Holographic heavy-baryons in the Witten-Sakai-Sugimoto model with the D0-D4 background

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Abstract

We extend the holographic analysis of light-baryon spectrum in [50] to the case involving the heavy flavors. With the construction of the Witten-Sakai-Sugimoto model in the D0-D4 background, we use the mechanism proposed in [59, 60, 61] by including two light and one heavy flavor branes, to describe the heavy-light baryons as heavy mesons bound to a flavor instanton. The background geometry of this model corresponds to an excited state in the dual field theory with nonzero glue condensate $\langle \text{Tr} F \wedge F \rangle = 8\pi^2 N_c \kappa$, or equivalently a $\theta$ angle, which is proportional to the number density of the D0-brane charge. At strongly coupled limit, this model shows that the heavy meson is always bound in the form of the zero mode of the flavor instanton in the fundamental representation. We systematically study the quantization for the effective Lagrangian of heavy-light baryons by employing the soliton picture, and derive the mass spectrum of heavy-light baryons in the situation with single- and double-heavy baryon. We find the difference in the mass spectrum becomes smaller if the density of D0-brane charge increases and the constraint of stable states of the heavy-light baryons is $1 < b < 3$. It indicates that baryon can not stably exist for sufficiently large density of D0 charge which is in agreement with the conclusions in the previous study of this model.

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1 Introduction

The heavy quark (c, b, t) is characterized by the heavy-quark symmetry in QCD [1] while the light quark (u, d, s) is dominated by the spontaneous breaking of chiral symmetry. As measured by [2, 3], the chiral doubling in heavy-light mesons [4, 5, 6, 7] combine these symmetries. Recently, flurry of experiments report that many multiquark exotics are incommensurate with quarkonia [8, 9, 10, 11, 12, 13, 14]. Accordingly, some new phenomena involving heavy-light multiquark states are strongly supported by these experimental results. On the other hand, the spontaneous parity violation in QCD has also been discussed with the running of the RHIC in many researches [15, 16, 17, 18, 19]. It is well known that the \( P \) or \( CP \) violation could be usually described by an nonzero \( \theta \) angle in the action of such theories. So when deconfinement happens in QCD, a metastable state with nonzero \( \theta \) angle may probably be produced in the hot and dense situation in RHIC, then the bubble forms with odd \( P \) or \( CP \) parity would soon decay into the true vacuum. For example, the chiral magnetic effect (CME) was proposed as a test of such phenomena [20, 21, 22]. Thus it is theoretically interesting to study the \( \theta \) dependence of some observables in QCD or in the gauge theory e.g. \( \theta \) dependence of the spectrum of the glueball [23], the phase diagram [24, 25] and the \( \theta \) dependence in the large \( N_c \) limit [26] (one can also review the details of the \( \theta \) dependence in [27]).

The holographic construction by the gauge/gravity duality offers a framework to investigate the aspects of the strongly coupled quantum field theory [28, 29] since QCD at low energy scale is non-perturbative. Using the D4-brane construction in Witten's [30], a concrete model was proposed by Sakai and Sugimoto [31, 32] which almost contains all necessary ingredients of QCD e.g. baryons [33, 34], quark matter, chiral/deconfinement phase transitions [35, 36, 37, 38], glueball spectrum and interaction [39, 40, 41, 42, 43]. Specifically, flavors are introduced into this model by a stack of \( N_f \) pairs of suitable D8/D8-branes as probes embedded in the \( N_c \) D4-branes background. The chiral quarks are in the fundamental representation of the color and flavor group which come from the massless spectrum of the open strings stretched between the color and flavor (D8/D8) branes. Since the flavor branes are connected, it provides a geometrical description for the spontaneous breaking of the chiral symmetry. Baryon in this model is the D4-brane warped on \( S^4 \) which is named as “baryon vertex” and it has been recognized as the instanton configuration of the gauge field on the worldvolume of the flavor branes [44]. In particular, the \( \theta \) angle in the dual theory is realized as the instantonic D-brane (D-instanton) holographically in the construction of the string theory [45]. Hence adding the D-instanton (D0-branes) to the background of Witten-Sakai-Sugimoto model involves the \( \theta \) dependence in the dual field theory [46, 47]. With the systematical study of the Witten-Sakai-Sugimoto model in the D0-D4 brane background (i.e. D0-D4/D8 brane system) in [48, 49], it provides a way to

\[2\] We will use “D4'-brane” to denote the baryon vertex in order to distinguish the \( N_c \) D4-branes in the following sections.
investigate the \( \theta \) dependence of QCD or Yang-Mills theory holographically \[50, 51, 52, 53\].

Since the baryon spectrum with light flavors can be reviewed in \[50, 51, 54, 55\] by using the Witten-Sakai-Sugimoto model with the D0-D4 brane background, naturally the purpose of this paper is to extend the analysis to involve the heavy flavors. As mentioned, the fundamental quarks in this model are represented as the fermion states created by the open strings stretched between the \( N_c \) D4-branes and \( N_f \) coincident D8/\( \overline{D8} \)-branes without length which are therefore massless states \[31, 58\]. So the fundamental quarks in this model can be named as “light quarks” and the \( N_f \) coincident flavor branes are certainly named as “light flavor branes”. In order to address the chiral and heavy-quark symmetries by the holographic duality, we follow the mechanism proposed in \[59, 60, 61\], that is to consider one pair of flavor brane as probe (named as “heavy flavor brane”) which is separated from the other \( N_f \) coincident flavor branes as shown in Figure 1. The string stretched between the heavy and light flavor branes (named as “HL-string”) produces massive multiplets. Hence the heavy-light mesons correspond to the low energy modes of these strings which can be approximated by the local vector fields in bi-fundamental representation in the vicinity of the light flavor branes. With the nonzero vacuum expectation value (vev), the heavy-light fields are massive and their mass comes from the moduli span by the dilaton fields in the action. This setup allows to describe the radial spectra of heavy-light multiplets, their pertinent vector and axial correlations and so on. The baryons with heavy flavor should be the form of instanton configurations in the worldvolume theory of D8/\( \overline{D8} \)-branes bound to heavy-light vector mesons. And this method will also develop the bound state approach in the context of the Skyrme model (e.g. \[62\]) holographically by involving the \( \theta \) dependence.

The outline of this paper is as follows. In section 2, we briefly review the D0-D4 background and its dual field theory. In section 3, we outline the geometrical setup and derive the heavy-light effective action through the Dirac-Born-Infeld (DBI) and the Chern-Simons (CS) actions of the D-branes. It shows the heavy-meson interactions to the instanton on the flavor branes. In section 4, we show the effective action in the double limit and a spin-1 vector meson is binding to the bulk instanton transmuting to spin-1/2. In section 5, we employ the quantization and show how to involve the heavy flavors in the spectrum additional to the previous works \[50, 51, 54, 55\] with a finite D0-brane charge or \( \theta \) angle. The derivation of baryon spectra with single and double-heavy quarks are explicitly given in this section. Section 6 is the summary. In the appendix, we briefly summarize the essential steps for the quantization of the light meson moduli without the heavy flavor branes which has already been studied in \[50\].

\(^3\)See also \[54, 55\] for a similar approach or the applications in \[56, 57\] in hydrodynamics with the model in \[48, 49\] while the D0-brane is not D-instanton.
Figure 1: Various brane configuration in the $\tau - U$ plane where $\tau$ is compactified on $S^1$. The bubble background (cigar) is produced by $N_c$ D4-branes with $N_0$ smeared D0-branes. The $N_f = 2$ light flavor D8/\overline{D8}-branes (L) living at antipodal position of the cigar are represented by the blue line. A pair of heavy flavor D8/\overline{D8}-brane (H) is separated from the light flavor branes which is represented by the red line. The massive state is produced by the string stretched between the light and heavy flavor branes (HL-string) which is represented by the green line in this figure.
2 The D0-D4 background and the dual field theory

In this section, we will review the D0-D4 background and its dual field theory by following [48].

In Einstein frame, the solution of D4-brane with smeared D0-brane in the IIA supergravity is given as,

\[ ds^2 = H_{4}^{-\frac{3}{8}} \left[ -H_{0}^{\frac{7}{8}} f(U) d\tau^2 + H_{0}^{\frac{5}{8}} \delta_{\mu\nu} dx^\mu dx^\nu \right] + H_{0}^{\frac{3}{8}} 4\left[ \frac{dU^2}{f(U)} + U^2 d\Omega_4^2 \right]. \] (2.1)

The direction \( \tau \) is compactified on a cycle with the period \( \beta \). The dilaton, Ramond-Ramond 2- and 4-form are given as,

\[ e^{-\Phi} = g_s^{-1} \left( \frac{H_4}{H_0^3} \right)^{\frac{1}{4}}, \quad f_2 = \frac{(2\pi l_s)^7 g_s N_0}{\omega_4 V_4} dU \wedge d\tau, \quad f_4 = \frac{(2\pi l_s)^3 N_c g_s}{\omega_4} \epsilon_4, \] (2.2)

where,

\[ H_4 = 1 + \frac{U_{Q4}^3}{U_3^3}, \quad H_0 = 1 + \frac{U_{Q0}^3}{U_3^3}, \quad f(U) = 1 - \frac{U_{KK}^3}{U_3^3}, \]
\[ U_{Q0}^3 = \frac{1}{2} \left( -U_{KK}^3 + \sqrt{U_{KK}^6 + \left( (2\pi l_s)^5 l_s^7 g_s \kappa N_c \right)^2} \right), \]
\[ U_{Q4}^3 = \frac{1}{2} \left( -U_{KK}^3 + \sqrt{U_{KK}^6 + \left( (2\pi l_s)^5 l_s^7 g_s N_c \right)^2} \right). \] (2.3)

We have used \( d\Omega_4, \epsilon_4 \) and \( \omega_4 = 8\pi^2/3 \) to represent the line element, the volume form and the volume of a unit \( S^4 \). \( U_{KK} \) is the horizon position of the radius coordinate and \( V_4 \) is the volume of the D4-branes. The numbers of D4- and D0-branes are denoted by \( N_c \) and \( N_0 \) respectively. D0-branes are smeared in the directions of \( x^0, x^1, x^2, x^3 \). So the number density of the D0-branes can be represented by \( N_0/V_4 \). In order to take account of the backreaction from the D0-branes as [47], we also require that \( N_0 \) is order of \( N_c \). In the large \( N_c \) limit, \( \kappa \) would be order of \( O(1) \) which is defined as \( \kappa = N_0/(N_c V_4) \).

In the string frame, making double Wick rotation and taking field limit i.e. \( \alpha' \to 0 \) with fixed \( U/\alpha' \) and \( U_{KK}/\alpha' \), then we obtain the D0-D4 bubble geometry, the metric becomes,

\[ ds^2 = \left( \frac{U}{R} \right)^{3/2} \left[ H_0^{1/2} \eta_{\mu\nu} dx^\mu dx^\nu + H_0^{-1/2} f(U) d\tau^2 \right] + H_0^{1/2} \left( \frac{R}{U} \right)^{3/2} \left[ \frac{dU^2}{f(U)} + U^2 d\Omega_4^2 \right], \] (2.4)

and the dilaton become,
\[ e^\Phi = g_s \left( \frac{U}{R} \right)^{3/4} H_0^{3/4}, \]  

(2.5)

where \( R^3 = \pi g_s l_s^3 N_c \) is the limit of \( U_{Q_4}^3 \). Here \( l_s \) is the length of the string and \( \alpha' = l_s^2 \). In the bubble geometry (2.4), the spacetime ends at \( U = U_{KK} \). In order to avoid the conical singularity at \( U_{KK} \), the period \( \beta \) of \( \tau \) must satisfy,

\[ \beta = \frac{4\pi}{3} U_{KK}^{-1/2} R^{3/2} b^{1/2}, \quad b \equiv H_0(U_{KK}). \]  

(2.6)

In the low-energy effective description, the dual theory is a five-dimensional \( U(N_c) \) Yang-Mills (YM) theory which lives inside the world volume of D4-brane. Since one direction of the D4-branes is compactified on a cycle \( \tau \), the four-dimensional Yang-Mills coupling could be obtained as studied in [30], which is relating the D4-brane tension, the five-dimensional Yang-Mills coupling constant \( g_5 \), then analyzing the relation of the five-dimensionally compactified theory and four dimensions on the \( \tau \) direction. Thus the resultantly four-dimensional Yang-Mills coupling is,

\[ g_{YM}^2 = \frac{g_5^2}{\beta} = \frac{4\pi^2 g_s l_s}{\beta}, \]  

(2.7)

then \( b \) and \( R^3 \) can be evaluated as,

\[ b = \frac{1}{2} \left[ 1 + (1 + C\beta^2)^{1/2} \right], \quad C \equiv \left( \frac{2\pi l_s^2}{\lambda^2} \right)^6 \frac{\lambda^2 R}{U_{KK}}, \quad R^3 = \frac{\beta \lambda l_s^2}{4\pi}, \]  

(2.8)

where the 't Hooft coupling \( \lambda \) is defined as \( \lambda = g_{YM}^2 N_c \). The Kaluza-Klein (KK) modes can be introduced by defining a mass scale \( M_{KK} = 2\pi/\beta \). The fermion and scalar become massive at the KK mass scale since the anti-periodic condition is imposed on the fermions [31]. Therefore, the massless modes of the open string dominate the dynamics in the low-energy theory which is described by a pure Yang-Mills theory. According to (2.6) and (2.8), we have the following relations,

\[ \beta = \frac{4\pi \lambda l_s^2}{9U_{KK}}, \quad M_{KK} = \frac{9U_{KK}}{2\lambda l_s^2 b}, \]  

(2.9)

Because \( b \geq 1 \) and \( U_{KK} \geq 2\lambda l_s^2 M_{KK}/9 \), \( \beta \) can be solved by using (2.8) and (2.9) as,

\[ \beta = \frac{4\pi \lambda l_s^2}{9U_{KK}} \frac{1}{1 - \left( \frac{2\pi l_s^2}{81U_{KK}^2} \right)^8 \lambda^4 R^2}, \quad b = \frac{1}{1 - \left( \frac{2\pi l_s^2}{81U_{KK}^2} \right)^8 \lambda^4 R^2}. \]  

(2.10)

Let us consider the effective action of a D4-brane with the smeared D0-branes in the background which takes the following form,

\[ S_{D_4} = -\mu_4 \text{Tr} \int d^4x d\tau e^{-\phi} \sqrt{-\det(G + 2\pi \alpha' F)} + \mu_4 \int C_5 + \frac{1}{2} (2\pi \alpha')^2 \mu_4 \int C_1 \wedge F \wedge F, \]  

(2.11)
Smeared D0-branes

$N_c$ D4-branes

$N_f$ D8/D8-branes

Baryon vertex D4'-branes

|          | 0 | 1 | 2 | 3 | 4(τ) | 5(U) | 6 | 7 | 8 | 9 |
|----------|---|---|---|---|------|------|---|---|---|---|
| Smeared D0-branes | = | = | = | = | -   | -    |   |   |   |   |
| $N_c$ D4-branes    | - | - | - | - | -    | -    |   |   |   |   |
| $N_f$ D8/D8-branes | - | - | - | - | -    | -    |   |   |   |   |
| Baryon vertex D4'-branes | - | - | - | - | -    | -    |   |   |   |   |

Table 1: The brane configurations: “=” denotes the smeared directions, “-” denotes the world volume directions.

where $\mu_4 = (2\pi)^{-4} l_s^{-5}$, $\phi = \Phi - \Phi_0$, $e^{\Phi_0} = g_s$ and $G$ is the induced metric on the world volume of D4-branes. $F$ is the gauge field strength on the D4-brane. $C_5$, $C_1$ is the Ramond-Ramond 5- and 1- form respectively and their field strengths are given in (2.2). The Yang-Mills action can be obtained by the leading-order expansion respected to small $F$ from the first term in (2.11) (i.e. the DBI action). In the bubble D0-D4 solution, we have $C_1 \sim \theta d\tau$ in (2.2), thus D0-branes are actually D-instantons (as shown in Table 1) and the last term in (2.12) could be integrated as,

$$\int_{S^1} C_1 \sim \theta \sim \tilde{\kappa}, \quad \int_{S^1 \times \mathbb{R}^4} C_1 \wedge F \wedge F \sim \theta \int_{\mathbb{R}^4} F \wedge F. \quad (2.12)$$

So the free parameter $\tilde{\kappa}$ (related to the $\theta$ angle in the dual field theory) has been introduced into the Witten-Sakai-Sugimoto model by this string theory background, however this background is not dual to the vacuum state of the gauge field theory. Similarly as in [45], in the dual field theory, some excited states with a constant homogeneous field strength background may be described in the D0-D4 model. The expectation value of $\text{Tr} F \wedge F$ can be evaluated as

$$\langle \text{Tr} F \wedge F \rangle = 8\pi^2 N_c \tilde{\kappa}. \quad (48, 50)$$

Then the deformed relations in the presence of D0-branes of the variables in QCD are given as follows,

$$R^3 = \frac{\lambda l_s^2}{2M_{KK}}, \quad g_s = \frac{\lambda}{2\pi M_{KK} N_c l_s}, \quad U_{KK} = \frac{2}{9} M_{KK} \lambda l_s^2 b. \quad (2.13)$$

3 Holographic effective action for heavy-light interaction

3.1 D-brane setup

The chiral symmetry $U_R (N_f) \times U_L (N_f)$ can be introduced into the D0-D4 system by adding a stack of probe $N_f$ D8-anti-D8 (D8/D8) branes into the background which are named as flavor branes. The spontaneously breaking of $U_R (N_f) \times U_L (N_f)$ symmetry to $U_V (N_f)$ in the dual field theory can be geometrically understood by the separately faraway D8/D8-branes combining near the bottom of the bubble at $U = U_{KK}$ (as shown by the blue lines in Figure 1). This can be verified by the appearance of massless Goldstones [63]. The brane configurations are illustrated in Table 1.
The induced metric on the probe D8/\overline{D8}-branes is,

\[ ds^2_{\text{D8}/\overline{\text{D8}}} = \left( \frac{U}{R} \right)^{3/2} H_0^{-1/2} \left[ f(U) + \left( \frac{R}{U} \right)^3 \frac{H_0}{f(U)} U^2 \right] d\tau^2 \]
\[ + \left( \frac{U}{R} \right)^{3/2} H_0^{1/2} \eta_{\mu\nu} dx^\mu dx^\nu + H_0^{1/2} \left( \frac{R}{U} \right)^{3/2} U^2 d\Omega_4^2. \]

where \( U' \) is the derivative with respect to \( \tau \). The action of the D8/\overline{D8}-branes can be obtained as,

\[ S_{\text{D8}/\overline{\text{D8}}} \propto \int d^4x dU H_0(U) U^4 \left[ f(U) + \left( \frac{R}{U} \right)^3 \frac{H_0}{f(U)} U^2 \right]^{1/2}, \]

then the equation of motion for \( U(\tau) \) can be derived as,

\[ \frac{d}{d\tau} \left( \frac{H_0(U) U^4 f(U)}{\left[ f(U) + \left( \frac{R}{U} \right)^3 \frac{H_0}{f(U)} U^2 \right]^{1/2}} \right) = 0, \]

which can be interpreted as the conservation of the energy. With the initial conditions \( U(\tau = 0) = U_0 \) and \( U'(\tau = 0) = 0 \), the generic formula of the embedding function \( \tau(U) \) can be solved as,

\[ \tau(U) = E(U_0) \int_{U_0}^{U} dU \frac{H_0^{1/2}(U) \left( \frac{R}{U} \right)^{3/2}}{f(U) \left[ H_0(U) U^8 f(U) - E^2(U_0) \right]^{1/2}}, \]

where \( E(U_0) = H_0(U_0) U_0^4 f^{1/2}(U_0) \) and \( U_0 \) denotes the connected position of the D8/\overline{D8}-branes. Following [31,48], we introduce the new coordinates \((r, \Theta)\) and \((y, z)\) which satisfy,

\[ y = r \cos \Theta, \quad z = r \sin \Theta, \]
\[ U^3 = U_{KK}^3 + U_{KK} r^2, \quad \Theta = \frac{2\pi}{\beta} \tau = \frac{3}{2} \frac{U_{KK}^{1/2}}{R^{3/2} H_0^{1/2}(U_{KK})}. \]

In this manuscript, we will consider the following configuration for the various flavor branes: the light flavor branes live at the antipodal position as in [31,48,49] which means they (D8/\overline{D8}-branes) are embedded at \( \Theta = \pm \frac{\pi}{2} \) respectively i.e. \( y = 0 \). The embedding function of the light flavor branes is \( \tau_L(U) = \frac{1}{4} \beta \) so that we have \( U^3 = U_{KK}^3 + U_{KK} z^2 \) on the light D8/\overline{D8}-branes.

Therefore the induced metric on them becomes,

\[ ^4\text{With the suitable boundary condition, } \tau_L(U) = \frac{1}{4} \beta \text{ is indeed a solution of (3.3) as discussed in [31,48,49].} \]
\[
\frac{ds_k^2}{ds_\text{Light-D8/D8}} = H_0^{1/2} \left( \frac{U}{R} \right)^{3/2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{4 U_{KK}}{9} \left( \frac{U}{R} \right)^{3/2} H_0^{1/2} dz^2 + H_0^{1/2} \left( \frac{R}{U} \right)^{3/2} U^2 d\Omega_4. \tag{3.6}
\]

For the heavy flavor branes, we have to choose another solution as \( \tau_H(U) \) from (3.4) with \( U_0 = U_H \neq U_{KK} \) since they must live at the non-antipodal position of the background.\(^5\) So the heavy flavor brane is separated from the light flavor branes with a finite separation at \( \tau_H(U_0) = 0 \) as shown in Figure 1. With the approach presented in [44, 50, 64], the light baryon spectrum in the D0-D4/D8 system has been studied in [50, 51, 54, 55]. In this paper, we extend our previous work by following [59, 60, 61] to study the heavy-light interaction and baryon spectrum with heavy flavors in the D0-D4/D8 system. Thus we consider \( N_f = 2 \) light flavor D8/D8-branes (L) and one pair of heavy (H) flavor brane as probe in the bubble D0-D4 geometry that spontaneously breaks chiral symmetry. The massive states on the light D8/D8-branes are produced by the heavy-light (HL) strings connecting heavy-light branes.

### 3.2 Yang-Mills and Chern-Simons action of the flavor branes

Since the baryon vertex lives inside the light flavor branes, the concern of this section is to study the effective dynamics of the baryons or mesons on the light flavor branes involving the heavy-light interaction. The lowest modes of the open string stretched between the heavy and light branes are attached to the baryon vertex as shown in Figure 1. In our D0-D4/D8 system, these string modes consist of longitudinal modes \( \Phi_a \) and the transverse modes \( \Psi \) near the light brane world volume. These fields acquire a nonzero vev at finite brane separation, which introduces the mass to the vector field [65]. These fields are always named as “bi-local”, however we will approximate them near the light flavor branes by local vector fields hence they are described by the standard DBI action. So this construction is distinct from the approaches presented in [66, 67, 68, 69, 70, 71, 72].

By keeping these in mind, let us consider action of the light flavor branes. For the D8-branes, the generic expansion of the DBI in the leading order can be written as,

\[
S_{\text{DBI}}^{\text{D8}} = -\frac{T_8 (2\pi\alpha')^2}{4} \int d^8 \xi \sqrt{\text{det} G} e^{-\Phi} \text{Tr} \left\{ F_{ab} F^{ab} - 2 D_a \phi^I D_D \phi^J + [\phi^I, \phi^J]^2 \right\}. \tag{3.7}
\]

where \( \phi^I \) is the transverse mode of the flavor branes and the index \( a, b \) runs over the flavor brane. Notice that there is only one transverse coordinate to the D8-brane, thus we define \( \phi^I \equiv \Psi \) to omit the index. The scalar field \( \Psi \) is traceless and adjoint representation additional

\(^5\)Since we are going to discuss the limit \( U_H, z_H \to \infty \) in the next section, an analytical solution for the embedding function of the heavy flavor brane could be \( \tau_H(U) = -\frac{2}{9} (\frac{U}{R})^{3/2} \frac{U_H}{U} \mathcal{F}_1 \left( \frac{1}{2}, \frac{2}{9}, \frac{25}{16}, \frac{U_H}{U} \right) + \frac{2\sqrt{\pi}}{3} \frac{U_H^{3/2}}{R^{1/2}} \mathcal{F}_1 \left( \frac{1}{2}, \frac{2}{9}, \frac{25}{16}, \frac{U_H}{U} \right) \) where \( \mathcal{F}_1 \) is the hypergeometric function. In this limit, the integral region on the heavy flavor brane is \( U > U_H \to \infty \), so we have \( f, H_0 \sim 1 \) to get this solution with (3.4).
to the adjoint gauge field $A_a$. Since the one pair of the heavy flavor brane is separated from the $N_f = 2$ light flavor branes with a string stretched between them, in string theory the world volume field can be combined in a superconnection. For the gauge field, we can use the following matrix-valued 1-form as,

$$A_a = \begin{pmatrix} A_a & \Phi_a \\ -\Phi_a & 0 \end{pmatrix},$$  \ (3.8)

where $A_a$ is $(N_f + 1) \times (N_f + 1)$ matrix-valued while $\Psi$ and $A_a$ are $N_f \times N_f$ valued. If all the flavor branes are coincident, the $\Phi_a$ multiplet is massless, otherwise $\Phi_a$ could be massive field.

The corresponding gauge field strength of (3.8) is,

$$F_{ab} = \begin{pmatrix} F_{ab} - \Phi_{[a} \Phi_{b]} & \partial_{[a} \Phi_{b]} + A_{[a} \Phi_{b]} \\ \partial_{[a} \Phi_{b]} + A_{[a} \Phi_{b]} & -\Phi_{[a} \Phi_{b]} \end{pmatrix},$$  \ (3.9)

Inserting the induced metric (3.6) with (2.5) into (3.7), we can write the DBI action as two parts,

$$S_{DBI}^{D8/D8} = S_{YM}^{D8} + S_{\Psi}.$$  \ (3.10)

The Yang-Mills part is calculated as,

$$S_{YM} = -2\tilde{T}U_{KK}^{-1} \int d^4xdz H_0^{1/2} / \sqrt{2} \text{Tr} \left[ \frac{1}{4} R^3 + \frac{1}{2} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} + \frac{9}{8} U_{KK}^3 \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} \right],$$  \ (3.11)

where $\mu, \nu$ run over 0, 1, 2, and 3 and

$$\tilde{T} = \frac{(2\pi\alpha')^2}{3g_s} T_{\omega_4} U_{KK}^{3/2} R^{3/2} = \frac{M_{KK}^2 \lambda N_c b^{3/2}}{486\pi^3}. \ (3.12)$$

In order to work with the dimensionless variables, we introduce the replacement $z \rightarrow z/U_{KK}$, $x^\mu \rightarrow x^\mu/M_{KK}$, $A_z \rightarrow A_z/U_{KK}$, $A_\mu \rightarrow A_\mu M_{KK}$ then (3.11) takes the following formulas,

$$S_{YM}^{D8/D8} = -\tilde{T} M_{KK}^{-2} \frac{9}{4b} \int d^4xdz H_0^{1/2} (U) \text{Tr} \left[ \frac{1}{2} U_{KK}^2 \mathcal{F}_{\mu\nu}^2 + \frac{U}{U_{KK}^3} \mathcal{F}_{\mu\nu}^2 \right],$$

$$= -a\lambda N_c b^{1/2} \int d^4xdz H_0^{1/2} (U) \text{Tr} \left[ \frac{1}{2} K (z)^{-1/3} \eta_{\mu\nu} \eta_{\rho\sigma} \mathcal{F}_{\mu\nu} \mathcal{F}_{\rho\sigma} + K (z) b \eta_{\mu\nu} \mathcal{F}_{\mu\nu} \mathcal{F}_{\rho\sigma} \right],$$  \ (3.13)

where $a = \frac{1}{216\pi^2}$ and $K (z) = 1 + z^2$. To see the dependence of $\lambda$, it would be convenient to employ the rescaling used in [44], which is

$$x^0, x^M \rightarrow (x^0, \lambda^{-1/2} x^M), \ (A_0, \Phi_0) \rightarrow (A_0, \Phi_0), \ (A_M, \Phi_M) \rightarrow (\lambda^{1/2} A_M, \lambda^{1/2} \Phi_M). \ (3.14)$$

\footnote{Working with this replacement is equivalent to work in the unit of $U_{KK} = M_{KK} = 1$ as [44] in this model.}
where $M, N$ run over $1, 2, 3, z$ and $i, j = 1, 2, 3$. Using (3.14) in the large $\lambda$ limit, (3.13) becomes,

\[
L_{YM}^{D8/D8} = -aN_c b^{3/2} \text{Tr} \left[ \frac{\lambda}{2} F_{MN}^2 - b z^2 \left( \frac{5}{12} - \frac{1}{4b} \right) F_{ij}^2 + \frac{b z^2}{2} \left( 1 + \frac{1}{b} \right) F_{iz}^2 - F_{0M}^2 \right] \\
\equiv aN_c b^{3/2} \mathcal{L}_M^H + aN_c b^{3/2} \lambda \mathcal{L}_0^H + aN_c b^{3/2} \mathcal{L}_1^H + \mathcal{O}(\lambda^{-1}) ,
\]

(3.15)

where $\mathcal{L}_M^H$ represents the Lagrangian for the light hadrons which has been derived in [50] and the explicit form of $\mathcal{L}_M^H$ can also be found in (A-1) (A-2) in the appendix. Substituting (3.15) for (3.9), we obtain

\[
\mathcal{L}_0^H = - \left( D_M \Phi_N^+ - D_N \Phi_M^+ \right) \left( D_M \Phi_N - D_N \Phi_M \right) + 2 \Phi_M^+ F^{MN} \Phi_N ,
\]

\[
\mathcal{L}_1^H = 2 \left( D_0 \Phi_M^+ - D_M \Phi_0^+ \right) \left( D_0 \Phi_M - D_M \Phi_0 \right) - 2 \Phi_0^+ F^{0M} \Phi_M - 2 \Phi_M^+ F^{0M} \Phi_M + \mathcal{L}_1^H ,
\]

(3.16)

where $D_M \Phi_N = \partial_M \Phi_N + A_M \Phi_N$ and

\[
\mathcal{L}_1^H = b z^2 \left( \frac{5}{6} - \frac{1}{2b} \right) \left( D_i \Phi_j - D_j \Phi_i \right) \left( D_i \Phi_j - D_j \Phi_i \right) - b z^2 \left( 1 + \frac{1}{b} \right) \left( D_i \Phi_2 - D_2 \Phi_i \right) \left( D_i \Phi_2 - D_2 \Phi_i \right) - b z^2 \left( \frac{5}{3} - \frac{1}{b} \right) \Phi_i^+ F^{ij} \Phi_j + b z^2 \left( 1 + \frac{1}{b} \right) \left( \Phi_i^+ F^{2i} \Phi_i + \text{c.c.} \right) .
\]

(3.17)

The action $S_{\Psi}$ in (3.10) is collected as,

\[
S_{\Psi} = - \frac{T_8 (2\pi)^2}{4} \int d^3 \xi \sqrt{-\det G} e^{-\Phi} \text{Tr} \left\{ -2 D_a \varphi^i D_a \varphi^i + [\varphi^i, \varphi^j]^2 \right\} \\
= \tilde{T}_8 \int d^4 x d z \sqrt{-\det G} e^{-\Phi} \text{Tr} \left\{ \frac{1}{2} D_a \Psi D_a \Psi - \frac{1}{4} |\Psi, \Psi|^2 \right\} .
\]

(3.18)

with $D_a \Psi = \partial_a \Psi + i [A_a, \Psi]$. According to [73], one can define the moduli by the extrema of the potential contribution or $[\Psi, [\Psi, \Psi]] = 0$ in (3.18). So the moduli solution of $\Psi$ for $N_f$ light branes separated from one heavy brane can be defined with a finite vev $v$ as,

\[
\Psi = \left( -\frac{v}{N_f}, 0 \right) ,
\]

(3.19)

With solution (3.19), we have

\[
S_{\Psi} = - \tilde{T} U^{-1}_{KK} v^2 \frac{(N_f + 1)^2}{N_f^2} \int d^4 x d z H_0^{3/2} U^2 \left( g_{zz} \Phi_z^+ \Phi_z + g_{\mu\nu} \Phi_\mu^+ \Phi_\nu \right) .
\]

(3.20)
Again, we introduce the dimensionless variable $\Phi_a$ by imposing $z \to zU_{KK}$, $x^\mu \to x^\mu/M_{KK}$, $A_z \to A_z/U_{KK}$, $A_\mu \to A_\mu M_{KK}$ which means it requires an additional replacement $v \to \frac{M_{KK}}{U_{KK}} v$.

Then using the $\lambda$ rescaling as in (3.14), we finally obtain,

$$ S_\Psi = -a N_c b^{3/2} \int d^4 x dz 2m_H^2 \Phi_M^\dagger \Phi_M + O(\lambda^{-1}), \quad (3.21) $$

where $m_H = \frac{1}{\sqrt{6}} N_f^{1/2} v$.

For a Dp-brane, there is a CS term in the total action whose standard form is,

$$ S_{CS}^{D_p} = \mu_p \int_{D_p} \sum_q C_{q+1} \wedge \text{Tr} e^{2\pi \alpha' F} = \mu_p \int_{D_p} \sum_n C_{p-2n+1} \wedge \frac{1}{n!} (2\pi \alpha')^n \text{Tr} F^n. \quad (3.22) $$

In our D0-D4 background, the non-vanished terms for the probe D8/D8-branes are,

$$ S_{CS}^{D8/D8} = \frac{1}{3!} (2\pi \alpha')^3 \mu_8 \int_{D8/D8} C_3 \wedge \text{Tr} [F^3] + 2\pi \alpha' \mu_8 \int_{D8/D8} C_7 \wedge \text{Tr} [F]. \quad (3.23) $$

The first term in (3.23) can be integrated out by using $dC_3 = f_4$ which has been given in (2.2), it yields a CS 5-form,

$$ S_{CS}^{D8/D8} = \frac{N_c}{24\pi^2} \int_{R^{11}} A_{5F} - \frac{1}{2} A^3 F + \frac{1}{10} A^5, \quad (3.24) $$

and this term is invariant under the $\lambda$ rescaling (3.14). However explicit calculations show that the second term in (3.23) becomes $O(\lambda^{-1})$ in the large $\lambda$ limit. So on the light flavor branes, only (3.24) in the CS term survives in strongly coupled limit. Inserting (3.8) and (3.9) into (3.24) with the dimensionless variables, (3.24) becomes,

$$ L_{CS}^{D8/D8} = L_{CS}^L (A) + L_{CS}^H, \quad (3.25) $$

where $L_{CS}^L (A)$ represents the CS term for the light hadrons given in (A-1) (A-2) which has been studied in [44, 50] and

$$ L_{CS}^H = -\frac{i N_c}{24\pi^2} \left( d\Phi^\dagger A d\Phi + d\Phi^\dagger A^2 d\Phi + \Phi^\dagger dA \Phi \right) $$

$$ -\frac{i N_c}{16\pi^2} \left( d\Phi^\dagger A^\dagger d\Phi + \Phi^\dagger A^\dagger A^2 d\Phi + \Phi^\dagger dA A \Phi \right) $$

$$ -\frac{5i N_c}{48\pi^2} \Phi^\dagger A^3 \Phi + O(\Phi^3, A). \quad (3.26) $$

Therefore the action for light-heavy interaction can be collected from (3.16) (3.17) (3.21) (3.26) on the light flavor branes.
4 The zero modes

In the limit of $\lambda \to \infty$ followed by $m_H \to \infty$, the heavy meson in bulk can be treated as the instanton configuration on the flavor branes which can be effectively treated as a spinor. And it forms a 4-dimensionally flavored zero-mode which can be interpreted as either a bound of heavy flavor or anti-heavy flavor in the spacetime of $\{x^\mu\}$. However, in the Skyrme model, the Wess-Zumino-Witten term is time-odd which carries opposite signs for heavy particles and anti-particles. While this is difficult for anti-particles by holography, it is remarkable.

4.1 Equations of motion

Let us consider the solution of the heavy meson field $\Phi_M$. Notice that $\Phi_M$ is independent of $\Phi_0$, so the equations of motion read from the action (3.16) (3.17) (3.21) (3.26),

$$D_M D_M \Phi_N - D_N D_M \Phi_M + 2F_{NM}\Phi_M + O(\lambda^{-1}) = 0.$$  \hspace{1cm} (4.1)

And the equation of motion for $\Phi_0$ is,

$$D_M (D_0 \Phi_M - D_M \Phi_0) - \mathcal{F}^{0M}\Phi_M - \frac{1}{64\pi^2ab^{3/2}}\epsilon_{MNPQ}\mathcal{K}_{MPNQ} + O(\lambda^{-1}) = 0,$$  \hspace{1cm} (4.2)

where the 4-form $\mathcal{K}_{MPNQ}$ is given as,

$$\mathcal{K}_{MPNQ} = \partial_M A_N \partial_P \Phi_Q + A_M A_N \partial_P \Phi_Q + \partial_M A_N A_P \Phi_Q + \frac{5}{6}A_M A_N A_P \Phi_Q.$$  \hspace{1cm} (4.3)

In the heavy quark limit, we follow [61] to redefine $\Phi_M = \phi_M e^{-im_H x^0}$ for particles while it follows the replacement $m_H \to -m_H$ for the anti-particle.

4.2 The double limit

It is very difficult to calculate all the contributions from the heavy meson field $\Phi_M$. Hence we consider the limit of $\lambda \to \infty$ followed by $m_H \to \infty$ (i.e. named as the “double limit”). So the leading contributions come from the light effective action presented in (2.5) which is of order $\lambda m_H^0$, while the next leading contributions come from the heavy-light interaction Lagrangian $\mathcal{L}_1^H$ in (3.16) and $\mathcal{L}_{CS}^H$ in (3.26) which is of order $\lambda^0 m_H$. The double limit is valid if we assume the heavy meson field $\Phi_M$ is very massive which means the separation of the heavy and light branes is very large as shown in Figure 1. So the straight pending string takes a value at $z_H^0$ which satisfies,

$$m_H = \lim_{z_H \to \infty} \int_0^{z_H} dz \sqrt{-g_{00}g_{zz}} \approx \frac{1}{\pi l_s^{1/2} U^{1/3} kK z_H^{2/3}} + O(z_H^0).$$  \hspace{1cm} (4.4)
It would be convenient to rewrite (4.4) with the dimensionless variables by the replacement
\[ m_H \rightarrow m_H M_{KK}, \ z_H \rightarrow z_H U_{KK}, \] then using (2.13) we have,
\[ \frac{m_H}{\lambda} = \frac{2b}{9\pi} \frac{2/3}{z_H}. \] (4.5)

According to the above discussion, the derivative of \( \Phi_M \) can be replaced by \( D_0 \Phi_M \rightarrow (D_0 \pm i m_H) \Phi_M \) with “-” for particle and “+” for anti-particle. Then we collect the order \( \lambda^0 m_H \) from our heavy-light action which is,
\[ L_{mH} = L_{1,m} + L_{CS,m}, \]
\[ L_{1,m} = ab^{3/2} N_c \left[ 4im_H \phi_M^\dagger D_0 \phi_M - 2i m_H \left( \phi_M^\dagger D_M \phi_M - c.c. \right) \right], \]
\[ L_{CS,m} = m_H N_c \epsilon_{MNPQ} \phi_M^\dagger F_{NP} \phi_Q = \frac{m_H N_c}{8\pi^2} \phi_M^\dagger \epsilon^{MNPQ} F_{MN} \phi_N. \] (4.6)

The equation of motion (4.2) suggests a considerable simplification \( D_M \Phi_M = 0 \) which implies that \( \Phi_M \) is covariantly transverse mode.

### 4.3 Vector to spinor

In the \( N_f = 2 \) case of the D0-D4/D8 system, the small size instanton is described by a flat-space 4-dimensional instanton solution of SU(2) Yang-mills theory [50] in the large \( \lambda \) limit, which is,
\[ A_{M}^{cl} = -\hat{\sigma}_{MN} \frac{x^N}{x^2 + \rho^2}, \]
\[ A_{0}^{cl} = -\frac{i}{8\pi^2 ab^{3/2} x^2} \left[ 1 - \frac{\rho^4}{(x^2 + \rho^2)^2} \right], \] (4.7)
where \( x^2 = (x^M - X^M)^2 \) and \( X^M \) is the constant. Notice that in (4.7), \( A_0^{cl} \) is Abelian while \( A_M^{cl} \) is non-Abelian. It carries a field strength,
\[ F_{MN} = \frac{2\hat{\sigma}_{MN} \rho^2}{(x^2 + \rho^2)^2}. \] (4.8)

By defining \( f_{MN} = \partial_{[M} \phi_N] + A_{[M} \phi_N] \), \( L_0^H \) in (3.16) can be rewritten as follows,
\[ L_0^H = -f_{MN}^\dagger f_{MN} + 2\phi_M^\dagger F_{MN} \phi_N \\
= -f_{MN}^\dagger f_{MN} + 2\epsilon_{MNPQ} \phi_M^\dagger D_N D_P \phi_Q \\
= -f_{MN}^\dagger f_{MN} + f_{MN}^\dagger \ast f_{MN} \\
= -\frac{1}{2} (f_{MN} - \ast f_{MN})^\dagger (f_{MN} - \ast f_{MN} ), \] (4.9)
where $\star$ represents the Hodge dual. Therefore the equations of motion (4.1) can be replaced by,

\begin{align*}
    f_{MN} - \star f_{MN} &= 0, \\
    D_M \phi_M &= 0,
\end{align*}

which is equivalent to

\begin{align*}
    \sigma_M D_M \psi = 0, \quad \text{with } \psi = \bar{\sigma}_M \phi_M.
\end{align*}

So $\phi_M$ can be solved from (4.10) as,

\begin{align*}
    \phi_M &= \bar{\sigma}_M \xi \frac{\rho}{(x^2 + \rho^2)^{3/2}} \equiv \bar{\sigma}_M f(x) \xi,
\end{align*}

which is in agreement with [61]. $\xi$ is a two-component spinor and the interplay of (4.11) is remarkable since it shows that a heavy vector meson holographically binds to an instantonic configuration in bulk which concludes that a vector zero mode is equivalently described by a spinor.

## 5 Quantization

The classical moduli of the bound instanton zero-mode should be quantized by slowly rotating and translating the bound state since it breaks rotational and translational symmetry. The quantization of the leading $\lambda N_c$ contribution can be found in [50] which is instantonic and standard, while the quantization of the sub-leading $\lambda^0 m_H$ contribution involving zero-modes in the D0-D4/D8 system is new. We will employ the quantization applied on D4/D8 as [61].

### 5.1 Collectivization

As in [59], we assume that the zero-modes slowly rotates, translates and deforms through

\begin{align*}
    \Phi_M &\to V [ a_I (t)] \Phi_M [ X^0 (t), Z (t), \rho (t), \chi (t) ], \\
    \Phi_0 &\to 0 + \delta \phi_0,
\end{align*}

where $X^0, Z$ is the center in the $x^i$ and $z$ directions respectively. $a_I$ is the $SU (2)$ gauge rotation. They are represented by $X^\alpha = (X^i, Z, \rho)$ with

\begin{align*}
    -i V^\dagger \partial_0 V = \Phi &= -\partial_t X^M A_M + \chi^i \Phi_i, \\
    \chi^i &= -i \text{Tr} (\tau^i a_I^{-1} \partial_t a_I).
\end{align*}
Here $a^i$'s carry the quantum numbers of isospin, the angular momentum and $\tau^i$'s are Pauli matrices. Since the equation (4.2) has to be satisfied, $\delta \phi_0$ is fixed at the next-leading order,

$$- D_M^2 \delta \phi_0 + D_M \bar{\sigma}_M \left[ \partial_i X^i \frac{\partial (f \chi)}{\partial X^i} + \partial_t \chi \right]$$

$$+ i \left( \partial_i X^\alpha \partial_\alpha \Phi_M - D_M \Phi \right) \bar{\sigma}_M \chi + \delta S_{CS} = 0. \quad (5.3)$$

For a general quantization of the ensuing moduli, we can solve (5.3) and then insert the solution back into the action.

### 5.2 Leading order of the heavy mass term

The heavy mass terms in the double limit are given in (4.6). Imposing (5.1) (5.2) on (4.6), the contributions to order $\lambda^0 m_H$ in $L_{1,m}$ come from three terms which are,

$$L_{1,m} = \frac{ab}{2} \left( 16i m_H \xi^\dagger \partial_t \xi f^2 + 16i m_H \xi^\dagger \xi A_0 f^2 - 16m_H f^2 \xi^\dagger \sigma_\mu \Phi \bar{\sigma}_\mu \xi \right), \quad (5.4)$$

where $A_0$ is the rescaled $U(1)$ gauge field. With the gauge field strength (4.8), the CS term in (4.6) can be written as,

$$L_{CS,m} = \frac{3m_H N_c}{\pi^2} \frac{f^2 \rho^2}{(x^2 + \rho^2)^2} \xi^\dagger \xi. \quad (5.5)$$

Notice that the third term in (5.4) vanishes owing to the identity $\sigma_\mu \tau^i \bar{\sigma}_\mu = 0$.

There is a Coulomb-like backreaction according to the coupling $\xi^\dagger \xi A_0$ in (5.4). To clarify this, let us introduce a Coulomb-like potential defined as $\varphi = -i A_0$. We collect all the $U(1)$ coupling from (4.6) up to $O(\lambda^0 m_H)$ as,

$$L_{U(1)} = -ab^{3/2} N_c \left[ \frac{1}{2} \left( \nabla \varphi \right)^2 + \varphi \left( \rho_0 - 16m_H f^2 \xi^\dagger \xi \right) \right], \quad (5.6)$$

where $\rho_0$ is the $U(1)$ “charge” which is given as,

$$\rho_0 = \frac{1}{64\pi^2 ab^{3/2} \epsilon_{MNPQ} F_{MN} F_{PQ}}. \quad (5.7)$$

Solving the equation of motion from (5.6) for $\varphi$, one obtains its onshell action as,

$$L_{U(1)} = L_{U(1)} [\rho_0] + 16ab^{3/2} N_c m_H f^2 \xi^\dagger \xi \left( -i A_0^d \right) - \frac{ab^{3/2} N_c}{24\pi^2 \rho^2} \left( 16m_H \xi^\dagger \xi \right)^2. \quad (5.8)$$

The last term is the Coulomb-like self-interaction which is repulsive and tantamount of fermion number repulsion in holography.
5.3 Moduli effective action

All the contributions up to \( \mathcal{O}(\lambda^0 m_H) \) in the effective moduli action can be collected from (5.4) (5.6) and (5.8). Let us summarize them as follows,

\[
\mathcal{L} = \mathcal{L}^L [a_I, X^\alpha] + 16im_Hab^{3/2}N_c\xi\partial_t\xi \int d^4x f^2 \\
- 16m_Hab^{3/2}N_c\xi\partial_t\xi \int d^4x \left[ iA_0^d f^2 - \frac{3}{16\pi^2ab^{3/2}(x^2 + \rho^2)^2} \right] \\
- \frac{ab^{3/2}N_c}{24\pi^2\rho^2} \left( 16m_H\xi\partial_t\xi \right)^2. \tag{5.9}
\]

Here \( \mathcal{L}^L \) refers to the effective action on the moduli space from the contribution of the light hadrons which is identical to the derivation in \[50\]. In (5.9), it shows the explicitly new contribution due to the bound heavy meson. In the leading order, the coupling of the light collective degrees of freedom should be a general reflection on heavy quark symmetry. However there is no such coupling in the order of \( \mathcal{O}(\lambda^0 m_H) \) in (5.9) to the heavy spinor degree of freedom \( \xi \). Notice that the coupling to the instanton size \( \rho \) does not upset this symmetry. In order to calculate (5.9), we follow the steps as \[12\] i.e. using the normalization \( \int d^4xf^2 = 1 \), inserting the explicit form of \( A_0^d \), and rescaling \( \xi \rightarrow \xi/\sqrt{16ab^{3/2}N_cm_H} \). Finally it yields,

\[
\mathcal{L} = \mathcal{L}^L [a_I, X^\alpha] + i\xi\partial_t\xi + \frac{3}{32\pi^2ab^{3/2}\rho^2}\xi\partial_t\xi - \frac{(\xi\partial_t\xi)^2}{24\pi^2ab^{3/2}\rho^2N_c}. \tag{5.10}
\]

It shows the zero-mode of the vector to the instanton transmutation of a massive spinor with a repulsively Coulomb-like self-interaction in the presence of the D0 charge. A negative mass term also means the heavy meson lowers its energy. So the preceding arguments are also suitable for an anti-heavy meson in the presence of an instanton with a positive mass term leading to (5.10). The energy of this meson rises in the presence of the instanton to order \( \lambda^0 \). It originates from the Chern-Simons term in the holographical action which is the analogue of the effects due to the Wess-Zumino-Witten term in the Skyrme model.

5.4 Heavy-light spectrum

The step to quantize the Lagrangian (5.9) follows the same discussion as those presented in \[50\] for \( \mathcal{L}^L [a_I, X^\alpha] \). We use \( \mathcal{H}^L [a_I, X^\alpha] \) to represent the Hamiltonian associated to \( \mathcal{L}^L [a_I, X^\alpha] \), then the Hamiltonian for (5.10) takes the following formula,

\[
\mathcal{H} = \mathcal{H}^L [a_I, X^\alpha] - \frac{3}{32\pi^2ab^{3/2}\rho^2}\xi\partial_t\xi + \frac{(\xi\partial_t\xi)^2}{24\pi^2ab^{3/2}\rho^2N_c}. \tag{5.11}
\]

And the quantization rule for the spinor \( \xi \) should be chosen as,
\[ \xi_i \xi_j^\dagger + \xi_j \xi_i^\dagger = \delta_{ij}. \] (5.12)

So the rotation of the spinor \( \xi \) is equivalent to a spatial rotation of the heavy vector meson field \( \phi_M \) since \( U^{-1} \bar{\sigma} M U = \Lambda_{MN} \bar{\sigma} M \), where \( U \) and \( \Lambda \) represents the rotation of a spinor and a vector respectively e.g. \( \xi \to U \xi, \phi_M \to \Lambda_{MN} \phi_N \). The parity of \( \xi \) is positive which is opposite to \( \phi_M \).

The spectrum of (5.11) follows the same discussion in [50]. Since the (5.11) contains only two terms proportional to \( \rho^{-2} \), by comparing (5.11) with the \( H^L [a_I, X^\alpha] \) presented in [50], the heavy-light spectrum can therefore be obtained by modifying \( Q \) as,

\[ Q = \frac{N_c}{40ab^{3/2} \pi^2} \to \frac{N_c}{40ab^{3/2} \pi^2} \left[ 1 - \frac{15}{4N_c} \xi \xi^\dagger + \frac{5}{3N_c^2} \left( \xi \xi^\dagger \right)^2 \right]. \] (5.13)

Let us use \( J \) and \( I \) to represent the spin and isospin, so they are related by

\[ \vec{J} = -\vec{I} + \vec{S} = -\vec{I} + \xi \vec{\tau} \frac{\pi}{2} \xi. \] (5.14)

And notice that we have \( J + I = 0 \) in the absence of the heavy-light meson as expected from the spin-flavor hedgehog character. The quantum states for a single bound state i.e. \( N_Q \equiv \xi \xi^\dagger = 1 \) and \( IJ^\pi \) assignments are labeled by,

\[ |N_Q, J_M, l_m, n_z, n_\rho > \quad \text{with} \quad IJ^\pi = l \left( \frac{l}{2} \pm \frac{1}{2} \right)^\pi. \] (5.15)

Here \( n_z, n_\rho = 0, 1, 2... \) represents the number of quanta associated to the collective motion and the radial breathing of the instanton core respectively. Following [44, 50], the spectrum of the bound heavy-light state in D0-D4/D8 system is,

\[ M_{NQ} = M_0 + N_Q m_H + M_{KK} \sqrt{\frac{3 - b}{3} (n_\rho + n_z + 1)} \]

\[ + M_{KK} \left[ \frac{(l + 1)^2 (3 - b)}{12} + \frac{3 - b}{15} N_c^2 \left( 1 - \frac{15}{4N_c} N_Q + \frac{5}{3N_c^2} N_Q^2 \right) \right]^{1/2}. \] (5.16)

\( M_{KK} \) is the Kaluza-Klein mass and \( M_0 = \frac{\lambda N_c b^{3/2}}{27 \pi} M_{KK} \).

**Single heavy-baryon spectrum**

The lowest heavy states with one heavy quark are characterized by \( N_Q = 1, l = \text{even}, N_c = 3 \) and \( n_z, n_\rho = 0, 1 \). So the mass spectrum is given as,

\[ M_{\text{single}} = M_0 + m_H + M_{KK} (n_\rho + n_z + 1) \sqrt{\frac{3 - b}{3}} \]

\[ + M_{KK} \left[ \frac{(l + 1)^2 (3 - b)}{12} - \frac{7}{180} (3 - b) \right]^{1/2}. \] (5.17)
Let us consider the states with \( n_z = n_{\rho} = 0 \) and identify the state with \( l = 0 \), the assignments \( IJ^\pi = 0^\frac{1}{2}^+ \) as the heavy-light iso-singlet \( \Lambda_Q \). Then we identify the state with \( l = 2 \) and the assignments \( IJ^\pi = 1^\frac{1}{2}^+, 1^\frac{3}{2}^+ \) as the heavy-light iso-triplet \( \Sigma_Q, \Sigma_Q^* \) respectively. Subtracting the nucleon mass \( M_N \) (which is identified as the state with \( l = 0 \) of the light-baryon spectrum) from \( (5.17) \), we have,

\[
M_{\Lambda_Q} - M_N - m_H \simeq -0.76\sqrt{3} - bM_{KK}, \\
M_{\Sigma_Q} - M_N - m_H \simeq -0.12\sqrt{3} - bM_{KK}, \\
M_{\Sigma_Q^*} - M_N - m_H \simeq -0.12\sqrt{3} - bM_{KK}.
\]

(5.18)

Thus we see the explicit dependence of D0 charge in the baryon spectrum in this model. Next we can study the excited heavy baryons with \( (5.17) \). Let us consider the low-lying breathing modes \( R \) \( (n_{\rho} = 1) \) with the even assignments \( IJ^\pi = 0^\frac{1}{2}^+, 1^\frac{1}{2}^+, 1^\frac{3}{2}^+ \) and the odd parity excited states \( O \) \( (n_z = 1) \) with the even assignments \( IJ^\pi = 0^\frac{3}{2}^-, 1^\frac{1}{2}^-, 1^\frac{3}{2}^- \). Using \( (5.17) \), we have \( (E = O, R) \),

\[
M_{\Lambda_Q} - M_N - m_H \simeq 0.23M_{\Lambda_Q} (b) + 0.77M_N (b) - 0.23m_H + m'_H, \\
M_{\Sigma_Q} - M_N - m_H \simeq 0.59M_{\Lambda_Q} (b) + 1.59M_N (b) + 0.59m_H + m'_H,
\]

(5.19)

where the holographically model-independent relations in \[61\],

\[
M_{\Lambda_Q'} = M_{\Lambda_Q} + m_{H'} - m_H, \\
M_{\Sigma_Q'} = 0.84M_N + m_{H'} + 0.16 (M_{\Lambda_Q} - m_H),
\]

(5.20)

has been imposed.

**Double-heavy baryons**

Since heavy baryons also contain anti-heavy quarks, let us return to the preceding arguments using the reduction \( \Phi_M = \phi_M e^{+i m_{H^0} x^0} \), in order to amount an anti-heavy-light meson. Most of the calculations are similar except for pertinent minus signs to the effective Lagrangian. In the form of a zero-mode, if we bind one heavy-light and one anti-heavy-light meson, the effective Lagrangian now reads,

\[
\mathcal{L} = \mathcal{L}^L \left[ a_L, X^\alpha \right] + i \xi_Q^\dagger \partial_t \xi_Q + \frac{3}{32\pi^2 a \bar{b}^{3/2} \bar{\rho}^2} \xi_Q^\dagger \xi_Q - i \xi_Q^\dagger \partial_t \xi_Q - \frac{3}{32\pi^2 a \bar{b}^{3/2} \bar{\rho}^2} \xi_Q^\dagger \xi_Q + \left( \xi_Q^\dagger \xi_Q - \xi_Q^\dagger \xi_Q^* \right)^2 \frac{24\pi^2 a \bar{b}^{3/2} \bar{\rho}^2 N_c}{24\pi^2 a \bar{b}^{3/2} \bar{\rho}^2 N_c}.
\]

(5.21)
The contributions of the mass from a heavy-light and anti-heavy-light meson are opposite as we have indicated. So the mass spectrum for baryons with $N_Q$ heavy quarks and $\bar{N}_Q$ anti-heavy quarks can be calculated as,

$$M_{Q\bar{Q}} = M_0 + (N_Q + N_{\bar{Q}}) m_H + M_{KK} \sqrt{\frac{3-b}{3}} (n_\rho + n_z + 1)$$

$$+ M_{KK} \left\{ \frac{(l+1)^2 (3-b)}{12} + \frac{3-b}{15} (1 - \frac{15 (N_Q - N_{\bar{Q}})}{4N_c} + \frac{5 (N_Q - N_{\bar{Q}})^2}{3N_c^2}) \right\}^{1/2}.$$  

(5.22)

The lowest state ($N_Q = N_{\bar{Q}} = 1, n_\rho = n_z = 0, l = 1$) with the assignments $IJ^\pi = \frac{1}{2}^+, \frac{3}{2}^+$ can be identified as pentaquark baryonic states and the masses are given as,

$$M_{Q\bar{Q}} (b) - M_N (b) - 2m_H = 0$$  

(5.23)

which does not depend on the D0 charge obviously.

For the excited pentaquark states, we identify the lowest state as $O$ with the odd parity, assignments $IJ^\pi = \frac{1}{2}^-, \frac{3}{2}^-$ and quantum number $N_Q = N_{\bar{Q}} = 1, n_\rho = 0, n_z = 1, l = 1$. The state with quantum number $N_Q = N_{\bar{Q}} = 1, n_\rho = 1, n_z = 0, l = 1$ and same assignments is identified as breathing or Roper $R$ pentaquarks as the ground state. So the mass relations for these states are given as ($E = O, R$),

$$M_{EQ\bar{Q}} (b) - M_N (b) - 2m_H \simeq 0.58 \sqrt{3-b} M_{KK}. $$  

(5.24)

On the other hand, the Delta-type pentaquarks can be identified as the states with quantum number $N_Q = N_{\bar{Q}} = 1, n_\rho = n_z = 0, l = 3$. Altogether, we have one $IJ^\pi = \frac{3}{2}^-, \frac{5}{2}^-$, two $IJ^\pi = \frac{3}{2}^-, \frac{3}{2}^-$, and one $IJ^\pi = \frac{3}{2}^-$ states, so the masses with heavy-flavors are given as,

$$M_{\Delta Q\bar{Q}} (b) - M_N (b) - 2m_H \simeq 0.42 \sqrt{3-b} M_{KK}. $$  

(5.25)

6 Summary

Using the Witten-Sakai-Sugimoto model in the D0-D4 background \[48, 49\] and the mechanism proposed in \[59, 60, 61\], we have extended the analysis in \[50, 51\] to involve the heavy flavors by a top-down holographic approach to the single- and double-heavy baryon spectra. The heavy-light interaction is introduced into this model by considering a pair of heavy flavor brane which is separated from the light flavor branes. The heavy baryon emerges from the zero mode of the reduced vector meson field to order $\lambda n_H^0$. The bind of the heavy and anti-heavy meson is equivalent as the instanton configurations of the gauge field on the flavor branes in leading.
order of $\lambda$, even in the presence of the Chern-Simons term. The smeared D0 charge has been
turned on in the D4-soliton background, so our calculation contains the excited states with
nonzero $\text{Tr} [F \wedge F]$ or a nonzero $\theta$ angle in the dual field theory. The $\theta$ dependence is through
a parameter $b$ (or $\tilde{\kappa}$) which is monotonically increasing with $\theta$.

Following the quantization in [61], the bound state moduli gives a rich spectrum. It contains
the coupled rotational, vibrational and translational modes. There are also some newly excited
states in the spectrum which are yet to be observed. The charmed pentaquark can be naturally
identified as a pair of degenerate heavy iso-doublets with $IJ^\pi = \frac{1}{2}^-\frac{3}{2}^-$ in the spectra when it
is extended to the double-heavy baryon case. Our calculation also shows the D0 charge moduli
in some new pentaquarks with hidden charm and bottom, and five Delta-like pentaquarks with
hidden charm in the spectra. Notice that our discussion returns to those in [61] if $b = 1$ i.e. no
D0 charge. Particularly for $b > 3$ case, we notice that the spectrum becomes complex which
indicates that baryons cannot be stable and it is in agreement with the previous study [50, 51]
of the holographic baryons in this model.

As in the most approaches of the gauge-gravity duality, our analysis is done in the large $N_c$
limit, and now with large $m_H$. Since the baryon spectrum demonstrates behavior of light baryons in [50, 51], we expect this model also captures the qualitative $\tilde{\kappa}$ (or $\theta$ angle) behavior, at least, for small $\tilde{\kappa}$ (or $\theta$ angle) in QCD-like theory when the heavy-light
interaction is involved. Although we have compared our results with the real-world nuclei or
quark states by setting $N_c = 3$ as [50, 51], there is still a long way to the realistic baryon
spectrum.

Appendix

In this Appendix, we collect the essential steps to quantize the light-baryon Lagrangian $L^L$
which is presented in this manuscript. The details can be systematically reviewed in [44, 50].
With the dimensionless variables, the explicit formula of $L^L [a_I, X^\alpha]$ is given as,

$$L^L = L^L_{YM} + L^L_{CS},$$

(A-1)

where

$$L^L_{YM} = -a\lambda N_c b^{1/2} \int d^4 x dz H_0 \text{Tr} \left[ \frac{1}{2} \frac{U_{KK}}{U} F_{\mu\nu} F^{\mu\nu} + \frac{U^3}{U_{KK}} b F_{\mu z} F^{\mu z} \right],$$

$$L^L_{CS} = \frac{N_c}{24\pi^2} \text{Tr} \left[ A \wedge F \wedge F - \frac{1}{2} A^2 \wedge F + \frac{1}{10} A^5 \right].$$

(A-2)

For the two-flavor case, the $U(2)$ gauge field $A$ can be decomposed to a $SU(2)$ part $A$ and a
$U(1)$ part $A$ as,
\[ A = A + \frac{1}{2} \hat{A}, \quad (A-3) \]

whose gauge field strength is,

\[ F = F + \frac{1}{2} \hat{F}. \quad (A-4) \]

In the large \( \lambda \) limit, imposing the \( \lambda \) rescale \((3.14)\), the EOM from \((A-2)\) can be obtained as,

\[ D_M F_{MN} + O(\lambda^{-1}) = 0, \quad D_M F_{0M} + \frac{1}{64\pi^2 ab^2/2} \epsilon_{MNPQ} \hat{F}_{MN} F_{PQ} + O(\lambda^{-1}) = 0 \]
\[ \partial_M \hat{F}_{0M} + O(\lambda^{-1}) = 0, \quad \partial_M \hat{F}_{0M} + \frac{1}{64\pi^2 ab^2/2} \epsilon_{MNPQ} \text{Tr}[F_{MN} F_{PQ}] + O(\lambda^{-1}) = 0, \quad (A-5) \]

and the solution is given in \((4.7)\).

In order to obtain the spectrum, we require the moduli of the solution to be time-dependent, i.e.

\[ X^\alpha, a^I \rightarrow X^\alpha(t), a^I(t). \quad (A-6) \]

Here \( a^I(t) \) refers to the \( SU(2) \) orientation. So the \( SU(2) \) gauge transformation also becomes time-dependent,

\[ A_M \rightarrow V \left( A_M^i - i \partial_M \right) V^{-1}, \]
\[ F_{MN} \rightarrow V \hat{F}_{MN} V^{-1}, \quad F_{0M} \rightarrow V \left( \hat{X}^\alpha \partial_\alpha A_M^i - D_M^i \Phi \right) V^{-1}, \quad (A-7) \]

where \( \Phi = -i V^\dagger \partial_0 V, \quad V^\dagger = V^{-1}. \)

The motion of the collective coordinates could be characterized by the effective Lagrangian in the moduli space. Up to \( O(\lambda^{-1}) \), it is

\[ L = \frac{1}{2} m_X g_{\alpha\beta} \dot{X}^\alpha \dot{X}^\beta - U(X^\alpha) + O(\lambda^{-1}) \]
\[ = \frac{1}{2} m_X \dot{X}^2 + \frac{1}{2} m_Z \dot{Z}^2 + \frac{1}{2} m_y \dot{y}_I \dot{y}_I - U(X^\alpha), \quad (A-8) \]

where the dot represents the derivative with respect to \( t \), \( g_{\alpha\beta} \) is the metric of the moduli space parameterized by \( X^\alpha \) which satisfies \( ds^2 = g_{\alpha\beta} dX^\alpha dX^\beta = d\hat{X}^2 + dZ^2 + 2 dy_I dy_I \) and \( \sum_{I=1}^4 y_I y_I = \rho^2 \). \( U(X^\alpha) \) is the effective potential associated to the onshell Lagrangian with the instanton solution \((4.7)\), i.e.

\[ \int d^3 x dz \mathcal{L}[a_I, X^\alpha]_{onshell} = -U(X^\alpha). \quad (A-9) \]
The baryon spectrum can be obtained by quantizing (A-8) (soliton) at rest. The quantization procedure is nothing but to replace the momenta in the Lagrangian to the corresponding differential operators which can act on the wave function of baryon states. So the quantized Hamiltonian associated to (A-8) is,

\[ H = H_X + H_Z + H_y, \]

\[ H_X = \frac{1}{2m_X} P_X^2 + M_0 = -\frac{1}{2m_X} \sum_{i=1}^{3} \partial^2 \partial X_i^2 + M_0, \]

\[ H_Z = \frac{1}{2m_Z} P_Z^2 + \frac{1}{2} m_Z \omega_Z^2 Z^2 = -\frac{1}{2m_Z} \partial Z^2 + \frac{1}{2} m_Z \omega_Z^2 Z^2, \]

\[ H_y = \frac{1}{2m_y} P_y^2 + \frac{1}{2} m_y \omega_y^2 \rho^2 + \frac{Q}{\rho^2} = -\frac{1}{2m_y} \sum_{l=1}^{4} \partial^2 \partial y_l^2 + \frac{1}{2} m_y \omega_y^2 \rho^2 + \frac{Q}{\rho^2}. \]  

(A-10)

In the unit of \( U_{KK} = M_{KK} = 1 \) or equivalently, with the replacement \( z \rightarrow zU_{KK}, \ x^\mu \rightarrow x^\mu/M_{KK}, \ \mathcal{A}_z \rightarrow \mathcal{A}_z/U_{KK}, \ \mathcal{A}_\mu \rightarrow \mathcal{A}_\mu M_{KK}, \) we have the following dimensionless values,

\[ M_0 = 8\pi^2 \lambda a b^{3/2} N_c, \ \omega_Z = \frac{1}{3} (3 - b), \ \omega_y = \frac{1}{12} (3 - b), \ Q = \frac{N_c}{40\pi^2 a b^{3/2}}. \]  

(A-11)

The eigenstates of \( H_Z \) are nothing but harmonic-oscillator states. The eigenfunctions of \( H_y \) are represented by \( T_{l}^{I}(a_{I}) R_{l,n_{y}}(\rho) \) where \( T_{l}^{I}(a_{I}) \) are the spherical harmonic functions on \( S^3 \). They are in the representations of \( (l, 2, l, 2) \) under the transformation of \( SO(4) = SU(2) \times SU(2)/Z_2 \). The former \( SU(2) \) corresponds to the isometric rotation while the latter is the space rotation in \( \{ x^i \} \). The states with \( I = J = \frac{1}{2} \) are described by this quantization, so the nucleon state is realized as the lowest state with \( l = 1, n_{\rho} = n_{z} = 0 \) of the Hamiltonian (A-10).

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