Spin Flavor Conversion of Neutrinos in Loop Quantum Gravity

G. Lambiase

a Dipartimento di Fisica "E.R. Caianiello" Università di Salerno, 84081 Baronissi (Sa), Italy.

b INFN - Gruppo Collegato di Salerno, Italy.

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Abstract: Loop quantum gravity theory incorporates a new scale length $L$ which induces a Lorentz invariance breakdown. This scale can be either an universal constant or can be fixed by the momentum of particles ($L \sim p^{-1}$). Effects of the scale parameter $L$ and helicity terms occurring in the dispersion relation of fermions are reviewed in the framework of spin-flip conversion of neutrino flavors.

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I. INTRODUCTION

Many attempts aimed to construct a theory of quantum gravity have shown that spacetime might have a non trivial topology at the Plank scale. The first to suggest that spacetime could have a foam-like structure was Wheeler in his seminal paper [1]. In the last years, the idea of quantum fluctuations of spacetime background has received a growing interest and today a large attention is addressed to study their effects on physical phenomena (see, for example, Refs. [2–9]).

The difficulties to build up a complete theory of quantum gravity has motivated the development of semiclassical approaches in which a Lorentz invariance breakdown occurs at the effective theory level. Deformations of the Lorentz invariance manifest by means of a slight deviation from the standard dispersion relations of particles propagating in the vacuum. Such modifications have been suggested in the paper [10], as well as in the framework of String Theory (D-branes) [12] and Loop Quantum Gravity [13–16]. These approaches foresee a dispersion relation in vacuum of particles of the form (we shall use natural units $c = 1 = \hbar$)

$$E^2 \approx p^2 + m^2 + f(M, p l_{pl}) ,$$

where $f(x)$ is a model dependent function, $M$ fixes a characteristic scale not necessarily determined by Planck length $l_{pl} \sim 10^{-19}$GeV$^{-1}$, and $p l_{pl} \ll 1$. As a consequence of Eq. (I.1), the quantum gravitational medium responds differently to the propagation of particles of different energies.

Recently, Alfaro, Morales-Téoctl, and Urrutia (AMU) have written a series of very important papers [15,16] in which it is developed a formalism based on loop quantum gravity [17]. In this theory a new length scale $L$ appears, $L \gg l_{pl}$, which separates the distances $d$ that manifest the quantum loop structure of space ($d \ll L$) from the continuous flat space ($d \gg L$). The new characteristic scale can be either an universal constant, or it can be fixed by the energy of the particle, $L \sim E^{-1}$. The latter case is called mobile scale.

In the AMU formalism, the dispersion relation of freely propagating fermions is given by [15]

$$E^2_\pm = A_p^2 p^2 + \eta p^2 \pm 2\lambda p + m^2 ,$$

where

$$A_p = 1 + \kappa_1 \left(\frac{l_{pl}}{L}\right) , \quad \eta = \kappa_3 l_{pl}^2 , \quad \lambda = \kappa_5 \frac{l_{pl}}{2L^2} .$$

The $\pm$ signs stand for the helicity of the propagating fermions, and $\kappa_i$ are unknown coefficients of the order $O(1)$. For photons, the dispersion relation derived in AMU theory is [16]

$$\omega^2_\pm = k^2 [A_\gamma^2 \pm 2\theta k] ,$$

In the framework of string theory, an alternative approach has been proposed by Kostelecky and collaborators. In their approach, even if the underlying theory has Lorentz and CPT invariance symmetries, the vacuum solution of the theory can spontaneously violate these symmetries [11].
where

\[ A_\gamma = 1 + \kappa_\gamma \left( \frac{l_{Pl}}{\mathcal{L}} \right)^{2+2\Upsilon}. \]  

(1.5)

Again ± signs stand for the helicity dependence of photons in the dispersion relation, \( \Upsilon = -1/2, 0, 1/2, 1, \ldots \), and \( k_\gamma \sim \mathcal{O}(1) \). It is worth noting that a similar result has been obtained by Gambini and Pullin [13] with \( A_\gamma = 1 \) and Ellis et al. [12] with the difference that the helicity dependence is absent.

On the experimental side, the present status eludes any possibility to probe directly quantum gravity effects. It has been suggested by Amelino-Camelia et al. [10] that \( \gamma \)-ray bursts might be a possible candidate to test the theories of quantum gravity due to their peculiar physical properties, i.e. the origin at cosmological distance and the high energy, which might make them sensitive to the additional terms occurring in (I.1). Alfaro and Palma [18] applies the AMU theory to the observed Greisen-Zatsepin-Kuz'min (GZK) limit anomaly and to the so called TeV-\( \gamma \) paradox, i.e. the detection of high-energy photons with a spectrum ranging up to 24 TeV from Mk 501. Assuming that no anomalies there exist, as recent works seem to indicate [19], they find that the favorite range for \( \mathcal{L} \) is

\[ 4.6 \times 10^{-17} \text{eV}^{-1} \lesssim \mathcal{L} \gtrsim 8.3 \times 10^{-18} \text{eV}^{-1}. \]  

(1.6)

Consequences of the dispersion relation (1.2) in other contexts have been studied in Refs. [20–22].

The aim of this paper is to investigate some consequences of fermion helicity terms appearing in the dispersion relation (1.2). Such a kind of analysis has been performed for photons (using Eq. (I.4)) by Gambini and Pullin [13], and Gleiser and Kazameh [23], which have shown the emergence of a birefringence effect. Here we study the effects of the helicity terms in spin flip conversion of neutrino flavors. In Refs. [15,22,24], the coefficients \( k_i \) appearing in (1.2) are taken flavor depending (hence responsible for the flavor mixing). To emphasize the role of the helicity terms, we assume that \( k_i \) are the same for all species of neutrinos \( (k_i^{(e)} = k_i^{(\mu)} = k_i^{(\tau)} = k_i) \).

II. SPIN FLAVOR OSCILLATIONS IN AMU THEORY

Before to study the spin flavor oscillation of neutrinos, some preliminary comments are in order. For the sake of convenience, we shall write Eq. (1.2) in the relativistic regime keeping out the relevant terms for our considerations. In this approximation the helicity operator is equal, up to the factor \( (m/E)^2 \ll 1 \), to the chiral operator [25]. We then get

\[ E_{L,R} \simeq p + \frac{m^2}{2p} \mp \lambda + F(p, \mathcal{L}, l_{Pl}, k_i), \]  

(II.1)

where the function \( F(x) \) contains all remaining contributions in (1.2), and it is diagonal in the chiral basis. The terms \( \pm \lambda \) change the effective energy of neutrinos depending if they are left-handed or right handed. This is a very peculiar aspect of the AMU theory since it gives rise, as we will see, to the occurrence of a resonance condition in the analysis of spin flavor oscillations.

As shown by Mikheyev, Smirnov, and Wolfenstein [26], when neutrinos propagate in a medium, their energy is shifted owing to the weak interaction with the background matter. The elastic scattering through charged current interaction gives the energy contribution \( 2\sqrt{2} G_F n_e \), whereas the neutral current interaction gives \( \sqrt{2} G_F n_n \), where \( G_F \) is the Fermi constant, and \( n_e \) (\( n_n \)) is the electron (neutron) density. The net contribution to the energy is therefore \( \sqrt{2} G_F (2n_e - n_n) \lesssim \sqrt{2} G_F n_e \) \( (n_n \lesssim n_e) \). Notice that for the Sun, a reasonable profile for \( n_e \) is \( n_e(r) = n_0 e^{-10.5 r/R_\odot} \), where \( n_0 = 85 N_A cm^{-3}, N_A \) is the Avogadro number [27,28]. At \( r = 0 \), one gets \( \sqrt{2} G_F n_e(0) \sim 10^{-12} \text{eV} \) [29]. For the Earth, \( \sqrt{2} G_F n_e(0) \sim 10^{-14} \text{eV} \) [29]. Besides, in order to have non vanishing off-diagonal elements in the effective Hamiltonian, we shall take into account of the interaction between neutrinos and an external magnetic field [30]

\[ \mathcal{L}_{\text{int}} = \bar{\nu}_\mu \sigma^{\mu\nu} F_{\mu\nu} \psi, \]  

(II.2)

where \( \mu \) is the magnetic momentum of the neutrino, \( F_{\mu\nu} \) is the electro-magnetic field tensor, and \( \sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu}, \gamma^{\nu}] \).

Collecting all that, the equation of evolution describing the conversion between two neutrino flavors is therefore [28,31]

\[ i \frac{d}{dr} \begin{pmatrix} \nu_e L \\ \nu_\mu L \\ \nu_e R \\ \nu_\mu R \end{pmatrix} = \mathcal{H} \begin{pmatrix} \nu_e L \\ \nu_\mu L \\ \nu_e R \\ \nu_\mu R \end{pmatrix}, \]  

(II.3)

2
where, in the chiral base, the matrix $\mathcal{H}$ is the effective Hamiltonian defined as \[26,29,28,32\]
\[
\mathcal{H} = \begin{pmatrix}
-\frac{\Delta m^2}{4p} \cos 2\theta - \lambda + \sqrt{2} G_F n_e & \frac{\Delta m^2}{4p} \sin 2\theta & \mu_e B & \mu B \\
\frac{\Delta m^2}{4p} \sin 2\theta & -\frac{\Delta m^2}{4p} \cos 2\theta - \lambda + \sqrt{2} G_F n_e & \mu B & \mu_{\mu\mu} B \\
\mu_e B & \mu B & -\frac{\Delta m^2}{4p} \cos 2\theta + \lambda & \frac{\Delta m^2}{4p} \sin 2\theta \\
\mu B & \mu_{\mu\mu} B & \frac{\Delta m^2}{4p} \cos 2\theta + \lambda & \frac{\Delta m^2}{4p} \sin 2\theta \\
\end{pmatrix},
\] (II.4)

up to terms proportional to identity matrix. Here $\Delta m^2 = m_2^2 - m_1^2$, $B$ is the magnetic field (the electric field is zero), and $\mu = \mu_{\mu\mu}$. In the more general case the effective Hamiltonian $\mathcal{H}$ does contain also a gravitational field term which is proportional to

$$A_G^\mu = \frac{1}{4} \varepsilon^{abcd}_\mu (\varepsilon_{\nu\tau \sigma} - \varepsilon_{\nu \sigma \tau}) \varepsilon^{\nu}_c \varepsilon^{\tau}_d$$

where $\varepsilon^\mu_\alpha$ are the vierbein fields and \(\varepsilon\) means the covariant derivative. Such a term vanishes for Schwarzschild-like geometry due to the spherical symmetry, so that no gravitational contributions arise in our analysis. We restrict to flavors $\nu_e - \nu_\mu$, but obviously this analysis works also for different neutrino flavors ($\nu_\mu - \nu_\tau$, $\nu_e - \nu_\tau$).

The terms giving the resonant condition of flavor spin-flip are the diagonal elements of the effective Hamiltonian (II.4). Hence

$$\nu_{eL} \rightarrow \nu_{\mu R} \quad 2\lambda + \frac{\Delta m^2}{2p} \cos 2\theta - \sqrt{2} G_F n_e (r_{res}) = 0,$$

(II.5)

$$\nu_{\mu L} \rightarrow \nu_{e R} \quad 2\lambda - \frac{\Delta m^2}{2p} \cos 2\theta - \sqrt{2} G_F n_e (r_{res}) = 0.$$

(II.6)

In both cases, we only have transitions from left-handed to right-handed neutrinos, the latter being sterile neutrinos do not have electroweak interaction with matter. Clearly, the resonant transitions do not occur simultaneously. From now on we assume that the neutrino mass hierarchy is $m_1 > m_2$ ($\Delta m^2 = -|\Delta m^2|$). Notice that expressions similar to Eqs. (II.5) and (II.6) hold also for Majorana neutrinos, with the difference that $\nu_{eL}, \nu_{\mu L} \rightarrow \nu_e, \nu_\mu$ and $\nu_{eR}, \nu_{\mu R} \rightarrow \bar{\nu}_e, \bar{\nu}_\mu$ in Eq. (II.3) \[28,33\].

Let us analyze the $\nu_{eL} \rightarrow \nu_{\mu R}$ transition. From Eq. (II.5), one gets $(E \sim p)$

$$|\cos 2\theta| \sim \frac{2E[2\lambda - \sqrt{2} G_F n_e (r_{res})]}{|\Delta m^2|} \lesssim 1.$$  

(II.7)

The transition between the two flavors $\nu_{eL} \rightarrow \nu_{\mu R}$ is determined by defining the effective mixing angle $\tilde{\theta}$ which diagonalizes the corresponding submatrix in (II.4)

$$\tan 2\tilde{\theta}(r) = \frac{4\mu B(r) E}{|\Delta m^2| \cos 2\theta - 4E\lambda + 2\sqrt{2} G_F E n_e (r)}.$$  

(II.8)

The resonance condition, for which neutrino oscillations are appreciably enhanced, occurs when the denominator of (II.8) vanishes. When $\lambda = 0$ there is no resonance unless the neutrino mass hierarchy is inverted ($m_2 > m_1$ and the usual resonance condition is restored). Flavor eigenstates and mass eigenstates are now related by the effective mixing angle

$$|\nu_e\rangle = \cos \tilde{\theta}|\nu_1\rangle + \sin \tilde{\theta}|\nu_2\rangle, \quad |\nu_\mu\rangle = -\sin \tilde{\theta}|\nu_1\rangle + \cos \tilde{\theta}|\nu_2\rangle$$

For adiabatic variation of the magnetic field\(^2\) and electron density, the conversion probability is \[34,35\]

$$P_{\nu_{eL} \rightarrow \nu_{\mu R}} = \frac{1}{2} (1 - \cos 2\tilde{\theta} \cos 2\theta),$$  

(II.11)

\(^2\)Unfortunately, the profile of magnetic fields in the core, radiation zone or convection zone of Sun is little known, except that
and at the resonance (\(\tilde{\theta} = \pi/4\)) it assumes the maximum value

\[
P_{\nu_e \rightarrow \nu_\mu} = \frac{1}{2}.
\]  

\(\text{(II.12)}\)

Let us now use Eq. (II.7) to determine the bounds on the scale parameter \(\mathcal{L}\) and the coefficient \(k_5\). We shall consider the cases in which 

i) \(\mathcal{L}\) is an universal constant, 

ii) \(\mathcal{L} \sim 10^{-15}\text{eV}^{-1}\) as recent results seem to suggest [18,22,24,36], and 

iii) \(\mathcal{L} \sim p^{-1}\) (mobile scale).

A. Solar Neutrinos

Due to the fact that the matter densities \(n_e\) varies with the distance \(r\), as well as the magnetic field \(B(r)\), the mass eigenstates evolve adiabatically only if the adiabaticity condition is satisfied at the resonance [29,28,32], that is

\[
\gamma \equiv \left| \frac{a_1 - a_2}{d\theta/dr} \right|_{res} \gg 1,
\]

\(\text{(II.13)}\)

where the effective mixing angle is defined by (II.8) and \(a_i\), \(i = 1, 2\), are the eigenvalues of the submatrix of evolution Eq. (II.4) corresponding to \(\nu_e \rightarrow \nu_\mu\) transition

\[
a_{1,2} = \frac{1}{2} \left\{ \sqrt{2} G_F n_e \pm \sqrt{\left( \frac{\Delta m^2}{2E} - 2\lambda + \sqrt{2} G_F n_e \right)^2 + 4\mu^2 |B\|^2} \right\}.
\]

\(\text{(II.14)}\)

The transition probability (II.11) is generalized by the formula [34]

\[
P_{\nu_e \rightarrow \nu_\mu} = \frac{1}{2} - \left( \frac{1}{2} - P_c \right) \cos 2\tilde{\theta} \cos 2\theta,
\]

\(\text{(II.15)}\)

where \(P_c = \exp[-2\pi\gamma \sin^2 \tilde{\theta} \cos \tilde{\theta} / \sin^2 2\tilde{\theta}]\) [29]. If \(\gamma \gg 1\) realizes, then the formula (II.11) is recovered.

Eq. (II.8) allows to calculate \((d\theta/dr)_{res}\), and by using (II.14), the adiabaticity condition (II.13) assumes the form

\[
\gamma^{-1} = \left( \frac{\sqrt{2} G_F}{8\mu^2 |B\|^2} \left| \frac{dn_e}{dr} \right| \right)_{res} \ll 1.
\]

\(\text{(II.16)}\)

The term containing \(dB/dr\) is absent since at the resonance its coefficient vanishes. Being \(B_\odot \gtrsim 10\text{T}\) in the solar convective zone [28], Eq. (II.16) reduces to

\[
e^{-10.5_{res}/R_\odot} \ll 0.1,
\]

\(\text{(II.17)}\)

they may be quite large. Similarly for Earth. In the case in which the magnetic field is homogeneous (constant), the conversion probability is (see, for example, the paper by Lim and Marciano in Ref. [30])

\[
P_{\nu_e \rightarrow \nu_\mu} = \frac{(2\mu B)^2}{(2\mu B)^2 + \left( \frac{\Delta m^2}{2E} \cos 2\theta - 2\lambda + \sqrt{2} G_F n_e \right)^2} \sin^2 \alpha,
\]

\(\text{(II.9)}\)

where

\[
\alpha = \sqrt{\left( \frac{\Delta m^2}{2E} \cos 2\theta - 2\lambda + \sqrt{2} G_F n_e \right)^2 + (2\mu B)^2 \Delta r}
\]

and \(\Delta r\) is the distance travelled by neutrinos. At the resonance it becomes

\[
P_{\nu_e \rightarrow \nu_\mu} = \sin^2 \mu B \Delta r.
\]

\(\text{(II.10)}\)

The neutrino magnetic momentum is \(\mu \sim 10^{-11}\mu_\odot \sim 6 \times 10^{-16}\text{eVT}^{-1}\) [28]. Taking the magnetic field of Earth \(B_\oplus \approx 5 \times 10^{-5}\text{T}\), constant over the region \(\Delta r \sim 2R_\oplus \approx 1.2 \times 10^7\text{m}\), it follows that the transition probability is \(P_{\nu_e \rightarrow \nu_\mu} \sim 3 \times 10^{-4}\), a value very small for giving an appreciable spin flavor rotation effect. For the Sun, the superficial magnetic field \(B_\odot \sim 10^{-1}\text{T}\) [28], and \(\Delta r \sim R_\odot \sim 7 \times 10^8\text{m}\) give \(P_{\nu_e \rightarrow \nu_\mu} \sim 0.04\). On the other hand, in the convective zone \(B_\odot \sim 10\text{T}\) [28], thus \(P_{\nu_e \rightarrow \nu_\mu} \sim 0.65\).
which is fulfilled by $r_{res} \gtrsim 0.4R_\odot$. Eq. (II.17) then assures that the mass eigenstates of neutrinos produced in the Sun evolve always adiabatically.

From (II.7) it follows the upper bound on $\lambda$

$$\lambda \lesssim \frac{1}{2} \left[ 10^{-12} e^{-10.5r_{res}/R_\odot} \text{eV} + \frac{\Delta m^2}{2E} \right].$$  \hspace{1cm} (II.18)

The best fit for neutrino oscillations induced by MSW effect is given by the following (experimental) values of the $\Delta m^2 - \sin^2 2\theta$ parameters [29]

$$\begin{align*}
|\Delta m^2| &\sim (3/10) \times 10^{-6} \text{eV}^2 & \sin^2 2\theta &\sim (0.6 \pm 1.3) \times 10^{-2} & \text{SMA} \\
|\Delta m^2| &\sim (1/20) \times 10^{-5} \text{eV}^2 & \sin^2 2\theta &\sim 0.5 \pm 0.9 & \text{LMA}
\end{align*}$$

where SMA and LMA stand for Small ($\sin^2 2\theta \ll 1$) and Large ($\sin^2 2\theta \lesssim 1$) Mixing Angle solution, respectively.

In what follows, estimations are carried out for solar neutrinos with the maximum energy $E \sim 15 \text{MeV}$, produced by $^8B$ and $\text{hep}$ reactions [29]. It is worth to point out that even though the flux of $^8B$ neutrinos produced in the Sun is much smaller than the fluxes of $pp$, $^7Be$, and $\text{pep}$ neutrinos, $^8B$ neutrinos give the major contribution to the event rates of experiments with a high energy detection threshold [29]. In fact, the results of Kamiokande and SuperKamiokande experiments are usually presented in terms of the measured flux of $^8B$ neutrinos [29].

The resonance occurring at $r_{res} \gtrsim 0.4R_\odot$ implies that the background matter density reduces and the dominant term in (II.18) is the massive one. For $k_5 \sim O(1)$, the lower bound on $L$ is $L \gtrsim 10^{-6} \text{eV}^{-1}$, which is nearly in the range of nuclear physics so that it can be discarded since no evidence of loop structure occurred at this scale. If $L \sim 10^{-18} \text{eV}^{-1}$, then the upper bound on $k_5$ is $k_5 < 10^{-25}$, which is a weak bound since the coefficient $k_5$ is expected to be of the order $O(1)$. Finally, in the case of mobile scale, one gets $k_5 \sim O(10) \div O(10^2)$ for the SMA solution, and $k_5 < O(10^2) \div O(10^3)$ for the LMA solution, that are the expected order of magnitude for $k_5$. Such ranges increase for neutrinos with energies $\lesssim 10 \text{MeV}$ (as, for instance, for $pp$, $^7Be$, $\text{pep}$ neutrinos).

Let us finally discuss the case of degenerate or massless neutrinos (or $\theta = \pi/4$). The effective mixing angle then reduces to

$$|\tan 2\theta| \sim \frac{\mu B_\odot}{\lambda},$$

where we have neglected the weak interaction of neutrinos with the background matter ($n_e \approx 0$). We consider neutrinos propagating in regions far from the Sun core, where the superficial magnetic field of the Sun is $B_\odot \sim 10^{-11} \text{T}$ [37]. Being the neutrino magnetic momentum $\mu \sim 10^{-11} \mu_B \sim 6 \times 10^{-16} \text{eVT}^{-1}$ [28], it follows that the magnetic energy of neutrinos is $\mu B_\odot \sim 6 \times 10^{-17} \text{eV}$. Since $k_5 \sim O(1)$ and taking $L \sim p^{-1}$, with $p \sim \text{MeV}$, one gets $|\tan 2\theta| \sim 1$, i.e. $\sin^2 2\theta \sim 0.5$. The transition probability (II.9) gives $F_{\nu_{\mu} \rightarrow \nu_{\mu}} = 0.16$.

\section*{B. Atmospheric Neutrinos}

Strong evidence in favor of atmospheric neutrino oscillations come from SK experiment [38], and the relevant values of $|\Delta m^2| - \sin^2 2\theta$ parameters are ($\nu_{\mu} - \nu_{\tau}$) [29,39]

$$\begin{align*}
|\Delta m^2| &\sim 5 \times 10^{-3} \text{eV}^2, & \sin^2 2\theta &\sim 1, \hspace{1cm} (II.19)
\end{align*}$$

which implies $\cos 2\theta < 1$. From Eq. (II.7) it follows ($E \sim \text{GeV}$) $L \gtrsim 10^{-9} \text{eV}^{-1}$ as $k_5 \sim O(1)$, which has to be discarded, as discussed before. If the scale length is fixed by $L \sim 10^{-18} \text{eV}^{-1}$, then the parameter $k_5$ is bounded by $k_5 \ll 10^{-16}$. Finally, in the case of mobile scale, one gets

$$k_5 < \frac{|\Delta m^2|}{4E^3\mu B_\odot} \sim 10^{-2}. \hspace{1cm} (II.20)$$

For neutrinos with sub-GeV energies, $E \gtrsim 100 \text{MeV}$, the coefficient $k_5$ in (II.20) is bounded from above by $k_5 \lesssim 10$, which agrees with expected order of $k_5 \sim O(1)$.

The channel of oscillation $\nu_{\mu} \rightarrow \nu_{\tau}$ in the accelerator LSND experiment, fits experimental data for the following region of parameters [40]

$$\begin{align*}
|\Delta m^2| &\sim \text{eV}^2, & \sin^2 2\theta &\approx (0.2 \div 3) \times 10^{-3}, \hspace{1cm} (II.21)
\end{align*}$$
thus $\cos 2\theta \approx 1$. The maximum neutrino energy is taken of the order $E \sim 10\text{MeV}$. As $k_5 \sim 1$, we find $L \sim 10^{-9}\text{eV}^{-1}$. For $L \sim 10^{-18}\text{eV}^{-1}$, it follows $k_5 \sim 10^{-17}$. Finally, for $L \sim E^{-1}$, the parameter $k_5$ is given by $k_5 \sim 10^4$. We have to point out that LSND experiments still remains unconfirmed, contrarily to the solar and atmospheric experiments which have been confirmed by other experiments.

The above results show that the helicity terms appearing in the dispersion relation (I.2) are the responsible for the occurrence of the resonance condition in the spin-flip conversion of neutrino flavors, whereas the background matter effects are negligible (or are absent). The estimations provide a narrow region of solar and atmospheric neutrino energies in which experiments might seek effects induced by helicity terms occurring in loop quantum gravity, as suggested by AMU’s theory.

III. CONCLUSION

Despite the concrete difficulties to probe quantum gravity effects occurring at the Planck scale, it is an intriguing suggestion that quantum gravity should predict a slight departure from Lorentz’s invariance, which manifests in a deformation of the dispersion relations of photons and fermions. Such results have been indeed suggested in loop quantum gravity [13,15,16] (and string theory [11,12]). This approach is endowed with a scale length characterizing the scale on which new effects are non trivial. The estimation of the order of magnitude of the new scale length and of parameters entering in the dispersion relations is certainly of current interest, as well as different scenarios where these effects become testable [18,41].

In this paper we have determined bounds on $L$ and $k_5$ (occurring in Eq. (I.2)) in the context of neutrinos oscillations. In the case of solar and atmospheric neutrino oscillations, the analysis performed suggests that the mobile scale $L \sim p^{-1}$ is the favorite one, with $k_5 \lesssim \mathcal{O}(1) \div \mathcal{O}(10)$, as expected. Thus, neutrino oscillations, seem to be very promising candidates (see also Refs. [15,22–24,42]) for bringing quantum gravity effects predicted by AMU theory to the realm of experimental evidence.

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