\mathcal{N}_{tot}^{J/\psi}(T,\mu_B)}{(\mathcal{N}_{O}(T,\mu_B)/2)^2} + \ldots,
$$

(2)

Where $T$ and $\mu_B$ are, respectively, the system temperature and baryonic chemical potential. The dots in (2) state for higher order terms with respect to the ratio $\mathcal{N}_{tot}^{J/\psi}(T,\mu_B)/(\mathcal{N}_{O}(T,\mu_B)/2)^2$, which can be safely neglected at $N_p > 100$. Here $N_O$ is the total open charm multiplicity calculated within the grand canonical ensemble of the equilibrium hadron gas model:

$$
N_O = V \sum_{j=D,D^*,D^{*\ast},...} n_j(T,\mu_B) \equiv V n_O(T,\mu_B).
$$

(3)

The sum in (3) runs over all known (anti)charmed particle species [28]. The total $J/\psi$ multiplicity $N_{J/\psi}^{tot}(T,\mu_B)$ includes the contribution of excited charmonium states decaying into $J/\psi$:

$$
N_{J/\psi}^{tot} = V \sum_{j=J/\psi,\chi_1,\chi_2,\psi'} R(j \rightarrow J/\psi) n_j(T,\mu_B) \equiv V n_{J/\psi}^{tot}(T,\mu_B).
$$

(4)

Here $R(j \rightarrow J/\psi)$ is the decay branching ratio of the charmonium $j$ into $J/\psi$: $R(J/\psi \rightarrow J/\psi) \equiv 1$, $R(\chi_1 \rightarrow J/\psi) \approx 0.27$, $R(\chi_2 \rightarrow J/\psi) \approx 0.14$ and $R(\psi' \rightarrow J/\psi) \approx 0.54$. The volume parameter $V$ is fixed by the condition of the baryon number conservation:

$$
N_p(b) = V \sum_{j=N,N,\Delta,\Delta,...} b_j n_j(T,\mu_B) \equiv V n_B(T,\mu_B).
$$

(5)

Here $b_j$ is the baryon number of particle type $j$ and the sum in (5) runs over all known (anti)baryon species [28]. The number $N_p(b)$ of nucleon participants at fixed impact parameter $b$ is found in Glauber’s approach (see e.g. Appendix of [24]).

The particle densities in (3–5) are found from:

$$
n_j(T,\mu_B) = \frac{d_j}{2\pi^2} \int_{0}^{\infty} k^2 dk \left[ \exp \left( \frac{\sqrt{m_j^2 + k^2 - \mu_j}}{T} \right) \mp 1 \right]^{-1},
$$

(6)

where $m_j$ and $d_j$ are, respectively, the mass and degeneracy factor of particle type $j$; $\mu_j = b_j \mu_B + s_j \mu_S + c_j \mu_C$ is the chemical potential with $s_j$ and $c_j$ being, respectively, the strangeness

\[2\text{In what follows we neglect fluctuations of the number of nucleon participants at fixed impact parameter.}\]
and charm of particle type \( j \). The strange and charm chemical potentials \( \mu_S \) and \( \mu_C \) are fixed by the requirement of zero net strangeness and charm in the system.

As was already discussed above, the analysis \([13]\) of the recent lattice data \([12]\) has shown that \( \psi' \) cannot exist in hot hadronic medium. To explain the observed \( \psi' \) multiplicity one has to assume that charmonia are formed close to the chemical freeze-out point. (For (semi)central Pb+Pb collisions at SPS the chemical freeze-out (almost) coincides with the hadronization.) The thermodynamical parameters \( T \) and \( \mu_B \) at chemical freeze-out can be estimated by fitting the hadron gas model to the hadron yield data in Pb+Pb collisions at SPS \([29\text{-}31]\). This procedure, however, provides only average (in a sense that will be defined later) values \( \bar{T} \) and \( \bar{\mu}_B \). In fact, the thermodynamic parameters are not the same for all events, but rather they are subject to fluctuations of dynamical origin.

Only a fraction of the initial energy of the incident nuclei is converted to thermal energy \( E_{th} \) (including the rest masses) of the produced hadron gas. The rest is redistributed to kinetic energy of the collective flow. The contribution of each nucleon participant to \( E_{th} \) is not fixed. It fluctuates in accord with a hitherto unknown dynamical mechanism.

Suppose one is able to measure both \( N_p \) and \( E_{th} \). Then, events can be grouped in bins with different thermal energy per participant, \( E_{th}/N_p \). Different thermal energies per net baryon number cannot be obtained at the same values of the temperature and baryonic chemical potential of the hadron gas. Therefore, fitting the hadron gas model to the hadron yield data in each bin, one would obtain different values of \( T \) and \( \mu_B \). In reality, however, such a selection of events is not feasible, therefore no experimental information about fluctuations of \( T \) and \( \mu_B \) is available. The procedure of \([29\text{-}31]\) has to mix events with different \( E_{th}/N_p \) and estimates average values \( \bar{T} \) and \( \bar{\mu}_B \). One can, however, make a reasonable assumption about the fluctuations. It was found that at different initial energies of colliding nuclei, the produced hadron gas chemically freezes-out at constant total thermal energy per hadron \([32]\):

\[
E_{th}(T, \mu_B)/N_h(T, \mu_B) = \text{const} \approx 1 \text{ GeV}.
\]  

Different initial energies result in different thermal energy per participant. Therefore, it is natural to assume that the same criterion is valid when \( E_{th}/N_p \) fluctuates at fixed initial energy.

At given \( T \) and \( \mu_B \), the thermal energy of the hadron gas can be found from

\[
E_{th}(T, \mu_B) = V \sum_j \varepsilon_j(T, \mu_B),
\]  

where \( \varepsilon_j(T, \mu_B) \) is the contribution of the hadron species \( j \) to the thermal energy density:

\[
\varepsilon_j(T, \mu_B) = \frac{d_j}{2\pi^2} \int_0^\infty k^2 dk \sqrt{m_j^2 + k^2} \left[ \exp\left( \frac{\sqrt{m_j^2 + k^2} - \mu_j}{T} \right) \pm 1 \right]^{-1}.
\]  

The number of hadrons \( N_h(T, \mu_B) \) is given by

\[
N_h(T, \mu_B) = V \sum_j n_j(T, \mu_B).
\]
The sum in equations (8) and (10) runs over all known stable hadron and resonance species [28].

Equation (7) defines the curve in the \((T, \mu_B)\) plane along which the freeze-out parameters fluctuate. To compare our model with the experiment the relation of each point on this curve to the transverse energy \(E_T\) of neutral hadrons at given impact parameter \(b\) is needed. The point \((\mathcal{T}, \mu_B)\) corresponds to the average value \(E_T\), which in the framework of wounded nucleon model [33] is proportional to the number of participant nucleons:

\[
\mathcal{E}_T = q N_p(b). \tag{11}
\]

The fluctuations of \(E_T\) are known to be Gaussian-distributed [34]:

\[
P_{MB}(E_T | b) = \frac{1}{\sqrt{2\pi q^2aN_p(b)}} \exp \left( - \frac{(E_T - qN_p(b))^2}{2q^2aN_p(b)} \right). \tag{12}
\]

The subscript ‘MB’ states for ‘minimum bias’. This probability distribution is valid for the event sample collected with the ‘minimum bias’ trigger, which includes all \(^3\) events having the given value of \(E_T\). The parameter values \(q = 0.274\) GeV and \(a = 1.27\) [35] are fixed from the minimum bias transverse energy distribution [3]. The deviations of \(E_T\) from its central value are mostly due to change of the number of neutral hadrons rather than due to change of average energy per hadron. Therefore, \(E_T\) is approximately proportional to the total multiplicity of final hadrons: \(E_T \sim N_{tot}(T, \mu_B)\). Due to resonance decay, \(N_{tot}(T, \mu_B)\) is larger than the number of particles in the hadron gas at freeze-out \(N_h(T, \mu_B)\). Namely, every term in the sum (10) should be multiplied by a factor \(g_j \geq 1:\)

\[
g_j = \begin{cases} 
1 & \text{for stable and weakly decaying particles;} \\
\sum_k kR(j \rightarrow kh) & \text{for resonances.} 
\end{cases} \tag{13}
\]

Here \(R(j \rightarrow kh)\) is the probability that the resonance \(j\) produces \(k\) hadrons in the final state.

Hence, the average number of \(J/\psi\) mesons, \(\langle J/\psi \rangle_{AB(b)}^{(E_T)}\), produced in an \(A + B\) collision at fixed impact parameter \(b\) and transverse energy \(E_T\) can be found in the following way:

1. \(T\) and \(\mu_B\) are found as a solution of coupled transcendental equations

\[
E_{th}(T, \mu_B)/N_h(T, \mu_B) = E_{th}(\mathcal{T}, \mu_B)/N_h(\mathcal{T}, \mu_B), \tag{14}
\]

\[
N_h(T, \mu_B)/N_{tot}(T, \mu_B) = E_T/E_T \tag{15}
\]

The first of the above equations expresses the freeze-out criterion (7): constant thermal energy per hadron. The second one appears due to the proportionality between the multiplicity of final particles and the measured neutral transverse energy.

2. The values of \(T\) and \(\mu_B\) found from (14) and (15) are substituted into the formula (2) giving the desired value of \(\langle J/\psi \rangle_{AB(b)}^{(E_T)}\).

\(^3\)In contrast, the dimuon trigger requires production of a dimuon pair.
In real experiment, the impact parameter is not fixed. The relevant quantity, the differential cross section of the $J/\psi$ production in collisions of nuclei A and B with respect to the transverse energy $E_T$, is obtained by integrating $\langle J/\psi \rangle^{(E_T)}_{AB(b)}$ over the impact parameter $b$:

$$ \frac{d\sigma_{AB \to J/\psi}}{dE_T} = 2\pi \int_0^\infty db \langle J/\psi \rangle^{(E_T)}_{AB(b)} P_{J/\psi}(E_T|b).$$  \hfill (16)

Here $P_{J/\psi}(E_T|b)$ is the probability to produce neutral transverse energy $E_T$ at fixed impact parameter $b$, provided that a $J/\psi$ particle is produced in the same event.

Formula (16) refers to the total number of produced $J/\psi$'s, while the measured quantity is the number of $\mu^+\mu^-$ pairs originating from their decay and satisfying certain kinematics conditions. Therefore, the cross-section (16) should be multiplied by the probability of $J/\psi$ decay into a dimuon pair $B_{\mu^+\mu^-}^{J/\psi} = (5.88 \pm 0.10)\%$ \cite{28} and by a factor $\eta$, the fraction of the dimuons satisfying the kinematical conditions of the NA50 spectrometer. Because the considered $J/\psi$ production mechanism is completely different from the “standard” production in hard collisions, the rapidity distribution of $J/\psi$'s may be very different from that in N+N collisions. Consequently, $\eta$ should not be necessarily equal to $\eta_{NN} \approx 0.24$ which is obtained from Schuler’s parameterization \cite{36} of proton-proton data. Therefore, $\eta$ will be treated as one more free parameter.

Finally, to make a comparison with the NA50 data, the $J/\psi$ cross section should be divided by the Drell-Yan cross section. Similarly to (16), it is found from

$$ \frac{d\sigma_{AB \to DY'}}{dE_T} = \int d^2b \langle DY' \rangle_{AB(b)} P_{DY}(E_T|b).$$  \hfill (17)

Here quantity $P_{DY}(E_T|b)$ is similar to $P_{J/\psi}(E_T|b)$, $\langle DY' \rangle_{AB(b)}$ is the average number of Drell-Yan pairs produced in an A+B collision at fixed impact parameter $b$. The prime means that only the pairs satisfying the kinematical conditions of the NA50 spectrometer are taken into account. Similarly to $c\bar{c}$ pairs, the number of Drell-Yan pairs is proportional to the number of primary nucleon-nucleon collisions:

$$ \langle DY' \rangle_{AB(b)} = AB \sigma_{DY'}^{NN} T_{AB}(b),$$  \hfill (18)

where $\sigma_{DY'}^{NN}$ is the production cross section of Drell-Yan pairs in nucleon-nucleon collisions. This quantity is isospin dependent, therefore the average value should be used:

$$ \sigma_{DY'}^{NN} = \frac{\sigma_{DY'}^{AB}}{AB}. $$  \hfill (19)

For the case of Pb+Pb collisions, $A = B = 208$ and $\sigma_{DY'}^{PbPb} = 1.49 \pm 0.13 \mu b$ \cite{37}.

Hence, the following formula will be used to fit the NA50 data:

$$ R(E_T) = \eta B_{\mu^+\mu^-}^{J/\psi} \left( \frac{d\sigma_{AB \to J/\psi}}{dE_T} \right) \left/ \left( \frac{d\sigma_{AB \to DY'}}{dE_T} \right) \right. $$  \hfill (20)

As the first step, we assume that both $P_{J/\psi}(E_T|b)$ and $P_{DY}(E_T|b)$ in (16) and (17) coincide with the 'minimum bias' probability (12). The result of the calculations is shown in Fig. 1 with the dotted line. Calculations with $T \equiv T$ and $\mu_B \equiv \mu_B$, i.e. ignoring the
fluctuations, were done for comparison (the thin dashed line). As is seen, the influence of fluctuations becomes essential only at $E_T > 100$. This can be explained as follows. If $E_T$ is not very large, $E_T < qN_p(0)$, the integration in \([10]\) runs over the region where $E_T/N_p > q$, as well as over the one with $E_T/N_p < q$. The central point of the Gaussian \([12]\), $E_T/N_p = q$, corresponds to $T = \mathbf{T}$ and $\mu_B = \mathbf{p}_B$. Deviations of $T$ and $\mu_B$ in both directions from their central values almost cancel each other. Therefore, the result does not differ essentially from that obtained at $T \equiv \mathbf{T}$ and $\mu_B \equiv \mathbf{p}_B$.

In contrast, when $E_T$ approaches the value $qN_p(0)$ or exceeds it, the cancellation does not take place any more. At large $E_T$, the region $E_T/N_p \lesssim q$ falls into nonphysical domain $N_p > N_p(0)$. Only the tail of the Gaussian \([12]\) with $E_T/N_p > q$ contributes to the integral \([10]\). This corresponds to fluctuational tail with $T > \mathbf{T}$ and $\mu_B < \mathbf{p}_B$.

Although both the thermal 'density' of $J/\psi$, $n_{J/\psi}^{\text{tot}}(T, \mu_B)$, as well as that of the open charm, $n_O(T, \mu_B)$, are very sensitive to the temperature, their ratio in the formula \([2]\),

$$\frac{N_{J/\psi}^{\text{tot}}(T, \mu_B)}{(N_O(T, \mu_B)/2)^2} = \frac{1}{V} \frac{n_{J/\psi}^{\text{tot}}(T, \mu_B)}{(n_O(T, \mu_B)/2)^2},$$

is much less sensitive. The additional $J/\psi$ suppression takes place mostly due to increasing of the system volume $V$. Smaller $\mu_B$ leads to smaller baryon density $n_B(T, \mu_B)$ in \([3]\). At fixed $N_p$ this results in a larger volume. Decreasing the number of $J/\psi$ with increasing volume at fixed number of $c\bar{c}$ pairs is intuitively clear: the larger is the volume the smaller is the probability that $c$ and $\bar{c}$ meet each other and form a quarkonium state.

As is seen from figure \([1]\), the suppression due to the $T$ and $\mu_B$ fluctuations is not strong enough to explain the experimental data at $E_T > 100$ GeV. The reason is that we have assumed that $P_{J/\psi}(E_T|b) \equiv P_{MB}(E_T|b)$. In fact, however, production of a $J/\psi$ particle costs a fraction of energy of the colliding nuclei. Therefore, the multiplicity of ordinary hadrons (and consequently $E_T$) in a $J/\psi$ event is, in average, smaller than that in a 'minimum bias' event (i.e. without registering $J/\psi$)\(^4\) at the same impact parameter. Hence, as was pointed out in \([10]\), the probability $P_{J/\psi}(E_T|b)$ is slightly different from $P_{MB}(E_T|b)$. Similarly to \([10]\), we assume that the nucleon pair that have produced a $J/\psi$ meson does not contribute to the transverse energy. Therefore, the probability $P_{J/\psi}(E_T|b)$ is obtained from formula \([12]\) by the replacement $N_p(b) \rightarrow N_p(b) - 2$. This induces a tiny (about 0.5%) shift of the $E_T$ probability distribution.

The transverse energy distribution of the Drell-Yan event sample $P_{DY}(E_T|b)$ should also differ from $P_{MB}(E_T|b)$ because of the energy loss for the Drell-Yan pair production. However, in the large $E_T$ region, where the mentioned difference is important, our result is to be compared with the data obtained from the minimum bias analysis. This means that the so called ”theoretical Drell-Yan” instead of the physical one was used as a reference for $J/\psi$ suppression pattern. This ”theoretical Drell-Yan” was obtained neglecting the $E_T$ loss in

\(^4\)The 'minimum bias' probability distribution $P_{MB}(E_T|b)$ is basically related to the events without $J/\psi$ or a Drell-Yan pair in the final state. Only a tiny fraction of Pb+Pb collisions results in production of charmonia or Drell-Yan pairs, therefore their influence on the shape of the minimum bias probability distribution is negligible.
the dimuon data sample. Therefore, to be consistent with the data presented by the NA50, we put $P_{DY}(E_T|b) \equiv P_{MB}(E_T|b)$.

The result of the calculations taking into account the modification of $P_{J/\psi}(E_T|b)$ (ignoring, for the moment, fluctuations of $T$ and $\mu_B$) is shown in figure 3 with the dash-dotted line. Again, a sizable effect (compare with the thin dashed line) is observed only at $E_T > 100$. This is explained by the following properties of the Gaussian distribution. Let us consider a ratio of two Gaussians. One of them is shifted with respect to another one:

$$\frac{\exp \left\{ -\frac{(x - (x_0 - \Delta))^2}{2} \right\}}{\exp \left\{ -\frac{(x - x_0)^2}{2} \right\}} = \exp \left\{ -2 \Delta \left[ x - (x_0 - \Delta) \right] \right\}. \quad (22)$$

If the shift $\Delta$ is small, the ratio (22) is very close to 1 near the central point $x_0$. But, due to exponential behavior, it differs essentially from unity at large distances from $x_0$, which corresponds to tails of the Gaussians. As was mentioned above, the shift of $P_{J/\psi}(E_T|b)$ with respect to $P_{MB}(E_T|b)$ is about 0.5%. Therefore, when the main contribution to the integral (16) comes from the vicinity of the central value of the Gaussian (12), the effect is small. In contrast, at large $E_T$, when the integration runs only over the tail of the Gaussian, the effect becomes essential.

Taking into account both effects, $T$ and $\mu_B$ fluctuations and $E_T$ loss in the $J/\psi$ event sample, one finds excellent agreement with the fitted data including high $E_T$ ($E_T > 100$ GeV) region (the thick solid line in figure 1). The free parameters $\sigma_{e\bar{e}}^{NN}$ and $\eta$ were fixed by fitting the model to the NA50 data.

As was mentioned above, the statistical coalescence model is not expected to describe small systems. This can be seen from the $\psi'$ data [18]. In the framework of this model the multiplicity of $\psi'$ is given by formula (3) with the replacement $N_{J/\psi} \rightarrow N_{\psi'}$. Consequently, the $\psi'$ to $J/\psi$ ratio as a function of the centrality should be constant and equal to its thermal equilibrium value. The experimental data [15] (see also a compilation in [18]) are consistent with this picture only at rather large numbers of participants [16]. Therefore our fit is restricted to the region

$$E_T > 27 \text{ GeV}, \quad (23)$$

which corresponds to $N_p \gtrsim 100$. Note that all NA50 data except the two leftmost points from the 1996 standard analysis set lie in the region (23).

To check the robustness of our model with respect to possible uncertainties we use two independent freeze-out parameter sets. The first one [29] has been obtained assuming strangeness and antistrangeness suppression by a factor $\gamma_s$: \n
$$\overline{T} = 158 \text{ MeV}, \quad \overline{\mu}_B = 238 \text{ MeV} \quad \gamma_s = 0.79. \quad (24)$$

In the second one [31] the complete strangeness equilibration has been supposed:

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5Only the complete (i.e. including $T$ and $\mu_B$ fluctuations and $E_T$ loss) model has been fitted to the data. The calculations shown by the thin dashed, dotted and dash-dotted lines in figure 1 has been done with the parameter set fixed from the complete model fit.
$$\mathcal{T} = 168 \text{ MeV}, \quad \overline{\mu}_B = 266 \text{ MeV} \quad \gamma_s \equiv 1. \quad (25)$$

The predicted effective $c\bar{c}$ production cross section as well as the fit quality appeared to be insensitive to the freeze-out parameters:

$$\sigma^{NN}_{c\bar{c}} = 35.7^{+10.8}_{-8.8} \mu b, \quad \chi^2/\text{dof} = 1.06 \quad (26)$$

for both sets (24) and (25). The parameter $\eta$ only slightly depends on $\mathcal{T}$ and $\overline{\mu}_B$:

$$\eta = 0.134^{+0.061}_{-0.042} \quad (27)$$

for the set (24), and

$$\eta = 0.126^{+0.059}_{-0.039} \quad (28)$$

for the set (25).

As it was already pointed out in our previous papers [19,24], the model predicts a rather large enhancement of the total charm (by a factor of about 3.5 within the rapidity window of the NA50 spectrometer). A direct measurement of the open charm in Pb+Pb collisions at the CERN SPS would allow for a test of this prediction.

Extrapolation of the model to low $E_T$ region reveals strong disagreement with the experimental data (about 3.5 standard deviations at the leftmost experimental point). This suggests that the charmonium production mechanism in central and peripheral Pb+Pb collisions may be of a very different nature. The ‘standard’ mechanism, formation of pre-resonant charmonium states in primary nucleon collisions and their subsequent suppression in nuclear medium, seems to dominate peripheral collisions. In contrast, the present statistical $c\bar{c}$ coalescence model assumes the formation of a deconfined quark-gluon medium in central collisions. This medium destroys all primary $c\bar{c}$ bound states. The final state charmonia are formed at a later stage of the reaction from $c$ and $\bar{c}$ quarks available in the medium. The distribution of $c$ and $\bar{c}$ over charmonia and open charm hadrons follows from the laws of statistical mechanics.

In conclusion, we have shown that the statistical $c\bar{c}$ coalescence model provides a quantitative description of the ‘anomalous $J/\psi$ suppression’ in Pb+Pb collisions at CERN SPS. In particular, the ‘second drop’ of the $J/\psi$ suppression pattern, at $E_T \approx 100 \text{ GeV}$, can be naturally explained within the model as the result of two simultaneous effects: the fluctuations of the freeze-out thermodynamic parameters and transverse energy losses in the dimuon events with respect to the minimum bias ones. The model predicts a rather strong enhancement of the open charm. Therefore a direct measurement of the open charm would be a crucial test of the statistical $c\bar{c}$ coalescence model at the SPS.

\footnote{A different possibility has been considered in [38]. It has been assumed that both direct and statistical coalescence mechanisms of $J/\psi$ production are present at all energies and at all collision centralities. No open charm enhancement has been permitted in [38]. Therefore the contribution of statistical coalescence mechanism has been found to be small at the SPS. Then, the primordial $J/\psi$’s that survived suppression in the quark-gluon plasma dominate even in the most central Pb+Pb collisions. Such an approach is, however, hardly able to describe the centrality dependence of the $J/\psi$ suppression pattern.}
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FIG. 1. The dependence of the $J/\psi$ over Drell-Yan ratio $R$ on the transverse energy. The points with error bars are the NA50 data. The lines correspond to different versions of the statistical $c \bar{c}$ coalescence model (see text for details) and to the normal nuclear suppression mechanism. The normal nuclear suppression curve is obtained at $\sigma_{abs} = 6.4$ mb, where $\sigma_{abs}$ is the absorption cross section of preresonant charmonia by sweeping nucleons. The vertical line shows the applicability domain of the statistical $c \bar{c}$ coalescence model $N_p > 100$. 