Explosive Z Pinch

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Abstract

We propose an explanation for the recently observed powerful contained explosion in a Z pinch experiment performed at Sandia National Laboratories. Our arguments are based on the assumption that a pure SU(2) Yang-Mills theory of scale $\sim 0.5\text{ MeV}$ is responsible for the emergence of the electron and its neutrino.
1 Introduction

Very recently, an unexpected powerful contained explosion was detected in a Z pinch experiment at Sandia National Laboratories. Preceding the explosion, an ion temperature $\sim 300$ keV was reached shortly after stagnation, and the energy radiated away in soft X-rays was observed to be 3 to 4 times the estimated kinetic energy released by the intersection of ions and electrons when accelerated towards the plasma axis in the course of implosion. In [1] a model was proposed which explains the rapid conversion of the magnetic-field energy to thermal energy of ions. The conversion mechanism employs short wavelength $m = 0$ magnetohydrodynamic (MHD) instabilities forming at stagnation which lead to viscous ion (and subsequently electron) heating. In this way, the observed imbalance between the kinetic energy of the implosion and of the energy radiated in soft X-ray emission was addressed.

The purpose of this Letter is to point out a likely reason for the contained explosion observed in high-temperature Z pinches at Sandia [2].

2 $SU(2)_e$ in its confining phase

Our analysis is based on the postulate that the emergence of the electron and its neutrino is due to pure SU(2) gauge dynamics subject to a Yang-Mills scale $\Lambda \sim m_e$. A pure Yang-Mills theory is defined solely in terms of gauge fields; no matter fields occur in the fundamental Lagrangian. Thermodynamically, SU(2) Yang-Mills theory comes in three phases. In order of decreasing temperature there is a deconfining, a preconfining, and a confining phase [3, 4]. Here we are only interested in the confining phase.

The excitations in that phase are single and selfintersecting center-vortex loops [3]. Each intersection point carries one unit of electric charge (each sign is equally likely) and a mass given by $m_e$: The mass spectrum thus is equidistant, $m_n = n \cdot m_e$. Two of the four flux tubes connected to an intersection point exhibit the same flux direction; the other two have the opposite flux. All flux tubes are infinitely thin. In a given flux-tube segment the direction of the flux is twofold degenerate: For a given flux-direction also the oppositely directed flux exists. Moreover, for a given soliton it is possible to go around the entire flux-system along a closed curve. This is why we identify each soliton with a spin-1/2 fermion. In the presence of propagating photons [3, 5, 6] the only stable excitations are the single and the one-fold selfintersecting center-vortex loop. The former is identified with the (electron-) neutrino while the latter represents the electron or the positron. Fig. 1 shows the distinct topologies of selfintersecting center-vortex loops up to intersection number $n = 3$. There is a charge-multiplicity factor $c_{n,k} \geq n + 1$ associated with the $k$th soliton of $n$ selfintersections: Each intersection point may carry charge $+$ or $-$, and we do not take into account any ordering of charges.

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1 Setting $c_{n,k} \equiv n + 1$ for $n > 2$ represents only a lower bound on the actual charge multiplicity.
Figure 1: The topologies of the solitonic excitations for an SU(2) Yang-Mills theory in the confining phase. Only the excitations with up to $n = 3$ selfintersections are depicted.

Modulo the charge multiplicities the number of distinct solitons is given by the number of distinct topologies of connected vacuum diagrams in a $\lambda\phi^4$ quantum field theory. This theory was investigated carefully in [7] and references therein. Up to $n = 6$ the number $W_n$ of topologically distinct and connected vacuum diagrams in a $\lambda\phi^4$-theory is given as follows [7]:

$$W_1 = 1, \quad W_2 = 2, \quad W_3 = 4, \quad W_4 = 10, \quad W_5 = 28, \quad W_6 = 97. \quad (1)$$

Let us proceed by assuming that all solitons are absolutely stable: Long-range interactions between the charges, mediated by photons, are assumed to be switched off and solitons are separated sufficiently such that no contact interactions can occur. The logarithm of the partition function $Z$ for the system of massive spin-1/2 states then takes the following form

$$\log Z = \sum_{n=0}^{\infty} \log Z_n, \quad \log Z_n = M_n V \int \frac{p^2 dp}{2\pi^2} \log \left(1 + e^{-\beta \omega_n(p)}\right) \quad (2)$$

where $\omega_n(p) = \sqrt{p^2 + (n \cdot m_e)^2}$, $V$ denotes the (thermodynamically large) volume of the system, and $\beta \equiv 1/T_e$ is the inverse (electron) temperature. The number $M_n$ represents the total multiplicity of solitons with $n$ selfintersections. It is given as

$$M_n = 2 \times \sum_{k=1}^{W_n} c_{n,k} \geq 2 W_n (n + 1) \quad (3)$$

where the factor 2 takes into account the spin degeneracy. The total pressure $P$ and the total energy density $\rho$ are obtained from $Z$ as

$$P = T \frac{\partial \log Z}{\partial V} = \sum_{n=1}^{\infty} P_n, \quad \rho = T \frac{\partial P}{\partial T} - P = \sum_{n=1}^{\infty} \rho_n \quad (4)$$

For example, at $n = 3$ the second and the third topology in Fig.1 have a charge multiplicity of $c_{3,2} = c_{3,3} = 6$ instead of 4.
Figure 2: Lower bound for the ratio of partial pressure $P_n$ to $T^4$ as a function of $n$ for three different temperatures: $T = 0.65 \, m_e$ (dark grey), $T = 0.7 \, m_e$ (grey), and $T = 0.75 \, m_e$ (light grey).

where $P_n$ and $\rho_n$ refer to the pressure and energy density of particles with mass $m_n = n \cdot m_e$ (solitons with $n$ selfintersections). Explicitly, we have

$$P_n = M_n T \int_0^\infty \frac{p^2 dp}{2\pi^2} \ln[1 + e^{-\beta \omega_n(p)}], \quad \rho_n = M_n \int_0^\infty \frac{p^2 dp}{2\pi^2} \frac{\omega_n(p)}{e^{\beta \omega_n(p)} + 1}. \quad (5)$$

For $n = 1$ we recover the usual expression for an electron-positron gas with chemical potential $\mu = 0$ at temperature $T$. The case $n = 0$ (neutrinos) is not important for the present discussion.

In [8] a lower bound $W_{n,\text{low}}$ for $W_n$ was obtained:

$$W_n > n! \, 3^{-n} \equiv W_{n,\text{low}}. \quad (6)$$

Using $W_{n,\text{low}}$ and setting $c_{n,k} \equiv n + 1$, one finds that $M_n$ is bounded from below by $M_{n,\text{low}} \equiv 2(n+1)! \, 3^{-n}$. We use $M_{n,\text{low}}$ to obtain a lower estimate for the partial pressure $P_n$ at large $n$. As is readily observed from Fig.2 the partial pressure $P_n$ exhibits asymptotic behavior. That is, the sum over $P_n$ seems to converge up to a (temperature dependent) critical value $n_c(T)$ but diverges when including contributions with $n > n_c(T)$. The asymptotic nature of the expansion of $P$ into $\sum_n P_n$ signals that an assumption made in deriving $P$ fails to hold for $n > n_c(T)$. Clearly, the assumption that up to arbitrarily large $n$ the associated solitons are stable and of mass $m_n = n \cdot m_e$ breaks down due to increasingly efficient contact interactions and annihilations. The decay or annihilation of large-$n$ solitons, however, modifies the spectral properties of small-$n$ excitations (increasing mass because of internal vibration and rotation) in such a way that the sum over partial pressures is likely to converge. The situation is somewhat reminiscent of Hilbert’s hotel story where an unpleasant situation occurring at a finite integer $n$ is resolved by pushing this integer to infinity. Namely, to accommodate a naked, small-$n$ excitation in the spectrum (win of energy over entropy in the partition function) it needs to be dressed by
additional internal energy which is released by the decay or annihilation of instable large-$n$ excitations. By induction this pushes $n_c(T)$ to infinity thus curing the apparent divergence of $\sum_n P_n$. For temperatures $T \leq 0.6 m_e$ it will be sufficient to restrict ourselves to $n \leq 6$ where the expansion in terms of $\textit{naked}$ excitations shows asymptotic convergence. A similar behavior is observed for the expansion of the energy density.

3 Explosive Z pinch

Let us first lay out the scenario and point to experimental benchmarks for the Z pinch dynamics observed at Sandia. A wire array is fed with a current of about 20 MA. By resistive heating the wire is transformed into a plasma column. At the same time the current builds up a strong magnetic field whose pressure is directed inwards (towards the plasma axis): The plasma column, whose maximal radius is about 5 mm, implodes until it stabilizes for about 5 ns (stagnation). Shortly before, at, and shortly after stagnation soft X-ray emission is detected allowing for an estimate of the electron temperature $T_e \sim 3$ keV. For the plasma implosion to stagnate the outward directed plasma pressure $P_p$ needs to be equal in magnitude to the inward directed magnetic pressure $P_m$ which is given as

$$P_m = -\frac{1}{2} \frac{I_p^2}{\pi R_s^2}$$

where $I_p$ is the plasma current, and $R_s$ denotes the radius of the plasma column at stagnation. Typically, one has $R_s \sim 1.5$ mm and $I_p \sim 20$ MA. This amounts to $P_m = -1.8 \times 10^{-12}$ MeV$^4$. For the electron density $n_e$ we take $n_e = Z n_i$ with $Z = 26$ and the ion density $n_i = 10^{26}$ m$^{-3}$ \[1\]. If one asserts that the plasma pressure $P_p$ is exclusively carried by electrons then the equilibrium condition $P_p + P_m = 0$ predicts an electron temperature at stagnation of $T_e \sim 31.55$ keV with an electron chemical potential $\mu_e = 11.4$ keV. Notice that $T_e$ is too high by a factor of 8.5 in comparison with the observed value $T_e = 3.6$ keV. Thus we conclude that ions play a substantial role in the pressure balance at stagnation. This point was made very explicit in \[1\].

According to \[1\] the magnetic-field energy prevailing at stagnation is converted into ion heat by $m = 0$ interchange MHD instabilities. This viscous heating mechanism increases the ion temperature to about $T_i \sim 300$ keV very rapidly (time scale set by Alfvén transit time $\tau_A \sim 1 \cdots 2$ ns). The increase of $T_i$ can be measured by an analysis of the Doppler broadening of the X-ray lines emitted by electron capture shortly after stagnation. Since the thermal equilibration time $\tau_e \sim 5$ ns for electrons is significantly larger than $\tau_A$ and since the ion-ion collision time $\tau_{ii}$ is only $\tau_{ii} \sim 37$ ps, $T_i$ will at first greatly exceed $T_e$.

We are interested in what happens once a time $\tau_e \sim 5$ ns has elapsed after stagnation. Ions will then have started to heat the electrons to $T_e \sim T_i \sim 300$ keV at least locally. According to our discussion in the Sec.\[2\] this will involve center-vortex
Figure 3: Plot of the truncated sums over partial pressures up to $N = 6$ (dark grey: $N = 1$; very light grey: $N = 6$) as functions of electron temperature $T_e$.

loops with a higher number of selfintersections. As a consequence, the equilibration time should decrease dramatically as compared to the conventional electron-gas picture: The electron temperature $T_e$ rises very rapidly at about $\tau_e \sim 5\text{ ns}$ after stagnation. In a conventional electron-photon plasma the Debye screening mass $m_D$ is given as

$$m_D \sim \sqrt{\frac{2}{T_e}} e \left( \frac{m_e T_e}{2 \pi} \right)^{3/4} \exp(-m_e/(2 T_e))$$

(8)

where $e = \sqrt{4\pi \alpha} \sim 1/3$ the electromagnetic coupling. This represents a lower bound for the photon's electric screening mass being generated in the plasma discussed in Sec.2. Due to $T_e$ being a sizable fraction of $m_e$ after the ion-induced heating $m_D$ is, according to Eq. (8), comparable to $T_e$. This means that starting at about $5\text{ ns}$ after stagnation no radiation is released by the then absolutely opaque plasma. At the same time, the plasma pressure will increase dramatically due to the presence of a large number of excitations with higher selfintersections.

Let us be more quantitative about this. Define the truncated sums for the pressure and for the energy density in the electronic system as $\bar{P}_N = \sum_{n=1}^{N} P_n$ and $\bar{\rho}_N = \sum_{n=1}^{N} \rho_n$, respectively. For $N = 6$ one is within the regime of asymptotic convergence for electron temperatures $< 0.6 m_e$. To make contact with the Z pinch experiment at Sandia we a priori would need to include a (small) chemical potential $\mu_e$ for preexisting electrons in the expression $\bar{P}_1$. However, we have checked that for $T_e > 30\text{ keV}$ the chemical potential can safely be neglected.

In Figs. 3 and 4 we present plots of $\bar{P}_N$ and $\bar{\rho}_N$ in dependence of $T_e$ for $N = 1, \cdots, 6$. While for temperatures $T_e \ll m_e$ no difference is visible (center-vortex loops with higher mass do practically not contribute) there is a clear enhancement of $\bar{P}_6$ and $\bar{\rho}_6$ with respect to $\bar{P}_1$ and $\bar{\rho}_1$ (contribution of electrons and positrons only) for $T_e \gtrsim 0.1\text{ MeV}$ already. The deviation from $\bar{P}_1$ and $\bar{\rho}_1$ keeps growing with increasing $T_e$. We consider $T_e \sim 0.3\text{ MeV}$ as an upper limit for the validity of our approximation (undressed and stable excitations). Going to higher temperatures would require the precise knowledge of $M_{n>6}$, see Eq. (3), and also would necessitate
Figure 4: Plot of the truncated sums over partial energy densities up to $N = 6$ (dark grey: $N = 1$; very light grey: $N = 6$) as functions of electron temperature $T_e$.

a consideration of finite widths and modified masses of the excited states in the evaluation of the partition function.

At $T_e = 0.25$ MeV we have $\bar{P}_6 \sim 3 \bar{P}_1$ and $\bar{\rho}_6 \sim 4.8 \bar{\rho}_1$, at $T_e = 0.3$ MeV we already have $\bar{P}_6 \sim 5 \bar{P}_1$ and $\bar{\rho}_6 \sim 9.4 \bar{\rho}_1$. Notice that at $T_e = 0.25$ MeV the ratio of $\bar{P}_1$ to the magnetic pressure $P_m$ at stagnation is: $\frac{\bar{P}_1}{P_m} \sim -4.4 \times 10^8$. We propose that $T_e$ reaching a value $\sim T_i \sim 0.3$ MeV is facilitated by the very existence of center-vortex loops with higher mass. This would initiate the final stage in the Z-pinch dynamics leading to explosion.

4 Conclusion

We have proposed a reason for the unexpected contained explosion of a Z pinch recently observed at Sandia National Laboratories. According to our scenario the presence of higher-mass center-vortex loops in the confining phase of SU(2)$_e$ accelerates the transit of thermal energy from ions to electrons and generates a larger pressure and energy density than expected from electron dynamics only. Once the electron temperature reaches a value of about 0.5 MeV a phase transition is expected to take place where all charged states condense into a new ground state (preconfining phase, ground state is superconducting [3]). In that phase the dual gauge boson (identified with the $Z_0$ vector boson of the Standard Model) albeit massive propagates [3, 4, 5].

On a qualitative level and more microscopically, we also predict that an extremely large (charge nonconserving) electron-positron multiplicity in the final state will be detected in hadron collisions at the LHC once the center-of-mass energy substantially exceeds the $Z_0$ mass, see also [5].
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