Planetary migration in evolving planetesimals discs

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Abstract. In the current paper, we further improved the model for the migration of planets introduced in Del Popolo et al. (2001) and extended to time-dependent planetesimal accretion disks in Del Popolo and Ekşi (2002). In the current study, the assumption of Del Popolo and Ekşi (2002), that the surface density in planetesimals is proportional to that of gas, is released. In order to obtain the evolution of planetesimal density, we use a method developed in Stepinski and Valageas (1997) which is able to simultaneously follow the evolution of gas and solid particles for up to $10^7$ yrs. Then, the disk model is coupled to migration model introduced in Del Popolo et al. (2001) in order to obtain the migration rate of the planet in the planetesimal. We find that the properties of solids known to exist in protoplanetary systems, together with reasonable density profiles for the disk, lead to a characteristic radius in the range $0.03 - 0.2$ AU for the final semi-major axis of the giant planet.

Key words: Planets and satellites: general

1. Introduction

The discovery of solar-like stars showing evidences for planets orbiting around them [Mayor & Queloz 1995, Marcy et al. 2000, Vogt et al. 2000, Butler et al. 2001] has greatly intensified the interest in understanding the formation and evolution of planetary systems, as well as the long-standing problem of the solar system origin.

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The extra-solar planets discovered so far are all more massive than Saturn, and most either orbit very close to their stars or travel on much more eccentric paths than any of the major planets in our Solar System.

It is difficult to explain the properties of these planets using the standard model for planet formation (Lissauer 1993; Boss 1995). Current theories (Mizuno 1980; Bodenheimer & Pollack 1986) predict that giant planets were formed by gas accretion onto massive ($\approx 15M_\oplus$) rocky cores which themselves were the result of the accumulation of a large number of icy planetesimals. The most favourable conditions for this process are found beyond the so-called “snow line” (Hayashi 1981; Sasselov & Lecar 2000). As a consequence, this standard model predicts nearly circular planetary orbits and giant planets distances $\geq 1$ AU from the central star where the temperature in the protostellar nebula is low enough for icy materials to condense (Boss 1995; Boss 1996; but see also Wuchterl 1993; Wuchterl 1996).

Therefore, in the case of close-in giants, it is very unlikely that such planets were formed at their present locations. Then, the most natural explanation for this paradox, and for planets on very short orbits, is that these planets have formed further away in the protoplanetary nebula and they have migrated afterwards to the small orbital distances at which they are observed (see DP1 and DP2 for a detailed discussion of migration mechanisms). In particular, in DP1 and DP2 we showed that dynamical friction between a planet and a planetesimals disk is an important mechanism for planet migration and we pointed out that some advantages of the model are:

a) Planet halt is naturally provided by the model.

b) It can explain planets found at heliocentric distances of $> 0.03 - 0.04$ AU, or planets having larger values of eccentricity.

c) It can explain metallicity enhancements observed in stars having planets in short-period orbits.

d) Radial migration is possible with moderate masses of planetesimal disks, in contrast with other models.

In DP1, following Opik 1976, it was assumed that the surface density in planetesimals $\Sigma_s$ varies as $\Sigma_s(r) = \Sigma_\odot (1AU/r)^{3/2}$, where $\Sigma_\odot$ is the surface density at 1 AU. In DP2 the previous assumption was substituted by a more reliable model for the disk, and in particular we used a time-dependent accretion disk, since it is widely accepted that the solar system at early phases in its evolution is well described by this kind of structure. An important assumption of DP2 was that the surface density in planetesimals remains proportional to that of gas: $\Sigma_s(R, t) \propto \Sigma(R, t)$. However, it is well-known that the distribution of planetesimals emerging from a turbulent disk does not necessarily reflect that of gas (e.g., Stepinski & Valageas 1996; Stepinski & Valageas 1997). Indeed, in addition
to gas-solid coupling, the evolution of the distribution of solids is also determined by coagulation, sedimentation, and evaporation/condensation. In order to take these effects into account we use the method developed in [Stepinski & Valageas 1997] which is able to simultaneously follow the evolution of gas and solid particles for up to $10^7$ yr. The main approximation used in this model is to associate one grain size to a given radius and time. Then, we use the radial distribution of planetesimals given by this model to evaluate the planet migration, which is calculated as in [Del Popolo et al. 2001].

This paper is organized as follows. In Sect. 2, we describe the disk model we use to obtain the distribution of the planetesimal. Then, in Sect. 3, we briefly review the migration model introduced in [Del Popolo et al. 2001]. Finally, we describe our results in Sect. 4 and Sect. 5 is devoted to conclusions.

2. Disk model

It is well-known that protostellar disks around young stellar objects are common: between 25% to 75% of young stellar objects in the Orion nebula seem to have disks with mass $10^{-3} M_\odot < M_d < 10^{-1} M_\odot$ and size $40 \pm 20$ AU [Beckwith & Sargent 1996]. Moreover, observations of circumstellar disks surrounding T Tauri stars support the view of disks having a limited life-span and characterized by continuous changes during their life. These evidences have led to a large consensus about the nebular origin of the Solar System. Moreover, it clearly appears necessary to model both the spatial and temporal changes of the disk (which cannot be handled by the minimum-mass model nor by steady-state models). Besides, one also needs to describe the global evolution of the solid material which constitutes, together with the gas, the protoplanetary disk. As usual, the time evolution of the surface density of the gas $\Sigma$ is given by the familiar equation (e.g., [Stepinski & Valageas 1997]):

$$\frac{\partial \Sigma}{\partial t} - \frac{3}{r} \frac{\partial}{\partial r} \left( r^{1/2} \frac{\partial}{\partial r} \left( r^{1/2} \nu_t \Sigma \right) \right) = 0$$

(1)

where $\nu_t$ is the turbulent viscosity. Since $\nu_t$ is not an explicit function of time, but instead depends only on the local disk quantities, it can be expressed as $\nu_t = \nu_t(\Sigma, r)$ and Eq. (1) can be solved subject to boundary conditions on the inner and outer edges of the disk. The opacity law needed to compute $\nu_t$ is obtained from [Ruden & Pollack 1991]. Then, Eq. (1) is solved by means of an implicit scheme. Note that the evolution of the gas is computed independently from the evolution of particles (which only make $\sim 1\%$ of the gas mass). Next, from $\Sigma(r, t)$ we can algebraically find all other gas disk variables.

1 The knowledge of this distribution and its time evolution is important to understand how planets form and in this paper it is a key issue since we wish to study the planet migration due to the interaction between planets and the local distribution of solid matter.
Next, as described in Stepinski & Valageas 1997 the evolution of the surface density of solid particles $\Sigma_s$ is given by:

$$\frac{\partial \Sigma_s}{\partial t} = \frac{3}{r} \frac{\partial}{\partial r} \left[ r^{1/2} \frac{\partial}{\partial r} \left( \nu_s \Sigma_s r^{1/2} \right) \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[ 2r \Sigma_s (\tau_\phi)_s \right].$$

(2)

The first diffusive term is similar to Eq.(1), where the effective viscosity $\nu_s$ is given by:

$$\nu_s = \frac{\nu}{S_c} \quad \text{with} \quad S_c = (1 + \Omega_k t_s) \sqrt{1 + \frac{v^2}{V_t^2}}.$$  

(3)

Here we introduced the Schmidt number $S_c$ which measures the coupling of the dust to the gas turbulence. We also used the relative velocity $v$ between a particle and the gas, the turbulent velocity $V_t$, the Keplerian angular velocity $\Omega_k$ and the so-called stopping time $t_s$. The average $\langle .. \rangle_s$ refers to the vertical averaging over the disk height weighted by the solid density. The dimensionless quantity $(\Omega_k t_s)$ measures the coupling of the solid particles to the gas. Therefore, the evolution of the dust radial distribution can be significantly different from the behaviour of the gas, depending on the particle size (see Stepinski & Valageas 1996 for a detailed study).

The second advective term in Eq.(2) is due to the lack of pressure support for the dust disk as compared with the gas disk. Thus, it is proportional to the azimuthal velocity difference $\tau_\phi$ between the dust and the gas. We refer the reader to Stepinski & Valageas 1997 for a more detailed presentation, see also Kornet et al. 2001.

In this article we are mainly interested in the distribution of solids at small radii hence we consider only one species of solid particles: high-temperature silicates with $T_{\text{evap}} = 1350$ K and a bulk density $\rho_{\text{bulk}} = 3.3$ g cm$^{-3}$. Thus, in our simplified model we follow the evolution of three distinct fluids: the gas, the vapour of silicates and the solid particles. In this way, we obtain the radial distribution of the planetesimal swarm after $10^7$ yr. Of course, at these late times when planetesimals have typically reached a size of a few km or larger, gravitational interactions should play a dominant role with respect to coagulation. However, if these interactions do not significantly affect the radial distribution of solids (note that the radial velocity of such large particles due to the interaction with the gas is negligible) we can still use the outcome of the fluid approach described above to study the migration of giant planets, as detailed below.

### 3. Migration model

In order to study the migration of giant planets, the model developed in DP1 is used (see also DP2). Since this model has already been described in these two papers, we only recall here the main points. We consider a planet revolving around a star of mass $M_\ast = 1M_\odot$. The equation of motion of the planet can be written as:

$$\ddot{r} = F_\odot + R$$

(4)
where the term $F_{\odot}$ represents the force per unit mass from the star, while $R$ is the dissipative force (i.e. the dynamical friction term—see Melita & Woolfson 1990).

In order to take into account dynamical friction in a disk structure, we use Binney (1977) model:

$$R = -k_{\parallel} v_{\parallel} e_{\parallel} - k_{\perp} v_{\perp} e_{\perp}$$

(5)

where $e_{\parallel}$ and $e_{\perp}$ are two unit vectors parallel and perpendicular to the disk plane and $k_{\parallel}$ and $k_{\perp}$ are given in Binney (1977) and DP1.

Since the damping of eccentricity and inclination is more rapid than radial migration (Ida 1990; Ida & Makino 1992; DP1), we only deal with radial migration and we assume that the planet has negligible inclination and eccentricity ($i_p \sim e_p \sim 0$) and that the initial distance to the star of the planet is 5.2 AU. We do not need to follow the evolution of the size distribution of planetesimals since for $m \ll M$ the frictional drag obtained does not depend on the mass $m$ of the planetesimals, because the velocity dispersion only depends on the mass $M$ of the giant planet. We merely use the planetesimal density $\rho_s$ reached after $10^7$ yrs, assuming that the height of the planetesimal disk does not evolve significantly.

4. Results

In this article, similarly to DP1 and DP2, we are mainly interested in studying the planet migration due to the interaction with planetesimals. Our model starts with a fully formed gaseous giant planet of 1 $M_J$ at 5.2 AU. For this reason we assume that the gas is almost dissipated when the planet starts its migration. 2 While the gas tends to be dissipated, (several evidences show that the disk lifetimes range from $10^5$ yr to $10^7$ yr, see Strom et al. 1993; Ruden & Pollack 1991), the coagulation process induces an increase of the density of solid particles with time (see Fig. 1) and gives rise to objects of increasing dimensions.

Once solids are in the form of planetesimals, the gas coupling becomes unimportant and the radial distribution of solids does not change any more. This is why we do not need to calculate its evolution for times longer than $10^7$ years. 3 Then the disk is populated by residual planetesimals for a longer period. Here it is important to emphasize that planetesimal formation is not independent from initial conditions. In particular, the final solid surface density depends in an intricate way on the initial disk mass $M_d$ and on the turbulent viscosity parameter $\alpha$, (see Stepinski & Valageas 1997 and Kornet et al. 2001).

2 Clearly the effect of the presence of gas should be that of accelerating the loss of angular momentum of the planet and to reduce the migration time.

3 Note however that the size distribution of planetesimals keeps changing due to their mutual gravitational interaction.
Table 1. Properties of the initial gas disk

| $M_d$ | $J_{50}$ | $\Sigma_1$ | $r_1$ | $\Sigma_2$ |
|-------|----------|-------------|-------|-------------|
| $10^{-1}$ | 911 | 22 | 200 | 100 |
| $10^{-2}$ | 85 | 1.7 | 200 | 100 |
| $10^{-3}$ | 5.5 | 1.2 | 50 | 30 |
| $10^{-4}$ | 0.46 | 0.2 | 50 | 2.8 |

In order to investigate the dependence of the giant planet migration on the properties of the protoplanetary disk we integrated the model introduced in the previous sections for several values of the initial disk surface density (i.e. several disk masses), and different values of $\alpha$. More precisely, as in Stepinski & Valageas 1997 we consider an initial gas surface density of the form:

$$\Sigma_0(r) = \Sigma_1 \left[1 + \left(\frac{r}{r_1}\right)^2\right]^{-3.78} + \Sigma_2 \left(\frac{r}{1\text{AU}}\right)^{-1.5}. \tag{6}$$

The quantities $\Sigma_1$, $r_1$ and $\Sigma_2$ are free parameters which we vary in order to study different disk masses. The values used are given in Tab. 1, where $M_d$ is the gas disk mass (in units of $M_\odot$), $J_{50}$ is the disk angular momentum (in units of $10^{50}$ g cm$^2$s$^{-1}$), $\Sigma_1$ and $\Sigma_2$ are in g cm$^{-2}$ and $r_1$ is in AU. The first term in Eq.(6) ensures that there is some mass up to large distances from the star, while the second term corresponds to the central concentration of the mass and determine the location of the evaporation radius. Note however that in any case the evaporation radius for the high-temperature silicates we study here remains of order 0.1 AU. As previously explained, we consider only one species of solids: high-temperature silicates with $T_{\text{evap}} = 1350$ K. We initialize the dust subdisk at time $t = 10^4$ yrs (i.e. after the gas distribution has relaxed towards a quasi-stationary state) by setting the solid surface density $\Sigma_s$ as: $\Sigma_s = 6 \times 10^{-3}\Sigma$ in order to account for cosmic abundance.

We show in Fig. 1 the evolution of the solid midplane density for the case $M_d = 10^{-1}M_\odot$, and for different values of $\alpha$. We can see a converged radial distribution of solids emerge at late times of order $10^6$ yr. Note that although the radial distribution of planetesimals depends on the value of $\alpha$, the total mass of solids in a disk is roughly independent of $\alpha$ since it remains approximately equal to the initial mass of solids in the disk. This means that solids are reshuffled within the disk but they are not lost into the star. $^4$ The value of $\alpha$ determines the radial distribution of solids: particles in a disk with a larger value of $\alpha$ (more turbulent disk) have larger inward radial velocities and consequently are locked into planetesimals closer to the star than particles in a less turbulent disk. Thus, the smaller the value of $\alpha$ the broader the final distribution of solids.

$^4$ In fact, solids initially located close to the evaporation radius are lost but they constitute a small percentage of the total solid material, which is predominantly located in the outer disk.
Fig. 1. (a) Evolution of solids for a disk with $M_d = 0.1 M_\odot$ and $\alpha = 0.1$ at $t = 10^4$ yrs (dashed line), $t = 10^6$ yrs (dotted line) and $t = 10^7$ yrs (solid line). (b) Same as Fig. 1a but with $\alpha = 0.01$. (c) Same as Fig. 1a but with $\alpha = 0.001$. (d) Same as Fig. 1a but with $\alpha = 0.0001$

The evaporation radius is located at $\approx 0.1 AU$ while the outer limit of converged $\rho_s$ is about 10 AU (it moves outward for smaller values of $\alpha$, e.g., $\approx 70$ AU for $\alpha = 10^{-4}$).

The coagulation process gives rise to solids of $10^6 - 10^7$ cm. In disks characterized by smaller values of $\alpha$, and thus a more extended distribution of solids, the range of sizes goes from $10^6 - 10^7$ cm at the evaporation radius down to $10^3 - 10^4$ cm at the outer limit. This is because the coagulation process is less efficient at larger radii where the solid density is smaller and the velocity dispersion of the dust decreases (along with the gas temperature which governs the turbulent velocity). At later times these solids will continue to increase their sizes, but will not change their radial position, as they are already large enough to have a negligible radial motion. We refer the reader to Stepinski & Valageas 1997 and Kornet et al. 2001 for more detailed discussions of the behaviour of protoplanetary disks.

Now, using the converged radial distribution of planetesimals derived in the previous section, we show in Fig. 2 the evolution of the semi-major axis $a(t)$ of a 1 $M_J$ planet
in a disk for $\alpha = 0.1$ and $M_d = 0.1$ (solid line), 0.01 (dotted line), 0.001 (short-dashed line), 0.0001$M$_\odot$ (long-dashed line), respectively. We recall here that the planet is initially located at 5.2 AU with $i_p \sim e_p \sim 0$.

It can be clearly seen that for a fixed value of $\alpha$ a disk of larger mass leads to a more rapid migration of the planet. This behaviour is quite natural since a more massive disk obviously yields a stronger frictional drag. Thus, in the cases $M = 0.1, 0.01M_\odot$ the planet migrates to $\simeq 0.08$ AU in $\simeq 1.5 \times 10^9$ yr and to $\simeq 0.03$ AU in $\simeq 2.5 \times 10^9$ yr, respectively. When the planet arrives at this distance the dynamical friction switches off and its migration stops. The stopping is simply due to the inner radius of the planetesimal disk. The latter is set by the evaporation radius. Indeed, solid bodies cannot condense at such small orbital radii $r \lesssim 0.1$ AU because the temperature is too high. Of course, this evaporation radius $r_{\text{evap}}$ depends on the properties of the solid grains we consider. For instance, for ice particles we have $r_{\text{evap}} \sim 1$ AU (e.g., Stepinski & Valageas 1997; Kornet et al. 2001). In this article we wish to understand the small orbital radii of observed planets, over the range $0.03 - 0.15$ AU. Therefore, we are interested in the inner regions of the disk where only high-temperature silicates with $T_{\text{evap}} \sim 1350$ K survive.

This is why we selected this component in this study. Then, the main point of Fig. 2 is that the properties of solids known to exist in protoplanetary systems, together with reasonable density profiles for the disk, lead to a characteristic radius in the range $0.03 - 0.2$ AU for the final semi-major axis of the giant planet. Note in particular that this process naturally explains why the migration stops at such radii.

For less massive disks, $M_d = 10^{-3}, 10^{-4}M_\odot$, the migration is slower and the planet has not enough time to migrate belows $\simeq 4$ AU. Note that we stop our calculation after $4 \times 10^9$ yr which is the typical age of protoplanetary disks like ours (the Sun is $\sim 4.5 \times 10^9$ yr old). Moreover, the planetesimal disk should have cleared out by this time. It is interesting to note that the planet moves closer to the central star in the case $M = 0.01M_\odot$ than in the case $M = 0.1M_\odot$. This is due to the fact that less massive disks usually have a smaller surface density which leads to a smaller temperature. This in turn implies a smaller evaporation radius so that the planet can move closer to the star. In order to study the effect of viscosity on migration, we performed three other calculations with $\alpha = 10^{-2}, 10^{-3}$ and $10^{-4}$, also plotted in Fig. 2. We can note that the dependence of the final radius on $\alpha$ is rather weak and non-monotonic because it competes with the dependence on the surface density whose precise value is an intricate function of $\alpha$. On the other hand, the opacity $\tau$ is given by $\tau \sim \kappa \Sigma$ where $\kappa$ is the Rosseland opacity, while the turbulent viscosity scales as $\nu_t \sim \alpha T_v / \Omega_k$, hence we obtain: $T_v^3 \propto \kappa \alpha \Sigma^2$.

\[5\] Indeed, we have the energy balance: $T_v^4 \propto \Sigma \nu_t \Omega_k^2$ where $T_v$ is the effective temperature, $\Sigma$ the gas surface density, $\nu_t$ the turbulent viscosity and $\Omega_k$ the Keplerian angular velocity. Besides, the mid-plane temperature $T_c$ obeys the relation $T_c^4 \sim \tau T_v^4$ where $\tau$ is the opacity. On the other hand, the opacity $\tau$ is given by $\tau \sim \kappa \Sigma$ where $\kappa$ is the Rosseland opacity, while the turbulent viscosity scales as $\nu_t \sim \alpha T_v / \Omega_k$, hence we obtain: $T_v^3 \propto \kappa \alpha \Sigma^2$. 
the other hand, the migration time usually increases going from $\alpha = 10^{-4}$ up to $\alpha = 0.1$. Indeed, as seen from Fig. 1 a smaller $\alpha$ can lead to a larger mid-plane solid density. This is partly due to the fact that the dust disk height is smaller because the turbulence measured by $\alpha$ is weaker.

The results displayed in Fig. 2 show that for $\alpha \lesssim 0.01$ a Jupiter-like planet can migrate to a very small distance from the parent star, $0.03 \, \text{AU} < r < 0.1 \, \text{AU}$, provided the disk mass is sufficiently large $M_d \gtrsim 10^{-3} M_\odot$. Only in the cases $M_d \lesssim 10^{-4} M_\odot$, or $M_d \lesssim 10^{-3} M_\odot$ with $\alpha \gtrsim 0.1$, the interaction with the planetesimal disk is too weak to yield a significant migration. Then, our results show that a final radius of $0.03 \, \text{AU} < r < 0.1 \, \text{AU}$ is a natural result, unless the planetesimal disk has been cleared off too early on (e.g., by gravitational scattering).

In summary, the present model predicts that, unless the disk mass is very small $M_d \lesssim 10^{-4} M_\odot$, planets tend to move close to the parent star and to pile up to distances of the order of 0.03 – 0.04 AU (see Kuchner & Lecar 2002). However, with some degree of fine-tuning it is also possible to find a planet at intermediate distances between its formation site and such small radii.

5. Conclusions

In the current paper, we further improved the model for the migration of planets introduced in DP1 and extended to time-dependent planetesimal accretion disks in DP2. After releasing the assumption of DP2 that the surface density of planetesimals is proportional to that of gas, a simplified model developed by Stepinski & Valageas (1996, 1997), that is able to simultaneously follow the evolution of gas and high-temperature silicates for up to $10^7 \, \text{yr}$, is used. Then we coupled this disk model to the migration model introduced in DP1 in order to obtain the migration rate of the planets in the planetesimal disk and to study how the migration rate depends on the disk mass, on its time evolution and on the dimensionless viscosity parameter $\alpha$.

We found that in the case of disks having a total mass of $M_d > 10^{-3} M_\odot$ planets can migrate inward over a large distance, while if $M < 10^{-3} M_\odot$ the planets remain almost at their initial position. On the other hand, for $M_d \sim 10^{-3} M_\odot$ a significant migration requires $\alpha \lesssim 10^{-2}$.

If the migration is efficient the planet usually ends up at a small radius in the range 0.03 – 0.1 AU which is simply set by the evaporation radius of the gaseous disk which gave rise at earlier times to the radial distribution of the planetesimal swarm. Therefore, our model provides a natural explanation for the small observed radii of extra-solar giant planets. In order to inhibit this process (so that Jupiter-like planets like our own remain at larger distances $\gtrsim 5 \, \text{AU}$) the planetesimal disk must be cleared off over a time-scale of
Fig. 2. (a) The evolution of the semi-major axis $a(t)$ of a Jupiter-mass planet, $M = 1 M_J$, in a planetesimal disk for $\alpha = 0.1$ and several values of $M_d$: 0.1 (solid line), 0.01 (dotted line), 0.001 (short-dashed line) and 0.0001$M_\odot$ (long-dashed line). (b) Same as Fig. (1a) but with $\alpha = 0.01$. (c) Same as Fig. (1a) but with $\alpha = 0.001$. (d) Same as Fig. (1a) but with $\alpha = 0.0001$.

In the order of or smaller than $10^9$ yr (depending on the properties of the disk) or the disk mass must be rather small (i.e. smaller than $1M_J$) which could suggest an alternative formation scenario for such a giant planet (i.e. not related with the disk itself).

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