A Compressed Coding Scheme for Evolutionary Algorithms in Mixed-Integer Programming: A Case Study on Multi-Objective Constrained Portfolio Optimization

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Abstract

A lot of real-world applications could be modeled as the Mixed-Integer Non-Linear Programming (MINLP) problems, and some prominent examples include portfolio optimization, resource allocation, image classification, as well as path planning. Actually, most of the models for these applications are non-convex and always involve some conflicting objectives. Hence, the Multi-Objective Evolutionary Algorithm (MOEA), which does not require the gradient information and is efficient at dealing with the multi-objective optimization problems, is adopted frequently for these problems. In this work, we discuss the coding scheme for MOEA in MINLP, and the major discussion focuses on the constrained portfolio optimization problem, which is a classic financial problem and could be naturally modeled as MINLP. As a result, the challenge, faced by a direct coding scheme for MOEA in MINLP, is pointed out that the searching in multiple search spaces is very complicated. Thus, a Compressed Coding Scheme (CCS), which converts an MINLP problem into a continuous problem, is proposed to address this challenge. The analyses and experiments on 20 portfolio

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benchmark instances, of which the number of available assets ranging from 31 to 2235, consistently indicate that CCS is not only efficient but also robust for dealing with the constrained multi-objective portfolio optimization.

**Keywords:** Evolutionary computations, multi-objective constrained portfolio optimization, mixed-integer programming, coding scheme

1. Introduction

The Mixed-Integer Non-Linear Programming (MINLP) model is significant for many real-world applications [25, 22, 35], ranging from the constrained portfolio optimization [33], path planning [43], resource allocation [3], to image classification [28]. For example, in the first two specific applications mentioned above, the selection (integer) and allocation (continuous) of the assets, and the number (integer) and angle (continuous) of the rotation of an aircraft are mixed-integer variables that should be dealt simultaneously.

Without loss of generality, the majority of the discussion in this work is about the constrained portfolio optimization, because it could be naturally modeled as an MINLP problem and it is one of the well-known financial problems [19]. To be specific, a portfolio optimization problem considers an optimal allocation of the limited fund in a series of risky assets, namely, securities, bonds, stocks, and derivatives. In practice, investors attempt to acquire the best-expected return in a given risk level or minimize the risk in an acceptable return range. In general, the expected return could be directly assessed by the profit. However, in terms of measuring the risk, there are different methods based on different assumptions of the markets, e.g., the Mean-Variance (MV) [28], the Value-at-Risk (VaR) [17], the Conditional Drawdown-at-Risk (CDaR) [1], the Conditional Value-at-Risk (CVaR) [18], and the Lower Partial Moments (LPM) [42]. The MV model, which plays a significant role in the progress of modern portfolio optimization [29], is studied as the basic model in this paper.

In the literature, some exact algorithms have been designed and used to solve the constrained portfolio optimization problems [4, 38]. Nonetheless, most of
them relax the cardinality constraint in varying degrees. On the other hand, Evolutionary Algorithms (EAs) have the ability to handle the strict cardinality constraint in portfolio problems, since they require little problem-specific knowledge, which probably makes them more robust to the specific problems with various mathematical features (e.g., they could tackle non-convex and discontinuous problems). Therefore, EAs could naturally tackle this problem.

There are many variations of portfolio problems based on the MV model. With regard to the objective function, they could be roughly divided into three categories as follows:

(i) **weighted formulation:** it combines the two objectives (risk and return) by using a weighting parameter and regards it as the final objective. To the best of our knowledge, it plays a dominant role in the single objective MV model for portfolio optimization [9, 14, 16].

(ii) **transforming objective functions:** it transforms one of the objectives into an equality or inequality constraint and considers the other as the final objective [37], or integrates the considered objectives into one with some criteria [34].

(iii) **multi-objective models:** generally, they regard the risk and return as two main aspects, and aim to find a set of trade-off solutions [26, 7, 31]. Furthermore, they can involve more than two objectives when considering more issues [36].

Following the above work, this paper considers an extended MV model that incorporates four real-world constraints [26]: (i) **cardinality constraint:** it restricts the number of assets in the portfolio result. (ii) **floor and ceiling constraint:** it determines the minimal and maximal quantities of every asset. (iii) **pre-assignment constraint:** it considers the preference of investors. (iv) **round lot constraint:** it demands the holding quantities of assets should be multiple of the minimal round lot. This constrained portfolio optimization has two layers of optimization [8]. In the first level, it aims to find the best selection
(combination) of available assets, which contains 0-1 integer variables, i.e., an asset is chosen or not. In the second level, it aims to find an optimal allocation of a finite fund, which contains continuous variables, i.e., the proportion of the fund assigned to each asset. Hence, this extended MV model can be transformed into an MINLP problem and is NP-hard \[38, 32\]. How to deal with 0-1 integer and continuous variables has become a key issue when using an EA. It is natural to use binary vectors to specify the selection of the assets and a real-valued vector to indicate the investment proportions. This kind of direct representation or coding scheme is the most popular strategy with EAs \[8, 27\]. However, the direct coding scheme also leads to challenges to algorithm design, and more specifically, (1) it is hard to reuse existing search operators in EAs since they are usually designed either for continuous or discrete variables, and (2) the search space becomes complicated and the optimal investment proportion in different combinations of the assets may be quite different.

Facing these challenges, this paper proposes a Compressed Coding Scheme (CCS), with which merely one real-valued vector is employed to represent the selection and allocation simultaneously. By this way, not only the reusing of the existing search operators is simplified, but also the multiple search spaces are integrated into one single search space. In fact, some literature has already mentioned the use of a real-valued vector to represent both discrete and continuous variables simultaneously \[11, 30, 20\]. But this article is the first time to discuss this coding scheme in-depth, and for the first time pointed out the above two advantages. Last but not least, some tailored search operators are proposed to enhance the performance of this coding scheme in this constrained portfolio optimization problem.

Further, it has been pointed out as a matter of fact that the objectives, viz. the expected return and risk of portfolios always conflict with each other \[33, 21\]. In such problems, the target is to find a set of solutions that could represent the best possible trade-off among the objectives, instead of identifying an optimal solution. Hence, CCS is integrated into three existing state-of-the-art Multi-objective Evolutionary Algorithms (MOEAs), i.e., the Decomposition based
Multi-objective Evolutionary Algorithm (MOEA/D) \cite{44}, the Non-dominated Sorting Genetic Algorithm (NSGA-II) \cite{13}, and the $\phi$ Metric Selection Evolutionary Multi-objective Algorithm (SMS-EMOA) \cite{5}. On a series of instances, we have conducted several simulation experiments, and experimental results demonstrate that MOEAs with CCS exhibit highly efficient capability and robustness in searching optimal solutions. These solutions are superior for their better diversities and shorter distances to the Pareto Front (PF).

The structure of this article is presented as follow. Section 2 introduces the formulation of the constrained multi-objective portfolio optimization problem. In Section 3 the direct and compressed coding schemes are presented. Further, the different search spaces of them are discussed. In Section 4 some reproduction operators and the repair method are introduced. Then, a complete algorithm framework, including CCS, the multi-objective selection method and coding scheme, is presented. Thereafter, some simulation experiments are presented in Section 5. Finally, Section 6 outlines the conclusions of this paper and presents some future research directions.

2. Mathematical Model

Before the definition of the constrained portfolio optimization model, we give the following notations.

\begin{align*}
N & \quad \text{the number of available assets} \\
K & \quad \text{the number of assets in a portfolio, i.e., the cardinality} \\
L & \quad \text{the number of assets in the pre-assignment set} \\
w_i & \quad \text{the proportion of capital invested in the } i\text{-th asset} \\
\rho_{ij} & \quad \text{the correlation coefficient of the returns of } i\text{-th and } j\text{-th assets} \\
\sigma_i & \quad \text{the standard deviation of } i\text{-th asset} \\
\sigma_{ij} & \quad \text{the covariance of } i\text{-th and } j\text{-th assets} \\
\mu_i & \quad \text{the expected return of the } i\text{-th asset} \\
v_i & \quad \text{the minimum trading lot of the } i\text{-th asset} \\
\epsilon_i & \quad \text{the lower limit on the investment of the } i\text{-th asset}
\end{align*}
the upper limit on the investment of the $i$-th asset

$y_i$  the multiple of the minimum trading lot in the $i$-th asset

$\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$

$s_i = \begin{cases} 
1, & \text{if the } i\text{-th } (i = 1, \ldots, N) \text{ asset is chosen} \\
0, & \text{otherwise} 
\end{cases}$

$z_i = \begin{cases} 
1, & \text{if the } i\text{-th asset is in the pre-assigned set} \\
0, & \text{otherwise} 
\end{cases}$

This paper considers the following bi-objective model \([6]\), which involves maximizing the return and minimizing the risk simultaneously. Meanwhile, it meets the four practical constraints \([26]\) mentioned above, namely, cardinality, quantity, pre-assignment, and round lot constraints.

\begin{align*}
\min f_1 &= \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij}, \\
\max f_2 &= \sum_{i=1}^{N} w_i \mu_i, \\
\text{subject to} & \quad \sum_{i=1}^{N} w_i = 1, \quad 0 \leq w_i \leq 1, \\
& \quad \sum_{i=1}^{N} s_i = K, \\
& \quad \epsilon_i s_i \leq w_i \leq \delta_i s_i, \quad i = 1, \ldots, N, \\
& \quad s_i \geq z_i, \quad i = 1, \ldots, N, \\
& \quad w_i = y_i v_i, \quad i = 1, \ldots, N, \quad y_i \in \mathbb{Z}_+, \\
& \quad s_i, z_i \in \{0, 1\}, \quad i = 1, \ldots, N,
\end{align*}

where Eqs. \([1]\) and \([2]\) are two respective objectives, minimizing the risk and maximizing the return, in portfolio optimization that conflict with each other.
Eq. (3) requires that all the capital should be invested in a valid portfolio. Eq. (4) is the cardinality constraint (i.e., $K$ assets are selected), and Eq. (5) is the floor and ceiling constraint, which restricts the investment proportion being allocated in the $i$-th asset should lie in $[\varepsilon_i, \delta_i]$. In addition, Eq. (6) represents that the $i$-th asset must be included in a portfolio ($z_i = 1$), when it is of interest for the investor. It is a pre-assignment constraint. Thereafter, Eq. (7) defines the round lot constraint. Finally, Eq. (8), which is the discrete constraint, represents that both the $s_i$ and $z_i$ must be binary.

In this multi-objective portfolio optimization problem, the ultimate goal is to find a set of efficient portfolios that no other solutions are better than them with respect to all the objectives at the same time [12]. These batch of efficient portfolios should consist of a efficient frontier, which is not only close to the PF but also well distributed. This is because once the efficient frontier is obtained, in general, the investors could determine which portfolio to be chosen according to their preferences. Hence, the diversity of the solutions in the efficient frontier is significant for the investors, who do not want to miss an interesting optimal portfolio with a certain return or risk.

### 3. Coding Schemes

The decision variables in the model presented in the last section include $s_i \in \{0, 1\}$ and $w_i \in [0, 1]$ for $i = 1, \cdots, N$. When using EAs to deal with such mixed variables, the solution representation or coding scheme becomes vital. This section firstly introduces a popular direct coding scheme (DCS) [26], and analyzes the properties and challenges of the corresponding search space. Then a new coding scheme, called compressed coding scheme (CCS), is introduced to eliminate the shortcomings of DCS.

**Direct Coding Scheme (DCS)** [26]

In DCS, a solution is represented by a vector as follows:
Figure 1: An illustration of DCS in portfolio optimization. Principally, there are two search spaces in a constrained portfolio optimization problem (discrete space and continuous space respectively). With DCS, an EA finds the selection and allocation of the assets directly in the actual solution spaces. In the discrete space, $s_1$ or $s_2$ is a point that presents the combinations of assets for selection, such as $\{2, 4\}$ with binary vector $\{0, 1, 0, 1, 0\}$. In the continuous space, $w_1$ or $w_2$ is a point that presents the weights of allocation, like real-valued vector $\{0.23, 0.77\}$.

$$c = \left(c_1, c_2, ..., c_N, ..., c_{2N}\right), \quad (9)$$

where $c_i \in [0, 1]$, $i = 1, ..., N$ and $c_i \in \{0, 1\}$, $i = N + 1, ..., 2N$. It is clear that a solution includes two parts, i.e., the selection vector $(c_1, \cdots, c_N)$ and the allocation vector $(c_{1+N}, \cdots, c_{2N})$ respectively. Fig. 1 illustrates this coding scheme.

As for the decoding process, a solution is converted into the selection of assets $s_i$ as in Eq. (10) and the allocation of the assets as in Eq. (11). An example of decoding is presented in Fig. 2.

$$s_i = \begin{cases} 
1, & \text{if } c_i = 1 \\
0, & \text{otherwise} 
\end{cases}, \quad i = 1, 2, ..., N \quad (10)$$

$$w_i = \frac{s_i c_{i+N}}{\sum_{j=1}^{N} s_j c_{j+N}}, \quad i = 1, 2, ..., N. \quad (11)$$

We now analyze the properties of the coding scheme. In order to show more concisely figures of the search spaces, three assets are randomly chosen out of
Figure 2: An example of decoding with DCS. The length of this solution is twice of the amount of available assets. Suppose $N = 5$ and $K = 2$ here, the decoding process can be divided into three steps. Step 1: The binary vector $\{0, 1, 0, 1, 0\}$ indicates the selection of assets is $s_2 = 1$ and $s_4 = 1$. Step 2: According to the selection $s$, the allocation vector in the solutions is $\{0.00, 0.28, 0.00, 0.96, 0.00\}$. Step 3: Finally, the portfolio $w$ is normalized as $\{0.00, 0.23, 0.00, 0.77, 0.00\}$.

2235, which is the largest problem that shall be introduced in Table 1, i.e., $N = 3$. And only tow assets are interested, i.e., $K = 2$. Furthermore, Eqs. (3) and (4) are neglected for a while. Fig. 3 illustrates the experimental results. It can be seen from the figure that when different assets combine together, the optimal investment proportion may be quite different. Take Figs. 3(d) and (e) as an example, when assets $\{1, 2\}$ are chosen, the best solutions are located in the bottom right corner of the search space, while when assets $\{1, 3\}$ are chosen, the best solutions are located in the upper left corner of the search space. This suggests that the search for the optimal solutions in allocation are strongly related to the combination of the assets. This phenomena also indicate that when using this coding scheme in EAs, it may be hard to optimize both the binary variables and the continuous variables simultaneously.

**Compressed Coding Scheme (CCS)**

This section introduces a new coding scheme, called compressed coding scheme (CCS), for evolutionary portfolio optimization. The basic idea is to use a real-valued vector with length $N$ to represent a solution, which is defined
So, the higher the values, the better the solutions. The three rows of figures from top to bottom are based on the portfolios with three different asset combinations, respectively \{1,2\}, \{1,3\} and \{2,3\}. The four columns of figures from left to right represent the search space of return, the top 20% points with regard to return, the search space of risk and the top 20% points with regard to risk. Notice that, the circles indicate the values, and the larger the circles, the higher the values.

Figure 3: An empirical analysis of DCS. The values of return and minus risk are normalized.

in Eq. (12).

\[ \mathbf{c} = (c_1, c_2, ..., c_N), \]  

(12)

where \( c_i \in [0, 1] \), and \( i = 1, 2, ..., N \). The length of the solution is the same as the number of assets. Fig. 4 shows that CCS represents the selection and allocation based on one string of real numbers in \([0, 1]^N\) for a multi-mapping, where one vector \( \mathbf{c} \) is able to represent both selection and allocation tasks simultaneously.

The values of a solution are utilized to represent not only the selection but also the allocation. More specifically, the values are sorted in descending order and the first \( K - L \) positions of the solution indicate the positions of the selected assets, and the \( L \) pre-assigned assets are selected as well. Then the \( K \) values of the solution are applied again to represent the weight. The decoding process
Figure 4: The illustration of CCS in portfolio optimization. Principally, there are two search spaces in a constrained portfolio optimization problem (discrete space and continuous space respectively). In the discrete space, $s_1$ or $s_2$ is a point that presents the combinations of assets for selection, such as $\{2, 4\}$. In the continuous space, $w_1$ or $w_2$ is a point that presents the weights of allocation, like $\{0.23, 0.77\}$.

works as follows. Firstly, a solution is converted into the selection of assets $s_1, \cdots, s_N$ where $s_i = 1$ if $c_i$ is in the $K - L$ largest ones or the $i$-th asset is pre-selected, otherwise $s_i = 0$. Then the allocation of the assets is given in Eq. (13). An example of decoding is presented in Fig. 5.

$$w_i = \frac{s_i c_i}{\sum_{j=1}^{N} s_j c_j}, \quad i = 1, 2, \ldots, N. \quad (13)$$

We now analyze the properties of CCS. The same experiment is conducted as in the above section except that CCS is used to replace DCS. Fig. 6 illustrates the experimental results. The results shown in Fig. 6 are different from those in Fig. 3. In Figs. 6(a) and (d), the best solutions lie in the bottom and right, and the worst solutions belong to the top and left. Further, these two solution areas are distinct instead of zigzag. The phenomena in DCS rarely happens in CCS. Moreover, This property is consistent in the other figures in Fig. 6. Therefore, in these integrated search spaces, finding the optimal solutions are probably easier than in the search spaces, which are multiple, of the direct coding scheme. In other word, it might be more efficient to optimize both the binary variables and the continuous variables simultaneously by employing CCS.
Figure 5: The example of decoding with CCS. In contrary to the decoding mentioned above, the length of this solution is as the same as the number of available assets and this solution is actually used twice. Suppose $N = 5$ and $K = 2$ here, the decoding process can be divided into three steps. Step 1: The values of $c_2$ and $c_4$ are higher than other genes, so the selection of assets is represented as $s_2 = 1$ and $s_4 = 1$. Step 2: Since the solution is utilized twice, the strings, which will interact with each other, are $\{0.00, 1.00, 0.00, 1.00, 0.00\}$ and $\{0.81, 0.91, 0.13, 0.91, 0.63\}$ respectively. So far, the allocation before normalization is $\{0.00, 0.91, 0.00, 0.91, 0.00\}$. Step 3: Finally, the portfolio $w$ is normalized as $\{0.00, 0.50, 0.00, 0.50, 0.00\}$.

4. CCS based Algorithm Framework

This section introduces how to deal with constrained portfolio optimization with CCS based EAs. Firstly, three search operators are presented. Then, a repair method is proposed to make all solutions feasible. Finally, a complete algorithm framework is given.

**Search Operators**

By using CCS, all solutions are represented as real-valued vectors. Therefore, the search operators for continuous variables can be directly employed here. As suggested in [26], this work firstly presents a basic differential evolution (DE) strategy [41], which never uses prior knowledge.

**O1** $c'_i := c_3i + F \times (c_1_i - c_2_i)$

where $c_1$, $c_2$ and $c_3$ are three solutions randomly selected from the population, $c'$ is the new solution, and $F$ is the scaling factor in DE.

With regard to the properties of the problems and CCS, we also propose two new search operators. Firstly, it is observed that the solutions of CCS prefer to
Figure 6: An empirical analysis of CCS. The values of return and minus risk are normalized. So the higher the values, the better the solutions. The three rows of figures from top to bottom are based on three times of randomly choosing 3 assets from 2235. The four columns of figures from left to right represent the search space of return, the top and last 20% points with regard to return, the search space of risk, and the top and last 20% points with regard to risk. Notice that, the circles indicate the values, and the larger the circles, the higher the values.

concentrate on equally distributed when the number of assets is large since the rank method for the gene values will reserve lots of large but similar values [10]. The following search operator utilizes this heuristic information.

**O2** \( c'_i := c_i^{r(1,2)} \),

where \( r(1, 2) \) is a random number in \([1, 2]\). This operator only changes the investment proportion, but keeps the combination of assets, because the rank of values for genes is not changed.

Secondly, a tailored operator that utilizes the known information of the portfolio optimization problem is proposed.

**O3** Swap the values of \( c_i \) and \( c_j \): \( c_i \leftrightarrow c_j \), where the asset \( i \) is randomly chosen from selected assets and the asset \( j \) is chosen by randomly using one of the following strategies:
• Randomly choose another asset from selected assets.
• Choose an asset which has the correlation coefficient \((\rho_{jj})\) value.
• Choose an asset which has the highest return \((\mu_j)\) value.
• Choose an asset which has the least correlation
  \((\sum_{i \in \{\text{selection}\}} \rho_{ij})\) with those \(K - 1\) assets already chosen.

**Constraints Handling Method**

New candidate solutions are repaired by using a strategy from [40] if the quantity and round lot constraints are violated. The procedure works as follows.

1. All weights that are smaller than the value of \(((\epsilon_i \mod v_i) + 1) \cdot v_i\) are adjusted by setting \(w_i := ((\epsilon_i \mod v_i) + 1) \cdot v_i\).
2. The weight are then adjusted to the nearest round lot level by setting \(w_i := w_i - (w_i \mod v_i)\).
3. The remaining amount of capital is added to the largest \(w_i\).

**Algorithm Framework**

This subsection introduces an algorithm framework for dealing with the constrained portfolio optimization problems by using the CCS strategy proposed in the last section, the search operators and the constraint handling method introduced in this section. The detailed pseudo-code of the algorithm framework is presented in Algorithm [1]

In line 1, the first generation population \(P\) is randomly initialized in \([0, 1]^N\), where \(N\) is the number of available assets and \(NP\) is the population size. For the fitness evaluation, each solution \(c\) in \(P\) is decoded by the coding scheme to a portfolio \(w\) (lines 2-4). The main iteration of the algorithm is described in lines 6-14. While the stopping criteria are not met \([24]\), the new candidate solution is generated with a search operator (line 9). The fitness of each new individual is assessed (line 10), and the new population \(P'\) is combined with \(P\) for comparison (line 11). Finally, the best individuals are selected by an MOEA selection strategy in each iteration as the next population \(P\).
Algorithm 1: Algorithm Framework

// initialization
1. Sample the initial population \( P \) randomly in \([0,1]^N\).

2. foreach \( c \in P \) do
   // decode and repair
   3. Decode \( c \) and repair it if the constraints are violated.
   // evaluation
   4. Evaluate \( c \).

5. end

6. while stopping criteria is not met do
   7. Set \( P' = \emptyset \).
   8. foreach \( c \in P \) do
      // variation
      9. Generate a new candidate \( c' \) from \( P \) by an operator randomly chosen from \( \{O1, O2, O3\} \)
      // decode and repair
      10. Decode \( c' \) and repair it if the constraints are violated.
      // evaluation
      11. Evaluate \( c' \)
      12. Set \( P' = P' \cup \{c'\} \).
   end
   // selection
   13. Select \( NP \) solutions to constitute the next population \( P \) from \( P \) and \( P' \).
14. end

5. Experiment Study

This section is devoted to the empirical study of the proposed CCS strategy and the new algorithm framework. This section is divided into three parts. First, the instances, parameters, and MOEAs are introduced. Second, the quality indicator adopted in this paper is presented. Finally, the MOEAs with CCS (MOEAs-CCS), MODEwAwL \[26\], and the MOEAs with the DCS (MOEAs-CCS), i.e., MOEAs with a Random Keys \[2\] based DCS strategy, are compared on the instances.
Experimental Settings

Five instances, which are either classic or established recently by the historical stock data from the Yahoo Finance website, are adopted. Table I presents the details of each data set. The details of the parameters, including population size, number of generations, scaling factor and crossover probability of DE, and the neighborhood size, in this study are shown in Table II. In addition, two constraint sets [26] are considered as follows:

(i) **Cardinality** $K = 10$, floor $\epsilon_i = 0.01$, ceiling $\delta_i = 1.0$, **pre-assignment** $z_{30} = 1$ and round lot $\nu_i = 0.008$,

(ii) **Cardinality** $K = 15$, floor $\epsilon_i = 0.01$, ceiling $\delta_i = 1.0$, **pre-assignment** $z_5 = 1$ and round lot $\nu_i = 0.008$.

Notice that, if not specified, the simulated experiments in this work are constructed with the first constraint set.

Majority of the MOEAs is based on three main frameworks, namely, the decomposition based framework, the Pareto domination based framework, and the indicator-based framework. Therefore, three widely-used MOEAs, namely, MOEA/D [44], NSGA-II [13], and SMS-EMOA [5], corresponding to the three frameworks, are utilized in this study.

*Decomposition based Multi-objective Evolutionary Algorithm (MOEA/D) [44].* MOEA/D is based on the decomposition framework. It uses the neighborhood information of each subproblem, which could be obtained from decomposing of a multi-objective optimization problem, to update a whole population simultaneously. As for this work, the Tchebycheff technique is implemented.

*Non-dominated Sorting Genetic Algorithm (NSGA-II) [13].* NSGA-II is a popular Pareto domination based MOEA. It uses Pareto domination relation-

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*Among the instances, D1, D2 and D3 are from [http://people.brunel.ac.uk/~mastjjb/jeb/orlib/portinfo.html](http://people.brunel.ac.uk/~mastjjb/jeb/orlib/portinfo.html), and D4 and D5 are from [http://satt.diegm.uniud.it/projects/portfolio-selection/](http://satt.diegm.uniud.it/projects/portfolio-selection/) (available at September 1st, 2018).*
Table 1: Five Benchmark Instances.

| Instance | Origin  | Name               | Asset Amount |
|----------|---------|--------------------|--------------|
| D1       | Hong Kong | Hang Seng          | 31           |
| D2       | Japan   | Nikkei             | 225          |
| D3       | Korea   | KOSPI Composite    | 562          |
| D4       | USA     | AMEX Composite     | 1893         |
| D5       | USA     | NASDAQ             | 2235         |

The $\varphi$ Metric Selection Evolutionary Multi-objective Algorithm (SMS-EMOA) \[5\]: SMS-EMOA is designed as a steady-state MOEA. It Pareto domination relationship and the hypervolume indicator to select promising solutions into the next generation.

In the subsection of the comparison study, Random Keys and CCS are incorporated into three MOEAs respective, and a Learning-Guided Multi-Objective Evolutionary Algorithm (MODEwAwL) \[26\] is implemented. The algorithm parameters are shown in Table 2.

Quality Indicators

Two performance metrics, which are well-known and frequently applied, are introduced in this paper. They are Inverted Generational Distance (IGD) \[39\], and Hypervolume \[45\]. Overall, the IGD and HV are general metrics for multi-objective problems, and they cover consideration of both proximity and diversity.

**Inverted Generational Distance (IGD) \[39\]**: The IGD evaluates the distances between every solution and the true PF. It is given as follow.

$$IGD = \frac{\sum_{i=1}^{|Q|} d_i}{|Q|},$$
Table 2: The parameter setting of all algorithms.

| Parameters                                           | Value   |
|------------------------------------------------------|---------|
| Population size \((NP)\)                            | 100     |
| Number of generations                               | 1000    |
| Scaling factor \((F \text{ in } DE)\)               | 0.5     |
| Crossover probability \((CR \text{ in } DE)\)       | 0.9     |
| Neighborhood size \((\text{only for } MOEA/D)\)     | 10      |

where \(Q\) is a set of obtained solutions, and \(d_i\) is the shortest Euclidean distance between \(i\)th solution and the PF. Notice that, the true PFs in these portfolio optimization problems are actually undiscovered. This is because they are highly constrained problem \[33\]. So, the best known unconstrained efficient frontiers (UCEFs) \[9\] are adopted instead of true PFs and they are available at: http://people.brunel.ac.uk/ mastjjb/jeb/orlib/portinfo.html (available at September 1st, 2018).

**Hypervolume Metric (HV)** \[45\]: HV, also known as the size of dominated space, is a quality indicator that rewards the convergence toward the PF as well as the representative distribution of points along the front. It normalizes the objective space and measures the volume of space, which is bounded by the obtained efficient solutions and a preference point \(r\). For each solution \(i \in Q\), a hypercube \(hc_i\) from solution \(i\) and the reference point \(r\) is measured. Generally, higher values of HV are preferable. However, in order to apply a uniform comparison as IGD, the HV, in this paper, is defined by

\[
HV = \text{volume}(\bigcup_{i=1}^{\lvert Q \rvert} (hc_r - hc_i)),
\]

where \(hc_r\) is the hypercube of the reference point. Hence, lower values are better with respect to this definition.

**Comparison Study**

The comparison study aims to answer the following questions.

- What is the performance of MOEAs with CCS?
Table 3: The table involves the results of seven algorithms with respect to IGD. First, A–G denote the algorithms MOEA/D-DCS, MOEA/D-CCS, NSGA-II-DCS, NSGA-II-CCS, SMS-DCS, SMS-CCS and MODEwAwL respectively. Second, the rank and mean rank of each algorithm on all instances are listed, specifically, lower mean rank means better performance. Third, the best result in each instance is remarked in gray. Finally, the superscripts of symbol ‘+’/’−’/’=’, such as 2, 4 and 6, correspond to algorithms B, D and F.

| Algo. | A   | B   | C   | D   | E   | F   | G   |
|-------|-----|-----|-----|-----|-----|-----|-----|
| Mean  | 2.43e–02 | 2.28e–02 | 5.55e–02 | 2.30e–02 | 2.27e–02 | 2.24e–02 | 2.78e–02 |
| Std   | 1.03e+03 | 4.34e+04 | 4.46e-02 | 6.45e+04 | 8.42e+04 | 7.64e+04 | 2.25e+03 |

Table 4: The table involves the results of seven algorithms with respect to IH. First, A–G denote the algorithms MOEA/D-DCS, MOEA/D-CCS, NSGA-II-DCS, NSGA-II-CCS, SMS-DCS, SMS-CCS and MODEwAwL respectively. Second, the rank and mean rank of each algorithm on all instances are listed, specially, lower mean rank means better performance. Third, the best result in each instance is remarked in gray. Finally, the superscripts of symbol ‘+’/’−’/’=’, such as 2, 4 and 6, correspond to algorithms B, D and F.

| Algo. | A   | B   | C   | D   | E   | F   | G   |
|-------|-----|-----|-----|-----|-----|-----|-----|
| Mean  | 6.42e–02 | 6.21e–02 | 1.29e–01 | 6.26e–02 | 6.62e–02 | 5.98e–02 | 6.65e–02 |
| Std   | 1.57e-03 | 8.28e+03 | 9.00e+02 | 8.68e-04 | 8.48e+04 | 6.61e+04 | 1.67e+03 |

| Algo. | A   | B   | C   | D   | E   | F   | G   |
|-------|-----|-----|-----|-----|-----|-----|-----|
| Mean  | 7.08e–02 | 5.74e–02 | 1.71e–01 | 5.79e–02 | 1.03e–01 | 6.48e–02 | 8.53e–02 |
| Std   | 1.29e+02 | 3.39e+03 | 5.12e-02 | 6.83e+03 | 3.30e-02 | 5.71e-02 | 9.73e+03 |

| Algo. | A   | B   | C   | D   | E   | F   | G   |
|-------|-----|-----|-----|-----|-----|-----|-----|
| Mean  | 5.92e–02 | 5.85e–02 | 1.73e–02 | 6.02e–02 | 6.62e–02 | 6.75e–02 | 1.49e–01 |
| Std   | 3.69e+03 | 5.08e+03 | 2.32e-02 | 3.50e+03 | 7.85e+03 | 8.56e+03 | 9.87e+02 |

| Algo. | A   | B   | C   | D   | E   | F   | G   |
|-------|-----|-----|-----|-----|-----|-----|-----|
| Mean  | 5.49e–02 | 5.39e–02 | 5.73e–02 | 5.17e–02 | 5.21e–02 | 5.02e–02 | 4.87e–02 |
| Std   | 2.99e+03 | 2.02e+03 | 5.65e-03 | 1.93e-03 | 2.74e+03 | 2.51e+03 | 2.46e-04 |

| Algo. | A   | B   | C   | D   | E   | F   | G   |
|-------|-----|-----|-----|-----|-----|-----|-----|
| Mean  | 6.21e–02 | 6.50e–02 | 7.71e–02 | 5.89e–02 | 5.87e–02 | 5.64e–02 | 6.99e–02 |
| Std   | 3.01e+03 | 4.47e+03 | 1.33e+02 | 1.40e+03 | 2.26e+03 | 1.78e+03 | 3.36e-02 |

| Algo. | A   | B   | C   | D   | E   | F   | G   |
|-------|-----|-----|-----|-----|-----|-----|-----|
| Mean  | 4.2e+00 | 3.6e+00 | 6.8e+00 | 3.0e+00 | 3.6e+00 | 2.4e+00 | 5.0e+00 |
| Std   | 3.0/2 | 2/3/4 | 5/0/0 | 2/3/0 | 2/3/0 | 5/0/0 | 2/3/0 |
| Mean  | 4/1/0 | 3/2/0 | 4/0/1 | 3/2/0 | 3/1/1 | 2/3/0 | 5/0/0 |
| Std   | 3/1/1 | 4/0/1 | 3/2/0 | 3/1/1 | 2/3/0 | 5/0/0 | 2/3/0 |
Figure 7: Performance of seven algorithms in terms of IGD, IH and the running time for D1-
D5. Specially, A-G denote the algorithms MOEA/D-DCS, MOEA/D-CCS, NSGA-II-DCS,
NSGA-II-CCS, SMS-DCS, SMS-CCS and MODEwAwL respectively.
Figure 8: Comparison of convergence of seven algorithms for IGD, HV over generations on five instances.
What is the contribution of the CCS strategy?

To answer the first question, MOEAs-CCS are compared with a state-of-the-art algorithm, i.e., MODEwAwL. To answer the second question, MOEAs-CCS are compared to MOEAs-DCS [11].

Table 3 shows that all the MOEAs-CCS are better than three MOEAs-DCS and MODEwAwL in terms of IGD. To be specific, three MOEAs-CCS win the first, second and third place among seven algorithms with respect to the mean rank. Further, they are statistically better than all the other algorithms with respect to the employed Wilcoxon rank-sum test [15], except the comparison between MOEA/D-CCS and SMS-DCS, in which the first one only beats the second one on two instances but fails on the other three.

Table 4 shows the results in terms of IH, and they are the same as in Table 3. To be specific, MOEAs-CCS are the top three algorithms out of seven with respect to the mean rank. In terms of the employed Wilcoxon rank-sum test, all the three algorithms are better than the others, except the comparison between MOEA/D-CCS and SMS-DCS.

Furthermore, the results of IGD, IH and the running time of all the seven algorithms are presented in Fig. 7. From left to right these figures illustrate the performance of each algorithm in terms of IGD, IH and running time, and from top to bottom these figures show the results on different instances. Obviously, NSGA-II-DCS is the worst algorithm of all. For example, on the D1, it has the worst variance and median value, and three unstable values out the box in terms of both IGD and IH. Meanwhile, the results of MOEAs-CCS keep stable on all the instances in terms of both IGD and IH, because the sizes of the result box are always small, and the results, with respect to the employed metrics, keep very low on all the instances.

As for the running time, there are two significant features. First, the combination of different coding schemes with one same MOEA generally does not
change the running time, except NSGA-II. Second, SMS is the fastest MOEA with respect to CPU time on the constrained portfolio optimization. The second feature could be quickly concluded by the diagrams in Fig 7 so it does not need too much explanation. However, the reason why NSGA-II-CCS is prominently faster than NSGA-II-RK is noticeable. It is said that the time complexity of NSGA-II is $O(MN^3)$ in the worst case, and $O(MN^2)$ in the best case. Specifically, the worst case is that each front contains only one solution, and the best case is that all the solutions are in the first front[13]. Hence, if a good efficient frontier is determined soon, the running time of NSGA-II will be significantly reduced. It is reflected from the side that CCS is better than DCS because CCS accelerate the convergence of NSGA-II. Here, the first feature is clearly understood.

Moreover, Fig. 8 shows that three MOEAs-CCS have a strong ability to search for optimal solutions on all the instances, because even if the result of one of the MOEAs-CCS is not optimal, it can achieve a good level, approximating the best in all the cases.

To conclude, the results above consistently indicate two significant points (i) MOEAs-CCS outperform MOEAs-DCS and MODEwAwL on the given instances in the quality of the obtained the results, and (ii) furthermore, MOEAs-CCS are more robust because the MOEAs perform similarly when using the CCS strategy.

More Experiments

So far, we only have implemented the algorithms on the five instances mentioned above with the first constraint set. The constructed experiments seem inadequate and not convincing, therefore, in the Appendix, we supplement some simulation experiments, including experiments on the five instances with the second constraint set and on additional fifteen data sets with both the first and the second constraint sets.

In this subsection, more experiments were introduced. As for the additional fifteen data sets, they are also established by the historical stock data from
Table 5: Additional Fifteen Benchmark Instances.

| Instance | Origin | Name        | Asset Amount |
|----------|--------|-------------|--------------|
| D6       | Germany| DAX 100     | 85           |
| D7       | UK     | FTSE 100    | 89           |
| D8       | USA    | S&P 100     | 98           |
| D9       | Australia | All ord     | 264          |
| D10      | Italy  | MIBTEL      | 167          |
| D11      | UK     | FTSE ACT250 | 128          |
| D12      | USA    | NASDAQ Bank | 380          |
| D13      | USA    | NASDAQ Biotech | 130     |
| D14      | USA    | NASDAQ Computer | 417   |
| D15      | USA    | NASDAQ Financial00 | 91   |
| D16      | USA    | NASDAQ Industrial | 808  |
| D17      | USA    | NASDAQ Telecom | 139  |
| D18      | USA    | NYSE US100  | 94           |
| D19      | USA    | NYSE World  | 170          |
| D20      | USA    | S&P 500     | 469          |

Table 6: The statistical results of the mean rank of The MOEAs-CCS and MOEAs-DCS. To simplify, the mean and variance values involve both IGD and IH of the corresponding three MOEAs.

| Algs.          | First Five Instances | Last Fifteen Instances |         |         |         |         |         |         |         |
|----------------|----------------------|------------------------|---------|---------|---------|---------|---------|---------|---------|
|                | 1st Constraints      | 2nd Constraints        | 1st Constraints | 2nd Constraints |
|                | Mean  | Var   | Mean  | Var   | Mean  | Var   | Mean  | Var   |         |         |
| CCS/MOEAs      | 2.77  | 0.12  | 2.73  | 0.15  | 2.85  | 0.87  | 3.05  | 1.03  |         |         |
| RK/MOEAs       | 4.83  | 2.17  | 4.87  | 2.03  | 4.95  | 2.60  | 4.25  | 3.02  |         |         |
Yahoo Finance. Table 5 shows the detail of the fifteen instances. All the experimental results acquired by the seven algorithms are the statistical averages of 20 independent runs with the same experiment configuration, except the constraint set. The details of the results are shown in the Appendix, and the statistical results are shown in Table 6.

On the whole, two quick conclusions for the simulated experiments could be drawn.

First, the MOEAs-CCS are always better than the MOEAs-DCS in terms of both IGD and IH, when the MOEA combined is the same one. For example, in Table 4 in the Appendix, both the mean rank of these algorithms and the employed Wilcoxon rank-sum test between each algorithm on the fifteen instances are easy to be distinguished. MOEA/D-CCS (2.5) is better than MOEA/D-DCS (3.9), NSGA-II-CCS (3.9) is better than NSGA-II-DCS (6.9), and SMS-CCS (2.8) is better than SMS-DCS (3.9), in terms of the mean rank on IGD. On the other hand, the comparison between MOEA/D-CCS and MOEA/D-DCS is 6/0/9, it between NSGA-II-CCS and NSGA-II-DCS is 13/0/2, and it between SMS-CCS and SMS-DCS is 5/0/10, where ‘X/Y/Z’ is a simplified count of ‘Better/Worse/Equal’ in terms of the employed Wilcoxon rank-sum test. It is clear that the MOEAs-CCS are hardly worse than the MOEAs-DCS.

Second, the best algorithm is always one of the three MOEAs-CCS. In all the result tables, which involve the MOEAs-CCS, the MOEAs-DCS and MOD-EwAwL, eight tables, the MOEA/D-CCS wins the first place 1 time, the NSGA-II-CCS does so 2 times, and the SMS-CCS does so 5 times.

Further, as for the statistical results, Table shows that the mean and variance values of the MOEAs-CCS are better than those of the MOEAs-DCS in all the cases. The better mean values suggest that the MOEAs-CCS outperform the MOEAs-DCS in searching the optimal solutions with the constrained portfolio optimization. And, the lower variance values imply that the MOEAs-CCS

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Available at: [http://satt.diegm.uniud.it/projects/portfolio-selection/](http://satt.diegm.uniud.it/projects/portfolio-selection/) (available at September 1st, 2018).
are more robust. In addition, the statistical results of MOEAwAwL are not listed, because they are worse than all MOEAs-CCS in many cases. And the variance seems meaningless since there is no variant of it.

6. Conclusions

This paper studies the portfolio optimization problems, which can be modeled as multi-objective optimization with mixed variables and constraints. In the literature, a variety of work has been done on how to deal with either multi-objective optimization problems or constrained optimization problems while mixed-variable optimization has not attracted much attention. It is not the first time using a real-valued vector to represent the mixed variables simultaneously in the constrained portfolio problems, but the difference between DCS and CCS is firstly discussed. It is pointed out that CCS could overcome the shortcoming, the complexity of multiple search spaces, of DCS. Moreover, two tailored reproduction operators are proposed since constrained portfolio optimization is a specific problem, which could be solved more efficiently with corresponding problem information. Then the coding scheme, the search operators, and a constraint handling strategy are integrated into three major multi-objective evolutionary algorithms. These new algorithms are conducted on 20 benchmark problems with different asset numbers. The comparison study with a state-of-the-art algorithm and MOEAs-DCS has demonstrated that the new coding strategy is promising for dealing with mixed-variable problems.

In the paper, CCS is applied to constrained portfolio optimization. There are a variety of other problems with mixed-variables worth exploring. This could be a direction for future work.

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