Vacuum polarization by a flat boundary in cosmic string spacetime

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Abstract
In this paper, we analyze the vacuum expectation values of the field squared and the energy–momentum tensor associated with a massive scalar field in a higher dimensional cosmic string spacetime, obeying Dirichlet or Neumann boundary conditions on the surface orthogonal to the string. In order to develop this analysis, the corresponding Green’s function is obtained. The Green’s function is given by the sum of two expressions: the first one corresponds to the standard Green’s function in the boundary-free cosmic string spacetime and the second contribution is induced by the boundary. The boundary-induced parts have opposite signs for Dirichlet and Neumann scalars. Because the analysis of vacuum polarization effects in the boundary-free cosmic string spacetime has been developed in the literature, here we are mainly interested in the calculations of the effects induced by the boundary. The boundary-induced parts have opposite signs for Dirichlet and Neumann scalars. Because the analysis of vacuum polarization effects in the boundary-free cosmic string spacetime has been developed in the literature, here we are mainly interested in the calculations of the effects induced by the boundary. In this way closed expressions for the corresponding expectation values are provided, as well as their asymptotic behavior in different limiting regions being investigated. We show that the non-trivial topology due to the cosmic string enhances the boundary-induced vacuum polarization effects for both the field squared and the energy–momentum tensor, compared to the case of a boundary in Minkowski spacetime. The presence of the cosmic string induces non-zero stress along the direction normal to the boundary. The corresponding vacuum force acting on the boundary is investigated.

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(Some figures in this article are in colour only in the electronic version)
1. Introduction

Cosmic strings are topologically stable gravitational defects which may have been created in the early Universe after Planck’s time by a vacuum phase transition [1, 2]. The gravitational field produced by a cosmic string may be approximated by a planar angle deficit in the two-dimensional subspace. The simplest theoretical model which describes a straight and infinitely long cosmic string is given by a Dirac-delta-type distribution for the energy–momentum tensor along the linear defect. Also this object can be described by classical field theory, coupling the energy–momentum tensor associated with the Maxwell–Higgs system investigated by Nielsen and Olesen in [3] with the Einstein equations [4, 5]. Although the recent observational data on the cosmic microwave background have ruled out cosmic strings as the primary source for primordial density perturbation, they are still candidate for the generation of a number of interesting physical effects such as gamma ray bursts [6], gravitational waves [7] and high-energy cosmic rays [8]. Moreover, in the framework of brane inflation [9–11], cosmic strings have attracted renewed interest, partly because a variant of their formation mechanism is proposed.

Although the geometry of the spacetime produced by an idealized cosmic string is locally flat, the planar angle deficit provides nonzero vacuum expectation values (VEVs) for different physical observables. In this context, the VEVs of the energy–momentum tensor have been calculated for scalar and fermionic fields in [12–16] and [17–20], respectively. Another type of vacuum polarization takes place when boundaries are present. In this sense, by imposing boundary conditions on quantum fields, additional shifts in the VEVs of physical quantities, such as the energy density and stresses, take place. This is the well-known Casimir effect (for a review, see [21]). The analysis of the Casimir effect in the idealized cosmic string spacetime has been developed for scalar [22], vector [23, 24] and fermionic fields [25, 26], obeying boundary conditions on cylindrical surfaces. The Casimir force for massless scalar fields subject to Dirichlet and Neumann boundary conditions in the setting of the conical piston has recently been investigated in [28]. Continuing along this line of investigation, in this paper we shall analyze the contribution on the vacuum polarization effects in a higher dimensional cosmic string spacetime induced by a scalar field obeying Dirichlet or Neumann boundary conditions on a surface orthogonal to the string. In addition to be a new perspective related to the Casimir effect, the present investigation may be relevant in the analysis of vacuum polarization effects induced by a brane in anti-de Sitter spacetime.

The paper is organized as follows. In section 2, we provide a general expression for the scalar Green’s function in a higher dimensional cosmic string spacetime admitting that the field obeys Dirichlet or Neumann conditions on a boundary orthogonal to the string. We shall see that this Green’s function is expressed in terms of two distinct contributions. The first one is the standard Green’s function for a massive scalar field in a boundary-free cosmic string spacetime, and the second contribution is due to the boundary condition obeyed by the field operator. The first contribution to the Green’s function is divergent at the coincidence limit and the second one is finite in this limit for points away from the boundary. Moreover, for specific values of the parameter associated with the planar angle deficit, the complete Green’s function can be expressed in a closed form in terms of a finite sum of the Macdonald functions. Because the analysis of the VEVs of physical quantities induced by the cosmic string has been developed in the literature by many authors, in sections 3 and 4 we calculate the contributions to the VEVs of the field squared and the energy–momentum tensor induced by the boundary. We shall see that near the boundary and for points outside the string, these

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3 Also vacuum polarization effects induced by a composite topological defect have been analyzed in [27].
contributions become more relevant than the contributions due to the cosmic string itself. In section 5, we summarize the most important results obtained. In this paper we shall use the units $\hbar = c = 1$.

2. Green’s function

2.1. Geometry of the problem and the heat kernel

We consider a massive scalar quantum field propagating in a $D$-dimensional cosmic string spacetime. By using the generalized cylindrical coordinates with the cosmic string on the subspace defined by $r = 0, r \geq 0$ being the radial polar coordinate, the corresponding metric tensor is defined by the line element

$$\text{d}s^2 = g_{ik} \, \text{d}x^i \, \text{d}x^k = -\text{d}t^2 + \text{d}r^2 + \alpha^2 r^2 \, \text{d}\phi^2 + \text{d}z^2 + \sum_{l=4}^{D-1} (\text{d}x^l)^2.$$  (1)

The coordinate system reads $x^i = (t, r, \phi, z, x^l)$, with $\phi \in [0, 2\pi]$, and $t, z, x^l \in (-\infty, \infty)$. The parameter $\alpha$, smaller than unity, codifies the presence of the string. In a four-dimensional spacetime, this parameter is related to the linear mass density of the string by $\alpha = 1 - 4G\mu$, with $G$ being the Newton gravitational constant. In this analysis, we shall admit the presence of extra coordinates, $x^l$, defined in a Euclidean $(D - 4)$-dimensional subspace.

For a scalar field propagating in an arbitrary curved spacetime, the field equation has the form

$$\left(\Box - m^2 + \xi R\right) \phi(x) = 0,$$  (2)

with $\Box$ denoting the covariant d’Alembertian and $R$ is the scalar curvature. In (2) we have introduced an arbitrary curvature coupling $\xi$. The minimal coupling corresponds to $\xi = 0$ and for the conformal one, $\xi = \xi_c = (D - 2)/(4(D - 1))$. We shall assume that the field obeys the Dirichlet boundary condition on the hypersurface orthogonal to the string and located at $z = 0$:

$$\phi(x) = 0, \quad z = 0.$$  (3)

The Green’s function associated with a massive scalar field in a curved spacetime obeys the second-order differential equation

$$\left(\Box - m^2 + \xi R\right) G(x, x') = -\delta^D(x - x'),$$  (4)

where $\delta^D(x, x')$ represents the bidensity Dirac distribution. This Green’s function can be obtained within the framework of the Schwinger–DeWitt formalism as follows:

$$G(x, x') = \int_0^{\infty} \text{d}s \, \mathcal{K}(x, x'; s),$$  (5)

where the heat kernel, $\mathcal{K}(x, x'; s)$, can be expressed in terms of a complete set of normalized eigenfunctions of the operator defined in (4) as follows:

$$\mathcal{K}(x, x'; s) = \sum_{\sigma} \Phi_{\sigma}(x) \Phi_{\sigma}^*(x') e^{-s\sigma^2},$$  (6)

with $\sigma^2$ being the corresponding positively defined eigenvalue.

Writing

$$\left(\Box - m^2 + \xi R\right) \Phi_{\sigma}(x) = -\sigma^2 \Phi_{\sigma}(x),$$  (7)
in the spacetime defined by the line element (1), a complete set of normalized solutions of the above equation, compatible with the boundary condition (3), can be specified in terms of a set of quantum numbers \((\omega, q, n, k_z, k_l)\), where \(n = 0, \pm 1, \pm 2, \ldots\), \((\omega, k_l) \in (\infty, \infty)\) and \((q, k_z) \geq 0\). These functions are given by
\[
\Phi_\sigma(x) = 2 \sqrt{q} \frac{\alpha}{(2\pi)^{(D-1)/2}} J_{|n|/\alpha}(qr) \sin(k_l z), \quad (8)
\]
where \(J_\nu(x)\) being the Bessel function and \(x = (x^4, \ldots, x^{D-1})\). The corresponding positively defined eigenvalue is given as
\[
\sigma^2 = \omega^2 + q^2 + k_z^2 + k_l^2 + m^2. \quad (9)
\]

The expression for the heat kernel is obtained from (8), after performing the integrals with the help of [29], we obtain
\[
K(x, x'; s) = \frac{2e^{-s\rho^2/4s}}{(4\pi s)^{D/2}} \sinh \left( \frac{zz'}{2s} \right) \sum_{n=-\infty}^{+\infty} e^{in\Delta \phi} I_{|n|/\alpha} \left( \frac{rr'}{2s} \right), \quad (10)
\]
where \(I_\nu(x)\) is the modified Bessel function and \(\Delta \rho^2 = -\Delta t^2 + r^2 + r'^2 + \Delta x^2 + z^2 + z'^2\). \(\Delta \phi = \phi - \phi', \Delta t = t - t', \) and \(\Delta x = x - x'\).

In general, it is not possible to provide a closed expression for the Green’s function by integrating over the variable \(s\) the heat kernel function (10), according to (5). However, for massless fields and for specific values of the parameter \(\alpha\), the corresponding Green’s functions can be expressed in terms of a finite sum of the associated Legendre functions and the Macdonald ones, respectively. These two different situations will be analyzed separately in the following subsections.

The case of a scalar field with the Neumann boundary condition, \(\partial_z \phi = 0\) at \(z = 0\), can be considered in a similar way. The corresponding eigenfunctions have the form (8) with the replacement \(\sin(k_z z) \rightarrow \cos(k_z z)\). The expression for the heat kernel is obtained from (10) with the replacement \(\sinh(z z'/(2s)) \rightarrow \cosh(z z'/(2s))\).

### 2.2. Special case

The analysis of vacuum polarization effects associated with a quantum scalar field in a cosmic string spacetime has been developed by many authors for the case where the parameter \(\alpha\) is equal to the inverse of an integer number \(p\), i.e. when \(\alpha = 1/p\) (see [13–15]). In this case, the corresponding Green’s function can be expressed in terms of the \(p\) images of the Minkowski spacetime function. Recently, the image method was also used in [22] to obtain a closed expression for massive scalar Green’s functions in a higher dimensional cosmic string spacetime obeying the Robin boundary condition on a cylindrical surface coaxial with the string. Here, in this subsection, we shall consider this specific situation, i.e. \(\alpha\) being the inverse of an integer number, to obtain the Green’s function in a closed form for the physical situation under consideration. For this case, the expression of the heat kernel can be further simplified with the help of the formula [30, 31]
\[
\sum_{n=-\infty}^{+\infty} e^{in\Delta \phi} I_{|n|/\alpha} (rr'/2s) = \frac{1}{p} \sum_{k=0}^{p-1} e^{i\frac{\Delta \phi}{p}} \cos \left( \frac{2\pi k}{p} \right). \quad (12)
\]

The corresponding heat kernel reads
\[
K(x, x'; s) = \frac{2e^{-sm^2}}{(4\pi s)^{D/2}} \sinh \left( \frac{zz'}{2s} \right) \sum_{k=0}^{p-1} e^{i\frac{\Delta \phi}{p}}, \quad (13)
\]
where
\[ V_k = -\Delta t^2 + \Delta x^2 + z^2 + r^2 + r'^2 - 2rr' \cos(\Delta \varphi / p + 2\pi k / p). \] (14)

Finally, substituting the above function into (5), with the help of [29], we obtain
\[ G(x, x') = \frac{m^{D-2}}{(2\pi)^{D/2}} \sum_{k=0}^{p-1} [f_{D/2-1}(mV_k(-)) - f_{D/2-1}(mV_k(+) )], \] (15)

where
\[ V_k(\pm) = [-\Delta t^2 + \Delta x^2 + (z \mp z')^2 + r^2 + r'^2 - 2rr' \cos(\Delta \varphi / p + 2\pi k / p)]^{1/2}. \] (16)

In (15) and in what follows, we use the notation
\[ f_\nu(x) = K_\nu(x) / x^\nu, \] (17)

\( K_\nu(x) \) being the Macdonald function. Expression (15) is further simplified in the case of a massless field. By using the asymptotic of the Macdonald function for small values of the argument [32], one obtains
\[ G(x, x') = \frac{\Gamma(D/2 - 1)}{4\pi^{D/2}} \sum_{k=0}^{p-1} [V_k^2 - V_{k(+) }^2 - V_{k(-)}^2]. \] (18)

We can see that the Green’s function (15) vanishes for \( z \) or \( z' \) being equal to zero. It can be presented as the sum of two different contributions as
\[ G(x, x') = G_{cs}(x, x') + G_b(x, x'). \] (19)

The first term on the right-hand side coincides with the Green’s function for a massive scalar field in the absence of the boundary. It is divergent at the coincidence limit and the divergence comes from the \( k = 0 \) term. The second contribution, \( G_b(x, x') \), is a consequence of the boundary condition imposed on the field. This contribution is finite at the coincidence limit for points outside the boundary.

The renormalized Green’s function, used for the evaluation of finite and well-defined VEVs, is given by subtracting from the corresponding Green’s function the Hadamard one. Because the cosmic string spacetime is locally flat, the Hadamard function coincides with the Green’s function in the Minkowski spacetime. For the above Green’s function, the renormalization procedure can be implemented explicitly by discarding the \( k = 0 \) component of the function \( G_{cs}(x, x') \).

The formulae for the Green’s function in the case of the Neumann boundary condition are obtained from (15) and (18) changing the sign of the terms with \( V_{k(+) } \). Consequently, the boundary-induced parts in the Green’s function, \( G_b(x, x') \), for Dirichlet and Neumann scalars differ by the sign. This is the case also for the corresponding parts in the VEVs of the field squared and the energy–momentum tensor discussed below in sections 3 and 4.

2.3. General case

For a general case where \( p = 1/\alpha \) is not an integer number, the Green’s function can be expressed in an integral form by substituting the heat kernel (10) into (5). After some intermediate steps, the Green’s function is presented in the form (19), where the boundary-free and boundary-induced parts are given by the expressions
\[ G_{cs}(x, x') = \frac{p}{(4\pi)^{D/2}} \sum_{n=-\infty}^{\infty} e^{in\Delta \varphi} \int_0^{\infty} dw \, w^{D/2-2} \, e^{-\frac{V_{k(-)} w - n^2}{\alpha}} \frac{I_{|n|\alpha}(rr'w/2)}{w}. \]
\[ G_b(x, x') = -\frac{P}{(4\pi)^{D/2}} \sum_{n=-\infty}^{+\infty} e^{i n \Delta \phi} \int_0^{\infty} dw \, w^{D/2-2} e^{-\frac{V_{\pm}}{w} w - \frac{w^2}{2}} I_{|n|p}(rr'/w) , \]  

where

\[ V_{\pm} = -\Delta t^2 + \Delta x^2 + (z \mp z')^2 + r^2 + r'^2 . \]  

We can also verify that the Green’s function vanishes for \( z \) or \( z' \) equal to zero. Here we can analyze the behavior of the both contributions in the coincidence limit. In this limit, \( V_{-1} = 2r^2 \), so due to the exponential behavior of the modified Bessel function for large arguments, the first integral diverges for large values of the variable \( w \). In the second contribution, \( V_{(+)} = 4z^2 + 2r^2 \); consequently, for \( z \neq 0 \) the integrand in the expression for \( G_b(x, x') \) goes to zero exponentially for large values of the integration variable.

We can also provide a closed expression for the Green’s function in the limit of a massless field. With the help of \([29]\) we obtain

\[ G(x, x') = \frac{e^{-i(D-3)\pi/2}}{(2\pi)^{(D+1)/2} (rr')^{D/2}-1} \sum_{n=-\infty}^{+\infty} e^{i n \Delta \phi} \left[ Q_{|n|p-1/2}(\cosh u_{(-)}) \frac{(\sinh u_{(-)})(D-3)/2}{(\sinh u_{(+)})(D-3)/2} - Q_{|n|p-1/2}(\cosh u_{(+)}) \right] . \]  

\( Q_{\nu}(x) \) being the associated Legendre function and

\[ \cosh u_{(\mp)} = \frac{V_{(\mp)}}{2rr'} \geq 1. \]  

Finally, we have to say that the renormalized Green’s function can be obtained by subtracting from the Green’s function the corresponding function in the Minkowski spacetime, which is given by \( G_{cs}(x, x') \) taking \( p = 1 \).

3. VEV of the field squared

This and the following sections will be devoted to the calculations of vacuum polarizations effects induced by the boundary. Two main calculations will be performed. The evaluation of the VEV of the field squared, in the first place, followed by the evaluation of the VEV of the energy–momentum tensor.

The VEV of the field squared is formally given by evaluating the Green’s function at the coincidence limit. In this analysis, the complete Green’s function is given by the sum of the Green’s function in the cosmic string spacetime in the absence of the boundary plus the boundary-induced part. In this way, we may write

\[ \langle \phi^2 \rangle = \langle \phi^2 \rangle_{cs} + \langle \phi^2 \rangle_b . \]  

However, because of the singular behavior of \( G_{cs}(x, x') \) at the coincidence limit, the renormalization procedure is needed for the first contribution of the above expression. The second contribution is finite at the coincidence limit for points outside the hypersurface \( z = 0 \). Because the VEV of the field squared in the cosmic string spacetime has been analyzed by many authors, here we are mainly interested in the analysis of the quantum effects induced by the boundary.

According to the previous section, we shall analyze the VEV of the field squared induced by the boundary for \( p \) being an integer number in the first part, and for the general case in the second one.
3.1. Special case

With $p$ being an integer number, the VEV of the field squared induced by the boundary is given simply by taking the coincidence limit of $G_b(x', x)$ given in (15). The result is presented by the expression

$$\langle \phi^2 \rangle_b = -\frac{m^{D-2}}{(2\pi)^{D/2}} \sum_{k=0}^{p-1} f_{D/2-1}(2m\sqrt{z^2 + r^2 s_k^2}),$$  \hspace{1cm} (25)$$

where

$$s_k = \sin(\pi k/p).$$  \hspace{1cm} (26)$$

As is seen, the boundary-induced part in the VEV is negative. In (25), the part with the term $k = 0$ is the corresponding VEV in the Minkowski spacetime and we can write

$$\langle \phi^2 \rangle_b = \langle \phi^2 \rangle_{b,(p=1)} + \langle \phi^2 \rangle_{b,C},$$  \hspace{1cm} (27)$$

where the second term on the right-hand side is the part of the VEV induced by the non-trivial topology of the cosmic string spacetime. Note that for the renormalized pure topological part, one has the expression (see [22])

$$\langle \phi^2 \rangle_{cs} = -\frac{m^{D-2}}{(2\pi)^{D/2}} \sum_{k=1}^{p-1} f_{D/2-1}(2mr s_k).$$  \hspace{1cm} (28)$$

The latter is always positive. It is of interest to note that the topological part in the total VEV, $\langle \phi^2 \rangle_{cs} + \langle \phi^2 \rangle_{b,C}$, vanishes on the boundary.

For $z \neq 0$ and for points far from the string, $r \gg |z|$, the dominant contribution to the boundary-induced part (25) comes from the $k = 0$ term and to the leading order we obtain

$$\langle \phi^2 \rangle_b \approx \langle \phi^2 \rangle_{b,(p=1)} = -\frac{m^{D-2}}{(2\pi)^{D/2}} f_{D/2-1}(2m|z|).$$  \hspace{1cm} (29)$$

Note that for $z \neq 0$ and for points on the string axis, $r = 0$, one has $\langle \phi^2 \rangle_{b,r=0} = p\langle \phi^2 \rangle_{b,(p=1)}$. In the limit $|z| \gg r$ and $m|z| \gg 1$, the leading-order term provides an exponentially suppressed behavior as

$$\langle \phi^2 \rangle_b \approx -\frac{p m^{D-3/2}}{2(4\pi)^{D/2}} e^{-2m|z|}.$$  \hspace{1cm} (30)$$

For a massless field, from (25) we obtain

$$\langle \phi^2 \rangle_b = -\frac{\Gamma(D/2 - 1)}{(4\pi)^{D/2}} \sum_{k=0}^{p-1} (z^2 + r^2 s_k^2)^{-(D/2 - 1)}.$$  \hspace{1cm} (31)$$

In this case, at large distances from the boundary, $|z| \gg r$, the boundary-induced VEV decays as a power law: $\langle \phi^2 \rangle_b \propto r^{D-D}$. In figure 1, we exhibit the behavior of (25) as a function of the dimensionless variables $mr$ and $mz$ for $D = 4$, $p = 3$.

As we have already explained and it is seen from the graph that for fixed $z$, the boundary-induced contribution is finite for $z \neq 0$ and $r = 0$. On the other hand, for a fixed non-vanishing radial coordinate, near the boundary it is dominated by the $k = 0$ term (see (29)). When $z$ goes to infinity, the VEV is exponentially suppressed.

In figure 2, we exhibit the behavior of (25) in the case of a $D = 4$ cosmic string as a function of the dimensionless variable $mr$ for three distinct values of $p = 2, 3, 4$ and for $mz = 0.5$. We can see that the effects induced by the string become more relevant for larger values of $p$. 

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3.2. General case

In the case of $p$ not being an integer number, the VEV of the field squared induced by the boundary is obtained by taking the coincidence limit of $G_b(x', x)$ given in (20). After a change of the integration variable, it can be written as

$$
\langle \phi^2 \rangle_b = -\frac{p r^2 - D}{4\pi^{D/2}} \int_0^\infty dy \ y^{D/2 - 2} \ e^{-(2z^2/r^2y+1)y-m^2y^2/(2y)} \ S_p(y),
$$

(32)
with

\[ S_p(y) = \sum_{n=0}^{\infty} I_{ap}(y), \tag{33} \]

where the prime means that the term with \( n = 0 \) should be halved. As is seen from (32), the boundary-induced VEV is negative for the general case of \( p \). Although not being possible to provide a closed expression for the above VEV, some limiting cases can be obtained. Due to the exponential decay, for \( |z| \gg r \) the dominant contribution comes from the region near the lower limit of the integration. In this region we may approximate \( S_p(y) \approx 1/2 \) and with the help of [29] we obtain \( \langle \phi^2 \rangle_b \approx p\langle \phi^2 \rangle_b^{(p=1)} \). We can see that for \( p > 1 \) the presence of the cosmic string increases the above results when compared with the corresponding ones in the absence of it.

We can find an integral representation for the VEV of the field squared by using the formula

\[ S_p(y) = \frac{1}{p} \sum_{k=0}^{p_0} e^{y \cos(2\pi k/p)} \frac{\sin(p\pi)}{2\pi} \int_0^\infty dx \frac{e^{-y \cosh x}}{\cosh(px) - \cos(p\pi)}, \tag{34} \]

where \( p_0 \) is an integer number defined by \( 2p_0 < p < 2p_0 + 2 \). For even values of \( p \), the corresponding formula is obtained from (34) by taking the limit. In this limit the second term on the right-hand side becomes \( e^{-y/(2p)} \). Formula (34) is obtained as a special case of the more general formula derived in [26].

Substituting (34) into (32), the integration over \( y \) is performed explicitly in terms of the Macdonald function and one obtains

\[ \langle \phi^2 \rangle_b = -\frac{2m^{D-2}}{(2\pi)^{D/2}} \left[ \sum_{k=0}^{p_0} f_{D/2-1}(2m\sqrt{z^2 + r^2s^2k}) \right] - \frac{p}{\pi} \sin(p\pi) \int_0^\infty dx \frac{f_{D/2-1}(2m\sqrt{z^2 + r^2 \cosh^2 x})}{\cosh(2px) - \cos(p\pi)} \]. \tag{35} \]

It can be seen that for integer values of \( p \), this result reduces to expression (25). On the string axis for the boundary-induced VEV, one has

\[ \langle \phi^2 \rangle_{b, r=0} = p\langle \phi^2 \rangle_{b}^{(p=1)}. \tag{36} \]

For points near the boundary, \( |z| \ll m^{-1}, |z| \ll r \), the main contribution to the boundary-induced VEV comes from the term \( k = 0 \) and to the leading order we find

\[ \langle \phi^2 \rangle_b \approx -\frac{\Gamma(D/2 - 1)}{(4\pi)^{D/2} |z|^{D-2}}. \tag{37} \]

This leading term does not depend on the angle deficit and it coincides with the corresponding term for a boundary in Minkowski spacetime. At large distances from the boundary, \( |z| \gg m^{-1} \), the boundary-induced part is exponentially suppressed and the VEV is dominated by the pure topological part \( \langle \phi^2 \rangle_{cs} \).

For a massless field, by using the relation \( f_0(y) \sim 2^{\nu-1}\Gamma(\nu)y^{-2\nu}, y \to 0 \), we obtain

\[ \langle \phi^2 \rangle_b = -\frac{2\Gamma(D/2 - 1)}{(4\pi)^{D/2}} \left[ \sum_{k=0}^{p_0} (z^2 + r^2s^2k)^{1-D/2} \right] - \frac{p}{\pi} \sin(p\pi) \int_0^\infty dx \frac{(z^2 + r^2 \cosh^2 x)^{1-D/2}}{\cosh(2px) - \cos(p\pi)} \]. \tag{38} \]
In this case, at large distances from the boundary, \( |z| \gg r \), the leading term in the corresponding asymptotic expansion has the form

\[
\langle \phi^2 \rangle_b \approx p \langle \phi^2 \rangle_b^{(p=1)} = -\frac{p \Gamma(D/2 - 1)}{(4\pi)^{D/2} |z|^{D-2}}.
\]  

(39)

Recall that near the boundary we have the behavior given by (37).

For a scalar field with the Neumann boundary condition at \( z = 0 \), the corresponding formulae for the VEV of the field squared are obtained from those given above by changing the sign of the boundary-induced part.

In figure 3, we plot \( \langle \phi^2 \rangle_b^C/m^2 \) as a function of the parameter \( p \) for \( mz = 0.5 \) and for several values of \( mr \) (numbers near the curves) in a four-dimensional spacetime (\( D = 4 \)). Note that for the first term on the right-hand side of (27), for \( mz = 0.5 \), one has \( \langle \phi^2 \rangle_b^{(p=1)} \approx -0.0152m^2 \).

We can also analyze the topological part in the boundary-induced VEV of the field squared by adding and subtracting from (32) the part corresponding to the Minkowski spacetime (\( p = 1 \)):

\[
\langle \phi^2 \rangle_b = -\frac{r^{2-D}}{(2\pi)^{D/2}} \int_0^\infty dy y^{D/2-2} e^{-(2z^2/r^2)y} \frac{m^2 y^2}{(2y)^D} \sum_{n=0}^\infty I_n(y)\]

\[
-\frac{r^{2-D}}{(2\pi)^{D/2}} \int_0^\infty dy y^{D/2-2} e^{-(2z^2/r^2+1)y} \frac{m^2 y^2}{(2y)^D} \sum_{n=0}^\infty \left[p I_{pn}(y) - I_n(y)\right].
\]  

(40)

The first contribution can be promptly obtained by noting that (see (34)) \( \sum_{n=0}^\infty I_n(y) = e^y/2 \). Of course, the corresponding expression coincides with (29) which is independent of \( r \) and diverges for \( z = 0 \).
An alternative representation for the topological part can be obtained by using the Abel–Plana formula (see, for instance, [33]) for the summation over \( n \) in the second term on the right-hand side of (40):

\[
\sum_{n=0}^{\infty} F(n) = \int_{0}^{\infty} du \ F(u) + i \int_{0}^{\infty} du \ \frac{F(iu) - F(-iu)}{e^{2\pi u} - 1}.
\]  

In our case, \( F(n) = pI_{np}(y) \). Now we can see that in the evaluation of the difference the terms coming from the first integral on the right-hand side of the Abel–Plana formula cancel out and one obtains

\[
\sum_{n=0}^{\infty} [pI_{np}(y) - I_n(y)] = \frac{2}{\pi} \int_{0}^{\infty} dv \ g(v, p) K_{iv}(y),
\]  

where we have introduced the notation

\[
g(v, p) = \sinh(\pi v) \left( \frac{1}{e^{2\pi v/p} - 1} - \frac{1}{e^{2\pi v} - 1} \right).
\]  

The respective contribution becomes

\[
\langle \phi^2 \rangle_C^{(B)} = -\frac{4\pi^{D-2}}{(2\pi)^{(D-1)/2}} \int_{0}^{\infty} dy y^{D/2-2} e^{-\left(2\pi^2/r^2\right)y^2-M^2r^2/(2y)} \int_{0}^{\infty} dv \ g(v, p) K_{iv}(y).
\]  

For \( r \neq 0 \), the above correction is finite at the boundary. Taking \( z = 0 \), for a massless field, the integral over \( y \) in (44) is evaluated in terms of the gamma function and we obtain

\[
\langle \phi^2 \rangle_C^{(B),z=0} = -\frac{8(4\pi)^{-(D+1)/2}}{\Gamma((D-1)/2)r^{D-2}} \int_{0}^{\infty} dv \ g(v, p) |\Gamma(D/2 - 1 - iv)|^2. 
\]  

For an even-dimensional spacetime, the modulus of the gamma function in this formula is expressed in terms of elementary functions and the integral can be explicitly evaluated. In particular, for \( D = 4 \) one has \( |\Gamma(1 - iv)|^2 = \pi v / \sinh(\pi v) \) and the integral above becomes equal to \( \pi (p^2 - 1)/24 \). In this case, we obtain

\[
\langle \phi^2 \rangle_C^{(B),z=0} = -\frac{p^2 - 1}{48\pi^2 r^2}.
\]  

This result is also obtained from (25), in the special case of integer \( p \). Taking \( D = 4 \), for a massless field we have

\[
\langle \phi^2 \rangle_C^{(B)} = -\frac{1}{16\pi^2} \sum_{k=1}^{p-1} \frac{1}{z^2 + r^2 \sin^2(\pi k/p)}. 
\]  

On the boundary, \( z = 0 \), the sum in this formula is explicitly evaluated, \( \sum_{k=1}^{p-1} \sin^{-2}(\pi k/p) = (p^2 - 1)/3 \), and (47) reduces to (46).

### 4. Energy–momentum tensor

Following the same line of investigation, in this section we are interested to calculate the contribution induced by the boundary on the VEV of the energy–momentum tensor. Similar to the case of the field squared, the energy–momentum tensor is presented in the decomposed form

\[
\langle T_{ik} \rangle = \langle T_{ik} \rangle_{cs} + \langle T_{ik} \rangle_{b},
\]
where $\langle T_{ik}\rangle_{cs}$ corresponds to the geometry of the cosmic string without boundaries. In order to evaluate the boundary-induced part, we shall use the following expression:

$$
\langle T_{ik}\rangle_b = \lim_{x' \to x} \partial_i \partial_k G_b(x, x') + [(\xi - 1/4) g_{ik} - \xi \partial_i \partial_k - \xi R_{ik}] \langle \phi^2 \rangle_b, \quad (49)
$$

where for the spacetime under consideration the Ricci tensor, $R_{ik}$, vanishes.

By using expression (35), for the covariant d’Alembertian appearing in (49) one obtains

$$
\Box \langle \phi^2 \rangle_b = -\frac{8m^0}{(2\pi)^{D/2}} \left\{ \sum_{l=0}^{p_0} \left[ 4m^2(z^2 + s_l^2r^2) f_{D/2+l}(u_1) - (1 + 2s_l^2) f_{D/2}(u_1) \right] \\
- \frac{p \sin(p\pi)}{2\pi} \int_0^\infty dx \left\{ \frac{w_1^2 + m^2r^2 \sinh^2(x)}{\cosh(px) - \cos(p\pi)} \right\},
$$

where

$$
w_1 = 2m\sqrt{z^2 + r^2s_l^2},
$$

$$
w_2 = 2m\sqrt{z^2 + r^2\cosh^2(x/2)}.
$$

In the evaluation of the derivative term $\partial_i \partial_k G_b(x, x')$, we need the expression for the sum $\sum_{n=-\infty}^{+\infty} n^2 I_{n|p}(y)$. For the summation of this series, we use the relation

$$
\sum_{n=-\infty}^{+\infty} n^2 I_{n|p}(y) = 2p^{-2}(y^2\partial_y^2 + 2y\partial_y - y^2)\delta_p(y), \quad (52)
$$

and formula (34). In the evaluation of the other derivative terms we can put $\phi = \phi'$ before differentiation. In this case, by using formula (34), for the boundary-induced part in the Green’s function, one obtains

$$
G_b(x, x')|_{\phi' = \phi} = -\frac{2m^{D-2}}{(2\pi)^{D/2}} \left\{ \sum_{l=0}^{p_0} f_{D/2-1}(m\sqrt{\nu_{l} + 2rr'\cos(2\pi l/p)}) \\
- \frac{p \sin(p\pi)}{2\pi} \int_0^\infty dx \frac{f_{D/2-1}(m\sqrt{\nu_{l}} + 2rr'\cosh x)}{\cosh(px) - \cos(p\pi)} \right\}. \quad (53)
$$

where $\nu_{l}$ is defined by (21).

Combining the formulae given above, for the separate components of the energy–momentum tensor, we obtain the following VEVs:

$$
\langle T_{ij} \rangle_b = -\frac{2m^0}{(2\pi)^{D/2}} \left[ \sum_{k=0}^{p_0} F_{ij}(2mr, 2mz, \cos x) - \frac{p}{\pi} \sin(p\pi) \int_0^\infty dx \frac{F_{ij}(2mr, 2mz, \cos x)}{\cosh(2px) - \cos(p\pi)} \right]. \quad (54)
$$

Here the functions are defined as

$$
F_0^0(u, v, w) = (4\xi - 1)(u^4w^2 + v^2) f_{D/2+1}(y) + [1 - (4\xi - 1)(1 + 2u^2)] f_{D/2}(y),
$$

$$
F_1^1(u, v, w) = (4\xi - 1)v^2 f_{D/2+1}(y) + 2[1 - 2\xi(1 + u^2)] f_{D/2}(y),
$$

$$
F_2^2(u, v, w) = [4\xi(w^4u^2 + v^2) - y^2] f_{D/2+1}(y) + 2[1 - 2\xi(1 + w^2)] f_{D/2}(y),
$$

$$
F_3^3(u, v, w) = (4\xi - 1)w^2[w^2u^2 f_{D/2+1}(y) - 2 f_{D/2}(y)],
$$

$$
F_4^4(u, v, w) = -(4\xi - 1)uw^2 f_{D/2+1}(y),
$$

$$
F_{ij}^{ij}(u, v, w) = -(4\xi - 1)uvw^2 f_{D/2+1}(y),
$$

$$
F_{ij}^{ij}(u, v, w) = -(4\xi - 1)uvw^2 f_{D/2+1}(y),
$$

$$
F_{ij}^{ij}(u, v, w) = -(4\xi - 1)uvw^2 f_{D/2+1}(y),
$$

where $\xi = 1$ and $D = 2$.\]
the indices 0, 1, 2, 3 correspond to the coordinates \( t, r, \varphi, z \), and
\[
y = \sqrt{u^2 + v^2 \omega^2}.
\]

(56)

For the components (no summation over \( l \)) \( \langle T^{i}_{l} \rangle_{b} \), \( l = 4, \ldots, D - 1 \), one has the relation \( \langle T^{i}_{l} \rangle_{b} = \langle T^{0}_{0} \rangle_{b} \). Of course, the latter relation is a direct consequence of the invariance of the problem with respect to the boosts along the directions \( x^l \), \( l = 4, \ldots, D - 1 \). Due to the presence of the boundary, the boost invariance along the \( z \)-axis is lost and, as a consequence, \( \langle T^{3}_{3} \rangle_{b} \neq \langle T^{0}_{0} \rangle_{b} \). As we see, the vacuum energy–momentum tensor is non-diagonal. Note that the off-diagonal component vanishes for \( z = 0 \) and \( r = 0 \), separately. In addition, on the string axis, \( r = 0 \), one has \( \langle T^{2}_{2} \rangle_{b} = \langle T^{1}_{1} \rangle_{b} \). For a scalar field with the Neumann boundary condition at \( z = 0 \), the corresponding formula for the VEV of the energy–momentum tensor is obtained from (54) by changing the sign of the boundary-induced part.

Due to the presence of the off-diagonal component, from the covariant conservation condition, \( \nabla_{\mu} \langle T_{\mu}^{\nu} \rangle = 0 \), for the energy–momentum tensor two non-trivial differential equations follow:
\[
\partial_{r} (r \langle T^{1}_{1} \rangle_{b}) + r \partial_{r} \langle T^{3}_{3} \rangle_{b} = \langle T^{2}_{2} \rangle_{b}
\]
and
\[
\partial_{r} \langle T^{3}_{3} \rangle_{b} = -\frac{1}{r} \partial_{r} (r \langle T^{1}_{1} \rangle_{b}).
\]

(57)

(58)

It can be checked that these relations are obeyed by the VEV of the energy–momentum tensor above.

The \( k = 0 \) term in (54) is the corresponding VEV for a flat boundary in Minkowski spacetime. The corresponding expression takes the form (no summation over \( l \))
\[
\langle T^{i}_{l} \rangle_{b}^{(p=1)} = -\frac{m D}{(2\pi)^{D/2}} \frac{1}{(4\xi - 1)(2mz)^{2}f_{D/2+1}(2m|z|) + 2(1 - 2\xi)f_{D/2}(2m|z|)}
\]
\[
\langle T^{0}_{0} \rangle_{b}^{(p=1)} = 0
\]

(59)

for \( l = 0, 1, 2, 4, \ldots, D - 1 \), and \( \langle T^{i}_{l} \rangle_{b}^{(p=1)} = 0 \) for the other components. Note that for a conformally coupled massless field \( \langle T^{1}_{1} \rangle_{b}^{(p=1)} = 0 \).

For points near the boundary, \( |z| \ll m^{-1} \), \( |z| \ll r \), one has
\[
\langle T^{i}_{l} \rangle_{b} \approx \langle T^{i}_{l} \rangle_{b}^{(p=1)} \approx -2(D - 1)(\xi - \xi_{c})\frac{\Gamma(D/2)\delta_{l}^{i}}{(4\pi)^{D/2}|z|^{D}},
\]
\[
\langle T^{0}_{0} \rangle_{b} \approx -\langle T^{0}_{0} \rangle_{b}^{(p=1)} \approx \frac{2m^{2}\Gamma(D/2)|z|^{2-D}\delta_{l}^{i}}{(4\pi)^{D/2}(D - 1)(D - 2)}
\]

(60)

(61)

The corresponding energy density, \( \langle T^{00} \rangle_{b} = -\langle T^{00} \rangle_{b} \), is negative for both minimally and conformally coupled scalar fields. For the components \( \langle T^{3}_{3} \rangle_{b} \) and \( \langle T^{1}_{1} \rangle_{b} \), the \( k = 0 \) term in (54) vanishes and these components are finite for \( r \neq 0 \). Note that for \( z \neq 0 \) the boundary-induced part in the VEVs is finite on the string axis, \( r = 0 \). By taking into account that the pure topological part diverges on the string we conclude that the latter dominates for points near the string.

At large distances from the string, \( r \gg m^{-1}, r \gg |z| \), the dominant contribution to the boundary-induced VEV (54) comes from the \( k = 0 \) term and, to the leading order, the VEVs coincide with the corresponding expressions for a boundary in Minkowski spacetime. In this limit the effects due to the non-trivial topology of the cosmic string are exponentially
suppressed. At large distances from the boundary, \( |z| \gg m^{-1} \), the boundary-induced VEVs in the components of the energy–momentum tensor are suppressed by the factor \( e^{-2m|z|} \).

In the case of a massless field, for the VEVs we obtain the expression

\[
[T^t_4]_b = -\frac{\Gamma(D/2)}{(4\pi)^{D/2}} \left[ \sum_{k=0}^{p_0} F^t_{(00)}(r, z, s_k) \frac{s_k^D}{(z^2 + r^2 s_k^2)^{D/2+1}} \right] \left[ \frac{1}{\cosh(2\pi r) - \cos(p\pi)} \right] .
\]

The functions for the separate components are defined by the expressions

\[
F^t_{(00)}(r, z, w) = \{(4\xi - 1)[(D - 2)w^2 - 1] + 1\}r^2 w^2 + [(4\xi - 1)(D - 1 - 2w^2) + 1]z^2,
\]

\[
F^t_{(01)}(r, z, w) = [2 - 4\xi(1 + w^2)]r^2 w^2 + [4\xi(D - 1 - w^2) - D + 2]z^2,
\]

\[
F^t_{(02)}(r, z, w) = [4\xi((D - 1)w^2 - 1) - D + 2]r^2 w^2 + [4\xi(D - 1 - w^2) - D + 2]z^2,
\]

\[
F^t_{(03)}(r, z, w) = (4\xi - 1)w^2[(D - 2)w^2 r^2 - 2z^2],
\]

\[
F^t_{(13)}(r, z, w) = -D(4\xi - 1)rz w^2.
\]

Now it can be checked that for a massless conformally coupled scalar field, the tensor \([T^t_4]_b\) is traceless. Note that for a massless field, the VEV \([T^t_4]_b^{(p=1)}\) is given by the right-hand side of (60) and vanishes for a conformally coupled field. The expression on the right-hand side of (60) gives the leading term in the asymptotic expansion of \([T^t_4]_b\) at large distances from the string, \( r \gg |z| \), for the components with \( l = 0 \) and \( D - 1 \). In the same limit, the other non-zero components decay as \( [T^t_4]_b \propto r^{-D+1} \) and \( [T^t_4]_b \propto r^{-D-1} \). At large distances from the boundary, \( |z| \gg r \), the boundary-induced part in the VEVs of the diagonal components behaves as (no summation over \( l \)) \( [T^t_4]_b \propto |z|^{-D} \), whereas for the off-diagonal component, one has \( [T^t_4]_b \propto |z|^{-D-1} \).

In the case of \( p \) being an integer number, the general formula (54) takes the form

\[
[T^t_4]_b = -\frac{m^D}{(2\pi)^{D/2}} \sum_{k=0}^{p-1} F^t_{(lz)}(2mr, 2mz, s_k),
\]

with the functions \( F^t_{(lz)}(u, v, w) \) defined in (55). Note that the corresponding expression for the boundary-free part has the form [22] (no summation over \( l \))

\[
[T^t_4]_cs = -\frac{m^D}{(2\pi)^{D/2}} \sum_{k=1}^{p_1} F_{cs,l}(2mr, s_k),
\]

where

\[
F_{cs,0}(u, w) = (1 - 4\xi)w^2 [f_{D/2+1}(uw) - [1 + 2(1 - 4\xi)w^2]f_{D/2}(uw)],
\]

\[
F_{cs,1}(u, w) = (4\xi w^2 - 1) f_{D/2}(uw),
\]

\[
F_{cs,2}(u, w) = (4\xi w^2 - 1) [f_{D/2}(uw) - u^2 w^2 f_{D/2+1}(uw)],
\]

and \( F_{cs,l}(u, w) = F_{cs,0}(u, w) \) for \( l = 3, \ldots, D - 1 \). For a massless field formulae (64) and (65) are simplified to (no summation over \( l \))

\[
[T^t_4]_cs = \frac{\Gamma(D/2)\delta^4}{2(4\pi)^{D/2}p^D} \sum_{k=1}^{p_1} \frac{F_{cs,l}(s_k)}{s_k^D},
\]
Figure 4. The boundary-induced part in the VEV of the energy density, $\langle T^{00}\rangle_b / m_4$, for minimally (left plot) and conformally (right plot) coupled scalar fields in a four-dimensional spacetime as a function of $m_r$ and $m_z$ for $p = 3$.

$$\langle T^i \rangle_b = - \frac{\Gamma(D/2)}{2(4\pi)^{D/2}} \frac{1}{s^2} \sum_{k=0}^{p-1} \frac{F_{(0)i}(r, z, s_k)}{(z^2 + r^2 s_k^2)^{(D-2)/2}}$$

with the functions

$$F_{c_{l,k}}(u) = (D - 2)(1 - 4\xi)u^2 - 1, \quad l = 0, 3, \ldots, D - 1,$$

$$F_{c_{l,1}}(u) = \frac{F_{c_{l,1}}(u)}{1 - \xi} = 4\xi u^2 - 1,$$

and the functions $F_{(0)i}(r, z, w)$ are given by (63). For a conformally coupled massless scalar field, the vacuum energy density $\langle T^{00} \rangle$ is always positive.

In the left/right panel of figure 4, the VEV of the energy density induced by the boundary is exhibited for a minimally/conformally coupled massive scalar field in a four-dimensional spacetime, as a function of $m_r$ and $m_z$ for $p = 3$. We can see that the energy density crucially depends on the curvature coupling parameter $\xi$.

The normal vacuum force acting on the boundary is finite for $r \neq 0$ and is determined by the component $\langle T^3 \rangle_{b,z=0}$ evaluated at $z = 0$:

$$\langle T^3 \rangle_{b,z=0} = \frac{2m^D}{(2\pi)^{D/2}} \left[ \sum_{k=1}^{p_0} s_k^2 F_3(2mr s_k) - \frac{p}{\pi} \sin(p\pi) \int_0^\infty dx \frac{F_3(2mr \cosh x \cosh^2 x)}{\cosh(2px) - \cos(p\pi)} \right],$$

where we have defined the function

$$F_3(u) = u^2 f_{D/2+1}(u) - 2 f_{D/2}(u).$$

The vacuum effective pressure on the boundary is given by $P = \langle T^3 \rangle_{b,z=0}$. Note that the dependence of the force on the curvature coupling parameter appears in the form of the factor...
Figure 5. The normal vacuum stress on the boundary, $\langle T_{3}^{3}\rangle_{b,z=0}/m^{D}$, for a minimally coupled scalar field in $D = 4$ as a function of $mr$ (left plot) for separate values of the parameter $p$ (numbers near the curves). In the right plot the same quantity is given as a function of $p$ for separate values of $mr$ (numbers near the curves).

For a massless field, expression (70) reduces to

$$\langle T_{3}^{3}\rangle_{b,z=0} = (D-2)\frac{(1-4\xi)\Gamma(D/2)}{4\pi^{D/2}r^{D}} \sum_{k=1}^{p_{0}} \frac{s_{k}^{2-D}}{\pi^{D/2}r^{D}}$$

$$= \frac{p}{\pi} \sin(p\pi) \int_{0}^{\infty} dx \frac{\cosh^{2-D} x}{\cosh(2px) - \cos(p\pi)}$$

(72)

In particular, for $p$ being an integer number and for $D = 4$ one has

$$\langle T_{3}^{3}\rangle_{b,z=0} = \frac{1-4\xi}{48\pi^{2}r^{4}}(p^{2} - 1).$$

(73)

For both minimally and conformally coupled scalar fields, the corresponding effective pressure is positive. In the left plot of figure 5 we have presented $\langle T_{3}^{3}\rangle_{b,z=0}/m^{D}$ for a minimally coupled scalar field in $D = 4$ as a function of $mr$ for separate values of the parameter $p$ (numbers near the curves). In the right plot, the same quantity is given as a function of $p$ for separate values of $mr$ (numbers near the curves).

5. Conclusion

In this paper, we have analyzed the effects induced by a flat boundary on the VEVs of the field squared and the energy–momentum tensor associated with a massive scalar field in a higher dimensional cosmic string spacetime. Although the analysis of the VEVs associated with quantum fields in the cosmic string spacetime taking into account the presence of boundaries is, in general, developed considering cylindrical surfaces coaxial with the string, here we decided to adopt a different geometry by considering a flat boundary surface orthogonal to the string. The condition imposed on the field at the boundary is Dirichlet one. For a scalar field with the Neumann boundary condition, the corresponding formulae for the VEVs of the field squared and the energy–momentum tensor are obtained from those given above by changing the sign of the boundary-induced parts. In the presence of the boundary, the VEVs obtained
are given in terms of a sum of two terms: the first one due to the cosmic string itself in the absence of the boundary and the second terms induced by the boundary. Because the analysis of the VEVs associated with scalar fields in a pure higher dimensional cosmic string spacetime has been developed in the literature, in this paper we were more interested to investigate the contribution induced by the boundary. Two distinct situations were considered: first, when the parameter which codifies the presence of the string is the inverse of an integer number, and the second for general values of this parameter. For the first case, the Green’s function is expressed in terms of a finite sum of the Macdonald functions, while for the second one, only an integral representation can be provided.

For integer values of the parameter $p$, the boundary-induced part in the VEV of the field squared is given by (25) and this part is always negative. Note that the pure topological part is positive and is given by expression (28). For the general case of the parameter $p$, we have provided the integral representation (35). For points on the string axis, the boundary-induced VEV in the field squared is related to the corresponding quantity in Minkowski spacetime by simple formula (36). For points near the boundary, the leading term in the asymptotic expansion of the VEV does not depend on the angle deficit and it coincides with the corresponding term for a boundary in Minkowski spacetime. At large distances from the boundary and for a massive field, the boundary-induced part is exponentially suppressed and the VEV of the field squared is dominated by the pure topological part. For a massless field, the boundary-induced VEV of the field squared is given by (38) and at large distances it decays as $|z|^{2-D}$.

Another important local characteristic of the vacuum state is the VEV of the energy–momentum tensor. Similar to the field squared, the vacuum energy–momentum tensor is decomposed into pure topological and boundary-induced parts. For the general case of the parameter $p$, the latter is non-diagonal and is given by expression (54). For a massless field, the corresponding formula is reduced to (62). In the case of $p$ being an integer number, the general formulae take the forms (64) and (68) for massive and massless fields, respectively. The corresponding VEV in the boundary-free cosmic string geometry is given by (65). For a conformally coupled massless scalar field, the total energy density is always positive. For points near the boundary, the VEVs of the energy density and the diagonal stresses along the directions parallel to the boundary diverge as $|z|^{-D}$ for a non-conformally coupled field. For a conformally coupled field, the leading terms vanish and these VEVs behave as $|z|^{2-D}$. The VEVs of the normal stress, $\langle T_{3\,3} \rangle_b$, and of the off-diagonal component, $\langle T_{1\,3} \rangle_b$, are finite on the boundary for points outside the string axis.

The normal vacuum force acting on the plate is determined by the component $\langle T_{3\,3} \rangle_b$ evaluated on the boundary. This force is determined by expressions (70) and (72) for massive and massless fields, respectively. In particular, for a massless field in four-dimensional spacetime, the corresponding effective pressure is given by (73) and it is positive for both conformal and minimal couplings. Note that for a flat boundary in Minkowski spacetime, the normal stress vanishes and the effective force in the geometry under consideration is induced by the presence of the string. Numerical examples presented show that the non-trivial topology due to the cosmic string enhances the vacuum polarization effects for both field squared and the energy–momentum tensor compared to the case of a boundary in Minkowski spacetime.

Before finishing this paper, we would like to say that the analysis developed here may be relevant in the investigation of vacuum polarization effects induced by a cosmic string in anti-de Sitter spacetime [34]. In fact for this geometry, adopting the Poincaré coordinate system, the coordinate along the string has a semi-infinite range and a flat boundary orthogonal to the string is present. It is also our future interest to analyze the vacuum polarization effects associated with quantum fields in this geometry.
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