The ultimate limit state of the underground circular tunnel segment lining

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Abstract. Circular tunnel segment lining with staggered joints in track tunnels of Prague Underground was found to be on the ultimate limit state; static analysis (mathematical modeling) is used to determine the causes which led to such situation. This situation is signaled by cracks and related deformations; lining load-limit coefficient can be used to determine the cause. Analysis is performed in the form of parametric study, where the variables are the values of geotechnical figures, the rigidity of the lining with staggered joints and the load of the lining. This paper focuses on analysis of reinforced concrete segmental lining Ø5.3 / 5.8 m (5 + 1 element) and cast iron Ø5.1 / 5.5 m (9 + 1 element). Parametric study using coefficient of loading limit for both of these cases in the Prague Underground leads to fast and relatively easy determination of the cause of the reaching of the ultimate limit state.

1. Introduction

In the track lines of Prague Underground, the circular tunnel lining (concrete or cast iron) with shifted staggered joints reached the emergency conditions of the load bearing capacity several times; it is a task of the static analysis (mathematical modelling) to assess the source of this emergency situation.

When the circular lining reaches the state of load bearing capacity, usually there is not enough time to elaborate complicated safety measures, especially if the reason of such changes is not known. Proposal for safety measures of the tunnel lining is usually based on the mathematical model analysis. The static model is prepared to correspond with real state as best as possible.

This article deals with analysis of selected reinforced concrete segmental lining Ø5.3/5.8 m (6 members) and cast iron lining Ø5.1/5.5 m (10 members). The goal of the parametric study is to assess the load bearing capacity of the lining and its margins. The reinforced concrete segmental lining was indicated with hair-like cracks parallel to the longitudinal track axis; the hypothesis was that the cracks were results of the impact pressure of the end of the shield in curve with small radius of the track. The cast iron lining was diagnosed with significant openings in abutting joints in the place, where lining was mounted on the concrete basement in the starting pit. Cast iron lining was backfilled, but compaction of the dusting was inadequate and after short time of the utilization the deformation arose.
2. The load bearing capacity analysis method

Designing the structure, we try to satisfy two basic but often contrasting conditions. We want to have the structure economical and safe too. Underground structures are loaded with continuous stress which creates its deformation. This state can be described mathematically in terms of structural mechanics. The durability of the structure is defined by three factors – placing the structure, load applied on the structure and natural environment surrounding the structure.

Static computation reveals the bearing-capacity of the lining considering rock-support interaction of different quality. For exact assessment of the load-bearing capacity, we use static parameters $M_i, N_i, T_i$ and then we can set measures to stop the additional destruction of lining.

Let $P_{U\text{max}}$ denote the limiting bearing-capacity of the cracks, $P_{L\text{max}}$ be the limiting loading capacity of the tunnel lining (Figure 1), and $P_i, P_j$ be the discreet loadings in points $i, j$ for the calculated static values $M_i, N_i, T_i$ and $M_j, N_j, T_j$, then

$$P(t) = f(M_i, N_i, T_i); P_j(t) = f(M_j, N_j, T_j)$$

(1)

If the limiting bearing-capacity of the tunnel lining is described by the function $P_{U\text{max}} = f(M_{U_i}, N_{U_i}, T_{U_i})$, then for time $(t)$ we can simply write equation

$$P_{U\text{max}}(t) = f(A_b, A_s, f_c, f_t) = f(M_{U_i}, N_{U_i}, T_{U_i})$$

(2)

where $A_b$ is the sectional area of concrete, $A_s$ is the sectional area of steel reinforcement, $f_c$ is the compressive strength of concrete, $f_t$ is the strength of steel in tension.

We compare values of the discrete loading $P_i, P_j$ and the bearing-capacity of the cracks described by equation (2). If $P_i(t)$ is less than $P_{U\text{max}}(t)$ then we conclude in equation

$$P_i(t) < P_{U\text{max}}(t)$$

(3)

We state that tunnel lining suits limiting bearing capacity of cracks. If loading $P_j(t)$ surpasses $P_{U\text{max}}(t)$, then it results into equation

$$P_j(t) > P_{U\text{max}}(t)$$

(4)

We state that the limiting bearing capacity of cracks is not satisfactory and construction is not suitable. The coefficient of the bearing-capacity of the lining $s_i$ is defined as the ratio of $P_{U\text{max}}(t)$ and $P_i(t)$

$$\frac{P_{U\text{max}}(t)}{P_i(t)} = s_i$$

(5)

The bearing-capacity of the tunnel lining is then calculated by the coefficient $s_i$ on the whole perimeter of the tunnel lining and in the places where the value of $s_i$ (5) is smaller than 1, $s_i < 1$, the construction is not satisfactory due to limiting bearing capacity of cracks. If the construction has more places with coefficient $s_i < 1$, then measures for prevention of increased cracks and deformations have to be adopted.
3. Analysed tunnel linings

In the parametric study, the coefficient of bearing capacity of two tunnel linings of Prague Underground, reinforced concrete segmental lining Ø5.3 / 5.8 m (5 + 1 member) and cast iron segmental lining Ø5.1 / 5.5 m (9 + 1 member) is assessed. In both cases shifted abutting joints were used and allowed limited deformations were kept. In the reinforced concrete segmental lining, hairline cracks parallel with longitudinal track axis arises. It has been supposed that the cracks were results of the impact pressure of the end of the shield in curve with small radius of the track.

Cast iron lining was assembled in open pit on the concrete basement of ca 4 m width and the pit backfilled by the soil showed significant openings of the abutting joints after commencing of operation of the underground.

Reinforced concrete segmental lining has outside diameter of 5.8 m and inside diameter of 5.3 m. Thickness of the lining is 0.25 m, width of the ring is 1.5 m, C50/60 XA1 concrete is used. Tubings are reinforced by steel (ØR6 – ØR12) and connected by bolts with dowels in abutting and radial joints. Shape of the ring is in Figure 2.

The cast iron lining Ø5.1/5.5 m (9+1 member) is made from C424-44, width of flange 195 mm and thickness of shell 20 mm. Thickness of the abutting and radial shells are 27 – 36 mm. In abutting shell of
tubing there are 4 openings of Ø 30 mm, radial shell has 6 openings of Ø 30 mm placed shifted to the radial joint axes.

Ring is assembled of 7 tubings Н-3-Л, two tubings of С-2-Л and one key tubing of К-2-Л. Weight of the whole ring is 5280 kg. According to the standard СНиП П-В.3-72 - Стальные конструкции-Нормы проектирования (Steel structures – designing code), cast iron СЧ24-44 has flexural compressive strength of 210 MPa, the flexural tensile strength is 80 MPa and shear strength is 60 MPa.

4. Polygonal computation method
Tunnel linings are modelled as polygons with many joints. Computation is made by general deformation method. Load of the rock on the lining is modelled as active loading in polygon vertices and passive counteraction of the rock is modelled by the Winkler’s springs. This method is universal and suitable to replace every shape of the construction and different geotechnical parameters. The computation gives us rather good information about inner forces in the lining, but doesn’t provide information about strength in arbitrary place in the rock. We only know reaction and stresses by which rock interacts with the lining. Results of computations depend on the boundary conditions, thus parametrical studies have to be conducted with different geotechnical parameters, values and shape of the loading. Stiffness (E-modulus of the concrete) is altered in the case of the reinforced concrete considering age and quality of the concrete.

The calculation runs in iterative cycles; in the first iterative cycle, all radial rock imitative rods are in function, and the calculations run until all tensile rods are removed and all pressed radial rods are in function only. The springs imitating rock have area and length value of 1. Their tightness relates to the $E_o$ deformation modulus of the rock. The characterization of the elastic rods can be written in the form

$$K_i = K_{(σ)} \frac{l_i + l_{ai}}{2} b$$

where $l_i$ is the length of the member of the polygon and $b$ is the width of the ring. The coefficient of elastic resistance of rock $K_{(σ)}$ depends not only on geotechnical parameters of the rock, but also on the shape of the construction and is usually determined according to B. G. Galerkin equation.

$$K_{(σ)} = \frac{E_o}{R(1 + ν)}$$

Let us assign the deformation and the movement of $i$ joint $U_i$, $V_i$, $θ_i$ and internal forces of $i$ joint $X_i$, $Y_i$, $M_i$ and outside loading $X'_i$, $Y'_i$. Then we can write the columnar matrix $\{P_i\}$ for the internal calculation parameter, and columnar matrix $\{P'_i\}$ for outside loading, where

$$\{P_i\} = \begin{bmatrix} U_i \\ V_i \\ θ_i \\ X_i \\ Y_i \\ M_i \end{bmatrix} ; \quad \{P'_i\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ X'_i \\ Y'_i \\ 0 \end{bmatrix}$$

We include the reactions of the supports as the internal forces for $i$ joint. Adding the deformation of elastic supports leads to the changes of the stiffness matrix coefficient $[K_i]$ which affects computation parameters. In general, load can be applied to any joint (joints have separate numbering, denoted with $j$). Matrix notation of the equation for calculating parameters for the joint $n$, including known boundary conditions, is in the form
\[
\{ P_o \} = \prod_{i=1}^{n} [K_i] \{ P_o \} + \sum_{i=1}^{n} \prod_{j=i+1}^{n} [K_j] \{ P_o \}
\]  

(9)

\{ P_o \} is column matrix of boundary calculation parameters.

\[ u = f (\tau, \gamma, r, R, \rho) \]  

(10)

where \( \tau \) – shear strength of the rock, \( \gamma \) – weight of the rock, \( r \) – radius of the circular boring, \( R \) – radius of the bearing rock ring, \( \rho \) – rheological property of the rock. Fenner – Pacher curve is given as

\[ \sigma_r = f (\sigma_{ro} \cdot \sigma_{min} \cdot u) \]  

(11)

where \( \sigma_{ro} \) is the highest value directly after assembling tight lining into bored face, \( \sigma_{min} \) is the lowest value of the strength \( \sigma_r \).

One part of the roof \( \sigma_o = \gamma h \) is carried by the arch in the roof, second part is acting as loading on the lining, which is consequently transposed as support to deforming bored face (value \( \sigma_r \)). Fenner –
Pacher curve can only be fully applied on bored face and tensioned lining that are symmetrical along vertical axis. Fenner – Pacher curve in asymmetric profiles is different for every point of lining. Fenner – Pacher curves can be obtained in each discrete point of lining for pre-selected ratio of the vertical and horizontal loading and particular elasticity/deformation modulus of rock by FEM mathematical modelling. Value of the loading with corresponding value of deformation of the bored face in the moment of the lining assembling would be obtained by using such curves. Fenner – Pacher curves for the reinforced concrete lining assembled by earth pressure balanced shield and for cast iron lining assembled by „erector” are shown in Figure 6. Curves differ significantly, earth pressure balanced shield doesn’t create big deformations of the rock, while using cast iron lining assembled by „erector” leads to bigger deformations. Smart deformation of the cast iron lining was ceased (see Figure 7) by grouting as depicted in Figure 6.

![Deformation and stress curve for „erector” and shield.](image)

**Figure 6.** Deformation and stress curve for „erector” and shield.

- a – elastic rock deformation,
- b – plastic rock deformation,
- A – cast iron assembling,
- C – line of ”rock-lining” state,
- D – state in long term equilibrium,
- R – area of loosened rock,
- 1 – value of original tenseness,
- 2 – beginning of the elastic plastic state of the rock,
- 3 – assembling of the lining,
- 4 – beginning of the activated lining after grouting,
- 5 – achievement short term state of equilibrium,
- 6 – beginning of the over passing of the limited bearing capacity of the lining, \( \Delta R \) increment of deformation %. 

\[
\sigma_0 = \gamma \times h
\]
5.2. Prague underground cast iron lining 5.1/5.5 m in inadequate dusting

Cast iron lining Ø5.1/5.5 m was mounted on the concrete basement in the starting pit 14.0 m deep. Design required compaction of the dusting especially on the side of the lining. Quality of the compaction required by design wasn’t delivered and during operation the deformation and opening joints in upper part of lining appeared. Clearance profile for railcars wasn’t affected. Tunnel lining state is depicted in Figure 7.

After indication of openings in joints of track tunnel, the grouting of the sides of cast iron lining was made (Figure 7). Deformations and opening of the joints immediately stopped. Calculation of the ring composed from tubings connected in abutting joints by numerous screws (radial joints are shifted) can be carried out as construction without joint. Sir Allen Muir Wood [4] uses for such case equation where moment of area is reduced:

\[ I = I_s + I_n \cdot \left( \frac{4}{n} \right)^2 \]  \( (12) \)

where \( I \) = reduced area-wise moment, \( I_s \) = area-wise moment of the force transmission zone, \( I_n \) = area-wise moment of complete section, \( n \) = number of segments \( n > 4 \), (small keysegment not counted), \( I_s \ll I \) lining of tubings with shifted radial joints and screws in abutting joints.

Parametric study analysis of the limiting deformation bearing capacity led to calculation of coefficients of loading limiting bearing capacity \( s_i \) with deformation modulus of rock \( E_o = 5, 50 \) a 100 MPa and with reduced area-wise moment according [4], equation (12). Results are depicted in Figure 8. Value of the limiting bearing capacity \( s = 1 \), (the bearing capacity of the lining isn’t reached) is depicted by grey solid line.

If the coefficient of loading limiting bearing capacity \( s_i \) is smaller than 1, openings in joints are registered. It means the bearing capacity of the lining is not sufficient at these spots and it is necessary to take mitigating action to ensure lining stability. Most dangerous is state with \( E_o = 5 \) MPa deformation modulus, value \( s_i \) reached \( E_{5MPa} = 0.24 – 0.32; \) \( E_{50MPa} = 0.44 – 0.64 \) and \( E_{100MPa} = 0.62 – \)
0.81. Analysis indicated several spots where coefficient of limiting bearing capacity $s_i$ is smaller than 1, or close to this value. Openings in joints facing to the rock wall aren’t visible, but must be considered at the time of stabilization of lining. If the coefficient of limiting bearing capacity $s_i$ exceeds 1, the lining has satisfactory load-bearing capacity.

5.3. Reinforced concrete segmental lining Ø5.3/5.8 m of Prague underground assembled by earth pressure balanced shield

Hairline cracks parallel to longitudinal track axis were found on the reinforced concrete segmental lining Ø5.3/5.8 m in Prague Underground in the altitude of horizontal axis of the lining. Two hypothesis of the cracks origin were proposed. One theory explained the origin of the cracks as a result of the impact pressure of the end of the shield in curve with small radius of the track. Minimum radius of the track tunnels is $R = 650$ m and contact of just the end of the shield with reinforced concrete lining is expected in this theory. Second theory explained the origin of the cracks as a result of hydraulic pressure on abutting joints of the lining in the time of shield moving. Such force is often asymmetric and in some cases pressure on tubing can reach extreme values.

5.3.1. Impact pressure of the end of the shield on reinforced concrete segmental lining. Lets assume load of the reinforced concrete segmental lining Ø5.3/5.8 m at the end of the shield according to Figure 9. The contact between the end of the shield and reinforced concrete lining is the most probable case in
curve with small radius of V.A metro line which is 650 m. According to the information from the construction site, the direction divergence of the shield face never exceeds 20 mm from the designed track.

Analyzing the contact of the shield end with lining with divergence of the shield face of 20 mm in radius 650 m, we found four theoretical positions of the shield, see Figure 10. It is assumed that contact of the shield end with lining takes place when shield rotation is deviated from designed track by $\alpha = 0^\circ 26' 25"$.

Shield face is deviated by 20 mm from designed track inside radius and shield end is in contact with reinforced concrete lining outwards of curve – option 1. Force on lining will be calculated as result of this state. For the geometry of the lining loaded by hydraulic pressure see Figure 9. The only force present is the force at contact between reinforced concrete lining and the shield end. Earth pressure force on the shield face is not included. Tubing’s ring is loaded with the grouting pressure of 3bars and forces of hydraulic pressure on abutting joints till the tubings are let out of shield.

![Figure 9. Contact of the shield end with reinforced concrete lining Ø5.3/5.8 m.](image)

![Figure 10. The shield maximum possible rotation in curve R = 650 m, resulting in divergency of the shield face of 20 mm from designed track.](image)

Option 1 – shield face is 20 mm deviated horizontally inwards the radius from designed track. Shield end is in contact with reinforced concrete lining outwards of curve.
Option 2 – shield face is 20 mm deviated horizontally outwards the radius from designed track. Shield end is in contact with reinforced concrete lining outwards of curve.

Option 3 – shield face is 20 mm deviated horizontally inwards the radius from designed track. Shield end is in contact with reinforced concrete lining inside of curve.

Option 4 – shield face is 20 mm deviated horizontally outwards the radius from designed track. Shield end is in contact with reinforced concrete lining inside of curve.

When radius is bored, 8 couples of hydraulic presses push on lining with maximum force $N_8 = 8 \times 2.43 = 19.46$ MN. We can state

$$\sum (N_i \times d_i) = P_s \times 2.909$$  \hspace{1cm} (13)$$

where $d_i$ is the distance of forces. Shield end is acting on the lining by force (simplified approach):

$$P_s = \frac{2.43 \times (3.259 + 4.23 + 5.011 + 5.418)}{2.909} = 12.319 \text{ MN}$$  \hspace{1cm} (14)$$

Head of the track tunnel is 15 – 35 m and if hairline cracks appeared immediately after moving ring from the shield, analysis would not include geostatic pressure. Thus maximum theoretical loading of shield end is $P_{sume} = 12.319$ MN.

Calculation of the cracks in loading lining by shield end according Figure 9 was carried out for three types of loading derived from maximum theoretical loading, which cannot be obviously achieved. Ring loading chosen was where $P_{sume} = 12.319$ MN.

| Table 1. Loading by shield end. |
|--------------------------------|
| $P_{1/20} = P_{sume} /20 = 0.61595$ MN |
| $P_{1/30} = P_{sume} /30 = 0.41063$ MN  \hspace{1cm} where $P_{sume} = 12.319$ MN |
| $P_{1/40} = P_{sume} /40 = 0.307975$ MN |

Figure 11. Computation of maximum force which the shield end is acting on lining Ø5.3/5.8 m.
Table 2. Size of the cracks.

| Name of file | $N_{Ed}$ [kN] | $N_{Rd}$ [kN] | $M_{Edy}$ [kNm] | $M_{Rdy}$ [kNm] | Use ID [%] | Calculation ID | Width of cracks $w$ [mm] | Use $w_{max}$ [%] | Review $w_{max}$ |
|--------------|----------------|----------------|-----------------|-----------------|-----------|----------------|-----------------------|-----------------|----------------|
| B12M24       | -403.75        | -13222.57      | 128.69          | 142.13          | 90.6      | OK            | 0.695                 | > 300           | NOK            |
| B12M25       | -268.95        | -13222.57      | 86.36           | 128.79          | 67.1      | OK            | 0.468                 | 234             | NOK            |
| B12M26       | -200.30        | -13222.57      | 64.76           | 121.90          | 53.2      | OK            | 0.352                 | 176             | NOK            |

Analysis results in conclusion that all three types of loading are satisfactory, but none satisfies limiting bearing capacity of cracks. The maximum allowed size of cracks is $w_{max} = 0.2$ mm. All analyzed cases breach such limit, thus we can state that origin of cracks as result of shield end contact on lining is possible.

5.3.2. Reinforced concrete lining loading by hydraulic press. Moving shield ahead, hydraulic presses set on last assembled tubings ring. In total shield is pressing on the ring by 16 couples of hydraulic cylinders, with angle 22.5° between each two couples. Each couple is pressing on the lining through steel plate (transferring sheet) with dimensions of 153 x 635mm, with technological maximum load on one steel plate up to force 2.433 MN. This force is not acting in the center of the tubing abutting joint (axis of tubing and axis of hydraulic press cylinder is shifted by 18.5 mm), so loading on tubings is eccentric.

The most loaded tubing is loaded with maximum three couples of hydraulic presses, resulting in force of

$$N_j = 3 \times 2.432849 = 7.3 \text{ MN}$$

(15)

All input data described above were applied in calculation of tubings by program ATENA (Červenka Consulting), where crack development was compared for different values of loading (size of cracks can be shown according to chosen preferences).

Task was modelled with nonlinear material model (3d_nonlinear_cementitious), fracture energy was derived from ModelCode 1990.

Boundary conditions of static stage of tubing model were set to follow real situation as close as possible. Loading was applied to the centre of each of the three steel plates.

Complete analysis gives more detailed data of cracks with size $> 0.15$mm with overall loading $\approx 7.3$ MN, which are depicted on Figure 12. Cracks in both cases are similar to cracks observed on lining in V.A track tunnel.

![Figure 12. Cracks > 0.15 mm by overall loading ≈ 7.3 MN applied on tubing (2.4 MN/1 couple press).](image)
6. Conclusions
The coefficient of the bearing capacity of the lining $s$ describes reserves in the bearing capacity of the lining and also shows the point where the ultimate state will be reached first. Cast iron lining has spots where cracks are faced outwards to rock and this method could find such spots. Analysis made for deformation modulus of rock $E_o = 5, 50 \text{ a } 100 \text{ MPa}$ and reduced area-wise moment by Muir Wood A. M. is shown in Figure 8, where one can simply identity values smaller than limiting coefficient of bearing capacity $s = 1$. Computation shows that most dangerous state is with deformation modulus $E_o = 5 \text{ MPa}$.

In reinforced concrete lining of tubings $\varnothing 5.3/5.8 \text{ m}$ in V.A line of Prague Underground, hairline cracks in the altitude of horizontal axis of the lining parallel to longitudinal track axis were identified early after assembling. As contribution to discussion about origin of the cracks, analysis of the hypothetical origin of cracks was made. We compared hypothesis of cracks origin as a result of horizontal force from shield end acting at time of moving shield in direction radius of track $R = 650 \text{ m}$ and hypothesis of crack origin as a result of pressure of shield’s hydraulic press cylinders. The result achieved were not decisive: none of both hypotheses is deemed absolutely correct. Cracks could be caused by relatively small force of the shield end and cracks $> 0.15 \text{ mm}$ could also appear after applying force of $7.3 \text{ MN}$ on tubing as depicted in Figure 12. As no decisive result is available, both theories may explain the cause of cracks and it is recommended to study this problem more deeply on both by mathematical and laboratory models.

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