Cylindrical and spherical dust-acoustic shock waves in a strongly coupled dusty plasma

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Abstract. Cylindrical and spherical dust-acoustic (DA) shock waves propagating in a strongly coupled dusty plasma are theoretically investigated. The generalized hydrodynamic model, in which highly charged dust are treated as strongly coupled, but electrons and ions are treated as weakly coupled, is employed. The modified Burgers equation, which is derived by using the reductive perturbation technique, is numerically solved to examine the effects of nonplanar cylindrical and spherical geometries on the basic features of the DA shock waves that are formed due to a strong correlation among highly negatively charged dust. It is shown that the effects of nonplanar cylindrical and spherical geometries significantly modify the basic properties of the DA shock structures. The implications of our results in laboratory dusty plasma experiments are briefly discussed.

Dust-acoustic (DA) waves, which were discovered by Rao et al [1] about two decades ago, are now found to be very common in both space and laboratory devices [1]–[7]. The linear properties of the DA waves, in which the inertia is provided by the dust particle mass and the restoring force, comes from the pressures of inertialess electrons and ions, are now understood from both theoretical and experimental points of view [1]–[14]. Nowadays, there is a great deal of interest in nonlinear DA waves because of their vital role in understanding the properties of the localized electrostatic perturbations in space and laboratory dusty plasmas [14]–[20]. The nonlinear DA waves have been investigated theoretically [1], [21]–[27], as well as experimentally [28]–[35] over the last few years. However, most of these theoretical works [1], [21]–[27] are limited to a weakly coupled dusty plasma, where the coupling parameter \( \Gamma = \Gamma_0 \exp(-\kappa) \), in which \( \Gamma_0 = z_d^2e^2/a_{d0}T_d \), \( \kappa = a_d/\lambda_D \), \( z_d \) is the number of electrons residing on the dust grain surface, \( e \) is the magnitude of the electron charge, \( a_d \approx n_{d0}^{-1/3} \) is the

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average inter-grain distance, $T_d$ is the dust temperature in units of the Boltzmann constant, $\lambda_d$ is the dusty plasma Debye radius, and $n_{d0}$ is the equilibrium dust number density) is much less than one ($\Gamma \ll 1$). It is well established that in most laboratory dusty plasmas (e.g. in rf and dc dusty plasma discharges), due to the high dust charge and the low dust temperature, the coupling parameter $\Gamma$ can be comparable to unity, or it can even be much larger than unity. It has been demonstrated by theoretical studies [36, 37] and laboratory experiments [38–40], that a dusty plasma (with few micron sized negatively charged dust grains) can readily go into a strongly coupled regime, and that under certain conditions on the value of $\Gamma$, the charged dust grains may organize themselves into dust crystal structures. It has also been observed by further laboratory experiments [41] that, due to the heating of dust gains, the dust crystals first melt and then vaporize, leading to the phase transitions (from solid to liquid, and then from liquid to gas). Motivated by these laboratory experiments [38–40], the dependence of $\Gamma$ on $\kappa$, and the condition (minimum value of $\Gamma$) for fluid–solid phase transition in Yukawa systems (dusty plasmas) have been theoretically investigated by a number of authors, e.g. Vaulina and Khrapak [42], Vaulina et al [43], etc. These laboratory dusty plasma experiments [38–41] and theoretical studies [36, 37, 42, 43], therefore, lead to the formation of various dusty plasma states ranging from the gaseous plasma to a liquid plasma, and from a liquid plasma to plasma crystals [38–41], [44] or vice versa [41–43], and provide an excellent opportunity not only for investigating the phase transitions of interest, but also for the study of the linear and nonlinear features of the DA and dust lattice waves.

Recently, DA shock waves in a strongly coupled dusty plasma have received much attention for understanding localized nonlinear structures in space and laboratory devices, and their role in melting the dust crystals. A number of theoretical investigations [45–48] have been made on the DA shock waves in a strongly coupled dusty plasma. However, most of the theoretical works on the DA shock waves [45–48] are based on one-dimensional (1D) planar geometry which may not be a realistic situation for laboratory devices, since the DA shock waves that may be observed in low-temperature dusty plasma discharges may not necessarily be bounded in one space dimension. Therefore, in this paper, we present an investigation of the nonplanar cylindrical and spherical DA shock waves in a strongly coupled dusty plasma. It was found that the strong dust correlation is a source of dissipation, and is responsible for the formation of cylindrical and spherical DA shock waves. The latter are found to be significantly different from those of the 1D planar DA shock waves.

We consider the nonlinear propagation of low-frequency nonplanar DA waves in an unmagnetized strongly coupled dusty plasma composed of strongly coupled negatively charged dust grains, and weakly coupled electrons and ions following the Boltzmann distribution. At equilibrium, we have $z_d n_{d0} + n_{e0} = n_{d0}$, where $n_{e0}$ ($n_{d0}$) is the equilibrium electron (ion) number density. The nonlinear dynamics of the low phase velocity (in comparison with the electron and ion thermal speeds) nonplanar DA waves propagating in our strongly coupled dusty plasma is governed by [49]

$$\frac{\partial n_d}{\partial t} + \frac{1}{r^v} \frac{\partial}{\partial r} (r^v n_d u_d) = 0,$$

$$D_t \left( m_d n_d D_t u_d - z_d e n_d \frac{\partial \phi}{\partial r} \right) = \frac{\eta}{r^v} \frac{\partial}{\partial r} \left( r^v \frac{\partial u_d}{\partial r} \right) + \left( \zeta + \frac{\eta}{3} \right) \frac{\partial}{\partial r} \left[ \frac{1}{r^v} \frac{\partial}{\partial r} (r^v u_d) \right],$$

$$\frac{1}{r^v} \frac{\partial}{\partial r} \left( r^v \frac{\partial \phi}{\partial r} \right) = 4\pi e \left[ n_{e0} \exp \left( \frac{e \phi}{T_e} \right) - n_{d0} \exp \left( -\frac{e \phi}{T_i} \right) + z_d n_d \right].$$

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where \( v = 0 \) for a 1D planar geometry and \( v = 1(2) \) for a nonplanar cylindrical (spherical) geometry, \( n_d \) is the dust number density, \( u_d \) is the dust fluid speed, \( \phi \) is the electrostatic wave potential, \( t(r) \) is the time (nonplanar space) variable, \( T_e(T_i) \) is the electron (ion) temperature in units of the Boltzmann constant, \( m_d \) is the dust grain mass, \( D_r = 1 + \tau_m \partial / \partial t, \) \( D_r = \partial / \partial t + u_d \partial / \partial r, \) \( \tau_m \) is the viscoelastic relaxation time, \( \eta(\zeta) \) is the bulk (shear) viscosity coefficient. There are various approaches for calculating these transport coefficients. These have been widely discussed in literature [49]–[52]. We note that in (2) we have neglected the dust thermal pressure term whose effect has been found to be insignificant [47] in the strongly coupled regime, since \( T_d / z_d T_i \ll 1 \).

To study cylindrical and spherical DA shock waves in a strongly coupled dusty plasma (described by (1)–(3)) by the reductive perturbation technique [53, 54], we first re-scale the stretched coordinates of Maxon and Vicelli [54] as

\[
R = -\epsilon (r + V_p t), \quad T = \epsilon^2 t,
\]

where \( \epsilon \) is a smallness parameter measuring the weakness of the dispersion, and \( V_p \) is the phase speed of the DA waves, and expand the variables \( n_d, u_d, \) and \( \phi \) about the unperturbed states in power series of \( \epsilon \), namely

\[
n_d = n_{d0} + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + \cdots,
\]

\[
u_d = \epsilon u_d^{(1)} + \epsilon^2 u_d^{(2)} + \cdots,
\]

\[
\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots.
\]

The stretched coordinates defined by (4) mean that we are interested to investigate the ingoing solutions [54] of (1)–(3).

Now, using (4)–(7) in (1)–(3), one can easily develop different sets of equations in various powers of \( \epsilon \). To the lowest order in \( \epsilon \), one can obtain a set of coupled equations for \( n_d^{(1)}, u_d^{(1)} \) and \( \phi^{(1)} \). The latter can be solved to obtain

\[
n_d^{(1)} = n_{d0} u_d^{(1)} / V_p = -n_{d0} z_d e \phi^{(1)} / m_d V_p^2,
\]

\[
V_p / C_d = \sqrt{\alpha - 1 / \alpha + \beta},
\]

where \( C_d = (z_d T_i / m_d)^{1/2}, \) \( \alpha = n_{d0} / n_{e0} \) and \( \beta = T_i / T_e \). Equation (9) represents the linear dispersion relation for the DA waves.

To the next-order in \( \epsilon \), one obtains another set of coupled equations for \( n_d^{(2)}, u_d^{(2)} \) and \( \phi^{(2)} \), which along with (8) and (9), can be reduced to a nonlinear dynamical equation

\[
\frac{\partial N}{\partial \tau} + \frac{\nu}{2\tau} N + AN \frac{\partial N}{\partial \xi} = C \frac{\partial^2 N}{\partial \xi^2},
\]

where \( N = n_d^{(1)} / n_{d0}, \) \( \tau = T \omega_{pd}, \) \( \xi = R / \lambda_{Dm}, \) \( \omega_{pd} = (4\pi n_{d0} z_d e^2 / m_d)^{1/2} \) and \( \lambda_{Dm} = (T_i / 4\pi z_d n_{d0} e^2)^{1/2} \). The nonlinear and dissipative coefficients \( A \) and \( C \) are, respectively,

\[
A = \left( \frac{\alpha - 1}{\alpha + \beta} \right)^{1/2} \left[ 1 + \frac{\alpha}{2} \left( \frac{1 + \beta}{\alpha + \beta} \right)^2 \right],
\]

\[
\lambda_{Dm} = (T_i / 4\pi z_d n_{d0} e^2)^{1/2}.
\]
The charge and the mass of the dust grain used in our numerical calculation correspond to a density compression (the shock height) is 10–20%, while the shock thickness is a fraction. Typical values of \( \eta_s = \eta / (m_d n_d \omega_d \alpha_d^2) \) is the normalized longitudinal viscosity coefficient. Typical values of \( \eta_s \) are [49, 50] \( \sim 1.04 \) for \( \Gamma = 1 \), \( \sim 0.08 \) for \( \Gamma = 10 \) and \( \sim 0.3 \) for \( \Gamma = 160 \).

Equation (10) is the Burgers equation modified by an additional term \((\nu/2\tau)N\) arising due to the effect of the nonplanar cylindrical \((\nu = 1)\) or spherical \((\nu = 2)\) geometry. An exact analytic solution of (10) is not possible. However, for 1D planar geometry \((\nu = 0)\) an exact stationary shock wave solution of (10) is possible, and is given by [55]

\[
N(\nu = 0) = \frac{N_m}{2} \left[ 1 - \tanh \left( \frac{\xi - U_0 \tau}{\Delta} \right) \right],
\]

where \( N_m = 2U_0/A \) is the shock height, \( \Delta = 2C/U_0 \) is the shock thickness, and \( U_0 \) is the shock wave speed (relative to the DA wave phase speed) normalized by \( C_d \). It is obvious from (11)–(13) that the formation of the DA shock wave is due to the strong correlation among negatively charged dust grains, and that such a DA shock wave is associated with the compression of the dust number density. The latter means that the dust particles are compressed, and as a result the dust number density increases. The basic features (the variation of the shock height and the shock thickness against different dusty plasma parameters) of 1D planar DA shock structures under different dusty plasma conditions have been studied elsewhere [45]–[48]. Our main interest here is to examine the effects of the nonplanar (cylindrical or spherical) geometry on these DA shock structures for typical laboratory dusty plasma conditions: \( n_d = 10^9 \text{ cm}^{-3}, \quad n_0 = 10^5 \text{ cm}^{-3}, \quad z_d = 3.78 \times 10^3, \quad T_e = 1 \text{ eV}, \quad T_i = 0.1 \text{ eV}, \quad T_d = 0.03 \text{ eV}, \quad m_d = 10^{-12} \text{ g} \) which correspond to \( \alpha \simeq 1.6, \beta = 0.1, \sigma_d \simeq 8 \times 10^{-5}, \Gamma \simeq 160, \eta_s = 0.3, \quad C_d = 24 \text{ cm s}^{-1}, \quad V_p = 14 \text{ cm s}^{-1}, \quad \lambda_{Dm} \sim 0.01 \text{ cm}, \quad a_d = 0.02 \text{ cm} \) and \( f = 1.78 \). We note that the nonlinear and dissipative coefficients \((A \text{ and } C)\) do not contain dust grain radius \( r_d \) explicitly. However, the charge and the mass of a dust grain are dependent on its size. The charge and the mass of the dust grain used in our numerical calculation correspond to a few micron sized dust grain, i.e. \( r_d \simeq 3 \mu \text{m} \).

These dusty plasma parameters allow us to approximate \( N_m \simeq 0.2 \) and \( \Delta \simeq 47 \) (for \( U_0 = 0.02 \)) or \( N_m \simeq 0.1 \) and \( \Delta \simeq 94 \) (for \( U_0 = 0.01 \)). This means that the maximum dust number density compression (the shock height) is 10–20\%, while the shock thickness is a fraction (\( \sim 47\lambda_{Dm} \) to \( \sim 94\lambda_{Dm} \)) of a centimeter (since \( \lambda_{Dm} \simeq 0.1 \)).

For the above-mentioned dusty plasma parameters, we have numerically solved (10), and have studied the effects of the cylindrical \((\nu = 1)\) and spherical \((\nu = 2)\) geometries on the time-dependent DA shock structures. Since for a large value of \( \tau \), the term \((\nu/2\tau)N\) is negligible, we start with a large (negative) value of \( \tau \) (namely \( \tau = -20 \)), and at this large value we choose (13) (which is the stationary solution of (10) without the term \((\nu/2\tau)N\) as our initial pulse. The results are depicted in figures 1 and 2. Figure 1 (figure 2) shows how the effect of the cylindrical (spherical) geometry modifies the DA shock structures. The numerical solutions of (10) (shown in figures 1 and 2) reveal that for a large value of \( \tau \) (e.g. \( \tau = -20 \)) the spherical and cylindrical shock waves are similar to the 1D planar ones. This is because for a large value of \( \tau \), the term \((\nu/2\tau)N\), which is due to the effect of the cylindrical \((\nu = 1)\) or spherical \((\nu = 1)\) geometry, is no longer dominant. However, as the value of \( \tau \) decreases, the term \((\nu/2\tau)N\) becomes dominant, and both the cylindrical and spherical DA shock structures significantly differ from the 1D planar DA shock wave. It is found that as the value of \( \tau \) decreases, the amplitude of these

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localized pulses increases. It is also found that the amplitude of cylindrical shock structures is larger than that of the 1D planar shock wave, but smaller than that of the spherical one.

To summarize, we have investigated the cylindrical and spherical DA shock waves in an unmagnetized dusty plasma composed of strongly coupled charged dust grains and weakly coupled electrons and ions. The number density of the latter follows the Boltzmann law. The dynamics of the cylindrical and spherical DA shock waves is governed by the modified Burgers equation (mBE). We have found that the effect of the strong correlations among highly charged dust grains is a source of dissipation (right-hand side of the mBE), and is responsible for the formation of the DA shock structures. The time evolution of the cylindrical and spherical DA

\[ n_i = 10^9 \text{ cm}^{-3}, \quad n_d = 10^5 \text{ cm}^{-3}, \quad z_d = 3.78 \times 10^3, \quad T_e = 1 \text{ eV}, \quad T_i = 0.1 \text{ eV}, \quad U_0 = 0.01. \]

\[ \text{We choose our initial pulse (in the form given by (13)) at } \tau = -20, \]

\[ \text{and show how its amplitude increases with decreasing the value of } \tau. \]
shock waves significantly differs from the 1D planar DA shock wave, and as time elapses, the amplitudes of the cylindrical and spherical DA shock waves increase. The amplitude of the cylindrical DA shock wave is larger than that of the 1D planar one, but smaller than that of the spherical DA shock wave. We hope that the results reported here may help us to understand the salient features of the cylindrical and spherical DA shock waves due to a balance between the nonlinearity and dissipation arising from the strong dust grain correlations. It may be added here that the effect of an external magnetic field on the cylindrical and spherical DA shock waves, which will allow the DA shock wave propagation oblique to the external magnetic field direction, may also be relevant to the nonlinear wave phenomena in laboratory dusty plasma experiments in the presence of powerful magnets.

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