Measurement Induced Dephasing and Suppression of Persistent Current in a Lattice Ring

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We investigate the effect of measurement on the persistent current density in a lattice ring penetrated by an Aharonov-Bohm flux and coupled to a quantum dot which is continuously monitored by a point contact detector. It is demonstrated by explicit analysis of the time-evolution of current density that the persistent current in the lattice ring is suppressed completely due to dephasing of quantum states induced by an analogous which-way-measurement that whether or not the electron is localized on the quantum dot. Practical experiments to observe the measurement induced dephasing is proposed.

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The properties of persistent currents in a mesoscopic ring induced by the threading Aharonov-Bohm(AB) magnetic flux can be affected dramatically by charge-transfer from the ring to a coupled quantum dot [1] and such phenomena have been well studied [1–7] with an in-line or side-branch quantum dot. Böttiker and Stafford show in Ref. [1] that the persistent current is varied by the charge transfer from a side branch quantum dot into a ring through a sequence of plateaus of diamagnetic and paramagnetic states while exhibits sharp resonances with an embedded quantum dot in the ring. Persistent current in the AB lattice ring coupled weakly with a quantum dot is influenced by the spin fluctuation [2] and unusual features of persistent-current behaviors resulted by the Kondo effect have been explored with a wide range of coupling constant [2]. The lattice ring is described by an ideal one-dimensional tight-binding model with N lattice sites and the quantum dot is treated as an Anderson impurity. The main interests of previous studies are focused on the influence of Kondo effect on the persistent current which is considered as the most convenient way to detect the Kondo screening cloud by experiments [3–7] in which one can measure directly the current to investigate the coherent transport through the quantum dot embedded in a lattice ring. Based on perturbation analysis Affleck and Simon [6] point out that the persistent current depends strongly on the ratio of the screening cloud size, which is the characteristic length scale associated with the bulk Kondo temperature, to the ring circumference. The persistent current is also a function of the total number of electrons and the magnetic flux threading the ring besides the ratio [7].

Cedraschi, Ponomarenko and Böttiker observe in Ref. [8] an interesting phenomenon that the persistent current in a mesoscopic ring is sensitive to its environment, which is modeled by an embedded quantum dot capacitatively coupled to an external circuit with a dissipative impedance. It is shown that the persistent current is strongly suppressed by zero-point quantum fluctuation with decreasing external impedance [8]. In the present paper, we study the measuring effect on the ground state of mesoscopic lattice ring coupled to a point contact through a side-branch quantum dot (see Fig.1) which is regarded as a device to measure the electron occupation of the dot. The point contact [9,10] consists of two reservoirs(emitter and collector) separated by an electrode as shown in Fig.1. The mesoscopic ring coupled to a side-branch quantum dot which is continuously monitored by the point contact. Electrons in the left and right reservoirs fill up to different Fermi Levels, $\mu_L, \mu_R$, respectively such that the electrons can flow through the electrode, for example, from the left to right reservoirs. We assume that the quantum dot has a single energy level $E_0$ and there are total $N$ sites in mesoscopic lattice-ring. $V_r$ represents the coupling between neighboring sites, and $V_0$ denotes coupling strength of the quantum dot with the nearest site-1. The charge current flowing through the point contact can be described by the well-known Landauer formula [11] seen to be $I = \frac{2e}{h} T (u_L - u_R)$, where $T$ is the transmission coefficient of the contact. The current through the point contact is sensitive to nearby electrostatic field and thus if an electron occupies the quantum dot, the current decreases due to the electrostatic repulsion. Therefore the point contact serves a measuring device to detect whether or not the quantum dot is occupied by an electron. We show that the persistent current in the mesoscopic ring will be strongly suppressed by the process to measure electron occupation of the dot no matter how weak the coupling between the point contact and the quantum dot is. In other words the quantum phase coherence in the mesoscopic ring which gives rise to the persistent current is completely destroyed by the continuous monitoring of the quantum dot by the detector to determine if the electron exists on it (a counterpart of the which-way-measurement in the two slits interference experiment).

The Hamiltonian describing the system is given by

$$H = H_{PC} + H_R + H_{int}. \tag{1}$$

where $H_{PC}$ represents the detector (quantum point contact)

$$H_{PC} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} \Omega_{lr} (a_r^\dagger a_l + h.c.) \tag{2}.$$
with $a_{l(r)}^\dagger$ and $a_{l(r)}$ being the creation and annihilation operators in the left (right) reservoir, and $\Omega_l$ denotes the hopping constant between the two reservoirs. The Hamiltonian for the lattice ring coupled with a quantum dot of a single energy level $E_0$ is

$$H_R = E_0 d_0^\dagger d_0 + \sum_{i=1}^{N} E_i d_i^\dagger d_i + V_c \left( e^{i 2 \pi \frac{\Phi}{\hbar} } d_1^\dagger d_N + h.c. \right) + \sum_{i=1}^{N-1} V_c \left( d_i^\dagger d_{i+1} + h.c. \right) + V_0 \left( d_0^\dagger d_0 + h.c. \right). \tag{3}$$

The time-dependent many-body state can be constructed with the hopping constant between the two reservoirs. The diagonal density-matrix element $\sigma_{ii}(t)$ indicates the probability of electron occupying the state of the lattice site $i$, while the off-diagonal elements $\sigma_{ij}(t)$ describes correlation between the states of the lattice sites $i, j$. The density-matrix elements can be found from $b(t)$ with the help of the Laplace transformation [12] $b(E) = \int b(t)e^{Et} dt$. The sum over energy levels of reservoirs which are assumed to be deeply inside the band, can be replaced by an integral, $\sum_{l} \rightarrow \int \rho_l \rho_r e^{E_l} dE_l$, with $\rho_l, \rho_r$ being the densities of states in the emitter and collector (reservoirs) of the point contact. We furthermore assume that the densities of states $\rho_l, \rho_r$, and the transition amplitude $\Omega_l$ are independent of energy [12]. The rate equations for density-matrix can be derived via the inverse Laplace transformation and the result is [9,10,12],

$$\dot{\sigma}_{ii} = -D_i \sigma_{ii} + D_i \sigma_{i-1,i} + i V_0 \left( \sigma_{i-1,i} - \sigma_{i,i} \right),$$

$$\dot{\sigma}_{i,i} = -D_{i,i} \sigma_{i,i} + D_{i,i} \sigma_{i-1,i-1} + i V_c \left( \sigma_{i-1,i-1} - \sigma_{i,i-1} \right) \tag{7.a}$$

for the diagonal elements and,

$$\dot{\sigma}_{i,i-1} = -D_{i,i-1} \sigma_{i,i-1} + D_{i,i-1} \sigma_{i,i-2} + i V_0 \left( \sigma_{i,i-2} - \sigma_{i,i-1} \right),$$

$$\dot{\sigma}_{i+1,i} = -D_{i+1,i} \sigma_{i+1,i} + D_{i+1,i} \sigma_{i+1,i+1} + i V_c \left( \sigma_{i+1,i+1} - \sigma_{i+1,i} \right), \tag{7.b}$$

where, the vacuum state $|0\rangle$ is defined such that there is no electron in the coupled system (lattice ring and dot) and electrons fill up to Fermi levels of the left and right reservoirs respectively. Substitution of the state $|\Psi(t)\rangle$ into time-dependent Schrödinger equation, $i \hbar \frac{d}{dt} |\Psi(t)\rangle = H |\Psi(t)\rangle$, yields equations for the coefficients $b(t)$ which are the probability amplitudes of the corresponding electron states. The density matrices $\sigma_{ij}^{(n)}(t)$ are defined from these amplitudes by tracing out the irrelevant degrees of freedom with $n$ being the number of electrons transmitting into the right reservoir of the point contact at time $t$ and the total density-matrix is

$$\sigma_{ij}(t) = \sum_n \sigma_{ij}^{(n)}(t) = b_i(t) b_j^\dagger(t) + \sum_{l,r} b_{ilr}(t) b_{jlr}^\dagger(t) + \sum_{l' \neq l, r' \neq r} b_{l'l'r'}(t) b_{j'l'r'}^\dagger(t) + \cdots. \tag{6}$$

with $d_0^\dagger (d_0)$ is the creation (annihilation) operator in the quantum dot and $d_i^\dagger (d_i)$ is the corresponding operator in the $i$-th lattice site $i = \{1, 2, \cdots N\}$. The magnetic flux $\Phi$ embraced by the lattice ring results in an AB phase $e^{i2 \pi \frac{\Phi}{\hbar}}$ with $\Phi_0 = \frac{\Phi}{\hbar}$ being the quantum unit of the flux, and $V_c$ represents hopping amplitude between the adjacent sites, while the hopping constant between the dot and site-1 of the lattice is denoted by $V_0$. The interaction Hamiltonian between the point contact and the quantum dot in the case that the electron occupies the quantum dot is seen to be

$$H_{int} = - \sum_{l,r} \delta \Omega_{lr} \left( a_l^\dagger a_l + h.c. \right) d_r^\dagger d_r. \tag{4}$$

The transition amplitude between the two reservoirs of the point contact decreases by a value of $\delta \Omega_{lr}$ due to the Coulomb repulsion of electron occupying the dot. In the following we study the dynamics of the quantum system with various initial conditions which indicate the result of measurements at time $t = 0$ and then evaluate the time-evolution of the quantum state in order to determine the effect of measurements on the current density in the ring. The time-dependent many-body state can be constructed by all possible electron states as

$$|\Psi(t)\rangle = |b_0(t) d_0^\dagger + \sum_{i=1}^{N} b_i(t) d_i^\dagger + \sum_{l,r} b_{ilr}(t) d_0^\dagger a_l^\dagger a_l \right. \tag{5}$$

$$+ \sum_{i=1}^{N} b_{ilr}(t) d_i^\dagger a_l^\dagger a_l + \cdots |0\rangle.$$
the ground state of electron in the lattice ring is in phase which gives rise to vanishing current

\[
\sigma_{1,i}^{(n)} = i\varepsilon_{i,1}\sigma_{1,i}^{(n)} - iV_0\sigma_{0,i}^{(n)} - D\sigma_{1,i}^{(n)} + D\sigma_{1,i}^{(n-1)}
\]

\[
+ iV_i\sigma_{1,i-1}^{(n)} + \sigma_{1,i+1}^{(n)} - \sigma_{2,i}^{(n)} - e^{-i2\pi\frac{\Delta}{v}}\sigma_{N,i}^{(n)},
\]

\[
\sigma_{N,i}^{(n)} = i\varepsilon_{N,N}\sigma_{N,i}^{(n)} - D\sigma_{N,i}^{(n)} + D\sigma_{N,i}^{(n-1)}
\]

\[
+ iV_i\sigma_{N,i-1}^{(n)} + \sigma_{N,i+1}^{(n)} - \sigma_{N-1,i}^{(n)} - e^{-i2\pi\frac{\Delta}{v}}\sigma_{1,i}^{(n)},
\]

\[
\sigma_{i,j}^{(n)} = i\varepsilon_{i,j}\sigma_{i,j}^{(n)} - D\sigma_{i,j}^{(n)} + D\sigma_{i,j}^{(n-1)}
\]

\[
+ iV_i\sigma_{i,j-1}^{(n)} + \sigma_{i,j+1}^{(n)} - \sigma_{i-1,j}^{(n)} - \sigma_{i+1,j}^{(n)} - \sigma_{i,j}^{(n)}
\]

\[
\sigma_{i,j}^{(n)} = \Omega_{i,i}^{(n)}
\]

for the off-diagonal elements, where \(\varepsilon_{i,j} = E_i - E_j\), \(i, j = \{2, 3 \cdots N - 1\}\), \(\Omega_i = \Omega_{ir} - \delta\Omega_{ir}\), \(D = 2\pi\Omega_i^2\rho_i\rho_r(\mu_L - \mu_R)\), \(D' = 2\pi\Omega_i^2\rho_i\rho_r(\mu_L - \mu_R)\), and the overhead dot denotes the time derivative. After the summation over \(n\) (see Eq. (6)), the rate equations are found in a compact form as

\[
\frac{d\sigma_{i,j}}{dt} = i\varepsilon_{i,j}\sigma_{i,j} + iV_{i-1,j}\sigma_{i-1,j} - iV_{i+1,j}\sigma_{i+1,j} + iV_{i,0}\sigma_{i,0} + iV_{N,0}\sigma_{N,0}
\]

\[
+ iV_i\sigma_{i,j-1} + \sigma_{i,j+1} - \sigma_{i-1,j} - \sigma_{i+1,j} - \sigma_{i,j} - \sigma_{i,j}^{(n)}
\]

\[
\sigma_{i,j}^{(n)} = \Omega_{i,i}^{(n)}
\]

where \(\Gamma = (\sqrt{D} - \sqrt{D'})^2\) and \(i, j = \{0, 1, \cdots N\}\). The boundary condition is obviously that \(i, j = N + 1 = 1\) and \(V_{1,0} = V_{N,0} = V_c e^{i2\pi\frac{\Delta}{v}}\) in view of the AB phase. Furthermore, \(V_i = V_{i,1} = V_{N,0} = V_c e^{i2\pi\frac{\Delta}{v}}\). From Eq. (8), it is seen that the detector (the last term of Eq. (8)) has effect only on the off-diagonal element with the subscripts either \(i\) or \(j\) being zero. For the case \(\Gamma = 0\) i.e. \(\Omega_{ir} = 0\), Eq. (8) reduces to the rate equations for the lattice ring with a side-coupled quantum dot discoupled from the detector. Based on the slave boson mean field approach [13], the oscillating persistent current for such a system has been found with the tight binding type Hamiltonian in Ref. [14].

The time-dependent current density in the lattice ring can be described by the dimensionless quantity (measured in the unit \(J_0 = 2eV_c/h\)) [15]

\[
J_{i,i+1} = \text{Im}(\sigma_{i,i+1}).
\]

which represents the current flows from the \(i + 1\) site to the \(i\) site in the lattice-ring. In stationary case, i.e. \(\frac{d\sigma_{i,j}}{dt} = 0\), Eqs. (8) possess a unique solution [9] with vanishing off-diagonal elements of the density matrix i.e.

\[
\sigma_{i,j}(t \to \infty) = 0, (i \neq j).
\]

which gives rise to vanishing current \(J = 0\) indicating the complete dephasing due to the effect of the detector. The physical interpretation is obvious, since when the detector is discoupled with the lattice-ring \(V_0 = 0\) the ground state of electron in the lattice ring is in phase coherent state which leads to the persistent current induced by the threading magnetic flux. This solution has been demonstrated by Cheung et al in the Ref. [16]. Then after the coupling is turned on \((V_0 \neq 0)\), the detector starts to monitor the coupled system and consequently destroys the phase coherence of electron in the ring. The persistent current in the ring vanishes at the asymptotic limit. To see this process clearly we in the following provide the numerical solution of Eq.(8) in order to exhibit the time-evolution of the off-diagonal elements of the density matrix. First of all we consider the case that the quantum dot is not occupied by the electron at time \(t = 0\), i.e. \(\sigma_{0,0}(t = 0) = 0\) as measured by the detector. Fig. 2 is a plot of the current density \(J_{i,i+1}\) (here \(i = 1\) for example) via time \(t\) with the initial condition that \(\sigma_{i,i}(t = 0) = 1/4\) for all \(i \neq 0\), and the other elements are zero, i.e. the electron occupies each site of ring with equal probability initially. For the case that the detector is discoupled with the quantum dot \((\Gamma = 0)\) at \(t > 0\), the current density oscillates with time shown in Fig. 2a since the electron is periodically scattered by the quantum dot which is considered as an impurity. While when the detector is turned on \((\Gamma \neq 0)\) at \(t > 0\) the current oscillation damps to zero in a short time period as shown in Fig. 2b. The situation is similar as that shown in Fig. 2a for the case that the electron is located initially at an any one site of the ring. An opposite situation is that the detector has detected the electron occupation of the quantum dot at time \(t = 0\). The initial condition for this case is \(\sigma_{0,0}(t = 0) = 1\), and \(\sigma_{i,j}(t = 0) = 0\) for all \(i, j \neq 0\). The numerical result shows that the time-evolution of the current density \(J_{1,2}(t)\) is again similar as in Fig. 2a for both the coupled and discoupled cases. Therefore the measurement induced suppression of the persistent current in the lattice ring is independent of the initial condition. It is also interesting to see the occupation probabilities of electron on the quantum dot which are different dramatically for the two cases of \(\Gamma = 0\) and \(\Gamma \neq 0\). In the former case the occupation probability oscillates with time (Fig. 3a) and in the later case the oscillation of the occupation probability damps to a value \(1/5\) (Fig. 3b) due to the measuring.

We conclude that measurement of detector leads to the suppression of persistent current in the mesoscopic ring. We emphasize that no matter how weak the coupling between the detector and the quantum dot is, the oscillation of current density damps to zero in the asymptotic limit. The measuring localization of electron, namely whether or not the electron exists at the quantum dot in our case, would destroy the phase coherence and the persistent current completely. In this paper we display an analytic evaluation of the measurement induced dephasing. The model described in Fig. 1 is, as a matter of fact, a practical set up of experiment in which the current density is used as a probe to test the effect of measurement on the quantum states.

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Figure Caption
Fig. 1. The mesoscopic ring coupled to a side-branch quantum dot which is continuous monitored by the quantum point contact placed near the quantum dot.

Fig. 2. The dimensionless current density \( J_{1,2} \) as a function of time evaluated from the numerical solution of Eqs. (8) for \( N = 4 \), \( V_0 = V_c = V \), \( \varepsilon_{i,j} = 0 \), and \( \Phi = 0.1\Phi_0 \) with the initial condition \( \sigma_{i,i}(t = 0) = 1/4 \), \( \sigma_{0,0}(t = 0) = 0 \) and \( \sigma_{i,j}(t = 0) = 0 (i \neq j) \). (a) The point contact is discoupled with the dot, \( \Gamma = 0 \). (b) The coupled case, \( \Gamma = V \).

Fig. 3. The time-evolution of the electron-occupation-probability of the quantum dot with initial condition \( \sigma_{0,0}(t = 0) = 1 \), and \( \sigma_{i,j}(t = 0) = 0 \) for all \( i, j \neq 0 \). (a) \( \sigma_{0,0}(t) \), for the discoupled case \( \Gamma = 0 \). (b) for the coupled case, \( \Gamma = V \).
0 20 40 60 80 100 120 140 160 180 200
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

probability

t ( in the units V )