Research Article

Analyzing the Solar Energy Data Using a New Anderson-Darling Test under Indeterminacy

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The generalization of the Anderson-Darling (AD) test under neutrosophic statistics is presented in this paper. We present the designing and operating procedure of neutrosophic Anderson-Darling when the quality of interest follows the neutrosophic normal distribution. The application of the proposed test is given using data from the renewable energy field. From the analysis of the data, it is concluded that the proposed test is effective and information to be applied when the data is recorded from the complex system in the renewable energy field.

1. Introduction

The use of an appropriate statistical technique depends on the nature of the data. The nonparametric statistical tests are applied for the analysis of the data if statistical tests indicate nonnormality in the data. The statistical techniques based on normality are applied when the data follows the normal distribution. Therefore, the diagnostic of the normality of the data is a basic and important step for the deep analysis of the data. Several statistical tests have been applied to test the normality of the data. Among the tests, Anderson-Darling (AD) has been widely applied to test the normality of the data. The AD test is applied to test the null hypothesis that the data follows the normal distribution versus the alternative hypothesis that the normal distribution is not a good choice for the data. Reference [1] applied the AD test for generalized Pareto distribution. Reference [2] worked on evaluating the performance of the AD test. [3] worked on the performance of the various statistical tests. Reference [4] worked on the computation aspects of the AD test. Reference [5] applied the AD test in risk internal models. Reference [6] worked on the ranking of statistical tests. More applications of the AD test can be seen in [7–10].

The accurate prediction and estimation of renewable energy depend on the correct statistical analysis of the data. The statistical test guides renewable energy experts to give an accurate estimation of the production and consumption of energy. Therefore, effective planning about the use and saving of energy depends on statistical tests. [11] worked on the prediction of solar energy. References [12, 13] modelled the wind data using the Weibull distribution. References [14, 15] provided the statistical analysis for the energy data. Some more applications of statistical methods can be seen in [16–21].

When the data has uncertain or fuzzy observations such as measuring the water level in a river and a lifetime of a product and predicting the solar energy and melting point of a component, the existing AD test cannot be applied for the analysis of the data. In this situation, the statistical test designed using the fuzzy logic is applied for testing the normality of the data. References [22, 23] worked on the Kolmogorov-Smirnov test based on fuzzy logic. Reference [24] worked on the application of fuzzy logic in decision making. For more details, the reader may see [26–31].

The neutrosophic logic introduced by [32] reduces to fuzzy logic if the measure of indeterminacy is not found. Neutrosophic logic, which is more flexible and informative than fuzzy logic, has many applications in the real world. Reference [33] showed the efficiency of neutrosophic logic over the fuzzy and interval-based approaches. More discussion
about the neutrosophic logic can be read in [34–45]. The
neutrosophic statistics which is worked on the neutrosophic
logic is introduced by [46]. This is the branch of statistics
which deals with the analysis of the data having the neutro-
sophic numbers. References [47, 48] introduced the analysis
based on neutrosophic numbers. Classical statistics is a spe-
cial case of neutrosophic statistics when no uncertainty is
found in the data. The neutrosophic statistics gives informa-
tion about the measure of indeterminacy that classical statis-
tics do not provide. Reference [49] introduced quality control
under neutrosophic statistics. References [50, 51] introduced
statistical tests of normality under neutrosophic statistics.
For more applications, the reader may read [52, 53].

A rich literature of the AD test under classical statistics
and fuzzy approach is available in the literature. The existing
AD tests are unable to provide the measure of indeterminacy
under an uncertain environment. Reference [54] developed
the operational process of the AD test using classical statis-
tics of the AD test under neutrosophic statistics. In this paper, we
will introduce the neutrosophic Anderson-Darling (NAD)
test. We will introduce the test statistics of the proposed test
under neutrosophic statistics. The necessary steps are given
to apply the proposed test under an uncertain environment.
We will discuss the efficiency of the proposed NAD test using
normal distribution or not. The operational process of the
proposed NAD is stated as follows.

Step 1. Select a random sample \( n_N \in [n_L, n_U] \). Compute the
neutrosophic averages of determined and indeterminate
parts of the data as \( \bar{a}_N = 1/n_N \sum_{i=1}^{n_N} a_i \) and \( \bar{b}_N = 1/n_N \sum_{i=1}^{n_N} b_i \) are means of determinate and
indeterminate parts, respectively. The neutrosophic standard
deviation (NSD) by following [47, 48] is given as

\[
s_N = \left( \frac{1}{n_N} \sum_{i=1}^{n_N} (X_{iN} - \bar{X}_N)^2 \right)^{1/2},
\]

where

\[
\bar{X}_N = \bar{a}_N + \bar{b}_N I_N; I_N \in [I_L, I_U].
\]

3. Proposed Test

The Anderson-Darling (AD) test under classical statistics is
applied to test the normality of the data having determined
values. We propose neutrosophic Anderson-Darling (NAD)
for testing the normality of the imprecise and indeterminate
data. The null hypothesis \( H_{0N} \) is that the given neutrosophic
data follows the neutrosophic normal distribution versus the
alternative hypothesis \( H_{1N} \) that the neutrosophic normal
distribution is not suitable. The normality test will lead the
energy expert either to use statistical analysis based on the

\[
\sum_{i=1}^{n_N} (X_{iN} - \bar{X}_N)^2 = \left[ \min \left( (a_i - \bar{a}_N)^2, ((a_i - \bar{a}_N) + 1 \times (b_i - \bar{b}_N)), (a_i - \bar{a}_N) + 1 \times (b_i - \bar{b}_N)^2 \right) \right],
\]

where

\[
X_{iN} = a_{iN} + b_{iN} I_N; I_N \in [I_L, I_U].
\]

The neutrosophic logic consists of three measures known as
the measure of truth, say \( T \); the measure of false, say \( F \); and
the measure of indeterminacy, say \( I \). The neutrosophic logic
is a generalization of fuzzy logic. Let \( a \) be the determined
part and indeterminate part of the data as

\[
\bar{a}_N = 1/n_N \sum_{i=1}^{n_N} a_i, \quad \bar{b}_N = 1/n_N \sum_{i=1}^{n_N} b_i
\]

and

\[
\bar{X}_N = \bar{a}_N + \bar{b}_N I_N; I_N \in [I_L, I_U].
\]
Step 4. Compute the cumulative probabilities using the following transformation:

\[ F_0(Z_N) = \Phi_N \left( \frac{X_N - \bar{X}_N}{s_N} \right), \quad I_N \epsilon [I_L, I_U], \]  

where \( \Phi_N(x_N) \) denotes the neutrosophic cumulative distribution function.

Step 5. Compute NAD using the following functional form:

\[ \text{NAD} = \sum_{i=1}^{N} \frac{1 - 2i}{n_N} \left( \ln \left( F_0 \left( Z_{N(i)} \right) \right) + \ln \left( 1 - F_0 \left( Z_{N(n+1-i)} \right) \right) \right) - n_N, \quad n_N \epsilon [n_L, n_U]. \]  

Step 6. Compute the critical value (CV) as follows:

\[ \text{CV} = \frac{0.752}{(1 + 0.75/n_N + 2.25/n_N^2)}, \quad n_N \epsilon [n_L, n_U]. \]  

Step 7. The null hypothesis \( H_{0N} \) will be accepted if NAD < CV.

4. Application of NAD Test

The application of the proposed NAD test is given with the help of solar data recorded from Riyadh sitatation, Saudi Arabia. The data is taken from [11]. According to [11], in order to predict solar radiation, the system will use historical observed data: the data of ten variables including temperature (T), average wind direction at 3 m (degree from the north), average wind speed at 3 m (m/s), Diffuse Horizontal Irradiance (DHI) (Wh/m²), Direct Normal Irradiance (DNI) (Wh/m²), Global Horizontal Irradiance (GHI) of the current day (Wh/m²), peak wind speed at 3 m (m/s), relative humidity (percent), station pressure (mB (hPa equivalent)), and next-day GHI (Wh/m²) (model output). The data is reported in Table 1. From Table 1, it can be seen that the solar data has neutrosophy. Therefore, the analysis of the data using the AD test under classical statistics may mislead the experimenters. In this situation, the use of the proposed NAD test will be quite effective and informative. The proposed NAD test on this data for variable T is implemented as follows.

Step 1. Select a random sample \( n_N \epsilon [12, 12] \). Compute the neutrosophic averages of determined and indeterminate parts of the data as \( \bar{a}_N = 1/n_N \sum_{i=1}^{n_N} a_i = (14.2 + \ldots + 19.5)/12 = 27.2 \) and \( \bar{b}_N = 1/n_N \sum_{i=1}^{n_N} b_i = (15.9 + \ldots + 21.9)/12 = 29.008 \).

Step 2. Compute the neutrosophic average of a neutrosophic random variable as \( \bar{X}_N = 27.2 + 29.008 I_N, I_N \epsilon [0, 1] \).

Step 3. Compute the neutrosophic standard deviation as follows:

\[ s_N = \sqrt{n_N \sum_{i=1}^{n_N} (X_{N(i)} - \bar{X}_N)^2} = s_N \epsilon [5.7606, 11.0750], \]

where \( \sum_{i=1}^{n_N} (X_{N(i)} - \bar{X}_N)^2 \) is given as

\[ \sum_{i=1}^{n_N} (X_{N(i)} - \bar{X}_N)^2 = \sum_{i=1}^{n_N} \left[ \min \left( (a_i - \bar{a}_N)^2, \left( (a_i - \bar{a}_N) \left( (a_i - \bar{a}_N) + 1 \times (b_i - \bar{b}_N) \right) \right), (a_i - \bar{a}_N) + 1 \times (b_i - \bar{b}_N)^2 \right) \right]. \]
Step 4. Compute the cumulative probabilities using the following transformation:

\[
F_0(Z_{n}) = \Phi_N \left( \frac{X_N - \bar{X}_N}{s_N} \right) = \Phi_N \left( \frac{[14.2, 15.9] - [27.2, 56.20]}{[5.7606, 11.0750]} \right), \ldots , \\
\Phi_N \left( \frac{[19.5, 21.9] - [27.2, 56.20]}{[5.7606, 11.0750]} \right), \quad I_N \epsilon [0, 1].
\]

(9)

Step 5. Compute NAD using the following functional form:

\[
\text{NAD} = \sum_{i=1}^{n_N} \frac{1 - 2i}{n_N} \left\{ \ln \left( F_0 \left( Z_{N(i)} \right) \right) + \ln \left( 1 - F_0 \left( Z_{N(n+1-i)} \right) \right) \right\} - n_N = \text{NAD} \epsilon [1.79, 49.26].
\]

(10)

Step 6. Compute the critical value (CV) as follows:

\[
\text{CV} = \frac{0.752}{\left( 1 + 0.75/n_N + 2.25/n_N^2 \right)} = 0.6975.
\]

(11)

Step 7. The null hypothesis \(H_0\) will be rejected as \(\text{NAD} > \text{CV}\). From the proposed NAD test, it is concluded that the variable temperature does not follow the normal distribution.

5. Comparative Study

As mentioned earlier, the proposed NAD test under neutrosophic statistics is the generalization of the AD test under classical statistics. The proposed NAD test reduces to an AD test under classical statistics if uncertainty does not exist. The indeterminate value of the NAD statistic is \(\text{NAD} \epsilon [1.79, 49.26]\). The neutrosophic form of NAD can be written as \(\text{NAD} = \text{AD} + 49.26I_N, \quad I_N \epsilon [0, 0.9636]\), where \(\text{AD} = 1.79\) shows the values of the AD test under classical statistics. The part \(49.26I_N\) shows the indeterminate part of the neutrosophic test. The proposed NAD test becomes the same as the AD test when \(I_N = 0\). From the study, it can be noted that the proposed test has the values in indeterminate interval. It means, under uncertainty, that the NAD test can take the values between 1.79 and 49.26. On the other hand, the existing AD test under classical statistics provides the determined value of the statistics. Therefore, the proposed test is more flexible than the existing test under uncertainty. In a neutrosophic analysis, the total probability can be more than one due to uncertainty which is called paraconsistent probability (see [46]). In addition, the proposed test provides the probability of indeterminacy that is 0.9636. The proposed NAD test can be interpreted as follows: under an indeterminate environment, the null hypothesis that the solar data follows the normal distribution will be accepted with the probability 0.95 and rejected with the probability 0.05, and the probability of indeterminacy is 0.9636. By comparing both tests, it can be seen that for the proposed, the sum of the probabilities is larger than 1 while in the existing test, the sum of probabilities is always equal to one. In addition, the proposed test provides information about the measure of indeterminacy while the existing test does not provide such information. The proposed test results in indeterminate intervals; therefore, the theory of the proposed test is the same as in [48]. From this comparison, it is concluded that the proposed test is quite informative, effective, and flexible to be applied for the renewable energy data as compared to the existing test under classical statistics.

6. Concluding Remarks

The existing AD test cannot be applied for testing the normality of the data in intervals, having neutrosophy and uncertainty. The generalization of the Anderson-Darling (AD) test under neutrosophic statistics that can be used to test the normality of such data was presented in this paper. We presented the designing and operational procedure of neutrosophic Anderson-Darling when the quality of interest followed the neutrosophic normal distribution. The application of the proposed test was given using data from the renewable energy field. From the analysis of the data, it was concluded that the proposed test is effective and information to be applied when the data is recorded from the complex system in the renewable energy field. The proposed test provides the results in indeterminate intervals that are required in dealing with the problem under uncertainty. We recommend that the renewable energy experts should apply the proposed test under an indeterminate environment. The proposed test for nonnormal distribution can be considered future research. Developing software to run the proposed test is also a fruitful area of future research. The application of the proposed test for big data can be considered future research.

Data Availability

The data is given in the paper.

Conflicts of Interest

The authors declare no conflict of interest regarding this paper.

Acknowledgments

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