Localization transition for light scattering by atoms in an external magnetic field

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A random three-dimensional (3D) ensemble of two-level atoms in a strong static magnetic field can localize light if its number density \( \rho \) exceeds a critical value \( \rho_c \simeq 0.1k_0^2 \), where \( k_0 \) is the wave number of light in the free space. Localized states arise in a narrow band of frequencies near the resonant frequency of an isolated atom. A critical exponent of the localization transition taking place upon varying the frequency of light at a constant \( \rho > \rho_c \) is estimated to be \( \nu = 1.57 \pm 0.07 \). This classifies the transition as an Anderson localization transition of 3D orthogonal universality class.

The search for Anderson localization of light in three-dimensional (3D) disordered media has been an active research direction since the mid-1980s when Sajeev John [1] and Philip Anderson [2] independently noticed that light could be localized by strong disorder in a way analogous to electron localization in disordered solids [3]. It was rapidly recognized that optical localization in a dielectric material is difficult to achieve because, on the one hand, of the way in which the disorder enters the optical wave equation (the position-dependent dielectric function \( \varepsilon(r) \) of the material multiplies the second-order time derivative of the electric field) and, on the other hand, of the relatively low values of \( \varepsilon \) of available transparent materials at optical frequencies (\( \varepsilon \lesssim 15 \)) [4]. As a consequence, the correlation length of disorder (or, equivalently, the typical size of individual scattering particles composing a disordered material) has to be of the order of the optical wavelength in order to achieve strong scattering and ultimately Anderson localization. Unfortunately, even the materials composed of particles with the largest available dielectric constants and sizes in a wide range around the optical wavelength did not allow for an indisputable observation of light localization in 3D thus far [5,7].

A random spatial arrangement of immobile atoms represents an alternative to dielectric media for reaching Anderson localization of light [8,9]. Indeed, the coherent backscattering of light, considered as a precursor of localization, was observed in cold atomic gases almost 20 years ago [10,11]. However, the vector character of light and the associated dipole-dipole interactions between atoms have been predicted to prevent Anderson localization in atomic systems [12,13]. A static external magnetic field partially suppresses the interatomic dipole-dipole interactions and induces a localization transition for light that is quasiresonant with a \( J_g = 0 \to (J_e = 1, m = \pm 1) \) transition \( (J_g \text{ and } J_e \text{ are the total angular momenta of the ground and excited states, respectively, and } m \text{ is the magnetic quantum number of the excited state}) \) [14]. This transition takes place in a dense medium that is made strongly anisotropic by the magnetic field, and in the presence of near-field couplings between atoms separated by less than a wavelength in distance. Questions thus arise concerning the nature of this transition: To which extent can it be considered a genuine, disorder-induced Anderson transition? What is its universality class? Does the anisotropy of the atomic medium in a strong magnetic field play any role? It is the purpose of this Letter to provide exhaustive answers to these questions and thereby motivate the experimental work on Anderson localization of light by cold atoms.

An ensemble of \( N \) identical two-level atoms (resonance frequency \( \omega_0, J_g = 0 \) for the ground state, \( J_e = 1 \) for the excited state) at positions \( \{r_j\}, j = 1, \ldots, N \), subjected to a constant external magnetic field \( B \parallel e_z \) and interacting with a free electromagnetic field, is described by the following Hamiltonian [15,17]:

\[
\hat{H} = \sum_{j=1}^{N} \sum_{m=-1}^{1} (\hbar\omega_0 + ge_B Bm) |e_{jm}\rangle \langle e_{jm}| + \sum_{\epsilon \pm k} \hbar c k \left( \hat{a}_{k\epsilon}^\dagger \hat{a}_{ke} + \frac{1}{2} \right) - \sum_{j=1}^{N} \hat{D}_j \cdot \hat{E}(r_j) + \frac{1}{2\epsilon_0} \sum_{j \neq n}^{N} \hat{D}_j \cdot \hat{D}_n \delta(r_j - r_n). \tag{1}
\]

Here we denote the atomic dipole operators by \( \hat{D}_j \), the electric displacement vector by \( e_0 \hat{E}(r) \), the photon creation and annihilation operators corresponding to a mode of the free electromagnetic field having a wave vector \( k \) and a polarization \( e \) by \( \hat{a}_{k\epsilon}^\dagger \) and \( \hat{a}_{ke} \), respectively. \( \mu_B \) is the Bohr magneton, and \( g_e \) is the Landé factor of the excited state. As discussed previously [14,18], the quasi-modes of the atomic subsystem can be found as eigenvectors of a \( 3N \times 3N \) Green’s matrix \( G \) with elements

\[
G_{e_{jm}e_{nm}} = (i - 2m\Delta) \delta_{e_{jm}e_{nm}} - \frac{2}{\hbar\Gamma_0} (1 - \delta_{e_{jm}e_{nm}})
\times \sum_{\mu,\nu} \langle \psi_{e_{jm}g_{n\epsilon}} | \psi_{e_{nm}g_{\epsilon}} \rangle \frac{e^{ik_0r_{jn}}}{r_{jn}^3}
\times \left\{ \delta_{\mu\nu} \left[ 1 - ik_0r_{jn} - (k_0r_{jn})^2 \right] \right.
\right.
\left.
\left. - \frac{v_{e_{jm}}^\mu v_{e_{nm}}^\nu}{r_{jn}^2} \left[ 3 - 3ik_0r_{jn} - (k_0r_{jn})^2 \right] \right\}. \tag{2}
\]
where \( k_0 = \omega_0/c \), \( \Delta = g_{\tau} \mu_B B / \hbar \Gamma_0 \) is the Zeeman shift in units of the spontaneous decay rate \( \Gamma_0 \), \( \mathbf{d}_{r_j,m} = \langle J_m | \hat{D}_j | J_0 \rangle \), and \( \mathbf{r}_{jn} = \mathbf{r}_j - \mathbf{r}_n \). The complex eigenvalues \( \lambda_n \) of the matrix \( G \) yield eigenfrequencies \( \omega_n = \omega_0 - (\Gamma_0/2) \text{Re} \lambda_n \) and decay rates \( \gamma_n/2 = (\Gamma_0/2) \text{Im} \lambda_n \) of quasimodes. The matrix \( G \) thus plays the role of an effective Hamiltonian of the open system of atoms interacting via the electromagnetic field.

In a strong magnetic field \( \Delta \gg 1 \) (still, \( \Delta \ll \omega_0 / \Gamma_0 \)), the eigenvalues \( \lambda_n \) split in three groups corresponding to transitions between the ground state and one of the three Zeeman sublevels \( (m = 0, \pm 1) \) which now have very different frequencies \( \omega_m = \omega_0 + m \Gamma_0 \Delta \) and hence decouple \([14]\). A group of eigenvalues around one of the three \( \omega_m \) can be found independently of the other two groups by diagonalizing a simplified \( N \times N \) Green’s matrix \( \mathcal{G}^{(m)} \). Indeed, consider the probability amplitude for a transfer of an excitation from an atom \( n \) in an excited state with, say, \( m = -1 \), to an atom \( j \) initially in the ground state. For the purpose of argument, assume that the atoms are far from each other: \( k_0 \varphi_{jn} \gg 1 \).

The atom \( n \) emits a photon having a frequency around \( \omega_{-1} = \omega_0 - \Gamma_0 \Delta \), a wave vector \( \mathbf{k} \parallel \mathbf{r}_{jn} \), and helicity \( \epsilon \perp \mathbf{k} \) with a probability amplitude proportional to a scalar product \( \epsilon^* \cdot \mathbf{d}_{g_{r_{jn}},m} \propto (1 - \epsilon \cos \theta_{jn}) \exp(-i \varphi_{jn}) \), where \( \epsilon = \pm 1 \) for right and left helicities, respectively, and \( \mathbf{r}_{jn} = \{ \mathbf{r}_{jn}, \theta_{jn}, \varphi_{jn} \} \). The photon will reach the atom \( j \) with a probability amplitude \( \propto \exp(ik_0 \varphi_{jn})/k_0 r_{jn} \) describing the propagation of a spherical wave, and will be absorbed on the transition \( J_g = 0 \rightarrow J_c = 1, m = -1 \) of this atom with a probability amplitude proportional to \( \epsilon \cdot \mathbf{d}_{r_{jn},g_{r_{jn}}} \propto (1 - \epsilon \cos \theta_{jn}) \exp(i \varphi_{jn}) \). The two other transitions corresponding to \( m = 0 \) and \( m = +1 \) cannot be excited because their resonant frequencies are too different from \( \omega_{-1} \). The probability amplitude \( \mathcal{G}_{jn}^{(-1)} \) of the excitation transfer is obtained by multiplying the probability amplitudes of emission by the atom \( n \), propagation from \( r_n \) to \( r_j \), and absorption by the atom \( j \neq n \), and then summing over \( \epsilon \):

\[
\mathcal{G}_{jn}^{(-1)} \propto \frac{\epsilon^*}{k_0 r_{jn}} \sum_{\epsilon = \pm 1} (1 - \epsilon \cos \theta_{jn})^2 \\
\approx \frac{\epsilon^*}{k_0 r_{jn}} (1 - \frac{1}{2} \sin^2 \theta_{jn}). \tag{3}
\]

Repeating all the reasonsing for \( m = +1 \), we obtain exactly the same equation for \( \mathcal{G}_{jn}^{(1)} \), whereas the result for \( \mathcal{G}_{jn}^{(0)} \) is different. Extending the above analysis to arbitrary \( r_{jn} \) by properly including near-field dipole-dipole interactions between the atoms and keeping trace of all numerical factors yields

\[
\mathcal{G}_{jn}^{(\pm 1)} = (i \mp 2 \Delta \delta_{jn}) + (1 - \delta_{jn}) \frac{3 \epsilon^*}{2 k_0 r_{jn}} \\
\times \left[ P(ik_0 r_{jn}) + Q(ik_0 r_{jn}) \sin^2 \theta_{jn} \right], \tag{4}
\]

where \( P(x) = 1 - 1/x + 1/x^2 \) and \( Q(x) = -1 + 3/x - 3/x^2 \). The effective anisotropy of the atomic medium in a strong magnetic field, which is not obvious from Eq. (2), now becomes evident because Eq. (4) contains an explicit dependence on the angle \( \theta_{jn} \) between \( r_{jn} \) and \( \mathbf{B} \).

Figure 1 illustrates the fact that the eigenvalues of the matrices (2) and (4) indeed coincide in the limit of large \( \Delta \). From here on we assume that the atoms are randomly distributed in a ball of radius \( R \) and volume \( V \) with an average density \( \rho = N/V \).

We will use Eq. (4) to study the localization transition for light that is quasiresonant with the transition between the ground state and one of the excited states corresponding to \( m = \pm 1 \) (there is no localization transition for \( m = 0 \) [14]). To identify the critical points (mobility edges), we use the approach developed in Ref. [19] for scalar waves. In brief, at a fixed (and sufficiently high) atomic number density \( \rho \) and for a set of different atom numbers \( N \), we compute the eigenvalues \( \lambda_n \) of the matrix (4) for ensembles of random realizations of disorder \( \{ r_{jm} \} \) and then estimate the probability density \( p(\ln g; \Re \Lambda, N) \) of the logarithm of the Thouless conductance \( g = \Im \Lambda_n / (\Re \Lambda_n - \Re \Lambda_{n-1}) \), where the angular brackets denote ensemble averaging. The small-\( g \) part of \( p(\ln g; \Re \Lambda, N) \) becomes independent of \( N \) at the critical points \( \Re \Lambda_c \). Instead of working with \( p(\ln g; \Re \Lambda, N) \), it is more convenient to analyze its low-rank (\( q \leq 0.05 \)) percentiles \( g_q \) defined by \( q = \int_0^{g_q} p(\ln g; \Re \Lambda, N) \, dq \). Figure 2(a) shows the third percentile as a function of
FIG. 2. (a) Third percentile of the logarithm of Thouless conductance $g$ as a function of frequency at a fixed number density of atoms $\rho/k_0^3 = 0.2$ for 10 different sizes of the atomic cloud: $N = 2000, 4000, 6000, 8000, 10000, 12000, 14000, 16000, 20000$ and $24000$ (symbols with error bars). At least $2 \times 10^7$ eigenvalues were calculated for each $N$ to compute the percentile. Vertical dashed lines indicate the approximate positions of mobility edges. (b) Fits (solid lines) of Eqs. (5) and (6) to numerical data for $N \geq 4000$ (symbols with error bars) around one of the mobility edges. The best-fit values of the mobility edge $\Re \Lambda_c$ and of the critical exponent $\nu$ are given on the graph.

FIG. 3. Best-fit values of the mobility edge $\Re \Lambda_c$ (a) and of the critical exponent $\nu$ (b) as functions of the rank $q$ of the analyzed percentile $\ln g_q$. Fits were performed for the data corresponding to $\rho/k_0^3 = 0.2$ and $\ln g_q$ within $\pm 1$ of an estimated crossing point of curves obtained for different $N$. The dashed solid lines show average values of $\Re \Lambda_c$ and $\nu$, respectively, with the values of the latter printed on the graphs. Grey areas visualize the errors of the averages.

Re$\Lambda$ for different $N$; the crossing points of lines corresponding to different $N$ provide approximate positions of mobility edges $\Re \Lambda_c$ shown by vertical dashed lines.

The finite-size scaling analysis of the localization transition consists in fitting the numerical data for $\ln g_q$ near a critical point by polynomials [19, 20]:

$$\ln g_q = \sum_{j_1=0}^{m_1} \sum_{j_2=0}^{n_2} a_{j_1,j_2} u_1(w)^{j_1} u_2(w)^{j_2}(k_0R)^{j_1/j_2} y, \quad (5)$$

$$u_i(w) = \sum_{j=0}^{m_i} b_{ij} w^j, \quad (6)$$

where $w = (\Re \Lambda - \Re \Lambda_c)/\Re \Lambda_c$, $\nu$ is a critical exponent of the localization transition, and $y < 0$ is an irrelevant exponent accounting for finite-size effects. $m_1 = 1$, $n_1 = 2$, $m_2 = n_2 = 1$ in Eqs. (5) and (6) are the minimum values that yield fits of acceptable quality and, at the same time, give consistent values of best-fit $\Re \Lambda_c$ and $\nu$ for all $q$ from 0.001 to 0.05. An example of fit is shown in Fig. 2(b) whereas Fig. 3 shows the best-fit values of the mobility edge, corresponding to $\Re \Lambda_c + 2m\Delta \simeq -1$ in Fig. 2(a), and of the critical exponent $\nu$ as functions of the rank $q$ of the considered percentile for $\rho/k_0^3 = 0.2$. The analysis of the second mobility edge $\Re \Lambda_c + 2m\Delta \simeq -2.4$ in Fig. 2(a)] is complicated by a stronger noise in the numerical data and does not yield reliable estimations of $\nu$ with an acceptable precision.

The best estimation of the critical exponent $\langle \nu \rangle = 1.57 \pm 0.07$ obtained by averaging results obtained for all $q = 0.001-0.05$ [see Fig. 3(b)], is consistent with the value expected for a localization transition of the 3D orthogonal universality class, typical for spinless time-reversal invariant systems [20, 21]. The Hamiltonian (1) is formally invariant under time reversal of the whole system “light + atoms + the magnet creating the magnetic field $B$” (remember that $B$ changes sign upon time reversal) [22]. However, for a constant $B$ that we consider, the subsystem “light + atoms” is not time-reversal invariant and
one might expect the localization transition to belong to the unitary universality class and have a different critical exponent [21,23]. To resolve this apparent contradiction, let us go back to the detailed consideration of the transfer of an excitation from an atom to a distant atom that we presented above to derive Eq. (3). The transfer is operated by photons of different helicities $\epsilon = \pm 1$ with amplitudes $A_{\rho j}^{(\epsilon)}$. It is not time-reversal invariant because the transfer of an excitation from the atom $j$ back to the atom $n$ would involve photons with different amplitudes: $A_{nj}^{(\epsilon)} \neq A_{jn}^{(\epsilon)}$. However, this exchange of photons has an additional symmetry imposing $A_{nj}^{(\epsilon)} = A_{jn}^{(-\epsilon)}$. In other words, the photons of positive (negative) helicity play the same role in the excitation transfer from one atom to another as the photons of negative (positive) helicity do for the transfer in the opposite direction. As a result, the effective Hamiltonian of the system of atoms in a strong magnetic field becomes time-reversal invariant in the limit $N \to \infty$, and the localization transition in this system belongs to the orthogonal universality class. The anisotropy induced in the atomic medium by the external magnetic field may play a role in determining the locations of mobility edges [21,22], but apparently does not modify the universality class of the localization transition, in agreement with previous theoretical results for the anisotropic Anderson model [26] and with experiments in cold-atom systems [27].

The analysis performed above for a single atomic density $\rho/k_0^3 = 0.2$ can be repeated for other densities as well. The full calculation is very time consuming but the mobility edges can be estimated from the results for a smaller number of random configurations $\{r_j\}$ and only two different $N$, see Fig. 4. For $0.1 \lesssim \rho/k_0^3 \lesssim 0.12$ the mobility edges are too close to be clearly distinguishable, and they disappear for $\rho/k_0^3 < \rho_c/k_0^3 \simeq 0.1$ (i.e., lines ln $g_q$ corresponding to different $N$ do not cross). The latter value also follows from polynomial fits in Fig. 4 as a minimum density at which localized states appear. It is slightly larger than $\rho_c/k_0^3 \simeq 0.08$ identified as the absolute localization threshold for scalar waves [28]. In our opinion, this difference reflects the residual dipole-dipole interactions which are partially suppressed but not fully eliminated by the magnetic field. As a consequence, Anderson localization requires a higher scatterer density and thus is more difficult to reach for light scattered by atoms in a magnetic field than for scalar waves. A the same time, the two transitions belong to the same universality class and are characterized by the same critical exponent $\nu$. The width $\Delta \omega$ of the frequency band where the states are localized increases with density $\rho$ (see the inset of Fig. 4). By fitting the dependence of $\Delta \omega$ on $\rho - \rho_c$ with a power law, we find

$$\Delta \omega \simeq \frac{3.6 \Gamma_0}{k_0^3} (\rho - \rho_c)^{2/3}.$$  

It is important to realize that in this work we take two limits in a well-defined order: first $B \to \infty$ and then $N \to \infty$. The first limit allows us to reduce the dimension of the considered random matrix $G$ and to work with a smaller matrix $G^{(\pm 1)}$, whereas the second one is implied by the finite-size scaling procedure. It would be interesting to check that modifying the order of the limits does not affect the results (in particular, the critical exponent $\nu$). In an experiment with cold atoms, neither $B \to \infty$ nor $N \to \infty$ is realizable, but it is nevertheless possible to achieve sufficiently strong fields $B$ to decouple transitions with different magnetic quantum numbers $m$ (see Ref. [28] for a discussion of a possible experiment).

In conclusion, quasiresonant light in a 3D ensemble of two-level atoms in a strong magnetic field exhibits a localization transition when its frequency is varied around $\omega = \omega_0 \pm \Gamma_0 \Delta$, provided that the number density of atoms $\rho$ exceeds a critical value $\rho_c \simeq 0.1k_0^3$. The width of the frequency range in which the states are localized increases with density $\rho$ as $(\rho - \rho_c)^{2/3}$ and remains smaller than or of the order of the natural line width $\Gamma_0$, at least for densities up to $\rho = 0.4k_0^3$. The estimated critical exponent of the localization transition is $\nu = 1.57 \pm 0.07$, which identifies it as an Anderson transition of the 3D orthogonal universality class. The transition belongs to this universality class despite the fact that the time-reversal symmetry of the system “light + atoms” is broken by the external magnetic field, which might have changed the universality class into the unitary one. However, the breakdown of the time-reversal symmetry is compensated by a symmetry between photons of opposite helicities propagating in opposite directions, restoring the time-reversal invariance on the level of the effective Hamiltonian [2] for $N \to \infty$. 

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**FIG. 4.** Localization phase diagram of an ensemble of two-level atoms in a strong magnetic field. The frequency on the vertical axis is measured from $\omega_m = \omega_0 + m\Gamma_0 \Delta$ (for $m = \pm 1$) in units of $\Gamma_0$. Symbols are mobility edges calculated from at least $10^7$ ($5.5 \times 10^6$) eigenvalues for $N = 8000$ ($16000$) at every $\rho$; lines are polynomial fits. The crossing point of the latter $\rho_c/k_0^3 \simeq 0.1$ is an estimation of the absolute localization threshold. Inset: the width of the frequency band of localized states (circles) obeys Eq. (7) (dashed line).
High atomic number densities needed for the observation of Anderson transition discussed in this Letter are attainable experimentally with atomic species providing manageable $J_g = 0 \rightarrow J_e = 1$ transitions, e.g., Yb [29]. Sr atoms can also be used [11, 30] as well as “artificial atoms” (quantum dots) which have an advantage of not suffering from the Doppler and recoil effects inherent for atomic systems [18].

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[1] S. John, Electromagnetic absorption in a disordered medium near a photon mobility edge, Phys. Rev. Lett. 53, 2169–2172 (1984).
[2] P.W. Anderson, The question of classical localization: A theory of white paint? Philos. Mag. B 52, 505–509 (1985).
[3] P.W. Anderson, Absence of diffusion in certain random lattices, Phys. Rev. 109, 1492–1505 (1958).
[4] S. John, Localization of light, Phys. Today 44(5), 32–40 (1991).
[5] T. van der Beek, P. Barthelemy, P.M. Johnson, D.S. Wiersma, and A. Lagendijk, Light transport through disordered layers of dense gallium arsenide submicron particles, Phys. Rev. B 85, 115401 (2012).
[6] T. Sperling, L. Schertel, M. Ackermann, G.J. Aubry, C.M. Aegerter, and G. Maret, Can 3D light localization be reached in ‘white paint’? New J. Phys. 18, 013039 (2016).
[7] S.E. Skipetrov and J.H. Page, Red light for Anderson localization, New J. Phys. 18, 021001 (2016).
[8] R. Kaiser, Cold atoms and multiple scattering, in Diffuse Waves in Complex Media, edited by J.-P. Fouque (Kluwer, Dordrecht, 1999), pp. 249–288.
[9] R. Kaiser, Quantum multiple scattering, J. Mod. Opt. 56, 2082 (2009).
[10] G. Labeyrie, F. de Tomasi, J.-C. Bernard, C. A. Müller, C. Miniatura, and R. Kaiser, Coherent backscattering of light by cold atoms, Phys. Rev. Lett. 83, 5266 (1999).
[11] Y. Bidel, B. Klappauf, J. C. Bernard, D. Delande, G. Labeyrie, C. Miniatura, D. Wilkowski, and R. Kaiser, Coherent light transport in a cold strontium cloud, Phys. Rev. Lett. 88, 203902 (2002).
[12] S.E. Skipetrov and I.M. Sokolov, Absence of Anderson localization of light in a random ensemble of point scatterers, Phys. Rev. Lett. 112, 023905 (2014).
[13] L. Bellando, A. Gero, E. Akkermans, and R. Kaiser, Cooperative effects and disorder: A scaling analysis of the spectrum of the effective atomic Hamiltonian, Phys. Rev. A 90, 063822 (2014).
[14] S.E. Skipetrov and I.M. Sokolov, Magnetic-field-driven localization of light in a cold-atom gas, Phys. Rev. Lett. 114, 053902 (2015).
[15] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, Photons and Atoms: Introduction to Quantum Electrodynamics (Wiley, New York, 1992).
[16] O. Morice, Y. Castin, and J. Dalibard, Refractive index of a dilute Bose gas, Phys. Rev. A 51, 3896 (1995).
[17] O. Sigwarth, G. Labeyrie, D. Delande and Ch. Miniatura, Multiple scattering of light in cold atomic clouds in a magnetic field, Phys. Rev. A 88, 033827 (2013).
[18] S.E. Skipetrov, I.M. Sokolov, and M.D. Havey, Control of light trapping in a large atomic system by a static magnetic field, Phys. Rev. A 94, 013825 (2016).
[19] S.E. Skipetrov, Finite-size scaling analysis of localization transition for scalar waves in a three-dimensional ensemble of resonant point scatterers, Phys. Rev. B 94, 064202 (2016).
[20] K. Slevin and T. Ohshima, Critical exponent for the Anderson transition in three-dimensional orthogonal universality class, New J. Phys. 16, 015012 (2014).
[21] F. Evers and A.D. Mirlin, Anderson transitions, Rev. Mod. Phys. 80, 1355 (2008).
[22] B.A. van Tiggelen and R. Maynard, Reciprocity and coherent backscattering of light, in Wave Propagation in Complex Media. The IMA Volumes in Mathematics and its Applications, Papanicolaou G. (Ed.), vol. 96 (Springer, New York, NY, 1998).
[23] K. Slevin and T. Ohshima, The Anderson transition: time reversal symmetry and universality, Phys. Rev. Lett. 78, 4083 (1997).
[24] A.A. Abrikosov, Anderson localization in strongly anisotropic metals, Phys. Rev. B 50, 1415 (1994).
[25] B.C. Kaas, B.A. van Tiggelen, and A. Lagendijk, Anisotropy and interference in wave transport: an analytic theory, Phys. Rev. Lett. 100, 123902 (2008).
[26] F. Milde, R.A. Römer, M. Schreiber, and V. Uski, Critical properties of the metal-insulator transition in anisotropic systems, Eur. Phys. J. B 15, 685 (2000).
[27] M. Lopez, J.-F. Clément, P. Szriftgiser, J.C. Garreau, and Dominique Delande, Experimental test of universality of the Anderson transition, Phys. Rev. Lett. 108, 095701 (2012).
[28] S.E. Skipetrov and I.M. Sokolov, Ioffe-Regel criterion of Anderson localization in the model of resonant point scatterers, Phys. Rev. Lett. 112, 023905 (2014).