Complex CKM matrix, spontaneous CP violation and generalized \( \mu-\tau \) symmetry

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Abstract

The multi-Higgs models having spontaneous CP violation (SPCV) and natural flavor conservation (NFC) lead to a real CKM matrix \( V \) contradicting current evidence in favour of a complex \( V \). This contradiction can be removed by using a generalized \( \mu-\tau \) (called 23) symmetry in place of the discrete symmetry conventionally used to obtain NFC. If 23 symmetry is exact then the Higgs induced flavour changing neutral currents (FCNC) vanish as in case of NFC. 23 breaking introduces SPCV, a phase in \( V \) and suppressed FCNC among quarks. The FCNC couplings \( F_{ij}^{d,u} \) between \( i \) and \( j \) generations show a hierarchy \( |F_{12}^{d,u}| < |F_{13}^{d,u}| < |F_{23}^{d,u}| \) with the result that the FCNC can have observable consequences in \( B \) mixing without conflicting with the \( K^0 - \bar{K}^0 \) mixing. Detailed fits to the quark masses and the CKM matrix are used to obtain the (complex) couplings \( F_{ij}^d \) and \( F_{ij}^u \). Combined constraints from flavour and CP violations in the \( K, B_d, B_s, D \) mesons are analyzed within the model. They allow (i) relatively light Higgs, 100-150 GeV (ii) measurable extra contributions to the magnitudes and phases of the \( B^0_{d,s} - \bar{B}^0_{d,s} \) mixing amplitudes and (iii) the \( D^0 - \bar{D}^0 \) mixing at the current sensitivity level.

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While the exact source of the observed CP violation is still unknown, it is now clear [1] that
the Cabibbo Kobayashi Maskawa (CKM) matrix $V$ describing the charged weak interactions
is complex. This follows from determination [2] of the angle $\gamma = -\text{Arg}(V_{ud}V_{cb}V_{cd}^*V_{ub}^*)$ using
the tree decays of the type $B \to D K$ which provide a clear evidence [1], for a complex $V$
even if one allows for arbitrary new physics (NP) contribution to the loop induced processes
in the standard model (SM). Explicit CP violation in SM automatically makes $V$ complex.
In contrast, the standard theories of spontaneous CP violation (SCPV) at the electroweak
scale [3] give a real CKM matrix. These theories contain two or more Higgs doublets and
lead to the Higgs induced flavour changing neutral current (FCNC). Discrete symmetry
conventionally imposed [3] in order to eliminate them automatically leads [4] to a real
$V$. Alternative possibility is not to eliminate FCNC through a discrete symmetry but suppress
them with a very heavy Higgs. This possibility also leads to a suppressed CP phase and
hence effectively a real CKM matrix if SCPV occurs at the electroweak scale [5]. In either
case, theories of SCPV need modifications in order to accommodate a complex CKM matrix.
We propose here one such modification which also implies verifiable rich phenomenology.

Instead of eliminating FCNC altogether, we use a discrete symmetry to selectively sup-
press them as was done in the past [6]. Such selective suppression [7] may even be needed as
there are several arguments favoring new physics contributions in the $B-\bar{B}$ mixing [8, 9, 10]
but not much in the $K$ system. Discrete symmetry we use is a generalization of the well-
known $\mu$-$\tau$ symmetry [11] to the quark sector. This symmetry, when softly b roken (1) leads
to spontaneous CP violation (2) can explain quark masses and entire complex CKM matrix
(3) gives rise to hierarchical FCNC with observable consequences in the $B-\bar{B}$ mixing.

Consider the SU(2) $\otimes$ U(1) model with two Higgs doublets $\phi_a$, ($a = 1, 2$) and a gener-
alized $\mu$-$\tau$ symmetry (to be called 23 symmetry) acting on fermion fields as
$f_2 \leftrightarrow f_3$ with $\phi_2 \to -\phi_2$. The Yukawa couplings are
\begin{equation}
-L = \bar{d}_L \Gamma^d_1 \phi^0_1 d_R + \bar{u}_L \Gamma^u_1 \phi^0_1 u_R + \text{H.c.},
\end{equation}
CP invariance makes $\Gamma^u,d_a$ real. Imposition of CP and the 23 invariance on the scalar potential
results in a CP conserving minimum. We achieve SCPV here by allowing soft breaking of 23
symmetry in the Higgs potential through a term $\mu_{12} \phi_1^\dagger \phi_2$ whose presence along with other
23 invariant terms violates CP spontaneously [12]. Without lose of generality we can assume
$\langle \phi^0_1 \rangle = v_1$ and $\langle \phi^0_2 \rangle = v_2 e^{i\alpha}$. This leads to the the quark mass matrices
$M^q (q= u,d)$:
\begin{equation}
M^d,u = \Gamma^d_{1} v_1^q + \Gamma^d_{2} v_2 e^{\pm i\alpha},
\end{equation}
with
\begin{equation}
\Gamma^d_{1} v_1 = \begin{pmatrix} X^q & A^q & A^q \\ A^q & B^q & C^q \end{pmatrix}, \Gamma^d_{2} v_2 = \begin{pmatrix} 0 & -A^q e_1^q & A^q e_2^q \\ -A^q e_1^q & C^q & B^q e_2^q \end{pmatrix}.
\end{equation}
In addition to imposing the 23 symmetry, we have also assumed that $M^q$ are symmetric as
would be the case in $SO(10)$ with appropriate Higgs representations.

Eqs. (2) and (3) give the following key features of the model
• The phase $\alpha$ in $M^q$ cannot be rotated away and leads to a complex $V$. This can be seen by considering Jarlskog invariant $Det[M^u M^{u\dagger}, M^d M^{d\dagger}]$ which is found to be non-zero as long as even one of $M^q$ is complex, i.e. $\epsilon_{1,2}^q \neq 0$ for $(q = u$ or $d)$. Thus unlike earlier models $[3]$, a complex CKM originates here from SCPV.

• An approximate $23$ symmetry ($|\epsilon_{1,2}^q| \ll 1$) can explain $[13]$ the quark masses and mixing. $V_{ub}$ and $V_{cb}$ vanish in the symmetric limit and quark masses and the Cabibbo angle can be reproduced with the hierarchy $[13]$:

$$|X^q| \ll |\sqrt{2}|A^q| \ll |B^q| \sim |C^q| \approx m_3^q / 2.$$  (4)

Small but non-zero $\epsilon_{1,2}^q$ generate $V_{cb}$, $V_{ub}$ and CP violation. Let

$$V^q_L M^q V^q_R = D^q,$$  (5)

where $D^q$ is the diagonal mass matrix with real and positive masses. The CKM matrix is given by $V \equiv V^u_L V^d_L$ and phases in $V^q_{LR}$ are chosen in such a way that $V$ has the standard form advocated in $[14]$. In the simplified case of CP conservation ($\alpha = 0$) and $\epsilon_1^q = 0$ one finds $[13]$

$$V_{L}^q = V_{R}^q = R_{23}(\pi/4) R_{23}(\theta_{23}^q) R_{13}(\theta_{13}^q) R_{12}(\theta_{12}^q),$$  (6)

with

$$\theta_{23}^q \approx -\epsilon_2^q / 2, \theta_{12}^q \approx \sqrt{-m_1^q / m_2^q}, \theta_{13}^q \approx m_2^q \theta_{12}^q \theta_{23}^q.$$  (7)

giving $V_{cb} \approx \epsilon_2^q - \epsilon_2^q / 2$, $V_{ub} \approx \epsilon_{12a} V_{cb}$. Thus $23$ breaking through $\epsilon_2^q$ not only generates $V_{cb}$ and $V_{ub}$ but also leads to relative hierarchy between them.

• Like other $2$ Higgs doublet models, eq. $[14]$ generates FCNC but they are linked here to $23$ breaking which also generates $V_{ub}, V_{cb}$. Both remain small if $23$ breaking is small. Eqs. $[13]$ can be manipulated to obtain

$$-\mathcal{L}_{FCNC} = \frac{(2\sqrt{2}G_F)^{1/2} m_b^q}{\sin \theta \cos \theta} F_{ij}^d \bar{d}_i L d_j R \phi_H + \text{H.C.},$$  (8)

where $\phi_H \equiv \cos \theta \phi_2^0 e^{-i\alpha} - \sin \theta \phi_1^0, \tan \theta = v_2 / v_1$ and

$$m_b F_{ij}^d \equiv (V^d_L \Gamma_{ij}^d v_2 e^{i\alpha} V^d_R)_{ij},$$  (9)

and we have introduced the physical third generation quark mass $m_b$ as an overall normalization to make $F_{ij}^d$ dimensionless. Analogous expressions hold in case of the up quarks. Eqs. $[13]$ and $[7]$ are used to show that

$$F_{12}^d \approx \frac{1}{2} \epsilon_2^d (\theta_{13}^d + 2 \theta_{12}^d \theta_{23}^d) \approx \pm 6.0 \times 10^{-4},$$

$$F_{13}^d \approx \frac{1}{2} \epsilon_2^d \theta_{12}^d \approx \pm 7.0 \times 10^{-3},$$

$$F_{23}^d \approx \frac{1}{2} \epsilon_2^d \approx \pm 4.0 \times 10^{-2}.$$  (10)
where the quoted numerical values follow from the approximate eqs.(7) by choosing $\epsilon_0^q \approx 2V_{cb}$. It is seen that $F^d_{ij}$ are suppressed if 23 symmetry is mildly broken, i.e. $|\epsilon_1,2| \ll 1$. Independent of this, they follow a hierarchy

$$|F^d_{12}| \ll |F^d_{13}| \ll |F^d_{23}|.$$  

(11)

Similar hierarchy holds in case of the up quarks also. This hierarchy is remarkable. The FCNC are suppressed most in the $K$ system where strong constraints on their existence already exist. In contrast, the flavour changing effects in the $B$ system can be more pronounced.

The strength and hierarchy of $F^q_{ij}$ can be probed through flavour changing transitions, particularly through $P^0 - \bar{P}^0$, ($P = K, B_d, B_s, D$) mixing. This mixing is generated in the SM at 1-loop level and thus can become comparable to the tree level FC effects in spite of the suppression in $F^q_{ij}$. $P^0 - \bar{P}^0$ mixing is induced by the element $M^P_{ij} \equiv \langle P^0|H_{eff}|\bar{P}^0 \rangle$. The effective Hamiltonian here contains two terms $H_{eff}^{SM} + H_{eff}^H$ where the second term is induced from the Higgs exchange. The $H_{eff}^H$ follows from eq.(8) in a straightforward manner:

$$H^H_{eff}(ij) = -\frac{2\sqrt{2}G_F m_0^2}{\sin^2 2\theta M^2_{\alpha} (F^d_{ij} C_\alpha^2 (\bar{d}_{iL} d_{jR})^2 + F^d_{ji} C_\alpha^2 (\bar{d}_{iR} d_{jL})^2 + 2F^d_{ij} F^d_{ji} C_\alpha^2 (\bar{d}_{iL} d_{jR}) (\bar{d}_{iR} d_{jL}))},$$  

(12)

where $ij = 12, 13, 23$ respectively denote $H_{eff}^H$ for the $K, B_d, B_s$ mesons. The model contains three real Higgs fields $H_\alpha$ whose masses $M_\alpha$ appear above. The real and imaginary parts of $\sqrt{2}\phi_H \equiv R + i I$ in eq.(8) are related to $H_\alpha$ through a $3 \times 3$ orthogonal matrix $O$ and one can write $\sqrt{2}\phi_H = (O_{R\alpha} + iO_{I\alpha})H_\alpha \equiv C_\alpha H_{\alpha}$ which defines the complex parameters $C_\alpha$ appearing in eq.(12).

Define $F^d_{ij} \equiv |F^d_{ij}| e^{i\delta_{ij}}, C_\alpha \equiv |C_\alpha| e^{i\eta_\alpha}$. Then using $|F^d_{ij}| = |F^q_{ij}|$ (following from symmetry of $M^q$) and the vacuum saturation approximation we obtain the Higgs contribution to $M_{12}(P)$ from eq.(12):

$$M^H_{12}(P) = \frac{\sqrt{2}G_F m_0^2 f^2 |M_P| C_\alpha^2 |F^d_{ij}|^2}{6 \sin^2 2\theta M^2_{\alpha}} Q_{ij} e^{i(s_{ij} - s_{ij})}$$  

(13)

with

$$Q_{ij} = \left[ A_P - 1 + 10 A_P \sin^2 (\frac{s_{ij} + s_{ij}}{2} + \eta_\alpha) \right]$$

and $A_P = \left( \frac{M_P}{m_0 + m_0} \right)^2$, ($P^0 \equiv \bar{ab}$). $\Delta M^H(P) \equiv 2|M^H_{12}(P)|$ following from eq.(13) depends on several unknown parameters in the Higgs sector while its phase is determined by the phases of $F^d_{ij}$ which depend only on parameters in $M^q$. For illustration, we retain the contribution of the lightest Higgs $\alpha \equiv H$ in eq.(13) and choose $M_H = 150 GeV, \sin^2 2\theta = 1, |C_H|^2 = 1/2, Q^P = 1/2Q_{max}$. The numerical values of $F^d_{ij}$ in eq.(10) then give

$$r^P \equiv \left| \frac{\Delta M^H(P)}{\Delta M^{exp}(P)} \right| \approx (0.25, 0.26, 0.11)$$
respectively for $P = B_d, B_s, K$. It follows that effect of the FCNC can be suppressed in the model without fine tuning or without having very heavy Higgs. But they need not be negligible and can imply some new contributions which can be looked for.

The new physics contribution to $M_{12}^{d,B_s} \equiv M_{12}^{d,s}$ has been parameterized in model independent studies \cite{8, 9, 10} by

$$M_{12}^{d,s} = M_{12}^{d,s;SM} (1 + \kappa^{d,s} e^{i\sigma^{d,s}}).$$

Unitarity of $V$, measurements of $|V_{us}|, |V_{cb}|, |V_{ub}|$ and the unarity angle $\gamma$-all through tree level processes have been used to determine the allowed ranges in $\Delta M_{SM}^d = 2|M_{12}^{d,s;SM}|$. The hadronic matrix elements entering $\Delta M_{SM}^d$ are determined using lattice results and we will specifically use predictions based on \cite{15}. The SM predictions along with the experimental determination of $\Delta M_{d,s}$ are used in \cite{10} to obtain

$$\rho^d \equiv \frac{\Delta M^d}{\Delta M_{SM}^d} = 0.97 \pm 0.39, \rho^s \equiv \frac{\Delta M^s}{\Delta M_{SM}^s} = 1.08 \pm 0.19. \quad (14)$$

Possible presence of new physics contribution is hinted by the phase $\phi_d$ of $M_{12}^d: \phi_d = 43.4^\circ \pm 2.5^\circ$ which differs from its value $53.4^\circ \pm 3.8^\circ$ in SM determined \cite{10} using $|V_{ub}|$ measured in inclusive $b \to u\ell\nu$. This implies a non-zero new physics contribution $\phi_d^{NP} = Arg(1 + \kappa_d e^{i\sigma_d}) = -(10.1 \pm 4.6)^\circ$ which in the present case can come from the Higgs exchanges. The values of $\rho_{d,s}$ and $\phi_d$ have been used to determine allowed ranges in the parameters $\kappa$ and $\sigma$. This is displayed in Fig.(1) in case of the $B_d$ mesons. We can confront these observations now with the specific predictions in the present case.

Our strategy is to first determine parameters in $M^q$ from the quark masses and mixing and then use them to determine $F_{ij}^q$ which are used to obtain information on $M_{12}^d(P)$. Since the number of parameters in $M^q$ is more than the observables, we do our analysis in two ways. First, we allow all parameters in $M^q$ to be free and determine them by minimizing $\chi^2$:

$$\chi^2 = \sum_{i=1,10} \left( \frac{E_i(x) - \bar{E}_i}{\delta E_i} \right)^2,$$

where $E_i(x)$ represent predictions of six quark masses, three moduli $|V_{us}|, |V_{cb}|, |V_{ub}|$ and the Jarlskog invariant $J$ calculated as functions of parameters of $M^q$. The quantities $\bar{E}_i \pm \delta E_i$ are their values determined from experiments. We choose quark masses at $M_Z$ given in \cite{16} and all the CKM elements except $|V_{ub}|$ as in \cite{14}. For the latter, we use the value $(4.4 \pm 0.3) \cdot 10^{-3}$ based on the determination \cite{10} from the inclusive $b$ decays. We find many solutions giving excellent fits with $\chi^2 \lesssim 10^{-7}$. One specific example is given in the table. The parameters of the table lead to

$$\begin{align*}
F_{12}^d &= (0.26 - 1.19 i) \cdot 10^{-4}, & F_{21}^d &= (0.096 + 1.22 i) \cdot 10^{-4}, \\
F_{13}^d &= -(0.53 + 5.2 i) \cdot 10^{-3}, & F_{31}^d &= -(5.1 + 0.85 i) \cdot 10^{-3}, \\
F_{23}^d &= (0.30 + 1.13 i) \cdot 10^{-2}, & F_{32}^d &= -(1.11 + 0.35 i) \cdot 10^{-2}, \\
F_{12}^s &= -(2.1 + 1.3 i) \cdot 10^{-4}, & F_{21}^s &= -(1.3 + 2.1 i) \cdot 10^{-4}.
\end{align*} \quad (15)$$
| X (GeV) | A (GeV) | B (GeV) | C (GeV) | $\epsilon_1$ | $\epsilon_2$ |
|---------|---------|---------|---------|------------|------------|
| up      | 0.0019  | 0.036   | 90.66   | −0.198     | −0.082     |
| down    | 0.0035  | 0.019   | 1.54    | 0.177      | −0.025     |

TABLE I: An example of fit to the quark masses and mixing angles corresponding to $\chi^2 = 1.4 \cdot 10^{-7}$ and $\alpha = -3.899$.

which are similar to the rough estimates in eq. (10).

The above fits strongly depend on some of the $\epsilon_{1,2}$ being non-zero since if they vanish then $|V_{ub}|, |V_{cb}|$ and CP violation also vanish. However, we could get excellent fits with $|\epsilon_{1,2}| < 0.2$ showing that approximately broken 23 symmetry provides a very good description of the quark spectrum. In an alternative analysis, we fixed $B^q, C^q$ from $|B^q - C^q| = m_{3q}, |B^q + C^q| = m_{2q}$ which correspond to 23 symmetric limit in the two generation case. This limit is found to be quite good and gives good fits with $\chi^2 \lesssim 1$ when it is minimized with respect to the remaining nine parameters.

The predictivity of the scheme comes from the fact that each set of parameters of $M^q$ determined as above completely fix (complex) FCNC strengths $F_{ij}^{u,d}$ in all 18 independent real quantities. We use these predicted values to calculate Higgs contribution to $M_{12}^P$ by randomly varying unknown parameters of eq.(13). We retain contribution of only one Higgs and vary its mass from 100-500 GeV. $|C_a|, \sin^2 2\theta$ and the phase $\eta_a$ are varied over their full range namely, $0 - 1$ and $0 - 2\pi$ respectively. We neglect the possible corrections to the vacuum saturation approximation but vary $f_{B_{d,s}}$ over the full 1$\sigma$ range: $f_{B_d} = 0.24 \pm 0.04$ GeV, $f_{B_s} = 0.2 \pm 0.025$ GeV. We require that (i) $\rho_{d,s}$ and $\phi_d$ lie in the allowed 1$\sigma$ range (ii) The Higgs contribution to the $D^0 - \bar{D^0}$ mixing amplitude satisfy the bound $|M_{12}^{DH}| < 2.2 \cdot 10^{-14}$ GeV derived in [14] from the BaBar and Belle measurements (iii) the Higgs contribution to the $K^0 - \bar{K^0}$ mass difference and to $\epsilon$ is an order of magnitude less than their central experimental values. Combined results of this analysis for several sets of allowed $F_{ij}^q$ are shown as scattered plot in Fig.(1).

The solid curves describe restrictions on $\kappa_d, \sigma_d$ following from eq.(14) and the measured value of $\phi_d$ in a model independent study. In the present case, the allowed values of $\kappa_d, \sigma_d$ are indirectly effected by restrictions coming from mixing of other mesons as well since the same set of Higgs parameters determine these mixings. Thus simultaneous imposition of the above mentioned constraints considerably restrict the allowed ranges in parameter space shown as scattered plot in Fig.(1). $\sigma_d$ is restricted in such a way that the Higgs contributes negatively to $\rho_d$ (in most parameter space) and reduces the value of $\rho_d$ compared to the SM. $\kappa_d$ and $\sigma_d$ are restricted in the range $0.2 < \kappa_d < 0.46, 185^\circ \lesssim \sigma_d \lesssim 229^\circ$ which correspond to $0.58 \lesssim \rho_d \lesssim 0.9$ and $\phi_{dNP}^s \approx -(5 - 15)^\circ$.

The right panel in Fig.(1) shows the predictions of $\kappa_s$ and possible new physics phase $\phi_{sNP}^s \equiv \text{Arg}(1 + \kappa_s e^{i\sigma_s})$ in $M_{12}^s$. The allowed values of $\kappa_s$ after the combined constraints from all sources are relatively small $\lesssim 0.1$. This also results in a small $\phi_{sNP}^s$ although the Higgs induced CP phase $\sigma_s$ could be large. $\phi_{sNP}^s$ may be approximately identified with
the CP violating phase $\phi_s$ in the semileptonic CP asymmetry of $B_s$ decay. The maximum allowed value $\phi_s^{NP} \approx \phi_s \approx 0.1$ in the model is much larger than the SM contribution ($\phi_s \approx (4.1 \pm 1.4) \cdot 10^{-3}$) and is consistent with the present value $|\phi_s| = 0.70^{+0.47}_{-0.39}$. Possible improvement in the value of $\phi_s$ at LHC would provide a crucial test of the model.

It is found that the $D^0 - \bar{D}^0$ mixing plays an important role in ruling out some of the regions in parameter space and in most of the allowed regions $|M_{12}^{DH}|$ remains close to the limit $2.2 \cdot 10^{-14}$ GeV.

The above considerations used the $|V_{ub}|$ determined from the inclusive $b \rightarrow u \ell \nu$ decay. We have repeated the analysis using the corresponding result $|V_{ub}| = (3.8 \pm 0.6) \cdot 10^{-3}$ from the exclusive decay. We find that regions in parameter space get shifted compared to Fig 1.

In summary, we have addressed the problem [18] of obtaining a phenomenological consistent picture of SCPV. This is an important issue in view of the fact the earlier theories of SCPV led to a real CKM matrix while recent observations need it to be complex. The proposed picture is phenomenologically consistent and does not need very heavy Higgs to suppress FCNC present in general multi Higgs models. The hierarchy in FCNC, eq. [11] obtained here through a discrete symmetry has observable consequences. The effect of Higgs is to reduce the $B_d^0 - \bar{B}_d^0$ mixing amplitude compared to the standard model prediction. The $D^0 - \bar{D}^0$ mixing can be close to the bound derived from observation [17]. The new contribution to the magnitude of $B_s^0 - \bar{B}_s^0$ mixing is small. The Higgs induced phase in this mixing is found to be relatively low but much larger than in the SM.

Noteworthy feature of the proposal is universality of the discrete symmetry used here.
The generalized $\mu$-\(\tau\) symmetry used here can explain large atmospheric mixing angle in the manner proposed in [13] on one hand and can also account for the desirable features of the quark mixing and CP violation as discussed here. Details of a unified description and constraints from other flavour and CP violating observables will be discussed elsewhere.

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