Research on Monopulse Angle Measurement Algorithm in Pseudolite Local Positioning System

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Abstract. The pseudo-satellite local positioning system can solve the problem of blind spots in satellite navigation systems, such as tunnels, underground parking lots, and indoor positioning. The method of direction-finding positioning can be combined with distance-finding positioning to improve positioning accuracy, and it can also independently realize positioning functions. The basis of direction-finding positioning is to accurately estimate the direction of the signal source. The sum-difference patterns of amplitude monopulse angle measurement algorithm has mature applications in the field of tracking radar, and can achieve accurate angle estimation. The sum-difference patterns of amplitude monopulse angle measurement algorithm is applied to a pseudo-satellite local positioning system. Through simulation experiments, the angle measurement performance of the algorithm under different conditions is analyzed.

1. INTRODUCTION

At present, satellite navigation and positioning systems are widely used, but in special scenarios such as mining areas in the valley, tunnels, and high-rise dense urban canyons, satellite signals cannot be received and accurate positioning services cannot be provided. The emergence of a pseudo-satellite local positioning system provides a possibility to solve the positioning problem in the above scenario. A pseudolite is a device that is arranged on the ground to emit signals similar to GNSS. Pseudolite stations established on the ground can not only enhance regional satellite navigation and positioning systems, but also improve the reliability and anti-interference ability of satellite positioning systems [1].

In the pseudo-satellite local positioning system, in addition to the pseudo-random code ranging or carrier phase ranging positioning method, a method of direction-finding positioning can also be introduced. On the one hand, the positioning function can be completed independently through direction finding, and it can also be combined with ranging positioning to improve positioning performance. Many studies have described angle of arrival (AOA) measurements. The first is using large aperture antennas, such as parabolic or horn antennas with channel detection systems [2]. However, this method has limited angular resolution. The second method is to use an antenna array and combine high-precision angle measurement algorithms, such as MUSIC, ESPRIT, but this requires calibration between each antenna, and the antenna structure is more complicated. Autrey [3] proposed an algorithm for estimating AOA through a passive synthetic aperture array, which is formed by the movement of a single array element. Yen [4] formed a circular passive synthetic array using a single array element. Paper [5] introduced an AOA measurement method using turntable, vector receiver, horn antenna and synthetic aperture processing.

Traditional AOA measurement algorithms, such as MUSIC, ESPRIT, since the essence of the algorithm is to use the phase information difference of the received signals of each antenna to estimate
the parameters. A new problem is introduced, that is, the time synchronization problem of each antenna, which requires accurate time differences between antennas, and it directly affects the performance of the classic AOA algorithm estimation. To avoid the impact of this problem, AOA estimation can be performed using the amplitude information of the signal.

The development of monopulse angle measurement technology in the field of tracking radar is very mature. The monopulse angle measurement algorithms can be divided into amplitude monopulse angle measurement and phase monopulse angle measurement according to different signal parameters. Among them, the amplitude monopulse angle measurement technology is a very mature method of accurate angle measurement. It not only has the advantages of small calculation volume, simple and reliable, but more importantly, the monopulse angle measurement technology has a high data rate and has strong interference ability. It was first used in the field of tracking radar. In one plane, two identical beams partially overlap, and the direction of overlap is the equal signal axis. By comparing the signals received by the two beams at the same time, the angular error signal of the target on this plane can be obtained, and then the error voltage is amplified and transformed and added to the driving motor to control the antenna to move in the direction of reducing error [6]. A monopulse radar system usually uses a monopulse antenna, which directly implements the sum and difference beams of the analog signal through a hardware structure, and then performs beam comparison and angle estimation. With the development of digital signal processing technology, the monopulse angle measurement algorithm can be completed by the processor, so that the angle measurement function can be realized by receiving signals using a common directional antenna.

In this paper, we mainly study the application performance and optimization of pseudo-satellite positioning system based on the amplitude sum-difference patterns of amplitude monopulse angle measurement algorithm. The second part introduces the traditional amplitude sum-difference patterns of amplitude monopulse angle measurement algorithm and analyzes it. The third part analyzes and compares algorithm performance under different conditions.

2. Method

![Figure 1. Sum-Difference Patterns of Amplitude Monopulse Angle Measurement Algorithm.](image)

As shown in the Figure 1, the two beams in the plane partially overlap each other. The deflection angle $\theta_0$ between the center axis of the two beams and the axis of the equal-intensity signal is also known. Assuming that the angle between the angle of arrival of the target transmitted signal and the axis of the equal-intensity signal is $\theta$ and the antenna beam pattern function is $F(\theta)$, the pattern functions of the two beams can be written as

$$F_1(\theta) = F(\theta_0 - \theta)$$
$$F_2(\theta) = F(\theta_0 + \theta)$$

The two received signals can be expressed as
\[ (u_1(\theta) = K_0 F_1(\theta) = K_0 F(\theta_0 - \theta) \]
\[ u_2(\theta) = K_0 F_2(\theta) = K_0 F(\theta_0 + \theta) \]

Among them, \( K_0 \) is the proportionality factor, which is related to factors such as antenna parameters, target distance, and target characteristics. It can be gotten from the above formula:
\[ \frac{u_1(\theta)}{u_2(\theta)} = \frac{f(\theta_0 - \theta)}{f(\theta_0 + \theta)} \]

The intensity ratio of the two received signals is determined by the angle of the two arriving signals relative to the receiving antenna, which is consistent with the conclusion of (6).

The sum \( u_\Sigma(\theta) \) and difference \( u_\Delta(\theta) \) can be calculated from \( u_1(\theta) \) and \( u_2(\theta) \) as follows.
\[ \{u_\Sigma(\theta) = u_1(\theta) + u_2(\theta) = K_0[F(\theta_0 + \theta) + F(\theta_0 - \theta)] \]
\[ \{u_\Delta(\theta) = u_1(\theta) - u_2(\theta) = K_0[F(\theta_0 + \theta) - F(\theta_0 - \theta)] \]

\( F_\Delta(\theta) = F(\theta_0 + \theta) + F(\theta_0 - \theta) \) is the sum pattern, \( F_\Delta(\theta) = F(\theta_0 + \theta) + F(\theta_0 - \theta) \) is the difference pattern. According to the Taylor expansion derivation, when \( \theta \) approaches zero (infinitely close to the central axis of the equal-strength signal, the echo signal has almost no deflection), \( F(\theta_0 + \theta) \) and \( F(\theta_0 - \theta) \) can be approximated as
\[ \{F(\theta_0 + \theta) = F(\theta_0) + F'(\theta_0)\theta + o(\theta^2) \approx F(\theta_0) + F'(\theta_0)\theta \]
\[ \{F(\theta_0 - \theta) = F(\theta_0) - F'(\theta_0)\theta + o(\theta^2) \approx F(\theta_0) - F'(\theta_0)\theta \]

Further derivation can be obtained:
\[ \{u_\Sigma(\theta) \approx 2K_0 F(\theta_0) \]
\[ \{u_\Delta(\theta) \approx 2K_0 F'(\theta_0)\theta \]

Finally, compare the amplitude of the sum signal with the amplitude of the difference signal:
\[ \frac{u_\Delta(\theta)}{u_\Sigma(\theta) \approx \frac{F'(\theta_0)}{F(\theta_0)} \theta = \rho \theta} \]

\( \rho \) is the normalized slope coefficient of the antenna pattern at the beam deflection angle \( \theta_0 \).

The deflection angle of the target echo signal can be calculated as:
\[ \theta = \frac{u_\Delta(\theta)}{u_\Sigma(\theta)} \rho \]

3. Performance Evaluation
A single carrier signal is used as the transmission signal, and its expression is
\[ T(t) = \sin(2\pi t) \]

Use a Gaussian antenna as the receiving antenna, its pattern function can be expressed as:
\[ f(\theta) = e^{-\frac{\theta_0^2}{\theta_0^2}} \]

\( \theta_{0.5} \) represents the half-power beam width of the antenna, which is determined by the structure of the antenna and the wavelength of the signal. Figure 2 shows the Gaussian antenna pattern when \( \theta_{0.5} = 10^\circ, 20^\circ, 30^\circ \).
3.1 Effect of Beam Width on Algorithm Performance

According to the analysis in Part 3, when using the sum-difference patterns of amplitude monopulse angle measurement algorithm, the antenna beam pattern is the basis and key of the algorithm, which directly affects the results and performance of the algorithm. An important indicator is the half-power beam width $\theta_{0.5}$. The following is a simulation experiment to study the performance of the algorithm under different half-power beam widths. In the simulation experiment, the beam angle is set to $20^\circ$.

It can be seen from Figure 3 that when the signal's arrival direction is close to the iso-signal axis direction, that is, the angle bisector of the two antennas, the angle measurement error is small; however, as the signal's arrival direction is away from the iso-signal axis direction, the algorithm error gradually increasing, this is due to the use of Taylor series expansion in the algorithm, which is consistent with the theory.

At the same time, when the beam angle is $2\theta_0 = 20^\circ$, the error curve of the angle measurement result can be obtained. The wider the half-power beam width, the better the algorithm's performance. When $\theta_{0.5} = 30^\circ$, within the range of $[-20^\circ, 20^\circ]$, the error of the estimation result of the algorithm is within $\pm 2^\circ$ and within the range of $[-10^\circ, 10^\circ]$, the error of the estimation result of the algorithm is within...
°1°1. When $\theta_{05} = 10^\circ$, the algorithm is in the range of [-5°, 5°], which can provide better estimation results. However, when the signal direction deviates from the direction of the iso-signal axis, the error of the algorithm's estimation result is large, and it cannot provide an effective angle estimation.

### 3.2 Effect of Beam Angle on Algorithm Performance

In the traditional sum-difference patterns of amplitude monopulse angle measurement algorithm, the deflection angle $\theta_0$ between the central axis of the two beams and the iso-signal axis, that is, the angle $2\theta_0$ between the central axes of the two beams needs to be determined in advance, which is also well understood in practice. This angle is the angle between the two antennas. Next, through simulation experiments, the performance of the angle measurement algorithm under different beam angle conditions are studied. In the simulation experiment, the half-power beam width is 20°.

![Figure 4. Algorithm Performance at Different Beam Angles.](image)

As shown in Figure 4, when the half-power beam width of the receiving antenna is 20°, the angle between the two antennas has a significant impact on the performance of the algorithm. The smaller the angle between the two antennas, the better the algorithm performance. This shows that in actual application, in order to obtain better angle estimation results, the angle between the two antennas should be minimized. However, due to the physical size limitation of the antenna, the angle between the two antenna beams cannot be arbitrarily small, so the antenna should be considered physical size and algorithm performance.

### 3.3 Effect of signal-to-noise ratio on algorithm performance

The above simulation conditions are ideal environments. In the real scene, the received signal is mixed with noise, and the intensity of the noise is measured by the signal-to-noise ratio (SNR). The meaning of SNR is the ratio of useful signal power to noise signal power, which can be expressed as

$$SNR = 10\log_{10} \left( \frac{P_s}{N} \right)$$

$P_s$ is the signal power and $N$ is the noise power. Next, the effects of different SNR on the angle measurement performance are simulated. Since the range of the SNR of the received signal of the ground-based pseudo-satellite positioning system is about [-23dB, -3dB], the cases where the SNR is 0dB, -10dB, and -20dB are simulated. In the simulation experiment, the half-power beam width is 20°, and the beam angle is $2\theta_0 = 20^\circ$. In order to more accurately represent the actual performance of the algorithm, Monte Carlo experiments are performed under the above condition, and the number of
experiments is set to 500. The root mean square error RMSE of the algorithm are compared under different SNR conditions.

![Figure 5. Monte Carlo Results under Different SNR.](image)

It can be seen from the figure 5 that when the signal arrival direction is $10^\circ$, the results of 500 Monte Carlo experiments show that under the condition of relatively poor SNR, the RMSE of the algorithm is about is $1.47^\circ$.

4. Conclusion
The method adopted in this paper is based on the direction finding and positioning in the pseudo-satellite local positioning system. The monopulse angle measurement algorithm used in the field of tracking radar is used in the pseudo-satellite positioning environment. Through theoretical analysis and simulation experiments, the influence of half-power beam width, antenna beam angle, and different SNR on the performance of the algorithm are studied separately, and it has reference significance for practical applications.

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