CP VIOLATION AND CKM PHASES FROM TIME-DEPENDENCES OF UNTAGGED $B_s$ DECAYS

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The $B_s$ system is analyzed in light of a possible width difference $\Delta \Gamma_s$ between its mass eigenstates. If $\Delta \Gamma_s$ is sizable, untagged $B_s$-meson decays may allow a probe of CP violation and moreover the extraction both of the Wolfenstein parameter $\eta$ and of the notoriously difficult to measure angle $\gamma$ of the unitarity triangle. To accomplish this ambitious task, time-dependent angular distributions for untagged $B_s$ decays into admixtures of CP eigenstates and channels that are caused by $b \to c \bar{s} q$ quark-level transitions play a key role. The work described here was done in collaboration with Isard Dunietz.

1 Introduction

The time-evolution due to $B^0_s - \bar{B}^0_s$ mixing is governed by the $B_s$ mass eigenstates $B_s^{\text{Heavy}}$ and $B_s^{\text{Light}}$ which are characterized by their mass eigenvalues $M^{(s)}_H$, $M^{(s)}_L$ and decay widths $\Gamma^{(s)}_H$, $\Gamma^{(s)}_L$. Because of these mixing effects, oscillatory $\Delta M_s t$ terms with $\Delta M_s \equiv M^{(s)}_H - M^{(s)}_L$ show up in the time-dependent transition rates $\Gamma(B^{0}_s(t) \to f)$ and $\Gamma(\bar{B}^{0}_s(t) \to f)$ describing decays of initially present $B^{0}_s$ and $\bar{B}^{0}_s$ mesons into a final state $f$, respectively. The “strength” of the $B^0_s - \bar{B}^0_s$ oscillations is measured by the mixing parameter $x_s \equiv \Delta \Gamma_s/\Gamma_s$, where $\Gamma_s \equiv (\Gamma^{(s)}_H + \Gamma^{(s)}_L)/2$. Within the Standard Model one expects $x_s = \mathcal{O}(20)$ implying very rapid $B^0_s - \bar{B}^0_s$ oscillations which require an excellent vertex resolution system to keep track of the $\Delta M_s t$ terms. That is obviously a formidable experimental task.

However, as pointed out by Dunietz,[3] it may not be necessary to trace the rapid $\Delta M_s t$ oscillations in order to obtain insights into the mechanism of CP violation. This remarkable feature is due to the expected sizable width difference $\Delta \Gamma_s \equiv \Gamma^{(s)}_H - \Gamma^{(s)}_L$. The major contributions to $\Delta \Gamma_s$, which may be as large as $\mathcal{O}(20\%)$ of the average decay width $\Gamma_s$, originate from $b \to \bar{c}c\bar{s}$ transitions into final states that are common both to $B^{0}_s$ and $\bar{B}^{0}_s$. Because of this width difference already untagged $B_s$ rates, which are defined by

$$\Gamma[f(t)] \equiv \Gamma(B^{0}_s(t) \to f) + \Gamma(\bar{B}^{0}_s(t) \to f),$$

may provide valuable information about the phase structure of the observable

$$\xi^{(s)}_f = \exp \left( -i \Theta^{(s)}_{M12} \frac{A(B^{0}_s \to f)}{A(\bar{B}^{0}_s \to f)} \right),$$

where $\Theta^{(s)}_{M12}$ is the weak $B^0_s - \bar{B}^0_s$ mixing phase. This can be seen nicely by writing Eq. (1) in a more explicit way as follows:

$$\Gamma[f(t)] \propto \left[ 1 + \left| \xi^{(s)}_f \right|^2 \right] \left[ e^{-\Gamma^{(s)}_{L,t}} + e^{-\Gamma^{(s)}_{H,t}} \right] - 2 \Re \xi^{(s)}_f \left[ e^{-\Gamma^{(s)}_{L,t}} - e^{-\Gamma^{(s)}_{H,t}} \right].$$

In this expression the rapid oscillatory $\Delta M_s t$ terms, which show up in the tagged rates, cancel. Therefore it depends only on the two exponents $e^{-\Gamma^{(s)}_{L,t}}$ and $e^{-\Gamma^{(s)}_{H,t}}$, where $\Gamma^{(s)}_{L,t}$ and $\Gamma^{(s)}_{H,t}$ can be determined e.g. from the angular distribution of the decay $B_s \to J/\psi \phi$. From an experimental point of view such untagged analyses are clearly much more promising than tagged ones in respect of efficiency, acceptance and purity.

2 A Transparent Example

In order to illustrate these untagged rates in more detail, let me discuss an estimate of the angle $\gamma$ of the usual “non-squashed” unitarity triangle of the Cabibbo–Kobayashi–Maskawa matrix (CKM matrix) using untagged $B_s \to K^+ K^-$ and $B_s \to K^0 \bar{K}^0$ decays. This approach has been proposed very recently by Dunietz and myself.[4] Using the SU(2) isospin symmetry of strong interactions to relate the QCD penguin contributions to these decays (electroweak penguins are color-suppressed...
in these modes and thus play a minor role), we obtain
\[
\Gamma[K^+K^-(t)] \propto |P'|^2 \left[ (1 - 2 |r| \cos \rho \cos \gamma + |r|^2 \cos^2 \gamma e^{-\Gamma_{K^0}^\gamma t} + |r|^2 \sin^2 \gamma e^{-\Gamma_{K^0}^\gamma t} \right] (4)
\]
and
\[
\Gamma[K^0\bar{K}^0(t)] \propto |P'|^2 e^{-\Gamma_{K^0}^\gamma t},
\]
where
\[
|r| e^{i\rho} = \frac{|T'|}{|P'|} e^{i(\delta_{\rho'} - \delta_{\rho})}.
\]

Here \( P' \) denotes the \( \bar{b} \rightarrow \bar{s} \) QCD penguin amplitude, \( T' \) is the color-allowed \( \bar{b} \rightarrow \bar{u}u\bar{s} \) tree amplitude, and \( \delta_{\rho'} \) and \( \delta_{\rho} \) are the corresponding CP-conserving strong phases. In order to determine \( \gamma \) from the untagged rates Eqs. (4) and (5), we need an additional input that is provided by the \( SU(3) \) flavor symmetry of strong interactions. If we neglect the color-suppressed current-current contributions to \( B^+ \rightarrow \pi^+\pi^0 \) we find
\[
|T'| \approx \lambda \frac{f_K}{f_{\pi}} \sqrt{2} |A(B^+ \rightarrow \pi^+\pi^0)|,
\]
where \( \lambda \) is the Wolfenstein parameter, \( f_K \) and \( f_{\pi} \) are the \( K \) and \( \pi \) meson decay constants, respectively, and \( A(B^+ \rightarrow \pi^+\pi^0) \) denotes the appropriately normalized \( B^+ \rightarrow \pi^+\pi^0 \) decay amplitude. Since \( |r| = |T'|/|P'| \) can be estimated with the help of Eq. (7) and allows the extraction of \( \gamma \) from the part of Eq. (4) evolving with the exponent \( e^{-\Gamma_{K^0}^\gamma t} \).

3 \hspace{1em} \( B_s \) Decays into Admixtures of CP Eigenstates

As we will see in a moment, one can even do better than in the previous section, i.e. without using an \( SU(3) \) flavor symmetry input, by considering the decays corresponding to \( B_s \rightarrow K\bar{K} \) where two vector mesons (or higher resonances) are present in the final states.

3.1 An Extraction of \( \gamma \) using Untagged \( B_s \rightarrow K^{*+}K^{*-} \) and \( B_s \rightarrow K^{*0}\bar{K}^{*0} \) Decays

The untagged angular distributions of these decays, which take the general form
\[
[f(\theta, \phi, \psi; t)] = \sum_k \left[ \bar{b}^{(k)}(t) + b^{(k)}(t) \right] g^{(k)}(\theta, \phi, \psi), \tag{8}
\]
provide many more observables than the untagged modes \( B_s \rightarrow K^{*+}K^{-} \) and \( B_s \rightarrow K^{*0}\bar{K}^{0*} \) discussed in Section 2. Here \( \theta, \phi \) and \( \psi \) are generic decay angles describing the kinematics of the decay products arising in the decay chain \( B_s \rightarrow K^* \rightarrow \pi K \rightarrow \pi K \). The observables \( \left[ \bar{b}^{(k)}(t) + b^{(k)}(t) \right] \) governing the time-evolution of the angular distribution Eq. (8) are given by real or imaginary parts of bilinear combinations of decay amplitudes that are of the following structure:
\[
\frac{\left( (K^*\bar{K}^*)_f |H_{\text{eff}}| B_s(t) \right)^*}{\left( (K^*\bar{K}^*)_f |H_{\text{eff}}| B_s(t) \right)} (9)
\]

In this expression \( f \) and \( \bar{f} \) are labels that define the relative polarizations of \( K^* \) and \( \bar{K}^* \) in final state configurations \( (K^*\bar{K}^*)_f \) (e.g. linear polarization states \( |0, \|, \bot \rangle \) with CP eigenvalues \( \eta_f^\text{CP} \):
\[
\langle CP \rangle \left( (K^*\bar{K}^*)_f \right) = \eta_f^\text{CP} \left( (K^*\bar{K}^*)_f \right). \tag{10}
\]

An analogous relation holds for \( \bar{f} \). The observables of the angular distributions for \( B_s \rightarrow K^{*+}K^{*-} \) and \( B_s \rightarrow K^{*0}\bar{K}^{*0} \) are given explicitly in Ref. Here the \( SU(2) \) isospin symmetry of strong interactions, the QCD penguin contributions of these decays can be related to each other. If one takes into account these relations and goes very carefully through the observables of the angular distributions, one finds that they allow the extraction of the CKM angle \( \gamma \) without any additional theoretical input. In particular no \( SU(3) \) symmetry arguments as in Section 2 are needed. The angular distributions provide moreover information about the hadronization dynamics of the corresponding decays, and the formalism developed for \( B_s \rightarrow K^{*+}K^{*-} \) applies also to \( B_s \rightarrow p\phi \phi \) if we perform a suitable replacement of variables. Since that channel is expected to be dominated by electroweak penguins, it may allow interesting insights into the physics of these operators.

3.2 The “Gold-plated” Transitions to Extract \( \eta \)

This subsection is devoted to an analysis of the untagged decays \( B_s \rightarrow D_{s*}^+D_s^- \) and \( B_s \rightarrow J/\psi \phi \).
which is the counterpart of the “gold-plated” mode $B_d \rightarrow J/\psi K_s$ to measure the angle $\beta$ of the unitarity triangle. These decays are dominated by a single CKM amplitude. Consequently the hadronic uncertainties cancel in the quantity $\xi_f^{(s)}$ defined by Eq. (8), which takes in that particular case the form

$$\xi_f^{(s)} = \exp(i \phi_{\text{CKM}}), \quad (11)$$

and the observables of the angular distributions simplify considerably. A characteristic feature of these angular distributions is interference between CP-even and CP-odd final state configurations leading to observables that are proportional to

$$\left( e^{-r_{L}^{(s)} t} - e^{-r_{H}^{(s)} t} \right) \sin \phi_{\text{CKM}}. \quad (12)$$

Here the CP-violating weak phase is given by $\phi_{\text{CKM}} = 2\lambda^2 \eta \approx O(0.03)$, where the Wolfenstein parameter $\eta$ fixes the height of the unitarity triangle. The observables of the angular distributions for both the color-allowed channel $B_s \rightarrow D_s^{(*)} D_s^{(*)}$ and the color-suppressed transition $B_s \rightarrow J/\psi \phi$ each provide sufficient information to determine the CP-violating weak phase $\phi_{\text{CKM}}$ from their untagged data samples thereby fixing the Wolfenstein parameter $\eta$. The extraction of $\phi_{\text{CKM}}$ is not as clean as that of $\beta$ from $B_d \rightarrow J/\psi K_s$. This is due to the smallness of $\phi_{\text{CKM}}$ with respect to $\beta$ enhancing the importance of the unmixed amplitudes proportional to the CKM factor $V_{ub} V_{us}$ which are similarly suppressed in both cases.

### 4 $B_s$ Decays caused by $b \rightarrow c u \bar{s}$

The $B_s$ decays discussed in this section are pure tree decays and probe the CKM angle $\gamma$ in a clean way. There are by now well-known strategies on the market using the time evolutions of such modes, e.g. $B_s \rightarrow D^0 \phi \rightarrow D_{s}^{*0} \phi$ and $B_s \rightarrow D_s^{*+} K^- \rightarrow D_{s}^{*+} K^- \rightarrow D_s^{*+} K^-$ to extract $\gamma$. However, in these strategies tagging is essential and the rapid $\Delta M_{f,t}$ oscillations have to be resolved which is an experimental challenge. The question what can be learned from untagged data samples of these decays, where the $\Delta M_{f,t}$ terms cancel, has been investigated by Dunietz in Ref. 20. In the untagged case the determination of $\gamma$ requires additional inputs: a measurement of the untagged $B_s \rightarrow D_{s}^{*+} \phi$ rate in the case of the color-suppressed modes $B_s \rightarrow D^{0} \phi$, and a theoretical input corresponding to the ratio of the unmixed rates $\Gamma(B_s^{0} \rightarrow D_{s}^{+} K^-)/\Gamma(B_s^{0} \rightarrow D_{s}^{-} \pi^+)$ in the case of the color-allowed decays $B_s \rightarrow D_s^{+} K^-$. This ratio can be estimated with the help of the “factorization” hypothesis which may work reasonably well for these color-allowed channels.

Interestingly the untagged data samples may exhibit CP-violating effects that are described by observables of the form

$$\Gamma[f(t)] - \Gamma[\bar{f}(t)] \propto \left( e^{-r_{L}^{(s)} t} - e^{-r_{H}^{(s)} t} \right) \sin \phi \sin \gamma. \quad (13)$$

Here $\phi$ is a CP-conserving strong phase shift and $\gamma$ is the usual angle of the unitarity triangle. Because of the sin $\phi$ factor, a non-trivial strong phase shift is essential in that case. Consequently the CP-violating observables Eq. (13) vanish within the factorization approximation predicting $\phi \in \{0, \pi\}$. Since factorization may be a reasonable working assumption for the color-allowed modes $B_s \rightarrow D_{s}^{+} K^- \rightarrow D_s^{+} K^-$, the CP-violating effects in their untagged data samples are expected to be very small. On the other hand, the factorization hypothesis is very questionable for the color-suppressed decays $B_s \rightarrow D^{0} \phi$ and sizable CP violation may show up in the corresponding untagged rates.

Concerning such CP-violating effects and the extraction of $\gamma$ from untagged rates, the decays $B_s \rightarrow D_{s}^{(*)} K^*$ and $B_s \rightarrow D_{s}^{(*)} \phi$ are expected to be more promising than the transitions discussed above. As was shown in Ref. 20, the time-dependencies of their untagged angular distributions allow a clean extraction of the CKM angle $\gamma$ without any additional input. The final state configurations of these decays are not admixtures of CP eigenstates as in Section 3. They can, however, be classified by their parity eigenvalues. A characteristic feature of the angular distributions are interferences between parity-even and parity-odd configurations that may lead to potentially large CP-violating effects in the untagged data samples even when all strong phase shifts vanish. An example of such an untagged CP-violating observable is the following quantity.
In that expression bilinear combinations of certain decay amplitudes (see Eq. 9) show up, \( f \in \{ 0, \| \} \) denotes a linear polarization state and \( \delta_f, \vartheta_f \) are CP-conserving phase shifts that are induced through strong final state interactions. For the details concerning the observable Eq. (4) – in particular the definition of the relevant charge-conjugate amplitudes \( A^c_{ij} \) and the quantities \( |R_f| \) – the reader is referred to Ref. 7.

Here I would like to emphasize only that the strong effects. For the details concerning the observables see M. Beneke, G. Buchalla and I. Dunietz, hep-ph/9605259; M. Beneke, these proceedings, hep-ph/9609214.

A similar comment applies also to the mode \( B_s \to D_s^*D_s^- \) discussed in Subsection 3.2.

5 Conclusions

The oscillatory \( \Delta M_s t \) terms arising from \( B_s^0 - \bar{B}_s^0 \) mixing, which may be too rapid to be resolved with present vertex technology, cancel in untagged rates of \( B_s \) decays that depend therefore only on the two exponents \( e^{-\Gamma_s^0 t} \) and \( e^{-\Gamma_{s}^* t} \). If the width difference \( \Delta \Gamma_s \) is sizable – as is expected from theoretical analyses – untagged \( B_s \) decays may allow the determination both of the CKM angle \( \gamma \) and of the Wolfenstein parameter \( \eta \) and may furthermore provide valuable insights into the mechanism of CP violation and the hadronization dynamics of the corresponding decays. To this end certain angular distributions may play a key role.

Compared to the tagged case, such untagged measurements are much more promising in view of efficiency, acceptance and purity. A lot of statistics is required, however, and the natural place for these experiments seems to be a hadron collider. Obviously the feasibility of untagged strategies to extract CKM phases depends crucially on a sizable width difference \( \Delta \Gamma_s \). Even if it should turn out to be too small for such untagged analyses, once \( \Delta \Gamma_s \neq 0 \) has been established experimentally, the formulae developed in Refs. 4, 7 have also to be used to determine CKM phases correctly from tagged measurements. In this sense we cannot lose and an exciting future concerning \( B_s \) decays may lie ahead of us!

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