Eigenstructure assignment for the position regulation of a fully-actuated marine craft

Christina Kazantzidou * Tristan Perez *; Francis Valentinis **

* Robotics and Autonomous Systems, School of Electrical Engineering and Computer Science, Queensland University of Technology (QUT), Brisbane QLD 4000, Australia
(e-mail: christina.kazantzidou@qut.edu.au, tristan.perez@qut.edu.au)
** Maritime Division, Defence Science and Technology Group, Fisherman’s Bend VIC 3207, Australia
(e-mail: Francis.Valentinis@dsto.defence.gov.au)

Abstract: In this paper, we adopt eigenstructure assignment in order to assist with the tuning of a nonlinear energy-based regulator for the positioning of a marine craft in the horizontal plane. The control law is designed using interconnection and damping assignment passivity-based control (IDA-PBC), which results in passive target dynamics that can be expressed as a port-Hamiltonian system (PHS). IDA-PBC has been applied before with success in a number of different applications. To date, however, there has been minimal development in either tuning tools or techniques that can analytically aid the designer in achieving the desired response characteristics. Good results can be achieved only with intuitive and meticulous manual tuning. By linearising the nonlinear target dynamics in PHS form, we demonstrate that the analysis of the eigenstructure, and consequently its assignment can significantly aid the tuning process. The approach provides a mechanism for simultaneously considering the frequency domain characteristics at a point of linearisation, as well as the time domain characteristics. A demonstration of the method is provided in the form of a design study for position regulation of an underwater vehicle in the horizontal plane.

Keywords: Marine Control Systems, Port-Hamiltonian Systems, Eigenstructure Assignment

1. INTRODUCTION

Mathematical models of marine craft often present a significant degree of uncertainty. This is because their dynamic response involves complex fluid-body and environmental interactions that are challenging to model over an extensive envelope of operating conditions. Passive energy-based nonlinear control law designs have proven to be effective in this application domain (see Perez et al. (2013)), with control laws being developed as a means of controlling a vehicle in the presence of such uncertainty. As the complexity of such designs increases, however, tuning becomes progressively more difficult. In this paper, we propose the use of eigenstructure assignment as a means to aid this tuning process.

There have been a number of interesting applications of energy-based approaches in the motion control of marine craft. Woolsey and Leonard (2002) adopt a Lagrangian rather than Hamiltonian approach, and consider the problem of stabilisation of fully-actuated underwater vehicles. Donaire and Perez (2010) design a control law to regulate the position of fully-actuated marine vehicles in three degrees of freedom (3DOF), namely, surge, sway, and yaw.

The problem of dynamic positioning of offshore vessels is addressed in Donaire and Perez (2012). The treatment of underactuation and more realistic hydrodynamics is described in Valentinis et al. (2015b), and the addition of energy-based guidance in Valentinis et al. (2015a).

In all of these examples, tuning of the target dynamics is achieved meticulously using designers’ intuition. In the case of designs with limited numbers of parameters such as that of Donaire and Perez (2010), this is very much achievable. As designs become more complex, however, as is the case in Valentinis et al. (2015b), the process becomes more difficult. Achieving good time domain performance can become a challenge of balancing an increasing number of often contradictory criteria.

Conventionally, tuning the target dynamics for a nonlinear control law has been conducted based primarily on nonlinear time domain objectives. Many designs, however, such as controllers for marine craft, can benefit from frequency domain considerations in tuning. For example, the design of a control law may need to reject certain wave excitation frequencies. In order to use certain bottom-looking sensors optimally, motion will need to be constrained to a range within a desired frequency band. Likewise, for a manned vessel, to ensure comfort for passengers, it is desirable to remain within a frequency band known to result in comfortable sailing.

---

* This work was partially supported by the Australian Research Council through the Discovery Project DP140100896.

1 Tristan Perez is also with the Institute for Future Environments (IFE) at QUT and the ARC Centre of Excellence in Mathematical and Statistical Frontiers (ACEMS).
Usually, these kinds of criteria are thought of as eigenvalue constraints. In reality, however, it is essential to consider both the target eigenvalues and eigenvectors simultaneously. This is important, because while tuning a system it is essential to identify modes with a particular identity and mode shape.

Whilst it is possible for a skilled designer to tune a system in the time domain to achieve these objectives, being able to utilise an approach based on eigenstructure has much more intuitive appeal. Likewise, thinking in terms of eigenstructure can simplify the gain optimisation process if a designer has an approach that allows for an analytical selection of gain values based on a desired eigenstructure.

In this paper, our main objective is to demonstrate how analysis and assignment of the eigenstructure of the target dynamics at a set point can aid the tuning of the regulation controller designed in Donaire and Perez (2010). We analyse the system to determine the eigenstructure assignments that can be achieved. To demonstrate the achievable outcomes, we consider the case study of a comparison of the position regulation performance of a number of nonlinear unmanned underwater vehicle control laws which are compared through simulations.

The paper is organised as follows. In Section 2, we briefly describe port-Hamiltonian systems with dissipation. The problem of position regulation of fully-actuated marine craft, and the controller design are analysed in Section 3. In Section 4, we show how to tune the stabilising controller and a way to investigate the corresponding eigenstructure assignment, which is provided in detail in Section 5. The analysis is illustrated with a case study in Section 6 with concluding remarks and future work presented in Section 7.

2. PORT-HAMILTONIAN SYSTEMS WITH DISSIPATION

In energy-based control, the objective is to shape the response of a system in such a way that the energy minimum is obtained in closed loop at the desired equilibrium. If a mechanical system is modeled as a port-Hamiltonian system (PHS), also referred to as a port-controlled Hamiltonian system in van der Schaft (2000), the interconnection and damping assignment passivity-based control (IDA-PBC) method can be used to enact such an energy-based control outcome, as described in Ortega et al. (2002).

This method shapes the total energy of the closed-loop system, i.e., the kinetic and potential energy, and injects damping. The dynamics of the open-loop system are matched with the dynamics of the closed-loop system to obtain the controller. For a survey on interconnection and damping assignment passivity-based control we refer the reader to Ortega and García-Canseco (2004) and the references therein.

We consider the input-output port-Hamiltonian system with dissipation

\[
\dot{x} = \left[ E(x) - F(x) \right] \frac{\partial H}{\partial x} + G(x) u, \quad (1)
\]

\[
y = G^T(x) \frac{\partial H}{\partial x},
\]

where the antisymmetric matrix \( E(x) = -E(x)^T \) describes the interconnection of the energy storing in the system, the positive semidefinite matrix \( F(x) \geq 0 \) describes the dissipation in the system and the matrix \( G(x) \) weighs the action of the input and defines the output, see for example van der Schaft (2006), Donaire and Perez (2012). Stability of the equilibrium point \( x^* \) at which the energy is minimised is guaranteed if: (i) \( H(x) \) is bounded from below, (ii) \( E(x) \) is antisymmetric, and (iii) \( F(x) \) is positive semidefinite, see for example Perez et al. (2013).

The objective is to use a controller such that the desired closed-loop system has the form

\[
\dot{x} = \left[ E_d(x) - F_d(x) \right] \frac{\partial H_d}{\partial x}, \quad (3)
\]

where \( E_d(x) = -E_d(x)^T, F_d(x) \geq 0, H_d(x) \) is bounded from below and the equilibrium point \( x^* \) minimises \( H_d(x) \), see for example Perez et al. (2013).

3. POSITION REGULATION OF FULLY-ACTUATED MARINE CRAFT IN 3DOF

For the position regulation problem of a marine craft in the horizontal plane, we consider three degrees of freedom in surge, sway, and yaw. The classical model of a marine craft is described by

\[
M \ddot{\nu} + C(\nu) \dot{\nu} + D(\nu) \nu = \tau, \quad (4)
\]

\[
\eta = R(\eta) \nu, \quad (5)
\]

where \( \eta \triangleq [n, e, \psi]^T \) is the generalised-position vector, \( \nu \triangleq [u, v, r]^T \) is the body-fixed velocity vector and \( \tau \) is the vector of total forces and moments, see for example Fossen (1994). The following terminology is commonly used and can be found in Fossen (2011). The mass matrix \( M \), which is symmetric, includes the rigid-body and the added mass matrix components, and has the structure:

\[
M \triangleq \begin{bmatrix}
M_{11} & 0 & 0 \\
0 & M_{22} & M_{23} \\
0 & M_{23} & M_{33}
\end{bmatrix}
\]

\[
\triangleq \begin{bmatrix}
m - X_u & 0 & 0 \\
0 & m - Y_v & m x_y - Y_z \\
0 & m x_y - Y_z & I_z - N_r
\end{bmatrix}, \quad (6)
\]

where \( m \) is the mass of the marine craft, \( x_y \) is the forward offset of the centre of gravity relative to a reference point, and \( X_u, Y_v, N_r \) are the added mass coefficients. The Coriolis-centripetal matrix, which is due to the rotation of the body-fixed reference frame with respect to the inertial reference frame, is antisymmetric and given by

\[
C(\nu) \triangleq \begin{bmatrix}
0 & 0 & -C_{13}(\nu) \\
0 & 0 & C_{23}(\nu) \\
C_{13}(\nu) & -C_{23}(\nu) & 0
\end{bmatrix}, \quad (7)
\]

where

\[
C_{13}(\nu) \triangleq (m - Y_v) u + (m x_y - Y_z) r = M_{22} v + M_{23} r,
\]

\[
C_{23}(\nu) \triangleq (m - X_u) u = M_{11} u.
\]

The damping matrix \( D(\nu) \) is assumed to be diagonally dominant and has the form:

\[
D(\nu) \triangleq D + D_n(\nu), \quad (8)
\]
where $\mathbf{D}$ is the linear damping component due to potential damping and possible skin friction, and $\mathbf{D}_n(\nu)$ is the nonlinear damping component due to quadratic damping and higher-order terms. Finally, the rotation matrix $\mathbf{R}(\eta)$ about the vertical axis (yaw) is given by

$$
\mathbf{R}(\eta) \triangleq \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

and it is an element in $SO(3)$, i.e., it is orthogonal and \( \det \mathbf{R}(\eta) = 1 \).

We now design a controller for the regulation of the position of a marine craft described by the nonlinear system:

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\mathbf{C}(x_1) - \mathbf{R}^T(x_2) \\ \mathbf{R}(x_2) \end{bmatrix} - \begin{bmatrix} \mathbf{D}(x_1) \\ 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x_1} \\ \frac{\partial H}{\partial x_2} \end{bmatrix} + \begin{bmatrix} \tau \end{bmatrix},
$$

where

$$
\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \triangleq \begin{bmatrix} \mathbf{M} \nu \\ \eta \end{bmatrix},
$$

and $\mathbf{C}(x_1) \triangleq \mathbf{C} (\mathbf{M}^{-1} x_1)$, which is antisymmetric, $\mathbf{D}(x_1) \triangleq \mathbf{D} (\mathbf{M}^{-1} x_1)$, which is positive definite, and the Hamiltonian is $\mathcal{H}(x_1, x_2) \triangleq \frac{1}{2} x_1^T \mathbf{M}^{-1} x_1$, which is the total kinetic energy, see for example Donaire and Perez (2012), Perez et al. (2013). The desired closed-loop system will have the form:

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\mathbf{C}(x_1) - \mathbf{R}^T(x_2) \\ \mathbf{R}(x_2) \end{bmatrix} - \begin{bmatrix} \mathbf{D}(x_1) \\ 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H_d}{\partial x_1} \\ \frac{\partial H_d}{\partial x_2} \end{bmatrix} + \begin{bmatrix} \tau \end{bmatrix},
$$

where $\mathbf{D}_d(x_1) > 0$ is the desired dissipation, which may be chosen to have a nonlinear component equal to the one of $\mathbf{D}(x_1)$, i.e.,

$$
\mathbf{D}_d(x_1) \triangleq \mathbf{D}_d + \mathbf{D}_n(x_1),
$$

where $\mathbf{D}_d > 0$ is the desired linear dissipation to be chosen. The desired Hamiltonian $\mathcal{H}_d(x_1, x_2)$, having a minimum at the desired equilibrium, is defined as

$$
\mathcal{H}_d(x_1, x_2) \triangleq \frac{1}{2} x_1^T \mathbf{M}^{-1} x_1 + \frac{1}{2} (x_2 - x_2^0)^T \mathbf{K} (x_2 - x_2^0),
$$

with $\mathbf{K} > 0$ to be chosen.

The associated matching problem corresponds to equating the right-hand side of (10) with the right-hand side of (12). This leads to the controller

$$
\tau = - \left[ \mathbf{D}_d(x_1) - \mathbf{D}_n(x_1) \right] \mathbf{M}^{-1} x_1 - \mathbf{R}^T(x_2) \mathbf{K} (x_2 - x_2^0)
$$

$$
= - (\mathbf{D}_d - \mathbf{D}) \mathbf{M}^{-1} x_1 - \mathbf{R}^T(x_2) \mathbf{K} (x_2 - x_2^0).
$$

The controller adds damping and reshapes the total energy of the system by adding potential energy that attracts the system to the desired position like a virtual spring. Since the desired Hamiltonian has a strict minimum at $x = (0, x_2^0)$, we obtain global asymptotic stability of the desired equilibrium by choosing the desired Hamiltonian as a Lyapunov function and applying the invariance principle, see for example Khalil (2000), Donaire and Perez (2012), Perez et al. (2013).

4. TUNING OF THE STABILISING CONTROLLER FOR POSITION REGULATION TO ZERO

The open literature provides little guidance on methods to tune the controller $\mathbf{r}$ in (15). We propose using a frequency domain approach, whereby the closed-loop system is linearised about the equilibrium point and the attainable eigenstructure assignment is investigated at this point.

The closed-loop system has the form $\dot{x} = f(x)$ and it has equilibrium point $x$. Without loss of generality, we may assume that $x_2^0 = 0$, so that $\mathbf{R}(x_2^0) = I_3$. This comes from a simple change of coordinates. Defining the deviations from the equilibrium as $\mathbf{\bar{x}} = x - x$, we may obtain the linearised system

$$
\dot{\mathbf{\bar{x}}} = \frac{\partial^T f(x)}{\partial x}|_{x=x} \mathbf{\bar{x}},
$$

where

$$
\frac{\partial^T f(x)}{\partial x}|_{x=x} = -\mathbf{D}_d \mathbf{M}^{-1} \frac{1}{2} (\mathbf{K} + \mathbf{K}^T) \mathbf{M}^{-1} 0 \triangleq \mathbf{A}_d.
$$

The matrix $\frac{1}{2} (\mathbf{K} + \mathbf{K}^T)$ is the symmetric part of $\mathbf{K}$. We assume that the matrix $\mathbf{K}$ to be chosen, apart from being positive definite, is also symmetric, i.e., $\mathbf{K} = \frac{1}{2} (\mathbf{K} + \mathbf{K}^T)$.

A way to investigate the eigenstructure assignment of the linearised closed-loop system and simultaneously tune the controller $\mathbf{r}$ is to firstly compute the null space of $[\mathbf{A}_d - \lambda \mathbf{I}_3]$, i.e., the polynomial matrix $\mathbf{X}(\lambda)$ such that $[\mathbf{A}_d - \lambda \mathbf{I}_3] \mathbf{X}(\lambda) = 0$. Then

$$
\begin{bmatrix} -\mathbf{D}_d \mathbf{M}^{-1} - \lambda \mathbf{I}_3 & -\mathbf{K} \\ \mathbf{M}^{-1} - \lambda \mathbf{I}_3 & -\mathbf{X}(\lambda) \end{bmatrix} = 0,
$$

or, equivalently,

$$
\mathbf{X}_1(\lambda) = \lambda \mathbf{M} \mathbf{X}_2(\lambda),
$$

$$
(\lambda^2 \mathbf{M} + \lambda \mathbf{D}_d + \mathbf{K}) \mathbf{X}_2(\lambda) = 0.
$$

If we choose $\mathbf{X}_2(\lambda)$ to be any nonsingular constant matrix, for example $\mathbf{X}_2(\lambda) = \mathbf{I}_3$, we obtain

$$
\mathbf{X}(\lambda) = \begin{bmatrix} \lambda \mathbf{M} \\ \mathbf{I}_3 \end{bmatrix},
$$

$$
\lambda^2 \mathbf{M} + \lambda \mathbf{D}_d + \mathbf{K} = 0.
$$

The matrix polynomial in the left-hand side of (22) has degree 2, which shows that we can assign six arbitrary closed-loop eigenvalues $\lambda_1, \ldots, \lambda_6$, located in the left-half complex plane to guarantee linear stability.\(^3\)

We notice that the determinant of (22) is the characteristic equation of the linearised closed-loop system. Indeed, the characteristic polynomial of the closed-loop system is

$$
\det (\lambda \mathbf{I}_3 - \mathbf{A}_d) = \det \begin{bmatrix} \lambda \mathbf{I}_3 + \mathbf{D}_d \mathbf{M}^{-1} \mathbf{K} \\ -\mathbf{M}^{-1} \lambda \mathbf{I}_3 \end{bmatrix}
$$

$$
= \det \begin{bmatrix} \lambda \mathbf{I}_3 + \mathbf{D}_d \mathbf{M}^{-1} + \frac{1}{\lambda} \mathbf{K} \mathbf{M}^{-1} \mathbf{K} \\ 0 \end{bmatrix} \det (\lambda \mathbf{I}_3)
$$

$$
= \det \left( \lambda^2 \mathbf{M} + \lambda \mathbf{D}_d + \mathbf{K} \right) \det (\mathbf{M}^{-1}) \det (\lambda \mathbf{I}_3).
$$

\(^3\) Linear stability does not guarantee nonlinear stability, however the latter is guaranteed by the PHS form of the target dynamics, which cannot be satisfied if the linearised system is unstable.
It can be shown that the matrix polynomial
\[
\lambda^2 I_3 - \lambda \text{diag}\{\lambda_1 + \lambda_2, \lambda_3 + \lambda_4, \lambda_5 + \lambda_6\} + \text{diag}\{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6\}
\]
has determinant \((\lambda - \lambda_1) \cdots (\lambda - \lambda_6)\).

Equating the matrix polynomial in (22) with the matrix polynomial (24) multiplied on the right by \(M\), we find
\[
D_d = -\text{diag}\{\lambda_1 + \lambda_2, \lambda_3 + \lambda_4, \lambda_5 + \lambda_6\} M, \quad (25)
\]
and the linearised closed-loop system will have the desired eigenvalues \(\lambda_1, \ldots, \lambda_6\), which we require to be negative real to ensure stability.

In order to guarantee that the matrix \(K\) is symmetric, the maximum number of eigenvalues that can be assigned is four, because the second and third elements in the diagonal matrices in (25)-(26) need to be the same, i.e., we should choose repeated eigenvalues: \(\lambda_1 = \lambda_3\) and \(\lambda_4 = \lambda_2\).

Consequently, the desired damping constant matrix \(D_d\) and the matrix \(K\) will have the following forms:
\[
D_d = -\begin{bmatrix}
\lambda_1 + \lambda_2 & 0 & 0 \\
0 & \lambda_3 + \lambda_4 & 0 \\
0 & 0 & \lambda_5 + \lambda_6
\end{bmatrix} M, \quad (27)
\]
\[
K = \begin{bmatrix}
\lambda_1 & 0 & 0 \\
0 & \lambda_3 & 0 \\
0 & 0 & \lambda_5
\end{bmatrix} M. \quad (28)
\]

Since both matrices are symmetric and products of a positive diagonal matrix by \(M\), they are both positive definite.

We now show how to compute the corresponding eigenvectors. We denote by \(m_i, e_i, i = 1, 2, 3\) the columns of \(M, I_3\), respectively. From (21), we may obtain a closed-loop eigenvector for a distinct eigenvalue \(\lambda\) by choosing one vector from
\[
\begin{bmatrix}
\lambda m_1 \\
e_1
\end{bmatrix}, \quad \begin{bmatrix}
\lambda m_2 \\
e_2
\end{bmatrix}, \quad \begin{bmatrix}
\lambda m_3 \\
e_3
\end{bmatrix}. \quad (29)
\]
This shows that the maximum geometric multiplicity of the eigenvalues is three. Consider the following equation:
\[
\begin{bmatrix}
-D_d M^{-1} - \lambda I_3 & -K \\
M^{-1} & -\lambda I_3
\end{bmatrix}
\begin{bmatrix}
m_i \\
e_i
\end{bmatrix} = 0, \quad (30)
\]
and differentiate it, obtaining
\[
\begin{bmatrix}
-D_d M^{-1} - \lambda I_3 & -K \\
M^{-1} & -\lambda I_3
\end{bmatrix}
\begin{bmatrix}
m_0 \\
e_i
\end{bmatrix} = \begin{bmatrix}
\lambda m_i \\
e_i
\end{bmatrix}, \quad (31)
\]
which shows that \([m_0]\) is a generalised eigenvector. Thus, the corresponding generalised eigenvectors are
\[
\begin{bmatrix}
m_1 \\
m_2 \\
m_3
\end{bmatrix}. \quad (32)
\]

Consequently, the closed-loop eigenvectors are given by an appropriate choice of six linearly independent vectors from (29) and (32) in such a way that a generalised eigenvector \([m_0]\) can be chosen, only if the eigenvector \(\begin{bmatrix}\lambda m_i \end{bmatrix}\) is chosen. The corresponding Jordan canonical form of \(A_d\) consists of \(1 \times 1\) and \(2 \times 2\) Jordan mini-blocks. We denote by \(J_1(\lambda)\) and \(J_2(\lambda)\) the \(1 \times 1\) and \(2 \times 2\) Jordan mini-blocks of \(\lambda\), respectively, i.e.,
\[
J_1(\lambda) \triangleq \lambda, \quad J_2(\lambda) \triangleq \begin{bmatrix}
\lambda & 1 \\
0 & \lambda
\end{bmatrix}. \quad (33)
\]

5. EIGENSTRUCTURE ASSIGNMENT

In this section, we provide the eigenstructure assignment that can be attained by all the combinations of eigenvalues and corresponding linearly independent vectors from (29) and (32). In particular, choosing different configurations of the closed-loop eigenvalues, we have the following cases. For each case, the matrices \(D_d\) and \(K\) are given together with the corresponding matrix \(V\) of closed-loop eigenvectors and Jordan canonical form \(J\).

- **Four eigenvalues with algebraic multiplicities \((1, 1, 2, 2)\)**
\[
D_d = -\text{diag}\{\lambda_1 + \lambda_2, \lambda_3 + \lambda_4, \lambda_5 + \lambda_6\} M, \quad (26)
\]
\[
K = \text{diag}\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} M, \quad (27)
\]
\[
V = \begin{bmatrix}
\lambda_1 m_1 & \lambda_2 m_1 & \lambda_3 m_2 & \lambda_4 m_2 & \lambda_5 m_3 & \lambda_6 m_3
\end{bmatrix}
\]
\[
J = \text{blkdiag}\{J_1(\lambda_1), J_1(\lambda_2), J_1(\lambda_3), J_1(\lambda_4)\};
\]

- **Three eigenvalues with algebraic multiplicities \((2, 2, 2)\)**
\[
D_d = -\text{diag}\{\lambda_1 + \lambda_2, \lambda_3 + \lambda_4, \lambda_5 + \lambda_6\} M, \quad (26)
\]
\[
K = \text{diag}\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} M, \quad (27)
\]
\[
V = \begin{bmatrix}
\lambda_1 m_1 & \lambda_2 m_1 & \lambda_3 m_2 & \lambda_4 m_2 & \lambda_5 m_3 & \lambda_6 m_3
\end{bmatrix}
\]
\[
J = \text{blkdiag}\{J_2(\lambda_1), J_1(\lambda_2), J_1(\lambda_3), J_1(\lambda_4)\};
\]

- **Three eigenvalues with algebraic multiplicities \((1, 3, 2)\)**
\[
D_d = -\text{diag}\{\lambda_1 + \lambda_2, \lambda_3 + \lambda_4, \lambda_5 + \lambda_6\} M, \quad (26)
\]
\[
K = \text{diag}\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} M, \quad (27)
\]
\[
V = \begin{bmatrix}
\lambda_1 m_1 & \lambda_2 m_1 & \lambda_3 m_2 & \lambda_4 m_2 & \lambda_5 m_3 & \lambda_6 m_3
\end{bmatrix}
\]
\[
J = \text{blkdiag}\{J_1(\lambda_1), J_1(\lambda_2), J_2(\lambda_3), J_2(\lambda_4)\};
\]

- **Three eigenvalues with algebraic multiplicities \((1, 1, 4)\)**
\[
D_d = -\text{diag}\{\lambda_1 + \lambda_2, 2 \lambda_3, 2 \lambda_3\} M, \quad (26)
\]
\[
K = \text{diag}\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} M, \quad (27)
\]
\[
V = \begin{bmatrix}
\lambda_1 m_1 & \lambda_2 m_1 & \lambda_3 m_2 & \lambda_4 m_2 & \lambda_5 m_3 & \lambda_6 m_3
\end{bmatrix}
\]
\[
J = \text{blkdiag}\{J_2(\lambda_1), J_2(\lambda_2), J_1(\lambda_3), J_1(\lambda_4)\};
\]

- **Two eigenvalues with algebraic multiplicities \((4, 2)\)**
\[
D_d = -(\lambda_1 + \lambda_2) M, \quad (26)
\]
\[
K = \lambda_1 \lambda_2 M, \quad (27)
\]
\[
V = \begin{bmatrix}
\lambda_1 m_1 & \lambda_1 m_2 & \lambda_1 m_3 & \lambda_1 m_4 & \lambda_2 m_2 & \lambda_2 m_3 & \lambda_2 m_4
\end{bmatrix}
\]
\[
J = \text{diag}\{J_1(\lambda_1), J_1(\lambda_2), J_1(\lambda_3), J_1(\lambda_4)\};
\]

- **Two eigenvalues with algebraic multiplicities \((3, 3)\)**
\[
D_d = -(\lambda_1 + \lambda_2) M, \quad (26)
\]
\[
K = \lambda_1 \lambda_2 M, \quad (27)
\]
\[
V = \begin{bmatrix}
\lambda_1 m_1 & \lambda_1 m_2 & \lambda_1 m_3 & \lambda_1 m_4 & \lambda_2 m_2 & \lambda_2 m_3 & \lambda_2 m_4
\end{bmatrix}
\]
\[
J = \text{diag}\{J_1(\lambda_1), J_1(\lambda_2), J_1(\lambda_3), J_1(\lambda_4)\};
\]
- **Two eigenvalues with algebraic multiplicities** $(2, 4)$

\[
D_d = - \text{diag} \{ 2 \lambda_1, 2 \lambda_2, 2 \lambda_2 \} M,
\]

\[
K = \text{diag} \{ \lambda_1^2, \lambda_2^2, \lambda_2^2 \} M,
\]

\[
V = \begin{bmatrix}
\lambda_1 m_1 & m_1 & \lambda_2 m_2 & m_2 & \lambda_2 m_3 & m_3 \\
e_1 & 0 & e_2 & 0 & e_3 & 0
\end{bmatrix},
\]

\[
J = \text{blkdiag} \{ J_2(\lambda_1), J_2(\lambda_2), J_2(\lambda_2) \};
\]

- **Two eigenvalues with algebraic multiplicities** $(1, 5)$

\[
D_d = - \text{diag} \{ \lambda_1 + \lambda_2, 2 \lambda_2, 2 \lambda_2 \} M,
\]

\[
K = \text{diag} \{ \lambda_1 \lambda_2, \lambda_2^2, \lambda_2^2 \} M,
\]

\[
V = \begin{bmatrix}
\lambda_1 m_1 & \lambda_2 m_1 & \lambda_2 m_2 & \lambda_2 m_3 & m_3 \\
e_1 & 0 & e_1 & 0 & e_3 & 0
\end{bmatrix},
\]

\[
J = \text{blkdiag} \{ J_1(\lambda_1), J_1(\lambda_2), J_2(\lambda_2), J_2(\lambda_2) \};
\]

- **One eigenvalue with algebraic multiplicity** $6$

\[
D_d = -2 \lambda M,
\]

\[
K = \lambda^2 M,
\]

\[
V = \begin{bmatrix}
\lambda m_1 & m_1 & \lambda m_2 & m_2 & \lambda m_3 & m_3 \\
e_1 & 0 & e_1 & 0 & e_3 & 0
\end{bmatrix},
\]

\[
J = \text{blkdiag} \{ J_2(\lambda), J_2(\lambda), J_2(\lambda) \}.
\]

6. CASE STUDY

We consider the model of an open-frame remotely operated underwater vehicle with a mass of $140 \text{ kg}$ from Donaire and Perez (2010). The vehicle has four thrusters in an $x$-type configuration that provides actuation in all of the degrees of freedom of interest, and each thruster can produce a maximum force of $150 \text{ N}$. The parameters of the model are

\[
M = \begin{bmatrix}
290 & 0 & 0 \\
0 & 404 & 50 \\
0 & 50 & 132
\end{bmatrix},
\]

\[
D(\nu) = D + D_u(\nu) = \begin{bmatrix}
95 & 0 & 0 \\
0 & 613 & 0 \\
0 & 0 & 105
\end{bmatrix} + \begin{bmatrix}
268 |u| & 0 & 0 \\
0 & 164 |v| & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]

In order to analyse the performance of the controller, we will compare the two cases where the closed-loop eigenvalues are the same or different for each element of the diagonal matrices in (27)-(28), i.e., when we have $2 \times 2$ or $1 \times 1$ Jordan blocks in the corresponding Jordan canonical form. It is adequate to compare for different values only the cases

**case I:** $D_d = -2 \lambda M$, $K = \lambda^2 M$,  

**case II:** $D_d = -(\lambda_1 + \lambda_2) M$, $K = \lambda_1 \lambda_2 M$.

Figures 1 and 2 show the state variables of the linearised closed-loop system, and Figures 3 and 4 show the state variables of the nonlinear closed-loop system, respectively.

Firstly, we select $\lambda = -0.1$, and $\lambda_1 = -0.05$, $\lambda_2 = -0.15$, so that $D_d = 0.2 M$ for both cases, while $K = 0.01 M$ in case I, and $K = 0.0075 M$ in case II. The values of $K$ are larger in case I and position regulation is achieved faster than in case II. Secondly, we select $\lambda = -0.2$ and $\lambda_1 = -0.1$, $\lambda_2 = -0.4$, so that $K = 0.04 M$ for both cases, while $D_d = 0.4 M$ in case I, and $D_d = 0.5 M$ in case II. Then the values of $D_d$ are smaller in case I and position regulation is again achieved faster than in all of the previous cases.

Consequently, the performance of the controller depends on the values of $K$, because if we increase it, the state variables asymptotically converge to the equilibrium point faster. If $K$ is the same for the cases I and II, i.e., $\lambda^2 = \lambda_1 \lambda_2$, the state variables asymptotically converge to the equilibrium point faster in case I.

The choice of eigenvalues was made in such a way that the controllers are tuned so that the saturation limit would not be reached for this kind of maneuvering.
7. CONCLUDING REMARKS AND FUTURE WORK

In this paper, we use a Hamiltonian system model of a marine craft for the position regulation problem in the horizontal plane and design an energy-based controller. The approach is based on stabilising the system at a desired position by shaping the energy and injecting damping. The closed-loop system is expressed in the port-Hamiltonian form, ensuring it is passive.

We demonstrate a systematic approach to tuning the nonlinear target dynamics. We do this by linearising the target dynamics at a single set point, which allows us to analyse and assign a suitable achievable eigenstructure. Once the desired characteristics are achieved in the linear closed loop, the performance of the nonlinear system with the same gains is examined. The analysis is illustrated by a case study based on the position regulation of an unmanned underwater vehicle in the horizontal plane.

In this paper, the only optimisation that we perform is at the single linearised operating point. This is a preliminary study to show the feasibility of the proposed approach. Given that the target dynamics is nonlinear, the response will vary when the system is further from this point. For this plant, the authors have observed that the structure of the target dynamics facilitates a response that will be sound across the operating envelope. This may not be the case for all plants, however. Exploration of how to reconcile an optimisation process, such as gain scheduling control, using more than a single linearisation point, and implementation on real marine vehicles are natural topics for future work.

REFERENCES

Donaire, A., and Perez, T. (2012). Dynamic positioning of marine craft using a port-Hamiltonian framework. *Automatica*, 48, 851-856.

Donaire, A., and Perez, T. (2010). Port-Hamiltonian theory of motion control for marine craft. In *Proceedings of the 8th IFAC Conference on Control Applications in Marine Systems*, Rostock, Germany, 201-206.

Fossen, T.I. (1994). *Guidance and Control of Ocean Marine Vehicles*. John Wiley & Sons, New York.

Fossen, T.I. (2011). *Handbook of Marine Craft Hydrodynamics and Motion Control*. John Wiley & Sons, Chichester, West Sussex.

Khalil, H. (2000). *Nonlinear Systems*. Prentice-Hall, New Jersey.

Ortega, R., and García-Canseco, E. (2004). Interconnection and damping assignment passivity-based control: a survey. *European Journal of Control*, 10(5), 432-450.

Ortega, R., and Spong, M. (1989). Adaptive motion control of rigid robots: a tutorial. *Automatica*, 25(6), 877-888.

Ortega, R., van der Schaft, A., Maschke, B., and Escober, G. (2002). Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems. *Automatica*, 38(4), 585-596.

Perez, T., Donaire, A., Renton, C., and Valentinis, F. (2013). Energy-based motion control of marine vehicles using interconnection and damping assignment passivity-based control - a survey. In *Proceedings of the 9th IFAC Conference on Control Applications in Marine Systems*, Osaka, Japan, 316-327.

Valentinis, F., Donaire A., and Perez, T. (2015a). Energy-based guidance of an underactuated unmanned underwater vehicle on a helical trajectory. *Control Engineering Practice*, 44, 138-156.

Valentinis, F., Donaire A., and Perez, T. (2015b). Energy-based motion control of a slender hull unmanned underwater vehicle. *Ocean Engineering*, 104, 604-616.

van der Schaft, A. (2000). *L2-Gain and Passivity Techniques in Nonlinear Control*. Springer-Verlag, London.

van der Schaft, A. (2006). Port-Hamiltonian systems: an introductory survey. In *Proceedings of the International Congress of Mathematicians*, Madrid, Spain, 1339-1365.

Woolsey, C.A., and Leonard N.E., (2002). Stabilizing underwater vehicle motion using internal rotors. *Automatica*, 38(12), 2053-2062.